

Computer Algebra Independent Integration Tests

Summer 2024

1-Algebraic-functions/1.2-Trinomial/1.2.1-Quadratic-
trinomial/1.2.1.2/90-1.2.1.2-b

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| 3.193 | $\int \frac{(d+ex)^{7/2}}{(a^2+2abx+b^2x^2)^3} dx$ | 1570 |
| 3.194 | $\int \frac{(d+ex)^{5/2}}{(a^2+2abx+b^2x^2)^3} dx$ | 1580 |
| 3.195 | $\int \frac{(d+ex)^{3/2}}{(a^2+2abx+b^2x^2)^3} dx$ | 1590 |
| 3.196 | $\int \frac{\sqrt{d+ex}}{(a^2+2abx+b^2x^2)^3} dx$ | 1601 |
| 3.197 | $\int \frac{1}{\sqrt{d+ex}(a^2+2abx+b^2x^2)^3} dx$ | 1612 |
| 3.198 | $\int \frac{1}{(d+ex)^{3/2}(a^2+2abx+b^2x^2)^3} dx$ | 1624 |
| 3.199 | $\int \frac{1}{(d+ex)^{5/2}(a^2+2abx+b^2x^2)^3} dx$ | 1638 |
| 3.200 | $\int \frac{1}{(d+ex)^{7/2}(a^2+2abx+b^2x^2)^3} dx$ | 1655 |
| 3.201 | $\int (d+ex)^3 \sqrt{cd^2+2cdex+ce^2x^2} dx$ | 1674 |

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| 3.202 | $\int (d + ex)\sqrt{cd^2 + 2cdex + ce^2x^2} dx$ | 1680 |
| 3.203 | $\int \frac{\sqrt{cd^2+2cdex+ce^2x^2}}{d+ex} dx$ | 1685 |
| 3.204 | $\int \frac{\sqrt{cd^2+2cdex+ce^2x^2}}{(d+ex)^3} dx$ | 1690 |
| 3.205 | $\int \frac{\sqrt{cd^2+2cdex+ce^2x^2}}{(d+ex)^5} dx$ | 1695 |
| 3.206 | $\int (d + ex)^2\sqrt{cd^2 + 2cdex + ce^2x^2} dx$ | 1700 |
| 3.207 | $\int \sqrt{cd^2 + 2cdex + ce^2x^2} dx$ | 1705 |
| 3.208 | $\int \frac{\sqrt{cd^2+2cdex+ce^2x^2}}{(d+ex)^2} dx$ | 1710 |
| 3.209 | $\int \frac{\sqrt{cd^2+2cdex+ce^2x^2}}{(d+ex)^4} dx$ | 1715 |
| 3.210 | $\int \frac{\sqrt{cd^2+2cdex+ce^2x^2}}{(d+ex)^6} dx$ | 1720 |
| 3.211 | $\int (d + ex)^3 (cd^2 + 2cdex + ce^2x^2)^{3/2} dx$ | 1725 |
| 3.212 | $\int (d + ex) (cd^2 + 2cdex + ce^2x^2)^{3/2} dx$ | 1731 |
| 3.213 | $\int \frac{(cd^2+2cdex+ce^2x^2)^{3/2}}{d+ex} dx$ | 1737 |
| 3.214 | $\int \frac{(cd^2+2cdex+ce^2x^2)^{3/2}}{(d+ex)^3} dx$ | 1742 |
| 3.215 | $\int \frac{(cd^2+2cdex+ce^2x^2)^{3/2}}{(d+ex)^5} dx$ | 1747 |
| 3.216 | $\int \frac{(cd^2+2cdex+ce^2x^2)^{3/2}}{(d+ex)^7} dx$ | 1752 |
| 3.217 | $\int (d + ex)^2 (cd^2 + 2cdex + ce^2x^2)^{3/2} dx$ | 1757 |
| 3.218 | $\int (cd^2 + 2cdex + ce^2x^2)^{3/2} dx$ | 1763 |
| 3.219 | $\int \frac{(cd^2+2cdex+ce^2x^2)^{3/2}}{(d+ex)^2} dx$ | 1769 |
| 3.220 | $\int \frac{(cd^2+2cdex+ce^2x^2)^{3/2}}{(d+ex)^4} dx$ | 1774 |
| 3.221 | $\int \frac{(cd^2+2cdex+ce^2x^2)^{3/2}}{(d+ex)^6} dx$ | 1779 |
| 3.222 | $\int \frac{(cd^2+2cdex+ce^2x^2)^{3/2}}{(d+ex)^8} dx$ | 1784 |
| 3.223 | $\int (d + ex) (cd^2 + 2cdex + ce^2x^2)^{5/2} dx$ | 1789 |
| 3.224 | $\int \frac{(cd^2+2cdex+ce^2x^2)^{5/2}}{d+ex} dx$ | 1795 |
| 3.225 | $\int \frac{(cd^2+2cdex+ce^2x^2)^{5/2}}{(d+ex)^3} dx$ | 1800 |
| 3.226 | $\int \frac{(cd^2+2cdex+ce^2x^2)^{5/2}}{(d+ex)^5} dx$ | 1806 |
| 3.227 | $\int \frac{(cd^2+2cdex+ce^2x^2)^{5/2}}{(d+ex)^7} dx$ | 1811 |
| 3.228 | $\int \frac{(cd^2+2cdex+ce^2x^2)^{5/2}}{(d+ex)^9} dx$ | 1816 |
| 3.229 | $\int (d + ex)^2 (cd^2 + 2cdex + ce^2x^2)^{5/2} dx$ | 1821 |
| 3.230 | $\int (cd^2 + 2cdex + ce^2x^2)^{5/2} dx$ | 1827 |
| 3.231 | $\int \frac{(cd^2+2cdex+ce^2x^2)^{5/2}}{(d+ex)^2} dx$ | 1833 |
| 3.232 | $\int \frac{(cd^2+2cdex+ce^2x^2)^{5/2}}{(d+ex)^4} dx$ | 1839 |

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| 3.233 | $\int \frac{(cd^2+2cdex+ce^2x^2)^{5/2}}{(d+ex)^6} dx$ | 1844 |
| 3.234 | $\int \frac{(cd^2+2cdex+ce^2x^2)^{5/2}}{(d+ex)^8} dx$ | 1849 |
| 3.235 | $\int \frac{(d+ex)^5}{\sqrt{cd^2+2cdex+ce^2x^2}} dx$ | 1854 |
| 3.236 | $\int \frac{(d+ex)^3}{\sqrt{cd^2+2cdex+ce^2x^2}} dx$ | 1860 |
| 3.237 | $\int \frac{d+ex}{\sqrt{cd^2+2cdex+ce^2x^2}} dx$ | 1866 |
| 3.238 | $\int \frac{1}{(d+ex)\sqrt{cd^2+2cdex+ce^2x^2}} dx$ | 1871 |
| 3.239 | $\int \frac{1}{(d+ex)^3\sqrt{cd^2+2cdex+ce^2x^2}} dx$ | 1876 |
| 3.240 | $\int \frac{(d+ex)^4}{\sqrt{cd^2+2cdex+ce^2x^2}} dx$ | 1881 |
| 3.241 | $\int \frac{(d+ex)^2}{\sqrt{cd^2+2cdex+ce^2x^2}} dx$ | 1887 |
| 3.242 | $\int \frac{1}{\sqrt{cd^2+2cdex+ce^2x^2}} dx$ | 1893 |
| 3.243 | $\int \frac{1}{(d+ex)^2\sqrt{cd^2+2cdex+ce^2x^2}} dx$ | 1898 |
| 3.244 | $\int \frac{1}{(d+ex)^4\sqrt{cd^2+2cdex+ce^2x^2}} dx$ | 1903 |
| 3.245 | $\int \frac{(d+ex)^5}{(cd^2+2cdex+ce^2x^2)^{3/2}} dx$ | 1908 |
| 3.246 | $\int \frac{(d+ex)^3}{(cd^2+2cdex+ce^2x^2)^{3/2}} dx$ | 1913 |
| 3.247 | $\int \frac{d+ex}{(cd^2+2cdex+ce^2x^2)^{3/2}} dx$ | 1918 |
| 3.248 | $\int \frac{1}{(d+ex)(cd^2+2cdex+ce^2x^2)^{3/2}} dx$ | 1923 |
| 3.249 | $\int \frac{1}{(d+ex)^3(cd^2+2cdex+ce^2x^2)^{3/2}} dx$ | 1928 |
| 3.250 | $\int \frac{(d+ex)^4}{(cd^2+2cdex+ce^2x^2)^{3/2}} dx$ | 1934 |
| 3.251 | $\int \frac{(d+ex)^2}{(cd^2+2cdex+ce^2x^2)^{3/2}} dx$ | 1940 |
| 3.252 | $\int \frac{1}{(cd^2+2cdex+ce^2x^2)^{3/2}} dx$ | 1945 |
| 3.253 | $\int \frac{1}{(d+ex)^2(cd^2+2cdex+ce^2x^2)^{3/2}} dx$ | 1950 |
| 3.254 | $\int \frac{(d+ex)^7}{(cd^2+2cdex+ce^2x^2)^{5/2}} dx$ | 1955 |
| 3.255 | $\int \frac{(d+ex)^5}{(cd^2+2cdex+ce^2x^2)^{5/2}} dx$ | 1961 |
| 3.256 | $\int \frac{(d+ex)^3}{(cd^2+2cdex+ce^2x^2)^{5/2}} dx$ | 1966 |
| 3.257 | $\int \frac{d+ex}{(cd^2+2cdex+ce^2x^2)^{5/2}} dx$ | 1971 |
| 3.258 | $\int \frac{1}{(d+ex)(cd^2+2cdex+ce^2x^2)^{5/2}} dx$ | 1976 |
| 3.259 | $\int \frac{(d+ex)^6}{(cd^2+2cdex+ce^2x^2)^{5/2}} dx$ | 1982 |
| 3.260 | $\int \frac{(d+ex)^4}{(cd^2+2cdex+ce^2x^2)^{5/2}} dx$ | 1988 |
| 3.261 | $\int \frac{(d+ex)^2}{(cd^2+2cdex+ce^2x^2)^{5/2}} dx$ | 1993 |
| 3.262 | $\int \frac{1}{(cd^2+2cdex+ce^2x^2)^{5/2}} dx$ | 1998 |
| 3.263 | $\int \frac{1}{(d+ex)^2(cd^2+2cdex+ce^2x^2)^{5/2}} dx$ | 2003 |

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| 3.264 | $\int \frac{1}{\left(\frac{be}{2c}+ex\right)\sqrt{\frac{b^2}{4c}+bx+cx^2}} dx$ | 2009 |
| 3.265 | $\int (d+ex)^4 \sqrt{a^2+2abx+b^2x^2} dx$ | 2014 |
| 3.266 | $\int (d+ex)^3 \sqrt{a^2+2abx+b^2x^2} dx$ | 2023 |
| 3.267 | $\int (d+ex)^2 \sqrt{a^2+2abx+b^2x^2} dx$ | 2033 |
| 3.268 | $\int (d+ex) \sqrt{a^2+2abx+b^2x^2} dx$ | 2040 |
| 3.269 | $\int \sqrt{a^2+2abx+b^2x^2} dx$ | 2046 |
| 3.270 | $\int \frac{\sqrt{a^2+2abx+b^2x^2}}{d+ex} dx$ | 2051 |
| 3.271 | $\int \frac{\sqrt{a^2+2abx+b^2x^2}}{(d+ex)^2} dx$ | 2056 |
| 3.272 | $\int \frac{\sqrt{a^2+2abx+b^2x^2}}{(d+ex)^3} dx$ | 2062 |
| 3.273 | $\int \frac{\sqrt{a^2+2abx+b^2x^2}}{(d+ex)^4} dx$ | 2067 |
| 3.274 | $\int \frac{\sqrt{a^2+2abx+b^2x^2}}{(d+ex)^5} dx$ | 2073 |
| 3.275 | $\int \frac{\sqrt{a^2+2abx+b^2x^2}}{(d+ex)^6} dx$ | 2079 |
| 3.276 | $\int (d+ex)^5 (a^2+2abx+b^2x^2)^{3/2} dx$ | 2085 |
| 3.277 | $\int (d+ex)^4 (a^2+2abx+b^2x^2)^{3/2} dx$ | 2094 |
| 3.278 | $\int (d+ex)^3 (a^2+2abx+b^2x^2)^{3/2} dx$ | 2104 |
| 3.279 | $\int (d+ex)^2 (a^2+2abx+b^2x^2)^{3/2} dx$ | 2112 |
| 3.280 | $\int (d+ex) (a^2+2abx+b^2x^2)^{3/2} dx$ | 2119 |
| 3.281 | $\int (a^2+2abx+b^2x^2)^{3/2} dx$ | 2126 |
| 3.282 | $\int \frac{(a^2+2abx+b^2x^2)^{3/2}}{d+ex} dx$ | 2132 |
| 3.283 | $\int \frac{(a^2+2abx+b^2x^2)^{3/2}}{(d+ex)^2} dx$ | 2138 |
| 3.284 | $\int \frac{(a^2+2abx+b^2x^2)^{3/2}}{(d+ex)^3} dx$ | 2144 |
| 3.285 | $\int \frac{(a^2+2abx+b^2x^2)^{3/2}}{(d+ex)^4} dx$ | 2150 |
| 3.286 | $\int \frac{(a^2+2abx+b^2x^2)^{3/2}}{(d+ex)^5} dx$ | 2156 |
| 3.287 | $\int \frac{(a^2+2abx+b^2x^2)^{3/2}}{(d+ex)^6} dx$ | 2163 |
| 3.288 | $\int \frac{(a^2+2abx+b^2x^2)^{3/2}}{(d+ex)^7} dx$ | 2170 |
| 3.289 | $\int \frac{(a^2+2abx+b^2x^2)^{3/2}}{(d+ex)^8} dx$ | 2177 |
| 3.290 | $\int \frac{(a^2+2abx+b^2x^2)^{3/2}}{(d+ex)^9} dx$ | 2184 |
| 3.291 | $\int (d+ex)^5 (a^2+2abx+b^2x^2)^{5/2} dx$ | 2191 |
| 3.292 | $\int (d+ex)^4 (a^2+2abx+b^2x^2)^{5/2} dx$ | 2200 |
| 3.293 | $\int (d+ex)^3 (a^2+2abx+b^2x^2)^{5/2} dx$ | 2210 |
| 3.294 | $\int (d+ex)^2 (a^2+2abx+b^2x^2)^{5/2} dx$ | 2219 |
| 3.295 | $\int (d+ex) (a^2+2abx+b^2x^2)^{5/2} dx$ | 2227 |
| 3.296 | $\int (a^2+2abx+b^2x^2)^{5/2} dx$ | 2234 |

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| 3.297 | $\int \frac{(a^2+2abx+b^2x^2)^{5/2}}{d+ex} dx$ | 2240 |
| 3.298 | $\int \frac{(a^2+2abx+b^2x^2)^{5/2}}{(d+ex)^2} dx$ | 2247 |
| 3.299 | $\int \frac{(a^2+2abx+b^2x^2)^{5/2}}{(d+ex)^3} dx$ | 2254 |
| 3.300 | $\int \frac{(a^2+2abx+b^2x^2)^{5/2}}{(d+ex)^4} dx$ | 2261 |
| 3.301 | $\int \frac{(a^2+2abx+b^2x^2)^{5/2}}{(d+ex)^5} dx$ | 2268 |
| 3.302 | $\int \frac{(a^2+2abx+b^2x^2)^{5/2}}{(d+ex)^6} dx$ | 2275 |
| 3.303 | $\int \frac{(a^2+2abx+b^2x^2)^{5/2}}{(d+ex)^7} dx$ | 2282 |
| 3.304 | $\int \frac{(a^2+2abx+b^2x^2)^{5/2}}{(d+ex)^8} dx$ | 2289 |
| 3.305 | $\int \frac{(a^2+2abx+b^2x^2)^{5/2}}{(d+ex)^9} dx$ | 2297 |
| 3.306 | $\int \frac{(a^2+2abx+b^2x^2)^{5/2}}{(d+ex)^{10}} dx$ | 2305 |
| 3.307 | $\int \frac{(a^2+2abx+b^2x^2)^{5/2}}{(d+ex)^{11}} dx$ | 2314 |
| 3.308 | $\int \frac{(a^2+2abx+b^2x^2)^{5/2}}{(d+ex)^{12}} dx$ | 2323 |
| 3.309 | $\int \frac{(d+ex)^4}{\sqrt{a^2+2abx+b^2x^2}} dx$ | 2332 |
| 3.310 | $\int \frac{(d+ex)^3}{\sqrt{a^2+2abx+b^2x^2}} dx$ | 2340 |
| 3.311 | $\int \frac{(d+ex)^2}{\sqrt{a^2+2abx+b^2x^2}} dx$ | 2347 |
| 3.312 | $\int \frac{d+ex}{\sqrt{a^2+2abx+b^2x^2}} dx$ | 2353 |
| 3.313 | $\int \frac{1}{\sqrt{a^2+2abx+b^2x^2}} dx$ | 2359 |
| 3.314 | $\int \frac{1}{(d+ex)\sqrt{a^2+2abx+b^2x^2}} dx$ | 2364 |
| 3.315 | $\int \frac{1}{(d+ex)^2\sqrt{a^2+2abx+b^2x^2}} dx$ | 2369 |
| 3.316 | $\int \frac{1}{(d+ex)^3\sqrt{a^2+2abx+b^2x^2}} dx$ | 2375 |
| 3.317 | $\int \frac{1}{(d+ex)^4\sqrt{a^2+2abx+b^2x^2}} dx$ | 2382 |
| 3.318 | $\int \frac{(d+ex)^4}{(a^2+2abx+b^2x^2)^{3/2}} dx$ | 2389 |
| 3.319 | $\int \frac{(d+ex)^3}{(a^2+2abx+b^2x^2)^{3/2}} dx$ | 2396 |
| 3.320 | $\int \frac{(d+ex)^2}{(a^2+2abx+b^2x^2)^{3/2}} dx$ | 2403 |
| 3.321 | $\int \frac{d+ex}{(a^2+2abx+b^2x^2)^{3/2}} dx$ | 2409 |
| 3.322 | $\int \frac{1}{(a^2+2abx+b^2x^2)^{3/2}} dx$ | 2414 |
| 3.323 | $\int \frac{1}{(d+ex)(a^2+2abx+b^2x^2)^{3/2}} dx$ | 2419 |
| 3.324 | $\int \frac{1}{(d+ex)^2(a^2+2abx+b^2x^2)^{3/2}} dx$ | 2425 |
| 3.325 | $\int \frac{1}{(d+ex)^3(a^2+2abx+b^2x^2)^{3/2}} dx$ | 2432 |
| 3.326 | $\int \frac{(d+ex)^6}{(a^2+2abx+b^2x^2)^{5/2}} dx$ | 2440 |

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| 3.327 | $\int \frac{(d+ex)^5}{(a^2+2abx+b^2x^2)^{5/2}} dx$ | 2448 |
| 3.328 | $\int \frac{(d+ex)^4}{(a^2+2abx+b^2x^2)^{5/2}} dx$ | 2455 |
| 3.329 | $\int \frac{(d+ex)^3}{(a^2+2abx+b^2x^2)^{5/2}} dx$ | 2462 |
| 3.330 | $\int \frac{(d+ex)^2}{(a^2+2abx+b^2x^2)^{5/2}} dx$ | 2469 |
| 3.331 | $\int \frac{d+ex}{(a^2+2abx+b^2x^2)^{5/2}} dx$ | 2475 |
| 3.332 | $\int \frac{1}{(a^2+2abx+b^2x^2)^{5/2}} dx$ | 2480 |
| 3.333 | $\int \frac{1}{(d+ex)(a^2+2abx+b^2x^2)^{5/2}} dx$ | 2485 |
| 3.334 | $\int \frac{1}{(d+ex)^2(a^2+2abx+b^2x^2)^{5/2}} dx$ | 2493 |
| 3.335 | $\int \frac{1}{(d+ex)^3(a^2+2abx+b^2x^2)^{5/2}} dx$ | 2501 |
| 3.336 | $\int (d+ex)(9+12x+4x^2)^{5/2} dx$ | 2510 |
| 3.337 | $\int (d+ex)(9+12x+4x^2)^{3/2} dx$ | 2516 |
| 3.338 | $\int (d+ex)\sqrt{9+12x+4x^2} dx$ | 2522 |
| 3.339 | $\int \frac{d+ex}{\sqrt{9+12x+4x^2}} dx$ | 2528 |
| 3.340 | $\int \frac{d+ex}{(9+12x+4x^2)^{3/2}} dx$ | 2533 |
| 3.341 | $\int \frac{d+ex}{(9+12x+4x^2)^{5/2}} dx$ | 2538 |
| 3.342 | $\int \frac{d+ex}{(9+12x+4x^2)^{7/2}} dx$ | 2543 |
| 3.343 | $\int \frac{1}{(d+ex)\sqrt{\frac{b^2}{4c}+bx+cx^2}} dx$ | 2548 |
| 3.344 | $\int (d+ex)^{5/2}\sqrt{a^2+2abx+b^2x^2} dx$ | 2554 |
| 3.345 | $\int (d+ex)^{3/2}\sqrt{a^2+2abx+b^2x^2} dx$ | 2560 |
| 3.346 | $\int \sqrt{d+ex}\sqrt{a^2+2abx+b^2x^2} dx$ | 2566 |
| 3.347 | $\int \frac{\sqrt{a^2+2abx+b^2x^2}}{\sqrt{d+ex}} dx$ | 2572 |
| 3.348 | $\int \frac{\sqrt{a^2+2abx+b^2x^2}}{(d+ex)^{3/2}} dx$ | 2578 |
| 3.349 | $\int \frac{\sqrt{a^2+2abx+b^2x^2}}{(d+ex)^{5/2}} dx$ | 2583 |
| 3.350 | $\int \frac{\sqrt{a^2+2abx+b^2x^2}}{(d+ex)^{7/2}} dx$ | 2589 |
| 3.351 | $\int (d+ex)^{5/2}(a^2+2abx+b^2x^2)^{3/2} dx$ | 2595 |
| 3.352 | $\int (d+ex)^{3/2}(a^2+2abx+b^2x^2)^{3/2} dx$ | 2602 |
| 3.353 | $\int \sqrt{d+ex}(a^2+2abx+b^2x^2)^{3/2} dx$ | 2609 |
| 3.354 | $\int \frac{(a^2+2abx+b^2x^2)^{3/2}}{\sqrt{d+ex}} dx$ | 2616 |
| 3.355 | $\int \frac{(a^2+2abx+b^2x^2)^{3/2}}{(d+ex)^{3/2}} dx$ | 2622 |
| 3.356 | $\int \frac{(a^2+2abx+b^2x^2)^{3/2}}{(d+ex)^{5/2}} dx$ | 2628 |
| 3.357 | $\int \frac{(a^2+2abx+b^2x^2)^{3/2}}{(d+ex)^{7/2}} dx$ | 2635 |
| 3.358 | $\int \frac{(a^2+2abx+b^2x^2)^{3/2}}{(d+ex)^{9/2}} dx$ | 2642 |

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| 3.359 | $\int \frac{(a^2+2abx+b^2x^2)^{3/2}}{(d+ex)^{11/2}} dx$ | 2649 |
| 3.360 | $\int (d+ex)^{5/2} (a^2+2abx+b^2x^2)^{5/2} dx$ | 2656 |
| 3.361 | $\int (d+ex)^{3/2} (a^2+2abx+b^2x^2)^{5/2} dx$ | 2664 |
| 3.362 | $\int \sqrt{d+ex} (a^2+2abx+b^2x^2)^{5/2} dx$ | 2672 |
| 3.363 | $\int \frac{(a^2+2abx+b^2x^2)^{5/2}}{\sqrt{d+ex}} dx$ | 2680 |
| 3.364 | $\int \frac{(a^2+2abx+b^2x^2)^{5/2}}{(d+ex)^{3/2}} dx$ | 2687 |
| 3.365 | $\int \frac{(a^2+2abx+b^2x^2)^{5/2}}{(d+ex)^{5/2}} dx$ | 2694 |
| 3.366 | $\int \frac{(a^2+2abx+b^2x^2)^{5/2}}{(d+ex)^{7/2}} dx$ | 2701 |
| 3.367 | $\int \frac{(a^2+2abx+b^2x^2)^{5/2}}{(d+ex)^{9/2}} dx$ | 2708 |
| 3.368 | $\int \frac{(a^2+2abx+b^2x^2)^{5/2}}{(d+ex)^{11/2}} dx$ | 2715 |
| 3.369 | $\int \frac{(a^2+2abx+b^2x^2)^{5/2}}{(d+ex)^{13/2}} dx$ | 2722 |
| 3.370 | $\int \frac{(a^2+2abx+b^2x^2)^{5/2}}{(d+ex)^{15/2}} dx$ | 2730 |
| 3.371 | $\int \frac{(d+ex)^{7/2}}{\sqrt{a^2+2abx+b^2x^2}} dx$ | 2738 |
| 3.372 | $\int \frac{(d+ex)^{5/2}}{\sqrt{a^2+2abx+b^2x^2}} dx$ | 2747 |
| 3.373 | $\int \frac{(d+ex)^{3/2}}{\sqrt{a^2+2abx+b^2x^2}} dx$ | 2755 |
| 3.374 | $\int \frac{\sqrt{d+ex}}{\sqrt{a^2+2abx+b^2x^2}} dx$ | 2762 |
| 3.375 | $\int \frac{1}{\sqrt{d+ex}\sqrt{a^2+2abx+b^2x^2}} dx$ | 2768 |
| 3.376 | $\int \frac{1}{(d+ex)^{3/2}\sqrt{a^2+2abx+b^2x^2}} dx$ | 2774 |
| 3.377 | $\int \frac{1}{(d+ex)^{5/2}\sqrt{a^2+2abx+b^2x^2}} dx$ | 2781 |
| 3.378 | $\int \frac{1}{(d+ex)^{7/2}\sqrt{a^2+2abx+b^2x^2}} dx$ | 2788 |
| 3.379 | $\int \frac{(d+ex)^{9/2}}{(a^2+2abx+b^2x^2)^{3/2}} dx$ | 2796 |
| 3.380 | $\int \frac{(d+ex)^{7/2}}{(a^2+2abx+b^2x^2)^{3/2}} dx$ | 2807 |
| 3.381 | $\int \frac{(d+ex)^{5/2}}{(a^2+2abx+b^2x^2)^{3/2}} dx$ | 2816 |
| 3.382 | $\int \frac{(d+ex)^{3/2}}{(a^2+2abx+b^2x^2)^{3/2}} dx$ | 2824 |
| 3.383 | $\int \frac{\sqrt{d+ex}}{(a^2+2abx+b^2x^2)^{3/2}} dx$ | 2831 |
| 3.384 | $\int \frac{1}{\sqrt{d+ex}(a^2+2abx+b^2x^2)^{3/2}} dx$ | 2838 |
| 3.385 | $\int \frac{1}{(d+ex)^{3/2}(a^2+2abx+b^2x^2)^{3/2}} dx$ | 2845 |
| 3.386 | $\int \frac{1}{(d+ex)^{5/2}(a^2+2abx+b^2x^2)^{3/2}} dx$ | 2854 |
| 3.387 | $\int \frac{1}{(d+ex)^{7/2}(a^2+2abx+b^2x^2)^{3/2}} dx$ | 2864 |
| 3.388 | $\int \frac{(d+ex)^{13/2}}{(a^2+2abx+b^2x^2)^{5/2}} dx$ | 2876 |

| | | |
|-------|---|------|
| 3.389 | $\int \frac{(d+ex)^{11/2}}{(a^2+2abx+b^2x^2)^{5/2}} dx$ | 2894 |
| 3.390 | $\int \frac{(d+ex)^{9/2}}{(a^2+2abx+b^2x^2)^{5/2}} dx$ | 2910 |
| 3.391 | $\int \frac{(d+ex)^{7/2}}{(a^2+2abx+b^2x^2)^{5/2}} dx$ | 2921 |
| 3.392 | $\int \frac{(d+ex)^{5/2}}{(a^2+2abx+b^2x^2)^{5/2}} dx$ | 2930 |
| 3.393 | $\int \frac{(d+ex)^{3/2}}{(a^2+2abx+b^2x^2)^{5/2}} dx$ | 2940 |
| 3.394 | $\int \frac{\sqrt{d+ex}}{(a^2+2abx+b^2x^2)^{5/2}} dx$ | 2950 |
| 3.395 | $\int \frac{1}{\sqrt{d+ex}(a^2+2abx+b^2x^2)^{5/2}} dx$ | 2960 |
| 3.396 | $\int \frac{1}{(d+ex)^{3/2}(a^2+2abx+b^2x^2)^{5/2}} dx$ | 2970 |
| 3.397 | $\int \frac{1}{(d+ex)^{5/2}(a^2+2abx+b^2x^2)^{5/2}} dx$ | 2982 |
| 3.398 | $\int \frac{1}{(d+ex)^{7/2}(a^2+2abx+b^2x^2)^{5/2}} dx$ | 2999 |
| 3.399 | $\int (ad+bdx)^m (a^2c+2abcx+b^2cx^2)^3 dx$ | 3017 |
| 3.400 | $\int (ad+bdx)^m (a^2c+2abcx+b^2cx^2)^2 dx$ | 3024 |
| 3.401 | $\int (ad+bdx)^m (a^2c+2abcx+b^2cx^2) dx$ | 3031 |
| 3.402 | $\int (ad+bdx)^m dx$ | 3037 |
| 3.403 | $\int \frac{(ad+bdx)^m}{a^2c+2abcx+b^2cx^2} dx$ | 3042 |
| 3.404 | $\int \frac{(ad+bdx)^m}{(a^2c+2abcx+b^2cx^2)^2} dx$ | 3048 |
| 3.405 | $\int \frac{(ad+bdx)^m}{(a^2c+2abcx+b^2cx^2)^3} dx$ | 3054 |
| 3.406 | $\int (ad+bdx)^m (a^2c+2abcx+b^2cx^2)^{3/2} dx$ | 3060 |
| 3.407 | $\int (ad+bdx)^m \sqrt{a^2c+2abcx+b^2cx^2} dx$ | 3066 |
| 3.408 | $\int \frac{(ad+bdx)^m}{\sqrt{a^2c+2abcx+b^2cx^2}} dx$ | 3071 |
| 3.409 | $\int \frac{(ad+bdx)^m}{(a^2c+2abcx+b^2cx^2)^{3/2}} dx$ | 3076 |
| 3.410 | $\int \frac{(ad+bdx)^m}{(a^2c+2abcx+b^2cx^2)^{5/2}} dx$ | 3082 |
| 3.411 | $\int (d+ex)^m (a^2+2abx+b^2x^2)^3 dx$ | 3088 |
| 3.412 | $\int (d+ex)^m (a^2+2abx+b^2x^2)^2 dx$ | 3098 |
| 3.413 | $\int (d+ex)^m (a^2+2abx+b^2x^2) dx$ | 3108 |
| 3.414 | $\int \frac{(d+ex)^m}{a^2+2abx+b^2x^2} dx$ | 3115 |
| 3.415 | $\int \frac{(d+ex)^m}{(a^2+2abx+b^2x^2)^2} dx$ | 3121 |
| 3.416 | $\int \frac{(d+ex)^m}{(a^2+2abx+b^2x^2)^3} dx$ | 3127 |
| 3.417 | $\int (d+ex)^m (a^2+2abx+b^2x^2)^{5/2} dx$ | 3133 |
| 3.418 | $\int (d+ex)^m (a^2+2abx+b^2x^2)^{3/2} dx$ | 3142 |
| 3.419 | $\int (d+ex)^m \sqrt{a^2+2abx+b^2x^2} dx$ | 3150 |
| 3.420 | $\int \frac{(d+ex)^m}{\sqrt{a^2+2abx+b^2x^2}} dx$ | 3156 |
| 3.421 | $\int \frac{(d+ex)^m}{(a^2+2abx+b^2x^2)^{3/2}} dx$ | 3161 |

3.422 $\int \frac{(d+ex)^m}{(a^2+2abx+b^2x^2)^{5/2}} dx \dots\dots\dots 3167$

3.423 $\int (d+ex)^3 (cd^2+2cdex+ce^2x^2)^p dx \dots\dots\dots 3173$

3.424 $\int (d+ex) (cd^2+2cdex+ce^2x^2)^p dx \dots\dots\dots 3179$

3.425 $\int \frac{(cd^2+2cdex+ce^2x^2)^p}{d+ex} dx \dots\dots\dots 3185$

3.426 $\int \frac{(cd^2+2cdex+ce^2x^2)^p}{(d+ex)^3} dx \dots\dots\dots 3190$

3.427 $\int \frac{(cd^2+2cdex+ce^2x^2)^p}{(d+ex)^5} dx \dots\dots\dots 3195$

3.428 $\int (d+ex)^4 (cd^2+2cdex+ce^2x^2)^p dx \dots\dots\dots 3201$

3.429 $\int (d+ex)^2 (cd^2+2cdex+ce^2x^2)^p dx \dots\dots\dots 3208$

3.430 $\int (cd^2+2cdex+ce^2x^2)^p dx \dots\dots\dots 3214$

3.431 $\int \frac{(cd^2+2cdex+ce^2x^2)^p}{(d+ex)^2} dx \dots\dots\dots 3219$

3.432 $\int \frac{(cd^2+2cdex+ce^2x^2)^p}{(d+ex)^4} dx \dots\dots\dots 3225$

3.433 $\int (d+ex)^3 (a^2+2abx+b^2x^2)^p dx \dots\dots\dots 3231$

3.434 $\int (d+ex)^2 (a^2+2abx+b^2x^2)^p dx \dots\dots\dots 3241$

3.435 $\int (d+ex) (a^2+2abx+b^2x^2)^p dx \dots\dots\dots 3249$

3.436 $\int (a^2+2abx+b^2x^2)^p dx \dots\dots\dots 3256$

3.437 $\int \frac{(a^2+2abx+b^2x^2)^p}{d+ex} dx \dots\dots\dots 3261$

3.438 $\int \frac{(a^2+2abx+b^2x^2)^p}{(d+ex)^2} dx \dots\dots\dots 3266$

3.439 $\int \frac{(a^2+2abx+b^2x^2)^p}{(d+ex)^3} dx \dots\dots\dots 3272$

3.440 $\int (d+ex)^{3/2} (a^2+2abx+b^2x^2)^p dx \dots\dots\dots 3278$

3.441 $\int \sqrt{d+ex} (a^2+2abx+b^2x^2)^p dx \dots\dots\dots 3284$

3.442 $\int \frac{(a^2+2abx+b^2x^2)^p}{\sqrt{d+ex}} dx \dots\dots\dots 3290$

3.443 $\int \frac{(a^2+2abx+b^2x^2)^p}{(d+ex)^{3/2}} dx \dots\dots\dots 3296$

3.444 $\int \frac{(a^2+2abx+b^2x^2)^p}{(d+ex)^{5/2}} dx \dots\dots\dots 3302$

3.445 $\int (d+ex)^m (cd^2+2cdex+ce^2x^2)^p dx \dots\dots\dots 3308$

3.446 $\int (d+ex)^p (cd^2+2cdex+ce^2x^2)^{-p} dx \dots\dots\dots 3314$

3.447 $\int (d+ex)^{-1-2p} (cd^2+2cdex+ce^2x^2)^p dx \dots\dots\dots 3319$

3.448 $\int (d+ex)^{-1+2p} (cd^2+2cdex+ce^2x^2)^{-p} dx \dots\dots\dots 3324$

3.449 $\int (1+x)^m (1+2x+x^2)^p dx \dots\dots\dots 3329$

3.450 $\int \left(\frac{be}{2c}+ex\right)^m \left(\frac{b^2}{4c}+bx+cx^2\right)^p dx \dots\dots\dots 3334$

3.451 $\int (d+ex)^m (a^2+2abx+b^2x^2)^p dx \dots\dots\dots 3340$

3.452 $\int (d+ex)^m (a^2+2abx+b^2x^2)^{5+p} dx \dots\dots\dots 3346$

3.453 $\int (d+ex)^{-3-2p} (a^2+2abx+b^2x^2)^p dx \dots\dots\dots 3351$

3.454 $\int (d+ex) (9+12x+4x^2)^p dx \dots\dots\dots 3357$

4 Appendix 3363

4.1 Listing of Grading functions 3363

4.2 Links to plain text integration problems used in this report for each CAS381

CHAPTER 1

INTRODUCTION

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This report gives the result of running the computer algebra independent integration test. The download section in on the main webpage contains links to download the problems in plain text format used for all CAS systems. The number of integrals in this report is [454]. This is test number [90].

1.1 Listing of CAS systems tested

The following are the CAS systems tested:

1. Mathematica 14 (January 9, 2024) on windows 10 pro.
2. Rubi 4.17.3 (Sept 25, 2023) on Mathematica 14 on windows 10m pro.
3. Maple 2024 (March 1, 2024) on windows 10 pro.
4. Maxima 5.47 (June 1, 2023) using Lisp SBCL 2.4.0 on Linux Manjaro 23.1.2 KDE via sagemath 10.3.
5. FriCAS 1.3.10 built with sbcl 2.3.11 (January 10, 2024) on Linux Manjaro 23.1.2 KDE via sagemath 10.3.
6. Giac/Xcas 1.9.0-99 on Linux via sagemath 10.3.
7. Sympy 1.12 using Python 3.11.6 (Nov 14 2023, 09:36:21) [GCC 13.2.1 20230801] on Linux Manjaro 23.1.2 KDE.
8. Mupad using Matlab 2021a with Symbolic Math Toolbox Version 8.7 on windows 10.
9. Reduce CSL rev 6687 (January 9, 2024) on Linux Manjaro 23.1.2 KDE.

Maxima and Fricas and Giac are called using Sagemath. This was done using Sagemath `integrate` command by changing the name of the algorithm to use the different CAS systems.

Sympy was run directly in Python not via sagemath.

Reduce was called directly.

1.2 Results

Important note: A number of problems in this test suite have no antiderivative in closed form. This means the antiderivative of these integrals can not be expressed in terms of elementary, special functions or Hypergeometric2F1 functions. RootSum and RootOf are not allowed. If a CAS returns the above integral unevaluated within the time limit, then the result is counted as passed and assigned an A grade.

However, if CAS times out, then it is assigned an F grade even if the integral is not integrable, as this implies CAS could not determine that the integral is not integrable in the time limit.

If a CAS returns an antiderivative to such an integral, it is assigned an A grade automatically and this special result is listed in the introduction section of each individual test report to make it easy to identify as this can be important result to investigate.

The results given in in the table below reflects the above.

| System | % solved | % Failed |
|-------------|----------------|---------------|
| Rubi | 100.00 (454) | 0.00 (0) |
| Mathematica | 100.00 (454) | 0.00 (0) |
| Maple | 96.48 (438) | 3.52 (16) |
| Fricas | 96.48 (438) | 3.52 (16) |
| Reduce | 95.59 (434) | 4.41 (20) |
| Giac | 86.78 (394) | 13.22 (60) |
| Mupad | 72.03 (327) | 27.97 (127) |
| Maxima | 68.72 (312) | 31.28 (142) |
| Sympy | 54.63 (248) | 45.37 (206) |

Table 1.1: Percentage solved for each CAS

The table below gives additional break down of the grading of quality of the antiderivatives generated by each CAS. The grading is given using the letters A,B,C and F with A being the best quality. The grading is accomplished by comparing the antiderivative generated with the optimal antiderivatives included in the test suite. The following table describes the meaning of these grades.

| grade | description |
|-------|---|
| A | Integral was solved and antiderivative is optimal in quality and leaf size. |
| B | Integral was solved and antiderivative is optimal in quality but leaf size is larger than twice the optimal antiderivatives leaf size. |
| C | Integral was solved and antiderivative is non-optimal in quality. This can be due to one or more of the following reasons <ol style="list-style-type: none"> 1. antiderivative contains a hypergeometric function and the optimal antiderivative does not. 2. antiderivative contains a special function and the optimal antiderivative does not. 3. antiderivative contains the imaginary unit and the optimal antiderivative does not. |
| F | Integral was not solved. Either the integral was returned unevaluated within the time limit, or it timed out, or CAS hanged or crashed or an exception was raised. |

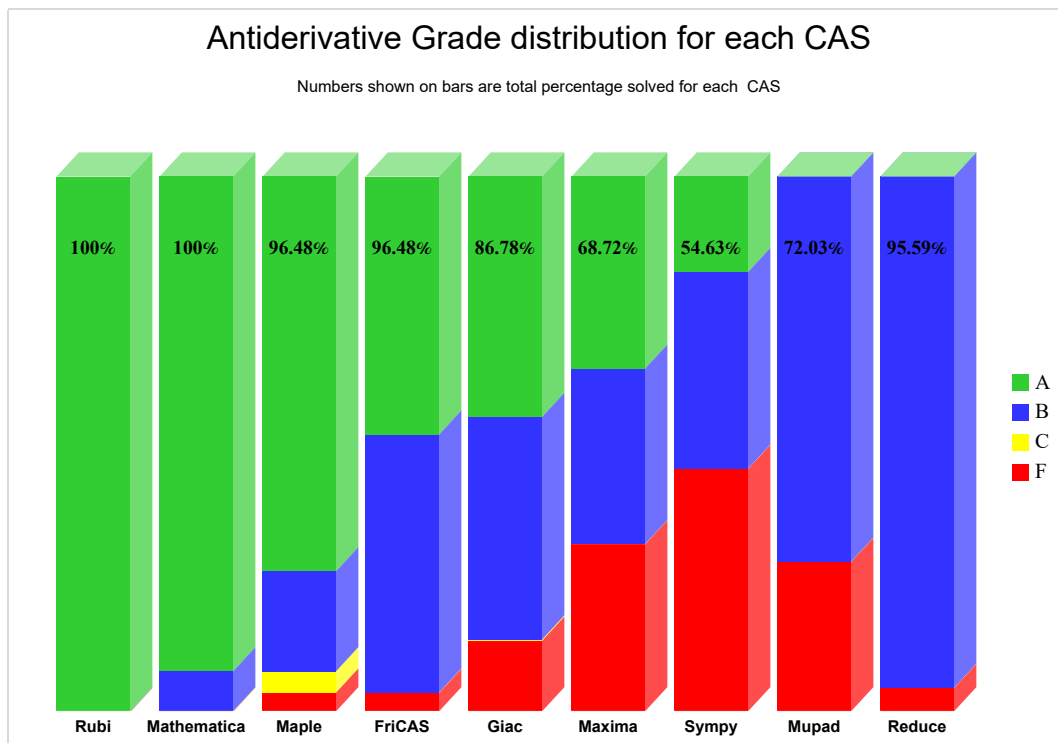
Table 1.2: Description of grading applied to integration result

Grading is implemented for all CAS systems. Based on the above, the following table summarizes the grading for this test suite.

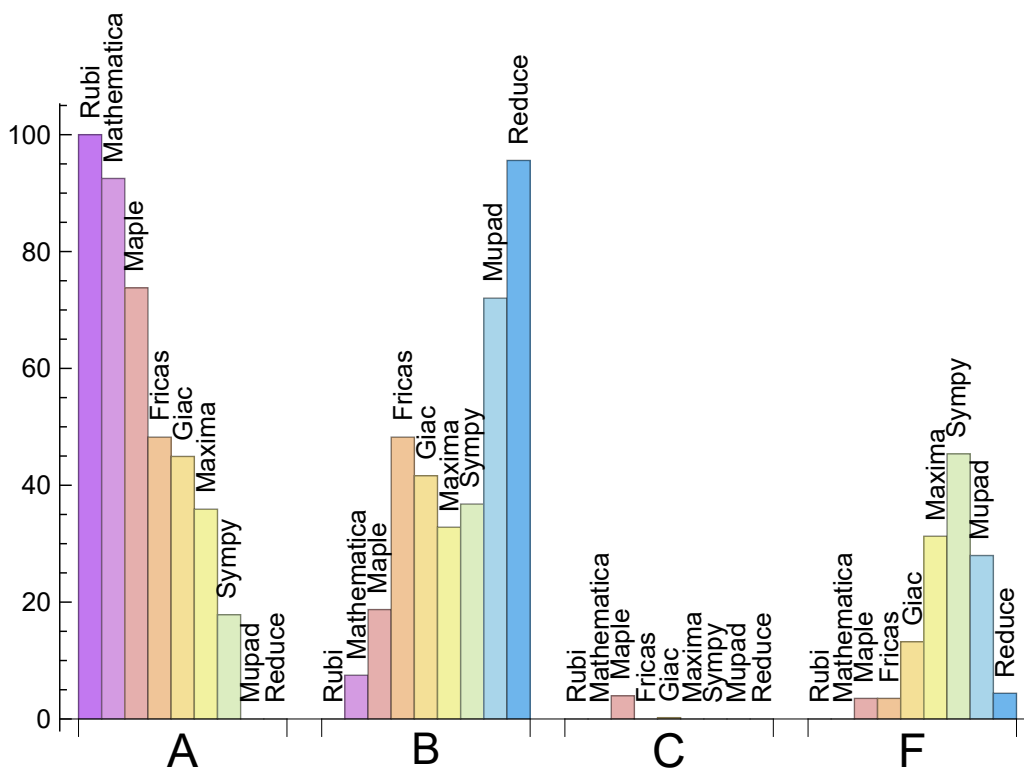
| System | % A grade | % B grade | % C grade | % F grade |
|-------------|-----------|-----------|-----------|-----------|
| Rubi | 100.000 | 0.000 | 0.000 | 0.000 |
| Mathematica | 92.511 | 7.489 | 0.000 | 0.000 |
| Maple | 73.789 | 18.722 | 3.965 | 3.524 |
| Fricas | 48.238 | 48.238 | 0.000 | 3.524 |
| Giac | 44.934 | 41.630 | 0.220 | 13.216 |
| Maxima | 35.903 | 32.819 | 0.000 | 31.278 |
| Sympy | 17.841 | 36.784 | 0.000 | 45.374 |
| Mupad | 0.000 | 72.026 | 0.000 | 27.974 |
| Reduce | 0.000 | 95.595 | 0.000 | 4.405 |

Table 1.3: Antiderivative Grade distribution of each CAS

The following is a Bar chart illustration of the data in the above table.



The figure below compares the grades of the CAS systems.



The following table shows the distribution of the different types of failures for each CAS. There are 3 types failures. The first is when CAS returns the input within the time limit, which means it could not solve it. This is the typical failure and given as **F**.

The second failure is due to time out. CAS could not solve the integral within the 3 minutes time limit which is assigned. This is assigned **F(-1)**.

The third is due to an exception generated, indicated as **F(-2)**. This most likely indicates an interface problem between sagemath and the CAS (applicable only to FriCAS, Maxima and Giac) or it could be an indication of an internal error in the CAS itself. This type of error requires more investigation to determine the cause.

| System | Number failed | Percentage normal failure | Percentage time-out failure | Percentage exception failure |
|-------------|---------------|---------------------------|-----------------------------|------------------------------|
| Rubi | 0 | 0.00 | 0.00 | 0.00 |
| Mathematica | 0 | 0.00 | 0.00 | 0.00 |
| Fricas | 16 | 100.00 | 0.00 | 0.00 |
| Maple | 16 | 100.00 | 0.00 | 0.00 |
| Reduce | 20 | 100.00 | 0.00 | 0.00 |
| Giac | 60 | 50.00 | 0.00 | 50.00 |
| Mupad | 127 | 0.00 | 100.00 | 0.00 |
| Maxima | 142 | 31.69 | 0.00 | 68.31 |
| Sympy | 206 | 72.82 | 22.82 | 4.37 |

Table 1.4: Failure statistics for each CAS

1.3 Time and leaf size Performance

The table below summarizes the performance of each CAS system in terms of time used and leaf size of results.

Mean size is the average leaf size produced by the CAS (before any normalization). The Normalized mean is relative to the mean size of the optimal anti-derivative given in the input files.

For example, if CAS has **Normalized mean** of 3, then the mean size of its leaf size is 3 times as large as the mean size of the optimal leaf size.

Median size is value of leaf size where half the values are larger than this and half are smaller (before any normalization). i.e. The Middle value.

Similarly the **Normalized median** is relative to the median leaf size of the optimal.

For example, if a CAS has Normalized median of 1.2, then its median is 1.2 as large as the median leaf size of the optimal.

| System | Mean time (sec) |
|-------------|-----------------|
| Maxima | 0.05 |
| Fricas | 0.10 |
| Giac | 0.20 |
| Reduce | 0.23 |
| Mathematica | 0.30 |
| Rubi | 0.40 |
| Maple | 1.36 |
| Sympy | 1.80 |
| Mupad | 5.34 |

Table 1.5: Time performance for each CAS

| System | Mean size | Normalized mean | Median size | Normalized median |
|-------------|-----------|-----------------|-------------|-------------------|
| Rubi | 91.56 | 0.94 | 69.50 | 1.00 |
| Mathematica | 114.51 | 1.11 | 72.00 | 0.89 |
| Maple | 146.00 | 1.29 | 78.00 | 0.97 |
| Maxima | 186.77 | 2.35 | 96.50 | 1.69 |
| Mupad | 186.88 | 2.05 | 80.00 | 1.46 |
| Giac | 265.91 | 2.26 | 174.00 | 1.58 |
| Reduce | 293.53 | 2.22 | 113.00 | 1.59 |
| Fricas | 313.06 | 2.46 | 116.50 | 1.82 |
| Sympy | 695.40 | 5.65 | 133.50 | 2.57 |

Table 1.6: Leaf size performance for each CAS

1.4 Performance based on number of rules Rubi used

This section shows how each CAS performed based on the number of rules Rubi needed to solve the same integral. One diagram is given for each CAS.

On the y axis is the percentage solved which Rubi itself needed the number of rules given the x axis. These plots show that as more rules are needed then most CAS system percentage of solving decreases which indicates the integral is becoming more complicated to solve.

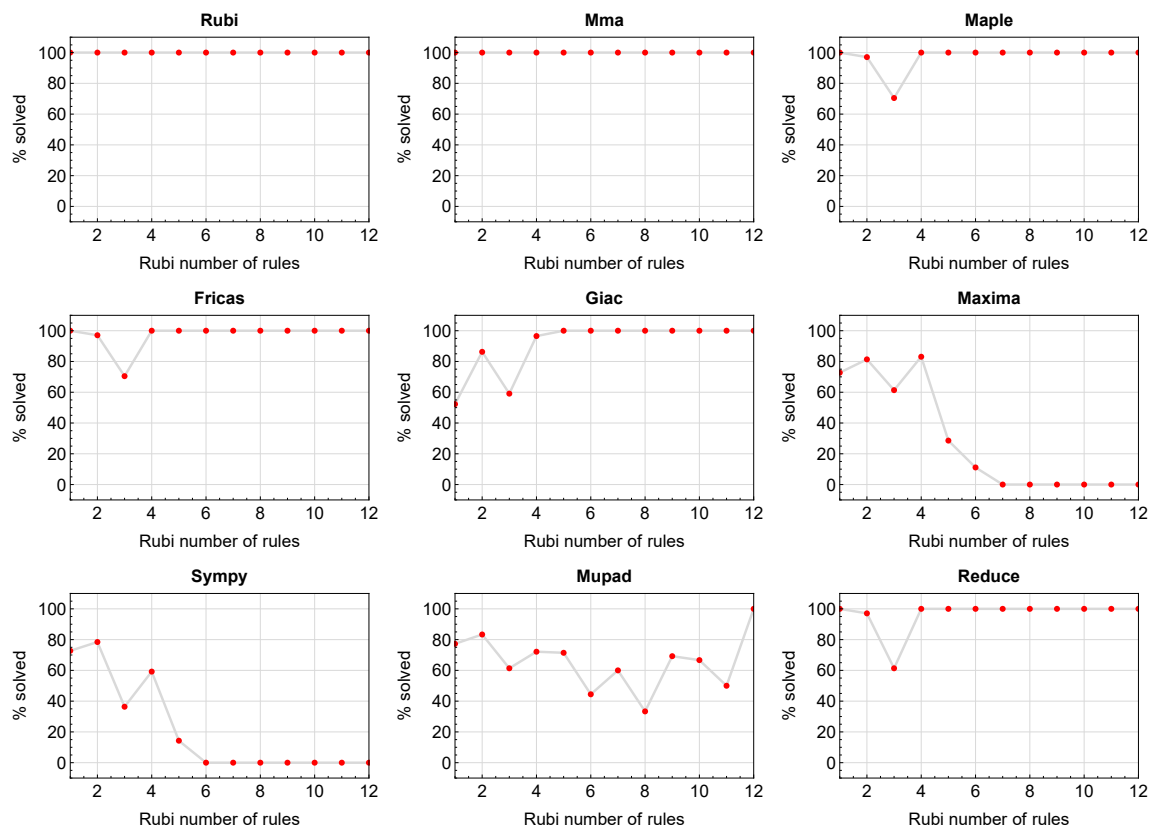


Figure 1.1: Solving statistics per number of Rubi rules used

1.5 Performance based on number of steps Rubi used

This section shows how each CAS performed based on the number of steps Rubi needed to solve the same integral. Note that the number of steps Rubi needed can be much higher than the number of rules, as the same rule could be used more than once.

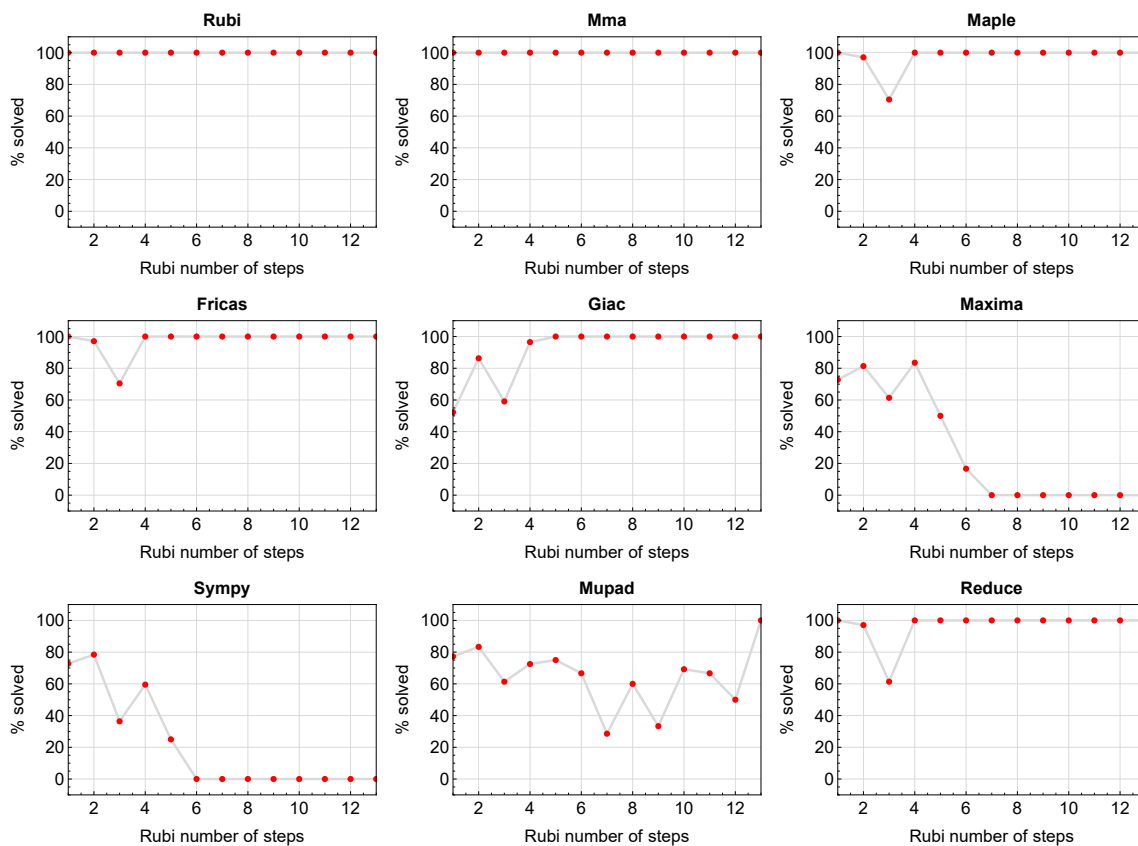


Figure 1.2: Solving statistics per number of Rubi steps used

The above diagram show that the percentage of solved intergals decreases for most CAS systems as the number of steps increases. As expected, for integrals that required less steps by Rubi, CAS systems had more success which indicates the integral was not as hard to solve. As Rubi needed more steps to solve the integral, the solved percentage decreased for most CAS systems which indicates the integral is becoming harder to solve.

1.6 Solved integrals histogram based on leaf size of result

The following shows the distribution of solved integrals for each CAS system based on leaf size of the antiderivatives produced by each CAS. It shows that most integrals solved produced leaf size less than about 100 to 150. The bin size used is 40.

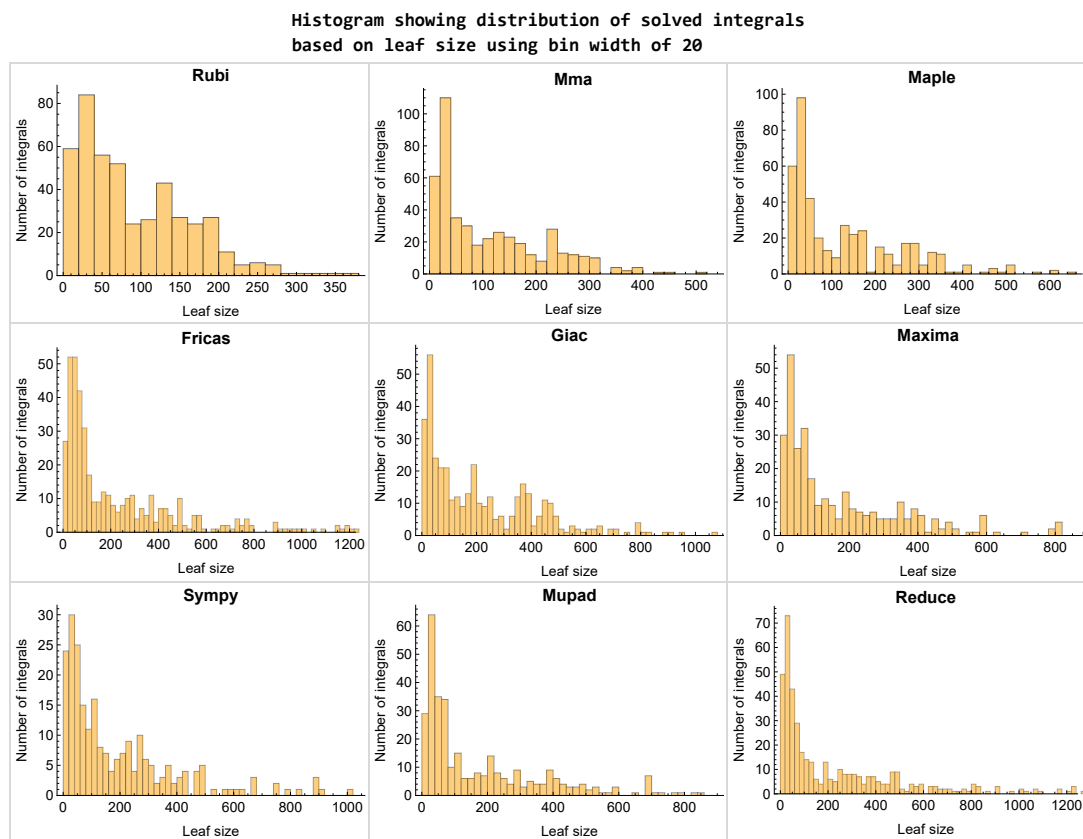


Figure 1.3: Solved integrals based on leaf size distribution

1.7 Solved integrals histogram based on CPU time used

The following shows the distribution of solved integrals for each CAS system based on CPU time used in seconds. The bin size used is 0.1 second.

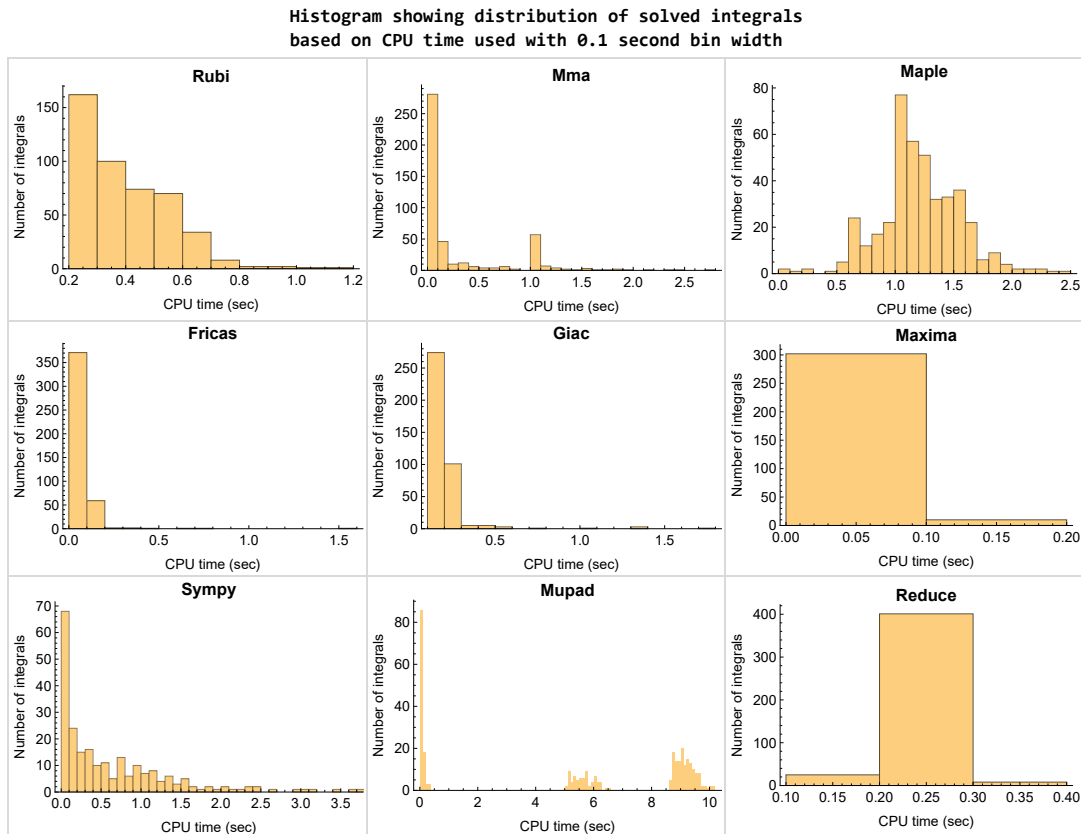


Figure 1.4: Solved integrals histogram based on CPU time used

1.8 Leaf size vs. CPU time used

The following shows the relation between the CPU time used to solve an integral and the leaf size of the antiderivative.

The result for Fracas, Maxima and Giac is shifted more to the right than the other CAS system due to the use of sagemath to call them, which causes an initial slight delay in the timing to start the integration due to overhead of starting a new process each time. This should also be taken into account when looking at the timing of these three CAS systems. Direct calls not using sagemath would result in faster timings, but current implementation uses sagemath as this makes testing much easier to do.

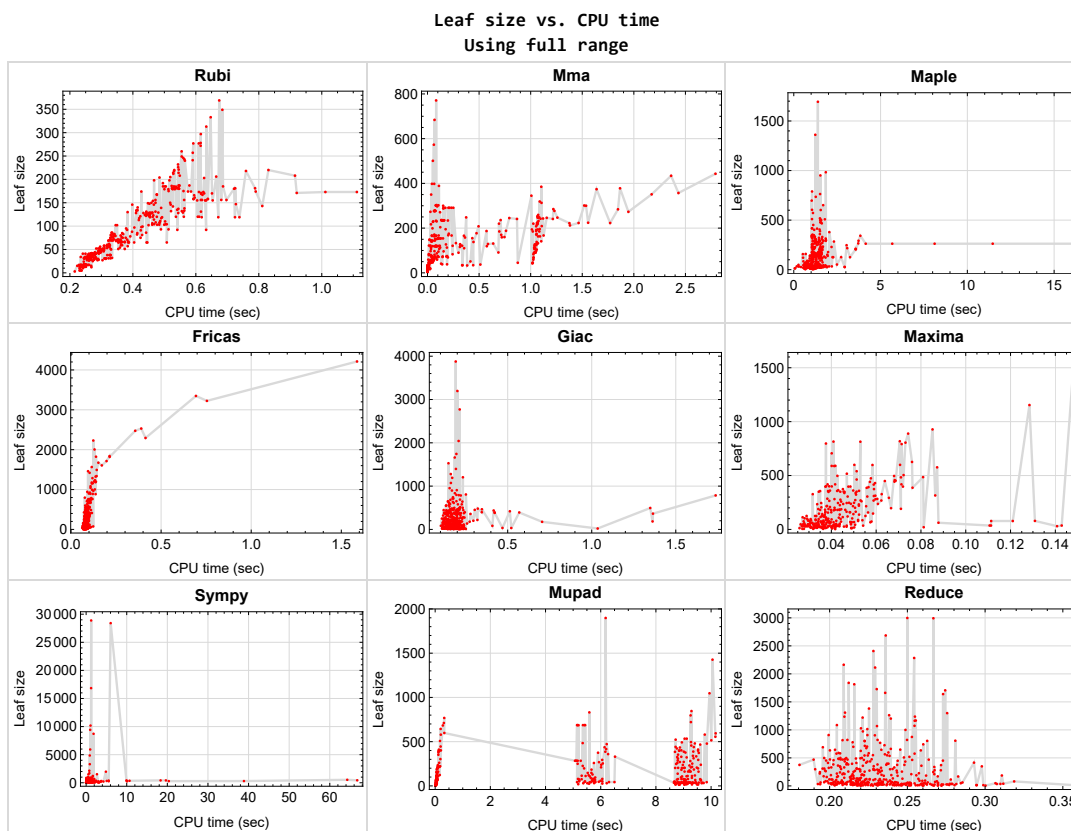


Figure 1.5: Leaf size vs. CPU time. Full range

1.9 list of integrals with no known antiderivative

{}

1.10 List of integrals solved by CAS but has no known antiderivative

Rubi {}

Mathematica {}

Maple {}

Maxima {}

Fricas {}

Sympy {}

Giac {}

Reduce {}

Mupad {}

1.11 list of integrals solved by CAS but failed verification

The following are integrals solved by CAS but the verification phase failed to verify the anti-derivative produced is correct. This does not necessarily mean that the anti-derivative is wrong as additional methods of verification might be needed, or more time is needed (3 minutes time limit was used). These integrals are listed here to make it possible to do further investigation to determine why the result could not be verified.

Rubi {}

Mathematica {}

Maple {2, 11, 268, 269, 270, 271, 272, 273, 274, 275, 338, 344, 345, 346, 347, 348, 349, 350}

Maxima Verification phase not currently implemented.

Fricas Verification phase not currently implemented.

Sympy Verification phase not currently implemented.

Giac Verification phase not currently implemented.

Reduce Verification phase not currently implemented.

Mupad Verification phase not currently implemented.

1.12 Timing

The command `AbsoluteTiming[]` was used in Mathematica to obtain the elapsed time for each integrate call. In Maple, the command `Usage` was used as in the following example

```
cpu_time := Usage(assign ('result_of_int',int(expr,x)),output='realtime')
```

For all other CAS systems, the elapsed time to complete each integral was found by taking the difference between the time after the call completed from the time before the call was made. This was done using Python's `time.time()` call.

All elapsed times shown are in seconds. A time limit of 3 CPU minutes was used for each integral. If the integrate command did not complete within this time limit, the integral was aborted and considered to have failed and assigned an F grade. The time used by failed integrals due to time out was not counted in the final statistics.

1.13 Verification

A verification phase was applied on the result of integration for **Rubi** and **Mathematica**.

Future version of this report will implement verification for the other CAS systems. For the integrals whose result was not run through a verification phase, it is assumed that the antiderivative was correct.

Verification phase also had 3 minutes time out. An integral whose result was not verified could still be correct, but further investigation is needed on those integrals. These integrals were marked in the summary table below and also in each integral separate section so they are easy to identify and locate.

1.14 Important notes about some of the results

Important note about Maxima results

Since tests were run in a batch mode, and using an automated script, then any integral where Maxima needed an interactive response from the user to answer a question during the evaluation of the integral will fail.

The exception raised is `ValueError`. Therefore Maxima results is lower than what would result if Maxima was run directly and each question was answered correctly.

The percentage of such failures were not counted for each test file, but for an example, for the `Timofeev` test file, there were about 14 such integrals out of total 705, or about 2 percent. This percentage can be higher or lower depending on the specific input test file.

Such integrals can be identified by looking at the output of the integration in each section for Maxima. The exception message will indicate the cause of error.

Maxima `integrate` was run using SageMath with the following settings set by default

```
'besselexpand : true'  
'display2d : false'  
'domain : complex'  
'keepfloat : true'  
'load(to_poly_solve)'  
'load(simplify_sum)'  
'load(abs_integrate)' 'load(diag)'
```

SageMath automatic loading of Maxima `abs_integrate` was found to cause some problems. So the following code was added to disable this effect.

```
from sage.interfaces.maxima_lib import maxima_lib  
maxima_lib.set('extra_definite_integration_methods', '[]')  
maxima_lib.set('extra_integration_methods', '[]')
```

See <https://ask.sagemath.org/question/43088/integrate-results-that-are-different-from-using-maxima/> for reference.

Important note about FriCAS result

There were few integrals which failed due to SageMath interface and not because FriCAS system could not do the integration.

These will fail With error `Exception raised: NotImplementedError`.

The number of such cases seems to be very small. About 1 or 2 percent of all integrals. These can be identified by looking at the exception message given in the result.

Important note about finding leaf size of antiderivative

For Mathematica, Rubi, and Maple, the builtin system function `LeafSize` was used to find the leaf size of each antiderivative.

The other CAS systems (SageMath and Sympy) do not have special builtin function for this purpose at this time. Therefore the leaf size for Fricas and Sympy antiderivative was determined using the following function, thanks to user `slelievre` at https://ask.sagemath.org/question/57123/could-we-have-a-leaf_count-function-in-base-sagemath/

```
def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)
```

For Sympy, which was called directly from Python, the following code was used to obtain the leafsize of its result

```
try:  
    # 1.7 is a fudge factor since it is low side from actual leaf count  
    leafCount = round(1.7*count_ops(anti))  
  
except Exception as ee:  
    leafCount = 1
```

Important note about Mupad results

Matlab's symbolic toolbox does not have a leaf count function to measure the size of the antiderivative. Maple was used to determine the leaf size of Mupad output by post processing Mupad result.

Currently no grading of the antiderivative for Mupad is implemented. If it can integrate the problem, it was assigned a B grade automatically as a placeholder. In the future, when grading function is implemented for Mupad, the tests will be rerun again.

The following is an example of using Matlab's symbolic toolbox (Mupad) to solve an integral

```
integrand = evalin(symengine, 'cos(x)*sin(x)')  
the_variable = evalin(symengine, 'x')  
anti = int(integrand,the_variable)
```

Which gives $\sin(x)^2/2$

1.15 Current tree layout of integration tests

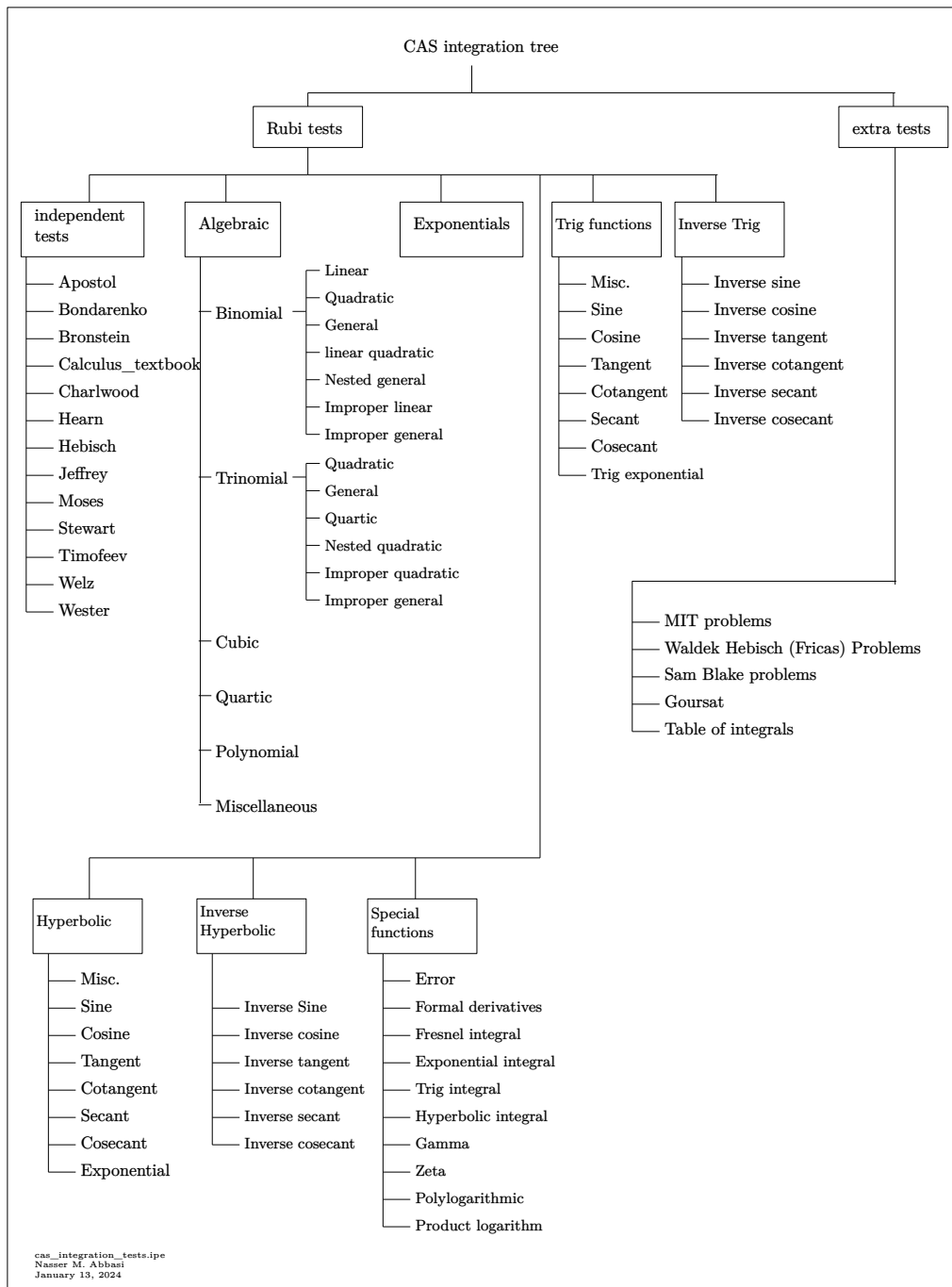
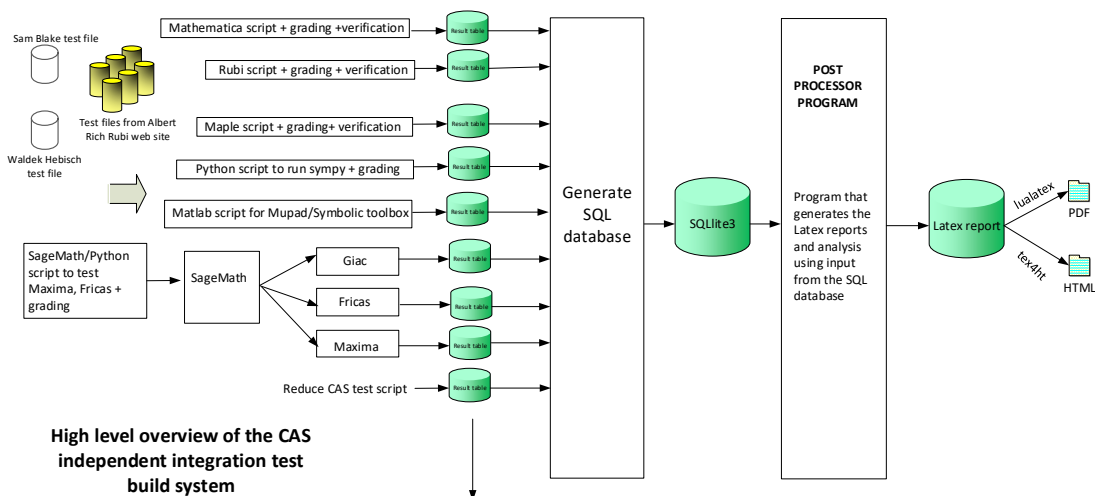


Figure 1.6: CAS integration tests tree

1.16 Design of the test system

The following diagram gives a high level view of the current test build system.



High level overview of the CAS independent integration test build system

One record (line) per one integral result. The line is CSV comma separated. This is description of each record

1. integer. the problem number.
2. integer. 0 for failed, 1 for passed, -1 for timeout, -2 for CAS specific exception. (this is not the grade field)
3. integer. Leaf size of result.
4. integer. Leaf size of the optimal antiderivative.
5. number. CPU time used to solve this integral. 0 if failed.
6. string. The integral in Latex format
7. string. The input used in CAS own syntax.
8. string. The result (antiderivative) produced by CAS in Latex format
9. string. The optimal antiderivative in Latex format.
10. integer. 0 or 1. Indicates if problem has known antiderivative or not
11. String. The result (antiderivative) in CAS own syntax.
12. String. The grade of the antiderivative. Can be "A", "B", "C", or "E"
13. String. Small string description of why the grade was given.
14. integer. 1 if result was verified or 0 if not verified. (For mma, rubi and maple only)

The following fields are present only in Rubi Table file

15. integer. Number of steps used.
16. integer. Number of rules used.
17. integer. Integrand leaf size.
18. real number. Ratio. Field 16 over field 17
19. String of form "{n,n,...}" which is list of the rules used by Rubi
20. String. The optimal antiderivative in Mathematica syntax

Nasser M. Abbasi
January 13, 2024
Design note

CHAPTER 2

DETAILED SUMMARY TABLES OF RESULTS

| | | |
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2.1 List of integrals sorted by grade for each CAS

| | |
|------------------|----|
| Rubi | 38 |
| Mma | 39 |
| Maple | 40 |
| Fricas | 41 |
| Maxima | 42 |
| Giac | 43 |
| Mupad | 44 |
| Sympy | 45 |
| Reduce | 46 |

Rubi

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 383, 384, 385, 386, 387, 388, 389, 390, 391, 392, 393, 394, 395, 396, 397, 398, 399, 400, 401, 402, 403, 404, 405, 406, 407, 408, 409, 410, 411, 412, 413, 414, 415, 416, 417, 418, 419, 420, 421, 422, 423, 424, 425, 426, 427, 428, 429, 430, 431, 432, 433, 434, 435, 436, 437, 438, 439, 440, 441, 442, 443, 444, 445, 446, 447, 448, 449, 450, 451, 452, 453, 454 }

B grade { }

C grade { }

F normal fail { }

F(-1) timedout fail { }

F(-2) exception fail { }

Mma

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 72, 73, 74, 75, 76, 77, 80, 81, 82, 83, 92, 93, 94, 95, 96, 97, 98, 99, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 130, 131, 133, 134, 135, 136, 137, 138, 139, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 305, 306, 307, 308, 309, 310, 311, 313, 315, 316, 317, 318, 319, 322, 323, 324, 325, 326, 327, 328, 330, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 383, 384, 385, 386, 387, 388, 389, 390, 391, 392, 393, 394, 395, 396, 397, 398, 399, 400, 401, 402, 403, 404, 405, 406, 407, 408, 409, 410, 411, 412, 413, 414, 415, 416, 417, 418, 419, 420, 421, 422, 423, 424, 425, 426, 427, 428, 429, 430, 431, 432, 433, 434, 435, 436, 437, 438, 439, 440, 441, 442, 443, 444, 445, 446, 447, 448, 449, 450, 451, 452, 453, 454 }

B grade { 54, 66, 67, 68, 69, 70, 71, 78, 79, 84, 85, 86, 87, 88, 89, 90, 91, 100, 101, 102, 103, 129, 132, 140, 271, 286, 303, 304, 312, 314, 320, 321, 329, 331 }

C grade { }

F normal fail { }

F(-1) timedout fail { }

F(-2) exception fail { }

Maple

A grade { 3, 4, 5, 6, 7, 8, 9, 11, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 56, 57, 58, 59, 60, 61, 63, 64, 65, 73, 74, 75, 76, 77, 81, 82, 83, 109, 110, 111, 112, 113, 114, 115, 116, 117, 120, 121, 123, 124, 125, 126, 127, 131, 134, 135, 136, 137, 138, 139, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 277, 278, 279, 280, 281, 282, 283, 284, 285, 287, 288, 289, 290, 296, 297, 298, 299, 300, 301, 302, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 326, 327, 328, 330, 331, 332, 333, 337, 339, 340, 341, 342, 343, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 383, 384, 385, 388, 389, 390, 399, 400, 401, 402, 403, 404, 405, 406, 407, 408, 409, 410, 419, 423, 424, 425, 426, 427, 428, 429, 430, 431, 432, 434, 435, 436, 445, 446, 448, 450, 453 }

B grade { 1, 2, 10, 12, 54, 55, 62, 66, 67, 68, 69, 70, 71, 72, 78, 79, 80, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 118, 119, 122, 128, 129, 130, 132, 133, 140, 141, 276, 286, 291, 292, 293, 294, 295, 303, 304, 305, 324, 325, 329, 334, 335, 336, 386, 387, 391, 392, 393, 394, 395, 396, 397, 398, 411, 412, 413, 417, 418, 433, 447 }

C grade { 268, 269, 270, 271, 272, 273, 274, 275, 338, 344, 345, 346, 347, 348, 349, 350, 449, 454 }

F normal fail { 414, 415, 416, 420, 421, 422, 437, 438, 439, 440, 441, 442, 443, 444, 451, 452 }

F(-1) timedout fail { }

F(-2) exception fail { }

Fricas

A grade { 3, 4, 5, 6, 7, 8, 14, 15, 16, 17, 18, 19, 22, 23, 24, 25, 26, 27, 31, 32, 33, 34, 35, 41, 42, 43, 44, 45, 52, 53, 56, 57, 58, 59, 60, 61, 63, 64, 73, 111, 112, 113, 114, 123, 134, 141, 142, 143, 144, 145, 149, 150, 151, 152, 153, 158, 159, 160, 161, 172, 173, 174, 202, 203, 204, 206, 207, 208, 209, 214, 215, 218, 219, 220, 221, 226, 227, 232, 233, 234, 236, 237, 238, 240, 241, 242, 243, 245, 246, 247, 250, 251, 252, 254, 255, 256, 259, 260, 261, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 288, 289, 290, 297, 298, 299, 300, 301, 302, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 318, 319, 320, 321, 322, 328, 330, 331, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 379, 380, 381, 382, 383, 390, 402, 403, 406, 407, 408, 419, 423, 424, 425, 426, 429, 430, 431, 434, 435, 436, 445, 446, 447, 448, 449, 450, 453, 454 }

B grade { 1, 2, 9, 10, 11, 12, 13, 20, 21, 28, 29, 30, 36, 37, 38, 39, 40, 46, 47, 48, 49, 50, 51, 54, 55, 62, 65, 66, 67, 68, 69, 70, 71, 72, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 115, 116, 117, 118, 119, 120, 121, 122, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 135, 136, 137, 138, 139, 140, 146, 147, 148, 154, 155, 156, 157, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 205, 210, 211, 212, 213, 216, 217, 222, 223, 224, 225, 228, 229, 230, 231, 235, 239, 244, 248, 249, 253, 257, 258, 262, 263, 286, 287, 291, 292, 293, 294, 295, 296, 303, 304, 305, 306, 317, 323, 324, 325, 326, 327, 329, 332, 333, 334, 335, 360, 378, 384, 385, 386, 387, 388, 389, 391, 392, 393, 394, 395, 396, 397, 398, 399, 400, 401, 404, 405, 409, 410, 411, 412, 413, 417, 418, 427, 428, 432, 433 }

C grade { }

F normal fail { 414, 415, 416, 420, 421, 422, 437, 438, 439, 440, 441, 442, 443, 444, 451, 452 }

F(-1) timedout fail { }

F(-2) exception fail { }

Maxima

A grade { 3, 4, 5, 6, 7, 8, 11, 14, 15, 16, 17, 18, 19, 22, 23, 24, 26, 27, 31, 32, 33, 34, 35, 36, 41, 42, 43, 44, 45, 48, 52, 56, 57, 58, 59, 60, 61, 63, 64, 73, 74, 75, 76, 109, 110, 111, 112, 113, 114, 120, 121, 123, 134, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 159, 160, 161, 202, 212, 213, 223, 224, 231, 237, 238, 239, 242, 243, 244, 247, 248, 252, 253, 257, 262, 264, 310, 311, 312, 313, 320, 321, 322, 330, 331, 332, 336, 337, 338, 339, 340, 341, 342, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 402, 403, 406, 407, 408, 409, 410, 413, 418, 419, 424, 425, 426, 427, 430, 431, 432, 433, 434, 435, 436, 445, 446, 447, 448, 449, 450, 454 }

B grade { 1, 2, 9, 10, 12, 13, 20, 21, 25, 28, 29, 30, 37, 38, 39, 40, 46, 47, 49, 50, 51, 53, 54, 55, 62, 65, 66, 67, 68, 69, 70, 71, 72, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 115, 116, 117, 118, 119, 122, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 135, 136, 137, 138, 139, 140, 158, 162, 163, 164, 165, 166, 167, 168, 169, 235, 236, 240, 241, 245, 246, 249, 250, 251, 254, 255, 256, 258, 259, 260, 261, 263, 265, 266, 267, 268, 269, 276, 277, 278, 279, 280, 281, 291, 292, 293, 294, 295, 296, 309, 318, 319, 326, 327, 328, 329, 360, 399, 400, 401, 404, 405, 411, 412, 417, 423, 428, 429 }

C grade { }

F normal fail { 371, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 383, 384, 385, 386, 387, 388, 389, 390, 391, 392, 393, 394, 395, 396, 397, 398, 414, 415, 416, 420, 421, 422, 437, 438, 439, 440, 441, 442, 443, 444, 451, 452, 453 }

F(-1) timedout fail { }

F(-2) exception fail { 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 203, 204, 205, 206, 207, 208, 209, 210, 211, 214, 215, 216, 217, 218, 219, 220, 221, 222, 225, 226, 227, 228, 229, 230, 232, 233, 234, 270, 271, 272, 273, 274, 275, 282, 283, 284, 285, 286, 287, 288, 289, 290, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 314, 315, 316, 317, 323, 324, 325, 333, 334, 335, 343 }

Giac

A grade { 3, 4, 6, 7, 8, 9, 14, 15, 16, 18, 19, 20, 22, 23, 24, 26, 27, 28, 29, 31, 32, 33, 35, 36, 37, 38, 39, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 56, 57, 58, 59, 61, 63, 64, 65, 73, 75, 76, 81, 82, 83, 109, 110, 111, 112, 113, 114, 115, 120, 121, 123, 124, 131, 134, 135, 136, 142, 143, 144, 145, 150, 151, 152, 153, 158, 172, 173, 174, 175, 176, 182, 183, 184, 185, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 213, 214, 215, 216, 218, 219, 220, 221, 222, 226, 227, 228, 232, 233, 234, 242, 267, 268, 269, 270, 271, 273, 274, 275, 282, 283, 284, 285, 288, 289, 290, 298, 299, 300, 301, 302, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 326, 327, 328, 330, 331, 332, 338, 339, 340, 341, 342, 346, 347, 348, 349, 350, 354, 355, 356, 357, 358, 359, 363, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 383, 384, 385, 390, 391, 392, 393, 394, 402, 407, 423, 424, 430, 436, 445, 446, 449 }

B grade { 1, 2, 10, 11, 12, 13, 17, 21, 25, 30, 34, 40, 53, 54, 55, 60, 62, 66, 67, 68, 69, 70, 71, 72, 74, 77, 78, 79, 80, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 116, 117, 118, 119, 122, 125, 126, 127, 128, 129, 130, 132, 133, 137, 138, 139, 140, 141, 146, 147, 148, 149, 154, 155, 156, 157, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 177, 178, 179, 180, 181, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 211, 212, 217, 223, 224, 225, 229, 230, 231, 265, 266, 272, 276, 277, 278, 279, 280, 281, 286, 287, 291, 292, 293, 294, 295, 296, 297, 303, 304, 305, 306, 307, 308, 324, 325, 329, 333, 334, 335, 336, 337, 344, 345, 351, 352, 353, 360, 361, 362, 364, 365, 386, 387, 388, 389, 395, 396, 397, 398, 399, 400, 401, 406, 411, 412, 413, 417, 418, 419, 428, 429, 433, 434, 435, 450, 454 }

C grade { 5 }

F normal fail { 403, 404, 405, 408, 409, 410, 414, 415, 416, 420, 421, 422, 425, 426, 427, 431, 432, 437, 438, 439, 440, 441, 442, 443, 444, 447, 448, 451, 452, 453 }

F(-1) timeout fail { }

F(-2) exception fail { 235, 236, 237, 238, 239, 240, 241, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 343 }

Mupad

A grade { }

B grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 209, 210, 212, 213, 215, 216, 218, 221, 222, 223, 224, 227, 228, 230, 234, 237, 238, 239, 242, 243, 244, 247, 248, 249, 252, 253, 256, 257, 258, 261, 262, 263, 264, 265, 266, 267, 268, 269, 272, 273, 274, 275, 280, 281, 286, 287, 288, 289, 290, 296, 303, 304, 305, 306, 307, 308, 312, 313, 321, 322, 329, 330, 331, 332, 337, 338, 339, 340, 341, 342, 347, 348, 349, 350, 354, 355, 356, 357, 358, 359, 363, 364, 365, 366, 367, 368, 369, 370, 399, 400, 401, 402, 403, 404, 405, 408, 411, 412, 413, 419, 423, 424, 425, 426, 427, 428, 429, 430, 431, 432, 433, 434, 435, 436, 445, 446, 449, 450, 453, 454 }

C grade { }

F normal fail { }

F(-1) timedout fail { 208, 211, 214, 217, 219, 220, 225, 226, 229, 231, 232, 233, 235, 236, 240, 241, 245, 246, 250, 251, 254, 255, 259, 260, 270, 271, 276, 277, 278, 279, 282, 283, 284, 285, 291, 292, 293, 294, 295, 297, 298, 299, 300, 301, 302, 309, 310, 311, 314, 315, 316, 317, 318, 319, 320, 323, 324, 325, 326, 327, 328, 333, 334, 335, 336, 343, 344, 345, 346, 351, 352, 353, 360, 361, 362, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 383, 384, 385, 386, 387, 388, 389, 390, 391, 392, 393, 394, 395, 396, 397, 398, 406, 407, 409, 410, 414, 415, 416, 417, 418, 420, 421, 422, 437, 438, 439, 440, 441, 442, 443, 444, 447, 448, 451, 452 }

F(-2) exception fail { }

Sympy

A grade { 4, 5, 6, 7, 15, 16, 17, 18, 19, 23, 24, 25, 26, 32, 33, 34, 35, 42, 43, 44, 45, 52, 53, 56, 57, 59, 60, 61, 63, 73, 74, 75, 108, 109, 110, 111, 112, 113, 121, 123, 142, 143, 144, 145, 148, 149, 150, 151, 159, 160, 169, 203, 213, 214, 219, 224, 226, 232, 237, 238, 242, 246, 247, 248, 250, 251, 255, 258, 259, 260, 264, 265, 266, 267, 268, 313, 337, 338, 339, 402, 425 }

B grade { 1, 2, 3, 8, 9, 10, 11, 12, 13, 14, 20, 21, 22, 27, 28, 29, 30, 31, 36, 37, 38, 39, 40, 41, 46, 47, 48, 49, 50, 51, 54, 55, 58, 62, 64, 65, 66, 67, 68, 69, 70, 71, 72, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 114, 115, 116, 117, 118, 119, 120, 122, 124, 125, 126, 127, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 146, 147, 152, 153, 154, 155, 156, 157, 158, 161, 162, 163, 164, 165, 166, 167, 168, 201, 202, 206, 207, 211, 212, 217, 218, 223, 225, 229, 230, 231, 235, 236, 240, 241, 245, 254, 256, 257, 269, 276, 277, 278, 279, 280, 281, 291, 292, 293, 294, 295, 296, 309, 310, 311, 312, 336, 399, 400, 401, 403, 404, 405, 411, 412, 413, 423, 424, 426, 427, 428, 429, 430, 435, 436, 454 }

C grade { }

F normal fail { 172, 173, 174, 175, 176, 177, 178, 185, 204, 205, 208, 209, 210, 215, 216, 220, 221, 222, 227, 228, 233, 234, 239, 243, 244, 249, 252, 253, 261, 262, 263, 270, 271, 272, 273, 274, 275, 282, 283, 284, 285, 286, 287, 288, 289, 290, 297, 298, 299, 300, 301, 302, 304, 305, 306, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 340, 341, 342, 343, 345, 346, 347, 348, 349, 351, 352, 353, 354, 355, 356, 357, 358, 359, 361, 362, 363, 364, 365, 366, 367, 372, 373, 374, 375, 376, 377, 380, 381, 382, 383, 384, 385, 386, 387, 393, 394, 395, 396, 397, 398, 406, 407, 408, 409, 410, 414, 415, 416, 418, 420, 431, 432, 433, 434, 437, 438, 439, 440, 441, 442, 443, 444, 445, 447, 448, 449, 450, 453 }

F(-1) timedout fail { 100, 101, 102, 103, 104, 105, 106, 107, 128, 170, 171, 179, 180, 181, 182, 183, 184, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 307, 344, 350, 360, 368, 369, 370, 371, 378, 379, 388, 389, 390, 391, 392 }

F(-2) exception fail { 303, 308, 417, 419, 421, 422, 446, 451, 452 }

Reduce

A grade { }

B grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 383, 384, 385, 386, 387, 388, 389, 390, 391, 392, 393, 394, 395, 396, 397, 398, 399, 400, 401, 402, 403, 404, 405, 406, 407, 408, 409, 410, 411, 412, 413, 417, 418, 419, 423, 424, 425, 426, 427, 428, 429, 430, 431, 432, 433, 434, 435, 436, 445, 449, 450, 454 }

C grade { }

F normal fail { 414, 415, 416, 420, 421, 422, 437, 438, 439, 440, 441, 442, 443, 444, 446, 447, 448, 451, 452, 453 }

F(-1) timedout fail { }

F(-2) exception fail { }

2.2 Detailed conclusion table per each integral for all CAS systems

Detailed conclusion table per each integral is given by the table below. The elapsed time is in seconds. For failed result it is given as **F(-1)** if the failure was due to timeout. It is given as **F(-2)** if the failure was due to an exception being raised, which could indicate a bug in the system. If the failure was due to integral not being evaluated within the time limit, then it is given as **F**.

In this table, the column **N.S.** means **normalized size** and is defined as $\frac{\text{antiderivative leaf size}}{\text{optimal antiderivative leaf size}}$. To make the table fit the page, the name **Mathematica** was abbreviated to **MMA**.

| Problem 1 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|------------|---------|-------|-------|-------|--------|--------|-------|-------|--------|-------|
| grade | N/A | A | A | B | B | B | B | B | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 15 | 15 | 15 | 45 | 47 | 47 | 51 | 47 | 44 | 47 |
| N.S. | 1 | 1.00 | 1.00 | 3.00 | 3.13 | 3.13 | 3.40 | 3.13 | 2.93 | 3.13 |
| time (sec) | N/A | 0.235 | 0.004 | 0.550 | 0.030 | 0.078 | 0.020 | 0.238 | 0.255 | 8.658 |

| Problem 2 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|------------|---------|-------|-------|-------|--------|--------|-------|-------|--------|-------|
| grade | N/A | A | A | B | B | B | B | B | B | B |
| verified | N/A | Yes | Yes | No | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 15 | 15 | 15 | 31 | 30 | 35 | 39 | 33 | 33 | 35 |
| N.S. | 1 | 1.00 | 1.00 | 2.07 | 2.00 | 2.33 | 2.60 | 2.20 | 2.20 | 2.33 |
| time (sec) | N/A | 0.237 | 0.003 | 0.289 | 0.028 | 0.078 | 0.019 | 0.225 | 0.281 | 0.047 |

| Problem 3 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|------------|---------|-------|-------|-------|--------|--------|-------|-------|--------|-------|
| grade | N/A | A | A | A | A | A | B | A | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 15 | 15 | 25 | 14 | 23 | 23 | 24 | 23 | 22 | 22 |
| N.S. | 1 | 1.00 | 1.67 | 0.93 | 1.53 | 1.53 | 1.60 | 1.53 | 1.47 | 1.47 |
| time (sec) | N/A | 0.223 | 0.000 | 0.082 | 0.031 | 0.069 | 0.020 | 0.245 | 0.234 | 0.035 |

| Problem 4 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|------------|---------|-------|-------|-------|--------|--------|-------|-------|--------|-------|
| grade | N/A | A | A | A | A | A | A | A | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 15 | 15 | 14 | 12 | 12 | 12 | 12 | 12 | 11 | 11 |
| N.S. | 1 | 1.00 | 0.93 | 0.80 | 0.80 | 0.80 | 0.80 | 0.80 | 0.73 | 0.73 |
| time (sec) | N/A | 0.230 | 0.001 | 0.667 | 0.026 | 0.076 | 0.022 | 0.238 | 0.257 | 0.024 |

| Problem 5 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|------------|---------|-------|-------|-------|--------|--------|-------|-------|--------|-------|
| grade | N/A | A | A | A | A | A | A | C | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 3 | 3 | 3 | 4 | 3 | 3 | 2 | 113 | 3 | 3 |
| N.S. | 1 | 1.00 | 1.00 | 1.33 | 1.00 | 1.00 | 0.67 | 37.67 | 1.00 | 1.00 |
| time (sec) | N/A | 0.215 | 0.000 | 0.665 | 0.042 | 0.068 | 0.025 | 0.239 | 0.272 | 0.006 |

| Problem 6 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|------------|---------|-------|-------|-------|--------|--------|-------|-------|--------|-------|
| grade | N/A | A | A | A | A | A | A | A | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 11 | 11 | 11 | 12 | 11 | 11 | 8 | 12 | 11 | 11 |
| N.S. | 1 | 1.00 | 1.00 | 1.09 | 1.00 | 1.00 | 0.73 | 1.09 | 1.00 | 1.00 |
| time (sec) | N/A | 0.233 | 0.001 | 0.608 | 0.049 | 0.086 | 0.025 | 0.235 | 0.255 | 0.029 |

| Problem 7 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|------------|---------|-------|-------|-------|--------|--------|-------|-------|--------|-------|
| grade | N/A | A | A | A | A | A | A | A | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 13 | 13 | 13 | 14 | 14 | 14 | 10 | 13 | 13 | 13 |
| N.S. | 1 | 1.00 | 1.00 | 1.08 | 1.08 | 1.08 | 0.77 | 1.00 | 1.00 | 1.00 |
| time (sec) | N/A | 0.237 | 0.004 | 0.624 | 0.038 | 0.070 | 0.058 | 0.177 | 0.237 | 0.041 |

| Problem 8 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|------------|---------|-------|-------|-------|--------|--------|-------|-------|--------|-------|
| grade | N/A | A | A | A | A | A | B | A | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 15 | 15 | 15 | 14 | 25 | 25 | 26 | 13 | 24 | 24 |
| N.S. | 1 | 1.00 | 1.00 | 0.93 | 1.67 | 1.67 | 1.73 | 0.87 | 1.60 | 1.60 |
| time (sec) | N/A | 0.244 | 0.003 | 0.742 | 0.037 | 0.077 | 0.081 | 0.221 | 0.273 | 0.038 |

| Problem 9 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|------------|---------|-------|-------|-------|--------|--------|-------|-------|--------|-------|
| grade | N/A | A | A | A | B | B | B | A | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 15 | 15 | 15 | 14 | 36 | 36 | 37 | 13 | 35 | 35 |
| N.S. | 1 | 1.00 | 1.00 | 0.93 | 2.40 | 2.40 | 2.47 | 0.87 | 2.33 | 2.33 |
| time (sec) | N/A | 0.247 | 0.004 | 0.626 | 0.033 | 0.092 | 0.114 | 0.199 | 0.260 | 8.656 |

| Problem 10 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|------------|---------|-------|-------|-------|--------|--------|-------|-------|--------|-------|
| grade | N/A | A | A | B | B | B | B | B | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 17 | 17 | 17 | 69 | 85 | 85 | 90 | 85 | 68 | 85 |
| N.S. | 1 | 1.00 | 1.00 | 4.06 | 5.00 | 5.00 | 5.29 | 5.00 | 4.00 | 5.00 |
| time (sec) | N/A | 0.261 | 0.003 | 1.060 | 0.033 | 0.100 | 0.023 | 0.240 | 0.218 | 0.050 |

| Problem 11 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|------------|---------|-------|-------|-------|--------|--------|-------|-------|--------|-------|
| grade | N/A | A | A | A | A | B | B | B | B | B |
| verified | N/A | Yes | Yes | No | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 17 | 17 | 17 | 31 | 30 | 71 | 80 | 60 | 57 | 71 |
| N.S. | 1 | 1.00 | 1.00 | 1.82 | 1.76 | 4.18 | 4.71 | 3.53 | 3.35 | 4.18 |
| time (sec) | N/A | 0.244 | 0.003 | 0.944 | 0.027 | 0.084 | 0.022 | 0.189 | 0.244 | 0.030 |

| Problem 12 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|------------|---------|-------|-------|-------|--------|--------|-------|-------|--------|-------|
| grade | N/A | A | A | B | B | B | B | B | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 17 | 17 | 17 | 47 | 68 | 57 | 60 | 57 | 46 | 57 |
| N.S. | 1 | 1.00 | 1.00 | 2.76 | 4.00 | 3.35 | 3.53 | 3.35 | 2.71 | 3.35 |
| time (sec) | N/A | 0.237 | 0.001 | 0.848 | 0.049 | 0.089 | 0.020 | 0.226 | 0.261 | 0.025 |

| Problem 13 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|------------|---------|-------|-------|-------|--------|--------|-------|-------|--------|-------|
| grade | N/A | A | A | A | B | B | B | B | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 17 | 17 | 17 | 27 | 43 | 43 | 46 | 43 | 35 | 43 |
| N.S. | 1 | 1.00 | 1.00 | 1.59 | 2.53 | 2.53 | 2.71 | 2.53 | 2.06 | 2.53 |
| time (sec) | N/A | 0.251 | 0.001 | 1.214 | 0.041 | 0.081 | 0.029 | 0.200 | 0.279 | 0.050 |

| Problem 14 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|------------|---------|-------|-------|-------|--------|--------|-------|-------|--------|-------|
| grade | N/A | A | A | A | A | A | B | A | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 17 | 17 | 17 | 16 | 29 | 29 | 29 | 15 | 24 | 24 |
| N.S. | 1 | 1.00 | 1.00 | 0.94 | 1.71 | 1.71 | 1.71 | 0.88 | 1.41 | 1.41 |
| time (sec) | N/A | 0.248 | 0.001 | 1.154 | 0.029 | 0.083 | 0.029 | 0.206 | 0.263 | 0.040 |

| Problem 15 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|------------|---------|-------|-------|-------|--------|--------|-------|-------|--------|-------|
| grade | N/A | A | A | A | A | A | A | A | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 17 | 17 | 16 | 14 | 16 | 16 | 15 | 16 | 13 | 13 |
| N.S. | 1 | 1.00 | 0.94 | 0.82 | 0.94 | 0.94 | 0.88 | 0.94 | 0.76 | 0.76 |
| time (sec) | N/A | 0.249 | 0.001 | 1.160 | 0.032 | 0.077 | 0.031 | 0.219 | 0.357 | 0.027 |

| Problem 16 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|------------|---------|-------|-------|-------|--------|--------|-------|-------|--------|-------|
| grade | N/A | A | A | A | A | A | A | A | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 5 | 5 | 5 | 6 | 5 | 5 | 3 | 5 | 5 | 5 |
| N.S. | 1 | 1.00 | 1.00 | 1.20 | 1.00 | 1.00 | 0.60 | 1.00 | 1.00 | 1.00 |
| time (sec) | N/A | 0.236 | 0.000 | 1.100 | 0.036 | 0.072 | 0.031 | 0.215 | 0.300 | 0.007 |

| Problem 17 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|------------|---------|-------|-------|-------|--------|--------|-------|-------|--------|-------|
| grade | N/A | A | A | A | A | A | A | B | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 13 | 13 | 13 | 14 | 13 | 13 | 10 | 27 | 13 | 13 |
| N.S. | 1 | 1.00 | 1.00 | 1.08 | 1.00 | 1.00 | 0.77 | 2.08 | 1.00 | 1.00 |
| time (sec) | N/A | 0.249 | 0.001 | 1.110 | 0.037 | 0.087 | 0.035 | 0.192 | 0.247 | 0.026 |

| Problem 18 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|------------|---------|-------|-------|-------|--------|--------|-------|-------|--------|-------|
| grade | N/A | A | A | A | A | A | A | A | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 15 | 15 | 15 | 16 | 16 | 16 | 12 | 15 | 15 | 15 |
| N.S. | 1 | 1.00 | 1.00 | 1.07 | 1.07 | 1.07 | 0.80 | 1.00 | 1.00 | 1.00 |
| time (sec) | N/A | 0.251 | 0.001 | 1.112 | 0.036 | 0.077 | 0.068 | 0.209 | 0.258 | 0.022 |

| Problem 19 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|------------|---------|-------|-------|-------|--------|--------|-------|-------|--------|-------|
| grade | N/A | A | A | A | A | A | A | A | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 17 | 17 | 17 | 16 | 27 | 27 | 27 | 15 | 26 | 26 |
| N.S. | 1 | 1.00 | 1.00 | 0.94 | 1.59 | 1.59 | 1.59 | 0.88 | 1.53 | 1.53 |
| time (sec) | N/A | 0.248 | 0.001 | 1.142 | 0.033 | 0.082 | 0.090 | 0.192 | 0.244 | 0.027 |

| Problem 20 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|------------|---------|-------|-------|-------|--------|--------|-------|-------|--------|-------|
| grade | N/A | A | A | A | B | B | B | A | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 17 | 17 | 17 | 16 | 38 | 38 | 39 | 15 | 37 | 37 |
| N.S. | 1 | 1.00 | 1.00 | 0.94 | 2.24 | 2.24 | 2.29 | 0.88 | 2.18 | 2.18 |
| time (sec) | N/A | 0.247 | 0.001 | 1.088 | 0.029 | 0.094 | 0.112 | 0.211 | 0.249 | 0.030 |

| Problem 21 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|------------|---------|-------|-------|-------|--------|--------|-------|-------|--------|-------|
| grade | N/A | A | A | A | B | B | B | B | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 17 | 17 | 17 | 16 | 37 | 37 | 39 | 37 | 35 | 43 |
| N.S. | 1 | 1.00 | 1.00 | 0.94 | 2.18 | 2.18 | 2.29 | 2.18 | 2.06 | 2.53 |
| time (sec) | N/A | 0.245 | 0.001 | 1.136 | 0.034 | 0.079 | 0.034 | 0.213 | 0.252 | 0.050 |

| Problem 22 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|------------|---------|-------|-------|-------|--------|--------|-------|-------|--------|-------|
| grade | N/A | A | A | A | A | A | B | A | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 17 | 17 | 17 | 16 | 26 | 26 | 24 | 26 | 24 | 24 |
| N.S. | 1 | 1.00 | 1.00 | 0.94 | 1.53 | 1.53 | 1.41 | 1.53 | 1.41 | 1.41 |
| time (sec) | N/A | 0.246 | 0.001 | 1.115 | 0.046 | 0.079 | 0.032 | 0.182 | 0.260 | 0.035 |

| Problem 23 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|------------|---------|-------|-------|-------|--------|--------|-------|-------|--------|-------|
| grade | N/A | A | A | A | A | A | A | A | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 17 | 17 | 16 | 14 | 15 | 15 | 12 | 15 | 13 | 13 |
| N.S. | 1 | 1.00 | 0.94 | 0.82 | 0.88 | 0.88 | 0.71 | 0.88 | 0.76 | 0.76 |
| time (sec) | N/A | 0.247 | 0.001 | 1.134 | 0.034 | 0.078 | 0.028 | 0.212 | 0.294 | 0.025 |

| Problem 24 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|------------|---------|-------|-------|-------|--------|--------|-------|-------|--------|-------|
| grade | N/A | A | A | A | A | A | A | A | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 5 | 5 | 5 | 6 | 5 | 5 | 2 | 5 | 5 | 5 |
| N.S. | 1 | 1.00 | 1.00 | 1.20 | 1.00 | 1.00 | 0.40 | 1.00 | 1.00 | 1.00 |
| time (sec) | N/A | 0.237 | 0.000 | 1.155 | 0.032 | 0.079 | 0.024 | 0.144 | 0.267 | 0.009 |

| Problem 25 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|------------|---------|-------|-------|-------|--------|--------|-------|-------|--------|-------|
| grade | N/A | A | A | A | B | A | A | B | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 13 | 13 | 16 | 14 | 29 | 13 | 12 | 29 | 13 | 13 |
| N.S. | 1 | 1.00 | 1.23 | 1.08 | 2.23 | 1.00 | 0.92 | 2.23 | 1.00 | 1.00 |
| time (sec) | N/A | 0.248 | 0.001 | 1.019 | 0.044 | 0.078 | 0.025 | 0.199 | 0.299 | 8.834 |

| Problem 26 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|------------|---------|-------|-------|-------|--------|--------|-------|-------|--------|-------|
| grade | N/A | A | A | A | A | A | A | A | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 15 | 15 | 15 | 16 | 15 | 15 | 14 | 15 | 15 | 15 |
| N.S. | 1 | 1.00 | 1.00 | 1.07 | 1.00 | 1.00 | 0.93 | 1.00 | 1.00 | 1.00 |
| time (sec) | N/A | 0.237 | 0.001 | 0.973 | 0.038 | 0.073 | 0.055 | 0.187 | 0.242 | 8.709 |

| Problem 27 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|------------|---------|-------|-------|-------|--------|--------|-------|-------|--------|-------|
| grade | N/A | A | A | A | A | A | B | A | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 17 | 17 | 17 | 16 | 27 | 27 | 31 | 15 | 26 | 29 |
| N.S. | 1 | 1.00 | 1.00 | 0.94 | 1.59 | 1.59 | 1.82 | 0.88 | 1.53 | 1.71 |
| time (sec) | N/A | 0.246 | 0.001 | 1.240 | 0.034 | 0.083 | 0.087 | 0.175 | 0.275 | 0.037 |

| Problem 28 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|------------|---------|-------|-------|-------|--------|--------|-------|-------|--------|-------|
| grade | N/A | A | A | A | B | B | B | A | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 17 | 17 | 17 | 16 | 39 | 39 | 44 | 15 | 37 | 41 |
| N.S. | 1 | 1.00 | 1.00 | 0.94 | 2.29 | 2.29 | 2.59 | 0.88 | 2.18 | 2.41 |
| time (sec) | N/A | 0.242 | 0.001 | 1.270 | 0.030 | 0.074 | 0.124 | 0.227 | 0.277 | 8.708 |

| Problem 29 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|------------|---------|-------|-------|-------|--------|--------|-------|-------|--------|-------|
| grade | N/A | A | A | A | B | B | B | A | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 17 | 17 | 17 | 16 | 51 | 51 | 58 | 15 | 48 | 53 |
| N.S. | 1 | 1.00 | 1.00 | 0.94 | 3.00 | 3.00 | 3.41 | 0.88 | 2.82 | 3.12 |
| time (sec) | N/A | 0.246 | 0.003 | 1.232 | 0.036 | 0.097 | 0.136 | 0.175 | 0.284 | 0.049 |

| Problem 30 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|------------|---------|-------|-------|-------|--------|--------|-------|-------|--------|-------|
| grade | N/A | A | A | A | B | B | B | B | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 17 | 17 | 17 | 16 | 37 | 37 | 46 | 37 | 35 | 43 |
| N.S. | 1 | 1.00 | 1.00 | 0.94 | 2.18 | 2.18 | 2.71 | 2.18 | 2.06 | 2.53 |
| time (sec) | N/A | 0.249 | 0.001 | 1.121 | 0.041 | 0.076 | 0.040 | 0.167 | 0.310 | 0.050 |

| Problem 31 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|------------|---------|-------|-------|-------|--------|--------|-------|-------|--------|-------|
| grade | N/A | A | A | A | A | A | B | A | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 17 | 17 | 17 | 16 | 26 | 26 | 29 | 26 | 24 | 24 |
| N.S. | 1 | 1.00 | 1.00 | 0.94 | 1.53 | 1.53 | 1.71 | 1.53 | 1.41 | 1.41 |
| time (sec) | N/A | 0.248 | 0.001 | 1.107 | 0.037 | 0.080 | 0.047 | 0.150 | 0.267 | 8.703 |

| Problem 32 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|------------|---------|-------|-------|-------|--------|--------|-------|-------|--------|-------|
| grade | N/A | A | A | A | A | A | A | A | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 17 | 17 | 16 | 14 | 15 | 15 | 15 | 15 | 13 | 13 |
| N.S. | 1 | 1.00 | 0.94 | 0.82 | 0.88 | 0.88 | 0.88 | 0.88 | 0.76 | 0.76 |
| time (sec) | N/A | 0.247 | 0.001 | 1.084 | 0.042 | 0.078 | 0.047 | 0.193 | 0.246 | 0.026 |

| Problem 33 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|------------|---------|-------|-------|-------|--------|--------|-------|-------|--------|-------|
| grade | N/A | A | A | A | A | A | A | A | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 5 | 5 | 5 | 6 | 5 | 5 | 3 | 5 | 5 | 5 |
| N.S. | 1 | 1.00 | 1.00 | 1.20 | 1.00 | 1.00 | 0.60 | 1.00 | 1.00 | 1.00 |
| time (sec) | N/A | 0.232 | 0.001 | 1.067 | 0.028 | 0.080 | 0.043 | 0.158 | 0.270 | 0.009 |

| Problem 34 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|------------|---------|-------|-------|-------|--------|--------|-------|-------|--------|-------|
| grade | N/A | A | A | A | A | A | A | B | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 13 | 13 | 13 | 14 | 13 | 13 | 17 | 128 | 13 | 13 |
| N.S. | 1 | 1.00 | 1.00 | 1.08 | 1.00 | 1.00 | 1.31 | 9.85 | 1.00 | 1.00 |
| time (sec) | N/A | 0.251 | 0.001 | 1.073 | 0.049 | 0.087 | 0.044 | 0.183 | 0.252 | 8.849 |

| Problem 35 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|------------|---------|-------|-------|-------|--------|--------|-------|-------|--------|-------|
| grade | N/A | A | A | A | A | A | A | A | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 15 | 15 | 15 | 16 | 19 | 19 | 17 | 15 | 15 | 19 |
| N.S. | 1 | 1.00 | 1.00 | 1.07 | 1.27 | 1.27 | 1.13 | 1.00 | 1.00 | 1.27 |
| time (sec) | N/A | 0.267 | 0.002 | 1.086 | 0.031 | 0.094 | 0.086 | 0.182 | 0.244 | 8.721 |

| Problem 36 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|------------|---------|-------|-------|-------|--------|--------|-------|-------|--------|-------|
| grade | N/A | A | A | A | A | B | B | A | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 17 | 17 | 17 | 16 | 30 | 33 | 36 | 29 | 26 | 35 |
| N.S. | 1 | 1.00 | 1.00 | 0.94 | 1.76 | 1.94 | 2.12 | 1.71 | 1.53 | 2.06 |
| time (sec) | N/A | 0.254 | 0.002 | 0.970 | 0.029 | 0.073 | 0.124 | 0.205 | 0.276 | 0.040 |

| Problem 37 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|------------|---------|-------|-------|-------|--------|--------|-------|-------|--------|-------|
| grade | N/A | A | A | A | B | B | B | A | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 17 | 17 | 17 | 16 | 47 | 47 | 51 | 15 | 37 | 49 |
| N.S. | 1 | 1.00 | 1.00 | 0.94 | 2.76 | 2.76 | 3.00 | 0.88 | 2.18 | 2.88 |
| time (sec) | N/A | 0.229 | 0.002 | 1.010 | 0.030 | 0.076 | 0.166 | 0.170 | 0.270 | 8.684 |

| Problem 38 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|------------|---------|-------|-------|-------|--------|--------|-------|-------|--------|-------|
| grade | N/A | A | A | A | B | B | B | A | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 17 | 17 | 17 | 16 | 61 | 61 | 66 | 15 | 48 | 63 |
| N.S. | 1 | 1.00 | 1.00 | 0.94 | 3.59 | 3.59 | 3.88 | 0.88 | 2.82 | 3.71 |
| time (sec) | N/A | 0.244 | 0.001 | 1.270 | 0.032 | 0.083 | 0.236 | 0.211 | 0.261 | 0.054 |

| Problem 39 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|------------|---------|-------|-------|-------|--------|--------|-------|-------|--------|-------|
| grade | N/A | A | A | A | B | B | B | A | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 17 | 17 | 17 | 16 | 75 | 75 | 82 | 15 | 59 | 77 |
| N.S. | 1 | 1.00 | 1.00 | 0.94 | 4.41 | 4.41 | 4.82 | 0.88 | 3.47 | 4.53 |
| time (sec) | N/A | 0.244 | 0.003 | 1.256 | 0.030 | 0.075 | 0.172 | 0.198 | 0.252 | 8.801 |

| Problem 40 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|------------|---------|-------|-------|-------|--------|--------|-------|-------|--------|-------|
| grade | N/A | A | A | A | B | B | B | B | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 17 | 17 | 17 | 16 | 37 | 37 | 46 | 37 | 35 | 43 |
| N.S. | 1 | 1.00 | 1.00 | 0.94 | 2.18 | 2.18 | 2.71 | 2.18 | 2.06 | 2.53 |
| time (sec) | N/A | 0.252 | 0.001 | 1.100 | 0.042 | 0.063 | 0.048 | 0.178 | 0.278 | 0.049 |

| Problem 41 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|------------|---------|-------|-------|-------|--------|--------|-------|-------|--------|-------|
| grade | N/A | A | A | A | A | A | B | A | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 17 | 17 | 17 | 16 | 26 | 26 | 29 | 26 | 24 | 24 |
| N.S. | 1 | 1.00 | 1.00 | 0.94 | 1.53 | 1.53 | 1.71 | 1.53 | 1.41 | 1.41 |
| time (sec) | N/A | 0.259 | 0.002 | 1.095 | 0.036 | 0.078 | 0.044 | 0.194 | 0.272 | 0.037 |

| Problem 42 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|------------|---------|-------|-------|-------|--------|--------|-------|-------|--------|-------|
| grade | N/A | A | A | A | A | A | A | A | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 17 | 17 | 16 | 14 | 15 | 15 | 15 | 15 | 13 | 13 |
| N.S. | 1 | 1.00 | 0.94 | 0.82 | 0.88 | 0.88 | 0.88 | 0.88 | 0.76 | 0.76 |
| time (sec) | N/A | 0.248 | 0.001 | 1.093 | 0.032 | 0.080 | 0.041 | 0.191 | 0.280 | 0.025 |

| Problem 43 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|------------|---------|-------|-------|-------|--------|--------|-------|-------|--------|-------|
| grade | N/A | A | A | A | A | A | A | A | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 5 | 5 | 5 | 6 | 5 | 5 | 3 | 5 | 5 | 5 |
| N.S. | 1 | 1.00 | 1.00 | 1.20 | 1.00 | 1.00 | 0.60 | 1.00 | 1.00 | 1.00 |
| time (sec) | N/A | 0.231 | 0.000 | 1.102 | 0.027 | 0.073 | 0.039 | 0.209 | 0.260 | 0.009 |

| Problem 44 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|------------|---------|-------|-------|-------|--------|--------|-------|-------|--------|-------|
| grade | N/A | A | A | A | A | A | A | A | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 13 | 13 | 13 | 14 | 13 | 13 | 17 | 14 | 13 | 13 |
| N.S. | 1 | 1.00 | 1.00 | 1.08 | 1.00 | 1.00 | 1.31 | 1.08 | 1.00 | 1.00 |
| time (sec) | N/A | 0.245 | 0.001 | 1.087 | 0.032 | 0.076 | 0.044 | 0.178 | 0.266 | 9.030 |

| Problem 45 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|------------|---------|-------|-------|-------|--------|--------|-------|-------|--------|-------|
| grade | N/A | A | A | A | A | A | A | A | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 15 | 15 | 15 | 16 | 19 | 19 | 17 | 15 | 15 | 19 |
| N.S. | 1 | 1.00 | 1.00 | 1.07 | 1.27 | 1.27 | 1.13 | 1.00 | 1.00 | 1.27 |
| time (sec) | N/A | 0.245 | 0.001 | 1.072 | 0.038 | 0.075 | 0.076 | 0.203 | 0.270 | 9.115 |

| Problem 46 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|------------|---------|-------|-------|-------|--------|--------|-------|-------|--------|-------|
| grade | N/A | A | A | A | B | B | B | A | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 17 | 17 | 17 | 16 | 33 | 33 | 36 | 26 | 26 | 35 |
| N.S. | 1 | 1.00 | 1.00 | 0.94 | 1.94 | 1.94 | 2.12 | 1.53 | 1.53 | 2.06 |
| time (sec) | N/A | 0.245 | 0.001 | 1.093 | 0.043 | 0.069 | 0.097 | 0.234 | 0.230 | 9.076 |

| Problem 47 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|------------|---------|-------|-------|-------|--------|--------|-------|-------|--------|-------|
| grade | N/A | A | A | A | B | B | B | A | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 17 | 17 | 17 | 16 | 47 | 47 | 51 | 15 | 37 | 49 |
| N.S. | 1 | 1.00 | 1.00 | 0.94 | 2.76 | 2.76 | 3.00 | 0.88 | 2.18 | 2.88 |
| time (sec) | N/A | 0.247 | 0.001 | 1.098 | 0.030 | 0.076 | 0.117 | 0.188 | 0.259 | 0.044 |

| Problem 48 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|------------|---------|-------|-------|-------|--------|--------|-------|-------|--------|-------|
| grade | N/A | A | A | A | A | B | B | A | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 17 | 17 | 17 | 16 | 30 | 61 | 66 | 29 | 48 | 63 |
| N.S. | 1 | 1.00 | 1.00 | 0.94 | 1.76 | 3.59 | 3.88 | 1.71 | 2.82 | 3.71 |
| time (sec) | N/A | 0.234 | 0.002 | 1.017 | 0.042 | 0.070 | 0.161 | 0.198 | 0.307 | 0.058 |

| Problem 49 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|------------|---------|-------|-------|-------|--------|--------|-------|-------|--------|-------|
| grade | N/A | A | A | A | B | B | B | A | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 17 | 17 | 17 | 16 | 75 | 75 | 82 | 15 | 59 | 77 |
| N.S. | 1 | 1.00 | 1.00 | 0.94 | 4.41 | 4.41 | 4.82 | 0.88 | 3.47 | 4.53 |
| time (sec) | N/A | 0.235 | 0.002 | 0.990 | 0.035 | 0.079 | 0.171 | 0.228 | 0.207 | 9.162 |

| Problem 50 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|------------|---------|-------|-------|-------|--------|--------|-------|-------|--------|-------|
| grade | N/A | A | A | A | B | B | B | A | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 17 | 17 | 17 | 16 | 89 | 89 | 97 | 15 | 70 | 91 |
| N.S. | 1 | 1.00 | 1.00 | 0.94 | 5.24 | 5.24 | 5.71 | 0.88 | 4.12 | 5.35 |
| time (sec) | N/A | 0.253 | 0.003 | 1.241 | 0.038 | 0.076 | 0.219 | 0.214 | 0.208 | 9.305 |

| Problem 51 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|------------|---------|-------|-------|-------|--------|--------|-------|-------|--------|-------|
| grade | N/A | A | A | A | B | B | B | A | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 17 | 17 | 17 | 16 | 103 | 103 | 112 | 15 | 81 | 105 |
| N.S. | 1 | 1.00 | 1.00 | 0.94 | 6.06 | 6.06 | 6.59 | 0.88 | 4.76 | 6.18 |
| time (sec) | N/A | 0.249 | 0.003 | 1.261 | 0.028 | 0.070 | 0.226 | 0.181 | 0.213 | 0.070 |

| Problem 52 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|------------|---------|-------|-------|-------|--------|--------|-------|-------|--------|-------|
| grade | N/A | A | A | A | A | A | A | A | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 10 | 10 | 10 | 11 | 10 | 10 | 7 | 11 | 10 | 10 |
| N.S. | 1 | 1.00 | 1.00 | 1.10 | 1.00 | 1.00 | 0.70 | 1.10 | 1.00 | 1.00 |
| time (sec) | N/A | 0.231 | 0.001 | 0.628 | 0.040 | 0.078 | 0.018 | 0.251 | 0.221 | 0.020 |

| Problem 53 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|------------|---------|-------|-------|-------|--------|--------|-------|-------|--------|-------|
| grade | N/A | A | A | A | B | A | A | B | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 10 | 10 | 10 | 11 | 22 | 10 | 7 | 22 | 10 | 10 |
| N.S. | 1 | 1.00 | 1.00 | 1.10 | 2.20 | 1.00 | 0.70 | 2.20 | 1.00 | 1.00 |
| time (sec) | N/A | 0.242 | 0.001 | 1.049 | 0.032 | 0.085 | 0.022 | 0.201 | 0.217 | 0.018 |

| Problem 54 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|------------|---------|-------|-------|-------|--------|--------|-------|-------|--------|-------|
| grade | N/A | A | B | B | B | B | B | B | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 65 | 65 | 148 | 157 | 156 | 156 | 168 | 170 | 172 | 144 |
| N.S. | 1 | 1.00 | 2.28 | 2.42 | 2.40 | 2.40 | 2.58 | 2.62 | 2.65 | 2.22 |
| time (sec) | N/A | 0.444 | 0.020 | 0.523 | 0.037 | 0.067 | 0.031 | 0.201 | 0.205 | 0.077 |

| Problem 55 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|------------|---------|-------|-------|-------|--------|--------|-------|-------|--------|-------|
| grade | N/A | A | A | B | B | B | B | B | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 65 | 65 | 122 | 121 | 124 | 124 | 133 | 130 | 131 | 115 |
| N.S. | 1 | 1.00 | 1.88 | 1.86 | 1.91 | 1.91 | 2.05 | 2.00 | 2.02 | 1.77 |
| time (sec) | N/A | 0.404 | 0.014 | 0.534 | 0.037 | 0.071 | 0.030 | 0.218 | 0.218 | 0.056 |

| Problem 56 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|------------|---------|-------|-------|-------|--------|--------|-------|-------|--------|-------|
| grade | N/A | A | A | A | A | A | A | A | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 65 | 65 | 79 | 84 | 81 | 81 | 87 | 89 | 90 | 74 |
| N.S. | 1 | 1.00 | 1.22 | 1.29 | 1.25 | 1.25 | 1.34 | 1.37 | 1.38 | 1.14 |
| time (sec) | N/A | 0.388 | 0.013 | 0.554 | 0.030 | 0.070 | 0.028 | 0.211 | 0.260 | 9.041 |

| Problem 57 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|------------|---------|-------|-------|-------|--------|--------|-------|-------|--------|-------|
| grade | N/A | A | A | A | A | A | A | A | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 38 | 38 | 46 | 48 | 48 | 48 | 49 | 49 | 49 | 47 |
| N.S. | 1 | 1.00 | 1.21 | 1.26 | 1.26 | 1.26 | 1.29 | 1.29 | 1.29 | 1.24 |
| time (sec) | N/A | 0.328 | 0.010 | 0.269 | 0.036 | 0.065 | 0.021 | 0.203 | 0.250 | 0.048 |

| Problem 58 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|------------|---------|-------|-------|-------|--------|--------|-------|-------|--------|-------|
| grade | N/A | A | A | A | A | A | B | A | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 14 | 14 | 22 | 13 | 20 | 20 | 19 | 20 | 21 | 20 |
| N.S. | 1 | 1.00 | 1.57 | 0.93 | 1.43 | 1.43 | 1.36 | 1.43 | 1.50 | 1.43 |
| time (sec) | N/A | 0.240 | 0.000 | 0.056 | 0.032 | 0.075 | 0.018 | 0.198 | 0.266 | 0.028 |

| Problem 59 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|------------|---------|-------|-------|-------|--------|--------|-------|-------|--------|-------|
| grade | N/A | A | A | A | A | A | A | A | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 49 | 50 | 43 | 56 | 60 | 62 | 44 | 60 | 72 | 62 |
| N.S. | 1 | 1.02 | 0.88 | 1.14 | 1.22 | 1.27 | 0.90 | 1.22 | 1.47 | 1.27 |
| time (sec) | N/A | 0.331 | 0.022 | 0.692 | 0.031 | 0.077 | 0.107 | 0.208 | 0.268 | 0.071 |

| Problem 60 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|------------|---------|-------|-------|-------|--------|--------|-------|-------|--------|-------|
| grade | N/A | A | A | A | A | A | A | B | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 51 | 51 | 47 | 63 | 67 | 92 | 60 | 114 | 112 | 71 |
| N.S. | 1 | 1.00 | 0.92 | 1.24 | 1.31 | 1.80 | 1.18 | 2.24 | 2.20 | 1.39 |
| time (sec) | N/A | 0.354 | 0.051 | 0.674 | 0.041 | 0.071 | 0.175 | 0.215 | 0.236 | 8.763 |

| Problem 61 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|------------|---------|-------|-------|-------|--------|--------|-------|-------|--------|-------|
| grade | N/A | A | A | A | A | A | A | A | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 59 | 59 | 48 | 66 | 80 | 100 | 80 | 69 | 113 | 77 |
| N.S. | 1 | 1.00 | 0.81 | 1.12 | 1.36 | 1.69 | 1.36 | 1.17 | 1.92 | 1.31 |
| time (sec) | N/A | 0.366 | 0.031 | 0.788 | 0.042 | 0.079 | 0.224 | 0.208 | 0.202 | 0.081 |

| Problem 62 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|------------|---------|-------|-------|-------|--------|--------|-------|-------|--------|-------|
| grade | N/A | A | A | B | B | B | B | B | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 28 | 28 | 53 | 60 | 84 | 84 | 88 | 59 | 69 | 80 |
| N.S. | 1 | 1.00 | 1.89 | 2.14 | 3.00 | 3.00 | 3.14 | 2.11 | 2.46 | 2.86 |
| time (sec) | N/A | 0.263 | 0.021 | 0.654 | 0.031 | 0.064 | 0.301 | 0.239 | 0.221 | 0.045 |

| Problem 63 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|------------|---------|-------|-------|-------|--------|--------|-------|-------|--------|-------|
| grade | N/A | A | A | A | A | A | A | A | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 65 | 65 | 55 | 62 | 98 | 98 | 104 | 96 | 95 | 96 |
| N.S. | 1 | 1.00 | 0.85 | 0.95 | 1.51 | 1.51 | 1.60 | 1.48 | 1.46 | 1.48 |
| time (sec) | N/A | 0.354 | 0.022 | 0.629 | 0.029 | 0.076 | 0.391 | 0.228 | 0.223 | 8.911 |

| Problem 64 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|------------|---------|-------|-------|-------|--------|--------|-------|-------|--------|-------|
| grade | N/A | A | A | A | A | A | B | A | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 65 | 65 | 55 | 62 | 109 | 109 | 116 | 61 | 106 | 107 |
| N.S. | 1 | 1.00 | 0.85 | 0.95 | 1.68 | 1.68 | 1.78 | 0.94 | 1.63 | 1.65 |
| time (sec) | N/A | 0.361 | 0.020 | 0.644 | 0.043 | 0.081 | 0.483 | 0.231 | 0.219 | 8.947 |

| Problem 65 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|------------|---------|-------|-------|-------|--------|--------|-------|-------|--------|-------|
| grade | N/A | A | A | A | B | B | B | A | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 65 | 65 | 55 | 62 | 120 | 120 | 128 | 61 | 117 | 118 |
| N.S. | 1 | 1.00 | 0.85 | 0.95 | 1.85 | 1.85 | 1.97 | 0.94 | 1.80 | 1.82 |
| time (sec) | N/A | 0.363 | 0.022 | 0.665 | 0.047 | 0.070 | 0.569 | 0.192 | 0.200 | 9.413 |

| Problem 66 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|------------|---------|-------|-------|-------|--------|--------|-------|-------|--------|-------|
| grade | N/A | A | B | B | B | B | B | B | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 119 | 119 | 398 | 417 | 418 | 418 | 462 | 470 | 472 | 402 |
| N.S. | 1 | 1.00 | 3.34 | 3.50 | 3.51 | 3.51 | 3.88 | 3.95 | 3.97 | 3.38 |
| time (sec) | N/A | 0.738 | 0.065 | 1.056 | 0.071 | 0.082 | 0.049 | 0.204 | 0.207 | 0.173 |

| Problem 67 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|------------|---------|-------|-------|-------|--------|--------|-------|-------|--------|-------|
| grade | N/A | A | B | B | B | B | B | B | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 119 | 119 | 350 | 353 | 360 | 360 | 401 | 396 | 397 | 340 |
| N.S. | 1 | 1.00 | 2.94 | 2.97 | 3.03 | 3.03 | 3.37 | 3.33 | 3.34 | 2.86 |
| time (sec) | N/A | 0.671 | 0.036 | 1.072 | 0.039 | 0.082 | 0.048 | 0.245 | 0.238 | 9.341 |

| Problem 68 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|------------|---------|-------|-------|-------|--------|--------|-------|-------|--------|-------|
| grade | N/A | A | B | B | B | B | B | B | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 119 | 119 | 273 | 288 | 285 | 285 | 318 | 321 | 322 | 271 |
| N.S. | 1 | 1.00 | 2.29 | 2.42 | 2.39 | 2.39 | 2.67 | 2.70 | 2.71 | 2.28 |
| time (sec) | N/A | 0.619 | 0.028 | 1.082 | 0.038 | 0.075 | 0.045 | 0.219 | 0.199 | 9.009 |

| Problem 69 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|------------|---------|-------|-------|-------|--------|--------|-------|-------|--------|-------|
| grade | N/A | A | B | B | B | B | B | B | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 92 | 92 | 217 | 222 | 225 | 225 | 243 | 245 | 247 | 208 |
| N.S. | 1 | 1.00 | 2.36 | 2.41 | 2.45 | 2.45 | 2.64 | 2.66 | 2.68 | 2.26 |
| time (sec) | N/A | 0.541 | 0.034 | 1.044 | 0.044 | 0.075 | 0.034 | 0.229 | 0.194 | 0.085 |

| Problem 70 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|------------|---------|-------|-------|-------|--------|--------|-------|-------|--------|-------|
| grade | N/A | A | B | B | B | B | B | B | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 65 | 65 | 148 | 157 | 156 | 156 | 168 | 170 | 172 | 144 |
| N.S. | 1 | 1.00 | 2.28 | 2.42 | 2.40 | 2.40 | 2.58 | 2.62 | 2.65 | 2.22 |
| time (sec) | N/A | 0.446 | 0.020 | 1.065 | 0.038 | 0.071 | 0.034 | 0.207 | 0.196 | 0.067 |

| Problem 71 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|------------|---------|-------|-------|-------|--------|--------|-------|-------|--------|-------|
| grade | N/A | A | B | B | B | B | B | B | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 38 | 38 | 84 | 94 | 96 | 96 | 100 | 97 | 97 | 88 |
| N.S. | 1 | 1.00 | 2.21 | 2.47 | 2.53 | 2.53 | 2.63 | 2.55 | 2.55 | 2.32 |
| time (sec) | N/A | 0.319 | 0.013 | 0.987 | 0.034 | 0.079 | 0.042 | 0.214 | 0.233 | 0.045 |

| Problem 72 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|------------|---------|-------|-------|-------|--------|--------|-------|-------|--------|-------|
| grade | N/A | A | A | B | B | B | B | B | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 14 | 14 | 14 | 43 | 53 | 42 | 42 | 42 | 43 | 42 |
| N.S. | 1 | 1.00 | 1.00 | 3.07 | 3.79 | 3.00 | 3.00 | 3.00 | 3.07 | 3.00 |
| time (sec) | N/A | 0.231 | 0.002 | 0.836 | 0.044 | 0.071 | 0.028 | 0.199 | 0.210 | 0.022 |

| Problem 73 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|------------|---------|-------|-------|-------|--------|--------|-------|-------|--------|-------|
| grade | N/A | A | A | A | A | A | A | A | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 98 | 98 | 115 | 168 | 177 | 179 | 136 | 184 | 208 | 189 |
| N.S. | 1 | 1.00 | 1.17 | 1.71 | 1.81 | 1.83 | 1.39 | 1.88 | 2.12 | 1.93 |
| time (sec) | N/A | 0.427 | 0.032 | 1.244 | 0.041 | 0.075 | 0.292 | 0.209 | 0.208 | 0.056 |

| Problem 74 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|------------|---------|-------|-------|-------|--------|--------|-------|-------|--------|-------|
| grade | N/A | A | A | A | A | B | A | B | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 104 | 104 | 165 | 175 | 183 | 267 | 155 | 245 | 297 | 203 |
| N.S. | 1 | 1.00 | 1.59 | 1.68 | 1.76 | 2.57 | 1.49 | 2.36 | 2.86 | 1.95 |
| time (sec) | N/A | 0.508 | 0.044 | 1.092 | 0.037 | 0.092 | 0.534 | 0.200 | 0.191 | 8.941 |

| Problem 75 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|------------|---------|-------|-------|-------|--------|--------|-------|-------|--------|-------|
| grade | N/A | A | A | A | A | B | A | A | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 103 | 103 | 167 | 172 | 191 | 291 | 185 | 183 | 330 | 196 |
| N.S. | 1 | 1.00 | 1.62 | 1.67 | 1.85 | 2.83 | 1.80 | 1.78 | 3.20 | 1.90 |
| time (sec) | N/A | 0.477 | 0.042 | 1.127 | 0.071 | 0.084 | 0.618 | 0.228 | 0.242 | 0.094 |

| Problem 76 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|------------|---------|-------|-------|-------|--------|--------|-------|-------|--------|-------|
| grade | N/A | A | A | A | A | B | B | A | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 103 | 103 | 163 | 178 | 201 | 292 | 209 | 177 | 309 | 204 |
| N.S. | 1 | 1.00 | 1.58 | 1.73 | 1.95 | 2.83 | 2.03 | 1.72 | 3.00 | 1.98 |
| time (sec) | N/A | 0.484 | 0.066 | 1.141 | 0.041 | 0.089 | 0.952 | 0.173 | 0.197 | 9.043 |

| Problem 77 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|------------|---------|-------|-------|-------|--------|--------|-------|-------|--------|-------|
| grade | N/A | A | A | A | B | B | B | B | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 111 | 111 | 119 | 176 | 220 | 268 | 230 | 281 | 269 | 213 |
| N.S. | 1 | 1.00 | 1.07 | 1.59 | 1.98 | 2.41 | 2.07 | 2.53 | 2.42 | 1.92 |
| time (sec) | N/A | 0.487 | 0.047 | 1.078 | 0.052 | 0.086 | 1.314 | 0.235 | 0.231 | 0.131 |

| Problem 78 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|------------|---------|-------|-------|-------|--------|--------|-------|-------|--------|-------|
| grade | N/A | A | B | B | B | B | B | B | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 28 | 28 | 140 | 161 | 215 | 215 | 236 | 180 | 192 | 203 |
| N.S. | 1 | 1.00 | 5.00 | 5.75 | 7.68 | 7.68 | 8.43 | 6.43 | 6.86 | 7.25 |
| time (sec) | N/A | 0.262 | 0.042 | 1.175 | 0.056 | 0.076 | 2.220 | 0.219 | 0.201 | 0.092 |

| Problem 79 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|------------|---------|-------|-------|-------|--------|--------|-------|-------|--------|-------|
| grade | N/A | A | B | B | B | B | B | B | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 58 | 58 | 144 | 171 | 236 | 236 | 255 | 184 | 240 | 226 |
| N.S. | 1 | 1.00 | 2.48 | 2.95 | 4.07 | 4.07 | 4.40 | 3.17 | 4.14 | 3.90 |
| time (sec) | N/A | 0.310 | 0.051 | 1.098 | 0.045 | 0.079 | 3.009 | 0.201 | 0.243 | 0.101 |

| Problem 80 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|------------|---------|-------|-------|-------|--------|--------|-------|-------|--------|-------|
| grade | N/A | A | A | B | B | B | B | B | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 89 | 102 | 144 | 171 | 247 | 247 | 267 | 184 | 251 | 237 |
| N.S. | 1 | 1.15 | 1.62 | 1.92 | 2.78 | 2.78 | 3.00 | 2.07 | 2.82 | 2.66 |
| time (sec) | N/A | 0.350 | 0.071 | 1.119 | 0.050 | 0.072 | 5.562 | 0.222 | 0.215 | 8.976 |

| Problem 81 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|------------|---------|-------|-------|-------|--------|--------|--------|-------|--------|-------|
| grade | N/A | A | A | A | B | B | B | A | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 117 | 117 | 144 | 171 | 258 | 258 | 279 | 184 | 262 | 248 |
| N.S. | 1 | 1.00 | 1.23 | 1.46 | 2.21 | 2.21 | 2.38 | 1.57 | 2.24 | 2.12 |
| time (sec) | N/A | 0.490 | 0.033 | 1.142 | 0.044 | 0.074 | 10.037 | 0.211 | 0.197 | 8.734 |

| Problem 82 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|------------|---------|-------|-------|-------|--------|--------|--------|-------|--------|-------|
| grade | N/A | A | A | A | B | B | B | A | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 119 | 119 | 144 | 171 | 269 | 269 | 291 | 184 | 273 | 259 |
| N.S. | 1 | 1.00 | 1.21 | 1.44 | 2.26 | 2.26 | 2.45 | 1.55 | 2.29 | 2.18 |
| time (sec) | N/A | 0.493 | 0.043 | 1.136 | 0.053 | 0.077 | 20.358 | 0.222 | 0.197 | 8.778 |

| Problem 83 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|------------|---------|-------|-------|-------|--------|--------|--------|-------|--------|-------|
| grade | N/A | A | A | A | B | B | B | A | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 119 | 119 | 144 | 171 | 280 | 280 | 303 | 184 | 284 | 270 |
| N.S. | 1 | 1.00 | 1.21 | 1.44 | 2.35 | 2.35 | 2.55 | 1.55 | 2.39 | 2.27 |
| time (sec) | N/A | 0.455 | 0.035 | 1.089 | 0.060 | 0.096 | 38.865 | 0.208 | 0.229 | 8.870 |

| Problem 84 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|------------|---------|-------|-------|-------|--------|--------|-------|-------|--------|-------|
| grade | N/A | A | B | B | B | B | B | B | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 173 | 173 | 771 | 789 | 797 | 797 | 884 | 906 | 908 | 768 |
| N.S. | 1 | 1.00 | 4.46 | 4.56 | 4.61 | 4.61 | 5.11 | 5.24 | 5.25 | 4.44 |
| time (sec) | N/A | 1.111 | 0.084 | 1.053 | 0.038 | 0.077 | 0.071 | 0.197 | 0.199 | 0.329 |

| Problem 85 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|------------|---------|-------|-------|-------|--------|--------|-------|-------|--------|-------|
| grade | N/A | A | B | B | B | B | B | B | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 173 | 173 | 684 | 697 | 706 | 706 | 796 | 798 | 799 | 683 |
| N.S. | 1 | 1.00 | 3.95 | 4.03 | 4.08 | 4.08 | 4.60 | 4.61 | 4.62 | 3.95 |
| time (sec) | N/A | 1.011 | 0.068 | 1.032 | 0.040 | 0.088 | 0.063 | 0.195 | 0.223 | 0.279 |

| Problem 86 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|------------|---------|-------|-------|-------|--------|--------|-------|-------|--------|-------|
| grade | N/A | A | B | B | B | B | B | B | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 171 | 171 | 573 | 604 | 599 | 599 | 677 | 689 | 690 | 579 |
| N.S. | 1 | 1.00 | 3.35 | 3.53 | 3.50 | 3.50 | 3.96 | 4.03 | 4.04 | 3.39 |
| time (sec) | N/A | 0.920 | 0.062 | 1.050 | 0.050 | 0.079 | 0.060 | 0.216 | 0.196 | 0.205 |

| Problem 87 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|------------|---------|-------|-------|-------|--------|--------|-------|-------|--------|-------|
| grade | N/A | A | B | B | B | B | B | B | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 143 | 143 | 501 | 510 | 517 | 517 | 580 | 579 | 581 | 492 |
| N.S. | 1 | 1.00 | 3.50 | 3.57 | 3.62 | 3.62 | 4.06 | 4.05 | 4.06 | 3.44 |
| time (sec) | N/A | 0.811 | 0.052 | 1.049 | 0.047 | 0.082 | 0.052 | 0.204 | 0.207 | 8.942 |

| Problem 88 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|------------|---------|-------|-------|-------|--------|--------|-------|-------|--------|-------|
| grade | N/A | A | B | B | B | B | B | B | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 119 | 119 | 398 | 417 | 418 | 418 | 462 | 470 | 472 | 402 |
| N.S. | 1 | 1.00 | 3.34 | 3.50 | 3.51 | 3.51 | 3.88 | 3.95 | 3.97 | 3.38 |
| time (sec) | N/A | 0.724 | 0.043 | 1.048 | 0.038 | 0.081 | 0.049 | 0.178 | 0.200 | 8.921 |

| Problem 89 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|------------|---------|-------|-------|-------|--------|--------|-------|-------|--------|-------|
| grade | N/A | A | B | B | B | B | B | B | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 92 | 92 | 276 | 325 | 327 | 327 | 364 | 362 | 363 | 308 |
| N.S. | 1 | 1.00 | 3.00 | 3.53 | 3.55 | 3.55 | 3.96 | 3.93 | 3.95 | 3.35 |
| time (sec) | N/A | 0.633 | 0.063 | 1.039 | 0.032 | 0.075 | 0.043 | 0.217 | 0.205 | 8.799 |

| Problem 90 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|------------|---------|-------|-------|-------|--------|--------|-------|-------|--------|-------|
| grade | N/A | A | B | B | B | B | B | B | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 65 | 65 | 199 | 232 | 234 | 234 | 252 | 253 | 254 | 214 |
| N.S. | 1 | 1.00 | 3.06 | 3.57 | 3.60 | 3.60 | 3.88 | 3.89 | 3.91 | 3.29 |
| time (sec) | N/A | 0.508 | 0.048 | 1.048 | 0.034 | 0.085 | 0.037 | 0.212 | 0.223 | 0.093 |

| Problem 91 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|------------|---------|-------|-------|-------|--------|--------|-------|-------|--------|-------|
| grade | N/A | A | B | B | B | B | B | B | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 38 | 38 | 122 | 140 | 142 | 142 | 148 | 145 | 145 | 126 |
| N.S. | 1 | 1.00 | 3.21 | 3.68 | 3.74 | 3.74 | 3.89 | 3.82 | 3.82 | 3.32 |
| time (sec) | N/A | 0.326 | 0.029 | 0.921 | 0.036 | 0.077 | 0.034 | 0.207 | 0.220 | 0.064 |

| Problem 92 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|------------|---------|-------|-------|-------|--------|--------|-------|-------|--------|-------|
| grade | N/A | A | A | B | B | B | B | B | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 14 | 14 | 14 | 65 | 97 | 64 | 66 | 64 | 65 | 64 |
| N.S. | 1 | 1.00 | 1.00 | 4.64 | 6.93 | 4.57 | 4.71 | 4.57 | 4.64 | 4.57 |
| time (sec) | N/A | 0.237 | 0.001 | 0.825 | 0.044 | 0.077 | 0.025 | 0.229 | 0.196 | 0.028 |

| Problem 93 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|------------|---------|-------|-------|-------|--------|--------|-------|-------|--------|-------|
| grade | N/A | A | A | B | B | B | B | B | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 146 | 146 | 230 | 333 | 349 | 351 | 296 | 375 | 411 | 385 |
| N.S. | 1 | 1.00 | 1.58 | 2.28 | 2.39 | 2.40 | 2.03 | 2.57 | 2.82 | 2.64 |
| time (sec) | N/A | 0.520 | 0.088 | 1.213 | 0.046 | 0.076 | 0.334 | 0.221 | 0.199 | 8.692 |

| Problem 94 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|------------|---------|-------|-------|-------|--------|--------|-------|-------|--------|-------|
| grade | N/A | A | A | B | B | B | B | B | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 156 | 156 | 302 | 345 | 357 | 496 | 311 | 446 | 549 | 523 |
| N.S. | 1 | 1.00 | 1.94 | 2.21 | 2.29 | 3.18 | 1.99 | 2.86 | 3.52 | 3.35 |
| time (sec) | N/A | 0.697 | 0.106 | 1.099 | 0.043 | 0.084 | 0.590 | 0.236 | 0.228 | 8.716 |

| Problem 95 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|------------|---------|-------|-------|-------|--------|--------|-------|-------|--------|-------|
| grade | N/A | A | A | B | B | B | B | B | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 158 | 158 | 303 | 343 | 364 | 548 | 340 | 362 | 625 | 441 |
| N.S. | 1 | 1.00 | 1.92 | 2.17 | 2.30 | 3.47 | 2.15 | 2.29 | 3.96 | 2.79 |
| time (sec) | N/A | 0.638 | 0.088 | 1.155 | 0.043 | 0.081 | 1.097 | 0.184 | 0.220 | 8.713 |

| Problem 96 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|------------|---------|-------|-------|-------|--------|--------|-------|-------|--------|-------|
| grade | N/A | A | A | B | B | B | B | B | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 156 | 156 | 302 | 344 | 374 | 576 | 367 | 356 | 645 | 393 |
| N.S. | 1 | 1.00 | 1.94 | 2.21 | 2.40 | 3.69 | 2.35 | 2.28 | 4.13 | 2.52 |
| time (sec) | N/A | 0.620 | 0.081 | 1.161 | 0.045 | 0.083 | 2.459 | 0.191 | 0.233 | 0.124 |

| Problem 97 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|------------|---------|-------|-------|-------|--------|--------|-------|-------|--------|-------|
| grade | N/A | A | A | B | B | B | B | B | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 155 | 155 | 301 | 343 | 387 | 571 | 394 | 516 | 634 | 387 |
| N.S. | 1 | 1.00 | 1.94 | 2.21 | 2.50 | 3.68 | 2.54 | 3.33 | 4.09 | 2.50 |
| time (sec) | N/A | 0.633 | 0.081 | 1.105 | 0.076 | 0.096 | 5.548 | 0.217 | 0.200 | 9.011 |

| Problem 98 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|------------|---------|-------|-------|-------|--------|--------|--------|-------|--------|-------|
| grade | N/A | A | A | B | B | B | B | B | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 155 | 155 | 297 | 348 | 397 | 542 | 420 | 351 | 576 | 399 |
| N.S. | 1 | 1.00 | 1.92 | 2.25 | 2.56 | 3.50 | 2.71 | 2.26 | 3.72 | 2.57 |
| time (sec) | N/A | 0.629 | 0.094 | 1.141 | 0.072 | 0.080 | 19.648 | 0.196 | 0.255 | 8.840 |

| Problem 99 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|------------|---------|-------|-------|-------|--------|--------|--------|-------|--------|-------|
| grade | N/A | A | A | B | B | B | B | B | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 167 | 167 | 233 | 342 | 416 | 492 | 439 | 359 | 498 | 353 |
| N.S. | 1 | 1.00 | 1.40 | 2.05 | 2.49 | 2.95 | 2.63 | 2.15 | 2.98 | 2.11 |
| time (sec) | N/A | 0.621 | 0.083 | 1.066 | 0.059 | 0.086 | 66.754 | 0.185 | 0.197 | 8.803 |

| Problem 100 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|-------------|---------|-------|-------|-------|--------|--------|-------|-------|--------|-------|
| grade | N/A | A | B | B | B | B | F(-1) | B | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 28 | 28 | 271 | 314 | 398 | 398 | 0 | 369 | 383 | 378 |
| N.S. | 1 | 1.00 | 9.68 | 11.21 | 14.21 | 14.21 | 0.00 | 13.18 | 13.68 | 13.50 |
| time (sec) | N/A | 0.267 | 0.122 | 1.071 | 0.048 | 0.079 | 0.000 | 0.187 | 0.202 | 0.119 |

| Problem 101 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|-------------|---------|-------|-------|-------|--------|--------|-------|-------|--------|-------|
| grade | N/A | A | B | B | B | B | F(-1) | B | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 58 | 58 | 277 | 335 | 430 | 430 | 0 | 375 | 453 | 410 |
| N.S. | 1 | 1.00 | 4.78 | 5.78 | 7.41 | 7.41 | 0.00 | 6.47 | 7.81 | 7.07 |
| time (sec) | N/A | 0.317 | 0.072 | 1.075 | 0.060 | 0.083 | 0.000 | 0.200 | 0.198 | 0.133 |

| Problem 102 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|-------------|---------|-------|-------|-------|--------|--------|-------|-------|--------|-------|
| grade | N/A | A | B | B | B | B | F(-1) | B | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 89 | 102 | 277 | 335 | 441 | 441 | 0 | 375 | 464 | 423 |
| N.S. | 1 | 1.15 | 3.11 | 3.76 | 4.96 | 4.96 | 0.00 | 4.21 | 5.21 | 4.75 |
| time (sec) | N/A | 0.345 | 0.070 | 1.050 | 0.057 | 0.085 | 0.000 | 0.179 | 0.222 | 8.759 |

| Problem 103 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|-------------|---------|-------|-------|-------|--------|--------|-------|-------|--------|-------|
| grade | N/A | A | B | B | B | B | F(-1) | B | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 120 | 146 | 277 | 335 | 452 | 452 | 0 | 375 | 475 | 434 |
| N.S. | 1 | 1.22 | 2.31 | 2.79 | 3.77 | 3.77 | 0.00 | 3.12 | 3.96 | 3.62 |
| time (sec) | N/A | 0.398 | 0.074 | 1.063 | 0.069 | 0.080 | 0.000 | 0.212 | 0.227 | 8.870 |

| Problem 104 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|-------------|---------|-------|-------|-------|--------|--------|-------|-------|--------|-------|
| grade | N/A | A | A | B | B | B | F(-1) | B | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 170 | 170 | 277 | 335 | 463 | 463 | 0 | 375 | 486 | 445 |
| N.S. | 1 | 1.00 | 1.63 | 1.97 | 2.72 | 2.72 | 0.00 | 2.21 | 2.86 | 2.62 |
| time (sec) | N/A | 0.627 | 0.067 | 1.075 | 0.051 | 0.092 | 0.000 | 0.191 | 0.206 | 0.198 |

| Problem 105 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|-------------|---------|-------|-------|-------|--------|--------|-------|-------|--------|-------|
| grade | N/A | A | A | B | B | B | F(-1) | B | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 173 | 173 | 277 | 335 | 474 | 474 | 0 | 375 | 497 | 456 |
| N.S. | 1 | 1.00 | 1.60 | 1.94 | 2.74 | 2.74 | 0.00 | 2.17 | 2.87 | 2.64 |
| time (sec) | N/A | 0.610 | 0.066 | 1.061 | 0.058 | 0.087 | 0.000 | 0.199 | 0.211 | 9.282 |

| Problem 106 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|-------------|---------|-------|-------|-------|--------|--------|-------|-------|--------|-------|
| grade | N/A | A | A | B | B | B | F(-1) | B | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 171 | 171 | 277 | 335 | 485 | 485 | 0 | 375 | 508 | 467 |
| N.S. | 1 | 1.00 | 1.62 | 1.96 | 2.84 | 2.84 | 0.00 | 2.19 | 2.97 | 2.73 |
| time (sec) | N/A | 0.617 | 0.073 | 1.072 | 0.081 | 0.079 | 0.000 | 0.239 | 0.248 | 9.607 |

| Problem 107 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|-------------|---------|-------|-------|-------|--------|--------|-------|-------|--------|-------|
| grade | N/A | A | A | B | B | B | F(-1) | B | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 173 | 173 | 277 | 335 | 496 | 496 | 0 | 375 | 519 | 478 |
| N.S. | 1 | 1.00 | 1.60 | 1.94 | 2.87 | 2.87 | 0.00 | 2.17 | 3.00 | 2.76 |
| time (sec) | N/A | 0.624 | 0.078 | 1.043 | 0.068 | 0.086 | 0.000 | 0.221 | 0.251 | 9.847 |

| Problem 108 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|-------------|---------|-------|-------|-------|--------|--------|-------|-------|--------|-------|
| grade | N/A | A | A | B | B | B | A | B | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 131 | 131 | 230 | 256 | 265 | 375 | 231 | 273 | 415 | 326 |
| N.S. | 1 | 1.00 | 1.76 | 1.95 | 2.02 | 2.86 | 1.76 | 2.08 | 3.17 | 2.49 |
| time (sec) | N/A | 0.616 | 0.056 | 1.115 | 0.039 | 0.086 | 0.481 | 0.220 | 0.293 | 8.773 |

| Problem 109 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|-------------|---------|-------|-------|-------|--------|--------|-------|-------|--------|-------|
| grade | N/A | A | A | A | A | B | A | A | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 108 | 104 | 166 | 175 | 184 | 268 | 155 | 188 | 297 | 203 |
| N.S. | 1 | 0.96 | 1.54 | 1.62 | 1.70 | 2.48 | 1.44 | 1.74 | 2.75 | 1.88 |
| time (sec) | N/A | 0.504 | 0.085 | 1.034 | 0.044 | 0.079 | 0.358 | 0.202 | 0.239 | 9.020 |

| Problem 110 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|-------------|---------|-------|-------|-------|--------|--------|-------|-------|--------|-------|
| grade | N/A | A | A | A | A | B | A | A | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 75 | 75 | 72 | 109 | 118 | 173 | 102 | 119 | 197 | 123 |
| N.S. | 1 | 1.00 | 0.96 | 1.45 | 1.57 | 2.31 | 1.36 | 1.59 | 2.63 | 1.64 |
| time (sec) | N/A | 0.424 | 0.036 | 1.056 | 0.037 | 0.085 | 0.260 | 0.185 | 0.220 | 0.072 |

| Problem 111 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|-------------|---------|-------|-------|-------|--------|--------|-------|-------|--------|-------|
| grade | N/A | A | A | A | A | A | A | A | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 51 | 51 | 47 | 63 | 67 | 92 | 60 | 65 | 112 | 71 |
| N.S. | 1 | 1.00 | 0.92 | 1.24 | 1.31 | 1.80 | 1.18 | 1.27 | 2.20 | 1.39 |
| time (sec) | N/A | 0.374 | 0.031 | 1.052 | 0.034 | 0.076 | 0.168 | 0.206 | 0.219 | 0.085 |

| Problem 112 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|-------------|---------|-------|-------|-------|--------|--------|-------|-------|--------|-------|
| grade | N/A | A | A | A | A | A | A | A | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 32 | 32 | 31 | 32 | 35 | 39 | 27 | 33 | 49 | 31 |
| N.S. | 1 | 1.00 | 0.97 | 1.00 | 1.09 | 1.22 | 0.84 | 1.03 | 1.53 | 0.97 |
| time (sec) | N/A | 0.334 | 0.009 | 0.947 | 0.034 | 0.087 | 0.089 | 0.204 | 0.205 | 9.024 |

| Problem 113 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|-------------|---------|-------|-------|-------|--------|--------|-------|-------|--------|-------|
| grade | N/A | A | A | A | A | A | A | A | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 12 | 12 | 12 | 13 | 13 | 13 | 10 | 12 | 12 | 12 |
| N.S. | 1 | 1.00 | 1.00 | 1.08 | 1.08 | 1.08 | 0.83 | 1.00 | 1.00 | 1.00 |
| time (sec) | N/A | 0.268 | 0.003 | 0.921 | 0.036 | 0.082 | 0.049 | 0.220 | 0.203 | 0.026 |

| Problem 114 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|-------------|---------|-------|-------|-------|--------|--------|-------|-------|--------|-------|
| grade | N/A | A | A | A | A | A | B | A | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 57 | 57 | 53 | 57 | 92 | 93 | 233 | 94 | 115 | 76 |
| N.S. | 1 | 1.00 | 0.93 | 1.00 | 1.61 | 1.63 | 4.09 | 1.65 | 2.02 | 1.33 |
| time (sec) | N/A | 0.383 | 0.022 | 1.234 | 0.033 | 0.085 | 0.338 | 0.202 | 0.201 | 8.926 |

| Problem 115 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|-------------|---------|-------|-------|-------|--------|--------|-------|-------|--------|-------|
| grade | N/A | A | A | A | B | B | B | A | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 81 | 81 | 66 | 82 | 208 | 241 | 406 | 154 | 459 | 182 |
| N.S. | 1 | 1.00 | 0.81 | 1.01 | 2.57 | 2.98 | 5.01 | 1.90 | 5.67 | 2.25 |
| time (sec) | N/A | 0.429 | 0.058 | 1.260 | 0.039 | 0.093 | 0.547 | 0.199 | 0.203 | 8.925 |

| Problem 116 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|-------------|---------|-------|-------|-------|--------|--------|-------|-------|--------|-------|
| grade | N/A | A | A | A | B | B | B | B | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 110 | 110 | 97 | 108 | 386 | 495 | 632 | 254 | 821 | 329 |
| N.S. | 1 | 1.00 | 0.88 | 0.98 | 3.51 | 4.50 | 5.75 | 2.31 | 7.46 | 2.99 |
| time (sec) | N/A | 0.502 | 0.068 | 1.302 | 0.056 | 0.090 | 0.831 | 0.176 | 0.211 | 8.921 |

| Problem 117 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|-------------|---------|-------|-------|-------|--------|--------|-------|-------|--------|-------|
| grade | N/A | A | A | A | B | B | B | B | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 133 | 133 | 120 | 131 | 599 | 753 | 882 | 350 | 1237 | 534 |
| N.S. | 1 | 1.00 | 0.90 | 0.98 | 4.50 | 5.66 | 6.63 | 2.63 | 9.30 | 4.02 |
| time (sec) | N/A | 0.563 | 0.094 | 1.293 | 0.071 | 0.102 | 1.119 | 0.193 | 0.255 | 9.042 |

| Problem 118 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|-------------|---------|-------|-------|-------|--------|--------|-------|-------|--------|-------|
| grade | N/A | A | A | B | B | B | B | B | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 160 | 156 | 301 | 344 | 374 | 577 | 367 | 356 | 645 | 393 |
| N.S. | 1 | 0.98 | 1.88 | 2.15 | 2.34 | 3.61 | 2.29 | 2.22 | 4.03 | 2.46 |
| time (sec) | N/A | 0.681 | 0.083 | 1.029 | 0.059 | 0.096 | 2.040 | 0.199 | 0.203 | 0.131 |

| Problem 119 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|-------------|---------|-------|-------|-------|--------|--------|-------|-------|--------|-------|
| grade | N/A | A | A | B | B | B | B | B | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 129 | 129 | 228 | 252 | 281 | 426 | 284 | 262 | 469 | 286 |
| N.S. | 1 | 1.00 | 1.77 | 1.95 | 2.18 | 3.30 | 2.20 | 2.03 | 3.64 | 2.22 |
| time (sec) | N/A | 0.559 | 0.052 | 1.044 | 0.041 | 0.091 | 1.378 | 0.171 | 0.190 | 8.790 |

| Problem 120 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|-------------|---------|-------|-------|-------|--------|--------|-------|-------|--------|-------|
| grade | N/A | A | A | A | A | B | B | A | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 103 | 103 | 166 | 178 | 201 | 292 | 209 | 177 | 309 | 204 |
| N.S. | 1 | 1.00 | 1.61 | 1.73 | 1.95 | 2.83 | 2.03 | 1.72 | 3.00 | 1.98 |
| time (sec) | N/A | 0.500 | 0.080 | 1.072 | 0.051 | 0.086 | 0.929 | 0.204 | 0.195 | 8.805 |

| Problem 121 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|-------------|---------|-------|-------|-------|--------|--------|-------|-------|--------|-------|
| grade | N/A | A | A | A | A | B | A | A | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 86 | 86 | 80 | 119 | 142 | 176 | 148 | 118 | 180 | 138 |
| N.S. | 1 | 1.00 | 0.93 | 1.38 | 1.65 | 2.05 | 1.72 | 1.37 | 2.09 | 1.60 |
| time (sec) | N/A | 0.442 | 0.032 | 1.030 | 0.050 | 0.089 | 0.543 | 0.202 | 0.244 | 8.789 |

| Problem 122 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|-------------|---------|-------|-------|-------|--------|--------|-------|-------|--------|-------|
| grade | N/A | A | A | B | B | B | B | B | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 28 | 28 | 53 | 66 | 84 | 84 | 88 | 59 | 69 | 80 |
| N.S. | 1 | 1.00 | 1.89 | 2.36 | 3.00 | 3.00 | 3.14 | 2.11 | 2.46 | 2.86 |
| time (sec) | N/A | 0.275 | 0.019 | 0.997 | 0.030 | 0.079 | 0.301 | 0.172 | 0.225 | 0.043 |

| Problem 123 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|-------------|---------|-------|-------|-------|--------|--------|-------|-------|--------|-------|
| grade | N/A | A | A | A | A | A | A | A | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 38 | 38 | 27 | 34 | 50 | 50 | 53 | 25 | 48 | 52 |
| N.S. | 1 | 1.00 | 0.71 | 0.89 | 1.32 | 1.32 | 1.39 | 0.66 | 1.26 | 1.37 |
| time (sec) | N/A | 0.328 | 0.008 | 0.977 | 0.037 | 0.089 | 0.163 | 0.192 | 0.199 | 0.032 |

| Problem 124 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|-------------|---------|-------|-------|-------|--------|--------|-------|-------|--------|-------|
| grade | N/A | A | A | A | B | B | B | A | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 14 | 14 | 14 | 13 | 35 | 35 | 37 | 12 | 34 | 37 |
| N.S. | 1 | 1.00 | 1.00 | 0.93 | 2.50 | 2.50 | 2.64 | 0.86 | 2.43 | 2.64 |
| time (sec) | N/A | 0.250 | 0.002 | 0.948 | 0.047 | 0.083 | 0.101 | 0.181 | 0.223 | 0.030 |

| Problem 125 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|-------------|---------|-------|-------|-------|--------|--------|-------|-------|--------|-------|
| grade | N/A | A | A | A | B | B | B | B | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 107 | 107 | 107 | 103 | 361 | 425 | 570 | 243 | 499 | 312 |
| N.S. | 1 | 1.00 | 1.00 | 0.96 | 3.37 | 3.97 | 5.33 | 2.27 | 4.66 | 2.92 |
| time (sec) | N/A | 0.484 | 0.034 | 1.279 | 0.050 | 0.098 | 0.745 | 0.191 | 0.221 | 9.365 |

| Problem 126 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|-------------|---------|-------|-------|-------|--------|--------|-------|-------|--------|-------|
| grade | N/A | A | A | A | B | B | B | B | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 132 | 132 | 121 | 132 | 598 | 751 | 882 | 288 | 1236 | 534 |
| N.S. | 1 | 1.00 | 0.92 | 1.00 | 4.53 | 5.69 | 6.68 | 2.18 | 9.36 | 4.05 |
| time (sec) | N/A | 0.555 | 0.083 | 1.338 | 0.058 | 0.103 | 1.171 | 0.161 | 0.209 | 9.117 |

| Problem 127 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|-------------|---------|-------|-------|-------|--------|--------|-------|-------|--------|-------|
| grade | N/A | A | A | A | B | B | B | B | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 170 | 170 | 154 | 165 | 889 | 1151 | 1217 | 458 | 1840 | 797 |
| N.S. | 1 | 1.00 | 0.91 | 0.97 | 5.23 | 6.77 | 7.16 | 2.69 | 10.82 | 4.69 |
| time (sec) | N/A | 0.651 | 0.150 | 1.397 | 0.074 | 0.116 | 1.660 | 0.199 | 0.212 | 9.258 |

| Problem 128 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|-------------|---------|-------|-------|-------|--------|--------|-------|-------|--------|-------|
| grade | N/A | A | A | B | B | B | F(-1) | B | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 212 | 208 | 195 | 572 | 626 | 943 | 0 | 586 | 1073 | 644 |
| N.S. | 1 | 0.98 | 0.92 | 2.70 | 2.95 | 4.45 | 0.00 | 2.76 | 5.06 | 3.04 |
| time (sec) | N/A | 0.915 | 0.175 | 1.105 | 0.076 | 0.090 | 0.000 | 0.158 | 0.255 | 0.199 |

| Problem 129 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|-------------|---------|-------|-------|-------|--------|--------|--------|-------|--------|-------|
| grade | N/A | A | B | B | B | B | B | B | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 181 | 181 | 389 | 451 | 504 | 732 | 524 | 463 | 816 | 508 |
| N.S. | 1 | 1.00 | 2.15 | 2.49 | 2.78 | 4.04 | 2.90 | 2.56 | 4.51 | 2.81 |
| time (sec) | N/A | 0.788 | 0.111 | 1.122 | 0.072 | 0.092 | 64.304 | 0.163 | 0.208 | 0.176 |

| Problem 130 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|-------------|---------|-------|-------|-------|--------|--------|--------|-------|--------|-------|
| grade | N/A | A | A | B | B | B | B | B | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 155 | 155 | 300 | 349 | 397 | 542 | 420 | 351 | 576 | 399 |
| N.S. | 1 | 1.00 | 1.94 | 2.25 | 2.56 | 3.50 | 2.71 | 2.26 | 3.72 | 2.57 |
| time (sec) | N/A | 0.653 | 0.152 | 1.107 | 0.048 | 0.091 | 18.337 | 0.165 | 0.204 | 8.848 |

| Problem 131 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|-------------|---------|-------|-------|-------|--------|--------|-------|-------|--------|-------|
| grade | N/A | A | A | A | B | B | B | A | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 138 | 138 | 171 | 261 | 310 | 372 | 326 | 264 | 375 | 261 |
| N.S. | 1 | 1.00 | 1.24 | 1.89 | 2.25 | 2.70 | 2.36 | 1.91 | 2.72 | 1.89 |
| time (sec) | N/A | 0.564 | 0.066 | 1.070 | 0.059 | 0.081 | 5.057 | 0.193 | 0.181 | 0.146 |

| Problem 132 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|-------------|---------|-------|-------|-------|--------|--------|-------|-------|--------|-------|
| grade | N/A | A | B | B | B | B | B | B | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 28 | 28 | 140 | 179 | 215 | 215 | 236 | 180 | 192 | 203 |
| N.S. | 1 | 1.00 | 5.00 | 6.39 | 7.68 | 7.68 | 8.43 | 6.43 | 6.86 | 7.25 |
| time (sec) | N/A | 0.277 | 0.037 | 1.070 | 0.038 | 0.085 | 2.331 | 0.187 | 0.222 | 8.698 |

| Problem 133 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|-------------|---------|-------|-------|-------|--------|--------|-------|-------|--------|-------|
| grade | N/A | A | A | B | B | B | B | B | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 58 | 58 | 97 | 121 | 160 | 160 | 172 | 114 | 159 | 154 |
| N.S. | 1 | 1.00 | 1.67 | 2.09 | 2.76 | 2.76 | 2.97 | 1.97 | 2.74 | 2.66 |
| time (sec) | N/A | 0.308 | 0.030 | 1.062 | 0.048 | 0.084 | 1.093 | 0.172 | 0.198 | 0.067 |

| Problem 134 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|-------------|---------|-------|-------|-------|--------|--------|-------|-------|--------|-------|
| grade | N/A | A | A | A | A | A | B | A | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 65 | 65 | 57 | 71 | 109 | 109 | 116 | 61 | 106 | 107 |
| N.S. | 1 | 1.00 | 0.88 | 1.09 | 1.68 | 1.68 | 1.78 | 0.94 | 1.63 | 1.65 |
| time (sec) | N/A | 0.385 | 0.020 | 1.070 | 0.038 | 0.073 | 0.490 | 0.192 | 0.204 | 0.056 |

| Problem 135 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|-------------|---------|-------|-------|-------|--------|--------|-------|-------|--------|-------|
| grade | N/A | A | A | A | B | B | B | A | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 38 | 38 | 27 | 35 | 72 | 72 | 76 | 25 | 70 | 74 |
| N.S. | 1 | 1.00 | 0.71 | 0.92 | 1.89 | 1.89 | 2.00 | 0.66 | 1.84 | 1.95 |
| time (sec) | N/A | 0.329 | 0.008 | 1.020 | 0.037 | 0.075 | 0.256 | 0.175 | 0.205 | 0.046 |

| Problem 136 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|-------------|---------|-------|-------|-------|--------|--------|-------|-------|--------|-------|
| grade | N/A | A | A | A | B | B | B | A | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 14 | 14 | 14 | 13 | 57 | 57 | 61 | 12 | 56 | 59 |
| N.S. | 1 | 1.00 | 1.00 | 0.93 | 4.07 | 4.07 | 4.36 | 0.86 | 4.00 | 4.21 |
| time (sec) | N/A | 0.241 | 0.002 | 0.958 | 0.028 | 0.074 | 0.149 | 0.180 | 0.205 | 0.044 |

| Problem 137 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|-------------|---------|-------|-------|-------|--------|--------|-------|-------|--------|-------|
| grade | N/A | A | A | A | B | B | B | B | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 155 | 155 | 152 | 147 | 805 | 923 | 1081 | 445 | 1086 | 721 |
| N.S. | 1 | 1.00 | 0.98 | 0.95 | 5.19 | 5.95 | 6.97 | 2.87 | 7.01 | 4.65 |
| time (sec) | N/A | 0.604 | 0.075 | 1.327 | 0.073 | 0.105 | 1.592 | 0.176 | 0.205 | 9.268 |

| Problem 138 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|-------------|---------|-------|-------|-------|--------|--------|-------|-------|--------|-------|
| grade | N/A | A | A | A | B | B | B | B | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 181 | 181 | 167 | 178 | 1154 | 1420 | 1518 | 473 | 2163 | 1047 |
| N.S. | 1 | 1.00 | 0.92 | 0.98 | 6.38 | 7.85 | 8.39 | 2.61 | 11.95 | 5.78 |
| time (sec) | N/A | 0.725 | 0.108 | 1.408 | 0.128 | 0.103 | 2.669 | 0.176 | 0.209 | 9.943 |

| Problem 139 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|-------------|---------|-------|-------|-------|--------|--------|-------|-------|--------|--------|
| grade | N/A | A | A | A | B | B | B | B | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 220 | 220 | 204 | 215 | 1558 | 2005 | 1974 | 714 | 2998 | 1427 |
| N.S. | 1 | 1.00 | 0.93 | 0.98 | 7.08 | 9.11 | 8.97 | 3.25 | 13.63 | 6.49 |
| time (sec) | N/A | 0.831 | 0.124 | 1.494 | 0.148 | 0.131 | 4.871 | 0.172 | 0.250 | 10.052 |

| Problem 140 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|-------------|---------|-------|-------|-------|--------|--------|-------|-------|--------|-------|
| grade | N/A | A | B | B | B | B | B | B | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 31 | 31 | 81 | 78 | 77 | 77 | 76 | 83 | 81 | 77 |
| N.S. | 1 | 1.00 | 2.61 | 2.52 | 2.48 | 2.48 | 2.45 | 2.68 | 2.61 | 2.48 |
| time (sec) | N/A | 0.329 | 0.016 | 0.724 | 0.034 | 0.079 | 0.033 | 0.169 | 0.239 | 0.064 |

| Problem 141 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|-------------|---------|-------|-------|-------|--------|--------|-------|-------|--------|-------|
| grade | N/A | A | A | B | A | A | B | B | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 31 | 31 | 59 | 56 | 53 | 53 | 58 | 59 | 57 | 55 |
| N.S. | 1 | 1.00 | 1.90 | 1.81 | 1.71 | 1.71 | 1.87 | 1.90 | 1.84 | 1.77 |
| time (sec) | N/A | 0.323 | 0.007 | 0.716 | 0.034 | 0.070 | 0.029 | 0.170 | 0.215 | 9.024 |

| Problem 142 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|-------------|---------|-------|-------|-------|--------|--------|-------|-------|--------|-------|
| grade | N/A | A | A | A | A | A | A | A | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 31 | 31 | 36 | 33 | 32 | 32 | 32 | 34 | 33 | 32 |
| N.S. | 1 | 1.00 | 1.16 | 1.06 | 1.03 | 1.03 | 1.03 | 1.10 | 1.06 | 1.03 |
| time (sec) | N/A | 0.315 | 0.005 | 0.170 | 0.033 | 0.073 | 0.019 | 0.155 | 0.192 | 0.045 |

| Problem 143 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|-------------|---------|-------|-------|-------|--------|--------|-------|-------|--------|-------|
| grade | N/A | A | A | A | A | A | A | A | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 30 | 30 | 30 | 26 | 26 | 31 | 20 | 27 | 36 | 24 |
| N.S. | 1 | 1.00 | 1.00 | 0.87 | 0.87 | 1.03 | 0.67 | 0.90 | 1.20 | 0.80 |
| time (sec) | N/A | 0.314 | 0.007 | 0.696 | 0.035 | 0.076 | 0.067 | 0.184 | 0.248 | 9.149 |

| Problem 144 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|-------------|---------|-------|-------|-------|--------|--------|-------|-------|--------|-------|
| grade | N/A | A | A | A | A | A | A | A | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 31 | 31 | 22 | 20 | 30 | 30 | 27 | 20 | 29 | 20 |
| N.S. | 1 | 1.00 | 0.71 | 0.65 | 0.97 | 0.97 | 0.87 | 0.65 | 0.94 | 0.65 |
| time (sec) | N/A | 0.312 | 0.008 | 0.718 | 0.039 | 0.075 | 0.125 | 0.163 | 0.194 | 0.042 |

| Problem 145 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|-------------|---------|-------|-------|-------|--------|--------|-------|-------|--------|-------|
| grade | N/A | A | A | A | A | A | A | A | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 31 | 31 | 22 | 20 | 40 | 40 | 37 | 20 | 39 | 20 |
| N.S. | 1 | 1.00 | 0.71 | 0.65 | 1.29 | 1.29 | 1.19 | 0.65 | 1.26 | 0.65 |
| time (sec) | N/A | 0.310 | 0.008 | 0.722 | 0.034 | 0.078 | 0.166 | 0.161 | 0.201 | 0.047 |

| Problem 146 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|-------------|---------|-------|-------|-------|--------|--------|-------|-------|--------|-------|
| grade | N/A | A | A | A | A | B | B | B | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 71 | 71 | 61 | 54 | 68 | 212 | 432 | 791 | 221 | 68 |
| N.S. | 1 | 1.00 | 0.86 | 0.76 | 0.96 | 2.99 | 6.08 | 11.14 | 3.11 | 0.96 |
| time (sec) | N/A | 0.382 | 0.055 | 0.841 | 0.039 | 0.084 | 0.491 | 0.209 | 0.194 | 9.066 |

| Problem 147 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|-------------|---------|-------|-------|-------|--------|--------|-------|-------|--------|-------|
| grade | N/A | A | A | A | A | B | B | B | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 71 | 71 | 61 | 54 | 68 | 174 | 355 | 558 | 180 | 68 |
| N.S. | 1 | 1.00 | 0.86 | 0.76 | 0.96 | 2.45 | 5.00 | 7.86 | 2.54 | 0.96 |
| time (sec) | N/A | 0.353 | 0.051 | 0.804 | 0.036 | 0.090 | 0.337 | 0.150 | 0.214 | 0.057 |

| Problem 148 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|-------------|---------|-------|-------|-------|--------|--------|-------|-------|--------|-------|
| grade | N/A | A | A | A | A | B | A | B | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 71 | 71 | 61 | 54 | 68 | 137 | 110 | 360 | 139 | 68 |
| N.S. | 1 | 1.00 | 0.86 | 0.76 | 0.96 | 1.93 | 1.55 | 5.07 | 1.96 | 0.96 |
| time (sec) | N/A | 0.353 | 0.049 | 0.785 | 0.036 | 0.077 | 0.763 | 0.168 | 0.215 | 9.080 |

| Problem 149 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|-------------|---------|-------|-------|-------|--------|--------|-------|-------|--------|-------|
| grade | N/A | A | A | A | A | A | A | B | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 71 | 71 | 61 | 54 | 68 | 99 | 110 | 200 | 98 | 68 |
| N.S. | 1 | 1.00 | 0.86 | 0.76 | 0.96 | 1.39 | 1.55 | 2.82 | 1.38 | 0.96 |
| time (sec) | N/A | 0.361 | 0.042 | 0.658 | 0.040 | 0.088 | 0.719 | 0.185 | 0.208 | 9.631 |

| Problem 150 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|-------------|---------|-------|-------|-------|--------|--------|-------|-------|--------|-------|
| grade | N/A | A | A | A | A | A | A | A | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 69 | 69 | 60 | 54 | 82 | 64 | 110 | 82 | 61 | 68 |
| N.S. | 1 | 1.00 | 0.87 | 0.78 | 1.19 | 0.93 | 1.59 | 1.19 | 0.88 | 0.99 |
| time (sec) | N/A | 0.351 | 0.047 | 0.631 | 0.036 | 0.084 | 0.552 | 0.175 | 0.194 | 0.056 |

| Problem 151 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|-------------|---------|-------|-------|-------|--------|--------|-------|-------|--------|-------|
| grade | N/A | A | A | A | A | A | A | A | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 67 | 67 | 59 | 55 | 75 | 73 | 94 | 84 | 62 | 67 |
| N.S. | 1 | 1.00 | 0.88 | 0.82 | 1.12 | 1.09 | 1.40 | 1.25 | 0.93 | 1.00 |
| time (sec) | N/A | 0.355 | 0.093 | 0.688 | 0.038 | 0.072 | 1.207 | 0.172 | 0.263 | 9.426 |

| Problem 152 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|-------------|---------|-------|-------|-------|--------|--------|-------|-------|--------|-------|
| grade | N/A | A | A | A | A | A | B | A | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 67 | 67 | 62 | 50 | 72 | 85 | 265 | 72 | 70 | 68 |
| N.S. | 1 | 1.00 | 0.93 | 0.75 | 1.07 | 1.27 | 3.96 | 1.07 | 1.04 | 1.01 |
| time (sec) | N/A | 0.386 | 0.051 | 0.685 | 0.044 | 0.083 | 0.309 | 0.196 | 0.212 | 0.059 |

| Problem 153 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|-------------|---------|-------|-------|-------|--------|--------|-------|-------|--------|-------|
| grade | N/A | A | A | A | A | A | B | A | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 69 | 69 | 61 | 56 | 65 | 95 | 389 | 68 | 81 | 62 |
| N.S. | 1 | 1.00 | 0.88 | 0.81 | 0.94 | 1.38 | 5.64 | 0.99 | 1.17 | 0.90 |
| time (sec) | N/A | 0.362 | 0.066 | 0.661 | 0.036 | 0.094 | 0.439 | 0.158 | 0.199 | 0.072 |

| Problem 154 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|-------------|---------|-------|-------|-------|--------|--------|-------|-------|--------|-------|
| grade | N/A | A | A | A | A | B | B | B | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 129 | 129 | 154 | 143 | 181 | 444 | 903 | 1661 | 480 | 112 |
| N.S. | 1 | 1.00 | 1.19 | 1.11 | 1.40 | 3.44 | 7.00 | 12.88 | 3.72 | 0.87 |
| time (sec) | N/A | 0.481 | 0.126 | 1.395 | 0.035 | 0.096 | 0.671 | 0.185 | 0.206 | 9.306 |

| Problem 155 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|-------------|---------|-------|-------|-------|--------|--------|-------|-------|--------|-------|
| grade | N/A | A | A | A | A | B | B | B | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 129 | 129 | 154 | 144 | 181 | 377 | 272 | 1204 | 405 | 112 |
| N.S. | 1 | 1.00 | 1.19 | 1.12 | 1.40 | 2.92 | 2.11 | 9.33 | 3.14 | 0.87 |
| time (sec) | N/A | 0.462 | 0.109 | 1.237 | 0.043 | 0.091 | 1.108 | 0.234 | 0.211 | 9.150 |

| Problem 156 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|-------------|---------|-------|-------|-------|--------|--------|-------|-------|--------|-------|
| grade | N/A | A | A | A | A | B | B | B | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 129 | 129 | 154 | 143 | 181 | 311 | 272 | 807 | 330 | 112 |
| N.S. | 1 | 1.00 | 1.19 | 1.11 | 1.40 | 2.41 | 2.11 | 6.26 | 2.56 | 0.87 |
| time (sec) | N/A | 0.443 | 0.104 | 1.176 | 0.044 | 0.084 | 1.068 | 0.251 | 0.201 | 0.042 |

| Problem 157 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|-------------|---------|-------|-------|-------|--------|--------|-------|-------|--------|-------|
| grade | N/A | A | A | A | A | B | B | B | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 129 | 129 | 154 | 143 | 181 | 245 | 272 | 470 | 255 | 112 |
| N.S. | 1 | 1.00 | 1.19 | 1.11 | 1.40 | 1.90 | 2.11 | 3.64 | 1.98 | 0.87 |
| time (sec) | N/A | 0.454 | 0.128 | 1.237 | 0.040 | 0.082 | 0.997 | 0.177 | 0.247 | 0.042 |

| Problem 158 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|-------------|---------|-------|-------|-------|--------|--------|-------|-------|--------|-------|
| grade | N/A | A | A | A | B | A | B | A | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 127 | 127 | 153 | 143 | 247 | 182 | 270 | 204 | 184 | 112 |
| N.S. | 1 | 1.00 | 1.20 | 1.13 | 1.94 | 1.43 | 2.13 | 1.61 | 1.45 | 0.88 |
| time (sec) | N/A | 0.455 | 0.097 | 1.165 | 0.048 | 0.093 | 0.910 | 0.169 | 0.262 | 0.041 |

| Problem 159 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|-------------|---------|-------|-------|-------|--------|--------|-------|-------|--------|-------|
| grade | N/A | A | A | A | A | A | A | B | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 123 | 123 | 151 | 145 | 189 | 192 | 224 | 240 | 186 | 153 |
| N.S. | 1 | 1.00 | 1.23 | 1.18 | 1.54 | 1.56 | 1.82 | 1.95 | 1.51 | 1.24 |
| time (sec) | N/A | 0.453 | 0.089 | 1.233 | 0.034 | 0.083 | 3.691 | 0.196 | 0.311 | 0.048 |

| Problem 160 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|-------------|---------|-------|-------|-------|--------|--------|-------|-------|--------|-------|
| grade | N/A | A | A | A | A | A | A | B | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 125 | 125 | 153 | 128 | 187 | 203 | 190 | 229 | 193 | 175 |
| N.S. | 1 | 1.00 | 1.22 | 1.02 | 1.50 | 1.62 | 1.52 | 1.83 | 1.54 | 1.40 |
| time (sec) | N/A | 0.446 | 0.103 | 1.207 | 0.044 | 0.086 | 3.488 | 0.124 | 0.240 | 9.215 |

| Problem 161 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|-------------|---------|-------|-------|-------|--------|--------|-------|-------|--------|-------|
| grade | N/A | A | A | A | A | A | B | B | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 125 | 125 | 154 | 131 | 189 | 213 | 1008 | 223 | 204 | 185 |
| N.S. | 1 | 1.00 | 1.23 | 1.05 | 1.51 | 1.70 | 8.06 | 1.78 | 1.63 | 1.48 |
| time (sec) | N/A | 0.442 | 0.108 | 1.232 | 0.039 | 0.089 | 0.490 | 0.196 | 0.221 | 9.376 |

| Problem 162 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|-------------|---------|-------|-------|-------|--------|--------|-------|-------|--------|-------|
| grade | N/A | A | A | A | B | B | B | B | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 187 | 187 | 291 | 276 | 350 | 729 | 495 | 2771 | 807 | 162 |
| N.S. | 1 | 1.00 | 1.56 | 1.48 | 1.87 | 3.90 | 2.65 | 14.82 | 4.32 | 0.87 |
| time (sec) | N/A | 0.580 | 0.250 | 1.424 | 0.051 | 0.099 | 1.492 | 0.214 | 0.281 | 9.232 |

| Problem 163 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|-------------|---------|-------|-------|-------|--------|--------|-------|-------|--------|-------|
| grade | N/A | A | A | A | B | B | B | B | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 185 | 185 | 291 | 276 | 350 | 635 | 495 | 2042 | 698 | 162 |
| N.S. | 1 | 1.00 | 1.57 | 1.49 | 1.89 | 3.43 | 2.68 | 11.04 | 3.77 | 0.88 |
| time (sec) | N/A | 0.530 | 0.203 | 1.393 | 0.034 | 0.090 | 1.396 | 0.208 | 0.215 | 9.260 |

| Problem 164 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|-------------|---------|-------|-------|-------|--------|--------|-------|-------|--------|-------|
| grade | N/A | A | A | A | B | B | B | B | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 187 | 187 | 291 | 276 | 350 | 541 | 495 | 1397 | 589 | 162 |
| N.S. | 1 | 1.00 | 1.56 | 1.48 | 1.87 | 2.89 | 2.65 | 7.47 | 3.15 | 0.87 |
| time (sec) | N/A | 0.548 | 0.231 | 1.286 | 0.044 | 0.092 | 1.268 | 0.192 | 0.206 | 0.055 |

| Problem 165 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|-------------|---------|-------|-------|-------|--------|--------|-------|-------|--------|-------|
| grade | N/A | A | A | A | B | B | B | B | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 187 | 187 | 291 | 276 | 350 | 447 | 495 | 836 | 480 | 162 |
| N.S. | 1 | 1.00 | 1.56 | 1.48 | 1.87 | 2.39 | 2.65 | 4.47 | 2.57 | 0.87 |
| time (sec) | N/A | 0.545 | 0.173 | 1.294 | 0.045 | 0.088 | 1.159 | 0.186 | 0.210 | 9.039 |

| Problem 166 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|-------------|---------|-------|-------|-------|--------|--------|-------|-------|--------|-------|
| grade | N/A | A | A | A | B | B | B | B | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 181 | 181 | 290 | 275 | 540 | 356 | 493 | 374 | 375 | 162 |
| N.S. | 1 | 1.00 | 1.60 | 1.52 | 2.98 | 1.97 | 2.72 | 2.07 | 2.07 | 0.90 |
| time (sec) | N/A | 0.538 | 0.159 | 1.224 | 0.051 | 0.088 | 1.109 | 0.122 | 0.248 | 9.167 |

| Problem 167 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|-------------|---------|-------|-------|-------|--------|--------|-------|-------|--------|-------|
| grade | N/A | A | A | A | B | B | B | B | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 179 | 179 | 288 | 278 | 358 | 365 | 416 | 479 | 377 | 231 |
| N.S. | 1 | 1.00 | 1.61 | 1.55 | 2.00 | 2.04 | 2.32 | 2.68 | 2.11 | 1.29 |
| time (sec) | N/A | 0.527 | 0.152 | 1.318 | 0.052 | 0.091 | 9.928 | 0.168 | 0.236 | 9.214 |

| Problem 168 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|-------------|---------|-------|-------|-------|--------|--------|--------|-------|--------|-------|
| grade | N/A | A | A | A | B | B | B | B | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 181 | 181 | 290 | 276 | 356 | 377 | 352 | 464 | 384 | 281 |
| N.S. | 1 | 1.00 | 1.60 | 1.52 | 1.97 | 2.08 | 1.94 | 2.56 | 2.12 | 1.55 |
| time (sec) | N/A | 0.524 | 0.157 | 1.332 | 0.035 | 0.097 | 10.692 | 0.205 | 0.211 | 9.247 |

| Problem 169 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|-------------|---------|-------|-------|-------|--------|--------|--------|-------|--------|-------|
| grade | N/A | A | A | A | B | B | A | B | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 179 | 179 | 291 | 248 | 356 | 388 | 301 | 457 | 395 | 322 |
| N.S. | 1 | 1.00 | 1.63 | 1.39 | 1.99 | 2.17 | 1.68 | 2.55 | 2.21 | 1.80 |
| time (sec) | N/A | 0.533 | 0.212 | 1.325 | 0.039 | 0.102 | 10.080 | 0.158 | 0.214 | 9.114 |

| Problem 170 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|-------------|---------|-------|-------|-------|--------|--------|-------|-------|--------|-------|
| grade | N/A | A | A | A | F(-2) | B | F(-1) | B | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 162 | 179 | 208 | 217 | 0 | 678 | 0 | 375 | 648 | 352 |
| N.S. | 1 | 1.10 | 1.28 | 1.34 | 0.00 | 4.19 | 0.00 | 2.31 | 4.00 | 2.17 |
| time (sec) | N/A | 0.513 | 0.495 | 3.079 | 0.000 | 0.096 | 0.000 | 0.154 | 0.245 | 0.115 |

| Problem 171 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|-------------|---------|-------|-------|-------|--------|--------|-------|-------|--------|-------|
| grade | N/A | A | A | A | F(-2) | B | F(-1) | B | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 137 | 150 | 158 | 167 | 0 | 486 | 0 | 270 | 449 | 235 |
| N.S. | 1 | 1.09 | 1.15 | 1.22 | 0.00 | 3.55 | 0.00 | 1.97 | 3.28 | 1.72 |
| time (sec) | N/A | 0.472 | 0.364 | 1.603 | 0.000 | 0.111 | 0.000 | 0.177 | 0.244 | 0.097 |

| Problem 172 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|-------------|---------|-------|-------|-------|--------|--------|-------|-------|--------|-------|
| grade | N/A | A | A | A | F(-2) | A | F | A | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 110 | 121 | 116 | 120 | 0 | 330 | 0 | 181 | 273 | 161 |
| N.S. | 1 | 1.10 | 1.05 | 1.09 | 0.00 | 3.00 | 0.00 | 1.65 | 2.48 | 1.46 |
| time (sec) | N/A | 0.424 | 0.314 | 1.599 | 0.000 | 0.103 | 0.000 | 0.190 | 0.226 | 0.109 |

| Problem 173 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|-------------|---------|-------|-------|-------|--------|--------|-------|-------|--------|-------|
| grade | N/A | A | A | A | F(-2) | A | F | A | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 85 | 92 | 83 | 93 | 0 | 210 | 0 | 113 | 127 | 109 |
| N.S. | 1 | 1.08 | 0.98 | 1.09 | 0.00 | 2.47 | 0.00 | 1.33 | 1.49 | 1.28 |
| time (sec) | N/A | 0.382 | 0.243 | 1.531 | 0.000 | 0.100 | 0.000 | 0.190 | 0.213 | 0.108 |

| Problem 174 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|-------------|---------|-------|-------|-------|--------|--------|-------|-------|--------|-------|
| grade | N/A | A | A | A | F(-2) | A | F | A | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 70 | 70 | 69 | 58 | 0 | 232 | 0 | 72 | 129 | 61 |
| N.S. | 1 | 1.00 | 0.99 | 0.83 | 0.00 | 3.31 | 0.00 | 1.03 | 1.84 | 0.87 |
| time (sec) | N/A | 0.347 | 0.198 | 1.395 | 0.000 | 0.099 | 0.000 | 0.155 | 0.247 | 9.629 |

| Problem 175 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|-------------|---------|-------|-------|-------|--------|--------|-------|-------|--------|-------|
| grade | N/A | A | A | A | F(-2) | B | F | A | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 76 | 76 | 75 | 64 | 0 | 280 | 0 | 87 | 156 | 74 |
| N.S. | 1 | 1.00 | 0.99 | 0.84 | 0.00 | 3.68 | 0.00 | 1.14 | 2.05 | 0.97 |
| time (sec) | N/A | 0.364 | 0.190 | 1.381 | 0.000 | 0.085 | 0.000 | 0.177 | 0.213 | 0.076 |

| Problem 176 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|-------------|---------|-------|-------|-------|--------|--------|-------|-------|--------|-------|
| grade | N/A | A | A | A | F(-2) | B | F | A | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 99 | 113 | 90 | 78 | 0 | 404 | 0 | 143 | 220 | 123 |
| N.S. | 1 | 1.14 | 0.91 | 0.79 | 0.00 | 4.08 | 0.00 | 1.44 | 2.22 | 1.24 |
| time (sec) | N/A | 0.419 | 0.301 | 1.409 | 0.000 | 0.107 | 0.000 | 0.194 | 0.205 | 0.167 |

| Problem 177 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|-------------|---------|-------|-------|-------|--------|--------|-------|-------|--------|-------|
| grade | N/A | A | A | A | F(-2) | B | F | B | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 124 | 149 | 125 | 121 | 0 | 762 | 0 | 216 | 493 | 161 |
| N.S. | 1 | 1.20 | 1.01 | 0.98 | 0.00 | 6.15 | 0.00 | 1.74 | 3.98 | 1.30 |
| time (sec) | N/A | 0.448 | 0.317 | 1.544 | 0.000 | 0.119 | 0.000 | 0.153 | 0.220 | 9.545 |

| Problem 178 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|-------------|---------|-------|-------|-------|--------|--------|-------|-------|--------|-------|
| grade | N/A | A | A | A | F(-2) | B | F | B | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 151 | 185 | 166 | 144 | 0 | 1198 | 0 | 298 | 804 | 198 |
| N.S. | 1 | 1.23 | 1.10 | 0.95 | 0.00 | 7.93 | 0.00 | 1.97 | 5.32 | 1.31 |
| time (sec) | N/A | 0.496 | 0.353 | 1.459 | 0.000 | 0.138 | 0.000 | 0.152 | 0.263 | 9.527 |

| Problem 179 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|-------------|---------|-------|-------|-------|--------|--------|-------|-------|--------|-------|
| grade | N/A | A | A | A | F(-2) | B | F(-1) | B | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 201 | 214 | 277 | 229 | 0 | 994 | 0 | 490 | 1000 | 495 |
| N.S. | 1 | 1.06 | 1.38 | 1.14 | 0.00 | 4.95 | 0.00 | 2.44 | 4.98 | 2.46 |
| time (sec) | N/A | 0.545 | 1.230 | 2.175 | 0.000 | 0.117 | 0.000 | 0.214 | 0.249 | 9.272 |

| Problem 180 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|-------------|---------|-------|-------|-------|--------|--------|-------|-------|--------|-------|
| grade | N/A | A | A | A | F(-2) | B | F(-1) | B | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 172 | 185 | 208 | 178 | 0 | 730 | 0 | 358 | 674 | 388 |
| N.S. | 1 | 1.08 | 1.21 | 1.03 | 0.00 | 4.24 | 0.00 | 2.08 | 3.92 | 2.26 |
| time (sec) | N/A | 0.502 | 1.053 | 1.977 | 0.000 | 0.108 | 0.000 | 0.206 | 0.224 | 9.355 |

| Problem 181 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|-------------|---------|-------|-------|-------|--------|--------|-------|-------|--------|-------|
| grade | N/A | A | A | A | F(-2) | B | F(-1) | B | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 145 | 156 | 160 | 151 | 0 | 498 | 0 | 245 | 383 | 303 |
| N.S. | 1 | 1.08 | 1.10 | 1.04 | 0.00 | 3.43 | 0.00 | 1.69 | 2.64 | 2.09 |
| time (sec) | N/A | 0.472 | 0.746 | 1.931 | 0.000 | 0.089 | 0.000 | 0.182 | 0.212 | 0.190 |

| Problem 182 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|-------------|---------|-------|-------|-------|--------|--------|-------|-------|--------|-------|
| grade | N/A | A | A | A | F(-2) | B | F(-1) | A | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 126 | 134 | 119 | 122 | 0 | 563 | 0 | 161 | 410 | 222 |
| N.S. | 1 | 1.06 | 0.94 | 0.97 | 0.00 | 4.47 | 0.00 | 1.28 | 3.25 | 1.76 |
| time (sec) | N/A | 0.415 | 0.575 | 1.830 | 0.000 | 0.103 | 0.000 | 0.158 | 0.217 | 0.183 |

| Problem 183 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|-------------|---------|-------|-------|-------|--------|--------|-------|-------|--------|-------|
| grade | N/A | A | A | A | F(-2) | B | F(-1) | A | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 136 | 140 | 129 | 119 | 0 | 666 | 0 | 185 | 470 | 209 |
| N.S. | 1 | 1.03 | 0.95 | 0.88 | 0.00 | 4.90 | 0.00 | 1.36 | 3.46 | 1.54 |
| time (sec) | N/A | 0.415 | 0.586 | 1.509 | 0.000 | 0.108 | 0.000 | 0.145 | 0.212 | 9.267 |

| Problem 184 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|-------------|---------|-------|-------|-------|--------|--------|-------|-------|--------|-------|
| grade | N/A | A | A | A | F(-2) | B | F(-1) | A | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 146 | 154 | 131 | 118 | 0 | 785 | 0 | 207 | 536 | 207 |
| N.S. | 1 | 1.05 | 0.90 | 0.81 | 0.00 | 5.38 | 0.00 | 1.42 | 3.67 | 1.42 |
| time (sec) | N/A | 0.464 | 0.637 | 1.457 | 0.000 | 0.112 | 0.000 | 0.158 | 0.239 | 9.482 |

| Problem 185 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|-------------|---------|-------|-------|-------|--------|--------|-------|-------|--------|-------|
| grade | N/A | A | A | A | F(-2) | B | F | A | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 147 | 168 | 128 | 130 | 0 | 884 | 0 | 231 | 597 | 218 |
| N.S. | 1 | 1.14 | 0.87 | 0.88 | 0.00 | 6.01 | 0.00 | 1.57 | 4.06 | 1.48 |
| time (sec) | N/A | 0.476 | 0.316 | 1.467 | 0.000 | 0.137 | 0.000 | 0.128 | 0.238 | 0.167 |

| Problem 186 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|-------------|---------|-------|-------|-------|--------|--------|-------|-------|--------|-------|
| grade | N/A | A | A | A | F(-2) | B | F(-1) | B | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 173 | 205 | 170 | 156 | 0 | 1184 | 0 | 326 | 702 | 294 |
| N.S. | 1 | 1.18 | 0.98 | 0.90 | 0.00 | 6.84 | 0.00 | 1.88 | 4.06 | 1.70 |
| time (sec) | N/A | 0.519 | 0.715 | 1.602 | 0.000 | 0.136 | 0.000 | 0.138 | 0.224 | 9.534 |

| Problem 187 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|-------------|---------|-------|-------|-------|--------|--------|-------|-------|--------|-------|
| grade | N/A | A | A | A | F(-2) | B | F(-1) | B | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 200 | 241 | 220 | 177 | 0 | 1820 | 0 | 432 | 1220 | 334 |
| N.S. | 1 | 1.20 | 1.10 | 0.88 | 0.00 | 9.10 | 0.00 | 2.16 | 6.10 | 1.67 |
| time (sec) | N/A | 0.590 | 1.026 | 1.599 | 0.000 | 0.215 | 0.000 | 0.208 | 0.221 | 9.462 |

| Problem 188 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|-------------|---------|-------|-------|-------|--------|--------|-------|-------|--------|-------|
| grade | N/A | A | A | A | F(-2) | B | F(-1) | B | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 229 | 277 | 285 | 200 | 0 | 2530 | 0 | 480 | 1814 | 373 |
| N.S. | 1 | 1.21 | 1.24 | 0.87 | 0.00 | 11.05 | 0.00 | 2.10 | 7.92 | 1.63 |
| time (sec) | N/A | 0.592 | 1.220 | 1.654 | 0.000 | 0.390 | 0.000 | 0.212 | 0.216 | 9.555 |

| Problem 189 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|-------------|---------|-------|-------|-------|--------|--------|-------|-------|--------|-------|
| grade | N/A | A | A | A | F(-2) | B | F(-1) | B | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 253 | 278 | 434 | 342 | 0 | 1606 | 0 | 784 | 1639 | 846 |
| N.S. | 1 | 1.10 | 1.72 | 1.35 | 0.00 | 6.35 | 0.00 | 3.10 | 6.48 | 3.34 |
| time (sec) | N/A | 0.615 | 2.361 | 3.835 | 0.000 | 0.171 | 0.000 | 0.221 | 0.273 | 9.289 |

| Problem 190 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|-------------|---------|-------|-------|-------|--------|--------|-------|-------|--------|-------|
| grade | N/A | A | A | A | F(-2) | B | F(-1) | B | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 224 | 249 | 351 | 292 | 0 | 1234 | 0 | 611 | 1163 | 711 |
| N.S. | 1 | 1.11 | 1.57 | 1.30 | 0.00 | 5.51 | 0.00 | 2.73 | 5.19 | 3.17 |
| time (sec) | N/A | 0.555 | 2.172 | 3.730 | 0.000 | 0.133 | 0.000 | 0.179 | 0.255 | 0.316 |

| Problem 191 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|-------------|---------|-------|-------|-------|--------|--------|-------|-------|--------|-------|
| grade | N/A | A | A | A | F(-2) | B | F(-1) | B | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 197 | 220 | 273 | 265 | 0 | 890 | 0 | 456 | 726 | 598 |
| N.S. | 1 | 1.12 | 1.39 | 1.35 | 0.00 | 4.52 | 0.00 | 2.31 | 3.69 | 3.04 |
| time (sec) | N/A | 0.532 | 1.947 | 3.714 | 0.000 | 0.121 | 0.000 | 0.214 | 0.235 | 0.332 |

| Problem 192 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|-------------|---------|-------|-------|-------|--------|--------|-------|-------|--------|-------|
| grade | N/A | A | A | A | F(-2) | B | F(-1) | B | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 178 | 198 | 212 | 211 | 0 | 1003 | 0 | 330 | 779 | 480 |
| N.S. | 1 | 1.11 | 1.19 | 1.19 | 0.00 | 5.63 | 0.00 | 1.85 | 4.38 | 2.70 |
| time (sec) | N/A | 0.467 | 1.381 | 3.566 | 0.000 | 0.132 | 0.000 | 0.198 | 0.215 | 9.411 |

| Problem 193 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|-------------|---------|-------|-------|-------|--------|--------|-------|-------|--------|-------|
| grade | N/A | A | A | A | F(-2) | B | F(-1) | B | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 188 | 204 | 223 | 207 | 0 | 1158 | 0 | 354 | 874 | 439 |
| N.S. | 1 | 1.09 | 1.19 | 1.10 | 0.00 | 6.16 | 0.00 | 1.88 | 4.65 | 2.34 |
| time (sec) | N/A | 0.485 | 1.464 | 3.578 | 0.000 | 0.121 | 0.000 | 0.202 | 0.253 | 9.374 |

| Problem 194 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|-------------|---------|-------|-------|-------|--------|--------|-------|-------|--------|-------|
| grade | N/A | A | A | A | F(-2) | B | F(-1) | B | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 198 | 218 | 222 | 205 | 0 | 1337 | 0 | 380 | 973 | 411 |
| N.S. | 1 | 1.10 | 1.12 | 1.04 | 0.00 | 6.75 | 0.00 | 1.92 | 4.91 | 2.08 |
| time (sec) | N/A | 0.531 | 1.376 | 3.561 | 0.000 | 0.142 | 0.000 | 0.182 | 0.221 | 9.528 |

| Problem 195 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|-------------|---------|-------|-------|-------|--------|--------|-------|-------|--------|-------|
| grade | N/A | A | A | A | F(-2) | B | F(-1) | B | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 208 | 232 | 223 | 219 | 0 | 1492 | 0 | 410 | 1068 | 398 |
| N.S. | 1 | 1.12 | 1.07 | 1.05 | 0.00 | 7.17 | 0.00 | 1.97 | 5.13 | 1.91 |
| time (sec) | N/A | 0.543 | 1.770 | 1.599 | 0.000 | 0.143 | 0.000 | 0.197 | 0.230 | 9.526 |

| Problem 196 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|-------------|---------|-------|-------|-------|--------|--------|-------|-------|--------|-------|
| grade | N/A | A | A | A | F(-2) | B | F(-1) | B | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 218 | 246 | 224 | 220 | 0 | 1673 | 0 | 432 | 1167 | 401 |
| N.S. | 1 | 1.13 | 1.03 | 1.01 | 0.00 | 7.67 | 0.00 | 1.98 | 5.35 | 1.84 |
| time (sec) | N/A | 0.560 | 1.557 | 1.647 | 0.000 | 0.152 | 0.000 | 0.136 | 0.255 | 0.208 |

| Problem 197 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|-------------|---------|-------|-------|-------|--------|--------|-------|-------|--------|-------|
| grade | N/A | A | A | A | F(-2) | B | F(-1) | B | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 213 | 260 | 220 | 219 | 0 | 1824 | 0 | 456 | 1262 | 252 |
| N.S. | 1 | 1.22 | 1.03 | 1.03 | 0.00 | 8.56 | 0.00 | 2.14 | 5.92 | 1.18 |
| time (sec) | N/A | 0.555 | 0.693 | 1.538 | 0.000 | 0.139 | 0.000 | 0.159 | 0.238 | 9.317 |

| Problem 198 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|-------------|---------|-------|-------|-------|--------|--------|-------|-------|--------|--------|
| grade | N/A | A | A | A | F(-2) | B | F(-1) | B | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 239 | 297 | 284 | 270 | 0 | 2294 | 0 | 579 | 1381 | 515 |
| N.S. | 1 | 1.24 | 1.19 | 1.13 | 0.00 | 9.60 | 0.00 | 2.42 | 5.78 | 2.15 |
| time (sec) | N/A | 0.616 | 1.846 | 1.643 | 0.000 | 0.414 | 0.000 | 0.227 | 0.225 | 10.009 |

| Problem 199 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|-------------|---------|-------|-------|-------|--------|--------|-------|-------|--------|--------|
| grade | N/A | A | A | A | F(-2) | B | F(-1) | B | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 266 | 333 | 357 | 291 | 0 | 3224 | 0 | 649 | 2113 | 555 |
| N.S. | 1 | 1.25 | 1.34 | 1.09 | 0.00 | 12.12 | 0.00 | 2.44 | 7.94 | 2.09 |
| time (sec) | N/A | 0.647 | 2.432 | 1.711 | 0.000 | 0.754 | 0.000 | 0.144 | 0.229 | 10.156 |

| Problem 200 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|-------------|---------|-------|-------|-------|--------|--------|-------|-------|--------|--------|
| grade | N/A | A | A | A | F(-2) | B | F(-1) | B | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 295 | 369 | 443 | 314 | 0 | 4212 | 0 | 898 | 2993 | 594 |
| N.S. | 1 | 1.25 | 1.50 | 1.06 | 0.00 | 14.28 | 0.00 | 3.04 | 10.15 | 2.01 |
| time (sec) | N/A | 0.674 | 2.791 | 1.839 | 0.000 | 1.587 | 0.000 | 0.168 | 0.267 | 10.165 |

| Problem 201 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|-------------|---------|-------|-------|-------|--------|--------|-------|-------|--------|-------|
| grade | N/A | A | A | A | F(-2) | B | B | A | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 34 | 34 | 27 | 24 | 0 | 74 | 187 | 21 | 45 | 239 |
| N.S. | 1 | 1.00 | 0.79 | 0.71 | 0.00 | 2.18 | 5.50 | 0.62 | 1.32 | 7.03 |
| time (sec) | N/A | 0.268 | 0.009 | 1.043 | 0.000 | 0.123 | 0.227 | 0.149 | 0.239 | 9.628 |

| Problem 202 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|-------------|---------|-------|-------|-------|--------|--------|-------|-------|--------|-------|
| grade | N/A | A | A | A | A | A | B | A | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 34 | 34 | 23 | 24 | 30 | 52 | 107 | 21 | 23 | 34 |
| N.S. | 1 | 1.00 | 0.68 | 0.71 | 0.88 | 1.53 | 3.15 | 0.62 | 0.68 | 1.00 |
| time (sec) | N/A | 0.256 | 0.007 | 0.791 | 0.041 | 0.080 | 0.116 | 0.116 | 0.231 | 9.693 |

| Problem 203 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|-------------|---------|-------|-------|-------|--------|--------|-------|-------|--------|-------|
| grade | N/A | A | A | A | F(-2) | A | A | A | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 28 | 28 | 17 | 16 | 0 | 31 | 37 | 24 | 4 | 15 |
| N.S. | 1 | 1.00 | 0.61 | 0.57 | 0.00 | 1.11 | 1.32 | 0.86 | 0.14 | 0.54 |
| time (sec) | N/A | 0.265 | 0.024 | 1.058 | 0.000 | 0.083 | 0.688 | 0.127 | 0.220 | 9.530 |

| Problem 204 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|-------------|---------|-------|-------|-------|--------|--------|-------|-------|--------|-------|
| grade | N/A | A | A | A | F(-2) | A | F | A | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 30 | 30 | 19 | 18 | 0 | 46 | 0 | 21 | 14 | 34 |
| N.S. | 1 | 1.00 | 0.63 | 0.60 | 0.00 | 1.53 | 0.00 | 0.70 | 0.47 | 1.13 |
| time (sec) | N/A | 0.267 | 0.007 | 1.066 | 0.000 | 0.079 | 0.000 | 0.135 | 0.295 | 9.177 |

| Problem 205 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|-------------|---------|-------|-------|-------|--------|--------|-------|-------|--------|-------|
| grade | N/A | A | A | A | F(-2) | B | F | A | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 34 | 34 | 27 | 24 | 0 | 68 | 0 | 21 | 36 | 34 |
| N.S. | 1 | 1.00 | 0.79 | 0.71 | 0.00 | 2.00 | 0.00 | 0.62 | 1.06 | 1.00 |
| time (sec) | N/A | 0.270 | 0.005 | 1.024 | 0.000 | 0.079 | 0.000 | 0.161 | 0.259 | 8.969 |

| Problem 206 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|-------------|---------|-------|-------|-------|--------|--------|-------|-------|--------|-------|
| grade | N/A | A | A | A | F(-2) | A | B | A | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 38 | 38 | 28 | 24 | 0 | 63 | 160 | 21 | 34 | 76 |
| N.S. | 1 | 1.00 | 0.74 | 0.63 | 0.00 | 1.66 | 4.21 | 0.55 | 0.89 | 2.00 |
| time (sec) | N/A | 0.279 | 0.009 | 0.970 | 0.000 | 0.091 | 0.741 | 0.114 | 0.308 | 9.603 |

| Problem 207 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|-------------|---------|-------|-------|-------|--------|--------|-------|-------|--------|-------|
| grade | N/A | A | A | A | F(-2) | A | B | A | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 36 | 36 | 31 | 40 | 0 | 41 | 71 | 36 | 12 | 33 |
| N.S. | 1 | 1.00 | 0.86 | 1.11 | 0.00 | 1.14 | 1.97 | 1.00 | 0.33 | 0.92 |
| time (sec) | N/A | 0.258 | 0.011 | 0.829 | 0.000 | 0.087 | 0.335 | 0.202 | 0.246 | 9.730 |

| Problem 208 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|-------------|---------|-------|-------|-------|--------|--------|-------|-------|--------|-------|
| grade | N/A | A | A | A | F(-2) | A | F | A | B | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 40 | 41 | 29 | 29 | 0 | 40 | 0 | 20 | 12 | 0 |
| N.S. | 1 | 1.02 | 0.72 | 0.72 | 0.00 | 1.00 | 0.00 | 0.50 | 0.30 | 0.00 |
| time (sec) | N/A | 0.282 | 0.005 | 1.102 | 0.000 | 0.086 | 0.000 | 0.123 | 0.252 | 0.000 |

| Problem 209 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|-------------|---------|-------|-------|-------|--------|--------|-------|-------|--------|-------|
| grade | N/A | A | A | A | F(-2) | A | F | A | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 39 | 38 | 27 | 24 | 0 | 57 | 0 | 21 | 25 | 34 |
| N.S. | 1 | 0.97 | 0.69 | 0.62 | 0.00 | 1.46 | 0.00 | 0.54 | 0.64 | 0.87 |
| time (sec) | N/A | 0.275 | 0.004 | 1.202 | 0.000 | 0.077 | 0.000 | 0.186 | 0.239 | 9.336 |

| Problem 210 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|-------------|---------|-------|-------|-------|--------|--------|-------|-------|--------|-------|
| grade | N/A | A | A | A | F(-2) | B | F | A | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 39 | 38 | 27 | 24 | 0 | 79 | 0 | 21 | 47 | 34 |
| N.S. | 1 | 0.97 | 0.69 | 0.62 | 0.00 | 2.03 | 0.00 | 0.54 | 1.21 | 0.87 |
| time (sec) | N/A | 0.271 | 0.005 | 1.461 | 0.000 | 0.081 | 0.000 | 0.143 | 0.216 | 9.463 |

| Problem 211 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|-------------|---------|-------|-------|-------|--------|--------|-------|-------|--------|-------|
| grade | N/A | A | A | A | F(-2) | B | B | B | B | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 34 | 34 | 27 | 25 | 0 | 103 | 277 | 133 | 68 | 0 |
| N.S. | 1 | 1.00 | 0.79 | 0.74 | 0.00 | 3.03 | 8.15 | 3.91 | 2.00 | 0.00 |
| time (sec) | N/A | 0.263 | 0.012 | 0.948 | 0.000 | 0.092 | 0.242 | 0.155 | 0.209 | 0.000 |

| Problem 212 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|-------------|---------|-------|-------|-------|--------|--------|-------|-------|--------|-------|
| grade | N/A | A | A | A | A | B | B | B | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 34 | 34 | 23 | 25 | 30 | 79 | 194 | 97 | 46 | 34 |
| N.S. | 1 | 1.00 | 0.68 | 0.74 | 0.88 | 2.32 | 5.71 | 2.85 | 1.35 | 1.00 |
| time (sec) | N/A | 0.266 | 0.012 | 0.903 | 0.027 | 0.095 | 0.148 | 0.191 | 0.233 | 9.424 |

| Problem 213 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|-------------|---------|-------|-------|-------|--------|--------|-------|-------|--------|-------|
| grade | N/A | A | A | A | A | B | A | A | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 31 | 31 | 20 | 25 | 27 | 55 | 39 | 51 | 24 | 16 |
| N.S. | 1 | 1.00 | 0.65 | 0.81 | 0.87 | 1.77 | 1.26 | 1.65 | 0.77 | 0.52 |
| time (sec) | N/A | 0.264 | 0.003 | 1.105 | 0.034 | 0.104 | 1.054 | 0.150 | 0.235 | 9.490 |

| Problem 214 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|-------------|---------|-------|-------|-------|--------|--------|-------|-------|--------|-------|
| grade | N/A | A | A | A | F(-2) | A | A | A | B | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 29 | 29 | 18 | 17 | 0 | 32 | 39 | 26 | 5 | 0 |
| N.S. | 1 | 1.00 | 0.62 | 0.59 | 0.00 | 1.10 | 1.34 | 0.90 | 0.17 | 0.00 |
| time (sec) | N/A | 0.264 | 0.026 | 1.083 | 0.000 | 0.089 | 1.312 | 0.162 | 0.228 | 0.000 |

| Problem 215 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|-------------|---------|-------|-------|-------|--------|--------|-------|-------|--------|-------|
| grade | N/A | A | A | A | F(-2) | A | F | A | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 32 | 32 | 25 | 25 | 0 | 47 | 0 | 21 | 15 | 35 |
| N.S. | 1 | 1.00 | 0.78 | 0.78 | 0.00 | 1.47 | 0.00 | 0.66 | 0.47 | 1.09 |
| time (sec) | N/A | 0.261 | 0.009 | 1.108 | 0.000 | 0.090 | 0.000 | 0.158 | 0.225 | 9.145 |

| Problem 216 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|-------------|---------|-------|-------|-------|--------|--------|-------|-------|--------|-------|
| grade | N/A | A | A | A | F(-2) | B | F | A | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 34 | 34 | 27 | 25 | 0 | 69 | 0 | 21 | 37 | 35 |
| N.S. | 1 | 1.00 | 0.79 | 0.74 | 0.00 | 2.03 | 0.00 | 0.62 | 1.09 | 1.03 |
| time (sec) | N/A | 0.260 | 0.006 | 1.135 | 0.000 | 0.087 | 0.000 | 0.177 | 0.233 | 9.668 |

| Problem 217 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|-------------|---------|-------|-------|-------|--------|--------|-------|-------|--------|-------|
| grade | N/A | A | A | A | F(-2) | B | B | B | B | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 39 | 39 | 28 | 25 | 0 | 91 | 197 | 90 | 57 | 0 |
| N.S. | 1 | 1.00 | 0.72 | 0.64 | 0.00 | 2.33 | 5.05 | 2.31 | 1.46 | 0.00 |
| time (sec) | N/A | 0.275 | 0.013 | 1.001 | 0.000 | 0.091 | 0.780 | 0.175 | 0.211 | 0.000 |

| Problem 218 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|-------------|---------|-------|-------|-------|--------|--------|-------|-------|--------|-------|
| grade | N/A | A | A | A | F(-2) | A | B | A | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 39 | 39 | 25 | 25 | 0 | 67 | 366 | 63 | 35 | 36 |
| N.S. | 1 | 1.00 | 0.64 | 0.64 | 0.00 | 1.72 | 9.38 | 1.62 | 0.90 | 0.92 |
| time (sec) | N/A | 0.258 | 0.001 | 0.872 | 0.000 | 0.091 | 0.797 | 0.145 | 0.210 | 0.040 |

| Problem 219 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|-------------|---------|-------|-------|-------|--------|--------|-------|-------|--------|-------|
| grade | N/A | A | A | A | F(-2) | A | A | A | B | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 37 | 37 | 33 | 40 | 0 | 43 | 73 | 43 | 13 | 0 |
| N.S. | 1 | 1.00 | 0.89 | 1.08 | 0.00 | 1.16 | 1.97 | 1.16 | 0.35 | 0.00 |
| time (sec) | N/A | 0.269 | 0.002 | 1.266 | 0.000 | 0.080 | 0.985 | 0.201 | 0.243 | 0.000 |

| Problem 220 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|-------------|---------|-------|-------|-------|--------|--------|-------|-------|--------|-------|
| grade | N/A | A | A | A | F(-2) | A | F | A | B | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 42 | 42 | 31 | 30 | 0 | 41 | 0 | 20 | 13 | 0 |
| N.S. | 1 | 1.00 | 0.74 | 0.71 | 0.00 | 0.98 | 0.00 | 0.48 | 0.31 | 0.00 |
| time (sec) | N/A | 0.278 | 0.004 | 1.286 | 0.000 | 0.091 | 0.000 | 0.139 | 0.242 | 0.000 |

| Problem 221 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|-------------|---------|-------|-------|-------|--------|--------|-------|-------|--------|-------|
| grade | N/A | A | A | A | F(-2) | A | F | A | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 41 | 39 | 27 | 25 | 0 | 58 | 0 | 21 | 26 | 35 |
| N.S. | 1 | 0.95 | 0.66 | 0.61 | 0.00 | 1.41 | 0.00 | 0.51 | 0.63 | 0.85 |
| time (sec) | N/A | 0.272 | 0.006 | 1.582 | 0.000 | 0.112 | 0.000 | 0.164 | 0.215 | 9.442 |

| Problem 222 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|-------------|---------|-------|-------|-------|--------|--------|-------|-------|--------|-------|
| grade | N/A | A | A | A | F(-2) | B | F | A | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 41 | 39 | 27 | 25 | 0 | 80 | 0 | 21 | 48 | 35 |
| N.S. | 1 | 0.95 | 0.66 | 0.61 | 0.00 | 1.95 | 0.00 | 0.51 | 1.17 | 0.85 |
| time (sec) | N/A | 0.276 | 0.006 | 1.918 | 0.000 | 0.090 | 0.000 | 0.174 | 0.214 | 9.337 |

| Problem 223 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|-------------|---------|-------|-------|-------|--------|--------|-------|-------|--------|-------|
| grade | N/A | A | A | A | A | B | B | B | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 34 | 34 | 23 | 27 | 30 | 117 | 287 | 149 | 70 | 19 |
| N.S. | 1 | 1.00 | 0.68 | 0.79 | 0.88 | 3.44 | 8.44 | 4.38 | 2.06 | 0.56 |
| time (sec) | N/A | 0.259 | 0.015 | 0.895 | 0.040 | 0.103 | 0.296 | 0.182 | 0.234 | 9.078 |

| Problem 224 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|-------------|---------|-------|-------|-------|--------|--------|-------|-------|--------|-------|
| grade | N/A | A | A | A | A | B | A | B | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 31 | 31 | 20 | 27 | 27 | 89 | 39 | 87 | 48 | 16 |
| N.S. | 1 | 1.00 | 0.65 | 0.87 | 0.87 | 2.87 | 1.26 | 2.81 | 1.55 | 0.52 |
| time (sec) | N/A | 0.262 | 0.004 | 1.142 | 0.034 | 0.094 | 1.527 | 0.191 | 0.201 | 9.308 |

| Problem 225 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|-------------|---------|-------|-------|-------|--------|--------|-------|-------|--------|-------|
| grade | N/A | A | A | A | F(-2) | B | B | B | B | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 32 | 32 | 21 | 27 | 0 | 61 | 201 | 69 | 26 | 0 |
| N.S. | 1 | 1.00 | 0.66 | 0.84 | 0.00 | 1.91 | 6.28 | 2.16 | 0.81 | 0.00 |
| time (sec) | N/A | 0.276 | 0.002 | 1.158 | 0.000 | 0.095 | 1.851 | 0.191 | 0.205 | 0.000 |

| Problem 226 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|-------------|---------|-------|-------|-------|--------|--------|-------|-------|--------|-------|
| grade | N/A | A | A | A | F(-2) | A | A | A | B | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 31 | 31 | 20 | 19 | 0 | 34 | 41 | 30 | 7 | 0 |
| N.S. | 1 | 1.00 | 0.65 | 0.61 | 0.00 | 1.10 | 1.32 | 0.97 | 0.23 | 0.00 |
| time (sec) | N/A | 0.282 | 0.004 | 1.335 | 0.000 | 0.094 | 2.947 | 0.161 | 0.220 | 0.000 |

| Problem 227 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|-------------|---------|-------|-------|-------|--------|--------|-------|-------|--------|-------|
| grade | N/A | A | A | A | F(-2) | A | F | A | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 32 | 32 | 25 | 27 | 0 | 49 | 0 | 21 | 17 | 37 |
| N.S. | 1 | 1.00 | 0.78 | 0.84 | 0.00 | 1.53 | 0.00 | 0.66 | 0.53 | 1.16 |
| time (sec) | N/A | 0.263 | 0.008 | 1.729 | 0.000 | 0.105 | 0.000 | 0.156 | 0.277 | 8.953 |

| Problem 228 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|-------------|---------|-------|-------|-------|--------|--------|-------|-------|--------|-------|
| grade | N/A | A | A | A | F(-2) | B | F | A | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 34 | 34 | 27 | 27 | 0 | 71 | 0 | 21 | 39 | 37 |
| N.S. | 1 | 1.00 | 0.79 | 0.79 | 0.00 | 2.09 | 0.00 | 0.62 | 1.15 | 1.09 |
| time (sec) | N/A | 0.261 | 0.008 | 2.937 | 0.000 | 0.082 | 0.000 | 0.122 | 0.217 | 8.837 |

| Problem 229 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|-------------|---------|-------|-------|-------|--------|--------|-------|-------|--------|-------|
| grade | N/A | A | A | A | F(-2) | B | B | B | B | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 41 | 41 | 28 | 27 | 0 | 131 | 241 | 127 | 81 | 0 |
| N.S. | 1 | 1.00 | 0.68 | 0.66 | 0.00 | 3.20 | 5.88 | 3.10 | 1.98 | 0.00 |
| time (sec) | N/A | 0.281 | 0.012 | 1.060 | 0.000 | 0.096 | 0.888 | 0.195 | 0.204 | 0.000 |

| Problem 230 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|-------------|---------|-------|-------|-------|--------|--------|-------|-------|--------|-------|
| grade | N/A | A | A | A | F(-2) | B | B | B | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 41 | 41 | 25 | 27 | 0 | 103 | 828 | 98 | 59 | 36 |
| N.S. | 1 | 1.00 | 0.61 | 0.66 | 0.00 | 2.51 | 20.20 | 2.39 | 1.44 | 0.88 |
| time (sec) | N/A | 0.260 | 0.002 | 0.981 | 0.000 | 0.092 | 1.415 | 0.198 | 0.225 | 8.801 |

| Problem 231 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|-------------|---------|-------|-------|-------|--------|--------|-------|-------|--------|-------|
| grade | N/A | A | A | A | A | B | B | B | B | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 41 | 41 | 26 | 27 | 35 | 75 | 371 | 89 | 37 | 0 |
| N.S. | 1 | 1.00 | 0.63 | 0.66 | 0.85 | 1.83 | 9.05 | 2.17 | 0.90 | 0.00 |
| time (sec) | N/A | 0.273 | 0.005 | 1.296 | 0.043 | 0.099 | 1.543 | 0.169 | 0.249 | 0.000 |

| Problem 232 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|-------------|---------|-------|-------|-------|--------------|--------|-------|-------|--------|--------------|
| grade | N/A | A | A | A | F(-2) | A | A | A | B | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 39 | 39 | 33 | 40 | 0 | 47 | 75 | 49 | 15 | 0 |
| N.S. | 1 | 1.00 | 0.85 | 1.03 | 0.00 | 1.21 | 1.92 | 1.26 | 0.38 | 0.00 |
| time (sec) | N/A | 0.266 | 0.002 | 1.480 | 0.000 | 0.086 | 2.025 | 0.214 | 0.223 | 0.000 |

| Problem 233 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|-------------|---------|-------|-------|-------|--------------|--------|----------|-------|--------|--------------|
| grade | N/A | A | A | A | F(-2) | A | F | A | B | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 42 | 44 | 31 | 32 | 0 | 43 | 0 | 20 | 15 | 0 |
| N.S. | 1 | 1.05 | 0.74 | 0.76 | 0.00 | 1.02 | 0.00 | 0.48 | 0.36 | 0.00 |
| time (sec) | N/A | 0.279 | 0.004 | 1.891 | 0.000 | 0.087 | 0.000 | 0.219 | 0.215 | 0.000 |

| Problem 234 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|-------------|---------|-------|-------|-------|--------------|--------|----------|-------|--------|-------|
| grade | N/A | A | A | A | F(-2) | A | F | A | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 41 | 41 | 27 | 27 | 0 | 60 | 0 | 21 | 28 | 37 |
| N.S. | 1 | 1.00 | 0.66 | 0.66 | 0.00 | 1.46 | 0.00 | 0.51 | 0.68 | 0.90 |
| time (sec) | N/A | 0.268 | 0.007 | 2.929 | 0.000 | 0.092 | 0.000 | 1.035 | 0.234 | 8.775 |

| Problem 235 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|-------------|---------|-------|-------|-------|--------|--------|-------|--------------|--------|--------------|
| grade | N/A | A | A | A | B | B | B | F(-2) | B | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 34 | 34 | 27 | 24 | 218 | 77 | 196 | 0 | 48 | 0 |
| N.S. | 1 | 1.00 | 0.79 | 0.71 | 6.41 | 2.26 | 5.76 | 0.00 | 1.41 | 0.00 |
| time (sec) | N/A | 0.253 | 0.007 | 1.207 | 0.038 | 0.080 | 0.262 | 0.000 | 0.251 | 0.000 |

| Problem 236 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|-------------|---------|-------|-------|-------|--------|--------|-------|-------|--------|-------|
| grade | N/A | A | A | A | B | A | B | F(-2) | B | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 34 | 34 | 27 | 24 | 146 | 55 | 114 | 0 | 26 | 0 |
| N.S. | 1 | 1.00 | 0.79 | 0.71 | 4.29 | 1.62 | 3.35 | 0.00 | 0.76 | 0.00 |
| time (sec) | N/A | 0.258 | 0.007 | 1.148 | 0.034 | 0.090 | 0.190 | 0.000 | 0.219 | 0.000 |

| Problem 237 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|-------------|---------|-------|-------|-------|--------|--------|-------|-------|--------|-------|
| grade | N/A | A | A | A | A | A | A | F(-2) | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 31 | 31 | 20 | 19 | 29 | 34 | 39 | 0 | 7 | 109 |
| N.S. | 1 | 1.00 | 0.65 | 0.61 | 0.94 | 1.10 | 1.26 | 0.00 | 0.23 | 3.52 |
| time (sec) | N/A | 0.246 | 0.002 | 1.138 | 0.028 | 0.090 | 0.148 | 0.000 | 0.218 | 9.110 |

| Problem 238 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|-------------|---------|-------|-------|-------|--------|--------|-------|-------|--------|-------|
| grade | N/A | A | A | A | A | A | A | F(-2) | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 29 | 29 | 18 | 17 | 19 | 49 | 41 | 0 | 17 | 27 |
| N.S. | 1 | 1.00 | 0.62 | 0.59 | 0.66 | 1.69 | 1.41 | 0.00 | 0.59 | 0.93 |
| time (sec) | N/A | 0.256 | 0.012 | 1.316 | 0.035 | 0.103 | 1.102 | 0.000 | 0.208 | 8.747 |

| Problem 239 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|-------------|---------|-------|-------|-------|--------|--------|-------|-------|--------|-------|
| grade | N/A | A | A | A | A | B | F | F(-2) | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 32 | 32 | 21 | 24 | 47 | 73 | 0 | 0 | 39 | 37 |
| N.S. | 1 | 1.00 | 0.66 | 0.75 | 1.47 | 2.28 | 0.00 | 0.00 | 1.22 | 1.16 |
| time (sec) | N/A | 0.256 | 0.012 | 1.309 | 0.028 | 0.091 | 0.000 | 0.000 | 0.230 | 8.786 |

| Problem 240 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|-------------|---------|-------|-------|-------|--------|--------|-------|-------|--------|-------|
| grade | N/A | A | A | A | B | A | B | F(-2) | B | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 41 | 38 | 27 | 24 | 182 | 66 | 219 | 0 | 37 | 0 |
| N.S. | 1 | 0.93 | 0.66 | 0.59 | 4.44 | 1.61 | 5.34 | 0.00 | 0.90 | 0.00 |
| time (sec) | N/A | 0.270 | 0.005 | 1.358 | 0.042 | 0.096 | 0.826 | 0.000 | 0.246 | 0.000 |

| Problem 241 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|-------------|---------|-------|-------|-------|--------|--------|-------|-------|--------|-------|
| grade | N/A | A | A | A | B | A | B | F(-2) | B | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 39 | 38 | 30 | 38 | 109 | 44 | 138 | 0 | 15 | 0 |
| N.S. | 1 | 0.97 | 0.77 | 0.97 | 2.79 | 1.13 | 3.54 | 0.00 | 0.38 | 0.00 |
| time (sec) | N/A | 0.277 | 0.002 | 1.264 | 0.038 | 0.090 | 0.717 | 0.000 | 0.231 | 0.000 |

| Problem 242 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|-------------|---------|-------|-------|-------|--------|--------|-------|-------|--------|-------|
| grade | N/A | A | A | A | A | A | A | A | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 39 | 39 | 28 | 27 | 18 | 42 | 26 | 34 | 15 | 29 |
| N.S. | 1 | 1.00 | 0.72 | 0.69 | 0.46 | 1.08 | 0.67 | 0.87 | 0.38 | 0.74 |
| time (sec) | N/A | 0.261 | 0.001 | 1.098 | 0.050 | 0.084 | 0.324 | 0.469 | 0.243 | 8.820 |

| Problem 243 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|-------------|---------|-------|-------|-------|--------|--------|-------|-------|--------|-------|
| grade | N/A | A | A | A | A | A | F | F(-2) | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 38 | 38 | 26 | 24 | 33 | 61 | 0 | 0 | 28 | 37 |
| N.S. | 1 | 1.00 | 0.68 | 0.63 | 0.87 | 1.61 | 0.00 | 0.00 | 0.74 | 0.97 |
| time (sec) | N/A | 0.271 | 0.006 | 1.472 | 0.039 | 0.088 | 0.000 | 0.000 | 0.218 | 9.160 |

| Problem 244 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|-------------|---------|-------|-------|-------|--------|--------|-------|-------|--------|-------|
| grade | N/A | A | A | A | A | B | F | F(-2) | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 38 | 38 | 27 | 24 | 61 | 85 | 0 | 0 | 50 | 37 |
| N.S. | 1 | 1.00 | 0.71 | 0.63 | 1.61 | 2.24 | 0.00 | 0.00 | 1.32 | 0.97 |
| time (sec) | N/A | 0.273 | 0.007 | 1.662 | 0.038 | 0.082 | 0.000 | 0.000 | 0.204 | 8.916 |

| Problem 245 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|-------------|---------|-------|-------|-------|--------|--------|-------|-------|--------|-------|
| grade | N/A | A | A | A | B | A | B | F(-2) | B | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 34 | 34 | 27 | 27 | 135 | 59 | 119 | 0 | 26 | 0 |
| N.S. | 1 | 1.00 | 0.79 | 0.79 | 3.97 | 1.74 | 3.50 | 0.00 | 0.76 | 0.00 |
| time (sec) | N/A | 0.248 | 0.006 | 1.287 | 0.037 | 0.099 | 0.274 | 0.000 | 0.211 | 0.000 |

| Problem 246 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|-------------|---------|-------|-------|-------|--------|--------|-------|-------|--------|-------|
| grade | N/A | A | A | A | B | A | A | F(-2) | B | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 31 | 31 | 20 | 19 | 64 | 38 | 42 | 0 | 7 | 0 |
| N.S. | 1 | 1.00 | 0.65 | 0.61 | 2.06 | 1.23 | 1.35 | 0.00 | 0.23 | 0.00 |
| time (sec) | N/A | 0.245 | 0.025 | 1.254 | 0.036 | 0.096 | 0.204 | 0.000 | 0.251 | 0.000 |

| Problem 247 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|-------------|---------|-------|-------|-------|--------|--------|-------|-------|--------|-------|
| grade | N/A | A | A | A | A | A | A | F(-2) | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 32 | 32 | 21 | 20 | 30 | 55 | 42 | 0 | 17 | 37 |
| N.S. | 1 | 1.00 | 0.66 | 0.62 | 0.94 | 1.72 | 1.31 | 0.00 | 0.53 | 1.16 |
| time (sec) | N/A | 0.246 | 0.004 | 1.057 | 0.030 | 0.082 | 0.186 | 0.000 | 0.251 | 8.795 |

| Problem 248 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|-------------|---------|-------|-------|-------|--------|--------|-------|-------|--------|-------|
| grade | N/A | A | A | A | A | B | A | F(-2) | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 31 | 31 | 20 | 27 | 47 | 83 | 42 | 0 | 39 | 37 |
| N.S. | 1 | 1.00 | 0.65 | 0.87 | 1.52 | 2.68 | 1.35 | 0.00 | 1.26 | 1.19 |
| time (sec) | N/A | 0.253 | 0.013 | 1.366 | 0.026 | 0.084 | 1.575 | 0.000 | 0.204 | 8.856 |

| Problem 249 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|-------------|---------|-------|-------|-------|--------|--------|-------|-------|--------|-------|
| grade | N/A | A | A | A | B | B | F | F(-2) | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 32 | 32 | 21 | 27 | 75 | 111 | 0 | 0 | 61 | 37 |
| N.S. | 1 | 1.00 | 0.66 | 0.84 | 2.34 | 3.47 | 0.00 | 0.00 | 1.91 | 1.16 |
| time (sec) | N/A | 0.252 | 0.018 | 1.409 | 0.039 | 0.082 | 0.000 | 0.000 | 0.207 | 8.930 |

| Problem 250 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|-------------|---------|-------|-------|-------|--------|--------|-------|-------|--------|-------|
| grade | N/A | A | A | A | B | A | A | F(-2) | B | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 39 | 41 | 33 | 40 | 99 | 48 | 223 | 0 | 15 | 0 |
| N.S. | 1 | 1.05 | 0.85 | 1.03 | 2.54 | 1.23 | 5.72 | 0.00 | 0.38 | 0.00 |
| time (sec) | N/A | 0.279 | 0.004 | 1.480 | 0.039 | 0.087 | 2.350 | 0.000 | 0.260 | 0.000 |

| Problem 251 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|-------------|---------|-------|-------|-------|--------|--------|-------|-------|--------|-------|
| grade | N/A | A | A | A | B | A | A | F(-2) | B | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 42 | 42 | 31 | 30 | 123 | 46 | 27 | 0 | 15 | 0 |
| N.S. | 1 | 1.00 | 0.74 | 0.71 | 2.93 | 1.10 | 0.64 | 0.00 | 0.36 | 0.00 |
| time (sec) | N/A | 0.270 | 0.004 | 1.285 | 0.036 | 0.089 | 1.668 | 0.000 | 0.231 | 0.000 |

| Problem 252 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|-------------|---------|-------|-------|-------|--------|--------|-------|-------|--------|-------|
| grade | N/A | A | A | A | A | A | F | F(-2) | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 41 | 41 | 25 | 27 | 18 | 69 | 0 | 0 | 28 | 37 |
| N.S. | 1 | 1.00 | 0.61 | 0.66 | 0.44 | 1.68 | 0.00 | 0.00 | 0.68 | 0.90 |
| time (sec) | N/A | 0.249 | 0.001 | 1.143 | 0.031 | 0.071 | 0.000 | 0.000 | 0.233 | 9.258 |

| Problem 253 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|-------------|---------|-------|-------|-------|--------|--------|-------|-------|--------|-------|
| grade | N/A | A | A | A | A | B | F | F(-2) | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 41 | 41 | 26 | 27 | 61 | 97 | 0 | 0 | 50 | 37 |
| N.S. | 1 | 1.00 | 0.63 | 0.66 | 1.49 | 2.37 | 0.00 | 0.00 | 1.22 | 0.90 |
| time (sec) | N/A | 0.272 | 0.013 | 1.504 | 0.031 | 0.081 | 0.000 | 0.000 | 0.208 | 9.257 |

| Problem 254 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|-------------|---------|-------|-------|-------|--------|--------|-------|-------|--------|-------|
| grade | N/A | A | A | A | B | A | B | F(-2) | B | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 34 | 34 | 23 | 27 | 268 | 59 | 119 | 0 | 26 | 0 |
| N.S. | 1 | 1.00 | 0.68 | 0.79 | 7.88 | 1.74 | 3.50 | 0.00 | 0.76 | 0.00 |
| time (sec) | N/A | 0.256 | 0.006 | 1.233 | 0.062 | 0.090 | 0.399 | 0.000 | 0.249 | 0.000 |

| Problem 255 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|-------------|---------|-------|-------|-------|--------|--------|-------|-------|--------|-------|
| grade | N/A | A | A | A | B | A | A | F(-2) | B | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 31 | 31 | 20 | 22 | 197 | 38 | 42 | 0 | 7 | 0 |
| N.S. | 1 | 1.00 | 0.65 | 0.71 | 6.35 | 1.23 | 1.35 | 0.00 | 0.23 | 0.00 |
| time (sec) | N/A | 0.247 | 0.002 | 1.214 | 0.067 | 0.074 | 0.304 | 0.000 | 0.260 | 0.000 |

| Problem 256 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|-------------|---------|-------|-------|-------|--------|--------|-------|-------|--------|-------|
| grade | N/A | A | A | A | B | A | B | F(-2) | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 32 | 32 | 21 | 20 | 159 | 55 | 70 | 0 | 17 | 37 |
| N.S. | 1 | 1.00 | 0.66 | 0.62 | 4.97 | 1.72 | 2.19 | 0.00 | 0.53 | 1.16 |
| time (sec) | N/A | 0.245 | 0.009 | 1.173 | 0.040 | 0.078 | 0.289 | 0.000 | 0.249 | 9.197 |

| Problem 257 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|-------------|---------|-------|-------|-------|--------|--------|-------|-------|--------|-------|
| grade | N/A | A | A | A | A | B | B | F(-2) | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 34 | 34 | 30 | 27 | 30 | 83 | 124 | 0 | 39 | 37 |
| N.S. | 1 | 1.00 | 0.88 | 0.79 | 0.88 | 2.44 | 3.65 | 0.00 | 1.15 | 1.09 |
| time (sec) | N/A | 0.234 | 0.009 | 1.048 | 0.032 | 0.073 | 0.397 | 0.000 | 0.312 | 9.204 |

| Problem 258 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|-------------|---------|-------|-------|-------|--------|--------|-------|-------|--------|-------|
| grade | N/A | A | A | A | B | B | A | F(-2) | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 31 | 31 | 20 | 27 | 75 | 111 | 42 | 0 | 61 | 37 |
| N.S. | 1 | 1.00 | 0.65 | 0.87 | 2.42 | 3.58 | 1.35 | 0.00 | 1.97 | 1.19 |
| time (sec) | N/A | 0.246 | 0.019 | 1.383 | 0.028 | 0.082 | 2.127 | 0.000 | 0.285 | 9.214 |

| Problem 259 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|-------------|---------|-------|-------|-------|--------|--------|-------|-------|--------|-------|
| grade | N/A | A | A | A | B | A | A | F(-2) | B | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 39 | 41 | 33 | 40 | 232 | 48 | 224 | 0 | 15 | 0 |
| N.S. | 1 | 1.05 | 0.85 | 1.03 | 5.95 | 1.23 | 5.74 | 0.00 | 0.38 | 0.00 |
| time (sec) | N/A | 0.267 | 0.003 | 1.700 | 0.052 | 0.082 | 4.256 | 0.000 | 0.203 | 0.000 |

| Problem 260 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|-------------|---------|-------|-------|-------|--------|--------|-------|-------|--------|-------|
| grade | N/A | A | A | A | B | A | A | F(-2) | B | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 42 | 42 | 31 | 30 | 449 | 46 | 29 | 0 | 15 | 0 |
| N.S. | 1 | 1.00 | 0.74 | 0.71 | 10.69 | 1.10 | 0.69 | 0.00 | 0.36 | 0.00 |
| time (sec) | N/A | 0.269 | 0.002 | 1.379 | 0.064 | 0.077 | 3.184 | 0.000 | 0.241 | 0.000 |

| Problem 261 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|-------------|---------|-------|-------|-------|--------|--------|-------|-------|--------|-------|
| grade | N/A | A | A | A | B | A | F | F(-2) | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 41 | 41 | 28 | 27 | 124 | 69 | 0 | 0 | 28 | 37 |
| N.S. | 1 | 1.00 | 0.68 | 0.66 | 3.02 | 1.68 | 0.00 | 0.00 | 0.68 | 0.90 |
| time (sec) | N/A | 0.269 | 0.005 | 1.302 | 0.033 | 0.078 | 0.000 | 0.000 | 0.201 | 9.553 |

| Problem 262 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|-------------|---------|-------|-------|-------|--------|--------|-------|-------|--------|-------|
| grade | N/A | A | A | A | A | B | F | F(-2) | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 41 | 41 | 25 | 27 | 18 | 97 | 0 | 0 | 50 | 37 |
| N.S. | 1 | 1.00 | 0.61 | 0.66 | 0.44 | 2.37 | 0.00 | 0.00 | 1.22 | 0.90 |
| time (sec) | N/A | 0.241 | 0.001 | 1.095 | 0.031 | 0.080 | 0.000 | 0.000 | 0.224 | 9.337 |

| Problem 263 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|-------------|---------|-------|-------|-------|--------|--------|-------|-------|--------|-------|
| grade | N/A | A | A | A | B | B | F | F(-2) | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 41 | 41 | 26 | 27 | 89 | 125 | 0 | 0 | 72 | 37 |
| N.S. | 1 | 1.00 | 0.63 | 0.66 | 2.17 | 3.05 | 0.00 | 0.00 | 1.76 | 0.90 |
| time (sec) | N/A | 0.266 | 0.011 | 1.437 | 0.038 | 0.083 | 0.000 | 0.000 | 0.203 | 9.064 |

| Problem 264 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|-------------|---------|-------|-------|-------|--------|--------|-------|-------|--------|-------|
| grade | N/A | A | A | A | A | A | A | F(-2) | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 27 | 27 | 21 | 20 | 33 | 49 | 56 | 0 | 20 | 25 |
| N.S. | 1 | 1.00 | 0.78 | 0.74 | 1.22 | 1.81 | 2.07 | 0.00 | 0.74 | 0.93 |
| time (sec) | N/A | 0.254 | 0.012 | 1.501 | 0.033 | 0.079 | 1.341 | 0.000 | 0.249 | 9.039 |

| Problem 265 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|-------------|---------|-------|-------|-------|--------|--------|-------|-------|--------|-------|
| grade | N/A | A | A | A | B | A | A | B | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 92 | 66 | 111 | 114 | 590 | 96 | 666 | 223 | 97 | 580 |
| N.S. | 1 | 0.72 | 1.21 | 1.24 | 6.41 | 1.04 | 7.24 | 2.42 | 1.05 | 6.30 |
| time (sec) | N/A | 0.354 | 1.041 | 0.915 | 0.042 | 0.069 | 1.997 | 0.160 | 0.239 | 9.771 |

| Problem 266 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|-------------|---------|-------|-------|-------|--------|--------|-------|-------|--------|-------|
| grade | N/A | A | A | A | B | A | A | B | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 92 | 66 | 89 | 90 | 399 | 69 | 464 | 173 | 73 | 377 |
| N.S. | 1 | 0.72 | 0.97 | 0.98 | 4.34 | 0.75 | 5.04 | 1.88 | 0.79 | 4.10 |
| time (sec) | N/A | 0.361 | 1.031 | 0.777 | 0.040 | 0.076 | 1.486 | 0.131 | 0.237 | 9.070 |

| Problem 267 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|-------------|---------|-------|-------|-------|--------|--------|-------|-------|--------|-------|
| grade | N/A | A | A | A | B | A | A | A | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 92 | 66 | 67 | 66 | 245 | 48 | 294 | 123 | 49 | 215 |
| N.S. | 1 | 0.72 | 0.73 | 0.72 | 2.66 | 0.52 | 3.20 | 1.34 | 0.53 | 2.34 |
| time (sec) | N/A | 0.375 | 1.027 | 0.737 | 0.047 | 0.071 | 1.200 | 0.141 | 0.202 | 9.311 |

| Problem 268 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|-------------|---------|-------|-------|-------|--------|--------|-------|-------|--------|-------|
| grade | N/A | A | A | C | B | A | A | A | B | B |
| verified | N/A | Yes | Yes | No | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 69 | 69 | 91 | 32 | 125 | 24 | 156 | 75 | 25 | 77 |
| N.S. | 1 | 1.00 | 1.32 | 0.46 | 1.81 | 0.35 | 2.26 | 1.09 | 0.36 | 1.12 |
| time (sec) | N/A | 0.322 | 0.687 | 0.612 | 0.033 | 0.080 | 0.958 | 0.146 | 0.220 | 9.001 |

| Problem 269 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|-------------|---------|-------|-------|-------|--------|--------|-------|-------|--------|-------|
| grade | N/A | A | A | C | B | A | B | A | B | B |
| verified | N/A | Yes | Yes | No | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 32 | 32 | 30 | 19 | 46 | 10 | 56 | 33 | 10 | 19 |
| N.S. | 1 | 1.00 | 0.94 | 0.59 | 1.44 | 0.31 | 1.75 | 1.03 | 0.31 | 0.59 |
| time (sec) | N/A | 0.265 | 0.008 | 0.543 | 0.028 | 0.073 | 0.465 | 0.140 | 0.231 | 8.875 |

| Problem 270 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|-------------|---------|-------|-------|-------|--------|--------|-------|-------|--------|-------|
| grade | N/A | A | A | C | F(-2) | A | F | A | B | F(-1) |
| verified | N/A | Yes | Yes | No | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 80 | 54 | 42 | 48 | 0 | 25 | 0 | 45 | 28 | 0 |
| N.S. | 1 | 0.68 | 0.52 | 0.60 | 0.00 | 0.31 | 0.00 | 0.56 | 0.35 | 0.00 |
| time (sec) | N/A | 0.359 | 1.017 | 0.879 | 0.000 | 0.080 | 0.000 | 0.122 | 0.227 | 0.000 |

| Problem 271 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|-------------|---------|-------|-------|-------|--------|--------|-------|-------|--------|-------|
| grade | N/A | A | B | C | F(-2) | A | F | A | B | F(-1) |
| verified | N/A | Yes | Yes | No | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 85 | 59 | 245 | 55 | 0 | 37 | 0 | 50 | 49 | 0 |
| N.S. | 1 | 0.69 | 2.88 | 0.65 | 0.00 | 0.44 | 0.00 | 0.59 | 0.58 | 0.00 |
| time (sec) | N/A | 0.362 | 0.793 | 0.962 | 0.000 | 0.082 | 0.000 | 0.142 | 0.232 | 0.000 |

| Problem 272 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|-------------|---------|-------|-------|-----------|--------------|--------|----------|-------|--------|-------|
| grade | N/A | A | A | C | F(-2) | A | F | B | B | B |
| verified | N/A | Yes | Yes | No | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 46 | 46 | 37 | 31 | 0 | 38 | 0 | 69 | 37 | 40 |
| N.S. | 1 | 1.00 | 0.80 | 0.67 | 0.00 | 0.83 | 0.00 | 1.50 | 0.80 | 0.87 |
| time (sec) | N/A | 0.291 | 0.512 | 1.150 | 0.000 | 0.075 | 0.000 | 0.151 | 0.231 | 8.924 |

| Problem 273 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|-------------|---------|-------|-------|-----------|--------------|--------|----------|-------|--------|-------|
| grade | N/A | A | A | C | F(-2) | A | F | A | B | B |
| verified | N/A | Yes | Yes | No | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 92 | 66 | 45 | 32 | 0 | 50 | 0 | 83 | 48 | 41 |
| N.S. | 1 | 0.72 | 0.49 | 0.35 | 0.00 | 0.54 | 0.00 | 0.90 | 0.52 | 0.45 |
| time (sec) | N/A | 0.373 | 0.875 | 1.300 | 0.000 | 0.088 | 0.000 | 0.136 | 0.252 | 9.031 |

| Problem 274 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|-------------|---------|-------|-------|-----------|--------------|--------|----------|-------|--------|-------|
| grade | N/A | A | A | C | F(-2) | A | F | A | B | B |
| verified | N/A | Yes | Yes | No | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 92 | 66 | 45 | 32 | 0 | 61 | 0 | 98 | 59 | 41 |
| N.S. | 1 | 0.72 | 0.49 | 0.35 | 0.00 | 0.66 | 0.00 | 1.07 | 0.64 | 0.45 |
| time (sec) | N/A | 0.366 | 1.020 | 1.381 | 0.000 | 0.086 | 0.000 | 0.152 | 0.215 | 9.095 |

| Problem 275 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|-------------|---------|-------|-------|-----------|--------------|--------|----------|-------|--------|-------|
| grade | N/A | A | A | C | F(-2) | A | F | A | B | B |
| verified | N/A | Yes | Yes | No | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 92 | 66 | 45 | 32 | 0 | 72 | 0 | 111 | 70 | 41 |
| N.S. | 1 | 0.72 | 0.49 | 0.35 | 0.00 | 0.78 | 0.00 | 1.21 | 0.76 | 0.45 |
| time (sec) | N/A | 0.374 | 1.019 | 1.645 | 0.000 | 0.093 | 0.000 | 0.133 | 0.218 | 9.440 |

| Problem 276 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|-------------|---------|-------|-------|-------|--------|--------|-------|-------|--------|--------------|
| grade | N/A | A | A | B | B | A | B | B | B | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 200 | 120 | 259 | 322 | 814 | 277 | 10178 | 528 | 305 | 0 |
| N.S. | 1 | 0.60 | 1.30 | 1.61 | 4.07 | 1.38 | 50.89 | 2.64 | 1.52 | 0.00 |
| time (sec) | N/A | 0.611 | 1.073 | 1.451 | 0.053 | 0.086 | 1.084 | 0.154 | 0.228 | 0.000 |

| Problem 277 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|-------------|---------|-------|-------|-------|--------|--------|-------|-------|--------|--------------|
| grade | N/A | A | A | A | B | A | B | B | B | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 200 | 120 | 215 | 264 | 589 | 225 | 5923 | 431 | 247 | 0 |
| N.S. | 1 | 0.60 | 1.08 | 1.32 | 2.94 | 1.12 | 29.62 | 2.16 | 1.24 | 0.00 |
| time (sec) | N/A | 0.562 | 1.060 | 1.225 | 0.040 | 0.077 | 1.007 | 0.147 | 0.267 | 0.000 |

| Problem 278 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|-------------|---------|-------|-------|-------|--------|--------|-------|-------|--------|--------------|
| grade | N/A | A | A | A | B | A | B | B | B | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 172 | 120 | 171 | 206 | 400 | 167 | 3305 | 337 | 189 | 0 |
| N.S. | 1 | 0.70 | 0.99 | 1.20 | 2.33 | 0.97 | 19.22 | 1.96 | 1.10 | 0.00 |
| time (sec) | N/A | 0.537 | 1.054 | 1.151 | 0.038 | 0.088 | 0.903 | 0.163 | 0.256 | 0.000 |

| Problem 279 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|-------------|---------|-------|-------|-------|--------|--------|-------|-------|--------|--------------|
| grade | N/A | A | A | A | B | A | B | B | B | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 114 | 93 | 127 | 148 | 245 | 124 | 1695 | 240 | 131 | 0 |
| N.S. | 1 | 0.82 | 1.11 | 1.30 | 2.15 | 1.09 | 14.87 | 2.11 | 1.15 | 0.00 |
| time (sec) | N/A | 0.489 | 1.039 | 1.066 | 0.035 | 0.078 | 0.796 | 0.146 | 0.215 | 0.000 |

| Problem 280 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|-------------|---------|-------|-------|-------|--------|--------|-------|-------|--------|-------|
| grade | N/A | A | A | A | B | A | B | B | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 71 | 71 | 83 | 90 | 125 | 69 | 745 | 145 | 73 | 42 |
| N.S. | 1 | 1.00 | 1.17 | 1.27 | 1.76 | 0.97 | 10.49 | 2.04 | 1.03 | 0.59 |
| time (sec) | N/A | 0.339 | 1.026 | 0.920 | 0.029 | 0.073 | 0.745 | 0.123 | 0.224 | 9.841 |

| Problem 281 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|-------------|---------|-------|-------|-------|--------|--------|-------|-------|--------|-------|
| grade | N/A | A | A | A | B | A | B | B | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 34 | 34 | 23 | 20 | 46 | 31 | 294 | 57 | 32 | 32 |
| N.S. | 1 | 1.00 | 0.68 | 0.59 | 1.35 | 0.91 | 8.65 | 1.68 | 0.94 | 0.94 |
| time (sec) | N/A | 0.282 | 0.009 | 0.850 | 0.027 | 0.078 | 0.748 | 0.116 | 0.215 | 9.658 |

| Problem 282 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|-------------|---------|-------|-------|-------|--------------|--------|----------|-------|--------|--------------|
| grade | N/A | A | A | A | F(-2) | A | F | A | B | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 166 | 102 | 92 | 141 | 0 | 115 | 0 | 176 | 132 | 0 |
| N.S. | 1 | 0.61 | 0.55 | 0.85 | 0.00 | 0.69 | 0.00 | 1.06 | 0.80 | 0.00 |
| time (sec) | N/A | 0.450 | 1.038 | 1.223 | 0.000 | 0.087 | 0.000 | 0.126 | 0.219 | 0.000 |

| Problem 283 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|-------------|---------|-------|-------|-------|--------------|--------|----------|-------|--------|--------------|
| grade | N/A | A | A | A | F(-2) | A | F | A | B | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 183 | 103 | 132 | 156 | 0 | 172 | 0 | 178 | 197 | 0 |
| N.S. | 1 | 0.56 | 0.72 | 0.85 | 0.00 | 0.94 | 0.00 | 0.97 | 1.08 | 0.00 |
| time (sec) | N/A | 0.461 | 1.071 | 1.283 | 0.000 | 0.072 | 0.000 | 0.274 | 0.233 | 0.000 |

| Problem 284 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|-------------|---------|-------|-------|-------|--------------|--------|----------|-------|--------|--------------|
| grade | N/A | A | A | A | F(-2) | A | F | A | B | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 186 | 106 | 131 | 161 | 0 | 188 | 0 | 172 | 209 | 0 |
| N.S. | 1 | 0.57 | 0.70 | 0.87 | 0.00 | 1.01 | 0.00 | 0.92 | 1.12 | 0.00 |
| time (sec) | N/A | 0.462 | 1.063 | 1.391 | 0.000 | 0.084 | 0.000 | 0.117 | 0.253 | 0.000 |

| Problem 285 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|-------------|---------|-------|-------|-------|--------------|--------|----------|-------|--------|--------------|
| grade | N/A | A | A | A | F(-2) | A | F | A | B | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 194 | 114 | 104 | 146 | 0 | 177 | 0 | 179 | 180 | 0 |
| N.S. | 1 | 0.59 | 0.54 | 0.75 | 0.00 | 0.91 | 0.00 | 0.92 | 0.93 | 0.00 |
| time (sec) | N/A | 0.467 | 1.059 | 1.454 | 0.000 | 0.078 | 0.000 | 0.146 | 0.215 | 0.000 |

| Problem 286 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|-------------|---------|-------|-------|-------|--------------|--------|----------|-------|--------|-------|
| grade | N/A | A | B | B | F(-2) | B | F | B | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 46 | 48 | 109 | 120 | 0 | 143 | 0 | 198 | 122 | 284 |
| N.S. | 1 | 1.04 | 2.37 | 2.61 | 0.00 | 3.11 | 0.00 | 4.30 | 2.65 | 6.17 |
| time (sec) | N/A | 0.297 | 1.042 | 1.655 | 0.000 | 0.078 | 0.000 | 0.159 | 0.232 | 5.121 |

| Problem 287 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|-------------|---------|-------|-------|-------|--------------|--------|----------|-------|--------|-------|
| grade | N/A | A | A | A | F(-2) | B | F | B | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 98 | 86 | 112 | 126 | 0 | 160 | 0 | 214 | 159 | 284 |
| N.S. | 1 | 0.88 | 1.14 | 1.29 | 0.00 | 1.63 | 0.00 | 2.18 | 1.62 | 2.90 |
| time (sec) | N/A | 0.332 | 1.042 | 2.273 | 0.000 | 0.085 | 0.000 | 0.158 | 0.205 | 5.100 |

| Problem 288 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|-------------|---------|-------|-------|-------|--------|--------|-------|-------|--------|-------|
| grade | N/A | A | A | A | F(-2) | A | F | A | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 143 | 120 | 112 | 126 | 0 | 171 | 0 | 229 | 170 | 284 |
| N.S. | 1 | 0.84 | 0.78 | 0.88 | 0.00 | 1.20 | 0.00 | 1.60 | 1.19 | 1.99 |
| time (sec) | N/A | 0.476 | 1.036 | 2.230 | 0.000 | 0.078 | 0.000 | 0.119 | 0.270 | 5.068 |

| Problem 289 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|-------------|---------|-------|-------|-------|--------|--------|-------|-------|--------|-------|
| grade | N/A | A | A | A | F(-2) | A | F | A | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 200 | 120 | 112 | 126 | 0 | 182 | 0 | 242 | 181 | 284 |
| N.S. | 1 | 0.60 | 0.56 | 0.63 | 0.00 | 0.91 | 0.00 | 1.21 | 0.90 | 1.42 |
| time (sec) | N/A | 0.463 | 1.034 | 2.728 | 0.000 | 0.083 | 0.000 | 0.134 | 0.235 | 5.166 |

| Problem 290 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|-------------|---------|-------|-------|-------|--------|--------|-------|-------|--------|-------|
| grade | N/A | A | A | A | F(-2) | A | F | A | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 200 | 120 | 112 | 126 | 0 | 193 | 0 | 257 | 192 | 284 |
| N.S. | 1 | 0.60 | 0.56 | 0.63 | 0.00 | 0.96 | 0.00 | 1.28 | 0.96 | 1.42 |
| time (sec) | N/A | 0.454 | 1.037 | 3.240 | 0.000 | 0.084 | 0.000 | 0.122 | 0.222 | 5.275 |

| Problem 291 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|-------------|---------|-------|-------|-------|--------|--------|--------|-------|--------|-------|
| grade | N/A | A | A | B | B | B | B | B | B | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 266 | 174 | 385 | 506 | 815 | 427 | 28886 | 785 | 489 | 0 |
| N.S. | 1 | 0.65 | 1.45 | 1.90 | 3.06 | 1.61 | 108.59 | 2.95 | 1.84 | 0.00 |
| time (sec) | N/A | 0.790 | 1.103 | 1.540 | 0.041 | 0.072 | 1.242 | 1.737 | 0.217 | 0.000 |

| Problem 292 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|-------------|---------|-------|-------|-------|--------|--------|-------|-------|--------|--------------|
| grade | N/A | A | A | B | B | B | B | B | B | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 219 | 147 | 319 | 414 | 588 | 360 | 16842 | 642 | 397 | 0 |
| N.S. | 1 | 0.67 | 1.46 | 1.89 | 2.68 | 1.64 | 76.90 | 2.93 | 1.81 | 0.00 |
| time (sec) | N/A | 0.727 | 1.087 | 1.342 | 0.041 | 0.083 | 1.223 | 0.230 | 0.237 | 0.000 |

| Problem 293 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|-------------|---------|-------|-------|-------|--------|--------|-------|-------|--------|--------------|
| grade | N/A | A | A | B | B | B | B | B | B | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 172 | 120 | 253 | 322 | 400 | 277 | 9421 | 500 | 305 | 0 |
| N.S. | 1 | 0.70 | 1.47 | 1.87 | 2.33 | 1.61 | 54.77 | 2.91 | 1.77 | 0.00 |
| time (sec) | N/A | 0.606 | 1.065 | 1.207 | 0.039 | 0.085 | 1.002 | 0.232 | 0.217 | 0.000 |

| Problem 294 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|-------------|---------|-------|-------|-------|--------|--------|-------|-------|--------|--------------|
| grade | N/A | A | A | B | B | B | B | B | B | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 125 | 93 | 187 | 230 | 245 | 197 | 4855 | 358 | 213 | 0 |
| N.S. | 1 | 0.74 | 1.50 | 1.84 | 1.96 | 1.58 | 38.84 | 2.86 | 1.70 | 0.00 |
| time (sec) | N/A | 0.513 | 1.053 | 1.119 | 0.043 | 0.077 | 0.900 | 0.279 | 0.210 | 0.000 |

| Problem 295 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|-------------|---------|-------|-------|-------|--------|--------|-------|-------|--------|--------------|
| grade | N/A | A | A | B | B | B | B | B | B | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 71 | 71 | 121 | 138 | 125 | 115 | 2147 | 218 | 121 | 0 |
| N.S. | 1 | 1.00 | 1.70 | 1.94 | 1.76 | 1.62 | 30.24 | 3.07 | 1.70 | 0.00 |
| time (sec) | N/A | 0.327 | 1.036 | 0.957 | 0.036 | 0.081 | 0.782 | 0.320 | 0.226 | 0.000 |

| Problem 296 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|-------------|---------|-------|-------|-------|--------|--------|-------|-------|--------|-------|
| grade | N/A | A | A | A | B | B | B | B | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 34 | 34 | 23 | 20 | 46 | 53 | 666 | 83 | 54 | 32 |
| N.S. | 1 | 1.00 | 0.68 | 0.59 | 1.35 | 1.56 | 19.59 | 2.44 | 1.59 | 0.94 |
| time (sec) | N/A | 0.260 | 0.010 | 0.874 | 0.032 | 0.080 | 1.340 | 0.409 | 0.270 | 0.025 |

| Problem 297 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|-------------|---------|-------|-------|-------|--------------|--------|----------|-------|--------|--------------|
| grade | N/A | A | A | A | F(-2) | A | F | B | B | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 254 | 150 | 185 | 298 | 0 | 259 | 0 | 399 | 301 | 0 |
| N.S. | 1 | 0.59 | 0.73 | 1.17 | 0.00 | 1.02 | 0.00 | 1.57 | 1.19 | 0.00 |
| time (sec) | N/A | 0.518 | 1.084 | 1.230 | 0.000 | 0.078 | 0.000 | 0.161 | 0.207 | 0.000 |

| Problem 298 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|-------------|---------|-------|-------|-------|--------------|--------|----------|-------|--------|--------------|
| grade | N/A | A | A | A | F(-2) | A | F | A | B | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 292 | 158 | 246 | 307 | 0 | 373 | 0 | 398 | 415 | 0 |
| N.S. | 1 | 0.54 | 0.84 | 1.05 | 0.00 | 1.28 | 0.00 | 1.36 | 1.42 | 0.00 |
| time (sec) | N/A | 0.638 | 1.156 | 1.279 | 0.000 | 0.101 | 0.000 | 0.136 | 0.224 | 0.000 |

| Problem 299 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|-------------|---------|-------|-------|-------|--------------|--------|----------|-------|--------|--------------|
| grade | N/A | A | A | A | F(-2) | A | F | A | B | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 295 | 161 | 248 | 301 | 0 | 416 | 0 | 390 | 470 | 0 |
| N.S. | 1 | 0.55 | 0.84 | 1.02 | 0.00 | 1.41 | 0.00 | 1.32 | 1.59 | 0.00 |
| time (sec) | N/A | 0.596 | 1.083 | 1.458 | 0.000 | 0.080 | 0.000 | 0.422 | 0.208 | 0.000 |

| Problem 300 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|-------------|---------|-------|-------|-------|--------|--------|-------|-------|--------|-------|
| grade | N/A | A | A | A | F(-2) | A | F | A | B | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 292 | 158 | 247 | 295 | 0 | 425 | 0 | 389 | 469 | 0 |
| N.S. | 1 | 0.54 | 0.85 | 1.01 | 0.00 | 1.46 | 0.00 | 1.33 | 1.61 | 0.00 |
| time (sec) | N/A | 0.592 | 1.076 | 1.668 | 0.000 | 0.092 | 0.000 | 0.124 | 0.264 | 0.000 |

| Problem 301 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|-------------|---------|-------|-------|-------|--------|--------|-------|-------|--------|-------|
| grade | N/A | A | A | A | F(-2) | A | F | A | B | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 292 | 158 | 243 | 298 | 0 | 412 | 0 | 383 | 434 | 0 |
| N.S. | 1 | 0.54 | 0.83 | 1.02 | 0.00 | 1.41 | 0.00 | 1.31 | 1.49 | 0.00 |
| time (sec) | N/A | 0.593 | 1.120 | 1.856 | 0.000 | 0.116 | 0.000 | 0.423 | 0.249 | 0.000 |

| Problem 302 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|-------------|---------|-------|-------|-------|--------|--------|-------|-------|--------|-------|
| grade | N/A | A | A | A | F(-2) | A | F | A | B | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 300 | 166 | 196 | 284 | 0 | 373 | 0 | 391 | 375 | 0 |
| N.S. | 1 | 0.55 | 0.65 | 0.95 | 0.00 | 1.24 | 0.00 | 1.30 | 1.25 | 0.00 |
| time (sec) | N/A | 0.595 | 1.088 | 2.313 | 0.000 | 0.085 | 0.000 | 0.569 | 0.243 | 0.000 |

| Problem 303 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|-------------|---------|-------|-------|-------|--------|--------|-------|-------|--------|-------|
| grade | N/A | A | B | B | F(-2) | B | F(-2) | B | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 46 | 48 | 218 | 247 | 0 | 300 | 0 | 419 | 279 | 687 |
| N.S. | 1 | 1.04 | 4.74 | 5.37 | 0.00 | 6.52 | 0.00 | 9.11 | 6.07 | 14.93 |
| time (sec) | N/A | 0.300 | 1.067 | 3.042 | 0.000 | 0.079 | 0.000 | 0.513 | 0.272 | 5.159 |

| Problem 304 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|-------------|---------|-------|-------|-------|--------|--------|-------|-------|--------|-------|
| grade | N/A | A | B | B | F(-2) | B | F | B | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 98 | 86 | 223 | 262 | 0 | 326 | 0 | 437 | 338 | 687 |
| N.S. | 1 | 0.88 | 2.28 | 2.67 | 0.00 | 3.33 | 0.00 | 4.46 | 3.45 | 7.01 |
| time (sec) | N/A | 0.334 | 1.061 | 4.144 | 0.000 | 0.075 | 0.000 | 0.417 | 0.267 | 5.436 |

| Problem 305 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|-------------|---------|-------|-------|-------|--------|--------|-------|-------|--------|-------|
| grade | N/A | A | A | B | F(-2) | B | F | B | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 149 | 130 | 223 | 262 | 0 | 337 | 0 | 452 | 349 | 687 |
| N.S. | 1 | 0.87 | 1.50 | 1.76 | 0.00 | 2.26 | 0.00 | 3.03 | 2.34 | 4.61 |
| time (sec) | N/A | 0.383 | 1.065 | 5.674 | 0.000 | 0.088 | 0.000 | 0.304 | 0.298 | 5.410 |

| Problem 306 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|-------------|---------|-------|-------|-------|--------|--------|-------|-------|--------|-------|
| grade | N/A | A | A | A | F(-2) | B | F | B | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 200 | 174 | 223 | 262 | 0 | 348 | 0 | 465 | 360 | 687 |
| N.S. | 1 | 0.87 | 1.12 | 1.31 | 0.00 | 1.74 | 0.00 | 2.32 | 1.80 | 3.44 |
| time (sec) | N/A | 0.427 | 1.065 | 8.114 | 0.000 | 0.080 | 0.000 | 0.347 | 0.259 | 5.356 |

| Problem 307 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|-------------|---------|-------|-------|--------|--------|--------|-------|-------|--------|-------|
| grade | N/A | A | A | A | F(-2) | A | F(-1) | B | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 308 | 174 | 223 | 262 | 0 | 359 | 0 | 480 | 371 | 686 |
| N.S. | 1 | 0.56 | 0.72 | 0.85 | 0.00 | 1.17 | 0.00 | 1.56 | 1.20 | 2.23 |
| time (sec) | N/A | 0.575 | 1.066 | 11.461 | 0.000 | 0.085 | 0.000 | 0.322 | 0.272 | 5.196 |

| Problem 308 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|-------------|---------|-------|-------|--------|--------|--------|-------|-------|--------|-------|
| grade | N/A | A | A | A | F(-2) | A | F(-2) | B | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 308 | 174 | 223 | 262 | 0 | 370 | 0 | 493 | 382 | 687 |
| N.S. | 1 | 0.56 | 0.72 | 0.85 | 0.00 | 1.20 | 0.00 | 1.60 | 1.24 | 2.23 |
| time (sec) | N/A | 0.585 | 1.064 | 16.160 | 0.000 | 0.080 | 0.000 | 0.213 | 0.231 | 5.142 |

| Problem 309 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|-------------|---------|-------|-------|-------|--------|--------|-------|-------|--------|-------|
| grade | N/A | A | A | A | B | A | B | A | B | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 222 | 123 | 130 | 220 | 348 | 181 | 746 | 274 | 208 | 0 |
| N.S. | 1 | 0.55 | 0.59 | 0.99 | 1.57 | 0.82 | 3.36 | 1.23 | 0.94 | 0.00 |
| time (sec) | N/A | 0.465 | 1.057 | 1.163 | 0.039 | 0.086 | 1.504 | 0.243 | 0.215 | 0.000 |

| Problem 310 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|-------------|---------|-------|-------|-------|--------|--------|-------|-------|--------|-------|
| grade | N/A | A | A | A | A | A | B | A | B | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 173 | 99 | 90 | 142 | 205 | 116 | 462 | 175 | 132 | 0 |
| N.S. | 1 | 0.57 | 0.52 | 0.82 | 1.18 | 0.67 | 2.67 | 1.01 | 0.76 | 0.00 |
| time (sec) | N/A | 0.431 | 1.047 | 0.954 | 0.036 | 0.083 | 1.194 | 0.705 | 0.217 | 0.000 |

| Problem 311 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|-------------|---------|-------|-------|-------|--------|--------|-------|-------|--------|-------|
| grade | N/A | A | A | A | A | A | B | A | B | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 124 | 75 | 59 | 87 | 96 | 63 | 265 | 96 | 72 | 0 |
| N.S. | 1 | 0.60 | 0.48 | 0.70 | 0.77 | 0.51 | 2.14 | 0.77 | 0.58 | 0.00 |
| time (sec) | N/A | 0.368 | 1.026 | 0.859 | 0.031 | 0.076 | 0.887 | 0.113 | 0.234 | 0.000 |

| Problem 312 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|-------------|---------|-------|-------|-------|--------|--------|-------|-------|--------|-------|
| grade | N/A | A | B | A | A | A | B | A | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 69 | 69 | 177 | 45 | 52 | 24 | 134 | 44 | 28 | 79 |
| N.S. | 1 | 1.00 | 2.57 | 0.65 | 0.75 | 0.35 | 1.94 | 0.64 | 0.41 | 1.14 |
| time (sec) | N/A | 0.336 | 0.475 | 0.730 | 0.032 | 0.090 | 0.766 | 0.145 | 0.220 | 5.710 |

| Problem 313 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|-------------|---------|-------|-------|-------|--------|--------|-------|-------|--------|-------|
| grade | N/A | A | A | A | A | A | A | A | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 35 | 35 | 26 | 25 | 12 | 10 | 24 | 17 | 10 | 19 |
| N.S. | 1 | 1.00 | 0.74 | 0.71 | 0.34 | 0.29 | 0.69 | 0.49 | 0.29 | 0.54 |
| time (sec) | N/A | 0.272 | 0.004 | 0.673 | 0.040 | 0.083 | 0.450 | 0.137 | 0.226 | 5.186 |

| Problem 314 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|-------------|---------|-------|-------|-------|--------|--------|-------|-------|--------|-------|
| grade | N/A | A | B | A | F(-2) | A | F | A | B | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 51 | 62 | 248 | 42 | 0 | 26 | 0 | 53 | 26 | 0 |
| N.S. | 1 | 1.22 | 4.86 | 0.82 | 0.00 | 0.51 | 0.00 | 1.04 | 0.51 | 0.00 |
| time (sec) | N/A | 0.325 | 0.375 | 1.063 | 0.000 | 0.079 | 0.000 | 0.140 | 0.232 | 0.000 |

| Problem 315 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|-------------|---------|-------|-------|-------|--------|--------|-------|-------|--------|-------|
| grade | N/A | A | A | A | F(-2) | A | F | A | B | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 131 | 82 | 247 | 81 | 0 | 92 | 0 | 100 | 115 | 0 |
| N.S. | 1 | 0.63 | 1.89 | 0.62 | 0.00 | 0.70 | 0.00 | 0.76 | 0.88 | 0.00 |
| time (sec) | N/A | 0.413 | 1.067 | 1.144 | 0.000 | 0.077 | 0.000 | 0.157 | 0.235 | 0.000 |

| Problem 316 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|-------------|---------|-------|-------|-------|--------------|--------|----------|-------|--------|--------------|
| grade | N/A | A | A | A | F(-2) | A | F | A | B | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 182 | 108 | 97 | 162 | 0 | 242 | 0 | 173 | 289 | 0 |
| N.S. | 1 | 0.59 | 0.53 | 0.89 | 0.00 | 1.33 | 0.00 | 0.95 | 1.59 | 0.00 |
| time (sec) | N/A | 0.485 | 1.053 | 1.286 | 0.000 | 0.091 | 0.000 | 0.141 | 0.220 | 0.000 |

| Problem 317 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|-------------|---------|-------|-------|-------|--------------|--------|----------|-------|--------|--------------|
| grade | N/A | A | A | A | F(-2) | B | F | A | B | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 231 | 132 | 124 | 256 | 0 | 425 | 0 | 251 | 499 | 0 |
| N.S. | 1 | 0.57 | 0.54 | 1.11 | 0.00 | 1.84 | 0.00 | 1.09 | 2.16 | 0.00 |
| time (sec) | N/A | 0.521 | 1.055 | 1.428 | 0.000 | 0.092 | 0.000 | 0.109 | 0.207 | 0.000 |

| Problem 318 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|-------------|---------|-------|-------|-------|--------|--------|----------|-------|--------|--------------|
| grade | N/A | A | A | A | B | A | F | A | B | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 210 | 128 | 174 | 214 | 397 | 292 | 0 | 216 | 330 | 0 |
| N.S. | 1 | 0.61 | 0.83 | 1.02 | 1.89 | 1.39 | 0.00 | 1.03 | 1.57 | 0.00 |
| time (sec) | N/A | 0.544 | 1.058 | 1.602 | 0.037 | 0.080 | 0.000 | 0.174 | 0.209 | 0.000 |

| Problem 319 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|-------------|---------|-------|-------|-------|--------|--------|----------|-------|--------|--------------|
| grade | N/A | A | A | A | B | A | F | A | B | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 161 | 104 | 125 | 154 | 237 | 188 | 0 | 136 | 209 | 0 |
| N.S. | 1 | 0.65 | 0.78 | 0.96 | 1.47 | 1.17 | 0.00 | 0.84 | 1.30 | 0.00 |
| time (sec) | N/A | 0.468 | 1.044 | 1.474 | 0.061 | 0.085 | 0.000 | 0.258 | 0.267 | 0.000 |

| Problem 320 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|-------------|---------|-------|-------|-------|--------|--------|----------|-------|--------|--------------|
| grade | N/A | A | B | A | A | A | F | A | B | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 117 | 85 | 241 | 92 | 113 | 99 | 0 | 84 | 113 | 0 |
| N.S. | 1 | 0.73 | 2.06 | 0.79 | 0.97 | 0.85 | 0.00 | 0.72 | 0.97 | 0.00 |
| time (sec) | N/A | 0.414 | 1.215 | 1.295 | 0.027 | 0.084 | 0.000 | 0.103 | 0.223 | 0.000 |

| Problem 321 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|-------------|---------|-------|-------|-------|--------|--------|----------|-------|--------|-------|
| grade | N/A | A | B | A | A | A | F | A | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 69 | 69 | 159 | 32 | 56 | 38 | 0 | 32 | 37 | 42 |
| N.S. | 1 | 1.00 | 2.30 | 0.46 | 0.81 | 0.55 | 0.00 | 0.46 | 0.54 | 0.61 |
| time (sec) | N/A | 0.323 | 0.720 | 1.214 | 0.039 | 0.072 | 0.000 | 0.521 | 0.222 | 5.645 |

| Problem 322 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|-------------|---------|-------|-------|-------|--------|--------|----------|-------|--------|-------|
| grade | N/A | A | A | A | A | A | F | A | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 34 | 34 | 23 | 20 | 14 | 24 | 0 | 20 | 23 | 30 |
| N.S. | 1 | 1.00 | 0.68 | 0.59 | 0.41 | 0.71 | 0.00 | 0.59 | 0.68 | 0.88 |
| time (sec) | N/A | 0.262 | 0.007 | 1.113 | 0.037 | 0.085 | 0.000 | 0.126 | 0.216 | 5.513 |

| Problem 323 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|-------------|---------|-------|-------|-------|--------------|--------|----------|-------|--------|--------------|
| grade | N/A | A | A | A | F(-2) | B | F | A | B | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 165 | 108 | 92 | 156 | 0 | 242 | 0 | 221 | 288 | 0 |
| N.S. | 1 | 0.65 | 0.56 | 0.95 | 0.00 | 1.47 | 0.00 | 1.34 | 1.75 | 0.00 |
| time (sec) | N/A | 0.472 | 1.046 | 1.544 | 0.000 | 0.083 | 0.000 | 0.118 | 0.228 | 0.000 |

| Problem 324 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|-------------|---------|-------|-------|-------|--------------|--------|----------|-------|--------|--------------|
| grade | N/A | A | A | B | F(-2) | B | F | B | B | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 217 | 135 | 141 | 330 | 0 | 494 | 0 | 321 | 821 | 0 |
| N.S. | 1 | 0.62 | 0.65 | 1.52 | 0.00 | 2.28 | 0.00 | 1.48 | 3.78 | 0.00 |
| time (sec) | N/A | 0.541 | 1.103 | 1.641 | 0.000 | 0.094 | 0.000 | 0.143 | 0.250 | 0.000 |

| Problem 325 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|-------------|---------|-------|-------|-------|--------------|--------|----------|-------|--------|--------------|
| grade | N/A | A | A | B | F(-2) | B | F | B | B | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 276 | 169 | 163 | 508 | 0 | 760 | 0 | 447 | 1202 | 0 |
| N.S. | 1 | 0.61 | 0.59 | 1.84 | 0.00 | 2.75 | 0.00 | 1.62 | 4.36 | 0.00 |
| time (sec) | N/A | 0.617 | 1.071 | 1.791 | 0.000 | 0.096 | 0.000 | 0.119 | 0.239 | 0.000 |

| Problem 326 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|-------------|---------|-------|-------|-------|--------|--------|----------|-------|--------|--------------|
| grade | N/A | A | A | A | B | B | F | A | B | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 302 | 180 | 313 | 379 | 576 | 572 | 0 | 389 | 634 | 0 |
| N.S. | 1 | 0.60 | 1.04 | 1.25 | 1.91 | 1.89 | 0.00 | 1.29 | 2.10 | 0.00 |
| time (sec) | N/A | 0.721 | 1.105 | 2.048 | 0.087 | 0.094 | 0.000 | 0.186 | 0.259 | 0.000 |

| Problem 327 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|-------------|---------|-------|-------|-------|--------|--------|----------|-------|--------|--------------|
| grade | N/A | A | A | A | B | B | F | A | B | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 253 | 156 | 242 | 291 | 443 | 412 | 0 | 281 | 434 | 0 |
| N.S. | 1 | 0.62 | 0.96 | 1.15 | 1.75 | 1.63 | 0.00 | 1.11 | 1.72 | 0.00 |
| time (sec) | N/A | 0.616 | 1.096 | 1.844 | 0.068 | 0.096 | 0.000 | 0.130 | 0.224 | 0.000 |

| Problem 328 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|-------------|---------|-------|-------|-------|--------|--------|----------|-------|--------|--------------|
| grade | N/A | A | A | A | B | A | F | A | B | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 209 | 137 | 138 | 202 | 323 | 267 | 0 | 200 | 269 | 0 |
| N.S. | 1 | 0.66 | 0.66 | 0.97 | 1.55 | 1.28 | 0.00 | 0.96 | 1.29 | 0.00 |
| time (sec) | N/A | 0.545 | 1.058 | 1.599 | 0.051 | 0.091 | 0.000 | 0.296 | 0.216 | 0.000 |

| Problem 329 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|-------------|---------|-------|-------|-------|--------|--------|----------|-------|--------|-------|
| grade | N/A | A | B | B | B | B | F | B | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 46 | 48 | 106 | 113 | 238 | 143 | 0 | 119 | 122 | 254 |
| N.S. | 1 | 1.04 | 2.30 | 2.46 | 5.17 | 3.11 | 0.00 | 2.59 | 2.65 | 5.52 |
| time (sec) | N/A | 0.295 | 1.036 | 1.456 | 0.049 | 0.077 | 0.000 | 0.149 | 0.214 | 6.076 |

| Problem 330 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|-------------|---------|-------|-------|-------|--------|--------|----------|-------|--------|-------|
| grade | N/A | A | A | A | A | A | F | A | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 125 | 91 | 69 | 69 | 115 | 98 | 0 | 69 | 95 | 79 |
| N.S. | 1 | 0.73 | 0.55 | 0.55 | 0.92 | 0.78 | 0.00 | 0.55 | 0.76 | 0.63 |
| time (sec) | N/A | 0.425 | 1.028 | 1.294 | 0.035 | 0.082 | 0.000 | 0.149 | 0.261 | 6.057 |

| Problem 331 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|-------------|---------|-------|-------|-------|--------|--------|----------|-------|--------|-------|
| grade | N/A | A | B | A | A | A | F | A | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 71 | 71 | 345 | 33 | 56 | 61 | 0 | 33 | 59 | 43 |
| N.S. | 1 | 1.00 | 4.86 | 0.46 | 0.79 | 0.86 | 0.00 | 0.46 | 0.83 | 0.61 |
| time (sec) | N/A | 0.333 | 1.007 | 1.122 | 0.033 | 0.099 | 0.000 | 0.120 | 0.250 | 6.300 |

| Problem 332 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|-------------|---------|-------|-------|-------|--------|--------|----------|-------|--------|-------|
| grade | N/A | A | A | A | A | B | F | A | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 34 | 34 | 23 | 20 | 14 | 46 | 0 | 20 | 45 | 30 |
| N.S. | 1 | 1.00 | 0.68 | 0.59 | 0.41 | 1.35 | 0.00 | 0.59 | 1.32 | 0.88 |
| time (sec) | N/A | 0.254 | 0.007 | 1.062 | 0.032 | 0.077 | 0.000 | 0.109 | 0.231 | 6.217 |

| Problem 333 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|-------------|---------|-------|-------|-------|--------------|--------|----------|-------|--------|--------------|
| grade | N/A | A | A | A | F(-2) | B | F | B | B | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 253 | 156 | 163 | 359 | 0 | 657 | 0 | 418 | 769 | 0 |
| N.S. | 1 | 0.62 | 0.64 | 1.42 | 0.00 | 2.60 | 0.00 | 1.65 | 3.04 | 0.00 |
| time (sec) | N/A | 0.563 | 1.090 | 1.502 | 0.000 | 0.099 | 0.000 | 0.151 | 0.232 | 0.000 |

| Problem 334 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|-------------|---------|-------|-------|-------|--------------|--------|----------|-------|--------|--------------|
| grade | N/A | A | A | B | F(-2) | B | F | B | B | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 307 | 185 | 167 | 651 | 0 | 1083 | 0 | 546 | 1706 | 0 |
| N.S. | 1 | 0.60 | 0.54 | 2.12 | 0.00 | 3.53 | 0.00 | 1.78 | 5.56 | 0.00 |
| time (sec) | N/A | 0.686 | 1.125 | 1.693 | 0.000 | 0.109 | 0.000 | 0.133 | 0.274 | 0.000 |

| Problem 335 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|-------------|---------|-------|-------|-------|--------------|--------|----------|-------|--------|--------------|
| grade | N/A | A | A | B | F(-2) | B | F | B | B | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 365 | 218 | 209 | 983 | 0 | 1565 | 0 | 683 | 2408 | 0 |
| N.S. | 1 | 0.60 | 0.57 | 2.69 | 0.00 | 4.29 | 0.00 | 1.87 | 6.60 | 0.00 |
| time (sec) | N/A | 0.759 | 1.107 | 1.845 | 0.000 | 0.115 | 0.000 | 0.159 | 0.228 | 0.000 |

| Problem 336 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|-------------|---------|-------|-------|-------|--------|--------|-------|-------|--------|--------------|
| grade | N/A | A | A | B | A | A | B | B | B | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 52 | 52 | 34 | 86 | 78 | 69 | 92 | 158 | 69 | 0 |
| N.S. | 1 | 1.00 | 0.65 | 1.65 | 1.50 | 1.33 | 1.77 | 3.04 | 1.33 | 0.00 |
| time (sec) | N/A | 0.290 | 0.443 | 0.852 | 0.148 | 0.083 | 0.649 | 0.113 | 0.240 | 0.000 |

| Problem 337 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|-------------|---------|-------|-------|-------|--------|--------|-------|-------|--------|-------|
| grade | N/A | A | A | A | A | A | A | B | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 52 | 52 | 33 | 62 | 78 | 44 | 66 | 110 | 45 | 67 |
| N.S. | 1 | 1.00 | 0.63 | 1.19 | 1.50 | 0.85 | 1.27 | 2.12 | 0.87 | 1.29 |
| time (sec) | N/A | 0.305 | 0.384 | 0.836 | 0.111 | 0.075 | 0.569 | 0.120 | 0.224 | 5.795 |

| Problem 338 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|-------------|---------|-------|-------|-----------|--------|--------|-------|-------|--------|-------|
| grade | N/A | A | A | C | A | A | A | A | B | B |
| verified | N/A | Yes | Yes | No | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 50 | 50 | 38 | 27 | 78 | 23 | 48 | 61 | 21 | 40 |
| N.S. | 1 | 1.00 | 0.76 | 0.54 | 1.56 | 0.46 | 0.96 | 1.22 | 0.42 | 0.80 |
| time (sec) | N/A | 0.297 | 0.142 | 0.414 | 0.121 | 0.087 | 0.642 | 0.131 | 0.272 | 5.764 |

| Problem 339 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|-------------|---------|-------|-------|-------|--------|--------|-------|-------|--------|-------|
| grade | N/A | A | A | A | A | A | A | A | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 56 | 56 | 41 | 40 | 30 | 20 | 46 | 39 | 23 | 46 |
| N.S. | 1 | 1.00 | 0.73 | 0.71 | 0.54 | 0.36 | 0.82 | 0.70 | 0.41 | 0.82 |
| time (sec) | N/A | 0.296 | 0.132 | 0.608 | 0.141 | 0.075 | 0.585 | 0.133 | 0.214 | 5.929 |

| Problem 340 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|-------------|---------|-------|-------|-------|--------|--------|----------|-------|--------|-------|
| grade | N/A | A | A | A | A | A | F | A | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 52 | 52 | 38 | 28 | 36 | 25 | 0 | 28 | 23 | 32 |
| N.S. | 1 | 1.00 | 0.73 | 0.54 | 0.69 | 0.48 | 0.00 | 0.54 | 0.44 | 0.62 |
| time (sec) | N/A | 0.295 | 0.190 | 0.878 | 0.111 | 0.084 | 0.000 | 0.144 | 0.219 | 0.050 |

| Problem 341 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|-------------|---------|-------|-------|-------|--------|--------|----------|-------|--------|-------|
| grade | N/A | A | A | A | A | A | F | A | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 52 | 52 | 34 | 28 | 36 | 35 | 0 | 28 | 34 | 32 |
| N.S. | 1 | 1.00 | 0.65 | 0.54 | 0.69 | 0.67 | 0.00 | 0.54 | 0.65 | 0.62 |
| time (sec) | N/A | 0.294 | 0.336 | 0.874 | 0.111 | 0.077 | 0.000 | 0.111 | 0.229 | 0.056 |

| Problem 342 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|-------------|---------|-------|-------|-------|--------|--------|----------|-------|--------|-------|
| grade | N/A | A | A | A | A | A | F | A | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 52 | 52 | 34 | 28 | 36 | 45 | 0 | 28 | 44 | 32 |
| N.S. | 1 | 1.00 | 0.65 | 0.54 | 0.69 | 0.87 | 0.00 | 0.54 | 0.85 | 0.62 |
| time (sec) | N/A | 0.284 | 0.339 | 0.825 | 0.143 | 0.079 | 0.000 | 0.154 | 0.234 | 0.060 |

| Problem 343 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|-------------|---------|-------|-------|-------|--------------|--------|----------|--------------|--------|--------------|
| grade | N/A | A | A | A | F(-2) | A | F | F(-2) | B | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 56 | 69 | 51 | 58 | 0 | 294 | 0 | 0 | 30 | 0 |
| N.S. | 1 | 1.23 | 0.91 | 1.04 | 0.00 | 5.25 | 0.00 | 0.00 | 0.54 | 0.00 |
| time (sec) | N/A | 0.331 | 0.416 | 1.473 | 0.000 | 0.100 | 0.000 | 0.000 | 0.237 | 0.000 |

| Problem 344 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|-------------|---------|-------|-------|-----------|--------|--------|--------------|-------|--------|--------------|
| grade | N/A | A | A | C | A | A | F(-1) | B | B | F(-1) |
| verified | N/A | Yes | Yes | No | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 96 | 70 | 48 | 33 | 93 | 93 | 0 | 354 | 92 | 0 |
| N.S. | 1 | 0.73 | 0.50 | 0.34 | 0.97 | 0.97 | 0.00 | 3.69 | 0.96 | 0.00 |
| time (sec) | N/A | 0.359 | 0.043 | 2.191 | 0.040 | 0.082 | 0.000 | 0.145 | 0.217 | 0.000 |

| Problem 345 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|-------------|---------|-------|-------|-----------|--------|--------|----------|-------|--------|--------------|
| grade | N/A | A | A | C | A | A | F | B | B | F(-1) |
| verified | N/A | Yes | Yes | No | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 96 | 70 | 48 | 33 | 69 | 69 | 0 | 228 | 68 | 0 |
| N.S. | 1 | 0.73 | 0.50 | 0.34 | 0.72 | 0.72 | 0.00 | 2.38 | 0.71 | 0.00 |
| time (sec) | N/A | 0.355 | 0.031 | 2.035 | 0.048 | 0.086 | 0.000 | 0.138 | 0.242 | 0.000 |

| Problem 346 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|-------------|---------|-------|-------|-----------|--------|--------|----------|-------|--------|--------------|
| grade | N/A | A | A | C | A | A | F | A | B | F(-1) |
| verified | N/A | Yes | Yes | No | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 96 | 70 | 48 | 33 | 46 | 46 | 0 | 124 | 44 | 0 |
| N.S. | 1 | 0.73 | 0.50 | 0.34 | 0.48 | 0.48 | 0.00 | 1.29 | 0.46 | 0.00 |
| time (sec) | N/A | 0.352 | 0.033 | 1.875 | 0.042 | 0.081 | 0.000 | 0.113 | 0.224 | 0.000 |

| Problem 347 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|-------------|---------|-------|-------|-----------|--------|--------|----------|-------|--------|-------|
| grade | N/A | A | A | C | A | A | F | A | B | B |
| verified | N/A | Yes | Yes | No | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 94 | 68 | 47 | 32 | 46 | 25 | 0 | 51 | 24 | 81 |
| N.S. | 1 | 0.72 | 0.50 | 0.34 | 0.49 | 0.27 | 0.00 | 0.54 | 0.26 | 0.86 |
| time (sec) | N/A | 0.358 | 0.034 | 1.422 | 0.038 | 0.081 | 0.000 | 0.113 | 0.204 | 5.758 |

| Problem 348 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|-------------|---------|-------|-------|-----------|--------|--------|----------|-------|--------|-------|
| grade | N/A | A | A | C | A | A | F | A | B | B |
| verified | N/A | Yes | Yes | No | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 92 | 66 | 48 | 32 | 25 | 35 | 0 | 52 | 26 | 58 |
| N.S. | 1 | 0.72 | 0.52 | 0.35 | 0.27 | 0.38 | 0.00 | 0.57 | 0.28 | 0.63 |
| time (sec) | N/A | 0.354 | 0.037 | 1.480 | 0.047 | 0.075 | 0.000 | 0.126 | 0.215 | 5.993 |

| Problem 349 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|-------------|---------|-------|-------|-----------|--------|--------|----------|-------|--------|-------|
| grade | N/A | A | A | C | A | A | F | A | B | B |
| verified | N/A | Yes | Yes | No | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 94 | 68 | 47 | 32 | 35 | 46 | 0 | 46 | 34 | 95 |
| N.S. | 1 | 0.72 | 0.50 | 0.34 | 0.37 | 0.49 | 0.00 | 0.49 | 0.36 | 1.01 |
| time (sec) | N/A | 0.347 | 0.033 | 1.465 | 0.054 | 0.083 | 0.000 | 0.131 | 0.262 | 5.801 |

| Problem 350 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|-------------|---------|-------|-------|-----------|--------|--------|--------------|-------|--------|-------|
| grade | N/A | A | A | C | A | A | F(-1) | A | B | B |
| verified | N/A | Yes | Yes | No | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 96 | 70 | 48 | 33 | 47 | 58 | 0 | 47 | 45 | 120 |
| N.S. | 1 | 0.73 | 0.50 | 0.34 | 0.49 | 0.60 | 0.00 | 0.49 | 0.47 | 1.25 |
| time (sec) | N/A | 0.358 | 0.038 | 1.490 | 0.049 | 0.098 | 0.000 | 0.115 | 0.223 | 5.563 |

| Problem 351 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|-------------|---------|-------|-------|-------|--------|--------|----------|-------|--------|--------------|
| grade | N/A | A | A | A | A | A | F | B | B | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 208 | 128 | 120 | 132 | 268 | 268 | 0 | 953 | 284 | 0 |
| N.S. | 1 | 0.62 | 0.58 | 0.63 | 1.29 | 1.29 | 0.00 | 4.58 | 1.37 | 0.00 |
| time (sec) | N/A | 0.461 | 0.110 | 1.490 | 0.059 | 0.089 | 0.000 | 0.154 | 0.232 | 0.000 |

| Problem 352 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|-------------|---------|-------|-------|-------|--------|--------|----------|-------|--------|--------------|
| grade | N/A | A | A | A | A | A | F | B | B | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 208 | 128 | 120 | 132 | 216 | 216 | 0 | 638 | 226 | 0 |
| N.S. | 1 | 0.62 | 0.58 | 0.63 | 1.04 | 1.04 | 0.00 | 3.07 | 1.09 | 0.00 |
| time (sec) | N/A | 0.450 | 0.081 | 1.549 | 0.050 | 0.089 | 0.000 | 0.147 | 0.257 | 0.000 |

| Problem 353 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|-------------|---------|-------|-------|-------|--------|--------|----------|-------|--------|--------------|
| grade | N/A | A | A | A | A | A | F | B | B | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 208 | 128 | 120 | 132 | 164 | 164 | 0 | 370 | 168 | 0 |
| N.S. | 1 | 0.62 | 0.58 | 0.63 | 0.79 | 0.79 | 0.00 | 1.78 | 0.81 | 0.00 |
| time (sec) | N/A | 0.460 | 0.078 | 1.752 | 0.037 | 0.082 | 0.000 | 0.127 | 0.283 | 0.000 |

| Problem 354 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|-------------|---------|-------|-------|-------|--------|--------|----------|-------|--------|-------|
| grade | N/A | A | A | A | A | A | F | A | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 204 | 124 | 119 | 132 | 164 | 115 | 0 | 161 | 114 | 198 |
| N.S. | 1 | 0.61 | 0.58 | 0.65 | 0.80 | 0.56 | 0.00 | 0.79 | 0.56 | 0.97 |
| time (sec) | N/A | 0.453 | 0.066 | 1.162 | 0.047 | 0.093 | 0.000 | 0.141 | 0.278 | 5.398 |

| Problem 355 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|-------------|---------|-------|-------|-------|--------|--------|----------|-------|--------|-------|
| grade | N/A | A | A | A | A | A | F | A | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 202 | 122 | 119 | 132 | 114 | 124 | 0 | 212 | 115 | 147 |
| N.S. | 1 | 0.60 | 0.59 | 0.65 | 0.56 | 0.61 | 0.00 | 1.05 | 0.57 | 0.73 |
| time (sec) | N/A | 0.442 | 0.072 | 1.142 | 0.045 | 0.082 | 0.000 | 0.156 | 0.226 | 5.529 |

| Problem 356 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|-------------|---------|-------|-------|-------|--------|--------|----------|-------|--------|-------|
| grade | N/A | A | A | A | A | A | F | A | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 204 | 124 | 119 | 108 | 125 | 136 | 0 | 201 | 122 | 184 |
| N.S. | 1 | 0.61 | 0.58 | 0.53 | 0.61 | 0.67 | 0.00 | 0.99 | 0.60 | 0.90 |
| time (sec) | N/A | 0.438 | 0.085 | 1.132 | 0.055 | 0.082 | 0.000 | 0.151 | 0.224 | 5.666 |

| Problem 357 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|-------------|---------|-------|-------|-------|--------|--------|----------|-------|--------|-------|
| grade | N/A | A | A | A | A | A | F | A | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 202 | 122 | 118 | 131 | 137 | 148 | 0 | 191 | 134 | 210 |
| N.S. | 1 | 0.60 | 0.58 | 0.65 | 0.68 | 0.73 | 0.00 | 0.95 | 0.66 | 1.04 |
| time (sec) | N/A | 0.451 | 0.086 | 1.131 | 0.052 | 0.079 | 0.000 | 0.154 | 0.241 | 6.013 |

| Problem 358 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|-------------|---------|-------|-------|-------|--------|--------|----------|-------|--------|-------|
| grade | N/A | A | A | A | A | A | F | A | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 204 | 124 | 119 | 132 | 147 | 158 | 0 | 188 | 145 | 239 |
| N.S. | 1 | 0.61 | 0.58 | 0.65 | 0.72 | 0.77 | 0.00 | 0.92 | 0.71 | 1.17 |
| time (sec) | N/A | 0.443 | 0.091 | 1.142 | 0.059 | 0.091 | 0.000 | 0.160 | 0.209 | 6.020 |

| Problem 359 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|-------------|---------|-------|-------|-------|--------|--------|----------|-------|--------|-------|
| grade | N/A | A | A | A | A | A | F | A | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 208 | 128 | 120 | 132 | 159 | 170 | 0 | 188 | 156 | 268 |
| N.S. | 1 | 0.62 | 0.58 | 0.63 | 0.76 | 0.82 | 0.00 | 0.90 | 0.75 | 1.29 |
| time (sec) | N/A | 0.434 | 0.082 | 1.094 | 0.048 | 0.088 | 0.000 | 0.140 | 0.213 | 5.938 |

| Problem 360 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|-------------|---------|-------|-------|-------|--------|--------|--------------|-------|--------|--------------|
| grade | N/A | A | A | A | B | B | F(-1) | B | B | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 320 | 186 | 235 | 289 | 497 | 497 | 0 | 1743 | 543 | 0 |
| N.S. | 1 | 0.58 | 0.73 | 0.90 | 1.55 | 1.55 | 0.00 | 5.45 | 1.70 | 0.00 |
| time (sec) | N/A | 0.547 | 0.147 | 1.440 | 0.043 | 0.095 | 0.000 | 0.195 | 0.225 | 0.000 |

| Problem 361 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|-------------|---------|-------|-------|-------|--------|--------|----------|-------|--------|--------------|
| grade | N/A | A | A | A | A | A | F | B | B | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 320 | 186 | 235 | 289 | 418 | 418 | 0 | 1192 | 451 | 0 |
| N.S. | 1 | 0.58 | 0.73 | 0.90 | 1.31 | 1.31 | 0.00 | 3.72 | 1.41 | 0.00 |
| time (sec) | N/A | 0.523 | 0.145 | 1.470 | 0.042 | 0.089 | 0.000 | 0.170 | 0.273 | 0.000 |

| Problem 362 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|-------------|---------|-------|-------|-------|--------|--------|----------|-------|--------|--------------|
| grade | N/A | A | A | A | A | A | F | B | B | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 318 | 184 | 235 | 289 | 338 | 338 | 0 | 713 | 359 | 0 |
| N.S. | 1 | 0.58 | 0.74 | 0.91 | 1.06 | 1.06 | 0.00 | 2.24 | 1.13 | 0.00 |
| time (sec) | N/A | 0.537 | 0.137 | 1.504 | 0.051 | 0.082 | 0.000 | 0.177 | 0.221 | 0.000 |

| Problem 363 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|-------------|---------|-------|-------|-------|--------|--------|----------|-------|--------|-------|
| grade | N/A | A | A | A | A | A | F | A | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 316 | 182 | 234 | 289 | 338 | 261 | 0 | 319 | 271 | 375 |
| N.S. | 1 | 0.58 | 0.74 | 0.91 | 1.07 | 0.83 | 0.00 | 1.01 | 0.86 | 1.19 |
| time (sec) | N/A | 0.544 | 0.198 | 1.154 | 0.046 | 0.082 | 0.000 | 0.168 | 0.234 | 5.904 |

| Problem 364 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|-------------|---------|-------|-------|-------|--------|--------|----------|-------|--------|-------|
| grade | N/A | A | A | A | A | A | F | B | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 314 | 180 | 234 | 289 | 261 | 271 | 0 | 476 | 273 | 294 |
| N.S. | 1 | 0.57 | 0.75 | 0.92 | 0.83 | 0.86 | 0.00 | 1.52 | 0.87 | 0.94 |
| time (sec) | N/A | 0.544 | 0.157 | 1.152 | 0.053 | 0.083 | 0.000 | 0.167 | 0.243 | 6.054 |

| Problem 365 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|-------------|---------|-------|-------|-------|--------|--------|----------|-------|--------|-------|
| grade | N/A | A | A | A | A | A | F | B | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 314 | 180 | 235 | 226 | 272 | 283 | 0 | 461 | 280 | 330 |
| N.S. | 1 | 0.57 | 0.75 | 0.72 | 0.87 | 0.90 | 0.00 | 1.47 | 0.89 | 1.05 |
| time (sec) | N/A | 0.525 | 0.144 | 1.210 | 0.051 | 0.084 | 0.000 | 0.170 | 0.267 | 6.519 |

| Problem 366 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|-------------|---------|-------|-------|-------|--------|--------|----------|-------|--------|-------|
| grade | N/A | A | A | A | A | A | F | A | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 316 | 182 | 236 | 215 | 283 | 294 | 0 | 457 | 291 | 359 |
| N.S. | 1 | 0.58 | 0.75 | 0.68 | 0.90 | 0.93 | 0.00 | 1.45 | 0.92 | 1.14 |
| time (sec) | N/A | 0.533 | 0.133 | 1.191 | 0.060 | 0.088 | 0.000 | 0.159 | 0.231 | 6.267 |

| Problem 367 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|-------------|---------|-------|-------|-------|--------|--------|----------|-------|--------|-------|
| grade | N/A | A | A | A | A | A | F | A | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 314 | 180 | 235 | 227 | 293 | 304 | 0 | 451 | 302 | 386 |
| N.S. | 1 | 0.57 | 0.75 | 0.72 | 0.93 | 0.97 | 0.00 | 1.44 | 0.96 | 1.23 |
| time (sec) | N/A | 0.546 | 0.137 | 1.220 | 0.066 | 0.088 | 0.000 | 0.155 | 0.240 | 6.167 |

| Problem 368 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|-------------|---------|-------|-------|-------|--------|--------|-------|-------|--------|-------|
| grade | N/A | A | A | A | A | A | F(-1) | A | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 314 | 180 | 235 | 282 | 305 | 316 | 0 | 441 | 313 | 416 |
| N.S. | 1 | 0.57 | 0.75 | 0.90 | 0.97 | 1.01 | 0.00 | 1.40 | 1.00 | 1.32 |
| time (sec) | N/A | 0.547 | 0.158 | 1.244 | 0.056 | 0.087 | 0.000 | 0.135 | 0.217 | 6.199 |

| Problem 369 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|-------------|---------|-------|-------|-------|--------|--------|-------|-------|--------|-------|
| grade | N/A | A | A | A | A | A | F(-1) | A | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 316 | 182 | 234 | 289 | 315 | 326 | 0 | 437 | 324 | 437 |
| N.S. | 1 | 0.58 | 0.74 | 0.91 | 1.00 | 1.03 | 0.00 | 1.38 | 1.03 | 1.38 |
| time (sec) | N/A | 0.531 | 0.150 | 1.213 | 0.086 | 0.096 | 0.000 | 0.190 | 0.233 | 6.140 |

| Problem 370 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|-------------|---------|-------|-------|-------|--------|--------|-------|-------|--------|-------|
| grade | N/A | A | A | A | A | A | F(-1) | A | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 318 | 184 | 235 | 289 | 327 | 338 | 0 | 437 | 335 | 472 |
| N.S. | 1 | 0.58 | 0.74 | 0.91 | 1.03 | 1.06 | 0.00 | 1.37 | 1.05 | 1.48 |
| time (sec) | N/A | 0.536 | 0.162 | 1.263 | 0.062 | 0.091 | 0.000 | 0.202 | 0.207 | 6.218 |

| Problem 371 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|-------------|---------|-------|-------|-------|--------|--------|-------|-------|--------|-------|
| grade | N/A | A | A | A | F | A | F(-1) | A | B | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 263 | 175 | 168 | 237 | 0 | 424 | 0 | 343 | 349 | 0 |
| N.S. | 1 | 0.67 | 0.64 | 0.90 | 0.00 | 1.61 | 0.00 | 1.30 | 1.33 | 0.00 |
| time (sec) | N/A | 0.513 | 0.201 | 1.536 | 0.000 | 0.091 | 0.000 | 0.196 | 0.207 | 0.000 |

| Problem 372 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|-------------|---------|-------|-------|-------|----------|--------|----------|-------|--------|--------------|
| grade | N/A | A | A | A | F | A | F | A | B | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 212 | 146 | 127 | 171 | 0 | 290 | 0 | 231 | 223 | 0 |
| N.S. | 1 | 0.69 | 0.60 | 0.81 | 0.00 | 1.37 | 0.00 | 1.09 | 1.05 | 0.00 |
| time (sec) | N/A | 0.467 | 0.179 | 1.072 | 0.000 | 0.092 | 0.000 | 0.196 | 0.209 | 0.000 |

| Problem 373 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|-------------|---------|-------|-------|-------|----------|--------|----------|-------|--------|--------------|
| grade | N/A | A | A | A | F | A | F | A | B | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 161 | 117 | 96 | 120 | 0 | 188 | 0 | 141 | 119 | 0 |
| N.S. | 1 | 0.73 | 0.60 | 0.75 | 0.00 | 1.17 | 0.00 | 0.88 | 0.74 | 0.00 |
| time (sec) | N/A | 0.420 | 0.156 | 1.047 | 0.000 | 0.095 | 0.000 | 0.161 | 0.229 | 0.000 |

| Problem 374 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|-------------|---------|-------|-------|-------|----------|--------|----------|-------|--------|--------------|
| grade | N/A | A | A | A | F | A | F | A | B | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 112 | 88 | 81 | 93 | 0 | 143 | 0 | 80 | 51 | 0 |
| N.S. | 1 | 0.79 | 0.72 | 0.83 | 0.00 | 1.28 | 0.00 | 0.71 | 0.46 | 0.00 |
| time (sec) | N/A | 0.397 | 0.068 | 1.065 | 0.000 | 0.090 | 0.000 | 0.162 | 0.231 | 0.000 |

| Problem 375 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|-------------|---------|-------|-------|-------|----------|--------|----------|-------|--------|--------------|
| grade | N/A | A | A | A | F | A | F | A | B | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 72 | 72 | 63 | 51 | 0 | 119 | 0 | 44 | 50 | 0 |
| N.S. | 1 | 1.00 | 0.88 | 0.71 | 0.00 | 1.65 | 0.00 | 0.61 | 0.69 | 0.00 |
| time (sec) | N/A | 0.339 | 0.052 | 1.037 | 0.000 | 0.079 | 0.000 | 0.158 | 0.241 | 0.000 |

| Problem 376 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|-------------|---------|-------|-------|-------|----------|--------|----------|-------|--------|--------------|
| grade | N/A | A | A | A | F | A | F | A | B | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 119 | 95 | 95 | 90 | 0 | 193 | 0 | 75 | 84 | 0 |
| N.S. | 1 | 0.80 | 0.80 | 0.76 | 0.00 | 1.62 | 0.00 | 0.63 | 0.71 | 0.00 |
| time (sec) | N/A | 0.379 | 0.106 | 1.062 | 0.000 | 0.087 | 0.000 | 0.176 | 0.197 | 0.000 |

| Problem 377 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|-------------|---------|-------|-------|-------|----------|--------|----------|-------|--------|--------------|
| grade | N/A | A | A | A | F | A | F | A | B | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 168 | 131 | 113 | 130 | 0 | 378 | 0 | 120 | 223 | 0 |
| N.S. | 1 | 0.78 | 0.67 | 0.77 | 0.00 | 2.25 | 0.00 | 0.71 | 1.33 | 0.00 |
| time (sec) | N/A | 0.443 | 0.188 | 1.099 | 0.000 | 0.102 | 0.000 | 0.174 | 0.231 | 0.000 |

| Problem 378 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|-------------|---------|-------|-------|-------|----------|--------|--------------|-------|--------|--------------|
| grade | N/A | A | A | A | F | B | F(-1) | A | B | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 219 | 167 | 133 | 202 | 0 | 686 | 0 | 190 | 437 | 0 |
| N.S. | 1 | 0.76 | 0.61 | 0.92 | 0.00 | 3.13 | 0.00 | 0.87 | 2.00 | 0.00 |
| time (sec) | N/A | 0.476 | 0.275 | 1.075 | 0.000 | 0.107 | 0.000 | 0.164 | 0.210 | 0.000 |

| Problem 379 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|-------------|---------|-------|-------|-------|----------|--------|--------------|-------|--------|--------------|
| grade | N/A | A | A | A | F | A | F(-1) | A | B | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 308 | 208 | 236 | 226 | 0 | 730 | 0 | 397 | 715 | 0 |
| N.S. | 1 | 0.68 | 0.77 | 0.73 | 0.00 | 2.37 | 0.00 | 1.29 | 2.32 | 0.00 |
| time (sec) | N/A | 0.549 | 0.709 | 1.638 | 0.000 | 0.107 | 0.000 | 0.346 | 0.223 | 0.000 |

| Problem 380 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|-------------|---------|-------|-------|-------|----------|--------|----------|-------|--------|--------------|
| grade | N/A | A | A | A | F | A | F | A | B | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 254 | 179 | 185 | 176 | 0 | 520 | 0 | 287 | 464 | 0 |
| N.S. | 1 | 0.70 | 0.73 | 0.69 | 0.00 | 2.05 | 0.00 | 1.13 | 1.83 | 0.00 |
| time (sec) | N/A | 0.500 | 0.696 | 1.552 | 0.000 | 0.097 | 0.000 | 0.198 | 0.253 | 0.000 |

| Problem 381 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|-------------|---------|-------|-------|-------|----------|--------|----------|-------|--------|--------------|
| grade | N/A | A | A | A | F | A | F | A | B | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 202 | 150 | 138 | 149 | 0 | 344 | 0 | 195 | 246 | 0 |
| N.S. | 1 | 0.74 | 0.68 | 0.74 | 0.00 | 1.70 | 0.00 | 0.97 | 1.22 | 0.00 |
| time (sec) | N/A | 0.459 | 0.446 | 1.573 | 0.000 | 0.103 | 0.000 | 0.213 | 0.232 | 0.000 |

| Problem 382 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|-------------|---------|-------|-------|-------|----------|--------|----------|-------|--------|--------------|
| grade | N/A | A | A | A | F | A | F | A | B | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 158 | 128 | 110 | 194 | 0 | 383 | 0 | 124 | 259 | 0 |
| N.S. | 1 | 0.81 | 0.70 | 1.23 | 0.00 | 2.42 | 0.00 | 0.78 | 1.64 | 0.00 |
| time (sec) | N/A | 0.410 | 0.355 | 1.541 | 0.000 | 0.103 | 0.000 | 0.164 | 0.220 | 0.000 |

| Problem 383 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|-------------|---------|-------|-------|-------|----------|--------|----------|-------|--------|--------------|
| grade | N/A | A | A | A | F | A | F | A | B | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 168 | 134 | 121 | 200 | 0 | 456 | 0 | 150 | 301 | 0 |
| N.S. | 1 | 0.80 | 0.72 | 1.19 | 0.00 | 2.71 | 0.00 | 0.89 | 1.79 | 0.00 |
| time (sec) | N/A | 0.412 | 0.336 | 1.467 | 0.000 | 0.091 | 0.000 | 0.200 | 0.219 | 0.000 |

| Problem 384 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|-------------|---------|-------|-------|-------|----------|--------|----------|-------|--------|--------------|
| grade | N/A | A | A | A | F | B | F | A | B | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 172 | 148 | 121 | 203 | 0 | 549 | 0 | 184 | 352 | 0 |
| N.S. | 1 | 0.86 | 0.70 | 1.18 | 0.00 | 3.19 | 0.00 | 1.07 | 2.05 | 0.00 |
| time (sec) | N/A | 0.453 | 0.243 | 1.503 | 0.000 | 0.104 | 0.000 | 1.362 | 0.228 | 0.000 |

| Problem 385 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|-------------|---------|-------|-------|-------|----------|--------|----------|-------|--------|--------------|
| grade | N/A | A | A | A | F | B | F | A | B | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 223 | 185 | 155 | 285 | 0 | 762 | 0 | 306 | 437 | 0 |
| N.S. | 1 | 0.83 | 0.70 | 1.28 | 0.00 | 3.42 | 0.00 | 1.37 | 1.96 | 0.00 |
| time (sec) | N/A | 0.504 | 0.435 | 1.493 | 0.000 | 0.112 | 0.000 | 0.169 | 0.226 | 0.000 |

| Problem 386 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|-------------|---------|-------|-------|-------|----------|--------|----------|-------|--------|--------------|
| grade | N/A | A | A | B | F | B | F | B | B | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 275 | 221 | 188 | 388 | 0 | 1206 | 0 | 388 | 804 | 0 |
| N.S. | 1 | 0.80 | 0.68 | 1.41 | 0.00 | 4.39 | 0.00 | 1.41 | 2.92 | 0.00 |
| time (sec) | N/A | 0.546 | 0.761 | 1.510 | 0.000 | 0.129 | 0.000 | 0.198 | 0.223 | 0.000 |

| Problem 387 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|-------------|---------|-------|-------|-------|----------|--------|----------|-------|--------|--------------|
| grade | N/A | A | A | B | F | B | F | B | B | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 329 | 257 | 241 | 518 | 0 | 1838 | 0 | 494 | 1299 | 0 |
| N.S. | 1 | 0.78 | 0.73 | 1.57 | 0.00 | 5.59 | 0.00 | 1.50 | 3.95 | 0.00 |
| time (sec) | N/A | 0.590 | 0.867 | 1.507 | 0.000 | 0.214 | 0.000 | 1.348 | 0.276 | 0.000 |

| Problem 388 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|-------------|---------|-------|-------|-------|----------|--------|--------------|-------|--------|--------------|
| grade | N/A | A | A | A | F | B | F(-1) | B | B | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 400 | 272 | 374 | 311 | 0 | 1286 | 0 | 651 | 1308 | 0 |
| N.S. | 1 | 0.68 | 0.94 | 0.78 | 0.00 | 3.22 | 0.00 | 1.63 | 3.27 | 0.00 |
| time (sec) | N/A | 0.615 | 1.639 | 1.613 | 0.000 | 0.119 | 0.000 | 0.195 | 0.210 | 0.000 |

| Problem 389 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|-------------|---------|-------|-------|-------|----------|--------|--------------|-------|--------|--------------|
| grade | N/A | A | A | A | F | B | F(-1) | B | B | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 346 | 243 | 300 | 260 | 0 | 968 | 0 | 498 | 907 | 0 |
| N.S. | 1 | 0.70 | 0.87 | 0.75 | 0.00 | 2.80 | 0.00 | 1.44 | 2.62 | 0.00 |
| time (sec) | N/A | 0.564 | 1.537 | 1.550 | 0.000 | 0.124 | 0.000 | 0.182 | 0.224 | 0.000 |

| Problem 390 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|-------------|---------|-------|-------|-------|----------|--------|--------------|-------|--------|--------------|
| grade | N/A | A | A | A | F | A | F(-1) | A | B | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 294 | 214 | 232 | 233 | 0 | 680 | 0 | 364 | 543 | 0 |
| N.S. | 1 | 0.73 | 0.79 | 0.79 | 0.00 | 2.31 | 0.00 | 1.24 | 1.85 | 0.00 |
| time (sec) | N/A | 0.528 | 1.076 | 1.678 | 0.000 | 0.108 | 0.000 | 1.364 | 0.222 | 0.000 |

| Problem 391 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|-------------|---------|-------|-------|-------|----------|--------|--------------|-------|--------|--------------|
| grade | N/A | A | A | B | F | B | F(-1) | A | B | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 250 | 192 | 185 | 467 | 0 | 765 | 0 | 251 | 583 | 0 |
| N.S. | 1 | 0.77 | 0.74 | 1.87 | 0.00 | 3.06 | 0.00 | 1.00 | 2.33 | 0.00 |
| time (sec) | N/A | 0.495 | 1.021 | 1.612 | 0.000 | 0.105 | 0.000 | 0.181 | 0.225 | 0.000 |

| Problem 392 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|-------------|---------|-------|-------|-------|----------|--------|--------------|-------|--------|--------------|
| grade | N/A | A | A | B | F | B | F(-1) | A | B | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 260 | 198 | 194 | 477 | 0 | 894 | 0 | 283 | 661 | 0 |
| N.S. | 1 | 0.76 | 0.75 | 1.83 | 0.00 | 3.44 | 0.00 | 1.09 | 2.54 | 0.00 |
| time (sec) | N/A | 0.516 | 1.012 | 1.572 | 0.000 | 0.108 | 0.000 | 0.178 | 0.239 | 0.000 |

| Problem 393 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|-------------|---------|-------|-------|-------|----------|--------|----------|-------|--------|--------------|
| grade | N/A | A | A | B | F | B | F | A | B | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 270 | 212 | 194 | 477 | 0 | 1043 | 0 | 321 | 742 | 0 |
| N.S. | 1 | 0.79 | 0.72 | 1.77 | 0.00 | 3.86 | 0.00 | 1.19 | 2.75 | 0.00 |
| time (sec) | N/A | 0.529 | 1.069 | 1.529 | 0.000 | 0.108 | 0.000 | 0.116 | 0.260 | 0.000 |

| Problem 394 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|-------------|---------|-------|-------|-------|----------|--------|----------|-------|--------|--------------|
| grade | N/A | A | A | B | F | B | F | A | B | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 280 | 226 | 192 | 500 | 0 | 1176 | 0 | 359 | 821 | 0 |
| N.S. | 1 | 0.81 | 0.69 | 1.79 | 0.00 | 4.20 | 0.00 | 1.28 | 2.93 | 0.00 |
| time (sec) | N/A | 0.544 | 1.054 | 1.580 | 0.000 | 0.115 | 0.000 | 0.129 | 0.231 | 0.000 |

| Problem 395 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|-------------|---------|-------|-------|-------|----------|--------|----------|-------|--------|--------------|
| grade | N/A | A | A | B | F | B | F | B | B | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 278 | 240 | 187 | 497 | 0 | 1325 | 0 | 391 | 903 | 0 |
| N.S. | 1 | 0.86 | 0.67 | 1.79 | 0.00 | 4.77 | 0.00 | 1.41 | 3.25 | 0.00 |
| time (sec) | N/A | 0.565 | 0.572 | 1.523 | 0.000 | 0.137 | 0.000 | 0.138 | 0.229 | 0.000 |

| Problem 396 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|-------------|---------|-------|-------|-------|----------|--------|----------|-------|--------|--------------|
| grade | N/A | A | A | B | F | B | F | B | B | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 329 | 277 | 247 | 602 | 0 | 1714 | 0 | 553 | 1014 | 0 |
| N.S. | 1 | 0.84 | 0.75 | 1.83 | 0.00 | 5.21 | 0.00 | 1.68 | 3.08 | 0.00 |
| time (sec) | N/A | 0.609 | 1.259 | 1.552 | 0.000 | 0.197 | 0.000 | 0.134 | 0.224 | 0.000 |

| Problem 397 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|-------------|---------|-------|-------|-------|----------|--------|----------|-------|--------|--------------|
| grade | N/A | A | A | B | F | B | F | B | B | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 381 | 313 | 301 | 763 | 0 | 2474 | 0 | 635 | 1661 | 0 |
| N.S. | 1 | 0.82 | 0.79 | 2.00 | 0.00 | 6.49 | 0.00 | 1.67 | 4.36 | 0.00 |
| time (sec) | N/A | 0.633 | 1.519 | 1.516 | 0.000 | 0.356 | 0.000 | 0.174 | 0.236 | 0.000 |

| Problem 398 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|-------------|---------|-------|-------|-------|----------|--------|----------|-------|--------|--------------|
| grade | N/A | A | A | B | F | B | F | B | B | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 435 | 349 | 378 | 951 | 0 | 3348 | 0 | 743 | 2284 | 0 |
| N.S. | 1 | 0.80 | 0.87 | 2.19 | 0.00 | 7.70 | 0.00 | 1.71 | 5.25 | 0.00 |
| time (sec) | N/A | 0.684 | 1.866 | 1.532 | 0.000 | 0.693 | 0.000 | 0.174 | 0.254 | 0.000 |

| Problem 399 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|-------------|---------|-------|-------|-------|--------|--------|-------|-------|--------|-------|
| grade | N/A | A | A | A | B | B | B | B | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 27 | 27 | 28 | 46 | 928 | 116 | 279 | 186 | 93 | 147 |
| N.S. | 1 | 1.00 | 1.04 | 1.70 | 34.37 | 4.30 | 10.33 | 6.89 | 3.44 | 5.44 |
| time (sec) | N/A | 0.288 | 0.009 | 1.546 | 0.085 | 0.077 | 0.969 | 0.122 | 0.258 | 5.662 |

| Problem 400 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|-------------|---------|-------|-------|-------|--------|--------|-------|-------|--------|-------|
| grade | N/A | A | A | A | B | B | B | B | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 27 | 27 | 28 | 46 | 457 | 88 | 214 | 138 | 71 | 109 |
| N.S. | 1 | 1.00 | 1.04 | 1.70 | 16.93 | 3.26 | 7.93 | 5.11 | 2.63 | 4.04 |
| time (sec) | N/A | 0.304 | 0.006 | 1.342 | 0.068 | 0.087 | 0.554 | 0.129 | 0.218 | 5.600 |

| Problem 401 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|-------------|---------|-------|-------|-------|--------|--------|-------|-------|--------|-------|
| grade | N/A | A | A | A | B | B | B | B | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 25 | 25 | 26 | 42 | 164 | 52 | 138 | 82 | 47 | 63 |
| N.S. | 1 | 1.00 | 1.04 | 1.68 | 6.56 | 2.08 | 5.52 | 3.28 | 1.88 | 2.52 |
| time (sec) | N/A | 0.273 | 0.005 | 0.765 | 0.051 | 0.096 | 0.330 | 0.156 | 0.208 | 5.684 |

| Problem 402 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|-------------|---------|-------|-------|-------|--------|--------|-------|-------|--------|-------|
| grade | N/A | A | A | A | A | A | A | A | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 24 | 24 | 23 | 24 | 24 | 23 | 29 | 24 | 24 | 24 |
| N.S. | 1 | 1.00 | 0.96 | 1.00 | 1.00 | 0.96 | 1.21 | 1.00 | 1.00 | 1.00 |
| time (sec) | N/A | 0.244 | 0.002 | 0.663 | 0.034 | 0.085 | 0.018 | 0.113 | 0.246 | 5.577 |

| Problem 403 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|-------------|---------|-------|-------|-------|--------|--------|-------|----------|--------|-------|
| grade | N/A | A | A | A | A | A | B | F | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 28 | 28 | 24 | 29 | 30 | 39 | 65 | 0 | 34 | 31 |
| N.S. | 1 | 1.00 | 0.86 | 1.04 | 1.07 | 1.39 | 2.32 | 0.00 | 1.21 | 1.11 |
| time (sec) | N/A | 0.291 | 0.006 | 1.452 | 0.041 | 0.076 | 0.357 | 0.000 | 0.265 | 5.761 |

| Problem 404 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|-------------|---------|-------|-------|-------|--------|--------|-------|----------|--------|-------|
| grade | N/A | A | A | A | B | B | B | F | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 30 | 30 | 28 | 29 | 68 | 103 | 148 | 0 | 80 | 53 |
| N.S. | 1 | 1.00 | 0.93 | 0.97 | 2.27 | 3.43 | 4.93 | 0.00 | 2.67 | 1.77 |
| time (sec) | N/A | 0.285 | 0.005 | 1.573 | 0.050 | 0.087 | 0.758 | 0.000 | 0.319 | 5.730 |

| Problem 405 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|-------------|---------|-------|-------|-------|--------|--------|-------|----------|--------|-------|
| grade | N/A | A | A | A | B | B | B | F | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 30 | 30 | 28 | 29 | 102 | 159 | 212 | 0 | 126 | 75 |
| N.S. | 1 | 1.00 | 0.93 | 0.97 | 3.40 | 5.30 | 7.07 | 0.00 | 4.20 | 2.50 |
| time (sec) | N/A | 0.289 | 0.006 | 1.878 | 0.049 | 0.082 | 1.737 | 0.000 | 0.257 | 5.308 |

| Problem 406 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|-------------|---------|-------|-------|-------|--------|--------|----------|-------|--------|--------------|
| grade | N/A | A | A | A | A | A | F | B | B | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 49 | 56 | 36 | 47 | 85 | 74 | 0 | 137 | 60 | 0 |
| N.S. | 1 | 1.14 | 0.73 | 0.96 | 1.73 | 1.51 | 0.00 | 2.80 | 1.22 | 0.00 |
| time (sec) | N/A | 0.325 | 0.016 | 1.277 | 0.054 | 0.090 | 0.000 | 0.143 | 0.242 | 0.000 |

| Problem 407 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|-------------|---------|-------|-------|-------|--------|--------|----------|-------|--------|--------------|
| grade | N/A | A | A | A | A | A | F | A | B | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 48 | 55 | 36 | 47 | 51 | 47 | 0 | 78 | 37 | 0 |
| N.S. | 1 | 1.15 | 0.75 | 0.98 | 1.06 | 0.98 | 0.00 | 1.62 | 0.77 | 0.00 |
| time (sec) | N/A | 0.326 | 0.008 | 1.082 | 0.042 | 0.089 | 0.000 | 0.132 | 0.217 | 0.000 |

| Problem 408 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|-------------|---------|-------|-------|-------|--------|--------|----------|----------|--------|-------|
| grade | N/A | A | A | A | A | A | F | F | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 46 | 46 | 34 | 33 | 20 | 48 | 0 | 0 | 22 | 51 |
| N.S. | 1 | 1.00 | 0.74 | 0.72 | 0.43 | 1.04 | 0.00 | 0.00 | 0.48 | 1.11 |
| time (sec) | N/A | 0.307 | 0.008 | 1.353 | 0.081 | 0.084 | 0.000 | 0.000 | 0.216 | 5.432 |

| Problem 409 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|-------------|---------|-------|-------|-------|--------|--------|----------|----------|--------|--------------|
| grade | N/A | A | A | A | A | B | F | F | B | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 52 | 59 | 36 | 40 | 54 | 125 | 0 | 0 | 59 | 0 |
| N.S. | 1 | 1.13 | 0.69 | 0.77 | 1.04 | 2.40 | 0.00 | 0.00 | 1.13 | 0.00 |
| time (sec) | N/A | 0.327 | 0.010 | 1.378 | 0.049 | 0.090 | 0.000 | 0.000 | 0.214 | 0.000 |

| Problem 410 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|-------------|---------|-------|-------|-------|--------|--------|----------|----------|--------|--------------|
| grade | N/A | A | A | A | A | B | F | F | B | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 54 | 59 | 36 | 40 | 88 | 181 | 0 | 0 | 105 | 0 |
| N.S. | 1 | 1.09 | 0.67 | 0.74 | 1.63 | 3.35 | 0.00 | 0.00 | 1.94 | 0.00 |
| time (sec) | N/A | 0.326 | 0.014 | 1.332 | 0.052 | 0.090 | 0.000 | 0.000 | 0.228 | 0.000 |

| Problem 411 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|-------------|---------|-------|-------|-------|--------|--------|--------|-------|--------|-------|
| grade | N/A | A | A | B | B | B | B | B | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 206 | 206 | 175 | 1694 | 818 | 2230 | 28410 | 3878 | 2686 | 1898 |
| N.S. | 1 | 1.00 | 0.85 | 8.22 | 3.97 | 10.83 | 137.91 | 18.83 | 13.04 | 9.21 |
| time (sec) | N/A | 0.665 | 0.206 | 1.389 | 0.071 | 0.124 | 6.043 | 0.191 | 0.236 | 6.179 |

| Problem 412 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|-------------|---------|-------|-------|-------|--------|--------|-------|-------|--------|-------|
| grade | N/A | A | A | B | B | B | B | B | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 142 | 142 | 121 | 735 | 392 | 901 | 8719 | 1528 | 1028 | 831 |
| N.S. | 1 | 1.00 | 0.85 | 5.18 | 2.76 | 6.35 | 61.40 | 10.76 | 7.24 | 5.85 |
| time (sec) | N/A | 0.527 | 0.119 | 1.266 | 0.057 | 0.099 | 1.832 | 0.148 | 0.220 | 5.588 |

| Problem 413 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|-------------|---------|-------|-------|-------|--------|--------|-------|-------|--------|-------|
| grade | N/A | A | A | B | A | B | B | B | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 78 | 78 | 67 | 159 | 138 | 237 | 1506 | 385 | 241 | 226 |
| N.S. | 1 | 1.00 | 0.86 | 2.04 | 1.77 | 3.04 | 19.31 | 4.94 | 3.09 | 2.90 |
| time (sec) | N/A | 0.392 | 0.085 | 0.687 | 0.044 | 0.094 | 0.559 | 0.143 | 0.207 | 5.347 |

| Problem 414 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|-------------|---------|-------|-------|----------|----------|----------|----------|----------|----------|--------------|
| grade | N/A | A | A | F | F | F | F | F | F | F(-1) |
| verified | N/A | Yes | Yes | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 51 | 51 | 52 | 0 | 0 | 0 | 0 | 0 | 1218 | 0 |
| N.S. | 1 | 1.00 | 1.02 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 23.88 | 0.00 |
| time (sec) | N/A | 0.329 | 0.046 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.215 | 0.000 |

| Problem 415 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|-------------|---------|-------|-------|----------|----------|----------|----------|----------|----------|--------------|
| grade | N/A | A | A | F | F | F | F | F | F | F(-1) |
| verified | N/A | Yes | Yes | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 53 | 53 | 54 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 1.00 | 1.02 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.318 | 0.053 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.248 | 0.000 |

| Problem 416 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|-------------|---------|-------|-------|-------|--------|--------|-------|-------|--------|-------|
| grade | N/A | A | A | F | F | F | F | F | F | F(-1) |
| verified | N/A | Yes | Yes | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 53 | 53 | 54 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 1.00 | 1.02 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.310 | 0.055 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.268 | 0.000 |

| Problem 417 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|-------------|---------|-------|-------|-------|--------|--------|-------|-------|--------|-------|
| grade | N/A | A | A | B | B | B | F(-2) | B | B | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 337 | 203 | 167 | 1361 | 792 | 1460 | 0 | 3197 | 1728 | 0 |
| N.S. | 1 | 0.60 | 0.50 | 4.04 | 2.35 | 4.33 | 0.00 | 9.49 | 5.13 | 0.00 |
| time (sec) | N/A | 0.626 | 0.233 | 1.240 | 0.071 | 0.095 | 0.000 | 0.201 | 0.230 | 0.000 |

| Problem 418 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|-------------|---------|-------|-------|-------|--------|--------|-------|-------|--------|-------|
| grade | N/A | A | A | B | A | B | F | B | B | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 219 | 139 | 113 | 402 | 305 | 496 | 0 | 1073 | 546 | 0 |
| N.S. | 1 | 0.63 | 0.52 | 1.84 | 1.39 | 2.26 | 0.00 | 4.90 | 2.49 | 0.00 |
| time (sec) | N/A | 0.490 | 0.123 | 1.178 | 0.047 | 0.095 | 0.000 | 0.162 | 0.237 | 0.000 |

| Problem 419 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|-------------|---------|-------|-------|-------|--------|--------|-------|-------|--------|-------|
| grade | N/A | A | A | A | A | A | F(-2) | B | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 101 | 75 | 59 | 62 | 62 | 83 | 0 | 180 | 75 | 129 |
| N.S. | 1 | 0.74 | 0.58 | 0.61 | 0.61 | 0.82 | 0.00 | 1.78 | 0.74 | 1.28 |
| time (sec) | N/A | 0.377 | 0.062 | 0.681 | 0.088 | 0.087 | 0.000 | 0.158 | 0.221 | 5.703 |

| Problem 420 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|-------------|---------|-------|-------|-------|--------|--------|-------|-------|--------|-------|
| grade | N/A | A | A | F | F | F | F | F | F | F(-1) |
| verified | N/A | Yes | Yes | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 76 | 76 | 67 | 0 | 0 | 0 | 0 | 0 | 94 | 0 |
| N.S. | 1 | 1.00 | 0.88 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 1.24 | 0.00 |
| time (sec) | N/A | 0.337 | 0.055 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.218 | 0.000 |

| Problem 421 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|-------------|---------|-------|-------|-------|--------|--------|-------|-------|--------|-------|
| grade | N/A | A | A | F | F | F | F(-2) | F | F | F(-1) |
| verified | N/A | Yes | Yes | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 79 | 79 | 72 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 1.00 | 0.91 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.339 | 0.069 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.226 | 0.000 |

| Problem 422 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|-------------|---------|-------|-------|-------|--------|--------|-------|-------|--------|-------|
| grade | N/A | A | A | F | F | F | F(-2) | F | F | F(-1) |
| verified | N/A | Yes | Yes | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 79 | 79 | 72 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 1.00 | 0.91 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.345 | 0.071 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.288 | 0.000 |

| Problem 423 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|-------------|---------|-------|-------|-------|--------|--------|-------|-------|--------|-------|
| grade | N/A | A | A | A | B | A | B | A | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 39 | 39 | 29 | 40 | 313 | 71 | 233 | 57 | 70 | 90 |
| N.S. | 1 | 1.00 | 0.74 | 1.03 | 8.03 | 1.82 | 5.97 | 1.46 | 1.79 | 2.31 |
| time (sec) | N/A | 0.276 | 0.008 | 1.504 | 0.060 | 0.096 | 0.363 | 0.136 | 0.234 | 5.640 |

| Problem 424 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|-------------|---------|-------|-------|-------|--------|--------|-------|-------|--------|-------|
| grade | N/A | A | A | A | A | A | B | A | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 39 | 39 | 29 | 38 | 37 | 47 | 139 | 37 | 48 | 57 |
| N.S. | 1 | 1.00 | 0.74 | 0.97 | 0.95 | 1.21 | 3.56 | 0.95 | 1.23 | 1.46 |
| time (sec) | N/A | 0.270 | 0.007 | 1.349 | 0.037 | 0.091 | 0.256 | 0.146 | 0.221 | 5.750 |

| Problem 425 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|-------------|---------|-------|-------|-------|--------|--------|-------|-------|--------|-------|
| grade | N/A | A | A | A | A | A | A | F | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 32 | 32 | 21 | 20 | 20 | 30 | 48 | 0 | 30 | 30 |
| N.S. | 1 | 1.00 | 0.66 | 0.62 | 0.62 | 0.94 | 1.50 | 0.00 | 0.94 | 0.94 |
| time (sec) | N/A | 0.263 | 0.002 | 1.566 | 0.040 | 0.093 | 0.155 | 0.000 | 0.247 | 5.367 |

| Problem 426 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|-------------|---------|-------|-------|-------|--------|--------|-------|-------|--------|-------|
| grade | N/A | A | A | A | A | A | B | F | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 39 | 39 | 25 | 29 | 45 | 70 | 100 | 0 | 67 | 52 |
| N.S. | 1 | 1.00 | 0.64 | 0.74 | 1.15 | 1.79 | 2.56 | 0.00 | 1.72 | 1.33 |
| time (sec) | N/A | 0.271 | 0.009 | 1.651 | 0.044 | 0.086 | 0.440 | 0.000 | 0.217 | 5.223 |

| Problem 427 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|-------------|---------|-------|-------|-------|--------|--------|-------|-------|--------|-------|
| grade | N/A | A | A | A | A | B | B | F | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 41 | 41 | 29 | 29 | 73 | 114 | 155 | 0 | 113 | 74 |
| N.S. | 1 | 1.00 | 0.71 | 0.71 | 1.78 | 2.78 | 3.78 | 0.00 | 2.76 | 1.80 |
| time (sec) | N/A | 0.282 | 0.005 | 1.695 | 0.058 | 0.083 | 0.935 | 0.000 | 0.218 | 5.211 |

| Problem 428 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|-------------|---------|-------|-------|-------|--------|--------|-------|-------|--------|-------|
| grade | N/A | A | A | A | B | B | B | B | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 40 | 49 | 29 | 41 | 483 | 82 | 306 | 193 | 82 | 115 |
| N.S. | 1 | 1.22 | 0.72 | 1.02 | 12.08 | 2.05 | 7.65 | 4.82 | 2.05 | 2.88 |
| time (sec) | N/A | 0.309 | 0.005 | 1.510 | 0.071 | 0.093 | 3.734 | 0.164 | 0.269 | 5.311 |

| Problem 429 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|-------------|---------|-------|-------|-------|--------|--------|-------|-------|--------|-------|
| grade | N/A | A | A | A | B | A | B | B | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 40 | 49 | 34 | 41 | 182 | 60 | 216 | 127 | 60 | 79 |
| N.S. | 1 | 1.22 | 0.85 | 1.02 | 4.55 | 1.50 | 5.40 | 3.18 | 1.50 | 1.98 |
| time (sec) | N/A | 0.303 | 0.009 | 1.462 | 0.046 | 0.097 | 2.495 | 0.143 | 0.220 | 5.177 |

| Problem 430 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|-------------|---------|-------|-------|-------|--------|--------|-------|-------|--------|-------|
| grade | N/A | A | A | A | A | A | B | A | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 38 | 47 | 27 | 28 | 32 | 36 | 116 | 59 | 38 | 45 |
| N.S. | 1 | 1.24 | 0.71 | 0.74 | 0.84 | 0.95 | 3.05 | 1.55 | 1.00 | 1.18 |
| time (sec) | N/A | 0.291 | 0.003 | 1.316 | 0.039 | 0.085 | 0.507 | 0.152 | 0.214 | 5.416 |

| Problem 431 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|-------------|---------|-------|-------|-------|--------|--------|----------|----------|--------|-------|
| grade | N/A | A | A | A | A | A | F | F | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 41 | 45 | 32 | 30 | 34 | 49 | 0 | 0 | 45 | 42 |
| N.S. | 1 | 1.10 | 0.78 | 0.73 | 0.83 | 1.20 | 0.00 | 0.00 | 1.10 | 1.02 |
| time (sec) | N/A | 0.299 | 0.006 | 1.601 | 0.043 | 0.089 | 0.000 | 0.000 | 0.206 | 5.143 |

| Problem 432 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|-------------|---------|-------|-------|-------|--------|--------|----------|----------|--------|-------|
| grade | N/A | A | A | A | A | B | F | F | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 41 | 50 | 29 | 30 | 66 | 95 | 0 | 0 | 91 | 64 |
| N.S. | 1 | 1.22 | 0.71 | 0.73 | 1.61 | 2.32 | 0.00 | 0.00 | 2.22 | 1.56 |
| time (sec) | N/A | 0.301 | 0.005 | 1.623 | 0.057 | 0.082 | 0.000 | 0.000 | 0.214 | 5.224 |

| Problem 433 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|-------------|---------|-------|-------|-------|--------|--------|----------|-------|--------|-------|
| grade | N/A | A | A | B | A | B | F | B | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 181 | 175 | 107 | 405 | 276 | 516 | 0 | 1279 | 564 | 484 |
| N.S. | 1 | 0.97 | 0.59 | 2.24 | 1.52 | 2.85 | 0.00 | 7.07 | 3.12 | 2.67 |
| time (sec) | N/A | 0.561 | 0.114 | 1.487 | 0.044 | 0.087 | 0.000 | 0.161 | 0.236 | 5.347 |

| Problem 434 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|-------------|---------|-------|-------|-------|--------|--------|----------|-------|--------|-------|
| grade | N/A | A | A | A | A | A | F | B | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 127 | 131 | 75 | 175 | 157 | 249 | 0 | 608 | 256 | 250 |
| N.S. | 1 | 1.03 | 0.59 | 1.38 | 1.24 | 1.96 | 0.00 | 4.79 | 2.02 | 1.97 |
| time (sec) | N/A | 0.472 | 0.099 | 1.466 | 0.056 | 0.087 | 0.000 | 0.152 | 0.212 | 5.164 |

| Problem 435 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|-------------|---------|-------|-------|-------|--------|--------|-------|-------|--------|-------|
| grade | N/A | A | A | A | A | A | B | B | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 76 | 80 | 54 | 65 | 78 | 96 | 604 | 224 | 93 | 112 |
| N.S. | 1 | 1.05 | 0.71 | 0.86 | 1.03 | 1.26 | 7.95 | 2.95 | 1.22 | 1.47 |
| time (sec) | N/A | 0.361 | 0.057 | 1.344 | 0.052 | 0.087 | 0.846 | 0.167 | 0.231 | 5.453 |

| Problem 436 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|-------------|---------|-------|-------|-------|--------|--------|-------|-------|--------|-------|
| grade | N/A | A | A | A | A | A | B | A | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 34 | 38 | 25 | 26 | 25 | 32 | 100 | 51 | 34 | 41 |
| N.S. | 1 | 1.12 | 0.74 | 0.76 | 0.74 | 0.94 | 2.94 | 1.50 | 1.00 | 1.21 |
| time (sec) | N/A | 0.269 | 0.004 | 1.329 | 0.037 | 0.077 | 0.480 | 0.173 | 0.202 | 5.598 |

| Problem 437 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|-------------|---------|-------|-------|----------|----------|----------|----------|----------|----------|--------------|
| grade | N/A | A | A | F | F | F | F | F | F | F(-1) |
| verified | N/A | Yes | Yes | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 71 | 78 | 62 | 0 | 0 | 0 | 0 | 0 | 129 | 0 |
| N.S. | 1 | 1.10 | 0.87 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 1.82 | 0.00 |
| time (sec) | N/A | 0.334 | 0.051 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.222 | 0.000 |

| Problem 438 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|-------------|---------|-------|-------|----------|----------|----------|----------|----------|----------|--------------|
| grade | N/A | A | A | F | F | F | F | F | F | F(-1) |
| verified | N/A | Yes | Yes | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 72 | 75 | 63 | 0 | 0 | 0 | 0 | 0 | 1322 | 0 |
| N.S. | 1 | 1.04 | 0.88 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 18.36 | 0.00 |
| time (sec) | N/A | 0.328 | 0.046 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.229 | 0.000 |

| Problem 439 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|-------------|---------|-------|-------|----------|----------|----------|----------|----------|----------|--------------|
| grade | N/A | A | A | F | F | F | F | F | F | F(-1) |
| verified | N/A | Yes | Yes | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 74 | 76 | 65 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 1.03 | 0.88 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.332 | 0.050 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.218 | 0.000 |

| Problem 440 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|-------------|---------|-------|-------|----------|----------|----------|----------|----------|----------|--------------|
| grade | N/A | A | A | F | F | F | F | F | F | F(-1) |
| verified | N/A | Yes | Yes | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 83 | 83 | 73 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 1.00 | 0.88 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.369 | 0.147 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.402 | 0.000 |

| Problem 441 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|-------------|---------|-------|-------|----------|----------|----------|----------|----------|----------|--------------|
| grade | N/A | A | A | F | F | F | F | F | F | F(-1) |
| verified | N/A | Yes | Yes | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 83 | 83 | 73 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 1.00 | 0.88 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.336 | 0.099 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.289 | 0.000 |

| Problem 442 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|-------------|---------|-------|-------|----------|----------|----------|----------|----------|----------|--------------|
| grade | N/A | A | A | F | F | F | F | F | F | F(-1) |
| verified | N/A | Yes | Yes | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 81 | 81 | 71 | 0 | 0 | 0 | 0 | 0 | 469 | 0 |
| N.S. | 1 | 1.00 | 0.88 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 5.79 | 0.00 |
| time (sec) | N/A | 0.341 | 0.108 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.258 | 0.000 |

| Problem 443 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|-------------|---------|-------|-------|----------|----------|----------|----------|----------|----------|--------------|
| grade | N/A | A | A | F | F | F | F | F | F | F(-1) |
| verified | N/A | Yes | Yes | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 81 | 81 | 71 | 0 | 0 | 0 | 0 | 0 | 1377 | 0 |
| N.S. | 1 | 1.00 | 0.88 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 17.00 | 0.00 |
| time (sec) | N/A | 0.330 | 0.145 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.274 | 0.000 |

| Problem 444 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|-------------|---------|-------|-------|----------|----------|----------|----------|----------|----------|--------------|
| grade | N/A | A | A | F | F | F | F | F | F | F(-1) |
| verified | N/A | Yes | Yes | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 83 | 83 | 73 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 1.00 | 0.88 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.342 | 0.182 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.295 | 0.000 |

| Problem 445 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|-------------|---------|-------|-------|-------|--------|--------|----------|-------|--------|-------|
| grade | N/A | A | A | A | A | A | F | A | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 43 | 43 | 32 | 44 | 43 | 39 | 0 | 62 | 46 | 43 |
| N.S. | 1 | 1.00 | 0.74 | 1.02 | 1.00 | 0.91 | 0.00 | 1.44 | 1.07 | 1.00 |
| time (sec) | N/A | 0.297 | 0.007 | 1.823 | 0.051 | 0.080 | 0.000 | 0.186 | 0.214 | 5.810 |

| Problem 446 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|-------------|---------|-------|-------|-------|--------|--------|--------------|-------|----------|-------|
| grade | N/A | A | A | A | A | A | F(-2) | A | F | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 44 | 44 | 33 | 44 | 29 | 30 | 0 | 63 | 34 | 43 |
| N.S. | 1 | 1.00 | 0.75 | 1.00 | 0.66 | 0.68 | 0.00 | 1.43 | 0.77 | 0.98 |
| time (sec) | N/A | 0.296 | 0.008 | 1.707 | 0.037 | 0.077 | 0.000 | 0.187 | 0.208 | 6.498 |

| Problem 447 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|-------------|---------|-------|-------|-------|--------|--------|----------|----------|----------|--------------|
| grade | N/A | A | A | B | A | A | F | F | F | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 41 | 41 | 30 | 95 | 13 | 13 | 0 | 0 | 51 | 0 |
| N.S. | 1 | 1.00 | 0.73 | 2.32 | 0.32 | 0.32 | 0.00 | 0.00 | 1.24 | 0.00 |
| time (sec) | N/A | 0.292 | 0.006 | 1.613 | 0.049 | 0.079 | 0.000 | 0.000 | 0.226 | 0.000 |

| Problem 448 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|-------------|---------|-------|-------|-------|--------|--------|----------|----------|----------|--------------|
| grade | N/A | A | A | A | A | A | F | F | F | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 43 | 43 | 32 | 74 | 15 | 15 | 0 | 0 | 64 | 0 |
| N.S. | 1 | 1.00 | 0.74 | 1.72 | 0.35 | 0.35 | 0.00 | 0.00 | 1.49 | 0.00 |
| time (sec) | N/A | 0.288 | 0.004 | 1.657 | 0.033 | 0.072 | 0.000 | 0.000 | 0.245 | 0.000 |

| Problem 449 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|-------------|---------|-------|-------|-------|--------|--------|----------|-------|--------|-------|
| grade | N/A | A | A | C | A | A | F | A | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 26 | 26 | 23 | 18 | 27 | 24 | 0 | 37 | 27 | 26 |
| N.S. | 1 | 1.00 | 0.88 | 0.69 | 1.04 | 0.92 | 0.00 | 1.42 | 1.04 | 1.00 |
| time (sec) | N/A | 0.261 | 0.005 | 1.151 | 0.042 | 0.087 | 0.000 | 0.134 | 0.224 | 6.059 |

| Problem 450 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|-------------|---------|-------|-------|-------|--------|--------|----------|-------|--------|-------|
| grade | N/A | A | A | A | A | A | F | B | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 50 | 57 | 54 | 52 | 79 | 63 | 0 | 110 | 66 | 51 |
| N.S. | 1 | 1.14 | 1.08 | 1.04 | 1.58 | 1.26 | 0.00 | 2.20 | 1.32 | 1.02 |
| time (sec) | N/A | 0.320 | 0.016 | 2.487 | 0.131 | 0.080 | 0.000 | 0.154 | 0.222 | 6.057 |

| Problem 451 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|-------------|---------|-------|-------|----------|----------|----------|--------------|----------|----------|--------------|
| grade | N/A | A | A | F | F | F | F(-2) | F | F | F(-1) |
| verified | N/A | Yes | Yes | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 77 | 85 | 75 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 1.10 | 0.97 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.366 | 0.045 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.249 | 0.000 |

| Problem 452 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|-------------|---------|-------|-------|-------|--------|--------|-------|-------|--------|-------|
| grade | N/A | A | A | F | F | F | F(-2) | F | F | F(-1) |
| verified | N/A | Yes | Yes | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 79 | 97 | 87 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 1.23 | 1.10 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.405 | 0.090 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 17.982 | 0.000 |

| Problem 453 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|-------------|---------|-------|-------|-------|--------|--------|-------|-------|--------|-------|
| grade | N/A | A | A | A | F | A | F | F | F | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 115 | 133 | 72 | 139 | 0 | 219 | 0 | 0 | 87 | 259 |
| N.S. | 1 | 1.16 | 0.63 | 1.21 | 0.00 | 1.90 | 0.00 | 0.00 | 0.76 | 2.25 |
| time (sec) | N/A | 0.411 | 0.071 | 1.953 | 0.000 | 0.092 | 0.000 | 0.000 | 0.242 | 5.754 |

| Problem 454 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|-------------|---------|-------|-------|-------|--------|--------|-------|-------|--------|-------|
| grade | N/A | A | A | C | A | A | B | B | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 60 | 60 | 48 | 43 | 65 | 64 | 321 | 148 | 63 | 87 |
| N.S. | 1 | 1.00 | 0.80 | 0.72 | 1.08 | 1.07 | 5.35 | 2.47 | 1.05 | 1.45 |
| time (sec) | N/A | 0.318 | 0.050 | 0.983 | 0.039 | 0.103 | 0.810 | 0.180 | 0.243 | 0.063 |

2.3 Detailed conclusion table specific for Rubi results

The following table is specific to Rubi only. It gives additional statistics for each integral. the column **steps** is the number of steps used by Rubi to obtain the antiderivative. The **rules** column is the number of unique rules used. The **integrand size** column is the leaf size of the integrand. Finally the ratio $\frac{\text{number of rules}}{\text{integrand size}}$ is also given. The larger this ratio is, the harder the integral is to solve. In this test file, problem number [189] had the largest ratio of [.428570999999999980]

Table 2.1: Rubi specific breakdown of results for each integral

| # | grade | number of steps used | number of unique rules | normalized antiderivative leaf size | integrand leaf size | $\frac{\text{number of rules}}{\text{integrand leaf size}}$ |
|----|-------|----------------------|------------------------|-------------------------------------|---------------------|---|
| 1 | A | 2 | 2 | 1.00 | 28 | 0.071 |
| 2 | A | 2 | 2 | 1.00 | 26 | 0.077 |
| 3 | A | 2 | 2 | 1.00 | 20 | 0.100 |
| 4 | A | 2 | 2 | 1.00 | 28 | 0.071 |
| 5 | A | 2 | 2 | 1.00 | 28 | 0.071 |
| 6 | A | 2 | 2 | 1.00 | 28 | 0.071 |
| 7 | A | 2 | 2 | 1.00 | 28 | 0.071 |
| 8 | A | 2 | 2 | 1.00 | 28 | 0.071 |
| 9 | A | 2 | 2 | 1.00 | 28 | 0.071 |
| 10 | A | 2 | 2 | 1.00 | 30 | 0.067 |
| 11 | A | 2 | 2 | 1.00 | 28 | 0.071 |
| 12 | A | 2 | 2 | 1.00 | 22 | 0.091 |
| 13 | A | 2 | 2 | 1.00 | 30 | 0.067 |
| 14 | A | 2 | 2 | 1.00 | 30 | 0.067 |
| 15 | A | 2 | 2 | 1.00 | 30 | 0.067 |
| 16 | A | 2 | 2 | 1.00 | 30 | 0.067 |
| 17 | A | 2 | 2 | 1.00 | 30 | 0.067 |
| 18 | A | 2 | 2 | 1.00 | 30 | 0.067 |
| 19 | A | 2 | 2 | 1.00 | 30 | 0.067 |
| 20 | A | 2 | 2 | 1.00 | 30 | 0.067 |
| 21 | A | 2 | 2 | 1.00 | 30 | 0.067 |

Continued on next page

Table 2.1 – continued from previous page

| # | grade | number of steps used | number of unique rules | normalized antiderivative leaf size | integrand leaf size | $\frac{\text{number of rules}}{\text{integrand leaf size}}$ |
|----|-------|----------------------|------------------------|-------------------------------------|---------------------|---|
| 22 | A | 2 | 2 | 1.00 | 30 | 0.067 |
| 23 | A | 2 | 2 | 1.00 | 30 | 0.067 |
| 24 | A | 2 | 2 | 1.00 | 30 | 0.067 |
| 25 | A | 2 | 2 | 1.00 | 28 | 0.071 |
| 26 | A | 2 | 2 | 1.00 | 22 | 0.091 |
| 27 | A | 2 | 2 | 1.00 | 30 | 0.067 |
| 28 | A | 2 | 2 | 1.00 | 30 | 0.067 |
| 29 | A | 2 | 2 | 1.00 | 30 | 0.067 |
| 30 | A | 2 | 2 | 1.00 | 30 | 0.067 |
| 31 | A | 2 | 2 | 1.00 | 30 | 0.067 |
| 32 | A | 2 | 2 | 1.00 | 30 | 0.067 |
| 33 | A | 2 | 2 | 1.00 | 30 | 0.067 |
| 34 | A | 2 | 2 | 1.00 | 30 | 0.067 |
| 35 | A | 2 | 2 | 1.00 | 30 | 0.067 |
| 36 | A | 2 | 2 | 1.00 | 28 | 0.071 |
| 37 | A | 2 | 2 | 1.00 | 22 | 0.091 |
| 38 | A | 2 | 2 | 1.00 | 30 | 0.067 |
| 39 | A | 2 | 2 | 1.00 | 30 | 0.067 |
| 40 | A | 2 | 2 | 1.00 | 30 | 0.067 |
| 41 | A | 2 | 2 | 1.00 | 30 | 0.067 |
| 42 | A | 2 | 2 | 1.00 | 30 | 0.067 |
| 43 | A | 2 | 2 | 1.00 | 30 | 0.067 |
| 44 | A | 2 | 2 | 1.00 | 30 | 0.067 |
| 45 | A | 2 | 2 | 1.00 | 30 | 0.067 |
| 46 | A | 2 | 2 | 1.00 | 30 | 0.067 |
| 47 | A | 2 | 2 | 1.00 | 30 | 0.067 |
| 48 | A | 2 | 2 | 1.00 | 28 | 0.071 |
| 49 | A | 2 | 2 | 1.00 | 22 | 0.091 |
| 50 | A | 2 | 2 | 1.00 | 30 | 0.067 |
| 51 | A | 2 | 2 | 1.00 | 30 | 0.067 |
| 52 | A | 1 | 1 | 1.00 | 7 | 0.143 |
| 53 | A | 2 | 2 | 1.00 | 24 | 0.083 |

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Table 2.1 – continued from previous page

| # | grade | number of steps used | number of unique rules | normalized antiderivative leaf size | integrand leaf size | $\frac{\text{number of rules}}{\text{integrand leaf size}}$ |
|----|-------|----------------------|------------------------|-------------------------------------|---------------------|---|
| 54 | A | 4 | 4 | 1.00 | 24 | 0.167 |
| 55 | A | 4 | 4 | 1.00 | 24 | 0.167 |
| 56 | A | 4 | 4 | 1.00 | 24 | 0.167 |
| 57 | A | 4 | 4 | 1.00 | 22 | 0.182 |
| 58 | A | 2 | 2 | 1.00 | 16 | 0.125 |
| 59 | A | 4 | 4 | 1.02 | 24 | 0.167 |
| 60 | A | 4 | 4 | 1.00 | 24 | 0.167 |
| 61 | A | 4 | 4 | 1.00 | 24 | 0.167 |
| 62 | A | 3 | 3 | 1.00 | 24 | 0.125 |
| 63 | A | 4 | 4 | 1.00 | 24 | 0.167 |
| 64 | A | 4 | 4 | 1.00 | 24 | 0.167 |
| 65 | A | 4 | 4 | 1.00 | 24 | 0.167 |
| 66 | A | 4 | 4 | 1.00 | 26 | 0.154 |
| 67 | A | 4 | 4 | 1.00 | 26 | 0.154 |
| 68 | A | 4 | 4 | 1.00 | 26 | 0.154 |
| 69 | A | 4 | 4 | 1.00 | 26 | 0.154 |
| 70 | A | 4 | 4 | 1.00 | 26 | 0.154 |
| 71 | A | 4 | 4 | 1.00 | 24 | 0.167 |
| 72 | A | 2 | 2 | 1.00 | 18 | 0.111 |
| 73 | A | 4 | 4 | 1.00 | 26 | 0.154 |
| 74 | A | 4 | 4 | 1.00 | 26 | 0.154 |
| 75 | A | 4 | 4 | 1.00 | 26 | 0.154 |
| 76 | A | 4 | 4 | 1.00 | 26 | 0.154 |
| 77 | A | 4 | 4 | 1.00 | 26 | 0.154 |
| 78 | A | 3 | 3 | 1.00 | 26 | 0.115 |
| 79 | A | 4 | 4 | 1.00 | 26 | 0.154 |
| 80 | A | 5 | 5 | 1.15 | 26 | 0.192 |
| 81 | A | 4 | 4 | 1.00 | 26 | 0.154 |
| 82 | A | 4 | 4 | 1.00 | 26 | 0.154 |
| 83 | A | 4 | 4 | 1.00 | 26 | 0.154 |
| 84 | A | 4 | 4 | 1.00 | 26 | 0.154 |
| 85 | A | 4 | 4 | 1.00 | 26 | 0.154 |

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Table 2.1 – continued from previous page

| # | grade | number of steps used | number of unique rules | normalized antiderivative leaf size | integrand leaf size | $\frac{\text{number of rules}}{\text{integrand leaf size}}$ |
|-----|-------|----------------------|------------------------|-------------------------------------|---------------------|---|
| 86 | A | 4 | 4 | 1.00 | 26 | 0.154 |
| 87 | A | 4 | 4 | 1.00 | 26 | 0.154 |
| 88 | A | 4 | 4 | 1.00 | 26 | 0.154 |
| 89 | A | 4 | 4 | 1.00 | 26 | 0.154 |
| 90 | A | 4 | 4 | 1.00 | 26 | 0.154 |
| 91 | A | 4 | 4 | 1.00 | 24 | 0.167 |
| 92 | A | 2 | 2 | 1.00 | 18 | 0.111 |
| 93 | A | 4 | 4 | 1.00 | 26 | 0.154 |
| 94 | A | 4 | 4 | 1.00 | 26 | 0.154 |
| 95 | A | 4 | 4 | 1.00 | 26 | 0.154 |
| 96 | A | 4 | 4 | 1.00 | 26 | 0.154 |
| 97 | A | 4 | 4 | 1.00 | 26 | 0.154 |
| 98 | A | 4 | 4 | 1.00 | 26 | 0.154 |
| 99 | A | 4 | 4 | 1.00 | 26 | 0.154 |
| 100 | A | 3 | 3 | 1.00 | 26 | 0.115 |
| 101 | A | 4 | 4 | 1.00 | 26 | 0.154 |
| 102 | A | 5 | 5 | 1.15 | 26 | 0.192 |
| 103 | A | 6 | 6 | 1.22 | 26 | 0.231 |
| 104 | A | 4 | 4 | 1.00 | 26 | 0.154 |
| 105 | A | 4 | 4 | 1.00 | 26 | 0.154 |
| 106 | A | 4 | 4 | 1.00 | 26 | 0.154 |
| 107 | A | 4 | 4 | 1.00 | 26 | 0.154 |
| 108 | A | 4 | 4 | 1.00 | 26 | 0.154 |
| 109 | A | 4 | 4 | 0.96 | 26 | 0.154 |
| 110 | A | 4 | 4 | 1.00 | 26 | 0.154 |
| 111 | A | 4 | 4 | 1.00 | 26 | 0.154 |
| 112 | A | 4 | 4 | 1.00 | 24 | 0.167 |
| 113 | A | 2 | 2 | 1.00 | 18 | 0.111 |
| 114 | A | 4 | 4 | 1.00 | 26 | 0.154 |
| 115 | A | 4 | 4 | 1.00 | 26 | 0.154 |
| 116 | A | 4 | 4 | 1.00 | 26 | 0.154 |
| 117 | A | 4 | 4 | 1.00 | 26 | 0.154 |

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Table 2.1 – continued from previous page

| # | grade | number of steps used | number of unique rules | normalized antiderivative leaf size | integrand leaf size | $\frac{\text{number of rules}}{\text{integrand leaf size}}$ |
|-----|-------|----------------------|------------------------|-------------------------------------|---------------------|---|
| 118 | A | 4 | 4 | 0.98 | 26 | 0.154 |
| 119 | A | 4 | 4 | 1.00 | 26 | 0.154 |
| 120 | A | 4 | 4 | 1.00 | 26 | 0.154 |
| 121 | A | 4 | 4 | 1.00 | 26 | 0.154 |
| 122 | A | 3 | 3 | 1.00 | 26 | 0.115 |
| 123 | A | 4 | 4 | 1.00 | 24 | 0.167 |
| 124 | A | 2 | 2 | 1.00 | 18 | 0.111 |
| 125 | A | 4 | 4 | 1.00 | 26 | 0.154 |
| 126 | A | 4 | 4 | 1.00 | 26 | 0.154 |
| 127 | A | 4 | 4 | 1.00 | 26 | 0.154 |
| 128 | A | 4 | 4 | 0.98 | 26 | 0.154 |
| 129 | A | 4 | 4 | 1.00 | 26 | 0.154 |
| 130 | A | 4 | 4 | 1.00 | 26 | 0.154 |
| 131 | A | 4 | 4 | 1.00 | 26 | 0.154 |
| 132 | A | 3 | 3 | 1.00 | 26 | 0.115 |
| 133 | A | 4 | 4 | 1.00 | 26 | 0.154 |
| 134 | A | 4 | 4 | 1.00 | 26 | 0.154 |
| 135 | A | 4 | 4 | 1.00 | 24 | 0.167 |
| 136 | A | 2 | 2 | 1.00 | 18 | 0.111 |
| 137 | A | 4 | 4 | 1.00 | 26 | 0.154 |
| 138 | A | 4 | 4 | 1.00 | 26 | 0.154 |
| 139 | A | 4 | 4 | 1.00 | 26 | 0.154 |
| 140 | A | 4 | 4 | 1.00 | 18 | 0.222 |
| 141 | A | 4 | 4 | 1.00 | 18 | 0.222 |
| 142 | A | 4 | 4 | 1.00 | 16 | 0.250 |
| 143 | A | 4 | 4 | 1.00 | 18 | 0.222 |
| 144 | A | 4 | 4 | 1.00 | 18 | 0.222 |
| 145 | A | 4 | 4 | 1.00 | 18 | 0.222 |
| 146 | A | 4 | 4 | 1.00 | 26 | 0.154 |
| 147 | A | 4 | 4 | 1.00 | 26 | 0.154 |
| 148 | A | 4 | 4 | 1.00 | 26 | 0.154 |
| 149 | A | 4 | 4 | 1.00 | 26 | 0.154 |

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Table 2.1 – continued from previous page

| # | grade | number of steps used | number of unique rules | normalized antiderivative leaf size | integrand leaf size | $\frac{\text{number of rules}}{\text{integrand leaf size}}$ |
|-----|-------|----------------------|------------------------|-------------------------------------|---------------------|---|
| 150 | A | 4 | 4 | 1.00 | 26 | 0.154 |
| 151 | A | 4 | 4 | 1.00 | 26 | 0.154 |
| 152 | A | 4 | 4 | 1.00 | 26 | 0.154 |
| 153 | A | 4 | 4 | 1.00 | 26 | 0.154 |
| 154 | A | 4 | 4 | 1.00 | 28 | 0.143 |
| 155 | A | 4 | 4 | 1.00 | 28 | 0.143 |
| 156 | A | 4 | 4 | 1.00 | 28 | 0.143 |
| 157 | A | 4 | 4 | 1.00 | 28 | 0.143 |
| 158 | A | 4 | 4 | 1.00 | 28 | 0.143 |
| 159 | A | 4 | 4 | 1.00 | 28 | 0.143 |
| 160 | A | 4 | 4 | 1.00 | 28 | 0.143 |
| 161 | A | 4 | 4 | 1.00 | 28 | 0.143 |
| 162 | A | 4 | 4 | 1.00 | 28 | 0.143 |
| 163 | A | 4 | 4 | 1.00 | 28 | 0.143 |
| 164 | A | 4 | 4 | 1.00 | 28 | 0.143 |
| 165 | A | 4 | 4 | 1.00 | 28 | 0.143 |
| 166 | A | 4 | 4 | 1.00 | 28 | 0.143 |
| 167 | A | 4 | 4 | 1.00 | 28 | 0.143 |
| 168 | A | 4 | 4 | 1.00 | 28 | 0.143 |
| 169 | A | 4 | 4 | 1.00 | 28 | 0.143 |
| 170 | A | 10 | 9 | 1.10 | 28 | 0.321 |
| 171 | A | 9 | 8 | 1.09 | 28 | 0.286 |
| 172 | A | 8 | 7 | 1.10 | 28 | 0.250 |
| 173 | A | 7 | 6 | 1.08 | 28 | 0.214 |
| 174 | A | 6 | 5 | 1.00 | 28 | 0.179 |
| 175 | A | 6 | 5 | 1.00 | 28 | 0.179 |
| 176 | A | 7 | 6 | 1.14 | 28 | 0.214 |
| 177 | A | 8 | 7 | 1.20 | 28 | 0.250 |
| 178 | A | 9 | 8 | 1.23 | 28 | 0.286 |
| 179 | A | 11 | 10 | 1.06 | 28 | 0.357 |
| 180 | A | 10 | 9 | 1.08 | 28 | 0.321 |
| 181 | A | 9 | 8 | 1.08 | 28 | 0.286 |

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Table 2.1 – continued from previous page

| # | grade | number of steps used | number of unique rules | normalized antiderivative leaf size | integrand leaf size | $\frac{\text{number of rules}}{\text{integrand leaf size}}$ |
|-----|-------|----------------------|------------------------|-------------------------------------|---------------------|---|
| 182 | A | 8 | 7 | 1.06 | 28 | 0.250 |
| 183 | A | 8 | 7 | 1.03 | 28 | 0.250 |
| 184 | A | 8 | 7 | 1.05 | 28 | 0.250 |
| 185 | A | 8 | 7 | 1.14 | 28 | 0.250 |
| 186 | A | 9 | 8 | 1.18 | 28 | 0.286 |
| 187 | A | 10 | 9 | 1.20 | 28 | 0.321 |
| 188 | A | 11 | 10 | 1.21 | 28 | 0.357 |
| 189 | A | 13 | 12 | 1.10 | 28 | 0.429 |
| 190 | A | 12 | 11 | 1.11 | 28 | 0.393 |
| 191 | A | 11 | 10 | 1.12 | 28 | 0.357 |
| 192 | A | 10 | 9 | 1.11 | 28 | 0.321 |
| 193 | A | 10 | 9 | 1.09 | 28 | 0.321 |
| 194 | A | 10 | 9 | 1.10 | 28 | 0.321 |
| 195 | A | 10 | 9 | 1.12 | 28 | 0.321 |
| 196 | A | 10 | 9 | 1.13 | 28 | 0.321 |
| 197 | A | 10 | 9 | 1.22 | 28 | 0.321 |
| 198 | A | 11 | 10 | 1.24 | 28 | 0.357 |
| 199 | A | 12 | 11 | 1.25 | 28 | 0.393 |
| 200 | A | 13 | 12 | 1.25 | 28 | 0.429 |
| 201 | A | 1 | 1 | 1.00 | 32 | 0.031 |
| 202 | A | 1 | 1 | 1.00 | 30 | 0.033 |
| 203 | A | 1 | 1 | 1.00 | 32 | 0.031 |
| 204 | A | 1 | 1 | 1.00 | 32 | 0.031 |
| 205 | A | 1 | 1 | 1.00 | 32 | 0.031 |
| 206 | A | 2 | 2 | 1.00 | 32 | 0.062 |
| 207 | A | 2 | 2 | 1.00 | 24 | 0.083 |
| 208 | A | 2 | 2 | 1.02 | 32 | 0.062 |
| 209 | A | 2 | 2 | 0.97 | 32 | 0.062 |
| 210 | A | 2 | 2 | 0.97 | 32 | 0.062 |
| 211 | A | 1 | 1 | 1.00 | 32 | 0.031 |
| 212 | A | 1 | 1 | 1.00 | 30 | 0.033 |
| 213 | A | 1 | 1 | 1.00 | 32 | 0.031 |

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Table 2.1 – continued from previous page

| # | grade | number of steps used | number of unique rules | normalized antiderivative leaf size | integrand leaf size | $\frac{\text{number of rules}}{\text{integrand leaf size}}$ |
|-----|-------|----------------------|------------------------|-------------------------------------|---------------------|---|
| 214 | A | 1 | 1 | 1.00 | 32 | 0.031 |
| 215 | A | 1 | 1 | 1.00 | 32 | 0.031 |
| 216 | A | 1 | 1 | 1.00 | 32 | 0.031 |
| 217 | A | 2 | 2 | 1.00 | 32 | 0.062 |
| 218 | A | 2 | 2 | 1.00 | 24 | 0.083 |
| 219 | A | 2 | 2 | 1.00 | 32 | 0.062 |
| 220 | A | 2 | 2 | 1.00 | 32 | 0.062 |
| 221 | A | 2 | 2 | 0.95 | 32 | 0.062 |
| 222 | A | 2 | 2 | 0.95 | 32 | 0.062 |
| 223 | A | 1 | 1 | 1.00 | 30 | 0.033 |
| 224 | A | 1 | 1 | 1.00 | 32 | 0.031 |
| 225 | A | 1 | 1 | 1.00 | 32 | 0.031 |
| 226 | A | 1 | 1 | 1.00 | 32 | 0.031 |
| 227 | A | 1 | 1 | 1.00 | 32 | 0.031 |
| 228 | A | 1 | 1 | 1.00 | 32 | 0.031 |
| 229 | A | 2 | 2 | 1.00 | 32 | 0.062 |
| 230 | A | 2 | 2 | 1.00 | 24 | 0.083 |
| 231 | A | 2 | 2 | 1.00 | 32 | 0.062 |
| 232 | A | 2 | 2 | 1.00 | 32 | 0.062 |
| 233 | A | 2 | 2 | 1.05 | 32 | 0.062 |
| 234 | A | 2 | 2 | 1.00 | 32 | 0.062 |
| 235 | A | 1 | 1 | 1.00 | 32 | 0.031 |
| 236 | A | 1 | 1 | 1.00 | 32 | 0.031 |
| 237 | A | 1 | 1 | 1.00 | 30 | 0.033 |
| 238 | A | 1 | 1 | 1.00 | 32 | 0.031 |
| 239 | A | 1 | 1 | 1.00 | 32 | 0.031 |
| 240 | A | 2 | 2 | 0.93 | 32 | 0.062 |
| 241 | A | 2 | 2 | 0.97 | 32 | 0.062 |
| 242 | A | 2 | 2 | 1.00 | 24 | 0.083 |
| 243 | A | 2 | 2 | 1.00 | 32 | 0.062 |
| 244 | A | 2 | 2 | 1.00 | 32 | 0.062 |
| 245 | A | 1 | 1 | 1.00 | 32 | 0.031 |

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Table 2.1 – continued from previous page

| # | grade | number of steps used | number of unique rules | normalized antiderivative leaf size | integrand leaf size | $\frac{\text{number of rules}}{\text{integrand leaf size}}$ |
|-----|-------|----------------------|------------------------|-------------------------------------|---------------------|---|
| 246 | A | 1 | 1 | 1.00 | 32 | 0.031 |
| 247 | A | 1 | 1 | 1.00 | 30 | 0.033 |
| 248 | A | 1 | 1 | 1.00 | 32 | 0.031 |
| 249 | A | 1 | 1 | 1.00 | 32 | 0.031 |
| 250 | A | 2 | 2 | 1.05 | 32 | 0.062 |
| 251 | A | 2 | 2 | 1.00 | 32 | 0.062 |
| 252 | A | 1 | 1 | 1.00 | 24 | 0.042 |
| 253 | A | 2 | 2 | 1.00 | 32 | 0.062 |
| 254 | A | 1 | 1 | 1.00 | 32 | 0.031 |
| 255 | A | 1 | 1 | 1.00 | 32 | 0.031 |
| 256 | A | 1 | 1 | 1.00 | 32 | 0.031 |
| 257 | A | 1 | 1 | 1.00 | 30 | 0.033 |
| 258 | A | 1 | 1 | 1.00 | 32 | 0.031 |
| 259 | A | 2 | 2 | 1.05 | 32 | 0.062 |
| 260 | A | 2 | 2 | 1.00 | 32 | 0.062 |
| 261 | A | 2 | 2 | 1.00 | 32 | 0.062 |
| 262 | A | 1 | 1 | 1.00 | 24 | 0.042 |
| 263 | A | 2 | 2 | 1.00 | 32 | 0.062 |
| 264 | A | 1 | 1 | 1.00 | 39 | 0.026 |
| 265 | A | 4 | 4 | 0.72 | 28 | 0.143 |
| 266 | A | 4 | 4 | 0.72 | 28 | 0.143 |
| 267 | A | 4 | 4 | 0.72 | 28 | 0.143 |
| 268 | A | 3 | 3 | 1.00 | 26 | 0.115 |
| 269 | A | 2 | 2 | 1.00 | 20 | 0.100 |
| 270 | A | 4 | 4 | 0.68 | 28 | 0.143 |
| 271 | A | 4 | 4 | 0.69 | 28 | 0.143 |
| 272 | A | 3 | 3 | 1.00 | 28 | 0.107 |
| 273 | A | 4 | 4 | 0.72 | 28 | 0.143 |
| 274 | A | 4 | 4 | 0.72 | 28 | 0.143 |
| 275 | A | 4 | 4 | 0.72 | 28 | 0.143 |
| 276 | A | 4 | 4 | 0.60 | 28 | 0.143 |
| 277 | A | 4 | 4 | 0.60 | 28 | 0.143 |

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Table 2.1 – continued from previous page

| # | grade | number of steps used | number of unique rules | normalized antiderivative leaf size | integrand leaf size | $\frac{\text{number of rules}}{\text{integrand leaf size}}$ |
|-----|-------|----------------------|------------------------|-------------------------------------|---------------------|---|
| 278 | A | 4 | 4 | 0.70 | 28 | 0.143 |
| 279 | A | 4 | 4 | 0.82 | 28 | 0.143 |
| 280 | A | 3 | 3 | 1.00 | 26 | 0.115 |
| 281 | A | 2 | 2 | 1.00 | 20 | 0.100 |
| 282 | A | 4 | 4 | 0.61 | 28 | 0.143 |
| 283 | A | 4 | 4 | 0.56 | 28 | 0.143 |
| 284 | A | 4 | 4 | 0.57 | 28 | 0.143 |
| 285 | A | 4 | 4 | 0.59 | 28 | 0.143 |
| 286 | A | 3 | 3 | 1.04 | 28 | 0.107 |
| 287 | A | 4 | 4 | 0.88 | 28 | 0.143 |
| 288 | A | 4 | 4 | 0.84 | 28 | 0.143 |
| 289 | A | 4 | 4 | 0.60 | 28 | 0.143 |
| 290 | A | 4 | 4 | 0.60 | 28 | 0.143 |
| 291 | A | 4 | 4 | 0.65 | 28 | 0.143 |
| 292 | A | 4 | 4 | 0.67 | 28 | 0.143 |
| 293 | A | 4 | 4 | 0.70 | 28 | 0.143 |
| 294 | A | 4 | 4 | 0.74 | 28 | 0.143 |
| 295 | A | 3 | 3 | 1.00 | 26 | 0.115 |
| 296 | A | 2 | 2 | 1.00 | 20 | 0.100 |
| 297 | A | 4 | 4 | 0.59 | 28 | 0.143 |
| 298 | A | 4 | 4 | 0.54 | 28 | 0.143 |
| 299 | A | 4 | 4 | 0.55 | 28 | 0.143 |
| 300 | A | 4 | 4 | 0.54 | 28 | 0.143 |
| 301 | A | 4 | 4 | 0.54 | 28 | 0.143 |
| 302 | A | 4 | 4 | 0.55 | 28 | 0.143 |
| 303 | A | 3 | 3 | 1.04 | 28 | 0.107 |
| 304 | A | 4 | 4 | 0.88 | 28 | 0.143 |
| 305 | A | 5 | 5 | 0.87 | 28 | 0.179 |
| 306 | A | 6 | 6 | 0.87 | 28 | 0.214 |
| 307 | A | 4 | 4 | 0.56 | 28 | 0.143 |
| 308 | A | 4 | 4 | 0.56 | 28 | 0.143 |
| 309 | A | 4 | 4 | 0.55 | 28 | 0.143 |

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Table 2.1 – continued from previous page

| # | grade | number of steps used | number of unique rules | normalized antiderivative leaf size | integrand leaf size | $\frac{\text{number of rules}}{\text{integrand leaf size}}$ |
|-----|-------|----------------------|------------------------|-------------------------------------|---------------------|---|
| 310 | A | 4 | 4 | 0.57 | 28 | 0.143 |
| 311 | A | 4 | 4 | 0.60 | 28 | 0.143 |
| 312 | A | 3 | 3 | 1.00 | 26 | 0.115 |
| 313 | A | 2 | 2 | 1.00 | 20 | 0.100 |
| 314 | A | 4 | 4 | 1.22 | 28 | 0.143 |
| 315 | A | 4 | 4 | 0.63 | 28 | 0.143 |
| 316 | A | 4 | 4 | 0.59 | 28 | 0.143 |
| 317 | A | 4 | 4 | 0.57 | 28 | 0.143 |
| 318 | A | 4 | 4 | 0.61 | 28 | 0.143 |
| 319 | A | 4 | 4 | 0.65 | 28 | 0.143 |
| 320 | A | 4 | 4 | 0.73 | 28 | 0.143 |
| 321 | A | 2 | 2 | 1.00 | 26 | 0.077 |
| 322 | A | 1 | 1 | 1.00 | 20 | 0.050 |
| 323 | A | 4 | 4 | 0.65 | 28 | 0.143 |
| 324 | A | 4 | 4 | 0.62 | 28 | 0.143 |
| 325 | A | 4 | 4 | 0.61 | 28 | 0.143 |
| 326 | A | 4 | 4 | 0.60 | 28 | 0.143 |
| 327 | A | 4 | 4 | 0.62 | 28 | 0.143 |
| 328 | A | 4 | 4 | 0.66 | 28 | 0.143 |
| 329 | A | 3 | 3 | 1.04 | 28 | 0.107 |
| 330 | A | 4 | 4 | 0.73 | 28 | 0.143 |
| 331 | A | 2 | 2 | 1.00 | 26 | 0.077 |
| 332 | A | 1 | 1 | 1.00 | 20 | 0.050 |
| 333 | A | 4 | 4 | 0.62 | 28 | 0.143 |
| 334 | A | 4 | 4 | 0.60 | 28 | 0.143 |
| 335 | A | 4 | 4 | 0.60 | 28 | 0.143 |
| 336 | A | 3 | 3 | 1.00 | 20 | 0.150 |
| 337 | A | 3 | 3 | 1.00 | 20 | 0.150 |
| 338 | A | 3 | 3 | 1.00 | 20 | 0.150 |
| 339 | A | 3 | 3 | 1.00 | 20 | 0.150 |
| 340 | A | 2 | 2 | 1.00 | 20 | 0.100 |
| 341 | A | 2 | 2 | 1.00 | 20 | 0.100 |

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Table 2.1 – continued from previous page

| # | grade | number of steps used | number of unique rules | normalized antiderivative leaf size | integrand leaf size | $\frac{\text{number of rules}}{\text{integrand leaf size}}$ |
|-----|-------|----------------------|------------------------|-------------------------------------|---------------------|---|
| 342 | A | 2 | 2 | 1.00 | 20 | 0.100 |
| 343 | A | 4 | 4 | 1.23 | 31 | 0.129 |
| 344 | A | 4 | 4 | 0.73 | 30 | 0.133 |
| 345 | A | 4 | 4 | 0.73 | 30 | 0.133 |
| 346 | A | 4 | 4 | 0.73 | 30 | 0.133 |
| 347 | A | 4 | 4 | 0.72 | 30 | 0.133 |
| 348 | A | 4 | 4 | 0.72 | 30 | 0.133 |
| 349 | A | 4 | 4 | 0.72 | 30 | 0.133 |
| 350 | A | 4 | 4 | 0.73 | 30 | 0.133 |
| 351 | A | 4 | 4 | 0.62 | 30 | 0.133 |
| 352 | A | 4 | 4 | 0.62 | 30 | 0.133 |
| 353 | A | 4 | 4 | 0.62 | 30 | 0.133 |
| 354 | A | 4 | 4 | 0.61 | 30 | 0.133 |
| 355 | A | 4 | 4 | 0.60 | 30 | 0.133 |
| 356 | A | 4 | 4 | 0.61 | 30 | 0.133 |
| 357 | A | 4 | 4 | 0.60 | 30 | 0.133 |
| 358 | A | 4 | 4 | 0.61 | 30 | 0.133 |
| 359 | A | 4 | 4 | 0.62 | 30 | 0.133 |
| 360 | A | 4 | 4 | 0.58 | 30 | 0.133 |
| 361 | A | 4 | 4 | 0.58 | 30 | 0.133 |
| 362 | A | 4 | 4 | 0.58 | 30 | 0.133 |
| 363 | A | 4 | 4 | 0.58 | 30 | 0.133 |
| 364 | A | 4 | 4 | 0.57 | 30 | 0.133 |
| 365 | A | 4 | 4 | 0.57 | 30 | 0.133 |
| 366 | A | 4 | 4 | 0.58 | 30 | 0.133 |
| 367 | A | 4 | 4 | 0.57 | 30 | 0.133 |
| 368 | A | 4 | 4 | 0.57 | 30 | 0.133 |
| 369 | A | 4 | 4 | 0.58 | 30 | 0.133 |
| 370 | A | 4 | 4 | 0.58 | 30 | 0.133 |
| 371 | A | 9 | 8 | 0.67 | 30 | 0.267 |
| 372 | A | 8 | 7 | 0.69 | 30 | 0.233 |
| 373 | A | 7 | 6 | 0.73 | 30 | 0.200 |

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Table 2.1 – continued from previous page

| # | grade | number of steps used | number of unique rules | normalized antiderivative leaf size | integrand leaf size | $\frac{\text{number of rules}}{\text{integrand leaf size}}$ |
|-----|-------|----------------------|------------------------|-------------------------------------|---------------------|---|
| 374 | A | 6 | 5 | 0.79 | 30 | 0.167 |
| 375 | A | 5 | 4 | 1.00 | 30 | 0.133 |
| 376 | A | 6 | 5 | 0.80 | 30 | 0.167 |
| 377 | A | 7 | 6 | 0.78 | 30 | 0.200 |
| 378 | A | 8 | 7 | 0.76 | 30 | 0.233 |
| 379 | A | 10 | 9 | 0.68 | 30 | 0.300 |
| 380 | A | 9 | 8 | 0.70 | 30 | 0.267 |
| 381 | A | 8 | 7 | 0.74 | 30 | 0.233 |
| 382 | A | 7 | 6 | 0.81 | 30 | 0.200 |
| 383 | A | 7 | 6 | 0.80 | 30 | 0.200 |
| 384 | A | 7 | 6 | 0.86 | 30 | 0.200 |
| 385 | A | 8 | 7 | 0.83 | 30 | 0.233 |
| 386 | A | 9 | 8 | 0.80 | 30 | 0.267 |
| 387 | A | 10 | 9 | 0.78 | 30 | 0.300 |
| 388 | A | 12 | 11 | 0.68 | 30 | 0.367 |
| 389 | A | 11 | 10 | 0.70 | 30 | 0.333 |
| 390 | A | 10 | 9 | 0.73 | 30 | 0.300 |
| 391 | A | 9 | 8 | 0.77 | 30 | 0.267 |
| 392 | A | 9 | 8 | 0.76 | 30 | 0.267 |
| 393 | A | 9 | 8 | 0.79 | 30 | 0.267 |
| 394 | A | 9 | 8 | 0.81 | 30 | 0.267 |
| 395 | A | 9 | 8 | 0.86 | 30 | 0.267 |
| 396 | A | 10 | 9 | 0.84 | 30 | 0.300 |
| 397 | A | 11 | 10 | 0.82 | 30 | 0.333 |
| 398 | A | 12 | 11 | 0.80 | 30 | 0.367 |
| 399 | A | 4 | 4 | 1.00 | 33 | 0.121 |
| 400 | A | 4 | 4 | 1.00 | 33 | 0.121 |
| 401 | A | 4 | 4 | 1.00 | 31 | 0.129 |
| 402 | A | 1 | 1 | 1.00 | 10 | 0.100 |
| 403 | A | 4 | 4 | 1.00 | 33 | 0.121 |
| 404 | A | 4 | 4 | 1.00 | 33 | 0.121 |
| 405 | A | 4 | 4 | 1.00 | 33 | 0.121 |

Continued on next page

Table 2.1 – continued from previous page

| # | grade | number of steps used | number of unique rules | normalized antiderivative leaf size | integrand leaf size | $\frac{\text{number of rules}}{\text{integrand leaf size}}$ |
|-----|-------|----------------------|------------------------|-------------------------------------|---------------------|---|
| 406 | A | 4 | 4 | 1.14 | 35 | 0.114 |
| 407 | A | 4 | 4 | 1.15 | 35 | 0.114 |
| 408 | A | 4 | 4 | 1.00 | 35 | 0.114 |
| 409 | A | 4 | 4 | 1.13 | 35 | 0.114 |
| 410 | A | 4 | 4 | 1.09 | 35 | 0.114 |
| 411 | A | 4 | 4 | 1.00 | 26 | 0.154 |
| 412 | A | 4 | 4 | 1.00 | 26 | 0.154 |
| 413 | A | 4 | 4 | 1.00 | 24 | 0.167 |
| 414 | A | 3 | 3 | 1.00 | 26 | 0.115 |
| 415 | A | 3 | 3 | 1.00 | 26 | 0.115 |
| 416 | A | 3 | 3 | 1.00 | 26 | 0.115 |
| 417 | A | 4 | 4 | 0.60 | 28 | 0.143 |
| 418 | A | 4 | 4 | 0.63 | 28 | 0.143 |
| 419 | A | 4 | 4 | 0.74 | 28 | 0.143 |
| 420 | A | 3 | 3 | 1.00 | 28 | 0.107 |
| 421 | A | 3 | 3 | 1.00 | 28 | 0.107 |
| 422 | A | 3 | 3 | 1.00 | 28 | 0.107 |
| 423 | A | 1 | 1 | 1.00 | 30 | 0.033 |
| 424 | A | 1 | 1 | 1.00 | 28 | 0.036 |
| 425 | A | 1 | 1 | 1.00 | 30 | 0.033 |
| 426 | A | 1 | 1 | 1.00 | 30 | 0.033 |
| 427 | A | 1 | 1 | 1.00 | 30 | 0.033 |
| 428 | A | 3 | 3 | 1.22 | 30 | 0.100 |
| 429 | A | 3 | 3 | 1.22 | 30 | 0.100 |
| 430 | A | 2 | 2 | 1.24 | 22 | 0.091 |
| 431 | A | 3 | 3 | 1.10 | 30 | 0.100 |
| 432 | A | 3 | 3 | 1.22 | 30 | 0.100 |
| 433 | A | 3 | 3 | 0.97 | 26 | 0.115 |
| 434 | A | 3 | 3 | 1.03 | 26 | 0.115 |
| 435 | A | 3 | 3 | 1.05 | 24 | 0.125 |
| 436 | A | 2 | 2 | 1.12 | 18 | 0.111 |
| 437 | A | 2 | 2 | 1.10 | 26 | 0.077 |

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Table 2.1 – continued from previous page

| # | grade | number of steps used | number of unique rules | normalized antiderivative leaf size | integrand leaf size | $\frac{\text{number of rules}}{\text{integrand leaf size}}$ |
|-----|-------|----------------------|------------------------|-------------------------------------|---------------------|---|
| 438 | A | 2 | 2 | 1.04 | 26 | 0.077 |
| 439 | A | 2 | 2 | 1.03 | 26 | 0.077 |
| 440 | A | 3 | 3 | 1.00 | 28 | 0.107 |
| 441 | A | 3 | 3 | 1.00 | 28 | 0.107 |
| 442 | A | 3 | 3 | 1.00 | 28 | 0.107 |
| 443 | A | 3 | 3 | 1.00 | 28 | 0.107 |
| 444 | A | 3 | 3 | 1.00 | 28 | 0.107 |
| 445 | A | 3 | 3 | 1.00 | 30 | 0.100 |
| 446 | A | 3 | 3 | 1.00 | 32 | 0.094 |
| 447 | A | 3 | 3 | 1.00 | 34 | 0.088 |
| 448 | A | 3 | 3 | 1.00 | 36 | 0.083 |
| 449 | A | 2 | 2 | 1.00 | 16 | 0.125 |
| 450 | A | 3 | 3 | 1.14 | 37 | 0.081 |
| 451 | A | 3 | 3 | 1.10 | 26 | 0.115 |
| 452 | A | 3 | 3 | 1.23 | 28 | 0.107 |
| 453 | A | 3 | 3 | 1.16 | 30 | 0.100 |
| 454 | A | 3 | 3 | 1.00 | 18 | 0.167 |

CHAPTER 3

LISTING OF INTEGRALS

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|------|---|-----|
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| 3.42 | $\int \frac{(d+ex)^7}{(cd^2+2cde x+ce^2x^2)^3} dx$ | 399 |
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| 3.44 | $\int \frac{(d+ex)^5}{(cd^2+2cde x+ce^2x^2)^3} dx$ | 409 |
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| 3.53 | $\int \frac{d+ex}{d^2+2dex+e^2x^2} dx$ | 457 |
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| | | |
|------|---|-----|
| 3.56 | $\int (d + ex)^2 (a^2 + 2abx + b^2x^2) dx$ | 476 |
| 3.57 | $\int (d + ex) (a^2 + 2abx + b^2x^2) dx$ | 482 |
| 3.58 | $\int (a^2 + 2abx + b^2x^2) dx$ | 488 |
| 3.59 | $\int \frac{a^2 + 2abx + b^2x^2}{d + ex} dx$ | 493 |
| 3.60 | $\int \frac{a^2 + 2abx + b^2x^2}{(d + ex)^2} dx$ | 499 |
| 3.61 | $\int \frac{a^2 + 2abx + b^2x^2}{(d + ex)^3} dx$ | 505 |
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| 3.81 | $\int \frac{(a^2 + 2abx + b^2x^2)^2}{(d + ex)^9} dx$ | 648 |
| 3.82 | $\int \frac{(a^2 + 2abx + b^2x^2)^2}{(d + ex)^{10}} dx$ | 655 |
| 3.83 | $\int \frac{(a^2 + 2abx + b^2x^2)^2}{(d + ex)^{11}} dx$ | 662 |
| 3.84 | $\int (d + ex)^8 (a^2 + 2abx + b^2x^2)^3 dx$ | 670 |
| 3.85 | $\int (d + ex)^7 (a^2 + 2abx + b^2x^2)^3 dx$ | 682 |
| 3.86 | $\int (d + ex)^6 (a^2 + 2abx + b^2x^2)^3 dx$ | 696 |
| 3.87 | $\int (d + ex)^5 (a^2 + 2abx + b^2x^2)^3 dx$ | 709 |
| 3.88 | $\int (d + ex)^4 (a^2 + 2abx + b^2x^2)^3 dx$ | 722 |
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|-------|---|-----|
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| 3.91 | $\int (d + ex) (a^2 + 2abx + b^2x^2)^3 dx$ | 751 |
| 3.92 | $\int (a^2 + 2abx + b^2x^2)^3 dx$ | 758 |
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| 3.100 | $\int \frac{(a^2 + 2abx + b^2x^2)^3}{(d + ex)^8} dx$ | 822 |
| 3.101 | $\int \frac{(a^2 + 2abx + b^2x^2)^3}{(d + ex)^9} dx$ | 829 |
| 3.102 | $\int \frac{(a^2 + 2abx + b^2x^2)^3}{(d + ex)^{10}} dx$ | 837 |
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| 3.113 | $\int \frac{1}{a^2 + 2abx + b^2x^2} dx$ | 917 |
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| 3.150 | $\int \frac{a^2+2abx+b^2x^2}{\sqrt{d+ex}} dx$ | 1179 |
| 3.151 | $\int \frac{a^2+2abx+b^2x^2}{(d+ex)^{3/2}} dx$ | 1185 |
| 3.152 | $\int \frac{a^2+2abx+b^2x^2}{(d+ex)^{5/2}} dx$ | 1191 |
| 3.153 | $\int \frac{a^2+2abx+b^2x^2}{(d+ex)^{7/2}} dx$ | 1197 |
| 3.154 | $\int (d+ex)^{7/2}(a^2+2abx+b^2x^2)^2 dx$ | 1203 |

| | | |
|-------|---|------|
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| 3.156 | $\int (d+ex)^{3/2} (a^2+2abx+b^2x^2)^2 dx$ | 1220 |
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| 3.161 | $\int \frac{(a^2+2abx+b^2x^2)^2}{(d+ex)^{7/2}} dx$ | 1258 |
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| 3.163 | $\int (d+ex)^{5/2} (a^2+2abx+b^2x^2)^3 dx$ | 1274 |
| 3.164 | $\int (d+ex)^{3/2} (a^2+2abx+b^2x^2)^3 dx$ | 1283 |
| 3.165 | $\int \sqrt{d+ex} (a^2+2abx+b^2x^2)^3 dx$ | 1292 |
| 3.166 | $\int \frac{(a^2+2abx+b^2x^2)^3}{\sqrt{d+ex}} dx$ | 1301 |
| 3.167 | $\int \frac{(a^2+2abx+b^2x^2)^3}{(d+ex)^{3/2}} dx$ | 1310 |
| 3.168 | $\int \frac{(a^2+2abx+b^2x^2)^3}{(d+ex)^{5/2}} dx$ | 1318 |
| 3.169 | $\int \frac{(a^2+2abx+b^2x^2)^3}{(d+ex)^{7/2}} dx$ | 1326 |
| 3.170 | $\int \frac{(d+ex)^{9/2}}{a^2+2abx+b^2x^2} dx$ | 1334 |
| 3.171 | $\int \frac{(d+ex)^{7/2}}{a^2+2abx+b^2x^2} dx$ | 1345 |
| 3.172 | $\int \frac{(d+ex)^{5/2}}{a^2+2abx+b^2x^2} dx$ | 1354 |
| 3.173 | $\int \frac{(d+ex)^{3/2}}{a^2+2abx+b^2x^2} dx$ | 1362 |
| 3.174 | $\int \frac{\sqrt{d+ex}}{a^2+2abx+b^2x^2} dx$ | 1369 |
| 3.175 | $\int \frac{1}{\sqrt{d+ex}(a^2+2abx+b^2x^2)} dx$ | 1375 |
| 3.176 | $\int \frac{1}{(d+ex)^{3/2}(a^2+2abx+b^2x^2)} dx$ | 1382 |
| 3.177 | $\int \frac{1}{(d+ex)^{5/2}(a^2+2abx+b^2x^2)} dx$ | 1389 |
| 3.178 | $\int \frac{1}{(d+ex)^{7/2}(a^2+2abx+b^2x^2)} dx$ | 1398 |
| 3.179 | $\int \frac{(d+ex)^{11/2}}{(a^2+2abx+b^2x^2)^2} dx$ | 1407 |
| 3.180 | $\int \frac{(d+ex)^{9/2}}{(a^2+2abx+b^2x^2)^2} dx$ | 1421 |
| 3.181 | $\int \frac{(d+ex)^{7/2}}{(a^2+2abx+b^2x^2)^2} dx$ | 1432 |
| 3.182 | $\int \frac{(d+ex)^{5/2}}{(a^2+2abx+b^2x^2)^2} dx$ | 1441 |
| 3.183 | $\int \frac{(d+ex)^{3/2}}{(a^2+2abx+b^2x^2)^2} dx$ | 1449 |
| 3.184 | $\int \frac{\sqrt{d+ex}}{(a^2+2abx+b^2x^2)^2} dx$ | 1457 |
| 3.185 | $\int \frac{1}{\sqrt{d+ex}(a^2+2abx+b^2x^2)^2} dx$ | 1465 |
| 3.186 | $\int \frac{1}{(d+ex)^{3/2}(a^2+2abx+b^2x^2)^2} dx$ | 1473 |

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| 3.187 | $\int \frac{1}{(d+ex)^{5/2}(a^2+2abx+b^2x^2)^2} dx$ | 1483 |
| 3.188 | $\int \frac{1}{(d+ex)^{7/2}(a^2+2abx+b^2x^2)^2} dx$ | 1494 |
| 3.189 | $\int \frac{(d+ex)^{15/2}}{(a^2+2abx+b^2x^2)^3} dx$ | 1508 |
| 3.190 | $\int \frac{(d+ex)^{13/2}}{(a^2+2abx+b^2x^2)^3} dx$ | 1528 |
| 3.191 | $\int \frac{(d+ex)^{11/2}}{(a^2+2abx+b^2x^2)^3} dx$ | 1546 |
| 3.192 | $\int \frac{(d+ex)^{9/2}}{(a^2+2abx+b^2x^2)^3} dx$ | 1560 |
| 3.193 | $\int \frac{(d+ex)^{7/2}}{(a^2+2abx+b^2x^2)^3} dx$ | 1570 |
| 3.194 | $\int \frac{(d+ex)^{5/2}}{(a^2+2abx+b^2x^2)^3} dx$ | 1580 |
| 3.195 | $\int \frac{(d+ex)^{3/2}}{(a^2+2abx+b^2x^2)^3} dx$ | 1590 |
| 3.196 | $\int \frac{\sqrt{d+ex}}{(a^2+2abx+b^2x^2)^3} dx$ | 1601 |
| 3.197 | $\int \frac{1}{\sqrt{d+ex}(a^2+2abx+b^2x^2)^3} dx$ | 1612 |
| 3.198 | $\int \frac{1}{(d+ex)^{3/2}(a^2+2abx+b^2x^2)^3} dx$ | 1624 |
| 3.199 | $\int \frac{1}{(d+ex)^{5/2}(a^2+2abx+b^2x^2)^3} dx$ | 1638 |
| 3.200 | $\int \frac{1}{(d+ex)^{7/2}(a^2+2abx+b^2x^2)^3} dx$ | 1655 |
| 3.201 | $\int (d+ex)^3 \sqrt{cd^2+2cdex+ce^2x^2} dx$ | 1674 |
| 3.202 | $\int (d+ex) \sqrt{cd^2+2cdex+ce^2x^2} dx$ | 1680 |
| 3.203 | $\int \frac{\sqrt{cd^2+2cdex+ce^2x^2}}{d+ex} dx$ | 1685 |
| 3.204 | $\int \frac{\sqrt{cd^2+2cdex+ce^2x^2}}{(d+ex)^3} dx$ | 1690 |
| 3.205 | $\int \frac{\sqrt{cd^2+2cdex+ce^2x^2}}{(d+ex)^5} dx$ | 1695 |
| 3.206 | $\int (d+ex)^2 \sqrt{cd^2+2cdex+ce^2x^2} dx$ | 1700 |
| 3.207 | $\int \sqrt{cd^2+2cdex+ce^2x^2} dx$ | 1705 |
| 3.208 | $\int \frac{\sqrt{cd^2+2cdex+ce^2x^2}}{(d+ex)^2} dx$ | 1710 |
| 3.209 | $\int \frac{\sqrt{cd^2+2cdex+ce^2x^2}}{(d+ex)^4} dx$ | 1715 |
| 3.210 | $\int \frac{\sqrt{cd^2+2cdex+ce^2x^2}}{(d+ex)^6} dx$ | 1720 |
| 3.211 | $\int (d+ex)^3 (cd^2+2cdex+ce^2x^2)^{3/2} dx$ | 1725 |
| 3.212 | $\int (d+ex) (cd^2+2cdex+ce^2x^2)^{3/2} dx$ | 1731 |
| 3.213 | $\int \frac{(cd^2+2cdex+ce^2x^2)^{3/2}}{d+ex} dx$ | 1737 |
| 3.214 | $\int \frac{(cd^2+2cdex+ce^2x^2)^{3/2}}{(d+ex)^3} dx$ | 1742 |
| 3.215 | $\int \frac{(cd^2+2cdex+ce^2x^2)^{3/2}}{(d+ex)^5} dx$ | 1747 |
| 3.216 | $\int \frac{(cd^2+2cdex+ce^2x^2)^{3/2}}{(d+ex)^7} dx$ | 1752 |
| 3.217 | $\int (d+ex)^2 (cd^2+2cdex+ce^2x^2)^{3/2} dx$ | 1757 |
| 3.218 | $\int (cd^2+2cdex+ce^2x^2)^{3/2} dx$ | 1763 |

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| 3.219 | $\int \frac{(cd^2+2cdex+ce^2x^2)^{3/2}}{(d+ex)^2} dx$ | 1769 |
| 3.220 | $\int \frac{(cd^2+2cdex+ce^2x^2)^{3/2}}{(d+ex)^4} dx$ | 1774 |
| 3.221 | $\int \frac{(cd^2+2cdex+ce^2x^2)^{3/2}}{(d+ex)^6} dx$ | 1779 |
| 3.222 | $\int \frac{(cd^2+2cdex+ce^2x^2)^{3/2}}{(d+ex)^8} dx$ | 1784 |
| 3.223 | $\int (d+ex)(cd^2+2cdex+ce^2x^2)^{5/2} dx$ | 1789 |
| 3.224 | $\int \frac{(cd^2+2cdex+ce^2x^2)^{5/2}}{d+ex} dx$ | 1795 |
| 3.225 | $\int \frac{(cd^2+2cdex+ce^2x^2)^{5/2}}{(d+ex)^3} dx$ | 1800 |
| 3.226 | $\int \frac{(cd^2+2cdex+ce^2x^2)^{5/2}}{(d+ex)^5} dx$ | 1806 |
| 3.227 | $\int \frac{(cd^2+2cdex+ce^2x^2)^{5/2}}{(d+ex)^7} dx$ | 1811 |
| 3.228 | $\int \frac{(cd^2+2cdex+ce^2x^2)^{5/2}}{(d+ex)^9} dx$ | 1816 |
| 3.229 | $\int (d+ex)^2 (cd^2+2cdex+ce^2x^2)^{5/2} dx$ | 1821 |
| 3.230 | $\int (cd^2+2cdex+ce^2x^2)^{5/2} dx$ | 1827 |
| 3.231 | $\int \frac{(cd^2+2cdex+ce^2x^2)^{5/2}}{(d+ex)^2} dx$ | 1833 |
| 3.232 | $\int \frac{(cd^2+2cdex+ce^2x^2)^{5/2}}{(d+ex)^4} dx$ | 1839 |
| 3.233 | $\int \frac{(cd^2+2cdex+ce^2x^2)^{5/2}}{(d+ex)^6} dx$ | 1844 |
| 3.234 | $\int \frac{(cd^2+2cdex+ce^2x^2)^{5/2}}{(d+ex)^8} dx$ | 1849 |
| 3.235 | $\int \frac{(d+ex)^5}{\sqrt{cd^2+2cdex+ce^2x^2}} dx$ | 1854 |
| 3.236 | $\int \frac{(d+ex)^3}{\sqrt{cd^2+2cdex+ce^2x^2}} dx$ | 1860 |
| 3.237 | $\int \frac{d+ex}{\sqrt{cd^2+2cdex+ce^2x^2}} dx$ | 1866 |
| 3.238 | $\int \frac{1}{(d+ex)\sqrt{cd^2+2cdex+ce^2x^2}} dx$ | 1871 |
| 3.239 | $\int \frac{1}{(d+ex)^3\sqrt{cd^2+2cdex+ce^2x^2}} dx$ | 1876 |
| 3.240 | $\int \frac{(d+ex)^4}{\sqrt{cd^2+2cdex+ce^2x^2}} dx$ | 1881 |
| 3.241 | $\int \frac{(d+ex)^2}{\sqrt{cd^2+2cdex+ce^2x^2}} dx$ | 1887 |
| 3.242 | $\int \frac{1}{\sqrt{cd^2+2cdex+ce^2x^2}} dx$ | 1893 |
| 3.243 | $\int \frac{1}{(d+ex)^2\sqrt{cd^2+2cdex+ce^2x^2}} dx$ | 1898 |
| 3.244 | $\int \frac{1}{(d+ex)^4\sqrt{cd^2+2cdex+ce^2x^2}} dx$ | 1903 |
| 3.245 | $\int \frac{(d+ex)^5}{(cd^2+2cdex+ce^2x^2)^{3/2}} dx$ | 1908 |
| 3.246 | $\int \frac{(d+ex)^3}{(cd^2+2cdex+ce^2x^2)^{3/2}} dx$ | 1913 |
| 3.247 | $\int \frac{d+ex}{(cd^2+2cdex+ce^2x^2)^{3/2}} dx$ | 1918 |
| 3.248 | $\int \frac{1}{(d+ex)(cd^2+2cdex+ce^2x^2)^{3/2}} dx$ | 1923 |
| 3.249 | $\int \frac{1}{(d+ex)^3(cd^2+2cdex+ce^2x^2)^{3/2}} dx$ | 1928 |

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| 3.250 | $\int \frac{(d+ex)^4}{(cd^2+2cdex+ce^2x^2)^{3/2}} dx$ | 1934 |
| 3.251 | $\int \frac{(d+ex)^2}{(cd^2+2cdex+ce^2x^2)^{3/2}} dx$ | 1940 |
| 3.252 | $\int \frac{1}{(cd^2+2cdex+ce^2x^2)^{3/2}} dx$ | 1945 |
| 3.253 | $\int \frac{1}{(d+ex)^2(cd^2+2cdex+ce^2x^2)^{3/2}} dx$ | 1950 |
| 3.254 | $\int \frac{(d+ex)^7}{(cd^2+2cdex+ce^2x^2)^{5/2}} dx$ | 1955 |
| 3.255 | $\int \frac{(d+ex)^5}{(cd^2+2cdex+ce^2x^2)^{5/2}} dx$ | 1961 |
| 3.256 | $\int \frac{(d+ex)^3}{(cd^2+2cdex+ce^2x^2)^{5/2}} dx$ | 1966 |
| 3.257 | $\int \frac{d+ex}{(cd^2+2cdex+ce^2x^2)^{5/2}} dx$ | 1971 |
| 3.258 | $\int \frac{1}{(d+ex)(cd^2+2cdex+ce^2x^2)^{5/2}} dx$ | 1976 |
| 3.259 | $\int \frac{(d+ex)^6}{(cd^2+2cdex+ce^2x^2)^{5/2}} dx$ | 1982 |
| 3.260 | $\int \frac{(d+ex)^4}{(cd^2+2cdex+ce^2x^2)^{5/2}} dx$ | 1988 |
| 3.261 | $\int \frac{(d+ex)^2}{(cd^2+2cdex+ce^2x^2)^{5/2}} dx$ | 1993 |
| 3.262 | $\int \frac{1}{(cd^2+2cdex+ce^2x^2)^{5/2}} dx$ | 1998 |
| 3.263 | $\int \frac{1}{(d+ex)^2(cd^2+2cdex+ce^2x^2)^{5/2}} dx$ | 2003 |
| 3.264 | $\int \frac{1}{\left(\frac{be}{2c}+ex\right)\sqrt{\frac{b^2}{4c}+bx+cx^2}} dx$ | 2009 |
| 3.265 | $\int (d+ex)^4\sqrt{a^2+2abx+b^2x^2} dx$ | 2014 |
| 3.266 | $\int (d+ex)^3\sqrt{a^2+2abx+b^2x^2} dx$ | 2023 |
| 3.267 | $\int (d+ex)^2\sqrt{a^2+2abx+b^2x^2} dx$ | 2033 |
| 3.268 | $\int (d+ex)\sqrt{a^2+2abx+b^2x^2} dx$ | 2040 |
| 3.269 | $\int \sqrt{a^2+2abx+b^2x^2} dx$ | 2046 |
| 3.270 | $\int \frac{\sqrt{a^2+2abx+b^2x^2}}{d+ex} dx$ | 2051 |
| 3.271 | $\int \frac{\sqrt{a^2+2abx+b^2x^2}}{(d+ex)^2} dx$ | 2056 |
| 3.272 | $\int \frac{\sqrt{a^2+2abx+b^2x^2}}{(d+ex)^3} dx$ | 2062 |
| 3.273 | $\int \frac{\sqrt{a^2+2abx+b^2x^2}}{(d+ex)^4} dx$ | 2067 |
| 3.274 | $\int \frac{\sqrt{a^2+2abx+b^2x^2}}{(d+ex)^5} dx$ | 2073 |
| 3.275 | $\int \frac{\sqrt{a^2+2abx+b^2x^2}}{(d+ex)^6} dx$ | 2079 |
| 3.276 | $\int (d+ex)^5(a^2+2abx+b^2x^2)^{3/2} dx$ | 2085 |
| 3.277 | $\int (d+ex)^4(a^2+2abx+b^2x^2)^{3/2} dx$ | 2094 |
| 3.278 | $\int (d+ex)^3(a^2+2abx+b^2x^2)^{3/2} dx$ | 2104 |
| 3.279 | $\int (d+ex)^2(a^2+2abx+b^2x^2)^{3/2} dx$ | 2112 |
| 3.280 | $\int (d+ex)(a^2+2abx+b^2x^2)^{3/2} dx$ | 2119 |
| 3.281 | $\int (a^2+2abx+b^2x^2)^{3/2} dx$ | 2126 |
| 3.282 | $\int \frac{(a^2+2abx+b^2x^2)^{3/2}}{d+ex} dx$ | 2132 |

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| 3.283 | $\int \frac{(a^2+2abx+b^2x^2)^{3/2}}{(d+ex)^2} dx$ | 2138 |
| 3.284 | $\int \frac{(a^2+2abx+b^2x^2)^{3/2}}{(d+ex)^3} dx$ | 2144 |
| 3.285 | $\int \frac{(a^2+2abx+b^2x^2)^{3/2}}{(d+ex)^4} dx$ | 2150 |
| 3.286 | $\int \frac{(a^2+2abx+b^2x^2)^{3/2}}{(d+ex)^5} dx$ | 2156 |
| 3.287 | $\int \frac{(a^2+2abx+b^2x^2)^{3/2}}{(d+ex)^6} dx$ | 2163 |
| 3.288 | $\int \frac{(a^2+2abx+b^2x^2)^{3/2}}{(d+ex)^7} dx$ | 2170 |
| 3.289 | $\int \frac{(a^2+2abx+b^2x^2)^{3/2}}{(d+ex)^8} dx$ | 2177 |
| 3.290 | $\int \frac{(a^2+2abx+b^2x^2)^{3/2}}{(d+ex)^9} dx$ | 2184 |
| 3.291 | $\int (d+ex)^5 (a^2+2abx+b^2x^2)^{5/2} dx$ | 2191 |
| 3.292 | $\int (d+ex)^4 (a^2+2abx+b^2x^2)^{5/2} dx$ | 2200 |
| 3.293 | $\int (d+ex)^3 (a^2+2abx+b^2x^2)^{5/2} dx$ | 2210 |
| 3.294 | $\int (d+ex)^2 (a^2+2abx+b^2x^2)^{5/2} dx$ | 2219 |
| 3.295 | $\int (d+ex) (a^2+2abx+b^2x^2)^{5/2} dx$ | 2227 |
| 3.296 | $\int (a^2+2abx+b^2x^2)^{5/2} dx$ | 2234 |
| 3.297 | $\int \frac{(a^2+2abx+b^2x^2)^{5/2}}{d+ex} dx$ | 2240 |
| 3.298 | $\int \frac{(a^2+2abx+b^2x^2)^{5/2}}{(d+ex)^2} dx$ | 2247 |
| 3.299 | $\int \frac{(a^2+2abx+b^2x^2)^{5/2}}{(d+ex)^3} dx$ | 2254 |
| 3.300 | $\int \frac{(a^2+2abx+b^2x^2)^{5/2}}{(d+ex)^4} dx$ | 2261 |
| 3.301 | $\int \frac{(a^2+2abx+b^2x^2)^{5/2}}{(d+ex)^5} dx$ | 2268 |
| 3.302 | $\int \frac{(a^2+2abx+b^2x^2)^{5/2}}{(d+ex)^6} dx$ | 2275 |
| 3.303 | $\int \frac{(a^2+2abx+b^2x^2)^{5/2}}{(d+ex)^7} dx$ | 2282 |
| 3.304 | $\int \frac{(a^2+2abx+b^2x^2)^{5/2}}{(d+ex)^8} dx$ | 2289 |
| 3.305 | $\int \frac{(a^2+2abx+b^2x^2)^{5/2}}{(d+ex)^9} dx$ | 2297 |
| 3.306 | $\int \frac{(a^2+2abx+b^2x^2)^{5/2}}{(d+ex)^{10}} dx$ | 2305 |
| 3.307 | $\int \frac{(a^2+2abx+b^2x^2)^{5/2}}{(d+ex)^{11}} dx$ | 2314 |
| 3.308 | $\int \frac{(a^2+2abx+b^2x^2)^{5/2}}{(d+ex)^{12}} dx$ | 2323 |
| 3.309 | $\int \frac{(d+ex)^4}{\sqrt{a^2+2abx+b^2x^2}} dx$ | 2332 |
| 3.310 | $\int \frac{(d+ex)^3}{\sqrt{a^2+2abx+b^2x^2}} dx$ | 2340 |
| 3.311 | $\int \frac{(d+ex)^2}{\sqrt{a^2+2abx+b^2x^2}} dx$ | 2347 |
| 3.312 | $\int \frac{d+ex}{\sqrt{a^2+2abx+b^2x^2}} dx$ | 2353 |

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| 3.313 | $\int \frac{1}{\sqrt{a^2+2abx+b^2x^2}} dx$ | 2359 |
| 3.314 | $\int \frac{1}{(d+ex)\sqrt{a^2+2abx+b^2x^2}} dx$ | 2364 |
| 3.315 | $\int \frac{1}{(d+ex)^2\sqrt{a^2+2abx+b^2x^2}} dx$ | 2369 |
| 3.316 | $\int \frac{1}{(d+ex)^3\sqrt{a^2+2abx+b^2x^2}} dx$ | 2375 |
| 3.317 | $\int \frac{1}{(d+ex)^4\sqrt{a^2+2abx+b^2x^2}} dx$ | 2382 |
| 3.318 | $\int \frac{(d+ex)^4}{(a^2+2abx+b^2x^2)^{3/2}} dx$ | 2389 |
| 3.319 | $\int \frac{(d+ex)^3}{(a^2+2abx+b^2x^2)^{3/2}} dx$ | 2396 |
| 3.320 | $\int \frac{(d+ex)^2}{(a^2+2abx+b^2x^2)^{3/2}} dx$ | 2403 |
| 3.321 | $\int \frac{d+ex}{(a^2+2abx+b^2x^2)^{3/2}} dx$ | 2409 |
| 3.322 | $\int \frac{1}{(a^2+2abx+b^2x^2)^{3/2}} dx$ | 2414 |
| 3.323 | $\int \frac{1}{(d+ex)(a^2+2abx+b^2x^2)^{3/2}} dx$ | 2419 |
| 3.324 | $\int \frac{1}{(d+ex)^2(a^2+2abx+b^2x^2)^{3/2}} dx$ | 2425 |
| 3.325 | $\int \frac{1}{(d+ex)^3(a^2+2abx+b^2x^2)^{3/2}} dx$ | 2432 |
| 3.326 | $\int \frac{(d+ex)^6}{(a^2+2abx+b^2x^2)^{5/2}} dx$ | 2440 |
| 3.327 | $\int \frac{(d+ex)^5}{(a^2+2abx+b^2x^2)^{5/2}} dx$ | 2448 |
| 3.328 | $\int \frac{(d+ex)^4}{(a^2+2abx+b^2x^2)^{5/2}} dx$ | 2455 |
| 3.329 | $\int \frac{(d+ex)^3}{(a^2+2abx+b^2x^2)^{5/2}} dx$ | 2462 |
| 3.330 | $\int \frac{(d+ex)^2}{(a^2+2abx+b^2x^2)^{5/2}} dx$ | 2469 |
| 3.331 | $\int \frac{d+ex}{(a^2+2abx+b^2x^2)^{5/2}} dx$ | 2475 |
| 3.332 | $\int \frac{1}{(a^2+2abx+b^2x^2)^{5/2}} dx$ | 2480 |
| 3.333 | $\int \frac{1}{(d+ex)(a^2+2abx+b^2x^2)^{5/2}} dx$ | 2485 |
| 3.334 | $\int \frac{1}{(d+ex)^2(a^2+2abx+b^2x^2)^{5/2}} dx$ | 2493 |
| 3.335 | $\int \frac{1}{(d+ex)^3(a^2+2abx+b^2x^2)^{5/2}} dx$ | 2501 |
| 3.336 | $\int (d+ex)(9+12x+4x^2)^{5/2} dx$ | 2510 |
| 3.337 | $\int (d+ex)(9+12x+4x^2)^{3/2} dx$ | 2516 |
| 3.338 | $\int (d+ex)\sqrt{9+12x+4x^2} dx$ | 2522 |
| 3.339 | $\int \frac{d+ex}{\sqrt{9+12x+4x^2}} dx$ | 2528 |
| 3.340 | $\int \frac{d+ex}{(9+12x+4x^2)^{3/2}} dx$ | 2533 |
| 3.341 | $\int \frac{d+ex}{(9+12x+4x^2)^{5/2}} dx$ | 2538 |
| 3.342 | $\int \frac{d+ex}{(9+12x+4x^2)^{7/2}} dx$ | 2543 |
| 3.343 | $\int \frac{1}{(d+ex)\sqrt{\frac{b^2}{4c}+bx+cx^2}} dx$ | 2548 |
| 3.344 | $\int (d+ex)^{5/2}\sqrt{a^2+2abx+b^2x^2} dx$ | 2554 |

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| 3.345 | $\int (d+ex)^{3/2} \sqrt{a^2+2abx+b^2x^2} dx$ | 2560 |
| 3.346 | $\int \sqrt{d+ex} \sqrt{a^2+2abx+b^2x^2} dx$ | 2566 |
| 3.347 | $\int \frac{\sqrt{a^2+2abx+b^2x^2}}{\sqrt{d+ex}} dx$ | 2572 |
| 3.348 | $\int \frac{\sqrt{a^2+2abx+b^2x^2}}{(d+ex)^{3/2}} dx$ | 2578 |
| 3.349 | $\int \frac{\sqrt{a^2+2abx+b^2x^2}}{(d+ex)^{5/2}} dx$ | 2583 |
| 3.350 | $\int \frac{\sqrt{a^2+2abx+b^2x^2}}{(d+ex)^{7/2}} dx$ | 2589 |
| 3.351 | $\int (d+ex)^{5/2} (a^2+2abx+b^2x^2)^{3/2} dx$ | 2595 |
| 3.352 | $\int (d+ex)^{3/2} (a^2+2abx+b^2x^2)^{3/2} dx$ | 2602 |
| 3.353 | $\int \sqrt{d+ex} (a^2+2abx+b^2x^2)^{3/2} dx$ | 2609 |
| 3.354 | $\int \frac{(a^2+2abx+b^2x^2)^{3/2}}{\sqrt{d+ex}} dx$ | 2616 |
| 3.355 | $\int \frac{(a^2+2abx+b^2x^2)^{3/2}}{(d+ex)^{3/2}} dx$ | 2622 |
| 3.356 | $\int \frac{(a^2+2abx+b^2x^2)^{3/2}}{(d+ex)^{5/2}} dx$ | 2628 |
| 3.357 | $\int \frac{(a^2+2abx+b^2x^2)^{3/2}}{(d+ex)^{7/2}} dx$ | 2635 |
| 3.358 | $\int \frac{(a^2+2abx+b^2x^2)^{3/2}}{(d+ex)^{9/2}} dx$ | 2642 |
| 3.359 | $\int \frac{(a^2+2abx+b^2x^2)^{3/2}}{(d+ex)^{11/2}} dx$ | 2649 |
| 3.360 | $\int (d+ex)^{5/2} (a^2+2abx+b^2x^2)^{5/2} dx$ | 2656 |
| 3.361 | $\int (d+ex)^{3/2} (a^2+2abx+b^2x^2)^{5/2} dx$ | 2664 |
| 3.362 | $\int \sqrt{d+ex} (a^2+2abx+b^2x^2)^{5/2} dx$ | 2672 |
| 3.363 | $\int \frac{(a^2+2abx+b^2x^2)^{5/2}}{\sqrt{d+ex}} dx$ | 2680 |
| 3.364 | $\int \frac{(a^2+2abx+b^2x^2)^{5/2}}{(d+ex)^{3/2}} dx$ | 2687 |
| 3.365 | $\int \frac{(a^2+2abx+b^2x^2)^{5/2}}{(d+ex)^{5/2}} dx$ | 2694 |
| 3.366 | $\int \frac{(a^2+2abx+b^2x^2)^{5/2}}{(d+ex)^{7/2}} dx$ | 2701 |
| 3.367 | $\int \frac{(a^2+2abx+b^2x^2)^{5/2}}{(d+ex)^{9/2}} dx$ | 2708 |
| 3.368 | $\int \frac{(a^2+2abx+b^2x^2)^{5/2}}{(d+ex)^{11/2}} dx$ | 2715 |
| 3.369 | $\int \frac{(a^2+2abx+b^2x^2)^{5/2}}{(d+ex)^{13/2}} dx$ | 2722 |
| 3.370 | $\int \frac{(a^2+2abx+b^2x^2)^{5/2}}{(d+ex)^{15/2}} dx$ | 2730 |
| 3.371 | $\int \frac{(d+ex)^{7/2}}{\sqrt{a^2+2abx+b^2x^2}} dx$ | 2738 |
| 3.372 | $\int \frac{(d+ex)^{5/2}}{\sqrt{a^2+2abx+b^2x^2}} dx$ | 2747 |
| 3.373 | $\int \frac{(d+ex)^{3/2}}{\sqrt{a^2+2abx+b^2x^2}} dx$ | 2755 |
| 3.374 | $\int \frac{\sqrt{d+ex}}{\sqrt{a^2+2abx+b^2x^2}} dx$ | 2762 |

| | | |
|-------|---|------|
| 3.375 | $\int \frac{1}{\sqrt{d+ex}\sqrt{a^2+2abx+b^2x^2}} dx$ | 2768 |
| 3.376 | $\int \frac{1}{(d+ex)^{3/2}\sqrt{a^2+2abx+b^2x^2}} dx$ | 2774 |
| 3.377 | $\int \frac{1}{(d+ex)^{5/2}\sqrt{a^2+2abx+b^2x^2}} dx$ | 2781 |
| 3.378 | $\int \frac{1}{(d+ex)^{7/2}\sqrt{a^2+2abx+b^2x^2}} dx$ | 2788 |
| 3.379 | $\int \frac{(d+ex)^{9/2}}{(a^2+2abx+b^2x^2)^{3/2}} dx$ | 2796 |
| 3.380 | $\int \frac{(d+ex)^{7/2}}{(a^2+2abx+b^2x^2)^{3/2}} dx$ | 2807 |
| 3.381 | $\int \frac{(d+ex)^{5/2}}{(a^2+2abx+b^2x^2)^{3/2}} dx$ | 2816 |
| 3.382 | $\int \frac{(d+ex)^{3/2}}{(a^2+2abx+b^2x^2)^{3/2}} dx$ | 2824 |
| 3.383 | $\int \frac{\sqrt{d+ex}}{(a^2+2abx+b^2x^2)^{3/2}} dx$ | 2831 |
| 3.384 | $\int \frac{1}{\sqrt{d+ex}(a^2+2abx+b^2x^2)^{3/2}} dx$ | 2838 |
| 3.385 | $\int \frac{1}{(d+ex)^{3/2}(a^2+2abx+b^2x^2)^{3/2}} dx$ | 2845 |
| 3.386 | $\int \frac{1}{(d+ex)^{5/2}(a^2+2abx+b^2x^2)^{3/2}} dx$ | 2854 |
| 3.387 | $\int \frac{1}{(d+ex)^{7/2}(a^2+2abx+b^2x^2)^{3/2}} dx$ | 2864 |
| 3.388 | $\int \frac{(d+ex)^{13/2}}{(a^2+2abx+b^2x^2)^{5/2}} dx$ | 2876 |
| 3.389 | $\int \frac{(d+ex)^{11/2}}{(a^2+2abx+b^2x^2)^{5/2}} dx$ | 2894 |
| 3.390 | $\int \frac{(d+ex)^{9/2}}{(a^2+2abx+b^2x^2)^{5/2}} dx$ | 2910 |
| 3.391 | $\int \frac{(d+ex)^{7/2}}{(a^2+2abx+b^2x^2)^{5/2}} dx$ | 2921 |
| 3.392 | $\int \frac{(d+ex)^{5/2}}{(a^2+2abx+b^2x^2)^{5/2}} dx$ | 2930 |
| 3.393 | $\int \frac{(d+ex)^{3/2}}{(a^2+2abx+b^2x^2)^{5/2}} dx$ | 2940 |
| 3.394 | $\int \frac{\sqrt{d+ex}}{(a^2+2abx+b^2x^2)^{5/2}} dx$ | 2950 |
| 3.395 | $\int \frac{1}{\sqrt{d+ex}(a^2+2abx+b^2x^2)^{5/2}} dx$ | 2960 |
| 3.396 | $\int \frac{1}{(d+ex)^{3/2}(a^2+2abx+b^2x^2)^{5/2}} dx$ | 2970 |
| 3.397 | $\int \frac{1}{(d+ex)^{5/2}(a^2+2abx+b^2x^2)^{5/2}} dx$ | 2982 |
| 3.398 | $\int \frac{1}{(d+ex)^{7/2}(a^2+2abx+b^2x^2)^{5/2}} dx$ | 2999 |
| 3.399 | $\int (ad + bdx)^m (a^2c + 2abcx + b^2cx^2)^3 dx$ | 3017 |
| 3.400 | $\int (ad + bdx)^m (a^2c + 2abcx + b^2cx^2)^2 dx$ | 3024 |
| 3.401 | $\int (ad + bdx)^m (a^2c + 2abcx + b^2cx^2) dx$ | 3031 |
| 3.402 | $\int (ad + bdx)^m dx$ | 3037 |
| 3.403 | $\int \frac{(ad+bdx)^m}{a^2c+2abcx+b^2cx^2} dx$ | 3042 |
| 3.404 | $\int \frac{(ad+bdx)^m}{(a^2c+2abcx+b^2cx^2)^2} dx$ | 3048 |
| 3.405 | $\int \frac{(ad+bdx)^m}{(a^2c+2abcx+b^2cx^2)^3} dx$ | 3054 |
| 3.406 | $\int (ad + bdx)^m (a^2c + 2abcx + b^2cx^2)^{3/2} dx$ | 3060 |

| | | |
|-------|---|------|
| 3.407 | $\int (ad + bdx)^m \sqrt{a^2c + 2abcx + b^2cx^2} dx$ | 3066 |
| 3.408 | $\int \frac{(ad+bdx)^m}{\sqrt{a^2c+2abcx+b^2cx^2}} dx$ | 3071 |
| 3.409 | $\int \frac{(ad+bdx)^m}{(a^2c+2abcx+b^2cx^2)^{3/2}} dx$ | 3076 |
| 3.410 | $\int \frac{(ad+bdx)^m}{(a^2c+2abcx+b^2cx^2)^{5/2}} dx$ | 3082 |
| 3.411 | $\int (d + ex)^m (a^2 + 2abx + b^2x^2)^3 dx$ | 3088 |
| 3.412 | $\int (d + ex)^m (a^2 + 2abx + b^2x^2)^2 dx$ | 3098 |
| 3.413 | $\int (d + ex)^m (a^2 + 2abx + b^2x^2) dx$ | 3108 |
| 3.414 | $\int \frac{(d+ex)^m}{a^2+2abx+b^2x^2} dx$ | 3115 |
| 3.415 | $\int \frac{(d+ex)^m}{(a^2+2abx+b^2x^2)^2} dx$ | 3121 |
| 3.416 | $\int \frac{(d+ex)^m}{(a^2+2abx+b^2x^2)^3} dx$ | 3127 |
| 3.417 | $\int (d + ex)^m (a^2 + 2abx + b^2x^2)^{5/2} dx$ | 3133 |
| 3.418 | $\int (d + ex)^m (a^2 + 2abx + b^2x^2)^{3/2} dx$ | 3142 |
| 3.419 | $\int (d + ex)^m \sqrt{a^2 + 2abx + b^2x^2} dx$ | 3150 |
| 3.420 | $\int \frac{(d+ex)^m}{\sqrt{a^2+2abx+b^2x^2}} dx$ | 3156 |
| 3.421 | $\int \frac{(d+ex)^m}{(a^2+2abx+b^2x^2)^{3/2}} dx$ | 3161 |
| 3.422 | $\int \frac{(d+ex)^m}{(a^2+2abx+b^2x^2)^{5/2}} dx$ | 3167 |
| 3.423 | $\int (d + ex)^3 (cd^2 + 2cdex + ce^2x^2)^p dx$ | 3173 |
| 3.424 | $\int (d + ex) (cd^2 + 2cdex + ce^2x^2)^p dx$ | 3179 |
| 3.425 | $\int \frac{(cd^2+2cdex+ce^2x^2)^p}{d+ex} dx$ | 3185 |
| 3.426 | $\int \frac{(cd^2+2cdex+ce^2x^2)^p}{(d+ex)^3} dx$ | 3190 |
| 3.427 | $\int \frac{(cd^2+2cdex+ce^2x^2)^p}{(d+ex)^5} dx$ | 3195 |
| 3.428 | $\int (d + ex)^4 (cd^2 + 2cdex + ce^2x^2)^p dx$ | 3201 |
| 3.429 | $\int (d + ex)^2 (cd^2 + 2cdex + ce^2x^2)^p dx$ | 3208 |
| 3.430 | $\int (cd^2 + 2cdex + ce^2x^2)^p dx$ | 3214 |
| 3.431 | $\int \frac{(cd^2+2cdex+ce^2x^2)^p}{(d+ex)^2} dx$ | 3219 |
| 3.432 | $\int \frac{(cd^2+2cdex+ce^2x^2)^p}{(d+ex)^4} dx$ | 3225 |
| 3.433 | $\int (d + ex)^3 (a^2 + 2abx + b^2x^2)^p dx$ | 3231 |
| 3.434 | $\int (d + ex)^2 (a^2 + 2abx + b^2x^2)^p dx$ | 3241 |
| 3.435 | $\int (d + ex) (a^2 + 2abx + b^2x^2)^p dx$ | 3249 |
| 3.436 | $\int (a^2 + 2abx + b^2x^2)^p dx$ | 3256 |
| 3.437 | $\int \frac{(a^2+2abx+b^2x^2)^p}{d+ex} dx$ | 3261 |
| 3.438 | $\int \frac{(a^2+2abx+b^2x^2)^p}{(d+ex)^2} dx$ | 3266 |
| 3.439 | $\int \frac{(a^2+2abx+b^2x^2)^p}{(d+ex)^3} dx$ | 3272 |
| 3.440 | $\int (d + ex)^{3/2} (a^2 + 2abx + b^2x^2)^p dx$ | 3278 |
| 3.441 | $\int \sqrt{d + ex} (a^2 + 2abx + b^2x^2)^p dx$ | 3284 |

| | | |
|-------|---|------|
| 3.442 | $\int \frac{(a^2+2abx+b^2x^2)^p}{\sqrt{d+ex}} dx$ | 3290 |
| 3.443 | $\int \frac{(a^2+2abx+b^2x^2)^p}{(d+ex)^{3/2}} dx$ | 3296 |
| 3.444 | $\int \frac{(a^2+2abx+b^2x^2)^p}{(d+ex)^{5/2}} dx$ | 3302 |
| 3.445 | $\int (d+ex)^m (cd^2+2cdex+ce^2x^2)^p dx$ | 3308 |
| 3.446 | $\int (d+ex)^p (cd^2+2cdex+ce^2x^2)^{-p} dx$ | 3314 |
| 3.447 | $\int (d+ex)^{-1-2p} (cd^2+2cdex+ce^2x^2)^p dx$ | 3319 |
| 3.448 | $\int (d+ex)^{-1+2p} (cd^2+2cdex+ce^2x^2)^{-p} dx$ | 3324 |
| 3.449 | $\int (1+x)^m (1+2x+x^2)^p dx$ | 3329 |
| 3.450 | $\int \left(\frac{be}{2c}+ex\right)^m \left(\frac{b^2}{4c}+bx+cx^2\right)^p dx$ | 3334 |
| 3.451 | $\int (d+ex)^m (a^2+2abx+b^2x^2)^p dx$ | 3340 |
| 3.452 | $\int (d+ex)^m (a^2+2abx+b^2x^2)^{5+p} dx$ | 3346 |
| 3.453 | $\int (d+ex)^{-3-2p} (a^2+2abx+b^2x^2)^p dx$ | 3351 |
| 3.454 | $\int (d+ex) (9+12x+4x^2)^p dx$ | 3357 |

3.1 $\int (d + ex)^2 (cd^2 + 2cdex + ce^2x^2) dx$

| | |
|---|-----|
| Optimal result | 191 |
| Mathematica [A] (verified) | 191 |
| Rubi [A] (verified) | 192 |
| Maple [B] (verified) | 193 |
| Fricas [B] (verification not implemented) | 193 |
| Sympy [B] (verification not implemented) | 194 |
| Maxima [B] (verification not implemented) | 194 |
| Giac [B] (verification not implemented) | 194 |
| Mupad [B] (verification not implemented) | 195 |
| Reduce [B] (verification not implemented) | 195 |

Optimal result

Integrand size = 28, antiderivative size = 15

$$\int (d + ex)^2 (cd^2 + 2cdex + ce^2x^2) dx = \frac{c(d + ex)^5}{5e}$$

output

```
1/5*c*(e*x+d)^5/e
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00

$$\int (d + ex)^2 (cd^2 + 2cdex + ce^2x^2) dx = \frac{c(d + ex)^5}{5e}$$

input

```
Integrate[(d + e*x)^2*(c*d^2 + 2*c*d*e*x + c*e^2*x^2),x]
```

output

```
(c*(d + e*x)^5)/(5*e)
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.071$, Rules used = {1098, 17}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (d + ex)^2 (cd^2 + 2cdex + ce^2x^2) dx$$

$$\downarrow 1098$$

$$\frac{\int c^2e^2(d + ex)^4 dx}{ce^2}$$

$$\downarrow 17$$

$$\frac{c(d + ex)^5}{5e}$$

input

```
Int[(d + e*x)^2*(c*d^2 + 2*c*d*e*x + c*e^2*x^2),x]
```

output

```
(c*(d + e*x)^5)/(5*e)
```

Defintions of rubi rules used

rule 17

```
Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] := Simp[c*((a + b*x)^(m + 1))/(b*(m + 1))], x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]
```

rule 1098

```
Int[((d_.) + (e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] := Simp[1/c^p Int[(d + e*x)^m*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 44 vs. $2(13) = 26$.

Time = 0.55 (sec) , antiderivative size = 45, normalized size of antiderivative = 3.00

| method | result | size |
|---------------|---|------|
| gosper | $\frac{x(e^4x^4+5de^3x^3+10d^2e^2x^2+10d^3ex+5d^4)c}{5}$ | 45 |
| default | $\frac{1}{5}e^4cx^5 + de^3cx^4 + 2d^2e^2cx^3 + 2cd^3ex^2 + cd^4x$ | 48 |
| norman | $\frac{1}{5}e^4cx^5 + de^3cx^4 + 2d^2e^2cx^3 + 2cd^3ex^2 + cd^4x$ | 48 |
| risch | $\frac{1}{5}e^4cx^5 + de^3cx^4 + 2d^2e^2cx^3 + 2cd^3ex^2 + cd^4x$ | 48 |
| parallelrisch | $\frac{1}{5}e^4cx^5 + de^3cx^4 + 2d^2e^2cx^3 + 2cd^3ex^2 + cd^4x$ | 48 |
| orering | $\frac{x(e^4x^4+5de^3x^3+10d^2e^2x^2+10d^3ex+5d^4)(x^2ce^2+2cdxe+c^2d^2)}{5(ex+d)^2}$ | 71 |

input `int((e*x+d)^2*(c*e^2*x^2+2*c*d*e*x+c*d^2),x,method=_RETURNVERBOSE)`

output `1/5*x*(e^4*x^4+5*d*e^3*x^3+10*d^2*e^2*x^2+10*d^3*e*x+5*d^4)*c`

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 47 vs. $2(13) = 26$.

Time = 0.08 (sec) , antiderivative size = 47, normalized size of antiderivative = 3.13

$$\int (d+ex)^2 (cd^2 + 2cdex + ce^2x^2) dx = \frac{1}{5}ce^4x^5 + cde^3x^4 + 2cd^2e^2x^3 + 2cd^3ex^2 + cd^4x$$

input `integrate((e*x+d)^2*(c*e^2*x^2+2*c*d*e*x+c*d^2),x, algorithm="fricas")`

output `1/5*c*e^4*x^5 + c*d*e^3*x^4 + 2*c*d^2*e^2*x^3 + 2*c*d^3*e*x^2 + c*d^4*x`

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 51 vs. $2(10) = 20$.

Time = 0.02 (sec) , antiderivative size = 51, normalized size of antiderivative = 3.40

$$\int (d + ex)^2 (cd^2 + 2cdex + ce^2x^2) dx = cd^4x + 2cd^3ex^2 + 2cd^2e^2x^3 + cde^3x^4 + \frac{ce^4x^5}{5}$$

input `integrate((e*x+d)**2*(c*e**2*x**2+2*c*d*e*x+c*d**2),x)`

output `c*d**4*x + 2*c*d**3*e*x**2 + 2*c*d**2*e**2*x**3 + c*d*e**3*x**4 + c*e**4*x**5/5`

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 47 vs. $2(13) = 26$.

Time = 0.03 (sec) , antiderivative size = 47, normalized size of antiderivative = 3.13

$$\int (d + ex)^2 (cd^2 + 2cdex + ce^2x^2) dx = \frac{1}{5} ce^4x^5 + cde^3x^4 + 2cd^2e^2x^3 + 2cd^3ex^2 + cd^4x$$

input `integrate((e*x+d)^2*(c*e^2*x^2+2*c*d*e*x+c*d^2),x, algorithm="maxima")`

output `1/5*c*e^4*x^5 + c*d*e^3*x^4 + 2*c*d^2*e^2*x^3 + 2*c*d^3*e*x^2 + c*d^4*x`

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 47 vs. $2(13) = 26$.

Time = 0.24 (sec) , antiderivative size = 47, normalized size of antiderivative = 3.13

$$\int (d + ex)^2 (cd^2 + 2cdex + ce^2x^2) dx = \frac{1}{5} ce^4x^5 + cde^3x^4 + 2cd^2e^2x^3 + 2cd^3ex^2 + cd^4x$$

input `integrate((e*x+d)^2*(c*e^2*x^2+2*c*d*e*x+c*d^2),x, algorithm="giac")`

output $1/5*c*e^{4*x^5} + c*d*e^{3*x^4} + 2*c*d^2*e^{2*x^3} + 2*c*d^3*e*x^2 + c*d^4*x$

Mupad [B] (verification not implemented)

Time = 8.66 (sec) , antiderivative size = 47, normalized size of antiderivative = 3.13

$$\int (d+ex)^2 (cd^2 + 2cdex + ce^2x^2) dx = cd^4x + 2cd^3ex^2 + 2cd^2e^2x^3 + cde^3x^4 + \frac{ce^4x^5}{5}$$

input `int((d + e*x)^2*(c*d^2 + c*e^2*x^2 + 2*c*d*e*x),x)`

output $(c*e^{4*x^5})/5 + c*d^4*x + 2*c*d^2*e^{2*x^3} + 2*c*d^3*e*x^2 + c*d*e^{3*x^4}$

Reduce [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 44, normalized size of antiderivative = 2.93

$$\int (d+ex)^2 (cd^2 + 2cdex + ce^2x^2) dx = \frac{cx(e^4x^4 + 5de^3x^3 + 10d^2e^2x^2 + 10d^3ex + 5d^4)}{5}$$

input `int((e*x+d)^2*(c*e^2*x^2+2*c*d*e*x+c*d^2),x)`

output $(c*x*(5*d**4 + 10*d**3*e*x + 10*d**2*e**2*x**2 + 5*d*e**3*x**3 + e**4*x**4))/5$

3.2 $\int (d + ex) (cd^2 + 2cdex + ce^2x^2) dx$

| | |
|---|-----|
| Optimal result | 196 |
| Mathematica [A] (verified) | 196 |
| Rubi [A] (verified) | 197 |
| Maple [B] (warning: unable to verify) | 198 |
| Fricas [B] (verification not implemented) | 198 |
| Sympy [B] (verification not implemented) | 199 |
| Maxima [B] (verification not implemented) | 199 |
| Giac [B] (verification not implemented) | 199 |
| Mupad [B] (verification not implemented) | 200 |
| Reduce [B] (verification not implemented) | 200 |

Optimal result

Integrand size = 26, antiderivative size = 15

$$\int (d + ex) (cd^2 + 2cdex + ce^2x^2) dx = \frac{c(d + ex)^4}{4e}$$

output `1/4*c*(e*x+d)^4/e`

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00

$$\int (d + ex) (cd^2 + 2cdex + ce^2x^2) dx = \frac{c(d + ex)^4}{4e}$$

input `Integrate[(d + e*x)*(c*d^2 + 2*c*d*e*x + c*e^2*x^2),x]`

output `(c*(d + e*x)^4)/(4*e)`

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {1098, 17}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (d + ex)(cd^2 + 2cdex + ce^2x^2) dx$$

$$\downarrow 1098$$

$$\frac{\int c^2e^2(d + ex)^3 dx}{ce^2}$$

$$\downarrow 17$$

$$\frac{c(d + ex)^4}{4e}$$

input

```
Int[(d + e*x)*(c*d^2 + 2*c*d*e*x + c*e^2*x^2), x]
```

output

```
(c*(d + e*x)^4)/(4*e)
```

Defintions of rubi rules used

rule 17

```
Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] := Simp[c*((a + b*x)^(m + 1))/(b*(m + 1))], x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]
```

rule 1098

```
Int[((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] := Simp[1/c^p Int[(d + e*x)^m*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Maple [B] (warning: unable to verify)

Leaf count of result is larger than twice the leaf count of optimal. 30 vs. $2(13) = 26$.

Time = 0.29 (sec) , antiderivative size = 31, normalized size of antiderivative = 2.07

| method | result | size |
|---------------|--|------|
| default | $\frac{(x^2ce^2+2cdxe+cd^2)^2}{4ce}$ | 31 |
| gospers | $\frac{x(e^3x^3+4de^2x^2+6d^2ex+4d^3)c}{4}$ | 34 |
| norman | $d^3cx + cdx^3e^2 + \frac{1}{4}cx^4e^3 + \frac{3}{2}cd^2ex^2$ | 36 |
| parallelrisch | $d^3cx + cdx^3e^2 + \frac{1}{4}cx^4e^3 + \frac{3}{2}cd^2ex^2$ | 36 |
| risch | $\frac{cx^4e^3}{4} + cdx^3e^2 + \frac{3cd^2ex^2}{2} + d^3cx + \frac{cd^4}{4e}$ | 45 |
| orering | $\frac{x(e^3x^3+4de^2x^2+6d^2ex+4d^3)(x^2ce^2+2cdxe+cd^2)}{4(ex+d)^2}$ | 60 |

input `int((e*x+d)*(c*e^2*x^2+2*c*d*e*x+c*d^2),x,method=_RETURNVERBOSE)`

output `1/4*(c*e^2*x^2+2*c*d*e*x+c*d^2)^2/c/e`

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 35 vs. $2(13) = 26$.

Time = 0.08 (sec) , antiderivative size = 35, normalized size of antiderivative = 2.33

$$\int (d + ex) (cd^2 + 2cdex + ce^2x^2) dx = \frac{1}{4}ce^3x^4 + cde^2x^3 + \frac{3}{2}cd^2ex^2 + cd^3x$$

input `integrate((e*x+d)*(c*e^2*x^2+2*c*d*e*x+c*d^2),x, algorithm="fricas")`

output `1/4*c*e^3*x^4 + c*d*e^2*x^3 + 3/2*c*d^2*e*x^2 + c*d^3*x`

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 39 vs. $2(10) = 20$.

Time = 0.02 (sec) , antiderivative size = 39, normalized size of antiderivative = 2.60

$$\int (d + ex) (cd^2 + 2cdex + ce^2x^2) dx = cd^3x + \frac{3cd^2ex^2}{2} + cde^2x^3 + \frac{ce^3x^4}{4}$$

input `integrate((e*x+d)*(c*e**2*x**2+2*c*d*e*x+c*d**2),x)`

output `c*d**3*x + 3*c*d**2*e*x**2/2 + c*d*e**2*x**3 + c*e**3*x**4/4`

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 30 vs. $2(13) = 26$.

Time = 0.03 (sec) , antiderivative size = 30, normalized size of antiderivative = 2.00

$$\int (d + ex) (cd^2 + 2cdex + ce^2x^2) dx = \frac{(ce^2x^2 + 2cdex + cd^2)^2}{4ce}$$

input `integrate((e*x+d)*(c*e^2*x^2+2*c*d*e*x+c*d^2),x, algorithm="maxima")`

output `1/4*(c*e^2*x^2 + 2*c*d*e*x + c*d^2)^2/(c*e)`

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 33 vs. $2(13) = 26$.

Time = 0.22 (sec) , antiderivative size = 33, normalized size of antiderivative = 2.20

$$\int (d + ex) (cd^2 + 2cdex + ce^2x^2) dx = \frac{1}{2} (ex^2 + 2dx)cd^2 + \frac{1}{4} (ex^2 + 2dx)^2ce$$

input `integrate((e*x+d)*(c*e^2*x^2+2*c*d*e*x+c*d^2),x, algorithm="giac")`

output $1/2*(e*x^2 + 2*d*x)*c*d^2 + 1/4*(e*x^2 + 2*d*x)^2*c*e$

Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 35, normalized size of antiderivative = 2.33

$$\int (d + ex) (cd^2 + 2cdex + ce^2x^2) dx = cd^3x + \frac{3cd^2ex^2}{2} + cde^2x^3 + \frac{ce^3x^4}{4}$$

input `int((d + e*x)*(c*d^2 + c*e^2*x^2 + 2*c*d*e*x),x)`

output $(c*e^3*x^4)/4 + c*d^3*x + (3*c*d^2*e*x^2)/2 + c*d*e^2*x^3$

Reduce [B] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 33, normalized size of antiderivative = 2.20

$$\int (d + ex) (cd^2 + 2cdex + ce^2x^2) dx = \frac{cx(e^3x^3 + 4de^2x^2 + 6d^2ex + 4d^3)}{4}$$

input `int((e*x+d)*(c*e^2*x^2+2*c*d*e*x+c*d^2),x)`

output $(c*x*(4*d**3 + 6*d**2*e*x + 4*d*e**2*x**2 + e**3*x**3))/4$

3.3 $\int (cd^2 + 2cdex + ce^2x^2) dx$

| | |
|---|-----|
| Optimal result | 201 |
| Mathematica [A] (verified) | 201 |
| Rubi [A] (verified) | 202 |
| Maple [A] (verified) | 203 |
| Fricas [A] (verification not implemented) | 203 |
| Sympy [B] (verification not implemented) | 204 |
| Maxima [A] (verification not implemented) | 204 |
| Giac [A] (verification not implemented) | 204 |
| Mupad [B] (verification not implemented) | 205 |
| Reduce [B] (verification not implemented) | 205 |

Optimal result

Integrand size = 20, antiderivative size = 15

$$\int (cd^2 + 2cdex + ce^2x^2) dx = \frac{c(d+ex)^3}{3e}$$

output

```
1/3*c*(e*x+d)^3/e
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.67

$$\int (cd^2 + 2cdex + ce^2x^2) dx = cd^2x + cdex^2 + \frac{1}{3}ce^2x^3$$

input

```
Integrate[c*d^2 + 2*c*d*e*x + c*e^2*x^2,x]
```

output

```
c*d^2*x + c*d*e*x^2 + (c*e^2*x^3)/3
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {1077, 17}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (cd^2 + 2cdex + ce^2x^2) dx$$

$$\downarrow 1077$$

$$\frac{\int (cxe^2 + cde)^2 dx}{ce^2}$$

$$\downarrow 17$$

$$\frac{c(d + ex)^3}{3e}$$

input `Int [c*d^2 + 2*c*d*e*x + c*e^2*x^2,x]`

output `(c*(d + e*x)^3)/(3*e)`

Defintions of rubi rules used

rule 17 `Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1))/(b*(m + 1)), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]`

rule 1077 `Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[1/c^p Int [(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]`

Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.93

| method | result | size |
|---------------|---|------|
| default | $\frac{c(ex+d)^3}{3e}$ | 14 |
| gosper | $\frac{x(e^2x^2+3dex+3d^2)c}{3}$ | 23 |
| norman | $cd^2x + cdx^2e + \frac{1}{3}ce^2x^3$ | 24 |
| risch | $cd^2x + cdx^2e + \frac{1}{3}ce^2x^3$ | 24 |
| parallelrisch | $cd^2x + cdx^2e + \frac{1}{3}ce^2x^3$ | 24 |
| parts | $cd^2x + cdx^2e + \frac{1}{3}ce^2x^3$ | 24 |
| orering | $\frac{x(e^2x^2+3dex+3d^2)(x^2ce^2+2cdxe+cd^2)}{3(ex+d)^2}$ | 49 |

input `int(c*e^2*x^2+2*c*d*e*x+c*d^2,x,method=_RETURNVERBOSE)`

output `1/3*c*(e*x+d)^3/e`

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.53

$$\int (cd^2 + 2cdex + ce^2x^2) dx = \frac{1}{3}ce^2x^3 + cdex^2 + cd^2x$$

input `integrate(c*e^2*x^2+2*c*d*e*x+c*d^2,x, algorithm="fricas")`

output `1/3*c*e^2*x^3 + c*d*e*x^2 + c*d^2*x`

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 24 vs. $2(10) = 20$.

Time = 0.02 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.60

$$\int (cd^2 + 2cdex + ce^2x^2) dx = cd^2x + cdex^2 + \frac{ce^2x^3}{3}$$

input `integrate(c*e**2*x**2+2*c*d*e*x+c*d**2,x)`

output `c*d**2*x + c*d*e*x**2 + c*e**2*x**3/3`

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.53

$$\int (cd^2 + 2cdex + ce^2x^2) dx = \frac{1}{3}ce^2x^3 + cdex^2 + cd^2x$$

input `integrate(c*e^2*x^2+2*c*d*e*x+c*d^2,x, algorithm="maxima")`

output `1/3*c*e^2*x^3 + c*d*e*x^2 + c*d^2*x`

Giac [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.53

$$\int (cd^2 + 2cdex + ce^2x^2) dx = \frac{1}{3}ce^2x^3 + cdex^2 + cd^2x$$

input `integrate(c*e^2*x^2+2*c*d*e*x+c*d^2,x, algorithm="giac")`

output `1/3*c*e^2*x^3 + c*d*e*x^2 + c*d^2*x`

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.47

$$\int (cd^2 + 2cdex + ce^2x^2) dx = \frac{cx(3d^2 + 3dex + e^2x^2)}{3}$$

input `int(c*d^2 + c*e^2*x^2 + 2*c*d*e*x,x)`output `(c*x*(3*d^2 + e^2*x^2 + 3*d*e*x))/3`**Reduce [B] (verification not implemented)**

Time = 0.23 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.47

$$\int (cd^2 + 2cdex + ce^2x^2) dx = \frac{cx(e^2x^2 + 3dex + 3d^2)}{3}$$

input `int(c*e^2*x^2+2*c*d*e*x+c*d^2,x)`output `(c*x*(3*d**2 + 3*d*e*x + e**2*x**2))/3`

3.4 $\int \frac{cd^2 + 2cdex + ce^2x^2}{d+ex} dx$

| | |
|---|-----|
| Optimal result | 206 |
| Mathematica [A] (verified) | 206 |
| Rubi [A] (verified) | 207 |
| Maple [A] (verified) | 208 |
| Fricas [A] (verification not implemented) | 208 |
| Sympy [A] (verification not implemented) | 209 |
| Maxima [A] (verification not implemented) | 209 |
| Giac [A] (verification not implemented) | 209 |
| Mupad [B] (verification not implemented) | 210 |
| Reduce [B] (verification not implemented) | 210 |

Optimal result

Integrand size = 28, antiderivative size = 15

$$\int \frac{cd^2 + 2cdex + ce^2x^2}{d + ex} dx = \frac{c(d + ex)^2}{2e}$$

output

```
1/2*c*(e*x+d)^2/e
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.93

$$\int \frac{cd^2 + 2cdex + ce^2x^2}{d + ex} dx = c \left(dx + \frac{ex^2}{2} \right)$$

input

```
Integrate[(c*d^2 + 2*c*d*e*x + c*e^2*x^2)/(d + e*x),x]
```

output

```
c*(d*x + (e*x^2)/2)
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.071$, Rules used = {1098, 17}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{cd^2 + 2cdex + ce^2x^2}{d + ex} dx$$

$$\downarrow 1098$$

$$\int \frac{c^2e^2(d + ex)dx}{ce^2}$$

$$\downarrow 17$$

$$\frac{c(d + ex)^2}{2e}$$

input

```
Int[(c*d^2 + 2*c*d*e*x + c*e^2*x^2)/(d + e*x),x]
```

output

```
(c*(d + e*x)^2)/(2*e)
```

Defintions of rubi rules used

rule 17

```
Int[(c_.)*((a_.) + (b_.)*(x_)^(m_.), x_Symbol] := Simp[c*((a + b*x)^(m + 1))/(b*(m + 1)), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]
```

rule 1098

```
Int[((d_.) + (e_.)*(x_)^(m_.))*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] := Simp[1/c^p Int[(d + e*x)^m*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Maple [A] (verified)

Time = 0.67 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.80

| method | result | size |
|-------------|--|------|
| gospers | $\frac{x(ex+2d)c}{2}$ | 12 |
| default | $c(\frac{1}{2}ex^2 + dx)$ | 13 |
| norman | $cdx + \frac{1}{2}ce x^2$ | 13 |
| risch | $cdx + \frac{1}{2}ce x^2$ | 13 |
| parallelsch | $cdx + \frac{1}{2}ce x^2$ | 13 |
| orering | $\frac{x(ex+2d)(x^2ce^2+2cdxe+cd^2)}{2(ex+d)^2}$ | 38 |

input `int((c*e^2*x^2+2*c*d*e*x+c*d^2)/(e*x+d),x,method=_RETURNVERBOSE)`

output `1/2*x*(e*x+2*d)*c`

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.80

$$\int \frac{cd^2 + 2cdex + ce^2x^2}{d + ex} dx = \frac{1}{2}cex^2 + cdx$$

input `integrate((c*e^2*x^2+2*c*d*e*x+c*d^2)/(e*x+d),x, algorithm="fricas")`

output `1/2*c*e*x^2 + c*d*x`

Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.80

$$\int \frac{cd^2 + 2cdex + ce^2x^2}{d + ex} dx = cdx + \frac{ce^2x^2}{2}$$

input `integrate((c**2*x**2+2*c*d*e*x+c*d**2)/(e*x+d),x)`output `c*d*x + c*e*x**2/2`**Maxima [A] (verification not implemented)**

Time = 0.03 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.80

$$\int \frac{cd^2 + 2cdex + ce^2x^2}{d + ex} dx = \frac{1}{2} cex^2 + cdx$$

input `integrate((c*e^2*x^2+2*c*d*e*x+c*d^2)/(e*x+d),x, algorithm="maxima")`output `1/2*c*e*x^2 + c*d*x`**Giac [A] (verification not implemented)**

Time = 0.24 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.80

$$\int \frac{cd^2 + 2cdex + ce^2x^2}{d + ex} dx = \frac{1}{2} cex^2 + cdx$$

input `integrate((c*e^2*x^2+2*c*d*e*x+c*d^2)/(e*x+d),x, algorithm="giac")`output `1/2*c*e*x^2 + c*d*x`

Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.73

$$\int \frac{cd^2 + 2cdex + ce^2x^2}{d + ex} dx = \frac{cx(2d + ex)}{2}$$

input `int((c*d^2 + c*e^2*x^2 + 2*c*d*e*x)/(d + e*x),x)`output `(c*x*(2*d + e*x))/2`**Reduce [B] (verification not implemented)**

Time = 0.26 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.73

$$\int \frac{cd^2 + 2cdex + ce^2x^2}{d + ex} dx = \frac{cx(ex + 2d)}{2}$$

input `int((c*e^2*x^2+2*c*d*e*x+c*d^2)/(e*x+d),x)`output `(c*x*(2*d + e*x))/2`

3.5 $\int \frac{cd^2+2cdex+ce^2x^2}{(d+ex)^2} dx$

| | |
|---|-----|
| Optimal result | 211 |
| Mathematica [A] (verified) | 211 |
| Rubi [A] (verified) | 212 |
| Maple [A] (verified) | 213 |
| Fricas [A] (verification not implemented) | 213 |
| Sympy [A] (verification not implemented) | 213 |
| Maxima [A] (verification not implemented) | 214 |
| Giac [C] (verification not implemented) | 214 |
| Mupad [B] (verification not implemented) | 215 |
| Reduce [B] (verification not implemented) | 215 |

Optimal result

Integrand size = 28, antiderivative size = 3

$$\int \frac{cd^2 + 2cdex + ce^2x^2}{(d + ex)^2} dx = cx$$

output `c*x`

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 3, normalized size of antiderivative = 1.00

$$\int \frac{cd^2 + 2cdex + ce^2x^2}{(d + ex)^2} dx = cx$$

input `Integrate[(c*d^2 + 2*c*d*e*x + c*e^2*x^2)/(d + e*x)^2,x]`

output `c*x`

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 3, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.071$, Rules used = {1098, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{cd^2 + 2cdex + ce^2x^2}{(d + ex)^2} dx$$

↓ 1098

$$\int \frac{c^2e^2dx}{ce^2}$$

↓ 24

$$cx$$

input

```
Int[(c*d^2 + 2*c*d*e*x + c*e^2*x^2)/(d + e*x)^2,x]
```

output

```
c*x
```

Defintions of rubi rules used

rule 24

```
Int[a_, x_Symbol] := Simp[a*x, x] /; FreeQ[a, x]
```

rule 1098

```
Int[((d_.) + (e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] := Simp[1/c^p Int[(d + e*x)^m*(b/2 + c*x)^(2*p), x], x] /; FreeQ[
{a, b, c, d, e, m}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Maple [A] (verified)

Time = 0.66 (sec) , antiderivative size = 4, normalized size of antiderivative = 1.33

| method | result | size |
|---------|--|------|
| default | cx | 4 |
| risch | cx | 4 |
| norman | $\frac{ce^2x^2+cdx}{ex+d}$ | 20 |
| orering | $\frac{x(x^2ce^2+2cdxe+cd^2)}{(ex+d)^2}$ | 30 |

input `int((c*e^2*x^2+2*c*d*e*x+c*d^2)/(e*x+d)^2,x,method=_RETURNVERBOSE)`

output `c*x`

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 3, normalized size of antiderivative = 1.00

$$\int \frac{cd^2 + 2cdex + ce^2x^2}{(d + ex)^2} dx = cx$$

input `integrate((c*e^2*x^2+2*c*d*e*x+c*d^2)/(e*x+d)^2,x, algorithm="fricas")`

output `c*x`

Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 2, normalized size of antiderivative = 0.67

$$\int \frac{cd^2 + 2cdex + ce^2x^2}{(d + ex)^2} dx = cx$$

input `integrate((c**2*x**2+2*c*d*e*x+c*d**2)/(e*x+d)**2,x)`

output

`c*x`**Maxima [A] (verification not implemented)**

Time = 0.04 (sec) , antiderivative size = 3, normalized size of antiderivative = 1.00

$$\int \frac{cd^2 + 2cdex + ce^2x^2}{(d + ex)^2} dx = cx$$

input

`integrate((c*e^2*x^2+2*c*d*e*x+c*d^2)/(e*x+d)^2,x, algorithm="maxima")`

output

`c*x`**Giac [C] (verification not implemented)**

Result contains higher order function than in optimal. Order 3 vs. order 1.

Time = 0.24 (sec) , antiderivative size = 113, normalized size of antiderivative = 37.67

$$\int \frac{cd^2 + 2cdex + ce^2x^2}{(d + ex)^2} dx = ce^2 \left(\frac{2d \log\left(\frac{|ex+d|}{(ex+d)^2|e|}\right)}{e^3} + \frac{ex+d}{e^3} - \frac{d^2}{(ex+d)e^3} \right) - 2cd \left(\frac{\log\left(\frac{|ex+d|}{(ex+d)^2|e|}\right)}{e} - \frac{d}{(ex+d)e} \right) - \frac{cd^2}{(ex+d)e}$$

input

`integrate((c*e^2*x^2+2*c*d*e*x+c*d^2)/(e*x+d)^2,x, algorithm="giac")`

output

`c*e^2*(2*d*log(abs(e*x + d)/((e*x + d)^2*abs(e)))/e^3 + (e*x + d)/e^3 - d^2/((e*x + d)*e^3)) - 2*c*d*(log(abs(e*x + d)/((e*x + d)^2*abs(e)))/e - d/((e*x + d)*e)) - c*d^2/((e*x + d)*e)`

Mupad [B] (verification not implemented)

Time = 0.01 (sec) , antiderivative size = 3, normalized size of antiderivative = 1.00

$$\int \frac{cd^2 + 2cdex + ce^2x^2}{(d + ex)^2} dx = cx$$

input `int((c*d^2 + c*e^2*x^2 + 2*c*d*e*x)/(d + e*x)^2,x)`

output `c*x`

Reduce [B] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 3, normalized size of antiderivative = 1.00

$$\int \frac{cd^2 + 2cdex + ce^2x^2}{(d + ex)^2} dx = cx$$

input `int((c*e^2*x^2+2*c*d*e*x+c*d^2)/(e*x+d)^2,x)`

output `c*x`

3.6 $\int \frac{cd^2 + 2cdex + ce^2x^2}{(d+ex)^3} dx$

| | |
|---|-----|
| Optimal result | 216 |
| Mathematica [A] (verified) | 216 |
| Rubi [A] (verified) | 217 |
| Maple [A] (verified) | 218 |
| Fricas [A] (verification not implemented) | 218 |
| Sympy [A] (verification not implemented) | 218 |
| Maxima [A] (verification not implemented) | 219 |
| Giac [A] (verification not implemented) | 219 |
| Mupad [B] (verification not implemented) | 219 |
| Reduce [B] (verification not implemented) | 220 |

Optimal result

Integrand size = 28, antiderivative size = 11

$$\int \frac{cd^2 + 2cdex + ce^2x^2}{(d+ex)^3} dx = \frac{c \log(d+ex)}{e}$$

output `c*ln(e*x+d)/e`

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00

$$\int \frac{cd^2 + 2cdex + ce^2x^2}{(d+ex)^3} dx = \frac{c \log(d+ex)}{e}$$

input `Integrate[(c*d^2 + 2*c*d*e*x + c*e^2*x^2)/(d + e*x)^3,x]`

output `(c*Log[d + e*x])/e`

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.071$, Rules used = {1098, 16}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{cd^2 + 2cdex + ce^2x^2}{(d + ex)^3} dx$$

↓ 1098

$$\frac{\int \frac{c^2e^2}{d+ex} dx}{ce^2}$$

↓ 16

$$\frac{c \log(d + ex)}{e}$$

input `Int[(c*d^2 + 2*c*d*e*x + c*e^2*x^2)/(d + e*x)^3,x]`

output `(c*Log[d + e*x])/e`

Defintions of rubi rules used

rule 16 `Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] := Simp[c*(Log[RemoveContent[a + b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]`

rule 1098 `Int[((d_.) + (e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] := Simp[1/c^p Int[(d + e*x)^m*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]`

Maple [A] (verified)

Time = 0.61 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.09

| method | result | size |
|--------------|-------------------------|------|
| default | $\frac{c \ln(ex+d)}{e}$ | 12 |
| norman | $\frac{c \ln(ex+d)}{e}$ | 12 |
| risch | $\frac{c \ln(ex+d)}{e}$ | 12 |
| parallelrisc | $\frac{c \ln(ex+d)}{e}$ | 12 |

input `int((c*e^2*x^2+2*c*d*e*x+c*d^2)/(e*x+d)^3,x,method=_RETURNVERBOSE)`output `c*ln(e*x+d)/e`**Fricas [A] (verification not implemented)**

Time = 0.09 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00

$$\int \frac{cd^2 + 2cdex + ce^2x^2}{(d + ex)^3} dx = \frac{c \log(ex + d)}{e}$$

input `integrate((c*e^2*x^2+2*c*d*e*x+c*d^2)/(e*x+d)^3,x, algorithm="fricas")`output `c*log(e*x + d)/e`**Sympy [A] (verification not implemented)**

Time = 0.02 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.73

$$\int \frac{cd^2 + 2cdex + ce^2x^2}{(d + ex)^3} dx = \frac{c \log(d + ex)}{e}$$

input `integrate((c*e**2*x**2+2*c*d*e*x+c*d**2)/(e*x+d)**3,x)`

output `c*log(d + e*x)/e`

Maxima [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00

$$\int \frac{cd^2 + 2cdex + ce^2x^2}{(d + ex)^3} dx = \frac{c \log(ex + d)}{e}$$

input `integrate((c*e^2*x^2+2*c*d*e*x+c*d^2)/(e*x+d)^3,x, algorithm="maxima")`

output `c*log(e*x + d)/e`

Giac [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.09

$$\int \frac{cd^2 + 2cdex + ce^2x^2}{(d + ex)^3} dx = \frac{c \log(|ex + d|)}{e}$$

input `integrate((c*e^2*x^2+2*c*d*e*x+c*d^2)/(e*x+d)^3,x, algorithm="giac")`

output `c*log(abs(e*x + d))/e`

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00

$$\int \frac{cd^2 + 2cdex + ce^2x^2}{(d + ex)^3} dx = \frac{c \ln(d + ex)}{e}$$

input `int((c*d^2 + c*e^2*x^2 + 2*c*d*e*x)/(d + e*x)^3,x)`

output `(c*log(d + e*x))/e`

Reduce [B] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00

$$\int \frac{cd^2 + 2cdex + ce^2x^2}{(d + ex)^3} dx = \frac{\log(ex + d) c}{e}$$

input `int((c*e^2*x^2+2*c*d*e*x+c*d^2)/(e*x+d)^3,x)`

output `(log(d + e*x)*c)/e`

3.7 $\int \frac{cd^2+2cdex+ce^2x^2}{(d+ex)^4} dx$

| | |
|---|-----|
| Optimal result | 221 |
| Mathematica [A] (verified) | 221 |
| Rubi [A] (verified) | 222 |
| Maple [A] (verified) | 223 |
| Fricas [A] (verification not implemented) | 223 |
| Sympy [A] (verification not implemented) | 224 |
| Maxima [A] (verification not implemented) | 224 |
| Giac [A] (verification not implemented) | 224 |
| Mupad [B] (verification not implemented) | 225 |
| Reduce [B] (verification not implemented) | 225 |

Optimal result

Integrand size = 28, antiderivative size = 13

$$\int \frac{cd^2 + 2cdex + ce^2x^2}{(d + ex)^4} dx = -\frac{c}{e(d + ex)}$$

output `-c/e/(e*x+d)`

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00

$$\int \frac{cd^2 + 2cdex + ce^2x^2}{(d + ex)^4} dx = -\frac{c}{e(d + ex)}$$

input `Integrate[(c*d^2 + 2*c*d*e*x + c*e^2*x^2)/(d + e*x)^4,x]`

output `-(c/(e*(d + e*x)))`

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.071$, Rules used = {1098, 17}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{cd^2 + 2cdex + ce^2x^2}{(d + ex)^4} dx$$

$$\downarrow 1098$$

$$\frac{\int \frac{c^2e^2}{(d+ex)^2} dx}{ce^2}$$

$$\downarrow 17$$

$$-\frac{c}{e(d + ex)}$$

input `Int[(c*d^2 + 2*c*d*e*x + c*e^2*x^2)/(d + e*x)^4,x]`

output `-(c/(e*(d + e*x)))`

Defintions of rubi rules used

rule 17 `Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] := Simp[c*((a + b*x)^(m + 1))/(b*(m + 1))], x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]`

rule 1098 `Int[((d_.) + (e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] := Simp[1/c^p Int[(d + e*x)^m*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]`

Maple [A] (verified)

Time = 0.62 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.08

| method | result | size |
|---------------|---|------|
| gospers | $-\frac{c}{e(ex+d)}$ | 14 |
| default | $-\frac{c}{e(ex+d)}$ | 14 |
| risch | $-\frac{c}{e(ex+d)}$ | 14 |
| parallelrisch | $-\frac{c}{e(ex+d)}$ | 14 |
| norman | $\frac{cdx+2ce^2x^2+\frac{e^2cx^3}{d}}{(ex+d)^3}$ | 32 |
| orering | $-\frac{x^2ce^2+2cdxe+cd^2}{(ex+d)^3e}$ | 33 |

input `int((c*e^2*x^2+2*c*d*e*x+c*d^2)/(e*x+d)^4,x,method=_RETURNVERBOSE)`

output `-c/e/(e*x+d)`

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.08

$$\int \frac{cd^2 + 2cdex + ce^2x^2}{(d + ex)^4} dx = -\frac{c}{e^2x + de}$$

input `integrate((c*e^2*x^2+2*c*d*e*x+c*d^2)/(e*x+d)^4,x, algorithm="fricas")`

output `-c/(e^2*x + d*e)`

Sympy [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.77

$$\int \frac{cd^2 + 2cdex + ce^2x^2}{(d + ex)^4} dx = -\frac{c}{de + e^2x}$$

input `integrate((c*e**2*x**2+2*c*d*e*x+c*d**2)/(e*x+d)**4,x)`output `-c/(d*e + e**2*x)`**Maxima [A] (verification not implemented)**

Time = 0.04 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.08

$$\int \frac{cd^2 + 2cdex + ce^2x^2}{(d + ex)^4} dx = -\frac{c}{e^2x + de}$$

input `integrate((c*e^2*x^2+2*c*d*e*x+c*d^2)/(e*x+d)^4,x, algorithm="maxima")`output `-c/(e^2*x + d*e)`**Giac [A] (verification not implemented)**

Time = 0.18 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00

$$\int \frac{cd^2 + 2cdex + ce^2x^2}{(d + ex)^4} dx = -\frac{c}{(ex + d)e}$$

input `integrate((c*e^2*x^2+2*c*d*e*x+c*d^2)/(e*x+d)^4,x, algorithm="giac")`output `-c/((e*x + d)*e)`

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00

$$\int \frac{cd^2 + 2cdex + ce^2x^2}{(d + ex)^4} dx = -\frac{c}{e(d + ex)}$$

input `int((c*d^2 + c*e^2*x^2 + 2*c*d*e*x)/(d + e*x)^4,x)`

output `-c/(e*(d + e*x))`

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00

$$\int \frac{cd^2 + 2cdex + ce^2x^2}{(d + ex)^4} dx = \frac{cx}{d(ex + d)}$$

input `int((c*e^2*x^2+2*c*d*e*x+c*d^2)/(e*x+d)^4,x)`

output `(c*x)/(d*(d + e*x))`

3.8 $\int \frac{cd^2 + 2cdex + ce^2x^2}{(d+ex)^5} dx$

| | |
|---|-----|
| Optimal result | 226 |
| Mathematica [A] (verified) | 226 |
| Rubi [A] (verified) | 227 |
| Maple [A] (verified) | 228 |
| Fricas [A] (verification not implemented) | 228 |
| Sympy [B] (verification not implemented) | 229 |
| Maxima [A] (verification not implemented) | 229 |
| Giac [A] (verification not implemented) | 229 |
| Mupad [B] (verification not implemented) | 230 |
| Reduce [B] (verification not implemented) | 230 |

Optimal result

Integrand size = 28, antiderivative size = 15

$$\int \frac{cd^2 + 2cdex + ce^2x^2}{(d+ex)^5} dx = -\frac{c}{2e(d+ex)^2}$$

output `-1/2*c/e/(e*x+d)^2`

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00

$$\int \frac{cd^2 + 2cdex + ce^2x^2}{(d+ex)^5} dx = -\frac{c}{2e(d+ex)^2}$$

input `Integrate[(c*d^2 + 2*c*d*e*x + c*e^2*x^2)/(d + e*x)^5,x]`

output `-1/2*c/(e*(d + e*x)^2)`

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.071$, Rules used = {1098, 17}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{cd^2 + 2cdex + ce^2x^2}{(d + ex)^5} dx$$

↓ 1098

$$\frac{\int \frac{c^2e^2}{(d+ex)^3} dx}{ce^2}$$

↓ 17

$$-\frac{c}{2e(d + ex)^2}$$

input `Int[(c*d^2 + 2*c*d*e*x + c*e^2*x^2)/(d + e*x)^5,x]`

output `-1/2*c/(e*(d + e*x)^2)`

Defintions of rubi rules used

rule 17 `Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] := Simp[c*((a + b*x)^(m + 1))/(b*(m + 1))], x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]`

rule 1098 `Int[((d_.) + (e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] := Simp[1/c^p Int[(d + e*x)^m*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]`

Maple [A] (verified)

Time = 0.74 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.93

| method | result | size |
|--------------|---|------|
| gospers | $-\frac{c}{2e(ex+d)^2}$ | 14 |
| default | $-\frac{c}{2e(ex+d)^2}$ | 14 |
| risch | $-\frac{c}{2e(ex+d)^2}$ | 14 |
| paralelrisch | $-\frac{c}{2e(ex+d)^2}$ | 14 |
| norman | $\frac{-\frac{cd^2}{2e} - \frac{ce^2x^2}{2} - cdx}{(ex+d)^4}$ | 31 |
| orering | $-\frac{x^2ce^2+2cdxe+cd^2}{2(ex+d)^4e}$ | 33 |

input `int((c*e^2*x^2+2*c*d*e*x+c*d^2)/(e*x+d)^5,x,method=_RETURNVERBOSE)`

output `-1/2*c/e/(e*x+d)^2`

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.67

$$\int \frac{cd^2 + 2cdex + ce^2x^2}{(d + ex)^5} dx = -\frac{c}{2(e^3x^2 + 2de^2x + d^2e)}$$

input `integrate((c*e^2*x^2+2*c*d*e*x+c*d^2)/(e*x+d)^5,x, algorithm="fricas")`

output `-1/2*c/(e^3*x^2 + 2*d*e^2*x + d^2*e)`

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 26 vs. $2(12) = 24$.

Time = 0.08 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.73

$$\int \frac{cd^2 + 2cdex + ce^2x^2}{(d + ex)^5} dx = -\frac{c}{2d^2e + 4de^2x + 2e^3x^2}$$

input `integrate((c*e**2*x**2+2*c*d*e*x+c*d**2)/(e*x+d)**5,x)`

output `-c/(2*d**2*e + 4*d*e**2*x + 2*e**3*x**2)`

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.67

$$\int \frac{cd^2 + 2cdex + ce^2x^2}{(d + ex)^5} dx = -\frac{c}{2(e^3x^2 + 2de^2x + d^2e)}$$

input `integrate((c*e^2*x^2+2*c*d*e*x+c*d^2)/(e*x+d)^5,x, algorithm="maxima")`

output `-1/2*c/(e^3*x^2 + 2*d*e^2*x + d^2*e)`

Giac [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.87

$$\int \frac{cd^2 + 2cdex + ce^2x^2}{(d + ex)^5} dx = -\frac{c}{2(ex + d)^2e}$$

input `integrate((c*e^2*x^2+2*c*d*e*x+c*d^2)/(e*x+d)^5,x, algorithm="giac")`

output `-1/2*c/((e*x + d)^2*e)`

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.60

$$\int \frac{cd^2 + 2cdex + ce^2x^2}{(d + ex)^5} dx = -\frac{c}{2e(d^2 + 2dex + e^2x^2)}$$

input `int((c*d^2 + c*e^2*x^2 + 2*c*d*e*x)/(d + e*x)^5,x)`output `-c/(2*e*(d^2 + e^2*x^2 + 2*d*e*x))`**Reduce [B] (verification not implemented)**

Time = 0.27 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.60

$$\int \frac{cd^2 + 2cdex + ce^2x^2}{(d + ex)^5} dx = -\frac{c}{2e(e^2x^2 + 2dex + d^2)}$$

input `int((c*e^2*x^2+2*c*d*e*x+c*d^2)/(e*x+d)^5,x)`output `(- c)/(2*e*(d**2 + 2*d*e*x + e**2*x**2))`

3.9 $\int \frac{cd^2 + 2cdex + ce^2x^2}{(d+ex)^6} dx$

| | |
|---|-----|
| Optimal result | 231 |
| Mathematica [A] (verified) | 231 |
| Rubi [A] (verified) | 232 |
| Maple [A] (verified) | 233 |
| Fricas [B] (verification not implemented) | 233 |
| Sympy [B] (verification not implemented) | 234 |
| Maxima [B] (verification not implemented) | 234 |
| Giac [A] (verification not implemented) | 234 |
| Mupad [B] (verification not implemented) | 235 |
| Reduce [B] (verification not implemented) | 235 |

Optimal result

Integrand size = 28, antiderivative size = 15

$$\int \frac{cd^2 + 2cdex + ce^2x^2}{(d+ex)^6} dx = -\frac{c}{3e(d+ex)^3}$$

output `-1/3*c/e/(e*x+d)^3`

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00

$$\int \frac{cd^2 + 2cdex + ce^2x^2}{(d+ex)^6} dx = -\frac{c}{3e(d+ex)^3}$$

input `Integrate[(c*d^2 + 2*c*d*e*x + c*e^2*x^2)/(d + e*x)^6,x]`

output `-1/3*c/(e*(d + e*x)^3)`

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.071$, Rules used = {1098, 17}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{cd^2 + 2cdex + ce^2x^2}{(d + ex)^6} dx$$

↓ 1098

$$\frac{\int \frac{c^2e^2}{(d+ex)^4} dx}{ce^2}$$

↓ 17

$$-\frac{c}{3e(d + ex)^3}$$

input `Int[(c*d^2 + 2*c*d*e*x + c*e^2*x^2)/(d + e*x)^6,x]`

output `-1/3*c/(e*(d + e*x)^3)`

Defintions of rubi rules used

rule 17 `Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] := Simp[c*((a + b*x)^(m + 1))/(b*(m + 1))], x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]`

rule 1098 `Int[((d_.) + (e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] := Simp[1/c^p Int[(d + e*x)^m*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]`

Maple [A] (verified)

Time = 0.63 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.93

| method | result | size |
|---------------|--|------|
| gospers | $-\frac{c}{3e(ex+d)^3}$ | 14 |
| default | $-\frac{c}{3e(ex+d)^3}$ | 14 |
| risch | $-\frac{c}{3e(ex+d)^3}$ | 14 |
| parallelrisch | $-\frac{c}{3e(ex+d)^3}$ | 14 |
| norman | $-\frac{\frac{cd^2}{3e} - \frac{ce^2x^2}{3} - \frac{2cdx}{3}}{(ex+d)^5}$ | 31 |
| orering | $-\frac{x^2ce^2+2cdxe+cd^2}{3(ex+d)^5e}$ | 33 |

input `int((c*e^2*x^2+2*c*d*e*x+c*d^2)/(e*x+d)^6,x,method=_RETURNVERBOSE)`

output `-1/3*c/e/(e*x+d)^3`

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 36 vs. 2(13) = 26.

Time = 0.09 (sec) , antiderivative size = 36, normalized size of antiderivative = 2.40

$$\int \frac{cd^2 + 2cdex + ce^2x^2}{(d + ex)^6} dx = -\frac{c}{3(e^4x^3 + 3de^3x^2 + 3d^2e^2x + d^3e)}$$

input `integrate((c*e^2*x^2+2*c*d*e*x+c*d^2)/(e*x+d)^6,x, algorithm="fricas")`

output `-1/3*c/(e^4*x^3 + 3*d*e^3*x^2 + 3*d^2*e^2*x + d^3*e)`

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 37 vs. $2(12) = 24$.

Time = 0.11 (sec) , antiderivative size = 37, normalized size of antiderivative = 2.47

$$\int \frac{cd^2 + 2cdex + ce^2x^2}{(d + ex)^6} dx = -\frac{c}{3d^3e + 9d^2e^2x + 9de^3x^2 + 3e^4x^3}$$

input `integrate((c*e**2*x**2+2*c*d*e*x+c*d**2)/(e*x+d)**6,x)`

output `-c/(3*d**3*e + 9*d**2*e**2*x + 9*d*e**3*x**2 + 3*e**4*x**3)`

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 36 vs. $2(13) = 26$.

Time = 0.03 (sec) , antiderivative size = 36, normalized size of antiderivative = 2.40

$$\int \frac{cd^2 + 2cdex + ce^2x^2}{(d + ex)^6} dx = -\frac{c}{3(e^4x^3 + 3de^3x^2 + 3d^2e^2x + d^3e)}$$

input `integrate((c*e^2*x^2+2*c*d*e*x+c*d^2)/(e*x+d)^6,x, algorithm="maxima")`

output `-1/3*c/(e^4*x^3 + 3*d*e^3*x^2 + 3*d^2*e^2*x + d^3*e)`

Giac [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.87

$$\int \frac{cd^2 + 2cdex + ce^2x^2}{(d + ex)^6} dx = -\frac{c}{3(ex + d)^3e}$$

input `integrate((c*e^2*x^2+2*c*d*e*x+c*d^2)/(e*x+d)^6,x, algorithm="giac")`

output `-1/3*c/((e*x + d)^3*e)`

Mupad [B] (verification not implemented)

Time = 8.66 (sec) , antiderivative size = 35, normalized size of antiderivative = 2.33

$$\int \frac{cd^2 + 2cdex + ce^2x^2}{(d + ex)^6} dx = -\frac{c}{3e(d^3 + 3d^2ex + 3de^2x^2 + e^3x^3)}$$

input `int((c*d^2 + c*e^2*x^2 + 2*c*d*e*x)/(d + e*x)^6,x)`output `-c/(3*e*(d^3 + e^3*x^3 + 3*d*e^2*x^2 + 3*d^2*e*x))`**Reduce [B] (verification not implemented)**

Time = 0.26 (sec) , antiderivative size = 35, normalized size of antiderivative = 2.33

$$\int \frac{cd^2 + 2cdex + ce^2x^2}{(d + ex)^6} dx = -\frac{c}{3e(e^3x^3 + 3de^2x^2 + 3d^2ex + d^3)}$$

input `int((c*e^2*x^2+2*c*d*e*x+c*d^2)/(e*x+d)^6,x)`output `(- c)/(3*e*(d**3 + 3*d**2*e*x + 3*d*e**2*x**2 + e**3*x**3))`

3.10 $\int (d + ex)^2 (cd^2 + 2cdex + ce^2x^2)^2 dx$

| | |
|---|-----|
| Optimal result | 236 |
| Mathematica [A] (verified) | 236 |
| Rubi [A] (verified) | 237 |
| Maple [B] (verified) | 238 |
| Fricas [B] (verification not implemented) | 238 |
| Sympy [B] (verification not implemented) | 239 |
| Maxima [B] (verification not implemented) | 239 |
| Giac [B] (verification not implemented) | 240 |
| Mupad [B] (verification not implemented) | 240 |
| Reduce [B] (verification not implemented) | 241 |

Optimal result

Integrand size = 30, antiderivative size = 17

$$\int (d + ex)^2 (cd^2 + 2cdex + ce^2x^2)^2 dx = \frac{c^2(d + ex)^7}{7e}$$

output

```
1/7*c^2*(e*x+d)^7/e
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00

$$\int (d + ex)^2 (cd^2 + 2cdex + ce^2x^2)^2 dx = \frac{c^2(d + ex)^7}{7e}$$

input

```
Integrate[(d + e*x)^2*(c*d^2 + 2*c*d*e*x + c*e^2*x^2)^2,x]
```

output

```
(c^2*(d + e*x)^7)/(7*e)
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = {1098, 17}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (d + ex)^2 (cd^2 + 2cdex + ce^2x^2)^2 dx$$

$$\downarrow 1098$$

$$\frac{\int c^4 e^4 (d + ex)^6 dx}{c^2 e^4}$$

$$\downarrow 17$$

$$\frac{c^2 (d + ex)^7}{7e}$$

input

```
Int[(d + e*x)^2*(c*d^2 + 2*c*d*e*x + c*e^2*x^2)^2,x]
```

output

```
(c^2*(d + e*x)^7)/(7*e)
```

Defintions of rubi rules used

rule 17

```
Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] := Simp[c*((a + b*x)^(m + 1))/(b*(m + 1))], x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]
```

rule 1098

```
Int[((d_.) + (e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] := Simp[1/c^p Int[(d + e*x)^m*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 68 vs. $2(15) = 30$.

Time = 1.06 (sec) , antiderivative size = 69, normalized size of antiderivative = 4.06

| method | result | size |
|---------------|---|------|
| gospers | $\frac{x(e^6x^6+7de^5x^5+21d^2e^4x^4+35d^3e^3x^3+35d^4e^2x^2+21d^5ex+7d^6)c^2}{7}$ | 69 |
| default | $\frac{1}{7}e^6c^2x^7 + de^5c^2x^6 + 3d^2c^2e^4x^5 + 5d^3e^3c^2x^4 + 5d^4c^2e^2x^3 + 3d^5ec^2x^2 + d^6c^2x$ | 86 |
| norman | $\frac{1}{7}e^6c^2x^7 + de^5c^2x^6 + 3d^2c^2e^4x^5 + 5d^3e^3c^2x^4 + 5d^4c^2e^2x^3 + 3d^5ec^2x^2 + d^6c^2x$ | 86 |
| risch | $\frac{1}{7}e^6c^2x^7 + de^5c^2x^6 + 3d^2c^2e^4x^5 + 5d^3e^3c^2x^4 + 5d^4c^2e^2x^3 + 3d^5ec^2x^2 + d^6c^2x$ | 86 |
| parallelrisch | $\frac{1}{7}e^6c^2x^7 + de^5c^2x^6 + 3d^2c^2e^4x^5 + 5d^3e^3c^2x^4 + 5d^4c^2e^2x^3 + 3d^5ec^2x^2 + d^6c^2x$ | 86 |
| orering | $\frac{x(e^6x^6+7de^5x^5+21d^2e^4x^4+35d^3e^3x^3+35d^4e^2x^2+21d^5ex+7d^6)(x^2ce^2+2cdxe+cd^2)^2}{7(ex+d)^4}$ | 95 |

input `int((e*x+d)^2*(c*e^2*x^2+2*c*d*e*x+c*d^2)^2,x,method=_RETURNVERBOSE)`

output `1/7*x*(e^6*x^6+7*d*e^5*x^5+21*d^2*e^4*x^4+35*d^3*e^3*x^3+35*d^4*e^2*x^2+21*d^5*e*x+7*d^6)*c^2`

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 85 vs. $2(15) = 30$.

Time = 0.10 (sec) , antiderivative size = 85, normalized size of antiderivative = 5.00

$$\int (d+ex)^2 (cd^2 + 2cdex + ce^2x^2)^2 dx = \frac{1}{7}c^2e^6x^7 + c^2de^5x^6 + 3c^2d^2e^4x^5 + 5c^2d^3e^3x^4 + 5c^2d^4e^2x^3 + 3c^2d^5ex^2 + c^2d^6x$$

input `integrate((e*x+d)^2*(c*e^2*x^2+2*c*d*e*x+c*d^2)^2,x, algorithm="fricas")`

output `1/7*c^2*e^6*x^7 + c^2*d*e^5*x^6 + 3*c^2*d^2*e^4*x^5 + 5*c^2*d^3*e^3*x^4 + 5*c^2*d^4*e^2*x^3 + 3*c^2*d^5*e*x^2 + c^2*d^6*x`

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 90 vs. $2(12) = 24$.

Time = 0.02 (sec) , antiderivative size = 90, normalized size of antiderivative = 5.29

$$\int (d + ex)^2 (cd^2 + 2cdex + ce^2x^2)^2 dx = c^2d^6x + 3c^2d^5ex^2 + 5c^2d^4e^2x^3 + 5c^2d^3e^3x^4 \\ + 3c^2d^2e^4x^5 + c^2de^5x^6 + \frac{c^2e^6x^7}{7}$$

input `integrate((e*x+d)**2*(c*e**2*x**2+2*c*d*e*x+c*d**2)**2,x)`

output `c**2*d**6*x + 3*c**2*d**5*e*x**2 + 5*c**2*d**4*e**2*x**3 + 5*c**2*d**3*e**3*x**4 + 3*c**2*d**2*e**4*x**5 + c**2*d*e**5*x**6 + c**2*e**6*x**7/7`

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 85 vs. $2(15) = 30$.

Time = 0.03 (sec) , antiderivative size = 85, normalized size of antiderivative = 5.00

$$\int (d + ex)^2 (cd^2 + 2cdex + ce^2x^2)^2 dx = \frac{1}{7}c^2e^6x^7 + c^2de^5x^6 + 3c^2d^2e^4x^5 + 5c^2d^3e^3x^4 \\ + 5c^2d^4e^2x^3 + 3c^2d^5ex^2 + c^2d^6x$$

input `integrate((e*x+d)^2*(c*e^2*x^2+2*c*d*e*x+c*d^2)^2,x, algorithm="maxima")`

output `1/7*c^2*e^6*x^7 + c^2*d*e^5*x^6 + 3*c^2*d^2*e^4*x^5 + 5*c^2*d^3*e^3*x^4 + 5*c^2*d^4*e^2*x^3 + 3*c^2*d^5*e*x^2 + c^2*d^6*x`

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 85 vs. $2(15) = 30$.

Time = 0.24 (sec) , antiderivative size = 85, normalized size of antiderivative = 5.00

$$\int (d + ex)^2 (cd^2 + 2cdex + ce^2x^2)^2 dx = \frac{1}{7} c^2 e^6 x^7 + c^2 d e^5 x^6 + 3 c^2 d^2 e^4 x^5 + 5 c^2 d^3 e^3 x^4 + 5 c^2 d^4 e^2 x^3 + 3 c^2 d^5 e x^2 + c^2 d^6 x$$

input `integrate((e*x+d)^2*(c*e^2*x^2+2*c*d*e*x+c*d^2)^2,x, algorithm="giac")`

output `1/7*c^2*e^6*x^7 + c^2*d*e^5*x^6 + 3*c^2*d^2*e^4*x^5 + 5*c^2*d^3*e^3*x^4 + 5*c^2*d^4*e^2*x^3 + 3*c^2*d^5*e*x^2 + c^2*d^6*x`

Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 85, normalized size of antiderivative = 5.00

$$\int (d + ex)^2 (cd^2 + 2cdex + ce^2x^2)^2 dx = c^2 d^6 x + 3 c^2 d^5 e x^2 + 5 c^2 d^4 e^2 x^3 + 5 c^2 d^3 e^3 x^4 + 3 c^2 d^2 e^4 x^5 + c^2 d e^5 x^6 + \frac{c^2 e^6 x^7}{7}$$

input `int((d + e*x)^2*(c*d^2 + c*e^2*x^2 + 2*c*d*e*x)^2,x)`

output `c^2*d^6*x + (c^2*e^6*x^7)/7 + 3*c^2*d^5*e*x^2 + c^2*d*e^5*x^6 + 5*c^2*d^4*e^2*x^3 + 5*c^2*d^3*e^3*x^4 + 3*c^2*d^2*e^4*x^5`

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 68, normalized size of antiderivative = 4.00

$$\int (d + ex)^2 (cd^2 + 2cdex + ce^2x^2)^2 dx$$

$$= \frac{c^2x(e^6x^6 + 7de^5x^5 + 21d^2e^4x^4 + 35d^3e^3x^3 + 35d^4e^2x^2 + 21d^5ex + 7d^6)}{7}$$

input `int((e*x+d)^2*(c*e^2*x^2+2*c*d*e*x+c*d^2)^2,x)`output `(c**2*x*(7*d**6 + 21*d**5*e*x + 35*d**4*e**2*x**2 + 35*d**3*e**3*x**3 + 21*d**2*e**4*x**4 + 7*d*e**5*x**5 + e**6*x**6))/7`

3.11 $\int (d + ex) (cd^2 + 2cdex + ce^2x^2)^2 dx$

| | |
|---|-----|
| Optimal result | 242 |
| Mathematica [A] (verified) | 242 |
| Rubi [A] (verified) | 243 |
| Maple [A] (warning: unable to verify) | 244 |
| Fricas [B] (verification not implemented) | 244 |
| Sympy [B] (verification not implemented) | 245 |
| Maxima [A] (verification not implemented) | 245 |
| Giac [B] (verification not implemented) | 245 |
| Mupad [B] (verification not implemented) | 246 |
| Reduce [B] (verification not implemented) | 246 |

Optimal result

Integrand size = 28, antiderivative size = 17

$$\int (d + ex) (cd^2 + 2cdex + ce^2x^2)^2 dx = \frac{c^2(d + ex)^6}{6e}$$

output

```
1/6*c^2*(e*x+d)^6/e
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00

$$\int (d + ex) (cd^2 + 2cdex + ce^2x^2)^2 dx = \frac{c^2(d + ex)^6}{6e}$$

input

```
Integrate[(d + e*x)*(c*d^2 + 2*c*d*e*x + c*e^2*x^2)^2,x]
```

output

```
(c^2*(d + e*x)^6)/(6*e)
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.071$, Rules used = {1098, 17}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (d + ex) (cd^2 + 2cdex + ce^2x^2)^2 dx$$

$$\downarrow 1098$$

$$\frac{\int c^4 e^4 (d + ex)^5 dx}{c^2 e^4}$$

$$\downarrow 17$$

$$\frac{c^2 (d + ex)^6}{6e}$$

input

```
Int[(d + e*x)*(c*d^2 + 2*c*d*e*x + c*e^2*x^2)^2,x]
```

output

```
(c^2*(d + e*x)^6)/(6*e)
```

Defintions of rubi rules used

rule 17

```
Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] := Simp[c*((a + b*x)^(m + 1))/(b*(m + 1)), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]
```

rule 1098

```
Int[((d_.) + (e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] := Simp[1/c^p Int[(d + e*x)^m*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```


Maple [A] (warning: unable to verify)

Time = 0.94 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.82

| method | result | size |
|---------------|---|------|
| default | $\frac{(x^2ce^2+2cdxe+cd^2)^3}{6ce}$ | 31 |
| gospers | $\frac{x(e^5x^5+6de^4x^4+15d^2e^3x^3+20d^3e^2x^2+15d^4ex+6d^5)c^2}{6}$ | 58 |
| norman | $c^2d^5x + dc^2x^5e^4 + \frac{1}{6}c^2x^6e^5 + \frac{5}{2}c^2d^2e^3x^4 + \frac{10}{3}c^2d^3e^2x^3 + \frac{5}{2}c^2d^4ex^2$ | 72 |
| parallelrisch | $c^2d^5x + dc^2x^5e^4 + \frac{1}{6}c^2x^6e^5 + \frac{5}{2}c^2d^2e^3x^4 + \frac{10}{3}c^2d^3e^2x^3 + \frac{5}{2}c^2d^4ex^2$ | 72 |
| risch | $\frac{c^2x^6e^5}{6} + dc^2x^5e^4 + \frac{5c^2d^2e^3x^4}{2} + \frac{10c^2d^3e^2x^3}{3} + \frac{5c^2d^4ex^2}{2} + c^2d^5x + \frac{c^2d^6}{6e}$ | 83 |
| orering | $\frac{x(e^5x^5+6de^4x^4+15d^2e^3x^3+20d^3e^2x^2+15d^4ex+6d^5)(x^2ce^2+2cdxe+cd^2)^2}{6(ex+d)^4}$ | 84 |

input `int((e*x+d)*(c*e^2*x^2+2*c*d*e*x+c*d^2)^2,x,method=_RETURNVERBOSE)`

output `1/6*(c*e^2*x^2+2*c*d*e*x+c*d^2)^3/c/e`

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 71 vs. 2(15) = 30.

Time = 0.08 (sec) , antiderivative size = 71, normalized size of antiderivative = 4.18

$$\int (d+ex)(cd^2+2cdex+ce^2x^2)^2 dx = \frac{1}{6}c^2e^5x^6 + c^2de^4x^5 + \frac{5}{2}c^2d^2e^3x^4 + \frac{10}{3}c^2d^3e^2x^3 + \frac{5}{2}c^2d^4ex^2 + c^2d^5x$$

input `integrate((e*x+d)*(c*e^2*x^2+2*c*d*e*x+c*d^2)^2,x, algorithm="fricas")`

output `1/6*c^2*e^5*x^6 + c^2*d*e^4*x^5 + 5/2*c^2*d^2*e^3*x^4 + 10/3*c^2*d^3*e^2*x^3 + 5/2*c^2*d^4*e*x^2 + c^2*d^5*x`

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 80 vs. $2(12) = 24$.

Time = 0.02 (sec) , antiderivative size = 80, normalized size of antiderivative = 4.71

$$\int (d + ex) (cd^2 + 2cdex + ce^2x^2)^2 dx = c^2d^5x + \frac{5c^2d^4ex^2}{2} + \frac{10c^2d^3e^2x^3}{3} + \frac{5c^2d^2e^3x^4}{2} + c^2de^4x^5 + \frac{c^2e^5x^6}{6}$$

input `integrate((e*x+d)*(c*e**2*x**2+2*c*d*e*x+c*d**2)**2,x)`

output `c**2*d**5*x + 5*c**2*d**4*e*x**2/2 + 10*c**2*d**3*e**2*x**3/3 + 5*c**2*d**2*e**3*x**4/2 + c**2*d*e**4*x**5 + c**2*e**5*x**6/6`

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.76

$$\int (d + ex) (cd^2 + 2cdex + ce^2x^2)^2 dx = \frac{(ce^2x^2 + 2cdex + cd^2)^3}{6ce}$$

input `integrate((e*x+d)*(c*e^2*x^2+2*c*d*e*x+c*d^2)^2,x, algorithm="maxima")`

output `1/6*(c*e^2*x^2 + 2*c*d*e*x + c*d^2)^3/(c*e)`

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 60 vs. $2(15) = 30$.

Time = 0.19 (sec) , antiderivative size = 60, normalized size of antiderivative = 3.53

$$\int (d + ex) (cd^2 + 2cdex + ce^2x^2)^2 dx = \frac{1}{2} (ex^2 + 2dx)c^2d^4 + \frac{1}{2} (ex^2 + 2dx)^2 c^2d^2e + \frac{1}{6} (ex^2 + 2dx)^3 c^2e^2$$

input `integrate((e*x+d)*(c*e^2*x^2+2*c*d*e*x+c*d^2)^2,x, algorithm="giac")`

output `1/2*(e*x^2 + 2*d*x)*c^2*d^4 + 1/2*(e*x^2 + 2*d*x)^2*c^2*d^2*e + 1/6*(e*x^2 + 2*d*x)^3*c^2*e^2`

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 71, normalized size of antiderivative = 4.18

$$\int (d + ex) (cd^2 + 2cdex + ce^2x^2)^2 dx = c^2 d^5 x + \frac{5c^2 d^4 e x^2}{2} + \frac{10c^2 d^3 e^2 x^3}{3} + \frac{5c^2 d^2 e^3 x^4}{2} + c^2 d e^4 x^5 + \frac{c^2 e^5 x^6}{6}$$

input `int((d + e*x)*(c*d^2 + c*e^2*x^2 + 2*c*d*e*x)^2,x)`

output `c^2*d^5*x + (c^2*e^5*x^6)/6 + (5*c^2*d^4*e*x^2)/2 + c^2*d*e^4*x^5 + (10*c^2*d^3*e^2*x^3)/3 + (5*c^2*d^2*e^3*x^4)/2`

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 57, normalized size of antiderivative = 3.35

$$\int (d + ex) (cd^2 + 2cdex + ce^2x^2)^2 dx = \frac{c^2x(e^5x^5 + 6de^4x^4 + 15d^2e^3x^3 + 20d^3e^2x^2 + 15d^4ex + 6d^5)}{6}$$

input `int((e*x+d)*(c*e^2*x^2+2*c*d*e*x+c*d^2)^2,x)`

output `(c**2*x*(6*d**5 + 15*d**4*e*x + 20*d**3*e**2*x**2 + 15*d**2*e**3*x**3 + 6*d*e**4*x**4 + e**5*x**5))/6`

3.12 $\int (cd^2 + 2cdex + ce^2x^2)^2 dx$

| | |
|---|-----|
| Optimal result | 247 |
| Mathematica [A] (verified) | 247 |
| Rubi [A] (verified) | 248 |
| Maple [B] (verified) | 249 |
| Fricas [B] (verification not implemented) | 249 |
| Sympy [B] (verification not implemented) | 250 |
| Maxima [B] (verification not implemented) | 250 |
| Giac [B] (verification not implemented) | 251 |
| Mupad [B] (verification not implemented) | 251 |
| Reduce [B] (verification not implemented) | 251 |

Optimal result

Integrand size = 22, antiderivative size = 17

$$\int (cd^2 + 2cdex + ce^2x^2)^2 dx = \frac{c^2(d + ex)^5}{5e}$$

output

```
1/5*c^2*(e*x+d)^5/e
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00

$$\int (cd^2 + 2cdex + ce^2x^2)^2 dx = \frac{c^2(d + ex)^5}{5e}$$

input

```
Integrate[(c*d^2 + 2*c*d*e*x + c*e^2*x^2)^2,x]
```

output

```
(c^2*(d + e*x)^5)/(5*e)
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {1077, 17}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (cd^2 + 2cdex + ce^2x^2)^2 dx$$

$$\downarrow 1077$$

$$\frac{\int (cxe^2 + cde)^4 dx}{c^2e^4}$$

$$\downarrow 17$$

$$\frac{c^2(d + ex)^5}{5e}$$

input `Int[(c*d^2 + 2*c*d*e*x + c*e^2*x^2)^2,x]`

output `(c^2*(d + e*x)^5)/(5*e)`

Defintions of rubi rules used

rule 17 `Int[(c_.)*((a_.) + (b_.)*(x_)^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1))/(b*(m + 1)), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]`

rule 1077 `Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[1/c^p Int[(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]`

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 46 vs. $2(15) = 30$.

Time = 0.85 (sec) , antiderivative size = 47, normalized size of antiderivative = 2.76

| method | result | size |
|---------------|---|------|
| gospers | $\frac{x(e^4x^4+5de^3x^3+10d^2e^2x^2+10d^3ex+5d^4)c^2}{5}$ | 47 |
| default | $\frac{1}{5}c^2x^5e^4 + dc^2x^4e^3 + 2x^3c^2d^2e^2 + 2x^2c^2d^3e + c^2d^4x$ | 58 |
| norman | $\frac{1}{5}c^2x^5e^4 + dc^2x^4e^3 + 2x^3c^2d^2e^2 + 2x^2c^2d^3e + c^2d^4x$ | 58 |
| risch | $\frac{1}{5}c^2x^5e^4 + dc^2x^4e^3 + 2x^3c^2d^2e^2 + 2x^2c^2d^3e + c^2d^4x$ | 58 |
| parallelrisch | $\frac{1}{5}c^2x^5e^4 + dc^2x^4e^3 + 2x^3c^2d^2e^2 + 2x^2c^2d^3e + c^2d^4x$ | 58 |
| orering | $\frac{x(e^4x^4+5de^3x^3+10d^2e^2x^2+10d^3ex+5d^4)(x^2ce^2+2cdxe+cd^2)^2}{5(ex+d)^4}$ | 73 |

input `int((c*e^2*x^2+2*c*d*e*x+c*d^2)^2,x,method=_RETURNVERBOSE)`

output `1/5*x*(e^4*x^4+5*d*e^3*x^3+10*d^2*e^2*x^2+10*d^3*e*x+5*d^4)*c^2`

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 57 vs. $2(15) = 30$.

Time = 0.09 (sec) , antiderivative size = 57, normalized size of antiderivative = 3.35

$$\int (cd^2 + 2cdex + ce^2x^2)^2 dx = \frac{1}{5}c^2e^4x^5 + c^2de^3x^4 + 2c^2d^2e^2x^3 + 2c^2d^3ex^2 + c^2d^4x$$

input `integrate((c*e^2*x^2+2*c*d*e*x+c*d^2)^2,x, algorithm="fricas")`

output `1/5*c^2*e^4*x^5 + c^2*d*e^3*x^4 + 2*c^2*d^2*e^2*x^3 + 2*c^2*d^3*e*x^2 + c^2*d^4*x`

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 60 vs. $2(12) = 24$.

Time = 0.02 (sec) , antiderivative size = 60, normalized size of antiderivative = 3.53

$$\int (cd^2 + 2cdex + ce^2x^2)^2 dx = c^2d^4x + 2c^2d^3ex^2 + 2c^2d^2e^2x^3 + c^2de^3x^4 + \frac{c^2e^4x^5}{5}$$

input `integrate((c**2*x**2+2*c*d*e*x+c*d**2)**2,x)`

output `c**2*d**4*x + 2*c**2*d**3*e*x**2 + 2*c**2*d**2*e**2*x**3 + c**2*d*e**3*x**4 + c**2*e**4*x**5/5`

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 68 vs. $2(15) = 30$.

Time = 0.05 (sec) , antiderivative size = 68, normalized size of antiderivative = 4.00

$$\int (cd^2 + 2cdex + ce^2x^2)^2 dx = \frac{1}{5}c^2e^4x^5 + c^2de^3x^4 + \frac{4}{3}c^2d^2e^2x^3 + c^2d^4x + \frac{2}{3}(ce^2x^3 + 3cdex^2)cd^2$$

input `integrate((c*e^2*x^2+2*c*d*e*x+c*d^2)^2,x, algorithm="maxima")`

output `1/5*c^2*e^4*x^5 + c^2*d*e^3*x^4 + 4/3*c^2*d^2*e^2*x^3 + c^2*d^4*x + 2/3*(c*e^2*x^3 + 3*c*d*e*x^2)*c*d^2`

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 57 vs. $2(15) = 30$.

Time = 0.23 (sec) , antiderivative size = 57, normalized size of antiderivative = 3.35

$$\int (cd^2 + 2cdex + ce^2x^2)^2 dx = \frac{1}{5} c^2 e^4 x^5 + c^2 d e^3 x^4 + 2 c^2 d^2 e^2 x^3 + 2 c^2 d^3 e x^2 + c^2 d^4 x$$

input `integrate((c*e^2*x^2+2*c*d*e*x+c*d^2)^2,x, algorithm="giac")`

output $\frac{1}{5}c^2e^4x^5 + c^2d^4x + 2c^2d^3ex^2 + 2c^2d^2e^2x^3 + 2c^2de^3x^4$

Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 57, normalized size of antiderivative = 3.35

$$\int (cd^2 + 2cdex + ce^2x^2)^2 dx = c^2 d^4 x + 2 c^2 d^3 e x^2 + 2 c^2 d^2 e^2 x^3 + c^2 d e^3 x^4 + \frac{c^2 e^4 x^5}{5}$$

input `int((c*d^2 + c*e^2*x^2 + 2*c*d*e*x)^2,x)`

output $c^2d^4x + (c^2e^4x^5)/5 + 2c^2d^3ex^2 + c^2d^2e^2x^3 + c^2de^3x^4$

Reduce [B] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 46, normalized size of antiderivative = 2.71

$$\int (cd^2 + 2cdex + ce^2x^2)^2 dx = \frac{c^2x(e^4x^4 + 5de^3x^3 + 10d^2e^2x^2 + 10d^3ex + 5d^4)}{5}$$

input `int((c*e^2*x^2+2*c*d*e*x+c*d^2)^2,x)`

output $(c^{**2}x(5d^{**4} + 10d^{**3}e^x + 10d^{**2}e^{**2}x^{**2} + 5de^{**3}x^{**3} + e^{**4}x^{**4}))/5$

$$3.13 \quad \int \frac{(cd^2 + 2cdex + ce^2x^2)^2}{d+ex} dx$$

| | |
|---|-----|
| Optimal result | 253 |
| Mathematica [A] (verified) | 253 |
| Rubi [A] (verified) | 254 |
| Maple [A] (verified) | 255 |
| Fricas [B] (verification not implemented) | 255 |
| Sympy [B] (verification not implemented) | 256 |
| Maxima [B] (verification not implemented) | 256 |
| Giac [B] (verification not implemented) | 256 |
| Mupad [B] (verification not implemented) | 257 |
| Reduce [B] (verification not implemented) | 257 |

Optimal result

Integrand size = 30, antiderivative size = 17

$$\int \frac{(cd^2 + 2cdex + ce^2x^2)^2}{d+ex} dx = \frac{c^2(d+ex)^4}{4e}$$

output $1/4*c^2*(e*x+d)^4/e$

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00

$$\int \frac{(cd^2 + 2cdex + ce^2x^2)^2}{d+ex} dx = \frac{c^2(d+ex)^4}{4e}$$

input `Integrate[(c*d^2 + 2*c*d*e*x + c*e^2*x^2)^2/(d + e*x),x]`

output $(c^2*(d + e*x)^4)/(4*e)$

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = {1098, 17}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(cd^2 + 2cdex + ce^2x^2)^2}{d + ex} dx$$

↓ 1098

$$\frac{\int c^4e^4(d + ex)^3 dx}{c^2e^4}$$

↓ 17

$$\frac{c^2(d + ex)^4}{4e}$$

input `Int[(c*d^2 + 2*c*d*e*x + c*e^2*x^2)^2/(d + e*x),x]`

output `(c^2*(d + e*x)^4)/(4*e)`

Defintions of rubi rules used

rule 17 `Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] := Simp[c*((a + b*x)^(m + 1))/(b*(m + 1)), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]`

rule 1098 `Int[((d_.) + (e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] := Simp[1/c^p Int[(d + e*x)^m*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]`

Maple [A] (verified)

Time = 1.21 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.59

| method | result | size |
|---------------|--|------|
| default | $\frac{c^2(e^2x^2+2dex+d^2)^2}{4e}$ | 27 |
| gospers | $\frac{x(e^3x^3+4de^2x^2+6d^2ex+4d^3)c^2}{4}$ | 36 |
| norman | $c^2d^3x + c^2de^2x^3 + \frac{1}{4}c^2x^4e^3 + \frac{3}{2}d^2ec^2x^2$ | 44 |
| parallemrisch | $c^2d^3x + c^2de^2x^3 + \frac{1}{4}c^2x^4e^3 + \frac{3}{2}d^2ec^2x^2$ | 44 |
| risch | $\frac{c^2x^4e^3}{4} + c^2de^2x^3 + \frac{3d^2ec^2x^2}{2} + c^2d^3x + \frac{c^2d^4}{4e}$ | 55 |
| orering | $\frac{x(e^3x^3+4de^2x^2+6d^2ex+4d^3)(x^2ce^2+2cdxe+cd^2)^2}{4(ex+d)^4}$ | 62 |

input `int((c*e^2*x^2+2*c*d*e*x+c*d^2)^2/(e*x+d),x,method=_RETURNVERBOSE)`

output `1/4*c^2*(e^2*x^2+2*d*e*x+d^2)^2/e`

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 43 vs. 2(15) = 30.

Time = 0.08 (sec) , antiderivative size = 43, normalized size of antiderivative = 2.53

$$\int \frac{(cd^2 + 2cdex + ce^2x^2)^2}{d + ex} dx = \frac{1}{4}c^2e^3x^4 + c^2de^2x^3 + \frac{3}{2}c^2d^2ex^2 + c^2d^3x$$

input `integrate((c*e^2*x^2+2*c*d*e*x+c*d^2)^2/(e*x+d),x, algorithm="fricas")`

output `1/4*c^2*e^3*x^4 + c^2*d*e^2*x^3 + 3/2*c^2*d^2*e*x^2 + c^2*d^3*x`

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 46 vs. $2(12) = 24$.

Time = 0.03 (sec) , antiderivative size = 46, normalized size of antiderivative = 2.71

$$\int \frac{(cd^2 + 2cdex + ce^2x^2)^2}{d + ex} dx = c^2d^3x + \frac{3c^2d^2ex^2}{2} + c^2de^2x^3 + \frac{c^2e^3x^4}{4}$$

input `integrate((c**2*x**2+2*c*d*e*x+c*d**2)**2/(e*x+d),x)`

output `c**2*d**3*x + 3*c**2*d**2*e*x**2/2 + c**2*d*e**2*x**3 + c**2*e**3*x**4/4`

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 43 vs. $2(15) = 30$.

Time = 0.04 (sec) , antiderivative size = 43, normalized size of antiderivative = 2.53

$$\int \frac{(cd^2 + 2cdex + ce^2x^2)^2}{d + ex} dx = \frac{1}{4}c^2e^3x^4 + c^2de^2x^3 + \frac{3}{2}c^2d^2ex^2 + c^2d^3x$$

input `integrate((c*e^2*x^2+2*c*d*e*x+c*d^2)^2/(e*x+d),x, algorithm="maxima")`

output `1/4*c^2*e^3*x^4 + c^2*d*e^2*x^3 + 3/2*c^2*d^2*e*x^2 + c^2*d^3*x`

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 43 vs. $2(15) = 30$.

Time = 0.20 (sec) , antiderivative size = 43, normalized size of antiderivative = 2.53

$$\int \frac{(cd^2 + 2cdex + ce^2x^2)^2}{d + ex} dx = \frac{1}{4}c^2e^3x^4 + c^2de^2x^3 + \frac{3}{2}c^2d^2ex^2 + c^2d^3x$$

input `integrate((c*e^2*x^2+2*c*d*e*x+c*d^2)^2/(e*x+d),x, algorithm="giac")`

output $1/4*c^2*e^3*x^4 + c^2*d*e^2*x^3 + 3/2*c^2*d^2*e*x^2 + c^2*d^3*x$

Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 43, normalized size of antiderivative = 2.53

$$\int \frac{(cd^2 + 2cdex + ce^2x^2)^2}{d + ex} dx = c^2 d^3 x + \frac{3c^2 d^2 e x^2}{2} + c^2 d e^2 x^3 + \frac{c^2 e^3 x^4}{4}$$

input $\text{int}((c*d^2 + c*e^2*x^2 + 2*c*d*e*x)^2/(d + e*x), x)$

output $c^2*d^3*x + (c^2*e^3*x^4)/4 + (3*c^2*d^2*e*x^2)/2 + c^2*d*e^2*x^3$

Reduce [B] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 35, normalized size of antiderivative = 2.06

$$\int \frac{(cd^2 + 2cdex + ce^2x^2)^2}{d + ex} dx = \frac{c^2 x (e^3 x^3 + 4d e^2 x^2 + 6d^2 e x + 4d^3)}{4}$$

input $\text{int}((c*e^2*x^2+2*c*d*e*x+c*d^2)^2/(e*x+d), x)$

output $(c**2*x*(4*d**3 + 6*d**2*e*x + 4*d*e**2*x**2 + e**3*x**3))/4$

$$3.14 \quad \int \frac{(cd^2 + 2cdex + ce^2x^2)^2}{(d+ex)^2} dx$$

| | |
|---|-----|
| Optimal result | 258 |
| Mathematica [A] (verified) | 258 |
| Rubi [A] (verified) | 259 |
| Maple [A] (verified) | 260 |
| Fricas [A] (verification not implemented) | 260 |
| Sympy [B] (verification not implemented) | 261 |
| Maxima [A] (verification not implemented) | 261 |
| Giac [A] (verification not implemented) | 261 |
| Mupad [B] (verification not implemented) | 262 |
| Reduce [B] (verification not implemented) | 262 |

Optimal result

Integrand size = 30, antiderivative size = 17

$$\int \frac{(cd^2 + 2cdex + ce^2x^2)^2}{(d+ex)^2} dx = \frac{c^2(d+ex)^3}{3e}$$

output

$$1/3*c^2*(e*x+d)^3/e$$

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00

$$\int \frac{(cd^2 + 2cdex + ce^2x^2)^2}{(d+ex)^2} dx = \frac{c^2(d+ex)^3}{3e}$$

input

$$\text{Integrate}[(c*d^2 + 2*c*d*e*x + c*e^2*x^2)^2/(d + e*x)^2, x]$$

output

$$(c^2*(d + e*x)^3)/(3*e)$$

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = {1098, 17}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(cd^2 + 2cdex + ce^2x^2)^2}{(d + ex)^2} dx$$

↓ 1098

$$\frac{\int c^4e^4(d + ex)^2 dx}{c^2e^4}$$

↓ 17

$$\frac{c^2(d + ex)^3}{3e}$$

input `Int[(c*d^2 + 2*c*d*e*x + c*e^2*x^2)^2/(d + e*x)^2,x]`

output `(c^2*(d + e*x)^3)/(3*e)`

Defintions of rubi rules used

rule 17 `Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] := Simp[c*((a + b*x)^(m + 1))/(b*(m + 1))], x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]`

rule 1098 `Int[((d_.) + (e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] := Simp[1/c^p Int[(d + e*x)^m*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]`

Maple [A] (verified)

Time = 1.15 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.94

| method | result | size |
|--------------|---|------|
| default | $\frac{c^2(ex+d)^3}{3e}$ | 16 |
| gospers | $\frac{x(e^2x^2+3dex+3d^2)c^2}{3}$ | 25 |
| paralelrisch | $\frac{1}{3}c^2e^2x^3 + c^2dex^2 + c^2d^2x$ | 30 |
| risch | $\frac{c^2e^2x^3}{3} + c^2dex^2 + c^2d^2x + \frac{c^2d^3}{3e}$ | 41 |
| orering | $\frac{x(e^2x^2+3dex+3d^2)(x^2ce^2+2cdxe+cd^2)^2}{3(ex+d)^4}$ | 51 |
| norman | $\frac{c^2d^3x + \frac{1}{3}c^2x^4e^3 + \frac{4}{3}c^2de^2x^3 + 2d^2ec^2x^2}{ex+d}$ | 53 |

input `int((c*e^2*x^2+2*c*d*e*x+c*d^2)^2/(e*x+d)^2,x,method=_RETURNVERBOSE)`

output `1/3*c^2*(e*x+d)^3/e`

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.71

$$\int \frac{(cd^2 + 2cdex + ce^2x^2)^2}{(d + ex)^2} dx = \frac{1}{3}c^2e^2x^3 + c^2dex^2 + c^2d^2x$$

input `integrate((c*e^2*x^2+2*c*d*e*x+c*d^2)^2/(e*x+d)^2,x, algorithm="fricas")`

output `1/3*c^2*e^2*x^3 + c^2*d*e*x^2 + c^2*d^2*x`

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 29 vs. $2(12) = 24$.

Time = 0.03 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.71

$$\int \frac{(cd^2 + 2cdex + ce^2x^2)^2}{(d + ex)^2} dx = c^2d^2x + c^2dex^2 + \frac{c^2e^2x^3}{3}$$

input `integrate((c**2*x**2+2*c*d*e*x+c*d**2)**2/(e*x+d)**2,x)`

output `c**2*d**2*x + c**2*d*e*x**2 + c**2*e**2*x**3/3`

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.71

$$\int \frac{(cd^2 + 2cdex + ce^2x^2)^2}{(d + ex)^2} dx = \frac{1}{3}c^2e^2x^3 + c^2dex^2 + c^2d^2x$$

input `integrate((c*e^2*x^2+2*c*d*e*x+c*d^2)^2/(e*x+d)^2,x, algorithm="maxima")`

output `1/3*c^2*e^2*x^3 + c^2*d*e*x^2 + c^2*d^2*x`

Giac [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.88

$$\int \frac{(cd^2 + 2cdex + ce^2x^2)^2}{(d + ex)^2} dx = \frac{(ex + d)^3 c^2}{3e}$$

input `integrate((c*e^2*x^2+2*c*d*e*x+c*d^2)^2/(e*x+d)^2,x, algorithm="giac")`

output `1/3*(e*x + d)^3*c^2/e`

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.41

$$\int \frac{(cd^2 + 2cdex + ce^2x^2)^2}{(d + ex)^2} dx = \frac{c^2 x (3d^2 + 3dex + e^2x^2)}{3}$$

input `int((c*d^2 + c*e^2*x^2 + 2*c*d*e*x)^2/(d + e*x)^2,x)`output `(c^2*x*(3*d^2 + e^2*x^2 + 3*d*e*x))/3`**Reduce [B] (verification not implemented)**

Time = 0.26 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.41

$$\int \frac{(cd^2 + 2cdex + ce^2x^2)^2}{(d + ex)^2} dx = \frac{c^2x(e^2x^2 + 3dex + 3d^2)}{3}$$

input `int((c*e^2*x^2+2*c*d*e*x+c*d^2)^2/(e*x+d)^2,x)`output `(c**2*x*(3*d**2 + 3*d*e*x + e**2*x**2))/3`

$$3.15 \quad \int \frac{(cd^2 + 2cdex + ce^2x^2)^2}{(d+ex)^3} dx$$

| | |
|---|-----|
| Optimal result | 263 |
| Mathematica [A] (verified) | 263 |
| Rubi [A] (verified) | 264 |
| Maple [A] (verified) | 265 |
| Fricas [A] (verification not implemented) | 265 |
| Sympy [A] (verification not implemented) | 266 |
| Maxima [A] (verification not implemented) | 266 |
| Giac [A] (verification not implemented) | 266 |
| Mupad [B] (verification not implemented) | 267 |
| Reduce [B] (verification not implemented) | 267 |

Optimal result

Integrand size = 30, antiderivative size = 17

$$\int \frac{(cd^2 + 2cdex + ce^2x^2)^2}{(d+ex)^3} dx = \frac{c^2(d+ex)^2}{2e}$$

output

```
1/2*c^2*(e*x+d)^2/e
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.94

$$\int \frac{(cd^2 + 2cdex + ce^2x^2)^2}{(d+ex)^3} dx = c^2 \left(dx + \frac{ex^2}{2} \right)$$

input

```
Integrate[(c*d^2 + 2*c*d*e*x + c*e^2*x^2)^2/(d + e*x)^3,x]
```

output

```
c^2*(d*x + (e*x^2)/2)
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = {1098, 17}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(cd^2 + 2cdex + ce^2x^2)^2}{(d + ex)^3} dx$$

↓ 1098

$$\frac{\int c^4 e^4 (d + ex) dx}{c^2 e^4}$$

↓ 17

$$\frac{c^2 (d + ex)^2}{2e}$$

input `Int[(c*d^2 + 2*c*d*e*x + c*e^2*x^2)^2/(d + e*x)^3,x]`

output `(c^2*(d + e*x)^2)/(2*e)`

Defintions of rubi rules used

rule 17 `Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] := Simp[c*((a + b*x)^(m + 1))/(b*(m + 1))], x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]`

rule 1098 `Int[((d_.) + (e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] := Simp[1/c^p Int[(d + e*x)^m*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]`

Maple [A] (verified)

Time = 1.16 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.82

| method | result | size |
|---------------|--|------|
| gospers | $\frac{x(ex+2d)c^2}{2}$ | 14 |
| default | $c^2(\frac{1}{2}ex^2 + dx)$ | 15 |
| risch | $\frac{1}{2}c^2ex^2 + c^2dx$ | 17 |
| parallelrisch | $\frac{1}{2}c^2ex^2 + c^2dx$ | 17 |
| orering | $\frac{x(ex+2d)(x^2ce^2+2cdxe+cd^2)^2}{2(ex+d)^4}$ | 40 |
| norman | $\frac{\frac{c^2x^4e^3}{2} - \frac{5c^2d^4}{2e} + 2c^2de^2x^3 - 4c^2d^3x}{(ex+d)^2}$ | 53 |

input `int((c*e^2*x^2+2*c*d*e*x+c*d^2)^2/(e*x+d)^3,x,method=_RETURNVERBOSE)`

output `1/2*x*(e*x+2*d)*c^2`

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.94

$$\int \frac{(cd^2 + 2cdex + ce^2x^2)^2}{(d + ex)^3} dx = \frac{1}{2}c^2ex^2 + c^2dx$$

input `integrate((c*e^2*x^2+2*c*d*e*x+c*d^2)^2/(e*x+d)^3,x, algorithm="fricas")`

output `1/2*c^2*e*x^2 + c^2*d*x`

Sympy [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.88

$$\int \frac{(cd^2 + 2cdex + ce^2x^2)^2}{(d + ex)^3} dx = c^2dx + \frac{c^2ex^2}{2}$$

input `integrate((c**2*x**2+2*c*d*e*x+c*d**2)**2/(e*x+d)**3,x)`output `c**2*d*x + c**2*e*x**2/2`**Maxima [A] (verification not implemented)**

Time = 0.03 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.94

$$\int \frac{(cd^2 + 2cdex + ce^2x^2)^2}{(d + ex)^3} dx = \frac{1}{2}c^2ex^2 + c^2dx$$

input `integrate((c*e^2*x^2+2*c*d*e*x+c*d^2)^2/(e*x+d)^3,x, algorithm="maxima")`output `1/2*c^2*e*x^2 + c^2*d*x`**Giac [A] (verification not implemented)**

Time = 0.22 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.94

$$\int \frac{(cd^2 + 2cdex + ce^2x^2)^2}{(d + ex)^3} dx = \frac{1}{2}c^2ex^2 + c^2dx$$

input `integrate((c*e^2*x^2+2*c*d*e*x+c*d^2)^2/(e*x+d)^3,x, algorithm="giac")`output `1/2*c^2*e*x^2 + c^2*d*x`

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.76

$$\int \frac{(cd^2 + 2cdex + ce^2x^2)^2}{(d + ex)^3} dx = \frac{c^2 x (2d + ex)}{2}$$

input `int((c*d^2 + c*e^2*x^2 + 2*c*d*e*x)^2/(d + e*x)^3,x)`output `(c^2*x*(2*d + e*x))/2`**Reduce [B] (verification not implemented)**

Time = 0.36 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.76

$$\int \frac{(cd^2 + 2cdex + ce^2x^2)^2}{(d + ex)^3} dx = \frac{c^2 x (ex + 2d)}{2}$$

input `int((c*e^2*x^2+2*c*d*e*x+c*d^2)^2/(e*x+d)^3,x)`output `(c**2*x*(2*d + e*x))/2`

$$3.16 \quad \int \frac{(cd^2 + 2cdex + ce^2x^2)^2}{(d+ex)^4} dx$$

| | |
|---|-----|
| Optimal result | 268 |
| Mathematica [A] (verified) | 268 |
| Rubi [A] (verified) | 269 |
| Maple [A] (verified) | 270 |
| Fricas [A] (verification not implemented) | 270 |
| Sympy [A] (verification not implemented) | 270 |
| Maxima [A] (verification not implemented) | 271 |
| Giac [A] (verification not implemented) | 271 |
| Mupad [B] (verification not implemented) | 271 |
| Reduce [B] (verification not implemented) | 272 |

Optimal result

Integrand size = 30, antiderivative size = 5

$$\int \frac{(cd^2 + 2cdex + ce^2x^2)^2}{(d+ex)^4} dx = c^2x$$

output `c^2*x`

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 5, normalized size of antiderivative = 1.00

$$\int \frac{(cd^2 + 2cdex + ce^2x^2)^2}{(d+ex)^4} dx = c^2x$$

input `Integrate[(c*d^2 + 2*c*d*e*x + c*e^2*x^2)^2/(d + e*x)^4,x]`

output `c^2*x`

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 5, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = {1098, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(cd^2 + 2cdex + ce^2x^2)^2}{(d + ex)^4} dx$$

↓ 1098

$$\frac{\int c^4 e^4 dx}{c^2 e^4}$$

↓ 24

$$c^2 x$$

input `Int[(c*d^2 + 2*c*d*e*x + c*e^2*x^2)^2/(d + e*x)^4,x]`

output `c^2*x`

Defintions of rubi rules used

rule 24 `Int[a_, x_Symbol] := Simp[a*x, x] /; FreeQ[a, x]`

rule 1098 `Int[((d_.) + (e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] := Simp[1/c^p Int[(d + e*x)^m*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]`

Maple [A] (verified)

Time = 1.10 (sec) , antiderivative size = 6, normalized size of antiderivative = 1.20

| method | result | size |
|---------|---|------|
| default | c^2x | 6 |
| risch | c^2x | 6 |
| orering | $\frac{x(x^2ce^2+2cdxe+cd^2)^2}{(ex+d)^4}$ | 32 |
| norman | $\frac{c^2x^4e^3 - \frac{3c^2d^4}{e} - 6d^2ec^2x^2 - 8c^2d^3x}{(ex+d)^3}$ | 52 |

input `int((c*e^2*x^2+2*c*d*e*x+c*d^2)^2/(e*x+d)^4,x,method=_RETURNVERBOSE)`output `c^2*x`**Fricas [A] (verification not implemented)**

Time = 0.07 (sec) , antiderivative size = 5, normalized size of antiderivative = 1.00

$$\int \frac{(cd^2 + 2cdex + ce^2x^2)^2}{(d + ex)^4} dx = c^2x$$

input `integrate((c*e^2*x^2+2*c*d*e*x+c*d^2)^2/(e*x+d)^4,x, algorithm="fricas")`output `c^2*x`**Sympy [A] (verification not implemented)**

Time = 0.03 (sec) , antiderivative size = 3, normalized size of antiderivative = 0.60

$$\int \frac{(cd^2 + 2cdex + ce^2x^2)^2}{(d + ex)^4} dx = c^2x$$

input `integrate((c*e**2*x**2+2*c*d*e*x+c*d**2)**2/(e*x+d)**4,x)`

output `c**2*x`

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 5, normalized size of antiderivative = 1.00

$$\int \frac{(cd^2 + 2cdex + ce^2x^2)^2}{(d + ex)^4} dx = c^2x$$

input `integrate((c*e^2*x^2+2*c*d*e*x+c*d^2)^2/(e*x+d)^4,x, algorithm="maxima")`

output `c^2*x`

Giac [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 5, normalized size of antiderivative = 1.00

$$\int \frac{(cd^2 + 2cdex + ce^2x^2)^2}{(d + ex)^4} dx = c^2x$$

input `integrate((c*e^2*x^2+2*c*d*e*x+c*d^2)^2/(e*x+d)^4,x, algorithm="giac")`

output `c^2*x`

Mupad [B] (verification not implemented)

Time = 0.01 (sec) , antiderivative size = 5, normalized size of antiderivative = 1.00

$$\int \frac{(cd^2 + 2cdex + ce^2x^2)^2}{(d + ex)^4} dx = c^2x$$

input `int((c*d^2 + c*e^2*x^2 + 2*c*d*e*x)^2/(d + e*x)^4,x)`

output `c^2*x`

Reduce [B] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 5, normalized size of antiderivative = 1.00

$$\int \frac{(cd^2 + 2cdex + ce^2x^2)^2}{(d + ex)^4} dx = c^2x$$

input `int((c*e^2*x^2+2*c*d*e*x+c*d^2)^2/(e*x+d)^4,x)`

output `c**2*x`

$$3.17 \quad \int \frac{(cd^2 + 2cdex + ce^2x^2)^2}{(d+ex)^5} dx$$

| | |
|---|-----|
| Optimal result | 273 |
| Mathematica [A] (verified) | 273 |
| Rubi [A] (verified) | 274 |
| Maple [A] (verified) | 275 |
| Fricas [A] (verification not implemented) | 275 |
| Sympy [A] (verification not implemented) | 275 |
| Maxima [A] (verification not implemented) | 276 |
| Giac [B] (verification not implemented) | 276 |
| Mupad [B] (verification not implemented) | 277 |
| Reduce [B] (verification not implemented) | 277 |

Optimal result

Integrand size = 30, antiderivative size = 13

$$\int \frac{(cd^2 + 2cdex + ce^2x^2)^2}{(d+ex)^5} dx = \frac{c^2 \log(d+ex)}{e}$$

output

```
c^2*ln(e*x+d)/e
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00

$$\int \frac{(cd^2 + 2cdex + ce^2x^2)^2}{(d+ex)^5} dx = \frac{c^2 \log(d+ex)}{e}$$

input

```
Integrate[(c*d^2 + 2*c*d*e*x + c*e^2*x^2)^2/(d + e*x)^5,x]
```

output

```
(c^2*Log[d + e*x])/e
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = {1098, 16}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(cd^2 + 2cdex + ce^2x^2)^2}{(d + ex)^5} dx$$

↓ 1098

$$\frac{\int \frac{c^4e^4}{d+ex} dx}{c^2e^4}$$

↓ 16

$$\frac{c^2 \log(d + ex)}{e}$$

input `Int[(c*d^2 + 2*c*d*e*x + c*e^2*x^2)^2/(d + e*x)^5,x]`

output `(c^2*Log[d + e*x])/e`

Defintions of rubi rules used

rule 16 `Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] := Simp[c*(Log[RemoveContent[a + b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]`

rule 1098 `Int[((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] := Simp[1/c^p Int[(d + e*x)^m*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]`

Maple [A] (verified)

Time = 1.11 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.08

| method | result | size |
|---------------|---------------------------|------|
| default | $\frac{c^2 \ln(ex+d)}{e}$ | 14 |
| norman | $\frac{c^2 \ln(ex+d)}{e}$ | 14 |
| risch | $\frac{c^2 \ln(ex+d)}{e}$ | 14 |
| parallelrisch | $\frac{c^2 \ln(ex+d)}{e}$ | 14 |

input `int((c*e^2*x^2+2*c*d*e*x+c*d^2)^2/(e*x+d)^5,x,method=_RETURNVERBOSE)`output `c^2*ln(e*x+d)/e`**Fricas [A] (verification not implemented)**

Time = 0.09 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00

$$\int \frac{(cd^2 + 2cdex + ce^2x^2)^2}{(d + ex)^5} dx = \frac{c^2 \log(ex + d)}{e}$$

input `integrate((c*e^2*x^2+2*c*d*e*x+c*d^2)^2/(e*x+d)^5,x, algorithm="fricas")`output `c^2*log(e*x + d)/e`**Sympy [A] (verification not implemented)**

Time = 0.03 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.77

$$\int \frac{(cd^2 + 2cdex + ce^2x^2)^2}{(d + ex)^5} dx = \frac{c^2 \log(d + ex)}{e}$$

input `integrate((c**2*x**2+2*c*d*e*x+c*d**2)**2/(e*x+d)**5,x)`

output `c**2*log(d + e*x)/e`

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00

$$\int \frac{(cd^2 + 2cdex + ce^2x^2)^2}{(d + ex)^5} dx = \frac{c^2 \log(ex + d)}{e}$$

input `integrate((c*e^2*x^2+2*c*d*e*x+c*d^2)^2/(e*x+d)^5,x, algorithm="maxima")`

output `c^2*log(e*x + d)/e`

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 27 vs. $2(13) = 26$.

Time = 0.19 (sec) , antiderivative size = 27, normalized size of antiderivative = 2.08

$$\int \frac{(cd^2 + 2cdex + ce^2x^2)^2}{(d + ex)^5} dx = -\frac{c^2 \log\left(\frac{|ex+d|}{(ex+d)^2|e|}\right)}{e}$$

input `integrate((c*e^2*x^2+2*c*d*e*x+c*d^2)^2/(e*x+d)^5,x, algorithm="giac")`

output `-c^2*log(abs(e*x + d)/((e*x + d)^2*abs(e)))/e`

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00

$$\int \frac{(cd^2 + 2cdex + ce^2x^2)^2}{(d + ex)^5} dx = \frac{c^2 \ln(d + ex)}{e}$$

input `int((c*d^2 + c*e^2*x^2 + 2*c*d*e*x)^2/(d + e*x)^5,x)`output `(c^2*log(d + e*x))/e`**Reduce [B] (verification not implemented)**

Time = 0.25 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00

$$\int \frac{(cd^2 + 2cdex + ce^2x^2)^2}{(d + ex)^5} dx = \frac{\log(ex + d) c^2}{e}$$

input `int((c*e^2*x^2+2*c*d*e*x+c*d^2)^2/(e*x+d)^5,x)`output `(log(d + e*x)*c**2)/e`

$$3.18 \quad \int \frac{(cd^2 + 2cdex + ce^2x^2)^2}{(d+ex)^6} dx$$

| | |
|---|-----|
| Optimal result | 278 |
| Mathematica [A] (verified) | 278 |
| Rubi [A] (verified) | 279 |
| Maple [A] (verified) | 280 |
| Fricas [A] (verification not implemented) | 280 |
| Sympy [A] (verification not implemented) | 281 |
| Maxima [A] (verification not implemented) | 281 |
| Giac [A] (verification not implemented) | 281 |
| Mupad [B] (verification not implemented) | 282 |
| Reduce [B] (verification not implemented) | 282 |

Optimal result

Integrand size = 30, antiderivative size = 15

$$\int \frac{(cd^2 + 2cdex + ce^2x^2)^2}{(d+ex)^6} dx = -\frac{c^2}{e(d+ex)}$$

output `-c^2/e/(e*x+d)`

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00

$$\int \frac{(cd^2 + 2cdex + ce^2x^2)^2}{(d+ex)^6} dx = -\frac{c^2}{e(d+ex)}$$

input `Integrate[(c*d^2 + 2*c*d*e*x + c*e^2*x^2)^2/(d + e*x)^6,x]`

output `-(c^2/(e*(d + e*x)))`

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = {1098, 17}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(cd^2 + 2cdex + ce^2x^2)^2}{(d + ex)^6} dx$$

↓ 1098

$$\frac{\int \frac{c^4e^4}{(d+ex)^2} dx}{c^2e^4}$$

↓ 17

$$-\frac{c^2}{e(d + ex)}$$

input

```
Int[(c*d^2 + 2*c*d*e*x + c*e^2*x^2)^2/(d + e*x)^6,x]
```

output

```
-(c^2/(e*(d + e*x)))
```

Defintions of rubi rules used

rule 17

```
Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1))/(b*(m + 1)), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]
```

rule 1098

```
Int[((d_.) + (e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[1/c^p Int[(d + e*x)^m*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Maple [A] (verified)

Time = 1.11 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.07

| method | result | size |
|---------------|---|------|
| gospers | $-\frac{c^2}{e(ex+d)}$ | 16 |
| default | $-\frac{c^2}{e(ex+d)}$ | 16 |
| risch | $-\frac{c^2}{e(ex+d)}$ | 16 |
| parallelrisch | $-\frac{c^2}{e(ex+d)}$ | 16 |
| orering | $-\frac{(x^2ce^2+2cdxe+cd^2)^2}{(ex+d)^5e}$ | 35 |
| norman | $-\frac{c^2d^4 - c^2x^4e^3 - 4c^2de^2x^3 - 6d^2ec^2x^2 - 4c^2d^3x}{(ex+d)^5}$ | 65 |

input `int((c*e^2*x^2+2*c*d*e*x+c*d^2)^2/(e*x+d)^6,x,method=_RETURNVERBOSE)`

output `-c^2/e/(e*x+d)`

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.07

$$\int \frac{(cd^2 + 2cdex + ce^2x^2)^2}{(d + ex)^6} dx = -\frac{c^2}{e^2x + de}$$

input `integrate((c*e^2*x^2+2*c*d*e*x+c*d^2)^2/(e*x+d)^6,x, algorithm="fricas")`

output `-c^2/(e^2*x + d*e)`

Sympy [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.80

$$\int \frac{(cd^2 + 2cdex + ce^2x^2)^2}{(d + ex)^6} dx = -\frac{c^2}{de + e^2x}$$

input `integrate((c**2*x**2+2*c*d*e*x+c*d**2)**2/(e*x+d)**6,x)`output `-c**2/(d*e + e**2*x)`**Maxima [A] (verification not implemented)**

Time = 0.04 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.07

$$\int \frac{(cd^2 + 2cdex + ce^2x^2)^2}{(d + ex)^6} dx = -\frac{c^2}{e^2x + de}$$

input `integrate((c*e^2*x^2+2*c*d*e*x+c*d^2)^2/(e*x+d)^6,x, algorithm="maxima")`output `-c^2/(e^2*x + d*e)`**Giac [A] (verification not implemented)**

Time = 0.21 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00

$$\int \frac{(cd^2 + 2cdex + ce^2x^2)^2}{(d + ex)^6} dx = -\frac{c^2}{(ex + d)e}$$

input `integrate((c*e^2*x^2+2*c*d*e*x+c*d^2)^2/(e*x+d)^6,x, algorithm="giac")`output `-c^2/((e*x + d)*e)`

Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00

$$\int \frac{(cd^2 + 2cdex + ce^2x^2)^2}{(d + ex)^6} dx = -\frac{c^2}{e(d + ex)}$$

input `int((c*d^2 + c*e^2*x^2 + 2*c*d*e*x)^2/(d + e*x)^6,x)`output `-c^2/(e*(d + e*x))`**Reduce [B] (verification not implemented)**

Time = 0.26 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00

$$\int \frac{(cd^2 + 2cdex + ce^2x^2)^2}{(d + ex)^6} dx = \frac{c^2x}{d(ex + d)}$$

input `int((c*e^2*x^2+2*c*d*e*x+c*d^2)^2/(e*x+d)^6,x)`output `(c**2*x)/(d*(d + e*x))`

$$3.19 \quad \int \frac{(cd^2 + 2cdex + ce^2x^2)^2}{(d+ex)^7} dx$$

| | |
|---|-----|
| Optimal result | 283 |
| Mathematica [A] (verified) | 283 |
| Rubi [A] (verified) | 284 |
| Maple [A] (verified) | 285 |
| Fricas [A] (verification not implemented) | 285 |
| Sympy [A] (verification not implemented) | 286 |
| Maxima [A] (verification not implemented) | 286 |
| Giac [A] (verification not implemented) | 286 |
| Mupad [B] (verification not implemented) | 287 |
| Reduce [B] (verification not implemented) | 287 |

Optimal result

Integrand size = 30, antiderivative size = 17

$$\int \frac{(cd^2 + 2cdex + ce^2x^2)^2}{(d+ex)^7} dx = -\frac{c^2}{2e(d+ex)^2}$$

output

$$-1/2*c^2/e/(e*x+d)^2$$

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00

$$\int \frac{(cd^2 + 2cdex + ce^2x^2)^2}{(d+ex)^7} dx = -\frac{c^2}{2e(d+ex)^2}$$

input

$$\text{Integrate}[(c*d^2 + 2*c*d*e*x + c*e^2*x^2)^2/(d + e*x)^7, x]$$

output

$$-1/2*c^2/(e*(d + e*x)^2)$$

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = {1098, 17}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(cd^2 + 2cdex + ce^2x^2)^2}{(d + ex)^7} dx$$

↓ 1098

$$\frac{\int \frac{c^4e^4}{(d+ex)^3} dx}{c^2e^4}$$

↓ 17

$$-\frac{c^2}{2e(d + ex)^2}$$

input

```
Int[(c*d^2 + 2*c*d*e*x + c*e^2*x^2)^2/(d + e*x)^7,x]
```

output

```
-1/2*c^2/(e*(d + e*x)^2)
```

Defintions of rubi rules used

rule 17

```
Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1))/(b*(m + 1)), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]
```

rule 1098

```
Int[((d_.) + (e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[1/c^p Int[(d + e*x)^m*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Maple [A] (verified)

Time = 1.14 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.94

| method | result | size |
|---------------|--|------|
| gospers | $-\frac{c^2}{2e(ex+d)^2}$ | 16 |
| default | $-\frac{c^2}{2e(ex+d)^2}$ | 16 |
| risch | $-\frac{c^2}{2e(ex+d)^2}$ | 16 |
| parallelrisch | $-\frac{c^2}{2e(ex+d)^2}$ | 16 |
| orering | $-\frac{(x^2ce^2+2cdxe+cd^2)^2}{2(ex+d)^6e}$ | 35 |
| norman | $\frac{-\frac{c^2d^4}{2e}-\frac{c^2x^4e^3}{2}-2c^2de^2x^3-3d^2ec^2x^2-2c^2d^3x}{(ex+d)^6}$ | 65 |

input `int((c*e^2*x^2+2*c*d*e*x+c*d^2)^2/(e*x+d)^7,x,method=_RETURNVERBOSE)`

output `-1/2*c^2/e/(e*x+d)^2`

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.59

$$\int \frac{(cd^2 + 2cdex + ce^2x^2)^2}{(d + ex)^7} dx = -\frac{c^2}{2(e^3x^2 + 2de^2x + d^2e)}$$

input `integrate((c*e^2*x^2+2*c*d*e*x+c*d^2)^2/(e*x+d)^7,x, algorithm="fricas")`

output `-1/2*c^2/(e^3*x^2 + 2*d*e^2*x + d^2*e)`

Sympy [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.59

$$\int \frac{(cd^2 + 2cdex + ce^2x^2)^2}{(d + ex)^7} dx = -\frac{c^2}{2d^2e + 4de^2x + 2e^3x^2}$$

input `integrate((c**2*x**2+2*c*d*e*x+c*d**2)**2/(e*x+d)**7,x)`output `-c**2/(2*d**2*e + 4*d*e**2*x + 2*e**3*x**2)`**Maxima [A] (verification not implemented)**

Time = 0.03 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.59

$$\int \frac{(cd^2 + 2cdex + ce^2x^2)^2}{(d + ex)^7} dx = -\frac{c^2}{2(e^3x^2 + 2de^2x + d^2e)}$$

input `integrate((c*e^2*x^2+2*c*d*e*x+c*d^2)^2/(e*x+d)^7,x, algorithm="maxima")`output `-1/2*c^2/(e^3*x^2 + 2*d*e^2*x + d^2*e)`**Giac [A] (verification not implemented)**

Time = 0.19 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.88

$$\int \frac{(cd^2 + 2cdex + ce^2x^2)^2}{(d + ex)^7} dx = -\frac{c^2}{2(ex + d)^2e}$$

input `integrate((c*e^2*x^2+2*c*d*e*x+c*d^2)^2/(e*x+d)^7,x, algorithm="giac")`output `-1/2*c^2/((e*x + d)^2*e)`

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.53

$$\int \frac{(cd^2 + 2cdex + ce^2x^2)^2}{(d + ex)^7} dx = -\frac{c^2}{2e(d^2 + 2dex + e^2x^2)}$$

input `int((c*d^2 + c*e^2*x^2 + 2*c*d*e*x)^2/(d + e*x)^7,x)`output `-c^2/(2*e*(d^2 + e^2*x^2 + 2*d*e*x))`**Reduce [B] (verification not implemented)**

Time = 0.24 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.53

$$\int \frac{(cd^2 + 2cdex + ce^2x^2)^2}{(d + ex)^7} dx = -\frac{c^2}{2e(e^2x^2 + 2dex + d^2)}$$

input `int((c*e^2*x^2+2*c*d*e*x+c*d^2)^2/(e*x+d)^7,x)`output `(- c**2)/(2*e*(d**2 + 2*d*e*x + e**2*x**2))`

$$3.20 \quad \int \frac{(cd^2 + 2cdex + ce^2x^2)^2}{(d+ex)^8} dx$$

| | |
|---|-----|
| Optimal result | 288 |
| Mathematica [A] (verified) | 288 |
| Rubi [A] (verified) | 289 |
| Maple [A] (verified) | 290 |
| Fricas [B] (verification not implemented) | 290 |
| Sympy [B] (verification not implemented) | 291 |
| Maxima [B] (verification not implemented) | 291 |
| Giac [A] (verification not implemented) | 291 |
| Mupad [B] (verification not implemented) | 292 |
| Reduce [B] (verification not implemented) | 292 |

Optimal result

Integrand size = 30, antiderivative size = 17

$$\int \frac{(cd^2 + 2cdex + ce^2x^2)^2}{(d+ex)^8} dx = -\frac{c^2}{3e(d+ex)^3}$$

output

$$-1/3*c^2/e/(e*x+d)^3$$

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00

$$\int \frac{(cd^2 + 2cdex + ce^2x^2)^2}{(d+ex)^8} dx = -\frac{c^2}{3e(d+ex)^3}$$

input

```
Integrate[(c*d^2 + 2*c*d*e*x + c*e^2*x^2)^2/(d + e*x)^8,x]
```

output

$$-1/3*c^2/(e*(d + e*x)^3)$$

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = {1098, 17}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(cd^2 + 2cdex + ce^2x^2)^2}{(d + ex)^8} dx$$

↓ 1098

$$\frac{\int \frac{c^4e^4}{(d+ex)^4} dx}{c^2e^4}$$

↓ 17

$$-\frac{c^2}{3e(d + ex)^3}$$

input

```
Int[(c*d^2 + 2*c*d*e*x + c*e^2*x^2)^2/(d + e*x)^8,x]
```

output

```
-1/3*c^2/(e*(d + e*x)^3)
```

Defintions of rubi rules used

rule 17

```
Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1))/(b*(m + 1)), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]
```

rule 1098

```
Int[((d_.) + (e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[1/c^p Int[(d + e*x)^m*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Maple [A] (verified)

Time = 1.09 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.94

| method | result | size |
|---------------|--|------|
| gosper | $-\frac{c^2}{3e(ex+d)^3}$ | 16 |
| default | $-\frac{c^2}{3e(ex+d)^3}$ | 16 |
| risch | $-\frac{c^2}{3e(ex+d)^3}$ | 16 |
| parallelrisch | $-\frac{c^2}{3e(ex+d)^3}$ | 16 |
| orering | $-\frac{(x^2ce^2+2cdxe+cd^2)^2}{3(ex+d)^7e}$ | 35 |
| norman | $\frac{-\frac{c^2d^4}{3e}-\frac{c^2x^4e^3}{3}-\frac{4c^2de^2x^3}{3}-2d^2ec^2x^2-\frac{4c^2d^3x}{3}}{(ex+d)^7}$ | 65 |

input `int((c*e^2*x^2+2*c*d*e*x+c*d^2)^2/(e*x+d)^8,x,method=_RETURNVERBOSE)`

output `-1/3*c^2/e/(e*x+d)^3`

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 38 vs. $2(15) = 30$.

Time = 0.09 (sec) , antiderivative size = 38, normalized size of antiderivative = 2.24

$$\int \frac{(cd^2 + 2cdex + ce^2x^2)^2}{(d + ex)^8} dx = -\frac{c^2}{3(e^4x^3 + 3de^3x^2 + 3d^2e^2x + d^3e)}$$

input `integrate((c*e^2*x^2+2*c*d*e*x+c*d^2)^2/(e*x+d)^8,x, algorithm="fricas")`

output `-1/3*c^2/(e^4*x^3 + 3*d*e^3*x^2 + 3*d^2*e^2*x + d^3*e)`

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 39 vs. $2(14) = 28$.

Time = 0.11 (sec) , antiderivative size = 39, normalized size of antiderivative = 2.29

$$\int \frac{(cd^2 + 2cdex + ce^2x^2)^2}{(d + ex)^8} dx = -\frac{c^2}{3d^3e + 9d^2e^2x + 9de^3x^2 + 3e^4x^3}$$

input `integrate((c**2*x**2+2*c*d*e*x+c*d**2)**2/(e*x+d)**8,x)`

output `-c**2/(3*d**3*e + 9*d**2*e**2*x + 9*d*e**3*x**2 + 3*e**4*x**3)`

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 38 vs. $2(15) = 30$.

Time = 0.03 (sec) , antiderivative size = 38, normalized size of antiderivative = 2.24

$$\int \frac{(cd^2 + 2cdex + ce^2x^2)^2}{(d + ex)^8} dx = -\frac{c^2}{3(e^4x^3 + 3de^3x^2 + 3d^2e^2x + d^3e)}$$

input `integrate((c*e^2*x^2+2*c*d*e*x+c*d^2)^2/(e*x+d)^8,x, algorithm="maxima")`

output `-1/3*c^2/(e^4*x^3 + 3*d*e^3*x^2 + 3*d^2*e^2*x + d^3*e)`

Giac [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.88

$$\int \frac{(cd^2 + 2cdex + ce^2x^2)^2}{(d + ex)^8} dx = -\frac{c^2}{3(ex + d)^3e}$$

input `integrate((c*e^2*x^2+2*c*d*e*x+c*d^2)^2/(e*x+d)^8,x, algorithm="giac")`

output `-1/3*c^2/((e*x + d)^3*e)`

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 37, normalized size of antiderivative = 2.18

$$\int \frac{(cd^2 + 2cdex + ce^2x^2)^2}{(d + ex)^8} dx = -\frac{c^2}{3e(d^3 + 3d^2ex + 3de^2x^2 + e^3x^3)}$$

input `int((c*d^2 + c*e^2*x^2 + 2*c*d*e*x)^2/(d + e*x)^8,x)`output `-c^2/(3*e*(d^3 + e^3*x^3 + 3*d*e^2*x^2 + 3*d^2*e*x))`**Reduce [B] (verification not implemented)**

Time = 0.25 (sec) , antiderivative size = 37, normalized size of antiderivative = 2.18

$$\int \frac{(cd^2 + 2cdex + ce^2x^2)^2}{(d + ex)^8} dx = -\frac{c^2}{3e(e^3x^3 + 3de^2x^2 + 3d^2ex + d^3)}$$

input `int((c*e^2*x^2+2*c*d*e*x+c*d^2)^2/(e*x+d)^8,x)`output `(- c**2)/(3*e*(d**3 + 3*d**2*e*x + 3*d*e**2*x**2 + e**3*x**3))`

3.21 $\int \frac{(d+ex)^5}{cd^2+2cdex+ce^2x^2} dx$

| | |
|---|-----|
| Optimal result | 293 |
| Mathematica [A] (verified) | 293 |
| Rubi [A] (verified) | 294 |
| Maple [A] (verified) | 295 |
| Fricas [B] (verification not implemented) | 295 |
| Sympy [B] (verification not implemented) | 296 |
| Maxima [B] (verification not implemented) | 296 |
| Giac [B] (verification not implemented) | 296 |
| Mupad [B] (verification not implemented) | 297 |
| Reduce [B] (verification not implemented) | 297 |

Optimal result

Integrand size = 30, antiderivative size = 17

$$\int \frac{(d+ex)^5}{cd^2+2cdex+ce^2x^2} dx = \frac{(d+ex)^4}{4ce}$$

output `1/4*(e*x+d)^4/c/e`

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00

$$\int \frac{(d+ex)^5}{cd^2+2cdex+ce^2x^2} dx = \frac{(d+ex)^4}{4ce}$$

input `Integrate[(d + e*x)^5/(c*d^2 + 2*c*d*e*x + c*e^2*x^2), x]`

output `(d + e*x)^4/(4*c*e)`

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = {1098, 17}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d + ex)^5}{cd^2 + 2cdex + ce^2x^2} dx$$

↓ 1098

$$ce^2 \int \frac{(d + ex)^3}{c^2e^2} dx$$

↓ 17

$$\frac{(d + ex)^4}{4ce}$$

input

```
Int[(d + e*x)^5/(c*d^2 + 2*c*d*e*x + c*e^2*x^2),x]
```

output

```
(d + e*x)^4/(4*c*e)
```

Defintions of rubi rules used

rule 17

```
Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] := Simp[c*((a + b*x)^(m + 1))/(b*(m + 1)), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]
```

rule 1098

```
Int[((d_.) + (e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] := Simp[1/c^p Int[(d + e*x)^m*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Maple [A] (verified)

Time = 1.14 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.94

| method | result | size |
|---------------|--|------|
| default | $\frac{(ex+d)^4}{4ce}$ | 16 |
| gosper | $\frac{x(e^3x^3+4de^2x^2+6d^2ex+4d^3)}{4c}$ | 36 |
| parallelrisch | $\frac{e^3x^4+4de^2x^3+6d^2ex^2+4d^3x}{4c}$ | 38 |
| risch | $\frac{e^3x^4}{4c} + \frac{e^2dx^3}{c} + \frac{3e d^2x^2}{2c} + \frac{d^3x}{c} + \frac{d^4}{4ec}$ | 55 |
| orering | $\frac{x(e^3x^3+4de^2x^2+6d^2ex+4d^3)(ex+d)^2}{4x^2ce^2+8cdxe+4cd^2}$ | 62 |
| norman | $\frac{-\frac{d^5}{ce} + \frac{e^4x^5}{4c} + \frac{5de^3x^4}{4c} + \frac{5d^2e^2x^3}{2c} + \frac{5d^3ex^2}{2c}}{ex+d}$ | 70 |

input `int((e*x+d)^5/(c*e^2*x^2+2*c*d*e*x+c*d^2),x,method=_RETURNVERBOSE)`

output `1/4*(e*x+d)^4/c/e`

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 37 vs. $2(15) = 30$.

Time = 0.08 (sec) , antiderivative size = 37, normalized size of antiderivative = 2.18

$$\int \frac{(d+ex)^5}{cd^2+2cdex+ce^2x^2} dx = \frac{e^3x^4+4de^2x^3+6d^2ex^2+4d^3x}{4c}$$

input `integrate((e*x+d)^5/(c*e^2*x^2+2*c*d*e*x+c*d^2),x, algorithm="fricas")`

output `1/4*(e^3*x^4+4*d*e^2*x^3+6*d^2*e*x^2+4*d^3*x)/c`

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 39 vs. $2(10) = 20$.

Time = 0.03 (sec) , antiderivative size = 39, normalized size of antiderivative = 2.29

$$\int \frac{(d+ex)^5}{cd^2+2cdex+ce^2x^2} dx = \frac{d^3x}{c} + \frac{3d^2ex^2}{2c} + \frac{de^2x^3}{c} + \frac{e^3x^4}{4c}$$

input `integrate((e*x+d)**5/(c*e**2*x**2+2*c*d*e*x+c*d**2),x)`

output `d**3*x/c + 3*d**2*e*x**2/(2*c) + d*e**2*x**3/c + e**3*x**4/(4*c)`

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 37 vs. $2(15) = 30$.

Time = 0.03 (sec) , antiderivative size = 37, normalized size of antiderivative = 2.18

$$\int \frac{(d+ex)^5}{cd^2+2cdex+ce^2x^2} dx = \frac{e^3x^4 + 4de^2x^3 + 6d^2ex^2 + 4d^3x}{4c}$$

input `integrate((e*x+d)^5/(c*e^2*x^2+2*c*d*e*x+c*d^2),x, algorithm="maxima")`

output `1/4*(e^3*x^4 + 4*d*e^2*x^3 + 6*d^2*e*x^2 + 4*d^3*x)/c`

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 37 vs. $2(15) = 30$.

Time = 0.21 (sec) , antiderivative size = 37, normalized size of antiderivative = 2.18

$$\int \frac{(d+ex)^5}{cd^2+2cdex+ce^2x^2} dx = \frac{e^3x^4 + 4de^2x^3 + 6d^2ex^2 + 4d^3x}{4c}$$

input `integrate((e*x+d)^5/(c*e^2*x^2+2*c*d*e*x+c*d^2),x, algorithm="giac")`

output $1/4*(e^3*x^4 + 4*d*e^2*x^3 + 6*d^2*e*x^2 + 4*d^3*x)/c$

Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 43, normalized size of antiderivative = 2.53

$$\int \frac{(d + ex)^5}{cd^2 + 2cdex + ce^2x^2} dx = \frac{d^3 x}{c} + \frac{e^3 x^4}{4c} + \frac{3d^2 e x^2}{2c} + \frac{d e^2 x^3}{c}$$

input `int((d + e*x)^5/(c*d^2 + c*e^2*x^2 + 2*c*d*e*x),x)`

output $(d^3*x)/c + (e^3*x^4)/(4*c) + (3*d^2*e*x^2)/(2*c) + (d*e^2*x^3)/c$

Reduce [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 35, normalized size of antiderivative = 2.06

$$\int \frac{(d + ex)^5}{cd^2 + 2cdex + ce^2x^2} dx = \frac{x(e^3x^3 + 4de^2x^2 + 6d^2ex + 4d^3)}{4c}$$

input `int((e*x+d)^5/(c*e^2*x^2+2*c*d*e*x+c*d^2),x)`

output $(x*(4*d**3 + 6*d**2*e*x + 4*d*e**2*x**2 + e**3*x**3))/(4*c)$

3.22 $\int \frac{(d+ex)^4}{cd^2+2cdex+ce^2x^2} dx$

| | |
|---|-----|
| Optimal result | 298 |
| Mathematica [A] (verified) | 298 |
| Rubi [A] (verified) | 299 |
| Maple [A] (verified) | 300 |
| Fricas [A] (verification not implemented) | 300 |
| Sympy [B] (verification not implemented) | 301 |
| Maxima [A] (verification not implemented) | 301 |
| Giac [A] (verification not implemented) | 301 |
| Mupad [B] (verification not implemented) | 302 |
| Reduce [B] (verification not implemented) | 302 |

Optimal result

Integrand size = 30, antiderivative size = 17

$$\int \frac{(d+ex)^4}{cd^2+2cdex+ce^2x^2} dx = \frac{(d+ex)^3}{3ce}$$

output

```
1/3*(e*x+d)^3/c/e
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00

$$\int \frac{(d+ex)^4}{cd^2+2cdex+ce^2x^2} dx = \frac{(d+ex)^3}{3ce}$$

input

```
Integrate[(d + e*x)^4/(c*d^2 + 2*c*d*e*x + c*e^2*x^2), x]
```

output

```
(d + e*x)^3/(3*c*e)
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = {1098, 17}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d + ex)^4}{cd^2 + 2cdex + ce^2x^2} dx$$

↓ 1098

$$ce^2 \int \frac{(d + ex)^2}{c^2e^2} dx$$

↓ 17

$$\frac{(d + ex)^3}{3ce}$$

input

```
Int[(d + e*x)^4/(c*d^2 + 2*c*d*e*x + c*e^2*x^2),x]
```

output

```
(d + e*x)^3/(3*c*e)
```

Defintions of rubi rules used

rule 17

```
Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] := Simp[c*((a + b*x)^(m + 1))/(b*(m + 1)), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]
```

rule 1098

```
Int[((d_.) + (e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] := Simp[1/c^p Int[(d + e*x)^m*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```


Maple [A] (verified)

Time = 1.12 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.94

| method | result | size |
|---------------|---|------|
| default | $\frac{(ex+d)^3}{3ce}$ | 16 |
| gosper | $\frac{x(e^2x^2+3dex+3d^2)}{3c}$ | 25 |
| parallelrisch | $\frac{e^2x^3+3dex^2+3d^2x}{3c}$ | 27 |
| risch | $\frac{e^2x^3}{3c} + \frac{edx^2}{c} + \frac{d^2x}{c} + \frac{d^3}{3ec}$ | 41 |
| orering | $\frac{x(e^2x^2+3dex+3d^2)(ex+d)^2}{3x^2ce^2+6cdxe+3cd^2}$ | 51 |
| norman | $\frac{-\frac{d^4}{ec} + \frac{e^3x^4}{3c} + \frac{2ed^2x^2}{c} + \frac{4e^2dx^3}{3c}}{ex+d}$ | 56 |

input `int((e*x+d)^4/(c*e^2*x^2+2*c*d*e*x+c*d^2),x,method=_RETURNVERBOSE)`output `1/3*(e*x+d)^3/c/e`**Fricas [A] (verification not implemented)**

Time = 0.08 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.53

$$\int \frac{(d+ex)^4}{cd^2+2cdex+ce^2x^2} dx = \frac{e^2x^3+3dex^2+3d^2x}{3c}$$

input `integrate((e*x+d)^4/(c*e^2*x^2+2*c*d*e*x+c*d^2),x, algorithm="fricas")`output `1/3*(e^2*x^3 + 3*d*e*x^2 + 3*d^2*x)/c`

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 24 vs. $2(10) = 20$.

Time = 0.03 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.41

$$\int \frac{(d + ex)^4}{cd^2 + 2cdex + ce^2x^2} dx = \frac{d^2x}{c} + \frac{dex^2}{c} + \frac{e^2x^3}{3c}$$

input `integrate((e*x+d)**4/(c*e**2*x**2+2*c*d*e*x+c*d**2),x)`

output `d**2*x/c + d*e*x**2/c + e**2*x**3/(3*c)`

Maxima [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.53

$$\int \frac{(d + ex)^4}{cd^2 + 2cdex + ce^2x^2} dx = \frac{e^2x^3 + 3dex^2 + 3d^2x}{3c}$$

input `integrate((e*x+d)^4/(c*e^2*x^2+2*c*d*e*x+c*d^2),x, algorithm="maxima")`

output `1/3*(e^2*x^3 + 3*d*e*x^2 + 3*d^2*x)/c`

Giac [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.53

$$\int \frac{(d + ex)^4}{cd^2 + 2cdex + ce^2x^2} dx = \frac{e^2x^3 + 3dex^2 + 3d^2x}{3c}$$

input `integrate((e*x+d)^4/(c*e^2*x^2+2*c*d*e*x+c*d^2),x, algorithm="giac")`

output `1/3*(e^2*x^3 + 3*d*e*x^2 + 3*d^2*x)/c`

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.41

$$\int \frac{(d + ex)^4}{cd^2 + 2cdex + ce^2x^2} dx = \frac{x(3d^2 + 3dex + e^2x^2)}{3c}$$

input `int((d + e*x)^4/(c*d^2 + c*e^2*x^2 + 2*c*d*e*x),x)`output `(x*(3*d^2 + e^2*x^2 + 3*d*e*x))/(3*c)`**Reduce [B] (verification not implemented)**

Time = 0.26 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.41

$$\int \frac{(d + ex)^4}{cd^2 + 2cdex + ce^2x^2} dx = \frac{x(e^2x^2 + 3dex + 3d^2)}{3c}$$

input `int((e*x+d)^4/(c*e^2*x^2+2*c*d*e*x+c*d^2),x)`output `(x*(3*d**2 + 3*d*e*x + e**2*x**2))/(3*c)`

3.23 $\int \frac{(d+ex)^3}{cd^2+2cdex+ce^2x^2} dx$

| | |
|---|-----|
| Optimal result | 303 |
| Mathematica [A] (verified) | 303 |
| Rubi [A] (verified) | 304 |
| Maple [A] (verified) | 305 |
| Fricas [A] (verification not implemented) | 305 |
| Sympy [A] (verification not implemented) | 306 |
| Maxima [A] (verification not implemented) | 306 |
| Giac [A] (verification not implemented) | 306 |
| Mupad [B] (verification not implemented) | 307 |
| Reduce [B] (verification not implemented) | 307 |

Optimal result

Integrand size = 30, antiderivative size = 17

$$\int \frac{(d+ex)^3}{cd^2+2cdex+ce^2x^2} dx = \frac{(d+ex)^2}{2ce}$$

output $1/2*(e*x+d)^2/c/e$

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.94

$$\int \frac{(d+ex)^3}{cd^2+2cdex+ce^2x^2} dx = \frac{dx + \frac{ex^2}{2}}{c}$$

input `Integrate[(d + e*x)^3/(c*d^2 + 2*c*d*e*x + c*e^2*x^2),x]`

output $(d*x + (e*x^2)/2)/c$

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = {1098, 17}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d + ex)^3}{cd^2 + 2cdex + ce^2x^2} dx$$

↓ 1098

$$ce^2 \int \frac{d + ex}{c^2e^2} dx$$

↓ 17

$$\frac{(d + ex)^2}{2ce}$$

input

```
Int[(d + e*x)^3/(c*d^2 + 2*c*d*e*x + c*e^2*x^2),x]
```

output

```
(d + e*x)^2/(2*c*e)
```

Defintions of rubi rules used

rule 17

```
Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] := Simp[c*((a + b*x)^(m + 1))/(b*(m + 1)), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]
```

rule 1098

```
Int[((d_.) + (e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] := Simp[1/c^p Int[(d + e*x)^m*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Maple [A] (verified)

Time = 1.13 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.82

| method | result | size |
|---------------|---|------|
| gospers | $\frac{x(ex+2d)}{2c}$ | 14 |
| default | $\frac{\frac{1}{2}ex^2+dx}{c}$ | 15 |
| parallelrisch | $\frac{ex^2+2dx}{2c}$ | 16 |
| risch | $\frac{ex^2}{2c} + \frac{xd}{c}$ | 17 |
| norman | $\frac{\frac{d^2x}{c} + \frac{e^2x^3}{2c} + \frac{3edx^2}{2c}}{ex+d}$ | 39 |
| orering | $\frac{x(ex+2d)(ex+d)^2}{2x^2ce^2+4cdxe+2cd^2}$ | 40 |

input `int((e*x+d)^3/(c*e^2*x^2+2*c*d*e*x+c*d^2),x,method=_RETURNVERBOSE)`

output `1/2*x*(e*x+2*d)/c`

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.88

$$\int \frac{(d+ex)^3}{cd^2+2cdex+ce^2x^2} dx = \frac{ex^2+2dx}{2c}$$

input `integrate((e*x+d)^3/(c*e^2*x^2+2*c*d*e*x+c*d^2),x, algorithm="fricas")`

output `1/2*(e*x^2 + 2*d*x)/c`

Sympy [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.71

$$\int \frac{(d + ex)^3}{cd^2 + 2cdex + ce^2x^2} dx = \frac{dx}{c} + \frac{ex^2}{2c}$$

input `integrate((e*x+d)**3/(c*e**2*x**2+2*c*d*e*x+c*d**2),x)`output `d*x/c + e*x**2/(2*c)`**Maxima [A] (verification not implemented)**

Time = 0.03 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.88

$$\int \frac{(d + ex)^3}{cd^2 + 2cdex + ce^2x^2} dx = \frac{ex^2 + 2 dx}{2 c}$$

input `integrate((e*x+d)^3/(c*e^2*x^2+2*c*d*e*x+c*d^2),x, algorithm="maxima")`output `1/2*(e*x^2 + 2*d*x)/c`**Giac [A] (verification not implemented)**

Time = 0.21 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.88

$$\int \frac{(d + ex)^3}{cd^2 + 2cdex + ce^2x^2} dx = \frac{ex^2 + 2 dx}{2 c}$$

input `integrate((e*x+d)^3/(c*e^2*x^2+2*c*d*e*x+c*d^2),x, algorithm="giac")`output `1/2*(e*x^2 + 2*d*x)/c`

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.76

$$\int \frac{(d + ex)^3}{cd^2 + 2cdex + ce^2x^2} dx = \frac{x(2d + ex)}{2c}$$

input `int((d + e*x)^3/(c*d^2 + c*e^2*x^2 + 2*c*d*e*x),x)`output `(x*(2*d + e*x))/(2*c)`**Reduce [B] (verification not implemented)**

Time = 0.29 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.76

$$\int \frac{(d + ex)^3}{cd^2 + 2cdex + ce^2x^2} dx = \frac{x(ex + 2d)}{2c}$$

input `int((e*x+d)^3/(c*e^2*x^2+2*c*d*e*x+c*d^2),x)`output `(x*(2*d + e*x))/(2*c)`

3.24 $\int \frac{(d+ex)^2}{cd^2+2cdex+ce^2x^2} dx$

| | |
|---|-----|
| Optimal result | 308 |
| Mathematica [A] (verified) | 308 |
| Rubi [A] (verified) | 309 |
| Maple [A] (verified) | 310 |
| Fricas [A] (verification not implemented) | 310 |
| Sympy [A] (verification not implemented) | 310 |
| Maxima [A] (verification not implemented) | 311 |
| Giac [A] (verification not implemented) | 311 |
| Mupad [B] (verification not implemented) | 311 |
| Reduce [B] (verification not implemented) | 312 |

Optimal result

Integrand size = 30, antiderivative size = 5

$$\int \frac{(d+ex)^2}{cd^2+2cdex+ce^2x^2} dx = \frac{x}{c}$$

output x/c

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 5, normalized size of antiderivative = 1.00

$$\int \frac{(d+ex)^2}{cd^2+2cdex+ce^2x^2} dx = \frac{x}{c}$$

input `Integrate[(d + e*x)^2/(c*d^2 + 2*c*d*e*x + c*e^2*x^2), x]`

output x/c

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 5, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = {1098, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d + ex)^2}{cd^2 + 2cdex + ce^2x^2} dx$$

↓ 1098

$$ce^2 \int \frac{1}{c^2e^2} dx$$

↓ 24

$$\frac{x}{c}$$

input `Int[(d + e*x)^2/(c*d^2 + 2*c*d*e*x + c*e^2*x^2),x]`

output `x/c`

Defintions of rubi rules used

rule 24 `Int[a_, x_Symbol] := Simp[a*x, x] /; FreeQ[a, x]`

rule 1098 `Int[((d_.) + (e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] := Simp[1/c^p Int[(d + e*x)^m*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]`

Maple [A] (verified)

Time = 1.16 (sec) , antiderivative size = 6, normalized size of antiderivative = 1.20

| method | result | size |
|---------|--|------|
| default | $\frac{x}{c}$ | 6 |
| risch | $\frac{x}{c}$ | 6 |
| norman | $\frac{\frac{ex^2}{c} + \frac{xd}{c}}{ex+d}$ | 24 |
| orering | $\frac{x(ex+d)^2}{x^2ce^2+2cdxe+cd^2}$ | 32 |

input `int((e*x+d)^2/(c*e^2*x^2+2*c*d*e*x+c*d^2),x,method=_RETURNVERBOSE)`output `x/c`**Fricas [A] (verification not implemented)**

Time = 0.08 (sec) , antiderivative size = 5, normalized size of antiderivative = 1.00

$$\int \frac{(d+ex)^2}{cd^2+2cdex+ce^2x^2} dx = \frac{x}{c}$$

input `integrate((e*x+d)^2/(c*e^2*x^2+2*c*d*e*x+c*d^2),x, algorithm="fricas")`output `x/c`**Sympy [A] (verification not implemented)**

Time = 0.02 (sec) , antiderivative size = 2, normalized size of antiderivative = 0.40

$$\int \frac{(d+ex)^2}{cd^2+2cdex+ce^2x^2} dx = \frac{x}{c}$$

input `integrate((e*x+d)**2/(c*e**2*x**2+2*c*d*e*x+c*d**2),x)`

output x/c **Maxima [A] (verification not implemented)**

Time = 0.03 (sec) , antiderivative size = 5, normalized size of antiderivative = 1.00

$$\int \frac{(d + ex)^2}{cd^2 + 2cdex + ce^2x^2} dx = \frac{x}{c}$$

input `integrate((e*x+d)^2/(c*e^2*x^2+2*c*d*e*x+c*d^2),x, algorithm="maxima")`output x/c **Giac [A] (verification not implemented)**

Time = 0.14 (sec) , antiderivative size = 5, normalized size of antiderivative = 1.00

$$\int \frac{(d + ex)^2}{cd^2 + 2cdex + ce^2x^2} dx = \frac{x}{c}$$

input `integrate((e*x+d)^2/(c*e^2*x^2+2*c*d*e*x+c*d^2),x, algorithm="giac")`output x/c **Mupad [B] (verification not implemented)**

Time = 0.01 (sec) , antiderivative size = 5, normalized size of antiderivative = 1.00

$$\int \frac{(d + ex)^2}{cd^2 + 2cdex + ce^2x^2} dx = \frac{x}{c}$$

input `int((d + e*x)^2/(c*d^2 + c*e^2*x^2 + 2*c*d*e*x),x)`output x/c

Reduce [B] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 5, normalized size of antiderivative = 1.00

$$\int \frac{(d + ex)^2}{cd^2 + 2cdex + ce^2x^2} dx = \frac{x}{c}$$

input `int((e*x+d)^2/(c*e^2*x^2+2*c*d*e*x+c*d^2),x)`

output `x/c`

3.25 $\int \frac{d+ex}{cd^2+2cdex+ce^2x^2} dx$

| | |
|---|-----|
| Optimal result | 313 |
| Mathematica [A] (verified) | 313 |
| Rubi [A] (verified) | 314 |
| Maple [A] (verified) | 315 |
| Fricas [A] (verification not implemented) | 315 |
| Sympy [A] (verification not implemented) | 315 |
| Maxima [B] (verification not implemented) | 316 |
| Giac [B] (verification not implemented) | 316 |
| Mupad [B] (verification not implemented) | 317 |
| Reduce [B] (verification not implemented) | 317 |

Optimal result

Integrand size = 28, antiderivative size = 13

$$\int \frac{d+ex}{cd^2+2cdex+ce^2x^2} dx = \frac{\log(d+ex)}{ce}$$

output `ln(e*x+d)/c/e`

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.23

$$\int \frac{d+ex}{cd^2+2cdex+ce^2x^2} dx = \frac{\log(cd+ce^2x)}{ce}$$

input `Integrate[(d + e*x)/(c*d^2 + 2*c*d*e*x + c*e^2*x^2),x]`

output `Log[c*d + c*e*x]/(c*e)`

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.071$, Rules used = {1098, 16}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{d + ex}{cd^2 + 2cdex + ce^2x^2} dx$$

↓ 1098

$$ce^2 \int \frac{1}{c^2e^2(d + ex)} dx$$

↓ 16

$$\frac{\log(d + ex)}{ce}$$

input

```
Int[(d + e*x)/(c*d^2 + 2*c*d*e*x + c*e^2*x^2), x]
```

output

```
Log[d + e*x]/(c*e)
```

Defintions of rubi rules used

rule 16

```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a + b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 1098

```
Int[((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[1/c^p Int[(d + e*x)^m*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Maple [A] (verified)

Time = 1.02 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.08

| method | result | size |
|---------------|------------------------|------|
| default | $\frac{\ln(ex+d)}{ce}$ | 14 |
| norman | $\frac{\ln(ex+d)}{ce}$ | 14 |
| risch | $\frac{\ln(ex+d)}{ce}$ | 14 |
| parallelrisch | $\frac{\ln(ex+d)}{ce}$ | 14 |

input `int((e*x+d)/(c*e^2*x^2+2*c*d*e*x+c*d^2),x,method=_RETURNVERBOSE)`output `ln(e*x+d)/c/e`**Fricas [A] (verification not implemented)**

Time = 0.08 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00

$$\int \frac{d + ex}{cd^2 + 2cdex + ce^2x^2} dx = \frac{\log(ex + d)}{ce}$$

input `integrate((e*x+d)/(c*e^2*x^2+2*c*d*e*x+c*d^2),x, algorithm="fricas")`output `log(e*x + d)/(c*e)`**Sympy [A] (verification not implemented)**

Time = 0.03 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.92

$$\int \frac{d + ex}{cd^2 + 2cdex + ce^2x^2} dx = \frac{\log(cd + cex)}{ce}$$

input `integrate((e*x+d)/(c*e**2*x**2+2*c*d*e*x+c*d**2),x)`

output `log(c*d + c*e*x)/(c*e)`

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 29 vs. 2(13) = 26.

Time = 0.04 (sec) , antiderivative size = 29, normalized size of antiderivative = 2.23

$$\int \frac{d + ex}{cd^2 + 2cdex + ce^2x^2} dx = \frac{\log(ce^2x^2 + 2cdex + cd^2)}{2ce}$$

input `integrate((e*x+d)/(c*e^2*x^2+2*c*d*e*x+c*d^2),x, algorithm="maxima")`

output `1/2*log(c*e^2*x^2 + 2*c*d*e*x + c*d^2)/(c*e)`

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 29 vs. 2(13) = 26.

Time = 0.20 (sec) , antiderivative size = 29, normalized size of antiderivative = 2.23

$$\int \frac{d + ex}{cd^2 + 2cdex + ce^2x^2} dx = \frac{\log(|cd^2 + (ex^2 + 2dx)ce|)}{2ce}$$

input `integrate((e*x+d)/(c*e^2*x^2+2*c*d*e*x+c*d^2),x, algorithm="giac")`

output `1/2*log(abs(c*d^2 + (e*x^2 + 2*d*x)*c*e))/(c*e)`

Mupad [B] (verification not implemented)

Time = 8.83 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00

$$\int \frac{d + ex}{cd^2 + 2cdex + ce^2x^2} dx = \frac{\ln(d + ex)}{ce}$$

input `int((d + e*x)/(c*d^2 + c*e^2*x^2 + 2*c*d*e*x),x)`

output `log(d + e*x)/(c*e)`

Reduce [B] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00

$$\int \frac{d + ex}{cd^2 + 2cdex + ce^2x^2} dx = \frac{\log(ex + d)}{ce}$$

input `int((e*x+d)/(c*e^2*x^2+2*c*d*e*x+c*d^2),x)`

output `log(d + e*x)/(c*e)`

3.26 $\int \frac{1}{cd^2 + 2cdex + ce^2x^2} dx$

| | |
|---|-----|
| Optimal result | 318 |
| Mathematica [A] (verified) | 318 |
| Rubi [A] (verified) | 319 |
| Maple [A] (verified) | 320 |
| Fricas [A] (verification not implemented) | 320 |
| Sympy [A] (verification not implemented) | 321 |
| Maxima [A] (verification not implemented) | 321 |
| Giac [A] (verification not implemented) | 321 |
| Mupad [B] (verification not implemented) | 322 |
| Reduce [B] (verification not implemented) | 322 |

Optimal result

Integrand size = 22, antiderivative size = 15

$$\int \frac{1}{cd^2 + 2cdex + ce^2x^2} dx = -\frac{1}{ce(d + ex)}$$

output

```
-1/c/e/(e*x+d)
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00

$$\int \frac{1}{cd^2 + 2cdex + ce^2x^2} dx = -\frac{1}{ce(d + ex)}$$

input

```
Integrate[(c*d^2 + 2*c*d*e*x + c*e^2*x^2)^(-1),x]
```

output

```
-(1/(c*e*(d + e*x)))
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {1077, 17}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{cd^2 + 2cdex + ce^2x^2} dx$$

$$\downarrow 1077$$

$$ce^2 \int \frac{1}{(cxe^2 + cde)^2} dx$$

$$\downarrow 17$$

$$-\frac{1}{ce(d + ex)}$$

input `Int[(c*d^2 + 2*c*d*e*x + c*e^2*x^2)^(-1),x]`

output `-(1/(c*e*(d + e*x)))`

Defintions of rubi rules used

rule 17 `Int[(c_.)*((a_.) + (b_.)*(x_)^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1))/(b*(m + 1)), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]`

rule 1077 `Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[1/c^p Int[(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]`

Maple [A] (verified)

Time = 0.97 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.07

| method | result | size |
|-------------|---------------------------------------|------|
| gospers | $-\frac{1}{ce(ex+d)}$ | 16 |
| default | $-\frac{1}{ce(ex+d)}$ | 16 |
| norman | $\frac{x}{cd(ex+d)}$ | 16 |
| risch | $-\frac{1}{ce(ex+d)}$ | 16 |
| parallelsch | $\frac{x}{cd(ex+d)}$ | 16 |
| orering | $-\frac{ex+d}{e(x^2ce^2+2cdxe+cd^2)}$ | 33 |

input `int(1/(c*e^2*x^2+2*c*d*e*x+c*d^2),x,method=_RETURNVERBOSE)`output `-1/c/e/(e*x+d)`**Fricas [A] (verification not implemented)**

Time = 0.07 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00

$$\int \frac{1}{cd^2 + 2cdex + ce^2x^2} dx = -\frac{1}{ce^2x + cde}$$

input `integrate(1/(c*e^2*x^2+2*c*d*e*x+c*d^2),x, algorithm="fricas")`output `-1/(c*e^2*x + c*d*e)`

Sympy [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.93

$$\int \frac{1}{cd^2 + 2cdex + ce^2x^2} dx = -\frac{1}{cde + ce^2x}$$

input `integrate(1/(c*e**2*x**2+2*c*d*e*x+c*d**2),x)`output `-1/(c*d*e + c*e**2*x)`**Maxima [A] (verification not implemented)**

Time = 0.04 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00

$$\int \frac{1}{cd^2 + 2cdex + ce^2x^2} dx = -\frac{1}{ce^2x + cde}$$

input `integrate(1/(c*e^2*x^2+2*c*d*e*x+c*d^2),x, algorithm="maxima")`output `-1/(c*e^2*x + c*d*e)`**Giac [A] (verification not implemented)**

Time = 0.19 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00

$$\int \frac{1}{cd^2 + 2cdex + ce^2x^2} dx = -\frac{1}{(ex + d)ce}$$

input `integrate(1/(c*e^2*x^2+2*c*d*e*x+c*d^2),x, algorithm="giac")`output `-1/((e*x + d)*c*e)`

Mupad [B] (verification not implemented)

Time = 8.71 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00

$$\int \frac{1}{cd^2 + 2cdex + ce^2x^2} dx = -\frac{1}{cxe^2 + cde}$$

input `int(1/(c*d^2 + c*e^2*x^2 + 2*c*d*e*x),x)`

output `-1/(c*d*e + c*e^2*x)`

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00

$$\int \frac{1}{cd^2 + 2cdex + ce^2x^2} dx = \frac{x}{cd(ex + d)}$$

input `int(1/(c*e^2*x^2+2*c*d*e*x+c*d^2),x)`

output `x/(c*d*(d + e*x))`

$$3.27 \quad \int \frac{1}{(d+ex)(cd^2+2cdex+ce^2x^2)} dx$$

| | |
|---|-----|
| Optimal result | 323 |
| Mathematica [A] (verified) | 323 |
| Rubi [A] (verified) | 324 |
| Maple [A] (verified) | 325 |
| Fricas [A] (verification not implemented) | 325 |
| Sympy [B] (verification not implemented) | 326 |
| Maxima [A] (verification not implemented) | 326 |
| Giac [A] (verification not implemented) | 326 |
| Mupad [B] (verification not implemented) | 327 |
| Reduce [B] (verification not implemented) | 327 |

Optimal result

Integrand size = 30, antiderivative size = 17

$$\int \frac{1}{(d+ex)(cd^2+2cdex+ce^2x^2)} dx = -\frac{1}{2ce(d+ex)^2}$$

output `-1/2/c/e/(e*x+d)^2`

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00

$$\int \frac{1}{(d+ex)(cd^2+2cdex+ce^2x^2)} dx = -\frac{1}{2ce(d+ex)^2}$$

input `Integrate[1/((d + e*x)*(c*d^2 + 2*c*d*e*x + c*e^2*x^2)),x]`

output `-1/2*1/(c*e*(d + e*x)^2)`

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = {1098, 17}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(d+ex)(cd^2+2cdex+ce^2x^2)} dx$$

↓ 1098

$$ce^2 \int \frac{1}{c^2e^2(d+ex)^3} dx$$

↓ 17

$$-\frac{1}{2ce(d+ex)^2}$$

input `Int[1/((d + e*x)*(c*d^2 + 2*c*d*e*x + c*e^2*x^2)),x]`

output `-1/2*1/(c*e*(d + e*x)^2)`

Defintions of rubi rules used

rule 17 `Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] := Simp[c*((a + b*x)^(m + 1))/(b*(m + 1)), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]`

rule 1098 `Int[((d_.) + (e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] := Simp[1/c^p Int[(d + e*x)^m*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]`

Maple [A] (verified)

Time = 1.24 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.94

| method | result | size |
|---------------|-------------------------------------|------|
| default | $-\frac{1}{2ce(ex+d)^2}$ | 16 |
| norman | $-\frac{1}{2ce(ex+d)^2}$ | 16 |
| risch | $-\frac{1}{2ce(ex+d)^2}$ | 16 |
| parallelrisch | $-\frac{1}{2ce(ex+d)^2}$ | 16 |
| gospers | $-\frac{1}{2ec(e^2x^2+2dex+d^2)}$ | 27 |
| orering | $-\frac{1}{2e(x^2ce^2+2cdxe+cd^2)}$ | 28 |

input `int(1/(e*x+d)/(c*e^2*x^2+2*c*d*e*x+c*d^2),x,method=_RETURNVERBOSE)`

output `-1/2/c/e/(e*x+d)^2`

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.59

$$\int \frac{1}{(d+ex)(cd^2+2cdex+ce^2x^2)} dx = -\frac{1}{2(cd^3+2cde^2x+cd^2e)}$$

input `integrate(1/(e*x+d)/(c*e^2*x^2+2*c*d*e*x+c*d^2),x, algorithm="fricas")`

output `-1/2/(c*e^3*x^2 + 2*c*d*e^2*x + c*d^2*e)`

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 31 vs. $2(14) = 28$.

Time = 0.09 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.82

$$\int \frac{1}{(d+ex)(cd^2+2cdex+ce^2x^2)} dx = -\frac{1}{2cd^2e+4cde^2x+2ce^3x^2}$$

input `integrate(1/(e*x+d)/(c*e**2*x**2+2*c*d*e*x+c*d**2),x)`

output `-1/(2*c*d**2*e + 4*c*d*e**2*x + 2*c*e**3*x**2)`

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.59

$$\int \frac{1}{(d+ex)(cd^2+2cdex+ce^2x^2)} dx = -\frac{1}{2(ce^3x^2+2cde^2x+cd^2e)}$$

input `integrate(1/(e*x+d)/(c*e^2*x^2+2*c*d*e*x+c*d^2),x, algorithm="maxima")`

output `-1/2/(c*e^3*x^2 + 2*c*d*e^2*x + c*d^2*e)`

Giac [A] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.88

$$\int \frac{1}{(d+ex)(cd^2+2cdex+ce^2x^2)} dx = -\frac{1}{2(ex+d)^2ce}$$

input `integrate(1/(e*x+d)/(c*e^2*x^2+2*c*d*e*x+c*d^2),x, algorithm="giac")`

output `-1/2/((e*x + d)^2*c*e)`

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.71

$$\int \frac{1}{(d + ex)(cd^2 + 2cdex + ce^2x^2)} dx = -\frac{1}{2cd^2e + 4cde^2x + 2ce^3x^2}$$

input `int(1/((d + e*x)*(c*d^2 + c*e^2*x^2 + 2*c*d*e*x)),x)`output `-1/(2*c*e^3*x^2 + 2*c*d^2*e + 4*c*d*e^2*x)`**Reduce [B] (verification not implemented)**

Time = 0.28 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.53

$$\int \frac{1}{(d + ex)(cd^2 + 2cdex + ce^2x^2)} dx = -\frac{1}{2ce(e^2x^2 + 2dex + d^2)}$$

input `int(1/(e*x+d)/(c*e^2*x^2+2*c*d*e*x+c*d^2),x)`output `(- 1)/(2*c*e*(d**2 + 2*d*e*x + e**2*x**2))`

3.28 $\int \frac{1}{(d+ex)^2(cd^2+2cdex+ce^2x^2)} dx$

| | |
|---|-----|
| Optimal result | 328 |
| Mathematica [A] (verified) | 328 |
| Rubi [A] (verified) | 329 |
| Maple [A] (verified) | 330 |
| Fricas [B] (verification not implemented) | 330 |
| Sympy [B] (verification not implemented) | 331 |
| Maxima [B] (verification not implemented) | 331 |
| Giac [A] (verification not implemented) | 331 |
| Mupad [B] (verification not implemented) | 332 |
| Reduce [B] (verification not implemented) | 332 |

Optimal result

Integrand size = 30, antiderivative size = 17

$$\int \frac{1}{(d+ex)^2(cd^2+2cdex+ce^2x^2)} dx = -\frac{1}{3ce(d+ex)^3}$$

output

`-1/3/c/e/(e*x+d)^3`

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00

$$\int \frac{1}{(d+ex)^2(cd^2+2cdex+ce^2x^2)} dx = -\frac{1}{3ce(d+ex)^3}$$

input

`Integrate[1/((d + e*x)^2*(c*d^2 + 2*c*d*e*x + c*e^2*x^2)), x]`

output

`-1/3*1/(c*e*(d + e*x)^3)`

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = {1098, 17}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(d+ex)^2(cd^2+2cdex+ce^2x^2)} dx$$

↓ 1098

$$ce^2 \int \frac{1}{c^2e^2(d+ex)^4} dx$$

↓ 17

$$-\frac{1}{3ce(d+ex)^3}$$

input `Int[1/((d + e*x)^2*(c*d^2 + 2*c*d*e*x + c*e^2*x^2)),x]`

output `-1/3*1/(c*e*(d + e*x)^3)`

Defintions of rubi rules used

rule 17 `Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] := Simp[c*((a + b*x)^(m + 1))/(b*(m + 1)), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]`

rule 1098 `Int[((d_.) + (e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] := Simp[1/c^p Int[(d + e*x)^m*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]`

Maple [A] (verified)

Time = 1.27 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.94

| method | result | size |
|---------------|---|------|
| default | $-\frac{1}{3ce(ex+d)^3}$ | 16 |
| norman | $-\frac{1}{3ce(ex+d)^3}$ | 16 |
| gospers | $-\frac{1}{3(ex+d)ec(e^2x^2+2dex+d^2)}$ | 34 |
| risch | $-\frac{1}{3(ex+d)ec(e^2x^2+2dex+d^2)}$ | 34 |
| parallelrisch | $-\frac{1}{3(ex+d)ec(e^2x^2+2dex+d^2)}$ | 34 |
| orering | $-\frac{1}{3(ex+d)e(x^2ce^2+2cdxe+cd^2)}$ | 35 |

input `int(1/(e*x+d)^2/(c*e^2*x^2+2*c*d*e*x+c*d^2),x,method=_RETURNVERBOSE)`

output `-1/3/c/e/(e*x+d)^3`

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 39 vs. 2(15) = 30.

Time = 0.07 (sec) , antiderivative size = 39, normalized size of antiderivative = 2.29

$$\int \frac{1}{(d+ex)^2(cd^2+2cdex+ce^2x^2)} dx = -\frac{1}{3(ce^4x^3+3cde^3x^2+3cd^2e^2x+cd^3e)}$$

input `integrate(1/(e*x+d)^2/(c*e^2*x^2+2*c*d*e*x+c*d^2),x, algorithm="fricas")`

output `-1/3/(c*e^4*x^3 + 3*c*d*e^3*x^2 + 3*c*d^2*e^2*x + c*d^3*e)`

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 44 vs. $2(14) = 28$.

Time = 0.12 (sec) , antiderivative size = 44, normalized size of antiderivative = 2.59

$$\int \frac{1}{(d+ex)^2 (cd^2 + 2cdex + ce^2x^2)} dx = -\frac{1}{3cd^3e + 9cd^2e^2x + 9cde^3x^2 + 3ce^4x^3}$$

input `integrate(1/(e*x+d)**2/(c*e**2*x**2+2*c*d*e*x+c*d**2),x)`

output `-1/(3*c*d**3*e + 9*c*d**2*e**2*x + 9*c*d*e**3*x**2 + 3*c*e**4*x**3)`

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 39 vs. $2(15) = 30$.

Time = 0.03 (sec) , antiderivative size = 39, normalized size of antiderivative = 2.29

$$\int \frac{1}{(d+ex)^2 (cd^2 + 2cdex + ce^2x^2)} dx = -\frac{1}{3(ce^4x^3 + 3cde^3x^2 + 3cd^2e^2x + cd^3e)}$$

input `integrate(1/(e*x+d)^2/(c*e^2*x^2+2*c*d*e*x+c*d^2),x, algorithm="maxima")`

output `-1/3/(c*e^4*x^3 + 3*c*d*e^3*x^2 + 3*c*d^2*e^2*x + c*d^3*e)`

Giac [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.88

$$\int \frac{1}{(d+ex)^2 (cd^2 + 2cdex + ce^2x^2)} dx = -\frac{1}{3(ex+d)^3 ce}$$

input `integrate(1/(e*x+d)^2/(c*e^2*x^2+2*c*d*e*x+c*d^2),x, algorithm="giac")`

output `-1/3/((e*x + d)^3*c*e)`

Mupad [B] (verification not implemented)

Time = 8.71 (sec) , antiderivative size = 41, normalized size of antiderivative = 2.41

$$\int \frac{1}{(d+ex)^2(cd^2+2cdex+ce^2x^2)} dx = -\frac{1}{3cd^3e+9cd^2e^2x+9cde^3x^2+3ce^4x^3}$$

input `int(1/((d + e*x)^2*(c*d^2 + c*e^2*x^2 + 2*c*d*e*x)),x)`output `-1/(3*c*e^4*x^3 + 3*c*d^3*e + 9*c*d^2*e^2*x + 9*c*d*e^3*x^2)`**Reduce [B] (verification not implemented)**

Time = 0.28 (sec) , antiderivative size = 37, normalized size of antiderivative = 2.18

$$\int \frac{1}{(d+ex)^2(cd^2+2cdex+ce^2x^2)} dx = -\frac{1}{3ce(e^3x^3+3de^2x^2+3d^2ex+d^3)}$$

input `int(1/(e*x+d)^2/(c*e^2*x^2+2*c*d*e*x+c*d^2),x)`output `(- 1)/(3*c*e*(d**3 + 3*d**2*e*x + 3*d*e**2*x**2 + e**3*x**3))`

$$3.29 \quad \int \frac{1}{(d+ex)^3 (cd^2+2cdex+ce^2x^2)} dx$$

| | |
|---|-----|
| Optimal result | 333 |
| Mathematica [A] (verified) | 333 |
| Rubi [A] (verified) | 334 |
| Maple [A] (verified) | 335 |
| Fricas [B] (verification not implemented) | 335 |
| Sympy [B] (verification not implemented) | 336 |
| Maxima [B] (verification not implemented) | 336 |
| Giac [A] (verification not implemented) | 337 |
| Mupad [B] (verification not implemented) | 337 |
| Reduce [B] (verification not implemented) | 337 |

Optimal result

Integrand size = 30, antiderivative size = 17

$$\int \frac{1}{(d+ex)^3 (cd^2+2cdex+ce^2x^2)} dx = -\frac{1}{4ce(d+ex)^4}$$

output `-1/4/c/e/(e*x+d)^4`

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00

$$\int \frac{1}{(d+ex)^3 (cd^2+2cdex+ce^2x^2)} dx = -\frac{1}{4ce(d+ex)^4}$$

input `Integrate[1/((d + e*x)^3*(c*d^2 + 2*c*d*e*x + c*e^2*x^2)),x]`

output `-1/4*1/(c*e*(d + e*x)^4)`

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = {1098, 17}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(d+ex)^3(cd^2+2cdex+ce^2x^2)} dx$$

↓ 1098

$$ce^2 \int \frac{1}{c^2e^2(d+ex)^5} dx$$

↓ 17

$$-\frac{1}{4ce(d+ex)^4}$$

input `Int[1/((d + e*x)^3*(c*d^2 + 2*c*d*e*x + c*e^2*x^2)),x]`

output `-1/4*1/(c*e*(d + e*x)^4)`

Defintions of rubi rules used

rule 17 `Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] := Simp[c*((a + b*x)^(m + 1))/(b*(m + 1)), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]`

rule 1098 `Int[((d_.) + (e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] := Simp[1/c^p Int[(d + e*x)^m*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]`

Maple [A] (verified)

Time = 1.23 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.94

| method | result | size |
|---------------|--|------|
| default | $-\frac{1}{4ce(ex+d)^4}$ | 16 |
| norman | $-\frac{1}{4ce(ex+d)^4}$ | 16 |
| gospers | $-\frac{1}{4(ex+d)^2 ec(e^2x^2+2dex+d^2)}$ | 34 |
| risch | $-\frac{1}{4(ex+d)^2 ec(e^2x^2+2dex+d^2)}$ | 34 |
| parallelrisch | $-\frac{1}{4(ex+d)^2 ec(e^2x^2+2dex+d^2)}$ | 34 |
| orering | $-\frac{1}{4(ex+d)^2 e(x^2ce^2+2cdxe+cd^2)}$ | 35 |

input `int(1/(e*x+d)^3/(c*e^2*x^2+2*c*d*e*x+c*d^2),x,method=_RETURNVERBOSE)`

output `-1/4/c/e/(e*x+d)^4`

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 51 vs. 2(15) = 30.

Time = 0.10 (sec) , antiderivative size = 51, normalized size of antiderivative = 3.00

$$\int \frac{1}{(d+ex)^3 (cd^2+2cdex+ce^2x^2)} dx$$

$$= -\frac{1}{4(ce^5x^4+4cde^4x^3+6cd^2e^3x^2+4cd^3e^2x+cd^4e)}$$

input `integrate(1/(e*x+d)^3/(c*e^2*x^2+2*c*d*e*x+c*d^2),x, algorithm="fricas")`

output `-1/4/(c*e^5*x^4+4*c*d*e^4*x^3+6*c*d^2*e^3*x^2+4*c*d^3*e^2*x+c*d^4*e)`

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 58 vs. $2(14) = 28$.

Time = 0.14 (sec) , antiderivative size = 58, normalized size of antiderivative = 3.41

$$\int \frac{1}{(d+ex)^3 (cd^2 + 2cdex + ce^2x^2)} dx$$

$$= -\frac{1}{4cd^4e + 16cd^3e^2x + 24cd^2e^3x^2 + 16cde^4x^3 + 4ce^5x^4}$$

input `integrate(1/(e*x+d)**3/(c*e**2*x**2+2*c*d*e*x+c*d**2),x)`

output `-1/(4*c*d**4*e + 16*c*d**3*e**2*x + 24*c*d**2*e**3*x**2 + 16*c*d*e**4*x**3 + 4*c*e**5*x**4)`

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 51 vs. $2(15) = 30$.

Time = 0.04 (sec) , antiderivative size = 51, normalized size of antiderivative = 3.00

$$\int \frac{1}{(d+ex)^3 (cd^2 + 2cdex + ce^2x^2)} dx$$

$$= -\frac{1}{4(ce^5x^4 + 4cde^4x^3 + 6cd^2e^3x^2 + 4cd^3e^2x + cd^4e)}$$

input `integrate(1/(e*x+d)^3/(c*e^2*x^2+2*c*d*e*x+c*d^2),x, algorithm="maxima")`

output `-1/4/(c*e^5*x^4 + 4*c*d*e^4*x^3 + 6*c*d^2*e^3*x^2 + 4*c*d^3*e^2*x + c*d^4*e)`

Giac [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.88

$$\int \frac{1}{(d+ex)^3 (cd^2 + 2cdex + ce^2x^2)} dx = -\frac{1}{4(ex+d)^4 ce}$$

input `integrate(1/(e*x+d)^3/(c*e^2*x^2+2*c*d*e*x+c*d^2),x, algorithm="giac")`output `-1/4/((e*x + d)^4*c*e)`**Mupad [B] (verification not implemented)**

Time = 0.05 (sec) , antiderivative size = 53, normalized size of antiderivative = 3.12

$$\int \frac{1}{(d+ex)^3 (cd^2 + 2cdex + ce^2x^2)} dx$$

$$= -\frac{1}{4cd^4e + 16cd^3e^2x + 24cd^2e^3x^2 + 16cde^4x^3 + 4ce^5x^4}$$

input `int(1/((d + e*x)^3*(c*d^2 + c*e^2*x^2 + 2*c*d*e*x)),x)`output `-1/(4*c*e^5*x^4 + 4*c*d^4*e + 24*c*d^2*e^3*x^2 + 16*c*d^3*e^2*x + 16*c*d*e^4*x^3)`**Reduce [B] (verification not implemented)**

Time = 0.28 (sec) , antiderivative size = 48, normalized size of antiderivative = 2.82

$$\int \frac{1}{(d+ex)^3 (cd^2 + 2cdex + ce^2x^2)} dx = -\frac{1}{4ce(e^4x^4 + 4de^3x^3 + 6d^2e^2x^2 + 4d^3ex + d^4)}$$

input `int(1/(e*x+d)^3/(c*e^2*x^2+2*c*d*e*x+c*d^2),x)`output `((- 1)/(4*c*e*(d**4 + 4*d**3*e*x + 6*d**2*e**2*x**2 + 4*d*e**3*x**3 + e**4*x**4))`

3.30
$$\int \frac{(d+ex)^7}{(cd^2+2cdex+ce^2x^2)^2} dx$$

| | |
|---|-----|
| Optimal result | 338 |
| Mathematica [A] (verified) | 338 |
| Rubi [A] (verified) | 339 |
| Maple [A] (verified) | 340 |
| Fricas [B] (verification not implemented) | 340 |
| Sympy [B] (verification not implemented) | 341 |
| Maxima [B] (verification not implemented) | 341 |
| Giac [B] (verification not implemented) | 341 |
| Mupad [B] (verification not implemented) | 342 |
| Reduce [B] (verification not implemented) | 342 |

Optimal result

Integrand size = 30, antiderivative size = 17

$$\int \frac{(d+ex)^7}{(cd^2+2cdex+ce^2x^2)^2} dx = \frac{(d+ex)^4}{4c^2e}$$

output `1/4*(e*x+d)^4/c^2/e`

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00

$$\int \frac{(d+ex)^7}{(cd^2+2cdex+ce^2x^2)^2} dx = \frac{(d+ex)^4}{4c^2e}$$

input `Integrate[(d + e*x)^7/(c*d^2 + 2*c*d*e*x + c*e^2*x^2)^2,x]`

output `(d + e*x)^4/(4*c^2*e)`

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = {1098, 17}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d + ex)^7}{(cd^2 + 2cdex + ce^2x^2)^2} dx$$

↓ 1098

$$c^2e^4 \int \frac{(d + ex)^3}{c^4e^4} dx$$

↓ 17

$$\frac{(d + ex)^4}{4c^2e}$$

input `Int[(d + e*x)^7/(c*d^2 + 2*c*d*e*x + c*e^2*x^2)^2,x]`

output `(d + e*x)^4/(4*c^2*e)`

Defintions of rubi rules used

rule 17 `Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] := Simp[c*((a + b*x)^(m + 1))/(b*(m + 1))], x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]`

rule 1098 `Int[((d_.) + (e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] := Simp[1/c^p Int[(d + e*x)^m*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]`

Maple [A] (verified)

Time = 1.12 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.94

| method | result | size |
|--------------|---|------|
| default | $\frac{(ex+d)^4}{4c^2e}$ | 16 |
| gospers | $\frac{x(e^3x^3+4de^2x^2+6d^2ex+4d^3)}{4c^2}$ | 36 |
| paralelrisch | $\frac{e^3x^4+4de^2x^3+6d^2ex^2+4d^3x}{4c^2}$ | 38 |
| risch | $\frac{e^3x^4}{4c^2} + \frac{e^2dx^3}{c^2} + \frac{3ed^2x^2}{2c^2} + \frac{d^3x}{c^2} + \frac{d^4}{4c^2e}$ | 55 |
| orering | $\frac{x(e^3x^3+4de^2x^2+6d^2ex+4d^3)(ex+d)^4}{4(x^2ce^2+2cdxe+cd^2)^2}$ | 62 |
| norman | $\frac{\frac{e^6x^7}{4c} + \frac{7de^5x^6}{4c} - \frac{17d^7}{2ec} + \frac{35e^3d^3x^4}{4c} + \frac{21e^4d^2x^5}{4c} - \frac{21d^5ex^2}{c} - \frac{49d^6x}{2c}}{c(ex+d)^3}$ | 96 |

input `int((e*x+d)^7/(c*e^2*x^2+2*c*d*e*x+c*d^2)^2,x,method=_RETURNVERBOSE)`output `1/4*(e*x+d)^4/c^2/e`**Fricas [B] (verification not implemented)**

Leaf count of result is larger than twice the leaf count of optimal. 37 vs. 2(15) = 30.

Time = 0.08 (sec) , antiderivative size = 37, normalized size of antiderivative = 2.18

$$\int \frac{(d+ex)^7}{(cd^2+2cdex+ce^2x^2)^2} dx = \frac{e^3x^4 + 4de^2x^3 + 6d^2ex^2 + 4d^3x}{4c^2}$$

input `integrate((e*x+d)^7/(c*e^2*x^2+2*c*d*e*x+c*d^2)^2,x, algorithm="fricas")`output `1/4*(e^3*x^4 + 4*d*e^2*x^3 + 6*d^2*e*x^2 + 4*d^3*x)/c^2`

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 46 vs. $2(12) = 24$.

Time = 0.04 (sec) , antiderivative size = 46, normalized size of antiderivative = 2.71

$$\int \frac{(d+ex)^7}{(cd^2+2cdex+ce^2x^2)^2} dx = \frac{d^3x}{c^2} + \frac{3d^2ex^2}{2c^2} + \frac{de^2x^3}{c^2} + \frac{e^3x^4}{4c^2}$$

input `integrate((e*x+d)**7/(c*e**2*x**2+2*c*d*e*x+c*d**2)**2,x)`

output `d**3*x/c**2 + 3*d**2*e*x**2/(2*c**2) + d*e**2*x**3/c**2 + e**3*x**4/(4*c**2)`

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 37 vs. $2(15) = 30$.

Time = 0.04 (sec) , antiderivative size = 37, normalized size of antiderivative = 2.18

$$\int \frac{(d+ex)^7}{(cd^2+2cdex+ce^2x^2)^2} dx = \frac{e^3x^4 + 4de^2x^3 + 6d^2ex^2 + 4d^3x}{4c^2}$$

input `integrate((e*x+d)^7/(c*e^2*x^2+2*c*d*e*x+c*d^2)^2,x, algorithm="maxima")`

output `1/4*(e^3*x^4 + 4*d*e^2*x^3 + 6*d^2*e*x^2 + 4*d^3*x)/c^2`

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 37 vs. $2(15) = 30$.

Time = 0.17 (sec) , antiderivative size = 37, normalized size of antiderivative = 2.18

$$\int \frac{(d+ex)^7}{(cd^2+2cdex+ce^2x^2)^2} dx = \frac{e^3x^4 + 4de^2x^3 + 6d^2ex^2 + 4d^3x}{4c^2}$$

input `integrate((e*x+d)^7/(c*e^2*x^2+2*c*d*e*x+c*d^2)^2,x, algorithm="giac")`

output `1/4*(e^3*x^4 + 4*d*e^2*x^3 + 6*d^2*e*x^2 + 4*d^3*x)/c^2`

Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 43, normalized size of antiderivative = 2.53

$$\int \frac{(d + ex)^7}{(cd^2 + 2cdex + ce^2x^2)^2} dx = \frac{d^3 x}{c^2} + \frac{e^3 x^4}{4c^2} + \frac{3d^2 ex^2}{2c^2} + \frac{de^2 x^3}{c^2}$$

input `int((d + e*x)^7/(c*d^2 + c*e^2*x^2 + 2*c*d*e*x)^2,x)`

output `(d^3*x)/c^2 + (e^3*x^4)/(4*c^2) + (3*d^2*e*x^2)/(2*c^2) + (d*e^2*x^3)/c^2`

Reduce [B] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 35, normalized size of antiderivative = 2.06

$$\int \frac{(d + ex)^7}{(cd^2 + 2cdex + ce^2x^2)^2} dx = \frac{x(e^3x^3 + 4de^2x^2 + 6d^2ex + 4d^3)}{4c^2}$$

input `int((e*x+d)^7/(c*e^2*x^2+2*c*d*e*x+c*d^2)^2,x)`

output `(x*(4*d**3 + 6*d**2*e*x + 4*d*e**2*x**2 + e**3*x**3))/(4*c**2)`

$$3.31 \quad \int \frac{(d+ex)^6}{(cd^2+2cdex+ce^2x^2)^2} dx$$

| | |
|---|-----|
| Optimal result | 343 |
| Mathematica [A] (verified) | 343 |
| Rubi [A] (verified) | 344 |
| Maple [A] (verified) | 345 |
| Fricas [A] (verification not implemented) | 345 |
| Sympy [B] (verification not implemented) | 346 |
| Maxima [A] (verification not implemented) | 346 |
| Giac [A] (verification not implemented) | 346 |
| Mupad [B] (verification not implemented) | 347 |
| Reduce [B] (verification not implemented) | 347 |

Optimal result

Integrand size = 30, antiderivative size = 17

$$\int \frac{(d+ex)^6}{(cd^2+2cdex+ce^2x^2)^2} dx = \frac{(d+ex)^3}{3c^2e}$$

output `1/3*(e*x+d)^3/c^2/e`

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00

$$\int \frac{(d+ex)^6}{(cd^2+2cdex+ce^2x^2)^2} dx = \frac{(d+ex)^3}{3c^2e}$$

input `Integrate[(d + e*x)^6/(c*d^2 + 2*c*d*e*x + c*e^2*x^2)^2,x]`

output `(d + e*x)^3/(3*c^2*e)`

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = {1098, 17}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d + ex)^6}{(cd^2 + 2cdex + ce^2x^2)^2} dx$$

↓ 1098

$$c^2e^4 \int \frac{(d + ex)^2}{c^4e^4} dx$$

↓ 17

$$\frac{(d + ex)^3}{3c^2e}$$

input `Int[(d + e*x)^6/(c*d^2 + 2*c*d*e*x + c*e^2*x^2)^2,x]`

output `(d + e*x)^3/(3*c^2*e)`

Defintions of rubi rules used

rule 17 `Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] := Simp[c*((a + b*x)^(m + 1))/(b*(m + 1))], x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]`

rule 1098 `Int[((d_.) + (e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] := Simp[1/c^p Int[(d + e*x)^m*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]`

Maple [A] (verified)

Time = 1.11 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.94

| method | result | size |
|---------------|--|------|
| default | $\frac{(ex+d)^3}{3c^2e}$ | 16 |
| gospers | $\frac{x(e^2x^2+3dex+3d^2)}{3c^2}$ | 25 |
| parallelrisch | $\frac{e^2x^3+3dex^2+3d^2x}{3c^2}$ | 27 |
| risch | $\frac{e^2x^3}{3c^2} + \frac{edx^2}{c^2} + \frac{d^2x}{c^2} + \frac{d^3}{3c^2e}$ | 41 |
| orering | $\frac{x(e^2x^2+3dex+3d^2)(ex+d)^4}{3(x^2ce^2+2cdxe+cd^2)^2}$ | 51 |
| norman | $\frac{\frac{e^5x^6}{3c} + \frac{2de^4x^5}{c} - \frac{19d^6}{3ec} + \frac{5e^3d^2x^4}{c} - \frac{15d^4ex^2}{c} - \frac{18d^5x}{c}}{c(ex+d)^3}$ | 82 |

input `int((e*x+d)^6/(c*e^2*x^2+2*c*d*e*x+c*d^2)^2,x,method=_RETURNVERBOSE)`

output `1/3*(e*x+d)^3/c^2/e`

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.53

$$\int \frac{(d+ex)^6}{(cd^2+2cdex+ce^2x^2)^2} dx = \frac{e^2x^3+3dex^2+3d^2x}{3c^2}$$

input `integrate((e*x+d)^6/(c*e^2*x^2+2*c*d*e*x+c*d^2)^2,x, algorithm="fricas")`

output `1/3*(e^2*x^3 + 3*d*e*x^2 + 3*d^2*x)/c^2`

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 29 vs. $2(12) = 24$.

Time = 0.05 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.71

$$\int \frac{(d + ex)^6}{(cd^2 + 2cdex + ce^2x^2)^2} dx = \frac{d^2x}{c^2} + \frac{dex^2}{c^2} + \frac{e^2x^3}{3c^2}$$

input `integrate((e*x+d)**6/(c*e**2*x**2+2*c*d*e*x+c*d**2)**2,x)`

output `d**2*x/c**2 + d*e*x**2/c**2 + e**2*x**3/(3*c**2)`

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.53

$$\int \frac{(d + ex)^6}{(cd^2 + 2cdex + ce^2x^2)^2} dx = \frac{e^2x^3 + 3dex^2 + 3d^2x}{3c^2}$$

input `integrate((e*x+d)^6/(c*e^2*x^2+2*c*d*e*x+c*d^2)^2,x, algorithm="maxima")`

output `1/3*(e^2*x^3 + 3*d*e*x^2 + 3*d^2*x)/c^2`

Giac [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.53

$$\int \frac{(d + ex)^6}{(cd^2 + 2cdex + ce^2x^2)^2} dx = \frac{e^2x^3 + 3dex^2 + 3d^2x}{3c^2}$$

input `integrate((e*x+d)^6/(c*e^2*x^2+2*c*d*e*x+c*d^2)^2,x, algorithm="giac")`

output `1/3*(e^2*x^3 + 3*d*e*x^2 + 3*d^2*x)/c^2`

Mupad [B] (verification not implemented)

Time = 8.70 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.41

$$\int \frac{(d + ex)^6}{(cd^2 + 2cde x + ce^2 x^2)^2} dx = \frac{x(3d^2 + 3dex + e^2 x^2)}{3c^2}$$

input `int((d + e*x)^6/(c*d^2 + c*e^2*x^2 + 2*c*d*e*x)^2,x)`output `(x*(3*d^2 + e^2*x^2 + 3*d*e*x))/(3*c^2)`**Reduce [B] (verification not implemented)**

Time = 0.27 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.41

$$\int \frac{(d + ex)^6}{(cd^2 + 2cde x + ce^2 x^2)^2} dx = \frac{x(e^2 x^2 + 3dex + 3d^2)}{3c^2}$$

input `int((e*x+d)^6/(c*e^2*x^2+2*c*d*e*x+c*d^2)^2,x)`output `(x*(3*d**2 + 3*d*e*x + e**2*x**2))/(3*c**2)`

$$3.32 \quad \int \frac{(d+ex)^5}{(cd^2+2cdex+ce^2x^2)^2} dx$$

| | |
|---|-----|
| Optimal result | 348 |
| Mathematica [A] (verified) | 348 |
| Rubi [A] (verified) | 349 |
| Maple [A] (verified) | 350 |
| Fricas [A] (verification not implemented) | 350 |
| Sympy [A] (verification not implemented) | 351 |
| Maxima [A] (verification not implemented) | 351 |
| Giac [A] (verification not implemented) | 351 |
| Mupad [B] (verification not implemented) | 352 |
| Reduce [B] (verification not implemented) | 352 |

Optimal result

Integrand size = 30, antiderivative size = 17

$$\int \frac{(d+ex)^5}{(cd^2+2cdex+ce^2x^2)^2} dx = \frac{(d+ex)^2}{2c^2e}$$

output `1/2*(e*x+d)^2/c^2/e`

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.94

$$\int \frac{(d+ex)^5}{(cd^2+2cdex+ce^2x^2)^2} dx = \frac{dx + \frac{ex^2}{2}}{c^2}$$

input `Integrate[(d + e*x)^5/(c*d^2 + 2*c*d*e*x + c*e^2*x^2)^2,x]`

output `(d*x + (e*x^2)/2)/c^2`

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = {1098, 17}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d + ex)^5}{(cd^2 + 2cdex + ce^2x^2)^2} dx$$

$$\downarrow 1098$$

$$c^2e^4 \int \frac{d + ex}{c^4e^4} dx$$

$$\downarrow 17$$

$$\frac{(d + ex)^2}{2c^2e}$$

input `Int[(d + e*x)^5/(c*d^2 + 2*c*d*e*x + c*e^2*x^2)^2,x]`

output `(d + e*x)^2/(2*c^2*e)`

Defintions of rubi rules used

rule 17 `Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] := Simp[c*((a + b*x)^(m + 1))/(b*(m + 1))], x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]`

rule 1098 `Int[((d_.) + (e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] := Simp[1/c^p Int[(d + e*x)^m*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]`

Maple [A] (verified)

Time = 1.08 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.82

| method | result | size |
|---------------|---|------|
| gospers | $\frac{x(ex+2d)}{2c^2}$ | 14 |
| default | $\frac{\frac{1}{2}ex^2+dx}{c^2}$ | 15 |
| parallelrisch | $\frac{ex^2+2dx}{2c^2}$ | 16 |
| risch | $\frac{ex^2}{2c^2} + \frac{dx}{c^2}$ | 17 |
| orering | $\frac{x(ex+2d)(ex+d)^4}{2(x^2ce^2+2cdxe+cd^2)^2}$ | 40 |
| norman | $\frac{-\frac{10d^3ex^2}{c} + \frac{e^4x^5}{2c} + \frac{5de^3x^4}{2c} - \frac{9d^5}{2ec} - \frac{25d^4x}{2c}}{c(ex+d)^3}$ | 68 |

input `int((e*x+d)^5/(c*e^2*x^2+2*c*d*e*x+c*d^2)^2,x,method=_RETURNVERBOSE)`

output `1/2*x*(e*x+2*d)/c^2`

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.88

$$\int \frac{(d+ex)^5}{(cd^2+2cdex+ce^2x^2)^2} dx = \frac{ex^2+2dx}{2c^2}$$

input `integrate((e*x+d)^5/(c*e^2*x^2+2*c*d*e*x+c*d^2)^2,x, algorithm="fricas")`

output `1/2*(e*x^2 + 2*d*x)/c^2`

Sympy [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.88

$$\int \frac{(d + ex)^5}{(cd^2 + 2cdex + ce^2x^2)^2} dx = \frac{dx}{c^2} + \frac{ex^2}{2c^2}$$

input `integrate((e*x+d)**5/(c*e**2*x**2+2*c*d*e*x+c*d**2)**2,x)`output `d*x/c**2 + e*x**2/(2*c**2)`**Maxima [A] (verification not implemented)**

Time = 0.04 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.88

$$\int \frac{(d + ex)^5}{(cd^2 + 2cdex + ce^2x^2)^2} dx = \frac{ex^2 + 2 dx}{2 c^2}$$

input `integrate((e*x+d)^5/(c*e^2*x^2+2*c*d*e*x+c*d^2)^2,x, algorithm="maxima")`output `1/2*(e*x^2 + 2*d*x)/c^2`**Giac [A] (verification not implemented)**

Time = 0.19 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.88

$$\int \frac{(d + ex)^5}{(cd^2 + 2cdex + ce^2x^2)^2} dx = \frac{ex^2 + 2 dx}{2 c^2}$$

input `integrate((e*x+d)^5/(c*e^2*x^2+2*c*d*e*x+c*d^2)^2,x, algorithm="giac")`output `1/2*(e*x^2 + 2*d*x)/c^2`

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.76

$$\int \frac{(d + ex)^5}{(cd^2 + 2cdex + ce^2x^2)^2} dx = \frac{x(2d + ex)}{2c^2}$$

input `int((d + e*x)^5/(c*d^2 + c*e^2*x^2 + 2*c*d*e*x)^2,x)`output `(x*(2*d + e*x))/(2*c^2)`**Reduce [B] (verification not implemented)**

Time = 0.25 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.76

$$\int \frac{(d + ex)^5}{(cd^2 + 2cdex + ce^2x^2)^2} dx = \frac{x(ex + 2d)}{2c^2}$$

input `int((e*x+d)^5/(c*e^2*x^2+2*c*d*e*x+c*d^2)^2,x)`output `(x*(2*d + e*x))/(2*c**2)`

3.33
$$\int \frac{(d+ex)^4}{(cd^2+2cdex+ce^2x^2)^2} dx$$

| | |
|---|-----|
| Optimal result | 353 |
| Mathematica [A] (verified) | 353 |
| Rubi [A] (verified) | 354 |
| Maple [A] (verified) | 355 |
| Fricas [A] (verification not implemented) | 355 |
| Sympy [A] (verification not implemented) | 355 |
| Maxima [A] (verification not implemented) | 356 |
| Giac [A] (verification not implemented) | 356 |
| Mupad [B] (verification not implemented) | 356 |
| Reduce [B] (verification not implemented) | 357 |

Optimal result

Integrand size = 30, antiderivative size = 5

$$\int \frac{(d + ex)^4}{(cd^2 + 2cdex + ce^2x^2)^2} dx = \frac{x}{c^2}$$

output

`x/c^2`

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 5, normalized size of antiderivative = 1.00

$$\int \frac{(d + ex)^4}{(cd^2 + 2cdex + ce^2x^2)^2} dx = \frac{x}{c^2}$$

input

`Integrate[(d + e*x)^4/(c*d^2 + 2*c*d*e*x + c*e^2*x^2)^2,x]`

output

`x/c^2`

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 5, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = {1098, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d + ex)^4}{(cd^2 + 2cdex + ce^2x^2)^2} dx$$

↓ 1098

$$c^2e^4 \int \frac{1}{c^4e^4} dx$$

↓ 24

$$\frac{x}{c^2}$$

input

```
Int[(d + e*x)^4/(c*d^2 + 2*c*d*e*x + c*e^2*x^2)^2,x]
```

output

```
x/c^2
```

Defintions of rubi rules used

rule 24

```
Int[a_, x_Symbol] := Simp[a*x, x] /; FreeQ[a, x]
```

rule 1098

```
Int[((d_.) + (e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] := Simp[1/c^p Int[(d + e*x)^m*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Maple [A] (verified)

Time = 1.07 (sec) , antiderivative size = 6, normalized size of antiderivative = 1.20

| method | result | size |
|---------|---|------|
| default | $\frac{x}{c^2}$ | 6 |
| risch | $\frac{x}{c^2}$ | 6 |
| orering | $\frac{x(ex+d)^4}{(x^2c^2e^2+2cdxe+cd^2)^2}$ | 32 |
| norman | $\frac{\frac{e^3x^4}{c} + \frac{d^3x}{c} + \frac{3e^2dx^3}{c} + \frac{3ed^2x^2}{c}}{c(ex+d)^3}$ | 55 |

input `int((e*x+d)^4/(c*e^2*x^2+2*c*d*e*x+c*d^2)^2,x,method=_RETURNVERBOSE)`

output `1/c^2*x`

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 5, normalized size of antiderivative = 1.00

$$\int \frac{(d+ex)^4}{(cd^2+2cdex+ce^2x^2)^2} dx = \frac{x}{c^2}$$

input `integrate((e*x+d)^4/(c*e^2*x^2+2*c*d*e*x+c*d^2)^2,x, algorithm="fricas")`

output `x/c^2`

Sympy [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 3, normalized size of antiderivative = 0.60

$$\int \frac{(d+ex)^4}{(cd^2+2cdex+ce^2x^2)^2} dx = \frac{x}{c^2}$$

input `integrate((e*x+d)**4/(c*e**2*x**2+2*c*d*e*x+c*d**2)**2,x)`

output `x/c**2`

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 5, normalized size of antiderivative = 1.00

$$\int \frac{(d + ex)^4}{(cd^2 + 2cdex + ce^2x^2)^2} dx = \frac{x}{c^2}$$

input `integrate((e*x+d)^4/(c*e^2*x^2+2*c*d*e*x+c*d^2)^2,x, algorithm="maxima")`

output `x/c^2`

Giac [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 5, normalized size of antiderivative = 1.00

$$\int \frac{(d + ex)^4}{(cd^2 + 2cdex + ce^2x^2)^2} dx = \frac{x}{c^2}$$

input `integrate((e*x+d)^4/(c*e^2*x^2+2*c*d*e*x+c*d^2)^2,x, algorithm="giac")`

output `x/c^2`

Mupad [B] (verification not implemented)

Time = 0.01 (sec) , antiderivative size = 5, normalized size of antiderivative = 1.00

$$\int \frac{(d + ex)^4}{(cd^2 + 2cdex + ce^2x^2)^2} dx = \frac{x}{c^2}$$

input `int((d + e*x)^4/(c*d^2 + c*e^2*x^2 + 2*c*d*e*x)^2,x)`

output `x/c^2`

Reduce [B] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 5, normalized size of antiderivative = 1.00

$$\int \frac{(d + ex)^4}{(cd^2 + 2cdex + ce^2x^2)^2} dx = \frac{x}{c^2}$$

input `int((e*x+d)^4/(c*e^2*x^2+2*c*d*e*x+c*d^2)^2,x)`

output `x/c**2`

$$3.34 \quad \int \frac{(d+ex)^3}{(cd^2+2cdex+ce^2x^2)^2} dx$$

| | |
|---|-----|
| Optimal result | 358 |
| Mathematica [A] (verified) | 358 |
| Rubi [A] (verified) | 359 |
| Maple [A] (verified) | 360 |
| Fricas [A] (verification not implemented) | 360 |
| Sympy [A] (verification not implemented) | 360 |
| Maxima [A] (verification not implemented) | 361 |
| Giac [B] (verification not implemented) | 361 |
| Mupad [B] (verification not implemented) | 362 |
| Reduce [B] (verification not implemented) | 362 |

Optimal result

Integrand size = 30, antiderivative size = 13

$$\int \frac{(d+ex)^3}{(cd^2+2cdex+ce^2x^2)^2} dx = \frac{\log(d+ex)}{c^2e}$$

output `ln(e*x+d)/c^2/e`

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00

$$\int \frac{(d+ex)^3}{(cd^2+2cdex+ce^2x^2)^2} dx = \frac{\log(d+ex)}{c^2e}$$

input `Integrate[(d + e*x)^3/(c*d^2 + 2*c*d*e*x + c*e^2*x^2)^2,x]`

output `Log[d + e*x]/(c^2*e)`

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = {1098, 16}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d + ex)^3}{(cd^2 + 2cdex + ce^2x^2)^2} dx$$

↓ 1098

$$c^2e^4 \int \frac{1}{c^4e^4(d + ex)} dx$$

↓ 16

$$\frac{\log(d + ex)}{c^2e}$$

input

```
Int[(d + e*x)^3/(c*d^2 + 2*c*d*e*x + c*e^2*x^2)^2,x]
```

output

```
Log[d + e*x]/(c^2*e)
```

Defintions of rubi rules used

rule 16

```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] := Simp[c*(Log[RemoveContent[a + b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 1098

```
Int[((d_.) + (e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] := Simp[1/c^p Int[(d + e*x)^m*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Maple [A] (verified)

Time = 1.07 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.08

| method | result | size |
|---------------|--------------------------|------|
| default | $\frac{\ln(ex+d)}{c^2e}$ | 14 |
| norman | $\frac{\ln(ex+d)}{c^2e}$ | 14 |
| risch | $\frac{\ln(ex+d)}{c^2e}$ | 14 |
| parallelrisch | $\frac{\ln(ex+d)}{c^2e}$ | 14 |

input `int((e*x+d)^3/(c*e^2*x^2+2*c*d*e*x+c*d^2)^2,x,method=_RETURNVERBOSE)`output `ln(e*x+d)/c^2/e`**Fricas [A] (verification not implemented)**

Time = 0.09 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00

$$\int \frac{(d+ex)^3}{(cd^2+2cdex+ce^2x^2)^2} dx = \frac{\log(ex+d)}{c^2e}$$

input `integrate((e*x+d)^3/(c*e^2*x^2+2*c*d*e*x+c*d^2)^2,x, algorithm="fricas")`output `log(e*x + d)/(c^2*e)`**Sympy [A] (verification not implemented)**

Time = 0.04 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.31

$$\int \frac{(d+ex)^3}{(cd^2+2cdex+ce^2x^2)^2} dx = \frac{\log(c^2d+c^2ex)}{c^2e}$$

input `integrate((e*x+d)**3/(c*e**2*x**2+2*c*d*e*x+c*d**2)**2,x)`

output `log(c**2*d + c**2*e*x)/(c**2*e)`

Maxima [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00

$$\int \frac{(d + ex)^3}{(cd^2 + 2cdex + ce^2x^2)^2} dx = \frac{\log(ex + d)}{c^2e}$$

input `integrate((e*x+d)^3/(c*e^2*x^2+2*c*d*e*x+c*d^2)^2,x, algorithm="maxima")`

output `log(e*x + d)/(c^2*e)`

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 128 vs. $2(13) = 26$.

Time = 0.18 (sec) , antiderivative size = 128, normalized size of antiderivative = 9.85

$$\int \frac{(d + ex)^3}{(cd^2 + 2cdex + ce^2x^2)^2} dx = \frac{\frac{d^2}{(cd^2 + (ex^2 + 2dx)ce)e} - \frac{\log\left(\frac{|cd^2 + (ex^2 + 2dx)ce||e|}{2(cd^2 + (ex^2 + 2dx)ce)^2e^2|c|}\right)}{ce}}{2c} - \frac{d^2}{2(cd^2 + (ex^2 + 2dx)ce)ce}$$

input `integrate((e*x+d)^3/(c*e^2*x^2+2*c*d*e*x+c*d^2)^2,x, algorithm="giac")`

output `1/2*(d^2/((c*d^2 + (e*x^2 + 2*d*x)*c*e)*e) - log(1/2*abs(c*d^2 + (e*x^2 + 2*d*x)*c*e)*abs(e)/((c*d^2 + (e*x^2 + 2*d*x)*c*e)^2*e^2*abs(c)))/(c*e))/c - 1/2*d^2/((c*d^2 + (e*x^2 + 2*d*x)*c*e)*c*e)`

Mupad [B] (verification not implemented)

Time = 8.85 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00

$$\int \frac{(d + ex)^3}{(cd^2 + 2cdex + ce^2x^2)^2} dx = \frac{\ln(d + ex)}{c^2 e}$$

input `int((d + e*x)^3/(c*d^2 + c*e^2*x^2 + 2*c*d*e*x)^2,x)`output `log(d + e*x)/(c^2*e)`**Reduce [B] (verification not implemented)**

Time = 0.25 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00

$$\int \frac{(d + ex)^3}{(cd^2 + 2cdex + ce^2x^2)^2} dx = \frac{\log(ex + d)}{c^2 e}$$

input `int((e*x+d)^3/(c*e^2*x^2+2*c*d*e*x+c*d^2)^2,x)`output `log(d + e*x)/(c**2*e)`

$$3.35 \quad \int \frac{(d+ex)^2}{(cd^2+2cdex+ce^2x^2)^2} dx$$

| | |
|---|-----|
| Optimal result | 363 |
| Mathematica [A] (verified) | 363 |
| Rubi [A] (verified) | 364 |
| Maple [A] (verified) | 365 |
| Fricas [A] (verification not implemented) | 365 |
| Sympy [A] (verification not implemented) | 366 |
| Maxima [A] (verification not implemented) | 366 |
| Giac [A] (verification not implemented) | 366 |
| Mupad [B] (verification not implemented) | 367 |
| Reduce [B] (verification not implemented) | 367 |

Optimal result

Integrand size = 30, antiderivative size = 15

$$\int \frac{(d+ex)^2}{(cd^2+2cdex+ce^2x^2)^2} dx = -\frac{1}{c^2e(d+ex)}$$

output

```
-1/c^2/e/(e*x+d)
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00

$$\int \frac{(d+ex)^2}{(cd^2+2cdex+ce^2x^2)^2} dx = -\frac{1}{c^2e(d+ex)}$$

input

```
Integrate[(d + e*x)^2/(c*d^2 + 2*c*d*e*x + c*e^2*x^2)^2,x]
```

output

```
-(1/(c^2*e*(d + e*x)))
```


Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = {1098, 17}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d + ex)^2}{(cd^2 + 2cdex + ce^2x^2)^2} dx$$

↓ 1098

$$c^2e^4 \int \frac{1}{c^4e^4(d + ex)^2} dx$$

↓ 17

$$-\frac{1}{c^2e(d + ex)}$$

input `Int[(d + e*x)^2/(c*d^2 + 2*c*d*e*x + c*e^2*x^2)^2,x]`

output `-(1/(c^2*e*(d + e*x)))`

Defintions of rubi rules used

rule 17 `Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] := Simp[c*((a + b*x)^(m + 1))/(b*(m + 1)), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]`

rule 1098 `Int[((d_.) + (e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] := Simp[1/c^p Int[(d + e*x)^m*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]`

Maple [A] (verified)

Time = 1.09 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.07

| method | result | size |
|---------------|--|------|
| gospers | $-\frac{1}{c^2 e^{ex+d}}$ | 16 |
| default | $-\frac{1}{c^2 e^{ex+d}}$ | 16 |
| risch | $-\frac{1}{c^2 e^{ex+d}}$ | 16 |
| parallelrisch | $-\frac{1}{c^2 e^{ex+d}}$ | 16 |
| orering | $-\frac{(ex+d)^3}{e(x^2 c e^2 + 2cdxe + c d^2)^2}$ | 35 |
| norman | $\frac{-\frac{d^2}{e} - \frac{e x^2}{c} - \frac{2xd}{c}}{c(ex+d)^3}$ | 40 |

input `int((e*x+d)^2/(c*e^2*x^2+2*c*d*e*x+c*d^2)^2,x,method=_RETURNVERBOSE)`

output `-1/c^2/e/(e*x+d)`

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.27

$$\int \frac{(d+ex)^2}{(cd^2+2cdex+ce^2x^2)^2} dx = -\frac{1}{c^2 e^2 x + c^2 de}$$

input `integrate((e*x+d)^2/(c*e^2*x^2+2*c*d*e*x+c*d^2)^2,x,algorithm="fricas")`

output `-1/(c^2*e^2*x + c^2*d*e)`

Sympy [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.13

$$\int \frac{(d + ex)^2}{(cd^2 + 2cdex + ce^2x^2)^2} dx = -\frac{1}{c^2de + c^2e^2x}$$

input `integrate((e*x+d)**2/(c*e**2*x**2+2*c*d*e*x+c*d**2)**2,x)`output `-1/(c**2*d*e + c**2*e**2*x)`**Maxima [A] (verification not implemented)**

Time = 0.03 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.27

$$\int \frac{(d + ex)^2}{(cd^2 + 2cdex + ce^2x^2)^2} dx = -\frac{1}{c^2e^2x + c^2de}$$

input `integrate((e*x+d)^2/(c*e^2*x^2+2*c*d*e*x+c*d^2)^2,x, algorithm="maxima")`output `-1/(c^2*e^2*x + c^2*d*e)`**Giac [A] (verification not implemented)**

Time = 0.18 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00

$$\int \frac{(d + ex)^2}{(cd^2 + 2cdex + ce^2x^2)^2} dx = -\frac{1}{(ex + d)c^2e}$$

input `integrate((e*x+d)^2/(c*e^2*x^2+2*c*d*e*x+c*d^2)^2,x, algorithm="giac")`output `-1/((e*x + d)*c^2*e)`

Mupad [B] (verification not implemented)

Time = 8.72 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.27

$$\int \frac{(d + ex)^2}{(cd^2 + 2cdex + ce^2x^2)^2} dx = -\frac{1}{x c^2 e^2 + d c^2 e}$$

input `int((d + e*x)^2/(c*d^2 + c*e^2*x^2 + 2*c*d*e*x)^2,x)`output `-1/(c^2*e^2*x + c^2*d*e)`**Reduce [B] (verification not implemented)**

Time = 0.24 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00

$$\int \frac{(d + ex)^2}{(cd^2 + 2cdex + ce^2x^2)^2} dx = \frac{x}{c^2d(ex + d)}$$

input `int((e*x+d)^2/(c*e^2*x^2+2*c*d*e*x+c*d^2)^2,x)`output `x/(c**2*d*(d + e*x))`

$$3.36 \quad \int \frac{d+ex}{(cd^2+2cdex+ce^2x^2)^2} dx$$

| | |
|---|-----|
| Optimal result | 368 |
| Mathematica [A] (verified) | 368 |
| Rubi [A] (verified) | 369 |
| Maple [A] (verified) | 370 |
| Fricas [B] (verification not implemented) | 370 |
| Sympy [B] (verification not implemented) | 371 |
| Maxima [A] (verification not implemented) | 371 |
| Giac [A] (verification not implemented) | 371 |
| Mupad [B] (verification not implemented) | 372 |
| Reduce [B] (verification not implemented) | 372 |

Optimal result

Integrand size = 28, antiderivative size = 17

$$\int \frac{d+ex}{(cd^2+2cdex+ce^2x^2)^2} dx = -\frac{1}{2c^2e(d+ex)^2}$$

output $-1/2/c^2/e/(e*x+d)^2$

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00

$$\int \frac{d+ex}{(cd^2+2cdex+ce^2x^2)^2} dx = -\frac{1}{2c^2e(d+ex)^2}$$

input `Integrate[(d + e*x)/(c*d^2 + 2*c*d*e*x + c*e^2*x^2)^2,x]`

output $-1/2*1/(c^2*e*(d + e*x)^2)$

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.071$, Rules used = {1098, 17}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{d + ex}{(cd^2 + 2cdex + ce^2x^2)^2} dx$$

$$\downarrow 1098$$

$$c^2 e^4 \int \frac{1}{c^4 e^4 (d + ex)^3} dx$$

$$\downarrow 17$$

$$-\frac{1}{2c^2 e (d + ex)^2}$$

input `Int[(d + e*x)/(c*d^2 + 2*c*d*e*x + c*e^2*x^2)^2,x]`

output `-1/2*1/(c^2*e*(d + e*x)^2)`

Defintions of rubi rules used

rule 17 `Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] := Simp[c*((a + b*x)^(m + 1))/(b*(m + 1)), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]`

rule 1098 `Int[((d_.) + (e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] := Simp[1/c^p Int[(d + e*x)^m*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]`

Maple [A] (verified)

Time = 0.97 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.94

| method | result | size |
|---------------|---|------|
| gospers | $-\frac{1}{2c^2e(ex+d)^2}$ | 16 |
| default | $-\frac{1}{2c^2e(ex+d)^2}$ | 16 |
| risch | $-\frac{1}{2c^2e(ex+d)^2}$ | 16 |
| parallelrisch | $-\frac{1}{2c^2e(ex+d)^2}$ | 16 |
| norman | $-\frac{\frac{d}{2ec} - \frac{x}{2c}}{c(ex+d)^{\frac{3}{2}}}$ | 28 |
| orering | $-\frac{(ex+d)^2}{2e(x^2ce^2+2cdxe+cd^2)^2}$ | 35 |

input `int((e*x+d)/(c*e^2*x^2+2*c*d*e*x+c*d^2)^2,x,method=_RETURNVERBOSE)`

output `-1/2/c^2/e/(e*x+d)^2`

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 33 vs. 2(15) = 30.

Time = 0.07 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.94

$$\int \frac{d+ex}{(cd^2+2cdex+ce^2x^2)^2} dx = -\frac{1}{2(c^2e^3x^2+2c^2de^2x+c^2d^2e)}$$

input `integrate((e*x+d)/(c*e^2*x^2+2*c*d*e*x+c*d^2)^2,x, algorithm="fricas")`

output `-1/2/(c^2*e^3*x^2 + 2*c^2*d*e^2*x + c^2*d^2*e)`

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 36 vs. $2(15) = 30$.

Time = 0.12 (sec) , antiderivative size = 36, normalized size of antiderivative = 2.12

$$\int \frac{d + ex}{(cd^2 + 2cdex + ce^2x^2)^2} dx = -\frac{1}{2c^2d^2e + 4c^2de^2x + 2c^2e^3x^2}$$

input `integrate((e*x+d)/(c*e**2*x**2+2*c*d*e*x+c*d**2)**2,x)`

output `-1/(2*c**2*d**2*e + 4*c**2*d*e**2*x + 2*c**2*e**3*x**2)`

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.76

$$\int \frac{d + ex}{(cd^2 + 2cdex + ce^2x^2)^2} dx = -\frac{1}{2(ce^2x^2 + 2cdex + cd^2)ce}$$

input `integrate((e*x+d)/(c*e^2*x^2+2*c*d*e*x+c*d^2)^2,x, algorithm="maxima")`

output `-1/2/((c*e^2*x^2 + 2*c*d*e*x + c*d^2)*c*e)`

Giac [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.71

$$\int \frac{d + ex}{(cd^2 + 2cdex + ce^2x^2)^2} dx = -\frac{1}{2(cd^2 + (ex^2 + 2dx)ce)ce}$$

input `integrate((e*x+d)/(c*e^2*x^2+2*c*d*e*x+c*d^2)^2,x, algorithm="giac")`

output `-1/2/((c*d^2 + (e*x^2 + 2*d*x)*c*e)*c*e)`

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 35, normalized size of antiderivative = 2.06

$$\int \frac{d + ex}{(cd^2 + 2cdex + ce^2x^2)^2} dx = -\frac{1}{2c^2d^2e + 4c^2de^2x + 2c^2e^3x^2}$$

input `int((d + e*x)/(c*d^2 + c*e^2*x^2 + 2*c*d*e*x)^2,x)`output `-1/(2*c^2*d^2*e + 2*c^2*e^3*x^2 + 4*c^2*d*e^2*x)`**Reduce [B] (verification not implemented)**

Time = 0.28 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.53

$$\int \frac{d + ex}{(cd^2 + 2cdex + ce^2x^2)^2} dx = -\frac{1}{2c^2e(e^2x^2 + 2dex + d^2)}$$

input `int((e*x+d)/(c*e^2*x^2+2*c*d*e*x+c*d^2)^2,x)`output `(- 1)/(2*c**2*e*(d**2 + 2*d*e*x + e**2*x**2))`

$$3.37 \quad \int \frac{1}{(cd^2 + 2cdex + ce^2x^2)^2} dx$$

| | |
|---|-----|
| Optimal result | 373 |
| Mathematica [A] (verified) | 373 |
| Rubi [A] (verified) | 374 |
| Maple [A] (verified) | 375 |
| Fricas [B] (verification not implemented) | 375 |
| Sympy [B] (verification not implemented) | 376 |
| Maxima [B] (verification not implemented) | 376 |
| Giac [A] (verification not implemented) | 376 |
| Mupad [B] (verification not implemented) | 377 |
| Reduce [B] (verification not implemented) | 377 |

Optimal result

Integrand size = 22, antiderivative size = 17

$$\int \frac{1}{(cd^2 + 2cdex + ce^2x^2)^2} dx = -\frac{1}{3c^2e(d + ex)^3}$$

output `-1/3/c^2/e/(e*x+d)^3`

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00

$$\int \frac{1}{(cd^2 + 2cdex + ce^2x^2)^2} dx = -\frac{1}{3c^2e(d + ex)^3}$$

input `Integrate[(c*d^2 + 2*c*d*e*x + c*e^2*x^2)^(-2),x]`

output `-1/3*1/(c^2*e*(d + e*x)^3)`

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {1077, 17}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(cd^2 + 2cdex + ce^2x^2)^2} dx$$

$$\downarrow 1077$$

$$c^2e^4 \int \frac{1}{(cxe^2 + cde)^4} dx$$

$$\downarrow 17$$

$$-\frac{1}{3c^2e(d + ex)^3}$$

input `Int[(c*d^2 + 2*c*d*e*x + c*e^2*x^2)^(-2),x]`

output `-1/3*1/(c^2*e*(d + e*x)^3)`

Defintions of rubi rules used

rule 17 `Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1))/(b*(m + 1))], x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]`

rule 1077 `Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[1/c^p Int[(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]`

Maple [A] (verified)

Time = 1.01 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.94

| method | result | size |
|---------------|---|------|
| default | $-\frac{1}{3c^2e(ex+d)^3}$ | 16 |
| norman | $-\frac{1}{3c^2e(ex+d)^3}$ | 16 |
| risch | $-\frac{1}{3c^2e(ex+d)^3}$ | 16 |
| parallelrisch | $-\frac{1}{3c^2e(ex+d)^3}$ | 16 |
| orering | $-\frac{ex+d}{3e(x^2ce^2+2cdxe+cd^2)^2}$ | 33 |
| gospers | $-\frac{1}{3(ex+d)(e^2x^2+2dex+d^2)ec^2}$ | 34 |

input `int(1/(c*e^2*x^2+2*c*d*e*x+c*d^2)^2,x,method=_RETURNVERBOSE)`

output `-1/3/c^2/e/(e*x+d)^3`

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 47 vs. 2(15) = 30.

Time = 0.08 (sec) , antiderivative size = 47, normalized size of antiderivative = 2.76

$$\int \frac{1}{(cd^2 + 2cdex + ce^2x^2)^2} dx = -\frac{1}{3(c^2e^4x^3 + 3c^2de^3x^2 + 3c^2d^2e^2x + c^2d^3e)}$$

input `integrate(1/(c*e^2*x^2+2*c*d*e*x+c*d^2)^2,x, algorithm="fricas")`

output `-1/3/(c^2*e^4*x^3 + 3*c^2*d*e^3*x^2 + 3*c^2*d^2*e^2*x + c^2*d^3*e)`

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 51 vs. $2(15) = 30$.

Time = 0.17 (sec) , antiderivative size = 51, normalized size of antiderivative = 3.00

$$\int \frac{1}{(cd^2 + 2cdex + ce^2x^2)^2} dx = -\frac{1}{3c^2d^3e + 9c^2d^2e^2x + 9c^2de^3x^2 + 3c^2e^4x^3}$$

input `integrate(1/(c*e**2*x**2+2*c*d*e*x+c*d**2)**2,x)`

output `-1/(3*c**2*d**3*e + 9*c**2*d**2*e**2*x + 9*c**2*d*e**3*x**2 + 3*c**2*e**4*x**3)`

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 47 vs. $2(15) = 30$.

Time = 0.03 (sec) , antiderivative size = 47, normalized size of antiderivative = 2.76

$$\int \frac{1}{(cd^2 + 2cdex + ce^2x^2)^2} dx = -\frac{1}{3(c^2e^4x^3 + 3c^2de^3x^2 + 3c^2d^2e^2x + c^2d^3e)}$$

input `integrate(1/(c*e^2*x^2+2*c*d*e*x+c*d^2)^2,x, algorithm="maxima")`

output `-1/3/(c^2*e^4*x^3 + 3*c^2*d*e^3*x^2 + 3*c^2*d^2*e^2*x + c^2*d^3*e)`

Giac [A] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.88

$$\int \frac{1}{(cd^2 + 2cdex + ce^2x^2)^2} dx = -\frac{1}{3(ex + d)^3c^2e}$$

input `integrate(1/(c*e^2*x^2+2*c*d*e*x+c*d^2)^2,x, algorithm="giac")`

output $-1/3/((e*x + d)^3*c^2*e)$

Mupad [B] (verification not implemented)

Time = 8.68 (sec) , antiderivative size = 49, normalized size of antiderivative = 2.88

$$\int \frac{1}{(cd^2 + 2cdex + ce^2x^2)^2} dx = -\frac{1}{3c^2d^3e + 9c^2d^2e^2x + 9c^2de^3x^2 + 3c^2e^4x^3}$$

input $\text{int}(1/(c*d^2 + c*e^2*x^2 + 2*c*d*e*x)^2,x)$

output $-1/(3*c^2*d^3*e + 3*c^2*e^4*x^3 + 9*c^2*d^2*e^2*x + 9*c^2*d*e^3*x^2)$

Reduce [B] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 37, normalized size of antiderivative = 2.18

$$\int \frac{1}{(cd^2 + 2cdex + ce^2x^2)^2} dx = -\frac{1}{3c^2e(e^3x^3 + 3de^2x^2 + 3d^2ex + d^3)}$$

input $\text{int}(1/(c*e^2*x^2+2*c*d*e*x+c*d^2)^2,x)$

output $(-1)/(3*c**2*e*(d**3 + 3*d**2*e*x + 3*d*e**2*x**2 + e**3*x**3))$

$$3.38 \quad \int \frac{1}{(d+ex)(cd^2+2cdex+ce^2x^2)^2} dx$$

| | |
|---|-----|
| Optimal result | 378 |
| Mathematica [A] (verified) | 378 |
| Rubi [A] (verified) | 379 |
| Maple [A] (verified) | 380 |
| Fricas [B] (verification not implemented) | 380 |
| Sympy [B] (verification not implemented) | 381 |
| Maxima [B] (verification not implemented) | 381 |
| Giac [A] (verification not implemented) | 382 |
| Mupad [B] (verification not implemented) | 382 |
| Reduce [B] (verification not implemented) | 382 |

Optimal result

Integrand size = 30, antiderivative size = 17

$$\int \frac{1}{(d+ex)(cd^2+2cdex+ce^2x^2)^2} dx = -\frac{1}{4c^2e(d+ex)^4}$$

output `-1/4/c^2/e/(e*x+d)^4`

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00

$$\int \frac{1}{(d+ex)(cd^2+2cdex+ce^2x^2)^2} dx = -\frac{1}{4c^2e(d+ex)^4}$$

input `Integrate[1/((d + e*x)*(c*d^2 + 2*c*d*e*x + c*e^2*x^2)^2), x]`

output `-1/4*1/(c^2*e*(d + e*x)^4)`

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = {1098, 17}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(d+ex)(cd^2+2cdex+ce^2x^2)^2} dx$$

↓ 1098

$$c^2e^4 \int \frac{1}{c^4e^4(d+ex)^5} dx$$

↓ 17

$$-\frac{1}{4c^2e(d+ex)^4}$$

input `Int[1/((d + e*x)*(c*d^2 + 2*c*d*e*x + c*e^2*x^2)^2),x]`

output `-1/4*1/(c^2*e*(d + e*x)^4)`

Defintions of rubi rules used

rule 17 `Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] := Simp[c*((a + b*x)^(m + 1))/(b*(m + 1)), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]`

rule 1098 `Int[((d_.) + (e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] := Simp[1/c^p Int[(d + e*x)^m*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]`

Maple [A] (verified)

Time = 1.27 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.94

| method | result | size |
|---------------|---|------|
| default | $-\frac{1}{4c^2e(ex+d)^4}$ | 16 |
| norman | $-\frac{1}{4c^2e(ex+d)^4}$ | 16 |
| gospers | $-\frac{1}{4ec^2(e^2x^2+2dex+d^2)^2}$ | 27 |
| orering | $-\frac{1}{4e(x^2ce^2+2cdxe+cd^2)^2}$ | 28 |
| risch | $-\frac{1}{4e(ex+d)^2c^2(e^2x^2+2dex+d^2)}$ | 34 |
| parallelrisch | $-\frac{1}{4e(ex+d)^2c^2(e^2x^2+2dex+d^2)}$ | 34 |

input `int(1/(e*x+d)/(c*e^2*x^2+2*c*d*e*x+c*d^2)^2,x,method=_RETURNVERBOSE)`

output `-1/4/c^2/e/(e*x+d)^4`

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 61 vs. 2(15) = 30.

Time = 0.08 (sec) , antiderivative size = 61, normalized size of antiderivative = 3.59

$$\int \frac{1}{(d+ex)(cd^2+2cdex+ce^2x^2)^2} dx$$

$$= -\frac{1}{4(c^2e^5x^4+4c^2de^4x^3+6c^2d^2e^3x^2+4c^2d^3e^2x+c^2d^4e)}$$

input `integrate(1/(e*x+d)/(c*e^2*x^2+2*c*d*e*x+c*d^2)^2,x, algorithm="fricas")`

output `-1/4/(c^2*e^5*x^4+4*c^2*d*e^4*x^3+6*c^2*d^2*e^3*x^2+4*c^2*d^3*e^2*x+c^2*d^4*e)`

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 66 vs. $2(15) = 30$.

Time = 0.24 (sec) , antiderivative size = 66, normalized size of antiderivative = 3.88

$$\int \frac{1}{(d+ex)(cd^2+2cdex+ce^2x^2)^2} dx$$

$$= -\frac{1}{4c^2d^4e + 16c^2d^3e^2x + 24c^2d^2e^3x^2 + 16c^2de^4x^3 + 4c^2e^5x^4}$$

input `integrate(1/(e*x+d)/(c*e**2*x**2+2*c*d*e*x+c*d**2)**2,x)`

output `-1/(4*c**2*d**4*e + 16*c**2*d**3*e**2*x + 24*c**2*d**2*e**3*x**2 + 16*c**2*d*e**4*x**3 + 4*c**2*e**5*x**4)`

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 61 vs. $2(15) = 30$.

Time = 0.03 (sec) , antiderivative size = 61, normalized size of antiderivative = 3.59

$$\int \frac{1}{(d+ex)(cd^2+2cdex+ce^2x^2)^2} dx$$

$$= -\frac{1}{4(c^2e^5x^4 + 4c^2de^4x^3 + 6c^2d^2e^3x^2 + 4c^2d^3e^2x + c^2d^4e)}$$

input `integrate(1/(e*x+d)/(c*e^2*x^2+2*c*d*e*x+c*d^2)^2,x, algorithm="maxima")`

output `-1/4/(c^2*e^5*x^4 + 4*c^2*d*e^4*x^3 + 6*c^2*d^2*e^3*x^2 + 4*c^2*d^3*e^2*x + c^2*d^4*e)`

Giac [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.88

$$\int \frac{1}{(d+ex)(cd^2+2cdex+ce^2x^2)^2} dx = -\frac{1}{4(ex+d)^4c^2e}$$

input `integrate(1/(e*x+d)/(c*e^2*x^2+2*c*d*e*x+c*d^2)^2,x, algorithm="giac")`output `-1/4/((e*x + d)^4*c^2*e)`**Mupad [B] (verification not implemented)**

Time = 0.05 (sec) , antiderivative size = 63, normalized size of antiderivative = 3.71

$$\int \frac{1}{(d+ex)(cd^2+2cdex+ce^2x^2)^2} dx$$

$$= -\frac{1}{4c^2d^4e + 16c^2d^3e^2x + 24c^2d^2e^3x^2 + 16c^2de^4x^3 + 4c^2e^5x^4}$$

input `int(1/((d + e*x)*(c*d^2 + c*e^2*x^2 + 2*c*d*e*x)^2),x)`output `-1/(4*c^2*d^4*e + 4*c^2*e^5*x^4 + 16*c^2*d^3*e^2*x + 16*c^2*d*e^4*x^3 + 24*c^2*d^2*e^3*x^2)`**Reduce [B] (verification not implemented)**

Time = 0.26 (sec) , antiderivative size = 48, normalized size of antiderivative = 2.82

$$\int \frac{1}{(d+ex)(cd^2+2cdex+ce^2x^2)^2} dx = -\frac{1}{4c^2e(e^4x^4+4de^3x^3+6d^2e^2x^2+4d^3ex+d^4)}$$

input `int(1/(e*x+d)/(c*e^2*x^2+2*c*d*e*x+c*d^2)^2,x)`output `(- 1)/(4*c**2*e*(d**4 + 4*d**3*e*x + 6*d**2*e**2*x**2 + 4*d*e**3*x**3 + e**4*x**4))`

3.39 $\int \frac{1}{(d+ex)^2 (cd^2+2cdex+ce^2x^2)^2} dx$

| | |
|---|-----|
| Optimal result | 383 |
| Mathematica [A] (verified) | 383 |
| Rubi [A] (verified) | 384 |
| Maple [A] (verified) | 385 |
| Fricas [B] (verification not implemented) | 385 |
| Sympy [B] (verification not implemented) | 386 |
| Maxima [B] (verification not implemented) | 386 |
| Giac [A] (verification not implemented) | 387 |
| Mupad [B] (verification not implemented) | 387 |
| Reduce [B] (verification not implemented) | 387 |

Optimal result

Integrand size = 30, antiderivative size = 17

$$\int \frac{1}{(d+ex)^2 (cd^2+2cdex+ce^2x^2)^2} dx = -\frac{1}{5c^2e(d+ex)^5}$$

output -1/5/c^2/e/(e*x+d)^5

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00

$$\int \frac{1}{(d+ex)^2 (cd^2+2cdex+ce^2x^2)^2} dx = -\frac{1}{5c^2e(d+ex)^5}$$

input Integrate[1/((d + e*x)^2*(c*d^2 + 2*c*d*e*x + c*e^2*x^2)^2), x]

output -1/5*1/(c^2*e*(d + e*x)^5)

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = {1098, 17}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(d+ex)^2 (cd^2+2cdex+ce^2x^2)^2} dx$$

↓ 1098

$$c^2e^4 \int \frac{1}{c^4e^4(d+ex)^6} dx$$

↓ 17

$$-\frac{1}{5c^2e(d+ex)^5}$$

input `Int[1/((d + e*x)^2*(c*d^2 + 2*c*d*e*x + c*e^2*x^2)^2),x]`

output `-1/5*1/(c^2*e*(d + e*x)^5)`

Defintions of rubi rules used

rule 17 `Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] := Simp[c*((a + b*x)^(m + 1))/(b*(m + 1)), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]`

rule 1098 `Int[((d_.) + (e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] := Simp[1/c^p Int[(d + e*x)^m*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]`

Maple [A] (verified)

Time = 1.26 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.94

| method | result | size |
|---------------|--|------|
| default | $-\frac{1}{5c^2e(ex+d)^5}$ | 16 |
| norman | $-\frac{1}{5c^2e(ex+d)^5}$ | 16 |
| gosper | $-\frac{1}{5(ex+d)e c^2(e^2x^2+2dex+d^2)^2}$ | 34 |
| risch | $-\frac{1}{5e(ex+d)^3c^2(e^2x^2+2dex+d^2)}$ | 34 |
| parallelrisch | $-\frac{1}{5e(ex+d)^3c^2(e^2x^2+2dex+d^2)}$ | 34 |
| orering | $-\frac{1}{5(ex+d)e(x^2c e^2+2cdxe+cd^2)^2}$ | 35 |

input `int(1/(e*x+d)^2/(c*e^2*x^2+2*c*d*e*x+c*d^2)^2,x,method=_RETURNVERBOSE)`

output `-1/5/c^2/e/(e*x+d)^5`

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 75 vs. 2(15) = 30.

Time = 0.08 (sec) , antiderivative size = 75, normalized size of antiderivative = 4.41

$$\int \frac{1}{(d+ex)^2 (cd^2+2cdex+ce^2x^2)^2} dx$$

$$= -\frac{1}{5(c^2e^6x^5+5c^2de^5x^4+10c^2d^2e^4x^3+10c^2d^3e^3x^2+5c^2d^4e^2x+c^2d^5e)}$$

input `integrate(1/(e*x+d)^2/(c*e^2*x^2+2*c*d*e*x+c*d^2)^2,x, algorithm="fricas")`

output `-1/5/(c^2*e^6*x^5+5*c^2*d*e^5*x^4+10*c^2*d^2*e^4*x^3+10*c^2*d^3*e^3*x^2+5*c^2*d^4*e^2*x+c^2*d^5*e)`

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 82 vs. $2(15) = 30$.

Time = 0.17 (sec) , antiderivative size = 82, normalized size of antiderivative = 4.82

$$\int \frac{1}{(d+ex)^2 (cd^2 + 2cde x + ce^2 x^2)^2} dx$$

$$= -\frac{1}{5c^2 d^5 e + 25c^2 d^4 e^2 x + 50c^2 d^3 e^3 x^2 + 50c^2 d^2 e^4 x^3 + 25c^2 d e^5 x^4 + 5c^2 e^6 x^5}$$

input `integrate(1/(e*x+d)**2/(c*e**2*x**2+2*c*d*e*x+c*d**2)**2,x)`

output `-1/(5*c**2*d**5*e + 25*c**2*d**4*e**2*x + 50*c**2*d**3*e**3*x**2 + 50*c**2*d**2*e**4*x**3 + 25*c**2*d*e**5*x**4 + 5*c**2*e**6*x**5)`

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 75 vs. $2(15) = 30$.

Time = 0.03 (sec) , antiderivative size = 75, normalized size of antiderivative = 4.41

$$\int \frac{1}{(d+ex)^2 (cd^2 + 2cde x + ce^2 x^2)^2} dx$$

$$= -\frac{1}{5(c^2 e^6 x^5 + 5c^2 d e^5 x^4 + 10c^2 d^2 e^4 x^3 + 10c^2 d^3 e^3 x^2 + 5c^2 d^4 e^2 x + c^2 d^5 e)}$$

input `integrate(1/(e*x+d)^2/(c*e^2*x^2+2*c*d*e*x+c*d^2)^2,x, algorithm="maxima")`

output `-1/5/(c^2*e^6*x^5 + 5*c^2*d*e^5*x^4 + 10*c^2*d^2*e^4*x^3 + 10*c^2*d^3*e^3*x^2 + 5*c^2*d^4*e^2*x + c^2*d^5*e)`

Giac [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.88

$$\int \frac{1}{(d+ex)^2 (cd^2+2cdex+ce^2x^2)^2} dx = -\frac{1}{5(ex+d)^5 c^2 e}$$

input `integrate(1/(e*x+d)^2/(c*e^2*x^2+2*c*d*e*x+c*d^2)^2,x, algorithm="giac")`output `-1/5/((e*x + d)^5*c^2*e)`**Mupad [B] (verification not implemented)**

Time = 8.80 (sec) , antiderivative size = 77, normalized size of antiderivative = 4.53

$$\int \frac{1}{(d+ex)^2 (cd^2+2cdex+ce^2x^2)^2} dx$$

$$= -\frac{1}{5c^2d^5e + 25c^2d^4e^2x + 50c^2d^3e^3x^2 + 50c^2d^2e^4x^3 + 25c^2de^5x^4 + 5c^2e^6x^5}$$

input `int(1/((d + e*x)^2*(c*d^2 + c*e^2*x^2 + 2*c*d*e*x)^2),x)`output `-1/(5*c^2*d^5*e + 5*c^2*e^6*x^5 + 25*c^2*d^4*e^2*x + 25*c^2*d*e^5*x^4 + 50*c^2*d^3*e^3*x^2 + 50*c^2*d^2*e^4*x^3)`**Reduce [B] (verification not implemented)**

Time = 0.25 (sec) , antiderivative size = 59, normalized size of antiderivative = 3.47

$$\int \frac{1}{(d+ex)^2 (cd^2+2cdex+ce^2x^2)^2} dx$$

$$= -\frac{1}{5c^2e(e^5x^5 + 5de^4x^4 + 10d^2e^3x^3 + 10d^3e^2x^2 + 5d^4ex + d^5)}$$

input `int(1/(e*x+d)^2/(c*e^2*x^2+2*c*d*e*x+c*d^2)^2,x)`

output $(-1)/(5c^2e(d^5 + 5d^4ex + 10d^3e^2x^2 + 10d^2e^3x^3 + 5de^4x^4 + e^5x^5))$

$$3.40 \quad \int \frac{(d+ex)^9}{(cd^2+2cdex+ce^2x^2)^3} dx$$

| | |
|---|-----|
| Optimal result | 389 |
| Mathematica [A] (verified) | 389 |
| Rubi [A] (verified) | 390 |
| Maple [A] (verified) | 391 |
| Fricas [B] (verification not implemented) | 391 |
| Sympy [B] (verification not implemented) | 392 |
| Maxima [B] (verification not implemented) | 392 |
| Giac [B] (verification not implemented) | 392 |
| Mupad [B] (verification not implemented) | 393 |
| Reduce [B] (verification not implemented) | 393 |

Optimal result

Integrand size = 30, antiderivative size = 17

$$\int \frac{(d+ex)^9}{(cd^2+2cdex+ce^2x^2)^3} dx = \frac{(d+ex)^4}{4c^3e}$$

output `1/4*(e*x+d)^4/c^3/e`

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00

$$\int \frac{(d+ex)^9}{(cd^2+2cdex+ce^2x^2)^3} dx = \frac{(d+ex)^4}{4c^3e}$$

input `Integrate[(d + e*x)^9/(c*d^2 + 2*c*d*e*x + c*e^2*x^2)^3,x]`

output `(d + e*x)^4/(4*c^3*e)`

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = {1098, 17}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d + ex)^9}{(cd^2 + 2cdex + ce^2x^2)^3} dx$$

$$\downarrow 1098$$

$$c^3e^6 \int \frac{(d + ex)^3}{c^6e^6} dx$$

$$\downarrow 17$$

$$\frac{(d + ex)^4}{4c^3e}$$

input `Int[(d + e*x)^9/(c*d^2 + 2*c*d*e*x + c*e^2*x^2)^3,x]`

output `(d + e*x)^4/(4*c^3*e)`

Defintions of rubi rules used

rule 17 `Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] := Simp[c*((a + b*x)^(m + 1))/(b*(m + 1))], x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]`

rule 1098 `Int[((d_.) + (e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] := Simp[1/c^p Int[(d + e*x)^m*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]`

Maple [A] (verified)

Time = 1.10 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.94

| method | result | size |
|---------------|--|------|
| default | $\frac{(ex+d)^4}{4c^3e}$ | 16 |
| gospers | $\frac{x(e^3x^3+4de^2x^2+6d^2ex+4d^3)}{4c^3}$ | 36 |
| parallelrisch | $\frac{e^3x^4+4de^2x^3+6d^2ex^2+4d^3x}{4c^3}$ | 38 |
| risch | $\frac{e^3x^4}{4c^3} + \frac{e^2dx^3}{c^3} + \frac{3ed^2x^2}{2c^3} + \frac{d^3x}{c^3} + \frac{d^4}{4c^3e}$ | 55 |
| orering | $\frac{x(e^3x^3+4de^2x^2+6d^2ex+4d^3)(ex+d)^6}{4(x^2ce^2+2cdxe+cd^2)^3}$ | 62 |
| norman | $\frac{-\frac{126d^5e^3x^4}{c} + \frac{e^8x^9}{4c} + \frac{9de^7x^8}{4c} - \frac{125d^9}{4ec} + \frac{21e^5d^3x^6}{c} + \frac{9e^6d^2x^7}{c} - \frac{294d^6e^2x^3}{c} - \frac{306d^7ex^2}{c} - \frac{621d^8x}{4c}}{c^2(ex+d)^5}$ | 124 |

input `int((e*x+d)^9/(c*e^2*x^2+2*c*d*e*x+c*d^2)^3,x,method=_RETURNVERBOSE)`output `1/4*(e*x+d)^4/c^3/e`**Fricas [B] (verification not implemented)**

Leaf count of result is larger than twice the leaf count of optimal. 37 vs. 2(15) = 30.

Time = 0.06 (sec) , antiderivative size = 37, normalized size of antiderivative = 2.18

$$\int \frac{(d+ex)^9}{(cd^2+2cdex+ce^2x^2)^3} dx = \frac{e^3x^4 + 4de^2x^3 + 6d^2ex^2 + 4d^3x}{4c^3}$$

input `integrate((e*x+d)^9/(c*e^2*x^2+2*c*d*e*x+c*d^2)^3,x, algorithm="fricas")`output `1/4*(e^3*x^4 + 4*d*e^2*x^3 + 6*d^2*e*x^2 + 4*d^3*x)/c^3`

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 46 vs. $2(12) = 24$.

Time = 0.05 (sec) , antiderivative size = 46, normalized size of antiderivative = 2.71

$$\int \frac{(d+ex)^9}{(cd^2+2cdex+ce^2x^2)^3} dx = \frac{d^3x}{c^3} + \frac{3d^2ex^2}{2c^3} + \frac{de^2x^3}{c^3} + \frac{e^3x^4}{4c^3}$$

input `integrate((e*x+d)**9/(c*e**2*x**2+2*c*d*e*x+c*d**2)**3,x)`

output `d**3*x/c**3 + 3*d**2*e*x**2/(2*c**3) + d*e**2*x**3/c**3 + e**3*x**4/(4*c**3)`

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 37 vs. $2(15) = 30$.

Time = 0.04 (sec) , antiderivative size = 37, normalized size of antiderivative = 2.18

$$\int \frac{(d+ex)^9}{(cd^2+2cdex+ce^2x^2)^3} dx = \frac{e^3x^4 + 4de^2x^3 + 6d^2ex^2 + 4d^3x}{4c^3}$$

input `integrate((e*x+d)^9/(c*e^2*x^2+2*c*d*e*x+c*d^2)^3,x, algorithm="maxima")`

output `1/4*(e^3*x^4 + 4*d*e^2*x^3 + 6*d^2*e*x^2 + 4*d^3*x)/c^3`

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 37 vs. $2(15) = 30$.

Time = 0.18 (sec) , antiderivative size = 37, normalized size of antiderivative = 2.18

$$\int \frac{(d+ex)^9}{(cd^2+2cdex+ce^2x^2)^3} dx = \frac{e^3x^4 + 4de^2x^3 + 6d^2ex^2 + 4d^3x}{4c^3}$$

input `integrate((e*x+d)^9/(c*e^2*x^2+2*c*d*e*x+c*d^2)^3,x, algorithm="giac")`

output `1/4*(e^3*x^4 + 4*d*e^2*x^3 + 6*d^2*e*x^2 + 4*d^3*x)/c^3`

Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 43, normalized size of antiderivative = 2.53

$$\int \frac{(d + ex)^9}{(cd^2 + 2cdex + ce^2x^2)^3} dx = \frac{d^3 x}{c^3} + \frac{e^3 x^4}{4c^3} + \frac{3d^2 ex^2}{2c^3} + \frac{de^2 x^3}{c^3}$$

input `int((d + e*x)^9/(c*d^2 + c*e^2*x^2 + 2*c*d*e*x)^3,x)`

output `(d^3*x)/c^3 + (e^3*x^4)/(4*c^3) + (3*d^2*e*x^2)/(2*c^3) + (d*e^2*x^3)/c^3`

Reduce [B] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 35, normalized size of antiderivative = 2.06

$$\int \frac{(d + ex)^9}{(cd^2 + 2cdex + ce^2x^2)^3} dx = \frac{x(e^3x^3 + 4de^2x^2 + 6d^2ex + 4d^3)}{4c^3}$$

input `int((e*x+d)^9/(c*e^2*x^2+2*c*d*e*x+c*d^2)^3,x)`

output `(x*(4*d**3 + 6*d**2*e*x + 4*d*e**2*x**2 + e**3*x**3))/(4*c**3)`

$$3.41 \quad \int \frac{(d+ex)^8}{(cd^2+2cdex+ce^2x^2)^3} dx$$

| | |
|---|-----|
| Optimal result | 394 |
| Mathematica [A] (verified) | 394 |
| Rubi [A] (verified) | 395 |
| Maple [A] (verified) | 396 |
| Fricas [A] (verification not implemented) | 396 |
| Sympy [B] (verification not implemented) | 397 |
| Maxima [A] (verification not implemented) | 397 |
| Giac [A] (verification not implemented) | 397 |
| Mupad [B] (verification not implemented) | 398 |
| Reduce [B] (verification not implemented) | 398 |

Optimal result

Integrand size = 30, antiderivative size = 17

$$\int \frac{(d+ex)^8}{(cd^2+2cdex+ce^2x^2)^3} dx = \frac{(d+ex)^3}{3c^3e}$$

output `1/3*(e*x+d)^3/c^3/e`

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00

$$\int \frac{(d+ex)^8}{(cd^2+2cdex+ce^2x^2)^3} dx = \frac{(d+ex)^3}{3c^3e}$$

input `Integrate[(d + e*x)^8/(c*d^2 + 2*c*d*e*x + c*e^2*x^2)^3,x]`

output `(d + e*x)^3/(3*c^3*e)`

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = {1098, 17}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d + ex)^8}{(cd^2 + 2cdex + ce^2x^2)^3} dx$$

$$\downarrow 1098$$

$$c^3 e^6 \int \frac{(d + ex)^2}{c^6 e^6} dx$$

$$\downarrow 17$$

$$\frac{(d + ex)^3}{3c^3 e}$$

input `Int[(d + e*x)^8/(c*d^2 + 2*c*d*e*x + c*e^2*x^2)^3,x]`

output `(d + e*x)^3/(3*c^3*e)`

Defintions of rubi rules used

rule 17 `Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] := Simp[c*((a + b*x)^(m + 1))/(b*(m + 1))], x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]`

rule 1098 `Int[((d_.) + (e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] := Simp[1/c^p Int[(d + e*x)^m*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]`

Maple [A] (verified)

Time = 1.10 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.94

| method | result | size |
|---------------|--|------|
| default | $\frac{(ex+d)^3}{3c^3e}$ | 16 |
| gospers | $\frac{x(e^2x^2+3dex+3d^2)}{3c^3}$ | 25 |
| parallelrisch | $\frac{e^2x^3+3dex^2+3d^2x}{3c^3}$ | 27 |
| risch | $\frac{e^2x^3}{3c^3} + \frac{edx^2}{c^3} + \frac{d^2x}{c^3} + \frac{d^3}{3c^3e}$ | 41 |
| orering | $\frac{x(e^2x^2+3dex+3d^2)(ex+d)^6}{3(x^2ce^2+2cdxe+cd^2)^3}$ | 51 |
| norman | $\frac{\frac{e^7x^8}{3c} + \frac{8de^6x^7}{3c} - \frac{55d^8}{3ec} + \frac{28e^5d^2x^6}{3c} - \frac{70d^4e^3x^4}{c} - \frac{168d^5e^2x^3}{c} - \frac{532d^6ex^2}{3c} - \frac{272d^7x}{3c}}{c^2(ex+d)^5}$ | 110 |

input `int((e*x+d)^8/(c*e^2*x^2+2*c*d*e*x+c*d^2)^3,x,method=_RETURNVERBOSE)`

output `1/3*(e*x+d)^3/c^3/e`

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.53

$$\int \frac{(d+ex)^8}{(cd^2+2cdex+ce^2x^2)^3} dx = \frac{e^2x^3+3dex^2+3d^2x}{3c^3}$$

input `integrate((e*x+d)^8/(c*e^2*x^2+2*c*d*e*x+c*d^2)^3,x, algorithm="fricas")`

output `1/3*(e^2*x^3 + 3*d*e*x^2 + 3*d^2*x)/c^3`

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 29 vs. $2(12) = 24$.

Time = 0.04 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.71

$$\int \frac{(d + ex)^8}{(cd^2 + 2cdex + ce^2x^2)^3} dx = \frac{d^2x}{c^3} + \frac{dex^2}{c^3} + \frac{e^2x^3}{3c^3}$$

input `integrate((e*x+d)**8/(c*e**2*x**2+2*c*d*e*x+c*d**2)**3,x)`

output `d**2*x/c**3 + d*e*x**2/c**3 + e**2*x**3/(3*c**3)`

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.53

$$\int \frac{(d + ex)^8}{(cd^2 + 2cdex + ce^2x^2)^3} dx = \frac{e^2x^3 + 3dex^2 + 3d^2x}{3c^3}$$

input `integrate((e*x+d)^8/(c*e^2*x^2+2*c*d*e*x+c*d^2)^3,x, algorithm="maxima")`

output `1/3*(e^2*x^3 + 3*d*e*x^2 + 3*d^2*x)/c^3`

Giac [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.53

$$\int \frac{(d + ex)^8}{(cd^2 + 2cdex + ce^2x^2)^3} dx = \frac{e^2x^3 + 3dex^2 + 3d^2x}{3c^3}$$

input `integrate((e*x+d)^8/(c*e^2*x^2+2*c*d*e*x+c*d^2)^3,x, algorithm="giac")`

output `1/3*(e^2*x^3 + 3*d*e*x^2 + 3*d^2*x)/c^3`

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.41

$$\int \frac{(d + ex)^8}{(cd^2 + 2cde x + ce^2 x^2)^3} dx = \frac{x(3d^2 + 3dex + e^2 x^2)}{3c^3}$$

input `int((d + e*x)^8/(c*d^2 + c*e^2*x^2 + 2*c*d*e*x)^3,x)`output `(x*(3*d^2 + e^2*x^2 + 3*d*e*x))/(3*c^3)`**Reduce [B] (verification not implemented)**

Time = 0.27 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.41

$$\int \frac{(d + ex)^8}{(cd^2 + 2cde x + ce^2 x^2)^3} dx = \frac{x(e^2 x^2 + 3dex + 3d^2)}{3c^3}$$

input `int((e*x+d)^8/(c*e^2*x^2+2*c*d*e*x+c*d^2)^3,x)`output `(x*(3*d**2 + 3*d*e*x + e**2*x**2))/(3*c**3)`

$$3.42 \quad \int \frac{(d+ex)^7}{(cd^2+2cdex+ce^2x^2)^3} dx$$

| | |
|---|-----|
| Optimal result | 399 |
| Mathematica [A] (verified) | 399 |
| Rubi [A] (verified) | 400 |
| Maple [A] (verified) | 401 |
| Fricas [A] (verification not implemented) | 401 |
| Sympy [A] (verification not implemented) | 402 |
| Maxima [A] (verification not implemented) | 402 |
| Giac [A] (verification not implemented) | 402 |
| Mupad [B] (verification not implemented) | 403 |
| Reduce [B] (verification not implemented) | 403 |

Optimal result

Integrand size = 30, antiderivative size = 17

$$\int \frac{(d+ex)^7}{(cd^2+2cdex+ce^2x^2)^3} dx = \frac{(d+ex)^2}{2c^3e}$$

output `1/2*(e*x+d)^2/c^3/e`

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.94

$$\int \frac{(d+ex)^7}{(cd^2+2cdex+ce^2x^2)^3} dx = \frac{dx + \frac{ex^2}{2}}{c^3}$$

input `Integrate[(d + e*x)^7/(c*d^2 + 2*c*d*e*x + c*e^2*x^2)^3,x]`

output `(d*x + (e*x^2)/2)/c^3`

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = {1098, 17}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d + ex)^7}{(cd^2 + 2cdex + ce^2x^2)^3} dx$$

$$\downarrow 1098$$

$$c^3e^6 \int \frac{d + ex}{c^6e^6} dx$$

$$\downarrow 17$$

$$\frac{(d + ex)^2}{2c^3e}$$

input `Int[(d + e*x)^7/(c*d^2 + 2*c*d*e*x + c*e^2*x^2)^3,x]`

output `(d + e*x)^2/(2*c^3*e)`

Defintions of rubi rules used

rule 17 `Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] := Simp[c*((a + b*x)^(m + 1))/(b*(m + 1))], x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]`

rule 1098 `Int[((d_.) + (e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] := Simp[1/c^p Int[(d + e*x)^m*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]`

Maple [A] (verified)

Time = 1.09 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.82

| method | result | size |
|---------------|--|------|
| gospers | $\frac{x(ex+2d)}{2c^3}$ | 14 |
| default | $\frac{\frac{1}{2}ex^2+dx}{c^3}$ | 15 |
| parallelrisch | $\frac{ex^2+2dx}{2c^3}$ | 16 |
| risch | $\frac{ex^2}{2c^3} + \frac{dx}{c^3}$ | 17 |
| orering | $\frac{x(ex+2d)(ex+d)^6}{2(x^2ce^2+2cdex+cd^2)^3}$ | 40 |
| norman | $\frac{-\frac{10d^7}{ec} - \frac{49d^6x}{c} + \frac{e^6x^7}{2c} + \frac{7de^5x^6}{2c} - \frac{35e^3d^3x^4}{c} - \frac{175d^4e^2x^3}{2c} - \frac{189d^5ex^2}{2c}}{c^2(ex+d)^5}$ | 96 |

input `int((e*x+d)^7/(c*e^2*x^2+2*c*d*e*x+c*d^2)^3,x,method=_RETURNVERBOSE)`

output `1/2*x*(e*x+2*d)/c^3`

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.88

$$\int \frac{(d+ex)^7}{(cd^2+2cdex+ce^2x^2)^3} dx = \frac{ex^2+2dx}{2c^3}$$

input `integrate((e*x+d)^7/(c*e^2*x^2+2*c*d*e*x+c*d^2)^3,x, algorithm="fricas")`

output `1/2*(e*x^2 + 2*d*x)/c^3`

Sympy [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.88

$$\int \frac{(d + ex)^7}{(cd^2 + 2cdex + ce^2x^2)^3} dx = \frac{dx}{c^3} + \frac{ex^2}{2c^3}$$

input `integrate((e*x+d)**7/(c*e**2*x**2+2*c*d*e*x+c*d**2)**3,x)`output `d*x/c**3 + e*x**2/(2*c**3)`**Maxima [A] (verification not implemented)**

Time = 0.03 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.88

$$\int \frac{(d + ex)^7}{(cd^2 + 2cdex + ce^2x^2)^3} dx = \frac{ex^2 + 2 dx}{2 c^3}$$

input `integrate((e*x+d)^7/(c*e^2*x^2+2*c*d*e*x+c*d^2)^3,x, algorithm="maxima")`output `1/2*(e*x^2 + 2*d*x)/c^3`**Giac [A] (verification not implemented)**

Time = 0.19 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.88

$$\int \frac{(d + ex)^7}{(cd^2 + 2cdex + ce^2x^2)^3} dx = \frac{ex^2 + 2 dx}{2 c^3}$$

input `integrate((e*x+d)^7/(c*e^2*x^2+2*c*d*e*x+c*d^2)^3,x, algorithm="giac")`output `1/2*(e*x^2 + 2*d*x)/c^3`

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.76

$$\int \frac{(d + ex)^7}{(cd^2 + 2cdex + ce^2x^2)^3} dx = \frac{x(2d + ex)}{2c^3}$$

input `int((d + e*x)^7/(c*d^2 + c*e^2*x^2 + 2*c*d*e*x)^3,x)`output `(x*(2*d + e*x))/(2*c^3)`**Reduce [B] (verification not implemented)**

Time = 0.28 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.76

$$\int \frac{(d + ex)^7}{(cd^2 + 2cdex + ce^2x^2)^3} dx = \frac{x(ex + 2d)}{2c^3}$$

input `int((e*x+d)^7/(c*e^2*x^2+2*c*d*e*x+c*d^2)^3,x)`output `(x*(2*d + e*x))/(2*c**3)`

$$3.43 \quad \int \frac{(d+ex)^6}{(cd^2+2cdex+ce^2x^2)^3} dx$$

| | |
|---|-----|
| Optimal result | 404 |
| Mathematica [A] (verified) | 404 |
| Rubi [A] (verified) | 405 |
| Maple [A] (verified) | 406 |
| Fricas [A] (verification not implemented) | 406 |
| Sympy [A] (verification not implemented) | 406 |
| Maxima [A] (verification not implemented) | 407 |
| Giac [A] (verification not implemented) | 407 |
| Mupad [B] (verification not implemented) | 407 |
| Reduce [B] (verification not implemented) | 408 |

Optimal result

Integrand size = 30, antiderivative size = 5

$$\int \frac{(d+ex)^6}{(cd^2+2cdex+ce^2x^2)^3} dx = \frac{x}{c^3}$$

output

x/c^3

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 5, normalized size of antiderivative = 1.00

$$\int \frac{(d+ex)^6}{(cd^2+2cdex+ce^2x^2)^3} dx = \frac{x}{c^3}$$

input

`Integrate[(d + e*x)^6/(c*d^2 + 2*c*d*e*x + c*e^2*x^2)^3,x]`

output

x/c^3

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 5, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = {1098, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d + ex)^6}{(cd^2 + 2cdex + ce^2x^2)^3} dx$$

↓ 1098

$$c^3 e^6 \int \frac{1}{c^6 e^6} dx$$

↓ 24

$$\frac{x}{c^3}$$

input

```
Int[(d + e*x)^6/(c*d^2 + 2*c*d*e*x + c*e^2*x^2)^3,x]
```

output

```
x/c^3
```

Defintions of rubi rules used

rule 24

```
Int[a_, x_Symbol] := Simp[a*x, x] /; FreeQ[a, x]
```

rule 1098

```
Int[((d_.) + (e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] := Simp[1/c^p Int[(d + e*x)^m*(b/2 + c*x)^(2*p), x], x] /; FreeQ[
{a, b, c, d, e, m}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Maple [A] (verified)

Time = 1.10 (sec) , antiderivative size = 6, normalized size of antiderivative = 1.20

| method | result | size |
|---------|---|------|
| default | $\frac{x}{c^3}$ | 6 |
| risch | $\frac{x}{c^3}$ | 6 |
| orering | $\frac{x(ex+d)^6}{(x^2ce^2+2cdxe+cd^2)^3}$ | 32 |
| norman | $\frac{\frac{e^5x^6}{c} - \frac{5d^6}{ec} - \frac{24d^5x}{c} - \frac{15e^3d^2x^4}{c} - \frac{40d^3e^2x^3}{c} - \frac{45d^4ex^2}{c}}{c^2(ex+d)^5}$ | 83 |

input `int((e*x+d)^6/(c*e^2*x^2+2*c*d*e*x+c*d^2)^3,x,method=_RETURNVERBOSE)`

output `x/c^3`

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 5, normalized size of antiderivative = 1.00

$$\int \frac{(d+ex)^6}{(cd^2+2cdex+ce^2x^2)^3} dx = \frac{x}{c^3}$$

input `integrate((e*x+d)^6/(c*e^2*x^2+2*c*d*e*x+c*d^2)^3,x, algorithm="fricas")`

output `x/c^3`

Sympy [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 3, normalized size of antiderivative = 0.60

$$\int \frac{(d+ex)^6}{(cd^2+2cdex+ce^2x^2)^3} dx = \frac{x}{c^3}$$

input `integrate((e*x+d)**6/(c*e**2*x**2+2*c*d*e*x+c*d**2)**3,x)`

output `x/c**3`

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 5, normalized size of antiderivative = 1.00

$$\int \frac{(d + ex)^6}{(cd^2 + 2cdex + ce^2x^2)^3} dx = \frac{x}{c^3}$$

input `integrate((e*x+d)^6/(c*e^2*x^2+2*c*d*e*x+c*d^2)^3,x, algorithm="maxima")`

output `x/c^3`

Giac [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 5, normalized size of antiderivative = 1.00

$$\int \frac{(d + ex)^6}{(cd^2 + 2cdex + ce^2x^2)^3} dx = \frac{x}{c^3}$$

input `integrate((e*x+d)^6/(c*e^2*x^2+2*c*d*e*x+c*d^2)^3,x, algorithm="giac")`

output `x/c^3`

Mupad [B] (verification not implemented)

Time = 0.01 (sec) , antiderivative size = 5, normalized size of antiderivative = 1.00

$$\int \frac{(d + ex)^6}{(cd^2 + 2cdex + ce^2x^2)^3} dx = \frac{x}{c^3}$$

input `int((d + e*x)^6/(c*d^2 + c*e^2*x^2 + 2*c*d*e*x)^3,x)`

output `x/c^3`

Reduce [B] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 5, normalized size of antiderivative = 1.00

$$\int \frac{(d + ex)^6}{(cd^2 + 2cdex + ce^2x^2)^3} dx = \frac{x}{c^3}$$

input `int((e*x+d)^6/(c*e^2*x^2+2*c*d*e*x+c*d^2)^3,x)`

output `x/c**3`

$$3.44 \quad \int \frac{(d+ex)^5}{(cd^2+2cdex+ce^2x^2)^3} dx$$

| | |
|---|-----|
| Optimal result | 409 |
| Mathematica [A] (verified) | 409 |
| Rubi [A] (verified) | 410 |
| Maple [A] (verified) | 411 |
| Fricas [A] (verification not implemented) | 411 |
| Sympy [A] (verification not implemented) | 411 |
| Maxima [A] (verification not implemented) | 412 |
| Giac [A] (verification not implemented) | 412 |
| Mupad [B] (verification not implemented) | 412 |
| Reduce [B] (verification not implemented) | 413 |

Optimal result

Integrand size = 30, antiderivative size = 13

$$\int \frac{(d+ex)^5}{(cd^2+2cdex+ce^2x^2)^3} dx = \frac{\log(d+ex)}{c^3e}$$

output `ln(e*x+d)/c^3/e`

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00

$$\int \frac{(d+ex)^5}{(cd^2+2cdex+ce^2x^2)^3} dx = \frac{\log(d+ex)}{c^3e}$$

input `Integrate[(d + e*x)^5/(c*d^2 + 2*c*d*e*x + c*e^2*x^2)^3,x]`

output `Log[d + e*x]/(c^3*e)`

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = {1098, 16}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d + ex)^5}{(cd^2 + 2cdex + ce^2x^2)^3} dx$$

$$\downarrow 1098$$

$$c^3e^6 \int \frac{1}{c^6e^6(d + ex)} dx$$

$$\downarrow 16$$

$$\frac{\log(d + ex)}{c^3e}$$

input

```
Int[(d + e*x)^5/(c*d^2 + 2*c*d*e*x + c*e^2*x^2)^3,x]
```

output

```
Log[d + e*x]/(c^3*e)
```

Defintions of rubi rules used

rule 16

```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] := Simp[c*(Log[RemoveContent[a + b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 1098

```
Int[((d_.) + (e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] := Simp[1/c^p Int[(d + e*x)^m*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Maple [A] (verified)

Time = 1.09 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.08

| method | result | size |
|---------------|--------------------------|------|
| default | $\frac{\ln(ex+d)}{c^3e}$ | 14 |
| norman | $\frac{\ln(ex+d)}{c^3e}$ | 14 |
| risch | $\frac{\ln(ex+d)}{c^3e}$ | 14 |
| parallelrisch | $\frac{\ln(ex+d)}{c^3e}$ | 14 |

input `int((e*x+d)^5/(c*e^2*x^2+2*c*d*e*x+c*d^2)^3,x,method=_RETURNVERBOSE)`

output `ln(e*x+d)/c^3/e`

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00

$$\int \frac{(d+ex)^5}{(cd^2+2cdex+ce^2x^2)^3} dx = \frac{\log(ex+d)}{c^3e}$$

input `integrate((e*x+d)^5/(c*e^2*x^2+2*c*d*e*x+c*d^2)^3,x, algorithm="fricas")`

output `log(e*x + d)/(c^3*e)`

Sympy [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.31

$$\int \frac{(d+ex)^5}{(cd^2+2cdex+ce^2x^2)^3} dx = \frac{\log(c^3d+c^3ex)}{c^3e}$$

input `integrate((e*x+d)**5/(c*e**2*x**2+2*c*d*e*x+c*d**2)**3,x)`

output `log(c**3*d + c**3*e*x)/(c**3*e)`

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00

$$\int \frac{(d + ex)^5}{(cd^2 + 2cdex + ce^2x^2)^3} dx = \frac{\log(ex + d)}{c^3e}$$

input `integrate((e*x+d)^5/(c*e^2*x^2+2*c*d*e*x+c*d^2)^3,x, algorithm="maxima")`

output `log(e*x + d)/(c^3*e)`

Giac [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.08

$$\int \frac{(d + ex)^5}{(cd^2 + 2cdex + ce^2x^2)^3} dx = \frac{\log(|ex + d|)}{c^3e}$$

input `integrate((e*x+d)^5/(c*e^2*x^2+2*c*d*e*x+c*d^2)^3,x, algorithm="giac")`

output `log(abs(e*x + d))/(c^3*e)`

Mupad [B] (verification not implemented)

Time = 9.03 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00

$$\int \frac{(d + ex)^5}{(cd^2 + 2cdex + ce^2x^2)^3} dx = \frac{\ln(d + ex)}{c^3e}$$

input `int((d + e*x)^5/(c*d^2 + c*e^2*x^2 + 2*c*d*e*x)^3,x)`

output `log(d + e*x)/(c^3*e)`

Reduce [B] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00

$$\int \frac{(d + ex)^5}{(cd^2 + 2cdex + ce^2x^2)^3} dx = \frac{\log(ex + d)}{c^3e}$$

input `int((e*x+d)^5/(c*e^2*x^2+2*c*d*e*x+c*d^2)^3,x)`

output `log(d + e*x)/(c**3*e)`

$$3.45 \quad \int \frac{(d+ex)^4}{(cd^2+2cdex+ce^2x^2)^3} dx$$

| | |
|---|-----|
| Optimal result | 414 |
| Mathematica [A] (verified) | 414 |
| Rubi [A] (verified) | 415 |
| Maple [A] (verified) | 416 |
| Fricas [A] (verification not implemented) | 416 |
| Sympy [A] (verification not implemented) | 417 |
| Maxima [A] (verification not implemented) | 417 |
| Giac [A] (verification not implemented) | 417 |
| Mupad [B] (verification not implemented) | 418 |
| Reduce [B] (verification not implemented) | 418 |

Optimal result

Integrand size = 30, antiderivative size = 15

$$\int \frac{(d+ex)^4}{(cd^2+2cdex+ce^2x^2)^3} dx = -\frac{1}{c^3e(d+ex)}$$

output `-1/c^3/e/(e*x+d)`

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00

$$\int \frac{(d+ex)^4}{(cd^2+2cdex+ce^2x^2)^3} dx = -\frac{1}{c^3e(d+ex)}$$

input `Integrate[(d + e*x)^4/(c*d^2 + 2*c*d*e*x + c*e^2*x^2)^3,x]`

output `-(1/(c^3*e*(d + e*x)))`

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = {1098, 17}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d + ex)^4}{(cd^2 + 2cdex + ce^2x^2)^3} dx$$

↓ 1098

$$c^3 e^6 \int \frac{1}{c^6 e^6 (d + ex)^2} dx$$

↓ 17

$$-\frac{1}{c^3 e (d + ex)}$$

input `Int[(d + e*x)^4/(c*d^2 + 2*c*d*e*x + c*e^2*x^2)^3,x]`

output `-(1/(c^3*e*(d + e*x)))`

Defintions of rubi rules used

rule 17 `Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] := Simp[c*((a + b*x)^(m + 1))/(b*(m + 1))], x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]`

rule 1098 `Int[((d_.) + (e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] := Simp[1/c^p Int[(d + e*x)^m*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]`

Maple [A] (verified)

Time = 1.07 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.07

| method | result | size |
|--------------|---|------|
| gospers | $-\frac{1}{c^3 e^{(ex+d)}}$ | 16 |
| default | $-\frac{1}{c^3 e^{(ex+d)}}$ | 16 |
| risch | $-\frac{1}{c^3 e^{(ex+d)}}$ | 16 |
| paralelrisch | $\frac{x}{d(ex+d)c^3}$ | 16 |
| orering | $-\frac{(ex+d)^5}{e^{(x^2 c e^2 + 2cdxe + c d^2)^3}}$ | 35 |
| norman | $\frac{-\frac{d^4}{ec} - \frac{e^3 x^4}{c} - \frac{4d^3 x}{c} - \frac{4e^2 d x^3}{c} - \frac{6e d^2 x^2}{c}}{c^2 (ex+d)^5}$ | 68 |

input `int((e*x+d)^4/(c*e^2*x^2+2*c*d*e*x+c*d^2)^3,x,method=_RETURNVERBOSE)`

output `-1/c^3/e/(e*x+d)`

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.27

$$\int \frac{(d+ex)^4}{(cd^2+2cdex+ce^2x^2)^3} dx = -\frac{1}{c^3 e^2 x + c^3 de}$$

input `integrate((e*x+d)^4/(c*e^2*x^2+2*c*d*e*x+c*d^2)^3,x, algorithm="fricas")`

output `-1/(c^3*e^2*x + c^3*d*e)`

Sympy [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.13

$$\int \frac{(d + ex)^4}{(cd^2 + 2cdex + ce^2x^2)^3} dx = -\frac{1}{c^3de + c^3e^2x}$$

input `integrate((e*x+d)**4/(c*e**2*x**2+2*c*d*e*x+c*d**2)**3,x)`output `-1/(c**3*d*e + c**3*e**2*x)`**Maxima [A] (verification not implemented)**

Time = 0.04 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.27

$$\int \frac{(d + ex)^4}{(cd^2 + 2cdex + ce^2x^2)^3} dx = -\frac{1}{c^3e^2x + c^3de}$$

input `integrate((e*x+d)^4/(c*e^2*x^2+2*c*d*e*x+c*d^2)^3,x, algorithm="maxima")`output `-1/(c^3*e^2*x + c^3*d*e)`**Giac [A] (verification not implemented)**

Time = 0.20 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00

$$\int \frac{(d + ex)^4}{(cd^2 + 2cdex + ce^2x^2)^3} dx = -\frac{1}{(ex + d)c^3e}$$

input `integrate((e*x+d)^4/(c*e^2*x^2+2*c*d*e*x+c*d^2)^3,x, algorithm="giac")`output `-1/((e*x + d)*c^3*e)`

Mupad [B] (verification not implemented)

Time = 9.12 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.27

$$\int \frac{(d + ex)^4}{(cd^2 + 2cdex + ce^2x^2)^3} dx = -\frac{1}{xc^3e^2 + dc^3e}$$

input `int((d + e*x)^4/(c*d^2 + c*e^2*x^2 + 2*c*d*e*x)^3,x)`output `-1/(c^3*e^2*x + c^3*d*e)`**Reduce [B] (verification not implemented)**

Time = 0.27 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00

$$\int \frac{(d + ex)^4}{(cd^2 + 2cdex + ce^2x^2)^3} dx = \frac{x}{c^3d(ex + d)}$$

input `int((e*x+d)^4/(c*e^2*x^2+2*c*d*e*x+c*d^2)^3,x)`output `x/(c**3*d*(d + e*x))`

$$3.46 \quad \int \frac{(d+ex)^3}{(cd^2+2cdex+ce^2x^2)^3} dx$$

| | |
|---|-----|
| Optimal result | 419 |
| Mathematica [A] (verified) | 419 |
| Rubi [A] (verified) | 420 |
| Maple [A] (verified) | 421 |
| Fricas [B] (verification not implemented) | 421 |
| Sympy [B] (verification not implemented) | 422 |
| Maxima [B] (verification not implemented) | 422 |
| Giac [A] (verification not implemented) | 422 |
| Mupad [B] (verification not implemented) | 423 |
| Reduce [B] (verification not implemented) | 423 |

Optimal result

Integrand size = 30, antiderivative size = 17

$$\int \frac{(d+ex)^3}{(cd^2+2cdex+ce^2x^2)^3} dx = -\frac{1}{2c^3e(d+ex)^2}$$

output

$$-1/2/c^3/e/(e*x+d)^2$$

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00

$$\int \frac{(d+ex)^3}{(cd^2+2cdex+ce^2x^2)^3} dx = -\frac{1}{2c^3e(d+ex)^2}$$

input

```
Integrate[(d + e*x)^3/(c*d^2 + 2*c*d*e*x + c*e^2*x^2)^3,x]
```

output

$$-1/2*1/(c^3*e*(d + e*x)^2)$$

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = {1098, 17}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d + ex)^3}{(cd^2 + 2cdex + ce^2x^2)^3} dx$$

↓ 1098

$$c^3 e^6 \int \frac{1}{c^6 e^6 (d + ex)^3} dx$$

↓ 17

$$-\frac{1}{2c^3 e (d + ex)^2}$$

input `Int[(d + e*x)^3/(c*d^2 + 2*c*d*e*x + c*e^2*x^2)^3,x]`

output `-1/2*1/(c^3*e*(d + e*x)^2)`

Defintions of rubi rules used

rule 17 `Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] := Simp[c*((a + b*x)^(m + 1))/(b*(m + 1))], x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]`

rule 1098 `Int[((d_.) + (e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] := Simp[1/c^p Int[(d + e*x)^m*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]`

Maple [A] (verified)

Time = 1.09 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.94

| method | result | size |
|---------------|---|------|
| gospers | $-\frac{1}{2c^3e(ex+d)^2}$ | 16 |
| default | $-\frac{1}{2c^3e(ex+d)^2}$ | 16 |
| risch | $-\frac{1}{2c^3e(ex+d)^2}$ | 16 |
| parallelrisch | $-\frac{1}{2c^3e(ex+d)^2}$ | 16 |
| orering | $-\frac{(ex+d)^4}{2e(x^2ce^2+2cdxe+cd^2)^3}$ | 35 |
| norman | $\frac{-\frac{d^3}{2ec} - \frac{e^2x^3}{2c} - \frac{3edx^2}{2c} - \frac{3d^2x}{2c}}{c^2(ex+d)^5}$ | 54 |

input `int((e*x+d)^3/(c*e^2*x^2+2*c*d*e*x+c*d^2)^3,x,method=_RETURNVERBOSE)`

output `-1/2/c^3/e/(e*x+d)^2`

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 33 vs. $2(15) = 30$.

Time = 0.07 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.94

$$\int \frac{(d+ex)^3}{(cd^2+2cdex+ce^2x^2)^3} dx = -\frac{1}{2(c^3e^3x^2+2c^3de^2x+c^3d^2e)}$$

input `integrate((e*x+d)^3/(c*e^2*x^2+2*c*d*e*x+c*d^2)^3,x, algorithm="fricas")`

output `-1/2/(c^3*e^3*x^2 + 2*c^3*d*e^2*x + c^3*d^2*e)`

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 36 vs. $2(15) = 30$.

Time = 0.10 (sec) , antiderivative size = 36, normalized size of antiderivative = 2.12

$$\int \frac{(d+ex)^3}{(cd^2+2cdex+ce^2x^2)^3} dx = -\frac{1}{2c^3d^2e+4c^3de^2x+2c^3e^3x^2}$$

input `integrate((e*x+d)**3/(c*e**2*x**2+2*c*d*e*x+c*d**2)**3,x)`

output `-1/(2*c**3*d**2*e + 4*c**3*d*e**2*x + 2*c**3*e**3*x**2)`

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 33 vs. $2(15) = 30$.

Time = 0.04 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.94

$$\int \frac{(d+ex)^3}{(cd^2+2cdex+ce^2x^2)^3} dx = -\frac{1}{2(c^3e^3x^2+2c^3de^2x+c^3d^2e)}$$

input `integrate((e*x+d)^3/(c*e^2*x^2+2*c*d*e*x+c*d^2)^3,x, algorithm="maxima")`

output `-1/2/(c^3*e^3*x^2 + 2*c^3*d*e^2*x + c^3*d^2*e)`

Giac [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.53

$$\int \frac{(d+ex)^3}{(cd^2+2cdex+ce^2x^2)^3} dx = -\frac{1}{2(d^2+(ex^2+2dx)e)c^3e}$$

input `integrate((e*x+d)^3/(c*e^2*x^2+2*c*d*e*x+c*d^2)^3,x, algorithm="giac")`

output `-1/2/((d^2 + (e*x^2 + 2*d*x)*e)*c^3*e)`

Mupad [B] (verification not implemented)

Time = 9.08 (sec) , antiderivative size = 35, normalized size of antiderivative = 2.06

$$\int \frac{(d + ex)^3}{(cd^2 + 2cdex + ce^2x^2)^3} dx = -\frac{1}{2c^3d^2e + 4c^3de^2x + 2c^3e^3x^2}$$

input `int((d + e*x)^3/(c*d^2 + c*e^2*x^2 + 2*c*d*e*x)^3,x)`output `-1/(2*c^3*d^2*e + 2*c^3*e^3*x^2 + 4*c^3*d*e^2*x)`**Reduce [B] (verification not implemented)**

Time = 0.23 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.53

$$\int \frac{(d + ex)^3}{(cd^2 + 2cdex + ce^2x^2)^3} dx = -\frac{1}{2c^3e(e^2x^2 + 2dex + d^2)}$$

input `int((e*x+d)^3/(c*e^2*x^2+2*c*d*e*x+c*d^2)^3,x)`output `(- 1)/(2*c**3*e*(d**2 + 2*d*e*x + e**2*x**2))`

$$3.47 \quad \int \frac{(d+ex)^2}{(cd^2+2cdex+ce^2x^2)^3} dx$$

| | |
|---|-----|
| Optimal result | 424 |
| Mathematica [A] (verified) | 424 |
| Rubi [A] (verified) | 425 |
| Maple [A] (verified) | 426 |
| Fricas [B] (verification not implemented) | 426 |
| Sympy [B] (verification not implemented) | 427 |
| Maxima [B] (verification not implemented) | 427 |
| Giac [A] (verification not implemented) | 427 |
| Mupad [B] (verification not implemented) | 428 |
| Reduce [B] (verification not implemented) | 428 |

Optimal result

Integrand size = 30, antiderivative size = 17

$$\int \frac{(d+ex)^2}{(cd^2+2cdex+ce^2x^2)^3} dx = -\frac{1}{3c^3e(d+ex)^3}$$

output

$$-1/3/c^3/e/(e*x+d)^3$$

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00

$$\int \frac{(d+ex)^2}{(cd^2+2cdex+ce^2x^2)^3} dx = -\frac{1}{3c^3e(d+ex)^3}$$

input

$$\text{Integrate}[(d + e*x)^2/(c*d^2 + 2*c*d*e*x + c*e^2*x^2)^3,x]$$

output

$$-1/3*1/(c^3*e*(d + e*x)^3)$$

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = {1098, 17}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d + ex)^2}{(cd^2 + 2cdex + ce^2x^2)^3} dx$$

$$\downarrow 1098$$

$$c^3 e^6 \int \frac{1}{c^6 e^6 (d + ex)^4} dx$$

$$\downarrow 17$$

$$-\frac{1}{3c^3 e (d + ex)^3}$$

input `Int[(d + e*x)^2/(c*d^2 + 2*c*d*e*x + c*e^2*x^2)^3,x]`

output `-1/3*1/(c^3*e*(d + e*x)^3)`

Defintions of rubi rules used

rule 17 `Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] := Simp[c*((a + b*x)^(m + 1))/(b*(m + 1))], x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]`

rule 1098 `Int[((d_.) + (e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] := Simp[1/c^p Int[(d + e*x)^m*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]`

Maple [A] (verified)

Time = 1.10 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.94

| method | result | size |
|-------------|---|------|
| gospers | $-\frac{1}{3c^3e(ex+d)^3}$ | 16 |
| default | $-\frac{1}{3c^3e(ex+d)^3}$ | 16 |
| risch | $-\frac{1}{3e(ex+d)(e^2x^2+2dex+d^2)c^3}$ | 34 |
| parallelsch | $-\frac{1}{3e(ex+d)(e^2x^2+2dex+d^2)c^3}$ | 34 |
| orering | $-\frac{(ex+d)^3}{3e(x^2ce^2+2cdxe+cd^2)^3}$ | 35 |
| norman | $\frac{-\frac{d^2}{3ec} - \frac{ex^2}{3c} - \frac{2xd}{3c}}{c^2(ex+d)^5}$ | 40 |

input `int((e*x+d)^2/(c*e^2*x^2+2*c*d*e*x+c*d^2)^3,x,method=_RETURNVERBOSE)`

output `-1/3/c^3/e/(e*x+d)^3`

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 47 vs. 2(15) = 30.

Time = 0.08 (sec) , antiderivative size = 47, normalized size of antiderivative = 2.76

$$\int \frac{(d+ex)^2}{(cd^2+2cdex+ce^2x^2)^3} dx = -\frac{1}{3(c^3e^4x^3+3c^3de^3x^2+3c^3d^2e^2x+c^3d^3e)}$$

input `integrate((e*x+d)^2/(c*e^2*x^2+2*c*d*e*x+c*d^2)^3,x, algorithm="fricas")`

output `-1/3/(c^3*e^4*x^3 + 3*c^3*d*e^3*x^2 + 3*c^3*d^2*e^2*x + c^3*d^3*e)`

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 51 vs. $2(15) = 30$.

Time = 0.12 (sec) , antiderivative size = 51, normalized size of antiderivative = 3.00

$$\int \frac{(d+ex)^2}{(cd^2+2cdex+ce^2x^2)^3} dx = -\frac{1}{3c^3d^3e+9c^3d^2e^2x+9c^3de^3x^2+3c^3e^4x^3}$$

input `integrate((e*x+d)**2/(c*e**2*x**2+2*c*d*e*x+c*d**2)**3,x)`

output `-1/(3*c**3*d**3*e + 9*c**3*d**2*e**2*x + 9*c**3*d*e**3*x**2 + 3*c**3*e**4*x**3)`

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 47 vs. $2(15) = 30$.

Time = 0.03 (sec) , antiderivative size = 47, normalized size of antiderivative = 2.76

$$\int \frac{(d+ex)^2}{(cd^2+2cdex+ce^2x^2)^3} dx = -\frac{1}{3(c^3e^4x^3+3c^3de^3x^2+3c^3d^2e^2x+c^3d^3e)}$$

input `integrate((e*x+d)^2/(c*e^2*x^2+2*c*d*e*x+c*d^2)^3,x, algorithm="maxima")`

output `-1/3/(c^3*e^4*x^3 + 3*c^3*d*e^3*x^2 + 3*c^3*d^2*e^2*x + c^3*d^3*e)`

Giac [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.88

$$\int \frac{(d+ex)^2}{(cd^2+2cdex+ce^2x^2)^3} dx = -\frac{1}{3(ex+d)^3c^3e}$$

input `integrate((e*x+d)^2/(c*e^2*x^2+2*c*d*e*x+c*d^2)^3,x, algorithm="giac")`

output $-1/3/((e*x + d)^3*c^3*e)$

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 49, normalized size of antiderivative = 2.88

$$\int \frac{(d + ex)^2}{(cd^2 + 2cdex + ce^2x^2)^3} dx = -\frac{1}{3c^3d^3e + 9c^3d^2e^2x + 9c^3de^3x^2 + 3c^3e^4x^3}$$

input $\text{int}((d + e*x)^2/(c*d^2 + c*e^2*x^2 + 2*c*d*e*x)^3,x)$

output $-1/(3*c^3*d^3*e + 3*c^3*e^4*x^3 + 9*c^3*d^2*e^2*x + 9*c^3*d*e^3*x^2)$

Reduce [B] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 37, normalized size of antiderivative = 2.18

$$\int \frac{(d + ex)^2}{(cd^2 + 2cdex + ce^2x^2)^3} dx = -\frac{1}{3c^3e(e^3x^3 + 3de^2x^2 + 3d^2ex + d^3)}$$

input $\text{int}((e*x+d)^2/(c*e^2*x^2+2*c*d*e*x+c*d^2)^3,x)$

output $(-1)/(3*c**3*e*(d**3 + 3*d**2*e*x + 3*d*e**2*x**2 + e**3*x**3))$

$$3.48 \quad \int \frac{d+ex}{(cd^2+2cdex+ce^2x^2)^3} dx$$

| | |
|---|-----|
| Optimal result | 429 |
| Mathematica [A] (verified) | 429 |
| Rubi [A] (verified) | 430 |
| Maple [A] (verified) | 431 |
| Fricas [B] (verification not implemented) | 431 |
| Sympy [B] (verification not implemented) | 432 |
| Maxima [A] (verification not implemented) | 432 |
| Giac [A] (verification not implemented) | 432 |
| Mupad [B] (verification not implemented) | 433 |
| Reduce [B] (verification not implemented) | 433 |

Optimal result

Integrand size = 28, antiderivative size = 17

$$\int \frac{d+ex}{(cd^2+2cdex+ce^2x^2)^3} dx = -\frac{1}{4c^3e(d+ex)^4}$$

output

```
-1/4/c^3/e/(e*x+d)^4
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00

$$\int \frac{d+ex}{(cd^2+2cdex+ce^2x^2)^3} dx = -\frac{1}{4c^3e(d+ex)^4}$$

input

```
Integrate[(d + e*x)/(c*d^2 + 2*c*d*e*x + c*e^2*x^2)^3,x]
```

output

```
-1/4*1/(c^3*e*(d + e*x)^4)
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.071$, Rules used = {1098, 17}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{d + ex}{(cd^2 + 2cdex + ce^2x^2)^3} dx$$

$$\downarrow 1098$$

$$c^3 e^6 \int \frac{1}{c^6 e^6 (d + ex)^5} dx$$

$$\downarrow 17$$

$$-\frac{1}{4c^3 e (d + ex)^4}$$

input

```
Int[(d + e*x)/(c*d^2 + 2*c*d*e*x + c*e^2*x^2)^3,x]
```

output

```
-1/4*1/(c^3*e*(d + e*x)^4)
```

Defintions of rubi rules used

rule 17

```
Int[(c_.)*((a_.) + (b_.)*(x_)^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1))/(b*(m + 1)), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]
```

rule 1098

```
Int[((d_.) + (e_.)*(x_)^(m_.))*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[1/c^p Int[(d + e*x)^m*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Maple [A] (verified)

Time = 1.02 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.94

| method | result | size |
|---------------|--|------|
| default | $-\frac{1}{4c^3e(ex+d)^4}$ | 16 |
| norman | $\frac{\frac{d}{4ec} - \frac{x}{4c}}{c^2(ex+d)^5}$ | 28 |
| gospers | $-\frac{1}{4(ex+d)^2(e^2x^2+2dex+d^2)c^3e}$ | 34 |
| risch | $-\frac{1}{4(ex+d)^2(e^2x^2+2dex+d^2)c^3e}$ | 34 |
| parallelrisch | $-\frac{1}{4(ex+d)^2(e^2x^2+2dex+d^2)c^3e}$ | 34 |
| orering | $-\frac{(ex+d)^2}{4e(x^2c^2e^2+2cdxe+c^2d^2)^3}$ | 35 |

input `int((e*x+d)/(c*e^2*x^2+2*c*d*e*x+c*d^2)^3,x,method=_RETURNVERBOSE)`

output `-1/4/c^3/e/(e*x+d)^4`

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 61 vs. 2(15) = 30.

Time = 0.07 (sec) , antiderivative size = 61, normalized size of antiderivative = 3.59

$$\int \frac{d+ex}{(cd^2+2cdex+ce^2x^2)^3} dx = -\frac{1}{4(c^3e^5x^4+4c^3de^4x^3+6c^3d^2e^3x^2+4c^3d^3e^2x+c^3d^4e)}$$

input `integrate((e*x+d)/(c*e^2*x^2+2*c*d*e*x+c*d^2)^3,x,algorithm="fricas")`

output `-1/4/(c^3*e^5*x^4+4*c^3*d*e^4*x^3+6*c^3*d^2*e^3*x^2+4*c^3*d^3*e^2*x+c^3*d^4*e)`

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 66 vs. $2(15) = 30$.

Time = 0.16 (sec) , antiderivative size = 66, normalized size of antiderivative = 3.88

$$\int \frac{d + ex}{(cd^2 + 2cdex + ce^2x^2)^3} dx$$

$$= -\frac{1}{4c^3d^4e + 16c^3d^3e^2x + 24c^3d^2e^3x^2 + 16c^3de^4x^3 + 4c^3e^5x^4}$$

input `integrate((e*x+d)/(c*e**2*x**2+2*c*d*e*x+c*d**2)**3,x)`

output `-1/(4*c**3*d**4*e + 16*c**3*d**3*e**2*x + 24*c**3*d**2*e**3*x**2 + 16*c**3*d*e**4*x**3 + 4*c**3*e**5*x**4)`

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.76

$$\int \frac{d + ex}{(cd^2 + 2cdex + ce^2x^2)^3} dx = -\frac{1}{4(ce^2x^2 + 2cdex + cd^2)^2ce}$$

input `integrate((e*x+d)/(c*e^2*x^2+2*c*d*e*x+c*d^2)^3,x, algorithm="maxima")`

output `-1/4/((c*e^2*x^2 + 2*c*d*e*x + c*d^2)^2*c*e)`

Giac [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.71

$$\int \frac{d + ex}{(cd^2 + 2cdex + ce^2x^2)^3} dx = -\frac{1}{4(cd^2 + (ex^2 + 2dx)ce)^2ce}$$

input `integrate((e*x+d)/(c*e^2*x^2+2*c*d*e*x+c*d^2)^3,x, algorithm="giac")`

output $-1/4/((c*d^2 + (e*x^2 + 2*d*x)*c*e)^2*c*e)$

Mupad [B] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 63, normalized size of antiderivative = 3.71

$$\int \frac{d + ex}{(cd^2 + 2cdex + ce^2x^2)^3} dx$$

$$= -\frac{1}{4c^3d^4e + 16c^3d^3e^2x + 24c^3d^2e^3x^2 + 16c^3de^4x^3 + 4c^3e^5x^4}$$

input `int((d + e*x)/(c*d^2 + c*e^2*x^2 + 2*c*d*e*x)^3,x)`

output $-1/(4*c^3*d^4*e + 4*c^3*e^5*x^4 + 16*c^3*d^3*e^2*x + 16*c^3*d*e^4*x^3 + 24*c^3*d^2*e^3*x^2)$

Reduce [B] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 48, normalized size of antiderivative = 2.82

$$\int \frac{d + ex}{(cd^2 + 2cdex + ce^2x^2)^3} dx = -\frac{1}{4c^3e(e^4x^4 + 4de^3x^3 + 6d^2e^2x^2 + 4d^3ex + d^4)}$$

input `int((e*x+d)/(c*e^2*x^2+2*c*d*e*x+c*d^2)^3,x)`

output $(-1)/(4*c**3*e*(d**4 + 4*d**3*e*x + 6*d**2*e**2*x**2 + 4*d*e**3*x**3 + e**4*x**4))$

$$3.49 \quad \int \frac{1}{(cd^2 + 2cdex + ce^2x^2)^3} dx$$

| | |
|---|-----|
| Optimal result | 434 |
| Mathematica [A] (verified) | 434 |
| Rubi [A] (verified) | 435 |
| Maple [A] (verified) | 436 |
| Fricas [B] (verification not implemented) | 436 |
| Sympy [B] (verification not implemented) | 437 |
| Maxima [B] (verification not implemented) | 437 |
| Giac [A] (verification not implemented) | 438 |
| Mupad [B] (verification not implemented) | 438 |
| Reduce [B] (verification not implemented) | 438 |

Optimal result

Integrand size = 22, antiderivative size = 17

$$\int \frac{1}{(cd^2 + 2cdex + ce^2x^2)^3} dx = -\frac{1}{5c^3e(d + ex)^5}$$

output

```
-1/5/c^3/e/(e*x+d)^5
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00

$$\int \frac{1}{(cd^2 + 2cdex + ce^2x^2)^3} dx = -\frac{1}{5c^3e(d + ex)^5}$$

input

```
Integrate[(c*d^2 + 2*c*d*e*x + c*e^2*x^2)^(-3),x]
```

output

```
-1/5*1/(c^3*e*(d + e*x)^5)
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {1077, 17}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(cd^2 + 2cdex + ce^2x^2)^3} dx$$

$$\downarrow 1077$$

$$c^3e^6 \int \frac{1}{(cxe^2 + cde)^6} dx$$

$$\downarrow 17$$

$$-\frac{1}{5c^3e(d+ex)^5}$$

input `Int[(c*d^2 + 2*c*d*e*x + c*e^2*x^2)^(-3),x]`

output `-1/5*1/(c^3*e*(d + e*x)^5)`

Defintions of rubi rules used

rule 17 `Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1))/(b*(m + 1))], x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]`

rule 1077 `Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[1/c^p Int[(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]`

Maple [A] (verified)

Time = 0.99 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.94

| method | result | size |
|---------------|---|------|
| default | $-\frac{1}{5c^3e(ex+d)^5}$ | 16 |
| norman | $-\frac{1}{5c^3e(ex+d)^5}$ | 16 |
| risch | $-\frac{1}{5c^3e(ex+d)^5}$ | 16 |
| parallelrisch | $-\frac{1}{5c^3e(ex+d)^5}$ | 16 |
| orering | $-\frac{ex+d}{5e(x^2ce^2+2cdxe+cd^2)^3}$ | 33 |
| gospers | $-\frac{1}{5(ex+d)(e^2x^2+2dex+d^2)^2c^3e}$ | 34 |

input `int(1/(c*e^2*x^2+2*c*d*e*x+c*d^2)^3,x,method=_RETURNVERBOSE)`

output `-1/5/c^3/e/(e*x+d)^5`

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 75 vs. 2(15) = 30.

Time = 0.08 (sec) , antiderivative size = 75, normalized size of antiderivative = 4.41

$$\int \frac{1}{(cd^2 + 2cdex + ce^2x^2)^3} dx$$

$$= -\frac{1}{5(c^3e^6x^5 + 5c^3de^5x^4 + 10c^3d^2e^4x^3 + 10c^3d^3e^3x^2 + 5c^3d^4e^2x + c^3d^5e)}$$

input `integrate(1/(c*e^2*x^2+2*c*d*e*x+c*d^2)^3,x, algorithm="fricas")`

output `-1/5/(c^3*e^6*x^5 + 5*c^3*d*e^5*x^4 + 10*c^3*d^2*e^4*x^3 + 10*c^3*d^3*e^3*x^2 + 5*c^3*d^4*e^2*x + c^3*d^5*e)`

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 82 vs. $2(15) = 30$.

Time = 0.17 (sec) , antiderivative size = 82, normalized size of antiderivative = 4.82

$$\int \frac{1}{(cd^2 + 2cdex + ce^2x^2)^3} dx$$

$$= -\frac{1}{5c^3d^5e + 25c^3d^4e^2x + 50c^3d^3e^3x^2 + 50c^3d^2e^4x^3 + 25c^3de^5x^4 + 5c^3e^6x^5}$$

input `integrate(1/(c*e**2*x**2+2*c*d*e*x+c*d**2)**3,x)`

output `-1/(5*c**3*d**5*e + 25*c**3*d**4*e**2*x + 50*c**3*d**3*e**3*x**2 + 50*c**3*d**2*e**4*x**3 + 25*c**3*d*e**5*x**4 + 5*c**3*e**6*x**5)`

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 75 vs. $2(15) = 30$.

Time = 0.04 (sec) , antiderivative size = 75, normalized size of antiderivative = 4.41

$$\int \frac{1}{(cd^2 + 2cdex + ce^2x^2)^3} dx$$

$$= -\frac{1}{5(c^3e^6x^5 + 5c^3de^5x^4 + 10c^3d^2e^4x^3 + 10c^3d^3e^3x^2 + 5c^3d^4e^2x + c^3d^5e)}$$

input `integrate(1/(c*e^2*x^2+2*c*d*e*x+c*d^2)^3,x, algorithm="maxima")`

output `-1/5/(c^3*e^6*x^5 + 5*c^3*d*e^5*x^4 + 10*c^3*d^2*e^4*x^3 + 10*c^3*d^3*e^3*x^2 + 5*c^3*d^4*e^2*x + c^3*d^5*e)`

Giac [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.88

$$\int \frac{1}{(cd^2 + 2cdex + ce^2x^2)^3} dx = -\frac{1}{5(ex + d)^5 c^3 e}$$

input `integrate(1/(c*e^2*x^2+2*c*d*e*x+c*d^2)^3,x, algorithm="giac")`output `-1/5/((e*x + d)^5*c^3*e)`**Mupad [B] (verification not implemented)**

Time = 9.16 (sec) , antiderivative size = 77, normalized size of antiderivative = 4.53

$$\int \frac{1}{(cd^2 + 2cdex + ce^2x^2)^3} dx$$

$$= -\frac{1}{5c^3d^5e + 25c^3d^4e^2x + 50c^3d^3e^3x^2 + 50c^3d^2e^4x^3 + 25c^3de^5x^4 + 5c^3e^6x^5}$$

input `int(1/(c*d^2 + c*e^2*x^2 + 2*c*d*e*x)^3,x)`output `-1/(5*c^3*d^5*e + 5*c^3*e^6*x^5 + 25*c^3*d^4*e^2*x + 25*c^3*d*e^5*x^4 + 50*c^3*d^3*e^3*x^2 + 50*c^3*d^2*e^4*x^3)`**Reduce [B] (verification not implemented)**

Time = 0.21 (sec) , antiderivative size = 59, normalized size of antiderivative = 3.47

$$\int \frac{1}{(cd^2 + 2cdex + ce^2x^2)^3} dx$$

$$= -\frac{1}{5c^3e(e^5x^5 + 5de^4x^4 + 10d^2e^3x^3 + 10d^3e^2x^2 + 5d^4ex + d^5)}$$

input `int(1/(c*e^2*x^2+2*c*d*e*x+c*d^2)^3,x)`

output $(-1)/(5c^3e(d^5 + 5d^4ex + 10d^3e^2x^2 + 10d^2e^3x^3 + 5de^4x^4 + e^5x^5))$

$$3.50 \quad \int \frac{1}{(d+ex)(cd^2+2cdex+ce^2x^2)^3} dx$$

| | |
|---|-----|
| Optimal result | 440 |
| Mathematica [A] (verified) | 440 |
| Rubi [A] (verified) | 441 |
| Maple [A] (verified) | 442 |
| Fricas [B] (verification not implemented) | 442 |
| Sympy [B] (verification not implemented) | 443 |
| Maxima [B] (verification not implemented) | 443 |
| Giac [A] (verification not implemented) | 444 |
| Mupad [B] (verification not implemented) | 444 |
| Reduce [B] (verification not implemented) | 444 |

Optimal result

Integrand size = 30, antiderivative size = 17

$$\int \frac{1}{(d+ex)(cd^2+2cdex+ce^2x^2)^3} dx = -\frac{1}{6c^3e(d+ex)^6}$$

output

```
-1/6/c^3/e/(e*x+d)^6
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00

$$\int \frac{1}{(d+ex)(cd^2+2cdex+ce^2x^2)^3} dx = -\frac{1}{6c^3e(d+ex)^6}$$

input

```
Integrate[1/((d + e*x)*(c*d^2 + 2*c*d*e*x + c*e^2*x^2)^3), x]
```

output

```
-1/6*1/(c^3*e*(d + e*x)^6)
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = {1098, 17}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(d + ex)(cd^2 + 2cdex + ce^2x^2)^3} dx$$

↓ 1098

$$c^3 e^6 \int \frac{1}{c^6 e^6 (d + ex)^7} dx$$

↓ 17

$$-\frac{1}{6c^3 e (d + ex)^6}$$

input `Int[1/((d + e*x)*(c*d^2 + 2*c*d*e*x + c*e^2*x^2)^3),x]`

output `-1/6*1/(c^3*e*(d + e*x)^6)`

Defintions of rubi rules used

rule 17 `Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] := Simp[c*((a + b*x)^(m + 1))/(b*(m + 1)), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]`

rule 1098 `Int[((d_.) + (e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] := Simp[1/c^p Int[(d + e*x)^m*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]`

Maple [A] (verified)

Time = 1.24 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.94

| method | result | size |
|---------------|---|------|
| default | $-\frac{1}{6c^3e(ex+d)^6}$ | 16 |
| norman | $-\frac{1}{6c^3e(ex+d)^6}$ | 16 |
| gosper | $-\frac{1}{6ec^3(e^2x^2+2dex+d^2)^3}$ | 27 |
| orering | $-\frac{1}{6e(x^2ce^2+2cdxe+cd^2)^3}$ | 28 |
| risch | $-\frac{1}{6e(ex+d)^2c^3(e^2x^2+2dex+d^2)^2}$ | 34 |
| parallelrisch | $-\frac{1}{6e(ex+d)^2c^3(e^2x^2+2dex+d^2)^2}$ | 34 |

input `int(1/(e*x+d)/(c*e^2*x^2+2*c*d*e*x+c*d^2)^3,x,method=_RETURNVERBOSE)`

output `-1/6/c^3/e/(e*x+d)^6`

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 89 vs. $2(15) = 30$.

Time = 0.08 (sec) , antiderivative size = 89, normalized size of antiderivative = 5.24

$$\int \frac{1}{(d+ex)(cd^2+2cdex+ce^2x^2)^3} dx =$$

$$-\frac{1}{6(c^3e^7x^6 + 6c^3de^6x^5 + 15c^3d^2e^5x^4 + 20c^3d^3e^4x^3 + 15c^3d^4e^3x^2 + 6c^3d^5e^2x + c^3d^6e)}$$

input `integrate(1/(e*x+d)/(c*e^2*x^2+2*c*d*e*x+c*d^2)^3,x, algorithm="fricas")`

output `-1/6/(c^3*e^7*x^6 + 6*c^3*d*e^6*x^5 + 15*c^3*d^2*e^5*x^4 + 20*c^3*d^3*e^4*x^3 + 15*c^3*d^4*e^3*x^2 + 6*c^3*d^5*e^2*x + c^3*d^6*e)`

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 97 vs. $2(15) = 30$.

Time = 0.22 (sec) , antiderivative size = 97, normalized size of antiderivative = 5.71

$$\int \frac{1}{(d+ex)(cd^2+2cdex+ce^2x^2)^3} dx = \frac{1}{6c^3d^6e + 36c^3d^5e^2x + 90c^3d^4e^3x^2 + 120c^3d^3e^4x^3 + 90c^3d^2e^5x^4 + 36c^3de^6x^5 + 6c^3e^7x^6}$$

input `integrate(1/(e*x+d)/(c*e**2*x**2+2*c*d*e*x+c*d**2)**3,x)`

output `-1/(6*c**3*d**6*e + 36*c**3*d**5*e**2*x + 90*c**3*d**4*e**3*x**2 + 120*c**3*d**3*e**4*x**3 + 90*c**3*d**2*e**5*x**4 + 36*c**3*d*e**6*x**5 + 6*c**3*e**7*x**6)`

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 89 vs. $2(15) = 30$.

Time = 0.04 (sec) , antiderivative size = 89, normalized size of antiderivative = 5.24

$$\int \frac{1}{(d+ex)(cd^2+2cdex+ce^2x^2)^3} dx = \frac{1}{6(c^3e^7x^6 + 6c^3de^6x^5 + 15c^3d^2e^5x^4 + 20c^3d^3e^4x^3 + 15c^3d^4e^3x^2 + 6c^3d^5e^2x + c^3d^6e)}$$

input `integrate(1/(e*x+d)/(c*e^2*x^2+2*c*d*e*x+c*d^2)^3,x, algorithm="maxima")`

output `-1/6/(c^3*e^7*x^6 + 6*c^3*d*e^6*x^5 + 15*c^3*d^2*e^5*x^4 + 20*c^3*d^3*e^4*x^3 + 15*c^3*d^4*e^3*x^2 + 6*c^3*d^5*e^2*x + c^3*d^6*e)`

Giac [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.88

$$\int \frac{1}{(d+ex)(cd^2+2cdex+ce^2x^2)^3} dx = -\frac{1}{6(ex+d)^6c^3e}$$

input `integrate(1/(e*x+d)/(c*e^2*x^2+2*c*d*e*x+c*d^2)^3,x, algorithm="giac")`output `-1/6/((e*x + d)^6*c^3*e)`**Mupad [B] (verification not implemented)**

Time = 9.31 (sec) , antiderivative size = 91, normalized size of antiderivative = 5.35

$$\int \frac{1}{(d+ex)(cd^2+2cdex+ce^2x^2)^3} dx =$$

$$-\frac{1}{6c^3d^6e + 36c^3d^5e^2x + 90c^3d^4e^3x^2 + 120c^3d^3e^4x^3 + 90c^3d^2e^5x^4 + 36c^3de^6x^5 + 6c^3e^7x^6}$$

input `int(1/((d + e*x)*(c*d^2 + c*e^2*x^2 + 2*c*d*e*x)^3),x)`output `-1/(6*c^3*d^6*e + 6*c^3*e^7*x^6 + 36*c^3*d^5*e^2*x + 36*c^3*d*e^6*x^5 + 90*c^3*d^4*e^3*x^2 + 120*c^3*d^3*e^4*x^3 + 90*c^3*d^2*e^5*x^4)`**Reduce [B] (verification not implemented)**

Time = 0.21 (sec) , antiderivative size = 70, normalized size of antiderivative = 4.12

$$\int \frac{1}{(d+ex)(cd^2+2cdex+ce^2x^2)^3} dx$$

$$= -\frac{1}{6c^3e(e^6x^6 + 6de^5x^5 + 15d^2e^4x^4 + 20d^3e^3x^3 + 15d^4e^2x^2 + 6d^5ex + d^6)}$$

input `int(1/(e*x+d)/(c*e^2*x^2+2*c*d*e*x+c*d^2)^3,x)`

output $(-1)/(6c^3e(d^6 + 6d^5ex + 15d^4e^2x^2 + 20d^3e^3x^3 + 15d^2e^4x^4 + 6de^5x^5 + e^6x^6))$

$$3.51 \quad \int \frac{1}{(d+ex)^2 (cd^2+2cdex+ce^2x^2)^3} dx$$

| | |
|---|-----|
| Optimal result | 446 |
| Mathematica [A] (verified) | 446 |
| Rubi [A] (verified) | 447 |
| Maple [A] (verified) | 448 |
| Fricas [B] (verification not implemented) | 448 |
| Sympy [B] (verification not implemented) | 449 |
| Maxima [B] (verification not implemented) | 449 |
| Giac [A] (verification not implemented) | 450 |
| Mupad [B] (verification not implemented) | 450 |
| Reduce [B] (verification not implemented) | 450 |

Optimal result

Integrand size = 30, antiderivative size = 17

$$\int \frac{1}{(d+ex)^2 (cd^2+2cdex+ce^2x^2)^3} dx = -\frac{1}{7c^3e(d+ex)^7}$$

output

```
-1/7/c^3/e/(e*x+d)^7
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00

$$\int \frac{1}{(d+ex)^2 (cd^2+2cdex+ce^2x^2)^3} dx = -\frac{1}{7c^3e(d+ex)^7}$$

input

```
Integrate[1/((d + e*x)^2*(c*d^2 + 2*c*d*e*x + c*e^2*x^2)^3),x]
```

output

```
-1/7*1/(c^3*e*(d + e*x)^7)
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = {1098, 17}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(d+ex)^2 (cd^2+2cdex+ce^2x^2)^3} dx$$

↓ 1098

$$c^3 e^6 \int \frac{1}{c^6 e^6 (d+ex)^8} dx$$

↓ 17

$$-\frac{1}{7c^3 e (d+ex)^7}$$

input `Int[1/((d + e*x)^2*(c*d^2 + 2*c*d*e*x + c*e^2*x^2)^3),x]`

output `-1/7*1/(c^3*e*(d + e*x)^7)`

Defintions of rubi rules used

rule 17 `Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] := Simp[c*((a + b*x)^(m + 1))/(b*(m + 1)), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]`

rule 1098 `Int[((d_.) + (e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] := Simp[1/c^p Int[(d + e*x)^m*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]`

Maple [A] (verified)

Time = 1.26 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.94

| method | result | size |
|---------------|---|------|
| default | $-\frac{1}{7c^3e(ex+d)^7}$ | 16 |
| norman | $-\frac{1}{7c^3e(ex+d)^7}$ | 16 |
| gosper | $-\frac{1}{7(ex+d)e c^3(e^2x^2+2dex+d^2)^3}$ | 34 |
| risch | $-\frac{1}{7e(ex+d)^3c^3(e^2x^2+2dex+d^2)^2}$ | 34 |
| parallelrisch | $-\frac{1}{7e(ex+d)^3c^3(e^2x^2+2dex+d^2)^2}$ | 34 |
| orering | $-\frac{1}{7(ex+d)e(x^2ce^2+2cdxe+cd^2)^3}$ | 35 |

input `int(1/(e*x+d)^2/(c*e^2*x^2+2*c*d*e*x+c*d^2)^3,x,method=_RETURNVERBOSE)`

output `-1/7/c^3/e/(e*x+d)^7`

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 103 vs. $2(15) = 30$.

Time = 0.07 (sec) , antiderivative size = 103, normalized size of antiderivative = 6.06

$$\int \frac{1}{(d+ex)^2(cd^2+2cdex+ce^2x^2)^3} dx =$$

$$-\frac{1}{7(c^3e^8x^7 + 7c^3de^7x^6 + 21c^3d^2e^6x^5 + 35c^3d^3e^5x^4 + 35c^3d^4e^4x^3 + 21c^3d^5e^3x^2 + 7c^3d^6e^2x + c^3d^7e)}$$

input `integrate(1/(e*x+d)^2/(c*e^2*x^2+2*c*d*e*x+c*d^2)^3,x, algorithm="fricas")`

output `-1/7/(c^3*e^8*x^7 + 7*c^3*d*e^7*x^6 + 21*c^3*d^2*e^6*x^5 + 35*c^3*d^3*e^5*x^4 + 35*c^3*d^4*e^4*x^3 + 21*c^3*d^5*e^3*x^2 + 7*c^3*d^6*e^2*x + c^3*d^7*e)`

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 112 vs. $2(15) = 30$.

Time = 0.23 (sec) , antiderivative size = 112, normalized size of antiderivative = 6.59

$$\int \frac{1}{(d+ex)^2 (cd^2 + 2cdex + ce^2x^2)^3} dx =$$

$$\frac{1}{7c^3d^7e + 49c^3d^6e^2x + 147c^3d^5e^3x^2 + 245c^3d^4e^4x^3 + 245c^3d^3e^5x^4 + 147c^3d^2e^6x^5 + 49c^3de^7x^6 + 7c^3e^8x^7}$$

input `integrate(1/(e*x+d)**2/(c*e**2*x**2+2*c*d*e*x+c*d**2)**3,x)`

output `-1/(7*c**3*d**7*e + 49*c**3*d**6*e**2*x + 147*c**3*d**5*e**3*x**2 + 245*c**3*d**4*e**4*x**3 + 245*c**3*d**3*e**5*x**4 + 147*c**3*d**2*e**6*x**5 + 49*c**3*d*e**7*x**6 + 7*c**3*e**8*x**7)`

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 103 vs. $2(15) = 30$.

Time = 0.03 (sec) , antiderivative size = 103, normalized size of antiderivative = 6.06

$$\int \frac{1}{(d+ex)^2 (cd^2 + 2cdex + ce^2x^2)^3} dx =$$

$$\frac{1}{7(c^3e^8x^7 + 7c^3de^7x^6 + 21c^3d^2e^6x^5 + 35c^3d^3e^5x^4 + 35c^3d^4e^4x^3 + 21c^3d^5e^3x^2 + 7c^3d^6e^2x + c^3d^7e)}$$

input `integrate(1/(e*x+d)^2/(c*e^2*x^2+2*c*d*e*x+c*d^2)^3,x, algorithm="maxima")`

output `-1/7/(c^3*e^8*x^7 + 7*c^3*d*e^7*x^6 + 21*c^3*d^2*e^6*x^5 + 35*c^3*d^3*e^5*x^4 + 35*c^3*d^4*e^4*x^3 + 21*c^3*d^5*e^3*x^2 + 7*c^3*d^6*e^2*x + c^3*d^7*e)`

Giac [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.88

$$\int \frac{1}{(d+ex)^2 (cd^2+2cdex+ce^2x^2)^3} dx = -\frac{1}{7(ex+d)^7 c^3 e}$$

input `integrate(1/(e*x+d)^2/(c*e^2*x^2+2*c*d*e*x+c*d^2)^3,x, algorithm="giac")`output `-1/7/((e*x + d)^7*c^3*e)`**Mupad [B] (verification not implemented)**

Time = 0.07 (sec) , antiderivative size = 105, normalized size of antiderivative = 6.18

$$\int \frac{1}{(d+ex)^2 (cd^2+2cdex+ce^2x^2)^3} dx = \frac{1}{7c^3d^7e + 49c^3d^6e^2x + 147c^3d^5e^3x^2 + 245c^3d^4e^4x^3 + 245c^3d^3e^5x^4 + 147c^3d^2e^6x^5 + 49c^3de^7x^6 + d^7}$$

input `int(1/((d + e*x)^2*(c*d^2 + c*e^2*x^2 + 2*c*d*e*x)^3),x)`output `-1/(7*c^3*d^7*e + 7*c^3*e^8*x^7 + 49*c^3*d^6*e^2*x + 49*c^3*d*e^7*x^6 + 14*7*c^3*d^5*e^3*x^2 + 245*c^3*d^4*e^4*x^3 + 245*c^3*d^3*e^5*x^4 + 147*c^3*d^2*e^6*x^5)`**Reduce [B] (verification not implemented)**

Time = 0.21 (sec) , antiderivative size = 81, normalized size of antiderivative = 4.76

$$\int \frac{1}{(d+ex)^2 (cd^2+2cdex+ce^2x^2)^3} dx = -\frac{1}{7c^3e(e^7x^7 + 7de^6x^6 + 21d^2e^5x^5 + 35d^3e^4x^4 + 35d^4e^3x^3 + 21d^5e^2x^2 + 7d^6ex + d^7)}$$

input `int(1/(e*x+d)^2/(c*e^2*x^2+2*c*d*e*x+c*d^2)^3,x)`

output $(-1)/(7c^3e(d^7 + 7d^6ex + 21d^5e^2x^2 + 35d^4e^3x^3 + 35d^3e^4x^4 + 21d^2e^5x^5 + 7de^6x^6 + e^7x^7))$

3.52 $\int \frac{1}{d+ex} dx$

| | |
|---|-----|
| Optimal result | 452 |
| Mathematica [A] (verified) | 452 |
| Rubi [A] (verified) | 453 |
| Maple [A] (verified) | 453 |
| Fricas [A] (verification not implemented) | 454 |
| Sympy [A] (verification not implemented) | 454 |
| Maxima [A] (verification not implemented) | 455 |
| Giac [A] (verification not implemented) | 455 |
| Mupad [B] (verification not implemented) | 455 |
| Reduce [B] (verification not implemented) | 456 |

Optimal result

Integrand size = 7, antiderivative size = 10

$$\int \frac{1}{d+ex} dx = \frac{\log(d+ex)}{e}$$

output `ln(e*x+d)/e`

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00

$$\int \frac{1}{d+ex} dx = \frac{\log(d+ex)}{e}$$

input `Integrate[(d + e*x)^(-1),x]`

output `Log[d + e*x]/e`

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {16}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{d + ex} dx$$

$$\downarrow 16$$

$$\frac{\log(d + ex)}{e}$$

input `Int[(d + e*x)^(-1),x]`

output `Log[d + e*x]/e`

Defintions of rubi rules used

rule 16 `Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a + b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]`

Maple [A] (verified)

Time = 0.63 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.10

| method | result | size |
|--------------|-----------------------|------|
| default | $\frac{\ln(ex+d)}{e}$ | 11 |
| norman | $\frac{\ln(ex+d)}{e}$ | 11 |
| risch | $\frac{\ln(ex+d)}{e}$ | 11 |
| parallelrisc | $\frac{\ln(ex+d)}{e}$ | 11 |

input `int(1/(e*x+d),x,method=_RETURNVERBOSE)`

output `ln(e*x+d)/e`

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00

$$\int \frac{1}{d+ex} dx = \frac{\log(ex+d)}{e}$$

input `integrate(1/(e*x+d),x, algorithm="fricas")`

output `log(e*x + d)/e`

Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.70

$$\int \frac{1}{d+ex} dx = \frac{\log(d+ex)}{e}$$

input `integrate(1/(e*x+d),x)`

output `log(d + e*x)/e`

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00

$$\int \frac{1}{d+ex} dx = \frac{\log(ex+d)}{e}$$

input `integrate(1/(e*x+d),x, algorithm="maxima")`

output `log(e*x + d)/e`

Giac [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.10

$$\int \frac{1}{d+ex} dx = \frac{\log(|ex+d|)}{e}$$

input `integrate(1/(e*x+d),x, algorithm="giac")`

output `log(abs(e*x + d))/e`

Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00

$$\int \frac{1}{d+ex} dx = \frac{\ln(d+ex)}{e}$$

input `int(1/(d + e*x),x)`

output `log(d + e*x)/e`

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00

$$\int \frac{1}{d+ex} dx = \frac{\log(ex+d)}{e}$$

input `int(1/(e*x+d),x)`

output `log(d + e*x)/e`

3.53 $\int \frac{d+ex}{d^2+2dex+e^2x^2} dx$

| | |
|---|-----|
| Optimal result | 457 |
| Mathematica [A] (verified) | 457 |
| Rubi [A] (verified) | 458 |
| Maple [A] (verified) | 459 |
| Fricas [A] (verification not implemented) | 459 |
| Sympy [A] (verification not implemented) | 459 |
| Maxima [B] (verification not implemented) | 460 |
| Giac [B] (verification not implemented) | 460 |
| Mupad [B] (verification not implemented) | 461 |
| Reduce [B] (verification not implemented) | 461 |

Optimal result

Integrand size = 24, antiderivative size = 10

$$\int \frac{d+ex}{d^2+2dex+e^2x^2} dx = \frac{\log(d+ex)}{e}$$

output `ln(e*x+d)/e`

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00

$$\int \frac{d+ex}{d^2+2dex+e^2x^2} dx = \frac{\log(d+ex)}{e}$$

input `Integrate[(d + e*x)/(d^2 + 2*d*e*x + e^2*x^2),x]`

output `Log[d + e*x]/e`

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {1098, 16}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{d + ex}{d^2 + 2dex + e^2x^2} dx$$

↓ 1098

$$e^2 \int \frac{1}{e^2(d + ex)} dx$$

↓ 16

$$\frac{\log(d + ex)}{e}$$

input

```
Int[(d + e*x)/(d^2 + 2*d*e*x + e^2*x^2),x]
```

output

```
Log[d + e*x]/e
```

Defintions of rubi rules used

rule 16

```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a + b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 1098

```
Int[((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[1/c^p Int[(d + e*x)^m*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Maple [A] (verified)

Time = 1.05 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.10

| method | result | size |
|---------------|-----------------------|------|
| default | $\frac{\ln(ex+d)}{e}$ | 11 |
| norman | $\frac{\ln(ex+d)}{e}$ | 11 |
| risch | $\frac{\ln(ex+d)}{e}$ | 11 |
| parallelrisch | $\frac{\ln(ex+d)}{e}$ | 11 |

input `int((e*x+d)/(e^2*x^2+2*d*e*x+d^2),x,method=_RETURNVERBOSE)`output `ln(e*x+d)/e`**Fricas [A] (verification not implemented)**

Time = 0.09 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00

$$\int \frac{d + ex}{d^2 + 2dex + e^2x^2} dx = \frac{\log(ex + d)}{e}$$

input `integrate((e*x+d)/(e^2*x^2+2*d*e*x+d^2),x, algorithm="fricas")`output `log(e*x + d)/e`**Sympy [A] (verification not implemented)**

Time = 0.02 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.70

$$\int \frac{d + ex}{d^2 + 2dex + e^2x^2} dx = \frac{\log(d + ex)}{e}$$

input `integrate((e*x+d)/(e**2*x**2+2*d*e*x+d**2),x)`

output `log(d + e*x)/e`

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 22 vs. 2(10) = 20.

Time = 0.03 (sec) , antiderivative size = 22, normalized size of antiderivative = 2.20

$$\int \frac{d + ex}{d^2 + 2dex + e^2x^2} dx = \frac{\log(e^2x^2 + 2dex + d^2)}{2e}$$

input `integrate((e*x+d)/(e^2*x^2+2*d*e*x+d^2),x, algorithm="maxima")`

output `1/2*log(e^2*x^2 + 2*d*e*x + d^2)/e`

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 22 vs. 2(10) = 20.

Time = 0.20 (sec) , antiderivative size = 22, normalized size of antiderivative = 2.20

$$\int \frac{d + ex}{d^2 + 2dex + e^2x^2} dx = \frac{\log(d^2 + (ex^2 + 2dx)e)}{2e}$$

input `integrate((e*x+d)/(e^2*x^2+2*d*e*x+d^2),x, algorithm="giac")`

output `1/2*log(d^2 + (e*x^2 + 2*d*x)*e)/e`

Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00

$$\int \frac{d + ex}{d^2 + 2dex + e^2x^2} dx = \frac{\ln(d + ex)}{e}$$

input `int((d + e*x)/(d^2 + e^2*x^2 + 2*d*e*x),x)`

output `log(d + e*x)/e`

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00

$$\int \frac{d + ex}{d^2 + 2dex + e^2x^2} dx = \frac{\log(ex + d)}{e}$$

input `int((e*x+d)/(e^2*x^2+2*d*e*x+d^2),x)`

output `log(d + e*x)/e`

3.54 $\int (d + ex)^4 (a^2 + 2abx + b^2x^2) dx$

| | |
|---|-----|
| Optimal result | 462 |
| Mathematica [B] (verified) | 462 |
| Rubi [A] (verified) | 463 |
| Maple [B] (verified) | 464 |
| Fricas [B] (verification not implemented) | 465 |
| Sympy [B] (verification not implemented) | 466 |
| Maxima [B] (verification not implemented) | 466 |
| Giac [B] (verification not implemented) | 467 |
| Mupad [B] (verification not implemented) | 467 |
| Reduce [B] (verification not implemented) | 468 |

Optimal result

Integrand size = 24, antiderivative size = 65

$$\int (d + ex)^4 (a^2 + 2abx + b^2x^2) dx = \frac{(bd - ae)^2(d + ex)^5}{5e^3} - \frac{b(bd - ae)(d + ex)^6}{3e^3} + \frac{b^2(d + ex)^7}{7e^3}$$

output

```
1/5*(-a*e+b*d)^2*(e*x+d)^5/e^3-1/3*b*(-a*e+b*d)*(e*x+d)^6/e^3+1/7*b^2*(e*x+d)^7/e^3
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 148 vs. $2(65) = 130$.

Time = 0.02 (sec) , antiderivative size = 148, normalized size of antiderivative = 2.28

$$\begin{aligned} \int (d+ex)^4 (a^2+2abx+b^2x^2) dx = & a^2d^4x + ad^3(bd+2ae)x^2 + \frac{1}{3}d^2(b^2d^2+8abde+6a^2e^2)x^3 \\ & + de(b^2d^2+3abde+a^2e^2)x^4 \\ & + \frac{1}{5}e^2(6b^2d^2+8abde+a^2e^2)x^5 \\ & + \frac{1}{3}be^3(2bd+ae)x^6 + \frac{1}{7}b^2e^4x^7 \end{aligned}$$

input `Integrate[(d + e*x)^4*(a^2 + 2*a*b*x + b^2*x^2),x]`

output $a^2d^4x + a^2d^3(bd + 2ae)x^2 + (d^2(b^2d^2 + 8abd^2e + 6a^2e^2)x^3)/3 + d^2e(b^2d^2 + 3abd^2e + a^2e^2)x^4 + (e^2(6b^2d^2 + 8abd^2e + a^2e^2)x^5)/5 + (b^2e^3(2bd + ae)x^6)/3 + (b^2e^4x^7)/7$

Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {1098, 27, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\begin{aligned} & \int (a^2 + 2abx + b^2x^2) (d + ex)^4 dx \\ & \quad \downarrow 1098 \\ & \int \frac{b^2(a + bx)^2(d + ex)^4 dx}{b^2} \\ & \quad \downarrow 27 \\ & \int (a + bx)^2(d + ex)^4 dx \\ & \quad \downarrow 49 \\ & \int \left(-\frac{2b(d + ex)^5(bd - ae)}{e^2} + \frac{(d + ex)^4(ae - bd)^2}{e^2} + \frac{b^2(d + ex)^6}{e^2} \right) dx \\ & \quad \downarrow 2009 \\ & -\frac{b(d + ex)^6(bd - ae)}{3e^3} + \frac{(d + ex)^5(bd - ae)^2}{5e^3} + \frac{b^2(d + ex)^7}{7e^3} \end{aligned}$$

input `Int[(d + e*x)^4*(a^2 + 2*a*b*x + b^2*x^2),x]`

output $((b*d - a*e)^2*(d + e*x)^5)/(5*e^3) - (b*(b*d - a*e)*(d + e*x)^6)/(3*e^3) + (b^2*(d + e*x)^7)/(7*e^3)$

Defintions of rubi rules used

rule 27 `Int[(a_)*(Fx_), x_Symbol] := Simp[a Int[Fx, x], x] /; FreeQ[a, x] && !MatchQ[Fx, (b_)*(Gx_)] /; FreeQ[b, x]`

rule 49 `Int[((a_.) + (b_.)*(x_.))^(m_.)*((c_.) + (d_.)*(x_.))^(n_.), x_Symbol] := Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] && IGtQ[m, 0] && IGtQ[m + n + 2, 0]`

rule 1098 `Int[((d_.) + (e_.)*(x_.))^(m_.)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] := Simp[1/c^p Int[(d + e*x)^m*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]`

rule 2009 `Int[u_, x_Symbol] := Simp[IntSum[u, x], x] /; SumQ[u]`

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 156 vs. 2(59) = 118.

Time = 0.52 (sec) , antiderivative size = 157, normalized size of antiderivative = 2.42

| method | result |
|--------------|---|
| norman | $\frac{b^2 e^4 x^7}{7} + \left(\frac{1}{3} a b e^4 + \frac{2}{3} b^2 d e^3\right) x^6 + \left(\frac{1}{5} a^2 e^4 + \frac{8}{5} a b d e^3 + \frac{6}{5} d^2 e^2 b^2\right) x^5 + (a^2 d e^3 + 3 a b d^2 e^2 + b^2 d^3 e) x^4 + \frac{6 a^2 d^2 e^2 + 8 a b d^3 e}{3} x^3 + \frac{2 a^2 d^3 e^2 + 6 a b d^4 e}{4} x^2 + \frac{2 a^2 d^4 e^2 + 6 a b d^5 e}{5} x + \frac{2 a^2 d^5 e^2 + 6 a b d^6 e}{6} + \frac{2 a^2 d^6 e^2 + 6 a b d^7 e}{7}$ |
| default | $\frac{b^2 e^4 x^7}{7} + \frac{(2 a b e^4 + 4 b^2 d e^3) x^6}{6} + \frac{(a^2 e^4 + 8 a b d e^3 + 6 d^2 e^2 b^2) x^5}{5} + \frac{(4 a^2 d e^3 + 12 a b d^2 e^2 + 4 b^2 d^3 e) x^4}{4} + \frac{(6 a^2 d^2 e^2 + 8 a b d^3 e) x^3}{3} + \frac{2 a^2 d^3 e^2 + 6 a b d^4 e}{4} x^2 + \frac{2 a^2 d^4 e^2 + 6 a b d^5 e}{5} x + \frac{2 a^2 d^5 e^2 + 6 a b d^6 e}{6} + \frac{2 a^2 d^6 e^2 + 6 a b d^7 e}{7}$ |
| risch | $\frac{1}{7} b^2 e^4 x^7 + \frac{1}{3} x^6 a b e^4 + \frac{2}{3} x^6 b^2 d e^3 + \frac{1}{5} x^5 a^2 e^4 + \frac{8}{5} x^5 a b d e^3 + \frac{6}{5} x^5 d^2 e^2 b^2 + a^2 d e^3 x^4 + 3 a b d^2 e^2 x^3 + \frac{2 a^2 d^3 e^2 + 6 a b d^4 e}{4} x^2 + \frac{2 a^2 d^4 e^2 + 6 a b d^5 e}{5} x + \frac{2 a^2 d^5 e^2 + 6 a b d^6 e}{6} + \frac{2 a^2 d^6 e^2 + 6 a b d^7 e}{7}$ |
| parallelrisc | $\frac{1}{7} b^2 e^4 x^7 + \frac{1}{3} x^6 a b e^4 + \frac{2}{3} x^6 b^2 d e^3 + \frac{1}{5} x^5 a^2 e^4 + \frac{8}{5} x^5 a b d e^3 + \frac{6}{5} x^5 d^2 e^2 b^2 + a^2 d e^3 x^4 + 3 a b d^2 e^2 x^3 + \frac{2 a^2 d^3 e^2 + 6 a b d^4 e}{4} x^2 + \frac{2 a^2 d^4 e^2 + 6 a b d^5 e}{5} x + \frac{2 a^2 d^5 e^2 + 6 a b d^6 e}{6} + \frac{2 a^2 d^6 e^2 + 6 a b d^7 e}{7}$ |
| gospers | $\frac{x(15 b^2 e^4 x^6 + 35 x^5 a b e^4 + 70 x^5 b^2 d e^3 + 21 x^4 a^2 e^4 + 168 x^4 a b d e^3 + 126 x^4 d^2 e^2 b^2 + 105 a^2 d e^3 x^3 + 315 a b d^2 e^2 x^3 + 105 b^2 d^3 e x^3 + 210 a^2 d^4 e^2 x^2 + 420 a b d^5 e x^2 + 105 a^2 d^5 e^2 x + 105 b^2 d^6 e x + 105 a^2 d^6 e^2 + 105 a b d^7 e)}{105}$ |
| orering | $\frac{x(15 b^2 e^4 x^6 + 35 x^5 a b e^4 + 70 x^5 b^2 d e^3 + 21 x^4 a^2 e^4 + 168 x^4 a b d e^3 + 126 x^4 d^2 e^2 b^2 + 105 a^2 d e^3 x^3 + 315 a b d^2 e^2 x^3 + 105 b^2 d^3 e x^3 + 210 a^2 d^4 e^2 x^2 + 420 a b d^5 e x^2 + 105 a^2 d^5 e^2 x + 105 b^2 d^6 e x + 105 a^2 d^6 e^2 + 105 a b d^7 e)}{105(bx+a)^2}$ |

input `int((e*x+d)^4*(b^2*x^2+2*a*b*x+a^2),x,method=_RETURNVERBOSE)`

output `1/7*b^2*e^4*x^7+(1/3*a*b*e^4+2/3*b^2*d*e^3)*x^6+(1/5*a^2*e^4+8/5*a*b*d*e^3+6/5*d^2*e^2*b^2)*x^5+(a^2*d*e^3+3*a*b*d^2*e^2+b^2*d^3*e)*x^4+(2*a^2*d^2*e^2+8/3*a*b*d^3*e+1/3*b^2*d^4)*x^3+(2*a^2*d^3*e+a*b*d^4)*x^2+a^2*d^4*x`

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 156 vs. $2(59) = 118$.

Time = 0.07 (sec) , antiderivative size = 156, normalized size of antiderivative = 2.40

$$\int (d + ex)^4 (a^2 + 2abx + b^2x^2) dx = \frac{1}{7} b^2 e^4 x^7 + a^2 d^4 x + \frac{1}{3} (2 b^2 d e^3 + a b e^4) x^6 + \frac{1}{5} (6 b^2 d^2 e^2 + 8 a b d e^3 + a^2 e^4) x^5 + (b^2 d^3 e + 3 a b d^2 e^2 + a^2 d e^3) x^4 + \frac{1}{3} (b^2 d^4 + 8 a b d^3 e + 6 a^2 d^2 e^2) x^3 + (a b d^4 + 2 a^2 d^3 e) x^2$$

input `integrate((e*x+d)^4*(b^2*x^2+2*a*b*x+a^2),x, algorithm="fricas")`

output `1/7*b^2*e^4*x^7 + a^2*d^4*x + 1/3*(2*b^2*d*e^3 + a*b*e^4)*x^6 + 1/5*(6*b^2*d^2*e^2 + 8*a*b*d*e^3 + a^2*e^4)*x^5 + (b^2*d^3*e + 3*a*b*d^2*e^2 + a^2*d*e^3)*x^4 + 1/3*(b^2*d^4 + 8*a*b*d^3*e + 6*a^2*d^2*e^2)*x^3 + (a*b*d^4 + 2*a^2*d^3*e)*x^2`

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 168 vs. $2(54) = 108$.

Time = 0.03 (sec) , antiderivative size = 168, normalized size of antiderivative = 2.58

$$\int (d + ex)^4 (a^2 + 2abx + b^2x^2) dx = a^2d^4x + \frac{b^2e^4x^7}{7} + x^6 \left(\frac{abe^4}{3} + \frac{2b^2de^3}{3} \right) + x^5 \left(\frac{a^2e^4}{5} + \frac{8abde^3}{5} + \frac{6b^2d^2e^2}{5} \right) + x^4 (a^2de^3 + 3abd^2e^2 + b^2d^3e) + x^3 \cdot \left(2a^2d^2e^2 + \frac{8abd^3e}{3} + \frac{b^2d^4}{3} \right) + x^2 \cdot (2a^2d^3e + abd^4)$$

input `integrate((e*x+d)**4*(b**2*x**2+2*a*b*x+a**2),x)`

output `a**2*d**4*x + b**2*e**4*x**7/7 + x**6*(a*b*e**4/3 + 2*b**2*d*e**3/3) + x**5*(a**2*e**4/5 + 8*a*b*d*e**3/5 + 6*b**2*d**2*e**2/5) + x**4*(a**2*d*e**3 + 3*a*b*d**2*e**2 + b**2*d**3*e) + x**3*(2*a**2*d**2*e**2 + 8*a*b*d**3*e/3 + b**2*d**4/3) + x**2*(2*a**2*d**3*e + a*b*d**4)`

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 156 vs. $2(59) = 118$.

Time = 0.04 (sec) , antiderivative size = 156, normalized size of antiderivative = 2.40

$$\int (d + ex)^4 (a^2 + 2abx + b^2x^2) dx = \frac{1}{7} b^2 e^4 x^7 + a^2 d^4 x + \frac{1}{3} (2 b^2 d e^3 + a b e^4) x^6 + \frac{1}{5} (6 b^2 d^2 e^2 + 8 a b d e^3 + a^2 e^4) x^5 + (b^2 d^3 e + 3 a b d^2 e^2 + a^2 d e^3) x^4 + \frac{1}{3} (b^2 d^4 + 8 a b d^3 e + 6 a^2 d^2 e^2) x^3 + (a b d^4 + 2 a^2 d^3 e) x^2$$

input `integrate((e*x+d)^4*(b^2*x^2+2*a*b*x+a^2),x, algorithm="maxima")`

output

```
1/7*b^2*e^4*x^7 + a^2*d^4*x + 1/3*(2*b^2*d*e^3 + a*b*e^4)*x^6 + 1/5*(6*b^2
*d^2*e^2 + 8*a*b*d*e^3 + a^2*e^4)*x^5 + (b^2*d^3*e + 3*a*b*d^2*e^2 + a^2*d
*e^3)*x^4 + 1/3*(b^2*d^4 + 8*a*b*d^3*e + 6*a^2*d^2*e^2)*x^3 + (a*b*d^4 + 2
*a^2*d^3*e)*x^2
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 170 vs. $2(59) = 118$.

Time = 0.20 (sec) , antiderivative size = 170, normalized size of antiderivative = 2.62

$$\int (d + ex)^4 (a^2 + 2abx + b^2x^2) dx = \frac{1}{7} b^2 e^4 x^7 + \frac{2}{3} b^2 d e^3 x^6 + \frac{1}{3} a b e^4 x^6 + \frac{6}{5} b^2 d^2 e^2 x^5$$

$$+ \frac{8}{5} a b d e^3 x^5 + \frac{1}{5} a^2 e^4 x^5 + b^2 d^3 e x^4 + 3 a b d^2 e^2 x^4$$

$$+ a^2 d e^3 x^4 + \frac{1}{3} b^2 d^4 x^3 + \frac{8}{3} a b d^3 e x^3$$

$$+ 2 a^2 d^2 e^2 x^3 + a b d^4 x^2 + 2 a^2 d^3 e x^2 + a^2 d^4 x$$

input

```
integrate((e*x+d)^4*(b^2*x^2+2*a*b*x+a^2),x, algorithm="giac")
```

output

```
1/7*b^2*e^4*x^7 + 2/3*b^2*d*e^3*x^6 + 1/3*a*b*e^4*x^6 + 6/5*b^2*d^2*e^2*x^
5 + 8/5*a*b*d*e^3*x^5 + 1/5*a^2*e^4*x^5 + b^2*d^3*e*x^4 + 3*a*b*d^2*e^2*x^
4 + a^2*d*e^3*x^4 + 1/3*b^2*d^4*x^3 + 8/3*a*b*d^3*e*x^3 + 2*a^2*d^2*e^2*x^
3 + a*b*d^4*x^2 + 2*a^2*d^3*e*x^2 + a^2*d^4*x
```

Mupad [B] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 144, normalized size of antiderivative = 2.22

$$\int (d + ex)^4 (a^2 + 2abx + b^2x^2) dx = x^3 \left(2a^2 d^2 e^2 + \frac{8abd^3e}{3} + \frac{b^2d^4}{3} \right)$$

$$+ x^5 \left(\frac{a^2e^4}{5} + \frac{8abde^3}{5} + \frac{6b^2d^2e^2}{5} \right) + a^2d^4x$$

$$+ \frac{b^2e^4x^7}{7} + ad^3x^2(2ae+bd) + \frac{be^3x^6(ae+2bd)}{3}$$

$$+ dex^4(a^2e^2 + 3abde + b^2d^2)$$

input `int((d + e*x)^4*(a^2 + b^2*x^2 + 2*a*b*x),x)`

output `x^3*((b^2*d^4)/3 + 2*a^2*d^2*e^2 + (8*a*b*d^3*e)/3) + x^5*((a^2*e^4)/5 + (6*b^2*d^2*e^2)/5 + (8*a*b*d*e^3)/5) + a^2*d^4*x + (b^2*e^4*x^7)/7 + a*d^3*x^2*(2*a*e + b*d) + (b*e^3*x^6*(a*e + 2*b*d))/3 + d*e*x^4*(a^2*e^2 + b^2*d^2 + 3*a*b*d*e)`

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 172, normalized size of antiderivative = 2.65

$$\int (d + ex)^4 (a^2 + 2abx + b^2x^2) dx$$

$$= \frac{x(15b^2e^4x^6 + 35abe^4x^5 + 70b^2de^3x^5 + 21a^2e^4x^4 + 168abd e^3x^4 + 126b^2d^2e^2x^4 + 105a^2de^3x^3 + 315abd^2e^2x^3 + 15b^2d^3e^2x^3 + 126b^2d^2e^2x^4 + 70b^2de^3x^5 + 15b^2e^4x^6)}{105}$$

input `int((e*x+d)^4*(b^2*x^2+2*a*b*x+a^2),x)`

output `(x*(105*a**2*d**4 + 210*a**2*d**3*e*x + 210*a**2*d**2*e**2*x**2 + 105*a**2*d*e**3*x**3 + 21*a**2*e**4*x**4 + 105*a*b*d**4*x + 280*a*b*d**3*e*x**2 + 315*a*b*d**2*e**2*x**3 + 168*a*b*d*e**3*x**4 + 35*a*b*e**4*x**5 + 35*b**2*d**4*x**2 + 105*b**2*d**3*e*x**3 + 126*b**2*d**2*e**2*x**4 + 70*b**2*d*e**3*x**5 + 15*b**2*e**4*x**6))/105`

3.55 $\int (d + ex)^3 (a^2 + 2abx + b^2x^2) dx$

| | |
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Optimal result

Integrand size = 24, antiderivative size = 65

$$\int (d + ex)^3 (a^2 + 2abx + b^2x^2) dx = \frac{(bd - ae)^2(d + ex)^4}{4e^3} - \frac{2b(bd - ae)(d + ex)^5}{5e^3} + \frac{b^2(d + ex)^6}{6e^3}$$

output

```
1/4*(-a*e+b*d)^2*(e*x+d)^4/e^3-2/5*b*(-a*e+b*d)*(e*x+d)^5/e^3+1/6*b^2*(e*x+d)^6/e^3
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 122, normalized size of antiderivative = 1.88

$$\int (d + ex)^3 (a^2 + 2abx + b^2x^2) dx = a^2d^3x + \frac{1}{2}ad^2(2bd + 3ae)x^2 + \frac{1}{3}d(b^2d^2 + 6abde + 3a^2e^2)x^3 + \frac{1}{4}e(3b^2d^2 + 6abde + a^2e^2)x^4 + \frac{1}{5}be^2(3bd + 2ae)x^5 + \frac{1}{6}b^2e^3x^6$$

input `Integrate[(d + e*x)^3*(a^2 + 2*a*b*x + b^2*x^2),x]`

output $a^2*d^3*x + (a*d^2*(2*b*d + 3*a*e)*x^2)/2 + (d*(b^2*d^2 + 6*a*b*d*e + 3*a^2*e^2)*x^3)/3 + (e*(3*b^2*d^2 + 6*a*b*d*e + a^2*e^2)*x^4)/4 + (b*e^2*(3*b*d + 2*a*e)*x^5)/5 + (b^2*e^3*x^6)/6$

Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {1098, 27, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\begin{aligned} & \int (a^2 + 2abx + b^2x^2) (d + ex)^3 dx \\ & \quad \downarrow 1098 \\ & \quad \int \frac{b^2(a + bx)^2(d + ex)^3 dx}{b^2} \\ & \quad \downarrow 27 \\ & \quad \int (a + bx)^2(d + ex)^3 dx \\ & \quad \downarrow 49 \\ & \quad \int \left(-\frac{2b(d + ex)^4(bd - ae)}{e^2} + \frac{(d + ex)^3(ae - bd)^2}{e^2} + \frac{b^2(d + ex)^5}{e^2} \right) dx \\ & \quad \downarrow 2009 \\ & \quad -\frac{2b(d + ex)^5(bd - ae)}{5e^3} + \frac{(d + ex)^4(bd - ae)^2}{4e^3} + \frac{b^2(d + ex)^6}{6e^3} \end{aligned}$$

input `Int[(d + e*x)^3*(a^2 + 2*a*b*x + b^2*x^2),x]`

output
$$\frac{(b*d - a*e)^2*(d + e*x)^4}{(4*e^3)} - \frac{(2*b*(b*d - a*e)*(d + e*x)^5)}{(5*e^3)} + \frac{(b^2*(d + e*x)^6)}{(6*e^3)}$$

Defintions of rubi rules used

rule 27
$$\text{Int}[(a_*)(Fx_), x_Symbol] \rightarrow \text{Simp}[a \text{ Int}[Fx, x], x] \text{ /; FreeQ}[a, x] \ \&\& \ !\text{MatchQ}[Fx, (b_*)(Gx_)] \text{ /; FreeQ}[b, x]$$

rule 49
$$\text{Int}[(a_.) + (b_.)*(x_)^{(m_.)}*((c_.) + (d_.)*(x_)^{(n_.)}), x_Symbol] \rightarrow \text{Int}[\text{ExpandIntegrand}[(a + b*x)^m*(c + d*x)^n, x], x] \text{ /; FreeQ}[\{a, b, c, d\}, x] \ \&\& \ \text{IGtQ}[m, 0] \ \&\& \ \text{IGtQ}[m + n + 2, 0]$$

rule 1098
$$\text{Int}[(d_.) + (e_.)*(x_)^{(m_.)}*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^{(p_.)}, x_Symbol] \rightarrow \text{Simp}[1/c^p \text{ Int}[(d + e*x)^m*(b/2 + c*x)^{(2*p)}, x], x] \text{ /; FreeQ}[\{a, b, c, d, e, m\}, x] \ \&\& \ \text{EqQ}[b^2 - 4*a*c, 0] \ \&\& \ \text{IntegerQ}[p]$$

rule 2009
$$\text{Int}[u_, x_Symbol] \rightarrow \text{Simp}[\text{IntSum}[u, x], x] \text{ /; SumQ}[u]$$

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 120 vs. $2(59) = 118$.

Time = 0.53 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.86

| method | result |
|--------------|---|
| norman | $\frac{b^2 e^3 x^6}{6} + \left(\frac{2}{5} a e^3 b + \frac{3}{5} d e^2 b^2\right) x^5 + \left(\frac{1}{4} a^2 e^3 + \frac{3}{2} a b d e^2 + \frac{3}{4} b^2 d^2 e\right) x^4 + (a^2 d e^2 + 2 a b d^2 e + \frac{1}{3} b^2 d^3) x^3 + \frac{1}{2} a^2 d^2 e^2 + \frac{1}{2} a b d^3 e + \frac{1}{6} b^2 d^4 e$ |
| default | $\frac{b^2 e^3 x^6}{6} + \frac{(2 a e^3 b + 3 d e^2 b^2) x^5}{5} + \frac{(a^2 e^3 + 6 a b d e^2 + 3 b^2 d^2 e) x^4}{4} + \frac{(3 a^2 d e^2 + 6 a b d^2 e + b^2 d^3) x^3}{3} + \frac{(3 a^2 d^2 e + 2 a b d^3) x^2}{2} + \frac{1}{6} b^2 d^4 e$ |
| risch | $\frac{1}{6} b^2 e^3 x^6 + \frac{2}{5} x^5 a e^3 b + \frac{3}{5} x^5 d e^2 b^2 + \frac{1}{4} x^4 a^2 e^3 + \frac{3}{2} x^4 a b d e^2 + \frac{3}{4} x^4 b^2 d^2 e + x^3 a^2 d e^2 + 2 x^3 a b d^2 e + \frac{1}{6} b^2 d^4 e$ |
| parallelrisc | $\frac{1}{6} b^2 e^3 x^6 + \frac{2}{5} x^5 a e^3 b + \frac{3}{5} x^5 d e^2 b^2 + \frac{1}{4} x^4 a^2 e^3 + \frac{3}{2} x^4 a b d e^2 + \frac{3}{4} x^4 b^2 d^2 e + x^3 a^2 d e^2 + 2 x^3 a b d^2 e + \frac{1}{6} b^2 d^4 e$ |
| gosper | $\frac{x(10 b^2 e^3 x^5 + 24 x^4 a e^3 b + 36 x^4 d e^2 b^2 + 15 x^3 a^2 e^3 + 90 x^3 a b d e^2 + 45 x^3 b^2 d^2 e + 60 x^2 a^2 d e^2 + 120 x^2 a b d^2 e + 20 x^2 b^2 d^3 + 90 x a^2 d^2 e + 60 a^2 d^3 e)}{60}$ |
| orering | $\frac{x(10 b^2 e^3 x^5 + 24 x^4 a e^3 b + 36 x^4 d e^2 b^2 + 15 x^3 a^2 e^3 + 90 x^3 a b d e^2 + 45 x^3 b^2 d^2 e + 60 x^2 a^2 d e^2 + 120 x^2 a b d^2 e + 20 x^2 b^2 d^3 + 90 x a^2 d^2 e + 60 a^2 d^3 e)}{60(bx+a)^2}$ |

input `int((e*x+d)^3*(b^2*x^2+2*a*b*x+a^2),x,method=_RETURNVERBOSE)`

output `1/6*b^2*e^3*x^6+(2/5*a*e^3*b+3/5*d*e^2*b^2)*x^5+(1/4*a^2*e^3+3/2*a*b*d*e^2+3/4*b^2*d^2*e)*x^4+(a^2*d*e^2+2*a*b*d^2*e+1/3*b^2*d^3)*x^3+(3/2*a^2*d^2*e+a*b*d^3)*x^2+a^2*d^3*x`

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 124 vs. $2(59) = 118$.

Time = 0.07 (sec) , antiderivative size = 124, normalized size of antiderivative = 1.91

$$\int (d + ex)^3 (a^2 + 2abx + b^2x^2) dx = \frac{1}{6} b^2 e^3 x^6 + a^2 d^3 x + \frac{1}{5} (3 b^2 d e^2 + 2 a b e^3) x^5 + \frac{1}{4} (3 b^2 d^2 e + 6 a b d e^2 + a^2 e^3) x^4 + \frac{1}{3} (b^2 d^3 + 6 a b d^2 e + 3 a^2 d e^2) x^3 + \frac{1}{2} (2 a b d^3 + 3 a^2 d^2 e) x^2$$

input `integrate((e*x+d)^3*(b^2*x^2+2*a*b*x+a^2),x, algorithm="fricas")`

output `1/6*b^2*e^3*x^6 + a^2*d^3*x + 1/5*(3*b^2*d*e^2 + 2*a*b*e^3)*x^5 + 1/4*(3*b^2*d^2*e + 6*a*b*d*e^2 + a^2*e^3)*x^4 + 1/3*(b^2*d^3 + 6*a*b*d^2*e + 3*a^2*d*e^2)*x^3 + 1/2*(2*a*b*d^3 + 3*a^2*d^2*e)*x^2`

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 133 vs. $2(56) = 112$.

Time = 0.03 (sec) , antiderivative size = 133, normalized size of antiderivative = 2.05

$$\int (d + ex)^3 (a^2 + 2abx + b^2x^2) dx = a^2d^3x + \frac{b^2e^3x^6}{6} + x^5 \cdot \left(\frac{2abe^3}{5} + \frac{3b^2de^2}{5} \right) \\ + x^4 \left(\frac{a^2e^3}{4} + \frac{3abde^2}{2} + \frac{3b^2d^2e}{4} \right) \\ + x^3 \left(a^2de^2 + 2abd^2e + \frac{b^2d^3}{3} \right) \\ + x^2 \cdot \left(\frac{3a^2d^2e}{2} + abd^3 \right)$$

input `integrate((e*x+d)**3*(b**2*x**2+2*a*b*x+a**2),x)`

output `a**2*d**3*x + b**2*e**3*x**6/6 + x**5*(2*a*b*e**3/5 + 3*b**2*d*e**2/5) + x**4*(a**2*e**3/4 + 3*a*b*d*e**2/2 + 3*b**2*d**2*e/4) + x**3*(a**2*d*e**2 + 2*a*b*d**2*e + b**2*d**3/3) + x**2*(3*a**2*d**2*e/2 + a*b*d**3)`

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 124 vs. 2(59) = 118.

Time = 0.04 (sec) , antiderivative size = 124, normalized size of antiderivative = 1.91

$$\int (d + ex)^3 (a^2 + 2abx + b^2x^2) dx = \frac{1}{6} b^2e^3x^6 + a^2d^3x + \frac{1}{5} (3b^2de^2 + 2abe^3)x^5 \\ + \frac{1}{4} (3b^2d^2e + 6abde^2 + a^2e^3)x^4 \\ + \frac{1}{3} (b^2d^3 + 6abd^2e + 3a^2de^2)x^3 \\ + \frac{1}{2} (2abd^3 + 3a^2d^2e)x^2$$

input `integrate((e*x+d)^3*(b^2*x^2+2*a*b*x+a^2),x, algorithm="maxima")`

output `1/6*b^2*e^3*x^6 + a^2*d^3*x + 1/5*(3*b^2*d*e^2 + 2*a*b*e^3)*x^5 + 1/4*(3*b^2*d^2*e + 6*a*b*d*e^2 + a^2*e^3)*x^4 + 1/3*(b^2*d^3 + 6*a*b*d^2*e + 3*a^2*d*e^2)*x^3 + 1/2*(2*a*b*d^3 + 3*a^2*d^2*e)*x^2`

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 130 vs. $2(59) = 118$.

Time = 0.22 (sec) , antiderivative size = 130, normalized size of antiderivative = 2.00

$$\begin{aligned} \int (d + ex)^3 (a^2 + 2abx + b^2x^2) dx = & \frac{1}{6} b^2 e^3 x^6 + \frac{3}{5} b^2 d e^2 x^5 + \frac{2}{5} a b e^3 x^5 + \frac{3}{4} b^2 d^2 e x^4 \\ & + \frac{3}{2} a b d e^2 x^4 + \frac{1}{4} a^2 e^3 x^4 + \frac{1}{3} b^2 d^3 x^3 + 2 a b d^2 e x^3 \\ & + a^2 d e^2 x^3 + a b d^3 x^2 + \frac{3}{2} a^2 d^2 e x^2 + a^2 d^3 x \end{aligned}$$

input `integrate((e*x+d)^3*(b^2*x^2+2*a*b*x+a^2),x, algorithm="giac")`

output `1/6*b^2*e^3*x^6 + 3/5*b^2*d*e^2*x^5 + 2/5*a*b*e^3*x^5 + 3/4*b^2*d^2*e*x^4 + 3/2*a*b*d*e^2*x^4 + 1/4*a^2*e^3*x^4 + 1/3*b^2*d^3*x^3 + 2*a*b*d^2*e*x^3 + a^2*d*e^2*x^3 + a*b*d^3*x^2 + 3/2*a^2*d^2*e*x^2 + a^2*d^3*x`

Mupad [B] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 115, normalized size of antiderivative = 1.77

$$\begin{aligned} \int (d + ex)^3 (a^2 + 2abx + b^2x^2) dx = & x^3 \left(a^2 d e^2 + 2 a b d^2 e + \frac{b^2 d^3}{3} \right) \\ & + x^4 \left(\frac{a^2 e^3}{4} + \frac{3 a b d e^2}{2} + \frac{3 b^2 d^2 e}{4} \right) \\ & + a^2 d^3 x + \frac{b^2 e^3 x^6}{6} + \frac{a d^2 x^2 (3 a e + 2 b d)}{2} \\ & + \frac{b e^2 x^5 (2 a e + 3 b d)}{5} \end{aligned}$$

input `int((d + e*x)^3*(a^2 + b^2*x^2 + 2*a*b*x),x)`

output `x^3*((b^2*d^3)/3 + a^2*d*e^2 + 2*a*b*d^2*e) + x^4*((a^2*e^3)/4 + (3*b^2*d^2*e)/4 + (3*a*b*d*e^2)/2) + a^2*d^3*x + (b^2*e^3*x^6)/6 + (a*d^2*x^2*(3*a*e + 2*b*d))/2 + (b*e^2*x^5*(2*a*e + 3*b*d))/5`

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 131, normalized size of antiderivative = 2.02

$$\int (d + ex)^3 (a^2 + 2abx + b^2x^2) dx$$

$$= \frac{x(10b^2e^3x^5 + 24abe^3x^4 + 36b^2de^2x^4 + 15a^2e^3x^3 + 90abd e^2x^3 + 45b^2d^2e x^3 + 60a^2d e^2x^2 + 120abd^2e x^2}{60}$$

input `int((e*x+d)^3*(b^2*x^2+2*a*b*x+a^2),x)`output `(x*(60*a**2*d**3 + 90*a**2*d**2*e*x + 60*a**2*d*e**2*x**2 + 15*a**2*e**3*x**3 + 60*a*b*d**3*x + 120*a*b*d**2*e*x**2 + 90*a*b*d*e**2*x**3 + 24*a*b*e**3*x**4 + 20*b**2*d**3*x**2 + 45*b**2*d**2*e*x**3 + 36*b**2*d*e**2*x**4 + 10*b**2*e**3*x**5))/60`

3.56 $\int (d + ex)^2 (a^2 + 2abx + b^2x^2) dx$

| | |
|---|-----|
| Optimal result | 476 |
| Mathematica [A] (verified) | 476 |
| Rubi [A] (verified) | 477 |
| Maple [A] (verified) | 478 |
| Fricas [A] (verification not implemented) | 479 |
| Sympy [A] (verification not implemented) | 479 |
| Maxima [A] (verification not implemented) | 480 |
| Giac [A] (verification not implemented) | 480 |
| Mupad [B] (verification not implemented) | 480 |
| Reduce [B] (verification not implemented) | 481 |

Optimal result

Integrand size = 24, antiderivative size = 65

$$\int (d + ex)^2 (a^2 + 2abx + b^2x^2) dx = \frac{(bd - ae)^2(a + bx)^3}{3b^3} + \frac{e(bd - ae)(a + bx)^4}{2b^3} + \frac{e^2(a + bx)^5}{5b^3}$$

output

```
1/3*(-a*e+b*d)^2*(b*x+a)^3/b^3+1/2*e*(-a*e+b*d)*(b*x+a)^4/b^3+1/5*e^2*(b*x+a)^5/b^3
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.22

$$\int (d + ex)^2 (a^2 + 2abx + b^2x^2) dx = a^2d^2x + ad(bd + ae)x^2 + \frac{1}{3}(b^2d^2 + 4abde + a^2e^2)x^3 + \frac{1}{2}be(bd + ae)x^4 + \frac{1}{5}b^2e^2x^5$$

input

```
Integrate[(d + e*x)^2*(a^2 + 2*a*b*x + b^2*x^2),x]
```

output

$$a^2 d^2 x + a d (b d + a e) x^2 + ((b^2 d^2 + 4 a b d e + a^2 e^2) x^3) / 3 + (b e (b d + a e) x^4) / 2 + (b^2 e^2 x^5) / 5$$

Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {1098, 27, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\begin{aligned} & \int (a^2 + 2abx + b^2x^2) (d + ex)^2 dx \\ & \quad \downarrow 1098 \\ & \int \frac{b^2(a + bx)^2(d + ex)^2 dx}{b^2} \\ & \quad \downarrow 27 \\ & \int (a + bx)^2(d + ex)^2 dx \\ & \quad \downarrow 49 \\ & \int \left(\frac{2e(a + bx)^3(bd - ae)}{b^2} + \frac{(a + bx)^2(bd - ae)^2}{b^2} + \frac{e^2(a + bx)^4}{b^2} \right) dx \\ & \quad \downarrow 2009 \\ & \frac{e(a + bx)^4(bd - ae)}{2b^3} + \frac{(a + bx)^3(bd - ae)^2}{3b^3} + \frac{e^2(a + bx)^5}{5b^3} \end{aligned}$$

input

```
Int[(d + e*x)^2*(a^2 + 2*a*b*x + b^2*x^2), x]
```

output

$$\frac{(b d - a e)^2 (a + b x)^3}{3 b^3} + \frac{e (b d - a e) (a + b x)^4}{2 b^3} + \frac{e^2 (a + b x)^5}{5 b^3}$$

Defintions of rubi rules used

rule 27 $\text{Int}[(a_*)(Fx_), x_Symbol] \rightarrow \text{Simp}[a \text{ Int}[Fx, x], x] /; \text{FreeQ}[a, x] \ \&\& \ !\text{MatchQ}[Fx, (b_*)(Gx_)] /; \text{FreeQ}[b, x]$

rule 49 $\text{Int}[((a_.) + (b_.)(x_))^{(m_.)} * ((c_.) + (d_.)(x_))^{(n_.)}, x_Symbol] \rightarrow \text{Int}[\text{ExpandIntegrand}[(a + b*x)^m * (c + d*x)^n, x], x] /; \text{FreeQ}[\{a, b, c, d\}, x] \ \&\& \ \text{IGtQ}[m, 0] \ \&\& \ \text{IGtQ}[m + n + 2, 0]$

rule 1098 $\text{Int}[((d_.) + (e_.)(x_))^{(m_.)} * ((a_) + (b_.)(x_) + (c_.)(x_)^2)^{(p_.)}, x_Symbol] \rightarrow \text{Simp}[1/c^p \text{ Int}[(d + e*x)^m * (b/2 + c*x)^{(2*p)}, x], x] /; \text{FreeQ}[\{a, b, c, d, e, m\}, x] \ \&\& \ \text{EqQ}[b^2 - 4*a*c, 0] \ \&\& \ \text{IntegerQ}[p]$

rule 2009 $\text{Int}[u_, x_Symbol] \rightarrow \text{Simp}[\text{IntSum}[u, x], x] /; \text{SumQ}[u]$

Maple [A] (verified)

Time = 0.55 (sec) , antiderivative size = 84, normalized size of antiderivative = 1.29

| method | result |
|--------------|---|
| norman | $\frac{b^2 e^2 x^5}{5} + \left(\frac{1}{2} a b e^2 + \frac{1}{2} d e b^2\right) x^4 + \left(\frac{1}{3} e^2 a^2 + \frac{4}{3} a b d e + \frac{1}{3} b^2 d^2\right) x^3 + (a^2 d e + a b d^2) x^2 + a^2 d^2 x$ |
| default | $\frac{b^2 e^2 x^5}{5} + \frac{(2 a b e^2 + 2 d e b^2) x^4}{4} + \frac{(e^2 a^2 + 4 a b d e + b^2 d^2) x^3}{3} + \frac{(2 a^2 d e + 2 a b d^2) x^2}{2} + a^2 d^2 x$ |
| risch | $\frac{1}{5} b^2 e^2 x^5 + \frac{1}{2} x^4 a b e^2 + \frac{1}{2} x^4 d e b^2 + \frac{1}{3} x^3 e^2 a^2 + \frac{4}{3} x^3 a b d e + \frac{1}{3} x^3 b^2 d^2 + d e a^2 x^2 + a b d^2 x^2 + a^2 d^2 x$ |
| paralelrisch | $\frac{1}{5} b^2 e^2 x^5 + \frac{1}{2} x^4 a b e^2 + \frac{1}{2} x^4 d e b^2 + \frac{1}{3} x^3 e^2 a^2 + \frac{4}{3} x^3 a b d e + \frac{1}{3} x^3 b^2 d^2 + d e a^2 x^2 + a b d^2 x^2 + a^2 d^2 x$ |
| gospers | $\frac{x(6 b^2 e^2 x^4 + 15 x^3 a b e^2 + 15 x^3 d e b^2 + 10 x^2 e^2 a^2 + 40 x^2 a b d e + 10 b^2 d^2 x^2 + 30 a^2 d e x + 30 a b d^2 x + 30 a^2 d^2)}{30}$ |
| orering | $\frac{x(6 b^2 e^2 x^4 + 15 x^3 a b e^2 + 15 x^3 d e b^2 + 10 x^2 e^2 a^2 + 40 x^2 a b d e + 10 b^2 d^2 x^2 + 30 a^2 d e x + 30 a b d^2 x + 30 a^2 d^2)(b^2 x^2 + 2 a b x + a^2)}{30(b x + a)^2}$ |

input $\text{int}((e*x+d)^2*(b^2*x^2+2*a*b*x+a^2), x, \text{method}=_RETURNVERBOSE)$

output $\frac{1}{5} b^2 e^2 x^5 + \left(\frac{1}{2} a b e^2 + \frac{1}{2} d e b^2\right) x^4 + \left(\frac{1}{3} e^2 a^2 + \frac{4}{3} a b d e + \frac{1}{3} b^2 d^2\right) x^3 + (a^2 d e + a b d^2) x^2 + a^2 d^2 x$

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.25

$$\int (d + ex)^2 (a^2 + 2abx + b^2x^2) dx = \frac{1}{5} b^2 e^2 x^5 + a^2 d^2 x + \frac{1}{2} (b^2 de + a b e^2) x^4 \\ + \frac{1}{3} (b^2 d^2 + 4 abde + a^2 e^2) x^3 + (abd^2 + a^2 de) x^2$$

input `integrate((e*x+d)^2*(b^2*x^2+2*a*b*x+a^2),x, algorithm="fricas")`output `1/5*b^2*e^2*x^5 + a^2*d^2*x + 1/2*(b^2*d*e + a*b*e^2)*x^4 + 1/3*(b^2*d^2 + 4*a*b*d*e + a^2*e^2)*x^3 + (a*b*d^2 + a^2*d*e)*x^2`**Sympy [A] (verification not implemented)**

Time = 0.03 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.34

$$\int (d + ex)^2 (a^2 + 2abx + b^2x^2) dx = a^2 d^2 x + \frac{b^2 e^2 x^5}{5} + x^4 \left(\frac{a b e^2}{2} + \frac{b^2 d e}{2} \right) \\ + x^3 \left(\frac{a^2 e^2}{3} + \frac{4 a b d e}{3} + \frac{b^2 d^2}{3} \right) + x^2 (a^2 d e + a b d^2)$$

input `integrate((e*x+d)**2*(b**2*x**2+2*a*b*x+a**2),x)`output `a**2*d**2*x + b**2*e**2*x**5/5 + x**4*(a*b*e**2/2 + b**2*d*e/2) + x**3*(a**2*e**2/3 + 4*a*b*d*e/3 + b**2*d**2/3) + x**2*(a**2*d*e + a*b*d**2)`

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.25

$$\int (d + ex)^2 (a^2 + 2abx + b^2x^2) dx = \frac{1}{5} b^2 e^2 x^5 + a^2 d^2 x + \frac{1}{2} (b^2 d e + a b e^2) x^4 + \frac{1}{3} (b^2 d^2 + 4 a b d e + a^2 e^2) x^3 + (a b d^2 + a^2 d e) x^2$$

input `integrate((e*x+d)^2*(b^2*x^2+2*a*b*x+a^2),x, algorithm="maxima")`

output `1/5*b^2*e^2*x^5 + a^2*d^2*x + 1/2*(b^2*d*e + a*b*e^2)*x^4 + 1/3*(b^2*d^2 + 4*a*b*d*e + a^2*e^2)*x^3 + (a*b*d^2 + a^2*d*e)*x^2`

Giac [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.37

$$\int (d + ex)^2 (a^2 + 2abx + b^2x^2) dx = \frac{1}{5} b^2 e^2 x^5 + \frac{1}{2} b^2 d e x^4 + \frac{1}{2} a b e^2 x^4 + \frac{1}{3} b^2 d^2 x^3 + \frac{4}{3} a b d e x^3 + \frac{1}{3} a^2 e^2 x^3 + a b d^2 x^2 + a^2 d e x^2 + a^2 d^2 x$$

input `integrate((e*x+d)^2*(b^2*x^2+2*a*b*x+a^2),x, algorithm="giac")`

output `1/5*b^2*e^2*x^5 + 1/2*b^2*d*e*x^4 + 1/2*a*b*e^2*x^4 + 1/3*b^2*d^2*x^3 + 4/3*a*b*d*e*x^3 + 1/3*a^2*e^2*x^3 + a*b*d^2*x^2 + a^2*d*e*x^2 + a^2*d^2*x`

Mupad [B] (verification not implemented)

Time = 9.04 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.14

$$\int (d + ex)^2 (a^2 + 2abx + b^2x^2) dx = x^3 \left(\frac{a^2 e^2}{3} + \frac{4 a b d e}{3} + \frac{b^2 d^2}{3} \right) + a^2 d^2 x + \frac{b^2 e^2 x^5}{5} + a d x^2 (a e + b d) + \frac{b e x^4 (a e + b d)}{2}$$

input `int((d + e*x)^2*(a^2 + b^2*x^2 + 2*a*b*x),x)`

output `x^3*((a^2*e^2)/3 + (b^2*d^2)/3 + (4*a*b*d*e)/3) + a^2*d^2*x + (b^2*e^2*x^5)/5 + a*d*x^2*(a*e + b*d) + (b*e*x^4*(a*e + b*d))/2`

Reduce [B] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.38

$$\int (d + ex)^2 (a^2 + 2abx + b^2x^2) dx$$

$$= \frac{x(6b^2e^2x^4 + 15ab e^2x^3 + 15b^2de x^3 + 10a^2e^2x^2 + 40abde x^2 + 10b^2d^2x^2 + 30a^2dex + 30ab d^2x + 30a^2d^2)}{30}$$

input `int((e*x+d)^2*(b^2*x^2+2*a*b*x+a^2),x)`

output `(x*(30*a**2*d**2 + 30*a**2*d*e*x + 10*a**2*e**2*x**2 + 30*a*b*d**2*x + 40*a*b*d*e*x**2 + 15*a*b*e**2*x**3 + 10*b**2*d**2*x**2 + 15*b**2*d*e*x**3 + 6*b**2*e**2*x**4))/30`

3.57 $\int (d + ex) (a^2 + 2abx + b^2x^2) dx$

| | |
|---|-----|
| Optimal result | 482 |
| Mathematica [A] (verified) | 482 |
| Rubi [A] (verified) | 483 |
| Maple [A] (verified) | 484 |
| Fricas [A] (verification not implemented) | 485 |
| Sympy [A] (verification not implemented) | 485 |
| Maxima [A] (verification not implemented) | 485 |
| Giac [A] (verification not implemented) | 486 |
| Mupad [B] (verification not implemented) | 486 |
| Reduce [B] (verification not implemented) | 486 |

Optimal result

Integrand size = 22, antiderivative size = 38

$$\int (d + ex) (a^2 + 2abx + b^2x^2) dx = \frac{(bd - ae)(a + bx)^3}{3b^2} + \frac{e(a + bx)^4}{4b^2}$$

output

```
1/3*(-a*e+b*d)*(b*x+a)^3/b^2+1/4*e*(b*x+a)^4/b^2
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.21

$$\int (d + ex) (a^2 + 2abx + b^2x^2) dx = \frac{1}{12}x(6a^2(2d + ex) + 4abx(3d + 2ex) + b^2x^2(4d + 3ex))$$

input

```
Integrate[(d + e*x)*(a^2 + 2*a*b*x + b^2*x^2),x]
```

output

```
(x*(6*a^2*(2*d + e*x) + 4*a*b*x*(3*d + 2*e*x) + b^2*x^2*(4*d + 3*e*x)))/12
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {1098, 27, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\begin{aligned}
 & \int (a^2 + 2abx + b^2x^2)(d + ex) dx \\
 & \quad \downarrow 1098 \\
 & \quad \int \frac{b^2(a + bx)^2(d + ex)dx}{b^2} \\
 & \quad \downarrow 27 \\
 & \quad \int (a + bx)^2(d + ex)dx \\
 & \quad \downarrow 49 \\
 & \quad \int \left(\frac{(a + bx)^2(bd - ae)}{b} + \frac{e(a + bx)^3}{b} \right) dx \\
 & \quad \downarrow 2009 \\
 & \quad \frac{(a + bx)^3(bd - ae)}{3b^2} + \frac{e(a + bx)^4}{4b^2}
 \end{aligned}$$

input `Int[(d + e*x)*(a^2 + 2*a*b*x + b^2*x^2),x]`

output `((b*d - a*e)*(a + b*x)^3)/(3*b^2) + (e*(a + b*x)^4)/(4*b^2)`

Definitions of rubi rules used

rule 27 $\text{Int}[(a_*)(Fx_), x_Symbol] \rightarrow \text{Simp}[a \text{ Int}[Fx, x], x] /; \text{FreeQ}[a, x] \ \&\& \ !\text{MatchQ}[Fx, (b_*)(Gx_)] /; \text{FreeQ}[b, x]$

rule 49 $\text{Int}[((a_.) + (b_.)(x_.))^{(m_.)} * ((c_.) + (d_.)(x_.))^{(n_.)}, x_Symbol] \rightarrow \text{Int}[\text{ExpandIntegrand}[(a + b*x)^m * (c + d*x)^n, x], x] /; \text{FreeQ}[\{a, b, c, d\}, x] \ \&\& \ \text{IGtQ}[m, 0] \ \&\& \ \text{IGtQ}[m + n + 2, 0]$

rule 1098 $\text{Int}[((d_.) + (e_.)(x_.))^{(m_.)} * ((a_) + (b_.)(x_) + (c_.)(x_)^2)^{(p_.)}, x_Symbol] \rightarrow \text{Simp}[1/c^p \text{ Int}[(d + e*x)^m * (b/2 + c*x)^{(2*p)}, x], x] /; \text{FreeQ}[\{a, b, c, d, e, m\}, x] \ \&\& \ \text{EqQ}[b^2 - 4*a*c, 0] \ \&\& \ \text{IntegerQ}[p]$

rule 2009 $\text{Int}[u_, x_Symbol] \rightarrow \text{Simp}[\text{IntSum}[u, x], x] /; \text{SumQ}[u]$

Maple [A] (verified)

Time = 0.27 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.26

| method | result | size |
|---------------|---|------|
| norman | $\frac{b^2 e x^4}{4} + \left(\frac{2}{3} a b e + \frac{1}{3} b^2 d\right) x^3 + \left(\frac{1}{2} a^2 e + d a b\right) x^2 + a^2 d x$ | 48 |
| default | $\frac{b^2 e x^4}{4} + \frac{(2 a b e + b^2 d) x^3}{3} + \frac{(a^2 e + 2 d a b) x^2}{2} + a^2 d x$ | 49 |
| gosper | $\frac{x(3 b^2 e x^3 + 8 a x^2 e b + 4 b^2 d x^2 + 6 a^2 e x + 12 a b d x + 12 a^2 d)}{12}$ | 50 |
| risch | $\frac{1}{4} b^2 e x^4 + \frac{2}{3} a b e x^3 + \frac{1}{3} b^2 d x^3 + \frac{1}{2} a^2 e x^2 + a b d x^2 + a^2 d x$ | 50 |
| parallelrisch | $\frac{1}{4} b^2 e x^4 + \frac{2}{3} a b e x^3 + \frac{1}{3} b^2 d x^3 + \frac{1}{2} a^2 e x^2 + a b d x^2 + a^2 d x$ | 50 |
| orering | $\frac{x(3 b^2 e x^3 + 8 a x^2 e b + 4 b^2 d x^2 + 6 a^2 e x + 12 a b d x + 12 a^2 d)(b^2 x^2 + 2 a b x + a^2)}{12(b x + a)^2}$ | 73 |

input $\text{int}((e*x+d)*(b^2*x^2+2*a*b*x+a^2), x, \text{method}=_RETURNVERBOSE)$

output $1/4*b^2*e*x^4+(2/3*a*b*e+1/3*b^2*d)*x^3+(1/2*a^2*e+d*a*b)*x^2+a^2*d*x$

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.26

$$\int (d+ex) (a^2+2abx+b^2x^2) dx = \frac{1}{4} b^2 ex^4 + a^2 dx + \frac{1}{3} (b^2 d + 2 abe) x^3 + \frac{1}{2} (2 abd + a^2 e) x^2$$

input `integrate((e*x+d)*(b^2*x^2+2*a*b*x+a^2),x, algorithm="fricas")`

output `1/4*b^2*e*x^4 + a^2*d*x + 1/3*(b^2*d + 2*a*b*e)*x^3 + 1/2*(2*a*b*d + a^2*e)*x^2`

Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.29

$$\int (d+ex) (a^2+2abx+b^2x^2) dx = a^2 dx + \frac{b^2 ex^4}{4} + x^3 \cdot \left(\frac{2abe}{3} + \frac{b^2 d}{3} \right) + x^2 \left(\frac{a^2 e}{2} + abd \right)$$

input `integrate((e*x+d)*(b**2*x**2+2*a*b*x+a**2),x)`

output `a**2*d*x + b**2*e*x**4/4 + x**3*(2*a*b*e/3 + b**2*d/3) + x**2*(a**2*e/2 + a*b*d)`

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.26

$$\int (d+ex) (a^2+2abx+b^2x^2) dx = \frac{1}{4} b^2 ex^4 + a^2 dx + \frac{1}{3} (b^2 d + 2 abe) x^3 + \frac{1}{2} (2 abd + a^2 e) x^2$$

input `integrate((e*x+d)*(b^2*x^2+2*a*b*x+a^2),x, algorithm="maxima")`

output `1/4*b^2*e*x^4 + a^2*d*x + 1/3*(b^2*d + 2*a*b*e)*x^3 + 1/2*(2*a*b*d + a^2*e)*x^2`

Giac [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.29

$$\int (d+ex)(a^2+2abx+b^2x^2) dx = \frac{1}{4}b^2ex^4 + \frac{1}{3}b^2dx^3 + \frac{2}{3}abex^3 + abdx^2 + \frac{1}{2}a^2ex^2 + a^2dx$$

input `integrate((e*x+d)*(b^2*x^2+2*a*b*x+a^2),x, algorithm="giac")`output `1/4*b^2*e*x^4 + 1/3*b^2*d*x^3 + 2/3*a*b*e*x^3 + a*b*d*x^2 + 1/2*a^2*e*x^2 + a^2*d*x`**Mupad [B] (verification not implemented)**

Time = 0.05 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.24

$$\int (d+ex)(a^2+2abx+b^2x^2) dx = x^2 \left(\frac{ea^2}{2} + bda \right) + x^3 \left(\frac{db^2}{3} + \frac{2aeb}{3} \right) + \frac{b^2ex^4}{4} + a^2dx$$

input `int((d + e*x)*(a^2 + b^2*x^2 + 2*a*b*x),x)`output `x^2*((a^2*e)/2 + a*b*d) + x^3*((b^2*d)/3 + (2*a*b*e)/3) + (b^2*e*x^4)/4 + a^2*d*x`**Reduce [B] (verification not implemented)**

Time = 0.25 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.29

$$\begin{aligned} & \int (d+ex)(a^2+2abx+b^2x^2) dx \\ &= \frac{x(3b^2ex^3 + 8abex^2 + 4b^2dx^2 + 6a^2ex + 12abdx + 12a^2d)}{12} \end{aligned}$$

input `int((e*x+d)*(b^2*x^2+2*a*b*x+a^2),x)`

output

```
(x*(12*a**2*d + 6*a**2*e*x + 12*a*b*d*x + 8*a*b*e*x**2 + 4*b**2*d*x**2 + 3
*b**2*e*x**3))/12
```

3.58 $\int (a^2 + 2abx + b^2x^2) dx$

| | |
|---|-----|
| Optimal result | 488 |
| Mathematica [A] (verified) | 488 |
| Rubi [A] (verified) | 489 |
| Maple [A] (verified) | 490 |
| Fricas [A] (verification not implemented) | 490 |
| Sympy [B] (verification not implemented) | 491 |
| Maxima [A] (verification not implemented) | 491 |
| Giac [A] (verification not implemented) | 491 |
| Mupad [B] (verification not implemented) | 492 |
| Reduce [B] (verification not implemented) | 492 |

Optimal result

Integrand size = 16, antiderivative size = 14

$$\int (a^2 + 2abx + b^2x^2) dx = \frac{(a + bx)^3}{3b}$$

output

```
1/3*(b*x+a)^3/b
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.57

$$\int (a^2 + 2abx + b^2x^2) dx = a^2x + abx^2 + \frac{b^2x^3}{3}$$

input

```
Integrate[a^2 + 2*a*b*x + b^2*x^2,x]
```

output

```
a^2*x + a*b*x^2 + (b^2*x^3)/3
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {1077, 17}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a^2 + 2abx + b^2x^2) dx$$

$$\downarrow 1077$$

$$\frac{\int (xb^2 + ab)^2 dx}{b^2}$$

$$\downarrow 17$$

$$\frac{(a + bx)^3}{3b}$$

input

```
Int[a^2 + 2*a*b*x + b^2*x^2,x]
```

output

```
(a + b*x)^3/(3*b)
```

Defintions of rubi rules used

rule 17

```
Int[(c_.)*((a_.) + (b_.)*(x_)^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1)
)/(b*(m + 1))), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]
```

rule 1077

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[1/c^p Int
[(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c}, x] && EqQ[b^2 - 4*a*c, 0] &&
IntegerQ[p]
```

Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.93

| method | result | size |
|---------------|--|------|
| default | $\frac{(bx+a)^3}{3b}$ | 13 |
| norman | $\frac{1}{3}b^2x^3 + abx^2 + a^2x$ | 21 |
| risch | $\frac{1}{3}b^2x^3 + abx^2 + a^2x$ | 21 |
| parallelrisch | $\frac{1}{3}b^2x^3 + abx^2 + a^2x$ | 21 |
| parts | $\frac{1}{3}b^2x^3 + abx^2 + a^2x$ | 21 |
| gospers | $\frac{x(b^2x^2+3abx+3a^2)}{3}$ | 22 |
| orering | $\frac{x(b^2x^2+3abx+3a^2)(b^2x^2+2abx+a^2)}{3(bx+a)^2}$ | 45 |

input `int(b^2*x^2+2*a*b*x+a^2,x,method=_RETURNVERBOSE)`output `1/3*(b*x+a)^3/b`**Fricas [A] (verification not implemented)**

Time = 0.08 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.43

$$\int (a^2 + 2abx + b^2x^2) dx = \frac{1}{3}b^2x^3 + abx^2 + a^2x$$

input `integrate(b^2*x^2+2*a*b*x+a^2,x, algorithm="fricas")`output `1/3*b^2*x^3 + a*b*x^2 + a^2*x`

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 19 vs. $2(8) = 16$.

Time = 0.02 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.36

$$\int (a^2 + 2abx + b^2x^2) dx = a^2x + abx^2 + \frac{b^2x^3}{3}$$

input `integrate(b**2*x**2+2*a*b*x+a**2,x)`

output `a**2*x + a*b*x**2 + b**2*x**3/3`

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.43

$$\int (a^2 + 2abx + b^2x^2) dx = \frac{1}{3}b^2x^3 + abx^2 + a^2x$$

input `integrate(b^2*x^2+2*a*b*x+a^2,x, algorithm="maxima")`

output `1/3*b^2*x^3 + a*b*x^2 + a^2*x`

Giac [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.43

$$\int (a^2 + 2abx + b^2x^2) dx = \frac{1}{3}b^2x^3 + abx^2 + a^2x$$

input `integrate(b^2*x^2+2*a*b*x+a^2,x, algorithm="giac")`

output `1/3*b^2*x^3 + a*b*x^2 + a^2*x`

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.43

$$\int (a^2 + 2abx + b^2x^2) dx = a^2x + abx^2 + \frac{b^2x^3}{3}$$

input `int(a^2 + b^2*x^2 + 2*a*b*x,x)`

output `a^2*x + (b^2*x^3)/3 + a*b*x^2`

Reduce [B] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.50

$$\int (a^2 + 2abx + b^2x^2) dx = \frac{x(b^2x^2 + 3abx + 3a^2)}{3}$$

input `int(b^2*x^2+2*a*b*x+a^2,x)`

output `(x*(3*a**2 + 3*a*b*x + b**2*x**2))/3`

3.59 $\int \frac{a^2 + 2abx + b^2x^2}{d + ex} dx$

| | |
|---|-----|
| Optimal result | 493 |
| Mathematica [A] (verified) | 493 |
| Rubi [A] (verified) | 494 |
| Maple [A] (verified) | 495 |
| Fricas [A] (verification not implemented) | 496 |
| Sympy [A] (verification not implemented) | 496 |
| Maxima [A] (verification not implemented) | 496 |
| Giac [A] (verification not implemented) | 497 |
| Mupad [B] (verification not implemented) | 497 |
| Reduce [B] (verification not implemented) | 498 |

Optimal result

Integrand size = 24, antiderivative size = 49

$$\int \frac{a^2 + 2abx + b^2x^2}{d + ex} dx = -\frac{b(bd - 2ae)x}{e^2} + \frac{b^2x^2}{2e} + \frac{(bd - ae)^2 \log(d + ex)}{e^3}$$

output

```
-b*(-2*a*e+b*d)*x/e^2+1/2*b^2*x^2/e+(-a*e+b*d)^2*ln(e*x+d)/e^3
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.88

$$\int \frac{a^2 + 2abx + b^2x^2}{d + ex} dx = \frac{bex(-2bd + 4ae + bex) + 2(bd - ae)^2 \log(d + ex)}{2e^3}$$

input

```
Integrate[(a^2 + 2*a*b*x + b^2*x^2)/(d + e*x),x]
```

output

```
(b*e*x*(-2*b*d + 4*a*e + b*e*x) + 2*(b*d - a*e)^2*Log[d + e*x])/(2*e^3)
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.02, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {1098, 27, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\begin{aligned}
 & \int \frac{a^2 + 2abx + b^2x^2}{d + ex} dx \\
 & \quad \downarrow \text{1098} \\
 & \int \frac{b^2(a+bx)^2}{d+ex} dx \\
 & \quad \downarrow \text{27} \\
 & \int \frac{(a + bx)^2}{d + ex} dx \\
 & \quad \downarrow \text{49} \\
 & \int \left(\frac{(ae - bd)^2}{e^2(d + ex)} - \frac{b(bd - ae)}{e^2} + \frac{b(a + bx)}{e} \right) dx \\
 & \quad \downarrow \text{2009} \\
 & \frac{(bd - ae)^2 \log(d + ex)}{e^3} - \frac{bx(bd - ae)}{e^2} + \frac{(a + bx)^2}{2e}
 \end{aligned}$$

input `Int[(a^2 + 2*a*b*x + b^2*x^2)/(d + e*x),x]`

output `-((b*(b*d - a*e)*x)/e^2) + (a + b*x)^2/(2*e) + ((b*d - a*e)^2*Log[d + e*x])/e^3`

Defintions of rubi rules used

rule 27 `Int[(a_)*(F x_), x_Symbol] := Simp[a Int[F x, x], x] /; FreeQ[a, x] && !MatchQ[F x, (b_)*(G x_) /; FreeQ[b, x]]`

rule 49 `Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] := Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] && IGtQ[m, 0] && IGtQ[m + n + 2, 0]`

rule 1098 `Int[((d_.) + (e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] := Simp[1/c^p Int[(d + e*x)^m*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]`

rule 2009 `Int[u_, x_Symbol] := Simp[IntSum[u, x], x] /; SumQ[u]`

Maple [A] (verified)

Time = 0.69 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.14

| method | result | size |
|---------------|---|------|
| default | $\frac{b(\frac{1}{2}be x^2 + 2aex - bdx)}{e^2} + \frac{(e^2 a^2 - 2abde + b^2 d^2) \ln(ex+d)}{e^3}$ | 56 |
| norman | $\frac{b(2ae - bd)x}{e^2} + \frac{b^2 x^2}{2e} + \frac{(e^2 a^2 - 2abde + b^2 d^2) \ln(ex+d)}{e^3}$ | 59 |
| parallelrisch | $\frac{b^2 e^2 x^2 + 2 \ln(ex+d) a^2 e^2 - 4 \ln(ex+d) abde + 2 \ln(ex+d) b^2 d^2 + 4xab e^2 - 2b^2 dex}{2e^3}$ | 73 |
| risch | $\frac{b^2 x^2}{2e} + \frac{2bax}{e} - \frac{b^2 dx}{e^2} + \frac{\ln(ex+d)a^2}{e} - \frac{2 \ln(ex+d)abd}{e^2} + \frac{\ln(ex+d)b^2 d^2}{e^3}$ | 74 |

input `int((b^2*x^2+2*a*b*x+a^2)/(e*x+d),x,method=_RETURNVERBOSE)`

output `b/e^2*(1/2*b*e*x^2+2*a*e*x-b*d*x)+(a^2*e^2-2*a*b*d*e+b^2*d^2)/e^3*ln(e*x+d)`

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.27

$$\int \frac{a^2 + 2abx + b^2x^2}{d + ex} dx = \frac{b^2e^2x^2 - 2(b^2de - 2abe^2)x + 2(b^2d^2 - 2abde + a^2e^2) \log(ex + d)}{2e^3}$$

input `integrate((b^2*x^2+2*a*b*x+a^2)/(e*x+d),x, algorithm="fricas")`output `1/2*(b^2*e^2*x^2 - 2*(b^2*d*e - 2*a*b*e^2)*x + 2*(b^2*d^2 - 2*a*b*d*e + a^2*e^2)*log(e*x + d))/e^3`**Sympy [A] (verification not implemented)**

Time = 0.11 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.90

$$\int \frac{a^2 + 2abx + b^2x^2}{d + ex} dx = \frac{b^2x^2}{2e} + x \left(\frac{2ab}{e} - \frac{b^2d}{e^2} \right) + \frac{(ae - bd)^2 \log(d + ex)}{e^3}$$

input `integrate((b**2*x**2+2*a*b*x+a**2)/(e*x+d),x)`output `b**2*x**2/(2*e) + x*(2*a*b/e - b**2*d/e**2) + (a*e - b*d)**2*log(d + e*x)/e**3`**Maxima [A] (verification not implemented)**

Time = 0.03 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.22

$$\int \frac{a^2 + 2abx + b^2x^2}{d + ex} dx = \frac{b^2ex^2 - 2(b^2d - 2abe)x + (b^2d^2 - 2abde + a^2e^2) \log(ex + d)}{2e^2}$$

input `integrate((b^2*x^2+2*a*b*x+a^2)/(e*x+d),x, algorithm="maxima")`

output

$$\frac{1}{2}(b^2 e x^2 - 2(b^2 d - 2 a b e)x)/e^2 + (b^2 d^2 - 2 a b d e + a^2 e^2) \log(e x + d)/e^3$$

Giac [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.22

$$\int \frac{a^2 + 2 a b x + b^2 x^2}{d + e x} dx = \frac{b^2 e x^2 - 2 b^2 d x + 4 a b e x}{2 e^2} + \frac{(b^2 d^2 - 2 a b d e + a^2 e^2) \log(|e x + d|)}{e^3}$$

input

```
integrate((b^2*x^2+2*a*b*x+a^2)/(e*x+d),x, algorithm="giac")
```

output

$$\frac{1}{2}(b^2 e x^2 - 2 b^2 d x + 4 a b e x)/e^2 + (b^2 d^2 - 2 a b d e + a^2 e^2) \log(\text{abs}(e x + d))/e^3$$

Mupad [B] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.27

$$\int \frac{a^2 + 2 a b x + b^2 x^2}{d + e x} dx = \frac{\ln(d + e x) (a^2 e^2 - 2 a b d e + b^2 d^2)}{e^3} - x \left(\frac{b^2 d}{e^2} - \frac{2 a b}{e} \right) + \frac{b^2 x^2}{2 e}$$

input

```
int((a^2 + b^2*x^2 + 2*a*b*x)/(d + e*x),x)
```

output

$$\frac{(\log(d + e x) * (a^2 e^2 + b^2 d^2 - 2 a b d e))}{e^3} - x * ((b^2 d)/e^2 - (2 a b)/e) + (b^2 x^2)/(2 e)$$

Reduce [B] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.47

$$\int \frac{a^2 + 2abx + b^2x^2}{d + ex} dx$$

$$= \frac{2 \log(ex + d) a^2 e^2 - 4 \log(ex + d) abde + 2 \log(ex + d) b^2 d^2 + 4ab e^2 x - 2b^2 dex + b^2 e^2 x^2}{2e^3}$$

input `int((b^2*x^2+2*a*b*x+a^2)/(e*x+d),x)`

output `(2*log(d + e*x)*a**2*e**2 - 4*log(d + e*x)*a*b*d*e + 2*log(d + e*x)*b**2*d**2 + 4*a*b*e**2*x - 2*b**2*d*e*x + b**2*e**2*x**2)/(2*e**3)`

3.60 $\int \frac{a^2 + 2abx + b^2x^2}{(d+ex)^2} dx$

| | |
|---|-----|
| Optimal result | 499 |
| Mathematica [A] (verified) | 499 |
| Rubi [A] (verified) | 500 |
| Maple [A] (verified) | 501 |
| Fricas [A] (verification not implemented) | 502 |
| Sympy [A] (verification not implemented) | 502 |
| Maxima [A] (verification not implemented) | 502 |
| Giac [B] (verification not implemented) | 503 |
| Mupad [B] (verification not implemented) | 503 |
| Reduce [B] (verification not implemented) | 504 |

Optimal result

Integrand size = 24, antiderivative size = 51

$$\int \frac{a^2 + 2abx + b^2x^2}{(d+ex)^2} dx = \frac{b^2x}{e^2} - \frac{(bd-ae)^2}{e^3(d+ex)} - \frac{2b(bd-ae)\log(d+ex)}{e^3}$$

output $b^2x/e^2 - (-a*e+b*d)^2/e^3/(e*x+d) - 2*b*(-a*e+b*d)*\ln(e*x+d)/e^3$

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.92

$$\int \frac{a^2 + 2abx + b^2x^2}{(d+ex)^2} dx = \frac{b^2ex - \frac{(bd-ae)^2}{d+ex} + 2b(-bd+ae)\log(d+ex)}{e^3}$$

input `Integrate[(a^2 + 2*a*b*x + b^2*x^2)/(d + e*x)^2,x]`

output $(b^2*e*x - (b*d - a*e)^2/(d + e*x) + 2*b*(-(b*d) + a*e)*\text{Log}[d + e*x])/e^3$

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {1098, 27, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\begin{aligned}
 & \int \frac{a^2 + 2abx + b^2x^2}{(d + ex)^2} dx \\
 & \quad \downarrow 1098 \\
 & \int \frac{b^2(a+bx)^2}{(d+ex)^2} dx \\
 & \quad \downarrow 27 \\
 & \int \frac{(a + bx)^2}{(d + ex)^2} dx \\
 & \quad \downarrow 49 \\
 & \int \left(-\frac{2b(bd - ae)}{e^2(d + ex)} + \frac{(ae - bd)^2}{e^2(d + ex)^2} + \frac{b^2}{e^2} \right) dx \\
 & \quad \downarrow 2009 \\
 & -\frac{(bd - ae)^2}{e^3(d + ex)} - \frac{2b(bd - ae) \log(d + ex)}{e^3} + \frac{b^2x}{e^2}
 \end{aligned}$$

input `Int[(a^2 + 2*a*b*x + b^2*x^2)/(d + e*x)^2,x]`

output `(b^2*x)/e^2 - (b*d - a*e)^2/(e^3*(d + e*x)) - (2*b*(b*d - a*e)*Log[d + e*x])/e^3`

Definitions of rubi rules used

rule 27 $\text{Int}[(a_*)(Fx_), x_Symbol] \rightarrow \text{Simp}[a \text{ Int}[Fx, x], x] /; \text{FreeQ}[a, x] \ \&\& \ !\text{MatchQ}[Fx, (b_*)(Gx_)] /; \text{FreeQ}[b, x]$

rule 49 $\text{Int}[((a_.) + (b_.)(x_))^{(m_.)}((c_.) + (d_.)(x_))^{(n_.)}, x_Symbol] \rightarrow \text{Int}[\text{ExpandIntegrand}[(a + b*x)^m*(c + d*x)^n, x], x] /; \text{FreeQ}[\{a, b, c, d\}, x] \ \&\& \ \text{IGtQ}[m, 0] \ \&\& \ \text{IGtQ}[m + n + 2, 0]$

rule 1098 $\text{Int}[((d_.) + (e_.)(x_))^{(m_.)}((a_) + (b_.)(x_) + (c_.)(x_)^2)^{(p_.)}, x_Symbol] \rightarrow \text{Simp}[1/c^p \text{ Int}[(d + e*x)^m*(b/2 + c*x)^{(2*p)}, x], x] /; \text{FreeQ}[\{a, b, c, d, e, m\}, x] \ \&\& \ \text{EqQ}[b^2 - 4*a*c, 0] \ \&\& \ \text{IntegerQ}[p]$

rule 2009 $\text{Int}[u_, x_Symbol] \rightarrow \text{Simp}[\text{IntSum}[u, x], x] /; \text{SumQ}[u]$

Maple [A] (verified)

Time = 0.67 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.24

| method | result | size |
|---------------|--|------|
| default | $\frac{b^2x}{e^2} + \frac{2b(ae-bd)\ln(ex+d)}{e^3} - \frac{e^2a^2-2abde+b^2d^2}{e^3(ex+d)}$ | 63 |
| norman | $\frac{\frac{b^2x^2}{e} - \frac{e^2a^2-2abde+2b^2d^2}{e^3}}{ex+d} + \frac{2b(ae-bd)\ln(ex+d)}{e^3}$ | 68 |
| risch | $\frac{b^2x}{e^2} + \frac{2b\ln(ex+d)a}{e^2} - \frac{2b^2\ln(ex+d)d}{e^3} - \frac{a^2}{e(ex+d)} + \frac{2abd}{e^2(ex+d)} - \frac{b^2d^2}{e^3(ex+d)}$ | 86 |
| parallelrisch | $\frac{2\ln(ex+d)xab e^2 - 2\ln(ex+d)xb^2de + b^2e^2x^2 + 2\ln(ex+d)abde - 2\ln(ex+d)b^2d^2 - e^2a^2 + 2abde - 2b^2d^2}{e^3(ex+d)}$ | 99 |

input $\text{int}((b^2*x^2+2*a*b*x+a^2)/(e*x+d)^2, x, \text{method}=_RETURNVERBOSE)$

output $b^2*x/e^2+2*b/e^3*(a*e-b*d)*\ln(e*x+d)-(a^2*e^2-2*a*b*d*e+b^2*d^2)/e^3/(e*x+d)$

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.80

$$\int \frac{a^2 + 2abx + b^2x^2}{(d + ex)^2} dx = \frac{b^2e^2x^2 + b^2dex - b^2d^2 + 2abde - a^2e^2 - 2(b^2d^2 - abde + (b^2de - abe^2)x) \log(ex + d)}{e^4x + de^3}$$

input `integrate((b^2*x^2+2*a*b*x+a^2)/(e*x+d)^2,x, algorithm="fricas")`output `(b^2*e^2*x^2 + b^2*d*e*x - b^2*d^2 + 2*a*b*d*e - a^2*e^2 - 2*(b^2*d^2 - a*b*d*e + (b^2*d*e - a*b*e^2)*x)*log(e*x + d))/(e^4*x + d*e^3)`**Sympy [A] (verification not implemented)**

Time = 0.17 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.18

$$\int \frac{a^2 + 2abx + b^2x^2}{(d + ex)^2} dx = \frac{b^2x}{e^2} + \frac{2b(ae - bd) \log(d + ex)}{e^3} + \frac{-a^2e^2 + 2abde - b^2d^2}{de^3 + e^4x}$$

input `integrate((b**2*x**2+2*a*b*x+a**2)/(e*x+d)**2,x)`output `b**2*x/e**2 + 2*b*(a*e - b*d)*log(d + e*x)/e**3 + (-a**2*e**2 + 2*a*b*d*e - b**2*d**2)/(d*e**3 + e**4*x)`**Maxima [A] (verification not implemented)**

Time = 0.04 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.31

$$\int \frac{a^2 + 2abx + b^2x^2}{(d + ex)^2} dx = \frac{b^2x}{e^2} - \frac{b^2d^2 - 2abde + a^2e^2}{e^4x + de^3} - \frac{2(b^2d - abe) \log(ex + d)}{e^3}$$

input `integrate((b^2*x^2+2*a*b*x+a^2)/(e*x+d)^2,x, algorithm="maxima")`

output

```
b^2*x/e^2 - (b^2*d^2 - 2*a*b*d*e + a^2*e^2)/(e^4*x + d*e^3) - 2*(b^2*d - a
*b*e)*log(e*x + d)/e^3
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 114 vs. $2(51) = 102$.

Time = 0.21 (sec) , antiderivative size = 114, normalized size of antiderivative = 2.24

$$\int \frac{a^2 + 2abx + b^2x^2}{(d + ex)^2} dx = b^2 \left(\frac{2d \log\left(\frac{|ex+d|}{(ex+d)^2|e|}\right)}{e^3} + \frac{ex+d}{e^3} - \frac{d^2}{(ex+d)e^3} \right) - \frac{2ab \left(\frac{\log\left(\frac{|ex+d|}{(ex+d)^2|e|}\right)}{e} - \frac{d}{(ex+d)e} \right)}{e} - \frac{a^2}{(ex+d)e}$$

input

```
integrate((b^2*x^2+2*a*b*x+a^2)/(e*x+d)^2,x, algorithm="giac")
```

output

```
b^2*(2*d*log(abs(e*x + d)/((e*x + d)^2*abs(e)))/e^3 + (e*x + d)/e^3 - d^2/
((e*x + d)*e^3)) - 2*a*b*(log(abs(e*x + d)/((e*x + d)^2*abs(e)))/e - d/((e
*x + d)*e))/e - a^2/((e*x + d)*e)
```

Mupad [B] (verification not implemented)

Time = 8.76 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.39

$$\int \frac{a^2 + 2abx + b^2x^2}{(d + ex)^2} dx = \frac{b^2x}{e^2} - \frac{a^2e^2 - 2abde + b^2d^2}{e(xe^3 + de^2)} - \frac{\ln(d + ex)(2b^2d - 2abe)}{e^3}$$

input

```
int((a^2 + b^2*x^2 + 2*a*b*x)/(d + e*x)^2,x)
```

output

```
(b^2*x)/e^2 - (a^2*e^2 + b^2*d^2 - 2*a*b*d*e)/(e*(d*e^2 + e^3*x)) - (log(d
+ e*x)*(2*b^2*d - 2*a*b*e))/e^3
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 112, normalized size of antiderivative = 2.20

$$\int \frac{a^2 + 2abx + b^2x^2}{(d + ex)^2} dx$$

$$= \frac{2 \log(ex + d) ab d^2 e + 2 \log(ex + d) abd e^2 x - 2 \log(ex + d) b^2 d^3 - 2 \log(ex + d) b^2 d^2 ex + a^2 e^3 x - 2abd e}{d e^3 (ex + d)}$$

input `int((b^2*x^2+2*a*b*x+a^2)/(e*x+d)^2,x)`output `(2*log(d + e*x)*a*b*d**2*e + 2*log(d + e*x)*a*b*d*e**2*x - 2*log(d + e*x)*b**2*d**3 - 2*log(d + e*x)*b**2*d**2*e*x + a**2*e**3*x - 2*a*b*d*e**2*x + 2*b**2*d**2*e*x + b**2*d*e**2*x**2)/(d*e**3*(d + e*x))`

3.61 $\int \frac{a^2+2abx+b^2x^2}{(d+ex)^3} dx$

| | |
|---|-----|
| Optimal result | 505 |
| Mathematica [A] (verified) | 505 |
| Rubi [A] (verified) | 506 |
| Maple [A] (verified) | 507 |
| Fricas [A] (verification not implemented) | 508 |
| Sympy [A] (verification not implemented) | 508 |
| Maxima [A] (verification not implemented) | 508 |
| Giac [A] (verification not implemented) | 509 |
| Mupad [B] (verification not implemented) | 509 |
| Reduce [B] (verification not implemented) | 510 |

Optimal result

Integrand size = 24, antiderivative size = 59

$$\int \frac{a^2 + 2abx + b^2x^2}{(d + ex)^3} dx = -\frac{(bd - ae)^2}{2e^3(d + ex)^2} + \frac{2b(bd - ae)}{e^3(d + ex)} + \frac{b^2 \log(d + ex)}{e^3}$$

output `-1/2*(-a*e+b*d)^2/e^3/(e*x+d)^2+2*b*(-a*e+b*d)/e^3/(e*x+d)+b^2*ln(e*x+d)/e^3`

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.81

$$\int \frac{a^2 + 2abx + b^2x^2}{(d + ex)^3} dx = \frac{\frac{(bd-ae)(3bd+ae+4bex)}{(d+ex)^2} + 2b^2 \log(d + ex)}{2e^3}$$

input `Integrate[(a^2 + 2*a*b*x + b^2*x^2)/(d + e*x)^3,x]`

output `((b*d - a*e)*(3*b*d + a*e + 4*b*e*x))/(d + e*x)^2 + 2*b^2*Log[d + e*x]/(2*e^3)`

Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {1098, 27, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\begin{aligned}
 & \int \frac{a^2 + 2abx + b^2x^2}{(d + ex)^3} dx \\
 & \quad \downarrow 1098 \\
 & \quad \int \frac{b^2(a+bx)^2}{(d+ex)^3} dx \\
 & \quad \quad \downarrow 27 \\
 & \quad \quad \int \frac{(a + bx)^2}{(d + ex)^3} dx \\
 & \quad \quad \quad \downarrow 49 \\
 & \quad \quad \quad \int \left(-\frac{2b(bd - ae)}{e^2(d + ex)^2} + \frac{(ae - bd)^2}{e^2(d + ex)^3} + \frac{b^2}{e^2(d + ex)} \right) dx \\
 & \quad \quad \quad \quad \downarrow 2009 \\
 & \quad \quad \quad \quad \frac{2b(bd - ae)}{e^3(d + ex)} - \frac{(bd - ae)^2}{2e^3(d + ex)^2} + \frac{b^2 \log(d + ex)}{e^3}
 \end{aligned}$$

input `Int[(a^2 + 2*a*b*x + b^2*x^2)/(d + e*x)^3,x]`

output `-1/2*(b*d - a*e)^2/(e^3*(d + e*x)^2) + (2*b*(b*d - a*e))/(e^3*(d + e*x)) + (b^2*Log[d + e*x])/e^3`

Definitions of rubi rules used

rule 27 $\text{Int}[(a_*)(Fx_), x_Symbol] \rightarrow \text{Simp}[a \text{ Int}[Fx, x], x] /; \text{FreeQ}[a, x] \ \&\& \ !\text{MatchQ}[Fx, (b_*)(Gx_)] /; \text{FreeQ}[b, x]$

rule 49 $\text{Int}[((a_.) + (b_.)(x_))^{(m_.)} * ((c_.) + (d_.)(x_))^{(n_.)}, x_Symbol] \rightarrow \text{Int}[\text{ExpandIntegrand}[(a + b*x)^m * (c + d*x)^n, x], x] /; \text{FreeQ}[\{a, b, c, d\}, x] \ \&\& \ \text{IGtQ}[m, 0] \ \&\& \ \text{IGtQ}[m + n + 2, 0]$

rule 1098 $\text{Int}[((d_.) + (e_.)(x_))^{(m_.)} * ((a_) + (b_.)(x_) + (c_.)(x_)^2)^{(p_.)}, x_Symbol] \rightarrow \text{Simp}[1/c^p \text{ Int}[(d + e*x)^m * (b/2 + c*x)^{(2*p)}, x], x] /; \text{FreeQ}[\{a, b, c, d, e, m\}, x] \ \&\& \ \text{EqQ}[b^2 - 4*a*c, 0] \ \&\& \ \text{IntegerQ}[p]$

rule 2009 $\text{Int}[u_, x_Symbol] \rightarrow \text{Simp}[\text{IntSum}[u, x], x] /; \text{SumQ}[u]$

Maple [A] (verified)

Time = 0.79 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.12

| method | result | size |
|--------------|--|------|
| risch | $\frac{-\frac{2b(ae-bd)x - e^2a^2 + 2abde - 3b^2d^2}{e^2}}{(ex+d)^2} + \frac{b^2 \ln(ex+d)}{e^3}$ | 66 |
| norman | $\frac{-\frac{e^2a^2 + 2abde - 3b^2d^2}{2e^3} - \frac{2(abe-b^2d)x}{e^2}}{(ex+d)^2} + \frac{b^2 \ln(ex+d)}{e^3}$ | 68 |
| default | $\frac{b^2 \ln(ex+d)}{e^3} - \frac{2b(ae-bd)}{e^3(ex+d)} - \frac{e^2a^2 - 2abde + b^2d^2}{2e^3(ex+d)^2}$ | 69 |
| parallelrisc | $\frac{2 \ln(ex+d)x^2b^2e^2 + 4 \ln(ex+d)x b^2de + 2 \ln(ex+d)b^2d^2 - 4xab e^2 + 4b^2dex - e^2a^2 - 2abde + 3b^2d^2}{2e^3(ex+d)^2}$ | 97 |

input $\text{int}((b^2*x^2 + 2*a*b*x + a^2)/(e*x+d)^3, x, \text{method}=_RETURNVERBOSE)$

output $(-2*b/e^2*(a*e-b*d)*x - 1/2*(a^2*e^2 + 2*a*b*d*e - 3*b^2*d^2)/e^3)/(e*x+d)^2 + b^2 * \ln(e*x+d)/e^3$

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.69

$$\int \frac{a^2 + 2abx + b^2x^2}{(d + ex)^3} dx$$

$$= \frac{3b^2d^2 - 2abde - a^2e^2 + 4(b^2de - abe^2)x + 2(b^2e^2x^2 + 2b^2dex + b^2d^2)\log(ex + d)}{2(e^5x^2 + 2de^4x + d^2e^3)}$$

input `integrate((b^2*x^2+2*a*b*x+a^2)/(e*x+d)^3,x, algorithm="fricas")`output `1/2*(3*b^2*d^2 - 2*a*b*d*e - a^2*e^2 + 4*(b^2*d*e - a*b*e^2)*x + 2*(b^2*e^2*x^2 + 2*b^2*d*e*x + b^2*d^2)*log(e*x + d))/(e^5*x^2 + 2*d*e^4*x + d^2*e^3)`**Sympy [A] (verification not implemented)**

Time = 0.22 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.36

$$\int \frac{a^2 + 2abx + b^2x^2}{(d + ex)^3} dx = \frac{b^2 \log(d + ex)}{e^3} + \frac{-a^2e^2 - 2abde + 3b^2d^2 + x(-4abe^2 + 4b^2de)}{2d^2e^3 + 4de^4x + 2e^5x^2}$$

input `integrate((b**2*x**2+2*a*b*x+a**2)/(e*x+d)**3,x)`output `b**2*log(d + e*x)/e**3 + (-a**2*e**2 - 2*a*b*d*e + 3*b**2*d**2 + x*(-4*a*b*e**2 + 4*b**2*d*e))/(2*d**2*e**3 + 4*d*e**4*x + 2*e**5*x**2)`**Maxima [A] (verification not implemented)**

Time = 0.04 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.36

$$\int \frac{a^2 + 2abx + b^2x^2}{(d + ex)^3} dx = \frac{3b^2d^2 - 2abde - a^2e^2 + 4(b^2de - abe^2)x}{2(e^5x^2 + 2de^4x + d^2e^3)} + \frac{b^2 \log(ex + d)}{e^3}$$

input `integrate((b^2*x^2+2*a*b*x+a^2)/(e*x+d)^3,x, algorithm="maxima")`

output

$$\frac{1}{2}(3b^2d^2 - 2abde - a^2e^2 + 4(b^2de - abe^2)x)/(e^5x^2 + 2de^4x + d^2e^3) + b^2\log(ex + d)/e^3$$

Giac [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.17

$$\int \frac{a^2 + 2abx + b^2x^2}{(d + ex)^3} dx = \frac{b^2 \log(|ex + d|)}{e^3} + \frac{4(b^2d - abe)x + \frac{3b^2d^2 - 2abde - a^2e^2}{e}}{2(ex + d)^2e^2}$$

input

```
integrate((b^2*x^2+2*a*b*x+a^2)/(e*x+d)^3,x, algorithm="giac")
```

output

$$b^2\log(\text{abs}(ex + d))/e^3 + 1/2*(4*(b^2*d - a*b*e)*x + (3*b^2*d^2 - 2*a*b*d*e - a^2*e^2)/e)/((e*x + d)^2*e^2)$$

Mupad [B] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.31

$$\int \frac{a^2 + 2abx + b^2x^2}{(d + ex)^3} dx = \frac{b^2 \ln(d + ex)}{e^3} - \frac{\frac{a^2e^2 + 2abde - 3b^2d^2}{2e^3} + \frac{2bx(ae - bd)}{e^2}}{d^2 + 2dex + e^2x^2}$$

input

```
int((a^2 + b^2*x^2 + 2*a*b*x)/(d + e*x)^3,x)
```

output

$$(b^2\log(d + e*x))/e^3 - ((a^2*e^2 - 3*b^2*d^2 + 2*a*b*d*e)/(2*e^3) + (2*b*x*(a*e - b*d))/e^2)/(d^2 + e^2*x^2 + 2*d*e*x)$$

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.92

$$\int \frac{a^2 + 2abx + b^2x^2}{(d + ex)^3} dx$$

$$= \frac{2 \log(ex + d) b^2 d^3 + 4 \log(ex + d) b^2 d^2 ex + 2 \log(ex + d) b^2 d e^2 x^2 - a^2 d e^2 + 2ab e^3 x^2 + b^2 d^3 - 2b^2 d e^2 x^2}{2d e^3 (e^2 x^2 + 2dex + d^2)}$$

input `int((b^2*x^2+2*a*b*x+a^2)/(e*x+d)^3,x)`output `(2*log(d + e*x)*b**2*d**3 + 4*log(d + e*x)*b**2*d**2*e*x + 2*log(d + e*x)*b**2*d*e**2*x**2 - a**2*d*e**2 + 2*a*b*e**3*x**2 + b**2*d**3 - 2*b**2*d*e**2*x**2)/(2*d*e**3*(d**2 + 2*d*e*x + e**2*x**2))`

$$3.62 \quad \int \frac{a^2 + 2abx + b^2x^2}{(d+ex)^4} dx$$

| | |
|---|-----|
| Optimal result | 511 |
| Mathematica [A] (verified) | 511 |
| Rubi [A] (verified) | 512 |
| Maple [B] (verified) | 513 |
| Fricas [B] (verification not implemented) | 513 |
| Sympy [B] (verification not implemented) | 514 |
| Maxima [B] (verification not implemented) | 514 |
| Giac [B] (verification not implemented) | 515 |
| Mupad [B] (verification not implemented) | 515 |
| Reduce [B] (verification not implemented) | 515 |

Optimal result

Integrand size = 24, antiderivative size = 28

$$\int \frac{a^2 + 2abx + b^2x^2}{(d+ex)^4} dx = \frac{(a+bx)^3}{3(bd-ae)(d+ex)^3}$$

output `1/3*(b*x+a)^3/(-a*e+b*d)/(e*x+d)^3`

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.89

$$\int \frac{a^2 + 2abx + b^2x^2}{(d+ex)^4} dx = -\frac{a^2e^2 + abe(d+3ex) + b^2(d^2 + 3dex + 3e^2x^2)}{3e^3(d+ex)^3}$$

input `Integrate[(a^2 + 2*a*b*x + b^2*x^2)/(d + e*x)^4,x]`

output `-1/3*(a^2*e^2 + a*b*e*(d + 3*e*x) + b^2*(d^2 + 3*d*e*x + 3*e^2*x^2))/(e^3*(d + e*x)^3)`

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {1098, 27, 48}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a^2 + 2abx + b^2x^2}{(d + ex)^4} dx$$

$$\downarrow 1098$$

$$\frac{\int \frac{b^2(a+bx)^2}{(d+ex)^4} dx}{b^2}$$

$$\downarrow 27$$

$$\int \frac{(a + bx)^2}{(d + ex)^4} dx$$

$$\downarrow 48$$

$$\frac{(a + bx)^3}{3(d + ex)^3(bd - ae)}$$

input `Int[(a^2 + 2*a*b*x + b^2*x^2)/(d + e*x)^4,x]`

output `(a + b*x)^3/(3*(b*d - a*e)*(d + e*x)^3)`

Defintions of rubi rules used

rule 27 `Int[(a_)*(Fx_), x_Symbol] :> Simp[a Int[Fx, x], x] /; FreeQ[a, x] && !MatchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]`

rule 48 `Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{a, b, c, d, m, n}, x] && EqQ[m + n + 2, 0] && NeQ[m, -1]`

rule 1098

```
Int[((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] := Simp[1/c^p Int[(d + e*x)^m*(b/2 + c*x)^(2*p), x], x] /; FreeQ[
{a, b, c, d, e, m}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 59 vs. $2(26) = 52$.

Time = 0.65 (sec) , antiderivative size = 60, normalized size of antiderivative = 2.14

| method | result | size |
|---------------|--|------|
| gospers | $-\frac{3b^2e^2x^2+3xab e^2+3b^2dex+e^2a^2+abde+b^2d^2}{3e^3(ex+d)^3}$ | 60 |
| risch | $-\frac{\frac{b^2x^2}{e} - \frac{b(ae+bd)x}{e^2} - \frac{e^2a^2+abde+b^2d^2}{3e^3}}{(ex+d)^3}$ | 60 |
| norman | $-\frac{\frac{b^2x^2}{e} - \frac{(abe+b^2d)x}{e^2} - \frac{e^2a^2+abde+b^2d^2}{3e^3}}{(ex+d)^3}$ | 62 |
| parallelrisch | $-\frac{3b^2e^2x^2-3xab e^2-3b^2dex-e^2a^2-abde-b^2d^2}{3e^3(ex+d)^3}$ | 63 |
| default | $-\frac{e^2a^2-2abde+b^2d^2}{3e^3(ex+d)^3} - \frac{b^2}{e^3(ex+d)} - \frac{b(ae-bd)}{e^3(ex+d)^2}$ | 71 |
| orering | $-\frac{(3b^2e^2x^2+3xab e^2+3b^2dex+e^2a^2+abde+b^2d^2)(b^2x^2+2abx+a^2)}{3e^3(bx+a)^2(ex+d)^3}$ | 83 |

input

```
int((b^2*x^2+2*a*b*x+a^2)/(e*x+d)^4,x,method=_RETURNVERBOSE)
```

output

```
-1/3*(3*b^2*e^2*x^2+3*a*b*e^2*x+3*b^2*d*e*x+a^2*e^2+a*b*d*e+b^2*d^2)/e^3/(
e*x+d)^3
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 84 vs. $2(26) = 52$.

Time = 0.06 (sec) , antiderivative size = 84, normalized size of antiderivative = 3.00

$$\int \frac{a^2 + 2abx + b^2x^2}{(d + ex)^4} dx = -\frac{3b^2e^2x^2 + b^2d^2 + abde + a^2e^2 + 3(b^2de + abe^2)x}{3(e^6x^3 + 3de^5x^2 + 3d^2e^4x + d^3e^3)}$$

input `integrate((b^2*x^2+2*a*b*x+a^2)/(e*x+d)^4,x, algorithm="fricas")`

output `-1/3*(3*b^2*e^2*x^2 + b^2*d^2 + a*b*d*e + a^2*e^2 + 3*(b^2*d*e + a*b*e^2)*x)/(e^6*x^3 + 3*d*e^5*x^2 + 3*d^2*e^4*x + d^3*e^3)`

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 88 vs. $2(20) = 40$.

Time = 0.30 (sec) , antiderivative size = 88, normalized size of antiderivative = 3.14

$$\int \frac{a^2 + 2abx + b^2x^2}{(d + ex)^4} dx = \frac{-a^2e^2 - abde - b^2d^2 - 3b^2e^2x^2 + x(-3abe^2 - 3b^2de)}{3d^3e^3 + 9d^2e^4x + 9de^5x^2 + 3e^6x^3}$$

input `integrate((b**2*x**2+2*a*b*x+a**2)/(e*x+d)**4,x)`

output `(-a**2*e**2 - a*b*d*e - b**2*d**2 - 3*b**2*e**2*x**2 + x*(-3*a*b*e**2 - 3*b**2*d*e))/(3*d**3*e**3 + 9*d**2*e**4*x + 9*d*e**5*x**2 + 3*e**6*x**3)`

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 84 vs. $2(26) = 52$.

Time = 0.03 (sec) , antiderivative size = 84, normalized size of antiderivative = 3.00

$$\int \frac{a^2 + 2abx + b^2x^2}{(d + ex)^4} dx = -\frac{3b^2e^2x^2 + b^2d^2 + abde + a^2e^2 + 3(b^2de + abe^2)x}{3(e^6x^3 + 3de^5x^2 + 3d^2e^4x + d^3e^3)}$$

input `integrate((b^2*x^2+2*a*b*x+a^2)/(e*x+d)^4,x, algorithm="maxima")`

output `-1/3*(3*b^2*e^2*x^2 + b^2*d^2 + a*b*d*e + a^2*e^2 + 3*(b^2*d*e + a*b*e^2)*x)/(e^6*x^3 + 3*d*e^5*x^2 + 3*d^2*e^4*x + d^3*e^3)`

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 59 vs. $2(26) = 52$.

Time = 0.24 (sec) , antiderivative size = 59, normalized size of antiderivative = 2.11

$$\int \frac{a^2 + 2abx + b^2x^2}{(d + ex)^4} dx = -\frac{3b^2e^2x^2 + 3b^2dex + 3abe^2x + b^2d^2 + abde + a^2e^2}{3(ex + d)^3e^3}$$

input `integrate((b^2*x^2+2*a*b*x+a^2)/(e*x+d)^4,x, algorithm="giac")`

output `-1/3*(3*b^2*e^2*x^2 + 3*b^2*d*e*x + 3*a*b*e^2*x + b^2*d^2 + a*b*d*e + a^2*e^2)/((e*x + d)^3*e^3)`

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 80, normalized size of antiderivative = 2.86

$$\int \frac{a^2 + 2abx + b^2x^2}{(d + ex)^4} dx = -\frac{\frac{a^2e^2+abde+b^2d^2}{3e^3} + \frac{b^2x^2}{e} + \frac{bx(ae+bd)}{e^2}}{d^3 + 3d^2ex + 3de^2x^2 + e^3x^3}$$

input `int((a^2 + b^2*x^2 + 2*a*b*x)/(d + e*x)^4,x)`

output `-((a^2*e^2 + b^2*d^2 + a*b*d*e)/(3*e^3) + (b^2*x^2)/e + (b*x*(a*e + b*d))/(e^2))/(d^3 + e^3*x^3 + 3*d*e^2*x^2 + 3*d^2*e*x)`

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 69, normalized size of antiderivative = 2.46

$$\int \frac{a^2 + 2abx + b^2x^2}{(d + ex)^4} dx = \frac{b^2e^2x^3 - 3abdex - a^2de - abd^2}{3de^2(e^3x^3 + 3de^2x^2 + 3d^2ex + d^3)}$$

input `int((b^2*x^2+2*a*b*x+a^2)/(e*x+d)^4,x)`

output $(-a^{**2}d*e - a*b*d^{**2} - 3*a*b*d*e*x + b^{**2}*e^{**2}*x^{**3})/(3*d*e^{**2}*(d^{**3} + 3*d^{**2}*e*x + 3*d*e^{**2}*x^{**2} + e^{**3}*x^{**3}))$

3.63 $\int \frac{a^2+2abx+b^2x^2}{(d+ex)^5} dx$

| | |
|---|-----|
| Optimal result | 517 |
| Mathematica [A] (verified) | 517 |
| Rubi [A] (verified) | 518 |
| Maple [A] (verified) | 519 |
| Fricas [A] (verification not implemented) | 520 |
| Sympy [A] (verification not implemented) | 520 |
| Maxima [A] (verification not implemented) | 521 |
| Giac [A] (verification not implemented) | 521 |
| Mupad [B] (verification not implemented) | 521 |
| Reduce [B] (verification not implemented) | 522 |

Optimal result

Integrand size = 24, antiderivative size = 65

$$\int \frac{a^2 + 2abx + b^2x^2}{(d + ex)^5} dx = -\frac{(bd - ae)^2}{4e^3(d + ex)^4} + \frac{2b(bd - ae)}{3e^3(d + ex)^3} - \frac{b^2}{2e^3(d + ex)^2}$$

output

```
-1/4*(-a*e+b*d)^2/e^3/(e*x+d)^4+2/3*b*(-a*e+b*d)/e^3/(e*x+d)^3-1/2*b^2/e^3/(e*x+d)^2
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.85

$$\int \frac{a^2 + 2abx + b^2x^2}{(d + ex)^5} dx = -\frac{3a^2e^2 + 2abe(d + 4ex) + b^2(d^2 + 4dex + 6e^2x^2)}{12e^3(d + ex)^4}$$

input

```
Integrate[(a^2 + 2*a*b*x + b^2*x^2)/(d + e*x)^5,x]
```

output

```
-1/12*(3*a^2*e^2 + 2*a*b*e*(d + 4*e*x) + b^2*(d^2 + 4*d*e*x + 6*e^2*x^2))/(e^3*(d + e*x)^4)
```

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {1098, 27, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\begin{aligned}
 & \int \frac{a^2 + 2abx + b^2x^2}{(d + ex)^5} dx \\
 & \quad \downarrow \text{1098} \\
 & \int \frac{b^2(a+bx)^2}{(d+ex)^5} dx \\
 & \quad \downarrow \text{27} \\
 & \int \frac{(a + bx)^2}{(d + ex)^5} dx \\
 & \quad \downarrow \text{53} \\
 & \int \left(-\frac{2b(bd - ae)}{e^2(d + ex)^4} + \frac{(ae - bd)^2}{e^2(d + ex)^5} + \frac{b^2}{e^2(d + ex)^3} \right) dx \\
 & \quad \downarrow \text{2009} \\
 & \frac{2b(bd - ae)}{3e^3(d + ex)^3} - \frac{(bd - ae)^2}{4e^3(d + ex)^4} - \frac{b^2}{2e^3(d + ex)^2}
 \end{aligned}$$

input `Int[(a^2 + 2*a*b*x + b^2*x^2)/(d + e*x)^5,x]`

output `-1/4*(b*d - a*e)^2/(e^3*(d + e*x)^4) + (2*b*(b*d - a*e))/(3*e^3*(d + e*x)^3) - b^2/(2*e^3*(d + e*x)^2)`

Definitions of rubi rules used

rule 27 $\text{Int}[(a_*)(Fx_), x_Symbol] \rightarrow \text{Simp}[a \text{ Int}[Fx, x], x] /; \text{FreeQ}[a, x] \ \&\& \ !\text{MatchQ}[Fx, (b_*)(Gx_)] /; \text{FreeQ}[b, x]$

rule 53 $\text{Int}[(a_*) + (b_*)(x_)^{(m_*)} * ((c_*) + (d_*)(x_)^{(n_*)}), x_Symbol] \rightarrow \text{Int}[\text{ExpandIntegrand}[(a + b*x)^m * (c + d*x)^n, x], x] /; \text{FreeQ}\{a, b, c, d, n\}, x] \ \&\& \ \text{IGtQ}[m, 0] \ \&\& \ (\ !\text{IntegerQ}[n] \ || \ (\text{EqQ}[c, 0] \ \&\& \ \text{LeQ}[7*m + 4*n + 4, 0]) \ || \ \text{LtQ}[9*m + 5*(n + 1), 0] \ || \ \text{GtQ}[m + n + 2, 0])$

rule 1098 $\text{Int}[(d_*) + (e_*)(x_)^{(m_*)} * ((a_*) + (b_*)(x_) + (c_*)(x_)^2)^{(p_*)}, x_Symbol] \rightarrow \text{Simp}[1/c^p \text{ Int}[(d + e*x)^m * (b/2 + c*x)^(2*p), x], x] /; \text{FreeQ}\{a, b, c, d, e, m\}, x] \ \&\& \ \text{EqQ}[b^2 - 4*a*c, 0] \ \&\& \ \text{IntegerQ}[p]$

rule 2009 $\text{Int}[u_, x_Symbol] \rightarrow \text{Simp}[\text{IntSum}[u, x], x] /; \text{SumQ}[u]$

Maple [A] (verified)

Time = 0.63 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.95

| method | result | size |
|---------------|---|------|
| gospers | $-\frac{6b^2e^2x^2+8xab^2e^2+4b^2dex+3e^2a^2+2abde+b^2d^2}{12e^3(ex+d)^4}$ | 62 |
| risch | $\frac{-\frac{b^2x^2}{2e} - \frac{b(2ae+bd)x}{3e^2} - \frac{3e^2a^2+2abde+b^2d^2}{12e^3}}{(ex+d)^4}$ | 63 |
| parallelrisch | $-\frac{6b^2e^3x^2-8abe^3x-4b^2de^2x-3a^2e^3-2abde^2-b^2d^2e}{12e^4(ex+d)^4}$ | 68 |
| default | $-\frac{2b(ae-bd)}{3e^3(ex+d)^3} - \frac{e^2a^2-2abde+b^2d^2}{4e^3(ex+d)^4} - \frac{b^2}{2e^3(ex+d)^2}$ | 71 |
| norman | $\frac{-\frac{b^2x^2}{2e} - \frac{(2abe^2+de b^2)x}{3e^3} - \frac{3a^2e^3+2abde^2+b^2d^2e}{12e^4}}{(ex+d)^4}$ | 71 |
| orering | $-\frac{(6b^2e^2x^2+8xab^2e^2+4b^2dex+3e^2a^2+2abde+b^2d^2)(b^2x^2+2abx+a^2)}{12e^3(bx+a)^2(ex+d)^4}$ | 85 |

input $\text{int}((b^2*x^2+2*a*b*x+a^2)/(e*x+d)^5, x, \text{method}=_RETURNVERBOSE)$

output

$$-1/12/e^3*(6*b^2*e^2*x^2+8*a*b*e^2*x+4*b^2*d*e*x+3*a^2*e^2+2*a*b*d*e+b^2*d^2)/(e*x+d)^4$$
Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 98, normalized size of antiderivative = 1.51

$$\int \frac{a^2 + 2abx + b^2x^2}{(d + ex)^5} dx = -\frac{6b^2e^2x^2 + b^2d^2 + 2abde + 3a^2e^2 + 4(b^2de + 2abe^2)x}{12(e^7x^4 + 4de^6x^3 + 6d^2e^5x^2 + 4d^3e^4x + d^4e^3)}$$

input

```
integrate((b^2*x^2+2*a*b*x+a^2)/(e*x+d)^5,x, algorithm="fricas")
```

output

$$-1/12*(6*b^2*e^2*x^2 + b^2*d^2 + 2*a*b*d*e + 3*a^2*e^2 + 4*(b^2*d*e + 2*a*b*e^2)*x)/(e^7*x^4 + 4*d*e^6*x^3 + 6*d^2*e^5*x^2 + 4*d^3*e^4*x + d^4*e^3)$$
Sympy [A] (verification not implemented)

Time = 0.39 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.60

$$\int \frac{a^2 + 2abx + b^2x^2}{(d + ex)^5} dx = \frac{-3a^2e^2 - 2abde - b^2d^2 - 6b^2e^2x^2 + x(-8abe^2 - 4b^2de)}{12d^4e^3 + 48d^3e^4x + 72d^2e^5x^2 + 48de^6x^3 + 12e^7x^4}$$

input

```
integrate((b**2*x**2+2*a*b*x+a**2)/(e*x+d)**5,x)
```

output

$$(-3*a**2*e**2 - 2*a*b*d*e - b**2*d**2 - 6*b**2*e**2*x**2 + x*(-8*a*b*e**2 - 4*b**2*d*e))/(12*d**4*e**3 + 48*d**3*e**4*x + 72*d**2*e**5*x**2 + 48*d*e**6*x**3 + 12*e**7*x**4)$$

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 98, normalized size of antiderivative = 1.51

$$\int \frac{a^2 + 2abx + b^2x^2}{(d + ex)^5} dx = -\frac{6b^2e^2x^2 + b^2d^2 + 2abde + 3a^2e^2 + 4(b^2de + 2abe^2)x}{12(e^7x^4 + 4de^6x^3 + 6d^2e^5x^2 + 4d^3e^4x + d^4e^3)}$$

input `integrate((b^2*x^2+2*a*b*x+a^2)/(e*x+d)^5,x, algorithm="maxima")`output `-1/12*(6*b^2*e^2*x^2 + b^2*d^2 + 2*a*b*d*e + 3*a^2*e^2 + 4*(b^2*d*e + 2*a*b*e^2)*x)/(e^7*x^4 + 4*d*e^6*x^3 + 6*d^2*e^5*x^2 + 4*d^3*e^4*x + d^4*e^3)`**Giac [A] (verification not implemented)**

Time = 0.23 (sec) , antiderivative size = 96, normalized size of antiderivative = 1.48

$$\int \frac{a^2 + 2abx + b^2x^2}{(d + ex)^5} dx = -\frac{\frac{3a^2}{(ex+d)^4} + \frac{6b^2}{(ex+d)^2e^2} - \frac{8b^2d}{(ex+d)^3e^2} + \frac{3b^2d^2}{(ex+d)^4e^2} + \frac{8ab}{(ex+d)^3e} - \frac{6abd}{(ex+d)^4e}}{12e}$$

input `integrate((b^2*x^2+2*a*b*x+a^2)/(e*x+d)^5,x, algorithm="giac")`output `-1/12*(3*a^2/(e*x + d)^4 + 6*b^2/((e*x + d)^2*e^2) - 8*b^2*d/((e*x + d)^3*e^2) + 3*b^2*d^2/((e*x + d)^4*e^2) + 8*a*b/((e*x + d)^3*e) - 6*a*b*d/((e*x + d)^4*e))/e`**Mupad [B] (verification not implemented)**

Time = 8.91 (sec) , antiderivative size = 96, normalized size of antiderivative = 1.48

$$\int \frac{a^2 + 2abx + b^2x^2}{(d + ex)^5} dx = -\frac{\frac{3a^2e^2+2abde+b^2d^2}{12e^3} + \frac{b^2x^2}{2e} + \frac{bx(2ae+bd)}{3e^2}}{d^4 + 4d^3ex + 6d^2e^2x^2 + 4de^3x^3 + e^4x^4}$$

input `int((a^2 + b^2*x^2 + 2*a*b*x)/(d + e*x)^5,x)`

output

```

-((3*a^2*e^2 + b^2*d^2 + 2*a*b*d*e)/(12*e^3) + (b^2*x^2)/(2*e) + (b*x*(2*a
*e + b*d))/(3*e^2))/(d^4 + e^4*x^4 + 4*d*e^3*x^3 + 6*d^2*e^2*x^2 + 4*d^3*e
*x)

```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.46

$$\int \frac{a^2 + 2abx + b^2x^2}{(d + ex)^5} dx = \frac{-6b^2e^2x^2 - 8abe^2x - 4b^2dex - 3a^2e^2 - 2abde - b^2d^2}{12e^3(e^4x^4 + 4de^3x^3 + 6d^2e^2x^2 + 4d^3ex + d^4)}$$

input

```

int((b^2*x^2+2*a*b*x+a^2)/(e*x+d)^5,x)

```

output

```

( - 3*a**2*e**2 - 2*a*b*d*e - 8*a*b*e**2*x - b**2*d**2 - 4*b**2*d*e*x - 6*
b**2*e**2*x**2)/(12*e**3*(d**4 + 4*d**3*e*x + 6*d**2*e**2*x**2 + 4*d*e**3*
x**3 + e**4*x**4))

```

3.64 $\int \frac{a^2+2abx+b^2x^2}{(d+ex)^6} dx$

| | |
|---|-----|
| Optimal result | 523 |
| Mathematica [A] (verified) | 523 |
| Rubi [A] (verified) | 524 |
| Maple [A] (verified) | 525 |
| Fricas [A] (verification not implemented) | 526 |
| Sympy [B] (verification not implemented) | 526 |
| Maxima [A] (verification not implemented) | 527 |
| Giac [A] (verification not implemented) | 527 |
| Mupad [B] (verification not implemented) | 527 |
| Reduce [B] (verification not implemented) | 528 |

Optimal result

Integrand size = 24, antiderivative size = 65

$$\int \frac{a^2 + 2abx + b^2x^2}{(d + ex)^6} dx = -\frac{(bd - ae)^2}{5e^3(d + ex)^5} + \frac{b(bd - ae)}{2e^3(d + ex)^4} - \frac{b^2}{3e^3(d + ex)^3}$$

```
output -1/5*(-a*e+b*d)^2/e^3/(e*x+d)^5+1/2*b*(-a*e+b*d)/e^3/(e*x+d)^4-1/3*b^2/e^3/(e*x+d)^3
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.85

$$\int \frac{a^2 + 2abx + b^2x^2}{(d + ex)^6} dx = -\frac{6a^2e^2 + 3abe(d + 5ex) + b^2(d^2 + 5dex + 10e^2x^2)}{30e^3(d + ex)^5}$$

```
input Integrate[(a^2 + 2*a*b*x + b^2*x^2)/(d + e*x)^6,x]
```

```
output -1/30*(6*a^2*e^2 + 3*a*b*e*(d + 5*e*x) + b^2*(d^2 + 5*d*e*x + 10*e^2*x^2))/(e^3*(d + e*x)^5)
```


Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {1098, 27, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\begin{aligned}
 & \int \frac{a^2 + 2abx + b^2x^2}{(d + ex)^6} dx \\
 & \quad \downarrow \text{1098} \\
 & \quad \int \frac{b^2(a+bx)^2}{(d+ex)^6} dx \\
 & \quad \quad \downarrow \text{27} \\
 & \quad \quad \int \frac{(a + bx)^2}{(d + ex)^6} dx \\
 & \quad \quad \quad \downarrow \text{53} \\
 & \quad \quad \quad \int \left(-\frac{2b(bd - ae)}{e^2(d + ex)^5} + \frac{(ae - bd)^2}{e^2(d + ex)^6} + \frac{b^2}{e^2(d + ex)^4} \right) dx \\
 & \quad \quad \quad \quad \downarrow \text{2009} \\
 & \quad \quad \quad \quad \frac{b(bd - ae)}{2e^3(d + ex)^4} - \frac{(bd - ae)^2}{5e^3(d + ex)^5} - \frac{b^2}{3e^3(d + ex)^3}
 \end{aligned}$$

input `Int[(a^2 + 2*a*b*x + b^2*x^2)/(d + e*x)^6, x]`

output `-1/5*(b*d - a*e)^2/(e^3*(d + e*x)^5) + (b*(b*d - a*e))/(2*e^3*(d + e*x)^4) - b^2/(3*e^3*(d + e*x)^3)`

Definitions of rubi rules used

rule 27 $\text{Int}[(a_*)(Fx_), x_Symbol] \rightarrow \text{Simp}[a \text{ Int}[Fx, x], x] /; \text{FreeQ}[a, x] \ \&\& \ !\text{MatchQ}[Fx, (b_*)(Gx_)] /; \text{FreeQ}[b, x]$

rule 53 $\text{Int}[((a_.) + (b_.)(x_))^{(m_.)} * ((c_.) + (d_.)(x_))^{(n_.)}, x_Symbol] \rightarrow \text{Int}[\text{ExpandIntegrand}[(a + b*x)^m * (c + d*x)^n, x], x] /; \text{FreeQ}[\{a, b, c, d, n\}, x] \ \&\& \ \text{IGtQ}[m, 0] \ \&\& \ (\ !\text{IntegerQ}[n] \ || \ (\text{EqQ}[c, 0] \ \&\& \ \text{LeQ}[7*m + 4*n + 4, 0]) \ || \ \text{LtQ}[9*m + 5*(n + 1), 0] \ || \ \text{GtQ}[m + n + 2, 0])$

rule 1098 $\text{Int}[((d_.) + (e_.)(x_))^{(m_.)} * ((a_.) + (b_.)(x_) + (c_.)(x_)^2)^{(p_.)}, x_Symbol] \rightarrow \text{Simp}[1/c^p \text{ Int}[(d + e*x)^m * (b/2 + c*x)^{(2*p)}, x], x] /; \text{FreeQ}[\{a, b, c, d, e, m\}, x] \ \&\& \ \text{EqQ}[b^2 - 4*a*c, 0] \ \&\& \ \text{IntegerQ}[p]$

rule 2009 $\text{Int}[u_, x_Symbol] \rightarrow \text{Simp}[\text{IntSum}[u, x], x] /; \text{SumQ}[u]$

Maple [A] (verified)

Time = 0.64 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.95

| method | result | size |
|---------------|---|------|
| gospers | $-\frac{10b^2e^2x^2+15xab e^2+5b^2dex+6e^2a^2+3abde+b^2d^2}{30e^3(ex+d)^5}$ | 62 |
| risch | $\frac{-\frac{b^2x^2}{3e} - \frac{b(3ae+bd)x}{6e^2} - \frac{6e^2a^2+3abde+b^2d^2}{30e^3}}{(ex+d)^5}$ | 63 |
| parallelrisch | $\frac{-10x^2b^2e^4-15abe^4x-5b^2de^3x-6a^2e^4-3abd e^3-d^2e^2b^2}{30e^5(ex+d)^5}$ | 70 |
| default | $-\frac{b^2}{3e^3(ex+d)^3} - \frac{b(ae-bd)}{2e^3(ex+d)^4} - \frac{e^2a^2-2abde+b^2d^2}{5e^3(ex+d)^5}$ | 71 |
| norman | $\frac{-\frac{b^2x^2}{3e} - \frac{(3ae^3b+de^2b^2)x}{6e^4} - \frac{6a^2e^4+3abd e^3+d^2e^2b^2}{30e^5}}{(ex+d)^5}$ | 75 |
| orering | $-\frac{(10b^2e^2x^2+15xab e^2+5b^2dex+6e^2a^2+3abde+b^2d^2)(b^2x^2+2abx+a^2)}{30e^3(bx+a)^2(ex+d)^5}$ | 85 |

input $\text{int}((b^2*x^2+2*a*b*x+a^2)/(e*x+d)^6, x, \text{method}=_RETURNVERBOSE)$

output

```
-1/30/e^3*(10*b^2*e^2*x^2+15*a*b*e^2*x+5*b^2*d*e*x+6*a^2*e^2+3*a*b*d*e+b^2*d^2)/(e*x+d)^5
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 109, normalized size of antiderivative = 1.68

$$\int \frac{a^2 + 2abx + b^2x^2}{(d + ex)^6} dx = -\frac{10b^2e^2x^2 + b^2d^2 + 3abde + 6a^2e^2 + 5(b^2de + 3abe^2)x}{30(e^8x^5 + 5de^7x^4 + 10d^2e^6x^3 + 10d^3e^5x^2 + 5d^4e^4x + d^5e^3)}$$

input

```
integrate((b^2*x^2+2*a*b*x+a^2)/(e*x+d)^6,x, algorithm="fricas")
```

output

```
-1/30*(10*b^2*e^2*x^2 + b^2*d^2 + 3*a*b*d*e + 6*a^2*e^2 + 5*(b^2*d*e + 3*a*b*e^2)*x)/(e^8*x^5 + 5*d*e^7*x^4 + 10*d^2*e^6*x^3 + 10*d^3*e^5*x^2 + 5*d^4*e^4*x + d^5*e^3)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 116 vs. 2(54) = 108.

Time = 0.48 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.78

$$\int \frac{a^2 + 2abx + b^2x^2}{(d + ex)^6} dx = \frac{-6a^2e^2 - 3abde - b^2d^2 - 10b^2e^2x^2 + x(-15abe^2 - 5b^2de)}{30d^5e^3 + 150d^4e^4x + 300d^3e^5x^2 + 300d^2e^6x^3 + 150de^7x^4 + 30e^8x^5}$$

input

```
integrate((b**2*x**2+2*a*b*x+a**2)/(e*x+d)**6,x)
```

output

```
(-6*a**2*e**2 - 3*a*b*d*e - b**2*d**2 - 10*b**2*e**2*x**2 + x*(-15*a*b*e**2 - 5*b**2*d*e))/(30*d**5*e**3 + 150*d**4*e**4*x + 300*d**3*e**5*x**2 + 300*d**2*e**6*x**3 + 150*d*e**7*x**4 + 30*e**8*x**5)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 109, normalized size of antiderivative = 1.68

$$\int \frac{a^2 + 2abx + b^2x^2}{(d + ex)^6} dx = -\frac{10b^2e^2x^2 + b^2d^2 + 3abde + 6a^2e^2 + 5(b^2de + 3abe^2)x}{30(e^8x^5 + 5de^7x^4 + 10d^2e^6x^3 + 10d^3e^5x^2 + 5d^4e^4x + d^5e^3)}$$

input `integrate((b^2*x^2+2*a*b*x+a^2)/(e*x+d)^6,x, algorithm="maxima")`

output `-1/30*(10*b^2*e^2*x^2 + b^2*d^2 + 3*a*b*d*e + 6*a^2*e^2 + 5*(b^2*d*e + 3*a*b*e^2)*x)/(e^8*x^5 + 5*d*e^7*x^4 + 10*d^2*e^6*x^3 + 10*d^3*e^5*x^2 + 5*d^4*e^4*x + d^5*e^3)`

Giac [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.94

$$\int \frac{a^2 + 2abx + b^2x^2}{(d + ex)^6} dx = -\frac{10b^2e^2x^2 + 5b^2dex + 15abe^2x + b^2d^2 + 3abde + 6a^2e^2}{30(ex + d)^5e^3}$$

input `integrate((b^2*x^2+2*a*b*x+a^2)/(e*x+d)^6,x, algorithm="giac")`

output `-1/30*(10*b^2*e^2*x^2 + 5*b^2*d*e*x + 15*a*b*e^2*x + b^2*d^2 + 3*a*b*d*e + 6*a^2*e^2)/((e*x + d)^5*e^3)`

Mupad [B] (verification not implemented)

Time = 8.95 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.65

$$\int \frac{a^2 + 2abx + b^2x^2}{(d + ex)^6} dx = -\frac{\frac{6a^2e^2+3abde+b^2d^2}{30e^3} + \frac{b^2x^2}{3e} + \frac{bx(3ae+bd)}{6e^2}}{d^5 + 5d^4ex + 10d^3e^2x^2 + 10d^2e^3x^3 + 5de^4x^4 + e^5x^5}$$

input `int((a^2 + b^2*x^2 + 2*a*b*x)/(d + e*x)^6,x)`

output

```

-((6*a^2*e^2 + b^2*d^2 + 3*a*b*d*e)/(30*e^3) + (b^2*x^2)/(3*e) + (b*x*(3*a
*e + b*d))/(6*e^2))/(d^5 + e^5*x^5 + 5*d*e^4*x^4 + 10*d^3*e^2*x^2 + 10*d^2
*e^3*x^3 + 5*d^4*e*x)

```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 106, normalized size of antiderivative = 1.63

$$\int \frac{a^2 + 2abx + b^2x^2}{(d + ex)^6} dx = \frac{-10b^2e^2x^2 - 15abe^2x - 5b^2dex - 6a^2e^2 - 3abde - b^2d^2}{30e^3(e^5x^5 + 5de^4x^4 + 10d^2e^3x^3 + 10d^3e^2x^2 + 5d^4ex + d^5)}$$

input

```

int((b^2*x^2+2*a*b*x+a^2)/(e*x+d)^6,x)

```

output

```

( - 6*a**2*e**2 - 3*a*b*d*e - 15*a*b*e**2*x - b**2*d**2 - 5*b**2*d*e*x - 1
0*b**2*e**2*x**2)/(30*e**3*(d**5 + 5*d**4*e*x + 10*d**3*e**2*x**2 + 10*d**
2*e**3*x**3 + 5*d*e**4*x**4 + e**5*x**5))

```

3.65 $\int \frac{a^2+2abx+b^2x^2}{(d+ex)^7} dx$

| | |
|---|-----|
| Optimal result | 529 |
| Mathematica [A] (verified) | 529 |
| Rubi [A] (verified) | 530 |
| Maple [A] (verified) | 531 |
| Fricas [B] (verification not implemented) | 532 |
| Sympy [B] (verification not implemented) | 532 |
| Maxima [B] (verification not implemented) | 533 |
| Giac [A] (verification not implemented) | 533 |
| Mupad [B] (verification not implemented) | 534 |
| Reduce [B] (verification not implemented) | 534 |

Optimal result

Integrand size = 24, antiderivative size = 65

$$\int \frac{a^2 + 2abx + b^2x^2}{(d + ex)^7} dx = -\frac{(bd - ae)^2}{6e^3(d + ex)^6} + \frac{2b(bd - ae)}{5e^3(d + ex)^5} - \frac{b^2}{4e^3(d + ex)^4}$$

```
output -1/6*(-a*e+b*d)^2/e^3/(e*x+d)^6+2/5*b*(-a*e+b*d)/e^3/(e*x+d)^5-1/4*b^2/e^3/(e*x+d)^4
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.85

$$\int \frac{a^2 + 2abx + b^2x^2}{(d + ex)^7} dx = -\frac{10a^2e^2 + 4abe(d + 6ex) + b^2(d^2 + 6dex + 15e^2x^2)}{60e^3(d + ex)^6}$$

```
input Integrate[(a^2 + 2*a*b*x + b^2*x^2)/(d + e*x)^7,x]
```

```
output -1/60*(10*a^2*e^2 + 4*a*b*e*(d + 6*e*x) + b^2*(d^2 + 6*d*e*x + 15*e^2*x^2))/(e^3*(d + e*x)^6)
```

Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {1098, 27, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\begin{aligned}
 & \int \frac{a^2 + 2abx + b^2x^2}{(d + ex)^7} dx \\
 & \quad \downarrow \text{1098} \\
 & \int \frac{b^2(a+bx)^2}{(d+ex)^7} dx \\
 & \quad \downarrow \text{27} \\
 & \int \frac{(a + bx)^2}{(d + ex)^7} dx \\
 & \quad \downarrow \text{53} \\
 & \int \left(-\frac{2b(bd - ae)}{e^2(d + ex)^6} + \frac{(ae - bd)^2}{e^2(d + ex)^7} + \frac{b^2}{e^2(d + ex)^5} \right) dx \\
 & \quad \downarrow \text{2009} \\
 & \frac{2b(bd - ae)}{5e^3(d + ex)^5} - \frac{(bd - ae)^2}{6e^3(d + ex)^6} - \frac{b^2}{4e^3(d + ex)^4}
 \end{aligned}$$

input `Int[(a^2 + 2*a*b*x + b^2*x^2)/(d + e*x)^7, x]`

output `-1/6*(b*d - a*e)^2/(e^3*(d + e*x)^6) + (2*b*(b*d - a*e))/(5*e^3*(d + e*x)^5) - b^2/(4*e^3*(d + e*x)^4)`

Defintions of rubi rules used

rule 27 `Int[(a_)*(Fx_), x_Symbol] := Simp[a Int[Fx, x], x] /; FreeQ[a, x] && !MatchQ[Fx, (b_)*(Gx_)] /; FreeQ[b, x]`

rule 53 `Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] := Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && IGtQ[m, 0] && (!IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])`

rule 1098 `Int[((d_.) + (e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] := Simp[1/c^p Int[(d + e*x)^m*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]`

rule 2009 `Int[u_, x_Symbol] := Simp[IntSum[u, x], x] /; SumQ[u]`

Maple [A] (verified)

Time = 0.66 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.95

| method | result | size |
|---------------|--|------|
| gospers | $-\frac{15b^2e^2x^2+24xab e^2+6b^2dex+10e^2a^2+4abde+b^2d^2}{60e^3(ex+d)^6}$ | 62 |
| risch | $-\frac{\frac{b^2x^2}{4e} - \frac{b(4ae+bd)x}{10e^2} - \frac{10e^2a^2+4abde+b^2d^2}{60e^3}}{(ex+d)^6}$ | 63 |
| parallelrisch | $-\frac{15b^2x^2e^5-24ab e^5x-6b^2de^4x-10a^2e^5-4abd e^4-b^2d^2e^3}{60e^6(ex+d)^6}$ | 70 |
| default | $-\frac{b^2}{4e^3(ex+d)^4} - \frac{2b(ae-bd)}{5e^3(ex+d)^5} - \frac{e^2a^2-2abde+b^2d^2}{6e^3(ex+d)^6}$ | 71 |
| norman | $-\frac{\frac{b^2x^2}{4e} - \frac{(4ab e^4+b^2de^3)x}{10e^5} - \frac{10a^2e^5+4abd e^4+b^2d^2e^3}{60e^6}}{(ex+d)^6}$ | 75 |
| orering | $-\frac{(15b^2e^2x^2+24xab e^2+6b^2dex+10e^2a^2+4abde+b^2d^2)(b^2x^2+2abx+a^2)}{60e^3(bx+a)^2(ex+d)^6}$ | 85 |

input `int((b^2*x^2+2*a*b*x+a^2)/(e*x+d)^7,x,method=_RETURNVERBOSE)`

output

$$-1/60/e^3*(15*b^2*e^2*x^2+24*a*b*e^2*x+6*b^2*d*e*x+10*a^2*e^2+4*a*b*d*e+b^2*d^2)/(e*x+d)^6$$

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 120 vs. 2(59) = 118.

Time = 0.07 (sec) , antiderivative size = 120, normalized size of antiderivative = 1.85

$$\int \frac{a^2 + 2abx + b^2x^2}{(d + ex)^7} dx$$

$$= -\frac{15b^2e^2x^2 + b^2d^2 + 4abde + 10a^2e^2 + 6(b^2de + 4abe^2)x}{60(e^9x^6 + 6de^8x^5 + 15d^2e^7x^4 + 20d^3e^6x^3 + 15d^4e^5x^2 + 6d^5e^4x + d^6e^3)}$$

input

```
integrate((b^2*x^2+2*a*b*x+a^2)/(e*x+d)^7,x, algorithm="fricas")
```

output

$$-1/60*(15*b^2*e^2*x^2 + b^2*d^2 + 4*a*b*d*e + 10*a^2*e^2 + 6*(b^2*d*e + 4*a*b*e^2)*x)/(e^9*x^6 + 6*d*e^8*x^5 + 15*d^2*e^7*x^4 + 20*d^3*e^6*x^3 + 15*d^4*e^5*x^2 + 6*d^5*e^4*x + d^6*e^3)$$

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 128 vs. 2(56) = 112.

Time = 0.57 (sec) , antiderivative size = 128, normalized size of antiderivative = 1.97

$$\int \frac{a^2 + 2abx + b^2x^2}{(d + ex)^7} dx$$

$$= \frac{-10a^2e^2 - 4abde - b^2d^2 - 15b^2e^2x^2 + x(-24abe^2 - 6b^2de)}{60d^6e^3 + 360d^5e^4x + 900d^4e^5x^2 + 1200d^3e^6x^3 + 900d^2e^7x^4 + 360de^8x^5 + 60e^9x^6}$$

input

```
integrate((b**2*x**2+2*a*b*x+a**2)/(e*x+d)**7,x)
```

output

$$(-10*a**2*e**2 - 4*a*b*d*e - b**2*d**2 - 15*b**2*e**2*x**2 + x*(-24*a*b*e**2 - 6*b**2*d*e))/(60*d**6*e**3 + 360*d**5*e**4*x + 900*d**4*e**5*x**2 + 1200*d**3*e**6*x**3 + 900*d**2*e**7*x**4 + 360*d*e**8*x**5 + 60*e**9*x**6)$$

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 120 vs. $2(59) = 118$.

Time = 0.05 (sec) , antiderivative size = 120, normalized size of antiderivative = 1.85

$$\int \frac{a^2 + 2abx + b^2x^2}{(d + ex)^7} dx$$

$$= -\frac{15b^2e^2x^2 + b^2d^2 + 4abde + 10a^2e^2 + 6(b^2de + 4abe^2)x}{60(e^9x^6 + 6de^8x^5 + 15d^2e^7x^4 + 20d^3e^6x^3 + 15d^4e^5x^2 + 6d^5e^4x + d^6e^3)}$$

input `integrate((b^2*x^2+2*a*b*x+a^2)/(e*x+d)^7,x, algorithm="maxima")`

output `-1/60*(15*b^2*e^2*x^2 + b^2*d^2 + 4*a*b*d*e + 10*a^2*e^2 + 6*(b^2*d*e + 4*a*b*e^2)*x)/(e^9*x^6 + 6*d*e^8*x^5 + 15*d^2*e^7*x^4 + 20*d^3*e^6*x^3 + 15*d^4*e^5*x^2 + 6*d^5*e^4*x + d^6*e^3)`

Giac [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.94

$$\int \frac{a^2 + 2abx + b^2x^2}{(d + ex)^7} dx = -\frac{15b^2e^2x^2 + 6b^2dex + 24abe^2x + b^2d^2 + 4abde + 10a^2e^2}{60(ex + d)^6e^3}$$

input `integrate((b^2*x^2+2*a*b*x+a^2)/(e*x+d)^7,x, algorithm="giac")`

output `-1/60*(15*b^2*e^2*x^2 + 6*b^2*d*e*x + 24*a*b*e^2*x + b^2*d^2 + 4*a*b*d*e + 10*a^2*e^2)/((e*x + d)^6*e^3)`

Mupad [B] (verification not implemented)

Time = 9.41 (sec) , antiderivative size = 118, normalized size of antiderivative = 1.82

$$\int \frac{a^2 + 2abx + b^2x^2}{(d + ex)^7} dx$$

$$= -\frac{\frac{10a^2e^2 + 4abde + b^2d^2}{60e^3} + \frac{b^2x^2}{4e} + \frac{bx(4ae + bd)}{10e^2}}{d^6 + 6d^5ex + 15d^4e^2x^2 + 20d^3e^3x^3 + 15d^2e^4x^4 + 6de^5x^5 + e^6x^6}$$

input `int((a^2 + b^2*x^2 + 2*a*b*x)/(d + e*x)^7,x)`output `-((10*a^2*e^2 + b^2*d^2 + 4*a*b*d*e)/(60*e^3) + (b^2*x^2)/(4*e) + (b*x*(4*a*e + b*d))/(10*e^2))/(d^6 + e^6*x^6 + 6*d*e^5*x^5 + 15*d^4*e^2*x^2 + 20*d^3*e^3*x^3 + 15*d^2*e^4*x^4 + 6*d^5*e*x)`**Reduce [B] (verification not implemented)**

Time = 0.20 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.80

$$\int \frac{a^2 + 2abx + b^2x^2}{(d + ex)^7} dx$$

$$= \frac{-15b^2e^2x^2 - 24ab e^2x - 6b^2dex - 10a^2e^2 - 4abde - b^2d^2}{60e^3(e^6x^6 + 6de^5x^5 + 15d^2e^4x^4 + 20d^3e^3x^3 + 15d^4e^2x^2 + 6d^5ex + d^6)}$$

input `int((b^2*x^2+2*a*b*x+a^2)/(e*x+d)^7,x)`output `(- 10*a**2*e**2 - 4*a*b*d*e - 24*a*b*e**2*x - b**2*d**2 - 6*b**2*d*e*x - 15*b**2*e**2*x**2)/(60*e**3*(d**6 + 6*d**5*e*x + 15*d**4*e**2*x**2 + 20*d**3*e**3*x**3 + 15*d**2*e**4*x**4 + 6*d*e**5*x**5 + e**6*x**6))`

3.66 $\int (d + ex)^6 (a^2 + 2abx + b^2x^2)^2 dx$

| | |
|---|-----|
| Optimal result | 535 |
| Mathematica [B] (verified) | 535 |
| Rubi [A] (verified) | 537 |
| Maple [B] (verified) | 538 |
| Fricas [B] (verification not implemented) | 539 |
| Sympy [B] (verification not implemented) | 540 |
| Maxima [B] (verification not implemented) | 541 |
| Giac [B] (verification not implemented) | 542 |
| Mupad [B] (verification not implemented) | 543 |
| Reduce [B] (verification not implemented) | 544 |

Optimal result

Integrand size = 26, antiderivative size = 119

$$\int (d + ex)^6 (a^2 + 2abx + b^2x^2)^2 dx = \frac{(bd - ae)^4(d + ex)^7}{7e^5} - \frac{b(bd - ae)^3(d + ex)^8}{2e^5} + \frac{2b^2(bd - ae)^2(d + ex)^9}{3e^5} - \frac{2b^3(bd - ae)(d + ex)^{10}}{5e^5} + \frac{b^4(d + ex)^{11}}{11e^5}$$

```
output 1/7*(-a*e+b*d)^4*(e*x+d)^7/e^5-1/2*b*(-a*e+b*d)^3*(e*x+d)^8/e^5+2/3*b^2*(-a*e+b*d)^2*(e*x+d)^9/e^5-2/5*b^3*(-a*e+b*d)*(e*x+d)^10/e^5+1/11*b^4*(e*x+d)^11/e^5
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 398 vs. 2(119) = 238.

Time = 0.07 (sec) , antiderivative size = 398, normalized size of antiderivative = 3.34

$$\int (d + ex)^6 (a^2 + 2abx + b^2x^2)^2 dx = a^4d^6x + a^3d^5(2bd + 3ae)x^2 + a^2d^4(2b^2d^2 + 8abde + 5a^2e^2)x^3 + ad^3(b^3d^3 + 9ab^2d^2e + 15a^2bde^2 + 5a^3e^3)x^4 + \frac{1}{5}d^2(b^4d^4 + 24ab^3d^3e + 90a^2b^2d^2e^2 + 80a^3bde^3 + 15a^4e^4)x^5 + de(b^4d^4 + 10ab^3d^3e + 20a^2b^2d^2e^2 + 10a^3bde^3 + a^4e^4)x^6 + \frac{1}{7}e^2(15b^4d^4 + 80ab^3d^3e + 90a^2b^2d^2e^2 + 24a^3bde^3 + a^4e^4)x^7 + \frac{1}{2}be^3(5b^3d^3 + 15ab^2d^2e + 9a^2bde^2 + a^3e^3)x^8 + \frac{1}{3}b^2e^4(5b^2d^2 + 8abde + 2a^2e^2)x^9 + \frac{1}{5}b^3e^5(3bd + 2ae)x^{10} + \frac{1}{11}b^4e^6x^{11}$$

input `Integrate[(d + e*x)^6*(a^2 + 2*a*b*x + b^2*x^2)^2,x]`

output `a^4*d^6*x + a^3*d^5*(2*b*d + 3*a*e)*x^2 + a^2*d^4*(2*b^2*d^2 + 8*a*b*d*e + 5*a^2*e^2)*x^3 + a*d^3*(b^3*d^3 + 9*a*b^2*d^2*e + 15*a^2*b*d*e^2 + 5*a^3*e^3)*x^4 + (d^2*(b^4*d^4 + 24*a*b^3*d^3*e + 90*a^2*b^2*d^2*e^2 + 80*a^3*b*d*e^3 + 15*a^4*e^4)*x^5)/5 + d*e*(b^4*d^4 + 10*a*b^3*d^3*e + 20*a^2*b^2*d^2*e^2 + 10*a^3*b*d*e^3 + a^4*e^4)*x^6 + (e^2*(15*b^4*d^4 + 80*a*b^3*d^3*e + 90*a^2*b^2*d^2*e^2 + 24*a^3*b*d*e^3 + a^4*e^4)*x^7)/7 + (b*e^3*(5*b^3*d^3 + 15*a*b^2*d^2*e + 9*a^2*b*d*e^2 + a^3*e^3)*x^8)/2 + (b^2*e^4*(5*b^2*d^2 + 8*a*b*d*e + 2*a^2*e^2)*x^9)/3 + (b^3*e^5*(3*b*d + 2*a*e)*x^10)/5 + (b^4*e^6*x^11)/11`

Rubi [A] (verified)

Time = 0.74 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {1098, 27, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\begin{aligned}
 & \int (a^2 + 2abx + b^2x^2)^2 (d + ex)^6 dx \\
 & \quad \downarrow 1098 \\
 & \quad \int \frac{b^4(a + bx)^4(d + ex)^6 dx}{b^4} \\
 & \quad \downarrow 27 \\
 & \quad \int (a + bx)^4(d + ex)^6 dx \\
 & \quad \downarrow 49 \\
 & \quad \int \left(-\frac{4b^3(d + ex)^9(bd - ae)}{e^4} + \frac{6b^2(d + ex)^8(bd - ae)^2}{e^4} - \frac{4b(d + ex)^7(bd - ae)^3}{e^4} + \frac{(d + ex)^6(ae - bd)^4}{e^4} + \frac{b^4(d + ex)^5}{e^4} \right) dx \\
 & \quad \downarrow 2009 \\
 & \quad -\frac{2b^3(d + ex)^{10}(bd - ae)}{5e^5} + \frac{2b^2(d + ex)^9(bd - ae)^2}{3e^5} - \frac{b(d + ex)^8(bd - ae)^3}{2e^5} + \frac{(d + ex)^7(bd - ae)^4}{7e^5} + \frac{b^4(d + ex)^{11}}{11e^5}
 \end{aligned}$$

input `Int[(d + e*x)^6*(a^2 + 2*a*b*x + b^2*x^2)^2,x]`

output `((b*d - a*e)^4*(d + e*x)^7)/(7*e^5) - (b*(b*d - a*e)^3*(d + e*x)^8)/(2*e^5) + (2*b^2*(b*d - a*e)^2*(d + e*x)^9)/(3*e^5) - (2*b^3*(b*d - a*e)*(d + e*x)^10)/(5*e^5) + (b^4*(d + e*x)^11)/(11*e^5)`

Definitions of rubi rules used

rule 27 $\text{Int}[(a_*)(Fx_), x_Symbol] \rightarrow \text{Simp}[a \text{ Int}[Fx, x], x] /; \text{FreeQ}[a, x] \ \&\& \ !\text{MatchQ}[Fx, (b_*)(Gx_)] /; \text{FreeQ}[b, x]$

rule 49 $\text{Int}[((a_.) + (b_.)(x_))^{(m_.)}((c_.) + (d_.)(x_))^{(n_.)}, x_Symbol] \rightarrow \text{Int}[\text{ExpandIntegrand}[(a + b*x)^m*(c + d*x)^n, x], x] /; \text{FreeQ}[\{a, b, c, d\}, x] \ \&\& \ \text{IGtQ}[m, 0] \ \&\& \ \text{IGtQ}[m + n + 2, 0]$

rule 1098 $\text{Int}[((d_.) + (e_.)(x_))^{(m_.)}((a_) + (b_.)(x_) + (c_.)(x_)^2)^{(p_.)}, x_Symbol] \rightarrow \text{Simp}[1/c^p \text{ Int}[(d + e*x)^m*(b/2 + c*x)^{(2*p)}, x], x] /; \text{FreeQ}[\{a, b, c, d, e, m\}, x] \ \&\& \ \text{EqQ}[b^2 - 4*a*c, 0] \ \&\& \ \text{IntegerQ}[p]$

rule 2009 $\text{Int}[u_, x_Symbol] \rightarrow \text{Simp}[\text{IntSum}[u, x], x] /; \text{SumQ}[u]$

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 416 vs. $2(109) = 218$.

Time = 1.06 (sec) , antiderivative size = 417, normalized size of antiderivative = 3.50

| method | result |
|--------------|--|
| norman | $\frac{b^4 e^6 x^{11}}{11} + \left(\frac{2}{5} a b^3 e^6 + \frac{3}{5} b^4 d e^5\right) x^{10} + \left(\frac{2}{3} a^2 b^2 e^6 + \frac{8}{3} a b^3 d e^5 + \frac{5}{3} b^4 d^2 e^4\right) x^9 + \left(\frac{1}{2} a^3 b e^6 + \frac{9}{2} a^2 b^2 d e^5\right) x^8 + \left(\frac{1}{3} a^4 e^6 + \frac{4}{3} a^3 b d e^5 + \frac{6}{3} a^2 b^2 d^2 e^4 + \frac{4}{3} a b^3 d^3 e^3 + \frac{1}{3} b^4 d^4 e^2\right) x^7 + \left(\frac{1}{4} a^5 e^6 + \frac{5}{4} a^4 b d e^5 + \frac{15}{4} a^3 b^2 d^2 e^4 + \frac{5}{4} a^2 b^3 d^3 e^3 + \frac{5}{4} a b^4 d^4 e^2 + \frac{1}{4} b^5 d^5 e\right) x^6 + \left(\frac{1}{5} a^6 e^6 + \frac{6}{5} a^5 b d e^5 + \frac{18}{5} a^4 b^2 d^2 e^4 + \frac{6}{5} a^3 b^3 d^3 e^3 + \frac{6}{5} a^2 b^4 d^4 e^2 + \frac{2}{5} a b^5 d^5 e\right) x^5 + \left(\frac{1}{6} a^7 e^6 + \frac{7}{6} a^6 b d e^5 + \frac{21}{6} a^5 b^2 d^2 e^4 + \frac{7}{6} a^4 b^3 d^3 e^3 + \frac{7}{6} a^3 b^4 d^4 e^2 + \frac{7}{6} a^2 b^5 d^5 e\right) x^4 + \left(\frac{1}{7} a^8 e^6 + \frac{8}{7} a^7 b d e^5 + \frac{28}{7} a^6 b^2 d^2 e^4 + \frac{8}{7} a^5 b^3 d^3 e^3 + \frac{8}{7} a^4 b^4 d^4 e^2 + \frac{8}{7} a^3 b^5 d^5 e\right) x^3 + \left(\frac{1}{8} a^9 e^6 + \frac{9}{8} a^8 b d e^5 + \frac{36}{8} a^7 b^2 d^2 e^4 + \frac{9}{8} a^6 b^3 d^3 e^3 + \frac{9}{8} a^5 b^4 d^4 e^2 + \frac{9}{8} a^4 b^5 d^5 e\right) x^2 + \left(\frac{1}{9} a^{10} e^6 + \frac{10}{9} a^9 b d e^5 + \frac{45}{9} a^8 b^2 d^2 e^4 + \frac{10}{9} a^7 b^3 d^3 e^3 + \frac{10}{9} a^6 b^4 d^4 e^2 + \frac{10}{9} a^5 b^5 d^5 e\right) x + \frac{1}{10} a^{11} e^6 + \frac{11}{10} a^{10} b d e^5 + \frac{55}{10} a^9 b^2 d^2 e^4 + \frac{11}{10} a^8 b^3 d^3 e^3 + \frac{11}{10} a^7 b^4 d^4 e^2 + \frac{11}{10} a^6 b^5 d^5 e$ |
| default | $\frac{b^4 e^6 x^{11}}{11} + \frac{(4 a b^3 e^6 + 6 b^4 d e^5) x^{10}}{10} + \frac{(6 a^2 b^2 e^6 + 24 a b^3 d e^5 + 15 b^4 d^2 e^4) x^9}{9} + \frac{(4 a^3 b e^6 + 36 a^2 b^2 d e^5 + 60 a b^3 d^2 e^4 + 20 b^4 d^3 e^3) x^8}{8}$ |
| risch | $\frac{8}{3} x^9 a b^3 d e^5 + \frac{9}{2} x^8 a^2 b^2 d e^5 + \frac{15}{2} x^8 a b^3 d^2 e^4 + \frac{24}{7} x^7 a^3 b d e^5 + \frac{90}{7} x^7 a^2 b^2 d^2 e^4 + \frac{80}{7} x^7 a b^3 d^3 e^3 + 1$ |
| parallelrisc | $\frac{8}{3} x^9 a b^3 d e^5 + \frac{9}{2} x^8 a^2 b^2 d e^5 + \frac{15}{2} x^8 a b^3 d^2 e^4 + \frac{24}{7} x^7 a^3 b d e^5 + \frac{90}{7} x^7 a^2 b^2 d^2 e^4 + \frac{80}{7} x^7 a b^3 d^3 e^3 + 1$ |
| gospers | $x(210 b^4 e^6 x^{10} + 924 x^9 a b^3 e^6 + 1386 x^9 b^4 d e^5 + 1540 x^8 a^2 b^2 e^6 + 6160 x^8 a b^3 d e^5 + 3850 x^8 b^4 d^2 e^4 + 1155 x^7 a^3 b e^6 + 10395 x^7 a^2 b^2 d e^5 + 5005 x^7 a b^3 d^2 e^4 + 1001 x^6 a^4 e^6 + 1101 x^6 a^3 b d e^5 + 3303 x^6 a^2 b^2 d^2 e^4 + 330 x^6 a b^3 d^3 e^3 + 33 x^5 a^5 e^6 + 330 x^5 a^4 b d e^5 + 1650 x^5 a^3 b^2 d^2 e^4 + 330 x^5 a^2 b^3 d^3 e^3 + 33 x^4 a^6 e^6 + 330 x^4 a^5 b d e^5 + 1650 x^4 a^4 b^2 d^2 e^4 + 330 x^4 a^3 b^3 d^3 e^3 + 33 x^3 a^7 e^6 + 330 x^3 a^6 b d e^5 + 1650 x^3 a^5 b^2 d^2 e^4 + 330 x^3 a^4 b^3 d^3 e^3 + 33 x^2 a^8 e^6 + 330 x^2 a^7 b d e^5 + 1650 x^2 a^6 b^2 d^2 e^4 + 330 x^2 a^5 b^3 d^3 e^3 + 33 x a^9 e^6 + 330 x a^8 b d e^5 + 1650 x a^7 b^2 d^2 e^4 + 330 x a^6 b^3 d^3 e^3 + 33 a^{10} e^6 + 330 a^9 b d e^5 + 1650 a^8 b^2 d^2 e^4 + 330 a^7 b^3 d^3 e^3 + 33 a^6 b^4 d^4 e^2 + 330 a^5 b^5 d^5 e$ |
| oring | $x(210 b^4 e^6 x^{10} + 924 x^9 a b^3 e^6 + 1386 x^9 b^4 d e^5 + 1540 x^8 a^2 b^2 e^6 + 6160 x^8 a b^3 d e^5 + 3850 x^8 b^4 d^2 e^4 + 1155 x^7 a^3 b e^6 + 10395 x^7 a^2 b^2 d e^5 + 5005 x^7 a b^3 d^2 e^4 + 1001 x^6 a^4 e^6 + 1101 x^6 a^3 b d e^5 + 3303 x^6 a^2 b^2 d^2 e^4 + 330 x^6 a b^3 d^3 e^3 + 33 x^5 a^5 e^6 + 330 x^5 a^4 b d e^5 + 1650 x^5 a^3 b^2 d^2 e^4 + 330 x^5 a^2 b^3 d^3 e^3 + 33 x^4 a^6 e^6 + 330 x^4 a^5 b d e^5 + 1650 x^4 a^4 b^2 d^2 e^4 + 330 x^4 a^3 b^3 d^3 e^3 + 33 x^3 a^7 e^6 + 330 x^3 a^6 b d e^5 + 1650 x^3 a^5 b^2 d^2 e^4 + 330 x^3 a^4 b^3 d^3 e^3 + 33 x^2 a^8 e^6 + 330 x^2 a^7 b d e^5 + 1650 x^2 a^6 b^2 d^2 e^4 + 330 x^2 a^5 b^3 d^3 e^3 + 33 x a^9 e^6 + 330 x a^8 b d e^5 + 1650 x a^7 b^2 d^2 e^4 + 330 x a^6 b^3 d^3 e^3 + 33 a^{10} e^6 + 330 a^9 b d e^5 + 1650 a^8 b^2 d^2 e^4 + 330 a^7 b^3 d^3 e^3 + 33 a^6 b^4 d^4 e^2 + 330 a^5 b^5 d^5 e$ |

input $\text{int}((e*x+d)^6*(b^2*x^2+2*a*b*x+a^2)^2,x,\text{method}=_RETURNVERBOSE)$

output

```
1/11*b^4*e^6*x^11+(2/5*a*b^3*e^6+3/5*b^4*d*e^5)*x^10+(2/3*a^2*b^2*e^6+8/3*
a*b^3*d*e^5+5/3*b^4*d^2*e^4)*x^9+(1/2*a^3*b*e^6+9/2*a^2*b^2*d*e^5+15/2*a*b
^3*d^2*e^4+5/2*b^4*d^3*e^3)*x^8+(1/7*a^4*e^6+24/7*a^3*b*d*e^5+90/7*a^2*b^2
*d^2*e^4+80/7*a*b^3*d^3*e^3+15/7*b^4*d^4*e^2)*x^7+(a^4*d*e^5+10*a^3*b*d^2*
e^4+20*a^2*b^2*d^3*e^3+10*a*b^3*d^4*e^2+b^4*d^5*e)*x^6+(3*a^4*d^2*e^4+16*a
^3*b*d^3*e^3+18*a^2*b^2*d^4*e^2+24/5*a*b^3*d^5*e+1/5*b^4*d^6)*x^5+(5*a^4*d
^3*e^3+15*a^3*b*d^4*e^2+9*a^2*b^2*d^5*e+a*b^3*d^6)*x^4+(5*a^4*d^4*e^2+8*a^
3*b*d^5*e+2*a^2*b^2*d^6)*x^3+(3*a^4*d^5*e+2*a^3*b*d^6)*x^2+d^6*a^4*x
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 418 vs. $2(109) = 218$.

Time = 0.08 (sec) , antiderivative size = 418, normalized size of antiderivative = 3.51

$$\int (d+ex)^6 (a^2+2abx+b^2x^2)^2 dx$$

$$= \frac{1}{11} b^4 e^6 x^{11} + a^4 d^6 x + \frac{1}{5} (3 b^4 d e^5 + 2 a b^3 e^6) x^{10} + \frac{1}{3} (5 b^4 d^2 e^4 + 8 a b^3 d e^5 + 2 a^2 b^2 e^6) x^9$$

$$+ \frac{1}{2} (5 b^4 d^3 e^3 + 15 a b^3 d^2 e^4 + 9 a^2 b^2 d e^5 + a^3 b e^6) x^8$$

$$+ \frac{1}{7} (15 b^4 d^4 e^2 + 80 a b^3 d^3 e^3 + 90 a^2 b^2 d^2 e^4 + 24 a^3 b d e^5 + a^4 e^6) x^7$$

$$+ (b^4 d^5 e + 10 a b^3 d^4 e^2 + 20 a^2 b^2 d^3 e^3 + 10 a^3 b d^2 e^4 + a^4 d e^5) x^6$$

$$+ \frac{1}{5} (b^4 d^6 + 24 a b^3 d^5 e + 90 a^2 b^2 d^4 e^2 + 80 a^3 b d^3 e^3 + 15 a^4 d^2 e^4) x^5$$

$$+ (a b^3 d^6 + 9 a^2 b^2 d^5 e + 15 a^3 b d^4 e^2 + 5 a^4 d^3 e^3) x^4$$

$$+ (2 a^2 b^2 d^6 + 8 a^3 b d^5 e + 5 a^4 d^4 e^2) x^3 + (2 a^3 b d^6 + 3 a^4 d^5 e) x^2$$

input

```
integrate((e*x+d)^6*(b^2*x^2+2*a*b*x+a^2)^2,x, algorithm="fricas")
```


output

```

1/11*b^4*e^6*x^11 + a^4*d^6*x + 1/5*(3*b^4*d*e^5 + 2*a*b^3*e^6)*x^10 + 1/3
*(5*b^4*d^2*e^4 + 8*a*b^3*d*e^5 + 2*a^2*b^2*e^6)*x^9 + 1/2*(5*b^4*d^3*e^3
+ 15*a*b^3*d^2*e^4 + 9*a^2*b^2*d*e^5 + a^3*b*e^6)*x^8 + 1/7*(15*b^4*d^4*e^
2 + 80*a*b^3*d^3*e^3 + 90*a^2*b^2*d^2*e^4 + 24*a^3*b*d*e^5 + a^4*e^6)*x^7
+ (b^4*d^5*e + 10*a*b^3*d^4*e^2 + 20*a^2*b^2*d^3*e^3 + 10*a^3*b*d^2*e^4 +
a^4*d*e^5)*x^6 + 1/5*(b^4*d^6 + 24*a*b^3*d^5*e + 90*a^2*b^2*d^4*e^2 + 80*a
^3*b*d^3*e^3 + 15*a^4*d^2*e^4)*x^5 + (a*b^3*d^6 + 9*a^2*b^2*d^5*e + 15*a^3
*b*d^4*e^2 + 5*a^4*d^3*e^3)*x^4 + (2*a^2*b^2*d^6 + 8*a^3*b*d^5*e + 5*a^4*d
^4*e^2)*x^3 + (2*a^3*b*d^6 + 3*a^4*d^5*e)*x^2

```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 462 vs. $2(105) = 210$.

Time = 0.05 (sec) , antiderivative size = 462, normalized size of antiderivative = 3.88

$$\begin{aligned}
\int (d + ex)^6 (a^2 + 2abx + b^2x^2)^2 dx &= a^4d^6x + \frac{b^4e^6x^{11}}{11} + x^{10} \cdot \left(\frac{2ab^3e^6}{5} + \frac{3b^4de^5}{5} \right) \\
&+ x^9 \cdot \left(\frac{2a^2b^2e^6}{3} + \frac{8ab^3de^5}{3} + \frac{5b^4d^2e^4}{3} \right) \\
&+ x^8 \left(\frac{a^3be^6}{2} + \frac{9a^2b^2de^5}{2} + \frac{15ab^3d^2e^4}{2} + \frac{5b^4d^3e^3}{2} \right) \\
&+ x^7 \left(\frac{a^4e^6}{7} + \frac{24a^3bde^5}{7} + \frac{90a^2b^2d^2e^4}{7} + \frac{80ab^3d^3e^3}{7} \right. \\
&\quad \left. + \frac{15b^4d^4e^2}{7} \right) + x^6 (a^4de^5 + 10a^3bd^2e^4 \\
&+ 20a^2b^2d^3e^3 + 10ab^3d^4e^2 + b^4d^5e) + x^5 \cdot \left(3a^4d^2e^4 \right. \\
&\quad \left. + 16a^3bd^3e^3 + 18a^2b^2d^4e^2 + \frac{24ab^3d^5e}{5} + \frac{b^4d^6}{5} \right) \\
&+ x^4 \cdot (5a^4d^3e^3 + 15a^3bd^4e^2 + 9a^2b^2d^5e + ab^3d^6) \\
&+ x^3 \cdot (5a^4d^4e^2 + 8a^3bd^5e + 2a^2b^2d^6) \\
&+ x^2 \cdot (3a^4d^5e + 2a^3bd^6)
\end{aligned}$$

input

```
integrate((e*x+d)**6*(b**2*x**2+2*a*b*x+a**2)**2,x)
```

output

```
a**4*d**6*x + b**4*e**6*x**11/11 + x**10*(2*a*b**3*e**6/5 + 3*b**4*d*e**5/
5) + x**9*(2*a**2*b**2*e**6/3 + 8*a*b**3*d*e**5/3 + 5*b**4*d**2*e**4/3) +
x**8*(a**3*b*e**6/2 + 9*a**2*b**2*d*e**5/2 + 15*a*b**3*d**2*e**4/2 + 5*b**
4*d**3*e**3/2) + x**7*(a**4*e**6/7 + 24*a**3*b*d*e**5/7 + 90*a**2*b**2*d**
2*e**4/7 + 80*a*b**3*d**3*e**3/7 + 15*b**4*d**4*e**2/7) + x**6*(a**4*d*e**
5 + 10*a**3*b*d**2*e**4 + 20*a**2*b**2*d**3*e**3 + 10*a*b**3*d**4*e**2 + b
**4*d**5*e) + x**5*(3*a**4*d**2*e**4 + 16*a**3*b*d**3*e**3 + 18*a**2*b**2*
d**4*e**2 + 24*a*b**3*d**5*e/5 + b**4*d**6/5) + x**4*(5*a**4*d**3*e**3 + 1
5*a**3*b*d**4*e**2 + 9*a**2*b**2*d**5*e + a*b**3*d**6) + x**3*(5*a**4*d**4
*e**2 + 8*a**3*b*d**5*e + 2*a**2*b**2*d**6) + x**2*(3*a**4*d**5*e + 2*a**3
*b*d**6)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 418 vs. $2(109) = 218$.

Time = 0.07 (sec) , antiderivative size = 418, normalized size of antiderivative = 3.51

$$\int (d + ex)^6 (a^2 + 2abx + b^2x^2)^2 dx$$

$$= \frac{1}{11} b^4 e^6 x^{11} + a^4 d^6 x + \frac{1}{5} (3 b^4 d e^5 + 2 a b^3 e^6) x^{10} + \frac{1}{3} (5 b^4 d^2 e^4 + 8 a b^3 d e^5 + 2 a^2 b^2 e^6) x^9$$

$$+ \frac{1}{2} (5 b^4 d^3 e^3 + 15 a b^3 d^2 e^4 + 9 a^2 b^2 d e^5 + a^3 b e^6) x^8$$

$$+ \frac{1}{7} (15 b^4 d^4 e^2 + 80 a b^3 d^3 e^3 + 90 a^2 b^2 d^2 e^4 + 24 a^3 b d e^5 + a^4 e^6) x^7$$

$$+ (b^4 d^5 e + 10 a b^3 d^4 e^2 + 20 a^2 b^2 d^3 e^3 + 10 a^3 b d^2 e^4 + a^4 d e^5) x^6$$

$$+ \frac{1}{5} (b^4 d^6 + 24 a b^3 d^5 e + 90 a^2 b^2 d^4 e^2 + 80 a^3 b d^3 e^3 + 15 a^4 d^2 e^4) x^5$$

$$+ (a b^3 d^6 + 9 a^2 b^2 d^5 e + 15 a^3 b d^4 e^2 + 5 a^4 d^3 e^3) x^4$$

$$+ (2 a^2 b^2 d^6 + 8 a^3 b d^5 e + 5 a^4 d^4 e^2) x^3 + (2 a^3 b d^6 + 3 a^4 d^5 e) x^2$$

input

```
integrate((e*x+d)^6*(b^2*x^2+2*a*b*x+a^2)^2,x, algorithm="maxima")
```

output

```

1/11*b^4*e^6*x^11 + a^4*d^6*x + 1/5*(3*b^4*d*e^5 + 2*a*b^3*e^6)*x^10 + 1/3
*(5*b^4*d^2*e^4 + 8*a*b^3*d*e^5 + 2*a^2*b^2*e^6)*x^9 + 1/2*(5*b^4*d^3*e^3
+ 15*a*b^3*d^2*e^4 + 9*a^2*b^2*d*e^5 + a^3*b*e^6)*x^8 + 1/7*(15*b^4*d^4*e^
2 + 80*a*b^3*d^3*e^3 + 90*a^2*b^2*d^2*e^4 + 24*a^3*b*d*e^5 + a^4*e^6)*x^7
+ (b^4*d^5*e + 10*a*b^3*d^4*e^2 + 20*a^2*b^2*d^3*e^3 + 10*a^3*b*d^2*e^4 +
a^4*d*e^5)*x^6 + 1/5*(b^4*d^6 + 24*a*b^3*d^5*e + 90*a^2*b^2*d^4*e^2 + 80*a
^3*b*d^3*e^3 + 15*a^4*d^2*e^4)*x^5 + (a*b^3*d^6 + 9*a^2*b^2*d^5*e + 15*a^3
*b*d^4*e^2 + 5*a^4*d^3*e^3)*x^4 + (2*a^2*b^2*d^6 + 8*a^3*b*d^5*e + 5*a^4*d
^4*e^2)*x^3 + (2*a^3*b*d^6 + 3*a^4*d^5*e)*x^2

```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 470 vs. $2(109) = 218$.

Time = 0.20 (sec) , antiderivative size = 470, normalized size of antiderivative = 3.95

$$\begin{aligned}
\int (d+ex)^6 (a^2+2abx+b^2x^2)^2 dx = & \frac{1}{11} b^4 e^6 x^{11} + \frac{3}{5} b^4 d e^5 x^{10} + \frac{2}{5} a b^3 e^6 x^{10} \\
& + \frac{5}{3} b^4 d^2 e^4 x^9 + \frac{8}{3} a b^3 d e^5 x^9 + \frac{2}{3} a^2 b^2 e^6 x^9 \\
& + \frac{5}{2} b^4 d^3 e^3 x^8 + \frac{15}{2} a b^3 d^2 e^4 x^8 + \frac{9}{2} a^2 b^2 d e^5 x^8 \\
& + \frac{1}{2} a^3 b e^6 x^8 + \frac{15}{7} b^4 d^4 e^2 x^7 + \frac{80}{7} a b^3 d^3 e^3 x^7 \\
& + \frac{90}{7} a^2 b^2 d^2 e^4 x^7 + \frac{24}{7} a^3 b d e^5 x^7 + \frac{1}{7} a^4 e^6 x^7 \\
& + b^4 d^5 e x^6 + 10 a b^3 d^4 e^2 x^6 + 20 a^2 b^2 d^3 e^3 x^6 \\
& + 10 a^3 b d^2 e^4 x^6 + a^4 d e^5 x^6 + \frac{1}{5} b^4 d^6 x^5 + \frac{24}{5} a b^3 d^5 e x^5 \\
& + 18 a^2 b^2 d^4 e^2 x^5 + 16 a^3 b d^3 e^3 x^5 + 3 a^4 d^2 e^4 x^5 \\
& + a b^3 d^6 x^4 + 9 a^2 b^2 d^5 e x^4 + 15 a^3 b d^4 e^2 x^4 \\
& + 5 a^4 d^3 e^3 x^4 + 2 a^2 b^2 d^6 x^3 + 8 a^3 b d^5 e x^3 \\
& + 5 a^4 d^4 e^2 x^3 + 2 a^3 b d^6 x^2 + 3 a^4 d^5 e x^2 + a^4 d^6 x
\end{aligned}$$

input

```

integrate((e*x+d)^6*(b^2*x^2+2*a*b*x+a^2)^2,x, algorithm="giac")

```

output

```

1/11*b^4*e^6*x^11 + 3/5*b^4*d*e^5*x^10 + 2/5*a*b^3*e^6*x^10 + 5/3*b^4*d^2*
e^4*x^9 + 8/3*a*b^3*d*e^5*x^9 + 2/3*a^2*b^2*e^6*x^9 + 5/2*b^4*d^3*e^3*x^8
+ 15/2*a*b^3*d^2*e^4*x^8 + 9/2*a^2*b^2*d*e^5*x^8 + 1/2*a^3*b*e^6*x^8 + 15/
7*b^4*d^4*e^2*x^7 + 80/7*a*b^3*d^3*e^3*x^7 + 90/7*a^2*b^2*d^2*e^4*x^7 + 24
/7*a^3*b*d*e^5*x^7 + 1/7*a^4*e^6*x^7 + b^4*d^5*e*x^6 + 10*a*b^3*d^4*e^2*x^
6 + 20*a^2*b^2*d^3*e^3*x^6 + 10*a^3*b*d^2*e^4*x^6 + a^4*d*e^5*x^6 + 1/5*b^
4*d^6*x^5 + 24/5*a*b^3*d^5*e*x^5 + 18*a^2*b^2*d^4*e^2*x^5 + 16*a^3*b*d^3*e
^3*x^5 + 3*a^4*d^2*e^4*x^5 + a*b^3*d^6*x^4 + 9*a^2*b^2*d^5*e*x^4 + 15*a^3*
b*d^4*e^2*x^4 + 5*a^4*d^3*e^3*x^4 + 2*a^2*b^2*d^6*x^3 + 8*a^3*b*d^5*e*x^3
+ 5*a^4*d^4*e^2*x^3 + 2*a^3*b*d^6*x^2 + 3*a^4*d^5*e*x^2 + a^4*d^6*x

```

Mupad [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 402, normalized size of antiderivative = 3.38

$$\begin{aligned}
\int (d + ex)^6 (a^2 + 2abx + b^2x^2)^2 dx = & x^5 \left(3a^4d^2e^4 + 16a^3bd^3e^3 + 18a^2b^2d^4e^2 \right. \\
& + \frac{24ab^3d^5e}{5} + \frac{b^4d^6}{5} \Big) + x^7 \left(\frac{a^4e^6}{7} + \frac{24a^3bde^5}{7} \right. \\
& \left. + \frac{90a^2b^2d^2e^4}{7} + \frac{80ab^3d^3e^3}{7} + \frac{15b^4d^4e^2}{7} \right) \\
& + x^4 \left(5a^4d^3e^3 + 15a^3bd^4e^2 + 9a^2b^2d^5e \right. \\
& \left. + ab^3d^6 \right) + x^8 \left(\frac{a^3be^6}{2} + \frac{9a^2b^2de^5}{2} \right. \\
& \left. + \frac{15ab^3d^2e^4}{2} + \frac{5b^4d^3e^3}{2} \right) \\
& + x^6 \left(a^4de^5 + 10a^3bd^2e^4 + 20a^2b^2d^3e^3 \right. \\
& \left. + 10ab^3d^4e^2 + b^4d^5e \right) + a^4d^6x + \frac{b^4e^6x^{11}}{11} \\
& + a^3d^5x^2(3ae + 2bd) + \frac{b^3e^5x^{10}(2ae + 3bd)}{5} \\
& + a^2d^4x^3(5a^2e^2 + 8abde + 2b^2d^2) \\
& + \frac{b^2e^4x^9(2a^2e^2 + 8abde + 5b^2d^2)}{3}
\end{aligned}$$

input

```
int((d + e*x)^6*(a^2 + b^2*x^2 + 2*a*b*x)^2,x)
```

output

```
x^5*((b^4*d^6)/5 + 3*a^4*d^2*e^4 + 16*a^3*b*d^3*e^3 + 18*a^2*b^2*d^4*e^2 +
(24*a*b^3*d^5*e)/5) + x^7*((a^4*e^6)/7 + (15*b^4*d^4*e^2)/7 + (80*a*b^3*d
^3*e^3)/7 + (90*a^2*b^2*d^2*e^4)/7 + (24*a^3*b*d*e^5)/7) + x^4*(a*b^3*d^6
+ 5*a^4*d^3*e^3 + 9*a^2*b^2*d^5*e + 15*a^3*b*d^4*e^2) + x^8*((a^3*b*e^6)/2
+ (5*b^4*d^3*e^3)/2 + (15*a*b^3*d^2*e^4)/2 + (9*a^2*b^2*d*e^5)/2) + x^6*(
a^4*d*e^5 + b^4*d^5*e + 10*a*b^3*d^4*e^2 + 10*a^3*b*d^2*e^4 + 20*a^2*b^2*d
^3*e^3) + a^4*d^6*x + (b^4*e^6*x^11)/11 + a^3*d^5*x^2*(3*a*e + 2*b*d) + (b
^3*e^5*x^10*(2*a*e + 3*b*d))/5 + a^2*d^4*x^3*(5*a^2*e^2 + 2*b^2*d^2 + 8*a*
b*d*e) + (b^2*e^4*x^9*(2*a^2*e^2 + 5*b^2*d^2 + 8*a*b*d*e))/3
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 472, normalized size of antiderivative = 3.97

$$\int (d + ex)^6 (a^2 + 2abx + b^2x^2)^2 dx$$

$$= \frac{x(210b^4e^6x^{10} + 924ab^3e^6x^9 + 1386b^4de^5x^9 + 1540a^2b^2e^6x^8 + 6160ab^3de^5x^8 + 3850b^4d^2e^4x^8 + 1155a^3b^4d^2e^4x^8 + 1155a^3b^4d^2e^4x^8 + 1155a^3b^4d^2e^4x^8)}{1}$$

input

```
int((e*x+d)^6*(b^2*x^2+2*a*b*x+a^2)^2,x)
```

output

```
(x*(2310*a**4*d**6 + 6930*a**4*d**5*e*x + 11550*a**4*d**4*e**2*x**2 + 1155
0*a**4*d**3*e**3*x**3 + 6930*a**4*d**2*e**4*x**4 + 2310*a**4*d*e**5*x**5 +
330*a**4*e**6*x**6 + 4620*a**3*b*d**6*x + 18480*a**3*b*d**5*e*x**2 + 3465
0*a**3*b*d**4*e**2*x**3 + 36960*a**3*b*d**3*e**3*x**4 + 23100*a**3*b*d**2*
e**4*x**5 + 7920*a**3*b*d*e**5*x**6 + 1155*a**3*b*e**6*x**7 + 4620*a**2*b*
*d**6*x**2 + 20790*a**2*b**2*d**5*e*x**3 + 41580*a**2*b**2*d**4*e**2*x**
4 + 46200*a**2*b**2*d**3*e**3*x**5 + 29700*a**2*b**2*d**2*e**4*x**6 + 1039
5*a**2*b**2*d*e**5*x**7 + 1540*a**2*b**2*e**6*x**8 + 2310*a*b**3*d**6*x**3
+ 11088*a*b**3*d**5*e*x**4 + 23100*a*b**3*d**4*e**2*x**5 + 26400*a*b**3*d
**3*e**3*x**6 + 17325*a*b**3*d**2*e**4*x**7 + 6160*a*b**3*d*e**5*x**8 + 92
4*a*b**3*e**6*x**9 + 462*b**4*d**6*x**4 + 2310*b**4*d**5*e*x**5 + 4950*b**
4*d**4*e**2*x**6 + 5775*b**4*d**3*e**3*x**7 + 3850*b**4*d**2*e**4*x**8 + 1
386*b**4*d*e**5*x**9 + 210*b**4*e**6*x**10))/2310
```

3.67 $\int (d + ex)^5 (a^2 + 2abx + b^2x^2)^2 dx$

| | |
|---|-----|
| Optimal result | 545 |
| Mathematica [B] (verified) | 545 |
| Rubi [A] (verified) | 547 |
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Optimal result

Integrand size = 26, antiderivative size = 119

$$\int (d + ex)^5 (a^2 + 2abx + b^2x^2)^2 dx = \frac{(bd - ae)^4(d + ex)^6}{6e^5} - \frac{4b(bd - ae)^3(d + ex)^7}{7e^5} + \frac{3b^2(bd - ae)^2(d + ex)^8}{4e^5} - \frac{4b^3(bd - ae)(d + ex)^9}{9e^5} + \frac{b^4(d + ex)^{10}}{10e^5}$$

output

```
1/6*(-a*e+b*d)^4*(e*x+d)^6/e^5-4/7*b*(-a*e+b*d)^3*(e*x+d)^7/e^5+3/4*b^2*(-a*e+b*d)^2*(e*x+d)^8/e^5-4/9*b^3*(-a*e+b*d)*(e*x+d)^9/e^5+1/10*b^4*(e*x+d)^10/e^5
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 350 vs. 2(119) = 238.

Time = 0.04 (sec) , antiderivative size = 350, normalized size of antiderivative = 2.94

$$\int (d + ex)^5 (a^2 + 2abx + b^2x^2)^2 dx = a^4 d^5 x + \frac{1}{2} a^3 d^4 (4bd + 5ae) x^2 + \frac{2}{3} a^2 d^3 (3b^2 d^2 + 10abde + 5a^2 e^2) x^3 + \frac{1}{2} a d^2 (2b^3 d^3 + 15ab^2 d^2 e + 20a^2 b d e^2 + 5a^3 e^3) x^4 + \frac{1}{5} d (b^4 d^4 + 20ab^3 d^3 e + 60a^2 b^2 d^2 e^2 + 40a^3 b d e^3 + 5a^4 e^4) x^5 + \frac{1}{6} e (5b^4 d^4 + 40ab^3 d^3 e + 60a^2 b^2 d^2 e^2 + 20a^3 b d e^3 + a^4 e^4) x^6 + \frac{2}{7} b e^2 (5b^3 d^3 + 20ab^2 d^2 e + 15a^2 b d e^2 + 2a^3 e^3) x^7 + \frac{1}{4} b^2 e^3 (5b^2 d^2 + 10abde + 3a^2 e^2) x^8 + \frac{1}{9} b^3 e^4 (5bd + 4ae) x^9 + \frac{1}{10} b^4 e^5 x^{10}$$

input `Integrate[(d + e*x)^5*(a^2 + 2*a*b*x + b^2*x^2)^2,x]`

output `a^4*d^5*x + (a^3*d^4*(4*b*d + 5*a*e)*x^2)/2 + (2*a^2*d^3*(3*b^2*d^2 + 10*a*b*d*e + 5*a^2*e^2)*x^3)/3 + (a*d^2*(2*b^3*d^3 + 15*a*b^2*d^2*e + 20*a^2*b*d*e^2 + 5*a^3*e^3)*x^4)/2 + (d*(b^4*d^4 + 20*a*b^3*d^3*e + 60*a^2*b^2*d^2*e^2 + 40*a^3*b*d*e^3 + 5*a^4*e^4)*x^5)/5 + (e*(5*b^4*d^4 + 40*a*b^3*d^3*e + 60*a^2*b^2*d^2*e^2 + 20*a^3*b*d*e^3 + a^4*e^4)*x^6)/6 + (2*b*e^2*(5*b^3*d^3 + 20*a*b^2*d^2*e + 15*a^2*b*d*e^2 + 2*a^3*e^3)*x^7)/7 + (b^2*e^3*(5*b^2*d^2 + 10*a*b*d*e + 3*a^2*e^2)*x^8)/4 + (b^3*e^4*(5*b*d + 4*a*e)*x^9)/9 + (b^4*e^5*x^10)/10`

Rubi [A] (verified)

Time = 0.67 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {1098, 27, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\begin{aligned}
 & \int (a^2 + 2abx + b^2x^2)^2 (d + ex)^5 dx \\
 & \quad \downarrow 1098 \\
 & \quad \int \frac{b^4(a + bx)^4 (d + ex)^5 dx}{b^4} \\
 & \quad \downarrow 27 \\
 & \quad \int (a + bx)^4 (d + ex)^5 dx \\
 & \quad \downarrow 49 \\
 & \quad \int \left(-\frac{4b^3(d + ex)^8 (bd - ae)}{e^4} + \frac{6b^2(d + ex)^7 (bd - ae)^2}{e^4} - \frac{4b(d + ex)^6 (bd - ae)^3}{e^4} + \frac{(d + ex)^5 (ae - bd)^4}{e^4} + \frac{b^4(d + ex)^4}{e^4} \right) dx \\
 & \quad \downarrow 2009 \\
 & \quad -\frac{4b^3(d + ex)^9 (bd - ae)}{9e^5} + \frac{3b^2(d + ex)^8 (bd - ae)^2}{4e^5} - \frac{4b(d + ex)^7 (bd - ae)^3}{7e^5} + \frac{(d + ex)^6 (bd - ae)^4}{6e^5} + \frac{b^4(d + ex)^{10}}{10e^5}
 \end{aligned}$$

input `Int[(d + e*x)^5*(a^2 + 2*a*b*x + b^2*x^2)^2,x]`

output `((b*d - a*e)^4*(d + e*x)^6)/(6*e^5) - (4*b*(b*d - a*e)^3*(d + e*x)^7)/(7*e^5) + (3*b^2*(b*d - a*e)^2*(d + e*x)^8)/(4*e^5) - (4*b^3*(b*d - a*e)*(d + e*x)^9)/(9*e^5) + (b^4*(d + e*x)^10)/(10*e^5)`

Defintions of rubi rules used

rule 27 `Int[(a_)*(Fx_), x_Symbol] := Simp[a Int[Fx, x], x] /; FreeQ[a, x] && !MatchQ[Fx, (b_)*(Gx_)] /; FreeQ[b, x]`

rule 49 `Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] := Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] && IGtQ[m, 0] && IGtQ[m + n + 2, 0]`

rule 1098 `Int[((d_.) + (e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] := Simp[1/c^p Int[(d + e*x)^m*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]`

rule 2009 `Int[u_, x_Symbol] := Simp[IntSum[u, x], x] /; SumQ[u]`

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 352 vs. 2(109) = 218.

Time = 1.07 (sec) , antiderivative size = 353, normalized size of antiderivative = 2.97

| method | result |
|--------------|---|
| norman | $\frac{b^4 e^5 x^{10}}{10} + \left(\frac{4}{9} a b^3 e^5 + \frac{5}{9} b^4 d e^4\right) x^9 + \left(\frac{3}{4} a^2 b^2 e^5 + \frac{5}{2} a b^3 d e^4 + \frac{5}{4} b^4 d^2 e^3\right) x^8 + \left(\frac{4}{7} a^3 b e^5 + \frac{30}{7} a^2 b^2 d e^4\right) x^7 + \left(\frac{4}{7} a^3 b e^5 + \frac{30}{7} a^2 b^2 d e^4\right) x^6 + \left(\frac{4}{7} a^3 b e^5 + \frac{30}{7} a^2 b^2 d e^4\right) x^5 + \left(\frac{4}{7} a^3 b e^5 + \frac{30}{7} a^2 b^2 d e^4\right) x^4 + \left(\frac{4}{7} a^3 b e^5 + \frac{30}{7} a^2 b^2 d e^4\right) x^3 + \left(\frac{4}{7} a^3 b e^5 + \frac{30}{7} a^2 b^2 d e^4\right) x^2 + \left(\frac{4}{7} a^3 b e^5 + \frac{30}{7} a^2 b^2 d e^4\right) x + \left(\frac{4}{7} a^3 b e^5 + \frac{30}{7} a^2 b^2 d e^4\right)$ |
| default | $\frac{b^4 e^5 x^{10}}{10} + \frac{(4 a b^3 e^5 + 5 b^4 d e^4) x^9}{9} + \frac{(6 a^2 b^2 e^5 + 20 a b^3 d e^4 + 10 b^4 d^2 e^3) x^8}{8} + \frac{(4 a^3 b e^5 + 30 a^2 b^2 d e^4 + 40 a b^3 d^2 e^3 + 10 b^4 d^3 e^2) x^7}{7}$ |
| risch | $\frac{5}{2} d^4 e a^4 x^2 + d^5 a^4 x + 10 x^4 a^3 b d^3 e^2 + \frac{15}{2} x^4 a^2 b^2 d^4 e + \frac{20}{3} x^3 a^3 b d^4 e + \frac{5}{2} x^8 a b^3 d e^4 + \frac{30}{7} x^7 a^2 b^2 d e^4$ |
| parallelrisc | $\frac{5}{2} d^4 e a^4 x^2 + d^5 a^4 x + 10 x^4 a^3 b d^3 e^2 + \frac{15}{2} x^4 a^2 b^2 d^4 e + \frac{20}{3} x^3 a^3 b d^4 e + \frac{5}{2} x^8 a b^3 d e^4 + \frac{30}{7} x^7 a^2 b^2 d e^4$ |
| gospers | $x(126 b^4 e^5 x^9 + 560 x^8 a b^3 e^5 + 700 x^8 b^4 d e^4 + 945 x^7 a^2 b^2 e^5 + 3150 x^7 a b^3 d e^4 + 1575 x^7 b^4 d^2 e^3 + 720 x^6 a^3 b e^5 + 5400 x^6 a^2 b^2 d e^4 + 720 x^5 a^3 b d e^4 + 5400 x^5 a^2 b^2 d^2 e^3 + 1260 x^4 a^3 b d^2 e^3 + 5400 x^4 a^2 b^2 d^3 e^2 + 1260 x^3 a^3 b d^3 e^2 + 5400 x^3 a^2 b^2 d^4 e + 1260 x^2 a^3 b d^4 e + 5400 x^2 a^2 b^2 d^5 e + 1260 x a^3 b d^5 e + 5400 a^2 b^2 d^6 e)$ |
| oring | $x(126 b^4 e^5 x^9 + 560 x^8 a b^3 e^5 + 700 x^8 b^4 d e^4 + 945 x^7 a^2 b^2 e^5 + 3150 x^7 a b^3 d e^4 + 1575 x^7 b^4 d^2 e^3 + 720 x^6 a^3 b e^5 + 5400 x^6 a^2 b^2 d e^4 + 720 x^5 a^3 b d e^4 + 5400 x^5 a^2 b^2 d^2 e^3 + 1260 x^4 a^3 b d^2 e^3 + 5400 x^4 a^2 b^2 d^3 e^2 + 1260 x^3 a^3 b d^3 e^2 + 5400 x^3 a^2 b^2 d^4 e + 1260 x^2 a^3 b d^4 e + 5400 x^2 a^2 b^2 d^5 e + 1260 x a^3 b d^5 e + 5400 a^2 b^2 d^6 e)$ |

input `int((e*x+d)^5*(b^2*x^2+2*a*b*x+a^2)^2,x,method=_RETURNVERBOSE)`

output

```
1/10*b^4*e^5*x^10+(4/9*a*b^3*e^5+5/9*b^4*d*e^4)*x^9+(3/4*a^2*b^2*e^5+5/2*a
*b^3*d*e^4+5/4*b^4*d^2*e^3)*x^8+(4/7*a^3*b*e^5+30/7*a^2*b^2*d*e^4+40/7*a*b
^3*d^2*e^3+10/7*b^4*d^3*e^2)*x^7+(1/6*a^4*e^5+10/3*a^3*b*d*e^4+10*a^2*b^2*
d^2*e^3+20/3*a*b^3*d^3*e^2+5/6*b^4*d^4*e)*x^6+(a^4*d*e^4+8*a^3*b*d^2*e^3+1
2*a^2*b^2*d^3*e^2+4*a*b^3*d^4*e+1/5*b^4*d^5)*x^5+(5/2*a^4*d^2*e^3+10*a^3*b
*d^3*e^2+15/2*a^2*b^2*d^4*e+a*b^3*d^5)*x^4+(10/3*a^4*d^3*e^2+20/3*a^3*b*d^
4*e+2*a^2*b^2*d^5)*x^3+(5/2*a^4*d^4*e+2*a^3*b*d^5)*x^2+d^5*a^4*x
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 360 vs. $2(109) = 218$.

Time = 0.08 (sec) , antiderivative size = 360, normalized size of antiderivative = 3.03

$$\int (d+ex)^5 (a^2+2abx+b^2x^2)^2 dx$$

$$= \frac{1}{10} b^4 e^5 x^{10} + a^4 d^5 x + \frac{1}{9} (5 b^4 d e^4 + 4 a b^3 e^5) x^9 + \frac{1}{4} (5 b^4 d^2 e^3 + 10 a b^3 d e^4 + 3 a^2 b^2 e^5) x^8$$

$$+ \frac{2}{7} (5 b^4 d^3 e^2 + 20 a b^3 d^2 e^3 + 15 a^2 b^2 d e^4 + 2 a^3 b e^5) x^7$$

$$+ \frac{1}{6} (5 b^4 d^4 e + 40 a b^3 d^3 e^2 + 60 a^2 b^2 d^2 e^3 + 20 a^3 b d e^4 + a^4 e^5) x^6$$

$$+ \frac{1}{5} (b^4 d^5 + 20 a b^3 d^4 e + 60 a^2 b^2 d^3 e^2 + 40 a^3 b d^2 e^3 + 5 a^4 d e^4) x^5$$

$$+ \frac{1}{2} (2 a b^3 d^5 + 15 a^2 b^2 d^4 e + 20 a^3 b d^3 e^2 + 5 a^4 d^2 e^3) x^4$$

$$+ \frac{2}{3} (3 a^2 b^2 d^5 + 10 a^3 b d^4 e + 5 a^4 d^3 e^2) x^3 + \frac{1}{2} (4 a^3 b d^5 + 5 a^4 d^4 e) x^2$$

input

```
integrate((e*x+d)^5*(b^2*x^2+2*a*b*x+a^2)^2,x, algorithm="fricas")
```

output

```
1/10*b^4*e^5*x^10 + a^4*d^5*x + 1/9*(5*b^4*d*e^4 + 4*a*b^3*e^5)*x^9 + 1/4*
(5*b^4*d^2*e^3 + 10*a*b^3*d*e^4 + 3*a^2*b^2*e^5)*x^8 + 2/7*(5*b^4*d^3*e^2
+ 20*a*b^3*d^2*e^3 + 15*a^2*b^2*d*e^4 + 2*a^3*b*e^5)*x^7 + 1/6*(5*b^4*d^4*
e + 40*a*b^3*d^3*e^2 + 60*a^2*b^2*d^2*e^3 + 20*a^3*b*d*e^4 + a^4*e^5)*x^6
+ 1/5*(b^4*d^5 + 20*a*b^3*d^4*e + 60*a^2*b^2*d^3*e^2 + 40*a^3*b*d^2*e^3 +
5*a^4*d*e^4)*x^5 + 1/2*(2*a*b^3*d^5 + 15*a^2*b^2*d^4*e + 20*a^3*b*d^3*e^2
+ 5*a^4*d^2*e^3)*x^4 + 2/3*(3*a^2*b^2*d^5 + 10*a^3*b*d^4*e + 5*a^4*d^3*e^2
)*x^3 + 1/2*(4*a^3*b*d^5 + 5*a^4*d^4*e)*x^2
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 401 vs. $2(107) = 214$.

Time = 0.05 (sec) , antiderivative size = 401, normalized size of antiderivative = 3.37

$$\int (d + ex)^5 (a^2 + 2abx + b^2x^2)^2 dx = a^4d^5x + \frac{b^4e^5x^{10}}{10} + x^9 \cdot \left(\frac{4ab^3e^5}{9} + \frac{5b^4de^4}{9} \right) \\ + x^8 \cdot \left(\frac{3a^2b^2e^5}{4} + \frac{5ab^3de^4}{2} + \frac{5b^4d^2e^3}{4} \right) + x^7 \\ \cdot \left(\frac{4a^3be^5}{7} + \frac{30a^2b^2de^4}{7} + \frac{40ab^3d^2e^3}{7} + \frac{10b^4d^3e^2}{7} \right) \\ + x^6 \left(\frac{a^4e^5}{6} + \frac{10a^3bde^4}{3} + 10a^2b^2d^2e^3 + \frac{20ab^3d^3e^2}{3} \right. \\ \left. + \frac{5b^4d^4e}{6} \right) + x^5 \left(a^4de^4 + 8a^3bd^2e^3 + 12a^2b^2d^3e^2 \right. \\ \left. + 4ab^3d^4e + \frac{b^4d^5}{5} \right) + x^4 \\ \cdot \left(\frac{5a^4d^2e^3}{2} + 10a^3bd^3e^2 + \frac{15a^2b^2d^4e}{2} + ab^3d^5 \right) \\ + x^3 \cdot \left(\frac{10a^4d^3e^2}{3} + \frac{20a^3bd^4e}{3} + 2a^2b^2d^5 \right) \\ + x^2 \cdot \left(\frac{5a^4d^4e}{2} + 2a^3bd^5 \right)$$

input `integrate((e*x+d)**5*(b**2*x**2+2*a*b*x+a**2)**2,x)`

output `a**4*d**5*x + b**4*e**5*x**10/10 + x**9*(4*a*b**3*e**5/9 + 5*b**4*d*e**4/9) + x**8*(3*a**2*b**2*e**5/4 + 5*a*b**3*d*e**4/2 + 5*b**4*d**2*e**3/4) + x**7*(4*a**3*b*e**5/7 + 30*a**2*b**2*d*e**4/7 + 40*a*b**3*d**2*e**3/7 + 10*b**4*d**3*e**2/7) + x**6*(a**4*e**5/6 + 10*a**3*b*d*e**4/3 + 10*a**2*b**2*d**2*e**3 + 20*a*b**3*d**3*e**2/3 + 5*b**4*d**4*e/6) + x**5*(a**4*d*e**4 + 8*a**3*b*d**2*e**3 + 12*a**2*b**2*d**3*e**2 + 4*a*b**3*d**4*e + b**4*d**5/5) + x**4*(5*a**4*d**2*e**3/2 + 10*a**3*b*d**3*e**2 + 15*a**2*b**2*d**4*e/2 + a*b**3*d**5) + x**3*(10*a**4*d**3*e**2/3 + 20*a**3*b*d**4*e/3 + 2*a**2*b**2*d**5) + x**2*(5*a**4*d**4*e/2 + 2*a**3*b*d**5)`

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 360 vs. $2(109) = 218$.

Time = 0.04 (sec) , antiderivative size = 360, normalized size of antiderivative = 3.03

$$\begin{aligned} & \int (d + ex)^5 (a^2 + 2abx + b^2x^2)^2 dx \\ &= \frac{1}{10} b^4 e^5 x^{10} + a^4 d^5 x + \frac{1}{9} (5 b^4 d e^4 + 4 a b^3 e^5) x^9 + \frac{1}{4} (5 b^4 d^2 e^3 + 10 a b^3 d e^4 + 3 a^2 b^2 e^5) x^8 \\ &+ \frac{2}{7} (5 b^4 d^3 e^2 + 20 a b^3 d^2 e^3 + 15 a^2 b^2 d e^4 + 2 a^3 b e^5) x^7 \\ &+ \frac{1}{6} (5 b^4 d^4 e + 40 a b^3 d^3 e^2 + 60 a^2 b^2 d^2 e^3 + 20 a^3 b d e^4 + a^4 e^5) x^6 \\ &+ \frac{1}{5} (b^4 d^5 + 20 a b^3 d^4 e + 60 a^2 b^2 d^3 e^2 + 40 a^3 b d^2 e^3 + 5 a^4 d e^4) x^5 \\ &+ \frac{1}{2} (2 a b^3 d^5 + 15 a^2 b^2 d^4 e + 20 a^3 b d^3 e^2 + 5 a^4 d^2 e^3) x^4 \\ &+ \frac{2}{3} (3 a^2 b^2 d^5 + 10 a^3 b d^4 e + 5 a^4 d^3 e^2) x^3 + \frac{1}{2} (4 a^3 b d^5 + 5 a^4 d^4 e) x^2 \end{aligned}$$

input `integrate((e*x+d)^5*(b^2*x^2+2*a*b*x+a^2)^2,x, algorithm="maxima")`

output `1/10*b^4*e^5*x^10 + a^4*d^5*x + 1/9*(5*b^4*d*e^4 + 4*a*b^3*e^5)*x^9 + 1/4*(5*b^4*d^2*e^3 + 10*a*b^3*d*e^4 + 3*a^2*b^2*e^5)*x^8 + 2/7*(5*b^4*d^3*e^2 + 20*a*b^3*d^2*e^3 + 15*a^2*b^2*d*e^4 + 2*a^3*b*e^5)*x^7 + 1/6*(5*b^4*d^4*e + 40*a*b^3*d^3*e^2 + 60*a^2*b^2*d^2*e^3 + 20*a^3*b*d*e^4 + a^4*e^5)*x^6 + 1/5*(b^4*d^5 + 20*a*b^3*d^4*e + 60*a^2*b^2*d^3*e^2 + 40*a^3*b*d^2*e^3 + 5*a^4*d*e^4)*x^5 + 1/2*(2*a*b^3*d^5 + 15*a^2*b^2*d^4*e + 20*a^3*b*d^3*e^2 + 5*a^4*d^2*e^3)*x^4 + 2/3*(3*a^2*b^2*d^5 + 10*a^3*b*d^4*e + 5*a^4*d^3*e^2)*x^3 + 1/2*(4*a^3*b*d^5 + 5*a^4*d^4*e)*x^2`

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 396 vs. $2(109) = 218$.

Time = 0.24 (sec) , antiderivative size = 396, normalized size of antiderivative = 3.33

$$\int (d + ex)^5 (a^2 + 2abx + b^2x^2)^2 dx = \frac{1}{10} b^4 e^5 x^{10} + \frac{5}{9} b^4 d e^4 x^9 + \frac{4}{9} a b^3 e^5 x^9 + \frac{5}{4} b^4 d^2 e^3 x^8$$

$$+ \frac{5}{2} a b^3 d e^4 x^8 + \frac{3}{4} a^2 b^2 e^5 x^8 + \frac{10}{7} b^4 d^3 e^2 x^7$$

$$+ \frac{40}{7} a b^3 d^2 e^3 x^7 + \frac{30}{7} a^2 b^2 d e^4 x^7 + \frac{4}{7} a^3 b e^5 x^7$$

$$+ \frac{5}{6} b^4 d^4 e x^6 + \frac{20}{3} a b^3 d^3 e^2 x^6 + 10 a^2 b^2 d^2 e^3 x^6$$

$$+ \frac{10}{3} a^3 b d e^4 x^6 + \frac{1}{6} a^4 e^5 x^6 + \frac{1}{5} b^4 d^5 x^5 + 4 a b^3 d^4 e x^5$$

$$+ 12 a^2 b^2 d^3 e^2 x^5 + 8 a^3 b d^2 e^3 x^5 + a^4 d e^4 x^5$$

$$+ a b^3 d^5 x^4 + \frac{15}{2} a^2 b^2 d^4 e x^4 + 10 a^3 b d^3 e^2 x^4$$

$$+ \frac{5}{2} a^4 d^2 e^3 x^4 + 2 a^2 b^2 d^5 x^3 + \frac{20}{3} a^3 b d^4 e x^3$$

$$+ \frac{10}{3} a^4 d^3 e^2 x^3 + 2 a^3 b d^5 x^2 + \frac{5}{2} a^4 d^4 e x^2 + a^4 d^5 x$$

input `integrate((e*x+d)^5*(b^2*x^2+2*a*b*x+a^2)^2,x, algorithm="giac")`

output `1/10*b^4*e^5*x^10 + 5/9*b^4*d*e^4*x^9 + 4/9*a*b^3*e^5*x^9 + 5/4*b^4*d^2*e^3*x^8 + 5/2*a*b^3*d*e^4*x^8 + 3/4*a^2*b^2*e^5*x^8 + 10/7*b^4*d^3*e^2*x^7 + 40/7*a*b^3*d^2*e^3*x^7 + 30/7*a^2*b^2*d*e^4*x^7 + 4/7*a^3*b*e^5*x^7 + 5/6*b^4*d^4*e*x^6 + 20/3*a*b^3*d^3*e^2*x^6 + 10*a^2*b^2*d^2*e^3*x^6 + 10/3*a^3*b*d*e^4*x^6 + 1/6*a^4*e^5*x^6 + 1/5*b^4*d^5*x^5 + 4*a*b^3*d^4*e*x^5 + 12*a^2*b^2*d^3*e^2*x^5 + 8*a^3*b*d^2*e^3*x^5 + a^4*d*e^4*x^5 + a*b^3*d^5*x^4 + 15/2*a^2*b^2*d^4*e*x^4 + 10*a^3*b*d^3*e^2*x^4 + 5/2*a^4*d^2*e^3*x^4 + 2*a^2*b^2*d^5*x^3 + 20/3*a^3*b*d^4*e*x^3 + 10/3*a^4*d^3*e^2*x^3 + 2*a^3*b*d^5*x^2 + 5/2*a^4*d^4*e*x^2 + a^4*d^5*x`

Mupad [B] (verification not implemented)

Time = 9.34 (sec) , antiderivative size = 340, normalized size of antiderivative = 2.86

$$\begin{aligned}
\int (d+ex)^5 (a^2+2abx+b^2x^2)^2 dx = & x^4 \left(\frac{5a^4d^2e^3}{2} + 10a^3bd^3e^2 + \frac{15a^2b^2d^4e}{2} + ab^3d^5 \right) \\
& + x^7 \left(\frac{4a^3be^5}{7} + \frac{30a^2b^2de^4}{7} + \frac{40ab^3d^2e^3}{7} \right. \\
& \quad \left. + \frac{10b^4d^3e^2}{7} \right) + x^5 \left(a^4de^4 + 8a^3bd^2e^3 \right. \\
& \quad \left. + 12a^2b^2d^3e^2 + 4ab^3d^4e + \frac{b^4d^5}{5} \right) \\
& + x^6 \left(\frac{a^4e^5}{6} + \frac{10a^3bde^4}{3} + 10a^2b^2d^2e^3 \right. \\
& \quad \left. + \frac{20ab^3d^3e^2}{3} + \frac{5b^4d^4e}{6} \right) + a^4d^5x + \frac{b^4e^5x^{10}}{10} \\
& + \frac{a^3d^4x^2(5ae+4bd)}{2} + \frac{b^3e^4x^9(4ae+5bd)}{9} \\
& + \frac{2a^2d^3x^3(5a^2e^2+10abde+3b^2d^2)}{3} \\
& + \frac{b^2e^3x^8(3a^2e^2+10abde+5b^2d^2)}{4}
\end{aligned}$$

input `int((d + e*x)^5*(a^2 + b^2*x^2 + 2*a*b*x)^2,x)`output
$$\begin{aligned}
& x^4*(a*b^3*d^5 + (5*a^4*d^2*e^3)/2 + (15*a^2*b^2*d^4*e)/2 + 10*a^3*b*d^3*e^2) \\
& + x^7*((4*a^3*b*e^5)/7 + (10*b^4*d^3*e^2)/7 + (40*a*b^3*d^2*e^3)/7 + (30*a^2*b^2*d*e^4)/7) \\
& + x^5*((b^4*d^5)/5 + a^4*d*e^4 + 8*a^3*b*d^2*e^3 + 12*a^2*b^2*d^3*e^2 + 4*a*b^3*d^4*e) \\
& + x^6*((a^4*e^5)/6 + (5*b^4*d^4*e)/6 + (20*a*b^3*d^3*e^2)/3 + 10*a^2*b^2*d^2*e^3 + (10*a^3*b*d*e^4)/3) \\
& + a^4*d^5*x + (b^4*e^5*x^10)/10 + (a^3*d^4*x^2*(5*a*e + 4*b*d))/2 + (b^3*e^4*x^9*(4*a*e + 5*b*d))/9 \\
& + (2*a^2*d^3*x^3*(5*a^2*e^2 + 3*b^2*d^2 + 10*a*b*d*e))/3 + (b^2*e^3*x^8*(3*a^2*e^2 + 5*b^2*d^2 + 10*a*b*d*e))/4
\end{aligned}$$

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 397, normalized size of antiderivative = 3.34

$$\int (d + ex)^5 (a^2 + 2abx + b^2x^2)^2 dx$$

$$= \frac{x(126b^4e^5x^9 + 560ab^3e^5x^8 + 700b^4de^4x^8 + 945a^2b^2e^5x^7 + 3150ab^3de^4x^7 + 1575b^4d^2e^3x^7 + 720a^3be^5x^6 + \dots)}{1260}$$

input `int((e*x+d)^5*(b^2*x^2+2*a*b*x+a^2)^2,x)`output `(x*(1260*a**4*d**5 + 3150*a**4*d**4*e*x + 4200*a**4*d**3*e**2*x**2 + 3150*a**4*d**2*e**3*x**3 + 1260*a**4*d*e**4*x**4 + 210*a**4*e**5*x**5 + 2520*a**3*b*d**5*x + 8400*a**3*b*d**4*e*x**2 + 12600*a**3*b*d**3*e**2*x**3 + 10800*a**3*b*d**2*e**3*x**4 + 4200*a**3*b*d*e**4*x**5 + 720*a**3*b*e**5*x**6 + 2520*a**2*b**2*d**5*x**2 + 9450*a**2*b**2*d**4*e*x**3 + 15120*a**2*b**2*d**3*e**2*x**4 + 12600*a**2*b**2*d**2*e**3*x**5 + 5400*a**2*b**2*d*e**4*x**6 + 945*a**2*b**2*e**5*x**7 + 1260*a*b**3*d**5*x**3 + 5040*a*b**3*d**4*e*x**4 + 8400*a*b**3*d**3*e**2*x**5 + 7200*a*b**3*d**2*e**3*x**6 + 3150*a*b**3*d*e**4*x**7 + 560*a*b**3*e**5*x**8 + 252*b**4*d**5*x**4 + 1050*b**4*d**4*e*x**5 + 1800*b**4*d**3*e**2*x**6 + 1575*b**4*d**2*e**3*x**7 + 700*b**4*d*e**4*x**8 + 126*b**4*e**5*x**9))/1260`

3.68 $\int (d + ex)^4 (a^2 + 2abx + b^2x^2)^2 dx$

| | |
|---|-----|
| Optimal result | 555 |
| Mathematica [B] (verified) | 555 |
| Rubi [A] (verified) | 556 |
| Maple [B] (verified) | 558 |
| Fricas [B] (verification not implemented) | 559 |
| Sympy [B] (verification not implemented) | 560 |
| Maxima [B] (verification not implemented) | 561 |
| Giac [B] (verification not implemented) | 562 |
| Mupad [B] (verification not implemented) | 563 |
| Reduce [B] (verification not implemented) | 563 |

Optimal result

Integrand size = 26, antiderivative size = 119

$$\int (d + ex)^4 (a^2 + 2abx + b^2x^2)^2 dx = \frac{(bd - ae)^4(a + bx)^5}{5b^5} + \frac{2e(bd - ae)^3(a + bx)^6}{3b^5} + \frac{6e^2(bd - ae)^2(a + bx)^7}{7b^5} + \frac{e^3(bd - ae)(a + bx)^8}{2b^5} + \frac{e^4(a + bx)^9}{9b^5}$$

```
output 1/5*(-a*e+b*d)^4*(b*x+a)^5/b^5+2/3*e*(-a*e+b*d)^3*(b*x+a)^6/b^5+6/7*e^2*(-a*e+b*d)^2*(b*x+a)^7/b^5+1/2*e^3*(-a*e+b*d)*(b*x+a)^8/b^5+1/9*e^4*(b*x+a)^9/b^5
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 273 vs. 2(119) = 238.

Time = 0.03 (sec) , antiderivative size = 273, normalized size of antiderivative = 2.29

$$\int (d + ex)^4 (a^2 + 2abx + b^2x^2)^2 dx = a^4d^4x + 2a^3d^3(bd + ae)x^2 + \frac{2}{3}a^2d^2(3b^2d^2 + 8abde + 3a^2e^2)x^3 + ad(b^3d^3 + 6ab^2d^2e + 6a^2bde^2 + a^3e^3)x^4 + \frac{1}{5}(b^4d^4 + 16ab^3d^3e + 36a^2b^2d^2e^2 + 16a^3bde^3 + a^4e^4)x^5 + \frac{2}{3}be(b^3d^3 + 6ab^2d^2e + 6a^2bde^2 + a^3e^3)x^6 + \frac{2}{7}b^2e^2(3b^2d^2 + 8abde + 3a^2e^2)x^7 + \frac{1}{2}b^3e^3(bd + ae)x^8 + \frac{1}{9}b^4e^4x^9$$

input

```
Integrate[(d + e*x)^4*(a^2 + 2*a*b*x + b^2*x^2)^2,x]
```

output

```
a^4*d^4*x + 2*a^3*d^3*(b*d + a*e)*x^2 + (2*a^2*d^2*(3*b^2*d^2 + 8*a*b*d*e + 3*a^2*e^2)*x^3)/3 + a*d*(b^3*d^3 + 6*a*b^2*d^2*e + 6*a^2*b*d*e^2 + a^3*e^3)*x^4 + ((b^4*d^4 + 16*a*b^3*d^3*e + 36*a^2*b^2*d^2*e^2 + 16*a^3*b*d*e^3 + a^4*e^4)*x^5)/5 + (2*b*e*(b^3*d^3 + 6*a*b^2*d^2*e + 6*a^2*b*d*e^2 + a^3*e^3)*x^6)/3 + (2*b^2*e^2*(3*b^2*d^2 + 8*a*b*d*e + 3*a^2*e^2)*x^7)/7 + (b^3*e^3*(b*d + a*e)*x^8)/2 + (b^4*e^4*x^9)/9
```

Rubi [A] (verified)

Time = 0.62 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {1098, 27, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a^2 + 2abx + b^2x^2)^2 (d + ex)^4 dx$$

↓ 1098

$$\frac{\int b^4(a+bx)^4(d+ex)^4 dx}{b^4}$$

↓ 27

$$\int (a+bx)^4(d+ex)^4 dx$$

↓ 49

$$\int \left(\frac{4e^3(a+bx)^7(bd-ae)}{b^4} + \frac{6e^2(a+bx)^6(bd-ae)^2}{b^4} + \frac{4e(a+bx)^5(bd-ae)^3}{b^4} + \frac{(a+bx)^4(bd-ae)^4}{b^4} + \frac{e^4(a+bx)^3(bd-ae)^5}{b^4} \right) dx$$

↓ 2009

$$\frac{e^3(a+bx)^8(bd-ae)}{2b^5} + \frac{6e^2(a+bx)^7(bd-ae)^2}{7b^5} + \frac{2e(a+bx)^6(bd-ae)^3}{3b^5} + \frac{(a+bx)^5(bd-ae)^4}{5b^5} + \frac{e^4(a+bx)^9}{9b^5}$$

input `Int[(d + e*x)^4*(a^2 + 2*a*b*x + b^2*x^2)^2,x]`

output `((b*d - a*e)^4*(a + b*x)^5)/(5*b^5) + (2*e*(b*d - a*e)^3*(a + b*x)^6)/(3*b^5) + (6*e^2*(b*d - a*e)^2*(a + b*x)^7)/(7*b^5) + (e^3*(b*d - a*e)*(a + b*x)^8)/(2*b^5) + (e^4*(a + b*x)^9)/(9*b^5)`

Defintions of rubi rules used

rule 27 `Int[(a_)*(F_x_), x_Symbol] := Simp[a Int[F_x, x], x] /; FreeQ[a, x] && !MatchQ[F_x, (b_)*(G_x_)] /; FreeQ[b, x]`

rule 49 `Int[((a_) + (b_)*(x_))^(m_)*((c_) + (d_)*(x_))^(n_), x_Symbol] := Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] && IGtQ[m, 0] && IGtQ[m + n + 2, 0]`

rule 1098 `Int[((d_) + (e_)*(x_))^(m_)*((a_) + (b_)*(x_) + (c_)*(x_)^2)^(p_), x_Symbol] := Simp[1/c^p Int[(d + e*x)^m*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]`

rule 2009

```
Int[u_, x_Symbol] := Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 287 vs. $2(109) = 218$.

Time = 1.08 (sec) , antiderivative size = 288, normalized size of antiderivative = 2.42

| method | result |
|---------------|---|
| norman | $\frac{b^4 e^4 x^9}{9} + \left(\frac{1}{2} a b^3 e^4 + \frac{1}{2} b^4 d e^3\right) x^8 + \left(\frac{6}{7} a^2 b^2 e^4 + \frac{16}{7} a b^3 d e^3 + \frac{6}{7} b^4 d^2 e^2\right) x^7 + \left(\frac{2}{3} a^3 b e^4 + 4 a^2 b^2 d e^3\right) x^6 + \left(\frac{2}{3} a^3 b^2 d e^3 + 4 a^2 b^3 d^2 e^2 + 2 a b^4 d^3 e\right) x^5 + \left(\frac{2}{3} a^3 b^3 d^2 e^2 + 4 a^2 b^4 d^3 e + 2 a b^5 d^4\right) x^4 + \left(\frac{2}{3} a^3 b^4 d^3 e + 4 a^2 b^5 d^4 + 2 a b^6 d^5\right) x^3 + \left(\frac{2}{3} a^3 b^5 d^4 + 4 a^2 b^6 d^5 + 2 a b^7 d^6\right) x^2 + \left(\frac{2}{3} a^3 b^6 d^5 + 4 a^2 b^7 d^6 + 2 a b^8 d^7\right) x + \frac{2}{3} a^3 b^7 d^6 + 4 a^2 b^8 d^7 + 2 a b^9 d^8$ |
| default | $\frac{b^4 e^4 x^9}{9} + \frac{(4 a b^3 e^4 + 4 b^4 d e^3) x^8}{8} + \frac{(6 a^2 b^2 e^4 + 16 a b^3 d e^3 + 6 b^4 d^2 e^2) x^7}{7} + \frac{(4 a^3 b e^4 + 24 a^2 b^2 d e^3 + 24 a b^3 d^2 e^2 + 4 b^4 d^3 e) x^6}{6} + \frac{(2 a^3 b^2 d e^3 + 4 a^2 b^3 d^2 e^2 + 2 a b^4 d^3 e) x^5}{5} + \frac{(2 a^3 b^3 d^2 e^2 + 4 a^2 b^4 d^3 e + 2 a b^5 d^4) x^4}{4} + \frac{(2 a^3 b^4 d^3 e + 4 a^2 b^5 d^4 + 2 a b^6 d^5) x^3}{3} + \frac{(2 a^3 b^5 d^4 + 4 a^2 b^6 d^5 + 2 a b^7 d^6) x^2}{2} + \frac{2 a^3 b^6 d^5 + 4 a^2 b^7 d^6 + 2 a b^8 d^7}{2}$ |
| risch | $\frac{1}{2} x^8 a b^3 e^4 + \frac{1}{2} x^8 b^4 d e^3 + \frac{6}{7} x^7 a^2 b^2 e^4 + \frac{6}{7} x^7 b^4 d^2 e^2 + \frac{2}{3} x^6 a^3 b e^4 + \frac{2}{3} x^6 b^4 d^3 e + 2 x^3 a^4 d^2 e^2 + \frac{1}{5} b^4 d^3 e$ |
| parallelrisch | $\frac{1}{2} x^8 a b^3 e^4 + \frac{1}{2} x^8 b^4 d e^3 + \frac{6}{7} x^7 a^2 b^2 e^4 + \frac{6}{7} x^7 b^4 d^2 e^2 + \frac{2}{3} x^6 a^3 b e^4 + \frac{2}{3} x^6 b^4 d^3 e + 2 x^3 a^4 d^2 e^2 + \frac{1}{5} b^4 d^3 e$ |
| gosper | $\frac{x(70 b^4 e^4 x^8 + 315 x^7 a b^3 e^4 + 315 x^7 b^4 d e^3 + 540 x^6 a^2 b^2 e^4 + 1440 x^6 a b^3 d e^3 + 540 x^6 b^4 d^2 e^2 + 420 x^5 a^3 b e^4 + 2520 x^5 a^2 b^2 d e^3 + 2520 x^5 a b^3 d^2 e^2 + 2520 x^5 b^4 d^3 e + 420 x^4 a^4 d^2 e^2 + 2520 x^4 a^3 b d^3 e + 2520 x^4 a^2 b^2 d^4 e + 2520 x^4 a b^3 d^5 e + 2520 x^4 b^4 d^6 e + 420 x^3 a^4 d^3 e^2 + 2520 x^3 a^3 b d^4 e^2 + 2520 x^3 a^2 b^2 d^5 e^2 + 2520 x^3 a b^3 d^6 e^2 + 2520 x^3 b^4 d^7 e^2 + 420 x^2 a^4 d^4 e^3 + 2520 x^2 a^3 b d^5 e^3 + 2520 x^2 a^2 b^2 d^6 e^3 + 2520 x^2 a b^3 d^7 e^3 + 2520 x^2 b^4 d^8 e^3 + 420 x a^4 d^5 e^4 + 2520 x a^3 b d^6 e^4 + 2520 x a^2 b^2 d^7 e^4 + 2520 x a b^3 d^8 e^4 + 2520 x b^4 d^9 e^4 + 420 a^4 d^6 e^5 + 2520 a^3 b d^7 e^5 + 2520 a^2 b^2 d^8 e^5 + 2520 a b^3 d^9 e^5 + 2520 b^4 d^{10} e^5 + 420 a^4 d^7 e^6 + 2520 a^3 b d^8 e^6 + 2520 a^2 b^2 d^9 e^6 + 2520 a b^3 d^{10} e^6 + 2520 b^4 d^{11} e^6 + 420 a^4 d^8 e^7 + 2520 a^3 b d^9 e^7 + 2520 a^2 b^2 d^{10} e^7 + 2520 a b^3 d^{11} e^7 + 2520 b^4 d^{12} e^7 + 420 a^4 d^9 e^8 + 2520 a^3 b d^{10} e^8 + 2520 a^2 b^2 d^{11} e^8 + 2520 a b^3 d^{12} e^8 + 2520 b^4 d^{13} e^8 + 420 a^4 d^{10} e^9 + 2520 a^3 b d^{11} e^9 + 2520 a^2 b^2 d^{12} e^9 + 2520 a b^3 d^{13} e^9 + 2520 b^4 d^{14} e^9 + 420 a^4 d^{11} e^{10} + 2520 a^3 b d^{12} e^{10} + 2520 a^2 b^2 d^{13} e^{10} + 2520 a b^3 d^{14} e^{10} + 2520 b^4 d^{15} e^{10} + 420 a^4 d^{12} e^{11} + 2520 a^3 b d^{13} e^{11} + 2520 a^2 b^2 d^{14} e^{11} + 2520 a b^3 d^{15} e^{11} + 2520 b^4 d^{16} e^{11} + 420 a^4 d^{13} e^{12} + 2520 a^3 b d^{14} e^{12} + 2520 a^2 b^2 d^{15} e^{12} + 2520 a b^3 d^{16} e^{12} + 2520 b^4 d^{17} e^{12} + 420 a^4 d^{14} e^{13} + 2520 a^3 b d^{15} e^{13} + 2520 a^2 b^2 d^{16} e^{13} + 2520 a b^3 d^{17} e^{13} + 2520 b^4 d^{18} e^{13} + 420 a^4 d^{15} e^{14} + 2520 a^3 b d^{16} e^{14} + 2520 a^2 b^2 d^{17} e^{14} + 2520 a b^3 d^{18} e^{14} + 2520 b^4 d^{19} e^{14} + 420 a^4 d^{16} e^{15} + 2520 a^3 b d^{17} e^{15} + 2520 a^2 b^2 d^{18} e^{15} + 2520 a b^3 d^{19} e^{15} + 2520 b^4 d^{20} e^{15} + 420 a^4 d^{17} e^{16} + 2520 a^3 b d^{18} e^{16} + 2520 a^2 b^2 d^{19} e^{16} + 2520 a b^3 d^{20} e^{16} + 2520 b^4 d^{21} e^{16} + 420 a^4 d^{18} e^{17} + 2520 a^3 b d^{19} e^{17} + 2520 a^2 b^2 d^{20} e^{17} + 2520 a b^3 d^{21} e^{17} + 2520 b^4 d^{22} e^{17} + 420 a^4 d^{19} e^{18} + 2520 a^3 b d^{20} e^{18} + 2520 a^2 b^2 d^{21} e^{18} + 2520 a b^3 d^{22} e^{18} + 2520 b^4 d^{23} e^{18} + 420 a^4 d^{20} e^{19} + 2520 a^3 b d^{21} e^{19} + 2520 a^2 b^2 d^{22} e^{19} + 2520 a b^3 d^{23} e^{19} + 2520 b^4 d^{24} e^{19} + 420 a^4 d^{21} e^{20} + 2520 a^3 b d^{22} e^{20} + 2520 a^2 b^2 d^{23} e^{20} + 2520 a b^3 d^{24} e^{20} + 2520 b^4 d^{25} e^{20} + 420 a^4 d^{22} e^{21} + 2520 a^3 b d^{23} e^{21} + 2520 a^2 b^2 d^{24} e^{21} + 2520 a b^3 d^{25} e^{21} + 2520 b^4 d^{26} e^{21} + 420 a^4 d^{23} e^{22} + 2520 a^3 b d^{24} e^{22} + 2520 a^2 b^2 d^{25} e^{22} + 2520 a b^3 d^{26} e^{22} + 2520 b^4 d^{27} e^{22} + 420 a^4 d^{24} e^{23} + 2520 a^3 b d^{25} e^{23} + 2520 a^2 b^2 d^{26} e^{23} + 2520 a b^3 d^{27} e^{23} + 2520 b^4 d^{28} e^{23} + 420 a^4 d^{25} e^{24} + 2520 a^3 b d^{26} e^{24} + 2520 a^2 b^2 d^{27} e^{24} + 2520 a b^3 d^{28} e^{24} + 2520 b^4 d^{29} e^{24} + 420 a^4 d^{26} e^{25} + 2520 a^3 b d^{27} e^{25} + 2520 a^2 b^2 d^{28} e^{25} + 2520 a b^3 d^{29} e^{25} + 2520 b^4 d^{30} e^{25} + 420 a^4 d^{27} e^{26} + 2520 a^3 b d^{28} e^{26} + 2520 a^2 b^2 d^{29} e^{26} + 2520 a b^3 d^{30} e^{26} + 2520 b^4 d^{31} e^{26} + 420 a^4 d^{28} e^{27} + 2520 a^3 b d^{29} e^{27} + 2520 a^2 b^2 d^{30} e^{27} + 2520 a b^3 d^{31} e^{27} + 2520 b^4 d^{32} e^{27} + 420 a^4 d^{29} e^{28} + 2520 a^3 b d^{30} e^{28} + 2520 a^2 b^2 d^{31} e^{28} + 2520 a b^3 d^{32} e^{28} + 2520 b^4 d^{33} e^{28} + 420 a^4 d^{30} e^{29} + 2520 a^3 b d^{31} e^{29} + 2520 a^2 b^2 d^{32} e^{29} + 2520 a b^3 d^{33} e^{29} + 2520 b^4 d^{34} e^{29} + 420 a^4 d^{31} e^{30} + 2520 a^3 b d^{32} e^{30} + 2520 a^2 b^2 d^{33} e^{30} + 2520 a b^3 d^{34} e^{30} + 2520 b^4 d^{35} e^{30} + 420 a^4 d^{32} e^{31} + 2520 a^3 b d^{33} e^{31} + 2520 a^2 b^2 d^{34} e^{31} + 2520 a b^3 d^{35} e^{31} + 2520 b^4 d^{36} e^{31} + 420 a^4 d^{33} e^{32} + 2520 a^3 b d^{34} e^{32} + 2520 a^2 b^2 d^{35} e^{32} + 2520 a b^3 d^{36} e^{32} + 2520 b^4 d^{37} e^{32} + 420 a^4 d^{34} e^{33} + 2520 a^3 b d^{35} e^{33} + 2520 a^2 b^2 d^{36} e^{33} + 2520 a b^3 d^{37} e^{33} + 2520 b^4 d^{38} e^{33} + 420 a^4 d^{35} e^{34} + 2520 a^3 b d^{36} e^{34} + 2520 a^2 b^2 d^{37} e^{34} + 2520 a b^3 d^{38} e^{34} + 2520 b^4 d^{39} e^{34} + 420 a^4 d^{36} e^{35} + 2520 a^3 b d^{37} e^{35} + 2520 a^2 b^2 d^{38} e^{35} + 2520 a b^3 d^{39} e^{35} + 2520 b^4 d^{40} e^{35} + 420 a^4 d^{37} e^{36} + 2520 a^3 b d^{38} e^{36} + 2520 a^2 b^2 d^{39} e^{36} + 2520 a b^3 d^{40} e^{36} + 2520 b^4 d^{41} e^{36} + 420 a^4 d^{38} e^{37} + 2520 a^3 b d^{39} e^{37} + 2520 a^2 b^2 d^{40} e^{37} + 2520 a b^3 d^{41} e^{37} + 2520 b^4 d^{42} e^{37} + 420 a^4 d^{39} e^{38} + 2520 a^3 b d^{40} e^{38} + 2520 a^2 b^2 d^{41} e^{38} + 2520 a b^3 d^{42} e^{38} + 2520 b^4 d^{43} e^{38} + 420 a^4 d^{40} e^{39} + 2520 a^3 b d^{41} e^{39} + 2520 a^2 b^2 d^{42} e^{39} + 2520 a b^3 d^{43} e^{39} + 2520 b^4 d^{44} e^{39} + 420 a^4 d^{41} e^{40} + 2520 a^3 b d^{42} e^{40} + 2520 a^2 b^2 d^{43} e^{40} + 2520 a b^3 d^{44} e^{40} + 2520 b^4 d^{45} e^{40} + 420 a^4 d^{42} e^{41} + 2520 a^3 b d^{43} e^{41} + 2520 a^2 b^2 d^{44} e^{41} + 2520 a b^3 d^{45} e^{41} + 2520 b^4 d^{46} e^{41} + 420 a^4 d^{43} e^{42} + 2520 a^3 b d^{44} e^{42} + 2520 a^2 b^2 d^{45} e^{42} + 2520 a b^3 d^{46} e^{42} + 2520 b^4 d^{47} e^{42} + 420 a^4 d^{44} e^{43} + 2520 a^3 b d^{45} e^{43} + 2520 a^2 b^2 d^{46} e^{43} + 2520 a b^3 d^{47} e^{43} + 2520 b^4 d^{48} e^{43} + 420 a^4 d^{45} e^{44} + 2520 a^3 b d^{46} e^{44} + 2520 a^2 b^2 d^{47} e^{44} + 2520 a b^3 d^{48} e^{44} + 2520 b^4 d^{49} e^{44} + 420 a^4 d^{46} e^{45} + 2520 a^3 b d^{47} e^{45} + 2520 a^2 b^2 d^{48} e^{45} + 2520 a b^3 d^{49} e^{45} + 2520 b^4 d^{50} e^{45} + 420 a^4 d^{47} e^{46} + 2520 a^3 b d^{48} e^{46} + 2520 a^2 b^2 d^{49} e^{46} + 2520 a b^3 d^{50} e^{46} + 2520 b^4 d^{51} e^{46} + 420 a^4 d^{48} e^{47} + 2520 a^3 b d^{49} e^{47} + 2520 a^2 b^2 d^{50} e^{47} + 2520 a b^3 d^{51} e^{47} + 2520 b^4 d^{52} e^{47} + 420 a^4 d^{49} e^{48} + 2520 a^3 b d^{50} e^{48} + 2520 a^2 b^2 d^{51} e^{48} + 2520 a b^3 d^{52} e^{48} + 2520 b^4 d^{53} e^{48} + 420 a^4 d^{50} e^{49} + 2520 a^3 b d^{51} e^{49} + 2520 a^2 b^2 d^{52} e^{49} + 2520 a b^3 d^{53} e^{49} + 2520 b^4 d^{54} e^{49} + 420 a^4 d^{51} e^{50} + 2520 a^3 b d^{52} e^{50} + 2520 a^2 b^2 d^{53} e^{50} + 2520 a b^3 d^{54} e^{50} + 2520 b^4 d^{55} e^{50} + 420 a^4 d^{52} e^{51} + 2520 a^3 b d^{53} e^{51} + 2520 a^2 b^2 d^{54} e^{51} + 2520 a b^3 d^{55} e^{51} + 2520 b^4 d^{56} e^{51} + 420 a^4 d^{53} e^{52} + 2520 a^3 b d^{54} e^{52} + 2520 a^2 b^2 d^{55} e^{52} + 2520 a b^3 d^{56} e^{52} + 2520 b^4 d^{57} e^{52} + 420 a^4 d^{54} e^{53} + 2520 a^3 b d^{55} e^{53} + 2520 a^2 b^2 d^{56} e^{53} + 2520 a b^3 d^{57} e^{53} + 2520 b^4 d^{58} e^{53} + 420 a^4 d^{55} e^{54} + 2520 a^3 b d^{56} e^{54} + 2520 a^2 b^2 d^{57} e^{54} + 2520 a b^3 d^{58} e^{54} + 2520 b^4 d^{59} e^{54} + 420 a^4 d^{56} e^{55} + 2520 a^3 b d^{57} e^{55} + 2520 a^2 b^2 d^{58} e^{55} + 2520 a b^3 d^{59} e^{55} + 2520 b^4 d^{60} e^{55} + 420 a^4 d^{57} e^{56} + 2520 a^3 b d^{58} e^{56} + 2520 a^2 b^2 d^{59} e^{56} + 2520 a b^3 d^{60} e^{56} + 2520 b^4 d^{61} e^{56} + 420 a^4 d^{58} e^{57} + 2520 a^3 b d^{59} e^{57} + 2520 a^2 b^2 d^{60} e^{57} + 2520 a b^3 d^{61} e^{57} + 2520 b^4 d^{62} e^{57} + 420 a^4 d^{59} e^{58} + 2520 a^3 b d^{60} e^{58} + 2520 a^2 b^2 d^{61} e^{58} + 2520 a b^3 d^{62} e^{58} + 2520 b^4 d^{63} e^{58} + 420 a^4 d^{60} e^{59} + 2520 a^3 b d^{61} e^{59} + 2520 a^2 b^2 d^{62} e^{59} + 2520 a b^3 d^{63} e^{59} + 2520 b^4 d^{64} e^{59} + 420 a^4 d^{61} e^{60} + 2520 a^3 b d^{62} e^{60} + 2520 a^2 b^2 d^{63} e^{60} + 2520 a b^3 d^{64} e^{60} + 2520 b^4 d^{65} e^{60} + 420 a^4 d^{62} e^{61} + 2520 a^3 b d^{63} e^{61} + 2520 a^2 b^2 d^{64} e^{61} + 2520 a b^3 d^{65} e^{61} + 2520 b^4 d^{66} e^{61} + 420 a^4 d^{63} e^{62} + 2520 a^3 b d^{64} e^{62} + 2520 a^2 b^2 d^{65} e^{62} + 2520 a b^3 d^{66} e^{62} + 2520 b^4 d^{67} e^{62} + 420 a^4 d^{64} e^{63} + 2520 a^3 b d^{65} e^{63} + 2520 a^2 b^2 d^{66} e^{63} + 2520 a b^3 d^{67} e^{63} + 2520 b^4 d^{68} e^{63} + 420 a^4 d^{65} e^{64} + 2520 a^3 b d^{66} e^{64} + 2520 a^2 b^2 d^{67} e^{64} + 2520 a b^3 d^{68} e^{64} + 2520 b^4 d^{69} e^{64} + 420 a^4 d^{66} e^{65} + 2520 a^3 b d^{67} e^{65} + 2520 a^2 b^2 d^{68} e^{65} + 2520 a b^3 d^{69} e^{65} + 2520 b^4 d^{70} e^{65} + 420 a^4 d^{67} e^{66} + 2520 a^3 b d^{68} e^{66} + 2520 a^2 b^2 d^{69} e^{66} + 2520 a b^3 d^{70} e^{66} + 2520 b^4 d^{71} e^{66} + 420 a^4 d^{68} e^{67} + 2520 a^3 b d^{69} e^{67} + 2520 a^2 b^2 d^{70} e^{67} + 2520 a b^3 d^{71} e^{67} + 2520 b^4 d^{72} e^{67} + 420 a^4 d^{69} e^{68} + 2520 a^3 b d^{70} e^{68} + 2520 a^2 b^2 d^{71} e^{68} + 2520 a b^3 d^{72} e^{68} + 2520 b^4 d^{73} e^{68} + 420 a^4 d^{70} e^{69} + 2520 a^3 b d^{71} e^{69} + 2520 a^2 b^2 d^{72} e^{69} + 2520 a b^3 d^{73} e^{69} + 2520 b^4 d^{74} e^{69} + 420 a^4 d^{71} e^{70} + 2520 a^3 b d^{72} e^{70} + 2520 a^2 b^2 d^{73} e^{70} + 2520 a b^3 d^{74} e^{70} + 2520 b^4 d^{75} e^{70} + 420 a^4 d^{72} e^{71} + 2520 a^3 b d^{73} e^{71} + 2520 a^2 b^2 d^{74} e^{71} + 2520 a b^3 d^{75} e^{71} + 2520 b^4 d^{76} e^{71} + 420 a^4 d^{73} e^{72} + 2520 a^3 b d^{74} e^{72} + 2520 a^2 b^2 d^{75} e^{72} + 2520 a b^3 d^{76} e^{72} + 2520 b^4 d^{77} e^{72} + 420 a^4 d^{74} e^{73} + 2520 a^3 b d^{75} e^{73} + 2520 a^2 b^2 d^{76} e^{73} + 2520 a b^3 d^{77} e^{73} + 2520 b^4 d^{78} e^{73} + 420 a^4 d^{75} e^{74} + 2520 a^3 b d^{76} e^{74} + 2520 a^2 b^2 d^{77} e^{74} + 2520 a b^3 d^{78} e^{74} + 2520 b^4 d^{79} e^{74} + 420 a^4 d^{76} e^{75} + 2520 a^3 b d^{77} e^{75} + 2520 a^2 b^2 d^{78} e^{75} + 2520 a b^3 d^{79} e^{75} + 2520 b^4 d^{80} e^{75} + 420 a^4 d^{77} e^{76} + 2520 a^3 b d^{78} e^{76} + 2520 a^2 b^2 d^{79} e^{76} + 2520 a b^3 d^{80} e^{76} + 2520 b^4 d^{81} e^{76} + 420 a^4 d^{78} e^{77} + 2520 a^3 b d^{79} e^{77} + 2520 a^2 b^2 d^{80} e^{77} + 2520 a b^3 d^{81} e^{77} + 2520 b^4 d^{82} e^{77} + 420 a^4 d^{79} e^{78} + 2520 a^3 b d^{80} e^{78} + 2520 a^2 b^2 d^{81} e^{78} + 2520 a b^3 d^{82} e^{78} + 2520 b^4 d^{83} e^{78} + 420 a^4 d^{80} e^{79} + 2520 a^3 b d^{81} e^{79} + 2520 a^2 b^2 d^{82} e^{79} + 2520 a b^3 d^{83} e^{79} + 2520 b^4 d^{84} e^{79} + 420 a^4 d^{81} e^{80} + 2520 a^3 b d^{82} e^{80} + 2520 a^2 b^2 d^{83} e^{80} + 2520 a b^3 d^{84} e^{80} + 2520 b^4 d^{85} e^{80} + 420 a^4 d^{82} e^{81} + 2520 a^3 b d^{83} e^{81} + 2520 a^2 b^2 d^{84} e^{81} + 2520 a b^3 d^{85} e^{81} + 2520 b^4 d^{86} e^{81} + 420 a^4 d^{83} e^{82} + 2520 a^3 b d^{84} e^{82} + 2520 a^2 b^2 d^{85} e^{82} + 2520 a b^3 d^{86} e^{82} + 2520 b^4 d^{87} e^{82} + 420 a^4 d^{84} e^{83} + 2520 a^3 b d^{85} e^{83} + 2520 a^2 b^2 d^{86} e^{83} + 2520 a b^3 d^{87} e^{83} + 2520 b^4 d^{88} e^{83} + 420 a^4 d^{85} e^{84} + 2520 a^3 b d^{86} e^{84} + 2520 a^2 b^2 d^{87} e^{84} + 2520 a b^3 d^{88} e^{84} + 2520 b^4 d^{89} e^{84} + 420 a^4 d^{86} e^{85} + 2520 a^3 b d^{87} e^{85} + 2520 a^2 b^2 d^{88} e^{85} + 2520 a b^3 d^{89} e^{85} + 2520 b^4 d^{90} e^{85} + 420 a^4 d^{87} e^{86} + 2520 a^3 b d^{88} e^{86} + 2520 a^2 b^2 d^{89} e^{86} + 2520 a b^3 d^{90} e^{86} + 2520 b^4 d^{91} e^{86} + 420 a^4 d^{88} e^{87} + 2520 a^3 b d^{89} e^{87} + 2520 a^2 b^2 d^{90} e^{87} + 2520 a b^3 d^{91} e^{87} + 2520 b^4 d^{92} e^{87} + 420 a^4 d^{89} e^{88} + 2520 a^3 b d^{90} e^{88} + 2520 a^2 b^2 d^{91} e^{88} + 2520 a b^3 d^{92} e^{88} + 2520 b^4 d^{93} e^{88} + 420 a^4 d^{90} e^{89} + 2520 a^3 b d^{91} e^{89} + 2520 a^2 b^2 d^{92} e^{89} + 2520 a b^3 d^{93} e^{89} + 2520 b^4 d^{94} e^{89} + 420 a^4 d^{91} e^{90} + 2520 a^3 b d^{92} e^{90} + 2520 a^2 b^2 d^{93} e^{90} + 2520 a b^3 d^{94} e^{90} + 2520 b^4 d^{95} e^{90} + 420 a^4 d^{92} e^{91} + 2520 a^3 b d^{93} e^{91} + 2520 a^2 b^2 d^{94} e^{91} + 2520 a b^3 d^{95} e^{91} + 2520 b^4 d^{96} e^{91} + 420 a^4 d^{93} e^{92} + 2520 a^3 b d^{94} e^{92} + 2520 a^2 b^2 d^{95} e^{92} + 2520 a b^3 d^{96} e^{92} + 2520 b^4 d^{97} e^{92} + 420 a^4 d^{94} e^{93} + 2520 a^3 b d^{95} e^{93} + 2520 a^2 b^2 d^{96} e^{93} + 2520 a b^3 d^{97} e^{93} + 2520 b^4 d^{98} e^{93} + 420 a^4 d^{95} e^{94} + 2520 a^3 b d^{96} e^{94} + 2520 a^2 b^2 d^{97} e^{94} + 2520 a b^3 d^{98} e^{94} + 2520 b^4 d^{99} e^{94} + 420 a^4 d^{96} e^{95} + 2520 a^3 b d^{97} e^{95} + 2520 a^2 b^2 d^{98} e^{95} + 2520 a b^3 d^{99} e^{95} + 2520 b^4 d^{100} e^{95} + 420 a^4 d^{97} e^{96} + 2520 a^3 b d^{98} e^{96} + 2520 a^2 b^2 d^{99} e^{96} + 2520 a b^3 d^{100} e^{96} + 2520 b^4 d^{101} e^{96} + 420 a^4 d^{98} e^{97} + 2520 a^3 b d^{99} e^{97} + 2520 a^2 b^2 d^{100} e^{97} + 2520 a b^3 d^{101} e^{97} + 2520 b^4 d^{102} e^{97} + 420 a^4 d^{99} e^{98} + 2520 a^3 b d^{100} e^{98} + 2520 a^2 b^2 d^{101} e^{98} + 2520 a b^3 d^{102} e^{98} + 2520 b^4 d^{103} e^{98} + 420 a^4 d^{100} e^{99} + 2520 a^3 b d^{101} e^{99} + 2520 a^2 b^2 d^{102} e^{99} + 2520 a b^3 d^{103} e^{99} + 2520 b^4 d^{104} e^{99} + 420 a^4 d^{101} e^{100} + 2520 a^3 b d^{102} e^{100} + 2520 a^2 b^2 d^{103} e^{100} + 2520 a b^3 d^{104} e^{100} + 2520 b^4 d^{105} e^{100} + 420 a^4 d^{102} e^{101} + 2520 a^3 b d^{103} e^{101} + 2520 a^2 b^2 d^{104} e^{101} + 2520 a b^3 d^{105} e^{101} + 2520 b^4 d^{106} e^{101} + 420 a^4 d^{103} e^{102} + 2520 a^3 b d^{104} e^{102} + 2520 a^2 b^2 d^{105} e^{102} + 2520 a b^3 d^{106} e^{102} + 2520 b^4 d^{107} e^{102} + 420 a^4 d^{104} e^{103} + 2520 a^3 b d^{105} e^{103} + 2520 a^2 b^2 d^{106} e^{103} + 2520 a b^3 d^{107} e^{103} + 2520 b^4 d^{108} e^{103} + 420 a^4 d^{105} e^{104} + 2520 a^3 b d^{106} e^{104} + 2520 a^2 b^2 d^{107} e^{104} + 2520 a b^3 d^{108} e^{104} + 2520 b^4 d^{109} e^{104} + 420 a^4 d^{106} e^{105} + 2520 a^3 b d^{107} e^{105} + 2520 a^2 b^2 d^{108} e^{105} + 2520 a b^3 d^{109} e^{105} + 2520 b^4 d^{110} e^{105} + 420 a^4 d^{107} e^{106} + 2520 a^3 b d^{108} e^{106} + 2520 a^2 b^2 d^{109} e^{106} + 2520 a b^3 d^{110} e^{106} + 2520 b^4 d^{111} e^{106} + 420 a^4 d^{108} e^{107} + 2520 a^3 b d^{109} e^{107} + 2520 a^2 b^2 d^{110} e^{107} + 2520 a b^3 d^{111} e^{107} + 2520 b^4 d^{112} e^{107} + 420 a^4 d^{109} e^{108} + 2520 a^3 b$ |

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 285 vs. $2(109) = 218$.

Time = 0.08 (sec) , antiderivative size = 285, normalized size of antiderivative = 2.39

$$\begin{aligned} & \int (d + ex)^4 (a^2 + 2abx + b^2x^2)^2 dx \\ &= \frac{1}{9} b^4 e^4 x^9 + a^4 d^4 x + \frac{1}{2} (b^4 d e^3 + a b^3 e^4) x^8 + \frac{2}{7} (3 b^4 d^2 e^2 + 8 a b^3 d e^3 + 3 a^2 b^2 e^4) x^7 \\ &+ \frac{2}{3} (b^4 d^3 e + 6 a b^3 d^2 e^2 + 6 a^2 b^2 d e^3 + a^3 b e^4) x^6 \\ &+ \frac{1}{5} (b^4 d^4 + 16 a b^3 d^3 e + 36 a^2 b^2 d^2 e^2 + 16 a^3 b d e^3 + a^4 e^4) x^5 \\ &+ (a b^3 d^4 + 6 a^2 b^2 d^3 e + 6 a^3 b d^2 e^2 + a^4 d e^3) x^4 \\ &+ \frac{2}{3} (3 a^2 b^2 d^4 + 8 a^3 b d^3 e + 3 a^4 d^2 e^2) x^3 + 2 (a^3 b d^4 + a^4 d^3 e) x^2 \end{aligned}$$

input `integrate((e*x+d)^4*(b^2*x^2+2*a*b*x+a^2)^2,x, algorithm="fricas")`

output `1/9*b^4*e^4*x^9 + a^4*d^4*x + 1/2*(b^4*d*e^3 + a*b^3*e^4)*x^8 + 2/7*(3*b^4*d^2*e^2 + 8*a*b^3*d*e^3 + 3*a^2*b^2*e^4)*x^7 + 2/3*(b^4*d^3*e + 6*a*b^3*d^2*e^2 + 6*a^2*b^2*d*e^3 + a^3*b*e^4)*x^6 + 1/5*(b^4*d^4 + 16*a*b^3*d^3*e + 36*a^2*b^2*d^2*e^2 + 16*a^3*b*d*e^3 + a^4*e^4)*x^5 + (a*b^3*d^4 + 6*a^2*b^2*d^3*e + 6*a^3*b*d^2*e^2 + a^4*d*e^3)*x^4 + 2/3*(3*a^2*b^2*d^4 + 8*a^3*b*d^3*e + 3*a^4*d^2*e^2)*x^3 + 2*(a^3*b*d^4 + a^4*d^3*e)*x^2`

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 318 vs. $2(105) = 210$.

Time = 0.05 (sec) , antiderivative size = 318, normalized size of antiderivative = 2.67

$$\int (d + ex)^4 (a^2 + 2abx + b^2x^2)^2 dx = a^4d^4x + \frac{b^4e^4x^9}{9} + x^8 \left(\frac{ab^3e^4}{2} + \frac{b^4de^3}{2} \right) + x^7$$

$$\cdot \left(\frac{6a^2b^2e^4}{7} + \frac{16ab^3de^3}{7} + \frac{6b^4d^2e^2}{7} \right) + x^6$$

$$\cdot \left(\frac{2a^3be^4}{3} + 4a^2b^2de^3 + 4ab^3d^2e^2 + \frac{2b^4d^3e}{3} \right)$$

$$+ x^5 \left(\frac{a^4e^4}{5} + \frac{16a^3bde^3}{5} + \frac{36a^2b^2d^2e^2}{5} + \frac{16ab^3d^3e}{5} + \frac{b^4d^4}{5} \right)$$

$$+ x^4 (a^4de^3 + 6a^3bd^2e^2 + 6a^2b^2d^3e + ab^3d^4)$$

$$+ x^3 \cdot \left(2a^4d^2e^2 + \frac{16a^3bd^3e}{3} + 2a^2b^2d^4 \right)$$

$$+ x^2 \cdot (2a^4d^3e + 2a^3bd^4)$$

input

```
integrate((e*x+d)**4*(b**2*x**2+2*a*b*x+a**2)**2,x)
```

output

```
a**4*d**4*x + b**4*e**4*x**9/9 + x**8*(a*b**3*e**4/2 + b**4*d*e**3/2) + x*
*7*(6*a**2*b**2*e**4/7 + 16*a*b**3*d*e**3/7 + 6*b**4*d**2*e**2/7) + x**6*(
2*a**3*b*e**4/3 + 4*a**2*b**2*d*e**3 + 4*a*b**3*d**2*e**2 + 2*b**4*d**3*e/
3) + x**5*(a**4*e**4/5 + 16*a**3*b*d*e**3/5 + 36*a**2*b**2*d**2*e**2/5 + 1
6*a*b**3*d**3*e/5 + b**4*d**4/5) + x**4*(a**4*d*e**3 + 6*a**3*b*d**2*e**2
+ 6*a**2*b**2*d**3*e + a*b**3*d**4) + x**3*(2*a**4*d**2*e**2 + 16*a**3*b*d
**3*e/3 + 2*a**2*b**2*d**4) + x**2*(2*a**4*d**3*e + 2*a**3*b*d**4)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 285 vs. $2(109) = 218$.

Time = 0.04 (sec) , antiderivative size = 285, normalized size of antiderivative = 2.39

$$\begin{aligned} & \int (d + ex)^4 (a^2 + 2abx + b^2x^2)^2 dx \\ &= \frac{1}{9} b^4 e^4 x^9 + a^4 d^4 x + \frac{1}{2} (b^4 d e^3 + a b^3 e^4) x^8 + \frac{2}{7} (3 b^4 d^2 e^2 + 8 a b^3 d e^3 + 3 a^2 b^2 e^4) x^7 \\ &+ \frac{2}{3} (b^4 d^3 e + 6 a b^3 d^2 e^2 + 6 a^2 b^2 d e^3 + a^3 b e^4) x^6 \\ &+ \frac{1}{5} (b^4 d^4 + 16 a b^3 d^3 e + 36 a^2 b^2 d^2 e^2 + 16 a^3 b d e^3 + a^4 e^4) x^5 \\ &+ (a b^3 d^4 + 6 a^2 b^2 d^3 e + 6 a^3 b d^2 e^2 + a^4 d e^3) x^4 \\ &+ \frac{2}{3} (3 a^2 b^2 d^4 + 8 a^3 b d^3 e + 3 a^4 d^2 e^2) x^3 + 2 (a^3 b d^4 + a^4 d^3 e) x^2 \end{aligned}$$

input `integrate((e*x+d)^4*(b^2*x^2+2*a*b*x+a^2)^2,x, algorithm="maxima")`

output `1/9*b^4*e^4*x^9 + a^4*d^4*x + 1/2*(b^4*d*e^3 + a*b^3*e^4)*x^8 + 2/7*(3*b^4*d^2*e^2 + 8*a*b^3*d*e^3 + 3*a^2*b^2*e^4)*x^7 + 2/3*(b^4*d^3*e + 6*a*b^3*d^2*e^2 + 6*a^2*b^2*d*e^3 + a^3*b*e^4)*x^6 + 1/5*(b^4*d^4 + 16*a*b^3*d^3*e + 36*a^2*b^2*d^2*e^2 + 16*a^3*b*d*e^3 + a^4*e^4)*x^5 + (a*b^3*d^4 + 6*a^2*b^2*d^3*e + 6*a^3*b*d^2*e^2 + a^4*d*e^3)*x^4 + 2/3*(3*a^2*b^2*d^4 + 8*a^3*b*d^3*e + 3*a^4*d^2*e^2)*x^3 + 2*(a^3*b*d^4 + a^4*d^3*e)*x^2`

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 321 vs. $2(109) = 218$.

Time = 0.22 (sec) , antiderivative size = 321, normalized size of antiderivative = 2.70

$$\int (d + ex)^4 (a^2 + 2abx + b^2x^2)^2 dx = \frac{1}{9}b^4e^4x^9 + \frac{1}{2}b^4de^3x^8 + \frac{1}{2}ab^3e^4x^8 + \frac{6}{7}b^4d^2e^2x^7$$

$$+ \frac{16}{7}ab^3de^3x^7 + \frac{6}{7}a^2b^2e^4x^7 + \frac{2}{3}b^4d^3ex^6$$

$$+ 4ab^3d^2e^2x^6 + 4a^2b^2de^3x^6 + \frac{2}{3}a^3be^4x^6 + \frac{1}{5}b^4d^4x^5$$

$$+ \frac{16}{5}ab^3d^3ex^5 + \frac{36}{5}a^2b^2d^2e^2x^5 + \frac{16}{5}a^3bde^3x^5$$

$$+ \frac{1}{5}a^4e^4x^5 + ab^3d^4x^4 + 6a^2b^2d^3ex^4 + 6a^3bd^2e^2x^4$$

$$+ a^4de^3x^4 + 2a^2b^2d^4x^3 + \frac{16}{3}a^3bd^3ex^3$$

$$+ 2a^4d^2e^2x^3 + 2a^3bd^4x^2 + 2a^4d^3ex^2 + a^4d^4x$$

input

```
integrate((e*x+d)^4*(b^2*x^2+2*a*b*x+a^2)^2,x, algorithm="giac")
```

output

```
1/9*b^4*e^4*x^9 + 1/2*b^4*d*e^3*x^8 + 1/2*a*b^3*e^4*x^8 + 6/7*b^4*d^2*e^2*
x^7 + 16/7*a*b^3*d*e^3*x^7 + 6/7*a^2*b^2*e^4*x^7 + 2/3*b^4*d^3*e*x^6 + 4*a
*b^3*d^2*e^2*x^6 + 4*a^2*b^2*d*e^3*x^6 + 2/3*a^3*b*e^4*x^6 + 1/5*b^4*d^4*x
^5 + 16/5*a*b^3*d^3*e*x^5 + 36/5*a^2*b^2*d^2*e^2*x^5 + 16/5*a^3*b*d*e^3*x
^5 + 1/5*a^4*e^4*x^5 + a*b^3*d^4*x^4 + 6*a^2*b^2*d^3*e*x^4 + 6*a^3*b*d^2*e
^2*x^4 + a^4*d*e^3*x^4 + 2*a^2*b^2*d^4*x^3 + 16/3*a^3*b*d^3*e*x^3 + 2*a^4*d
^2*e^2*x^3 + 2*a^3*b*d^4*x^2 + 2*a^4*d^3*e*x^2 + a^4*d^4*x
```

Mupad [B] (verification not implemented)

Time = 9.01 (sec) , antiderivative size = 271, normalized size of antiderivative = 2.28

$$\int (d + ex)^4 (a^2 + 2abx + b^2x^2)^2 dx = x^5 \left(\frac{a^4 e^4}{5} + \frac{16 a^3 b d e^3}{5} + \frac{36 a^2 b^2 d^2 e^2}{5} + \frac{16 a b^3 d^3 e}{5} + \frac{b^4 d^4}{5} \right) + x^4 (a^4 d e^3 + 6 a^3 b d^2 e^2 + 6 a^2 b^2 d^3 e + a b^3 d^4) + x^6 \left(\frac{2 a^3 b e^4}{3} + 4 a^2 b^2 d e^3 + 4 a b^3 d^2 e^2 + \frac{2 b^4 d^3 e}{3} \right) + a^4 d^4 x + \frac{b^4 e^4 x^9}{9} + 2 a^3 d^3 x^2 (a e + b d) + \frac{b^3 e^3 x^8 (a e + b d)}{2} + \frac{2 a^2 d^2 x^3 (3 a^2 e^2 + 8 a b d e + 3 b^2 d^2)}{3} + \frac{2 b^2 e^2 x^7 (3 a^2 e^2 + 8 a b d e + 3 b^2 d^2)}{7}$$

input `int((d + e*x)^4*(a^2 + b^2*x^2 + 2*a*b*x)^2,x)`output `x^5*((a^4*e^4)/5 + (b^4*d^4)/5 + (36*a^2*b^2*d^2*e^2)/5 + (16*a*b^3*d^3*e)/5 + (16*a^3*b*d*e^3)/5) + x^4*(a*b^3*d^4 + a^4*d*e^3 + 6*a^2*b^2*d^3*e + 6*a^3*b*d^2*e^2) + x^6*((2*a^3*b*e^4)/3 + (2*b^4*d^3*e)/3 + 4*a*b^3*d^2*e^2 + 4*a^2*b^2*d*e^3) + a^4*d^4*x + (b^4*e^4*x^9)/9 + 2*a^3*d^3*x^2*(a*e + b*d) + (b^3*e^3*x^8*(a*e + b*d))/2 + (2*a^2*d^2*x^3*(3*a^2*e^2 + 3*b^2*d^2 + 8*a*b*d*e))/3 + (2*b^2*e^2*x^7*(3*a^2*e^2 + 3*b^2*d^2 + 8*a*b*d*e))/7`**Reduce [B] (verification not implemented)**

Time = 0.20 (sec) , antiderivative size = 322, normalized size of antiderivative = 2.71

$$\int (d + ex)^4 (a^2 + 2abx + b^2x^2)^2 dx = \frac{x(70b^4e^4x^8 + 315ab^3e^4x^7 + 315b^4de^3x^7 + 540a^2b^2e^4x^6 + 1440ab^3de^3x^6 + 540b^4d^2e^2x^6 + 420a^3be^4x^5 + \dots)}{1}$$

input `int((e*x+d)^4*(b^2*x^2+2*a*b*x+a^2)^2,x)`

output `(x*(630*a**4*d**4 + 1260*a**4*d**3*e*x + 1260*a**4*d**2*e**2*x**2 + 630*a*
*4*d*e**3*x**3 + 126*a**4*e**4*x**4 + 1260*a**3*b*d**4*x + 3360*a**3*b*d**
3*e*x**2 + 3780*a**3*b*d**2*e**2*x**3 + 2016*a**3*b*d*e**3*x**4 + 420*a**3
*b*e**4*x**5 + 1260*a**2*b**2*d**4*x**2 + 3780*a**2*b**2*d**3*e*x**3 + 453
6*a**2*b**2*d**2*e**2*x**4 + 2520*a**2*b**2*d*e**3*x**5 + 540*a**2*b**2*e*
*4*x**6 + 630*a*b**3*d**4*x**3 + 2016*a*b**3*d**3*e*x**4 + 2520*a*b**3*d**
2*e**2*x**5 + 1440*a*b**3*d*e**3*x**6 + 315*a*b**3*e**4*x**7 + 126*b**4*d*
*4*x**4 + 420*b**4*d**3*e*x**5 + 540*b**4*d**2*e**2*x**6 + 315*b**4*d*e**3
*x**7 + 70*b**4*e**4*x**8))/630`

3.69 $\int (d + ex)^3 (a^2 + 2abx + b^2x^2)^2 dx$

| | |
|---|-----|
| Optimal result | 565 |
| Mathematica [B] (verified) | 566 |
| Rubi [A] (verified) | 566 |
| Maple [B] (verified) | 568 |
| Fricas [B] (verification not implemented) | 568 |
| Sympy [B] (verification not implemented) | 569 |
| Maxima [B] (verification not implemented) | 570 |
| Giac [B] (verification not implemented) | 571 |
| Mupad [B] (verification not implemented) | 572 |
| Reduce [B] (verification not implemented) | 572 |

Optimal result

Integrand size = 26, antiderivative size = 92

$$\int (d + ex)^3 (a^2 + 2abx + b^2x^2)^2 dx = \frac{(bd - ae)^3(a + bx)^5}{5b^4} + \frac{e(bd - ae)^2(a + bx)^6}{2b^4} + \frac{3e^2(bd - ae)(a + bx)^7}{7b^4} + \frac{e^3(a + bx)^8}{8b^4}$$

output

```
1/5*(-a*e+b*d)^3*(b*x+a)^5/b^4+1/2*e*(-a*e+b*d)^2*(b*x+a)^6/b^4+3/7*e^2*(-a*e+b*d)*(b*x+a)^7/b^4+1/8*e^3*(b*x+a)^8/b^4
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 217 vs. $2(92) = 184$.

Time = 0.03 (sec) , antiderivative size = 217, normalized size of antiderivative = 2.36

$$\int (d + ex)^3 (a^2 + 2abx + b^2x^2)^2 dx = a^4d^3x + \frac{1}{2}a^3d^2(4bd + 3ae)x^2 + a^2d(2b^2d^2 + 4abde + a^2e^2)x^3 + \frac{1}{4}a(4b^3d^3 + 18ab^2d^2e + 12a^2bde^2 + a^3e^3)x^4 + \frac{1}{5}b(b^3d^3 + 12ab^2d^2e + 18a^2bde^2 + 4a^3e^3)x^5 + \frac{1}{2}b^2e(b^2d^2 + 4abde + 2a^2e^2)x^6 + \frac{1}{7}b^3e^2(3bd + 4ae)x^7 + \frac{1}{8}b^4e^3x^8$$

input

```
Integrate[(d + e*x)^3*(a^2 + 2*a*b*x + b^2*x^2)^2,x]
```

output

```
a^4*d^3*x + (a^3*d^2*(4*b*d + 3*a*e)*x^2)/2 + a^2*d*(2*b^2*d^2 + 4*a*b*d*e + a^2*e^2)*x^3 + (a*(4*b^3*d^3 + 18*a*b^2*d^2*e + 12*a^2*b*d*e^2 + a^3*e^3)*x^4)/4 + (b*(b^3*d^3 + 12*a*b^2*d^2*e + 18*a^2*b*d*e^2 + 4*a^3*e^3)*x^5)/5 + (b^2*e*(b^2*d^2 + 4*a*b*d*e + 2*a^2*e^2)*x^6)/2 + (b^3*e^2*(3*b*d + 4*a*e)*x^7)/7 + (b^4*e^3*x^8)/8
```

Rubi [A] (verified)

Time = 0.54 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {1098, 27, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a^2 + 2abx + b^2x^2)^2 (d + ex)^3 dx$$

↓ 1098

$$\begin{aligned}
 & \frac{\int b^4(a+bx)^4(d+ex)^3 dx}{b^4} \\
 & \quad \downarrow 27 \\
 & \int (a+bx)^4(d+ex)^3 dx \\
 & \quad \downarrow 49 \\
 & \int \left(\frac{3e^2(a+bx)^6(bd-ae)}{b^3} + \frac{3e(a+bx)^5(bd-ae)^2}{b^3} + \frac{(a+bx)^4(bd-ae)^3}{b^3} + \frac{e^3(a+bx)^7}{b^3} \right) dx \\
 & \quad \downarrow 2009 \\
 & \frac{3e^2(a+bx)^7(bd-ae)}{7b^4} + \frac{e(a+bx)^6(bd-ae)^2}{2b^4} + \frac{(a+bx)^5(bd-ae)^3}{5b^4} + \frac{e^3(a+bx)^8}{8b^4}
 \end{aligned}$$

input `Int[(d + e*x)^3*(a^2 + 2*a*b*x + b^2*x^2)^2,x]`

output `((b*d - a*e)^3*(a + b*x)^5)/(5*b^4) + (e*(b*d - a*e)^2*(a + b*x)^6)/(2*b^4) + (3*e^2*(b*d - a*e)*(a + b*x)^7)/(7*b^4) + (e^3*(a + b*x)^8)/(8*b^4)`

Defintions of rubi rules used

rule 27 `Int[(a_)*(F_x_), x_Symbol] := Simp[a Int[F_x, x], x] /; FreeQ[a, x] && !MatchQ[F_x, (b_)*(G_x_)] /; FreeQ[b, x]`

rule 49 `Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] := Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] && IGtQ[m, 0] && IGtQ[m + n + 2, 0]`

rule 1098 `Int[((d_.) + (e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] := Simp[1/c^p Int[(d + e*x)^m*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]`

rule 2009 `Int[u_, x_Symbol] := Simp[IntSum[u, x], x] /; SumQ[u]`

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 221 vs. $2(84) = 168$.

Time = 1.04 (sec) , antiderivative size = 222, normalized size of antiderivative = 2.41

| method | result |
|---------------|---|
| norman | $\frac{e^3 b^4 x^8}{8} + \left(\frac{4}{7} a b^3 e^3 + \frac{3}{7} b^4 d e^2\right) x^7 + \left(a^2 b^2 e^3 + 2 d e^2 a b^3 + \frac{1}{2} d^2 e b^4\right) x^6 + \left(\frac{4}{5} a^3 b e^3 + \frac{18}{5} a^2 b^2 d e^2\right) x^5 + \left(\frac{4}{5} a^3 b e^3 + \frac{18}{5} a^2 b^2 d e^2 + 12 d^2 e a b^3 + b^4 d^3\right) x^4 + \left(\frac{4}{5} a^3 b e^3 + \frac{18}{5} a^2 b^2 d e^2 + 12 d^2 e a b^3 + b^4 d^3\right) x^3 + \left(\frac{4}{5} a^3 b e^3 + \frac{18}{5} a^2 b^2 d e^2 + 12 d^2 e a b^3 + b^4 d^3\right) x^2 + \left(\frac{4}{5} a^3 b e^3 + \frac{18}{5} a^2 b^2 d e^2 + 12 d^2 e a b^3 + b^4 d^3\right) x + \left(\frac{4}{5} a^3 b e^3 + \frac{18}{5} a^2 b^2 d e^2 + 12 d^2 e a b^3 + b^4 d^3\right)$ |
| default | $\frac{e^3 b^4 x^8}{8} + \frac{(4 a b^3 e^3 + 3 b^4 d e^2) x^7}{7} + \frac{(6 a^2 b^2 e^3 + 12 d e^2 a b^3 + 3 d^2 e b^4) x^6}{6} + \frac{(4 a^3 b e^3 + 18 a^2 b^2 d e^2 + 12 d^2 e a b^3 + b^4 d^3) x^5}{5} + \frac{(4 a^3 b e^3 + 18 a^2 b^2 d e^2 + 12 d^2 e a b^3 + b^4 d^3) x^4}{4} + \frac{(4 a^3 b e^3 + 18 a^2 b^2 d e^2 + 12 d^2 e a b^3 + b^4 d^3) x^3}{3} + \frac{(4 a^3 b e^3 + 18 a^2 b^2 d e^2 + 12 d^2 e a b^3 + b^4 d^3) x^2}{2} + \frac{(4 a^3 b e^3 + 18 a^2 b^2 d e^2 + 12 d^2 e a b^3 + b^4 d^3) x}{1} + \frac{(4 a^3 b e^3 + 18 a^2 b^2 d e^2 + 12 d^2 e a b^3 + b^4 d^3)}{1}$ |
| risch | $\frac{1}{8} e^3 b^4 x^8 + \frac{4}{7} x^7 a b^3 e^3 + \frac{3}{7} x^7 b^4 d e^2 + x^6 a^2 b^2 e^3 + 2 x^6 d e^2 a b^3 + \frac{1}{2} x^6 d^2 e b^4 + \frac{4}{5} x^5 a^3 b e^3 + \frac{18}{5} x^5 a^2 b^2 d e^2 + \frac{12}{5} x^5 d^2 e a b^3 + \frac{1}{5} x^5 b^4 d^3 + \frac{4}{5} x^4 a^3 b e^3 + \frac{18}{5} x^4 a^2 b^2 d e^2 + \frac{12}{5} x^4 d^2 e a b^3 + \frac{1}{5} x^4 b^4 d^3 + \frac{4}{5} x^3 a^3 b e^3 + \frac{18}{5} x^3 a^2 b^2 d e^2 + \frac{12}{5} x^3 d^2 e a b^3 + \frac{1}{5} x^3 b^4 d^3 + \frac{4}{5} x^2 a^3 b e^3 + \frac{18}{5} x^2 a^2 b^2 d e^2 + \frac{12}{5} x^2 d^2 e a b^3 + \frac{1}{5} x^2 b^4 d^3 + \frac{4}{5} x a^3 b e^3 + \frac{18}{5} x a^2 b^2 d e^2 + \frac{12}{5} x d^2 e a b^3 + \frac{1}{5} x b^4 d^3 + \frac{4}{5} a^3 b e^3 + \frac{18}{5} a^2 b^2 d e^2 + \frac{12}{5} d^2 e a b^3 + \frac{1}{5} b^4 d^3$ |
| parallelrisch | $\frac{1}{8} e^3 b^4 x^8 + \frac{4}{7} x^7 a b^3 e^3 + \frac{3}{7} x^7 b^4 d e^2 + x^6 a^2 b^2 e^3 + 2 x^6 d e^2 a b^3 + \frac{1}{2} x^6 d^2 e b^4 + \frac{4}{5} x^5 a^3 b e^3 + \frac{18}{5} x^5 a^2 b^2 d e^2 + \frac{12}{5} x^5 d^2 e a b^3 + \frac{1}{5} x^5 b^4 d^3 + \frac{4}{5} x^4 a^3 b e^3 + \frac{18}{5} x^4 a^2 b^2 d e^2 + \frac{12}{5} x^4 d^2 e a b^3 + \frac{1}{5} x^4 b^4 d^3 + \frac{4}{5} x^3 a^3 b e^3 + \frac{18}{5} x^3 a^2 b^2 d e^2 + \frac{12}{5} x^3 d^2 e a b^3 + \frac{1}{5} x^3 b^4 d^3 + \frac{4}{5} x^2 a^3 b e^3 + \frac{18}{5} x^2 a^2 b^2 d e^2 + \frac{12}{5} x^2 d^2 e a b^3 + \frac{1}{5} x^2 b^4 d^3 + \frac{4}{5} x a^3 b e^3 + \frac{18}{5} x a^2 b^2 d e^2 + \frac{12}{5} x d^2 e a b^3 + \frac{1}{5} x b^4 d^3 + \frac{4}{5} a^3 b e^3 + \frac{18}{5} a^2 b^2 d e^2 + \frac{12}{5} d^2 e a b^3 + \frac{1}{5} b^4 d^3$ |
| gospers | $x(35e^3b^4x^7+160x^6ab^3e^3+120x^6b^4de^2+280x^5a^2b^2e^3+560x^5de^2ab^3+140x^5d^2eb^4+224x^4a^3be^3+1008x^4a^2b^2de^2+672x^4d^2eab^3+112x^4b^4d^3)+\frac{1}{5}x^5(4a^3be^3+18a^2b^2de^2+12d^2eab^3+b^4d^3)+\frac{1}{3}x^3(4a^3be^3+18a^2b^2de^2+12d^2eab^3+b^4d^3)+\frac{1}{2}x^2(4a^3be^3+18a^2b^2de^2+12d^2eab^3+b^4d^3)+\frac{1}{1}x(4a^3be^3+18a^2b^2de^2+12d^2eab^3+b^4d^3)+\frac{1}{1}(4a^3be^3+18a^2b^2de^2+12d^2eab^3+b^4d^3)$ |
| orering | $x(35e^3b^4x^7+160x^6ab^3e^3+120x^6b^4de^2+280x^5a^2b^2e^3+560x^5de^2ab^3+140x^5d^2eb^4+224x^4a^3be^3+1008x^4a^2b^2de^2+672x^4d^2eab^3+112x^4b^4d^3)+\frac{1}{5}x^5(4a^3be^3+18a^2b^2de^2+12d^2eab^3+b^4d^3)+\frac{1}{3}x^3(4a^3be^3+18a^2b^2de^2+12d^2eab^3+b^4d^3)+\frac{1}{2}x^2(4a^3be^3+18a^2b^2de^2+12d^2eab^3+b^4d^3)+\frac{1}{1}x(4a^3be^3+18a^2b^2de^2+12d^2eab^3+b^4d^3)+\frac{1}{1}(4a^3be^3+18a^2b^2de^2+12d^2eab^3+b^4d^3)$ |

input `int((e*x+d)^3*(b^2*x^2+2*a*b*x+a^2)^2,x,method=_RETURNVERBOSE)`

output `1/8*e^3*b^4*x^8+(4/7*a*b^3*e^3+3/7*b^4*d*e^2)*x^7+(a^2*b^2*e^3+2*d*e^2*a*b^3+1/2*d^2*e*b^4)*x^6+(4/5*a^3*b*e^3+18/5*a^2*b^2*d*e^2+12/5*d^2*e*a*b^3+1/5*b^4*d^3)*x^5+(1/4*e^3*a^4+3*d*e^2*a^3*b+9/2*d^2*e*a^2*b^2+a*b^3*d^3)*x^4+(a^4*d*e^2+4*a^3*b*d^2*e+2*a^2*b^2*d^3)*x^3+(3/2*d^2*e*a^4+2*a^3*b*d^3)*x^2+a^4*d^3*x`

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 225 vs. $2(84) = 168$.

Time = 0.08 (sec) , antiderivative size = 225, normalized size of antiderivative = 2.45

$$\int (d + ex)^3 (a^2 + 2abx + b^2x^2)^2 dx = \frac{1}{8} b^4 e^3 x^8 + a^4 d^3 x + \frac{1}{7} (3b^4 d e^2 + 4ab^3 e^3) x^7$$

$$+ \frac{1}{2} (b^4 d^2 e + 4ab^3 d e^2 + 2a^2 b^2 e^3) x^6$$

$$+ \frac{1}{5} (b^4 d^3 + 12ab^3 d^2 e + 18a^2 b^2 d e^2 + 4a^3 b e^3) x^5$$

$$+ \frac{1}{4} (4ab^3 d^3 + 18a^2 b^2 d^2 e + 12a^3 b d e^2 + a^4 e^3) x^4$$

$$+ (2a^2 b^2 d^3 + 4a^3 b d^2 e + a^4 d e^2) x^3$$

$$+ \frac{1}{2} (4a^3 b d^3 + 3a^4 d^2 e) x^2$$

input `integrate((e*x+d)^3*(b^2*x^2+2*a*b*x+a^2)^2,x, algorithm="fricas")`

output `1/8*b^4*e^3*x^8 + a^4*d^3*x + 1/7*(3*b^4*d*e^2 + 4*a*b^3*e^3)*x^7 + 1/2*(b^4*d^2*e + 4*a*b^3*d*e^2 + 2*a^2*b^2*e^3)*x^6 + 1/5*(b^4*d^3 + 12*a*b^3*d^2*e + 18*a^2*b^2*d*e^2 + 4*a^3*b*e^3)*x^5 + 1/4*(4*a*b^3*d^3 + 18*a^2*b^2*d^2*e + 12*a^3*b*d*e^2 + a^4*e^3)*x^4 + (2*a^2*b^2*d^3 + 4*a^3*b*d^2*e + a^4*d*e^2)*x^3 + 1/2*(4*a^3*b*d^3 + 3*a^4*d^2*e)*x^2`

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 243 vs. 2(80) = 160.

Time = 0.03 (sec) , antiderivative size = 243, normalized size of antiderivative = 2.64

$$\int (d + ex)^3 (a^2 + 2abx + b^2x^2)^2 dx = a^4 d^3 x + \frac{b^4 e^3 x^8}{8} + x^7 \cdot \left(\frac{4ab^3 e^3}{7} + \frac{3b^4 d e^2}{7} \right)$$

$$+ x^6 \left(a^2 b^2 e^3 + 2ab^3 d e^2 + \frac{b^4 d^2 e}{2} \right) + x^5$$

$$\cdot \left(\frac{4a^3 b e^3}{5} + \frac{18a^2 b^2 d e^2}{5} + \frac{12ab^3 d^2 e}{5} + \frac{b^4 d^3}{5} \right)$$

$$+ x^4 \left(\frac{a^4 e^3}{4} + 3a^3 b d e^2 + \frac{9a^2 b^2 d^2 e}{2} + ab^3 d^3 \right)$$

$$+ x^3 (a^4 d e^2 + 4a^3 b d^2 e + 2a^2 b^2 d^3)$$

$$+ x^2 \cdot \left(\frac{3a^4 d^2 e}{2} + 2a^3 b d^3 \right)$$

input `integrate((e*x+d)**3*(b**2*x**2+2*a*b*x+a**2)**2,x)`

output `a**4*d**3*x + b**4*e**3*x**8/8 + x**7*(4*a*b**3*e**3/7 + 3*b**4*d*e**2/7) + x**6*(a**2*b**2*e**3 + 2*a*b**3*d*e**2 + b**4*d**2*e/2) + x**5*(4*a**3*b*e**3/5 + 18*a**2*b**2*d*e**2/5 + 12*a*b**3*d**2*e/5 + b**4*d**3/5) + x**4*(a**4*e**3/4 + 3*a**3*b*d*e**2 + 9*a**2*b**2*d**2*e/2 + a*b**3*d**3) + x**3*(a**4*d*e**2 + 4*a**3*b*d**2*e + 2*a**2*b**2*d**3) + x**2*(3*a**4*d**2*e/2 + 2*a**3*b*d**3)`

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 225 vs. $2(84) = 168$.

Time = 0.04 (sec) , antiderivative size = 225, normalized size of antiderivative = 2.45

$$\int (d+ex)^3 (a^2+2abx+b^2x^2)^2 dx = \frac{1}{8} b^4 e^3 x^8 + a^4 d^3 x + \frac{1}{7} (3b^4 d e^2 + 4ab^3 e^3) x^7 + \frac{1}{2} (b^4 d^2 e + 4ab^3 d e^2 + 2a^2 b^2 e^3) x^6 + \frac{1}{5} (b^4 d^3 + 12ab^3 d^2 e + 18a^2 b^2 d e^2 + 4a^3 b e^3) x^5 + \frac{1}{4} (4ab^3 d^3 + 18a^2 b^2 d^2 e + 12a^3 b d e^2 + a^4 e^3) x^4 + (2a^2 b^2 d^3 + 4a^3 b d^2 e + a^4 d e^2) x^3 + \frac{1}{2} (4a^3 b d^3 + 3a^4 d^2 e) x^2$$

input `integrate((e*x+d)^3*(b^2*x^2+2*a*b*x+a^2)^2,x, algorithm="maxima")`

output `1/8*b^4*e^3*x^8 + a^4*d^3*x + 1/7*(3*b^4*d*e^2 + 4*a*b^3*e^3)*x^7 + 1/2*(b^4*d^2*e + 4*a*b^3*d*e^2 + 2*a^2*b^2*e^3)*x^6 + 1/5*(b^4*d^3 + 12*a*b^3*d^2*e + 18*a^2*b^2*d*e^2 + 4*a^3*b*e^3)*x^5 + 1/4*(4*a*b^3*d^3 + 18*a^2*b^2*d^2*e + 12*a^3*b*d*e^2 + a^4*e^3)*x^4 + (2*a^2*b^2*d^3 + 4*a^3*b*d^2*e + a^4*d*e^2)*x^3 + 1/2*(4*a^3*b*d^3 + 3*a^4*d^2*e)*x^2`

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 245 vs. $2(84) = 168$.

Time = 0.23 (sec) , antiderivative size = 245, normalized size of antiderivative = 2.66

$$\begin{aligned} \int (d+ex)^3 (a^2+2abx+b^2x^2)^2 dx = & \frac{1}{8}b^4e^3x^8 + \frac{3}{7}b^4de^2x^7 + \frac{4}{7}ab^3e^3x^7 + \frac{1}{2}b^4d^2ex^6 \\ & + 2ab^3de^2x^6 + a^2b^2e^3x^6 + \frac{1}{5}b^4d^3x^5 \\ & + \frac{12}{5}ab^3d^2ex^5 + \frac{18}{5}a^2b^2de^2x^5 + \frac{4}{5}a^3be^3x^5 \\ & + ab^3d^3x^4 + \frac{9}{2}a^2b^2d^2ex^4 + 3a^3bde^2x^4 \\ & + \frac{1}{4}a^4e^3x^4 + 2a^2b^2d^3x^3 + 4a^3bd^2ex^3 \\ & + a^4de^2x^3 + 2a^3bd^3x^2 + \frac{3}{2}a^4d^2ex^2 + a^4d^3x \end{aligned}$$

input `integrate((e*x+d)^3*(b^2*x^2+2*a*b*x+a^2)^2,x, algorithm="giac")`

output `1/8*b^4*e^3*x^8 + 3/7*b^4*d*e^2*x^7 + 4/7*a*b^3*e^3*x^7 + 1/2*b^4*d^2*e*x^6 + 2*a*b^3*d*e^2*x^6 + a^2*b^2*e^3*x^6 + 1/5*b^4*d^3*x^5 + 12/5*a*b^3*d^2*e*x^5 + 18/5*a^2*b^2*d*e^2*x^5 + 4/5*a^3*b*e^3*x^5 + a*b^3*d^3*x^4 + 9/2*a^2*b^2*d^2*e*x^4 + 3*a^3*b*d*e^2*x^4 + 1/4*a^4*e^3*x^4 + 2*a^2*b^2*d^3*x^3 + 4*a^3*b*d^2*e*x^3 + a^4*d*e^2*x^3 + 2*a^3*b*d^3*x^2 + 3/2*a^4*d^2*e*x^2 + a^4*d^3*x`

Mupad [B] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 208, normalized size of antiderivative = 2.26

$$\int (d + ex)^3 (a^2 + 2abx + b^2x^2)^2 dx = x^4 \left(\frac{a^4 e^3}{4} + 3a^3 b d e^2 + \frac{9a^2 b^2 d^2 e}{2} + a b^3 d^3 \right) + x^5 \left(\frac{4a^3 b e^3}{5} + \frac{18a^2 b^2 d e^2}{5} + \frac{12a b^3 d^2 e}{5} + \frac{b^4 d^3}{5} \right) + a^4 d^3 x + \frac{b^4 e^3 x^8}{8} + \frac{a^3 d^2 x^2 (3ae + 4bd)}{2} + \frac{b^3 e^2 x^7 (4ae + 3bd)}{7} + a^2 d x^3 (a^2 e^2 + 4abde + 2b^2 d^2) + \frac{b^2 e x^6 (2a^2 e^2 + 4abde + b^2 d^2)}{2}$$

input `int((d + e*x)^3*(a^2 + b^2*x^2 + 2*a*b*x)^2,x)`output `x^4*((a^4*e^3)/4 + a*b^3*d^3 + (9*a^2*b^2*d^2*e)/2 + 3*a^3*b*d*e^2) + x^5*((b^4*d^3)/5 + (4*a^3*b*e^3)/5 + (18*a^2*b^2*d*e^2)/5 + (12*a*b^3*d^2*e)/5) + a^4*d^3*x + (b^4*e^3*x^8)/8 + (a^3*d^2*x^2*(3*a*e + 4*b*d))/2 + (b^3*e^2*x^7*(4*a*e + 3*b*d))/7 + a^2*d*x^3*(a^2*e^2 + 2*b^2*d^2 + 4*a*b*d*e) + (b^2*e*x^6*(2*a^2*e^2 + b^2*d^2 + 4*a*b*d*e))/2`**Reduce [B] (verification not implemented)**

Time = 0.19 (sec) , antiderivative size = 247, normalized size of antiderivative = 2.68

$$\int (d + ex)^3 (a^2 + 2abx + b^2x^2)^2 dx = \frac{x(35b^4e^3x^7 + 160ab^3e^3x^6 + 120b^4de^2x^6 + 280a^2b^2e^3x^5 + 560ab^3de^2x^5 + 140b^4d^2e^2x^5 + 224a^3be^3x^4 + 1$$

input `int((e*x+d)^3*(b^2*x^2+2*a*b*x+a^2)^2,x)`

output

```
(x*(280*a**4*d**3 + 420*a**4*d**2*e*x + 280*a**4*d*e**2*x**2 + 70*a**4*e**3*x**3 + 560*a**3*b*d**3*x + 1120*a**3*b*d**2*e*x**2 + 840*a**3*b*d*e**2*x**3 + 224*a**3*b*e**3*x**4 + 560*a**2*b**2*d**3*x**2 + 1260*a**2*b**2*d**2*e*x**3 + 1008*a**2*b**2*d*e**2*x**4 + 280*a**2*b**2*e**3*x**5 + 280*a*b**3*d**3*x**3 + 672*a*b**3*d**2*e*x**4 + 560*a*b**3*d*e**2*x**5 + 160*a*b**3*e**3*x**6 + 56*b**4*d**3*x**4 + 140*b**4*d**2*e*x**5 + 120*b**4*d*e**2*x**6 + 35*b**4*e**3*x**7))/280
```

3.70 $\int (d + ex)^2 (a^2 + 2abx + b^2x^2)^2 dx$

| | |
|---|-----|
| Optimal result | 574 |
| Mathematica [B] (verified) | 574 |
| Rubi [A] (verified) | 575 |
| Maple [B] (verified) | 576 |
| Fricas [B] (verification not implemented) | 577 |
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| Maxima [B] (verification not implemented) | 578 |
| Giac [B] (verification not implemented) | 579 |
| Mupad [B] (verification not implemented) | 580 |
| Reduce [B] (verification not implemented) | 580 |

Optimal result

Integrand size = 26, antiderivative size = 65

$$\int (d + ex)^2 (a^2 + 2abx + b^2x^2)^2 dx = \frac{(bd - ae)^2(a + bx)^5}{5b^3} + \frac{e(bd - ae)(a + bx)^6}{3b^3} + \frac{e^2(a + bx)^7}{7b^3}$$

output

```
1/5*(-a*e+b*d)^2*(b*x+a)^5/b^3+1/3*e*(-a*e+b*d)*(b*x+a)^6/b^3+1/7*e^2*(b*x+a)^7/b^3
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 148 vs. 2(65) = 130.

Time = 0.02 (sec) , antiderivative size = 148, normalized size of antiderivative = 2.28

$$\int (d + ex)^2 (a^2 + 2abx + b^2x^2)^2 dx = a^4d^2x + a^3d(2bd + ae)x^2 + \frac{1}{3}a^2(6b^2d^2 + 8abde + a^2e^2)x^3 + ab(b^2d^2 + 3abde + a^2e^2)x^4 + \frac{1}{5}b^2(b^2d^2 + 8abde + 6a^2e^2)x^5 + \frac{1}{3}b^3e(bd + 2ae)x^6 + \frac{1}{7}b^4e^2x^7$$

input `Integrate[(d + e*x)^2*(a^2 + 2*a*b*x + b^2*x^2)^2,x]`

output $a^4*d^2*x + a^3*d*(2*b*d + a*e)*x^2 + (a^2*(6*b^2*d^2 + 8*a*b*d*e + a^2*e^2)*x^3)/3 + a*b*(b^2*d^2 + 3*a*b*d*e + a^2*e^2)*x^4 + (b^2*(b^2*d^2 + 8*a*b*d*e + 6*a^2*e^2)*x^5)/5 + (b^3*e*(b*d + 2*a*e)*x^6)/3 + (b^4*e^2*x^7)/7$

Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {1098, 27, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\begin{aligned} & \int (a^2 + 2abx + b^2x^2)^2 (d + ex)^2 dx \\ & \quad \downarrow 1098 \\ & \quad \int \frac{b^4(a + bx)^4(d + ex)^2 dx}{b^4} \\ & \quad \downarrow 27 \\ & \quad \int (a + bx)^4(d + ex)^2 dx \\ & \quad \downarrow 49 \\ & \quad \int \left(\frac{2e(a + bx)^5(bd - ae)}{b^2} + \frac{(a + bx)^4(bd - ae)^2}{b^2} + \frac{e^2(a + bx)^6}{b^2} \right) dx \\ & \quad \downarrow 2009 \\ & \quad \frac{e(a + bx)^6(bd - ae)}{3b^3} + \frac{(a + bx)^5(bd - ae)^2}{5b^3} + \frac{e^2(a + bx)^7}{7b^3} \end{aligned}$$

input `Int[(d + e*x)^2*(a^2 + 2*a*b*x + b^2*x^2)^2,x]`

```
output ((b*d - a*e)^2*(a + b*x)^5)/(5*b^3) + (e*(b*d - a*e)*(a + b*x)^6)/(3*b^3)
+ (e^2*(a + b*x)^7)/(7*b^3)
```

Defintions of rubi rules used

```
rule 27 Int[(a_)*(Fx_), x_Symbol] := Simp[a Int[Fx, x], x] /; FreeQ[a, x] && !Ma
tchQ[Fx, (b_)*(Gx_)] /; FreeQ[b, x]
```

```
rule 49 Int[((a_.) + (b_.)*(x_.))^(m_.)*((c_.) + (d_.)*(x_.))^(n_.), x_Symbol] := Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

```
rule 1098 Int[((d_.) + (e_.)*(x_.))^(m_.)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] := Simp[1/c^p Int[(d + e*x)^m*(b/2 + c*x)^(2*p), x], x] /; FreeQ[
{a, b, c, d, e, m}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

```
rule 2009 Int[u_, x_Symbol] := Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 156 vs. 2(59) = 118.

Time = 1.06 (sec) , antiderivative size = 157, normalized size of antiderivative = 2.42

| method | result |
|--------------|--|
| norman | $\frac{e^2 b^4 x^7}{7} + \left(\frac{2}{3} e^2 a b^3 + \frac{1}{3} b^4 d e\right) x^6 + \left(\frac{6}{5} a^2 b^2 e^2 + \frac{8}{5} a b^3 d e + \frac{1}{5} d^2 b^4\right) x^5 + (e^2 a^3 b + 3 d e a^2 b^2 + a b^3 d) x^4 + \frac{(4 e^2 a^3 b + 12 d e a^2 b^2 + 4 a b^3 d^2) x^4}{4} + \frac{(a^4 e^2 + 8 d e a^3 b + 6 d^2 a^2 b^2 + 4 a^2 b^3 d + 3 a b^4 d^2) x^3}{3}$ |
| default | $\frac{e^2 b^4 x^7}{7} + \frac{(4 e^2 a b^3 + 2 b^4 d e) x^6}{6} + \frac{(6 a^2 b^2 e^2 + 8 a b^3 d e + d^2 b^4) x^5}{5} + \frac{(4 e^2 a^3 b + 12 d e a^2 b^2 + 4 a b^3 d^2) x^4}{4} + \frac{(a^4 e^2 + 8 d e a^3 b + 6 d^2 a^2 b^2 + 4 a^2 b^3 d + 3 a b^4 d^2) x^3}{3}$ |
| risch | $\frac{1}{7} e^2 b^4 x^7 + \frac{2}{3} x^6 e^2 a b^3 + \frac{1}{3} x^6 b^4 d e + \frac{6}{5} x^5 a^2 b^2 e^2 + \frac{8}{5} x^5 a b^3 d e + \frac{1}{5} x^5 d^2 b^4 + a^3 b e^2 x^4 + 3 a^2 b^2 d e x^4$ |
| parallelrisc | $\frac{1}{7} e^2 b^4 x^7 + \frac{2}{3} x^6 e^2 a b^3 + \frac{1}{3} x^6 b^4 d e + \frac{6}{5} x^5 a^2 b^2 e^2 + \frac{8}{5} x^5 a b^3 d e + \frac{1}{5} x^5 d^2 b^4 + a^3 b e^2 x^4 + 3 a^2 b^2 d e x^4$ |
| gospers | $\frac{x(15 e^2 b^4 x^6 + 70 x^5 e^2 a b^3 + 35 x^5 b^4 d e + 126 x^4 a^2 b^2 e^2 + 168 x^4 a b^3 d e + 21 x^4 d^2 b^4 + 105 a^3 b e^2 x^3 + 315 a^2 b^2 d e x^3 + 105 a b^3 d^2 x^3 + 35 a^4 e^2 x^2 + 210 a^3 d e x^2 + 105 a^2 d^2 x^2 + 35 a b^4 d^2 x + 35 a^2 b^3 d^2 x + 35 a^3 b^2 d^2 x + 35 a^4 b d^2 x + 35 a^5 d^2 x + 35 a^6 d^2)}{105}$ |
| orering | $\frac{x(15 e^2 b^4 x^6 + 70 x^5 e^2 a b^3 + 35 x^5 b^4 d e + 126 x^4 a^2 b^2 e^2 + 168 x^4 a b^3 d e + 21 x^4 d^2 b^4 + 105 a^3 b e^2 x^3 + 315 a^2 b^2 d e x^3 + 105 a b^3 d^2 x^3 + 35 a^4 e^2 x^2 + 210 a^3 d e x^2 + 105 a^2 d^2 x^2 + 35 a b^4 d^2 x + 35 a^2 b^3 d^2 x + 35 a^3 b^2 d^2 x + 35 a^4 b d^2 x + 35 a^5 d^2 x + 35 a^6 d^2)}{105(bx+a)^4}$ |

input `int((e*x+d)^2*(b^2*x^2+2*a*b*x+a^2)^2,x,method=_RETURNVERBOSE)`

output `1/7*e^2*b^4*x^7+(2/3*e^2*a*b^3+1/3*b^4*d*e)*x^6+(6/5*a^2*b^2*e^2+8/5*a*b^3*d*e+1/5*d^2*b^4)*x^5+(a^3*b*e^2+3*a^2*b^2*d*e+a*b^3*d^2)*x^4+(1/3*a^4*e^2+8/3*d*e*a^3*b+2*d^2*a^2*b^2)*x^3+(a^4*d*e+2*a^3*b*d^2)*x^2+a^4*d^2*x`

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 156 vs. $2(59) = 118$.

Time = 0.07 (sec) , antiderivative size = 156, normalized size of antiderivative = 2.40

$$\begin{aligned} \int (d+ex)^2 (a^2+2abx+b^2x^2)^2 dx &= \frac{1}{7} b^4 e^2 x^7 + a^4 d^2 x + \frac{1}{3} (b^4 d e + 2 a b^3 e^2) x^6 \\ &+ \frac{1}{5} (b^4 d^2 + 8 a b^3 d e + 6 a^2 b^2 e^2) x^5 \\ &+ (a b^3 d^2 + 3 a^2 b^2 d e + a^3 b e^2) x^4 \\ &+ \frac{1}{3} (6 a^2 b^2 d^2 + 8 a^3 b d e + a^4 e^2) x^3 \\ &+ (2 a^3 b d^2 + a^4 d e) x^2 \end{aligned}$$

input `integrate((e*x+d)^2*(b^2*x^2+2*a*b*x+a^2)^2,x, algorithm="fricas")`

output `1/7*b^4*e^2*x^7 + a^4*d^2*x + 1/3*(b^4*d*e + 2*a*b^3*e^2)*x^6 + 1/5*(b^4*d^2 + 8*a*b^3*d*e + 6*a^2*b^2*e^2)*x^5 + (a*b^3*d^2 + 3*a^2*b^2*d*e + a^3*b*e^2)*x^4 + 1/3*(6*a^2*b^2*d^2 + 8*a^3*b*d*e + a^4*e^2)*x^3 + (2*a^3*b*d^2 + a^4*d*e)*x^2`

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 168 vs. $2(54) = 108$.

Time = 0.03 (sec) , antiderivative size = 168, normalized size of antiderivative = 2.58

$$\begin{aligned} \int (d + ex)^2 (a^2 + 2abx + b^2x^2)^2 dx = & a^4d^2x + \frac{b^4e^2x^7}{7} + x^6 \cdot \left(\frac{2ab^3e^2}{3} + \frac{b^4de}{3} \right) \\ & + x^5 \cdot \left(\frac{6a^2b^2e^2}{5} + \frac{8ab^3de}{5} + \frac{b^4d^2}{5} \right) \\ & + x^4 (a^3be^2 + 3a^2b^2de + ab^3d^2) \\ & + x^3 \left(\frac{a^4e^2}{3} + \frac{8a^3bde}{3} + 2a^2b^2d^2 \right) \\ & + x^2 (a^4de + 2a^3bd^2) \end{aligned}$$

input `integrate((e*x+d)**2*(b**2*x**2+2*a*b*x+a**2)**2,x)`

output `a**4*d**2*x + b**4*e**2*x**7/7 + x**6*(2*a*b**3*e**2/3 + b**4*d*e/3) + x**5*(6*a**2*b**2*e**2/5 + 8*a*b**3*d*e/5 + b**4*d**2/5) + x**4*(a**3*b*e**2 + 3*a**2*b**2*d*e + a*b**3*d**2) + x**3*(a**4*e**2/3 + 8*a**3*b*d*e/3 + 2*a**2*b**2*d**2) + x**2*(a**4*d*e + 2*a**3*b*d**2)`

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 156 vs. $2(59) = 118$.

Time = 0.04 (sec) , antiderivative size = 156, normalized size of antiderivative = 2.40

$$\begin{aligned} \int (d + ex)^2 (a^2 + 2abx + b^2x^2)^2 dx = & \frac{1}{7} b^4 e^2 x^7 + a^4 d^2 x + \frac{1}{3} (b^4 d e + 2 a b^3 e^2) x^6 \\ & + \frac{1}{5} (b^4 d^2 + 8 a b^3 d e + 6 a^2 b^2 e^2) x^5 \\ & + (a b^3 d^2 + 3 a^2 b^2 d e + a^3 b e^2) x^4 \\ & + \frac{1}{3} (6 a^2 b^2 d^2 + 8 a^3 b d e + a^4 e^2) x^3 \\ & + (2 a^3 b d^2 + a^4 d e) x^2 \end{aligned}$$

input `integrate((e*x+d)^2*(b^2*x^2+2*a*b*x+a^2)^2,x, algorithm="maxima")`

output

```
1/7*b^4*e^2*x^7 + a^4*d^2*x + 1/3*(b^4*d*e + 2*a*b^3*e^2)*x^6 + 1/5*(b^4*d^2 + 8*a*b^3*d*e + 6*a^2*b^2*e^2)*x^5 + (a*b^3*d^2 + 3*a^2*b^2*d*e + a^3*b*e^2)*x^4 + 1/3*(6*a^2*b^2*d^2 + 8*a^3*b*d*e + a^4*e^2)*x^3 + (2*a^3*b*d^2 + a^4*d*e)*x^2
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 170 vs. $2(59) = 118$.

Time = 0.21 (sec) , antiderivative size = 170, normalized size of antiderivative = 2.62

$$\int (d + ex)^2 (a^2 + 2abx + b^2x^2)^2 dx = \frac{1}{7}b^4e^2x^7 + \frac{1}{3}b^4dex^6 + \frac{2}{3}ab^3e^2x^6 + \frac{1}{5}b^4d^2x^5 + \frac{8}{5}ab^3dex^5 + \frac{6}{5}a^2b^2e^2x^5 + ab^3d^2x^4 + 3a^2b^2dex^4 + a^3be^2x^4 + 2a^2b^2d^2x^3 + \frac{8}{3}a^3bdex^3 + \frac{1}{3}a^4e^2x^3 + 2a^3bd^2x^2 + a^4dex^2 + a^4d^2x$$

input

```
integrate((e*x+d)^2*(b^2*x^2+2*a*b*x+a^2)^2,x, algorithm="giac")
```

output

```
1/7*b^4*e^2*x^7 + 1/3*b^4*d*e*x^6 + 2/3*a*b^3*e^2*x^6 + 1/5*b^4*d^2*x^5 + 8/5*a*b^3*d*e*x^5 + 6/5*a^2*b^2*e^2*x^5 + a*b^3*d^2*x^4 + 3*a^2*b^2*d*e*x^4 + a^3*b*e^2*x^4 + 2*a^2*b^2*d^2*x^3 + 8/3*a^3*b*d*e*x^3 + 1/3*a^4*e^2*x^3 + 2*a^3*b*d^2*x^2 + a^4*d*e*x^2 + a^4*d^2*x
```


Mupad [B] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 144, normalized size of antiderivative = 2.22

$$\int (d + ex)^2 (a^2 + 2abx + b^2x^2)^2 dx = x^3 \left(\frac{a^4 e^2}{3} + \frac{8a^3 b d e}{3} + 2a^2 b^2 d^2 \right) + x^5 \left(\frac{6a^2 b^2 e^2}{5} + \frac{8a b^3 d e}{5} + \frac{b^4 d^2}{5} \right) + a^4 d^2 x + \frac{b^4 e^2 x^7}{7} + a^3 d x^2 (a e + 2b d) + \frac{b^3 e x^6 (2a e + b d)}{3} + a b x^4 (a^2 e^2 + 3a b d e + b^2 d^2)$$

input `int((d + e*x)^2*(a^2 + b^2*x^2 + 2*a*b*x)^2,x)`output `x^3*((a^4*e^2)/3 + 2*a^2*b^2*d^2 + (8*a^3*b*d*e)/3) + x^5*((b^4*d^2)/5 + (6*a^2*b^2*e^2)/5 + (8*a*b^3*d*e)/5) + a^4*d^2*x + (b^4*e^2*x^7)/7 + a^3*d*x^2*(a*e + 2*b*d) + (b^3*e*x^6*(2*a*e + b*d))/3 + a*b*x^4*(a^2*e^2 + b^2*d^2 + 3*a*b*d*e)`**Reduce [B] (verification not implemented)**

Time = 0.20 (sec) , antiderivative size = 172, normalized size of antiderivative = 2.65

$$\int (d + ex)^2 (a^2 + 2abx + b^2x^2)^2 dx = \frac{x(15b^4e^2x^6 + 70ab^3e^2x^5 + 35b^4dex^5 + 126a^2b^2e^2x^4 + 168ab^3dex^4 + 21b^4d^2x^4 + 105a^3be^2x^3 + 315a^2b^2dex^3 + 105a^2b^2d^2x^3 + 126a^2b^2de^2x^2 + 105a^2b^2d^2e^2x^2 + 168a^2b^2d^2ex^2 + 70a^2b^2d^2e^2x^2 + 21b^4d^2x^2 + 35b^4d^2ex^2 + 15b^4d^2e^2x^2)}{105}$$

input `int((e*x+d)^2*(b^2*x^2+2*a*b*x+a^2)^2,x)`output `(x*(105*a**4*d**2 + 105*a**4*d*e*x + 35*a**4*e**2*x**2 + 210*a**3*b*d**2*x + 280*a**3*b*d*e*x**2 + 105*a**3*b*e**2*x**3 + 210*a**2*b**2*d**2*x**2 + 315*a**2*b**2*d*e*x**3 + 126*a**2*b**2*e**2*x**4 + 105*a*b**3*d**2*x**3 + 168*a*b**3*d*e*x**4 + 70*a*b**3*e**2*x**5 + 21*b**4*d**2*x**4 + 35*b**4*d*e*x**5 + 15*b**4*e**2*x**6))/105`

3.71 $\int (d + ex) (a^2 + 2abx + b^2x^2)^2 dx$

| | |
|---|-----|
| Optimal result | 581 |
| Mathematica [B] (verified) | 581 |
| Rubi [A] (verified) | 582 |
| Maple [B] (verified) | 583 |
| Fricas [B] (verification not implemented) | 584 |
| Sympy [B] (verification not implemented) | 584 |
| Maxima [B] (verification not implemented) | 585 |
| Giac [B] (verification not implemented) | 585 |
| Mupad [B] (verification not implemented) | 586 |
| Reduce [B] (verification not implemented) | 586 |

Optimal result

Integrand size = 24, antiderivative size = 38

$$\int (d + ex) (a^2 + 2abx + b^2x^2)^2 dx = \frac{(bd - ae)(a + bx)^5}{5b^2} + \frac{e(a + bx)^6}{6b^2}$$

output `1/5*(-a*e+b*d)*(b*x+a)^5/b^2+1/6*e*(b*x+a)^6/b^2`

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 84 vs. $2(38) = 76$.

Time = 0.01 (sec) , antiderivative size = 84, normalized size of antiderivative = 2.21

$$\begin{aligned} \int (d + ex) (a^2 + 2abx + b^2x^2)^2 dx = & \frac{1}{30}x(15a^4(2d + ex) + 20a^3bx(3d + 2ex) \\ & + 15a^2b^2x^2(4d + 3ex) + 6ab^3x^3(5d + 4ex) \\ & + b^4x^4(6d + 5ex)) \end{aligned}$$

input `Integrate[(d + e*x)*(a^2 + 2*a*b*x + b^2*x^2)^2,x]`

output `(x*(15*a^4*(2*d + e*x) + 20*a^3*b*x*(3*d + 2*e*x) + 15*a^2*b^2*x^2*(4*d + 3*e*x) + 6*a*b^3*x^3*(5*d + 4*e*x) + b^4*x^4*(6*d + 5*e*x)))/30`

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {1098, 27, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\begin{aligned}
 & \int (a^2 + 2abx + b^2x^2)^2 (d + ex) dx \\
 & \quad \downarrow 1098 \\
 & \int \frac{b^4(a + bx)^4(d + ex)dx}{b^4} \\
 & \quad \downarrow 27 \\
 & \int (a + bx)^4(d + ex)dx \\
 & \quad \downarrow 49 \\
 & \int \left(\frac{(a + bx)^4(bd - ae)}{b} + \frac{e(a + bx)^5}{b} \right) dx \\
 & \quad \downarrow 2009 \\
 & \frac{(a + bx)^5(bd - ae)}{5b^2} + \frac{e(a + bx)^6}{6b^2}
 \end{aligned}$$

input `Int[(d + e*x)*(a^2 + 2*a*b*x + b^2*x^2)^2,x]`

output `((b*d - a*e)*(a + b*x)^5)/(5*b^2) + (e*(a + b*x)^6)/(6*b^2)`

Defintions of rubi rules used

rule 27 `Int[(a_)*(F x_), x_Symbol] := Simp[a Int[F x, x], x] /; FreeQ[a, x] && !MatchQ[F x, (b_)*(G x_) /; FreeQ[b, x]]`

rule 49 `Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] := Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] && IGtQ[m, 0] && IGtQ[m + n + 2, 0]`

rule 1098 `Int[((d_.) + (e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] := Simp[1/c^p Int[(d + e*x)^m*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]`

rule 2009 `Int[u_, x_Symbol] := Simp[IntSum[u, x], x] /; SumQ[u]`

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 93 vs. 2(34) = 68.

Time = 0.99 (sec) , antiderivative size = 94, normalized size of antiderivative = 2.47

| method | result |
|--------------|---|
| norman | $\frac{eb^4x^6}{6} + \left(\frac{4}{5}ea b^3 + \frac{1}{5}db^4\right)x^5 + \left(\frac{3}{2}a^2b^2e + ab^3d\right)x^4 + \left(\frac{4}{3}a^3eb + 2a^2db^2\right)x^3 + \left(\frac{1}{2}ea^4 + 2a^3bd\right)x^2 + a^4dx$ |
| default | $\frac{eb^4x^6}{6} + \frac{(4ea b^3 + db^4)x^5}{5} + \frac{(6a^2b^2e + 4ab^3d)x^4}{4} + \frac{(4a^3eb + 6a^2db^2)x^3}{3} + \frac{(ea^4 + 4a^3bd)x^2}{2} + a^4dx$ |
| gospers | $\frac{x(5eb^4x^5 + 24x^4ea b^3 + 6dx^4b^4 + 45x^3a^2b^2e + 30ab^3dx^3 + 40x^2a^3eb + 60a^2b^2dx^2 + 15a^4ex + 60a^3bdx + 30a^4d)}{30}$ |
| risch | $\frac{1}{6}eb^4x^6 + \frac{4}{5}x^5ea b^3 + \frac{1}{5}x^5db^4 + \frac{3}{2}x^4a^2b^2e + x^4ab^3d + \frac{4}{3}a^3be x^3 + 2a^2b^2dx^3 + \frac{1}{2}a^4ex^2 + 2a^3bdx^2 + a^4dx$ |
| parallelrisc | $\frac{1}{6}eb^4x^6 + \frac{4}{5}x^5ea b^3 + \frac{1}{5}x^5db^4 + \frac{3}{2}x^4a^2b^2e + x^4ab^3d + \frac{4}{3}a^3be x^3 + 2a^2b^2dx^3 + \frac{1}{2}a^4ex^2 + 2a^3bdx^2 + a^4dx$ |
| orering | $\frac{x(5eb^4x^5 + 24x^4ea b^3 + 6dx^4b^4 + 45x^3a^2b^2e + 30ab^3dx^3 + 40x^2a^3eb + 60a^2b^2dx^2 + 15a^4ex + 60a^3bdx + 30a^4d)(b^2x^2 + 2abx + a^2)}{30(bx + a)^4}$ |

input `int((e*x+d)*(b^2*x^2+2*a*b*x+a^2)^2,x,method=_RETURNVERBOSE)`

output

```
1/6*e*b^4*x^6+(4/5*e*a*b^3+1/5*d*b^4)*x^5+(3/2*a^2*b^2*e+a*b^3*d)*x^4+(4/3
*a^3*e*b+2*a^2*d*b^2)*x^3+(1/2*e*a^4+2*a^3*b*d)*x^2+a^4*d*x
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 96 vs. $2(34) = 68$.

Time = 0.08 (sec) , antiderivative size = 96, normalized size of antiderivative = 2.53

$$\int (d + ex) (a^2 + 2abx + b^2x^2)^2 dx = \frac{1}{6} b^4 ex^6 + a^4 dx + \frac{1}{5} (b^4 d + 4 ab^3 e) x^5$$

$$+ \frac{1}{2} (2 ab^3 d + 3 a^2 b^2 e) x^4$$

$$+ \frac{2}{3} (3 a^2 b^2 d + 2 a^3 b e) x^3 + \frac{1}{2} (4 a^3 b d + a^4 e) x^2$$

input

```
integrate((e*x+d)*(b^2*x^2+2*a*b*x+a^2)^2,x, algorithm="fricas")
```

output

```
1/6*b^4*e*x^6 + a^4*d*x + 1/5*(b^4*d + 4*a*b^3*e)*x^5 + 1/2*(2*a*b^3*d + 3
*a^2*b^2*e)*x^4 + 2/3*(3*a^2*b^2*d + 2*a^3*b*e)*x^3 + 1/2*(4*a^3*b*d + a^4
*e)*x^2
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 100 vs. $2(32) = 64$.

Time = 0.04 (sec) , antiderivative size = 100, normalized size of antiderivative = 2.63

$$\int (d + ex) (a^2 + 2abx + b^2x^2)^2 dx = a^4 dx + \frac{b^4 ex^6}{6} + x^5 \cdot \left(\frac{4ab^3 e}{5} + \frac{b^4 d}{5} \right)$$

$$+ x^4 \cdot \left(\frac{3a^2 b^2 e}{2} + ab^3 d \right) + x^3$$

$$\cdot \left(\frac{4a^3 b e}{3} + 2a^2 b^2 d \right) + x^2 \left(\frac{a^4 e}{2} + 2a^3 b d \right)$$

input

```
integrate((e*x+d)*(b**2*x**2+2*a*b*x+a**2)**2,x)
```

output

```
a**4*d*x + b**4*e*x**6/6 + x**5*(4*a*b**3*e/5 + b**4*d/5) + x**4*(3*a**2*b
**2*e/2 + a*b**3*d) + x**3*(4*a**3*b*e/3 + 2*a**2*b**2*d) + x**2*(a**4*e/2
+ 2*a**3*b*d)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 96 vs. $2(34) = 68$.

Time = 0.03 (sec) , antiderivative size = 96, normalized size of antiderivative = 2.53

$$\int (d + ex) (a^2 + 2abx + b^2x^2)^2 dx = \frac{1}{6} b^4 ex^6 + a^4 dx + \frac{1}{5} (b^4 d + 4 ab^3 e) x^5$$

$$+ \frac{1}{2} (2 ab^3 d + 3 a^2 b^2 e) x^4$$

$$+ \frac{2}{3} (3 a^2 b^2 d + 2 a^3 b e) x^3 + \frac{1}{2} (4 a^3 b d + a^4 e) x^2$$

input

```
integrate((e*x+d)*(b^2*x^2+2*a*b*x+a^2)^2,x, algorithm="maxima")
```

output

```
1/6*b^4*e*x^6 + a^4*d*x + 1/5*(b^4*d + 4*a*b^3*e)*x^5 + 1/2*(2*a*b^3*d + 3
*a^2*b^2*e)*x^4 + 2/3*(3*a^2*b^2*d + 2*a^3*b*e)*x^3 + 1/2*(4*a^3*b*d + a^4
*e)*x^2
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 97 vs. $2(34) = 68$.

Time = 0.21 (sec) , antiderivative size = 97, normalized size of antiderivative = 2.55

$$\int (d + ex) (a^2 + 2abx + b^2x^2)^2 dx = \frac{1}{6} b^4 ex^6 + \frac{1}{5} b^4 dx^5 + \frac{4}{5} ab^3 ex^5 + ab^3 dx^4 + \frac{3}{2} a^2 b^2 ex^4$$

$$+ 2 a^2 b^2 dx^3 + \frac{4}{3} a^3 b e x^3 + 2 a^3 b d x^2 + \frac{1}{2} a^4 ex^2 + a^4 dx$$

input

```
integrate((e*x+d)*(b^2*x^2+2*a*b*x+a^2)^2,x, algorithm="giac")
```

output

$$\frac{1}{6}b^4ex^6 + \frac{1}{5}b^4d^2x^5 + \frac{4}{5}ab^3ex^5 + ab^3d^2x^4 + \frac{3}{2}a^2b^2ex^4 + 2a^2b^2d^2x^3 + \frac{4}{3}a^3b^2ex^3 + 2a^3b^2d^2x^2 + \frac{1}{2}a^4ex^2 + a^4d^2x$$

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 88, normalized size of antiderivative = 2.32

$$\int (d + ex) (a^2 + 2abx + b^2x^2)^2 dx = x^2 \left(\frac{ea^4}{2} + 2bda^3 \right) + x^5 \left(\frac{db^4}{5} + \frac{4aeb^3}{5} \right) + \frac{b^4ex^6}{6} + a^4dx + \frac{2a^2bx^3(2ae + 3bd)}{3} + \frac{ab^2x^4(3ae + 2bd)}{2}$$

input

$$\text{int}((d + e*x)*(a^2 + b^2*x^2 + 2*a*b*x)^2, x)$$

output

$$x^2*((a^4e)/2 + 2*a^3*b*d) + x^5*((b^4*d)/5 + (4*a*b^3*e)/5) + (b^4*ex^6)/6 + a^4*d*x + (2*a^2*b*x^3*(2*a*e + 3*b*d))/3 + (a*b^2*x^4*(3*a*e + 2*b*d))/2$$

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 97, normalized size of antiderivative = 2.55

$$\int (d + ex) (a^2 + 2abx + b^2x^2)^2 dx = \frac{x(5b^4ex^5 + 24ab^3ex^4 + 6b^4dx^4 + 45a^2b^2ex^3 + 30ab^3dx^3 + 40a^3bex^2 + 60a^2b^2dx^2 + 15a^4ex + 60a^3bd)}{30}$$

input

$$\text{int}((e*x+d)*(b^2*x^2+2*a*b*x+a^2)^2, x)$$

output

$$(x*(30*a**4*d + 15*a**4*e*x + 60*a**3*b*d*x + 40*a**3*b*e*x**2 + 60*a**2*b**2*d*x**2 + 45*a**2*b**2*e*x**3 + 30*a*b**3*d*x**3 + 24*a*b**3*e*x**4 + 6*b**4*d*x**4 + 5*b**4*e*x**5))/30$$

3.72 $\int (a^2 + 2abx + b^2x^2)^2 dx$

| | |
|---|-----|
| Optimal result | 587 |
| Mathematica [A] (verified) | 587 |
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Optimal result

Integrand size = 18, antiderivative size = 14

$$\int (a^2 + 2abx + b^2x^2)^2 dx = \frac{(a + bx)^5}{5b}$$

output

```
1/5*(b*x+a)^5/b
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00

$$\int (a^2 + 2abx + b^2x^2)^2 dx = \frac{(a + bx)^5}{5b}$$

input

```
Integrate[(a^2 + 2*a*b*x + b^2*x^2)^2,x]
```

output

```
(a + b*x)^5/(5*b)
```


Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {1077, 17}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a^2 + 2abx + b^2x^2)^2 dx$$

$$\downarrow 1077$$

$$\frac{\int (xb^2 + ab)^4 dx}{b^4}$$

$$\downarrow 17$$

$$\frac{(a + bx)^5}{5b}$$

input `Int[(a^2 + 2*a*b*x + b^2*x^2)^2,x]`

output `(a + b*x)^5/(5*b)`

Defintions of rubi rules used

rule 17 `Int[(c_.)*((a_.) + (b_.)*(x_)^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1))/(b*(m + 1)), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]`

rule 1077 `Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[1/c^p Int[(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]`

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 42 vs. $2(12) = 24$.

Time = 0.84 (sec) , antiderivative size = 43, normalized size of antiderivative = 3.07

| method | result | size |
|-------------|--|------|
| default | $\frac{1}{5}b^4x^5 + ab^3x^4 + 2a^2b^2x^3 + 2a^3bx^2 + a^4x$ | 43 |
| norman | $\frac{1}{5}b^4x^5 + ab^3x^4 + 2a^2b^2x^3 + 2a^3bx^2 + a^4x$ | 43 |
| risch | $\frac{1}{5}b^4x^5 + ab^3x^4 + 2a^2b^2x^3 + 2a^3bx^2 + a^4x$ | 43 |
| parallelsch | $\frac{1}{5}b^4x^5 + ab^3x^4 + 2a^2b^2x^3 + 2a^3bx^2 + a^4x$ | 43 |
| gosper | $\frac{x(b^4x^4 + 5ab^3x^3 + 10a^2b^2x^2 + 10a^3bx + 5a^4)}{5}$ | 44 |
| orering | $\frac{x(b^4x^4 + 5ab^3x^3 + 10a^2b^2x^2 + 10a^3bx + 5a^4)(b^2x^2 + 2abx + a^2)^2}{5(bx+a)^4}$ | 69 |

input `int((b^2*x^2+2*a*b*x+a^2)^2,x,method=_RETURNVERBOSE)`

output `1/5*b^4*x^5+a*b^3*x^4+2*a^2*b^2*x^3+2*a^3*b*x^2+a^4*x`

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 42 vs. $2(12) = 24$.

Time = 0.07 (sec) , antiderivative size = 42, normalized size of antiderivative = 3.00

$$\int (a^2 + 2abx + b^2x^2)^2 dx = \frac{1}{5}b^4x^5 + ab^3x^4 + 2a^2b^2x^3 + 2a^3bx^2 + a^4x$$

input `integrate((b^2*x^2+2*a*b*x+a^2)^2,x, algorithm="fricas")`

output `1/5*b^4*x^5 + a*b^3*x^4 + 2*a^2*b^2*x^3 + 2*a^3*b*x^2 + a^4*x`

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 42 vs. $2(8) = 16$.

Time = 0.03 (sec) , antiderivative size = 42, normalized size of antiderivative = 3.00

$$\int (a^2 + 2abx + b^2x^2)^2 dx = a^4x + 2a^3bx^2 + 2a^2b^2x^3 + ab^3x^4 + \frac{b^4x^5}{5}$$

input `integrate((b**2*x**2+2*a*b*x+a**2)**2,x)`

output `a**4*x + 2*a**3*b*x**2 + 2*a**2*b**2*x**3 + a*b**3*x**4 + b**4*x**5/5`

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 53 vs. $2(12) = 24$.

Time = 0.04 (sec) , antiderivative size = 53, normalized size of antiderivative = 3.79

$$\int (a^2 + 2abx + b^2x^2)^2 dx = \frac{1}{5}b^4x^5 + ab^3x^4 + \frac{4}{3}a^2b^2x^3 + a^4x + \frac{2}{3}(b^2x^3 + 3abx^2)a^2$$

input `integrate((b^2*x^2+2*a*b*x+a^2)^2,x, algorithm="maxima")`

output `1/5*b^4*x^5 + a*b^3*x^4 + 4/3*a^2*b^2*x^3 + a^4*x + 2/3*(b^2*x^3 + 3*a*b*x^2)*a^2`

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 42 vs. $2(12) = 24$.

Time = 0.20 (sec) , antiderivative size = 42, normalized size of antiderivative = 3.00

$$\int (a^2 + 2abx + b^2x^2)^2 dx = \frac{1}{5}b^4x^5 + ab^3x^4 + 2a^2b^2x^3 + 2a^3bx^2 + a^4x$$

input `integrate((b^2*x^2+2*a*b*x+a^2)^2,x, algorithm="giac")`

output $1/5*b^4*x^5 + a*b^3*x^4 + 2*a^2*b^2*x^3 + 2*a^3*b*x^2 + a^4*x$

Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 42, normalized size of antiderivative = 3.00

$$\int (a^2 + 2abx + b^2x^2)^2 dx = a^4x + 2a^3bx^2 + 2a^2b^2x^3 + ab^3x^4 + \frac{b^4x^5}{5}$$

input $\text{int}((a^2 + b^2*x^2 + 2*a*b*x)^2, x)$

output $a^4*x + (b^4*x^5)/5 + 2*a^3*b*x^2 + a*b^3*x^4 + 2*a^2*b^2*x^3$

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 43, normalized size of antiderivative = 3.07

$$\int (a^2 + 2abx + b^2x^2)^2 dx = \frac{x(b^4x^4 + 5ab^3x^3 + 10a^2b^2x^2 + 10a^3bx + 5a^4)}{5}$$

input $\text{int}(b^2*x^2+2*a*b*x+a^2)^2, x)$

output $(x*(5*a**4 + 10*a**3*b*x + 10*a**2*b**2*x**2 + 5*a*b**3*x**3 + b**4*x**4)) / 5$

3.73 $\int \frac{(a^2+2abx+b^2x^2)^2}{d+ex} dx$

| | |
|---|-----|
| Optimal result | 592 |
| Mathematica [A] (verified) | 592 |
| Rubi [A] (verified) | 593 |
| Maple [A] (verified) | 594 |
| Fricas [A] (verification not implemented) | 595 |
| Sympy [A] (verification not implemented) | 595 |
| Maxima [A] (verification not implemented) | 596 |
| Giac [A] (verification not implemented) | 596 |
| Mupad [B] (verification not implemented) | 597 |
| Reduce [B] (verification not implemented) | 597 |

Optimal result

Integrand size = 26, antiderivative size = 98

$$\int \frac{(a^2 + 2abx + b^2x^2)^2}{d + ex} dx = -\frac{b(bd - ae)^3x}{e^4} + \frac{(bd - ae)^2(a + bx)^2}{2e^3} - \frac{(bd - ae)(a + bx)^3}{3e^2} + \frac{(a + bx)^4}{4e} + \frac{(bd - ae)^4 \log(d + ex)}{e^5}$$

output `-b*(-a*e+b*d)^3*x/e^4+1/2*(-a*e+b*d)^2*(b*x+a)^2/e^3-1/3*(-a*e+b*d)*(b*x+a)^3/e^2+1/4*(b*x+a)^4/e+(-a*e+b*d)^4*ln(e*x+d)/e^5`

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 115, normalized size of antiderivative = 1.17

$$\int \frac{(a^2 + 2abx + b^2x^2)^2}{d + ex} dx = \frac{bex(48a^3e^3 + 36a^2be^2(-2d + ex) + 8ab^2e(6d^2 - 3dex + 2e^2x^2) + b^3(-12d^3 + 6d^2ex - 4de^2x^2 + 3e^3x^3))}{12e^5}$$

input `Integrate[(a^2 + 2*a*b*x + b^2*x^2)^2/(d + e*x),x]`

output

$$(b*e*x*(48*a^3*e^3 + 36*a^2*b*e^2*(-2*d + e*x) + 8*a*b^2*e*(6*d^2 - 3*d*e*x + 2*e^2*x^2) + b^3*(-12*d^3 + 6*d^2*e*x - 4*d*e^2*x^2 + 3*e^3*x^3)) + 12*(b*d - a*e)^4*Log[d + e*x])/(12*e^5)$$

Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 98, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {1098, 27, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a^2 + 2abx + b^2x^2)^2}{d + ex} dx$$

↓ 1098

$$\int \frac{b^4(a+bx)^4}{d+ex} dx$$

↓ 27

$$\int \frac{(a + bx)^4}{d + ex} dx$$

↓ 49

$$\int \left(\frac{(ae - bd)^4}{e^4(d + ex)} - \frac{b(bd - ae)^3}{e^4} + \frac{b(a + bx)(bd - ae)^2}{e^3} - \frac{b(a + bx)^2(bd - ae)}{e^2} + \frac{b(a + bx)^3}{e} \right) dx$$

↓ 2009

$$\frac{(bd - ae)^4 \log(d + ex)}{e^5} - \frac{bx(bd - ae)^3}{e^4} + \frac{(a + bx)^2(bd - ae)^2}{2e^3} - \frac{(a + bx)^3(bd - ae)}{3e^2} + \frac{(a + bx)^4}{4e}$$

input

$$\text{Int}[(a^2 + 2*a*b*x + b^2*x^2)^2/(d + e*x), x]$$

output

$$-((b*(b*d - a*e)^3*x)/e^4) + ((b*d - a*e)^2*(a + b*x)^2)/(2*e^3) - ((b*d - a*e)*(a + b*x)^3)/(3*e^2) + (a + b*x)^4/(4*e) + ((b*d - a*e)^4*Log[d + e*x])/e^5$$

Defintions of rubi rules used

rule 27 `Int[(a_)*(F x_), x_Symbol] := Simp[a Int[F x, x], x] /; FreeQ[a, x] && !MatchQ[F x, (b_)*(G x_) /; FreeQ[b, x]]`

rule 49 `Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] := Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] && IGtQ[m, 0] && IGtQ[m + n + 2, 0]`

rule 1098 `Int[((d_.) + (e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] := Simp[1/c^p Int[(d + e*x)^m*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]`

rule 2009 `Int[u_, x_Symbol] := Simp[IntSum[u, x], x] /; SumQ[u]`

Maple [A] (verified)

Time = 1.24 (sec) , antiderivative size = 168, normalized size of antiderivative = 1.71

| method | result |
|--------------|---|
| norman | $\frac{b(4e^3a^3 - 6a^2bde^2 + 4ab^2d^2e - b^3d^3)x}{e^4} + \frac{b^4x^4}{4e} + \frac{b^2(6e^2a^2 - 4abde + b^2d^2)x^2}{2e^3} + \frac{b^3(4ae - bd)x^3}{3e^2} + \frac{(a^4e^4 - 4a^3bde^3 + 6a^2b^2d^2e^2 - 4ab^3d^3e + b^4d^4)x}{12e^5}$ |
| default | $b \left(\frac{b^3x^4e^3}{4} + \frac{(2ae - bd)b^2e^2 + 2b^2e^3a}{3}x^3 + \frac{(2(2ae - bd)ab e^2 + be(2e^2a^2 - 2abde + b^2d^2))x^2}{2} + (2ae - bd)(2e^2a^2 - 2abde + b^2d^2)x \right) \frac{1}{e^4} + \frac{\ln(ex + d)}{e}$ |
| risch | $\frac{b^4x^4}{4e} + \frac{4b^3x^3a}{3e} - \frac{b^4x^3d}{3e^2} + \frac{3b^2x^2a^2}{e} - \frac{2b^3x^2ad}{e^2} + \frac{b^4x^2d^2}{2e^3} + \frac{4ba^3x}{e} - \frac{6b^2a^2dx}{e^2} + \frac{4b^3ad^2x}{e^3} - \frac{b^4d^3x}{e^4} + \frac{\ln(ex + d)}{e}$ |
| parallelrisc | $\frac{3b^4x^4e^4 + 16x^3ab^3e^4 - 4x^3b^4de^3 + 36x^2a^2b^2e^4 - 24x^2ab^3de^3 + 6x^2b^4d^2e^2 + 12\ln(ex + d)a^4e^4 - 48\ln(ex + d)a^3bde^3 + 72\ln(ex + d)a^2b^2d^2e^2 - 48\ln(ex + d)abd^3e + b^4d^4}{12e^5}$ |

input `int((b^2*x^2+2*a*b*x+a^2)^2/(e*x+d), x, method=_RETURNVERBOSE)`

output `b*(4*a^3*e^3-6*a^2*b*d*e^2+4*a*b^2*d^2*e-b^3*d^3)/e^4*x+1/4/e*b^4*x^4+1/2/e^3*b^2*(6*a^2*e^2-4*a*b*d*e+b^2*d^2)*x^2+1/3/e^2*b^3*(4*a*e-b*d)*x^3+(a^4*e^4-4*a^3*b*d*e^3+6*a^2*b^2*d^2*e^2-4*a*b^3*d^3*e+b^4*d^4)/e^5*ln(e*x+d)`

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 179, normalized size of antiderivative = 1.83

$$\int \frac{(a^2 + 2abx + b^2x^2)^2}{d + ex} dx$$

$$= \frac{3b^4e^4x^4 - 4(b^4de^3 - 4ab^3e^4)x^3 + 6(b^4d^2e^2 - 4ab^3de^3 + 6a^2b^2e^4)x^2 - 12(b^4d^3e - 4ab^3d^2e^2 + 6a^2b^2de^3) - 12e^5 \log(dx + e)}{12e^5}$$

input `integrate((b^2*x^2+2*a*b*x+a^2)^2/(e*x+d),x, algorithm="fricas")`output `1/12*(3*b^4*e^4*x^4 - 4*(b^4*d*e^3 - 4*a*b^3*e^4)*x^3 + 6*(b^4*d^2*e^2 - 4*a*b^3*d*e^3 + 6*a^2*b^2*e^4)*x^2 - 12*(b^4*d^3*e - 4*a*b^3*d^2*e^2 + 6*a^2*b^2*d*e^3 - 4*a^3*b*e^4)*x + 12*(b^4*d^4 - 4*a*b^3*d^3*e + 6*a^2*b^2*d^2*e^2 - 4*a^3*b*d*e^3 + a^4*e^4)*log(e*x + d))/e^5`**Sympy [A] (verification not implemented)**

Time = 0.29 (sec) , antiderivative size = 136, normalized size of antiderivative = 1.39

$$\int \frac{(a^2 + 2abx + b^2x^2)^2}{d + ex} dx = \frac{b^4x^4}{4e} + x^3 \cdot \left(\frac{4ab^3}{3e} - \frac{b^4d}{3e^2} \right) + x^2 \cdot \left(\frac{3a^2b^2}{e} - \frac{2ab^3d}{e^2} + \frac{b^4d^2}{2e^3} \right)$$

$$+ x \left(\frac{4a^3b}{e} - \frac{6a^2b^2d}{e^2} + \frac{4ab^3d^2}{e^3} - \frac{b^4d^3}{e^4} \right)$$

$$+ \frac{(ae - bd)^4 \log(d + ex)}{e^5}$$

input `integrate((b**2*x**2+2*a*b*x+a**2)**2/(e*x+d),x)`output `b**4*x**4/(4*e) + x**3*(4*a*b**3/(3*e) - b**4*d/(3*e**2)) + x**2*(3*a**2*b**2/e - 2*a*b**3*d/e**2 + b**4*d**2/(2*e**3)) + x*(4*a**3*b/e - 6*a**2*b**2*d/e**2 + 4*a*b**3*d**2/e**3 - b**4*d**3/e**4) + (a*e - b*d)**4*log(d + e*x)/e**5`

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 177, normalized size of antiderivative = 1.81

$$\int \frac{(a^2 + 2abx + b^2x^2)^2}{d + ex} dx$$

$$= \frac{3b^4e^3x^4 - 4(b^4de^2 - 4ab^3e^3)x^3 + 6(b^4d^2e - 4ab^3de^2 + 6a^2b^2e^3)x^2 - 12(b^4d^3 - 4ab^3d^2e + 6a^2b^2de^2 - 4a^3bde^3 + a^4e^4) \log(ex + d)}{e^5}$$

input `integrate((b^2*x^2+2*a*b*x+a^2)^2/(e*x+d),x, algorithm="maxima")`

output `1/12*(3*b^4*e^3*x^4 - 4*(b^4*d*e^2 - 4*a*b^3*e^3)*x^3 + 6*(b^4*d^2*e - 4*a*b^3*d*e^2 + 6*a^2*b^2*e^3)*x^2 - 12*(b^4*d^3 - 4*a*b^3*d^2*e + 6*a^2*b^2*d*e^2 - 4*a^3*b*d*e^3)*x)/e^4 + (b^4*d^4 - 4*a*b^3*d^3*e + 6*a^2*b^2*d^2*e^2 - 4*a^3*b*d*e^3 + a^4*e^4)*log(e*x + d)/e^5`

Giac [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 184, normalized size of antiderivative = 1.88

$$\int \frac{(a^2 + 2abx + b^2x^2)^2}{d + ex} dx$$

$$= \frac{3b^4e^3x^4 - 4b^4de^2x^3 + 16ab^3e^3x^3 + 6b^4d^2ex^2 - 24ab^3de^2x^2 + 36a^2b^2e^3x^2 - 12b^4d^3x + 48ab^3d^2ex - 72a^2b^2d^2e^2x + 48a^3bde^3x}{e^5} + \frac{12e^4}{e^5} (b^4d^4 - 4ab^3d^3e + 6a^2b^2d^2e^2 - 4a^3bde^3 + a^4e^4) \log(|ex + d|)$$

input `integrate((b^2*x^2+2*a*b*x+a^2)^2/(e*x+d),x, algorithm="giac")`

output `1/12*(3*b^4*e^3*x^4 - 4*b^4*d*e^2*x^3 + 16*a*b^3*e^3*x^3 + 6*b^4*d^2*e*x^2 - 24*a*b^3*d*e^2*x^2 + 36*a^2*b^2*e^3*x^2 - 12*b^4*d^3*x + 48*a*b^3*d^2*e*x - 72*a^2*b^2*d^2*e^2*x + 48*a^3*b*d*e^3*x)/e^4 + (b^4*d^4 - 4*a*b^3*d^3*e + 6*a^2*b^2*d^2*e^2 - 4*a^3*b*d*e^3 + a^4*e^4)*log(abs(e*x + d))/e^5`

Mupad [B] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 189, normalized size of antiderivative = 1.93

$$\int \frac{(a^2 + 2abx + b^2x^2)^2}{d + ex} dx$$

$$= x^3 \left(\frac{4ab^3}{3e} - \frac{b^4d}{3e^2} \right) + x \left(\frac{d \left(\frac{d \left(\frac{4ab^3}{e} - \frac{b^4d}{e^2} \right) - 6a^2b^2}{e} \right) + 4a^3b}{e} \right)$$

$$- x^2 \left(\frac{d \left(\frac{4ab^3}{e} - \frac{b^4d}{e^2} \right) - 3a^2b^2}{2e} \right)$$

$$+ \frac{\ln(d + ex) (a^4e^4 - 4a^3bd e^3 + 6a^2b^2d^2e^2 - 4ab^3d^3e + b^4d^4)}{e^5} + \frac{b^4x^4}{4e}$$

input `int((a^2 + b^2*x^2 + 2*a*b*x)^2/(d + e*x), x)`output `x^3*((4*a*b^3)/(3*e) - (b^4*d)/(3*e^2)) + x*((d*((d*((4*a*b^3)/e - (b^4*d)/e^2))/e - (6*a^2*b^2)/e))/e + (4*a^3*b)/e) - x^2*((d*((4*a*b^3)/e - (b^4*d)/e^2))/(2*e) - (3*a^2*b^2)/e) + (log(d + e*x)*(a^4*e^4 + b^4*d^4 + 6*a^2*b^2*d^2*e^2 - 4*a*b^3*d^3*e - 4*a^3*b*d*e^3))/e^5 + (b^4*x^4)/(4*e)`**Reduce [B] (verification not implemented)**

Time = 0.21 (sec) , antiderivative size = 208, normalized size of antiderivative = 2.12

$$\int \frac{(a^2 + 2abx + b^2x^2)^2}{d + ex} dx$$

$$= \frac{12 \log(ex + d) a^4 e^4 - 48 \log(ex + d) a^3 b d e^3 + 72 \log(ex + d) a^2 b^2 d^2 e^2 - 48 \log(ex + d) a b^3 d^3 e + 12 \log(ex + d) b^4 d^4}{e^5} + \frac{b^4 x^4}{4e}$$

input `int((b^2*x^2+2*a*b*x+a^2)^2/(e*x+d), x)`

output

```
(12*log(d + e*x)*a**4*e**4 - 48*log(d + e*x)*a**3*b*d*e**3 + 72*log(d + e*
x)*a**2*b**2*d**2*e**2 - 48*log(d + e*x)*a*b**3*d**3*e + 12*log(d + e*x)*b
**4*d**4 + 48*a**3*b*e**4*x - 72*a**2*b**2*d*e**3*x + 36*a**2*b**2*e**4*x*
*2 + 48*a*b**3*d**2*e**2*x - 24*a*b**3*d*e**3*x**2 + 16*a*b**3*e**4*x**3 -
12*b**4*d**3*e*x + 6*b**4*d**2*e**2*x**2 - 4*b**4*d*e**3*x**3 + 3*b**4*e*
*4*x**4)/(12*e**5)
```

3.74
$$\int \frac{(a^2+2abx+b^2x^2)^2}{(d+ex)^2} dx$$

| | |
|---|-----|
| Optimal result | 599 |
| Mathematica [A] (verified) | 599 |
| Rubi [A] (verified) | 600 |
| Maple [A] (verified) | 601 |
| Fricas [B] (verification not implemented) | 602 |
| Sympy [A] (verification not implemented) | 602 |
| Maxima [A] (verification not implemented) | 603 |
| Giac [B] (verification not implemented) | 603 |
| Mupad [B] (verification not implemented) | 604 |
| Reduce [B] (verification not implemented) | 605 |

Optimal result

Integrand size = 26, antiderivative size = 104

$$\int \frac{(a^2 + 2abx + b^2x^2)^2}{(d + ex)^2} dx = \frac{6b^2(bd - ae)^2x}{e^4} - \frac{(bd - ae)^4}{e^5(d + ex)} - \frac{2b^3(bd - ae)(d + ex)^2}{e^5} + \frac{b^4(d + ex)^3}{3e^5} - \frac{4b(bd - ae)^3 \log(d + ex)}{e^5}$$

output

```
6*b^2*(-a*e+b*d)^2*x/e^4-(-a*e+b*d)^4/e^5/(e*x+d)-2*b^3*(-a*e+b*d)*(e*x+d)
^2/e^5+1/3*b^4*(e*x+d)^3/e^5-4*b*(-a*e+b*d)^3*ln(e*x+d)/e^5
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 165, normalized size of antiderivative = 1.59

$$\int \frac{(a^2 + 2abx + b^2x^2)^2}{(d + ex)^2} dx = \frac{12a^3bde^3 - 3a^4e^4 + 18a^2b^2e^2(-d^2 + dex + e^2x^2) + 6ab^3e(2d^3 - 4d^2ex - 3de^2x^2 + e^3x^3) + b^4(-3d^4 + 9d^3ex + 6d^2e^2x^2 - 3de^3x^3 + e^4x^4)}{3e^5(d + ex)}$$

input

```
Integrate[(a^2 + 2*a*b*x + b^2*x^2)^2/(d + e*x)^2,x]
```

output

```
(12*a^3*b*d*e^3 - 3*a^4*e^4 + 18*a^2*b^2*e^2*(-d^2 + d*e*x + e^2*x^2) + 6*
a*b^3*e*(2*d^3 - 4*d^2*e*x - 3*d*e^2*x^2 + e^3*x^3) + b^4*(-3*d^4 + 9*d^3*
e*x + 6*d^2*e^2*x^2 - 2*d*e^3*x^3 + e^4*x^4) - 12*b*(b*d - a*e)^3*(d + e*x
)*Log[d + e*x])/(3*e^5*(d + e*x))
```

Rubi [A] (verified)

Time = 0.51 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {1098, 27, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a^2 + 2abx + b^2x^2)^2}{(d + ex)^2} dx$$

$$\downarrow 1098$$

$$\int \frac{b^4(a+bx)^4}{(d+ex)^2} dx$$

$$\downarrow 27$$

$$\int \frac{(a + bx)^4}{(d + ex)^2} dx$$

$$\downarrow 49$$

$$\int \left(-\frac{4b^3(d + ex)(bd - ae)}{e^4} + \frac{6b^2(bd - ae)^2}{e^4} - \frac{4b(bd - ae)^3}{e^4(d + ex)} + \frac{(ae - bd)^4}{e^4(d + ex)^2} + \frac{b^4(d + ex)^2}{e^4} \right) dx$$

$$\downarrow 2009$$

$$-\frac{2b^3(d + ex)^2(bd - ae)}{e^5} + \frac{6b^2x(bd - ae)^2}{e^4} - \frac{(bd - ae)^4}{e^5(d + ex)} - \frac{4b(bd - ae)^3 \log(d + ex)}{e^5} + \frac{b^4(d + ex)^3}{3e^5}$$

input

```
Int[(a^2 + 2*a*b*x + b^2*x^2)^2/(d + e*x)^2,x]
```


output

```
b^2/e^4*(1/3*b^2*e^2*x^3+2*x^2*a*b*e^2-b^2*d*e*x^2+6*a^2*e^2*x-8*a*b*d*e*x
+3*b^2*d^2*x)+4*b/e^5*(a^3*e^3-3*a^2*b*d*e^2+3*a*b^2*d^2*e-b^3*d^3)*ln(e*x
+d)-(a^4*e^4-4*a^3*b*d*e^3+6*a^2*b^2*d^2*e^2-4*a*b^3*d^3*e+b^4*d^4)/e^5/(e
*x+d)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 267 vs. $2(102) = 204$.

Time = 0.09 (sec) , antiderivative size = 267, normalized size of antiderivative = 2.57

$$\int \frac{(a^2 + 2abx + b^2x^2)^2}{(d + ex)^2} dx$$

$$= \frac{b^4e^4x^4 - 3b^4d^4 + 12ab^3d^3e - 18a^2b^2d^2e^2 + 12a^3bde^3 - 3a^4e^4 - 2(b^4de^3 - 3ab^3e^4)x^3 + 6(b^4d^2e^2 - 3ab^3d^2e^2 - 3a^2b^2d^2e^2)x^2 + 6(a^3bde^3 - 3a^2b^2d^2e^2 - a^3b^2d^2e^2)x + 6(a^3bde^3 - 3a^2b^2d^2e^2 - a^3b^2d^2e^2)}{(d + ex)^2}$$

input

```
integrate((b^2*x^2+2*a*b*x+a^2)^2/(e*x+d)^2,x, algorithm="fricas")
```

output

```
1/3*(b^4*e^4*x^4 - 3*b^4*d^4 + 12*a*b^3*d^3*e - 18*a^2*b^2*d^2*e^2 + 12*a^
3*b*d*e^3 - 3*a^4*e^4 - 2*(b^4*d*e^3 - 3*a*b^3*e^4)*x^3 + 6*(b^4*d^2*e^2 -
3*a*b^3*d*e^3 + 3*a^2*b^2*e^4)*x^2 + 3*(3*b^4*d^3*e - 8*a*b^3*d^2*e^2 + 6
*a^2*b^2*d*e^3)*x - 12*(b^4*d^4 - 3*a*b^3*d^3*e + 3*a^2*b^2*d^2*e^2 - a^3*
b*d*e^3 + (b^4*d^3*e - 3*a*b^3*d^2*e^2 + 3*a^2*b^2*d*e^3 - a^3*b*e^4)*x)*1
og(e*x + d))/(e^6*x + d*e^5)
```

Sympy [A] (verification not implemented)

Time = 0.53 (sec) , antiderivative size = 155, normalized size of antiderivative = 1.49

$$\int \frac{(a^2 + 2abx + b^2x^2)^2}{(d + ex)^2} dx = \frac{b^4x^3}{3e^2} + \frac{4b(ae - bd)^3 \log(d + ex)}{e^5} + x^2$$

$$\cdot \left(\frac{2ab^3}{e^2} - \frac{b^4d}{e^3} \right) + x \left(\frac{6a^2b^2}{e^2} - \frac{8ab^3d}{e^3} + \frac{3b^4d^2}{e^4} \right)$$

$$+ \frac{-a^4e^4 + 4a^3bde^3 - 6a^2b^2d^2e^2 + 4ab^3d^3e - b^4d^4}{de^5 + e^6x}$$

input

```
integrate((b**2*x**2+2*a*b*x+a**2)**2/(e*x+d)**2,x)
```

output

```
b**4*x**3/(3*e**2) + 4*b*(a*e - b*d)**3*log(d + e*x)/e**5 + x**2*(2*a*b**3
/e**2 - b**4*d/e**3) + x*(6*a**2*b**2/e**2 - 8*a*b**3*d/e**3 + 3*b**4*d**2
/e**4) + (-a**4*e**4 + 4*a**3*b*d*e**3 - 6*a**2*b**2*d**2*e**2 + 4*a*b**3*
d**3*e - b**4*d**4)/(d*e**5 + e**6*x)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 183, normalized size of antiderivative = 1.76

$$\int \frac{(a^2 + 2abx + b^2x^2)^2}{(d + ex)^2} dx$$

$$= -\frac{b^4d^4 - 4ab^3d^3e + 6a^2b^2d^2e^2 - 4a^3bde^3 + a^4e^4}{e^6x + de^5}$$

$$+ \frac{b^4e^2x^3 - 3(b^4de - 2ab^3e^2)x^2 + 3(3b^4d^2 - 8ab^3de + 6a^2b^2e^2)x}{3e^4}$$

$$- \frac{4(b^4d^3 - 3ab^3d^2e + 3a^2b^2de^2 - a^3be^3) \log(ex + d)}{e^5}$$

input

```
integrate((b^2*x^2+2*a*b*x+a^2)^2/(e*x+d)^2,x, algorithm="maxima")
```

output

```
-(b^4*d^4 - 4*a*b^3*d^3*e + 6*a^2*b^2*d^2*e^2 - 4*a^3*b*d*e^3 + a^4*e^4)/(
e^6*x + d*e^5) + 1/3*(b^4*e^2*x^3 - 3*(b^4*d*e - 2*a*b^3*e^2)*x^2 + 3*(3*b
^4*d^2 - 8*a*b^3*d*e + 6*a^2*b^2*e^2)*x)/e^4 - 4*(b^4*d^3 - 3*a*b^3*d^2*e
+ 3*a^2*b^2*d*e^2 - a^3*b*e^3)*log(e*x + d)/e^5
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 245 vs. 2(102) = 204.

Time = 0.20 (sec) , antiderivative size = 245, normalized size of antiderivative = 2.36

$$\int \frac{(a^2 + 2abx + b^2x^2)^2}{(d + ex)^2} dx = \frac{\left(b^4 - \frac{6(b^4de - ab^3e^2)}{(ex+d)e} + \frac{18(b^4d^2e^2 - 2ab^3de^3 + a^2b^2e^4)}{(ex+d)^2e^2}\right)(ex + d)^3}{3e^5}$$

$$+ \frac{4(b^4d^3 - 3ab^3d^2e + 3a^2b^2de^2 - a^3be^3) \log\left(\frac{|ex+d|}{(ex+d)^2|e|}\right)}{e^5}$$

$$- \frac{\frac{b^4d^4e^3}{ex+d} - \frac{4ab^3d^3e^4}{ex+d} + \frac{6a^2b^2d^2e^5}{ex+d} - \frac{4a^3bde^6}{ex+d} + \frac{a^4e^7}{ex+d}}{e^8}$$

input `integrate((b^2*x^2+2*a*b*x+a^2)^2/(e*x+d)^2,x, algorithm="giac")`

output `1/3*(b^4 - 6*(b^4*d*e - a*b^3*e^2)/((e*x + d)*e) + 18*(b^4*d^2*e^2 - 2*a*b^3*d*e^3 + a^2*b^2*e^4)/((e*x + d)^2*e^2))*(e*x + d)^3/e^5 + 4*(b^4*d^3 - 3*a*b^3*d^2*e + 3*a^2*b^2*d*e^2 - a^3*b*e^3)*log(abs(e*x + d)/((e*x + d)^2*abs(e)))/e^5 - (b^4*d^4*e^3/(e*x + d) - 4*a*b^3*d^3*e^4/(e*x + d) + 6*a^2*b^2*d^2*e^5/(e*x + d) - 4*a^3*b*d*e^6/(e*x + d) + a^4*e^7/(e*x + d))/e^8`

Mupad [B] (verification not implemented)

Time = 8.94 (sec) , antiderivative size = 203, normalized size of antiderivative = 1.95

$$\int \frac{(a^2 + 2abx + b^2x^2)^2}{(d + ex)^2} dx$$

$$= x^2 \left(\frac{2ab^3}{e^2} - \frac{b^4d}{e^3} \right) - x \left(\frac{2d \left(\frac{4ab^3}{e^2} - \frac{2b^4d}{e^3} \right)}{e} - \frac{6a^2b^2}{e^2} + \frac{b^4d^2}{e^4} \right)$$

$$+ \frac{b^4x^3}{3e^2} - \frac{\ln(d + ex) (-4a^3be^3 + 12a^2b^2de^2 - 12ab^3d^2e + 4b^4d^3)}{e^5}$$

$$- \frac{a^4e^4 - 4a^3bde^3 + 6a^2b^2d^2e^2 - 4ab^3d^3e + b^4d^4}{e(xe^5 + de^4)}$$

input `int((a^2 + b^2*x^2 + 2*a*b*x)^2/(d + e*x)^2,x)`

output `x^2*((2*a*b^3)/e^2 - (b^4*d)/e^3) - x*((2*d*((4*a*b^3)/e^2 - (2*b^4*d)/e^3))/e - (6*a^2*b^2)/e^2 + (b^4*d^2)/e^4) + (b^4*x^3)/(3*e^2) - (log(d + e*x)*(4*b^4*d^3 - 4*a^3*b*e^3 + 12*a^2*b^2*d*e^2 - 12*a*b^3*d^2*e))/e^5 - (a^4*e^4 + b^4*d^4 + 6*a^2*b^2*d^2*e^2 - 4*a*b^3*d^3*e - 4*a^3*b*d*e^3)/(e*(d*e^4 + e^5*x))`

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 297, normalized size of antiderivative = 2.86

$$\int \frac{(a^2 + 2abx + b^2x^2)^2}{(d + ex)^2} dx$$

$$= \frac{12 \log(ex + d) a^3 b d^2 e^3 + 12 \log(ex + d) a^3 b d e^4 x - 36 \log(ex + d) a^2 b^2 d^3 e^2 - 36 \log(ex + d) a^2 b^2 d^2 e^3 x + \dots}{(d + ex)^2}$$

input `int((b^2*x^2+2*a*b*x+a^2)^2/(e*x+d)^2,x)`output `(12*log(d + e*x)*a**3*b*d**2*e**3 + 12*log(d + e*x)*a**3*b*d*e**4*x - 36*log(d + e*x)*a**2*b**2*d**3*e**2 - 36*log(d + e*x)*a**2*b**2*d**2*e**3*x + 36*log(d + e*x)*a*b**3*d**4*e + 36*log(d + e*x)*a*b**3*d**3*e**2*x - 12*log(d + e*x)*b**4*d**5 - 12*log(d + e*x)*b**4*d**4*e*x + 3*a**4*e**5*x - 12*a**3*b*d*e**4*x + 36*a**2*b**2*d**2*e**3*x + 18*a**2*b**2*d*e**4*x**2 - 36*a*b**3*d**3*e**2*x - 18*a*b**3*d**2*e**3*x**2 + 6*a*b**3*d*e**4*x**3 + 12*b**4*d**4*e*x + 6*b**4*d**3*e**2*x**2 - 2*b**4*d**2*e**3*x**3 + b**4*d*e**4*x**4)/(3*d*e**5*(d + e*x))`

3.75 $\int \frac{(a^2+2abx+b^2x^2)^2}{(d+ex)^3} dx$

| | |
|---|-----|
| Optimal result | 606 |
| Mathematica [A] (verified) | 606 |
| Rubi [A] (verified) | 607 |
| Maple [A] (verified) | 608 |
| Fricas [B] (verification not implemented) | 609 |
| Sympy [A] (verification not implemented) | 609 |
| Maxima [A] (verification not implemented) | 610 |
| Giac [A] (verification not implemented) | 610 |
| Mupad [B] (verification not implemented) | 611 |
| Reduce [B] (verification not implemented) | 611 |

Optimal result

Integrand size = 26, antiderivative size = 103

$$\int \frac{(a^2 + 2abx + b^2x^2)^2}{(d + ex)^3} dx = -\frac{b^3(3bd - 4ae)x}{e^4} + \frac{b^4x^2}{2e^3} - \frac{(bd - ae)^4}{2e^5(d + ex)^2} + \frac{4b(bd - ae)^3}{e^5(d + ex)} + \frac{6b^2(bd - ae)^2 \log(d + ex)}{e^5}$$

output `-b^3*(-4*a*e+3*b*d)*x/e^4+1/2*b^4*x^2/e^3-1/2*(-a*e+b*d)^4/e^5/(e*x+d)^2+4*b*(-a*e+b*d)^3/e^5/(e*x+d)+6*b^2*(-a*e+b*d)^2*ln(e*x+d)/e^5`

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 167, normalized size of antiderivative = 1.62

$$\int \frac{(a^2 + 2abx + b^2x^2)^2}{(d + ex)^3} dx = \frac{-a^4e^4 - 4a^3be^3(d + 2ex) + 6a^2b^2de^2(3d + 4ex) + 4ab^3e(-5d^3 - 4d^2ex + 4de^2x^2 + 2e^3x^3) + b^4(7d^4 + 20d^3ex - 15d^2e^2x^2 - 4de^3x^3 + e^4x^4)}{2e^5(d + ex)^2}$$

input `Integrate[(a^2 + 2*a*b*x + b^2*x^2)^2/(d + e*x)^3,x]`

output

$$\begin{aligned} & (-a^4e^4) - 4a^3b^3e^3(d + 2ex) + 6a^2b^2d^2e^2(3d + 4ex) + 4ab^3e^3(-5d^3 - 4d^2ex + 4de^2x^2 + 2e^3x^3) + b^4(7d^4 + 2d^3ex - 11d^2e^2x^2 - 4de^3x^3 + e^4x^4) + 12b^2(bd - ae)^2(d + ex)^2 \text{Log}[d + ex] / (2e^5(d + ex)^2) \end{aligned}$$

Rubi [A] (verified)

Time = 0.48 (sec) , antiderivative size = 103, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {1098, 27, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\begin{aligned} & \int \frac{(a^2 + 2abx + b^2x^2)^2}{(d + ex)^3} dx \\ & \quad \downarrow \text{1098} \\ & \int \frac{b^4(a+bx)^4}{(d+ex)^3} dx \\ & \quad \downarrow \text{27} \\ & \int \frac{(a + bx)^4}{(d + ex)^3} dx \\ & \quad \downarrow \text{49} \\ & \int \left(-\frac{b^3(3bd - 4ae)}{e^4} + \frac{6b^2(bd - ae)^2}{e^4(d + ex)} - \frac{4b(bd - ae)^3}{e^4(d + ex)^2} + \frac{(ae - bd)^4}{e^4(d + ex)^3} + \frac{b^4x}{e^3} \right) dx \\ & \quad \downarrow \text{2009} \\ & -\frac{b^3x(3bd - 4ae)}{e^4} + \frac{6b^2(bd - ae)^2 \log(d + ex)}{e^5} + \frac{4b(bd - ae)^3}{e^5(d + ex)} - \frac{(bd - ae)^4}{2e^5(d + ex)^2} + \frac{b^4x^2}{2e^3} \end{aligned}$$

input

$$\text{Int}[(a^2 + 2a*b*x + b^2*x^2)^2/(d + e*x)^3, x]$$

output

$$-\frac{(b^3(3bd - 4ae)x)/e^4 + (b^4x^2)/(2e^3) - (bd - ae)^4/(2e^5(d + ex)^2) + (4b(bd - ae)^3)/(e^5(d + ex)) + (6b^2(bd - ae)^2 \text{Log}[d + ex])/e^5}$$

Defintions of rubi rules used

rule 27

$$\text{Int}[(a_*)(Fx_), x_Symbol] \rightarrow \text{Simp}[a \text{ Int}[Fx, x], x] /; \text{FreeQ}[a, x] \ \&\& \ !\text{MatchQ}[Fx, (b_*)(Gx_)] /; \text{FreeQ}[b, x]$$

rule 49

$$\text{Int}[((a_.) + (b_.)(x_))^{(m_.)}*((c_.) + (d_.)(x_))^{(n_.)}, x_Symbol] \rightarrow \text{Int}[\text{ExpandIntegrand}[(a + b*x)^m*(c + d*x)^n, x], x] /; \text{FreeQ}\{a, b, c, d\}, x \ \&\& \ \text{IGtQ}[m, 0] \ \&\& \ \text{IGtQ}[m + n + 2, 0]$$

rule 1098

$$\text{Int}(((d_.) + (e_.)(x_))^{(m_.)}*((a_) + (b_.)(x_) + (c_.)(x_)^2)^{(p_.)}, x_Symbol] \rightarrow \text{Simp}[1/c^p \text{ Int}[(d + ex)^m*(b/2 + c*x)^(2*p), x], x] /; \text{FreeQ}\{a, b, c, d, e, m\}, x \ \&\& \ \text{EqQ}[b^2 - 4*a*c, 0] \ \&\& \ \text{IntegerQ}[p]$$

rule 2009

$$\text{Int}[u_, x_Symbol] \rightarrow \text{Simp}[\text{IntSum}[u, x], x] /; \text{SumQ}[u]$$

Maple [A] (verified)

Time = 1.13 (sec) , antiderivative size = 172, normalized size of antiderivative = 1.67

| method | result |
|--------------|---|
| default | $\frac{b^3(\frac{1}{2}be x^2+4aex-3bdx)}{e^4} + \frac{6b^2(e^2a^2-2abde+b^2d^2) \ln(ex+d)}{e^5} - \frac{4b(e^3a^3-3a^2bde^2+3ab^2d^2e-b^3d^3)}{e^5(ex+d)} - \frac{a^4e^4-4a^3bde^3}{e^5(ex+d)}$ |
| norman | $\frac{-a^4e^4+4a^3bde^3-18a^2b^2d^2e^2+36ab^3d^3e-18b^4d^4}{2e^5} + \frac{b^4x^4}{2e} - \frac{2(2a^3be^3-6a^2b^2de^2+12d^2ea b^3-6b^4d^3)x}{e^4} + \frac{2b^3(2ae-bd)x^3}{e^2} + \frac{6b^2(e^2a^2-2abde+b^2d^2) \ln(ex+d)}{(ex+d)^2}$ |
| risch | $\frac{b^4x^2}{2e^3} + \frac{4b^3ax}{e^3} - \frac{3b^4dx}{e^4} + \frac{(-4a^3be^3+12a^2b^2de^2-12d^2ea b^3+4b^4d^3)x - a^4e^4+4a^3bde^3-18a^2b^2d^2e^2+20ab^3d^3e-7b^4d^4}{e^4(ex+d)^2} +$ |
| parallelrisc | $\frac{b^4x^4e^4+12 \ln(ex+d)x^2a^2b^2e^4-24 \ln(ex+d)x^2ab^3de^3+12 \ln(ex+d)x^2b^4d^2e^2+8x^3ab^3e^4-4x^3b^4de^3+24 \ln(ex+d)xa^2b^2de^3}{e^4(ex+d)^2}$ |

input

$$\text{int}((b^2*x^2+2*a*b*x+a^2)^2/(e*x+d)^3,x,\text{method}=_RETURNVERBOSE)$$

output

```
b^3/e^4*(1/2*b*e*x^2+4*a*e*x-3*b*d*x)+6*b^2/e^5*(a^2*e^2-2*a*b*d*e+b^2*d^2)*ln(e*x+d)-4*b/e^5*(a^3*e^3-3*a^2*b*d*e^2+3*a*b^2*d^2*e-b^3*d^3)/(e*x+d)-1/2*(a^4*e^4-4*a^3*b*d*e^3+6*a^2*b^2*d^2*e^2-4*a*b^3*d^3*e+b^4*d^4)/e^5/(e*x+d)^2
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 291 vs. $2(99) = 198$.

Time = 0.08 (sec) , antiderivative size = 291, normalized size of antiderivative = 2.83

$$\int \frac{(a^2 + 2abx + b^2x^2)^2}{(d + ex)^3} dx$$

$$= \frac{b^4e^4x^4 + 7b^4d^4 - 20ab^3d^3e + 18a^2b^2d^2e^2 - 4a^3bde^3 - a^4e^4 - 4(b^4de^3 - 2ab^3e^4)x^3 - (11b^4d^2e^2 - 16ab^3d^2e^3)x^2 + 2(b^4d^3e - 8a^2b^3d^2e^2 + 12a^2b^2d^2e^3 - 4a^3b^2de^4)x + 12(b^4d^4 - 2a^2b^3d^3e + a^2b^2d^2e^2 + (b^4d^2e^2 - 2a^2b^3d^2e^3 + a^2b^2e^4)x^2 + 2(b^4d^3e - 2a^2b^3d^2e^2 + a^2b^2d^2e^3)x) \log(ex + d)}{(e^7x^2 + 2de^6x + d^2e^5)}$$

input

```
integrate((b^2*x^2+2*a*b*x+a^2)^2/(e*x+d)^3,x, algorithm="fricas")
```

output

```
1/2*(b^4*e^4*x^4 + 7*b^4*d^4 - 20*a*b^3*d^3*e + 18*a^2*b^2*d^2*e^2 - 4*a^3*b*d*e^3 - a^4*e^4 - 4*(b^4*d^3*e - 2*a*b^3*d^2*e^2 + 12*a^2*b^2*d^2*e^3 - 16*a*b^3*d^2*e^3)*x^2 + 2*(b^4*d^3*e - 8*a*b^3*d^2*e^2 + 12*a^2*b^2*d^2*e^3 - 4*a^3*b^2*d^2*e^4)*x + 12*(b^4*d^4 - 2*a*b^3*d^3*e + a^2*b^2*d^2*e^2 + (b^4*d^2*e^2 - 2*a^2*b^3*d^2*e^3 + a^2*b^2*e^4)*x^2 + 2*(b^4*d^3*e - 2*a*b^3*d^2*e^2 + a^2*b^2*d^2*e^3)*x)*log(e*x + d))/(e^7*x^2 + 2*d*e^6*x + d^2*e^5)
```

Sympy [A] (verification not implemented)

Time = 0.62 (sec) , antiderivative size = 185, normalized size of antiderivative = 1.80

$$\int \frac{(a^2 + 2abx + b^2x^2)^2}{(d + ex)^3} dx = \frac{b^4x^2}{2e^3} + \frac{6b^2(ae - bd)^2 \log(d + ex)}{e^5} + x \left(\frac{4ab^3}{e^3} - \frac{3b^4d}{e^4} \right) + \frac{-a^4e^4 - 4a^3bde^3 + 18a^2b^2d^2e^2 - 20ab^3d^3e + 7b^4d^4 + x(-8a^3be^4 + 24a^2b^2de^3 - 24ab^3d^2e^2 + 8b^4d^3e)}{2d^2e^5 + 4de^6x + 2e^7x^2}$$

input

```
integrate((b**2*x**2+2*a*b*x+a**2)**2/(e*x+d)**3,x)
```

output

```
b**4*x**2/(2*e**3) + 6*b**2*(a*e - b*d)**2*log(d + e*x)/e**5 + x*(4*a*b**3
/e**3 - 3*b**4*d/e**4) + (-a**4*e**4 - 4*a**3*b*d*e**3 + 18*a**2*b**2*d**2
*e**2 - 20*a*b**3*d**3*e + 7*b**4*d**4 + x*(-8*a**3*b*e**4 + 24*a**2*b**2*
d*e**3 - 24*a*b**3*d**2*e**2 + 8*b**4*d**3*e))/(2*d**2*e**5 + 4*d*e**6*x +
2*e**7*x**2)
```

Maxima [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 191, normalized size of antiderivative = 1.85

$$\int \frac{(a^2 + 2abx + b^2x^2)^2}{(d + ex)^3} dx$$

$$= \frac{7b^4d^4 - 20ab^3d^3e + 18a^2b^2d^2e^2 - 4a^3bde^3 - a^4e^4 + 8(b^4d^3e - 3ab^3d^2e^2 + 3a^2b^2de^3 - a^3be^4)x}{2(e^7x^2 + 2de^6x + d^2e^5)} + \frac{b^4ex^2 - 2(3b^4d - 4ab^3e)x}{2e^4} + \frac{6(b^4d^2 - 2ab^3de + a^2b^2e^2) \log(ex + d)}{e^5}$$

input

```
integrate((b^2*x^2+2*a*b*x+a^2)^2/(e*x+d)^3,x, algorithm="maxima")
```

output

```
1/2*(7*b^4*d^4 - 20*a*b^3*d^3*e + 18*a^2*b^2*d^2*e^2 - 4*a^3*b*d*e^3 - a^4
*e^4 + 8*(b^4*d^3*e - 3*a*b^3*d^2*e^2 + 3*a^2*b^2*d*e^3 - a^3*b*e^4)*x)/(e
^7*x^2 + 2*d*e^6*x + d^2*e^5) + 1/2*(b^4*e*x^2 - 2*(3*b^4*d - 4*a*b^3*e)*x
)/e^4 + 6*(b^4*d^2 - 2*a*b^3*d*e + a^2*b^2*e^2)*log(e*x + d)/e^5
```

Giac [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 183, normalized size of antiderivative = 1.78

$$\int \frac{(a^2 + 2abx + b^2x^2)^2}{(d + ex)^3} dx$$

$$= \frac{6(b^4d^2 - 2ab^3de + a^2b^2e^2) \log(|ex + d|)}{e^5} + \frac{b^4e^3x^2 - 6b^4de^2x + 8ab^3e^3x}{2e^6} + \frac{7b^4d^4 - 20ab^3d^3e + 18a^2b^2d^2e^2 - 4a^3bde^3 - a^4e^4 + 8(b^4d^3e - 3ab^3d^2e^2 + 3a^2b^2de^3 - a^3be^4)x}{2(ex + d)^2e^5}$$

input

```
integrate((b^2*x^2+2*a*b*x+a^2)^2/(e*x+d)^3,x, algorithm="giac")
```

output

```
6*(b^4*d^2 - 2*a*b^3*d*e + a^2*b^2*e^2)*log(abs(e*x + d))/e^5 + 1/2*(b^4*e^3*x^2 - 6*b^4*d*e^2*x + 8*a*b^3*e^3*x)/e^6 + 1/2*(7*b^4*d^4 - 20*a*b^3*d^3*e + 18*a^2*b^2*d^2*e^2 - 4*a^3*b*d*e^3 - a^4*e^4 + 8*(b^4*d^3*e - 3*a*b^3*d^2*e^2 + 3*a^2*b^2*d*e^3 - a^3*b*e^4)*x)/((e*x + d)^2*e^5)
```

Mupad [B] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 196, normalized size of antiderivative = 1.90

$$\int \frac{(a^2 + 2abx + b^2x^2)^2}{(d + ex)^3} dx = x \left(\frac{4ab^3}{e^3} - \frac{3b^4d}{e^4} \right) - \frac{\frac{a^4e^4 + 4a^3bde^3 - 18a^2b^2d^2e^2 + 20ab^3d^3e - 7b^4d^4}{2e} - x(-4a^3be^3 + 12a^2b^2de^2 - 12ab^3d^2e + 4b^4d^3)}{d^2e^4 + 2de^5x + e^6x^2} + \frac{b^4x^2}{2e^3} + \frac{\ln(d + ex)(6a^2b^2e^2 - 12ab^3de + 6b^4d^2)}{e^5}$$

input

```
int((a^2 + b^2*x^2 + 2*a*b*x)^2/(d + e*x)^3,x)
```

output

```
x*((4*a*b^3)/e^3 - (3*b^4*d)/e^4) - ((a^4*e^4 - 7*b^4*d^4 - 18*a^2*b^2*d^2*e^2 + 20*a*b^3*d^3*e + 4*a^3*b*d*e^3)/(2*e) - x*(4*b^4*d^3 - 4*a^3*b*e^3 + 12*a^2*b^2*d*e^2 - 12*a*b^3*d^2*e))/(d^2*e^4 + e^6*x^2 + 2*d*e^5*x) + (b^4*x^2)/(2*e^3) + (log(d + e*x)*(6*b^4*d^2 + 6*a^2*b^2*e^2 - 12*a*b^3*d*e))/e^5
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 330, normalized size of antiderivative = 3.20

$$\int \frac{(a^2 + 2abx + b^2x^2)^2}{(d + ex)^3} dx = \frac{12 \log(ex + d) a^2 b^2 d^3 e^2 + 24 \log(ex + d) a^2 b^2 d^2 e^3 x + 12 \log(ex + d) a^2 b^2 d e^4 x^2 - 24 \log(ex + d) a b^3 d^4 e - \dots}{(d + ex)^3}$$

input

```
int((b^2*x^2+2*a*b*x+a^2)^2/(e*x+d)^3,x)
```


output

```
(12*log(d + e*x)*a**2*b**2*d**3*e**2 + 24*log(d + e*x)*a**2*b**2*d**2*e**3
*x + 12*log(d + e*x)*a**2*b**2*d**4*x**2 - 24*log(d + e*x)*a*b**3*d**4*e
- 48*log(d + e*x)*a*b**3*d**3*e**2*x - 24*log(d + e*x)*a*b**3*d**2*e**3*x
**2 + 12*log(d + e*x)*b**4*d**5 + 24*log(d + e*x)*b**4*d**4*e*x + 12*log(d
+ e*x)*b**4*d**3*e**2*x**2 - a**4*d*e**4 + 4*a**3*b*e**5*x**2 + 6*a**2*b*
*2*d**3*e**2 - 12*a**2*b**2*d**4*x**2 - 12*a*b**3*d**4*e + 24*a*b**3*d**
2*e**3*x**2 + 8*a*b**3*d**4*x**3 + 6*b**4*d**5 - 12*b**4*d**3*e**2*x**2
- 4*b**4*d**2*e**3*x**3 + b**4*d**4*x**4)/(2*d**5*(d**2 + 2*d*e*x + e
**2*x**2))
```

3.76 $\int \frac{(a^2+2abx+b^2x^2)^2}{(d+ex)^4} dx$

| | |
|---|-----|
| Optimal result | 613 |
| Mathematica [A] (verified) | 613 |
| Rubi [A] (verified) | 614 |
| Maple [A] (verified) | 615 |
| Fricas [B] (verification not implemented) | 616 |
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| Maxima [A] (verification not implemented) | 617 |
| Giac [A] (verification not implemented) | 617 |
| Mupad [B] (verification not implemented) | 618 |
| Reduce [B] (verification not implemented) | 618 |

Optimal result

Integrand size = 26, antiderivative size = 103

$$\int \frac{(a^2 + 2abx + b^2x^2)^2}{(d + ex)^4} dx = \frac{b^4x}{e^4} - \frac{(bd - ae)^4}{3e^5(d + ex)^3} + \frac{2b(bd - ae)^3}{e^5(d + ex)^2} - \frac{6b^2(bd - ae)^2}{e^5(d + ex)} - \frac{4b^3(bd - ae) \log(d + ex)}{e^5}$$

output `b^4*x/e^4-1/3*(-a*e+b*d)^4/e^5/(e*x+d)^3+2*b*(-a*e+b*d)^3/e^5/(e*x+d)^2-6*b^2*(-a*e+b*d)^2/e^5/(e*x+d)-4*b^3*(-a*e+b*d)*ln(e*x+d)/e^5`

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 163, normalized size of antiderivative = 1.58

$$\int \frac{(a^2 + 2abx + b^2x^2)^2}{(d + ex)^4} dx = \frac{a^4e^4 + 2a^3be^3(d + 3ex) + 6a^2b^2e^2(d^2 + 3dex + 3e^2x^2) - 2ab^3de(11d^2 + 27dex + 18e^2x^2) + b^4(13d^4 + 3e^5(d + ex)^3)}{3e^5(d + ex)^3}$$

input `Integrate[(a^2 + 2*a*b*x + b^2*x^2)^2/(d + e*x)^4,x]`

output

$$-1/3*(a^4*e^4 + 2*a^3*b*e^3*(d + 3*e*x) + 6*a^2*b^2*e^2*(d^2 + 3*d*e*x + 3*e^2*x^2) - 2*a*b^3*d*e*(11*d^2 + 27*d*e*x + 18*e^2*x^2) + b^4*(13*d^4 + 27*d^3*e*x + 9*d^2*e^2*x^2 - 9*d*e^3*x^3 - 3*e^4*x^4) + 12*b^3*(b*d - a*e)*(d + e*x)^3*Log[d + e*x])/(e^5*(d + e*x)^3)$$

Rubi [A] (verified)

Time = 0.48 (sec) , antiderivative size = 103, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {1098, 27, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\begin{aligned} & \int \frac{(a^2 + 2abx + b^2x^2)^2}{(d + ex)^4} dx \\ & \quad \downarrow 1098 \\ & \int \frac{b^4(a+bx)^4}{(d+ex)^4} dx \\ & \quad \downarrow 27 \\ & \int \frac{(a + bx)^4}{(d + ex)^4} dx \\ & \quad \downarrow 49 \\ & \int \left(-\frac{4b^3(bd - ae)}{e^4(d + ex)} + \frac{6b^2(bd - ae)^2}{e^4(d + ex)^2} - \frac{4b(bd - ae)^3}{e^4(d + ex)^3} + \frac{(ae - bd)^4}{e^4(d + ex)^4} + \frac{b^4}{e^4} \right) dx \\ & \quad \downarrow 2009 \\ & -\frac{4b^3(bd - ae) \log(d + ex)}{e^5} - \frac{6b^2(bd - ae)^2}{e^5(d + ex)} + \frac{2b(bd - ae)^3}{e^5(d + ex)^2} - \frac{(bd - ae)^4}{3e^5(d + ex)^3} + \frac{b^4x}{e^4} \end{aligned}$$

input

$$\text{Int}[(a^2 + 2*a*b*x + b^2*x^2)^2/(d + e*x)^4, x]$$

```
output (b^4*x)/e^4 - (b*d - a*e)^4/(3*e^5*(d + e*x)^3) + (2*b*(b*d - a*e)^3)/(e^5*(d + e*x)^2) - (6*b^2*(b*d - a*e)^2)/(e^5*(d + e*x)) - (4*b^3*(b*d - a*e)*Log[d + e*x])/e^5
```

Defintions of rubi rules used

```
rule 27 Int[(a_)*(F x_), x_Symbol] := Simp[a Int[F x, x], x] /; FreeQ[a, x] && !MatchQ[F x, (b_)*(G x_) /; FreeQ[b, x]]
```

```
rule 49 Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] := Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] && IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

```
rule 1098 Int[((d_.) + (e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] := Simp[1/c^p Int[(d + e*x)^m*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

```
rule 2009 Int[u_, x_Symbol] := Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 1.14 (sec) , antiderivative size = 178, normalized size of antiderivative = 1.73

| method | result |
|--------------|---|
| default | $\frac{b^4 x}{e^4} - \frac{a^4 e^4 - 4a^3 b d e^3 + 6a^2 b^2 d^2 e^2 - 4a b^3 d^3 e + b^4 d^4}{3e^5 (ex+d)^3} + \frac{4b^3 (ae-bd) \ln(ex+d)}{e^5} - \frac{6b^2 (e^2 a^2 - 2abde + b^2 d^2)}{e^5 (ex+d)} - \frac{2b (e^3 a^3 - 3a^2 b d e^2 + 3a b^2 d^2 e - b^3 d^3)}{e^5 (ex+d)}$ |
| norman | $\frac{\frac{b^4 x^4}{e} - a^4 e^4 + 2a^3 b d e^3 + 6a^2 b^2 d^2 e^2 - 22a b^3 d^3 e + 22b^4 d^4}{3e^5} - \frac{3(2a^2 b^2 e^2 - 4a b^3 d e + 4d^2 b^4) x^2}{e^3 (ex+d)^3} - \frac{(2a^3 b e^3 + 6a^2 b^2 d e^2 - 18d^2 e a b^3 + 18b^4 d^3) x}{e^4}$ |
| risch | $\frac{b^4 x}{e^4} + \frac{(-6a^2 b^2 e^3 + 12d e^2 a b^3 - 6d^2 e b^4) x^2 - 2b (e^3 a^3 + 3a^2 b d e^2 - 9a b^2 d^2 e + 5b^3 d^3) x - a^4 e^4 + 2a^3 b d e^3 + 6a^2 b^2 d^2 e^2 - 22a b^3 d^3 e + 22b^4 d^4}{e^4 (ex+d)^3}$ |
| parallelrisc | $\frac{-a^4 e^4 + 36 \ln(ex+d) x^2 a b^3 d e^3 + 36 \ln(ex+d) x a b^3 d^2 e^2 - 2a^3 b d e^3 + 22a b^3 d^3 e - 6a^2 b^2 d^2 e^2 - 22b^4 d^4 - 36 \ln(ex+d) x b^4 d^3 e - 36 \ln(ex+d) b^4 d^4}{e^4}$ |

```
input int((b^2*x^2+2*a*b*x+a^2)^2/(e*x+d)^4,x,method=_RETURNVERBOSE)
```

output

```
b^4*x/e^4-1/3*(a^4*e^4-4*a^3*b*d*e^3+6*a^2*b^2*d^2*e^2-4*a*b^3*d^3*e+b^4*d^4)/e^5/(e*x+d)^3+4*b^3/e^5*(a*e-b*d)*ln(e*x+d)-6*b^2/e^5*(a^2*e^2-2*a*b*d*e+b^2*d^2)/(e*x+d)-2*b/e^5*(a^3*e^3-3*a^2*b*d*e^2+3*a*b^2*d^2*e-b^3*d^3)/(e*x+d)^2
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 292 vs. $2(101) = 202$.

Time = 0.09 (sec) , antiderivative size = 292, normalized size of antiderivative = 2.83

$$\int \frac{(a^2 + 2abx + b^2x^2)^2}{(d + ex)^4} dx = \frac{3b^4e^4x^4 + 9b^4de^3x^3 - 13b^4d^4 + 22ab^3d^3e - 6a^2b^2d^2e^2 - 2a^3bde^3 - a^4e^4 - 9(b^4d^2e^2 - 4ab^3de^3 + 2a^2b^2d^2e^2)}{(d + ex)^4}$$

input

```
integrate((b^2*x^2+2*a*b*x+a^2)^2/(e*x+d)^4,x, algorithm="fricas")
```

output

```
1/3*(3*b^4*e^4*x^4 + 9*b^4*d*e^3*x^3 - 13*b^4*d^4 + 22*a*b^3*d^3*e - 6*a^2*b^2*d^2*e^2 - 2*a^3*b*d*e^3 - a^4*e^4 - 9*(b^4*d^2*e^2 - 4*a*b^3*d*e^3 + 2*a^2*b^2*d^2*e^2)*x^2 - 3*(9*b^4*d^3*e - 18*a*b^3*d^2*e^2 + 6*a^2*b^2*d^2*e^3 + 2*a^3*b*d*e^4)*x - 12*(b^4*d^4 - a*b^3*d^3*e + (b^4*d*e^3 - a*b^3*d^2*e^2)*x^3 + 3*(b^4*d^2*e^2 - a*b^3*d^2*e^3)*x^2 + 3*(b^4*d^3*e - a*b^3*d^2*e^2)*x)*log(e*x + d))/(e^8*x^3 + 3*d*e^7*x^2 + 3*d^2*e^6*x + d^3*e^5)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 209 vs. $2(94) = 188$.

Time = 0.95 (sec) , antiderivative size = 209, normalized size of antiderivative = 2.03

$$\int \frac{(a^2 + 2abx + b^2x^2)^2}{(d + ex)^4} dx = \frac{b^4x}{e^4} + \frac{4b^3(ae - bd) \log(d + ex)}{e^5} + \frac{-a^4e^4 - 2a^3bde^3 - 6a^2b^2d^2e^2 + 22ab^3d^3e - 13b^4d^4 + x^2(-18a^2b^2e^4 + 36ab^3de^3 - 18b^4d^2e^2) + x(-6a^3bde^3 + 6a^2b^2d^2e^2) + 3d^2e^6x + d^3e^5}{3d^3e^5 + 9d^2e^6x + 9de^7x^2 + 3e^8x^3}$$

input

```
integrate((b**2*x**2+2*a*b*x+a**2)**2/(e*x+d)**4,x)
```

output

```
b**4*x/e**4 + 4*b**3*(a*e - b*d)*log(d + e*x)/e**5 + (-a**4*e**4 - 2*a**3*
b*d*e**3 - 6*a**2*b**2*d**2*e**2 + 22*a*b**3*d**3*e - 13*b**4*d**4 + x**2*
(-18*a**2*b**2*e**4 + 36*a*b**3*d*e**3 - 18*b**4*d**2*e**2) + x*(-6*a**3*b
*e**4 - 18*a**2*b**2*d*e**3 + 54*a*b**3*d**2*e**2 - 30*b**4*d**3*e))/(3*d*
*3*e**5 + 9*d**2*e**6*x + 9*d*e**7*x**2 + 3*e**8*x**3)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 201, normalized size of antiderivative = 1.95

$$\int \frac{(a^2 + 2abx + b^2x^2)^2}{(d + ex)^4} dx = \frac{b^4x}{e^4} - \frac{13b^4d^4 - 22ab^3d^3e + 6a^2b^2d^2e^2 + 2a^3bde^3 + a^4e^4 + 18(b^4d^2e^2 - 2ab^3de^3 + a^2b^2e^4)x^2 + 6(5b^4d^3e - 3(e^8x^3 + 3de^7x^2 + 3d^2e^6x + d^3e^5))}{3(e^8x^3 + 3de^7x^2 + 3d^2e^6x + d^3e^5)} - \frac{4(b^4d - ab^3e)\log(ex + d)}{e^5}$$

input

```
integrate((b^2*x^2+2*a*b*x+a^2)^2/(e*x+d)^4,x, algorithm="maxima")
```

output

```
b^4*x/e^4 - 1/3*(13*b^4*d^4 - 22*a*b^3*d^3*e + 6*a^2*b^2*d^2*e^2 + 2*a^3*b
*d*e^3 + a^4*e^4 + 18*(b^4*d^2*e^2 - 2*a*b^3*d*e^3 + a^2*b^2*e^4)*x^2 + 6*
(5*b^4*d^3*e - 9*a*b^3*d^2*e^2 + 3*a^2*b^2*d*e^3 + a^3*b*e^4)*x)/(e^8*x^3
+ 3*d*e^7*x^2 + 3*d^2*e^6*x + d^3*e^5) - 4*(b^4*d - a*b^3*e)*log(e*x + d)/
e^5
```

Giac [A] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 177, normalized size of antiderivative = 1.72

$$\int \frac{(a^2 + 2abx + b^2x^2)^2}{(d + ex)^4} dx = \frac{b^4x}{e^4} - \frac{4(b^4d - ab^3e)\log(|ex + d|)}{e^5} - \frac{13b^4d^4 - 22ab^3d^3e + 6a^2b^2d^2e^2 + 2a^3bde^3 + a^4e^4 + 18(b^4d^2e^2 - 2ab^3de^3 + a^2b^2e^4)x^2 + 6(5b^4d^3e - 3(ex + d)^3e^5)}{3(ex + d)^3e^5}$$

input

```
integrate((b^2*x^2+2*a*b*x+a^2)^2/(e*x+d)^4,x, algorithm="giac")
```

output

```
b^4*x/e^4 - 4*(b^4*d - a*b^3*e)*log(abs(e*x + d))/e^5 - 1/3*(13*b^4*d^4 -
22*a*b^3*d^3*e + 6*a^2*b^2*d^2*e^2 + 2*a^3*b*d*e^3 + a^4*e^4 + 18*(b^4*d^2
*e^2 - 2*a*b^3*d*e^3 + a^2*b^2*e^4)*x^2 + 6*(5*b^4*d^3*e - 9*a*b^3*d^2*e^2
+ 3*a^2*b^2*d*e^3 + a^3*b*e^4)*x)/((e*x + d)^3*e^5)
```

Mupad [B] (verification not implemented)

Time = 9.04 (sec) , antiderivative size = 204, normalized size of antiderivative = 1.98

$$\int \frac{(a^2 + 2abx + b^2x^2)^2}{(d + ex)^4} dx = \frac{b^4 x}{e^4} - \frac{\ln(d + ex) (4b^4 d - 4ab^3 e)}{e^5} - \frac{a^4 e^4 + 2a^3 b d e^3 + 6a^2 b^2 d^2 e^2 - 22a b^3 d^3 e + 13b^4 d^4}{3e} + x(2a^3 b e^3 + 6a^2 b^2 d e^2 - 18a b^3 d^2 e + 10b^4 d^3) + x^2(6a^2 b^2 d^2 e^2 - 12a b^3 d^3 e + 6a^2 b^4 d^4) + x^3(3a^3 b^2 d^2 e^2 - 12a^2 b^3 d^3 e + 6a^3 b^4 d^4) + x^4(4a^4 d^3 e^3 - 12a^3 b d^4 e^2 + 6a^4 d^5 e) + x^5(5a^4 d^4 e^4 - 12a^3 b d^5 e^3 + 6a^4 d^6 e^2) + x^6(6a^4 d^5 e^5 - 12a^3 b d^6 e^4 + 6a^4 d^7 e^3) + x^7(7a^4 d^6 e^6 - 12a^3 b d^7 e^5 + 6a^4 d^8 e^4) + x^8(8a^4 d^7 e^7 - 12a^3 b d^8 e^6 + 6a^4 d^9 e^5) + x^9(9a^4 d^8 e^8 - 12a^3 b d^9 e^7 + 6a^4 d^{10} e^6) + x^{10}(10a^4 d^9 e^9 - 12a^3 b d^{10} e^8 + 6a^4 d^{11} e^7) + x^{11}(11a^4 d^{10} e^{10} - 12a^3 b d^{11} e^9 + 6a^4 d^{12} e^8) + x^{12}(12a^4 d^{11} e^{11} - 12a^3 b d^{12} e^{10} + 6a^4 d^{13} e^9) + x^{13}(13a^4 d^{12} e^{12} - 12a^3 b d^{13} e^{11} + 6a^4 d^{14} e^{10}) + x^{14}(14a^4 d^{13} e^{13} - 12a^3 b d^{14} e^{12} + 6a^4 d^{15} e^{11}) + x^{15}(15a^4 d^{14} e^{14} - 12a^3 b d^{15} e^{13} + 6a^4 d^{16} e^{12}) + x^{16}(16a^4 d^{15} e^{15} - 12a^3 b d^{16} e^{14} + 6a^4 d^{17} e^{13}) + x^{17}(17a^4 d^{16} e^{16} - 12a^3 b d^{17} e^{15} + 6a^4 d^{18} e^{14}) + x^{18}(18a^4 d^{17} e^{17} - 12a^3 b d^{18} e^{16} + 6a^4 d^{19} e^{15}) + x^{19}(19a^4 d^{18} e^{18} - 12a^3 b d^{19} e^{17} + 6a^4 d^{20} e^{16}) + x^{20}(20a^4 d^{19} e^{19} - 12a^3 b d^{20} e^{18} + 6a^4 d^{21} e^{17}) + x^{21}(21a^4 d^{20} e^{20} - 12a^3 b d^{21} e^{19} + 6a^4 d^{22} e^{18}) + x^{22}(22a^4 d^{21} e^{21} - 12a^3 b d^{22} e^{20} + 6a^4 d^{23} e^{19}) + x^{23}(23a^4 d^{22} e^{22} - 12a^3 b d^{23} e^{21} + 6a^4 d^{24} e^{20}) + x^{24}(24a^4 d^{23} e^{23} - 12a^3 b d^{24} e^{22} + 6a^4 d^{25} e^{21}) + x^{25}(25a^4 d^{24} e^{24} - 12a^3 b d^{25} e^{23} + 6a^4 d^{26} e^{22}) + x^{26}(26a^4 d^{25} e^{25} - 12a^3 b d^{26} e^{24} + 6a^4 d^{27} e^{23}) + x^{27}(27a^4 d^{26} e^{26} - 12a^3 b d^{27} e^{25} + 6a^4 d^{28} e^{24}) + x^{28}(28a^4 d^{27} e^{27} - 12a^3 b d^{28} e^{26} + 6a^4 d^{29} e^{25}) + x^{29}(29a^4 d^{28} e^{28} - 12a^3 b d^{29} e^{27} + 6a^4 d^{30} e^{26}) + x^{30}(30a^4 d^{29} e^{29} - 12a^3 b d^{30} e^{28} + 6a^4 d^{31} e^{27}) + x^{31}(31a^4 d^{30} e^{30} - 12a^3 b d^{31} e^{29} + 6a^4 d^{32} e^{28}) + x^{32}(32a^4 d^{31} e^{31} - 12a^3 b d^{32} e^{30} + 6a^4 d^{33} e^{29}) + x^{33}(33a^4 d^{32} e^{32} - 12a^3 b d^{33} e^{31} + 6a^4 d^{34} e^{30}) + x^{34}(34a^4 d^{33} e^{33} - 12a^3 b d^{34} e^{32} + 6a^4 d^{35} e^{31}) + x^{35}(35a^4 d^{34} e^{34} - 12a^3 b d^{35} e^{33} + 6a^4 d^{36} e^{32}) + x^{36}(36a^4 d^{35} e^{35} - 12a^3 b d^{36} e^{34} + 6a^4 d^{37} e^{33}) + x^{37}(37a^4 d^{36} e^{36} - 12a^3 b d^{37} e^{35} + 6a^4 d^{38} e^{34}) + x^{38}(38a^4 d^{37} e^{37} - 12a^3 b d^{38} e^{36} + 6a^4 d^{39} e^{35}) + x^{39}(39a^4 d^{38} e^{38} - 12a^3 b d^{39} e^{37} + 6a^4 d^{40} e^{36}) + x^{40}(40a^4 d^{39} e^{39} - 12a^3 b d^{40} e^{38} + 6a^4 d^{41} e^{37}) + x^{41}(41a^4 d^{40} e^{40} - 12a^3 b d^{41} e^{39} + 6a^4 d^{42} e^{38}) + x^{42}(42a^4 d^{41} e^{41} - 12a^3 b d^{42} e^{40} + 6a^4 d^{43} e^{39}) + x^{43}(43a^4 d^{42} e^{42} - 12a^3 b d^{43} e^{41} + 6a^4 d^{44} e^{40}) + x^{44}(44a^4 d^{43} e^{43} - 12a^3 b d^{44} e^{42} + 6a^4 d^{45} e^{41}) + x^{45}(45a^4 d^{44} e^{44} - 12a^3 b d^{45} e^{43} + 6a^4 d^{46} e^{42}) + x^{46}(46a^4 d^{45} e^{45} - 12a^3 b d^{46} e^{44} + 6a^4 d^{47} e^{43}) + x^{47}(47a^4 d^{46} e^{46} - 12a^3 b d^{47} e^{45} + 6a^4 d^{48} e^{44}) + x^{48}(48a^4 d^{47} e^{47} - 12a^3 b d^{48} e^{46} + 6a^4 d^{49} e^{45}) + x^{49}(49a^4 d^{48} e^{48} - 12a^3 b d^{49} e^{47} + 6a^4 d^{50} e^{46}) + x^{50}(50a^4 d^{49} e^{49} - 12a^3 b d^{50} e^{48} + 6a^4 d^{51} e^{47}) + x^{51}(51a^4 d^{50} e^{50} - 12a^3 b d^{51} e^{49} + 6a^4 d^{52} e^{48}) + x^{52}(52a^4 d^{51} e^{51} - 12a^3 b d^{52} e^{50} + 6a^4 d^{53} e^{49}) + x^{53}(53a^4 d^{52} e^{52} - 12a^3 b d^{53} e^{51} + 6a^4 d^{54} e^{50}) + x^{54}(54a^4 d^{53} e^{53} - 12a^3 b d^{54} e^{52} + 6a^4 d^{55} e^{51}) + x^{55}(55a^4 d^{54} e^{54} - 12a^3 b d^{55} e^{53} + 6a^4 d^{56} e^{52}) + x^{56}(56a^4 d^{55} e^{55} - 12a^3 b d^{56} e^{54} + 6a^4 d^{57} e^{53}) + x^{57}(57a^4 d^{56} e^{56} - 12a^3 b d^{57} e^{55} + 6a^4 d^{58} e^{54}) + x^{58}(58a^4 d^{57} e^{57} - 12a^3 b d^{58} e^{56} + 6a^4 d^{59} e^{55}) + x^{59}(59a^4 d^{58} e^{58} - 12a^3 b d^{59} e^{57} + 6a^4 d^{60} e^{56}) + x^{60}(60a^4 d^{59} e^{59} - 12a^3 b d^{60} e^{58} + 6a^4 d^{61} e^{57}) + x^{61}(61a^4 d^{60} e^{60} - 12a^3 b d^{61} e^{59} + 6a^4 d^{62} e^{58}) + x^{62}(62a^4 d^{61} e^{61} - 12a^3 b d^{62} e^{60} + 6a^4 d^{63} e^{59}) + x^{63}(63a^4 d^{62} e^{62} - 12a^3 b d^{63} e^{61} + 6a^4 d^{64} e^{60}) + x^{64}(64a^4 d^{63} e^{63} - 12a^3 b d^{64} e^{62} + 6a^4 d^{65} e^{61}) + x^{65}(65a^4 d^{64} e^{64} - 12a^3 b d^{65} e^{63} + 6a^4 d^{66} e^{62}) + x^{66}(66a^4 d^{65} e^{65} - 12a^3 b d^{66} e^{64} + 6a^4 d^{67} e^{63}) + x^{67}(67a^4 d^{66} e^{66} - 12a^3 b d^{67} e^{65} + 6a^4 d^{68} e^{64}) + x^{68}(68a^4 d^{67} e^{67} - 12a^3 b d^{68} e^{66} + 6a^4 d^{69} e^{65}) + x^{69}(69a^4 d^{68} e^{68} - 12a^3 b d^{69} e^{67} + 6a^4 d^{70} e^{66}) + x^{70}(70a^4 d^{69} e^{69} - 12a^3 b d^{70} e^{68} + 6a^4 d^{71} e^{67}) + x^{71}(71a^4 d^{70} e^{70} - 12a^3 b d^{71} e^{69} + 6a^4 d^{72} e^{68}) + x^{72}(72a^4 d^{71} e^{71} - 12a^3 b d^{72} e^{70} + 6a^4 d^{73} e^{69}) + x^{73}(73a^4 d^{72} e^{72} - 12a^3 b d^{73} e^{71} + 6a^4 d^{74} e^{70}) + x^{74}(74a^4 d^{73} e^{73} - 12a^3 b d^{74} e^{72} + 6a^4 d^{75} e^{71}) + x^{75}(75a^4 d^{74} e^{74} - 12a^3 b d^{75} e^{73} + 6a^4 d^{76} e^{72}) + x^{76}(76a^4 d^{75} e^{75} - 12a^3 b d^{76} e^{74} + 6a^4 d^{77} e^{73}) + x^{77}(77a^4 d^{76} e^{76} - 12a^3 b d^{77} e^{75} + 6a^4 d^{78} e^{74}) + x^{78}(78a^4 d^{77} e^{77} - 12a^3 b d^{78} e^{76} + 6a^4 d^{79} e^{75}) + x^{79}(79a^4 d^{78} e^{78} - 12a^3 b d^{79} e^{77} + 6a^4 d^{80} e^{76}) + x^{80}(80a^4 d^{79} e^{79} - 12a^3 b d^{80} e^{78} + 6a^4 d^{81} e^{77}) + x^{81}(81a^4 d^{80} e^{80} - 12a^3 b d^{81} e^{79} + 6a^4 d^{82} e^{78}) + x^{82}(82a^4 d^{81} e^{81} - 12a^3 b d^{82} e^{80} + 6a^4 d^{83} e^{79}) + x^{83}(83a^4 d^{82} e^{82} - 12a^3 b d^{83} e^{81} + 6a^4 d^{84} e^{80}) + x^{84}(84a^4 d^{83} e^{83} - 12a^3 b d^{84} e^{82} + 6a^4 d^{85} e^{81}) + x^{85}(85a^4 d^{84} e^{84} - 12a^3 b d^{85} e^{83} + 6a^4 d^{86} e^{82}) + x^{86}(86a^4 d^{85} e^{85} - 12a^3 b d^{86} e^{84} + 6a^4 d^{87} e^{83}) + x^{87}(87a^4 d^{86} e^{86} - 12a^3 b d^{87} e^{85} + 6a^4 d^{88} e^{84}) + x^{88}(88a^4 d^{87} e^{87} - 12a^3 b d^{88} e^{86} + 6a^4 d^{89} e^{85}) + x^{89}(89a^4 d^{88} e^{88} - 12a^3 b d^{89} e^{87} + 6a^4 d^{90} e^{86}) + x^{90}(90a^4 d^{89} e^{89} - 12a^3 b d^{90} e^{88} + 6a^4 d^{91} e^{87}) + x^{91}(91a^4 d^{90} e^{90} - 12a^3 b d^{91} e^{89} + 6a^4 d^{92} e^{88}) + x^{92}(92a^4 d^{91} e^{91} - 12a^3 b d^{92} e^{90} + 6a^4 d^{93} e^{89}) + x^{93}(93a^4 d^{92} e^{92} - 12a^3 b d^{93} e^{91} + 6a^4 d^{94} e^{90}) + x^{94}(94a^4 d^{93} e^{93} - 12a^3 b d^{94} e^{92} + 6a^4 d^{95} e^{91}) + x^{95}(95a^4 d^{94} e^{94} - 12a^3 b d^{95} e^{93} + 6a^4 d^{96} e^{92}) + x^{96}(96a^4 d^{95} e^{95} - 12a^3 b d^{96} e^{94} + 6a^4 d^{97} e^{93}) + x^{97}(97a^4 d^{96} e^{96} - 12a^3 b d^{97} e^{95} + 6a^4 d^{98} e^{94}) + x^{98}(98a^4 d^{97} e^{97} - 12a^3 b d^{98} e^{96} + 6a^4 d^{99} e^{95}) + x^{99}(99a^4 d^{98} e^{98} - 12a^3 b d^{99} e^{97} + 6a^4 d^{100} e^{96}) + x^{100}(100a^4 d^{99} e^{99} - 12a^3 b d^{100} e^{98} + 6a^4 d^{101} e^{97}) + x^{101}(101a^4 d^{100} e^{100} - 12a^3 b d^{101} e^{99} + 6a^4 d^{102} e^{98}) + x^{102}(102a^4 d^{101} e^{101} - 12a^3 b d^{102} e^{100} + 6a^4 d^{103} e^{99}) + x^{103}(103a^4 d^{102} e^{102} - 12a^3 b d^{103} e^{101} + 6a^4 d^{104} e^{100}) + x^{104}(104a^4 d^{103} e^{103} - 12a^3 b d^{104} e^{102} + 6a^4 d^{105} e^{101}) + x^{105}(105a^4 d^{104} e^{104} - 12a^3 b d^{105} e^{103} + 6a^4 d^{106} e^{102}) + x^{106}(106a^4 d^{105} e^{105} - 12a^3 b d^{106} e^{104} + 6a^4 d^{107} e^{103}) + x^{107}(107a^4 d^{106} e^{106} - 12a^3 b d^{107} e^{105} + 6a^4 d^{108} e^{104}) + x^{108}(108a^4 d^{107} e^{107} - 12a^3 b d^{108} e^{106} + 6a^4 d^{109} e^{105}) + x^{109}(109a^4 d^{108} e^{108} - 12a^3 b d^{109} e^{107} + 6a^4 d^{110} e^{106}) + x^{110}(110a^4 d^{109} e^{109} - 12a^3 b d^{110} e^{108} + 6a^4 d^{111} e^{107}) + x^{111}(111a^4 d^{110} e^{110} - 12a^3 b d^{111} e^{109} + 6a^4 d^{112} e^{108}) + x^{112}(112a^4 d^{111} e^{111} - 12a^3 b d^{112} e^{110} + 6a^4 d^{113} e^{109}) + x^{113}(113a^4 d^{112} e^{112} - 12a^3 b d^{113} e^{111} + 6a^4 d^{114} e^{110}) + x^{114}(114a^4 d^{113} e^{113} - 12a^3 b d^{114} e^{112} + 6a^4 d^{115} e^{111}) + x^{115}(115a^4 d^{114} e^{114} - 12a^3 b d^{115} e^{113} + 6a^4 d^{116} e^{112}) + x^{116}(116a^4 d^{115} e^{115} - 12a^3 b d^{116} e^{114} + 6a^4 d^{117} e^{113}) + x^{117}(117a^4 d^{116} e^{116} - 12a^3 b d^{117} e^{115} + 6a^4 d^{118} e^{114}) + x^{118}(118a^4 d^{117} e^{117} - 12a^3 b d^{118} e^{116} + 6a^4 d^{119} e^{115}) + x^{119}(119a^4 d^{118} e^{118} - 12a^3 b d^{119} e^{117} + 6a^4 d^{120} e^{116}) + x^{120}(120a^4 d^{119} e^{119} - 12a^3 b d^{120} e^{118} + 6a^4 d^{121} e^{117}) + x^{121}(121a^4 d^{120} e^{120} - 12a^3 b d^{121} e^{119} + 6a^4 d^{122} e^{118}) + x^{122}(122a^4 d^{121} e^{121} - 12a^3 b d^{122} e^{120} + 6a^4 d^{123} e^{119}) + x^{123}(123a^4 d^{122} e^{122} - 12a^3 b d^{123} e^{121} + 6a^4 d^{124} e^{120}) + x^{124}(124a^4 d^{123} e^{123} - 12a^3 b d^{124} e^{122} + 6a^4 d^{125} e^{121}) + x^{125}(125a^4 d^{124} e^{124} - 12a^3 b d^{125} e^{123} + 6a^4 d^{126} e^{122}) + x^{126}(126a^4 d^{125} e^{125} - 12a^3 b d^{126} e^{124} + 6a^4 d^{127} e^{123}) + x^{127}(127a^4 d^{126} e^{126} - 12a^3 b d^{127} e^{125} + 6a^4 d^{128} e^{124}) + x^{128}(128a^4 d^{127} e^{127} - 12a^3 b d^{128} e^{126} + 6a^4 d^{129} e^{125}) + x^{129}(129a^4 d^{128} e^{128} - 12a^3 b d^{129} e^{127} + 6a^4 d^{130} e^{126}) + x^{130}(130a^4 d^{129} e^{129} - 12a^3 b d^{130} e^{128} + 6a^4 d^{131} e^{127}) + x^{131}(131a^4 d^{130} e^{130} - 12a^3 b d^{131} e^{129} + 6a^4 d^{132} e^{128}) + x^{132}(132a^4 d^{131} e^{131} - 12a^3 b d^{132} e^{130} + 6a^4 d^{133} e^{129}) + x^{133}(133a^4 d^{132} e^{132} - 12a^3 b d^{133} e^{131} + 6a^4 d^{134} e^{130}) + x^{134}(134a^4 d^{133} e^{133} - 12a^3 b d^{134} e^{132} + 6a^4 d^{135} e^{131}) + x^{135}(135a^4 d^{134} e^{134} - 12a^3 b d^{135} e^{133} + 6a^4 d^{136} e^{132}) + x^{136}(136a^4 d^{135} e^{135} - 12a^3 b d^{136} e^{134} + 6a^4 d^{137} e^{133}) + x^{137}(137a^4 d^{136} e^{136} - 12a^3 b d^{137} e^{135} + 6a^4 d^{138} e^{134}) + x^{138}(138a^4 d^{137} e^{137} - 12a^3 b d^{138} e^{136} + 6a^4 d^{139} e^{135}) + x^{139}(139a^4 d^{138} e^{138} - 12a^3 b d^{139} e^{137} + 6a^4 d^{140} e^{136}) + x^{140}(140a^4 d^{139} e^{139} - 12a^3 b d^{140} e^{138} + 6a^4 d^{141} e^{137}) + x^{141}(141a^4 d^{140} e^{140} - 12a^3 b d^{141} e^{139} + 6a^4 d^{142} e^{138}) + x^{142}(142a^4 d^{141} e^{141} - 12a^3 b d^{142} e^{140} + 6a^4 d^{143} e^{139}) + x^{143}(143a^4 d^{142} e^{142} - 12a^3 b d^{143} e^{141} + 6a^4 d^{144} e^{140}) + x^{144}(144a^4 d^{143} e^{143} - 12a^3 b d^{144} e^{142} + 6a^4 d^{145} e^{141}) + x^{145}(145a^4 d^{144} e^{144} - 12a^3 b d^{145} e^{143} + 6a^4 d^{146} e^{142}) + x^{146}(146a^4 d^{145} e^{145} - 12a^3 b d^{146} e^{144} + 6a^4 d^{147} e^{143}) + x^{147}(147a^4 d^{146} e^{146} - 12a^3 b d^{147} e^{145} + 6a^4 d^{148} e^{144}) + x^{148}(148a^4 d^{147} e^{147} - 12a^3 b d^{148} e^{146} + 6a^4 d^{149} e^{145}) + x^{149}(149a^4 d^{148} e^{148} - 12a^3 b d^{149} e^{147} + 6a^4 d^{150} e^{146}) + x^{150}(150a^4 d^{149} e^{149} - 12a^3 b d^{150} e^{148} + 6a^4 d^{151} e^{147}) + x^{151}(151a^4 d^{150} e^{150} - 12a^3 b d^{151} e^{149} + 6a^4 d^{152} e^{148}) + x^{152}(152a^4 d^{151} e^{151} - 12a^3 b d^{152} e^{150} + 6a^4 d^{153} e^{149}) + x^{153}(153a^4 d^{152} e^{152} - 12a^3 b d^{153} e^{151} + 6a^4 d^{154} e^{150}) + x^{154}(154a^4 d^{153} e^{153} - 12a^3 b d^{154} e^{152} + 6a^4 d^{155} e^{151}) + x^{155}(155a^4 d^{154} e^{154} - 12a^3 b d^{155} e^{153} + 6a^4 d^{156} e^{152}) + x^{156}(156a^4 d^{155} e^{155} - 12a^3 b d^{156} e^{154} + 6a^4 d^{157} e^{153}) + x^{157}(157a^4 d^{156} e^{156} - 12a^3 b d^{157} e^{155} + 6a^4 d^{158} e^{154}) + x^{158}(158a^4 d^{157} e^{157} - 12a^3 b d^{158} e^{156} + 6a^4 d^{159} e^{155}) + x^{159}(159a^4 d^{158} e^{158} - 12a^3 b d^{159} e^{157} + 6a^4 d^{160} e^{156}) + x^{160}(160a^4 d^{159} e^{159} - 12a^3 b d^{160} e^{158} + 6a^4 d^{161} e^{157}) + x^{161}(161a^4 d^{160} e^{160} - 12a^3 b d^{161} e^{159} + 6a^4 d^{162} e^{158}) + x^{162}(162a^4 d^{161} e^{161} - 12a^3 b d^{162} e^{160} + 6a^4 d^{163} e^{159}) + x^{163}(163a^4 d^{162} e^{162} - 12a^3 b d^{163} e^{161} + 6a^4 d^{164} e^{160}) + x^{164}(164a^4 d^{163} e^{163} - 12a^3 b d^{164} e^{162} + 6a^4 d^{165} e^{161}) + x^{165}(165a^4 d^{164} e^{164} - 12a^3 b d^{165} e^{163} + 6a^4 d^{166} e^{162}) + x^{166}(166a^4 d^{165} e^{165} - 12a^3 b d^{166} e^{164} + 6a^4 d^{167} e^{163}) + x^{167}(167a^4 d^{166} e^{166} - 12a^3 b d^{167} e^{165} + 6a^4 d^{168} e^{164}) + x^{168}(168a^4 d^{167} e^{167} - 12a^3 b d^{168} e^{166} + 6a^4 d^{169} e^{165}) + x^{169}(169a^4 d^{168} e^{168} - 12a^3 b d^{169} e^{167} + 6a^4 d^{170} e^{166}) + x^{170}(170a^4 d^{169} e^{169} - 12a^3 b d^{170} e^{168} + 6a^4 d^{171} e^{167}) + x^{171}(171a^4 d^{170} e^{170} - 12a^3 b d^{171} e^{169} + 6a^4 d^{172} e^{168}) + x^{172}(172a^4 d^{171} e^{171} - 12a^3 b d^{172} e^{170} + 6a^4 d^{173} e^{169}) + x^{173}(173a^4 d^{172} e^{172} - 12a^3 b d^{173} e^{171} + 6a^4 d^{174} e^{170}) + x^{174}(174a^4 d^{173} e^{173} - 12a^3 b d^{174} e^{172} + 6a^4 d^{175} e^{171}) + x^{175}(175a^4 d^{174} e^{174} - 12a^3 b d^{175} e^{173} + 6a^4 d^{176} e^{172}) + x^{176}(176a^4 d^{175} e^{175} - 12a^3 b d^{176} e^{174} + 6a^4 d^{177} e^{173}) + x^{177}(177a^4 d^{176} e^{176} - 12a^3 b d^{177} e^{175} + 6a^4 d^{178} e^{174}) + x^{178}(178a^4 d^{177} e^{177} - 12a^3 b d^{178} e^{176} + 6a^4 d^{179} e^{175}) + x^{179}(179a^4 d^{178} e^{178} - 12a^3 b d^{179} e^{177} + 6a^4 d^{180} e^{176}) + x^{180}(180a^4 d^{179} e^{179} - 12a^3 b d^{180} e^{178} + 6a^4 d^{181} e^{177}) + x^{181}(181a^4 d^{180} e^{180} - 12a^3 b d^{181} e^{179} + 6a^4 d^{182} e^{178}) + x^{182}(182a^4 d^{181} e^{181} - 12a^3 b d^{182} e^{180} + 6a^4 d^{183} e^{179}) + x^{183}(183a^4 d^{182} e^{182} - 12a^3 b d^{183} e^{181} + 6a^4 d^{184} e^{180}) + x^{184}(184a^4 d^{183} e^{183} - 12a^3 b d^{184} e^{182} + 6a^4 d^{185} e^{181}) + x^{185}(185a^4 d^{184} e^{184} - 12a^3 b d^{185} e^{183} + 6a^4 d^{186} e^{182}) + x^{186}(186a^4 d^{185} e^{185} - 12a^3 b d^{186} e^{184} + 6a^4 d^{187} e^{183}) + x^{187}(187a^4 d^{186} e^{186} - 12a^3 b d^{187} e^{185} + 6a^4 d^{188} e^{184}) + x^{188}(188a^4 d^{187} e^{187} - 12a^3 b d^{188} e^{186} + 6a^4 d^{189} e^{185}) + x^{189}(189a^4 d^{188} e^{188} - 12a^3 b d^{189} e^{187} + 6a^4 d^{190} e^{186}) + x^{190}(190a^4 d^{189} e^{189} - 12a^3 b d^{190} e^{188} + 6a^4 d^{191} e^{187}) + x^{191}(191a^4 d^{190} e^{190} - 12a^3 b d^{191} e^{189} + 6a^4 d^{192} e^{188}) + x^{192}(192a^4 d^{191} e^{191} - 12a^3 b d^{192} e^{190} + 6a^4 d^{193} e^{189}) + x^{193}(193a^4 d^{192} e^{192} - 12a^3 b d^{193} e^{191} + 6a^4 d^{194} e^{190}) + x^{194}(194a^4 d^{193} e^{193} - 12a^3 b d^{194} e^{192} + 6a^4 d^{195} e^{191}) + x^{195}(195a^4 d^{194} e^{194} - 12a^3 b d^{195} e^{193} + 6a^4 d^{196} e^{192}) + x^{196}(196a^4 d^{195} e^{195} - 12a^3 b d^{196} e^{194} + 6a^4 d^{197} e^{193}) + x^{197}(197a^4 d^{196} e^{196} - 12a^3 b d^{197} e^{195} + 6a^4 d^{198} e^{194}) + x^{198}(198a^4 d^{197} e^{197} - 12a^3 b d^{198} e^{196} + 6a^4 d^{199} e^{195}) + x^{199}(199a^4 d^{198} e^{198} - 12a^3 b d^{199} e^{197} + 6a^4 d^{200} e^{196}) + x^{200}(200a^4 d^{199} e^{199} - 12a^3 b d^{200} e^{198} + 6a^4 d^{201} e^{197}) + x^{201}(201a^4 d^{200} e^{200} - 12a^3 b d^{201} e^{199} + 6a^4 d^{202} e^{198}) + x^{202}(202a^4 d^{201} e^{201} -$$

output

```
(12*log(d + e*x)*a*b**3*d**4*e + 36*log(d + e*x)*a*b**3*d**3*e**2*x + 36*log(d + e*x)*a*b**3*d**2*e**3*x**2 + 12*log(d + e*x)*a*b**3*d*e**4*x**3 - 12*log(d + e*x)*b**4*d**5 - 36*log(d + e*x)*b**4*d**4*e*x - 36*log(d + e*x)*b**4*d**3*e**2*x**2 - 12*log(d + e*x)*b**4*d**2*e**3*x**3 - a**4*d*e**4 - 2*a**3*b*d**2*e**3 - 6*a**3*b*d*e**4*x + 6*a**2*b**2*e**5*x**3 + 10*a*b**3*d**4*e + 18*a*b**3*d**3*e**2*x - 12*a*b**3*d*e**4*x**3 - 10*b**4*d**5 - 18*b**4*d**4*e*x + 12*b**4*d**2*e**3*x**3 + 3*b**4*d*e**4*x**4)/(3*d*e**5*(d**3 + 3*d**2*e*x + 3*d*e**2*x**2 + e**3*x**3))
```


3.77
$$\int \frac{(a^2 + 2abx + b^2x^2)^2}{(d+ex)^5} dx$$

| | |
|---|-----|
| Optimal result | 620 |
| Mathematica [A] (verified) | 620 |
| Rubi [A] (verified) | 621 |
| Maple [A] (verified) | 622 |
| Fricas [B] (verification not implemented) | 623 |
| Sympy [B] (verification not implemented) | 623 |
| Maxima [B] (verification not implemented) | 624 |
| Giac [B] (verification not implemented) | 624 |
| Mupad [B] (verification not implemented) | 625 |
| Reduce [B] (verification not implemented) | 625 |

Optimal result

Integrand size = 26, antiderivative size = 111

$$\int \frac{(a^2 + 2abx + b^2x^2)^2}{(d + ex)^5} dx = -\frac{(bd - ae)^4}{4e^5(d + ex)^4} + \frac{4b(bd - ae)^3}{3e^5(d + ex)^3} - \frac{3b^2(bd - ae)^2}{e^5(d + ex)^2} + \frac{4b^3(bd - ae)}{e^5(d + ex)} + \frac{b^4 \log(d + ex)}{e^5}$$

output

```
-1/4*(-a*e+b*d)^4/e^5/(e*x+d)^4+4/3*b*(-a*e+b*d)^3/e^5/(e*x+d)^3-3*b^2*(-a
*e+b*d)^2/e^5/(e*x+d)^2+4*b^3*(-a*e+b*d)/e^5/(e*x+d)+b^4*ln(e*x+d)/e^5
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.07

$$\int \frac{(a^2 + 2abx + b^2x^2)^2}{(d + ex)^5} dx = \frac{(bd-ae)(3a^3e^3+a^2be^2(7d+16ex)+ab^2e(13d^2+40dex+36e^2x^2))+b^3(25d^3+88d^2ex+108de^2x^2+48e^3x^3)}{(d+ex)^4} + 12b^4 \log(d + ex)$$

= $\frac{\hspace{10em}}{12e^5}$

input

```
Integrate[(a^2 + 2*a*b*x + b^2*x^2)^2/(d + e*x)^5,x]
```

output

$$\frac{(((b*d - a*e)*(3*a^3*e^3 + a^2*b*e^2*(7*d + 16*e*x) + a*b^2*e*(13*d^2 + 40*d*e*x + 36*e^2*x^2) + b^3*(25*d^3 + 88*d^2*e*x + 108*d*e^2*x^2 + 48*e^3*x^3)))/(d + e*x)^4 + 12*b^4*Log[d + e*x])/(12*e^5)}$$

Rubi [A] (verified)

Time = 0.49 (sec) , antiderivative size = 111, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {1098, 27, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a^2 + 2abx + b^2x^2)^2}{(d + ex)^5} dx$$

↓ 1098

$$\frac{\int \frac{b^4(a+bx)^4}{(d+ex)^5} dx}{b^4}$$

↓ 27

$$\int \frac{(a + bx)^4}{(d + ex)^5} dx$$

↓ 49

$$\int \left(-\frac{4b^3(bd - ae)}{e^4(d + ex)^2} + \frac{6b^2(bd - ae)^2}{e^4(d + ex)^3} - \frac{4b(bd - ae)^3}{e^4(d + ex)^4} + \frac{(ae - bd)^4}{e^4(d + ex)^5} + \frac{b^4}{e^4(d + ex)} \right) dx$$

↓ 2009

$$\frac{4b^3(bd - ae)}{e^5(d + ex)} - \frac{3b^2(bd - ae)^2}{e^5(d + ex)^2} + \frac{4b(bd - ae)^3}{3e^5(d + ex)^3} - \frac{(bd - ae)^4}{4e^5(d + ex)^4} + \frac{b^4 \log(d + ex)}{e^5}$$

input

$$\text{Int}[(a^2 + 2*a*b*x + b^2*x^2)^2/(d + e*x)^5, x]$$

output

$$-1/4*(b*d - a*e)^4/(e^5*(d + e*x)^4) + (4*b*(b*d - a*e)^3)/(3*e^5*(d + e*x)^3) - (3*b^2*(b*d - a*e)^2)/(e^5*(d + e*x)^2) + (4*b^3*(b*d - a*e))/(e^5*(d + e*x)) + (b^4*Log[d + e*x])/e^5$$

Defintions of rubi rules used

rule 27 `Int[(a_)*(Fx_), x_Symbol] := Simp[a Int[Fx, x], x] /; FreeQ[a, x] && !MatchQ[Fx, (b_)*(Gx_)] /; FreeQ[b, x]`

rule 49 `Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] := Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] && IGtQ[m, 0] && IGtQ[m + n + 2, 0]`

rule 1098 `Int[((d_.) + (e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] := Simp[1/c^p Int[(d + e*x)^m*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]`

rule 2009 `Int[u_, x_Symbol] := Simp[IntSum[u, x], x] /; SumQ[u]`

Maple [A] (verified)

Time = 1.08 (sec) , antiderivative size = 176, normalized size of antiderivative = 1.59

| method | result |
|---------------|--|
| risch | $\frac{-\frac{4b^3(ae-bd)x^3}{e^2} - \frac{3b^2(e^2a^2+2abde-3b^2d^2)x^2}{e^3} - \frac{2b(2e^3a^3+3a^2bde^2+6ab^2d^2e-11b^3d^3)x}{3e^4} - \frac{3a^4e^4+4a^3bde^3+6a^2b^2d^2e^2+12ab^3d^3e-12e^5}{12e^5}}{(ex+d)^4}$ |
| norman | $\frac{-\frac{3a^4e^4+4a^3bde^3+6a^2b^2d^2e^2+12ab^3d^3e-25b^4d^4}{12e^5} - \frac{4(eab^3-db^4)x^3}{e^2} - \frac{3(a^2b^2e^2+2ab^3de-3d^2b^4)x^2}{e^3} - \frac{2(2a^3be^3+3a^2b^2de^2+6d^2eab^3)}{3e^4}}{(ex+d)^4}$ |
| default | $-\frac{4b(e^3a^3-3a^2bde^2+3ab^2d^2e-b^3d^3)}{3e^5(ex+d)^3} - \frac{a^4e^4-4a^3bde^3+6a^2b^2d^2e^2-4ab^3d^3e+b^4d^4}{4e^5(ex+d)^4} + \frac{b^4 \ln(ex+d)}{e^5} - \frac{4b^3(ae-bd)}{e^5(ex+d)}$ |
| parallelrisch | $\frac{12 \ln(ex+d)x^4b^4e^4+48 \ln(ex+d)x^3b^4de^3+72 \ln(ex+d)x^2b^4d^2e^2-48x^3ab^3e^4+48x^3b^4de^3+48 \ln(ex+d)xb^4d^3e-36x^2a^2b^2e^2}{(ex+d)^4}$ |

input `int((b^2*x^2+2*a*b*x+a^2)^2/(e*x+d)^5,x,method=_RETURNVERBOSE)`

output

$$\frac{(-4b^3(ae-bd)/e^{2x^3-3b^2(a^2e^2+2abde-3b^2d^2)}/e^{3x^2-2/3} + b(2a^3e^3+3a^2bde^2+6a^2b^2d^2e-11b^3d^3)/e^{4x-1}/12(3a^4e^4+4a^3bde^3+6a^2b^2d^2e^2+12ab^3d^3e-25b^4d^4)/e^5)/(ex+d)^4 + b^4 \ln(ex+d)/e^5}$$
Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 268 vs. $2(107) = 214$.

Time = 0.09 (sec) , antiderivative size = 268, normalized size of antiderivative = 2.41

$$\int \frac{(a^2 + 2abx + b^2x^2)^2}{(d + ex)^5} dx = \frac{25b^4d^4 - 12ab^3d^3e - 6a^2b^2d^2e^2 - 4a^3bde^3 - 3a^4e^4 + 48(b^4de^3 - ab^3e^4)x^3 + 36(3b^4d^2e^2 - 2ab^3de^3 - 12(e^9x^4 + 4$$

input

```
integrate((b^2*x^2+2*a*b*x+a^2)^2/(e*x+d)^5,x, algorithm="fricas")
```

output

$$\frac{1}{12} \frac{(25b^4d^4 - 12a^2b^3d^3e - 6a^2b^2d^2e^2 - 4a^3bde^3 - 3a^4e^4 + 48(b^4de^3 - ab^3e^4)x^3 + 36(3b^4d^2e^2 - 2a^2b^3d^3e - a^2b^2e^4)x^2 + 8(11b^4d^3e - 6a^2b^3d^2e^2 - 3a^3b^2d^2e^3 - 2a^3b^2e^4)x + 12(b^4e^4x^4 + 4b^4d^3e^3x^3 + 6b^4d^2e^2x^2 + 4b^4d^3e^3ex + b^4d^4) \log(ex + d))}{(e^9x^4 + 4d^3e^8x^3 + 6d^2e^7x^2 + 4d^3e^6x + d^4e^5)}$$
Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 230 vs. $2(100) = 200$.

Time = 1.31 (sec) , antiderivative size = 230, normalized size of antiderivative = 2.07

$$\int \frac{(a^2 + 2abx + b^2x^2)^2}{(d + ex)^5} dx = \frac{b^4 \log(d + ex)}{e^5} + \frac{-3a^4e^4 - 4a^3bde^3 - 6a^2b^2d^2e^2 - 12ab^3d^3e + 25b^4d^4 + x^3(-48ab^3e^4 + 48b^4de^3) + x^2(-36a^2b^2e^4 - 72a^3b^2d^2e^3)}{12d^4e^5 + 48d^3e^6x + 72d^2e^7x^2 + 48de^8x^3}$$

input

```
integrate((b**2*x**2+2*a*b*x+a**2)**2/(e*x+d)**5,x)
```

output

```
b**4*log(d + e*x)/e**5 + (-3*a**4*e**4 - 4*a**3*b*d*e**3 - 6*a**2*b**2*d**2*e**2 - 12*a*b**3*d**3*e + 25*b**4*d**4 + x**3*(-48*a*b**3*e**4 + 48*b**4*d*e**3) + x**2*(-36*a**2*b**2*e**4 - 72*a*b**3*d*e**3 + 108*b**4*d**2*e**2) + x*(-16*a**3*b*e**4 - 24*a**2*b**2*d*e**3 - 48*a*b**3*d**2*e**2 + 88*b**4*d**3*e))/(12*d**4*e**5 + 48*d**3*e**6*x + 72*d**2*e**7*x**2 + 48*d*e**8*x**3 + 12*e**9*x**4)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 220 vs. $2(107) = 214$.

Time = 0.05 (sec) , antiderivative size = 220, normalized size of antiderivative = 1.98

$$\int \frac{(a^2 + 2abx + b^2x^2)^2}{(d + ex)^5} dx$$

$$= \frac{25b^4d^4 - 12ab^3d^3e - 6a^2b^2d^2e^2 - 4a^3bde^3 - 3a^4e^4 + 48(b^4de^3 - ab^3e^4)x^3 + 36(3b^4d^2e^2 - 2ab^3de^3 - 12(e^9x^4 + 4de^8x^3 + 6d^2e^7x^2 + 4d^3e^6x + d^4e^5) + \frac{b^4 \log(ex + d)}{e^5}}$$

input

```
integrate((b^2*x^2+2*a*b*x+a^2)^2/(e*x+d)^5,x, algorithm="maxima")
```

output

```
1/12*(25*b^4*d^4 - 12*a*b^3*d^3*e - 6*a^2*b^2*d^2*e^2 - 4*a^3*b*d*e^3 - 3*a^4*e^4 + 48*(b^4*d*e^3 - a*b^3*e^4)*x^3 + 36*(3*b^4*d^2*e^2 - 2*a*b^3*d*e^3 - a^2*b^2*e^4)*x^2 + 8*(11*b^4*d^3*e - 6*a*b^3*d^2*e^2 - 3*a^2*b^2*d*e^3 - 2*a^3*b*e^4)*x)/(e^9*x^4 + 4*d*e^8*x^3 + 6*d^2*e^7*x^2 + 4*d^3*e^6*x + d^4*e^5) + b^4*log(e*x + d)/e^5
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 281 vs. $2(107) = 214$.

Time = 0.24 (sec) , antiderivative size = 281, normalized size of antiderivative = 2.53

$$\int \frac{(a^2 + 2abx + b^2x^2)^2}{(d + ex)^5} dx = -\frac{b^4 \log\left(\frac{|ex+d|}{(ex+d)^2|e|}\right)}{e^5}$$

$$+ \frac{48b^4de^{15}}{ex+d} - \frac{36b^4d^2e^{15}}{(ex+d)^2} + \frac{16b^4d^3e^{15}}{(ex+d)^3} - \frac{3b^4d^4e^{15}}{(ex+d)^4} - \frac{48ab^3e^{16}}{ex+d} + \frac{72ab^3de^{16}}{(ex+d)^2} - \frac{48ab^3d^2e^{16}}{(ex+d)^3} + \frac{12ab^3d^3e^{16}}{(ex+d)^4} - \frac{36a^2b^2e^{17}}{(ex+d)^2} + \frac{48a^3be^{17}}{(ex+d)^3}$$

12 e²⁰

input `integrate((b^2*x^2+2*a*b*x+a^2)^2/(e*x+d)^5,x, algorithm="giac")`

output
$$-b^4 \log(\operatorname{abs}(e*x + d)/((e*x + d)^2 \operatorname{abs}(e)))/e^5 + 1/12*(48*b^4*d*e^{15}/(e*x + d) - 36*b^4*d^2*e^{15}/(e*x + d)^2 + 16*b^4*d^3*e^{15}/(e*x + d)^3 - 3*b^4*d^4*e^{15}/(e*x + d)^4 - 48*a*b^3*e^{16}/(e*x + d) + 72*a*b^3*d*e^{16}/(e*x + d)^2 - 48*a*b^3*d^2*e^{16}/(e*x + d)^3 + 12*a*b^3*d^3*e^{16}/(e*x + d)^4 - 36*a^2*b^2*e^{17}/(e*x + d)^2 + 48*a^2*b^2*d*e^{17}/(e*x + d)^3 - 18*a^2*b^2*d^2*e^{17}/(e*x + d)^4 - 16*a^3*b*e^{18}/(e*x + d)^3 + 12*a^3*b*d*e^{18}/(e*x + d)^4 - 3*a^4*e^{19}/(e*x + d)^4)/e^{20}$$

Mupad [B] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 213, normalized size of antiderivative = 1.92

$$\int \frac{(a^2 + 2abx + b^2x^2)^2}{(d + ex)^5} dx = \frac{b^4 \ln(d + ex)}{e^5} - \frac{3a^4e^4 + 4a^3bde^3 + 6a^2b^2d^2e^2 + 12ab^3d^3e - 25b^4d^4}{12e^5} + \frac{3x^2(a^2b^2e^2 + 2ab^3de - 3b^4d^2)}{e^3} + \frac{2x(2a^3be^3 + 3a^2b^2de^2 + 6ab^3d^2e - 11b^4d^3)}{3e^4} - \frac{d^4 + 4d^3ex + 6d^2e^2x^2 + 4de^3x^3 + e^4x^4}{d^4 + 4d^3ex + 6d^2e^2x^2 + 4de^3x^3 + e^4x^4}$$

input `int((a^2 + b^2*x^2 + 2*a*b*x)^2/(d + e*x)^5,x)`

output
$$(b^4 \log(d + e*x))/e^5 - ((3*a^4*e^4 - 25*b^4*d^4 + 6*a^2*b^2*d^2*e^2 + 12*a*b^3*d^3*e + 4*a^3*b*d*e^3)/(12*e^5) + (3*x^2*(a^2*b^2*e^2 - 3*b^4*d^2 + 2*a*b^3*d*e))/e^3 + (2*x*(2*a^3*b*e^3 - 11*b^4*d^3 + 3*a^2*b^2*d*e^2 + 6*a*b^3*d^2*e))/(3*e^4) + (4*b^3*x^3*(a*e - b*d))/e^2)/(d^4 + e^4*x^4 + 4*d*e^3*x^3 + 6*d^2*e^2*x^2 + 4*d^3*e*x)$$

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 269, normalized size of antiderivative = 2.42

$$\int \frac{(a^2 + 2abx + b^2x^2)^2}{(d + ex)^5} dx = \frac{12 \log(ex + d) b^4 d^5 + 48 \log(ex + d) b^4 d^4 ex + 72 \log(ex + d) b^4 d^3 e^2 x^2 + 48 \log(ex + d) b^4 d^2 e^3 x^3 + 12 \log(ex + d) b^4 d e^4 x^4 + 12 a^4 e^4 + 4 a^3 b d e^3 + 6 a^2 b^2 d^2 e^2 + 12 a b^3 d^3 e - 25 b^4 d^4}{12 e^5} + \frac{3 x^2 (a^2 b^2 e^2 + 2 a b^3 d e - 3 b^4 d^2)}{e^3} + \frac{2 x (2 a^3 b e^3 + 3 a^2 b^2 d e^2 + 6 a b^3 d^2 e - 11 b^4 d^3)}{3 e^4} - \frac{d^4 + 4 d^3 e x + 6 d^2 e^2 x^2 + 4 d e^3 x^3 + e^4 x^4}{d^4 + 4 d^3 e x + 6 d^2 e^2 x^2 + 4 d e^3 x^3 + e^4 x^4}$$

input `int((b^2*x^2+2*a*b*x+a^2)^2/(e*x+d)^5,x)`

output `(12*log(d + e*x)*b**4*d**5 + 48*log(d + e*x)*b**4*d**4*e*x + 72*log(d + e*x)*b**4*d**3*e**2*x**2 + 48*log(d + e*x)*b**4*d**2*e**3*x**3 + 12*log(d + e*x)*b**4*d*e**4*x**4 - 3*a**4*d*e**4 - 4*a**3*b*d**2*e**3 - 16*a**3*b*d*e**4*x - 6*a**2*b**2*d**3*e**2 - 24*a**2*b**2*d**2*e**3*x - 36*a**2*b**2*d*e**4*x**2 + 12*a*b**3*e**5*x**4 + 13*b**4*d**5 + 40*b**4*d**4*e*x + 36*b**4*d**3*e**2*x**2 - 12*b**4*d*e**4*x**4)/(12*d*e**5*(d**4 + 4*d**3*e*x + 6*d**2*e**2*x**2 + 4*d*e**3*x**3 + e**4*x**4))`

$$3.78 \quad \int \frac{(a^2 + 2abx + b^2x^2)^2}{(d + ex)^6} dx$$

| | |
|---|-----|
| Optimal result | 627 |
| Mathematica [B] (verified) | 627 |
| Rubi [A] (verified) | 628 |
| Maple [B] (verified) | 629 |
| Fricas [B] (verification not implemented) | 630 |
| Sympy [B] (verification not implemented) | 630 |
| Maxima [B] (verification not implemented) | 631 |
| Giac [B] (verification not implemented) | 631 |
| Mupad [B] (verification not implemented) | 632 |
| Reduce [B] (verification not implemented) | 632 |

Optimal result

Integrand size = 26, antiderivative size = 28

$$\int \frac{(a^2 + 2abx + b^2x^2)^2}{(d + ex)^6} dx = \frac{(a + bx)^5}{5(bd - ae)(d + ex)^5}$$

output `1/5*(b*x+a)^5/(-a*e+b*d)/(e*x+d)^5`

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 140 vs. 2(28) = 56.

Time = 0.04 (sec) , antiderivative size = 140, normalized size of antiderivative = 5.00

$$\int \frac{(a^2 + 2abx + b^2x^2)^2}{(d + ex)^6} dx = \frac{a^4e^4 + a^3be^3(d + 5ex) + a^2b^2e^2(d^2 + 5dex + 10e^2x^2) + ab^3e(d^3 + 5d^2ex + 10de^2x^2 + 10e^3x^3) + b^4(d^4 - 4d^3ex + 6d^2e^2x^2 - 4de^3x^3 + e^4x^4)}{5e^5(d + ex)^5}$$

input `Integrate[(a^2 + 2*a*b*x + b^2*x^2)^2/(d + e*x)^6,x]`

output

$$\begin{aligned} & -1/5*(a^4*e^4 + a^3*b*e^3*(d + 5*e*x) + a^2*b^2*e^2*(d^2 + 5*d*e*x + 10*e^2*x^2) \\ & + a*b^3*e*(d^3 + 5*d^2*e*x + 10*d*e^2*x^2 + 10*e^3*x^3) + b^4*(d^4 \\ & + 5*d^3*e*x + 10*d^2*e^2*x^2 + 10*d*e^3*x^3 + 5*e^4*x^4))/(e^5*(d + e*x)^5) \end{aligned}$$
Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.115$, Rules used = {1098, 27, 48}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\begin{aligned} & \int \frac{(a^2 + 2abx + b^2x^2)^2}{(d + ex)^6} dx \\ & \quad \downarrow 1098 \\ & \int \frac{b^4(a+bx)^4}{(d+ex)^6} dx \\ & \quad \downarrow 27 \\ & \int \frac{(a + bx)^4}{(d + ex)^6} dx \\ & \quad \downarrow 48 \\ & \frac{(a + bx)^5}{5(d + ex)^5(bd - ae)} \end{aligned}$$

input

$$\text{Int}[(a^2 + 2*a*b*x + b^2*x^2)^2/(d + e*x)^6,x]$$

output

$$(a + b*x)^5/(5*(b*d - a*e)*(d + e*x)^5)$$

Defintions of rubi rules used

rule 27 `Int[(a_)*(Fx_), x_Symbol] := Simp[a Int[Fx, x], x] /; FreeQ[a, x] && !MatchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]`

rule 48 `Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] := Simp[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{a, b, c, d, m, n}, x] && EqQ[m + n + 2, 0] && NeQ[m, -1]`

rule 1098 `Int[((d_.) + (e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] := Simp[1/c^p Int[(d + e*x)^m*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]`

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 160 vs. 2(26) = 52.

Time = 1.18 (sec) , antiderivative size = 161, normalized size of antiderivative = 5.75

| method | result |
|---------------|---|
| risch | $\frac{-\frac{b^4 x^4}{e} - \frac{2b^3(ae+bd)x^3}{e^2} - \frac{2b^2(e^2 a^2 + abde + b^2 d^2)x^2}{e^3} - \frac{b(e^3 a^3 + a^2 bde^2 + a b^2 d^2 e + b^3 d^3)x}{e^4} - \frac{a^4 e^4 + a^3 bde^3 + a^2 b^2 d^2 e^2 + a b^3 d^3 e + b^4 d^4}{5e^5}}{(ex+d)^5}$ |
| norman | $\frac{-\frac{b^4 x^4}{e} - \frac{2(ea b^3 + d b^4)x^3}{e^2} - \frac{2(a^2 b^2 e^2 + a b^3 de + d^2 b^4)x^2}{e^3} - \frac{(a^3 b e^3 + a^2 b^2 d e^2 + d^2 e a b^3 + b^4 d^3)x}{e^4} - \frac{a^4 e^4 + a^3 bde^3 + a^2 b^2 d^2 e^2 + a b^3 d^3 e + b^4 d^4}{5e^5}}{(ex+d)^5}$ |
| gospers | $\frac{-5b^4 x^4 e^4 + 10x^3 a b^3 e^4 + 10x^3 b^4 d e^3 + 10x^2 a^2 b^2 e^4 + 10x^2 a b^3 d e^3 + 10x^2 b^4 d^2 e^2 + 5x a^3 b e^4 + 5x a^2 b^2 d e^3 + 5x a b^3 d^2 e^2 + 5x b^4 d^3 e}{5e^5(ex+d)^5}$ |
| default | $\frac{2b^2(e^2 a^2 - 2abde + b^2 d^2)}{e^5(ex+d)^3} - \frac{b(e^3 a^3 - 3a^2 bde^2 + 3a b^2 d^2 e - b^3 d^3)}{e^5(ex+d)^4} - \frac{a^4 e^4 - 4a^3 bde^3 + 6a^2 b^2 d^2 e^2 - 4a b^3 d^3 e + b^4 d^4}{5e^5(ex+d)^5} - \frac{b}{e^5(e^2 a^2 - 2abde + b^2 d^2)}$ |
| parallelrisch | $\frac{-5b^4 x^4 e^4 - 10x^3 a b^3 e^4 - 10x^3 b^4 d e^3 - 10x^2 a^2 b^2 e^4 - 10x^2 a b^3 d e^3 - 10x^2 b^4 d^2 e^2 - 5x a^3 b e^4 - 5x a^2 b^2 d e^3 - 5x a b^3 d^2 e^2 - 5x b^4 d^3 e}{5e^5(ex+d)^5}$ |
| orering | $\frac{-(5b^4 x^4 e^4 + 10x^3 a b^3 e^4 + 10x^3 b^4 d e^3 + 10x^2 a^2 b^2 e^4 + 10x^2 a b^3 d e^3 + 10x^2 b^4 d^2 e^2 + 5x a^3 b e^4 + 5x a^2 b^2 d e^3 + 5x a b^3 d^2 e^2 + 5x b^4 d^3 e)}{5e^5(bx+a)^4(ex+d)^5}$ |

input `int((b^2*x^2+2*a*b*x+a^2)^2/(e*x+d)^6,x,method=_RETURNVERBOSE)`

output

$$\frac{(-1/e*b^4*x^4-2*b^3*(a*e+b*d)/e^2*x^3-2*b^2*(a^2*e^2+a*b*d*e+b^2*d^2)/e^3*x^2-b*(a^3*e^3+a^2*b*d*e^2+a*b^2*d^2*e+b^3*d^3)/e^4*x-1/5*(a^4*e^4+a^3*b*d*e^3+a^2*b^2*d^2*e^2+a*b^3*d^3*e+b^4*d^4)/e^5)/(e*x+d)^5}$$
Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 215 vs. $2(26) = 52$.

Time = 0.08 (sec) , antiderivative size = 215, normalized size of antiderivative = 7.68

$$\int \frac{(a^2 + 2abx + b^2x^2)^2}{(d + ex)^6} dx = \frac{5b^4e^4x^4 + b^4d^4 + ab^3d^3e + a^2b^2d^2e^2 + a^3bde^3 + a^4e^4 + 10(b^4de^3 + ab^3e^4)x^3 + 10(b^4d^2e^2 + ab^3de^3 + a^2b^2d^2e^4)x^2 + 5(b^4d^3e + a^2b^2d^2e^3 + a^3bde^4)x + a^4e^4}{5(e^{10}x^5 + 5de^9x^4 + 10d^2e^8x^3 + 10d^3e^7x^2 + 5d^4e^6x + d^5e^5)}$$

input

```
integrate((b^2*x^2+2*a*b*x+a^2)^2/(e*x+d)^6,x, algorithm="fricas")
```

output

$$\frac{-1/5*(5*b^4*e^4*x^4 + b^4*d^4 + a*b^3*d^3*e + a^2*b^2*d^2*e^2 + a^3*b*d*e^3 + a^4*e^4 + 10*(b^4*d*e^3 + a*b^3*e^4)*x^3 + 10*(b^4*d^2*e^2 + a*b^3*d*e^3 + a^2*b^2*e^4)*x^2 + 5*(b^4*d^3*e + a*b^3*d^2*e^2 + a^2*b^2*d*e^3 + a^3*b*e^4)*x)/(e^{10}*x^5 + 5*d*e^9*x^4 + 10*d^2*e^8*x^3 + 10*d^3*e^7*x^2 + 5*d^4*e^6*x + d^5*e^5)}$$
Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 236 vs. $2(20) = 40$.

Time = 2.22 (sec) , antiderivative size = 236, normalized size of antiderivative = 8.43

$$\int \frac{(a^2 + 2abx + b^2x^2)^2}{(d + ex)^6} dx = \frac{-a^4e^4 - a^3bde^3 - a^2b^2d^2e^2 - ab^3d^3e - b^4d^4 - 5b^4e^4x^4 + x^3(-10ab^3e^4 - 10b^4de^3) + x^2(-10a^2b^2e^4 - 10a^3bde^3) + x(-10a^2b^2d^2e^3 - 10a^3bde^4) + a^4e^4}{5d^5e^5 + 25d^4e^6x + 50d^3e^7x^2 + 50d^2e^8x^3 + 25de^9x^4}$$

input

```
integrate((b**2*x**2+2*a*b*x+a**2)**2/(e*x+d)**6,x)
```

output

```
(-a**4*e**4 - a**3*b*d*e**3 - a**2*b**2*d**2*e**2 - a*b**3*d**3*e - b**4*d
**4 - 5*b**4*e**4*x**4 + x**3*(-10*a*b**3*e**4 - 10*b**4*d*e**3) + x**2*(-
10*a**2*b**2*e**4 - 10*a*b**3*d*e**3 - 10*b**4*d**2*e**2) + x*(-5*a**3*b*e
**4 - 5*a**2*b**2*d*e**3 - 5*a*b**3*d**2*e**2 - 5*b**4*d**3*e)) / (5*d**5*e
**5 + 25*d**4*e**6*x + 50*d**3*e**7*x**2 + 50*d**2*e**8*x**3 + 25*d*e**9*x
**4 + 5*e**10*x**5)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 215 vs. $2(26) = 52$.

Time = 0.06 (sec) , antiderivative size = 215, normalized size of antiderivative = 7.68

$$\int \frac{(a^2 + 2abx + b^2x^2)^2}{(d + ex)^6} dx =$$

$$\frac{5b^4e^4x^4 + b^4d^4 + ab^3d^3e + a^2b^2d^2e^2 + a^3bde^3 + a^4e^4 + 10(b^4de^3 + ab^3e^4)x^3 + 10(b^4d^2e^2 + ab^3de^3 + a^2b^2d^2e^2)x^2 + 10(b^4de^3 + ab^3e^4)x + 5(b^4d^2e^2 + ab^3de^3 + a^2b^2d^2e^2)}{5(e^{10}x^5 + 5de^9x^4 + 10d^2e^8x^3 + 10d^3e^7x^2 + 5d^4e^6x + d^5e^5)}$$

input

```
integrate((b^2*x^2+2*a*b*x+a^2)^2/(e*x+d)^6,x, algorithm="maxima")
```

output

```
-1/5*(5*b^4*e^4*x^4 + b^4*d^4 + a*b^3*d^3*e + a^2*b^2*d^2*e^2 + a^3*b*d*e
^3 + a^4*e^4 + 10*(b^4*d*e^3 + a*b^3*e^4)*x^3 + 10*(b^4*d^2*e^2 + a*b^3*d*e
^3 + a^2*b^2*e^4)*x^2 + 5*(b^4*d^3*e + a*b^3*d^2*e^2 + a^2*b^2*d*e^3 + a^3
*b*e^4)*x)/(e^10*x^5 + 5*d*e^9*x^4 + 10*d^2*e^8*x^3 + 10*d^3*e^7*x^2 + 5*d
^4*e^6*x + d^5*e^5)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 180 vs. $2(26) = 52$.

Time = 0.22 (sec) , antiderivative size = 180, normalized size of antiderivative = 6.43

$$\int \frac{(a^2 + 2abx + b^2x^2)^2}{(d + ex)^6} dx =$$

$$\frac{5b^4e^4x^4 + 10b^4de^3x^3 + 10ab^3e^4x^3 + 10b^4d^2e^2x^2 + 10ab^3de^3x^2 + 10a^2b^2e^4x^2 + 5b^4d^3ex + 5ab^3d^2e^2x + 5a^2b^2d^2e^2}{5(ex + d)^5e^5}$$

input `integrate((b^2*x^2+2*a*b*x+a^2)^2/(e*x+d)^6,x, algorithm="giac")`

output
$$-1/5*(5*b^4*e^4*x^4 + 10*b^4*d*e^3*x^3 + 10*a*b^3*e^4*x^3 + 10*b^4*d^2*e^2*x^2 + 10*a*b^3*d*e^3*x^2 + 10*a^2*b^2*e^4*x^2 + 5*b^4*d^3*e*x + 5*a*b^3*d^2*e^2*x + 5*a^2*b^2*d*e^3*x + 5*a^3*b*e^4*x + b^4*d^4 + a*b^3*d^3*e + a^2*b^2*d^2*e^2 + a^3*b*d*e^3 + a^4*e^4)/((e*x + d)^5*e^5)$$

Mupad [B] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 203, normalized size of antiderivative = 7.25

$$\int \frac{(a^2 + 2abx + b^2x^2)^2}{(d + ex)^6} dx = -\frac{\frac{a^4 e^4 + a^3 b d e^3 + a^2 b^2 d^2 e^2 + a b^3 d^3 e + b^4 d^4}{5 e^5} + \frac{b^4 x^4}{e} + \frac{2 b^3 x^3 (a e + b d)}{e^2} + \frac{b x (a^3 e^3 + a^2 b d e^2 + a b^2 d^2 e + b^3 d^3)}{e^4} + \frac{2 b^2 x^2 (a^2 e^2 + a b d e)}{e^3}}{d^5 + 5 d^4 e x + 10 d^3 e^2 x^2 + 10 d^2 e^3 x^3 + 5 d e^4 x^4 + e^5 x^5}$$

input `int((a^2 + b^2*x^2 + 2*a*b*x)^2/(d + e*x)^6,x)`

output
$$-((a^4*e^4 + b^4*d^4 + a^2*b^2*d^2*e^2 + a*b^3*d^3*e + a^3*b*d*e^3)/(5*e^5) + (b^4*x^4)/e + (2*b^3*x^3*(a*e + b*d))/e^2 + (b*x*(a^3*e^3 + b^3*d^3 + a*b^2*d^2*e + a^2*b*d*e^2))/e^4 + (2*b^2*x^2*(a^2*e^2 + b^2*d^2 + a*b*d*e))/e^3)/(d^5 + e^5*x^5 + 5*d*e^4*x^4 + 10*d^3*e^2*x^2 + 10*d^2*e^3*x^3 + 5*d^4*e*x)$$

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 192, normalized size of antiderivative = 6.86

$$\int \frac{(a^2 + 2abx + b^2x^2)^2}{(d + ex)^6} dx = \frac{b^4 e^4 x^5 - 10 a b^3 d e^3 x^3 - 10 a^2 b^2 d e^3 x^2 - 10 a b^3 d^2 e^2 x^2 - 5 a^3 b d e^3 x - 5 a^2 b^2 d^2 e^2 x - 5 a b^3 d^3 e x - a^4 d e^3 - a^5}{5 d e^4 (e^5 x^5 + 5 d e^4 x^4 + 10 d^2 e^3 x^3 + 10 d^3 e^2 x^2 + 5 d^4 e x + d^5)}$$

input `int((b^2*x^2+2*a*b*x+a^2)^2/(e*x+d)^6,x)`

output

```
( - a**4*d**3 - a**3*b*d**2*e**2 - 5*a**3*b*d*e**3*x - a**2*b**2*d**3*e
- 5*a**2*b**2*d**2*e**2*x - 10*a**2*b**2*d*e**3*x**2 - a*b**3*d**4 - 5*a*b
**3*d**3*e*x - 10*a*b**3*d**2*e**2*x**2 - 10*a*b**3*d*e**3*x**3 + b**4*e**
4*x**5)/(5*d*e**4*(d**5 + 5*d**4*e*x + 10*d**3*e**2*x**2 + 10*d**2*e**3*x*
*3 + 5*d*e**4*x**4 + e**5*x**5))
```

3.79 $\int \frac{(a^2+2abx+b^2x^2)^2}{(d+ex)^7} dx$

| | |
|---|-----|
| Optimal result | 634 |
| Mathematica [B] (verified) | 634 |
| Rubi [A] (verified) | 635 |
| Maple [B] (verified) | 637 |
| Fricas [B] (verification not implemented) | 637 |
| Sympy [B] (verification not implemented) | 638 |
| Maxima [B] (verification not implemented) | 638 |
| Giac [B] (verification not implemented) | 639 |
| Mupad [B] (verification not implemented) | 639 |
| Reduce [B] (verification not implemented) | 640 |

Optimal result

Integrand size = 26, antiderivative size = 58

$$\int \frac{(a^2 + 2abx + b^2x^2)^2}{(d + ex)^7} dx = \frac{(a + bx)^5}{6(bd - ae)(d + ex)^6} + \frac{b(a + bx)^5}{30(bd - ae)^2(d + ex)^5}$$

output $1/6*(b*x+a)^5/(-a*e+b*d)/(e*x+d)^6+1/30*b*(b*x+a)^5/(-a*e+b*d)^2/(e*x+d)^5$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 144 vs. 2(58) = 116.

Time = 0.05 (sec) , antiderivative size = 144, normalized size of antiderivative = 2.48

$$\int \frac{(a^2 + 2abx + b^2x^2)^2}{(d + ex)^7} dx = \frac{-5a^4e^4 + 4a^3be^3(d + 6ex) + 3a^2b^2e^2(d^2 + 6dex + 15e^2x^2) + 2ab^3e(d^3 + 6d^2ex + 15de^2x^2 + 20e^3x^3) + b^4e^4}{30e^5(d + ex)^6}$$

input `Integrate[(a^2 + 2*a*b*x + b^2*x^2)^2/(d + e*x)^7,x]`

output

$$\begin{aligned} & -1/30*(5*a^4*e^4 + 4*a^3*b*e^3*(d + 6*e*x) + 3*a^2*b^2*e^2*(d^2 + 6*d*e*x \\ & + 15*e^2*x^2) + 2*a*b^3*e*(d^3 + 6*d^2*e*x + 15*d*e^2*x^2 + 20*e^3*x^3) + \\ & b^4*(d^4 + 6*d^3*e*x + 15*d^2*e^2*x^2 + 20*d*e^3*x^3 + 15*e^4*x^4))/(e^5*(\\ & d + e*x)^6) \end{aligned}$$

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {1098, 27, 55, 48}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\begin{aligned} & \int \frac{(a^2 + 2abx + b^2x^2)^2}{(d + ex)^7} dx \\ & \quad \downarrow 1098 \\ & \int \frac{b^4(a+bx)^4}{(d+ex)^7} dx \\ & \quad \downarrow 27 \\ & \int \frac{(a + bx)^4}{(d + ex)^7} dx \\ & \quad \downarrow 55 \\ & \frac{b \int \frac{(a+bx)^4}{(d+ex)^6} dx}{6(bd - ae)} + \frac{(a + bx)^5}{6(d + ex)^6(bd - ae)} \\ & \quad \downarrow 48 \\ & \frac{b(a + bx)^5}{30(d + ex)^5(bd - ae)^2} + \frac{(a + bx)^5}{6(d + ex)^6(bd - ae)} \end{aligned}$$

input

$$\text{Int}[(a^2 + 2*a*b*x + b^2*x^2)^2/(d + e*x)^7, x]$$

output $(a + b*x)^5/(6*(b*d - a*e)*(d + e*x)^6) + (b*(a + b*x)^5)/(30*(b*d - a*e)^2*(d + e*x)^5)$

Defintions of rubi rules used

rule 27 `Int[(a_)*(Fx_), x_Symbol] := Simp[a Int[Fx, x], x] /; FreeQ[a, x] && !MatchQ[Fx, (b_)*(Gx_)] /; FreeQ[b, x]`

rule 48 `Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] := Simp[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{a, b, c, d, m, n}, x] && EqQ[m + n + 2, 0] && NeQ[m, -1]`

rule 55 `Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] := Simp[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*(Simplify[m + n + 2]/((b*c - a*d)*(m + 1))) Int[(a + b*x)^Simplify[m + 1]*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && ILtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] && !(LtQ[m, -1] && LtQ[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimplerQ[m, 1] || !SumSimplerQ[n, 1])`

rule 1098 `Int[((d_.) + (e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] := Simp[1/c^p Int[(d + e*x)^m*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]`

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 170 vs. 2(54) = 108.

Time = 1.10 (sec) , antiderivative size = 171, normalized size of antiderivative = 2.95

| method | result |
|--------------|--|
| risch | $\frac{-\frac{b^4 x^4}{2e} - \frac{2b^3(2ae+bd)x^3}{3e^2} - \frac{b^2(3e^2 a^2 + 2abde + b^2 d^2)x^2}{2e^3} - \frac{b(4e^3 a^3 + 3a^2 bde^2 + 2a b^2 d^2 e + b^3 d^3)x}{5e^4}}{(ex+d)^6} - \frac{5a^4 e^4 + 4a^3 bde^3 + 3a^2 b^2 d^2 e^2 + 2a b^3 d^3}{30e^5}$ |
| gospers | $\frac{15b^4 x^4 e^4 + 40x^3 a b^3 e^4 + 20x^3 b^4 d e^3 + 45x^2 a^2 b^2 e^4 + 30x^2 a b^3 d e^3 + 15x^2 b^4 d^2 e^2 + 24x a^3 b e^4 + 18x a^2 b^2 d e^3 + 12x a b^3 d^2 e^2 + 6x}{30e^5 (ex+d)^6}$ |
| default | $\frac{4b^3(ae-bd)}{3e^5(ex+d)^3} - \frac{3b^2(e^2 a^2 - 2abde + b^2 d^2)}{2e^5(ex+d)^4} - \frac{4b(e^3 a^3 - 3a^2 bde^2 + 3a b^2 d^2 e - b^3 d^3)}{5e^5(ex+d)^5} - \frac{b^4}{2e^5(ex+d)^2} - \frac{a^4 e^4 - 4a^3 bde^3 + 6a^2 b^2 d^2 e^2 - 4a b^3 d^3 e + b^4 d^4}{6e^5}$ |
| norman | $\frac{-\frac{b^4 x^4}{2e} - \frac{2(2e^2 a b^3 + b^4 de)x^3}{3e^3} - \frac{(3a^2 b^2 e^3 + 2d e^2 a b^3 + d^2 e b^4)x^2}{2e^4} - \frac{(4a^3 b e^4 + 3a^2 b^2 d e^3 + 2a b^3 d^2 e^2 + b^4 d^3 e)x}{5e^5}}{(ex+d)^6} - \frac{5a^4 e^5 + 4a^3 bde^4 + 3a^2 b^2 d^2 e^3 + 2a b^3 d^3 e + b^4 d^4}{30e^5}$ |
| paralelrisch | $\frac{-15b^4 x^4 e^5 - 40a b^3 e^5 x^3 - 20b^4 d e^4 x^3 - 45a^2 b^2 e^5 x^2 - 30a b^3 d e^4 x^2 - 15b^4 d^2 e^3 x^2 - 24a^3 b e^5 x - 18a^2 b^2 d e^4 x - 12a b^3 d^2 e^3 x - 6b^4 d^3}{30e^6 (ex+d)^6}$ |
| orering | $\frac{(15b^4 x^4 e^4 + 40x^3 a b^3 e^4 + 20x^3 b^4 d e^3 + 45x^2 a^2 b^2 e^4 + 30x^2 a b^3 d e^3 + 15x^2 b^4 d^2 e^2 + 24x a^3 b e^4 + 18x a^2 b^2 d e^3 + 12x a b^3 d^2 e^2 + 6x)}{30e^5 (bx+a)^4 (ex+d)^6}$ |

```
input int((b^2*x^2+2*a*b*x+a^2)^2/(e*x+d)^7,x,method=_RETURNVERBOSE)
```

```
output (-1/2/e*b^4*x^4-2/3/e^2*b^3*(2*a*e+b*d)*x^3-1/2/e^3*b^2*(3*a^2*e^2+2*a*b*d*e+b^2*d^2)*x^2-1/5*b/e^4*(4*a^3*e^3+3*a^2*b*d*e^2+2*a*b^2*d^2*e+b^3*d^3)*x-1/30/e^5*(5*a^4*e^4+4*a^3*b*d*e^3+3*a^2*b^2*d^2*e^2+2*a*b^3*d^3*e+b^4*d^4))/(e*x+d)^6
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 236 vs. 2(54) = 108.

Time = 0.08 (sec) , antiderivative size = 236, normalized size of antiderivative = 4.07

$$\int \frac{(a^2 + 2abx + b^2x^2)^2}{(d + ex)^7} dx = \frac{15 b^4 e^4 x^4 + b^4 d^4 + 2 a b^3 d^3 e + 3 a^2 b^2 d^2 e^2 + 4 a^3 b d e^3 + 5 a^4 e^4 + 20 (b^4 d e^3 + 2 a b^3 e^4) x^3 + 15 (b^4 d^2 e^2 + 2 a b^3 d e^3 + 3 a^2 b^2 d^2 e^2) x^2 + 10 (a^3 b d e^3 + 2 a^2 b^2 d^2 e^2) x + 5 (a^4 e^4 + 4 a^3 b d e^3 + 3 a^2 b^2 d^2 e^2 + 2 a b^3 d^3 e + b^4 d^4)}{30 (e^{11} x^6 + 6 d e^{10} x^5 + 15 d^2 e^9 x^4 + 20 d^3 e^8 x^3 + 15 d^4 e^7)}$$

```
input integrate((b^2*x^2+2*a*b*x+a^2)^2/(e*x+d)^7,x, algorithm="fricas")
```

output

$$\frac{-1/30*(15*b^4*e^4*x^4 + b^4*d^4 + 2*a*b^3*d^3*e + 3*a^2*b^2*d^2*e^2 + 4*a^3*b*d*e^3 + 5*a^4*e^4 + 20*(b^4*d*e^3 + 2*a*b^3*e^4)*x^3 + 15*(b^4*d^2*e^2 + 2*a*b^3*d*e^3 + 3*a^2*b^2*e^4)*x^2 + 6*(b^4*d^3*e + 2*a*b^3*d^2*e^2 + 3*a^2*b^2*d*e^3 + 4*a^3*b*e^4)*x)/(e^11*x^6 + 6*d*e^10*x^5 + 15*d^2*e^9*x^4 + 20*d^3*e^8*x^3 + 15*d^4*e^7*x^2 + 6*d^5*e^6*x + d^6*e^5)}$$

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 255 vs. $2(46) = 92$.

Time = 3.01 (sec) , antiderivative size = 255, normalized size of antiderivative = 4.40

$$\int \frac{(a^2 + 2abx + b^2x^2)^2}{(d + ex)^7} dx = \frac{-5a^4e^4 - 4a^3bde^3 - 3a^2b^2d^2e^2 - 2ab^3d^3e - b^4d^4 - 15b^4e^4x^4 + x^3(-40ab^3e^4 - 20b^4de^3) + x^2(-45a^2b^2e^4}{30d^6e^5 + 180d^5e^6x + 450d^4e^7x^2 + 600d^3e^8x^3 + 450d^2e^9x^4 + 20d^3e^8x^3 + 15d^4e^7x^2 + 6d^5e^6x + d^6e^5}$$

input

```
integrate((b**2*x**2+2*a*b*x+a**2)**2/(e*x+d)**7,x)
```

output

$$\frac{(-5*a**4*e**4 - 4*a**3*b*d*e**3 - 3*a**2*b**2*d**2*e**2 - 2*a*b**3*d**3*e - b**4*d**4 - 15*b**4*e**4*x**4 + x**3*(-40*a*b**3*e**4 - 20*b**4*d*e**3) + x**2*(-45*a**2*b**2*e**4 - 30*a*b**3*d*e**3 - 15*b**4*d**2*e**2) + x*(-44*a**3*b*e**4 - 18*a**2*b**2*d*e**3 - 12*a*b**3*d**2*e**2 - 6*b**4*d**3*e))/(30*d**6*e**5 + 180*d**5*e**6*x + 450*d**4*e**7*x**2 + 600*d**3*e**8*x**3 + 450*d**2*e**9*x**4 + 180*d**e**10*x**5 + 30*e**11*x**6)}$$

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 236 vs. $2(54) = 108$.

Time = 0.05 (sec) , antiderivative size = 236, normalized size of antiderivative = 4.07

$$\int \frac{(a^2 + 2abx + b^2x^2)^2}{(d + ex)^7} dx = \frac{15b^4e^4x^4 + b^4d^4 + 2ab^3d^3e + 3a^2b^2d^2e^2 + 4a^3bde^3 + 5a^4e^4 + 20(b^4de^3 + 2ab^3e^4)x^3 + 15(b^4d^2e^2 + 2ab^3de^3 + 3a^2b^2d^2e^2)x^2 + 6(b^4d^3e + 2ab^3d^2e^2 + 3a^2b^2de^3 + 4a^3be^4)x}{30(e^{11}x^6 + 6de^{10}x^5 + 15d^2e^9x^4 + 20d^3e^8x^3 + 15d^4e^7x^2 + 6d^5e^6x + d^6e^5)}$$

input `integrate((b^2*x^2+2*a*b*x+a^2)^2/(e*x+d)^7,x, algorithm="maxima")`

output
$$-1/30*(15*b^4*e^4*x^4 + b^4*d^4 + 2*a*b^3*d^3*e + 3*a^2*b^2*d^2*e^2 + 4*a^3*b*d*e^3 + 5*a^4*e^4 + 20*(b^4*d*e^3 + 2*a*b^3*e^4)*x^3 + 15*(b^4*d^2*e^2 + 2*a*b^3*d*e^3 + 3*a^2*b^2*e^4)*x^2 + 6*(b^4*d^3*e + 2*a*b^3*d^2*e^2 + 3*a^2*b^2*d*e^3 + 4*a^3*b*e^4)*x)/(e^11*x^6 + 6*d*e^10*x^5 + 15*d^2*e^9*x^4 + 20*d^3*e^8*x^3 + 15*d^4*e^7*x^2 + 6*d^5*e^6*x + d^6*e^5)$$

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 184 vs. $2(54) = 108$.

Time = 0.20 (sec) , antiderivative size = 184, normalized size of antiderivative = 3.17

$$\int \frac{(a^2 + 2abx + b^2x^2)^2}{(d + ex)^7} dx = \frac{15b^4e^4x^4 + 20b^4de^3x^3 + 40ab^3e^4x^3 + 15b^4d^2e^2x^2 + 30ab^3de^3x^2 + 45a^2b^2e^4x^2 + 6b^4d^3ex + 12ab^3d^2e^3}{30(e^6x + d)^6e^5}$$

input `integrate((b^2*x^2+2*a*b*x+a^2)^2/(e*x+d)^7,x, algorithm="giac")`

output
$$-1/30*(15*b^4*e^4*x^4 + 20*b^4*d*e^3*x^3 + 40*a*b^3*e^4*x^3 + 15*b^4*d^2*e^2*x^2 + 30*a*b^3*d*e^3*x^2 + 45*a^2*b^2*e^4*x^2 + 6*b^4*d^3*e*x + 12*a*b^3*d^2*e^2*x + 18*a^2*b^2*d*e^3*x + 24*a^3*b*e^4*x + b^4*d^4 + 2*a*b^3*d^3*e + 3*a^2*b^2*d^2*e^2 + 4*a^3*b*d*e^3 + 5*a^4*e^4)/((e*x + d)^6*e^5)$$

Mupad [B] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 226, normalized size of antiderivative = 3.90

$$\int \frac{(a^2 + 2abx + b^2x^2)^2}{(d + ex)^7} dx = \frac{\frac{5a^4e^4 + 4a^3bde^3 + 3a^2b^2d^2e^2 + 2ab^3d^3e + b^4d^4}{30e^5} + \frac{b^4x^4}{2e} + \frac{2b^3x^3(2ae + bd)}{3e^2} + \frac{bx(4a^3e^3 + 3a^2bde^2 + 2ab^2d^2e + b^3d^3)}{5e^4} + \frac{b^2x^2}{d^6 + 6d^5ex + 15d^4e^2x^2 + 20d^3e^3x^3 + 15d^2e^4x^4 + 6de^5x^5 + e^6x^6}}$$

input `int((a^2 + b^2*x^2 + 2*a*b*x)^2/(d + e*x)^7,x)`

output
$$\begin{aligned} & -((5a^4e^4 + b^4d^4 + 3a^2b^2d^2e^2 + 2ab^3d^3e + 4a^3bd^2e^3) / (30e^5) + (b^4x^4)/(2e) + (2b^3x^3(2ae + bd))/(3e^2) + (bx(4a^3e^3 + b^3d^3 + 2ab^2d^2e + 3a^2bd^2e^2))/(5e^4) + (b^2x^2(3a^2e^2 + b^2d^2 + 2abd^2e))/(2e^3)) / (d^6 + e^6x^6 + 6d^5e^5x^5 + 15d^4e^4x^4 + 20d^3e^3x^3 + 15d^2e^4x^4 + 6d^5e^5x^5) \end{aligned}$$

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 240, normalized size of antiderivative = 4.14

$$\int \frac{(a^2 + 2abx + b^2x^2)^2}{(d + ex)^7} dx = \frac{-15b^4e^4x^4 - 40ab^3e^4x^3 - 20b^4de^3x^3 - 45a^2b^2e^4x^2 - 30ab^3de^3x^2 - 15b^4d^2e^2x^2 - 24a^3be^4x - 18a^2b^2de^3}{30e^5(e^6x^6 + 6de^5x^5 + 15d^2e^4x^4 + 20d^3e^3x^3 + 15d^4e^4x^4 + 6d^5e^5x^5 + e^6x^6)}$$

input `int((b^2*x^2+2*a*b*x+a^2)^2/(e*x+d)^7,x)`

output
$$\begin{aligned} & (-5a^4e^4 - 4a^3bd^2e^3 - 24a^3b^2d^2e^3x - 3a^2b^3d^3e^3 - 18a^2b^2d^2e^3x - 45a^2b^2e^4x^2 - 2ab^3d^3e^3 - 12ab^3d^2e^2x - 30ab^3de^3x^2 - 40ab^3e^4x^3 - b^4d^4 - 6b^4d^3e^3x - 15b^4d^2e^2x^2 - 20b^4de^3x^3 - 15b^4e^4x^4) / (30e^5(d^6 + 6d^5ex + 15d^4e^2x^2 + 20d^3e^3x^3 + 15d^2e^4x^4 + 6de^5x^5 + e^6x^6)) \end{aligned}$$

3.80 $\int \frac{(a^2+2abx+b^2x^2)^2}{(d+ex)^8} dx$

| | |
|---|-----|
| Optimal result | 641 |
| Mathematica [A] (verified) | 641 |
| Rubi [A] (verified) | 642 |
| Maple [B] (verified) | 644 |
| Fricas [B] (verification not implemented) | 644 |
| Sympy [B] (verification not implemented) | 645 |
| Maxima [B] (verification not implemented) | 645 |
| Giac [B] (verification not implemented) | 646 |
| Mupad [B] (verification not implemented) | 646 |
| Reduce [B] (verification not implemented) | 647 |

Optimal result

Integrand size = 26, antiderivative size = 89

$$\int \frac{(a^2 + 2abx + b^2x^2)^2}{(d + ex)^8} dx = \frac{(a + bx)^5}{7(bd - ae)(d + ex)^7} + \frac{b(a + bx)^5}{21(bd - ae)^2(d + ex)^6} + \frac{b^2(a + bx)^5}{105(bd - ae)^3(d + ex)^5}$$

output 1/7*(b*x+a)^5/(-a*e+b*d)/(e*x+d)^7+1/21*b*(b*x+a)^5/(-a*e+b*d)^2/(e*x+d)^6+1/105*b^2*(b*x+a)^5/(-a*e+b*d)^3/(e*x+d)^5

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 144, normalized size of antiderivative = 1.62

$$\int \frac{(a^2 + 2abx + b^2x^2)^2}{(d + ex)^8} dx = \frac{15a^4e^4 + 10a^3be^3(d + 7ex) + 6a^2b^2e^2(d^2 + 7dex + 21e^2x^2) + 3ab^3e(d^3 + 7d^2ex + 21de^2x^2 + 35e^3x^3) - b^4(d^4 + 7d^3ex + 21d^2e^2x^2 + 35de^3x^3 + 7e^4x^4)}{105e^5(d + ex)^7}$$

input Integrate[(a^2 + 2*a*b*x + b^2*x^2)^2/(d + e*x)^8,x]

output

$$-1/105*(15*a^4*e^4 + 10*a^3*b*e^3*(d + 7*e*x) + 6*a^2*b^2*e^2*(d^2 + 7*d*e*x + 21*e^2*x^2) + 3*a*b^3*e*(d^3 + 7*d^2*e*x + 21*d*e^2*x^2 + 35*e^3*x^3) + b^4*(d^4 + 7*d^3*e*x + 21*d^2*e^2*x^2 + 35*d*e^3*x^3 + 35*e^4*x^4))/(e^5*(d + e*x)^7)$$

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 102, normalized size of antiderivative = 1.15, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.192$, Rules used = {1098, 27, 55, 55, 48}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\begin{aligned} & \int \frac{(a^2 + 2abx + b^2x^2)^2}{(d + ex)^8} dx \\ & \quad \downarrow 1098 \\ & \int \frac{b^4(a+bx)^4}{(d+ex)^8} dx \\ & \quad \downarrow 27 \\ & \int \frac{(a + bx)^4}{(d + ex)^8} dx \\ & \quad \downarrow 55 \\ & \frac{2b \int \frac{(a+bx)^4}{(d+ex)^7} dx}{7(bd - ae)} + \frac{(a + bx)^5}{7(d + ex)^7(bd - ae)} \\ & \quad \downarrow 55 \\ & \frac{2b \left(\frac{b \int \frac{(a+bx)^4}{(d+ex)^6} dx}{6(bd - ae)} + \frac{(a+bx)^5}{6(d+ex)^6(bd - ae)} \right)}{7(bd - ae)} + \frac{(a + bx)^5}{7(d + ex)^7(bd - ae)} \\ & \quad \downarrow 48 \\ & \frac{(a + bx)^5}{7(d + ex)^7(bd - ae)} + \frac{2b \left(\frac{b(a+bx)^5}{30(d+ex)^5(bd - ae)^2} + \frac{(a+bx)^5}{6(d+ex)^6(bd - ae)} \right)}{7(bd - ae)} \end{aligned}$$

input `Int[(a^2 + 2*a*b*x + b^2*x^2)^2/(d + e*x)^8,x]`

output `(a + b*x)^5/(7*(b*d - a*e)*(d + e*x)^7) + (2*b*((a + b*x)^5/(6*(b*d - a*e)*
*(d + e*x)^6) + (b*(a + b*x)^5)/(30*(b*d - a*e)^2*(d + e*x)^5)))/(7*(b*d -
a*e))`

Defintions of rubi rules used

rule 27 `Int[(a_)*(F_x_), x_Symbol] := Simp[a Int[F_x, x], x] /; FreeQ[a, x] && !Ma
tchQ[F_x, (b_)*(G_x_)] /; FreeQ[b, x]`

rule 48 `Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] := Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && EqQ[m + n + 2, 0] && NeQ[m, -1]`

rule 55 `Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] := Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*(S
implify[m + n + 2]/((b*c - a*d)*(m + 1))) Int[(a + b*x)^Simplify[m + 1]*
(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && ILtQ[Simplify[m + n +
2], 0] && NeQ[m, -1] && !(LtQ[m, -1] && LtQ[n, -1] && (EqQ[a, 0] || (NeQ[
c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimplerQ[m, 1] || !SumSimp
lerQ[n, 1])`

rule 1098 `Int[((d_.) + (e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] := Simp[1/c^p Int[(d + e*x)^m*(b/2 + c*x)^(2*p), x], x] /; FreeQ[
{a, b, c, d, e, m}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]`

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 170 vs. 2(83) = 166.

Time = 1.12 (sec) , antiderivative size = 171, normalized size of antiderivative = 1.92

| method | result |
|---------------|--|
| risch | $\frac{-\frac{b^4 x^4}{3e} - \frac{b^3(3ae+bd)x^3}{3e^2} - \frac{b^2(6e^2 a^2+3abde+b^2 d^2)x^2}{5e^3} - \frac{b(10e^3 a^3+6a^2 bd e^2+3a b^2 d^2 e+b^3 d^3)x}{15e^4} - \frac{15a^4 e^4+10a^3 bd e^3+6a^2 b^2 d^2 e^2+3a b^3 d^3}{105e^5}}{(ex+d)^7}$ |
| gospers | $\frac{-35b^4 x^4 e^4+105x^3 a b^3 e^4+35x^3 b^4 d e^3+126x^2 a^2 b^2 e^4+63x^2 a b^3 d e^3+21x^2 b^4 d^2 e^2+70x a^3 b e^4+42x a^2 b^2 d e^3+21x a b^3 d^2 e^2+70a^4 e^4}{105e^5(ex+d)^7}$ |
| default | $-\frac{b^4}{3e^5(ex+d)^3} - \frac{b^3(ae-bd)}{e^5(ex+d)^4} - \frac{a^4 e^4-4a^3 bd e^3+6a^2 b^2 d^2 e^2-4a b^3 d^3 e+b^4 d^4}{7e^5(ex+d)^7} - \frac{6b^2(e^2 a^2-2abde+b^2 d^2)}{5e^5(ex+d)^5} - \frac{2b(e^3 a^3-3a^2 b d e^2+b^2 d^2 e)}{e^5(ex+d)^5}$ |
| parallelrisch | $\frac{-35b^4 x^4 e^6-105a b^3 e^6 x^3-35b^4 d e^5 x^3-126a^2 b^2 e^6 x^2-63a b^3 d e^5 x^2-21b^4 d^2 e^4 x^2-70a^3 b e^6 x-42a^2 b^2 d e^5 x-21a b^3 d^2 e^4 x-70a^4 e^6}{105e^7(ex+d)^7}$ |
| norman | $\frac{-\frac{b^4 x^4}{3e} - \frac{(3a b^3 e^3+b^4 d e^2)x^3}{3e^4} - \frac{(6a^2 b^2 e^4+3a b^3 d e^3+b^4 d^2 e^2)x^2}{5e^5} - \frac{(10a^3 b e^5+6a^2 b^2 d e^4+3a b^3 d^2 e^3+b^4 d^3 e^2)x}{15e^6} - \frac{15a^4 e^6+10a^3 bd e^5+6a^2 b^2 d^2 e^4+3a b^3 d^3 e^3+b^4 d^4 e^2}{105e^7}}{(ex+d)^7}$ |
| orering | $\frac{-(35b^4 x^4 e^4+105x^3 a b^3 e^4+35x^3 b^4 d e^3+126x^2 a^2 b^2 e^4+63x^2 a b^3 d e^3+21x^2 b^4 d^2 e^2+70x a^3 b e^4+42x a^2 b^2 d e^3+21x a b^3 d^2 e^2+70a^4 e^4)}{105e^5(bx+a)^4(ex+d)^7}$ |

input `int((b^2*x^2+2*a*b*x+a^2)^2/(e*x+d)^8,x,method=_RETURNVERBOSE)`

output
$$\frac{(-1/3/e*b^4*x^4-1/3/e^2*b^3*(3*a*e+b*d)*x^3-1/5*b^2/e^3*(6*a^2*e^2+3*a*b*d*e+b^2*d^2)*x^2-1/15*b/e^4*(10*a^3*e^3+6*a^2*b*d*e^2+3*a*b^2*d^2*e+b^3*d^3)*x-1/105/e^5*(15*a^4*e^4+10*a^3*b*d*e^3+6*a^2*b^2*d^2*e^2+3*a*b^3*d^3*e+b^4*d^4))/(e*x+d)^7}$$

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 247 vs. 2(83) = 166.

Time = 0.07 (sec) , antiderivative size = 247, normalized size of antiderivative = 2.78

$$\int \frac{(a^2 + 2abx + b^2x^2)^2}{(d + ex)^8} dx = \frac{35 b^4 e^4 x^4 + b^4 d^4 + 3 a b^3 d^3 e + 6 a^2 b^2 d^2 e^2 + 10 a^3 b d e^3 + 15 a^4 e^4 + 35 (b^4 d e^3 + 3 a b^3 e^4) x^3 + 21 (b^4 d^2 e^2 + 3 a b^3 d e^3 + 3 a^2 b^2 d^2 e) x^2 + 7 (a^3 b d e^3 + 3 a^2 b^2 d^2 e) x + 7 a^4 e^4}{105 (e^{12} x^7 + 7 d e^{11} x^6 + 21 d^2 e^{10} x^5 + 35 d^3 e^9 x^4 + 35 d^4 e^8 x^3 - \dots)}$$

input `integrate((b^2*x^2+2*a*b*x+a^2)^2/(e*x+d)^8,x, algorithm="fricas")`

output

```
-1/105*(35*b^4*e^4*x^4 + b^4*d^4 + 3*a*b^3*d^3*e + 6*a^2*b^2*d^2*e^2 + 10*
a^3*b*d*e^3 + 15*a^4*e^4 + 35*(b^4*d*e^3 + 3*a*b^3*e^4)*x^3 + 21*(b^4*d^2*
e^2 + 3*a*b^3*d*e^3 + 6*a^2*b^2*e^4)*x^2 + 7*(b^4*d^3*e + 3*a*b^3*d^2*e^2
+ 6*a^2*b^2*d*e^3 + 10*a^3*b*e^4)*x)/(e^12*x^7 + 7*d*e^11*x^6 + 21*d^2*e^1
0*x^5 + 35*d^3*e^9*x^4 + 35*d^4*e^8*x^3 + 21*d^5*e^7*x^2 + 7*d^6*e^6*x + d
^7*e^5)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 267 vs. $2(73) = 146$.

Time = 5.56 (sec) , antiderivative size = 267, normalized size of antiderivative = 3.00

$$\int \frac{(a^2 + 2abx + b^2x^2)^2}{(d + ex)^8} dx$$

$$= \frac{-15a^4e^4 - 10a^3bde^3 - 6a^2b^2d^2e^2 - 3ab^3d^3e - b^4d^4 - 35b^4e^4x^4 + x^3(-105ab^3e^4 - 35b^4de^3) + x^2(-126a^2b^2d^2e^3 - 63a^2b^3d^2e^2 - 21b^4d^2e^2) + x(-70a^3b^2de^4 - 42a^3b^2d^2e^3 - 21a^3b^3d^2e^2 - 7b^4d^3e) + 105a^4e^4}{105d^7e^5 + 735d^6e^6x + 2205d^5e^7x^2 + 3675d^4e^8x^3 + 3675d^3e^9x^4 + 2205d^2e^{10}x^5 + 735de^{11}x^6 + 105e^{12}x^7}$$

input

```
integrate((b**2*x**2+2*a*b*x+a**2)**2/(e*x+d)**8,x)
```

output

```
(-15*a**4*e**4 - 10*a**3*b*d*e**3 - 6*a**2*b**2*d**2*e**2 - 3*a*b**3*d**3*
e - b**4*d**4 - 35*b**4*e**4*x**4 + x**3*(-105*a*b**3*e**4 - 35*b**4*d*e**
3) + x**2*(-126*a**2*b**2*d**2*e**3 - 63*a*b**3*d*e**3 - 21*b**4*d**2*e**2) + x
*(-70*a**3*b*e**4 - 42*a**2*b**2*d*e**3 - 21*a*b**3*d**2*e**2 - 7*b**4*d**
3*e))/(105*d**7*e**5 + 735*d**6*e**6*x + 2205*d**5*e**7*x**2 + 3675*d**4*e
**8*x**3 + 3675*d**3*e**9*x**4 + 2205*d**2*e**10*x**5 + 735*d*e**11*x**6 +
105*e**12*x**7)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 247 vs. $2(83) = 166$.

Time = 0.05 (sec) , antiderivative size = 247, normalized size of antiderivative = 2.78

$$\int \frac{(a^2 + 2abx + b^2x^2)^2}{(d + ex)^8} dx =$$

$$-\frac{35b^4e^4x^4 + b^4d^4 + 3ab^3d^3e + 6a^2b^2d^2e^2 + 10a^3bde^3 + 15a^4e^4 + 35(b^4de^3 + 3ab^3e^4)x^3 + 21(b^4d^2e^2 + 3a^2b^2de^3 + 6a^2b^3d^2e^2)x^2 + 7(b^4d^3e + 3a^2b^2d^2e^2 + 6a^2b^3d^2e^2)x + 105a^4e^4}{105(e^{12}x^7 + 7de^{11}x^6 + 21d^2e^{10}x^5 + 35d^3e^9x^4 + 35d^4e^8x^3 + 2205d^2e^{10}x^5 + 735de^{11}x^6 + 105e^{12}x^7)}$$

input `integrate((b^2*x^2+2*a*b*x+a^2)^2/(e*x+d)^8,x, algorithm="maxima")`

output
$$-1/105*(35*b^4*e^4*x^4 + b^4*d^4 + 3*a*b^3*d^3*e + 6*a^2*b^2*d^2*e^2 + 10*a^3*b*d*e^3 + 15*a^4*e^4 + 35*(b^4*d*e^3 + 3*a*b^3*e^4)*x^3 + 21*(b^4*d^2*e^2 + 3*a*b^3*d*e^3 + 6*a^2*b^2*e^4)*x^2 + 7*(b^4*d^3*e + 3*a*b^3*d^2*e^2 + 6*a^2*b^2*d*e^3 + 10*a^3*b*e^4)*x)/(e^12*x^7 + 7*d*e^11*x^6 + 21*d^2*e^10*x^5 + 35*d^3*e^9*x^4 + 35*d^4*e^8*x^3 + 21*d^5*e^7*x^2 + 7*d^6*e^6*x + d^7*e^5)$$

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 184 vs. 2(83) = 166.

Time = 0.22 (sec) , antiderivative size = 184, normalized size of antiderivative = 2.07

$$\int \frac{(a^2 + 2abx + b^2x^2)^2}{(d + ex)^8} dx = \frac{35b^4e^4x^4 + 35b^4de^3x^3 + 105ab^3e^4x^3 + 21b^4d^2e^2x^2 + 63ab^3de^3x^2 + 126a^2b^2e^4x^2 + 7b^4d^3ex + 21ab^3d^2e^3}{105(ex + d)^7e^5}$$

input `integrate((b^2*x^2+2*a*b*x+a^2)^2/(e*x+d)^8,x, algorithm="giac")`

output
$$-1/105*(35*b^4*e^4*x^4 + 35*b^4*d*e^3*x^3 + 105*a*b^3*e^4*x^3 + 21*b^4*d^2*e^2*x^2 + 63*a*b^3*d*e^3*x^2 + 126*a^2*b^2*e^4*x^2 + 7*b^4*d^3*e*x + 21*a*b^3*d^2*e^2*x + 42*a^2*b^2*d*e^3*x + 70*a^3*b*e^4*x + b^4*d^4 + 3*a*b^3*d^3*e + 6*a^2*b^2*d^2*e^2 + 10*a^3*b*d*e^3 + 15*a^4*e^4)/((e*x + d)^7*e^5)$$

Mupad [B] (verification not implemented)

Time = 8.98 (sec) , antiderivative size = 237, normalized size of antiderivative = 2.66

$$\int \frac{(a^2 + 2abx + b^2x^2)^2}{(d + ex)^8} dx = -\frac{\frac{15a^4e^4 + 10a^3bde^3 + 6a^2b^2d^2e^2 + 3ab^3d^3e + b^4d^4}{105e^5} + \frac{b^4x^4}{3e} + \frac{b^3x^3(3ae + bd)}{3e^2} + \frac{bx(10a^3e^3 + 6a^2bde^2 + 3ab^2d^2e + b^3d^3)}{15e^4} + \frac{b^2x^2}{e^5}}{d^7 + 7d^6ex + 21d^5e^2x^2 + 35d^4e^3x^3 + 35d^3e^4x^4 + 21d^2e^5x^5 + 7d^6e^6x^6 + e^7x^7}$$

3.81 $\int \frac{(a^2+2abx+b^2x^2)^2}{(d+ex)^9} dx$

| | |
|---|-----|
| Optimal result | 648 |
| Mathematica [A] (verified) | 648 |
| Rubi [A] (verified) | 649 |
| Maple [A] (verified) | 650 |
| Fricas [B] (verification not implemented) | 651 |
| Sympy [B] (verification not implemented) | 652 |
| Maxima [B] (verification not implemented) | 652 |
| Giac [A] (verification not implemented) | 653 |
| Mupad [B] (verification not implemented) | 653 |
| Reduce [B] (verification not implemented) | 654 |

Optimal result

Integrand size = 26, antiderivative size = 117

$$\int \frac{(a^2 + 2abx + b^2x^2)^2}{(d + ex)^9} dx = -\frac{(bd - ae)^4}{8e^5(d + ex)^8} + \frac{4b(bd - ae)^3}{7e^5(d + ex)^7} - \frac{b^2(bd - ae)^2}{e^5(d + ex)^6} + \frac{4b^3(bd - ae)}{5e^5(d + ex)^5} - \frac{b^4}{4e^5(d + ex)^4}$$

output

```
-1/8*(-a*e+b*d)^4/e^5/(e*x+d)^8+4/7*b*(-a*e+b*d)^3/e^5/(e*x+d)^7-b^2*(-a*e+b*d)^2/e^5/(e*x+d)^6+4/5*b^3*(-a*e+b*d)/e^5/(e*x+d)^5-1/4*b^4/e^5/(e*x+d)^4
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 144, normalized size of antiderivative = 1.23

$$\int \frac{(a^2 + 2abx + b^2x^2)^2}{(d + ex)^9} dx = \frac{-35a^4e^4 + 20a^3be^3(d + 8ex) + 10a^2b^2e^2(d^2 + 8dex + 28e^2x^2) + 4ab^3e(d^3 + 8d^2ex + 28de^2x^2 + 56e^3x^3) - b^4(d^4 + 8d^3ex + 28d^2e^2x^2 + 56de^3x^3 + 56e^4x^4)}{280e^5(d + ex)^8}$$

input

```
Integrate[(a^2 + 2*a*b*x + b^2*x^2)^2/(d + e*x)^9,x]
```

output

$$\frac{-1/280*(35*a^4*e^4 + 20*a^3*b*e^3*(d + 8*e*x) + 10*a^2*b^2*e^2*(d^2 + 8*d*e*x + 28*e^2*x^2) + 4*a*b^3*e*(d^3 + 8*d^2*e*x + 28*d*e^2*x^2 + 56*e^3*x^3) + b^4*(d^4 + 8*d^3*e*x + 28*d^2*e^2*x^2 + 56*d*e^3*x^3 + 70*e^4*x^4))/(e^5*(d + e*x)^8)}$$

Rubi [A] (verified)

Time = 0.49 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {1098, 27, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\begin{aligned} & \int \frac{(a^2 + 2abx + b^2x^2)^2}{(d + ex)^9} dx \\ & \quad \downarrow \text{1098} \\ & \int \frac{b^4(a+bx)^4}{(d+ex)^9} dx \\ & \quad \downarrow \text{27} \\ & \int \frac{(a + bx)^4}{(d + ex)^9} dx \\ & \quad \downarrow \text{53} \\ & \int \left(-\frac{4b^3(bd - ae)}{e^4(d + ex)^6} + \frac{6b^2(bd - ae)^2}{e^4(d + ex)^7} - \frac{4b(bd - ae)^3}{e^4(d + ex)^8} + \frac{(ae - bd)^4}{e^4(d + ex)^9} + \frac{b^4}{e^4(d + ex)^5} \right) dx \\ & \quad \downarrow \text{2009} \\ & \frac{4b^3(bd - ae)}{5e^5(d + ex)^5} - \frac{b^2(bd - ae)^2}{e^5(d + ex)^6} + \frac{4b(bd - ae)^3}{7e^5(d + ex)^7} - \frac{(bd - ae)^4}{8e^5(d + ex)^8} - \frac{b^4}{4e^5(d + ex)^4} \end{aligned}$$

input

$$\text{Int}[(a^2 + 2*a*b*x + b^2*x^2)^2/(d + e*x)^9, x]$$

output

$$-1/8*(b*d - a*e)^4/(e^5*(d + e*x)^8) + (4*b*(b*d - a*e)^3)/(7*e^5*(d + e*x)^7) - (b^2*(b*d - a*e)^2)/(e^5*(d + e*x)^6) + (4*b^3*(b*d - a*e))/(5*e^5*(d + e*x)^5) - b^4/(4*e^5*(d + e*x)^4)$$
Defintions of rubi rules used

rule 27

```
Int[(a_)*(F x_), x_Symbol] := Simp[a Int[F x, x], x] /; FreeQ[a, x] && !MatchQ[F x, (b_)*(G x_)] /; FreeQ[b, x]
```

rule 53

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] := Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && IGtQ[m, 0] && (!IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 1098

```
Int[((d_.) + (e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] := Simp[1/c^p Int[(d + e*x)^m*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

rule 2009

```
Int[u_, x_Symbol] := Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 1.14 (sec) , antiderivative size = 171, normalized size of antiderivative = 1.46

| method | result |
|---------------|---|
| risch | $\frac{-\frac{b^4 x^4}{4e} - \frac{b^3(4ae+bd)x^3}{5e^2} - \frac{b^2(10e^2 a^2 + 4abde + b^2 d^2)x^2}{10e^3} - \frac{b(20e^3 a^3 + 10a^2 bd e^2 + 4a b^2 d^2 e + b^3 d^3)x}{35e^4} - \frac{35a^4 e^4 + 20a^3 bd e^3 + 10a^2 b^2 d^2 e^2 + 4a b^3 d^3 e + b^4 d^4}{280e^5}}{(ex+d)^8}$ |
| gospers | $-\frac{70b^4 x^4 e^4 + 224x^3 a b^3 e^4 + 56x^3 b^4 d e^3 + 280x^2 a^2 b^2 e^4 + 112x^2 a b^3 d e^3 + 28x^2 b^4 d^2 e^2 + 160x a^3 b e^4 + 80x a^2 b^2 d e^3 + 32x a b^3 d^2 e^2}{280e^5 (ex+d)^8}$ |
| default | $-\frac{b^4}{4e^5 (ex+d)^4} - \frac{4b(e^3 a^3 - 3a^2 bd e^2 + 3a b^2 d^2 e - b^3 d^3)}{7e^5 (ex+d)^7} - \frac{4b^3(ae-bd)}{5e^5 (ex+d)^5} - \frac{a^4 e^4 - 4a^3 bd e^3 + 6a^2 b^2 d^2 e^2 - 4a b^3 d^3 e + b^4 d^4}{8e^5 (ex+d)^8}$ |
| parallelrisch | $-\frac{70b^4 x^4 e^7 - 224a b^3 e^7 x^3 - 56b^4 d e^6 x^3 - 280a^2 b^2 e^7 x^2 - 112a b^3 d e^6 x^2 - 28b^4 d^2 e^5 x^2 - 160a^3 b e^7 x - 80a^2 b^2 d e^6 x - 32a b^3 d^2 e^5 x}{280e^8 (ex+d)^8}$ |
| norman | $-\frac{b^4 x^4}{4e} - \frac{(4a b^3 e^4 + b^4 d e^3)x^3}{5e^5} - \frac{(10a^2 b^2 e^5 + 4a b^3 d e^4 + b^4 d^2 e^3)x^2}{10e^6} - \frac{(20a^3 b e^6 + 10a^2 b^2 d e^5 + 4a b^3 d^2 e^4 + b^4 d^3 e^3)x}{35e^7} - \frac{35e^7 a^4 + 20a^3 bd e^6}{(ex+d)^8}$ |
| orering | $-\frac{(70b^4 x^4 e^4 + 224x^3 a b^3 e^4 + 56x^3 b^4 d e^3 + 280x^2 a^2 b^2 e^4 + 112x^2 a b^3 d e^3 + 28x^2 b^4 d^2 e^2 + 160x a^3 b e^4 + 80x a^2 b^2 d e^3 + 32x a b^3 d^2 e^2}{280e^5 (bx+a)^4 (ex+d)^8}$ |

input `int((b^2*x^2+2*a*b*x+a^2)^2/(e*x+d)^9,x,method=_RETURNVERBOSE)`

output
$$\frac{(-1/4/e*b^4*x^4-1/5*b^3/e^2*(4*a*e+b*d)*x^3-1/10*b^2/e^3*(10*a^2*e^2+4*a*b*d*e+b^2*d^2)*x^2-1/35*b/e^4*(20*a^3*e^3+10*a^2*b*d*e^2+4*a*b^2*d^2*e+b^3*d^3)*x-1/280/e^5*(35*a^4*e^4+20*a^3*b*d*e^3+10*a^2*b^2*d^2*e^2+4*a*b^3*d^3*e+b^4*d^4))/(e*x+d)^8}$$

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 258 vs. $2(109) = 218$.

Time = 0.07 (sec) , antiderivative size = 258, normalized size of antiderivative = 2.21

$$\int \frac{(a^2 + 2abx + b^2x^2)^2}{(d + ex)^9} dx =$$

$$-\frac{70b^4e^4x^4 + b^4d^4 + 4ab^3d^3e + 10a^2b^2d^2e^2 + 20a^3bde^3 + 35a^4e^4 + 56(b^4de^3 + 4ab^3e^4)x^3 + 28(b^4d^2e^2}{280(e^{13}x^8 + 8de^{12}x^7 + 28d^2e^{11}x^6 + 56d^3e^{10}x^5 + 70d^4e^9x^4 + 56$$

input `integrate((b^2*x^2+2*a*b*x+a^2)^2/(e*x+d)^9,x, algorithm="fricas")`

output

```
-1/280*(70*b^4*e^4*x^4 + b^4*d^4 + 4*a*b^3*d^3*e + 10*a^2*b^2*d^2*e^2 + 20
*a^3*b*d*e^3 + 35*a^4*e^4 + 56*(b^4*d*e^3 + 4*a*b^3*e^4)*x^3 + 28*(b^4*d^2
*e^2 + 4*a*b^3*d*e^3 + 10*a^2*b^2*e^4)*x^2 + 8*(b^4*d^3*e + 4*a*b^3*d^2*e^
2 + 10*a^2*b^2*d*e^3 + 20*a^3*b*e^4)*x)/(e^13*x^8 + 8*d*e^12*x^7 + 28*d^2*
e^11*x^6 + 56*d^3*e^10*x^5 + 70*d^4*e^9*x^4 + 56*d^5*e^8*x^3 + 28*d^6*e^7*
x^2 + 8*d^7*e^6*x + d^8*e^5)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 279 vs. $2(104) = 208$.

Time = 10.04 (sec) , antiderivative size = 279, normalized size of antiderivative = 2.38

$$\int \frac{(a^2 + 2abx + b^2x^2)^2}{(d + ex)^9} dx$$

$$= \frac{-35a^4e^4 - 20a^3bde^3 - 10a^2b^2d^2e^2 - 4ab^3d^3e - b^4d^4 - 70b^4e^4x^4 + x^3(-224ab^3e^4 - 56b^4de^3) + x^2(-280a^2b^2d^2e^4 - 112a^2b^2d^2e^4 - 112a^2b^2d^2e^4 - 28b^4d^2e^2) + x(-160a^3bde^4 - 80a^3bde^4 - 32a^3bde^4 - 8b^4d^3e) + (-35a^4e^4 - 20a^3bde^3 - 10a^2b^2d^2e^2 - 4ab^3d^3e - b^4d^4 - 70b^4e^4x^4 + x^3(-224ab^3e^4 - 56b^4de^3) + x^2(-280a^2b^2d^2e^4 - 112a^2b^2d^2e^4 - 112a^2b^2d^2e^4 - 28b^4d^2e^2) + x(-160a^3bde^4 - 80a^3bde^4 - 32a^3bde^4 - 8b^4d^3e))}{280d^8e^5 + 2240d^7e^6x + 7840d^6e^7x^2 + 15680d^5e^8x^3 + 19600d^4e^9x^4 + 15680d^3e^10x^5 + 7840d^2e^11x^6 + 2240de^12x^7 + 280e^13x^8}$$

input

```
integrate((b**2*x**2+2*a*b*x+a**2)**2/(e*x+d)**9,x)
```

output

```
(-35*a**4*e**4 - 20*a**3*b*d*e**3 - 10*a**2*b**2*d**2*e**2 - 4*a*b**3*d**3
*e - b**4*d**4 - 70*b**4*e**4*x**4 + x**3*(-224*a*b**3*e**4 - 56*b**4*d*e*
*3) + x**2*(-280*a**2*b**2*d**2*e**4 - 112*a*b**3*d*e**3 - 28*b**4*d**2*e**2) +
x*(-160*a**3*b*e**4 - 80*a**2*b**2*d*e**3 - 32*a*b**3*d**2*e**2 - 8*b**4*
d**3*e))/(280*d**8*e**5 + 2240*d**7*e**6*x + 7840*d**6*e**7*x**2 + 15680*d
**5*e**8*x**3 + 19600*d**4*e**9*x**4 + 15680*d**3*e**10*x**5 + 7840*d**2*
e**11*x**6 + 2240*d*e**12*x**7 + 280*e**13*x**8)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 258 vs. $2(109) = 218$.

Time = 0.04 (sec) , antiderivative size = 258, normalized size of antiderivative = 2.21

$$\int \frac{(a^2 + 2abx + b^2x^2)^2}{(d + ex)^9} dx =$$

$$-\frac{70b^4e^4x^4 + b^4d^4 + 4ab^3d^3e + 10a^2b^2d^2e^2 + 20a^3bde^3 + 35a^4e^4 + 56(b^4de^3 + 4ab^3e^4)x^3 + 28(b^4d^2e^2 + 4ab^3de^3 + 10a^2b^2d^2e^4)x^2 + 8(b^4d^3e + 4ab^3d^2e^2 + 10a^2b^2d^2e^4)x + (-35a^4e^4 - 20a^3bde^3 - 10a^2b^2d^2e^2 - 4ab^3d^3e - b^4d^4 - 70b^4e^4x^4 + x^3(-224ab^3e^4 - 56b^4de^3) + x^2(-280a^2b^2d^2e^4 - 112a^2b^2d^2e^4 - 112a^2b^2d^2e^4 - 28b^4d^2e^2) + x(-160a^3bde^4 - 80a^3bde^4 - 32a^3bde^4 - 8b^4d^3e))}{280(e^{13}x^8 + 8de^{12}x^7 + 28d^2e^{11}x^6 + 56d^3e^{10}x^5 + 70d^4e^9x^4 + 15680d^3e^{10}x^5 + 7840d^2e^{11}x^6 + 2240de^{12}x^7 + 280e^{13}x^8)}$$

input `integrate((b^2*x^2+2*a*b*x+a^2)^2/(e*x+d)^9,x, algorithm="maxima")`

output
$$-1/280*(70*b^4*e^4*x^4 + b^4*d^4 + 4*a*b^3*d^3*e + 10*a^2*b^2*d^2*e^2 + 20*a^3*b*d*e^3 + 35*a^4*e^4 + 56*(b^4*d*e^3 + 4*a*b^3*e^4)*x^3 + 28*(b^4*d^2*e^2 + 4*a*b^3*d*e^3 + 10*a^2*b^2*d*e^4)*x^2 + 8*(b^4*d^3*e + 4*a*b^3*d^2*e^2 + 10*a^2*b^2*d*e^3 + 20*a^3*b*d*e^4)*x)/(e^13*x^8 + 8*d*e^12*x^7 + 28*d^2*e^11*x^6 + 56*d^3*e^10*x^5 + 70*d^4*e^9*x^4 + 56*d^5*e^8*x^3 + 28*d^6*e^7*x^2 + 8*d^7*e^6*x + d^8*e^5)$$

Giac [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 184, normalized size of antiderivative = 1.57

$$\int \frac{(a^2 + 2abx + b^2x^2)^2}{(d + ex)^9} dx = \frac{70b^4e^4x^4 + 56b^4de^3x^3 + 224ab^3e^4x^3 + 28b^4d^2e^2x^2 + 112ab^3de^3x^2 + 280a^2b^2e^4x^2 + 8b^4d^3ex + 32ab^3d^3e}{280(ex + d)^8e^5}$$

input `integrate((b^2*x^2+2*a*b*x+a^2)^2/(e*x+d)^9,x, algorithm="giac")`

output
$$-1/280*(70*b^4*e^4*x^4 + 56*b^4*d*e^3*x^3 + 224*a*b^3*e^4*x^3 + 28*b^4*d^2*e^2*x^2 + 112*a*b^3*d*e^3*x^2 + 280*a^2*b^2*d*e^4*x^2 + 8*b^4*d^3*e*x + 32*a*b^3*d^2*e^2*x + 80*a^2*b^2*d*e^3*x + 160*a^3*b*d*e^4*x + b^4*d^4 + 4*a*b^3*d^3*e + 10*a^2*b^2*d^2*e^2 + 20*a^3*b*d*e^3 + 35*a^4*e^4)/((e*x + d)^8*e^5)$$

Mupad [B] (verification not implemented)

Time = 8.73 (sec) , antiderivative size = 248, normalized size of antiderivative = 2.12

$$\int \frac{(a^2 + 2abx + b^2x^2)^2}{(d + ex)^9} dx = \frac{\frac{35a^4e^4 + 20a^3bde^3 + 10a^2b^2d^2e^2 + 4ab^3d^3e + b^4d^4}{280e^5} + \frac{b^4x^4}{4e} + \frac{b^3x^3(4ae + bd)}{5e^2} + \frac{bx(20a^3e^3 + 10a^2bde^2 + 4ab^2d^2e + b^3d^3)}{35e^4} + \frac{b^2}{280e^5}}{d^8 + 8d^7ex + 28d^6e^2x^2 + 56d^5e^3x^3 + 70d^4e^4x^4 + 56d^3e^5x^5 + 28d^2e^6x^6 + 8de^7x^7}$$

3.82 $\int \frac{(a^2+2abx+b^2x^2)^2}{(d+ex)^{10}} dx$

| | |
|---|-----|
| Optimal result | 655 |
| Mathematica [A] (verified) | 655 |
| Rubi [A] (verified) | 656 |
| Maple [A] (verified) | 657 |
| Fricas [B] (verification not implemented) | 658 |
| Sympy [B] (verification not implemented) | 659 |
| Maxima [B] (verification not implemented) | 660 |
| Giac [A] (verification not implemented) | 660 |
| Mupad [B] (verification not implemented) | 661 |
| Reduce [B] (verification not implemented) | 661 |

Optimal result

Integrand size = 26, antiderivative size = 119

$$\int \frac{(a^2 + 2abx + b^2x^2)^2}{(d + ex)^{10}} dx = -\frac{(bd - ae)^4}{9e^5(d + ex)^9} + \frac{b(bd - ae)^3}{2e^5(d + ex)^8} - \frac{6b^2(bd - ae)^2}{7e^5(d + ex)^7} + \frac{2b^3(bd - ae)}{3e^5(d + ex)^6} - \frac{b^4}{5e^5(d + ex)^5}$$

output

```
-1/9*(-a*e+b*d)^4/e^5/(e*x+d)^9+1/2*b*(-a*e+b*d)^3/e^5/(e*x+d)^8-6/7*b^2*(-a*e+b*d)^2/e^5/(e*x+d)^7+2/3*b^3*(-a*e+b*d)/e^5/(e*x+d)^6-1/5*b^4/e^5/(e*x+d)^5
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 144, normalized size of antiderivative = 1.21

$$\int \frac{(a^2 + 2abx + b^2x^2)^2}{(d + ex)^{10}} dx = \frac{70a^4e^4 + 35a^3be^3(d + 9ex) + 15a^2b^2e^2(d^2 + 9dex + 36e^2x^2) + 5ab^3e(d^3 + 9d^2ex + 36de^2x^2 + 84e^3x^3) - 630e^5(d + ex)^9}{630e^5(d + ex)^9}$$

input

```
Integrate[(a^2 + 2*a*b*x + b^2*x^2)^2/(d + e*x)^10,x]
```

output

$$\frac{-1/630*(70*a^4*e^4 + 35*a^3*b*e^3*(d + 9*e*x) + 15*a^2*b^2*e^2*(d^2 + 9*d*e*x + 36*e^2*x^2) + 5*a*b^3*e*(d^3 + 9*d^2*e*x + 36*d*e^2*x^2 + 84*e^3*x^3) + b^4*(d^4 + 9*d^3*e*x + 36*d^2*e^2*x^2 + 84*d*e^3*x^3 + 126*e^4*x^4))/(e^5*(d + e*x)^9)}$$

Rubi [A] (verified)

Time = 0.49 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {1098, 27, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\begin{aligned} & \int \frac{(a^2 + 2abx + b^2x^2)^2}{(d + ex)^{10}} dx \\ & \quad \downarrow \text{1098} \\ & \int \frac{b^4(a+bx)^4}{(d+ex)^{10}} dx \\ & \quad \downarrow \text{27} \\ & \int \frac{(a + bx)^4}{(d + ex)^{10}} dx \\ & \quad \downarrow \text{53} \\ & \int \left(-\frac{4b^3(bd - ae)}{e^4(d + ex)^7} + \frac{6b^2(bd - ae)^2}{e^4(d + ex)^8} - \frac{4b(bd - ae)^3}{e^4(d + ex)^9} + \frac{(ae - bd)^4}{e^4(d + ex)^{10}} + \frac{b^4}{e^4(d + ex)^6} \right) dx \\ & \quad \downarrow \text{2009} \\ & \frac{2b^3(bd - ae)}{3e^5(d + ex)^6} - \frac{6b^2(bd - ae)^2}{7e^5(d + ex)^7} + \frac{b(bd - ae)^3}{2e^5(d + ex)^8} - \frac{(bd - ae)^4}{9e^5(d + ex)^9} - \frac{b^4}{5e^5(d + ex)^5} \end{aligned}$$

input

$$\text{Int}[(a^2 + 2*a*b*x + b^2*x^2)^2/(d + e*x)^10,x]$$

output

$$-1/9*(b*d - a*e)^4/(e^5*(d + e*x)^9) + (b*(b*d - a*e)^3)/(2*e^5*(d + e*x)^8) - (6*b^2*(b*d - a*e)^2)/(7*e^5*(d + e*x)^7) + (2*b^3*(b*d - a*e))/(3*e^5*(d + e*x)^6) - b^4/(5*e^5*(d + e*x)^5)$$
Defintions of rubi rules used

rule 27

$$\text{Int}[(a_*)(Fx_), x_Symbol] \rightarrow \text{Simp}[a \text{ Int}[Fx, x], x] /; \text{FreeQ}[a, x] \ \&\& \ !\text{MatchQ}[Fx, (b_*)(Gx_)] /; \text{FreeQ}[b, x]$$

rule 53

$$\text{Int}[((a_.) + (b_.)(x_))^{(m_.)}*((c_.) + (d_.)(x_))^{(n_.)}, x_Symbol] \rightarrow \text{Int}[\text{ExpandIntegrand}[(a + b*x)^m*(c + d*x)^n, x], x] /; \text{FreeQ}[\{a, b, c, d, n\}, x] \ \&\& \ \text{IGtQ}[m, 0] \ \&\& \ (\ !\text{IntegerQ}[n] \ || \ (\text{EqQ}[c, 0] \ \&\& \ \text{LeQ}[7*m + 4*n + 4, 0]) \ || \ \text{LtQ}[9*m + 5*(n + 1), 0] \ || \ \text{GtQ}[m + n + 2, 0])$$

rule 1098

$$\text{Int}[((d_.) + (e_.)(x_))^{(m_.)}*((a_) + (b_.)(x_) + (c_.)(x_)^2)^{(p_.)}, x_Symbol] \rightarrow \text{Simp}[1/c^p \text{ Int}[(d + e*x)^m*(b/2 + c*x)^{(2*p)}, x], x] /; \text{FreeQ}[\{a, b, c, d, e, m\}, x] \ \&\& \ \text{EqQ}[b^2 - 4*a*c, 0] \ \&\& \ \text{IntegerQ}[p]$$

rule 2009

$$\text{Int}[u_, x_Symbol] \rightarrow \text{Simp}[\text{IntSum}[u, x], x] /; \text{SumQ}[u]$$
Maple [A] (verified)

Time = 1.14 (sec) , antiderivative size = 171, normalized size of antiderivative = 1.44

| method | result |
|--------------|---|
| risch | $\frac{-\frac{b^4 x^4}{5e} - \frac{2b^3(5ae+bd)x^3}{15e^2} - \frac{2b^2(15e^2a^2+5abde+b^2d^2)x^2}{35e^3} - \frac{b(35e^3a^3+15a^2bde^2+5ab^2d^2e+b^3d^3)x}{70e^4} - \frac{70a^4e^4+35a^3bde^3+15a^2b^2d^2e^2+630e^5}{630e^5}}{(ex+d)^9}$ |
| gosper | $-\frac{126b^4x^4e^4+420x^3ab^3e^4+84x^3b^4de^3+540x^2a^2b^2e^4+180x^2ab^3de^3+36x^2b^4d^2e^2+315xa^3be^4+135xa^2b^2de^3+45xab^3d^2e^2}{630e^5(ex+d)^9}$ |
| default | $-\frac{a^4e^4-4a^3bde^3+6a^2b^2d^2e^2-4ab^3d^3e+b^4d^4}{9e^5(ex+d)^9} - \frac{6b^2(e^2a^2-2abde+b^2d^2)}{7e^5(ex+d)^7} - \frac{b^4}{5e^5(ex+d)^5} - \frac{b(e^3a^3-3a^2bde^2+3ab^2d^2e)}{2e^5(ex+d)^8}$ |
| parallelrisc | $-\frac{126b^4x^4e^8-420ab^3e^8x^3-84b^4de^7x^3-540a^2b^2e^8x^2-180ab^3de^7x^2-36b^4d^2e^6x^2-315a^3be^8x-135a^2b^2de^7x-45ab^3d^2e^6x}{630e^9(ex+d)^9}$ |
| norman | $-\frac{b^4x^4}{5e} - \frac{2(5ab^3e^5+b^4de^4)x^3}{15e^6} - \frac{2(15a^2b^2e^6+5ab^3de^5+b^4d^2e^4)x^2}{35e^7} - \frac{(35a^3be^7+15a^2b^2de^6+5ab^3d^2e^5+b^4d^3e^4)x}{70e^8} - \frac{70a^4e^8+35a^3bde^7}{630e^9}$ |
| orering | $-\frac{(126b^4x^4e^4+420x^3ab^3e^4+84x^3b^4de^3+540x^2a^2b^2e^4+180x^2ab^3de^3+36x^2b^4d^2e^2+315xa^3be^4+135xa^2b^2de^3+45xab^3d^2e^2)}{630e^5(bx+a)^4(ex+d)^9}$ |

input `int((b^2*x^2+2*a*b*x+a^2)^2/(e*x+d)^10,x,method=_RETURNVERBOSE)`

output
$$\frac{(-1/5/e*b^4*x^4-2/15/e^2*b^3*(5*a*e+b*d)*x^3-2/35*b^2/e^3*(15*a^2*e^2+5*a*b*d*e+b^2*d^2)*x^2-1/70*b/e^4*(35*a^3*e^3+15*a^2*b*d*e^2+5*a*b^2*d^2*e+b^3*d^3)*x-1/630/e^5*(70*a^4*e^4+35*a^3*b*d*e^3+15*a^2*b^2*d^2*e^2+5*a*b^3*d^3*e+b^4*d^4))/(e*x+d)^9}$$

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 269 vs. 2(109) = 218.

Time = 0.08 (sec) , antiderivative size = 269, normalized size of antiderivative = 2.26

$$\int \frac{(a^2 + 2abx + b^2x^2)^2}{(d + ex)^{10}} dx =$$

$$-\frac{126b^4e^4x^4 + b^4d^4 + 5ab^3d^3e + 15a^2b^2d^2e^2 + 35a^3bde^3 + 70a^4e^4 + 84(b^4de^3 + 5ab^3e^4)x^3 + 36(b^4d^2e^2 + 3ab^3de^3 + 3a^2b^2d^2e^2)x^2 + 126a^3bde^3 + 135a^2b^2de^3 + 45ab^3d^2e^2 + 45a^4e^4}{630(e^{14}x^9 + 9de^{13}x^8 + 36d^2e^{12}x^7 + 84d^3e^{11}x^6 + 126d^4e^{10}x^5 + 126d^5e^9)}$$

input `integrate((b^2*x^2+2*a*b*x+a^2)^2/(e*x+d)^10,x, algorithm="fricas")`

output

```
-1/630*(126*b^4*e^4*x^4 + b^4*d^4 + 5*a*b^3*d^3*e + 15*a^2*b^2*d^2*e^2 + 3
5*a^3*b*d*e^3 + 70*a^4*e^4 + 84*(b^4*d*e^3 + 5*a*b^3*e^4)*x^3 + 36*(b^4*d^
2*e^2 + 5*a*b^3*d*e^3 + 15*a^2*b^2*e^4)*x^2 + 9*(b^4*d^3*e + 5*a*b^3*d^2*e
^2 + 15*a^2*b^2*d*e^3 + 35*a^3*b*e^4)*x)/(e^14*x^9 + 9*d*e^13*x^8 + 36*d^2
*e^12*x^7 + 84*d^3*e^11*x^6 + 126*d^4*e^10*x^5 + 126*d^5*e^9*x^4 + 84*d^6*
e^8*x^3 + 36*d^7*e^7*x^2 + 9*d^8*e^6*x + d^9*e^5)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 291 vs. $2(105) = 210$.

Time = 20.36 (sec) , antiderivative size = 291, normalized size of antiderivative = 2.45

$$\int \frac{(a^2 + 2abx + b^2x^2)^2}{(d + ex)^{10}} dx$$

$$= \frac{-70a^4e^4 - 35a^3bde^3 - 15a^2b^2d^2e^2 - 5ab^3d^3e - b^4d^4 - 126b^4e^4x^4 + x^3(-420ab^3e^4 - 84b^4de^3) + x^2(-540a^2b^2d^2e^4 - 180a^2b^2d^2e^3 - 36b^4d^2e^2) + x(-315a^3b^3e^4 - 135a^3b^2d^3e^3 - 45a^3b^2d^2e^2 - 9b^4d^3e)}{630d^9e^5 + 5670d^8e^6x + 22680d^7e^7x^2 + 52920d^6e^8x^3 + 79380d^5e^9x^4 + 79380d^4e^{10}x^5 + 52920d^3e^{11}x^6 + 22680d^2e^{12}x^7 + 5670de^{13}x^8 + 630e^{14}x^9}$$

input

```
integrate((b**2*x**2+2*a*b*x+a**2)**2/(e*x+d)**10,x)
```

output

```
(-70*a**4*e**4 - 35*a**3*b*d*e**3 - 15*a**2*b**2*d**2*e**2 - 5*a*b**3*d**3
*e - b**4*d**4 - 126*b**4*e**4*x**4 + x**3*(-420*a*b**3*e**4 - 84*b**4*d*e
**3) + x**2*(-540*a**2*b**2*d**2*e**4 - 180*a**2*b**2*d**2*e**3 - 36*b**4*d**2*e**2)
+ x*(-315*a**3*b**3*e**4 - 135*a**3*b**2*d**3*e**3 - 45*a**3*b**2*d**2*e**2 - 9*b**
4*d**3*e))/(630*d**9*e**5 + 5670*d**8*e**6*x + 22680*d**7*e**7*x**2 + 5292
0*d**6*e**8*x**3 + 79380*d**5*e**9*x**4 + 79380*d**4*e**10*x**5 + 52920*d*
**3*e**11*x**6 + 22680*d**2*e**12*x**7 + 5670*d*e**13*x**8 + 630*e**14*x**9
)
```


Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 269 vs. $2(109) = 218$.

Time = 0.05 (sec) , antiderivative size = 269, normalized size of antiderivative = 2.26

$$\int \frac{(a^2 + 2abx + b^2x^2)^2}{(d + ex)^{10}} dx =$$

$$-\frac{126b^4e^4x^4 + b^4d^4 + 5ab^3d^3e + 15a^2b^2d^2e^2 + 35a^3bde^3 + 70a^4e^4 + 84(b^4de^3 + 5ab^3e^4)x^3 + 36(b^4d^2e^2 + 5a^3b^3d^3e + 15a^2b^2d^2e^2 + 35a^3bde^3 + 70a^4e^4)x^2 + 9(b^4d^3e + 5a^3b^3d^2e^2 + 15a^2b^2d^2e^3 + 35a^3b^3e^4)x}{630(e^{14}x^9 + 9de^{13}x^8 + 36d^2e^{12}x^7 + 84d^3e^{11}x^6 + 126d^4e^{10}x^5 + 126d^5e^9x^4 + 84d^6e^8x^3 + 36d^7e^7x^2 + 9d^8e^6x + d^9e^5)}$$

input `integrate((b^2*x^2+2*a*b*x+a^2)^2/(e*x+d)^10,x, algorithm="maxima")`

output `-1/630*(126*b^4*e^4*x^4 + b^4*d^4 + 5*a*b^3*d^3*e + 15*a^2*b^2*d^2*e^2 + 35*a^3*b*d*e^3 + 70*a^4*e^4 + 84*(b^4*d*e^3 + 5*a*b^3*e^4)*x^3 + 36*(b^4*d^2*e^2 + 5*a*b^3*d*e^3 + 15*a^2*b^2*e^4)*x^2 + 9*(b^4*d^3*e + 5*a*b^3*d^2*e^2 + 15*a^2*b^2*d*e^3 + 35*a^3*b^3*e^4)*x)/(e^14*x^9 + 9*d*e^13*x^8 + 36*d^2*e^12*x^7 + 84*d^3*e^11*x^6 + 126*d^4*e^10*x^5 + 126*d^5*e^9*x^4 + 84*d^6*e^8*x^3 + 36*d^7*e^7*x^2 + 9*d^8*e^6*x + d^9*e^5)`

Giac [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 184, normalized size of antiderivative = 1.55

$$\int \frac{(a^2 + 2abx + b^2x^2)^2}{(d + ex)^{10}} dx =$$

$$-\frac{126b^4e^4x^4 + 84b^4de^3x^3 + 420ab^3e^4x^3 + 36b^4d^2e^2x^2 + 180ab^3de^3x^2 + 540a^2b^2e^4x^2 + 9b^4d^3ex + 45a^3b^3d^3e + 15a^2b^2d^2e^2 + 35a^3b^3d^3e + 70a^4e^4}{630(ex + d)^9e^5}$$

input `integrate((b^2*x^2+2*a*b*x+a^2)^2/(e*x+d)^10,x, algorithm="giac")`

output `-1/630*(126*b^4*e^4*x^4 + 84*b^4*d*e^3*x^3 + 420*a*b^3*e^4*x^3 + 36*b^4*d^2*e^2*x^2 + 180*a*b^3*d*e^3*x^2 + 540*a^2*b^2*e^4*x^2 + 9*b^4*d^3*e*x + 45*a*b^3*d^3*e + 15*a^2*b^2*d^2*e^2 + 35*a^3*b^3*d^3*e + 70*a^4*e^4)/((e*x + d)^9*e^5)`

Mupad [B] (verification not implemented)

Time = 8.78 (sec) , antiderivative size = 259, normalized size of antiderivative = 2.18

$$\int \frac{(a^2 + 2abx + b^2x^2)^2}{(d + ex)^{10}} dx =$$

$$-\frac{70a^4e^4 + 35a^3bde^3 + 15a^2b^2d^2e^2 + 5ab^3d^3e + b^4d^4}{630e^5} + \frac{b^4x^4}{5e} + \frac{2b^3x^3(5ae + bd)}{15e^2} + \frac{bx(35a^3e^3 + 15a^2bde^2 + 5ab^2d^2e + b^3d^3)}{70e^4} + \dots$$

$$-\frac{d^9 + 9d^8ex + 36d^7e^2x^2 + 84d^6e^3x^3 + 126d^5e^4x^4 + 126d^4e^5x^5 + 84d^3e^6x^6 + 36d^2e^7x^7 + \dots}{630e^5}$$

input `int((a^2 + b^2*x^2 + 2*a*b*x)^2/(d + e*x)^10,x)`output `-((70*a^4*e^4 + b^4*d^4 + 15*a^2*b^2*d^2*e^2 + 5*a*b^3*d^3*e + 35*a^3*b*d*e^3)/(630*e^5) + (b^4*x^4)/(5*e) + (2*b^3*x^3*(5*a*e + b*d))/(15*e^2) + (b*x*(35*a^3*e^3 + b^3*d^3 + 5*a*b^2*d^2*e + 15*a^2*b*d*e^2))/(70*e^4) + (2*b^2*x^2*(15*a^2*e^2 + b^2*d^2 + 5*a*b*d*e))/(35*e^3))/(d^9 + e^9*x^9 + 9*d*e^8*x^8 + 36*d^7*e^2*x^2 + 84*d^6*e^3*x^3 + 126*d^5*e^4*x^4 + 126*d^4*e^5*x^5 + 84*d^3*e^6*x^6 + 36*d^2*e^7*x^7 + 9*d^8*e*x)`**Reduce [B] (verification not implemented)**

Time = 0.20 (sec) , antiderivative size = 273, normalized size of antiderivative = 2.29

$$\int \frac{(a^2 + 2abx + b^2x^2)^2}{(d + ex)^{10}} dx$$

$$= \frac{-126b^4e^4x^4 - 420ab^3e^4x^3 - 84b^4de^3x^3 - 540a^2b^2e^4x^2 - 180ab^3de^3x^2 - 36b^4d^2e^2x^2 - 315a^3be^4x - 135a^4e^4}{630e^5(e^9x^9 + 9de^8x^8 + 36d^2e^7x^7 + 84d^3e^6x^6 + 126d^4e^5x^5 + 126d^5e^4x^4 + 84d^6e^3x^3 + 36d^7e^2x^2 + 9d^8ex + d^9)}$$

input `int((b^2*x^2+2*a*b*x+a^2)^2/(e*x+d)^10,x)`output `(- 70*a**4*e**4 - 35*a**3*b*d*e**3 - 315*a**3*b*e**4*x - 15*a**2*b**2*d**2*e**2 - 135*a**2*b**2*d*e**3*x - 540*a**2*b**2*e**4*x**2 - 5*a*b**3*d**3*e - 45*a*b**3*d**2*e**2*x - 180*a*b**3*d*e**3*x**2 - 420*a*b**3*e**4*x**3 - b**4*d**4 - 9*b**4*d**3*e*x - 36*b**4*d**2*e**2*x**2 - 84*b**4*d*e**3*x**3 - 126*b**4*e**4*x**4)/(630*e**5*(d**9 + 9*d**8*e*x + 36*d**7*e**2*x**2 + 84*d**6*e**3*x**3 + 126*d**5*e**4*x**4 + 126*d**4*e**5*x**5 + 84*d**3*e**6*x**6 + 36*d**2*e**7*x**7 + 9*d*e**8*x**8 + e**9*x**9))`

3.83
$$\int \frac{(a^2+2abx+b^2x^2)^2}{(d+ex)^{11}} dx$$

| | |
|---|-----|
| Optimal result | 662 |
| Mathematica [A] (verified) | 662 |
| Rubi [A] (verified) | 663 |
| Maple [A] (verified) | 664 |
| Fricas [B] (verification not implemented) | 665 |
| Sympy [B] (verification not implemented) | 666 |
| Maxima [B] (verification not implemented) | 667 |
| Giac [A] (verification not implemented) | 667 |
| Mupad [B] (verification not implemented) | 668 |
| Reduce [B] (verification not implemented) | 668 |

Optimal result

Integrand size = 26, antiderivative size = 119

$$\int \frac{(a^2 + 2abx + b^2x^2)^2}{(d + ex)^{11}} dx = -\frac{(bd - ae)^4}{10e^5(d + ex)^{10}} + \frac{4b(bd - ae)^3}{9e^5(d + ex)^9} - \frac{3b^2(bd - ae)^2}{4e^5(d + ex)^8} + \frac{4b^3(bd - ae)}{7e^5(d + ex)^7} - \frac{b^4}{6e^5(d + ex)^6}$$

output

```
-1/10*(-a*e+b*d)^4/e^5/(e*x+d)^10+4/9*b*(-a*e+b*d)^3/e^5/(e*x+d)^9-3/4*b^2*(-a*e+b*d)^2/e^5/(e*x+d)^8+4/7*b^3*(-a*e+b*d)/e^5/(e*x+d)^7-1/6*b^4/e^5/(e*x+d)^6
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 144, normalized size of antiderivative = 1.21

$$\int \frac{(a^2 + 2abx + b^2x^2)^2}{(d + ex)^{11}} dx = \frac{-126a^4e^4 + 56a^3be^3(d + 10ex) + 21a^2b^2e^2(d^2 + 10dex + 45e^2x^2) + 6ab^3e(d^3 + 10d^2ex + 45de^2x^2 + 120d^2ex^2 + 120de^2x^3 + 60e^3x^4)}{1260e^5(d + ex)^{10}}$$

input

```
Integrate[(a^2 + 2*a*b*x + b^2*x^2)^2/(d + e*x)^11,x]
```

output

$$\frac{-1/1260*(126*a^4*e^4 + 56*a^3*b*e^3*(d + 10*e*x) + 21*a^2*b^2*e^2*(d^2 + 10*d*e*x + 45*e^2*x^2) + 6*a*b^3*e*(d^3 + 10*d^2*e*x + 45*d*e^2*x^2 + 120*e^3*x^3) + b^4*(d^4 + 10*d^3*e*x + 45*d^2*e^2*x^2 + 120*d*e^3*x^3 + 210*e^4*x^4))/(e^5*(d + e*x)^10)}$$

Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {1098, 27, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\begin{aligned} & \int \frac{(a^2 + 2abx + b^2x^2)^2}{(d + ex)^{11}} dx \\ & \quad \downarrow 1098 \\ & \quad \int \frac{b^4(a+bx)^4}{(d+ex)^{11}} dx \\ & \quad \quad \downarrow 27 \\ & \quad \quad \int \frac{(a + bx)^4}{(d + ex)^{11}} dx \\ & \quad \quad \quad \downarrow 53 \\ & \quad \quad \quad \int \left(-\frac{4b^3(bd - ae)}{e^4(d + ex)^8} + \frac{6b^2(bd - ae)^2}{e^4(d + ex)^9} - \frac{4b(bd - ae)^3}{e^4(d + ex)^{10}} + \frac{(ae - bd)^4}{e^4(d + ex)^{11}} + \frac{b^4}{e^4(d + ex)^7} \right) dx \\ & \quad \quad \quad \quad \downarrow 2009 \\ & \quad \quad \quad \quad \frac{4b^3(bd - ae)}{7e^5(d + ex)^7} - \frac{3b^2(bd - ae)^2}{4e^5(d + ex)^8} + \frac{4b(bd - ae)^3}{9e^5(d + ex)^9} - \frac{(bd - ae)^4}{10e^5(d + ex)^{10}} - \frac{b^4}{6e^5(d + ex)^6} \end{aligned}$$

input

$$\text{Int}[(a^2 + 2*a*b*x + b^2*x^2)^2/(d + e*x)^11,x]$$

output

$$-1/10*(b*d - a*e)^4/(e^5*(d + e*x)^{10}) + (4*b*(b*d - a*e)^3)/(9*e^5*(d + e*x)^9) - (3*b^2*(b*d - a*e)^2)/(4*e^5*(d + e*x)^8) + (4*b^3*(b*d - a*e))/(7*e^5*(d + e*x)^7) - b^4/(6*e^5*(d + e*x)^6)$$
Defintions of rubi rules used

rule 27

$$\text{Int}[(a_*)(Fx_), x_Symbol] \rightarrow \text{Simp}[a \text{ Int}[Fx, x], x] /; \text{FreeQ}[a, x] \ \&\& \ !\text{MatchQ}[Fx, (b_*)(Gx_)] /; \text{FreeQ}[b, x]$$

rule 53

$$\text{Int}[((a_.) + (b_.)(x_))^{(m_.)}*((c_.) + (d_.)(x_))^{(n_.)}, x_Symbol] \rightarrow \text{Int}[\text{ExpandIntegrand}[(a + b*x)^m*(c + d*x)^n, x], x] /; \text{FreeQ}[\{a, b, c, d, n\}, x] \ \&\& \ \text{IGtQ}[m, 0] \ \&\& \ (\ !\text{IntegerQ}[n] \ || \ (\text{EqQ}[c, 0] \ \&\& \ \text{LeQ}[7*m + 4*n + 4, 0]) \ || \ \text{LtQ}[9*m + 5*(n + 1), 0] \ || \ \text{GtQ}[m + n + 2, 0])$$

rule 1098

$$\text{Int}[((d_.) + (e_.)(x_))^{(m_.)}*((a_) + (b_.)(x_) + (c_.)(x_)^2)^{(p_.)}, x_Symbol] \rightarrow \text{Simp}[1/c^p \text{ Int}[(d + e*x)^m*(b/2 + c*x)^{(2*p)}, x], x] /; \text{FreeQ}[\{a, b, c, d, e, m\}, x] \ \&\& \ \text{EqQ}[b^2 - 4*a*c, 0] \ \&\& \ \text{IntegerQ}[p]$$

rule 2009

$$\text{Int}[u_, x_Symbol] \rightarrow \text{Simp}[\text{IntSum}[u, x], x] /; \text{SumQ}[u]$$
Maple [A] (verified)

Time = 1.09 (sec) , antiderivative size = 171, normalized size of antiderivative = 1.44

| method | result |
|---------------|--|
| risch | $\frac{-\frac{b^4 x^4}{6e} - \frac{2b^3(6ae+bd)x^3}{21e^2} - \frac{b^2(21e^2 a^2 + 6abde + b^2 d^2)x^2}{28e^3} - \frac{b(56e^3 a^3 + 21a^2 bde^2 + 6a b^2 d^2 e + b^3 d^3)x}{126e^4} - \frac{126a^4 e^4 + 56a^3 bde^3 + 21a^2 b^2 d^2 e^2 + 1260e^5}{(ex+d)^{10}}$ |
| gospers | $-\frac{210b^4 x^4 e^4 + 720x^3 a b^3 e^4 + 120x^3 b^4 d e^3 + 945x^2 a^2 b^2 e^4 + 270x^2 a b^3 d e^3 + 45x^2 b^4 d^2 e^2 + 560x a^3 b e^4 + 210x a^2 b^2 d e^3 + 60x a b^3 d^2 e^2 + 1260e^5 (ex+d)^{10}}$ |
| default | $-\frac{4b(e^3 a^3 - 3a^2 bde^2 + 3ab^2 d^2 e - b^3 d^3)}{9e^5 (ex+d)^9} - \frac{4b^3 (ae-bd)}{7e^5 (ex+d)^7} - \frac{3b^2 (e^2 a^2 - 2abde + b^2 d^2)}{4e^5 (ex+d)^8} - \frac{a^4 e^4 - 4a^3 bde^3 + 6a^2 b^2 d^2 e^2 - 4ab^3 d^3}{10e^5 (ex+d)^{10}}$ |
| parallelrisch | $-\frac{210b^4 x^4 e^9 - 720a b^3 e^9 x^3 - 120b^4 d e^8 x^3 - 945a^2 b^2 e^9 x^2 - 270a b^3 d e^8 x^2 - 45b^4 d^2 e^7 x^2 - 560a^3 b e^9 x - 210a^2 b^2 d e^8 x - 60a b^3 d^2 e^7}{1260e^{10} (ex+d)^{10}}$ |
| norman | $-\frac{b^4 x^4}{6e} - \frac{2(6ab^3 e^6 + b^4 d e^5)x^3}{21e^7} - \frac{(21a^2 b^2 e^7 + 6ab^3 d e^6 + b^4 d^2 e^5)x^2}{28e^8} - \frac{(56a^3 b e^8 + 21a^2 b^2 d e^7 + 6ab^3 d^2 e^6 + b^4 d^3 e^5)x}{126e^9} - \frac{126a^4 e^9 + 56a^3 bde^8 + 21a^2 b^2 d^2 e^7 + 1260e^{10}}{(ex+d)^{10}}$ |
| orering | $-\frac{(210b^4 x^4 e^4 + 720x^3 a b^3 e^4 + 120x^3 b^4 d e^3 + 945x^2 a^2 b^2 e^4 + 270x^2 a b^3 d e^3 + 45x^2 b^4 d^2 e^2 + 560x a^3 b e^4 + 210x a^2 b^2 d e^3 + 60x a b^3 d^2 e^2 + 1260e^5 (bx+a)^4 (ex+d)^{10}}$ |

input `int((b^2*x^2+2*a*b*x+a^2)^2/(e*x+d)^11,x,method=_RETURNVERBOSE)`

output
$$\frac{(-1/6/e*b^4*x^4-2/21*b^3/e^2*(6*a*e+b*d)*x^3-1/28*b^2/e^3*(21*a^2*e^2+6*a*b*d*e+b^2*d^2)*x^2-1/126*b/e^4*(56*a^3*e^3+21*a^2*b*d*e^2+6*a*b^2*d^2*e+b^3*d^3)*x-1/1260/e^5*(126*a^4*e^4+56*a^3*b*d*e^3+21*a^2*b^2*d^2*e^2+6*a*b^3*d^3*e+b^4*d^4))/(e*x+d)^{10}}$$

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 280 vs. 2(109) = 218.

Time = 0.10 (sec) , antiderivative size = 280, normalized size of antiderivative = 2.35

$$\int \frac{(a^2 + 2abx + b^2x^2)^2}{(d + ex)^{11}} dx =$$

$$-\frac{210b^4 e^4 x^4 + b^4 d^4 + 6ab^3 d^3 e + 21a^2 b^2 d^2 e^2 + 56a^3 b d e^3 + 126a^4 e^4 + 120(b^4 d e^3 + 6ab^3 e^4)x^3 + 45(b^4 d^2 e^2 + 6ab^3 d^3 e + 126a^4 e^4)x^2 + 1260e^5 (bx+a)^4 (ex+d)^{10}}$$

input `integrate((b^2*x^2+2*a*b*x+a^2)^2/(e*x+d)^11,x, algorithm="fricas")`

output

```
-1/1260*(210*b^4*e^4*x^4 + b^4*d^4 + 6*a*b^3*d^3*e + 21*a^2*b^2*d^2*e^2 +
56*a^3*b*d*e^3 + 126*a^4*e^4 + 120*(b^4*d*e^3 + 6*a*b^3*e^4)*x^3 + 45*(b^4
*d^2*e^2 + 6*a*b^3*d*e^3 + 21*a^2*b^2*e^4)*x^2 + 10*(b^4*d^3*e + 6*a*b^3*d
^2*e^2 + 21*a^2*b^2*d*e^3 + 56*a^3*b*e^4)*x)/(e^15*x^10 + 10*d*e^14*x^9 +
45*d^2*e^13*x^8 + 120*d^3*e^12*x^7 + 210*d^4*e^11*x^6 + 252*d^5*e^10*x^5 +
210*d^6*e^9*x^4 + 120*d^7*e^8*x^3 + 45*d^8*e^7*x^2 + 10*d^9*e^6*x + d^10*
e^5)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 303 vs. $2(107) = 214$.

Time = 38.86 (sec) , antiderivative size = 303, normalized size of antiderivative = 2.55

$$\int \frac{(a^2 + 2abx + b^2x^2)^2}{(d + ex)^{11}} dx$$

$$= \frac{-126a^4e^4 - 56a^3bde^3 - 21a^2b^2d^2e^2 - 6ab^3d^3e - b^4d^4 - 210b^4e^4x^4 + x^3(-720ab^3e^4 - 120b^4de^3) + x^2(-945a^2b^2e^4 - 270ab^3de^3 - 45b^4d^2e^2) + x(-560a^3be^4 - 210a^2b^2de^3 - 60ab^3d^2e^2 - 10b^4d^3e)}{1260d^{10}e^5 + 12600d^9e^6x + 56700d^8e^7x^2 + 151200d^7e^8x^3 + 264600d^6e^9x^4 + 317520d^5e^{10}x^5 + 264600d^4e^{11}x^6 + 151200d^3e^{12}x^7 + 56700d^2e^{13}x^8 + 12600de^{14}x^9 + 1260e^{15}x^{10}}$$

input

```
integrate((b**2*x**2+2*a*b*x+a**2)**2/(e*x+d)**11,x)
```

output

```
(-126*a**4*e**4 - 56*a**3*b*d*e**3 - 21*a**2*b**2*d**2*e**2 - 6*a*b**3*d**
3*e - b**4*d**4 - 210*b**4*e**4*x**4 + x**3*(-720*a*b**3*e**4 - 120*b**4*d
*e**3) + x**2*(-945*a**2*b**2*e**4 - 270*a*b**3*d*e**3 - 45*b**4*d**2*e**2
) + x*(-560*a**3*b*e**4 - 210*a**2*b**2*d*e**3 - 60*a*b**3*d**2*e**2 - 10*
b**4*d**3*e))/(1260*d**10*e**5 + 12600*d**9*e**6*x + 56700*d**8*e**7*x**2
+ 151200*d**7*e**8*x**3 + 264600*d**6*e**9*x**4 + 317520*d**5*e**10*x**5 +
264600*d**4*e**11*x**6 + 151200*d**3*e**12*x**7 + 56700*d**2*e**13*x**8 +
12600*d*e**14*x**9 + 1260*e**15*x**10)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 280 vs. $2(109) = 218$.

Time = 0.06 (sec) , antiderivative size = 280, normalized size of antiderivative = 2.35

$$\int \frac{(a^2 + 2abx + b^2x^2)^2}{(d + ex)^{11}} dx =$$

$$-\frac{210b^4e^4x^4 + b^4d^4 + 6ab^3d^3e + 21a^2b^2d^2e^2 + 56a^3bde^3 + 126a^4e^4 + 120(b^4de^3 + 6ab^3e^4)x^3 + 45(b^4d^2e^2 + 6a^2b^3d^2e^3 + 21a^2b^2d^2e^4)x^2 + 10(b^4d^3e + 6a^2b^3d^3e^2 + 21a^2b^2d^3e^3 + 56a^3bde^4)x}{1260(e^{15}x^{10} + 10de^{14}x^9 + 45d^2e^{13}x^8 + 120d^3e^{12}x^7 + 210d^4e^{11}x^6 + 252d^5e^{10}x^5 + 210d^6e^9x^4 + 120d^7e^8x^3 + 45d^8e^7x^2 + 10d^9e^6x + d^{10}e^5)}$$

input `integrate((b^2*x^2+2*a*b*x+a^2)^2/(e*x+d)^11,x, algorithm="maxima")`

output
$$-1/1260*(210*b^4*e^4*x^4 + b^4*d^4 + 6*a*b^3*d^3*e + 21*a^2*b^2*d^2*e^2 + 56*a^3*b*d*e^3 + 126*a^4*e^4 + 120*(b^4*d*e^3 + 6*a*b^3*e^4)*x^3 + 45*(b^4*d^2*e^2 + 6*a*b^3*d^2*e^3 + 21*a^2*b^2*d^2*e^4)*x^2 + 10*(b^4*d^3*e + 6*a*b^3*d^3*e^2 + 21*a^2*b^2*d^3*e^3 + 56*a^3*b*d^3*e^4)*x)/(e^{15}*x^{10} + 10*d*e^{14}*x^9 + 45*d^2*e^{13}*x^8 + 120*d^3*e^{12}*x^7 + 210*d^4*e^{11}*x^6 + 252*d^5*e^{10}*x^5 + 210*d^6*e^9*x^4 + 120*d^7*e^8*x^3 + 45*d^8*e^7*x^2 + 10*d^9*e^6*x + d^{10}*e^5)$$

Giac [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 184, normalized size of antiderivative = 1.55

$$\int \frac{(a^2 + 2abx + b^2x^2)^2}{(d + ex)^{11}} dx =$$

$$-\frac{210b^4e^4x^4 + 120b^4de^3x^3 + 720ab^3e^4x^3 + 45b^4d^2e^2x^2 + 270ab^3de^3x^2 + 945a^2b^2e^4x^2 + 10b^4d^3ex + 60a^2b^3d^2e^2x + 210a^2b^2d^2e^3x + 560a^3bde^4x + b^4d^4 + 6a^2b^3d^3e + 21a^2b^2d^2e^2 + 56a^3bde^3 + 126a^4e^4}{1260(ex + d)^{10}e^5}$$

input `integrate((b^2*x^2+2*a*b*x+a^2)^2/(e*x+d)^11,x, algorithm="giac")`

output
$$-1/1260*(210*b^4*e^4*x^4 + 120*b^4*d*e^3*x^3 + 720*a*b^3*e^4*x^3 + 45*b^4*d^2*e^2*x^2 + 270*a*b^3*d^2*e^3*x^2 + 945*a^2*b^2*d^2*e^4*x^2 + 10*b^4*d^3*e*x + 60*a*b^3*d^2*e^2*x + 210*a^2*b^2*d^2*e^3*x + 560*a^3*b*d^3*e^4*x + b^4*d^4 + 6*a^2*b^3*d^3*e + 21*a^2*b^2*d^2*e^2 + 56*a^3*b*d^3*e^3 + 126*a^4*e^4)/((e*x + d)^{10}*e^5)$$

Mupad [B] (verification not implemented)

Time = 8.87 (sec) , antiderivative size = 270, normalized size of antiderivative = 2.27

$$\int \frac{(a^2 + 2abx + b^2x^2)^2}{(d + ex)^{11}} dx =$$

$$-\frac{\frac{126a^4e^4 + 56a^3bde^3 + 21a^2b^2d^2e^2 + 6ab^3d^3e + b^4d^4}{1260e^5} + \frac{b^4x^4}{6e} + \frac{2b^3x^3(6ae + bd)}{21e^2} + \frac{bx(56a^3e^3 + 21a^2bde^2 + 6ab^2d^2e + b^3d^3)}{126e^4}}{d^{10} + 10d^9ex + 45d^8e^2x^2 + 120d^7e^3x^3 + 210d^6e^4x^4 + 252d^5e^5x^5 + 210d^4e^6x^6 + 120d^3e^7x^7 + \dots}$$

input `int((a^2 + b^2*x^2 + 2*a*b*x)^2/(d + e*x)^11,x)`

output

```

-((126*a^4*e^4 + b^4*d^4 + 21*a^2*b^2*d^2*e^2 + 6*a*b^3*d^3*e + 56*a^3*b*d
*e^3)/(1260*e^5) + (b^4*x^4)/(6*e) + (2*b^3*x^3*(6*a*e + b*d))/(21*e^2) +
(b*x*(56*a^3*e^3 + b^3*d^3 + 6*a*b^2*d^2*e + 21*a^2*b*d*e^2))/(126*e^4) +
(b^2*x^2*(21*a^2*e^2 + b^2*d^2 + 6*a*b*d*e))/(28*e^3))/(d^10 + e^10*x^10 +
10*d*e^9*x^9 + 45*d^8*e^2*x^2 + 120*d^7*e^3*x^3 + 210*d^6*e^4*x^4 + 252*d
^5*e^5*x^5 + 210*d^4*e^6*x^6 + 120*d^3*e^7*x^7 + 45*d^2*e^8*x^8 + 10*d^9*e
*x)

```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 284, normalized size of antiderivative = 2.39

$$\int \frac{(a^2 + 2abx + b^2x^2)^2}{(d + ex)^{11}} dx$$

$$= \frac{-210b^4e^4x^4 - 720ab^3e^4x^3 - 120b^4de^3x^3 - 945a^2b^2e^4x^2 - 270ab^3de^3x^2 - 45b^4d^2e^2x^2 - 560a^3be^4x - 210a^4e^4}{1260e^5(e^{10}x^{10} + 10de^9x^9 + 45d^2e^8x^8 + 120d^3e^7x^7 + 210d^4e^6x^6 + 252d^5e^5x^5 + \dots)}$$

input `int((b^2*x^2+2*a*b*x+a^2)^2/(e*x+d)^11,x)`

output

```
( - 126*a**4*e**4 - 56*a**3*b*d*e**3 - 560*a**3*b*e**4*x - 21*a**2*b**2*d*
*2*e**2 - 210*a**2*b**2*d*e**3*x - 945*a**2*b**2*e**4*x**2 - 6*a*b**3*d**3
*e - 60*a*b**3*d**2*e**2*x - 270*a*b**3*d*e**3*x**2 - 720*a*b**3*e**4*x**3
- b**4*d**4 - 10*b**4*d**3*e*x - 45*b**4*d**2*e**2*x**2 - 120*b**4*d*e**3
*x**3 - 210*b**4*e**4*x**4)/(1260*e**5*(d**10 + 10*d**9*e*x + 45*d**8*e**2
*x**2 + 120*d**7*e**3*x**3 + 210*d**6*e**4*x**4 + 252*d**5*e**5*x**5 + 210
*d**4*e**6*x**6 + 120*d**3*e**7*x**7 + 45*d**2*e**8*x**8 + 10*d*e**9*x**9
+ e**10*x**10))
```

3.84 $\int (d + ex)^8 (a^2 + 2abx + b^2x^2)^3 dx$

| | |
|---|-----|
| Optimal result | 670 |
| Mathematica [B] (verified) | 671 |
| Rubi [A] (verified) | 672 |
| Maple [B] (verified) | 674 |
| Fricas [B] (verification not implemented) | 675 |
| Sympy [B] (verification not implemented) | 675 |
| Maxima [B] (verification not implemented) | 676 |
| Giac [B] (verification not implemented) | 677 |
| Mupad [B] (verification not implemented) | 679 |
| Reduce [B] (verification not implemented) | 680 |

Optimal result

Integrand size = 26, antiderivative size = 173

$$\int (d + ex)^8 (a^2 + 2abx + b^2x^2)^3 dx = \frac{(bd - ae)^6 (d + ex)^9}{9e^7} - \frac{3b(bd - ae)^5 (d + ex)^{10}}{5e^7} + \frac{15b^2(bd - ae)^4 (d + ex)^{11}}{11e^7} - \frac{5b^3(bd - ae)^3 (d + ex)^{12}}{3e^7} + \frac{15b^4(bd - ae)^2 (d + ex)^{13}}{13e^7} - \frac{3b^5(bd - ae)(d + ex)^{14}}{7e^7} + \frac{b^6 (d + ex)^{15}}{15e^7}$$

output

```
1/9*(-a*e+b*d)^6*(e*x+d)^9/e^7-3/5*b*(-a*e+b*d)^5*(e*x+d)^10/e^7+15/11*b^2
*(-a*e+b*d)^4*(e*x+d)^11/e^7-5/3*b^3*(-a*e+b*d)^3*(e*x+d)^12/e^7+15/13*b^4
*(-a*e+b*d)^2*(e*x+d)^13/e^7-3/7*b^5*(-a*e+b*d)*(e*x+d)^14/e^7+1/15*b^6*(e
*x+d)^15/e^7
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 771 vs. $2(173) = 346$.

Time = 0.08 (sec) , antiderivative size = 771, normalized size of antiderivative = 4.46

$$\begin{aligned}
 \int (d + ex)^8 (a^2 + 2abx + b^2x^2)^3 dx = & a^6 d^8 x + a^5 d^7 (3bd + 4ae)x^2 \\
 & + \frac{1}{3} a^4 d^6 (15b^2 d^2 + 48abde + 28a^2 e^2) x^3 \\
 & + a^3 d^5 (5b^3 d^3 + 30ab^2 d^2 e + 42a^2 bde^2 + 14a^3 e^3) x^4 \\
 & + \frac{1}{5} a^2 d^4 (15b^4 d^4 + 160ab^3 d^3 e + 420a^2 b^2 d^2 e^2 \\
 & \quad + 336a^3 bde^3 + 70a^4 e^4) x^5 \\
 & + \frac{1}{3} ad^3 (3b^5 d^5 + 60ab^4 d^4 e + 280a^2 b^3 d^3 e^2 \\
 & \quad + 420a^3 b^2 d^2 e^3 + 210a^4 bde^4 + 28a^5 e^5) x^6 \\
 & + \frac{1}{7} d^2 (b^6 d^6 + 48ab^5 d^5 e + 420a^2 b^4 d^4 e^2 \\
 & \quad + 1120a^3 b^3 d^3 e^3 + 1050a^4 b^2 d^2 e^4 + 336a^5 bde^5 \\
 & \quad + 28a^6 e^6) x^7 \\
 & + de (b^6 d^6 + 21ab^5 d^5 e + 105a^2 b^4 d^4 e^2 + 175a^3 b^3 d^3 e^3 \\
 & + 105a^4 b^2 d^2 e^4 + 21a^5 bde^5 + a^6 e^6) x^8 + \frac{1}{9} e^2 (28b^6 d^6 \\
 & + 336ab^5 d^5 e + 1050a^2 b^4 d^4 e^2 + 1120a^3 b^3 d^3 e^3 \\
 & + 420a^4 b^2 d^2 e^4 + 48a^5 bde^5 + a^6 e^6) x^9 \\
 & + \frac{1}{5} be^3 (28b^5 d^5 + 210ab^4 d^4 e + 420a^2 b^3 d^3 e^2 \\
 & \quad + 280a^3 b^2 d^2 e^3 + 60a^4 bde^4 + 3a^5 e^5) x^{10} \\
 & + \frac{1}{11} b^2 e^4 (70b^4 d^4 + 336ab^3 d^3 e + 420a^2 b^2 d^2 e^2 \\
 & \quad + 160a^3 bde^3 + 15a^4 e^4) x^{11} \\
 & + \frac{1}{3} b^3 e^5 (14b^3 d^3 + 42ab^2 d^2 e + 30a^2 bde^2 + 5a^3 e^3) x^{12} \\
 & + \frac{1}{13} b^4 e^6 (28b^2 d^2 + 48abde + 15a^2 e^2) x^{13} \\
 & + \frac{1}{7} b^5 e^7 (4bd + 3ae)x^{14} + \frac{1}{15} b^6 e^8 x^{15}
 \end{aligned}$$

input

```
Integrate[(d + e*x)^8*(a^2 + 2*a*b*x + b^2*x^2)^3,x]
```

output

```

a^6*d^8*x + a^5*d^7*(3*b*d + 4*a*e)*x^2 + (a^4*d^6*(15*b^2*d^2 + 48*a*b*d*
e + 28*a^2*e^2)*x^3)/3 + a^3*d^5*(5*b^3*d^3 + 30*a*b^2*d^2*e + 42*a^2*b*d*
e^2 + 14*a^3*e^3)*x^4 + (a^2*d^4*(15*b^4*d^4 + 160*a*b^3*d^3*e + 420*a^2*b
^2*d^2*e^2 + 336*a^3*b*d*e^3 + 70*a^4*e^4)*x^5)/5 + (a*d^3*(3*b^5*d^5 + 60
*a*b^4*d^4*e + 280*a^2*b^3*d^3*e^2 + 420*a^3*b^2*d^2*e^3 + 210*a^4*b*d*e^4
+ 28*a^5*e^5)*x^6)/3 + (d^2*(b^6*d^6 + 48*a*b^5*d^5*e + 420*a^2*b^4*d^4*e
^2 + 1120*a^3*b^3*d^3*e^3 + 1050*a^4*b^2*d^2*e^4 + 336*a^5*b*d*e^5 + 28*a^
6*e^6)*x^7)/7 + d*e*(b^6*d^6 + 21*a*b^5*d^5*e + 105*a^2*b^4*d^4*e^2 + 175*
a^3*b^3*d^3*e^3 + 105*a^4*b^2*d^2*e^4 + 21*a^5*b*d*e^5 + a^6*e^6)*x^8 + (e
^2*(28*b^6*d^6 + 336*a*b^5*d^5*e + 1050*a^2*b^4*d^4*e^2 + 1120*a^3*b^3*d^3
*e^3 + 420*a^4*b^2*d^2*e^4 + 48*a^5*b*d*e^5 + a^6*e^6)*x^9)/9 + (b*e^3*(28
*b^5*d^5 + 210*a*b^4*d^4*e + 420*a^2*b^3*d^3*e^2 + 280*a^3*b^2*d^2*e^3 + 6
0*a^4*b*d*e^4 + 3*a^5*e^5)*x^10)/5 + (b^2*e^4*(70*b^4*d^4 + 336*a*b^3*d^3*
e + 420*a^2*b^2*d^2*e^2 + 160*a^3*b*d*e^3 + 15*a^4*e^4)*x^11)/11 + (b^3*e^
5*(14*b^3*d^3 + 42*a*b^2*d^2*e + 30*a^2*b*d*e^2 + 5*a^3*e^3)*x^12)/3 + (b^
4*e^6*(28*b^2*d^2 + 48*a*b*d*e + 15*a^2*e^2)*x^13)/13 + (b^5*e^7*(4*b*d +
3*a*e)*x^14)/7 + (b^6*e^8*x^15)/15

```

Rubi [A] (verified)

Time = 1.11 (sec) , antiderivative size = 173, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {1098, 27, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\begin{aligned}
 & \int (a^2 + 2abx + b^2x^2)^3 (d + ex)^8 dx \\
 & \quad \downarrow 1098 \\
 & \frac{\int b^6(a + bx)^6(d + ex)^8 dx}{b^6} \\
 & \quad \downarrow 27 \\
 & \int (a + bx)^6(d + ex)^8 dx \\
 & \quad \downarrow 49
 \end{aligned}$$

$$\int \left(-\frac{6b^5(d+ex)^{13}(bd-ae)}{e^6} + \frac{15b^4(d+ex)^{12}(bd-ae)^2}{e^6} - \frac{20b^3(d+ex)^{11}(bd-ae)^3}{e^6} + \frac{15b^2(d+ex)^{10}(bd-ae)^4}{e^6} \right. \\ \left. - \frac{3b^5(d+ex)^{14}(bd-ae)}{7e^7} + \frac{15b^4(d+ex)^{13}(bd-ae)^2}{13e^7} - \frac{5b^3(d+ex)^{12}(bd-ae)^3}{3e^7} + \frac{15b^2(d+ex)^{11}(bd-ae)^4}{11e^7} - \frac{3b(d+ex)^{10}(bd-ae)^5}{5e^7} + \frac{(d+ex)^9(bd-ae)^6}{9e^7} + \frac{b^6(d+ex)^{15}}{15e^7} \right)$$

↓ 2009

input `Int[(d + e*x)^8*(a^2 + 2*a*b*x + b^2*x^2)^3,x]`

output `((b*d - a*e)^6*(d + e*x)^9)/(9*e^7) - (3*b*(b*d - a*e)^5*(d + e*x)^10)/(5*e^7) + (15*b^2*(b*d - a*e)^4*(d + e*x)^11)/(11*e^7) - (5*b^3*(b*d - a*e)^3*(d + e*x)^12)/(3*e^7) + (15*b^4*(b*d - a*e)^2*(d + e*x)^13)/(13*e^7) - (3*b^5*(b*d - a*e)*(d + e*x)^14)/(7*e^7) + (b^6*(d + e*x)^15)/(15*e^7)`

Defintions of rubi rules used

rule 27 `Int[(a_)*(F_x_), x_Symbol] := Simp[a Int[F_x, x], x] /; FreeQ[a, x] && !MatchQ[F_x, (b_)*(G_x_) /; FreeQ[b, x]]`

rule 49 `Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] := Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] && IGtQ[m, 0] && IGtQ[m + n + 2, 0]`

rule 1098 `Int[((d_.) + (e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] := Simp[1/c^p Int[(d + e*x)^m*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]`

rule 2009 `Int[u_, x_Symbol] := Simp[IntSum[u, x], x] /; SumQ[u]`

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 788 vs. $2(159) = 318$.

Time = 1.05 (sec) , antiderivative size = 789, normalized size of antiderivative = 4.56

| method | result |
|---------------|--|
| norman | $a^6 d^8 x + (4a^6 d^7 e + 3a^5 b d^8) x^2 + (\frac{28}{3} a^6 e^2 d^6 + 16a^5 b d^7 e + 5a^4 b^2 d^8) x^3 + (14a^6 d^5 e^3 + 42a^5 b d^6 e^2 + 30a^4 b^2 d^7 e x^4 + \frac{28}{9} x^9 b^6 e^2 d^6 + 4x^7 a^6 d^2 e^6 + \frac{28}{3} x^6 a^6 d^3 e^5 + x^6 a b^5 d^8 + 14x^5 a^6 d^4 e^4 + 3x^5 a^2 b^4 d^7 e^3 + 30a^4 b^2 d^7 e x^4 + \frac{28}{9} x^9 b^6 e^2 d^6 + 4x^7 a^6 d^2 e^6 + \frac{28}{3} x^6 a^6 d^3 e^5 + x^6 a b^5 d^8 + 14x^5 a^6 d^4 e^4 + 3x^5 a^2 b^4 d^7 e^3$ |
| default | $\frac{b^6 e^8 x^{15}}{15} + \frac{(6a b^5 e^8 + 8b^6 d e^7) x^{14}}{14} + \frac{(15a^2 b^4 e^8 + 48a b^5 d e^7 + 28b^6 d^2 e^6) x^{13}}{13} + \frac{(20a^3 b^3 e^8 + 120a^2 b^4 d e^7 + 168a b^5 d^2 e^6 + 56a^6 d^8) x^{12}}{12}$ |
| risch | $30a^4 b^2 d^7 e x^4 + \frac{28}{9} x^9 b^6 e^2 d^6 + 4x^7 a^6 d^2 e^6 + \frac{28}{3} x^6 a^6 d^3 e^5 + x^6 a b^5 d^8 + 14x^5 a^6 d^4 e^4 + 3x^5 a^2 b^4 d^7 e^3$ |
| parallelrisch | $30a^4 b^2 d^7 e x^4 + \frac{28}{9} x^9 b^6 e^2 d^6 + 4x^7 a^6 d^2 e^6 + \frac{28}{3} x^6 a^6 d^3 e^5 + x^6 a b^5 d^8 + 14x^5 a^6 d^4 e^4 + 3x^5 a^2 b^4 d^7 e^3$ |
| gospers | $x(3003b^6 e^8 x^{14} + 19305x^{13} a b^5 e^8 + 25740x^{13} b^6 d e^7 + 51975x^{12} a^2 b^4 e^8 + 166320x^{12} a b^5 d e^7 + 97020x^{12} b^6 d^2 e^6 + 75075x^{11} a^3 b^3 e^8$ |
| orering | $x(3003b^6 e^8 x^{14} + 19305x^{13} a b^5 e^8 + 25740x^{13} b^6 d e^7 + 51975x^{12} a^2 b^4 e^8 + 166320x^{12} a b^5 d e^7 + 97020x^{12} b^6 d^2 e^6 + 75075x^{11} a^3 b^3 e^8$ |

```
input int((e*x+d)^8*(b^2*x^2+2*a*b*x+a^2)^3,x,method=_RETURNVERBOSE)
```

```
output a^6*d^8*x+(4*a^6*d^7*e+3*a^5*b*d^8)*x^2+(28/3*a^6*e^2*d^6+16*a^5*b*d^7*e+5*a^4*b^2*d^8)*x^3+(14*a^6*d^5*e^3+42*a^5*b*d^6*e^2+30*a^4*b^2*d^7*e+5*a^3*b^3*d^8)*x^4+(14*a^6*d^4*e^4+336/5*a^5*b*d^5*e^3+84*a^4*b^2*e^2*d^6+32*a^3*b^3*d^7*e+3*a^2*b^4*d^8)*x^5+(28/3*a^6*d^3*e^5+70*a^5*b*d^4*e^4+140*a^4*b^2*d^5*e^3+280/3*a^3*b^3*e^2*d^6+20*a^2*b^4*d^7*e+a*b^5*d^8)*x^6+(4*a^6*d^2*e^6+48*a^5*b*d^3*e^5+150*a^4*b^2*d^4*e^4+160*a^3*b^3*d^5*e^3+60*a^2*b^4*d^6*e^2+48/7*a*b^5*d^7*e+1/7*b^6*d^8)*x^7+(a^6*d*e^7+21*a^5*b*d^2*e^6+105*a^4*b^2*d^3*e^5+175*a^3*b^3*d^4*e^4+105*a^2*b^4*d^5*e^3+21*a*b^5*d^6*e^2+b^6*d^7*e)*x^8+(1/9*a^6*e^8+16/3*a^5*b*d*e^7+140/3*a^4*b^2*d^2*e^6+1120/9*a^3*b^3*d^3*e^5+350/3*a^2*b^4*d^4*e^4+112/3*a*b^5*d^5*e^3+28/9*b^6*d^6)*x^9+(3/5*a^5*b*e^8+12*a^4*b^2*d*e^7+56*a^3*b^3*d^2*e^6+84*a^2*b^4*d^3*e^5+42*a*b^5*d^4*e^4+28/5*b^6*d^5*e^3)*x^10+(15/11*a^4*b^2*e^8+160/11*a^3*b^3*d*e^7+420/11*a^2*b^4*d^2*e^6+336/11*a*b^5*d^3*e^5+70/11*b^6*d^4*e^4)*x^11+(5/3*a^3*b^3*e^8+10*a^2*b^4*d*e^7+14*a*b^5*d^2*e^6+14/3*b^6*d^3*e^5)*x^12+(15/13*a^2*b^4*e^8+48/13*a*b^5*d*e^7+28/13*b^6*d^2*e^6)*x^13+(3/7*a*b^5*d^8+4/7*b^6*d*e^7)*x^14+1/15*b^6*d^8*x^15
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 797 vs. $2(159) = 318$.

Time = 0.08 (sec) , antiderivative size = 797, normalized size of antiderivative = 4.61

$$\int (d + ex)^8 (a^2 + 2abx + b^2x^2)^3 dx = \text{Too large to display}$$

input `integrate((e*x+d)^8*(b^2*x^2+2*a*b*x+a^2)^3,x, algorithm="fricas")`

output

```
1/15*b^6*e^8*x^15 + a^6*d^8*x + 1/7*(4*b^6*d*e^7 + 3*a*b^5*e^8)*x^14 + 1/1
3*(28*b^6*d^2*e^6 + 48*a*b^5*d*e^7 + 15*a^2*b^4*e^8)*x^13 + 1/3*(14*b^6*d^
3*e^5 + 42*a*b^5*d^2*e^6 + 30*a^2*b^4*d*e^7 + 5*a^3*b^3*e^8)*x^12 + 1/11*(
70*b^6*d^4*e^4 + 336*a*b^5*d^3*e^5 + 420*a^2*b^4*d^2*e^6 + 160*a^3*b^3*d*e
^7 + 15*a^4*b^2*e^8)*x^11 + 1/5*(28*b^6*d^5*e^3 + 210*a*b^5*d^4*e^4 + 420*
a^2*b^4*d^3*e^5 + 280*a^3*b^3*d^2*e^6 + 60*a^4*b^2*d*e^7 + 3*a^5*b*e^8)*x^
10 + 1/9*(28*b^6*d^6*e^2 + 336*a*b^5*d^5*e^3 + 1050*a^2*b^4*d^4*e^4 + 1120
*a^3*b^3*d^3*e^5 + 420*a^4*b^2*d^2*e^6 + 48*a^5*b*d*e^7 + a^6*e^8)*x^9 + (
b^6*d^7*e + 21*a*b^5*d^6*e^2 + 105*a^2*b^4*d^5*e^3 + 175*a^3*b^3*d^4*e^4 +
105*a^4*b^2*d^3*e^5 + 21*a^5*b*d^2*e^6 + a^6*d*e^7)*x^8 + 1/7*(b^6*d^8 +
48*a*b^5*d^7*e + 420*a^2*b^4*d^6*e^2 + 1120*a^3*b^3*d^5*e^3 + 1050*a^4*b^2
*d^4*e^4 + 336*a^5*b*d^3*e^5 + 28*a^6*d^2*e^6)*x^7 + 1/3*(3*a*b^5*d^8 + 60
*a^2*b^4*d^7*e + 280*a^3*b^3*d^6*e^2 + 420*a^4*b^2*d^5*e^3 + 210*a^5*b*d^4
*e^4 + 28*a^6*d^3*e^5)*x^6 + 1/5*(15*a^2*b^4*d^8 + 160*a^3*b^3*d^7*e + 420
*a^4*b^2*d^6*e^2 + 336*a^5*b*d^5*e^3 + 70*a^6*d^4*e^4)*x^5 + (5*a^3*b^3*d^
8 + 30*a^4*b^2*d^7*e + 42*a^5*b*d^6*e^2 + 14*a^6*d^5*e^3)*x^4 + 1/3*(15*a^
4*b^2*d^8 + 48*a^5*b*d^7*e + 28*a^6*d^6*e^2)*x^3 + (3*a^5*b*d^8 + 4*a^6*d^
7*e)*x^2
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 884 vs. $2(158) = 316$.

Time = 0.07 (sec) , antiderivative size = 884, normalized size of antiderivative = 5.11

$$\int (d + ex)^8 (a^2 + 2abx + b^2x^2)^3 dx = \text{Too large to display}$$

input `integrate((e*x+d)**8*(b**2*x**2+2*a*b*x+a**2)**3,x)`

output

```

a**6*d**8*x + b**6*e**8*x**15/15 + x**14*(3*a*b**5*e**8/7 + 4*b**6*d*e**7/
7) + x**13*(15*a**2*b**4*e**8/13 + 48*a*b**5*d*e**7/13 + 28*b**6*d**2*e**6
/13) + x**12*(5*a**3*b**3*e**8/3 + 10*a**2*b**4*d*e**7 + 14*a*b**5*d**2*e*
*6 + 14*b**6*d**3*e**5/3) + x**11*(15*a**4*b**2*e**8/11 + 160*a**3*b**3*d*
e**7/11 + 420*a**2*b**4*d**2*e**6/11 + 336*a*b**5*d**3*e**5/11 + 70*b**6*d
**4*e**4/11) + x**10*(3*a**5*b*e**8/5 + 12*a**4*b**2*d*e**7 + 56*a**3*b**3
*d**2*e**6 + 84*a**2*b**4*d**3*e**5 + 42*a*b**5*d**4*e**4 + 28*b**6*d**5*e
**3/5) + x**9*(a**6*e**8/9 + 16*a**5*b*d*e**7/3 + 140*a**4*b**2*d**2*e**6/
3 + 1120*a**3*b**3*d**3*e**5/9 + 350*a**2*b**4*d**4*e**4/3 + 112*a*b**5*d*
*5*e**3/3 + 28*b**6*d**6*e**2/9) + x**8*(a**6*d*e**7 + 21*a**5*b*d**2*e**6
+ 105*a**4*b**2*d**3*e**5 + 175*a**3*b**3*d**4*e**4 + 105*a**2*b**4*d**5*
e**3 + 21*a*b**5*d**6*e**2 + b**6*d**7*e) + x**7*(4*a**6*d**2*e**6 + 48*a*
*5*b*d**3*e**5 + 150*a**4*b**2*d**4*e**4 + 160*a**3*b**3*d**5*e**3 + 60*a*
*2*b**4*d**6*e**2 + 48*a*b**5*d**7*e/7 + b**6*d**8/7) + x**6*(28*a**6*d**3
*e**5/3 + 70*a**5*b*d**4*e**4 + 140*a**4*b**2*d**5*e**3 + 280*a**3*b**3*d*
*6*e**2/3 + 20*a**2*b**4*d**7*e + a*b**5*d**8) + x**5*(14*a**6*d**4*e**4 +
336*a**5*b*d**5*e**3/5 + 84*a**4*b**2*d**6*e**2 + 32*a**3*b**3*d**7*e + 3
*a**2*b**4*d**8) + x**4*(14*a**6*d**5*e**3 + 42*a**5*b*d**6*e**2 + 30*a**4
*b**2*d**7*e + 5*a**3*b**3*d**8) + x**3*(28*a**6*d**6*e**2/3 + 16*a**5*b*d
**7*e + 5*a**4*b**2*d**8) + x**2*(4*a**6*d**7*e + 3*a**5*b*d**8)

```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 797 vs. $2(159) = 318$.

Time = 0.04 (sec) , antiderivative size = 797, normalized size of antiderivative = 4.61

$$\int (d + ex)^8 (a^2 + 2abx + b^2x^2)^3 dx = \text{Too large to display}$$

input

```
integrate((e*x+d)^8*(b^2*x^2+2*a*b*x+a^2)^3,x, algorithm="maxima")
```

output

```

1/15*b^6*e^8*x^15 + a^6*d^8*x + 1/7*(4*b^6*d*e^7 + 3*a*b^5*e^8)*x^14 + 1/1
3*(28*b^6*d^2*e^6 + 48*a*b^5*d*e^7 + 15*a^2*b^4*e^8)*x^13 + 1/3*(14*b^6*d^
3*e^5 + 42*a*b^5*d^2*e^6 + 30*a^2*b^4*d*e^7 + 5*a^3*b^3*e^8)*x^12 + 1/11*(
70*b^6*d^4*e^4 + 336*a*b^5*d^3*e^5 + 420*a^2*b^4*d^2*e^6 + 160*a^3*b^3*d*e
^7 + 15*a^4*b^2*e^8)*x^11 + 1/5*(28*b^6*d^5*e^3 + 210*a*b^5*d^4*e^4 + 420*
a^2*b^4*d^3*e^5 + 280*a^3*b^3*d^2*e^6 + 60*a^4*b^2*d*e^7 + 3*a^5*b*e^8)*x^
10 + 1/9*(28*b^6*d^6*e^2 + 336*a*b^5*d^5*e^3 + 1050*a^2*b^4*d^4*e^4 + 1120
*a^3*b^3*d^3*e^5 + 420*a^4*b^2*d^2*e^6 + 48*a^5*b*d*e^7 + a^6*e^8)*x^9 + (
b^6*d^7*e + 21*a*b^5*d^6*e^2 + 105*a^2*b^4*d^5*e^3 + 175*a^3*b^3*d^4*e^4 +
105*a^4*b^2*d^3*e^5 + 21*a^5*b*d^2*e^6 + a^6*d*e^7)*x^8 + 1/7*(b^6*d^8 +
48*a*b^5*d^7*e + 420*a^2*b^4*d^6*e^2 + 1120*a^3*b^3*d^5*e^3 + 1050*a^4*b^2
*d^4*e^4 + 336*a^5*b*d^3*e^5 + 28*a^6*d^2*e^6)*x^7 + 1/3*(3*a*b^5*d^8 + 60
*a^2*b^4*d^7*e + 280*a^3*b^3*d^6*e^2 + 420*a^4*b^2*d^5*e^3 + 210*a^5*b*d^4
*e^4 + 28*a^6*d^3*e^5)*x^6 + 1/5*(15*a^2*b^4*d^8 + 160*a^3*b^3*d^7*e + 420
*a^4*b^2*d^6*e^2 + 336*a^5*b*d^5*e^3 + 70*a^6*d^4*e^4)*x^5 + (5*a^3*b^3*d^
8 + 30*a^4*b^2*d^7*e + 42*a^5*b*d^6*e^2 + 14*a^6*d^5*e^3)*x^4 + 1/3*(15*a^
4*b^2*d^8 + 48*a^5*b*d^7*e + 28*a^6*d^6*e^2)*x^3 + (3*a^5*b*d^8 + 4*a^6*d^
7*e)*x^2

```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 906 vs. $2(159) = 318$.

Time = 0.20 (sec) , antiderivative size = 906, normalized size of antiderivative = 5.24

$$\int (d + ex)^8 (a^2 + 2abx + b^2x^2)^3 dx = \text{Too large to display}$$

input

```
integrate((e*x+d)^8*(b^2*x^2+2*a*b*x+a^2)^3,x, algorithm="giac")
```

output

$$\begin{aligned}
& 1/15*b^6*e^8*x^15 + 4/7*b^6*d*e^7*x^14 + 3/7*a*b^5*e^8*x^14 + 28/13*b^6*d^2*e^6*x^13 + 48/13*a*b^5*d*e^7*x^13 + 15/13*a^2*b^4*e^8*x^13 + 14/3*b^6*d^3*e^5*x^12 + 14*a*b^5*d^2*e^6*x^12 + 10*a^2*b^4*d*e^7*x^12 + 5/3*a^3*b^3*e^8*x^12 + 70/11*b^6*d^4*e^4*x^11 + 336/11*a*b^5*d^3*e^5*x^11 + 420/11*a^2*b^4*d^2*e^6*x^11 + 160/11*a^3*b^3*d*e^7*x^11 + 15/11*a^4*b^2*e^8*x^11 + 28/5*b^6*d^5*e^3*x^10 + 42*a*b^5*d^4*e^4*x^10 + 84*a^2*b^4*d^3*e^5*x^10 + 56*a^3*b^3*d^2*e^6*x^10 + 12*a^4*b^2*d*e^7*x^10 + 3/5*a^5*b*e^8*x^10 + 28/9*b^6*d^6*e^2*x^9 + 112/3*a*b^5*d^5*e^3*x^9 + 350/3*a^2*b^4*d^4*e^4*x^9 + 1120/9*a^3*b^3*d^3*e^5*x^9 + 140/3*a^4*b^2*d^2*e^6*x^9 + 16/3*a^5*b*d*e^7*x^9 + 1/9*a^6*e^8*x^9 + b^6*d^7*e*x^8 + 21*a*b^5*d^6*e^2*x^8 + 105*a^2*b^4*d^5*e^3*x^8 + 175*a^3*b^3*d^4*e^4*x^8 + 105*a^4*b^2*d^3*e^5*x^8 + 21*a^5*b*d^2*e^6*x^8 + a^6*d*e^7*x^8 + 1/7*b^6*d^8*x^7 + 48/7*a*b^5*d^7*e*x^7 + 60*a^2*b^4*d^6*e^2*x^7 + 160*a^3*b^3*d^5*e^3*x^7 + 150*a^4*b^2*d^4*e^4*x^7 + 48*a^5*b*d^3*e^5*x^7 + 4*a^6*d^2*e^6*x^7 + a*b^5*d^8*x^6 + 20*a^2*b^4*d^7*e*x^6 + 280/3*a^3*b^3*d^6*e^2*x^6 + 140*a^4*b^2*d^5*e^3*x^6 + 70*a^5*b*d^4*e^4*x^6 + 28/3*a^6*d^3*e^5*x^6 + 3*a^2*b^4*d^8*x^5 + 32*a^3*b^3*d^7*e*x^5 + 84*a^4*b^2*d^6*e^2*x^5 + 336/5*a^5*b*d^5*e^3*x^5 + 14*a^6*d^4*e^4*x^5 + 5*a^3*b^3*d^8*x^4 + 30*a^4*b^2*d^7*e*x^4 + 42*a^5*b*d^6*e^2*x^4 + 14*a^6*d^5*e^3*x^4 + 5*a^4*b^2*d^8*x^3 + 16*a^5*b*d^7*e*x^3 + 28/3*a^6*d^6*e^2*x^3 + 3*a^5*b*d^8*x^2 + 4*a^6*d^7*e*x^2 + a^6*d^8*x
\end{aligned}$$

Mupad [B] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 768, normalized size of antiderivative = 4.44

$$\begin{aligned}
& \int (d + ex)^8 (a^2 + 2abx + b^2x^2)^3 dx \\
&= x^7 \left(4a^6 d^2 e^6 + 48a^5 b d^3 e^5 + 150a^4 b^2 d^4 e^4 + 160a^3 b^3 d^5 e^3 + 60a^2 b^4 d^6 e^2 \right. \\
&\quad \left. + \frac{48ab^5 d^7 e}{7} + \frac{b^6 d^8}{7} \right) + x^9 \left(\frac{a^6 e^8}{9} + \frac{16a^5 b d e^7}{3} + \frac{140a^4 b^2 d^2 e^6}{3} + \frac{1120a^3 b^3 d^3 e^5}{9} \right. \\
&\quad \left. + \frac{350a^2 b^4 d^4 e^4}{3} + \frac{112ab^5 d^5 e^3}{3} + \frac{28b^6 d^6 e^2}{9} \right) \\
&\quad + x^5 \left(14a^6 d^4 e^4 + \frac{336a^5 b d^5 e^3}{5} + 84a^4 b^2 d^6 e^2 + 32a^3 b^3 d^7 e + 3a^2 b^4 d^8 \right) \\
&\quad + x^{11} \left(\frac{15a^4 b^2 e^8}{11} + \frac{160a^3 b^3 d e^7}{11} + \frac{420a^2 b^4 d^2 e^6}{11} + \frac{336ab^5 d^3 e^5}{11} + \frac{70b^6 d^4 e^4}{11} \right) \\
&\quad + x^6 \left(\frac{28a^6 d^3 e^5}{3} + 70a^5 b d^4 e^4 + 140a^4 b^2 d^5 e^3 + \frac{280a^3 b^3 d^6 e^2}{3} + 20a^2 b^4 d^7 e \right. \\
&\quad \left. + ab^5 d^8 \right) + x^{10} \left(\frac{3a^5 b e^8}{5} + 12a^4 b^2 d e^7 + 56a^3 b^3 d^2 e^6 + 84a^2 b^4 d^3 e^5 \right. \\
&\quad \left. + 42ab^5 d^4 e^4 + \frac{28b^6 d^5 e^3}{5} \right) + x^8 (a^6 d e^7 + 21a^5 b d^2 e^6 + 105a^4 b^2 d^3 e^5 \\
&\quad + 175a^3 b^3 d^4 e^4 + 105a^2 b^4 d^5 e^3 + 21ab^5 d^6 e^2 + b^6 d^7 e) \\
&\quad + a^6 d^8 x + \frac{b^6 e^8 x^{15}}{15} + a^3 d^5 x^4 (14a^3 e^3 + 42a^2 b d e^2 + 30ab^2 d^2 e + 5b^3 d^3) \\
&\quad + \frac{b^3 e^5 x^{12} (5a^3 e^3 + 30a^2 b d e^2 + 42ab^2 d^2 e + 14b^3 d^3)}{3} \\
&\quad + a^5 d^7 x^2 (4ae + 3bd) + \frac{b^5 e^7 x^{14} (3ae + 4bd)}{7} \\
&\quad + \frac{a^4 d^6 x^3 (28a^2 e^2 + 48ab d e + 15b^2 d^2)}{3} + \frac{b^4 e^6 x^{13} (15a^2 e^2 + 48ab d e + 28b^2 d^2)}{13}
\end{aligned}$$

input `int((d + e*x)^8*(a^2 + b^2*x^2 + 2*a*b*x)^3,x)`

output

```
x^7*((b^6*d^8)/7 + 4*a^6*d^2*e^6 + 48*a^5*b*d^3*e^5 + 60*a^2*b^4*d^6*e^2 +
160*a^3*b^3*d^5*e^3 + 150*a^4*b^2*d^4*e^4 + (48*a*b^5*d^7*e)/7) + x^9*((a
^6*e^8)/9 + (28*b^6*d^6*e^2)/9 + (112*a*b^5*d^5*e^3)/3 + (350*a^2*b^4*d^4*
e^4)/3 + (1120*a^3*b^3*d^3*e^5)/9 + (140*a^4*b^2*d^2*e^6)/3 + (16*a^5*b*d*
e^7)/3) + x^5*(3*a^2*b^4*d^8 + 14*a^6*d^4*e^4 + 32*a^3*b^3*d^7*e + (336*a^
5*b*d^5*e^3)/5 + 84*a^4*b^2*d^6*e^2) + x^11*((15*a^4*b^2*e^8)/11 + (70*b^6
*d^4*e^4)/11 + (336*a*b^5*d^3*e^5)/11 + (160*a^3*b^3*d*e^7)/11 + (420*a^2*
b^4*d^2*e^6)/11) + x^6*(a*b^5*d^8 + (28*a^6*d^3*e^5)/3 + 20*a^2*b^4*d^7*e
+ 70*a^5*b*d^4*e^4 + (280*a^3*b^3*d^6*e^2)/3 + 140*a^4*b^2*d^5*e^3) + x^10
*((3*a^5*b*e^8)/5 + (28*b^6*d^5*e^3)/5 + 42*a*b^5*d^4*e^4 + 12*a^4*b^2*d*e
^7 + 84*a^2*b^4*d^3*e^5 + 56*a^3*b^3*d^2*e^6) + x^8*(a^6*d*e^7 + b^6*d^7*e
+ 21*a*b^5*d^6*e^2 + 21*a^5*b*d^2*e^6 + 105*a^2*b^4*d^5*e^3 + 175*a^3*b^3
*d^4*e^4 + 105*a^4*b^2*d^3*e^5) + a^6*d^8*x + (b^6*e^8*x^15)/15 + a^3*d^5*
x^4*(14*a^3*e^3 + 5*b^3*d^3 + 30*a*b^2*d^2*e + 42*a^2*b*d*e^2) + (b^3*e^5*
x^12*(5*a^3*e^3 + 14*b^3*d^3 + 42*a*b^2*d^2*e + 30*a^2*b*d*e^2))/3 + a^5*d
^7*x^2*(4*a*e + 3*b*d) + (b^5*e^7*x^14*(3*a*e + 4*b*d))/7 + (a^4*d^6*x^3*(
28*a^2*e^2 + 15*b^2*d^2 + 48*a*b*d*e))/3 + (b^4*e^6*x^13*(15*a^2*e^2 + 28*
b^2*d^2 + 48*a*b*d*e))/13
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 908, normalized size of antiderivative = 5.25

$$\int (d + ex)^8 (a^2 + 2abx + b^2x^2)^3 dx = \text{Too large to display}$$

input

```
int((e*x+d)^8*(b^2*x^2+2*a*b*x+a^2)^3,x)
```

output

```
(x*(45045*a**6*d**8 + 180180*a**6*d**7*e*x + 420420*a**6*d**6*e**2*x**2 +
630630*a**6*d**5*e**3*x**3 + 630630*a**6*d**4*e**4*x**4 + 420420*a**6*d**3
*e**5*x**5 + 180180*a**6*d**2*e**6*x**6 + 45045*a**6*d*e**7*x**7 + 5005*a
*6*e**8*x**8 + 135135*a**5*b*d**8*x + 720720*a**5*b*d**7*e*x**2 + 1891890*
a**5*b*d**6*e**2*x**3 + 3027024*a**5*b*d**5*e**3*x**4 + 3153150*a**5*b*d**
4*e**4*x**5 + 2162160*a**5*b*d**3*e**5*x**6 + 945945*a**5*b*d**2*e**6*x**7
+ 240240*a**5*b*d*e**7*x**8 + 27027*a**5*b*e**8*x**9 + 225225*a**4*b**2*d
**8*x**2 + 1351350*a**4*b**2*d**7*e*x**3 + 3783780*a**4*b**2*d**6*e**2*x**
4 + 6306300*a**4*b**2*d**5*e**3*x**5 + 6756750*a**4*b**2*d**4*e**4*x**6 +
4729725*a**4*b**2*d**3*e**5*x**7 + 2102100*a**4*b**2*d**2*e**6*x**8 + 5405
40*a**4*b**2*d*e**7*x**9 + 61425*a**4*b**2*e**8*x**10 + 225225*a**3*b**3*d
**8*x**3 + 1441440*a**3*b**3*d**7*e*x**4 + 4204200*a**3*b**3*d**6*e**2*x**
5 + 7207200*a**3*b**3*d**5*e**3*x**6 + 7882875*a**3*b**3*d**4*e**4*x**7 +
5605600*a**3*b**3*d**3*e**5*x**8 + 2522520*a**3*b**3*d**2*e**6*x**9 + 6552
00*a**3*b**3*d*e**7*x**10 + 75075*a**3*b**3*e**8*x**11 + 135135*a**2*b**4*
d**8*x**4 + 900900*a**2*b**4*d**7*e*x**5 + 2702700*a**2*b**4*d**6*e**2*x**
6 + 4729725*a**2*b**4*d**5*e**3*x**7 + 5255250*a**2*b**4*d**4*e**4*x**8 +
3783780*a**2*b**4*d**3*e**5*x**9 + 1719900*a**2*b**4*d**2*e**6*x**10 + 450
450*a**2*b**4*d*e**7*x**11 + 51975*a**2*b**4*e**8*x**12 + 45045*a*b**5*d**
8*x**5 + 308880*a*b**5*d**7*e*x**6 + 945945*a*b**5*d**6*e**2*x**7 + 168...
```

3.85 $\int (d + ex)^7 (a^2 + 2abx + b^2x^2)^3 dx$

| | |
|---|-----|
| Optimal result | 682 |
| Mathematica [B] (verified) | 683 |
| Rubi [A] (verified) | 684 |
| Maple [B] (verified) | 686 |
| Fricas [B] (verification not implemented) | 687 |
| Sympy [B] (verification not implemented) | 688 |
| Maxima [B] (verification not implemented) | 690 |
| Giac [B] (verification not implemented) | 692 |
| Mupad [B] (verification not implemented) | 693 |
| Reduce [B] (verification not implemented) | 694 |

Optimal result

Integrand size = 26, antiderivative size = 173

$$\int (d + ex)^7 (a^2 + 2abx + b^2x^2)^3 dx = \frac{(bd - ae)^6(d + ex)^8}{8e^7} - \frac{2b(bd - ae)^5(d + ex)^9}{3e^7} + \frac{3b^2(bd - ae)^4(d + ex)^{10}}{2e^7} - \frac{20b^3(bd - ae)^3(d + ex)^{11}}{11e^7} + \frac{5b^4(bd - ae)^2(d + ex)^{12}}{4e^7} - \frac{6b^5(bd - ae)(d + ex)^{13}}{13e^7} + \frac{b^6(d + ex)^{14}}{14e^7}$$

output

```
1/8*(-a*e+b*d)^6*(e*x+d)^8/e^7-2/3*b*(-a*e+b*d)^5*(e*x+d)^9/e^7+3/2*b^2*(-a*e+b*d)^4*(e*x+d)^10/e^7-20/11*b^3*(-a*e+b*d)^3*(e*x+d)^11/e^7+5/4*b^4*(-a*e+b*d)^2*(e*x+d)^12/e^7-6/13*b^5*(-a*e+b*d)*(e*x+d)^13/e^7+1/14*b^6*(e*x+d)^14/e^7
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 684 vs. $2(173) = 346$.

Time = 0.07 (sec) , antiderivative size = 684, normalized size of antiderivative = 3.95

$$\begin{aligned}
 \int (d + ex)^7 (a^2 + 2abx + b^2x^2)^3 dx = & a^6 d^7 x + \frac{1}{2} a^5 d^6 (6bd + 7ae) x^2 \\
 & + a^4 d^5 (5b^2 d^2 + 14abde + 7a^2 e^2) x^3 \\
 & + \frac{1}{4} a^3 d^4 (20b^3 d^3 + 105ab^2 d^2 e + 126a^2 bde^2 \\
 & \quad + 35a^3 e^3) x^4 + a^2 d^3 (3b^4 d^4 + 28ab^3 d^3 e \\
 & \quad + 63a^2 b^2 d^2 e^2 + 42a^3 bde^3 + 7a^4 e^4) x^5 \\
 & + \frac{1}{2} ad^2 (2b^5 d^5 + 35ab^4 d^4 e + 140a^2 b^3 d^3 e^2 \\
 & \quad + 175a^3 b^2 d^2 e^3 + 70a^4 bde^4 + 7a^5 e^5) x^6 \\
 & + \frac{1}{7} d (b^6 d^6 + 42ab^5 d^5 e + 315a^2 b^4 d^4 e^2 + 700a^3 b^3 d^3 e^3 \\
 & \quad + 525a^4 b^2 d^2 e^4 + 126a^5 bde^5 + 7a^6 e^6) x^7 \\
 & + \frac{1}{8} e (7b^6 d^6 + 126ab^5 d^5 e + 525a^2 b^4 d^4 e^2 \\
 & \quad + 700a^3 b^3 d^3 e^3 + 315a^4 b^2 d^2 e^4 + 42a^5 bde^5 + a^6 e^6) x^8 \\
 & + \frac{1}{3} be^2 (7b^5 d^5 + 70ab^4 d^4 e + 175a^2 b^3 d^3 e^2 \\
 & \quad + 140a^3 b^2 d^2 e^3 + 35a^4 bde^4 + 2a^5 e^5) x^9 \\
 & + \frac{1}{2} b^2 e^3 (7b^4 d^4 + 42ab^3 d^3 e + 63a^2 b^2 d^2 e^2 \\
 & \quad + 28a^3 bde^3 + 3a^4 e^4) x^{10} + \frac{1}{11} b^3 e^4 (35b^3 d^3 \\
 & \quad + 126ab^2 d^2 e + 105a^2 bde^2 + 20a^3 e^3) x^{11} \\
 & + \frac{1}{4} b^4 e^5 (7b^2 d^2 + 14abde + 5a^2 e^2) x^{12} \\
 & + \frac{1}{13} b^5 e^6 (7bd + 6ae) x^{13} + \frac{1}{14} b^6 e^7 x^{14}
 \end{aligned}$$

input

```
Integrate[(d + e*x)^7*(a^2 + 2*a*b*x + b^2*x^2)^3,x]
```


output

```

a^6*d^7*x + (a^5*d^6*(6*b*d + 7*a*e)*x^2)/2 + a^4*d^5*(5*b^2*d^2 + 14*a*b*
d*e + 7*a^2*e^2)*x^3 + (a^3*d^4*(20*b^3*d^3 + 105*a*b^2*d^2*e + 126*a^2*b*
d*e^2 + 35*a^3*e^3)*x^4)/4 + a^2*d^3*(3*b^4*d^4 + 28*a*b^3*d^3*e + 63*a^2*
b^2*d^2*e^2 + 42*a^3*b*d*e^3 + 7*a^4*e^4)*x^5 + (a*d^2*(2*b^5*d^5 + 35*a*b
^4*d^4*e + 140*a^2*b^3*d^3*e^2 + 175*a^3*b^2*d^2*e^3 + 70*a^4*b*d*e^4 + 7*
a^5*e^5)*x^6)/2 + (d*(b^6*d^6 + 42*a*b^5*d^5*e + 315*a^2*b^4*d^4*e^2 + 700
*a^3*b^3*d^3*e^3 + 525*a^4*b^2*d^2*e^4 + 126*a^5*b*d*e^5 + 7*a^6*e^6)*x^7)
/7 + (e*(7*b^6*d^6 + 126*a*b^5*d^5*e + 525*a^2*b^4*d^4*e^2 + 700*a^3*b^3*d
^3*e^3 + 315*a^4*b^2*d^2*e^4 + 42*a^5*b*d*e^5 + a^6*e^6)*x^8)/8 + (b*e^2*(
7*b^5*d^5 + 70*a*b^4*d^4*e + 175*a^2*b^3*d^3*e^2 + 140*a^3*b^2*d^2*e^3 + 3
5*a^4*b*d*e^4 + 2*a^5*e^5)*x^9)/3 + (b^2*e^3*(7*b^4*d^4 + 42*a*b^3*d^3*e +
63*a^2*b^2*d^2*e^2 + 28*a^3*b*d*e^3 + 3*a^4*e^4)*x^10)/2 + (b^3*e^4*(35*b
^3*d^3 + 126*a*b^2*d^2*e + 105*a^2*b*d*e^2 + 20*a^3*e^3)*x^11)/11 + (b^4*e
^5*(7*b^2*d^2 + 14*a*b*d*e + 5*a^2*e^2)*x^12)/4 + (b^5*e^6*(7*b*d + 6*a*e)
*x^13)/13 + (b^6*e^7*x^14)/14

```

Rubi [A] (verified)

Time = 1.01 (sec) , antiderivative size = 173, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {1098, 27, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\begin{aligned}
 & \int (a^2 + 2abx + b^2x^2)^3 (d + ex)^7 dx \\
 & \quad \downarrow 1098 \\
 & \quad \int \frac{b^6(a + bx)^6(d + ex)^7 dx}{b^6} \\
 & \quad \downarrow 27 \\
 & \quad \int (a + bx)^6(d + ex)^7 dx \\
 & \quad \downarrow 49 \\
 & \int \left(-\frac{6b^5(d + ex)^{12}(bd - ae)}{e^6} + \frac{15b^4(d + ex)^{11}(bd - ae)^2}{e^6} - \frac{20b^3(d + ex)^{10}(bd - ae)^3}{e^6} + \frac{15b^2(d + ex)^9(bd - ae)^4}{e^6} \right) dx
 \end{aligned}$$

$$\begin{array}{c} \downarrow \text{2009} \\ \frac{6b^5(d+ex)^{13}(bd-ae)}{13e^7} + \frac{5b^4(d+ex)^{12}(bd-ae)^2}{4e^7} - \frac{20b^3(d+ex)^{11}(bd-ae)^3}{11e^7} + \\ \frac{3b^2(d+ex)^{10}(bd-ae)^4}{2e^7} - \frac{2b(d+ex)^9(bd-ae)^5}{3e^7} + \frac{(d+ex)^8(bd-ae)^6}{8e^7} + \frac{b^6(d+ex)^{14}}{14e^7} \end{array}$$

input `Int[(d + e*x)^7*(a^2 + 2*a*b*x + b^2*x^2)^3,x]`

output `((b*d - a*e)^6*(d + e*x)^8)/(8*e^7) - (2*b*(b*d - a*e)^5*(d + e*x)^9)/(3*e^7) + (3*b^2*(b*d - a*e)^4*(d + e*x)^10)/(2*e^7) - (20*b^3*(b*d - a*e)^3*(d + e*x)^11)/(11*e^7) + (5*b^4*(b*d - a*e)^2*(d + e*x)^12)/(4*e^7) - (6*b^5*(b*d - a*e)*(d + e*x)^13)/(13*e^7) + (b^6*(d + e*x)^14)/(14*e^7)`

Defintions of rubi rules used

rule 27 `Int[(a_)*(F_x_), x_Symbol] := Simp[a Int[F_x, x], x] /; FreeQ[a, x] && !MatchQ[F_x, (b_)*(G_x_) /; FreeQ[b, x]]`

rule 49 `Int[((a_) + (b_)*(x_))^(m_)*((c_) + (d_)*(x_))^(n_), x_Symbol] := Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] && IGtQ[m, 0] && IGtQ[m + n + 2, 0]`

rule 1098 `Int[((d_) + (e_)*(x_))^(m_)*((a_) + (b_)*(x_) + (c_)*(x_)^2)^(p_), x_Symbol] := Simp[1/c^p Int[(d + e*x)^m*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]`

rule 2009 `Int[u_, x_Symbol] := Simp[IntSum[u, x], x] /; SumQ[u]`

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 706 vs. $2(159) = 318$.

Time = 0.09 (sec) , antiderivative size = 706, normalized size of antiderivative = 4.08

$$\begin{aligned}
& \int (d + ex)^7 (a^2 + 2abx + b^2x^2)^3 dx \\
&= \frac{1}{14} b^6 e^7 x^{14} + a^6 d^7 x + \frac{1}{13} (7b^6 d e^6 + 6ab^5 e^7) x^{13} + \frac{1}{4} (7b^6 d^2 e^5 + 14ab^5 d e^6 + 5a^2 b^4 e^7) x^{12} \\
&+ \frac{1}{11} (35b^6 d^3 e^4 + 126ab^5 d^2 e^5 + 105a^2 b^4 d e^6 + 20a^3 b^3 e^7) x^{11} \\
&+ \frac{1}{2} (7b^6 d^4 e^3 + 42ab^5 d^3 e^4 + 63a^2 b^4 d^2 e^5 + 28a^3 b^3 d e^6 + 3a^4 b^2 e^7) x^{10} \\
&+ \frac{1}{3} (7b^6 d^5 e^2 + 70ab^5 d^4 e^3 + 175a^2 b^4 d^3 e^4 + 140a^3 b^3 d^2 e^5 + 35a^4 b^2 d e^6 + 2a^5 b e^7) x^9 \\
&+ \frac{1}{8} (7b^6 d^6 e + 126ab^5 d^5 e^2 + 525a^2 b^4 d^4 e^3 + 700a^3 b^3 d^3 e^4 + 315a^4 b^2 d^2 e^5 + 42a^5 b d e^6 + a^6 e^7) x^8 \\
&+ \frac{1}{7} (b^6 d^7 + 42ab^5 d^6 e + 315a^2 b^4 d^5 e^2 + 700a^3 b^3 d^4 e^3 + 525a^4 b^2 d^3 e^4 + 126a^5 b d^2 e^5 + 7a^6 d e^6) x^7 \\
&+ \frac{1}{2} (2ab^5 d^7 + 35a^2 b^4 d^6 e + 140a^3 b^3 d^5 e^2 + 175a^4 b^2 d^4 e^3 + 70a^5 b d^3 e^4 + 7a^6 d^2 e^5) x^6 \\
&+ (3a^2 b^4 d^7 + 28a^3 b^3 d^6 e + 63a^4 b^2 d^5 e^2 + 42a^5 b d^4 e^3 + 7a^6 d^3 e^4) x^5 \\
&+ \frac{1}{4} (20a^3 b^3 d^7 + 105a^4 b^2 d^6 e + 126a^5 b d^5 e^2 + 35a^6 d^4 e^3) x^4 \\
&+ (5a^4 b^2 d^7 + 14a^5 b d^6 e + 7a^6 d^5 e^2) x^3 + \frac{1}{2} (6a^5 b d^7 + 7a^6 d^6 e) x^2
\end{aligned}$$

input `integrate((e*x+d)^7*(b^2*x^2+2*a*b*x+a^2)^3,x, algorithm="fricas")`

output

```

1/14*b^6*e^7*x^14 + a^6*d^7*x + 1/13*(7*b^6*d*e^6 + 6*a*b^5*e^7)*x^13 + 1/
4*(7*b^6*d^2*e^5 + 14*a*b^5*d*e^6 + 5*a^2*b^4*e^7)*x^12 + 1/11*(35*b^6*d^3
*e^4 + 126*a*b^5*d^2*e^5 + 105*a^2*b^4*d*e^6 + 20*a^3*b^3*e^7)*x^11 + 1/2*
(7*b^6*d^4*e^3 + 42*a*b^5*d^3*e^4 + 63*a^2*b^4*d^2*e^5 + 28*a^3*b^3*d*e^6
+ 3*a^4*b^2*e^7)*x^10 + 1/3*(7*b^6*d^5*e^2 + 70*a*b^5*d^4*e^3 + 175*a^2*b^
4*d^3*e^4 + 140*a^3*b^3*d^2*e^5 + 35*a^4*b^2*d*e^6 + 2*a^5*b*e^7)*x^9 + 1/
8*(7*b^6*d^6*e + 126*a*b^5*d^5*e^2 + 525*a^2*b^4*d^4*e^3 + 700*a^3*b^3*d^3
*e^4 + 315*a^4*b^2*d^2*e^5 + 42*a^5*b*d*e^6 + a^6*e^7)*x^8 + 1/7*(b^6*d^7
+ 42*a*b^5*d^6*e + 315*a^2*b^4*d^5*e^2 + 700*a^3*b^3*d^4*e^3 + 525*a^4*b^2
*d^3*e^4 + 126*a^5*b*d^2*e^5 + 7*a^6*d*e^6)*x^7 + 1/2*(2*a*b^5*d^7 + 35*a^
2*b^4*d^6*e + 140*a^3*b^3*d^5*e^2 + 175*a^4*b^2*d^4*e^3 + 70*a^5*b*d^3*e^4
+ 7*a^6*d^2*e^5)*x^6 + (3*a^2*b^4*d^7 + 28*a^3*b^3*d^6*e + 63*a^4*b^2*d^5
*e^2 + 42*a^5*b*d^4*e^3 + 7*a^6*d^3*e^4)*x^5 + 1/4*(20*a^3*b^3*d^7 + 105*a^
4*b^2*d^6*e + 126*a^5*b*d^5*e^2 + 35*a^6*d^4*e^3)*x^4 + (5*a^4*b^2*d^7 +
14*a^5*b*d^6*e + 7*a^6*d^5*e^2)*x^3 + 1/2*(6*a^5*b*d^7 + 7*a^6*d^6*e)*x^2

```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 796 vs. $2(158) = 316$.

Time = 0.06 (sec) , antiderivative size = 796, normalized size of antiderivative = 4.60

$$\begin{aligned}
 \int (d+ex)^7 (a^2+2abx+b^2x^2)^3 dx = & a^6 d^7 x + \frac{b^6 e^7 x^{14}}{14} + x^{13} \cdot \left(\frac{6ab^5 e^7}{13} + \frac{7b^6 d e^6}{13} \right) \\
 & + x^{12} \cdot \left(\frac{5a^2 b^4 e^7}{4} + \frac{7ab^5 d e^6}{2} + \frac{7b^6 d^2 e^5}{4} \right) + x^{11} \\
 & \cdot \left(\frac{20a^3 b^3 e^7}{11} + \frac{105a^2 b^4 d e^6}{11} + \frac{126ab^5 d^2 e^5}{11} \right. \\
 & \quad \left. + \frac{35b^6 d^3 e^4}{11} \right) + x^{10} \cdot \left(\frac{3a^4 b^2 e^7}{2} + 14a^3 b^3 d e^6 \right. \\
 & \quad \left. + \frac{63a^2 b^4 d^2 e^5}{2} + 21ab^5 d^3 e^4 + \frac{7b^6 d^4 e^3}{2} \right) \\
 & + x^9 \cdot \left(\frac{2a^5 b e^7}{3} + \frac{35a^4 b^2 d e^6}{3} + \frac{140a^3 b^3 d^2 e^5}{3} \right. \\
 & \quad \left. + \frac{175a^2 b^4 d^3 e^4}{3} + \frac{70ab^5 d^4 e^3}{3} + \frac{7b^6 d^5 e^2}{3} \right) \\
 & + x^8 \left(\frac{a^6 e^7}{8} + \frac{21a^5 b d e^6}{4} + \frac{315a^4 b^2 d^2 e^5}{8} \right. \\
 & \quad \left. + \frac{175a^3 b^3 d^3 e^4}{2} + \frac{525a^2 b^4 d^4 e^3}{8} + \frac{63ab^5 d^5 e^2}{4} \right. \\
 & \quad \left. + \frac{7b^6 d^6 e}{8} \right) + x^7 \left(a^6 d e^6 + 18a^5 b d^2 e^5 + 75a^4 b^2 d^3 e^4 \right. \\
 & \quad \left. + 100a^3 b^3 d^4 e^3 + 45a^2 b^4 d^5 e^2 + 6ab^5 d^6 e + \frac{b^6 d^7}{7} \right) \\
 & + x^6 \cdot \left(\frac{7a^6 d^2 e^5}{2} + 35a^5 b d^3 e^4 + \frac{175a^4 b^2 d^4 e^3}{2} \right. \\
 & \quad \left. + 70a^3 b^3 d^5 e^2 + \frac{35a^2 b^4 d^6 e}{2} + ab^5 d^7 \right) + x^5 \\
 & \cdot \left(7a^6 d^3 e^4 + 42a^5 b d^4 e^3 + 63a^4 b^2 d^5 e^2 + 28a^3 b^3 d^6 e \right. \\
 & \quad \left. + 3a^2 b^4 d^7 \right) + x^4 \\
 & \cdot \left(\frac{35a^6 d^4 e^3}{4} + \frac{63a^5 b d^5 e^2}{2} + \frac{105a^4 b^2 d^6 e}{4} + 5a^3 b^3 d^7 \right) \\
 & + x^3 \cdot \left(7a^6 d^5 e^2 + 14a^5 b d^6 e + 5a^4 b^2 d^7 \right) \\
 & + x^2 \cdot \left(\frac{7a^6 d^6 e}{2} + 3a^5 b d^7 \right)
 \end{aligned}$$

input `integrate((e*x+d)**7*(b**2*x**2+2*a*b*x+a**2)**3,x)`

output

```

a**6*d**7*x + b**6*e**7*x**14/14 + x**13*(6*a*b**5*e**7/13 + 7*b**6*d*e**6
/13) + x**12*(5*a**2*b**4*e**7/4 + 7*a*b**5*d*e**6/2 + 7*b**6*d**2*e**5/4)
+ x**11*(20*a**3*b**3*e**7/11 + 105*a**2*b**4*d*e**6/11 + 126*a*b**5*d**2
*e**5/11 + 35*b**6*d**3*e**4/11) + x**10*(3*a**4*b**2*e**7/2 + 14*a**3*b**
3*d*e**6 + 63*a**2*b**4*d**2*e**5/2 + 21*a*b**5*d**3*e**4 + 7*b**6*d**4*e*
*3/2) + x**9*(2*a**5*b*e**7/3 + 35*a**4*b**2*d*e**6/3 + 140*a**3*b**3*d**2
*e**5/3 + 175*a**2*b**4*d**3*e**4/3 + 70*a*b**5*d**4*e**3/3 + 7*b**6*d**5*
e**2/3) + x**8*(a**6*e**7/8 + 21*a**5*b*d*e**6/4 + 315*a**4*b**2*d**2*e**5
/8 + 175*a**3*b**3*d**3*e**4/2 + 525*a**2*b**4*d**4*e**3/8 + 63*a*b**5*d**
5*e**2/4 + 7*b**6*d**6*e/8) + x**7*(a**6*d*e**6 + 18*a**5*b*d**2*e**5 + 75
*a**4*b**2*d**3*e**4 + 100*a**3*b**3*d**4*e**3 + 45*a**2*b**4*d**5*e**2 +
6*a*b**5*d**6*e + b**6*d**7/7) + x**6*(7*a**6*d**2*e**5/2 + 35*a**5*b*d**3
*e**4 + 175*a**4*b**2*d**4*e**3/2 + 70*a**3*b**3*d**5*e**2 + 35*a**2*b**4*
d**6*e/2 + a*b**5*d**7) + x**5*(7*a**6*d**3*e**4 + 42*a**5*b*d**4*e**3 + 6
3*a**4*b**2*d**5*e**2 + 28*a**3*b**3*d**6*e + 3*a**2*b**4*d**7) + x**4*(35
*a**6*d**4*e**3/4 + 63*a**5*b*d**5*e**2/2 + 105*a**4*b**2*d**6*e/4 + 5*a**
3*b**3*d**7) + x**3*(7*a**6*d**5*e**2 + 14*a**5*b*d**6*e + 5*a**4*b**2*d**
7) + x**2*(7*a**6*d**6*e/2 + 3*a**5*b*d**7)

```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 706 vs. $2(159) = 318$.

Time = 0.04 (sec) , antiderivative size = 706, normalized size of antiderivative = 4.08

$$\int (d + ex)^7 (a^2 + 2abx + b^2x^2)^3 dx$$

$$= \frac{1}{14} b^6 e^7 x^{14} + a^6 d^7 x + \frac{1}{13} (7b^6 d e^6 + 6ab^5 e^7) x^{13} + \frac{1}{4} (7b^6 d^2 e^5 + 14ab^5 d e^6 + 5a^2 b^4 e^7) x^{12}$$

$$+ \frac{1}{11} (35b^6 d^3 e^4 + 126ab^5 d^2 e^5 + 105a^2 b^4 d e^6 + 20a^3 b^3 e^7) x^{11}$$

$$+ \frac{1}{2} (7b^6 d^4 e^3 + 42ab^5 d^3 e^4 + 63a^2 b^4 d^2 e^5 + 28a^3 b^3 d e^6 + 3a^4 b^2 e^7) x^{10}$$

$$+ \frac{1}{3} (7b^6 d^5 e^2 + 70ab^5 d^4 e^3 + 175a^2 b^4 d^3 e^4 + 140a^3 b^3 d^2 e^5 + 35a^4 b^2 d e^6 + 2a^5 b e^7) x^9$$

$$+ \frac{1}{8} (7b^6 d^6 e + 126ab^5 d^5 e^2 + 525a^2 b^4 d^4 e^3 + 700a^3 b^3 d^3 e^4 + 315a^4 b^2 d^2 e^5 + 42a^5 b d e^6 + a^6 e^7) x^8$$

$$+ \frac{1}{7} (b^6 d^7 + 42ab^5 d^6 e + 315a^2 b^4 d^5 e^2 + 700a^3 b^3 d^4 e^3 + 525a^4 b^2 d^3 e^4 + 126a^5 b d^2 e^5 + 7a^6 d e^6) x^7$$

$$+ \frac{1}{2} (2ab^5 d^7 + 35a^2 b^4 d^6 e + 140a^3 b^3 d^5 e^2 + 175a^4 b^2 d^4 e^3 + 70a^5 b d^3 e^4 + 7a^6 d^2 e^5) x^6$$

$$+ (3a^2 b^4 d^7 + 28a^3 b^3 d^6 e + 63a^4 b^2 d^5 e^2 + 42a^5 b d^4 e^3 + 7a^6 d^3 e^4) x^5$$

$$+ \frac{1}{4} (20a^3 b^3 d^7 + 105a^4 b^2 d^6 e + 126a^5 b d^5 e^2 + 35a^6 d^4 e^3) x^4$$

$$+ (5a^4 b^2 d^7 + 14a^5 b d^6 e + 7a^6 d^5 e^2) x^3 + \frac{1}{2} (6a^5 b d^7 + 7a^6 d^6 e) x^2$$

input `integrate((e*x+d)^7*(b^2*x^2+2*a*b*x+a^2)^3,x, algorithm="maxima")`

output `1/14*b^6*e^7*x^14 + a^6*d^7*x + 1/13*(7*b^6*d*e^6 + 6*a*b^5*e^7)*x^13 + 1/4*(7*b^6*d^2*e^5 + 14*a*b^5*d*e^6 + 5*a^2*b^4*e^7)*x^12 + 1/11*(35*b^6*d^3*e^4 + 126*a*b^5*d^2*e^5 + 105*a^2*b^4*d*e^6 + 20*a^3*b^3*e^7)*x^11 + 1/2*(7*b^6*d^4*e^3 + 42*a*b^5*d^3*e^4 + 63*a^2*b^4*d^2*e^5 + 28*a^3*b^3*d*e^6 + 3*a^4*b^2*e^7)*x^10 + 1/3*(7*b^6*d^5*e^2 + 70*a*b^5*d^4*e^3 + 175*a^2*b^4*d^3*e^4 + 140*a^3*b^3*d^2*e^5 + 35*a^4*b^2*d*e^6 + 2*a^5*b*e^7)*x^9 + 1/8*(7*b^6*d^6*e + 126*a*b^5*d^5*e^2 + 525*a^2*b^4*d^4*e^3 + 700*a^3*b^3*d^3*e^4 + 315*a^4*b^2*d^2*e^5 + 42*a^5*b*d*e^6 + a^6*e^7)*x^8 + 1/7*(b^6*d^7 + 42*a*b^5*d^6*e + 315*a^2*b^4*d^5*e^2 + 700*a^3*b^3*d^4*e^3 + 525*a^4*b^2*d^3*e^4 + 126*a^5*b*d^2*e^5 + 7*a^6*d*e^6)*x^7 + 1/2*(2*a*b^5*d^7 + 35*a^2*b^4*d^6*e + 140*a^3*b^3*d^5*e^2 + 175*a^4*b^2*d^4*e^3 + 70*a^5*b*d^3*e^4 + 7*a^6*d^2*e^5)*x^6 + (3*a^2*b^4*d^7 + 28*a^3*b^3*d^6*e + 63*a^4*b^2*d^5*e^2 + 42*a^5*b*d^4*e^3 + 7*a^6*d^3*e^4)*x^5 + 1/4*(20*a^3*b^3*d^7 + 105*a^4*b^2*d^6*e + 126*a^5*b*d^5*e^2 + 35*a^6*d^4*e^3)*x^4 + (5*a^4*b^2*d^7 + 14*a^5*b*d^6*e + 7*a^6*d^5*e^2)*x^3 + 1/2*(6*a^5*b*d^7 + 7*a^6*d^6*e)*x^2`

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 798 vs. $2(159) = 318$.

Time = 0.19 (sec) , antiderivative size = 798, normalized size of antiderivative = 4.61

$$\int (d + ex)^7 (a^2 + 2abx + b^2x^2)^3 dx = \text{Too large to display}$$

input `integrate((e*x+d)^7*(b^2*x^2+2*a*b*x+a^2)^3,x, algorithm="giac")`

output

```
1/14*b^6*e^7*x^14 + 7/13*b^6*d*e^6*x^13 + 6/13*a*b^5*e^7*x^13 + 7/4*b^6*d^2*
e^5*x^12 + 7/2*a*b^5*d*e^6*x^12 + 5/4*a^2*b^4*e^7*x^12 + 35/11*b^6*d^3*e^4*
x^11 + 126/11*a*b^5*d^2*e^5*x^11 + 105/11*a^2*b^4*d*e^6*x^11 + 20/11*a^3*
b^3*e^7*x^11 + 7/2*b^6*d^4*e^3*x^10 + 21*a*b^5*d^3*e^4*x^10 + 63/2*a^2*b^4*
d^2*e^5*x^10 + 14*a^3*b^3*d*e^6*x^10 + 3/2*a^4*b^2*e^7*x^10 + 7/3*b^6*d^5*
e^2*x^9 + 70/3*a*b^5*d^4*e^3*x^9 + 175/3*a^2*b^4*d^3*e^4*x^9 + 140/3*a^3*
b^3*d^2*e^5*x^9 + 35/3*a^4*b^2*d*e^6*x^9 + 2/3*a^5*b*e^7*x^9 + 7/8*b^6*d^6*
e*x^8 + 63/4*a*b^5*d^5*e^2*x^8 + 525/8*a^2*b^4*d^4*e^3*x^8 + 175/2*a^3*
b^3*d^3*e^4*x^8 + 315/8*a^4*b^2*d^2*e^5*x^8 + 21/4*a^5*b*d*e^6*x^8 + 1/8*a^6*
e^7*x^8 + 1/7*b^6*d^7*x^7 + 6*a*b^5*d^6*e*x^7 + 45*a^2*b^4*d^5*e^2*x^7 +
100*a^3*b^3*d^4*e^3*x^7 + 75*a^4*b^2*d^3*e^4*x^7 + 18*a^5*b*d^2*e^5*x^7 +
a^6*d*e^6*x^7 + a*b^5*d^7*x^6 + 35/2*a^2*b^4*d^6*e*x^6 + 70*a^3*b^3*d^5*
e^2*x^6 + 175/2*a^4*b^2*d^4*e^3*x^6 + 35*a^5*b*d^3*e^4*x^6 + 7/2*a^6*d^2*
e^5*x^6 + 3*a^2*b^4*d^7*x^5 + 28*a^3*b^3*d^6*e*x^5 + 63*a^4*b^2*d^5*e^2*x^5 +
42*a^5*b*d^4*e^3*x^5 + 7*a^6*d^3*e^4*x^5 + 5*a^3*b^3*d^7*x^4 + 105/4*a^4*
b^2*d^6*e*x^4 + 63/2*a^5*b*d^5*e^2*x^4 + 35/4*a^6*d^4*e^3*x^4 + 5*a^4*b^2*
d^7*x^3 + 14*a^5*b*d^6*e*x^3 + 7*a^6*d^5*e^2*x^3 + 3*a^5*b*d^7*x^2 + 7/2*
a^6*d^6*e*x^2 + a^6*d^7*x
```

Mupad [B] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 683, normalized size of antiderivative = 3.95

$$\begin{aligned}
\int (d + ex)^7 (a^2 + 2abx + b^2x^2)^3 dx = & x^5 (7a^6 d^3 e^4 + 42a^5 b d^4 e^3 + 63a^4 b^2 d^5 e^2 \\
& + 28a^3 b^3 d^6 e + 3a^2 b^4 d^7) \\
& + x^{10} \left(\frac{3a^4 b^2 e^7}{2} + 14a^3 b^3 d e^6 + \frac{63a^2 b^4 d^2 e^5}{2} \right. \\
& \left. + 21ab^5 d^3 e^4 + \frac{7b^6 d^4 e^3}{2} \right) \\
& + x^6 \left(\frac{7a^6 d^2 e^5}{2} + 35a^5 b d^3 e^4 + \frac{175a^4 b^2 d^4 e^3}{2} \right. \\
& \left. + 70a^3 b^3 d^5 e^2 + \frac{35a^2 b^4 d^6 e}{2} + ab^5 d^7 \right) \\
& + x^9 \left(\frac{2a^5 b e^7}{3} + \frac{35a^4 b^2 d e^6}{3} + \frac{140a^3 b^3 d^2 e^5}{3} \right. \\
& \left. + \frac{175a^2 b^4 d^3 e^4}{3} + \frac{70ab^5 d^4 e^3}{3} + \frac{7b^6 d^5 e^2}{3} \right) \\
& + x^7 \left(a^6 d e^6 + 18a^5 b d^2 e^5 + 75a^4 b^2 d^3 e^4 \right. \\
& \left. + 100a^3 b^3 d^4 e^3 + 45a^2 b^4 d^5 e^2 + 6ab^5 d^6 e \right. \\
& \left. + \frac{b^6 d^7}{7} \right) + x^8 \left(\frac{a^6 e^7}{8} + \frac{21a^5 b d e^6}{4} \right. \\
& \left. + \frac{315a^4 b^2 d^2 e^5}{8} + \frac{175a^3 b^3 d^3 e^4}{2} + \frac{525a^2 b^4 d^4 e^3}{8} \right. \\
& \left. + \frac{63ab^5 d^5 e^2}{4} + \frac{7b^6 d^6 e}{8} \right) + x^4 \left(\frac{35a^6 d^4 e^3}{4} \right. \\
& \left. + \frac{63a^5 b d^5 e^2}{2} + \frac{105a^4 b^2 d^6 e}{4} + 5a^3 b^3 d^7 \right) \\
& + x^{11} \left(\frac{20a^3 b^3 e^7}{11} + \frac{105a^2 b^4 d e^6}{11} + \frac{126ab^5 d^2 e^5}{11} \right. \\
& \left. + \frac{35b^6 d^3 e^4}{11} \right) + a^6 d^7 x + \frac{b^6 e^7 x^{14}}{14} \\
& + \frac{a^5 d^6 x^2 (7ae + 6bd)}{2} + \frac{b^5 e^6 x^{13} (6ae + 7bd)}{13} \\
& + a^4 d^5 x^3 (7a^2 e^2 + 14abde + 5b^2 d^2) \\
& + \frac{b^4 e^5 x^{12} (5a^2 e^2 + 14abde + 7b^2 d^2)}{4}
\end{aligned}$$

input `int((d + e*x)^7*(a^2 + b^2*x^2 + 2*a*b*x)^3,x)`

output

$$\begin{aligned} & x^5*(3*a^2*b^4*d^7 + 7*a^6*d^3*e^4 + 28*a^3*b^3*d^6*e + 42*a^5*b*d^4*e^3 + \\ & 63*a^4*b^2*d^5*e^2) + x^{10}*((3*a^4*b^2*e^7)/2 + (7*b^6*d^4*e^3)/2 + 21*a* \\ & b^5*d^3*e^4 + 14*a^3*b^3*d*e^6 + (63*a^2*b^4*d^2*e^5)/2) + x^6*(a*b^5*d^7 \\ & + (7*a^6*d^2*e^5)/2 + (35*a^2*b^4*d^6*e)/2 + 35*a^5*b*d^3*e^4 + 70*a^3*b^3 \\ & *d^5*e^2 + (175*a^4*b^2*d^4*e^3)/2) + x^9*((2*a^5*b*e^7)/3 + (7*b^6*d^5*e^ \\ & 2)/3 + (70*a*b^5*d^4*e^3)/3 + (35*a^4*b^2*d*e^6)/3 + (175*a^2*b^4*d^3*e^4) \\ & /3 + (140*a^3*b^3*d^2*e^5)/3) + x^7*((b^6*d^7)/7 + a^6*d*e^6 + 18*a^5*b*d^ \\ & 2*e^5 + 45*a^2*b^4*d^5*e^2 + 100*a^3*b^3*d^4*e^3 + 75*a^4*b^2*d^3*e^4 + 6* \\ & a*b^5*d^6*e) + x^8*((a^6*e^7)/8 + (7*b^6*d^6*e)/8 + (63*a*b^5*d^5*e^2)/4 + \\ & (525*a^2*b^4*d^4*e^3)/8 + (175*a^3*b^3*d^3*e^4)/2 + (315*a^4*b^2*d^2*e^5) \\ & /8 + (21*a^5*b*d*e^6)/4) + x^4*(5*a^3*b^3*d^7 + (35*a^6*d^4*e^3)/4 + (105* \\ & a^4*b^2*d^6*e)/4 + (63*a^5*b*d^5*e^2)/2) + x^{11}*((20*a^3*b^3*e^7)/11 + (35 \\ & *b^6*d^3*e^4)/11 + (126*a*b^5*d^2*e^5)/11 + (105*a^2*b^4*d*e^6)/11) + a^6* \\ & d^7*x + (b^6*e^7*x^{14})/14 + (a^5*d^6*x^2*(7*a*e + 6*b*d))/2 + (b^5*e^6*x^{1 \\ & 3*(6*a*e + 7*b*d)})/13 + a^4*d^5*x^3*(7*a^2*e^2 + 5*b^2*d^2 + 14*a*b*d*e) + \\ & (b^4*e^5*x^{12*(5*a^2*e^2 + 7*b^2*d^2 + 14*a*b*d*e)})/4 \end{aligned}$$

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 799, normalized size of antiderivative = 4.62

$$\int (d + ex)^7 (a^2 + 2abx + b^2x^2)^3 dx$$

$$= \frac{x(1716b^6e^7x^{13} + 11088ab^5e^7x^{12} + 12936b^6de^6x^{12} + 30030a^2b^4e^7x^{11} + 84084ab^5de^6x^{11} + 42042b^6d^2e^5x^{10} + 11088a^2b^3e^7x^9 + 12936ab^4de^6x^9 + 30030a^3b^2e^7x^8 + 84084a^2b^3de^6x^8 + 42042ab^4d^2e^5x^7 + 11088a^4b^2e^7x^6 + 12936a^3b^3de^6x^6 + 30030a^4b^2d^2e^5x^5 + 84084a^5b^2e^7x^4 + 12936a^4b^3de^6x^4 + 42042a^5b^2d^2e^5x^3 + 11088a^6b^2e^7x^2 + 12936a^5b^3de^6x^2 + 42042a^6b^2d^2e^5x + 11088a^7e^7x + 12936a^6bde^6x + 42042a^7d^2e^5x + 11088a^8e^7 + 12936a^7bde^6 + 42042a^8d^2e^5 + 11088a^9e^7 + 12936a^8bde^6 + 42042a^9d^2e^5 + 11088a^{10}e^7 + 12936a^9bde^6 + 42042a^{10}d^2e^5 + 11088a^{11}e^7 + 12936a^{10}bde^6 + 42042a^{11}d^2e^5 + 11088a^{12}e^7 + 12936a^{11}bde^6 + 42042a^{12}d^2e^5 + 11088a^{13}e^7 + 12936a^{12}bde^6 + 42042a^{13}d^2e^5 + 11088a^{14}e^7 + 12936a^{13}bde^6 + 42042a^{14}d^2e^5 + 11088a^{15}e^7 + 12936a^{14}bde^6 + 42042a^{15}d^2e^5 + 11088a^{16}e^7 + 12936a^{15}bde^6 + 42042a^{16}d^2e^5 + 11088a^{17}e^7 + 12936a^{16}bde^6 + 42042a^{17}d^2e^5 + 11088a^{18}e^7 + 12936a^{17}bde^6 + 42042a^{18}d^2e^5 + 11088a^{19}e^7 + 12936a^{18}bde^6 + 42042a^{19}d^2e^5 + 11088a^{20}e^7 + 12936a^{19}bde^6 + 42042a^{20}d^2e^5 + 11088a^{21}e^7 + 12936a^{20}bde^6 + 42042a^{21}d^2e^5 + 11088a^{22}e^7 + 12936a^{21}bde^6 + 42042a^{22}d^2e^5 + 11088a^{23}e^7 + 12936a^{22}bde^6 + 42042a^{23}d^2e^5 + 11088a^{24}e^7 + 12936a^{23}bde^6 + 42042a^{24}d^2e^5 + 11088a^{25}e^7 + 12936a^{24}bde^6 + 42042a^{25}d^2e^5 + 11088a^{26}e^7 + 12936a^{25}bde^6 + 42042a^{26}d^2e^5 + 11088a^{27}e^7 + 12936a^{26}bde^6 + 42042a^{27}d^2e^5 + 11088a^{28}e^7 + 12936a^{27}bde^6 + 42042a^{28}d^2e^5 + 11088a^{29}e^7 + 12936a^{28}bde^6 + 42042a^{29}d^2e^5 + 11088a^{30}e^7 + 12936a^{29}bde^6 + 42042a^{30}d^2e^5 + 11088a^{31}e^7 + 12936a^{30}bde^6 + 42042a^{31}d^2e^5 + 11088a^{32}e^7 + 12936a^{31}bde^6 + 42042a^{32}d^2e^5 + 11088a^{33}e^7 + 12936a^{32}bde^6 + 42042a^{33}d^2e^5 + 11088a^{34}e^7 + 12936a^{33}bde^6 + 42042a^{34}d^2e^5 + 11088a^{35}e^7 + 12936a^{34}bde^6 + 42042a^{35}d^2e^5 + 11088a^{36}e^7 + 12936a^{35}bde^6 + 42042a^{36}d^2e^5 + 11088a^{37}e^7 + 12936a^{36}bde^6 + 42042a^{37}d^2e^5 + 11088a^{38}e^7 + 12936a^{37}bde^6 + 42042a^{38}d^2e^5 + 11088a^{39}e^7 + 12936a^{38}bde^6 + 42042a^{39}d^2e^5 + 11088a^{40}e^7 + 12936a^{39}bde^6 + 42042a^{40}d^2e^5 + 11088a^{41}e^7 + 12936a^{40}bde^6 + 42042a^{41}d^2e^5 + 11088a^{42}e^7 + 12936a^{41}bde^6 + 42042a^{42}d^2e^5 + 11088a^{43}e^7 + 12936a^{42}bde^6 + 42042a^{43}d^2e^5 + 11088a^{44}e^7 + 12936a^{43}bde^6 + 42042a^{44}d^2e^5 + 11088a^{45}e^7 + 12936a^{44}bde^6 + 42042a^{45}d^2e^5 + 11088a^{46}e^7 + 12936a^{45}bde^6 + 42042a^{46}d^2e^5 + 11088a^{47}e^7 + 12936a^{46}bde^6 + 42042a^{47}d^2e^5 + 11088a^{48}e^7 + 12936a^{47}bde^6 + 42042a^{48}d^2e^5 + 11088a^{49}e^7 + 12936a^{48}bde^6 + 42042a^{49}d^2e^5 + 11088a^{50}e^7 + 12936a^{49}bde^6 + 42042a^{50}d^2e^5 + 11088a^{51}e^7 + 12936a^{50}bde^6 + 42042a^{51}d^2e^5 + 11088a^{52}e^7 + 12936a^{51}bde^6 + 42042a^{52}d^2e^5 + 11088a^{53}e^7 + 12936a^{52}bde^6 + 42042a^{53}d^2e^5 + 11088a^{54}e^7 + 12936a^{53}bde^6 + 42042a^{54}d^2e^5 + 11088a^{55}e^7 + 12936a^{54}bde^6 + 42042a^{55}d^2e^5 + 11088a^{56}e^7 + 12936a^{55}bde^6 + 42042a^{56}d^2e^5 + 11088a^{57}e^7 + 12936a^{56}bde^6 + 42042a^{57}d^2e^5 + 11088a^{58}e^7 + 12936a^{57}bde^6 + 42042a^{58}d^2e^5 + 11088a^{59}e^7 + 12936a^{58}bde^6 + 42042a^{59}d^2e^5 + 11088a^{60}e^7 + 12936a^{59}bde^6 + 42042a^{60}d^2e^5 + 11088a^{61}e^7 + 12936a^{60}bde^6 + 42042a^{61}d^2e^5 + 11088a^{62}e^7 + 12936a^{61}bde^6 + 42042a^{62}d^2e^5 + 11088a^{63}e^7 + 12936a^{62}bde^6 + 42042a^{63}d^2e^5 + 11088a^{64}e^7 + 12936a^{63}bde^6 + 42042a^{64}d^2e^5 + 11088a^{65}e^7 + 12936a^{64}bde^6 + 42042a^{65}d^2e^5 + 11088a^{66}e^7 + 12936a^{65}bde^6 + 42042a^{66}d^2e^5 + 11088a^{67}e^7 + 12936a^{66}bde^6 + 42042a^{67}d^2e^5 + 11088a^{68}e^7 + 12936a^{67}bde^6 + 42042a^{68}d^2e^5 + 11088a^{69}e^7 + 12936a^{68}bde^6 + 42042a^{69}d^2e^5 + 11088a^{70}e^7 + 12936a^{69}bde^6 + 42042a^{70}d^2e^5 + 11088a^{71}e^7 + 12936a^{70}bde^6 + 42042a^{71}d^2e^5 + 11088a^{72}e^7 + 12936a^{71}bde^6 + 42042a^{72}d^2e^5 + 11088a^{73}e^7 + 12936a^{72}bde^6 + 42042a^{73}d^2e^5 + 11088a^{74}e^7 + 12936a^{73}bde^6 + 42042a^{74}d^2e^5 + 11088a^{75}e^7 + 12936a^{74}bde^6 + 42042a^{75}d^2e^5 + 11088a^{76}e^7 + 12936a^{75}bde^6 + 42042a^{76}d^2e^5 + 11088a^{77}e^7 + 12936a^{76}bde^6 + 42042a^{77}d^2e^5 + 11088a^{78}e^7 + 12936a^{77}bde^6 + 42042a^{78}d^2e^5 + 11088a^{79}e^7 + 12936a^{78}bde^6 + 42042a^{79}d^2e^5 + 11088a^{80}e^7 + 12936a^{79}bde^6 + 42042a^{80}d^2e^5 + 11088a^{81}e^7 + 12936a^{80}bde^6 + 42042a^{81}d^2e^5 + 11088a^{82}e^7 + 12936a^{81}bde^6 + 42042a^{82}d^2e^5 + 11088a^{83}e^7 + 12936a^{82}bde^6 + 42042a^{83}d^2e^5 + 11088a^{84}e^7 + 12936a^{83}bde^6 + 42042a^{84}d^2e^5 + 11088a^{85}e^7 + 12936a^{84}bde^6 + 42042a^{85}d^2e^5 + 11088a^{86}e^7 + 12936a^{85}bde^6 + 42042a^{86}d^2e^5 + 11088a^{87}e^7 + 12936a^{86}bde^6 + 42042a^{87}d^2e^5 + 11088a^{88}e^7 + 12936a^{87}bde^6 + 42042a^{88}d^2e^5 + 11088a^{89}e^7 + 12936a^{88}bde^6 + 42042a^{89}d^2e^5 + 11088a^{90}e^7 + 12936a^{89}bde^6 + 42042a^{90}d^2e^5 + 11088a^{91}e^7 + 12936a^{90}bde^6 + 42042a^{91}d^2e^5 + 11088a^{92}e^7 + 12936a^{91}bde^6 + 42042a^{92}d^2e^5 + 11088a^{93}e^7 + 12936a^{92}bde^6 + 42042a^{93}d^2e^5 + 11088a^{94}e^7 + 12936a^{93}bde^6 + 42042a^{94}d^2e^5 + 11088a^{95}e^7 + 12936a^{94}bde^6 + 42042a^{95}d^2e^5 + 11088a^{96}e^7 + 12936a^{95}bde^6 + 42042a^{96}d^2e^5 + 11088a^{97}e^7 + 12936a^{96}bde^6 + 42042a^{97}d^2e^5 + 11088a^{98}e^7 + 12936a^{97}bde^6 + 42042a^{98}d^2e^5 + 11088a^{99}e^7 + 12936a^{98}bde^6 + 42042a^{99}d^2e^5 + 11088a^{100}e^7 + 12936a^{99}bde^6 + 42042a^{100}d^2e^5 + 11088a^{101}e^7 + 12936a^{100}bde^6 + 42042a^{101}d^2e^5 + 11088a^{102}e^7 + 12936a^{101}bde^6 + 42042a^{102}d^2e^5 + 11088a^{103}e^7 + 12936a^{102}bde^6 + 42042a^{103}d^2e^5 + 11088a^{104}e^7 + 12936a^{103}bde^6 + 42042a^{104}d^2e^5 + 11088a^{105}e^7 + 12936a^{104}bde^6 + 42042a^{105}d^2e^5 + 11088a^{106}e^7 + 12936a^{105}bde^6 + 42042a^{106}d^2e^5 + 11088a^{107}e^7 + 12936a^{106}bde^6 + 42042a^{107}d^2e^5 + 11088a^{108}e^7 + 12936a^{107}bde^6 + 42042a^{108}d^2e^5 + 11088a^{109}e^7 + 12936a^{108}bde^6 + 42042a^{109}d^2e^5 + 11088a^{110}e^7 + 12936a^{109}bde^6 + 42042a^{110}d^2e^5 + 11088a^{111}e^7 + 12936a^{110}bde^6 + 42042a^{111}d^2e^5 + 11088a^{112}e^7 + 12936a^{111}bde^6 + 42042a^{112}d^2e^5 + 11088a^{113}e^7 + 12936a^{112}bde^6 + 42042a^{113}d^2e^5 + 11088a^{114}e^7 + 12936a^{113}bde^6 + 42042a^{114}d^2e^5 + 11088a^{115}e^7 + 12936a^{114}bde^6 + 42042a^{115}d^2e^5 + 11088a^{116}e^7 + 12936a^{115}bde^6 + 42042a^{116}d^2e^5 + 11088a^{117}e^7 + 12936a^{116}bde^6 + 42042a^{117}d^2e^5 + 11088a^{118}e^7 + 12936a^{117}bde^6 + 42042a^{118}d^2e^5 + 11088a^{119}e^7 + 12936a^{118}bde^6 + 42042a^{119}d^2e^5 + 11088a^{120}e^7 + 12936a^{119}bde^6 + 42042a^{120}d^2e^5 + 11088a^{121}e^7 + 12936a^{120}bde^6 + 42042a^{121}d^2e^5 + 11088a^{122}e^7 + 12936a^{121}bde^6 + 42042a^{122}d^2e^5 + 11088a^{123}e^7 + 12936a^{122}bde^6 + 42042a^{123}d^2e^5 + 11088a^{124}e^7 + 12936a^{123}bde^6 + 42042a^{124}d^2e^5 + 11088a^{125}e^7 + 12936a^{124}bde^6 + 42042a^{125}d^2e^5 + 11088a^{126}e^7 + 12936a^{125}bde^6 + 42042a^{126}d^2e^5 + 11088a^{127}e^7 + 12936a^{126}bde^6 + 42042a^{127}d^2e^5 + 11088a^{128}e^7 + 12936a^{127}bde^6 + 42042a^{128}d^2e^5 + 11088a^{129}e^7 + 12936a^{128}bde^6 + 42042a^{129}d^2e^5 + 11088a^{130}e^7 + 12936a^{129}bde^6 + 42042a^{130}d^2e^5 + 11088a^{131}e^7 + 12936a^{130}bde^6 + 42042a^{131}d^2e^5 + 11088a^{132}e^7 + 12936a^{131}bde^6 + 42042a^{132}d^2e^5 + 11088a^{133}e^7 + 12936a^{132}bde^6 + 42042a^{133}d^2e^5 + 11088a^{134}e^7 + 12936a^{133}bde^6 + 42042a^{134}d^2e^5 + 11088a^{135}e^7 + 12936a^{134}bde^6 + 42042a^{135}d^2e^5 + 11088a^{136}e^7 + 12936a^{135}bde^6 + 42042a^{136}d^2e^5 + 11088a^{137}e^7 + 12936a^{136}bde^6 + 42042a^{137}d^2e^5 + 11088a^{138}e^7 + 12936a^{137}bde^6 + 42042a^{138}d^2e^5 + 11088a^{139}e^7 + 12936a^{138}bde^6 + 42042a^{139}d^2e^5 + 11088a^{140}e^7 + 12936a^{139}bde^6 + 42042a^{140}d^2e^5 + 11088a^{141}e^7 + 12936a^{140}bde^6 + 42042a^{141}d^2e^5 + 11088a^{142}e^7 + 12936a^{141}bde^6 + 42042a^{142}d^2e^5 + 11088a^{143}e^7 + 12936a^{142}bde^6 + 42042a^{143}d^2e^5 + 11088a^{144}e^7 + 12936a^{143}bde^6 + 42042a^{144}d^2e^5 + 11088a^{145}e^7 + 12936a^{144}bde^6 + 42042a^{145}d^2e^5 + 11088a^{146}e^7 + 12936a^{145}bde^6 + 42042a^{146}d^2e^5 + 11088a^{147}e^7 + 12936a^{146}bde^6 + 42042a^{147}d^2e^5 + 11088a^{148}e^7 + 12936a^{147}bde^6 + 42042a^{148}d^2e^5 + 11088a^{149}e^7 + 12936a^{148}bde^6 + 42042a^{149}d^2e^5 + 11088a^{150}e^7 + 12936a^{149}bde^6 + 42042a^{150}d^2e^5 + 11088a^{151}e^7 + 12936a^{150}bde^6 + 42042a^{151}d^2e^5 + 11088a^{152}e^7 + 12936a^{151}bde^6 + 42042a^{152}d^2e^5 + 11088a^{153}e^7 + 12936a^{152}bde^6 + 42042a^{153}d^2e^5 + 11088a^{154}e^7 + 12936a^{153}bde^6 + 42042a^{154}d^2e^5 + 11088a^{155}e^7 + 12936a^{154}bde^6 + 42042a^{155}d^2e^5 + 11088a^{156}e^7 + 12936a^{155}bde^6 + 42042a^{156}d^2e^5 + 11088a^{157}e^7 + 12936a^{156}bde^6 + 42042a^{157}d^2e^5 + 11088a^{158}e^7 + 12936a^{157}bde^6 + 42042a^{158}d^2e^5 + 11088a^{159}e^7 + 12936a^{158}bde^6 + 42042a^{159}d^2e^5 + 11088a^{160}e^7 + 12936a^{159}bde^6 + 42042a^{160}d^2e^5 + 11088a^{161}e^7 + 12936a^{160}bde^6 + 42042a^{161}d^2e^5 + 11088a^{162}e^7 + 12936a^{161}bde^6 + 42042a^{162}d^2e^5 + 11088a^{163}e^7 + 12936a^{162}bde^6 + 42042a^{163}d^2e^5 + 11088a^{164}e^7 + 12936a^{163}bde^6 + 42042a^{164}d^2e^5 + 11088a^{165}e^7 + 12936a^{164}bde^6 + 42042a^{165}d^2e^5 + 11088a^{166}e^7 + 12936a^{165}bde^6 + 42042a^{166}d^2e^5 + 11088a^{167}e^7 + 12936a^{166}bde^6 + 42042a^{167}d^2e^5 + 11088a^{168}e^7 + 12936a^{167}bde^6 + 42042a^{168}d^2e^5 + 11088a^{169}e^7 + 12936a^{168}bde^6 + 42042a^{169}d^2e^5 + 11088a^{170}e^7 + 12936a^{169}bde^6 + 42042a^{170}d^2e^5 + 11088a^{171}e^7 + 12936a^{170}bde^6 + 42042a^{171}d^2e^5 + 11088a^{172}e^7 + 12936a^{171}bde^6 + 42042a^{172}d^2e^5 + 11088a^{173}e^7 + 12936a^{172}bde^6 + 42042a^{173}d^2e^5 + 11088a^{174}e^7 + 12936a^{173}bde^6 + 42042a^{174}d^2e^5 + 11088a^{175}e^7 + 12936a^{174}bde^6 + 42042a^{175}d^2e^5 + 11088a^{176}e^7 + 12936a^{175}bde^6 + 42042a^{176}d^2e^5 + 11088a^{177}e^7 + 12936a^{176}bde^6 + 42042a^{177}d^2e^5 + 11088a^{178}e^7 + 12936a^{177}bde^6 + 42042a^{178}d^2e^5 + 11088a^{179}e^7 + 12936a^{178}bde^6 + 42042a^{179}d^2e^5 + 11088a^{180}e^7 + 12936a^{179}bde^6 + 42042a^{180}d^2e^5 + 11088a^{181}e^7 + 12936a^{180}bde^6 + 42042a^{181}d^2e^5 + 11088a^{182}e^7 + 12936a^{181}bde^6 + 42042a^{182}d^2e^5 + 11088a^{183}e^7 + 12936a^{182}bde^6 + 42042a^{183}d^2e^5 + 11088a^{184}e^7 + 12936a^{183}bde^6 + 42042a^{184}d^2e^5 + 11088a^{185}e^7 + 12936a^{184}bde^6 + 42042a^{185}d^2e^5 + 11088a^{186}e^7 + 12936a^{185}bde^6 + 42042a^{186}d^2e^5 + 11088a^{187}e^7 + 12936a^{186}bde^6 + 42042a^{187}d^2e^5 + 11088a^{188}e^7 + 12936a^{187}bde^6 + 42042a^{188}d^2e^5 + 11088a^{189}e^7 + 12936a^{188}bde^6 + 42042a^{189}d^2e^5 + 11088a^{190}e^7 + 12936a^{189}bde^6 + 42042a^{190}d^2e^5 + 11088a^{191}e^7 + 12936a^{190}bde^6 + 42042a^{191}d^2e^5 + 11088a^{192}e^7 + 12936a^{191}bde^6 + 42042a^{192}d^2e^5 + 11088a^{193}e^7 + 12936a^{192}bde^6 + 42042a^{193}d^2e^5 + 11088a^{194}e^7 + 12936a^{193}bde^6 + 42042a^{194}d^2e^5 + 11088a^{195}e^7 + 12936a^{194}bde^6 + 42042a^{195}d^2e^5 + 11088a^{196}e^7 + 12936a^{195}bde^6 + 42042a^{196}d^2e^5 + 11088a^{197}e^7 + 12936a^{196}bde^6 + 42042a^{197}d^2e^5 + 11088a^{198}e^7 + 12936a^{197}bde^6 + 42042a^{198}d^2e^5 + 11088a^{199}e^7 + 12936a^{198}bde^6 + 42042a^{199}d^2e^5 + 11088a^{200}e^7 + 12936a^{199}bde^6 + 42042a^{200}d^2e^5 + 11088a^{201}e^7 + 12936a^{200}bde^6 + 42042a^{201}d^2e^5 + 11088a^{202}e^7 + 12936a^{201}bde^6 + 42042a^{202}d^2e^5 + 11088a^{203}e^7 + 12936a^{202}bde^6 + 42042a^{203}d^2e^5 + 11088a^{204}e^7 + 12936a^{203}bde^6 + 42042a^{204}d^2e^5 + 11088a^{205}e^7 + 12936a^{204}bde^6 + 42042a^{205}d^2e^5 + 11088a^{206}e^7 + 12936a^{205}bde^6 + 42042a^{206}d^2e^5 + 11088a^{207}e^7 + 12936a^{206}bde^6 + 42042a^{207}d^2e^5 + 11088a^{208}e^7 + 12936a^{207}bde^6 + 42042a^{208}d^2e^5 + 11088a^{209}e^7 + 12936a^{208}bde^6 + 42042a^{209}d^2e^5 + 11088a^{210}e^7 + 12936a^{209}bde^6 + 42042a^{210}d^2e^5 + 11088a^{211}e^7 + 12936a^{210}bde^6 + 42042a^{211}d^2e^5 + 11088a^{212}e^7 + 12936a^{211}bde^6 + 42042a^{212}d^2e^5 + 11088a^{213}e^7 + 12936a^{212}bde^6 + 42042a^{213}d^2e^5 + 11088a^{214}e^7 + 12936a^{213}bde^6 + 42042a^{214}d^2e^5 + 11088a^{215}e^7 + 12936a^{214}bde^6 + 42042a^{215}d^2e^5 + 11088a^{216}e^7 + 12936a^{215}bde^6 + 42042a^{216}d^2e^5 + 11088a^{217}e^7 + 12936a^{216}bde^6 + 42042a^{217}d^2e^5 + 11088a^{218}e^7 + 1$$

output

```
(x*(24024*a**6*d**7 + 84084*a**6*d**6*e*x + 168168*a**6*d**5*e**2*x**2 + 2
10210*a**6*d**4*e**3*x**3 + 168168*a**6*d**3*e**4*x**4 + 84084*a**6*d**2*e
**5*x**5 + 24024*a**6*d*e**6*x**6 + 3003*a**6*e**7*x**7 + 72072*a**5*b*d**
7*x + 336336*a**5*b*d**6*e*x**2 + 756756*a**5*b*d**5*e**2*x**3 + 1009008*a
**5*b*d**4*e**3*x**4 + 840840*a**5*b*d**3*e**4*x**5 + 432432*a**5*b*d**2*e
**5*x**6 + 126126*a**5*b*d*e**6*x**7 + 16016*a**5*b*e**7*x**8 + 120120*a**
4*b**2*d**7*x**2 + 630630*a**4*b**2*d**6*e*x**3 + 1513512*a**4*b**2*d**5*e
**2*x**4 + 2102100*a**4*b**2*d**4*e**3*x**5 + 1801800*a**4*b**2*d**3*e**4*
x**6 + 945945*a**4*b**2*d**2*e**5*x**7 + 280280*a**4*b**2*d*e**6*x**8 + 36
036*a**4*b**2*e**7*x**9 + 120120*a**3*b**3*d**7*x**3 + 672672*a**3*b**3*d
**6*e*x**4 + 1681680*a**3*b**3*d**5*e**2*x**5 + 2402400*a**3*b**3*d**4*e**3
*x**6 + 2102100*a**3*b**3*d**3*e**4*x**7 + 1121120*a**3*b**3*d**2*e**5*x**
8 + 336336*a**3*b**3*d*e**6*x**9 + 43680*a**3*b**3*e**7*x**10 + 72072*a**2
*b**4*d**7*x**4 + 420420*a**2*b**4*d**6*e*x**5 + 1081080*a**2*b**4*d**5*e
**2*x**6 + 1576575*a**2*b**4*d**4*e**3*x**7 + 1401400*a**2*b**4*d**3*e**4*x
**8 + 756756*a**2*b**4*d**2*e**5*x**9 + 229320*a**2*b**4*d*e**6*x**10 + 30
030*a**2*b**4*e**7*x**11 + 24024*a*b**5*d**7*x**5 + 144144*a*b**5*d**6*e*x
**6 + 378378*a*b**5*d**5*e**2*x**7 + 560560*a*b**5*d**4*e**3*x**8 + 504504
*a*b**5*d**3*e**4*x**9 + 275184*a*b**5*d**2*e**5*x**10 + 84084*a*b**5*d*e
**6*x**11 + 11088*a*b**5*e**7*x**12 + 3432*b**6*d**7*x**6 + 21021*b**6*d...
```

3.86 $\int (d + ex)^6 (a^2 + 2abx + b^2x^2)^3 dx$

| | |
|---|-----|
| Optimal result | 696 |
| Mathematica [B] (verified) | 697 |
| Rubi [A] (verified) | 698 |
| Maple [B] (verified) | 700 |
| Fricas [B] (verification not implemented) | 701 |
| Sympy [B] (verification not implemented) | 702 |
| Maxima [B] (verification not implemented) | 703 |
| Giac [B] (verification not implemented) | 704 |
| Mupad [B] (verification not implemented) | 706 |
| Reduce [B] (verification not implemented) | 707 |

Optimal result

Integrand size = 26, antiderivative size = 171

$$\int (d + ex)^6 (a^2 + 2abx + b^2x^2)^3 dx = \frac{(bd - ae)^6(a + bx)^7}{7b^7} + \frac{3e(bd - ae)^5(a + bx)^8}{4b^7} + \frac{5e^2(bd - ae)^4(a + bx)^9}{3b^7} + \frac{2e^3(bd - ae)^3(a + bx)^{10}}{b^7} + \frac{15e^4(bd - ae)^2(a + bx)^{11}}{11b^7} + \frac{e^5(bd - ae)(a + bx)^{12}}{2b^7} + \frac{e^6(a + bx)^{13}}{13b^7}$$

output

```
1/7*(-a*e+b*d)^6*(b*x+a)^7/b^7+3/4*e*(-a*e+b*d)^5*(b*x+a)^8/b^7+5/3*e^2*(-a*e+b*d)^4*(b*x+a)^9/b^7+2*e^3*(-a*e+b*d)^3*(b*x+a)^10/b^7+15/11*e^4*(-a*e+b*d)^2*(b*x+a)^11/b^7+1/2*e^5*(-a*e+b*d)*(b*x+a)^12/b^7+1/13*e^6*(b*x+a)^13/b^7
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 573 vs. $2(171) = 342$.

Time = 0.06 (sec) , antiderivative size = 573, normalized size of antiderivative = 3.35

$$\begin{aligned}
 \int (d + ex)^6 (a^2 + 2abx + b^2x^2)^3 dx = & a^6d^6x + 3a^5d^5(bd + ae)x^2 \\
 & + a^4d^4(5b^2d^2 + 12abde + 5a^2e^2)x^3 \\
 & + \frac{5}{2}a^3d^3(2b^3d^3 + 9ab^2d^2e + 9a^2bde^2 + 2a^3e^3)x^4 \\
 & + 3a^2d^2(b^4d^4 + 8ab^3d^3e + 15a^2b^2d^2e^2 + 8a^3bde^3 \\
 & \quad + a^4e^4)x^5 + ad(b^5d^5 + 15ab^4d^4e + 50a^2b^3d^3e^2 \\
 & \quad + 50a^3b^2d^2e^3 + 15a^4bde^4 + a^5e^5)x^6 \\
 & + \frac{1}{7}(b^6d^6 + 36ab^5d^5e + 225a^2b^4d^4e^2 + 400a^3b^3d^3e^3 \\
 & \quad + 225a^4b^2d^2e^4 + 36a^5bde^5 + a^6e^6)x^7 \\
 & + \frac{3}{4}be(b^5d^5 + 15ab^4d^4e + 50a^2b^3d^3e^2 + 50a^3b^2d^2e^3 \\
 & \quad + 15a^4bde^4 + a^5e^5)x^8 + \frac{5}{3}b^2e^2(b^4d^4 + 8ab^3d^3e \\
 & \quad + 15a^2b^2d^2e^2 + 8a^3bde^3 + a^4e^4)x^9 \\
 & + b^3e^3(2b^3d^3 + 9ab^2d^2e + 9a^2bde^2 + 2a^3e^3)x^{10} \\
 & + \frac{3}{11}b^4e^4(5b^2d^2 + 12abde + 5a^2e^2)x^{11} \\
 & + \frac{1}{2}b^5e^5(bd + ae)x^{12} + \frac{1}{13}b^6e^6x^{13}
 \end{aligned}$$

input

```
Integrate[(d + e*x)^6*(a^2 + 2*a*b*x + b^2*x^2)^3,x]
```

output

```

a^6*d^6*x + 3*a^5*d^5*(b*d + a*e)*x^2 + a^4*d^4*(5*b^2*d^2 + 12*a*b*d*e +
5*a^2*e^2)*x^3 + (5*a^3*d^3*(2*b^3*d^3 + 9*a*b^2*d^2*e + 9*a^2*b*d*e^2 + 2
*a^3*e^3)*x^4)/2 + 3*a^2*d^2*(b^4*d^4 + 8*a*b^3*d^3*e + 15*a^2*b^2*d^2*e^2
+ 8*a^3*b*d*e^3 + a^4*e^4)*x^5 + a*d*(b^5*d^5 + 15*a*b^4*d^4*e + 50*a^2*b
^3*d^3*e^2 + 50*a^3*b^2*d^2*e^3 + 15*a^4*b*d*e^4 + a^5*e^5)*x^6 + ((b^6*d^
6 + 36*a*b^5*d^5*e + 225*a^2*b^4*d^4*e^2 + 400*a^3*b^3*d^3*e^3 + 225*a^4*b
^2*d^2*e^4 + 36*a^5*b*d*e^5 + a^6*e^6)*x^7)/7 + (3*b*e*(b^5*d^5 + 15*a*b^4
*d^4*e + 50*a^2*b^3*d^3*e^2 + 50*a^3*b^2*d^2*e^3 + 15*a^4*b*d*e^4 + a^5*e^
5)*x^8)/4 + (5*b^2*e^2*(b^4*d^4 + 8*a*b^3*d^3*e + 15*a^2*b^2*d^2*e^2 + 8*a
^3*b*d*e^3 + a^4*e^4)*x^9)/3 + b^3*e^3*(2*b^3*d^3 + 9*a*b^2*d^2*e + 9*a^2*
b*d*e^2 + 2*a^3*e^3)*x^10 + (3*b^4*e^4*(5*b^2*d^2 + 12*a*b*d*e + 5*a^2*e^2
)*x^11)/11 + (b^5*e^5*(b*d + a*e)*x^12)/2 + (b^6*e^6*x^13)/13

```

Rubi [A] (verified)

Time = 0.92 (sec) , antiderivative size = 171, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {1098, 27, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\begin{aligned}
 & \int (a^2 + 2abx + b^2x^2)^3 (d + ex)^6 dx \\
 & \quad \downarrow 1098 \\
 & \quad \int \frac{b^6(a + bx)^6 (d + ex)^6 dx}{b^6} \\
 & \quad \quad \downarrow 27 \\
 & \quad \quad \int (a + bx)^6 (d + ex)^6 dx \\
 & \quad \quad \quad \downarrow 49 \\
 & \quad \quad \quad \int \left(\frac{6e^5(a + bx)^{11}(bd - ae)}{b^6} + \frac{15e^4(a + bx)^{10}(bd - ae)^2}{b^6} + \frac{20e^3(a + bx)^9(bd - ae)^3}{b^6} + \frac{15e^2(a + bx)^8(bd - ae)^4}{b^6} \right) dx \\
 & \quad \quad \quad \quad \downarrow 2009
 \end{aligned}$$

$$\frac{e^5(a+bx)^{12}(bd-ae)}{3b^7} + \frac{15e^4(a+bx)^{11}(bd-ae)^2}{4b^7} + \frac{2e^3(a+bx)^{10}(bd-ae)^3}{7b^7} + \frac{5e^2(a+bx)^9(bd-ae)^4}{3b^7} + \frac{3e(a+bx)^8(bd-ae)^5}{4b^7} + \frac{(a+bx)^7(bd-ae)^6}{7b^7} + \frac{e^6(a+bx)^{13}}{13b^7}$$

input `Int[(d + e*x)^6*(a^2 + 2*a*b*x + b^2*x^2)^3,x]`

output `((b*d - a*e)^6*(a + b*x)^7)/(7*b^7) + (3*e*(b*d - a*e)^5*(a + b*x)^8)/(4*b^7) + (5*e^2*(b*d - a*e)^4*(a + b*x)^9)/(3*b^7) + (2*e^3*(b*d - a*e)^3*(a + b*x)^10)/b^7 + (15*e^4*(b*d - a*e)^2*(a + b*x)^11)/(11*b^7) + (e^5*(b*d - a*e)*(a + b*x)^12)/(2*b^7) + (e^6*(a + b*x)^13)/(13*b^7)`

Defintions of rubi rules used

rule 27 `Int[(a_)*(F_x_), x_Symbol] := Simp[a Int[F_x, x], x] /; FreeQ[a, x] && !MatchQ[F_x, (b_)*(G_x_)] /; FreeQ[b, x]`

rule 49 `Int[((a_) + (b_)*(x_))^(m_)*((c_) + (d_)*(x_))^(n_), x_Symbol] := Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] && IGtQ[m, 0] && IGtQ[m + n + 2, 0]`

rule 1098 `Int[((d_) + (e_)*(x_))^(m_)*((a_) + (b_)*(x_) + (c_)*(x_)^2)^(p_), x_Symbol] := Simp[1/c^p Int[(d + e*x)^m*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]`

rule 2009 `Int[u_, x_Symbol] := Simp[IntSum[u, x], x] /; SumQ[u]`

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 603 vs. $2(159) = 318$.

Time = 1.05 (sec) , antiderivative size = 604, normalized size of antiderivative = 3.53

| method | result |
|---------------|---|
| norman | $\frac{b^6 e^6 x^{13}}{13} + \left(\frac{1}{2} a b^5 e^6 + \frac{1}{2} b^6 d e^5\right) x^{12} + \left(\frac{15}{11} a^2 b^4 e^6 + \frac{36}{11} a b^5 d e^5 + \frac{15}{11} b^6 d^2 e^4\right) x^{11} + (2a^3 b^3 e^6 + 9a^2 b^4 d e^5 + 15a b^5 d^2 e^4 + 6b^6 d^3 e^3) x^{10} + (5/3 a^4 b^2 e^6 + 40/3 a^3 b^3 d e^5 + 25 a^2 b^4 d^2 e^4 + 40/3 a b^5 d^3 e^3 + 5/3 b^6 d^4 e^2) x^9 + (3/4 a^5 b e^6 + 45/4 a^4 b^2 d e^5 + 75/2 a^3 b^3 d^2 e^4 + 75/2 a^2 b^4 d^3 e^3 + 45/4 a b^5 d^4 e^2 + 3/4 b^6 d^5 e) x^8 + (1/7 a^6 e^6 + 36/7 a^5 b d e^5 + 225/7 a^4 b^2 d^2 e^4 + 40/7 a^3 b^3 d^3 e^3 + 225/7 a^2 b^4 d^4 e^2 + 36/7 a b^5 d^5 e + 1/7 b^6 d^6) x^7 + (a^6 d e^5 + 15 a^5 b d^2 e^4 + 50 a^4 b^2 d^3 e^3 + 50 a^3 b^3 d^4 e^2 + 15 a^2 b^4 d^5 e + a b^5 d^6) x^6 + (3 a^6 d^2 e^4 + 24 a^5 b d^3 e^3 + 45 a^4 b^2 d^4 e^2 + 24 a^3 b^3 d^5 e + 3 a^2 b^4 d^6) x^5 + (5 a^6 d^3 e^3 + 45/2 a^5 b d^4 e^2 + 45/2 a^4 b^2 d^5 e + 5 a^3 b^3 d^6) x^4 + (5 a^6 d^4 e^2 + 12 a^5 b d^5 e + 5 a^4 b^2 d^6) x^3 + (3 a^6 d^5 e + 3 a^5 b d^6) x^2 + a^6 d^6 x$ |
| default | $\frac{b^6 e^6 x^{13}}{13} + \frac{(6 a b^5 e^6 + 6 b^6 d e^5) x^{12}}{12} + \frac{(15 a^2 b^4 e^6 + 36 a b^5 d e^5 + 15 b^6 d^2 e^4) x^{11}}{11} + \frac{(20 a^3 b^3 e^6 + 90 a^2 b^4 d e^5 + 90 a b^5 d^2 e^4 + 20 b^6 d^3 e^3) x^{10}}{10} + \dots$ |
| risch | $\frac{1}{2} x^{12} a b^5 e^6 + \frac{1}{2} x^{12} b^6 d e^5 + \frac{15}{11} x^{11} a^2 b^4 e^6 + 15 a^2 b^4 d^5 e x^6 + 24 a^5 b d^3 e^3 x^5 + 45 a^4 b^2 d^4 e^2 x^5 + 24 a^3 b^3 d^5 e x^4 + \dots$ |
| parallelrisch | $\frac{1}{2} x^{12} a b^5 e^6 + \frac{1}{2} x^{12} b^6 d e^5 + \frac{15}{11} x^{11} a^2 b^4 e^6 + 15 a^2 b^4 d^5 e x^6 + 24 a^5 b d^3 e^3 x^5 + 45 a^4 b^2 d^4 e^2 x^5 + 24 a^3 b^3 d^5 e x^4 + \dots$ |
| gospers | $x(924 b^6 e^6 x^{12} + 6006 x^{11} a b^5 e^6 + 6006 x^{11} b^6 d e^5 + 16380 x^{10} a^2 b^4 e^6 + 39312 x^{10} a b^5 d e^5 + 16380 x^{10} b^6 d^2 e^4 + 24024 a^3 b^3 e^6 x^9 + 108 \dots)$ |
| orering | $x(924 b^6 e^6 x^{12} + 6006 x^{11} a b^5 e^6 + 6006 x^{11} b^6 d e^5 + 16380 x^{10} a^2 b^4 e^6 + 39312 x^{10} a b^5 d e^5 + 16380 x^{10} b^6 d^2 e^4 + 24024 a^3 b^3 e^6 x^9 + 108 \dots)$ |

input

```
int((e*x+d)^6*(b^2*x^2+2*a*b*x+a^2)^3,x,method=_RETURNVERBOSE)
```

output

```
1/13*b^6*e^6*x^13+(1/2*a*b^5*e^6+1/2*b^6*d*e^5)*x^12+(15/11*a^2*b^4*e^6+36/11*a*b^5*d*e^5+15/11*b^6*d^2*e^4)*x^11+(2*a^3*b^3*e^6+9*a^2*b^4*d*e^5+9*a*b^5*d^2*e^4+2*b^6*d^3*e^3)*x^10+(5/3*a^4*b^2*e^6+40/3*a^3*b^3*d*e^5+25*a^2*b^4*d^2*e^4+40/3*a*b^5*d^3*e^3+5/3*b^6*d^4*e^2)*x^9+(3/4*a^5*b*e^6+45/4*a^4*b^2*d*e^5+75/2*a^3*b^3*d^2*e^4+75/2*a^2*b^4*d^3*e^3+45/4*a*b^5*d^4*e^2+3/4*b^6*d^5*e)*x^8+(1/7*a^6*e^6+36/7*a^5*b*d*e^5+225/7*a^4*b^2*d^2*e^4+40/7*a^3*b^3*d^3*e^3+225/7*a^2*b^4*d^4*e^2+36/7*a*b^5*d^5*e+1/7*b^6*d^6)*x^7+(a^6*d*e^5+15*a^5*b*d^2*e^4+50*a^4*b^2*d^3*e^3+50*a^3*b^3*d^4*e^2+15*a^2*b^4*d^5*e+a*b^5*d^6)*x^6+(3*a^6*d^2*e^4+24*a^5*b*d^3*e^3+45*a^4*b^2*d^4*e^2+24*a^3*b^3*d^5*e+3*a^2*b^4*d^6)*x^5+(5*a^6*d^3*e^3+45/2*a^5*b*d^4*e^2+45/2*a^4*b^2*d^5*e+5*a^3*b^3*d^6)*x^4+(5*a^6*d^4*e^2+12*a^5*b*d^5*e+5*a^4*b^2*d^6)*x^3+(3*a^6*d^5*e+3*a^5*b*d^6)*x^2+a^6*d^6*x
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 599 vs. $2(159) = 318$.

Time = 0.08 (sec) , antiderivative size = 599, normalized size of antiderivative = 3.50

$$\int (d+ex)^6 (a^2+2abx+b^2x^2)^3 dx$$

$$= \frac{1}{13} b^6 e^6 x^{13} + a^6 d^6 x + \frac{1}{2} (b^6 d e^5 + a b^5 e^6) x^{12} + \frac{3}{11} (5 b^6 d^2 e^4 + 12 a b^5 d e^5 + 5 a^2 b^4 e^6) x^{11}$$

$$+ (2 b^6 d^3 e^3 + 9 a b^5 d^2 e^4 + 9 a^2 b^4 d e^5 + 2 a^3 b^3 e^6) x^{10}$$

$$+ \frac{5}{3} (b^6 d^4 e^2 + 8 a b^5 d^3 e^3 + 15 a^2 b^4 d^2 e^4 + 8 a^3 b^3 d e^5 + a^4 b^2 e^6) x^9$$

$$+ \frac{3}{4} (b^6 d^5 e + 15 a b^5 d^4 e^2 + 50 a^2 b^4 d^3 e^3 + 50 a^3 b^3 d^2 e^4 + 15 a^4 b^2 d e^5 + a^5 b e^6) x^8$$

$$+ \frac{1}{7} (b^6 d^6 + 36 a b^5 d^5 e + 225 a^2 b^4 d^4 e^2 + 400 a^3 b^3 d^3 e^3 + 225 a^4 b^2 d^2 e^4 + 36 a^5 b d e^5 + a^6 e^6) x^7$$

$$+ (a b^5 d^6 + 15 a^2 b^4 d^5 e + 50 a^3 b^3 d^4 e^2 + 50 a^4 b^2 d^3 e^3 + 15 a^5 b d^2 e^4 + a^6 d e^5) x^6$$

$$+ 3 (a^2 b^4 d^6 + 8 a^3 b^3 d^5 e + 15 a^4 b^2 d^4 e^2 + 8 a^5 b d^3 e^3 + a^6 d^2 e^4) x^5$$

$$+ \frac{5}{2} (2 a^3 b^3 d^6 + 9 a^4 b^2 d^5 e + 9 a^5 b d^4 e^2 + 2 a^6 d^3 e^3) x^4$$

$$+ (5 a^4 b^2 d^6 + 12 a^5 b d^5 e + 5 a^6 d^4 e^2) x^3 + 3 (a^5 b d^6 + a^6 d^5 e) x^2$$

input `integrate((e*x+d)^6*(b^2*x^2+2*a*b*x+a^2)^3,x, algorithm="fricas")`

output `1/13*b^6*e^6*x^13 + a^6*d^6*x + 1/2*(b^6*d*e^5 + a*b^5*e^6)*x^12 + 3/11*(5*b^6*d^2*e^4 + 12*a*b^5*d*e^5 + 5*a^2*b^4*e^6)*x^11 + (2*b^6*d^3*e^3 + 9*a*b^5*d^2*e^4 + 9*a^2*b^4*d*e^5 + 2*a^3*b^3*e^6)*x^10 + 5/3*(b^6*d^4*e^2 + 8*a*b^5*d^3*e^3 + 15*a^2*b^4*d^2*e^4 + 8*a^3*b^3*d*e^5 + a^4*b^2*e^6)*x^9 + 3/4*(b^6*d^5*e + 15*a*b^5*d^4*e^2 + 50*a^2*b^4*d^3*e^3 + 50*a^3*b^3*d^2*e^4 + 15*a^4*b^2*d*e^5 + a^5*b*e^6)*x^8 + 1/7*(b^6*d^6 + 36*a*b^5*d^5*e + 225*a^2*b^4*d^4*e^2 + 400*a^3*b^3*d^3*e^3 + 225*a^4*b^2*d^2*e^4 + 36*a^5*b*d*e^5 + a^6*e^6)*x^7 + (a*b^5*d^6 + 15*a^2*b^4*d^5*e + 50*a^3*b^3*d^4*e^2 + 50*a^4*b^2*d^3*e^3 + 15*a^5*b*d^2*e^4 + a^6*d*e^5)*x^6 + 3*(a^2*b^4*d^6 + 8*a^3*b^3*d^5*e + 15*a^4*b^2*d^4*e^2 + 8*a^5*b*d^3*e^3 + a^6*d^2*e^4)*x^5 + 5/2*(2*a^3*b^3*d^6 + 9*a^4*b^2*d^5*e + 9*a^5*b*d^4*e^2 + 2*a^6*d^3*e^3)*x^4 + (5*a^4*b^2*d^6 + 12*a^5*b*d^5*e + 5*a^6*d^4*e^2)*x^3 + 3*(a^5*b*d^6 + a^6*d^5*e)*x^2`

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 677 vs. $2(155) = 310$.

Time = 0.06 (sec) , antiderivative size = 677, normalized size of antiderivative = 3.96

$$\begin{aligned}
 \int (d + ex)^6 (a^2 + 2abx + b^2x^2)^3 dx = & a^6 d^6 x + \frac{b^6 e^6 x^{13}}{13} + x^{12} \left(\frac{ab^5 e^6}{2} + \frac{b^6 d e^5}{2} \right) + x^{11} \\
 & \cdot \left(\frac{15a^2 b^4 e^6}{11} + \frac{36ab^5 d e^5}{11} + \frac{15b^6 d^2 e^4}{11} \right) + x^{10} \\
 & \cdot (2a^3 b^3 e^6 + 9a^2 b^4 d e^5 + 9ab^5 d^2 e^4 + 2b^6 d^3 e^3) \\
 & + x^9 \cdot \left(\frac{5a^4 b^2 e^6}{3} + \frac{40a^3 b^3 d e^5}{3} + 25a^2 b^4 d^2 e^4 \right. \\
 & \qquad \qquad \qquad \left. + \frac{40ab^5 d^3 e^3}{3} + \frac{5b^6 d^4 e^2}{3} \right) \\
 & + x^8 \cdot \left(\frac{3a^5 b e^6}{4} + \frac{45a^4 b^2 d e^5}{4} + \frac{75a^3 b^3 d^2 e^4}{2} \right. \\
 & \qquad \qquad \qquad \left. + \frac{75a^2 b^4 d^3 e^3}{2} + \frac{45ab^5 d^4 e^2}{4} + \frac{3b^6 d^5 e}{4} \right) \\
 & + x^7 \left(\frac{a^6 e^6}{7} + \frac{36a^5 b d e^5}{7} + \frac{225a^4 b^2 d^2 e^4}{7} \right. \\
 & \qquad \qquad \qquad \left. + \frac{400a^3 b^3 d^3 e^3}{7} + \frac{225a^2 b^4 d^4 e^2}{7} + \frac{36ab^5 d^5 e}{7} + \frac{b^6 d^6}{7} \right) \\
 & + x^6 (a^6 d e^5 + 15a^5 b d^2 e^4 + 50a^4 b^2 d^3 e^3 + 50a^3 b^3 d^4 e^2 \\
 & \qquad \qquad \qquad + 15a^2 b^4 d^5 e + ab^5 d^6) + x^5 \cdot (3a^6 d^2 e^4 + 24a^5 b d^3 e^3 \\
 & \qquad \qquad \qquad + 45a^4 b^2 d^4 e^2 + 24a^3 b^3 d^5 e + 3a^2 b^4 d^6) + x^4 \\
 & \cdot \left(5a^6 d^3 e^3 + \frac{45a^5 b d^4 e^2}{2} + \frac{45a^4 b^2 d^5 e}{2} + 5a^3 b^3 d^6 \right) \\
 & + x^3 \cdot (5a^6 d^4 e^2 + 12a^5 b d^5 e + 5a^4 b^2 d^6) \\
 & + x^2 \cdot (3a^6 d^5 e + 3a^5 b d^6)
 \end{aligned}$$

input

```
integrate((e*x+d)**6*(b**2*x**2+2*a*b*x+a**2)**3,x)
```

output

```

a**6*d**6*x + b**6*e**6*x**13/13 + x**12*(a*b**5*e**6/2 + b**6*d*e**5/2) +
x**11*(15*a**2*b**4*e**6/11 + 36*a*b**5*d*e**5/11 + 15*b**6*d**2*e**4/11)
+ x**10*(2*a**3*b**3*e**6 + 9*a**2*b**4*d*e**5 + 9*a*b**5*d**2*e**4 + 2*b
**6*d**3*e**3) + x**9*(5*a**4*b**2*e**6/3 + 40*a**3*b**3*d*e**5/3 + 25*a**
2*b**4*d**2*e**4 + 40*a*b**5*d**3*e**3/3 + 5*b**6*d**4*e**2/3) + x**8*(3*a
**5*b*e**6/4 + 45*a**4*b**2*d*e**5/4 + 75*a**3*b**3*d**2*e**4/2 + 75*a**2*
b**4*d**3*e**3/2 + 45*a*b**5*d**4*e**2/4 + 3*b**6*d**5*e/4) + x**7*(a**6*e
**6/7 + 36*a**5*b*d*e**5/7 + 225*a**4*b**2*d**2*e**4/7 + 400*a**3*b**3*d**
3*e**3/7 + 225*a**2*b**4*d**4*e**2/7 + 36*a*b**5*d**5*e/7 + b**6*d**6/7) +
x**6*(a**6*d*e**5 + 15*a**5*b*d**2*e**4 + 50*a**4*b**2*d**3*e**3 + 50*a**
3*b**3*d**4*e**2 + 15*a**2*b**4*d**5*e + a*b**5*d**6) + x**5*(3*a**6*d**2*
e**4 + 24*a**5*b*d**3*e**3 + 45*a**4*b**2*d**4*e**2 + 24*a**3*b**3*d**5*e
+ 3*a**2*b**4*d**6) + x**4*(5*a**6*d**3*e**3 + 45*a**5*b*d**4*e**2/2 + 45*
a**4*b**2*d**5*e/2 + 5*a**3*b**3*d**6) + x**3*(5*a**6*d**4*e**2 + 12*a**5*
b*d**5*e + 5*a**4*b**2*d**6) + x**2*(3*a**6*d**5*e + 3*a**5*b*d**6)

```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 599 vs. $2(159) = 318$.

Time = 0.05 (sec) , antiderivative size = 599, normalized size of antiderivative = 3.50

$$\begin{aligned}
& \int (d + ex)^6 (a^2 + 2abx + b^2x^2)^3 dx \\
&= \frac{1}{13} b^6 e^6 x^{13} + a^6 d^6 x + \frac{1}{2} (b^6 d e^5 + a b^5 e^6) x^{12} + \frac{3}{11} (5 b^6 d^2 e^4 + 12 a b^5 d e^5 + 5 a^2 b^4 e^6) x^{11} \\
&+ (2 b^6 d^3 e^3 + 9 a b^5 d^2 e^4 + 9 a^2 b^4 d e^5 + 2 a^3 b^3 e^6) x^{10} \\
&+ \frac{5}{3} (b^6 d^4 e^2 + 8 a b^5 d^3 e^3 + 15 a^2 b^4 d^2 e^4 + 8 a^3 b^3 d e^5 + a^4 b^2 e^6) x^9 \\
&+ \frac{3}{4} (b^6 d^5 e + 15 a b^5 d^4 e^2 + 50 a^2 b^4 d^3 e^3 + 50 a^3 b^3 d^2 e^4 + 15 a^4 b^2 d e^5 + a^5 b e^6) x^8 \\
&+ \frac{1}{7} (b^6 d^6 + 36 a b^5 d^5 e + 225 a^2 b^4 d^4 e^2 + 400 a^3 b^3 d^3 e^3 + 225 a^4 b^2 d^2 e^4 + 36 a^5 b d e^5 + a^6 e^6) x^7 \\
&+ (a b^5 d^6 + 15 a^2 b^4 d^5 e + 50 a^3 b^3 d^4 e^2 + 50 a^4 b^2 d^3 e^3 + 15 a^5 b d^2 e^4 + a^6 d e^5) x^6 \\
&+ 3 (a^2 b^4 d^6 + 8 a^3 b^3 d^5 e + 15 a^4 b^2 d^4 e^2 + 8 a^5 b d^3 e^3 + a^6 d^2 e^4) x^5 \\
&+ \frac{5}{2} (2 a^3 b^3 d^6 + 9 a^4 b^2 d^5 e + 9 a^5 b d^4 e^2 + 2 a^6 d^3 e^3) x^4 \\
&+ (5 a^4 b^2 d^6 + 12 a^5 b d^5 e + 5 a^6 d^4 e^2) x^3 + 3 (a^5 b d^6 + a^6 d^5 e) x^2
\end{aligned}$$

input

```

integrate((e*x+d)^6*(b^2*x^2+2*a*b*x+a^2)^3,x, algorithm="maxima")

```

output

```

1/13*b^6*e^6*x^13 + a^6*d^6*x + 1/2*(b^6*d*e^5 + a*b^5*e^6)*x^12 + 3/11*(5
*b^6*d^2*e^4 + 12*a*b^5*d*e^5 + 5*a^2*b^4*e^6)*x^11 + (2*b^6*d^3*e^3 + 9*a
*b^5*d^2*e^4 + 9*a^2*b^4*d*e^5 + 2*a^3*b^3*e^6)*x^10 + 5/3*(b^6*d^4*e^2 +
8*a*b^5*d^3*e^3 + 15*a^2*b^4*d^2*e^4 + 8*a^3*b^3*d*e^5 + a^4*b^2*e^6)*x^9
+ 3/4*(b^6*d^5*e + 15*a*b^5*d^4*e^2 + 50*a^2*b^4*d^3*e^3 + 50*a^3*b^3*d^2*
e^4 + 15*a^4*b^2*d*e^5 + a^5*b*e^6)*x^8 + 1/7*(b^6*d^6 + 36*a*b^5*d^5*e +
225*a^2*b^4*d^4*e^2 + 400*a^3*b^3*d^3*e^3 + 225*a^4*b^2*d^2*e^4 + 36*a^5*b
*d*e^5 + a^6*e^6)*x^7 + (a*b^5*d^6 + 15*a^2*b^4*d^5*e + 50*a^3*b^3*d^4*e^2
+ 50*a^4*b^2*d^3*e^3 + 15*a^5*b*d^2*e^4 + a^6*d*e^5)*x^6 + 3*(a^2*b^4*d^6
+ 8*a^3*b^3*d^5*e + 15*a^4*b^2*d^4*e^2 + 8*a^5*b*d^3*e^3 + a^6*d^2*e^4)*x
^5 + 5/2*(2*a^3*b^3*d^6 + 9*a^4*b^2*d^5*e + 9*a^5*b*d^4*e^2 + 2*a^6*d^3*e^
3)*x^4 + (5*a^4*b^2*d^6 + 12*a^5*b*d^5*e + 5*a^6*d^4*e^2)*x^3 + 3*(a^5*b*d
^6 + a^6*d^5*e)*x^2

```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 689 vs. $2(159) = 318$.

Time = 0.22 (sec) , antiderivative size = 689, normalized size of antiderivative = 4.03

$$\begin{aligned}
 \int (d + ex)^6 (a^2 + 2abx + b^2x^2)^3 dx = & \frac{1}{13} b^6 e^6 x^{13} + \frac{1}{2} b^6 d e^5 x^{12} + \frac{1}{2} a b^5 e^6 x^{12} \\
 & + \frac{15}{11} b^6 d^2 e^4 x^{11} + \frac{36}{11} a b^5 d e^5 x^{11} + \frac{15}{11} a^2 b^4 e^6 x^{11} \\
 & + 2 b^6 d^3 e^3 x^{10} + 9 a b^5 d^2 e^4 x^{10} + 9 a^2 b^4 d e^5 x^{10} \\
 & + 2 a^3 b^3 e^6 x^{10} + \frac{5}{3} b^6 d^4 e^2 x^9 + \frac{40}{3} a b^5 d^3 e^3 x^9 \\
 & + 25 a^2 b^4 d^2 e^4 x^9 + \frac{40}{3} a^3 b^3 d e^5 x^9 + \frac{5}{3} a^4 b^2 e^6 x^9 \\
 & + \frac{3}{4} b^6 d^5 e x^8 + \frac{45}{4} a b^5 d^4 e^2 x^8 + \frac{75}{2} a^2 b^4 d^3 e^3 x^8 \\
 & + \frac{75}{2} a^3 b^3 d^2 e^4 x^8 + \frac{45}{4} a^4 b^2 d e^5 x^8 + \frac{3}{4} a^5 b e^6 x^8 \\
 & + \frac{1}{7} b^6 d^6 x^7 + \frac{36}{7} a b^5 d^5 e x^7 + \frac{225}{7} a^2 b^4 d^4 e^2 x^7 \\
 & + \frac{400}{7} a^3 b^3 d^3 e^3 x^7 + \frac{225}{7} a^4 b^2 d^2 e^4 x^7 + \frac{36}{7} a^5 b d e^5 x^7 \\
 & + \frac{1}{7} a^6 e^6 x^7 + a b^5 d^6 x^6 + 15 a^2 b^4 d^5 e x^6 \\
 & + 50 a^3 b^3 d^4 e^2 x^6 + 50 a^4 b^2 d^3 e^3 x^6 + 15 a^5 b d^2 e^4 x^6 \\
 & + a^6 d e^5 x^6 + 3 a^2 b^4 d^6 x^5 + 24 a^3 b^3 d^5 e x^5 \\
 & + 45 a^4 b^2 d^4 e^2 x^5 + 24 a^5 b d^3 e^3 x^5 + 3 a^6 d^2 e^4 x^5 \\
 & + 5 a^3 b^3 d^6 x^4 + \frac{45}{2} a^4 b^2 d^5 e x^4 + \frac{45}{2} a^5 b d^4 e^2 x^4 \\
 & + 5 a^6 d^3 e^3 x^4 + 5 a^4 b^2 d^6 x^3 + 12 a^5 b d^5 e x^3 \\
 & + 5 a^6 d^4 e^2 x^3 + 3 a^5 b d^6 x^2 + 3 a^6 d^5 e x^2 + a^6 d^6 x
 \end{aligned}$$

input `integrate((e*x+d)^6*(b^2*x^2+2*a*b*x+a^2)^3,x, algorithm="giac")`

output

```

1/13*b^6*e^6*x^13 + 1/2*b^6*d*e^5*x^12 + 1/2*a*b^5*e^6*x^12 + 15/11*b^6*d^
2*e^4*x^11 + 36/11*a*b^5*d*e^5*x^11 + 15/11*a^2*b^4*e^6*x^11 + 2*b^6*d^3*e
^3*x^10 + 9*a*b^5*d^2*e^4*x^10 + 9*a^2*b^4*d*e^5*x^10 + 2*a^3*b^3*e^6*x^10
+ 5/3*b^6*d^4*e^2*x^9 + 40/3*a*b^5*d^3*e^3*x^9 + 25*a^2*b^4*d^2*e^4*x^9 +
40/3*a^3*b^3*d*e^5*x^9 + 5/3*a^4*b^2*e^6*x^9 + 3/4*b^6*d^5*e*x^8 + 45/4*a
*b^5*d^4*e^2*x^8 + 75/2*a^2*b^4*d^3*e^3*x^8 + 75/2*a^3*b^3*d^2*e^4*x^8 + 4
5/4*a^4*b^2*d*e^5*x^8 + 3/4*a^5*b*d*e^6*x^8 + 1/7*b^6*d^6*x^7 + 36/7*a*b^5*d
^5*e*x^7 + 225/7*a^2*b^4*d^4*e^2*x^7 + 400/7*a^3*b^3*d^3*e^3*x^7 + 225/7*a
^4*b^2*d^2*e^4*x^7 + 36/7*a^5*b*d*e^5*x^7 + 1/7*a^6*e^6*x^7 + a*b^5*d^6*x^
6 + 15*a^2*b^4*d^5*e*x^6 + 50*a^3*b^3*d^4*e^2*x^6 + 50*a^4*b^2*d^3*e^3*x^6
+ 15*a^5*b*d^2*e^4*x^6 + a^6*d*e^5*x^6 + 3*a^2*b^4*d^6*x^5 + 24*a^3*b^3*d
^5*e*x^5 + 45*a^4*b^2*d^4*e^2*x^5 + 24*a^5*b*d^3*e^3*x^5 + 3*a^6*d^2*e^4*x
^5 + 5*a^3*b^3*d^6*x^4 + 45/2*a^4*b^2*d^5*e*x^4 + 45/2*a^5*b*d^4*e^2*x^4 +
5*a^6*d^3*e^3*x^4 + 5*a^4*b^2*d^6*x^3 + 12*a^5*b*d^5*e*x^3 + 5*a^6*d^4*e^
2*x^3 + 3*a^5*b*d^6*x^2 + 3*a^6*d^5*e*x^2 + a^6*d^6*x

```

Mupad [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 579, normalized size of antiderivative = 3.39

$$\begin{aligned}
& \int (d + ex)^6 (a^2 + 2abx + b^2x^2)^3 dx \\
&= x^6 (a^6 d e^5 + 15 a^5 b d^2 e^4 + 50 a^4 b^2 d^3 e^3 + 50 a^3 b^3 d^4 e^2 + 15 a^2 b^4 d^5 e + a b^5 d^6) \\
&+ x^8 \left(\frac{3 a^5 b e^6}{4} + \frac{45 a^4 b^2 d e^5}{4} + \frac{75 a^3 b^3 d^2 e^4}{2} + \frac{75 a^2 b^4 d^3 e^3}{2} + \frac{45 a b^5 d^4 e^2}{4} + \frac{3 b^6 d^5 e}{4} \right) \\
&+ x^5 (3 a^6 d^2 e^4 + 24 a^5 b d^3 e^3 + 45 a^4 b^2 d^4 e^2 + 24 a^3 b^3 d^5 e + 3 a^2 b^4 d^6) \\
&+ x^9 \left(\frac{5 a^4 b^2 e^6}{3} + \frac{40 a^3 b^3 d e^5}{3} + 25 a^2 b^4 d^2 e^4 + \frac{40 a b^5 d^3 e^3}{3} + \frac{5 b^6 d^4 e^2}{3} \right) + x^7 \left(\frac{a^6 e^6}{7} \right. \\
&+ \left. \frac{36 a^5 b d e^5}{7} + \frac{225 a^4 b^2 d^2 e^4}{7} + \frac{400 a^3 b^3 d^3 e^3}{7} + \frac{225 a^2 b^4 d^4 e^2}{7} + \frac{36 a b^5 d^5 e}{7} + \frac{b^6 d^6}{7} \right) \\
&+ a^6 d^6 x + \frac{b^6 e^6 x^{13}}{13} + \frac{5 a^3 d^3 x^4 (2 a^3 e^3 + 9 a^2 b d e^2 + 9 a b^2 d^2 e + 2 b^3 d^3)}{2} \\
&+ b^3 e^3 x^{10} (2 a^3 e^3 + 9 a^2 b d e^2 + 9 a b^2 d^2 e + 2 b^3 d^3) + 3 a^5 d^5 x^2 (a e + b d) \\
&+ \frac{b^5 e^5 x^{12} (a e + b d)}{2} + a^4 d^4 x^3 (5 a^2 e^2 + 12 a b d e + 5 b^2 d^2) \\
&+ \frac{3 b^4 e^4 x^{11} (5 a^2 e^2 + 12 a b d e + 5 b^2 d^2)}{11}
\end{aligned}$$

input `int((d + e*x)^6*(a^2 + b^2*x^2 + 2*a*b*x)^3,x)`

output $x^6*(a*b^5*d^6 + a^6*d*e^5 + 15*a^2*b^4*d^5*e + 15*a^5*b*d^2*e^4 + 50*a^3*b^3*d^4*e^2 + 50*a^4*b^2*d^3*e^3) + x^8*((3*a^5*b*e^6)/4 + (3*b^6*d^5*e)/4 + (45*a*b^5*d^4*e^2)/4 + (45*a^4*b^2*d*e^5)/4 + (75*a^2*b^4*d^3*e^3)/2 + (75*a^3*b^3*d^2*e^4)/2) + x^5*(3*a^2*b^4*d^6 + 3*a^6*d^2*e^4 + 24*a^3*b^3*d^5*e + 24*a^5*b*d^3*e^3 + 45*a^4*b^2*d^4*e^2) + x^9*((5*a^4*b^2*e^6)/3 + (5*b^6*d^4*e^2)/3 + (40*a*b^5*d^3*e^3)/3 + (40*a^3*b^3*d*e^5)/3 + 25*a^2*b^4*d^2*e^4) + x^7*((a^6*e^6)/7 + (b^6*d^6)/7 + (225*a^2*b^4*d^4*e^2)/7 + (400*a^3*b^3*d^3*e^3)/7 + (225*a^4*b^2*d^2*e^4)/7 + (36*a*b^5*d^5*e)/7 + (36*a^5*b*d*e^5)/7) + a^6*d^6*x + (b^6*e^6*x^13)/13 + (5*a^3*d^3*x^4*(2*a^3*e^3 + 2*b^3*d^3 + 9*a*b^2*d^2*e + 9*a^2*b*d*e^2))/2 + b^3*e^3*x^10*(2*a^3*e^3 + 2*b^3*d^3 + 9*a*b^2*d^2*e + 9*a^2*b*d*e^2) + 3*a^5*d^5*x^2*(a*e + b*d) + (b^5*e^5*x^12*(a*e + b*d))/2 + a^4*d^4*x^3*(5*a^2*e^2 + 5*b^2*d^2 + 12*a*b*d*e) + (3*b^4*e^4*x^11*(5*a^2*e^2 + 5*b^2*d^2 + 12*a*b*d*e))/11$

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 690, normalized size of antiderivative = 4.04

$$\int (d + ex)^6 (a^2 + 2abx + b^2x^2)^3 dx$$

$$= \frac{x(924b^6e^6x^{12} + 6006ab^5e^6x^{11} + 6006b^6de^5x^{11} + 16380a^2b^4e^6x^{10} + 39312ab^5de^5x^{10} + 16380b^6d^2e^4x^{10} + \dots)}{1}$$

input `int((e*x+d)^6*(b^2*x^2+2*a*b*x+a^2)^3,x)`

output

```
(x*(12012*a**6*d**6 + 36036*a**6*d**5*e*x + 60060*a**6*d**4*e**2*x**2 + 60060*a**6*d**3*e**3*x**3 + 36036*a**6*d**2*e**4*x**4 + 12012*a**6*d*e**5*x**5 + 1716*a**6*e**6*x**6 + 36036*a**5*b*d**6*x + 144144*a**5*b*d**5*e*x**2 + 270270*a**5*b*d**4*e**2*x**3 + 288288*a**5*b*d**3*e**3*x**4 + 180180*a**5*b*d**2*e**4*x**5 + 61776*a**5*b*d*e**5*x**6 + 9009*a**5*b*e**6*x**7 + 60060*a**4*b**2*d**6*x**2 + 270270*a**4*b**2*d**5*e*x**3 + 540540*a**4*b**2*d**4*e**2*x**4 + 600600*a**4*b**2*d**3*e**3*x**5 + 386100*a**4*b**2*d**2*e**4*x**6 + 135135*a**4*b**2*d*e**5*x**7 + 20020*a**4*b**2*e**6*x**8 + 60060*a**3*b**3*d**6*x**3 + 288288*a**3*b**3*d**5*e*x**4 + 600600*a**3*b**3*d**4*e**2*x**5 + 686400*a**3*b**3*d**3*e**3*x**6 + 450450*a**3*b**3*d**2*e**4*x**7 + 160160*a**3*b**3*d*e**5*x**8 + 24024*a**3*b**3*e**6*x**9 + 36036*a**2*b**4*d**6*x**4 + 180180*a**2*b**4*d**5*e*x**5 + 386100*a**2*b**4*d**4*e**2*x**6 + 450450*a**2*b**4*d**3*e**3*x**7 + 300300*a**2*b**4*d**2*e**4*x**8 + 108108*a**2*b**4*d*e**5*x**9 + 16380*a**2*b**4*e**6*x**10 + 12012*a*b**5*d**6*x**5 + 61776*a*b**5*d**5*e*x**6 + 135135*a*b**5*d**4*e**2*x**7 + 160160*a*b**5*d**3*e**3*x**8 + 108108*a*b**5*d**2*e**4*x**9 + 39312*a*b**5*d*e**5*x**10 + 6006*a*b**5*e**6*x**11 + 1716*b**6*d**6*x**6 + 9009*b**6*d**5*e*x**7 + 20020*b**6*d**4*e**2*x**8 + 24024*b**6*d**3*e**3*x**9 + 16380*b**6*d**2*e**4*x**10 + 6006*b**6*d*e**5*x**11 + 924*b**6*e**6*x**12))/12012
```

3.87 $\int (d + ex)^5 (a^2 + 2abx + b^2x^2)^3 dx$

| | |
|---|-----|
| Optimal result | 709 |
| Mathematica [B] (verified) | 710 |
| Rubi [A] (verified) | 711 |
| Maple [B] (verified) | 712 |
| Fricas [B] (verification not implemented) | 713 |
| Sympy [B] (verification not implemented) | 714 |
| Maxima [B] (verification not implemented) | 716 |
| Giac [B] (verification not implemented) | 717 |
| Mupad [B] (verification not implemented) | 719 |
| Reduce [B] (verification not implemented) | 720 |

Optimal result

Integrand size = 26, antiderivative size = 143

$$\int (d + ex)^5 (a^2 + 2abx + b^2x^2)^3 dx = \frac{(bd - ae)^5(a + bx)^7}{7b^6} + \frac{5e(bd - ae)^4(a + bx)^8}{8b^6} + \frac{10e^2(bd - ae)^3(a + bx)^9}{9b^6} + \frac{e^3(bd - ae)^2(a + bx)^{10}}{b^6} + \frac{5e^4(bd - ae)(a + bx)^{11}}{11b^6} + \frac{e^5(a + bx)^{12}}{12b^6}$$

output

```
1/7*(-a*e+b*d)^5*(b*x+a)^7/b^6+5/8*e*(-a*e+b*d)^4*(b*x+a)^8/b^6+10/9*e^2*(-a*e+b*d)^3*(b*x+a)^9/b^6+e^3*(-a*e+b*d)^2*(b*x+a)^10/b^6+5/11*e^4*(-a*e+b*d)*(b*x+a)^11/b^6+1/12*e^5*(b*x+a)^12/b^6
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 501 vs. $2(143) = 286$.

Time = 0.05 (sec) , antiderivative size = 501, normalized size of antiderivative = 3.50

$$\int (d + ex)^5 (a^2 + 2abx + b^2x^2)^3 dx = a^6 d^5 x + \frac{1}{2} a^5 d^4 (6bd + 5ae) x^2 + \frac{5}{3} a^4 d^3 (3b^2 d^2 + 6abde + 2a^2 e^2) x^3 + \frac{5}{4} a^3 d^2 (4b^3 d^3 + 15ab^2 d^2 e + 12a^2 bde^2 + 2a^3 e^3) x^4 + a^2 d (3b^4 d^4 + 20ab^3 d^3 e + 30a^2 b^2 d^2 e^2 + 12a^3 bde^3 + a^4 e^4) x^5 + \frac{1}{6} a (6b^5 d^5 + 75ab^4 d^4 e + 200a^2 b^3 d^3 e^2 + 150a^3 b^2 d^2 e^3 + 30a^4 bde^4 + a^5 e^5) x^6 + \frac{1}{7} b (b^5 d^5 + 30ab^4 d^4 e + 150a^2 b^3 d^3 e^2 + 200a^3 b^2 d^2 e^3 + 75a^4 bde^4 + 6a^5 e^5) x^7 + \frac{5}{8} b^2 e (b^4 d^4 + 12ab^3 d^3 e + 30a^2 b^2 d^2 e^2 + 20a^3 bde^3 + 3a^4 e^4) x^8 + \frac{5}{9} b^3 e^2 (2b^3 d^3 + 12ab^2 d^2 e + 15a^2 bde^2 + 4a^3 e^3) x^9 + \frac{1}{2} b^4 e^3 (2b^2 d^2 + 6abde + 3a^2 e^2) x^{10} + \frac{1}{11} b^5 e^4 (5bd + 6ae) x^{11} + \frac{1}{12} b^6 e^5 x^{12}$$

input `Integrate[(d + e*x)^5*(a^2 + 2*a*b*x + b^2*x^2)^3,x]`

output `a^6*d^5*x + (a^5*d^4*(6*b*d + 5*a*e)*x^2)/2 + (5*a^4*d^3*(3*b^2*d^2 + 6*a*b*d*e + 2*a^2*e^2)*x^3)/3 + (5*a^3*d^2*(4*b^3*d^3 + 15*a*b^2*d^2*e + 12*a^2*b*d*e^2 + 2*a^3*e^3)*x^4)/4 + a^2*d*(3*b^4*d^4 + 20*a*b^3*d^3*e + 30*a^2*b^2*d^2*e^2 + 12*a^3*b*d*e^3 + a^4*e^4)*x^5 + (a*(6*b^5*d^5 + 75*a*b^4*d^4*e + 200*a^2*b^3*d^3*e^2 + 150*a^3*b^2*d^2*e^3 + 30*a^4*b*d*e^4 + a^5*e^5)*x^6)/6 + (b*(b^5*d^5 + 30*a*b^4*d^4*e + 150*a^2*b^3*d^3*e^2 + 200*a^3*b^2*d^2*e^3 + 75*a^4*b*d*e^4 + 6*a^5*e^5)*x^7)/7 + (5*b^2*e*(b^4*d^4 + 12*a*b^3*d^3*e + 30*a^2*b^2*d^2*e^2 + 20*a^3*b*d*e^3 + 3*a^4*e^4)*x^8)/8 + (5*b^3*e^2*(2*b^3*d^3 + 12*a*b^2*d^2*e + 15*a^2*b*d*e^2 + 4*a^3*e^3)*x^9)/9 + (b^4*e^3*(2*b^2*d^2 + 6*a*b*d*e + 3*a^2*e^2)*x^10)/2 + (b^5*e^4*(5*b*d + 6*a*e)*x^11)/11 + (b^6*e^5*x^12)/12`

Rubi [A] (verified)

Time = 0.81 (sec) , antiderivative size = 143, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {1098, 27, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a^2 + 2abx + b^2x^2)^3 (d + ex)^5 dx$$

$$\downarrow 1098$$

$$\frac{\int b^6 (a + bx)^6 (d + ex)^5 dx}{b^6}$$

$$\downarrow 27$$

$$\int (a + bx)^6 (d + ex)^5 dx$$

$$\downarrow 49$$

$$\int \left(\frac{5e^4 (a + bx)^{10} (bd - ae)}{b^5} + \frac{10e^3 (a + bx)^9 (bd - ae)^2}{b^5} + \frac{10e^2 (a + bx)^8 (bd - ae)^3}{b^5} + \frac{5e (a + bx)^7 (bd - ae)^4}{b^5} + \dots \right) dx$$

$$\downarrow 2009$$

$$\frac{5e^4 (a + bx)^{11} (bd - ae)}{11b^6} + \frac{e^3 (a + bx)^{10} (bd - ae)^2}{b^6} + \frac{10e^2 (a + bx)^9 (bd - ae)^3}{9b^6} + \frac{5e (a + bx)^8 (bd - ae)^4}{8b^6} + \frac{(a + bx)^7 (bd - ae)^5}{7b^6} + \frac{e^5 (a + bx)^{12}}{12b^6}$$

input `Int[(d + e*x)^5*(a^2 + 2*a*b*x + b^2*x^2)^3,x]`

output $((b*d - a*e)^5*(a + b*x)^7)/(7*b^6) + (5*e*(b*d - a*e)^4*(a + b*x)^8)/(8*b^6) + (10*e^2*(b*d - a*e)^3*(a + b*x)^9)/(9*b^6) + (e^3*(b*d - a*e)^2*(a + b*x)^10)/b^6 + (5*e^4*(b*d - a*e)*(a + b*x)^11)/(11*b^6) + (e^5*(a + b*x)^12)/(12*b^6)$

Definitions of rubi rules used

rule 27 $\text{Int}[(a_*)(Fx_), x_Symbol] \rightarrow \text{Simp}[a \text{ Int}[Fx, x], x] /; \text{FreeQ}[a, x] \ \&\& \ !\text{MatchQ}[Fx, (b_*)(Gx_)] /; \text{FreeQ}[b, x]$

rule 49 $\text{Int}[((a_.) + (b_.)(x_))^{(m_.)}((c_.) + (d_.)(x_))^{(n_.)}, x_Symbol] \rightarrow \text{Int}[\text{ExpandIntegrand}[(a + b*x)^m*(c + d*x)^n, x], x] /; \text{FreeQ}[\{a, b, c, d\}, x] \ \&\& \ \text{IGtQ}[m, 0] \ \&\& \ \text{IGtQ}[m + n + 2, 0]$

rule 1098 $\text{Int}[((d_.) + (e_.)(x_))^{(m_.)}((a_) + (b_.)(x_) + (c_.)(x_)^2)^{(p_.)}, x_Symbol] \rightarrow \text{Simp}[1/c^p \text{ Int}[(d + e*x)^m*(b/2 + c*x)^{(2*p)}, x], x] /; \text{FreeQ}[\{a, b, c, d, e, m\}, x] \ \&\& \ \text{EqQ}[b^2 - 4*a*c, 0] \ \&\& \ \text{IntegerQ}[p]$

rule 2009 $\text{Int}[u_, x_Symbol] \rightarrow \text{Simp}[\text{IntSum}[u, x], x] /; \text{SumQ}[u]$

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 509 vs. $2(133) = 266$.

Time = 1.05 (sec) , antiderivative size = 510, normalized size of antiderivative = 3.57

| method | result |
|---------------|---|
| norman | $\frac{e^5 b^6 x^{12}}{12} + \left(\frac{6}{11} e^5 a b^5 + \frac{5}{11} d e^4 b^6\right) x^{11} + \left(\frac{3}{2} e^5 a^2 b^4 + 3 d e^4 a b^5 + d^2 e^3 b^6\right) x^{10} + \left(\frac{20}{9} e^5 a^3 b^3 + \frac{25}{3} a^2 d e^4\right) x^9 + \left(\frac{5}{11} e^5 a^4 b^2 + \frac{15}{8} d e^4 a^3 b\right) x^8 + \left(\frac{5}{11} e^5 a^5 b + \frac{15}{8} d e^4 a^2 b^2 + \frac{5}{8} d^2 e^3 a b^3\right) x^7 + \left(\frac{5}{11} e^5 a^6 + \frac{15}{8} d e^4 a^3 b^2 + \frac{5}{8} d^2 e^3 a^2 b^3\right) x^6 + \left(\frac{5}{11} e^5 a^7 + \frac{15}{8} d e^4 a^4 b + \frac{5}{8} d^2 e^3 a^3 b^2\right) x^5 + \left(\frac{5}{11} e^5 a^8 + \frac{15}{8} d e^4 a^5 + \frac{5}{8} d^2 e^3 a^4 b\right) x^4 + \left(\frac{5}{11} e^5 a^9 + \frac{15}{8} d e^4 a^6 + \frac{5}{8} d^2 e^3 a^5 b\right) x^3 + \left(\frac{5}{11} e^5 a^{10} + \frac{15}{8} d e^4 a^7 + \frac{5}{8} d^2 e^3 a^6 b\right) x^2 + \left(\frac{5}{11} e^5 a^{11} + \frac{15}{8} d e^4 a^8 + \frac{5}{8} d^2 e^3 a^7 b\right) x + \frac{5}{11} e^5 a^{12} + \frac{15}{8} d e^4 a^9 + \frac{5}{8} d^2 e^3 a^8 b$ |
| default | $\frac{e^5 b^6 x^{12}}{12} + \frac{(6e^5 a b^5 + 5d e^4 b^6)x^{11}}{11} + \frac{(15e^5 a^2 b^4 + 30d e^4 a b^5 + 10d^2 e^3 b^6)x^{10}}{10} + \frac{(20e^5 a^3 b^3 + 75a^2 b^4 d e^4 + 60d^2 e^3 a b^5 + 10d^3 e^2 b^6)x^9}{9}$ |
| risch | $\frac{5}{11} x^{11} d e^4 b^6 + \frac{3}{2} x^{10} e^5 a^2 b^4 + x^{10} d^2 e^3 b^6 + \frac{20}{9} x^9 e^5 a^3 b^3 + \frac{10}{9} x^9 d^3 e^2 b^6 + \frac{15}{8} x^8 e^5 a^4 b^2 + \frac{5}{8} x^8 d^4 e b^6$ |
| parallelrisch | $\frac{5}{11} x^{11} d e^4 b^6 + \frac{3}{2} x^{10} e^5 a^2 b^4 + x^{10} d^2 e^3 b^6 + \frac{20}{9} x^9 e^5 a^3 b^3 + \frac{10}{9} x^9 d^3 e^2 b^6 + \frac{15}{8} x^8 e^5 a^4 b^2 + \frac{5}{8} x^8 d^4 e b^6$ |
| gosper | $x(462e^5 b^6 x^{11} + 3024x^{10} e^5 a b^5 + 2520x^{10} d e^4 b^6 + 8316x^9 e^5 a^2 b^4 + 16632x^9 d e^4 a b^5 + 5544x^9 d^2 e^3 b^6 + 12320x^8 e^5 a^3 b^3 + 46200x^8 d e^4 a^2 b^2 + 12320x^8 d^2 e^3 a b^3 + 46200x^7 e^5 a^4 b + 12320x^7 d e^4 a^3 b^2 + 46200x^7 d^2 e^3 a^2 b^3 + 46200x^6 e^5 a^5 + 12320x^6 d e^4 a^4 b + 46200x^6 d^2 e^3 a^3 b^2 + 46200x^5 e^5 a^6 + 12320x^5 d e^4 a^5 + 46200x^5 d^2 e^3 a^4 b + 46200x^4 e^5 a^7 + 12320x^4 d e^4 a^6 + 46200x^4 d^2 e^3 a^5 b + 46200x^3 e^5 a^8 + 12320x^3 d e^4 a^7 + 46200x^3 d^2 e^3 a^6 b + 46200x^2 e^5 a^9 + 12320x^2 d e^4 a^8 + 46200x^2 d^2 e^3 a^7 b + 46200x e^5 a^{10} + 12320x d e^4 a^9 + 46200x d^2 e^3 a^8 b + 46200e^5 a^{12} + 12320d e^4 a^{11} + 46200d^2 e^3 a^{10} b)$ |
| orering | $x(462e^5 b^6 x^{11} + 3024x^{10} e^5 a b^5 + 2520x^{10} d e^4 b^6 + 8316x^9 e^5 a^2 b^4 + 16632x^9 d e^4 a b^5 + 5544x^9 d^2 e^3 b^6 + 12320x^8 e^5 a^3 b^3 + 46200x^8 d e^4 a^2 b^2 + 12320x^8 d^2 e^3 a b^3 + 46200x^7 e^5 a^4 b + 12320x^7 d e^4 a^3 b^2 + 46200x^7 d^2 e^3 a^2 b^3 + 46200x^6 e^5 a^5 + 12320x^6 d e^4 a^4 b + 46200x^6 d^2 e^3 a^3 b^2 + 46200x^5 e^5 a^6 + 12320x^5 d e^4 a^5 + 46200x^5 d^2 e^3 a^4 b + 46200x^4 e^5 a^7 + 12320x^4 d e^4 a^6 + 46200x^4 d^2 e^3 a^5 b + 46200x^3 e^5 a^8 + 12320x^3 d e^4 a^7 + 46200x^3 d^2 e^3 a^6 b + 46200x^2 e^5 a^9 + 12320x^2 d e^4 a^8 + 46200x^2 d^2 e^3 a^7 b + 46200x e^5 a^{10} + 12320x d e^4 a^9 + 46200x d^2 e^3 a^8 b + 46200e^5 a^{12} + 12320d e^4 a^{11} + 46200d^2 e^3 a^{10} b)$ |

input $\text{int}((e*x+d)^5*(b^2*x^2+2*a*b*x+a^2)^3, x, \text{method}=_RETURNVERBOSE)$

output

```

1/12*e^5*b^6*x^12+(6/11*e^5*a*b^5+5/11*d*e^4*b^6)*x^11+(3/2*e^5*a^2*b^4+3*
d*e^4*a*b^5+d^2*e^3*b^6)*x^10+(20/9*e^5*a^3*b^3+25/3*a^2*b^4*d*e^4+20/3*d^
2*e^3*a*b^5+10/9*d^3*e^2*b^6)*x^9+(15/8*e^5*a^4*b^2+25/2*d*e^4*a^3*b^3+75/
4*d^2*e^3*a^2*b^4+15/2*d^3*e^2*a*b^5+5/8*d^4*e*b^6)*x^8+(6/7*a^5*b*e^5+75/
7*d*e^4*a^4*b^2+200/7*d^2*e^3*a^3*b^3+150/7*d^3*e^2*a^2*b^4+30/7*d^4*e*a*b
^5+1/7*d^5*b^6)*x^7+(1/6*e^5*a^6+5*d*e^4*a^5*b+25*d^2*e^3*a^4*b^2+100/3*d^
3*e^2*a^3*b^3+25/2*d^4*e*a^2*b^4+d^5*a*b^5)*x^6+(a^6*d*e^4+12*a^5*b*d^2*e^
3+30*a^4*b^2*d^3*e^2+20*a^3*b^3*d^4*e+3*a^2*b^4*d^5)*x^5+(5/2*d^2*e^3*a^6+
15*d^3*e^2*a^5*b+75/4*d^4*e*a^4*b^2+5*d^5*a^3*b^3)*x^4+(10/3*d^3*e^2*a^6+1
0*d^4*e*a^5*b+5*d^5*a^4*b^2)*x^3+(5/2*d^4*e*a^6+3*d^5*a^5*b)*x^2+d^5*a^6*x

```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 517 vs. $2(133) = 266$.

Time = 0.08 (sec) , antiderivative size = 517, normalized size of antiderivative = 3.62

$$\begin{aligned}
& \int (d + ex)^5 (a^2 + 2abx + b^2x^2)^3 dx \\
&= \frac{1}{12} b^6 e^5 x^{12} + a^6 d^5 x + \frac{1}{11} (5 b^6 d e^4 + 6 a b^5 e^5) x^{11} + \frac{1}{2} (2 b^6 d^2 e^3 + 6 a b^5 d e^4 + 3 a^2 b^4 e^5) x^{10} \\
&+ \frac{5}{9} (2 b^6 d^3 e^2 + 12 a b^5 d^2 e^3 + 15 a^2 b^4 d e^4 + 4 a^3 b^3 e^5) x^9 \\
&+ \frac{5}{8} (b^6 d^4 e + 12 a b^5 d^3 e^2 + 30 a^2 b^4 d^2 e^3 + 20 a^3 b^3 d e^4 + 3 a^4 b^2 e^5) x^8 \\
&+ \frac{1}{7} (b^6 d^5 + 30 a b^5 d^4 e + 150 a^2 b^4 d^3 e^2 + 200 a^3 b^3 d^2 e^3 + 75 a^4 b^2 d e^4 + 6 a^5 b e^5) x^7 \\
&+ \frac{1}{6} (6 a b^5 d^5 + 75 a^2 b^4 d^4 e + 200 a^3 b^3 d^3 e^2 + 150 a^4 b^2 d^2 e^3 + 30 a^5 b d e^4 + a^6 e^5) x^6 \\
&+ (3 a^2 b^4 d^5 + 20 a^3 b^3 d^4 e + 30 a^4 b^2 d^3 e^2 + 12 a^5 b d^2 e^3 + a^6 d e^4) x^5 \\
&+ \frac{5}{4} (4 a^3 b^3 d^5 + 15 a^4 b^2 d^4 e + 12 a^5 b d^3 e^2 + 2 a^6 d^2 e^3) x^4 \\
&+ \frac{5}{3} (3 a^4 b^2 d^5 + 6 a^5 b d^4 e + 2 a^6 d^3 e^2) x^3 + \frac{1}{2} (6 a^5 b d^5 + 5 a^6 d^4 e) x^2
\end{aligned}$$

input

```

integrate((e*x+d)^5*(b^2*x^2+2*a*b*x+a^2)^3,x, algorithm="fricas")

```

output

```

1/12*b^6*e^5*x^12 + a^6*d^5*x + 1/11*(5*b^6*d*e^4 + 6*a*b^5*e^5)*x^11 + 1/
2*(2*b^6*d^2*e^3 + 6*a*b^5*d*e^4 + 3*a^2*b^4*e^5)*x^10 + 5/9*(2*b^6*d^3*e^
2 + 12*a*b^5*d^2*e^3 + 15*a^2*b^4*d*e^4 + 4*a^3*b^3*e^5)*x^9 + 5/8*(b^6*d^
4*e + 12*a*b^5*d^3*e^2 + 30*a^2*b^4*d^2*e^3 + 20*a^3*b^3*d*e^4 + 3*a^4*b^2
*e^5)*x^8 + 1/7*(b^6*d^5 + 30*a*b^5*d^4*e + 150*a^2*b^4*d^3*e^2 + 200*a^3*
b^3*d^2*e^3 + 75*a^4*b^2*d*e^4 + 6*a^5*b*e^5)*x^7 + 1/6*(6*a*b^5*d^5 + 75*
a^2*b^4*d^4*e + 200*a^3*b^3*d^3*e^2 + 150*a^4*b^2*d^2*e^3 + 30*a^5*b*d*e^4
+ a^6*e^5)*x^6 + (3*a^2*b^4*d^5 + 20*a^3*b^3*d^4*e + 30*a^4*b^2*d^3*e^2 +
12*a^5*b*d^2*e^3 + a^6*d*e^4)*x^5 + 5/4*(4*a^3*b^3*d^5 + 15*a^4*b^2*d^4*e
+ 12*a^5*b*d^3*e^2 + 2*a^6*d^2*e^3)*x^4 + 5/3*(3*a^4*b^2*d^5 + 6*a^5*b*d^
4*e + 2*a^6*d^3*e^2)*x^3 + 1/2*(6*a^5*b*d^5 + 5*a^6*d^4*e)*x^2

```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 580 vs. $2(129) = 258$.

Time = 0.05 (sec) , antiderivative size = 580, normalized size of antiderivative = 4.06

$$\begin{aligned}
 \int (d + ex)^5 (a^2 + 2abx + b^2x^2)^3 dx = & a^6 d^5 x + \frac{b^6 e^5 x^{12}}{12} + x^{11} \cdot \left(\frac{6ab^5 e^5}{11} + \frac{5b^6 d e^4}{11} \right) \\
 & + x^{10} \cdot \left(\frac{3a^2 b^4 e^5}{2} + 3ab^5 d e^4 + b^6 d^2 e^3 \right) + x^9 \\
 & \cdot \left(\frac{20a^3 b^3 e^5}{9} + \frac{25a^2 b^4 d e^4}{3} + \frac{20ab^5 d^2 e^3}{3} + \frac{10b^6 d^3 e^2}{9} \right) \\
 & + x^8 \cdot \left(\frac{15a^4 b^2 e^5}{8} + \frac{25a^3 b^3 d e^4}{2} + \frac{75a^2 b^4 d^2 e^3}{4} \right. \\
 & \quad \left. + \frac{15ab^5 d^3 e^2}{2} + \frac{5b^6 d^4 e}{8} \right) \\
 & + x^7 \cdot \left(\frac{6a^5 b e^5}{7} + \frac{75a^4 b^2 d e^4}{7} + \frac{200a^3 b^3 d^2 e^3}{7} \right. \\
 & \quad \left. + \frac{150a^2 b^4 d^3 e^2}{7} + \frac{30ab^5 d^4 e}{7} + \frac{b^6 d^5}{7} \right) \\
 & + x^6 \left(\frac{a^6 e^5}{6} + 5a^5 b d e^4 + 25a^4 b^2 d^2 e^3 + \frac{100a^3 b^3 d^3 e^2}{3} \right. \\
 & \quad \left. + \frac{25a^2 b^4 d^4 e}{2} + ab^5 d^5 \right) + x^5 (a^6 d e^4 + 12a^5 b d^2 e^3 \\
 & \quad + 30a^4 b^2 d^3 e^2 + 20a^3 b^3 d^4 e + 3a^2 b^4 d^5) + x^4 \\
 & \cdot \left(\frac{5a^6 d^2 e^3}{2} + 15a^5 b d^3 e^2 + \frac{75a^4 b^2 d^4 e}{4} + 5a^3 b^3 d^5 \right) \\
 & + x^3 \cdot \left(\frac{10a^6 d^3 e^2}{3} + 10a^5 b d^4 e + 5a^4 b^2 d^5 \right) \\
 & + x^2 \cdot \left(\frac{5a^6 d^4 e}{2} + 3a^5 b d^5 \right)
 \end{aligned}$$

input `integrate((e*x+d)**5*(b**2*x**2+2*a*b*x+a**2)**3,x)`

output

```

a**6*d**5*x + b**6*e**5*x**12/12 + x**11*(6*a*b**5*e**5/11 + 5*b**6*d*e**4
/11) + x**10*(3*a**2*b**4*e**5/2 + 3*a*b**5*d*e**4 + b**6*d**2*e**3) + x**
9*(20*a**3*b**3*e**5/9 + 25*a**2*b**4*d*e**4/3 + 20*a*b**5*d**2*e**3/3 + 1
0*b**6*d**3*e**2/9) + x**8*(15*a**4*b**2*e**5/8 + 25*a**3*b**3*d*e**4/2 +
75*a**2*b**4*d**2*e**3/4 + 15*a*b**5*d**3*e**2/2 + 5*b**6*d**4*e/8) + x**7
*(6*a**5*b*e**5/7 + 75*a**4*b**2*d*e**4/7 + 200*a**3*b**3*d**2*e**3/7 + 15
0*a**2*b**4*d**3*e**2/7 + 30*a*b**5*d**4*e/7 + b**6*d**5/7) + x**6*(a**6*e
**5/6 + 5*a**5*b*d*e**4 + 25*a**4*b**2*d**2*e**3 + 100*a**3*b**3*d**3*e**2
/3 + 25*a**2*b**4*d**4*e/2 + a*b**5*d**5) + x**5*(a**6*d*e**4 + 12*a**5*b*
d**2*e**3 + 30*a**4*b**2*d**3*e**2 + 20*a**3*b**3*d**4*e + 3*a**2*b**4*d**
5) + x**4*(5*a**6*d**2*e**3/2 + 15*a**5*b*d**3*e**2 + 75*a**4*b**2*d**4*e/
4 + 5*a**3*b**3*d**5) + x**3*(10*a**6*d**3*e**2/3 + 10*a**5*b*d**4*e + 5*a
**4*b**2*d**5) + x**2*(5*a**6*d**4*e/2 + 3*a**5*b*d**5)

```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 517 vs. $2(133) = 266$.

Time = 0.05 (sec) , antiderivative size = 517, normalized size of antiderivative = 3.62

$$\begin{aligned}
& \int (d + ex)^5 (a^2 + 2abx + b^2x^2)^3 dx \\
&= \frac{1}{12} b^6 e^5 x^{12} + a^6 d^5 x + \frac{1}{11} (5b^6 d e^4 + 6ab^5 e^5) x^{11} + \frac{1}{2} (2b^6 d^2 e^3 + 6ab^5 d e^4 + 3a^2 b^4 e^5) x^{10} \\
&+ \frac{5}{9} (2b^6 d^3 e^2 + 12ab^5 d^2 e^3 + 15a^2 b^4 d e^4 + 4a^3 b^3 e^5) x^9 \\
&+ \frac{5}{8} (b^6 d^4 e + 12ab^5 d^3 e^2 + 30a^2 b^4 d^2 e^3 + 20a^3 b^3 d e^4 + 3a^4 b^2 e^5) x^8 \\
&+ \frac{1}{7} (b^6 d^5 + 30ab^5 d^4 e + 150a^2 b^4 d^3 e^2 + 200a^3 b^3 d^2 e^3 + 75a^4 b^2 d e^4 + 6a^5 b e^5) x^7 \\
&+ \frac{1}{6} (6ab^5 d^5 + 75a^2 b^4 d^4 e + 200a^3 b^3 d^3 e^2 + 150a^4 b^2 d^2 e^3 + 30a^5 b d e^4 + a^6 e^5) x^6 \\
&+ (3a^2 b^4 d^5 + 20a^3 b^3 d^4 e + 30a^4 b^2 d^3 e^2 + 12a^5 b d^2 e^3 + a^6 d e^4) x^5 \\
&+ \frac{5}{4} (4a^3 b^3 d^5 + 15a^4 b^2 d^4 e + 12a^5 b d^3 e^2 + 2a^6 d^2 e^3) x^4 \\
&+ \frac{5}{3} (3a^4 b^2 d^5 + 6a^5 b d^4 e + 2a^6 d^3 e^2) x^3 + \frac{1}{2} (6a^5 b d^5 + 5a^6 d^4 e) x^2
\end{aligned}$$

input

```

integrate((e*x+d)^5*(b^2*x^2+2*a*b*x+a^2)^3,x, algorithm="maxima")

```

output

$$\begin{aligned}
& 1/12*b^6*e^5*x^{12} + a^6*d^5*x + 1/11*(5*b^6*d*e^4 + 6*a*b^5*e^5)*x^{11} + 1/ \\
& 2*(2*b^6*d^2*e^3 + 6*a*b^5*d*e^4 + 3*a^2*b^4*e^5)*x^{10} + 5/9*(2*b^6*d^3*e^ \\
& 2 + 12*a*b^5*d^2*e^3 + 15*a^2*b^4*d*e^4 + 4*a^3*b^3*e^5)*x^9 + 5/8*(b^6*d^ \\
& 4*e + 12*a*b^5*d^3*e^2 + 30*a^2*b^4*d^2*e^3 + 20*a^3*b^3*d*e^4 + 3*a^4*b^2 \\
& *e^5)*x^8 + 1/7*(b^6*d^5 + 30*a*b^5*d^4*e + 150*a^2*b^4*d^3*e^2 + 200*a^3* \\
& b^3*d^2*e^3 + 75*a^4*b^2*d*e^4 + 6*a^5*b*e^5)*x^7 + 1/6*(6*a*b^5*d^5 + 75* \\
& a^2*b^4*d^4*e + 200*a^3*b^3*d^3*e^2 + 150*a^4*b^2*d^2*e^3 + 30*a^5*b*d*e^4 \\
& + a^6*e^5)*x^6 + (3*a^2*b^4*d^5 + 20*a^3*b^3*d^4*e + 30*a^4*b^2*d^3*e^2 + \\
& 12*a^5*b*d^2*e^3 + a^6*d*e^4)*x^5 + 5/4*(4*a^3*b^3*d^5 + 15*a^4*b^2*d^4*e \\
& + 12*a^5*b*d^3*e^2 + 2*a^6*d^2*e^3)*x^4 + 5/3*(3*a^4*b^2*d^5 + 6*a^5*b*d^ \\
& 4*e + 2*a^6*d^3*e^2)*x^3 + 1/2*(6*a^5*b*d^5 + 5*a^6*d^4*e)*x^2
\end{aligned}$$

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 579 vs. $2(133) = 266$.

Time = 0.20 (sec) , antiderivative size = 579, normalized size of antiderivative = 4.05

$$\begin{aligned}
 \int (d + ex)^5 (a^2 + 2abx + b^2x^2)^3 dx = & \frac{1}{12} b^6 e^5 x^{12} + \frac{5}{11} b^6 d e^4 x^{11} + \frac{6}{11} a b^5 e^5 x^{11} \\
 & + b^6 d^2 e^3 x^{10} + 3 a b^5 d e^4 x^{10} + \frac{3}{2} a^2 b^4 e^5 x^{10} \\
 & + \frac{10}{9} b^6 d^3 e^2 x^9 + \frac{20}{3} a b^5 d^2 e^3 x^9 + \frac{25}{3} a^2 b^4 d e^4 x^9 \\
 & + \frac{20}{9} a^3 b^3 e^5 x^9 + \frac{5}{8} b^6 d^4 e x^8 + \frac{15}{2} a b^5 d^3 e^2 x^8 \\
 & + \frac{75}{4} a^2 b^4 d^2 e^3 x^8 + \frac{25}{2} a^3 b^3 d e^4 x^8 + \frac{15}{8} a^4 b^2 e^5 x^8 \\
 & + \frac{1}{7} b^6 d^5 x^7 + \frac{30}{7} a b^5 d^4 e x^7 + \frac{150}{7} a^2 b^4 d^3 e^2 x^7 \\
 & + \frac{200}{7} a^3 b^3 d^2 e^3 x^7 + \frac{75}{7} a^4 b^2 d e^4 x^7 + \frac{6}{7} a^5 b e^5 x^7 \\
 & + a b^5 d^5 x^6 + \frac{25}{2} a^2 b^4 d^4 e x^6 + \frac{100}{3} a^3 b^3 d^3 e^2 x^6 \\
 & + 25 a^4 b^2 d^2 e^3 x^6 + 5 a^5 b d e^4 x^6 + \frac{1}{6} a^6 e^5 x^6 \\
 & + 3 a^2 b^4 d^5 x^5 + 20 a^3 b^3 d^4 e x^5 + 30 a^4 b^2 d^3 e^2 x^5 \\
 & + 12 a^5 b d^2 e^3 x^5 + a^6 d e^4 x^5 + 5 a^3 b^3 d^5 x^4 \\
 & + \frac{75}{4} a^4 b^2 d^4 e x^4 + 15 a^5 b d^3 e^2 x^4 + \frac{5}{2} a^6 d^2 e^3 x^4 \\
 & + 5 a^4 b^2 d^5 x^3 + 10 a^5 b d^4 e x^3 + \frac{10}{3} a^6 d^3 e^2 x^3 \\
 & + 3 a^5 b d^5 x^2 + \frac{5}{2} a^6 d^4 e x^2 + a^6 d^5 x
 \end{aligned}$$

input `integrate((e*x+d)^5*(b^2*x^2+2*a*b*x+a^2)^3,x, algorithm="giac")`

output

```

1/12*b^6*e^5*x^12 + 5/11*b^6*d*e^4*x^11 + 6/11*a*b^5*e^5*x^11 + b^6*d^2*e^
3*x^10 + 3*a*b^5*d*e^4*x^10 + 3/2*a^2*b^4*e^5*x^10 + 10/9*b^6*d^3*e^2*x^9
+ 20/3*a*b^5*d^2*e^3*x^9 + 25/3*a^2*b^4*d*e^4*x^9 + 20/9*a^3*b^3*e^5*x^9 +
5/8*b^6*d^4*e*x^8 + 15/2*a*b^5*d^3*e^2*x^8 + 75/4*a^2*b^4*d^2*e^3*x^8 + 2
5/2*a^3*b^3*d*e^4*x^8 + 15/8*a^4*b^2*e^5*x^8 + 1/7*b^6*d^5*x^7 + 30/7*a*b^
5*d^4*e*x^7 + 150/7*a^2*b^4*d^3*e^2*x^7 + 200/7*a^3*b^3*d^2*e^3*x^7 + 75/7
*a^4*b^2*d*e^4*x^7 + 6/7*a^5*b*e^5*x^7 + a*b^5*d^5*x^6 + 25/2*a^2*b^4*d^4*
e*x^6 + 100/3*a^3*b^3*d^3*e^2*x^6 + 25*a^4*b^2*d^2*e^3*x^6 + 5*a^5*b*d*e^4
*x^6 + 1/6*a^6*e^5*x^6 + 3*a^2*b^4*d^5*x^5 + 20*a^3*b^3*d^4*e*x^5 + 30*a^4
*b^2*d^3*e^2*x^5 + 12*a^5*b*d^2*e^3*x^5 + a^6*d*e^4*x^5 + 5*a^3*b^3*d^5*x^
4 + 75/4*a^4*b^2*d^4*e*x^4 + 15*a^5*b*d^3*e^2*x^4 + 5/2*a^6*d^2*e^3*x^4 +
5*a^4*b^2*d^5*x^3 + 10*a^5*b*d^4*e*x^3 + 10/3*a^6*d^3*e^2*x^3 + 3*a^5*b*d^
5*x^2 + 5/2*a^6*d^4*e*x^2 + a^6*d^5*x

```

Mupad [B] (verification not implemented)

Time = 8.94 (sec) , antiderivative size = 492, normalized size of antiderivative = 3.44

$$\begin{aligned}
& \int (d + ex)^5 (a^2 + 2abx + b^2x^2)^3 dx \\
&= x^5 (a^6 d e^4 + 12 a^5 b d^2 e^3 + 30 a^4 b^2 d^3 e^2 + 20 a^3 b^3 d^4 e + 3 a^2 b^4 d^5) \\
&+ x^8 \left(\frac{15 a^4 b^2 e^5}{8} + \frac{25 a^3 b^3 d e^4}{2} + \frac{75 a^2 b^4 d^2 e^3}{4} + \frac{15 a b^5 d^3 e^2}{2} + \frac{5 b^6 d^4 e}{8} \right) \\
&+ x^6 \left(\frac{a^6 e^5}{6} + 5 a^5 b d e^4 + 25 a^4 b^2 d^2 e^3 + \frac{100 a^3 b^3 d^3 e^2}{3} + \frac{25 a^2 b^4 d^4 e}{2} + a b^5 d^5 \right) \\
&+ x^7 \left(\frac{6 a^5 b e^5}{7} + \frac{75 a^4 b^2 d e^4}{7} + \frac{200 a^3 b^3 d^2 e^3}{7} + \frac{150 a^2 b^4 d^3 e^2}{7} + \frac{30 a b^5 d^4 e}{7} + \frac{b^6 d^5}{7} \right) \\
&+ a^6 d^5 x + \frac{b^6 e^5 x^{12}}{12} + \frac{5 a^3 d^2 x^4 (2 a^3 e^3 + 12 a^2 b d e^2 + 15 a b^2 d^2 e + 4 b^3 d^3)}{4} \\
&+ \frac{5 b^3 e^2 x^9 (4 a^3 e^3 + 15 a^2 b d e^2 + 12 a b^2 d^2 e + 2 b^3 d^3)}{9} \\
&+ \frac{a^5 d^4 x^2 (5 a e + 6 b d)}{2} + \frac{b^5 e^4 x^{11} (6 a e + 5 b d)}{11} \\
&+ \frac{5 a^4 d^3 x^3 (2 a^2 e^2 + 6 a b d e + 3 b^2 d^2)}{3} + \frac{b^4 e^3 x^{10} (3 a^2 e^2 + 6 a b d e + 2 b^2 d^2)}{2}
\end{aligned}$$

input

```
int((d + e*x)^5*(a^2 + b^2*x^2 + 2*a*b*x)^3,x)
```

output

```
x^5*(a^6*d*e^4 + 3*a^2*b^4*d^5 + 20*a^3*b^3*d^4*e + 12*a^5*b*d^2*e^3 + 30*
a^4*b^2*d^3*e^2) + x^8*((5*b^6*d^4*e)/8 + (15*a^4*b^2*e^5)/8 + (15*a*b^5*d
^3*e^2)/2 + (25*a^3*b^3*d*e^4)/2 + (75*a^2*b^4*d^2*e^3)/4) + x^6*((a^6*e^5
)/6 + a*b^5*d^5 + (25*a^2*b^4*d^4*e)/2 + (100*a^3*b^3*d^3*e^2)/3 + 25*a^4*
b^2*d^2*e^3 + 5*a^5*b*d*e^4) + x^7*((b^6*d^5)/7 + (6*a^5*b*e^5)/7 + (75*a^
4*b^2*d*e^4)/7 + (150*a^2*b^4*d^3*e^2)/7 + (200*a^3*b^3*d^2*e^3)/7 + (30*a
*b^5*d^4*e)/7) + a^6*d^5*x + (b^6*e^5*x^12)/12 + (5*a^3*d^2*x^4*(2*a^3*e^3
+ 4*b^3*d^3 + 15*a*b^2*d^2*e + 12*a^2*b*d*e^2))/4 + (5*b^3*e^2*x^9*(4*a^3
*e^3 + 2*b^3*d^3 + 12*a*b^2*d^2*e + 15*a^2*b*d*e^2))/9 + (a^5*d^4*x^2*(5*a
*e + 6*b*d))/2 + (b^5*e^4*x^11*(6*a*e + 5*b*d))/11 + (5*a^4*d^3*x^3*(2*a^2
*e^2 + 3*b^2*d^2 + 6*a*b*d*e))/3 + (b^4*e^3*x^10*(3*a^2*e^2 + 2*b^2*d^2 +
6*a*b*d*e))/2
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 581, normalized size of antiderivative = 4.06

$$\int (d + ex)^5 (a^2 + 2abx + b^2x^2)^3 dx$$

$$= \frac{x(462b^6e^5x^{11} + 3024ab^5e^5x^{10} + 2520b^6de^4x^{10} + 8316a^2b^4e^5x^9 + 16632ab^5de^4x^9 + 5544b^6d^2e^3x^9 + 1232a^2b^4d^2e^3x^8 + 16632ab^5de^3x^8 + 5544b^6d^2e^2x^8 + 1232a^2b^4d^2e^2x^7 + 16632ab^5de^2x^7 + 5544b^6d^2e^2x^7 + 1232a^2b^4d^2e^2x^6 + 16632ab^5de^2x^6 + 5544b^6d^2e^2x^6 + 1232a^2b^4d^2e^2x^5 + 16632ab^5de^2x^5 + 5544b^6d^2e^2x^5 + 1232a^2b^4d^2e^2x^4 + 16632ab^5de^2x^4 + 5544b^6d^2e^2x^4 + 1232a^2b^4d^2e^2x^3 + 16632ab^5de^2x^3 + 5544b^6d^2e^2x^3 + 1232a^2b^4d^2e^2x^2 + 16632ab^5de^2x^2 + 5544b^6d^2e^2x^2 + 1232a^2b^4d^2e^2x + 16632ab^5de^2x + 5544b^6d^2e^2x + 1232a^2b^4d^2e^2)}{12}$$

input

```
int((e*x+d)^5*(b^2*x^2+2*a*b*x+a^2)^3,x)
```

output

```
(x*(5544*a**6*d**5 + 13860*a**6*d**4*e*x + 18480*a**6*d**3*e**2*x**2 + 13860*a**6*d**2*e**3*x**3 + 5544*a**6*d*e**4*x**4 + 924*a**6*e**5*x**5 + 16632*a**5*b*d**5*x + 55440*a**5*b*d**4*e*x**2 + 83160*a**5*b*d**3*e**2*x**3 + 66528*a**5*b*d**2*e**3*x**4 + 27720*a**5*b*d*e**4*x**5 + 4752*a**5*b*e**5*x**6 + 27720*a**4*b**2*d**5*x**2 + 103950*a**4*b**2*d**4*e*x**3 + 166320*a**4*b**2*d**3*e**2*x**4 + 138600*a**4*b**2*d**2*e**3*x**5 + 59400*a**4*b**2*d*e**4*x**6 + 10395*a**4*b**2*e**5*x**7 + 27720*a**3*b**3*d**5*x**3 + 110880*a**3*b**3*d**4*e*x**4 + 184800*a**3*b**3*d**3*e**2*x**5 + 158400*a**3*b**3*d**2*e**3*x**6 + 69300*a**3*b**3*d*e**4*x**7 + 12320*a**3*b**3*e**5*x**8 + 16632*a**2*b**4*d**5*x**4 + 69300*a**2*b**4*d**4*e*x**5 + 118800*a**2*b**4*d**3*e**2*x**6 + 103950*a**2*b**4*d**2*e**3*x**7 + 46200*a**2*b**4*d*e**4*x**8 + 8316*a**2*b**4*e**5*x**9 + 5544*a*b**5*d**5*x**5 + 23760*a*b**5*d**4*e*x**6 + 41580*a*b**5*d**3*e**2*x**7 + 36960*a*b**5*d**2*e**3*x**8 + 16632*a*b**5*d*e**4*x**9 + 3024*a*b**5*e**5*x**10 + 792*b**6*d**5*x**6 + 3465*b**6*d**4*e*x**7 + 6160*b**6*d**3*e**2*x**8 + 5544*b**6*d**2*e**3*x**9 + 2520*b**6*d*e**4*x**10 + 462*b**6*e**5*x**11))/5544
```

3.88 $\int (d + ex)^4 (a^2 + 2abx + b^2x^2)^3 dx$

| | |
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| Optimal result | 722 |
| Mathematica [B] (verified) | 722 |
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Optimal result

Integrand size = 26, antiderivative size = 119

$$\int (d + ex)^4 (a^2 + 2abx + b^2x^2)^3 dx = \frac{(bd - ae)^4(a + bx)^7}{7b^5} + \frac{e(bd - ae)^3(a + bx)^8}{2b^5} + \frac{2e^2(bd - ae)^2(a + bx)^9}{3b^5} + \frac{2e^3(bd - ae)(a + bx)^{10}}{5b^5} + \frac{e^4(a + bx)^{11}}{11b^5}$$

```
output 1/7*(-a*e+b*d)^4*(b*x+a)^7/b^5+1/2*e*(-a*e+b*d)^3*(b*x+a)^8/b^5+2/3*e^2*(-a*e+b*d)^2*(b*x+a)^9/b^5+2/5*e^3*(-a*e+b*d)*(b*x+a)^10/b^5+1/11*e^4*(b*x+a)^11/b^5
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 398 vs. 2(119) = 238.

Time = 0.04 (sec) , antiderivative size = 398, normalized size of antiderivative = 3.34

$$\begin{aligned}
 \int (d + ex)^4 (a^2 + 2abx + b^2x^2)^3 dx = & a^6d^4x + a^5d^3(3bd + 2ae)x^2 \\
 & + a^4d^2(5b^2d^2 + 8abde + 2a^2e^2)x^3 \\
 & + a^3d(5b^3d^3 + 15ab^2d^2e + 9a^2bde^2 + a^3e^3)x^4 \\
 & + \frac{1}{5}a^2(15b^4d^4 + 80ab^3d^3e + 90a^2b^2d^2e^2 \\
 & \quad + 24a^3bde^3 + a^4e^4)x^5 + ab(b^4d^4 + 10ab^3d^3e \\
 & \quad + 20a^2b^2d^2e^2 + 10a^3bde^3 + a^4e^4)x^6 + \frac{1}{7}b^2(b^4d^4 \\
 & \quad + 24ab^3d^3e + 90a^2b^2d^2e^2 + 80a^3bde^3 + 15a^4e^4)x^7 \\
 & + \frac{1}{2}b^3e(b^3d^3 + 9ab^2d^2e + 15a^2bde^2 + 5a^3e^3)x^8 \\
 & + \frac{1}{3}b^4e^2(2b^2d^2 + 8abde + 5a^2e^2)x^9 \\
 & + \frac{1}{5}b^5e^3(2bd + 3ae)x^{10} + \frac{1}{11}b^6e^4x^{11}
 \end{aligned}$$

input `Integrate[(d + e*x)^4*(a^2 + 2*a*b*x + b^2*x^2)^3,x]`

output `a^6*d^4*x + a^5*d^3*(3*b*d + 2*a*e)*x^2 + a^4*d^2*(5*b^2*d^2 + 8*a*b*d*e + 2*a^2*e^2)*x^3 + a^3*d*(5*b^3*d^3 + 15*a*b^2*d^2*e + 9*a^2*b*d*e^2 + a^3*e^3)*x^4 + (a^2*(15*b^4*d^4 + 80*a*b^3*d^3*e + 90*a^2*b^2*d^2*e^2 + 24*a^3*b*d*e^3 + a^4*e^4)*x^5)/5 + a*b*(b^4*d^4 + 10*a*b^3*d^3*e + 20*a^2*b^2*d^2*e^2 + 10*a^3*b*d*e^3 + a^4*e^4)*x^6 + (b^2*(b^4*d^4 + 24*a*b^3*d^3*e + 90*a^2*b^2*d^2*e^2 + 80*a^3*b*d*e^3 + 15*a^4*e^4)*x^7)/7 + (b^3*e*(b^3*d^3 + 9*a*b^2*d^2*e + 15*a^2*b*d*e^2 + 5*a^3*e^3)*x^8)/2 + (b^4*e^2*(2*b^2*d^2 + 8*a*b*d*e + 5*a^2*e^2)*x^9)/3 + (b^5*e^3*(2*b*d + 3*a*e)*x^10)/5 + (b^6*e^4*x^11)/11`

Rubi [A] (verified)

Time = 0.72 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {1098, 27, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a^2 + 2abx + b^2x^2)^3 (d + ex)^4 dx$$

$$\downarrow 1098$$

$$\frac{\int b^6 (a + bx)^6 (d + ex)^4 dx}{b^6}$$

$$\downarrow 27$$

$$\int (a + bx)^6 (d + ex)^4 dx$$

$$\downarrow 49$$

$$\int \left(\frac{4e^3 (a + bx)^9 (bd - ae)}{b^4} + \frac{6e^2 (a + bx)^8 (bd - ae)^2}{b^4} + \frac{4e (a + bx)^7 (bd - ae)^3}{b^4} + \frac{(a + bx)^6 (bd - ae)^4}{b^4} + \frac{e^4 (a + bx)^5 (bd - ae)^5}{b^4} \right) dx$$

$$\downarrow 2009$$

$$\frac{2e^3 (a + bx)^{10} (bd - ae)}{5b^5} + \frac{2e^2 (a + bx)^9 (bd - ae)^2}{3b^5} + \frac{e (a + bx)^8 (bd - ae)^3}{2b^5} + \frac{(a + bx)^7 (bd - ae)^4}{7b^5} + \frac{e^4 (a + bx)^{11}}{11b^5}$$

input `Int[(d + e*x)^4*(a^2 + 2*a*b*x + b^2*x^2)^3,x]`

output `((b*d - a*e)^4*(a + b*x)^7)/(7*b^5) + (e*(b*d - a*e)^3*(a + b*x)^8)/(2*b^5) + (2*e^2*(b*d - a*e)^2*(a + b*x)^9)/(3*b^5) + (2*e^3*(b*d - a*e)*(a + b*x)^10)/(5*b^5) + (e^4*(a + b*x)^11)/(11*b^5)`

output

```
1/11*e^4*b^6*x^11+(3/5*e^4*a*b^5+2/5*d*e^3*b^6)*x^10+(5/3*e^4*a^2*b^4+8/3*
d*e^3*a*b^5+2/3*d^2*e^2*b^6)*x^9+(5/2*e^4*a^3*b^3+15/2*d*e^3*a^2*b^4+9/2*d
^2*e^2*a*b^5+1/2*d^3*e*b^6)*x^8+(15/7*e^4*a^4*b^2+80/7*d*e^3*a^3*b^3+90/7*
d^2*e^2*a^2*b^4+24/7*d^3*e*a*b^5+1/7*d^4*b^6)*x^7+(a^5*b*e^4+10*a^4*b^2*d*
e^3+20*a^3*b^3*d^2*e^2+10*a^2*b^4*d^3*e+a*b^5*d^4)*x^6+(1/5*e^4*a^6+24/5*d
*e^3*a^5*b+18*d^2*e^2*a^4*b^2+16*d^3*e*a^3*b^3+3*a^2*b^4*d^4)*x^5+(a^6*d*e
^3+9*a^5*b*d^2*e^2+15*a^4*b^2*d^3*e+5*a^3*b^3*d^4)*x^4+(2*a^6*d^2*e^2+8*a^
5*b*d^3*e+5*a^4*b^2*d^4)*x^3+(2*a^6*d^3*e+3*a^5*b*d^4)*x^2+d^4*a^6*x
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 418 vs. $2(109) = 218$.

Time = 0.08 (sec) , antiderivative size = 418, normalized size of antiderivative = 3.51

$$\int (d+ex)^4 (a^2+2abx+b^2x^2)^3 dx$$

$$= \frac{1}{11} b^6 e^4 x^{11} + a^6 d^4 x + \frac{1}{5} (2b^6 d e^3 + 3ab^5 e^4) x^{10} + \frac{1}{3} (2b^6 d^2 e^2 + 8ab^5 d e^3 + 5a^2 b^4 e^4) x^9$$

$$+ \frac{1}{2} (b^6 d^3 e + 9ab^5 d^2 e^2 + 15a^2 b^4 d e^3 + 5a^3 b^3 e^4) x^8$$

$$+ \frac{1}{7} (b^6 d^4 + 24ab^5 d^3 e + 90a^2 b^4 d^2 e^2 + 80a^3 b^3 d e^3 + 15a^4 b^2 e^4) x^7$$

$$+ (ab^5 d^4 + 10a^2 b^4 d^3 e + 20a^3 b^3 d^2 e^2 + 10a^4 b^2 d e^3 + a^5 b e^4) x^6$$

$$+ \frac{1}{5} (15a^2 b^4 d^4 + 80a^3 b^3 d^3 e + 90a^4 b^2 d^2 e^2 + 24a^5 b d e^3 + a^6 e^4) x^5$$

$$+ (5a^3 b^3 d^4 + 15a^4 b^2 d^3 e + 9a^5 b d^2 e^2 + a^6 d e^3) x^4$$

$$+ (5a^4 b^2 d^4 + 8a^5 b d^3 e + 2a^6 d^2 e^2) x^3 + (3a^5 b d^4 + 2a^6 d^3 e) x^2$$

input

```
integrate((e*x+d)^4*(b^2*x^2+2*a*b*x+a^2)^3,x, algorithm="fricas")
```

output

```

1/11*b^6*e^4*x^11 + a^6*d^4*x + 1/5*(2*b^6*d*e^3 + 3*a*b^5*e^4)*x^10 + 1/3
*(2*b^6*d^2*e^2 + 8*a*b^5*d*e^3 + 5*a^2*b^4*e^4)*x^9 + 1/2*(b^6*d^3*e + 9*
a*b^5*d^2*e^2 + 15*a^2*b^4*d*e^3 + 5*a^3*b^3*e^4)*x^8 + 1/7*(b^6*d^4 + 24*
a*b^5*d^3*e + 90*a^2*b^4*d^2*e^2 + 80*a^3*b^3*d*e^3 + 15*a^4*b^2*e^4)*x^7
+ (a*b^5*d^4 + 10*a^2*b^4*d^3*e + 20*a^3*b^3*d^2*e^2 + 10*a^4*b^2*d*e^3 +
a^5*b*e^4)*x^6 + 1/5*(15*a^2*b^4*d^4 + 80*a^3*b^3*d^3*e + 90*a^4*b^2*d^2*e
^2 + 24*a^5*b*d*e^3 + a^6*e^4)*x^5 + (5*a^3*b^3*d^4 + 15*a^4*b^2*d^3*e + 9
*a^5*b*d^2*e^2 + a^6*d*e^3)*x^4 + (5*a^4*b^2*d^4 + 8*a^5*b*d^3*e + 2*a^6*d
^2*e^2)*x^3 + (3*a^5*b*d^4 + 2*a^6*d^3*e)*x^2

```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 462 vs. $2(105) = 210$.

Time = 0.05 (sec) , antiderivative size = 462, normalized size of antiderivative = 3.88

$$\begin{aligned}
\int (d + ex)^4 (a^2 + 2abx + b^2x^2)^3 dx &= a^6 d^4 x + \frac{b^6 e^4 x^{11}}{11} + x^{10} \cdot \left(\frac{3ab^5 e^4}{5} + \frac{2b^6 d e^3}{5} \right) \\
&+ x^9 \cdot \left(\frac{5a^2 b^4 e^4}{3} + \frac{8ab^5 d e^3}{3} + \frac{2b^6 d^2 e^2}{3} \right) + x^8 \\
&\cdot \left(\frac{5a^3 b^3 e^4}{2} + \frac{15a^2 b^4 d e^3}{2} + \frac{9ab^5 d^2 e^2}{2} + \frac{b^6 d^3 e}{2} \right) \\
&+ x^7 \cdot \left(\frac{15a^4 b^2 e^4}{7} + \frac{80a^3 b^3 d e^3}{7} + \frac{90a^2 b^4 d^2 e^2}{7} \right. \\
&\quad \left. + \frac{24ab^5 d^3 e}{7} + \frac{b^6 d^4}{7} \right) + x^6 (a^5 b e^4 + 10a^4 b^2 d e^3 \\
&\quad + 20a^3 b^3 d^2 e^2 + 10a^2 b^4 d^3 e + ab^5 d^4) + x^5 \left(\frac{a^6 e^4}{5} \right. \\
&\quad \left. + \frac{24a^5 b d e^3}{5} + 18a^4 b^2 d^2 e^2 + 16a^3 b^3 d^3 e + 3a^2 b^4 d^4 \right) \\
&+ x^4 (a^6 d e^3 + 9a^5 b d^2 e^2 + 15a^4 b^2 d^3 e + 5a^3 b^3 d^4) \\
&+ x^3 \cdot (2a^6 d^2 e^2 + 8a^5 b d^3 e + 5a^4 b^2 d^4) \\
&+ x^2 \cdot (2a^6 d^3 e + 3a^5 b d^4)
\end{aligned}$$

input

```

integrate((e*x+d)**4*(b**2*x**2+2*a*b*x+a**2)**3,x)

```

output

```
a**6*d**4*x + b**6*e**4*x**11/11 + x**10*(3*a*b**5*e**4/5 + 2*b**6*d*e**3/5) + x**9*(5*a**2*b**4*e**4/3 + 8*a*b**5*d*e**3/3 + 2*b**6*d**2*e**2/3) + x**8*(5*a**3*b**3*e**4/2 + 15*a**2*b**4*d*e**3/2 + 9*a*b**5*d**2*e**2/2 + b**6*d**3*e/2) + x**7*(15*a**4*b**2*e**4/7 + 80*a**3*b**3*d*e**3/7 + 90*a**2*b**4*d**2*e**2/7 + 24*a*b**5*d**3*e/7 + b**6*d**4/7) + x**6*(a**5*b*e**4 + 10*a**4*b**2*d*e**3 + 20*a**3*b**3*d**2*e**2 + 10*a**2*b**4*d**3*e + a*b**5*d**4) + x**5*(a**6*e**4/5 + 24*a**5*b*d*e**3/5 + 18*a**4*b**2*d**2*e**2 + 16*a**3*b**3*d**3*e + 3*a**2*b**4*d**4) + x**4*(a**6*d*e**3 + 9*a**5*b*d**2*e**2 + 15*a**4*b**2*d**3*e + 5*a**3*b**3*d**4) + x**3*(2*a**6*d**2*e**2 + 8*a**5*b*d**3*e + 5*a**4*b**2*d**4) + x**2*(2*a**6*d**3*e + 3*a**5*b*d**4)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 418 vs. $2(109) = 218$.

Time = 0.04 (sec) , antiderivative size = 418, normalized size of antiderivative = 3.51

$$\int (d + ex)^4 (a^2 + 2abx + b^2x^2)^3 dx$$

$$= \frac{1}{11} b^6 e^4 x^{11} + a^6 d^4 x + \frac{1}{5} (2b^6 d e^3 + 3ab^5 e^4) x^{10} + \frac{1}{3} (2b^6 d^2 e^2 + 8ab^5 d e^3 + 5a^2 b^4 e^4) x^9$$

$$+ \frac{1}{2} (b^6 d^3 e + 9ab^5 d^2 e^2 + 15a^2 b^4 d e^3 + 5a^3 b^3 e^4) x^8$$

$$+ \frac{1}{7} (b^6 d^4 + 24ab^5 d^3 e + 90a^2 b^4 d^2 e^2 + 80a^3 b^3 d e^3 + 15a^4 b^2 e^4) x^7$$

$$+ (ab^5 d^4 + 10a^2 b^4 d^3 e + 20a^3 b^3 d^2 e^2 + 10a^4 b^2 d e^3 + a^5 b e^4) x^6$$

$$+ \frac{1}{5} (15a^2 b^4 d^4 + 80a^3 b^3 d^3 e + 90a^4 b^2 d^2 e^2 + 24a^5 b d e^3 + a^6 e^4) x^5$$

$$+ (5a^3 b^3 d^4 + 15a^4 b^2 d^3 e + 9a^5 b d^2 e^2 + a^6 d e^3) x^4$$

$$+ (5a^4 b^2 d^4 + 8a^5 b d^3 e + 2a^6 d^2 e^2) x^3 + (3a^5 b d^4 + 2a^6 d^3 e) x^2$$

input

```
integrate((e*x+d)^4*(b^2*x^2+2*a*b*x+a^2)^3,x, algorithm="maxima")
```

output

```

1/11*b^6*e^4*x^11 + a^6*d^4*x + 1/5*(2*b^6*d*e^3 + 3*a*b^5*e^4)*x^10 + 1/3
*(2*b^6*d^2*e^2 + 8*a*b^5*d*e^3 + 5*a^2*b^4*e^4)*x^9 + 1/2*(b^6*d^3*e + 9*
a*b^5*d^2*e^2 + 15*a^2*b^4*d*e^3 + 5*a^3*b^3*e^4)*x^8 + 1/7*(b^6*d^4 + 24*
a*b^5*d^3*e + 90*a^2*b^4*d^2*e^2 + 80*a^3*b^3*d*e^3 + 15*a^4*b^2*e^4)*x^7
+ (a*b^5*d^4 + 10*a^2*b^4*d^3*e + 20*a^3*b^3*d^2*e^2 + 10*a^4*b^2*d*e^3 +
a^5*b*e^4)*x^6 + 1/5*(15*a^2*b^4*d^4 + 80*a^3*b^3*d^3*e + 90*a^4*b^2*d^2*e
^2 + 24*a^5*b*d*e^3 + a^6*e^4)*x^5 + (5*a^3*b^3*d^4 + 15*a^4*b^2*d^3*e + 9
*a^5*b*d^2*e^2 + a^6*d*e^3)*x^4 + (5*a^4*b^2*d^4 + 8*a^5*b*d^3*e + 2*a^6*d
^2*e^2)*x^3 + (3*a^5*b*d^4 + 2*a^6*d^3*e)*x^2

```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 470 vs. $2(109) = 218$.

Time = 0.18 (sec) , antiderivative size = 470, normalized size of antiderivative = 3.95

$$\begin{aligned}
\int (d+ex)^4 (a^2+2abx+b^2x^2)^3 dx = & \frac{1}{11} b^6 e^4 x^{11} + \frac{2}{5} b^6 d e^3 x^{10} + \frac{3}{5} a b^5 e^4 x^{10} + \frac{2}{3} b^6 d^2 e^2 x^9 \\
& + \frac{8}{3} a b^5 d e^3 x^9 + \frac{5}{3} a^2 b^4 e^4 x^9 + \frac{1}{2} b^6 d^3 e x^8 \\
& + \frac{9}{2} a b^5 d^2 e^2 x^8 + \frac{15}{2} a^2 b^4 d e^3 x^8 + \frac{5}{2} a^3 b^3 e^4 x^8 \\
& + \frac{1}{7} b^6 d^4 x^7 + \frac{24}{7} a b^5 d^3 e x^7 + \frac{90}{7} a^2 b^4 d^2 e^2 x^7 \\
& + \frac{80}{7} a^3 b^3 d e^3 x^7 + \frac{15}{7} a^4 b^2 e^4 x^7 + a b^5 d^4 x^6 \\
& + 10 a^2 b^4 d^3 e x^6 + 20 a^3 b^3 d^2 e^2 x^6 + 10 a^4 b^2 d e^3 x^6 \\
& + a^5 b e^4 x^6 + 3 a^2 b^4 d^4 x^5 + 16 a^3 b^3 d^3 e x^5 \\
& + 18 a^4 b^2 d^2 e^2 x^5 + \frac{24}{5} a^5 b d e^3 x^5 + \frac{1}{5} a^6 e^4 x^5 \\
& + 5 a^3 b^3 d^4 x^4 + 15 a^4 b^2 d^3 e x^4 + 9 a^5 b d^2 e^2 x^4 \\
& + a^6 d e^3 x^4 + 5 a^4 b^2 d^4 x^3 + 8 a^5 b d^3 e x^3 \\
& + 2 a^6 d^2 e^2 x^3 + 3 a^5 b d^4 x^2 + 2 a^6 d^3 e x^2 + a^6 d^4 x
\end{aligned}$$

input

```

integrate((e*x+d)^4*(b^2*x^2+2*a*b*x+a^2)^3,x, algorithm="giac")

```

output

```

1/11*b^6*e^4*x^11 + 2/5*b^6*d*e^3*x^10 + 3/5*a*b^5*e^4*x^10 + 2/3*b^6*d^2*
e^2*x^9 + 8/3*a*b^5*d*e^3*x^9 + 5/3*a^2*b^4*e^4*x^9 + 1/2*b^6*d^3*e*x^8 +
9/2*a*b^5*d^2*e^2*x^8 + 15/2*a^2*b^4*d*e^3*x^8 + 5/2*a^3*b^3*e^4*x^8 + 1/7
*b^6*d^4*x^7 + 24/7*a*b^5*d^3*e*x^7 + 90/7*a^2*b^4*d^2*e^2*x^7 + 80/7*a^3*
b^3*d*e^3*x^7 + 15/7*a^4*b^2*e^4*x^7 + a*b^5*d^4*x^6 + 10*a^2*b^4*d^3*e*x^
6 + 20*a^3*b^3*d^2*e^2*x^6 + 10*a^4*b^2*d*e^3*x^6 + a^5*b*e^4*x^6 + 3*a^2*
b^4*d^4*x^5 + 16*a^3*b^3*d^3*e*x^5 + 18*a^4*b^2*d^2*e^2*x^5 + 24/5*a^5*b*d
*e^3*x^5 + 1/5*a^6*e^4*x^5 + 5*a^3*b^3*d^4*x^4 + 15*a^4*b^2*d^3*e*x^4 + 9*
a^5*b*d^2*e^2*x^4 + a^6*d*e^3*x^4 + 5*a^4*b^2*d^4*x^3 + 8*a^5*b*d^3*e*x^3
+ 2*a^6*d^2*e^2*x^3 + 3*a^5*b*d^4*x^2 + 2*a^6*d^3*e*x^2 + a^6*d^4*x

```

Mupad [B] (verification not implemented)

Time = 8.92 (sec) , antiderivative size = 402, normalized size of antiderivative = 3.38

$$\begin{aligned}
\int (d+ex)^4 (a^2+2abx+b^2x^2)^3 dx = & x^5 \left(\frac{a^6 e^4}{5} + \frac{24 a^5 b d e^3}{5} + 18 a^4 b^2 d^2 e^2 + 16 a^3 b^3 d^3 e \right. \\
& \left. + 3 a^2 b^4 d^4 \right) \\
& + x^7 \left(\frac{15 a^4 b^2 e^4}{7} + \frac{80 a^3 b^3 d e^3}{7} + \frac{90 a^2 b^4 d^2 e^2}{7} \right. \\
& \left. + \frac{24 a b^5 d^3 e}{7} + \frac{b^6 d^4}{7} \right) + x^4 (a^6 d e^3 + 9 a^5 b d^2 e^2 \\
& + 15 a^4 b^2 d^3 e + 5 a^3 b^3 d^4) + x^8 \left(\frac{5 a^3 b^3 e^4}{2} \right. \\
& \left. + \frac{15 a^2 b^4 d e^3}{2} + \frac{9 a b^5 d^2 e^2}{2} + \frac{b^6 d^3 e}{2} \right) \\
& + x^6 (a^5 b e^4 + 10 a^4 b^2 d e^3 + 20 a^3 b^3 d^2 e^2 \\
& + 10 a^2 b^4 d^3 e + a b^5 d^4) + a^6 d^4 x + \frac{b^6 e^4 x^{11}}{11} \\
& + a^5 d^3 x^2 (2 a e + 3 b d) + \frac{b^5 e^3 x^{10} (3 a e + 2 b d)}{5} \\
& + a^4 d^2 x^3 (2 a^2 e^2 + 8 a b d e + 5 b^2 d^2) \\
& + \frac{b^4 e^2 x^9 (5 a^2 e^2 + 8 a b d e + 2 b^2 d^2)}{3}
\end{aligned}$$

input

```
int((d + e*x)^4*(a^2 + b^2*x^2 + 2*a*b*x)^3,x)
```

output

```
x^5*((a^6*e^4)/5 + 3*a^2*b^4*d^4 + 16*a^3*b^3*d^3*e + 18*a^4*b^2*d^2*e^2 +
(24*a^5*b*d*e^3)/5) + x^7*((b^6*d^4)/7 + (15*a^4*b^2*e^4)/7 + (80*a^3*b^3
*d*e^3)/7 + (90*a^2*b^4*d^2*e^2)/7 + (24*a*b^5*d^3*e)/7) + x^4*(a^6*d*e^3
+ 5*a^3*b^3*d^4 + 15*a^4*b^2*d^3*e + 9*a^5*b*d^2*e^2) + x^8*((b^6*d^3*e)/2
+ (5*a^3*b^3*e^4)/2 + (9*a*b^5*d^2*e^2)/2 + (15*a^2*b^4*d*e^3)/2) + x^6*(
a*b^5*d^4 + a^5*b*e^4 + 10*a^2*b^4*d^3*e + 10*a^4*b^2*d*e^3 + 20*a^3*b^3*d
^2*e^2) + a^6*d^4*x + (b^6*e^4*x^11)/11 + a^5*d^3*x^2*(2*a*e + 3*b*d) + (b
^5*e^3*x^10*(3*a*e + 2*b*d))/5 + a^4*d^2*x^3*(2*a^2*e^2 + 5*b^2*d^2 + 8*a*
b*d*e) + (b^4*e^2*x^9*(5*a^2*e^2 + 2*b^2*d^2 + 8*a*b*d*e))/3
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 472, normalized size of antiderivative = 3.97

$$\int (d + ex)^4 (a^2 + 2abx + b^2x^2)^3 dx$$

$$= \frac{x(210b^6e^4x^{10} + 1386ab^5e^4x^9 + 924b^6de^3x^9 + 3850a^2b^4e^4x^8 + 6160ab^5de^3x^8 + 1540b^6d^2e^2x^8 + 5775a^3b^5e^3x^7 + 2100a^2b^4d^2e^2x^7 + 1386ab^5d^2e^2x^7 + 6160a^2b^4de^3x^6 + 1386ab^5de^3x^6 + 330b^6d^2e^2x^6 + 1155b^6d^2e^2x^6 + 924b^6de^3x^5 + 210b^6e^4x^5)}{2310}$$

input

```
int((e*x+d)^4*(b^2*x^2+2*a*b*x+a^2)^3,x)
```

output

```
(x*(2310*a**6*d**4 + 4620*a**6*d**3*e*x + 4620*a**6*d**2*e**2*x**2 + 2310*
a**6*d*e**3*x**3 + 462*a**6*e**4*x**4 + 6930*a**5*b*d**4*x + 18480*a**5*b*
d**3*e*x**2 + 20790*a**5*b*d**2*e**2*x**3 + 11088*a**5*b*d*e**3*x**4 + 231
0*a**5*b*e**4*x**5 + 11550*a**4*b**2*d**4*x**2 + 34650*a**4*b**2*d**3*e*x*
*3 + 41580*a**4*b**2*d**2*e**2*x**4 + 23100*a**4*b**2*d*e**3*x**5 + 4950*a
**4*b**2*e**4*x**6 + 11550*a**3*b**3*d**4*x**3 + 36960*a**3*b**3*d**3*e*x*
*4 + 46200*a**3*b**3*d**2*e**2*x**5 + 26400*a**3*b**3*d*e**3*x**6 + 5775*a
**3*b**3*e**4*x**7 + 6930*a**2*b**4*d**4*x**4 + 23100*a**2*b**4*d**3*e*x**
5 + 29700*a**2*b**4*d**2*e**2*x**6 + 17325*a**2*b**4*d*e**3*x**7 + 3850*a*
*2*b**4*e**4*x**8 + 2310*a*b**5*d**4*x**5 + 7920*a*b**5*d**3*e*x**6 + 1039
5*a*b**5*d**2*e**2*x**7 + 6160*a*b**5*d*e**3*x**8 + 1386*a*b**5*e**4*x**9
+ 330*b**6*d**4*x**6 + 1155*b**6*d**3*e*x**7 + 1540*b**6*d**2*e**2*x**8 +
924*b**6*d*e**3*x**9 + 210*b**6*e**4*x**10))/2310
```


3.89 $\int (d + ex)^3 (a^2 + 2abx + b^2x^2)^3 dx$

| | |
|---|-----|
| Optimal result | 732 |
| Mathematica [B] (verified) | 733 |
| Rubi [A] (verified) | 733 |
| Maple [B] (verified) | 735 |
| Fricas [B] (verification not implemented) | 735 |
| Sympy [B] (verification not implemented) | 737 |
| Maxima [B] (verification not implemented) | 738 |
| Giac [B] (verification not implemented) | 738 |
| Mupad [B] (verification not implemented) | 740 |
| Reduce [B] (verification not implemented) | 741 |

Optimal result

Integrand size = 26, antiderivative size = 92

$$\int (d + ex)^3 (a^2 + 2abx + b^2x^2)^3 dx = \frac{(bd - ae)^3(a + bx)^7}{7b^4} + \frac{3e(bd - ae)^2(a + bx)^8}{8b^4} + \frac{e^2(bd - ae)(a + bx)^9}{3b^4} + \frac{e^3(a + bx)^{10}}{10b^4}$$

output

```
1/7*(-a*e+b*d)^3*(b*x+a)^7/b^4+3/8*e*(-a*e+b*d)^2*(b*x+a)^8/b^4+1/3*e^2*(-a*e+b*d)*(b*x+a)^9/b^4+1/10*e^3*(b*x+a)^10/b^4
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 276 vs. $2(92) = 184$.

Time = 0.06 (sec) , antiderivative size = 276, normalized size of antiderivative = 3.00

$$\int (d + ex)^3 (a^2 + 2abx + b^2x^2)^3 dx = \frac{1}{840}x(210a^6(4d^3 + 6d^2ex + 4de^2x^2 + e^3x^3) + 252a^5bx(10d^3 + 20d^2ex + 15de^2x^2 + 4e^3x^3) + 210a^4b^2x^2(20d^3 + 45d^2ex + 36de^2x^2 + 10e^3x^3) + 120a^3b^3x^3(35d^3 + 84d^2ex + 70de^2x^2 + 20e^3x^3) + 45a^2b^4x^4(56d^3 + 140d^2ex + 120de^2x^2 + 35e^3x^3) + 10ab^5x^5(84d^3 + 216d^2ex + 189de^2x^2 + 56e^3x^3) + b^6x^6(120d^3 + 315d^2ex + 280de^2x^2 + 84e^3x^3))$$

input

```
Integrate[(d + e*x)^3*(a^2 + 2*a*b*x + b^2*x^2)^3,x]
```

output

```
(x*(210*a^6*(4*d^3 + 6*d^2*e*x + 4*d*e^2*x^2 + e^3*x^3) + 252*a^5*b*x*(10*d^3 + 20*d^2*e*x + 15*d*e^2*x^2 + 4*e^3*x^3) + 210*a^4*b^2*x^2*(20*d^3 + 45*d^2*e*x + 36*d*e^2*x^2 + 10*e^3*x^3) + 120*a^3*b^3*x^3*(35*d^3 + 84*d^2*e*x + 70*d*e^2*x^2 + 20*e^3*x^3) + 45*a^2*b^4*x^4*(56*d^3 + 140*d^2*e*x + 120*d*e^2*x^2 + 35*e^3*x^3) + 10*a*b^5*x^5*(84*d^3 + 216*d^2*e*x + 189*d*e^2*x^2 + 56*e^3*x^3) + b^6*x^6*(120*d^3 + 315*d^2*e*x + 280*d*e^2*x^2 + 84*e^3*x^3)))/840
```

Rubi [A] (verified)

Time = 0.63 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {1098, 27, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a^2 + 2abx + b^2x^2)^3 (d + ex)^3 dx$$

↓ 1098

$$\frac{\int b^6(a+bx)^6(d+ex)^3 dx}{b^6}$$

↓ 27

$$\int (a+bx)^6(d+ex)^3 dx$$

↓ 49

$$\int \left(\frac{3e^2(a+bx)^8(bd-ae)}{b^3} + \frac{3e(a+bx)^7(bd-ae)^2}{b^3} + \frac{(a+bx)^6(bd-ae)^3}{b^3} + \frac{e^3(a+bx)^9}{b^3} \right) dx$$

↓ 2009

$$\frac{e^2(a+bx)^9(bd-ae)}{3b^4} + \frac{3e(a+bx)^8(bd-ae)^2}{8b^4} + \frac{(a+bx)^7(bd-ae)^3}{7b^4} + \frac{e^3(a+bx)^{10}}{10b^4}$$

input `Int[(d + e*x)^3*(a^2 + 2*a*b*x + b^2*x^2)^3,x]`

output `((b*d - a*e)^3*(a + b*x)^7)/(7*b^4) + (3*e*(b*d - a*e)^2*(a + b*x)^8)/(8*b^4) + (e^2*(b*d - a*e)*(a + b*x)^9)/(3*b^4) + (e^3*(a + b*x)^10)/(10*b^4)`

Defintions of rubi rules used

rule 27 `Int[(a_)*(F_x_), x_Symbol] := Simp[a Int[F_x, x], x] /; FreeQ[a, x] && !MatchQ[F_x, (b_)*(G_x_)] /; FreeQ[b, x]`

rule 49 `Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] := Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] && IGtQ[m, 0] && IGtQ[m + n + 2, 0]`

rule 1098 `Int[((d_.) + (e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] := Simp[1/c^p Int[(d + e*x)^m*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]`

rule 2009 `Int[u_, x_Symbol] := Simp[IntSum[u, x], x] /; SumQ[u]`

Time = 0.07 (sec) , antiderivative size = 327, normalized size of antiderivative = 3.55

$$\begin{aligned}
 & \int (d + ex)^3 (a^2 + 2abx + b^2x^2)^3 dx \\
 &= \frac{1}{10} b^6 e^3 x^{10} + a^6 d^3 x + \frac{1}{3} (b^6 d e^2 + 2 a b^5 e^3) x^9 + \frac{3}{8} (b^6 d^2 e + 6 a b^5 d e^2 + 5 a^2 b^4 e^3) x^8 \\
 &+ \frac{1}{7} (b^6 d^3 + 18 a b^5 d^2 e + 45 a^2 b^4 d e^2 + 20 a^3 b^3 e^3) x^7 \\
 &+ \frac{1}{2} (2 a b^5 d^3 + 15 a^2 b^4 d^2 e + 20 a^3 b^3 d e^2 + 5 a^4 b^2 e^3) x^6 \\
 &+ \frac{3}{5} (5 a^2 b^4 d^3 + 20 a^3 b^3 d^2 e + 15 a^4 b^2 d e^2 + 2 a^5 b e^3) x^5 \\
 &+ \frac{1}{4} (20 a^3 b^3 d^3 + 45 a^4 b^2 d^2 e + 18 a^5 b d e^2 + a^6 e^3) x^4 \\
 &+ (5 a^4 b^2 d^3 + 6 a^5 b d^2 e + a^6 d e^2) x^3 + \frac{3}{2} (2 a^5 b d^3 + a^6 d^2 e) x^2
 \end{aligned}$$

input `integrate((e*x+d)^3*(b^2*x^2+2*a*b*x+a^2)^3,x, algorithm="fricas")`

output

```

1/10*b^6*e^3*x^10 + a^6*d^3*x + 1/3*(b^6*d*e^2 + 2*a*b^5*e^3)*x^9 + 3/8*(b
^6*d^2*e + 6*a*b^5*d*e^2 + 5*a^2*b^4*e^3)*x^8 + 1/7*(b^6*d^3 + 18*a*b^5*d
^2*e + 45*a^2*b^4*d*e^2 + 20*a^3*b^3*e^3)*x^7 + 1/2*(2*a*b^5*d^3 + 15*a^2*b
^4*d^2*e + 20*a^3*b^3*d*e^2 + 5*a^4*b^2*e^3)*x^6 + 3/5*(5*a^2*b^4*d^3 + 20
*a^3*b^3*d^2*e + 15*a^4*b^2*d*e^2 + 2*a^5*b*e^3)*x^5 + 1/4*(20*a^3*b^3*d^3
+ 45*a^4*b^2*d^2*e + 18*a^5*b*d*e^2 + a^6*e^3)*x^4 + (5*a^4*b^2*d^3 + 6*a
^5*b*d^2*e + a^6*d*e^2)*x^3 + 3/2*(2*a^5*b*d^3 + a^6*d^2*e)*x^2

```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 364 vs. $2(80) = 160$.

Time = 0.04 (sec) , antiderivative size = 364, normalized size of antiderivative = 3.96

$$\int (d + ex)^3 (a^2 + 2abx + b^2x^2)^3 dx = a^6 d^3 x + \frac{b^6 e^3 x^{10}}{10} + x^9 \cdot \left(\frac{2ab^5 e^3}{3} + \frac{b^6 d e^2}{3} \right) \\ + x^8 \cdot \left(\frac{15a^2 b^4 e^3}{8} + \frac{9ab^5 d e^2}{4} + \frac{3b^6 d^2 e}{8} \right) + x^7 \\ \cdot \left(\frac{20a^3 b^3 e^3}{7} + \frac{45a^2 b^4 d e^2}{7} + \frac{18ab^5 d^2 e}{7} + \frac{b^6 d^3}{7} \right) + x^6 \\ \cdot \left(\frac{5a^4 b^2 e^3}{2} + 10a^3 b^3 d e^2 + \frac{15a^2 b^4 d^2 e}{2} + ab^5 d^3 \right) \\ + x^5 \cdot \left(\frac{6a^5 b e^3}{5} + 9a^4 b^2 d e^2 + 12a^3 b^3 d^2 e + 3a^2 b^4 d^3 \right) \\ + x^4 \left(\frac{a^6 e^3}{4} + \frac{9a^5 b d e^2}{2} + \frac{45a^4 b^2 d^2 e}{4} + 5a^3 b^3 d^3 \right) \\ + x^3 (a^6 d e^2 + 6a^5 b d^2 e + 5a^4 b^2 d^3) \\ + x^2 \cdot \left(\frac{3a^6 d^2 e}{2} + 3a^5 b d^3 \right)$$

input `integrate((e*x+d)**3*(b**2*x**2+2*a*b*x+a**2)**3,x)`

output `a**6*d**3*x + b**6*e**3*x**10/10 + x**9*(2*a*b**5*e**3/3 + b**6*d*e**2/3) + x**8*(15*a**2*b**4*e**3/8 + 9*a*b**5*d*e**2/4 + 3*b**6*d**2*e/8) + x**7*(20*a**3*b**3*e**3/7 + 45*a**2*b**4*d*e**2/7 + 18*a*b**5*d**2*e/7 + b**6*d**3/7) + x**6*(5*a**4*b**2*e**3/2 + 10*a**3*b**3*d*e**2 + 15*a**2*b**4*d**2*e/2 + a*b**5*d**3) + x**5*(6*a**5*b*e**3/5 + 9*a**4*b**2*d*e**2 + 12*a**3*b**3*d**2*e + 3*a**2*b**4*d**3) + x**4*(a**6*e**3/4 + 9*a**5*b*d*e**2/2 + 45*a**4*b**2*d**2*e/4 + 5*a**3*b**3*d**3) + x**3*(a**6*d*e**2 + 6*a**5*b*d**2*e + 5*a**4*b**2*d**3) + x**2*(3*a**6*d**2*e/2 + 3*a**5*b*d**3)`

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 327 vs. $2(84) = 168$.

Time = 0.03 (sec) , antiderivative size = 327, normalized size of antiderivative = 3.55

$$\begin{aligned} & \int (d + ex)^3 (a^2 + 2abx + b^2x^2)^3 dx \\ &= \frac{1}{10} b^6 e^3 x^{10} + a^6 d^3 x + \frac{1}{3} (b^6 d e^2 + 2 a b^5 e^3) x^9 + \frac{3}{8} (b^6 d^2 e + 6 a b^5 d e^2 + 5 a^2 b^4 e^3) x^8 \\ &+ \frac{1}{7} (b^6 d^3 + 18 a b^5 d^2 e + 45 a^2 b^4 d e^2 + 20 a^3 b^3 e^3) x^7 \\ &+ \frac{1}{2} (2 a b^5 d^3 + 15 a^2 b^4 d^2 e + 20 a^3 b^3 d e^2 + 5 a^4 b^2 e^3) x^6 \\ &+ \frac{3}{5} (5 a^2 b^4 d^3 + 20 a^3 b^3 d^2 e + 15 a^4 b^2 d e^2 + 2 a^5 b e^3) x^5 \\ &+ \frac{1}{4} (20 a^3 b^3 d^3 + 45 a^4 b^2 d^2 e + 18 a^5 b d e^2 + a^6 e^3) x^4 \\ &+ (5 a^4 b^2 d^3 + 6 a^5 b d^2 e + a^6 d e^2) x^3 + \frac{3}{2} (2 a^5 b d^3 + a^6 d^2 e) x^2 \end{aligned}$$

input `integrate((e*x+d)^3*(b^2*x^2+2*a*b*x+a^2)^3,x, algorithm="maxima")`

output `1/10*b^6*e^3*x^10 + a^6*d^3*x + 1/3*(b^6*d*e^2 + 2*a*b^5*e^3)*x^9 + 3/8*(b^6*d^2*e + 6*a*b^5*d*e^2 + 5*a^2*b^4*e^3)*x^8 + 1/7*(b^6*d^3 + 18*a*b^5*d^2*e + 45*a^2*b^4*d*e^2 + 20*a^3*b^3*e^3)*x^7 + 1/2*(2*a*b^5*d^3 + 15*a^2*b^4*d^2*e + 20*a^3*b^3*d*e^2 + 5*a^4*b^2*e^3)*x^6 + 3/5*(5*a^2*b^4*d^3 + 20*a^3*b^3*d^2*e + 15*a^4*b^2*d*e^2 + 2*a^5*b*e^3)*x^5 + 1/4*(20*a^3*b^3*d^3 + 45*a^4*b^2*d^2*e + 18*a^5*b*d*e^2 + a^6*e^3)*x^4 + (5*a^4*b^2*d^3 + 6*a^5*b*d^2*e + a^6*d*e^2)*x^3 + 3/2*(2*a^5*b*d^3 + a^6*d^2*e)*x^2`

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 362 vs. $2(84) = 168$.

Time = 0.22 (sec) , antiderivative size = 362, normalized size of antiderivative = 3.93

$$\int (d + ex)^3 (a^2 + 2abx + b^2x^2)^3 dx = \frac{1}{10} b^6 e^3 x^{10} + \frac{1}{3} b^6 d e^2 x^9 + \frac{2}{3} a b^5 e^3 x^9$$

$$+ \frac{3}{8} b^6 d^2 e x^8 + \frac{9}{4} a b^5 d e^2 x^8 + \frac{15}{8} a^2 b^4 e^3 x^8$$

$$+ \frac{1}{7} b^6 d^3 x^7 + \frac{18}{7} a b^5 d^2 e x^7 + \frac{45}{7} a^2 b^4 d e^2 x^7$$

$$+ \frac{20}{7} a^3 b^3 e^3 x^7 + a b^5 d^3 x^6 + \frac{15}{2} a^2 b^4 d^2 e x^6$$

$$+ 10 a^3 b^3 d e^2 x^6 + \frac{5}{2} a^4 b^2 e^3 x^6 + 3 a^2 b^4 d^3 x^5$$

$$+ 12 a^3 b^3 d^2 e x^5 + 9 a^4 b^2 d e^2 x^5 + \frac{6}{5} a^5 b e^3 x^5$$

$$+ 5 a^3 b^3 d^3 x^4 + \frac{45}{4} a^4 b^2 d^2 e x^4 + \frac{9}{2} a^5 b d e^2 x^4$$

$$+ \frac{1}{4} a^6 e^3 x^4 + 5 a^4 b^2 d^3 x^3 + 6 a^5 b d^2 e x^3$$

$$+ a^6 d e^2 x^3 + 3 a^5 b d^3 x^2 + \frac{3}{2} a^6 d^2 e x^2 + a^6 d^3 x$$

input `integrate((e*x+d)^3*(b^2*x^2+2*a*b*x+a^2)^3,x, algorithm="giac")`

output `1/10*b^6*e^3*x^10 + 1/3*b^6*d*e^2*x^9 + 2/3*a*b^5*e^3*x^9 + 3/8*b^6*d^2*e*x^8 + 9/4*a*b^5*d*e^2*x^8 + 15/8*a^2*b^4*e^3*x^8 + 1/7*b^6*d^3*x^7 + 18/7*a*b^5*d^2*e*x^7 + 45/7*a^2*b^4*d*e^2*x^7 + 20/7*a^3*b^3*e^3*x^7 + a*b^5*d^3*x^6 + 15/2*a^2*b^4*d^2*e*x^6 + 10*a^3*b^3*d*e^2*x^6 + 5/2*a^4*b^2*e^3*x^6 + 3*a^2*b^4*d^3*x^5 + 12*a^3*b^3*d^2*e*x^5 + 9*a^4*b^2*d*e^2*x^5 + 6/5*a^5*b*e^3*x^5 + 5*a^3*b^3*d^3*x^4 + 45/4*a^4*b^2*d^2*e*x^4 + 9/2*a^5*b*d*e^2*x^4 + 1/4*a^6*e^3*x^4 + 5*a^4*b^2*d^3*x^3 + 6*a^5*b*d^2*e*x^3 + a^6*d*e^2*x^3 + 3*a^5*b*d^3*x^2 + 3/2*a^6*d^2*e*x^2 + a^6*d^3*x`

Mupad [B] (verification not implemented)

Time = 8.80 (sec) , antiderivative size = 308, normalized size of antiderivative = 3.35

$$\begin{aligned}
\int (d+ex)^3 (a^2+2abx+b^2x^2)^3 dx = & x^5 \left(\frac{6a^5be^3}{5} + 9a^4b^2de^2 + 12a^3b^3d^2e + 3a^2b^4d^3 \right) \\
& + x^6 \left(\frac{5a^4b^2e^3}{2} + 10a^3b^3de^2 + \frac{15a^2b^4d^2e}{2} \right. \\
& \left. + ab^5d^3 \right) + x^4 \left(\frac{a^6e^3}{4} + \frac{9a^5bde^2}{2} + \frac{45a^4b^2d^2e}{4} \right. \\
& \left. + 5a^3b^3d^3 \right) + x^7 \left(\frac{20a^3b^3e^3}{7} + \frac{45a^2b^4de^2}{7} \right. \\
& \left. + \frac{18ab^5d^2e}{7} + \frac{b^6d^3}{7} \right) + a^6d^3x + \frac{b^6e^3x^{10}}{10} \\
& + \frac{3a^5d^2x^2(ae+2bd)}{2} + \frac{b^5e^2x^9(2ae+bd)}{3} \\
& + a^4dx^3(a^2e^2+6abde+5b^2d^2) \\
& + \frac{3b^4ex^8(5a^2e^2+6abde+b^2d^2)}{8}
\end{aligned}$$

input `int((d + e*x)^3*(a^2 + b^2*x^2 + 2*a*b*x)^3,x)`output `x^5*((6*a^5*b*e^3)/5 + 3*a^2*b^4*d^3 + 12*a^3*b^3*d^2*e + 9*a^4*b^2*d*e^2) + x^6*(a*b^5*d^3 + (5*a^4*b^2*e^3)/2 + (15*a^2*b^4*d^2*e)/2 + 10*a^3*b^3*d*e^2) + x^4*((a^6*e^3)/4 + 5*a^3*b^3*d^3 + (45*a^4*b^2*d^2*e)/4 + (9*a^5*b*d*e^2)/2) + x^7*((b^6*d^3)/7 + (20*a^3*b^3*e^3)/7 + (45*a^2*b^4*d*e^2)/7 + (18*a*b^5*d^2*e)/7) + a^6*d^3*x + (b^6*e^3*x^10)/10 + (3*a^5*d^2*x^2*(a*e + 2*b*d))/2 + (b^5*e^2*x^9*(2*a*e + b*d))/3 + a^4*d*x^3*(a^2*e^2 + 5*b^2*d^2 + 6*a*b*d*e) + (3*b^4*e*x^8*(5*a^2*e^2 + b^2*d^2 + 6*a*b*d*e))/8`

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 363, normalized size of antiderivative = 3.95

$$\int (d + ex)^3 (a^2 + 2abx + b^2x^2)^3 dx$$

$$= \frac{x(84b^6e^3x^9 + 560ab^5e^3x^8 + 280b^6de^2x^8 + 1575a^2b^4e^3x^7 + 1890ab^5de^2x^7 + 315b^6d^2ex^7 + 2400a^3b^3e^3x^6$$

input `int((e*x+d)^3*(b^2*x^2+2*a*b*x+a^2)^3,x)`

output

```
(x*(840*a**6*d**3 + 1260*a**6*d**2*e*x + 840*a**6*d**e**2*x**2 + 210*a**6*e**3*x**3 + 2520*a**5*b*d**3*x + 5040*a**5*b*d**2*e*x**2 + 3780*a**5*b*d*e**2*x**3 + 1008*a**5*b*e**3*x**4 + 4200*a**4*b**2*d**3*x**2 + 9450*a**4*b**2*d**2*e*x**3 + 7560*a**4*b**2*d**e**2*x**4 + 2100*a**4*b**2*e**3*x**5 + 4200*a**3*b**3*d**3*x**3 + 10080*a**3*b**3*d**2*e*x**4 + 8400*a**3*b**3*d*e**2*x**5 + 2400*a**3*b**3*e**3*x**6 + 2520*a**2*b**4*d**3*x**4 + 6300*a**2*b**4*d**2*e*x**5 + 5400*a**2*b**4*d*e**2*x**6 + 1575*a**2*b**4*e**3*x**7 + 840*a*b**5*d**3*x**5 + 2160*a*b**5*d**2*e*x**6 + 1890*a*b**5*d*e**2*x**7 + 560*a*b**5*e**3*x**8 + 120*b**6*d**3*x**6 + 315*b**6*d**2*e*x**7 + 280*b**6*d**e**2*x**8 + 84*b**6*e**3*x**9))/840
```

3.90 $\int (d + ex)^2 (a^2 + 2abx + b^2x^2)^3 dx$

| | |
|---|-----|
| Optimal result | 742 |
| Mathematica [B] (verified) | 743 |
| Rubi [A] (verified) | 743 |
| Maple [B] (verified) | 745 |
| Fricas [B] (verification not implemented) | 745 |
| Sympy [B] (verification not implemented) | 747 |
| Maxima [B] (verification not implemented) | 747 |
| Giac [B] (verification not implemented) | 748 |
| Mupad [B] (verification not implemented) | 749 |
| Reduce [B] (verification not implemented) | 750 |

Optimal result

Integrand size = 26, antiderivative size = 65

$$\int (d + ex)^2 (a^2 + 2abx + b^2x^2)^3 dx = \frac{(bd - ae)^2(a + bx)^7}{7b^3} + \frac{e(bd - ae)(a + bx)^8}{4b^3} + \frac{e^2(a + bx)^9}{9b^3}$$

output

```
1/7*(-a*e+b*d)^2*(b*x+a)^7/b^3+1/4*e*(-a*e+b*d)*(b*x+a)^8/b^3+1/9*e^2*(b*x+a)^9/b^3
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 199 vs. $2(65) = 130$.

Time = 0.05 (sec) , antiderivative size = 199, normalized size of antiderivative = 3.06

$$\int (d + ex)^2 (a^2 + 2abx + b^2x^2)^3 dx = \frac{1}{252}x(84a^6(3d^2 + 3dex + e^2x^2) + 126a^5bx(6d^2 + 8dex + 3e^2x^2) + 126a^4b^2x^2(10d^2 + 15dex + 6e^2x^2) + 84a^3b^3x^3(15d^2 + 24dex + 10e^2x^2) + 36a^2b^4x^4(21d^2 + 35dex + 15e^2x^2) + 9ab^5x^5(28d^2 + 48dex + 21e^2x^2) + b^6x^6(36d^2 + 63dex + 28e^2x^2))$$

input

```
Integrate[(d + e*x)^2*(a^2 + 2*a*b*x + b^2*x^2)^3,x]
```

output

```
(x*(84*a^6*(3*d^2 + 3*d*e*x + e^2*x^2) + 126*a^5*b*x*(6*d^2 + 8*d*e*x + 3*e^2*x^2) + 126*a^4*b^2*x^2*(10*d^2 + 15*d*e*x + 6*e^2*x^2) + 84*a^3*b^3*x^3*(15*d^2 + 24*d*e*x + 10*e^2*x^2) + 36*a^2*b^4*x^4*(21*d^2 + 35*d*e*x + 15*e^2*x^2) + 9*a*b^5*x^5*(28*d^2 + 48*d*e*x + 21*e^2*x^2) + b^6*x^6*(36*d^2 + 63*d*e*x + 28*e^2*x^2)))/252
```

Rubi [A] (verified)

Time = 0.51 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {1098, 27, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a^2 + 2abx + b^2x^2)^3 (d + ex)^2 dx$$

$$\downarrow 1098$$

$$\frac{\int b^6(a + bx)^6(d + ex)^2 dx}{b^6}$$

$$\begin{array}{c}
 \downarrow 27 \\
 \int (a + bx)^6 (d + ex)^2 dx \\
 \downarrow 49 \\
 \int \left(\frac{2e(a + bx)^7 (bd - ae)}{b^2} + \frac{(a + bx)^6 (bd - ae)^2}{b^2} + \frac{e^2 (a + bx)^8}{b^2} \right) dx \\
 \downarrow 2009 \\
 \frac{e(a + bx)^8 (bd - ae)}{4b^3} + \frac{(a + bx)^7 (bd - ae)^2}{7b^3} + \frac{e^2 (a + bx)^9}{9b^3}
 \end{array}$$

input `Int[(d + e*x)^2*(a^2 + 2*a*b*x + b^2*x^2)^3,x]`

output `((b*d - a*e)^2*(a + b*x)^7)/(7*b^3) + (e*(b*d - a*e)*(a + b*x)^8)/(4*b^3) + (e^2*(a + b*x)^9)/(9*b^3)`

Defintions of rubi rules used

rule 27 `Int[(a_)*(Fx_), x_Symbol] := Simp[a Int[Fx, x], x] /; FreeQ[a, x] && !MatchQ[Fx, (b_)*(Gx_)] /; FreeQ[b, x]`

rule 49 `Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] := Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] && IGtQ[m, 0] && IGtQ[m + n + 2, 0]`

rule 1098 `Int[((d_.) + (e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] := Simp[1/c^p Int[(d + e*x)^m*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]`

rule 2009 `Int[u_, x_Symbol] := Simp[IntSum[u, x], x] /; SumQ[u]`

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 231 vs. 2(59) = 118.

Time = 1.05 (sec) , antiderivative size = 232, normalized size of antiderivative = 3.57

| method | result |
|---------------|---|
| norman | $\frac{e^2 b^6 x^9}{9} + \left(\frac{3}{4} e^2 a b^5 + \frac{1}{4} d e b^6\right) x^8 + \left(\frac{15}{7} e^2 a^2 b^4 + \frac{12}{7} d e a b^5 + \frac{1}{7} d^2 b^6\right) x^7 + \left(\frac{10}{3} e^2 a^3 b^3 + 5 d e a^2 b^4 + \frac{1}{3} d^2 a b^5\right) x^6 + \left(\frac{3}{2} e^2 a^4 b^2 + 4 d e a^3 b^3 + \frac{1}{2} d^2 a^2 b^4\right) x^5 + \left(\frac{1}{2} e^2 a^5 b + \frac{1}{2} d e a^4 b^2 + \frac{1}{6} d^2 a^3 b^3\right) x^4 + \left(\frac{1}{6} e^2 a^6 + \frac{1}{6} d e a^5 b + \frac{1}{24} d^2 a^4 b^2\right) x^3 + \left(\frac{1}{24} e^2 a^7 + \frac{1}{24} d e a^6 b + \frac{1}{72} d^2 a^5 b^2\right) x^2 + \frac{1}{72} e^2 a^8 + \frac{1}{72} d e a^7 b + \frac{1}{216} d^2 a^6 b^2$ |
| default | $\frac{e^2 b^6 x^9}{9} + \frac{(6e^2 a b^5 + 2de b^6)x^8}{8} + \frac{(15e^2 a^2 b^4 + 12dea b^5 + d^2 b^6)x^7}{7} + \frac{(20e^2 a^3 b^3 + 30de a^2 b^4 + 6a b^5 d^2)x^6}{6} + \frac{(15e^2 a^4 b^2 + 4e^2 a^5 b + 4d e a^4 b^2 + 2d^2 a^3 b^3)x^5}{5} + \frac{(3e^2 a^6 + 3d e a^5 b + d^2 a^4 b^2)x^4}{4} + \frac{(e^2 a^7 + d e a^6 b + d^2 a^5 b^2)x^3}{3} + \frac{e^2 a^8 + d e a^7 b + d^2 a^6 b^2}{24}$ |
| risch | $\frac{1}{9} e^2 b^6 x^9 + \frac{3}{4} x^8 e^2 a b^5 + \frac{1}{4} x^8 d e b^6 + \frac{15}{7} x^7 e^2 a^2 b^4 + \frac{12}{7} x^7 d e a b^5 + \frac{1}{7} x^7 d^2 b^6 + \frac{10}{3} x^6 e^2 a^3 b^3 + 5 x^6 d e a^2 b^4 + \frac{1}{3} x^6 d^2 a b^5 + \frac{3}{2} x^5 e^2 a^4 b^2 + 4 x^5 d e a^3 b^3 + \frac{1}{2} x^5 d^2 a^2 b^4 + \frac{1}{2} x^4 e^2 a^5 b + \frac{1}{2} x^4 d e a^4 b^2 + \frac{1}{6} x^4 d^2 a^3 b^3 + \frac{1}{6} x^3 e^2 a^6 + \frac{1}{6} x^3 d e a^5 b + \frac{1}{24} x^3 d^2 a^4 b^2 + \frac{1}{24} x^2 e^2 a^7 + \frac{1}{24} x^2 d e a^6 b + \frac{1}{72} x^2 d^2 a^5 b^2 + \frac{1}{72} e^2 a^8 + \frac{1}{72} d e a^7 b + \frac{1}{216} d^2 a^6 b^2$ |
| parallelrisch | $\frac{1}{9} e^2 b^6 x^9 + \frac{3}{4} x^8 e^2 a b^5 + \frac{1}{4} x^8 d e b^6 + \frac{15}{7} x^7 e^2 a^2 b^4 + \frac{12}{7} x^7 d e a b^5 + \frac{1}{7} x^7 d^2 b^6 + \frac{10}{3} x^6 e^2 a^3 b^3 + 5 x^6 d e a^2 b^4 + \frac{1}{3} x^6 d^2 a b^5 + \frac{3}{2} x^5 e^2 a^4 b^2 + 4 x^5 d e a^3 b^3 + \frac{1}{2} x^5 d^2 a^2 b^4 + \frac{1}{2} x^4 e^2 a^5 b + \frac{1}{2} x^4 d e a^4 b^2 + \frac{1}{6} x^4 d^2 a^3 b^3 + \frac{1}{6} x^3 e^2 a^6 + \frac{1}{6} x^3 d e a^5 b + \frac{1}{24} x^3 d^2 a^4 b^2 + \frac{1}{24} x^2 e^2 a^7 + \frac{1}{24} x^2 d e a^6 b + \frac{1}{72} x^2 d^2 a^5 b^2 + \frac{1}{72} e^2 a^8 + \frac{1}{72} d e a^7 b + \frac{1}{216} d^2 a^6 b^2$ |
| gospers | $x(28e^2 b^6 x^8 + 189x^7 e^2 a b^5 + 63x^7 d e b^6 + 540x^6 e^2 a^2 b^4 + 432x^6 d e a b^5 + 36x^6 d^2 b^6 + 840x^5 e^2 a^3 b^3 + 1260x^5 d e a^2 b^4 + 252x^5 a b^5 d^2 + 126x^4 e^2 a^4 b^2 + 84x^4 d e a^3 b^3 + 21x^4 d^2 a^2 b^4 + 126x^3 e^2 a^5 b + 126x^3 d e a^4 b^2 + 21x^3 d^2 a^3 b^3 + 126x^2 e^2 a^6 + 126x^2 d e a^5 b + 21x^2 d^2 a^4 b^2 + 126x e^2 a^7 + 126x d e a^6 b + 21x d^2 a^5 b^2 + 126e^2 a^8 + 126d e a^7 b + 21d^2 a^6 b^2)$ |
| orering | $x(28e^2 b^6 x^8 + 189x^7 e^2 a b^5 + 63x^7 d e b^6 + 540x^6 e^2 a^2 b^4 + 432x^6 d e a b^5 + 36x^6 d^2 b^6 + 840x^5 e^2 a^3 b^3 + 1260x^5 d e a^2 b^4 + 252x^5 a b^5 d^2 + 126x^4 e^2 a^4 b^2 + 84x^4 d e a^3 b^3 + 21x^4 d^2 a^2 b^4 + 126x^3 e^2 a^5 b + 126x^3 d e a^4 b^2 + 21x^3 d^2 a^3 b^3 + 126x^2 e^2 a^6 + 126x^2 d e a^5 b + 21x^2 d^2 a^4 b^2 + 126x e^2 a^7 + 126x d e a^6 b + 21x d^2 a^5 b^2 + 126e^2 a^8 + 126d e a^7 b + 21d^2 a^6 b^2)$ |

input `int((e*x+d)^2*(b^2*x^2+2*a*b*x+a^2)^3,x,method=_RETURNVERBOSE)`

output `1/9*e^2*b^6*x^9+(3/4*e^2*a*b^5+1/4*d*e*b^6)*x^8+(15/7*e^2*a^2*b^4+12/7*d*e*a*b^5+1/7*d^2*b^6)*x^7+(10/3*e^2*a^3*b^3+5*d*e*a^2*b^4+a*b^5*d^2)*x^6+(3*a^4*b^2*e^2+8*a^3*b^3*d*e+3*a^2*b^4*d^2)*x^5+(3/2*e^2*a^5*b+15/2*d*e*a^4*b^2+5*d^2*a^3*b^3)*x^4+(1/3*e^2*a^6+4*d*e*a^5*b+5*d^2*a^4*b^2)*x^3+(a^6*d*e+3*a^5*b*d^2)*x^2+d^2*a^6*x`

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 234 vs. 2(59) = 118.

Time = 0.08 (sec) , antiderivative size = 234, normalized size of antiderivative = 3.60

$$\int (d + ex)^2 (a^2 + 2abx + b^2x^2)^3 dx = \frac{1}{9} b^6 e^2 x^9 + a^6 d^2 x + \frac{1}{4} (b^6 d e + 3 a b^5 e^2) x^8 + \frac{1}{7} (b^6 d^2 + 12 a b^5 d e + 15 a^2 b^4 e^2) x^7 + \frac{1}{3} (3 a b^5 d^2 + 15 a^2 b^4 d e + 10 a^3 b^3 e^2) x^6 + (3 a^2 b^4 d^2 + 8 a^3 b^3 d e + 3 a^4 b^2 e^2) x^5 + \frac{1}{2} (10 a^3 b^3 d^2 + 15 a^4 b^2 d e + 3 a^5 b e^2) x^4 + \frac{1}{3} (15 a^4 b^2 d^2 + 12 a^5 b d e + a^6 e^2) x^3 + (3 a^5 b d^2 + a^6 d e) x^2$$

input

```
integrate((e*x+d)^2*(b^2*x^2+2*a*b*x+a^2)^3,x, algorithm="fricas")
```

output

```
1/9*b^6*e^2*x^9 + a^6*d^2*x + 1/4*(b^6*d*e + 3*a*b^5*e^2)*x^8 + 1/7*(b^6*d^2 + 12*a*b^5*d*e + 15*a^2*b^4*e^2)*x^7 + 1/3*(3*a*b^5*d^2 + 15*a^2*b^4*d*e + 10*a^3*b^3*e^2)*x^6 + (3*a^2*b^4*d^2 + 8*a^3*b^3*d*e + 3*a^4*b^2*e^2)*x^5 + 1/2*(10*a^3*b^3*d^2 + 15*a^4*b^2*d*e + 3*a^5*b*e^2)*x^4 + 1/3*(15*a^4*b^2*d^2 + 12*a^5*b*d*e + a^6*e^2)*x^3 + (3*a^5*b*d^2 + a^6*d*e)*x^2
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 252 vs. $2(54) = 108$.

Time = 0.04 (sec) , antiderivative size = 252, normalized size of antiderivative = 3.88

$$\int (d + ex)^2 (a^2 + 2abx + b^2x^2)^3 dx = a^6d^2x + \frac{b^6e^2x^9}{9} + x^8 \cdot \left(\frac{3ab^5e^2}{4} + \frac{b^6de}{4} \right) \\ + x^7 \cdot \left(\frac{15a^2b^4e^2}{7} + \frac{12ab^5de}{7} + \frac{b^6d^2}{7} \right) \\ + x^6 \cdot \left(\frac{10a^3b^3e^2}{3} + 5a^2b^4de + ab^5d^2 \right) \\ + x^5 \cdot (3a^4b^2e^2 + 8a^3b^3de + 3a^2b^4d^2) \\ + x^4 \cdot \left(\frac{3a^5be^2}{2} + \frac{15a^4b^2de}{2} + 5a^3b^3d^2 \right) \\ + x^3 \left(\frac{a^6e^2}{3} + 4a^5bde + 5a^4b^2d^2 \right) \\ + x^2 (a^6de + 3a^5bd^2)$$

input

```
integrate((e*x+d)**2*(b**2*x**2+2*a*b*x+a**2)**3,x)
```

output

```
a**6*d**2*x + b**6*e**2*x**9/9 + x**8*(3*a*b**5*e**2/4 + b**6*d*e/4) + x**7*(15*a**2*b**4*e**2/7 + 12*a*b**5*d*e/7 + b**6*d**2/7) + x**6*(10*a**3*b**3*e**2/3 + 5*a**2*b**4*d*e + a*b**5*d**2) + x**5*(3*a**4*b**2*e**2 + 8*a**3*b**3*d*e + 3*a**2*b**4*d**2) + x**4*(3*a**5*b*e**2/2 + 15*a**4*b**2*d*e/2 + 5*a**3*b**3*d**2) + x**3*(a**6*e**2/3 + 4*a**5*b*d*e + 5*a**4*b**2*d**2) + x**2*(a**6*d*e + 3*a**5*b*d**2)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 234 vs. $2(59) = 118$.

Time = 0.03 (sec) , antiderivative size = 234, normalized size of antiderivative = 3.60

$$\int (d + ex)^2 (a^2 + 2abx + b^2x^2)^3 dx = \frac{1}{9} b^6 e^2 x^9 + a^6 d^2 x + \frac{1}{4} (b^6 de + 3 ab^5 e^2) x^8 + \frac{1}{7} (b^6 d^2 + 12 ab^5 de + 15 a^2 b^4 e^2) x^7 + \frac{1}{3} (3 ab^5 d^2 + 15 a^2 b^4 de + 10 a^3 b^3 e^2) x^6 + (3 a^2 b^4 d^2 + 8 a^3 b^3 de + 3 a^4 b^2 e^2) x^5 + \frac{1}{2} (10 a^3 b^3 d^2 + 15 a^4 b^2 de + 3 a^5 b e^2) x^4 + \frac{1}{3} (15 a^4 b^2 d^2 + 12 a^5 b de + a^6 e^2) x^3 + (3 a^5 b d^2 + a^6 de) x^2$$

input

```
integrate((e*x+d)^2*(b^2*x^2+2*a*b*x+a^2)^3,x, algorithm="maxima")
```

output

```
1/9*b^6*e^2*x^9 + a^6*d^2*x + 1/4*(b^6*d*e + 3*a*b^5*e^2)*x^8 + 1/7*(b^6*d^2 + 12*a*b^5*d*e + 15*a^2*b^4*e^2)*x^7 + 1/3*(3*a*b^5*d^2 + 15*a^2*b^4*d*e + 10*a^3*b^3*e^2)*x^6 + (3*a^2*b^4*d^2 + 8*a^3*b^3*d*e + 3*a^4*b^2*e^2)*x^5 + 1/2*(10*a^3*b^3*d^2 + 15*a^4*b^2*d*e + 3*a^5*b*e^2)*x^4 + 1/3*(15*a^4*b^2*d^2 + 12*a^5*b*d*e + a^6*e^2)*x^3 + (3*a^5*b*d^2 + a^6*d*e)*x^2
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 253 vs. 2(59) = 118.

Time = 0.21 (sec) , antiderivative size = 253, normalized size of antiderivative = 3.89

$$\int (d + ex)^2 (a^2 + 2abx + b^2x^2)^3 dx = \frac{1}{9} b^6 e^2 x^9 + \frac{1}{4} b^6 dex^8 + \frac{3}{4} ab^5 e^2 x^8 + \frac{1}{7} b^6 d^2 x^7 + \frac{12}{7} ab^5 dex^7 + \frac{15}{7} a^2 b^4 e^2 x^7 + ab^5 d^2 x^6 + 5 a^2 b^4 dex^6 + \frac{10}{3} a^3 b^3 e^2 x^6 + 3 a^2 b^4 d^2 x^5 + 8 a^3 b^3 dex^5 + 3 a^4 b^2 e^2 x^5 + 5 a^3 b^3 d^2 x^4 + \frac{15}{2} a^4 b^2 dex^4 + \frac{3}{2} a^5 b e^2 x^4 + 5 a^4 b^2 d^2 x^3 + 4 a^5 b dex^3 + \frac{1}{3} a^6 e^2 x^3 + 3 a^5 b d^2 x^2 + a^6 dex^2 + a^6 d^2 x$$

input `integrate((e*x+d)^2*(b^2*x^2+2*a*b*x+a^2)^3,x, algorithm="giac")`

output
$$\begin{aligned} & 1/9*b^6*e^2*x^9 + 1/4*b^6*d*e*x^8 + 3/4*a*b^5*e^2*x^8 + 1/7*b^6*d^2*x^7 + \\ & 12/7*a*b^5*d*e*x^7 + 15/7*a^2*b^4*e^2*x^7 + a*b^5*d^2*x^6 + 5*a^2*b^4*d*e* \\ & x^6 + 10/3*a^3*b^3*e^2*x^6 + 3*a^2*b^4*d^2*x^5 + 8*a^3*b^3*d*e*x^5 + 3*a^4 \\ & *b^2*e^2*x^5 + 5*a^3*b^3*d^2*x^4 + 15/2*a^4*b^2*d*e*x^4 + 3/2*a^5*b*e^2*x^ \\ & 4 + 5*a^4*b^2*d^2*x^3 + 4*a^5*b*d*e*x^3 + 1/3*a^6*e^2*x^3 + 3*a^5*b*d^2*x^ \\ & 2 + a^6*d*e*x^2 + a^6*d^2*x \end{aligned}$$

Mupad [B] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 214, normalized size of antiderivative = 3.29

$$\begin{aligned} \int (d + ex)^2 (a^2 + 2abx + b^2x^2)^3 dx = & x^3 \left(\frac{a^6 e^2}{3} + 4a^5 b d e + 5a^4 b^2 d^2 \right) \\ & + x^7 \left(\frac{15a^2 b^4 e^2}{7} + \frac{12a b^5 d e}{7} + \frac{b^6 d^2}{7} \right) \\ & + a^6 d^2 x + \frac{b^6 e^2 x^9}{9} + a^5 d x^2 (a e + 3 b d) \\ & + \frac{b^5 e x^8 (3 a e + b d)}{4} \\ & + \frac{a^3 b x^4 (3 a^2 e^2 + 15 a b d e + 10 b^2 d^2)}{2} \\ & + \frac{a b^3 x^6 (10 a^2 e^2 + 15 a b d e + 3 b^2 d^2)}{3} \\ & + a^2 b^2 x^5 (3 a^2 e^2 + 8 a b d e + 3 b^2 d^2) \end{aligned}$$

input `int((d + e*x)^2*(a^2 + b^2*x^2 + 2*a*b*x)^3,x)`

output
$$\begin{aligned} & x^3*((a^6*e^2)/3 + 5*a^4*b^2*d^2 + 4*a^5*b*d*e) + x^7*((b^6*d^2)/7 + (15*a \\ & ^2*b^4*e^2)/7 + (12*a*b^5*d*e)/7) + a^6*d^2*x + (b^6*e^2*x^9)/9 + a^5*d*x^ \\ & 2*(a*e + 3*b*d) + (b^5*e*x^8*(3*a*e + b*d))/4 + (a^3*b*x^4*(3*a^2*e^2 + 10 \\ & *b^2*d^2 + 15*a*b*d*e))/2 + (a*b^3*x^6*(10*a^2*e^2 + 3*b^2*d^2 + 15*a*b*d* \\ & e))/3 + a^2*b^2*x^5*(3*a^2*e^2 + 3*b^2*d^2 + 8*a*b*d*e) \end{aligned}$$

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 254, normalized size of antiderivative = 3.91

$$\int (d + ex)^2 (a^2 + 2abx + b^2x^2)^3 dx$$

$$= \frac{x(28b^6e^2x^8 + 189ab^5e^2x^7 + 63b^6dex^7 + 540a^2b^4e^2x^6 + 432ab^5dex^6 + 36b^6d^2x^6 + 840a^3b^3e^2x^5 + 1260a^2b^4dex^5 + 540a^3b^3dex^4 + 1260a^4b^2d^2x^4 + 1890a^4b^2d^2ex^3 + 756a^4b^2e^2x^4 + 1260a^3b^3d^2x^3 + 2016a^3b^3d^2ex^4 + 840a^3b^3e^2x^5 + 756a^2b^4d^2x^4 + 1260a^2b^4d^2ex^5 + 540a^2b^4e^2x^6 + 252ab^5d^2x^5 + 432ab^5dex^6 + 189ab^5e^2x^7 + 36b^6d^2x^6 + 63b^6dex^7 + 28b^6e^2x^8)}{252}$$

input `int((e*x+d)^2*(b^2*x^2+2*a*b*x+a^2)^3,x)`output `(x*(252*a**6*d**2 + 252*a**6*d*e*x + 84*a**6*e**2*x**2 + 756*a**5*b*d**2*x + 1008*a**5*b*d*e*x**2 + 378*a**5*b*e**2*x**3 + 1260*a**4*b**2*d**2*x**2 + 1890*a**4*b**2*d*e*x**3 + 756*a**4*b**2*e**2*x**4 + 1260*a**3*b**3*d**2*x**3 + 2016*a**3*b**3*d*e*x**4 + 840*a**3*b**3*e**2*x**5 + 756*a**2*b**4*d**2*x**4 + 1260*a**2*b**4*d*e*x**5 + 540*a**2*b**4*e**2*x**6 + 252*a*b**5*d**2*x**5 + 432*a*b**5*d*e*x**6 + 189*a*b**5*e**2*x**7 + 36*b**6*d**2*x**6 + 63*b**6*d*e*x**7 + 28*b**6*e**2*x**8))/252`

3.91 $\int (d + ex) (a^2 + 2abx + b^2x^2)^3 dx$

| | |
|---|-----|
| Optimal result | 751 |
| Mathematica [B] (verified) | 751 |
| Rubi [A] (verified) | 752 |
| Maple [B] (verified) | 753 |
| Fricas [B] (verification not implemented) | 754 |
| Sympy [B] (verification not implemented) | 754 |
| Maxima [B] (verification not implemented) | 755 |
| Giac [B] (verification not implemented) | 756 |
| Mupad [B] (verification not implemented) | 756 |
| Reduce [B] (verification not implemented) | 757 |

Optimal result

Integrand size = 24, antiderivative size = 38

$$\int (d + ex) (a^2 + 2abx + b^2x^2)^3 dx = \frac{(bd - ae)(a + bx)^7}{7b^2} + \frac{e(a + bx)^8}{8b^2}$$

output

```
1/7*(-a*e+b*d)*(b*x+a)^7/b^2+1/8*e*(b*x+a)^8/b^2
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 122 vs. 2(38) = 76.

Time = 0.03 (sec) , antiderivative size = 122, normalized size of antiderivative = 3.21

$$\begin{aligned} \int (d + ex) (a^2 + 2abx + b^2x^2)^3 dx = & \frac{1}{56}x(28a^6(2d + ex) + 56a^5bx(3d + 2ex) \\ & + 70a^4b^2x^2(4d + 3ex) + 56a^3b^3x^3(5d + 4ex) \\ & + 28a^2b^4x^4(6d + 5ex) + 8ab^5x^5(7d + 6ex) \\ & + b^6x^6(8d + 7ex)) \end{aligned}$$

input

```
Integrate[(d + e*x)*(a^2 + 2*a*b*x + b^2*x^2)^3,x]
```

output

$$(x*(28*a^6*(2*d + e*x) + 56*a^5*b*x*(3*d + 2*e*x) + 70*a^4*b^2*x^2*(4*d + 3*e*x) + 56*a^3*b^3*x^3*(5*d + 4*e*x) + 28*a^2*b^4*x^4*(6*d + 5*e*x) + 8*a*b^5*x^5*(7*d + 6*e*x) + b^6*x^6*(8*d + 7*e*x)))/56$$
Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {1098, 27, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\begin{aligned} & \int (a^2 + 2abx + b^2x^2)^3 (d + ex) dx \\ & \quad \downarrow 1098 \\ & \frac{\int b^6(a + bx)^6(d + ex)dx}{b^6} \\ & \quad \downarrow 27 \\ & \int (a + bx)^6(d + ex)dx \\ & \quad \downarrow 49 \\ & \int \left(\frac{(a + bx)^6(bd - ae)}{b} + \frac{e(a + bx)^7}{b} \right) dx \\ & \quad \downarrow 2009 \\ & \frac{(a + bx)^7(bd - ae)}{7b^2} + \frac{e(a + bx)^8}{8b^2} \end{aligned}$$

input

$$\text{Int}[(d + e*x)*(a^2 + 2*a*b*x + b^2*x^2)^3, x]$$

output

$$((b*d - a*e)*(a + b*x)^7)/(7*b^2) + (e*(a + b*x)^8)/(8*b^2)$$

Defintions of rubi rules used

rule 27 `Int[(a_)*(Fx_), x_Symbol] := Simp[a Int[Fx, x], x] /; FreeQ[a, x] && !MatchQ[Fx, (b_)*(Gx_)] /; FreeQ[b, x]`

rule 49 `Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] := Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] && IGtQ[m, 0] && IGtQ[m + n + 2, 0]`

rule 1098 `Int[((d_.) + (e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] := Simp[1/c^p Int[(d + e*x)^m*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]`

rule 2009 `Int[u_, x_Symbol] := Simp[IntSum[u, x], x] /; SumQ[u]`

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 139 vs. 2(34) = 68.

Time = 0.92 (sec) , antiderivative size = 140, normalized size of antiderivative = 3.68

| method | result |
|--------------|---|
| norman | $\frac{b^6 e x^8}{8} + \left(\frac{6}{7} e a b^5 + \frac{1}{7} d b^6\right) x^7 + \left(\frac{5}{2} e a^2 b^4 + d a b^5\right) x^6 + \left(4 e a^3 b^3 + 3 d a^2 b^4\right) x^5 + \left(\frac{15}{4} e a^4 b^2 + 5 d a^3 b^3\right) x^4 + \left(\frac{3}{2} e a^5 b + 3 d a^4 b^2\right) x^3 + \left(\frac{3}{8} e a^6 + \frac{3}{4} d a^5\right) x^2 + \frac{3}{8} e a^7 + \frac{3}{4} d a^6 x + \frac{3}{8} e x^8 + \frac{6}{7} e a b^5 x^7 + \frac{1}{7} e a^2 b^4 x^6 + \frac{5}{2} e a^3 b^3 x^5 + \frac{15}{4} e a^4 b^2 x^4 + \frac{3}{2} e a^5 b x^3 + \frac{3}{8} e a^6 x^2 + \frac{3}{8} e x^8$ |
| default | $\frac{b^6 e x^8}{8} + \frac{(6 e a b^5 + d b^6) x^7}{7} + \frac{(15 e a^2 b^4 + 6 d a b^5) x^6}{6} + \frac{(20 e a^3 b^3 + 15 d a^2 b^4) x^5}{5} + \frac{(15 e a^4 b^2 + 20 a^3 d b^3) x^4}{4} + \frac{(6 a^5 b e + 3 d a^4 b^2) x^3}{3} + \frac{3 a^6 e + 3 d a^5}{8} x^2 + \frac{3 e a^7 + 3 d a^6}{8} x + \frac{3 e x^8}{8}$ |
| gospers | $\frac{x(7 b^6 e x^7 + 48 x^6 e a b^5 + 8 b^6 d x^6 + 140 x^5 e a^2 b^4 + 56 a b^5 d x^5 + 224 a^3 b^3 e x^4 + 168 a^2 b^4 d x^4 + 210 x^3 e a^4 b^2 + 280 a^3 b^3 d x^3 + 112 a^5 b e x^2 + 3 a^6 e + 3 d a^5) x^2 + 3 e a^7 + 3 d a^6}{56}$ |
| risch | $\frac{1}{8} b^6 e x^8 + \frac{6}{7} x^7 e a b^5 + \frac{1}{7} x^7 d b^6 + \frac{5}{2} x^6 e a^2 b^4 + a b^5 d x^6 + 4 a^3 b^3 e x^5 + 3 a^2 b^4 d x^5 + \frac{15}{4} x^4 e a^4 b^2 + \frac{3}{2} x^3 e a^5 b + \frac{3}{8} x^2 e a^6 + \frac{3}{8} e a^7 + \frac{3}{8} e x^8$ |
| parallelrisc | $\frac{1}{8} b^6 e x^8 + \frac{6}{7} x^7 e a b^5 + \frac{1}{7} x^7 d b^6 + \frac{5}{2} x^6 e a^2 b^4 + a b^5 d x^6 + 4 a^3 b^3 e x^5 + 3 a^2 b^4 d x^5 + \frac{15}{4} x^4 e a^4 b^2 + \frac{3}{2} x^3 e a^5 b + \frac{3}{8} x^2 e a^6 + \frac{3}{8} e a^7 + \frac{3}{8} e x^8$ |
| orering | $\frac{x(7 b^6 e x^7 + 48 x^6 e a b^5 + 8 b^6 d x^6 + 140 x^5 e a^2 b^4 + 56 a b^5 d x^5 + 224 a^3 b^3 e x^4 + 168 a^2 b^4 d x^4 + 210 x^3 e a^4 b^2 + 280 a^3 b^3 d x^3 + 112 a^5 b e x^2 + 3 a^6 e + 3 d a^5)}{56(bx+a)^6}$ |

input `int((e*x+d)*(b^2*x^2+2*a*b*x+a^2)^3,x,method=_RETURNVERBOSE)`

output

```
1/8*b^6*e*x^8+(6/7*e*a*b^5+1/7*d*b^6)*x^7+(5/2*e*a^2*b^4+d*a*b^5)*x^6+(4*a^3*b^3*e+3*a^2*b^4*d)*x^5+(15/4*e*a^4*b^2+5*a^3*d*b^3)*x^4+(2*a^5*b*e+5*a^4*b^2*d)*x^3+(1/2*e*a^6+3*a^5*b*d)*x^2+a^6*d*x
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 142 vs. $2(34) = 68$.

Time = 0.08 (sec) , antiderivative size = 142, normalized size of antiderivative = 3.74

$$\begin{aligned} \int (d + ex) (a^2 + 2abx + b^2x^2)^3 dx &= \frac{1}{8} b^6 ex^8 + a^6 dx + \frac{1}{7} (b^6 d + 6 ab^5 e) x^7 \\ &+ \frac{1}{2} (2 ab^5 d + 5 a^2 b^4 e) x^6 \\ &+ (3 a^2 b^4 d + 4 a^3 b^3 e) x^5 + \frac{5}{4} (4 a^3 b^3 d + 3 a^4 b^2 e) x^4 \\ &+ (5 a^4 b^2 d + 2 a^5 b e) x^3 + \frac{1}{2} (6 a^5 b d + a^6 e) x^2 \end{aligned}$$

input

```
integrate((e*x+d)*(b^2*x^2+2*a*b*x+a^2)^3,x, algorithm="fricas")
```

output

```
1/8*b^6*e*x^8 + a^6*d*x + 1/7*(b^6*d + 6*a*b^5*e)*x^7 + 1/2*(2*a*b^5*d + 5*a^2*b^4*e)*x^6 + (3*a^2*b^4*d + 4*a^3*b^3*e)*x^5 + 5/4*(4*a^3*b^3*d + 3*a^4*b^2*e)*x^4 + (5*a^4*b^2*d + 2*a^5*b*e)*x^3 + 1/2*(6*a^5*b*d + a^6*e)*x^2
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 148 vs. $2(32) = 64$.

Time = 0.03 (sec) , antiderivative size = 148, normalized size of antiderivative = 3.89

$$\begin{aligned} \int (d + ex) (a^2 + 2abx + b^2x^2)^3 dx &= a^6 dx + \frac{b^6 ex^8}{8} + x^7 \cdot \left(\frac{6ab^5 e}{7} + \frac{b^6 d}{7} \right) + x^6 \\ &\cdot \left(\frac{5a^2 b^4 e}{2} + ab^5 d \right) + x^5 \cdot (4a^3 b^3 e + 3a^2 b^4 d) \\ &+ x^4 \cdot \left(\frac{15a^4 b^2 e}{4} + 5a^3 b^3 d \right) + x^3 \\ &\cdot (2a^5 b e + 5a^4 b^2 d) + x^2 \left(\frac{a^6 e}{2} + 3a^5 b d \right) \end{aligned}$$

input `integrate((e*x+d)*(b**2*x**2+2*a*b*x+a**2)**3,x)`

output `a**6*d*x + b**6*e*x**8/8 + x**7*(6*a*b**5*e/7 + b**6*d/7) + x**6*(5*a**2*b**4*e/2 + a*b**5*d) + x**5*(4*a**3*b**3*e + 3*a**2*b**4*d) + x**4*(15*a**4*b**2*e/4 + 5*a**3*b**3*d) + x**3*(2*a**5*b*e + 5*a**4*b**2*d) + x**2*(a**6*e/2 + 3*a**5*b*d)`

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 142 vs. $2(34) = 68$.

Time = 0.04 (sec) , antiderivative size = 142, normalized size of antiderivative = 3.74

$$\begin{aligned} \int (d + ex) (a^2 + 2abx + b^2x^2)^3 dx &= \frac{1}{8} b^6 ex^8 + a^6 dx + \frac{1}{7} (b^6 d + 6 ab^5 e) x^7 \\ &+ \frac{1}{2} (2 ab^5 d + 5 a^2 b^4 e) x^6 \\ &+ (3 a^2 b^4 d + 4 a^3 b^3 e) x^5 + \frac{5}{4} (4 a^3 b^3 d + 3 a^4 b^2 e) x^4 \\ &+ (5 a^4 b^2 d + 2 a^5 b e) x^3 + \frac{1}{2} (6 a^5 b d + a^6 e) x^2 \end{aligned}$$

input `integrate((e*x+d)*(b^2*x^2+2*a*b*x+a^2)^3,x, algorithm="maxima")`

output `1/8*b^6*e*x^8 + a^6*d*x + 1/7*(b^6*d + 6*a*b^5*e)*x^7 + 1/2*(2*a*b^5*d + 5*a^2*b^4*e)*x^6 + (3*a^2*b^4*d + 4*a^3*b^3*e)*x^5 + 5/4*(4*a^3*b^3*d + 3*a^4*b^2*e)*x^4 + (5*a^4*b^2*d + 2*a^5*b*e)*x^3 + 1/2*(6*a^5*b*d + a^6*e)*x^2`

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 145 vs. $2(34) = 68$.

Time = 0.21 (sec) , antiderivative size = 145, normalized size of antiderivative = 3.82

$$\int (d + ex) (a^2 + 2abx + b^2x^2)^3 dx = \frac{1}{8} b^6 ex^8 + \frac{1}{7} b^6 dx^7 + \frac{6}{7} ab^5 ex^7 + ab^5 dx^6 + \frac{5}{2} a^2 b^4 ex^6 + 3 a^2 b^4 dx^5 + 4 a^3 b^3 ex^5 + 5 a^3 b^3 dx^4 + \frac{15}{4} a^4 b^2 ex^4 + 5 a^4 b^2 dx^3 + 2 a^5 b ex^3 + 3 a^5 b dx^2 + \frac{1}{2} a^6 ex^2 + a^6 dx$$

input `integrate((e*x+d)*(b^2*x^2+2*a*b*x+a^2)^3,x, algorithm="giac")`

output `1/8*b^6*e*x^8 + 1/7*b^6*d*x^7 + 6/7*a*b^5*e*x^7 + a*b^5*d*x^6 + 5/2*a^2*b^4*e*x^6 + 3*a^2*b^4*d*x^5 + 4*a^3*b^3*e*x^5 + 5*a^3*b^3*d*x^4 + 15/4*a^4*b^2*e*x^4 + 5*a^4*b^2*d*x^3 + 2*a^5*b*e*x^3 + 3*a^5*b*d*x^2 + 1/2*a^6*e*x^2 + a^6*d*x`

Mupad [B] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 126, normalized size of antiderivative = 3.32

$$\int (d + ex) (a^2 + 2abx + b^2x^2)^3 dx = x^2 \left(\frac{e a^6}{2} + 3 b d a^5 \right) + x^7 \left(\frac{d b^6}{7} + \frac{6 a e b^5}{7} \right) + \frac{b^6 e x^8}{8} + a^6 dx + a^4 b x^3 (2 a e + 5 b d) + \frac{a b^4 x^6 (5 a e + 2 b d)}{2} + \frac{5 a^3 b^2 x^4 (3 a e + 4 b d)}{4} + a^2 b^3 x^5 (4 a e + 3 b d)$$

input `int((d + e*x)*(a^2 + b^2*x^2 + 2*a*b*x)^3,x)`

output `x^2*((a^6*e)/2 + 3*a^5*b*d) + x^7*((b^6*d)/7 + (6*a*b^5*e)/7) + (b^6*e*x^8)/8 + a^6*d*x + a^4*b*x^3*(2*a*e + 5*b*d) + (a*b^4*x^6*(5*a*e + 2*b*d))/2 + (5*a^3*b^2*x^4*(3*a*e + 4*b*d))/4 + a^2*b^3*x^5*(4*a*e + 3*b*d)`

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 145, normalized size of antiderivative = 3.82

$$\int (d + ex) (a^2 + 2abx + b^2x^2)^3 dx$$

$$= \frac{x(7b^6ex^7 + 48ab^5ex^6 + 8b^6dx^6 + 140a^2b^4ex^5 + 56ab^5dx^5 + 224a^3b^3ex^4 + 168a^2b^4dx^4 + 210a^4b^2ex^3 - \dots}{56}$$

input `int((e*x+d)*(b^2*x^2+2*a*b*x+a^2)^3,x)`output `(x*(56*a**6*d + 28*a**6*e*x + 168*a**5*b*d*x + 112*a**5*b*e*x**2 + 280*a**4*b**2*d*x**2 + 210*a**4*b**2*e*x**3 + 280*a**3*b**3*d*x**3 + 224*a**3*b**3*e*x**4 + 168*a**2*b**4*d*x**4 + 140*a**2*b**4*e*x**5 + 56*a*b**5*d*x**5 + 48*a*b**5*e*x**6 + 8*b**6*d*x**6 + 7*b**6*e*x**7))/56`

3.92 $\int (a^2 + 2abx + b^2x^2)^3 dx$

| | |
|---|-----|
| Optimal result | 758 |
| Mathematica [A] (verified) | 758 |
| Rubi [A] (verified) | 759 |
| Maple [B] (verified) | 760 |
| Fricas [B] (verification not implemented) | 760 |
| Sympy [B] (verification not implemented) | 761 |
| Maxima [B] (verification not implemented) | 761 |
| Giac [B] (verification not implemented) | 762 |
| Mupad [B] (verification not implemented) | 762 |
| Reduce [B] (verification not implemented) | 762 |

Optimal result

Integrand size = 18, antiderivative size = 14

$$\int (a^2 + 2abx + b^2x^2)^3 dx = \frac{(a + bx)^7}{7b}$$

output `1/7*(b*x+a)^7/b`

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00

$$\int (a^2 + 2abx + b^2x^2)^3 dx = \frac{(a + bx)^7}{7b}$$

input `Integrate[(a^2 + 2*a*b*x + b^2*x^2)^3,x]`

output `(a + b*x)^7/(7*b)`

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {1077, 17}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a^2 + 2abx + b^2x^2)^3 dx$$

$$\downarrow 1077$$

$$\frac{\int (xb^2 + ab)^6 dx}{b^6}$$

$$\downarrow 17$$

$$\frac{(a + bx)^7}{7b}$$

input `Int[(a^2 + 2*a*b*x + b^2*x^2)^3,x]`

output `(a + b*x)^7/(7*b)`

Defintions of rubi rules used

rule 17 `Int[(c_.)*((a_.) + (b_.)*(x_)^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1))/(b*(m + 1)), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]`

rule 1077 `Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[1/c^p Int[(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]`

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 64 vs. $2(12) = 24$.

Time = 0.82 (sec) , antiderivative size = 65, normalized size of antiderivative = 4.64

| method | result | size |
|--------------|--|------|
| default | $\frac{1}{7}b^6x^7 + ab^5x^6 + 3a^2b^4x^5 + 5a^3b^3x^4 + 5a^4b^2x^3 + 3bx^2a^5 + a^6x$ | 65 |
| norman | $\frac{1}{7}b^6x^7 + ab^5x^6 + 3a^2b^4x^5 + 5a^3b^3x^4 + 5a^4b^2x^3 + 3bx^2a^5 + a^6x$ | 65 |
| risch | $\frac{1}{7}b^6x^7 + ab^5x^6 + 3a^2b^4x^5 + 5a^3b^3x^4 + 5a^4b^2x^3 + 3bx^2a^5 + a^6x$ | 65 |
| parallelrisc | $\frac{1}{7}b^6x^7 + ab^5x^6 + 3a^2b^4x^5 + 5a^3b^3x^4 + 5a^4b^2x^3 + 3bx^2a^5 + a^6x$ | 65 |
| gospers | $\frac{x(b^6x^6 + 7ab^5x^5 + 21a^2b^4x^4 + 35a^3b^3x^3 + 35a^4b^2x^2 + 21a^5bx + 7a^6)}{7}$ | 66 |
| orering | $\frac{x(b^6x^6 + 7ab^5x^5 + 21a^2b^4x^4 + 35a^3b^3x^3 + 35a^4b^2x^2 + 21a^5bx + 7a^6)(b^2x^2 + 2abx + a^2)^3}{7(bx+a)^6}$ | 91 |

input `int((b^2*x^2+2*a*b*x+a^2)^3,x,method=_RETURNVERBOSE)`

output $\frac{1}{7}b^6x^7 + ab^5x^6 + 3a^2b^4x^5 + 5a^3b^3x^4 + 5a^4b^2x^3 + 3bx^2a^5 + a^6x$

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 64 vs. $2(12) = 24$.

Time = 0.08 (sec) , antiderivative size = 64, normalized size of antiderivative = 4.57

$$\int (a^2 + 2abx + b^2x^2)^3 dx = \frac{1}{7}b^6x^7 + ab^5x^6 + 3a^2b^4x^5 + 5a^3b^3x^4 + 5a^4b^2x^3 + 3a^5bx^2 + a^6x$$

input `integrate((b^2*x^2+2*a*b*x+a^2)^3,x, algorithm="fricas")`

output $\frac{1}{7}b^6x^7 + ab^5x^6 + 3a^2b^4x^5 + 5a^3b^3x^4 + 5a^4b^2x^3 + 3a^5bx^2 + a^6x$

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 66 vs. $2(8) = 16$.

Time = 0.02 (sec) , antiderivative size = 66, normalized size of antiderivative = 4.71

$$\int (a^2 + 2abx + b^2x^2)^3 dx = a^6x + 3a^5bx^2 + 5a^4b^2x^3 + 5a^3b^3x^4 + 3a^2b^4x^5 + ab^5x^6 + \frac{b^6x^7}{7}$$

input `integrate((b**2*x**2+2*a*b*x+a**2)**3,x)`

output `a**6*x + 3*a**5*b*x**2 + 5*a**4*b**2*x**3 + 5*a**3*b**3*x**4 + 3*a**2*b**4*x**5 + a*b**5*x**6 + b**6*x**7/7`

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 97 vs. $2(12) = 24$.

Time = 0.04 (sec) , antiderivative size = 97, normalized size of antiderivative = 6.93

$$\int (a^2 + 2abx + b^2x^2)^3 dx = \frac{1}{7} b^6 x^7 + ab^5 x^6 + \frac{12}{5} a^2 b^4 x^5 + 2 a^3 b^3 x^4 + a^6 x + (b^2 x^3 + 3 abx^2) a^4 + \frac{1}{5} (3 b^4 x^5 + 15 ab^3 x^4 + 20 a^2 b^2 x^3) a^2$$

input `integrate((b^2*x^2+2*a*b*x+a^2)^3,x, algorithm="maxima")`

output `1/7*b^6*x^7 + a*b^5*x^6 + 12/5*a^2*b^4*x^5 + 2*a^3*b^3*x^4 + a^6*x + (b^2*x^3 + 3*a*b*x^2)*a^4 + 1/5*(3*b^4*x^5 + 15*a*b^3*x^4 + 20*a^2*b^2*x^3)*a^2`

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 64 vs. $2(12) = 24$.

Time = 0.23 (sec) , antiderivative size = 64, normalized size of antiderivative = 4.57

$$\int (a^2 + 2abx + b^2x^2)^3 dx = \frac{1}{7}b^6x^7 + ab^5x^6 + 3a^2b^4x^5 + 5a^3b^3x^4 + 5a^4b^2x^3 + 3a^5bx^2 + a^6x$$

input `integrate((b^2*x^2+2*a*b*x+a^2)^3,x, algorithm="giac")`

output `1/7*b^6*x^7 + a*b^5*x^6 + 3*a^2*b^4*x^5 + 5*a^3*b^3*x^4 + 5*a^4*b^2*x^3 + 3*a^5*b*x^2 + a^6*x`

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 64, normalized size of antiderivative = 4.57

$$\int (a^2 + 2abx + b^2x^2)^3 dx = a^6x + 3a^5bx^2 + 5a^4b^2x^3 + 5a^3b^3x^4 + 3a^2b^4x^5 + ab^5x^6 + \frac{b^6x^7}{7}$$

input `int((a^2 + b^2*x^2 + 2*a*b*x)^3,x)`

output `a^6*x + (b^6*x^7)/7 + 3*a^5*b*x^2 + a*b^5*x^6 + 5*a^4*b^2*x^3 + 5*a^3*b^3*x^4 + 3*a^2*b^4*x^5`

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 65, normalized size of antiderivative = 4.64

$$\int (a^2 + 2abx + b^2x^2)^3 dx = \frac{x(b^6x^6 + 7ab^5x^5 + 21a^2b^4x^4 + 35a^3b^3x^3 + 35a^4b^2x^2 + 21a^5bx + 7a^6)}{7}$$

input `int((b^2*x^2+2*a*b*x+a^2)^3,x)`

output `(x*(7*a**6 + 21*a**5*b*x + 35*a**4*b**2*x**2 + 35*a**3*b**3*x**3 + 21*a**2*b**4*x**4 + 7*a*b**5*x**5 + b**6*x**6))/7`

3.93 $\int \frac{(a^2+2abx+b^2x^2)^3}{d+ex} dx$

| | |
|---|-----|
| Optimal result | 764 |
| Mathematica [A] (verified) | 764 |
| Rubi [A] (verified) | 765 |
| Maple [B] (verified) | 767 |
| Fricas [B] (verification not implemented) | 767 |
| Sympy [B] (verification not implemented) | 768 |
| Maxima [B] (verification not implemented) | 769 |
| Giac [B] (verification not implemented) | 769 |
| Mupad [B] (verification not implemented) | 771 |
| Reduce [B] (verification not implemented) | 772 |

Optimal result

Integrand size = 26, antiderivative size = 146

$$\int \frac{(a^2 + 2abx + b^2x^2)^3}{d + ex} dx = -\frac{b(bd - ae)^5x}{e^6} + \frac{(bd - ae)^4(a + bx)^2}{2e^5} - \frac{(bd - ae)^3(a + bx)^3}{3e^4} + \frac{(bd - ae)^2(a + bx)^4}{4e^3} - \frac{(bd - ae)(a + bx)^5}{5e^2} + \frac{(a + bx)^6}{6e} + \frac{(bd - ae)^6 \log(d + ex)}{e^7}$$

output

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-b*(-a*e+b*d)^5*x/e^6+1/2*(-a*e+b*d)^4*(b*x+a)^2/e^5-1/3*(-a*e+b*d)^3*(b*x+a)^3/e^4+1/4*(-a*e+b*d)^2*(b*x+a)^4/e^3-1/5*(-a*e+b*d)*(b*x+a)^5/e^2+1/6*(b*x+a)^6/e+(-a*e+b*d)^6*ln(e*x+d)/e^7
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Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 230, normalized size of antiderivative = 1.58

$$\int \frac{(a^2 + 2abx + b^2x^2)^3}{d + ex} dx = \frac{bex(360a^5e^5 + 450a^4be^4(-2d + ex) + 200a^3b^2e^3(6d^2 - 3dex + 2e^2x^2) + 75a^2b^3e^2(-12d^3 + 6d^2ex - 4de^2x^2) + 15ab^4e(-12d^4 + 6d^3ex - 4d^2e^2x^2 + 2de^3x^3) + b^5e^2(-12d^5 + 6d^4ex - 4d^3e^2x^2 + 2d^2e^3x^3 - de^4x^4) + b^6e^3(-12d^6 + 6d^5ex - 4d^4e^2x^2 + 2d^3e^3x^3 - d^2e^4x^4 + de^5x^5) + b^7e^4(-12d^7 + 6d^6ex - 4d^5e^2x^2 + 2d^4e^3x^3 - d^3e^4x^4 + d^2e^5x^5 - de^6x^6) + b^8e^5(-12d^8 + 6d^7ex - 4d^6e^2x^2 + 2d^5e^3x^3 - d^4e^4x^4 + d^3e^5x^5 - d^2e^6x^6 + de^7x^7) + b^9e^6(-12d^9 + 6d^8ex - 4d^7e^2x^2 + 2d^6e^3x^3 - d^5e^4x^4 + d^4e^5x^5 - d^3e^6x^6 + d^2e^7x^7 - de^8x^8) + b^{10}e^7(-12d^{10} + 6d^9ex - 4d^8e^2x^2 + 2d^7e^3x^3 - d^6e^4x^4 + d^5e^5x^5 - d^4e^6x^6 + d^3e^7x^7 - d^2e^8x^8 + de^9x^9) + b^{11}e^8(-12d^{11} + 6d^{10}ex - 4d^9e^2x^2 + 2d^8e^3x^3 - d^7e^4x^4 + d^6e^5x^5 - d^5e^6x^6 + d^4e^7x^7 - d^3e^8x^8 + d^2e^9x^9 - de^{10}x^{10}) + b^{12}e^9(-12d^{12} + 6d^{11}ex - 4d^{10}e^2x^2 + 2d^9e^3x^3 - d^8e^4x^4 + d^7e^5x^5 - d^6e^6x^6 + d^5e^7x^7 - d^4e^8x^8 + d^3e^9x^9 - d^2e^{10}x^{10} + de^{11}x^{11}) + b^{13}e^{10}(-12d^{13} + 6d^{12}ex - 4d^{11}e^2x^2 + 2d^{10}e^3x^3 - d^9e^4x^4 + d^8e^5x^5 - d^7e^6x^6 + d^6e^7x^7 - d^5e^8x^8 + d^4e^9x^9 - d^3e^{10}x^{10} + d^2e^{11}x^{11} - de^{12}x^{12}) + b^{14}e^{11}(-12d^{14} + 6d^{13}ex - 4d^{12}e^2x^2 + 2d^{11}e^3x^3 - d^{10}e^4x^4 + d^9e^5x^5 - d^8e^6x^6 + d^7e^7x^7 - d^6e^8x^8 + d^5e^9x^9 - d^4e^{10}x^{10} + d^3e^{11}x^{11} - d^2e^{12}x^{12} + de^{13}x^{13}) + b^{15}e^{12}(-12d^{15} + 6d^{14}ex - 4d^{13}e^2x^2 + 2d^{12}e^3x^3 - d^{11}e^4x^4 + d^{10}e^5x^5 - d^9e^6x^6 + d^8e^7x^7 - d^7e^8x^8 + d^6e^9x^9 - d^5e^{10}x^{10} + d^4e^{11}x^{11} - d^3e^{12}x^{12} + d^2e^{13}x^{13} - de^{14}x^{14}) + b^{16}e^{13}(-12d^{16} + 6d^{15}ex - 4d^{14}e^2x^2 + 2d^{13}e^3x^3 - d^{12}e^4x^4 + d^{11}e^5x^5 - d^{10}e^6x^6 + d^9e^7x^7 - d^8e^8x^8 + d^7e^9x^9 - d^6e^{10}x^{10} + d^5e^{11}x^{11} - d^4e^{12}x^{12} + d^3e^{13}x^{13} - d^2e^{14}x^{14} + de^{15}x^{15}) + b^{17}e^{14}(-12d^{17} + 6d^{16}ex - 4d^{15}e^2x^2 + 2d^{14}e^3x^3 - d^{13}e^4x^4 + d^{12}e^5x^5 - d^{11}e^6x^6 + d^{10}e^7x^7 - d^9e^8x^8 + d^8e^9x^9 - d^7e^{10}x^{10} + d^6e^{11}x^{11} - d^5e^{12}x^{12} + d^4e^{13}x^{13} - d^3e^{14}x^{14} + d^2e^{15}x^{15} - de^{16}x^{16}) + b^{18}e^{15}(-12d^{18} + 6d^{17}ex - 4d^{16}e^2x^2 + 2d^{15}e^3x^3 - d^{14}e^4x^4 + d^{13}e^5x^5 - d^{12}e^6x^6 + d^{11}e^7x^7 - d^{10}e^8x^8 + d^9e^9x^9 - d^8e^{10}x^{10} + d^7e^{11}x^{11} - d^6e^{12}x^{12} + d^5e^{13}x^{13} - d^4e^{14}x^{14} + d^3e^{15}x^{15} - d^2e^{16}x^{16} + de^{17}x^{17}) + b^{19}e^{16}(-12d^{19} + 6d^{18}ex - 4d^{17}e^2x^2 + 2d^{16}e^3x^3 - d^{15}e^4x^4 + d^{14}e^5x^5 - d^{13}e^6x^6 + d^{12}e^7x^7 - d^{11}e^8x^8 + d^{10}e^9x^9 - d^9e^{10}x^{10} + d^8e^{11}x^{11} - d^7e^{12}x^{12} + d^6e^{13}x^{13} - d^5e^{14}x^{14} + d^4e^{15}x^{15} - d^3e^{16}x^{16} + d^2e^{17}x^{17} - de^{18}x^{18}) + b^{20}e^{17}(-12d^{20} + 6d^{19}ex - 4d^{18}e^2x^2 + 2d^{17}e^3x^3 - d^{16}e^4x^4 + d^{15}e^5x^5 - d^{14}e^6x^6 + d^{13}e^7x^7 - d^{12}e^8x^8 + d^{11}e^9x^9 - d^{10}e^{10}x^{10} + d^9e^{11}x^{11} - d^8e^{12}x^{12} + d^7e^{13}x^{13} - d^6e^{14}x^{14} + d^5e^{15}x^{15} - d^4e^{16}x^{16} + d^3e^{17}x^{17} - d^2e^{18}x^{18} + de^{19}x^{19}) + b^{21}e^{18}(-12d^{21} + 6d^{20}ex - 4d^{19}e^2x^2 + 2d^{18}e^3x^3 - d^{17}e^4x^4 + d^{16}e^5x^5 - d^{15}e^6x^6 + d^{14}e^7x^7 - d^{13}e^8x^8 + d^{12}e^9x^9 - d^{11}e^{10}x^{10} + d^{10}e^{11}x^{11} - d^9e^{12}x^{12} + d^8e^{13}x^{13} - d^7e^{14}x^{14} + d^6e^{15}x^{15} - d^5e^{16}x^{16} + d^4e^{17}x^{17} - d^3e^{18}x^{18} + d^2e^{19}x^{19} - de^{20}x^{20}) + b^{22}e^{19}(-12d^{22} + 6d^{21}ex - 4d^{20}e^2x^2 + 2d^{19}e^3x^3 - d^{18}e^4x^4 + d^{17}e^5x^5 - d^{16}e^6x^6 + d^{15}e^7x^7 - d^{14}e^8x^8 + d^{13}e^9x^9 - d^{12}e^{10}x^{10} + d^{11}e^{11}x^{11} - d^{10}e^{12}x^{12} + d^9e^{13}x^{13} - d^8e^{14}x^{14} + d^7e^{15}x^{15} - d^6e^{16}x^{16} + d^5e^{17}x^{17} - d^4e^{18}x^{18} + d^3e^{19}x^{19} - d^2e^{20}x^{20} + de^{21}x^{21}) + b^{23}e^{20}(-12d^{23} + 6d^{22}ex - 4d^{21}e^2x^2 + 2d^{20}e^3x^3 - d^{19}e^4x^4 + d^{18}e^5x^5 - d^{17}e^6x^6 + d^{16}e^7x^7 - d^{15}e^8x^8 + d^{14}e^9x^9 - d^{13}e^{10}x^{10} + d^{12}e^{11}x^{11} - d^{11}e^{12}x^{12} + d^{10}e^{13}x^{13} - d^9e^{14}x^{14} + d^8e^{15}x^{15} - d^7e^{16}x^{16} + d^6e^{17}x^{17} - d^5e^{18}x^{18} + d^4e^{19}x^{19} - d^3e^{20}x^{20} + d^2e^{21}x^{21} - de^{22}x^{22}) + b^{24}e^{21}(-12d^{24} + 6d^{23}ex - 4d^{22}e^2x^2 + 2d^{21}e^3x^3 - d^{20}e^4x^4 + d^{19}e^5x^5 - d^{18}e^6x^6 + d^{17}e^7x^7 - d^{16}e^8x^8 + d^{15}e^9x^9 - d^{14}e^{10}x^{10} + d^{13}e^{11}x^{11} - d^{12}e^{12}x^{12} + d^{11}e^{13}x^{13} - d^{10}e^{14}x^{14} + d^9e^{15}x^{15} - d^8e^{16}x^{16} + d^7e^{17}x^{17} - d^6e^{18}x^{18} + d^5e^{19}x^{19} - d^4e^{20}x^{20} + d^3e^{21}x^{21} - d^2e^{22}x^{22} + de^{23}x^{23}) + b^{25}e^{22}(-12d^{25} + 6d^{24}ex - 4d^{23}e^2x^2 + 2d^{22}e^3x^3 - d^{21}e^4x^4 + d^{20}e^5x^5 - d^{19}e^6x^6 + d^{18}e^7x^7 - d^{17}e^8x^8 + d^{16}e^9x^9 - d^{15}e^{10}x^{10} + d^{14}e^{11}x^{11} - d^{13}e^{12}x^{12} + d^{12}e^{13}x^{13} - d^{11}e^{14}x^{14} + d^{10}e^{15}x^{15} - d^9e^{16}x^{16} + d^8e^{17}x^{17} - d^7e^{18}x^{18} + d^6e^{19}x^{19} - d^5e^{20}x^{20} + d^4e^{21}x^{21} - d^3e^{22}x^{22} + d^2e^{23}x^{23} - de^{24}x^{24}) + b^{26}e^{23}(-12d^{26} + 6d^{25}ex - 4d^{24}e^2x^2 + 2d^{23}e^3x^3 - d^{22}e^4x^4 + d^{21}e^5x^5 - d^{20}e^6x^6 + d^{19}e^7x^7 - d^{18}e^8x^8 + d^{17}e^9x^9 - d^{16}e^{10}x^{10} + d^{15}e^{11}x^{11} - d^{14}e^{12}x^{12} + d^{13}e^{13}x^{13} - d^{12}e^{14}x^{14} + d^{11}e^{15}x^{15} - d^{10}e^{16}x^{16} + d^9e^{17}x^{17} - d^8e^{18}x^{18} + d^7e^{19}x^{19} - d^6e^{20}x^{20} + d^5e^{21}x^{21} - d^4e^{22}x^{22} + d^3e^{23}x^{23} - d^2e^{24}x^{24} + de^{25}x^{25}) + b^{27}e^{24}(-12d^{27} + 6d^{26}ex - 4d^{25}e^2x^2 + 2d^{24}e^3x^3 - d^{23}e^4x^4 + d^{22}e^5x^5 - d^{21}e^6x^6 + d^{20}e^7x^7 - d^{19}e^8x^8 + d^{18}e^9x^9 - d^{17}e^{10}x^{10} + d^{16}e^{11}x^{11} - d^{15}e^{12}x^{12} + d^{14}e^{13}x^{13} - d^{13}e^{14}x^{14} + d^{12}e^{15}x^{15} - d^{11}e^{16}x^{16} + d^{10}e^{17}x^{17} - d^9e^{18}x^{18} + d^8e^{19}x^{19} - d^7e^{20}x^{20} + d^6e^{21}x^{21} - d^5e^{22}x^{22} + d^4e^{23}x^{23} - d^3e^{24}x^{24} + d^2e^{25}x^{25} - de^{26}x^{26}) + b^{28}e^{25}(-12d^{28} + 6d^{27}ex - 4d^{26}e^2x^2 + 2d^{25}e^3x^3 - d^{24}e^4x^4 + d^{23}e^5x^5 - d^{22}e^6x^6 + d^{21}e^7x^7 - d^{20}e^8x^8 + d^{19}e^9x^9 - d^{18}e^{10}x^{10} + d^{17}e^{11}x^{11} - d^{16}e^{12}x^{12} + d^{15}e^{13}x^{13} - d^{14}e^{14}x^{14} + d^{13}e^{15}x^{15} - d^{12}e^{16}x^{16} + d^{11}e^{17}x^{17} - d^{10}e^{18}x^{18} + d^9e^{19}x^{19} - d^8e^{20}x^{20} + d^7e^{21}x^{21} - d^6e^{22}x^{22} + d^5e^{23}x^{23} - d^4e^{24}x^{24} + d^3e^{25}x^{25} - d^2e^{26}x^{26} + de^{27}x^{27}) + b^{29}e^{26}(-12d^{29} + 6d^{28}ex - 4d^{27}e^2x^2 + 2d^{26}e^3x^3 - d^{25}e^4x^4 + d^{24}e^5x^5 - d^{23}e^6x^6 + d^{22}e^7x^7 - d^{21}e^8x^8 + d^{20}e^9x^9 - d^{19}e^{10}x^{10} + d^{18}e^{11}x^{11} - d^{17}e^{12}x^{12} + d^{16}e^{13}x^{13} - d^{15}e^{14}x^{14} + d^{14}e^{15}x^{15} - d^{13}e^{16}x^{16} + d^{12}e^{17}x^{17} - d^{11}e^{18}x^{18} + d^{10}e^{19}x^{19} - d^9e^{20}x^{20} + d^8e^{21}x^{21} - d^7e^{22}x^{22} + d^6e^{23}x^{23} - d^5e^{24}x^{24} + d^4e^{25}x^{25} - d^3e^{26}x^{26} + d^2e^{27}x^{27} - de^{28}x^{28}) + b^{30}e^{27}(-12d^{30} + 6d^{29}ex - 4d^{28}e^2x^2 + 2d^{27}e^3x^3 - d^{26}e^4x^4 + d^{25}e^5x^5 - d^{24}e^6x^6 + d^{23}e^7x^7 - d^{22}e^8x^8 + d^{21}e^9x^9 - d^{20}e^{10}x^{10} + d^{19}e^{11}x^{11} - d^{18}e^{12}x^{12} + d^{17}e^{13}x^{13} - d^{16}e^{14}x^{14} + d^{15}e^{15}x^{15} - d^{14}e^{16}x^{16} + d^{13}e^{17}x^{17} - d^{12}e^{18}x^{18} + d^{11}e^{19}x^{19} - d^{10}e^{20}x^{20} + d^9e^{21}x^{21} - d^8e^{22}x^{22} + d^7e^{23}x^{23} - d^6e^{24}x^{24} + d^5e^{25}x^{25} - d^4e^{26}x^{26} + d^3e^{27}x^{27} - d^2e^{28}x^{28} + de^{29}x^{29}) + b^{31}e^{28}(-12d^{31} + 6d^{30}ex - 4d^{29}e^2x^2 + 2d^{28}e^3x^3 - d^{27}e^4x^4 + d^{26}e^5x^5 - d^{25}e^6x^6 + d^{24}e^7x^7 - d^{23}e^8x^8 + d^{22}e^9x^9 - d^{21}e^{10}x^{10} + d^{20}e^{11}x^{11} - d^{19}e^{12}x^{12} + d^{18}e^{13}x^{13} - d^{17}e^{14}x^{14} + d^{16}e^{15}x^{15} - d^{15}e^{16}x^{16} + d^{14}e^{17}x^{17} - d^{13}e^{18}x^{18} + d^{12}e^{19}x^{19} - d^{11}e^{20}x^{20} + d^{10}e^{21}x^{21} - d^9e^{22}x^{22} + d^8e^{23}x^{23} - d^7e^{24}x^{24} + d^6e^{25}x^{25} - d^5e^{26}x^{26} + d^4e^{27}x^{27} - d^3e^{28}x^{28} + d^2e^{29}x^{29} - de^{30}x^{30}) + b^{32}e^{29}(-12d^{32} + 6d^{31}ex - 4d^{30}e^2x^2 + 2d^{29}e^3x^3 - d^{28}e^4x^4 + d^{27}e^5x^5 - d^{26}e^6x^6 + d^{25}e^7x^7 - d^{24}e^8x^8 + d^{23}e^9x^9 - d^{22}e^{10}x^{10} + d^{21}e^{11}x^{11} - d^{20}e^{12}x^{12} + d^{19}e^{13}x^{13} - d^{18}e^{14}x^{14} + d^{17}e^{15}x^{15} - d^{16}e^{16}x^{16} + d^{15}e^{17}x^{17} - d^{14}e^{18}x^{18} + d^{13}e^{19}x^{19} - d^{12}e^{20}x^{20} + d^{11}e^{21}x^{21} - d^{10}e^{22}x^{22} + d^9e^{23}x^{23} - d^8e^{24}x^{24} + d^7e^{25}x^{25} - d^6e^{26}x^{26} + d^5e^{27}x^{27} - d^4e^{28}x^{28} + d^3e^{29}x^{29} - d^2e^{30}x^{30} + de^{31}x^{31}) + b^{33}e^{30}(-12d^{33} + 6d^{32}ex - 4d^{31}e^2x^2 + 2d^{30}e^3x^3 - d^{29}e^4x^4 + d^{28}e^5x^5 - d^{27}e^6x^6 + d^{26}e^7x^7 - d^{25}e^8x^8 + d^{24}e^9x^9 - d^{23}e^{10}x^{10} + d^{22}e^{11}x^{11} - d^{21}e^{12}x^{12} + d^{20}e^{13}x^{13} - d^{19}e^{14}x^{14} + d^{18}e^{15}x^{15} - d^{17}e^{16}x^{16} + d^{16}e^{17}x^{17} - d^{15}e^{18}x^{18} + d^{14}e^{19}x^{19} - d^{13}e^{20}x^{20} + d^{12}e^{21}x^{21} - d^{11}e^{22}x^{22} + d^{10}e^{23}x^{23} - d^9e^{24}x^{24} + d^8e^{25}x^{25} - d^7e^{26}x^{26} + d^6e^{27}x^{27} - d^5e^{28}x^{28} + d^4e^{29}x^{29} - d^3e^{30}x^{30} + d^2e^{31}x^{31} - de^{32}x^{32}) + b^{34}e^{31}(-12d^{34} + 6d^{33}ex - 4d^{32}e^2x^2 + 2d^{31}e^3x^3 - d^{30}e^4x^4 + d^{29}e^5x^5 - d^{28}e^6x^6 + d^{27}e^7x^7 - d^{26}e^8x^8 + d^{25}e^9x^9 - d^{24}e^{10}x^{10} + d^{23}e^{11}x^{11} - d^{22}e^{12}x^{12} + d^{21}e^{13}x^{13} - d^{20}e^{14}x^{14} + d^{19}e^{15}x^{15} - d^{18}e^{16}x^{16} + d^{17}e^{17}x^{17} - d^{16}e^{18}x^{18} + d^{15}e^{19}x^{19} - d^{14}e^{20}x^{20} + d^{13}e^{21}x^{21} - d^{12}e^{22}x^{22} + d^{11}e^{23}x^{23} - d^{10}e^{24}x^{24} + d^9e^{25}x^{25} - d^8e^{26}x^{26} + d^7e^{27}x^{27} - d^6e^{28}x^{28} + d^5e^{29}x^{29} - d^4e^{30}x^{30} + d^3e^{31}x^{31} - d^2e^{32}x^{32} + de^{33}x^{33}) + b^{35}e^{32}(-12d^{35} + 6d^{34}ex - 4d^{33}e^2x^2 + 2d^{32}e^3x^3 - d^{31}e^4x^4 + d^{30}e^5x^5 - d^{29}e^6x^6 + d^{28}e^7x^7 - d^{27}e^8x^8 + d^{26}e^9x^9 - d^{25}e^{10}x^{10} + d^{24}e^{11}x^{11} - d^{23}e^{12}x^{12} + d^{22}e^{13}x^{13} - d^{21}e^{14}x^{14} + d^{20}e^{15}x^{15} - d^{19}e^{16}x^{16} + d^{18}e^{17}x^{17} - d^{17}e^{18}x^{18} + d^{16}e^{19}x^{19} - d^{15}e^{20}x^{20} + d^{14}e^{21}x^{21} - d^{13}e^{22}x^{22} + d^{12}e^{23}x^{23} - d^{11}e^{24}x^{24} + d^{10}e^{25}x^{25} - d^9e^{26}x^{26} + d^8e^{27}x^{27} - d^7e^{28}x^{28} + d^6e^{29}x^{29} - d^5e^{30}x^{30} + d^4e^{31}x^{31} - d^3e^{32}x^{32} + d^2e^{33}x^{33} - de^{34}x^{34}) + b^{36}e^{33}(-12d^{36} + 6d^{35}ex - 4d^{34}e^2x^2 + 2d^{33}e^3x^3 - d^{32}e^4x^4 + d^{31}e^5x^5 - d^{30}e^6x^6 + d^{29}e^7x^7 - d^{28}e^8x^8 + d^{27}e^9x^9 - d^{26}e^{10}x^{10} + d^{25}e^{11}x^{11} - d^{24}e^{12}x^{12} + d^{23}e^{13}x^{13} - d^{22}e^{14}x^{14} + d^{21}e^{15}x^{15} - d^{20}e^{16}x^{16} + d^{19}e^{17}x^{17} - d^{18}e^{18}x^{18} + d^{17}e^{19}x^{19} - d^{16}e^{20}x^{20} + d^{15}e^{21}x^{21} - d^{14}e^{22}x^{22} + d^{13}e^{23}x^{23} - d^{12}e^{24}x^{24} + d^{11}e^{25}x^{25} - d^{10}e^{26}x^{26} + d^9e^{27}x^{27} - d^8e^{28}x^{28} + d^7e^{29}x^{29} - d^6e^{30}x^{30} + d^5e^{31}x^{31} - d^4e^{32}x^{32} + d^3e^{33}x^{33} - d^2e^{34}x^{34} + de^{35}x^{35}) + b^{37}e^{34}(-12d^{37} + 6d^{36}ex - 4d^{35}e^2x^2 + 2d^{34}e^3x^3 - d^{33}e^4x^4 + d^{32}e^5x^5 - d^{31}e^6x^6 + d^{30}e^7x^7 - d^{29}e^8x^8 + d^{28}e^9x^9 - d^{27}e^{10}x^{10} + d^{26}e^{11}x^{11} - d^{25}e^{12}x^{12} + d^{24}e^{13}x^{13} - d^{23}e^{14}x^{14} + d^{22}e^{15}x^{15} - d^{21}e^{16}x^{16} + d^{20}e^{17}x^{17} - d^{19}e^{18}x^{18} + d^{18}e^{19}x^{19} - d^{17}e^{20}x^{20} + d^{16}e^{21}x^{21} - d^{15}e^{22}x^{22} + d^{14}e^{23}x^{23} - d^{13}e^{24}x^{24} + d^{12}e^{25}x^{25} - d^{11}e^{26}x^{26} + d^{10}e^{27}x^{27} - d^9e^{28}x^{28} + d^8e^{29}x^{29} - d^7e^{30}x^{30} + d^6e^{31}x^{31} - d^5e^{32}x^{32} + d^4e^{33}x^{33} - d^3e^{34}x^{34} + d^2e^{35}x^{35} - de^{36}x^{36}) + b^{38}e^{35}(-12d^{38} + 6d^{37}ex - 4d^{36}e^2x^2 + 2d^{35}e^3x^3 - d^{34}e^4x^4 + d^{33}e^5x^5 - d^{32}e^6x^6 + d^{31}e^7x^7 - d^{30}e^8x^8 + d^{29}e^9x^9 - d^{28}e^{10}x^{10} + d^{27}e^{11}x^{11} - d^{26}e^{12}x^{12} + d^{25}e^{13}x^{13} - d^{24}e^{14}x^{14} + d^{23}e^{15}x^{15} - d^{22}e^{16}x^{16} + d^{21}e^{17}x^{17} - d^{20}e^{18}x^{18} + d^{19}e^{19}x^{19} - d^{18}e^{20}x^{20} + d^{17}e^{21}x^{21} - d^{16}e^{22}x^{22} + d^{15}e^{23}x^{23} - d^{14}e^{24}x^{24} + d^{13}e^{25}x^{25} - d^{12}e^{26}x^{26} + d^{11}e^{27}x^{27} - d^{10}e^{28}x^{28} + d^9e^{29}x^{29} - d^8e^{30}x^{30} + d^7e^{31}x^{31} - d^6e^{32}x^{32} + d^5e^{33}x^{33} - d^4e^{34}x^{34} + d^3e^{35}x^{35} - d^2e^{36}x^{36} + de^{37}x^{37}) + b^{39}e^{36}(-12d^{39} + 6d^{38}ex - 4d^{37}e^2x^2 + 2d^{36}e^3x^3 - d^{35}e^4x^4 + d^{34}e^5x^5 - d^{33}e^6x^6 + d^{32}e^7x^7 - d^{31}e^8x^8 + d^{30}e^9x^9 - d^{29}e^{10}x^{10} + d^{28}e^{11}x^{11} - d^{27}e^{12}x^{12} + d^{26}e^{13}x^{13} - d^{25}e^{14}x^{14} + d^{24}e^{15}x^{15} - d^{23}e^{16}x^{16} + d^{22}e^{17}x^{17} - d^{21}e^{18}x^{18} + d^{20}e^{19}x^{19} - d^{19}e^{20}x^{20} + d^{18}e^{21}x^{21} - d^{17}e^{22}x^{22} + d^{16}e^{23}x^{23} - d^{15}e^{24}x^{24} + d^{14}e^{25}x^{25} - d^{13}e^{26}x^{26} + d^{12}e^{27}x^{27} - d^{11}e^{28}x^{28} + d^{10}e^{29}x^{29} - d^9e^{30}x^{30} + d^8e^{31}x^{31} - d^7e^{32}x^{32} + d^6e^{33}x^{33} - d^5e^{34}x^{34} + d^4e^{35}x^{35} - d^3e^{36}x^{36} + d^2e^{37}x^{37} - de^{38}x^{38}) + b^{40}e^{37}(-12d^{40} + 6d^{39}ex - 4d^{38}e^2x^2 + 2d^{37}e^3x^3 - d^{36}e^4x^4 + d^{35}e^5x^5 - d^{34}e^6x^6 + d^{33}e^7x^7 - d^{32}e^8x^8 + d^{31}e^9x^9 - d^{30}e^{10}x^{10} + d^{29}e^{11}x^{11} - d^{28}e^{12}x^{12} + d^{27}e^{13}x^{13} - d^{26}e^{14}x^{14} + d^{25}e^{15}x^{15} - d^{24}e^{16}x^{16} + d^{23}e^{17}x^{17} - d^{22}e^{18}x^{18} + d^{21}e^{19}x^{19} - d^{20}e^{20}x^{20} + d^{19}e^{21}x^{21} - d^{18}e^{22}x^{22} + d^{17}e^{23}x^{23} - d^{16}e^{24}x^{24} + d^{15}e^{25}x^{25} - d^{14}e^{26}x^{26} + d^{13}e^{27}x^{27} - d^{12}e^{28}x^{28} + d^{11}e^{29}x^{29} - d^{10}e^{30}x^{30} + d^9e^{31}x^{31} - d^8e^{32}x^{32} + d^7e^{33}x^{33} - d^6e^{34}x^{34} + d^5e^{35}x^{35} - d$$

input `Integrate[(a^2 + 2*a*b*x + b^2*x^2)^3/(d + e*x),x]`

output $(b^6 e^7 (360 a^5 e^5 + 450 a^4 b e^4 (-2d + ex) + 200 a^3 b^2 e^3 (6d^2 - 3d ex + 2e^2 x^2) + 75 a^2 b^3 e^2 (-12d^3 + 6d^2 ex - 4d e^2 x^2 + 3e^3 x^3) + 6 a b^4 e (60d^4 - 30d^3 ex + 20d^2 e^2 x^2 - 15d e^3 x^3 + 12e^4 x^4) + b^5 (-60d^5 + 30d^4 ex - 20d^3 e^2 x^2 + 15d^2 e^3 x^3 - 12d e^4 x^4 + 10e^5 x^5)) + 60(bd - ae)^6 \text{Log}[d + ex]) / (60 e^7)$

Rubi [A] (verified)

Time = 0.52 (sec) , antiderivative size = 146, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {1098, 27, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a^2 + 2abx + b^2x^2)^3}{d + ex} dx$$

$$\downarrow 1098$$

$$\int \frac{b^6 (a+bx)^6}{d+ex} dx$$

$$\downarrow 27$$

$$\int \frac{(a+bx)^6}{d+ex} dx$$

$$\downarrow 49$$

$$\int \left(\frac{(ae - bd)^6}{e^6(d + ex)} - \frac{b(bd - ae)^5}{e^6} + \frac{b(a + bx)(bd - ae)^4}{e^5} - \frac{b(a + bx)^2(bd - ae)^3}{e^4} + \frac{b(a + bx)^3(bd - ae)^2}{e^3} - \frac{b(a + bx)^4(bd - ae)}{e^2} + \frac{b(a + bx)^5}{e} \right) dx$$

$$\downarrow 2009$$

$$\frac{(bd - ae)^6 \log(d + ex)}{e^7} - \frac{bx(bd - ae)^5}{e^6} + \frac{(a + bx)^2(bd - ae)^4}{2e^5} - \frac{(a + bx)^3(bd - ae)^3}{3e^4} + \frac{(a + bx)^4(bd - ae)^2}{4e^3} - \frac{(a + bx)^5(bd - ae)}{5e^2} + \frac{(a + bx)^6}{6e}$$

input `Int[(a^2 + 2*a*b*x + b^2*x^2)^3/(d + e*x),x]`

output `-((b*(b*d - a*e)^5*x)/e^6) + ((b*d - a*e)^4*(a + b*x)^2)/(2*e^5) - ((b*d - a*e)^3*(a + b*x)^3)/(3*e^4) + ((b*d - a*e)^2*(a + b*x)^4)/(4*e^3) - ((b*d - a*e)*(a + b*x)^5)/(5*e^2) + (a + b*x)^6/(6*e) + ((b*d - a*e)^6*Log[d + e*x])/e^7`

Defintions of rubi rules used

rule 27 `Int[(a_)*(Fx_), x_Symbol] := Simp[a Int[Fx, x], x] /; FreeQ[a, x] && !MatchQ[Fx, (b_)*(Gx_)] /; FreeQ[b, x]`

rule 49 `Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] := Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] && IGtQ[m, 0] && IGtQ[m + n + 2, 0]`

rule 1098 `Int[((d_.) + (e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] := Simp[1/c^p Int[(d + e*x)^m*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]`

rule 2009 `Int[u_, x_Symbol] := Simp[IntSum[u, x], x] /; SumQ[u]`

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 332 vs. $2(136) = 272$.

Time = 1.21 (sec) , antiderivative size = 333, normalized size of antiderivative = 2.28

| method | result |
|---------------|---|
| norman | $\frac{b(6e^5a^5-15a^4bd e^4+20a^3b^2d^2e^3-15a^2b^3d^3e^2+6ab^4d^4e-b^5d^5)x}{e^6} + \frac{b^6x^6}{6e} + \frac{b^5(6ae-bd)x^5}{5e^2} + \frac{b^2(15a^4e^4-20a^3bd e^3+15a^2b^2d^2e^2-6ab^3d^3e+b^4d^4)}{e^5}$ |
| risch | $-\frac{3b^5x^4ad}{2e^2} - \frac{5b^4x^3a^2d}{e^2} + \frac{2b^5x^3ad^2}{e^3} - \frac{10b^3x^2a^3d}{e^2} + \frac{15b^4x^2a^2d^2}{2e^3} - \frac{3b^5x^2ad^3}{e^4} - \frac{15b^2a^4dx}{e^2} + \frac{20b^3a^3d^2x}{e^3} - \frac{15b^4a^2d^3x}{e^4}$ |
| parallelrisch | $120x^3ab^5d^2e^4-600x^2a^3b^3de^5+450x^2a^2b^4d^2e^4-180x^2ab^5d^3e^3-900xa^4b^2de^5+1200xa^3b^3d^2e^4-900xa^2b^4d^3e^3+360xab^5d^2e^4$ |
| default | $b\left(\frac{b^5x^6e^5}{6} + \frac{((2ae-bd)b^2e^2+be(3abe^2-deb^2))b^2e^2+b^3e^3(ab e^2+de b^2)}{5}x^5 + \frac{((2ae-bd)(3abe^2-deb^2)+be(3e^2a^2-3abde+b^2d^2))}{5}x^4\right)$ |

```
input int((b^2*x^2+2*a*b*x+a^2)^3/(e*x+d), x, method=_RETURNVERBOSE)
```

```
output b*(6*a^5*e^5-15*a^4*b*d*e^4+20*a^3*b^2*d^2*e^3-15*a^2*b^3*d^3*e^2+6*a*b^4*d^4*e-b^5*d^5)/e^6*x+1/6/e*b^6*x^6+1/5*b^5/e^2*(6*a*e-b*d)*x^5+1/2/e^5*b^2*(15*a^4*e^4-20*a^3*b*d*e^3+15*a^2*b^2*d^2*e^2-6*a*b^3*d^3*e+b^4*d^4)*x^2+1/3/e^4*b^3*(20*a^3*e^3-15*a^2*b*d*e^2+6*a*b^2*d^2*e-b^3*d^3)*x^3+1/4/e^3*b^4*(15*a^2*e^2-6*a*b*d*e+b^2*d^2)*x^4+(a^6*e^6-6*a^5*b*d*e^5+15*a^4*b^2*d^2*e^4-20*a^3*b^3*d^3*e^3+15*a^2*b^4*d^4*e^2-6*a*b^5*d^5*e+b^6*d^6)/e^7*ln(e*x+d)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 351 vs. $2(136) = 272$.

Time = 0.08 (sec) , antiderivative size = 351, normalized size of antiderivative = 2.40

$$\int \frac{(a^2 + 2abx + b^2x^2)^3}{d + ex} dx$$

$$= \frac{10b^6e^6x^6 - 12(b^6de^5 - 6ab^5e^6)x^5 + 15(b^6d^2e^4 - 6ab^5de^5 + 15a^2b^4e^6)x^4 - 20(b^6d^3e^3 - 6ab^5d^2e^4 + 15a^2b^4d^3e^2 - 6ab^5d^4e + b^6d^5)x^3 + 15(b^6d^4e^2 - 6ab^5d^3e^2 + 15a^2b^4d^4e - 6ab^5d^5 + b^6d^6)x^2 - 20(b^6d^5e - 6ab^5d^4e + 15a^2b^4d^5 - 6ab^5d^6 + b^6d^7)x + 15(b^6d^6 - 6ab^5d^5 + 15a^2b^4d^6 - 6ab^5d^7 + b^6d^8) \ln(d + ex)}{e^7}$$

```
input integrate((b^2*x^2+2*a*b*x+a^2)^3/(e*x+d), x, algorithm="fricas")
```

output

```
1/60*(10*b^6*e^6*x^6 - 12*(b^6*d*e^5 - 6*a*b^5*e^6)*x^5 + 15*(b^6*d^2*e^4
- 6*a*b^5*d*e^5 + 15*a^2*b^4*e^6)*x^4 - 20*(b^6*d^3*e^3 - 6*a*b^5*d^2*e^4
+ 15*a^2*b^4*d*e^5 - 20*a^3*b^3*e^6)*x^3 + 30*(b^6*d^4*e^2 - 6*a*b^5*d^3*e
^3 + 15*a^2*b^4*d^2*e^4 - 20*a^3*b^3*d*e^5 + 15*a^4*b^2*e^6)*x^2 - 60*(b^6
*d^5*e - 6*a*b^5*d^4*e^2 + 15*a^2*b^4*d^3*e^3 - 20*a^3*b^3*d^2*e^4 + 15*a^
4*b^2*d*e^5 - 6*a^5*b*e^6)*x + 60*(b^6*d^6 - 6*a*b^5*d^5*e + 15*a^2*b^4*d^
4*e^2 - 20*a^3*b^3*d^3*e^3 + 15*a^4*b^2*d^2*e^4 - 6*a^5*b*d*e^5 + a^6*e^6)
*log(e*x + d))/e^7
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 296 vs. $2(124) = 248$.

Time = 0.33 (sec) , antiderivative size = 296, normalized size of antiderivative = 2.03

$$\int \frac{(a^2 + 2abx + b^2x^2)^3}{d + ex} dx = \frac{b^6x^6}{6e} + x^5 \cdot \left(\frac{6ab^5}{5e} - \frac{b^6d}{5e^2} \right) + x^4 \cdot \left(\frac{15a^2b^4}{4e} - \frac{3ab^5d}{2e^2} + \frac{b^6d^2}{4e^3} \right) + x^3 \cdot \left(\frac{20a^3b^3}{3e} - \frac{5a^2b^4d}{e^2} + \frac{2ab^5d^2}{e^3} - \frac{b^6d^3}{3e^4} \right) + x^2 \cdot \left(\frac{15a^4b^2}{2e} - \frac{10a^3b^3d}{e^2} + \frac{15a^2b^4d^2}{2e^3} - \frac{3ab^5d^3}{e^4} + \frac{b^6d^4}{2e^5} \right) + x \cdot \left(\frac{6a^5b}{e} - \frac{15a^4b^2d}{e^2} + \frac{20a^3b^3d^2}{e^3} - \frac{15a^2b^4d^3}{e^4} + \frac{6ab^5d^4}{e^5} - \frac{b^6d^5}{e^6} \right) + \frac{(ae - bd)^6 \log(d + ex)}{e^7}$$

input

```
integrate((b**2*x**2+2*a*b*x+a**2)**3/(e*x+d),x)
```

output

```
b**6*x**6/(6*e) + x**5*(6*a*b**5/(5*e) - b**6*d/(5*e**2)) + x**4*(15*a**2*
b**4/(4*e) - 3*a*b**5*d/(2*e**2) + b**6*d**2/(4*e**3)) + x**3*(20*a**3*b**
3/(3*e) - 5*a**2*b**4*d/e**2 + 2*a*b**5*d**2/e**3 - b**6*d**3/(3*e**4)) +
x**2*(15*a**4*b**2/(2*e) - 10*a**3*b**3*d/e**2 + 15*a**2*b**4*d**2/(2*e**3
) - 3*a*b**5*d**3/e**4 + b**6*d**4/(2*e**5)) + x*(6*a**5*b/e - 15*a**4*b**
2*d/e**2 + 20*a**3*b**3*d**2/e**3 - 15*a**2*b**4*d**3/e**4 + 6*a*b**5*d**4
/e**5 - b**6*d**5/e**6) + (a*e - b*d)**6*log(d + e*x)/e**7
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 349 vs. $2(136) = 272$.

Time = 0.05 (sec) , antiderivative size = 349, normalized size of antiderivative = 2.39

$$\int \frac{(a^2 + 2abx + b^2x^2)^3}{d + ex} dx$$

$$= \frac{10b^6e^5x^6 - 12(b^6de^4 - 6ab^5e^5)x^5 + 15(b^6d^2e^3 - 6ab^5de^4 + 15a^2b^4e^5)x^4 - 20(b^6d^3e^2 - 6ab^5d^2e^3 + 15a^2b^4d^3e^2 - 20a^3b^3d^3e^3 + 15a^4b^2d^2e^4 - 6a^5bde^5 + a^6e^6) \log(ex + d)}{e^7}$$

input `integrate((b^2*x^2+2*a*b*x+a^2)^3/(e*x+d),x, algorithm="maxima")`

output `1/60*(10*b^6*e^5*x^6 - 12*(b^6*d*e^4 - 6*a*b^5*e^5)*x^5 + 15*(b^6*d^2*e^3 - 6*a*b^5*d*e^4 + 15*a^2*b^4*e^5)*x^4 - 20*(b^6*d^3*e^2 - 6*a*b^5*d^2*e^3 + 15*a^2*b^4*d*e^4 - 20*a^3*b^3*e^5)*x^3 + 30*(b^6*d^4*e - 6*a*b^5*d^3*e^2 + 15*a^2*b^4*d^2*e^3 - 20*a^3*b^3*d*e^4 + 15*a^4*b^2*e^5)*x^2 - 60*(b^6*d^5 - 6*a*b^5*d^4*e + 15*a^2*b^4*d^3*e^2 - 20*a^3*b^3*d^2*e^3 + 15*a^4*b^2*d*e^4 - 6*a^5*b*d*e^5)*x)/e^6 + (b^6*d^6 - 6*a*b^5*d^5*e + 15*a^2*b^4*d^4*e^2 - 20*a^3*b^3*d^3*e^3 + 15*a^4*b^2*d^2*e^4 - 6*a^5*b*d*e^5 + a^6*e^6)*log(e*x + d)/e^7`

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 375 vs. $2(136) = 272$.

Time = 0.22 (sec) , antiderivative size = 375, normalized size of antiderivative = 2.57

$$\int \frac{(a^2 + 2abx + b^2x^2)^3}{d + ex} dx$$

$$= \frac{10b^6e^5x^6 - 12b^6de^4x^5 + 72ab^5e^5x^5 + 15b^6d^2e^3x^4 - 90ab^5de^4x^4 + 225a^2b^4e^5x^4 - 20b^6d^3e^2x^3 + 120ab^5d^3e^2x^3 - 20a^3b^3d^3e^3x^3 + 15a^4b^2d^2e^4x^2 - 6a^5bde^5x^2 + a^6e^6x^2 - 60a^5bde^5x + 60a^6e^6x - 60a^5bde^5 + 60a^6e^6) \log(|ex + d|)}{e^7}$$

input `integrate((b^2*x^2+2*a*b*x+a^2)^3/(e*x+d),x, algorithm="giac")`

output

```
1/60*(10*b^6*e^5*x^6 - 12*b^6*d*e^4*x^5 + 72*a*b^5*e^5*x^5 + 15*b^6*d^2*e^
3*x^4 - 90*a*b^5*d*e^4*x^4 + 225*a^2*b^4*e^5*x^4 - 20*b^6*d^3*e^2*x^3 + 12
0*a*b^5*d^2*e^3*x^3 - 300*a^2*b^4*d*e^4*x^3 + 400*a^3*b^3*e^5*x^3 + 30*b^6
*d^4*e*x^2 - 180*a*b^5*d^3*e^2*x^2 + 450*a^2*b^4*d^2*e^3*x^2 - 600*a^3*b^3
*d*e^4*x^2 + 450*a^4*b^2*e^5*x^2 - 60*b^6*d^5*x + 360*a*b^5*d^4*e*x - 900*
a^2*b^4*d^3*e^2*x + 1200*a^3*b^3*d^2*e^3*x - 900*a^4*b^2*d*e^4*x + 360*a^5
*b*e^5*x)/e^6 + (b^6*d^6 - 6*a*b^5*d^5*e + 15*a^2*b^4*d^4*e^2 - 20*a^3*b^3
*d^3*e^3 + 15*a^4*b^2*d^2*e^4 - 6*a^5*b*d*e^5 + a^6*e^6)*log(abs(e*x + d))
/e^7
```

Mupad [B] (verification not implemented)

Time = 8.69 (sec) , antiderivative size = 385, normalized size of antiderivative = 2.64

$$\begin{aligned}
 & \int \frac{(a^2 + 2abx + b^2x^2)^3}{d + ex} dx \\
 &= x^5 \left(\frac{6ab^5}{5e} - \frac{b^6d}{5e^2} \right) + x^3 \left(\frac{d \left(\frac{d \left(\frac{6ab^5}{e} - \frac{b^6d}{e^2} \right) - \frac{15a^2b^4}{e} \right)}{3e} + \frac{20a^3b^3}{3e} \right) \\
 & \quad \left(\frac{d \left(\frac{d \left(\frac{d \left(\frac{6ab^5}{e} - \frac{b^6d}{e^2} \right) - \frac{15a^2b^4}{e} \right) + \frac{20a^3b^3}{e}}{e} \right) - \frac{15a^4b^2}{e}}{e} \right) \\
 & + x \left(\frac{d \left(\frac{d \left(\frac{d \left(\frac{6ab^5}{e} - \frac{b^6d}{e^2} \right) - \frac{15a^2b^4}{e} \right) + \frac{20a^3b^3}{e}}{e} \right) - \frac{15a^4b^2}{e}}{e} \right) + \frac{6a^5b}{e} \\
 & - x^4 \left(\frac{d \left(\frac{6ab^5}{e} - \frac{b^6d}{e^2} \right) - \frac{15a^2b^4}{4e}}{4e} \right) \\
 & - x^2 \left(\frac{d \left(\frac{d \left(\frac{d \left(\frac{6ab^5}{e} - \frac{b^6d}{e^2} \right) - \frac{15a^2b^4}{e} \right) + \frac{20a^3b^3}{e}}{e} \right) - \frac{15a^4b^2}{2e}}{2e} \right) \\
 & + \frac{\ln(d + ex) (a^6e^6 - 6a^5bde^5 + 15a^4b^2d^2e^4 - 20a^3b^3d^3e^3 + 15a^2b^4d^4e^2 - 6ab^5d^5e + b^6d^6)}{e^7} \\
 & + \frac{b^6x^6}{6e}
 \end{aligned}$$

input `int((a^2 + b^2*x^2 + 2*a*b*x)^3/(d + e*x), x)`

output `x^5*((6*a*b^5)/(5*e) - (b^6*d)/(5*e^2)) + x^3*((d*((d*((6*a*b^5)/e - (b^6*d)/e^2))/e - (15*a^2*b^4)/e))/(3*e) + (20*a^3*b^3)/(3*e)) + x*((d*((d*((d*((d*((6*a*b^5)/e - (b^6*d)/e^2))/e - (15*a^2*b^4)/e))/e - (20*a^3*b^3)/e))/e - (15*a^4*b^2)/e))/e + (6*a^5*b)/e) - x^4*((d*((6*a*b^5)/e - (b^6*d)/e^2))/(4*e) - (15*a^2*b^4)/(4*e)) - x^2*((d*((d*((d*((6*a*b^5)/e - (b^6*d)/e^2))/e - (15*a^2*b^4)/e))/e + (20*a^3*b^3)/e))/(2*e) - (15*a^4*b^2)/(2*e)) + (log(d + e*x)*(a^6*e^6 + b^6*d^6 + 15*a^2*b^4*d^4*e^2 - 20*a^3*b^3*d^3*e^3 + 15*a^4*b^2*d^2*e^4 - 6*a*b^5*d^5*e - 6*a^5*b*d*e^5))/e^7 + (b^6*x^6)/(6*e)`

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 411, normalized size of antiderivative = 2.82

$$\int \frac{(a^2 + 2abx + b^2x^2)^3}{d + ex} dx$$

$$= \frac{450a^2b^4d^2e^4x^2 - 300a^2b^4de^5x^3 + 360ab^5d^4e^2x - 180ab^5d^3e^3x^2 + 120ab^5d^2e^4x^3 - 90ab^5de^5x^4 - 360 \log(d + ex)(a^6e^6 + b^6d^6 + 15a^2b^4d^4e^2 - 20a^3b^3d^3e^3 + 15a^4b^2d^2e^4 - 6ab^5d^5e - 6a^5bd^5e^5)}{6e^7}$$

input `int((b^2*x^2+2*a*b*x+a^2)^3/(e*x+d), x)`

output `(60*log(d + e*x)*a**6*e**6 - 360*log(d + e*x)*a**5*b*d*e**5 + 900*log(d + e*x)*a**4*b**2*d**2*e**4 - 1200*log(d + e*x)*a**3*b**3*d**3*e**3 + 900*log(d + e*x)*a**2*b**4*d**4*e**2 - 360*log(d + e*x)*a*b**5*d**5*e + 60*log(d + e*x)*b**6*d**6 + 360*a**5*b*e**6*x - 900*a**4*b**2*d*e**5*x + 450*a**4*b**2*e**6*x**2 + 1200*a**3*b**3*d**2*e**4*x - 600*a**3*b**3*d*e**5*x**2 + 400*a**3*b**3*e**6*x**3 - 900*a**2*b**4*d**3*e**3*x + 450*a**2*b**4*d**2*e**4*x**2 - 300*a**2*b**4*d*e**5*x**3 + 225*a**2*b**4*e**6*x**4 + 360*a*b**5*d**4*e**2*x - 180*a*b**5*d**3*e**3*x**2 + 120*a*b**5*d**2*e**4*x**3 - 90*a*b**5*d*e**5*x**4 + 72*a*b**5*e**6*x**5 - 60*b**6*d**5*e*x + 30*b**6*d**4*e**2*x**2 - 20*b**6*d**3*e**3*x**3 + 15*b**6*d**2*e**4*x**4 - 12*b**6*d*e**5*x**5 + 10*b**6*e**6*x**6)/(60*e**7)`

3.94 $\int \frac{(a^2+2abx+b^2x^2)^3}{(d+ex)^2} dx$

| | |
|---|-----|
| Optimal result | 773 |
| Mathematica [A] (verified) | 773 |
| Rubi [A] (verified) | 774 |
| Maple [B] (verified) | 776 |
| Fricas [B] (verification not implemented) | 776 |
| Sympy [B] (verification not implemented) | 777 |
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| Giac [B] (verification not implemented) | 779 |
| Mupad [B] (verification not implemented) | 780 |
| Reduce [B] (verification not implemented) | 781 |

Optimal result

Integrand size = 26, antiderivative size = 156

$$\int \frac{(a^2 + 2abx + b^2x^2)^3}{(d + ex)^2} dx = \frac{15b^2(bd - ae)^4x}{e^6} - \frac{(bd - ae)^6}{e^7(d + ex)} - \frac{10b^3(bd - ae)^3(d + ex)^2}{e^7} + \frac{5b^4(bd - ae)^2(d + ex)^3}{e^7} - \frac{3b^5(bd - ae)(d + ex)^4}{2e^7} + \frac{b^6(d + ex)^5}{5e^7} - \frac{6b(bd - ae)^5 \log(d + ex)}{e^7}$$

output

```
15*b^2*(-a*e+b*d)^4*x/e^6-(-a*e+b*d)^6/e^7/(e*x+d)-10*b^3*(-a*e+b*d)^3*(e*x+d)^2/e^7+5*b^4*(-a*e+b*d)^2*(e*x+d)^3/e^7-3/2*b^5*(-a*e+b*d)*(e*x+d)^4/e^7+1/5*b^6*(e*x+d)^5/e^7-6*b*(-a*e+b*d)^5*ln(e*x+d)/e^7
```

Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 302, normalized size of antiderivative = 1.94

$$\int \frac{(a^2 + 2abx + b^2x^2)^3}{(d + ex)^2} dx = \frac{60a^5bde^5 - 10a^6e^6 + 150a^4b^2e^4(-d^2 + dex + e^2x^2) + 100a^3b^3e^3(2d^3 - 4d^2ex - 3de^2x^2 + e^3x^3) + 50a^2b^4e^2(-d^2 + dex + e^2x^2) + 10a^2b^3e^3(2d^2 - 4dex + e^2x^2) + 10ab^4e^4(-d + ex + e^2x) + 5b^5e^5(-d + ex) + 5b^6e^6}{e^7}$$

input `Integrate[(a^2 + 2*a*b*x + b^2*x^2)^3/(d + e*x)^2,x]`

output $(60*a^5*b*d*e^5 - 10*a^6*e^6 + 150*a^4*b^2*e^4*(-d^2 + d*e*x + e^2*x^2) + 100*a^3*b^3*e^3*(2*d^3 - 4*d^2*e*x - 3*d*e^2*x^2 + e^3*x^3) + 50*a^2*b^4*e^2*(-3*d^4 + 9*d^3*e*x + 6*d^2*e^2*x^2 - 2*d*e^3*x^3 + e^4*x^4) + 5*a*b^5*e*(12*d^5 - 48*d^4*e*x - 30*d^3*e^2*x^2 + 10*d^2*e^3*x^3 - 5*d*e^4*x^4 + 3*e^5*x^5) + b^6*(-10*d^6 + 50*d^5*e*x + 30*d^4*e^2*x^2 - 10*d^3*e^3*x^3 + 5*d^2*e^4*x^4 - 3*d*e^5*x^5 + 2*e^6*x^6) - 60*b*(b*d - a*e)^5*(d + e*x)*\text{Log}[d + e*x])/(10*e^7*(d + e*x))$

Rubi [A] (verified)

Time = 0.70 (sec) , antiderivative size = 156, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {1098, 27, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a^2 + 2abx + b^2x^2)^3}{(d + ex)^2} dx$$

$$\downarrow 1098$$

$$\frac{\int \frac{b^6(a+bx)^6}{(d+ex)^2} dx}{b^6}$$

$$\downarrow 27$$

$$\int \frac{(a + bx)^6}{(d + ex)^2} dx$$

$$\downarrow 49$$

$$\int \left(-\frac{6b^5(d + ex)^3(bd - ae)}{e^6} + \frac{15b^4(d + ex)^2(bd - ae)^2}{e^6} - \frac{20b^3(d + ex)(bd - ae)^3}{e^6} + \frac{15b^2(bd - ae)^4}{e^6} - \frac{6b(bd - ae)^5}{e^6(d + ex)} \right) dx$$

$$\downarrow 2009$$

$$-\frac{3b^5(d+ex)^4(bd-ae)}{2e^7} + \frac{5b^4(d+ex)^3(bd-ae)^2}{e^6} - \frac{10b^3(d+ex)^2(bd-ae)^3}{e^7} + \frac{15b^2x(bd-ae)^4}{e^6} - \frac{(bd-ae)^6}{e^7(d+ex)} - \frac{e^7}{6b(bd-ae)^5 \log(d+ex)} + \frac{e^7}{5e^7} \frac{(d+ex)^5}{5e^7}$$

input `Int[(a^2 + 2*a*b*x + b^2*x^2)^3/(d + e*x)^2,x]`

output `(15*b^2*(b*d - a*e)^4*x)/e^6 - (b*d - a*e)^6/(e^7*(d + e*x)) - (10*b^3*(b*d - a*e)^3*(d + e*x)^2)/e^7 + (5*b^4*(b*d - a*e)^2*(d + e*x)^3)/e^7 - (3*b^5*(b*d - a*e)*(d + e*x)^4)/(2*e^7) + (b^6*(d + e*x)^5)/(5*e^7) - (6*b*(b*d - a*e)^5*Log[d + e*x])/e^7`

Defintions of rubi rules used

rule 27 `Int[(a_)*(F_x_), x_Symbol] := Simp[a Int[F_x, x], x] /; FreeQ[a, x] && !MatchQ[F_x, (b_)*(G_x_)] /; FreeQ[b, x]`

rule 49 `Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] := Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] && IGtQ[m, 0] && IGtQ[m + n + 2, 0]`

rule 1098 `Int[((d_.) + (e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] := Simp[1/c^p Int[(d + e*x)^m*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]`

rule 2009 `Int[u_, x_Symbol] := Simp[IntSum[u, x], x] /; SumQ[u]`

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 344 vs. 2(152) = 304.

Time = 1.10 (sec) , antiderivative size = 345, normalized size of antiderivative = 2.21

| method | result |
|--------------|--|
| norman | $\frac{b^3(10e^3a^3-10a^2bde^2+5ab^2d^2e-b^3d^3)x^3}{e^4} + \frac{(a^6e^6-6a^5bde^5+30a^4b^2d^2e^4-60a^3b^3d^3e^3+60a^2b^4d^4e^2-30ab^5d^5e+6b^6d^6)x}{de^6} + \frac{b^6x^6}{5e} + \dots$ |
| default | $\frac{b^2(\frac{1}{5}b^4x^5e^4+\frac{3}{2}x^4ab^3e^4-\frac{1}{2}x^4b^4de^3+5x^3a^2b^2e^4-4x^3ab^3de^3+x^3b^4d^2e^2+10x^2a^3be^4-15x^2a^2b^2de^3+9x^2ab^3d^2e^2-2x^2b^4d^2e^2)}{e^6}$ |
| risch | $-\frac{4b^5x^3ad}{e^3} - \frac{15b^4x^2a^2d}{e^3} - \frac{15a^4b^2d^2}{e^3(ex+d)} + \frac{20a^3b^3d^3}{e^4(ex+d)} - \frac{15a^2b^4d^4}{e^5(ex+d)} + \frac{6ab^5d^5}{e^6(ex+d)} + \frac{9b^5x^2ad^2}{e^4} - \frac{40b^3a^3dx}{e^3} + \frac{45b^4a^2}{e^4}$ |
| paralelrisch | $\frac{300ab^5d^5e+60a^5bde^5+600a^3b^3d^3e^3-300a^4b^2d^2e^4-600\ln(ex+d)xa^2b^4d^3e^3-600a^2b^4d^4e^2+60\ln(ex+d)xa^5be^6-60\ln(ex+d)}$ |

```
input int((b^2*x^2+2*a*b*x+a^2)^3/(e*x+d)^2,x,method=_RETURNVERBOSE)
```

```
output (b^3*(10*a^3*e^3-10*a^2*b*d*e^2+5*a*b^2*d^2*e-b^3*d^3)/e^4*x^3+(a^6*e^6-6*a^5*b*d*e^5+30*a^4*b^2*d^2*e^4-60*a^3*b^3*d^3*e^3+60*a^2*b^4*d^4*e^2-30*a*b^5*d^5*e+6*b^6*d^6)/d/e^6*x+1/5/e*b^6*x^6+3*b^2*(5*a^4*e^4-10*a^3*b*d*e^3+10*a^2*b^2*d^2*e^2-5*a*b^3*d^3*e+b^4*d^4)/e^5*x^2+1/2*b^4*(10*a^2*e^2-5*a*b*d*e+b^2*d^2)/e^3*x^4+3/10*b^5*(5*a*e-b*d)/e^2*x^5)/(e*x+d)+6*b/e^7*(a^5*e^5-5*a^4*b*d*e^4+10*a^3*b^2*d^2*e^3-10*a^2*b^3*d^3*e^2+5*a*b^4*d^4*e-b^5*d^5)*ln(e*x+d)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 496 vs. 2(152) = 304.

Time = 0.08 (sec) , antiderivative size = 496, normalized size of antiderivative = 3.18

$$\int \frac{(a^2 + 2abx + b^2x^2)^3}{(d + ex)^2} dx$$

$$= \frac{2b^6e^6x^6 - 10b^6d^6 + 60ab^5d^5e - 150a^2b^4d^4e^2 + 200a^3b^3d^3e^3 - 150a^4b^2d^2e^4 + 60a^5bde^5 - 10a^6e^6 - 3(\dots)}{\dots}$$

```
input integrate((b^2*x^2+2*a*b*x+a^2)^3/(e*x+d)^2,x, algorithm="fricas")
```

output

```

1/10*(2*b^6*e^6*x^6 - 10*b^6*d^6 + 60*a*b^5*d^5*e - 150*a^2*b^4*d^4*e^2 +
200*a^3*b^3*d^3*e^3 - 150*a^4*b^2*d^2*e^4 + 60*a^5*b*d*e^5 - 10*a^6*e^6 -
3*(b^6*d*e^5 - 5*a*b^5*e^6)*x^5 + 5*(b^6*d^2*e^4 - 5*a*b^5*d*e^5 + 10*a^2*
b^4*e^6)*x^4 - 10*(b^6*d^3*e^3 - 5*a*b^5*d^2*e^4 + 10*a^2*b^4*d*e^5 - 10*a
^3*b^3*e^6)*x^3 + 30*(b^6*d^4*e^2 - 5*a*b^5*d^3*e^3 + 10*a^2*b^4*d^2*e^4 -
10*a^3*b^3*d*e^5 + 5*a^4*b^2*e^6)*x^2 + 10*(5*b^6*d^5*e - 24*a*b^5*d^4*e^
2 + 45*a^2*b^4*d^3*e^3 - 40*a^3*b^3*d^2*e^4 + 15*a^4*b^2*d*e^5)*x - 60*(b^
6*d^6 - 5*a*b^5*d^5*e + 10*a^2*b^4*d^4*e^2 - 10*a^3*b^3*d^3*e^3 + 5*a^4*b^
2*d^2*e^4 - a^5*b*d*e^5 + (b^6*d^5*e - 5*a*b^5*d^4*e^2 + 10*a^2*b^4*d^3*e^
3 - 10*a^3*b^3*d^2*e^4 + 5*a^4*b^2*d*e^5 - a^5*b*e^6)*x)*log(e*x + d))/(e^
8*x + d*e^7)

```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 311 vs. $2(143) = 286$.

Time = 0.59 (sec) , antiderivative size = 311, normalized size of antiderivative = 1.99

$$\begin{aligned}
& \int \frac{(a^2 + 2abx + b^2x^2)^3}{(d + ex)^2} dx \\
&= \frac{b^6x^5}{5e^2} + \frac{6b(ae - bd)^5 \log(d + ex)}{e^7} + x^4 \cdot \left(\frac{3ab^5}{2e^2} - \frac{b^6d}{2e^3} \right) + x^3 \\
&\quad \cdot \left(\frac{5a^2b^4}{e^2} - \frac{4ab^5d}{e^3} + \frac{b^6d^2}{e^4} \right) + x^2 \cdot \left(\frac{10a^3b^3}{e^2} - \frac{15a^2b^4d}{e^3} + \frac{9ab^5d^2}{e^4} - \frac{2b^6d^3}{e^5} \right) \\
&\quad + x \left(\frac{15a^4b^2}{e^2} - \frac{40a^3b^3d}{e^3} + \frac{45a^2b^4d^2}{e^4} - \frac{24ab^5d^3}{e^5} + \frac{5b^6d^4}{e^6} \right) \\
&\quad + \frac{-a^6e^6 + 6a^5bde^5 - 15a^4b^2d^2e^4 + 20a^3b^3d^3e^3 - 15a^2b^4d^4e^2 + 6ab^5d^5e - b^6d^6}{de^7 + e^8x}
\end{aligned}$$

input

```
integrate((b**2*x**2+2*a*b*x+a**2)**3/(e*x+d)**2,x)
```

output

```

b**6*x**5/(5*e**2) + 6*b*(a*e - b*d)**5*log(d + e*x)/e**7 + x**4*(3*a*b**5
/(2*e**2) - b**6*d/(2*e**3)) + x**3*(5*a**2*b**4/e**2 - 4*a*b**5*d/e**3 +
b**6*d**2/e**4) + x**2*(10*a**3*b**3/e**2 - 15*a**2*b**4*d/e**3 + 9*a*b**5
*d**2/e**4 - 2*b**6*d**3/e**5) + x*(15*a**4*b**2/e**2 - 40*a**3*b**3*d/e**
3 + 45*a**2*b**4*d**2/e**4 - 24*a*b**5*d**3/e**5 + 5*b**6*d**4/e**6) + (-a
**6*e**6 + 6*a**5*b*d*e**5 - 15*a**4*b**2*d**2*e**4 + 20*a**3*b**3*d**3*e
**3 - 15*a**2*b**4*d**4*e**2 + 6*a*b**5*d**5*e - b**6*d**6)/(d*e**7 + e**8*
x)

```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 357 vs. $2(152) = 304$.

Time = 0.04 (sec) , antiderivative size = 357, normalized size of antiderivative = 2.29

$$\int \frac{(a^2 + 2abx + b^2x^2)^3}{(d + ex)^2} dx$$

$$= -\frac{b^6d^6 - 6ab^5d^5e + 15a^2b^4d^4e^2 - 20a^3b^3d^3e^3 + 15a^4b^2d^2e^4 - 6a^5bde^5 + a^6e^6}{e^8x + de^7}$$

$$+ \frac{2b^6e^4x^5 - 5(b^6de^3 - 3ab^5e^4)x^4 + 10(b^6d^2e^2 - 4ab^5de^3 + 5a^2b^4e^4)x^3 - 10(2b^6d^3e - 9ab^5d^2e^2 + 15a^2b^4d^3e^3 - 10a^3b^3d^2e^4 + 5a^4b^2de^4 - a^5be^5)\log(ex + d)}{e^7}$$

input

```

integrate((b^2*x^2+2*a*b*x+a^2)^3/(e*x+d)^2,x, algorithm="maxima")

```

output

```

-(b^6*d^6 - 6*a*b^5*d^5*e + 15*a^2*b^4*d^4*e^2 - 20*a^3*b^3*d^3*e^3 + 15*a
^4*b^2*d^2*e^4 - 6*a^5*b*d*e^5 + a^6*e^6)/(e^8*x + d*e^7) + 1/10*(2*b^6*e^
4*x^5 - 5*(b^6*d*e^3 - 3*a*b^5*e^4)*x^4 + 10*(b^6*d^2*e^2 - 4*a*b^5*d*e^3
+ 5*a^2*b^4*e^4)*x^3 - 10*(2*b^6*d^3*e - 9*a*b^5*d^2*e^2 + 15*a^2*b^4*d*d*e^
3 - 10*a^3*b^3*d^2*e^4)*x^2 + 10*(5*b^6*d^4 - 24*a*b^5*d^3*e + 45*a^2*b^4*d^2*
e^2 - 40*a^3*b^3*d*d*e^3 + 15*a^4*b^2*e^4)*x)/e^6 - 6*(b^6*d^5 - 5*a*b^5*d^4
*e + 10*a^2*b^4*d^3*e^2 - 10*a^3*b^3*d^2*e^3 + 5*a^4*b^2*d*d*e^4 - a^5*b*e^5
)*log(e*x + d)/e^7

```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 446 vs. $2(152) = 304$.

Time = 0.24 (sec) , antiderivative size = 446, normalized size of antiderivative = 2.86

$$\int \frac{(a^2 + 2abx + b^2x^2)^3}{(d + ex)^2} dx$$

$$= \frac{\left(2b^6 - \frac{15(b^6de - ab^5e^2)}{(ex+d)e} + \frac{50(b^6d^2e^2 - 2ab^5de^3 + a^2b^4e^4)}{(ex+d)^2e^2} - \frac{100(b^6d^3e^3 - 3ab^5d^2e^4 + 3a^2b^4de^5 - a^3b^3e^6)}{(ex+d)^3e^3} + \frac{150(b^6d^4e^4 - 4ab^5d^3e^5 + 6a^2b^4d^2e^6 - 4a^3b^3de^7 + a^4b^2e^8)}{(ex+d)^4e^4}\right)}{10e^7}$$

$$+ \frac{6(b^6d^5 - 5ab^5d^4e + 10a^2b^4d^3e^2 - 10a^3b^3d^2e^3 + 5a^4b^2de^4 - a^5be^5) \log\left(\frac{|ex+d|}{(ex+d)^2|e|}\right)}{e^7}$$

$$- \frac{\frac{b^6d^6e^5}{ex+d} - \frac{6ab^5d^5e^6}{ex+d} + \frac{15a^2b^4d^4e^7}{ex+d} - \frac{20a^3b^3d^3e^8}{ex+d} + \frac{15a^4b^2d^2e^9}{ex+d} - \frac{6a^5bde^{10}}{ex+d} + \frac{a^6e^{11}}{ex+d}}{e^{12}}$$

input `integrate((b^2*x^2+2*a*b*x+a^2)^3/(e*x+d)^2,x, algorithm="giac")`

output `1/10*(2*b^6 - 15*(b^6*d*e - a*b^5*e^2)/((e*x + d)*e) + 50*(b^6*d^2*e^2 - 2*a*b^5*d*e^3 + a^2*b^4*e^4)/((e*x + d)^2*e^2) - 100*(b^6*d^3*e^3 - 3*a*b^5*d^2*e^4 + 3*a^2*b^4*d*e^5 - a^3*b^3*e^6)/((e*x + d)^3*e^3) + 150*(b^6*d^4*e^4 - 4*a*b^5*d^3*e^5 + 6*a^2*b^4*d^2*e^6 - 4*a^3*b^3*d*e^7 + a^4*b^2*e^8)/((e*x + d)^4*e^4))*(e*x + d)^5/e^7 + 6*(b^6*d^5 - 5*a*b^5*d^4*e + 10*a^2*b^4*d^3*e^2 - 10*a^3*b^3*d^2*e^3 + 5*a^4*b^2*d*e^4 - a^5*b*e^5)*log(abs(e*x + d)/((e*x + d)^2*abs(e)))/e^7 - (b^6*d^6*e^5/(e*x + d) - 6*a*b^5*d^5*e^6/(e*x + d) + 15*a^2*b^4*d^4*e^7/(e*x + d) - 20*a^3*b^3*d^3*e^8/(e*x + d) + 15*a^4*b^2*d^2*e^9/(e*x + d) - 6*a^5*b*d*e^10/(e*x + d) + a^6*e^11/(e*x + d))/e^12`

Mupad [B] (verification not implemented)

Time = 8.72 (sec) , antiderivative size = 523, normalized size of antiderivative = 3.35

$$\begin{aligned}
& \int \frac{(a^2 + 2abx + b^2x^2)^3}{(d + ex)^2} dx \\
&= x^4 \left(\frac{3ab^5}{2e^2} - \frac{b^6d}{2e^3} \right) - x^3 \left(\frac{2d \left(\frac{6ab^5}{e^2} - \frac{2b^6d}{e^3} \right)}{3e} - \frac{5a^2b^4}{e^2} + \frac{b^6d^2}{3e^4} \right) \\
&+ x^2 \left(\frac{10a^3b^3}{e^2} + \frac{d \left(\frac{2d \left(\frac{6ab^5}{e^2} - \frac{2b^6d}{e^3} \right)}{e} - \frac{15a^2b^4}{e^2} + \frac{b^6d^2}{e^4} \right)}{e} - \frac{d^2 \left(\frac{6ab^5}{e^2} - \frac{2b^6d}{e^3} \right)}{2e^2} \right) \\
&+ x \left(\frac{d^2 \left(\frac{2d \left(\frac{6ab^5}{e^2} - \frac{2b^6d}{e^3} \right)}{e} - \frac{15a^2b^4}{e^2} + \frac{b^6d^2}{e^4} \right)}{e^2} \right) \\
&- \left(\frac{2d \left(\frac{20a^3b^3}{e^2} + \frac{2d \left(\frac{2d \left(\frac{6ab^5}{e^2} - \frac{2b^6d}{e^3} \right)}{e} - \frac{15a^2b^4}{e^2} + \frac{b^6d^2}{e^4} \right)}{e} - \frac{d^2 \left(\frac{6ab^5}{e^2} - \frac{2b^6d}{e^3} \right)}{e^2} \right)}{e} + \frac{15a^4b^2}{e^2} \right) \\
&- \frac{\ln(d + ex) (-6a^5be^5 + 30a^4b^2de^4 - 60a^3b^3d^2e^3 + 60a^2b^4d^3e^2 - 30ab^5d^4e + 6b^6d^5)}{e^7} \\
&+ \frac{b^6x^5}{5e^2} \\
&- \frac{a^6e^6 - 6a^5bde^5 + 15a^4b^2d^2e^4 - 20a^3b^3d^3e^3 + 15a^2b^4d^4e^2 - 6ab^5d^5e + b^6d^6}{e(xe^7 + de^6)}
\end{aligned}$$

input

```
int((a^2 + b^2*x^2 + 2*a*b*x)^3/(d + e*x)^2,x)
```

output

```
x^4*((3*a*b^5)/(2*e^2) - (b^6*d)/(2*e^3)) - x^3*((2*d*((6*a*b^5)/e^2 - (2*
b^6*d)/e^3))/(3*e) - (5*a^2*b^4)/e^2 + (b^6*d^2)/(3*e^4)) + x^2*((10*a^3*b
^3)/e^2 + (d*((2*d*((6*a*b^5)/e^2 - (2*b^6*d)/e^3))/e - (15*a^2*b^4)/e^2 +
(b^6*d^2)/e^4))/e - (d^2*((6*a*b^5)/e^2 - (2*b^6*d)/e^3))/(2*e^2)) + x*((
d^2*((2*d*((6*a*b^5)/e^2 - (2*b^6*d)/e^3))/e - (15*a^2*b^4)/e^2 + (b^6*d^2
)/e^4))/e^2 - (2*d*((20*a^3*b^3)/e^2 + (2*d*((2*d*((6*a*b^5)/e^2 - (2*b^6*
d)/e^3))/e - (15*a^2*b^4)/e^2 + (b^6*d^2)/e^4))/e - (d^2*((6*a*b^5)/e^2 -
(2*b^6*d)/e^3))/e^2))/e + (15*a^4*b^2)/e^2) - (log(d + e*x)*(6*b^6*d^5 - 6
*a^5*b*e^5 + 30*a^4*b^2*d*e^4 + 60*a^2*b^4*d^3*e^2 - 60*a^3*b^3*d^2*e^3 -
30*a*b^5*d^4*e))/e^7 + (b^6*x^5)/(5*e^2) - (a^6*e^6 + b^6*d^6 + 15*a^2*b^4
*d^4*e^2 - 20*a^3*b^3*d^3*e^3 + 15*a^4*b^2*d^2*e^4 - 6*a*b^5*d^5*e - 6*a^5
*b*d*e^5)/(e*(d*e^6 + e^7*x))
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 549, normalized size of antiderivative = 3.52

$$\int \frac{(a^2 + 2abx + b^2x^2)^3}{(d + ex)^2} dx$$

$$= \frac{60 \log(ex + d) a^5 b d^2 e^5 - 300 \log(ex + d) a^4 b^2 d^3 e^4 + 600 \log(ex + d) a^3 b^3 d^4 e^3 - 600 \log(ex + d) a^2 b^4 d^5 e^2}{(d + ex)^2}$$

input

```
int((b^2*x^2+2*a*b*x+a^2)^3/(e*x+d)^2,x)
```

output

```
(60*log(d + e*x)*a**5*b*d**2*e**5 + 60*log(d + e*x)*a**5*b*d*e**6*x - 300*
log(d + e*x)*a**4*b**2*d**3*e**4 - 300*log(d + e*x)*a**4*b**2*d**2*e**5*x
+ 600*log(d + e*x)*a**3*b**3*d**4*e**3 + 600*log(d + e*x)*a**3*b**3*d**3*e
**4*x - 600*log(d + e*x)*a**2*b**4*d**5*e**2 - 600*log(d + e*x)*a**2*b**4*
d**4*e**3*x + 300*log(d + e*x)*a*b**5*d**6*e + 300*log(d + e*x)*a*b**5*d**
5*e**2*x - 60*log(d + e*x)*b**6*d**7 - 60*log(d + e*x)*b**6*d**6*e*x + 10*
a**6*e**7*x - 60*a**5*b*d*e**6*x + 300*a**4*b**2*d**2*e**5*x + 150*a**4*b*
**2*d*e**6*x**2 - 600*a**3*b**3*d**3*e**4*x - 300*a**3*b**3*d**2*e**5*x**2
+ 100*a**3*b**3*d*e**6*x**3 + 600*a**2*b**4*d**4*e**3*x + 300*a**2*b**4*d
**3*e**4*x**2 - 100*a**2*b**4*d**2*e**5*x**3 + 50*a**2*b**4*d*e**6*x**4 - 3
00*a*b**5*d**5*e**2*x - 150*a*b**5*d**4*e**3*x**2 + 50*a*b**5*d**3*e**4*x
**3 - 25*a*b**5*d**2*e**5*x**4 + 15*a*b**5*d*e**6*x**5 + 60*b**6*d**6*e*x
+ 30*b**6*d**5*e**2*x**2 - 10*b**6*d**4*e**3*x**3 + 5*b**6*d**3*e**4*x**4 -
3*b**6*d**2*e**5*x**5 + 2*b**6*d*e**6*x**6)/(10*d*e**7*(d + e*x))
```


input `Integrate[(a^2 + 2*a*b*x + b^2*x^2)^3/(d + e*x)^3,x]`

output
$$\frac{(-2*a^6*e^6 - 12*a^5*b*e^5*(d + 2*e*x) + 30*a^4*b^2*d*e^4*(3*d + 4*e*x) + 40*a^3*b^3*e^3*(-5*d^3 - 4*d^2*e*x + 4*d*e^2*x^2 + 2*e^3*x^3) + 30*a^2*b^4*e^2*(7*d^4 + 2*d^3*e*x - 11*d^2*e^2*x^2 - 4*d*e^3*x^3 + e^4*x^4) + 4*a*b^5*e*(-27*d^5 + 6*d^4*e*x + 63*d^3*e^2*x^2 + 20*d^2*e^3*x^3 - 5*d*e^4*x^4 + 2*e^5*x^5) + b^6*(22*d^6 - 16*d^5*e*x - 68*d^4*e^2*x^2 - 20*d^3*e^3*x^3 + 5*d^2*e^4*x^4 - 2*d*e^5*x^5 + e^6*x^6) + 60*b^2*(b*d - a*e)^4*(d + e*x)^2 * \text{Log}[d + e*x]}{(4*e^7*(d + e*x)^2)}$$

Rubi [A] (verified)

Time = 0.64 (sec) , antiderivative size = 158, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {1098, 27, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\begin{aligned} & \int \frac{(a^2 + 2abx + b^2x^2)^3}{(d + ex)^3} dx \\ & \quad \downarrow 1098 \\ & \quad \int \frac{b^6(a+bx)^6}{(d+ex)^3} dx \\ & \quad \quad \downarrow 27 \\ & \quad \quad \int \frac{(a + bx)^6}{(d + ex)^3} dx \\ & \quad \quad \quad \downarrow 49 \\ & \quad \quad \quad \int \left(-\frac{6b^5(d + ex)^2(bd - ae)}{e^6} + \frac{15b^4(d + ex)(bd - ae)^2}{e^6} - \frac{20b^3(bd - ae)^3}{e^6} + \frac{15b^2(bd - ae)^4}{e^6(d + ex)} - \frac{6b(bd - ae)^5}{e^6(d + ex)^2} + \frac{(bd - ae)^6}{e^6(d + ex)^3} \right) dx \\ & \quad \quad \quad \quad \downarrow 2009 \end{aligned}$$

$$-\frac{2b^5(d+ex)^3(bd-ae)}{e^7} + \frac{15b^4(d+ex)^2(bd-ae)^2}{e^7} - \frac{20b^3x(bd-ae)^3}{e^7} + \frac{15b^2(bd-ae)^4 \log(d+ex)}{e^7} + \frac{6b(bd-ae)^5}{e^7(d+ex)} - \frac{(bd-ae)^6}{2e^7(d+ex)^2} + \frac{e^6}{4e^7}(d+ex)^4$$

input `Int[(a^2 + 2*a*b*x + b^2*x^2)^3/(d + e*x)^3,x]`

output `(-20*b^3*(b*d - a*e)^3*x)/e^6 - (b*d - a*e)^6/(2*e^7*(d + e*x)^2) + (6*b*(b*d - a*e)^5)/(e^7*(d + e*x)) + (15*b^4*(b*d - a*e)^2*(d + e*x)^2)/(2*e^7) - (2*b^5*(b*d - a*e)*(d + e*x)^3)/e^7 + (b^6*(d + e*x)^4)/(4*e^7) + (15*b^2*(b*d - a*e)^4*Log[d + e*x])/e^7`

Defintions of rubi rules used

rule 27 `Int[(a_)*(F_x_), x_Symbol] := Simp[a Int[F_x, x], x] /; FreeQ[a, x] && !MatchQ[F_x, (b_)*(G_x_) /; FreeQ[b, x]]`

rule 49 `Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] := Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] && IGtQ[m, 0] && IGtQ[m + n + 2, 0]`

rule 1098 `Int[((d_.) + (e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] := Simp[1/c^p Int[(d + e*x)^m*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]`

rule 2009 `Int[u_, x_Symbol] := Simp[IntSum[u, x], x] /; SumQ[u]`

output

```

1/4*(b^6*e^6*x^6 + 22*b^6*d^6 - 108*a*b^5*d^5*e + 210*a^2*b^4*d^4*e^2 - 20
0*a^3*b^3*d^3*e^3 + 90*a^4*b^2*d^2*e^4 - 12*a^5*b*d*e^5 - 2*a^6*e^6 - 2*(b
^6*d*e^5 - 4*a*b^5*e^6)*x^5 + 5*(b^6*d^2*e^4 - 4*a*b^5*d*e^5 + 6*a^2*b^4*e
^6)*x^4 - 20*(b^6*d^3*e^3 - 4*a*b^5*d^2*e^4 + 6*a^2*b^4*d*e^5 - 4*a^3*b^3*
e^6)*x^3 - 2*(34*b^6*d^4*e^2 - 126*a*b^5*d^3*e^3 + 165*a^2*b^4*d^2*e^4 - 8
0*a^3*b^3*d*e^5)*x^2 - 4*(4*b^6*d^5*e - 6*a*b^5*d^4*e^2 - 15*a^2*b^4*d^3*e
^3 + 40*a^3*b^3*d^2*e^4 - 30*a^4*b^2*d*e^5 + 6*a^5*b*e^6)*x + 60*(b^6*d^6
- 4*a*b^5*d^5*e + 6*a^2*b^4*d^4*e^2 - 4*a^3*b^3*d^3*e^3 + a^4*b^2*d^2*e^4
+ (b^6*d^4*e^2 - 4*a*b^5*d^3*e^3 + 6*a^2*b^4*d^2*e^4 - 4*a^3*b^3*d*e^5 + a
^4*b^2*e^6)*x^2 + 2*(b^6*d^5*e - 4*a*b^5*d^4*e^2 + 6*a^2*b^4*d^3*e^3 - 4*a
^3*b^3*d^2*e^4 + a^4*b^2*d*e^5)*x)*log(e*x + d))/(e^9*x^2 + 2*d*e^8*x + d
^2*e^7)

```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 340 vs. $2(144) = 288$.

Time = 1.10 (sec) , antiderivative size = 340, normalized size of antiderivative = 2.15

$$\begin{aligned}
\int \frac{(a^2 + 2abx + b^2x^2)^3}{(d + ex)^3} dx &= \frac{b^6x^4}{4e^3} + \frac{15b^2(ae - bd)^4 \log(d + ex)}{e^7} + x^3 \cdot \left(\frac{2ab^5}{e^3} - \frac{b^6d}{e^4} \right) \\
&+ x^2 \cdot \left(\frac{15a^2b^4}{2e^3} - \frac{9ab^5d}{e^4} + \frac{3b^6d^2}{e^5} \right) + x \left(\frac{20a^3b^3}{e^3} - \frac{45a^2b^4d}{e^4} + \frac{36ab^5d^2}{e^5} - \frac{10b^6d^3}{e^6} \right) \\
&+ \frac{-a^6e^6 - 6a^5bde^5 + 45a^4b^2d^2e^4 - 100a^3b^3d^3e^3 + 105a^2b^4d^4e^2 - 54ab^5d^5e + 11b^6d^6 + x(-12a^5be^6 + 60a^4b^2d^2e^5 - 120a^3b^3d^3e^4 + 120a^2b^4d^4e^3 - 60ab^5d^5e^2 + 12b^6d^6e)}{2d^2e^7 + 4de^8x + 2e^9x^2}
\end{aligned}$$

input

```
integrate((b**2*x**2+2*a*b*x+a**2)**3/(e*x+d)**3,x)
```

output

```

b**6*x**4/(4*e**3) + 15*b**2*(a*e - b*d)**4*log(d + e*x)/e**7 + x**3*(2*a*
b**5/e**3 - b**6*d/e**4) + x**2*(15*a**2*b**4/(2*e**3) - 9*a*b**5*d/e**4 +
3*b**6*d**2/e**5) + x*(20*a**3*b**3/e**3 - 45*a**2*b**4*d/e**4 + 36*a*b**
5*d**2/e**5 - 10*b**6*d**3/e**6) + (-a**6*e**6 - 6*a**5*b*d*e**5 + 45*a**4
*b**2*d**2*e**4 - 100*a**3*b**3*d**3*e**3 + 105*a**2*b**4*d**4*e**2 - 54*a
*b**5*d**5*e + 11*b**6*d**6 + x*(-12*a**5*b*e**6 + 60*a**4*b**2*d*e**5 - 1
20*a**3*b**3*d**2*e**4 + 120*a**2*b**4*d**3*e**3 - 60*a*b**5*d**4*e**2 + 1
2*b**6*d**5*e))/(2*d**2*e**7 + 4*d*e**8*x + 2*e**9*x**2)

```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 364 vs. $2(152) = 304$.

Time = 0.04 (sec) , antiderivative size = 364, normalized size of antiderivative = 2.30

$$\int \frac{(a^2 + 2abx + b^2x^2)^3}{(d + ex)^3} dx$$

$$= \frac{11b^6d^6 - 54ab^5d^5e + 105a^2b^4d^4e^2 - 100a^3b^3d^3e^3 + 45a^4b^2d^2e^4 - 6a^5bde^5 - a^6e^6 + 12(b^6d^5e - 5ab^5d^4e^2 + 10a^2b^4d^3e^3 - 10a^3b^3d^2e^4 + 5a^4b^2d^1e^5 - a^5b^1d^0e^6)x}{2(e^9x^2 + 2de^8x + d^2e^7)} + \frac{b^6e^3x^4 - 4(b^6de^2 - 2ab^5e^3)x^3 + 6(2b^6d^2e - 6ab^5de^2 + 5a^2b^4e^3)x^2 - 4(10b^6d^3 - 36ab^5d^2e + 45a^2b^4d^1e^3)x - 4(10b^6d^3 - 36ab^5d^2e + 45a^2b^4d^1e^3)}{4e^6} + \frac{15(b^6d^4 - 4ab^5d^3e + 6a^2b^4d^2e^2 - 4a^3b^3de^3 + a^4b^2e^4) \log(ex + d)}{e^7}$$

input `integrate((b^2*x^2+2*a*b*x+a^2)^3/(e*x+d)^3,x, algorithm="maxima")`

output $\frac{1}{2} \cdot \frac{(11b^6d^6 - 54a^2b^5d^5e + 105a^4b^4d^4e^2 - 100a^3b^3d^3e^3 + 45a^4b^2d^2e^4 - 6a^5b^1d^1e^5 - a^6e^6 + 12(b^6d^5e - 5a^2b^5d^4e^2 + 10a^2b^4d^3e^3 - 10a^3b^3d^2e^4 + 5a^4b^2d^1e^5 - a^5b^1d^0e^6)x)}{(e^9x^2 + 2de^8x + d^2e^7)} + \frac{1}{4} \cdot \frac{(b^6e^3x^4 - 4(b^6de^2 - 2ab^5e^3)x^3 + 6(2b^6d^2e - 6ab^5de^2 + 5a^2b^4e^3)x^2 - 4(10b^6d^3 - 36ab^5d^2e + 45a^2b^4d^1e^3)x - 4(10b^6d^3 - 36ab^5d^2e + 45a^2b^4d^1e^3))}{e^6} + \frac{15(b^6d^4 - 4ab^5d^3e + 6a^2b^4d^2e^2 - 4a^3b^3de^3 + a^4b^2e^4) \log(ex + d)}{e^7}$

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 362 vs. $2(152) = 304$.

Time = 0.18 (sec) , antiderivative size = 362, normalized size of antiderivative = 2.29

$$\int \frac{(a^2 + 2abx + b^2x^2)^3}{(d + ex)^3} dx$$

$$= \frac{15(b^6d^4 - 4ab^5d^3e + 6a^2b^4d^2e^2 - 4a^3b^3de^3 + a^4b^2e^4) \log(|ex + d|)}{e^7} + \frac{11b^6d^6 - 54ab^5d^5e + 105a^2b^4d^4e^2 - 100a^3b^3d^3e^3 + 45a^4b^2d^2e^4 - 6a^5bde^5 - a^6e^6 + 12(b^6d^5e - 5ab^5d^4e^2 + 10a^2b^4d^3e^3 - 10a^3b^3d^2e^4 + 5a^4b^2d^1e^5 - a^5b^1d^0e^6)x}{2(ex + d)^2e^7} + \frac{b^6e^9x^4 - 4b^6de^8x^3 + 8ab^5e^9x^3 + 12b^6d^2e^7x^2 - 36ab^5de^8x^2 + 30a^2b^4e^9x^2 - 40b^6d^3e^6x + 144ab^5d^2e^7}{4e^{12}}$$

input `integrate((b^2*x^2+2*a*b*x+a^2)^3/(e*x+d)^3,x, algorithm="giac")`

output $15*(b^6*d^4 - 4*a*b^5*d^3*e + 6*a^2*b^4*d^2*e^2 - 4*a^3*b^3*d*e^3 + a^4*b^2*e^4)*\log(\text{abs}(e*x + d))/e^7 + 1/2*(11*b^6*d^6 - 54*a*b^5*d^5*e + 105*a^2*b^4*d^4*e^2 - 100*a^3*b^3*d^3*e^3 + 45*a^4*b^2*d^2*e^4 - 6*a^5*b*d*e^5 - a^6*e^6 + 12*(b^6*d^5*e - 5*a*b^5*d^4*e^2 + 10*a^2*b^4*d^3*e^3 - 10*a^3*b^3*d^2*e^4 + 5*a^4*b^2*d*e^5 - a^5*b*e^6)*x)/((e*x + d)^2*e^7) + 1/4*(b^6*e^9*x^4 - 4*b^6*d*e^8*x^3 + 8*a*b^5*e^9*x^3 + 12*b^6*d^2*e^7*x^2 - 36*a*b^5*d*e^8*x^2 + 30*a^2*b^4*e^9*x^2 - 40*b^6*d^3*e^6*x + 144*a*b^5*d^2*e^7*x - 180*a^2*b^4*d*e^8*x + 80*a^3*b^3*e^9*x)/e^{12}$

Mupad [B] (verification not implemented)

Time = 8.71 (sec) , antiderivative size = 441, normalized size of antiderivative = 2.79

$$\int \frac{(a^2 + 2abx + b^2x^2)^3}{(d + ex)^3} dx$$

$$= x \left(\frac{20a^3b^3}{e^3} - \frac{b^6d^3}{e^6} + \frac{3d \left(\frac{3d \left(\frac{6ab^5}{e^3} - \frac{3b^6d}{e^4} \right) - \frac{15a^2b^4}{e^3} + \frac{3b^6d^2}{e^5} \right)}{e} - \frac{3d^2 \left(\frac{6ab^5}{e^3} - \frac{3b^6d}{e^4} \right)}{e^2} \right)$$

$$- \frac{a^6e^6 + 6a^5bde^5 - 45a^4b^2d^2e^4 + 100a^3b^3d^3e^3 - 105a^2b^4d^4e^2 + 54ab^5d^5e - 11b^6d^6}{2e} - \frac{x(-6a^5be^5 + 30a^4b^2de^4 - 60a^3b^3d^2e^3 + 30a^2b^4d^3e^2 - 6a^3b^3d^2e^2 + 6a^4b^2de^4 - 60a^3b^3d^2e^3 + 90a^2b^4d^2e^2 - 60ab^5d^3e + 15b^6d^4)}{d^2e^6 + 2de^7x + e^8x^2}$$

$$+ x^3 \left(\frac{2ab^5}{e^3} - \frac{b^6d}{e^4} \right) - x^2 \left(\frac{3d \left(\frac{6ab^5}{e^3} - \frac{3b^6d}{e^4} \right)}{2e} - \frac{15a^2b^4}{2e^3} + \frac{3b^6d^2}{2e^5} \right)$$

$$+ \frac{\ln(d + ex) (15a^4b^2e^4 - 60a^3b^3de^3 + 90a^2b^4d^2e^2 - 60ab^5d^3e + 15b^6d^4)}{e^7}$$

$$+ \frac{b^6x^4}{4e^3}$$

input `int((a^2 + b^2*x^2 + 2*a*b*x)^3/(d + e*x)^3,x)`

3.96
$$\int \frac{(a^2 + 2abx + b^2x^2)^3}{(d + ex)^4} dx$$

| | |
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| Optimal result | 790 |
| Mathematica [A] (verified) | 790 |
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Optimal result

Integrand size = 26, antiderivative size = 156

$$\int \frac{(a^2 + 2abx + b^2x^2)^3}{(d + ex)^4} dx = \frac{15b^4(bd - ae)^2x}{e^6} - \frac{(bd - ae)^6}{3e^7(d + ex)^3} + \frac{3b(bd - ae)^5}{e^7(d + ex)^2} - \frac{15b^2(bd - ae)^4}{e^7(d + ex)} - \frac{3b^5(bd - ae)(d + ex)^2}{e^7} + \frac{b^6(d + ex)^3}{3e^7} - \frac{20b^3(bd - ae)^3 \log(d + ex)}{e^7}$$

output

```
15*b^4*(-a*e+b*d)^2*x/e^6-1/3*(-a*e+b*d)^6/e^7/(e*x+d)^3+3*b*(-a*e+b*d)^5/e^7/(e*x+d)^2-15*b^2*(-a*e+b*d)^4/e^7/(e*x+d)-3*b^5*(-a*e+b*d)*(e*x+d)^2/e^7+1/3*b^6*(e*x+d)^3/e^7-20*b^3*(-a*e+b*d)^3*ln(e*x+d)/e^7
```

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 302, normalized size of antiderivative = 1.94

$$\int \frac{(a^2 + 2abx + b^2x^2)^3}{(d + ex)^4} dx = \frac{-a^6e^6 - 3a^5be^5(d + 3ex) - 15a^4b^2e^4(d^2 + 3dex + 3e^2x^2) + 10a^3b^3de^3(11d^2 + 27dex + 18e^2x^2) + 15a^2b^4e^2(d + ex)^3 + 3ab^5e(d + ex)^2 + b^6e(d + ex)}{(d + ex)^4}$$

input `Integrate[(a^2 + 2*a*b*x + b^2*x^2)^3/(d + e*x)^4,x]`

output
$$\frac{-(a^6 e^6) - 3a^5 b e^5 (d + 3e x) - 15a^4 b^2 e^4 (d^2 + 3d e x + 3e^2 x^2) + 10a^3 b^3 d e^3 (11d^2 + 27d e x + 18e^2 x^2) + 15a^2 b^4 e^2 (-13d^4 - 27d^3 e x - 9d^2 e^2 x^2 + 9d e^3 x^3 + 3e^4 x^4) + 3a b^5 e (47d^5 + 81d^4 e x - 9d^3 e^2 x^2 - 63d^2 e^3 x^3 - 15d e^4 x^4 + 3e^5 x^5) + b^6 (-37d^6 - 51d^5 e x + 39d^4 e^2 x^2 + 73d^3 e^3 x^3 + 15d^2 e^4 x^4 - 3d e^5 x^5 + e^6 x^6) - 60b^3 (b d - a e)^3 (d + e x)^3 \text{Log}[d + e x]}{(3e^7 (d + e x)^3)}$$

Rubi [A] (verified)

Time = 0.62 (sec) , antiderivative size = 156, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {1098, 27, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a^2 + 2abx + b^2 x^2)^3}{(d + ex)^4} dx$$

$$\downarrow 1098$$

$$\frac{\int \frac{b^6 (a+bx)^6}{(d+ex)^4} dx}{b^6}$$

$$\downarrow 27$$

$$\int \frac{(a + bx)^6}{(d + ex)^4} dx$$

$$\downarrow 49$$

$$\int \left(-\frac{6b^5(d+ex)(bd-ae)}{e^6} + \frac{15b^4(bd-ae)^2}{e^6} - \frac{20b^3(bd-ae)^3}{e^6(d+ex)} + \frac{15b^2(bd-ae)^4}{e^6(d+ex)^2} - \frac{6b(bd-ae)^5}{e^6(d+ex)^3} + \frac{(ae-bd)^6}{e^6(d+ex)^4} \right) dx$$

$$\downarrow 2009$$

$$-\frac{3b^5(d+ex)^2(bd-ae)}{e^7} + \frac{15b^4x(bd-ae)^2}{e^6} - \frac{20b^3(bd-ae)^3 \log(d+ex)}{e^7} - \frac{15b^2(bd-ae)^4}{e^7(d+ex)} + \frac{3b(bd-ae)^5}{e^7(d+ex)^2} - \frac{(bd-ae)^6}{3e^7(d+ex)^3} + \frac{b^6(d+ex)^3}{3e^7}$$

input `Int[(a^2 + 2*a*b*x + b^2*x^2)^3/(d + e*x)^4,x]`

output `(15*b^4*(b*d - a*e)^2*x)/e^6 - (b*d - a*e)^6/(3*e^7*(d + e*x)^3) + (3*b*(b*d - a*e)^5)/(e^7*(d + e*x)^2) - (15*b^2*(b*d - a*e)^4)/(e^7*(d + e*x)) - (3*b^5*(b*d - a*e)*(d + e*x)^2)/e^7 + (b^6*(d + e*x)^3)/(3*e^7) - (20*b^3*(b*d - a*e)^3*Log[d + e*x])/e^7`

Defintions of rubi rules used

rule 27 `Int[(a_)*(F_x_), x_Symbol] := Simp[a Int[F_x, x], x] /; FreeQ[a, x] && !MatchQ[F_x, (b_)*(G_x_)] /; FreeQ[b, x]`

rule 49 `Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] := Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] && IGtQ[m, 0] && IGtQ[m + n + 2, 0]`

rule 1098 `Int[((d_.) + (e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] := Simp[1/c^p Int[(d + e*x)^m*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]`

rule 2009 `Int[u_, x_Symbol] := Simp[IntSum[u, x], x] /; SumQ[u]`

output

```

1/3*(b^6*e^6*x^6 - 37*b^6*d^6 + 141*a*b^5*d^5*e - 195*a^2*b^4*d^4*e^2 + 11
0*a^3*b^3*d^3*e^3 - 15*a^4*b^2*d^2*e^4 - 3*a^5*b*d*e^5 - a^6*e^6 - 3*(b^6*
d*e^5 - 3*a*b^5*e^6)*x^5 + 15*(b^6*d^2*e^4 - 3*a*b^5*d*e^5 + 3*a^2*b^4*e^6
)*x^4 + (73*b^6*d^3*e^3 - 189*a*b^5*d^2*e^4 + 135*a^2*b^4*d*e^5)*x^3 + 3*(
13*b^6*d^4*e^2 - 9*a*b^5*d^3*e^3 - 45*a^2*b^4*d^2*e^4 + 60*a^3*b^3*d*e^5 -
15*a^4*b^2*e^6)*x^2 - 3*(17*b^6*d^5*e - 81*a*b^5*d^4*e^2 + 135*a^2*b^4*d^
3*e^3 - 90*a^3*b^3*d^2*e^4 + 15*a^4*b^2*d*e^5 + 3*a^5*b*e^6)*x - 60*(b^6*d
^6 - 3*a*b^5*d^5*e + 3*a^2*b^4*d^4*e^2 - a^3*b^3*d^3*e^3 + (b^6*d^3*e^3 -
3*a*b^5*d^2*e^4 + 3*a^2*b^4*d*e^5 - a^3*b^3*e^6)*x^3 + 3*(b^6*d^4*e^2 - 3*
a*b^5*d^3*e^3 + 3*a^2*b^4*d^2*e^4 - a^3*b^3*d*e^5)*x^2 + 3*(b^6*d^5*e - 3*
a*b^5*d^4*e^2 + 3*a^2*b^4*d^3*e^3 - a^3*b^3*d^2*e^4)*x)*log(e*x + d))/(e^1
0*x^3 + 3*d*e^9*x^2 + 3*d^2*e^8*x + d^3*e^7)

```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 367 vs. $2(143) = 286$.

Time = 2.46 (sec) , antiderivative size = 367, normalized size of antiderivative = 2.35

$$\int \frac{(a^2 + 2abx + b^2x^2)^3}{(d + ex)^4} dx = \frac{b^6x^3}{3e^4} + \frac{20b^3(ae - bd)^3 \log(d + ex)}{e^7}$$

$$+ x^2 \cdot \left(\frac{3ab^5}{e^4} - \frac{2b^6d}{e^5} \right) + x \left(\frac{15a^2b^4}{e^4} - \frac{24ab^5d}{e^5} + \frac{10b^6d^2}{e^6} \right)$$

$$+ \frac{-a^6e^6 - 3a^5bde^5 - 15a^4b^2d^2e^4 + 110a^3b^3d^3e^3 - 195a^2b^4d^4e^2 + 141ab^5d^5e - 37b^6d^6 + x^2(-45a^4b^2e^6 + \dots)}{3}$$

input

```
integrate((b**2*x**2+2*a*b*x+a**2)**3/(e*x+d)**4,x)
```

output

```

b**6*x**3/(3*e**4) + 20*b**3*(a*e - b*d)**3*log(d + e*x)/e**7 + x**2*(3*a*
b**5/e**4 - 2*b**6*d/e**5) + x*(15*a**2*b**4/e**4 - 24*a*b**5*d/e**5 + 10*
b**6*d**2/e**6) + (-a**6*e**6 - 3*a**5*b*d*e**5 - 15*a**4*b**2*d**2*e**4 +
110*a**3*b**3*d**3*e**3 - 195*a**2*b**4*d**4*e**2 + 141*a*b**5*d**5*e - 3
7*b**6*d**6 + x**2*(-45*a**4*b**2*e**6 + 180*a**3*b**3*d*e**5 - 270*a**2*b
**4*d**2*e**4 + 180*a*b**5*d**3*e**3 - 45*b**6*d**4*e**2) + x*(-9*a**5*b*e
**6 - 45*a**4*b**2*d*e**5 + 270*a**3*b**3*d**2*e**4 - 450*a**2*b**4*d**3*e
**3 + 315*a*b**5*d**4*e**2 - 81*b**6*d**5*e))/(3*d**3*e**7 + 9*d**2*e**8*x
+ 9*d*e**9*x**2 + 3*e**10*x**3)

```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 374 vs. $2(152) = 304$.

Time = 0.05 (sec) , antiderivative size = 374, normalized size of antiderivative = 2.40

$$\int \frac{(a^2 + 2abx + b^2x^2)^3}{(d + ex)^4} dx =$$

$$-\frac{37b^6d^6 - 141ab^5d^5e + 195a^2b^4d^4e^2 - 110a^3b^3d^3e^3 + 15a^4b^2d^2e^4 + 3a^5bde^5 + a^6e^6 + 45(b^6d^4e^2 - 4a^6e^6)}{3(e^{10}x^3 - 3e^6)}$$

$$+ \frac{b^6e^2x^3 - 3(2b^6de - 3ab^5e^2)x^2 + 3(10b^6d^2 - 24ab^5de + 15a^2b^4e^2)x}{3e^6}$$

$$- \frac{20(b^6d^3 - 3ab^5d^2e + 3a^2b^4de^2 - a^3b^3e^3) \log(ex + d)}{e^7}$$

input

```
integrate((b^2*x^2+2*a*b*x+a^2)^3/(e*x+d)^4,x, algorithm="maxima")
```

output

```
-1/3*(37*b^6*d^6 - 141*a*b^5*d^5*e + 195*a^2*b^4*d^4*e^2 - 110*a^3*b^3*d^3
*e^3 + 15*a^4*b^2*d^2*e^4 + 3*a^5*b*d*e^5 + a^6*e^6 + 45*(b^6*d^4*e^2 - 4*
a*b^5*d^3*e^3 + 6*a^2*b^4*d^2*e^4 - 4*a^3*b^3*d*e^5 + a^4*b^2*e^6)*x^2 + 9
*(9*b^6*d^5*e - 35*a*b^5*d^4*e^2 + 50*a^2*b^4*d^3*e^3 - 30*a^3*b^3*d^2*e^4
+ 5*a^4*b^2*d*e^5 + a^5*b*e^6)*x)/(e^10*x^3 + 3*d*e^9*x^2 + 3*d^2*e^8*x +
d^3*e^7) + 1/3*(b^6*e^2*x^3 - 3*(2*b^6*d*e - 3*a*b^5*e^2)*x^2 + 3*(10*b^6
*d^2 - 24*a*b^5*d*e + 15*a^2*b^4*e^2)*x)/e^6 - 20*(b^6*d^3 - 3*a*b^5*d^2*e
+ 3*a^2*b^4*d*e^2 - a^3*b^3*e^3)*log(e*x + d)/e^7
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 356 vs. $2(152) = 304$.

Time = 0.19 (sec) , antiderivative size = 356, normalized size of antiderivative = 2.28

$$\int \frac{(a^2 + 2abx + b^2x^2)^3}{(d + ex)^4} dx = -\frac{20(b^6d^3 - 3ab^5d^2e + 3a^2b^4de^2 - a^3b^3e^3) \log(|ex + d|)}{e^7}$$

$$-\frac{37b^6d^6 - 141ab^5d^5e + 195a^2b^4d^4e^2 - 110a^3b^3d^3e^3 + 15a^4b^2d^2e^4 + 3a^5bde^5 + a^6e^6 + 45(b^6d^4e^2 - 4a^6e^6)}{3e^{12}}$$

$$+ \frac{b^6e^8x^3 - 6b^6de^7x^2 + 9ab^5e^8x^2 + 30b^6d^2e^6x - 72ab^5de^7x + 45a^2b^4e^8x}{3e^{12}}$$

input `integrate((b^2*x^2+2*a*b*x+a^2)^3/(e*x+d)^4,x, algorithm="giac")`

output
$$\begin{aligned} & -20*(b^6*d^3 - 3*a*b^5*d^2*e + 3*a^2*b^4*d*e^2 - a^3*b^3*e^3)*\log(\text{abs}(e*x \\ & + d))/e^7 - 1/3*(37*b^6*d^6 - 141*a*b^5*d^5*e + 195*a^2*b^4*d^4*e^2 - 110* \\ & a^3*b^3*d^3*e^3 + 15*a^4*b^2*d^2*e^4 + 3*a^5*b*d*e^5 + a^6*e^6 + 45*(b^6*d \\ & ^4*e^2 - 4*a*b^5*d^3*e^3 + 6*a^2*b^4*d^2*e^4 - 4*a^3*b^3*d*e^5 + a^4*b^2*e \\ & ^6)*x^2 + 9*(9*b^6*d^5*e - 35*a*b^5*d^4*e^2 + 50*a^2*b^4*d^3*e^3 - 30*a^3* \\ & b^3*d^2*e^4 + 5*a^4*b^2*d*e^5 + a^5*b*e^6)*x)/((e*x + d)^3*e^7) + 1/3*(b^6 \\ & *e^8*x^3 - 6*b^6*d*e^7*x^2 + 9*a*b^5*e^8*x^2 + 30*b^6*d^2*e^6*x - 72*a*b^5 \\ & *d*e^7*x + 45*a^2*b^4*e^8*x)/e^{12} \end{aligned}$$

Mupad [B] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 393, normalized size of antiderivative = 2.52

$$\begin{aligned} \int \frac{(a^2 + 2abx + b^2x^2)^3}{(d + ex)^4} dx &= x^2 \left(\frac{3ab^5}{e^4} - \frac{2b^6d}{e^5} \right) \\ & - \frac{x^2 (15a^4b^2e^5 - 60a^3b^3de^4 + 90a^2b^4d^2e^3 - 60ab^5d^3e^2 + 15b^6d^4e) + \frac{a^6e^6 + 3a^5bde^5 + 15a^4b^2d^2e^4 - 110a^3b^3d^3e^3 - 15a^2b^4d^4e^2 + 3a^5b^5d^5e - 3a^6e^6}{d^3e^6} + \frac{4d \left(\frac{6ab^5}{e^4} - \frac{4b^6d}{e^5} \right) - \frac{15a^2b^4}{e^4} + \frac{6b^6d^2}{e^6}}{e} - \frac{\ln(d + ex) (-20a^3b^3e^3 + 60a^2b^4de^2 - 60ab^5d^2e + 20b^6d^3) + \frac{b^6x^3}{3e^4}}{e^7} \end{aligned}$$

input `int((a^2 + b^2*x^2 + 2*a*b*x)^3/(d + e*x)^4,x)`

output
$$\begin{aligned} & x^2*((3*a*b^5)/e^4 - (2*b^6*d)/e^5) - (x^2*(15*b^6*d^4*e + 15*a^4*b^2*e^5 \\ & - 60*a*b^5*d^3*e^2 - 60*a^3*b^3*d*e^4 + 90*a^2*b^4*d^2*e^3) + (a^6*e^6 + 3 \\ & 7*b^6*d^6 + 195*a^2*b^4*d^4*e^2 - 110*a^3*b^3*d^3*e^3 + 15*a^4*b^2*d^2*e^4 \\ & - 141*a*b^5*d^5*e + 3*a^5*b*d*e^5)/(3*e) + x*(27*b^6*d^5 + 3*a^5*b*e^5 + \\ & 15*a^4*b^2*d*e^4 + 150*a^2*b^4*d^3*e^2 - 90*a^3*b^3*d^2*e^3 - 105*a*b^5*d^4 \\ & 4*e))/(d^3*e^6 + e^9*x^3 + 3*d^2*e^7*x + 3*d*e^8*x^2) - x*((4*d*((6*a*b^5) \\ & /e^4 - (4*b^6*d)/e^5))/e - (15*a^2*b^4)/e^4 + (6*b^6*d^2)/e^6) - (\log(d + \\ & e*x)*(20*b^6*d^3 - 20*a^3*b^3*e^3 + 60*a^2*b^4*d*e^2 - 60*a*b^5*d^2*e))/e^7 \\ & + (b^6*x^3)/(3*e^4) \end{aligned}$$

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 645, normalized size of antiderivative = 4.13

$$\int \frac{(a^2 + 2abx + b^2x^2)^3}{(d + ex)^4} dx$$

$$= \frac{-a^6 d e^6 + 60 \log(ex + d) a^3 b^3 d^4 e^3 - 180 \log(ex + d) a^2 b^4 d^5 e^2 + 180 \log(ex + d) a b^5 d^6 e - 180 \log(ex + d) a^6 d^7}{(d + ex)^4}$$

input `int((b^2*x^2+2*a*b*x+a^2)^3/(e*x+d)^4,x)`

output

```
(60*log(d + e*x)*a**3*b**3*d**4*e**3 + 180*log(d + e*x)*a**3*b**3*d**3*e**4*x + 180*log(d + e*x)*a**3*b**3*d**2*e**5*x**2 + 60*log(d + e*x)*a**3*b**3*d**e**6*x**3 - 180*log(d + e*x)*a**2*b**4*d**5*e**2 - 540*log(d + e*x)*a**2*b**4*d**4*e**3*x - 540*log(d + e*x)*a**2*b**4*d**3*e**4*x**2 - 180*log(d + e*x)*a**2*b**4*d**2*e**5*x**3 + 180*log(d + e*x)*a*b**5*d**6*e + 540*log(d + e*x)*a*b**5*d**5*e**2*x + 540*log(d + e*x)*a*b**5*d**4*e**3*x**2 + 180*log(d + e*x)*a*b**5*d**3*e**4*x**3 - 60*log(d + e*x)*b**6*d**7 - 180*log(d + e*x)*b**6*d**6*e*x - 180*log(d + e*x)*b**6*d**5*e**2*x**2 - 60*log(d + e*x)*b**6*d**4*e**3*x**3 - a**6*d*e**6 - 3*a**5*b*d**2*e**5 - 9*a**5*b*d*e**6*x + 15*a**4*b**2*e**7*x**3 + 50*a**3*b**3*d**4*e**3 + 90*a**3*b**3*d**3*e**4*x - 60*a**3*b**3*d*e**6*x**3 - 150*a**2*b**4*d**5*e**2 - 270*a**2*b**4*d**4*e**3*x + 180*a**2*b**4*d**2*e**5*x**3 + 45*a**2*b**4*d*e**6*x**4 + 150*a*b**5*d**6*e + 270*a*b**5*d**5*e**2*x - 180*a*b**5*d**3*e**4*x**3 - 45*a*b**5*d**2*e**5*x**4 + 9*a*b**5*d*e**6*x**5 - 50*b**6*d**7 - 90*b**6*d**6*e*x + 60*b**6*d**4*e**3*x**3 + 15*b**6*d**3*e**4*x**4 - 3*b**6*d**2*e**5*x**5 + b**6*d*e**6*x**6)/(3*d*e**7*(d**3 + 3*d**2*e*x + 3*d*e**2*x**2 + e**3*x**3))
```

3.97
$$\int \frac{(a^2+2abx+b^2x^2)^3}{(d+ex)^5} dx$$

| | |
|---|-----|
| Optimal result | 798 |
| Mathematica [A] (verified) | 798 |
| Rubi [A] (verified) | 799 |
| Maple [B] (verified) | 801 |
| Fricas [B] (verification not implemented) | 801 |
| Sympy [B] (verification not implemented) | 802 |
| Maxima [B] (verification not implemented) | 803 |
| Giac [B] (verification not implemented) | 803 |
| Mupad [B] (verification not implemented) | 804 |
| Reduce [B] (verification not implemented) | 805 |

Optimal result

Integrand size = 26, antiderivative size = 155

$$\int \frac{(a^2 + 2abx + b^2x^2)^3}{(d + ex)^5} dx = -\frac{b^5(5bd - 6ae)x}{e^6} + \frac{b^6x^2}{2e^5} - \frac{(bd - ae)^6}{4e^7(d + ex)^4} + \frac{2b(bd - ae)^5}{e^7(d + ex)^3} - \frac{15b^2(bd - ae)^4}{2e^7(d + ex)^2} + \frac{20b^3(bd - ae)^3}{e^7(d + ex)} + \frac{15b^4(bd - ae)^2 \log(d + ex)}{e^7}$$

output

```
-b^5*(-6*a*e+5*b*d)*x/e^6+1/2*b^6*x^2/e^5-1/4*(-a*e+b*d)^6/e^7/(e*x+d)^4+2
*b*(-a*e+b*d)^5/e^7/(e*x+d)^3-15/2*b^2*(-a*e+b*d)^4/e^7/(e*x+d)^2+20*b^3*(
-a*e+b*d)^3/e^7/(e*x+d)+15*b^4*(-a*e+b*d)^2*ln(e*x+d)/e^7
```

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 301, normalized size of antiderivative = 1.94

$$\int \frac{(a^2 + 2abx + b^2x^2)^3}{(d + ex)^5} dx = \frac{a^6e^6 + 2a^5be^5(d + 4ex) + 5a^4b^2e^4(d^2 + 4dex + 6e^2x^2) + 20a^3b^3e^3(d^3 + 4d^2ex + 6de^2x^2 + 4e^3x^3) - 5a^2b^4e^2(d^4 + 4d^3ex + 6d^2e^2x^2 + 4de^3x^3 + e^4x^4)}{e^7(d + ex)^4}$$

input `Integrate[(a^2 + 2*a*b*x + b^2*x^2)^3/(d + e*x)^5,x]`

output
$$\frac{-1/4*(a^6*e^6 + 2*a^5*b*e^5*(d + 4*e*x) + 5*a^4*b^2*e^4*(d^2 + 4*d*e*x + 6*e^2*x^2) + 20*a^3*b^3*e^3*(d^3 + 4*d^2*e*x + 6*d*e^2*x^2 + 4*e^3*x^3) - 5*a^2*b^4*d*e^2*(25*d^3 + 88*d^2*e*x + 108*d*e^2*x^2 + 48*e^3*x^3) + 2*a*b^5*e*(77*d^5 + 248*d^4*e*x + 252*d^3*e^2*x^2 + 48*d^2*e^3*x^3 - 48*d*e^4*x^4 - 12*e^5*x^5) - b^6*(57*d^6 + 168*d^5*e*x + 132*d^4*e^2*x^2 - 32*d^3*e^3*x^3 - 68*d^2*e^4*x^4 - 12*d*e^5*x^5 + 2*e^6*x^6) - 60*b^4*(b*d - a*e)^2*(d + e*x)^4*\text{Log}[d + e*x]}{(e^7*(d + e*x)^4)}$$

Rubi [A] (verified)

Time = 0.63 (sec) , antiderivative size = 155, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {1098, 27, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\begin{aligned} & \int \frac{(a^2 + 2abx + b^2x^2)^3}{(d + ex)^5} dx \\ & \quad \downarrow 1098 \\ & \quad \int \frac{b^6(a+bx)^6}{(d+ex)^5} dx \\ & \quad \quad \downarrow 27 \\ & \quad \quad \int \frac{(a + bx)^6}{(d + ex)^5} dx \\ & \quad \quad \quad \downarrow 49 \\ & \quad \quad \quad \int \left(-\frac{b^5(5bd - 6ae)}{e^6} + \frac{15b^4(bd - ae)^2}{e^6(d + ex)} - \frac{20b^3(bd - ae)^3}{e^6(d + ex)^2} + \frac{15b^2(bd - ae)^4}{e^6(d + ex)^3} - \frac{6b(bd - ae)^5}{e^6(d + ex)^4} + \frac{(ae - bd)^6}{e^6(d + ex)^5} + \frac{b^6x}{e^5} \right) dx \\ & \quad \quad \quad \quad \downarrow 2009 \end{aligned}$$

$$-\frac{b^5x(5bd - 6ae)}{e^6} + \frac{15b^4(bd - ae)^2 \log(d + ex)}{e^7} + \frac{20b^3(bd - ae)^3}{e^7(d + ex)} - \frac{15b^2(bd - ae)^4}{2e^7(d + ex)^2} + \frac{2b(bd - ae)^5}{e^7(d + ex)^3} - \frac{(bd - ae)^6}{4e^7(d + ex)^4} + \frac{b^6x^2}{2e^5}$$

input `Int[(a^2 + 2*a*b*x + b^2*x^2)^3/(d + e*x)^5,x]`

output `-((b^5*(5*b*d - 6*a*e)*x)/e^6) + (b^6*x^2)/(2*e^5) - (b*d - a*e)^6/(4*e^7*(d + e*x)^4) + (2*b*(b*d - a*e)^5)/(e^7*(d + e*x)^3) - (15*b^2*(b*d - a*e)^4)/(2*e^7*(d + e*x)^2) + (20*b^3*(b*d - a*e)^3)/(e^7*(d + e*x)) + (15*b^4*(b*d - a*e)^2*Log[d + e*x])/e^7`

Defintions of rubi rules used

rule 27 `Int[(a_)*(F_x_), x_Symbol] := Simp[a Int[F_x, x], x] /; FreeQ[a, x] && !MatchQ[F_x, (b_)*(G_x_)] /; FreeQ[b, x]`

rule 49 `Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] := Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] && IGtQ[m, 0] && IGtQ[m + n + 2, 0]`

rule 1098 `Int[((d_.) + (e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] := Simp[1/c^p Int[(d + e*x)^m*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]`

rule 2009 `Int[u_, x_Symbol] := Simp[IntSum[u, x], x] /; SumQ[u]`

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 342 vs. $2(149) = 298$.

Time = 1.10 (sec) , antiderivative size = 343, normalized size of antiderivative = 2.21

| method | result |
|---------------|--|
| default | $\frac{b^5(\frac{1}{2}be^x+6aex-5bdx)}{e^6} - \frac{2b(e^5a^5-5a^4bde^4+10a^3b^2d^2e^3-10a^2b^3d^3e^2+5ab^4d^4e-b^5d^5)}{e^7(ex+d)^3} - \frac{a^6e^6-6a^5bde^5+15a^4b^2d^2e^4-20a^3b^3d^3e^3+15a^2b^4d^4e-15a^2b^5d^5+15a^2b^6d^6}{e^4}$ |
| norman | $\frac{-a^6e^6+2a^5bde^5+5a^4b^2d^2e^4+20a^3b^3d^3e^3-125a^2b^4d^4e^2+250ab^5d^5e-125b^6d^6}{4e^7} + \frac{b^6x^6}{2e} - \frac{4(5e^3a^3b^3-15d^2a^2b^4+30d^2eab^5-15d^3b^6)}{e^4}$ |
| risch | $\frac{b^6x^2}{2e^5} + \frac{6b^5ax}{e^5} - \frac{5b^6dx}{e^6} + \frac{(-20e^5a^3b^3+60a^2b^4de^4-60d^2e^3ab^5+20d^3e^2b^6)x^3 - \frac{15b^2e(a^4e^4+4a^3bde^3-18a^2b^2d^2e^2+20ab^3d^2e-15b^4d^2)}{2}}{e^7}$ |
| parallelrisch | $\frac{-250ab^5d^5e-2a^5bde^5-20a^3b^3d^3e^3-5a^4b^2d^2e^4+240\ln(ex+d)xa^2b^4d^3e^3+125a^2b^4d^4e^2+360\ln(ex+d)x^2b^6d^4e^2+240\ln(ex+d)x^3b^6d^4e^2}{e^7}$ |

input `int((b^2*x^2+2*a*b*x+a^2)^3/(e*x+d)^5,x,method=_RETURNVERBOSE)`

output
$$\frac{b^5}{e^6} \left(\frac{1}{2} b e^x + 6 a e x - 5 b d x \right) - \frac{2 b}{e^7} \left(a^5 e^5 - 5 a^4 b d e^4 + 10 a^3 b^2 d^2 e^3 - 10 a^2 b^3 d^3 e^2 + 5 a b^4 d^4 e - b^5 d^5 \right) / (e x + d)^3 - \frac{1}{4} \left(a^6 e^6 - 6 a^5 b d e^5 + 15 a^4 b^2 d^2 e^4 - 20 a^3 b^3 d^3 e^3 + 15 a^2 b^4 d^4 e^2 - 20 a^2 b^5 d^5 e + b^6 d^6 \right) / e^7 / (e x + d)^4 + \frac{15 b^4}{e^7} \left(a^2 e^2 - 2 a b d e + b^2 d^2 \right) * \ln(e x + d) - \frac{20 b^3}{e^7} \left(a^3 e^3 - 3 a^2 b d e^2 + 3 a a b^2 d^2 e - b^3 d^3 \right) / (e x + d) - \frac{15}{2} \frac{b^2}{e^7} \left(a^4 e^4 - 4 a^3 b d e^3 + 6 a^2 b^2 d^2 e^2 - 4 a a b^3 d^3 e + b^4 d^4 \right) / (e x + d)^2$$

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 571 vs. $2(149) = 298$.

Time = 0.10 (sec) , antiderivative size = 571, normalized size of antiderivative = 3.68

$$\int \frac{(a^2 + 2abx + b^2x^2)^3}{(d + ex)^5} dx$$

$$= \frac{2b^6e^6x^6 + 57b^6d^6 - 154ab^5d^5e + 125a^2b^4d^4e^2 - 20a^3b^3d^3e^3 - 5a^4b^2d^2e^4 - 2a^5bde^5 - a^6e^6 - 12(b^6de^5}{e^7}$$

input `integrate((b^2*x^2+2*a*b*x+a^2)^3/(e*x+d)^5,x, algorithm="fricas")`

output

```
1/4*(2*b^6*e^6*x^6 + 57*b^6*d^6 - 154*a*b^5*d^5*e + 125*a^2*b^4*d^4*e^2 -
20*a^3*b^3*d^3*e^3 - 5*a^4*b^2*d^2*e^4 - 2*a^5*b*d*e^5 - a^6*e^6 - 12*(b^6
*d*e^5 - 2*a*b^5*e^6)*x^5 - 4*(17*b^6*d^2*e^4 - 24*a*b^5*d*e^5)*x^4 - 16*(
2*b^6*d^3*e^3 + 6*a*b^5*d^2*e^4 - 15*a^2*b^4*d*e^5 + 5*a^3*b^3*e^6)*x^3 +
6*(22*b^6*d^4*e^2 - 84*a*b^5*d^3*e^3 + 90*a^2*b^4*d^2*e^4 - 20*a^3*b^3*d*e
^5 - 5*a^4*b^2*e^6)*x^2 + 4*(42*b^6*d^5*e - 124*a*b^5*d^4*e^2 + 110*a^2*b^
4*d^3*e^3 - 20*a^3*b^3*d^2*e^4 - 5*a^4*b^2*d*e^5 - 2*a^5*b*e^6)*x + 60*(b^
6*d^6 - 2*a*b^5*d^5*e + a^2*b^4*d^4*e^2 + (b^6*d^2*e^4 - 2*a*b^5*d*e^5 + a
^2*b^4*e^6)*x^4 + 4*(b^6*d^3*e^3 - 2*a*b^5*d^2*e^4 + a^2*b^4*d*e^5)*x^3 +
6*(b^6*d^4*e^2 - 2*a*b^5*d^3*e^3 + a^2*b^4*d^2*e^4)*x^2 + 4*(b^6*d^5*e -
2*a*b^5*d^4*e^2 + a^2*b^4*d^3*e^3)*x)*log(e*x + d))/(e^11*x^4 + 4*d*e^10*x^
3 + 6*d^2*e^9*x^2 + 4*d^3*e^8*x + d^4*e^7)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 394 vs. $2(143) = 286$.

Time = 5.55 (sec) , antiderivative size = 394, normalized size of antiderivative = 2.54

$$\int \frac{(a^2 + 2abx + b^2x^2)^3}{(d + ex)^5} dx = \frac{b^6x^2}{2e^5} + \frac{15b^4(ae - bd)^2 \log(d + ex)}{e^7} + x \left(\frac{6ab^5}{e^5} - \frac{5b^6d}{e^6} \right) + \frac{-a^6e^6 - 2a^5bde^5 - 5a^4b^2d^2e^4 - 20a^3b^3d^3e^3 + 125a^2b^4d^4e^2 - 154ab^5d^5e + 57b^6d^6 + x^3(-80a^3b^3e^6 + 240a^2b^4d^4e^5 - 240a^3b^5d^5e^4 + 80b^6d^3e^3) + x^2(-30a^4b^2e^6 - 120a^3b^3d^3e^5 + 540a^2b^4d^2e^4 - 600ab^5d^3e^3 + 210b^6d^4e^2) + x(-8a^5b^2e^6 - 20a^4b^3d^3e^5 - 80a^3b^4d^4e^4 + 440a^2b^5d^5e^3 - 520ab^6d^6e^2 + 188b^7d^7e)}{4d^4e^7 + 16d^3e^8x + 24d^2e^9x^2 + 16d^3e^10x^3 + 4e^11x^4}$$

input

```
integrate((b**2*x**2+2*a*b*x+a**2)**3/(e*x+d)**5,x)
```

output

```
b**6*x**2/(2*e**5) + 15*b**4*(a*e - b*d)**2*log(d + e*x)/e**7 + x*(6*a*b**
5/e**5 - 5*b**6*d/e**6) + (-a**6*e**6 - 2*a**5*b*d*e**5 - 5*a**4*b**2*d**2
*e**4 - 20*a**3*b**3*d**3*e**3 + 125*a**2*b**4*d**4*e**2 - 154*a*b**5*d**5
*e + 57*b**6*d**6 + x**3*(-80*a**3*b**3*e**6 + 240*a**2*b**4*d*e**5 - 240*
a*b**5*d**2*e**4 + 80*b**6*d**3*e**3) + x**2*(-30*a**4*b**2*e**6 - 120*a**
3*b**3*d*e**5 + 540*a**2*b**4*d**2*e**4 - 600*a*b**5*d**3*e**3 + 210*b**6*
d**4*e**2) + x*(-8*a**5*b**2*e**6 - 20*a**4*b**2*d*e**5 - 80*a**3*b**3*d**2*
e**4 + 440*a**2*b**4*d**3*e**3 - 520*a*b**5*d**4*e**2 + 188*b**6*d**5*e))/
(4*d**4*e**7 + 16*d**3*e**8*x + 24*d**2*e**9*x**2 + 16*d*e**10*x**3 + 4*e**
11*x**4)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 387 vs. $2(149) = 298$.

Time = 0.08 (sec) , antiderivative size = 387, normalized size of antiderivative = 2.50

$$\int \frac{(a^2 + 2abx + b^2x^2)^3}{(d + ex)^5} dx$$

$$= \frac{57b^6d^6 - 154ab^5d^5e + 125a^2b^4d^4e^2 - 20a^3b^3d^3e^3 - 5a^4b^2d^2e^4 - 2a^5bde^5 - a^6e^6 + 80(b^6d^3e^3 - 3ab^5d^2e^2 + \frac{b^6ex^2 - 2(5b^6d - 6ab^5e)x}{2e^6} + \frac{15(b^6d^2 - 2ab^5de + a^2b^4e^2)\log(ex + d)}{e^7}}{e^7}$$

input `integrate((b^2*x^2+2*a*b*x+a^2)^3/(e*x+d)^5,x, algorithm="maxima")`

output

```
1/4*(57*b^6*d^6 - 154*a*b^5*d^5*e + 125*a^2*b^4*d^4*e^2 - 20*a^3*b^3*d^3*e^3 - 5*a^4*b^2*d^2*e^4 - 2*a^5*b*d*e^5 - a^6*e^6 + 80*(b^6*d^3*e^3 - 3*a*b^5*d^2*e^2 + 3*a^2*b^4*d*e^5 - a^3*b^3*e^6)*x^3 + 30*(7*b^6*d^4*e^2 - 20*a*b^5*d^3*e^3 + 18*a^2*b^4*d^2*e^4 - 4*a^3*b^3*d*e^5 - a^4*b^2*e^6)*x^2 + 4*(47*b^6*d^5*e - 130*a*b^5*d^4*e^2 + 110*a^2*b^4*d^3*e^3 - 20*a^3*b^3*d^2*e^4 - 5*a^4*b^2*d*e^5 - 2*a^5*b*e^6)*x)/(e^11*x^4 + 4*d*e^10*x^3 + 6*d^2*e^9*x^2 + 4*d^3*e^8*x + d^4*e^7) + 1/2*(b^6*e*x^2 - 2*(5*b^6*d - 6*a*b^5*e)*x)/e^6 + 15*(b^6*d^2 - 2*a*b^5*d*e + a^2*b^4*e^2)*log(e*x + d)/e^7
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 516 vs. $2(149) = 298$.

Time = 0.22 (sec) , antiderivative size = 516, normalized size of antiderivative = 3.33

$$\int \frac{(a^2 + 2abx + b^2x^2)^3}{(d + ex)^5} dx$$

$$= \frac{\left(b^6 - \frac{12(b^6de - ab^5e^2)}{(ex+d)e}\right)(ex + d)^2}{2e^7} - \frac{15(b^6d^2 - 2ab^5de + a^2b^4e^2)\log\left(\frac{|ex+d|}{(ex+d)^2|e|}\right)}{e^7}$$

$$+ \frac{80b^6d^3e^{29}}{ex+d} - \frac{30b^6d^4e^{29}}{(ex+d)^2} + \frac{8b^6d^5e^{29}}{(ex+d)^3} - \frac{b^6d^6e^{29}}{(ex+d)^4} - \frac{240ab^5d^2e^{30}}{ex+d} + \frac{120ab^5d^3e^{30}}{(ex+d)^2} - \frac{40ab^5d^4e^{30}}{(ex+d)^3} + \frac{6ab^5d^5e^{30}}{(ex+d)^4} + \frac{240a^2b^4de^{31}}{ex+d}$$

input `integrate((b^2*x^2+2*a*b*x+a^2)^3/(e*x+d)^5,x, algorithm="giac")`

output

```

1/2*(b^6 - 12*(b^6*d*e - a*b^5*e^2)/((e*x + d)*e))*(e*x + d)^2/e^7 - 15*(b
^6*d^2 - 2*a*b^5*d*e + a^2*b^4*e^2)*log(abs(e*x + d)/((e*x + d)^2*abs(e)))
/e^7 + 1/4*(80*b^6*d^3*e^29/(e*x + d) - 30*b^6*d^4*e^29/(e*x + d)^2 + 8*b
^6*d^5*e^29/(e*x + d)^3 - b^6*d^6*e^29/(e*x + d)^4 - 240*a*b^5*d^2*e^30/(e*
x + d) + 120*a*b^5*d^3*e^30/(e*x + d)^2 - 40*a*b^5*d^4*e^30/(e*x + d)^3 +
6*a*b^5*d^5*e^30/(e*x + d)^4 + 240*a^2*b^4*d*e^31/(e*x + d) - 180*a^2*b^4*
d^2*e^31/(e*x + d)^2 + 80*a^2*b^4*d^3*e^31/(e*x + d)^3 - 15*a^2*b^4*d^4*e
^31/(e*x + d)^4 - 80*a^3*b^3*e^32/(e*x + d) + 120*a^3*b^3*d*e^32/(e*x + d)^
2 - 80*a^3*b^3*d^2*e^32/(e*x + d)^3 + 20*a^3*b^3*d^3*e^32/(e*x + d)^4 - 30
*a^4*b^2*e^33/(e*x + d)^2 + 40*a^4*b^2*d*e^33/(e*x + d)^3 - 15*a^4*b^2*d^2
*e^33/(e*x + d)^4 - 8*a^5*b*d*e^34/(e*x + d)^3 + 6*a^5*b*d*e^34/(e*x + d)^4
- a^6*e^35/(e*x + d)^4)/e^36

```

Mupad [B] (verification not implemented)

Time = 9.01 (sec) , antiderivative size = 387, normalized size of antiderivative = 2.50

$$\int \frac{(a^2 + 2abx + b^2x^2)^3}{(d + ex)^5} dx = x \left(\frac{6ab^5}{e^5} - \frac{5b^6d}{e^6} \right)$$

$$\frac{x^2 \left(\frac{15a^4b^2e^5}{2} + 30a^3b^3de^4 - 135a^2b^4d^2e^3 + 150ab^5d^3e^2 - \frac{105b^6d^4e}{2} \right) + \frac{a^6e^6 + 2a^5bde^5 + 5a^4b^2d^2e^4 + 20a^3b^3d^3e^3 + 15a^2b^4d^4e^2 + 6a^2b^5d^5e + 2a^5b^2d^2e^4 - 47b^6d^5 + 5a^4b^2d^2e^4 - 110a^2b^4d^3e^2 + 20a^3b^3d^2e^3 + 130a^2b^5d^4e}{(4e)} + x \left(\frac{2a^5b^2e^5 - 47b^6d^5 + 5a^4b^2d^2e^4 - 110a^2b^4d^3e^2 + 20a^3b^3d^2e^3 + 130a^2b^5d^4e}{4e} \right) + x^3 \left(\frac{20a^3b^3e^5 - 20b^6d^3e^2 + 60a^2b^5d^2e^3 - 60a^2b^4d^4e^4}{d^4e^6 + e^{10}x^4 + 4d^3e^7x + 4d^2e^9x^3 + 6d^2e^8x^2} \right) + \frac{b^6x^2}{2e^5} + \frac{\ln(d + ex) (15a^2b^4e^2 - 30ab^5de + 15b^6d^2)}{e^7}$$

input

```
int((a^2 + b^2*x^2 + 2*a*b*x)^3/(d + e*x)^5,x)
```

output

```

x*((6*a*b^5)/e^5 - (5*b^6*d)/e^6) - (x^2*((15*a^4*b^2*e^5)/2 - (105*b^6*d
^4*e)/2 + 150*a*b^5*d^3*e^2 + 30*a^3*b^3*d*e^4 - 135*a^2*b^4*d^2*e^3) + (a
^6*e^6 - 57*b^6*d^6 - 125*a^2*b^4*d^4*e^2 + 20*a^3*b^3*d^3*e^3 + 5*a^4*b^2*
d^2*e^4 + 154*a*b^5*d^5*e + 2*a^5*b*d*e^5)/(4*e) + x*(2*a^5*b^2*e^5 - 47*b^6
*d^5 + 5*a^4*b^2*d^2*e^4 - 110*a^2*b^4*d^3*e^2 + 20*a^3*b^3*d^2*e^3 + 130*a*
b^5*d^4*e) + x^3*(20*a^3*b^3*e^5 - 20*b^6*d^3*e^2 + 60*a*b^5*d^2*e^3 - 60*
a^2*b^4*d^4*e^4)/(d^4*e^6 + e^10*x^4 + 4*d^3*e^7*x + 4*d^2*e^9*x^3 + 6*d^2*e
^8*x^2) + (b^6*x^2)/(2*e^5) + (log(d + e*x)*(15*b^6*d^2 + 15*a^2*b^4*e^2 -
30*a*b^5*d*e))/e^7

```


3.98 $\int \frac{(a^2+2abx+b^2x^2)^3}{(d+ex)^6} dx$

| | |
|---|-----|
| Optimal result | 806 |
| Mathematica [A] (verified) | 806 |
| Rubi [A] (verified) | 807 |
| Maple [B] (verified) | 809 |
| Fricas [B] (verification not implemented) | 809 |
| Sympy [B] (verification not implemented) | 810 |
| Maxima [B] (verification not implemented) | 811 |
| Giac [B] (verification not implemented) | 811 |
| Mupad [B] (verification not implemented) | 812 |
| Reduce [B] (verification not implemented) | 813 |

Optimal result

Integrand size = 26, antiderivative size = 155

$$\int \frac{(a^2 + 2abx + b^2x^2)^3}{(d + ex)^6} dx = \frac{b^6x}{e^6} - \frac{(bd - ae)^6}{5e^7(d + ex)^5} + \frac{3b(bd - ae)^5}{2e^7(d + ex)^4} - \frac{5b^2(bd - ae)^4}{e^7(d + ex)^3} + \frac{10b^3(bd - ae)^3}{e^7(d + ex)^2} - \frac{15b^4(bd - ae)^2}{e^7(d + ex)} - \frac{6b^5(bd - ae) \log(d + ex)}{e^7}$$

output

```
b^6*x/e^6-1/5*(-a*e+b*d)^6/e^7/(e*x+d)^5+3/2*b*(-a*e+b*d)^5/e^7/(e*x+d)^4-5*b^2*(-a*e+b*d)^4/e^7/(e*x+d)^3+10*b^3*(-a*e+b*d)^3/e^7/(e*x+d)^2-15*b^4*(-a*e+b*d)^2/e^7/(e*x+d)-6*b^5*(-a*e+b*d)*ln(e*x+d)/e^7
```

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 297, normalized size of antiderivative = 1.92

$$\int \frac{(a^2 + 2abx + b^2x^2)^3}{(d + ex)^6} dx = \frac{2a^6e^6 + 3a^5be^5(d + 5ex) + 5a^4b^2e^4(d^2 + 5dex + 10e^2x^2) + 10a^3b^3e^3(d^3 + 5d^2ex + 10de^2x^2 + 10e^3x^3)}{e^7(d + ex)^5}$$

input `Integrate[(a^2 + 2*a*b*x + b^2*x^2)^3/(d + e*x)^6,x]`

output
$$\begin{aligned} & -1/10*(2*a^6*e^6 + 3*a^5*b*e^5*(d + 5*e*x) + 5*a^4*b^2*e^4*(d^2 + 5*d*e*x \\ & + 10*e^2*x^2) + 10*a^3*b^3*e^3*(d^3 + 5*d^2*e*x + 10*d*e^2*x^2 + 10*e^3*x^3) \\ & + 30*a^2*b^4*e^2*(d^4 + 5*d^3*e*x + 10*d^2*e^2*x^2 + 10*d*e^3*x^3 + 5*e^4*x^4) - a*b^5*d*e*(137*d^4 + 625*d^3*e*x + 1100*d^2*e^2*x^2 + 900*d*e^3*x^3 \\ & + 300*e^4*x^4) + b^6*(87*d^6 + 375*d^5*e*x + 600*d^4*e^2*x^2 + 400*d^3*e^3*x^3 + 50*d^2*e^4*x^4 - 50*d*e^5*x^5 - 10*e^6*x^6) + 60*b^5*(b*d - a*e) \\ & *(d + e*x)^5*\text{Log}[d + e*x])/(e^7*(d + e*x)^5) \end{aligned}$$

Rubi [A] (verified)

Time = 0.63 (sec) , antiderivative size = 155, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {1098, 27, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\begin{aligned} & \int \frac{(a^2 + 2abx + b^2x^2)^3}{(d + ex)^6} dx \\ & \quad \downarrow 1098 \\ & \quad \int \frac{b^6(a+bx)^6}{(d+ex)^6} dx \\ & \quad \quad \downarrow 27 \\ & \quad \quad \int \frac{(a + bx)^6}{(d + ex)^6} dx \\ & \quad \quad \quad \downarrow 49 \\ & \quad \quad \quad \int \left(-\frac{6b^5(bd - ae)}{e^6(d + ex)} + \frac{15b^4(bd - ae)^2}{e^6(d + ex)^2} - \frac{20b^3(bd - ae)^3}{e^6(d + ex)^3} + \frac{15b^2(bd - ae)^4}{e^6(d + ex)^4} - \frac{6b(bd - ae)^5}{e^6(d + ex)^5} + \frac{(ae - bd)^6}{e^6(d + ex)^6} + \frac{b^6}{e^6} \right) dx \\ & \quad \quad \quad \quad \downarrow 2009 \end{aligned}$$

$$-\frac{6b^5(bd - ae)\log(d + ex)}{e^7} - \frac{15b^4(bd - ae)^2}{e^7(d + ex)} + \frac{10b^3(bd - ae)^3}{e^7(d + ex)^2} - \frac{5b^2(bd - ae)^4}{e^7(d + ex)^3} + \frac{3b(bd - ae)^5}{2e^7(d + ex)^4} - \frac{(bd - ae)^6}{5e^7(d + ex)^5} + \frac{b^6x}{e^6}$$

input `Int[(a^2 + 2*a*b*x + b^2*x^2)^3/(d + e*x)^6,x]`

output `(b^6*x)/e^6 - (b*d - a*e)^6/(5*e^7*(d + e*x)^5) + (3*b*(b*d - a*e)^5)/(2*e^7*(d + e*x)^4) - (5*b^2*(b*d - a*e)^4)/(e^7*(d + e*x)^3) + (10*b^3*(b*d - a*e)^3)/(e^7*(d + e*x)^2) - (15*b^4*(b*d - a*e)^2)/(e^7*(d + e*x)) - (6*b^5*(b*d - a*e)*Log[d + e*x])/e^7`

Defintions of rubi rules used

rule 27 `Int[(a_)*(F_x_), x_Symbol] := Simp[a Int[F_x, x], x] /; FreeQ[a, x] && !MatchQ[F_x, (b_)*(G_x_)] /; FreeQ[b, x]`

rule 49 `Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] := Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] && IGtQ[m, 0] && IGtQ[m + n + 2, 0]`

rule 1098 `Int[((d_.) + (e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] := Simp[1/c^p Int[(d + e*x)^m*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]`

rule 2009 `Int[u_, x_Symbol] := Simp[IntSum[u, x], x] /; SumQ[u]`

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 347 vs. 2(151) = 302.

Time = 1.14 (sec) , antiderivative size = 348, normalized size of antiderivative = 2.25

| method | result |
|---------------|--|
| risch | $\frac{b^6 x}{e^6} + \frac{(-15e^5 a^2 b^4 + 30d e^4 a b^5 - 15d^2 e^3 b^6)x^4 - 10b^3 e^2 (e^3 a^3 + 3a^2 b d e^2 - 9a b^2 d^2 e + 5b^3 d^3)x^3 - 5b^2 e (a^4 e^4 + 2a^3 b d e^3 + 6a^2 b^2 d^2 e^2 - 4a b^3 d^3 e + b^4 d^4)x^2 - 3b (e^5 a^5 - 5a^4 b d e^4 + 10a^3 b^2 d^2 e^3 - 10a^2 b^3 d^3 e^2 + 5a b^4 d^4 e - b^5 d^5)x - 10(e^3 a^3 b^3 + 3a^2 b^2 d^2 e^2 - 137a b^5 d^5 e + 137b^6 d^6)x^4 - 10(e^3 a^3 b^3 + 3a^2 b^2 d^2 e^2 - 137a b^5 d^5 e + 137b^6 d^6)x^3 - 5(3e^2 a^2 b^4 - 6d e a b^5 + 6d^2 b^6)x^2 - 10(e^3 a^3 b^3 + 3a^2 b^2 d^2 e^2 - 137a b^5 d^5 e + 137b^6 d^6)x - 10(e^3 a^3 b^3 + 3a^2 b^2 d^2 e^2 - 137a b^5 d^5 e + 137b^6 d^6)}{e^7 (e x + d)^3}$ |
| default | $\frac{b^6 x}{e^6} - \frac{5b^2 (a^4 e^4 - 4a^3 b d e^3 + 6a^2 b^2 d^2 e^2 - 4a b^3 d^3 e + b^4 d^4)}{e^7 (e x + d)^3} - \frac{3b (e^5 a^5 - 5a^4 b d e^4 + 10a^3 b^2 d^2 e^3 - 10a^2 b^3 d^3 e^2 + 5a b^4 d^4 e - b^5 d^5)}{2e^7 (e x + d)^4}$ |
| norman | $\frac{b^6 x^6}{e} - \frac{2a^6 e^6 + 3a^5 b d e^5 + 5a^4 b^2 d^2 e^4 + 10a^3 b^3 d^3 e^3 + 30a^2 b^4 d^4 e^2 - 137a b^5 d^5 e + 137b^6 d^6}{10e^7} - \frac{5(3e^2 a^2 b^4 - 6d e a b^5 + 6d^2 b^6)x^4}{e^3} - \frac{10(e^3 a^3 b^3 + 3a^2 b^2 d^2 e^2 - 137a b^5 d^5 e + 137b^6 d^6)x^3}{e^3}$ |
| parallelrisch | $\frac{137a b^5 d^5 e - 3a^5 b d e^5 - 10a^3 b^3 d^3 e^3 - 5a^4 b^2 d^2 e^4 - 30a^2 b^4 d^4 e^2 - 600 \ln(e x + d)x^2 b^6 d^4 e^2 - 300 \ln(e x + d)x b^6 d^5 e - 300 \ln(e x + d)x^4}{e^7}$ |

input

```
int((b^2*x^2+2*a*b*x+a^2)^3/(e*x+d)^6,x,method=_RETURNVERBOSE)
```

output

```
b^6*x/e^6+((-15*a^2*b^4*e^5+30*a*b^5*d*e^4-15*b^6*d^2*e^3)*x^4-10*b^3*e^2*(a^3*e^3+3*a^2*b*d*e^2-9*a*b^2*d^2*e+5*b^3*d^3)*x^3-5*b^2*e*(a^4*e^4+2*a^3*b*d*e^3+6*a^2*b^2*d^2*e^2-22*a*b^3*d^3*e+13*b^4*d^4)*x^2-1/2*b*(3*a^5*e^5+5*a^4*b*d*e^4+10*a^3*b^2*d^2*e^3+30*a^2*b^3*d^3*e^2-125*a*b^4*d^4*e+77*b^5*d^5)*x-1/10/e*(2*a^6*e^6+3*a^5*b*d*e^5+5*a^4*b^2*d^2*e^4+10*a^3*b^3*d^3*e^3+30*a^2*b^4*d^4*e^2-137*a*b^5*d^5*e+87*b^6*d^6))/e^6/(e*x+d)^5+6*b^5/e^6*ln(e*x+d)*a-6*b^6/e^7*ln(e*x+d)*d
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 542 vs. 2(151) = 302.

Time = 0.08 (sec) , antiderivative size = 542, normalized size of antiderivative = 3.50

$$\int \frac{(a^2 + 2abx + b^2x^2)^3}{(d + ex)^6} dx$$

$$= \frac{10 b^6 e^6 x^6 + 50 b^6 d e^5 x^5 - 87 b^6 d^6 + 137 a b^5 d^5 e - 30 a^2 b^4 d^4 e^2 - 10 a^3 b^3 d^3 e^3 - 5 a^4 b^2 d^2 e^4 - 3 a^5 b d e^5 - 2 a^6}{e^7}$$

input

```
integrate((b^2*x^2+2*a*b*x+a^2)^3/(e*x+d)^6,x, algorithm="fricas")
```

output

```

1/10*(10*b^6*e^6*x^6 + 50*b^6*d*e^5*x^5 - 87*b^6*d^2 + 137*a*b^5*d^5*e - 3
0*a^2*b^4*d^4*e^2 - 10*a^3*b^3*d^3*e^3 - 5*a^4*b^2*d^2*e^4 - 3*a^5*b*d*e^5
- 2*a^6*e^6 - 50*(b^6*d^2*e^4 - 6*a*b^5*d*e^5 + 3*a^2*b^4*e^6)*x^4 - 100*
(4*b^6*d^3*e^3 - 9*a*b^5*d^2*e^4 + 3*a^2*b^4*d*e^5 + a^3*b^3*e^6)*x^3 - 50
*(12*b^6*d^4*e^2 - 22*a*b^5*d^3*e^3 + 6*a^2*b^4*d^2*e^4 + 2*a^3*b^3*d*e^5
+ a^4*b^2*e^6)*x^2 - 5*(75*b^6*d^5*e - 125*a*b^5*d^4*e^2 + 30*a^2*b^4*d^3*
e^3 + 10*a^3*b^3*d^2*e^4 + 5*a^4*b^2*d*e^5 + 3*a^5*b*e^6)*x - 60*(b^6*d^6
- a*b^5*d^5*e + (b^6*d*e^5 - a*b^5*e^6)*x^5 + 5*(b^6*d^2*e^4 - a*b^5*d*e^5
)*x^4 + 10*(b^6*d^3*e^3 - a*b^5*d^2*e^4)*x^3 + 10*(b^6*d^4*e^2 - a*b^5*d^3
*e^3)*x^2 + 5*(b^6*d^5*e - a*b^5*d^4*e^2)*x)*log(e*x + d))/(e^12*x^5 + 5*d
*e^11*x^4 + 10*d^2*e^10*x^3 + 10*d^3*e^9*x^2 + 5*d^4*e^8*x + d^5*e^7)

```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 420 vs. $2(143) = 286$.

Time = 19.65 (sec) , antiderivative size = 420, normalized size of antiderivative = 2.71

$$\int \frac{(a^2 + 2abx + b^2x^2)^3}{(d + ex)^6} dx = \frac{b^6x}{e^6} + \frac{6b^5(ae - bd) \log(d + ex)}{e^7} + \frac{-2a^6e^6 - 3a^5bde^5 - 5a^4b^2d^2e^4 - 10a^3b^3d^3e^3 - 30a^2b^4d^4e^2 + 137ab^5d^5e - 87b^6d^6 + x^4(-150a^2b^4e^6 + 300a^3b^3d^3e^3 - 150b^6d^2e^4) + x^3(-100a^3b^3e^6 - 30a^2b^4d^4e^2 + 90a^3b^3d^3e^3 - 500b^6d^3e^3) + x^2(-50a^4b^2e^6 - 100a^3b^3d^3e^5 - 300a^2b^4d^2e^4 + 1100a^3b^3d^3e^3 - 650b^6d^4e^2) + x(-15a^5b^5e^6 - 25a^4b^2d^2e^5 - 50a^3b^3d^2e^4 - 150a^2b^4d^3e^3 + 625a^3b^3d^3e^3 + 625a^4b^2d^2e^4 - 385b^6d^5e))/(10d^5e^7 + 50d^4e^8x + 100d^3e^9x^2 + 100d^2e^10x^3 + 50d^4e^11x^4 + 10e^12x^5)$$

input

```
integrate((b**2*x**2+2*a*b*x+a**2)**3/(e*x+d)**6,x)
```

output

```

b**6*x/e**6 + 6*b**5*(a*e - b*d)*log(d + e*x)/e**7 + (-2*a**6*e**6 - 3*a**
5*b*d*e**5 - 5*a**4*b**2*d**2*e**4 - 10*a**3*b**3*d**3*e**3 - 30*a**2*b**4
*d**4*e**2 + 137*a*b**5*d**5*e - 87*b**6*d**6 + x**4*(-150*a**2*b**4*e**6
+ 300*a*b**5*d*e**5 - 150*b**6*d**2*e**4) + x**3*(-100*a**3*b**3*e**6 - 30
0*a**2*b**4*d*e**5 + 900*a*b**5*d**2*e**4 - 500*b**6*d**3*e**3) + x**2*(-5
0*a**4*b**2*e**6 - 100*a**3*b**3*d*e**5 - 300*a**2*b**4*d**2*e**4 + 1100*a
*b**5*d**3*e**3 - 650*b**6*d**4*e**2) + x*(-15*a**5*b*e**6 - 25*a**4*b**2*
d*e**5 - 50*a**3*b**3*d**2*e**4 - 150*a**2*b**4*d**3*e**3 + 625*a*b**5*d**
4*e**2 - 385*b**6*d**5*e))/(10*d**5*e**7 + 50*d**4*e**8*x + 100*d**3*e**9*
x**2 + 100*d**2*e**10*x**3 + 50*d*e**11*x**4 + 10*e**12*x**5)

```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 397 vs. $2(151) = 302$.

Time = 0.07 (sec) , antiderivative size = 397, normalized size of antiderivative = 2.56

$$\int \frac{(a^2 + 2abx + b^2x^2)^3}{(d + ex)^6} dx = \frac{b^6x}{e^6} - \frac{87b^6d^6 - 137ab^5d^5e + 30a^2b^4d^4e^2 + 10a^3b^3d^3e^3 + 5a^4b^2d^2e^4 + 3a^5bde^5 + 2a^6e^6 + 150(b^6d^2e^4 - 2ab^5d^2e^2 + a^2b^4d^2e^2)}{e^7} - \frac{6(b^6d - ab^5e) \log(ex + d)}{e^7}$$

input

```
integrate((b^2*x^2+2*a*b*x+a^2)^3/(e*x+d)^6,x, algorithm="maxima")
```

output

```
b^6*x/e^6 - 1/10*(87*b^6*d^6 - 137*a*b^5*d^5*e + 30*a^2*b^4*d^4*e^2 + 10*a^3*b^3*d^3*e^3 + 5*a^4*b^2*d^2*e^4 + 3*a^5*b*d*e^5 + 2*a^6*e^6 + 150*(b^6*d^2*e^4 - 2*a*b^5*d^2*e^2 + a^2*b^4*d^2*e^2)*x^4 + 100*(5*b^6*d^3*e^3 - 9*a*b^5*d^2*e^4 + 3*a^2*b^4*d*e^5 + a^3*b^3*e^6)*x^3 + 50*(13*b^6*d^4*e^2 - 22*a*b^5*d^3*e^3 + 6*a^2*b^4*d^2*e^4 + 2*a^3*b^3*d*e^5 + a^4*b^2*e^6)*x^2 + 5*(7*b^6*d^5*e - 125*a*b^5*d^4*e^2 + 30*a^2*b^4*d^3*e^3 + 10*a^3*b^3*d^2*e^4 + 5*a^4*b^2*d*e^5 + 3*a^5*b*e^6)*x)/(e^12*x^5 + 5*d*e^11*x^4 + 10*d^2*e^10*x^3 + 10*d^3*e^9*x^2 + 5*d^4*e^8*x + d^5*e^7) - 6*(b^6*d - a*b^5*e)*log(e*x + d)/e^7
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 351 vs. $2(151) = 302$.

Time = 0.20 (sec) , antiderivative size = 351, normalized size of antiderivative = 2.26

$$\int \frac{(a^2 + 2abx + b^2x^2)^3}{(d + ex)^6} dx = \frac{b^6x}{e^6} - \frac{6(b^6d - ab^5e) \log(|ex + d|)}{e^7} - \frac{87b^6d^6 - 137ab^5d^5e + 30a^2b^4d^4e^2 + 10a^3b^3d^3e^3 + 5a^4b^2d^2e^4 + 3a^5bde^5 + 2a^6e^6 + 150(b^6d^2e^4 - 2ab^5d^2e^2 + a^2b^4d^2e^2)}{e^7}$$

input

```
integrate((b^2*x^2+2*a*b*x+a^2)^3/(e*x+d)^6,x, algorithm="giac")
```


output

$$\begin{aligned} & b^6 x / e^6 - 6(b^6 d - a b^5 e) \log(\operatorname{abs}(e x + d)) / e^7 - 1/10(87 b^6 d^6 - \\ & 137 a b^5 d^5 e + 30 a^2 b^4 d^4 e^2 + 10 a^3 b^3 d^3 e^3 + 5 a^4 b^2 d^2 e^4 + 3 a^5 b d e^5 + 2 a^6 e^6 + 150(b^6 d^2 e^4 - 2 a b^5 d e^5 + a^2 b^4 e^6) x^4 + 100(5 b^6 d^3 e^3 - 9 a b^5 d^2 e^4 + 3 a^2 b^4 d e^5 + a^3 b^3 e^6) x^3 + 50(13 b^6 d^4 e^2 - 22 a b^5 d^3 e^3 + 6 a^2 b^4 d^2 e^4 + 2 a^3 b^3 d e^5 + a^4 b^2 e^6) x^2 + 5(77 b^6 d^5 e - 125 a b^5 d^4 e^2 + 30 a^2 b^4 d^3 e^3 + 10 a^3 b^3 d^2 e^4 + 5 a^4 b^2 d e^5 + 3 a^5 b e^6) x) / ((e x + d)^5 e^7) \end{aligned}$$

Mupad [B] (verification not implemented)

Time = 8.84 (sec) , antiderivative size = 399, normalized size of antiderivative = 2.57

$$\int \frac{(a^2 + 2abx + b^2x^2)^3}{(d + ex)^6} dx = \frac{b^6 x}{e^6} - \frac{\ln(d + ex)(6b^6 d - 6ab^5 e)}{e^7} - \frac{x^2(5a^4 b^2 e^5 + 10a^3 b^3 d e^4 + 30a^2 b^4 d^2 e^3 - 110ab^5 d^3 e^2 + 65b^6 d^4 e) + x^4(15a^2 b^4 e^5 - 30ab^5 d e^4 + \dots)}{\dots}$$

input

$$\operatorname{int}((a^2 + b^2 x^2 + 2 a b x)^3 / (d + e x)^6, x)$$

output

$$\begin{aligned} & (b^6 x) / e^6 - (\log(d + e x) * (6 b^6 d - 6 a b^5 e)) / e^7 - (x^2 * (65 b^6 d^4 e + 5 a^4 b^2 e^5 - 110 a b^5 d^3 e^2 + 10 a^3 b^3 d e^4 + 30 a^2 b^4 d^2 e^3) + x^4 * (15 a^2 b^4 e^5 + 15 b^6 d^2 e^3 - 30 a b^5 d e^4) + (2 a^6 e^6 + 87 b^6 d^6 + 30 a^2 b^4 d^4 e^2 + 10 a^3 b^3 d^3 e^3 + 5 a^4 b^2 d^2 e^4 - 137 a b^5 d^5 e + 3 a^5 b d e^5) / (10 e) + x * ((77 b^6 d^5) / 2 + (3 a^5 b e^5) / 2 + (5 a^4 b^2 d e^4) / 2 + 15 a^2 b^4 d^3 e^2 + 5 a^3 b^3 d^2 e^3 - (125 a b^5 d^4 e) / 2) + x^3 * (10 a^3 b^3 e^5 + 50 b^6 d^3 e^2 - 90 a b^5 d^2 e^3 + 30 a^2 b^4 d e^4) / (d^5 e^6 + e^{11} x^5 + 5 d^4 e^7 x + 5 d e^{10} x^4 + 10 d^3 e^8 x^2 + 10 d^2 e^9 x^3) \end{aligned}$$

Reduce [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 576, normalized size of antiderivative = 3.72

$$\int \frac{(a^2 + 2abx + b^2x^2)^3}{(d + ex)^6} dx$$

$$= \frac{-2a^6de^6 + 60\log(ex + d)ab^5d^6e - 300\log(ex + d)b^6d^6ex - 15a^5bd^6e^6x - 25a^4b^2d^2e^5x - 50a^4b^2de^6x^2}{(d + ex)^6}$$

input `int((b^2*x^2+2*a*b*x+a^2)^3/(e*x+d)^6,x)`

output `(60*log(d + e*x)*a*b**5*d**6*e + 300*log(d + e*x)*a*b**5*d**5*e**2*x + 600*log(d + e*x)*a*b**5*d**4*e**3*x**2 + 600*log(d + e*x)*a*b**5*d**3*e**4*x**3 + 300*log(d + e*x)*a*b**5*d**2*e**5*x**4 + 60*log(d + e*x)*a*b**5*d**6*x**5 - 60*log(d + e*x)*b**6*d**7 - 300*log(d + e*x)*b**6*d**6*e*x - 600*log(d + e*x)*b**6*d**5*e**2*x**2 - 600*log(d + e*x)*b**6*d**4*e**3*x**3 - 300*log(d + e*x)*b**6*d**3*e**4*x**4 - 60*log(d + e*x)*b**6*d**2*e**5*x**5 - 2*a**6*d*e**6 - 3*a**5*b*d**2*e**5 - 15*a**5*b*d*e**6*x - 5*a**4*b**2*d**3*e**4 - 25*a**4*b**2*d**2*e**5*x - 50*a**4*b**2*d*e**6*x**2 - 10*a**3*b**3*d**4*e**3 - 50*a**3*b**3*d**3*e**4*x - 100*a**3*b**3*d**2*e**5*x**2 - 100*a**3*b**3*d*e**6*x**3 + 30*a**2*b**4*e**7*x**5 + 77*a*b**5*d**6*e + 325*a*b**5*d**5*e**2*x + 500*a*b**5*d**4*e**3*x**2 + 300*a*b**5*d**3*e**4*x**3 - 60*a*b**5*d*e**6*x**5 - 77*b**6*d**7 - 325*b**6*d**6*e*x - 500*b**6*d**5*e**2*x**2 - 300*b**6*d**4*e**3*x**3 + 60*b**6*d**2*e**5*x**5 + 10*b**6*d*e**6*x**6)/(10*d*e**7*(d**5 + 5*d**4*e*x + 10*d**3*e**2*x**2 + 10*d**2*e**3*x**3 + 5*d*e**4*x**4 + e**5*x**5))`

3.99 $\int \frac{(a^2+2abx+b^2x^2)^3}{(d+ex)^7} dx$

| | |
|---|-----|
| Optimal result | 814 |
| Mathematica [A] (verified) | 814 |
| Rubi [A] (verified) | 815 |
| Maple [B] (verified) | 817 |
| Fricas [B] (verification not implemented) | 817 |
| Sympy [B] (verification not implemented) | 818 |
| Maxima [B] (verification not implemented) | 819 |
| Giac [B] (verification not implemented) | 819 |
| Mupad [B] (verification not implemented) | 820 |
| Reduce [B] (verification not implemented) | 821 |

Optimal result

Integrand size = 26, antiderivative size = 167

$$\int \frac{(a^2 + 2abx + b^2x^2)^3}{(d + ex)^7} dx = -\frac{(bd - ae)^6}{6e^7(d + ex)^6} + \frac{6b(bd - ae)^5}{5e^7(d + ex)^5} - \frac{15b^2(bd - ae)^4}{4e^7(d + ex)^4} + \frac{20b^3(bd - ae)^3}{3e^7(d + ex)^3} - \frac{15b^4(bd - ae)^2}{2e^7(d + ex)^2} + \frac{6b^5(bd - ae)}{e^7(d + ex)} + \frac{b^6 \log(d + ex)}{e^7}$$

output

```
-1/6*(-a*e+b*d)^6/e^7/(e*x+d)^6+6/5*b*(-a*e+b*d)^5/e^7/(e*x+d)^5-15/4*b^2*(-a*e+b*d)^4/e^7/(e*x+d)^4+20/3*b^3*(-a*e+b*d)^3/e^7/(e*x+d)^3-15/2*b^4*(-a*e+b*d)^2/e^7/(e*x+d)^2+6*b^5*(-a*e+b*d)/e^7/(e*x+d)+b^6*ln(e*x+d)/e^7
```

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 233, normalized size of antiderivative = 1.40

$$\int \frac{(a^2 + 2abx + b^2x^2)^3}{(d + ex)^7} dx = \frac{(bd-ae)(10a^5e^5+2a^4be^4(11d+36ex)+a^3b^2e^3(37d^2+162dex+225e^2x^2)+a^2b^3e^2(57d^3+282d^2ex+525de^2x^2+400e^3x^3)+ab^4e(87d^4+462d^3ex+60e^4x^2)+b^5e^2(57d^5+282d^4ex+525d^3e^2x^2+400d^2e^3x^3)+b^6e^3(87d^6+462d^5ex+60e^2d^4x^2+60e^3d^3x^3+60e^4d^2x^4+60e^5dx^5+60e^6x^6))}{(d+ex)^6}$$

input `Integrate[(a^2 + 2*a*b*x + b^2*x^2)^3/(d + e*x)^7,x]`

output `((((b*d - a*e)*(10*a^5*e^5 + 2*a^4*b*e^4*(11*d + 36*e*x) + a^3*b^2*e^3*(37*d^2 + 162*d*e*x + 225*e^2*x^2) + a^2*b^3*e^2*(57*d^3 + 282*d^2*e*x + 525*d*e^2*x^2 + 400*e^3*x^3) + a*b^4*e*(87*d^4 + 462*d^3*e*x + 975*d^2*e^2*x^2 + 1000*d*e^3*x^3 + 450*e^4*x^4) + b^5*(147*d^5 + 822*d^4*e*x + 1875*d^3*e^2*x^2 + 2200*d^2*e^3*x^3 + 1350*d*e^4*x^4 + 360*e^5*x^5)))/(d + e*x)^6 + 60*b^6*Log[d + e*x])/(60*e^7)`

Rubi [A] (verified)

Time = 0.62 (sec) , antiderivative size = 167, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {1098, 27, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a^2 + 2abx + b^2x^2)^3}{(d + ex)^7} dx$$

$$\downarrow 1098$$

$$\int \frac{b^6(a+bx)^6}{(d+ex)^7} dx$$

$$\downarrow 27$$

$$\int \frac{(a + bx)^6}{(d + ex)^7} dx$$

$$\downarrow 49$$

$$\int \left(-\frac{6b^5(bd - ae)}{e^6(d + ex)^2} + \frac{15b^4(bd - ae)^2}{e^6(d + ex)^3} - \frac{20b^3(bd - ae)^3}{e^6(d + ex)^4} + \frac{15b^2(bd - ae)^4}{e^6(d + ex)^5} - \frac{6b(bd - ae)^5}{e^6(d + ex)^6} + \frac{(ae - bd)^6}{e^6(d + ex)^7} + \frac{1}{e^6(d + ex)^7} \right) dx$$

$$\downarrow 2009$$

$$\frac{6b^5(bd - ae)}{e^7(d + ex)} - \frac{15b^4(bd - ae)^2}{2e^7(d + ex)^2} + \frac{20b^3(bd - ae)^3}{3e^7(d + ex)^3} - \frac{15b^2(bd - ae)^4}{4e^7(d + ex)^4} + \frac{6b(bd - ae)^5}{5e^7(d + ex)^5} - \frac{(bd - ae)^6}{6e^7(d + ex)^6} + \frac{b^6 \log(d + ex)}{e^7}$$

input `Int[(a^2 + 2*a*b*x + b^2*x^2)^3/(d + e*x)^7,x]`

output `-1/6*(b*d - a*e)^6/(e^7*(d + e*x)^6) + (6*b*(b*d - a*e)^5)/(5*e^7*(d + e*x)^5) - (15*b^2*(b*d - a*e)^4)/(4*e^7*(d + e*x)^4) + (20*b^3*(b*d - a*e)^3)/(3*e^7*(d + e*x)^3) - (15*b^4*(b*d - a*e)^2)/(2*e^7*(d + e*x)^2) + (6*b^5*(b*d - a*e))/(e^7*(d + e*x)) + (b^6*Log[d + e*x])/e^7`

Defintions of rubi rules used

rule 27 `Int[(a_)*(F_x_), x_Symbol] := Simp[a Int[F_x, x], x] /; FreeQ[a, x] && !MatchQ[F_x, (b_)*(G_x_)] /; FreeQ[b, x]`

rule 49 `Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] := Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] && IGtQ[m, 0] && IGtQ[m + n + 2, 0]`

rule 1098 `Int[((d_.) + (e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] := Simp[1/c^p Int[(d + e*x)^m*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]`

rule 2009 `Int[u_, x_Symbol] := Simp[IntSum[u, x], x] /; SumQ[u]`

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 341 vs. $2(157) = 314$.

Time = 1.07 (sec) , antiderivative size = 342, normalized size of antiderivative = 2.05

| method | result |
|---------------|---|
| risch | $\frac{-\frac{6b^5(ae-bd)x^5}{e^2} - \frac{15b^4(e^2a^2+2abde-3b^2d^2)x^4}{2e^3} - \frac{10b^3(2e^3a^3+3a^2bde^2+6ab^2d^2e-11b^3d^3)x^3}{3e^4} - \frac{5b^2(3a^4e^4+4a^3bde^3+6a^2b^2d^2e^2+12ab^3d^3e-13b^4d^4)x^2}{4e^5} - \frac{10a^6e^6+12a^5bde^5+15a^4b^2d^2e^4+20a^3b^3d^3e^3+30a^2b^4d^4e^2+60ab^5d^5e-147b^6d^6}{60e^7} - \frac{6(eab^5-db^6)x^5}{e^2} - \frac{15(e^2a^2b^4+2deab^5-3d^2b^6)x^4}{2e^3}}$ |
| norman | |
| default | $-\frac{20b^3(e^3a^3-3a^2bde^2+3ab^2d^2e-b^3d^3)}{3e^7(ex+d)^3} - \frac{15b^2(a^4e^4-4a^3bde^3+6a^2b^2d^2e^2-4ab^3d^3e+b^4d^4)}{4e^7(ex+d)^4} + \frac{b^6 \ln(ex+d)}{e^7} - \frac{6b(e^5a^6-6a^5bde^5-12a^4b^2d^2e^4-20a^3b^3d^3e^3-15a^4b^2d^2e^4-30a^2b^4d^4e^2+900 \ln(ex+d)x^2b^6d^4e^2+360 \ln(ex+d)xb^6d^5e+900 \ln(ex+d)b^6d^6)}{e^7}$ |
| parallelrisch | |

input

```
int((b^2*x^2+2*a*b*x+a^2)^3/(e*x+d)^7,x,method=_RETURNVERBOSE)
```

output

```
(-6*b^5*(a*e-b*d)/e^2*x^5-15/2*b^4*(a^2*e^2+2*a*b*d*e-3*b^2*d^2)/e^3*x^4-10/3*b^3*(2*a^3*e^3+3*a^2*b*d*e^2+6*a*b^2*d^2*e-11*b^3*d^3)/e^4*x^3-5/4*b^2*(3*a^4*e^4+4*a^3*b*d*e^3+6*a^2*b^2*d^2*e^2+12*a*b^3*d^3*e-25*b^4*d^4)/e^5*x^2-1/10*b*(12*a^5*e^5+15*a^4*b*d*e^4+20*a^3*b^2*d^2*e^3+30*a^2*b^3*d^3*e^2+60*a*b^4*d^4*e-137*b^5*d^5)/e^6*x-1/60*(10*a^6*e^6+12*a^5*b*d*e^5+15*a^4*b^2*d^2*e^4+20*a^3*b^3*d^3*e^3+30*a^2*b^4*d^4*e^2+60*a*b^5*d^5*e-147*b^6*d^6)/e^7)/(e*x+d)^6+b^6*ln(e*x+d)/e^7
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 492 vs. $2(157) = 314$.

Time = 0.09 (sec) , antiderivative size = 492, normalized size of antiderivative = 2.95

$$\int \frac{(a^2 + 2abx + b^2x^2)^3}{(d + ex)^7} dx$$

$$= \frac{147b^6d^6 - 60ab^5d^5e - 30a^2b^4d^4e^2 - 20a^3b^3d^3e^3 - 15a^4b^2d^2e^4 - 12a^5bde^5 - 10a^6e^6 + 360(b^6de^5 - ab^5d^6)}{(d + ex)^7}$$

input

```
integrate((b^2*x^2+2*a*b*x+a^2)^3/(e*x+d)^7,x, algorithm="fricas")
```

output

```
1/60*(147*b^6*d^6 - 60*a*b^5*d^5*e - 30*a^2*b^4*d^4*e^2 - 20*a^3*b^3*d^3*e^3 - 15*a^4*b^2*d^2*e^4 - 12*a^5*b*d*e^5 - 10*a^6*e^6 + 360*(b^6*d*e^5 - a*b^5*e^6)*x^5 + 450*(3*b^6*d^2*e^4 - 2*a*b^5*d*e^5 - a^2*b^4*e^6)*x^4 + 200*(11*b^6*d^3*e^3 - 6*a*b^5*d^2*e^4 - 3*a^2*b^4*d*e^5 - 2*a^3*b^3*e^6)*x^3 + 75*(25*b^6*d^4*e^2 - 12*a*b^5*d^3*e^3 - 6*a^2*b^4*d^2*e^4 - 4*a^3*b^3*d*e^5 - 3*a^4*b^2*e^6)*x^2 + 6*(137*b^6*d^5*e - 60*a*b^5*d^4*e^2 - 30*a^2*b^4*d^3*e^3 - 20*a^3*b^3*d^2*e^4 - 15*a^4*b^2*d*e^5 - 12*a^5*b*e^6)*x + 60*(b^6*e^6*x^6 + 6*b^6*d*e^5*x^5 + 15*b^6*d^2*e^4*x^4 + 20*b^6*d^3*e^3*x^3 + 15*b^6*d^4*e^2*x^2 + 6*b^6*d^5*e*x + b^6*d^6)*log(e*x + d)/(e^13*x^6 + 6*d*e^12*x^5 + 15*d^2*e^11*x^4 + 20*d^3*e^10*x^3 + 15*d^4*e^9*x^2 + 6*d^5*e^8*x + d^6*e^7)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 439 vs. $2(153) = 306$.

Time = 66.75 (sec) , antiderivative size = 439, normalized size of antiderivative = 2.63

$$\int \frac{(a^2 + 2abx + b^2x^2)^3}{(d + ex)^7} dx = \frac{b^6 \log(d + ex)}{e^7} + \frac{-10a^6e^6 - 12a^5bde^5 - 15a^4b^2d^2e^4 - 20a^3b^3d^3e^3 - 30a^2b^4d^4e^2 - 60ab^5d^5e + 147b^6d^6 + x^5(-360ab^5e^6 - 360b^6d^5e^5) + x^4(-450a^2b^4e^6 - 900ab^5d^5e^5 + 1350b^6d^4e^4) + x^3(-400a^3b^3e^6 - 600a^2b^4d^4e^5 - 1200ab^5d^3e^4 + 2200b^6d^2e^3) + x^2(-225a^4b^2e^6 - 300a^3b^3d^3e^5 - 450a^2b^4d^2e^4 - 900ab^5d^3e^3 + 1875b^6d^4e^2) + x(-72a^5b^5e^6 - 90a^4b^4d^4e^5 - 120a^3b^3d^3e^4 - 180a^2b^4d^2e^3 - 360ab^5d^4e^2 + 822b^6d^5e)}{(60d^6e^7 + 360d^5e^8x + 900d^4e^9x^2 + 1200d^3e^10x^3 + 900d^2e^11x^4 + 360de^12x^5 + 60e^13x^6)}$$

input

```
integrate((b**2*x**2+2*a*b*x+a**2)**3/(e*x+d)**7,x)
```

output

```
b**6*log(d + e*x)/e**7 + (-10*a**6*e**6 - 12*a**5*b*d*e**5 - 15*a**4*b**2*d**2*e**4 - 20*a**3*b**3*d**3*e**3 - 30*a**2*b**4*d**4*e**2 - 60*a*b**5*d**5*e + 147*b**6*d**6 + x**5*(-360*a*b**5*e**6 + 360*b**6*d*e**5) + x**4*(-450*a**2*b**4*e**6 - 900*a*b**5*d*e**5 + 1350*b**6*d**2*e**4) + x**3*(-400*a**3*b**3*e**6 - 600*a**2*b**4*d*e**5 - 1200*a*b**5*d**2*e**4 + 2200*b**6*d**3*e**3) + x**2*(-225*a**4*b**2*e**6 - 300*a**3*b**3*d*e**5 - 450*a**2*b**4*d**2*e**4 - 900*a*b**5*d**3*e**3 + 1875*b**6*d**4*e**2) + x*(-72*a**5*b*e**6 - 90*a**4*b**2*d*e**5 - 120*a**3*b**3*d**2*e**4 - 180*a**2*b**4*d**3*e**3 - 360*a*b**5*d**4*e**2 + 822*b**6*d**5*e))/(60*d**6*e**7 + 360*d**5*e**8*x + 900*d**4*e**9*x**2 + 1200*d**3*e**10*x**3 + 900*d**2*e**11*x**4 + 360*d*e**12*x**5 + 60*e**13*x**6)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 416 vs. $2(157) = 314$.

Time = 0.06 (sec) , antiderivative size = 416, normalized size of antiderivative = 2.49

$$\int \frac{(a^2 + 2abx + b^2x^2)^3}{(d + ex)^7} dx$$

$$= \frac{147 b^6 d^6 - 60 a b^5 d^5 e - 30 a^2 b^4 d^4 e^2 - 20 a^3 b^3 d^3 e^3 - 15 a^4 b^2 d^2 e^4 - 12 a^5 b d e^5 - 10 a^6 e^6 + 360 (b^6 d e^5 - a b^5 d^2 e^4 + 450 (3 b^6 d^2 e^4 - 2 a b^5 d^3 e^5 - a^2 b^4 e^6) x^4 + 200 (11 b^6 d^3 e^3 - 6 a b^5 d^2 e^4 - 3 a^2 b^4 d e^5 - 2 a^3 b^3 e^6) x^3 + 75 (25 b^6 d^4 e^2 - 12 a b^5 d^3 e^3 - 6 a^2 b^4 d^2 e^4 - 4 a^3 b^3 d e^5 - 3 a^4 b^2 e^6) x^2 + 6 (137 b^6 d^5 e - 60 a b^5 d^4 e^2 - 30 a^2 b^4 d^3 e^3 - 20 a^3 b^3 d^2 e^4 - 15 a^4 b^2 d e^5 - 12 a^5 b e^6) x}{e^7} + \frac{b^6 \log(ex + d)}{e^7}$$

input

```
integrate((b^2*x^2+2*a*b*x+a^2)^3/(e*x+d)^7,x, algorithm="maxima")
```

output

```
1/60*(147*b^6*d^6 - 60*a*b^5*d^5*e - 30*a^2*b^4*d^4*e^2 - 20*a^3*b^3*d^3*e^3 - 15*a^4*b^2*d^2*e^4 - 12*a^5*b*d*e^5 - 10*a^6*e^6 + 360*(b^6*d*e^5 - a*b^5*e^6)*x^5 + 450*(3*b^6*d^2*e^4 - 2*a*b^5*d*e^5 - a^2*b^4*e^6)*x^4 + 200*(11*b^6*d^3*e^3 - 6*a*b^5*d^2*e^4 - 3*a^2*b^4*d*e^5 - 2*a^3*b^3*e^6)*x^3 + 75*(25*b^6*d^4*e^2 - 12*a*b^5*d^3*e^3 - 6*a^2*b^4*d^2*e^4 - 4*a^3*b^3*d*e^5 - 3*a^4*b^2*e^6)*x^2 + 6*(137*b^6*d^5*e - 60*a*b^5*d^4*e^2 - 30*a^2*b^4*d^3*e^3 - 20*a^3*b^3*d^2*e^4 - 15*a^4*b^2*d*e^5 - 12*a^5*b*e^6)*x)/(e^13*x^6 + 6*d*e^12*x^5 + 15*d^2*e^11*x^4 + 20*d^3*e^10*x^3 + 15*d^4*e^9*x^2 + 6*d^5*e^8*x + d^6*e^7) + b^6*log(e*x + d)/e^7
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 359 vs. $2(157) = 314$.

Time = 0.18 (sec) , antiderivative size = 359, normalized size of antiderivative = 2.15

$$\int \frac{(a^2 + 2abx + b^2x^2)^3}{(d + ex)^7} dx = \frac{b^6 \log(|ex + d|)}{e^7} + \frac{360 (b^6 d e^4 - a b^5 e^5) x^5 + 450 (3 b^6 d^2 e^3 - 2 a b^5 d e^4 - a^2 b^4 e^5) x^4 + 200 (11 b^6 d^3 e^2 - 6 a b^5 d^2 e^3 - 3 a^2 b^4 d e^4 - 2 a^3 b^3 e^5) x^3 + 75 (25 b^6 d^4 e - 12 a b^5 d^3 e^2 - 6 a^2 b^4 d^2 e^3 - 4 a^3 b^3 d e^4 - 3 a^4 b^2 e^5) x^2 + 6 (137 b^6 d^5 - 60 a b^5 d^4 e - 30 a^2 b^4 d^3 e^2 - 20 a^3 b^3 d^2 e^3 - 15 a^4 b^2 d e^4 - 12 a^5 b e^5) x}{e^7}$$

input

```
integrate((b^2*x^2+2*a*b*x+a^2)^3/(e*x+d)^7,x, algorithm="giac")
```


output

```

b^6*log(abs(e*x + d))/e^7 + 1/60*(360*(b^6*d*e^4 - a*b^5*e^5)*x^5 + 450*(3
*b^6*d^2*e^3 - 2*a*b^5*d*e^4 - a^2*b^4*e^5)*x^4 + 200*(11*b^6*d^3*e^2 - 6*
a*b^5*d^2*e^3 - 3*a^2*b^4*d*e^4 - 2*a^3*b^3*e^5)*x^3 + 75*(25*b^6*d^4*e -
12*a*b^5*d^3*e^2 - 6*a^2*b^4*d^2*e^3 - 4*a^3*b^3*d*e^4 - 3*a^4*b^2*e^5)*x^
2 + 6*(137*b^6*d^5 - 60*a*b^5*d^4*e - 30*a^2*b^4*d^3*e^2 - 20*a^3*b^3*d^2*
e^3 - 15*a^4*b^2*d*e^4 - 12*a^5*b*e^5)*x + (147*b^6*d^6 - 60*a*b^5*d^5*e -
30*a^2*b^4*d^4*e^2 - 20*a^3*b^3*d^3*e^3 - 15*a^4*b^2*d^2*e^4 - 12*a^5*b*d
*e^5 - 10*a^6*e^6)/e)/((e*x + d)^6*e^6)

```

Mupad [B] (verification not implemented)

Time = 8.80 (sec) , antiderivative size = 353, normalized size of antiderivative = 2.11

$$\int \frac{(a^2 + 2abx + b^2x^2)^3}{(d + ex)^7} dx = \frac{b^6 \ln(d + ex)}{e^7} - \frac{x^5(6ab^5e^6 - 6b^6de^5)}{e^7} + x^2 \left(\frac{15a^4b^2e^6}{4} + 5a^3b^3de^5 + \frac{15a^2b^4d^2e^4}{2} + 15ab^5d^3e^3 - \frac{125b^6d^4e^2}{4} \right) + x^4 \left(\frac{15a^4b^2e^6}{4} - \frac{125b^6d^4e^2}{4} + 15a^3b^3de^5 + 5a^2b^4d^2e^4 + 15a^2b^4d^2e^4 \right) + x^4 \left(\frac{15a^2b^4d^2e^4}{2} + x^4 \left(\frac{15a^2b^4d^2e^4}{2} - \frac{45b^6d^2e^4}{2} + 15a^2b^4d^2e^4 \right) + x \left(\frac{6a^5b^2e^6}{5} - \frac{137b^6d^5e}{10} + 6a^2b^4d^3e^3 + 2a^3b^3d^2e^4 + \frac{3a^4b^2d^2e^5}{2} + 3a^2b^4d^3e^3 + 2a^3b^3d^2e^4 \right) + \frac{a^6e^6}{6} - \frac{49b^6d^6}{20} + x^3 \left(\frac{20a^3b^3e^6}{3} - \frac{110b^6d^3e^3}{3} + 20a^2b^5d^2e^4 + 10a^2b^4d^4e^5 \right) + \frac{a^2b^4d^4e^2}{2} + \frac{a^3b^3d^3e^3}{3} + \frac{a^4b^2d^2e^4}{4} + ab^5d^5e + \frac{a^5b^2d^2e^5}{5} \right) / (e^7(d + ex)^6)$$

input

```
int((a^2 + b^2*x^2 + 2*a*b*x)^3/(d + e*x)^7,x)
```

output

```

(b^6*log(d + e*x))/e^7 - (x^5*(6*a*b^5*e^6 - 6*b^6*d*e^5) + x^2*((15*a^4*b
^2*e^6)/4 - (125*b^6*d^4*e^2)/4 + 15*a*b^5*d^3*e^3 + 5*a^3*b^3*d*e^5 + (15
*a^2*b^4*d^2*e^4)/2) + x^4*((15*a^2*b^4*d^2*e^4)/2 - (45*b^6*d^2*e^4)/2 + 15*a
*b^5*d*e^5) + x*((6*a^5*b^2*e^6)/5 - (137*b^6*d^5*e)/10 + 6*a^2*b^4*d^3*e^3 +
(3*a^4*b^2*d^2*e^5)/2 + 3*a^2*b^4*d^3*e^3 + 2*a^3*b^3*d^2*e^4) + (a^6*e^6)/6
- (49*b^6*d^6)/20 + x^3*((20*a^3*b^3*e^6)/3 - (110*b^6*d^3*e^3)/3 + 20*a*
b^5*d^2*e^4 + 10*a^2*b^4*d^4*e^5) + (a^2*b^4*d^4*e^2)/2 + (a^3*b^3*d^3*e^3)/
3 + (a^4*b^2*d^2*e^4)/4 + a*b^5*d^5*e + (a^5*b^2*d^2*e^5)/5)/(e^7*(d + e*x)^6)

```


output

$$\begin{aligned} & -1/7*(a^6*e^6 + a^5*b*e^5*(d + 7*e*x) + a^4*b^2*e^4*(d^2 + 7*d*e*x + 21*e^2*x^2) \\ & + a^3*b^3*e^3*(d^3 + 7*d^2*e*x + 21*d*e^2*x^2 + 35*e^3*x^3) + a^2*b^4*e^2*(d^4 + 7*d^3*e*x \\ & + 21*d^2*e^2*x^2 + 35*d*e^3*x^3 + 35*e^4*x^4) + a*b^5*e*(d^5 + 7*d^4*e*x + 21*d^3*e^2*x^2 \\ & + 35*d^2*e^3*x^3 + 35*d*e^4*x^4 + 21*e^5*x^5) + b^6*(d^6 + 7*d^5*e*x + 21*d^4*e^2*x^2 + 35*d^3*e^3*x^3 + 35*d^2*e^4*x^4 \\ & + 21*d*e^5*x^5 + 7*e^6*x^6))/(e^7*(d + e*x)^7) \end{aligned}$$

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.115$, Rules used = {1098, 27, 48}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\begin{aligned} & \int \frac{(a^2 + 2abx + b^2x^2)^3}{(d + ex)^8} dx \\ & \quad \downarrow 1098 \\ & \int \frac{b^6(a+bx)^6}{(d+ex)^8} dx \\ & \quad \downarrow 27 \\ & \int \frac{(a + bx)^6}{(d + ex)^8} dx \\ & \quad \downarrow 48 \\ & \frac{(a + bx)^7}{7(d + ex)^7(bd - ae)} \end{aligned}$$

input

$$\text{Int}[(a^2 + 2*a*b*x + b^2*x^2)^3/(d + e*x)^8,x]$$

output

$$(a + b*x)^7/(7*(b*d - a*e)*(d + e*x)^7)$$

Defintions of rubi rules used

rule 27 `Int[(a_)*(Fx_), x_Symbol] := Simp[a Int[Fx, x], x] /; FreeQ[a, x] && !MatchQ[Fx, (b_)*(Gx_)] /; FreeQ[b, x]`

rule 48 `Int[((a_.) + (b_.)*(x_)^(m_.))*((c_.) + (d_.)*(x_)^(n_.), x_Symbol] := Simp[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{a, b, c, d, m, n}, x] && EqQ[m + n + 2, 0] && NeQ[m, -1]`

rule 1098 `Int[((d_.) + (e_.)*(x_)^(m_.))*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] := Simp[1/c^p Int[(d + e*x)^m*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]`

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 313 vs. 2(26) = 52.

Time = 1.07 (sec) , antiderivative size = 314, normalized size of antiderivative = 11.21

| method | result |
|---------------|--|
| risch | $\frac{-\frac{b^6 x^6}{e} - \frac{3b^5(ae+bd)x^5}{e^2} - \frac{5b^4(e^2 a^2 + abde + b^2 d^2)x^4}{e^3} - \frac{5b^3(e^3 a^3 + a^2 bde^2 + a b^2 d^2 e + b^3 d^3)x^3}{e^4} - \frac{3b^2(a^4 e^4 + a^3 bde^3 + a^2 b^2 d^2 e^2 + a b^3 d^3 e + b^4 d^4)}{e^5}}{(ex+d)^8}$ |
| norman | $\frac{-\frac{b^6 x^6}{e} - \frac{3(ea b^5 + d b^6)x^5}{e^2} - \frac{5(e^2 a^2 b^4 + dea b^5 + d^2 b^6)x^4}{e^3} - \frac{5(e^3 a^3 b^3 + d e^2 a^2 b^4 + d^2 e a b^5 + d^3 b^6)x^3}{e^4} - \frac{3(e^4 a^4 b^2 + d e^3 a^3 b^3 + d^2 e^2 a^2 b^4 + d^3 e a b^5 + d^4 b^6)}{e^5}}{(ex+d)^8}$ |
| default | $\frac{5b^4(e^2 a^2 - 2abde + b^2 d^2)}{e^7(ex+d)^3} - \frac{5b^3(e^3 a^3 - 3a^2 bde^2 + 3a b^2 d^2 e - b^3 d^3)}{e^7(ex+d)^4} - \frac{a^6 e^6 - 6a^5 bde^5 + 15a^4 b^2 d^2 e^4 - 20a^3 b^3 d^3 e^3 + 15a^2 b^4 d^4 - 6a b^5 d^5 + b^6 d^6}{7e^7(ex+d)^7}$ |
| gospers | $-7x^6 b^6 e^6 + 21x^5 a b^5 e^6 + 21x^5 b^6 d e^5 + 35x^4 a^2 b^4 e^6 + 35x^4 a b^5 d e^5 + 35x^4 b^6 d^2 e^4 + 35x^3 a^3 b^3 e^6 + 35x^3 a^2 b^4 d e^5 + 35x^3 a b^5 d^2 e^4 + 35x^3 b^6 d^3 e^3 + 35x^2 a^4 b^2 e^6 + 35x^2 a^3 b^3 d e^5 + 35x^2 a^2 b^4 d^2 e^4 + 35x^2 a b^5 d^3 e^3 + 35x^2 b^6 d^4 e^2 + 35x a^5 b d e^6 + 35x a^4 b^2 d^2 e^5 + 35x a^3 b^3 d^3 e^4 + 35x a^2 b^4 d^4 e^3 + 35x a b^5 d^5 e^2 + 35x b^6 d^6 e^1 + 7a^6 e^6 - 6a^5 bde^5 + 15a^4 b^2 d^2 e^4 - 20a^3 b^3 d^3 e^3 + 15a^2 b^4 d^4 - 6a b^5 d^5 + b^6 d^6$ |
| parallelrisch | $-7x^6 b^6 e^6 - 21x^5 a b^5 e^6 - 21x^5 b^6 d e^5 - 35x^4 a^2 b^4 e^6 - 35x^4 a b^5 d e^5 - 35x^4 b^6 d^2 e^4 - 35x^3 a^3 b^3 e^6 - 35x^3 a^2 b^4 d e^5 - 35x^3 a b^5 d^2 e^4 - 35x^3 b^6 d^3 e^3 - 35x^2 a^4 b^2 e^6 - 35x^2 a^3 b^3 d e^5 - 35x^2 a^2 b^4 d^2 e^4 - 35x^2 a b^5 d^3 e^3 - 35x^2 b^6 d^4 e^2 - 35x a^5 b d e^6 - 35x a^4 b^2 d^2 e^5 - 35x a^3 b^3 d^3 e^4 - 35x a^2 b^4 d^4 e^3 - 35x a b^5 d^5 e^2 - 35x b^6 d^6 e^1 + 7a^6 e^6 - 6a^5 bde^5 + 15a^4 b^2 d^2 e^4 - 20a^3 b^3 d^3 e^3 + 15a^2 b^4 d^4 - 6a b^5 d^5 + b^6 d^6$ |
| orering | $-(7x^6 b^6 e^6 + 21x^5 a b^5 e^6 + 21x^5 b^6 d e^5 + 35x^4 a^2 b^4 e^6 + 35x^4 a b^5 d e^5 + 35x^4 b^6 d^2 e^4 + 35x^3 a^3 b^3 e^6 + 35x^3 a^2 b^4 d e^5 + 35x^3 a b^5 d^2 e^4 + 35x^3 b^6 d^3 e^3 + 35x^2 a^4 b^2 e^6 + 35x^2 a^3 b^3 d e^5 + 35x^2 a^2 b^4 d^2 e^4 + 35x^2 a b^5 d^3 e^3 + 35x^2 b^6 d^4 e^2 + 35x a^5 b d e^6 + 35x a^4 b^2 d^2 e^5 + 35x a^3 b^3 d^3 e^4 + 35x a^2 b^4 d^4 e^3 + 35x a b^5 d^5 e^2 + 35x b^6 d^6 e^1 + 7a^6 e^6 - 6a^5 bde^5 + 15a^4 b^2 d^2 e^4 - 20a^3 b^3 d^3 e^3 + 15a^2 b^4 d^4 - 6a b^5 d^5 + b^6 d^6)$ |

input `int((b^2*x^2+2*a*b*x+a^2)^3/(e*x+d)^8,x,method=_RETURNVERBOSE)`

output

```
(-1/e*b^6*x^6-3*b^5*(a*e+b*d)/e^2*x^5-5*b^4*(a^2*e^2+a*b*d*e+b^2*d^2)/e^3*
x^4-5*b^3*(a^3*e^3+a^2*b*d*e^2+a*b^2*d^2*e+b^3*d^3)/e^4*x^3-3*b^2*(a^4*e^4
+a^3*b*d*e^3+a^2*b^2*d^2*e^2+a*b^3*d^3*e+b^4*d^4)/e^5*x^2-b*(a^5*e^5+a^4*b
*d*e^4+a^3*b^2*d^2*e^3+a^2*b^3*d^3*e^2+a*b^4*d^4*e+b^5*d^5)/e^6*x-1/7*(a^6
*e^6+a^5*b*d*e^5+a^4*b^2*d^2*e^4+a^3*b^3*d^3*e^3+a^2*b^4*d^4*e^2+a*b^5*d^5
*e+b^6*d^6)/e^7)/(e*x+d)^7
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 398 vs. $2(26) = 52$.

Time = 0.08 (sec) , antiderivative size = 398, normalized size of antiderivative = 14.21

$$\int \frac{(a^2 + 2abx + b^2x^2)^3}{(d + ex)^8} dx =$$

$$\frac{7b^6e^6x^6 + b^6d^6 + ab^5d^5e + a^2b^4d^4e^2 + a^3b^3d^3e^3 + a^4b^2d^2e^4 + a^5bde^5 + a^6e^6 + 21(b^6de^5 + ab^5e^6)x^5 + \dots}{(d + ex)^8}$$

input

```
integrate((b^2*x^2+2*a*b*x+a^2)^3/(e*x+d)^8,x, algorithm="fricas")
```

output

```
-1/7*(7*b^6*e^6*x^6 + b^6*d^6 + a*b^5*d^5*e + a^2*b^4*d^4*e^2 + a^3*b^3*d^
3*e^3 + a^4*b^2*d^2*e^4 + a^5*b*d*e^5 + a^6*e^6 + 21*(b^6*d*e^5 + a*b^5*e^
6)*x^5 + 35*(b^6*d^2*e^4 + a*b^5*d*e^5 + a^2*b^4*e^6)*x^4 + 35*(b^6*d^3*e^
3 + a*b^5*d^2*e^4 + a^2*b^4*d*e^5 + a^3*b^3*e^6)*x^3 + 21*(b^6*d^4*e^2 + a
*b^5*d^3*e^3 + a^2*b^4*d^2*e^4 + a^3*b^3*d*e^5 + a^4*b^2*e^6)*x^2 + 7*(b^6
*d^5*e + a*b^5*d^4*e^2 + a^2*b^4*d^3*e^3 + a^3*b^3*d^2*e^4 + a^4*b^2*d*e^5
+ a^5*b*e^6)*x)/(e^14*x^7 + 7*d*e^13*x^6 + 21*d^2*e^12*x^5 + 35*d^3*e^11*
x^4 + 35*d^4*e^10*x^3 + 21*d^5*e^9*x^2 + 7*d^6*e^8*x + d^7*e^7)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(a^2 + 2abx + b^2x^2)^3}{(d + ex)^8} dx = \text{Timed out}$$

input `integrate((b**2*x**2+2*a*b*x+a**2)**3/(e*x+d)**8,x)`

output Timed out

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 398 vs. 2(26) = 52.

Time = 0.05 (sec) , antiderivative size = 398, normalized size of antiderivative = 14.21

$$\int \frac{(a^2 + 2abx + b^2x^2)^3}{(d + ex)^8} dx = \frac{7b^6e^6x^6 + b^6d^6 + ab^5d^5e + a^2b^4d^4e^2 + a^3b^3d^3e^3 + a^4b^2d^2e^4 + a^5bde^5 + a^6e^6 + 21(b^6de^5 + ab^5e^6)x^5 + \dots}{(e^7x^7 + 7d^6e^6x^6 + 21d^5e^5x^5 + 35d^4e^4x^4 + 35d^3e^3x^3 + 21d^2e^2x^2 + 7de^1x + d^7e^7)}$$

input `integrate((b^2*x^2+2*a*b*x+a^2)^3/(e*x+d)^8,x, algorithm="maxima")`

output `-1/7*(7*b^6*e^6*x^6 + b^6*d^6 + a*b^5*d^5*e + a^2*b^4*d^4*e^2 + a^3*b^3*d^3*e^3 + a^4*b^2*d^2*e^4 + a^5*b*d*e^5 + a^6*e^6 + 21*(b^6*d*e^5 + a*b^5*e^6)*x^5 + 35*(b^6*d^2*e^4 + a*b^5*d*e^5 + a^2*b^4*e^6)*x^4 + 35*(b^6*d^3*e^3 + a*b^5*d^2*e^4 + a^2*b^4*d*e^5 + a^3*b^3*e^6)*x^3 + 21*(b^6*d^4*e^2 + a*b^5*d^3*e^3 + a^2*b^4*d^2*e^4 + a^3*b^3*d*e^5 + a^4*b^2*e^6)*x^2 + 7*(b^6*d^5*e + a*b^5*d^4*e^2 + a^2*b^4*d^3*e^3 + a^3*b^3*d^2*e^4 + a^4*b^2*d*e^5 + a^5*b*e^6)*x)/(e^14*x^7 + 7*d*e^13*x^6 + 21*d^2*e^12*x^5 + 35*d^3*e^11*x^4 + 35*d^4*e^10*x^3 + 21*d^5*e^9*x^2 + 7*d^6*e^8*x + d^7*e^7)`

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 369 vs. $2(26) = 52$.

Time = 0.19 (sec) , antiderivative size = 369, normalized size of antiderivative = 13.18

$$\int \frac{(a^2 + 2abx + b^2x^2)^3}{(d + ex)^8} dx =$$

$$\frac{7b^6e^6x^6 + 21b^6de^5x^5 + 21ab^5e^6x^5 + 35b^6d^2e^4x^4 + 35ab^5de^5x^4 + 35a^2b^4e^6x^4 + 35b^6d^3e^3x^3 + 35ab^5d^4e^2x^2 + 21a^2b^3d^3e^3x^2 + 21a^2b^4d^2e^4x^2 + 21a^3b^3d^2e^5x^2 + 21a^4b^2d^2e^6x^2 + 7b^6d^5e^6x + 7a^2b^4d^3e^3x + 7a^3b^3d^2e^4x + 7a^4b^2d^2e^5x + b^6d^6 + a^6e^6 + a^5bde^5 + a^4b^2d^2e^4 + a^3b^3d^3e^3 + a^2b^4d^4e^2 + a^3b^3d^3e^3 + a^4b^2d^2e^4 + a^5bd^2e^5 + a^6e^6}{(ex + d)^7e^7}$$

input `integrate((b^2*x^2+2*a*b*x+a^2)^3/(e*x+d)^8,x, algorithm="giac")`

output

```
-1/7*(7*b^6*e^6*x^6 + 21*b^6*d*e^5*x^5 + 21*a*b^5*e^6*x^5 + 35*b^6*d^2*e^4*x^4 + 35*a*b^5*d*e^5*x^4 + 35*a^2*b^4*e^6*x^4 + 35*b^6*d^3*e^3*x^3 + 35*a*b^5*d^2*e^4*x^3 + 35*a^2*b^4*d*e^5*x^3 + 35*a^3*b^3*e^6*x^3 + 21*b^6*d^4*e^2*x^2 + 21*a*b^5*d^3*e^3*x^2 + 21*a^2*b^4*d^2*e^4*x^2 + 21*a^3*b^3*d^2*e^5*x^2 + 21*a^4*b^2*d^2*e^6*x^2 + 7*b^6*d^5*e^6*x + 7*a*b^5*d^4*e^2*x + 7*a^2*b^4*d^3*e^3*x + 7*a^3*b^3*d^2*e^4*x + 7*a^4*b^2*d^2*e^5*x + 7*a^5*b*d^2*e^6*x + b^6*d^6 + a*b^5*d^5*e + a^2*b^4*d^4*e^2 + a^3*b^3*d^3*e^3 + a^4*b^2*d^2*e^4 + a^5*b*d^2*e^5 + a^6*e^6)/((e*x + d)^7*e^7)
```

Mupad [B] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 378, normalized size of antiderivative = 13.50

$$\int \frac{(a^2 + 2abx + b^2x^2)^3}{(d + ex)^8} dx =$$

$$\frac{a^6e^6 + a^5bde^5 + a^4b^2d^2e^4 + a^3b^3d^3e^3 + a^2b^4d^4e^2 + ab^5d^5e + b^6d^6}{7e^7} + \frac{b^6x^6}{e} + \frac{5b^3x^3(a^3e^3 + a^2bde^2 + ab^2d^2e + b^3d^3)}{e^4} + \frac{bx(a^5e^5 + a^4bde^4 + a^3b^2d^2e^3 + a^2b^3d^3e^2 + ab^4d^4e + b^5d^5)}{d^7 + 7d^6ex + 21d^5e^2x^2 + 35d^4e^3x^3}$$

input `int((a^2 + b^2*x^2 + 2*a*b*x)^3/(d + e*x)^8,x)`

output

$$-\left(\frac{a^6 e^6 + b^6 d^6 + a^2 b^4 d^4 e^2 + a^3 b^3 d^3 e^3 + a^4 b^2 d^2 e^4 + a b^5 d^5 e + a^5 b d e^5}{7 e^7} + \frac{b^6 x^6}{e} + \frac{5 b^3 x^3 (a^3 e^3 + b^3 d^3 + a b^2 d^2 e + a^2 b d e^2)}{e^4} + \frac{b x (a^5 e^5 + b^5 d^5 + a^2 b^3 d^3 e^2 + a^3 b^2 d^2 e^3 + a b^4 d^4 e + a^4 b d e^4)}{e^6} + \frac{3 b^5 x^5 (a e + b d)}{e^2} + \frac{3 b^2 x^2 (a^4 e^4 + b^4 d^4 + a^2 b^2 d^2 e^2 + a b^3 d^3 e + a^3 b d e^3)}{e^5} + \frac{5 b^4 x^4 (a^2 e^2 + b^2 d^2 + a b d e)}{e^3}\right) / (d^7 + e^7 x^7 + 7 d e^6 x^6 + 21 d^5 e^2 x^2 + 35 d^4 e^3 x^3 + 35 d^3 e^4 x^4 + 21 d^2 e^5 x^5 + 7 d^6 e x)$$
Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 383, normalized size of antiderivative = 13.68

$$\int \frac{(a^2 + 2abx + b^2x^2)^3}{(d + ex)^8} dx$$

$$= \frac{b^6 e^6 x^7 - 21 a b^5 d e^5 x^5 - 35 a^2 b^4 d e^5 x^4 - 35 a b^5 d^2 e^4 x^4 - 35 a^3 b^3 d e^5 x^3 - 35 a^2 b^4 d^2 e^4 x^3 - 35 a b^5 d^3 e^3 x^3 - 21 a^4 b^2 d^2 e^4 x^2 - 21 a^3 b^3 d^3 e^3 x^2 - 21 a^2 b^4 d^4 e^2 x^2 - 21 a b^5 d^5 e x - 21 a^2 b^4 d^4 e^2 x - 21 a^3 b^3 d^3 e^3 x - 21 a^4 b^2 d^2 e^4 x - 21 a^5 b d e^5}{(d^7 + 7 d e^6 x + 21 d^5 e^2 x^2 + 35 d^4 e^3 x^3 + 35 d^3 e^4 x^4 + 21 d^2 e^5 x^5 + 7 d^6 e x + e^7 x^7)}$$

input

`int((b^2*x^2+2*a*b*x+a^2)^3/(e*x+d)^8,x)`

output

$$\left(- a^{**6} d^{**5} - a^{**5} b d^{**2} e^{**4} - 7 a^{**5} b d e^{**5} x - a^{**4} b^{**2} d^{**3} e^{**3} x^3 - 7 a^{**4} b^{**2} d^{**2} e^{**4} x - 21 a^{**4} b^{**2} d e^{**5} x^{**2} - a^{**3} b^{**3} d^{**4} e^{**2} x^{**2} - 7 a^{**3} b^{**3} d^{**3} e^{**3} x - 21 a^{**3} b^{**3} d^{**2} e^{**4} x^{**2} - 35 a^{**3} b^{**3} d e^{**5} x^{**3} - a^{**2} b^{**4} d^{**5} e - 7 a^{**2} b^{**4} d^{**4} e^{**2} x - 21 a^{**2} b^{**4} d^{**3} e^{**3} x^{**2} - 35 a^{**2} b^{**4} d^{**2} e^{**4} x^{**3} - 35 a^{**2} b^{**4} d e^{**5} x^{**4} - a^{**1} b^{**5} d^{**6} - 7 a^{**1} b^{**5} d^{**5} e x - 21 a^{**1} b^{**5} d^{**4} e^{**2} x^{**2} - 35 a^{**1} b^{**5} d^{**3} e^{**3} x^{**3} - 35 a^{**1} b^{**5} d^{**2} e^{**4} x^{**4} - 21 a^{**1} b^{**5} d e^{**5} x^{**5} + b^{**6} e^{**6} x^{**7} \right) / (7 d^{**6} e^{**6} (d^{**7} + 7 d^{**6} e x + 21 d^{**5} e^{**2} x^{**2} + 35 d^{**4} e^{**3} x^{**3} + 35 d^{**3} e^{**4} x^{**4} + 21 d^{**2} e^{**5} x^{**5} + 7 d e^{**6} x^{**6} + e^{**7} x^{**7}))$$

3.101 $\int \frac{(a^2+2abx+b^2x^2)^3}{(d+ex)^9} dx$

| | |
|---|-----|
| Optimal result | 829 |
| Mathematica [B] (verified) | 829 |
| Rubi [A] (verified) | 830 |
| Maple [B] (verified) | 832 |
| Fricas [B] (verification not implemented) | 832 |
| Sympy [F(-1)] | 833 |
| Maxima [B] (verification not implemented) | 833 |
| Giac [B] (verification not implemented) | 834 |
| Mupad [B] (verification not implemented) | 835 |
| Reduce [B] (verification not implemented) | 835 |

Optimal result

Integrand size = 26, antiderivative size = 58

$$\int \frac{(a^2 + 2abx + b^2x^2)^3}{(d + ex)^9} dx = \frac{(a + bx)^7}{8(bd - ae)(d + ex)^8} + \frac{b(a + bx)^7}{56(bd - ae)^2(d + ex)^7}$$

output `1/8*(b*x+a)^7/(-a*e+b*d)/(e*x+d)^8+1/56*b*(b*x+a)^7/(-a*e+b*d)^2/(e*x+d)^7`

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 277 vs. 2(58) = 116.

Time = 0.07 (sec) , antiderivative size = 277, normalized size of antiderivative = 4.78

$$\int \frac{(a^2 + 2abx + b^2x^2)^3}{(d + ex)^9} dx = \frac{7a^6e^6 + 6a^5be^5(d + 8ex) + 5a^4b^2e^4(d^2 + 8dex + 28e^2x^2) + 4a^3b^3e^3(d^3 + 8d^2ex + 28de^2x^2 + 56e^3x^3) + \dots}{(d + ex)^9}$$

input `Integrate[(a^2 + 2*a*b*x + b^2*x^2)^3/(d + e*x)^9,x]`

output

```
-1/56*(7*a^6*e^6 + 6*a^5*b*e^5*(d + 8*e*x) + 5*a^4*b^2*e^4*(d^2 + 8*d*e*x
+ 28*e^2*x^2) + 4*a^3*b^3*e^3*(d^3 + 8*d^2*e*x + 28*d*e^2*x^2 + 56*e^3*x^3
) + 3*a^2*b^4*e^2*(d^4 + 8*d^3*e*x + 28*d^2*e^2*x^2 + 56*d*e^3*x^3 + 70*e^
4*x^4) + 2*a*b^5*e*(d^5 + 8*d^4*e*x + 28*d^3*e^2*x^2 + 56*d^2*e^3*x^3 + 70
*d*e^4*x^4 + 56*e^5*x^5) + b^6*(d^6 + 8*d^5*e*x + 28*d^4*e^2*x^2 + 56*d^3*
e^3*x^3 + 70*d^2*e^4*x^4 + 56*d*e^5*x^5 + 28*e^6*x^6))/(e^7*(d + e*x)^8)
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {1098, 27, 55, 48}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\begin{aligned}
 & \int \frac{(a^2 + 2abx + b^2x^2)^3}{(d + ex)^9} dx \\
 & \quad \downarrow 1098 \\
 & \quad \int \frac{b^6(a+bx)^6}{(d+ex)^9} dx \\
 & \quad \quad \downarrow 27 \\
 & \quad \int \frac{(a + bx)^6}{(d + ex)^9} dx \\
 & \quad \quad \downarrow 55 \\
 & \quad \frac{b \int \frac{(a+bx)^6}{(d+ex)^8} dx}{8(bd - ae)} + \frac{(a + bx)^7}{8(d + ex)^8(bd - ae)} \\
 & \quad \quad \downarrow 48 \\
 & \quad \frac{b(a + bx)^7}{56(d + ex)^7(bd - ae)^2} + \frac{(a + bx)^7}{8(d + ex)^8(bd - ae)}
 \end{aligned}$$

input

```
Int[(a^2 + 2*a*b*x + b^2*x^2)^3/(d + e*x)^9,x]
```

output $(a + b*x)^7/(8*(b*d - a*e)*(d + e*x)^8) + (b*(a + b*x)^7)/(56*(b*d - a*e)^2*(d + e*x)^7)$

Defintions of rubi rules used

rule 27 $\text{Int}[(a_*)(Fx_), x_Symbol] \rightarrow \text{Simp}[a \text{ Int}[Fx, x], x] /; \text{FreeQ}[a, x] \ \&\& \ !\text{MatchQ}[Fx, (b_*)(Gx_)] /; \text{FreeQ}[b, x]$

rule 48 $\text{Int}[(a_.) + (b_.)*(x_)^{(m_.)}*((c_.) + (d_.)*(x_)^{(n_.)}), x_Symbol] \rightarrow \text{Simp}[(a + b*x)^{(m + 1)}*((c + d*x)^{(n + 1)} / ((b*c - a*d)*(m + 1))), x] /; \text{FreeQ}[\{a, b, c, d, m, n\}, x] \ \&\& \ \text{EqQ}[m + n + 2, 0] \ \&\& \ \text{NeQ}[m, -1]$

rule 55 $\text{Int}[(a_.) + (b_.)*(x_)^{(m_.)}*((c_.) + (d_.)*(x_)^{(n_.)}), x_Symbol] \rightarrow \text{Simp}[(a + b*x)^{(m + 1)}*((c + d*x)^{(n + 1)} / ((b*c - a*d)*(m + 1))), x] - \text{Simp}[d*(\text{Simplify}[m + n + 2] / ((b*c - a*d)*(m + 1))) \text{Int}[(a + b*x)^{\text{Simplify}[m + 1]}*(c + d*x)^n, x], x] /; \text{FreeQ}[\{a, b, c, d, m, n\}, x] \ \&\& \ \text{ILtQ}[\text{Simplify}[m + n + 2], 0] \ \&\& \ \text{NeQ}[m, -1] \ \&\& \ !(\text{LtQ}[m, -1] \ \&\& \ \text{LtQ}[n, -1] \ \&\& \ (\text{EqQ}[a, 0] \ || \ (\text{NeQ}[c, 0] \ \&\& \ \text{LtQ}[m - n, 0] \ \&\& \ \text{IntegerQ}[n]))) \ \&\& \ (\text{SumSimplerQ}[m, 1] \ || \ !\text{SumSimplerQ}[n, 1])$

rule 1098 $\text{Int}[(d_.) + (e_.)*(x_)^{(m_.)}*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^{(p_.)}, x_Symbol] \rightarrow \text{Simp}[1/c^p \text{Int}[(d + e*x)^m*(b/2 + c*x)^{(2*p)}, x], x] /; \text{FreeQ}[\{a, b, c, d, e, m\}, x] \ \&\& \ \text{EqQ}[b^2 - 4*a*c, 0] \ \&\& \ \text{IntegerQ}[p]$

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 334 vs. 2(54) = 108.

Time = 1.08 (sec) , antiderivative size = 335, normalized size of antiderivative = 5.78

| method | result |
|--------------|---|
| risch | $-\frac{b^6 x^6}{2e} - \frac{b^5(2ae+bd)x^5}{e^2} - \frac{5b^4(3e^2 a^2+2abde+b^2 d^2)x^4}{4e^3} - \frac{b^3(4e^3 a^3+3a^2 bde^2+2a b^2 d^2 e+b^3 d^3)x^3}{e^4} - \frac{b^2(5a^4 e^4+4a^3 bde^3+3a^2 b^2 d^2 e^2+2a b^3 d^3 e+b^4 d^4)x^2}{2e^5} - \frac{b(5a^5 e^5+5a^4 bde^4+10a^3 b^2 d^2 e^3-10a^2 b^3 d^3 e^2+5a b^4 d^4 e-b^5 d^5)x}{7e^7} - \frac{4b^3(e^3 a^3+3a^2 bde^2+2a b^2 d^2 e+b^3 d^3)x}{7e^7}$ |
| default | $-\frac{2b^5(ae-bd)}{e^7(ex+d)^3} - \frac{15b^4(e^2 a^2-2abde+b^2 d^2)}{4e^7(ex+d)^4} - \frac{6b(e^5 a^5-5a^4 bde^4+10a^3 b^2 d^2 e^3-10a^2 b^3 d^3 e^2+5a b^4 d^4 e-b^5 d^5)}{7e^7(ex+d)^7} - \frac{4b^3(e^3 a^3+3a^2 bde^2+2a b^2 d^2 e+b^3 d^3)x}{7e^7}$ |
| norman | $-\frac{b^6 x^6}{2e} - \frac{(2e^2 a b^5+de b^6)x^5}{e^3} - \frac{5(3e^3 a^2 b^4+2d e^2 a b^5+d^2 e b^6)x^4}{4e^4} - \frac{(4e^4 a^3 b^3+3d e^3 a^2 b^4+2d^2 e^2 a b^5+d^3 e b^6)x^3}{e^5} - \frac{(5e^5 a^4 b^2+4d e^4 a^3 b^3+3d^2 e^3 a^2 b^4+2d^3 e^2 a b^5+d^4 e b^6)x^2}{2e^6} - \frac{b(5a^5 e^5+5a^4 bde^4+10a^3 b^2 d^2 e^3-10a^2 b^3 d^3 e^2+5a b^4 d^4 e-b^5 d^5)x}{7e^7}$ |
| gosper | $-\frac{28x^6 b^6 e^6+112x^5 a b^5 e^6+56x^5 b^6 d e^5+210x^4 a^2 b^4 e^6+140x^4 a b^5 d e^5+70x^4 b^6 d^2 e^4+224x^3 a^3 b^3 e^6+168x^3 a^2 b^4 d e^5+112x^3 a b^5 d^2 e^4+56x^3 b^6 d^3 e^3+28x^2 a^4 b^2 e^6+168x^2 a^3 b^3 d e^5+112x^2 a^2 b^4 d^2 e^4+56x^2 a b^5 d^3 e^3+28x^2 b^6 d^4 e^2}{e^7} - \frac{b(5a^5 e^5+5a^4 bde^4+10a^3 b^2 d^2 e^3-10a^2 b^3 d^3 e^2+5a b^4 d^4 e-b^5 d^5)x}{7e^7}$ |
| paralelrisch | $-\frac{28b^6 x^6 e^7-112a b^5 e^7 x^5-56b^6 d e^6 x^5-210a^2 b^4 e^7 x^4-140a b^5 d e^6 x^4-70b^6 d^2 e^5 x^4-224a^3 b^3 e^7 x^3-168a^2 b^4 d e^6 x^3-112a b^5 d^2 e^5 x^3-56x^3 b^6 d^3 e^3+28x^2 a^4 b^2 e^6+168x^2 a^3 b^3 d e^5+112x^2 a^2 b^4 d^2 e^4+56x^2 a b^5 d^3 e^3+28x^2 b^6 d^4 e^2}{e^7} - \frac{b(5a^5 e^5+5a^4 bde^4+10a^3 b^2 d^2 e^3-10a^2 b^3 d^3 e^2+5a b^4 d^4 e-b^5 d^5)x}{7e^7}$ |
| orering | $-\frac{(28x^6 b^6 e^6+112x^5 a b^5 e^6+56x^5 b^6 d e^5+210x^4 a^2 b^4 e^6+140x^4 a b^5 d e^5+70x^4 b^6 d^2 e^4+224x^3 a^3 b^3 e^6+168x^3 a^2 b^4 d e^5+112x^3 a b^5 d^2 e^4+56x^3 b^6 d^3 e^3+28x^2 a^4 b^2 e^6+168x^2 a^3 b^3 d e^5+112x^2 a^2 b^4 d^2 e^4+56x^2 a b^5 d^3 e^3+28x^2 b^6 d^4 e^2)}{e^7} - \frac{b(5a^5 e^5+5a^4 bde^4+10a^3 b^2 d^2 e^3-10a^2 b^3 d^3 e^2+5a b^4 d^4 e-b^5 d^5)x}{7e^7}$ |

input `int((b^2*x^2+2*a*b*x+a^2)^3/(e*x+d)^9,x,method=_RETURNVERBOSE)`

output
$$\frac{(-1/2/e*b^6*x^6-b^5/e^2*(2*a*e+b*d)*x^5-5/4/e^3*b^4*(3*a^2*e^2+2*a*b*d*e+b^2*d^2)*x^4-1/e^4*b^3*(4*a^3*e^3+3*a^2*b*d*e^2+2*a*b^2*d^2*e+b^3*d^3)*x^3-1/2/e^5*b^2*(5*a^4*e^4+4*a^3*b*d*e^3+3*a^2*b^2*d^2*e^2+2*a*b^3*d^3*e+b^4*d^4)*x^2-1/7*b/e^6*(6*a^5*e^5+5*a^4*b*d*e^4+4*a^3*b^2*d^2*e^3+3*a^2*b^3*d^3*e^2+2*a*b^4*d^4*e+b^5*d^5)*x-1/56/e^7*(7*a^6*e^6+6*a^5*b*d*e^5+5*a^4*b^2*d^2*e^4+4*a^3*b^3*d^3*e^3+3*a^2*b^4*d^4*e^2+2*a*b^5*d^5*e+b^6*d^6))/(e*x+d)^8}$$

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 430 vs. 2(54) = 108.

Time = 0.08 (sec) , antiderivative size = 430, normalized size of antiderivative = 7.41

$$\int \frac{(a^2 + 2abx + b^2x^2)^3}{(d + ex)^9} dx =$$

$$-\frac{28 b^6 e^6 x^6 + b^6 d^6 + 2 a b^5 d^5 e + 3 a^2 b^4 d^4 e^2 + 4 a^3 b^3 d^3 e^3 + 5 a^4 b^2 d^2 e^4 + 6 a^5 b d e^5 + 7 a^6 e^6 + 56 (b^6 d e^5 + 2$$

input `integrate((b^2*x^2+2*a*b*x+a^2)^3/(e*x+d)^9,x, algorithm="fricas")`

output
$$\frac{-1/56*(28*b^6*e^6*x^6 + b^6*d^6 + 2*a*b^5*d^5*e + 3*a^2*b^4*d^4*e^2 + 4*a^3*b^3*d^3*e^3 + 5*a^4*b^2*d^2*e^4 + 6*a^5*b*d*e^5 + 7*a^6*e^6 + 56*(b^6*d*e^5 + 2*a*b^5*e^6)*x^5 + 70*(b^6*d^2*e^4 + 2*a*b^5*d*e^5 + 3*a^2*b^4*e^6)*x^4 + 56*(b^6*d^3*e^3 + 2*a*b^5*d^2*e^4 + 3*a^2*b^4*d*e^5 + 4*a^3*b^3*e^6)*x^3 + 28*(b^6*d^4*e^2 + 2*a*b^5*d^3*e^3 + 3*a^2*b^4*d^2*e^4 + 4*a^3*b^3*d*e^5 + 5*a^4*b^2*e^6)*x^2 + 8*(b^6*d^5*e + 2*a*b^5*d^4*e^2 + 3*a^2*b^4*d^3*e^3 + 4*a^3*b^3*d^2*e^4 + 5*a^4*b^2*d*e^5 + 6*a^5*b*e^6)*x)/(e^15*x^8 + 8*d*e^14*x^7 + 28*d^2*e^13*x^6 + 56*d^3*e^12*x^5 + 70*d^4*e^11*x^4 + 56*d^5*e^10*x^3 + 28*d^6*e^9*x^2 + 8*d^7*e^8*x + d^8*e^7)}$$

Sympy [F(-1)]

Timed out.

$$\int \frac{(a^2 + 2abx + b^2x^2)^3}{(d + ex)^9} dx = \text{Timed out}$$

input `integrate((b**2*x**2+2*a*b*x+a**2)**3/(e*x+d)**9,x)`

output `Timed out`

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 430 vs. $2(54) = 108$.

Time = 0.06 (sec) , antiderivative size = 430, normalized size of antiderivative = 7.41

$$\int \frac{(a^2 + 2abx + b^2x^2)^3}{(d + ex)^9} dx = \frac{28 b^6 e^6 x^6 + b^6 d^6 + 2 a b^5 d^5 e + 3 a^2 b^4 d^4 e^2 + 4 a^3 b^3 d^3 e^3 + 5 a^4 b^2 d^2 e^4 + 6 a^5 b d e^5 + 7 a^6 e^6 + 56 (b^6 d e^5 + 2$$

input `integrate((b^2*x^2+2*a*b*x+a^2)^3/(e*x+d)^9,x, algorithm="maxima")`

output

```
-1/56*(28*b^6*e^6*x^6 + b^6*d^6 + 2*a*b^5*d^5*e + 3*a^2*b^4*d^4*e^2 + 4*a^3*b^3*d^3*e^3 + 5*a^4*b^2*d^2*e^4 + 6*a^5*b*d*e^5 + 7*a^6*e^6 + 56*(b^6*d*e^5 + 2*a*b^5*e^6)*x^5 + 70*(b^6*d^2*e^4 + 2*a*b^5*d*e^5 + 3*a^2*b^4*e^6)*x^4 + 56*(b^6*d^3*e^3 + 2*a*b^5*d^2*e^4 + 3*a^2*b^4*d*e^5 + 4*a^3*b^3*e^6)*x^3 + 28*(b^6*d^4*e^2 + 2*a*b^5*d^3*e^3 + 3*a^2*b^4*d^2*e^4 + 4*a^3*b^3*d*e^5 + 5*a^4*b^2*e^6)*x^2 + 8*(b^6*d^5*e + 2*a*b^5*d^4*e^2 + 3*a^2*b^4*d^3*e^3 + 4*a^3*b^3*d^2*e^4 + 5*a^4*b^2*d*e^5 + 6*a^5*b*e^6)*x)/(e^15*x^8 + 8*d*e^14*x^7 + 28*d^2*e^13*x^6 + 56*d^3*e^12*x^5 + 70*d^4*e^11*x^4 + 56*d^5*e^10*x^3 + 28*d^6*e^9*x^2 + 8*d^7*e^8*x + d^8*e^7)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 375 vs. $2(54) = 108$.

Time = 0.20 (sec) , antiderivative size = 375, normalized size of antiderivative = 6.47

$$\int \frac{(a^2 + 2abx + b^2x^2)^3}{(d + ex)^9} dx =$$

$$\frac{28b^6e^6x^6 + 56b^6de^5x^5 + 112ab^5e^6x^5 + 70b^6d^2e^4x^4 + 140ab^5de^5x^4 + 210a^2b^4e^6x^4 + 56b^6d^3e^3x^3 + 112a^3b^3e^6x^3 + 28b^6d^4e^2x^2 + 56a^2b^4d^2e^4x^2 + 112a^3b^3d^2e^4x^2 + 84a^2b^4d^2e^4x^2 + 112a^3b^3d^2e^4x^2 + 140a^4b^2e^6x^2 + 8b^6d^5e^5x + 16a^2b^4d^5e^5x + 16a^3b^3d^5e^5x + 24a^2b^4d^3e^3x + 32a^3b^3d^2e^4x + 40a^4b^2d^2e^5x + 48a^5b^2e^6x + b^6d^6 + 2a^2b^4d^6 + 3a^2b^4d^4e^2 + 4a^3b^3d^3e^3 + 5a^4b^2d^2e^4 + 6a^5b^2d^2e^4 + 7a^6e^6)/((ex + d)^8e^7)$$

input

```
integrate((b^2*x^2+2*a*b*x+a^2)^3/(e*x+d)^9,x, algorithm="giac")
```

output

```
-1/56*(28*b^6*e^6*x^6 + 56*b^6*d*e^5*x^5 + 112*a*b^5*e^6*x^5 + 70*b^6*d^2*e^4*x^4 + 140*a*b^5*d*e^5*x^4 + 210*a^2*b^4*e^6*x^4 + 56*b^6*d^3*e^3*x^3 + 112*a*b^5*d^2*e^4*x^3 + 168*a^2*b^4*d*e^5*x^3 + 224*a^3*b^3*e^6*x^3 + 28*b^6*d^4*e^2*x^2 + 56*a*b^5*d^3*e^3*x^2 + 84*a^2*b^4*d^2*e^4*x^2 + 112*a^3*b^3*d^2*e^5*x^2 + 140*a^4*b^2*e^6*x^2 + 8*b^6*d^5*e^5*x + 16*a*b^5*d^4*e^2*x + 24*a^2*b^4*d^3*e^3*x + 32*a^3*b^3*d^2*e^4*x + 40*a^4*b^2*d^2*e^5*x + 48*a^5*b^2*e^6*x + b^6*d^6 + 2*a*b^5*d^5*e + 3*a^2*b^4*d^4*e^2 + 4*a^3*b^3*d^3*e^3 + 5*a^4*b^2*d^2*e^4 + 6*a^5*b^2*d^2*e^4 + 7*a^6*e^6)/((e*x + d)^8*e^7)
```

Mupad [B] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 410, normalized size of antiderivative = 7.07

$$\int \frac{(a^2 + 2abx + b^2x^2)^3}{(d + ex)^9} dx =$$

$$-\frac{7a^6e^6 + 6a^5bde^5 + 5a^4b^2d^2e^4 + 4a^3b^3d^3e^3 + 3a^2b^4d^4e^2 + 2ab^5d^5e + b^6d^6}{56e^7} + \frac{b^6x^6}{2e} + \frac{b^3x^3(4a^3e^3 + 3a^2bde^2 + 2ab^2d^2e + b^3d^3)}{e^4} +$$

$$d^8 + 8d^7ex + 28d^6e^2x^2 + 5$$

input `int((a^2 + b^2*x^2 + 2*a*b*x)^3/(d + e*x)^9,x)`

output

$$-((7*a^6*e^6 + b^6*d^6 + 3*a^2*b^4*d^4*e^2 + 4*a^3*b^3*d^3*e^3 + 5*a^4*b^2*d^2*e^4 + 2*a*b^5*d^5*e + 6*a^5*b*d*e^5)/(56*e^7) + (b^6*x^6)/(2*e) + (b^3*x^3*(4*a^3*e^3 + b^3*d^3 + 2*a*b^2*d^2*e + 3*a^2*b*d*e^2))/e^4 + (b*x*(6*a^5*e^5 + b^5*d^5 + 3*a^2*b^3*d^3*e^2 + 4*a^3*b^2*d^2*e^3 + 2*a*b^4*d^4*e + 5*a^4*b*d*e^4))/(7*e^6) + (b^5*x^5*(2*a*e + b*d))/e^2 + (b^2*x^2*(5*a^4*e^4 + b^4*d^4 + 3*a^2*b^2*d^2*e^2 + 2*a*b^3*d^3*e + 4*a^3*b*d*e^3))/(2*e^5) + (5*b^4*x^4*(3*a^2*e^2 + b^2*d^2 + 2*a*b*d*e))/(4*e^3))/(d^8 + e^8*x^8 + 8*d*e^7*x^7 + 28*d^6*e^2*x^2 + 56*d^5*e^3*x^3 + 70*d^4*e^4*x^4 + 56*d^3*e^5*x^5 + 28*d^2*e^6*x^6 + 8*d^7*e*x)$$

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 453, normalized size of antiderivative = 7.81

$$\int \frac{(a^2 + 2abx + b^2x^2)^3}{(d + ex)^9} dx$$

$$= \frac{-28b^6e^6x^6 - 112ab^5e^6x^5 - 56b^6de^5x^5 - 210a^2b^4e^6x^4 - 140ab^5de^5x^4 - 70b^6d^2e^4x^4 - 224a^3b^3e^6x^3 - 16$$

input `int((b^2*x^2+2*a*b*x+a^2)^3/(e*x+d)^9,x)`

output

```
( - 7*a**6*e**6 - 6*a**5*b*d*e**5 - 48*a**5*b*e**6*x - 5*a**4*b**2*d**2*e*
*4 - 40*a**4*b**2*d*e**5*x - 140*a**4*b**2*e**6*x**2 - 4*a**3*b**3*d**3*e*
*3 - 32*a**3*b**3*d**2*e**4*x - 112*a**3*b**3*d*e**5*x**2 - 224*a**3*b**3*
e**6*x**3 - 3*a**2*b**4*d**4*e**2 - 24*a**2*b**4*d**3*e**3*x - 84*a**2*b**
4*d**2*e**4*x**2 - 168*a**2*b**4*d*e**5*x**3 - 210*a**2*b**4*e**6*x**4 - 2
*a*b**5*d**5*e - 16*a*b**5*d**4*e**2*x - 56*a*b**5*d**3*e**3*x**2 - 112*a*
b**5*d**2*e**4*x**3 - 140*a*b**5*d*e**5*x**4 - 112*a*b**5*e**6*x**5 - b**6
*d**6 - 8*b**6*d**5*e*x - 28*b**6*d**4*e**2*x**2 - 56*b**6*d**3*e**3*x**3
- 70*b**6*d**2*e**4*x**4 - 56*b**6*d*e**5*x**5 - 28*b**6*e**6*x**6)/(56*e*
*7*(d**8 + 8*d**7*e*x + 28*d**6*e**2*x**2 + 56*d**5*e**3*x**3 + 70*d**4*e*
*4*x**4 + 56*d**3*e**5*x**5 + 28*d**2*e**6*x**6 + 8*d*e**7*x**7 + e**8*x**
8))
```

3.102 $\int \frac{(a^2+2abx+b^2x^2)^3}{(d+ex)^{10}} dx$

| | |
|---|-----|
| Optimal result | 837 |
| Mathematica [B] (verified) | 837 |
| Rubi [A] (verified) | 838 |
| Maple [B] (verified) | 840 |
| Fricas [B] (verification not implemented) | 841 |
| Sympy [F(-1)] | 841 |
| Maxima [B] (verification not implemented) | 842 |
| Giac [B] (verification not implemented) | 842 |
| Mupad [B] (verification not implemented) | 843 |
| Reduce [B] (verification not implemented) | 844 |

Optimal result

Integrand size = 26, antiderivative size = 89

$$\int \frac{(a^2 + 2abx + b^2x^2)^3}{(d + ex)^{10}} dx = \frac{(a + bx)^7}{9(bd - ae)(d + ex)^9} + \frac{b(a + bx)^7}{36(bd - ae)^2(d + ex)^8} + \frac{b^2(a + bx)^7}{252(bd - ae)^3(d + ex)^7}$$

output

```
1/9*(b*x+a)^7/(-a*e+b*d)/(e*x+d)^9+1/36*b*(b*x+a)^7/(-a*e+b*d)^2/(e*x+d)^8
+1/252*b^2*(b*x+a)^7/(-a*e+b*d)^3/(e*x+d)^7
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 277 vs. 2(89) = 178.

Time = 0.07 (sec) , antiderivative size = 277, normalized size of antiderivative = 3.11

$$\int \frac{(a^2 + 2abx + b^2x^2)^3}{(d + ex)^{10}} dx = \frac{28a^6e^6 + 21a^5be^5(d + 9ex) + 15a^4b^2e^4(d^2 + 9dex + 36e^2x^2) + 10a^3b^3e^3(d^3 + 9d^2ex + 36de^2x^2 + 84e^3x^3) + 5a^2b^4e^2(d^4 + 12d^3ex + 48d^2e^2x^2 + 96de^3x^3 + 64e^4x^4)}{252e^7(d + ex)^7}$$

input `Integrate[(a^2 + 2*a*b*x + b^2*x^2)^3/(d + e*x)^10,x]`

output
$$\frac{-1/252*(28*a^6*e^6 + 21*a^5*b*e^5*(d + 9*e*x) + 15*a^4*b^2*e^4*(d^2 + 9*d*e*x + 36*e^2*x^2) + 10*a^3*b^3*e^3*(d^3 + 9*d^2*e*x + 36*d*e^2*x^2 + 84*e^3*x^3) + 6*a^2*b^4*e^2*(d^4 + 9*d^3*e*x + 36*d^2*e^2*x^2 + 84*d*e^3*x^3 + 126*e^4*x^4) + 3*a*b^5*e*(d^5 + 9*d^4*e*x + 36*d^3*e^2*x^2 + 84*d^2*e^3*x^3 + 126*d*e^4*x^4 + 126*e^5*x^5) + b^6*(d^6 + 9*d^5*e*x + 36*d^4*e^2*x^2 + 84*d^3*e^3*x^3 + 126*d^2*e^4*x^4 + 126*d*e^5*x^5 + 84*e^6*x^6))/(e^7*(d + e*x)^9)}$$

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 102, normalized size of antiderivative = 1.15, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.192$, Rules used = {1098, 27, 55, 55, 48}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\begin{aligned} & \int \frac{(a^2 + 2abx + b^2x^2)^3}{(d + ex)^{10}} dx \\ & \quad \downarrow \text{1098} \\ & \int \frac{b^6(a+bx)^6}{(d+ex)^{10}} dx \\ & \quad \downarrow \text{27} \\ & \int \frac{(a + bx)^6}{(d + ex)^{10}} dx \\ & \quad \downarrow \text{55} \\ & \frac{2b \int \frac{(a+bx)^6}{(d+ex)^9} dx}{9(bd - ae)} + \frac{(a + bx)^7}{9(d + ex)^9(bd - ae)} \\ & \quad \downarrow \text{55} \end{aligned}$$

$$\frac{2b \left(\frac{b \int \frac{(a+bx)^6 dx}{(d+ex)^8} + \frac{(a+bx)^7}{8(d+ex)^8(bd-ae)}}{8(bd-ae)} \right)}{9(bd-ae)} + \frac{(a+bx)^7}{9(d+ex)^9(bd-ae)}$$

↓ 48

$$\frac{(a+bx)^7}{9(d+ex)^9(bd-ae)} + \frac{2b \left(\frac{b(a+bx)^7}{56(d+ex)^7(bd-ae)^2} + \frac{(a+bx)^7}{8(d+ex)^8(bd-ae)} \right)}{9(bd-ae)}$$

input `Int[(a^2 + 2*a*b*x + b^2*x^2)^3/(d + e*x)^10,x]`

output `(a + b*x)^7/(9*(b*d - a*e)*(d + e*x)^9) + (2*b*((a + b*x)^7/(8*(b*d - a*e)*
*(d + e*x)^8) + (b*(a + b*x)^7)/(56*(b*d - a*e)^2*(d + e*x)^7)))/(9*(b*d -
a*e))`

Defintions of rubi rules used

rule 27 `Int[(a_)*(F_x_), x_Symbol] := Simp[a Int[F_x, x], x] /; FreeQ[a, x] && !Ma
tchQ[F_x, (b_)*(G_x_) /; FreeQ[b, x]]`

rule 48 `Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] := Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && EqQ[m + n + 2, 0] && NeQ[m, -1]`

rule 55 `Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] := Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*(S
implify[m + n + 2]/((b*c - a*d)*(m + 1))) Int[(a + b*x)^Simplify[m + 1]*(
c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && ILtQ[Simplify[m + n +
2], 0] && NeQ[m, -1] && !(LtQ[m, -1] && LtQ[n, -1] && (EqQ[a, 0] || (NeQ[
c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimplerQ[m, 1] || !SumSimp
lerQ[n, 1])`

rule 1098

```
Int[((d_.) + (e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] := Simp[1/c^p Int[(d + e*x)^m*(b/2 + c*x)^(2*p), x], x] /; FreeQ[
{a, b, c, d, e, m}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 334 vs. 2(83) = 166.

Time = 1.05 (sec) , antiderivative size = 335, normalized size of antiderivative = 3.76

| method | result |
|--------------|--|
| risch | $\frac{-\frac{b^6 x^6}{3e} - \frac{b^5(3ae+bd)x^5}{2e^2} - \frac{b^4(6e^2 a^2+3abde+b^2 d^2)x^4}{2e^3} - \frac{b^3(10e^3 a^3+6a^2 b d e^2+3a b^2 d^2 e+b^3 d^3)x^3}{3e^4} - \frac{b^2(15a^4 e^4+10a^3 b d e^3+6a^2 b^2 d^2 e^2+b^3 d^3)}{7e^5}}$ |
| default | $-\frac{b^6}{3e^7(e x+d)^3} - \frac{3b^5(ae-bd)}{2e^7(e x+d)^4} - \frac{a^6 e^6-6a^5 b d e^5+15a^4 b^2 d^2 e^4-20a^3 b^3 d^3 e^3+15a^2 b^4 d^4 e^2-6a b^5 d^5 e+b^6 d^6}{9e^7(e x+d)^3} - \frac{15b^2(a^4 e^4-b^3 d^3)}{7e^5}$ |
| norman | $\frac{-\frac{b^6 x^6}{3e} - \frac{(3e^3 a b^5+d e^2 b^6)x^5}{2e^4} - \frac{(6e^4 a^2 b^4+3d e^3 a b^5+d^2 e^2 b^6)x^4}{2e^5} - \frac{(10e^5 a^3 b^3+6a^2 b^4 d e^4+3d^2 e^3 a b^5+d^3 e^2 b^6)x^3}{3e^6} - \frac{(15a^4 b^2 e^6+10a^3 b d e^5+b^3 d^3)}{7e^5}}$ |
| gospers | $-\frac{84x^6 b^6 e^6+378x^5 a b^5 e^6+126x^5 b^6 d e^5+756x^4 a^2 b^4 e^6+378x^4 a b^5 d e^5+126x^4 b^6 d^2 e^4+840x^3 a^3 b^3 e^6+504x^3 a^2 b^4 d e^5+252x^3 a b^5 d^2 e^4}{(e x+d)^9}$ |
| paralelrisch | $-\frac{84b^6 x^6 e^8-378a b^5 e^8 x^5-126b^6 d e^7 x^5-756a^2 b^4 e^8 x^4-378a b^5 d e^7 x^4-126b^6 d^2 e^6 x^4-840a^3 b^3 e^8 x^3-504a^2 b^4 d e^7 x^3-252a b^5 d^2 e^6 x^3}{(e x+d)^9}$ |
| orering | $-\frac{(84x^6 b^6 e^6+378x^5 a b^5 e^6+126x^5 b^6 d e^5+756x^4 a^2 b^4 e^6+378x^4 a b^5 d e^5+126x^4 b^6 d^2 e^4+840x^3 a^3 b^3 e^6+504x^3 a^2 b^4 d e^5+252x^3 a b^5 d^2 e^4)}{(e x+d)^9}$ |

input

```
int((b^2*x^2+2*a*b*x+a^2)^3/(e*x+d)^10,x,method=_RETURNVERBOSE)
```

output

```
(-1/3/e*b^6*x^6-1/2*b^5/e^2*(3*a*e+b*d)*x^5-1/2/e^3*b^4*(6*a^2*e^2+3*a*b*d
*e+b^2*d^2)*x^4-1/3/e^4*b^3*(10*a^3*e^3+6*a^2*b*d*e^2+3*a*b^2*d^2*e+b^3*d^
3)*x^3-1/7*b^2/e^5*(15*a^4*e^4+10*a^3*b*d*e^3+6*a^2*b^2*d^2*e^2+3*a*b^3*d^
3*e+b^4*d^4)*x^2-1/28*b/e^6*(21*a^5*e^5+15*a^4*b*d*e^4+10*a^3*b^2*d^2*e^3+
6*a^2*b^3*d^3*e^2+3*a*b^4*d^4*e+b^5*d^5)*x-1/252/e^7*(28*a^6*e^6+21*a^5*b*
d*e^5+15*a^4*b^2*d^2*e^4+10*a^3*b^3*d^3*e^3+6*a^2*b^4*d^4*e^2+3*a*b^5*d^5*
e+b^6*d^6))/(e*x+d)^9
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 441 vs. $2(83) = 166$.

Time = 0.08 (sec) , antiderivative size = 441, normalized size of antiderivative = 4.96

$$\int \frac{(a^2 + 2abx + b^2x^2)^3}{(d + ex)^{10}} dx =$$

$$\frac{84b^6e^6x^6 + b^6d^6 + 3ab^5d^5e + 6a^2b^4d^4e^2 + 10a^3b^3d^3e^3 + 15a^4b^2d^2e^4 + 21a^5bde^5 + 28a^6e^6 + 126(b^6d^6 + 3ab^5d^5e + 6a^2b^4d^4e^2 + 10a^3b^3d^3e^3 + 15a^4b^2d^2e^4 + 21a^5bde^5 + 28a^6e^6)}{(d + ex)^{10}}$$

input `integrate((b^2*x^2+2*a*b*x+a^2)^3/(e*x+d)^10,x, algorithm="fricas")`

output `-1/252*(84*b^6*e^6*x^6 + b^6*d^6 + 3*a*b^5*d^5*e + 6*a^2*b^4*d^4*e^2 + 10*a^3*b^3*d^3*e^3 + 15*a^4*b^2*d^2*e^4 + 21*a^5*b*d*e^5 + 28*a^6*e^6 + 126*(b^6*d*e^5 + 3*a*b^5*e^6)*x^5 + 126*(b^6*d^2*e^4 + 3*a*b^5*d*e^5 + 6*a^2*b^4*e^6)*x^4 + 84*(b^6*d^3*e^3 + 3*a*b^5*d^2*e^4 + 6*a^2*b^4*d*e^5 + 10*a^3*b^3*e^6)*x^3 + 36*(b^6*d^4*e^2 + 3*a*b^5*d^3*e^3 + 6*a^2*b^4*d^2*e^4 + 10*a^3*b^3*d*e^5 + 15*a^4*b^2*e^6)*x^2 + 9*(b^6*d^5*e + 3*a*b^5*d^4*e^2 + 6*a^2*b^4*d^3*e^3 + 10*a^3*b^3*d^2*e^4 + 15*a^4*b^2*d*e^5 + 21*a^5*b*e^6)*x)/(e^16*x^9 + 9*d*e^15*x^8 + 36*d^2*e^14*x^7 + 84*d^3*e^13*x^6 + 126*d^4*e^12*x^5 + 126*d^5*e^11*x^4 + 84*d^6*e^10*x^3 + 36*d^7*e^9*x^2 + 9*d^8*e^8*x + d^9*e^7)`

Sympy [F(-1)]

Timed out.

$$\int \frac{(a^2 + 2abx + b^2x^2)^3}{(d + ex)^{10}} dx = \text{Timed out}$$

input `integrate((b**2*x**2+2*a*b*x+a**2)**3/(e*x+d)**10,x)`

output `Timed out`

output

```
-1/252*(84*b^6*e^6*x^6 + 126*b^6*d*e^5*x^5 + 378*a*b^5*e^6*x^5 + 126*b^6*d^2*e^4*x^4 + 378*a*b^5*d*e^5*x^4 + 756*a^2*b^4*e^6*x^4 + 84*b^6*d^3*e^3*x^3 + 252*a*b^5*d^2*e^4*x^3 + 504*a^2*b^4*d*e^5*x^3 + 840*a^3*b^3*e^6*x^3 + 36*b^6*d^4*e^2*x^2 + 108*a*b^5*d^3*e^3*x^2 + 216*a^2*b^4*d^2*e^4*x^2 + 360*a^3*b^3*d*e^5*x^2 + 540*a^4*b^2*e^6*x^2 + 9*b^6*d^5*e*x + 27*a*b^5*d^4*e^2*x + 54*a^2*b^4*d^3*e^3*x + 90*a^3*b^3*d^2*e^4*x + 135*a^4*b^2*d*e^5*x + 189*a^5*b*e^6*x + b^6*d^6 + 3*a*b^5*d^5*e + 6*a^2*b^4*d^4*e^2 + 10*a^3*b^3*d^3*e^3 + 15*a^4*b^2*d^2*e^4 + 21*a^5*b*d*e^5 + 28*a^6*e^6)/((e*x + d)^9*e^7)
```

Mupad [B] (verification not implemented)

Time = 8.76 (sec) , antiderivative size = 423, normalized size of antiderivative = 4.75

$$\int \frac{(a^2 + 2abx + b^2x^2)^3}{(d + ex)^{10}} dx =$$

$$-\frac{28a^6e^6 + 21a^5bde^5 + 15a^4b^2d^2e^4 + 10a^3b^3d^3e^3 + 6a^2b^4d^4e^2 + 3ab^5d^5e + b^6d^6}{252e^7} + \frac{b^6x^6}{3e} + \frac{b^3x^3(10a^3e^3 + 6a^2bde^2 + 3ab^2d^2e + b^3d^3)}{3e^4} + \frac{d^9 + 9d^8ex + 36d^7e^2x^2 + 84d^6e^3x^3 + 126d^5e^4x^4 + 126d^4e^5x^5 + 84d^3e^6x^6 + 36d^2e^7x^7 + 9d^8e^8x^8 + 36d^7e^9x^9}{(d + ex)^{10}}$$

input

```
int((a^2 + b^2*x^2 + 2*a*b*x)^3/(d + e*x)^10,x)
```

output

```
-((28*a^6*e^6 + b^6*d^6 + 6*a^2*b^4*d^4*e^2 + 10*a^3*b^3*d^3*e^3 + 15*a^4*b^2*d^2*e^4 + 3*a*b^5*d^5*e + 21*a^5*b*d*e^5)/(252*e^7) + (b^6*x^6)/(3*e) + (b^3*x^3*(10*a^3*e^3 + b^3*d^3 + 3*a*b^2*d^2*e + 6*a^2*b*d*e^2))/(3*e^4) + (b*x*(21*a^5*e^5 + b^5*d^5 + 6*a^2*b^3*d^3*e^2 + 10*a^3*b^2*d^2*e^3 + 3*a*b^4*d^4*e + 15*a^4*b*d*e^4))/(28*e^6) + (b^5*x^5*(3*a*e + b*d))/(2*e^2) + (b^2*x^2*(15*a^4*e^4 + b^4*d^4 + 6*a^2*b^2*d^2*e^2 + 3*a*b^3*d^3*e + 10*a^3*b*d*e^3))/(7*e^5) + (b^4*x^4*(6*a^2*e^2 + b^2*d^2 + 3*a*b*d*e))/(2*e^3))/(d^9 + e^9*x^9 + 9*d*e^8*x^8 + 36*d^7*e^2*x^2 + 84*d^6*e^3*x^3 + 126*d^5*e^4*x^4 + 126*d^4*e^5*x^5 + 84*d^3*e^6*x^6 + 36*d^2*e^7*x^7 + 9*d^8*e^8*x^8 + 36*d^7*e^9*x^9)
```


Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 464, normalized size of antiderivative = 5.21

$$\int \frac{(a^2 + 2abx + b^2x^2)^3}{(d + ex)^{10}} dx$$

$$= \frac{-84b^6e^6x^6 - 378ab^5e^6x^5 - 126b^6de^5x^5 - 756a^2b^4e^6x^4 - 378ab^5de^5x^4 - 126b^6d^2e^4x^4 - 840a^3b^3e^6x^3 - \dots}{(252e^7(d^9 + 9d^8ex + 36d^7e^2x^2 + 84d^6e^3x^3 + 126d^5e^4x^4 + 126d^4e^5x^5 + 84d^3e^6x^6 + 36d^2e^7x^7 + 9de^8x^8 + e^9x^9))}$$

input `int((b^2*x^2+2*a*b*x+a^2)^3/(e*x+d)^10,x)`output `(- 28*a**6*e**6 - 21*a**5*b*d*e**5 - 189*a**5*b*e**6*x - 15*a**4*b**2*d**2*e**4 - 135*a**4*b**2*d*e**5*x - 540*a**4*b**2*e**6*x**2 - 10*a**3*b**3*d**3*e**3 - 90*a**3*b**3*d**2*e**4*x - 360*a**3*b**3*d*e**5*x**2 - 840*a**3*b**3*e**6*x**3 - 6*a**2*b**4*d**4*e**2 - 54*a**2*b**4*d**3*e**3*x - 216*a**2*b**4*d**2*e**4*x**2 - 504*a**2*b**4*d*e**5*x**3 - 756*a**2*b**4*e**6*x**4 - 3*a*b**5*d**5*e - 27*a*b**5*d**4*e**2*x - 108*a*b**5*d**3*e**3*x**2 - 252*a*b**5*d**2*e**4*x**3 - 378*a*b**5*d*e**5*x**4 - 378*a*b**5*e**6*x**5 - b**6*d**6 - 9*b**6*d**5*e*x - 36*b**6*d**4*e**2*x**2 - 84*b**6*d**3*e**3*x**3 - 126*b**6*d**2*e**4*x**4 - 126*b**6*d*e**5*x**5 - 84*b**6*e**6*x**6)/(252*e**7*(d**9 + 9*d**8*e*x + 36*d**7*e**2*x**2 + 84*d**6*e**3*x**3 + 126*d**5*e**4*x**4 + 126*d**4*e**5*x**5 + 84*d**3*e**6*x**6 + 36*d**2*e**7*x**7 + 9*d*e**8*x**8 + e**9*x**9))`

input `Integrate[(a^2 + 2*a*b*x + b^2*x^2)^3/(d + e*x)^11,x]`

output
$$\frac{-1/840*(84*a^6*e^6 + 56*a^5*b*e^5*(d + 10*e*x) + 35*a^4*b^2*e^4*(d^2 + 10*d*e*x + 45*e^2*x^2) + 20*a^3*b^3*e^3*(d^3 + 10*d^2*e*x + 45*d*e^2*x^2 + 120*e^3*x^3) + 10*a^2*b^4*e^2*(d^4 + 10*d^3*e*x + 45*d^2*e^2*x^2 + 120*d*e^3*x^3 + 210*e^4*x^4) + 4*a*b^5*e*(d^5 + 10*d^4*e*x + 45*d^3*e^2*x^2 + 120*d^2*e^3*x^3 + 210*d*e^4*x^4 + 252*e^5*x^5) + b^6*(d^6 + 10*d^5*e*x + 45*d^4*e^2*x^2 + 120*d^3*e^3*x^3 + 210*d^2*e^4*x^4 + 252*d*e^5*x^5 + 210*e^6*x^6))}{(e^7*(d + e*x)^10)}$$

Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 146, normalized size of antiderivative = 1.22, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {1098, 27, 55, 55, 55, 48}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\begin{aligned} & \int \frac{(a^2 + 2abx + b^2x^2)^3}{(d + ex)^{11}} dx \\ & \quad \downarrow 1098 \\ & \quad \int \frac{b^6(a+bx)^6}{(d+ex)^{11}} dx \\ & \quad \quad \downarrow 27 \\ & \quad \quad \int \frac{(a + bx)^6}{(d + ex)^{11}} dx \\ & \quad \quad \quad \downarrow 55 \\ & \quad \quad \quad \frac{3b \int \frac{(a+bx)^6}{(d+ex)^{10}} dx}{10(bd - ae)} + \frac{(a + bx)^7}{10(d + ex)^{10}(bd - ae)} \\ & \quad \quad \quad \quad \downarrow 55 \end{aligned}$$

$$\begin{aligned}
& \frac{3b \left(\frac{2b \int \frac{(a+bx)^6 dx}{(d+ex)^9} + \frac{(a+bx)^7}{9(d+ex)^9(bd-ae)}}{10(bd-ae)} + \frac{(a+bx)^7}{10(d+ex)^{10}(bd-ae)} \right)}{10(bd-ae)} \\
& \quad \downarrow 55 \\
& \frac{3b \left(\frac{2b \left(\frac{b \int \frac{(a+bx)^6 dx}{(d+ex)^8} + \frac{(a+bx)^7}{8(d+ex)^8(bd-ae)} \right)}{9(bd-ae)} + \frac{(a+bx)^7}{9(d+ex)^9(bd-ae)} \right)}{10(bd-ae)} + \frac{(a+bx)^7}{10(d+ex)^{10}(bd-ae)} \\
& \quad \downarrow 48 \\
& \frac{(a+bx)^7}{10(d+ex)^{10}(bd-ae)} + \frac{3b \left(\frac{(a+bx)^7}{9(d+ex)^9(bd-ae)} + \frac{2b \left(\frac{b(a+bx)^7}{56(d+ex)^7(bd-ae)^2} + \frac{(a+bx)^7}{8(d+ex)^8(bd-ae)} \right)}{9(bd-ae)} \right)}{10(bd-ae)}
\end{aligned}$$

input `Int[(a^2 + 2*a*b*x + b^2*x^2)^3/(d + e*x)^11,x]`

output `(a + b*x)^7/(10*(b*d - a*e)*(d + e*x)^10) + (3*b*((a + b*x)^7/(9*(b*d - a*e)*(d + e*x)^9) + (2*b*((a + b*x)^7/(8*(b*d - a*e)*(d + e*x)^8) + (b*(a + b*x)^7)/(56*(b*d - a*e)^2*(d + e*x)^7)))/(9*(b*d - a*e)))/(10*(b*d - a*e))`

Defintions of rubi rules used

rule 27 `Int[(a_)*(F_x_), x_Symbol] := Simp[a Int[F_x, x], x] /; FreeQ[a, x] && !MatchQ[F_x, (b_)*(G_x_)] /; FreeQ[b, x]`

rule 48 `Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] := Simp[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{a, b, c, d, m, n}, x] && EqQ[m + n + 2, 0] && NeQ[m, -1]`

rule 55

```
Int[((a_.) + (b_.)*(x_)^(m_))*((c_.) + (d_.)*(x_)^(n_)), x_Symbol] := Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*(S
implify[m + n + 2]/((b*c - a*d)*(m + 1))) Int[(a + b*x)^Simplify[m + 1]*(
c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && ILtQ[Simplify[m + n +
2], 0] && NeQ[m, -1] && !(LtQ[m, -1] && LtQ[n, -1] && (EqQ[a, 0] || (NeQ[
c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimplerQ[m, 1] || !SumSimp
lerQ[n, 1])
```

rule 1098

```
Int[((d_.) + (e_.)*(x_)^(m_.))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] := Simp[1/c^p Int[(d + e*x)^m*(b/2 + c*x)^(2*p), x], x] /; FreeQ[
{a, b, c, d, e, m}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 334 vs. 2(112) = 224.

Time = 1.06 (sec) , antiderivative size = 335, normalized size of antiderivative = 2.79

| method | result |
|---------------|---|
| risch | $-\frac{b^6 x^6}{4e} - \frac{3b^5(4ae+bd)x^5}{10e^2} - \frac{b^4(10e^2a^2+4abde+b^2d^2)x^4}{4e^3} - \frac{b^3(20e^3a^3+10a^2bde^2+4ab^2d^2e+b^3d^3)x^3}{7e^4} - \frac{3b^2(35a^4e^4+20a^3bde^3+10a^2b^2d^2e^2+5ab^3d^3e+b^4d^4)x^2}{56e^5} - \frac{3b(5a^5e^5+5a^4bde^4+10a^3b^2d^2e^3-10a^2b^3d^3e^2+5ab^4d^4e-b^5d^5)x}{3e^7} - \frac{20b^3(e^3a^3-3a^2bde^2+3ab^2d^2e-b^3d^3)x}{7e^7}$ |
| default | $-\frac{b^6}{4e^7(e^2x^2+2ex+d)^4} - \frac{2b(e^5a^5-5a^4bde^4+10a^3b^2d^2e^3-10a^2b^3d^3e^2+5ab^4d^4e-b^5d^5)}{3e^7(e^2x^2+2ex+d)^9} - \frac{20b^3(e^3a^3-3a^2bde^2+3ab^2d^2e-b^3d^3)x}{7e^7(e^2x^2+2ex+d)^7}$ |
| norman | $-\frac{b^6 x^6}{4e} - \frac{3(4e^4 a b^5 + d e^3 b^6) x^5}{10e^5} - \frac{(10e^5 a^2 b^4 + 4d e^4 a b^5 + d^2 e^3 b^6) x^4}{4e^6} - \frac{(20a^3 b^3 e^6 + 10a^2 b^4 d e^5 + 4a b^5 d^2 e^4 + b^6 d^3 e^3) x^3}{7e^7} - \frac{3(35a^4 b^2 e^7 + 20a^3 b d e^6 + 10a^2 b^2 d^2 e^5 + 5a b^3 d^3 e^4 + b^4 d^4) x^2}{56e^5}$ |
| gospers | $-\frac{210x^6 b^6 e^6 + 1008x^5 a b^5 e^6 + 252x^5 b^6 d e^5 + 2100x^4 a^2 b^4 e^6 + 840x^4 a b^5 d e^5 + 210x^4 b^6 d^2 e^4 + 2400x^3 a^3 b^3 e^6 + 1200x^3 a^2 b^4 d e^5 + 1200x^3 a b^5 d^2 e^4 + 1200x^3 b^6 d^3 e^3}{4e^7}$ |
| parallelrisch | $-\frac{210b^6 x^6 e^9 - 1008a b^5 e^9 x^5 - 252b^6 d e^8 x^5 - 2100a^2 b^4 e^9 x^4 - 840a b^5 d e^8 x^4 - 210b^6 d^2 e^7 x^4 - 2400a^3 b^3 e^9 x^3 - 1200a^2 b^4 d e^8 x^3 - 1200a b^5 d^2 e^7 x^3 - 1200b^6 d^3 e^6 x^3}{4e^7}$ |
| orering | $-\frac{(210x^6 b^6 e^6 + 1008x^5 a b^5 e^6 + 252x^5 b^6 d e^5 + 2100x^4 a^2 b^4 e^6 + 840x^4 a b^5 d e^5 + 210x^4 b^6 d^2 e^4 + 2400x^3 a^3 b^3 e^6 + 1200x^3 a^2 b^4 d e^5 + 1200x^3 a b^5 d^2 e^4 + 1200x^3 b^6 d^3 e^3) x^3}{4e^7}$ |

input

```
int((b^2*x^2+2*a*b*x+a^2)^3/(e*x+d)^11,x,method=_RETURNVERBOSE)
```

output

```
(-1/4/e*b^6*x^6-3/10*b^5/e^2*(4*a*e+b*d)*x^5-1/4/e^3*b^4*(10*a^2*e^2+4*a*b*d*e+b^2*d^2)*x^4-1/7*b^3/e^4*(20*a^3*e^3+10*a^2*b*d*e^2+4*a*b^2*d^2*e+b^3*d^3)*x^3-3/56*b^2/e^5*(35*a^4*e^4+20*a^3*b*d*e^3+10*a^2*b^2*d^2*e^2+4*a*b^3*d^3*e+b^4*d^4)*x^2-1/84*b/e^6*(56*a^5*e^5+35*a^4*b*d*e^4+20*a^3*b^2*d^2*e^3+10*a^2*b^3*d^3*e^2+4*a*b^4*d^4*e+b^5*d^5)*x-1/840/e^7*(84*a^6*e^6+56*a^5*b*d*e^5+35*a^4*b^2*d^2*e^4+20*a^3*b^3*d^3*e^3+10*a^2*b^4*d^4*e^2+4*a*b^5*d^5*e+b^6*d^6))/(e*x+d)^10
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 452 vs. $2(112) = 224$.

Time = 0.08 (sec) , antiderivative size = 452, normalized size of antiderivative = 3.77

$$\int \frac{(a^2 + 2abx + b^2x^2)^3}{(d + ex)^{11}} dx = \frac{210b^6e^6x^6 + b^6d^6 + 4ab^5d^5e + 10a^2b^4d^4e^2 + 20a^3b^3d^3e^3 + 35a^4b^2d^2e^4 + 56a^5bde^5 + 84a^6e^6 + 252(b^6d^6e^5 + 4a^2b^4d^4e^5 + 10a^3b^3d^3e^4 + 20a^4b^2d^2e^3 + 10a^5bd^2e^2 + 4a^6d^2e)}{(e^7x^{10} + 10d^6e^6x^9 + 45d^5e^5x^8 + 120d^4e^4x^7 + 210d^3e^3x^6 + 252d^2e^2x^5 + 210d^2e^2x^4 + 120d^2e^2x^3 + 45d^2e^2x^2 + 10d^2e^2x + d^2e^2)}$$

input

```
integrate((b^2*x^2+2*a*b*x+a^2)^3/(e*x+d)^11,x, algorithm="fricas")
```

output

```
-1/840*(210*b^6*e^6*x^6 + b^6*d^6 + 4*a*b^5*d^5*e + 10*a^2*b^4*d^4*e^2 + 20*a^3*b^3*d^3*e^3 + 35*a^4*b^2*d^2*e^4 + 56*a^5*b*d*e^5 + 84*a^6*e^6 + 252*(b^6*d^6*e^5 + 4*a^2*b^4*d^4*e^5 + 10*a^3*b^3*d^3*e^4 + 20*a^4*b^2*d^2*e^3 + 10*a^5*b*d^2*e^2 + 4*a^6*d^2*e^2 + 20*a^3*b^3*d^3*e^5 + 35*a^4*b^2*d^2*e^6)*x^2 + 10*(b^6*d^5*e + 4*a*b^5*d^4*e^2 + 20*a^2*b^4*d^3*e^3 + 20*a^3*b^3*d^2*e^4 + 35*a^4*b^2*d^2*e^5 + 56*a^5*b*d^2*e^6)*x)/(e^17*x^10 + 10*d^6*e^6*x^9 + 45*d^5*e^5*x^8 + 120*d^4*e^4*x^7 + 210*d^3*e^3*x^6 + 252*d^2*e^2*x^5 + 210*d^2*e^2*x^4 + 120*d^2*e^2*x^3 + 45*d^2*e^2*x^2 + 10*d^2*e^2*x + d^2*e^2)
```


Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 375 vs. $2(112) = 224$.

Time = 0.21 (sec) , antiderivative size = 375, normalized size of antiderivative = 3.12

$$\int \frac{(a^2 + 2abx + b^2x^2)^3}{(d + ex)^{11}} dx =$$

$$\frac{210 b^6 e^6 x^6 + 252 b^6 d e^5 x^5 + 1008 a b^5 e^6 x^5 + 210 b^6 d^2 e^4 x^4 + 840 a b^5 d e^5 x^4 + 2100 a^2 b^4 e^6 x^4 + 120 b^6 d^3 e^3 x^3}{-}$$

input `integrate((b^2*x^2+2*a*b*x+a^2)^3/(e*x+d)^11,x, algorithm="giac")`

output

$$\frac{-1/840*(210*b^6*e^6*x^6 + 252*b^6*d*e^5*x^5 + 1008*a*b^5*e^6*x^5 + 210*b^6*d^2*e^4*x^4 + 840*a*b^5*d*e^5*x^4 + 2100*a^2*b^4*e^6*x^4 + 120*b^6*d^3*e^3*x^3 + 480*a*b^5*d^2*e^4*x^3 + 1200*a^2*b^4*d*e^5*x^3 + 2400*a^3*b^3*e^6*x^3 + 45*b^6*d^4*e^2*x^2 + 180*a*b^5*d^3*e^3*x^2 + 450*a^2*b^4*d^2*e^4*x^2 + 900*a^3*b^3*d*e^5*x^2 + 1575*a^4*b^2*e^6*x^2 + 10*b^6*d^5*e*x + 40*a*b^5*d^4*e^2*x + 100*a^2*b^4*d^3*e^3*x + 200*a^3*b^3*d^2*e^4*x + 350*a^4*b^2*d*e^5*x + 560*a^5*b*e^6*x + b^6*d^6 + 4*a*b^5*d^5*e + 10*a^2*b^4*d^4*e^2 + 20*a^3*b^3*d^3*e^3 + 35*a^4*b^2*d^2*e^4 + 56*a^5*b*d*e^5 + 84*a^6*e^6)/((e*x + d)^10*e^7)}$$
Mupad [B] (verification not implemented)

Time = 8.87 (sec) , antiderivative size = 434, normalized size of antiderivative = 3.62

$$\int \frac{(a^2 + 2abx + b^2x^2)^3}{(d + ex)^{11}} dx =$$

$$\frac{84 a^6 e^6 + 56 a^5 b d e^5 + 35 a^4 b^2 d^2 e^4 + 20 a^3 b^3 d^3 e^3 + 10 a^2 b^4 d^4 e^2 + 4 a b^5 d^5 e + b^6 d^6}{840 e^7} + \frac{b^6 x^6}{4 e} + \frac{b^3 x^3 (20 a^3 e^3 + 10 a^2 b d e^2 + 4 a b^2 d^2 e + b^3 d^3)}{7 e^4}$$

$$\frac{1}{d^{10} + 10 d^9 e x + 45 d^8 e^2 x^2 + 120 d^7 e^3 x^3}$$

input `int((a^2 + b^2*x^2 + 2*a*b*x)^3/(d + e*x)^11,x)`

output

```

-((84*a^6*e^6 + b^6*d^6 + 10*a^2*b^4*d^4*e^2 + 20*a^3*b^3*d^3*e^3 + 35*a^4
*b^2*d^2*e^4 + 4*a*b^5*d^5*e + 56*a^5*b*d*e^5)/(840*e^7) + (b^6*x^6)/(4*e)
+ (b^3*x^3*(20*a^3*e^3 + b^3*d^3 + 4*a*b^2*d^2*e + 10*a^2*b*d*e^2))/(7*e^
4) + (b*x*(56*a^5*e^5 + b^5*d^5 + 10*a^2*b^3*d^3*e^2 + 20*a^3*b^2*d^2*e^3
+ 4*a*b^4*d^4*e + 35*a^4*b*d*e^4))/(84*e^6) + (3*b^5*x^5*(4*a*e + b*d))/(1
0*e^2) + (3*b^2*x^2*(35*a^4*e^4 + b^4*d^4 + 10*a^2*b^2*d^2*e^2 + 4*a*b^3*d
^3*e + 20*a^3*b*d*e^3))/(56*e^5) + (b^4*x^4*(10*a^2*e^2 + b^2*d^2 + 4*a*b*
d*e))/(4*e^3))/(d^10 + e^10*x^10 + 10*d*e^9*x^9 + 45*d^8*e^2*x^2 + 120*d^7
*e^3*x^3 + 210*d^6*e^4*x^4 + 252*d^5*e^5*x^5 + 210*d^4*e^6*x^6 + 120*d^3*e
^7*x^7 + 45*d^2*e^8*x^8 + 10*d^9*e*x)

```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 475, normalized size of antiderivative = 3.96

$$\int \frac{(a^2 + 2abx + b^2x^2)^3}{(d + ex)^{11}} dx$$

$$= \frac{-210b^6e^6x^6 - 1008ab^5e^6x^5 - 252b^6de^5x^5 - 2100a^2b^4e^6x^4 - 840ab^5de^5x^4 - 210b^6d^2e^4x^4 - 2400a^3b^3e^6x^3 - 2100a^2b^4e^5x^3 - 1008ab^5d^2e^4x^3 - 450a^3b^3d^2e^4x^3 - 1200a^2b^4de^5x^3 - 2100a^2b^4e^6x^3 - 4a^3b^5d^5e - 40a^2b^5d^4e^2x - 180ab^5d^3e^3x^2 - 480ab^5d^2e^4x^3 - 840ab^5de^5x^4 - 1008ab^5e^6x^5 - b^6d^6 - 10b^6d^5ex - 45b^6d^4e^2x^2 - 120b^6d^3e^3x^3 - 210b^6d^2e^4x^4 - 252b^6de^5x^5 - 210b^6e^6x^6)/(840e^7(d^{10} + 10d^9ex + 45d^8e^2x^2 + 120d^7e^3x^3 + 210d^6e^4x^4 + 252d^5e^5x^5 + 210d^4e^6x^6 + 120d^3e^7x^7 + 45d^2e^8x^8 + 10d^9ex + e^{10}x^{10}))}$$

input

```
int((b^2*x^2+2*a*b*x+a^2)^3/(e*x+d)^11,x)
```

output

```

( - 84*a**6*e**6 - 56*a**5*b*d*e**5 - 560*a**5*b*e**6*x - 35*a**4*b**2*d**
2*e**4 - 350*a**4*b**2*d*e**5*x - 1575*a**4*b**2*e**6*x**2 - 20*a**3*b**3*
d**3*e**3 - 200*a**3*b**3*d**2*e**4*x - 900*a**3*b**3*d*e**5*x**2 - 2400*a
**3*b**3*e**6*x**3 - 10*a**2*b**4*d**4*e**2 - 100*a**2*b**4*d**3*e**3*x -
450*a**2*b**4*d**2*e**4*x**2 - 1200*a**2*b**4*d*e**5*x**3 - 2100*a**2*b**4
*e**6*x**4 - 4*a*b**5*d**5*e - 40*a*b**5*d**4*e**2*x - 180*a*b**5*d**3*e**
3*x**2 - 480*a*b**5*d**2*e**4*x**3 - 840*a*b**5*d*e**5*x**4 - 1008*a*b**5*
e**6*x**5 - b**6*d**6 - 10*b**6*d**5*e*x - 45*b**6*d**4*e**2*x**2 - 120*b
**6*d**3*e**3*x**3 - 210*b**6*d**2*e**4*x**4 - 252*b**6*d*e**5*x**5 - 210*b
**6*e**6*x**6)/(840*e**7*(d**10 + 10*d**9*e*x + 45*d**8*e**2*x**2 + 120*d
**7*e**3*x**3 + 210*d**6*e**4*x**4 + 252*d**5*e**5*x**5 + 210*d**4*e**6*x**
6 + 120*d**3*e**7*x**7 + 45*d**2*e**8*x**8 + 10*d**9*e*x**9 + e**10*x**10)
)

```

3.104 $\int \frac{(a^2+2abx+b^2x^2)^3}{(d+ex)^{12}} dx$

| | |
|---|-----|
| Optimal result | 853 |
| Mathematica [A] (verified) | 854 |
| Rubi [A] (verified) | 854 |
| Maple [B] (verified) | 856 |
| Fricas [B] (verification not implemented) | 857 |
| Sympy [F(-1)] | 857 |
| Maxima [B] (verification not implemented) | 858 |
| Giac [B] (verification not implemented) | 858 |
| Mupad [B] (verification not implemented) | 859 |
| Reduce [B] (verification not implemented) | 860 |

Optimal result

Integrand size = 26, antiderivative size = 170

$$\int \frac{(a^2 + 2abx + b^2x^2)^3}{(d + ex)^{12}} dx = -\frac{(bd - ae)^6}{11e^7(d + ex)^{11}} + \frac{3b(bd - ae)^5}{5e^7(d + ex)^{10}} - \frac{5b^2(bd - ae)^4}{3e^7(d + ex)^9} + \frac{5b^3(bd - ae)^3}{2e^7(d + ex)^8} - \frac{15b^4(bd - ae)^2}{7e^7(d + ex)^7} + \frac{b^5(bd - ae)}{e^7(d + ex)^6} - \frac{b^6}{5e^7(d + ex)^5}$$

output

```
-1/11*(-a*e+b*d)^6/e^7/(e*x+d)^11+3/5*b*(-a*e+b*d)^5/e^7/(e*x+d)^10-5/3*b^2*(-a*e+b*d)^4/e^7/(e*x+d)^9+5/2*b^3*(-a*e+b*d)^3/e^7/(e*x+d)^8-15/7*b^4*(-a*e+b*d)^2/e^7/(e*x+d)^7+b^5*(-a*e+b*d)/e^7/(e*x+d)^6-1/5*b^6/e^7/(e*x+d)^5
```

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 277, normalized size of antiderivative = 1.63

$$\int \frac{(a^2 + 2abx + b^2x^2)^3}{(d + ex)^{12}} dx =$$

$$\frac{210a^6e^6 + 126a^5be^5(d + 11ex) + 70a^4b^2e^4(d^2 + 11dex + 55e^2x^2) + 35a^3b^3e^3(d^3 + 11d^2ex + 55de^2x^2 +$$

input `Integrate[(a^2 + 2*a*b*x + b^2*x^2)^3/(d + e*x)^12,x]`

output `-1/2310*(210*a^6*e^6 + 126*a^5*b*e^5*(d + 11*e*x) + 70*a^4*b^2*e^4*(d^2 + 11*d*e*x + 55*e^2*x^2) + 35*a^3*b^3*e^3*(d^3 + 11*d^2*e*x + 55*d*e^2*x^2 + 165*e^3*x^3) + 15*a^2*b^4*e^2*(d^4 + 11*d^3*e*x + 55*d^2*e^2*x^2 + 165*d*e^3*x^3 + 330*e^4*x^4) + 5*a*b^5*e*(d^5 + 11*d^4*e*x + 55*d^3*e^2*x^2 + 165*d^2*e^3*x^3 + 330*d*e^4*x^4 + 462*e^5*x^5) + b^6*(d^6 + 11*d^5*e*x + 55*d^4*e^2*x^2 + 165*d^3*e^3*x^3 + 330*d^2*e^4*x^4 + 462*d*e^5*x^5 + 462*e^6*x^6))/(e^7*(d + e*x)^11)`

Rubi [A] (verified)

Time = 0.63 (sec) , antiderivative size = 170, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {1098, 27, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a^2 + 2abx + b^2x^2)^3}{(d + ex)^{12}} dx$$

$$\downarrow 1098$$

$$\frac{\int \frac{b^6(a+bx)^6}{(d+ex)^{12}} dx}{b^6}$$

$$\downarrow 27$$

$$\int \frac{(a + bx)^6}{(d + ex)^{12}} dx$$

↓ 53

$$\int \left(-\frac{6b^5(bd - ae)}{e^6(d + ex)^7} + \frac{15b^4(bd - ae)^2}{e^6(d + ex)^8} - \frac{20b^3(bd - ae)^3}{e^6(d + ex)^9} + \frac{15b^2(bd - ae)^4}{e^6(d + ex)^{10}} - \frac{6b(bd - ae)^5}{e^6(d + ex)^{11}} + \frac{(ae - bd)^6}{e^6(d + ex)^{12}} + \frac{1}{e^6(d + ex)^{12}} \right) dx$$

↓ 2009

$$\frac{b^5(bd - ae)}{e^7(d + ex)^6} - \frac{15b^4(bd - ae)^2}{7e^7(d + ex)^7} + \frac{5b^3(bd - ae)^3}{2e^7(d + ex)^8} - \frac{5b^2(bd - ae)^4}{3e^7(d + ex)^9} + \frac{3b(bd - ae)^5}{5e^7(d + ex)^{10}} - \frac{(bd - ae)^6}{11e^7(d + ex)^{11}} - \frac{b^6}{5e^7(d + ex)^5}$$

input `Int[(a^2 + 2*a*b*x + b^2*x^2)^3/(d + e*x)^12,x]`

output `-1/11*(b*d - a*e)^6/(e^7*(d + e*x)^11) + (3*b*(b*d - a*e)^5)/(5*e^7*(d + e*x)^10) - (5*b^2*(b*d - a*e)^4)/(3*e^7*(d + e*x)^9) + (5*b^3*(b*d - a*e)^3)/(2*e^7*(d + e*x)^8) - (15*b^4*(b*d - a*e)^2)/(7*e^7*(d + e*x)^7) + (b^5*(b*d - a*e))/(e^7*(d + e*x)^6) - b^6/(5*e^7*(d + e*x)^5)`

Defintions of rubi rules used

rule 27 `Int[(a_)*(F_x_), x_Symbol] := Simp[a Int[F_x, x], x] /; FreeQ[a, x] && !MatchQ[F_x, (b_)*(G_x_)] /; FreeQ[b, x]`

rule 53 `Int[((a_) + (b_)*(x_))^(m_)*((c_) + (d_)*(x_))^(n_), x_Symbol] := Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && IGtQ[m, 0] && (!IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])`

rule 1098 `Int[((d_) + (e_)*(x_))^(m_)*((a_) + (b_)*(x_) + (c_)*(x_)^2)^(p_), x_Symbol] := Simp[1/c^p Int[(d + e*x)^m*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]`

rule 2009

```
Int[u_, x_Symbol] := Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 334 vs. 2(158) = 316.

Time = 1.08 (sec) , antiderivative size = 335, normalized size of antiderivative = 1.97

| method | result |
|--------------|--|
| risch | $-\frac{b^6 x^6}{5e} - \frac{b^5(5ae+bd)x^5}{5e^2} - \frac{b^4(15e^2 a^2 + 5abde + b^2 d^2)x^4}{7e^3} - \frac{b^3(35e^3 a^3 + 15a^2 bde^2 + 5ab^2 d^2 e + b^3 d^3)x^3}{14e^4} - \frac{b^2(70a^4 e^4 + 35a^3 bde^3 + 15a^2 b^2 d^2 e^2 + 5ab^3 d^3 e + b^4 d^4)x^2}{42e^5}$ |
| default | $-\frac{a^6 e^6 - 6a^5 b d e^5 + 15a^4 b^2 d^2 e^4 - 20a^3 b^3 d^3 e^3 + 15a^2 b^4 d^4 e^2 - 6a b^5 d^5 e + b^6 d^6}{11e^7 (ex+d)^{11}} - \frac{5b^2 (a^4 e^4 - 4a^3 b d e^3 + 6a^2 b^2 d^2 e^2 - 4a b^3 d^3 e + b^4 d^4)}{3e^7 (ex+d)^9}$ |
| norman | $-\frac{b^6 x^6}{5e} - \frac{(5e^5 a b^5 + d e^4 b^6)x^5}{5e^6} - \frac{(15a^2 b^4 e^6 + 5a b^5 d e^5 + b^6 d^2 e^4)x^4}{7e^7} - \frac{(35a^3 b^3 e^7 + 15a^2 b^4 d e^6 + 5a b^5 d^2 e^5 + b^6 d^3 e^4)x^3}{14e^8} - \frac{(70a^4 b^2 e^8 + 35a^3 b^3 d e^7 + 15a^2 b^4 d^2 e^6 + 5ab^5 d^3 e^5 + b^6 d^4 e^4)x^2}{42e^9}$ |
| gospers | $-\frac{462x^6 b^6 e^6 + 2310x^5 a b^5 e^6 + 462x^5 b^6 d e^5 + 4950x^4 a^2 b^4 e^6 + 1650x^4 a b^5 d e^5 + 330x^4 b^6 d^2 e^4 + 5775x^3 a^3 b^3 e^6 + 2475x^3 a^2 b^4 d e^5 + 12375x^3 a b^5 d^2 e^4 + 4725x^3 b^6 d^3 e^3 + 1575x^2 a^4 e^4 + 1575x^2 a^3 b d e^3 + 1575x^2 a^2 b^2 d^2 e^2 + 1575x^2 a b^3 d^3 e + 1575x^2 b^4 d^4}{(ex+d)^{11}}$ |
| parallelrisc | $-\frac{462b^6 x^6 e^{10} - 2310a b^5 e^{10} x^5 - 462b^6 d e^9 x^5 - 4950a^2 b^4 e^{10} x^4 - 1650a b^5 d e^9 x^4 - 330b^6 d^2 e^8 x^4 - 5775a^3 b^3 e^{10} x^3 - 2475a^2 b^4 d e^9 x^3 - 12375a b^5 d^2 e^8 x^3 - 4725x^3 b^6 d^3 e^7 - 1575x^2 a^4 e^4 - 1575x^2 a^3 b d e^3 - 1575x^2 a^2 b^2 d^2 e^2 - 1575x^2 a b^3 d^3 e + 1575x^2 b^4 d^4}{(ex+d)^{11}}$ |
| orering | $-\frac{(462x^6 b^6 e^6 + 2310x^5 a b^5 e^6 + 462x^5 b^6 d e^5 + 4950x^4 a^2 b^4 e^6 + 1650x^4 a b^5 d e^5 + 330x^4 b^6 d^2 e^4 + 5775x^3 a^3 b^3 e^6 + 2475x^3 a^2 b^4 d e^5 + 12375x^3 a b^5 d^2 e^4 + 4725x^3 b^6 d^3 e^3 + 1575x^2 a^4 e^4 + 1575x^2 a^3 b d e^3 + 1575x^2 a^2 b^2 d^2 e^2 + 1575x^2 a b^3 d^3 e + 1575x^2 b^4 d^4)}{(ex+d)^{11}}$ |

input

```
int((b^2*x^2+2*a*b*x+a^2)^3/(e*x+d)^12,x,method=_RETURNVERBOSE)
```

output

```
(-1/5/e*b^6*x^6-1/5*b^5/e^2*(5*a*e+b*d)*x^5-1/7*b^4/e^3*(15*a^2*e^2+5*a*b*d*e+b^2*d^2)*x^4-1/14/e^4*b^3*(35*a^3*e^3+15*a^2*b*d*e^2+5*a*b^2*d^2*e+b^3*d^3)*x^3-1/42*b^2/e^5*(70*a^4*e^4+35*a^3*b*d*e^3+15*a^2*b^2*d^2*e^2+5*a*b^3*d^3*e+b^4*d^4)*x^2-1/210*b/e^6*(126*a^5*e^5+70*a^4*b*d*e^4+35*a^3*b^2*d^2*e^3+15*a^2*b^3*d^3*e^2+5*a*b^4*d^4*e+b^5*d^5)*x-1/2310/e^7*(210*a^6*e^6+126*a^5*b*d*e^5+70*a^4*b^2*d^2*e^4+35*a^3*b^3*d^3*e^3+15*a^2*b^4*d^4*e^2+5*a*b^5*d^5*e+b^6*d^6))/(e*x+d)^11
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 463 vs. $2(158) = 316$.

Time = 0.09 (sec) , antiderivative size = 463, normalized size of antiderivative = 2.72

$$\int \frac{(a^2 + 2abx + b^2x^2)^3}{(d + ex)^{12}} dx =$$

$$\frac{462b^6e^6x^6 + b^6d^6 + 5ab^5d^5e + 15a^2b^4d^4e^2 + 35a^3b^3d^3e^3 + 70a^4b^2d^2e^4 + 126a^5bde^5 + 210a^6e^6 + 462$$

input `integrate((b^2*x^2+2*a*b*x+a^2)^3/(e*x+d)^12,x, algorithm="fricas")`

output

```
-1/2310*(462*b^6*e^6*x^6 + b^6*d^6 + 5*a*b^5*d^5*e + 15*a^2*b^4*d^4*e^2 +
35*a^3*b^3*d^3*e^3 + 70*a^4*b^2*d^2*e^4 + 126*a^5*b*d*e^5 + 210*a^6*e^6 +
462*(b^6*d*e^5 + 5*a*b^5*e^6)*x^5 + 330*(b^6*d^2*e^4 + 5*a*b^5*d*e^5 + 15*
a^2*b^4*e^6)*x^4 + 165*(b^6*d^3*e^3 + 5*a*b^5*d^2*e^4 + 15*a^2*b^4*d*e^5 +
35*a^3*b^3*e^6)*x^3 + 55*(b^6*d^4*e^2 + 5*a*b^5*d^3*e^3 + 15*a^2*b^4*d^2*
e^4 + 35*a^3*b^3*d*e^5 + 70*a^4*b^2*e^6)*x^2 + 11*(b^6*d^5*e + 5*a*b^5*d^4*
e^2 + 15*a^2*b^4*d^3*e^3 + 35*a^3*b^3*d^2*e^4 + 70*a^4*b^2*d*e^5 + 126*a^
5*b*e^6)*x)/(e^18*x^11 + 11*d*e^17*x^10 + 55*d^2*e^16*x^9 + 165*d^3*e^15*x^
^8 + 330*d^4*e^14*x^7 + 462*d^5*e^13*x^6 + 462*d^6*e^12*x^5 + 330*d^7*e^11
*x^4 + 165*d^8*e^10*x^3 + 55*d^9*e^9*x^2 + 11*d^10*e^8*x + d^11*e^7)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(a^2 + 2abx + b^2x^2)^3}{(d + ex)^{12}} dx = \text{Timed out}$$

input `integrate((b**2*x**2+2*a*b*x+a**2)**3/(e*x+d)**12,x)`

output `Timed out`

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 463 vs. $2(158) = 316$.

Time = 0.05 (sec) , antiderivative size = 463, normalized size of antiderivative = 2.72

$$\int \frac{(a^2 + 2abx + b^2x^2)^3}{(d + ex)^{12}} dx =$$

$$462 b^6 e^6 x^6 + b^6 d^6 + 5 a b^5 d^5 e + 15 a^2 b^4 d^4 e^2 + 35 a^3 b^3 d^3 e^3 + 70 a^4 b^2 d^2 e^4 + 126 a^5 b d e^5 + 210 a^6 e^6 + 462$$

input `integrate((b^2*x^2+2*a*b*x+a^2)^3/(e*x+d)^12,x, algorithm="maxima")`

output

$$\begin{aligned} & -1/2310*(462*b^6*e^6*x^6 + b^6*d^6 + 5*a*b^5*d^5*e + 15*a^2*b^4*d^4*e^2 + \\ & 35*a^3*b^3*d^3*e^3 + 70*a^4*b^2*d^2*e^4 + 126*a^5*b*d*e^5 + 210*a^6*e^6 + \\ & 462*(b^6*d*e^5 + 5*a*b^5*e^6)*x^5 + 330*(b^6*d^2*e^4 + 5*a*b^5*d*e^5 + 15* \\ & a^2*b^4*e^6)*x^4 + 165*(b^6*d^3*e^3 + 5*a*b^5*d^2*e^4 + 15*a^2*b^4*d*e^5 + \\ & 35*a^3*b^3*e^6)*x^3 + 55*(b^6*d^4*e^2 + 5*a*b^5*d^3*e^3 + 15*a^2*b^4*d^2* \\ & e^4 + 35*a^3*b^3*d*e^5 + 70*a^4*b^2*e^6)*x^2 + 11*(b^6*d^5*e + 5*a*b^5*d^4 \\ & *e^2 + 15*a^2*b^4*d^3*e^3 + 35*a^3*b^3*d^2*e^4 + 70*a^4*b^2*d*e^5 + 126*a^ \\ & 5*b*e^6)*x)/(e^18*x^11 + 11*d*e^17*x^10 + 55*d^2*e^16*x^9 + 165*d^3*e^15*x \\ & ^8 + 330*d^4*e^14*x^7 + 462*d^5*e^13*x^6 + 462*d^6*e^12*x^5 + 330*d^7*e^11 \\ & *x^4 + 165*d^8*e^10*x^3 + 55*d^9*e^9*x^2 + 11*d^10*e^8*x + d^11*e^7) \end{aligned}$$
Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 375 vs. $2(158) = 316$.

Time = 0.19 (sec) , antiderivative size = 375, normalized size of antiderivative = 2.21

$$\int \frac{(a^2 + 2abx + b^2x^2)^3}{(d + ex)^{12}} dx =$$

$$462 b^6 e^6 x^6 + 462 b^6 d e^5 x^5 + 2310 a b^5 e^6 x^5 + 330 b^6 d^2 e^4 x^4 + 1650 a b^5 d e^5 x^4 + 4950 a^2 b^4 e^6 x^4 + 165 b^6 d^3 e^6$$

input `integrate((b^2*x^2+2*a*b*x+a^2)^3/(e*x+d)^12,x, algorithm="giac")`

output

```
-1/2310*(462*b^6*e^6*x^6 + 462*b^6*d*e^5*x^5 + 2310*a*b^5*e^6*x^5 + 330*b^6*d^2*e^4*x^4 + 1650*a*b^5*d*e^5*x^4 + 4950*a^2*b^4*e^6*x^4 + 165*b^6*d^3*e^3*x^3 + 825*a*b^5*d^2*e^4*x^3 + 2475*a^2*b^4*d*e^5*x^3 + 5775*a^3*b^3*e^6*x^3 + 55*b^6*d^4*e^2*x^2 + 275*a*b^5*d^3*e^3*x^2 + 825*a^2*b^4*d^2*e^4*x^2 + 1925*a^3*b^3*d*e^5*x^2 + 3850*a^4*b^2*e^6*x^2 + 11*b^6*d^5*e*x + 55*a*b^5*d^4*e^2*x + 165*a^2*b^4*d^3*e^3*x + 385*a^3*b^3*d^2*e^4*x + 770*a^4*b^2*d*e^5*x + 1386*a^5*b*e^6*x + b^6*d^6 + 5*a*b^5*d^5*e + 15*a^2*b^4*d^4*e^2 + 35*a^3*b^3*d^3*e^3 + 70*a^4*b^2*d^2*e^4 + 126*a^5*b*d*e^5 + 210*a^6*e^6)/(e*x + d)^11*e^7)
```

Mupad [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 445, normalized size of antiderivative = 2.62

$$\int \frac{(a^2 + 2abx + b^2x^2)^3}{(d + ex)^{12}} dx =$$

$$-\frac{210a^6e^6 + 126a^5bde^5 + 70a^4b^2d^2e^4 + 35a^3b^3d^3e^3 + 15a^2b^4d^4e^2 + 5ab^5d^5e + b^6d^6}{2310e^7} + \frac{b^6x^6}{5e} + \frac{b^3x^3(35a^3e^3 + 15a^2bde^2 + 5ab^2d^2e)}{14e^4} + \frac{d^{11} + 11d^{10}ex + 55d^9e^2x^2 + 165d^8e^3x^3 + 330d^7e^4x^4 + 462d^6e^5x^5 + 462d^5e^6x^6 + 330d^4e^7x^7 + 165d^3e^8x^8 + 55d^2e^9x^9 + 11d^{10}ex}{(d + ex)^{12}}$$

input

```
int((a^2 + b^2*x^2 + 2*a*b*x)^3/(d + e*x)^12,x)
```

output

```
-((210*a^6*e^6 + b^6*d^6 + 15*a^2*b^4*d^4*e^2 + 35*a^3*b^3*d^3*e^3 + 70*a^4*b^2*d^2*e^4 + 5*a*b^5*d^5*e + 126*a^5*b*d*e^5)/(2310*e^7) + (b^6*x^6)/(5*e) + (b^3*x^3*(35*a^3*e^3 + b^3*d^3 + 5*a*b^2*d^2*e + 15*a^2*b*d*e^2))/(14*e^4) + (b*x*(126*a^5*e^5 + b^5*d^5 + 15*a^2*b^3*d^3*e^2 + 35*a^3*b^2*d^2*e^3 + 5*a*b^4*d^4*e + 70*a^4*b*d*e^4))/(210*e^6) + (b^5*x^5*(5*a*e + b*d))/(5*e^2) + (b^2*x^2*(70*a^4*e^4 + b^4*d^4 + 15*a^2*b^2*d^2*e^2 + 5*a*b^3*d^3*e + 35*a^3*b*d*e^3))/(42*e^5) + (b^4*x^4*(15*a^2*e^2 + b^2*d^2 + 5*a*b*d*e))/(7*e^3))/(d^11 + e^11*x^11 + 11*d*e^10*x^10 + 55*d^9*e^2*x^2 + 165*d^8*e^3*x^3 + 330*d^7*e^4*x^4 + 462*d^6*e^5*x^5 + 462*d^5*e^6*x^6 + 330*d^4*e^7*x^7 + 165*d^3*e^8*x^8 + 55*d^2*e^9*x^9 + 11*d^10*e*x)
```


Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 486, normalized size of antiderivative = 2.86

$$\int \frac{(a^2 + 2abx + b^2x^2)^3}{(d + ex)^{12}} dx$$

$$= \frac{-462b^6e^6x^6 - 2310ab^5e^6x^5 - 462b^6de^5x^5 - 4950a^2b^4e^6x^4 - 1650ab^5de^5x^4 - 330b^6d^2e^4x^4 - 5775a^3b^3e^6}{(d + ex)^{11}}$$

input `int((b^2*x^2+2*a*b*x+a^2)^3/(e*x+d)^12,x)`

output

```
( - 210*a**6*e**6 - 126*a**5*b*d*e**5 - 1386*a**5*b*e**6*x - 70*a**4*b**2*d**2*e**4 - 770*a**4*b**2*d*e**5*x - 3850*a**4*b**2*e**6*x**2 - 35*a**3*b**3*d**3*e**3 - 385*a**3*b**3*d**2*e**4*x - 1925*a**3*b**3*d*e**5*x**2 - 5775*a**3*b**3*e**6*x**3 - 15*a**2*b**4*d**4*e**2 - 165*a**2*b**4*d**3*e**3*x - 825*a**2*b**4*d**2*e**4*x**2 - 2475*a**2*b**4*d*e**5*x**3 - 4950*a**2*b**4*e**6*x**4 - 5*a*b**5*d**5*e - 55*a*b**5*d**4*e**2*x - 275*a*b**5*d**3*e**3*x**2 - 825*a*b**5*d**2*e**4*x**3 - 1650*a*b**5*d*e**5*x**4 - 2310*a*b**5*e**6*x**5 - b**6*d**6 - 11*b**6*d**5*e*x - 55*b**6*d**4*e**2*x**2 - 165*b**6*d**3*e**3*x**3 - 330*b**6*d**2*e**4*x**4 - 462*b**6*d*e**5*x**5 - 462*b**6*e**6*x**6)/(2310*e**7*(d**11 + 11*d**10*e*x + 55*d**9*e**2*x**2 + 165*d**8*e**3*x**3 + 330*d**7*e**4*x**4 + 462*d**6*e**5*x**5 + 462*d**5*e**6*x**6 + 330*d**4*e**7*x**7 + 165*d**3*e**8*x**8 + 55*d**2*e**9*x**9 + 11*d*e**10*x**10 + e**11*x**11))
```

3.105 $\int \frac{(a^2+2abx+b^2x^2)^3}{(d+ex)^{13}} dx$

| | |
|---|-----|
| Optimal result | 861 |
| Mathematica [A] (verified) | 862 |
| Rubi [A] (verified) | 862 |
| Maple [B] (verified) | 864 |
| Fricas [B] (verification not implemented) | 865 |
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| Giac [B] (verification not implemented) | 866 |
| Mupad [B] (verification not implemented) | 867 |
| Reduce [B] (verification not implemented) | 868 |

Optimal result

Integrand size = 26, antiderivative size = 173

$$\int \frac{(a^2 + 2abx + b^2x^2)^3}{(d + ex)^{13}} dx = -\frac{(bd - ae)^6}{12e^7(d + ex)^{12}} + \frac{6b(bd - ae)^5}{11e^7(d + ex)^{11}} - \frac{3b^2(bd - ae)^4}{2e^7(d + ex)^{10}} + \frac{20b^3(bd - ae)^3}{9e^7(d + ex)^9} - \frac{15b^4(bd - ae)^2}{8e^7(d + ex)^8} + \frac{6b^5(bd - ae)}{7e^7(d + ex)^7} - \frac{b^6}{6e^7(d + ex)^6}$$

output

```
-1/12*(-a*e+b*d)^6/e^7/(e*x+d)^12+6/11*b*(-a*e+b*d)^5/e^7/(e*x+d)^11-3/2*b^2*(-a*e+b*d)^4/e^7/(e*x+d)^10+20/9*b^3*(-a*e+b*d)^3/e^7/(e*x+d)^9-15/8*b^4*(-a*e+b*d)^2/e^7/(e*x+d)^8+6/7*b^5*(-a*e+b*d)/e^7/(e*x+d)^7-1/6*b^6/e^7/(e*x+d)^6
```

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 277, normalized size of antiderivative = 1.60

$$\int \frac{(a^2 + 2abx + b^2x^2)^3}{(d + ex)^{13}} dx = \frac{462a^6e^6 + 252a^5be^5(d + 12ex) + 126a^4b^2e^4(d^2 + 12dex + 66e^2x^2) + 56a^3b^3e^3(d^3 + 12d^2ex + 66de^2x^2)}{\dots}$$

input

```
Integrate[(a^2 + 2*a*b*x + b^2*x^2)^3/(d + e*x)^13,x]
```

output

```
-1/5544*(462*a^6*e^6 + 252*a^5*b*e^5*(d + 12*e*x) + 126*a^4*b^2*e^4*(d^2 + 12*d*e*x + 66*e^2*x^2) + 56*a^3*b^3*e^3*(d^3 + 12*d^2*e*x + 66*d*e^2*x^2 + 220*e^3*x^3) + 21*a^2*b^4*e^2*(d^4 + 12*d^3*e*x + 66*d^2*e^2*x^2 + 220*d*e^3*x^3 + 495*e^4*x^4) + 6*a*b^5*e*(d^5 + 12*d^4*e*x + 66*d^3*e^2*x^2 + 220*d^2*e^3*x^3 + 495*d*e^4*x^4 + 792*e^5*x^5) + b^6*(d^6 + 12*d^5*e*x + 66*d^4*e^2*x^2 + 220*d^3*e^3*x^3 + 495*d^2*e^4*x^4 + 792*d*e^5*x^5 + 924*e^6*x^6))/(e^7*(d + e*x)^12)
```

Rubi [A] (verified)

Time = 0.61 (sec) , antiderivative size = 173, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {1098, 27, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a^2 + 2abx + b^2x^2)^3}{(d + ex)^{13}} dx$$

↓ 1098

$$\frac{\int \frac{b^6(a+bx)^6}{(d+ex)^{13}} dx}{b^6}$$

↓ 27

$$\int \frac{(a + bx)^6}{(d + ex)^{13}} dx$$

↓ 53

$$\int \left(-\frac{6b^5(bd - ae)}{e^6(d + ex)^8} + \frac{15b^4(bd - ae)^2}{e^6(d + ex)^9} - \frac{20b^3(bd - ae)^3}{e^6(d + ex)^{10}} + \frac{15b^2(bd - ae)^4}{e^6(d + ex)^{11}} - \frac{6b(bd - ae)^5}{e^6(d + ex)^{12}} + \frac{(ae - bd)^6}{e^6(d + ex)^{13}} + \frac{1}{e^6(d + ex)^{13}} \right) dx$$

↓ 2009

$$\frac{6b^5(bd - ae)}{7e^7(d + ex)^7} - \frac{15b^4(bd - ae)^2}{8e^7(d + ex)^8} + \frac{20b^3(bd - ae)^3}{9e^7(d + ex)^9} - \frac{3b^2(bd - ae)^4}{2e^7(d + ex)^{10}} + \frac{6b(bd - ae)^5}{11e^7(d + ex)^{11}} - \frac{(bd - ae)^6}{12e^7(d + ex)^{12}} - \frac{b^6}{6e^7(d + ex)^6}$$

input `Int[(a^2 + 2*a*b*x + b^2*x^2)^3/(d + e*x)^13,x]`

output `-1/12*(b*d - a*e)^6/(e^7*(d + e*x)^12) + (6*b*(b*d - a*e)^5)/(11*e^7*(d + e*x)^11) - (3*b^2*(b*d - a*e)^4)/(2*e^7*(d + e*x)^10) + (20*b^3*(b*d - a*e)^3)/(9*e^7*(d + e*x)^9) - (15*b^4*(b*d - a*e)^2)/(8*e^7*(d + e*x)^8) + (6*b^5*(b*d - a*e))/(7*e^7*(d + e*x)^7) - b^6/(6*e^7*(d + e*x)^6)`

Defintions of rubi rules used

rule 27 `Int[(a_)*(F_x_), x_Symbol] := Simp[a Int[F_x, x], x] /; FreeQ[a, x] && !MatchQ[F_x, (b_)*(G_x_)] /; FreeQ[b, x]`

rule 53 `Int[((a_) + (b_)*(x_))^(m_)*((c_) + (d_)*(x_))^(n_), x_Symbol] := Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && IGtQ[m, 0] && (!IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])`

rule 1098 `Int[((d_) + (e_)*(x_))^(m_)*((a_) + (b_)*(x_) + (c_)*(x_)^2)^(p_), x_Symbol] := Simp[1/c^p Int[(d + e*x)^m*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]`

rule 2009

```
Int[u_, x_Symbol] := Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 334 vs. 2(159) = 318.

Time = 1.06 (sec) , antiderivative size = 335, normalized size of antiderivative = 1.94

| method | result |
|---------------|--|
| risch | $-\frac{b^6 x^6}{6e} - \frac{b^5(6ae+bd)x^5}{7e^2} - \frac{5b^4(21e^2 a^2+6abde+b^2 d^2)x^4}{56e^3} - \frac{5b^3(56e^3 a^3+21a^2 b d e^2+6a b^2 d^2 e+b^3 d^3)x^3}{126e^4} - \frac{b^2(126a^4 e^4+56a^3 b d e^3+21a^2 b^2 d^2 e^2+6a b^3 d^3 e+b^4 d^4)x^2}{84e^5}$ |
| default | $-\frac{6b(e^5 a^5-5a^4 b d e^4+10a^3 b^2 d^2 e^3-10a^2 b^3 d^3 e^2+5a b^4 d^4 e-b^5 d^5)}{11e^7(ex+d)^{11}} - \frac{a^6 e^6-6a^5 b d e^5+15a^4 b^2 d^2 e^4-20a^3 b^3 d^3 e^3+15a^2 b^4 d^4 e^2-6a b^5 d^5 e+b^6 d^6}{12e^7(ex+d)^{12}}$ |
| norman | $-\frac{b^6 x^6}{6e} - \frac{(6a b^5 e^6+b^6 d e^5)x^5}{7e^7} - \frac{5(21a^2 b^4 e^7+6a b^5 d e^6+b^6 d^2 e^5)x^4}{56e^8} - \frac{5(56a^3 b^3 e^8+21a^2 b^4 d e^7+6a b^5 d^2 e^6+b^6 d^3 e^5)x^3}{126e^9} - \frac{(126a^4 b^2 e^9+21a^3 b^3 d e^8+6a^2 b^4 d^2 e^7+6a b^5 d^3 e^6+b^6 d^4 e^5)x^2}{84e^{10}}$ |
| gospers | $-\frac{924x^6 b^6 e^6+4752x^5 a b^5 e^6+792x^5 b^6 d e^5+10395x^4 a^2 b^4 e^6+2970x^4 a b^5 d e^5+495x^4 b^6 d^2 e^4+12320x^3 a^3 b^3 e^6+4620x^3 a^2 b^4 d e^5-924b^6 x^6 e^{11}-4752a b^5 e^{11} x^5-792b^6 d e^{10} x^5-10395a^2 b^4 e^{11} x^4-2970a b^5 d e^{10} x^4-495b^6 d^2 e^9 x^4-12320a^3 b^3 e^{11} x^3-4620a^2 b^4 d e^5}{(924x^6 b^6 e^6+4752x^5 a b^5 e^6+792x^5 b^6 d e^5+10395x^4 a^2 b^4 e^6+2970x^4 a b^5 d e^5+495x^4 b^6 d^2 e^4+12320x^3 a^3 b^3 e^6+4620x^3 a^2 b^4 d e^5-924b^6 x^6 e^{11}-4752a b^5 e^{11} x^5-792b^6 d e^{10} x^5-10395a^2 b^4 e^{11} x^4-2970a b^5 d e^{10} x^4-495b^6 d^2 e^9 x^4-12320a^3 b^3 e^{11} x^3-4620a^2 b^4 d e^5)}{12e^7(ex+d)^{12}}$ |
| parallelrisch | $-\frac{924x^6 b^6 e^6+4752x^5 a b^5 e^6+792x^5 b^6 d e^5+10395x^4 a^2 b^4 e^6+2970x^4 a b^5 d e^5+495x^4 b^6 d^2 e^4+12320x^3 a^3 b^3 e^6+4620x^3 a^2 b^4 d e^5-924b^6 x^6 e^{11}-4752a b^5 e^{11} x^5-792b^6 d e^{10} x^5-10395a^2 b^4 e^{11} x^4-2970a b^5 d e^{10} x^4-495b^6 d^2 e^9 x^4-12320a^3 b^3 e^{11} x^3-4620a^2 b^4 d e^5}{(924x^6 b^6 e^6+4752x^5 a b^5 e^6+792x^5 b^6 d e^5+10395x^4 a^2 b^4 e^6+2970x^4 a b^5 d e^5+495x^4 b^6 d^2 e^4+12320x^3 a^3 b^3 e^6+4620x^3 a^2 b^4 d e^5-924b^6 x^6 e^{11}-4752a b^5 e^{11} x^5-792b^6 d e^{10} x^5-10395a^2 b^4 e^{11} x^4-2970a b^5 d e^{10} x^4-495b^6 d^2 e^9 x^4-12320a^3 b^3 e^{11} x^3-4620a^2 b^4 d e^5)}$ |
| orering | $-\frac{(924x^6 b^6 e^6+4752x^5 a b^5 e^6+792x^5 b^6 d e^5+10395x^4 a^2 b^4 e^6+2970x^4 a b^5 d e^5+495x^4 b^6 d^2 e^4+12320x^3 a^3 b^3 e^6+4620x^3 a^2 b^4 d e^5-924b^6 x^6 e^{11}-4752a b^5 e^{11} x^5-792b^6 d e^{10} x^5-10395a^2 b^4 e^{11} x^4-2970a b^5 d e^{10} x^4-495b^6 d^2 e^9 x^4-12320a^3 b^3 e^{11} x^3-4620a^2 b^4 d e^5)}{(924x^6 b^6 e^6+4752x^5 a b^5 e^6+792x^5 b^6 d e^5+10395x^4 a^2 b^4 e^6+2970x^4 a b^5 d e^5+495x^4 b^6 d^2 e^4+12320x^3 a^3 b^3 e^6+4620x^3 a^2 b^4 d e^5-924b^6 x^6 e^{11}-4752a b^5 e^{11} x^5-792b^6 d e^{10} x^5-10395a^2 b^4 e^{11} x^4-2970a b^5 d e^{10} x^4-495b^6 d^2 e^9 x^4-12320a^3 b^3 e^{11} x^3-4620a^2 b^4 d e^5)}$ |

input

```
int((b^2*x^2+2*a*b*x+a^2)^3/(e*x+d)^13,x,method=_RETURNVERBOSE)
```

output

```
(-1/6/e*b^6*x^6-1/7*b^5/e^2*(6*a*e+b*d)*x^5-5/56*b^4/e^3*(21*a^2*e^2+6*a*b*d*e+b^2*d^2)*x^4-5/126*b^3/e^4*(56*a^3*e^3+21*a^2*b*d*e^2+6*a*b^2*d^2*e+b^3*d^3)*x^3-1/84*b^2/e^5*(126*a^4*e^4+56*a^3*b*d*e^3+21*a^2*b^2*d^2*e^2+6*a*b^3*d^3*e+b^4*d^4)*x^2-1/462*b/e^6*(252*a^5*e^5+126*a^4*b*d*e^4+56*a^3*b^2*d^2*e^3+21*a^2*b^3*d^3*e^2+6*a*b^4*d^4*e+b^5*d^5)*x-1/5544/e^7*(462*a^6*e^6+252*a^5*b*d*e^5+126*a^4*b^2*d^2*e^4+56*a^3*b^3*d^3*e^3+21*a^2*b^4*d^4*e^2+6*a*b^5*d^5*e+b^6*d^6))/(e*x+d)^12
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 474 vs. $2(159) = 318$.

Time = 0.09 (sec) , antiderivative size = 474, normalized size of antiderivative = 2.74

$$\int \frac{(a^2 + 2abx + b^2x^2)^3}{(d + ex)^{13}} dx = \frac{924b^6e^6x^6 + b^6d^6 + 6ab^5d^5e + 21a^2b^4d^4e^2 + 56a^3b^3d^3e^3 + 126a^4b^2d^2e^4 + 252a^5bde^5 + 462a^6e^6 + 792a^7e^7}{5544}$$

input `integrate((b^2*x^2+2*a*b*x+a^2)^3/(e*x+d)^13,x, algorithm="fricas")`

output `-1/5544*(924*b^6*e^6*x^6 + b^6*d^6 + 6*a*b^5*d^5*e + 21*a^2*b^4*d^4*e^2 + 56*a^3*b^3*d^3*e^3 + 126*a^4*b^2*d^2*e^4 + 252*a^5*b*d*e^5 + 462*a^6*e^6 + 792*(b^6*d*e^5 + 6*a*b^5*e^6)*x^5 + 495*(b^6*d^2*e^4 + 6*a*b^5*d*e^5 + 21*a^2*b^4*e^6)*x^4 + 220*(b^6*d^3*e^3 + 6*a*b^5*d^2*e^4 + 21*a^2*b^4*d*e^5 + 56*a^3*b^3*e^6)*x^3 + 66*(b^6*d^4*e^2 + 6*a*b^5*d^3*e^3 + 21*a^2*b^4*d^2*e^4 + 56*a^3*b^3*d*e^5 + 126*a^4*b^2*e^6)*x^2 + 12*(b^6*d^5*e + 6*a*b^5*d^4*e^2 + 21*a^2*b^4*d^3*e^3 + 56*a^3*b^3*d^2*e^4 + 126*a^4*b^2*d*e^5 + 252*a^5*b*d*e^6)*x)/(e^19*x^12 + 12*d*e^18*x^11 + 66*d^2*e^17*x^10 + 220*d^3*e^16*x^9 + 495*d^4*e^15*x^8 + 792*d^5*e^14*x^7 + 924*d^6*e^13*x^6 + 792*d^7*e^12*x^5 + 495*d^8*e^11*x^4 + 220*d^9*e^10*x^3 + 66*d^10*e^9*x^2 + 12*d^11*e^8*x + d^12*e^7)`

Sympy [F(-1)]

Timed out.

$$\int \frac{(a^2 + 2abx + b^2x^2)^3}{(d + ex)^{13}} dx = \text{Timed out}$$

input `integrate((b**2*x**2+2*a*b*x+a**2)**3/(e*x+d)**13,x)`

output `Timed out`

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 474 vs. $2(159) = 318$.

Time = 0.06 (sec) , antiderivative size = 474, normalized size of antiderivative = 2.74

$$\int \frac{(a^2 + 2abx + b^2x^2)^3}{(d + ex)^{13}} dx = \frac{924b^6e^6x^6 + b^6d^6 + 6ab^5d^5e + 21a^2b^4d^4e^2 + 56a^3b^3d^3e^3 + 126a^4b^2d^2e^4 + 252a^5bde^5 + 462a^6e^6 + 792a^7d^7e^7}{5544}$$

input `integrate((b^2*x^2+2*a*b*x+a^2)^3/(e*x+d)^13,x, algorithm="maxima")`

output `-1/5544*(924*b^6*e^6*x^6 + b^6*d^6 + 6*a*b^5*d^5*e + 21*a^2*b^4*d^4*e^2 + 56*a^3*b^3*d^3*e^3 + 126*a^4*b^2*d^2*e^4 + 252*a^5*b*d*e^5 + 462*a^6*e^6 + 792*(b^6*d*e^5 + 6*a*b^5*e^6)*x^5 + 495*(b^6*d^2*e^4 + 6*a*b^5*d*e^5 + 21*a^2*b^4*e^6)*x^4 + 220*(b^6*d^3*e^3 + 6*a*b^5*d^2*e^4 + 21*a^2*b^4*d*e^5 + 56*a^3*b^3*e^6)*x^3 + 66*(b^6*d^4*e^2 + 6*a*b^5*d^3*e^3 + 21*a^2*b^4*d^2*e^4 + 56*a^3*b^3*d*e^5 + 126*a^4*b^2*e^6)*x^2 + 12*(b^6*d^5*e + 6*a*b^5*d^4*e^2 + 21*a^2*b^4*d^3*e^3 + 56*a^3*b^3*d^2*e^4 + 126*a^4*b^2*d*e^5 + 252*a^5*b*d*e^6)*x)/(e^19*x^12 + 12*d*e^18*x^11 + 66*d^2*e^17*x^10 + 220*d^3*e^16*x^9 + 495*d^4*e^15*x^8 + 792*d^5*e^14*x^7 + 924*d^6*e^13*x^6 + 792*d^7*e^12*x^5 + 495*d^8*e^11*x^4 + 220*d^9*e^10*x^3 + 66*d^10*e^9*x^2 + 12*d^11*e^8*x + d^12*e^7)`

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 375 vs. $2(159) = 318$.

Time = 0.20 (sec) , antiderivative size = 375, normalized size of antiderivative = 2.17

$$\int \frac{(a^2 + 2abx + b^2x^2)^3}{(d + ex)^{13}} dx = \frac{924b^6e^6x^6 + 792b^6de^5x^5 + 4752ab^5e^6x^5 + 495b^6d^2e^4x^4 + 2970ab^5de^5x^4 + 10395a^2b^4e^6x^4 + 220b^6d^3e^7}{5544}$$

input `integrate((b^2*x^2+2*a*b*x+a^2)^3/(e*x+d)^13,x, algorithm="giac")`

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 497, normalized size of antiderivative = 2.87

$$\int \frac{(a^2 + 2abx + b^2x^2)^3}{(d + ex)^{13}} dx$$

$$= \frac{-924b^6e^6x^6 - 4752ab^5e^6x^5 - 792b^6de^5x^5 - 10395a^2b^4e^6x^4 - 2970ab^5de^5x^4 - 495b^6d^2e^4x^4 - 12320a^3b^3d^2e^3x^3 - 21a^2b^4d^2e^2x^3 - 252a^2b^4d^3e^3x^3 - 1386a^2b^4d^2e^4x^2 - 4620a^2b^4de^5x^3 - 10395a^2b^4e^6x^4 - 6ab^5d^5e - 72ab^5d^4e^2x - 396ab^5d^3e^3x^2 - 1320ab^5d^2e^4x^3 - 2970ab^5de^5x^4 - 4752ab^5e^6x^5 - b^6d^6 - 12b^6d^5ex - 66b^6d^4e^2x^2 - 220b^6d^3e^3x^3 - 495b^6d^2e^4x^4 - 792b^6de^5x^5 - 924b^6e^6x^6}{(5544e^7(d^{12} + 12d^{11}ex + 66d^{10}e^2x^2 + 220d^9e^3x^3 + 495d^8e^4x^4 + 792d^7e^5x^5 + 924d^6e^6x^6 + 792d^5e^7x^7 + 495d^4e^8x^8 + 220d^3e^9x^9 + 66d^2e^{10}x^{10} + 12de^{11}x^{11} + e^{12}x^{12}))}$$

input `int((b^2*x^2+2*a*b*x+a^2)^3/(e*x+d)^13,x)`output `(- 462*a**6*e**6 - 252*a**5*b*d*e**5 - 3024*a**5*b*e**6*x - 126*a**4*b**2*d**2*e**4 - 1512*a**4*b**2*d*e**5*x - 8316*a**4*b**2*e**6*x**2 - 56*a**3*b**3*d**3*e**3 - 672*a**3*b**3*d**2*e**4*x - 3696*a**3*b**3*d*e**5*x**2 - 12320*a**3*b**3*e**6*x**3 - 21*a**2*b**4*d**4*e**2 - 252*a**2*b**4*d**3*e**3*x - 1386*a**2*b**4*d**2*e**4*x**2 - 4620*a**2*b**4*d*e**5*x**3 - 10395*a**2*b**4*e**6*x**4 - 6*a*b**5*d**5*e - 72*a*b**5*d**4*e**2*x - 396*a*b**5*d**3*e**3*x**2 - 1320*a*b**5*d**2*e**4*x**3 - 2970*a*b**5*d*e**5*x**4 - 4752*a*b**5*e**6*x**5 - b**6*d**6 - 12*b**6*d**5*e*x - 66*b**6*d**4*e**2*x**2 - 220*b**6*d**3*e**3*x**3 - 495*b**6*d**2*e**4*x**4 - 792*b**6*d*e**5*x**5 - 924*b**6*e**6*x**6)/(5544*e**7*(d**12 + 12*d**11*e*x + 66*d**10*e**2*x**2 + 220*d**9*e**3*x**3 + 495*d**8*e**4*x**4 + 792*d**7*e**5*x**5 + 924*d**6*e**6*x**6 + 792*d**5*e**7*x**7 + 495*d**4*e**8*x**8 + 220*d**3*e**9*x**9 + 66*d**2*e**10*x**10 + 12*d*e**11*x**11 + e**12*x**12))`

3.106 $\int \frac{(a^2+2abx+b^2x^2)^3}{(d+ex)^{14}} dx$

| | |
|---|-----|
| Optimal result | 869 |
| Mathematica [A] (verified) | 870 |
| Rubi [A] (verified) | 870 |
| Maple [B] (verified) | 872 |
| Fricas [B] (verification not implemented) | 873 |
| Sympy [F(-1)] | 873 |
| Maxima [B] (verification not implemented) | 874 |
| Giac [B] (verification not implemented) | 874 |
| Mupad [B] (verification not implemented) | 875 |
| Reduce [B] (verification not implemented) | 876 |

Optimal result

Integrand size = 26, antiderivative size = 171

$$\int \frac{(a^2 + 2abx + b^2x^2)^3}{(d + ex)^{14}} dx = -\frac{(bd - ae)^6}{13e^7(d + ex)^{13}} + \frac{b(bd - ae)^5}{2e^7(d + ex)^{12}} - \frac{15b^2(bd - ae)^4}{11e^7(d + ex)^{11}} + \frac{2b^3(bd - ae)^3}{e^7(d + ex)^{10}} - \frac{5b^4(bd - ae)^2}{3e^7(d + ex)^9} + \frac{3b^5(bd - ae)}{4e^7(d + ex)^8} - \frac{b^6}{7e^7(d + ex)^7}$$

output

```
-1/13*(-a*e+b*d)^6/e^7/(e*x+d)^13+1/2*b*(-a*e+b*d)^5/e^7/(e*x+d)^12-15/11*
b^2*(-a*e+b*d)^4/e^7/(e*x+d)^11+2*b^3*(-a*e+b*d)^3/e^7/(e*x+d)^10-5/3*b^4*
(-a*e+b*d)^2/e^7/(e*x+d)^9+3/4*b^5*(-a*e+b*d)/e^7/(e*x+d)^8-1/7*b^6/e^7/(e
*x+d)^7
```

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 277, normalized size of antiderivative = 1.62

$$\int \frac{(a^2 + 2abx + b^2x^2)^3}{(d + ex)^{14}} dx = \frac{924a^6e^6 + 462a^5be^5(d + 13ex) + 210a^4b^2e^4(d^2 + 13dex + 78e^2x^2) + 84a^3b^3e^3(d^3 + 13d^2ex + 78de^2x^2)}{\dots}$$

input

```
Integrate[(a^2 + 2*a*b*x + b^2*x^2)^3/(d + e*x)^14,x]
```

output

```
-1/12012*(924*a^6*e^6 + 462*a^5*b*e^5*(d + 13*e*x) + 210*a^4*b^2*e^4*(d^2 + 13*d*e*x + 78*e^2*x^2) + 84*a^3*b^3*e^3*(d^3 + 13*d^2*e*x + 78*d*e^2*x^2 + 286*e^3*x^3) + 28*a^2*b^4*e^2*(d^4 + 13*d^3*e*x + 78*d^2*e^2*x^2 + 286*d*e^3*x^3 + 715*e^4*x^4) + 7*a*b^5*e*(d^5 + 13*d^4*e*x + 78*d^3*e^2*x^2 + 286*d^2*e^3*x^3 + 715*d*e^4*x^4 + 1287*e^5*x^5) + b^6*(d^6 + 13*d^5*e*x + 78*d^4*e^2*x^2 + 286*d^3*e^3*x^3 + 715*d^2*e^4*x^4 + 1287*d*e^5*x^5 + 1716*e^6*x^6))/(e^7*(d + e*x)^13)
```

Rubi [A] (verified)

Time = 0.62 (sec) , antiderivative size = 171, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {1098, 27, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a^2 + 2abx + b^2x^2)^3}{(d + ex)^{14}} dx$$

↓ 1098

$$\frac{\int \frac{b^6(a+bx)^6}{(d+ex)^{14}} dx}{b^6}$$

↓ 27

$$\int \frac{(a + bx)^6}{(d + ex)^{14}} dx$$

↓ 53

$$\int \left(-\frac{6b^5(bd - ae)}{e^6(d + ex)^9} + \frac{15b^4(bd - ae)^2}{e^6(d + ex)^{10}} - \frac{20b^3(bd - ae)^3}{e^6(d + ex)^{11}} + \frac{15b^2(bd - ae)^4}{e^6(d + ex)^{12}} - \frac{6b(bd - ae)^5}{e^6(d + ex)^{13}} + \frac{(ae - bd)^6}{e^6(d + ex)^{14}} + \frac{1}{e^6(d + ex)^{14}} \right) dx$$

↓ 2009

$$\frac{3b^5(bd - ae)}{4e^7(d + ex)^8} - \frac{5b^4(bd - ae)^2}{3e^7(d + ex)^9} + \frac{2b^3(bd - ae)^3}{e^7(d + ex)^{10}} - \frac{15b^2(bd - ae)^4}{11e^7(d + ex)^{11}} + \frac{b(bd - ae)^5}{2e^7(d + ex)^{12}} - \frac{(bd - ae)^6}{13e^7(d + ex)^{13}} - \frac{b^6}{7e^7(d + ex)^7}$$

input `Int[(a^2 + 2*a*b*x + b^2*x^2)^3/(d + e*x)^14,x]`

output `-1/13*(b*d - a*e)^6/(e^7*(d + e*x)^13) + (b*(b*d - a*e)^5)/(2*e^7*(d + e*x)^12) - (15*b^2*(b*d - a*e)^4)/(11*e^7*(d + e*x)^11) + (2*b^3*(b*d - a*e)^3)/(e^7*(d + e*x)^10) - (5*b^4*(b*d - a*e)^2)/(3*e^7*(d + e*x)^9) + (3*b^5*(b*d - a*e))/(4*e^7*(d + e*x)^8) - b^6/(7*e^7*(d + e*x)^7)`

Defintions of rubi rules used

rule 27 `Int[(a_)*(Fx_), x_Symbol] := Simp[a Int[Fx, x], x] /; FreeQ[a, x] && !MatchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]`

rule 53 `Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] := Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && IGtQ[m, 0] && (!IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])`

rule 1098 `Int[((d_.) + (e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] := Simp[1/c^p Int[(d + e*x)^m*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]`

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 485 vs. $2(159) = 318$.

Time = 0.08 (sec) , antiderivative size = 485, normalized size of antiderivative = 2.84

$$\int \frac{(a^2 + 2abx + b^2x^2)^3}{(d + ex)^{14}} dx = \frac{1716b^6e^6x^6 + b^6d^6 + 7ab^5d^5e + 28a^2b^4d^4e^2 + 84a^3b^3d^3e^3 + 210a^4b^2d^2e^4 + 462a^5bde^5 + 924a^6e^6 + 12012(e^{20}x^{13} + \dots)}{12012(e^{20}x^{13} + \dots)}$$

input `integrate((b^2*x^2+2*a*b*x+a^2)^3/(e*x+d)^14,x, algorithm="fricas")`

output `-1/12012*(1716*b^6*e^6*x^6 + b^6*d^6 + 7*a*b^5*d^5*e + 28*a^2*b^4*d^4*e^2 + 84*a^3*b^3*d^3*e^3 + 210*a^4*b^2*d^2*e^4 + 462*a^5*b*d*e^5 + 924*a^6*e^6 + 1287*(b^6*d*e^5 + 7*a*b^5*e^6)*x^5 + 715*(b^6*d^2*e^4 + 7*a*b^5*d*e^5 + 28*a^2*b^4*e^6)*x^4 + 286*(b^6*d^3*e^3 + 7*a*b^5*d^2*e^4 + 28*a^2*b^4*d*e^5 + 84*a^3*b^3*e^6)*x^3 + 78*(b^6*d^4*e^2 + 7*a*b^5*d^3*e^3 + 28*a^2*b^4*d^2*e^4 + 84*a^3*b^3*d*e^5 + 210*a^4*b^2*e^6)*x^2 + 13*(b^6*d^5*e + 7*a*b^5*d^4*e^2 + 28*a^2*b^4*d^3*e^3 + 84*a^3*b^3*d^2*e^4 + 210*a^4*b^2*d*e^5 + 462*a^5*b*e^6)*x)/(e^20*x^13 + 13*d*e^19*x^12 + 78*d^2*e^18*x^11 + 286*d^3*e^17*x^10 + 715*d^4*e^16*x^9 + 1287*d^5*e^15*x^8 + 1716*d^6*e^14*x^7 + 1716*d^7*e^13*x^6 + 1287*d^8*e^12*x^5 + 715*d^9*e^11*x^4 + 286*d^10*e^10*x^3 + 78*d^11*e^9*x^2 + 13*d^12*e^8*x + d^13*e^7)`

Sympy [F(-1)]

Timed out.

$$\int \frac{(a^2 + 2abx + b^2x^2)^3}{(d + ex)^{14}} dx = \text{Timed out}$$

input `integrate((b**2*x**2+2*a*b*x+a**2)**3/(e*x+d)**14,x)`

output `Timed out`

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 485 vs. $2(159) = 318$.

Time = 0.08 (sec) , antiderivative size = 485, normalized size of antiderivative = 2.84

$$\int \frac{(a^2 + 2abx + b^2x^2)^3}{(d + ex)^{14}} dx = \frac{1716b^6e^6x^6 + b^6d^6 + 7ab^5d^5e + 28a^2b^4d^4e^2 + 84a^3b^3d^3e^3 + 210a^4b^2d^2e^4 + 462a^5bde^5 + 924a^6e^6 + 12012(e^{20}x^{13} + \dots)}{12012(e^{20}x^{13} + \dots)}$$

input `integrate((b^2*x^2+2*a*b*x+a^2)^3/(e*x+d)^14,x, algorithm="maxima")`

output `-1/12012*(1716*b^6*e^6*x^6 + b^6*d^6 + 7*a*b^5*d^5*e + 28*a^2*b^4*d^4*e^2 + 84*a^3*b^3*d^3*e^3 + 210*a^4*b^2*d^2*e^4 + 462*a^5*b*d*e^5 + 924*a^6*e^6 + 1287*(b^6*d*e^5 + 7*a*b^5*e^6)*x^5 + 715*(b^6*d^2*e^4 + 7*a*b^5*d*e^5 + 28*a^2*b^4*d*e^6)*x^4 + 286*(b^6*d^3*e^3 + 7*a*b^5*d^2*e^4 + 28*a^2*b^4*d*e^5 + 84*a^3*b^3*d*e^6)*x^3 + 78*(b^6*d^4*e^2 + 7*a*b^5*d^3*e^3 + 28*a^2*b^4*d^2*e^4 + 84*a^3*b^3*d*e^5 + 210*a^4*b^2*d*e^6)*x^2 + 13*(b^6*d^5*e + 7*a*b^5*d^4*e^2 + 28*a^2*b^4*d^3*e^3 + 84*a^3*b^3*d^2*e^4 + 210*a^4*b^2*d*e^5 + 462*a^5*b*d*e^6)*x)/(e^20*x^13 + 13*d*e^19*x^12 + 78*d^2*e^18*x^11 + 286*d^3*e^17*x^10 + 715*d^4*e^16*x^9 + 1287*d^5*e^15*x^8 + 1716*d^6*e^14*x^7 + 1716*d^7*e^13*x^6 + 1287*d^8*e^12*x^5 + 715*d^9*e^11*x^4 + 286*d^10*e^10*x^3 + 78*d^11*e^9*x^2 + 13*d^12*e^8*x + d^13*e^7)`

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 375 vs. $2(159) = 318$.

Time = 0.24 (sec) , antiderivative size = 375, normalized size of antiderivative = 2.19

$$\int \frac{(a^2 + 2abx + b^2x^2)^3}{(d + ex)^{14}} dx = \frac{1716b^6e^6x^6 + 1287b^6de^5x^5 + 9009ab^5e^6x^5 + 715b^6d^2e^4x^4 + 5005ab^5de^5x^4 + 20020a^2b^4e^6x^4 + 286b^6d^6 + \dots}{12012(e^{20}x^{13} + \dots)}$$

input `integrate((b^2*x^2+2*a*b*x+a^2)^3/(e*x+d)^14,x, algorithm="giac")`

output

```
-1/12012*(1716*b^6*e^6*x^6 + 1287*b^6*d*e^5*x^5 + 9009*a*b^5*e^6*x^5 + 715
*b^6*d^2*e^4*x^4 + 5005*a*b^5*d*e^5*x^4 + 20020*a^2*b^4*e^6*x^4 + 286*b^6*
d^3*e^3*x^3 + 2002*a*b^5*d^2*e^4*x^3 + 8008*a^2*b^4*d*e^5*x^3 + 24024*a^3*
b^3*e^6*x^3 + 78*b^6*d^4*e^2*x^2 + 546*a*b^5*d^3*e^3*x^2 + 2184*a^2*b^4*d^
2*e^4*x^2 + 6552*a^3*b^3*d*e^5*x^2 + 16380*a^4*b^2*e^6*x^2 + 13*b^6*d^5*e*
x + 91*a*b^5*d^4*e^2*x + 364*a^2*b^4*d^3*e^3*x + 1092*a^3*b^3*d^2*e^4*x +
2730*a^4*b^2*d*e^5*x + 6006*a^5*b*e^6*x + b^6*d^6 + 7*a*b^5*d^5*e + 28*a^2
*b^4*d^4*e^2 + 84*a^3*b^3*d^3*e^3 + 210*a^4*b^2*d^2*e^4 + 462*a^5*b*d*e^5
+ 924*a^6*e^6)/((e*x + d)^13*e^7)
```

Mupad [B] (verification not implemented)

Time = 9.61 (sec) , antiderivative size = 467, normalized size of antiderivative = 2.73

$$\int \frac{(a^2 + 2abx + b^2x^2)^3}{(d + ex)^{14}} dx =$$

$$-\frac{\frac{924 a^6 e^6 + 462 a^5 b d e^5 + 210 a^4 b^2 d^2 e^4 + 84 a^3 b^3 d^3 e^3 + 28 a^2 b^4 d^4 e^2 + 7 a b^5 d^5 e + b^6 d^6}{12012 e^7} + \frac{b^6 x^6}{7 e} + \frac{b^3 x^3 (84 a^3 e^3 + 28 a^2 b d e^2 + 7 a b^2 d^2 e)}{42 e^4}}{d^{13} + 13 d^{12} e x + 78 d^{11} e^2 x^2 + 286 d^{10} e^3 x^3 + 715 d^9 e^4 x^4 + 1287 d^8 e^5 x^5 + 1716 d^7 e^6 x^6 + 1287 d^6 e^7 x^7 + 924 d^5 e^8 x^8 + 462 d^4 e^9 x^9 + 286 d^3 e^{10} x^{10} + 13 d^2 e^{11} x^{11} + d e^{12} x^{12} + e^{13} x^{13}}$$

input

```
int((a^2 + b^2*x^2 + 2*a*b*x)^3/(d + e*x)^14,x)
```

output

```
-((924*a^6*e^6 + b^6*d^6 + 28*a^2*b^4*d^4*e^2 + 84*a^3*b^3*d^3*e^3 + 210*a
^4*b^2*d^2*e^4 + 7*a*b^5*d^5*e + 462*a^5*b*d*e^5)/(12012*e^7) + (b^6*x^6)/
(7*e) + (b^3*x^3*(84*a^3*e^3 + b^3*d^3 + 7*a*b^2*d^2*e + 28*a^2*b*d*e^2))/
(42*e^4) + (b*x*(462*a^5*e^5 + b^5*d^5 + 28*a^2*b^3*d^3*e^2 + 84*a^3*b^2*d
^2*e^3 + 7*a*b^4*d^4*e + 210*a^4*b*d*e^4))/(924*e^6) + (3*b^5*x^5*(7*a*e +
b*d))/(28*e^2) + (b^2*x^2*(210*a^4*e^4 + b^4*d^4 + 28*a^2*b^2*d^2*e^2 + 7
*a*b^3*d^3*e + 84*a^3*b*d*e^3))/(154*e^5) + (5*b^4*x^4*(28*a^2*e^2 + b^2*d
^2 + 7*a*b*d*e))/(84*e^3))/(d^13 + e^13*x^13 + 13*d*e^12*x^12 + 78*d^11*e^
2*x^2 + 286*d^10*e^3*x^3 + 715*d^9*e^4*x^4 + 1287*d^8*e^5*x^5 + 1716*d^7*e
^6*x^6 + 1287*d^6*e^7*x^7 + 924*d^5*e^8*x^8 + 462*d^4*e^9*x^9 + 286*d^3*e
^10*x^10 + 13*d^2*e^11*x^11 + d*e^12*x^12 + e^13*x^13)
```


Reduce [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 508, normalized size of antiderivative = 2.97

$$\int \frac{(a^2 + 2abx + b^2x^2)^3}{(d + ex)^{14}} dx$$

$$= \frac{-1716b^6e^6x^6 - 9009ab^5e^6x^5 - 1287b^6de^5x^5 - 20020a^2b^4e^6x^4 - 5005ab^5de^5x^4 - 715b^6d^2e^4x^4 - 24024a^3b^3e^6x^3 - 28a^2b^4d^2e^4x^3 - 364a^2b^4d^3e^3x^3 - 2184a^2b^4d^2e^4x^2 - 8008a^2b^4de^5x^3 - 20020a^2b^4e^6x^4 - 7ab^5d^5e - 91ab^5d^4e^2x - 546ab^5d^3e^3x^2 - 2002ab^5d^2e^4x^3 - 5005ab^5de^5x^4 - 9009ab^5e^6x^5 - b^6d^6 - 13b^6d^5ex - 78b^6d^4e^2x^2 - 286b^6d^3e^3x^3 - 715b^6d^2e^4x^4 - 1287b^6de^5x^5 - 1716b^6e^6x^6)/(12012e^7(d^{13} + 13d^{12}ex + 78d^{11}e^2x^2 + 286d^{10}e^3x^3 + 715d^9e^4x^4 + 1287d^8e^5x^5 + 1716d^7e^6x^6 + 1716d^6e^7x^7 + 1287d^5e^8x^8 + 715d^4e^9x^9 + 286d^3e^{10}x^{10} + 78d^2e^{11}x^{11} + 13de^{12}x^{12} + e^{13}x^{13}))$$

input `int((b^2*x^2+2*a*b*x+a^2)^3/(e*x+d)^14,x)`output `(- 924*a**6*e**6 - 462*a**5*b*d*e**5 - 6006*a**5*b*e**6*x - 210*a**4*b**2*d**2*e**4 - 2730*a**4*b**2*d*e**5*x - 16380*a**4*b**2*e**6*x**2 - 84*a**3*b**3*d**3*e**3 - 1092*a**3*b**3*d**2*e**4*x - 6552*a**3*b**3*d*e**5*x**2 - 24024*a**3*b**3*e**6*x**3 - 28*a**2*b**4*d**4*e**2 - 364*a**2*b**4*d**3*e**3*x - 2184*a**2*b**4*d**2*e**4*x**2 - 8008*a**2*b**4*d*e**5*x**3 - 20020*a**2*b**4*e**6*x**4 - 7*a*b**5*d**5*e - 91*a*b**5*d**4*e**2*x - 546*a*b**5*d**3*e**3*x**2 - 2002*a*b**5*d**2*e**4*x**3 - 5005*a*b**5*d*e**5*x**4 - 9009*a*b**5*e**6*x**5 - b**6*d**6 - 13*b**6*d**5*e*x - 78*b**6*d**4*e**2*x**2 - 286*b**6*d**3*e**3*x**3 - 715*b**6*d**2*e**4*x**4 - 1287*b**6*d*e**5*x**5 - 1716*b**6*e**6*x**6)/(12012*e**7*(d**13 + 13*d**12*e*x + 78*d**11*e**2*x**2 + 286*d**10*e**3*x**3 + 715*d**9*e**4*x**4 + 1287*d**8*e**5*x**5 + 1716*d**7*e**6*x**6 + 1716*d**6*e**7*x**7 + 1287*d**5*e**8*x**8 + 715*d**4*e**9*x**9 + 286*d**3*e**10*x**10 + 78*d**2*e**11*x**11 + 13*d*e**12*x**12 + e**13*x**13))`

3.107 $\int \frac{(a^2+2abx+b^2x^2)^3}{(d+ex)^{15}} dx$

| | |
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Optimal result

Integrand size = 26, antiderivative size = 173

$$\int \frac{(a^2 + 2abx + b^2x^2)^3}{(d + ex)^{15}} dx = -\frac{(bd - ae)^6}{14e^7(d + ex)^{14}} + \frac{6b(bd - ae)^5}{13e^7(d + ex)^{13}} - \frac{5b^2(bd - ae)^4}{4e^7(d + ex)^{12}} + \frac{20b^3(bd - ae)^3}{11e^7(d + ex)^{11}} - \frac{3b^4(bd - ae)^2}{2e^7(d + ex)^{10}} + \frac{2b^5(bd - ae)}{3e^7(d + ex)^9} - \frac{b^6}{8e^7(d + ex)^8}$$

output

```
-1/14*(-a*e+b*d)^6/e^7/(e*x+d)^14+6/13*b*(-a*e+b*d)^5/e^7/(e*x+d)^13-5/4*b^2*(-a*e+b*d)^4/e^7/(e*x+d)^12+20/11*b^3*(-a*e+b*d)^3/e^7/(e*x+d)^11-3/2*b^4*(-a*e+b*d)^2/e^7/(e*x+d)^10+2/3*b^5*(-a*e+b*d)/e^7/(e*x+d)^9-1/8*b^6/e^7/(e*x+d)^8
```

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 277, normalized size of antiderivative = 1.60

$$\int \frac{(a^2 + 2abx + b^2x^2)^3}{(d + ex)^{15}} dx = \frac{1716a^6e^6 + 792a^5be^5(d + 14ex) + 330a^4b^2e^4(d^2 + 14dex + 91e^2x^2) + 120a^3b^3e^3(d^3 + 14d^2ex + 91de^2x^2 + 364e^3x^3) + 36a^2b^4e^2(d^4 + 14d^3ex + 91d^2e^2x^2 + 364de^3x^3 + 1001e^4x^4) + 8ab^5e(d^5 + 14d^4ex + 91d^3e^2x^2 + 364d^2e^3x^3 + 1001de^4x^4 + 2002e^5x^5) + b^6(d^6 + 14d^5ex + 91d^4e^2x^2 + 364d^3e^3x^3 + 1001d^2e^4x^4 + 2002de^5x^5 + 3003e^6x^6)}{(e^7(d + ex)^{14}}$$

input

```
Integrate[(a^2 + 2*a*b*x + b^2*x^2)^3/(d + e*x)^15,x]
```

output

```
-1/24024*(1716*a^6*e^6 + 792*a^5*b*e^5*(d + 14*e*x) + 330*a^4*b^2*e^4*(d^2 + 14*d*e*x + 91*e^2*x^2) + 120*a^3*b^3*e^3*(d^3 + 14*d^2*e*x + 91*d*e^2*x^2 + 364*e^3*x^3) + 36*a^2*b^4*e^2*(d^4 + 14*d^3*e*x + 91*d^2*e^2*x^2 + 364*d*e^3*x^3 + 1001*e^4*x^4) + 8*a*b^5*e*(d^5 + 14*d^4*e*x + 91*d^3*e^2*x^2 + 364*d^2*e^3*x^3 + 1001*d*e^4*x^4 + 2002*e^5*x^5) + b^6*(d^6 + 14*d^5*e*x + 91*d^4*e^2*x^2 + 364*d^3*e^3*x^3 + 1001*d^2*e^4*x^4 + 2002*d*e^5*x^5 + 3003*e^6*x^6))/(e^7*(d + e*x)^14)
```

Rubi [A] (verified)

Time = 0.62 (sec) , antiderivative size = 173, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {1098, 27, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a^2 + 2abx + b^2x^2)^3}{(d + ex)^{15}} dx$$

$$\downarrow 1098$$

$$\frac{\int \frac{b^6(a+bx)^6}{(d+ex)^{15}} dx}{b^6}$$

$$\downarrow 27$$

$$\int \frac{(a + bx)^6}{(d + ex)^{15}} dx$$

↓ 53

$$\int \left(-\frac{6b^5(bd - ae)}{e^6(d + ex)^{10}} + \frac{15b^4(bd - ae)^2}{e^6(d + ex)^{11}} - \frac{20b^3(bd - ae)^3}{e^6(d + ex)^{12}} + \frac{15b^2(bd - ae)^4}{e^6(d + ex)^{13}} - \frac{6b(bd - ae)^5}{e^6(d + ex)^{14}} + \frac{(ae - bd)^6}{e^6(d + ex)^{15}} + \frac{1}{e^6(d + ex)^{15}} \right) dx$$

↓ 2009

$$\frac{2b^5(bd - ae)}{3e^7(d + ex)^9} - \frac{3b^4(bd - ae)^2}{2e^7(d + ex)^{10}} + \frac{20b^3(bd - ae)^3}{11e^7(d + ex)^{11}} - \frac{5b^2(bd - ae)^4}{4e^7(d + ex)^{12}} + \frac{6b(bd - ae)^5}{13e^7(d + ex)^{13}} - \frac{(bd - ae)^6}{14e^7(d + ex)^{14}} - \frac{b^6}{8e^7(d + ex)^8}$$

input `Int[(a^2 + 2*a*b*x + b^2*x^2)^3/(d + e*x)^15,x]`

output `-1/14*(b*d - a*e)^6/(e^7*(d + e*x)^14) + (6*b*(b*d - a*e)^5)/(13*e^7*(d + e*x)^13) - (5*b^2*(b*d - a*e)^4)/(4*e^7*(d + e*x)^12) + (20*b^3*(b*d - a*e)^3)/(11*e^7*(d + e*x)^11) - (3*b^4*(b*d - a*e)^2)/(2*e^7*(d + e*x)^10) + (2*b^5*(b*d - a*e))/(3*e^7*(d + e*x)^9) - b^6/(8*e^7*(d + e*x)^8)`

Defintions of rubi rules used

rule 27 `Int[(a_)*(Fx_), x_Symbol] := Simp[a Int[Fx, x], x] /; FreeQ[a, x] && !MatchQ[Fx, (b_)*(Gx_)] /; FreeQ[b, x]`

rule 53 `Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] := Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && IGtQ[m, 0] && (!IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])`

rule 1098 `Int[((d_.) + (e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] := Simp[1/c^p Int[(d + e*x)^m*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]`

rule 2009

```
Int[u_, x_Symbol] := Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 334 vs. 2(159) = 318.

Time = 1.04 (sec) , antiderivative size = 335, normalized size of antiderivative = 1.94

| method | result |
|--------------|---|
| risch | $-\frac{b^6 x^6}{8e} - \frac{b^5(8ae+bd)x^5}{12e^2} - \frac{b^4(36e^2 a^2+8abde+b^2 d^2)x^4}{24e^3} - \frac{b^3(120e^3 a^3+36a^2 bde^2+8a b^2 d^2 e+b^3 d^3)x^3}{66e^4} - \frac{b^2(330a^4 e^4+120a^3 bde^3+36a^2 b^2 d^2 e^2+8a b^3 d^3 e+b^4 d^4)x^2}{264e^5}$ |
| default | $-\frac{20b^3(e^3 a^3-3a^2 bde^2+3a b^2 d^2 e-b^3 d^3)}{11e^7(e^2 x+d)^{11}} - \frac{5b^2(a^4 e^4-4a^3 bde^3+6a^2 b^2 d^2 e^2-4a b^3 d^3 e+b^4 d^4)}{4e^7(e^2 x+d)^{12}} - \frac{6b(e^5 a^5-5a^4 bde^4+10a^3 b^2 d^2 e^3-5a^2 b^3 d^3 e^2+5a b^4 d^4 e-b^5 d^5)}{11e^7(e^2 x+d)^{13}}$ |
| norman | $-\frac{1716a^6 e^{13}+792a^5 bde^{12}+330a^4 b^2 d^2 e^{11}+120a^3 b^3 d^3 e^{10}+36a^2 b^4 d^4 e^9+8a b^5 d^5 e^8+b^6 d^6 e^7}{24024e^{14}} - \frac{(792a^5 b e^{12}+330a^4 b^2 d e^{11}+120a^3 b^3 d^2 e^{10}+36a^2 b^4 d^3 e^9+8a b^5 d^4 e^8+b^6 d^5 e^7)}{1716e^{13}}$ |
| gospers | $-\frac{3003x^6 b^6 e^6+16016x^5 a b^5 e^6+2002x^5 b^6 d e^5+36036x^4 a^2 b^4 e^6+8008x^4 a b^5 d e^5+1001x^4 b^6 d^2 e^4+43680x^3 a^3 b^3 e^6+13104x^3 a^2 b^4 d e^5+16016x^3 a b^5 d^2 e^4+16016x^3 b^6 d^3 e^3+16016x^2 a^2 b^4 d^2 e^4+16016x^2 a b^5 d^3 e^3+16016x^2 b^6 d^4 e^2+16016x a^3 b^3 d^3 e^3+16016x a^2 b^4 d^4 e^2+16016x a b^5 d^5 e+16016x b^6 d^6 e}{24024e^{14}}$ |
| parallelrisc | $-\frac{3003b^6 x^6 e^{13}-16016a b^5 e^{13} x^5-2002b^6 d e^{12} x^5-36036a^2 b^4 e^{13} x^4-8008a b^5 d e^{12} x^4-1001b^6 d^2 e^{11} x^4-43680a^3 b^3 e^{13} x^3-13104a^2 b^4 d e^{12} x^3-16016a b^5 d^2 e^{11} x^3-16016x^2 a^2 b^4 d^2 e^4-16016x^2 a b^5 d^3 e^3-16016x^2 b^6 d^4 e^2-16016x a^3 b^3 d^3 e^3-16016x a^2 b^4 d^4 e^2-16016x a b^5 d^5 e-16016x b^6 d^6 e}{24024e^{14}}$ |
| orering | $-\frac{(3003x^6 b^6 e^6+16016x^5 a b^5 e^6+2002x^5 b^6 d e^5+36036x^4 a^2 b^4 e^6+8008x^4 a b^5 d e^5+1001x^4 b^6 d^2 e^4+43680x^3 a^3 b^3 e^6+13104x^3 a^2 b^4 d e^5+16016x^3 a b^5 d^2 e^4+16016x^3 b^6 d^3 e^3+16016x^2 a^2 b^4 d^2 e^4+16016x^2 a b^5 d^3 e^3+16016x^2 b^6 d^4 e^2+16016x a^3 b^3 d^3 e^3+16016x a^2 b^4 d^4 e^2+16016x a b^5 d^5 e+16016x b^6 d^6 e)}{24024e^{14}}$ |

input

```
int((b^2*x^2+2*a*b*x+a^2)^3/(e*x+d)^15,x,method=_RETURNVERBOSE)
```

output

```
(-1/8/e*b^6*x^6-1/12*b^5/e^2*(8*a*e+b*d)*x^5-1/24*b^4/e^3*(36*a^2*e^2+8*a*b*d*e+b^2*d^2)*x^4-1/66*b^3/e^4*(120*a^3*e^3+36*a^2*b*d*e^2+8*a*b^2*d^2*e+b^3*d^3)*x^3-1/264*b^2/e^5*(330*a^4*e^4+120*a^3*b*d*e^3+36*a^2*b^2*d^2*e^2+8*a*b^3*d^3*e+b^4*d^4)*x^2-1/1716*b/e^6*(792*a^5*e^5+330*a^4*b*d*e^4+120*a^3*b^2*d^2*e^3+36*a^2*b^3*d^3*e^2+8*a*b^4*d^4*e+b^5*d^5)*x-1/24024/e^7*(1716*a^6*e^6+792*a^5*b*d*e^5+330*a^4*b^2*d^2*e^4+120*a^3*b^3*d^3*e^3+36*a^2*b^4*d^4*e^2+8*a*b^5*d^5*e+b^6*d^6))/(e*x+d)^14
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 496 vs. $2(159) = 318$.

Time = 0.09 (sec) , antiderivative size = 496, normalized size of antiderivative = 2.87

$$\int \frac{(a^2 + 2abx + b^2x^2)^3}{(d + ex)^{15}} dx = \frac{3003b^6e^6x^6 + b^6d^6 + 8ab^5d^5e + 36a^2b^4d^4e^2 + 120a^3b^3d^3e^3 + 330a^4b^2d^2e^4 + 792a^5bde^5 + 1716a^6e^6 + 24024(e^{21}x^{14} + 14de^{20}x^{13} + 91d^2e^{19}x^{12} + 364d^3e^{18}x^{11} + 1001d^4e^{17}x^{10} + 2002d^5e^{16}x^9 + 3003d^6e^{15}x^8 + 3432d^7e^{14}x^7 + 3003d^8e^{13}x^6 + 2002d^9e^{12}x^5 + 1001d^{10}e^{11}x^4 + 364d^{11}e^{10}x^3 + 91d^{12}e^9x^2 + 14d^{13}e^8x + d^{14}e^7)}{24024(e^{21}x^{14} + 14de^{20}x^{13} + 91d^2e^{19}x^{12} + 364d^3e^{18}x^{11} + 1001d^4e^{17}x^{10} + 2002d^5e^{16}x^9 + 3003d^6e^{15}x^8 + 3432d^7e^{14}x^7 + 3003d^8e^{13}x^6 + 2002d^9e^{12}x^5 + 1001d^{10}e^{11}x^4 + 364d^{11}e^{10}x^3 + 91d^{12}e^9x^2 + 14d^{13}e^8x + d^{14}e^7)}$$

input `integrate((b^2*x^2+2*a*b*x+a^2)^3/(e*x+d)^15,x, algorithm="fricas")`

output `-1/24024*(3003*b^6*e^6*x^6 + b^6*d^6 + 8*a*b^5*d^5*e + 36*a^2*b^4*d^4*e^2 + 120*a^3*b^3*d^3*e^3 + 330*a^4*b^2*d^2*e^4 + 792*a^5*b*d*e^5 + 1716*a^6*e^6 + 2002*(b^6*d*e^5 + 8*a*b^5*e^6)*x^5 + 1001*(b^6*d^2*e^4 + 8*a*b^5*d*e^5 + 36*a^2*b^4*e^6)*x^4 + 364*(b^6*d^3*e^3 + 8*a*b^5*d^2*e^4 + 36*a^2*b^4*d*e^5 + 120*a^3*b^3*e^6)*x^3 + 91*(b^6*d^4*e^2 + 8*a*b^5*d^3*e^3 + 36*a^2*b^4*d^2*e^4 + 120*a^3*b^3*d*e^5 + 330*a^4*b^2*e^6)*x^2 + 14*(b^6*d^5*e + 8*a*b^5*d^4*e^2 + 36*a^2*b^4*d^3*e^3 + 120*a^3*b^3*d^2*e^4 + 330*a^4*b^2*d*e^5 + 792*a^5*b*e^6)*x)/(e^21*x^14 + 14*d*e^20*x^13 + 91*d^2*e^19*x^12 + 364*d^3*e^18*x^11 + 1001*d^4*e^17*x^10 + 2002*d^5*e^16*x^9 + 3003*d^6*e^15*x^8 + 3432*d^7*e^14*x^7 + 3003*d^8*e^13*x^6 + 2002*d^9*e^12*x^5 + 1001*d^10*e^11*x^4 + 364*d^11*e^10*x^3 + 91*d^12*e^9*x^2 + 14*d^13*e^8*x + d^14*e^7)`

Sympy [F(-1)]

Timed out.

$$\int \frac{(a^2 + 2abx + b^2x^2)^3}{(d + ex)^{15}} dx = \text{Timed out}$$

input `integrate((b**2*x**2+2*a*b*x+a**2)**3/(e*x+d)**15,x)`

output `Timed out`

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 496 vs. $2(159) = 318$.

Time = 0.07 (sec) , antiderivative size = 496, normalized size of antiderivative = 2.87

$$\int \frac{(a^2 + 2abx + b^2x^2)^3}{(d + ex)^{15}} dx = \frac{3003b^6e^6x^6 + b^6d^6 + 8ab^5d^5e + 36a^2b^4d^4e^2 + 120a^3b^3d^3e^3 + 330a^4b^2d^2e^4 + 792a^5bde^5 + 1716a^6e^6 + 24024(e^{21}x^{14} + 14de^{20}x^{13} + 91d^2e^{19}x^{12} + 364d^3e^{18}x^{11} + 1001d^4e^{17}x^{10} + 2002d^5e^{16}x^9 + 3003d^6e^{15}x^8 + 3432d^7e^{14}x^7 + 3003d^8e^{13}x^6 + 2002d^9e^{12}x^5 + 1001d^{10}e^{11}x^4 + 364d^{11}e^{10}x^3 + 91d^{12}e^9x^2 + 14d^{13}e^8x + d^{14}e^7)}{24024(e^{21}x^{14} + 14de^{20}x^{13} + 91d^2e^{19}x^{12} + 364d^3e^{18}x^{11} + 1001d^4e^{17}x^{10} + 2002d^5e^{16}x^9 + 3003d^6e^{15}x^8 + 3432d^7e^{14}x^7 + 3003d^8e^{13}x^6 + 2002d^9e^{12}x^5 + 1001d^{10}e^{11}x^4 + 364d^{11}e^{10}x^3 + 91d^{12}e^9x^2 + 14d^{13}e^8x + d^{14}e^7)}$$

input `integrate((b^2*x^2+2*a*b*x+a^2)^3/(e*x+d)^15,x, algorithm="maxima")`

output `-1/24024*(3003*b^6*e^6*x^6 + b^6*d^6 + 8*a*b^5*d^5*e + 36*a^2*b^4*d^4*e^2 + 120*a^3*b^3*d^3*e^3 + 330*a^4*b^2*d^2*e^4 + 792*a^5*b*d*e^5 + 1716*a^6*e^6 + 24024*(b^6*d*e^5 + 8*a*b^5*e^6)*x^5 + 1001*(b^6*d^2*e^4 + 8*a*b^5*d*e^5 + 36*a^2*b^4*e^6)*x^4 + 364*(b^6*d^3*e^3 + 8*a*b^5*d^2*e^4 + 36*a^2*b^4*d*e^5 + 120*a^3*b^3*e^6)*x^3 + 91*(b^6*d^4*e^2 + 8*a*b^5*d^3*e^3 + 36*a^2*b^4*d^2*e^4 + 120*a^3*b^3*d*e^5 + 330*a^4*b^2*e^6)*x^2 + 14*(b^6*d^5*e + 8*a*b^5*d^4*e^2 + 36*a^2*b^4*d^3*e^3 + 120*a^3*b^3*d^2*e^4 + 330*a^4*b^2*d*e^5 + 792*a^5*b*e^6)*x)/(e^21*x^14 + 14*d*e^20*x^13 + 91*d^2*e^19*x^12 + 364*d^3*e^18*x^11 + 1001*d^4*e^17*x^10 + 2002*d^5*e^16*x^9 + 3003*d^6*e^15*x^8 + 3432*d^7*e^14*x^7 + 3003*d^8*e^13*x^6 + 2002*d^9*e^12*x^5 + 1001*d^10*e^11*x^4 + 364*d^11*e^10*x^3 + 91*d^12*e^9*x^2 + 14*d^13*e^8*x + d^14*e^7)`

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 375 vs. $2(159) = 318$.

Time = 0.22 (sec) , antiderivative size = 375, normalized size of antiderivative = 2.17

$$\int \frac{(a^2 + 2abx + b^2x^2)^3}{(d + ex)^{15}} dx = \frac{3003b^6e^6x^6 + 2002b^6de^5x^5 + 16016ab^5e^6x^5 + 1001b^6d^2e^4x^4 + 8008ab^5de^5x^4 + 36036a^2b^4e^6x^4 + 364d^11e^{10}x^3 + 91d^{12}e^9x^2 + 14d^{13}e^8x + d^{14}e^7}{24024(e^{21}x^{14} + 14de^{20}x^{13} + 91d^2e^{19}x^{12} + 364d^3e^{18}x^{11} + 1001d^4e^{17}x^{10} + 2002d^5e^{16}x^9 + 3003d^6e^{15}x^8 + 3432d^7e^{14}x^7 + 3003d^8e^{13}x^6 + 2002d^9e^{12}x^5 + 1001d^{10}e^{11}x^4 + 364d^{11}e^{10}x^3 + 91d^{12}e^9x^2 + 14d^{13}e^8x + d^{14}e^7)}$$

input `integrate((b^2*x^2+2*a*b*x+a^2)^3/(e*x+d)^15,x, algorithm="giac")`

output

```
-1/24024*(3003*b^6*e^6*x^6 + 2002*b^6*d*e^5*x^5 + 16016*a*b^5*e^6*x^5 + 10
01*b^6*d^2*e^4*x^4 + 8008*a*b^5*d*e^5*x^4 + 36036*a^2*b^4*e^6*x^4 + 364*b^
6*d^3*e^3*x^3 + 2912*a*b^5*d^2*e^4*x^3 + 13104*a^2*b^4*d*e^5*x^3 + 43680*a
^3*b^3*e^6*x^3 + 91*b^6*d^4*e^2*x^2 + 728*a*b^5*d^3*e^3*x^2 + 3276*a^2*b^4
*d^2*e^4*x^2 + 10920*a^3*b^3*d*e^5*x^2 + 30030*a^4*b^2*e^6*x^2 + 14*b^6*d^
5*e*x + 112*a*b^5*d^4*e^2*x + 504*a^2*b^4*d^3*e^3*x + 1680*a^3*b^3*d^2*e^4
*x + 4620*a^4*b^2*d*e^5*x + 11088*a^5*b*e^6*x + b^6*d^6 + 8*a*b^5*d^5*e +
36*a^2*b^4*d^4*e^2 + 120*a^3*b^3*d^3*e^3 + 330*a^4*b^2*d^2*e^4 + 792*a^5*b
*d*e^5 + 1716*a^6*e^6)/((e*x + d)^14*e^7)
```

Mupad [B] (verification not implemented)

Time = 9.85 (sec) , antiderivative size = 478, normalized size of antiderivative = 2.76

$$\int \frac{(a^2 + 2abx + b^2x^2)^3}{(d + ex)^{15}} dx =$$

$$-\frac{\frac{1716 a^6 e^6 + 792 a^5 b d e^5 + 330 a^4 b^2 d^2 e^4 + 120 a^3 b^3 d^3 e^3 + 36 a^2 b^4 d^4 e^2 + 8 a b^5 d^5 e + b^6 d^6}{24024 e^7} + \frac{b^6 x^6}{8 e} + \frac{b^3 x^3 (120 a^3 e^3 + 36 a^2 b d e^2 + 8 a b^2 d^2 e)}{66 e^4}}{d^{14} + 14 d^{13} e x + 91 d^{12} e^2 x^2 + 364 d^{11} e^3 x^3 + 1001 d^{10} e^4 x^4 + 2002 d^9 e^5 x^5}$$

input

```
int((a^2 + b^2*x^2 + 2*a*b*x)^3/(d + e*x)^15,x)
```

output

```
-((1716*a^6*e^6 + b^6*d^6 + 36*a^2*b^4*d^4*e^2 + 120*a^3*b^3*d^3*e^3 + 330
*a^4*b^2*d^2*e^4 + 8*a*b^5*d^5*e + 792*a^5*b*d*e^5)/(24024*e^7) + (b^6*x^6
)/(8*e) + (b^3*x^3*(120*a^3*e^3 + b^3*d^3 + 8*a*b^2*d^2*e + 36*a^2*b*d*e^2
))/(66*e^4) + (b*x*(792*a^5*e^5 + b^5*d^5 + 36*a^2*b^3*d^3*e^2 + 120*a^3*b
^2*d^2*e^3 + 8*a*b^4*d^4*e + 330*a^4*b*d*e^4))/(1716*e^6) + (b^5*x^5*(8*a*
e + b*d))/(12*e^2) + (b^2*x^2*(330*a^4*e^4 + b^4*d^4 + 36*a^2*b^2*d^2*e^2
+ 8*a*b^3*d^3*e + 120*a^3*b*d*e^3))/(264*e^5) + (b^4*x^4*(36*a^2*e^2 + b^2
*d^2 + 8*a*b*d*e))/(24*e^3))/(d^14 + e^14*x^14 + 14*d*e^13*x^13 + 91*d^12*
e^2*x^2 + 364*d^11*e^3*x^3 + 1001*d^10*e^4*x^4 + 2002*d^9*e^5*x^5 + 3003*d
^8*e^6*x^6 + 3432*d^7*e^7*x^7 + 3003*d^6*e^8*x^8 + 2002*d^5*e^9*x^9 + 1001
*d^4*e^10*x^10 + 364*d^3*e^11*x^11 + 91*d^2*e^12*x^12 + 14*d^13*e*x)
```


Reduce [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 519, normalized size of antiderivative = 3.00

$$\int \frac{(a^2 + 2abx + b^2x^2)^3}{(d + ex)^{15}} dx$$

$$= \frac{-3003b^6e^6x^6 - 16016ab^5e^6x^5 - 2002b^6de^5x^5 - 36036a^2b^4e^6x^4 - 8008ab^5de^5x^4 - 1001b^6d^2e^4x^4 - 4368$$

input `int((b^2*x^2+2*a*b*x+a^2)^3/(e*x+d)^15,x)`

output

```
( - 1716*a**6*e**6 - 792*a**5*b*d*e**5 - 11088*a**5*b*e**6*x - 330*a**4*b**2*d**2*e**4 - 4620*a**4*b**2*d*e**5*x - 30030*a**4*b**2*e**6*x**2 - 120*a**3*b**3*d**3*e**3 - 1680*a**3*b**3*d**2*e**4*x - 10920*a**3*b**3*d*e**5*x**2 - 43680*a**3*b**3*e**6*x**3 - 36*a**2*b**4*d**4*e**2 - 504*a**2*b**4*d**3*e**3*x - 3276*a**2*b**4*d**2*e**4*x**2 - 13104*a**2*b**4*d*e**5*x**3 - 36036*a**2*b**4*e**6*x**4 - 8*a*b**5*d**5*e - 112*a*b**5*d**4*e**2*x - 728*a*b**5*d**3*e**3*x**2 - 2912*a*b**5*d**2*e**4*x**3 - 8008*a*b**5*d*e**5*x**4 - 16016*a*b**5*e**6*x**5 - b**6*d**6 - 14*b**6*d**5*e*x - 91*b**6*d**4*e**2*x**2 - 364*b**6*d**3*e**3*x**3 - 1001*b**6*d**2*e**4*x**4 - 2002*b**6*d*e**5*x**5 - 3003*b**6*e**6*x**6)/(24024*e**7*(d**14 + 14*d**13*e*x + 91*d**12*e**2*x**2 + 364*d**11*e**3*x**3 + 1001*d**10*e**4*x**4 + 2002*d**9*e**5*x**5 + 3003*d**8*e**6*x**6 + 3432*d**7*e**7*x**7 + 3003*d**6*e**8*x**8 + 2002*d**5*e**9*x**9 + 1001*d**4*e**10*x**10 + 364*d**3*e**11*x**11 + 91*d**2*e**12*x**12 + 14*d*e**13*x**13 + e**14*x**14))
```

3.108 $\int \frac{(d+ex)^5}{a^2+2abx+b^2x^2} dx$

| | |
|---|-----|
| Optimal result | 885 |
| Mathematica [A] (verified) | 885 |
| Rubi [A] (verified) | 886 |
| Maple [B] (verified) | 888 |
| Fricas [B] (verification not implemented) | 888 |
| Sympy [A] (verification not implemented) | 889 |
| Maxima [B] (verification not implemented) | 890 |
| Giac [B] (verification not implemented) | 890 |
| Mupad [B] (verification not implemented) | 891 |
| Reduce [B] (verification not implemented) | 892 |

Optimal result

Integrand size = 26, antiderivative size = 131

$$\int \frac{(d+ex)^5}{a^2+2abx+b^2x^2} dx = \frac{10e^2(bd-ae)^3x}{b^5} - \frac{(bd-ae)^5}{b^6(a+bx)} + \frac{5e^3(bd-ae)^2(a+bx)^2}{b^6} + \frac{5e^4(bd-ae)(a+bx)^3}{3b^6} + \frac{e^5(a+bx)^4}{4b^6} + \frac{5e(bd-ae)^4 \log(a+bx)}{b^6}$$

output

```
10*e^2*(-a*e+b*d)^3*x/b^5-(-a*e+b*d)^5/b^6/(b*x+a)+5*e^3*(-a*e+b*d)^2*(b*x+a)^2/b^6+5/3*e^4*(-a*e+b*d)*(b*x+a)^3/b^6+1/4*e^5*(b*x+a)^4/b^6+5*e*(-a*e+b*d)^4*ln(b*x+a)/b^6
```

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 230, normalized size of antiderivative = 1.76

$$\int \frac{(d+ex)^5}{a^2+2abx+b^2x^2} dx = \frac{12a^5e^5 - 12a^4be^4(5d+4ex) + 30a^3b^2e^3(4d^2+6dex-e^2x^2) + 10a^2b^3e^2(-12d^3-24d^2ex+12de^2x^2+e^3x^3)}{b^6}$$

input `Integrate[(d + e*x)^5/(a^2 + 2*a*b*x + b^2*x^2),x]`

output $(12*a^5*e^5 - 12*a^4*b*e^4*(5*d + 4*e*x) + 30*a^3*b^2*e^3*(4*d^2 + 6*d*e*x - e^2*x^2) + 10*a^2*b^3*e^2*(-12*d^3 - 24*d^2*e*x + 12*d*e^2*x^2 + e^3*x^3) - 5*a*b^4*e*(-12*d^4 - 24*d^3*e*x + 36*d^2*e^2*x^2 + 8*d*e^3*x^3 + e^4*x^4) + b^5*(-12*d^5 + 120*d^3*e^2*x^2 + 60*d^2*e^3*x^3 + 20*d*e^4*x^4 + 3*e^5*x^5) + 60*e*(b*d - a*e)^4*(a + b*x)*\text{Log}[a + b*x])/(12*b^6*(a + b*x))$

Rubi [A] (verified)

Time = 0.62 (sec) , antiderivative size = 131, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {1098, 27, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d + ex)^5}{a^2 + 2abx + b^2x^2} dx$$

$$\downarrow 1098$$

$$b^2 \int \frac{(d + ex)^5}{b^2(a + bx)^2} dx$$

$$\downarrow 27$$

$$\int \frac{(d + ex)^5}{(a + bx)^2} dx$$

$$\downarrow 49$$

$$\int \left(\frac{5e^4(a + bx)^2(bd - ae)}{b^5} + \frac{10e^3(a + bx)(bd - ae)^2}{b^5} + \frac{10e^2(bd - ae)^3}{b^5} + \frac{5e(bd - ae)^4}{b^5(a + bx)} + \frac{(bd - ae)^5}{b^5(a + bx)^2} + \frac{e^5(a + bx)}{b^5} \right) dx$$

$$\downarrow 2009$$

$$\frac{5e^4(a + bx)^3(bd - ae)}{3b^6} + \frac{5e^3(a + bx)^2(bd - ae)^2}{b^6} - \frac{(bd - ae)^5}{b^6(a + bx)} + \frac{5e(bd - ae)^4 \log(a + bx)}{b^6} + \frac{e^5(a + bx)^4}{4b^6} + \frac{10e^2x(bd - ae)^3}{b^5}$$

input `Int[(d + e*x)^5/(a^2 + 2*a*b*x + b^2*x^2),x]`

output `(10*e^2*(b*d - a*e)^3*x)/b^5 - (b*d - a*e)^5/(b^6*(a + b*x)) + (5*e^3*(b*d - a*e)^2*(a + b*x)^2)/b^6 + (5*e^4*(b*d - a*e)*(a + b*x)^3)/(3*b^6) + (e^5*(a + b*x)^4)/(4*b^6) + (5*e*(b*d - a*e)^4*Log[a + b*x])/b^6`

Defintions of rubi rules used

rule 27 `Int[(a_)*(Fx_), x_Symbol] := Simp[a Int[Fx, x], x] /; FreeQ[a, x] && !MatchQ[Fx, (b_)*(Gx_)] /; FreeQ[b, x]`

rule 49 `Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] := Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] && IGtQ[m, 0] && IGtQ[m + n + 2, 0]`

rule 1098 `Int[((d_.) + (e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] := Simp[1/c^p Int[(d + e*x)^m*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]`

rule 2009 `Int[u_, x_Symbol] := Simp[IntSum[u, x], x] /; SumQ[u]`

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 255 vs. 2(127) = 254.

Time = 1.12 (sec) , antiderivative size = 256, normalized size of antiderivative = 1.95

| method | result |
|---------------|---|
| norman | $\frac{e^5 x^5}{4b} - \frac{5e^2(e^3 a^3 - 4a^2 b d e^2 + 6a b^2 d^2 e - 4b^3 d^3)}{2b^4} x^2 + \frac{5e^3(e^2 a^2 - 4ab d e + 6b^2 d^2)}{6b^3} x^3 - \frac{5e^4(ae - 4bd)x^4}{12b^2} - \frac{(5e^5 a^5 - 20a^4 b d e^4 + 30a^3 b^2 d^2 e^3 - 20a^2 b^3 d^3 e^2 + 10a b^4 d^4 e - b^5 d^5)}{a b^5}$ |
| default | $-\frac{e^2(-\frac{1}{4}b^3 x^4 e^3 + \frac{2}{3}a b^2 e^3 x^3 - \frac{5}{3}b^3 d e^2 x^3 - \frac{3}{2}a^2 b e^3 x^2 + 5a b^2 d e^2 x^2 - 5b^3 d^2 e x^2 + 4e^3 a^3 x - 15a^2 b d e^2 x + 20a b^2 d^2 e x - 10b^3 d^3 x)}{b^5}$ |
| risch | $\frac{e^5 x^4}{4b^2} - \frac{2e^5 a x^3}{3b^3} + \frac{5e^4 d x^3}{3b^2} + \frac{3e^5 a^2 x^2}{2b^4} - \frac{5e^4 a d x^2}{b^3} + \frac{5e^3 d^2 x^2}{b^2} - \frac{4e^5 a^3 x}{b^5} + \frac{15e^4 a^2 d x}{b^4} - \frac{20e^3 a d^2 x}{b^3} + \frac{10e^2 d^3 x}{b^2}$ |
| parallelrisch | $-240 \ln(bx+a) x a^3 b^2 d e^4 - 240 \ln(bx+a) x a b^4 d^3 e^2 - 12b^5 d^5 + 360 \ln(bx+a) x a^2 b^3 d^2 e^3 - 240a^4 b d e^4 + 360a^3 b^2 d^2 e^3 - 240a^2 b^3 d^3 e^2$ |

```
input int((e*x+d)^5/(b^2*x^2+2*a*b*x+a^2),x,method=_RETURNVERBOSE)
```

```
output (1/4*e^5/b*x^5-5/2*e^2*(a^3*e^3-4*a^2*b*d*e^2+6*a*b^2*d^2*e-4*b^3*d^3)/b^4*x^2+5/6*e^3*(a^2*e^2-4*a*b*d*e+6*b^2*d^2)/b^3*x^3-5/12*e^4*(a*e-4*b*d)/b^2*x^4-(5*a^5*e^5-20*a^4*b*d*e^4+30*a^3*b^2*d^2*e^3-20*a^2*b^3*d^3*e^2+5*a*b^4*d^4*e-b^5*d^5)/a/b^5*x)/(b*x+a)+5/b^6*e*(a^4*e^4-4*a^3*b*d*e^3+6*a^2*b^2*d^2*e^2-4*a*b^3*d^3*e+b^4*d^4)*ln(b*x+a)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 375 vs. 2(127) = 254.

Time = 0.09 (sec) , antiderivative size = 375, normalized size of antiderivative = 2.86

$$\int \frac{(d + ex)^5}{a^2 + 2abx + b^2x^2} dx = \frac{3b^5e^5x^5 - 12b^5d^5 + 60ab^4d^4e - 120a^2b^3d^3e^2 + 120a^3b^2d^2e^3 - 60a^4bde^4 + 12a^5e^5 + 5(4b^5de^4 - ab^4e^5)}{a^2 + 2abx + b^2x^2}$$

```
input integrate((e*x+d)^5/(b^2*x^2+2*a*b*x+a^2),x, algorithm="fricas")
```

output

```
1/12*(3*b^5*e^5*x^5 - 12*b^5*d^5 + 60*a*b^4*d^4*e - 120*a^2*b^3*d^3*e^2 +
120*a^3*b^2*d^2*e^3 - 60*a^4*b*d*e^4 + 12*a^5*e^5 + 5*(4*b^5*d*e^4 - a*b^4
*e^5)*x^4 + 10*(6*b^5*d^2*e^3 - 4*a*b^4*d*e^4 + a^2*b^3*e^5)*x^3 + 30*(4*b
^5*d^3*e^2 - 6*a*b^4*d^2*e^3 + 4*a^2*b^3*d*e^4 - a^3*b^2*e^5)*x^2 + 12*(10
*a*b^4*d^3*e^2 - 20*a^2*b^3*d^2*e^3 + 15*a^3*b^2*d*e^4 - 4*a^4*b*e^5)*x +
60*(a*b^4*d^4*e - 4*a^2*b^3*d^3*e^2 + 6*a^3*b^2*d^2*e^3 - 4*a^4*b*d*e^4 +
a^5*e^5 + (b^5*d^4*e - 4*a*b^4*d^3*e^2 + 6*a^2*b^3*d^2*e^3 - 4*a^3*b^2*d*e
^4 + a^4*b*e^5)*x)*log(b*x + a))/(b^7*x + a*b^6)
```

Sympy [A] (verification not implemented)

Time = 0.48 (sec) , antiderivative size = 231, normalized size of antiderivative = 1.76

$$\int \frac{(d+ex)^5}{a^2+2abx+b^2x^2} dx = x^3 \left(-\frac{2ae^5}{3b^3} + \frac{5de^4}{3b^2} \right) + x^2 \cdot \left(\frac{3a^2e^5}{2b^4} - \frac{5ade^4}{b^3} + \frac{5d^2e^3}{b^2} \right) + x \left(-\frac{4a^3e^5}{b^5} + \frac{15a^2de^4}{b^4} - \frac{20ad^2e^3}{b^3} + \frac{10d^3e^2}{b^2} \right) + \frac{a^5e^5 - 5a^4bde^4 + 10a^3b^2d^2e^3 - 10a^2b^3d^3e^2 + 5ab^4d^4e - b^5d^5}{ab^6 + b^7x} + \frac{e^5x^4}{4b^2} + \frac{5e(ae - bd)^4 \log(a + bx)}{b^6}$$

input

```
integrate((e*x+d)**5/(b**2*x**2+2*a*b*x+a**2),x)
```

output

```
x**3*(-2*a*e**5/(3*b**3) + 5*d*e**4/(3*b**2)) + x**2*(3*a**2*e**5/(2*b**4)
- 5*a*d*e**4/b**3 + 5*d**2*e**3/b**2) + x*(-4*a**3*e**5/b**5 + 15*a**2*d*
e**4/b**4 - 20*a*d**2*e**3/b**3 + 10*d**3*e**2/b**2) + (a**5*e**5 - 5*a**4
*b*d*e**4 + 10*a**3*b**2*d**2*e**3 - 10*a**2*b**3*d**3*e**2 + 5*a*b**4*d**
4*e - b**5*d**5)/(a*b**6 + b**7*x) + e**5*x**4/(4*b**2) + 5*e*(a*e - b*d)*
**4*log(a + b*x)/b**6
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 265 vs. $2(127) = 254$.

Time = 0.04 (sec) , antiderivative size = 265, normalized size of antiderivative = 2.02

$$\int \frac{(d+ex)^5}{a^2+2abx+b^2x^2} dx$$

$$= -\frac{b^5d^5 - 5ab^4d^4e + 10a^2b^3d^3e^2 - 10a^3b^2d^2e^3 + 5a^4bde^4 - a^5e^5}{b^7x + ab^6}$$

$$+ \frac{3b^3e^5x^4 + 4(5b^3de^4 - 2ab^2e^5)x^3 + 6(10b^3d^2e^3 - 10ab^2de^4 + 3a^2be^5)x^2 + 12(10b^3d^3e^2 - 20ab^2d^2e^3 + 5(b^4d^4e - 4ab^3d^3e^2 + 6a^2b^2d^2e^3 - 4a^3bde^4 + a^4e^5)\log(bx+a)}{12b^5}$$

$$+ \frac{5(b^4d^4e - 4ab^3d^3e^2 + 6a^2b^2d^2e^3 - 4a^3bde^4 + a^4e^5)\log(bx+a)}{b^6}$$

input `integrate((e*x+d)^5/(b^2*x^2+2*a*b*x+a^2),x, algorithm="maxima")`

output
$$-(b^5d^5 - 5a^2b^4d^4e + 10a^2b^3d^3e^2 - 10a^3b^2d^2e^3 + 5a^4bde^4 - a^5e^5)/(b^7x + ab^6) + 1/12*(3b^3e^5x^4 + 4*(5b^3d^3e^2 - 2a^2b^2e^5)*x^3 + 6*(10b^3d^2e^3 - 10a^2b^2de^4 + 3a^2be^5)*x^2 + 12*(10b^3d^3e^2 - 20a^2b^2d^2e^3 + 5*(b^4d^4e - 4a^3bde^4 + a^4e^5)*\log(bx+a))/b^5 + 5*(b^4d^4e - 4a^3b^3d^3e^2 + 6a^2b^2d^2e^3 - 4a^3bde^4 + a^4e^5)*\log(bx+a)/b^6$$

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 273 vs. $2(127) = 254$.

Time = 0.22 (sec) , antiderivative size = 273, normalized size of antiderivative = 2.08

$$\int \frac{(d+ex)^5}{a^2+2abx+b^2x^2} dx$$

$$= \frac{5(b^4d^4e - 4ab^3d^3e^2 + 6a^2b^2d^2e^3 - 4a^3bde^4 + a^4e^5)\log(|bx+a|)}{b^6}$$

$$- \frac{b^5d^5 - 5ab^4d^4e + 10a^2b^3d^3e^2 - 10a^3b^2d^2e^3 + 5a^4bde^4 - a^5e^5}{(bx+a)b^6}$$

$$+ \frac{3b^6e^5x^4 + 20b^6de^4x^3 - 8ab^5e^5x^3 + 60b^6d^2e^3x^2 - 60ab^5de^4x^2 + 18a^2b^4e^5x^2 + 120b^6d^3e^2x - 240ab^5a}{12b^8}$$

input `integrate((e*x+d)^5/(b^2*x^2+2*a*b*x+a^2),x, algorithm="giac")`

output $5*(b^4*d^4*e - 4*a*b^3*d^3*e^2 + 6*a^2*b^2*d^2*e^3 - 4*a^3*b*d*e^4 + a^4*e^5)*\log(\text{abs}(b*x + a))/b^6 - (b^5*d^5 - 5*a*b^4*d^4*e + 10*a^2*b^3*d^3*e^2 - 10*a^3*b^2*d^2*e^3 + 5*a^4*b*d*e^4 - a^5*e^5)/((b*x + a)*b^6) + 1/12*(3*b^6*e^5*x^4 + 20*b^6*d*e^4*x^3 - 8*a*b^5*e^5*x^3 + 60*b^6*d^2*e^3*x^2 - 60*a*b^5*d*e^4*x^2 + 18*a^2*b^4*e^5*x^2 + 120*b^6*d^3*e^2*x - 240*a*b^5*d^2*e^3*x + 180*a^2*b^4*d*e^4*x - 48*a^3*b^3*e^5*x)/b^8$

Mupad [B] (verification not implemented)

Time = 8.77 (sec) , antiderivative size = 326, normalized size of antiderivative = 2.49

$$\int \frac{(d + ex)^5}{a^2 + 2abx + b^2x^2} dx$$

$$= x \left(\frac{10d^3e^2}{b^2} - \frac{2a \left(\frac{2a \left(\frac{2ae^5}{b^3} - \frac{5de^4}{b^2} \right)}{b} - \frac{a^2e^5}{b^4} + \frac{10d^2e^3}{b^2} \right)}{b} + \frac{a^2 \left(\frac{2ae^5}{b^3} - \frac{5de^4}{b^2} \right)}{b^2} \right)$$

$$- x^3 \left(\frac{2ae^5}{3b^3} - \frac{5de^4}{3b^2} \right) + x^2 \left(\frac{a \left(\frac{2ae^5}{b^3} - \frac{5de^4}{b^2} \right)}{b} - \frac{a^2e^5}{2b^4} + \frac{5d^2e^3}{b^2} \right)$$

$$+ \frac{\ln(a + bx) (5a^4e^5 - 20a^3bde^4 + 30a^2b^2d^2e^3 - 20ab^3d^3e^2 + 5b^4d^4e)}{b^6}$$

$$+ \frac{a^5e^5 - 5a^4bde^4 + 10a^3b^2d^2e^3 - 10a^2b^3d^3e^2 + 5ab^4d^4e - b^5d^5}{b(xb^6 + ab^5)} + \frac{e^5x^4}{4b^2}$$

input `int((d + e*x)^5/(a^2 + b^2*x^2 + 2*a*b*x),x)`

output

```
x*((10*d^3*e^2)/b^2 - (2*a*((2*a*((2*a*e^5)/b^3 - (5*d*e^4)/b^2))/b - (a^2
*e^5)/b^4 + (10*d^2*e^3)/b^2))/b + (a^2*((2*a*e^5)/b^3 - (5*d*e^4)/b^2))/b
^2) - x^3*((2*a*e^5)/(3*b^3) - (5*d*e^4)/(3*b^2)) + x^2*((a*((2*a*e^5)/b^3
- (5*d*e^4)/b^2))/b - (a^2*e^5)/(2*b^4) + (5*d^2*e^3)/b^2) + (log(a + b*x
)*(5*a^4*e^5 + 5*b^4*d^4*e - 20*a*b^3*d^3*e^2 + 30*a^2*b^2*d^2*e^3 - 20*a^
3*b*d*e^4))/b^6 + (a^5*e^5 - b^5*d^5 - 10*a^2*b^3*d^3*e^2 + 10*a^3*b^2*d^2
*e^3 + 5*a*b^4*d^4*e - 5*a^4*b*d*e^4)/(b*(a*b^5 + b^6*x)) + (e^5*x^4)/(4*b
^2)
```

Reduce [B] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 415, normalized size of antiderivative = 3.17

$$\int \frac{(d + ex)^5}{a^2 + 2abx + b^2x^2} dx$$

$$= \frac{-240 \log(bx + a) a^5 b d e^4 + 60 \log(bx + a) a^5 b e^5 x + 360 \log(bx + a) a^4 b^2 d^2 e^3 - 240 \log(bx + a) a^3 b^3 d^3 e^2}{12 a^6 b^6 (a + b x)}$$

input

```
int((e*x+d)^5/(b^2*x^2+2*a*b*x+a^2),x)
```

output

```
(60*log(a + b*x)*a**6*e**5 - 240*log(a + b*x)*a**5*b*d*e**4 + 60*log(a + b
*x)*a**5*b*e**5*x + 360*log(a + b*x)*a**4*b**2*d**2*e**3 - 240*log(a + b*x
)*a**4*b**2*d*e**4*x - 240*log(a + b*x)*a**3*b**3*d**3*e**2 + 360*log(a +
b*x)*a**3*b**3*d**2*e**3*x + 60*log(a + b*x)*a**2*b**4*d**4*e - 240*log(a
+ b*x)*a**2*b**4*d**3*e**2*x + 60*log(a + b*x)*a*b**5*d**4*e*x - 60*a**5*b
**5*x + 240*a**4*b**2*d*e**4*x - 30*a**4*b**2*e**5*x**2 - 360*a**3*b**3*
d**2*e**3*x + 120*a**3*b**3*d*e**4*x**2 + 10*a**3*b**3*e**5*x**3 + 240*a**
2*b**4*d**3*e**2*x - 180*a**2*b**4*d**2*e**3*x**2 - 40*a**2*b**4*d*e**4*x*
*3 - 5*a**2*b**4*e**5*x**4 - 60*a*b**5*d**4*e*x + 120*a*b**5*d**3*e**2*x**
2 + 60*a*b**5*d**2*e**3*x**3 + 20*a*b**5*d*e**4*x**4 + 3*a*b**5*e**5*x**5
+ 12*b**6*d**5*x)/(12*a*b**6*(a + b*x))
```

3.109 $\int \frac{(d+ex)^4}{a^2+2abx+b^2x^2} dx$

| | |
|---|-----|
| Optimal result | 893 |
| Mathematica [A] (verified) | 893 |
| Rubi [A] (verified) | 894 |
| Maple [A] (verified) | 895 |
| Fricas [B] (verification not implemented) | 896 |
| Sympy [A] (verification not implemented) | 896 |
| Maxima [A] (verification not implemented) | 897 |
| Giac [A] (verification not implemented) | 897 |
| Mupad [B] (verification not implemented) | 898 |
| Reduce [B] (verification not implemented) | 899 |

Optimal result

Integrand size = 26, antiderivative size = 108

$$\int \frac{(d+ex)^4}{a^2+2abx+b^2x^2} dx = \frac{e^2(6b^2d^2 - 8abde + 3a^2e^2)x}{b^4} + \frac{e^3(2bd - ae)x^2}{b^3} + \frac{e^4x^3}{3b^2} - \frac{(bd - ae)^4}{b^5(a + bx)} + \frac{4e(bd - ae)^3 \log(a + bx)}{b^5}$$

output

```
e^2*(3*a^2*e^2-8*a*b*d*e+6*b^2*d^2)*x/b^4+e^3*(-a*e+2*b*d)*x^2/b^3+1/3*e^4*x^3/b^2-(-a*e+b*d)^4/b^5/(b*x+a)+4*e*(-a*e+b*d)^3*ln(b*x+a)/b^5
```

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 166, normalized size of antiderivative = 1.54

$$\int \frac{(d+ex)^4}{a^2+2abx+b^2x^2} dx = \frac{-3a^4e^4 + 3a^3be^3(4d + 3ex) + 6a^2b^2e^2(-3d^2 - 4dex + e^2x^2) - 2ab^3e(-6d^3 - 9d^2ex + 9de^2x^2 + e^3x^3) + 3b^5(a + bx)}{3b^5(a + bx)}$$

input

```
Integrate[(d + e*x)^4/(a^2 + 2*a*b*x + b^2*x^2), x]
```

output

$$(-3a^4e^4 + 3a^3be^3(4d + 3ex) + 6a^2b^2e^2(-3d^2 - 4dex + e^2x^2) - 2ab^3e(-6d^3 - 9d^2ex + 9de^2x^2 + e^3x^3) + b^4(-3d^4 + 18d^2e^2x^2 + 6de^3x^3 + e^4x^4) - 12e(-(bd) + ae)^3(a + bx) \cdot \text{Log}[a + bx]) / (3b^5(a + bx))$$

Rubi [A] (verified)

Time = 0.50 (sec) , antiderivative size = 104, normalized size of antiderivative = 0.96, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {1098, 27, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\begin{aligned} & \int \frac{(d + ex)^4}{a^2 + 2abx + b^2x^2} dx \\ & \quad \downarrow 1098 \\ & b^2 \int \frac{(d + ex)^4}{b^2(a + bx)^2} dx \\ & \quad \downarrow 27 \\ & \int \frac{(d + ex)^4}{(a + bx)^2} dx \\ & \quad \downarrow 49 \\ & \int \left(\frac{4e^3(a + bx)(bd - ae)}{b^4} + \frac{6e^2(bd - ae)^2}{b^4} + \frac{4e(bd - ae)^3}{b^4(a + bx)} + \frac{(bd - ae)^4}{b^4(a + bx)^2} + \frac{e^4(a + bx)^2}{b^4} \right) dx \\ & \quad \downarrow 2009 \\ & \frac{2e^3(a + bx)^2(bd - ae)}{b^5} - \frac{(bd - ae)^4}{b^5(a + bx)} + \frac{4e(bd - ae)^3 \log(a + bx)}{b^5} + \frac{e^4(a + bx)^3}{3b^5} + \frac{6e^2x(bd - ae)^2}{b^4} \end{aligned}$$

input

$$\text{Int}[(d + ex)^4/(a^2 + 2a*bx + b^2*x^2), x]$$

output $(6e^{2(bd - a^2x)}/b^4 - (bd - a^2x)/(b^5(a + bx)) + (2e^{3(bd - a^2x)}(a + bx)^2)/b^5 + (e^{4(a + bx)^3}/(3b^5) + (4e^{(bd - a^2x)^3} \log[a + bx])/b^5$

Defintions of rubi rules used

rule 27 $\text{Int}[(a_*)(Fx_), x_Symbol] \rightarrow \text{Simp}[a \text{ Int}[Fx, x], x] /; \text{FreeQ}[a, x] \ \&\& \ !\text{MatchQ}[Fx, (b_*)(Gx_)] /; \text{FreeQ}[b, x]$

rule 49 $\text{Int}[((a_.) + (b_.)(x_.))^{(m_.)}((c_.) + (d_.)(x_.))^{(n_.)}, x_Symbol] \rightarrow \text{Int}[\text{ExpandIntegrand}[(a + bx)^m(c + dx)^n, x], x] /; \text{FreeQ}\{a, b, c, d\}, x \ \&\& \ \text{IGtQ}[m, 0] \ \&\& \ \text{IGtQ}[m + n + 2, 0]$

rule 1098 $\text{Int}[((d_.) + (e_.)(x_.))^{(m_.)}((a_) + (b_.)(x_) + (c_.)(x_)^2)^{(p_.)}, x_Symbol] \rightarrow \text{Simp}[1/c^p \text{ Int}[(d + ex)^m(b + cx)^{(2p)}, x], x] /; \text{FreeQ}\{a, b, c, d, e, m\}, x \ \&\& \ \text{EqQ}[b^2 - 4ac, 0] \ \&\& \ \text{IntegerQ}[p]$

rule 2009 $\text{Int}[u_, x_Symbol] \rightarrow \text{Simp}[\text{IntSum}[u, x], x] /; \text{SumQ}[u]$

Maple [A] (verified)

Time = 1.03 (sec) , antiderivative size = 175, normalized size of antiderivative = 1.62

| method | result |
|--------------|--|
| default | $\frac{e^2(\frac{1}{3}b^2e^2x^3 - x^2ab e^2 + 2b^2de x^2 + 3a^2e^2x - 8abdex + 6b^2d^2x)}{b^4} - \frac{a^4e^4 - 4a^3bde^3 + 6a^2b^2d^2e^2 - 4ab^3d^3e + b^4d^4}{b^5(bx+a)} - \frac{4e(e^3a^3 - 3a^2bd e^2 + \dots)}{b^5(bx+a)}$ |
| norman | $\frac{(4a^4e^4 - 12a^3bde^3 + 12a^2b^2d^2e^2 - 4ab^3d^3e + b^4d^4)x + \frac{e^4x^4}{3b} + \frac{2e^2(e^2a^2 - 3abde + 3b^2d^2)x^2}{b^3} - \frac{2e^3(ae - 3bd)x^3}{3b^2}}{bx+a} - \frac{4e(e^3a^3 - 3a^2bd e^2 + \dots)}{b^5(bx+a)}$ |
| risch | $\frac{e^4x^3}{3b^2} - \frac{e^4x^2a}{b^3} + \frac{2e^3dx^2}{b^2} + \frac{3e^4a^2x}{b^4} - \frac{8e^3adx}{b^3} + \frac{6e^2d^2x}{b^2} - \frac{a^4e^4}{b^5(bx+a)} + \frac{4a^3de^3}{b^4(bx+a)} - \frac{6a^2d^2e^2}{b^3(bx+a)} + \frac{4ad^3e}{b^2(bx+a)} - \dots$ |
| parallelrisc | $-\frac{b^4x^4e^4 + 2x^3ab^3e^4 - 6x^3b^4de^3 + 12\ln(bx+a)xa^3be^4 - 36\ln(bx+a)xa^2b^2de^3 + 36\ln(bx+a)xab^3d^2e^2 - 12\ln(bx+a)xb^4d^3}{b^5(bx+a)}$ |

input $\text{int}((e*x+d)^4/(b^2*x^2+2*a*b*x+a^2), x, \text{method}=_RETURNVERBOSE)$

output

```
e^2/b^4*(1/3*b^2*e^2*x^3-x^2*a*b*e^2+2*b^2*d*e*x^2+3*a^2*e^2*x-8*a*b*d*e*x
+6*b^2*d^2*x)-1/b^5*(a^4*e^4-4*a^3*b*d*e^3+6*a^2*b^2*d^2*e^2-4*a*b^3*d^3*e
+b^4*d^4)/(b*x+a)-4/b^5*e*(a^3*e^3-3*a^2*b*d*e^2+3*a*b^2*d^2*e-b^3*d^3)*ln
(b*x+a)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 268 vs. $2(106) = 212$.

Time = 0.08 (sec) , antiderivative size = 268, normalized size of antiderivative = 2.48

$$\int \frac{(d+ex)^4}{a^2+2abx+b^2x^2} dx$$

$$= \frac{b^4e^4x^4 - 3b^4d^4 + 12ab^3d^3e - 18a^2b^2d^2e^2 + 12a^3bde^3 - 3a^4e^4 + 2(3b^4de^3 - ab^3e^4)x^3 + 6(3b^4d^2e^2 - 3$$

input

```
integrate((e*x+d)^4/(b^2*x^2+2*a*b*x+a^2),x, algorithm="fricas")
```

output

```
1/3*(b^4*e^4*x^4 - 3*b^4*d^4 + 12*a*b^3*d^3*e - 18*a^2*b^2*d^2*e^2 + 12*a^
3*b*d*e^3 - 3*a^4*e^4 + 2*(3*b^4*d*e^3 - a*b^3*e^4)*x^3 + 6*(3*b^4*d^2*e^2
- 3*a*b^3*d*e^3 + a^2*b^2*e^4)*x^2 + 3*(6*a*b^3*d^2*e^2 - 8*a^2*b^2*d*e^3
+ 3*a^3*b*e^4)*x + 12*(a*b^3*d^3*e - 3*a^2*b^2*d^2*e^2 + 3*a^3*b*d*e^3 -
a^4*e^4 + (b^4*d^3*e - 3*a*b^3*d^2*e^2 + 3*a^2*b^2*d*e^3 - a^3*b*e^4)*x)*l
og(b*x + a))/(b^6*x + a*b^5)
```

Sympy [A] (verification not implemented)

Time = 0.36 (sec) , antiderivative size = 155, normalized size of antiderivative = 1.44

$$\int \frac{(d+ex)^4}{a^2+2abx+b^2x^2} dx = x^2 \left(-\frac{ae^4}{b^3} + \frac{2de^3}{b^2} \right) + x \left(\frac{3a^2e^4}{b^4} - \frac{8ade^3}{b^3} + \frac{6d^2e^2}{b^2} \right)$$

$$+ \frac{-a^4e^4 + 4a^3bde^3 - 6a^2b^2d^2e^2 + 4ab^3d^3e - b^4d^4}{ab^5 + b^6x}$$

$$+ \frac{e^4x^3}{3b^2} - \frac{4e(ae - bd)^3 \log(a + bx)}{b^5}$$

input

```
integrate((e*x+d)**4/(b**2*x**2+2*a*b*x+a**2),x)
```

output

```
x**2*(-a***4/b**3 + 2*d***3/b**2) + x*(3*a**2*e***4/b**4 - 8*a*d*e***3/b**
3 + 6*d**2*e***2/b**2) + (-a**4*e***4 + 4*a**3*b*d*e***3 - 6*a**2*b**2*d**2*e
**2 + 4*a*b**3*d**3*e - b**4*d**4)/(a*b**5 + b**6*x) + e**4*x**3/(3*b**2)
- 4*e*(a*e - b*d)**3*log(a + b*x)/b**5
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 184, normalized size of antiderivative = 1.70

$$\int \frac{(d+ex)^4}{a^2+2abx+b^2x^2} dx$$

$$= -\frac{b^4d^4 - 4ab^3d^3e + 6a^2b^2d^2e^2 - 4a^3bde^3 + a^4e^4}{b^6x + ab^5}$$

$$+ \frac{b^2e^4x^3 + 3(2b^2de^3 - abe^4)x^2 + 3(6b^2d^2e^2 - 8abde^3 + 3a^2e^4)x}{3b^4}$$

$$+ \frac{4(b^3d^3e - 3ab^2d^2e^2 + 3a^2bde^3 - a^3e^4) \log(bx+a)}{b^5}$$

input

```
integrate((e*x+d)^4/(b^2*x^2+2*a*b*x+a^2),x, algorithm="maxima")
```

output

```
-(b^4*d^4 - 4*a*b^3*d^3*e + 6*a^2*b^2*d^2*e^2 - 4*a^3*b*d*e^3 + a^4*e^4)/(
b^6*x + a*b^5) + 1/3*(b^2*e^4*x^3 + 3*(2*b^2*d*e^3 - a*b*e^4)*x^2 + 3*(6*b
^2*d^2*e^2 - 8*a*b*d*e^3 + 3*a^2*e^4)*x)/b^4 + 4*(b^3*d^3*e - 3*a*b^2*d^2*
e^2 + 3*a^2*b*d*e^3 - a^3*e^4)*log(b*x + a)/b^5
```

Giac [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 188, normalized size of antiderivative = 1.74

$$\int \frac{(d+ex)^4}{a^2+2abx+b^2x^2} dx$$

$$= \frac{4(b^3d^3e - 3ab^2d^2e^2 + 3a^2bde^3 - a^3e^4) \log(|bx+a|)}{b^5}$$

$$+ \frac{b^4e^4x^3 + 6b^4de^3x^2 - 3ab^3e^4x^2 + 18b^4d^2e^2x - 24ab^3de^3x + 9a^2b^2e^4x}{3b^6}$$

$$- \frac{b^4d^4 - 4ab^3d^3e + 6a^2b^2d^2e^2 - 4a^3bde^3 + a^4e^4}{(bx+a)b^5}$$

input `integrate((e*x+d)^4/(b^2*x^2+2*a*b*x+a^2),x, algorithm="giac")`

output $4*(b^3*d^3*e - 3*a*b^2*d^2*e^2 + 3*a^2*b*d*e^3 - a^3*e^4)*\log(\text{abs}(b*x + a)) / b^5 + 1/3*(b^4*e^4*x^3 + 6*b^4*d*e^3*x^2 - 3*a*b^3*e^4*x^2 + 18*b^4*d^2*e^2*x - 24*a*b^3*d*e^3*x + 9*a^2*b^2*e^4*x) / b^6 - (b^4*d^4 - 4*a*b^3*d^3*e + 6*a^2*b^2*d^2*e^2 - 4*a^3*b*d*e^3 + a^4*e^4) / ((b*x + a)*b^5)$

Mupad [B] (verification not implemented)

Time = 9.02 (sec) , antiderivative size = 203, normalized size of antiderivative = 1.88

$$\int \frac{(d + ex)^4}{a^2 + 2abx + b^2x^2} dx = x \left(\frac{2a \left(\frac{2ae^4}{b^3} - \frac{4de^3}{b^2} \right)}{b} - \frac{a^2 e^4}{b^4} + \frac{6d^2 e^2}{b^2} \right) - x^2 \left(\frac{ae^4}{b^3} - \frac{2de^3}{b^2} \right) + \frac{e^4 x^3}{3b^2} - \frac{\ln(a + bx) (4a^3 e^4 - 12a^2 b d e^3 + 12a b^2 d^2 e^2 - 4b^3 d^3 e)}{b^5} - \frac{a^4 e^4 - 4a^3 b d e^3 + 6a^2 b^2 d^2 e^2 - 4a b^3 d^3 e + b^4 d^4}{b(x b^5 + a b^4)}$$

input `int((d + e*x)^4/(a^2 + b^2*x^2 + 2*a*b*x),x)`

output $x*((2*a*((2*a*e^4)/b^3 - (4*d*e^3)/b^2))/b - (a^2*e^4)/b^4 + (6*d^2*e^2)/b^2) - x^2*((a*e^4)/b^3 - (2*d*e^3)/b^2) + (e^4*x^3)/(3*b^2) - (\log(a + b*x))*(4*a^3*e^4 - 4*b^3*d^3*e + 12*a*b^2*d^2*e^2 - 12*a^2*b*d*e^3)/b^5 - (a^4*e^4 + b^4*d^4 + 6*a^2*b^2*d^2*e^2 - 4*a*b^3*d^3*e - 4*a^3*b*d*e^3)/(b*(a*b^4 + b^5*x))$

3.110 $\int \frac{(d+ex)^3}{a^2+2abx+b^2x^2} dx$

| | |
|---|-----|
| Optimal result | 900 |
| Mathematica [A] (verified) | 900 |
| Rubi [A] (verified) | 901 |
| Maple [A] (verified) | 902 |
| Fricas [B] (verification not implemented) | 903 |
| Sympy [A] (verification not implemented) | 903 |
| Maxima [A] (verification not implemented) | 904 |
| Giac [A] (verification not implemented) | 904 |
| Mupad [B] (verification not implemented) | 905 |
| Reduce [B] (verification not implemented) | 905 |

Optimal result

Integrand size = 26, antiderivative size = 75

$$\int \frac{(d+ex)^3}{a^2+2abx+b^2x^2} dx = \frac{e^2(3bd-2ae)x}{b^3} + \frac{e^3x^2}{2b^2} - \frac{(bd-ae)^3}{b^4(a+bx)} + \frac{3e(bd-ae)^2 \log(a+bx)}{b^4}$$

output

```
e^2*(-2*a*e+3*b*d)*x/b^3+1/2*e^3*x^2/b^2-(-a*e+b*d)^3/b^4/(b*x+a)+3*e*(-a*
e+b*d)^2*ln(b*x+a)/b^4
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.96

$$\int \frac{(d+ex)^3}{a^2+2abx+b^2x^2} dx = \frac{2be^2(3bd-2ae)x + b^2e^3x^2 - \frac{2(bd-ae)^3}{a+bx} + 6e(bd-ae)^2 \log(a+bx)}{2b^4}$$

input

```
Integrate[(d + e*x)^3/(a^2 + 2*a*b*x + b^2*x^2),x]
```

output

```
(2*b*e^2*(3*b*d - 2*a*e)*x + b^2*e^3*x^2 - (2*(b*d - a*e)^3)/(a + b*x) + 6
*e*(b*d - a*e)^2*Log[a + b*x])/(2*b^4)
```

Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {1098, 27, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\begin{aligned}
 & \int \frac{(d+ex)^3}{a^2+2abx+b^2x^2} dx \\
 & \quad \downarrow \text{1098} \\
 & b^2 \int \frac{(d+ex)^3}{b^2(a+bx)^2} dx \\
 & \quad \downarrow \text{27} \\
 & \int \frac{(d+ex)^3}{(a+bx)^2} dx \\
 & \quad \downarrow \text{49} \\
 & \int \left(\frac{e^2(3bd-2ae)}{b^3} + \frac{3e(bd-ae)^2}{b^3(a+bx)} + \frac{(bd-ae)^3}{b^3(a+bx)^2} + \frac{e^3x}{b^2} \right) dx \\
 & \quad \downarrow \text{2009} \\
 & -\frac{(bd-ae)^3}{b^4(a+bx)} + \frac{3e(bd-ae)^2 \log(a+bx)}{b^4} + \frac{e^2x(3bd-2ae)}{b^3} + \frac{e^3x^2}{2b^2}
 \end{aligned}$$

input `Int[(d + e*x)^3/(a^2 + 2*a*b*x + b^2*x^2), x]`

output `(e^2*(3*b*d - 2*a*e)*x)/b^3 + (e^3*x^2)/(2*b^2) - (b*d - a*e)^3/(b^4*(a + b*x)) + (3*e*(b*d - a*e)^2*Log[a + b*x])/b^4`

Defintions of rubi rules used

rule 27 `Int[(a_)*(F_x_), x_Symbol] := Simp[a Int[F_x, x], x] /; FreeQ[a, x] && !MatchQ[F_x, (b_)*(G_x_)] /; FreeQ[b, x]`

rule 49 `Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] := Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] && IGtQ[m, 0] && IGtQ[m + n + 2, 0]`

rule 1098 `Int[((d_.) + (e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] := Simp[1/c^p Int[(d + e*x)^m*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]`

rule 2009 `Int[u_, x_Symbol] := Simp[IntSum[u, x], x] /; SumQ[u]`

Maple [A] (verified)

Time = 1.06 (sec) , antiderivative size = 109, normalized size of antiderivative = 1.45

| method | result |
|---------------|---|
| default | $-\frac{e^2(-\frac{1}{2}be^2x^2+2aex-3bdx)}{b^3} - \frac{-e^3a^3+3a^2bde^2-3ab^2d^2e+b^3d^3}{b^4(bx+a)} + \frac{3e(e^2a^2-2abde+b^2d^2)\ln(bx+a)}{b^4}$ |
| norman | $\frac{3e^3a^3-6a^2bde^2+3ab^2d^2e-b^3d^3}{b^4} + \frac{e^3x^3}{2b} - \frac{3e^2(ae-2bd)x^2}{2b^2} + \frac{3e(e^2a^2-2abde+b^2d^2)\ln(bx+a)}{b^4}$ |
| risch | $\frac{e^3x^2}{2b^2} - \frac{2e^3ax}{b^3} + \frac{3e^2dx}{b^2} + \frac{e^3a^3}{b^4(bx+a)} - \frac{3a^2de^2}{b^3(bx+a)} + \frac{3ad^2e}{b^2(bx+a)} - \frac{d^3}{b(bx+a)} + \frac{3e^3\ln(bx+a)a^2}{b^4} - \frac{6e^2\ln(bx+a)ad}{b^3}$ |
| parallelrisch | $\frac{e^3x^3+6\ln(bx+a)xa^2be^3-12\ln(bx+a)xab^2de^2+6\ln(bx+a)xb^3d^2e-3x^2ab^2e^3+6x^2b^3de^2+6\ln(bx+a)a^3e^3-12\ln(bx+a)}{2b^4(bx+a)}$ |

input `int((e*x+d)^3/(b^2*x^2+2*a*b*x+a^2), x, method=_RETURNVERBOSE)`

output `-e^2/b^3*(-1/2*b*e*x^2+2*a*e*x-3*b*d*x)-1/b^4*(-a^3*e^3+3*a^2*b*d*e^2-3*a*b^2*d^2*e+b^3*d^3)/(b*x+a)+3/b^4*e*(a^2*e^2-2*a*b*d*e+b^2*d^2)*ln(b*x+a)`

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 173 vs. $2(73) = 146$.

Time = 0.09 (sec) , antiderivative size = 173, normalized size of antiderivative = 2.31

$$\int \frac{(d+ex)^3}{a^2+2abx+b^2x^2} dx = \frac{b^3e^3x^3 - 2b^3d^3 + 6ab^2d^2e - 6a^2bde^2 + 2a^3e^3 + 3(2b^3de^2 - ab^2e^3)x^2 + 2(3ab^2de^2 - 2a^2be^3)x + 6(ab^2d^2e - a^2bde^2 + a^3e^3)}{2(b^5x + ab^4)}$$

input `integrate((e*x+d)^3/(b^2*x^2+2*a*b*x+a^2),x, algorithm="fricas")`

output `1/2*(b^3*e^3*x^3 - 2*b^3*d^3 + 6*a*b^2*d^2*e - 6*a^2*b*d*e^2 + 2*a^3*e^3 + 3*(2*b^3*d*e^2 - a*b^2*e^3)*x^2 + 2*(3*a*b^2*d*e^2 - 2*a^2*b*e^3)*x + 6*(a*b^2*d^2*e - 2*a^2*b*d*e^2 + a^3*e^3 + (b^3*d^2*e - 2*a*b^2*d*e^2 + a^2*b*e^3)*x)*log(b*x + a))/(b^5*x + a*b^4)`

Sympy [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 102, normalized size of antiderivative = 1.36

$$\int \frac{(d+ex)^3}{a^2+2abx+b^2x^2} dx = x \left(-\frac{2ae^3}{b^3} + \frac{3de^2}{b^2} \right) + \frac{a^3e^3 - 3a^2bde^2 + 3ab^2d^2e - b^3d^3}{ab^4 + b^5x} + \frac{e^3x^2}{2b^2} + \frac{3e(ae - bd)^2 \log(a + bx)}{b^4}$$

input `integrate((e*x+d)**3/(b**2*x**2+2*a*b*x+a**2),x)`

output `x*(-2*a*e**3/b**3 + 3*d*e**2/b**2) + (a**3*e**3 - 3*a**2*b*d*e**2 + 3*a*b*d**2*e - b**3*d**3)/(a*b**4 + b**5*x) + e**3*x**2/(2*b**2) + 3*e*(a*e - b*d)**2*log(a + b*x)/b**4`

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 118, normalized size of antiderivative = 1.57

$$\int \frac{(d+ex)^3}{a^2+2abx+b^2x^2} dx = -\frac{b^3d^3-3ab^2d^2e+3a^2bde^2-a^3e^3}{b^5x+ab^4} + \frac{be^3x^2+2(3bde^2-2ae^3)x}{2b^3} + \frac{3(b^2d^2e-2abde^2+a^2e^3)\log(bx+a)}{b^4}$$

input `integrate((e*x+d)^3/(b^2*x^2+2*a*b*x+a^2),x, algorithm="maxima")`

output `-(b^3*d^3 - 3*a*b^2*d^2*e + 3*a^2*b*d*e^2 - a^3*e^3)/(b^5*x + a*b^4) + 1/2*(b*e^3*x^2 + 2*(3*b*d*e^2 - 2*a*e^3)*x)/b^3 + 3*(b^2*d^2*e - 2*a*b*d*e^2 + a^2*e^3)*log(b*x + a)/b^4`

Giac [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.59

$$\int \frac{(d+ex)^3}{a^2+2abx+b^2x^2} dx = \frac{3(b^2d^2e-2abde^2+a^2e^3)\log(|bx+a|)}{b^4} + \frac{b^2e^3x^2+6b^2de^2x-4abe^3x}{2b^4} - \frac{b^3d^3-3ab^2d^2e+3a^2bde^2-a^3e^3}{(bx+a)b^4}$$

input `integrate((e*x+d)^3/(b^2*x^2+2*a*b*x+a^2),x, algorithm="giac")`

output `3*(b^2*d^2*e - 2*a*b*d*e^2 + a^2*e^3)*log(abs(b*x + a))/b^4 + 1/2*(b^2*e^3*x^2 + 6*b^2*d*e^2*x - 4*a*b*e^3*x)/b^4 - (b^3*d^3 - 3*a*b^2*d^2*e + 3*a^2*b*d*e^2 - a^3*e^3)/((b*x + a)*b^4)`

Mupad [B] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 123, normalized size of antiderivative = 1.64

$$\int \frac{(d+ex)^3}{a^2+2abx+b^2x^2} dx = \frac{\ln(a+bx)(3a^2e^3-6abde^2+3b^2d^2e)}{b^4} - x\left(\frac{2ae^3}{b^3} - \frac{3de^2}{b^2}\right) + \frac{a^3e^3-3a^2bde^2+3ab^2d^2e-b^3d^3}{b(xb^4+ab^3)} + \frac{e^3x^2}{2b^2}$$

input `int((d + e*x)^3/(a^2 + b^2*x^2 + 2*a*b*x), x)`output `(log(a + b*x)*(3*a^2*e^3 + 3*b^2*d^2*e - 6*a*b*d*e^2))/b^4 - x*((2*a*e^3)/b^3 - (3*d*e^2)/b^2) + (a^3*e^3 - b^3*d^3 + 3*a*b^2*d^2*e - 3*a^2*b*d*e^2)/(b*(a*b^3 + b^4*x)) + (e^3*x^2)/(2*b^2)`**Reduce [B] (verification not implemented)**

Time = 0.22 (sec) , antiderivative size = 197, normalized size of antiderivative = 2.63

$$\int \frac{(d+ex)^3}{a^2+2abx+b^2x^2} dx = \frac{6 \log(bx+a) a^4 e^3 - 12 \log(bx+a) a^3 b d e^2 + 6 \log(bx+a) a^3 b e^3 x + 6 \log(bx+a) a^2 b^2 d^2 e - 12 \log(bx+a) a^2 b d e^2 x + 6 \log(bx+a) a^2 b^2 d^2 e x^2 - 6 a^3 b^2 d e^2 x^2 + 6 a^2 b^3 d e^2 x^2 - 3 a^3 b^2 d e^2 x^3 - 6 a^2 b^3 d e^2 x^3 + 6 a^2 b^3 d e^2 x^3 + a^3 b^3 e^3 x^3 + 2 b^4 d^3 x^3}{(2 a^4 b^4 (a + b x))}$$

input `int((e*x+d)^3/(b^2*x^2+2*a*b*x+a^2), x)`output `(6*log(a + b*x)*a**4*e**3 - 12*log(a + b*x)*a**3*b*d*e**2 + 6*log(a + b*x)*a**3*b*e**3*x + 6*log(a + b*x)*a**2*b**2*d**2*e - 12*log(a + b*x)*a**2*b**2*d*e**2*x + 6*log(a + b*x)*a*b**3*d**2*e*x - 6*a**3*b*e**3*x + 12*a**2*b**2*d*e**2*x - 3*a**2*b**2*e**3*x**2 - 6*a*b**3*d**2*e*x + 6*a*b**3*d*e**2*x**2 + a*b**3*e**3*x**3 + 2*b**4*d**3*x)/(2*a*b**4*(a + b*x))`

3.111 $\int \frac{(d+ex)^2}{a^2+2abx+b^2x^2} dx$

| | |
|---|-----|
| Optimal result | 906 |
| Mathematica [A] (verified) | 906 |
| Rubi [A] (verified) | 907 |
| Maple [A] (verified) | 908 |
| Fricas [A] (verification not implemented) | 909 |
| Sympy [A] (verification not implemented) | 909 |
| Maxima [A] (verification not implemented) | 909 |
| Giac [A] (verification not implemented) | 910 |
| Mupad [B] (verification not implemented) | 910 |
| Reduce [B] (verification not implemented) | 911 |

Optimal result

Integrand size = 26, antiderivative size = 51

$$\int \frac{(d+ex)^2}{a^2+2abx+b^2x^2} dx = \frac{e^2x}{b^2} - \frac{(bd-ae)^2}{b^3(a+bx)} + \frac{2e(bd-ae)\log(a+bx)}{b^3}$$

output $e^2x/b^2 - (-a*e+b*d)^2/b^3/(b*x+a) + 2*e*(-a*e+b*d)*\ln(b*x+a)/b^3$

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.92

$$\int \frac{(d+ex)^2}{a^2+2abx+b^2x^2} dx = \frac{be^2x - \frac{(bd-ae)^2}{a+bx} + 2e(bd-ae)\log(a+bx)}{b^3}$$

input `Integrate[(d + e*x)^2/(a^2 + 2*a*b*x + b^2*x^2),x]`

output $(b*e^2*x - (b*d - a*e)^2/(a + b*x) + 2*e*(b*d - a*e)*\text{Log}[a + b*x])/b^3$

Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {1098, 27, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\begin{aligned}
 & \int \frac{(d+ex)^2}{a^2+2abx+b^2x^2} dx \\
 & \quad \downarrow \text{1098} \\
 & b^2 \int \frac{(d+ex)^2}{b^2(a+bx)^2} dx \\
 & \quad \downarrow \text{27} \\
 & \int \frac{(d+ex)^2}{(a+bx)^2} dx \\
 & \quad \downarrow \text{49} \\
 & \int \left(\frac{2e(bd-ae)}{b^2(a+bx)} + \frac{(bd-ae)^2}{b^2(a+bx)^2} + \frac{e^2}{b^2} \right) dx \\
 & \quad \downarrow \text{2009} \\
 & -\frac{(bd-ae)^2}{b^3(a+bx)} + \frac{2e(bd-ae)\log(a+bx)}{b^3} + \frac{e^2x}{b^2}
 \end{aligned}$$

input `Int[(d + e*x)^2/(a^2 + 2*a*b*x + b^2*x^2), x]`

output `(e^2*x)/b^2 - (b*d - a*e)^2/(b^3*(a + b*x)) + (2*e*(b*d - a*e)*Log[a + b*x])/b^3`

Defintions of rubi rules used

rule 27 `Int[(a_)*(Fx), x_Symbol] := Simp[a Int[Fx, x], x] /; FreeQ[a, x] && !MatchQ[Fx, (b_)*(Gx) /; FreeQ[b, x]]`

rule 49 `Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] := Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] && IGtQ[m, 0] && IGtQ[m + n + 2, 0]`

rule 1098 `Int[((d_.) + (e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] := Simp[1/c^p Int[(d + e*x)^m*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]`

rule 2009 `Int[u_, x_Symbol] := Simp[IntSum[u, x], x] /; SumQ[u]`

Maple [A] (verified)

Time = 1.05 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.24

| method | result | size |
|---------------|---|------|
| default | $\frac{e^2 x}{b^2} - \frac{e^2 a^2 - 2abde + b^2 d^2}{b^3 (bx+a)} - \frac{2e(ae-bd) \ln(bx+a)}{b^3}$ | 63 |
| norman | $\frac{\frac{e^2 x^2}{b} - \frac{2e^2 a^2 - 2abde + b^2 d^2}{b^3}}{bx+a} - \frac{2e(ae-bd) \ln(bx+a)}{b^3}$ | 68 |
| risch | $\frac{e^2 x}{b^2} - \frac{e^2 a^2}{b^3 (bx+a)} + \frac{2ade}{b^2 (bx+a)} - \frac{d^2}{b (bx+a)} - \frac{2e^2 \ln(bx+a)a}{b^3} + \frac{2e \ln(bx+a)d}{b^2}$ | 86 |
| parallelrisch | $-\frac{2 \ln(bx+a) x a b e^2 - 2 \ln(bx+a) x b^2 d e - b^2 e^2 x^2 + 2 \ln(bx+a) a^2 e^2 - 2 \ln(bx+a) a b d e + 2 e^2 a^2 - 2 a b d e + b^2 d^2}{b^3 (bx+a)}$ | 100 |

input `int((e*x+d)^2/(b^2*x^2+2*a*b*x+a^2), x, method=_RETURNVERBOSE)`

output `e^2*x/b^2-1/b^3*(a^2*e^2-2*a*b*d*e+b^2*d^2)/(b*x+a)-2/b^3*e*(a*e-b*d)*ln(b*x+a)`

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.80

$$\int \frac{(d+ex)^2}{a^2+2abx+b^2x^2} dx = \frac{b^2e^2x^2 + abe^2x - b^2d^2 + 2abde - a^2e^2 + 2(abde - a^2e^2 + (b^2de - abe^2)x) \log(bx+a)}{b^4x + ab^3}$$

input `integrate((e*x+d)^2/(b^2*x^2+2*a*b*x+a^2),x, algorithm="fricas")`output `(b^2*e^2*x^2 + a*b*e^2*x - b^2*d^2 + 2*a*b*d*e - a^2*e^2 + 2*(a*b*d*e - a^2*e^2 + (b^2*d*e - a*b*e^2)*x)*log(b*x + a))/(b^4*x + a*b^3)`**Sympy [A] (verification not implemented)**

Time = 0.17 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.18

$$\int \frac{(d+ex)^2}{a^2+2abx+b^2x^2} dx = \frac{-a^2e^2 + 2abde - b^2d^2}{ab^3 + b^4x} + \frac{e^2x}{b^2} - \frac{2e(ae - bd) \log(a+bx)}{b^3}$$

input `integrate((e*x+d)**2/(b**2*x**2+2*a*b*x+a**2),x)`output `(-a**2*e**2 + 2*a*b*d*e - b**2*d**2)/(a*b**3 + b**4*x) + e**2*x/b**2 - 2*e*(a*e - b*d)*log(a + b*x)/b**3`**Maxima [A] (verification not implemented)**

Time = 0.03 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.31

$$\int \frac{(d+ex)^2}{a^2+2abx+b^2x^2} dx = \frac{e^2x}{b^2} - \frac{b^2d^2 - 2abde + a^2e^2}{b^4x + ab^3} + \frac{2(bde - ae^2) \log(bx+a)}{b^3}$$

input `integrate((e*x+d)^2/(b^2*x^2+2*a*b*x+a^2),x, algorithm="maxima")`

output

$$e^{2x}/b^2 - (b^2d^2 - 2abde + a^2e^2)/(b^4x + ab^3) + 2(bde - a^2e^2)\log(bx + a)/b^3$$
Giac [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.27

$$\int \frac{(d + ex)^2}{a^2 + 2abx + b^2x^2} dx = \frac{e^2x}{b^2} + \frac{2(bde - ae^2)\log(|bx + a|)}{b^3} - \frac{b^2d^2 - 2abde + a^2e^2}{(bx + a)b^3}$$

input

```
integrate((e*x+d)^2/(b^2*x^2+2*a*b*x+a^2),x, algorithm="giac")
```

output

$$e^{2x}/b^2 + 2(bde - a^2e^2)\log(\text{abs}(bx + a))/b^3 - (b^2d^2 - 2abde + a^2e^2)/((bx + a)b^3)$$
Mupad [B] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.39

$$\int \frac{(d + ex)^2}{a^2 + 2abx + b^2x^2} dx = \frac{e^2x}{b^2} - \frac{a^2e^2 - 2abde + b^2d^2}{b(xb^3 + ab^2)} - \frac{\ln(a + bx)(2ae^2 - 2bde)}{b^3}$$

input

```
int((d + e*x)^2/(a^2 + b^2*x^2 + 2*a*b*x),x)
```

output

$$(e^{2x})/b^2 - (a^2e^2 + b^2d^2 - 2abde)/(b(a^2 + b^3x)) - (\log(a + bx)(2ae^2 - 2bde))/b^3$$

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 112, normalized size of antiderivative = 2.20

$$\int \frac{(d + ex)^2}{a^2 + 2abx + b^2x^2} dx$$

$$= \frac{-2 \log(bx + a) a^3 e^2 + 2 \log(bx + a) a^2 b d e - 2 \log(bx + a) a^2 b e^2 x + 2 \log(bx + a) a b^2 d e x + 2 a^2 b e^2 x - 2 a^2 b^2 d e x^2}{a b^3 (bx + a)}$$

input `int((e*x+d)^2/(b^2*x^2+2*a*b*x+a^2),x)`output `(- 2*log(a + b*x)*a**3*e**2 + 2*log(a + b*x)*a**2*b*d*e - 2*log(a + b*x)*a**2*b*e**2*x + 2*log(a + b*x)*a*b**2*d*e*x + 2*a**2*b*e**2*x - 2*a*b**2*d*e*x + a*b**2*e**2*x**2 + b**3*d**2*x)/(a*b**3*(a + b*x))`

3.112 $\int \frac{d+ex}{a^2+2abx+b^2x^2} dx$

| | |
|---|-----|
| Optimal result | 912 |
| Mathematica [A] (verified) | 912 |
| Rubi [A] (verified) | 913 |
| Maple [A] (verified) | 914 |
| Fricas [A] (verification not implemented) | 915 |
| Sympy [A] (verification not implemented) | 915 |
| Maxima [A] (verification not implemented) | 915 |
| Giac [A] (verification not implemented) | 916 |
| Mupad [B] (verification not implemented) | 916 |
| Reduce [B] (verification not implemented) | 916 |

Optimal result

Integrand size = 24, antiderivative size = 32

$$\int \frac{d+ex}{a^2+2abx+b^2x^2} dx = -\frac{bd-ae}{b^2(a+bx)} + \frac{e \log(a+bx)}{b^2}$$

output

```
-(-a*e+b*d)/b^2/(b*x+a)+e*ln(b*x+a)/b^2
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.97

$$\int \frac{d+ex}{a^2+2abx+b^2x^2} dx = \frac{-bd+ae}{b^2(a+bx)} + \frac{e \log(a+bx)}{b^2}$$

input

```
Integrate[(d + e*x)/(a^2 + 2*a*b*x + b^2*x^2), x]
```

output

```
(-(b*d) + a*e)/(b^2*(a + b*x)) + (e*Log[a + b*x])/b^2
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {1098, 27, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\begin{aligned}
 & \int \frac{d + ex}{a^2 + 2abx + b^2x^2} dx \\
 & \quad \downarrow \text{1098} \\
 & b^2 \int \frac{d + ex}{b^2(a + bx)^2} dx \\
 & \quad \downarrow \text{27} \\
 & \int \frac{d + ex}{(a + bx)^2} dx \\
 & \quad \downarrow \text{49} \\
 & \int \left(\frac{bd - ae}{b(a + bx)^2} + \frac{e}{b(a + bx)} \right) dx \\
 & \quad \downarrow \text{2009} \\
 & \frac{e \log(a + bx)}{b^2} - \frac{bd - ae}{b^2(a + bx)}
 \end{aligned}$$

input `Int[(d + e*x)/(a^2 + 2*a*b*x + b^2*x^2),x]`

output `-((b*d - a*e)/(b^2*(a + b*x))) + (e*Log[a + b*x])/b^2`

Definitions of rubi rules used

rule 27 $\text{Int}[(a_*)(Fx_), x_Symbol] \rightarrow \text{Simp}[a \text{ Int}[Fx, x], x] /; \text{FreeQ}[a, x] \ \&\& \ !\text{MatchQ}[Fx, (b_*)(Gx_)] /; \text{FreeQ}[b, x]$

rule 49 $\text{Int}[((a_.) + (b_.)(x_))^{(m_.)}((c_.) + (d_.)(x_))^{(n_.)}, x_Symbol] \rightarrow \text{Int}[\text{ExpandIntegrand}[(a + b*x)^m*(c + d*x)^n, x], x] /; \text{FreeQ}\{a, b, c, d\}, x \ \&\& \ \text{IGtQ}[m, 0] \ \&\& \ \text{IGtQ}[m + n + 2, 0]$

rule 1098 $\text{Int}[((d_.) + (e_.)(x_))^{(m_.)}((a_) + (b_.)(x_) + (c_.)(x_)^2)^{(p_.)}, x_Symbol] \rightarrow \text{Simp}[1/c^p \text{ Int}[(d + e*x)^m*(b/2 + c*x)^{(2*p)}, x], x] /; \text{FreeQ}\{a, b, c, d, e, m\}, x \ \&\& \ \text{EqQ}[b^2 - 4*a*c, 0] \ \&\& \ \text{IntegerQ}[p]$

rule 2009 $\text{Int}[u_, x_Symbol] \rightarrow \text{Simp}[\text{IntSum}[u, x], x] /; \text{SumQ}[u]$

Maple [A] (verified)

Time = 0.95 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.00

| method | result | size |
|---------------|--|------|
| norman | $\frac{ae-bd}{b^2(bx+a)} + \frac{e \ln(bx+a)}{b^2}$ | 32 |
| default | $-\frac{-ae+bd}{b^2(bx+a)} + \frac{e \ln(bx+a)}{b^2}$ | 33 |
| risch | $\frac{ae}{b^2(bx+a)} - \frac{d}{b(bx+a)} + \frac{e \ln(bx+a)}{b^2}$ | 39 |
| parallelrisch | $\frac{\ln(bx+a)xb + \ln(bx+a)ae + ae - bd}{b^2(bx+a)}$ | 39 |

input $\text{int}((e*x+d)/(b^2*x^2+2*a*b*x+a^2), x, \text{method}=_RETURNVERBOSE)$

output $(a*e-b*d)/b^2/(b*x+a)+e*\ln(b*x+a)/b^2$

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.22

$$\int \frac{d + ex}{a^2 + 2abx + b^2x^2} dx = -\frac{bd - ae - (bex + ae) \log (bx + a)}{b^3x + ab^2}$$

input `integrate((e*x+d)/(b^2*x^2+2*a*b*x+a^2),x, algorithm="fricas")`output `-(b*d - a*e - (b*e*x + a*e)*log(b*x + a))/(b^3*x + a*b^2)`**Sympy [A] (verification not implemented)**

Time = 0.09 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.84

$$\int \frac{d + ex}{a^2 + 2abx + b^2x^2} dx = \frac{ae - bd}{ab^2 + b^3x} + \frac{e \log (a + bx)}{b^2}$$

input `integrate((e*x+d)/(b**2*x**2+2*a*b*x+a**2),x)`output `(a*e - b*d)/(a*b**2 + b**3*x) + e*log(a + b*x)/b**2`**Maxima [A] (verification not implemented)**

Time = 0.03 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.09

$$\int \frac{d + ex}{a^2 + 2abx + b^2x^2} dx = -\frac{bd - ae}{b^3x + ab^2} + \frac{e \log (bx + a)}{b^2}$$

input `integrate((e*x+d)/(b^2*x^2+2*a*b*x+a^2),x, algorithm="maxima")`output `-(b*d - a*e)/(b^3*x + a*b^2) + e*log(b*x + a)/b^2`

Giac [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.03

$$\int \frac{d + ex}{a^2 + 2abx + b^2x^2} dx = \frac{e \log(|bx + a|)}{b^2} - \frac{bd - ae}{(bx + a)b^2}$$

input `integrate((e*x+d)/(b^2*x^2+2*a*b*x+a^2),x, algorithm="giac")`output `e*log(abs(b*x + a))/b^2 - (b*d - a*e)/((b*x + a)*b^2)`**Mupad [B] (verification not implemented)**

Time = 9.02 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.97

$$\int \frac{d + ex}{a^2 + 2abx + b^2x^2} dx = \frac{ae - bd}{b^2(a + bx)} + \frac{e \ln(a + bx)}{b^2}$$

input `int((d + e*x)/(a^2 + b^2*x^2 + 2*a*b*x),x)`output `(a*e - b*d)/(b^2*(a + b*x)) + (e*log(a + b*x))/b^2`**Reduce [B] (verification not implemented)**

Time = 0.21 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.53

$$\int \frac{d + ex}{a^2 + 2abx + b^2x^2} dx = \frac{\log(bx + a) a^2 e + \log(bx + a) abex - abex + b^2 dx}{a b^2 (bx + a)}$$

input `int((e*x+d)/(b^2*x^2+2*a*b*x+a^2),x)`output `(log(a + b*x)*a**2*e + log(a + b*x)*a*b*e*x - a*b*e*x + b**2*d*x)/(a*b**2*(a + b*x))`

3.113 $\int \frac{1}{a^2+2abx+b^2x^2} dx$

| | |
|---|-----|
| Optimal result | 917 |
| Mathematica [A] (verified) | 917 |
| Rubi [A] (verified) | 918 |
| Maple [A] (verified) | 919 |
| Fricas [A] (verification not implemented) | 919 |
| Sympy [A] (verification not implemented) | 920 |
| Maxima [A] (verification not implemented) | 920 |
| Giac [A] (verification not implemented) | 920 |
| Mupad [B] (verification not implemented) | 921 |
| Reduce [B] (verification not implemented) | 921 |

Optimal result

Integrand size = 18, antiderivative size = 12

$$\int \frac{1}{a^2 + 2abx + b^2x^2} dx = -\frac{1}{b(a + bx)}$$

output `-1/b/(b*x+a)`

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00

$$\int \frac{1}{a^2 + 2abx + b^2x^2} dx = -\frac{1}{b(a + bx)}$$

input `Integrate[(a^2 + 2*a*b*x + b^2*x^2)^(-1),x]`

output `-(1/(b*(a + b*x)))`

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {1077, 17}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{a^2 + 2abx + b^2x^2} dx$$

$$\downarrow 1077$$

$$b^2 \int \frac{1}{(xb^2 + ab)^2} dx$$

$$\downarrow 17$$

$$-\frac{1}{b(a + bx)}$$

input `Int[(a^2 + 2*a*b*x + b^2*x^2)^(-1),x]`

output `-(1/(b*(a + b*x)))`

Defintions of rubi rules used

rule 17 `Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1))/(b*(m + 1)), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]`

rule 1077 `Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[1/c^p Int[(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]`

Maple [A] (verified)

Time = 0.92 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.08

| method | result | size |
|--------------|------------------------------------|------|
| gosper | $-\frac{1}{b(bx+a)}$ | 13 |
| default | $-\frac{1}{b(bx+a)}$ | 13 |
| norman | $\frac{x}{a(bx+a)}$ | 13 |
| risch | $-\frac{1}{b(bx+a)}$ | 13 |
| parallelrisc | $-\frac{1}{b(bx+a)}$ | 13 |
| orering | $-\frac{bx+a}{b(b^2x^2+2abx+a^2)}$ | 29 |

input `int(1/(b^2*x^2+2*a*b*x+a^2),x,method=_RETURNVERBOSE)`output `-1/b/(b*x+a)`**Fricas [A] (verification not implemented)**

Time = 0.08 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.08

$$\int \frac{1}{a^2 + 2abx + b^2x^2} dx = -\frac{1}{b^2x + ab}$$

input `integrate(1/(b^2*x^2+2*a*b*x+a^2),x, algorithm="fricas")`output `-1/(b^2*x + a*b)`

Sympy [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.83

$$\int \frac{1}{a^2 + 2abx + b^2x^2} dx = -\frac{1}{ab + b^2x}$$

input `integrate(1/(b**2*x**2+2*a*b*x+a**2),x)`output `-1/(a*b + b**2*x)`**Maxima [A] (verification not implemented)**

Time = 0.04 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.08

$$\int \frac{1}{a^2 + 2abx + b^2x^2} dx = -\frac{1}{b^2x + ab}$$

input `integrate(1/(b^2*x^2+2*a*b*x+a^2),x, algorithm="maxima")`output `-1/(b^2*x + a*b)`**Giac [A] (verification not implemented)**

Time = 0.22 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00

$$\int \frac{1}{a^2 + 2abx + b^2x^2} dx = -\frac{1}{(bx + a)b}$$

input `integrate(1/(b^2*x^2+2*a*b*x+a^2),x, algorithm="giac")`output `-1/((b*x + a)*b)`

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00

$$\int \frac{1}{a^2 + 2abx + b^2x^2} dx = -\frac{1}{b(a + bx)}$$

input `int(1/(a^2 + b^2*x^2 + 2*a*b*x),x)`

output `-1/(b*(a + b*x))`

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00

$$\int \frac{1}{a^2 + 2abx + b^2x^2} dx = \frac{x}{a(bx + a)}$$

input `int(1/(b^2*x^2+2*a*b*x+a^2),x)`

output `x/(a*(a + b*x))`

3.114 $\int \frac{1}{(d+ex)(a^2+2abx+b^2x^2)} dx$

| | |
|---|-----|
| Optimal result | 922 |
| Mathematica [A] (verified) | 922 |
| Rubi [A] (verified) | 923 |
| Maple [A] (verified) | 924 |
| Fricas [A] (verification not implemented) | 925 |
| Sympy [B] (verification not implemented) | 925 |
| Maxima [A] (verification not implemented) | 926 |
| Giac [A] (verification not implemented) | 926 |
| Mupad [B] (verification not implemented) | 927 |
| Reduce [B] (verification not implemented) | 927 |

Optimal result

Integrand size = 26, antiderivative size = 57

$$\int \frac{1}{(d+ex)(a^2+2abx+b^2x^2)} dx = -\frac{1}{(bd-ae)(a+bx)} - \frac{e \log(a+bx)}{(bd-ae)^2} + \frac{e \log(d+ex)}{(bd-ae)^2}$$

output `-1/(-a*e+b*d)/(b*x+a)-e*ln(b*x+a)/(-a*e+b*d)^2+e*ln(e*x+d)/(-a*e+b*d)^2`

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.93

$$\int \frac{1}{(d+ex)(a^2+2abx+b^2x^2)} dx = \frac{-bd+ae-e(a+bx)\log(a+bx)+e(a+bx)\log(d+ex)}{(bd-ae)^2(a+bx)}$$

input `Integrate[1/((d+e*x)*(a^2+2*a*b*x+b^2*x^2)),x]`

output `(-(b*d)+a*e-e*(a+b*x)*Log[a+b*x]+e*(a+b*x)*Log[d+e*x])/((b*d-a*e)^2*(a+b*x))`

Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {1098, 27, 54, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\begin{aligned}
 & \int \frac{1}{(a^2 + 2abx + b^2x^2)(d + ex)} dx \\
 & \quad \downarrow 1098 \\
 & b^2 \int \frac{1}{b^2(a + bx)^2(d + ex)} dx \\
 & \quad \downarrow 27 \\
 & \int \frac{1}{(a + bx)^2(d + ex)} dx \\
 & \quad \downarrow 54 \\
 & \int \left(\frac{e^2}{(d + ex)(bd - ae)^2} - \frac{be}{(a + bx)(bd - ae)^2} + \frac{b}{(a + bx)^2(bd - ae)} \right) dx \\
 & \quad \downarrow 2009 \\
 & -\frac{1}{(a + bx)(bd - ae)} - \frac{e \log(a + bx)}{(bd - ae)^2} + \frac{e \log(d + ex)}{(bd - ae)^2}
 \end{aligned}$$

input `Int[1/((d + e*x)*(a^2 + 2*a*b*x + b^2*x^2)),x]`

output `-(1/((b*d - a*e)*(a + b*x))) - (e*Log[a + b*x])/(b*d - a*e)^2 + (e*Log[d + e*x])/(b*d - a*e)^2`

Definitions of rubi rules used

rule 27 $\text{Int}[(a_*)(Fx_), x_Symbol] \rightarrow \text{Simp}[a \text{ Int}[Fx, x], x] /; \text{FreeQ}[a, x] \ \&\& \ !\text{MatchQ}[Fx, (b_*)(Gx_)] /; \text{FreeQ}[b, x]$

rule 54 $\text{Int}[((a_) + (b_*)(x_))^{(m_)*}((c_.) + (d_.)(x_))^{(n_.)}, x_Symbol] \rightarrow \text{Int}[\text{ExpandIntegrand}[(a + b*x)^m*(c + d*x)^n, x], x] /; \text{FreeQ}[\{a, b, c, d\}, x] \ \&\& \ \text{ILtQ}[m, 0] \ \&\& \ \text{IntegerQ}[n] \ \&\& \ !(\text{IGtQ}[n, 0] \ \&\& \ \text{LtQ}[m + n + 2, 0])$

rule 1098 $\text{Int}[((d_.) + (e_.)(x_))^{(m_.)*}((a_) + (b_.)(x_) + (c_.)(x_)^2)^{(p_.)}, x_Symbol] \rightarrow \text{Simp}[1/c^p \text{ Int}[(d + e*x)^m*(b/2 + c*x)^{(2*p)}, x], x] /; \text{FreeQ}[\{a, b, c, d, e, m\}, x] \ \&\& \ \text{EqQ}[b^2 - 4*a*c, 0] \ \&\& \ \text{IntegerQ}[p]$

rule 2009 $\text{Int}[u_, x_Symbol] \rightarrow \text{Simp}[\text{IntSum}[u, x], x] /; \text{SumQ}[u]$

Maple [A] (verified)

Time = 1.23 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.00

| method | result | size |
|---------------|--|------|
| default | $\frac{1}{(ae-bd)(bx+a)} - \frac{e \ln(bx+a)}{(ae-bd)^2} + \frac{e \ln(ex+d)}{(ae-bd)^2}$ | 57 |
| risch | $\frac{1}{(ae-bd)(bx+a)} + \frac{e \ln(-ex-d)}{e^2 a^2 - 2abde + b^2 d^2} - \frac{e \ln(bx+a)}{e^2 a^2 - 2abde + b^2 d^2}$ | 86 |
| norman | $-\frac{bx}{a(ae-bd)(bx+a)} + \frac{e \ln(ex+d)}{e^2 a^2 - 2abde + b^2 d^2} - \frac{e \ln(bx+a)}{e^2 a^2 - 2abde + b^2 d^2}$ | 89 |
| parallelrisch | $-\frac{\ln(bx+a)xabe - \ln(ex+d)xabe + \ln(bx+a)a^2e - \ln(ex+d)a^2e + abex - b^2 dx}{(e^2 a^2 - 2abde + b^2 d^2)(bx+a)a}$ | 95 |

input $\text{int}(1/(e*x+d)/(b^2*x^2+2*a*b*x+a^2), x, \text{method}=_RETURNVERBOSE)$

output $1/(a*e-b*d)/(b*x+a) - e/(a*e-b*d)^2 * \ln(b*x+a) + e/(a*e-b*d)^2 * \ln(e*x+d)$

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 93, normalized size of antiderivative = 1.63

$$\int \frac{1}{(d+ex)(a^2+2abx+b^2x^2)} dx$$

$$= -\frac{bd-ae+(bex+ae)\log(bx+a)-(bex+ae)\log(ex+d)}{ab^2d^2-2a^2bde+a^3e^2+(b^3d^2-2ab^2de+a^2be^2)x}$$

input `integrate(1/(e*x+d)/(b^2*x^2+2*a*b*x+a^2),x, algorithm="fricas")`

output `-(b*d - a*e + (b*e*x + a*e)*log(b*x + a) - (b*e*x + a*e)*log(e*x + d))/(a*b^2*d^2 - 2*a^2*b*d*e + a^3*e^2 + (b^3*d^2 - 2*a*b^2*d*e + a^2*b*e^2)*x)`

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 233 vs. 2(46) = 92.

Time = 0.34 (sec) , antiderivative size = 233, normalized size of antiderivative = 4.09

$$\int \frac{1}{(d+ex)(a^2+2abx+b^2x^2)} dx$$

$$= \frac{e \log \left(x + \frac{-\frac{a^3e^4}{(ae-bd)^2} + \frac{3a^2bde^3}{(ae-bd)^2} - \frac{3ab^2d^2e^2}{(ae-bd)^2} + ae^2 + \frac{b^3d^3e}{(ae-bd)^2} + bde}{(ae-bd)^2} \right)}{(ae-bd)^2}$$

$$- \frac{e \log \left(x + \frac{\frac{a^3e^4}{(ae-bd)^2} - \frac{3a^2bde^3}{(ae-bd)^2} + \frac{3ab^2d^2e^2}{(ae-bd)^2} + ae^2 - \frac{b^3d^3e}{(ae-bd)^2} + bde}{2be^2} \right)}{(ae-bd)^2} + \frac{1}{a^2e - abd + x(abe - b^2d)}$$

input `integrate(1/(e*x+d)/(b**2*x**2+2*a*b*x+a**2),x)`

output

```
e*log(x + (-a**3*e**4/(a*e - b*d)**2 + 3*a**2*b*d*e**3/(a*e - b*d)**2 - 3*
a*b**2*d**2*e**2/(a*e - b*d)**2 + a*e**2 + b**3*d**3*e/(a*e - b*d)**2 + b*
d*e)/(2*b*e**2))/(a*e - b*d)**2 - e*log(x + (a**3*e**4/(a*e - b*d)**2 - 3*
a**2*b*d*e**3/(a*e - b*d)**2 + 3*a*b**2*d**2*e**2/(a*e - b*d)**2 + a*e**2
- b**3*d**3*e/(a*e - b*d)**2 + b*d*e)/(2*b*e**2))/(a*e - b*d)**2 + 1/(a**2
*e - a*b*d + x*(a*b*e - b**2*d))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.61

$$\int \frac{1}{(d+ex)(a^2+2abx+b^2x^2)} dx = -\frac{e \log (bx+a)}{b^2 d^2-2 abde+a^2 e^2} + \frac{e \log (ex+d)}{b^2 d^2-2 abde+a^2 e^2} - \frac{1}{abd-a^2 e+(b^2 d-abe)x}$$

input

```
integrate(1/(e*x+d)/(b^2*x^2+2*a*b*x+a^2),x, algorithm="maxima")
```

output

```
-e*log(b*x + a)/(b^2*d^2 - 2*a*b*d*e + a^2*e^2) + e*log(e*x + d)/(b^2*d^2
- 2*a*b*d*e + a^2*e^2) - 1/(a*b*d - a^2*e + (b^2*d - a*b*e)*x)
```

Giac [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.65

$$\int \frac{1}{(d+ex)(a^2+2abx+b^2x^2)} dx = -\frac{be \log (|bx+a|)}{b^3 d^2-2 ab^2 de+a^2 be^2} + \frac{e^2 \log (|ex+d|)}{b^2 d^2 e-2 abde^2+a^2 e^3} - \frac{1}{(bd-ae)(bx+a)}$$

input

```
integrate(1/(e*x+d)/(b^2*x^2+2*a*b*x+a^2),x, algorithm="giac")
```

output

```
-b*e*log(abs(b*x + a))/(b^3*d^2 - 2*a*b^2*d*e + a^2*b*e^2) + e^2*log(abs(e
*x + d))/(b^2*d^2*e - 2*a*b*d*e^2 + a^2*e^3) - 1/((b*d - a*e)*(b*x + a))
```

Mupad [B] (verification not implemented)

Time = 8.93 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.33

$$\int \frac{1}{(d+ex)(a^2+2abx+b^2x^2)} dx = \frac{1}{(ae-bd)(a+bx)} - \frac{2e \operatorname{atanh}\left(\frac{a^2e^2-b^2d^2}{(ae-bd)^2} + \frac{2bex}{ae-bd}\right)}{(ae-bd)^2}$$

input `int(1/((d + e*x)*(a^2 + b^2*x^2 + 2*a*b*x)),x)`output `1/((a*e - b*d)*(a + b*x)) - (2*e*atanh((a^2*e^2 - b^2*d^2)/(a*e - b*d)^2 + (2*b*e*x)/(a*e - b*d)))/(a*e - b*d)^2`**Reduce [B] (verification not implemented)**

Time = 0.20 (sec) , antiderivative size = 115, normalized size of antiderivative = 2.02

$$\int \frac{1}{(d+ex)(a^2+2abx+b^2x^2)} dx = \frac{-\log(bx+a)a^2e - \log(bx+a)abex + \log(ex+d)a^2e + \log(ex+d)abex - abex + b^2dx}{a(a^2be^2x - 2ab^2dex + b^3d^2x + a^3e^2 - 2a^2bde + ab^2d^2)}$$

input `int(1/(e*x+d)/(b^2*x^2+2*a*b*x+a^2),x)`output `(- log(a + b*x)*a**2*e - log(a + b*x)*a*b*e*x + log(d + e*x)*a**2*e + log(d + e*x)*a*b*e*x - a*b*e*x + b**2*d*x)/(a*(a**3*e**2 - 2*a**2*b*d*e + a**2*b*e**2*x + a*b**2*d**2 - 2*a*b**2*d*e*x + b**3*d**2*x))`

3.115 $\int \frac{1}{(d+ex)^2(a^2+2abx+b^2x^2)} dx$

| | |
|---|-----|
| Optimal result | 928 |
| Mathematica [A] (verified) | 928 |
| Rubi [A] (verified) | 929 |
| Maple [A] (verified) | 930 |
| Fricas [B] (verification not implemented) | 931 |
| Sympy [B] (verification not implemented) | 931 |
| Maxima [B] (verification not implemented) | 932 |
| Giac [A] (verification not implemented) | 933 |
| Mupad [B] (verification not implemented) | 933 |
| Reduce [B] (verification not implemented) | 934 |

Optimal result

Integrand size = 26, antiderivative size = 81

$$\int \frac{1}{(d+ex)^2(a^2+2abx+b^2x^2)} dx = -\frac{b}{(bd-ae)^2(a+bx)} - \frac{e}{(bd-ae)^2(d+ex)} - \frac{2be \log(a+bx)}{(bd-ae)^3} + \frac{2be \log(d+ex)}{(bd-ae)^3}$$

output

```
-b/(-a*e+b*d)^2/(b*x+a)-e/(-a*e+b*d)^2/(e*x+d)-2*b*e*ln(b*x+a)/(-a*e+b*d)^3+2*b*e*ln(e*x+d)/(-a*e+b*d)^3
```

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.81

$$\int \frac{1}{(d+ex)^2(a^2+2abx+b^2x^2)} dx = \frac{\frac{b(-bd+ae)}{a+bx} + \frac{e(-bd+ae)}{d+ex} - 2be \log(a+bx) + 2be \log(d+ex)}{(bd-ae)^3}$$

input

```
Integrate[1/((d + e*x)^2*(a^2 + 2*a*b*x + b^2*x^2)),x]
```

output $((b*(-(b*d) + a*e))/(a + b*x) + (e*(-(b*d) + a*e))/(d + e*x) - 2*b*e*Log[a + b*x] + 2*b*e*Log[d + e*x])/(b*d - a*e)^3$

Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {1098, 27, 54, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a^2 + 2abx + b^2x^2)(d + ex)^2} dx$$

↓ 1098

$$b^2 \int \frac{1}{b^2(a + bx)^2(d + ex)^2} dx$$

↓ 27

$$\int \frac{1}{(a + bx)^2(d + ex)^2} dx$$

↓ 54

$$\int \left(-\frac{2b^2e}{(a + bx)(bd - ae)^3} + \frac{b^2}{(a + bx)^2(bd - ae)^2} + \frac{2be^2}{(d + ex)(bd - ae)^3} + \frac{e^2}{(d + ex)^2(bd - ae)^2} \right) dx$$

↓ 2009

$$-\frac{b}{(a + bx)(bd - ae)^2} - \frac{e}{(d + ex)(bd - ae)^2} - \frac{2be \log(a + bx)}{(bd - ae)^3} + \frac{2be \log(d + ex)}{(bd - ae)^3}$$

input `Int[1/((d + e*x)^2*(a^2 + 2*a*b*x + b^2*x^2)),x]`

output $-(b/((b*d - a*e)^2*(a + b*x))) - e/((b*d - a*e)^2*(d + e*x)) - (2*b*e*Log[a + b*x])/(b*d - a*e)^3 + (2*b*e*Log[d + e*x])/(b*d - a*e)^3$

Defintions of rubi rules used

rule 27 $\text{Int}[(a_*)(Fx_), x_Symbol] \rightarrow \text{Simp}[a \text{ Int}[Fx, x], x] /; \text{FreeQ}[a, x] \ \&\& \ !\text{MatchQ}[Fx, (b_*)(Gx_)] /; \text{FreeQ}[b, x]$

rule 54 $\text{Int}[(a_*) + (b_*)(x_)]^{(m_*)} * ((c_*) + (d_*)(x_)]^{(n_*)}, x_Symbol] \rightarrow \text{Int}[\text{ExpandIntegrand}[(a + b*x)^m * (c + d*x)^n, x], x] /; \text{FreeQ}[\{a, b, c, d\}, x] \ \&\& \ \text{IntegerQ}[m, 0] \ \&\& \ \text{IntegerQ}[n] \ \&\& \ !(\text{IGtQ}[n, 0] \ \&\& \ \text{LtQ}[m + n + 2, 0])$

rule 1098 $\text{Int}[(d_*) + (e_*)(x_)]^{(m_*)} * ((a_*) + (b_*)(x_*) + (c_*)(x_*)^2)^{(p_*)}, x_Symbol] \rightarrow \text{Simp}[1/c^p \text{ Int}[(d + e*x)^m * (b/2 + c*x)^(2*p), x], x] /; \text{FreeQ}[\{a, b, c, d, e, m\}, x] \ \&\& \ \text{EqQ}[b^2 - 4*a*c, 0] \ \&\& \ \text{IntegerQ}[p]$

rule 2009 $\text{Int}[u_, x_Symbol] \rightarrow \text{Simp}[\text{IntSum}[u, x], x] /; \text{SumQ}[u]$

Maple [A] (verified)

Time = 1.26 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.01

| method | result |
|---------------|---|
| default | $-\frac{b}{(ae-bd)^2(bx+a)} + \frac{2be \ln(bx+a)}{(ae-bd)^3} - \frac{e}{(ae-bd)^2(e+x+d)} - \frac{2be \ln(ex+d)}{(ae-bd)^3}$ |
| risch | $\frac{-\frac{2bex}{e^2a^2-2abde+b^2d^2} - \frac{ae+bd}{e^2a^2-2abde+b^2d^2}}{(bx+a)(ex+d)} + \frac{2be \ln(-bx-a)}{e^3a^3-3a^2bde^2+3ab^2d^2e-b^3d^3} - \frac{2be \ln(ex+d)}{e^3a^3-3a^2bde^2+3ab^2d^2e-b^3d^3}$ |
| norman | $\frac{-\frac{ab e^2 - de b^2}{eb(e^2a^2-2abde+b^2d^2)} - \frac{2bex}{e^2a^2-2abde+b^2d^2}}{(bx+a)(ex+d)} + \frac{2be \ln(bx+a)}{e^3a^3-3a^2bde^2+3ab^2d^2e-b^3d^3} - \frac{2be \ln(ex+d)}{e^3a^3-3a^2bde^2+3ab^2d^2e-b^3d^3}$ |
| parallelrisch | $\frac{2 \ln(bx+a)x^2b^3e^3 - 2 \ln(ex+d)x^2b^3e^3 + 2 \ln(bx+a)xa b^2e^3 + 2 \ln(bx+a)x b^3d e^2 - 2 \ln(ex+d)xa b^2e^3 - 2 \ln(ex+d)x b^3d e^2 + 2 \ln(ex+d)xb^3d e^2 + 2 \ln(ex+d)xb^3d e^2}{(e^3a^3-3a^2bde^2+3ab^2d^2e-b^3d^3)(bx+a)(ex+d)be}$ |

input `int(1/(e*x+d)^2/(b^2*x^2+2*a*b*x+a^2), x, method=_RETURNVERBOSE)`

output
$$-b/(a*e-b*d)^2/(b*x+a)+2*b/(a*e-b*d)^3*e*\ln(b*x+a)-e/(a*e-b*d)^2/(e*x+d)-*b/(a*e-b*d)^3*e*\ln(e*x+d)$$

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 241 vs. $2(81) = 162$.

Time = 0.09 (sec) , antiderivative size = 241, normalized size of antiderivative = 2.98

$$\int \frac{1}{(d+ex)^2(a^2+2abx+b^2x^2)} dx =$$

$$\frac{b^2d^2 - a^2e^2 + 2(b^2de - abe^2)x + 2(b^2e^2x^2 + abde + (b^2de + abe^2)x) \log(bx + a) - 2(b^2e^2x^2 + abde + ab^3d^4 - 3a^2b^2d^3e + 3a^3bd^2e^2 - a^4de^3 + (b^4d^3e - 3ab^3d^2e^2 + 3a^2b^2de^3 - a^3be^4)x^2 + (b^4d^4 - 2ab^3d^3e + 2a^2b^2d^2e^2 - a^3bd^2e^3 - a^4de^4)x}{ab^3d^4 - 3a^2b^2d^3e + 3a^3bd^2e^2 - a^4de^3 + (b^4d^3e - 3ab^3d^2e^2 + 3a^2b^2de^3 - a^3be^4)x^2 + (b^4d^4 - 2ab^3d^3e + 2a^2b^2d^2e^2 - a^3bd^2e^3 - a^4de^4)x}$$

input `integrate(1/(e*x+d)^2/(b^2*x^2+2*a*b*x+a^2),x, algorithm="fricas")`

output
$$\frac{-(b^2d^2 - a^2e^2 + 2(b^2d^2e - a^2b^2e^2)x + 2(b^2e^2x^2 + a^2b^2d^2e + (b^2d^2e + a^2b^2e^2)x) \log(bx + a) - 2(b^2e^2x^2 + a^2b^2d^2e + (b^2d^2e + a^2b^2e^2)x) \log(ex + d))}{(a^2b^3d^4 - 3a^2b^2d^3e + 3a^3bd^2e^2 - a^4de^3 + (b^4d^3e - 3ab^3d^2e^2 + 3a^2b^2de^3 - a^3be^4)x^2 + (b^4d^4 - 2a^2b^3d^3e + 2a^3bd^2e^2 - a^4de^4)x}$$

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 406 vs. $2(70) = 140$.

Time = 0.55 (sec) , antiderivative size = 406, normalized size of antiderivative = 5.01

$$\int \frac{1}{(d+ex)^2(a^2+2abx+b^2x^2)} dx$$

$$= -\frac{2be \log\left(x + \frac{-\frac{2a^4be^5}{(ae-bd)^3} + \frac{8a^3b^2de^4}{(ae-bd)^3} - \frac{12a^2b^3d^2e^3}{(ae-bd)^3} + \frac{8ab^4d^3e^2}{(ae-bd)^3} + 2abe^2 - \frac{2b^5d^4e}{(ae-bd)^3} + 2b^2de}{4b^2e^2}\right)}{(ae-bd)^3}$$

$$+ \frac{2be \log\left(x + \frac{\frac{2a^4be^5}{(ae-bd)^3} - \frac{8a^3b^2de^4}{(ae-bd)^3} + \frac{12a^2b^3d^2e^3}{(ae-bd)^3} - \frac{8ab^4d^3e^2}{(ae-bd)^3} + 2abe^2 + \frac{2b^5d^4e}{(ae-bd)^3} + 2b^2de}{4b^2e^2}\right)}{(ae-bd)^3}$$

$$+ \frac{-ae - bd - 2bex}{a^3de^2 - 2a^2bd^2e + ab^2d^3 + x^2(a^2be^3 - 2ab^2de^2 + b^3d^2e) + x(a^3e^3 - a^2bde^2 - ab^2d^2e + b^3d^3)}$$

input `integrate(1/(e*x+d)**2/(b**2*x**2+2*a*b*x+a**2),x)`

output

```
-2*b*e*log(x + (-2*a**4*b*e**5/(a*e - b*d)**3 + 8*a**3*b**2*d*e**4/(a*e -
b*d)**3 - 12*a**2*b**3*d**2*e**3/(a*e - b*d)**3 + 8*a*b**4*d**3*e**2/(a*e
- b*d)**3 + 2*a*b*e**2 - 2*b**5*d**4*e/(a*e - b*d)**3 + 2*b**2*d*e)/(4*b**
2*e**2))/(a*e - b*d)**3 + 2*b*e*log(x + (2*a**4*b*e**5/(a*e - b*d)**3 - 8*
a**3*b**2*d*e**4/(a*e - b*d)**3 + 12*a**2*b**3*d**2*e**3/(a*e - b*d)**3 -
8*a*b**4*d**3*e**2/(a*e - b*d)**3 + 2*a*b*e**2 + 2*b**5*d**4*e/(a*e - b*d)
**3 + 2*b**2*d*e)/(4*b**2*e**2))/(a*e - b*d)**3 + (-a*e - b*d - 2*b*e*x)/(
a**3*d*e**2 - 2*a**2*b*d**2*e + a*b**2*d**3 + x**2*(a**2*b*e**3 - 2*a*b**2
*d*e**2 + b**3*d**2*e) + x*(a**3*e**3 - a**2*b*d*e**2 - a*b**2*d**2*e + b
**3*d**3))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 208 vs. $2(81) = 162$.

Time = 0.04 (sec) , antiderivative size = 208, normalized size of antiderivative = 2.57

$$\int \frac{1}{(d+ex)^2(a^2+2abx+b^2x^2)} dx$$

$$= -\frac{2be \log(bx+a)}{b^3d^3 - 3ab^2d^2e + 3a^2bde^2 - a^3e^3} + \frac{2be \log(ex+d)}{b^3d^3 - 3ab^2d^2e + 3a^2bde^2 - a^3e^3}$$

$$-\frac{2bex + bd + ae}{ab^2d^3 - 2a^2bd^2e + a^3de^2 + (b^3d^2e - 2ab^2de^2 + a^2be^3)x^2 + (b^3d^3 - ab^2d^2e - a^2bde^2 + a^3e^3)x}$$

input

```
integrate(1/(e*x+d)^2/(b^2*x^2+2*a*b*x+a^2),x, algorithm="maxima")
```

output

```
-2*b*e*log(b*x + a)/(b^3*d^3 - 3*a*b^2*d^2*e + 3*a^2*b*d*e^2 - a^3*e^3) +
2*b*e*log(e*x + d)/(b^3*d^3 - 3*a*b^2*d^2*e + 3*a^2*b*d*e^2 - a^3*e^3) - (
2*b*e*x + b*d + a*e)/(a*b^2*d^3 - 2*a^2*b*d^2*e + a^3*d*e^2 + (b^3*d^2*e -
2*a*b^2*d*e^2 + a^2*b*e^3)*x^2 + (b^3*d^3 - a*b^2*d^2*e - a^2*b*d*e^2 + a
^3*e^3)*x)
```

Giac [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 154, normalized size of antiderivative = 1.90

$$\int \frac{1}{(d+ex)^2(a^2+2abx+b^2x^2)} dx = -\frac{2be^2 \log\left(\left|b - \frac{bd}{ex+d} + \frac{ae}{ex+d}\right|\right)}{b^3d^3e - 3ab^2d^2e^2 + 3a^2bde^3 - a^3e^4} - \frac{e^3}{(b^2d^2e^2 - 2abde^3 + a^2e^4)(ex+d)} - \frac{b^2e}{(bd-ae)^3\left(b - \frac{bd}{ex+d} + \frac{ae}{ex+d}\right)}$$

input `integrate(1/(e*x+d)^2/(b^2*x^2+2*a*b*x+a^2),x, algorithm="giac")`output `-2*b*e^2*log(abs(b - b*d/(e*x + d) + a*e/(e*x + d)))/(b^3*d^3*e - 3*a*b^2*d^2*e^2 + 3*a^2*b*d*e^3 - a^3*e^4) - e^3/((b^2*d^2*e^2 - 2*a*b*d*e^3 + a^2*e^4)*(e*x + d)) - b^2*e/((b*d - a*e)^3*(b - b*d/(e*x + d) + a*e/(e*x + d)))`**Mupad [B] (verification not implemented)**

Time = 8.93 (sec) , antiderivative size = 182, normalized size of antiderivative = 2.25

$$\int \frac{1}{(d+ex)^2(a^2+2abx+b^2x^2)} dx = \frac{4be \operatorname{atanh}\left(\frac{a^3e^3 - a^2bde^2 - ab^2d^2e + b^3d^3}{(ae-bd)^3} + \frac{2bex(a^2e^2 - 2abde + b^2d^2)}{(ae-bd)^3}\right)}{(ae-bd)^3} - \frac{\frac{ae+bd}{a^2e^2 - 2abde + b^2d^2} + \frac{2bex}{a^2e^2 - 2abde + b^2d^2}}{bex^2 + (ae+bd)x + ad}$$

input `int(1/((d + e*x)^2*(a^2 + b^2*x^2 + 2*a*b*x)),x)`output `(4*b*e*atanh((a^3*e^3 + b^3*d^3 - a*b^2*d^2*e - a^2*b*d*e^2)/(a*e - b*d)^3 + (2*b*e*x*(a^2*e^2 + b^2*d^2 - 2*a*b*d*e))/(a*e - b*d)^3))/(a*e - b*d)^3 - ((a*e + b*d)/(a^2*e^2 + b^2*d^2 - 2*a*b*d*e) + (2*b*e*x)/(a^2*e^2 + b^2*d^2 - 2*a*b*d*e))/(a*d + x*(a*e + b*d) + b*e*x^2)`

3.116 $\int \frac{1}{(d+ex)^3(a^2+2abx+b^2x^2)} dx$

| | |
|---|-----|
| Optimal result | 935 |
| Mathematica [A] (verified) | 935 |
| Rubi [A] (verified) | 936 |
| Maple [A] (verified) | 937 |
| Fricas [B] (verification not implemented) | 938 |
| Sympy [B] (verification not implemented) | 939 |
| Maxima [B] (verification not implemented) | 940 |
| Giac [B] (verification not implemented) | 940 |
| Mupad [B] (verification not implemented) | 941 |
| Reduce [B] (verification not implemented) | 942 |

Optimal result

Integrand size = 26, antiderivative size = 110

$$\int \frac{1}{(d+ex)^3(a^2+2abx+b^2x^2)} dx = -\frac{b^2}{(bd-ae)^3(a+bx)} - \frac{e}{2(bd-ae)^2(d+ex)^2} - \frac{2be}{(bd-ae)^3(d+ex)} - \frac{3b^2e \log(a+bx)}{(bd-ae)^4} + \frac{3b^2e \log(d+ex)}{(bd-ae)^4}$$

```
output -b^2/(-a*e+b*d)^3/(b*x+a)-1/2*e/(-a*e+b*d)^2/(e*x+d)^2-2*b*e/(-a*e+b*d)^3/
(e*x+d)-3*b^2*e*ln(b*x+a)/(-a*e+b*d)^4+3*b^2*e*ln(e*x+d)/(-a*e+b*d)^4
```

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 97, normalized size of antiderivative = 0.88

$$\int \frac{1}{(d+ex)^3(a^2+2abx+b^2x^2)} dx = -\frac{\frac{2b^2(bd-ae)}{a+bx} + \frac{e(bd-ae)^2}{(d+ex)^2} + \frac{4be(bd-ae)}{d+ex} + 6b^2e \log(a+bx) - 6b^2e \log(d+ex)}{2(bd-ae)^4}$$

input `Integrate[1/((d + e*x)^3*(a^2 + 2*a*b*x + b^2*x^2)),x]`

output `-1/2*((2*b^2*(b*d - a*e))/(a + b*x) + (e*(b*d - a*e)^2)/(d + e*x)^2 + (4*b*e*(b*d - a*e))/(d + e*x) + 6*b^2*e*Log[a + b*x] - 6*b^2*e*Log[d + e*x])/ (b*d - a*e)^4`

Rubi [A] (verified)

Time = 0.50 (sec) , antiderivative size = 110, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {1098, 27, 54, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a^2 + 2abx + b^2x^2)(d + ex)^3} dx$$

$$\downarrow 1098$$

$$b^2 \int \frac{1}{b^2(a + bx)^2(d + ex)^3} dx$$

$$\downarrow 27$$

$$\int \frac{1}{(a + bx)^2(d + ex)^3} dx$$

$$\downarrow 54$$

$$\int \left(-\frac{3b^3e}{(a + bx)(bd - ae)^4} + \frac{b^3}{(a + bx)^2(bd - ae)^3} + \frac{3b^2e^2}{(d + ex)(bd - ae)^4} + \frac{2be^2}{(d + ex)^2(bd - ae)^3} + \frac{e^2}{(d + ex)^3(bd - ae)^2} \right) dx$$

$$\downarrow 2009$$

$$-\frac{b^2}{(a + bx)(bd - ae)^3} - \frac{3b^2e \log(a + bx)}{(bd - ae)^4} + \frac{3b^2e \log(d + ex)}{e(bd - ae)^4} - \frac{2be}{(d + ex)(bd - ae)^3} - \frac{e}{2(d + ex)^2(bd - ae)^2}$$

input `Int[1/((d + e*x)^3*(a^2 + 2*a*b*x + b^2*x^2)),x]`

```
output
-(b^2/((b*d - a*e)^3*(a + b*x))) - e/(2*(b*d - a*e)^2*(d + e*x)^2) - (2*b*
e)/((b*d - a*e)^3*(d + e*x)) - (3*b^2*e*Log[a + b*x])/(b*d - a*e)^4 + (3*b
^2*e*Log[d + e*x])/(b*d - a*e)^4
```

Defintions of rubi rules used

```
rule 27 Int[(a_)*(Fx_), x_Symbol] := Simp[a Int[Fx, x], x] /; FreeQ[a, x] && !Ma
tchQ[Fx, (b_)*(Gx_)] /; FreeQ[b, x]
```

```
rule 54 Int[((a_) + (b_)*(x_))^(m_)*((c_) + (d_)*(x_))^(n_), x_Symbol] := Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x] /; FreeQ[{a, b, c, d}, x] &&
!LtQ[m, 0] && IntegerQ[n] && !(IGtQ[n, 0] && LtQ[m + n + 2, 0])
```

```
rule 1098 Int[((d_) + (e_)*(x_))^(m_)*((a_) + (b_)*(x_) + (c_)*(x_)^2)^(p_), x_
Symbol] := Simp[1/c^p Int[(d + e*x)^m*(b/2 + c*x)^(2*p), x], x] /; FreeQ[
{a, b, c, d, e, m}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

```
rule 2009 Int[u_, x_Symbol] := Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 1.30 (sec) , antiderivative size = 108, normalized size of antiderivative = 0.98

| method | result |
|---------------|--|
| default | $\frac{b^2}{(ae-bd)^3(bx+a)} - \frac{3b^2e \ln(bx+a)}{(ae-bd)^4} - \frac{e}{2(ae-bd)^2(ex+d)^2} + \frac{3b^2e \ln(ex+d)}{(ae-bd)^4} + \frac{2eb}{(ae-bd)^3(ex+d)}$ |
| risch | $\frac{3b^2e^2x^2}{e^3a^3-3a^2bd^2+3ab^2d^2e-b^3d^3} + \frac{3(ae+3bd)ebx}{2(e^3a^3-3a^2bd^2+3ab^2d^2e-b^3d^3)} - \frac{e^2a^2-5abde-2b^2d^2}{2(e^3a^3-3a^2bd^2+3ab^2d^2e-b^3d^3)} + \frac{3eb^2}{a^4e^4-4a^3bde^3+6a^2b^2d^2e^2-4ab^3d^3}$ |
| norman | $\frac{3b^2e^2x^2}{e^3a^3-3a^2bd^2+3ab^2d^2e-b^3d^3} + \frac{-a^2be^4+5ab^2de^3+2b^3d^2e^2}{2e^2b(e^3a^3-3a^2bd^2+3ab^2d^2e-b^3d^3)} + \frac{(3ab^2e^4+9b^3de^3)x}{2e^2b(e^3a^3-3a^2bd^2+3ab^2d^2e-b^3d^3)} - \frac{3eb^2}{a^4e^4-4a^3bde^3+6a^2b^2d^2e^2-4ab^3d^3}$ |
| parallelrisch | $- \frac{12 \ln(ex+d)xa b^3de^4 - 6 \ln(ex+d)ab^3d^2e^3 - 6a^2b^2de^4 + 3ab^3d^2e^3 + 12 \ln(bx+a)xa b^3de^4 - 6xab^3de^4 + 6 \ln(bx+a)x^2a b^3de^4}{(ex+d)^2(bx+a)}$ |

input `int(1/(e*x+d)^3/(b^2*x^2+2*a*b*x+a^2),x,method=_RETURNVERBOSE)`

output `b^2/(a*e-b*d)^3/(b*x+a)-3*b^2/(a*e-b*d)^4*e*ln(b*x+a)-1/2*e/(a*e-b*d)^2/(e*x+d)^2+3*b^2/(a*e-b*d)^4*e*ln(e*x+d)+2*e/(a*e-b*d)^3*b/(e*x+d)`

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 495 vs. $2(108) = 216$.

Time = 0.09 (sec) , antiderivative size = 495, normalized size of antiderivative = 4.50

$$\int \frac{1}{(d+ex)^3(a^2+2abx+b^2x^2)} dx = \frac{2b^3d^3 + 3ab^2d^2e - 6a^2bde^2 + a^3e^3 + 6(b^3de^2 - ab^2e^3)x^2 + 3(3b^3d^2e - 2ab^2de^2 - a^2be^3)x + 6(b^3e^3 - 2ab^2d^2e + a^2bde^2 - a^3e^3)}{2(ab^4d^6 - 4a^2b^3d^5e + 6a^3b^2d^4e^2 - 4a^4bd^3e^3 + a^5d^2e^4 + (b^5d^4e^2 - 4ab^4d^3e^3 + 6a^2b^3d^2e^4 - 4a^3b^2de^5)x + 6(b^3e^3 - 2ab^2d^2e + a^2bde^2 - a^3e^3))}$$

input `integrate(1/(e*x+d)^3/(b^2*x^2+2*a*b*x+a^2),x, algorithm="fricas")`

output `-1/2*(2*b^3*d^3 + 3*a*b^2*d^2*e - 6*a^2*b*d*e^2 + a^3*e^3 + 6*(b^3*d*e^2 - a*b^2*e^3)*x^2 + 3*(3*b^3*d^2*e - 2*a*b^2*d*e^2 - a^2*b*e^3)*x + 6*(b^3*e^3 - 2*a*b^2*d^2*e + (2*b^3*d*e^2 + a*b^2*e^3)*x^2 + (b^3*d^2*e + 2*a*b^2*d*e^2)*x)*log(b*x + a) - 6*(b^3*e^3*x^3 + a*b^2*d^2*e + (2*b^3*d*e^2 + a*b^2*e^3)*x^2 + (b^3*d^2*e + 2*a*b^2*d*e^2)*x)*log(e*x + d))/(a*b^4*d^6 - 4*a^2*b^3*d^5*e + 6*a^3*b^2*d^4*e^2 - 4*a^4*b*d^3*e^3 + a^5*d^2*e^4 + (b^5*d^4*e^2 - 4*a*b^4*d^3*e^3 + 6*a^2*b^3*d^2*e^4 - 4*a^3*b^2*d*e^5 + a^4*b*e^6)*x^3 + (2*b^5*d^5*e - 7*a*b^4*d^4*e^2 + 8*a^2*b^3*d^3*e^3 - 2*a^3*b^2*d^2*e^4 - 2*a^4*b*d*e^5 + a^5*e^6)*x^2 + (b^5*d^6 - 2*a*b^4*d^5*e - 2*a^2*b^3*d^4*e^2 + 8*a^3*b^2*d^3*e^3 - 7*a^4*b*d^2*e^4 + 2*a^5*d*e^5)*x)`

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 632 vs. $2(97) = 194$.

Time = 0.83 (sec) , antiderivative size = 632, normalized size of antiderivative = 5.75

$$\int \frac{1}{(d+ex)^3 (a^2+2abx+b^2x^2)} dx$$

$$= \frac{3b^2e \log \left(x + \frac{-\frac{3a^5b^2e^6}{(ae-bd)^4} + \frac{15a^4b^3de^5}{(ae-bd)^4} - \frac{30a^3b^4d^2e^4}{(ae-bd)^4} + \frac{30a^2b^5d^3e^3}{(ae-bd)^4} - \frac{15ab^6d^4e^2}{(ae-bd)^4} + 3ab^2e^2 + \frac{3b^7d^5e}{(ae-bd)^4} + 3b^3de}{6b^3e^2} \right)}{(ae-bd)^4}$$

$$- \frac{3b^2e \log \left(x + \frac{\frac{3a^5b^2e^6}{(ae-bd)^4} - \frac{15a^4b^3de^5}{(ae-bd)^4} + \frac{30a^3b^4d^2e^4}{(ae-bd)^4} - \frac{30a^2b^5d^3e^3}{(ae-bd)^4} + \frac{15ab^6d^4e^2}{(ae-bd)^4} + 3ab^2e^2 - \frac{3b^7d^5e}{(ae-bd)^4} + 3b^3de}{6b^3e^2} \right)}{(ae-bd)^4}$$

$$+ \frac{-a^2e^2 + 5abde + 2b^2d^2 + 2a^4d^2e^3 - 6a^3bd^3e^2 + 6a^2b^2d^4e - 2ab^3d^5 + x^3 \cdot (2a^3be^5 - 6a^2b^2de^4 + 6ab^3d^2e^3 - 2b^4d^3e^2) + x^2 \cdot (2a^4e^5 - 6a^3bd^3e^2 + 6a^2b^2d^4e - 2ab^3d^5) + x \cdot (2a^4e^5 - 6a^3bd^3e^2 + 6a^2b^2d^4e - 2ab^3d^5) + 2a^4e^5}{(ae-bd)^4}$$

input `integrate(1/(e*x+d)**3/(b**2*x**2+2*a*b*x+a**2), x)`

output

```

3*b**2*e*log(x + (-3*a**5*b**2*e**6/(a*e - b*d)**4 + 15*a**4*b**3*d*e**5/(
a*e - b*d)**4 - 30*a**3*b**4*d**2*e**4/(a*e - b*d)**4 + 30*a**2*b**5*d**3*
e**3/(a*e - b*d)**4 - 15*a*b**6*d**4*e**2/(a*e - b*d)**4 + 3*a*b**2*e**2 +
3*b**7*d**5*e/(a*e - b*d)**4 + 3*b**3*d*e)/(6*b**3*e**2))/(a*e - b*d)**4
- 3*b**2*e*log(x + (3*a**5*b**2*e**6/(a*e - b*d)**4 - 15*a**4*b**3*d*e**5/
(a*e - b*d)**4 + 30*a**3*b**4*d**2*e**4/(a*e - b*d)**4 - 30*a**2*b**5*d**3
*e**3/(a*e - b*d)**4 + 15*a*b**6*d**4*e**2/(a*e - b*d)**4 + 3*a*b**2*e**2
- 3*b**7*d**5*e/(a*e - b*d)**4 + 3*b**3*d*e)/(6*b**3*e**2))/(a*e - b*d)**4
+ (-a**2*e**2 + 5*a*b*d*e + 2*b**2*d**2 + 6*b**2*e**2*x**2 + x*(3*a*b*e**
2 + 9*b**2*d*e))/(2*a**4*d**2*e**3 - 6*a**3*b*d**3*e**2 + 6*a**2*b**2*d**4
*e - 2*a*b**3*d**5 + x**3*(2*a**3*b*e**5 - 6*a**2*b**2*d*e**4 + 6*a*b**3*d
**2*e**3 - 2*b**4*d**3*e**2) + x**2*(2*a**4*e**5 - 2*a**3*b*d*e**4 - 6*a**
2*b**2*d**2*e**3 + 10*a*b**3*d**3*e**2 - 4*b**4*d**4*e) + x*(4*a**4*d*e**4
- 10*a**3*b*d**2*e**3 + 6*a**2*b**2*d**3*e**2 + 2*a*b**3*d**4*e - 2*b**4*
d**5))

```


Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 386 vs. $2(108) = 216$.

Time = 0.06 (sec) , antiderivative size = 386, normalized size of antiderivative = 3.51

$$\int \frac{1}{(d+ex)^3 (a^2+2abx+b^2x^2)} dx = -\frac{3b^2e \log (bx+a)}{b^4d^4-4ab^3d^3e+6a^2b^2d^2e^2-4a^3bde^3+a^4e^4}$$

$$+\frac{3b^2e \log (ex+d)}{b^4d^4-4ab^3d^3e+6a^2b^2d^2e^2-4a^3bde^3+a^4e^4}$$

$$-\frac{6b^2e^2x^2+2b^2d^2+5abde-a^3be^5}{2(ab^3d^5-3a^2b^2d^4e+3a^3bd^3e^2-a^4d^2e^3+(b^4d^3e^2-3ab^3d^2e^3+3a^2b^2de^4-a^3be^5)x^3+(2b^4d^4e-5$$

input `integrate(1/(e*x+d)^3/(b^2*x^2+2*a*b*x+a^2),x, algorithm="maxima")`

output `-3*b^2*e*log(b*x + a)/(b^4*d^4 - 4*a*b^3*d^3*e + 6*a^2*b^2*d^2*e^2 - 4*a^3*b*d*e^3 + a^4*e^4) + 3*b^2*e*log(e*x + d)/(b^4*d^4 - 4*a*b^3*d^3*e + 6*a^2*b^2*d^2*e^2 - 4*a^3*b*d*e^3 + a^4*e^4) - 1/2*(6*b^2*e^2*x^2 + 2*b^2*d^2 + 5*a*b*d*e - a^2*e^2 + 3*(3*b^2*d*e + a*b*e^2)*x)/(a*b^3*d^5 - 3*a^2*b^2*d^4*e + 3*a^3*b*d^3*e^2 - a^4*d^2*e^3 + (b^4*d^3*e^2 - 3*a*b^3*d^2*e^3 + 3*a^2*b^2*d*e^4 - a^3*b*e^5)*x^3 + (2*b^4*d^4*e - 5*a*b^3*d^3*e^2 + 3*a^2*b^2*d^2*e^3 + a^3*b*d*e^4 - a^4*e^5)*x^2 + (b^4*d^5 - a*b^3*d^4*e - 3*a^2*b^2*d^3*e^2 + 5*a^3*b*d^2*e^3 - 2*a^4*d*e^4)*x)`

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 254 vs. $2(108) = 216$.

Time = 0.18 (sec) , antiderivative size = 254, normalized size of antiderivative = 2.31

$$\int \frac{1}{(d+ex)^3 (a^2+2abx+b^2x^2)} dx = -\frac{3b^3e \log (|bx+a|)}{b^5d^4-4ab^4d^3e+6a^2b^3d^2e^2-4a^3b^2de^3+a^4be^4}$$

$$+\frac{3b^2e^2 \log (|ex+d|)}{b^4d^4e-4ab^3d^3e^2+6a^2b^2d^2e^3-4a^3bde^4+a^4e^5}$$

$$-\frac{2b^3d^3+3ab^2d^2e-6a^2bde^2+a^3e^3+6(b^3de^2-ab^2e^3)x^2+3(3b^3d^2e-2ab^2de^2-a^2be^3)x}{2(bd-ae)^4(bx+a)(ex+d)^2}$$

input `integrate(1/(e*x+d)^3/(b^2*x^2+2*a*b*x+a^2),x, algorithm="giac")`

output

$$-3*b^3*e*\log(\text{abs}(b*x + a))/(b^5*d^4 - 4*a*b^4*d^3*e + 6*a^2*b^3*d^2*e^2 - 4*a^3*b^2*d*e^3 + a^4*b*e^4) + 3*b^2*e^2*\log(\text{abs}(e*x + d))/(b^4*d^4*e - 4*a*b^3*d^3*e^2 + 6*a^2*b^2*d^2*e^3 - 4*a^3*b*d*e^4 + a^4*e^5) - 1/2*(2*b^3*d^3 + 3*a*b^2*d^2*e - 6*a^2*b*d*e^2 + a^3*e^3 + 6*(b^3*d*e^2 - a*b^2*e^3))*x^2 + 3*(3*b^3*d^2*e - 2*a*b^2*d*e^2 - a^2*b*e^3)*x)/((b*d - a*e)^4*(b*x + a)*(e*x + d)^2)$$
Mupad [B] (verification not implemented)

Time = 8.92 (sec) , antiderivative size = 329, normalized size of antiderivative = 2.99

$$\int \frac{1}{(d + ex)^3 (a^2 + 2abx + b^2x^2)} dx$$

$$= \frac{\frac{-a^2 e^2 + 5abde + 2b^2 d^2}{2(a^3 e^3 - 3a^2 bde^2 + 3ab^2 d^2 e - b^3 d^3)} + \frac{3bx(ae^2 + 3bde)}{2(a^3 e^3 - 3a^2 bde^2 + 3ab^2 d^2 e - b^3 d^3)} + \frac{3b^2 e^2 x^2}{a^3 e^3 - 3a^2 bde^2 + 3ab^2 d^2 e - b^3 d^3}}{x(bd^2 + 2aed) + ad^2 + x^2(ae^2 + 2bde) + be^2 x^3} - \frac{6b^2 e \operatorname{atanh}\left(\frac{a^4 e^4 - 2a^3 bde^3 + 2ab^3 d^3 e - b^4 d^4}{(ae - bd)^4} + \frac{2bex(a^3 e^3 - 3a^2 bde^2 + 3ab^2 d^2 e - b^3 d^3)}{(ae - bd)^4}\right)}{(ae - bd)^4}$$

input

`int(1/((d + e*x)^3*(a^2 + b^2*x^2 + 2*a*b*x)),x)`

output

$$\begin{aligned} & ((2*b^2*d^2 - a^2*e^2 + 5*a*b*d*e)/(2*(a^3*e^3 - b^3*d^3 + 3*a*b^2*d^2*e - 3*a^2*b*d*e^2)) + (3*b*x*(a*e^2 + 3*b*d*e))/(2*(a^3*e^3 - b^3*d^3 + 3*a*b^2*d^2*e - 3*a^2*b*d*e^2)) + (3*b^2*e^2*x^2)/(a^3*e^3 - b^3*d^3 + 3*a*b^2*d^2*e - 3*a^2*b*d*e^2))/(x*(b*d^2 + 2*a*d*e) + a*d^2 + x^2*(a*e^2 + 2*b*d*e) + b*e^2*x^3) - (6*b^2*e*\operatorname{atanh}((a^4*e^4 - b^4*d^4 + 2*a*b^3*d^3*e - 2*a^3*b*d*e^3)/(a*e - b*d)^4 + (2*b*e*x*(a^3*e^3 - b^3*d^3 + 3*a*b^2*d^2*e - 3*a^2*b*d*e^2))/(a*e - b*d)^4))/(a*e - b*d)^4 \end{aligned}$$

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 821, normalized size of antiderivative = 7.46

$$\int \frac{1}{(d+ex)^3 (a^2+2abx+b^2x^2)} dx = \text{Too large to display}$$

input `int(1/(e*x+d)^3/(b^2*x^2+2*a*b*x+a^2),x)`

output

```
( - 6*log(a + b*x)*a**2*b**2*d**2*e**2 - 12*log(a + b*x)*a**2*b**2*d*e**3*x
- 6*log(a + b*x)*a**2*b**2*e**4*x**2 - 12*log(a + b*x)*a*b**3*d**3*e - 3
0*log(a + b*x)*a*b**3*d**2*e**2*x - 24*log(a + b*x)*a*b**3*d*e**3*x**2 - 6
*log(a + b*x)*a*b**3*e**4*x**3 - 12*log(a + b*x)*b**4*d**3*e*x - 24*log(a
+ b*x)*b**4*d**2*e**2*x**2 - 12*log(a + b*x)*b**4*d*e**3*x**3 + 6*log(d +
e*x)*a**2*b**2*d**2*e**2 + 12*log(d + e*x)*a**2*b**2*d*e**3*x + 6*log(d +
e*x)*a**2*b**2*e**4*x**2 + 12*log(d + e*x)*a*b**3*d**3*e + 30*log(d + e*x)
*a*b**3*d**2*e**2*x + 24*log(d + e*x)*a*b**3*d*e**3*x**2 + 6*log(d + e*x)*
a*b**3*e**4*x**3 + 12*log(d + e*x)*b**4*d**3*e*x + 24*log(d + e*x)*b**4*d
**2*e**2*x**2 + 12*log(d + e*x)*b**4*d*e**3*x**3 - a**4*e**4 + 4*a**3*b*d*e
**3 + 3*a**3*b*e**4*x + 3*a**2*b**2*d**2*e**2 - 2*a*b**3*d**3*e + 9*a*b**3
*d**2*e**2*x - 6*a*b**3*e**4*x**3 - 4*b**4*d**4 - 12*b**4*d**3*e*x + 6*b**
4*d*e**3*x**3)/(2*(a**6*d**2*e**5 + 2*a**6*d*e**6*x + a**6*e**7*x**2 - 2*a
**5*b*d**3*e**4 - 3*a**5*b*d**2*e**5*x + a**5*b*e**7*x**3 - 2*a**4*b**2*d
**4*e**3 - 6*a**4*b**2*d**3*e**4*x - 6*a**4*b**2*d**2*e**5*x**2 - 2*a**4*b
**2*d*e**6*x**3 + 8*a**3*b**3*d**5*e**2 + 14*a**3*b**3*d**4*e**3*x + 4*a**3
*b**3*d**3*e**4*x**2 - 2*a**3*b**3*d**2*e**5*x**3 - 7*a**2*b**4*d**6*e - 6
*a**2*b**4*d**5*e**2*x + 9*a**2*b**4*d**4*e**3*x**2 + 8*a**2*b**4*d**3*e**
4*x**3 + 2*a*b**5*d**7 - 3*a*b**5*d**6*e*x - 12*a*b**5*d**5*e**2*x**2 - 7*
a*b**5*d**4*e**3*x**3 + 2*b**6*d**7*x + 4*b**6*d**6*e*x**2 + 2*b**6*d**...
```

3.117 $\int \frac{1}{(d+ex)^4(a^2+2abx+b^2x^2)} dx$

| | |
|---|-----|
| Optimal result | 943 |
| Mathematica [A] (verified) | 943 |
| Rubi [A] (verified) | 944 |
| Maple [A] (verified) | 945 |
| Fricas [B] (verification not implemented) | 946 |
| Sympy [B] (verification not implemented) | 947 |
| Maxima [B] (verification not implemented) | 948 |
| Giac [B] (verification not implemented) | 948 |
| Mupad [B] (verification not implemented) | 949 |
| Reduce [B] (verification not implemented) | 950 |

Optimal result

Integrand size = 26, antiderivative size = 133

$$\int \frac{1}{(d+ex)^4(a^2+2abx+b^2x^2)} dx = -\frac{b^3}{(bd-ae)^4(a+bx)} - \frac{e}{3(bd-ae)^2(d+ex)^3} - \frac{be}{(bd-ae)^3(d+ex)^2} - \frac{3b^2e}{(bd-ae)^4(d+ex)} - \frac{4b^3e \log(a+bx)}{(bd-ae)^5} + \frac{4b^3e \log(d+ex)}{(bd-ae)^5}$$

output

```
-b^3/(-a*e+b*d)^4/(b*x+a)-1/3*e/(-a*e+b*d)^2/(e*x+d)^3-b*e/(-a*e+b*d)^3/(e*x+d)^2-3*b^2*e/(-a*e+b*d)^4/(e*x+d)-4*b^3*e*ln(b*x+a)/(-a*e+b*d)^5+4*b^3*e*ln(e*x+d)/(-a*e+b*d)^5
```

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 120, normalized size of antiderivative = 0.90

$$\int \frac{1}{(d+ex)^4(a^2+2abx+b^2x^2)} dx = \frac{-\frac{3b^3(bd-ae)}{a+bx} + \frac{e(-bd+ae)^3}{(d+ex)^3} - \frac{3be(bd-ae)^2}{(d+ex)^2} - \frac{9b^2e(bd-ae)}{d+ex} - 12b^3e \log(a+bx) + 12b^3e \log(d+ex)}{3(bd-ae)^5}$$

input `Integrate[1/((d + e*x)^4*(a^2 + 2*a*b*x + b^2*x^2)),x]`

output
$$\frac{((-3*b^3*(b*d - a*e))/(a + b*x) + (e*(-(b*d) + a*e)^3)/(d + e*x)^3 - (3*b*e*(b*d - a*e)^2)/(d + e*x)^2 - (9*b^2*e*(b*d - a*e))/(d + e*x) - 12*b^3*e*Log[a + b*x] + 12*b^3*e*Log[d + e*x])/(3*(b*d - a*e)^5)}$$

Rubi [A] (verified)

Time = 0.56 (sec) , antiderivative size = 133, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {1098, 27, 54, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\begin{aligned} & \int \frac{1}{(a^2 + 2abx + b^2x^2)(d + ex)^4} dx \\ & \quad \downarrow 1098 \\ & b^2 \int \frac{1}{b^2(a + bx)^2(d + ex)^4} dx \\ & \quad \downarrow 27 \\ & \int \frac{1}{(a + bx)^2(d + ex)^4} dx \\ & \quad \downarrow 54 \\ & \int \left(-\frac{4b^4e}{(a + bx)(bd - ae)^5} + \frac{b^4}{(a + bx)^2(bd - ae)^4} + \frac{4b^3e^2}{(d + ex)(bd - ae)^5} + \frac{3b^2e^2}{(d + ex)^2(bd - ae)^4} + \frac{2be^2}{(d + ex)^3(bd - ae)^3} \right. \\ & \quad \downarrow 2009 \\ & \left. -\frac{b^3}{(a + bx)(bd - ae)^4} - \frac{4b^3e \log(a + bx)}{(bd - ae)^5} + \frac{4b^3e \log(d + ex)}{(bd - ae)^5} - \frac{3b^2e}{(d + ex)(bd - ae)^4} - \frac{b^3}{(d + ex)^2(bd - ae)^3} - \frac{3b^2e}{3(d + ex)^3(bd - ae)^2} \right) dx \end{aligned}$$

input `Int[1/((d + e*x)^4*(a^2 + 2*a*b*x + b^2*x^2)),x]`

output
$$-\frac{b^3}{(bd - ae)^4(a + bx)} - \frac{e}{3(bd - ae)^2(d + ex)^3} - \frac{b^3e}{(bd - ae)^3(d + ex)^2} - \frac{3b^2e}{(bd - ae)^4(d + ex)} - \frac{4b^3e \operatorname{Log}[a + bx]}{(bd - ae)^5} + \frac{4b^3e \operatorname{Log}[d + ex]}{(bd - ae)^5}$$

Defintions of rubi rules used

rule 27
$$\operatorname{Int}[(a_*)(Fx_), x_Symbol] \rightarrow \operatorname{Simp}[a \operatorname{Int}[Fx, x], x] /; \operatorname{FreeQ}[a, x] \ \&\& \ !\operatorname{MatchQ}[Fx, (b_*)(Gx_)] /; \operatorname{FreeQ}[b, x]$$

rule 54
$$\operatorname{Int}[(a_*) + (b_*)(x_)^m * ((c_*) + (d_*)(x_)^n), x_Symbol] \rightarrow \operatorname{Int}[\operatorname{ExpandIntegrand}[(a + bx)^m(c + dx)^n, x], x] /; \operatorname{FreeQ}\{a, b, c, d\}, x \ \&\& \ \operatorname{ILtQ}[m, 0] \ \&\& \ \operatorname{IntegerQ}[n] \ \&\& \ !(\operatorname{IGtQ}[n, 0] \ \&\& \ \operatorname{LtQ}[m + n + 2, 0])$$

rule 1098
$$\operatorname{Int}[(d_*) + (e_*)(x_)^m * ((a_*) + (b_*)(x_) + (c_*)(x_)^2)^p, x_Symbol] \rightarrow \operatorname{Simp}[1/c^p \operatorname{Int}[(d + ex)^m(b/2 + cx)^{2p}], x] /; \operatorname{FreeQ}\{a, b, c, d, e, m\}, x \ \&\& \ \operatorname{EqQ}[b^2 - 4ac, 0] \ \&\& \ \operatorname{IntegerQ}[p]$$

rule 2009
$$\operatorname{Int}[u_, x_Symbol] \rightarrow \operatorname{Simp}[\operatorname{IntSum}[u, x], x] /; \operatorname{SumQ}[u]$$

Maple [A] (verified)

Time = 1.29 (sec) , antiderivative size = 131, normalized size of antiderivative = 0.98

| method | result |
|---------------|---|
| default | $-\frac{b^3}{(ae-bd)^4(bx+a)} + \frac{4b^3e \ln(bx+a)}{(ae-bd)^5} - \frac{e}{3(ae-bd)^2(ex+d)^3} - \frac{4b^3e \ln(ex+d)}{(ae-bd)^5} - \frac{3eb^2}{(ae-bd)^4(ex+d)} + \frac{eb}{(ae-bd)^3(ex+d)}$ |
| risch | $-\frac{4b^3e^3x^3}{a^4e^4-4a^3bde^3+6a^2b^2d^2e^2-4ab^3d^3e+b^4d^4} - \frac{2e^2(ae+5bd)b^2x^2}{a^4e^4-4a^3bde^3+6a^2b^2d^2e^2-4ab^3d^3e+b^4d^4} + \frac{2(e^2a^2-8abde-11b^2d^2)be}{3(a^4e^4-4a^3bde^3+6a^2b^2d^2e^2-4ab^3d^3e+b^4d^4)} + \frac{(ex+d)^3(bx+a)}{(ex+d)^3(bx+a)}$ |
| norman | $-\frac{4b^3e^3x^3}{a^4e^4-4a^3bde^3+6a^2b^2d^2e^2-4ab^3d^3e+b^4d^4} + \frac{(-2ab^3e^5-10b^4de^4)x^2}{e^2b(a^4e^4-4a^3bde^3+6a^2b^2d^2e^2-4ab^3d^3e+b^4d^4)} + \frac{-a^3be^6+5a^2b^2de^5-13ab^3d^2e^4}{3e^3b(a^4e^4-4a^3bde^3+6a^2b^2d^2e^2-4ab^3d^3e+b^4d^4)} + \frac{(ex+d)^3(bx+a)}{(ex+d)^3(bx+a)}$ |
| parallelrisch | $-6xa b^4 d^2 e^5 + 12 \ln(bx+a) x^3 a b^4 e^7 + 36 \ln(bx+a) x^3 b^5 d e^6 - 12 \ln(ex+d) x^3 a b^4 e^7 - 36 \ln(ex+d) x^3 b^5 d e^6 + 36 \ln(bx+a) x^2 b^5 d^2 e^6$ |

input `int(1/(e*x+d)^4/(b^2*x^2+2*a*b*x+a^2),x,method=_RETURNVERBOSE)`

output
$$-\frac{b^3}{(a e - b d)^4 (b x + a)} + \frac{4 b^3}{(a e - b d)^5 e \ln(b x + a)} - \frac{1}{3} \frac{e}{(a e - b d)^2 (e x + d)^3} - \frac{4 b^3}{(a e - b d)^5 e \ln(e x + d)} - \frac{3 e}{(a e - b d)^4 b^2 (e x + d)} + \frac{e}{(a e - b d)^3 b (e x + d)^2}$$

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 753 vs. $2(131) = 262$.

Time = 0.10 (sec) , antiderivative size = 753, normalized size of antiderivative = 5.66

$$\int \frac{1}{(d + ex)^4 (a^2 + 2abx + b^2x^2)} dx = \frac{3b^4d^4 + 10ab^3d^3e - 18a^2b^2d^2e^2 + 6a^3bde^3 - a^4e^4 + 12(b^4de^3 - ab^3e^4)x^3 + 6(5b^4d^2e^2 - 4ab^3d^2e^2 - 3ab^5d^8 - 5a^2b^4d^7e + 10a^3b^3d^6e^2 - 10a^4b^2d^5e^3 + 5a^5bd^4e^4 - a^6d^3e^5 + (b^6d^5e^3 - 5ab^5d^4e^4 + 10a^2b^4d^3e^5 - 10a^3b^3d^2e^6 + 5a^4b^2d^2e^7 - a^5bde^8)x^2 + (3b^4d^2e^2 + ab^3d^2e^3)x^2 + (b^4d^3e + 3ab^3d^2e^2)x \log(bx + a) - 12(b^4e^4x^4 + ab^3d^3e + (3b^4d^3e^3 + ab^3e^4)x^3 + 3(b^4d^2e^2 + ab^3d^2e^3)x^2 + (b^4d^3e + 3ab^3d^2e^2)x) \log(ex + d)}{(a^5b^5d^8 - 5a^2b^4d^7e + 10a^3b^3d^6e^2 - 10a^4b^2d^5e^3 + 5a^5bd^4e^4 - a^6d^3e^5 + (b^6d^5e^3 - 5ab^5d^4e^4 + 10a^2b^4d^3e^5 - 10a^3b^3d^2e^6 + 5a^4b^2d^2e^7 - a^5bde^8)x^4 + (3b^6d^6e^2 - 14a^2b^5d^5e^3 + 25a^2b^4d^4e^4 - 20a^3b^3d^3e^5 + 5a^4b^2d^2e^6 + 2a^5bd^2e^7 - a^6e^8)x^3 + 3(b^6d^7e - 4a^2b^5d^6e^2 + 5a^2b^4d^5e^3 - 5a^4b^2d^3e^5 + 4a^5bd^2e^6 - a^6d^2e^7)x^2 + (b^6d^8 - 2a^2b^5d^7e - 5a^2b^4d^6e^2 + 20a^3b^3d^5e^3 - 25a^4b^2d^4e^4 + 14a^5bd^3e^5 - 3a^6d^2e^6)x}$$

input `integrate(1/(e*x+d)^4/(b^2*x^2+2*a*b*x+a^2),x, algorithm="fricas")`

output
$$-\frac{1}{3} \frac{(3b^4d^4 + 10a^2b^3d^3e - 18a^2b^2d^2e^2 + 6a^3bde^3 - a^4e^4 + 12(b^4d^2e^2 - 4ab^3d^2e^2 - a^2b^2e^4)x^3 + 6(5b^4d^2e^2 - 4ab^3d^2e^2 - a^2b^2e^4)x^2 + 2(11b^4d^3e - 3a^2b^3d^2e^2 - 9a^2b^2d^2e^3 + a^3b^2e^4)x + 12(b^4e^4x^4 + ab^3d^3e + (3b^4d^3e^3 + ab^3e^4)x^3 + 3(b^4d^2e^2 + ab^3d^2e^3)x^2 + (b^4d^3e + 3ab^3d^2e^2)x) \log(bx + a) - 12(b^4e^4x^4 + ab^3d^3e + (3b^4d^3e^3 + ab^3e^4)x^3 + 3(b^4d^2e^2 + ab^3d^2e^3)x^2 + (b^4d^3e + 3ab^3d^2e^2)x) \log(ex + d)}{(a^5b^5d^8 - 5a^2b^4d^7e + 10a^3b^3d^6e^2 - 10a^4b^2d^5e^3 + 5a^5bd^4e^4 - a^6d^3e^5 + (b^6d^5e^3 - 5ab^5d^4e^4 + 10a^2b^4d^3e^5 - 10a^3b^3d^2e^6 + 5a^4b^2d^2e^7 - a^5bde^8)x^4 + (3b^6d^6e^2 - 14a^2b^5d^5e^3 + 25a^2b^4d^4e^4 - 20a^3b^3d^3e^5 + 5a^4b^2d^2e^6 + 2a^5bd^2e^7 - a^6e^8)x^3 + 3(b^6d^7e - 4a^2b^5d^6e^2 + 5a^2b^4d^5e^3 - 5a^4b^2d^3e^5 + 4a^5bd^2e^6 - a^6d^2e^7)x^2 + (b^6d^8 - 2a^2b^5d^7e - 5a^2b^4d^6e^2 + 20a^3b^3d^5e^3 - 25a^4b^2d^4e^4 + 14a^5bd^3e^5 - 3a^6d^2e^6)x}$$

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 882 vs. $2(117) = 234$.

Time = 1.12 (sec) , antiderivative size = 882, normalized size of antiderivative = 6.63

$$\int \frac{1}{(d+ex)^4(a^2+2abx+b^2x^2)} dx = \text{Too large to display}$$

input `integrate(1/(e*x+d)**4/(b**2*x**2+2*a*b*x+a**2),x)`

output

```
-4*b**3*e*log(x + (-4*a**6*b**3*e**7/(a*e - b*d)**5 + 24*a**5*b**4*d*e**6/
(a*e - b*d)**5 - 60*a**4*b**5*d**2*e**5/(a*e - b*d)**5 + 80*a**3*b**6*d**3
*e**4/(a*e - b*d)**5 - 60*a**2*b**7*d**4*e**3/(a*e - b*d)**5 + 24*a*b**8*d
**5*e**2/(a*e - b*d)**5 + 4*a*b**3*e**2 - 4*b**9*d**6*e/(a*e - b*d)**5 + 4
*b**4*d*e)/(8*b**4*e**2))/(a*e - b*d)**5 + 4*b**3*e*log(x + (4*a**6*b**3*e
**7/(a*e - b*d)**5 - 24*a**5*b**4*d*e**6/(a*e - b*d)**5 + 60*a**4*b**5*d**
2*e**5/(a*e - b*d)**5 - 80*a**3*b**6*d**3*e**4/(a*e - b*d)**5 + 60*a**2*b**
7*d**4*e**3/(a*e - b*d)**5 - 24*a*b**8*d**5*e**2/(a*e - b*d)**5 + 4*a*b**
3*e**2 + 4*b**9*d**6*e/(a*e - b*d)**5 + 4*b**4*d*e)/(8*b**4*e**2))/(a*e -
b*d)**5 + (-a**3*e**3 + 5*a**2*b*d*e**2 - 13*a*b**2*d**2*e - 3*b**3*d**3 -
12*b**3*e**3*x**3 + x**2*(-6*a*b**2*e**3 - 30*b**3*d*e**2) + x*(2*a**2*b*
e**3 - 16*a*b**2*d*e**2 - 22*b**3*d**2*e))/(3*a**5*d**3*e**4 - 12*a**4*b*d
**4*e**3 + 18*a**3*b**2*d**5*e**2 - 12*a**2*b**3*d**6*e + 3*a*b**4*d**7 +
x**4*(3*a**4*b*e**7 - 12*a**3*b**2*d*e**6 + 18*a**2*b**3*d**2*e**5 - 12*a*
b**4*d**3*e**4 + 3*b**5*d**4*e**3) + x**3*(3*a**5*e**7 - 3*a**4*b*d*e**6 -
18*a**3*b**2*d**2*e**5 + 42*a**2*b**3*d**3*e**4 - 33*a*b**4*d**4*e**3 + 9
*b**5*d**5*e**2) + x**2*(9*a**5*d*e**6 - 27*a**4*b*d**2*e**5 + 18*a**3*b**
2*d**3*e**4 + 18*a**2*b**3*d**4*e**3 - 27*a*b**4*d**5*e**2 + 9*b**5*d**6*e
) + x*(9*a**5*d**2*e**5 - 33*a**4*b*d**3*e**4 + 42*a**3*b**2*d**4*e**3 - 1
8*a**2*b**3*d**5*e**2 - 3*a*b**4*d**6*e + 3*b**5*d**7))
```


Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 599 vs. $2(131) = 262$.

Time = 0.07 (sec) , antiderivative size = 599, normalized size of antiderivative = 4.50

$$\int \frac{1}{(d+ex)^4 (a^2+2abx+b^2x^2)} dx$$

$$= -\frac{4b^3e \log(bx+a)}{b^5d^5 - 5ab^4d^4e + 10a^2b^3d^3e^2 - 10a^3b^2d^2e^3 + 5a^4bde^4 - a^5e^5}$$

$$+ \frac{4b^3e \log(ex+d)}{b^5d^5 - 5ab^4d^4e + 10a^2b^3d^3e^2 - 10a^3b^2d^2e^3 + 5a^4bde^4 - a^5e^5}$$

$$- \frac{3(ab^4d^7 - 4a^2b^3d^6e + 6a^3b^2d^5e^2 - 4a^4bd^4e^3 + a^5d^3e^4 + (b^5d^4e^3 - 4ab^4d^3e^4 + 6a^2b^3d^2e^5 - 4a^3b^2de^6$$

input `integrate(1/(e*x+d)^4/(b^2*x^2+2*a*b*x+a^2),x, algorithm="maxima")`

output

```
-4*b^3*e*log(b*x + a)/(b^5*d^5 - 5*a*b^4*d^4*e + 10*a^2*b^3*d^3*e^2 - 10*a^3*b^2*d^2*e^3 + 5*a^4*b*d*e^4 - a^5*e^5) + 4*b^3*e*log(e*x + d)/(b^5*d^5 - 5*a*b^4*d^4*e + 10*a^2*b^3*d^3*e^2 - 10*a^3*b^2*d^2*e^3 + 5*a^4*b*d*e^4 - a^5*e^5) - 1/3*(12*b^3*e^3*x^3 + 3*b^3*d^3 + 13*a*b^2*d^2*e - 5*a^2*b*d*e^2 + a^3*e^3 + 6*(5*b^3*d*e^2 + a*b^2*e^3)*x^2 + 2*(11*b^3*d^2*e + 8*a*b^2*d*e^2 - a^2*b*e^3)*x)/(a*b^4*d^7 - 4*a^2*b^3*d^6*e + 6*a^3*b^2*d^5*e^2 - 4*a^4*b*d^4*e^3 + a^5*d^3*e^4 + (b^5*d^4*e^3 - 4*a*b^4*d^3*e^4 + 6*a^2*b^3*d^2*e^5 - 4*a^3*b^2*d*e^6 + a^4*b*e^7)*x^4 + (3*b^5*d^5*e^2 - 11*a*b^4*d^4*e^3 + 14*a^2*b^3*d^3*e^4 - 6*a^3*b^2*d^2*e^5 - a^4*b*d*e^6 + a^5*e^7)*x^3 + 3*(b^5*d^6*e - 3*a*b^4*d^5*e^2 + 2*a^2*b^3*d^4*e^3 + 2*a^3*b^2*d^3*e^4 - 3*a^4*b*d^2*e^5 + a^5*d*e^6)*x^2 + (b^5*d^7 - a*b^4*d^6*e - 6*a^2*b^3*d^5*e^2 + 14*a^3*b^2*d^4*e^3 - 11*a^4*b*d^3*e^4 + 3*a^5*d^2*e^5)*x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 350 vs. $2(131) = 262$.

Time = 0.19 (sec) , antiderivative size = 350, normalized size of antiderivative = 2.63

$$\int \frac{1}{(d+ex)^4 (a^2+2abx+b^2x^2)} dx$$

$$= -\frac{4b^4e \log(|bx+a|)}{b^6d^5 - 5ab^5d^4e + 10a^2b^4d^3e^2 - 10a^3b^3d^2e^3 + 5a^4b^2de^4 - a^5be^5}$$

$$+ \frac{4b^3e^2 \log(|ex+d|)}{b^5d^5e - 5ab^4d^4e^2 + 10a^2b^3d^3e^3 - 10a^3b^2d^2e^4 + 5a^4bde^5 - a^5e^6}$$

$$- \frac{3b^4d^4 + 10ab^3d^3e - 18a^2b^2d^2e^2 + 6a^3bde^3 - a^4e^4 + 12(b^4de^3 - ab^3e^4)x^3 + 6(5b^4d^2e^2 - 4ab^3de^3 - a^4e^4)}{3(bd-ae)^5(bx+a)(ex+d)^3}$$

input `integrate(1/(e*x+d)^4/(b^2*x^2+2*a*b*x+a^2),x, algorithm="giac")`

output `-4*b^4*e*log(abs(b*x + a))/(b^6*d^5 - 5*a*b^5*d^4*e + 10*a^2*b^4*d^3*e^2 - 10*a^3*b^3*d^2*e^3 + 5*a^4*b^2*d*e^4 - a^5*b*e^5) + 4*b^3*e^2*log(abs(e*x + d))/(b^5*d^5*e - 5*a*b^4*d^4*e^2 + 10*a^2*b^3*d^3*e^3 - 10*a^3*b^2*d^2*e^4 + 5*a^4*b*d*e^5 - a^5*e^6) - 1/3*(3*b^4*d^4 + 10*a*b^3*d^3*e - 18*a^2*b^2*d^2*e^2 + 6*a^3*b*d*e^3 - a^4*e^4 + 12*(b^4*d*e^3 - a*b^3*e^4)*x^3 + 6*(5*b^4*d^2*e^2 - 4*a*b^3*d*e^3 - a^2*b^2*e^4)*x^2 + 2*(11*b^4*d^3*e - 3*a*b^3*d^2*e^2 - 9*a^2*b^2*d*e^3 + a^3*b*e^4)*x)/((b*d - a*e)^5*(b*x + a)*(e*x + d)^3)`

Mupad [B] (verification not implemented)

Time = 9.04 (sec) , antiderivative size = 534, normalized size of antiderivative = 4.02

$$\int \frac{1}{(d+ex)^4 (a^2+2abx+b^2x^2)} dx$$

$$= \frac{8b^3e \operatorname{atanh}\left(\frac{a^5e^5-3a^4bde^4+2a^3b^2d^2e^3+2a^2b^3d^3e^2-3ab^4d^4e+b^5d^5}{(ae-bd)^5}\right) + \frac{2bex(a^4e^4-4a^3bde^3+6a^2b^2d^2e^2-4ab^3d^3e+b^4d^4)}{(ae-bd)^5}}{(ae-bd)^5}$$

$$- \frac{\frac{a^3e^3-5a^2bde^2+13ab^2d^2e+3b^3d^3}{3(a^4e^4-4a^3bde^3+6a^2b^2d^2e^2-4ab^3d^3e+b^4d^4)}}{x^3(ae^3+3bde^2)} + \frac{4b^3e^3x^3}{x^2(3bd^2e+3ade^2)} + \frac{2b^2x^2(ae^3+5bde^2)}{ad^3+x(bd^3+3ade^2)}$$

input `int(1/((d + e*x)^4*(a^2 + b^2*x^2 + 2*a*b*x)),x)`

output

```
(8*b^3*e*atanh((a^5*e^5 + b^5*d^5 + 2*a^2*b^3*d^3*e^2 + 2*a^3*b^2*d^2*e^3
- 3*a*b^4*d^4*e - 3*a^4*b*d*e^4)/(a*e - b*d)^5 + (2*b*e*x*(a^4*e^4 + b^4*d
^4 + 6*a^2*b^2*d^2*e^2 - 4*a*b^3*d^3*e - 4*a^3*b*d*e^3))/(a*e - b*d)^5))/(
a*e - b*d)^5 - ((a^3*e^3 + 3*b^3*d^3 + 13*a*b^2*d^2*e - 5*a^2*b*d*e^2)/(3*
(a^4*e^4 + b^4*d^4 + 6*a^2*b^2*d^2*e^2 - 4*a*b^3*d^3*e - 4*a^3*b*d*e^3)) +
(4*b^3*e^3*x^3)/(a^4*e^4 + b^4*d^4 + 6*a^2*b^2*d^2*e^2 - 4*a*b^3*d^3*e -
4*a^3*b*d*e^3) + (2*b^2*x^2*(a*e^3 + 5*b*d*e^2))/(a^4*e^4 + b^4*d^4 + 6*a^
2*b^2*d^2*e^2 - 4*a*b^3*d^3*e - 4*a^3*b*d*e^3) + (2*b*x*(11*b^2*d^2*e - a^
2*e^3 + 8*a*b*d*e^2))/(3*(a^4*e^4 + b^4*d^4 + 6*a^2*b^2*d^2*e^2 - 4*a*b^3*
d^3*e - 4*a^3*b*d*e^3)))/(x^3*(a*e^3 + 3*b*d*e^2) + x^2*(3*a*d*e^2 + 3*b*d
^2*e) + a*d^3 + x*(b*d^3 + 3*a*d^2*e) + b*e^3*x^4)
```

Reduce [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 1237, normalized size of antiderivative = 9.30

$$\int \frac{1}{(d+ex)^4(a^2+2abx+b^2x^2)} dx = \text{Too large to display}$$

input

```
int(1/(e*x+d)^4/(b^2*x^2+2*a*b*x+a^2),x)
```

output

```
(12*log(a + b*x)*a**2*b**3*d**3*e**2 + 36*log(a + b*x)*a**2*b**3*d**2*e**3
*x + 36*log(a + b*x)*a**2*b**3*d**4*x**2 + 12*log(a + b*x)*a**2*b**3*e**
5*x**3 + 36*log(a + b*x)*a*b**4*d**4*e + 120*log(a + b*x)*a*b**4*d**3*e**2
*x + 144*log(a + b*x)*a*b**4*d**2*e**3*x**2 + 72*log(a + b*x)*a*b**4*d**e**
4*x**3 + 12*log(a + b*x)*a*b**4*e**5*x**4 + 36*log(a + b*x)*b**5*d**4*e*x
+ 108*log(a + b*x)*b**5*d**3*e**2*x**2 + 108*log(a + b*x)*b**5*d**2*e**3*x
**3 + 36*log(a + b*x)*b**5*d**4*x**4 - 12*log(d + e*x)*a**2*b**3*d**3*e*
*2 - 36*log(d + e*x)*a**2*b**3*d**2*e**3*x - 36*log(d + e*x)*a**2*b**3*d*e
**4*x**2 - 12*log(d + e*x)*a**2*b**3*e**5*x**3 - 36*log(d + e*x)*a*b**4*d*
*4*e - 120*log(d + e*x)*a*b**4*d**3*e**2*x - 144*log(d + e*x)*a*b**4*d**2*
e**3*x**2 - 72*log(d + e*x)*a*b**4*d**4*x**3 - 12*log(d + e*x)*a*b**4*e*
*5*x**4 - 36*log(d + e*x)*b**5*d**4*e*x - 108*log(d + e*x)*b**5*d**3*e**2*
x**2 - 108*log(d + e*x)*b**5*d**2*e**3*x**3 - 36*log(d + e*x)*b**5*d**4*
x**4 - a**5*e**5 + 3*a**4*b*d**4 + 2*a**4*b*e**5*x - 12*a**3*b**2*d**4
*x - 6*a**3*b**2*e**5*x**2 - 32*a**2*b**3*d**3*e**2 - 24*a**2*b**3*d**2*e*
*3*x - 6*a**2*b**3*d**4*x**2 + 21*a*b**4*d**4*e - 20*a*b**4*d**3*e**2*x
- 42*a*b**4*d**2*e**3*x**2 + 12*a*b**4*e**5*x**4 + 9*b**5*d**5 + 54*b**5*d
**4*e*x + 54*b**5*d**3*e**2*x**2 - 12*b**5*d**4*x**4)/(3*(a**7*d**3*e**6
+ 3*a**7*d**2*e**7*x + 3*a**7*d**e**8*x**2 + a**7*e**9*x**3 - 2*a**6*b*d**
4*e**5 - 5*a**6*b*d**3*e**6*x - 3*a**6*b*d**2*e**7*x**2 + a**6*b*d**e**8...
```

3.118
$$\int \frac{(d+ex)^6}{(a^2+2abx+b^2x^2)^2} dx$$

| | |
|---|-----|
| Optimal result | 952 |
| Mathematica [A] (verified) | 952 |
| Rubi [A] (verified) | 953 |
| Maple [B] (verified) | 955 |
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| Giac [B] (verification not implemented) | 957 |
| Mupad [B] (verification not implemented) | 958 |
| Reduce [B] (verification not implemented) | 959 |

Optimal result

Integrand size = 26, antiderivative size = 160

$$\int \frac{(d+ex)^6}{(a^2+2abx+b^2x^2)^2} dx = \frac{e^4(15b^2d^2 - 24abde + 10a^2e^2)x}{b^6} + \frac{e^5(3bd - 2ae)x^2}{b^5} + \frac{e^6x^3}{3b^4} - \frac{(bd - ae)^6}{3b^7(a + bx)^3} - \frac{3e(bd - ae)^5}{b^7(a + bx)^2} - \frac{15e^2(bd - ae)^4}{b^7(a + bx)} + \frac{20e^3(bd - ae)^3 \log(a + bx)}{b^7}$$

output `e^4*(10*a^2*e^2-24*a*b*d*e+15*b^2*d^2)*x/b^6+e^5*(-2*a*e+3*b*d)*x^2/b^5+1/3*e^6*x^3/b^4-1/3*(-a*e+b*d)^6/b^7/(b*x+a)^3-3*e*(-a*e+b*d)^5/b^7/(b*x+a)^2-15*e^2*(-a*e+b*d)^4/b^7/(b*x+a)+20*e^3*(-a*e+b*d)^3*ln(b*x+a)/b^7`

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 301, normalized size of antiderivative = 1.88

$$\int \frac{(d+ex)^6}{(a^2+2abx+b^2x^2)^2} dx = \frac{-37a^6e^6 + 3a^5be^5(47d - 17ex) + 3a^4b^2e^4(-65d^2 + 81dex + 13e^2x^2) + a^3b^3e^3(110d^3 - 405d^2ex - 27de^2}{(a^2+2abx+b^2x^2)^2}$$

input `Integrate[(d + e*x)^6/(a^2 + 2*a*b*x + b^2*x^2)^2,x]`

output $(-37*a^6*e^6 + 3*a^5*b*e^5*(47*d - 17*e*x) + 3*a^4*b^2*e^4*(-65*d^2 + 81*d*e*x + 13*e^2*x^2) + a^3*b^3*e^3*(110*d^3 - 405*d^2*e*x - 27*d*e^2*x^2 + 73*e^3*x^3) + 3*a^2*b^4*e^2*(-5*d^4 + 90*d^3*e*x - 45*d^2*e^2*x^2 - 63*d*e^3*x^3 + 5*e^4*x^4) - 3*a*b^5*e*(d^5 + 15*d^4*e*x - 60*d^3*e^2*x^2 - 45*d^2*e^3*x^3 + 15*d*e^4*x^4 + e^5*x^5) + b^6*(-d^6 - 9*d^5*e*x - 45*d^4*e^2*x^2 + 45*d^2*e^4*x^4 + 9*d*e^5*x^5 + e^6*x^6) - 60*e^3*(-(b*d) + a*e)^3*(a + b*x)^3*\text{Log}[a + b*x])/(3*b^7*(a + b*x)^3)$

Rubi [A] (verified)

Time = 0.68 (sec) , antiderivative size = 156, normalized size of antiderivative = 0.98, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {1098, 27, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d + ex)^6}{(a^2 + 2abx + b^2x^2)^2} dx$$

$$\downarrow 1098$$

$$b^4 \int \frac{(d + ex)^6}{b^4(a + bx)^4} dx$$

$$\downarrow 27$$

$$\int \frac{(d + ex)^6}{(a + bx)^4} dx$$

$$\downarrow 49$$

$$\int \left(\frac{6e^5(a + bx)(bd - ae)}{b^6} + \frac{15e^4(bd - ae)^2}{b^6} + \frac{20e^3(bd - ae)^3}{b^6(a + bx)} + \frac{15e^2(bd - ae)^4}{b^6(a + bx)^2} + \frac{6e(bd - ae)^5}{b^6(a + bx)^3} + \frac{(bd - ae)^6}{b^6(a + bx)^4} \right) dx$$

$$\downarrow 2009$$

$$\frac{3e^5(a+bx)^2(bd-ae)}{b^7} + \frac{20e^3(bd-ae)^3 \log(a+bx)}{b^7} - \frac{15e^2(bd-ae)^4}{b^7(a+bx)} - \frac{3e(bd-ae)^5}{b^7(a+bx)^2} - \frac{(bd-ae)^6}{3b^7(a+bx)^3} + \frac{e^6(a+bx)^3}{3b^7} + \frac{15e^4x(bd-ae)^2}{b^6}$$

input `Int[(d + e*x)^6/(a^2 + 2*a*b*x + b^2*x^2)^2,x]`

output `(15*e^4*(b*d - a*e)^2*x)/b^6 - (b*d - a*e)^6/(3*b^7*(a + b*x)^3) - (3*e*(b*d - a*e)^5)/(b^7*(a + b*x)^2) - (15*e^2*(b*d - a*e)^4)/(b^7*(a + b*x)) + (3*e^5*(b*d - a*e)*(a + b*x)^2)/b^7 + (e^6*(a + b*x)^3)/(3*b^7) + (20*e^3*(b*d - a*e)^3*Log[a + b*x])/b^7`

Defintions of rubi rules used

rule 27 `Int[(a_)*(F_x_), x_Symbol] := Simp[a Int[F_x, x], x] /; FreeQ[a, x] && !MatchQ[F_x, (b_)*(G_x_)] /; FreeQ[b, x]`

rule 49 `Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] := Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] && IGtQ[m, 0] && IGtQ[m + n + 2, 0]`

rule 1098 `Int[((d_.) + (e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] := Simp[1/c^p Int[(d + e*x)^m*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]`

rule 2009 `Int[u_, x_Symbol] := Simp[IntSum[u, x], x] /; SumQ[u]`

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 343 vs. 2(156) = 312.

Time = 1.03 (sec) , antiderivative size = 344, normalized size of antiderivative = 2.15

| method | result |
|--------------|--|
| norman | $\frac{-\frac{110a^6e^6-330a^5bde^5+330a^4b^2d^2e^4-110a^3b^3d^3e^3+15a^2b^4d^4e^2+3ab^5d^5e+b^6d^6}{3b^7} + \frac{e^6x^6}{3b} - \frac{3(20a^4e^6-60a^3bde^5+60a^2b^2d^2e^4-20ab^3d^3e^3+15a^2b^4d^4e^2-3ab^5d^5e+b^6d^6)}{b^5} (bx+a)^3}{(bx+a)^3}$ |
| default | $\frac{e^4(\frac{1}{3}b^2e^2x^3-2x^2ab e^2+3b^2de x^2+10a^2e^2x-24abdex+15b^2d^2x)}{b^6} - \frac{a^6e^6-6a^5bde^5+15a^4b^2d^2e^4-20a^3b^3d^3e^3+15a^2b^4d^4e^2-3ab^5d^5e+b^6d^6}{3b^7(bx+a)^3}$ |
| risch | $\frac{e^6x^3}{3b^4} - \frac{2e^6x^2a}{b^5} + \frac{3e^5dx^2}{b^4} + \frac{10e^6a^2x}{b^6} - \frac{24e^5adx}{b^5} + \frac{15e^4d^2x}{b^4} + \frac{(-15a^4be^6+60a^3de^5b^2-90a^2d^2e^4b^3+60ab^4d^3e^3-3a^2b^5d^4e^2+b^6d^5)}{b^5(bx+a)^3}$ |
| parallelrisc | $- \frac{3ab^5d^5e-330a^5bde^5-110a^3b^3d^3e^3-540\ln(bx+a)x a^4b^2de^5+540\ln(bx+a)x a^3b^3d^2e^4+180\ln(bx+a)x^3 a b^5d^2e^4+330a^4b^6d^3e^3}{b^6(bx+a)^3}$ |

```
input int((e*x+d)^6/(b^2*x^2+2*a*b*x+a^2)^2,x,method=_RETURNVERBOSE)
```

```
output (-1/3*(110*a^6*e^6-330*a^5*b*d*e^5+330*a^4*b^2*d^2*e^4-110*a^3*b^3*d^3*e^3+15*a^2*b^4*d^4*e^2+3*a*b^5*d^5*e+b^6*d^6)/b^7+1/3*e^6/b*x^6-3*(20*a^4*e^6-60*a^3*b*d*e^5+60*a^2*b^2*d^2*e^4-20*a*b^3*d^3*e^3+5*b^4*d^4*e^2)/b^5*x^2-3*(30*a^5*e^6-90*a^4*b*d*e^5+90*a^3*b^2*d^2*e^4-30*a^2*b^3*d^3*e^3+5*a*b^4*d^4*e^2+b^5*d^5*e)/b^6*x+5*e^4*(a^2*e^2-3*a*b*d*e+3*b^2*d^2)/b^3*x^4-e^5*(a*e-3*b*d)/b^2*x^5)/(b*x+a)^3-20/b^7*e^3*(a^3*e^3-3*a^2*b*d*e^2+3*a*b^2*d^2*e-b^3*d^3)*ln(b*x+a)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 577 vs. 2(156) = 312.

Time = 0.10 (sec) , antiderivative size = 577, normalized size of antiderivative = 3.61

$$\int \frac{(d+ex)^6}{(a^2+2abx+b^2x^2)^2} dx = \frac{b^6e^6x^6 - b^6d^6 - 3ab^5d^5e - 15a^2b^4d^4e^2 + 110a^3b^3d^3e^3 - 195a^4b^2d^2e^4 + 141a^5bde^5 - 37a^6e^6 + 3(3b^6de^5 - 3ab^5d^4e^4 + 3a^2b^4d^3e^3 - 3a^3b^3d^2e^2 + 3a^4b^2de - 3a^5bd + 3a^6)}{b^6(bx+a)^3} + \ln(bx+a)$$

```
input integrate((e*x+d)^6/(b^2*x^2+2*a*b*x+a^2)^2,x, algorithm="fricas")
```


output

```

1/3*(b^6*e^6*x^6 - b^6*d^6 - 3*a*b^5*d^5*e - 15*a^2*b^4*d^4*e^2 + 110*a^3*
b^3*d^3*e^3 - 195*a^4*b^2*d^2*e^4 + 141*a^5*b*d*e^5 - 37*a^6*e^6 + 3*(3*b^
6*d*e^5 - a*b^5*e^6)*x^5 + 15*(3*b^6*d^2*e^4 - 3*a*b^5*d*e^5 + a^2*b^4*e^6
)*x^4 + (135*a*b^5*d^2*e^4 - 189*a^2*b^4*d*e^5 + 73*a^3*b^3*e^6)*x^3 - 3*(
15*b^6*d^4*e^2 - 60*a*b^5*d^3*e^3 + 45*a^2*b^4*d^2*e^4 + 9*a^3*b^3*d*e^5 -
13*a^4*b^2*e^6)*x^2 - 3*(3*b^6*d^5*e + 15*a*b^5*d^4*e^2 - 90*a^2*b^4*d^3*
e^3 + 135*a^3*b^3*d^2*e^4 - 81*a^4*b^2*d*e^5 + 17*a^5*b*e^6)*x + 60*(a^3*b
^3*d^3*e^3 - 3*a^4*b^2*d^2*e^4 + 3*a^5*b*d*e^5 - a^6*e^6 + (b^6*d^3*e^3 -
3*a*b^5*d^2*e^4 + 3*a^2*b^4*d*e^5 - a^3*b^3*e^6)*x^3 + 3*(a*b^5*d^3*e^3 -
3*a^2*b^4*d^2*e^4 + 3*a^3*b^3*d*e^5 - a^4*b^2*e^6)*x^2 + 3*(a^2*b^4*d^3*e^
3 - 3*a^3*b^3*d^2*e^4 + 3*a^4*b^2*d*e^5 - a^5*b*e^6)*x)*log(b*x + a))/(b^1
0*x^3 + 3*a*b^9*x^2 + 3*a^2*b^8*x + a^3*b^7)

```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 367 vs. $2(153) = 306$.

Time = 2.04 (sec) , antiderivative size = 367, normalized size of antiderivative = 2.29

$$\int \frac{(d+ex)^6}{(a^2+2abx+b^2x^2)^2} dx = x^2 \left(-\frac{2ae^6}{b^5} + \frac{3de^5}{b^4} \right) + x \left(\frac{10a^2e^6}{b^6} - \frac{24ade^5}{b^5} + \frac{15d^2e^4}{b^4} \right) + \frac{-37a^6e^6 + 141a^5bde^5 - 195a^4b^2d^2e^4 + 110a^3b^3d^3e^3 - 15a^2b^4d^4e^2 - 3ab^5d^5e - b^6d^6 + x^2(-45a^4b^2e^6 + 30a^5b^3de^5 - 15a^6d^3e^3 - 3a^7d^4e^2 + 3a^8d^5e - 3a^9d^6)}{b^7} + \frac{e^6x^3}{3b^4} - \frac{20e^3(ae-bd)^3 \log(a+bx)}{b^7}$$

input

```
integrate((e*x+d)**6/(b**2*x**2+2*a*b*x+a**2)**2,x)
```

output

```

x**2*(-2*a*e**6/b**5 + 3*d*e**5/b**4) + x*(10*a**2*e**6/b**6 - 24*a*d*e**5
/b**5 + 15*d**2*e**4/b**4) + (-37*a**6*e**6 + 141*a**5*b*d*e**5 - 195*a**4
*b**2*d**2*e**4 + 110*a**3*b**3*d**3*e**3 - 15*a**2*b**4*d**4*e**2 - 3*a*b
**5*d**5*e - b**6*d**6 + x**2*(-45*a**4*b**2*e**6 + 180*a**3*b**3*d*e**5 -
270*a**2*b**4*d**2*e**4 + 180*a*b**5*d**3*e**3 - 45*b**6*d**4*e**2) + x*(
-81*a**5*b*e**6 + 315*a**4*b**2*d*e**5 - 450*a**3*b**3*d**2*e**4 + 270*a**
2*b**4*d**3*e**3 - 45*a*b**5*d**4*e**2 - 9*b**6*d**5*e))/(3*a**3*b**7 + 9*
a**2*b**8*x + 9*a*b**9*x**2 + 3*b**10*x**3) + e**6*x**3/(3*b**4) - 20*e**3
*(a*e - b*d)**3*log(a + b*x)/b**7

```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 374 vs. $2(156) = 312$.

Time = 0.06 (sec) , antiderivative size = 374, normalized size of antiderivative = 2.34

$$\int \frac{(d+ex)^6}{(a^2+2abx+b^2x^2)^2} dx =$$

$$\frac{b^6d^6 + 3ab^5d^5e + 15a^2b^4d^4e^2 - 110a^3b^3d^3e^3 + 195a^4b^2d^2e^4 - 141a^5bde^5 + 37a^6e^6 + 45(b^6d^4e^2 - 4ab^5d^3e^3 + 6a^2b^4d^2e^4 - 4a^3b^3d^2e^5 + a^4b^2de^6)x^2 + 9(b^6d^5e + 5a^2b^5d^4e^2 - 30a^2b^4d^3e^3 + 50a^3b^3d^2e^4 - 35a^4b^2de^5 + 9a^5bde^6)x}{3(b^{10}x^3 + 3a^2b^8x^2 + 3a^2b^7x + a^3b^7)} + \frac{b^2e^6x^3 + 3(3b^2de^5 - 2abe^6)x^2 + 3(15b^2d^2e^4 - 24abde^5 + 10a^2e^6)x}{3b^6} + \frac{20(b^3d^3e^3 - 3ab^2d^2e^4 + 3a^2bde^5 - a^3e^6) \log(bx+a)}{b^7}$$

input `integrate((e*x+d)^6/(b^2*x^2+2*a*b*x+a^2)^2,x, algorithm="maxima")`

output `-1/3*(b^6*d^6 + 3*a*b^5*d^5*e + 15*a^2*b^4*d^4*e^2 - 110*a^3*b^3*d^3*e^3 + 195*a^4*b^2*d^2*e^4 - 141*a^5*b*d*e^5 + 37*a^6*e^6 + 45*(b^6*d^4*e^2 - 4*a*b^5*d^3*e^3 + 6*a^2*b^4*d^2*e^4 - 4*a^3*b^3*d*e^5 + a^4*b^2*e^6)*x^2 + 9*(b^6*d^5*e + 5*a*b^5*d^4*e^2 - 30*a^2*b^4*d^3*e^3 + 50*a^3*b^3*d^2*e^4 - 35*a^4*b^2*d*e^5 + 9*a^5*b*e^6)*x)/(b^10*x^3 + 3*a*b^8*x^2 + 3*a^2*b^7*x + a^3*b^7) + 1/3*(b^2*e^6*x^3 + 3*(3*b^2*d*e^5 - 2*a*b*e^6)*x^2 + 3*(15*b^2*d^2*e^4 - 24*a*b*d*e^5 + 10*a^2*e^6)*x)/b^6 + 20*(b^3*d^3*e^3 - 3*a*b^2*d^2*e^4 + 3*a^2*b*d*e^5 - a^3*e^6)*log(b*x + a)/b^7`

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 356 vs. $2(156) = 312$.

Time = 0.20 (sec) , antiderivative size = 356, normalized size of antiderivative = 2.22

$$\int \frac{(d+ex)^6}{(a^2+2abx+b^2x^2)^2} dx = \frac{20(b^3d^3e^3 - 3ab^2d^2e^4 + 3a^2bde^5 - a^3e^6) \log(|bx+a|)}{b^7}$$

$$\frac{b^6d^6 + 3ab^5d^5e + 15a^2b^4d^4e^2 - 110a^3b^3d^3e^3 + 195a^4b^2d^2e^4 - 141a^5bde^5 + 37a^6e^6 + 45(b^6d^4e^2 - 4ab^5d^3e^3 + 6a^2b^4d^2e^4 - 4a^3b^3d^2e^5 + a^4b^2de^6)x^2 + 9(b^6d^5e + 5a^2b^5d^4e^2 - 30a^2b^4d^3e^3 + 50a^3b^3d^2e^4 - 35a^4b^2de^5 + 9a^5bde^6)x}{3b^{12}} + \frac{b^8e^6x^3 + 9ab^8de^5x^2 - 6ab^7e^6x^2 + 45b^8d^2e^4x - 72ab^7de^5x + 30a^2b^6e^6x}{3b^{12}}$$

input `integrate((e*x+d)^6/(b^2*x^2+2*a*b*x+a^2)^2,x, algorithm="giac")`

output
$$20*(b^3*d^3*e^3 - 3*a*b^2*d^2*e^4 + 3*a^2*b*d*e^5 - a^3*e^6)*\log(\text{abs}(b*x + a))/b^7 - 1/3*(b^6*d^6 + 3*a*b^5*d^5*e + 15*a^2*b^4*d^4*e^2 - 110*a^3*b^3*d^3*e^3 + 195*a^4*b^2*d^2*e^4 - 141*a^5*b*d*e^5 + 37*a^6*e^6 + 45*(b^6*d^4*e^2 - 4*a*b^5*d^3*e^3 + 6*a^2*b^4*d^2*e^4 - 4*a^3*b^3*d*e^5 + a^4*b^2*e^6)*x^2 + 9*(b^6*d^5*e + 5*a*b^5*d^4*e^2 - 30*a^2*b^4*d^3*e^3 + 50*a^3*b^3*d^2*e^4 - 35*a^4*b^2*d*e^5 + 9*a^5*b*e^6)*x)/((b*x + a)^3*b^7) + 1/3*(b^8*e^6*x^3 + 9*b^8*d*e^5*x^2 - 6*a*b^7*e^6*x^2 + 45*b^8*d^2*e^4*x - 72*a*b^7*d*e^5*x + 30*a^2*b^6*e^6*x)/b^{12}$$

Mupad [B] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 393, normalized size of antiderivative = 2.46

$$\int \frac{(d+ex)^6}{(a^2+2abx+b^2x^2)^2} dx = x \left(\frac{4a \left(\frac{4ae^6}{b^5} - \frac{6de^5}{b^4} \right)}{b} - \frac{6a^2e^6}{b^6} + \frac{15d^2e^4}{b^4} \right) - \frac{x^2 (15a^4be^6 - 60a^3b^2de^5 + 90a^2b^3d^2e^4 - 60ab^4d^3e^3 + 15b^5d^4e^2) + \frac{37a^6e^6 - 141a^5bde^5 + 195a^4b^2d^2e^4 - 110a^3b^3d^3e^3 + 15a^2b^4d^4e^2 - 60a^2b^2d^2e^4 - 60a^3b^3d^3e^3 + 150a^4b^2d^2e^4 - 105a^4bde^5}{b^7} + \frac{e^6x^3}{3b^4}}{a^3b^6 + \dots} - x^2 \left(\frac{2ae^6}{b^5} - \frac{3de^5}{b^4} \right) - \frac{\ln(a+bx) (20a^3e^6 - 60a^2bde^5 + 60ab^2d^2e^4 - 20b^3d^3e^3)}{b^7} + \frac{e^6x^3}{3b^4}$$

input `int((d + e*x)^6/(a^2 + b^2*x^2 + 2*a*b*x)^2,x)`

output
$$x*((4*a*((4*a*e^6)/b^5 - (6*d*e^5)/b^4))/b - (6*a^2*e^6)/b^6 + (15*d^2*e^4)/b^4) - (x^2*(15*a^4*b*e^6 + 15*b^5*d^4*e^2 - 60*a*b^4*d^3*e^3 - 60*a^3*b^2*d^2*e^5 + 90*a^2*b^3*d^2*e^4) + (37*a^6*e^6 + b^6*d^6 + 15*a^2*b^4*d^4*e^2 - 110*a^3*b^3*d^3*e^3 + 195*a^4*b^2*d^2*e^4 + 3*a*b^5*d^5*e - 141*a^5*b*d*e^5)/(3*b) + x*(27*a^5*e^6 + 3*b^5*d^5*e + 15*a*b^4*d^4*e^2 - 90*a^2*b^3*d^3*e^3 + 150*a^3*b^2*d^2*e^4 - 105*a^4*b*d*e^5))/(a^3*b^6 + b^9*x^3 + 3*a^2*b^7*x + 3*a*b^8*x^2) - x^2*((2*a*e^6)/b^5 - (3*d*e^5)/b^4) - (\log(a + b*x)*(20*a^3*e^6 - 20*b^3*d^3*e^3 + 60*a*b^2*d^2*e^4 - 60*a^2*b*d*e^5))/b^7 + (e^6*x^3)/(3*b^4)$$

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 645, normalized size of antiderivative = 4.03

$$\int \frac{(d + ex)^6}{(a^2 + 2abx + b^2x^2)^2} dx$$

$$= \frac{-180 \log(bx + a) a^5 b^2 e^6 x^2 + 60 \log(bx + a) a^4 b^3 d^3 e^3 - 60 \log(bx + a) a^4 b^3 e^6 x^3 + 270 a^5 b^2 d e^5 x - 270 a^4 b^3 d^2 e^2 x^2}{(3a^2 b^2 x^2 + b^3 x^3)}$$

input `int((e*x+d)^6/(b^2*x^2+2*a*b*x+a^2)^2,x)`

output

```
( - 60*log(a + b*x)*a**7*e**6 + 180*log(a + b*x)*a**6*b*d*e**5 - 180*log(a + b*x)*a**6*b*e**6*x - 180*log(a + b*x)*a**5*b**2*d**2*e**4 + 540*log(a + b*x)*a**5*b**2*d*e**5*x - 180*log(a + b*x)*a**5*b**2*e**6*x**2 + 60*log(a + b*x)*a**4*b**3*d**3*e**3 - 540*log(a + b*x)*a**4*b**3*d**2*e**4*x + 540*log(a + b*x)*a**4*b**3*d*e**5*x**2 - 60*log(a + b*x)*a**4*b**3*e**6*x**3 + 180*log(a + b*x)*a**3*b**4*d**3*e**3*x - 540*log(a + b*x)*a**3*b**4*d**2*e**4*x**2 + 180*log(a + b*x)*a**3*b**4*d*e**5*x**3 + 180*log(a + b*x)*a**2*b**5*d**3*e**3*x**2 - 180*log(a + b*x)*a**2*b**5*d**2*e**4*x**3 + 60*log(a + b*x)*a*b**6*d**3*e**3*x**3 - 50*a**7*e**6 + 150*a**6*b*d*e**5 - 90*a**6*b*e**6*x - 150*a**5*b**2*d**2*e**4 + 270*a**5*b**2*d*e**5*x + 50*a**4*b**3*d**3*e**3 - 270*a**4*b**3*d**2*e**4*x + 60*a**4*b**3*e**6*x**3 + 90*a**3*b**4*d**3*e**3*x - 180*a**3*b**4*d*e**5*x**3 + 15*a**3*b**4*e**6*x**4 - 3*a**2*b**5*d**5*e + 180*a**2*b**5*d**2*e**4*x**3 - 45*a**2*b**5*d*e**5*x**4 - 3*a**2*b**5*e**6*x**5 - a*b**6*d**6 - 9*a*b**6*d**5*e*x - 60*a*b**6*d**3*e**3*x**3 + 45*a*b**6*d**2*e**4*x**4 + 9*a*b**6*d*e**5*x**5 + a*b**6*e**6*x**6 + 15*b**7*d**4*e**2*x**3)/(3*a*b**7*(a**3 + 3*a**2*b*x + 3*a*b**2*x**2 + b**3*x**3))
```

3.119 $\int \frac{(d+ex)^5}{(a^2+2abx+b^2x^2)^2} dx$

| | |
|---|-----|
| Optimal result | 960 |
| Mathematica [A] (verified) | 960 |
| Rubi [A] (verified) | 961 |
| Maple [B] (verified) | 962 |
| Fricas [B] (verification not implemented) | 963 |
| Sympy [B] (verification not implemented) | 964 |
| Maxima [B] (verification not implemented) | 964 |
| Giac [B] (verification not implemented) | 965 |
| Mupad [B] (verification not implemented) | 966 |
| Reduce [B] (verification not implemented) | 966 |

Optimal result

Integrand size = 26, antiderivative size = 129

$$\int \frac{(d+ex)^5}{(a^2+2abx+b^2x^2)^2} dx = \frac{e^4(5bd-4ae)x}{b^5} + \frac{e^5x^2}{2b^4} - \frac{(bd-ae)^5}{3b^6(a+bx)^3} - \frac{5e(bd-ae)^4}{2b^6(a+bx)^2} - \frac{10e^2(bd-ae)^3}{b^6(a+bx)} + \frac{10e^3(bd-ae)^2 \log(a+bx)}{b^6}$$

output

```
e^4*(-4*a*e+5*b*d)*x/b^5+1/2*e^5*x^2/b^4-1/3*(-a*e+b*d)^5/b^6/(b*x+a)^3-5/2*e*(-a*e+b*d)^4/b^6/(b*x+a)^2-10*e^2*(-a*e+b*d)^3/b^6/(b*x+a)+10*e^3*(-a*e+b*d)^2*ln(b*x+a)/b^6
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 228, normalized size of antiderivative = 1.77

$$\int \frac{(d+ex)^5}{(a^2+2abx+b^2x^2)^2} dx = \frac{47a^5e^5 + a^4be^4(-130d + 81ex) + a^3b^2e^3(110d^2 - 270dex - 9e^2x^2) - a^2b^3e^2(20d^3 - 270d^2ex + 90de^2x^2)}{b^6}$$

input

```
Integrate[(d + e*x)^5/(a^2 + 2*a*b*x + b^2*x^2)^2,x]
```

output

$$(47*a^5*e^5 + a^4*b*e^4*(-130*d + 81*e*x) + a^3*b^2*e^3*(110*d^2 - 270*d*e*x - 9*e^2*x^2) - a^2*b^3*e^2*(20*d^3 - 270*d^2*e*x + 90*d*e^2*x^2 + 63*e^3*x^3) - 5*a*b^4*e*(d^4 + 12*d^3*e*x - 36*d^2*e^2*x^2 - 18*d*e^3*x^3 + 3*e^4*x^4) + b^5*(-2*d^5 - 15*d^4*e*x - 60*d^3*e^2*x^2 + 30*d*e^4*x^4 + 3*e^5*x^5) + 60*e^3*(b*d - a*e)^2*(a + b*x)^3*Log[a + b*x])/(6*b^6*(a + b*x)^3)$$
Rubi [A] (verified)

Time = 0.56 (sec) , antiderivative size = 129, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {1098, 27, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d + ex)^5}{(a^2 + 2abx + b^2x^2)^2} dx$$

$$\downarrow 1098$$

$$b^4 \int \frac{(d + ex)^5}{b^4(a + bx)^4} dx$$

$$\downarrow 27$$

$$\int \frac{(d + ex)^5}{(a + bx)^4} dx$$

$$\downarrow 49$$

$$\int \left(\frac{e^4(5bd - 4ae)}{b^5} + \frac{10e^3(bd - ae)^2}{b^5(a + bx)} + \frac{10e^2(bd - ae)^3}{b^5(a + bx)^2} + \frac{5e(bd - ae)^4}{b^5(a + bx)^3} + \frac{(bd - ae)^5}{b^5(a + bx)^4} + \frac{e^5x}{b^4} \right) dx$$

$$\downarrow 2009$$

$$\frac{10e^3(bd - ae)^2 \log(a + bx)}{b^6} - \frac{10e^2(bd - ae)^3}{b^6(a + bx)} - \frac{5e(bd - ae)^4}{2b^6(a + bx)^2} - \frac{(bd - ae)^5}{3b^6(a + bx)^3} + \frac{e^4x(5bd - 4ae)}{b^5} + \frac{e^5x^2}{2b^4}$$

input

$$\text{Int}[(d + e*x)^5/(a^2 + 2*a*b*x + b^2*x^2)^2, x]$$

```
output (e^4*(5*b*d - 4*a*e)*x)/b^5 + (e^5*x^2)/(2*b^4) - (b*d - a*e)^5/(3*b^6*(a + b*x)^3) - (5*e*(b*d - a*e)^4)/(2*b^6*(a + b*x)^2) - (10*e^2*(b*d - a*e)^3)/(b^6*(a + b*x)) + (10*e^3*(b*d - a*e)^2*Log[a + b*x])/b^6
```

Defintions of rubi rules used

```
rule 27 Int[(a_)*(Fx_), x_Symbol] := Simp[a Int[Fx, x], x] /; FreeQ[a, x] && !MatchQ[Fx, (b_)*(Gx_)] /; FreeQ[b, x]
```

```
rule 49 Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] := Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] && IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

```
rule 1098 Int[((d_.) + (e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] := Simp[1/c^p Int[(d + e*x)^m*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

```
rule 2009 Int[u_, x_Symbol] := Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 251 vs. 2(123) = 246.

Time = 1.04 (sec) , antiderivative size = 252, normalized size of antiderivative = 1.95

| method | result |
|--------------|--|
| default | $-\frac{e^4(-\frac{1}{2}be^2x^2+4aex-5bdx)}{b^5} - \frac{-e^5a^5+5a^4bde^4-10a^3b^2d^2e^3+10a^2b^3d^3e^2-5ab^4d^4e+b^5d^5}{3b^6(bx+a)^3} + \frac{10e^2(e^3a^3-3a^2bde^2+3a^2b^2d^2e-3a^2b^3d^3e^2)}{b^6(bx+a)}$ |
| norman | $\frac{(30a^3e^5-60a^2bde^4+30ab^2d^2e^3-10b^3d^3e^2)x^2}{b^4} + \frac{110e^5a^5-220a^4bde^4+110a^3b^2d^2e^3-20a^2b^3d^3e^2-5ab^4d^4e-2b^5d^5}{6b^6} + \frac{e^5x^5}{2b} + \frac{(90a^4e^5-10a^3bde^4+10a^2b^2d^2e^3-10ab^3d^3e^2)}{(bx+a)^3}$ |
| risch | $\frac{e^5x^2}{2b^4} - \frac{4e^5ax}{b^5} + \frac{5e^4dx}{b^4} + \frac{(10a^3be^5-30a^2b^2de^4+30ab^3d^2e^3-10b^4d^3e^2)x^2}{b^5(b^2x^2+2abx+a^2)} + \frac{5e(7a^4e^4-20a^3bde^3+18a^2b^2d^2e^2-4ab^3d^3e-5a^2b^4d^4e-2b^5d^5)}{b^5(b^2x^2+2abx+a^2)(bx+a)}$ |
| parallelrisc | $-\frac{360\ln(bx+a)xa^3b^2de^4-540a^3b^2de^4x-2b^5d^5-120\ln(bx+a)x^3ab^4de^4+180\ln(bx+a)xa^2b^3d^2e^3+60\ln(bx+a)x^3a^2b^3e^5}{b^6(bx+a)}$ |

input `int((e*x+d)^5/(b^2*x^2+2*a*b*x+a^2)^2,x,method=_RETURNVERBOSE)`

output `-e^4/b^5*(-1/2*b*e*x^2+4*a*e*x-5*b*d*x)-1/3/b^6*(-a^5*e^5+5*a^4*b*d*e^4-10*a^3*b^2*d^2*e^3+10*a^2*b^3*d^3*e^2-5*a*b^4*d^4*e+b^5*d^5)/(b*x+a)^3+10/b^6*e^2*(a^3*e^3-3*a^2*b*d*e^2+3*a*b^2*d^2*e-b^3*d^3)/(b*x+a)+10/b^6*e^3*(a^2*e^2-2*a*b*d*e+b^2*d^2)*ln(b*x+a)-5/2/b^6*e*(a^4*e^4-4*a^3*b*d*e^3+6*a^2*b^2*d^2*e^2-4*a*b^3*d^3*e+b^4*d^4)/(b*x+a)^2`

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 426 vs. $2(123) = 246$.

Time = 0.09 (sec) , antiderivative size = 426, normalized size of antiderivative = 3.30

$$\int \frac{(d+ex)^5}{(a^2+2abx+b^2x^2)^2} dx$$

$$= \frac{3b^5e^5x^5 - 2b^5d^5 - 5ab^4d^4e - 20a^2b^3d^3e^2 + 110a^3b^2d^2e^3 - 130a^4bde^4 + 47a^5e^5 + 15(2b^5de^4 - ab^4e^5)x}{(b^9x^3 + 3a^2b^8x^2 + 3a^2b^7x + a^3b^6)}$$

input `integrate((e*x+d)^5/(b^2*x^2+2*a*b*x+a^2)^2,x, algorithm="fricas")`

output `1/6*(3*b^5*e^5*x^5 - 2*b^5*d^5 - 5*a*b^4*d^4*e - 20*a^2*b^3*d^3*e^2 + 110*a^3*b^2*d^2*e^3 - 130*a^4*b*d*e^4 + 47*a^5*e^5 + 15*(2*b^5*d*e^4 - a*b^4*e^5)*x^4 + 9*(10*a*b^4*d*e^4 - 7*a^2*b^3*e^5)*x^3 - 3*(20*b^5*d^3*e^2 - 60*a*b^4*d^2*e^3 + 30*a^2*b^3*d*e^4 + 3*a^3*b^2*e^5)*x^2 - 3*(5*b^5*d^4*e + 20*a*b^4*d^3*e^2 - 90*a^2*b^3*d^2*e^3 + 90*a^3*b^2*d*e^4 - 27*a^4*b*e^5)*x + 60*(a^3*b^2*d^2*e^3 - 2*a^4*b*d*e^4 + a^5*e^5 + (b^5*d^2*e^3 - 2*a*b^4*d*e^4 + a^2*b^3*e^5)*x^3 + 3*(a*b^4*d^2*e^3 - 2*a^2*b^3*d*e^4 + a^3*b^2*e^5)*x^2 + 3*(a^2*b^3*d^2*e^3 - 2*a^3*b^2*d*e^4 + a^4*b*e^5)*x)*log(b*x + a))/(b^9*x^3 + 3*a^2*b^8*x^2 + 3*a^2*b^7*x + a^3*b^6)`

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 284 vs. $2(119) = 238$.

Time = 1.38 (sec) , antiderivative size = 284, normalized size of antiderivative = 2.20

$$\int \frac{(d+ex)^5}{(a^2+2abx+b^2x^2)^2} dx = x \left(-\frac{4ae^5}{b^5} + \frac{5de^4}{b^4} \right) + \frac{47a^5e^5 - 130a^4bde^4 + 110a^3b^2d^2e^3 - 20a^2b^3d^3e^2 - 5ab^4d^4e - 2b^5d^5 + x^2 \cdot (60a^3b^2e^5 - 180a^2b^3de^4 + 180ab^4d^2e^3 - 60b^5d^5)}{6a^3b^6 + 18a^2b^7x + 18ab^8x^2} + \frac{e^5x^2}{2b^4} + \frac{10e^3(ae-bd)^2 \log(a+bx)}{b^6}$$

input

```
integrate((e*x+d)**5/(b**2*x**2+2*a*b*x+a**2)**2,x)
```

output

```
x*(-4*a*e**5/b**5 + 5*d*e**4/b**4) + (47*a**5*e**5 - 130*a**4*b*d*e**4 + 10*a**3*b**2*d**2*e**3 - 20*a**2*b**3*d**3*e**2 - 5*a*b**4*d**4*e - 2*b**5*d**5 + x**2*(60*a**3*b**2*e**5 - 180*a**2*b**3*d*e**4 + 180*a*b**4*d**2*e**3 - 60*b**5*d**3*e**2) + x*(105*a**4*b*e**5 - 300*a**3*b**2*d*e**4 + 270*a**2*b**3*d**2*e**3 - 60*a*b**4*d**3*e**2 - 15*b**5*d**4*e))/(6*a**3*b**6 + 18*a**2*b**7*x + 18*a*b**8*x**2 + 6*b**9*x**3) + e**5*x**2/(2*b**4) + 10*e**3*(a*e - b*d)**2*log(a + b*x)/b**6
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 281 vs. $2(123) = 246$.

Time = 0.04 (sec) , antiderivative size = 281, normalized size of antiderivative = 2.18

$$\int \frac{(d+ex)^5}{(a^2+2abx+b^2x^2)^2} dx = -\frac{2b^5d^5 + 5ab^4d^4e + 20a^2b^3d^3e^2 - 110a^3b^2d^2e^3 + 130a^4bde^4 - 47a^5e^5 + 60(b^5d^3e^2 - 3ab^4d^2e^3 + 3a^2b^3de^4 - 60b^5d^5)}{6(b^9x^3 + 3ab^8x^2 + 3a^2b^7x - 6a^3b^6)} + \frac{be^5x^2 + 2(5bde^4 - 4ae^5)x}{2b^5} + \frac{10(b^2d^2e^3 - 2abde^4 + a^2e^5) \log(bx+a)}{b^6}$$

input

```
integrate((e*x+d)^5/(b^2*x^2+2*a*b*x+a^2)^2,x, algorithm="maxima")
```

output

```
-1/6*(2*b^5*d^5 + 5*a*b^4*d^4*e + 20*a^2*b^3*d^3*e^2 - 110*a^3*b^2*d^2*e^3
+ 130*a^4*b*d*e^4 - 47*a^5*e^5 + 60*(b^5*d^3*e^2 - 3*a*b^4*d^2*e^3 + 3*a^
2*b^3*d*e^4 - a^3*b^2*e^5)*x^2 + 15*(b^5*d^4*e + 4*a*b^4*d^3*e^2 - 18*a^2*
b^3*d^2*e^3 + 20*a^3*b^2*d*e^4 - 7*a^4*b*e^5)*x)/(b^9*x^3 + 3*a*b^8*x^2 +
3*a^2*b^7*x + a^3*b^6) + 1/2*(b*e^5*x^2 + 2*(5*b*d*e^4 - 4*a*e^5)*x)/b^5 +
10*(b^2*d^2*e^3 - 2*a*b*d*e^4 + a^2*e^5)*log(b*x + a)/b^6
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 262 vs. $2(123) = 246$.

Time = 0.17 (sec) , antiderivative size = 262, normalized size of antiderivative = 2.03

$$\int \frac{(d+ex)^5}{(a^2+2abx+b^2x^2)^2} dx$$

$$= \frac{10(b^2d^2e^3 - 2abde^4 + a^2e^5) \log(|bx+a|)}{b^6} + \frac{b^4e^5x^2 + 10b^4de^4x - 8ab^3e^5x}{2b^8}$$

$$- \frac{2b^5d^5 + 5ab^4d^4e + 20a^2b^3d^3e^2 - 110a^3b^2d^2e^3 + 130a^4bde^4 - 47a^5e^5 + 60(b^5d^3e^2 - 3ab^4d^2e^3 + 3a^2b^3de^4 - a^3b^2e^5)}{6(bx+a)^3b^6}$$

input

```
integrate((e*x+d)^5/(b^2*x^2+2*a*b*x+a^2)^2,x, algorithm="giac")
```

output

```
10*(b^2*d^2*e^3 - 2*a*b*d*e^4 + a^2*e^5)*log(abs(b*x + a))/b^6 + 1/2*(b^4*
e^5*x^2 + 10*b^4*d*e^4*x - 8*a*b^3*e^5*x)/b^8 - 1/6*(2*b^5*d^5 + 5*a*b^4*d
^4*e + 20*a^2*b^3*d^3*e^2 - 110*a^3*b^2*d^2*e^3 + 130*a^4*b*d*e^4 - 47*a^5
*e^5 + 60*(b^5*d^3*e^2 - 3*a*b^4*d^2*e^3 + 3*a^2*b^3*d*e^4 - a^3*b^2*e^5)*
x^2 + 15*(b^5*d^4*e + 4*a*b^4*d^3*e^2 - 18*a^2*b^3*d^2*e^3 + 20*a^3*b^2*d*
e^4 - 7*a^4*b*e^5)*x)/((b*x + a)^3*b^6)
```


output

```
(60*log(a + b*x)*a**6*e**5 - 120*log(a + b*x)*a**5*b*d*e**4 + 180*log(a +
b*x)*a**5*b*e**5*x + 60*log(a + b*x)*a**4*b**2*d**2*e**3 - 360*log(a + b*x
)*a**4*b**2*d*e**4*x + 180*log(a + b*x)*a**4*b**2*e**5*x**2 + 180*log(a +
b*x)*a**3*b**3*d**2*e**3*x - 360*log(a + b*x)*a**3*b**3*d*e**4*x**2 + 60*l
og(a + b*x)*a**3*b**3*e**5*x**3 + 180*log(a + b*x)*a**2*b**4*d**2*e**3*x**
2 - 120*log(a + b*x)*a**2*b**4*d*e**4*x**3 + 60*log(a + b*x)*a*b**5*d**2*e
**3*x**3 + 50*a**6*e**5 - 100*a**5*b*d*e**4 + 90*a**5*b*e**5*x + 50*a**4*b
**2*d**2*e**3 - 180*a**4*b**2*d*e**4*x + 90*a**3*b**3*d**2*e**3*x - 60*a**
3*b**3*e**5*x**3 - 5*a**2*b**4*d**4*e + 120*a**2*b**4*d*e**4*x**3 - 15*a**
2*b**4*e**5*x**4 - 2*a*b**5*d**5 - 15*a*b**5*d**4*e*x - 60*a*b**5*d**2*e**
3*x**3 + 30*a*b**5*d*e**4*x**4 + 3*a*b**5*e**5*x**5 + 20*b**6*d**3*e**2*x*
*3)/(6*a*b**6*(a**3 + 3*a**2*b*x + 3*a*b**2*x**2 + b**3*x**3))
```

3.120 $\int \frac{(d+ex)^4}{(a^2+2abx+b^2x^2)^2} dx$

| | |
|---|-----|
| Optimal result | 968 |
| Mathematica [A] (verified) | 968 |
| Rubi [A] (verified) | 969 |
| Maple [A] (verified) | 970 |
| Fricas [B] (verification not implemented) | 971 |
| Sympy [B] (verification not implemented) | 971 |
| Maxima [A] (verification not implemented) | 972 |
| Giac [A] (verification not implemented) | 972 |
| Mupad [B] (verification not implemented) | 973 |
| Reduce [B] (verification not implemented) | 973 |

Optimal result

Integrand size = 26, antiderivative size = 103

$$\int \frac{(d+ex)^4}{(a^2+2abx+b^2x^2)^2} dx = \frac{e^4x}{b^4} - \frac{(bd-ae)^4}{3b^5(a+bx)^3} - \frac{2e(bd-ae)^3}{b^5(a+bx)^2} - \frac{6e^2(bd-ae)^2}{b^5(a+bx)} + \frac{4e^3(bd-ae)\log(a+bx)}{b^5}$$

output `e^4*x/b^4-1/3*(-a*e+b*d)^4/b^5/(b*x+a)^3-2*e*(-a*e+b*d)^3/b^5/(b*x+a)^2-6*e^2*(-a*e+b*d)^2/b^5/(b*x+a)+4*e^3*(-a*e+b*d)*ln(b*x+a)/b^5`

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 166, normalized size of antiderivative = 1.61

$$\int \frac{(d+ex)^4}{(a^2+2abx+b^2x^2)^2} dx = \frac{-13a^4e^4 + a^3be^3(22d - 27ex) - 3a^2b^2e^2(2d^2 - 18dex + 3e^2x^2) + ab^3e(-2d^3 - 18d^2ex + 36de^2x^2 + 9e^3x^3)}{3b^5(a+bx)^3}$$

input `Integrate[(d + e*x)^4/(a^2 + 2*a*b*x + b^2*x^2)^2,x]`

output

$$\begin{aligned} & (-13a^4e^4 + a^3b^3e^3(22d - 27ex) - 3a^2b^2e^2(2d^2 - 18d^2ex \\ & + 3e^2x^2) + ab^3e^3(-2d^3 - 18d^2ex + 36d^2e^2x^2 + 9e^3x^3) - \\ & b^4(d^4 + 6d^3ex + 18d^2e^2x^2 - 3e^4x^4) - 12e^3(-(b^4d) + a^4e \\ &)*(a + bx)^3\text{Log}[a + bx]) / (3b^5(a + bx)^3) \end{aligned}$$
Rubi [A] (verified)

Time = 0.50 (sec) , antiderivative size = 103, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {1098, 27, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\begin{aligned} & \int \frac{(d + ex)^4}{(a^2 + 2abx + b^2x^2)^2} dx \\ & \quad \downarrow \text{1098} \\ & b^4 \int \frac{(d + ex)^4}{b^4(a + bx)^4} dx \\ & \quad \downarrow \text{27} \\ & \int \frac{(d + ex)^4}{(a + bx)^4} dx \\ & \quad \downarrow \text{49} \\ & \int \left(\frac{4e^3(bd - ae)}{b^4(a + bx)} + \frac{6e^2(bd - ae)^2}{b^4(a + bx)^2} + \frac{4e(bd - ae)^3}{b^4(a + bx)^3} + \frac{(bd - ae)^4}{b^4(a + bx)^4} + \frac{e^4}{b^4} \right) dx \\ & \quad \downarrow \text{2009} \\ & \frac{4e^3(bd - ae) \log(a + bx)}{b^5} - \frac{6e^2(bd - ae)^2}{b^5(a + bx)} - \frac{2e(bd - ae)^3}{b^5(a + bx)^2} - \frac{(bd - ae)^4}{3b^5(a + bx)^3} + \frac{e^4x}{b^4} \end{aligned}$$

input

$$\text{Int}[(d + e*x)^4/(a^2 + 2*a*b*x + b^2*x^2)^2, x]$$

```
output (e^4*x)/b^4 - (b*d - a*e)^4/(3*b^5*(a + b*x)^3) - (2*e*(b*d - a*e)^3)/(b^5*(a + b*x)^2) - (6*e^2*(b*d - a*e)^2)/(b^5*(a + b*x)) + (4*e^3*(b*d - a*e)*Log[a + b*x])/b^5
```

Defintions of rubi rules used

```
rule 27 Int[(a_)*(F_x_), x_Symbol] := Simp[a Int[F_x, x], x] /; FreeQ[a, x] && !MatchQ[F_x, (b_)*(G_x_)] /; FreeQ[b, x]
```

```
rule 49 Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] := Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] && IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

```
rule 1098 Int[((d_.) + (e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] := Simp[1/c^p Int[(d + e*x)^m*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

```
rule 2009 Int[u_, x_Symbol] := Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 1.07 (sec) , antiderivative size = 178, normalized size of antiderivative = 1.73

| method | result |
|---------------|--|
| default | $\frac{e^4 x}{b^4} - \frac{a^4 e^4 - 4a^3 b d e^3 + 6a^2 b^2 d^2 e^2 - 4a b^3 d^3 e + b^4 d^4}{3b^5 (bx+a)^3} - \frac{6e^2 (e^2 a^2 - 2abde + b^2 d^2)}{b^5 (bx+a)} - \frac{4e^3 (ae - bd) \ln(bx+a)}{b^5} + \frac{2e (e^3 a^3 - 3a^2 b d e^2 + 3a b^2 d^2 e - b^3 d^3)}{b^5}$ |
| norman | $\frac{e^4 x^4}{b} - \frac{22a^4 e^4 - 22a^3 b d e^3 + 6a^2 b^2 d^2 e^2 + 2a b^3 d^3 e + b^4 d^4}{3b^5} - \frac{3(4a^2 e^4 - 4abd e^3 + 2d^2 e^2 b^2) x^2}{b^3 (bx+a)^3} - \frac{(18e^4 a^3 - 18a^2 b d e^3 + 6a b^2 d^2 e^2 + 2d^3 e b^3) x}{b^4}$ |
| risch | $\frac{e^4 x}{b^4} + \frac{(-6a^2 b e^4 + 12a b^2 d e^3 - 6b^3 d^2 e^2) x^2 - 2e (5e^3 a^3 - 9a^2 b d e^2 + 3a b^2 d^2 e + b^3 d^3) x - 13a^4 e^4 - 22a^3 b d e^3 + 6a^2 b^2 d^2 e^2 + 2a b^3 d^3 e}{b^4 (b^2 x^2 + 2abx + a^2) (bx+a)}$ |
| parallelrisch | $-\frac{22a^4 e^4 - 22a^3 b d e^3 + 2a b^3 d^3 e + 6a^2 b^2 d^2 e^2 + 12 \ln(bx+a) x^3 a b^3 e^4 - 12 \ln(bx+a) x^3 b^4 d e^3 + 36 \ln(bx+a) x^2 a^2 b^2 e^4 + b^4 d^4 - 36 \ln(bx+a) x^2 a b^3 d e^3}{b^5}$ |

```
input int((e*x+d)^4/(b^2*x^2+2*a*b*x+a^2)^2,x,method=_RETURNVERBOSE)
```

output

```
e^4*x/b^4-1/3/b^5*(a^4*e^4-4*a^3*b*d*e^3+6*a^2*b^2*d^2*e^2-4*a*b^3*d^3*e+b^4*d^4)/(b*x+a)^3-6/b^5*e^2*(a^2*e^2-2*a*b*d*e+b^2*d^2)/(b*x+a)-4/b^5*e^3*(a*e-b*d)*ln(b*x+a)+2/b^5*e*(a^3*e^3-3*a^2*b*d*e^2+3*a*b^2*d^2*e-b^3*d^3)/(b*x+a)^2
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 292 vs. 2(101) = 202.

Time = 0.09 (sec) , antiderivative size = 292, normalized size of antiderivative = 2.83

$$\int \frac{(d+ex)^4}{(a^2+2abx+b^2x^2)^2} dx$$

$$= \frac{3b^4e^4x^4 + 9ab^3e^4x^3 - b^4d^4 - 2ab^3d^3e - 6a^2b^2d^2e^2 + 22a^3bde^3 - 13a^4e^4 - 9(2b^4d^2e^2 - 4ab^3de^3 + a^2b^2d^2e^2)}{(a^2+2abx+b^2x^2)^2}$$

input

```
integrate((e*x+d)^4/(b^2*x^2+2*a*b*x+a^2)^2,x, algorithm="fricas")
```

output

```
1/3*(3*b^4*e^4*x^4 + 9*a*b^3*e^4*x^3 - b^4*d^4 - 2*a*b^3*d^3*e - 6*a^2*b^2*d^2*e^2 + 22*a^3*b*d*e^3 - 13*a^4*e^4 - 9*(2*b^4*d^2*e^2 - 4*a*b^3*d*e^3 + a^2*b^2*e^4)*x^2 - 3*(2*b^4*d^3*e + 6*a*b^3*d^2*e^2 - 18*a^2*b^2*d*e^3 + 9*a^3*b*e^4)*x + 12*(a^3*b*d*e^3 - a^4*e^4 + (b^4*d*e^3 - a*b^3*e^4)*x^3 + 3*(a*b^3*d*e^3 - a^2*b^2*e^4)*x^2 + 3*(a^2*b^2*d*e^3 - a^3*b*e^4)*x)*log(b*x + a)/(b^8*x^3 + 3*a*b^7*x^2 + 3*a^2*b^6*x + a^3*b^5)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 209 vs. 2(94) = 188.

Time = 0.93 (sec) , antiderivative size = 209, normalized size of antiderivative = 2.03

$$\int \frac{(d+ex)^4}{(a^2+2abx+b^2x^2)^2} dx$$

$$= \frac{-13a^4e^4 + 22a^3bde^3 - 6a^2b^2d^2e^2 - 2ab^3d^3e - b^4d^4 + x^2(-18a^2b^2e^4 + 36ab^3de^3 - 18b^4d^2e^2) + x(-30a^3b^2de^3 + 12a^2b^3d^2e^2 - 6ab^4d^3e - 6a^2b^2d^2e^2) + \frac{e^4x}{b^4} - \frac{4e^3(ae-bd)\log(a+bx)}{b^5}}{3a^3b^5 + 9a^2b^6x + 9ab^7x^2 + 3b^8x^3}$$

input `integrate((e*x+d)**4/(b**2*x**2+2*a*b*x+a**2)**2,x)`

output `(-13*a**4*e**4 + 22*a**3*b*d*e**3 - 6*a**2*b**2*d**2*e**2 - 2*a*b**3*d**3*e - b**4*d**4 + x**2*(-18*a**2*b**2*e**4 + 36*a*b**3*d*e**3 - 18*b**4*d**2*e**2) + x*(-30*a**3*b*e**4 + 54*a**2*b**2*d*e**3 - 18*a*b**3*d**2*e**2 - 6*b**4*d**3*e))/(3*a**3*b**5 + 9*a**2*b**6*x + 9*a*b**7*x**2 + 3*b**8*x**3) + e**4*x/b**4 - 4*e**3*(a*e - b*d)*log(a + b*x)/b**5`

Maxima [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 201, normalized size of antiderivative = 1.95

$$\int \frac{(d+ex)^4}{(a^2+2abx+b^2x^2)^2} dx = \frac{e^4x}{b^4} - \frac{b^4d^4 + 2ab^3d^3e + 6a^2b^2d^2e^2 - 22a^3bde^3 + 13a^4e^4 + 18(b^4d^2e^2 - 2ab^3de^3 + a^2b^2e^4)x^2 + 6(b^4d^3e + 3b^4d^2e^2 - 2ab^3de^3 + a^2b^2e^4)x + 4(bde^3 - ae^4)\log(bx+a)}{3(b^8x^3 + 3ab^7x^2 + 3a^2b^6x + a^3b^5)} + \frac{4(bde^3 - ae^4)\log(bx+a)}{b^5}$$

input `integrate((e*x+d)^4/(b^2*x^2+2*a*b*x+a^2)^2,x, algorithm="maxima")`

output `e^4*x/b^4 - 1/3*(b^4*d^4 + 2*a*b^3*d^3*e + 6*a^2*b^2*d^2*e^2 - 22*a^3*b*d*e^3 + 13*a^4*e^4 + 18*(b^4*d^2*e^2 - 2*a*b^3*d*e^3 + a^2*b^2*e^4)*x^2 + 6*(b^4*d^3*e + 3*a*b^3*d^2*e^2 - 9*a^2*b^2*d*e^3 + 5*a^3*b*e^4)*x)/(b^8*x^3 + 3*a*b^7*x^2 + 3*a^2*b^6*x + a^3*b^5) + 4*(b*d*e^3 - a*e^4)*log(b*x + a)/b^5`

Giac [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 177, normalized size of antiderivative = 1.72

$$\int \frac{(d+ex)^4}{(a^2+2abx+b^2x^2)^2} dx = \frac{e^4x}{b^4} + \frac{4(bde^3 - ae^4)\log(|bx+a|)}{b^5} - \frac{b^4d^4 + 2ab^3d^3e + 6a^2b^2d^2e^2 - 22a^3bde^3 + 13a^4e^4 + 18(b^4d^2e^2 - 2ab^3de^3 + a^2b^2e^4)x^2 + 6(b^4d^3e + 3b^4d^2e^2 - 2ab^3de^3 + a^2b^2e^4)x + 4(bde^3 - ae^4)\log(bx+a)}{3(bx+a)^3b^5}$$

output

```
( - 12*log(a + b*x)*a**5*e**4 + 12*log(a + b*x)*a**4*b*d*e**3 - 36*log(a +
b*x)*a**4*b*e**4*x + 36*log(a + b*x)*a**3*b**2*d*e**3*x - 36*log(a + b*x)
*a**3*b**2*e**4*x**2 + 36*log(a + b*x)*a**2*b**3*d*e**3*x**2 - 12*log(a +
b*x)*a**2*b**3*e**4*x**3 + 12*log(a + b*x)*a*b**4*d*e**3*x**3 - 10*a**5*e*
**4 + 10*a**4*b*d*e**3 - 18*a**4*b*e**4*x + 18*a**3*b**2*d*e**3*x - 2*a**2*
b**3*d**3*e + 12*a**2*b**3*e**4*x**3 - a*b**4*d**4 - 6*a*b**4*d**3*e*x - 1
2*a*b**4*d*e**3*x**3 + 3*a*b**4*e**4*x**4 + 6*b**5*d**2*e**2*x**3)/(3*a*b*
*5*(a**3 + 3*a**2*b*x + 3*a*b**2*x**2 + b**3*x**3))
```

3.121
$$\int \frac{(d+ex)^3}{(a^2+2abx+b^2x^2)^2} dx$$

| | |
|---|-----|
| Optimal result | 975 |
| Mathematica [A] (verified) | 975 |
| Rubi [A] (verified) | 976 |
| Maple [A] (verified) | 977 |
| Fricas [B] (verification not implemented) | 978 |
| Sympy [A] (verification not implemented) | 978 |
| Maxima [A] (verification not implemented) | 979 |
| Giac [A] (verification not implemented) | 979 |
| Mupad [B] (verification not implemented) | 980 |
| Reduce [B] (verification not implemented) | 980 |

Optimal result

Integrand size = 26, antiderivative size = 86

$$\int \frac{(d+ex)^3}{(a^2+2abx+b^2x^2)^2} dx = -\frac{(bd-ae)^3}{3b^4(a+bx)^3} - \frac{3e(bd-ae)^2}{2b^4(a+bx)^2} - \frac{3e^2(bd-ae)}{b^4(a+bx)} + \frac{e^3 \log(a+bx)}{b^4}$$

output

$$-1/3*(-a*e+b*d)^3/b^4/(b*x+a)^3-3/2*e*(-a*e+b*d)^2/b^4/(b*x+a)^2-3*e^2*(-a*e+b*d)/b^4/(b*x+a)+e^3*\ln(b*x+a)/b^4$$

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.93

$$\int \frac{(d+ex)^3}{(a^2+2abx+b^2x^2)^2} dx = \frac{-(bd-ae)(11a^2e^2+abe(5d+27ex)+b^2(2d^2+9dex+18e^2x^2))}{(a+bx)^3} + 6e^3 \log(a+bx)}{6b^4}$$

input

`Integrate[(d + e*x)^3/(a^2 + 2*a*b*x + b^2*x^2)^2,x]`

output

$$\frac{-(((b*d - a*e)*(11*a^2*e^2 + a*b*e*(5*d + 27*e*x) + b^2*(2*d^2 + 9*d*e*x + 18*e^2*x^2)))/(a + b*x)^3 + 6*e^3*\text{Log}[a + b*x])/(6*b^4)}$$

Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {1098, 27, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\begin{aligned} & \int \frac{(d + ex)^3}{(a^2 + 2abx + b^2x^2)^2} dx \\ & \quad \downarrow 1098 \\ & b^4 \int \frac{(d + ex)^3}{b^4(a + bx)^4} dx \\ & \quad \downarrow 27 \\ & \int \frac{(d + ex)^3}{(a + bx)^4} dx \\ & \quad \downarrow 49 \\ & \int \left(\frac{3e^2(bd - ae)}{b^3(a + bx)^2} + \frac{3e(bd - ae)^2}{b^3(a + bx)^3} + \frac{(bd - ae)^3}{b^3(a + bx)^4} + \frac{e^3}{b^3(a + bx)} \right) dx \\ & \quad \downarrow 2009 \\ & -\frac{3e^2(bd - ae)}{b^4(a + bx)} - \frac{3e(bd - ae)^2}{2b^4(a + bx)^2} - \frac{(bd - ae)^3}{3b^4(a + bx)^3} + \frac{e^3 \log(a + bx)}{b^4} \end{aligned}$$

input

$$\text{Int}[(d + e*x)^3/(a^2 + 2*a*b*x + b^2*x^2)^2, x]$$

output

$$-1/3*(b*d - a*e)^3/(b^4*(a + b*x)^3) - (3*e*(b*d - a*e)^2)/(2*b^4*(a + b*x)^2) - (3*e^2*(b*d - a*e))/(b^4*(a + b*x)) + (e^3*\text{Log}[a + b*x])/b^4$$

Defintions of rubi rules used

rule 27 `Int[(a_)*(Fx_), x_Symbol] := Simp[a Int[Fx, x], x] /; FreeQ[a, x] && !MatchQ[Fx, (b_)*(Gx_)] /; FreeQ[b, x]`

rule 49 `Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] := Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] && IGtQ[m, 0] && IGtQ[m + n + 2, 0]`

rule 1098 `Int[((d_.) + (e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] := Simp[1/c^p Int[(d + e*x)^m*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]`

rule 2009 `Int[u_, x_Symbol] := Simp[IntSum[u, x], x] /; SumQ[u]`

Maple [A] (verified)

Time = 1.03 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.38

| method | result |
|---------------|--|
| norman | $\frac{11e^3a^3 - 6a^2bde^2 - 3ab^2d^2e - 2b^3d^3}{6b^4} + \frac{3(ae^3 - bde^2)x^2}{b^2} + \frac{3(3a^2e^3 - 2abde^2 - b^2d^2e)x}{2b^3} + \frac{e^3 \ln(bx+a)}{b^4}$ |
| default | $-\frac{e^3a^3 + 3a^2bde^2 - 3ab^2d^2e + b^3d^3}{3b^4(bx+a)^3} + \frac{3e^2(ae-bd)}{b^4(bx+a)} + \frac{e^3 \ln(bx+a)}{b^4} - \frac{3e(e^2a^2 - 2abde + b^2d^2)}{2b^4(bx+a)^2}$ |
| risch | $\frac{3e^2(ae-bd)x^2}{b^2} + \frac{3e(3e^2a^2 - 2abde - b^2d^2)x}{2b^3} + \frac{11e^3a^3 - 6a^2bde^2 - 3ab^2d^2e - 2b^3d^3}{6b^4} + \frac{e^3 \ln(bx+a)}{b^4}$ |
| parallelrisch | $\frac{6 \ln(bx+a)x^3b^3e^3 + 18 \ln(bx+a)x^2ab^2e^3 + 18 \ln(bx+a)xa^2be^3 + 18x^2a^2b^2e^3 - 18x^2b^3de^2 + 6 \ln(bx+a)a^3e^3 + 27a^2be^3x - 18xa}{6b^4(b^2x^2 + 2abx + a^2)(bx+a)}$ |

input `int((e*x+d)^3/(b^2*x^2+2*a*b*x+a^2)^2,x,method=_RETURNVERBOSE)`

output `(1/6*(11*a^3*e^3-6*a^2*b*d*e^2-3*a*b^2*d^2*e-2*b^3*d^3)/b^4+3*(a*e^3-b*d*e^2)/b^2*x^2+3/2*(3*a^2*e^3-2*a*b*d*e^2-b^2*d^2*e)/b^3*x)/(b*x+a)^3+e^3*ln(b*x+a)/b^4`

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 176 vs. $2(82) = 164$.

Time = 0.09 (sec) , antiderivative size = 176, normalized size of antiderivative = 2.05

$$\int \frac{(d+ex)^3}{(a^2+2abx+b^2x^2)^2} dx = \frac{2b^3d^3 + 3ab^2d^2e + 6a^2bde^2 - 11a^3e^3 + 18(b^3de^2 - ab^2e^3)x^2 + 9(b^3d^2e + 2ab^2de^2 - 3a^2be^3)x - 6(b^3d^2e^2 - ab^2e^3)}{6(b^7x^3 + 3ab^6x^2 + 3a^2b^5x + a^3b^4)}$$

input `integrate((e*x+d)^3/(b^2*x^2+2*a*b*x+a^2)^2,x, algorithm="fricas")`

output `-1/6*(2*b^3*d^3 + 3*a*b^2*d^2*e + 6*a^2*b*d*e^2 - 11*a^3*e^3 + 18*(b^3*d*e^2 - a*b^2*e^3)*x^2 + 9*(b^3*d^2*e + 2*a*b^2*d*e^2 - 3*a^2*b*e^3)*x - 6*(b^3*d^2*e^2 - ab^2*e^3)*x^3 + 3*a*b^2*e^3*x^2 + 3*a^2*b*e^3*x + a^3*e^3)*log(b*x + a)/(b^7*x^3 + 3*a*b^6*x^2 + 3*a^2*b^5*x + a^3*b^4)`

Sympy [A] (verification not implemented)

Time = 0.54 (sec) , antiderivative size = 148, normalized size of antiderivative = 1.72

$$\int \frac{(d+ex)^3}{(a^2+2abx+b^2x^2)^2} dx = \frac{11a^3e^3 - 6a^2bde^2 - 3ab^2d^2e - 2b^3d^3 + x^2 \cdot (18ab^2e^3 - 18b^3de^2) + x(27a^2be^3 - 18ab^2de^2 - 9b^3d^2e)}{6a^3b^4 + 18a^2b^5x + 18ab^6x^2 + 6b^7x^3} + \frac{e^3 \log(a+bx)}{b^4}$$

input `integrate((e*x+d)**3/(b**2*x**2+2*a*b*x+a**2)**2,x)`

output `(11*a**3*e**3 - 6*a**2*b*d*e**2 - 3*a*b**2*d**2*e - 2*b**3*d**3 + x**2*(18*a*b**2*e**3 - 18*b**3*d*e**2) + x*(27*a**2*b*e**3 - 18*a*b**2*d*e**2 - 9*b**3*d**2*e))/(6*a**3*b**4 + 18*a**2*b**5*x + 18*a*b**6*x**2 + 6*b**7*x**3) + e**3*log(a + b*x)/b**4`

Maxima [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 142, normalized size of antiderivative = 1.65

$$\int \frac{(d+ex)^3}{(a^2+2abx+b^2x^2)^2} dx =$$

$$-\frac{2b^3d^3+3ab^2d^2e+6a^2bde^2-11a^3e^3+18(b^3de^2-ab^2e^3)x^2+9(b^3d^2e+2ab^2de^2-3a^2be^3)x}{6(b^7x^3+3ab^6x^2+3a^2b^5x+a^3b^4)}$$

$$+\frac{e^3 \log(bx+a)}{b^4}$$

input `integrate((e*x+d)^3/(b^2*x^2+2*a*b*x+a^2)^2,x, algorithm="maxima")`output `-1/6*(2*b^3*d^3+3*a*b^2*d^2*e+6*a^2*b*d*e^2-11*a^3*e^3+18*(b^3*d*e^2-a*b^2*e^3)*x^2+9*(b^3*d^2*e+2*a*b^2*d*e^2-3*a^2*b*e^3)*x)/(b^7*x^3+3*a*b^6*x^2+3*a^2*b^5*x+a^3*b^4)+e^3*log(b*x+a)/b^4`**Giac [A] (verification not implemented)**

Time = 0.20 (sec) , antiderivative size = 118, normalized size of antiderivative = 1.37

$$\int \frac{(d+ex)^3}{(a^2+2abx+b^2x^2)^2} dx$$

$$= \frac{e^3 \log(|bx+a|)}{b^4}$$

$$-\frac{18(b^2de^2-abe^3)x^2+9(b^2d^2e+2abde^2-3a^2e^3)x+\frac{2b^3d^3+3ab^2d^2e+6a^2bde^2-11a^3e^3}{b}}{6(bx+a)^3b^3}$$

input `integrate((e*x+d)^3/(b^2*x^2+2*a*b*x+a^2)^2,x, algorithm="giac")`output `e^3*log(abs(b*x+a))/b^4-1/6*(18*(b^2*d*e^2-a*b*e^3)*x^2+9*(b^2*d^2*e+2*a*b*d*e^2-3*a^2*e^3)*x+(2*b^3*d^3+3*a*b^2*d^2*e+6*a^2*b*d*e^2-11*a^3*e^3)/b)/((b*x+a)^3*b^3)`

Mupad [B] (verification not implemented)

Time = 8.79 (sec) , antiderivative size = 138, normalized size of antiderivative = 1.60

$$\int \frac{(d+ex)^3}{(a^2+2abx+b^2x^2)^2} dx$$

$$= \frac{e^3 \ln(a+bx)}{b^4} - \frac{-11a^3e^3+6a^2bde^2+3ab^2d^2e+2b^3d^3}{6b^4} + \frac{3x(-3a^2e^3+2abde^2+b^2d^2e)}{2b^3} - \frac{3e^2x^2(ae-bd)}{b^2}$$

$$- \frac{a^3+3a^2bx+3ab^2x^2+b^3x^3}{a^3+3a^2bx+3ab^2x^2+b^3x^3}$$

input `int((d + e*x)^3/(a^2 + b^2*x^2 + 2*a*b*x)^2,x)`output `(e^3*log(a + b*x))/b^4 - ((2*b^3*d^3 - 11*a^3*e^3 + 3*a*b^2*d^2*e + 6*a^2*b*d*e^2)/(6*b^4) + (3*x*(b^2*d^2*e - 3*a^2*e^3 + 2*a*b*d*e^2))/(2*b^3) - (3*e^2*x^2*(a*e - b*d))/b^2)/(a^3 + b^3*x^3 + 3*a*b^2*x^2 + 3*a^2*b*x)`**Reduce [B] (verification not implemented)**

Time = 0.24 (sec) , antiderivative size = 180, normalized size of antiderivative = 2.09

$$\int \frac{(d+ex)^3}{(a^2+2abx+b^2x^2)^2} dx$$

$$= \frac{6 \log(bx+a) a^4 e^3 + 18 \log(bx+a) a^3 b e^3 x + 18 \log(bx+a) a^2 b^2 e^3 x^2 + 6 \log(bx+a) a b^3 e^3 x^3 + 5 a^4 e^3 + 6 a b^4 (b^3 x^3 + 3 a b^2 x^2 + 3 a^2 b x + a^3)}{6 a b^4 (b^3 x^3 + 3 a b^2 x^2 + 3 a^2 b x + a^3)}$$

input `int((e*x+d)^3/(b^2*x^2+2*a*b*x+a^2)^2,x)`output `(6*log(a + b*x)*a**4*e**3 + 18*log(a + b*x)*a**3*b*e**3*x + 18*log(a + b*x)*a**2*b**2*e**3*x**2 + 6*log(a + b*x)*a*b**3*e**3*x**3 + 5*a**4*e**3 + 9*a**3*b*e**3*x - 3*a**2*b**2*d**2*e - 2*a*b**3*d**3 - 9*a*b**3*d**2*e*x - 6*a*b**3*e**3*x**3 + 6*b**4*d*e**2*x**3)/(6*a*b**4*(a**3 + 3*a**2*b*x + 3*a*b**2*x**2 + b**3*x**3))`

$$3.122 \quad \int \frac{(d+ex)^2}{(a^2+2abx+b^2x^2)^2} dx$$

| | |
|---|-----|
| Optimal result | 981 |
| Mathematica [A] (verified) | 981 |
| Rubi [A] (verified) | 982 |
| Maple [B] (verified) | 983 |
| Fricas [B] (verification not implemented) | 983 |
| Sympy [B] (verification not implemented) | 984 |
| Maxima [B] (verification not implemented) | 984 |
| Giac [B] (verification not implemented) | 985 |
| Mupad [B] (verification not implemented) | 985 |
| Reduce [B] (verification not implemented) | 985 |

Optimal result

Integrand size = 26, antiderivative size = 28

$$\int \frac{(d+ex)^2}{(a^2+2abx+b^2x^2)^2} dx = -\frac{(d+ex)^3}{3(bd-ae)(a+bx)^3}$$

output

$$-1/3*(e*x+d)^3/(-a*e+b*d)/(b*x+a)^3$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.89

$$\int \frac{(d+ex)^2}{(a^2+2abx+b^2x^2)^2} dx = -\frac{a^2e^2 + abe(d+3ex) + b^2(d^2+3dex+3e^2x^2)}{3b^3(a+bx)^3}$$

input

$$\text{Integrate}[(d + e*x)^2/(a^2 + 2*a*b*x + b^2*x^2)^2,x]$$

output

$$-1/3*(a^2*e^2 + a*b*e*(d + 3*e*x) + b^2*(d^2 + 3*d*e*x + 3*e^2*x^2))/(b^3*(a + b*x)^3)$$

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.115$, Rules used = {1098, 27, 48}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d + ex)^2}{(a^2 + 2abx + b^2x^2)^2} dx$$

↓ 1098

$$b^4 \int \frac{(d + ex)^2}{b^4(a + bx)^4} dx$$

↓ 27

$$\int \frac{(d + ex)^2}{(a + bx)^4} dx$$

↓ 48

$$-\frac{(d + ex)^3}{3(a + bx)^3(bd - ae)}$$

input `Int[(d + e*x)^2/(a^2 + 2*a*b*x + b^2*x^2)^2,x]`

output `-1/3*(d + e*x)^3/((b*d - a*e)*(a + b*x)^3)`

Defintions of rubi rules used

rule 27 `Int[(a_)*(F_x_), x_Symbol] :> Simp[a Int[F_x, x], x] /; FreeQ[a, x] && !MatchQ[F_x, (b_)*(G_x_)] /; FreeQ[b, x]`

rule 48 `Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{a, b, c, d, m, n}, x] && EqQ[m + n + 2, 0] && NeQ[m, -1]`

rule 1098

```
Int[((d_.) + (e_.)*(x_.))^(m_.)*((a_.) + (b_.)*(x_.) + (c_.)*(x_.)^2)^(p_.), x_
Symbol] := Simp[1/c^p Int[(d + e*x)^m*(b/2 + c*x)^(2*p), x], x] /; FreeQ[
{a, b, c, d, e, m}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 65 vs. 2(26) = 52.

Time = 1.00 (sec) , antiderivative size = 66, normalized size of antiderivative = 2.36

| method | result | size |
|--------------|--|------|
| norman | $\frac{-\frac{e^2 x^2}{b} + \frac{(-a e^2 - b d e)x}{b^2} + \frac{-e^2 a^2 - a b d e - b^2 d^2}{3 b^3}}{(b x + a)^3}$ | 66 |
| default | $-\frac{e^2 a^2 - 2 a b d e + b^2 d^2}{3 b^3 (b x + a)^3} - \frac{e^2}{b^3 (b x + a)} + \frac{e(a e - b d)}{b^3 (b x + a)^2}$ | 70 |
| orering | $-\frac{(3 b^2 e^2 x^2 + 3 x a b e^2 + 3 b^2 d e x + e^2 a^2 + a b d e + b^2 d^2)(b x + a)}{3 b^3 (b^2 x^2 + 2 a b x + a^2)^2}$ | 76 |
| gospers | $-\frac{3 b^2 e^2 x^2 + 3 x a b e^2 + 3 b^2 d e x + e^2 a^2 + a b d e + b^2 d^2}{3 (b x + a) (b^2 x^2 + 2 a b x + a^2) b^3}$ | 78 |
| risch | $\frac{-\frac{e^2 x^2}{b} - \frac{e(a e + b d)x}{b^2} - \frac{e^2 a^2 + a b d e + b^2 d^2}{3 b^3}}{(b x + a) (b^2 x^2 + 2 a b x + a^2)}$ | 78 |
| parallelrisc | $\frac{-3 b^2 e^2 x^2 - 3 x a b e^2 - 3 b^2 d e x - e^2 a^2 - a b d e - b^2 d^2}{3 b^3 (b^2 x^2 + 2 a b x + a^2) (b x + a)}$ | 81 |

input

```
int((e*x+d)^2/(b^2*x^2+2*a*b*x+a^2)^2,x,method=_RETURNVERBOSE)
```

output

```
(-e^2/b*x^2+(-a*e^2-b*d*e)/b^2*x+1/3*(-a^2*e^2-a*b*d*e-b^2*d^2)/b^3)/(b*x+a)^3
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 84 vs. 2(26) = 52.

Time = 0.08 (sec) , antiderivative size = 84, normalized size of antiderivative = 3.00

$$\int \frac{(d + ex)^2}{(a^2 + 2abx + b^2x^2)^2} dx = -\frac{3b^2e^2x^2 + b^2d^2 + abde + a^2e^2 + 3(b^2de + abe^2)x}{3(b^6x^3 + 3ab^5x^2 + 3a^2b^4x + a^3b^3)}$$

input `integrate((e*x+d)^2/(b^2*x^2+2*a*b*x+a^2)^2,x, algorithm="fricas")`

output `-1/3*(3*b^2*e^2*x^2 + b^2*d^2 + a*b*d*e + a^2*e^2 + 3*(b^2*d*e + a*b*e^2)*x)/(b^6*x^3 + 3*a*b^5*x^2 + 3*a^2*b^4*x + a^3*b^3)`

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 88 vs. $2(22) = 44$.

Time = 0.30 (sec) , antiderivative size = 88, normalized size of antiderivative = 3.14

$$\int \frac{(d+ex)^2}{(a^2+2abx+b^2x^2)^2} dx = \frac{-a^2e^2 - abde - b^2d^2 - 3b^2e^2x^2 + x(-3abe^2 - 3b^2de)}{3a^3b^3 + 9a^2b^4x + 9ab^5x^2 + 3b^6x^3}$$

input `integrate((e*x+d)**2/(b**2*x**2+2*a*b*x+a**2)**2,x)`

output `(-a**2*e**2 - a*b*d*e - b**2*d**2 - 3*b**2*e**2*x**2 + x*(-3*a*b*e**2 - 3*b**2*d*e))/(3*a**3*b**3 + 9*a**2*b**4*x + 9*a*b**5*x**2 + 3*b**6*x**3)`

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 84 vs. $2(26) = 52$.

Time = 0.03 (sec) , antiderivative size = 84, normalized size of antiderivative = 3.00

$$\int \frac{(d+ex)^2}{(a^2+2abx+b^2x^2)^2} dx = -\frac{3b^2e^2x^2 + b^2d^2 + abde + a^2e^2 + 3(b^2de + abe^2)x}{3(b^6x^3 + 3ab^5x^2 + 3a^2b^4x + a^3b^3)}$$

input `integrate((e*x+d)^2/(b^2*x^2+2*a*b*x+a^2)^2,x, algorithm="maxima")`

output `-1/3*(3*b^2*e^2*x^2 + b^2*d^2 + a*b*d*e + a^2*e^2 + 3*(b^2*d*e + a*b*e^2)*x)/(b^6*x^3 + 3*a*b^5*x^2 + 3*a^2*b^4*x + a^3*b^3)`

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 59 vs. $2(26) = 52$.

Time = 0.17 (sec) , antiderivative size = 59, normalized size of antiderivative = 2.11

$$\int \frac{(d+ex)^2}{(a^2+2abx+b^2x^2)^2} dx = -\frac{3b^2e^2x^2+3b^2dex+3abe^2x+b^2d^2+abde+a^2e^2}{3(bx+a)^3b^3}$$

input `integrate((e*x+d)^2/(b^2*x^2+2*a*b*x+a^2)^2,x, algorithm="giac")`

output `-1/3*(3*b^2*e^2*x^2+3*b^2*d*e*x+3*a*b*e^2*x+b^2*d^2+a*b*d*e+a^2*e^2)/((b*x+a)^3*b^3)`

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 80, normalized size of antiderivative = 2.86

$$\int \frac{(d+ex)^2}{(a^2+2abx+b^2x^2)^2} dx = -\frac{a^2e^2+abde+b^2d^2}{3b^3} + \frac{e^2x^2}{b} + \frac{ex(ae+bd)}{b^2}$$

input `int((d+e*x)^2/(a^2+b^2*x^2+2*a*b*x)^2,x)`

output `-((a^2*e^2+b^2*d^2+a*b*d*e)/(3*b^3)+(e^2*x^2)/b+(e*x*(a*e+b*d))/b^2)/(a^3+b^3*x^3+3*a*b^2*x^2+3*a^2*b*x)`

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 69, normalized size of antiderivative = 2.46

$$\int \frac{(d+ex)^2}{(a^2+2abx+b^2x^2)^2} dx = \frac{b^2e^2x^3-3abdex-a^2de-abd^2}{3ab^2(b^3x^3+3ab^2x^2+3a^2bx+a^3)}$$

input `int((e*x+d)^2/(b^2*x^2+2*a*b*x+a^2)^2,x)`

output $(-a^{**2}d*e - a*b*d^{**2} - 3*a*b*d*e*x + b^{**2}*e^{**2}*x^{**3})/(3*a*b^{**2}*(a^{**3} + 3*a^{**2}*b*x + 3*a*b^{**2}*x^{**2} + b^{**3}*x^{**3}))$

3.123 $\int \frac{d+ex}{(a^2+2abx+b^2x^2)^2} dx$

| | |
|---|-----|
| Optimal result | 987 |
| Mathematica [A] (verified) | 987 |
| Rubi [A] (verified) | 988 |
| Maple [A] (verified) | 989 |
| Fricas [A] (verification not implemented) | 990 |
| Sympy [A] (verification not implemented) | 990 |
| Maxima [A] (verification not implemented) | 991 |
| Giac [A] (verification not implemented) | 991 |
| Mupad [B] (verification not implemented) | 991 |
| Reduce [B] (verification not implemented) | 992 |

Optimal result

Integrand size = 24, antiderivative size = 38

$$\int \frac{d+ex}{(a^2+2abx+b^2x^2)^2} dx = -\frac{bd-ae}{3b^2(a+bx)^3} - \frac{e}{2b^2(a+bx)^2}$$

output `-1/3*(-a*e+b*d)/b^2/(b*x+a)^3-1/2*e/b^2/(b*x+a)^2`

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.71

$$\int \frac{d+ex}{(a^2+2abx+b^2x^2)^2} dx = -\frac{2bd+ae+3bex}{6b^2(a+bx)^3}$$

input `Integrate[(d + e*x)/(a^2 + 2*a*b*x + b^2*x^2)^2,x]`

output `-1/6*(2*b*d + a*e + 3*b*e*x)/(b^2*(a + b*x)^3)`

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {1098, 27, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\begin{aligned}
 & \int \frac{d + ex}{(a^2 + 2abx + b^2x^2)^2} dx \\
 & \quad \downarrow \text{1098} \\
 & b^4 \int \frac{d + ex}{b^4(a + bx)^4} dx \\
 & \quad \downarrow \text{27} \\
 & \int \frac{d + ex}{(a + bx)^4} dx \\
 & \quad \downarrow \text{53} \\
 & \int \left(\frac{bd - ae}{b(a + bx)^4} + \frac{e}{b(a + bx)^3} \right) dx \\
 & \quad \downarrow \text{2009} \\
 & -\frac{bd - ae}{3b^2(a + bx)^3} - \frac{e}{2b^2(a + bx)^2}
 \end{aligned}$$

input `Int[(d + e*x)/(a^2 + 2*a*b*x + b^2*x^2)^2,x]`

output `-1/3*(b*d - a*e)/(b^2*(a + b*x)^3) - e/(2*b^2*(a + b*x)^2)`

Definitions of rubi rules used

rule 27 $\text{Int}[(a_*)(Fx_), x_Symbol] \rightarrow \text{Simp}[a \text{ Int}[Fx, x], x] /; \text{FreeQ}[a, x] \ \&\& \ !\text{MatchQ}[Fx, (b_*)(Gx_)] /; \text{FreeQ}[b, x]$

rule 53 $\text{Int}[(a_.) + (b_.)*(x_)^{(m_.)}*((c_.) + (d_.)*(x_)^{(n_.)}), x_Symbol] \rightarrow \text{Int}[\text{ExpandIntegrand}[(a + b*x)^m*(c + d*x)^n, x], x] /; \text{FreeQ}\{a, b, c, d, n\}, x] \ \&\& \ \text{IGtQ}[m, 0] \ \&\& \ (\ !\text{IntegerQ}[n] \ || \ (\text{EqQ}[c, 0] \ \&\& \ \text{LeQ}[7*m + 4*n + 4, 0]) \ || \ \text{LtQ}[9*m + 5*(n + 1), 0] \ || \ \text{GtQ}[m + n + 2, 0])$

rule 1098 $\text{Int}[(d_.) + (e_.)*(x_)^{(m_.)}*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^{(p_.)}, x_Symbol] \rightarrow \text{Simp}[1/c^p \text{ Int}[(d + e*x)^m*(b/2 + c*x)^(2*p), x], x] /; \text{FreeQ}\{a, b, c, d, e, m\}, x] \ \&\& \ \text{EqQ}[b^2 - 4*a*c, 0] \ \&\& \ \text{IntegerQ}[p]$

rule 2009 $\text{Int}[u_, x_Symbol] \rightarrow \text{Simp}[\text{IntSum}[u, x], x] /; \text{SumQ}[u]$

Maple [A] (verified)

Time = 0.98 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.89

| method | result | size |
|--------------|--|------|
| norman | $\frac{-\frac{ex}{2b} + \frac{-abe-2b^2d}{6b^3}}{(bx+a)^3}$ | 34 |
| default | $-\frac{-ae+bd}{3b^2(bx+a)^3} - \frac{e}{2b^2(bx+a)^2}$ | 35 |
| orering | $-\frac{(3bex+ae+2bd)(bx+a)}{6b^2(b^2x^2+2abx+a^2)^2}$ | 42 |
| gospers | $-\frac{3bex+ae+2bd}{6b^2(bx+a)(b^2x^2+2abx+a^2)}$ | 44 |
| risch | $\frac{-\frac{ex}{2b} - \frac{ae+2bd}{6b^2}}{(bx+a)(b^2x^2+2abx+a^2)}$ | 48 |
| parallelrisc | $\frac{-3b^2ex-abe-2b^2d}{6b^3(b^2x^2+2abx+a^2)(bx+a)}$ | 50 |

input $\text{int}((e*x+d)/(b^2*x^2+2*a*b*x+a^2)^2, x, \text{method}=_RETURNVERBOSE)$

output $(-1/2*ex/b+1/6*(-a*b*e-2*b^2*d)/b^3)/(b*x+a)^3$

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.32

$$\int \frac{d + ex}{(a^2 + 2abx + b^2x^2)^2} dx = -\frac{3bex + 2bd + ae}{6(b^5x^3 + 3ab^4x^2 + 3a^2b^3x + a^3b^2)}$$

input `integrate((e*x+d)/(b^2*x^2+2*a*b*x+a^2)^2,x, algorithm="fricas")`

output $-1/6*(3*b*e*x + 2*b*d + a*e)/(b^5*x^3 + 3*a*b^4*x^2 + 3*a^2*b^3*x + a^3*b^2)$

Sympy [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.39

$$\int \frac{d + ex}{(a^2 + 2abx + b^2x^2)^2} dx = \frac{-ae - 2bd - 3bex}{6a^3b^2 + 18a^2b^3x + 18ab^4x^2 + 6b^5x^3}$$

input `integrate((e*x+d)/(b**2*x**2+2*a*b*x+a**2)**2,x)`

output $(-a*e - 2*b*d - 3*b*e*x)/(6*a**3*b**2 + 18*a**2*b**3*x + 18*a*b**4*x**2 + 6*b**5*x**3)$

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.32

$$\int \frac{d + ex}{(a^2 + 2abx + b^2x^2)^2} dx = -\frac{3bex + 2bd + ae}{6(b^5x^3 + 3ab^4x^2 + 3a^2b^3x + a^3b^2)}$$

input `integrate((e*x+d)/(b^2*x^2+2*a*b*x+a^2)^2,x, algorithm="maxima")`output `-1/6*(3*b*e*x + 2*b*d + a*e)/(b^5*x^3 + 3*a*b^4*x^2 + 3*a^2*b^3*x + a^3*b^2)`**Giac [A] (verification not implemented)**

Time = 0.19 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.66

$$\int \frac{d + ex}{(a^2 + 2abx + b^2x^2)^2} dx = -\frac{3bex + 2bd + ae}{6(bx + a)^3b^2}$$

input `integrate((e*x+d)/(b^2*x^2+2*a*b*x+a^2)^2,x, algorithm="giac")`output `-1/6*(3*b*e*x + 2*b*d + a*e)/((b*x + a)^3*b^2)`**Mupad [B] (verification not implemented)**

Time = 0.03 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.37

$$\int \frac{d + ex}{(a^2 + 2abx + b^2x^2)^2} dx = -\frac{\frac{ae+2bd}{6b^2} + \frac{ex}{2b}}{a^3 + 3a^2bx + 3ab^2x^2 + b^3x^3}$$

input `int((d + e*x)/(a^2 + b^2*x^2 + 2*a*b*x)^2,x)`output `-((a*e + 2*b*d)/(6*b^2) + (e*x)/(2*b))/(a^3 + b^3*x^3 + 3*a*b^2*x^2 + 3*a^2*b*x)`

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.26

$$\int \frac{d + ex}{(a^2 + 2abx + b^2x^2)^2} dx = \frac{-3bex - ae - 2bd}{6b^2(b^3x^3 + 3ab^2x^2 + 3a^2bx + a^3)}$$

input `int((e*x+d)/(b^2*x^2+2*a*b*x+a^2)^2,x)`

output `(- a*e - 2*b*d - 3*b*e*x)/(6*b**2*(a**3 + 3*a**2*b*x + 3*a*b**2*x**2 + b**3*x**3))`

$$3.124 \quad \int \frac{1}{(a^2 + 2abx + b^2x^2)^2} dx$$

| | |
|---|-----|
| Optimal result | 993 |
| Mathematica [A] (verified) | 993 |
| Rubi [A] (verified) | 994 |
| Maple [A] (verified) | 995 |
| Fricas [B] (verification not implemented) | 995 |
| Sympy [B] (verification not implemented) | 996 |
| Maxima [B] (verification not implemented) | 996 |
| Giac [A] (verification not implemented) | 996 |
| Mupad [B] (verification not implemented) | 997 |
| Reduce [B] (verification not implemented) | 997 |

Optimal result

Integrand size = 18, antiderivative size = 14

$$\int \frac{1}{(a^2 + 2abx + b^2x^2)^2} dx = -\frac{1}{3b(a + bx)^3}$$

output

```
-1/3/b/(b*x+a)^3
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00

$$\int \frac{1}{(a^2 + 2abx + b^2x^2)^2} dx = -\frac{1}{3b(a + bx)^3}$$

input

```
Integrate[(a^2 + 2*a*b*x + b^2*x^2)^(-2), x]
```

output

```
-1/3*1/(b*(a + b*x)^3)
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {1077, 17}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a^2 + 2abx + b^2x^2)^2} dx$$

$$\downarrow 1077$$

$$b^4 \int \frac{1}{(xb^2 + ab)^4} dx$$

$$\downarrow 17$$

$$-\frac{1}{3b(a + bx)^3}$$

input `Int[(a^2 + 2*a*b*x + b^2*x^2)^(-2),x]`

output `-1/3*1/(b*(a + b*x)^3)`

Defintions of rubi rules used

rule 17 `Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1))/(b*(m + 1))], x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]`

rule 1077 `Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[1/c^p Int[(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]`

Maple [A] (verified)

Time = 0.95 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.93

| method | result | size |
|---------------|--|------|
| default | $-\frac{1}{3b(bx+a)^3}$ | 13 |
| norman | $-\frac{1}{3b(bx+a)^3}$ | 13 |
| risch | $-\frac{1}{3b(bx+a)^3}$ | 13 |
| orering | $-\frac{bx+a}{3b(b^2x^2+2abx+a^2)^2}$ | 29 |
| gosper | $-\frac{1}{3(bx+a)(b^2x^2+2abx+a^2)b}$ | 31 |
| parallelrisch | $-\frac{1}{3(bx+a)(b^2x^2+2abx+a^2)b}$ | 31 |

input `int(1/(b^2*x^2+2*a*b*x+a^2)^2,x,method=_RETURNVERBOSE)`

output `-1/3/b/(b*x+a)^3`

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 35 vs. 2(12) = 24.

Time = 0.08 (sec) , antiderivative size = 35, normalized size of antiderivative = 2.50

$$\int \frac{1}{(a^2 + 2abx + b^2x^2)^2} dx = -\frac{1}{3(b^4x^3 + 3ab^3x^2 + 3a^2b^2x + a^3b)}$$

input `integrate(1/(b^2*x^2+2*a*b*x+a^2)^2,x, algorithm="fricas")`

output `-1/3/(b^4*x^3 + 3*a*b^3*x^2 + 3*a^2*b^2*x + a^3*b)`

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 37 vs. $2(12) = 24$.

Time = 0.10 (sec) , antiderivative size = 37, normalized size of antiderivative = 2.64

$$\int \frac{1}{(a^2 + 2abx + b^2x^2)^2} dx = -\frac{1}{3a^3b + 9a^2b^2x + 9ab^3x^2 + 3b^4x^3}$$

input `integrate(1/(b**2*x**2+2*a*b*x+a**2)**2,x)`

output `-1/(3*a**3*b + 9*a**2*b**2*x + 9*a*b**3*x**2 + 3*b**4*x**3)`

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 35 vs. $2(12) = 24$.

Time = 0.05 (sec) , antiderivative size = 35, normalized size of antiderivative = 2.50

$$\int \frac{1}{(a^2 + 2abx + b^2x^2)^2} dx = -\frac{1}{3(b^4x^3 + 3ab^3x^2 + 3a^2b^2x + a^3b)}$$

input `integrate(1/(b^2*x^2+2*a*b*x+a^2)^2,x, algorithm="maxima")`

output `-1/3/(b^4*x^3 + 3*a*b^3*x^2 + 3*a^2*b^2*x + a^3*b)`

Giac [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.86

$$\int \frac{1}{(a^2 + 2abx + b^2x^2)^2} dx = -\frac{1}{3(bx + a)^3b}$$

input `integrate(1/(b^2*x^2+2*a*b*x+a^2)^2,x, algorithm="giac")`

output `-1/3/((b*x + a)^3*b)`

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 37, normalized size of antiderivative = 2.64

$$\int \frac{1}{(a^2 + 2abx + b^2x^2)^2} dx = -\frac{1}{3a^3b + 9a^2b^2x + 9ab^3x^2 + 3b^4x^3}$$

input `int(1/(a^2 + b^2*x^2 + 2*a*b*x)^2,x)`output `-1/(3*a^3*b + 3*b^4*x^3 + 9*a^2*b^2*x + 9*a*b^3*x^2)`**Reduce [B] (verification not implemented)**

Time = 0.22 (sec) , antiderivative size = 34, normalized size of antiderivative = 2.43

$$\int \frac{1}{(a^2 + 2abx + b^2x^2)^2} dx = -\frac{1}{3b(b^3x^3 + 3ab^2x^2 + 3a^2bx + a^3)}$$

input `int(1/(b^2*x^2+2*a*b*x+a^2)^2,x)`output `(- 1)/(3*b*(a**3 + 3*a**2*b*x + 3*a*b**2*x**2 + b**3*x**3))`

3.125 $\int \frac{1}{(d+ex)(a^2+2abx+b^2x^2)^2} dx$

| | |
|---|------|
| Optimal result | 998 |
| Mathematica [A] (verified) | 998 |
| Rubi [A] (verified) | 999 |
| Maple [A] (verified) | 1000 |
| Fricas [B] (verification not implemented) | 1001 |
| Sympy [B] (verification not implemented) | 1002 |
| Maxima [B] (verification not implemented) | 1003 |
| Giac [B] (verification not implemented) | 1003 |
| Mupad [B] (verification not implemented) | 1004 |
| Reduce [B] (verification not implemented) | 1005 |

Optimal result

Integrand size = 26, antiderivative size = 107

$$\int \frac{1}{(d+ex)(a^2+2abx+b^2x^2)^2} dx = -\frac{1}{3(bd-ae)(a+bx)^3} + \frac{e}{2(bd-ae)^2(a+bx)^2} - \frac{e^2}{(bd-ae)^3(a+bx)} - \frac{e^3 \log(a+bx)}{(bd-ae)^4} + \frac{e^3 \log(d+ex)}{(bd-ae)^4}$$

output

```
-1/3/(-a*e+b*d)/(b*x+a)^3+1/2*e/(-a*e+b*d)^2/(b*x+a)^2-e^2/(-a*e+b*d)^3/(b*x+a)-e^3*ln(b*x+a)/(-a*e+b*d)^4+e^3*ln(e*x+d)/(-a*e+b*d)^4
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.00

$$\int \frac{1}{(d+ex)(a^2+2abx+b^2x^2)^2} dx = \frac{1}{3(-bd+ae)(a+bx)^3} + \frac{e}{2(bd-ae)^2(a+bx)^2} - \frac{e^2}{(bd-ae)^3(a+bx)} - \frac{e^3 \log(a+bx)}{(bd-ae)^4} + \frac{e^3 \log(d+ex)}{(bd-ae)^4}$$

input `Integrate[1/((d + e*x)*(a^2 + 2*a*b*x + b^2*x^2)^2),x]`

output $\frac{1}{3*(-(b*d) + a*e)*(a + b*x)^3} + \frac{e}{2*(b*d - a*e)^2*(a + b*x)^2} - \frac{e^2}{(b*d - a*e)^3*(a + b*x)} - \frac{(e^3*\text{Log}[a + b*x])}{(b*d - a*e)^4} + \frac{(e^3*\text{Log}[d + e*x])}{(b*d - a*e)^4}$

Rubi [A] (verified)

Time = 0.48 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {1098, 27, 54, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a^2 + 2abx + b^2x^2)^2 (d + ex)} dx$$

↓ 1098

$$b^4 \int \frac{1}{b^4(a + bx)^4(d + ex)} dx$$

↓ 27

$$\int \frac{1}{(a + bx)^4(d + ex)} dx$$

↓ 54

$$\int \left(\frac{e^4}{(d + ex)(bd - ae)^4} - \frac{be^3}{(a + bx)(bd - ae)^4} + \frac{be^2}{(a + bx)^2(bd - ae)^3} - \frac{be}{(a + bx)^3(bd - ae)^2} + \frac{b}{(a + bx)^4(bd - ae)} \right) dx$$

↓ 2009

$$-\frac{e^3 \log(a + bx)}{(bd - ae)^4} + \frac{e^3 \log(d + ex)}{(bd - ae)^4} - \frac{e^2}{(a + bx)(bd - ae)^3} + \frac{e}{2(a + bx)^2(bd - ae)^2} - \frac{1}{3(a + bx)^3(bd - ae)}$$

input `Int[1/((d + e*x)*(a^2 + 2*a*b*x + b^2*x^2)^2),x]`

output
$$-1/3*1/((b*d - a*e)*(a + b*x)^3) + e/(2*(b*d - a*e)^2*(a + b*x)^2) - e^2/((b*d - a*e)^3*(a + b*x)) - (e^3*Log[a + b*x])/(b*d - a*e)^4 + (e^3*Log[d + e*x])/(b*d - a*e)^4$$

Defintions of rubi rules used

rule 27
$$\text{Int}[(a_*)(Fx_), x_Symbol] \rightarrow \text{Simp}[a \text{ Int}[Fx, x], x] /; \text{FreeQ}[a, x] \ \&\& \ !\text{MatchQ}[Fx, (b_*)(Gx_)] /; \text{FreeQ}[b, x]$$

rule 54
$$\text{Int}[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] \rightarrow \text{Int}[\text{ExpandIntegrand}[(a + b*x)^m*(c + d*x)^n, x], x] /; \text{FreeQ}[\{a, b, c, d\}, x] \ \&\& \ \text{ILtQ}[m, 0] \ \&\& \ \text{IntegerQ}[n] \ \&\& \ !(\text{IGtQ}[n, 0] \ \&\& \ \text{LtQ}[m + n + 2, 0])$$

rule 1098
$$\text{Int}(((d_.) + (e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] \rightarrow \text{Simp}[1/c^p \text{ Int}[(d + e*x)^m*(b/2 + c*x)^(2*p), x], x] /; \text{FreeQ}[\{a, b, c, d, e, m\}, x] \ \&\& \ \text{EqQ}[b^2 - 4*a*c, 0] \ \&\& \ \text{IntegerQ}[p]$$

rule 2009
$$\text{Int}[u_, x_Symbol] \rightarrow \text{Simp}[\text{IntSum}[u, x], x] /; \text{SumQ}[u]$$

Maple [A] (verified)

Time = 1.28 (sec) , antiderivative size = 103, normalized size of antiderivative = 0.96

| method | result |
|---------------|---|
| default | $\frac{1}{3(ae-bd)(bx+a)^3} + \frac{e}{2(ae-bd)^2(bx+a)^2} + \frac{e^2}{(ae-bd)^3(bx+a)} - \frac{e^3 \ln(bx+a)}{(ae-bd)^4} + \frac{e^3 \ln(ex+d)}{(ae-bd)^4}$ |
| norman | $\frac{\frac{b^2 e^2 x^2}{e^3 a^3 - 3a^2 b d e^2 + 3a b^2 d^2 e - b^3 d^3} + \frac{11a^2 b^3 e^2 - 7a b^4 d e + 2b^5 d^2}{6b^3 (e^3 a^3 - 3a^2 b d e^2 + 3a b^2 d^2 e - b^3 d^3)} + \frac{(5e^2 a b^3 - b^4 d e) x}{2b^2 (e^3 a^3 - 3a^2 b d e^2 + 3a b^2 d^2 e - b^3 d^3)}}{(bx+a)^3} + \frac{e^3 \ln(ex+d)}{a^4 e^4 - 4a^3 b d e^3 - 6a^2 b^2 d^2 e^2 + 6a b^3 d^3 e - 6a^2 b^4 d^4}$ |
| risch | $\frac{\frac{b^2 e^2 x^2}{e^3 a^3 - 3a^2 b d e^2 + 3a b^2 d^2 e - b^3 d^3} + \frac{(5ae-bd) b e x}{2e^3 a^3 - 6a^2 b d e^2 + 6a b^2 d^2 e - 2b^3 d^3} + \frac{11e^2 a^2 - 7abde + 2b^2 d^2}{6e^3 a^3 - 18a^2 b d e^2 + 18a b^2 d^2 e - 6b^3 d^3}}{(b^2 x^2 + 2abx + a^2)(bx+a)} - \frac{e^3 \ln(ex+d)}{a^4 e^4 - 4a^3 b d e^3 + 6a^2 b^2 d^2 e^2 - 6a b^3 d^3 e + 6a^2 b^4 d^4}$ |
| parallelrisch | $- \frac{-9d^2 e a b^5 + 18d e^2 a^2 b^4 + 2d^3 b^6 + 18x a b^5 d e^2 + 18 \ln(bx+a) x^2 a b^5 e^3 - 18 \ln(ex+d) x^2 a b^5 e^3 + 18 \ln(bx+a) x a^2 b^4 e^3 - 18 \ln(ex+d) x a^2 b^4 e^3}{6(a^4 e^4 - 4a^3 b d e^3 + 6a^2 b^2 d^2 e^2 - 6a b^3 d^3 e + 6a^2 b^4 d^4)}$ |

input
$$\text{int}(1/(e*x+d)/(b^2*x^2+2*a*b*x+a^2)^2, x, \text{method}=_RETURNVERBOSE)$$

output

```
1/3/(a*e-b*d)/(b*x+a)^3+1/2*e/(a*e-b*d)^2/(b*x+a)^2+e^2/(a*e-b*d)^3/(b*x+a)
)-e^3/(a*e-b*d)^4*ln(b*x+a)+e^3/(a*e-b*d)^4*ln(e*x+d)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 425 vs. $2(103) = 206$.

Time = 0.10 (sec) , antiderivative size = 425, normalized size of antiderivative = 3.97

$$\int \frac{1}{(d+ex)(a^2+2abx+b^2x^2)^2} dx =$$

$$\frac{2b^3d^3 - 9ab^2d^2e + 18a^2bde^2 - 11a^3e^3 + 6(b^3de^2 - ab^2e^3)x^2 - 3(b^3d^2e - 6ab^2de^2 + 5a^2bde^3)x + 6(a^3b^4d^4 - 4a^4b^3d^3e + 6a^5b^2d^2e^2 - 4a^6bde^3 + a^7e^4 + (b^7d^4 - 4ab^6d^3e + 6a^2b^5d^2e^2 - 4a^3b^4de^3 + a^4b^3e^4)) \log(bx+a) - 6(a^3b^4d^4 - 4a^4b^3d^3e + 6a^5b^2d^2e^2 - 4a^6bde^3 + a^7e^4)}{6(a^3b^4d^4 - 4a^4b^3d^3e + 6a^5b^2d^2e^2 - 4a^6bde^3 + a^7e^4 + (b^7d^4 - 4ab^6d^3e + 6a^2b^5d^2e^2 - 4a^3b^4de^3 + a^4b^3e^4))}$$

input

```
integrate(1/(e*x+d)/(b^2*x^2+2*a*b*x+a^2)^2,x, algorithm="fricas")
```

output

```
-1/6*(2*b^3*d^3 - 9*a*b^2*d^2*e + 18*a^2*b*d*e^2 - 11*a^3*e^3 + 6*(b^3*d*e
^2 - a*b^2*e^3)*x^2 - 3*(b^3*d^2*e - 6*a*b^2*d*e^2 + 5*a^2*b*e^3)*x + 6*(b
^3*e^3*x^3 + 3*a*b^2*e^3*x^2 + 3*a^2*b*e^3*x + a^3*e^3)*log(b*x + a) - 6*(
b^3*e^3*x^3 + 3*a*b^2*e^3*x^2 + 3*a^2*b*e^3*x + a^3*e^3)*log(e*x + d))/(a^
3*b^4*d^4 - 4*a^4*b^3*d^3*e + 6*a^5*b^2*d^2*e^2 - 4*a^6*b*d*e^3 + a^7*e^4
+ (b^7*d^4 - 4*a*b^6*d^3*e + 6*a^2*b^5*d^2*e^2 - 4*a^3*b^4*d*e^3 + a^4*b^3
*e^4)*x^3 + 3*(a*b^6*d^4 - 4*a^2*b^5*d^3*e + 6*a^3*b^4*d^2*e^2 - 4*a^4*b^3
*d*e^3 + a^5*b^2*e^4)*x^2 + 3*(a^2*b^5*d^4 - 4*a^3*b^4*d^3*e + 6*a^4*b^3*d
^2*e^2 - 4*a^5*b^2*d*e^3 + a^6*b*e^4)*x)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 570 vs. $2(88) = 176$.

Time = 0.74 (sec) , antiderivative size = 570, normalized size of antiderivative = 5.33

$$\int \frac{1}{(d+ex)(a^2+2abx+b^2x^2)^2} dx$$

$$= \frac{e^3 \log\left(x + \frac{-\frac{a^5e^8}{(ae-bd)^4} + \frac{5a^4bde^7}{(ae-bd)^4} - \frac{10a^3b^2d^2e^6}{(ae-bd)^4} + \frac{10a^2b^3d^3e^5}{(ae-bd)^4} - \frac{5ab^4d^4e^4}{(ae-bd)^4} + ae^4 + \frac{b^5d^5e^3}{(ae-bd)^4} + bde^3}{2be^4}\right)}{(ae-bd)^4}$$

$$- \frac{e^3 \log\left(x + \frac{\frac{a^5e^8}{(ae-bd)^4} - \frac{5a^4bde^7}{(ae-bd)^4} + \frac{10a^3b^2d^2e^6}{(ae-bd)^4} - \frac{10a^2b^3d^3e^5}{(ae-bd)^4} + \frac{5ab^4d^4e^4}{(ae-bd)^4} + ae^4 - \frac{b^5d^5e^3}{(ae-bd)^4} + bde^3}{2be^4}\right)}{(ae-bd)^4}$$

$$+ \frac{11a^2e^2 - 7abde + 2b^2d^2 + 6b^2e^2}{6a^6e^3 - 18a^5bde^2 + 18a^4b^2d^2e - 6a^3b^3d^3 + x^3 \cdot (6a^3b^3e^3 - 18a^2b^4de^2 + 18ab^5d^2e - 6b^6d^3) + x^2 \cdot (18a^4$$

input `integrate(1/(e*x+d)/(b**2*x**2+2*a*b*x+a**2)**2,x)`

output `e**3*log(x + (-a**5*e**8/(a*e - b*d)**4 + 5*a**4*b*d*e**7/(a*e - b*d)**4 - 10*a**3*b**2*d**2*e**6/(a*e - b*d)**4 + 10*a**2*b**3*d**3*e**5/(a*e - b*d)**4 - 5*a*b**4*d**4*e**4/(a*e - b*d)**4 + a*e**4 + b**5*d**5*e**3/(a*e - b*d)**4 + b*d*e**3)/(2*b*e**4))/(a*e - b*d)**4 - e**3*log(x + (a**5*e**8/(a*e - b*d)**4 - 5*a**4*b*d*e**7/(a*e - b*d)**4 + 10*a**3*b**2*d**2*e**6/(a*e - b*d)**4 - 10*a**2*b**3*d**3*e**5/(a*e - b*d)**4 + 5*a*b**4*d**4*e**4/(a*e - b*d)**4 + a*e**4 - b**5*d**5*e**3/(a*e - b*d)**4 + b*d*e**3)/(2*b*e**4))/(a*e - b*d)**4 + (11*a**2*e**2 - 7*a*b*d*e + 2*b**2*d**2 + 6*b**2*e**2*x**2 + x*(15*a*b*e**2 - 3*b**2*d*e))/(6*a**6*e**3 - 18*a**5*b*d*e**2 + 18*a**4*b**2*d**2*e - 6*a**3*b**3*d**3 + x**3*(6*a**3*b**3*e**3 - 18*a**2*b**4*d*e**2 + 18*a*b**5*d**2*e - 6*b**6*d**3) + x**2*(18*a**4*b**2*e**3 - 54*a**3*b**3*d*e**2 + 54*a**2*b**4*d**2*e - 18*a*b**5*d**3) + x*(18*a**5*b**5*e**3 - 54*a**4*b**2*d*e**2 + 54*a**3*b**3*d**2*e - 18*a**2*b**4*d**3))`

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 361 vs. $2(103) = 206$.

Time = 0.05 (sec) , antiderivative size = 361, normalized size of antiderivative = 3.37

$$\int \frac{1}{(d+ex)(a^2+2abx+b^2x^2)^2} dx = -\frac{e^3 \log (bx+a)}{b^4 d^4 - 4 ab^3 d^3 e + 6 a^2 b^2 d^2 e^2 - 4 a^3 b d e^3 + a^4 e^4}$$

$$+ \frac{e^3 \log (ex+d)}{b^4 d^4 - 4 ab^3 d^3 e + 6 a^2 b^2 d^2 e^2 - 4 a^3 b d e^3 + a^4 e^4}$$

$$- \frac{6 b^2 e^2 x^2 + 2 b^2 d^2 - 7 abde + 11 a^2 e^2 - 3 (a^3 b^3 d^3 - 3 a^4 b^2 d^2 e + 3 a^5 b d e^2 - a^6 e^3 + (b^6 d^3 - 3 ab^5 d^2 e + 3 a^2 b^4 d e^2 - a^3 b^3 e^3) x^3 + 3 (ab^5 d^3 - 3 a^2 b^4 d^2 e + 3 a^3 b^3 d e^2 - a^4 b^2 e^3) x^2 + 3 (a^2 b^4 d^3 - 3 a^3 b^3 d^2 e + 3 a^4 b^2 d e^2 - a^5 b e^3) x)}{6 (b^4 d^4 - 4 ab^3 d^3 e + 6 a^2 b^2 d^2 e^2 - 4 a^3 b d e^3 + a^4 e^4)}$$

input `integrate(1/(e*x+d)/(b^2*x^2+2*a*b*x+a^2)^2,x, algorithm="maxima")`

output `-e^3*log(b*x + a)/(b^4*d^4 - 4*a*b^3*d^3*e + 6*a^2*b^2*d^2*e^2 - 4*a^3*b*d*e^3 + a^4*e^4) + e^3*log(e*x + d)/(b^4*d^4 - 4*a*b^3*d^3*e + 6*a^2*b^2*d^2*e^2 - 4*a^3*b*d*e^3 + a^4*e^4) - 1/6*(6*b^2*e^2*x^2 + 2*b^2*d^2 - 7*a*b*d*e + 11*a^2*e^2 - 3*(b^2*d*e - 5*a*b*e^2)*x)/(a^3*b^3*d^3 - 3*a^4*b^2*d^2*e + 3*a^5*b*d*e^2 - a^6*e^3 + (b^6*d^3 - 3*a*b^5*d^2*e + 3*a^2*b^4*d*e^2 - a^3*b^3*e^3)*x^3 + 3*(a*b^5*d^3 - 3*a^2*b^4*d^2*e + 3*a^3*b^3*d*e^2 - a^4*b^2*e^3)*x^2 + 3*(a^2*b^4*d^3 - 3*a^3*b^3*d^2*e + 3*a^4*b^2*d*e^2 - a^5*b*e^3)*x)`

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 243 vs. $2(103) = 206$.

Time = 0.19 (sec) , antiderivative size = 243, normalized size of antiderivative = 2.27

$$\int \frac{1}{(d+ex)(a^2+2abx+b^2x^2)^2} dx = -\frac{be^3 \log (|bx+a|)}{b^5 d^4 - 4 ab^4 d^3 e + 6 a^2 b^3 d^2 e^2 - 4 a^3 b^2 d e^3 + a^4 b e^4}$$

$$+ \frac{e^4 \log (|ex+d|)}{b^4 d^4 e - 4 ab^3 d^3 e^2 + 6 a^2 b^2 d^2 e^3 - 4 a^3 b d e^4 + a^4 e^5}$$

$$- \frac{2 b^3 d^3 - 9 ab^2 d^2 e + 18 a^2 b d e^2 - 11 a^3 e^3 + 6 (b^3 d e^2 - ab^2 e^3) x^2 - 3 (b^3 d^2 e - 6 ab^2 d e^2 + 5 a^2 b e^3) x}{6 (bd - ae)^4 (bx + a)^3}$$

input `integrate(1/(e*x+d)/(b^2*x^2+2*a*b*x+a^2)^2,x, algorithm="giac")`

output

```
-b*e^3*log(abs(b*x + a))/(b^5*d^4 - 4*a*b^4*d^3*e + 6*a^2*b^3*d^2*e^2 - 4*
a^3*b^2*d*e^3 + a^4*b*e^4) + e^4*log(abs(e*x + d))/(b^4*d^4*e - 4*a*b^3*d^
3*e^2 + 6*a^2*b^2*d^2*e^3 - 4*a^3*b*d*e^4 + a^4*e^5) - 1/6*(2*b^3*d^3 - 9*
a*b^2*d^2*e + 18*a^2*b*d*e^2 - 11*a^3*e^3 + 6*(b^3*d*e^2 - a*b^2*e^3))*x^2
- 3*(b^3*d^2*e - 6*a*b^2*d*e^2 + 5*a^2*b*e^3)*x)/((b*d - a*e)^4*(b*x + a)^
3)
```

Mupad [B] (verification not implemented)

Time = 9.36 (sec) , antiderivative size = 312, normalized size of antiderivative = 2.92

$$\int \frac{1}{(d+ex)(a^2+2abx+b^2x^2)^2} dx$$

$$= \frac{\frac{11a^2e^2-7abde+2b^2d^2}{6(a^3e^3-3a^2bde^2+3ab^2d^2e-b^3d^3)} - \frac{ex(b^2d-5abe)}{2(a^3e^3-3a^2bde^2+3ab^2d^2e-b^3d^3)} + \frac{b^2e^2x^2}{a^3e^3-3a^2bde^2+3ab^2d^2e-b^3d^3}}{a^3+3a^2bx+3ab^2x^2+b^3x^3}$$

$$- \frac{2e^3 \operatorname{atanh}\left(\frac{a^4e^4-2a^3bde^3+2ab^3d^3e-b^4d^4}{(ae-bd)^4} + \frac{2bex(a^3e^3-3a^2bde^2+3ab^2d^2e-b^3d^3)}{(ae-bd)^4}\right)}{(ae-bd)^4}$$

input

```
int(1/((d + e*x)*(a^2 + b^2*x^2 + 2*a*b*x)^2),x)
```

output

```
((11*a^2*e^2 + 2*b^2*d^2 - 7*a*b*d*e)/(6*(a^3*e^3 - b^3*d^3 + 3*a*b^2*d^2*
e - 3*a^2*b*d*e^2)) - (e*x*(b^2*d - 5*a*b*e))/(2*(a^3*e^3 - b^3*d^3 + 3*a*
b^2*d^2*e - 3*a^2*b*d*e^2)) + (b^2*e^2*x^2)/(a^3*e^3 - b^3*d^3 + 3*a*b^2*d
^2*e - 3*a^2*b*d*e^2))/(a^3 + b^3*x^3 + 3*a*b^2*x^2 + 3*a^2*b*x) - (2*e^3*
atanh((a^4*e^4 - b^4*d^4 + 2*a*b^3*d^3*e - 2*a^3*b*d*e^3)/(a*e - b*d)^4 +
(2*b*e*x*(a^3*e^3 - b^3*d^3 + 3*a*b^2*d^2*e - 3*a^2*b*d*e^2))/(a*e - b*d)^
4))/(a*e - b*d)^4
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 499, normalized size of antiderivative = 4.66

$$\int \frac{1}{(d+ex)(a^2+2abx+b^2x^2)^2} dx$$

$$= \frac{-6 \log(bx+a) a^4 e^3 - 18 \log(bx+a) a^3 b e^3 x - 18 \log(bx+a) a^2 b^2 e^3 x^2 - 6 \log(bx+a) a b^3 e^3 x^3 + 6 \log(e)}{6a(a^4 b^3 e^4 x^3 - 4a^3 b^4 d e^3 x^3 + 6a^2 b^5 d^2 e^2 x^3 - 4a b^6 d^3 e x^3 + b^7 d^4 x^3 + 3a^5 b^2 e^4 x^2 - 12a^4 b^3 e^3 x^2 - 6a^3 b^4 d e^3 x^2 + 6a^2 b^5 d^2 e^2 x^2 - 4a b^6 d^3 e x^2 + b^7 d^4 x^2 + 3a^5 b^2 e^4 x - 12a^4 b^3 e^3 x - 6a^3 b^4 d e^3 x + 6a^2 b^5 d^2 e^2 x - 4a b^6 d^3 e x + b^7 d^4 x + 3a^5 b^2 e^4 - 12a^4 b^3 e^3 - 6a^3 b^4 d e^3 + 6a^2 b^5 d^2 e^2 - 4a b^6 d^3 e + b^7 d^4)}$$

input `int(1/(e*x+d)/(b^2*x^2+2*a*b*x+a^2)^2,x)`output `(- 6*log(a + b*x)*a**4*e**3 - 18*log(a + b*x)*a**3*b*e**3*x - 18*log(a + b*x)*a**2*b**2*e**3*x**2 - 6*log(a + b*x)*a*b**3*e**3*x**3 + 6*log(d + e*x)*a**4*e**3 + 18*log(d + e*x)*a**3*b*e**3*x + 18*log(d + e*x)*a**2*b**2*e**3*x**2 + 6*log(d + e*x)*a*b**3*e**3*x**3 + 9*a**4*e**3 - 16*a**3*b*d*e**2 + 9*a**3*b*e**3*x + 9*a**2*b**2*d**2*e - 12*a**2*b**2*d*e**2*x - 2*a*b**3*d**3 + 3*a*b**3*d**2*e*x - 2*a*b**3*e**3*x**3 + 2*b**4*d*e**2*x**3)/(6*a*(a**7*e**4 - 4*a**6*b*d*e**3 + 3*a**6*b*e**4*x + 6*a**5*b**2*d**2*e**2 - 12*a**5*b**2*d*e**3*x + 3*a**5*b**2*e**4*x**2 - 4*a**4*b**3*d**3*e + 18*a**4*b**3*d**2*e**2*x - 12*a**4*b**3*d*e**3*x**2 + a**4*b**3*e**4*x**3 + a**3*b**4*d**4 - 12*a**3*b**4*d**3*e*x + 18*a**3*b**4*d**2*e**2*x**2 - 4*a**3*b**4*d*e**3*x**3 + 3*a**2*b**5*d**4*x - 12*a**2*b**5*d**3*e*x**2 + 6*a**2*b**5*d**2*e**2*x**3 + 3*a*b**6*d**4*x**2 - 4*a*b**6*d**3*e*x**3 + b**7*d**4*x**3))`

3.126 $\int \frac{1}{(d+ex)^2(a^2+2abx+b^2x^2)^2} dx$

| | |
|---|------|
| Optimal result | 1006 |
| Mathematica [A] (verified) | 1006 |
| Rubi [A] (verified) | 1007 |
| Maple [A] (verified) | 1008 |
| Fricas [B] (verification not implemented) | 1009 |
| Sympy [B] (verification not implemented) | 1010 |
| Maxima [B] (verification not implemented) | 1011 |
| Giac [B] (verification not implemented) | 1011 |
| Mupad [B] (verification not implemented) | 1012 |
| Reduce [B] (verification not implemented) | 1013 |

Optimal result

Integrand size = 26, antiderivative size = 132

$$\int \frac{1}{(d+ex)^2(a^2+2abx+b^2x^2)^2} dx = -\frac{b}{3(bd-ae)^2(a+bx)^3} + \frac{be}{(bd-ae)^3(a+bx)^2} - \frac{3be^2}{(bd-ae)^4(a+bx)} - \frac{e^3}{(bd-ae)^4(d+ex)} - \frac{4be^3 \log(a+bx)}{(bd-ae)^5} + \frac{4be^3 \log(d+ex)}{(bd-ae)^5}$$

output

```
-1/3*b/(-a*e+b*d)^2/(b*x+a)^3+b*e/(-a*e+b*d)^3/(b*x+a)^2-3*b*e^2/(-a*e+b*d)^4/(b*x+a)-e^3/(-a*e+b*d)^4/(e*x+d)-4*b*e^3*ln(b*x+a)/(-a*e+b*d)^5+4*b*e^3*ln(e*x+d)/(-a*e+b*d)^5
```

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 121, normalized size of antiderivative = 0.92

$$\int \frac{1}{(d+ex)^2(a^2+2abx+b^2x^2)^2} dx = \frac{-\frac{b(bd-ae)^3}{(a+bx)^3} + \frac{3be(bd-ae)^2}{(a+bx)^2} - \frac{9be^2(bd-ae)}{a+bx} + \frac{3e^3(-bd+ae)}{d+ex} - 12be^3 \log(a+bx) + 12be^3 \log(d+ex)}{3(bd-ae)^5}$$

input `Integrate[1/((d + e*x)^2*(a^2 + 2*a*b*x + b^2*x^2)^2),x]`

output
$$\left(-\frac{(b(bd - ae))^3}{(a + bx)^3} + \frac{3b^2e(bd - ae)^2}{(a + bx)^2} - (9b^2e^2(bd - ae))/(a + bx) + \frac{3e^3(-bd + ae)}{(d + ex)} - 12b^2e^3 \frac{\text{Log}[a + bx] + 12b^2e^3 \text{Log}[d + ex]}{(3(bd - ae))^5} \right)$$

Rubi [A] (verified)

Time = 0.56 (sec) , antiderivative size = 132, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {1098, 27, 54, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\begin{aligned} & \int \frac{1}{(a^2 + 2abx + b^2x^2)^2 (d + ex)^2} dx \\ & \quad \downarrow 1098 \\ & b^4 \int \frac{1}{b^4(a + bx)^4 (d + ex)^2} dx \\ & \quad \downarrow 27 \\ & \int \frac{1}{(a + bx)^4 (d + ex)^2} dx \\ & \quad \downarrow 54 \\ & \int \left(-\frac{4b^2e^3}{(a + bx)(bd - ae)^5} + \frac{3b^2e^2}{(a + bx)^2(bd - ae)^4} - \frac{2b^2e}{(a + bx)^3(bd - ae)^3} + \frac{b^2}{(a + bx)^4(bd - ae)^2} + \frac{4be^4}{(d + ex)(bd - ae)} \right) dx \\ & \quad \downarrow 2009 \\ & -\frac{e^3}{(d + ex)(bd - ae)^4} - \frac{4be^3 \log(a + bx)}{(bd - ae)^5} + \frac{4be^3 \log(d + ex)}{(bd - ae)^5} - \frac{3be^2}{(a + bx)(bd - ae)^4} + \\ & \quad \frac{1}{(a + bx)^2(bd - ae)^3} - \frac{1}{3(a + bx)^3(bd - ae)^2} \end{aligned}$$

input `Int[1/((d + e*x)^2*(a^2 + 2*a*b*x + b^2*x^2)^2),x]`

output

$$-1/3*b/((b*d - a*e)^2*(a + b*x)^3) + (b*e)/((b*d - a*e)^3*(a + b*x)^2) - (3*b*e^2)/((b*d - a*e)^4*(a + b*x)) - e^3/((b*d - a*e)^4*(d + e*x)) - (4*b*e^3*Log[a + b*x])/(b*d - a*e)^5 + (4*b*e^3*Log[d + e*x])/(b*d - a*e)^5$$

Defintions of rubi rules used

rule 27

$$\text{Int}[(a_*)(Fx_), x_Symbol] \rightarrow \text{Simp}[a \text{ Int}[Fx, x], x] /; \text{FreeQ}[a, x] \ \&\& \ !\text{MatchQ}[Fx, (b_)*(Gx_)] /; \text{FreeQ}[b, x]$$

rule 54

$$\text{Int}[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] \rightarrow \text{Int}[\text{ExpandIntegrand}[(a + b*x)^m*(c + d*x)^n, x], x] /; \text{FreeQ}[\{a, b, c, d\}, x] \ \&\& \ \text{ILtQ}[m, 0] \ \&\& \ \text{IntegerQ}[n] \ \&\& \ !(\text{IGtQ}[n, 0] \ \&\& \ \text{LtQ}[m + n + 2, 0])$$

rule 1098

$$\text{Int}(((d_.) + (e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] \rightarrow \text{Simp}[1/c^p \text{ Int}[(d + e*x)^m*(b/2 + c*x)^(2*p), x], x] /; \text{FreeQ}[\{a, b, c, d, e, m\}, x] \ \&\& \ \text{EqQ}[b^2 - 4*a*c, 0] \ \&\& \ \text{IntegerQ}[p]$$

rule 2009

$$\text{Int}[u_, x_Symbol] \rightarrow \text{Simp}[\text{IntSum}[u, x], x] /; \text{SumQ}[u]$$

Maple [A] (verified)

Time = 1.34 (sec) , antiderivative size = 132, normalized size of antiderivative = 1.00

| method | result |
|---------------|---|
| default | $-\frac{b}{3(ae-bd)^2(bx+a)^3} + \frac{4be^3 \ln(bx+a)}{(ae-bd)^5} - \frac{3be^2}{(ae-bd)^4(bx+a)} - \frac{be}{(ae-bd)^3(bx+a)^2} - \frac{e^3}{(ae-bd)^4(ex+d)} - \frac{4be^3 \ln(ex+a)}{(ae-bd)^5}$ |
| risch | $-\frac{4b^3e^3x^3}{a^4e^4-4a^3bde^3+6a^2b^2d^2e^2-4ab^3d^3e+b^4d^4} - \frac{2b^2(5ae+bd)e^2x^2}{a^4e^4-4a^3bde^3+6a^2b^2d^2e^2-4ab^3d^3e+b^4d^4} - \frac{2(11e^2a^2+8abde-b^2d^2)be}{3(a^4e^4-4a^3bde^3+6a^2b^2d^2e^2-4ab^3d^3e)} \frac{1}{(ex+d)(b^2x^2+2abx+a^2)(bx+a)}$ |
| norman | $-\frac{4b^3e^3x^3}{a^4e^4-4a^3bde^3+6a^2b^2d^2e^2-4ab^3d^3e+b^4d^4} + \frac{(-10ab^4e^4-2b^5de^3)x^2}{b^2e(a^4e^4-4a^3bde^3+6a^2b^2d^2e^2-4ab^3d^3e+b^4d^4)} + \frac{-3e^4a^3b^3-13de^3a^2b^4+5d^2e}{3b^3e(a^4e^4-4a^3bde^3+6a^2b^2d^2e^2-4ab^3d^3e+b^4d^4)} \frac{1}{(bx+a)^3(ex+d)}$ |
| parallelrisch | $-\frac{10a^3b^4de^4+18a^2b^5d^2e^3-6ab^6d^3e^2-3a^4b^3e^5+36 \ln(bx+a)x^2ab^6de^4-36 \ln(ex+d)x^2ab^6de^4+36 \ln(bx+a)a^2b^5de^4-12a^3b^4de^4}{(bx+a)^3(ex+d)}$ |

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 882 vs. $2(117) = 234$.

Time = 1.17 (sec) , antiderivative size = 882, normalized size of antiderivative = 6.68

$$\int \frac{1}{(d+ex)^2 (a^2+2abx+b^2x^2)^2} dx = \text{Too large to display}$$

input `integrate(1/(e*x+d)**2/(b**2*x**2+2*a*b*x+a**2)**2,x)`

output

```
-4*b**e**3*log(x + (-4*a**6*b**e**9/(a*e - b*d)**5 + 24*a**5*b**2*d**e**8/(a*
e - b*d)**5 - 60*a**4*b**3*d**2*e**7/(a*e - b*d)**5 + 80*a**3*b**4*d**3*e*
**6/(a*e - b*d)**5 - 60*a**2*b**5*d**4*e**5/(a*e - b*d)**5 + 24*a*b**6*d**5
**e**4/(a*e - b*d)**5 + 4*a*b**e**4 - 4*b**7*d**6*e**3/(a*e - b*d)**5 + 4*b*
**2*d**e**3)/(8*b**2*e**4))/(a*e - b*d)**5 + 4*b**e**3*log(x + (4*a**6*b**e**9
/(a*e - b*d)**5 - 24*a**5*b**2*d**e**8/(a*e - b*d)**5 + 60*a**4*b**3*d**2*e
**7/(a*e - b*d)**5 - 80*a**3*b**4*d**3*e**6/(a*e - b*d)**5 + 60*a**2*b**5*
d**4*e**5/(a*e - b*d)**5 - 24*a*b**6*d**5*e**4/(a*e - b*d)**5 + 4*a*b**e**4
+ 4*b**7*d**6*e**3/(a*e - b*d)**5 + 4*b**2*d**e**3)/(8*b**2*e**4))/(a*e -
b*d)**5 + (-3*a**3*e**3 - 13*a**2*b*d**e**2 + 5*a*b**2*d**2*e - b**3*d**3 -
12*b**3*e**3*x**3 + x**2*(-30*a*b**2*e**3 - 6*b**3*d**e**2) + x*(-22*a**2*
b**e**3 - 16*a*b**2*d**e**2 + 2*b**3*d**2*e))/(3*a**7*d**e**4 - 12*a**6*b*d**
2*e**3 + 18*a**5*b**2*d**3*e**2 - 12*a**4*b**3*d**4*e + 3*a**3*b**4*d**5 +
x**4*(3*a**4*b**3*e**5 - 12*a**3*b**4*d**e**4 + 18*a**2*b**5*d**2*e**3 - 1
2*a*b**6*d**3*e**2 + 3*b**7*d**4*e) + x**3*(9*a**5*b**2*e**5 - 33*a**4*b**
3*d**e**4 + 42*a**3*b**4*d**2*e**3 - 18*a**2*b**5*d**3*e**2 - 3*a*b**6*d**4
*e + 3*b**7*d**5) + x**2*(9*a**6*b**e**5 - 27*a**5*b**2*d**e**4 + 18*a**4*b*
**3*d**2*e**3 + 18*a**3*b**4*d**3*e**2 - 27*a**2*b**5*d**4*e + 9*a*b**6*d**
5) + x*(3*a**7*e**5 - 3*a**6*b*d**e**4 - 18*a**5*b**2*d**2*e**3 + 42*a**4*b
**3*d**3*e**2 - 33*a**3*b**4*d**4*e + 9*a**2*b**5*d**5))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 598 vs. $2(130) = 260$.

Time = 0.06 (sec) , antiderivative size = 598, normalized size of antiderivative = 4.53

$$\int \frac{1}{(d+ex)^2 (a^2+2abx+b^2x^2)^2} dx$$

$$= -\frac{4be^3 \log(bx+a)}{b^5d^5 - 5ab^4d^4e + 10a^2b^3d^3e^2 - 10a^3b^2d^2e^3 + 5a^4bde^4 - a^5e^5}$$

$$+ \frac{4be^3 \log(ex+d)}{b^5d^5 - 5ab^4d^4e + 10a^2b^3d^3e^2 - 10a^3b^2d^2e^3 + 5a^4bde^4 - a^5e^5}$$

$$- \frac{3(a^3b^4d^5 - 4a^4b^3d^4e + 6a^5b^2d^3e^2 - 4a^6bd^2e^3 + a^7de^4 + (b^7d^4e - 4ab^6d^3e^2 + 6a^2b^5d^2e^3 - 4a^3b^4de^4 +$$

input `integrate(1/(e*x+d)^2/(b^2*x^2+2*a*b*x+a^2)^2,x, algorithm="maxima")`

output

```
-4*b*e^3*log(b*x + a)/(b^5*d^5 - 5*a*b^4*d^4*e + 10*a^2*b^3*d^3*e^2 - 10*a^3*b^2*d^2*e^3 + 5*a^4*b*d*e^4 - a^5*e^5) + 4*b*e^3*log(e*x + d)/(b^5*d^5 - 5*a*b^4*d^4*e + 10*a^2*b^3*d^3*e^2 - 10*a^3*b^2*d^2*e^3 + 5*a^4*b*d*e^4 - a^5*e^5) - 1/3*(12*b^3*e^3*x^3 + b^3*d^3 - 5*a*b^2*d^2*e + 13*a^2*b*d*e^2 + 3*a^3*e^3 + 6*(b^3*d*e^2 + 5*a*b^2*e^3)*x^2 - 2*(b^3*d^2*e - 8*a*b^2*d*e^2 - 11*a^2*b*e^3)*x)/(a^3*b^4*d^5 - 4*a^4*b^3*d^4*e + 6*a^5*b^2*d^3*e^2 - 4*a^6*b*d^2*e^3 + a^7*d*e^4 + (b^7*d^4*e - 4*a*b^6*d^3*e^2 + 6*a^2*b^5*d^2*e^3 - 4*a^3*b^4*d*e^4 + a^4*b^3*e^5)*x^4 + (b^7*d^5 - a*b^6*d^4*e - 6*a^2*b^5*d^3*e^2 + 14*a^3*b^4*d^2*e^3 - 11*a^4*b^3*d*e^4 + 3*a^5*b^2*e^5)*x^3 + 3*(a*b^6*d^5 - 3*a^2*b^5*d^4*e + 2*a^3*b^4*d^3*e^2 + 2*a^4*b^3*d^2*e^3 - 3*a^5*b^2*d*e^4 + a^6*b*e^5)*x^2 + (3*a^2*b^5*d^5 - 11*a^3*b^4*d^4*e + 14*a^4*b^3*d^3*e^2 - 6*a^5*b^2*d^2*e^3 - a^6*b*d*e^4 + a^7*e^5)*x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 288 vs. $2(130) = 260$.

Time = 0.16 (sec) , antiderivative size = 288, normalized size of antiderivative = 2.18

$$\int \frac{1}{(d+ex)^2 (a^2+2abx+b^2x^2)^2} dx$$

$$= \frac{e^7}{(b^4d^4e^4 - 4ab^3d^3e^5 + 6a^2b^2d^2e^6 - 4a^3bde^7 + a^4e^8)(ex+d)}$$

$$- \frac{4be^4 \log\left(\left|b - \frac{bd}{ex+d} + \frac{ae}{ex+d}\right|\right)}{b^5d^5e - 5ab^4d^4e^2 + 10a^2b^3d^3e^3 - 10a^3b^2d^2e^4 + 5a^4bde^5 - a^5e^6}$$

$$- \frac{13b^4e^3 - \frac{30(b^4de^4 - ab^3e^5)}{(ex+d)e} + \frac{18(b^4d^2e^5 - 2ab^3de^6 + a^2b^2e^7)}{(ex+d)^2e^2}}{3(bd - ae)^5 \left(b - \frac{bd}{ex+d} + \frac{ae}{ex+d}\right)^3}$$

input `integrate(1/(e*x+d)^2/(b^2*x^2+2*a*b*x+a^2)^2,x, algorithm="giac")`

output `-e^7/((b^4*d^4*e^4 - 4*a*b^3*d^3*e^5 + 6*a^2*b^2*d^2*e^6 - 4*a^3*b*d*e^7 + a^4*e^8)*(e*x + d)) - 4*b*e^4*log(abs(b - b*d/(e*x + d) + a*e/(e*x + d)))/ (b^5*d^5*e - 5*a*b^4*d^4*e^2 + 10*a^2*b^3*d^3*e^3 - 10*a^3*b^2*d^2*e^4 + 5*a^4*b*d*e^5 - a^5*e^6) - 1/3*(13*b^4*e^3 - 30*(b^4*d*e^4 - a*b^3*e^5)/((e*x + d)*e) + 18*(b^4*d^2*e^5 - 2*a*b^3*d*e^6 + a^2*b^2*e^7)/((e*x + d)^2*e^2))/((b*d - a*e)^5*(b - b*d/(e*x + d) + a*e/(e*x + d))^3)`

Mupad [B] (verification not implemented)

Time = 9.12 (sec) , antiderivative size = 534, normalized size of antiderivative = 4.05

$$\int \frac{1}{(d+ex)^2 (a^2+2abx+b^2x^2)^2} dx$$

$$= \frac{8be^3 \operatorname{atanh}\left(\frac{a^5e^5 - 3a^4bde^4 + 2a^3b^2d^2e^3 + 2a^2b^3d^3e^2 - 3ab^4d^4e + b^5d^5}{(ae-bd)^5} + \frac{2bex(a^4e^4 - 4a^3bde^3 + 6a^2b^2d^2e^2 - 4ab^3d^3e + b^4d^4)}{(ae-bd)^5}\right)}{(ae-bd)^5}$$

$$- \frac{\frac{3a^3e^3 + 13a^2bde^2 - 5ab^2d^2e + b^3d^3}{3(a^4e^4 - 4a^3bde^3 + 6a^2b^2d^2e^2 - 4ab^3d^3e + b^4d^4)}}{x^3(db^3 + 3aeb^2) + x^2(3ea^2b + 3dab^2) + a^3d + x(ea^3 + 3a^2b)}$$

input `int(1/((d + e*x)^2*(a^2 + b^2*x^2 + 2*a*b*x)^2),x)`

output

```
(8*b*e^3*atanh((a^5*e^5 + b^5*d^5 + 2*a^2*b^3*d^3*e^2 + 2*a^3*b^2*d^2*e^3
- 3*a*b^4*d^4*e - 3*a^4*b*d*e^4)/(a*e - b*d)^5 + (2*b*e*x*(a^4*e^4 + b^4*d
^4 + 6*a^2*b^2*d^2*e^2 - 4*a*b^3*d^3*e - 4*a^3*b*d*e^3))/(a*e - b*d)^5))/(
a*e - b*d)^5 - ((3*a^3*e^3 + b^3*d^3 - 5*a*b^2*d^2*e + 13*a^2*b*d*e^2)/(3*
(a^4*e^4 + b^4*d^4 + 6*a^2*b^2*d^2*e^2 - 4*a*b^3*d^3*e - 4*a^3*b*d*e^3)) +
(4*b^3*e^3*x^3)/(a^4*e^4 + b^4*d^4 + 6*a^2*b^2*d^2*e^2 - 4*a*b^3*d^3*e -
4*a^3*b*d*e^3) + (2*e^2*x^2*(b^3*d + 5*a*b^2*e))/(a^4*e^4 + b^4*d^4 + 6*a^
2*b^2*d^2*e^2 - 4*a*b^3*d^3*e - 4*a^3*b*d*e^3) + (2*e*x*(11*a^2*b*e^2 - b^
3*d^2 + 8*a*b^2*d*e))/(3*(a^4*e^4 + b^4*d^4 + 6*a^2*b^2*d^2*e^2 - 4*a*b^3*
d^3*e - 4*a^3*b*d*e^3)))/(x^3*(b^3*d + 3*a*b^2*e) + x^2*(3*a*b^2*d + 3*a^
2*b*e) + a^3*d + x*(a^3*e + 3*a^2*b*d) + b^3*e*x^4)
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 1236, normalized size of antiderivative = 9.36

$$\int \frac{1}{(d + ex)^2 (a^2 + 2abx + b^2x^2)^2} dx = \text{Too large to display}$$

input

```
int(1/(e*x+d)^2/(b^2*x^2+2*a*b*x+a^2)^2,x)
```

output

```
(36*log(a + b*x)*a**4*b*d***4 + 36*log(a + b*x)*a**4*b***5*x + 12*log(a
+ b*x)*a**3*b**2*d**2***3 + 120*log(a + b*x)*a**3*b**2*d***4*x + 108*log
(a + b*x)*a**3*b**2***5*x**2 + 36*log(a + b*x)*a**2*b**3*d**2***3*x + 14
4*log(a + b*x)*a**2*b**3*d***4*x**2 + 108*log(a + b*x)*a**2*b**3***5*x**
3 + 36*log(a + b*x)*a*b**4*d**2***3*x**2 + 72*log(a + b*x)*a*b**4*d***4*
x**3 + 36*log(a + b*x)*a*b**4***5*x**4 + 12*log(a + b*x)*b**5*d**2***3*x
**3 + 12*log(a + b*x)*b**5*d***4*x**4 - 36*log(d + e*x)*a**4*b*d***4 - 3
6*log(d + e*x)*a**4*b***5*x - 12*log(d + e*x)*a**3*b**2*d**2***3 - 120*l
og(d + e*x)*a**3*b**2*d***4*x - 108*log(d + e*x)*a**3*b**2***5*x**2 - 36
*log(d + e*x)*a**2*b**3*d**2***3*x - 144*log(d + e*x)*a**2*b**3*d***4*x*
*2 - 108*log(d + e*x)*a**2*b**3***5*x**3 - 36*log(d + e*x)*a*b**4*d**2***
3*x**2 - 72*log(d + e*x)*a*b**4*d***4*x**3 - 36*log(d + e*x)*a*b**4***5
*x**4 - 12*log(d + e*x)*b**5*d**2***3*x**3 - 12*log(d + e*x)*b**5*d***4*
x**4 - 9*a**5***5 - 21*a**4*b*d***4 - 54*a**4*b***5*x + 32*a**3*b**2*d*
**2***3 + 20*a**3*b**2*d***4*x - 54*a**3*b**2***5*x**2 + 24*a**2*b**3*d*
**2***3*x + 42*a**2*b**3*d***4*x**2 - 3*a*b**4*d***4*e + 12*a*b**4*d**3***e
**2*x + 6*a*b**4*d**2***3*x**2 + 12*a*b**4***5*x**4 + b**5*d**5 - 2*b**5*
d**4*e*x + 6*b**5*d**3***2*x**2 - 12*b**5*d***4*x**4)/(3*(3*a**9*d***6
+ 3*a**9***7*x - 14*a**8*b*d**2***5 - 5*a**8*b*d***6*x + 9*a**8*b***7*
x**2 + 25*a**7*b**2*d**3***4 - 17*a**7*b**2*d**2***5*x - 33*a**7*b**2...
```

3.127 $\int \frac{1}{(d+ex)^3(a^2+2abx+b^2x^2)^2} dx$

| | |
|---|------|
| Optimal result | 1015 |
| Mathematica [A] (verified) | 1016 |
| Rubi [A] (verified) | 1016 |
| Maple [A] (verified) | 1018 |
| Fricas [B] (verification not implemented) | 1018 |
| Sympy [B] (verification not implemented) | 1019 |
| Maxima [B] (verification not implemented) | 1020 |
| Giac [B] (verification not implemented) | 1021 |
| Mupad [B] (verification not implemented) | 1022 |
| Reduce [B] (verification not implemented) | 1023 |

Optimal result

Integrand size = 26, antiderivative size = 170

$$\int \frac{1}{(d+ex)^3(a^2+2abx+b^2x^2)^2} dx = -\frac{b^2}{3(bd-ae)^3(a+bx)^3} + \frac{3b^2e}{2(bd-ae)^4(a+bx)^2} - \frac{6b^2e^2}{(bd-ae)^5(a+bx)} - \frac{e^3}{4be^3} - \frac{2(bd-ae)^4(d+ex)^2}{(bd-ae)^5(d+ex)} - \frac{10b^2e^3 \log(a+bx)}{(bd-ae)^6} + \frac{10b^2e^3 \log(d+ex)}{(bd-ae)^6}$$

output

```
-1/3*b^2/(-a*e+b*d)^3/(b*x+a)^3+3/2*b^2*e/(-a*e+b*d)^4/(b*x+a)^2-6*b^2*e^2/(-a*e+b*d)^5/(b*x+a)-1/2*e^3/(-a*e+b*d)^4/(e*x+d)^2-4*b*e^3/(-a*e+b*d)^5/(e*x+d)-10*b^2*e^3*ln(b*x+a)/(-a*e+b*d)^6+10*b^2*e^3*ln(e*x+d)/(-a*e+b*d)^6
```

Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 154, normalized size of antiderivative = 0.91

$$\int \frac{1}{(d+ex)^3 (a^2+2abx+b^2x^2)^2} dx = \frac{\frac{2b^2(bd-ae)^3}{(a+bx)^3} - \frac{9b^2e(bd-ae)^2}{(a+bx)^2} + \frac{36b^2e^2(bd-ae)}{a+bx} + \frac{3e^3(bd-ae)^2}{(d+ex)^2} + \frac{24be^3(bd-ae)}{d+ex} + 60b^2e^3 \log(a+bx) - 60b^2e^3 \log(d+ex)}{6(bd-ae)^6}$$

input

```
Integrate[1/((d + e*x)^3*(a^2 + 2*a*b*x + b^2*x^2)^2),x]
```

output

```
-1/6*((2*b^2*(b*d - a*e)^3)/(a + b*x)^3 - (9*b^2*e*(b*d - a*e)^2)/(a + b*x)^2 + (36*b^2*e^2*(b*d - a*e))/(a + b*x) + (3*e^3*(b*d - a*e)^2)/(d + e*x)^2 + (24*b*e^3*(b*d - a*e))/(d + e*x) + 60*b^2*e^3*Log[a + b*x] - 60*b^2*e^3*Log[d + e*x])/(b*d - a*e)^6
```

Rubi [A] (verified)

Time = 0.65 (sec) , antiderivative size = 170, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {1098, 27, 54, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\begin{aligned} & \int \frac{1}{(a^2+2abx+b^2x^2)^2 (d+ex)^3} dx \\ & \quad \downarrow 1098 \\ & b^4 \int \frac{1}{b^4(a+bx)^4 (d+ex)^3} dx \\ & \quad \downarrow 27 \\ & \int \frac{1}{(a+bx)^4 (d+ex)^3} dx \\ & \quad \downarrow 54 \end{aligned}$$

$$\int \left(-\frac{10b^3e^3}{(a+bx)(bd-ae)^6} + \frac{6b^3e^2}{(a+bx)^2(bd-ae)^5} - \frac{3b^3e}{(a+bx)^3(bd-ae)^4} + \frac{b^3}{(a+bx)^4(bd-ae)^3} + \frac{10b^2e^4}{(d+ex)(bd-ae)^2} \right)$$

↓ 2009

$$-\frac{10b^2e^3 \log(a+bx)}{(bd-ae)^6} + \frac{10b^2e^3 \log(d+ex)}{(bd-ae)^6} - \frac{6b^2e^2}{(a+bx)(bd-ae)^5} + \frac{3b^2e}{2(a+bx)^2(bd-ae)^4} - \frac{b^2}{3(a+bx)^3(bd-ae)^3} - \frac{4be^3}{(d+ex)(bd-ae)^5} - \frac{e^3}{2(d+ex)^2(bd-ae)^4}$$

input `Int[1/((d + e*x)^3*(a^2 + 2*a*b*x + b^2*x^2)^2), x]`

output `-1/3*b^2/((b*d - a*e)^3*(a + b*x)^3) + (3*b^2*e)/(2*(b*d - a*e)^4*(a + b*x)^2) - (6*b^2*e^2)/((b*d - a*e)^5*(a + b*x)) - e^3/(2*(b*d - a*e)^4*(d + e*x)^2) - (4*b*e^3)/((b*d - a*e)^5*(d + e*x)) - (10*b^2*e^3*Log[a + b*x])/((b*d - a*e)^6 + (10*b^2*e^3*Log[d + e*x])/(b*d - a*e)^6)`

Defintions of rubi rules used

rule 27 `Int[(a_)*(Fx_), x_Symbol] := Simp[a Int[Fx, x], x] /; FreeQ[a, x] && !MatchQ[Fx, (b_)*(Gx_)] /; FreeQ[b, x]`

rule 54 `Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] := Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] && ILtQ[m, 0] && IntegerQ[n] && !(IGtQ[n, 0] && LtQ[m + n + 2, 0])`

rule 1098 `Int[((d_.) + (e_.)*(x_))^(m_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] := Simp[1/c^p Int[(d + e*x)^m*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]`

rule 2009 `Int[u_, x_Symbol] := Simp[IntSum[u, x], x] /; SumQ[u]`

Maple [A] (verified)

Time = 1.40 (sec) , antiderivative size = 165, normalized size of antiderivative = 0.97

| method | result |
|--------------|---|
| default | $\frac{b^2}{3(ae-bd)^3(bx+a)^3} - \frac{10b^2e^3 \ln(bx+a)}{(ae-bd)^6} + \frac{6b^2e^2}{(ae-bd)^5(bx+a)} + \frac{3b^2e}{2(ae-bd)^4(bx+a)^2} - \frac{e^3}{2(ae-bd)^4(ex+d)^2} + \frac{10b^2e^3 \ln(bx+a)}{(ae-bd)^6}$ |
| risch | $\frac{10b^4e^4x^4}{e^5a^5-5a^4bde^4+10a^3b^2d^2e^3-10a^2b^3d^3e^2+5ab^4d^4e-b^5d^5} + \frac{5e^3b^3(5ae+3bd)x^3}{e^5a^5-5a^4bde^4+10a^3b^2d^2e^3-10a^2b^3d^3e^2+5ab^4d^4e-b^5d^5} + \frac{10b^4e^4x^4}{3(e^5a^5-5a^4bde^4+10a^3b^2d^2e^3-10a^2b^3d^3e^2+5ab^4d^4e-b^5d^5)}$ |
| norman | $\frac{10b^4e^4x^4}{e^5a^5-5a^4bde^4+10a^3b^2d^2e^3-10a^2b^3d^3e^2+5ab^4d^4e-b^5d^5} + \frac{(25a^5b^5e^6+15b^6de^5)x^3}{e^2b^2(e^5a^5-5a^4bde^4+10a^3b^2d^2e^3-10a^2b^3d^3e^2+5ab^4d^4e-b^5d^5)} + \frac{10b^4e^4x^4}{6b^3e^2(e^5a^5-5a^4bde^4+10a^3b^2d^2e^3-10a^2b^3d^3e^2+5ab^4d^4e-b^5d^5)}$ |
| paralelrisch | $-\frac{60 \ln(bx+a)x^5b^8e^7-60 \ln(ex+d)x^5b^8e^7-60x^4ab^7e^7+60x^4b^8de^6-150x^3a^2b^6e^7+90x^3b^8d^2e^5-110x^2a^3b^5e^7+20x^2b^8d^3e^4}{1}$ |

input `int(1/(e*x+d)^3/(b^2*x^2+2*a*b*x+a^2)^2,x,method=_RETURNVERBOSE)`

output `1/3*b^2/(a*e-b*d)^3/(b*x+a)^3-10*b^2/(a*e-b*d)^6*e^3*ln(b*x+a)+6*b^2/(a*e-b*d)^5*e^2/(b*x+a)+3/2*b^2/(a*e-b*d)^4*e/(b*x+a)^2-1/2*e^3/(a*e-b*d)^4/(e*x+d)^2+10*b^2/(a*e-b*d)^6*e^3*ln(e*x+d)+4*e^3/(a*e-b*d)^5*b/(e*x+d)`

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1151 vs. 2(164) = 328.

Time = 0.12 (sec) , antiderivative size = 1151, normalized size of antiderivative = 6.77

$$\int \frac{1}{(d+ex)^3(a^2+2abx+b^2x^2)^2} dx = \text{Too large to display}$$

input `integrate(1/(e*x+d)^3/(b^2*x^2+2*a*b*x+a^2)^2,x, algorithm="fricas")`

output

```

-1/6*(2*b^5*d^5 - 15*a*b^4*d^4*e + 60*a^2*b^3*d^3*e^2 - 20*a^3*b^2*d^2*e^3
- 30*a^4*b*d*e^4 + 3*a^5*e^5 + 60*(b^5*d*e^4 - a*b^4*e^5)*x^4 + 30*(3*b^5
*d^2*e^3 + 2*a*b^4*d*e^4 - 5*a^2*b^3*e^5)*x^3 + 10*(2*b^5*d^3*e^2 + 21*a*b
^4*d^2*e^3 - 12*a^2*b^3*d*e^4 - 11*a^3*b^2*e^5)*x^2 - 5*(b^5*d^4*e - 12*a*
b^4*d^3*e^2 - 24*a^2*b^3*d^2*e^3 + 32*a^3*b^2*d*e^4 + 3*a^4*b*e^5)*x + 60*
(b^5*e^5*x^5 + a^3*b^2*d^2*e^3 + (2*b^5*d*e^4 + 3*a*b^4*e^5)*x^4 + (b^5*d^
2*e^3 + 6*a*b^4*d*e^4 + 3*a^2*b^3*e^5)*x^3 + (3*a*b^4*d^2*e^3 + 6*a^2*b^3*
d*e^4 + a^3*b^2*e^5)*x^2 + (3*a^2*b^3*d^2*e^3 + 2*a^3*b^2*d*e^4)*x)*log(b*
x + a) - 60*(b^5*e^5*x^5 + a^3*b^2*d^2*e^3 + (2*b^5*d*e^4 + 3*a*b^4*e^5)*x
^4 + (b^5*d^2*e^3 + 6*a*b^4*d*e^4 + 3*a^2*b^3*e^5)*x^3 + (3*a*b^4*d^2*e^3
+ 6*a^2*b^3*d*e^4 + a^3*b^2*e^5)*x^2 + (3*a^2*b^3*d^2*e^3 + 2*a^3*b^2*d*e^
4)*x)*log(e*x + d))/(a^3*b^6*d^8 - 6*a^4*b^5*d^7*e + 15*a^5*b^4*d^6*e^2 -
20*a^6*b^3*d^5*e^3 + 15*a^7*b^2*d^4*e^4 - 6*a^8*b*d^3*e^5 + a^9*d^2*e^6 +
(b^9*d^6*e^2 - 6*a*b^8*d^5*e^3 + 15*a^2*b^7*d^4*e^4 - 20*a^3*b^6*d^3*e^5 +
15*a^4*b^5*d^2*e^6 - 6*a^5*b^4*d*e^7 + a^6*b^3*e^8)*x^5 + (2*b^9*d^7*e -
9*a*b^8*d^6*e^2 + 12*a^2*b^7*d^5*e^3 + 5*a^3*b^6*d^4*e^4 - 30*a^4*b^5*d^3*
e^5 + 33*a^5*b^4*d^2*e^6 - 16*a^6*b^3*d*e^7 + 3*a^7*b^2*e^8)*x^4 + (b^9*d^
8 - 18*a^2*b^7*d^6*e^2 + 52*a^3*b^6*d^5*e^3 - 60*a^4*b^5*d^4*e^4 + 24*a^5*
b^4*d^3*e^5 + 10*a^6*b^3*d^2*e^6 - 12*a^7*b^2*d*e^7 + 3*a^8*b*e^8)*x^3 + (
3*a*b^8*d^8 - 12*a^2*b^7*d^7*e + 10*a^3*b^6*d^6*e^2 + 24*a^4*b^5*d^5*e^...

```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1217 vs. $2(153) = 306$.

Time = 1.66 (sec) , antiderivative size = 1217, normalized size of antiderivative = 7.16

$$\int \frac{1}{(d+ex)^3 (a^2+2abx+b^2x^2)^2} dx = \text{Too large to display}$$

input

```
integrate(1/(e*x+d)**3/(b**2*x**2+2*a*b*x+a**2)**2,x)
```


output

```

10*b**2*e**3*log(x + (-10*a**7*b**2*e**10/(a*e - b*d)**6 + 70*a**6*b**3*d*
e**9/(a*e - b*d)**6 - 210*a**5*b**4*d**2*e**8/(a*e - b*d)**6 + 350*a**4*b*
*5*d**3*e**7/(a*e - b*d)**6 - 350*a**3*b**6*d**4*e**6/(a*e - b*d)**6 + 210
*a**2*b**7*d**5*e**5/(a*e - b*d)**6 - 70*a*b**8*d**6*e**4/(a*e - b*d)**6 +
10*a*b**2*e**4 + 10*b**9*d**7*e**3/(a*e - b*d)**6 + 10*b**3*d*e**3)/(20*b
**3*e**4))/(a*e - b*d)**6 - 10*b**2*e**3*log(x + (10*a**7*b**2*e**10/(a*e
- b*d)**6 - 70*a**6*b**3*d*e**9/(a*e - b*d)**6 + 210*a**5*b**4*d**2*e**8/(
a*e - b*d)**6 - 350*a**4*b**5*d**3*e**7/(a*e - b*d)**6 + 350*a**3*b**6*d**
4*e**6/(a*e - b*d)**6 - 210*a**2*b**7*d**5*e**5/(a*e - b*d)**6 + 70*a*b**8
*d**6*e**4/(a*e - b*d)**6 + 10*a*b**2*e**4 - 10*b**9*d**7*e**3/(a*e - b*d)
**6 + 10*b**3*d*e**3)/(20*b**3*e**4))/(a*e - b*d)**6 + (-3*a**4*e**4 + 27*
a**3*b*d*e**3 + 47*a**2*b**2*d**2*e**2 - 13*a*b**3*d**3*e + 2*b**4*d**4 +
60*b**4*e**4*x**4 + x**3*(150*a*b**3*e**4 + 90*b**4*d*e**3) + x**2*(110*a
*2*b**2*e**4 + 230*a*b**3*d*e**3 + 20*b**4*d**2*e**2) + x*(15*a**3*b*e**4
+ 175*a**2*b**2*d*e**3 + 55*a*b**3*d**2*e**2 - 5*b**4*d**3*e))/(6*a**8*d**
2*e**5 - 30*a**7*b*d**3*e**4 + 60*a**6*b**2*d**4*e**3 - 60*a**5*b**3*d**5*
e**2 + 30*a**4*b**4*d**6*e - 6*a**3*b**5*d**7 + x**5*(6*a**5*b**3*e**7 - 3
0*a**4*b**4*d*e**6 + 60*a**3*b**5*d**2*e**5 - 60*a**2*b**6*d**3*e**4 + 30*
a*b**7*d**4*e**3 - 6*b**8*d**5*e**2) + x**4*(18*a**6*b**2*e**7 - 78*a**5*b
**3*d*e**6 + 120*a**4*b**4*d**2*e**5 - 60*a**3*b**5*d**3*e**4 - 30*a**2...

```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 889 vs. $2(164) = 328$.

Time = 0.07 (sec) , antiderivative size = 889, normalized size of antiderivative = 5.23

$$\int \frac{1}{(d + ex)^3 (a^2 + 2abx + b^2x^2)^2} dx = \text{Too large to display}$$

input

```
integrate(1/(e*x+d)^3/(b^2*x^2+2*a*b*x+a^2)^2,x, algorithm="maxima")
```

output

```

-10*b^2*e^3*log(b*x + a)/(b^6*d^6 - 6*a*b^5*d^5*e + 15*a^2*b^4*d^4*e^2 - 2
0*a^3*b^3*d^3*e^3 + 15*a^4*b^2*d^2*e^4 - 6*a^5*b*d*e^5 + a^6*e^6) + 10*b^2
*e^3*log(e*x + d)/(b^6*d^6 - 6*a*b^5*d^5*e + 15*a^2*b^4*d^4*e^2 - 20*a^3*b
^3*d^3*e^3 + 15*a^4*b^2*d^2*e^4 - 6*a^5*b*d*e^5 + a^6*e^6) - 1/6*(60*b^4*e
^4*x^4 + 2*b^4*d^4 - 13*a*b^3*d^3*e + 47*a^2*b^2*d^2*e^2 + 27*a^3*b*d*e^3
- 3*a^4*e^4 + 30*(3*b^4*d*e^3 + 5*a*b^3*e^4)*x^3 + 10*(2*b^4*d^2*e^2 + 23*
a*b^3*d*e^3 + 11*a^2*b^2*e^4)*x^2 - 5*(b^4*d^3*e - 11*a*b^3*d^2*e^2 - 35*a
^2*b^2*d*e^3 - 3*a^3*b*e^4)*x)/(a^3*b^5*d^7 - 5*a^4*b^4*d^6*e + 10*a^5*b^3
*d^5*e^2 - 10*a^6*b^2*d^4*e^3 + 5*a^7*b*d^3*e^4 - a^8*d^2*e^5 + (b^8*d^5*e
^2 - 5*a*b^7*d^4*e^3 + 10*a^2*b^6*d^3*e^4 - 10*a^3*b^5*d^2*e^5 + 5*a^4*b^4
*d*e^6 - a^5*b^3*e^7)*x^5 + (2*b^8*d^6*e - 7*a*b^7*d^5*e^2 + 5*a^2*b^6*d^4
*e^3 + 10*a^3*b^5*d^3*e^4 - 20*a^4*b^4*d^2*e^5 + 13*a^5*b^3*d*e^6 - 3*a^6*
b^2*e^7)*x^4 + (b^8*d^7 + a*b^7*d^6*e - 17*a^2*b^6*d^5*e^2 + 35*a^3*b^5*d^
4*e^3 - 25*a^4*b^4*d^3*e^4 - a^5*b^3*d^2*e^5 + 9*a^6*b^2*d*e^6 - 3*a^7*b*e
^7)*x^3 + (3*a*b^7*d^7 - 9*a^2*b^6*d^6*e + a^3*b^5*d^5*e^2 + 25*a^4*b^4*d^
4*e^3 - 35*a^5*b^3*d^3*e^4 + 17*a^6*b^2*d^2*e^5 - a^7*b*d*e^6 - a^8*e^7)*x
^2 + (3*a^2*b^6*d^7 - 13*a^3*b^5*d^6*e + 20*a^4*b^4*d^5*e^2 - 10*a^5*b^3*d
^4*e^3 - 5*a^6*b^2*d^3*e^4 + 7*a^7*b*d^2*e^5 - 2*a^8*d*e^6)*x)

```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 458 vs. $2(164) = 328$.

Time = 0.20 (sec) , antiderivative size = 458, normalized size of antiderivative = 2.69

$$\begin{aligned}
& \int \frac{1}{(d+ex)^3 (a^2+2abx+b^2x^2)^2} dx \\
&= -\frac{10b^3e^3 \log(|bx+a|)}{b^7d^6 - 6ab^6d^5e + 15a^2b^5d^4e^2 - 20a^3b^4d^3e^3 + 15a^4b^3d^2e^4 - 6a^5b^2de^5 + a^6be^6} \\
&+ \frac{10b^2e^4 \log(|ex+d|)}{b^6d^6e - 6ab^5d^5e^2 + 15a^2b^4d^4e^3 - 20a^3b^3d^3e^4 + 15a^4b^2d^2e^5 - 6a^5bde^6 + a^6e^7} \\
&- \frac{2b^5d^5 - 15ab^4d^4e + 60a^2b^3d^3e^2 - 20a^3b^2d^2e^3 - 30a^4bde^4 + 3a^5e^5 + 60(b^5de^4 - ab^4e^5)x^4 + 30(3b^5d^5e^5 - 15ab^4d^4e^4 + 60a^2b^3d^3e^3 - 20a^3b^2d^2e^2 + 30a^4bde^1 - a^5e^0)x^3 + 10(2b^4d^4e^4 - 23ab^3d^3e^3 + 11a^2b^2d^2e^2 - 3a^3bd^1e^1 + a^4d^0e^0)x^2 + 5(b^4d^3e^3 - 11ab^3d^2e^2 + 35a^2b^2d^1e^1 - 3a^3bd^0e^0)x}{(d+ex)^3 (a^2+2abx+b^2x^2)^2}
\end{aligned}$$

input

```
integrate(1/(e*x+d)^3/(b^2*x^2+2*a*b*x+a^2)^2,x, algorithm="giac")
```

output

```

-10*b^3*e^3*log(abs(b*x + a))/(b^7*d^6 - 6*a*b^6*d^5*e + 15*a^2*b^5*d^4*e^
2 - 20*a^3*b^4*d^3*e^3 + 15*a^4*b^3*d^2*e^4 - 6*a^5*b^2*d*e^5 + a^6*b*e^6)
+ 10*b^2*e^4*log(abs(e*x + d))/(b^6*d^6*e - 6*a*b^5*d^5*e^2 + 15*a^2*b^4*
d^4*e^3 - 20*a^3*b^3*d^3*e^4 + 15*a^4*b^2*d^2*e^5 - 6*a^5*b*d*e^6 + a^6*e^
7) - 1/6*(2*b^5*d^5 - 15*a*b^4*d^4*e + 60*a^2*b^3*d^3*e^2 - 20*a^3*b^2*d^2
*e^3 - 30*a^4*b*d*e^4 + 3*a^5*e^5 + 60*(b^5*d*e^4 - a*b^4*e^5)*x^4 + 30*(3
*b^5*d^2*e^3 + 2*a*b^4*d*e^4 - 5*a^2*b^3*e^5)*x^3 + 10*(2*b^5*d^3*e^2 + 21
*a*b^4*d^2*e^3 - 12*a^2*b^3*d*e^4 - 11*a^3*b^2*e^5)*x^2 - 5*(b^5*d^4*e - 1
2*a*b^4*d^3*e^2 - 24*a^2*b^3*d^2*e^3 + 32*a^3*b^2*d*e^4 + 3*a^4*b*e^5)*x)/
((b*d - a*e)^6*(b*x + a)^3*(e*x + d)^2)

```

Mupad [B] (verification not implemented)

Time = 9.26 (sec) , antiderivative size = 797, normalized size of antiderivative = 4.69

$$\int \frac{1}{(d+ex)^3 (a^2+2abx+b^2x^2)^2} dx$$

$$= \frac{-3a^4e^4+27a^3bde^3+47a^2b^2d^2e^2-13ab^3d^3e+2b^4d^4}{6(a^5e^5-5a^4bde^4+10a^3b^2d^2e^3-10a^2b^3d^3e^2+5ab^4d^4e-b^5d^5)} + \frac{5ex(3a^3be^3+35a^2b^2de^2+11ab^3d^2e-b^4d^3)}{6(a^5e^5-5a^4bde^4+10a^3b^2d^2e^3-10a^2b^3d^3e^2+5ab^4d^4e-b^5d^5)}$$

$$- \frac{20b^2e^3 \operatorname{atanh}\left(\frac{a^6e^6-4a^5bde^5+5a^4b^2d^2e^4-5a^2b^4d^4e^2+4ab^5d^5e-b^6d^6}{(ae-bd)^6} + \frac{x^2(a^3e^2+6a^2bde+3ab^2d^2)+x^3(3a^2b^2d^2+6ab^3de+3a^2b^3d^2)+x^4(3a^3b^2d^2+6a^2b^3de+3ab^4d^2)+x^5(3a^4b^2d^2+6a^3b^3de+3a^2b^4d^2)+x^6(3a^5b^2d^2+6a^4b^3de+3a^3b^4d^2)+x^7(3a^4b^3d^2+6a^3b^4de+3a^2b^5d^2)+x^8(3a^5b^3d^2+6a^4b^4de+3a^3b^5d^2)+x^9(3a^4b^4d^2+6a^3b^5de+3a^2b^6d^2)+x^{10}(3a^5b^5d^2+6a^4b^6de+3a^3b^7d^2)+x^{11}(3a^4b^6d^2+6a^3b^7de+3a^2b^8d^2)+x^{12}(3a^5b^7d^2+6a^4b^8de+3a^3b^9d^2)+x^{13}(3a^4b^8d^2+6a^3b^9de+3a^2b^{10}d^2)+x^{14}(3a^5b^9d^2+6a^4b^{10}de+3a^3b^{11}d^2)+x^{15}(3a^4b^{10}d^2+6a^3b^{11}de+3a^2b^{12}d^2)+x^{16}(3a^5b^{11}d^2+6a^4b^{12}de+3a^3b^{13}d^2)+x^{17}(3a^4b^{12}d^2+6a^3b^{13}de+3a^2b^{14}d^2)+x^{18}(3a^5b^{13}d^2+6a^4b^{14}de+3a^3b^{15}d^2)+x^{19}(3a^4b^{14}d^2+6a^3b^{15}de+3a^2b^{16}d^2)+x^{20}(3a^5b^{15}d^2+6a^4b^{16}de+3a^3b^{17}d^2)+x^{21}(3a^4b^{16}d^2+6a^3b^{17}de+3a^2b^{18}d^2)+x^{22}(3a^5b^{17}d^2+6a^4b^{18}de+3a^3b^{19}d^2)+x^{23}(3a^4b^{18}d^2+6a^3b^{19}de+3a^2b^{20}d^2)+x^{24}(3a^5b^{19}d^2+6a^4b^{20}de+3a^3b^{21}d^2)+x^{25}(3a^4b^{20}d^2+6a^3b^{21}de+3a^2b^{22}d^2)+x^{26}(3a^5b^{21}d^2+6a^4b^{22}de+3a^3b^{23}d^2)+x^{27}(3a^4b^{22}d^2+6a^3b^{23}de+3a^2b^{24}d^2)+x^{28}(3a^5b^{23}d^2+6a^4b^{24}de+3a^3b^{25}d^2)+x^{29}(3a^4b^{24}d^2+6a^3b^{25}de+3a^2b^{26}d^2)+x^{30}(3a^5b^{25}d^2+6a^4b^{26}de+3a^3b^{27}d^2)+x^{31}(3a^4b^{26}d^2+6a^3b^{27}de+3a^2b^{28}d^2)+x^{32}(3a^5b^{27}d^2+6a^4b^{28}de+3a^3b^{29}d^2)+x^{33}(3a^4b^{28}d^2+6a^3b^{29}de+3a^2b^{30}d^2)+x^{34}(3a^5b^{29}d^2+6a^4b^{30}de+3a^3b^{31}d^2)+x^{35}(3a^4b^{30}d^2+6a^3b^{31}de+3a^2b^{32}d^2)+x^{36}(3a^5b^{31}d^2+6a^4b^{32}de+3a^3b^{33}d^2)+x^{37}(3a^4b^{32}d^2+6a^3b^{33}de+3a^2b^{34}d^2)+x^{38}(3a^5b^{33}d^2+6a^4b^{34}de+3a^3b^{35}d^2)+x^{39}(3a^4b^{34}d^2+6a^3b^{35}de+3a^2b^{36}d^2)+x^{40}(3a^5b^{35}d^2+6a^4b^{36}de+3a^3b^{37}d^2)+x^{41}(3a^4b^{36}d^2+6a^3b^{37}de+3a^2b^{38}d^2)+x^{42}(3a^5b^{37}d^2+6a^4b^{38}de+3a^3b^{39}d^2)+x^{43}(3a^4b^{38}d^2+6a^3b^{39}de+3a^2b^{40}d^2)+x^{44}(3a^5b^{39}d^2+6a^4b^{40}de+3a^3b^{41}d^2)+x^{45}(3a^4b^{40}d^2+6a^3b^{41}de+3a^2b^{42}d^2)+x^{46}(3a^5b^{41}d^2+6a^4b^{42}de+3a^3b^{43}d^2)+x^{47}(3a^4b^{42}d^2+6a^3b^{43}de+3a^2b^{44}d^2)+x^{48}(3a^5b^{43}d^2+6a^4b^{44}de+3a^3b^{45}d^2)+x^{49}(3a^4b^{44}d^2+6a^3b^{45}de+3a^2b^{46}d^2)+x^{50}(3a^5b^{45}d^2+6a^4b^{46}de+3a^3b^{47}d^2)+x^{51}(3a^4b^{46}d^2+6a^3b^{47}de+3a^2b^{48}d^2)+x^{52}(3a^5b^{47}d^2+6a^4b^{48}de+3a^3b^{49}d^2)+x^{53}(3a^4b^{48}d^2+6a^3b^{49}de+3a^2b^{50}d^2)+x^{54}(3a^5b^{49}d^2+6a^4b^{50}de+3a^3b^{51}d^2)+x^{55}(3a^4b^{50}d^2+6a^3b^{51}de+3a^2b^{52}d^2)+x^{56}(3a^5b^{51}d^2+6a^4b^{52}de+3a^3b^{53}d^2)+x^{57}(3a^4b^{52}d^2+6a^3b^{53}de+3a^2b^{54}d^2)+x^{58}(3a^5b^{53}d^2+6a^4b^{54}de+3a^3b^{55}d^2)+x^{59}(3a^4b^{54}d^2+6a^3b^{55}de+3a^2b^{56}d^2)+x^{60}(3a^5b^{55}d^2+6a^4b^{56}de+3a^3b^{57}d^2)+x^{61}(3a^4b^{56}d^2+6a^3b^{57}de+3a^2b^{58}d^2)+x^{62}(3a^5b^{57}d^2+6a^4b^{58}de+3a^3b^{59}d^2)+x^{63}(3a^4b^{58}d^2+6a^3b^{59}de+3a^2b^{60}d^2)+x^{64}(3a^5b^{59}d^2+6a^4b^{60}de+3a^3b^{61}d^2)+x^{65}(3a^4b^{60}d^2+6a^3b^{61}de+3a^2b^{62}d^2)+x^{66}(3a^5b^{61}d^2+6a^4b^{62}de+3a^3b^{63}d^2)+x^{67}(3a^4b^{62}d^2+6a^3b^{63}de+3a^2b^{64}d^2)+x^{68}(3a^5b^{63}d^2+6a^4b^{64}de+3a^3b^{65}d^2)+x^{69}(3a^4b^{64}d^2+6a^3b^{65}de+3a^2b^{66}d^2)+x^{70}(3a^5b^{65}d^2+6a^4b^{66}de+3a^3b^{67}d^2)+x^{71}(3a^4b^{66}d^2+6a^3b^{67}de+3a^2b^{68}d^2)+x^{72}(3a^5b^{67}d^2+6a^4b^{68}de+3a^3b^{69}d^2)+x^{73}(3a^4b^{68}d^2+6a^3b^{69}de+3a^2b^{70}d^2)+x^{74}(3a^5b^{69}d^2+6a^4b^{70}de+3a^3b^{71}d^2)+x^{75}(3a^4b^{70}d^2+6a^3b^{71}de+3a^2b^{72}d^2)+x^{76}(3a^5b^{71}d^2+6a^4b^{72}de+3a^3b^{73}d^2)+x^{77}(3a^4b^{72}d^2+6a^3b^{73}de+3a^2b^{74}d^2)+x^{78}(3a^5b^{73}d^2+6a^4b^{74}de+3a^3b^{75}d^2)+x^{79}(3a^4b^{74}d^2+6a^3b^{75}de+3a^2b^{76}d^2)+x^{80}(3a^5b^{75}d^2+6a^4b^{76}de+3a^3b^{77}d^2)+x^{81}(3a^4b^{76}d^2+6a^3b^{77}de+3a^2b^{78}d^2)+x^{82}(3a^5b^{77}d^2+6a^4b^{78}de+3a^3b^{79}d^2)+x^{83}(3a^4b^{78}d^2+6a^3b^{79}de+3a^2b^{80}d^2)+x^{84}(3a^5b^{79}d^2+6a^4b^{80}de+3a^3b^{81}d^2)+x^{85}(3a^4b^{80}d^2+6a^3b^{81}de+3a^2b^{82}d^2)+x^{86}(3a^5b^{81}d^2+6a^4b^{82}de+3a^3b^{83}d^2)+x^{87}(3a^4b^{82}d^2+6a^3b^{83}de+3a^2b^{84}d^2)+x^{88}(3a^5b^{83}d^2+6a^4b^{84}de+3a^3b^{85}d^2)+x^{89}(3a^4b^{84}d^2+6a^3b^{85}de+3a^2b^{86}d^2)+x^{90}(3a^5b^{85}d^2+6a^4b^{86}de+3a^3b^{87}d^2)+x^{91}(3a^4b^{86}d^2+6a^3b^{87}de+3a^2b^{88}d^2)+x^{92}(3a^5b^{87}d^2+6a^4b^{88}de+3a^3b^{89}d^2)+x^{93}(3a^4b^{88}d^2+6a^3b^{89}de+3a^2b^{90}d^2)+x^{94}(3a^5b^{89}d^2+6a^4b^{90}de+3a^3b^{91}d^2)+x^{95}(3a^4b^{90}d^2+6a^3b^{91}de+3a^2b^{92}d^2)+x^{96}(3a^5b^{91}d^2+6a^4b^{92}de+3a^3b^{93}d^2)+x^{97}(3a^4b^{92}d^2+6a^3b^{93}de+3a^2b^{94}d^2)+x^{98}(3a^5b^{93}d^2+6a^4b^{94}de+3a^3b^{95}d^2)+x^{99}(3a^4b^{94}d^2+6a^3b^{95}de+3a^2b^{96}d^2)+x^{100}(3a^5b^{95}d^2+6a^4b^{96}de+3a^3b^{97}d^2)+x^{101}(3a^4b^{96}d^2+6a^3b^{97}de+3a^2b^{98}d^2)+x^{102}(3a^5b^{97}d^2+6a^4b^{98}de+3a^3b^{99}d^2)+x^{103}(3a^4b^{98}d^2+6a^3b^{99}de+3a^2b^{100}d^2)+x^{104}(3a^5b^{99}d^2+6a^4b^{100}de+3a^3b^{101}d^2)+x^{105}(3a^4b^{100}d^2+6a^3b^{101}de+3a^2b^{102}d^2)+x^{106}(3a^5b^{101}d^2+6a^4b^{102}de+3a^3b^{103}d^2)+x^{107}(3a^4b^{102}d^2+6a^3b^{103}de+3a^2b^{104}d^2)+x^{108}(3a^5b^{103}d^2+6a^4b^{104}de+3a^3b^{105}d^2)+x^{109}(3a^4b^{104}d^2+6a^3b^{105}de+3a^2b^{106}d^2)+x^{110}(3a^5b^{105}d^2+6a^4b^{106}de+3a^3b^{107}d^2)+x^{111}(3a^4b^{106}d^2+6a^3b^{107}de+3a^2b^{108}d^2)+x^{112}(3a^5b^{107}d^2+6a^4b^{108}de+3a^3b^{109}d^2)+x^{113}(3a^4b^{108}d^2+6a^3b^{109}de+3a^2b^{110}d^2)+x^{114}(3a^5b^{109}d^2+6a^4b^{110}de+3a^3b^{111}d^2)+x^{115}(3a^4b^{110}d^2+6a^3b^{111}de+3a^2b^{112}d^2)+x^{116}(3a^5b^{111}d^2+6a^4b^{112}de+3a^3b^{113}d^2)+x^{117}(3a^4b^{112}d^2+6a^3b^{113}de+3a^2b^{114}d^2)+x^{118}(3a^5b^{113}d^2+6a^4b^{114}de+3a^3b^{115}d^2)+x^{119}(3a^4b^{114}d^2+6a^3b^{115}de+3a^2b^{116}d^2)+x^{120}(3a^5b^{115}d^2+6a^4b^{116}de+3a^3b^{117}d^2)+x^{121}(3a^4b^{116}d^2+6a^3b^{117}de+3a^2b^{118}d^2)+x^{122}(3a^5b^{117}d^2+6a^4b^{118}de+3a^3b^{119}d^2)+x^{123}(3a^4b^{118}d^2+6a^3b^{119}de+3a^2b^{120}d^2)+x^{124}(3a^5b^{119}d^2+6a^4b^{120}de+3a^3b^{121}d^2)+x^{125}(3a^4b^{120}d^2+6a^3b^{121}de+3a^2b^{122}d^2)+x^{126}(3a^5b^{121}d^2+6a^4b^{122}de+3a^3b^{123}d^2)+x^{127}(3a^4b^{122}d^2+6a^3b^{123}de+3a^2b^{124}d^2)+x^{128}(3a^5b^{123}d^2+6a^4b^{124}de+3a^3b^{125}d^2)+x^{129}(3a^4b^{124}d^2+6a^3b^{125}de+3a^2b^{126}d^2)+x^{130}(3a^5b^{125}d^2+6a^4b^{126}de+3a^3b^{127}d^2)+x^{131}(3a^4b^{126}d^2+6a^3b^{127}de+3a^2b^{128}d^2)+x^{132}(3a^5b^{127}d^2+6a^4b^{128}de+3a^3b^{129}d^2)+x^{133}(3a^4b^{128}d^2+6a^3b^{129}de+3a^2b^{130}d^2)+x^{134}(3a^5b^{129}d^2+6a^4b^{130}de+3a^3b^{131}d^2)+x^{135}(3a^4b^{130}d^2+6a^3b^{131}de+3a^2b^{132}d^2)+x^{136}(3a^5b^{131}d^2+6a^4b^{132}de+3a^3b^{133}d^2)+x^{137}(3a^4b^{132}d^2+6a^3b^{133}de+3a^2b^{134}d^2)+x^{138}(3a^5b^{133}d^2+6a^4b^{134}de+3a^3b^{135}d^2)+x^{139}(3a^4b^{134}d^2+6a^3b^{135}de+3a^2b^{136}d^2)+x^{140}(3a^5b^{135}d^2+6a^4b^{136}de+3a^3b^{137}d^2)+x^{141}(3a^4b^{136}d^2+6a^3b^{137}de+3a^2b^{138}d^2)+x^{142}(3a^5b^{137}d^2+6a^4b^{138}de+3a^3b^{139}d^2)+x^{143}(3a^4b^{138}d^2+6a^3b^{139}de+3a^2b^{140}d^2)+x^{144}(3a^5b^{139}d^2+6a^4b^{140}de+3a^3b^{141}d^2)+x^{145}(3a^4b^{140}d^2+6a^3b^{141}de+3a^2b^{142}d^2)+x^{146}(3a^5b^{141}d^2+6a^4b^{142}de+3a^3b^{143}d^2)+x^{147}(3a^4b^{142}d^2+6a^3b^{143}de+3a^2b^{144}d^2)+x^{148}(3a^5b^{143}d^2+6a^4b^{144}de+3a^3b^{145}d^2)+x^{149}(3a^4b^{144}d^2+6a^3b^{145}de+3a^2b^{146}d^2)+x^{150}(3a^5b^{145}d^2+6a^4b^{146}de+3a^3b^{147}d^2)+x^{151}(3a^4b^{146}d^2+6a^3b^{147}de+3a^2b^{148}d^2)+x^{152}(3a^5b^{147}d^2+6a^4b^{148}de+3a^3b^{149}d^2)+x^{153}(3a^4b^{148}d^2+6a^3b^{149}de+3a^2b^{150}d^2)+x^{154}(3a^5b^{149}d^2+6a^4b^{150}de+3a^3b^{151}d^2)+x^{155}(3a^4b^{150}d^2+6a^3b^{151}de+3a^2b^{152}d^2)+x^{156}(3a^5b^{151}d^2+6a^4b^{152}de+3a^3b^{153}d^2)+x^{157}(3a^4b^{152}d^2+6a^3b^{153}de+3a^2b^{154}d^2)+x^{158}(3a^5b^{153}d^2+6a^4b^{154}de+3a^3b^{155}d^2)+x^{159}(3a^4b^{154}d^2+6a^3b^{155}de+3a^2b^{156}d^2)+x^{160}(3a^5b^{155}d^2+6a^4b^{156}de+3a^3b^{157}d^2)+x^{161}(3a^4b^{156}d^2+6a^3b^{157}de+3a^2b^{158}d^2)+x^{162}(3a^5b^{157}d^2+6a^4b^{158}de+3a^3b^{159}d^2)+x^{163}(3a^4b^{158}d^2+6a^3b^{159}de+3a^2b^{160}d^2)+x^{164}(3a^5b^{159}d^2+6a^4b^{160}de+3a^3b^{161}d^2)+x^{165}(3a^4b^{160}d^2+6a^3b^{161}de+3a^2b^{162}d^2)+x^{166}(3a^5b^{161}d^2+6a^4b^{162}de+3a^3b^{163}d^2)+x^{167}(3a^4b^{162}d^2+6a^3b^{163}de+3a^2b^{164}d^2)+x^{168}(3a^5b^{163}d^2+6a^4b^{164}de+3a^3b^{165}d^2)+x^{169}(3a^4b^{164}d^2+6a^3b^{165}de+3a^2b^{166}d^2)+x^{170}(3a^5b^{165}d^2+6a^4b^{166}de+3a^3b^{167}d^2)+x^{171}(3a^4b^{166}d^2+6a^3b^{167}de+3a^2b^{168}d^2)+x^{172}(3a^5b^{167}d^2+6a^4b^{168}de+3a^3b^{169}d^2)+x^{173}(3a^4b^{168}d^2+6a^3b^{169}de+3a^2b^{170}d^2)+x^{174}(3a^5b^{169}d^2+6a^4b^{170}de+3a^3b^{171}d^2)+x^{175}(3a^4b^{170}d^2+6a^3b^{171}de+3a^2b^{172}d^2)+x^{176}(3a^5b^{171}d^2+6a^4b^{172}de+3a^3b^{173}d^2)+x^{177}(3a^4b^{172}d^2+6a^3b^{173}de+3a^2b^{174}d^2)+x^{178}(3a^5b^{173}d^2+6a^4b^{174}de+3a^3b^{175}d^2)+x^{179}(3a^4b^{174}d^2+6a^3b^{175}de+3a^2b^{176}d^2)+x^{180}(3a^5b^{175}d^2+6a^4b^{176}de+3a^3b^{177}d^2)+x^{181}(3a^4b^{176}d^2+6a^3b^{177}de+3a^2b^{178}d^2)+x^{182}(3a^5b^{177}d^2+6a^4b^{178}de+3a^3b^{179}d^2)+x^{183}(3a^4b^{178}d^2+6a^3b^{179}de+3a^2b^{180}d^2)+x^{184}(3a^5b^{179}d^2+6a^4b^{180}de+3a^3b^{181}d^2)+x^{185}(3a^4b^{180}d^2+6a^3b^{181}de+3a^2b^{182}d^2)+x^{186}(3a^5b^{181}d^2+6a^4b^{182}de+3a^3b^{183}d^2)+x^{187}(3a^4b^{182}d^2+6a^3b^{183}de+3a^2b^{184}d^2)+x^{188}(3a^5b^{183}d^2+6a^4b^{184}de+3a^3b^{185}d^2)+x^{189}(3a^4b^{184}d^2+6a^3b^{185}de+3a^2b^{186}d^2)+x^{190}(3a^5b^{185}d^2+6a^4b^{186}de+3a^3b^{187}d^2)+x^{191}(3a^4b^{186}d^2+6a^3b^{187}de+3a^2b^{188}d^2)+x^{192}(3a^5b^{187}d^2+6a^4b^{188}de+3a^3b^{189}d^2)+x^{193}(3a^4b^{188}d^2+6a^3b^{189}de+3a^2b^{190}d^2)+x^{194}(3a^5b^{189}d^2+6a^4b^{190}de+3a^3b^{191}d^2)+x^{195}(3a^4b^{190}d^2+6a^3b^{191}de+3a^2b^{192}d^2)+x^{196}(3a^5b^{191}d^2+6a^4b^{192}de+3a^3b^{193}d^2)+x^{197}(3a^4b^{192}d^2+6a^3b^{193}de+3a^2b^{194}d^2)+x^{198}(3a^5b^{193}d^2+6a^4b^{194}de+3a^3b^{195}d^2)+x^{199}(3a^4b^{194}d^2+6a^3b^{195}de+3a^2b^{196}d^2)+x^{200}(3a^5b^{195}d^2+6a^4b^{196}de+3a^3b^{197}d^2)+x^{201}(3a^4b^{196}d^2+6a^3b^{197}de+3a^2b^{198}d^2)+x^{202}(3a^5b^{197}d^2+6a^4b^{198}de+3a^3b^{199}d^2)+x^{203}(3a^4b^{198}d^2+6a^3b^{199}de+3a^2b^{200}d^2)+x^{204}(3a^5b^{199}d^2+6a^4b^{200}de+3a^3b^{201}d^2)+x^{205}(3a^4b^{200}d^2+6a^3b^{201}de+3a^2b^{202}d^2)+x^{206}(3a^5b^{201}d^2+6a^4b^{202}de+3a^3b^{203}d^2)+x^{207}(3a^4b^{202}d^2+6a^3b^{203}de+3a^2b^{204}d^2)+x^{208}(3a^5b^{203}d^2+6a^4b^{204}de+3a^3b^{205}d^2)+x^{209}(3a^4b^{204}d^2+6a^3b^{205}de+3a^2b^{206}d^2)+x^{210}(3a^5b^{205}d^2+6a^4b^{206}de+3a^3b^{207}d^2)+x^{211}(3a^4b^{206}d^2+6a^3b^{207}de+3a^2b^{208}d^2)+x^{212}(3a^5b^{207}d^2+6a^4b^{208}de+3a^3b^{209}d^2)+x^{213}(3a^4b^{208}d^2+6a^3b^{209}de+3a^2b^{210}d^2)+x^{214}(3a^5b^{209}d^2+6a^4b^{210}de+3a^3b^{211}d^2)+x^{215}(3a^4b^{210}d^2+6a^3b^{211}de+3a^2b^{212}d^2)+x^{216}(3a^5b^{211}d^2+6a^4b^{212}de+3a^3b^{213}d^2)+x^{217}(3a^4b^{212}d^2+6a^3b^{213}de+3a^2b^{214}d^2)+x^{218}(3a^5b^{213}d^2+6a^4b^{214}de+3a^3b^{215}d^2)+x^{219}(3a^4b^{214}d^2+6a^3b^{215}de+3a^2b^{216}d^2)+x^{220}(3a^5b^{215}d^2+6a^4b^{216}de+3a^3b^{217}d^2)+x^{221}(3a^4b^{216}d^2+6a^3b^{217}de+3a^2b^{218}d^2)+x^{222}(3a^5b^{217}d^2+6a^4b^{218}de+3a^3b^{219}d^2)+x^{223}(3a^4b^{218}d^2+6a^3b^{219}de+3a^2b^{220}d^2)+x^{224}(3a^5b^{219}d^2+6a^4b^{220}de+3a^3b^{221}d^2)+x^{225}(3a^4b^{220}d^2+6a^3b^{221}de+3a^2b^{222}d^2)+x^{226}(3a^5b^{221}d^2+6a^4b^{222}de+3a^3b^{223}d^2)+x^{227}(3a^4b^{222}d^2+6a^3b^{223}de+3a^2b^{224}d^2)+x^{228}(3a^5b^{223}d^2+6a^4b^{224}de+3a^3b^{225}d^2)+x^{229}(3a^4b^{224}d^2+6a^3b^{225}de+3a^2b^{226}d^2)+x^{230}(3a^5b^{225}d^2+6a^4b^{226}de+3a^3b^{227}d^2)+x^{231}(3a^4b^{226}d^2+6a^3b^{227}de+3a^2b^{228}d^2)+x^{232}(3a^5b^{227}d^2+6a^4b^{228}de+3a^3b^{229}d^2)+x^{233}(3a^4b^{228}d^2+6a^3b^{229}de+3a^2b^{230}d^2)+x^{234}(3a^5b^{229}d^2+6a^4b^{230}de+3a^3b^{231}d^2)+x^{235}(3a^4b^{230}d^2+6a^3b^{231}de+3a^2b^{232}d^2)+x^{236}(3a^5b^{231}d^2+6a^4b^{232}de+3a^3b^{233}d^2)+x^{237}(3a^4b^{232}d^2+6a^3b^{233}de+3a^2b^{234}d^2)+x^{238}(3a^5b^{233}d^2+6a^4b^{234}de+3a^3b^{235}d^2)+x^{239}(3a^4b^{234}d^2+6a^3b^{235}de+3a^2b^{236}d^2)+x^{240}(3a^5b^{235}d^2+6a^4b^{236}de+3a^3b^{237}d^2)+x^{241}(3a^4b^{236}d^2+6a^3b^{237}de+3a^2b^{238}d^2)+x^{242}(3a^5b^{237}d^2+6a^4b^{238}de+3a^3b^{239}d^2)+x^{243}(3a^4b^{238}d^2+6a^3b^{239}de+3a^2b^{240}d^2)+x^{244}(3a^5b^{239}d^2+6a^4b^{240}de+3a^3b^{241}d^2)+x^{245}(3a^4b^{240}d^2+6a^3b^{241}de+3a^2b^{242}d^2)+x^{246}(3a^5b^{241}d^2+6a^4b^{242}de+3a^3b^{243}d^2)+x^{247}(3a^4b^{242}d^2+6a^3b^{243}de+3a^2b^{244}d^2)+x^{248}(3a^5b^{243}d^2+6a^4b^{244}de+3a^3b^{245}d^2)+x^{249}(3a^4b^{244}d^2+6a^3b^{245}de+3a^2b^{246}d^2)+x^{250}(3a^5b^{245}d^2+6a^4b^{246}de+3a^3b^{247}d^2)+x^{251}(3a^4b^{246}d^2+6a^3b^{247}de+3a^2b^{248}d^2)+x^{252}(3a^5b^{247}d^2+6a^4b^{248}de+3a^3b^{249}d^2)+x^{253}(3a^4b^{248}d^2+6a^3b^{249}de+3a^2b^{250}d^2)+x^{254}(3a^5b^{249}d^2+6a^4b^{250}de+3a^3b^{251}d^2)+x^{255}(3a^4b^{250}d^2+6a^3b^{251}de+3a^2b^{252}d^2)+x^{256}(3a^5b^{251}d^2+6a^4b^{252}de+3$$

output

```

((2*b^4*d^4 - 3*a^4*e^4 + 47*a^2*b^2*d^2*e^2 - 13*a*b^3*d^3*e + 27*a^3*b*d
*e^3)/(6*(a^5*e^5 - b^5*d^5 - 10*a^2*b^3*d^3*e^2 + 10*a^3*b^2*d^2*e^3 + 5*
a*b^4*d^4*e - 5*a^4*b*d*e^4)) + (5*e*x*(3*a^3*b*e^3 - b^4*d^3 + 35*a^2*b^2
*d*e^2 + 11*a*b^3*d^2*e))/(6*(a^5*e^5 - b^5*d^5 - 10*a^2*b^3*d^3*e^2 + 10*
a^3*b^2*d^2*e^3 + 5*a*b^4*d^4*e - 5*a^4*b*d*e^4)) + (10*b^4*e^4*x^4)/(a^5*
e^5 - b^5*d^5 - 10*a^2*b^3*d^3*e^2 + 10*a^3*b^2*d^2*e^3 + 5*a*b^4*d^4*e -
5*a^4*b*d*e^4) + (5*e^2*x^2*(2*b^4*d^2 + 11*a^2*b^2*e^2 + 23*a*b^3*d*e))/(
3*(a^5*e^5 - b^5*d^5 - 10*a^2*b^3*d^3*e^2 + 10*a^3*b^2*d^2*e^3 + 5*a*b^4*d
^4*e - 5*a^4*b*d*e^4)) + (5*b*e^2*x^3*(5*a*b^2*e^2 + 3*b^3*d*e))/(a^5*e^5
- b^5*d^5 - 10*a^2*b^3*d^3*e^2 + 10*a^3*b^2*d^2*e^3 + 5*a*b^4*d^4*e - 5*a^
4*b*d*e^4))/(x^2*(a^3*e^2 + 3*a*b^2*d^2 + 6*a^2*b*d*e) + x^3*(b^3*d^2 + 3*
a^2*b*e^2 + 6*a*b^2*d*e) + x*(3*a^2*b*d^2 + 2*a^3*d*e) + x^4*(3*a*b^2*e^2
+ 2*b^3*d*e) + a^3*d^2 + b^3*e^2*x^5) - (20*b^2*e^3*atanh((a^6*e^6 - b^6*d
^6 - 5*a^2*b^4*d^4*e^2 + 5*a^4*b^2*d^2*e^4 + 4*a*b^5*d^5*e - 4*a^5*b*d*e^5
)/(a*e - b*d)^6 + (2*b*e*x*(a^5*e^5 - b^5*d^5 - 10*a^2*b^3*d^3*e^2 + 10*a^
3*b^2*d^2*e^3 + 5*a*b^4*d^4*e - 5*a^4*b*d*e^4))/(a*e - b*d)^6))/(a*e - b*d
)^6

```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 1840, normalized size of antiderivative = 10.82

$$\int \frac{1}{(d + ex)^3 (a^2 + 2abx + b^2x^2)^2} dx = \text{Too large to display}$$

input

```
int(1/(e*x+d)^3/(b^2*x^2+2*a*b*x+a^2)^2,x)
```

output

```
( - 180*log(a + b*x)*a**4*b**2*d**2*e**4 - 360*log(a + b*x)*a**4*b**2*d*e*
*5*x - 180*log(a + b*x)*a**4*b**2*e**6*x**2 - 120*log(a + b*x)*a**3*b**3*d
**3*e**3 - 780*log(a + b*x)*a**3*b**3*d**2*e**4*x - 1200*log(a + b*x)*a**3
*b**3*d*e**5*x**2 - 540*log(a + b*x)*a**3*b**3*e**6*x**3 - 360*log(a + b*x
)*a**2*b**4*d**3*e**3*x - 1260*log(a + b*x)*a**2*b**4*d**2*e**4*x**2 - 144
0*log(a + b*x)*a**2*b**4*d*e**5*x**3 - 540*log(a + b*x)*a**2*b**4*e**6*x**
4 - 360*log(a + b*x)*a*b**5*d**3*e**3*x**2 - 900*log(a + b*x)*a*b**5*d**2*
e**4*x**3 - 720*log(a + b*x)*a*b**5*d*e**5*x**4 - 180*log(a + b*x)*a*b**5*
e**6*x**5 - 120*log(a + b*x)*b**6*d**3*e**3*x**3 - 240*log(a + b*x)*b**6*d
**2*e**4*x**4 - 120*log(a + b*x)*b**6*d*e**5*x**5 + 180*log(d + e*x)*a**4*
b**2*d**2*e**4 + 360*log(d + e*x)*a**4*b**2*d*e**5*x + 180*log(d + e*x)*a*
**4*b**2*e**6*x**2 + 120*log(d + e*x)*a**3*b**3*d**3*e**3 + 780*log(d + e*x
)*a**3*b**3*d**2*e**4*x + 1200*log(d + e*x)*a**3*b**3*d*e**5*x**2 + 540*lo
g(d + e*x)*a**3*b**3*e**6*x**3 + 360*log(d + e*x)*a**2*b**4*d**3*e**3*x +
1260*log(d + e*x)*a**2*b**4*d**2*e**4*x**2 + 1440*log(d + e*x)*a**2*b**4*d
**5*x**3 + 540*log(d + e*x)*a**2*b**4*e**6*x**4 + 360*log(d + e*x)*a*b**
5*d**3*e**3*x**2 + 900*log(d + e*x)*a*b**5*d**2*e**4*x**3 + 720*log(d + e*
x)*a*b**5*d*e**5*x**4 + 180*log(d + e*x)*a*b**5*e**6*x**5 + 120*log(d + e*
x)*b**6*d**3*e**3*x**3 + 240*log(d + e*x)*b**6*d**2*e**4*x**4 + 120*log(d
+ e*x)*b**6*d*e**5*x**5 - 9*a**6*e**6 + 84*a**5*b*d*e**5 + 45*a**5*b*e...
```

$$3.128 \quad \int \frac{(d+ex)^8}{(a^2+2abx+b^2x^2)^3} dx$$

| | |
|---|------|
| Optimal result | 1025 |
| Mathematica [A] (verified) | 1026 |
| Rubi [A] (verified) | 1026 |
| Maple [B] (verified) | 1028 |
| Fricas [B] (verification not implemented) | 1028 |
| Sympy [F(-1)] | 1029 |
| Maxima [B] (verification not implemented) | 1030 |
| Giac [B] (verification not implemented) | 1031 |
| Mupad [B] (verification not implemented) | 1032 |
| Reduce [B] (verification not implemented) | 1033 |

Optimal result

Integrand size = 26, antiderivative size = 212

$$\int \frac{(d+ex)^8}{(a^2+2abx+b^2x^2)^3} dx = \frac{e^6(28b^2d^2 - 48abde + 21a^2e^2)x}{b^8} + \frac{e^7(4bd - 3ae)x^2}{b^7} + \frac{e^8x^3}{3b^6} - \frac{(bd - ae)^8}{5b^9(a+bx)^5} - \frac{2e(bd - ae)^7}{b^9(a+bx)^4} - \frac{28e^2(bd - ae)^6}{3b^9(a+bx)^3} - \frac{28e^3(bd - ae)^5}{b^9(a+bx)^2} - \frac{70e^4(bd - ae)^4}{b^9(a+bx)} + \frac{56e^5(bd - ae)^3 \log(a+bx)}{b^9}$$

output

```
e^6*(21*a^2*e^2-48*a*b*d*e+28*b^2*d^2)*x/b^8+e^7*(-3*a*e+4*b*d)*x^2/b^7+1/3*e^8*x^3/b^6-1/5*(-a*e+b*d)^8/b^9/(b*x+a)^5-2*e*(-a*e+b*d)^7/b^9/(b*x+a)^4-28/3*e^2*(-a*e+b*d)^6/b^9/(b*x+a)^3-28*e^3*(-a*e+b*d)^5/b^9/(b*x+a)^2-70*e^4*(-a*e+b*d)^4/b^9/(b*x+a)+56*e^5*(-a*e+b*d)^3*ln(b*x+a)/b^9
```

Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 195, normalized size of antiderivative = 0.92

$$\int \frac{(d+ex)^8}{(a^2+2abx+b^2x^2)^3} dx$$

$$= \frac{15be^6(28b^2d^2 - 48abde + 21a^2e^2)x + 15b^2e^7(4bd - 3ae)x^2 + 5b^3e^8x^3 - \frac{3(bd-ae)^8}{(a+bx)^5} + \frac{30e(-bd+ae)^7}{(a+bx)^4} - \frac{140e^2(bd-ae)^6}{(a+bx)^3} + \frac{420e^3(-bd+ae)^5}{(a+bx)^2} - \frac{1050e^4(bd-ae)^4}{(a+bx)} + 840e^5(bd-ae)^3 \operatorname{Log}[a+bx]}{15b^9}$$

input

```
Integrate[(d + e*x)^8/(a^2 + 2*a*b*x + b^2*x^2)^3,x]
```

output

```
(15*b*e^6*(28*b^2*d^2 - 48*a*b*d*e + 21*a^2*e^2)*x + 15*b^2*e^7*(4*b*d - 3*a*e)*x^2 + 5*b^3*e^8*x^3 - (3*(b*d - a*e)^8)/(a + b*x)^5 + (30*e*(-(b*d) + a*e)^7)/(a + b*x)^4 - (140*e^2*(b*d - a*e)^6)/(a + b*x)^3 + (420*e^3*(-(b*d) + a*e)^5)/(a + b*x)^2 - (1050*e^4*(b*d - a*e)^4)/(a + b*x) + 840*e^5*(b*d - a*e)^3*Log[a + b*x])/(15*b^9)
```

Rubi [A] (verified)

Time = 0.91 (sec) , antiderivative size = 208, normalized size of antiderivative = 0.98, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {1098, 27, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex)^8}{(a^2+2abx+b^2x^2)^3} dx$$

$$\downarrow 1098$$

$$b^6 \int \frac{(d+ex)^8}{b^6(a+bx)^6} dx$$

$$\downarrow 27$$

$$\int \frac{(d+ex)^8}{(a+bx)^6} dx$$

$$\downarrow 49$$

$$\int \left(\frac{8e^7(a+bx)(bd-ae)}{b^8} + \frac{28e^6(bd-ae)^2}{b^8} + \frac{56e^5(bd-ae)^3}{b^8(a+bx)} + \frac{70e^4(bd-ae)^4}{b^8(a+bx)^2} + \frac{56e^3(bd-ae)^5}{b^8(a+bx)^3} + \frac{28e^2(bd-ae)^6}{b^8(a+bx)^4} \right)$$

↓ 2009

$$\frac{4e^7(a+bx)^2(bd-ae)}{b^9} + \frac{56e^5(bd-ae)^3 \log(a+bx)}{b^9} - \frac{70e^4(bd-ae)^4}{b^9(a+bx)} - \frac{28e^3(bd-ae)^5}{b^9(a+bx)^2} - \frac{28e^2(bd-ae)^6}{3b^9(a+bx)^3} - \frac{2e(bd-ae)^7}{b^9(a+bx)^4} - \frac{(bd-ae)^8}{5b^9(a+bx)^5} + \frac{e^8(a+bx)^3}{3b^9} + \frac{28e^6x(bd-ae)^2}{b^8}$$

input `Int[(d + e*x)^8/(a^2 + 2*a*b*x + b^2*x^2)^3,x]`

output `(28*e^6*(b*d - a*e)^2*x)/b^8 - (b*d - a*e)^8/(5*b^9*(a + b*x)^5) - (2*e*(b*d - a*e)^7)/(b^9*(a + b*x)^4) - (28*e^2*(b*d - a*e)^6)/(3*b^9*(a + b*x)^3) - (28*e^3*(b*d - a*e)^5)/(b^9*(a + b*x)^2) - (70*e^4*(b*d - a*e)^4)/(b^9*(a + b*x)) + (4*e^7*(b*d - a*e)*(a + b*x)^2)/b^9 + (e^8*(a + b*x)^3)/(3*b^9) + (56*e^5*(b*d - a*e)^3*Log[a + b*x])/b^9`

Defintions of rubi rules used

rule 27 `Int[(a_)*(F_x_), x_Symbol] := Simp[a Int[F_x, x], x] /; FreeQ[a, x] && !MatchQ[F_x, (b_)*(G_x_)] /; FreeQ[b, x]`

rule 49 `Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] := Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] && IGtQ[m, 0] && IGtQ[m + n + 2, 0]`

rule 1098 `Int[((d_.) + (e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] := Simp[1/c^p Int[(d + e*x)^m*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]`

rule 2009 `Int[u_, x_Symbol] := Simp[IntSum[u, x], x] /; SumQ[u]`

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 571 vs. 2(206) = 412.

Time = 1.10 (sec) , antiderivative size = 572, normalized size of antiderivative = 2.70

| method | result |
|--------------|---|
| norman | $\frac{-1918a^8e^8 - 5754a^7bde^7 + 5754a^6b^2d^2e^6 - 1918b^3d^3e^5a^5 + 210b^4d^4e^4a^4 + 42b^5d^5e^3a^3 + 14b^6d^6e^2a^2 + 6b^7d^7ea + 3b^8d^8 + \frac{e^8x^8}{3b} - \frac{5(56a^4e^8}{15b^9}}$ |
| default | $\frac{e^6(\frac{1}{3}b^2e^2x^3 - 3x^2abe^2 + 4b^2de^2x^2 + 21a^2e^2x - 48abdex + 28b^2d^2x)}{b^8} - \frac{a^8e^8 - 8a^7bde^7 + 28a^6b^2d^2e^6 - 56b^3d^3e^5a^5 + 70b^4d^4e^4a^4}{5b^9(bx+a)^5}$ |
| risch | $\frac{e^8x^3}{3b^6} - \frac{3e^8x^2a}{b^7} + \frac{4e^7dx^2}{b^6} + \frac{21e^8a^2x}{b^8} - \frac{48e^7adx}{b^7} + \frac{28e^6d^2x}{b^6} + \frac{(-70a^4b^3e^8 + 280a^3de^7b^4 - 420a^2d^2e^6b^5 + 280ab^6d^3}{b^6}$ |
| parallelrisc | Expression too large to display |

```
input int((e*x+d)^8/(b^2*x^2+2*a*b*x+a^2)^3,x,method=_RETURNVERBOSE)
```

```
output (-1/15*(1918*a^8*e^8-5754*a^7*b*d*e^7+5754*a^6*b^2*d^2*e^6-1918*a^5*b^3*d^3*e^5+210*a^4*b^4*d^4*e^4+42*a^3*b^5*d^5*e^3+14*a^2*b^6*d^6*e^2+6*a*b^7*d^7*e+3*b^8*d^8)/b^9+1/3*e^8/b*x^8-5*(56*a^4*e^8-168*a^3*b*d*e^7+168*a^2*b^2*d^2*e^6-56*a*b^3*d^3*e^5+14*b^4*d^4*e^4)/b^5*x^4-2*(420*a^5*e^8-1260*a^4*b*d*e^7+1260*a^3*b^2*d^2*e^6-420*a^2*b^3*d^3*e^5+70*a*b^4*d^4*e^4+14*b^5*d^5*e^3)/b^6*x^3-2/3*(1540*a^6*e^8-4620*a^5*b*d*e^7+4620*a^4*b^2*d^2*e^6-1540*a^3*b^3*d^3*e^5+210*a^2*b^4*d^4*e^4+42*a*b^5*d^5*e^3+14*b^6*d^6*e^2)/b^7*x^2-1/3*(1750*a^7*e^8-5250*a^6*b*d*e^7+5250*a^5*b^2*d^2*e^6-1750*a^4*b^3*d^3*e^5+210*a^3*b^4*d^4*e^4+42*a^2*b^5*d^5*e^3+14*a*b^6*d^6*e^2+6*b^7*d^7*e)/b^8*x+28/3*e^6*(a^2*e^2-3*a*b*d*e+3*b^2*d^2)/b^3*x^6-4/3*e^7*(a*e-3*b*d)/b^2*x^7)/(b*x+a)^5-56/b^9*e^5*(a^3*e^3-3*a^2*b*d*e^2+3*a*b^2*d^2*e-b^3*d^3)*ln(b*x+a)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 943 vs. 2(206) = 412.

Time = 0.09 (sec) , antiderivative size = 943, normalized size of antiderivative = 4.45

$$\int \frac{(d + ex)^8}{(a^2 + 2abx + b^2x^2)^3} dx = \text{Too large to display}$$

input `integrate((e*x+d)^8/(b^2*x^2+2*a*b*x+a^2)^3,x, algorithm="fricas")`

output

$$\frac{1}{15} \cdot (5b^8e^8x^8 - 3b^8d^8 - 6ab^7d^7e - 14a^2b^6d^6e^2 - 42a^3b^5d^5e^3 - 210a^4b^4d^4e^4 + 1918a^5b^3d^3e^5 - 3654a^6b^2d^2e^6 + 2754a^7bd^1e^7 - 743a^8e^8 + 20(3b^8d^7e - ab^7e^8))x^7 + 140(3b^8d^2e^6 - 3ab^7d^7e + a^2b^6e^8)x^6 + 25(84ab^7d^2e^6 - 120a^2b^6d^7e + 47a^3b^5e^8)x^5 - 25(42b^8d^4e^4 - 168ab^7d^3e^5 + 84a^2b^6d^2e^6 + 96a^3b^5d^7e - 67a^4b^4e^8)x^4 - 10(42b^8d^5e^3 + 210ab^7d^4e^4 - 1260a^2b^6d^3e^5 + 1680a^3b^5d^2e^6 - 780a^4b^4d^7e + 85a^5b^3e^8)x^3 - 10(14b^8d^6e^2 + 42ab^7d^5e^3 + 210a^2b^6d^4e^4 - 1540a^3b^5d^3e^5 + 2520a^4b^4d^2e^6 - 1620a^5b^3d^7e + 365a^6b^2e^8)x^2 - 5(6b^8d^7e + 14ab^7d^6e^2 + 42a^2b^6d^5e^3 + 210a^3b^5d^4e^4 - 1750a^4b^4d^3e^5 + 3150a^5b^3d^2e^6 - 2250a^6b^2d^7e + 575a^7be^8)x + 840(a^5b^3d^3e^5 - 3a^6b^2d^2e^6 + 3a^7bd^7e - a^8e^8 + (b^8d^3e^5 - 3ab^7d^2e^6 + 3a^2b^6d^7e - a^3b^5e^8)x^5 + 5(ab^7d^3e^5 - 3a^2b^6d^2e^6 + 3a^3b^5d^7e - a^4b^4e^8)x^4 + 10(a^2b^6d^3e^5 - 3a^3b^5d^2e^6 + 3a^4b^4d^7e - a^5b^3e^8)x^3 + 10(a^3b^5d^3e^5 - 3a^4b^4d^2e^6 + 3a^5b^3d^7e - a^6b^2e^8)x^2 + 5(a^4b^4d^3e^5 - 3a^5b^3d^2e^6 + 3a^6b^2d^7e - a^7be^8)x) \cdot \log(bx + a) / (b^{14}x^5 + 5a^2b^{13}x^4 + 10a^2b^{12}x^3 + 10a^3b^{11}x^2 + 5a^4b^{10}x + a^5b^9)$$

Sympy [F(-1)]

Timed out.

$$\int \frac{(d + ex)^8}{(a^2 + 2abx + b^2x^2)^3} dx = \text{Timed out}$$

input `integrate((e*x+d)**8/(b**2*x**2+2*a*b*x+a**2)**3,x)`

output Timed out

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 626 vs. $2(206) = 412$.

Time = 0.08 (sec) , antiderivative size = 626, normalized size of antiderivative = 2.95

$$\int \frac{(d+ex)^8}{(a^2+2abx+b^2x^2)^3} dx =$$

$$\frac{3b^8d^8 + 6ab^7d^7e + 14a^2b^6d^6e^2 + 42a^3b^5d^5e^3 + 210a^4b^4d^4e^4 - 1918a^5b^3d^3e^5 + 3654a^6b^2d^2e^6 - 2754a^7bd^2e^7 + 743a^8e^8 + 1050(b^8d^4e^4 - 4a^3b^7d^3e^5 + 6a^2b^6d^2e^6 - 4a^4b^5d^4e^4 - 30a^2b^6d^3e^5 + 50a^3b^5d^2e^6 - 35a^4b^4d^2e^7 + 9a^5b^3e^8)x^3 + 140(b^8d^6e^2 + 3a^3b^7d^5e^3 + 15a^2b^6d^4e^4 - 110a^3b^5d^3e^5 + 195a^4b^4d^2e^6 - 141a^5b^3d^2e^7 + 37a^6b^2e^8)x^2 + 10(3b^8d^7e + 7a^3b^7d^6e^2 + 21a^2b^6d^5e^3 + 105a^3b^5d^4e^4 - 875a^4b^4d^3e^5 + 1617a^5b^3d^2e^6 - 1197a^6b^2d^2e^7 + 319a^7b^2e^8)x}{3b^8} + \frac{56(b^3d^3e^5 - 3ab^2d^2e^6 + 3a^2bde^7 - a^3e^8) \log(bx+a)}{b^9}$$

input

```
integrate((e*x+d)^8/(b^2*x^2+2*a*b*x+a^2)^3,x, algorithm="maxima")
```

output

```
-1/15*(3*b^8*d^8 + 6*a*b^7*d^7*e + 14*a^2*b^6*d^6*e^2 + 42*a^3*b^5*d^5*e^3 + 210*a^4*b^4*d^4*e^4 - 1918*a^5*b^3*d^3*e^5 + 3654*a^6*b^2*d^2*e^6 - 2754*a^7*b*d^2*e^7 + 743*a^8*e^8 + 1050*(b^8*d^4*e^4 - 4*a^3*b^7*d^3*e^5 + 6*a^2*b^6*d^2*e^6 - 4*a^4*b^5*d^4*e^4 - 30*a^2*b^6*d^3*e^5 + 50*a^3*b^5*d^2*e^6 - 35*a^4*b^4*d^2*e^7 + 9*a^5*b^3*e^8)*x^3 + 140*(b^8*d^6*e^2 + 3*a^3*b^7*d^5*e^3 + 15*a^2*b^6*d^4*e^4 - 110*a^3*b^5*d^3*e^5 + 195*a^4*b^4*d^2*e^6 - 141*a^5*b^3*d^2*e^7 + 37*a^6*b^2*e^8)*x^2 + 10*(3*b^8*d^7*e + 7*a^3*b^7*d^6*e^2 + 21*a^2*b^6*d^5*e^3 + 105*a^3*b^5*d^4*e^4 - 875*a^4*b^4*d^3*e^5 + 1617*a^5*b^3*d^2*e^6 - 1197*a^6*b^2*d^2*e^7 + 319*a^7*b^2*e^8)*x)/(b^14*x^5 + 5*a*b^13*x^4 + 10*a^2*b^12*x^3 + 10*a^3*b^11*x^2 + 5*a^4*b^10*x + a^5*b^9) + 1/3*(b^2*e^8*x^3 + 3*(4*b^2*d^2*e^7 - 3*a*b*e^8)*x^2 + 3*(28*b^2*d^2*e^6 - 48*a*b*d^2*e^7 + 21*a^2*e^8)*x)/b^8 + 56*(b^3*d^3*e^5 - 3*a*b^2*d^2*e^6 + 3*a^2*b*d^2*e^7 - a^3*e^8)*log(b*x + a)/b^9
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 586 vs. $2(206) = 412$.

Time = 0.16 (sec) , antiderivative size = 586, normalized size of antiderivative = 2.76

$$\int \frac{(d+ex)^8}{(a^2+2abx+b^2x^2)^3} dx = \frac{56(b^3d^3e^5 - 3ab^2d^2e^6 + 3a^2bde^7 - a^3e^8) \log(|bx+a|)}{b^9} - \frac{3b^8d^8 + 6ab^7d^7e + 14a^2b^6d^6e^2 + 42a^3b^5d^5e^3 + 210a^4b^4d^4e^4 - 1918a^5b^3d^3e^5 + 3654a^6b^2d^2e^6 - 2754a^7bd^2e^7 + 743a^8e^8 + 1050(b^8d^4e^4 - 4a^3b^7d^3e^5 + 6a^2b^6d^2e^6 - 4a^3b^5de^7 + a^4b^4e^8)x^4 + 420(b^8d^5e^3 + 5a^3b^7d^4e^4 - 30a^2b^6d^3e^5 + 50a^3b^5d^2e^6 - 35a^4b^4de^7 + 9a^5b^3e^8)x^3 + 140(b^8d^6e^2 + 3a^3b^7d^5e^3 + 15a^2b^6d^4e^4 - 110a^3b^5d^3e^5 + 195a^4b^4d^2e^6 - 141a^5b^3de^7 + 37a^6b^2e^8)x^2 + 10(3b^8d^7e + 7a^3b^7d^6e^2 + 21a^2b^6d^5e^3 + 105a^3b^5d^4e^4 - 875a^4b^4d^3e^5 + 1617a^5b^3d^2e^6 - 1197a^6b^2de^7 + 319a^7be^8)x}{3b^{18}} + \frac{b^{12}e^8x^3 + 12b^{12}de^7x^2 - 9ab^{11}e^8x^2 + 84b^{12}d^2e^6x - 144ab^{11}de^7x + 63a^2b^{10}e^8x}{3b^{18}}$$

input

```
integrate((e*x+d)^8/(b^2*x^2+2*a*b*x+a^2)^3,x, algorithm="giac")
```

output

```
56*(b^3*d^3*e^5 - 3*a*b^2*d^2*e^6 + 3*a^2*b*d*e^7 - a^3*e^8)*log(abs(b*x +
a))/b^9 - 1/15*(3*b^8*d^8 + 6*a*b^7*d^7*e + 14*a^2*b^6*d^6*e^2 + 42*a^3*b
^5*d^5*e^3 + 210*a^4*b^4*d^4*e^4 - 1918*a^5*b^3*d^3*e^5 + 3654*a^6*b^2*d^2
*e^6 - 2754*a^7*b*d*e^7 + 743*a^8*e^8 + 1050*(b^8*d^4*e^4 - 4*a*b^7*d^3*e^
5 + 6*a^2*b^6*d^2*e^6 - 4*a^3*b^5*d*e^7 + a^4*b^4*e^8)*x^4 + 420*(b^8*d^5*
e^3 + 5*a*b^7*d^4*e^4 - 30*a^2*b^6*d^3*e^5 + 50*a^3*b^5*d^2*e^6 - 35*a^4*b
^4*d*e^7 + 9*a^5*b^3*e^8)*x^3 + 140*(b^8*d^6*e^2 + 3*a*b^7*d^5*e^3 + 15*a^
2*b^6*d^4*e^4 - 110*a^3*b^5*d^3*e^5 + 195*a^4*b^4*d^2*e^6 - 141*a^5*b^3*d*
e^7 + 37*a^6*b^2*e^8)*x^2 + 10*(3*b^8*d^7*e + 7*a*b^7*d^6*e^2 + 21*a^2*b^6
*d^5*e^3 + 105*a^3*b^5*d^4*e^4 - 875*a^4*b^4*d^3*e^5 + 1617*a^5*b^3*d^2*e^
6 - 1197*a^6*b^2*d*e^7 + 319*a^7*b*e^8)*x)/((b*x + a)^5*b^9) + 1/3*(b^12*e
^8*x^3 + 12*b^12*d*e^7*x^2 - 9*a*b^11*e^8*x^2 + 84*b^12*d^2*e^6*x - 144*a*
b^11*d*e^7*x + 63*a^2*b^10*e^8*x)/b^18
```

Mupad [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 644, normalized size of antiderivative = 3.04

$$\int \frac{(d+ex)^8}{(a^2+2abx+b^2x^2)^3} dx = x \left(\frac{6a \left(\frac{6ae^8}{b^7} - \frac{8de^7}{b^6} \right)}{b} - \frac{15a^2e^8}{b^8} + \frac{28d^2e^6}{b^6} \right) \\ - x^4 (70a^4b^3e^8 - 280a^3b^4de^7 + 420a^2b^5d^2e^6 - 280ab^6d^3e^5 + 70b^7d^4e^4) + \frac{743a^8e^8 - 2754a^7bde^7 + 3654a^6b^2d^2e^6 + 6a^5b^3d^3e^5 + 3654a^4b^4d^4e^4 - 2754a^3b^5d^5e^3 + 210a^2b^6d^6e^2 + 42ab^7d^7e - 743a^8e^8}{15b} \\ - x^2 \left(\frac{3ae^8}{b^7} - \frac{4de^7}{b^6} \right) - \frac{\ln(a+bx) (56a^3e^8 - 168a^2bde^7 + 168ab^2d^2e^6 - 56b^3d^3e^5)}{b^9} + \frac{e^8x^3}{3b^6}$$

input

```
int((d + e*x)^8/(a^2 + b^2*x^2 + 2*a*b*x)^3,x)
```

output

```
x*((6*a*((6*a*e^8)/b^7 - (8*d*e^7)/b^6))/b - (15*a^2*e^8)/b^8 + (28*d^2*e^6)/b^6) - (x^4*(70*a^4*b^3*e^8 + 70*b^7*d^4*e^4 - 280*a*b^6*d^3*e^5 - 280*a^3*b^4*d*e^7 + 420*a^2*b^5*d^2*e^6) + (743*a^8*e^8 + 3*b^8*d^8 + 14*a^2*b^6*d^6*e^2 + 42*a^3*b^5*d^5*e^3 + 210*a^4*b^4*d^4*e^4 - 1918*a^5*b^3*d^3*e^5 + 3654*a^6*b^2*d^2*e^6 + 6*a*b^7*d^7*e - 2754*a^7*b*d*e^7))/(15*b) + x*((638*a^7*e^8)/3 + 2*b^7*d^7*e + (14*a*b^6*d^6*e^2)/3 + 14*a^2*b^5*d^5*e^3 + 70*a^3*b^4*d^4*e^4 - (1750*a^4*b^3*d^3*e^5)/3 + 1078*a^5*b^2*d^2*e^6 - 798*a^6*b*d*e^7) + x^3*(252*a^5*b^2*e^8 + 28*b^7*d^5*e^3 + 140*a*b^6*d^4*e^4 - 980*a^4*b^3*d*e^7 - 840*a^2*b^5*d^3*e^5 + 1400*a^3*b^4*d^2*e^6) + x^2*((1036*a^6*b*e^8)/3 + (28*b^7*d^6*e^2)/3 + 28*a*b^6*d^5*e^3 - 1316*a^5*b^2*d*e^7 + 140*a^2*b^5*d^4*e^4 - (3080*a^3*b^4*d^3*e^5)/3 + 1820*a^4*b^3*d^2*e^6))/(a^5*b^8 + b^13*x^5 + 5*a^4*b^9*x + 5*a*b^12*x^4 + 10*a^3*b^10*x^2 + 10*a^2*b^11*x^3) - x^2*((3*a*e^8)/b^7 - (4*d*e^7)/b^6) - (log(a + b*x)*(56*a^3*e^8 - 56*b^3*d^3*e^5 + 168*a*b^2*d^2*e^6 - 168*a^2*b*d*e^7))/b^9 + (e^8*x^3)/(3*b^6)
```

Reduce [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 1073, normalized size of antiderivative = 5.06

$$\int \frac{(d + ex)^8}{(a^2 + 2abx + b^2x^2)^3} dx = \text{Too large to display}$$

input `int((e*x+d)^8/(b^2*x^2+2*a*b*x+a^2)^3,x)`

output

```
( - 840*log(a + b*x)*a**9*e**8 + 2520*log(a + b*x)*a**8*b*d*e**7 - 4200*log(a + b*x)*a**8*b*e**8*x - 2520*log(a + b*x)*a**7*b**2*d**2*e**6 + 12600*log(a + b*x)*a**7*b**2*d*e**7*x - 8400*log(a + b*x)*a**7*b**2*e**8*x**2 + 840*log(a + b*x)*a**6*b**3*d**3*e**5 - 12600*log(a + b*x)*a**6*b**3*d**2*e**6*x + 25200*log(a + b*x)*a**6*b**3*d*e**7*x**2 - 8400*log(a + b*x)*a**6*b**3*e**8*x**3 + 4200*log(a + b*x)*a**5*b**4*d**3*e**5*x - 25200*log(a + b*x)*a**5*b**4*d**2*e**6*x**2 + 25200*log(a + b*x)*a**5*b**4*d*e**7*x**3 - 4200*log(a + b*x)*a**5*b**4*e**8*x**4 + 8400*log(a + b*x)*a**4*b**5*d**3*e**5*x**2 - 25200*log(a + b*x)*a**4*b**5*d**2*e**6*x**3 + 12600*log(a + b*x)*a**4*b**5*d*e**7*x**4 - 840*log(a + b*x)*a**4*b**5*e**8*x**5 + 8400*log(a + b*x)*a**3*b**6*d**3*e**5*x**3 - 12600*log(a + b*x)*a**3*b**6*d**2*e**6*x**4 + 2520*log(a + b*x)*a**3*b**6*d*e**7*x**5 + 4200*log(a + b*x)*a**2*b**7*d**3*e**5*x**4 - 2520*log(a + b*x)*a**2*b**7*d**2*e**6*x**5 + 840*log(a + b*x)*a*b**8*d**3*e**5*x**5 - 1078*a**9*e**8 + 3234*a**8*b*d*e**7 - 4550*a**8*b*e**8*x - 3234*a**7*b**2*d**2*e**6 + 13650*a**7*b**2*d*e**7*x - 7000*a**7*b**2*e**8*x**2 + 1078*a**6*b**3*d**3*e**5 - 13650*a**6*b**3*d**2*e**6*x + 21000*a**6*b**3*d*e**7*x**2 - 4200*a**6*b**3*e**8*x**3 + 4550*a**5*b**4*d**3*e**5*x - 21000*a**5*b**4*d**2*e**6*x**2 + 12600*a**5*b**4*d*e**7*x**3 - 42*a**4*b**5*d**5*e**3 + 7000*a**4*b**5*d**3*e**5*x**2 - 12600*a**4*b**5*d**2*e**6*x**3 + 840*a**4*b**5*e**8*x**5 - 14*a**3*b**6*d**6*e**...
```

3.129
$$\int \frac{(d+ex)^7}{(a^2+2abx+b^2x^2)^3} dx$$

| | |
|---|------|
| Optimal result | 1034 |
| Mathematica [B] (verified) | 1035 |
| Rubi [A] (verified) | 1035 |
| Maple [B] (verified) | 1037 |
| Fricas [B] (verification not implemented) | 1038 |
| Sympy [B] (verification not implemented) | 1038 |
| Maxima [B] (verification not implemented) | 1039 |
| Giac [B] (verification not implemented) | 1040 |
| Mupad [B] (verification not implemented) | 1041 |
| Reduce [B] (verification not implemented) | 1042 |

Optimal result

Integrand size = 26, antiderivative size = 181

$$\int \frac{(d+ex)^7}{(a^2+2abx+b^2x^2)^3} dx = \frac{e^6(7bd-6ae)x}{b^7} + \frac{e^7x^2}{2b^6} - \frac{(bd-ae)^7}{5b^8(a+bx)^5} - \frac{7e(bd-ae)^6}{4b^8(a+bx)^4} - \frac{7e^2(bd-ae)^5}{b^8(a+bx)^3} - \frac{35e^3(bd-ae)^4}{2b^8(a+bx)^2} - \frac{35e^4(bd-ae)^3}{b^8(a+bx)} + \frac{21e^5(bd-ae)^2 \log(a+bx)}{b^8}$$

output

```
e^6*(-6*a*e+7*b*d)*x/b^7+1/2*e^7*x^2/b^6-1/5*(-a*e+b*d)^7/b^8/(b*x+a)^5-7/4*e*(-a*e+b*d)^6/b^8/(b*x+a)^4-7*e^2*(-a*e+b*d)^5/b^8/(b*x+a)^3-35/2*e^3*(-a*e+b*d)^4/b^8/(b*x+a)^2-35*e^4*(-a*e+b*d)^3/b^8/(b*x+a)+21*e^5*(-a*e+b*d)^2*ln(b*x+a)/b^8
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 389 vs. $2(181) = 362$.

Time = 0.11 (sec) , antiderivative size = 389, normalized size of antiderivative = 2.15

$$\int \frac{(d+ex)^7}{(a^2+2abx+b^2x^2)^3} dx$$

$$= \frac{459a^7e^7 + 3a^6be^6(-406d + 625ex) + a^5b^2e^5(959d^2 - 5250dex + 2700e^2x^2) + 5a^4b^3e^4(-28d^3 + 875d^2ex$$

input `Integrate[(d + e*x)^7/(a^2 + 2*a*b*x + b^2*x^2)^3,x]`

output $(459*a^7*e^7 + 3*a^6*b*e^6*(-406*d + 625*e*x) + a^5*b^2*e^5*(959*d^2 - 5250*d*e*x + 2700*e^2*x^2) + 5*a^4*b^3*e^4*(-28*d^3 + 875*d^2*e*x - 1680*d*e^2*x^2 + 260*e^3*x^3) - 5*a^3*b^4*e^3*(7*d^4 + 140*d^3*e*x - 1540*d^2*e^2*x^2 + 1120*d*e^3*x^3 + 80*e^4*x^4) - a^2*b^5*e^2*(14*d^5 + 175*d^4*e*x + 1400*d^3*e^2*x^2 - 6300*d^2*e^3*x^3 + 700*d*e^4*x^4 + 500*e^5*x^5) - 7*a*b^6*e*(d^6 + 10*d^5*e*x + 50*d^4*e^2*x^2 + 200*d^3*e^3*x^3 - 300*d^2*e^4*x^4 - 100*d*e^5*x^5 + 10*e^6*x^6) - b^7*(4*d^7 + 35*d^6*e*x + 140*d^5*e^2*x^2 + 350*d^4*e^3*x^3 + 700*d^3*e^4*x^4 - 140*d*e^6*x^6 - 10*e^7*x^7) + 420*e^5*(b*d - a*e)^2*(a + b*x)^5*Log[a + b*x])/(20*b^8*(a + b*x)^5)$

Rubi [A] (verified)

Time = 0.79 (sec) , antiderivative size = 181, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {1098, 27, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex)^7}{(a^2+2abx+b^2x^2)^3} dx$$

$$\downarrow 1098$$

$$b^6 \int \frac{(d+ex)^7}{b^6(a+bx)^6} dx$$

$$\int \frac{(d+ex)^7}{(a+bx)^6} dx$$

$$\int \left(\frac{e^6(7bd-6ae)}{b^7} + \frac{21e^5(bd-ae)^2}{b^7(a+bx)} + \frac{35e^4(bd-ae)^3}{b^7(a+bx)^2} + \frac{35e^3(bd-ae)^4}{b^7(a+bx)^3} + \frac{21e^2(bd-ae)^5}{b^7(a+bx)^4} + \frac{7e(bd-ae)^6}{b^7(a+bx)^5} + \frac{e^7}{b^7} \right) dx$$

$$\frac{21e^5(bd-ae)^2 \log(a+bx)}{b^8} - \frac{35e^4(bd-ae)^3}{b^8(a+bx)} - \frac{35e^3(bd-ae)^4}{2b^8(a+bx)^2} - \frac{7e^2(bd-ae)^5}{b^8(a+bx)^3} - \frac{7e(bd-ae)^6}{4b^8(a+bx)^4} - \frac{(bd-ae)^7}{5b^8(a+bx)^5} + \frac{e^6x(7bd-6ae)}{b^7} + \frac{e^7x^2}{2b^6}$$

input `Int[(d + e*x)^7/(a^2 + 2*a*b*x + b^2*x^2)^3,x]`

output `(e^6*(7*b*d - 6*a*e)*x)/b^7 + (e^7*x^2)/(2*b^6) - (b*d - a*e)^7/(5*b^8*(a + b*x)^5) - (7*e*(b*d - a*e)^6)/(4*b^8*(a + b*x)^4) - (7*e^2*(b*d - a*e)^5)/(b^8*(a + b*x)^3) - (35*e^3*(b*d - a*e)^4)/(2*b^8*(a + b*x)^2) - (35*e^4*(b*d - a*e)^3)/(b^8*(a + b*x)) + (21*e^5*(b*d - a*e)^2*Log[a + b*x])/b^8`

Defintions of rubi rules used

rule 27 `Int[(a_)*(F_x_), x_Symbol] := Simp[a Int[F_x, x], x] /; FreeQ[a, x] && !MatchQ[F_x, (b_)*(G_x_)] /; FreeQ[b, x]`

rule 49 `Int[((a_) + (b_)*(x_))^(m_)*((c_) + (d_)*(x_))^(n_), x_Symbol] := Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] && IGtQ[m, 0] && IGtQ[m + n + 2, 0]`

rule 1098 `Int[((d_) + (e_)*(x_))^(m_)*((a_) + (b_)*(x_) + (c_)*(x_)^2)^(p_), x_Symbol] := Simp[1/c^p Int[(d + e*x)^m*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]`

```
rule 2009 Int[u_, x_Symbol] := Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 450 vs. 2(173) = 346.

Time = 1.12 (sec) , antiderivative size = 451, normalized size of antiderivative = 2.49

| method | result |
|--------------|--|
| default | $-\frac{e^6(-\frac{1}{2}be^7x^2+6acx-7bdx)}{b^7} - \frac{-a^7e^7+7a^6bd^2e^6-21a^5b^2d^2e^5+35b^3d^3e^4a^4-35b^4d^4e^3a^3+21b^5d^5e^2a^2-7b^6d^6ea+b^7d^7}{5b^8(bx+a)^5} +$ |
| norman | $\frac{959a^7e^7-1918a^6bd^2e^6+959a^5b^2d^2e^5-140b^3d^3e^4a^4-35b^4d^4e^3a^3-14b^5d^5e^2a^2-7b^6d^6ea-4b^7d^7}{20b^8} + \frac{e^7x^7}{2b} + \frac{5(21a^3e^7-42a^2bd^2e^6+21a^2b^2d^2e^5-7a^2b^3d^3e^4+7a^2b^4d^4e^3-7a^2b^5d^5e^2+7a^2b^6d^6ea+b^7d^7)}{b^4}$ |
| risch | $\frac{e^7x^2}{2b^6} - \frac{6e^7ax}{b^7} + \frac{7e^6dx}{b^6} + \frac{(35a^3b^3e^7-105a^2b^4de^6+105ab^5d^2e^5-35b^6d^3e^4)x^4 + \frac{35b^2e^3(7a^4e^4-20a^3bd^2e^3+18a^2b^2d^2e^2-4a^2b^3d^3e+4a^2b^4d^4e-4a^2b^5d^5e+4a^2b^6d^6ea+b^7d^7)}{2}}{2}$ |
| parallelrisc | $\frac{-4b^7d^7-15400a^4b^3de^6x^2-8750a^5b^2de^6x-4200a^2b^5de^6x^4-12600a^3b^4de^6x^3-1918a^6bd^2e^6+959a^5b^2d^2e^5+2100x^4ab^6d^2e^5}{2}$ |

```
input int((e*x+d)^7/(b^2*x^2+2*a*b*x+a^2)^3,x,method=_RETURNVERBOSE)
```

```
output -e^6/b^7*(-1/2*b*e*x^2+6*a*e*x-7*b*d*x)-1/5/b^8*(-a^7*e^7+7*a^6*b*d*e^6-21*a^5*b^2*d^2*e^5+35*a^4*b^3*d^3*e^4-35*a^3*b^4*d^4*e^3+21*a^2*b^5*d^5*e^2-7*a*b^6*d^6*e+b^7*d^7)/(b*x+a)^5+7/b^8*e^2*(a^5*e^5-5*a^4*b*d*e^4+10*a^3*b^2*d^2*e^3-10*a^2*b^3*d^3*e^2+5*a*b^4*d^4*e-b^5*d^5)/(b*x+a)^3+35/b^8*e^4*(a^3*e^3-3*a^2*b*d*e^2+3*a*b^2*d^2*e-b^3*d^3)/(b*x+a)+21/b^8*e^5*(a^2*e^2-2*a*b*d*e+b^2*d^2)*ln(b*x+a)-35/2/b^8*e^3*(a^4*e^4-4*a^3*b*d*e^3+6*a^2*b^2*d^2*e^2-4*a*b^3*d^3*e+b^4*d^4)/(b*x+a)^2-7/4/b^8*e*(a^6*e^6-6*a^5*b*d*e^5+15*a^4*b^2*d^2*e^4-20*a^3*b^3*d^3*e^3+15*a^2*b^4*d^4*e^2-6*a*b^5*d^5*e+b^6*d^6)/(b*x+a)^4
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 732 vs. $2(173) = 346$.

Time = 0.09 (sec) , antiderivative size = 732, normalized size of antiderivative = 4.04

$$\int \frac{(d + ex)^7}{(a^2 + 2abx + b^2x^2)^3} dx$$

$$= \frac{10 b^7 e^7 x^7 - 4 b^7 d^7 - 7 a b^6 d^6 e - 14 a^2 b^5 d^5 e^2 - 35 a^3 b^4 d^4 e^3 - 140 a^4 b^3 d^3 e^4 + 959 a^5 b^2 d^2 e^5 - 1218 a^6 b d e^6 + \dots}{(a^2 + 2abx + b^2x^2)^3}$$

input `integrate((e*x+d)^7/(b^2*x^2+2*a*b*x+a^2)^3,x, algorithm="fricas")`

output

```
1/20*(10*b^7*e^7*x^7 - 4*b^7*d^7 - 7*a*b^6*d^6*e - 14*a^2*b^5*d^5*e^2 - 35
*a^3*b^4*d^4*e^3 - 140*a^4*b^3*d^3*e^4 + 959*a^5*b^2*d^2*e^5 - 1218*a^6*b*
d*e^6 + 459*a^7*e^7 + 70*(2*b^7*d*e^6 - a*b^6*e^7)*x^6 + 100*(7*a*b^6*d*e^
6 - 5*a^2*b^5*e^7)*x^5 - 100*(7*b^7*d^3*e^4 - 21*a*b^6*d^2*e^5 + 7*a^2*b^5
*d*e^6 + 4*a^3*b^4*e^7)*x^4 - 50*(7*b^7*d^4*e^3 + 28*a*b^6*d^3*e^4 - 126*a
^2*b^5*d^2*e^5 + 112*a^3*b^4*d*e^6 - 26*a^4*b^3*e^7)*x^3 - 10*(14*b^7*d^5*
e^2 + 35*a*b^6*d^4*e^3 + 140*a^2*b^5*d^3*e^4 - 770*a^3*b^4*d^2*e^5 + 840*a
^4*b^3*d*e^6 - 270*a^5*b^2*e^7)*x^2 - 5*(7*b^7*d^6*e + 14*a*b^6*d^5*e^2 +
35*a^2*b^5*d^4*e^3 + 140*a^3*b^4*d^3*e^4 - 875*a^4*b^3*d^2*e^5 + 1050*a^5*
b^2*d*e^6 - 375*a^6*b*e^7)*x + 420*(a^5*b^2*d^2*e^5 - 2*a^6*b*d*e^6 + a^7*
e^7 + (b^7*d^2*e^5 - 2*a*b^6*d*e^6 + a^2*b^5*e^7)*x^5 + 5*(a*b^6*d^2*e^5 -
2*a^2*b^5*d*e^6 + a^3*b^4*e^7)*x^4 + 10*(a^2*b^5*d^2*e^5 - 2*a^3*b^4*d*e^
6 + a^4*b^3*e^7)*x^3 + 10*(a^3*b^4*d^2*e^5 - 2*a^4*b^3*d*e^6 + a^5*b^2*e^7
)*x^2 + 5*(a^4*b^3*d^2*e^5 - 2*a^5*b^2*d*e^6 + a^6*b*e^7)*x)*log(b*x + a)
/(b^13*x^5 + 5*a*b^12*x^4 + 10*a^2*b^11*x^3 + 10*a^3*b^10*x^2 + 5*a^4*b^9*
x + a^5*b^8)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 524 vs. $2(168) = 336$.

Time = 64.30 (sec) , antiderivative size = 524, normalized size of antiderivative = 2.90

$$\int \frac{(d+ex)^7}{(a^2+2abx+b^2x^2)^3} dx = x \left(-\frac{6ae^7}{b^7} + \frac{7de^6}{b^6} \right) + \frac{459a^7e^7 - 1218a^6bde^6 + 959a^5b^2d^2e^5 - 140a^4b^3d^3e^4 - 35a^3b^4d^4e^3 - 14a^2b^5d^5e^2 - 7ab^6d^6e - 4b^7d^7}{b^8} + \frac{e^7x^2}{2b^6} + \frac{21e^5(ae-bd)^2 \log(a+bx)}{b^8}$$

input

```
integrate((e*x+d)**7/(b**2*x**2+2*a*b*x+a**2)**3,x)
```

output

```
x*(-6*a*e**7/b**7 + 7*d*e**6/b**6) + (459*a**7*e**7 - 1218*a**6*b*d*e**6 +
959*a**5*b**2*d**2*e**5 - 140*a**4*b**3*d**3*e**4 - 35*a**3*b**4*d**4*e**
3 - 14*a**2*b**5*d**5*e**2 - 7*a*b**6*d**6*e - 4*b**7*d**7 + x**4*(700*a**
3*b**4*e**7 - 2100*a**2*b**5*d*e**6 + 2100*a*b**6*d**2*e**5 - 700*b**7*d**
3*e**4) + x**3*(2450*a**4*b**3*e**7 - 7000*a**3*b**4*d*e**6 + 6300*a**2*b*
*5*d**2*e**5 - 1400*a*b**6*d**3*e**4 - 350*b**7*d**4*e**3) + x**2*(3290*a*
*5*b**2*e**7 - 9100*a**4*b**3*d*e**6 + 7700*a**3*b**4*d**2*e**5 - 1400*a**
2*b**5*d**3*e**4 - 350*a*b**6*d**4*e**3 - 140*b**7*d**5*e**2) + x*(1995*a*
*6*b*e**7 - 5390*a**5*b**2*d*e**6 + 4375*a**4*b**3*d**2*e**5 - 700*a**3*b*
*4*d**3*e**4 - 175*a**2*b**5*d**4*e**3 - 70*a*b**6*d**5*e**2 - 35*b**7*d**
6*e))/(20*a**5*b**8 + 100*a**4*b**9*x + 200*a**3*b**10*x**2 + 200*a**2*b**
11*x**3 + 100*a*b**12*x**4 + 20*b**13*x**5) + e**7*x**2/(2*b**6) + 21*e**5
*(a*e - b*d)**2*log(a + b*x)/b**8
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 504 vs. $2(173) = 346$.

Time = 0.07 (sec) , antiderivative size = 504, normalized size of antiderivative = 2.78

$$\int \frac{(d+ex)^7}{(a^2+2abx+b^2x^2)^3} dx = \frac{4b^7d^7 + 7ab^6d^6e + 14a^2b^5d^5e^2 + 35a^3b^4d^4e^3 + 140a^4b^3d^3e^4 - 959a^5b^2d^2e^5 + 1218a^6bde^6 - 459a^7e^7}{b^8} + \frac{be^7x^2 + 2(7bde^6 - 6ae^7)x}{2b^7} + \frac{21(b^2d^2e^5 - 2abde^6 + a^2e^7) \log(bx+a)}{b^8}$$

input `integrate((e*x+d)^7/(b^2*x^2+2*a*b*x+a^2)^3,x, algorithm="maxima")`

output

$$\begin{aligned}
 & -1/20*(4*b^7*d^7 + 7*a*b^6*d^6*e + 14*a^2*b^5*d^5*e^2 + 35*a^3*b^4*d^4*e^3 \\
 & + 140*a^4*b^3*d^3*e^4 - 959*a^5*b^2*d^2*e^5 + 1218*a^6*b*d*e^6 - 459*a^7* \\
 & e^7 + 700*(b^7*d^3*e^4 - 3*a*b^6*d^2*e^5 + 3*a^2*b^5*d*e^6 - a^3*b^4*e^7)* \\
 & x^4 + 350*(b^7*d^4*e^3 + 4*a*b^6*d^3*e^4 - 18*a^2*b^5*d^2*e^5 + 20*a^3*b^4 \\
 & *d*e^6 - 7*a^4*b^3*e^7)*x^3 + 70*(2*b^7*d^5*e^2 + 5*a*b^6*d^4*e^3 + 20*a^2 \\
 & *b^5*d^3*e^4 - 110*a^3*b^4*d^2*e^5 + 130*a^4*b^3*d*e^6 - 47*a^5*b^2*e^7)*x \\
 & ^2 + 35*(b^7*d^6*e + 2*a*b^6*d^5*e^2 + 5*a^2*b^5*d^4*e^3 + 20*a^3*b^4*d^3* \\
 & e^4 - 125*a^4*b^3*d^2*e^5 + 154*a^5*b^2*d*e^6 - 57*a^6*b*e^7)*x)/(b^13*x^5 \\
 & + 5*a*b^12*x^4 + 10*a^2*b^11*x^3 + 10*a^3*b^10*x^2 + 5*a^4*b^9*x + a^5*b^8) \\
 & + 1/2*(b*e^7*x^2 + 2*(7*b*d*e^6 - 6*a*e^7)*x)/b^7 + 21*(b^2*d^2*e^5 - 2 \\
 & *a*b*d*e^6 + a^2*e^7)*\log(b*x + a)/b^8
 \end{aligned}$$

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 463 vs. $2(173) = 346$.

Time = 0.16 (sec) , antiderivative size = 463, normalized size of antiderivative = 2.56

$$\begin{aligned}
 & \int \frac{(d+ex)^7}{(a^2+2abx+b^2x^2)^3} dx \\
 & = \frac{21(b^2d^2e^5 - 2abde^6 + a^2e^7) \log(|bx+a|)}{b^8} + \frac{b^6e^7x^2 + 14b^6de^6x - 12ab^5e^7x}{2b^{12}} \\
 & \quad - \frac{4b^7d^7 + 7ab^6d^6e + 14a^2b^5d^5e^2 + 35a^3b^4d^4e^3 + 140a^4b^3d^3e^4 - 959a^5b^2d^2e^5 + 1218a^6bde^6 - 459a^7e^7}{b^8}
 \end{aligned}$$

input `integrate((e*x+d)^7/(b^2*x^2+2*a*b*x+a^2)^3,x, algorithm="giac")`

output

```

21*(b^2*d^2*e^5 - 2*a*b*d*e^6 + a^2*e^7)*log(abs(b*x + a))/b^8 + 1/2*(b^6*
e^7*x^2 + 14*b^6*d*e^6*x - 12*a*b^5*e^7*x)/b^12 - 1/20*(4*b^7*d^7 + 7*a*b^
6*d^6*e + 14*a^2*b^5*d^5*e^2 + 35*a^3*b^4*d^4*e^3 + 140*a^4*b^3*d^3*e^4 -
959*a^5*b^2*d^2*e^5 + 1218*a^6*b*d*e^6 - 459*a^7*e^7 + 700*(b^7*d^3*e^4 -
3*a*b^6*d^2*e^5 + 3*a^2*b^5*d*e^6 - a^3*b^4*e^7)*x^4 + 350*(b^7*d^4*e^3 +
4*a*b^6*d^3*e^4 - 18*a^2*b^5*d^2*e^5 + 20*a^3*b^4*d*e^6 - 7*a^4*b^3*e^7)*x
^3 + 70*(2*b^7*d^5*e^2 + 5*a*b^6*d^4*e^3 + 20*a^2*b^5*d^3*e^4 - 110*a^3*b^
4*d^2*e^5 + 130*a^4*b^3*d*e^6 - 47*a^5*b^2*e^7)*x^2 + 35*(b^7*d^6*e + 2*a*
b^6*d^5*e^2 + 5*a^2*b^5*d^4*e^3 + 20*a^3*b^4*d^3*e^4 - 125*a^4*b^3*d^2*e^5
+ 154*a^5*b^2*d*e^6 - 57*a^6*b*e^7)*x)/((b*x + a)^5*b^8)

```

Mupad [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 508, normalized size of antiderivative = 2.81

$$\int \frac{(d+ex)^7}{(a^2+2abx+b^2x^2)^3} dx = \frac{e^7 x^2}{2b^6} - \frac{-459a^7e^7+1218a^6bde^6-959a^5b^2d^2e^5+140a^4b^3d^3e^4+35a^3b^4d^4e^3+14a^2b^5d^5e^2+7ab^6d^6e+4b^7d^7}{20b} + x \left(-\frac{399a^6e^7}{4} + \frac{539a^5}{2} \right) - x \left(\frac{6ae^7}{b^7} - \frac{7de^6}{b^6} \right) + \frac{\ln(a+bx)(21a^2e^7-42abd e^6+21b^2d^2e^5)}{b^8}$$

input

```
int((d + e*x)^7/(a^2 + b^2*x^2 + 2*a*b*x)^3,x)
```

output

```

(e^7*x^2)/(2*b^6) - ((4*b^7*d^7 - 459*a^7*e^7 + 14*a^2*b^5*d^5*e^2 + 35*a^
3*b^4*d^4*e^3 + 140*a^4*b^3*d^3*e^4 - 959*a^5*b^2*d^2*e^5 + 7*a*b^6*d^6*e
+ 1218*a^6*b*d*e^6)/(20*b) + x*((7*b^6*d^6*e)/4 - (399*a^6*e^7)/4 + (7*a*b
^5*d^5*e^2)/2 + (35*a^2*b^4*d^4*e^3)/4 + 35*a^3*b^3*d^3*e^4 - (875*a^4*b^2
*d^2*e^5)/4 + (539*a^5*b*d*e^6)/2) + x^3*((35*b^6*d^4*e^3)/2 - (245*a^4*b^
2*e^7)/2 + 70*a*b^5*d^3*e^4 + 350*a^3*b^3*d*e^6 - 315*a^2*b^4*d^2*e^5) + x
^2*(7*b^6*d^5*e^2 - (329*a^5*b*e^7)/2 + (35*a*b^5*d^4*e^3)/2 + 455*a^4*b^2
*d*e^6 + 70*a^2*b^4*d^3*e^4 - 385*a^3*b^3*d^2*e^5) - x^4*(35*a^3*b^3*e^7 -
35*b^6*d^3*e^4 + 105*a*b^5*d^2*e^5 - 105*a^2*b^4*d*e^6)/(a^5*b^7 + b^12*
x^5 + 5*a^4*b^8*x + 5*a*b^11*x^4 + 10*a^3*b^9*x^2 + 10*a^2*b^10*x^3) - x*(
(6*a*e^7)/b^7 - (7*d*e^6)/b^6) + (log(a + b*x)*(21*a^2*e^7 + 21*b^2*d^2*e^
5 - 42*a*b*d*e^6))/b^8

```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 816, normalized size of antiderivative = 4.51

$$\int \frac{(d + ex)^7}{(a^2 + 2abx + b^2x^2)^3} dx = \text{Too large to display}$$

input `int((e*x+d)^7/(b^2*x^2+2*a*b*x+a^2)^3,x)`

output

```
(420*log(a + b*x)*a**8*e**7 - 840*log(a + b*x)*a**7*b*d*e**6 + 2100*log(a
+ b*x)*a**7*b*e**7*x + 420*log(a + b*x)*a**6*b**2*d**2*e**5 - 4200*log(a +
b*x)*a**6*b**2*d*e**6*x + 4200*log(a + b*x)*a**6*b**2*e**7*x**2 + 2100*log
(a + b*x)*a**5*b**3*d**2*e**5*x - 8400*log(a + b*x)*a**5*b**3*d*e**6*x**2
+ 4200*log(a + b*x)*a**5*b**3*e**7*x**3 + 4200*log(a + b*x)*a**4*b**4*d**
2*e**5*x**2 - 8400*log(a + b*x)*a**4*b**4*d*e**6*x**3 + 2100*log(a + b*x)*
a**4*b**4*e**7*x**4 + 4200*log(a + b*x)*a**3*b**5*d**2*e**5*x**3 - 4200*log
(a + b*x)*a**3*b**5*d*e**6*x**4 + 420*log(a + b*x)*a**3*b**5*e**7*x**5 +
2100*log(a + b*x)*a**2*b**6*d**2*e**5*x**4 - 840*log(a + b*x)*a**2*b**6*d
e**6*x**5 + 420*log(a + b*x)*a*b**7*d**2*e**5*x**5 + 539*a**8*e**7 - 1078*
a**7*b*d*e**6 + 2275*a**7*b*e**7*x + 539*a**6*b**2*d**2*e**5 - 4550*a**6*b
**2*d*e**6*x + 3500*a**6*b**2*e**7*x**2 + 2275*a**5*b**3*d**2*e**5*x - 700
0*a**5*b**3*d*e**6*x**2 + 2100*a**5*b**3*e**7*x**3 - 35*a**4*b**4*d**4*e**
3 + 3500*a**4*b**4*d**2*e**5*x**2 - 4200*a**4*b**4*d*e**6*x**3 - 14*a**3*b
**5*d**5*e**2 - 175*a**3*b**5*d**4*e**3*x + 2100*a**3*b**5*d**2*e**5*x**3
- 420*a**3*b**5*e**7*x**5 - 7*a**2*b**6*d**6*e - 70*a**2*b**6*d**5*e**2*x
- 350*a**2*b**6*d**4*e**3*x**2 + 840*a**2*b**6*d*e**6*x**5 - 70*a**2*b**6*
e**7*x**6 - 4*a*b**7*d**7 - 35*a*b**7*d**6*e*x - 140*a*b**7*d**5*e**2*x**2
- 350*a*b**7*d**4*e**3*x**3 - 420*a*b**7*d**2*e**5*x**5 + 140*a*b**7*d*e
**6*x**6 + 10*a*b**7*e**7*x**7 + 140*b**8*d**3*e**4*x**5)/(20*a*b**8*(a...
```

3.130 $\int \frac{(d+ex)^6}{(a^2+2abx+b^2x^2)^3} dx$

| | |
|---|------|
| Optimal result | 1043 |
| Mathematica [A] (verified) | 1043 |
| Rubi [A] (verified) | 1044 |
| Maple [B] (verified) | 1046 |
| Fricas [B] (verification not implemented) | 1046 |
| Sympy [B] (verification not implemented) | 1047 |
| Maxima [B] (verification not implemented) | 1048 |
| Giac [B] (verification not implemented) | 1048 |
| Mupad [B] (verification not implemented) | 1049 |
| Reduce [B] (verification not implemented) | 1050 |

Optimal result

Integrand size = 26, antiderivative size = 155

$$\int \frac{(d+ex)^6}{(a^2+2abx+b^2x^2)^3} dx = \frac{e^6x}{b^6} - \frac{(bd-ae)^6}{5b^7(a+bx)^5} - \frac{3e(bd-ae)^5}{2b^7(a+bx)^4} - \frac{5e^2(bd-ae)^4}{b^7(a+bx)^3} - \frac{10e^3(bd-ae)^3}{b^7(a+bx)^2} - \frac{15e^4(bd-ae)^2}{b^7(a+bx)} + \frac{6e^5(bd-ae)\log(a+bx)}{b^7}$$

output

```
e^6*x/b^6-1/5*(-a*e+b*d)^6/b^7/(b*x+a)^5-3/2*e*(-a*e+b*d)^5/b^7/(b*x+a)^4-
5*e^2*(-a*e+b*d)^4/b^7/(b*x+a)^3-10*e^3*(-a*e+b*d)^3/b^7/(b*x+a)^2-15*e^4*
(-a*e+b*d)^2/b^7/(b*x+a)+6*e^5*(-a*e+b*d)*ln(b*x+a)/b^7
```

Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 300, normalized size of antiderivative = 1.94

$$\int \frac{(d+ex)^6}{(a^2+2abx+b^2x^2)^3} dx = \frac{87a^6e^6 + a^5be^5(-137d + 375ex) + 5a^4b^2e^4(6d^2 - 125dex + 120e^2x^2) + 10a^3b^3e^3(d^3 + 15d^2ex - 110dex^2 + 35d^2e^2x^2 - 15d^2e^3x^3 + 35d^2e^4x^4 - 15d^2e^5x^5 + 35d^2e^6x^6) + 6a^2b^4e^2(d^4 + 4d^3ex + 6d^2e^2x^2 + 4d^2e^3x^3 + 6d^2e^4x^4 + 4d^2e^5x^5 + 6d^2e^6x^6) + 2ab^5e(d^5 + 5d^4ex + 10d^3e^2x^2 + 10d^3e^3x^3 + 10d^3e^4x^4 + 5d^3e^5x^5 + 6d^3e^6x^6) + b^6e^6(d^6 + 6d^5ex + 15d^4e^2x^2 + 20d^4e^3x^3 + 15d^4e^4x^4 + 6d^4e^5x^5 + 6d^4e^6x^6)}{(a+bx)^7}$$

input `Integrate[(d + e*x)^6/(a^2 + 2*a*b*x + b^2*x^2)^3,x]`

output
$$\begin{aligned} & -1/10*(87*a^6*e^6 + a^5*b*e^5*(-137*d + 375*e*x) + 5*a^4*b^2*e^4*(6*d^2 - \\ & 125*d*e*x + 120*e^2*x^2) + 10*a^3*b^3*e^3*(d^3 + 15*d^2*e*x - 110*d*e^2*x^2 \\ & 2 + 40*e^3*x^3) + 5*a^2*b^4*e^2*(d^4 + 10*d^3*e*x + 60*d^2*e^2*x^2 - 180*d \\ & *e^3*x^3 + 10*e^4*x^4) + a*b^5*e*(3*d^5 + 25*d^4*e*x + 100*d^3*e^2*x^2 + 3 \\ & 00*d^2*e^3*x^3 - 300*d*e^4*x^4 - 50*e^5*x^5) + b^6*(2*d^6 + 15*d^5*e*x + 5 \\ & 0*d^4*e^2*x^2 + 100*d^3*e^3*x^3 + 150*d^2*e^4*x^4 - 10*e^6*x^6) + 60*e^5*(\\ & -(b*d) + a*e)*(a + b*x)^5*\text{Log}[a + b*x])/(b^7*(a + b*x)^5) \end{aligned}$$

Rubi [A] (verified)

Time = 0.65 (sec) , antiderivative size = 155, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {1098, 27, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\begin{aligned} & \int \frac{(d + ex)^6}{(a^2 + 2abx + b^2x^2)^3} dx \\ & \quad \downarrow 1098 \\ & b^6 \int \frac{(d + ex)^6}{b^6(a + bx)^6} dx \\ & \quad \downarrow 27 \\ & \int \frac{(d + ex)^6}{(a + bx)^6} dx \\ & \quad \downarrow 49 \\ & \int \left(\frac{6e^5(bd - ae)}{b^6(a + bx)} + \frac{15e^4(bd - ae)^2}{b^6(a + bx)^2} + \frac{20e^3(bd - ae)^3}{b^6(a + bx)^3} + \frac{15e^2(bd - ae)^4}{b^6(a + bx)^4} + \frac{6e(bd - ae)^5}{b^6(a + bx)^5} + \frac{(bd - ae)^6}{b^6(a + bx)^6} + \frac{e^6}{b^6} \right) dx \\ & \quad \downarrow 2009 \end{aligned}$$

$$\frac{6e^5(bd - ae)\log(a + bx)}{b^7} - \frac{15e^4(bd - ae)^2}{b^7(a + bx)} - \frac{10e^3(bd - ae)^3}{b^7(a + bx)^2} - \frac{5e^2(bd - ae)^4}{b^7(a + bx)^3} - \frac{3e(bd - ae)^5}{2b^7(a + bx)^4} - \frac{(bd - ae)^6}{5b^7(a + bx)^5} + \frac{e^6x}{b^6}$$

input `Int[(d + e*x)^6/(a^2 + 2*a*b*x + b^2*x^2)^3,x]`

output `(e^6*x)/b^6 - (b*d - a*e)^6/(5*b^7*(a + b*x)^5) - (3*e*(b*d - a*e)^5)/(2*b^7*(a + b*x)^4) - (5*e^2*(b*d - a*e)^4)/(b^7*(a + b*x)^3) - (10*e^3*(b*d - a*e)^3)/(b^7*(a + b*x)^2) - (15*e^4*(b*d - a*e)^2)/(b^7*(a + b*x)) + (6*e^5*(b*d - a*e)*Log[a + b*x])/b^7`

Defintions of rubi rules used

rule 27 `Int[(a_)*(F_x_), x_Symbol] := Simp[a Int[F_x, x], x] /; FreeQ[a, x] && !MatchQ[F_x, (b_)*(G_x_)] /; FreeQ[b, x]`

rule 49 `Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] := Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] && IGtQ[m, 0] && IGtQ[m + n + 2, 0]`

rule 1098 `Int[((d_.) + (e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] := Simp[1/c^p Int[(d + e*x)^m*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]`

rule 2009 `Int[u_, x_Symbol] := Simp[IntSum[u, x], x] /; SumQ[u]`

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 348 vs. 2(151) = 302.

Time = 1.11 (sec) , antiderivative size = 349, normalized size of antiderivative = 2.25

| method | result |
|---------------|---|
| default | $\frac{e^6 x}{b^6} - \frac{a^6 e^6 - 6a^5 b d e^5 + 15a^4 b^2 d^2 e^4 - 20a^3 b^3 d^3 e^3 + 15a^2 b^4 d^4 e^2 - 6a b^5 d^5 e + b^6 d^6}{5b^7 (bx+a)^5} - \frac{5e^2 (a^4 e^4 - 4a^3 b d e^3 + 6a^2 b^2 d^2 e^2 - 4a b^3 d^3 e + b^4 d^4)}{b^7 (bx+a)^3}$ |
| norman | $\frac{e^6 x^6}{b} - \frac{137a^6 e^6 - 137a^5 b d e^5 + 30a^4 b^2 d^2 e^4 + 10a^3 b^3 d^3 e^3 + 5a^2 b^4 d^4 e^2 + 3a b^5 d^5 e + 2b^6 d^6}{10b^7} - \frac{5(6a^2 e^6 - 6abd e^5 + 3d^2 e^4 b^2)x^4 - 10(9e^6 a^3 - 9a^2 b d e^5 + 3a^2 b^2 d^2 e^4 - 3a b^3 d^3 e^3 + 3a^2 b^4 d^4 e^2 - 3a b^5 d^5 e + b^6 d^6)}{b^3}$ |
| risch | $\frac{e^6 x}{b^6} + \frac{(-15a^2 b^3 e^6 + 30a b^4 d e^5 - 15d^2 e^4 b^5)x^4 - 10b^2 e^3 (5e^3 a^3 - 9a^2 b d e^2 + 3a b^2 d^2 e + b^3 d^3)x^3 - 5b e^2 (13a^4 e^4 - 22a^3 b d e^3 + 6a^2 b^2 d^2 e^2 - 4a b^3 d^3 e + b^4 d^4)}{b^6}$ |
| parallelrisch | $-\frac{3a b^5 d^5 e - 137a^5 b d e^5 + 10a^3 b^3 d^3 e^3 - 300 \ln(bx+a) x a^4 b^2 d e^5 - 300 \ln(bx+a) x^4 a b^5 d e^5 + 30a^4 b^2 d^2 e^4 + 5a^2 b^4 d^4 e^2 + 2b^6 d^6 - 6a^5 b d e^5 + 30a^4 b^2 d^2 e^4 - 20a^3 b^3 d^3 e^3 + 15a^2 b^4 d^4 e^2 - 6a b^5 d^5 e + b^6 d^6}{b^6}$ |

input

```
int((e*x+d)^6/(b^2*x^2+2*a*b*x+a^2)^3,x,method=_RETURNVERBOSE)
```

output

```
e^6*x/b^6-1/5/b^7*(a^6*e^6-6*a^5*b*d*e^5+15*a^4*b^2*d^2*e^4-20*a^3*b^3*d^3*e^3+15*a^2*b^4*d^4*e^2-6*a*b^5*d^5*e+b^6*d^6)/(b*x+a)^5-5/b^7*e^2*(a^4*e^4-4*a^3*b*d*e^3+6*a^2*b^2*d^2*e^2-4*a*b^3*d^3*e+b^4*d^4)/(b*x+a)^3-15/b^7*e^4*(a^2*e^2-2*a*b*d*e+b^2*d^2)/(b*x+a)-6/b^7*e^5*(a*e-b*d)*ln(b*x+a)+10/b^7*e^3*(a^3*e^3-3*a^2*b*d*e^2+3*a*b^2*d^2*e-b^3*d^3)/(b*x+a)^2+3/2/b^7*e*(a^5*e^5-5*a^4*b*d*e^4+10*a^3*b^2*d^2*e^3-10*a^2*b^3*d^3*e^2+5*a*b^4*d^4*e-b^5*d^5)/(b*x+a)^4
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 542 vs. 2(151) = 302.

Time = 0.09 (sec) , antiderivative size = 542, normalized size of antiderivative = 3.50

$$\int \frac{(d + ex)^6}{(a^2 + 2abx + b^2x^2)^3} dx = \frac{10 b^6 e^6 x^6 + 50 a b^5 e^6 x^5 - 2 b^6 d^6 - 3 a b^5 d^5 e - 5 a^2 b^4 d^4 e^2 - 10 a^3 b^3 d^3 e^3 - 30 a^4 b^2 d^2 e^4 + 137 a^5 b d e^5 - 87 a^6}{b^6}$$

input

```
integrate((e*x+d)^6/(b^2*x^2+2*a*b*x+a^2)^3,x, algorithm="fricas")
```

output

```
1/10*(10*b^6*e^6*x^6 + 50*a*b^5*e^6*x^5 - 2*b^6*d^6 - 3*a*b^5*d^5*e - 5*a^
2*b^4*d^4*e^2 - 10*a^3*b^3*d^3*e^3 - 30*a^4*b^2*d^2*e^4 + 137*a^5*b*d*e^5
- 87*a^6*e^6 - 50*(3*b^6*d^2*e^4 - 6*a*b^5*d*e^5 + a^2*b^4*e^6)*x^4 - 100*
(b^6*d^3*e^3 + 3*a*b^5*d^2*e^4 - 9*a^2*b^4*d*e^5 + 4*a^3*b^3*e^6)*x^3 - 50
*(b^6*d^4*e^2 + 2*a*b^5*d^3*e^3 + 6*a^2*b^4*d^2*e^4 - 22*a^3*b^3*d*e^5 + 1
2*a^4*b^2*e^6)*x^2 - 5*(3*b^6*d^5*e + 5*a*b^5*d^4*e^2 + 10*a^2*b^4*d^3*e^3
+ 30*a^3*b^3*d^2*e^4 - 125*a^4*b^2*d*e^5 + 75*a^5*b*e^6)*x + 60*(a^5*b*d*
e^5 - a^6*e^6 + (b^6*d*e^5 - a*b^5*e^6)*x^5 + 5*(a*b^5*d*e^5 - a^2*b^4*e^6
)*x^4 + 10*(a^2*b^4*d*e^5 - a^3*b^3*e^6)*x^3 + 10*(a^3*b^3*d*e^5 - a^4*b^2
*e^6)*x^2 + 5*(a^4*b^2*d*e^5 - a^5*b*e^6)*x)*log(b*x + a))/(b^12*x^5 + 5*a
*b^11*x^4 + 10*a^2*b^10*x^3 + 10*a^3*b^9*x^2 + 5*a^4*b^8*x + a^5*b^7)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 420 vs. $2(143) = 286$.

Time = 18.34 (sec) , antiderivative size = 420, normalized size of antiderivative = 2.71

$$\int \frac{(d+ex)^6}{(a^2+2abx+b^2x^2)^3} dx$$

$$= \frac{-87a^6e^6 + 137a^5bde^5 - 30a^4b^2d^2e^4 - 10a^3b^3d^3e^3 - 5a^2b^4d^4e^2 - 3ab^5d^5e - 2b^6d^6 + x^4(-150a^2b^4e^6 + 300a^3b^3d^3e^3 - 150a^4b^2d^2e^4 - 10a^5bde^5 - 87a^6e^6)}{b^7} + \frac{e^6x}{b^6} - \frac{6e^5(ae - bd)\log(a + bx)}{b^7}$$

input

```
integrate((e*x+d)**6/(b**2*x**2+2*a*b*x+a**2)**3,x)
```

output

```
(-87*a**6*e**6 + 137*a**5*b*d*e**5 - 30*a**4*b**2*d**2*e**4 - 10*a**3*b**3
*d**3*e**3 - 5*a**2*b**4*d**4*e**2 - 3*a*b**5*d**5*e - 2*b**6*d**6 + x**4*
(-150*a**2*b**4*e**6 + 300*a*b**5*d*e**5 - 150*b**6*d**2*e**4) + x**3*(-50
0*a**3*b**3*e**6 + 900*a**2*b**4*d*e**5 - 300*a*b**5*d**2*e**4 - 100*b**6*
d**3*e**3) + x**2*(-650*a**4*b**2*e**6 + 1100*a**3*b**3*d*e**5 - 300*a**2*
b**4*d**2*e**4 - 100*a*b**5*d**3*e**3 - 50*b**6*d**4*e**2) + x*(-385*a**5*
b*e**6 + 625*a**4*b**2*d*e**5 - 150*a**3*b**3*d**2*e**4 - 50*a**2*b**4*d**
3*e**3 - 25*a*b**5*d**4*e**2 - 15*b**6*d**5*e))/(10*a**5*b**7 + 50*a**4*b*
*8*x + 100*a**3*b**9*x**2 + 100*a**2*b**10*x**3 + 50*a*b**11*x**4 + 10*b**
12*x**5) + e**6*x/b**6 - 6*e**5*(a*e - b*d)*log(a + b*x)/b**7
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 397 vs. $2(151) = 302$.

Time = 0.05 (sec) , antiderivative size = 397, normalized size of antiderivative = 2.56

$$\int \frac{(d+ex)^6}{(a^2+2abx+b^2x^2)^3} dx = \frac{e^6x}{b^6} - \frac{2b^6d^6 + 3ab^5d^5e + 5a^2b^4d^4e^2 + 10a^3b^3d^3e^3 + 30a^4b^2d^2e^4 - 137a^5bde^5 + 87a^6e^6 + 150(b^6d^2e^4 - 2ab^5de^3 + 5a^2b^4d^2e^2 - 22a^3b^3de^2 + 13a^4b^2de^2 + 5(3b^6d^5e + 5ab^5d^4e^2 + 10a^2b^4d^3e^3 + 30a^3b^3d^2e^4 - 125a^4b^2d^2e^5 + 77a^5bde^6) * x)}{(b^{12}x^5 + 5a^2b^{11}x^4 + 10a^4b^9x^3 + 10a^6b^8x^2 + 5a^8b^7x + a^{10}b^6)} + \frac{6(bde^5 - ae^6) \log(|bx+a|)}{b^7}$$

input

```
integrate((e*x+d)^6/(b^2*x^2+2*a*b*x+a^2)^3,x, algorithm="maxima")
```

output

```
e^6*x/b^6 - 1/10*(2*b^6*d^6 + 3*a*b^5*d^5*e + 5*a^2*b^4*d^4*e^2 + 10*a^3*b^3*d^3*e^3 + 30*a^4*b^2*d^2*e^4 - 137*a^5*b*d*e^5 + 87*a^6*e^6 + 150*(b^6*d^2*e^4 - 2*a*b^5*d*e^3 + a^2*b^4*d^2*e^2)*x^4 + 100*(b^6*d^3*e^3 + 3*a*b^5*d^2*e^2 - 9*a^2*b^4*d*e^2 + 5*a^3*b^3*d^2*e^2)*x^3 + 50*(b^6*d^4*e^2 + 2*a*b^5*d^3*e^2 + 6*a^2*b^4*d^2*e^2 - 22*a^3*b^3*d^2*e^2 + 13*a^4*b^2*d^2*e^2)*x^2 + 5*(3*b^6*d^5*e + 5*a*b^5*d^4*e^2 + 10*a^2*b^4*d^3*e^3 + 30*a^3*b^3*d^2*e^4 - 125*a^4*b^2*d^2*e^5 + 77*a^5*b*d^2*e^6)*x)/(b^12*x^5 + 5*a^2*b^11*x^4 + 10*a^4*b^9*x^3 + 10*a^6*b^8*x^2 + 5*a^8*b^7*x + a^10*b^6) + 6*(b*d*e^5 - a*e^6)*log(b*x + a)/b^7
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 351 vs. $2(151) = 302$.

Time = 0.17 (sec) , antiderivative size = 351, normalized size of antiderivative = 2.26

$$\int \frac{(d+ex)^6}{(a^2+2abx+b^2x^2)^3} dx = \frac{e^6x}{b^6} + \frac{6(bde^5 - ae^6) \log(|bx+a|)}{b^7} - \frac{2b^6d^6 + 3ab^5d^5e + 5a^2b^4d^4e^2 + 10a^3b^3d^3e^3 + 30a^4b^2d^2e^4 - 137a^5bde^5 + 87a^6e^6 + 150(b^6d^2e^4 - 2ab^5de^3 + 5a^2b^4d^2e^2 - 22a^3b^3de^2 + 13a^4b^2de^2 + 5(3b^6d^5e + 5ab^5d^4e^2 + 10a^2b^4d^3e^3 + 30a^3b^3d^2e^4 - 125a^4b^2d^2e^5 + 77a^5bde^6) * x)}{(b^{12}x^5 + 5a^2b^{11}x^4 + 10a^4b^9x^3 + 10a^6b^8x^2 + 5a^8b^7x + a^{10}b^6)}$$

input

```
integrate((e*x+d)^6/(b^2*x^2+2*a*b*x+a^2)^3,x, algorithm="giac")
```

output

```
e^6*x/b^6 + 6*(b*d*e^5 - a*e^6)*log(abs(b*x + a))/b^7 - 1/10*(2*b^6*d^6 +
3*a*b^5*d^5*e + 5*a^2*b^4*d^4*e^2 + 10*a^3*b^3*d^3*e^3 + 30*a^4*b^2*d^2*e^
4 - 137*a^5*b*d*e^5 + 87*a^6*e^6 + 150*(b^6*d^2*e^4 - 2*a*b^5*d*e^5 + a^2*
b^4*e^6)*x^4 + 100*(b^6*d^3*e^3 + 3*a*b^5*d^2*e^4 - 9*a^2*b^4*d*e^5 + 5*a^
3*b^3*e^6)*x^3 + 50*(b^6*d^4*e^2 + 2*a*b^5*d^3*e^3 + 6*a^2*b^4*d^2*e^4 - 2
2*a^3*b^3*d*e^5 + 13*a^4*b^2*e^6)*x^2 + 5*(3*b^6*d^5*e + 5*a*b^5*d^4*e^2 +
10*a^2*b^4*d^3*e^3 + 30*a^3*b^3*d^2*e^4 - 125*a^4*b^2*d*e^5 + 77*a^5*b*e^
6)*x)/((b*x + a)^5*b^7)
```

Mupad [B] (verification not implemented)

Time = 8.85 (sec) , antiderivative size = 399, normalized size of antiderivative = 2.57

$$\int \frac{(d+ex)^6}{(a^2+2abx+b^2x^2)^3} dx = \frac{e^6 x}{b^6} - \frac{\ln(a+bx)(6ae^6 - 6bde^5)}{b^7} - \frac{x^2(65a^4be^6 - 110a^3b^2de^5 + 30a^2b^3d^2e^4 + 10ab^4d^3e^3 + 5b^5d^4e^2) + x^4(15a^2b^3e^6 - 30ab^4de^5 + \dots)}{a^5b^6 + b^{11}x^5 + 5a^4b^7x + 5a^3b^8x^2 + 10a^2b^9x^3}$$

input

```
int((d + e*x)^6/(a^2 + b^2*x^2 + 2*a*b*x)^3,x)
```

output

```
(e^6*x)/b^6 - (log(a + b*x)*(6*a*e^6 - 6*b*d*e^5))/b^7 - (x^2*(65*a^4*b*e^
6 + 5*b^5*d^4*e^2 + 10*a*b^4*d^3*e^3 - 110*a^3*b^2*d*e^5 + 30*a^2*b^3*d^2*
e^4) + x^4*(15*a^2*b^3*e^6 + 15*b^5*d^2*e^4 - 30*a*b^4*d*e^5) + (87*a^6*e^
6 + 2*b^6*d^6 + 5*a^2*b^4*d^4*e^2 + 10*a^3*b^3*d^3*e^3 + 30*a^4*b^2*d^2*e^
4 + 3*a*b^5*d^5*e - 137*a^5*b*d*e^5)/(10*b) + x*((77*a^5*e^6)/2 + (3*b^5*d
^5*e)/2 + (5*a*b^4*d^4*e^2)/2 + 5*a^2*b^3*d^3*e^3 + 15*a^3*b^2*d^2*e^4 - (
125*a^4*b*d*e^5)/2) + x^3*(50*a^3*b^2*e^6 + 10*b^5*d^3*e^3 + 30*a*b^4*d^2*
e^4 - 90*a^2*b^3*d*e^5))/(a^5*b^6 + b^11*x^5 + 5*a^4*b^7*x + 5*a^3*b^8*x^
4 + 10*a^2*b^9*x^3)
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 576, normalized size of antiderivative = 3.72

$$\int \frac{(d + ex)^6}{(a^2 + 2abx + b^2x^2)^3} dx$$

$$= \frac{-600 \log(bx + a) a^5 b^2 e^6 x^2 - 600 \log(bx + a) a^4 b^3 e^6 x^3 + 325 a^5 b^2 d e^5 x - 50 a^3 b^4 d^3 e^3 x + 300 a^3 b^4 d e^5 x^3 -$$

input `int((e*x+d)^6/(b^2*x^2+2*a*b*x+a^2)^3,x)`

output `(- 60*log(a + b*x)*a**7*e**6 + 60*log(a + b*x)*a**6*b*d*e**5 - 300*log(a + b*x)*a**6*b*e**6*x + 300*log(a + b*x)*a**5*b**2*d*e**5*x - 600*log(a + b*x)*a**5*b**2*e**6*x**2 + 600*log(a + b*x)*a**4*b**3*d*e**5*x**2 - 600*log(a + b*x)*a**4*b**3*e**6*x**3 + 600*log(a + b*x)*a**3*b**4*d*e**5*x**3 - 300*log(a + b*x)*a**3*b**4*e**6*x**4 + 300*log(a + b*x)*a**2*b**5*d*e**5*x**4 - 60*log(a + b*x)*a**2*b**5*e**6*x**5 + 60*log(a + b*x)*a*b**6*d*e**5*x**5 - 77*a**7*e**6 + 77*a**6*b*d*e**5 - 325*a**6*b*e**6*x + 325*a**5*b**2*d*e**5*x - 500*a**5*b**2*e**6*x**2 - 10*a**4*b**3*d**3*e**3 + 500*a**4*b**3*d*e**5*x**2 - 300*a**4*b**3*e**6*x**3 - 5*a**3*b**4*d**4*e**2 - 50*a**3*b**4*d**3*e**3*x + 300*a**3*b**4*d*e**5*x**3 - 3*a**2*b**5*d**5*e - 25*a**2*b**5*d**4*e**2*x - 100*a**2*b**5*d**3*e**3*x**2 + 60*a**2*b**5*e**6*x**5 - 2*a*b**6*d**6 - 15*a*b**6*d**5*e*x - 50*a*b**6*d**4*e**2*x**2 - 100*a*b**6*d**3*e**3*x**3 - 60*a*b**6*d*e**5*x**5 + 10*a*b**6*e**6*x**6 + 30*b**7*d**2*e**4*x**5)/(10*a*b**7*(a**5 + 5*a**4*b*x + 10*a**3*b**2*x**2 + 10*a**2*b**3*x**3 + 5*a*b**4*x**4 + b**5*x**5))`

3.131
$$\int \frac{(d+ex)^5}{(a^2+2abx+b^2x^2)^3} dx$$

| | |
|---|------|
| Optimal result | 1051 |
| Mathematica [A] (verified) | 1051 |
| Rubi [A] (verified) | 1052 |
| Maple [A] (verified) | 1053 |
| Fricas [B] (verification not implemented) | 1054 |
| Sympy [B] (verification not implemented) | 1055 |
| Maxima [B] (verification not implemented) | 1055 |
| Giac [A] (verification not implemented) | 1056 |
| Mupad [B] (verification not implemented) | 1056 |
| Reduce [B] (verification not implemented) | 1057 |

Optimal result

Integrand size = 26, antiderivative size = 138

$$\int \frac{(d+ex)^5}{(a^2+2abx+b^2x^2)^3} dx = -\frac{(bd-ae)^5}{5b^6(a+bx)^5} - \frac{5e(bd-ae)^4}{4b^6(a+bx)^4} - \frac{10e^2(bd-ae)^3}{3b^6(a+bx)^3} - \frac{5e^3(bd-ae)^2}{b^6(a+bx)^2} - \frac{5e^4(bd-ae)}{b^6(a+bx)} + \frac{e^5 \log(a+bx)}{b^6}$$

output

```
-1/5*(-a*e+b*d)^5/b^6/(b*x+a)^5-5/4*e*(-a*e+b*d)^4/b^6/(b*x+a)^4-10/3*e^2*(-a*e+b*d)^3/b^6/(b*x+a)^3-5*e^3*(-a*e+b*d)^2/b^6/(b*x+a)^2-5*e^4*(-a*e+b*d)/b^6/(b*x+a)+e^5*ln(b*x+a)/b^6
```

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 171, normalized size of antiderivative = 1.24

$$\int \frac{(d+ex)^5}{(a^2+2abx+b^2x^2)^3} dx = \frac{(bd-ae)(137a^4e^4+a^3be^3(77d+625ex)+a^2b^2e^2(47d^2+325dex+1100e^2x^2)+ab^3e(27d^3+175d^2e)+b^4e^2(9d^4+50dex+110e^2x^2)+b^5e^3(3d^5+15d^4e+15d^3e^2+5d^2e^3)+b^6e^4(d^5+5d^4e+10d^3e^2+10d^2e^3+5de^4+e^5))}{60b^6(a+bx)^5} + \frac{e^5 \log(a+bx)}{b^6}$$

input `Integrate[(d + e*x)^5/(a^2 + 2*a*b*x + b^2*x^2)^3,x]`

output
$$-1/60*((b*d - a*e)*(137*a^4*e^4 + a^3*b*e^3*(77*d + 625*e*x) + a^2*b^2*e^2*(47*d^2 + 325*d*e*x + 1100*e^2*x^2) + a*b^3*e*(27*d^3 + 175*d^2*e*x + 500*d*e^2*x^2 + 900*e^3*x^3) + b^4*(12*d^4 + 75*d^3*e*x + 200*d^2*e^2*x^2 + 300*d*e^3*x^3 + 300*e^4*x^4)))/(b^6*(a + b*x)^5) + (e^5*Log[a + b*x])/b^6$$

Rubi [A] (verified)

Time = 0.56 (sec) , antiderivative size = 138, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {1098, 27, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\begin{aligned} & \int \frac{(d + ex)^5}{(a^2 + 2abx + b^2x^2)^3} dx \\ & \quad \downarrow 1098 \\ & b^6 \int \frac{(d + ex)^5}{b^6(a + bx)^6} dx \\ & \quad \downarrow 27 \\ & \int \frac{(d + ex)^5}{(a + bx)^6} dx \\ & \quad \downarrow 49 \\ & \int \left(\frac{5e^4(bd - ae)}{b^5(a + bx)^2} + \frac{10e^3(bd - ae)^2}{b^5(a + bx)^3} + \frac{10e^2(bd - ae)^3}{b^5(a + bx)^4} + \frac{5e(bd - ae)^4}{b^5(a + bx)^5} + \frac{(bd - ae)^5}{b^5(a + bx)^6} + \frac{e^5}{b^5(a + bx)} \right) dx \\ & \quad \downarrow 2009 \\ & -\frac{5e^4(bd - ae)}{b^6(a + bx)} - \frac{5e^3(bd - ae)^2}{b^6(a + bx)^2} - \frac{10e^2(bd - ae)^3}{3b^6(a + bx)^3} - \frac{5e(bd - ae)^4}{4b^6(a + bx)^4} - \frac{(bd - ae)^5}{5b^6(a + bx)^5} + \frac{e^5 \log(a + bx)}{b^6} \end{aligned}$$

input `Int[(d + e*x)^5/(a^2 + 2*a*b*x + b^2*x^2)^3,x]`

output

$$-1/5*(b*d - a*e)^5/(b^6*(a + b*x)^5) - (5*e*(b*d - a*e)^4)/(4*b^6*(a + b*x)^4) - (10*e^2*(b*d - a*e)^3)/(3*b^6*(a + b*x)^3) - (5*e^3*(b*d - a*e)^2)/(b^6*(a + b*x)^2) - (5*e^4*(b*d - a*e))/(b^6*(a + b*x)) + (e^5*Log[a + b*x])/b^6$$

Defintions of rubi rules used

rule 27

$$\text{Int}[(a_)*(Fx_), x_Symbol] \text{ :> Simp}[a \text{ Int}[Fx, x], x] \text{ /; FreeQ}[a, x] \ \&\& \ !\text{MatchQ}[Fx, (b_)*(Gx_)] \text{ /; FreeQ}[b, x]$$

rule 49

$$\text{Int}[((a_.) + (b_.)*(x_))^m*((c_.) + (d_.)*(x_))^n, x_Symbol] \text{ :> Int}[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] \text{ /; FreeQ}[\{a, b, c, d\}, x] \ \&\& \ \text{IGtQ}[m, 0] \ \&\& \ \text{IGtQ}[m + n + 2, 0]$$

rule 1098

$$\text{Int}[((d_.) + (e_.)*(x_))^m*((a_.) + (b_.)*(x_.) + (c_.)*(x_.)^2)^p, x_Symbol] \text{ :> Simp}[1/c^p \text{ Int}[(d + e*x)^m*(b/2 + c*x)^(2*p), x], x] \text{ /; FreeQ}[\{a, b, c, d, e, m\}, x] \ \&\& \ \text{EqQ}[b^2 - 4*a*c, 0] \ \&\& \ \text{IntegerQ}[p]$$

rule 2009

$$\text{Int}[u_, x_Symbol] \text{ :> Simp}[\text{IntSum}[u, x], x] \text{ /; SumQ}[u]$$

Maple [A] (verified)

Time = 1.07 (sec) , antiderivative size = 261, normalized size of antiderivative = 1.89

| method | result |
|--------------|--|
| norman | $\frac{137e^5a^5 - 60a^4bde^4 - 30a^3b^2d^2e^3 - 20a^2b^3d^3e^2 - 15ab^4d^4e - 12b^5d^5}{60b^6} + \frac{5(ae^5 - de^4b)x^4}{b^2} + \frac{5(3a^2e^5 - 2ade^4b - d^2e^3b^2)x^3}{b^3} + \frac{5(11a^3e^5 - 6a^2bde^4 - 3a^2b^2d^2e^3 - 3ab^3d^3e^2 - 3a^2b^4d^4e - 3ab^5d^5)}{(bx+a)^5}$ |
| default | $-\frac{e^5a^5 + 5a^4bde^4 - 10a^3b^2d^2e^3 + 10a^2b^3d^3e^2 - 5ab^4d^4e + b^5d^5}{5b^6(bx+a)^5} + \frac{10e^2(e^3a^3 - 3a^2bde^2 + 3ab^2d^2e - b^3d^3)}{3b^6(bx+a)^3} + \frac{5e^4(ae - bd)}{b^6(bx+a)}$ |
| risch | $\frac{5e^4(ae - bd)x^4}{b^2} + \frac{5e^3(3e^2a^2 - 2abde - b^2d^2)x^3}{b^3} + \frac{5e^2(11e^3a^3 - 6a^2bde^2 - 3ab^2d^2e - 2b^3d^3)x^2}{3b^4} + \frac{5e(25a^4e^4 - 12a^3bde^3 - 6a^2b^2d^2e^2 - 4ab^3d^3)}{(bx+a)(b^2x^2 + 2abx + a^2)^2} + \frac{12b^5}{(bx+a)(b^2x^2 + 2abx + a^2)^2}$ |
| parallelrisc | $-\frac{300a^3b^2de^4x - 12b^5d^5 + 600 \ln(bx+a)x^3a^2b^3e^5 - 60a^4bde^4 - 30a^3b^2d^2e^3 + 625a^4be^5x - 75b^5d^4ex - 20a^2b^3d^3e^2 - 15ab^4d^4e + 12b^5d^5}{(bx+a)^5}$ |

input `int((e*x+d)^5/(b^2*x^2+2*a*b*x+a^2)^3,x,method=_RETURNVERBOSE)`

output
$$\frac{(1/60*(137*a^5*e^5-60*a^4*b*d*e^4-30*a^3*b^2*d^2*e^3-20*a^2*b^3*d^3*e^2-15*a*b^4*d^4*e-12*b^5*d^5)/b^6+5*(a*e^5-b*d*e^4)/b^2*x^4+5*(3*a^2*e^5-2*a*b*d*e^4-b^2*d^2*e^3)/b^3*x^3+5/3*(11*a^3*e^5-6*a^2*b*d*e^4-3*a*b^2*d^2*e^3-2*b^3*d^3*e^2)/b^4*x^2+5/12*(25*a^4*e^5-12*a^3*b*d*e^4-6*a^2*b^2*d^2*e^3-4*a*b^3*d^3*e^2-3*b^4*d^4*e)/b^5*x)/(b*x+a)^5+e^5*\ln(b*x+a)/b^6$$

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 372 vs. $2(132) = 264$.

Time = 0.08 (sec) , antiderivative size = 372, normalized size of antiderivative = 2.70

$$\int \frac{(d+ex)^5}{(a^2+2abx+b^2x^2)^3} dx = \frac{12b^5d^5 + 15ab^4d^4e + 20a^2b^3d^3e^2 + 30a^3b^2d^2e^3 + 60a^4bde^4 - 137a^5e^5 + 300(b^5de^4 - ab^4e^5)x^4 + 300(b^5d^2e^3 + 2ab^4d^2e^4 - 3a^2b^3d^2e^5)x^3 + 100(2b^5d^3e^2 + 3ab^4d^2e^3 + 6a^2b^3d^2e^4 - 11a^3b^2d^2e^5)x^2 + 25(3b^5d^4e + 4ab^4d^3e^2 + 6a^2b^3d^2e^3 + 12a^3b^2d^2e^4 - 25a^4b^2e^5)x - 60(b^5e^5x^5 + 5ab^4e^5x^4 + 10a^2b^3e^5x^3 + 10a^3b^2e^5x^2 + 5a^4be^5x + a^5e^5)*\log(bx+a)}{(b^{11}x^5 + 5a*b^{10}x^4 + 10a^2*b^9x^3 + 10a^3*b^8x^2 + 5a^4*b^7x + a^5*b^6)}$$

input `integrate((e*x+d)^5/(b^2*x^2+2*a*b*x+a^2)^3,x, algorithm="fricas")`

output
$$\frac{-1/60*(12*b^5*d^5 + 15*a*b^4*d^4*e + 20*a^2*b^3*d^3*e^2 + 30*a^3*b^2*d^2*e^3 + 60*a^4*b*d*e^4 - 137*a^5*e^5 + 300*(b^5*d^2*e^4 - a*b^4*e^5)*x^4 + 300*(b^5*d^2*e^3 + 2*a*b^4*d^2*e^4 - 3*a^2*b^3*d^2*e^5)*x^3 + 100*(2*b^5*d^3*e^2 + 3*a*b^4*d^2*e^3 + 6*a^2*b^3*d^2*e^4 - 11*a^3*b^2*d^2*e^5)*x^2 + 25*(3*b^5*d^4*e + 4*a*b^4*d^3*e^2 + 6*a^2*b^3*d^2*e^3 + 12*a^3*b^2*d^2*e^4 - 25*a^4*b^2*e^5)*x - 60*(b^5*e^5*x^5 + 5*a*b^4*e^5*x^4 + 10*a^2*b^3*e^5*x^3 + 10*a^3*b^2*e^5*x^2 + 5*a^4*b^2*e^5*x + a^5*e^5)*\log(b*x + a)}{(b^{11}*x^5 + 5*a*b^{10}*x^4 + 10*a^2*b^9*x^3 + 10*a^3*b^8*x^2 + 5*a^4*b^7*x + a^5*b^6)}$$

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 326 vs. $2(126) = 252$.

Time = 5.06 (sec) , antiderivative size = 326, normalized size of antiderivative = 2.36

$$\int \frac{(d+ex)^5}{(a^2+2abx+b^2x^2)^3} dx = \frac{137a^5e^5 - 60a^4bde^4 - 30a^3b^2d^2e^3 - 20a^2b^3d^3e^2 - 15ab^4d^4e - 12b^5d^5 + x^4 \cdot (300ab^4e^5 - 300b^5de^4) + x^3 \cdot (300ab^4e^5 - 300b^5de^4) + x^2 \cdot (900a^2b^3e^5 - 600ab^4de^4 - 300b^5d^2e^3) + x \cdot (625a^4be^5 - 300a^3b^2de^4 - 150a^2b^3d^2e^3 - 100ab^4d^3e^2 - 75b^5d^4e)}{60a^5b^6 + 300a^4b^7x + 600a^3b^8x^2 + 600a^2b^9x^3 + 300ab^{10}x^4 + 60b^{11}x^5} + \frac{e^5 \log(a+bx)}{b^6}$$

input `integrate((e*x+d)**5/(b**2*x**2+2*a*b*x+a**2)**3,x)`

output `(137*a**5*e**5 - 60*a**4*b*d*e**4 - 30*a**3*b**2*d**2*e**3 - 20*a**2*b**3*d**3*e**2 - 15*a*b**4*d**4*e - 12*b**5*d**5 + x**4*(300*a*b**4*e**5 - 300*b**5*d*e**4) + x**3*(900*a**2*b**3*e**5 - 600*a*b**4*d*e**4 - 300*b**5*d**2*e**3) + x**2*(1100*a**3*b**2*e**5 - 600*a**2*b**3*d*e**4 - 300*a*b**4*d**2*e**3 - 200*b**5*d**3*e**2) + x*(625*a**4*b*e**5 - 300*a**3*b**2*d*e**4 - 150*a**2*b**3*d**2*e**3 - 100*a*b**4*d**3*e**2 - 75*b**5*d**4*e))/(60*a**5*b**6 + 300*a**4*b**7*x + 600*a**3*b**8*x**2 + 600*a**2*b**9*x**3 + 300*a*b**10*x**4 + 60*b**11*x**5) + e**5*log(a + b*x)/b**6`

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 310 vs. $2(132) = 264$.

Time = 0.06 (sec) , antiderivative size = 310, normalized size of antiderivative = 2.25

$$\int \frac{(d+ex)^5}{(a^2+2abx+b^2x^2)^3} dx = \frac{12b^5d^5 + 15ab^4d^4e + 20a^2b^3d^3e^2 + 30a^3b^2d^2e^3 + 60a^4bde^4 - 137a^5e^5 + 300(b^5de^4 - ab^4e^5)x^4 + 300(b^5de^4 - ab^4e^5)x^3 + 300(b^5de^4 - ab^4e^5)x^2 + 300(b^5de^4 - ab^4e^5)x + 300(b^5de^4 - ab^4e^5)}{60(b^6 + 300ab^7x + 600a^2b^8x^2 + 600a^3b^9x^3 + 300a^4b^{10}x^4 + 60a^5b^{11}x^5)} + \frac{e^5 \log(bx+a)}{b^6}$$

input `integrate((e*x+d)^5/(b^2*x^2+2*a*b*x+a^2)^3,x, algorithm="maxima")`

output

```
-1/60*(12*b^5*d^5 + 15*a*b^4*d^4*e + 20*a^2*b^3*d^3*e^2 + 30*a^3*b^2*d^2*e^3 + 60*a^4*b*d*e^4 - 137*a^5*e^5 + 300*(b^5*d*e^4 - a*b^4*e^5)*x^4 + 300*(b^5*d^2*e^3 + 2*a*b^4*d*e^4 - 3*a^2*b^3*e^5)*x^3 + 100*(2*b^5*d^3*e^2 + 3*a*b^4*d^2*e^3 + 6*a^2*b^3*d*e^4 - 11*a^3*b^2*e^5)*x^2 + 25*(3*b^5*d^4*e + 4*a*b^4*d^3*e^2 + 6*a^2*b^3*d^2*e^3 + 12*a^3*b^2*d*e^4 - 25*a^4*b*e^5)*x)/(b^11*x^5 + 5*a*b^10*x^4 + 10*a^2*b^9*x^3 + 10*a^3*b^8*x^2 + 5*a^4*b^7*x + a^5*b^6) + e^5*log(b*x + a)/b^6
```

Giac [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 264, normalized size of antiderivative = 1.91

$$\int \frac{(d+ex)^5}{(a^2+2abx+b^2x^2)^3} dx = \frac{e^5 \log(|bx+a|)}{b^6} - \frac{300(b^4de^4 - ab^3e^5)x^4 + 300(b^4d^2e^3 + 2ab^3de^4 - 3a^2b^2e^5)x^3 + 100(2b^4d^3e^2 + 3ab^3d^2e^3 + 6a^2b^2de^4 -$$

input

```
integrate((e*x+d)^5/(b^2*x^2+2*a*b*x+a^2)^3,x, algorithm="giac")
```

output

```
e^5*log(abs(b*x + a))/b^6 - 1/60*(300*(b^4*d*e^4 - a*b^3*e^5)*x^4 + 300*(b^4*d^2*e^3 + 2*a*b^3*d*e^4 - 3*a^2*b^2*e^5)*x^3 + 100*(2*b^4*d^3*e^2 + 3*a*b^3*d^2*e^3 + 6*a^2*b^2*d*e^4 - 11*a^3*b*e^5)*x^2 + 25*(3*b^4*d^4*e + 4*a*b^3*d^3*e^2 + 6*a^2*b^2*d^2*e^3 + 12*a^3*b*d*e^4 - 25*a^4*e^5)*x + (12*b^5*d^5 + 15*a*b^4*d^4*e + 20*a^2*b^3*d^3*e^2 + 30*a^3*b^2*d^2*e^3 + 60*a^4*b*d*e^4 - 137*a^5*e^5)/b)/((b*x + a)^5*b^5)
```

Mupad [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 261, normalized size of antiderivative = 1.89

$$\int \frac{(d+ex)^5}{(a^2+2abx+b^2x^2)^3} dx = \frac{e^5 \ln(a+bx)}{b^6} - \frac{x \left(-\frac{125a^4be^5}{12} + 5a^3b^2de^4 + \frac{5a^2b^3d^2e^3}{2} + \frac{5ab^4d^3e^2}{3} + \frac{5b^5d^4e}{4} \right) - x^4(5ab^4e^5 - 5b^5de^4) + x^3(-15a^2b^3e^5 + 15a^3b^2de^4 - 5a^4e^5)}{b^6}$$

input `int((d + e*x)^5/(a^2 + b^2*x^2 + 2*a*b*x)^3,x)`

output
$$\begin{aligned} & (e^5 \log(a + b*x))/b^6 - (x*((5*b^5*d^4*e)/4 - (125*a^4*b*e^5)/12 + (5*a*b^4*d^3*e^2)/3 + 5*a^3*b^2*d*e^4 + (5*a^2*b^3*d^2*e^3)/2) - x^4*(5*a*b^4*e^5 - 5*b^5*d*e^4) + x^3*(5*b^5*d^2*e^3 - 15*a^2*b^3*e^5 + 10*a*b^4*d*e^4) - \\ & (137*a^5*e^5)/60 + (b^5*d^5)/5 + x^2*((10*b^5*d^3*e^2)/3 - (55*a^3*b^2*e^5)/3 + 5*a*b^4*d^2*e^3 + 10*a^2*b^3*d*e^4) + (a^2*b^3*d^3*e^2)/3 + (a^3*b^2*d^2*e^3)/2 + (a*b^4*d^4*e)/4 + a^4*b*d*e^4/(b^6*(a + b*x)^5) \end{aligned}$$

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 375, normalized size of antiderivative = 2.72

$$\int \frac{(d + ex)^5}{(a^2 + 2abx + b^2x^2)^3} dx$$

$$= \frac{60 \log(bx + a) a^6 e^5 + 300 \log(bx + a) a^5 b e^5 x + 600 \log(bx + a) a^4 b^2 e^5 x^2 + 600 \log(bx + a) a^3 b^3 e^5 x^3 + 300 \log(bx + a) a^2 b^4 e^5 x^4 + 60 \log(bx + a) a b^5 e^5 x^5 + 77 a^6 e^5 + 325 a^5 b e^5 x - 30 a^4 b^2 d e^5 x^2 + 500 a^4 b^2 e^5 x^2 - 20 a^3 b^3 d e^5 x^2 - 150 a^3 b^3 d e^5 x^2 + 300 a^3 b^3 e^5 x^3 - 15 a^2 b^4 d e^5 x - 100 a^2 b^4 d e^5 x^2 - 300 a^2 b^4 d e^5 x^2 + 12 a b^5 d e^5 - 75 a b^5 d e^5 x - 200 a b^5 d e^5 x^2 - 300 a b^5 d e^5 x^3 - 60 a b^5 e^5 x^5 + 60 b^6 d e^5 x^5}{(60 a^6 b^6 (a^5 + 5 a^4 b x + 10 a^3 b^2 x^2 + 10 a^2 b^3 x^3 + 5 a b^4 x^4 + b^5 x^5))}$$

input `int((e*x+d)^5/(b^2*x^2+2*a*b*x+a^2)^3,x)`

output
$$\begin{aligned} & (60*\log(a + b*x)*a**6*e**5 + 300*\log(a + b*x)*a**5*b*e**5*x + 600*\log(a + \\ & b*x)*a**4*b**2*e**5*x**2 + 600*\log(a + b*x)*a**3*b**3*e**5*x**3 + 300*\log(a + \\ & b*x)*a**2*b**4*e**5*x**4 + 60*\log(a + b*x)*a*b**5*e**5*x**5 + 77*a**6* \\ & e**5 + 325*a**5*b*e**5*x - 30*a**4*b**2*d**2*e**3 + 500*a**4*b**2*e**5*x** \\ & 2 - 20*a**3*b**3*d**3*e**2 - 150*a**3*b**3*d**2*e**3*x + 300*a**3*b**3*e** \\ & 5*x**3 - 15*a**2*b**4*d**4*e - 100*a**2*b**4*d**3*e**2*x - 300*a**2*b**4*d \\ & **2*e**3*x**2 - 12*a*b**5*d**5 - 75*a*b**5*d**4*e*x - 200*a*b**5*d**3*e**2 \\ & *x**2 - 300*a*b**5*d**2*e**3*x**3 - 60*a*b**5*e**5*x**5 + 60*b**6*d*e**4*x \\ & **5)/(60*a*b**6*(a**5 + 5*a**4*b*x + 10*a**3*b**2*x**2 + 10*a**2*b**3*x**3 \\ & + 5*a*b**4*x**4 + b**5*x**5)) \end{aligned}$$

$$3.132 \quad \int \frac{(d+ex)^4}{(a^2+2abx+b^2x^2)^3} dx$$

| | |
|---|------|
| Optimal result | 1058 |
| Mathematica [B] (verified) | 1058 |
| Rubi [A] (verified) | 1059 |
| Maple [B] (verified) | 1060 |
| Fricas [B] (verification not implemented) | 1061 |
| Sympy [B] (verification not implemented) | 1061 |
| Maxima [B] (verification not implemented) | 1062 |
| Giac [B] (verification not implemented) | 1062 |
| Mupad [B] (verification not implemented) | 1063 |
| Reduce [B] (verification not implemented) | 1063 |

Optimal result

Integrand size = 26, antiderivative size = 28

$$\int \frac{(d+ex)^4}{(a^2+2abx+b^2x^2)^3} dx = -\frac{(d+ex)^5}{5(bd-ae)(a+bx)^5}$$

output `-1/5*(e*x+d)^5/(-a*e+b*d)/(b*x+a)^5`

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 140 vs. 2(28) = 56.

Time = 0.04 (sec) , antiderivative size = 140, normalized size of antiderivative = 5.00

$$\int \frac{(d+ex)^4}{(a^2+2abx+b^2x^2)^3} dx = \frac{a^4e^4 + a^3be^3(d+5ex) + a^2b^2e^2(d^2+5dex+10e^2x^2) + ab^3e(d^3+5d^2ex+10de^2x^2+10e^3x^3) + b^4(d^4 - 5b^5(a+bx)^5)}{5b^5(a+bx)^5}$$

input `Integrate[(d + e*x)^4/(a^2 + 2*a*b*x + b^2*x^2)^3,x]`

output

```
-1/5*(a^4*e^4 + a^3*b*e^3*(d + 5*e*x) + a^2*b^2*e^2*(d^2 + 5*d*e*x + 10*e^2*x^2) + a*b^3*e*(d^3 + 5*d^2*e*x + 10*d*e^2*x^2 + 10*e^3*x^3) + b^4*(d^4 + 5*d^3*e*x + 10*d^2*e^2*x^2 + 10*d*e^3*x^3 + 5*e^4*x^4))/(b^5*(a + b*x)^5)
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.115$, Rules used = {1098, 27, 48}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d + ex)^4}{(a^2 + 2abx + b^2x^2)^3} dx$$

$$\downarrow 1098$$

$$b^6 \int \frac{(d + ex)^4}{b^6(a + bx)^6} dx$$

$$\downarrow 27$$

$$\int \frac{(d + ex)^4}{(a + bx)^6} dx$$

$$\downarrow 48$$

$$-\frac{(d + ex)^5}{5(a + bx)^5(bd - ae)}$$

input

```
Int[(d + e*x)^4/(a^2 + 2*a*b*x + b^2*x^2)^3,x]
```

output

```
-1/5*(d + e*x)^5/((b*d - a*e)*(a + b*x)^5)
```


Defintions of rubi rules used

rule 27 $\text{Int}[(a_*)(Fx_), x_Symbol] \rightarrow \text{Simp}[a \text{ Int}[Fx, x], x] /; \text{FreeQ}[a, x] \ \&\& \ !\text{MatchQ}[Fx, (b_*)(Gx_)] /; \text{FreeQ}[b, x]$

rule 48 $\text{Int}[((a_.) + (b_.)*(x_))^{(m_.)}*((c_.) + (d_.)*(x_))^{(n_.)}, x_Symbol] \rightarrow \text{Simp}[(a + b*x)^{(m + 1)}*((c + d*x)^{(n + 1)} / ((b*c - a*d)*(m + 1))), x] /; \text{FreeQ}[\{a, b, c, d, m, n\}, x] \ \&\& \ \text{EqQ}[m + n + 2, 0] \ \&\& \ \text{NeQ}[m, -1]$

rule 1098 $\text{Int}[((d_.) + (e_.)*(x_))^{(m_.)}*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^{(p_.)}, x_Symbol] \rightarrow \text{Simp}[1/c^p \text{ Int}[(d + e*x)^m*(b/2 + c*x)^(2*p), x], x] /; \text{FreeQ}[\{a, b, c, d, e, m\}, x] \ \&\& \ \text{EqQ}[b^2 - 4*a*c, 0] \ \&\& \ \text{IntegerQ}[p]$

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 178 vs. 2(26) = 52.

Time = 1.07 (sec) , antiderivative size = 179, normalized size of antiderivative = 6.39

| method | result |
|---------------|---|
| risch | $\frac{-\frac{e^4 x^4}{b} - \frac{2e^3(ae+bd)x^3}{b^2} - \frac{2e^2(e^2 a^2 + abde + b^2 d^2)x^2}{b^3} - \frac{e(e^3 a^3 + a^2 bde^2 + a b^2 d^2 e + b^3 d^3)x}{b^4} - \frac{a^4 e^4 + a^3 bde^3 + a^2 b^2 d^2 e^2 + a b^3 d^3 e + b^4 d^4}{5b^5}}{(bx+a)(b^2x^2+2abx+a^2)^2}$ |
| norman | $\frac{-\frac{e^4 x^4}{b} + \frac{2(-e^4 a - bde^3)x^3}{b^2} + \frac{2(-a^2 e^4 - abde^3 - d^2 e^2 b^2)x^2}{b^3} + \frac{(-e^4 a^3 - a^2 d e^3 b - a d^2 e^2 b^2 - d^3 e b^3)x}{b^4} + \frac{-a^4 e^4 - a^3 bde^3 - a^2 b^2 d^2 e^2 - a b^3 d^3 e - b^4 d^4}{5b^5}}{(bx+a)^5}$ |
| default | $-\frac{a^4 e^4 - 4a^3 bde^3 + 6a^2 b^2 d^2 e^2 - 4a b^3 d^3 e + b^4 d^4}{5b^5 (bx+a)^5} - \frac{2e^2(e^2 a^2 - 2abde + b^2 d^2)}{b^5 (bx+a)^3} - \frac{e^4}{b^5 (bx+a)} + \frac{2e^3(ae-bd)}{b^5 (bx+a)^2} + \frac{e(e^3 a^3 - 3a^2 bde^2 + a^2 b^2 d^2 e - b^3 d^3 e^2)}{b^5 (bx+a)^4}$ |
| orering | $-\frac{(5b^4 x^4 e^4 + 10x^3 a b^3 e^4 + 10x^3 b^4 d e^3 + 10x^2 a^2 b^2 e^4 + 10x^2 a b^3 d e^3 + 10x^2 b^4 d^2 e^2 + 5x a^3 b e^4 + 5x a^2 b^2 d e^3 + 5x a b^3 d^2 e^2 + 5x b^4 d^3 e)}{5b^5 (b^2 x^2 + 2abx + a^2)^3}$ |
| gosper | $-\frac{5b^4 x^4 e^4 + 10x^3 a b^3 e^4 + 10x^3 b^4 d e^3 + 10x^2 a^2 b^2 e^4 + 10x^2 a b^3 d e^3 + 10x^2 b^4 d^2 e^2 + 5x a^3 b e^4 + 5x a^2 b^2 d e^3 + 5x a b^3 d^2 e^2 + 5x b^4 d^3 e}{5(bx+a)(b^2x^2+2abx+a^2)^2 b^5}$ |
| parallelrisch | $\frac{-5b^4 x^4 e^4 - 10x^3 a b^3 e^4 - 10x^3 b^4 d e^3 - 10x^2 a^2 b^2 e^4 - 10x^2 a b^3 d e^3 - 10x^2 b^4 d^2 e^2 - 5x a^3 b e^4 - 5x a^2 b^2 d e^3 - 5x a b^3 d^2 e^2 - 5x b^4 d^3 e}{5b^5 (b^2 x^2 + 2abx + a^2)^2 (bx+a)}$ |

input $\text{int}((e*x+d)^4/(b^2*x^2+2*a*b*x+a^2)^3, x, \text{method}=_RETURNVERBOSE)$

output

$$\frac{(-e^4/b^4x^4 - 2e^3(ae+bd)/b^2x^3 - 2e^2(a^2e^2+abd^2+ab^2d^2)/b^3x^2 - e(a^3e^3+a^2bd^2+ab^2d^2e+b^3d^3)/b^4x - 1/5(a^4e^4+a^3bd^2e^3+a^2b^2d^2e^2+ab^3d^3e+b^4d^4)/b^5)/(bx+a)/(b^2x^2+2abx+a^2)^2}$$

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 215 vs. $2(26) = 52$.

Time = 0.09 (sec) , antiderivative size = 215, normalized size of antiderivative = 7.68

$$\int \frac{(d+ex)^4}{(a^2+2abx+b^2x^2)^3} dx = \frac{5b^4e^4x^4 + b^4d^4 + ab^3d^3e + a^2b^2d^2e^2 + a^3bde^3 + a^4e^4 + 10(b^4de^3 + ab^3e^4)x^3 + 10(b^4d^2e^2 + ab^3de^3 + a^2b^2d^2e^2)x^2 + 5(b^4d^3e + a^2b^2d^2e^2 + a^3bde^3)x + a^4e^4}{5(b^{10}x^5 + 5ab^9x^4 + 10a^2b^8x^3 + 10a^3b^7x^2 + 5a^4b^6x + a^5)}$$

input

```
integrate((e*x+d)^4/(b^2*x^2+2*a*b*x+a^2)^3,x, algorithm="fricas")
```

output

$$\frac{-1/5(5b^4e^4x^4 + b^4d^4 + ab^3d^3e + a^2b^2d^2e^2 + a^3bd^2e^3 + a^4e^4 + 10(b^4d^2e^2 + ab^3d^2e^2 + a^2b^2d^2e^2)x^2 + 5(b^4d^3e + a^2b^2d^2e^2 + a^3bde^3)x + a^4e^4)/(b^{10}x^5 + 5a^2b^8x^3 + 10a^3b^7x^2 + 5a^4b^6x + a^5b^5)}$$

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 236 vs. $2(22) = 44$.

Time = 2.33 (sec) , antiderivative size = 236, normalized size of antiderivative = 8.43

$$\int \frac{(d+ex)^4}{(a^2+2abx+b^2x^2)^3} dx = \frac{-a^4e^4 - a^3bde^3 - a^2b^2d^2e^2 - ab^3d^3e - b^4d^4 - 5b^4e^4x^4 + x^3(-10ab^3e^4 - 10b^4de^3) + x^2(-10a^2b^2e^4 - 10a^3bd^2e^3) + x(-10a^2b^2d^2e^2 - 10a^3bde^3) + a^4e^4}{5a^5b^5 + 25a^4b^6x + 50a^3b^7x^2 + 50a^2b^8x^3 + 25ab^9x^4}$$

input

```
integrate((e*x+d)**4/(b**2*x**2+2*a*b*x+a**2)**3,x)
```

output

```
(-a**4*e**4 - a**3*b*d*e**3 - a**2*b**2*d**2*e**2 - a*b**3*d**3*e - b**4*d
**4 - 5*b**4*e**4*x**4 + x**3*(-10*a*b**3*e**4 - 10*b**4*d*e**3) + x**2*(-
10*a**2*b**2*e**4 - 10*a*b**3*d*e**3 - 10*b**4*d**2*e**2) + x*(-5*a**3*b*e
**4 - 5*a**2*b**2*d*e**3 - 5*a*b**3*d**2*e**2 - 5*b**4*d**3*e)) / (5*a**5*b
**5 + 25*a**4*b**6*x + 50*a**3*b**7*x**2 + 50*a**2*b**8*x**3 + 25*a*b**9*x
**4 + 5*b**10*x**5)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 215 vs. $2(26) = 52$.

Time = 0.04 (sec) , antiderivative size = 215, normalized size of antiderivative = 7.68

$$\int \frac{(d+ex)^4}{(a^2+2abx+b^2x^2)^3} dx = \frac{5b^4e^4x^4 + b^4d^4 + ab^3d^3e + a^2b^2d^2e^2 + a^3bde^3 + a^4e^4 + 10(b^4de^3 + ab^3e^4)x^3 + 10(b^4d^2e^2 + ab^3de^3 + a^2b^2d^2e^2)x^2 + 10(b^4d^3e + ab^3d^2e^2 + a^2b^2d^2e^2 + a^3bde^3)x + 5a^4e^4}{5(b^{10}x^5 + 5ab^9x^4 + 10a^2b^8x^3 + 10a^3b^7x^2 + 5a^4b^6x + a^5b^5)}$$

input

```
integrate((e*x+d)^4/(b^2*x^2+2*a*b*x+a^2)^3,x, algorithm="maxima")
```

output

```
-1/5*(5*b^4*e^4*x^4 + b^4*d^4 + a*b^3*d^3*e + a^2*b^2*d^2*e^2 + a^3*b*d*e
^3 + a^4*e^4 + 10*(b^4*d*e^3 + a*b^3*e^4)*x^3 + 10*(b^4*d^2*e^2 + a*b^3*d*e
^3 + a^2*b^2*e^4)*x^2 + 5*(b^4*d^3*e + a*b^3*d^2*e^2 + a^2*b^2*d^2*e^2 + a^3
*b*e^4)*x) / (b^10*x^5 + 5*a*b^9*x^4 + 10*a^2*b^8*x^3 + 10*a^3*b^7*x^2 + 5*a
^4*b^6*x + a^5*b^5)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 180 vs. $2(26) = 52$.

Time = 0.19 (sec) , antiderivative size = 180, normalized size of antiderivative = 6.43

$$\int \frac{(d+ex)^4}{(a^2+2abx+b^2x^2)^3} dx = \frac{5b^4e^4x^4 + 10b^4de^3x^3 + 10ab^3e^4x^3 + 10b^4d^2e^2x^2 + 10ab^3de^3x^2 + 10a^2b^2e^4x^2 + 5b^4d^3ex + 5ab^3d^2e^2x + 5a^2b^2d^2e^2 + a^3bde^3 + a^4e^4}{5(bx+a)^5b^5}$$

input `integrate((e*x+d)^4/(b^2*x^2+2*a*b*x+a^2)^3,x, algorithm="giac")`

output
$$-1/5*(5*b^4*e^4*x^4 + 10*b^4*d*e^3*x^3 + 10*a*b^3*e^4*x^3 + 10*b^4*d^2*e^2*x^2 + 10*a*b^3*d*e^3*x^2 + 10*a^2*b^2*e^4*x^2 + 5*b^4*d^3*e*x + 5*a*b^3*d^2*e^2*x + 5*a^2*b^2*d*e^3*x + 5*a^3*b*e^4*x + b^4*d^4 + a*b^3*d^3*e + a^2*b^2*d^2*e^2 + a^3*b*d*e^3 + a^4*e^4)/((b*x + a)^5*b^5)$$

Mupad [B] (verification not implemented)

Time = 8.70 (sec) , antiderivative size = 203, normalized size of antiderivative = 7.25

$$\int \frac{(d+ex)^4}{(a^2+2abx+b^2x^2)^3} dx = -\frac{\frac{a^4e^4+a^3bde^3+a^2b^2d^2e^2+ab^3d^3e+b^4d^4}{5b^5} + \frac{e^4x^4}{b} + \frac{2e^3x^3(ae+bd)}{b^2} + \frac{ex(a^3e^3+a^2bde^2+ab^2d^2e+b^3d^3)}{b^4} + \frac{2e^2x^2(a^2e^2+abde)}{b^3}}{a^5+5a^4bx+10a^3b^2x^2+10a^2b^3x^3+5ab^4x^4+b^5x^5}$$

input `int((d + e*x)^4/(a^2 + b^2*x^2 + 2*a*b*x)^3,x)`

output
$$-((a^4*e^4 + b^4*d^4 + a^2*b^2*d^2*e^2 + a*b^3*d^3*e + a^3*b*d*e^3)/(5*b^5) + (e^4*x^4)/b + (2*e^3*x^3*(a*e + b*d))/b^2 + (e*x*(a^3*e^3 + b^3*d^3 + a*b^2*d^2*e + a^2*b*d*e^2))/b^4 + (2*e^2*x^2*(a^2*e^2 + b^2*d^2 + a*b*d*e))/b^3)/(a^5 + b^5*x^5 + 5*a*b^4*x^4 + 10*a^3*b^2*x^2 + 10*a^2*b^3*x^3 + 5*a^4*b*x)$$

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 192, normalized size of antiderivative = 6.86

$$\int \frac{(d+ex)^4}{(a^2+2abx+b^2x^2)^3} dx = \frac{b^4e^4x^5 - 10ab^3de^3x^3 - 10a^2b^2de^3x^2 - 10ab^3d^2e^2x^2 - 5a^3bde^3x - 5a^2b^2d^2e^2x - 5ab^3d^3ex - a^4de^3 - a^5e^4}{5ab^4(b^5x^5 + 5ab^4x^4 + 10a^2b^3x^3 + 10a^3b^2x^2 + 5a^4bx + a^5)}$$

input `int((e*x+d)^4/(b^2*x^2+2*a*b*x+a^2)^3,x)`

output

```
( - a**4*d**3 - a**3*b*d**2*e**2 - 5*a**3*b*d*e**3*x - a**2*b**2*d**3*e
- 5*a**2*b**2*d**2*e**2*x - 10*a**2*b**2*d*e**3*x**2 - a*b**3*d**4 - 5*a*b
**3*d**3*e*x - 10*a*b**3*d**2*e**2*x**2 - 10*a*b**3*d*e**3*x**3 + b**4*e**
4*x**5)/(5*a*b**4*(a**5 + 5*a**4*b*x + 10*a**3*b**2*x**2 + 10*a**2*b**3*x*
*3 + 5*a*b**4*x**4 + b**5*x**5))
```

3.133 $\int \frac{(d+ex)^3}{(a^2+2abx+b^2x^2)^3} dx$

| | |
|---|------|
| Optimal result | 1065 |
| Mathematica [A] (verified) | 1065 |
| Rubi [A] (verified) | 1066 |
| Maple [B] (verified) | 1067 |
| Fricas [B] (verification not implemented) | 1068 |
| Sympy [B] (verification not implemented) | 1069 |
| Maxima [B] (verification not implemented) | 1069 |
| Giac [B] (verification not implemented) | 1070 |
| Mupad [B] (verification not implemented) | 1070 |
| Reduce [B] (verification not implemented) | 1071 |

Optimal result

Integrand size = 26, antiderivative size = 58

$$\int \frac{(d+ex)^3}{(a^2+2abx+b^2x^2)^3} dx = -\frac{(d+ex)^4}{5(bd-ae)(a+bx)^5} + \frac{e(d+ex)^4}{20(bd-ae)^2(a+bx)^4}$$

output

$$-1/5*(e*x+d)^4/(-a*e+b*d)/(b*x+a)^5+1/20*e*(e*x+d)^4/(-a*e+b*d)^2/(b*x+a)^4$$

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.67

$$\int \frac{(d+ex)^3}{(a^2+2abx+b^2x^2)^3} dx = \frac{a^3e^3 + a^2be^2(2d+5ex) + ab^2e(3d^2+10dex+10e^2x^2) + b^3(4d^3+15d^2ex+20de^2x^2+10e^3x^3)}{20b^4(a+bx)^5}$$

input

$$\text{Integrate}[(d+e*x)^3/(a^2+2*a*b*x+b^2*x^2)^3,x]$$

output

$$-1/20*(a^3*e^3 + a^2*b*e^2*(2*d + 5*e*x) + a*b^2*e*(3*d^2 + 10*d*e*x + 10*e^2*x^2) + b^3*(4*d^3 + 15*d^2*e*x + 20*d*e^2*x^2 + 10*e^3*x^3))/(b^4*(a + b*x)^5)$$

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {1098, 27, 55, 48}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\begin{aligned} & \int \frac{(d+ex)^3}{(a^2+2abx+b^2x^2)^3} dx \\ & \quad \downarrow 1098 \\ & b^6 \int \frac{(d+ex)^3}{b^6(a+bx)^6} dx \\ & \quad \downarrow 27 \\ & \int \frac{(d+ex)^3}{(a+bx)^6} dx \\ & \quad \downarrow 55 \\ & \frac{e \int \frac{(d+ex)^3}{(a+bx)^5} dx}{5(bd-ae)} - \frac{(d+ex)^4}{5(a+bx)^5(bd-ae)} \\ & \quad \downarrow 48 \\ & \frac{e(d+ex)^4}{20(a+bx)^4(bd-ae)^2} - \frac{(d+ex)^4}{5(a+bx)^5(bd-ae)} \end{aligned}$$

input

$$\text{Int}[(d + e*x)^3/(a^2 + 2*a*b*x + b^2*x^2)^3,x]$$

output

$$-1/5*(d + e*x)^4/((b*d - a*e)*(a + b*x)^5) + (e*(d + e*x)^4)/(20*(b*d - a*e)^2*(a + b*x)^4)$$

Defintions of rubi rules used

rule 27 `Int[(a_)*(F_x_), x_Symbol] := Simp[a Int[F_x, x], x] /; FreeQ[a, x] && !MatchQ[F_x, (b_)*(G_x_) /; FreeQ[b, x]]`

rule 48 `Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] := Simp[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{a, b, c, d, m, n}, x] && EqQ[m + n + 2, 0] && NeQ[m, -1]`

rule 55 `Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] := Simp[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*(Simplify[m + n + 2]/((b*c - a*d)*(m + 1))) Int[(a + b*x)^Simplify[m + 1]*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && ILtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] && !(LtQ[m, -1] && LtQ[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimplerQ[m, 1] || !SumSimplerQ[n, 1])`

rule 1098 `Int[((d_.) + (e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] := Simp[1/c^p Int[(d + e*x)^m*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]`

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 120 vs. $2(54) = 108$.

Time = 1.06 (sec) , antiderivative size = 121, normalized size of antiderivative = 2.09

| method | result | size |
|---------------|--|------|
| default | $-\frac{-e^3 a^3 + 3a^2 b d e^2 - 3a b^2 d^2 e + b^3 d^3}{5b^4 (bx+a)^5} + \frac{e^2 (ae-bd)}{b^4 (bx+a)^3} - \frac{e^3}{2b^4 (bx+a)^2} - \frac{3e(e^2 a^2 - 2abde + b^2 d^2)}{4b^4 (bx+a)^4}$ | 12 |
| norman | $-\frac{e^3 x^3}{2b} + \frac{(-a e^3 b - 2d e^2 b^2) x^2}{2b^3} + \frac{(-a^2 b e^3 - 2a b^2 d e^2 - 3b^3 d^2 e) x}{4b^4} + \frac{-a^3 b e^3 - 2a^2 b^2 d e^2 - 3d^2 e a b^3 - 4b^4 d^3}{20b^5}$ | 12 |
| risch | $\frac{-\frac{e^3 x^3}{2b} - \frac{e^2 (ae+2bd)x^2}{2b^2} - \frac{e(e^2 a^2 + 2abde + 3b^2 d^2)x}{4b^3} - \frac{e^3 a^3 + 2a^2 b d e^2 + 3a b^2 d^2 e + 4b^3 d^3}{20b^4}}{(bx+a)(b^2 x^2 + 2abx + a^2)^2}$ | 12 |
| orering | $-\frac{(10e^3 x^3 b^3 + 10x^2 a b^2 e^3 + 20x^2 b^3 d e^2 + 5a^2 b e^3 x + 10x a b^2 d e^2 + 15b^3 d^2 e x + e^3 a^3 + 2a^2 b d e^2 + 3a b^2 d^2 e + 4b^3 d^3)(bx+a)}{20b^4 (b^2 x^2 + 2abx + a^2)^3}$ | 13 |
| gospers | $-\frac{10e^3 x^3 b^3 + 10x^2 a b^2 e^3 + 20x^2 b^3 d e^2 + 5a^2 b e^3 x + 10x a b^2 d e^2 + 15b^3 d^2 e x + e^3 a^3 + 2a^2 b d e^2 + 3a b^2 d^2 e + 4b^3 d^3}{20b^4 (bx+a)(b^2 x^2 + 2abx + a^2)^2}$ | 13 |
| parallelrisch | $\frac{-10e^3 x^3 b^4 - 10a b^3 e^3 x^2 - 20b^4 d e^2 x^2 - 5a^2 b^2 e^3 x - 10a b^3 d e^2 x - 15b^4 d^2 e x - a^3 b e^3 - 2a^2 b^2 d e^2 - 3d^2 e a b^3 - 4b^4 d^3}{20b^5 (b^2 x^2 + 2abx + a^2)^2 (bx+a)}$ | 13 |

```
input int((e*x+d)^3/(b^2*x^2+2*a*b*x+a^2)^3,x,method=_RETURNVERBOSE)
```

```
output -1/5*(-a^3*e^3+3*a^2*b*d*e^2-3*a*b^2*d^2*e+b^3*d^3)/b^4/(b*x+a)^5+e^2*(a*e-b*d)/b^4/(b*x+a)^3-1/2*e^3/b^4/(b*x+a)^2-3/4/b^4*e*(a^2*e^2-2*a*b*d*e+b^2*d^2)/(b*x+a)^4
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 160 vs. 2(54) = 108.

Time = 0.08 (sec) , antiderivative size = 160, normalized size of antiderivative = 2.76

$$\int \frac{(d+ex)^3}{(a^2+2abx+b^2x^2)^3} dx = \frac{10b^3e^3x^3 + 4b^3d^3 + 3ab^2d^2e + 2a^2bde^2 + a^3e^3 + 10(2b^3de^2 + ab^2e^3)x^2 + 5(3b^3d^2e + 2ab^2de^2 + a^2b^3d^2)}{20(b^9x^5 + 5ab^8x^4 + 10a^2b^7x^3 + 10a^3b^6x^2 + 5a^4b^5x + a^5b^4)}$$

```
input integrate((e*x+d)^3/(b^2*x^2+2*a*b*x+a^2)^3,x, algorithm="fricas")
```

```
output -1/20*(10*b^3*e^3*x^3 + 4*b^3*d^3 + 3*a*b^2*d^2*e + 2*a^2*b*d*e^2 + a^3*e^3 + 10*(2*b^3*d*e^2 + a*b^2*e^3)*x^2 + 5*(3*b^3*d^2*e + 2*a*b^2*d*e^2 + a^2*b^3*d^2)*x)/(b^9*x^5 + 5*a*b^8*x^4 + 10*a^2*b^7*x^3 + 10*a^3*b^6*x^2 + 5*a^4*b^5*x + a^5*b^4)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 172 vs. $2(46) = 92$.

Time = 1.09 (sec) , antiderivative size = 172, normalized size of antiderivative = 2.97

$$\int \frac{(d+ex)^3}{(a^2+2abx+b^2x^2)^3} dx = \frac{-a^3e^3 - 2a^2bde^2 - 3ab^2d^2e - 4b^3d^3 - 10b^3e^3x^3 + x^2(-10ab^2e^3 - 20b^3de^2) + x(-5a^2be^3 - 10ab^2de^2 - 10a^2b^2e^2) + 5a^2b^2e^2x^2 + 5ab^3de^2 + 5a^3e^3x}{20a^5b^4 + 100a^4b^5x + 200a^3b^6x^2 + 200a^2b^7x^3 + 100ab^8x^4 + 20b^9x^5}$$

input `integrate((e*x+d)**3/(b**2*x**2+2*a*b*x+a**2)**3,x)`

output `(-a**3*e**3 - 2*a**2*b*d*e**2 - 3*a*b**2*d**2*e - 4*b**3*d**3 - 10*b**3*e**3*x**3 + x**2*(-10*a*b**2*e**3 - 20*b**3*d*e**2) + x*(-5*a**2*b*e**3 - 10*a*b**2*d*e**2 - 15*b**3*d**2*e))/(20*a**5*b**4 + 100*a**4*b**5*x + 200*a**3*b**6*x**2 + 200*a**2*b**7*x**3 + 100*a*b**8*x**4 + 20*b**9*x**5)`

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 160 vs. $2(54) = 108$.

Time = 0.05 (sec) , antiderivative size = 160, normalized size of antiderivative = 2.76

$$\int \frac{(d+ex)^3}{(a^2+2abx+b^2x^2)^3} dx = \frac{-10b^3e^3x^3 + 4b^3d^3 + 3ab^2d^2e + 2a^2bde^2 + a^3e^3 + 10(2b^3de^2 + ab^2e^3)x^2 + 5(3b^3d^2e + 2ab^2de^2 + a^2b^2e^2)x + 5a^2b^2e^2x^2 + 5ab^3de^2 + 5a^3e^3x}{20(b^9x^5 + 5ab^8x^4 + 10a^2b^7x^3 + 10a^3b^6x^2 + 5a^4b^5x + a^5b^4)}$$

input `integrate((e*x+d)^3/(b^2*x^2+2*a*b*x+a^2)^3,x, algorithm="maxima")`

output `-1/20*(10*b^3*e^3*x^3 + 4*b^3*d^3 + 3*a*b^2*d^2*e + 2*a^2*b*d*e^2 + a^3*e^3 + 10*(2*b^3*d*e^2 + a*b^2*e^3)*x^2 + 5*(3*b^3*d^2*e + 2*a*b^2*d*e^2 + a^2*b^2*e^3)*x)/(b^9*x^5 + 5*a*b^8*x^4 + 10*a^2*b^7*x^3 + 10*a^3*b^6*x^2 + 5*a^4*b^5*x + a^5*b^4)`

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 114 vs. $2(54) = 108$.

Time = 0.17 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.97

$$\int \frac{(d+ex)^3}{(a^2+2abx+b^2x^2)^3} dx = \frac{10b^3e^3x^3 + 20b^3de^2x^2 + 10ab^2e^3x^2 + 15b^3d^2ex + 10ab^2de^2x + 5a^2be^3x + 4b^3d^3 + 3ab^2d^2e + 2a^2bd^2}{20(bx+a)^5b^4}$$

input `integrate((e*x+d)^3/(b^2*x^2+2*a*b*x+a^2)^3,x, algorithm="giac")`

output `-1/20*(10*b^3*e^3*x^3 + 20*b^3*d*e^2*x^2 + 10*a*b^2*e^3*x^2 + 15*b^3*d^2*e*x + 10*a*b^2*d*e^2*x + 5*a^2*b*e^3*x + 4*b^3*d^3 + 3*a*b^2*d^2*e + 2*a^2*b*d*e^2 + a^3*e^3)/((b*x + a)^5*b^4)`

Mupad [B] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 154, normalized size of antiderivative = 2.66

$$\int \frac{(d+ex)^3}{(a^2+2abx+b^2x^2)^3} dx = -\frac{\frac{a^3e^3+2a^2bde^2+3ab^2d^2e+4b^3d^3}{20b^4} + \frac{e^3x^3}{2b} + \frac{ex(a^2e^2+2abde+3b^2d^2)}{4b^3} + \frac{e^2x^2(ae+2bd)}{2b^2}}{a^5 + 5a^4bx + 10a^3b^2x^2 + 10a^2b^3x^3 + 5ab^4x^4 + b^5x^5}$$

input `int((d + e*x)^3/(a^2 + b^2*x^2 + 2*a*b*x)^3,x)`

output `-((a^3*e^3 + 4*b^3*d^3 + 3*a*b^2*d^2*e + 2*a^2*b*d*e^2)/(20*b^4) + (e^3*x^3)/(2*b) + (e*x*(a^2*e^2 + 3*b^2*d^2 + 2*a*b*d*e))/(4*b^3) + (e^2*x^2*(a*e + 2*b*d))/(2*b^2))/(a^5 + b^5*x^5 + 5*a*b^4*x^4 + 10*a^3*b^2*x^2 + 10*a^2*b^3*x^3 + 5*a^4*b*x)`

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 159, normalized size of antiderivative = 2.74

$$\int \frac{(d + ex)^3}{(a^2 + 2abx + b^2x^2)^3} dx$$

$$= \frac{-10b^3e^3x^3 - 10ab^2e^3x^2 - 20b^3de^2x^2 - 5a^2be^3x - 10ab^2de^2x - 15b^3d^2ex - a^3e^3 - 2a^2bde^2 - 3ab^2d^2e}{20b^4(b^5x^5 + 5ab^4x^4 + 10a^2b^3x^3 + 10a^3b^2x^2 + 5a^4bx + a^5)}$$

input `int((e*x+d)^3/(b^2*x^2+2*a*b*x+a^2)^3,x)`output `(- a**3*e**3 - 2*a**2*b*d*e**2 - 5*a**2*b*e**3*x - 3*a*b**2*d**2*e - 10*a*b**2*d*e**2*x - 10*a*b**2*e**3*x**2 - 4*b**3*d**3 - 15*b**3*d**2*e*x - 20*b**3*d*e**2*x**2 - 10*b**3*e**3*x**3)/(20*b**4*(a**5 + 5*a**4*b*x + 10*a**3*b**2*x**2 + 10*a**2*b**3*x**3 + 5*a*b**4*x**4 + b**5*x**5))`

$$3.134 \quad \int \frac{(d+ex)^2}{(a^2+2abx+b^2x^2)^3} dx$$

| | |
|---|------|
| Optimal result | 1072 |
| Mathematica [A] (verified) | 1072 |
| Rubi [A] (verified) | 1073 |
| Maple [A] (verified) | 1074 |
| Fricas [A] (verification not implemented) | 1075 |
| Sympy [B] (verification not implemented) | 1075 |
| Maxima [A] (verification not implemented) | 1076 |
| Giac [A] (verification not implemented) | 1076 |
| Mupad [B] (verification not implemented) | 1076 |
| Reduce [B] (verification not implemented) | 1077 |

Optimal result

Integrand size = 26, antiderivative size = 65

$$\int \frac{(d+ex)^2}{(a^2+2abx+b^2x^2)^3} dx = -\frac{(bd-ae)^2}{5b^3(a+bx)^5} - \frac{e(bd-ae)}{2b^3(a+bx)^4} - \frac{e^2}{3b^3(a+bx)^3}$$

output

```
-1/5*(-a*e+b*d)^2/b^3/(b*x+a)^5-1/2*e*(-a*e+b*d)/b^3/(b*x+a)^4-1/3*e^2/b^3/(b*x+a)^3
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.88

$$\int \frac{(d+ex)^2}{(a^2+2abx+b^2x^2)^3} dx = -\frac{a^2e^2+abe(3d+5ex)+b^2(6d^2+15dex+10e^2x^2)}{30b^3(a+bx)^5}$$

input

```
Integrate[(d + e*x)^2/(a^2 + 2*a*b*x + b^2*x^2)^3,x]
```

output

```
-1/30*(a^2*e^2 + a*b*e*(3*d + 5*e*x) + b^2*(6*d^2 + 15*d*e*x + 10*e^2*x^2))/(b^3*(a + b*x)^5)
```

Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {1098, 27, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\begin{aligned}
 & \int \frac{(d+ex)^2}{(a^2+2abx+b^2x^2)^3} dx \\
 & \quad \downarrow \text{1098} \\
 & b^6 \int \frac{(d+ex)^2}{b^6(a+bx)^6} dx \\
 & \quad \downarrow \text{27} \\
 & \int \frac{(d+ex)^2}{(a+bx)^6} dx \\
 & \quad \downarrow \text{53} \\
 & \int \left(\frac{2e(bd-ae)}{b^2(a+bx)^5} + \frac{(bd-ae)^2}{b^2(a+bx)^6} + \frac{e^2}{b^2(a+bx)^4} \right) dx \\
 & \quad \downarrow \text{2009} \\
 & -\frac{e(bd-ae)}{2b^3(a+bx)^4} - \frac{(bd-ae)^2}{5b^3(a+bx)^5} - \frac{e^2}{3b^3(a+bx)^3}
 \end{aligned}$$

input `Int[(d + e*x)^2/(a^2 + 2*a*b*x + b^2*x^2)^3,x]`

output `-1/5*(b*d - a*e)^2/(b^3*(a + b*x)^5) - (e*(b*d - a*e))/(2*b^3*(a + b*x)^4) - e^2/(3*b^3*(a + b*x)^3)`

Defintions of rubi rules used

rule 27 $\text{Int}[(a_*)(Fx_), x_Symbol] \rightarrow \text{Simp}[a \text{ Int}[Fx, x], x] /; \text{FreeQ}[a, x] \ \&\& \ !\text{MatchQ}[Fx, (b_*)(Gx_)] /; \text{FreeQ}[b, x]$

rule 53 $\text{Int}[(a_.) + (b_.)*(x_)^{(m_.)}*((c_.) + (d_.)*(x_)^{(n_.)}), x_Symbol] \rightarrow \text{Int}[\text{ExpandIntegrand}[(a + b*x)^m*(c + d*x)^n, x], x] /; \text{FreeQ}[\{a, b, c, d, n\}, x] \ \&\& \ \text{IGtQ}[m, 0] \ \&\& \ (\ !\text{IntegerQ}[n] \ || \ (\text{EqQ}[c, 0] \ \&\& \ \text{LeQ}[7*m + 4*n + 4, 0]) \ || \ \text{LtQ}[9*m + 5*(n + 1), 0] \ || \ \text{GtQ}[m + n + 2, 0])$

rule 1098 $\text{Int}[(d_.) + (e_.)*(x_)^{(m_.)}*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^{(p_.)}, x_Symbol] \rightarrow \text{Simp}[1/c^p \text{ Int}[(d + e*x)^m*(b/2 + c*x)^(2*p), x], x] /; \text{FreeQ}[\{a, b, c, d, e, m\}, x] \ \&\& \ \text{EqQ}[b^2 - 4*a*c, 0] \ \&\& \ \text{IntegerQ}[p]$

rule 2009 $\text{Int}[u_, x_Symbol] \rightarrow \text{Simp}[\text{IntSum}[u, x], x] /; \text{SumQ}[u]$

Maple [A] (verified)

Time = 1.07 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.09

| method | result | size |
|---------------|--|------|
| default | $-\frac{e^2 a^2 - 2abde + b^2 d^2}{5b^3 (bx+a)^5} - \frac{e^2}{3b^3 (bx+a)^3} + \frac{e(ae-bd)}{2b^3 (bx+a)^4}$ | 71 |
| norman | $-\frac{e^2 x^2}{3b} + \frac{(-a b^2 e^2 - 3b^3 de)x}{6b^4} + \frac{-a^2 b^2 e^2 - 3a b^3 de - 6d^2 b^4}{30b^5}$ | 77 |
| orering | $-\frac{(10b^2 e^2 x^2 + 5xab e^2 + 15b^2 dex + e^2 a^2 + 3abde + 6b^2 d^2)(bx+a)}{30b^3 (b^2 x^2 + 2abx + a^2)^3}$ | 78 |
| gospers | $-\frac{10b^2 e^2 x^2 + 5xab e^2 + 15b^2 dex + e^2 a^2 + 3abde + 6b^2 d^2}{30b^3 (bx+a)(b^2 x^2 + 2abx + a^2)^2}$ | 80 |
| risch | $-\frac{e^2 x^2}{3b} - \frac{e(ae+3bd)x}{6b^2} - \frac{e^2 a^2 + 3abde + 6b^2 d^2}{30b^3}$ | 81 |
| parallelrisch | $-\frac{10e^2 x^2 b^4 - 5a b^3 e^2 x - 15b^4 dex - a^2 b^2 e^2 - 3a b^3 de - 6d^2 b^4}{30b^5 (b^2 x^2 + 2abx + a^2)^2 (bx+a)}$ | 88 |

input $\text{int}((e*x+d)^2/(b^2*x^2+2*a*b*x+a^2)^3, x, \text{method}=_RETURNVERBOSE)$

output
$$-1/5*(a^2*e^2-2*a*b*d*e+b^2*d^2)/b^3/(b*x+a)^5-1/3*e^2/b^3/(b*x+a)^3+1/2/b^3*e*(a*e-b*d)/(b*x+a)^4$$

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 109, normalized size of antiderivative = 1.68

$$\int \frac{(d+ex)^2}{(a^2+2abx+b^2x^2)^3} dx$$

$$= -\frac{10b^2e^2x^2 + 6b^2d^2 + 3abde + a^2e^2 + 5(3b^2de + abe^2)x}{30(b^8x^5 + 5ab^7x^4 + 10a^2b^6x^3 + 10a^3b^5x^2 + 5a^4b^4x + a^5b^3)}$$

input `integrate((e*x+d)^2/(b^2*x^2+2*a*b*x+a^2)^3,x, algorithm="fricas")`

output
$$-1/30*(10*b^2*e^2*x^2 + 6*b^2*d^2 + 3*a*b*d*e + a^2*e^2 + 5*(3*b^2*d*e + a*b*e^2)*x)/(b^8*x^5 + 5*a*b^7*x^4 + 10*a^2*b^6*x^3 + 10*a^3*b^5*x^2 + 5*a^4*b^4*x + a^5*b^3)$$

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 116 vs. 2(56) = 112.

Time = 0.49 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.78

$$\int \frac{(d+ex)^2}{(a^2+2abx+b^2x^2)^3} dx$$

$$= \frac{-a^2e^2 - 3abde - 6b^2d^2 - 10b^2e^2x^2 + x(-5abe^2 - 15b^2de)}{30a^5b^3 + 150a^4b^4x + 300a^3b^5x^2 + 300a^2b^6x^3 + 150ab^7x^4 + 30b^8x^5}$$

input `integrate((e*x+d)**2/(b**2*x**2+2*a*b*x+a**2)**3,x)`

output
$$(-a**2*e**2 - 3*a*b*d*e - 6*b**2*d**2 - 10*b**2*e**2*x**2 + x*(-5*a*b*e**2 - 15*b**2*d*e))/(30*a**5*b**3 + 150*a**4*b**4*x + 300*a**3*b**5*x**2 + 300*a**2*b**6*x**3 + 150*a*b**7*x**4 + 30*b**8*x**5)$$

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 109, normalized size of antiderivative = 1.68

$$\int \frac{(d+ex)^2}{(a^2+2abx+b^2x^2)^3} dx = -\frac{10b^2e^2x^2+6b^2d^2+3abde+a^2e^2+5(3b^2de+abe^2)x}{30(b^8x^5+5ab^7x^4+10a^2b^6x^3+10a^3b^5x^2+5a^4b^4x+a^5b^3)}$$

input `integrate((e*x+d)^2/(b^2*x^2+2*a*b*x+a^2)^3,x, algorithm="maxima")`output `-1/30*(10*b^2*e^2*x^2 + 6*b^2*d^2 + 3*a*b*d*e + a^2*e^2 + 5*(3*b^2*d*e + a*b*e^2)*x)/(b^8*x^5 + 5*a*b^7*x^4 + 10*a^2*b^6*x^3 + 10*a^3*b^5*x^2 + 5*a^4*b^4*x + a^5*b^3)`**Giac [A] (verification not implemented)**

Time = 0.19 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.94

$$\int \frac{(d+ex)^2}{(a^2+2abx+b^2x^2)^3} dx = -\frac{10b^2e^2x^2+15b^2dex+5abe^2x+6b^2d^2+3abde+a^2e^2}{30(bx+a)^5b^3}$$

input `integrate((e*x+d)^2/(b^2*x^2+2*a*b*x+a^2)^3,x, algorithm="giac")`output `-1/30*(10*b^2*e^2*x^2 + 15*b^2*d*e*x + 5*a*b*e^2*x + 6*b^2*d^2 + 3*a*b*d*e + a^2*e^2)/((b*x + a)^5*b^3)`**Mupad [B] (verification not implemented)**

Time = 0.06 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.65

$$\int \frac{(d+ex)^2}{(a^2+2abx+b^2x^2)^3} dx = -\frac{\frac{a^2e^2+3abde+6b^2d^2}{30b^3} + \frac{e^2x^2}{3b} + \frac{ex(ae+3bd)}{6b^2}}{a^5+5a^4bx+10a^3b^2x^2+10a^2b^3x^3+5ab^4x^4+b^5x^5}$$

input `int((d + e*x)^2/(a^2 + b^2*x^2 + 2*a*b*x)^3,x)`

output

```

-((a^2*e^2 + 6*b^2*d^2 + 3*a*b*d*e)/(30*b^3) + (e^2*x^2)/(3*b) + (e*x*(a*e
+ 3*b*d))/(6*b^2))/(a^5 + b^5*x^5 + 5*a*b^4*x^4 + 10*a^3*b^2*x^2 + 10*a^2
*b^3*x^3 + 5*a^4*b*x)

```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 106, normalized size of antiderivative = 1.63

$$\int \frac{(d + ex)^2}{(a^2 + 2abx + b^2x^2)^3} dx = \frac{-10b^2e^2x^2 - 5abe^2x - 15b^2dex - a^2e^2 - 3abde - 6b^2d^2}{30b^3(b^5x^5 + 5ab^4x^4 + 10a^2b^3x^3 + 10a^3b^2x^2 + 5a^4bx + a^5)}$$

input

```

int((e*x+d)^2/(b^2*x^2+2*a*b*x+a^2)^3,x)

```

output

```

( - a**2*e**2 - 3*a*b*d*e - 5*a*b*e**2*x - 6*b**2*d**2 - 15*b**2*d*e*x - 1
0*b**2*e**2*x**2)/(30*b**3*(a**5 + 5*a**4*b*x + 10*a**3*b**2*x**2 + 10*a**
2*b**3*x**3 + 5*a*b**4*x**4 + b**5*x**5))

```

3.135 $\int \frac{d+ex}{(a^2+2abx+b^2x^2)^3} dx$

| | |
|---|------|
| Optimal result | 1078 |
| Mathematica [A] (verified) | 1078 |
| Rubi [A] (verified) | 1079 |
| Maple [A] (verified) | 1080 |
| Fricas [B] (verification not implemented) | 1081 |
| Sympy [B] (verification not implemented) | 1081 |
| Maxima [B] (verification not implemented) | 1082 |
| Giac [A] (verification not implemented) | 1082 |
| Mupad [B] (verification not implemented) | 1082 |
| Reduce [B] (verification not implemented) | 1083 |

Optimal result

Integrand size = 24, antiderivative size = 38

$$\int \frac{d+ex}{(a^2+2abx+b^2x^2)^3} dx = -\frac{bd-ae}{5b^2(a+bx)^5} - \frac{e}{4b^2(a+bx)^4}$$

output `-1/5*(-a*e+b*d)/b^2/(b*x+a)^5-1/4*e/b^2/(b*x+a)^4`

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.71

$$\int \frac{d+ex}{(a^2+2abx+b^2x^2)^3} dx = -\frac{4bd+ae+5bex}{20b^2(a+bx)^5}$$

input `Integrate[(d + e*x)/(a^2 + 2*a*b*x + b^2*x^2)^3,x]`

output `-1/20*(4*b*d + a*e + 5*b*e*x)/(b^2*(a + b*x)^5)`

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {1098, 27, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\begin{aligned}
 & \int \frac{d + ex}{(a^2 + 2abx + b^2x^2)^3} dx \\
 & \quad \downarrow \text{1098} \\
 & b^6 \int \frac{d + ex}{b^6(a + bx)^6} dx \\
 & \quad \downarrow \text{27} \\
 & \int \frac{d + ex}{(a + bx)^6} dx \\
 & \quad \downarrow \text{53} \\
 & \int \left(\frac{bd - ae}{b(a + bx)^6} + \frac{e}{b(a + bx)^5} \right) dx \\
 & \quad \downarrow \text{2009} \\
 & -\frac{bd - ae}{5b^2(a + bx)^5} - \frac{e}{4b^2(a + bx)^4}
 \end{aligned}$$

input `Int[(d + e*x)/(a^2 + 2*a*b*x + b^2*x^2)^3,x]`

output `-1/5*(b*d - a*e)/(b^2*(a + b*x)^5) - e/(4*b^2*(a + b*x)^4)`

Definitions of rubi rules used

rule 27 $\text{Int}[(a_*)(Fx_), x_Symbol] \rightarrow \text{Simp}[a \text{ Int}[Fx, x], x] /; \text{FreeQ}[a, x] \ \&\& \ !\text{MatchQ}[Fx, (b_*)(Gx_)] /; \text{FreeQ}[b, x]$

rule 53 $\text{Int}[(a_.) + (b_.)*(x_)^{(m_.)}*((c_.) + (d_.)*(x_)^{(n_.)}), x_Symbol] \rightarrow \text{Int}[\text{ExpandIntegrand}[(a + b*x)^m*(c + d*x)^n, x], x] /; \text{FreeQ}[\{a, b, c, d, n\}, x] \ \&\& \ \text{IGtQ}[m, 0] \ \&\& \ (\ !\text{IntegerQ}[n] \ || \ (\text{EqQ}[c, 0] \ \&\& \ \text{LeQ}[7*m + 4*n + 4, 0]) \ || \ \text{LtQ}[9*m + 5*(n + 1), 0] \ || \ \text{GtQ}[m + n + 2, 0])$

rule 1098 $\text{Int}[(d_.) + (e_.)*(x_)^{(m_.)}*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^{(p_.)}, x_Symbol] \rightarrow \text{Simp}[1/c^p \text{ Int}[(d + e*x)^m*(b/2 + c*x)^{(2*p)}, x], x] /; \text{FreeQ}[\{a, b, c, d, e, m\}, x] \ \&\& \ \text{EqQ}[b^2 - 4*a*c, 0] \ \&\& \ \text{IntegerQ}[p]$

rule 2009 $\text{Int}[u_, x_Symbol] \rightarrow \text{Simp}[\text{IntSum}[u, x], x] /; \text{SumQ}[u]$

Maple [A] (verified)

Time = 1.02 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.92

| method | result | size |
|---------------|---|------|
| default | $-\frac{-ae+bd}{5b^2(bx+a)^5} - \frac{e}{4b^2(bx+a)^4}$ | 35 |
| norman | $\frac{-\frac{ex}{4b} + \frac{-ab^3e-4db^4}{20b^5}}{(bx+a)^5}$ | 36 |
| orering | $-\frac{(5bex+ae+4bd)(bx+a)}{20b^2(b^2x^2+2abx+a^2)^3}$ | 42 |
| gospers | $-\frac{5bex+ae+4bd}{20b^2(bx+a)(b^2x^2+2abx+a^2)^2}$ | 44 |
| risch | $\frac{-\frac{ex}{4b} - \frac{ae+4bd}{20b^2}}{(bx+a)(b^2x^2+2abx+a^2)^2}$ | 48 |
| parallelrisch | $\frac{-5exb^4-ab^3e-4db^4}{20b^5(b^2x^2+2abx+a^2)^2(bx+a)}$ | 52 |

input $\text{int}((e*x+d)/(b^2*x^2+2*a*b*x+a^2)^3, x, \text{method}=_RETURNVERBOSE)$

output $-1/5*(-a*e+b*d)/b^2/(b*x+a)^5-1/4*e/b^2/(b*x+a)^4$

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 72 vs. $2(34) = 68$.

Time = 0.07 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.89

$$\int \frac{d + ex}{(a^2 + 2abx + b^2x^2)^3} dx$$

$$= -\frac{5bex + 4bd + ae}{20(b^7x^5 + 5ab^6x^4 + 10a^2b^5x^3 + 10a^3b^4x^2 + 5a^4b^3x + a^5b^2)}$$

input `integrate((e*x+d)/(b^2*x^2+2*a*b*x+a^2)^3,x, algorithm="fricas")`

output $-1/20*(5*b*e*x + 4*b*d + a*e)/(b^7*x^5 + 5*a*b^6*x^4 + 10*a^2*b^5*x^3 + 10*a^3*b^4*x^2 + 5*a^4*b^3*x + a^5*b^2)$

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 76 vs. $2(32) = 64$.

Time = 0.26 (sec) , antiderivative size = 76, normalized size of antiderivative = 2.00

$$\int \frac{d + ex}{(a^2 + 2abx + b^2x^2)^3} dx$$

$$= \frac{-ae - 4bd - 5bex}{20a^5b^2 + 100a^4b^3x + 200a^3b^4x^2 + 200a^2b^5x^3 + 100ab^6x^4 + 20b^7x^5}$$

input `integrate((e*x+d)/(b**2*x**2+2*a*b*x+a**2)**3,x)`

output $(-a*e - 4*b*d - 5*b*e*x)/(20*a**5*b**2 + 100*a**4*b**3*x + 200*a**3*b**4*x**2 + 200*a**2*b**5*x**3 + 100*a*b**6*x**4 + 20*b**7*x**5)$

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 72 vs. $2(34) = 68$.

Time = 0.04 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.89

$$\int \frac{d + ex}{(a^2 + 2abx + b^2x^2)^3} dx = -\frac{5bex + 4bd + ae}{20(b^7x^5 + 5ab^6x^4 + 10a^2b^5x^3 + 10a^3b^4x^2 + 5a^4b^3x + a^5b^2)}$$

input `integrate((e*x+d)/(b^2*x^2+2*a*b*x+a^2)^3,x, algorithm="maxima")`

output `-1/20*(5*b*e*x + 4*b*d + a*e)/(b^7*x^5 + 5*a*b^6*x^4 + 10*a^2*b^5*x^3 + 10*a^3*b^4*x^2 + 5*a^4*b^3*x + a^5*b^2)`

Giac [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.66

$$\int \frac{d + ex}{(a^2 + 2abx + b^2x^2)^3} dx = -\frac{5bex + 4bd + ae}{20(bx + a)^5b^2}$$

input `integrate((e*x+d)/(b^2*x^2+2*a*b*x+a^2)^3,x, algorithm="giac")`

output `-1/20*(5*b*e*x + 4*b*d + a*e)/((b*x + a)^5*b^2)`

Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.95

$$\int \frac{d + ex}{(a^2 + 2abx + b^2x^2)^3} dx = -\frac{\frac{ae+4bd}{20b^2} + \frac{ex}{4b}}{a^5 + 5a^4bx + 10a^3b^2x^2 + 10a^2b^3x^3 + 5ab^4x^4 + b^5x^5}$$

input `int((d + e*x)/(a^2 + b^2*x^2 + 2*a*b*x)^3,x)`

output

$$-\left(\frac{a^3 b^2 x^2 + 10 a^2 b^3 x^3 + 5 a^4 b x}{(a^5 + b^5 x^5 + 5 a b^4 x^4 + 10 a^3 b^2 x^2 + 10 a^2 b^3 x^3 + 5 a^4 b x)} + \frac{e x}{4 b}\right)$$
Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.84

$$\int \frac{d + e x}{(a^2 + 2 a b x + b^2 x^2)^3} dx = \frac{-5 b e x - a e - 4 b d}{20 b^2 (b^5 x^5 + 5 a b^4 x^4 + 10 a^2 b^3 x^3 + 10 a^3 b^2 x^2 + 5 a^4 b x + a^5)}$$

input

$$\text{int}((e*x+d)/(b^2*x^2+2*a*b*x+a^2)^3,x)$$

output

$$\left(\frac{-a e - 4 b d - 5 b e x}{20 b^2 (a^5 + 5 a^4 b x + 10 a^3 b^2 x^2 + 10 a^2 b^3 x^3 + 5 a b^4 x^4 + b^5 x^5)}\right)$$

$$3.136 \quad \int \frac{1}{(a^2 + 2abx + b^2x^2)^3} dx$$

| | |
|---|------|
| Optimal result | 1084 |
| Mathematica [A] (verified) | 1084 |
| Rubi [A] (verified) | 1085 |
| Maple [A] (verified) | 1086 |
| Fricas [B] (verification not implemented) | 1086 |
| Sympy [B] (verification not implemented) | 1087 |
| Maxima [B] (verification not implemented) | 1087 |
| Giac [A] (verification not implemented) | 1087 |
| Mupad [B] (verification not implemented) | 1088 |
| Reduce [B] (verification not implemented) | 1088 |

Optimal result

Integrand size = 18, antiderivative size = 14

$$\int \frac{1}{(a^2 + 2abx + b^2x^2)^3} dx = -\frac{1}{5b(a + bx)^5}$$

output `-1/5/b/(b*x+a)^5`

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00

$$\int \frac{1}{(a^2 + 2abx + b^2x^2)^3} dx = -\frac{1}{5b(a + bx)^5}$$

input `Integrate[(a^2 + 2*a*b*x + b^2*x^2)^(-3), x]`

output `-1/5*1/(b*(a + b*x)^5)`

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {1077, 17}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a^2 + 2abx + b^2x^2)^3} dx$$

$$\downarrow 1077$$

$$b^6 \int \frac{1}{(xb^2 + ab)^6} dx$$

$$\downarrow 17$$

$$-\frac{1}{5b(a + bx)^5}$$

input `Int[(a^2 + 2*a*b*x + b^2*x^2)^(-3),x]`

output `-1/5*1/(b*(a + b*x)^5)`

Defintions of rubi rules used

rule 17 `Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1))/(b*(m + 1)), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]`

rule 1077 `Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[1/c^p Int[(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]`

Maple [A] (verified)

Time = 0.96 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.93

| method | result | size |
|--------------|--|------|
| default | $-\frac{1}{5b(bx+a)^5}$ | 13 |
| norman | $-\frac{1}{5b(bx+a)^5}$ | 13 |
| risch | $-\frac{1}{5b(bx+a)^5}$ | 13 |
| orering | $-\frac{bx+a}{5b(b^2x^2+2abx+a^2)^3}$ | 29 |
| gosper | $-\frac{1}{5(bx+a)(b^2x^2+2abx+a^2)^2b}$ | 31 |
| parallelrisc | $-\frac{1}{5(bx+a)(b^2x^2+2abx+a^2)^2b}$ | 31 |

input `int(1/(b^2*x^2+2*a*b*x+a^2)^3,x,method=_RETURNVERBOSE)`

output `-1/5/b/(b*x+a)^5`

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 57 vs. $2(12) = 24$.

Time = 0.07 (sec) , antiderivative size = 57, normalized size of antiderivative = 4.07

$$\int \frac{1}{(a^2 + 2abx + b^2x^2)^3} dx = -\frac{1}{5(b^6x^5 + 5ab^5x^4 + 10a^2b^4x^3 + 10a^3b^3x^2 + 5a^4b^2x + a^5b)}$$

input `integrate(1/(b^2*x^2+2*a*b*x+a^2)^3,x, algorithm="fricas")`

output `-1/5/(b^6*x^5 + 5*a*b^5*x^4 + 10*a^2*b^4*x^3 + 10*a^3*b^3*x^2 + 5*a^4*b^2*x + a^5*b)`

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 61 vs. $2(12) = 24$.

Time = 0.15 (sec) , antiderivative size = 61, normalized size of antiderivative = 4.36

$$\int \frac{1}{(a^2 + 2abx + b^2x^2)^3} dx = -\frac{1}{5a^5b + 25a^4b^2x + 50a^3b^3x^2 + 50a^2b^4x^3 + 25ab^5x^4 + 5b^6x^5}$$

input `integrate(1/(b**2*x**2+2*a*b*x+a**2)**3,x)`

output `-1/(5*a**5*b + 25*a**4*b**2*x + 50*a**3*b**3*x**2 + 50*a**2*b**4*x**3 + 25*a*b**5*x**4 + 5*b**6*x**5)`

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 57 vs. $2(12) = 24$.

Time = 0.03 (sec) , antiderivative size = 57, normalized size of antiderivative = 4.07

$$\int \frac{1}{(a^2 + 2abx + b^2x^2)^3} dx = -\frac{1}{5(b^6x^5 + 5ab^5x^4 + 10a^2b^4x^3 + 10a^3b^3x^2 + 5a^4b^2x + a^5b)}$$

input `integrate(1/(b^2*x^2+2*a*b*x+a^2)^3,x, algorithm="maxima")`

output `-1/5/(b^6*x^5 + 5*a*b^5*x^4 + 10*a^2*b^4*x^3 + 10*a^3*b^3*x^2 + 5*a^4*b^2*x + a^5*b)`

Giac [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.86

$$\int \frac{1}{(a^2 + 2abx + b^2x^2)^3} dx = -\frac{1}{5(bx + a)^5b}$$

input `integrate(1/(b^2*x^2+2*a*b*x+a^2)^3,x, algorithm="giac")`

output $-1/5/((b*x + a)^{5*b})$

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 59, normalized size of antiderivative = 4.21

$$\int \frac{1}{(a^2 + 2abx + b^2x^2)^3} dx$$

$$= -\frac{1}{5a^5b + 25a^4b^2x + 50a^3b^3x^2 + 50a^2b^4x^3 + 25ab^5x^4 + 5b^6x^5}$$

input `int(1/(a^2 + b^2*x^2 + 2*a*b*x)^3,x)`

output $-1/(5*a^5*b + 5*b^6*x^5 + 25*a^4*b^2*x + 25*a*b^5*x^4 + 50*a^3*b^3*x^2 + 50*a^2*b^4*x^3)$

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 56, normalized size of antiderivative = 4.00

$$\int \frac{1}{(a^2 + 2abx + b^2x^2)^3} dx = -\frac{1}{5b(b^5x^5 + 5ab^4x^4 + 10a^2b^3x^3 + 10a^3b^2x^2 + 5a^4bx + a^5)}$$

input `int(1/(b^2*x^2+2*a*b*x+a^2)^3,x)`

output $(-1)/(5*b*(a**5 + 5*a**4*b*x + 10*a**3*b**2*x**2 + 10*a**2*b**3*x**3 + 5*a*b**4*x**4 + b**5*x**5))$

3.137 $\int \frac{1}{(d+ex)(a^2+2abx+b^2x^2)^3} dx$

| | |
|---|------|
| Optimal result | 1089 |
| Mathematica [A] (verified) | 1090 |
| Rubi [A] (verified) | 1090 |
| Maple [A] (verified) | 1092 |
| Fricas [B] (verification not implemented) | 1092 |
| Sympy [B] (verification not implemented) | 1093 |
| Maxima [B] (verification not implemented) | 1094 |
| Giac [B] (verification not implemented) | 1095 |
| Mupad [B] (verification not implemented) | 1096 |
| Reduce [B] (verification not implemented) | 1097 |

Optimal result

Integrand size = 26, antiderivative size = 155

$$\int \frac{1}{(d+ex)(a^2+2abx+b^2x^2)^3} dx = -\frac{1}{5(bd-ae)(a+bx)^5} + \frac{e}{4(bd-ae)^2(a+bx)^4} - \frac{e^2}{3(bd-ae)^3(a+bx)^3} + \frac{e^4}{2(bd-ae)^4(a+bx)^2} - \frac{e^5 \log(a+bx)}{(bd-ae)^6} + \frac{e^5 \log(d+ex)}{(bd-ae)^6}$$

output

```
-1/5/(-a*e+b*d)/(b*x+a)^5+1/4*e/(-a*e+b*d)^2/(b*x+a)^4-1/3*e^2/(-a*e+b*d)^3/(b*x+a)^3+1/2*e^3/(-a*e+b*d)^4/(b*x+a)^2-e^4/(-a*e+b*d)^5/(b*x+a)-e^5*ln(b*x+a)/(-a*e+b*d)^6+e^5*ln(e*x+d)/(-a*e+b*d)^6
```

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 152, normalized size of antiderivative = 0.98

$$\int \frac{1}{(d+ex)(a^2+2abx+b^2x^2)^3} dx$$

$$= \frac{-12(bd-ae)^5 + 15e(bd-ae)^4(a+bx) + 20e^2(-bd+ae)^3(a+bx)^2 + 30e^3(bd-ae)^2(a+bx)^3 + 60e^4(-bd+ae)(a+bx)^4 - 60e^5(a+bx)^5 \operatorname{Log}[a+bx] + 60e^5(a+bx)^5 \operatorname{Log}[d+ex]}{60(bd-ae)^6(a+bx)^5}$$

input

```
Integrate[1/((d + e*x)*(a^2 + 2*a*b*x + b^2*x^2)^3),x]
```

output

```
(-12*(b*d - a*e)^5 + 15*e*(b*d - a*e)^4*(a + b*x) + 20*e^2*(-(b*d) + a*e)^3*(a + b*x)^2 + 30*e^3*(b*d - a*e)^2*(a + b*x)^3 + 60*e^4*(-(b*d) + a*e)*(a + b*x)^4 - 60*e^5*(a + b*x)^5*Log[a + b*x] + 60*e^5*(a + b*x)^5*Log[d + e*x])/(60*(b*d - a*e)^6*(a + b*x)^5)
```

Rubi [A] (verified)

Time = 0.60 (sec) , antiderivative size = 155, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {1098, 27, 54, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a^2 + 2abx + b^2x^2)^3 (d + ex)} dx$$

$$\downarrow 1098$$

$$b^6 \int \frac{1}{b^6(a + bx)^6(d + ex)} dx$$

$$\downarrow 27$$

$$\int \frac{1}{(a + bx)^6(d + ex)} dx$$

$$\downarrow 54$$

$$\int \left(\frac{e^6}{(d+ex)(bd-ae)^6} - \frac{be^5}{(a+bx)(bd-ae)^6} + \frac{be^4}{(a+bx)^2(bd-ae)^5} - \frac{be^3}{(a+bx)^3(bd-ae)^4} + \frac{be^2}{(a+bx)^4(bd-ae)^3} \right)$$

↓ 2009

$$-\frac{e^5 \log(a+bx)}{(bd-ae)^6} + \frac{e^5 \log(d+ex)}{(bd-ae)^6} - \frac{e^4}{(a+bx)(bd-ae)^5} + \frac{e^3}{2(a+bx)^2(bd-ae)^4} - \frac{e^2}{3(a+bx)^3(bd-ae)^3} + \frac{e}{4(a+bx)^4(bd-ae)^2} - \frac{1}{5(a+bx)^5(bd-ae)}$$

input `Int[1/((d + e*x)*(a^2 + 2*a*b*x + b^2*x^2)^3), x]`

output `-1/5*1/((b*d - a*e)*(a + b*x)^5) + e/(4*(b*d - a*e)^2*(a + b*x)^4) - e^2/(3*(b*d - a*e)^3*(a + b*x)^3) + e^3/(2*(b*d - a*e)^4*(a + b*x)^2) - e^4/((b*d - a*e)^5*(a + b*x)) - (e^5*Log[a + b*x])/(b*d - a*e)^6 + (e^5*Log[d + e*x])/(b*d - a*e)^6`

Defintions of rubi rules used

rule 27 `Int[(a_)*(F_x_), x_Symbol] := Simp[a Int[F_x, x], x] /; FreeQ[a, x] && !MatchQ[F_x, (b_)*(G_x_) /; FreeQ[b, x]]`

rule 54 `Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] := Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] && ILtQ[m, 0] && IntegerQ[n] && !(IGtQ[n, 0] && LtQ[m + n + 2, 0])`

rule 1098 `Int[((d_.) + (e_.)*(x_))^(m_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] := Simp[1/c^p Int[(d + e*x)^m*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]`

rule 2009 `Int[u_, x_Symbol] := Simp[IntSum[u, x], x] /; SumQ[u]`

Maple [A] (verified)

Time = 1.33 (sec) , antiderivative size = 147, normalized size of antiderivative = 0.95

| method | result |
|---------------|--|
| default | $\frac{1}{5(ae-bd)(bx+a)^5} + \frac{e}{4(ae-bd)^2(bx+a)^4} + \frac{e^2}{3(ae-bd)^3(bx+a)^3} + \frac{e^3}{2(ae-bd)^4(bx+a)^2} + \frac{e^4}{(ae-bd)^5(bx+a)} - \frac{e^5 \ln(bx+a)}{(ae-bd)^5}$ |
| parallelrisch | $-\frac{300a^4b^6de^4 - 300a^3b^7d^2e^3 + 200a^2b^8d^3e^2 - 75ab^9d^4e + 100xab^9d^3e^2 + 300 \ln(bx+a)x^4ab^9e^5 - 300 \ln(ex+d)x^4ab^9e^5 + 600 \ln(ex+d)x^4ab^9e^5}{(ae-bd)^5}$ |
| risch | $\frac{b^4e^4x^4}{e^5a^5 - 5a^4bde^4 + 10a^3b^2d^2e^3 - 10a^2b^3d^3e^2 + 5ab^4d^4e - b^5d^5} + \frac{(9ae-bd)b^3e^3x^3}{2e^5a^5 - 10a^4bde^4 + 20a^3b^2d^2e^3 - 20a^2b^3d^3e^2 + 10ab^4d^4e - 2b^5d^5} + \frac{6e^5a^5 - 5a^4bde^4 + 10a^3b^2d^2e^3 - 10a^2b^3d^3e^2 + 5ab^4d^4e - b^5d^5}{e^5a^5 - 5a^4bde^4 + 10a^3b^2d^2e^3 - 10a^2b^3d^3e^2 + 5ab^4d^4e - b^5d^5}$ |
| norman | $\frac{b^4e^4x^4}{e^5a^5 - 5a^4bde^4 + 10a^3b^2d^2e^3 - 10a^2b^3d^3e^2 + 5ab^4d^4e - b^5d^5} + \frac{137a^4b^5e^4 - 163a^3b^6de^3 + 137a^2b^7d^2e^2 - 63ab^8d^3e + 12b^9d^4}{60b^5(e^5a^5 - 5a^4bde^4 + 10a^3b^2d^2e^3 - 10a^2b^3d^3e^2 + 5ab^4d^4e - b^5d^5)} + \frac{2b^2(e^5a^5 - 5a^4bde^4 + 10a^3b^2d^2e^3 - 10a^2b^3d^3e^2 + 5ab^4d^4e - b^5d^5)}{60b^5(e^5a^5 - 5a^4bde^4 + 10a^3b^2d^2e^3 - 10a^2b^3d^3e^2 + 5ab^4d^4e - b^5d^5)}$ |

input

```
int(1/(e*x+d)/(b^2*x^2+2*a*b*x+a^2)^3,x,method=_RETURNVERBOSE)
```

output

```
1/5/(a*e-b*d)/(b*x+a)^5+1/4*e/(a*e-b*d)^2/(b*x+a)^4+1/3*e^2/(a*e-b*d)^3/(b*x+a)^3+1/2*e^3/(a*e-b*d)^4/(b*x+a)^2+e^4/(a*e-b*d)^5/(b*x+a)-e^5/(a*e-b*d)^6*ln(b*x+a)+e^5/(a*e-b*d)^6*ln(e*x+d)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 923 vs. 2(147) = 294.

Time = 0.10 (sec) , antiderivative size = 923, normalized size of antiderivative = 5.95

$$\int \frac{1}{(d+ex)(a^2+2abx+b^2x^2)^3} dx = \text{Too large to display}$$

input

```
integrate(1/(e*x+d)/(b^2*x^2+2*a*b*x+a^2)^3,x,algorithm="fricas")
```

output

```

-1/60*(12*b^5*d^5 - 75*a*b^4*d^4*e + 200*a^2*b^3*d^3*e^2 - 300*a^3*b^2*d^2
*e^3 + 300*a^4*b*d*e^4 - 137*a^5*e^5 + 60*(b^5*d*e^4 - a*b^4*e^5)*x^4 - 30
*(b^5*d^2*e^3 - 10*a*b^4*d*e^4 + 9*a^2*b^3*e^5)*x^3 + 10*(2*b^5*d^3*e^2 -
15*a*b^4*d^2*e^3 + 60*a^2*b^3*d*e^4 - 47*a^3*b^2*e^5)*x^2 - 5*(3*b^5*d^4*e
- 20*a*b^4*d^3*e^2 + 60*a^2*b^3*d^2*e^3 - 120*a^3*b^2*d*e^4 + 77*a^4*b*e^
5)*x + 60*(b^5*e^5*x^5 + 5*a*b^4*e^5*x^4 + 10*a^2*b^3*e^5*x^3 + 10*a^3*b^2
*e^5*x^2 + 5*a^4*b*e^5*x + a^5*e^5)*log(b*x + a) - 60*(b^5*e^5*x^5 + 5*a*b
^4*e^5*x^4 + 10*a^2*b^3*e^5*x^3 + 10*a^3*b^2*e^5*x^2 + 5*a^4*b*e^5*x + a^5
*e^5)*log(e*x + d)/(a^5*b^6*d^6 - 6*a^6*b^5*d^5*e + 15*a^7*b^4*d^4*e^2 -
20*a^8*b^3*d^3*e^3 + 15*a^9*b^2*d^2*e^4 - 6*a^10*b*d*e^5 + a^11*e^6 + (b^1
1*d^6 - 6*a*b^10*d^5*e + 15*a^2*b^9*d^4*e^2 - 20*a^3*b^8*d^3*e^3 + 15*a^4*
b^7*d^2*e^4 - 6*a^5*b^6*d*e^5 + a^6*b^5*e^6)*x^5 + 5*(a*b^10*d^6 - 6*a^2*b
^9*d^5*e + 15*a^3*b^8*d^4*e^2 - 20*a^4*b^7*d^3*e^3 + 15*a^5*b^6*d^2*e^4 -
6*a^6*b^5*d*e^5 + a^7*b^4*e^6)*x^4 + 10*(a^2*b^9*d^6 - 6*a^3*b^8*d^5*e + 1
5*a^4*b^7*d^4*e^2 - 20*a^5*b^6*d^3*e^3 + 15*a^6*b^5*d^2*e^4 - 6*a^7*b^4*d*
e^5 + a^8*b^3*e^6)*x^3 + 10*(a^3*b^8*d^6 - 6*a^4*b^7*d^5*e + 15*a^5*b^6*d^
4*e^2 - 20*a^6*b^5*d^3*e^3 + 15*a^7*b^4*d^2*e^4 - 6*a^8*b^3*d*e^5 + a^9*b^
2*e^6)*x^2 + 5*(a^4*b^7*d^6 - 6*a^5*b^6*d^5*e + 15*a^6*b^5*d^4*e^2 - 20*a^
7*b^4*d^3*e^3 + 15*a^8*b^3*d^2*e^4 - 6*a^9*b^2*d*e^5 + a^10*b*e^6)*x)

```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1081 vs. $2(129) = 258$.

Time = 1.59 (sec) , antiderivative size = 1081, normalized size of antiderivative = 6.97

$$\int \frac{1}{(d+ex)(a^2+2abx+b^2x^2)^3} dx = \text{Too large to display}$$

input

```
integrate(1/(e*x+d)/(b**2*x**2+2*a*b*x+a**2)**3,x)
```

output

```
e**5*log(x + (-a**7*e**12/(a*e - b*d)**6 + 7*a**6*b*d*e**11/(a*e - b*d)**6
- 21*a**5*b**2*d**2*e**10/(a*e - b*d)**6 + 35*a**4*b**3*d**3*e**9/(a*e -
b*d)**6 - 35*a**3*b**4*d**4*e**8/(a*e - b*d)**6 + 21*a**2*b**5*d**5*e**7/(
a*e - b*d)**6 - 7*a*b**6*d**6*e**6/(a*e - b*d)**6 + a**6 + b**7*d**7*e**
5/(a*e - b*d)**6 + b*d*e**5)/(2*b*e**6))/(a*e - b*d)**6 - e**5*log(x + (a*
*7*e**12/(a*e - b*d)**6 - 7*a**6*b*d*e**11/(a*e - b*d)**6 + 21*a**5*b**2*d
**2*e**10/(a*e - b*d)**6 - 35*a**4*b**3*d**3*e**9/(a*e - b*d)**6 + 35*a**3
*b**4*d**4*e**8/(a*e - b*d)**6 - 21*a**2*b**5*d**5*e**7/(a*e - b*d)**6 + 7
*a*b**6*d**6*e**6/(a*e - b*d)**6 + a**6 - b**7*d**7*e**5/(a*e - b*d)**6
+ b*d*e**5)/(2*b*e**6))/(a*e - b*d)**6 + (137*a**4*e**4 - 163*a**3*b*d*e**
3 + 137*a**2*b**2*d**2*e**2 - 63*a*b**3*d**3*e + 12*b**4*d**4 + 60*b**4*e*
*4*x**4 + x**3*(270*a*b**3*e**4 - 30*b**4*d*e**3) + x**2*(470*a**2*b**2*e*
*4 - 130*a*b**3*d*e**3 + 20*b**4*d**2*e**2) + x*(385*a**3*b*e**4 - 215*a**
2*b**2*d*e**3 + 85*a*b**3*d**2*e**2 - 15*b**4*d**3*e))/(60*a**10*e**5 - 30
0*a**9*b*d*e**4 + 600*a**8*b**2*d**2*e**3 - 600*a**7*b**3*d**3*e**2 + 300*
a**6*b**4*d**4*e - 60*a**5*b**5*d**5 + x**5*(60*a**5*b**5*e**5 - 300*a**4*
b**6*d*e**4 + 600*a**3*b**7*d**2*e**3 - 600*a**2*b**8*d**3*e**2 + 300*a*b*
*9*d**4*e - 60*b**10*d**5) + x**4*(300*a**6*b**4*e**5 - 1500*a**5*b**5*d*
e**4 + 3000*a**4*b**6*d**2*e**3 - 3000*a**3*b**7*d**3*e**2 + 1500*a**2*b**8
*d**4*e - 300*a*b**9*d**5) + x**3*(600*a**7*b**3*e**5 - 3000*a**6*b**4*...
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 805 vs. $2(147) = 294$.

Time = 0.07 (sec) , antiderivative size = 805, normalized size of antiderivative = 5.19

$$\int \frac{1}{(d+ex)(a^2+2abx+b^2x^2)^3} dx$$

$$= -\frac{e^5 \log(bx+a)}{b^6 d^6 - 6ab^5 d^5 e + 15a^2 b^4 d^4 e^2 - 20a^3 b^3 d^3 e^3 + 15a^4 b^2 d^2 e^4 - 6a^5 b d e^5 + a^6 e^6}$$

$$+ \frac{e^5 \log(ex+d)}{b^6 d^6 - 6ab^5 d^5 e + 15a^2 b^4 d^4 e^2 - 20a^3 b^3 d^3 e^3 + 15a^4 b^2 d^2 e^4 - 6a^5 b d e^5 + a^6 e^6}$$

$$- \frac{60(a^5 b^5 d^5 - 5a^6 b^4 d^4 e + 10a^7 b^3 d^3 e^2 - 10a^8 b^2 d^2 e^3 + 5a^9 b d e^4 - a^{10} e^5 + (b^{10} d^5 - 5ab^9 d^4 e + 10a^2 b^8 d^3 e^2 - 10a^3 b^7 d^2 e^3 + 5a^4 b^6 d e^4 - a^5 b^5 e^5))}{(b^6 d^6 - 6ab^5 d^5 e + 15a^2 b^4 d^4 e^2 - 20a^3 b^3 d^3 e^3 + 15a^4 b^2 d^2 e^4 - 6a^5 b d e^5 + a^6 e^6)^2}$$

input

```
integrate(1/(e*x+d)/(b^2*x^2+2*a*b*x+a^2)^3,x, algorithm="maxima")
```

output

```
-e^5*log(b*x + a)/(b^6*d^6 - 6*a*b^5*d^5*e + 15*a^2*b^4*d^4*e^2 - 20*a^3*b^3*d^3*e^3 + 15*a^4*b^2*d^2*e^4 - 6*a^5*b*d*e^5 + a^6*e^6) + e^5*log(e*x + d)/(b^6*d^6 - 6*a*b^5*d^5*e + 15*a^2*b^4*d^4*e^2 - 20*a^3*b^3*d^3*e^3 + 15*a^4*b^2*d^2*e^4 - 6*a^5*b*d*e^5 + a^6*e^6) - 1/60*(60*b^4*e^4*x^4 + 12*b^4*d^4 - 63*a*b^3*d^3*e + 137*a^2*b^2*d^2*e^2 - 163*a^3*b*d*e^3 + 137*a^4*e^4 - 30*(b^4*d*e^3 - 9*a*b^3*e^4)*x^3 + 10*(2*b^4*d^2*e^2 - 13*a*b^3*d*e^3 + 47*a^2*b^2*e^4)*x^2 - 5*(3*b^4*d^3*e - 17*a*b^3*d^2*e^2 + 43*a^2*b^2*d*e^3 - 77*a^3*b*e^4)*x)/(a^5*b^5*d^5 - 5*a^6*b^4*d^4*e + 10*a^7*b^3*d^3*e^2 - 10*a^8*b^2*d^2*e^3 + 5*a^9*b*d*e^4 - a^10*e^5 + (b^10*d^5 - 5*a*b^9*d^4*e + 10*a^2*b^8*d^3*e^2 - 10*a^3*b^7*d^2*e^3 + 5*a^4*b^6*d*e^4 - a^5*b^5*d^4*e^5)*x^5 + 5*(a*b^9*d^5 - 5*a^2*b^8*d^4*e + 10*a^3*b^7*d^3*e^2 - 10*a^4*b^6*d^2*e^3 + 5*a^5*b^5*d*e^4 - a^6*b^4*e^5)*x^4 + 10*(a^2*b^8*d^5 - 5*a^3*b^7*d^4*e + 10*a^4*b^6*d^3*e^2 - 10*a^5*b^5*d^2*e^3 + 5*a^6*b^4*d*e^4 - a^7*b^3*e^5)*x^3 + 10*(a^3*b^7*d^5 - 5*a^4*b^6*d^4*e + 10*a^5*b^5*d^3*e^2 - 10*a^6*b^4*d^2*e^3 + 5*a^7*b^3*d*e^4 - a^8*b^2*e^5)*x^2 + 5*(a^4*b^6*d^5 - 5*a^5*b^5*d^4*e + 10*a^6*b^4*d^3*e^2 - 10*a^7*b^3*d^2*e^3 + 5*a^8*b^2*d*e^4 - a^9*b*e^5)*x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 445 vs. $2(147) = 294$.

Time = 0.18 (sec) , antiderivative size = 445, normalized size of antiderivative = 2.87

$$\int \frac{1}{(d+ex)(a^2+2abx+b^2x^2)^3} dx$$

$$= -\frac{be^5 \log(|bx+a|)}{b^7d^6 - 6ab^6d^5e + 15a^2b^5d^4e^2 - 20a^3b^4d^3e^3 + 15a^4b^3d^2e^4 - 6a^5b^2de^5 + a^6be^6} + \frac{e^6 \log(|ex+d|)}{b^6d^6e - 6ab^5d^5e^2 + 15a^2b^4d^4e^3 - 20a^3b^3d^3e^4 + 15a^4b^2d^2e^5 - 6a^5bde^6 + a^6e^7} - \frac{12b^5d^5 - 75ab^4d^4e + 200a^2b^3d^3e^2 - 300a^3b^2d^2e^3 + 300a^4bde^4 - 137a^5e^5 + 60(b^5de^4 - ab^4e^5)x^4 - 300a^6b^4d^3e^2 + 300a^7b^3d^2e^3 - 100a^8b^2de^4 + 100a^9bde^5 - 100a^{10}e^6}{(b^6d^6e - 6ab^5d^5e^2 + 15a^2b^4d^4e^3 - 20a^3b^3d^3e^4 + 15a^4b^2d^2e^5 - 6a^5bde^6 + a^6e^7)}$$

input

```
integrate(1/(e*x+d)/(b^2*x^2+2*a*b*x+a^2)^3,x, algorithm="giac")
```

output

```
-b*e^5*log(abs(b*x + a))/(b^7*d^6 - 6*a*b^6*d^5*e + 15*a^2*b^5*d^4*e^2 - 20*a^3*b^4*d^3*e^3 + 15*a^4*b^3*d^2*e^4 - 6*a^5*b^2*d*e^5 + a^6*b*e^6) + e^6*log(abs(e*x + d))/(b^6*d^6*e - 6*a*b^5*d^5*e^2 + 15*a^2*b^4*d^4*e^3 - 20*a^3*b^3*d^3*e^4 + 15*a^4*b^2*d^2*e^5 - 6*a^5*b*d*e^6 + a^6*e^7) - 1/60*(12*b^5*d^5 - 75*a*b^4*d^4*e + 200*a^2*b^3*d^3*e^2 - 300*a^3*b^2*d^2*e^3 + 300*a^4*b*d*e^4 - 137*a^5*e^5 + 60*(b^5*d*e^4 - a*b^4*e^5)*x^4 - 30*(b^5*d^2*e^3 - 10*a*b^4*d*e^4 + 9*a^2*b^3*e^5)*x^3 + 10*(2*b^5*d^3*e^2 - 15*a*b^4*d^2*e^3 + 60*a^2*b^3*d*e^4 - 47*a^3*b^2*e^5)*x^2 - 5*(3*b^5*d^4*e - 20*a*b^4*d^3*e^2 + 60*a^2*b^3*d^2*e^3 - 120*a^3*b^2*d*e^4 + 77*a^4*b*e^5)*x)/((b*d - a*e)^6*(b*x + a)^5)
```

Mupad [B] (verification not implemented)

Time = 9.27 (sec) , antiderivative size = 721, normalized size of antiderivative = 4.65

$$\int \frac{1}{(d + ex)(a^2 + 2abx + b^2x^2)^3} dx$$

$$= \frac{137a^4e^4 - 163a^3bde^3 + 137a^2b^2d^2e^2 - 63ab^3d^3e + 12b^4d^4}{60(a^5e^5 - 5a^4bde^4 + 10a^3b^2d^2e^3 - 10a^2b^3d^3e^2 + 5ab^4d^4e - b^5d^5)} - \frac{ex(-77a^3be^3 + 43a^2b^2de^2 - 17ab^3d^2e + 3b^4d^3)}{12(a^5e^5 - 5a^4bde^4 + 10a^3b^2d^2e^3 - 10a^2b^3d^3e^2 + 5ab^4d^4e - b^5d^5)}$$

$$- \frac{2e^5 \operatorname{atanh}\left(\frac{a^6e^6 - 4a^5bde^5 + 5a^4b^2d^2e^4 - 5a^2b^4d^4e^2 + 4ab^5d^5e - b^6d^6}{(ae - bd)^6}\right) + \frac{2bex(a^5e^5 - 5a^4bde^4 + 10a^3b^2d^2e^3 - 10a^2b^3d^3e^2 + 5ab^4d^4e - b^5d^5)}{(ae - bd)^6}}{(ae - bd)^6}$$

input

```
int(1/((d + e*x)*(a^2 + b^2*x^2 + 2*a*b*x)^3), x)
```

output

```

((137*a^4*e^4 + 12*b^4*d^4 + 137*a^2*b^2*d^2*e^2 - 63*a*b^3*d^3*e - 163*a^3*b*d*e^3)/(60*(a^5*e^5 - b^5*d^5 - 10*a^2*b^3*d^3*e^2 + 10*a^3*b^2*d^2*e^3 + 5*a*b^4*d^4*e - 5*a^4*b*d*e^4)) - (e*x*(3*b^4*d^3 - 77*a^3*b*e^3 + 43*a^2*b^2*d*e^2 - 17*a*b^3*d^2*e))/(12*(a^5*e^5 - b^5*d^5 - 10*a^2*b^3*d^3*e^2 + 10*a^3*b^2*d^2*e^3 + 5*a*b^4*d^4*e - 5*a^4*b*d*e^4)) + (b^4*e^4*x^4)/(a^5*e^5 - b^5*d^5 - 10*a^2*b^3*d^3*e^2 + 10*a^3*b^2*d^2*e^3 + 5*a*b^4*d^4*e - 5*a^4*b*d*e^4) - (e^3*x^3*(b^4*d - 9*a*b^3*e))/(2*(a^5*e^5 - b^5*d^5 - 10*a^2*b^3*d^3*e^2 + 10*a^3*b^2*d^2*e^3 + 5*a*b^4*d^4*e - 5*a^4*b*d*e^4)) + (e^2*x^2*(2*b^4*d^2 + 47*a^2*b^2*e^2 - 13*a*b^3*d*e))/(6*(a^5*e^5 - b^5*d^5 - 10*a^2*b^3*d^3*e^2 + 10*a^3*b^2*d^2*e^3 + 5*a*b^4*d^4*e - 5*a^4*b*d*e^4)))/(a^5 + b^5*x^5 + 5*a*b^4*x^4 + 10*a^3*b^2*x^2 + 10*a^2*b^3*x^3 + 5*a^4*b*x) - (2*e^5*atanh((a^6*e^6 - b^6*d^6 - 5*a^2*b^4*d^4*e^2 + 5*a^4*b^2*d^2*e^4 + 4*a*b^5*d^5*e - 4*a^5*b*d*e^5)/(a*e - b*d))^6 + (2*b*e*x*(a^5*e^5 - b^5*d^5 - 10*a^2*b^3*d^3*e^2 + 10*a^3*b^2*d^2*e^3 + 5*a*b^4*d^4*e - 5*a^4*b*d*e^4))/(a*e - b*d))^6)/(a*e - b*d)^6

```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 1086, normalized size of antiderivative = 7.01

$$\int \frac{1}{(d+ex)(a^2+2abx+b^2x^2)^3} dx = \text{Too large to display}$$

input

```
int(1/(e*x+d)/(b^2*x^2+2*a*b*x+a^2)^3,x)
```

output

```
( - 60*log(a + b*x)*a**6*e**5 - 300*log(a + b*x)*a**5*b*e**5*x - 600*log(a
+ b*x)*a**4*b**2*e**5*x**2 - 600*log(a + b*x)*a**3*b**3*e**5*x**3 - 300*log(a
+ b*x)*a**2*b**4*e**5*x**4 - 60*log(a + b*x)*a*b**5*e**5*x**5 + 60*log
(d + e*x)*a**6*e**5 + 300*log(d + e*x)*a**5*b*e**5*x + 600*log(d + e*x)*a
**4*b**2*e**5*x**2 + 600*log(d + e*x)*a**3*b**3*e**5*x**3 + 300*log(d + e
*x)*a**2*b**4*e**5*x**4 + 60*log(d + e*x)*a*b**5*e**5*x**5 + 125*a**6*e**5
- 288*a**5*b*d*e**4 + 325*a**5*b*e**5*x + 300*a**4*b**2*d**2*e**3 - 540*a
**4*b**2*d*e**4*x + 350*a**4*b**2*e**5*x**2 - 200*a**3*b**3*d**3*e**2 + 300
*a**3*b**3*d**2*e**3*x - 480*a**3*b**3*d*e**4*x**2 + 150*a**3*b**3*e**5*x
**3 + 75*a**2*b**4*d**4*e - 100*a**2*b**4*d**3*e**2*x + 150*a**2*b**4*d**2
e**3*x**2 - 180*a**2*b**4*d*e**4*x**3 - 12*a*b**5*d**5 + 15*a*b**5*d**4*e
x - 20*a*b**5*d**3*e**2*x**2 + 30*a*b**5*d**2*e**3*x**3 - 12*a*b**5*e**5*x
**5 + 12*b**6*d*e**4*x**5)/(60*a*(a**11*e**6 - 6*a**10*b*d*e**5 + 5*a**10*
b*e**6*x + 15*a**9*b**2*d**2*e**4 - 30*a**9*b**2*d*e**5*x + 10*a**9*b**2*
e**6*x**2 - 20*a**8*b**3*d**3*e**3 + 75*a**8*b**3*d**2*e**4*x - 60*a**8*b
**3*d*e**5*x**2 + 10*a**8*b**3*e**6*x**3 + 15*a**7*b**4*d**4*e**2 - 100*a**7
*b**4*d**3*e**3*x + 150*a**7*b**4*d**2*e**4*x**2 - 60*a**7*b**4*d*e**5*x**
3 + 5*a**7*b**4*e**6*x**4 - 6*a**6*b**5*d**5*e + 75*a**6*b**5*d**4*e**2*x
- 200*a**6*b**5*d**3*e**3*x**2 + 150*a**6*b**5*d**2*e**4*x**3 - 30*a**6*b
**5*d*e**5*x**4 + a**6*b**5*e**6*x**5 + a**5*b**6*d**6 - 30*a**5*b**6*d...
```

3.138 $\int \frac{1}{(d+ex)^2(a^2+2abx+b^2x^2)^3} dx$

| | |
|---|------|
| Optimal result | 1099 |
| Mathematica [A] (verified) | 1100 |
| Rubi [A] (verified) | 1100 |
| Maple [A] (verified) | 1102 |
| Fricas [B] (verification not implemented) | 1102 |
| Sympy [B] (verification not implemented) | 1103 |
| Maxima [B] (verification not implemented) | 1104 |
| Giac [B] (verification not implemented) | 1105 |
| Mupad [B] (verification not implemented) | 1106 |
| Reduce [B] (verification not implemented) | 1107 |

Optimal result

Integrand size = 26, antiderivative size = 181

$$\int \frac{1}{(d+ex)^2(a^2+2abx+b^2x^2)^3} dx = -\frac{b}{5(bd-ae)^2(a+bx)^5} + \frac{be}{2(bd-ae)^3(a+bx)^4} - \frac{be^2}{(bd-ae)^4(a+bx)^3} + \frac{2be^3}{(bd-ae)^5(a+bx)^2} - \frac{(bd-ae)^6(a+bx)}{5be^4} - \frac{(bd-ae)^6(d+ex)}{e^5} - \frac{6be^5 \log(a+bx)}{(bd-ae)^7} + \frac{6be^5 \log(d+ex)}{(bd-ae)^7}$$

output

```
-1/5*b/(-a*e+b*d)^2/(b*x+a)^5+1/2*b*e/(-a*e+b*d)^3/(b*x+a)^4-b*e^2/(-a*e+b*d)^4/(b*x+a)^3+2*b*e^3/(-a*e+b*d)^5/(b*x+a)^2-5*b*e^4/(-a*e+b*d)^6/(b*x+a)-e^5/(-a*e+b*d)^6/(e*x+d)-6*b*e^5*ln(b*x+a)/(-a*e+b*d)^7+6*b*e^5*ln(e*x+d)/(-a*e+b*d)^7
```


Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 167, normalized size of antiderivative = 0.92

$$\int \frac{1}{(d+ex)^2 (a^2+2abx+b^2x^2)^3} dx$$

$$= \frac{-\frac{2b(bd-ae)^5}{(a+bx)^5} + \frac{5be(bd-ae)^4}{(a+bx)^4} - \frac{10be^2(bd-ae)^3}{(a+bx)^3} + \frac{20be^3(bd-ae)^2}{(a+bx)^2} - \frac{50be^4(bd-ae)}{a+bx} + \frac{10e^5(-bd+ae)}{d+ex} - 60be^5 \log(a+bx) + 60e^5 \log(d+ex)}{10(bd-ae)^7}$$

input

```
Integrate[1/((d + e*x)^2*(a^2 + 2*a*b*x + b^2*x^2)^3),x]
```

output

```
((-2*b*(b*d - a*e)^5)/(a + b*x)^5 + (5*b*e*(b*d - a*e)^4)/(a + b*x)^4 - (10*b*e^2*(b*d - a*e)^3)/(a + b*x)^3 + (20*b*e^3*(b*d - a*e)^2)/(a + b*x)^2 - (50*b*e^4*(b*d - a*e))/(a + b*x) + (10*e^5*(-(b*d) + a*e))/(d + e*x) - 60*b*e^5*Log[a + b*x] + 60*b*e^5*Log[d + e*x])/(10*(b*d - a*e)^7)
```

Rubi [A] (verified)

Time = 0.73 (sec) , antiderivative size = 181, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {1098, 27, 54, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a^2+2abx+b^2x^2)^3 (d+ex)^2} dx$$

$$\downarrow 1098$$

$$b^6 \int \frac{1}{b^6(a+bx)^6 (d+ex)^2} dx$$

$$\downarrow 27$$

$$\int \frac{1}{(a+bx)^6 (d+ex)^2} dx$$

$$\downarrow 54$$

$$\int \left(-\frac{6b^2e^5}{(a+bx)(bd-ae)^7} + \frac{5b^2e^4}{(a+bx)^2(bd-ae)^6} - \frac{4b^2e^3}{(a+bx)^3(bd-ae)^5} + \frac{3b^2e^2}{(a+bx)^4(bd-ae)^4} - \frac{2b^2e}{(a+bx)^5(bd-ae)^3} \right) dx$$

↓ 2009

$$-\frac{e^5}{(d+ex)(bd-ae)^6} - \frac{6be^5 \log(a+bx)}{2be^3(bd-ae)^6} - \frac{6be^5 \log(d+ex)}{be^2(bd-ae)^7} - \frac{5be^4}{(a+bx)(bd-ae)^6} + \frac{1}{(a+bx)^2(bd-ae)^5} - \frac{1}{(a+bx)^3(bd-ae)^4} + \frac{1}{2(a+bx)^4(bd-ae)^3} - \frac{1}{5(a+bx)^5(bd-ae)^2}$$

input `Int[1/((d + e*x)^2*(a^2 + 2*a*b*x + b^2*x^2)^3),x]`

output `-1/5*b/((b*d - a*e)^2*(a + b*x)^5) + (b*e)/(2*(b*d - a*e)^3*(a + b*x)^4) - (b*e^2)/((b*d - a*e)^4*(a + b*x)^3) + (2*b*e^3)/((b*d - a*e)^5*(a + b*x)^2) - (5*b*e^4)/((b*d - a*e)^6*(a + b*x)) - e^5/((b*d - a*e)^6*(d + e*x)) - (6*b*e^5*Log[a + b*x])/((b*d - a*e)^7) + (6*b*e^5*Log[d + e*x])/((b*d - a*e)^7)`

Defintions of rubi rules used

rule 27 `Int[(a_)*(F_x_), x_Symbol] := Simp[a Int[F_x, x], x] /; FreeQ[a, x] && !MatchQ[F_x, (b_)*(G_x_)] /; FreeQ[b, x]`

rule 54 `Int[((a_) + (b_.)*(x_)^(m_))*((c_.) + (d_.)*(x_)^(n_.)), x_Symbol] := Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] && ILtQ[m, 0] && IntegerQ[n] && !(IGtQ[n, 0] && LtQ[m + n + 2, 0])`

rule 1098 `Int[((d_.) + (e_.)*(x_)^(m_.))*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] := Simp[1/c^p Int[(d + e*x)^m*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]`

rule 2009 `Int[u_, x_Symbol] := Simp[IntSum[u, x], x] /; SumQ[u]`

Maple [A] (verified)

Time = 1.41 (sec) , antiderivative size = 178, normalized size of antiderivative = 0.98

| method | result |
|-------------|---|
| default | $-\frac{b}{5(ae-bd)^2(bx+a)^5} + \frac{6be^5 \ln(bx+a)}{(ae-bd)^7} - \frac{5be^4}{(ae-bd)^6(bx+a)} - \frac{2be^3}{(ae-bd)^5(bx+a)^2} - \frac{be^2}{(ae-bd)^4(bx+a)^3} - \frac{be}{2(ae-bd)^3(bx+a)^4}$ |
| parallelsch | $\frac{240x^4 a b^{10} d e^6 + 330x^3 a^2 b^9 d e^6 + 150x^3 a b^{10} d^2 e^5 + 130x^2 a^3 b^8 d e^6 + 300x^2 a^2 b^9 d^2 e^5 - 50x^2 a b^{10} d^3 e^4 - 85x a^4 b^7 d e^6 + 300x a^3 b^8 d^2 e^5 - 50x a^2 b^9 d^3 e^4 - 20x a^3 b^4 d^4 e^2 - 6a b^5 d^5 e + b^6 d^6}{a^6 e^6 - 6a^5 b d e^5 + 15a^4 b^2 d^2 e^4 - 20a^3 b^3 d^3 e^3 + 15a^2 b^4 d^4 e^2 - 6a b^5 d^5 e + b^6 d^6} - \frac{3b^4(9ae+bd)e^4 x^4}{a^6 e^6 - 6a^5 b d e^5 + 15a^4 b^2 d^2 e^4 - 20a^3 b^3 d^3 e^3 + 15a^2 b^4 d^4 e^2 - 6a b^5 d^5 e + b^6 d^6}$ |
| risch | $-\frac{6b^5 e^5 x^5}{a^6 e^6 - 6a^5 b d e^5 + 15a^4 b^2 d^2 e^4 - 20a^3 b^3 d^3 e^3 + 15a^2 b^4 d^4 e^2 - 6a b^5 d^5 e + b^6 d^6} + \frac{(-27a b^6 e^6 - 3b^7 d e^5) x^4}{b^2 e (a^6 e^6 - 6a^5 b d e^5 + 15a^4 b^2 d^2 e^4 - 20a^3 b^3 d^3 e^3 + 15a^2 b^4 d^4 e^2 - 6a b^5 d^5 e + b^6 d^6)}$ |
| norman | $-\frac{6b^5 e^5 x^5}{a^6 e^6 - 6a^5 b d e^5 + 15a^4 b^2 d^2 e^4 - 20a^3 b^3 d^3 e^3 + 15a^2 b^4 d^4 e^2 - 6a b^5 d^5 e + b^6 d^6} + \frac{(-27a b^6 e^6 - 3b^7 d e^5) x^4}{b^2 e (a^6 e^6 - 6a^5 b d e^5 + 15a^4 b^2 d^2 e^4 - 20a^3 b^3 d^3 e^3 + 15a^2 b^4 d^4 e^2 - 6a b^5 d^5 e + b^6 d^6)}$ |

input

```
int(1/(e*x+d)^2/(b^2*x^2+2*a*b*x+a^2)^3,x,method=_RETURNVERBOSE)
```

output

```
-1/5*b/(a*e-b*d)^2/(b*x+a)^5+6*b/(a*e-b*d)^7*e^5*ln(b*x+a)-5*b/(a*e-b*d)^6
*e^4/(b*x+a)-2*b/(a*e-b*d)^5*e^3/(b*x+a)^2-b/(a*e-b*d)^4*e^2/(b*x+a)^3-1/2
*b/(a*e-b*d)^3*e/(b*x+a)^4-e^5/(a*e-b*d)^6/(e*x+d)-6*b/(a*e-b*d)^7*e^5*ln(
e*x+d)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1420 vs. 2(177) = 354.

Time = 0.10 (sec) , antiderivative size = 1420, normalized size of antiderivative = 7.85

$$\int \frac{1}{(d+ex)^2 (a^2+2abx+b^2x^2)^3} dx = \text{Too large to display}$$

input

```
integrate(1/(e*x+d)^2/(b^2*x^2+2*a*b*x+a^2)^3,x, algorithm="fricas")
```

output

```

-1/10*(2*b^6*d^6 - 15*a*b^5*d^5*e + 50*a^2*b^4*d^4*e^2 - 100*a^3*b^3*d^3*e
^3 + 150*a^4*b^2*d^2*e^4 - 77*a^5*b*d*e^5 - 10*a^6*e^6 + 60*(b^6*d*e^5 - a
*b^5*e^6)*x^5 + 30*(b^6*d^2*e^4 + 8*a*b^5*d*e^5 - 9*a^2*b^4*e^6)*x^4 - 10*
(b^6*d^3*e^3 - 15*a*b^5*d^2*e^4 - 33*a^2*b^4*d*e^5 + 47*a^3*b^3*e^6)*x^3 +
5*(b^6*d^4*e^2 - 10*a*b^5*d^3*e^3 + 60*a^2*b^4*d^2*e^4 + 26*a^3*b^3*d*e^5
- 77*a^4*b^2*e^6)*x^2 - (3*b^6*d^5*e - 25*a*b^5*d^4*e^2 + 100*a^2*b^4*d^3
*e^3 - 300*a^3*b^3*d^2*e^4 + 85*a^4*b^2*d*e^5 + 137*a^5*b*e^6)*x + 60*(b^6
*e^6*x^6 + a^5*b*d*e^5 + (b^6*d*e^5 + 5*a*b^5*e^6)*x^5 + 5*(a*b^5*d*e^5 +
2*a^2*b^4*e^6)*x^4 + 10*(a^2*b^4*d*e^5 + a^3*b^3*e^6)*x^3 + 5*(2*a^3*b^3*d
*e^5 + a^4*b^2*e^6)*x^2 + (5*a^4*b^2*d*e^5 + a^5*b*e^6)*x)*log(b*x + a) -
60*(b^6*e^6*x^6 + a^5*b*d*e^5 + (b^6*d*e^5 + 5*a*b^5*e^6)*x^5 + 5*(a*b^5*d
*e^5 + 2*a^2*b^4*e^6)*x^4 + 10*(a^2*b^4*d*e^5 + a^3*b^3*e^6)*x^3 + 5*(2*a^
3*b^3*d*e^5 + a^4*b^2*e^6)*x^2 + (5*a^4*b^2*d*e^5 + a^5*b*e^6)*x)*log(e*x
+ d))/(a^5*b^7*d^8 - 7*a^6*b^6*d^7*e + 21*a^7*b^5*d^6*e^2 - 35*a^8*b^4*d^5
*e^3 + 35*a^9*b^3*d^4*e^4 - 21*a^10*b^2*d^3*e^5 + 7*a^11*b*d^2*e^6 - a^12*
d*e^7 + (b^12*d^7*e - 7*a*b^11*d^6*e^2 + 21*a^2*b^10*d^5*e^3 - 35*a^3*b^9*
d^4*e^4 + 35*a^4*b^8*d^3*e^5 - 21*a^5*b^7*d^2*e^6 + 7*a^6*b^6*d*e^7 - a^7*
b^5*e^8)*x^6 + (b^12*d^8 - 2*a*b^11*d^7*e - 14*a^2*b^10*d^6*e^2 + 70*a^3*b^
9*d^5*e^3 - 140*a^4*b^8*d^4*e^4 + 154*a^5*b^7*d^3*e^5 - 98*a^6*b^6*d^2*e^
6 + 34*a^7*b^5*d*e^7 - 5*a^8*b^4*e^8)*x^5 + 5*(a*b^11*d^8 - 5*a^2*b^10*...

```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1518 vs. $2(162) = 324$.

Time = 2.67 (sec) , antiderivative size = 1518, normalized size of antiderivative = 8.39

$$\int \frac{1}{(d+ex)^2 (a^2+2abx+b^2x^2)^3} dx = \text{Too large to display}$$

input

```
integrate(1/(e*x+d)**2/(b**2*x**2+2*a*b*x+a**2)**3,x)
```

output

```

-6*b**5*log(x + (-6*a**8*b**13/(a*e - b*d)**7 + 48*a**7*b**2*d**12/(
a*e - b*d)**7 - 168*a**6*b**3*d**2*e**11/(a*e - b*d)**7 + 336*a**5*b**4*d
**3*e**10/(a*e - b*d)**7 - 420*a**4*b**5*d**4*e**9/(a*e - b*d)**7 + 336*a**
3*b**6*d**5*e**8/(a*e - b*d)**7 - 168*a**2*b**7*d**6*e**7/(a*e - b*d)**7 +
48*a*b**8*d**7*e**6/(a*e - b*d)**7 + 6*a*b*e**6 - 6*b**9*d**8*e**5/(a*e -
b*d)**7 + 6*b**2*d**e**5)/(12*b**2*e**6))/(a*e - b*d)**7 + 6*b*e**5*log(x
+ (6*a**8*b**13/(a*e - b*d)**7 - 48*a**7*b**2*d**12/(a*e - b*d)**7 + 1
68*a**6*b**3*d**2*e**11/(a*e - b*d)**7 - 336*a**5*b**4*d**3*e**10/(a*e - b
*d)**7 + 420*a**4*b**5*d**4*e**9/(a*e - b*d)**7 - 336*a**3*b**6*d**5*e**8/
(a*e - b*d)**7 + 168*a**2*b**7*d**6*e**7/(a*e - b*d)**7 - 48*a*b**8*d**7*e
**6/(a*e - b*d)**7 + 6*a*b*e**6 + 6*b**9*d**8*e**5/(a*e - b*d)**7 + 6*b**2
*d**e**5)/(12*b**2*e**6))/(a*e - b*d)**7 + (-10*a**5*e**5 - 87*a**4*b*d**e**
4 + 63*a**3*b**2*d**2*e**3 - 37*a**2*b**3*d**3*e**2 + 13*a*b**4*d**4*e - 2
*b**5*d**5 - 60*b**5*e**5*x**5 + x**4*(-270*a*b**4*e**5 - 30*b**5*d**e**4)
+ x**3*(-470*a**2*b**3*e**5 - 140*a*b**4*d**e**4 + 10*b**5*d**2*e**3) + x**
2*(-385*a**3*b**2*e**5 - 255*a**2*b**3*d**e**4 + 45*a*b**4*d**2*e**3 - 5*b
**5*d**3*e**2) + x*(-137*a**4*b**e**5 - 222*a**3*b**2*d**e**4 + 78*a**2*b**3*
d**2*e**3 - 22*a*b**4*d**3*e**2 + 3*b**5*d**4*e))/(10*a**11*d**e**6 - 60*a
**10*b*d**2*e**5 + 150*a**9*b**2*d**3*e**4 - 200*a**8*b**3*d**4*e**3 + 150*
a**7*b**4*d**5*e**2 - 60*a**6*b**5*d**6*e + 10*a**5*b**6*d**7 + x**6(1...

```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1154 vs. $2(177) = 354$.

Time = 0.13 (sec) , antiderivative size = 1154, normalized size of antiderivative = 6.38

$$\int \frac{1}{(d + ex)^2 (a^2 + 2abx + b^2x^2)^3} dx = \text{Too large to display}$$

input

```
integrate(1/(e*x+d)^2/(b^2*x^2+2*a*b*x+a^2)^3,x, algorithm="maxima")
```

output

```

-6*b*e^5*log(b*x + a)/(b^7*d^7 - 7*a*b^6*d^6*e + 21*a^2*b^5*d^5*e^2 - 35*a^3*b^4*d^4*e^3 + 35*a^4*b^3*d^3*e^4 - 21*a^5*b^2*d^2*e^5 + 7*a^6*b*d*e^6 - a^7*e^7) + 6*b*e^5*log(e*x + d)/(b^7*d^7 - 7*a*b^6*d^6*e + 21*a^2*b^5*d^5*e^2 - 35*a^3*b^4*d^4*e^3 + 35*a^4*b^3*d^3*e^4 - 21*a^5*b^2*d^2*e^5 + 7*a^6*b*d*e^6 - a^7*e^7) - 1/10*(60*b^5*e^5*x^5 + 2*b^5*d^5 - 13*a*b^4*d^4*e + 37*a^2*b^3*d^3*e^2 - 63*a^3*b^2*d^2*e^3 + 87*a^4*b*d*e^4 + 10*a^5*e^5 + 30*(b^5*d*e^4 + 9*a*b^4*e^5)*x^4 - 10*(b^5*d^2*e^3 - 14*a*b^4*d*e^4 - 47*a^2*b^3*e^5)*x^3 + 5*(b^5*d^3*e^2 - 9*a*b^4*d^2*e^3 + 51*a^2*b^3*d*e^4 + 77*a^3*b^2*e^5)*x^2 - (3*b^5*d^4*e - 22*a*b^4*d^3*e^2 + 78*a^2*b^3*d^2*e^3 - 222*a^3*b^2*d*e^4 - 137*a^4*b*e^5)*x)/(a^5*b^6*d^7 - 6*a^6*b^5*d^6*e + 15*a^7*b^4*d^5*e^2 - 20*a^8*b^3*d^4*e^3 + 15*a^9*b^2*d^3*e^4 - 6*a^10*b*d^2*e^5 + a^11*d*e^6 + (b^11*d^6*e - 6*a*b^10*d^5*e^2 + 15*a^2*b^9*d^4*e^3 - 20*a^3*b^8*d^3*e^4 + 15*a^4*b^7*d^2*e^5 - 6*a^5*b^6*d*e^6 + a^6*b^5*e^7)*x^6 + (b^11*d^7 - a*b^10*d^6*e - 15*a^2*b^9*d^5*e^2 + 55*a^3*b^8*d^4*e^3 - 85*a^4*b^7*d^3*e^4 + 69*a^5*b^6*d^2*e^5 - 29*a^6*b^5*d*e^6 + 5*a^7*b^4*e^7)*x^5 + 5*(a*b^10*d^7 - 4*a^2*b^9*d^6*e + 3*a^3*b^8*d^5*e^2 + 10*a^4*b^7*d^4*e^3 - 25*a^5*b^6*d^3*e^4 + 24*a^6*b^5*d^2*e^5 - 11*a^7*b^4*d*e^6 + 2*a^8*b^3*e^7)*x^4 + 10*(a^2*b^9*d^7 - 5*a^3*b^8*d^6*e + 9*a^4*b^7*d^5*e^2 - 5*a^5*b^6*d^4*e^3 - 5*a^6*b^5*d^3*e^4 + 9*a^7*b^4*d^2*e^5 - 5*a^8*b^3*d*e^6 + a^9*b^2*e^7)*x^3 + 5*(2*a^3*b^8*d^7 - 11*a^4*b^7*d^6*e + 24*a^5*b^6*d^...

```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 473 vs. $2(177) = 354$.

Time = 0.18 (sec) , antiderivative size = 473, normalized size of antiderivative = 2.61

$$\int \frac{1}{(d+ex)^2 (a^2+2abx+b^2x^2)^3} dx =$$

$$\frac{e^{11}}{(b^6d^6e^6 - 6ab^5d^5e^7 + 15a^2b^4d^4e^8 - 20a^3b^3d^3e^9 + 15a^4b^2d^2e^{10} - 6a^5bde^{11} + a^6e^{12})(ex+d)} - \frac{6be^6 \log\left(\left|b - \frac{bd}{ex+d} + \frac{ae}{ex+d}\right|\right)}{(b^7d^7e - 7ab^6d^6e^2 + 21a^2b^5d^5e^3 - 35a^3b^4d^4e^4 + 35a^4b^3d^3e^5 - 21a^5b^2d^2e^6 + 7a^6bde^7 - a^7e^8)} - \frac{87b^6e^5 - \frac{385(b^6de^6 - ab^5e^7)}{(ex+d)e} + \frac{650(b^6d^2e^7 - 2ab^5de^8 + a^2b^4e^9)}{(ex+d)^2e^2} - \frac{500(b^6d^3e^8 - 3ab^5d^2e^9 + 3a^2b^4de^{10} - a^3b^3e^{11})}{(ex+d)^3e^3} + \frac{150(b^6d^4e^9 - 4a^2b^5d^3e^{10} + 4a^3b^4d^2e^{11} - a^4b^3de^{12})}{(ex+d)^4e^4}}{10(bd - ae)^7 \left(b - \frac{bd}{ex+d} + \frac{ae}{ex+d}\right)^5}$$

input

```
integrate(1/(e*x+d)^2/(b^2*x^2+2*a*b*x+a^2)^3,x, algorithm="giac")
```


output

```
(12*b*e^5*atanh((a^7*e^7 + b^7*d^7 + 9*a^2*b^5*d^5*e^2 - 5*a^3*b^4*d^4*e^3
- 5*a^4*b^3*d^3*e^4 + 9*a^5*b^2*d^2*e^5 - 5*a*b^6*d^6*e - 5*a^6*b*d*e^6)/
(a*e - b*d)^7 + (2*b*e*x*(a^6*e^6 + b^6*d^6 + 15*a^2*b^4*d^4*e^2 - 20*a^3*
b^3*d^3*e^3 + 15*a^4*b^2*d^2*e^4 - 6*a*b^5*d^5*e - 6*a^5*b*d*e^5))/(a*e -
b*d)^7))/(a*e - b*d)^7 - ((10*a^5*e^5 + 2*b^5*d^5 + 37*a^2*b^3*d^3*e^2 - 6
3*a^3*b^2*d^2*e^3 - 13*a*b^4*d^4*e + 87*a^4*b*d*e^4)/(10*(a^6*e^6 + b^6*d^
6 + 15*a^2*b^4*d^4*e^2 - 20*a^3*b^3*d^3*e^3 + 15*a^4*b^2*d^2*e^4 - 6*a*b^5
*d^5*e - 6*a^5*b*d*e^5)) + (e^3*x^3*(47*a^2*b^3*e^2 - b^5*d^2 + 14*a*b^4*d
*e))/(a^6*e^6 + b^6*d^6 + 15*a^2*b^4*d^4*e^2 - 20*a^3*b^3*d^3*e^3 + 15*a^4
*b^2*d^2*e^4 - 6*a*b^5*d^5*e - 6*a^5*b*d*e^5) + (e^2*x^2*(b^5*d^3 + 77*a^3
*b^2*e^3 + 51*a^2*b^3*d*e^2 - 9*a*b^4*d^2*e))/(2*(a^6*e^6 + b^6*d^6 + 15*a
^2*b^4*d^4*e^2 - 20*a^3*b^3*d^3*e^3 + 15*a^4*b^2*d^2*e^4 - 6*a*b^5*d^5*e -
6*a^5*b*d*e^5)) + (6*b^5*e^5*x^5)/(a^6*e^6 + b^6*d^6 + 15*a^2*b^4*d^4*e^2
- 20*a^3*b^3*d^3*e^3 + 15*a^4*b^2*d^2*e^4 - 6*a*b^5*d^5*e - 6*a^5*b*d*e^5
) + (3*e^4*x^4*(b^5*d + 9*a*b^4*e))/(a^6*e^6 + b^6*d^6 + 15*a^2*b^4*d^4*e^
2 - 20*a^3*b^3*d^3*e^3 + 15*a^4*b^2*d^2*e^4 - 6*a*b^5*d^5*e - 6*a^5*b*d*e^
5) + (e*x*(137*a^4*b*e^4 - 3*b^5*d^4 + 222*a^3*b^2*d*e^3 - 78*a^2*b^3*d^2*
e^2 + 22*a*b^4*d^3*e))/(10*(a^6*e^6 + b^6*d^6 + 15*a^2*b^4*d^4*e^2 - 20*a^
3*b^3*d^3*e^3 + 15*a^4*b^2*d^2*e^4 - 6*a*b^5*d^5*e - 6*a^5*b*d*e^5)))/(x^5
*(b^5*d + 5*a*b^4*e) + x^3*(10*a^2*b^3*d + 10*a^3*b^2*e) + a^5*d + x*(a...
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 2163, normalized size of antiderivative = 11.95

$$\int \frac{1}{(d + ex)^2 (a^2 + 2abx + b^2x^2)^3} dx = \text{Too large to display}$$

input

```
int(1/(e*x+d)^2/(b^2*x^2+2*a*b*x+a^2)^3,x)
```


output

```
(300*log(a + b*x)*a**6*b*d**e**6 + 300*log(a + b*x)*a**6*b**e**7*x + 60*log(a + b*x)*a**5*b**2*d**2*e**5 + 1560*log(a + b*x)*a**5*b**2*d**e**6*x + 1500*log(a + b*x)*a**5*b**2*e**7*x**2 + 300*log(a + b*x)*a**4*b**3*d**2*e**5*x + 3300*log(a + b*x)*a**4*b**3*d**e**6*x**2 + 3000*log(a + b*x)*a**4*b**3*e**7*x**3 + 600*log(a + b*x)*a**3*b**4*d**2*e**5*x**2 + 3600*log(a + b*x)*a**3*b**4*d**e**6*x**3 + 3000*log(a + b*x)*a**3*b**4*e**7*x**4 + 600*log(a + b*x)*a**2*b**5*d**2*e**5*x**3 + 2100*log(a + b*x)*a**2*b**5*d**e**6*x**4 + 1500*log(a + b*x)*a**2*b**5*e**7*x**5 + 300*log(a + b*x)*a*b**6*d**2*e**5*x**4 + 600*log(a + b*x)*a*b**6*d**e**6*x**5 + 300*log(a + b*x)*a*b**6*e**7*x**6 + 60*log(a + b*x)*b**7*d**2*e**5*x**5 + 60*log(a + b*x)*b**7*d**e**6*x**6 - 300*log(d + e*x)*a**6*b*d**e**6 - 300*log(d + e*x)*a**6*b**e**7*x - 60*log(d + e*x)*a**5*b**2*d**2*e**5 - 1560*log(d + e*x)*a**5*b**2*d**e**6*x - 1500*log(d + e*x)*a**5*b**2*e**7*x**2 - 300*log(d + e*x)*a**4*b**3*d**2*e**5*x - 3300*log(d + e*x)*a**4*b**3*d**e**6*x**2 - 3000*log(d + e*x)*a**4*b**3*e**7*x**3 - 600*log(d + e*x)*a**3*b**4*d**2*e**5*x**2 - 3600*log(d + e*x)*a**3*b**4*d**e**6*x**3 - 3000*log(d + e*x)*a**3*b**4*e**7*x**4 - 600*log(d + e*x)*a**2*b**5*d**2*e**5*x**3 - 2100*log(d + e*x)*a**2*b**5*d**e**6*x**4 - 1500*log(d + e*x)*a**2*b**5*e**7*x**5 - 300*log(d + e*x)*a*b**6*d**2*e**5*x**4 - 600*log(d + e*x)*a*b**6*d**e**6*x**5 - 300*log(d + e*x)*a*b**6*e**7*x**6 - 60*log(d + e*x)*b**7*d**2*e**5*x**5 - 60*log(d + e*x)*b**...
```

3.139 $\int \frac{1}{(d+ex)^3(a^2+2abx+b^2x^2)^3} dx$

| | |
|---|------|
| Optimal result | 1109 |
| Mathematica [A] (verified) | 1110 |
| Rubi [A] (verified) | 1110 |
| Maple [A] (verified) | 1112 |
| Fricas [B] (verification not implemented) | 1112 |
| Sympy [B] (verification not implemented) | 1113 |
| Maxima [B] (verification not implemented) | 1114 |
| Giac [B] (verification not implemented) | 1115 |
| Mupad [B] (verification not implemented) | 1116 |
| Reduce [B] (verification not implemented) | 1117 |

Optimal result

Integrand size = 26, antiderivative size = 220

$$\int \frac{1}{(d+ex)^3(a^2+2abx+b^2x^2)^3} dx = -\frac{b^2}{5(bd-ae)^3(a+bx)^5} + \frac{3b^2e}{4(bd-ae)^4(a+bx)^4} - \frac{2b^2e^2}{(bd-ae)^5(a+bx)^3} + \frac{5b^2e^3}{(bd-ae)^6(a+bx)^2} - \frac{15b^2e^4}{(bd-ae)^7(a+bx)} - \frac{e^5}{6be^5} - \frac{2(bd-ae)^6(d+ex)^2}{(bd-ae)^7(d+ex)} - \frac{21b^2e^5 \log(a+bx)}{(bd-ae)^8} + \frac{21b^2e^5 \log(d+ex)}{(bd-ae)^8}$$

output

```
-1/5*b^2/(-a*e+b*d)^3/(b*x+a)^5+3/4*b^2*e/(-a*e+b*d)^4/(b*x+a)^4-2*b^2*e^2/(-a*e+b*d)^5/(b*x+a)^3+5*b^2*e^3/(-a*e+b*d)^6/(b*x+a)^2-15*b^2*e^4/(-a*e+b*d)^7/(b*x+a)-1/2*e^5/(-a*e+b*d)^6/(e*x+d)^2-6*b^2*e^5/(-a*e+b*d)^7/(e*x+d)-21*b^2*e^5*ln(b*x+a)/(-a*e+b*d)^8+21*b^2*e^5*ln(e*x+d)/(-a*e+b*d)^8
```

Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 204, normalized size of antiderivative = 0.93

$$\int \frac{1}{(d+ex)^3 (a^2+2abx+b^2x^2)^3} dx = \frac{\frac{4b^2(bd-ae)^5}{(a+bx)^5} - \frac{15b^2e(bd-ae)^4}{(a+bx)^4} + \frac{40b^2e^2(bd-ae)^3}{(a+bx)^3} - \frac{100b^2e^3(bd-ae)^2}{(a+bx)^2} + \frac{300b^2e^4(bd-ae)}{a+bx} + \frac{10e^5(bd-ae)^2}{(d+ex)^2} + \frac{120be^5(bd-ae)}{d+ex} + \frac{20(bd-ae)^8}{20(bd-ae)^8}$$

input

```
Integrate[1/((d + e*x)^3*(a^2 + 2*a*b*x + b^2*x^2)^3),x]
```

output

```
-1/20*((4*b^2*(b*d - a*e)^5)/(a + b*x)^5 - (15*b^2*e*(b*d - a*e)^4)/(a + b*x)^4 + (40*b^2*e^2*(b*d - a*e)^3)/(a + b*x)^3 - (100*b^2*e^3*(b*d - a*e)^2)/(a + b*x)^2 + (300*b^2*e^4*(b*d - a*e))/(a + b*x) + (10*e^5*(b*d - a*e)^2)/(d + e*x)^2 + (120*b*e^5*(b*d - a*e))/(d + e*x) + 420*b^2*e^5*Log[a + b*x] - 420*b^2*e^5*Log[d + e*x])/(b*d - a*e)^8
```

Rubi [A] (verified)Time = 0.83 (sec) , antiderivative size = 220, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {1098, 27, 54, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\begin{aligned} & \int \frac{1}{(a^2+2abx+b^2x^2)^3 (d+ex)^3} dx \\ & \quad \downarrow \text{1098} \\ & b^6 \int \frac{1}{b^6(a+bx)^6 (d+ex)^3} dx \\ & \quad \downarrow \text{27} \\ & \int \frac{1}{(a+bx)^6 (d+ex)^3} dx \\ & \quad \downarrow \text{54} \end{aligned}$$

$$\int \left(-\frac{21b^3e^5}{(a+bx)(bd-ae)^8} + \frac{15b^3e^4}{(a+bx)^2(bd-ae)^7} - \frac{10b^3e^3}{(a+bx)^3(bd-ae)^6} + \frac{6b^3e^2}{(a+bx)^4(bd-ae)^5} - \frac{3b^3e}{(a+bx)^5(bd-ae)^4} \right)$$

↓ 2009

$$-\frac{21b^2e^5 \log(a+bx)}{(bd-ae)^8} + \frac{21b^2e^5 \log(d+ex)}{(bd-ae)^8} - \frac{15b^2e^4}{(a+bx)(bd-ae)^7} + \frac{5b^2e^3}{(a+bx)^2(bd-ae)^6} - \frac{2b^2e^2}{(a+bx)^3(bd-ae)^5} + \frac{3b^2e}{4(a+bx)^4(bd-ae)^4} - \frac{5(a+bx)^5(bd-ae)^3}{e^5} - \frac{6be^5}{(d+ex)(bd-ae)^7} - \frac{2(d+ex)^2(bd-ae)^6}{e^5}$$

input `Int[1/((d + e*x)^3*(a^2 + 2*a*b*x + b^2*x^2)^3),x]`

output `-1/5*b^2/((b*d - a*e)^3*(a + b*x)^5) + (3*b^2*e)/(4*(b*d - a*e)^4*(a + b*x)^4) - (2*b^2*e^2)/((b*d - a*e)^5*(a + b*x)^3) + (5*b^2*e^3)/((b*d - a*e)^6*(a + b*x)^2) - (15*b^2*e^4)/((b*d - a*e)^7*(a + b*x)) - e^5/(2*(b*d - a*e)^6*(d + e*x)^2) - (6*b*e^5)/((b*d - a*e)^7*(d + e*x)) - (21*b^2*e^5*Log[a + b*x])/((b*d - a*e)^8) + (21*b^2*e^5*Log[d + e*x])/((b*d - a*e)^8)`

Defintions of rubi rules used

rule 27 `Int[(a_)*(F_x_), x_Symbol] := Simp[a Int[F_x, x], x] /; FreeQ[a, x] && !MatchQ[F_x, (b_)*(G_x_)] /; FreeQ[b, x]`

rule 54 `Int[((a_) + (b_)*(x_))^(m_)*((c_) + (d_)*(x_))^(n_), x_Symbol] := Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] && ILtQ[m, 0] && IntegerQ[n] && !(IGtQ[n, 0] && LtQ[m + n + 2, 0])`

rule 1098 `Int[((d_) + (e_)*(x_))^(m_)*((a_) + (b_)*(x_) + (c_)*(x_)^2)^(p_), x_Symbol] := Simp[1/c^p Int[(d + e*x)^m*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]`

rule 2009 `Int[u_, x_Symbol] := Simp[IntSum[u, x], x] /; SumQ[u]`

Maple [A] (verified)

Time = 1.49 (sec) , antiderivative size = 215, normalized size of antiderivative = 0.98

| method | result |
|--------------|---|
| default | $\frac{b^2}{5(ae-bd)^3(bx+a)^5} - \frac{21b^2e^5 \ln(bx+a)}{(ae-bd)^8} + \frac{15b^2e^4}{(ae-bd)^7(bx+a)} + \frac{5b^2e^3}{(ae-bd)^6(bx+a)^2} + \frac{2b^2e^2}{(ae-bd)^5(bx+a)^3} + \frac{3b^2e}{4(ae-bd)^4(bx+a)^4}$ |
| risch | Expression too large to display |
| norman | Expression too large to display |
| parallelrisc | Expression too large to display |

input `int(1/(e*x+d)^3/(b^2*x^2+2*a*b*x+a^2)^3,x,method=_RETURNVERBOSE)`

output $\frac{1}{5}b^2/(a*e-b*d)^3/(b*x+a)^5 - 21*b^2/(a*e-b*d)^8*e^5*\ln(b*x+a) + 15*b^2/(a*e-b*d)^7*e^4/(b*x+a) + 5*b^2/(a*e-b*d)^6*e^3/(b*x+a)^2 + 2*b^2/(a*e-b*d)^5*e^2/(b*x+a)^3 + 3/4*b^2/(a*e-b*d)^4*e/(b*x+a)^4 - 1/2*e^5/(a*e-b*d)^6/(e*x+d)^2 + 21*b^2/(a*e-b*d)^8*e^5*\ln(e*x+d) + 6*e^5/(a*e-b*d)^7*b/(e*x+d)$

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2005 vs. 2(214) = 428.

Time = 0.13 (sec) , antiderivative size = 2005, normalized size of antiderivative = 9.11

$$\int \frac{1}{(d+ex)^3(a^2+2abx+b^2x^2)^3} dx = \text{Too large to display}$$

input `integrate(1/(e*x+d)^3/(b^2*x^2+2*a*b*x+a^2)^3,x,algorithm="fricas")`

output

```

-1/20*(4*b^7*d^7 - 35*a*b^6*d^6*e + 140*a^2*b^5*d^5*e^2 - 350*a^3*b^4*d^4*
e^3 + 700*a^4*b^3*d^3*e^4 - 329*a^5*b^2*d^2*e^5 - 140*a^6*b*d*e^6 + 10*a^7
*e^7 + 420*(b^7*d*e^6 - a*b^6*e^7)*x^6 + 630*(b^7*d^2*e^5 + 2*a*b^6*d*e^6
- 3*a^2*b^5*e^7)*x^5 + 70*(2*b^7*d^3*e^4 + 39*a*b^6*d^2*e^5 + 6*a^2*b^5*d*
e^6 - 47*a^3*b^4*e^7)*x^4 - 35*(b^7*d^4*e^3 - 20*a*b^6*d^3*e^4 - 126*a^2*b
^5*d^2*e^5 + 68*a^3*b^4*d*e^6 + 77*a^4*b^3*e^7)*x^3 + 7*(2*b^7*d^5*e^2 - 2
5*a*b^6*d^4*e^3 + 200*a^2*b^5*d^3*e^4 + 430*a^3*b^4*d^2*e^5 - 470*a^4*b^3*
d*e^6 - 137*a^5*b^2*e^7)*x^2 - 7*(b^7*d^6*e - 10*a*b^6*d^5*e^2 + 50*a^2*b^
5*d^4*e^3 - 200*a^3*b^4*d^3*e^4 - 65*a^4*b^3*d^2*e^5 + 214*a^5*b^2*d*e^6 +
10*a^6*b*e^7)*x + 420*(b^7*e^7*x^7 + a^5*b^2*d^2*e^5 + (2*b^7*d*e^6 + 5*a
*b^6*e^7)*x^6 + (b^7*d^2*e^5 + 10*a*b^6*d*e^6 + 10*a^2*b^5*e^7)*x^5 + 5*(a
*b^6*d^2*e^5 + 4*a^2*b^5*d*e^6 + 2*a^3*b^4*e^7)*x^4 + 5*(2*a^2*b^5*d^2*e^5
+ 4*a^3*b^4*d*e^6 + a^4*b^3*e^7)*x^3 + (10*a^3*b^4*d^2*e^5 + 10*a^4*b^3*d
*e^6 + a^5*b^2*e^7)*x^2 + (5*a^4*b^3*d^2*e^5 + 2*a^5*b^2*d*e^6)*x)*log(b*x
+ a) - 420*(b^7*e^7*x^7 + a^5*b^2*d^2*e^5 + (2*b^7*d*e^6 + 5*a*b^6*e^7)*x
^6 + (b^7*d^2*e^5 + 10*a*b^6*d*e^6 + 10*a^2*b^5*e^7)*x^5 + 5*(a*b^6*d^2*e^
5 + 4*a^2*b^5*d*e^6 + 2*a^3*b^4*e^7)*x^4 + 5*(2*a^2*b^5*d^2*e^5 + 4*a^3*b^
4*d*e^6 + a^4*b^3*e^7)*x^3 + (10*a^3*b^4*d^2*e^5 + 10*a^4*b^3*d*e^6 + a^5*
b^2*e^7)*x^2 + (5*a^4*b^3*d^2*e^5 + 2*a^5*b^2*d*e^6)*x)*log(e*x + d))/(a^5
*b^8*d^10 - 8*a^6*b^7*d^9*e + 28*a^7*b^6*d^8*e^2 - 56*a^8*b^5*d^7*e^3 +...

```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1974 vs. $2(201) = 402$.

Time = 4.87 (sec) , antiderivative size = 1974, normalized size of antiderivative = 8.97

$$\int \frac{1}{(d + ex)^3 (a^2 + 2abx + b^2x^2)^3} dx = \text{Too large to display}$$

input

```
integrate(1/(e*x+d)**3/(b**2*x**2+2*a*b*x+a**2)**3,x)
```

output

```

21*b**2*e**5*log(x + (-21*a**9*b**2*e**14/(a*e - b*d)**8 + 189*a**8*b**3*d
e**13/(a*e - b*d)**8 - 756*a**7*b**4*d**2*e**12/(a*e - b*d)**8 + 1764*a**
6*b**5*d**3*e**11/(a*e - b*d)**8 - 2646*a**5*b**6*d**4*e**10/(a*e - b*d)**
8 + 2646*a**4*b**7*d**5*e**9/(a*e - b*d)**8 - 1764*a**3*b**8*d**6*e**8/(a*
e - b*d)**8 + 756*a**2*b**9*d**7*e**7/(a*e - b*d)**8 - 189*a*b**10*d**8*e*
*6/(a*e - b*d)**8 + 21*a*b**2*e**6 + 21*b**11*d**9*e**5/(a*e - b*d)**8 + 2
1*b**3*d*e**5)/(42*b**3*e**6))/(a*e - b*d)**8 - 21*b**2*e**5*log(x + (21*a
**9*b**2*e**14/(a*e - b*d)**8 - 189*a**8*b**3*d*e**13/(a*e - b*d)**8 + 756
*a**7*b**4*d**2*e**12/(a*e - b*d)**8 - 1764*a**6*b**5*d**3*e**11/(a*e - b*
d)**8 + 2646*a**5*b**6*d**4*e**10/(a*e - b*d)**8 - 2646*a**4*b**7*d**5*e**
9/(a*e - b*d)**8 + 1764*a**3*b**8*d**6*e**8/(a*e - b*d)**8 - 756*a**2*b**9
*d**7*e**7/(a*e - b*d)**8 + 189*a*b**10*d**8*e**6/(a*e - b*d)**8 + 21*a*b*
*2*e**6 - 21*b**11*d**9*e**5/(a*e - b*d)**8 + 21*b**3*d*e**5)/(42*b**3*e**
6))/(a*e - b*d)**8 + (-10*a**6*e**6 + 130*a**5*b*d*e**5 + 459*a**4*b**2*d*
*2*e**4 - 241*a**3*b**3*d**3*e**3 + 109*a**2*b**4*d**4*e**2 - 31*a*b**5*d*
*5*e + 4*b**6*d**6 + 420*b**6*e**6*x**6 + x**5*(1890*a*b**5*e**6 + 630*b**
6*d*e**5) + x**4*(3290*a**2*b**4*e**6 + 2870*a*b**5*d*e**5 + 140*b**6*d**2
*e**4) + x**3*(2695*a**3*b**3*e**6 + 5075*a**2*b**4*d*e**5 + 665*a*b**5*d*
*2*e**4 - 35*b**6*d**3*e**3) + x**2*(959*a**4*b**2*e**6 + 4249*a**3*b**3*d
e**5 + 1239*a**2*b**4*d**2*e**4 - 161*a*b**5*d**3*e**3 + 14*b**6*d**4*...

```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1558 vs. $2(214) = 428$.

Time = 0.15 (sec) , antiderivative size = 1558, normalized size of antiderivative = 7.08

$$\int \frac{1}{(d + ex)^3 (a^2 + 2abx + b^2x^2)^3} dx = \text{Too large to display}$$

input

```
integrate(1/(e*x+d)^3/(b^2*x^2+2*a*b*x+a^2)^3,x, algorithm="maxima")
```

output

```

-21*b^2*e^5*log(b*x + a)/(b^8*d^8 - 8*a*b^7*d^7*e + 28*a^2*b^6*d^6*e^2 - 5
6*a^3*b^5*d^5*e^3 + 70*a^4*b^4*d^4*e^4 - 56*a^5*b^3*d^3*e^5 + 28*a^6*b^2*d
^2*e^6 - 8*a^7*b*d*e^7 + a^8*e^8) + 21*b^2*e^5*log(e*x + d)/(b^8*d^8 - 8*a
*b^7*d^7*e + 28*a^2*b^6*d^6*e^2 - 56*a^3*b^5*d^5*e^3 + 70*a^4*b^4*d^4*e^4
- 56*a^5*b^3*d^3*e^5 + 28*a^6*b^2*d^2*e^6 - 8*a^7*b*d*e^7 + a^8*e^8) - 1/2
0*(420*b^6*e^6*x^6 + 4*b^6*d^6 - 31*a*b^5*d^5*e + 109*a^2*b^4*d^4*e^2 - 24
1*a^3*b^3*d^3*e^3 + 459*a^4*b^2*d^2*e^4 + 130*a^5*b*d*e^5 - 10*a^6*e^6 + 6
30*(b^6*d*e^5 + 3*a*b^5*e^6)*x^5 + 70*(2*b^6*d^2*e^4 + 41*a*b^5*d*e^5 + 47
*a^2*b^4*e^6)*x^4 - 35*(b^6*d^3*e^3 - 19*a*b^5*d^2*e^4 - 145*a^2*b^4*d*e^5
- 77*a^3*b^3*e^6)*x^3 + 7*(2*b^6*d^4*e^2 - 23*a*b^5*d^3*e^3 + 177*a^2*b^4
*d^2*e^4 + 607*a^3*b^3*d*e^5 + 137*a^4*b^2*e^6)*x^2 - 7*(b^6*d^5*e - 9*a*b
^5*d^4*e^2 + 41*a^2*b^4*d^3*e^3 - 159*a^3*b^3*d^2*e^4 - 224*a^4*b^2*d*e^5
- 10*a^5*b*e^6)*x)/(a^5*b^7*d^9 - 7*a^6*b^6*d^8*e + 21*a^7*b^5*d^7*e^2 - 3
5*a^8*b^4*d^6*e^3 + 35*a^9*b^3*d^5*e^4 - 21*a^10*b^2*d^4*e^5 + 7*a^11*b*d
^3*e^6 - a^12*d^2*e^7 + (b^12*d^7*e^2 - 7*a*b^11*d^6*e^3 + 21*a^2*b^10*d^5*
e^4 - 35*a^3*b^9*d^4*e^5 + 35*a^4*b^8*d^3*e^6 - 21*a^5*b^7*d^2*e^7 + 7*a^6
*b^6*d*e^8 - a^7*b^5*e^9)*x^7 + (2*b^12*d^8*e - 9*a*b^11*d^7*e^2 + 7*a^2*b
^10*d^6*e^3 + 35*a^3*b^9*d^5*e^4 - 105*a^4*b^8*d^4*e^5 + 133*a^5*b^7*d^3*e
^6 - 91*a^6*b^6*d^2*e^7 + 33*a^7*b^5*d*e^8 - 5*a^8*b^4*e^9)*x^6 + (b^12*d
^9 + 3*a*b^11*d^8*e - 39*a^2*b^10*d^7*e^2 + 105*a^3*b^9*d^6*e^3 - 105*a^...

```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 714 vs. $2(214) = 428$.

Time = 0.17 (sec) , antiderivative size = 714, normalized size of antiderivative = 3.25

$$\int \frac{1}{(d+ex)^3 (a^2+2abx+b^2x^2)^3} dx =$$

$$\frac{21b^3e^5 \log(|bx+a|)}{b^9d^8 - 8ab^8d^7e + 28a^2b^7d^6e^2 - 56a^3b^6d^5e^3 + 70a^4b^5d^4e^4 - 56a^5b^4d^3e^5 + 28a^6b^3d^2e^6 - 8a^7b^2de^7 + a^8e^8} + \frac{21b^2e^6 \log(|ex+d|)}{b^8d^8e - 8ab^7d^7e^2 + 28a^2b^6d^6e^3 - 56a^3b^5d^5e^4 + 70a^4b^4d^4e^5 - 56a^5b^3d^3e^6 + 28a^6b^2d^2e^7 - 8a^7bde^8 + 4b^7d^7 - 35ab^6d^6e + 140a^2b^5d^5e^2 - 350a^3b^4d^4e^3 + 700a^4b^3d^3e^4 - 329a^5b^2d^2e^5 - 140a^6bde^6 + 10a^7e^7 - a^8e^8}$$

input

```
integrate(1/(e*x+d)^3/(b^2*x^2+2*a*b*x+a^2)^3,x, algorithm="giac")
```


output

```

-21*b^3*e^5*log(abs(b*x + a))/(b^9*d^8 - 8*a*b^8*d^7*e + 28*a^2*b^7*d^6*e^
2 - 56*a^3*b^6*d^5*e^3 + 70*a^4*b^5*d^4*e^4 - 56*a^5*b^4*d^3*e^5 + 28*a^6*
b^3*d^2*e^6 - 8*a^7*b^2*d*e^7 + a^8*b*e^8) + 21*b^2*e^6*log(abs(e*x + d))/
(b^8*d^8*e - 8*a*b^7*d^7*e^2 + 28*a^2*b^6*d^6*e^3 - 56*a^3*b^5*d^5*e^4 + 7
0*a^4*b^4*d^4*e^5 - 56*a^5*b^3*d^3*e^6 + 28*a^6*b^2*d^2*e^7 - 8*a^7*b*d*e^
8 + a^8*e^9) - 1/20*(4*b^7*d^7 - 35*a*b^6*d^6*e + 140*a^2*b^5*d^5*e^2 - 35
0*a^3*b^4*d^4*e^3 + 700*a^4*b^3*d^3*e^4 - 329*a^5*b^2*d^2*e^5 - 140*a^6*b*
d*e^6 + 10*a^7*e^7 + 420*(b^7*d*e^6 - a*b^6*e^7)*x^6 + 630*(b^7*d^2*e^5 +
2*a*b^6*d*e^6 - 3*a^2*b^5*e^7)*x^5 + 70*(2*b^7*d^3*e^4 + 39*a*b^6*d^2*e^5
+ 6*a^2*b^5*d*e^6 - 47*a^3*b^4*e^7)*x^4 - 35*(b^7*d^4*e^3 - 20*a*b^6*d^3*e
^4 - 126*a^2*b^5*d^2*e^5 + 68*a^3*b^4*d*e^6 + 77*a^4*b^3*e^7)*x^3 + 7*(2*b
^7*d^5*e^2 - 25*a*b^6*d^4*e^3 + 200*a^2*b^5*d^3*e^4 + 430*a^3*b^4*d^2*e^5
- 470*a^4*b^3*d*e^6 - 137*a^5*b^2*e^7)*x^2 - 7*(b^7*d^6*e - 10*a*b^6*d^5*e
^2 + 50*a^2*b^5*d^4*e^3 - 200*a^3*b^4*d^3*e^4 - 65*a^4*b^3*d^2*e^5 + 214*a
^5*b^2*d*e^6 + 10*a^6*b*e^7)*x)/((b*d - a*e)^8*(b*x + a)^5*(e*x + d)^2)

```

Mupad [B] (verification not implemented)

Time = 10.05 (sec) , antiderivative size = 1427, normalized size of antiderivative = 6.49

$$\int \frac{1}{(d + ex)^3 (a^2 + 2abx + b^2x^2)^3} dx = \text{Too large to display}$$

input

```
int(1/((d + e*x)^3*(a^2 + b^2*x^2 + 2*a*b*x)^3),x)
```

output

```

((4*b^6*d^6 - 10*a^6*e^6 + 109*a^2*b^4*d^4*e^2 - 241*a^3*b^3*d^3*e^3 + 459
*a^4*b^2*d^2*e^4 - 31*a*b^5*d^5*e + 130*a^5*b*d*e^5)/(20*(a^7*e^7 - b^7*d^
7 - 21*a^2*b^5*d^5*e^2 + 35*a^3*b^4*d^4*e^3 - 35*a^4*b^3*d^3*e^4 + 21*a^5*
b^2*d^2*e^5 + 7*a*b^6*d^6*e - 7*a^6*b*d*e^6)) + (7*e^3*x^3*(77*a^3*b^3*e^3
- b^6*d^3 + 145*a^2*b^4*d*e^2 + 19*a*b^5*d^2*e))/(4*(a^7*e^7 - b^7*d^7 -
21*a^2*b^5*d^5*e^2 + 35*a^3*b^4*d^4*e^3 - 35*a^4*b^3*d^3*e^4 + 21*a^5*b^2*
d^2*e^5 + 7*a*b^6*d^6*e - 7*a^6*b*d*e^6)) + (21*b^6*e^6*x^6)/(a^7*e^7 - b^
7*d^7 - 21*a^2*b^5*d^5*e^2 + 35*a^3*b^4*d^4*e^3 - 35*a^4*b^3*d^3*e^4 + 21*
a^5*b^2*d^2*e^5 + 7*a*b^6*d^6*e - 7*a^6*b*d*e^6) + (7*e^2*x^2*(2*b^6*d^4 +
137*a^4*b^2*e^4 + 607*a^3*b^3*d*e^3 + 177*a^2*b^4*d^2*e^2 - 23*a*b^5*d^3*
e))/(20*(a^7*e^7 - b^7*d^7 - 21*a^2*b^5*d^5*e^2 + 35*a^3*b^4*d^4*e^3 - 35*
a^4*b^3*d^3*e^4 + 21*a^5*b^2*d^2*e^5 + 7*a*b^6*d^6*e - 7*a^6*b*d*e^6)) + (
7*e^4*x^4*(2*b^6*d^2 + 47*a^2*b^4*e^2 + 41*a*b^5*d*e))/(2*(a^7*e^7 - b^7*d
^7 - 21*a^2*b^5*d^5*e^2 + 35*a^3*b^4*d^4*e^3 - 35*a^4*b^3*d^3*e^4 + 21*a^5
*b^2*d^2*e^5 + 7*a*b^6*d^6*e - 7*a^6*b*d*e^6)) + (7*e*x*(10*a^5*b*e^5 - b^
6*d^5 + 224*a^4*b^2*d*e^4 - 41*a^2*b^4*d^3*e^2 + 159*a^3*b^3*d^2*e^3 + 9*a
*b^5*d^4*e))/(20*(a^7*e^7 - b^7*d^7 - 21*a^2*b^5*d^5*e^2 + 35*a^3*b^4*d^4*
e^3 - 35*a^4*b^3*d^3*e^4 + 21*a^5*b^2*d^2*e^5 + 7*a*b^6*d^6*e - 7*a^6*b*d*
e^6)) + (63*b*e^4*x^5*(3*a*b^4*e^2 + b^5*d*e))/(2*(a^7*e^7 - b^7*d^7 - 21*
a^2*b^5*d^5*e^2 + 35*a^3*b^4*d^4*e^3 - 35*a^4*b^3*d^3*e^4 + 21*a^5*b^2*...

```

Reduce [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 2998, normalized size of antiderivative = 13.63

$$\int \frac{1}{(d+ex)^3 (a^2+2abx+b^2x^2)^3} dx = \text{Too large to display}$$

input

```
int(1/(e*x+d)^3/(b^2*x^2+2*a*b*x+a^2)^3,x)
```

output

```
( - 2100*log(a + b*x)*a**6*b**2*d**2*e**6 - 4200*log(a + b*x)*a**6*b**2*d*
e**7*x - 2100*log(a + b*x)*a**6*b**2*e**8*x**2 - 840*log(a + b*x)*a**5*b**
3*d**3*e**5 - 12180*log(a + b*x)*a**5*b**3*d**2*e**6*x - 21840*log(a + b*x
)*a**5*b**3*d*e**7*x**2 - 10500*log(a + b*x)*a**5*b**3*e**8*x**3 - 4200*lo
g(a + b*x)*a**4*b**4*d**3*e**5*x - 29400*log(a + b*x)*a**4*b**4*d**2*e**6*
x**2 - 46200*log(a + b*x)*a**4*b**4*d*e**7*x**3 - 21000*log(a + b*x)*a**4*
b**4*e**8*x**4 - 8400*log(a + b*x)*a**3*b**5*d**3*e**5*x**2 - 37800*log(a
+ b*x)*a**3*b**5*d**2*e**6*x**3 - 50400*log(a + b*x)*a**3*b**5*d*e**7*x**4
- 21000*log(a + b*x)*a**3*b**5*e**8*x**5 - 8400*log(a + b*x)*a**2*b**6*d*
*3*e**5*x**3 - 27300*log(a + b*x)*a**2*b**6*d**2*e**6*x**4 - 29400*log(a +
b*x)*a**2*b**6*d*e**7*x**5 - 10500*log(a + b*x)*a**2*b**6*e**8*x**6 - 420
0*log(a + b*x)*a*b**7*d**3*e**5*x**4 - 10500*log(a + b*x)*a*b**7*d**2*e**6
*x**5 - 8400*log(a + b*x)*a*b**7*d*e**7*x**6 - 2100*log(a + b*x)*a*b**7*e*
*8*x**7 - 840*log(a + b*x)*b**8*d**3*e**5*x**5 - 1680*log(a + b*x)*b**8*d*
*2*e**6*x**6 - 840*log(a + b*x)*b**8*d*e**7*x**7 + 2100*log(d + e*x)*a**6*
b**2*d**2*e**6 + 4200*log(d + e*x)*a**6*b**2*d*e**7*x + 2100*log(d + e*x)*
a**6*b**2*e**8*x**2 + 840*log(d + e*x)*a**5*b**3*d**3*e**5 + 12180*log(d +
e*x)*a**5*b**3*d**2*e**6*x + 21840*log(d + e*x)*a**5*b**3*d*e**7*x**2 + 1
0500*log(d + e*x)*a**5*b**3*e**8*x**3 + 4200*log(d + e*x)*a**4*b**4*d**3*e
**5*x + 29400*log(d + e*x)*a**4*b**4*d**2*e**6*x**2 + 46200*log(d + e*x...
```

3.140 $\int (d + ex)(9 + 12x + 4x^2)^3 dx$

| | |
|---|------|
| Optimal result | 1119 |
| Mathematica [B] (verified) | 1119 |
| Rubi [A] (verified) | 1120 |
| Maple [B] (verified) | 1121 |
| Fricas [B] (verification not implemented) | 1122 |
| Sympy [B] (verification not implemented) | 1122 |
| Maxima [B] (verification not implemented) | 1123 |
| Giac [B] (verification not implemented) | 1123 |
| Mupad [B] (verification not implemented) | 1124 |
| Reduce [B] (verification not implemented) | 1124 |

Optimal result

Integrand size = 18, antiderivative size = 31

$$\int (d + ex)(9 + 12x + 4x^2)^3 dx = \frac{1}{28}(2d - 3e)(3 + 2x)^7 + \frac{1}{32}e(3 + 2x)^8$$

output

```
1/28*(2*d-3*e)*(3+2*x)^7+1/32*e*(3+2*x)^8
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 81 vs. $2(31) = 62$.

Time = 0.02 (sec) , antiderivative size = 81, normalized size of antiderivative = 2.61

$$\int (d + ex)(9 + 12x + 4x^2)^3 dx = 729dx + \frac{729}{2}(4d + e)x^2 + 324(5d + 3e)x^3 + 135(8d + 9e)x^4 + 432(d + 2e)x^5 + 24(4d + 15e)x^6 + \frac{64}{7}(d + 9e)x^7 + 8ex^8$$

input

```
Integrate[(d + e*x)*(9 + 12*x + 4*x^2)^3,x]
```

output

$$729*d*x + (729*(4*d + e)*x^2)/2 + 324*(5*d + 3*e)*x^3 + 135*(8*d + 9*e)*x^4 + 432*(d + 2*e)*x^5 + 24*(4*d + 15*e)*x^6 + (64*(d + 9*e)*x^7)/7 + 8*e*x^8$$

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {1098, 27, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\begin{aligned} & \int (4x^2 + 12x + 9)^3 (d + ex) dx \\ & \quad \downarrow 1098 \\ & \frac{1}{64} \int 64(2x + 3)^6 (d + ex) dx \\ & \quad \downarrow 27 \\ & \int (2x + 3)^6 (d + ex) dx \\ & \quad \downarrow 49 \\ & \int \left(\frac{1}{2}(2x + 3)^6 (2d - 3e) + \frac{1}{2}e(2x + 3)^7 \right) dx \\ & \quad \downarrow 2009 \\ & \frac{1}{28}(2x + 3)^7 (2d - 3e) + \frac{1}{32}e(2x + 3)^8 \end{aligned}$$

input

$$\text{Int}[(d + e*x)*(9 + 12*x + 4*x^2)^3, x]$$

output

$$((2*d - 3*e)*(3 + 2*x)^7)/28 + (e*(3 + 2*x)^8)/32$$

Defintions of rubi rules used

rule 27 `Int[(a_)*(Fx_), x_Symbol] := Simp[a Int[Fx, x], x] /; FreeQ[a, x] && !MatchQ[Fx, (b_)*(Gx_)] /; FreeQ[b, x]`

rule 49 `Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] := Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] && IGtQ[m, 0] && IGtQ[m + n + 2, 0]`

rule 1098 `Int[((d_.) + (e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] := Simp[1/c^p Int[(d + e*x)^m*(b + c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]`

rule 2009 `Int[u_, x_Symbol] := Simp[IntSum[u, x], x] /; SumQ[u]`

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 77 vs. 2(27) = 54.

Time = 0.72 (sec) , antiderivative size = 78, normalized size of antiderivative = 2.52

| method | result |
|---------------|---|
| norman | $8e x^8 + \left(\frac{64d}{7} + \frac{576e}{7}\right) x^7 + (96d + 360e) x^6 + (432d + 864e) x^5 + (1080d + 1215e) x^4 + (1620d + 2025e) x^3 + (112e x^7 + 128d x^6 + 1152e x^6 + 1344d x^5 + 5040x^5e + 6048d x^4 + 12096x^4e + 15120d x^3 + 17010x^3e + 22680d x^2 + 13608e x^2 + 2025d x + 1350e) x^2 + 1350d x + 1350e$ |
| gospers | $\frac{x(112e x^7 + 128d x^6 + 1152e x^6 + 1344d x^5 + 5040x^5e + 6048d x^4 + 12096x^4e + 15120d x^3 + 17010x^3e + 22680d x^2 + 13608e x^2 + 2025d x + 1350e)}{14}$ |
| default | $8e x^8 + \frac{(64d+576e)x^7}{7} + \frac{(576d+2160e)x^6}{6} + \frac{(2160d+4320e)x^5}{5} + \frac{(4320d+4860e)x^4}{4} + \frac{(4860d+2916e)x^3}{3} + \frac{(2916d+1350e)x^2}{2} + 1350d x + 1350e$ |
| risch | $8e x^8 + \frac{64}{7}x^7d + \frac{576}{7}e x^7 + 96d x^6 + 360e x^6 + 432d x^5 + 864x^5e + 1080d x^4 + 1215x^4e + 1620d x^3 + 2025x^3e + 112e x^7 + 128d x^6 + 1152e x^6 + 1344d x^5 + 5040x^5e + 6048d x^4 + 12096x^4e + 15120d x^3 + 17010x^3e + 22680d x^2 + 13608e x^2 + 2025d x + 1350e$ |
| parallelrisch | $8e x^8 + \frac{64}{7}x^7d + \frac{576}{7}e x^7 + 96d x^6 + 360e x^6 + 432d x^5 + 864x^5e + 1080d x^4 + 1215x^4e + 1620d x^3 + 2025x^3e + 112e x^7 + 128d x^6 + 1152e x^6 + 1344d x^5 + 5040x^5e + 6048d x^4 + 12096x^4e + 15120d x^3 + 17010x^3e + 22680d x^2 + 13608e x^2 + 2025d x + 1350e$ |
| orering | $\frac{x(112e x^7 + 128d x^6 + 1152e x^6 + 1344d x^5 + 5040x^5e + 6048d x^4 + 12096x^4e + 15120d x^3 + 17010x^3e + 22680d x^2 + 13608e x^2 + 2025d x + 1350e)}{14(2x+3)^6}$ |

input `int((e*x+d)*(4*x^2+12*x+9)^3,x,method=_RETURNVERBOSE)`

output

```
8*e*x^8+(64/7*d+576/7*e)*x^7+(96*d+360*e)*x^6+(432*d+864*e)*x^5+(1080*d+12
15*e)*x^4+(1620*d+972*e)*x^3+(1458*d+729/2*e)*x^2+729*d*x
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 77 vs. $2(27) = 54$.

Time = 0.08 (sec) , antiderivative size = 77, normalized size of antiderivative = 2.48

$$\int (d + ex) (9 + 12x + 4x^2)^3 dx = 8ex^8 + \frac{64}{7}(d + 9e)x^7 + 24(4d + 15e)x^6$$

$$+ 432(d + 2e)x^5 + 135(8d + 9e)x^4$$

$$+ 324(5d + 3e)x^3 + \frac{729}{2}(4d + e)x^2 + 729dx$$

input

```
integrate((e*x+d)*(4*x^2+12*x+9)^3,x, algorithm="fricas")
```

output

```
8*e*x^8 + 64/7*(d + 9*e)*x^7 + 24*(4*d + 15*e)*x^6 + 432*(d + 2*e)*x^5 + 1
35*(8*d + 9*e)*x^4 + 324*(5*d + 3*e)*x^3 + 729/2*(4*d + e)*x^2 + 729*d*x
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 76 vs. $2(24) = 48$.

Time = 0.03 (sec) , antiderivative size = 76, normalized size of antiderivative = 2.45

$$\int (d + ex) (9 + 12x + 4x^2)^3 dx = 729dx + 8ex^8 + x^7 \cdot \left(\frac{64d}{7} + \frac{576e}{7} \right) + x^6 \cdot (96d + 360e)$$

$$+ x^5 \cdot (432d + 864e) + x^4 \cdot (1080d + 1215e)$$

$$+ x^3 \cdot (1620d + 972e) + x^2 \cdot \left(1458d + \frac{729e}{2} \right)$$

input

```
integrate((e*x+d)*(4*x**2+12*x+9)**3,x)
```

output

```
729*d*x + 8*e*x**8 + x**7*(64*d/7 + 576*e/7) + x**6*(96*d + 360*e) + x**5*
(432*d + 864*e) + x**4*(1080*d + 1215*e) + x**3*(1620*d + 972*e) + x**2*(1
458*d + 729*e/2)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 77 vs. $2(27) = 54$.

Time = 0.03 (sec) , antiderivative size = 77, normalized size of antiderivative = 2.48

$$\int (d + ex) (9 + 12x + 4x^2)^3 dx = 8ex^8 + \frac{64}{7}(d + 9e)x^7 + 24(4d + 15e)x^6 \\ + 432(d + 2e)x^5 + 135(8d + 9e)x^4 \\ + 324(5d + 3e)x^3 + \frac{729}{2}(4d + e)x^2 + 729dx$$

input `integrate((e*x+d)*(4*x^2+12*x+9)^3,x, algorithm="maxima")`

output `8*e*x^8 + 64/7*(d + 9*e)*x^7 + 24*(4*d + 15*e)*x^6 + 432*(d + 2*e)*x^5 + 135*(8*d + 9*e)*x^4 + 324*(5*d + 3*e)*x^3 + 729/2*(4*d + e)*x^2 + 729*d*x`

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 83 vs. $2(27) = 54$.

Time = 0.17 (sec) , antiderivative size = 83, normalized size of antiderivative = 2.68

$$\int (d + ex) (9 + 12x + 4x^2)^3 dx = 8ex^8 + \frac{64}{7}dx^7 + \frac{576}{7}ex^7 + 96dx^6 + 360ex^6 \\ + 432dx^5 + 864ex^5 + 1080dx^4 + 1215ex^4 \\ + 1620dx^3 + 972ex^3 + 1458dx^2 + \frac{729}{2}ex^2 + 729dx$$

input `integrate((e*x+d)*(4*x^2+12*x+9)^3,x, algorithm="giac")`

output `8*e*x^8 + 64/7*d*x^7 + 576/7*e*x^7 + 96*d*x^6 + 360*e*x^6 + 432*d*x^5 + 864*e*x^5 + 1080*d*x^4 + 1215*e*x^4 + 1620*d*x^3 + 972*e*x^3 + 1458*d*x^2 + 729/2*e*x^2 + 729*d*x`

Mupad [B] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 77, normalized size of antiderivative = 2.48

$$\int (d + ex) (9 + 12x + 4x^2)^3 dx = 8ex^8 + \left(\frac{64d}{7} + \frac{576e}{7}\right)x^7 + (96d + 360e)x^6 \\ + (432d + 864e)x^5 + (1080d + 1215e)x^4 \\ + (1620d + 972e)x^3 + \left(1458d + \frac{729e}{2}\right)x^2 + 729dx$$

input `int((d + e*x)*(12*x + 4*x^2 + 9)^3,x)`output `x^6*(96*d + 360*e) + x^7*((64*d)/7 + (576*e)/7) + x^5*(432*d + 864*e) + x^4*(1458*d + (729*e)/2) + x^3*(1080*d + 1215*e) + x^2*(1620*d + 972*e) + 729*d*x + 8*e*x^8`**Reduce [B] (verification not implemented)**

Time = 0.24 (sec) , antiderivative size = 81, normalized size of antiderivative = 2.61

$$\int (d + ex) (9 + 12x + 4x^2)^3 dx \\ = \frac{x(112ex^7 + 128dx^6 + 1152ex^6 + 1344dx^5 + 5040ex^5 + 6048dx^4 + 12096ex^4 + 15120dx^3 + 17010ex^3)}{14}$$

input `int((e*x+d)*(4*x^2+12*x+9)^3,x)`output `(x*(128*d*x**6 + 1344*d*x**5 + 6048*d*x**4 + 15120*d*x**3 + 22680*d*x**2 + 20412*d*x + 10206*d + 112*e*x**7 + 1152*e*x**6 + 5040*e*x**5 + 12096*e*x**4 + 17010*e*x**3 + 13608*e*x**2 + 5103*e*x))/14`

3.141 $\int (d + ex)(9 + 12x + 4x^2)^2 dx$

| | |
|---|------|
| Optimal result | 1125 |
| Mathematica [A] (verified) | 1125 |
| Rubi [A] (verified) | 1126 |
| Maple [B] (verified) | 1127 |
| Fricas [A] (verification not implemented) | 1128 |
| Sympy [B] (verification not implemented) | 1128 |
| Maxima [A] (verification not implemented) | 1129 |
| Giac [B] (verification not implemented) | 1129 |
| Mupad [B] (verification not implemented) | 1130 |
| Reduce [B] (verification not implemented) | 1130 |

Optimal result

Integrand size = 18, antiderivative size = 31

$$\int (d + ex)(9 + 12x + 4x^2)^2 dx = \frac{1}{20}(2d - 3e)(3 + 2x)^5 + \frac{1}{24}e(3 + 2x)^6$$

output `1/20*(2*d-3*e)*(3+2*x)^5+1/24*e*(3+2*x)^6`

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.90

$$\int (d + ex)(9 + 12x + 4x^2)^2 dx = 81dx + \frac{27}{2}(8d + 3e)x^2 + 72(d + e)x^3 + 6(4d + 9e)x^4 + \frac{16}{5}(d + 6e)x^5 + \frac{8ex^6}{3}$$

input `Integrate[(d + e*x)*(9 + 12*x + 4*x^2)^2,x]`

output `81*d*x + (27*(8*d + 3*e)*x^2)/2 + 72*(d + e)*x^3 + 6*(4*d + 9*e)*x^4 + (16*(d + 6*e)*x^5)/5 + (8*e*x^6)/3`

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {1098, 27, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (4x^2 + 12x + 9)^2 (d + ex) dx$$

$$\downarrow 1098$$

$$\frac{1}{16} \int 16(2x + 3)^4 (d + ex) dx$$

$$\downarrow 27$$

$$\int (2x + 3)^4 (d + ex) dx$$

$$\downarrow 49$$

$$\int \left(\frac{1}{2}(2x + 3)^4 (2d - 3e) + \frac{1}{2}e(2x + 3)^5 \right) dx$$

$$\downarrow 2009$$

$$\frac{1}{20}(2x + 3)^5 (2d - 3e) + \frac{1}{24}e(2x + 3)^6$$

input `Int[(d + e*x)*(9 + 12*x + 4*x^2)^2,x]`

output `((2*d - 3*e)*(3 + 2*x)^5)/20 + (e*(3 + 2*x)^6)/24`

Defintions of rubi rules used

rule 27 `Int[(a_)*(Fx), x_Symbol] := Simp[a Int[Fx, x], x] /; FreeQ[a, x] && !MatchQ[Fx, (b_)*(Gx) /; FreeQ[b, x]]`

rule 49 `Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] := Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] && IGtQ[m, 0] && IGtQ[m + n + 2, 0]`

rule 1098 `Int[((d_.) + (e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] := Simp[1/c^p Int[(d + e*x)^m*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]`

rule 2009 `Int[u_, x_Symbol] := Simp[IntSum[u, x], x] /; SumQ[u]`

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 55 vs. 2(27) = 54.

Time = 0.72 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.81

| method | result | size |
|---------------|---|------|
| norman | $\frac{8ex^6}{3} + \left(\frac{16d}{5} + \frac{96e}{5}\right)x^5 + (24d + 54e)x^4 + (72d + 72e)x^3 + \left(108d + \frac{81e}{2}\right)x^2 + 81dx$ | 56 |
| gospers | $\frac{x(80x^5e + 96dx^4 + 576x^4e + 720dx^3 + 1620x^3e + 2160dx^2 + 2160ex^2 + 3240dx + 1215ex + 2430d)}{30}$ | 58 |
| default | $\frac{8ex^6}{3} + \frac{(16d+96e)x^5}{5} + \frac{(96d+216e)x^4}{4} + \frac{(216d+216e)x^3}{3} + \frac{(216d+81e)x^2}{2} + 81dx$ | 60 |
| risch | $\frac{8}{3}ex^6 + \frac{16}{5}dx^5 + \frac{96}{5}x^5e + 24dx^4 + 54x^4e + 72dx^3 + 72x^3e + 108dx^2 + \frac{81}{2}ex^2 + 81dx$ | 60 |
| parallelrisch | $\frac{8}{3}ex^6 + \frac{16}{5}dx^5 + \frac{96}{5}x^5e + 24dx^4 + 54x^4e + 72dx^3 + 72x^3e + 108dx^2 + \frac{81}{2}ex^2 + 81dx$ | 60 |
| orering | $\frac{x(80x^5e + 96dx^4 + 576x^4e + 720dx^3 + 1620x^3e + 2160dx^2 + 2160ex^2 + 3240dx + 1215ex + 2430d)(4x^2 + 12x + 9)^2}{30(2x+3)^4}$ | 77 |

input `int((e*x+d)*(4*x^2+12*x+9)^2,x,method=_RETURNVERBOSE)`

output $8/3*e*x^6+(16/5*d+96/5*e)*x^5+(24*d+54*e)*x^4+(72*d+72*e)*x^3+(108*d+81/2*e)*x^2+81*d*x$

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.71

$$\int (d + ex) (9 + 12x + 4x^2)^2 dx = \frac{8}{3} ex^6 + \frac{16}{5} (d + 6e)x^5 + 6(4d + 9e)x^4 + 72(d + e)x^3 + \frac{27}{2} (8d + 3e)x^2 + 81 dx$$

input `integrate((e*x+d)*(4*x^2+12*x+9)^2,x, algorithm="fricas")`

output $8/3*e*x^6 + 16/5*(d + 6*e)*x^5 + 6*(4*d + 9*e)*x^4 + 72*(d + e)*x^3 + 27/2*(8*d + 3*e)*x^2 + 81*d*x$

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 58 vs. $2(24) = 48$.

Time = 0.03 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.87

$$\int (d + ex) (9 + 12x + 4x^2)^2 dx = 81dx + \frac{8ex^6}{3} + x^5 \cdot \left(\frac{16d}{5} + \frac{96e}{5} \right) + x^4 \cdot (24d + 54e) + x^3 \cdot (72d + 72e) + x^2 \cdot \left(108d + \frac{81e}{2} \right)$$

input `integrate((e*x+d)*(4*x**2+12*x+9)**2,x)`

output $81*d*x + 8*e*x**6/3 + x**5*(16*d/5 + 96*e/5) + x**4*(24*d + 54*e) + x**3*(72*d + 72*e) + x**2*(108*d + 81*e/2)$

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.71

$$\int (d + ex) (9 + 12x + 4x^2)^2 dx = \frac{8}{3} ex^6 + \frac{16}{5} (d + 6e)x^5 + 6(4d + 9e)x^4 + 72(d + e)x^3 + \frac{27}{2} (8d + 3e)x^2 + 81 dx$$

input `integrate((e*x+d)*(4*x^2+12*x+9)^2,x, algorithm="maxima")`

output `8/3*e*x^6 + 16/5*(d + 6*e)*x^5 + 6*(4*d + 9*e)*x^4 + 72*(d + e)*x^3 + 27/2*(8*d + 3*e)*x^2 + 81*d*x`

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 59 vs. $2(27) = 54$.

Time = 0.17 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.90

$$\int (d + ex) (9 + 12x + 4x^2)^2 dx = \frac{8}{3} ex^6 + \frac{16}{5} dx^5 + \frac{96}{5} ex^5 + 24 dx^4 + 54 ex^4 + 72 dx^3 + 72 ex^3 + 108 dx^2 + \frac{81}{2} ex^2 + 81 dx$$

input `integrate((e*x+d)*(4*x^2+12*x+9)^2,x, algorithm="giac")`

output `8/3*e*x^6 + 16/5*d*x^5 + 96/5*e*x^5 + 24*d*x^4 + 54*e*x^4 + 72*d*x^3 + 72*e*x^3 + 108*d*x^2 + 81/2*e*x^2 + 81*d*x`

Mupad [B] (verification not implemented)

Time = 9.02 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.77

$$\int (d + ex) (9 + 12x + 4x^2)^2 dx = \frac{8ex^6}{3} + \left(\frac{16d}{5} + \frac{96e}{5}\right) x^5 + (24d + 54e) x^4 + (72d + 72e) x^3 + \left(108d + \frac{81e}{2}\right) x^2 + 81dx$$

input `int((d + e*x)*(12*x + 4*x^2 + 9)^2,x)`

output `x^4*(24*d + 54*e) + x^5*((16*d)/5 + (96*e)/5) + x^3*(72*d + 72*e) + x^2*(108*d + (81*e)/2) + 81*d*x + (8*e*x^6)/3`

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.84

$$\int (d + ex) (9 + 12x + 4x^2)^2 dx = \frac{x(80ex^5 + 96dx^4 + 576ex^4 + 720dx^3 + 1620ex^3 + 2160dx^2 + 2160ex^2 + 3240dx + 1215ex + 2430d)}{30}$$

input `int((e*x+d)*(4*x^2+12*x+9)^2,x)`

output `(x*(96*d*x**4 + 720*d*x**3 + 2160*d*x**2 + 3240*d*x + 2430*d + 80*e*x**5 + 576*e*x**4 + 1620*e*x**3 + 2160*e*x**2 + 1215*e*x))/30`

3.142 $\int (d + ex) (9 + 12x + 4x^2) dx$

| | |
|---|------|
| Optimal result | 1131 |
| Mathematica [A] (verified) | 1131 |
| Rubi [A] (verified) | 1132 |
| Maple [A] (verified) | 1133 |
| Fricas [A] (verification not implemented) | 1134 |
| Sympy [A] (verification not implemented) | 1134 |
| Maxima [A] (verification not implemented) | 1134 |
| Giac [A] (verification not implemented) | 1135 |
| Mupad [B] (verification not implemented) | 1135 |
| Reduce [B] (verification not implemented) | 1135 |

Optimal result

Integrand size = 16, antiderivative size = 31

$$\int (d + ex) (9 + 12x + 4x^2) dx = \frac{1}{12}(2d - 3e)(3 + 2x)^3 + \frac{1}{16}e(3 + 2x)^4$$

output `1/12*(2*d-3*e)*(3+2*x)^3+1/16*e*(3+2*x)^4`

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.16

$$\int (d + ex) (9 + 12x + 4x^2) dx = 9dx + \frac{3}{2}(4d + 3e)x^2 + \frac{4}{3}(d + 3e)x^3 + ex^4$$

input `Integrate[(d + e*x)*(9 + 12*x + 4*x^2), x]`

output `9*d*x + (3*(4*d + 3*e)*x^2)/2 + (4*(d + 3*e)*x^3)/3 + e*x^4`

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {1098, 27, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\begin{aligned}
 & \int (4x^2 + 12x + 9)(d + ex) dx \\
 & \quad \downarrow \text{1098} \\
 & \frac{1}{4} \int 4(2x + 3)^2(d + ex) dx \\
 & \quad \downarrow \text{27} \\
 & \int (2x + 3)^2(d + ex) dx \\
 & \quad \downarrow \text{49} \\
 & \int \left(\frac{1}{2}(2x + 3)^2(2d - 3e) + \frac{1}{2}e(2x + 3)^3 \right) dx \\
 & \quad \downarrow \text{2009} \\
 & \frac{1}{12}(2x + 3)^3(2d - 3e) + \frac{1}{16}e(2x + 3)^4
 \end{aligned}$$

input `Int[(d + e*x)*(9 + 12*x + 4*x^2),x]`

output `((2*d - 3*e)*(3 + 2*x)^3)/12 + (e*(3 + 2*x)^4)/16`

Definitions of rubi rules used

rule 27 `Int[(a_)*(Fx_), x_Symbol] := Simp[a Int[Fx, x], x] /; FreeQ[a, x] && !MatchQ[Fx, (b_)*(Gx_)] /; FreeQ[b, x]`

rule 49 `Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] := Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] && IGtQ[m, 0] && IGtQ[m + n + 2, 0]`

rule 1098 `Int[((d_.) + (e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] := Simp[1/c^p Int[(d + e*x)^m*(b + c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]`

rule 2009 `Int[u_, x_Symbol] := Simp[IntSum[u, x], x] /; SumQ[u]`

Maple [A] (verified)

Time = 0.17 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.06

| method | result | size |
|--------------|---|------|
| norman | $x^4 e + \left(\frac{4d}{3} + 4e\right) x^3 + \left(6d + \frac{9e}{2}\right) x^2 + 9dx$ | 33 |
| gospers | $\frac{x(6x^3 e + 8d x^2 + 24e x^2 + 36dx + 27ex + 54d)}{6}$ | 34 |
| default | $x^4 e + \frac{(4d+12e)x^3}{3} + \frac{(12d+9e)x^2}{2} + 9dx$ | 35 |
| risch | $x^4 e + \frac{4}{3}d x^3 + 4x^3 e + 6d x^2 + \frac{9}{2}e x^2 + 9dx$ | 35 |
| parallelrisc | $x^4 e + \frac{4}{3}d x^3 + 4x^3 e + 6d x^2 + \frac{9}{2}e x^2 + 9dx$ | 35 |
| orering | $\frac{x(6x^3 e + 8d x^2 + 24e x^2 + 36dx + 27ex + 54d)(4x^2 + 12x + 9)}{6(2x+3)^2}$ | 51 |

input `int((e*x+d)*(4*x^2+12*x+9),x,method=_RETURNVERBOSE)`

output `x^4*e+(4/3*d+4*e)*x^3+(6*d+9/2*e)*x^2+9*d*x`

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.03

$$\int (d + ex) (9 + 12x + 4x^2) dx = ex^4 + \frac{4}{3} (d + 3e)x^3 + \frac{3}{2} (4d + 3e)x^2 + 9dx$$

input `integrate((e*x+d)*(4*x^2+12*x+9),x, algorithm="fricas")`output `e*x^4 + 4/3*(d + 3*e)*x^3 + 3/2*(4*d + 3*e)*x^2 + 9*d*x`**Sympy [A] (verification not implemented)**

Time = 0.02 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.03

$$\int (d + ex) (9 + 12x + 4x^2) dx = 9dx + ex^4 + x^3 \cdot \left(\frac{4d}{3} + 4e \right) + x^2 \cdot \left(6d + \frac{9e}{2} \right)$$

input `integrate((e*x+d)*(4*x**2+12*x+9),x)`output `9*d*x + e*x**4 + x**3*(4*d/3 + 4*e) + x**2*(6*d + 9*e/2)`**Maxima [A] (verification not implemented)**

Time = 0.03 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.03

$$\int (d + ex) (9 + 12x + 4x^2) dx = ex^4 + \frac{4}{3} (d + 3e)x^3 + \frac{3}{2} (4d + 3e)x^2 + 9dx$$

input `integrate((e*x+d)*(4*x^2+12*x+9),x, algorithm="maxima")`output `e*x^4 + 4/3*(d + 3*e)*x^3 + 3/2*(4*d + 3*e)*x^2 + 9*d*x`

Giac [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.10

$$\int (d + ex) (9 + 12x + 4x^2) dx = ex^4 + \frac{4}{3} dx^3 + 4ex^3 + 6dx^2 + \frac{9}{2} ex^2 + 9dx$$

input `integrate((e*x+d)*(4*x^2+12*x+9),x, algorithm="giac")`

output `e*x^4 + 4/3*d*x^3 + 4*e*x^3 + 6*d*x^2 + 9/2*e*x^2 + 9*d*x`

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.03

$$\int (d + ex) (9 + 12x + 4x^2) dx = ex^4 + \left(\frac{4d}{3} + 4e\right) x^3 + \left(6d + \frac{9e}{2}\right) x^2 + 9dx$$

input `int((d + e*x)*(12*x + 4*x^2 + 9),x)`

output `x^3*((4*d)/3 + 4*e) + x^2*(6*d + (9*e)/2) + 9*d*x + e*x^4`

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.06

$$\int (d + ex) (9 + 12x + 4x^2) dx = \frac{x(6ex^3 + 8dx^2 + 24ex^2 + 36dx + 27ex + 54d)}{6}$$

input `int((e*x+d)*(4*x^2+12*x+9),x)`

output `(x*(8*d*x**2 + 36*d*x + 54*d + 6*e*x**3 + 24*e*x**2 + 27*e*x))/6`

3.143 $\int \frac{d+ex}{9+12x+4x^2} dx$

| | |
|---|------|
| Optimal result | 1136 |
| Mathematica [A] (verified) | 1136 |
| Rubi [A] (verified) | 1137 |
| Maple [A] (verified) | 1138 |
| Fricas [A] (verification not implemented) | 1139 |
| Sympy [A] (verification not implemented) | 1139 |
| Maxima [A] (verification not implemented) | 1139 |
| Giac [A] (verification not implemented) | 1140 |
| Mupad [B] (verification not implemented) | 1140 |
| Reduce [B] (verification not implemented) | 1140 |

Optimal result

Integrand size = 18, antiderivative size = 30

$$\int \frac{d+ex}{9+12x+4x^2} dx = -\frac{2d-3e}{4(3+2x)} + \frac{1}{4}e \log(3+2x)$$

output

```
-1/4*(2*d-3*e)/(3+2*x)+1/4*e*ln(3+2*x)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.00

$$\int \frac{d+ex}{9+12x+4x^2} dx = \frac{-2d+3e}{4(3+2x)} + \frac{1}{4}e \log(3+2x)$$

input

```
Integrate[(d + e*x)/(9 + 12*x + 4*x^2),x]
```

output

```
(-2*d + 3*e)/(4*(3 + 2*x)) + (e*Log[3 + 2*x])/4
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {1098, 27, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\begin{aligned}
 & \int \frac{d + ex}{4x^2 + 12x + 9} dx \\
 & \quad \downarrow \text{1098} \\
 & 4 \int \frac{d + ex}{4(2x + 3)^2} dx \\
 & \quad \downarrow \text{27} \\
 & \int \frac{d + ex}{(2x + 3)^2} dx \\
 & \quad \downarrow \text{49} \\
 & \int \left(\frac{2d - 3e}{2(2x + 3)^2} + \frac{e}{2(2x + 3)} \right) dx \\
 & \quad \downarrow \text{2009} \\
 & \frac{1}{4} e \log(2x + 3) - \frac{2d - 3e}{4(2x + 3)}
 \end{aligned}$$

input `Int[(d + e*x)/(9 + 12*x + 4*x^2),x]`

output `-1/4*(2*d - 3*e)/(3 + 2*x) + (e*Log[3 + 2*x])/4`

Definitions of rubi rules used

rule 27 $\text{Int}[(a_*)(Fx_), x_Symbol] \rightarrow \text{Simp}[a \text{ Int}[Fx, x], x] /; \text{FreeQ}[a, x] \ \&\& \ !\text{MatchQ}[Fx, (b_*)(Gx_)] /; \text{FreeQ}[b, x]$

rule 49 $\text{Int}[((a_.) + (b_.)(x_))^{(m_.)} * ((c_.) + (d_.)(x_))^{(n_.)}, x_Symbol] \rightarrow \text{Int}[\text{ExpandIntegrand}[(a + b*x)^m * (c + d*x)^n, x], x] /; \text{FreeQ}[\{a, b, c, d\}, x] \ \&\& \ \text{IGtQ}[m, 0] \ \&\& \ \text{IGtQ}[m + n + 2, 0]$

rule 1098 $\text{Int}[((d_.) + (e_.)(x_))^{(m_.)} * ((a_) + (b_.)(x_) + (c_.)(x_)^2)^{(p_.)}, x_Symbol] \rightarrow \text{Simp}[1/c^p \text{ Int}[(d + e*x)^m * (b/2 + c*x)^{(2*p)}, x], x] /; \text{FreeQ}[\{a, b, c, d, e, m\}, x] \ \&\& \ \text{EqQ}[b^2 - 4*a*c, 0] \ \&\& \ \text{IntegerQ}[p]$

rule 2009 $\text{Int}[u_, x_Symbol] \rightarrow \text{Simp}[\text{IntSum}[u, x], x] /; \text{SumQ}[u]$

Maple [A] (verified)

Time = 0.70 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.87

| method | result | size |
|---------------|--|------|
| norman | $-\frac{d}{2} + \frac{3e}{4} + \frac{e \ln(2x+3)}{4}$ | 26 |
| default | $-\frac{d}{2} - \frac{3e}{4} + \frac{e \ln(2x+3)}{4}$ | 27 |
| risch | $-\frac{d}{4(x+\frac{3}{2})} + \frac{3e}{8(x+\frac{3}{2})} + \frac{e \ln(2x+3)}{4}$ | 27 |
| parallelrisch | $\frac{2e \ln(x+\frac{3}{2})x + 3e \ln(x+\frac{3}{2}) - 2d + 3e}{8x+12}$ | 32 |
| meijerg | $\frac{e \left(-\frac{2x}{3(1+\frac{2x}{3})} + \ln(1+\frac{2x}{3}) \right)}{4} + \frac{dx}{9+6x}$ | 33 |

input $\text{int}((e*x+d)/(4*x^2+12*x+9), x, \text{method}=_RETURNVERBOSE)$

output $(-1/2*d+3/4*e)/(2*x+3)+1/4*e*\ln(2*x+3)$

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.03

$$\int \frac{d + ex}{9 + 12x + 4x^2} dx = \frac{(2ex + 3e) \log(2x + 3) - 2d + 3e}{4(2x + 3)}$$

input `integrate((e*x+d)/(4*x^2+12*x+9),x, algorithm="fricas")`output `1/4*((2*e*x + 3*e)*log(2*x + 3) - 2*d + 3*e)/(2*x + 3)`**Sympy [A] (verification not implemented)**

Time = 0.07 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.67

$$\int \frac{d + ex}{9 + 12x + 4x^2} dx = \frac{e \log(2x + 3)}{4} + \frac{-2d + 3e}{8x + 12}$$

input `integrate((e*x+d)/(4*x**2+12*x+9),x)`output `e*log(2*x + 3)/4 + (-2*d + 3*e)/(8*x + 12)`**Maxima [A] (verification not implemented)**

Time = 0.04 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.87

$$\int \frac{d + ex}{9 + 12x + 4x^2} dx = \frac{1}{4} e \log(2x + 3) - \frac{2d - 3e}{4(2x + 3)}$$

input `integrate((e*x+d)/(4*x^2+12*x+9),x, algorithm="maxima")`output `1/4*e*log(2*x + 3) - 1/4*(2*d - 3*e)/(2*x + 3)`

Giac [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.90

$$\int \frac{d + ex}{9 + 12x + 4x^2} dx = \frac{1}{4} e \log(|2x + 3|) - \frac{2d - 3e}{4(2x + 3)}$$

input `integrate((e*x+d)/(4*x^2+12*x+9),x, algorithm="giac")`output `1/4*e*log(abs(2*x + 3)) - 1/4*(2*d - 3*e)/(2*x + 3)`**Mupad [B] (verification not implemented)**

Time = 9.15 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.80

$$\int \frac{d + ex}{9 + 12x + 4x^2} dx = \frac{e \ln\left(x + \frac{3}{2}\right)}{4} - \frac{\frac{d}{2} - \frac{3e}{4}}{2x + 3}$$

input `int((d + e*x)/(12*x + 4*x^2 + 9),x)`output `(e*log(x + 3/2))/4 - (d/2 - (3*e)/4)/(2*x + 3)`**Reduce [B] (verification not implemented)**

Time = 0.25 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.20

$$\int \frac{d + ex}{9 + 12x + 4x^2} dx = \frac{6 \log(2x + 3) ex + 9 \log(2x + 3) e + 4dx - 6ex}{24x + 36}$$

input `int((e*x+d)/(4*x^2+12*x+9),x)`output `(6*log(2*x + 3)*e*x + 9*log(2*x + 3)*e + 4*d*x - 6*e*x)/(12*(2*x + 3))`

$$3.144 \quad \int \frac{d+ex}{(9+12x+4x^2)^2} dx$$

| | |
|---|------|
| Optimal result | 1141 |
| Mathematica [A] (verified) | 1141 |
| Rubi [A] (verified) | 1142 |
| Maple [A] (verified) | 1143 |
| Fricas [A] (verification not implemented) | 1144 |
| Sympy [A] (verification not implemented) | 1144 |
| Maxima [A] (verification not implemented) | 1144 |
| Giac [A] (verification not implemented) | 1145 |
| Mupad [B] (verification not implemented) | 1145 |
| Reduce [B] (verification not implemented) | 1145 |

Optimal result

Integrand size = 18, antiderivative size = 31

$$\int \frac{d+ex}{(9+12x+4x^2)^2} dx = -\frac{2d-3e}{12(3+2x)^3} - \frac{e}{8(3+2x)^2}$$

output `-1/12*(2*d-3*e)/(3+2*x)^3-1/8*e/(3+2*x)^2`

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.71

$$\int \frac{d+ex}{(9+12x+4x^2)^2} dx = -\frac{4d+3e+6ex}{24(3+2x)^3}$$

input `Integrate[(d + e*x)/(9 + 12*x + 4*x^2)^2,x]`

output `-1/24*(4*d + 3*e + 6*e*x)/(3 + 2*x)^3`

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {1098, 27, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\begin{aligned}
 & \int \frac{d + ex}{(4x^2 + 12x + 9)^2} dx \\
 & \quad \downarrow \text{1098} \\
 & 16 \int \frac{d + ex}{16(2x + 3)^4} dx \\
 & \quad \downarrow \text{27} \\
 & \int \frac{d + ex}{(2x + 3)^4} dx \\
 & \quad \downarrow \text{53} \\
 & \int \left(\frac{2d - 3e}{2(2x + 3)^4} + \frac{e}{2(2x + 3)^3} \right) dx \\
 & \quad \downarrow \text{2009} \\
 & -\frac{2d - 3e}{12(2x + 3)^3} - \frac{e}{8(2x + 3)^2}
 \end{aligned}$$

input `Int[(d + e*x)/(9 + 12*x + 4*x^2)^2,x]`

output `-1/12*(2*d - 3*e)/(3 + 2*x)^3 - e/(8*(3 + 2*x)^2)`

Definitions of rubi rules used

rule 27 $\text{Int}[(a_*)(Fx_), x_Symbol] \rightarrow \text{Simp}[a \text{ Int}[Fx, x], x] /; \text{FreeQ}[a, x] \ \&\& \ !\text{MatchQ}[Fx, (b_*)(Gx_)] /; \text{FreeQ}[b, x]$

rule 53 $\text{Int}[((a_.) + (b_.)(x_))^{(m_.)}((c_.) + (d_.)(x_))^{(n_.)}, x_Symbol] \rightarrow \text{Int}[\text{ExpandIntegrand}[(a + b*x)^m*(c + d*x)^n, x], x] /; \text{FreeQ}\{a, b, c, d, n\}, x] \ \&\& \ \text{IGtQ}[m, 0] \ \&\& \ (\ !\text{IntegerQ}[n] \ || \ (\text{EqQ}[c, 0] \ \&\& \ \text{LeQ}[7*m + 4*n + 4, 0]) \ || \ \text{LtQ}[9*m + 5*(n + 1), 0] \ || \ \text{GtQ}[m + n + 2, 0])$

rule 1098 $\text{Int}[((d_.) + (e_.)(x_))^{(m_.)}((a_.) + (b_.)(x_) + (c_.)(x_)^2)^{(p_.)}, x_Symbol] \rightarrow \text{Simp}[1/c^p \text{ Int}[(d + e*x)^m*(b/2 + c*x)^{(2*p)}, x], x] /; \text{FreeQ}\{a, b, c, d, e, m\}, x] \ \&\& \ \text{EqQ}[b^2 - 4*a*c, 0] \ \&\& \ \text{IntegerQ}[p]$

rule 2009 $\text{Int}[u_, x_Symbol] \rightarrow \text{Simp}[\text{IntSum}[u, x], x] /; \text{SumQ}[u]$

Maple [A] (verified)

Time = 0.72 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.65

| method | result | size |
|---------------|---|------|
| norman | $\frac{-\frac{1}{4}ex - \frac{1}{8}e - \frac{1}{6}d}{(2x+3)^3}$ | 20 |
| risch | $\frac{-\frac{1}{4}ex - \frac{1}{8}e - \frac{1}{6}d}{(2x+3)^3}$ | 21 |
| default | $-\frac{e}{8(2x+3)^2} - \frac{\frac{d}{2} - \frac{3e}{4}}{3(2x+3)^3}$ | 28 |
| orering | $-\frac{(2x+3)(6ex+4d+3e)}{24(4x^2+12x+9)^2}$ | 31 |
| gospers | $-\frac{6ex+4d+3e}{24(2x+3)(4x^2+12x+9)}$ | 33 |
| parallelrisch | $\frac{-6ex-4d-3e}{24(4x^2+12x+9)(2x+3)}$ | 33 |
| meijerg | $\frac{e x^2 \left(\frac{2x}{3} + 3\right)}{486 \left(1 + \frac{2x}{3}\right)^3} + \frac{dx \left(\frac{4}{9}x^2 + 2x + 3\right)}{243 \left(1 + \frac{2x}{3}\right)^3}$ | 41 |

input $\text{int}((e*x+d)/(4*x^2+12*x+9)^2, x, \text{method}=_RETURNVERBOSE)$

output $(-1/4*e*x-1/8*e-1/6*d)/(2*x+3)^3$

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.97

$$\int \frac{d + ex}{(9 + 12x + 4x^2)^2} dx = -\frac{6ex + 4d + 3e}{24(8x^3 + 36x^2 + 54x + 27)}$$

input `integrate((e*x+d)/(4*x^2+12*x+9)^2,x, algorithm="fricas")`

output $-1/24*(6*e*x + 4*d + 3*e)/(8*x^3 + 36*x^2 + 54*x + 27)$

Sympy [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.87

$$\int \frac{d + ex}{(9 + 12x + 4x^2)^2} dx = \frac{-4d - 6ex - 3e}{192x^3 + 864x^2 + 1296x + 648}$$

input `integrate((e*x+d)/(4*x**2+12*x+9)**2,x)`

output $(-4*d - 6*e*x - 3*e)/(192*x**3 + 864*x**2 + 1296*x + 648)$

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.97

$$\int \frac{d + ex}{(9 + 12x + 4x^2)^2} dx = -\frac{6ex + 4d + 3e}{24(8x^3 + 36x^2 + 54x + 27)}$$

input `integrate((e*x+d)/(4*x^2+12*x+9)^2,x, algorithm="maxima")`

output $-1/24*(6*e*x + 4*d + 3*e)/(8*x^3 + 36*x^2 + 54*x + 27)$

Giac [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.65

$$\int \frac{d + ex}{(9 + 12x + 4x^2)^2} dx = -\frac{6ex + 4d + 3e}{24(2x + 3)^3}$$

input `integrate((e*x+d)/(4*x^2+12*x+9)^2,x, algorithm="giac")`output `-1/24*(6*e*x + 4*d + 3*e)/(2*x + 3)^3`**Mupad [B] (verification not implemented)**

Time = 0.04 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.65

$$\int \frac{d + ex}{(9 + 12x + 4x^2)^2} dx = -\frac{4d + 3e + 6ex}{24(2x + 3)^3}$$

input `int((d + e*x)/(12*x + 4*x^2 + 9)^2,x)`output `-(4*d + 3*e + 6*e*x)/(24*(2*x + 3)^3)`**Reduce [B] (verification not implemented)**

Time = 0.19 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.94

$$\int \frac{d + ex}{(9 + 12x + 4x^2)^2} dx = \frac{-6ex - 4d - 3e}{192x^3 + 864x^2 + 1296x + 648}$$

input `int((e*x+d)/(4*x^2+12*x+9)^2,x)`output `(- 4*d - 6*e*x - 3*e)/(24*(8*x**3 + 36*x**2 + 54*x + 27))`

3.145 $\int \frac{d+ex}{(9+12x+4x^2)^3} dx$

| | |
|---|------|
| Optimal result | 1146 |
| Mathematica [A] (verified) | 1146 |
| Rubi [A] (verified) | 1147 |
| Maple [A] (verified) | 1148 |
| Fricas [A] (verification not implemented) | 1149 |
| Sympy [A] (verification not implemented) | 1149 |
| Maxima [A] (verification not implemented) | 1150 |
| Giac [A] (verification not implemented) | 1150 |
| Mupad [B] (verification not implemented) | 1150 |
| Reduce [B] (verification not implemented) | 1151 |

Optimal result

Integrand size = 18, antiderivative size = 31

$$\int \frac{d + ex}{(9 + 12x + 4x^2)^3} dx = -\frac{2d - 3e}{20(3 + 2x)^5} - \frac{e}{16(3 + 2x)^4}$$

output `-1/20*(2*d-3*e)/(3+2*x)^5-1/16*e/(3+2*x)^4`

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.71

$$\int \frac{d + ex}{(9 + 12x + 4x^2)^3} dx = -\frac{8d + e(3 + 10x)}{80(3 + 2x)^5}$$

input `Integrate[(d + e*x)/(9 + 12*x + 4*x^2)^3,x]`

output `-1/80*(8*d + e*(3 + 10*x))/(3 + 2*x)^5`

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {1098, 27, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\begin{aligned}
 & \int \frac{d + ex}{(4x^2 + 12x + 9)^3} dx \\
 & \quad \downarrow \text{1098} \\
 & 64 \int \frac{d + ex}{64(2x + 3)^6} dx \\
 & \quad \downarrow \text{27} \\
 & \int \frac{d + ex}{(2x + 3)^6} dx \\
 & \quad \downarrow \text{53} \\
 & \int \left(\frac{2d - 3e}{2(2x + 3)^6} + \frac{e}{2(2x + 3)^5} \right) dx \\
 & \quad \downarrow \text{2009} \\
 & -\frac{2d - 3e}{20(2x + 3)^5} - \frac{e}{16(2x + 3)^4}
 \end{aligned}$$

input `Int[(d + e*x)/(9 + 12*x + 4*x^2)^3,x]`

output `-1/20*(2*d - 3*e)/(3 + 2*x)^5 - e/(16*(3 + 2*x)^4)`

Definitions of rubi rules used

rule 27 $\text{Int}[(a_*)(Fx_), x_Symbol] \rightarrow \text{Simp}[a \text{ Int}[Fx, x], x] /; \text{FreeQ}[a, x] \ \&\& \ !\text{MatchQ}[Fx, (b_*)(Gx_)] /; \text{FreeQ}[b, x]$

rule 53 $\text{Int}[(a_.) + (b_.)*(x_)^{(m_.)}*((c_.) + (d_.)*(x_)^{(n_.)}), x_Symbol] \rightarrow \text{Int}[\text{ExpandIntegrand}[(a + b*x)^m*(c + d*x)^n, x], x] /; \text{FreeQ}[\{a, b, c, d, n\}, x] \ \&\& \ \text{IGtQ}[m, 0] \ \&\& \ (\ !\text{IntegerQ}[n] \ || \ (\text{EqQ}[c, 0] \ \&\& \ \text{LeQ}[7*m + 4*n + 4, 0]) \ || \ \text{LtQ}[9*m + 5*(n + 1), 0] \ || \ \text{GtQ}[m + n + 2, 0])$

rule 1098 $\text{Int}[(d_.) + (e_.)*(x_)^{(m_.)}*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^{(p_.)}, x_Symbol] \rightarrow \text{Simp}[1/c^p \text{ Int}[(d + e*x)^m*(b/2 + c*x)^(2*p), x], x] /; \text{FreeQ}[\{a, b, c, d, e, m\}, x] \ \&\& \ \text{EqQ}[b^2 - 4*a*c, 0] \ \&\& \ \text{IntegerQ}[p]$

rule 2009 $\text{Int}[u_, x_Symbol] \rightarrow \text{Simp}[\text{IntSum}[u, x], x] /; \text{SumQ}[u]$

Maple [A] (verified)

Time = 0.72 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.65

| method | result | size |
|---------------|---|------|
| norman | $\frac{-\frac{1}{8}ex - \frac{3}{80}e - \frac{1}{10}d}{(2x+3)^5}$ | 20 |
| risch | $\frac{-\frac{1}{8}ex - \frac{3}{80}e - \frac{1}{10}d}{(2x+3)^5}$ | 21 |
| default | $-\frac{e}{16(2x+3)^4} - \frac{\frac{d}{2} - \frac{3e}{4}}{5(2x+3)^5}$ | 28 |
| orering | $-\frac{(2x+3)(10ex+8d+3e)}{80(4x^2+12x+9)^3}$ | 31 |
| gosper | $-\frac{10ex+8d+3e}{80(2x+3)(4x^2+12x+9)^2}$ | 33 |
| parallelrisch | $\frac{-20ex-16d-6e}{160(4x^2+12x+9)^2(2x+3)}$ | 33 |
| meijerg | $\frac{e x^2 (\frac{8}{27} x^3 + \frac{20}{9} x^2 + \frac{20}{3} x + 10)}{14580(1 + \frac{2x}{3})^5} + \frac{dx (\frac{16}{81} x^4 + \frac{40}{27} x^3 + \frac{40}{9} x^2 + \frac{20}{3} x + 5)}{3645(1 + \frac{2x}{3})^5}$ | 61 |

input $\text{int}((e*x+d)/(4*x^2+12*x+9)^3, x, \text{method}=_RETURNVERBOSE)$

output $(-1/8*e*x-3/80*e-1/10*d)/(2*x+3)^5$

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.29

$$\int \frac{d + ex}{(9 + 12x + 4x^2)^3} dx = -\frac{10ex + 8d + 3e}{80(32x^5 + 240x^4 + 720x^3 + 1080x^2 + 810x + 243)}$$

input `integrate((e*x+d)/(4*x^2+12*x+9)^3,x, algorithm="fricas")`

output $-1/80*(10*e*x + 8*d + 3*e)/(32*x^5 + 240*x^4 + 720*x^3 + 1080*x^2 + 810*x + 243)$

Sympy [A] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.19

$$\int \frac{d + ex}{(9 + 12x + 4x^2)^3} dx = \frac{-8d - 10ex - 3e}{2560x^5 + 19200x^4 + 57600x^3 + 86400x^2 + 64800x + 19440}$$

input `integrate((e*x+d)/(4*x**2+12*x+9)**3,x)`

output $(-8*d - 10*e*x - 3*e)/(2560*x**5 + 19200*x**4 + 57600*x**3 + 86400*x**2 + 64800*x + 19440)$

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.29

$$\int \frac{d + ex}{(9 + 12x + 4x^2)^3} dx = -\frac{10ex + 8d + 3e}{80(32x^5 + 240x^4 + 720x^3 + 1080x^2 + 810x + 243)}$$

input `integrate((e*x+d)/(4*x^2+12*x+9)^3,x, algorithm="maxima")`output `-1/80*(10*e*x + 8*d + 3*e)/(32*x^5 + 240*x^4 + 720*x^3 + 1080*x^2 + 810*x + 243)`**Giac [A] (verification not implemented)**

Time = 0.16 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.65

$$\int \frac{d + ex}{(9 + 12x + 4x^2)^3} dx = -\frac{10ex + 8d + 3e}{80(2x + 3)^5}$$

input `integrate((e*x+d)/(4*x^2+12*x+9)^3,x, algorithm="giac")`output `-1/80*(10*e*x + 8*d + 3*e)/(2*x + 3)^5`**Mupad [B] (verification not implemented)**

Time = 0.05 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.65

$$\int \frac{d + ex}{(9 + 12x + 4x^2)^3} dx = -\frac{8d + 3e + 10ex}{80(2x + 3)^5}$$

input `int((d + e*x)/(12*x + 4*x^2 + 9)^3,x)`output `-(8*d + 3*e + 10*e*x)/(80*(2*x + 3)^5)`

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.26

$$\int \frac{d + ex}{(9 + 12x + 4x^2)^3} dx = \frac{-10ex - 8d - 3e}{2560x^5 + 19200x^4 + 57600x^3 + 86400x^2 + 64800x + 19440}$$

input `int((e*x+d)/(4*x^2+12*x+9)^3,x)`

output `(- 8*d - 10*e*x - 3*e)/(80*(32*x**5 + 240*x**4 + 720*x**3 + 1080*x**2 + 810*x + 243))`

3.146 $\int (d + ex)^{7/2} (a^2 + 2abx + b^2x^2) dx$

| | |
|---|------|
| Optimal result | 1152 |
| Mathematica [A] (verified) | 1152 |
| Rubi [A] (verified) | 1153 |
| Maple [A] (verified) | 1154 |
| Fricas [B] (verification not implemented) | 1155 |
| Sympy [B] (verification not implemented) | 1155 |
| Maxima [A] (verification not implemented) | 1156 |
| Giac [B] (verification not implemented) | 1157 |
| Mupad [B] (verification not implemented) | 1158 |
| Reduce [B] (verification not implemented) | 1158 |

Optimal result

Integrand size = 26, antiderivative size = 71

$$\int (d + ex)^{7/2} (a^2 + 2abx + b^2x^2) dx = \frac{2(bd - ae)^2(d + ex)^{9/2}}{9e^3} - \frac{4b(bd - ae)(d + ex)^{11/2}}{11e^3} + \frac{2b^2(d + ex)^{13/2}}{13e^3}$$

output

$2/9*(-a*e+b*d)^2*(e*x+d)^(9/2)/e^3-4/11*b*(-a*e+b*d)*(e*x+d)^(11/2)/e^3+2/13*b^2*(e*x+d)^(13/2)/e^3$

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.86

$$\int (d + ex)^{7/2} (a^2 + 2abx + b^2x^2) dx = \frac{2(d + ex)^{9/2} (143a^2e^2 + 26abe(-2d + 9ex) + b^2(8d^2 - 36dex + 99e^2x^2))}{1287e^3}$$

input

`Integrate[(d + e*x)^(7/2)*(a^2 + 2*a*b*x + b^2*x^2),x]`

output

$$(2*(d + e*x)^(9/2)*(143*a^2*e^2 + 26*a*b*e*(-2*d + 9*e*x) + b^2*(8*d^2 - 36*d*e*x + 99*e^2*x^2)))/(1287*e^3)$$

Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {1098, 27, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\begin{aligned} & \int (a^2 + 2abx + b^2x^2)(d + ex)^{7/2} dx \\ & \quad \downarrow 1098 \\ & \int \frac{b^2(a + bx)^2(d + ex)^{7/2} dx}{b^2} \\ & \quad \downarrow 27 \\ & \int (a + bx)^2(d + ex)^{7/2} dx \\ & \quad \downarrow 53 \\ & \int \left(-\frac{2b(d + ex)^{9/2}(bd - ae)}{e^2} + \frac{(d + ex)^{7/2}(ae - bd)^2}{e^2} + \frac{b^2(d + ex)^{11/2}}{e^2} \right) dx \\ & \quad \downarrow 2009 \\ & -\frac{4b(d + ex)^{11/2}(bd - ae)}{11e^3} + \frac{2(d + ex)^{9/2}(bd - ae)^2}{9e^3} + \frac{2b^2(d + ex)^{13/2}}{13e^3} \end{aligned}$$

input

$$\text{Int}[(d + e*x)^(7/2)*(a^2 + 2*a*b*x + b^2*x^2), x]$$

output

$$(2*(b*d - a*e)^2*(d + e*x)^(9/2))/(9*e^3) - (4*b*(b*d - a*e)*(d + e*x)^(11/2))/(11*e^3) + (2*b^2*(d + e*x)^(13/2))/(13*e^3)$$

Defintions of rubi rules used

rule 27 $\text{Int}[(a_*)(Fx_), x_Symbol] \rightarrow \text{Simp}[a \text{ Int}[Fx, x], x] /; \text{FreeQ}[a, x] \ \&\& \ !\text{MatchQ}[Fx, (b_*)(Gx_)] /; \text{FreeQ}[b, x]$

rule 53 $\text{Int}[(a_.) + (b_.)*(x_))^{(m_.)}*((c_.) + (d_.)*(x_))^{(n_.)}, x_Symbol] \rightarrow \text{Int}[\text{ExpandIntegrand}[(a + b*x)^m*(c + d*x)^n, x], x] /; \text{FreeQ}[\{a, b, c, d, n\}, x] \ \&\& \ \text{IGtQ}[m, 0] \ \&\& \ (\ !\text{IntegerQ}[n] \ || \ (\text{EqQ}[c, 0] \ \&\& \ \text{LeQ}[7*m + 4*n + 4, 0]) \ || \ \text{LtQ}[9*m + 5*(n + 1), 0] \ || \ \text{GtQ}[m + n + 2, 0])$

rule 1098 $\text{Int}[(d_.) + (e_.)*(x_))^{(m_.)}*((a_.) + (b_.)*(x_.) + (c_.)*(x_.)^2)^{(p_.)}, x_Symbol] \rightarrow \text{Simp}[1/c^p \text{ Int}[(d + e*x)^m*(b/2 + c*x)^(2*p), x], x] /; \text{FreeQ}[\{a, b, c, d, e, m\}, x] \ \&\& \ \text{EqQ}[b^2 - 4*a*c, 0] \ \&\& \ \text{IntegerQ}[p]$

rule 2009 $\text{Int}[u_, x_Symbol] \rightarrow \text{Simp}[\text{IntSum}[u, x], x] /; \text{SumQ}[u]$

Maple [A] (verified)

Time = 0.84 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.76

| method | result |
|------------------|---|
| pseudoelliptic | $\frac{2(ex+d)^{\frac{9}{2}} \left(\left(\frac{9}{13}b^2x^2 + \frac{18}{11}abx + a^2 \right) e^2 - \frac{4 \left(\frac{9bx}{13} + a \right) bde}{11} + \frac{8b^2d^2}{143} \right)}{9e^3}$ |
| gospers | $\frac{2(ex+d)^{\frac{9}{2}} (99b^2e^2x^2 + 234xab e^2 - 36b^2dex + 143e^2a^2 - 52abde + 8b^2d^2)}{1287e^3}$ |
| derivativdivides | $\frac{\frac{2b^2(ex+d)^{\frac{13}{2}}}{13} + \frac{2(2abe - 2b^2d)(ex+d)^{\frac{11}{2}}}{11} + \frac{2(e^2a^2 - 2abde + b^2d^2)(ex+d)^{\frac{9}{2}}}{9}}{e^3}$ |
| default | $\frac{2b^2(ex+d)^{\frac{13}{2}}}{13} + \frac{2(2abe - 2b^2d)(ex+d)^{\frac{11}{2}}}{11} + \frac{2(e^2a^2 - 2abde + b^2d^2)(ex+d)^{\frac{9}{2}}}{9}}{e^3}$ |
| orering | $\frac{2(99b^2e^2x^2 + 234xab e^2 - 36b^2dex + 143e^2a^2 - 52abde + 8b^2d^2)(ex+d)^{\frac{9}{2}} (b^2x^2 + 2abx + a^2)}{1287e^3 (bx+a)^2}$ |
| trager | $\frac{2(99b^2e^6x^6 + 234ab e^6x^5 + 360b^2d e^5x^5 + 143a^2e^6x^4 + 884abd e^5x^4 + 458b^2d^2e^4x^4 + 572a^2d e^5x^3 + 1196abd^2e^4x^3 + 212b^2d^3e^4x^3 + 1196abd^2e^4x^3 + 212b^2d^3e^4x^3)}{1287e^3 (bx+a)^2}$ |
| risch | $\frac{2(99b^2e^6x^6 + 234ab e^6x^5 + 360b^2d e^5x^5 + 143a^2e^6x^4 + 884abd e^5x^4 + 458b^2d^2e^4x^4 + 572a^2d e^5x^3 + 1196abd^2e^4x^3 + 212b^2d^3e^4x^3)}{1287e^3 (bx+a)^2}$ |

input `int((e*x+d)^(7/2)*(b^2*x^2+2*a*b*x+a^2),x,method=_RETURNVERBOSE)`

output $2/9*(e*x+d)^{(9/2)}*((9/13*b^2*x^2+18/11*a*b*x+a^2)*e^{-2}-4/11*(9/13*b*x+a)*b*d*e+8/143*b^2*d^2)/e^3$

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 212 vs. 2(59) = 118.

Time = 0.08 (sec) , antiderivative size = 212, normalized size of antiderivative = 2.99

$$\int (d+ex)^{7/2} (a^2+2abx+b^2x^2) dx = \frac{2(99b^2e^6x^6+8b^2d^6-52abd^5e+143a^2d^4e^2+18(20b^2de^5+13abe^6)x^5+(458b^2d^2e^4+884a^2d^2e^5+143a^2e^6)x^4+4(53b^2d^3e^3+299a*b*d^2e^4+143a^2*d*e^5)x^3+3(b^2*d^4*e^2+208a*b*d^3*e^3+286a^2*d^2*e^4)x^2-2*(2*b^2*d^5*e-13a*b*d^4*e^2-286a^2*d^3*e^3)*x)*\sqrt{e*x+d}}{e^3}$$

input `integrate((e*x+d)^(7/2)*(b^2*x^2+2*a*b*x+a^2),x, algorithm="fricas")`

output $2/1287*(99*b^2*e^6*x^6+8*b^2*d^6-52*a*b*d^5*e+143*a^2*d^4*e^2+18*(20*b^2*d*e^5+13*a*b*e^6)*x^5+(458*b^2*d^2*e^4+884*a*b*d*e^5+143*a^2*e^6)*x^4+4*(53*b^2*d^3*e^3+299*a*b*d^2*e^4+143*a^2*d*e^5)*x^3+3*(b^2*d^4*e^2+208*a*b*d^3*e^3+286*a^2*d^2*e^4)*x^2-2*(2*b^2*d^5*e-13*a*b*d^4*e^2-286*a^2*d^3*e^3)*x)*\sqrt{e*x+d}/e^3$

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 432 vs. 2(65) = 130.

Time = 0.49 (sec) , antiderivative size = 432, normalized size of antiderivative = 6.08

$$\int (d+ex)^{7/2} (a^2+2abx+b^2x^2) dx = \begin{cases} \frac{2a^2d^4\sqrt{d+ex}}{9e} + \frac{8a^2d^3x\sqrt{d+ex}}{9} + \frac{4a^2d^2ex^2\sqrt{d+ex}}{3} + \frac{8a^2de^2x^3\sqrt{d+ex}}{9} + \frac{2a^2e^3x^4\sqrt{d+ex}}{9} - \frac{8abd^5\sqrt{d+ex}}{99e^2} + \frac{4abd^4}{9} \\ d^{\frac{7}{2}} \left(a^2x + abx^2 + \frac{b^2x^3}{3} \right) \end{cases}$$

input `integrate((e*x+d)**(7/2)*(b**2*x**2+2*a*b*x+a**2),x)`

output `Piecewise((2*a**2*d**4*sqrt(d + e*x)/(9*e) + 8*a**2*d**3*x*sqrt(d + e*x)/9 + 4*a**2*d**2*e*x**2*sqrt(d + e*x)/3 + 8*a**2*d*e**2*x**3*sqrt(d + e*x)/9 + 2*a**2*e**3*x**4*sqrt(d + e*x)/9 - 8*a*b*d**5*sqrt(d + e*x)/(99*e**2) + 4*a*b*d**4*x*sqrt(d + e*x)/(99*e) + 32*a*b*d**3*x**2*sqrt(d + e*x)/33 + 184*a*b*d**2*e*x**3*sqrt(d + e*x)/99 + 136*a*b*d*e**2*x**4*sqrt(d + e*x)/99 + 4*a*b*e**3*x**5*sqrt(d + e*x)/11 + 16*b**2*d**6*sqrt(d + e*x)/(1287*e**3) - 8*b**2*d**5*x*sqrt(d + e*x)/(1287*e**2) + 2*b**2*d**4*x**2*sqrt(d + e*x)/(429*e) + 424*b**2*d**3*x**3*sqrt(d + e*x)/1287 + 916*b**2*d**2*e*x**4*sqrt(d + e*x)/1287 + 80*b**2*d*e**2*x**5*sqrt(d + e*x)/143 + 2*b**2*e**3*x**6*sqrt(d + e*x)/13, Ne(e, 0)), (d**(7/2)*(a**2*x + a*b*x**2 + b**2*x**3/3), True))`

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.96

$$\int (d + ex)^{7/2} (a^2 + 2abx + b^2x^2) dx = \frac{2 \left(99 (ex + d)^{\frac{13}{2}} b^2 - 234 (b^2d - abe)(ex + d)^{\frac{11}{2}} + 143 (b^2d^2 - 2abde + a^2e^2)(ex + d)^{\frac{9}{2}} \right)}{1287 e^3}$$

input `integrate((e*x+d)^(7/2)*(b^2*x^2+2*a*b*x+a^2),x, algorithm="maxima")`

output `2/1287*(99*(e*x + d)^(13/2)*b^2 - 234*(b^2*d - a*b*e)*(e*x + d)^(11/2) + 143*(b^2*d^2 - 2*a*b*d*e + a^2*e^2)*(e*x + d)^(9/2))/e^3`

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 791 vs. $2(59) = 118$.

Time = 0.21 (sec) , antiderivative size = 791, normalized size of antiderivative = 11.14

$$\int (d + ex)^{7/2} (a^2 + 2abx + b^2x^2) dx = \text{Too large to display}$$

input `integrate((e*x+d)^(7/2)*(b^2*x^2+2*a*b*x+a^2),x, algorithm="giac")`

output

```
2/45045*(45045*sqrt(e*x + d)*a^2*d^4 + 60060*((e*x + d)^(3/2) - 3*sqrt(e*x
+ d)*d)*a^2*d^3 + 30030*((e*x + d)^(3/2) - 3*sqrt(e*x + d)*d)*a*b*d^4/e +
18018*(3*(e*x + d)^(5/2) - 10*(e*x + d)^(3/2)*d + 15*sqrt(e*x + d)*d^2)*a
^2*d^2 + 3003*(3*(e*x + d)^(5/2) - 10*(e*x + d)^(3/2)*d + 15*sqrt(e*x + d)
*d^2)*b^2*d^4/e^2 + 24024*(3*(e*x + d)^(5/2) - 10*(e*x + d)^(3/2)*d + 15*s
qrt(e*x + d)*d^2)*a*b*d^3/e + 5148*(5*(e*x + d)^(7/2) - 21*(e*x + d)^(5/2)
*d + 35*(e*x + d)^(3/2)*d^2 - 35*sqrt(e*x + d)*d^3)*a^2*d + 5148*(5*(e*x +
d)^(7/2) - 21*(e*x + d)^(5/2)*d + 35*(e*x + d)^(3/2)*d^2 - 35*sqrt(e*x +
d)*d^3)*b^2*d^3/e^2 + 15444*(5*(e*x + d)^(7/2) - 21*(e*x + d)^(5/2)*d + 35
*(e*x + d)^(3/2)*d^2 - 35*sqrt(e*x + d)*d^3)*a*b*d^2/e + 143*(35*(e*x + d)
^(9/2) - 180*(e*x + d)^(7/2)*d + 378*(e*x + d)^(5/2)*d^2 - 420*(e*x + d)^(
3/2)*d^3 + 315*sqrt(e*x + d)*d^4)*a^2 + 858*(35*(e*x + d)^(9/2) - 180*(e*x
+ d)^(7/2)*d + 378*(e*x + d)^(5/2)*d^2 - 420*(e*x + d)^(3/2)*d^3 + 315*sq
rt(e*x + d)*d^4)*b^2*d^2/e^2 + 1144*(35*(e*x + d)^(9/2) - 180*(e*x + d)^(7
/2)*d + 378*(e*x + d)^(5/2)*d^2 - 420*(e*x + d)^(3/2)*d^3 + 315*sqrt(e*x +
d)*d^4)*a*b*d/e + 260*(63*(e*x + d)^(11/2) - 385*(e*x + d)^(9/2)*d + 990*
(e*x + d)^(7/2)*d^2 - 1386*(e*x + d)^(5/2)*d^3 + 1155*(e*x + d)^(3/2)*d^4
- 693*sqrt(e*x + d)*d^5)*b^2*d/e^2 + 130*(63*(e*x + d)^(11/2) - 385*(e*x +
d)^(9/2)*d + 990*(e*x + d)^(7/2)*d^2 - 1386*(e*x + d)^(5/2)*d^3 + 1155*(e
*x + d)^(3/2)*d^4 - 693*sqrt(e*x + d)*d^5)*a*b/e + 15*(231*(e*x + d)^(1...
```

Mupad [B] (verification not implemented)

Time = 9.07 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.96

$$\int (d + ex)^{7/2} (a^2 + 2abx + b^2x^2) dx = \frac{2(d + ex)^{9/2} (99b^2(d + ex)^2 + 143a^2e^2 + 143b^2d^2 - 234b^2d(d + ex) + 234abe(d + ex))}{1287e^3}$$

input `int((d + e*x)^(7/2)*(a^2 + b^2*x^2 + 2*a*b*x), x)`output `(2*(d + e*x)^(9/2)*(99*b^2*(d + e*x)^2 + 143*a^2*e^2 + 143*b^2*d^2 - 234*b^2*d*(d + e*x) + 234*a*b*e*(d + e*x) - 286*a*b*d*e))/(1287*e^3)`**Reduce [B] (verification not implemented)**

Time = 0.19 (sec) , antiderivative size = 221, normalized size of antiderivative = 3.11

$$\int (d + ex)^{7/2} (a^2 + 2abx + b^2x^2) dx = \frac{2\sqrt{ex + d} (99b^2e^6x^6 + 234abe^6x^5 + 360b^2de^5x^5 + 143a^2e^6x^4 + 884abd e^5x^4 + 458b^2d^2e^4x^4 + 26a^2e^4x^3 + 572a^2de^5x^3 + 143a^2e^6x^4 - 52abde^5e + 624abd^4e^2x + 1196abd^3e^3x^2 + 884abd^4e^5x^4 + 234abd^5e^6x^5 + 8b^2d^6 - 4b^2d^5ex + 3b^2d^4e^2x^2 + 212b^2d^3e^3x^3 + 458b^2d^2e^4x^4 + 360b^2de^5x^5 + 99b^2e^6x^6))}{1287e^3}$$

input `int((e*x+d)^(7/2)*(b^2*x^2+2*a*b*x+a^2), x)`output `(2*sqrt(d + e*x)*(143*a**2*d**4*e**2 + 572*a**2*d**3*e**3*x + 858*a**2*d**2*e**4*x**2 + 572*a**2*d*e**5*x**3 + 143*a**2*e**6*x**4 - 52*a*b*d**5*e + 26*a*b*d**4*e**2*x + 624*a*b*d**3*e**3*x**2 + 1196*a*b*d**2*e**4*x**3 + 884*a*b*d*e**5*x**4 + 234*a*b*e**6*x**5 + 8*b**2*d**6 - 4*b**2*d**5*e*x + 3*b**2*d**4*e**2*x**2 + 212*b**2*d**3*e**3*x**3 + 458*b**2*d**2*e**4*x**4 + 360*b**2*d*e**5*x**5 + 99*b**2*e**6*x**6))/(1287*e**3)`

3.147 $\int (d + ex)^{5/2} (a^2 + 2abx + b^2x^2) dx$

| | |
|---|------|
| Optimal result | 1159 |
| Mathematica [A] (verified) | 1159 |
| Rubi [A] (verified) | 1160 |
| Maple [A] (verified) | 1161 |
| Fricas [B] (verification not implemented) | 1162 |
| Sympy [B] (verification not implemented) | 1162 |
| Maxima [A] (verification not implemented) | 1163 |
| Giac [B] (verification not implemented) | 1163 |
| Mupad [B] (verification not implemented) | 1164 |
| Reduce [B] (verification not implemented) | 1165 |

Optimal result

Integrand size = 26, antiderivative size = 71

$$\int (d + ex)^{5/2} (a^2 + 2abx + b^2x^2) dx = \frac{2(bd - ae)^2(d + ex)^{7/2}}{7e^3} - \frac{4b(bd - ae)(d + ex)^{9/2}}{9e^3} + \frac{2b^2(d + ex)^{11/2}}{11e^3}$$

output

```
2/7*(-a*e+b*d)^2*(e*x+d)^(7/2)/e^3-4/9*b*(-a*e+b*d)*(e*x+d)^(9/2)/e^3+2/11
*b^2*(e*x+d)^(11/2)/e^3
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.86

$$\int (d + ex)^{5/2} (a^2 + 2abx + b^2x^2) dx = \frac{2(d + ex)^{7/2} (99a^2e^2 + 22abe(-2d + 7ex) + b^2(8d^2 - 28dex + 63e^2x^2))}{693e^3}$$

input

```
Integrate[(d + e*x)^(5/2)*(a^2 + 2*a*b*x + b^2*x^2),x]
```

output

$$(2*(d + e*x)^(7/2)*(99*a^2*e^2 + 22*a*b*e*(-2*d + 7*e*x) + b^2*(8*d^2 - 28*d*e*x + 63*e^2*x^2)))/(693*e^3)$$

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {1098, 27, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\begin{aligned} & \int (a^2 + 2abx + b^2x^2)(d + ex)^{5/2} dx \\ & \quad \downarrow 1098 \\ & \int \frac{b^2(a + bx)^2(d + ex)^{5/2} dx}{b^2} \\ & \quad \downarrow 27 \\ & \int (a + bx)^2(d + ex)^{5/2} dx \\ & \quad \downarrow 53 \\ & \int \left(-\frac{2b(d + ex)^{7/2}(bd - ae)}{e^2} + \frac{(d + ex)^{5/2}(ae - bd)^2}{e^2} + \frac{b^2(d + ex)^{9/2}}{e^2} \right) dx \\ & \quad \downarrow 2009 \\ & -\frac{4b(d + ex)^{9/2}(bd - ae)}{9e^3} + \frac{2(d + ex)^{7/2}(bd - ae)^2}{7e^3} + \frac{2b^2(d + ex)^{11/2}}{11e^3} \end{aligned}$$

input

$$\text{Int}[(d + e*x)^(5/2)*(a^2 + 2*a*b*x + b^2*x^2), x]$$

output

$$(2*(b*d - a*e)^2*(d + e*x)^(7/2))/(7*e^3) - (4*b*(b*d - a*e)*(d + e*x)^(9/2))/(9*e^3) + (2*b^2*(d + e*x)^(11/2))/(11*e^3)$$

Defintions of rubi rules used

rule 27 $\text{Int}[(a_*)(Fx_), x_Symbol] \rightarrow \text{Simp}[a \text{ Int}[Fx, x], x] /; \text{FreeQ}[a, x] \ \&\& \ !\text{MatchQ}[Fx, (b_*)(Gx_)] /; \text{FreeQ}[b, x]$

rule 53 $\text{Int}[(a_.) + (b_.)*(x_))^{(m_.)}*((c_.) + (d_.)*(x_))^{(n_.)}, x_Symbol] \rightarrow \text{Int}[\text{ExpandIntegrand}[(a + b*x)^m*(c + d*x)^n, x], x] /; \text{FreeQ}[\{a, b, c, d, n\}, x] \ \&\& \ \text{IGtQ}[m, 0] \ \&\& \ (\ !\text{IntegerQ}[n] \ || \ (\text{EqQ}[c, 0] \ \&\& \ \text{LeQ}[7*m + 4*n + 4, 0]) \ || \ \text{LtQ}[9*m + 5*(n + 1), 0] \ || \ \text{GtQ}[m + n + 2, 0])$

rule 1098 $\text{Int}[(d_.) + (e_.)*(x_))^{(m_.)}*((a_.) + (b_.)*(x_.) + (c_.)*(x_.)^2)^{(p_.)}, x_Symbol] \rightarrow \text{Simp}[1/c^p \text{ Int}[(d + e*x)^m*(b/2 + c*x)^(2*p), x], x] /; \text{FreeQ}[\{a, b, c, d, e, m\}, x] \ \&\& \ \text{EqQ}[b^2 - 4*a*c, 0] \ \&\& \ \text{IntegerQ}[p]$

rule 2009 $\text{Int}[u_, x_Symbol] \rightarrow \text{Simp}[\text{IntSum}[u, x], x] /; \text{SumQ}[u]$

Maple [A] (verified)

Time = 0.80 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.76

| method | result |
|------------------|--|
| pseudoelliptic | $\frac{2(ex+d)^{\frac{7}{2}} \left(\left(\frac{7}{11}b^2x^2 + \frac{14}{9}abx + a^2 \right) e^2 - \frac{4 \left(\frac{7bx}{11} + a \right) bde}{9} + \frac{8b^2d^2}{99} \right)}{7e^3}$ |
| gospers | $\frac{2(ex+d)^{\frac{7}{2}} (63b^2e^2x^2 + 154xab e^2 - 28b^2dex + 99e^2a^2 - 44abde + 8b^2d^2)}{693e^3}$ |
| derivativdivides | $\frac{\frac{2b^2(ex+d)^{\frac{11}{2}}}{11} + \frac{2(2abe - 2b^2d)(ex+d)^{\frac{9}{2}}}{9} + \frac{2(e^2a^2 - 2abde + b^2d^2)(ex+d)^{\frac{7}{2}}}{7}}{e^3}$ |
| default | $\frac{2b^2(ex+d)^{\frac{11}{2}}}{11} + \frac{2(2abe - 2b^2d)(ex+d)^{\frac{9}{2}}}{9} + \frac{2(e^2a^2 - 2abde + b^2d^2)(ex+d)^{\frac{7}{2}}}{7}}{e^3}$ |
| orering | $\frac{2(63b^2e^2x^2 + 154xab e^2 - 28b^2dex + 99e^2a^2 - 44abde + 8b^2d^2)(ex+d)^{\frac{7}{2}} (b^2x^2 + 2abx + a^2)}{693e^3(bx+a)^2}$ |
| trager | $\frac{2(63b^2e^5x^5 + 154ab e^5x^4 + 161b^2d e^4x^4 + 99a^2e^5x^3 + 418abd e^4x^3 + 113b^2d^2e^3x^3 + 297a^2d e^4x^2 + 330abd^2e^3x^2 + 3b^2d^3e^2)}{693e^3}$ |
| risch | $\frac{2(63b^2e^5x^5 + 154ab e^5x^4 + 161b^2d e^4x^4 + 99a^2e^5x^3 + 418abd e^4x^3 + 113b^2d^2e^3x^3 + 297a^2d e^4x^2 + 330abd^2e^3x^2 + 3b^2d^3e^2)}{693e^3}$ |

input `int((e*x+d)^(5/2)*(b^2*x^2+2*a*b*x+a^2),x,method=_RETURNVERBOSE)`

output `2/7*(e*x+d)^(7/2)*((7/11*b^2*x^2+14/9*a*b*x+a^2)*e^-4-4/9*(7/11*b*x+a)*b*d*e+8/99*b^2*d^2)/e^3`

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 174 vs. 2(59) = 118.

Time = 0.09 (sec) , antiderivative size = 174, normalized size of antiderivative = 2.45

$$\int (d + ex)^{5/2} (a^2 + 2abx + b^2x^2) dx = \frac{2(63b^2e^5x^5 + 8b^2d^5 - 44abd^4e + 99a^2d^3e^2 + 7(23b^2de^4 + 22abe^5)x^4 + (113b^2d^2e^3 + 418abd^2e^2)x^3 + 3(b^2d^3e^2 + 110a*b*d^2*e^3 + 99a^2*d*e^4)x^2 - (4b^2*d^4*e - 22a*b*d^3*e^2 - 297a^2*d^2*e^3)x)*\sqrt{e*x + d}}{e^3}$$

input `integrate((e*x+d)^(5/2)*(b^2*x^2+2*a*b*x+a^2),x, algorithm="fricas")`

output `2/693*(63*b^2*e^5*x^5 + 8*b^2*d^5 - 44*a*b*d^4*e + 99*a^2*d^3*e^2 + 7*(23*b^2*d*e^4 + 22*a*b*e^5)*x^4 + (113*b^2*d^2*e^3 + 418*a*b*d*e^4 + 99*a^2*e^5)*x^3 + 3*(b^2*d^3*e^2 + 110*a*b*d^2*e^3 + 99*a^2*d*e^4)*x^2 - (4*b^2*d^4*e - 22*a*b*d^3*e^2 - 297*a^2*d^2*e^3)*x)*sqrt(e*x + d)/e^3`

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 355 vs. 2(65) = 130.

Time = 0.34 (sec) , antiderivative size = 355, normalized size of antiderivative = 5.00

$$\int (d + ex)^{5/2} (a^2 + 2abx + b^2x^2) dx = \left\{ \begin{array}{l} \frac{2a^2d^3\sqrt{d+ex}}{7e} + \frac{6a^2d^2x\sqrt{d+ex}}{7} + \frac{6a^2dex^2\sqrt{d+ex}}{7} + \frac{2a^2e^2x^3\sqrt{d+ex}}{7} - \frac{8abd^4\sqrt{d+ex}}{63e^2} + \frac{4abd^3x\sqrt{d+ex}}{63e} + \frac{20abd^2x^2}{21} \\ d^{\frac{5}{2}} \left(a^2x + abx^2 + \frac{b^2x^3}{3} \right) \end{array} \right.$$

input `integrate((e*x+d)**(5/2)*(b**2*x**2+2*a*b*x+a**2),x)`

output

```
Piecewise((2*a**2*d**3*sqrt(d + e*x)/(7*e) + 6*a**2*d**2*x*sqrt(d + e*x)/7
+ 6*a**2*d*e*x**2*sqrt(d + e*x)/7 + 2*a**2*e**2*x**3*sqrt(d + e*x)/7 - 8*
a*b*d**4*sqrt(d + e*x)/(63*e**2) + 4*a*b*d**3*x*sqrt(d + e*x)/(63*e) + 20*
a*b*d**2*x**2*sqrt(d + e*x)/21 + 76*a*b*d*e*x**3*sqrt(d + e*x)/63 + 4*a*b*
e**2*x**4*sqrt(d + e*x)/9 + 16*b**2*d**5*sqrt(d + e*x)/(693*e**3) - 8*b**2
*d**4*x*sqrt(d + e*x)/(693*e**2) + 2*b**2*d**3*x**2*sqrt(d + e*x)/(231*e)
+ 226*b**2*d**2*x**3*sqrt(d + e*x)/693 + 46*b**2*d*e*x**4*sqrt(d + e*x)/99
+ 2*b**2*e**2*x**5*sqrt(d + e*x)/11, Ne(e, 0)), (d**(5/2)*(a**2*x + a*b*x
**2 + b**2*x**3/3), True))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.96

$$\int (d + ex)^{5/2} (a^2 + 2abx + b^2x^2) dx = \frac{2 \left(63 (ex + d)^{\frac{11}{2}} b^2 - 154 (b^2d - abe)(ex + d)^{\frac{9}{2}} + 99 (b^2d^2 - 2abde + a^2e^2)(ex + d)^{\frac{7}{2}} \right)}{693 e^3}$$

input

```
integrate((e*x+d)^(5/2)*(b^2*x^2+2*a*b*x+a^2),x, algorithm="maxima")
```

output

```
2/693*(63*(e*x + d)^(11/2)*b^2 - 154*(b^2*d - a*b*e)*(e*x + d)^(9/2) + 99*
(b^2*d^2 - 2*a*b*d*e + a^2*e^2)*(e*x + d)^(7/2))/e^3
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 558 vs. 2(59) = 118.

Time = 0.15 (sec) , antiderivative size = 558, normalized size of antiderivative = 7.86

$$\int (d + ex)^{5/2} (a^2 + 2abx + b^2x^2) dx = \text{Too large to display}$$

input

```
integrate((e*x+d)^(5/2)*(b^2*x^2+2*a*b*x+a^2),x, algorithm="giac")
```


output

```

2/3465*(3465*sqrt(e*x + d)*a^2*d^3 + 3465*((e*x + d)^(3/2) - 3*sqrt(e*x +
d)*d)*a^2*d^2 + 2310*((e*x + d)^(3/2) - 3*sqrt(e*x + d)*d)*a*b*d^3/e + 693
*(3*(e*x + d)^(5/2) - 10*(e*x + d)^(3/2)*d + 15*sqrt(e*x + d)*d^2)*a^2*d +
231*(3*(e*x + d)^(5/2) - 10*(e*x + d)^(3/2)*d + 15*sqrt(e*x + d)*d^2)*b^2
*d^3/e^2 + 1386*(3*(e*x + d)^(5/2) - 10*(e*x + d)^(3/2)*d + 15*sqrt(e*x +
d)*d^2)*a*b*d^2/e + 99*(5*(e*x + d)^(7/2) - 21*(e*x + d)^(5/2)*d + 35*(e*x
+ d)^(3/2)*d^2 - 35*sqrt(e*x + d)*d^3)*a^2 + 297*(5*(e*x + d)^(7/2) - 21*
(e*x + d)^(5/2)*d + 35*(e*x + d)^(3/2)*d^2 - 35*sqrt(e*x + d)*d^3)*b^2*d^2
/e^2 + 594*(5*(e*x + d)^(7/2) - 21*(e*x + d)^(5/2)*d + 35*(e*x + d)^(3/2)*
d^2 - 35*sqrt(e*x + d)*d^3)*a*b*d/e + 33*(35*(e*x + d)^(9/2) - 180*(e*x +
d)^(7/2)*d + 378*(e*x + d)^(5/2)*d^2 - 420*(e*x + d)^(3/2)*d^3 + 315*sqrt(
e*x + d)*d^4)*b^2*d/e^2 + 22*(35*(e*x + d)^(9/2) - 180*(e*x + d)^(7/2)*d +
378*(e*x + d)^(5/2)*d^2 - 420*(e*x + d)^(3/2)*d^3 + 315*sqrt(e*x + d)*d^4
)*a*b/e + 5*(63*(e*x + d)^(11/2) - 385*(e*x + d)^(9/2)*d + 990*(e*x + d)^(
7/2)*d^2 - 1386*(e*x + d)^(5/2)*d^3 + 1155*(e*x + d)^(3/2)*d^4 - 693*sqrt(
e*x + d)*d^5)*b^2/e^2)/e

```

Mupad [B] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.96

$$\int (d + ex)^{5/2} (a^2 + 2abx + b^2x^2) dx = \frac{2(d + ex)^{7/2} (63b^2(d + ex)^2 + 99a^2e^2 + 99b^2d^2 - 154b^2d(d + ex) + 154abe(d + ex) - 198abd^2e) + 154ab^2e^2(d + ex)^2 - 154ab^2e^2d(d + ex) + 154ab^2e^2d^2 - 154ab^2e^2d^3 + 154ab^2e^2d^4 - 154ab^2e^2d^5}{693e^3}$$

input

```
int((d + e*x)^(5/2)*(a^2 + b^2*x^2 + 2*a*b*x),x)
```

output

```

(2*(d + e*x)^(7/2)*(63*b^2*(d + e*x)^2 + 99*a^2*e^2 + 99*b^2*d^2 - 154*b^2
*d*(d + e*x) + 154*a*b*e*(d + e*x) - 198*a*b*d*e))/(693*e^3)

```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 180, normalized size of antiderivative = 2.54

$$\int (d + ex)^{5/2} (a^2 + 2abx + b^2x^2) dx = \frac{2\sqrt{ex + d} (63b^2e^5x^5 + 154ab e^5x^4 + 161b^2d e^4x^4 + 99a^2e^5x^3 + 418abd e^4x^3 + 113b^2d^2e^3x^3 + 2$$

input `int((e*x+d)^(5/2)*(b^2*x^2+2*a*b*x+a^2),x)`output `(2*sqrt(d + e*x)*(99*a**2*d**3*e**2 + 297*a**2*d**2*e**3*x + 297*a**2*d*e**4*x**2 + 99*a**2*e**5*x**3 - 44*a*b*d**4*e + 22*a*b*d**3*e**2*x + 330*a*b*d**2*e**3*x**2 + 418*a*b*d*e**4*x**3 + 154*a*b*e**5*x**4 + 8*b**2*d**5 - 4*b**2*d**4*e*x + 3*b**2*d**3*e**2*x**2 + 113*b**2*d**2*e**3*x**3 + 161*b**2*d*e**4*x**4 + 63*b**2*e**5*x**5))/(693*e**3)`

3.148 $\int (d + ex)^{3/2} (a^2 + 2abx + b^2x^2) dx$

| | |
|---|------|
| Optimal result | 1166 |
| Mathematica [A] (verified) | 1166 |
| Rubi [A] (verified) | 1167 |
| Maple [A] (verified) | 1168 |
| Fricas [B] (verification not implemented) | 1169 |
| Sympy [A] (verification not implemented) | 1169 |
| Maxima [A] (verification not implemented) | 1170 |
| Giac [B] (verification not implemented) | 1170 |
| Mupad [B] (verification not implemented) | 1171 |
| Reduce [B] (verification not implemented) | 1171 |

Optimal result

Integrand size = 26, antiderivative size = 71

$$\int (d + ex)^{3/2} (a^2 + 2abx + b^2x^2) dx = \frac{2(bd - ae)^2(d + ex)^{5/2}}{5e^3} - \frac{4b(bd - ae)(d + ex)^{7/2}}{7e^3} + \frac{2b^2(d + ex)^{9/2}}{9e^3}$$

output

```
2/5*(-a*e+b*d)^2*(e*x+d)^(5/2)/e^3-4/7*b*(-a*e+b*d)*(e*x+d)^(7/2)/e^3+2/9*
b^2*(e*x+d)^(9/2)/e^3
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.86

$$\int (d + ex)^{3/2} (a^2 + 2abx + b^2x^2) dx = \frac{2(d + ex)^{5/2} (63a^2e^2 + 18abe(-2d + 5ex) + b^2(8d^2 - 20dex + 35e^2x^2))}{315e^3}$$

input

```
Integrate[(d + e*x)^(3/2)*(a^2 + 2*a*b*x + b^2*x^2),x]
```

output

$$(2*(d + e*x)^(5/2)*(63*a^2*e^2 + 18*a*b*e*(-2*d + 5*e*x) + b^2*(8*d^2 - 20*d*e*x + 35*e^2*x^2)))/(315*e^3)$$

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {1098, 27, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\begin{aligned} & \int (a^2 + 2abx + b^2x^2)(d + ex)^{3/2} dx \\ & \quad \downarrow 1098 \\ & \frac{\int b^2(a + bx)^2(d + ex)^{3/2} dx}{b^2} \\ & \quad \downarrow 27 \\ & \int (a + bx)^2(d + ex)^{3/2} dx \\ & \quad \downarrow 53 \\ & \int \left(-\frac{2b(d + ex)^{5/2}(bd - ae)}{e^2} + \frac{(d + ex)^{3/2}(ae - bd)^2}{e^2} + \frac{b^2(d + ex)^{7/2}}{e^2} \right) dx \\ & \quad \downarrow 2009 \\ & -\frac{4b(d + ex)^{7/2}(bd - ae)}{7e^3} + \frac{2(d + ex)^{5/2}(bd - ae)^2}{5e^3} + \frac{2b^2(d + ex)^{9/2}}{9e^3} \end{aligned}$$

input

$$\text{Int}[(d + e*x)^(3/2)*(a^2 + 2*a*b*x + b^2*x^2), x]$$

output

$$(2*(b*d - a*e)^2*(d + e*x)^(5/2))/(5*e^3) - (4*b*(b*d - a*e)*(d + e*x)^(7/2))/(7*e^3) + (2*b^2*(d + e*x)^(9/2))/(9*e^3)$$

Defintions of rubi rules used

rule 27 $\text{Int}[(a_*)(Fx_), x_Symbol] \rightarrow \text{Simp}[a \text{ Int}[Fx, x], x] /; \text{FreeQ}[a, x] \ \&\& \ !\text{MatchQ}[Fx, (b_*)(Gx_)] /; \text{FreeQ}[b, x]$

rule 53 $\text{Int}[(a_*) + (b_*)(x_)^{(m_*)} * ((c_*) + (d_*)(x_)^{(n_*)}), x_Symbol] \rightarrow \text{Int}[\text{ExpandIntegrand}[(a + b*x)^m * (c + d*x)^n, x], x] /; \text{FreeQ}[\{a, b, c, d, n\}, x] \ \&\& \ \text{IGtQ}[m, 0] \ \&\& \ (\ !\text{IntegerQ}[n] \ || \ (\text{EqQ}[c, 0] \ \&\& \ \text{LeQ}[7*m + 4*n + 4, 0]) \ || \ \text{LtQ}[9*m + 5*(n + 1), 0] \ || \ \text{GtQ}[m + n + 2, 0])$

rule 1098 $\text{Int}[(d_*) + (e_*)(x_)^{(m_*)} * ((a_*) + (b_*)(x_) + (c_*)(x_)^2)^{(p_*)}, x_Symbol] \rightarrow \text{Simp}[1/c^p \text{ Int}[(d + e*x)^m * (b/2 + c*x)^(2*p), x], x] /; \text{FreeQ}[\{a, b, c, d, e, m\}, x] \ \&\& \ \text{EqQ}[b^2 - 4*a*c, 0] \ \&\& \ \text{IntegerQ}[p]$

rule 2009 $\text{Int}[u_, x_Symbol] \rightarrow \text{Simp}[\text{IntSum}[u, x], x] /; \text{SumQ}[u]$

Maple [A] (verified)

Time = 0.78 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.76

| method | result |
|------------------|---|
| pseudoelliptic | $\frac{2 \left(\left(\frac{5}{9} b^2 x^2 + \frac{10}{7} abx + a^2 \right) e^2 - \frac{4b \left(\frac{5bx}{9} + a \right) de}{7} + \frac{8b^2 d^2}{63} \right) (ex+d)^{\frac{5}{2}}}{5e^3}$ |
| gospers | $\frac{2(ex+d)^{\frac{5}{2}} (35b^2e^2x^2 + 90xab e^2 - 20b^2dex + 63e^2a^2 - 36abde + 8b^2d^2)}{315e^3}$ |
| derivativdivides | $\frac{\frac{2b^2(ex+d)^{\frac{9}{2}}}{9} + \frac{2(2abe-2b^2d)(ex+d)^{\frac{7}{2}}}{7} + \frac{2(e^2a^2-2abde+b^2d^2)(ex+d)^{\frac{5}{2}}}{5}}{e^3}$ |
| default | $\frac{2b^2(ex+d)^{\frac{9}{2}}}{9} + \frac{2(2abe-2b^2d)(ex+d)^{\frac{7}{2}}}{7} + \frac{2(e^2a^2-2abde+b^2d^2)(ex+d)^{\frac{5}{2}}}{5}}{e^3}$ |
| oring | $\frac{2(35b^2e^2x^2 + 90xab e^2 - 20b^2dex + 63e^2a^2 - 36abde + 8b^2d^2)(ex+d)^{\frac{5}{2}}(b^2x^2 + 2abx + a^2)}{315e^3(bx+a)^2}$ |
| trager | $\frac{2(35b^2e^4x^4 + 90ab e^4x^3 + 50b^2d e^3x^3 + 63a^2e^4x^2 + 144abd e^3x^2 + 3b^2d^2e^2x^2 + 126a^2d e^3x + 18abd^2e^2x - 4b^2d^3ex + 63a^2d^2)}{315e^3}$ |
| risch | $\frac{2(35b^2e^4x^4 + 90ab e^4x^3 + 50b^2d e^3x^3 + 63a^2e^4x^2 + 144abd e^3x^2 + 3b^2d^2e^2x^2 + 126a^2d e^3x + 18abd^2e^2x - 4b^2d^3ex + 63a^2d^2)}{315e^3}$ |

input `int((e*x+d)^(3/2)*(b^2*x^2+2*a*b*x+a^2),x,method=_RETURNVERBOSE)`

output $2/5*((5/9*b^2*x^2+10/7*a*b*x+a^2)*e^2-4/7*b*(5/9*b*x+a)*d*e+8/63*b^2*d^2)*(e*x+d)^(5/2)/e^3$

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 137 vs. $2(59) = 118$.

Time = 0.08 (sec) , antiderivative size = 137, normalized size of antiderivative = 1.93

$$\int (d+ex)^{3/2} (a^2+2abx+b^2x^2) dx = \frac{2(35b^2e^4x^4+8b^2d^4-36abd^3e+63a^2d^2e^2+10(5b^2de^3+9abe^4)x^3+3(b^2d^2e^2+48abde^3-315e^3)}{315e^3}$$

input `integrate((e*x+d)^(3/2)*(b^2*x^2+2*a*b*x+a^2),x, algorithm="fricas")`

output $2/315*(35*b^2*e^4*x^4+8*b^2*d^4-36*a*b*d^3*e+63*a^2*d^2*e^2+10*(5*b^2*d*e^3+9*a*b*e^4)*x^3+3*(b^2*d^2*e^2+48*a*b*d*e^3+21*a^2*e^4)*x^2-2*(2*b^2*d^3*e-9*a*b*d^2*e^2-63*a^2*d*e^3)*x)*sqrt(e*x+d)/e^3$

Sympy [A] (verification not implemented)

Time = 0.76 (sec) , antiderivative size = 110, normalized size of antiderivative = 1.55

$$\int (d+ex)^{3/2} (a^2+2abx+b^2x^2) dx = \begin{cases} \frac{2\left(\frac{b^2(d+ex)^{\frac{9}{2}}}{9e^2} + \frac{(d+ex)^{\frac{7}{2}} \cdot (2abe-2b^2d)}{7e^2} + \frac{(d+ex)^{\frac{5}{2}} (a^2e^2-2abde+b^2d^2)}{5e^2}\right)}{e} & \text{for } e \neq 0 \\ d^{\frac{3}{2}} \left(a^2x + abx^2 + \frac{b^2x^3}{3}\right) & \text{otherwise} \end{cases}$$

input `integrate((e*x+d)**(3/2)*(b**2*x**2+2*a*b*x+a**2),x)`

output

```
Piecewise((2*(b**2*(d + e*x)**(9/2)/(9*e**2) + (d + e*x)**(7/2)*(2*a*b*e -
2*b**2*d)/(7*e**2) + (d + e*x)**(5/2)*(a**2*e**2 - 2*a*b*d*e + b**2*d**2)
/(5*e**2))/e, Ne(e, 0)), (d**(3/2)*(a**2*x + a*b*x**2 + b**2*x**3/3), True
))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.96

$$\int (d + ex)^{3/2} (a^2 + 2abx + b^2x^2) dx = \frac{2 \left(35 (ex + d)^{9/2} b^2 - 90 (b^2d - abe)(ex + d)^{7/2} + 63 (b^2d^2 - 2abde + a^2e^2)(ex + d)^{5/2} \right)}{315 e^3}$$

input

```
integrate((e*x+d)^(3/2)*(b^2*x^2+2*a*b*x+a^2),x, algorithm="maxima")
```

output

```
2/315*(35*(e*x + d)^(9/2)*b^2 - 90*(b^2*d - a*b*e)*(e*x + d)^(7/2) + 63*(b
^2*d^2 - 2*a*b*d*e + a^2*e^2)*(e*x + d)^(5/2))/e^3
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 360 vs. 2(59) = 118.

Time = 0.17 (sec) , antiderivative size = 360, normalized size of antiderivative = 5.07

$$\int (d + ex)^{3/2} (a^2 + 2abx + b^2x^2) dx = \frac{2 \left(315 \sqrt{ex + d} a^2 d^2 + 210 \left((ex + d)^{3/2} - 3 \sqrt{ex + d} d \right) a^2 d + \frac{210 \left((ex + d)^{3/2} - 3 \sqrt{ex + d} d \right) a b d^2}{e} + 21 \left(3 \right) \right)}{315 e^3}$$

input

```
integrate((e*x+d)^(3/2)*(b^2*x^2+2*a*b*x+a^2),x, algorithm="giac")
```

output

```
2/315*(315*sqrt(e*x + d)*a^2*d^2 + 210*((e*x + d)^(3/2) - 3*sqrt(e*x + d)*
d)*a^2*d + 210*((e*x + d)^(3/2) - 3*sqrt(e*x + d)*d)*a*b*d^2/e + 21*(3*(e
x + d)^(5/2) - 10*(e*x + d)^(3/2)*d + 15*sqrt(e*x + d)*d^2)*a^2 + 21*(3*(e
*x + d)^(5/2) - 10*(e*x + d)^(3/2)*d + 15*sqrt(e*x + d)*d^2)*b^2*d^2/e^2 +
84*(3*(e*x + d)^(5/2) - 10*(e*x + d)^(3/2)*d + 15*sqrt(e*x + d)*d^2)*a*b*
d/e + 18*(5*(e*x + d)^(7/2) - 21*(e*x + d)^(5/2)*d + 35*(e*x + d)^(3/2)*d^
2 - 35*sqrt(e*x + d)*d^3)*b^2*d/e^2 + 18*(5*(e*x + d)^(7/2) - 21*(e*x + d)
^(5/2)*d + 35*(e*x + d)^(3/2)*d^2 - 35*sqrt(e*x + d)*d^3)*a*b/e + (35*(e*x
+ d)^(9/2) - 180*(e*x + d)^(7/2)*d + 378*(e*x + d)^(5/2)*d^2 - 420*(e*x +
d)^(3/2)*d^3 + 315*sqrt(e*x + d)*d^4)*b^2/e^2)/e
```

Mupad [B] (verification not implemented)

Time = 9.08 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.96

$$\int (d + ex)^{3/2} (a^2 + 2abx + b^2x^2) dx = \frac{2(d + ex)^{5/2} (35b^2(d + ex)^2 + 63a^2e^2 + 63b^2d^2 - 90b^2d(d + ex) + 90abe(d + ex) - 126a^2bde)}{315e^3}$$

input

```
int((d + e*x)^(3/2)*(a^2 + b^2*x^2 + 2*a*b*x), x)
```

output

```
(2*(d + e*x)^(5/2)*(35*b^2*(d + e*x)^2 + 63*a^2*e^2 + 63*b^2*d^2 - 90*b^2*
d*(d + e*x) + 90*a*b*e*(d + e*x) - 126*a*b*d*e))/(315*e^3)
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 139, normalized size of antiderivative = 1.96

$$\int (d + ex)^{3/2} (a^2 + 2abx + b^2x^2) dx = \frac{2\sqrt{ex + d} (35b^2e^4x^4 + 90abe^4x^3 + 50b^2de^3x^3 + 63a^2e^4x^2 + 144abd^2e^3x^2 + 3b^2d^2e^2x^2 + 126a^2bde^2x + 126a^2bde)}{315e^3}$$

input

```
int((e*x+d)^(3/2)*(b^2*x^2+2*a*b*x+a^2), x)
```


output

$$\frac{(2\sqrt{d + ex})(63a^2d^2e^2 + 126a^2de^3x + 63a^2e^4x^2 - 36abd^3e + 18abd^2e^2x + 144abd^3e^3x^2 + 90abde^4x^3 + 8b^2d^4 - 4b^2d^3ex + 3b^2d^2e^2x^2 + 50b^2de^3x^3 + 35b^2e^4x^4)}{(315e^3)}$$

3.149 $\int \sqrt{d + ex}(a^2 + 2abx + b^2x^2) dx$

| | |
|---|------|
| Optimal result | 1173 |
| Mathematica [A] (verified) | 1173 |
| Rubi [A] (verified) | 1174 |
| Maple [A] (verified) | 1175 |
| Fricas [A] (verification not implemented) | 1176 |
| Sympy [A] (verification not implemented) | 1176 |
| Maxima [A] (verification not implemented) | 1177 |
| Giac [B] (verification not implemented) | 1177 |
| Mupad [B] (verification not implemented) | 1178 |
| Reduce [B] (verification not implemented) | 1178 |

Optimal result

Integrand size = 26, antiderivative size = 71

$$\int \sqrt{d + ex}(a^2 + 2abx + b^2x^2) dx = \frac{2(bd - ae)^2(d + ex)^{3/2}}{3e^3} - \frac{4b(bd - ae)(d + ex)^{5/2}}{5e^3} + \frac{2b^2(d + ex)^{7/2}}{7e^3}$$

output

```
2/3*(-a*e+b*d)^2*(e*x+d)^(3/2)/e^3-4/5*b*(-a*e+b*d)*(e*x+d)^(5/2)/e^3+2/7*
b^2*(e*x+d)^(7/2)/e^3
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.86

$$\int \sqrt{d + ex}(a^2 + 2abx + b^2x^2) dx = \frac{2(d + ex)^{3/2} (35a^2e^2 + 14abe(-2d + 3ex) + b^2(8d^2 - 12dex + 15e^2x^2))}{105e^3}$$

input

```
Integrate[Sqrt[d + e*x]*(a^2 + 2*a*b*x + b^2*x^2), x]
```

output

$$(2*(d + e*x)^{(3/2)}*(35*a^2*e^2 + 14*a*b*e*(-2*d + 3*e*x) + b^2*(8*d^2 - 12*d*e*x + 15*e^2*x^2)))/(105*e^3)$$

Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {1098, 27, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\begin{aligned} & \int (a^2 + 2abx + b^2x^2) \sqrt{d + ex} dx \\ & \quad \downarrow 1098 \\ & \int \frac{b^2(a + bx)^2 \sqrt{d + ex} dx}{b^2} \\ & \quad \downarrow 27 \\ & \int (a + bx)^2 \sqrt{d + ex} dx \\ & \quad \downarrow 53 \\ & \int \left(-\frac{2b(d + ex)^{3/2}(bd - ae)}{e^2} + \frac{\sqrt{d + ex}(ae - bd)^2}{e^2} + \frac{b^2(d + ex)^{5/2}}{e^2} \right) dx \\ & \quad \downarrow 2009 \\ & -\frac{4b(d + ex)^{5/2}(bd - ae)}{5e^3} + \frac{2(d + ex)^{3/2}(bd - ae)^2}{3e^3} + \frac{2b^2(d + ex)^{7/2}}{7e^3} \end{aligned}$$

input

$$\text{Int}[\text{Sqrt}[d + e*x]*(a^2 + 2*a*b*x + b^2*x^2), x]$$

output

$$(2*(b*d - a*e)^2*(d + e*x)^{(3/2)})/(3*e^3) - (4*b*(b*d - a*e)*(d + e*x)^{(5/2)})/(5*e^3) + (2*b^2*(d + e*x)^{(7/2)})/(7*e^3)$$

Defintions of rubi rules used

rule 27 $\text{Int}[(a_*)(Fx_), x_Symbol] \rightarrow \text{Simp}[a \text{ Int}[Fx, x], x] /; \text{FreeQ}[a, x] \ \&\& \ !\text{MatchQ}[Fx, (b_*)(Gx_)] /; \text{FreeQ}[b, x]$

rule 53 $\text{Int}[((a_.) + (b_.)(x_))^{(m_.)} * ((c_.) + (d_.)(x_))^{(n_.)}, x_Symbol] \rightarrow \text{Int}[\text{ExpandIntegrand}[(a + b*x)^m * (c + d*x)^n, x], x] /; \text{FreeQ}[\{a, b, c, d, n\}, x] \ \&\& \ \text{IGtQ}[m, 0] \ \&\& \ (\ !\text{IntegerQ}[n] \ || \ (\text{EqQ}[c, 0] \ \&\& \ \text{LeQ}[7*m + 4*n + 4, 0]) \ || \ \text{LtQ}[9*m + 5*(n + 1), 0] \ || \ \text{GtQ}[m + n + 2, 0])$

rule 1098 $\text{Int}[((d_.) + (e_.)(x_))^{(m_.)} * ((a_.) + (b_.)(x_) + (c_.)(x_)^2)^{(p_.)}, x_Symbol] \rightarrow \text{Simp}[1/c^p \text{ Int}[(d + e*x)^m * (b/2 + c*x)^(2*p), x], x] /; \text{FreeQ}[\{a, b, c, d, e, m\}, x] \ \&\& \ \text{EqQ}[b^2 - 4*a*c, 0] \ \&\& \ \text{IntegerQ}[p]$

rule 2009 $\text{Int}[u_, x_Symbol] \rightarrow \text{Simp}[\text{IntSum}[u, x], x] /; \text{SumQ}[u]$

Maple [A] (verified)

Time = 0.66 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.76

| method | result | size |
|-------------------|---|------|
| pseudoelliptic | $\frac{2(ex+d)^{\frac{3}{2}} \left(\left(\frac{3}{7}b^2x^2 + \frac{6}{5}abx + a^2 \right) e^2 - \frac{4b \left(\frac{3bx}{7} + a \right) de}{5} + \frac{8b^2d^2}{35} \right)}{3e^3}$ | 54 |
| gospers | $\frac{2(ex+d)^{\frac{3}{2}} (15b^2e^2x^2 + 42xab e^2 - 12b^2dex + 35e^2a^2 - 28abde + 8b^2d^2)}{105e^3}$ | 63 |
| derivativedivides | $\frac{\frac{2b^2(ex+d)^{\frac{7}{2}}}{7} + \frac{2(2abe-2b^2d)(ex+d)^{\frac{5}{2}}}{5} + \frac{2(e^2a^2-2abde+b^2d^2)(ex+d)^{\frac{3}{2}}}{3}}{e^3}$ | 70 |
| default | $\frac{\frac{2b^2(ex+d)^{\frac{7}{2}}}{7} + \frac{2(2abe-2b^2d)(ex+d)^{\frac{5}{2}}}{5} + \frac{2(e^2a^2-2abde+b^2d^2)(ex+d)^{\frac{3}{2}}}{3}}{e^3}$ | 70 |
| orering | $\frac{2(15b^2e^2x^2 + 42xab e^2 - 12b^2dex + 35e^2a^2 - 28abde + 8b^2d^2)(ex+d)^{\frac{3}{2}} (b^2x^2 + 2abx + a^2)}{105e^3 (bx+a)^2}$ | 86 |
| trager | $\frac{2(15b^2e^3x^3 + 42a e^3b x^2 + 3d e^2b^2x^2 + 35a^2e^3x + 14abd e^2x - 4b^2d^2ex + 35a^2d e^2 - 28ab d^2e + 8b^2d^3) \sqrt{ex+d}}{105e^3}$ | 100 |
| risch | $\frac{2(15b^2e^3x^3 + 42a e^3b x^2 + 3d e^2b^2x^2 + 35a^2e^3x + 14abd e^2x - 4b^2d^2ex + 35a^2d e^2 - 28ab d^2e + 8b^2d^3) \sqrt{ex+d}}{105e^3}$ | 100 |

input `int((e*x+d)^(1/2)*(b^2*x^2+2*a*b*x+a^2),x,method=_RETURNVERBOSE)`

output $\frac{2}{3}(e*x+d)^{3/2} * ((3/7*b^2*x^2+6/5*a*b*x+a^2)*e^{-2}-4/5*b*(3/7*b*x+a)*d*e+8/35*b^2*d^2)/e^3$

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.39

$$\int \sqrt{d+ex}(a^2+2abx+b^2x^2) dx$$

$$= \frac{2(15b^2e^3x^3+8b^2d^3-28abd^2e+35a^2de^2+3(b^2de^2+14abe^3)x^2-(4b^2d^2e-14abde^2-35a^2e^3)x)\sqrt{d+ex}}{105e^3}$$

input `integrate((e*x+d)^(1/2)*(b^2*x^2+2*a*b*x+a^2),x, algorithm="fricas")`

output $\frac{2}{105} * (15*b^2*e^3*x^3 + 8*b^2*d^3 - 28*a*b*d^2*e + 35*a^2*d*e^2 + 3*(b^2*d*e^2 + 14*a*b*e^3)*x^2 - (4*b^2*d^2*e - 14*a*b*d*e^2 - 35*a^2*e^3)*x) * \text{sqrt}(e*x + d) / e^3$

Sympy [A] (verification not implemented)

Time = 0.72 (sec) , antiderivative size = 110, normalized size of antiderivative = 1.55

$$\int \sqrt{d+ex}(a^2+2abx+b^2x^2) dx$$

$$= \begin{cases} \frac{2 \left(\frac{b^2(d+ex)^{7/2}}{7e^2} + \frac{(d+ex)^{5/2} \cdot (2abe-2b^2d)}{5e^2} + \frac{(d+ex)^{3/2} (a^2e^2-2abde+b^2d^2)}{3e^2} \right)}{e} & \text{for } e \neq 0 \\ \sqrt{d} \left(a^2x + abx^2 + \frac{b^2x^3}{3} \right) & \text{otherwise} \end{cases}$$

input `integrate((e*x+d)**(1/2)*(b**2*x**2+2*a*b*x+a**2),x)`

output

```
Piecewise((2*(b**2*(d + e*x)**(7/2)/(7*e**2) + (d + e*x)**(5/2)*(2*a*b*e -
2*b**2*d)/(5*e**2) + (d + e*x)**(3/2)*(a**2*e**2 - 2*a*b*d*e + b**2*d**2)
/(3*e**2))/e, Ne(e, 0)), (sqrt(d)*(a**2*x + a*b*x**2 + b**2*x**3/3), True)
)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.96

$$\int \sqrt{d+ex}(a^2+2abx+b^2x^2) dx$$

$$= \frac{2 \left(15(ex+d)^{\frac{7}{2}}b^2 - 42(b^2d - abe)(ex+d)^{\frac{5}{2}} + 35(b^2d^2 - 2abde + a^2e^2)(ex+d)^{\frac{3}{2}} \right)}{105e^3}$$

input

```
integrate((e*x+d)^(1/2)*(b^2*x^2+2*a*b*x+a^2),x, algorithm="maxima")
```

output

```
2/105*(15*(e*x + d)^(7/2)*b^2 - 42*(b^2*d - a*b*e)*(e*x + d)^(5/2) + 35*(b
^2*d^2 - 2*a*b*d*e + a^2*e^2)*(e*x + d)^(3/2))/e^3
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 200 vs. $2(59) = 118$.

Time = 0.18 (sec) , antiderivative size = 200, normalized size of antiderivative = 2.82

$$\int \sqrt{d+ex}(a^2+2abx+b^2x^2) dx$$

$$= \frac{2 \left(105 \sqrt{ex+da^2d} + 35 \left((ex+d)^{\frac{3}{2}} - 3\sqrt{ex+dd} \right) a^2 + \frac{70 \left((ex+d)^{\frac{3}{2}} - 3\sqrt{ex+dd} \right) abd}{e} + \frac{7 \left(3(ex+d)^{\frac{5}{2}} - 10(ex+d)^{\frac{3}{2}}d \right)}{e^2} \right)}{105}$$

input

```
integrate((e*x+d)^(1/2)*(b^2*x^2+2*a*b*x+a^2),x, algorithm="giac")
```

output

```
2/105*(105*sqrt(e*x + d)*a^2*d + 35*((e*x + d)^(3/2) - 3*sqrt(e*x + d)*
a^2 + 70*((e*x + d)^(3/2) - 3*sqrt(e*x + d)*a*b*d/e + 7*(3*(e*x + d)^(5
/2) - 10*(e*x + d)^(3/2)*d + 15*sqrt(e*x + d)*d^2)*b^2*d/e^2 + 14*(3*(e*x
+ d)^(5/2) - 10*(e*x + d)^(3/2)*d + 15*sqrt(e*x + d)*d^2)*a*b/e + 3*(5*(e*
x + d)^(7/2) - 21*(e*x + d)^(5/2)*d + 35*(e*x + d)^(3/2)*d^2 - 35*sqrt(e*x
+ d)*d^3)*b^2/e^2)/e
```

Mupad [B] (verification not implemented)

Time = 9.63 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.96

$$\int \sqrt{d+ex}(a^2+2abx+b^2x^2) dx$$

$$= \frac{2(d+ex)^{3/2}(15b^2(d+ex)^2+35a^2e^2+35b^2d^2-42b^2d(d+ex)+42abe(d+ex)-70abde)}{105e^3}$$

input

```
int((d + e*x)^(1/2)*(a^2 + b^2*x^2 + 2*a*b*x), x)
```

output

```
(2*(d + e*x)^(3/2)*(15*b^2*(d + e*x)^2 + 35*a^2*e^2 + 35*b^2*d^2 - 42*b^2*
d*(d + e*x) + 42*a*b*e*(d + e*x) - 70*a*b*d*e))/(105*e^3)
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 98, normalized size of antiderivative = 1.38

$$\int \sqrt{d+ex}(a^2+2abx+b^2x^2) dx$$

$$= \frac{2\sqrt{ex+d}(15b^2e^3x^3+42abe^3x^2+3b^2de^2x^2+35a^2e^3x+14abd e^2x-4b^2d^2ex+35a^2de^2-28abd^2e+105e^3)}{105e^3}$$

input

```
int((e*x+d)^(1/2)*(b^2*x^2+2*a*b*x+a^2), x)
```

output

```
(2*sqrt(d + e*x)*(35*a**2*d*e**2 + 35*a**2*e**3*x - 28*a*b*d**2*e + 14*a*b
*d*e**2*x + 42*a*b*e**3*x**2 + 8*b**2*d**3 - 4*b**2*d**2*e*x + 3*b**2*d*e*
**2*x**2 + 15*b**2*e**3*x**3))/(105*e**3)
```

3.150 $\int \frac{a^2+2abx+b^2x^2}{\sqrt{d+ex}} dx$

| | |
|---|------|
| Optimal result | 1179 |
| Mathematica [A] (verified) | 1179 |
| Rubi [A] (verified) | 1180 |
| Maple [A] (verified) | 1181 |
| Fricas [A] (verification not implemented) | 1182 |
| Sympy [A] (verification not implemented) | 1182 |
| Maxima [A] (verification not implemented) | 1183 |
| Giac [A] (verification not implemented) | 1183 |
| Mupad [B] (verification not implemented) | 1184 |
| Reduce [B] (verification not implemented) | 1184 |

Optimal result

Integrand size = 26, antiderivative size = 69

$$\int \frac{a^2 + 2abx + b^2x^2}{\sqrt{d + ex}} dx = \frac{2(bd - ae)^2 \sqrt{d + ex}}{e^3} - \frac{4b(bd - ae)(d + ex)^{3/2}}{3e^3} + \frac{2b^2(d + ex)^{5/2}}{5e^3}$$

output `2*(-a*e+b*d)^2*(e*x+d)^(1/2)/e^3-4/3*b*(-a*e+b*d)*(e*x+d)^(3/2)/e^3+2/5*b^2*(e*x+d)^(5/2)/e^3`

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.87

$$\int \frac{a^2 + 2abx + b^2x^2}{\sqrt{d + ex}} dx = \frac{2\sqrt{d + ex}(15a^2e^2 + 10abe(-2d + ex) + b^2(8d^2 - 4dex + 3e^2x^2))}{15e^3}$$

input `Integrate[(a^2 + 2*a*b*x + b^2*x^2)/Sqrt[d + e*x], x]`

output `(2*Sqrt[d + e*x]*(15*a^2*e^2 + 10*a*b*e*(-2*d + e*x) + b^2*(8*d^2 - 4*d*e*x + 3*e^2*x^2)))/(15*e^3)`

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {1098, 27, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\begin{aligned}
 & \int \frac{a^2 + 2abx + b^2x^2}{\sqrt{d + ex}} dx \\
 & \quad \downarrow \text{1098} \\
 & \int \frac{b^2(a+bx)^2}{\sqrt{d+ex} b^2} dx \\
 & \quad \downarrow \text{27} \\
 & \int \frac{(a + bx)^2}{\sqrt{d + ex}} dx \\
 & \quad \downarrow \text{53} \\
 & \int \left(-\frac{2b\sqrt{d+ex}(bd-ae)}{e^2} + \frac{(ae-bd)^2}{e^2\sqrt{d+ex}} + \frac{b^2(d+ex)^{3/2}}{e^2} \right) dx \\
 & \quad \downarrow \text{2009} \\
 & -\frac{4b(d+ex)^{3/2}(bd-ae)}{3e^3} + \frac{2\sqrt{d+ex}(bd-ae)^2}{e^3} + \frac{2b^2(d+ex)^{5/2}}{5e^3}
 \end{aligned}$$

input `Int[(a^2 + 2*a*b*x + b^2*x^2)/Sqrt[d + e*x],x]`

output `(2*(b*d - a*e)^2*Sqrt[d + e*x])/e^3 - (4*b*(b*d - a*e)*(d + e*x)^(3/2))/(3*e^3) + (2*b^2*(d + e*x)^(5/2))/(5*e^3)`

Defintions of rubi rules used

rule 27 $\text{Int}[(a_)(Fx_), x_Symbol] \rightarrow \text{Simp}[a \text{ Int}[Fx, x], x] /; \text{FreeQ}[a, x] \ \&\& \ !\text{MatchQ}[Fx, (b_)(Gx_)] /; \text{FreeQ}[b, x]$

rule 53 $\text{Int}[(a_ + (b_)(x_))^{(m_)}((c_ + (d_)(x_))^{(n_)}), x_Symbol] \rightarrow \text{Int}[\text{ExpandIntegrand}[(a + b*x)^m*(c + d*x)^n, x], x] /; \text{FreeQ}\{a, b, c, d, n\}, x] \ \&\& \ \text{IGtQ}[m, 0] \ \&\& \ (\ !\text{IntegerQ}[n] \ || \ (\text{EqQ}[c, 0] \ \&\& \ \text{LeQ}[7*m + 4*n + 4, 0]) \ || \ \text{LtQ}[9*m + 5*(n + 1), 0] \ || \ \text{GtQ}[m + n + 2, 0])$

rule 1098 $\text{Int}[(d_ + (e_)(x_))^{(m_)}((a_ + (b_)(x_ + (c_)(x_)^2)^{(p_)}), x_Symbol] \rightarrow \text{Simp}[1/c^p \text{ Int}[(d + e*x)^m*(b/2 + c*x)^(2*p), x], x] /; \text{FreeQ}\{a, b, c, d, e, m\}, x] \ \&\& \ \text{EqQ}[b^2 - 4*a*c, 0] \ \&\& \ \text{IntegerQ}[p]$

rule 2009 $\text{Int}[u_, x_Symbol] \rightarrow \text{Simp}[\text{IntSum}[u, x], x] /; \text{SumQ}[u]$

Maple [A] (verified)

Time = 0.63 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.78

| method | result | size |
|-------------------|--|------|
| pseudoelliptic | $\frac{2\left(\left(\frac{1}{5}b^2x^2 + \frac{2}{3}abx + a^2\right)e^2 - \frac{4b\left(\frac{bx}{5} + a\right)de}{3} + \frac{8b^2d^2}{15}\right)\sqrt{ex+d}}{e^3}$ | 54 |
| derivativedivides | $\frac{\frac{2(ex+d)^{\frac{5}{2}}b^2}{5} + \frac{4(ae-bd)b(ex+d)^{\frac{3}{2}}}{3} + 2(ae-bd)^2\sqrt{ex+d}}{e^3}$ | 55 |
| default | $\frac{\frac{2(ex+d)^{\frac{5}{2}}b^2}{5} + \frac{4(ae-bd)b(ex+d)^{\frac{3}{2}}}{3} + 2(ae-bd)^2\sqrt{ex+d}}{e^3}$ | 55 |
| gosper | $\frac{2(3b^2e^2x^2 + 10xab e^2 - 4b^2dex + 15e^2a^2 - 20abde + 8b^2d^2)\sqrt{ex+d}}{15e^3}$ | 63 |
| trager | $\frac{2(3b^2e^2x^2 + 10xab e^2 - 4b^2dex + 15e^2a^2 - 20abde + 8b^2d^2)\sqrt{ex+d}}{15e^3}$ | 63 |
| risch | $\frac{2(3b^2e^2x^2 + 10xab e^2 - 4b^2dex + 15e^2a^2 - 20abde + 8b^2d^2)\sqrt{ex+d}}{15e^3}$ | 63 |
| orering | $\frac{2(3b^2e^2x^2 + 10xab e^2 - 4b^2dex + 15e^2a^2 - 20abde + 8b^2d^2)\sqrt{ex+d}(b^2x^2 + 2abx + a^2)}{15e^3(bx+a)^2}$ | 86 |

input `int((b^2*x^2+2*a*b*x+a^2)/(e*x+d)^(1/2),x,method=_RETURNVERBOSE)`

output `2*((1/5*b^2*x^2+2/3*a*b*x+a^2)*e^2-4/3*b*(1/5*b*x+a)*d*e+8/15*b^2*d^2)*(e*x+d)^(1/2)/e^3`

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.93

$$\int \frac{a^2 + 2abx + b^2x^2}{\sqrt{d+ex}} dx$$

$$= \frac{2(3b^2e^2x^2 + 8b^2d^2 - 20abde + 15a^2e^2 - 2(2b^2de - 5abe^2)x)\sqrt{ex+d}}{15e^3}$$

input `integrate((b^2*x^2+2*a*b*x+a^2)/(e*x+d)^(1/2),x, algorithm="fricas")`

output `2/15*(3*b^2*e^2*x^2 + 8*b^2*d^2 - 20*a*b*d*e + 15*a^2*e^2 - 2*(2*b^2*d*e - 5*a*b*e^2)*x)*sqrt(e*x + d)/e^3`

Sympy [A] (verification not implemented)

Time = 0.55 (sec) , antiderivative size = 110, normalized size of antiderivative = 1.59

$$\int \frac{a^2 + 2abx + b^2x^2}{\sqrt{d+ex}} dx$$

$$= \begin{cases} \frac{2a^2\sqrt{d+ex} + \frac{4ab\left(-d\sqrt{d+ex} + \frac{(d+ex)^{\frac{3}{2}}}{3}\right)}{e} + \frac{2b^2\left(d^2\sqrt{d+ex} - \frac{2d(d+ex)^{\frac{3}{2}}}{3} + \frac{(d+ex)^{\frac{5}{2}}}{5}\right)}{e^2}}{e} & \text{for } e \neq 0 \\ \frac{a^2x+abx^2+\frac{b^2x^3}{3}}{\sqrt{d}} & \text{otherwise} \end{cases}$$

input `integrate((b**2*x**2+2*a*b*x+a**2)/(e*x+d)**(1/2),x)`

output

```
Piecewise(((2*a**2*sqrt(d + e*x) + 4*a*b*(-d*sqrt(d + e*x) + (d + e*x)**(3/2)/3)/e + 2*b**2*(d**2*sqrt(d + e*x) - 2*d*(d + e*x)**(3/2)/3 + (d + e*x)**(5/2)/5)/e**2)/e, Ne(e, 0)), ((a**2*x + a*b*x**2 + b**2*x**3/3)/sqrt(d), True))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.19

$$\int \frac{a^2 + 2abx + b^2x^2}{\sqrt{d + ex}} dx$$

$$= \frac{2 \left(15 \sqrt{ex + d} a^2 + \frac{10 \left((ex+d)^{\frac{3}{2}} - 3 \sqrt{ex+dd} \right) ab}{e} + \frac{\left(3 (ex+d)^{\frac{5}{2}} - 10 (ex+d)^{\frac{3}{2}} d + 15 \sqrt{ex+dd^2} \right) b^2}{e^2} \right)}{15 e}$$

input

```
integrate((b^2*x^2+2*a*b*x+a^2)/(e*x+d)^(1/2),x, algorithm="maxima")
```

output

```
2/15*(15*sqrt(e*x + d)*a^2 + 10*((e*x + d)^(3/2) - 3*sqrt(e*x + d)*d)*a*b/e + (3*(e*x + d)^(5/2) - 10*(e*x + d)^(3/2)*d + 15*sqrt(e*x + d)*d^2)*b^2/e^2)/e
```

Giac [A] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.19

$$\int \frac{a^2 + 2abx + b^2x^2}{\sqrt{d + ex}} dx$$

$$= \frac{2 \left(15 \sqrt{ex + d} a^2 + \frac{10 \left((ex+d)^{\frac{3}{2}} - 3 \sqrt{ex+dd} \right) ab}{e} + \frac{\left(3 (ex+d)^{\frac{5}{2}} - 10 (ex+d)^{\frac{3}{2}} d + 15 \sqrt{ex+dd^2} \right) b^2}{e^2} \right)}{15 e}$$

input

```
integrate((b^2*x^2+2*a*b*x+a^2)/(e*x+d)^(1/2),x, algorithm="giac")
```

output

```
2/15*(15*sqrt(e*x + d)*a^2 + 10*((e*x + d)^(3/2) - 3*sqrt(e*x + d)*d)*a*b/e + (3*(e*x + d)^(5/2) - 10*(e*x + d)^(3/2)*d + 15*sqrt(e*x + d)*d^2)*b^2/e^2)/e
```

Mupad [B] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.99

$$\int \frac{a^2 + 2abx + b^2x^2}{\sqrt{d + ex}} dx$$

$$= \frac{2\sqrt{d + ex} (3b^2(d + ex)^2 + 15a^2e^2 + 15b^2d^2 - 10b^2d(d + ex) + 10abe(d + ex) - 30abde)}{15e^3}$$

input `int((a^2 + b^2*x^2 + 2*a*b*x)/(d + e*x)^(1/2),x)`output `(2*(d + e*x)^(1/2)*(3*b^2*(d + e*x)^2 + 15*a^2*e^2 + 15*b^2*d^2 - 10*b^2*d*(d + e*x) + 10*a*b*e*(d + e*x) - 30*a*b*d*e))/(15*e^3)`**Reduce [B] (verification not implemented)**

Time = 0.19 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.88

$$\int \frac{a^2 + 2abx + b^2x^2}{\sqrt{d + ex}} dx$$

$$= \frac{2\sqrt{ex + d} (3b^2e^2x^2 + 10abe^2x - 4b^2dex + 15a^2e^2 - 20abde + 8b^2d^2)}{15e^3}$$

input `int((b^2*x^2+2*a*b*x+a^2)/(e*x+d)^(1/2),x)`output `(2*sqrt(d + e*x)*(15*a**2*e**2 - 20*a*b*d*e + 10*a*b*e**2*x + 8*b**2*d**2 - 4*b**2*d*e*x + 3*b**2*e**2*x**2))/(15*e**3)`

$$3.151 \quad \int \frac{a^2 + 2abx + b^2x^2}{(d+ex)^{3/2}} dx$$

| | |
|---|------|
| Optimal result | 1185 |
| Mathematica [A] (verified) | 1185 |
| Rubi [A] (verified) | 1186 |
| Maple [A] (verified) | 1187 |
| Fricas [A] (verification not implemented) | 1188 |
| Sympy [A] (verification not implemented) | 1188 |
| Maxima [A] (verification not implemented) | 1189 |
| Giac [A] (verification not implemented) | 1189 |
| Mupad [B] (verification not implemented) | 1190 |
| Reduce [B] (verification not implemented) | 1190 |

Optimal result

Integrand size = 26, antiderivative size = 67

$$\int \frac{a^2 + 2abx + b^2x^2}{(d+ex)^{3/2}} dx = -\frac{2(bd - ae)^2}{e^3\sqrt{d+ex}} - \frac{4b(bd - ae)\sqrt{d+ex}}{e^3} + \frac{2b^2(d+ex)^{3/2}}{3e^3}$$

output

```
-2*(-a*e+b*d)^2/e^3/(e*x+d)^(1/2)-4*b*(-a*e+b*d)*(e*x+d)^(1/2)/e^3+2/3*b^2
*(e*x+d)^(3/2)/e^3
```

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.88

$$\int \frac{a^2 + 2abx + b^2x^2}{(d+ex)^{3/2}} dx = \frac{2(-3a^2e^2 + 6abe(2d+ex) + b^2(-8d^2 - 4dex + e^2x^2))}{3e^3\sqrt{d+ex}}$$

input

```
Integrate[(a^2 + 2*a*b*x + b^2*x^2)/(d + e*x)^(3/2),x]
```

output

```
(2*(-3*a^2*e^2 + 6*a*b*e*(2*d + e*x) + b^2*(-8*d^2 - 4*d*e*x + e^2*x^2)))/
(3*e^3*Sqrt[d + e*x])
```

Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {1098, 27, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\begin{aligned}
 & \int \frac{a^2 + 2abx + b^2x^2}{(d + ex)^{3/2}} dx \\
 & \quad \downarrow \text{1098} \\
 & \int \frac{b^2(a+bx)^2}{(d+ex)^{3/2}} dx \\
 & \quad \downarrow \text{27} \\
 & \int \frac{(a + bx)^2}{(d + ex)^{3/2}} dx \\
 & \quad \downarrow \text{53} \\
 & \int \left(-\frac{2b(bd - ae)}{e^2\sqrt{d + ex}} + \frac{(ae - bd)^2}{e^2(d + ex)^{3/2}} + \frac{b^2\sqrt{d + ex}}{e^2} \right) dx \\
 & \quad \downarrow \text{2009} \\
 & -\frac{4b\sqrt{d + ex}(bd - ae)}{e^3} - \frac{2(bd - ae)^2}{e^3\sqrt{d + ex}} + \frac{2b^2(d + ex)^{3/2}}{3e^3}
 \end{aligned}$$

input `Int[(a^2 + 2*a*b*x + b^2*x^2)/(d + e*x)^(3/2),x]`

output `(-2*(b*d - a*e)^2)/(e^3*Sqrt[d + e*x]) - (4*b*(b*d - a*e)*Sqrt[d + e*x])/e^3 + (2*b^2*(d + e*x)^(3/2))/(3*e^3)`

Defintions of rubi rules used

rule 27 $\text{Int}[(a_*)(Fx_), x_Symbol] \rightarrow \text{Simp}[a \text{ Int}[Fx, x], x] /; \text{FreeQ}[a, x] \ \&\& \ !\text{MatchQ}[Fx, (b_*)(Gx_)] /; \text{FreeQ}[b, x]$

rule 53 $\text{Int}[(a_.) + (b_.)*(x_))^{(m_.)}*((c_.) + (d_.)*(x_))^{(n_.)}, x_Symbol] \rightarrow \text{Int}[\text{ExpandIntegrand}[(a + b*x)^m*(c + d*x)^n, x], x] /; \text{FreeQ}[\{a, b, c, d, n\}, x] \ \&\& \ \text{IGtQ}[m, 0] \ \&\& \ (\ !\text{IntegerQ}[n] \ || \ (\text{EqQ}[c, 0] \ \&\& \ \text{LeQ}[7*m + 4*n + 4, 0]) \ || \ \text{LtQ}[9*m + 5*(n + 1), 0] \ || \ \text{GtQ}[m + n + 2, 0])$

rule 1098 $\text{Int}[(d_.) + (e_.)*(x_))^{(m_.)}*((a_.) + (b_.)*(x_.) + (c_.)*(x_.)^2)^{(p_.)}, x_Symbol] \rightarrow \text{Simp}[1/c^p \text{ Int}[(d + e*x)^m*(b/2 + c*x)^(2*p), x], x] /; \text{FreeQ}[\{a, b, c, d, e, m\}, x] \ \&\& \ \text{EqQ}[b^2 - 4*a*c, 0] \ \&\& \ \text{IntegerQ}[p]$

rule 2009 $\text{Int}[u_, x_Symbol] \rightarrow \text{Simp}[\text{IntSum}[u, x], x] /; \text{SumQ}[u]$

Maple [A] (verified)

Time = 0.69 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.82

| method | result | size |
|-------------------|--|------|
| pseudoelliptic | $\frac{2(e^2x^2 - 4dex - 8d^2)b^2}{3\sqrt{ex+d}e^3} + 8e\left(\frac{ex}{2} + d\right)ab - 2e^2a^2$ | 55 |
| risch | $\frac{2b(bex + 6ae - 5bd)\sqrt{ex+d}}{3e^3} - \frac{2(e^2a^2 - 2abde + b^2d^2)}{e^3\sqrt{ex+d}}$ | 61 |
| gosper | $-\frac{2(-b^2e^2x^2 - 6xab e^2 + 4b^2dex + 3e^2a^2 - 12abde + 8b^2d^2)}{3\sqrt{ex+d}e^3}$ | 63 |
| trager | $-\frac{2(-b^2e^2x^2 - 6xab e^2 + 4b^2dex + 3e^2a^2 - 12abde + 8b^2d^2)}{3\sqrt{ex+d}e^3}$ | 63 |
| derivativedivides | $\frac{2(ex+d)^{\frac{3}{2}}b^2}{3} + 4abe\sqrt{ex+d} - 4b^2d\sqrt{ex+d} - \frac{2(e^2a^2 - 2abde + b^2d^2)}{\sqrt{ex+d}}$ | 74 |
| default | $\frac{2(ex+d)^{\frac{3}{2}}b^2}{3} + 4abe\sqrt{ex+d} - 4b^2d\sqrt{ex+d} - \frac{2(e^2a^2 - 2abde + b^2d^2)}{\sqrt{ex+d}}$ | 74 |
| orering | $-\frac{2(-b^2e^2x^2 - 6xab e^2 + 4b^2dex + 3e^2a^2 - 12abde + 8b^2d^2)(b^2x^2 + 2abx + a^2)}{3e^3(bx+a)^2\sqrt{ex+d}}$ | 86 |

input `int((b^2*x^2+2*a*b*x+a^2)/(e*x+d)^(3/2),x,method=_RETURNVERBOSE)`

output `2/3*((e^2*x^2-4*d*e*x-8*d^2)*b^2+12*e*(1/2*e*x+d)*a*b-3*e^2*a^2)/(e*x+d)^(1/2)/e^3`

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.09

$$\int \frac{a^2 + 2abx + b^2x^2}{(d + ex)^{3/2}} dx = \frac{2(b^2e^2x^2 - 8b^2d^2 + 12abde - 3a^2e^2 - 2(2b^2de - 3abe^2)x)\sqrt{ex + d}}{3(e^4x + de^3)}$$

input `integrate((b^2*x^2+2*a*b*x+a^2)/(e*x+d)^(3/2),x, algorithm="fricas")`

output `2/3*(b^2*e^2*x^2 - 8*b^2*d^2 + 12*a*b*d*e - 3*a^2*e^2 - 2*(2*b^2*d*e - 3*a*b*e^2)*x)*sqrt(e*x + d)/(e^4*x + d*e^3)`

Sympy [A] (verification not implemented)

Time = 1.21 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.40

$$\int \frac{a^2 + 2abx + b^2x^2}{(d + ex)^{3/2}} dx = \begin{cases} \frac{2\left(\frac{b^2(d+ex)^{\frac{3}{2}}}{3e^2} + \frac{\sqrt{d+ex}(2abe-2b^2d)}{e^2} - \frac{(ae-bd)^2}{e^2\sqrt{d+ex}}\right)}{e} & \text{for } e \neq 0 \\ \frac{a^2x+abx^2+\frac{b^2x^3}{3}}{d^{\frac{3}{2}}} & \text{otherwise} \end{cases}$$

input `integrate((b**2*x**2+2*a*b*x+a**2)/(e*x+d)**(3/2),x)`

output `Piecewise((2*(b**2*(d + e*x)**(3/2)/(3*e**2) + sqrt(d + e*x)*(2*a*b*e - 2*b**2*d)/e**2 - (a*e - b*d)**2/(e**2*sqrt(d + e*x)))/e, Ne(e, 0)), ((a**2*x + a*b*x**2 + b**2*x**3/3)/d**(3/2), True))`

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.12

$$\int \frac{a^2 + 2abx + b^2x^2}{(d + ex)^{3/2}} dx = \frac{2 \left(\frac{(ex+d)^{\frac{3}{2}} b^2 - 6(b^2d - abe)\sqrt{ex+d}}{e^2} - \frac{3(b^2d^2 - 2abde + a^2e^2)}{\sqrt{ex+de^2}} \right)}{3e}$$

input `integrate((b^2*x^2+2*a*b*x+a^2)/(e*x+d)^(3/2),x, algorithm="maxima")`

output `2/3*(((e*x + d)^(3/2)*b^2 - 6*(b^2*d - a*b*e)*sqrt(e*x + d))/e^2 - 3*(b^2*d^2 - 2*a*b*d*e + a^2*e^2)/(sqrt(e*x + d)*e^2))/e`

Giac [A] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 84, normalized size of antiderivative = 1.25

$$\int \frac{a^2 + 2abx + b^2x^2}{(d + ex)^{3/2}} dx = -\frac{2(b^2d^2 - 2abde + a^2e^2)}{\sqrt{ex + d}e^3} + \frac{2 \left((ex + d)^{\frac{3}{2}} b^2 e^6 - 6\sqrt{ex + d} b^2 d e^6 + 6\sqrt{ex + d} a b e^7 \right)}{3e^9}$$

input `integrate((b^2*x^2+2*a*b*x+a^2)/(e*x+d)^(3/2),x, algorithm="giac")`

output `-2*(b^2*d^2 - 2*a*b*d*e + a^2*e^2)/(sqrt(e*x + d)*e^3) + 2/3*((e*x + d)^(3/2)*b^2*e^6 - 6*sqrt(e*x + d)*b^2*d*e^6 + 6*sqrt(e*x + d)*a*b*e^7)/e^9`

Mupad [B] (verification not implemented)

Time = 9.43 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.00

$$\int \frac{a^2 + 2abx + b^2x^2}{(d + ex)^{3/2}} dx = \frac{\frac{2b^2(d+ex)^2}{3} - 2a^2e^2 - 2b^2d^2 - 4b^2d(d+ex) + 4abe(d+ex) + 4abde}{e^3\sqrt{d+ex}}$$

input `int((a^2 + b^2*x^2 + 2*a*b*x)/(d + e*x)^(3/2),x)`output `((2*b^2*(d + e*x)^2)/3 - 2*a^2*e^2 - 2*b^2*d^2 - 4*b^2*d*(d + e*x) + 4*a*b*e*(d + e*x) + 4*a*b*d*e)/(e^3*(d + e*x)^(1/2))`**Reduce [B] (verification not implemented)**

Time = 0.26 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.93

$$\int \frac{a^2 + 2abx + b^2x^2}{(d + ex)^{3/2}} dx = \frac{\frac{2}{3}b^2e^2x^2 + 4abe^2x - \frac{8}{3}b^2dex - 2a^2e^2 + 8abde - \frac{16}{3}b^2d^2}{\sqrt{ex + d}e^3}$$

input `int((b^2*x^2+2*a*b*x+a^2)/(e*x+d)^(3/2),x)`output `(2*(- 3*a**2*e**2 + 12*a*b*d*e + 6*a*b*e**2*x - 8*b**2*d**2 - 4*b**2*d*e*x + b**2*e**2*x**2))/(3*sqrt(d + e*x)*e**3)`

$$3.152 \quad \int \frac{a^2 + 2abx + b^2x^2}{(d+ex)^{5/2}} dx$$

| | |
|---|------|
| Optimal result | 1191 |
| Mathematica [A] (verified) | 1191 |
| Rubi [A] (verified) | 1192 |
| Maple [A] (verified) | 1193 |
| Fricas [A] (verification not implemented) | 1194 |
| Sympy [B] (verification not implemented) | 1194 |
| Maxima [A] (verification not implemented) | 1195 |
| Giac [A] (verification not implemented) | 1195 |
| Mupad [B] (verification not implemented) | 1196 |
| Reduce [B] (verification not implemented) | 1196 |

Optimal result

Integrand size = 26, antiderivative size = 67

$$\int \frac{a^2 + 2abx + b^2x^2}{(d+ex)^{5/2}} dx = -\frac{2(bd - ae)^2}{3e^3(d+ex)^{3/2}} + \frac{4b(bd - ae)}{e^3\sqrt{d+ex}} + \frac{2b^2\sqrt{d+ex}}{e^3}$$

output

```
-2/3*(-a*e+b*d)^2/e^3/(e*x+d)^(3/2)+4*b*(-a*e+b*d)/e^3/(e*x+d)^(1/2)+2*b^2
*(e*x+d)^(1/2)/e^3
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.93

$$\int \frac{a^2 + 2abx + b^2x^2}{(d+ex)^{5/2}} dx = \frac{-2a^2e^2 - 4abe(2d + 3ex) + 2b^2(8d^2 + 12dex + 3e^2x^2)}{3e^3(d+ex)^{3/2}}$$

input

```
Integrate[(a^2 + 2*a*b*x + b^2*x^2)/(d + e*x)^(5/2),x]
```

output

```
(-2*a^2*e^2 - 4*a*b*e*(2*d + 3*e*x) + 2*b^2*(8*d^2 + 12*d*e*x + 3*e^2*x^2)
)/(3*e^3*(d + e*x)^(3/2))
```

Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {1098, 27, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\begin{aligned}
 & \int \frac{a^2 + 2abx + b^2x^2}{(d + ex)^{5/2}} dx \\
 & \quad \downarrow \text{1098} \\
 & \int \frac{b^2(a+bx)^2}{(d+ex)^{5/2}} dx \\
 & \quad \downarrow \text{27} \\
 & \int \frac{(a + bx)^2}{(d + ex)^{5/2}} dx \\
 & \quad \downarrow \text{53} \\
 & \int \left(-\frac{2b(bd - ae)}{e^2(d + ex)^{3/2}} + \frac{(ae - bd)^2}{e^2(d + ex)^{5/2}} + \frac{b^2}{e^2\sqrt{d + ex}} \right) dx \\
 & \quad \downarrow \text{2009} \\
 & \frac{4b(bd - ae)}{e^3\sqrt{d + ex}} - \frac{2(bd - ae)^2}{3e^3(d + ex)^{3/2}} + \frac{2b^2\sqrt{d + ex}}{e^3}
 \end{aligned}$$

input `Int[(a^2 + 2*a*b*x + b^2*x^2)/(d + e*x)^(5/2),x]`

output `(-2*(b*d - a*e)^2)/(3*e^3*(d + e*x)^(3/2)) + (4*b*(b*d - a*e))/(e^3*Sqrt[d + e*x]) + (2*b^2*Sqrt[d + e*x])/e^3`

Defintions of rubi rules used

rule 27 `Int[(a_)*(Fx_), x_Symbol] := Simp[a Int[Fx, x], x] /; FreeQ[a, x] && !MatchQ[Fx, (b_)*(Gx_)] /; FreeQ[b, x]`

rule 53 `Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] := Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && IGtQ[m, 0] && (!IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])`

rule 1098 `Int[((d_.) + (e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] := Simp[1/c^p Int[(d + e*x)^m*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]`

rule 2009 `Int[u_, x_Symbol] := Simp[IntSum[u, x], x] /; SumQ[u]`

Maple [A] (verified)

Time = 0.68 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.75

| method | result | size |
|-------------------|--|------|
| risch | $\frac{2b^2\sqrt{ex+d}}{e^3} - \frac{2(6bex+ae+5bd)(ae-bd)}{3e^3(ex+d)^{\frac{3}{2}}}$ | 50 |
| pseudoelliptic | $-\frac{2((-3b^2x^2+6abx+a^2)e^2+4bd(-3bx+a)e-8b^2d^2)}{3(ex+d)^{\frac{3}{2}}e^3}$ | 54 |
| gosper | $-\frac{2(-3b^2e^2x^2+6xab e^2-12b^2dex+e^2a^2+4abde-8b^2d^2)}{3(ex+d)^{\frac{3}{2}}e^3}$ | 62 |
| trager | $-\frac{2(-3b^2e^2x^2+6xab e^2-12b^2dex+e^2a^2+4abde-8b^2d^2)}{3(ex+d)^{\frac{3}{2}}e^3}$ | 62 |
| derivativedivides | $\frac{2b^2\sqrt{ex+d}-\frac{4b(ae-bd)}{\sqrt{ex+d}}-\frac{2(e^2a^2-2abde+b^2d^2)}{3(ex+d)^{\frac{3}{2}}}}{e^3}$ | 66 |
| default | $\frac{2b^2\sqrt{ex+d}-\frac{4b(ae-bd)}{\sqrt{ex+d}}-\frac{2(e^2a^2-2abde+b^2d^2)}{3(ex+d)^{\frac{3}{2}}}}{e^3}$ | 66 |
| orering | $-\frac{2(-3b^2e^2x^2+6xab e^2-12b^2dex+e^2a^2+4abde-8b^2d^2)(b^2x^2+2abx+a^2)}{3e^3(bx+a)^2(ex+d)^{\frac{3}{2}}}$ | 85 |

input `int((b^2*x^2+2*a*b*x+a^2)/(e*x+d)^(5/2),x,method=_RETURNVERBOSE)`

output `2*b^2*(e*x+d)^(1/2)/e^3-2/3*(6*b*e*x+a*e+5*b*d)*(a*e-b*d)/e^3/(e*x+d)^(3/2)`

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.27

$$\int \frac{a^2 + 2abx + b^2x^2}{(d + ex)^{5/2}} dx = \frac{2(3b^2e^2x^2 + 8b^2d^2 - 4abde - a^2e^2 + 6(2b^2de - abe^2)x)\sqrt{ex + d}}{3(e^5x^2 + 2de^4x + d^2e^3)}$$

input `integrate((b^2*x^2+2*a*b*x+a^2)/(e*x+d)^(5/2),x, algorithm="fricas")`

output `2/3*(3*b^2*e^2*x^2 + 8*b^2*d^2 - 4*a*b*d*e - a^2*e^2 + 6*(2*b^2*d*e - a*b*e^2)*x)*sqrt(e*x + d)/(e^5*x^2 + 2*d*e^4*x + d^2*e^3)`

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 265 vs. 2(61) = 122.

Time = 0.31 (sec) , antiderivative size = 265, normalized size of antiderivative = 3.96

$$\int \frac{a^2 + 2abx + b^2x^2}{(d + ex)^{5/2}} dx = \left\{ \begin{array}{l} -\frac{2a^2e^2}{3de^3\sqrt{d+ex}+3e^4x\sqrt{d+ex}} - \frac{8abde}{3de^3\sqrt{d+ex}+3e^4x\sqrt{d+ex}} - \frac{12abe^2x}{3de^3\sqrt{d+ex}+3e^4x\sqrt{d+ex}} + \frac{16b^2d^2}{3de^3\sqrt{d+ex}} \\ \frac{a^2x+abx^2+\frac{b^2x^3}{3}}{d^{\frac{5}{2}}} \end{array} \right.$$

input `integrate((b**2*x**2+2*a*b*x+a**2)/(e*x+d)**(5/2),x)`

output

```
Piecewise((-2*a**2*e**2/(3*d*e**3*sqrt(d + e*x) + 3*e**4*x*sqrt(d + e*x))
- 8*a*b*d*e/(3*d*e**3*sqrt(d + e*x) + 3*e**4*x*sqrt(d + e*x)) - 12*a*b*e**
2*x/(3*d*e**3*sqrt(d + e*x) + 3*e**4*x*sqrt(d + e*x)) + 16*b**2*d**2/(3*d*
e**3*sqrt(d + e*x) + 3*e**4*x*sqrt(d + e*x)) + 24*b**2*d*e*x/(3*d*e**3*sq
r
t(d + e*x) + 3*e**4*x*sqrt(d + e*x)) + 6*b**2*e**2*x**2/(3*d*e**3*sqrt(d +
e*x) + 3*e**4*x*sqrt(d + e*x)), Ne(e, 0)), ((a**2*x + a*b*x**2 + b**2*x**
3/3)/d**(5/2), True))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.07

$$\int \frac{a^2 + 2abx + b^2x^2}{(d + ex)^{5/2}} dx = \frac{2 \left(\frac{3\sqrt{ex+db^2}}{e^2} - \frac{b^2d^2 - 2abde + a^2e^2 - 6(b^2d - abe)(ex+d)}{(ex+d)^{\frac{3}{2}}e^2} \right)}{3e}$$

input

```
integrate((b^2*x^2+2*a*b*x+a^2)/(e*x+d)^(5/2),x, algorithm="maxima")
```

output

```
2/3*(3*sqrt(e*x + d)*b^2/e^2 - (b^2*d^2 - 2*a*b*d*e + a^2*e^2 - 6*(b^2*d -
a*b*e)*(e*x + d))/((e*x + d)^(3/2)*e^2))/e
```

Giac [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.07

$$\int \frac{a^2 + 2abx + b^2x^2}{(d + ex)^{5/2}} dx = \frac{2\sqrt{ex + db^2}}{e^3} + \frac{2(6(ex + d)b^2d - b^2d^2 - 6(ex + d)abe + 2abde - a^2e^2)}{3(ex + d)^{\frac{3}{2}}e^3}$$

input

```
integrate((b^2*x^2+2*a*b*x+a^2)/(e*x+d)^(5/2),x, algorithm="giac")
```

output

```
2*sqrt(e*x + d)*b^2/e^3 + 2/3*(6*(e*x + d)*b^2*d - b^2*d^2 - 6*(e*x + d)*a
*b*e + 2*a*b*d*e - a^2*e^2)/((e*x + d)^(3/2)*e^3)
```


Mupad [B] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.01

$$\int \frac{a^2 + 2abx + b^2x^2}{(d + ex)^{5/2}} dx = \frac{6b^2(d + ex)^2 - 2a^2e^2 - 2b^2d^2 + 12b^2d(d + ex) - 12abe(d + ex) + 4abde}{3e^3(d + ex)^{3/2}}$$

input `int((a^2 + b^2*x^2 + 2*a*b*x)/(d + e*x)^(5/2),x)`output `(6*b^2*(d + e*x)^2 - 2*a^2*e^2 - 2*b^2*d^2 + 12*b^2*d*(d + e*x) - 12*a*b*e*(d + e*x) + 4*a*b*d*e)/(3*e^3*(d + e*x)^(3/2))`**Reduce [B] (verification not implemented)**

Time = 0.21 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.04

$$\int \frac{a^2 + 2abx + b^2x^2}{(d + ex)^{5/2}} dx = \frac{2b^2e^2x^2 - 4abe^2x + 8b^2dex - \frac{2}{3}a^2e^2 - \frac{8}{3}abde + \frac{16}{3}b^2d^2}{\sqrt{ex + d}e^3(ex + d)}$$

input `int((b^2*x^2+2*a*b*x+a^2)/(e*x+d)^(5/2),x)`output `(2*(- a**2*e**2 - 4*a*b*d*e - 6*a*b*e**2*x + 8*b**2*d**2 + 12*b**2*d*e*x + 3*b**2*e**2*x**2))/(3*sqrt(d + e*x)*e**3*(d + e*x))`

$$3.153 \quad \int \frac{a^2 + 2abx + b^2x^2}{(d+ex)^{7/2}} dx$$

| | |
|---|------|
| Optimal result | 1197 |
| Mathematica [A] (verified) | 1197 |
| Rubi [A] (verified) | 1198 |
| Maple [A] (verified) | 1199 |
| Fricas [A] (verification not implemented) | 1200 |
| Sympy [B] (verification not implemented) | 1200 |
| Maxima [A] (verification not implemented) | 1201 |
| Giac [A] (verification not implemented) | 1201 |
| Mupad [B] (verification not implemented) | 1202 |
| Reduce [B] (verification not implemented) | 1202 |

Optimal result

Integrand size = 26, antiderivative size = 69

$$\int \frac{a^2 + 2abx + b^2x^2}{(d+ex)^{7/2}} dx = -\frac{2(bd-ae)^2}{5e^3(d+ex)^{5/2}} + \frac{4b(bd-ae)}{3e^3(d+ex)^{3/2}} - \frac{2b^2}{e^3\sqrt{d+ex}}$$

output

```
-2/5*(-a*e+b*d)^2/e^3/(e*x+d)^(5/2)+4/3*b*(-a*e+b*d)/e^3/(e*x+d)^(3/2)-2*b^2/e^3/(e*x+d)^(1/2)
```

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.88

$$\int \frac{a^2 + 2abx + b^2x^2}{(d+ex)^{7/2}} dx = -\frac{2(3a^2e^2 + 2abe(2d+5ex) + b^2(8d^2 + 20dex + 15e^2x^2))}{15e^3(d+ex)^{5/2}}$$

input

```
Integrate[(a^2 + 2*a*b*x + b^2*x^2)/(d + e*x)^(7/2),x]
```

output

```
(-2*(3*a^2*e^2 + 2*a*b*e*(2*d + 5*e*x) + b^2*(8*d^2 + 20*d*e*x + 15*e^2*x^2))/(15*e^3*(d + e*x)^(5/2))
```

Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {1098, 27, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\begin{aligned}
 & \int \frac{a^2 + 2abx + b^2x^2}{(d + ex)^{7/2}} dx \\
 & \quad \downarrow \text{1098} \\
 & \int \frac{b^2(a+bx)^2}{(d+ex)^{7/2}} dx \\
 & \quad \downarrow \text{27} \\
 & \int \frac{(a + bx)^2}{(d + ex)^{7/2}} dx \\
 & \quad \downarrow \text{53} \\
 & \int \left(-\frac{2b(bd - ae)}{e^2(d + ex)^{5/2}} + \frac{(ae - bd)^2}{e^2(d + ex)^{7/2}} + \frac{b^2}{e^2(d + ex)^{3/2}} \right) dx \\
 & \quad \downarrow \text{2009} \\
 & \frac{4b(bd - ae)}{3e^3(d + ex)^{3/2}} - \frac{2(bd - ae)^2}{5e^3(d + ex)^{5/2}} - \frac{2b^2}{e^3\sqrt{d + ex}}
 \end{aligned}$$

input `Int[(a^2 + 2*a*b*x + b^2*x^2)/(d + e*x)^(7/2), x]`

output `(-2*(b*d - a*e)^2)/(5*e^3*(d + e*x)^(5/2)) + (4*b*(b*d - a*e))/(3*e^3*(d + e*x)^(3/2)) - (2*b^2)/(e^3*sqrt[d + e*x])`

Defintions of rubi rules used

rule 27 $\text{Int}[(a_*)(Fx_), x_Symbol] \rightarrow \text{Simp}[a \text{ Int}[Fx, x], x] /; \text{FreeQ}[a, x] \ \&\& \ !\text{MatchQ}[Fx, (b_*)(Gx_)] /; \text{FreeQ}[b, x]$

rule 53 $\text{Int}[((a_.) + (b_.)(x_))^{(m_.)}*((c_.) + (d_.)(x_))^{(n_.)}, x_Symbol] \rightarrow \text{Int}[\text{ExpandIntegrand}[(a + b*x)^m*(c + d*x)^n, x], x] /; \text{FreeQ}[\{a, b, c, d, n\}, x] \ \&\& \ \text{IGtQ}[m, 0] \ \&\& \ (\ !\text{IntegerQ}[n] \ || \ (\text{EqQ}[c, 0] \ \&\& \ \text{LeQ}[7*m + 4*n + 4, 0]) \ || \ \text{LtQ}[9*m + 5*(n + 1), 0] \ || \ \text{GtQ}[m + n + 2, 0])$

rule 1098 $\text{Int}[((d_.) + (e_.)(x_))^{(m_.)}*((a_) + (b_.)(x_) + (c_.)(x_)^2)^{(p_.)}, x_Symbol] \rightarrow \text{Simp}[1/c^p \text{ Int}[(d + e*x)^m*(b/2 + c*x)^(2*p), x], x] /; \text{FreeQ}[\{a, b, c, d, e, m\}, x] \ \&\& \ \text{EqQ}[b^2 - 4*a*c, 0] \ \&\& \ \text{IntegerQ}[p]$

rule 2009 $\text{Int}[u_, x_Symbol] \rightarrow \text{Simp}[\text{IntSum}[u, x], x] /; \text{SumQ}[u]$

Maple [A] (verified)

Time = 0.66 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.81

| method | result | size |
|------------------|--|------|
| pseudoelliptic | $\frac{2(-15b^2x^2 - 10abx - 3a^2)e^2 - \frac{8bd(5bx+a)e}{15} - \frac{16b^2d^2}{15}}{(ex+d)^{\frac{5}{2}}e^3}$ | 56 |
| gospers | $-\frac{2(15b^2e^2x^2 + 10xabe^2 + 20b^2dex + 3e^2a^2 + 4abde + 8b^2d^2)}{15(ex+d)^{\frac{5}{2}}e^3}$ | 63 |
| trager | $-\frac{2(15b^2e^2x^2 + 10xabe^2 + 20b^2dex + 3e^2a^2 + 4abde + 8b^2d^2)}{15(ex+d)^{\frac{5}{2}}e^3}$ | 63 |
| derivativdivides | $-\frac{2(e^2a^2 - 2abde + b^2d^2)}{5(ex+d)^{\frac{5}{2}}} - \frac{2b^2}{\sqrt{ex+d}} - \frac{4b(ae-bd)}{3(ex+d)^{\frac{3}{2}}}$ | 67 |
| default | $-\frac{2(e^2a^2 - 2abde + b^2d^2)}{5(ex+d)^{\frac{5}{2}}} - \frac{2b^2}{\sqrt{ex+d}} - \frac{4b(ae-bd)}{3(ex+d)^{\frac{3}{2}}}$ | 67 |
| orering | $-\frac{2(15b^2e^2x^2 + 10xabe^2 + 20b^2dex + 3e^2a^2 + 4abde + 8b^2d^2)(b^2x^2 + 2abx + a^2)}{15e^3(bx+a)^2(ex+d)^{\frac{5}{2}}}$ | 86 |

input `int((b^2*x^2+2*a*b*x+a^2)/(e*x+d)^(7/2),x,method=_RETURNVERBOSE)`

output `2/15*((-15*b^2*x^2-10*a*b*x-3*a^2)*e^2-4*b*d*(5*b*x+a)*e-8*b^2*d^2)/(e*x+d)^(5/2)/e^3`

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.38

$$\int \frac{a^2 + 2abx + b^2x^2}{(d + ex)^{7/2}} dx = \frac{2(15b^2e^2x^2 + 8b^2d^2 + 4abde + 3a^2e^2 + 10(2b^2de + abe^2)x)\sqrt{ex + d}}{15(e^6x^3 + 3de^5x^2 + 3d^2e^4x + d^3e^3)}$$

input `integrate((b^2*x^2+2*a*b*x+a^2)/(e*x+d)^(7/2),x, algorithm="fricas")`

output `-2/15*(15*b^2*e^2*x^2 + 8*b^2*d^2 + 4*a*b*d*e + 3*a^2*e^2 + 10*(2*b^2*d*e + a*b*e^2)*x)*sqrt(e*x + d)/(e^6*x^3 + 3*d*e^5*x^2 + 3*d^2*e^4*x + d^3*e^3)`

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 389 vs. $2(63) = 126$.

Time = 0.44 (sec) , antiderivative size = 389, normalized size of antiderivative = 5.64

$$\int \frac{a^2 + 2abx + b^2x^2}{(d + ex)^{7/2}} dx = \left\{ \begin{array}{l} -\frac{6a^2e^2}{15d^2e^3\sqrt{d+ex}+30de^4x\sqrt{d+ex}+15e^5x^2\sqrt{d+ex}} - \frac{8abde}{15d^2e^3\sqrt{d+ex}+30de^4x\sqrt{d+ex}+15e^5x^2\sqrt{d+ex}} - \frac{1}{d^{7/2}} \\ \frac{a^2x+abx^2+\frac{b^2x^3}{3}}{d^{7/2}} \end{array} \right.$$

input `integrate((b**2*x**2+2*a*b*x+a**2)/(e*x+d)**(7/2),x)`

output

```
Piecewise((-6*a**2*e**2/(15*d**2*e**3*sqrt(d + e*x) + 30*d*e**4*x*sqrt(d + e*x) + 15*e**5*x**2*sqrt(d + e*x)) - 8*a*b*d*e/(15*d**2*e**3*sqrt(d + e*x) + 30*d*e**4*x*sqrt(d + e*x) + 15*e**5*x**2*sqrt(d + e*x)) - 20*a*b*e**2*x/(15*d**2*e**3*sqrt(d + e*x) + 30*d*e**4*x*sqrt(d + e*x) + 15*e**5*x**2*sqrt(d + e*x)) - 16*b**2*d**2/(15*d**2*e**3*sqrt(d + e*x) + 30*d*e**4*x*sqrt(d + e*x) + 15*e**5*x**2*sqrt(d + e*x)) - 40*b**2*d*e*x/(15*d**2*e**3*sqrt(d + e*x) + 30*d*e**4*x*sqrt(d + e*x) + 15*e**5*x**2*sqrt(d + e*x)) - 30*b**2*e**2*x**2/(15*d**2*e**3*sqrt(d + e*x) + 30*d*e**4*x*sqrt(d + e*x) + 15*e**5*x**2*sqrt(d + e*x)), Ne(e, 0)), ((a**2*x + a*b*x**2 + b**2*x**3/3)/d**(7/2), True))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.94

$$\int \frac{a^2 + 2abx + b^2x^2}{(d + ex)^{7/2}} dx = \frac{2(15(ex + d)^2b^2 + 3b^2d^2 - 6abde + 3a^2e^2 - 10(b^2d - abe)(ex + d))}{15(ex + d)^{5/2}e^3}$$

input

```
integrate((b^2*x^2+2*a*b*x+a^2)/(e*x+d)^(7/2),x, algorithm="maxima")
```

output

```
-2/15*(15*(e*x + d)^2*b^2 + 3*b^2*d^2 - 6*a*b*d*e + 3*a^2*e^2 - 10*(b^2*d - a*b*e)*(e*x + d))/((e*x + d)^(5/2)*e^3)
```

Giac [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.99

$$\int \frac{a^2 + 2abx + b^2x^2}{(d + ex)^{7/2}} dx = \frac{2(15(ex + d)^2b^2 - 10(ex + d)b^2d + 3b^2d^2 + 10(ex + d)abe - 6abde + 3a^2e^2)}{15(ex + d)^{5/2}e^3}$$

input

```
integrate((b^2*x^2+2*a*b*x+a^2)/(e*x+d)^(7/2),x, algorithm="giac")
```

output

$$-2/15*(15*(e*x + d)^2*b^2 - 10*(e*x + d)*b^2*d + 3*b^2*d^2 + 10*(e*x + d)*a*b*e - 6*a*b*d*e + 3*a^2*e^2)/((e*x + d)^(5/2)*e^3)$$

Mupad [B] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.90

$$\int \frac{a^2 + 2abx + b^2x^2}{(d + ex)^{7/2}} dx = \frac{6a^2e^2 + 8abde + 20abe^2x + 16b^2d^2 + 40b^2dex + 30b^2e^2x^2}{15e^3(d + ex)^{5/2}}$$

input

$$\text{int}((a^2 + b^2*x^2 + 2*a*b*x)/(d + e*x)^(7/2), x)$$

output

$$-(6*a^2*e^2 + 16*b^2*d^2 + 30*b^2*e^2*x^2 + 20*a*b*e^2*x + 40*b^2*d*e*x + 8*a*b*d*e)/(15*e^3*(d + e*x)^(5/2))$$

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.17

$$\int \frac{a^2 + 2abx + b^2x^2}{(d + ex)^{7/2}} dx = \frac{-2b^2e^2x^2 - \frac{4}{3}abe^2x - \frac{8}{3}b^2dex - \frac{2}{5}a^2e^2 - \frac{8}{15}abde - \frac{16}{15}b^2d^2}{\sqrt{ex + d}e^3(e^2x^2 + 2dex + d^2)}$$

input

$$\text{int}((b^2*x^2+2*a*b*x+a^2)/(e*x+d)^(7/2), x)$$

output

$$(2*(-3*a**2*e**2 - 4*a*b*d*e - 10*a*b*e**2*x - 8*b**2*d**2 - 20*b**2*d*e*x - 15*b**2*e**2*x**2))/(15*sqrt(d + e*x)*e**3*(d**2 + 2*d*e*x + e**2*x**2))$$

3.154 $\int (d + ex)^{7/2} (a^2 + 2abx + b^2x^2)^2 dx$

| | |
|---|------|
| Optimal result | 1203 |
| Mathematica [A] (verified) | 1203 |
| Rubi [A] (verified) | 1204 |
| Maple [A] (verified) | 1206 |
| Fricas [B] (verification not implemented) | 1206 |
| Sympy [B] (verification not implemented) | 1207 |
| Maxima [A] (verification not implemented) | 1208 |
| Giac [B] (verification not implemented) | 1209 |
| Mupad [B] (verification not implemented) | 1210 |
| Reduce [B] (verification not implemented) | 1210 |

Optimal result

Integrand size = 28, antiderivative size = 129

$$\int (d + ex)^{7/2} (a^2 + 2abx + b^2x^2)^2 dx = \frac{2(bd - ae)^4(d + ex)^{9/2}}{9e^5} - \frac{8b(bd - ae)^3(d + ex)^{11/2}}{11e^5} + \frac{12b^2(bd - ae)^2(d + ex)^{13/2}}{13e^5} - \frac{8b^3(bd - ae)(d + ex)^{15/2}}{15e^5} + \frac{2b^4(d + ex)^{17/2}}{17e^5}$$

output

```
2/9*(-a*e+b*d)^4*(e*x+d)^(9/2)/e^5-8/11*b*(-a*e+b*d)^3*(e*x+d)^(11/2)/e^5+
12/13*b^2*(-a*e+b*d)^2*(e*x+d)^(13/2)/e^5-8/15*b^3*(-a*e+b*d)*(e*x+d)^(15/
2)/e^5+2/17*b^4*(e*x+d)^(17/2)/e^5
```

Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 154, normalized size of antiderivative = 1.19

$$\int (d + ex)^{7/2} (a^2 + 2abx + b^2x^2)^2 dx = \frac{2(d + ex)^{9/2} (12155a^4e^4 + 4420a^3be^3(-2d + 9ex) + 510a^2b^2e^2(8d^2 - 36dex + 99e^2x^2) + 68a$$

input `Integrate[(d + e*x)^(7/2)*(a^2 + 2*a*b*x + b^2*x^2)^2,x]`

output $(2*(d + e*x)^{(9/2)}*(12155*a^4*e^4 + 4420*a^3*b*e^3*(-2*d + 9*e*x) + 510*a^2*b^2*e^2*(8*d^2 - 36*d*e*x + 99*e^2*x^2) + 68*a*b^3*e*(-16*d^3 + 72*d^2*e*x - 198*d*e^2*x^2 + 429*e^3*x^3) + b^4*(128*d^4 - 576*d^3*e*x + 1584*d^2*e^2*x^2 - 3432*d*e^3*x^3 + 6435*e^4*x^4)))/(109395*e^5)$

Rubi [A] (verified)

Time = 0.48 (sec) , antiderivative size = 129, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {1098, 27, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a^2 + 2abx + b^2x^2)^2 (d + ex)^{7/2} dx$$

$$\downarrow 1098$$

$$\frac{\int b^4(a + bx)^4(d + ex)^{7/2} dx}{b^4}$$

$$\downarrow 27$$

$$\int (a + bx)^4(d + ex)^{7/2} dx$$

$$\downarrow 53$$

$$\int \left(-\frac{4b^3(d + ex)^{13/2}(bd - ae)}{e^4} + \frac{6b^2(d + ex)^{11/2}(bd - ae)^2}{e^4} - \frac{4b(d + ex)^{9/2}(bd - ae)^3}{e^4} + \frac{(d + ex)^{7/2}(ae - bd)^4}{e^4} \right) dx$$

$$\downarrow 2009$$

$$-\frac{8b^3(d + ex)^{15/2}(bd - ae)}{15e^5} + \frac{12b^2(d + ex)^{13/2}(bd - ae)^2}{13e^5} - \frac{8b(d + ex)^{11/2}(bd - ae)^3}{11e^5} + \frac{2(d + ex)^{9/2}(bd - ae)^4}{9e^5} + \frac{2b^4(d + ex)^{17/2}}{17e^5}$$

input `Int[(d + e*x)^(7/2)*(a^2 + 2*a*b*x + b^2*x^2)^2,x]`

output `(2*(b*d - a*e)^4*(d + e*x)^(9/2))/(9*e^5) - (8*b*(b*d - a*e)^3*(d + e*x)^(11/2))/(11*e^5) + (12*b^2*(b*d - a*e)^2*(d + e*x)^(13/2))/(13*e^5) - (8*b^3*(b*d - a*e)*(d + e*x)^(15/2))/(15*e^5) + (2*b^4*(d + e*x)^(17/2))/(17*e^5)`

Defintions of rubi rules used

rule 27 `Int[(a_)*(Fx_), x_Symbol] := Simp[a Int[Fx, x], x] /; FreeQ[a, x] && !MatchQ[Fx, (b_)*(Gx_)] /; FreeQ[b, x]`

rule 53 `Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] := Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && IGtQ[m, 0] && (!IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])`

rule 1098 `Int[((d_.) + (e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] := Simp[1/c^p Int[(d + e*x)^m*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]`

rule 2009 `Int[u_, x_Symbol] := Simp[IntSum[u, x], x] /; SumQ[u]`

Maple [A] (verified)

Time = 1.40 (sec) , antiderivative size = 143, normalized size of antiderivative = 1.11

| method | result |
|-------------------|--|
| pseudoelliptic | $2 \left(\frac{\frac{9}{17} b^4 x^4 + \frac{12}{5} a b^3 x^3 + \frac{54}{13} a^2 b^2 x^2 + \frac{36}{11} a^3 b x + a^4}{e^4} - \frac{8bd \left(\frac{33}{85} b^3 x^3 + \frac{99}{65} a b^2 x^2 + \frac{27}{13} a^2 b x + a^3 \right) e^3}{11} + \frac{48b^2 \left(\frac{33}{85} b^2 x^2 + \frac{6}{5} a b x + a^2 \right) a}{143} \right) \frac{1}{9e^5}$ |
| derivativedivides | $\frac{\frac{2b^4(e^4 x^4 + d^4) \frac{17}{2}}{17} + \frac{4(2abe - 2b^2 d) b^2 (e^4 x^4 + d^4) \frac{15}{2}}{15}}{e^5} + \frac{2 \left(2(e^2 a^2 - 2abde + b^2 d^2) b^2 + (2abe - 2b^2 d)^2 \right) (e^4 x^4 + d^4) \frac{13}{2}}{13} + \frac{4(e^2 a^2 - 2abde + b^2 d^2) (2)}{11}$ |
| default | $\frac{\frac{2b^4(e^4 x^4 + d^4) \frac{17}{2}}{17} + \frac{4(2abe - 2b^2 d) b^2 (e^4 x^4 + d^4) \frac{15}{2}}{15}}{e^5} + \frac{2 \left(2(e^2 a^2 - 2abde + b^2 d^2) b^2 + (2abe - 2b^2 d)^2 \right) (e^4 x^4 + d^4) \frac{13}{2}}{13} + \frac{4(e^2 a^2 - 2abde + b^2 d^2) (2)}{11}$ |
| gospers | $\frac{2(e^4 x^4 + d^4)^{\frac{9}{2}} (6435b^4 x^4 e^4 + 29172x^3 a b^3 e^4 - 3432x^3 b^4 d e^3 + 50490x^2 a^2 b^2 e^4 - 13464x^2 a b^3 d e^3 + 1584x^2 b^4 d^2 e^2 + 39780x a^3 b e^4 - 18360x a^2 b^2 d e^3 + 12155a^4 d^4 e^4 + 12155a^4 d^4 e^4)}{1093}$ |
| orering | $2(6435b^4 x^4 e^4 + 29172x^3 a b^3 e^4 - 3432x^3 b^4 d e^3 + 50490x^2 a^2 b^2 e^4 - 13464x^2 a b^3 d e^3 + 1584x^2 b^4 d^2 e^2 + 39780x a^3 b e^4 - 18360x a^2 b^2 d e^3 + 12155a^4 d^4 e^4 + 12155a^4 d^4 e^4)$ |
| trager | $2(6435e^8 b^4 x^8 + 29172a b^3 e^8 x^7 + 22308b^4 d e^7 x^7 + 50490a^2 b^2 e^8 x^6 + 103224a b^3 d e^7 x^6 + 26466b^4 d^2 e^6 x^6 + 39780a^3 b e^8 x^5 + 12155a^4 d^4 e^4 + 12155a^4 d^4 e^4)$ |
| risch | $2(6435e^8 b^4 x^8 + 29172a b^3 e^8 x^7 + 22308b^4 d e^7 x^7 + 50490a^2 b^2 e^8 x^6 + 103224a b^3 d e^7 x^6 + 26466b^4 d^2 e^6 x^6 + 39780a^3 b e^8 x^5 + 12155a^4 d^4 e^4 + 12155a^4 d^4 e^4)$ |

input `int((e*x+d)^(7/2)*(b^2*x^2+2*a*b*x+a^2)^2,x,method=_RETURNVERBOSE)`

output
$$\frac{2}{9} \left(\frac{9}{17} b^4 x^4 + \frac{12}{5} a b^3 x^3 + \frac{54}{13} a^2 b^2 x^2 + \frac{36}{11} a^3 b x + a^4 \right) e^4 - \frac{8}{11} b d \left(\frac{33}{85} b^3 x^3 + \frac{99}{65} a b^2 x^2 + \frac{27}{13} a^2 b x + a^3 \right) e^3 + \frac{48}{143} b^2 \left(\frac{33}{85} b^2 x^2 + \frac{6}{5} a b x + a^2 \right) a e^2 - \frac{64}{715} b^3 d^3 \left(\frac{9}{17} b^2 x^2 + 2 a b x + a^2 \right) e^2 - \frac{64}{715} b^3 d^3 \left(\frac{9}{17} b^2 x^2 + 2 a b x + a^2 \right) e^2 + \frac{128}{12155} b^4 d^4 (e^4 x^4 + d^4) e^5$$

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 444 vs. 2(109) = 218.

Time = 0.10 (sec) , antiderivative size = 444, normalized size of antiderivative = 3.44

$$\int (d + ex)^{7/2} (a^2 + 2abx + b^2 x^2)^2 dx = \frac{2(6435b^4 e^8 x^8 + 128b^4 d^8 - 1088ab^3 d^7 e + 4080a^2 b^2 d^6 e^2 - 8840a^3 b d^5 e^3 + 12155a^4 d^4 e^4 + 12155a^4 d^4 e^4)}{1093}$$

input `integrate((e*x+d)^(7/2)*(b^2*x^2+2*a*b*x+a^2)^2,x, algorithm="fricas")`

output

```
2/109395*(6435*b^4*e^8*x^8 + 128*b^4*d^8 - 1088*a*b^3*d^7*e + 4080*a^2*b^2
*d^6*e^2 - 8840*a^3*b*d^5*e^3 + 12155*a^4*d^4*e^4 + 1716*(13*b^4*d*e^7 + 1
7*a*b^3*e^8)*x^7 + 66*(401*b^4*d^2*e^6 + 1564*a*b^3*d*e^7 + 765*a^2*b^2*e^
8)*x^6 + 36*(303*b^4*d^3*e^5 + 3502*a*b^3*d^2*e^6 + 5100*a^2*b^2*d*e^7 + 1
105*a^3*b*e^8)*x^5 + 5*(7*b^4*d^4*e^4 + 10880*a*b^3*d^3*e^5 + 46716*a^2*b^
2*d^2*e^6 + 30056*a^3*b*d*e^7 + 2431*a^4*e^8)*x^4 - 20*(2*b^4*d^5*e^3 - 17
*a*b^3*d^4*e^4 - 5406*a^2*b^2*d^3*e^5 - 10166*a^3*b*d^2*e^6 - 2431*a^4*d*e
^7)*x^3 + 6*(8*b^4*d^6*e^2 - 68*a*b^3*d^5*e^3 + 255*a^2*b^2*d^4*e^4 + 1768
0*a^3*b*d^3*e^5 + 12155*a^4*d^2*e^6)*x^2 - 4*(16*b^4*d^7*e - 136*a*b^3*d^6
*e^2 + 510*a^2*b^2*d^5*e^3 - 1105*a^3*b*d^4*e^4 - 12155*a^4*d^3*e^5)*x)*sq
rt(e*x + d)/e^5
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 903 vs. $2(119) = 238$.

Time = 0.67 (sec) , antiderivative size = 903, normalized size of antiderivative = 7.00

$$\int (d + ex)^{7/2} (a^2 + 2abx + b^2x^2)^2 dx = \text{Too large to display}$$

input

```
integrate((e*x+d)**(7/2)*(b**2*x**2+2*a*b*x+a**2)**2,x)
```

output

```
Piecewise((2*a**4*d**4*sqrt(d + e*x)/(9*e) + 8*a**4*d**3*x*sqrt(d + e*x)/9
+ 4*a**4*d**2*e*x**2*sqrt(d + e*x)/3 + 8*a**4*d*e**2*x**3*sqrt(d + e*x)/9
+ 2*a**4*e**3*x**4*sqrt(d + e*x)/9 - 16*a**3*b*d**5*sqrt(d + e*x)/(99*e**
2) + 8*a**3*b*d**4*x*sqrt(d + e*x)/(99*e) + 64*a**3*b*d**3*x**2*sqrt(d + e
*x)/33 + 368*a**3*b*d**2*e*x**3*sqrt(d + e*x)/99 + 272*a**3*b*d*e**2*x**4*
sqrt(d + e*x)/99 + 8*a**3*b*e**3*x**5*sqrt(d + e*x)/11 + 32*a**2*b**2*d**6
*sqrt(d + e*x)/(429*e**3) - 16*a**2*b**2*d**5*x*sqrt(d + e*x)/(429*e**2) +
4*a**2*b**2*d**4*x**2*sqrt(d + e*x)/(143*e) + 848*a**2*b**2*d**3*x**3*sqr
t(d + e*x)/429 + 1832*a**2*b**2*d**2*e*x**4*sqrt(d + e*x)/429 + 480*a**2*b
**2*d*e**2*x**5*sqrt(d + e*x)/143 + 12*a**2*b**2*e**3*x**6*sqrt(d + e*x)/1
3 - 128*a*b**3*d**7*sqrt(d + e*x)/(6435*e**4) + 64*a*b**3*d**6*x*sqrt(d +
e*x)/(6435*e**3) - 16*a*b**3*d**5*x**2*sqrt(d + e*x)/(2145*e**2) + 8*a*b**
3*d**4*x**3*sqrt(d + e*x)/(1287*e) + 1280*a*b**3*d**3*x**4*sqrt(d + e*x)/1
287 + 1648*a*b**3*d**2*e*x**5*sqrt(d + e*x)/715 + 368*a*b**3*d*e**2*x**6*s
qrt(d + e*x)/195 + 8*a*b**3*e**3*x**7*sqrt(d + e*x)/15 + 256*b**4*d**8*sqr
t(d + e*x)/(109395*e**5) - 128*b**4*d**7*x*sqrt(d + e*x)/(109395*e**4) + 3
2*b**4*d**6*x**2*sqrt(d + e*x)/(36465*e**3) - 16*b**4*d**5*x**3*sqrt(d + e
*x)/(21879*e**2) + 14*b**4*d**4*x**4*sqrt(d + e*x)/(21879*e) + 2424*b**4*d
**3*x**5*sqrt(d + e*x)/12155 + 1604*b**4*d**2*e*x**6*sqrt(d + e*x)/3315 +
104*b**4*d*e**2*x**7*sqrt(d + e*x)/255 + 2*b**4*e**3*x**8*sqrt(d + e*x)...
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 181, normalized size of antiderivative = 1.40

$$\int (d + ex)^{7/2} (a^2 + 2abx + b^2x^2)^2 dx = \frac{2 \left(6435 (ex + d)^{17/2} b^4 - 29172 (b^4d - ab^3e)(ex + d)^{15/2} + 50490 (b^4d^2 - 2ab^3de + a^2b^2e^2)(ex + d)^{13/2} - 39780 (b^4d^3 - 3a^2b^3d^2e + 3a^2b^2d^2e^2 - a^3b^3e^3)(ex + d)^{11/2} + 12155 (b^4d^4 - 4a^2b^3d^3e + 6a^2b^2d^2e^2 - 4a^3b^3d^2e^2 - 4a^4e^4)(ex + d)^{9/2} \right)}{e^5}$$

input

```
integrate((e*x+d)^(7/2)*(b^2*x^2+2*a*b*x+a^2)^2,x, algorithm="maxima")
```

output

```
2/109395*(6435*(e*x + d)^(17/2)*b^4 - 29172*(b^4*d - a*b^3*e)*(e*x + d)^(1
5/2) + 50490*(b^4*d^2 - 2*a*b^3*d*e + a^2*b^2*e^2)*(e*x + d)^(13/2) - 3978
0*(b^4*d^3 - 3*a*b^3*d^2*e + 3*a^2*b^2*d^2*e^2 - a^3*b^3*e^3)*(e*x + d)^(11/2)
+ 12155*(b^4*d^4 - 4*a*b^3*d^3*e + 6*a^2*b^2*d^2*e^2 - 4*a^3*b^3*d^2*e^2 + a^
4*e^4)*(e*x + d)^(9/2))/e^5
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1661 vs. $2(109) = 218$.

Time = 0.18 (sec) , antiderivative size = 1661, normalized size of antiderivative = 12.88

$$\int (d + ex)^{7/2} (a^2 + 2abx + b^2x^2)^2 dx = \text{Too large to display}$$

input `integrate((e*x+d)^(7/2)*(b^2*x^2+2*a*b*x+a^2)^2,x, algorithm="giac")`

output

```
2/765765*(765765*sqrt(e*x + d)*a^4*d^4 + 1021020*((e*x + d)^(3/2) - 3*sqrt
(e*x + d)*d)*a^4*d^3 + 1021020*((e*x + d)^(3/2) - 3*sqrt(e*x + d)*d)*a^3*b
*d^4/e + 306306*(3*(e*x + d)^(5/2) - 10*(e*x + d)^(3/2)*d + 15*sqrt(e*x +
d)*d^2)*a^4*d^2 + 306306*(3*(e*x + d)^(5/2) - 10*(e*x + d)^(3/2)*d + 15*sq
rt(e*x + d)*d^2)*a^2*b^2*d^4/e^2 + 816816*(3*(e*x + d)^(5/2) - 10*(e*x + d
)^(3/2)*d + 15*sqrt(e*x + d)*d^2)*a^3*b*d^3/e + 87516*(5*(e*x + d)^(7/2) -
21*(e*x + d)^(5/2)*d + 35*(e*x + d)^(3/2)*d^2 - 35*sqrt(e*x + d)*d^3)*a^4
*d + 87516*(5*(e*x + d)^(7/2) - 21*(e*x + d)^(5/2)*d + 35*(e*x + d)^(3/2)*
d^2 - 35*sqrt(e*x + d)*d^3)*a*b^3*d^4/e^3 + 525096*(5*(e*x + d)^(7/2) - 21
*(e*x + d)^(5/2)*d + 35*(e*x + d)^(3/2)*d^2 - 35*sqrt(e*x + d)*d^3)*a^2*b^
2*d^3/e^2 + 525096*(5*(e*x + d)^(7/2) - 21*(e*x + d)^(5/2)*d + 35*(e*x + d
)^(3/2)*d^2 - 35*sqrt(e*x + d)*d^3)*a^3*b*d^2/e + 2431*(35*(e*x + d)^(9/2)
- 180*(e*x + d)^(7/2)*d + 378*(e*x + d)^(5/2)*d^2 - 420*(e*x + d)^(3/2)*d
^3 + 315*sqrt(e*x + d)*d^4)*a^4 + 2431*(35*(e*x + d)^(9/2) - 180*(e*x + d)
^(7/2)*d + 378*(e*x + d)^(5/2)*d^2 - 420*(e*x + d)^(3/2)*d^3 + 315*sqrt(e*
x + d)*d^4)*b^4*d^4/e^4 + 38896*(35*(e*x + d)^(9/2) - 180*(e*x + d)^(7/2)*
d + 378*(e*x + d)^(5/2)*d^2 - 420*(e*x + d)^(3/2)*d^3 + 315*sqrt(e*x + d)*
d^4)*a*b^3*d^3/e^3 + 87516*(35*(e*x + d)^(9/2) - 180*(e*x + d)^(7/2)*d + 3
78*(e*x + d)^(5/2)*d^2 - 420*(e*x + d)^(3/2)*d^3 + 315*sqrt(e*x + d)*d^4)*
a^2*b^2*d^2/e^2 + 38896*(35*(e*x + d)^(9/2) - 180*(e*x + d)^(7/2)*d + 3...
```

Mupad [B] (verification not implemented)

Time = 9.31 (sec) , antiderivative size = 112, normalized size of antiderivative = 0.87

$$\int (d+ex)^{7/2} (a^2+2abx+b^2x^2)^2 dx = \frac{2b^4(d+ex)^{17/2}}{17e^5} - \frac{(8b^4d-8ab^3e)(d+ex)^{15/2}}{15e^5} + \frac{2(ae-bd)^4(d+ex)^{9/2}}{9e^5} + \frac{12b^2(ae-bd)^2(d+ex)^{13/2}}{13e^5} + \frac{8b(ae-bd)^3(d+ex)^{11/2}}{11e^5}$$

input `int((d + e*x)^(7/2)*(a^2 + b^2*x^2 + 2*a*b*x)^2,x)`output `(2*b^4*(d + e*x)^(17/2))/(17*e^5) - ((8*b^4*d - 8*a*b^3*e)*(d + e*x)^(15/2))/(15*e^5) + (2*(a*e - b*d)^4*(d + e*x)^(9/2))/(9*e^5) + (12*b^2*(a*e - b*d)^2*(d + e*x)^(13/2))/(13*e^5) + (8*b*(a*e - b*d)^3*(d + e*x)^(11/2))/(11*e^5)`**Reduce [B] (verification not implemented)**

Time = 0.21 (sec) , antiderivative size = 480, normalized size of antiderivative = 3.72

$$\int (d+ex)^{7/2} (a^2+2abx+b^2x^2)^2 dx = \frac{2\sqrt{ex+d}(6435b^4e^8x^8 + 29172ab^3e^8x^7 + 22308b^4de^7x^7 + 50490a^2b^2e^8x^6 + 103224ab^3de^7x^6 + \dots)}{11e^5}$$

input `int((e*x+d)^(7/2)*(b^2*x^2+2*a*b*x+a^2)^2,x)`

output

```
(2*sqrt(d + e*x)*(12155*a**4*d**4*e**4 + 48620*a**4*d**3*e**5*x + 72930*a*
**4*d**2*e**6*x**2 + 48620*a**4*d*e**7*x**3 + 12155*a**4*e**8*x**4 - 8840*a
**3*b*d**5*e**3 + 4420*a**3*b*d**4*e**4*x + 106080*a**3*b*d**3*e**5*x**2 +
  203320*a**3*b*d**2*e**6*x**3 + 150280*a**3*b*d*e**7*x**4 + 39780*a**3*b*e
**8*x**5 + 4080*a**2*b**2*d**6*e**2 - 2040*a**2*b**2*d**5*e**3*x + 1530*a*
**2*b**2*d**4*e**4*x**2 + 108120*a**2*b**2*d**3*e**5*x**3 + 233580*a**2*b**
2*d**2*e**6*x**4 + 183600*a**2*b**2*d*e**7*x**5 + 50490*a**2*b**2*e**8*x**
6 - 1088*a*b**3*d**7*e + 544*a*b**3*d**6*e**2*x - 408*a*b**3*d**5*e**3*x**
2 + 340*a*b**3*d**4*e**4*x**3 + 54400*a*b**3*d**3*e**5*x**4 + 126072*a*b**
3*d**2*e**6*x**5 + 103224*a*b**3*d*e**7*x**6 + 29172*a*b**3*e**8*x**7 + 12
8*b**4*d**8 - 64*b**4*d**7*e*x + 48*b**4*d**6*e**2*x**2 - 40*b**4*d**5*e**
3*x**3 + 35*b**4*d**4*e**4*x**4 + 10908*b**4*d**3*e**5*x**5 + 26466*b**4*d
**2*e**6*x**6 + 22308*b**4*d*e**7*x**7 + 6435*b**4*e**8*x**8))/(109395*e**
5)
```


3.155 $\int (d + ex)^{5/2} (a^2 + 2abx + b^2x^2)^2 dx$

| | |
|---|------|
| Optimal result | 1212 |
| Mathematica [A] (verified) | 1212 |
| Rubi [A] (verified) | 1213 |
| Maple [A] (verified) | 1215 |
| Fricas [B] (verification not implemented) | 1215 |
| Sympy [B] (verification not implemented) | 1216 |
| Maxima [A] (verification not implemented) | 1217 |
| Giac [B] (verification not implemented) | 1217 |
| Mupad [B] (verification not implemented) | 1218 |
| Reduce [B] (verification not implemented) | 1219 |

Optimal result

Integrand size = 28, antiderivative size = 129

$$\int (d + ex)^{5/2} (a^2 + 2abx + b^2x^2)^2 dx = \frac{2(bd - ae)^4(d + ex)^{7/2}}{7e^5} - \frac{8b(bd - ae)^3(d + ex)^{9/2}}{9e^5} + \frac{12b^2(bd - ae)^2(d + ex)^{11/2}}{11e^5} - \frac{8b^3(bd - ae)(d + ex)^{13/2}}{13e^5} + \frac{2b^4(d + ex)^{15/2}}{15e^5}$$

output

$$\frac{2}{7}*(-a*e+b*d)^4*(e*x+d)^(7/2)/e^5-8/9*b*(-a*e+b*d)^3*(e*x+d)^(9/2)/e^5+12/11*b^2*(-a*e+b*d)^2*(e*x+d)^(11/2)/e^5-8/13*b^3*(-a*e+b*d)*(e*x+d)^(13/2)/e^5+2/15*b^4*(e*x+d)^(15/2)/e^5$$

Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 154, normalized size of antiderivative = 1.19

$$\int (d + ex)^{5/2} (a^2 + 2abx + b^2x^2)^2 dx = \frac{2(d + ex)^{7/2} (6435a^4e^4 + 2860a^3be^3(-2d + 7ex) + 390a^2b^2e^2(8d^2 - 28dex + 63e^2x^2) + 60ab^3e(8d^2 - 28dex + 63e^2x^2) + 60ab^4d^2)}{15e^5}$$

input `Integrate[(d + e*x)^(5/2)*(a^2 + 2*a*b*x + b^2*x^2)^2,x]`

output $(2*(d + e*x)^{(7/2)}*(6435*a^4*e^4 + 2860*a^3*b*e^3*(-2*d + 7*e*x) + 390*a^2*b^2*e^2*(8*d^2 - 28*d*e*x + 63*e^2*x^2) + 60*a*b^3*e*(-16*d^3 + 56*d^2*e*x - 126*d*e^2*x^2 + 231*e^3*x^3) + b^4*(128*d^4 - 448*d^3*e*x + 1008*d^2*e^2*x^2 - 1848*d*e^3*x^3 + 3003*e^4*x^4)))/(45045*e^5)$

Rubi [A] (verified)

Time = 0.46 (sec) , antiderivative size = 129, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {1098, 27, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\begin{aligned}
 & \int (a^2 + 2abx + b^2x^2)^2 (d + ex)^{5/2} dx \\
 & \quad \downarrow 1098 \\
 & \quad \int \frac{b^4(a + bx)^4(d + ex)^{5/2} dx}{b^4} \\
 & \quad \quad \downarrow 27 \\
 & \quad \quad \int (a + bx)^4(d + ex)^{5/2} dx \\
 & \quad \quad \quad \downarrow 53 \\
 & \quad \quad \quad \int \left(-\frac{4b^3(d + ex)^{11/2}(bd - ae)}{e^4} + \frac{6b^2(d + ex)^{9/2}(bd - ae)^2}{e^4} - \frac{4b(d + ex)^{7/2}(bd - ae)^3}{e^4} + \frac{(d + ex)^{5/2}(ae - bd)^4}{e^4} \right) \\
 & \quad \quad \quad \quad \downarrow 2009 \\
 & \quad \quad \quad \quad -\frac{8b^3(d + ex)^{13/2}(bd - ae)}{13e^5} + \frac{12b^2(d + ex)^{11/2}(bd - ae)^2}{11e^5} - \frac{8b(d + ex)^{9/2}(bd - ae)^3}{9e^5} + \\
 & \quad \quad \quad \quad \quad \frac{2(d + ex)^{7/2}(bd - ae)^4}{7e^5} + \frac{2b^4(d + ex)^{15/2}}{15e^5}
 \end{aligned}$$

input `Int[(d + e*x)^(5/2)*(a^2 + 2*a*b*x + b^2*x^2)^2,x]`

output `(2*(b*d - a*e)^4*(d + e*x)^(7/2))/(7*e^5) - (8*b*(b*d - a*e)^3*(d + e*x)^(9/2))/(9*e^5) + (12*b^2*(b*d - a*e)^2*(d + e*x)^(11/2))/(11*e^5) - (8*b^3*(b*d - a*e)*(d + e*x)^(13/2))/(13*e^5) + (2*b^4*(d + e*x)^(15/2))/(15*e^5)`

Defintions of rubi rules used

rule 27 `Int[(a_)*(F_x_), x_Symbol] := Simp[a Int[F_x, x], x] /; FreeQ[a, x] && !MatchQ[F_x, (b_)*(G_x_)] /; FreeQ[b, x]`

rule 53 `Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] := Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && IGtQ[m, 0] && (!IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])`

rule 1098 `Int[((d_.) + (e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] := Simp[1/c^p Int[(d + e*x)^m*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]`

rule 2009 `Int[u_, x_Symbol] := Simp[IntSum[u, x], x] /; SumQ[u]`

output

```
2/45045*(3003*b^4*e^7*x^7 + 128*b^4*d^7 - 960*a*b^3*d^6*e + 3120*a^2*b^2*d^5*e^2 - 5720*a^3*b*d^4*e^3 + 6435*a^4*d^3*e^4 + 231*(31*b^4*d*e^6 + 60*a*b^3*e^7)*x^6 + 63*(71*b^4*d^2*e^5 + 540*a*b^3*d*e^6 + 390*a^2*b^2*e^7)*x^5 + 35*(b^4*d^3*e^4 + 636*a*b^3*d^2*e^5 + 1794*a^2*b^2*d*e^6 + 572*a^3*b*e^7)*x^4 - 5*(8*b^4*d^4*e^3 - 60*a*b^3*d^3*e^4 - 8814*a^2*b^2*d^2*e^5 - 10868*a^3*b*d*e^6 - 1287*a^4*e^7)*x^3 + 3*(16*b^4*d^5*e^2 - 120*a*b^3*d^4*e^3 + 390*a^2*b^2*d^3*e^4 + 14300*a^3*b*d^2*e^5 + 6435*a^4*d*e^6)*x^2 - (64*b^4*d^6*e - 480*a*b^3*d^5*e^2 + 1560*a^2*b^2*d^4*e^3 - 2860*a^3*b*d^3*e^4 - 19305*a^4*d^2*e^5)*x)*sqrt(e*x + d)/e^5
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 272 vs. $2(119) = 238$.

Time = 1.11 (sec) , antiderivative size = 272, normalized size of antiderivative = 2.11

$$\int (d + ex)^{5/2} (a^2 + 2abx + b^2x^2)^2 dx = \frac{2 \left(\frac{b^4(d+ex)^{15/2}}{15e^4} + \frac{(d+ex)^{13/2} \cdot (4ab^3e - 4b^4d)}{13e^4} + \frac{(d+ex)^{11/2} \cdot (6a^2b^2e^2 - 12ab^3de + 6b^4d^2)}{11e^4} + \frac{(d+ex)^{9/2} \cdot (4a^3be^3 - 12a^2b^2de^2 + 12ab^3d^2e - 4b^4d^3)}{9e^4} \right)}{e} + \frac{d^{5/2} \left(a^4x + 2a^3bx^2 + 2a^2b^2x^3 + ab^3x^4 + \frac{b^4x^5}{5} \right)}{5}$$

input

```
integrate((e*x+d)**(5/2)*(b**2*x**2+2*a*b*x+a**2)**2,x)
```

output

```
Piecewise((2*(b**4*(d + e*x)**(15/2)/(15*e**4) + (d + e*x)**(13/2)*(4*a*b**3*e - 4*b**4*d)/(13*e**4) + (d + e*x)**(11/2)*(6*a**2*b**2*e**2 - 12*a*b**3*d*e + 6*b**4*d**2)/(11*e**4) + (d + e*x)**(9/2)*(4*a**3*b*e**3 - 12*a**2*b**2*d*e**2 + 12*a*b**3*d**2*e - 4*b**4*d**3)/(9*e**4) + (d + e*x)**(7/2)*(a**4*e**4 - 4*a**3*b*d*e**3 + 6*a**2*b**2*d**2*e**2 - 4*a*b**3*d**3*e + b**4*d**4)/(7*e**4))/e, Ne(e, 0)), (d**(5/2)*(a**4*x + 2*a**3*b*x**2 + 2*a**2*b**2*x**3 + a*b**3*x**4 + b**4*x**5/5), True))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 181, normalized size of antiderivative = 1.40

$$\int (d + ex)^{5/2} (a^2 + 2abx + b^2x^2)^2 dx = \frac{2 \left(3003 (ex + d)^{\frac{15}{2}} b^4 - 13860 (b^4d - ab^3e)(ex + d)^{\frac{13}{2}} + 24570 (b^4d^2 - 2ab^3de + a^2b^2e^2)(ex + d)^{\frac{11}{2}} - 20020 (b^4d^3 - 3a*b^3*d^2*e + 3*a^2*b^2*d*e^2 - a^3*b*e^3)(ex + d)^{\frac{9}{2}} + 6435 (b^4d^4 - 4a*b^3*d^3*e + 6*a^2*b^2*d^2*e^2 - 4*a^3*b*d*e^3 + a^4*e^4)(ex + d)^{\frac{7}{2}} \right)}{e^5}$$

input `integrate((e*x+d)^(5/2)*(b^2*x^2+2*a*b*x+a^2)^2,x, algorithm="maxima")`

output `2/45045*(3003*(e*x + d)^(15/2)*b^4 - 13860*(b^4*d - a*b^3*e)*(e*x + d)^(13/2) + 24570*(b^4*d^2 - 2*a*b^3*d*e + a^2*b^2*e^2)*(e*x + d)^(11/2) - 20020*(b^4*d^3 - 3*a*b^3*d^2*e + 3*a^2*b^2*d*e^2 - a^3*b*e^3)*(e*x + d)^(9/2) + 6435*(b^4*d^4 - 4*a*b^3*d^3*e + 6*a^2*b^2*d^2*e^2 - 4*a^3*b*d*e^3 + a^4*e^4)*(e*x + d)^(7/2))/e^5`

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1204 vs. 2(109) = 218.

Time = 0.23 (sec) , antiderivative size = 1204, normalized size of antiderivative = 9.33

$$\int (d + ex)^{5/2} (a^2 + 2abx + b^2x^2)^2 dx = \text{Too large to display}$$

input `integrate((e*x+d)^(5/2)*(b^2*x^2+2*a*b*x+a^2)^2,x, algorithm="giac")`

output

```

2/45045*(45045*sqrt(e*x + d)*a^4*d^3 + 45045*((e*x + d)^(3/2) - 3*sqrt(e*x
+ d)*d)*a^4*d^2 + 60060*((e*x + d)^(3/2) - 3*sqrt(e*x + d)*d)*a^3*b*d^3/e
+ 9009*(3*(e*x + d)^(5/2) - 10*(e*x + d)^(3/2)*d + 15*sqrt(e*x + d)*d^2)*
a^4*d + 18018*(3*(e*x + d)^(5/2) - 10*(e*x + d)^(3/2)*d + 15*sqrt(e*x + d)
*d^2)*a^2*b^2*d^3/e^2 + 36036*(3*(e*x + d)^(5/2) - 10*(e*x + d)^(3/2)*d +
15*sqrt(e*x + d)*d^2)*a^3*b*d^2/e + 1287*(5*(e*x + d)^(7/2) - 21*(e*x + d)
^(5/2)*d + 35*(e*x + d)^(3/2)*d^2 - 35*sqrt(e*x + d)*d^3)*a^4 + 5148*(5*(e
*x + d)^(7/2) - 21*(e*x + d)^(5/2)*d + 35*(e*x + d)^(3/2)*d^2 - 35*sqrt(e*
x + d)*d^3)*a*b^3*d^3/e^3 + 23166*(5*(e*x + d)^(7/2) - 21*(e*x + d)^(5/2)*
d + 35*(e*x + d)^(3/2)*d^2 - 35*sqrt(e*x + d)*d^3)*a^2*b^2*d^2/e^2 + 15444
*(5*(e*x + d)^(7/2) - 21*(e*x + d)^(5/2)*d + 35*(e*x + d)^(3/2)*d^2 - 35*s
qrt(e*x + d)*d^3)*a^3*b*d/e + 143*(35*(e*x + d)^(9/2) - 180*(e*x + d)^(7/2)
)*d + 378*(e*x + d)^(5/2)*d^2 - 420*(e*x + d)^(3/2)*d^3 + 315*sqrt(e*x + d)
*d^4)*b^4*d^3/e^4 + 1716*(35*(e*x + d)^(9/2) - 180*(e*x + d)^(7/2)*d + 37
8*(e*x + d)^(5/2)*d^2 - 420*(e*x + d)^(3/2)*d^3 + 315*sqrt(e*x + d)*d^4)*a
*b^3*d^2/e^3 + 2574*(35*(e*x + d)^(9/2) - 180*(e*x + d)^(7/2)*d + 378*(e*x
+ d)^(5/2)*d^2 - 420*(e*x + d)^(3/2)*d^3 + 315*sqrt(e*x + d)*d^4)*a^2*b^2
*d/e^2 + 572*(35*(e*x + d)^(9/2) - 180*(e*x + d)^(7/2)*d + 378*(e*x + d)^(
5/2)*d^2 - 420*(e*x + d)^(3/2)*d^3 + 315*sqrt(e*x + d)*d^4)*a^3*b/e + 195*
(63*(e*x + d)^(11/2) - 385*(e*x + d)^(9/2)*d + 990*(e*x + d)^(7/2)*d^2 ...

```

Mupad [B] (verification not implemented)

Time = 9.15 (sec) , antiderivative size = 112, normalized size of antiderivative = 0.87

$$\begin{aligned}
\int (d + ex)^{5/2} (a^2 + 2abx + b^2x^2)^2 dx &= \frac{2b^4(d + ex)^{15/2}}{15e^5} \\
&- \frac{(8b^4d - 8ab^3e)(d + ex)^{13/2}}{13e^5} + \frac{2(ae - bd)^4(d + ex)^{7/2}}{7e^5} \\
&+ \frac{12b^2(ae - bd)^2(d + ex)^{11/2}}{11e^5} + \frac{8b(ae - bd)^3(d + ex)^{9/2}}{9e^5}
\end{aligned}$$

input

```
int((d + e*x)^(5/2)*(a^2 + b^2*x^2 + 2*a*b*x)^2,x)
```

output

```

(2*b^4*(d + e*x)^(15/2))/(15*e^5) - ((8*b^4*d - 8*a*b^3*e)*(d + e*x)^(13/2)
)/(13*e^5) + (2*(a*e - b*d)^4*(d + e*x)^(7/2))/(7*e^5) + (12*b^2*(a*e - b
*d)^2*(d + e*x)^(11/2))/(11*e^5) + (8*b*(a*e - b*d)^3*(d + e*x)^(9/2))/(9*
e^5)

```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 405, normalized size of antiderivative = 3.14

$$\int (d + ex)^{5/2} (a^2 + 2abx + b^2x^2)^2 dx = \frac{2\sqrt{ex + d}(3003b^4e^7x^7 + 13860ab^3e^7x^6 + 7161b^4de^6x^6 + 24570a^2b^2e^7x^5 + 34020ab^3de^6x^5 + \dots)}{45045e^{5/2}}$$

input

```
int((e*x+d)^(5/2)*(b^2*x^2+2*a*b*x+a^2)^2,x)
```

output

```
(2*sqrt(d + e*x)*(6435*a**4*d**3*e**4 + 19305*a**4*d**2*e**5*x + 19305*a**4*d*e**6*x**2 + 6435*a**4*e**7*x**3 - 5720*a**3*b*d**4*e**3 + 2860*a**3*b*d**3*e**4*x + 42900*a**3*b*d**2*e**5*x**2 + 54340*a**3*b*d*e**6*x**3 + 20020*a**3*b*e**7*x**4 + 3120*a**2*b**2*d**5*e**2 - 1560*a**2*b**2*d**4*e**3*x + 1170*a**2*b**2*d**3*e**4*x**2 + 44070*a**2*b**2*d**2*e**5*x**3 + 62790*a**2*b**2*d*e**6*x**4 + 24570*a**2*b**2*e**7*x**5 - 960*a*b**3*d**6*e + 480*a*b**3*d**5*e**2*x - 360*a*b**3*d**4*e**3*x**2 + 300*a*b**3*d**3*e**4*x**3 + 22260*a*b**3*d**2*e**5*x**4 + 34020*a*b**3*d*e**6*x**5 + 13860*a*b**3*e**7*x**6 + 128*b**4*d**7 - 64*b**4*d**6*e*x + 48*b**4*d**5*e**2*x**2 - 40*b**4*d**4*e**3*x**3 + 35*b**4*d**3*e**4*x**4 + 4473*b**4*d**2*e**5*x**5 + 7161*b**4*d*e**6*x**6 + 3003*b**4*e**7*x**7))/(45045*e**5)
```


3.156 $\int (d + ex)^{3/2} (a^2 + 2abx + b^2x^2)^2 dx$

| | |
|---|------|
| Optimal result | 1220 |
| Mathematica [A] (verified) | 1220 |
| Rubi [A] (verified) | 1221 |
| Maple [A] (verified) | 1223 |
| Fricas [B] (verification not implemented) | 1223 |
| Sympy [B] (verification not implemented) | 1224 |
| Maxima [A] (verification not implemented) | 1225 |
| Giac [B] (verification not implemented) | 1225 |
| Mupad [B] (verification not implemented) | 1226 |
| Reduce [B] (verification not implemented) | 1227 |

Optimal result

Integrand size = 28, antiderivative size = 129

$$\int (d + ex)^{3/2} (a^2 + 2abx + b^2x^2)^2 dx = \frac{2(bd - ae)^4(d + ex)^{5/2}}{5e^5} - \frac{8b(bd - ae)^3(d + ex)^{7/2}}{7e^5} + \frac{4b^2(bd - ae)^2(d + ex)^{9/2}}{3e^5} - \frac{8b^3(bd - ae)(d + ex)^{11/2}}{11e^5} + \frac{2b^4(d + ex)^{13/2}}{13e^5}$$

output

$$\frac{2}{5}(-a+bd)^4(e^2x+d)^{5/2}/e^5 - \frac{8}{7}b(-a+bd)^3(e^2x+d)^{7/2}/e^5 + \frac{4}{3}b^2(-a+bd)^2(e^2x+d)^{9/2}/e^5 - \frac{8}{11}b^3(-a+bd)(e^2x+d)^{11/2}/e^5 + \frac{2}{13}b^4(e^2x+d)^{13/2}/e^5$$

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 154, normalized size of antiderivative = 1.19

$$\int (d + ex)^{3/2} (a^2 + 2abx + b^2x^2)^2 dx = \frac{2(d + ex)^{5/2} (3003a^4e^4 + 1716a^3be^3(-2d + 5ex) + 286a^2b^2e^2(8d^2 - 20dex + 35e^2x^2) + 52ab^3e(8d^2 - 20dex + 35e^2x^2) + 52ab^4d^2)}{11e^5}$$

input `Integrate[(d + e*x)^(3/2)*(a^2 + 2*a*b*x + b^2*x^2)^2,x]`

output $(2*(d + e*x)^{(5/2)}*(3003*a^4*e^4 + 1716*a^3*b*e^3*(-2*d + 5*e*x) + 286*a^2*b^2*e^2*(8*d^2 - 20*d*e*x + 35*e^2*x^2) + 52*a*b^3*e*(-16*d^3 + 40*d^2*e*x - 70*d*e^2*x^2 + 105*e^3*x^3) + b^4*(128*d^4 - 320*d^3*e*x + 560*d^2*e^2*x^2 - 840*d*e^3*x^3 + 1155*e^4*x^4)))/(15015*e^5)$

Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 129, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {1098, 27, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a^2 + 2abx + b^2x^2)^2 (d + ex)^{3/2} dx$$

$$\downarrow 1098$$

$$\frac{\int b^4(a + bx)^4(d + ex)^{3/2} dx}{b^4}$$

$$\downarrow 27$$

$$\int (a + bx)^4(d + ex)^{3/2} dx$$

$$\downarrow 53$$

$$\int \left(-\frac{4b^3(d + ex)^{9/2}(bd - ae)}{e^4} + \frac{6b^2(d + ex)^{7/2}(bd - ae)^2}{e^4} - \frac{4b(d + ex)^{5/2}(bd - ae)^3}{e^4} + \frac{(d + ex)^{3/2}(ae - bd)^4}{e^4} \right) dx$$

$$\downarrow 2009$$

$$-\frac{8b^3(d + ex)^{11/2}(bd - ae)}{11e^5} + \frac{4b^2(d + ex)^{9/2}(bd - ae)^2}{3e^5} - \frac{8b(d + ex)^{7/2}(bd - ae)^3}{7e^5} + \frac{2(d + ex)^{5/2}(bd - ae)^4}{5e^5} + \frac{2b^4(d + ex)^{13/2}}{13e^5}$$

input `Int[(d + e*x)^(3/2)*(a^2 + 2*a*b*x + b^2*x^2)^2,x]`

output `(2*(b*d - a*e)^4*(d + e*x)^(5/2))/(5*e^5) - (8*b*(b*d - a*e)^3*(d + e*x)^(7/2))/(7*e^5) + (4*b^2*(b*d - a*e)^2*(d + e*x)^(9/2))/(3*e^5) - (8*b^3*(b*d - a*e)*(d + e*x)^(11/2))/(11*e^5) + (2*b^4*(d + e*x)^(13/2))/(13*e^5)`

Defintions of rubi rules used

rule 27 `Int[(a_)*(Fx_), x_Symbol] := Simp[a Int[Fx, x], x] /; FreeQ[a, x] && !MatchQ[Fx, (b_)*(Gx_)] /; FreeQ[b, x]`

rule 53 `Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] := Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && IGtQ[m, 0] && (!IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])`

rule 1098 `Int[((d_.) + (e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] := Simp[1/c^p Int[(d + e*x)^m*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]`

rule 2009 `Int[u_, x_Symbol] := Simp[IntSum[u, x], x] /; SumQ[u]`

Maple [A] (verified)

Time = 1.18 (sec) , antiderivative size = 143, normalized size of antiderivative = 1.11

| method | result |
|-------------------|--|
| pseudoelliptic | $\frac{2(ex+d)^{\frac{5}{2}} \left(\left(\frac{5}{13} b^4 x^4 + \frac{20}{11} a b^3 x^3 + \frac{10}{3} a^2 b^2 x^2 + \frac{20}{7} a^3 b x + a^4 \right) e^4 - \frac{8 \left(\frac{35}{143} b^3 x^3 + \frac{35}{33} a b^2 x^2 + \frac{5}{3} a^2 b x + a^3 \right) b d e^3}{7} + \frac{16 b^2 d^2 \left(\frac{35}{143} b^2 x^2 + \frac{20}{11} a b x + a^2 \right) e^2}{7} \right)}{5e^5}$ |
| derivativedivides | $\frac{\frac{2b^4(ex+d)^{\frac{13}{2}}}{13} + \frac{4(2abe-2b^2d)b^2(ex+d)^{\frac{11}{2}}}{11} + \frac{2(2(e^2a^2-2abde+b^2d^2)b^2+(2abe-2b^2d)^2)(ex+d)^{\frac{9}{2}}}{9} + \frac{4(e^2a^2-2abde+b^2d^2)(2a^2b^2+2ab^2d)}{7}}{e^5}$ |
| default | $\frac{\frac{2b^4(ex+d)^{\frac{13}{2}}}{13} + \frac{4(2abe-2b^2d)b^2(ex+d)^{\frac{11}{2}}}{11} + \frac{2(2(e^2a^2-2abde+b^2d^2)b^2+(2abe-2b^2d)^2)(ex+d)^{\frac{9}{2}}}{9} + \frac{4(e^2a^2-2abde+b^2d^2)(2a^2b^2+2ab^2d)}{7}}{e^5}$ |
| gospers | $\frac{2(ex+d)^{\frac{5}{2}} (1155b^4x^4e^4+5460x^3ab^3e^4-840x^3b^4de^3+10010x^2a^2b^2e^4-3640x^2ab^3de^3+560x^2b^4d^2e^2+8580xa^3be^4-5720xa^2b^2de^3+14300a^2b^2d^2e)}{15015e^5}$ |
| orering | $\frac{2(1155b^4x^4e^4+5460x^3ab^3e^4-840x^3b^4de^3+10010x^2a^2b^2e^4-3640x^2ab^3de^3+560x^2b^4d^2e^2+8580xa^3be^4-5720xa^2b^2de^3+14300a^2b^2d^2e)}{15015e^5}$ |
| trager | $\frac{2(1155b^4e^6x^6+5460ab^3e^6x^5+1470b^4de^5x^5+10010a^2b^2e^6x^4+7280ab^3de^5x^4+35b^4d^2e^4x^4+8580a^3be^6x^3+14300a^2b^2d^2e^3+14300a^2b^2d^2e)}{15015e^5}$ |
| risch | $\frac{2(1155b^4e^6x^6+5460ab^3e^6x^5+1470b^4de^5x^5+10010a^2b^2e^6x^4+7280ab^3de^5x^4+35b^4d^2e^4x^4+8580a^3be^6x^3+14300a^2b^2d^2e^3+14300a^2b^2d^2e)}{15015e^5}$ |

input `int((e*x+d)^(3/2)*(b^2*x^2+2*a*b*x+a^2)^2,x,method=_RETURNVERBOSE)`

output `2/5*(e*x+d)^(5/2)*((5/13*b^4*x^4+20/11*a*b^3*x^3+10/3*a^2*b^2*x^2+20/7*a^3*b*x+a^4)*e^4-8/7*(35/143*b^3*x^3+35/33*a*b^2*x^2+5/3*a^2*b*x+a^3)*b*d*e^3+16/21*b^2*d^2*(35/143*b^2*x^2+10/11*a*b*x+a^2)*e^2-64/231*b^3*(5/13*b*x+a^2)*d^3*e+128/3003*b^4*d^4)/e^5`

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 311 vs. 2(109) = 218.

Time = 0.08 (sec) , antiderivative size = 311, normalized size of antiderivative = 2.41

$$\int (d+ex)^{3/2} (a^2+2abx+b^2x^2)^2 dx = \frac{2(1155b^4e^6x^6+128b^4d^6-832ab^3d^5e+2288a^2b^2d^4e^2-3432a^3bd^3e^3+3003a^4d^2e^4+2100a^5de^3+14300a^2b^2d^2e^3+14300a^2b^2d^2e)}{15015e^5}$$

input `integrate((e*x+d)^(3/2)*(b^2*x^2+2*a*b*x+a^2)^2,x, algorithm="fricas")`

output

```
2/15015*(1155*b^4*e^6*x^6 + 128*b^4*d^6 - 832*a*b^3*d^5*e + 2288*a^2*b^2*d^4*e^2 - 3432*a^3*b*d^3*e^3 + 3003*a^4*d^2*e^4 + 210*(7*b^4*d*e^5 + 26*a*b^3*e^6)*x^5 + 35*(b^4*d^2*e^4 + 208*a*b^3*d*e^5 + 286*a^2*b^2*e^6)*x^4 - 20*(2*b^4*d^3*e^3 - 13*a*b^3*d^2*e^4 - 715*a^2*b^2*d*e^5 - 429*a^3*b*e^6)*x^3 + 3*(16*b^4*d^4*e^2 - 104*a*b^3*d^3*e^3 + 286*a^2*b^2*d^2*e^4 + 4576*a^3*b*d*e^5 + 1001*a^4*e^6)*x^2 - 2*(32*b^4*d^5*e - 208*a*b^3*d^4*e^2 + 572*a^2*b^2*d^3*e^3 - 858*a^3*b*d^2*e^4 - 3003*a^4*d*e^5)*x)*sqrt(e*x + d)/e^5
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 272 vs. $2(119) = 238$.

Time = 1.07 (sec) , antiderivative size = 272, normalized size of antiderivative = 2.11

$$\int (d + ex)^{3/2} (a^2 + 2abx + b^2x^2)^2 dx = \frac{2 \left(\frac{b^4(d+ex)^{13/2}}{13e^4} + \frac{(d+ex)^{11/2} \cdot (4ab^3e - 4b^4d)}{11e^4} + \frac{(d+ex)^{9/2} \cdot (6a^2b^2e^2 - 12ab^3de + 6b^4d^2)}{9e^4} + \frac{(d+ex)^{7/2} \cdot (4a^3be^3 - 12a^2b^2de^2 + 12ab^3d^2e - 4b^4d^3)}{7e^4} \right)}{e} + d^2 \left(a^4x + 2a^3bx^2 + 2a^2b^2x^3 + ab^3x^4 + \frac{b^4x^5}{5} \right)$$

input

```
integrate((e*x+d)**(3/2)*(b**2*x**2+2*a*b*x+a**2)**2,x)
```

output

```
Piecewise((2*(b**4*(d + e*x)**(13/2)/(13*e**4) + (d + e*x)**(11/2)*(4*a*b**3*e - 4*b**4*d)/(11*e**4) + (d + e*x)**(9/2)*(6*a**2*b**2*e**2 - 12*a*b**3*d*e + 6*b**4*d**2)/(9*e**4) + (d + e*x)**(7/2)*(4*a**3*b*e**3 - 12*a**2*b**2*d*e**2 + 12*a*b**3*d**2*e - 4*b**4*d**3)/(7*e**4) + (d + e*x)**(5/2)*(a**4*e**4 - 4*a**3*b*d*e**3 + 6*a**2*b**2*d**2*e**2 - 4*a*b**3*d**3*e + b**4*d**4)/(5*e**4))/e, Ne(e, 0)), (d**(3/2)*(a**4*x + 2*a**3*b*x**2 + 2*a**2*b**2*x**3 + a*b**3*x**4 + b**4*x**5/5), True))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 181, normalized size of antiderivative = 1.40

$$\int (d + ex)^{3/2} (a^2 + 2abx + b^2x^2)^2 dx = \frac{2 \left(1155 (ex + d)^{\frac{13}{2}} b^4 - 5460 (b^4d - ab^3e)(ex + d)^{\frac{11}{2}} + 10010 (b^4d^2 - 2ab^3de + a^2b^2e^2)(ex + d)^{\frac{9}{2}} - 8580 (b^4d^3 - 3ab^3d^2e + 3a^2b^2d^2e^2 - a^3b^2e^3)(ex + d)^{\frac{7}{2}} + 3003 (b^4d^4 - 4ab^3d^3e + 6a^2b^2d^2e^2 - 4a^3b^2d^2e^3 + a^4e^4)(ex + d)^{\frac{5}{2}} \right)}{e^5}$$

input

```
integrate((e*x+d)^(3/2)*(b^2*x^2+2*a*b*x+a^2)^2,x, algorithm="maxima")
```

output

```
2/15015*(1155*(e*x + d)^(13/2)*b^4 - 5460*(b^4*d - a*b^3*e)*(e*x + d)^(11/2) + 10010*(b^4*d^2 - 2*a*b^3*d*e + a^2*b^2*e^2)*(e*x + d)^(9/2) - 8580*(b^4*d^3 - 3*a*b^3*d^2*e + 3*a^2*b^2*d^2*e^2 - a^3*b^2*e^3)*(e*x + d)^(7/2) + 3003*(b^4*d^4 - 4*a*b^3*d^3*e + 6*a^2*b^2*d^2*e^2 - 4*a^3*b^2*d^2*e^3 + a^4*e^4)*(e*x + d)^(5/2))/e^5
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 807 vs. 2(109) = 218.

Time = 0.25 (sec) , antiderivative size = 807, normalized size of antiderivative = 6.26

$$\int (d + ex)^{3/2} (a^2 + 2abx + b^2x^2)^2 dx = \text{Too large to display}$$

input

```
integrate((e*x+d)^(3/2)*(b^2*x^2+2*a*b*x+a^2)^2,x, algorithm="giac")
```

output

```

2/45045*(45045*sqrt(e*x + d)*a^4*d^2 + 30030*((e*x + d)^(3/2) - 3*sqrt(e*x
+ d)*d)*a^4*d + 60060*((e*x + d)^(3/2) - 3*sqrt(e*x + d)*d)*a^3*b*d^2/e +
3003*(3*(e*x + d)^(5/2) - 10*(e*x + d)^(3/2)*d + 15*sqrt(e*x + d)*d^2)*a^
4 + 18018*(3*(e*x + d)^(5/2) - 10*(e*x + d)^(3/2)*d + 15*sqrt(e*x + d)*d^2
)*a^2*b^2*d^2/e^2 + 24024*(3*(e*x + d)^(5/2) - 10*(e*x + d)^(3/2)*d + 15*s
qrt(e*x + d)*d^2)*a^3*b*d/e + 5148*(5*(e*x + d)^(7/2) - 21*(e*x + d)^(5/2)
*d + 35*(e*x + d)^(3/2)*d^2 - 35*sqrt(e*x + d)*d^3)*a*b^3*d^2/e^3 + 15444*
(5*(e*x + d)^(7/2) - 21*(e*x + d)^(5/2)*d + 35*(e*x + d)^(3/2)*d^2 - 35*sq
rt(e*x + d)*d^3)*a^2*b^2*d/e^2 + 5148*(5*(e*x + d)^(7/2) - 21*(e*x + d)^(5
/2)*d + 35*(e*x + d)^(3/2)*d^2 - 35*sqrt(e*x + d)*d^3)*a^3*b/e + 143*(35*(
e*x + d)^(9/2) - 180*(e*x + d)^(7/2)*d + 378*(e*x + d)^(5/2)*d^2 - 420*(e*
x + d)^(3/2)*d^3 + 315*sqrt(e*x + d)*d^4)*b^4*d^2/e^4 + 1144*(35*(e*x + d)
^(9/2) - 180*(e*x + d)^(7/2)*d + 378*(e*x + d)^(5/2)*d^2 - 420*(e*x + d)^(
3/2)*d^3 + 315*sqrt(e*x + d)*d^4)*a*b^3*d/e^3 + 858*(35*(e*x + d)^(9/2) -
180*(e*x + d)^(7/2)*d + 378*(e*x + d)^(5/2)*d^2 - 420*(e*x + d)^(3/2)*d^3
+ 315*sqrt(e*x + d)*d^4)*a^2*b^2/e^2 + 130*(63*(e*x + d)^(11/2) - 385*(e*x
+ d)^(9/2)*d + 990*(e*x + d)^(7/2)*d^2 - 1386*(e*x + d)^(5/2)*d^3 + 1155*
(e*x + d)^(3/2)*d^4 - 693*sqrt(e*x + d)*d^5)*b^4*d/e^4 + 260*(63*(e*x + d)
^(11/2) - 385*(e*x + d)^(9/2)*d + 990*(e*x + d)^(7/2)*d^2 - 1386*(e*x + d)
^(5/2)*d^3 + 1155*(e*x + d)^(3/2)*d^4 - 693*sqrt(e*x + d)*d^5)*a*b^3/e^...

```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 112, normalized size of antiderivative = 0.87

$$\begin{aligned}
\int (d + ex)^{3/2} (a^2 + 2abx + b^2x^2)^2 dx &= \frac{2b^4(d + ex)^{13/2}}{13e^5} \\
&- \frac{(8b^4d - 8ab^3e)(d + ex)^{11/2}}{11e^5} + \frac{2(ae - bd)^4(d + ex)^{5/2}}{5e^5} \\
&+ \frac{4b^2(ae - bd)^2(d + ex)^{9/2}}{3e^5} + \frac{8b(ae - bd)^3(d + ex)^{7/2}}{7e^5}
\end{aligned}$$

input

```
int((d + e*x)^(3/2)*(a^2 + b^2*x^2 + 2*a*b*x)^2,x)
```

output

```

(2*b^4*(d + e*x)^(13/2))/(13*e^5) - ((8*b^4*d - 8*a*b^3*e)*(d + e*x)^(11/2
))/((11*e^5) + (2*(a*e - b*d)^4*(d + e*x)^(5/2))/(5*e^5) + (4*b^2*(a*e - b*
d)^2*(d + e*x)^(9/2))/(3*e^5) + (8*b*(a*e - b*d)^3*(d + e*x)^(7/2))/(7*e^5
)

```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 330, normalized size of antiderivative = 2.56

$$\int (d + ex)^{3/2} (a^2 + 2abx + b^2x^2)^2 dx = \frac{2\sqrt{ex + d} (1155b^4e^6x^6 + 5460ab^3e^6x^5 + 1470b^4de^5x^5 + 10010a^2b^2e^6x^4 + 7280ab^3de^5x^4 + 3003a^4d^2e^4x^3 - 3432a^3bd^3e^3x^3 + 1716a^3bd^2e^4x^2 + 13728a^3bd^2e^5x^2 + 8580a^3b^2e^6x^3 + 2288a^2b^2d^4e^2x^2 - 1144a^2b^2d^3e^3x + 858a^2b^2d^2e^4x^2 + 14300a^2b^2d^2e^5x^3 + 10010a^2b^2e^6x^4 - 832ab^3d^5e + 416ab^3d^4e^2x - 312ab^3d^3e^3x^2 + 260ab^3d^2e^4x^3 + 7280ab^3d^2e^5x^4 + 5460ab^3e^6x^5 + 128b^4d^6 - 64b^4d^5ex + 48b^4d^4e^2x^2 - 40b^4d^3e^3x^3 + 35b^4d^2e^4x^4 + 1470b^4de^5x^5 + 1155b^4e^6x^6)}{(15015e^5)}$$

input

```
int((e*x+d)^(3/2)*(b^2*x^2+2*a*b*x+a^2)^2,x)
```

output

```
(2*sqrt(d + e*x)*(3003*a**4*d**2*e**4 + 6006*a**4*d*e**5*x + 3003*a**4*e**6*x**2 - 3432*a**3*b*d**3*e**3 + 1716*a**3*b*d**2*e**4*x + 13728*a**3*b*d*e**5*x**2 + 8580*a**3*b*e**6*x**3 + 2288*a**2*b**2*d**4*e**2 - 1144*a**2*b**2*d**3*e**3*x + 858*a**2*b**2*d**2*e**4*x**2 + 14300*a**2*b**2*d*e**5*x**3 + 10010*a**2*b**2*e**6*x**4 - 832*a*b**3*d**5*e + 416*a*b**3*d**4*e**2*x - 312*a*b**3*d**3*e**3*x**2 + 260*a*b**3*d**2*e**4*x**3 + 7280*a*b**3*d*e**5*x**4 + 5460*a*b**3*e**6*x**5 + 128*b**4*d**6 - 64*b**4*d**5*e*x + 48*b**4*d**4*e**2*x**2 - 40*b**4*d**3*e**3*x**3 + 35*b**4*d**2*e**4*x**4 + 1470*b**4*d*e**5*x**5 + 1155*b**4*e**6*x**6))/(15015*e**5)
```


3.157 $\int \sqrt{d + ex}(a^2 + 2abx + b^2x^2)^2 dx$

| | |
|---|------|
| Optimal result | 1228 |
| Mathematica [A] (verified) | 1228 |
| Rubi [A] (verified) | 1229 |
| Maple [A] (verified) | 1231 |
| Fricas [B] (verification not implemented) | 1231 |
| Sympy [B] (verification not implemented) | 1232 |
| Maxima [A] (verification not implemented) | 1233 |
| Giac [B] (verification not implemented) | 1233 |
| Mupad [B] (verification not implemented) | 1234 |
| Reduce [B] (verification not implemented) | 1235 |

Optimal result

Integrand size = 28, antiderivative size = 129

$$\int \sqrt{d + ex}(a^2 + 2abx + b^2x^2)^2 dx = \frac{2(bd - ae)^4(d + ex)^{3/2}}{3e^5} - \frac{8b(bd - ae)^3(d + ex)^{5/2}}{5e^5} + \frac{12b^2(bd - ae)^2(d + ex)^{7/2}}{7e^5} - \frac{8b^3(bd - ae)(d + ex)^{9/2}}{9e^5} + \frac{2b^4(d + ex)^{11/2}}{11e^5}$$

output

$$\frac{2}{3}*(-a*e+b*d)^4*(e*x+d)^(3/2)/e^5-8/5*b*(-a*e+b*d)^3*(e*x+d)^(5/2)/e^5+12/7*b^2*(-a*e+b*d)^2*(e*x+d)^(7/2)/e^5-8/9*b^3*(-a*e+b*d)*(e*x+d)^(9/2)/e^5+2/11*b^4*(e*x+d)^(11/2)/e^5$$

Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 154, normalized size of antiderivative = 1.19

$$\int \sqrt{d + ex}(a^2 + 2abx + b^2x^2)^2 dx = \frac{2(d + ex)^{3/2} (1155a^4e^4 + 924a^3be^3(-2d + 3ex) + 198a^2b^2e^2(8d^2 - 12dex + 15e^2x^2) + 44ab^3e(-16d^3 + 24d^2ex - 12dex^2 + 2e^2x^3) + 11b^4e^2(-16d^4 + 32d^3ex - 12d^2ex^2 + 2e^2x^3))}{3465e^5}$$

input `Integrate[Sqrt[d + e*x]*(a^2 + 2*a*b*x + b^2*x^2)^2,x]`

output $(2*(d + e*x)^{(3/2)}*(1155*a^4*e^4 + 924*a^3*b*e^3*(-2*d + 3*e*x) + 198*a^2*b^2*e^2*(8*d^2 - 12*d*e*x + 15*e^2*x^2) + 44*a*b^3*e*(-16*d^3 + 24*d^2*e*x - 30*d*e^2*x^2 + 35*e^3*x^3) + b^4*(128*d^4 - 192*d^3*e*x + 240*d^2*e^2*x^2 - 280*d*e^3*x^3 + 315*e^4*x^4)))/(3465*e^5)$

Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 129, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {1098, 27, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a^2 + 2abx + b^2x^2)^2 \sqrt{d + ex} dx$$

$$\downarrow 1098$$

$$\frac{\int b^4(a + bx)^4 \sqrt{d + ex} dx}{b^4}$$

$$\downarrow 27$$

$$\int (a + bx)^4 \sqrt{d + ex} dx$$

$$\downarrow 53$$

$$\int \left(-\frac{4b^3(d + ex)^{7/2}(bd - ae)}{e^4} + \frac{6b^2(d + ex)^{5/2}(bd - ae)^2}{e^4} - \frac{4b(d + ex)^{3/2}(bd - ae)^3}{e^4} + \frac{\sqrt{d + ex}(ae - bd)^4}{e^4} + b \right) dx$$

$$\downarrow 2009$$

$$-\frac{8b^3(d + ex)^{9/2}(bd - ae)}{9e^5} + \frac{12b^2(d + ex)^{7/2}(bd - ae)^2}{7e^5} - \frac{8b(d + ex)^{5/2}(bd - ae)^3}{5e^5} + \frac{2(d + ex)^{3/2}(bd - ae)^4}{3e^5} + \frac{2b^4(d + ex)^{11/2}}{11e^5}$$

input `Int[Sqrt[d + e*x]*(a^2 + 2*a*b*x + b^2*x^2)^2,x]`

output `(2*(b*d - a*e)^4*(d + e*x)^(3/2))/(3*e^5) - (8*b*(b*d - a*e)^3*(d + e*x)^(5/2))/(5*e^5) + (12*b^2*(b*d - a*e)^2*(d + e*x)^(7/2))/(7*e^5) - (8*b^3*(b*d - a*e)*(d + e*x)^(9/2))/(9*e^5) + (2*b^4*(d + e*x)^(11/2))/(11*e^5)`

Defintions of rubi rules used

rule 27 `Int[(a_)*(Fx_), x_Symbol] := Simp[a Int[Fx, x], x] /; FreeQ[a, x] && !MatchQ[Fx, (b_)*(Gx_)] /; FreeQ[b, x]`

rule 53 `Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] := Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && IGtQ[m, 0] && (!IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])`

rule 1098 `Int[((d_.) + (e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] := Simp[1/c^p Int[(d + e*x)^m*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]`

rule 2009 `Int[u_, x_Symbol] := Simp[IntSum[u, x], x] /; SumQ[u]`

Maple [A] (verified)

Time = 1.24 (sec) , antiderivative size = 143, normalized size of antiderivative = 1.11

| method | result |
|-------------------|---|
| pseudoelliptic | $2 \left(\frac{\left(\frac{3}{11} b^4 x^4 + \frac{4}{3} a b^3 x^3 + \frac{18}{7} a^2 b^2 x^2 + \frac{12}{5} a^3 b x + a^4 \right) e^4 - \frac{8 \left(\frac{5}{33} b^3 x^3 + \frac{5}{7} a b^2 x^2 + \frac{9}{7} a^2 b x + a^3 \right) b d e^3}{5} + \frac{48 \left(\frac{5}{33} b^2 x^2 + \frac{2}{3} a b x + a^2 \right) b^2 d^2 e^2}{35} \right) \frac{1}{3e^5}$ |
| derivativedivides | $\frac{\frac{2b^4(e^4x^4 + d^4)}{11} + \frac{4(2abe - 2b^2d)b^2(e^4x^3 + d^3)}{9} + \frac{2(2(e^2a^2 - 2abde + b^2d^2)b^2 + (2abe - 2b^2d)^2)(e^4x^2 + d^2)}{7} + \frac{4(e^2a^2 - 2abde + b^2d^2)(2ab^2d^2 + d^4)}{5}}{e^5}$ |
| default | $\frac{2b^4(e^4x^4 + d^4)}{11} + \frac{4(2abe - 2b^2d)b^2(e^4x^3 + d^3)}{9} + \frac{2(2(e^2a^2 - 2abde + b^2d^2)b^2 + (2abe - 2b^2d)^2)(e^4x^2 + d^2)}{7} + \frac{4(e^2a^2 - 2abde + b^2d^2)(2ab^2d^2 + d^4)}{5}}{e^5}$ |
| gospers | $\frac{2(e^4x^4 + d^4)^{\frac{3}{2}} (315b^4x^4e^4 + 1540x^3ab^3e^4 - 280x^3b^4de^3 + 2970x^2a^2b^2e^4 - 1320x^2ab^3de^3 + 240x^2b^4d^2e^2 + 2772xa^3be^4 - 2376x^2b^2d^2e^2 + 128d^4)}{3465e^5}$ |
| oring | $\frac{2(315b^4x^4e^4 + 1540x^3ab^3e^4 - 280x^3b^4de^3 + 2970x^2a^2b^2e^4 - 1320x^2ab^3de^3 + 240x^2b^4d^2e^2 + 2772xa^3be^4 - 2376x^2b^2d^2e^2 + 128d^4)}{3465e^5}$ |
| trager | $\frac{2(315b^4e^5x^5 + 1540ab^3e^5x^4 + 35b^4de^4x^4 + 2970a^2b^2e^5x^3 + 220ab^3de^4x^3 - 40b^4d^2e^3x^3 + 2772a^3be^5x^2 + 594a^2b^2de^4x^2 + 128d^4)}{e^5}$ |
| risch | $\frac{2(315b^4e^5x^5 + 1540ab^3e^5x^4 + 35b^4de^4x^4 + 2970a^2b^2e^5x^3 + 220ab^3de^4x^3 - 40b^4d^2e^3x^3 + 2772a^3be^5x^2 + 594a^2b^2de^4x^2 + 128d^4)}{e^5}$ |

input `int((e*x+d)^(1/2)*(b^2*x^2+2*a*b*x+a^2)^2,x,method=_RETURNVERBOSE)`

output
$$\frac{2}{3} \left(\frac{3}{11} b^4 x^4 + \frac{4}{3} a b^3 x^3 + \frac{18}{7} a^2 b^2 x^2 + \frac{12}{5} a^3 b x + a^4 \right) e^4 - \frac{8}{5} \left(\frac{5}{33} b^3 x^3 + \frac{5}{7} a b^2 x^2 + \frac{9}{7} a^2 b x + a^3 \right) b d e^3 + \frac{48}{35} \left(\frac{5}{33} b^2 x^2 + \frac{2}{3} a b x + a^2 \right) b^2 d^2 e^2 + \frac{2}{3} a b^2 d^2 e^2 - \frac{64}{105} (3/11 b^4 x^4 + 4/3 a b^3 x^3 + 18/7 a^2 b^2 x^2 + 12/5 a^3 b x + a^4) e^4 - \frac{128}{1155} b^4 d^4 e^4 \right) \frac{1}{e^5}$$

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 245 vs. 2(109) = 218.

Time = 0.08 (sec) , antiderivative size = 245, normalized size of antiderivative = 1.90

$$\int \sqrt{d + ex} (a^2 + 2abx + b^2x^2)^2 dx$$

$$= \frac{2(315b^4e^5x^5 + 128b^4d^5 - 704ab^3d^4e + 1584a^2b^2d^3e^2 - 1848a^3bd^2e^3 + 1155a^4de^4 + 35(b^4de^4 + 44ab^3d^4e^2 + 128d^4))}{e^5}$$

input `integrate((e*x+d)^(1/2)*(b^2*x^2+2*a*b*x+a^2)^2,x, algorithm="fricas")`

output

```
2/3465*(315*b^4*e^5*x^5 + 128*b^4*d^5 - 704*a*b^3*d^4*e + 1584*a^2*b^2*d^3
*e^2 - 1848*a^3*b*d^2*e^3 + 1155*a^4*d*e^4 + 35*(b^4*d*e^4 + 44*a*b^3*e^5)
*x^4 - 10*(4*b^4*d^2*e^3 - 22*a*b^3*d*e^4 - 297*a^2*b^2*e^5)*x^3 + 6*(8*b^
4*d^3*e^2 - 44*a*b^3*d^2*e^3 + 99*a^2*b^2*d*e^4 + 462*a^3*b*e^5)*x^2 - (64
*b^4*d^4*e - 352*a*b^3*d^3*e^2 + 792*a^2*b^2*d^2*e^3 - 924*a^3*b*d*e^4 - 1
155*a^4*e^5)*x)*sqrt(e*x + d)/e^5
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 272 vs. $2(119) = 238$.

Time = 1.00 (sec) , antiderivative size = 272, normalized size of antiderivative = 2.11

$$\int \sqrt{d+ex}(a^2+2abx+b^2x^2)^2 dx$$

$$= \frac{2 \left(\frac{b^4(d+ex)^{\frac{11}{2}}}{11e^4} + \frac{(d+ex)^{\frac{9}{2}} \cdot (4ab^3e-4b^4d)}{9e^4} + \frac{(d+ex)^{\frac{7}{2}} \cdot (6a^2b^2e^2-12ab^3de+6b^4d^2)}{7e^4} + \frac{(d+ex)^{\frac{5}{2}} \cdot (4a^3be^3-12a^2b^2de^2+12ab^3d^2e-4b^4d^3)}{5e^4} + \frac{(d+ex)^{\frac{3}{2}} \cdot (a^4e^4)}{3e^4} \right)}{e} + \sqrt{d} \left(a^4x + 2a^3bx^2 + 2a^2b^2x^3 + ab^3x^4 + \frac{b^4x^5}{5} \right)$$

input

```
integrate((e*x+d)**(1/2)*(b**2*x**2+2*a*b*x+a**2)**2,x)
```

output

```
Piecewise((2*(b**4*(d + e*x)**(11/2)/(11*e**4) + (d + e*x)**(9/2)*(4*a*b**
3*e - 4*b**4*d)/(9*e**4) + (d + e*x)**(7/2)*(6*a**2*b**2*e**2 - 12*a*b**3*
d*e + 6*b**4*d**2)/(7*e**4) + (d + e*x)**(5/2)*(4*a**3*b*e**3 - 12*a**2*b*
**2*d*e**2 + 12*a*b**3*d**2*e - 4*b**4*d**3)/(5*e**4) + (d + e*x)**(3/2)*(a
**4*e**4 - 4*a**3*b*d*e**3 + 6*a**2*b**2*d**2*e**2 - 4*a*b**3*d**3*e + b**
4*d**4)/(3*e**4))/e, Ne(e, 0)), (sqrt(d)*(a**4*x + 2*a**3*b*x**2 + 2*a**2*
b**2*x**3 + a*b**3*x**4 + b**4*x**5/5), True))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 181, normalized size of antiderivative = 1.40

$$\int \sqrt{d+ex}(a^2+2abx+b^2x^2)^2 dx$$

$$= \frac{2 \left(315 (ex+d)^{\frac{11}{2}} b^4 - 1540 (b^4 d - ab^3 e)(ex+d)^{\frac{9}{2}} + 2970 (b^4 d^2 - 2ab^3 de + a^2 b^2 e^2)(ex+d)^{\frac{7}{2}} - 2772 (b^4 d^3 - 3ab^3 d^2 e + 3a^2 b^2 d e^2 - a^3 b e^3)(ex+d)^{\frac{5}{2}} + 1155 (b^4 d^4 - 4ab^3 d^3 e + 6a^2 b^2 d^2 e^2 - 4a^3 b d e^3 + a^4 e^4)(ex+d)^{\frac{3}{2}} \right)}{e^5}$$

input

```
integrate((e*x+d)^(1/2)*(b^2*x^2+2*a*b*x+a^2)^2,x, algorithm="maxima")
```

output

```
2/3465*(315*(e*x + d)^(11/2)*b^4 - 1540*(b^4*d - a*b^3*e)*(e*x + d)^(9/2)
+ 2970*(b^4*d^2 - 2*a*b^3*d*e + a^2*b^2*e^2)*(e*x + d)^(7/2) - 2772*(b^4*d
^3 - 3*a*b^3*d^2*e + 3*a^2*b^2*d*e^2 - a^3*b*e^3)*(e*x + d)^(5/2) + 1155*(
b^4*d^4 - 4*a*b^3*d^3*e + 6*a^2*b^2*d^2*e^2 - 4*a^3*b*d*e^3 + a^4*e^4)*(e*
x + d)^(3/2))/e^5
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 470 vs. 2(109) = 218.

Time = 0.18 (sec) , antiderivative size = 470, normalized size of antiderivative = 3.64

$$\int \sqrt{d+ex}(a^2+2abx+b^2x^2)^2 dx$$

$$= \frac{2 \left(3465 \sqrt{ex+da} d^4 + 1155 \left((ex+d)^{\frac{3}{2}} - 3 \sqrt{ex+dd} \right) a^4 + \frac{4620 \left((ex+d)^{\frac{3}{2}} - 3 \sqrt{ex+dd} \right) a^3 b d}{e} + \frac{1386 \left(3 (ex+d)^{\frac{5}{2}} - 1 \right) a^2 b^2 d^2}{e^2} \right)}{e^5}$$

input

```
integrate((e*x+d)^(1/2)*(b^2*x^2+2*a*b*x+a^2)^2,x, algorithm="giac")
```

output

```
2/3465*(3465*sqrt(e*x + d)*a^4*d + 1155*((e*x + d)^(3/2) - 3*sqrt(e*x + d)
*d)*a^4 + 4620*((e*x + d)^(3/2) - 3*sqrt(e*x + d)*d)*a^3*b*d/e + 1386*(3*(
e*x + d)^(5/2) - 10*(e*x + d)^(3/2)*d + 15*sqrt(e*x + d)*d^2)*a^2*b^2*d/e^
2 + 924*(3*(e*x + d)^(5/2) - 10*(e*x + d)^(3/2)*d + 15*sqrt(e*x + d)*d^2)*
a^3*b/e + 396*(5*(e*x + d)^(7/2) - 21*(e*x + d)^(5/2)*d + 35*(e*x + d)^(3/
2)*d^2 - 35*sqrt(e*x + d)*d^3)*a*b^3*d/e^3 + 594*(5*(e*x + d)^(7/2) - 21*(
e*x + d)^(5/2)*d + 35*(e*x + d)^(3/2)*d^2 - 35*sqrt(e*x + d)*d^3)*a^2*b^2/
e^2 + 11*(35*(e*x + d)^(9/2) - 180*(e*x + d)^(7/2)*d + 378*(e*x + d)^(5/2)
*d^2 - 420*(e*x + d)^(3/2)*d^3 + 315*sqrt(e*x + d)*d^4)*b^4*d/e^4 + 44*(35
*(e*x + d)^(9/2) - 180*(e*x + d)^(7/2)*d + 378*(e*x + d)^(5/2)*d^2 - 420*(
e*x + d)^(3/2)*d^3 + 315*sqrt(e*x + d)*d^4)*a*b^3/e^3 + 5*(63*(e*x + d)^(1
1/2) - 385*(e*x + d)^(9/2)*d + 990*(e*x + d)^(7/2)*d^2 - 1386*(e*x + d)^(5
/2)*d^3 + 1155*(e*x + d)^(3/2)*d^4 - 693*sqrt(e*x + d)*d^5)*b^4/e^4)/e
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 112, normalized size of antiderivative = 0.87

$$\int \sqrt{d+ex}(a^2+2abx+b^2x^2)^2 dx = \frac{2b^4(d+ex)^{11/2}}{11e^5} - \frac{(8b^4d-8ab^3e)(d+ex)^{9/2}}{9e^5} + \frac{2(ae-bd)^4(d+ex)^{3/2}}{3e^5} + \frac{12b^2(ae-bd)^2(d+ex)^{7/2}}{7e^5} + \frac{8b(ae-bd)^3(d+ex)^{5/2}}{5e^5}$$

input

```
int((d + e*x)^(1/2)*(a^2 + b^2*x^2 + 2*a*b*x)^2,x)
```

output

```
(2*b^4*(d + e*x)^(11/2))/(11*e^5) - ((8*b^4*d - 8*a*b^3*e)*(d + e*x)^(9/2)
)/(9*e^5) + (2*(a*e - b*d)^4*(d + e*x)^(3/2))/(3*e^5) + (12*b^2*(a*e - b*d)
)^2*(d + e*x)^(7/2))/(7*e^5) + (8*b*(a*e - b*d)^3*(d + e*x)^(5/2))/(5*e^5)
```

Reduce [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 255, normalized size of antiderivative = 1.98

$$\int \sqrt{d+ex}(a^2+2abx+b^2x^2)^2 dx$$

$$= \frac{2\sqrt{ex+d}(315b^4e^5x^5 + 1540ab^3e^5x^4 + 35b^4de^4x^4 + 2970a^2b^2e^5x^3 + 220ab^3de^4x^3 - 40b^4d^2e^3x^3 + 2772$$

input `int((e*x+d)^(1/2)*(b^2*x^2+2*a*b*x+a^2)^2,x)`output `(2*sqrt(d + e*x)*(1155*a**4*d*e**4 + 1155*a**4*e**5*x - 1848*a**3*b*d**2*e**3 + 924*a**3*b*d*e**4*x + 2772*a**3*b*e**5*x**2 + 1584*a**2*b**2*d**3*e**2 - 792*a**2*b**2*d**2*e**3*x + 594*a**2*b**2*d*e**4*x**2 + 2970*a**2*b**2*e**5*x**3 - 704*a*b**3*d**4*e + 352*a*b**3*d**3*e**2*x - 264*a*b**3*d**2*e**3*x**2 + 220*a*b**3*d*e**4*x**3 + 1540*a*b**3*e**5*x**4 + 128*b**4*d**5 - 64*b**4*d**4*e*x + 48*b**4*d**3*e**2*x**2 - 40*b**4*d**2*e**3*x**3 + 35*b**4*d*e**4*x**4 + 315*b**4*e**5*x**5))/(3465*e**5)`

3.158 $\int \frac{(a^2+2abx+b^2x^2)^2}{\sqrt{d+ex}} dx$

| | |
|---|------|
| Optimal result | 1236 |
| Mathematica [A] (verified) | 1236 |
| Rubi [A] (verified) | 1237 |
| Maple [A] (verified) | 1239 |
| Fricas [A] (verification not implemented) | 1239 |
| Sympy [B] (verification not implemented) | 1240 |
| Maxima [B] (verification not implemented) | 1241 |
| Giac [A] (verification not implemented) | 1241 |
| Mupad [B] (verification not implemented) | 1242 |
| Reduce [B] (verification not implemented) | 1242 |

Optimal result

Integrand size = 28, antiderivative size = 127

$$\int \frac{(a^2 + 2abx + b^2x^2)^2}{\sqrt{d + ex}} dx = \frac{2(bd - ae)^4 \sqrt{d + ex}}{e^5} - \frac{8b(bd - ae)^3 (d + ex)^{3/2}}{3e^5} + \frac{12b^2(bd - ae)^2 (d + ex)^{5/2}}{5e^5} - \frac{8b^3(bd - ae)(d + ex)^{7/2}}{7e^5} + \frac{2b^4(d + ex)^{9/2}}{9e^5}$$

output

```
2*(-a*e+b*d)^4*(e*x+d)^(1/2)/e^5-8/3*b*(-a*e+b*d)^3*(e*x+d)^(3/2)/e^5+12/5
*b^2*(-a*e+b*d)^2*(e*x+d)^(5/2)/e^5-8/7*b^3*(-a*e+b*d)*(e*x+d)^(7/2)/e^5+
/9*b^4*(e*x+d)^(9/2)/e^5
```

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 153, normalized size of antiderivative = 1.20

$$\int \frac{(a^2 + 2abx + b^2x^2)^2}{\sqrt{d + ex}} dx = \frac{2\sqrt{d + ex}(315a^4e^4 + 420a^3be^3(-2d + ex) + 126a^2b^2e^2(8d^2 - 4dex + 3e^2x^2) + 36ab^3e(-16d^3 + 8d^2ex - 315e^5))}{315e^5}$$

input `Integrate[(a^2 + 2*a*b*x + b^2*x^2)^2/Sqrt[d + e*x],x]`

output $(2\sqrt{d + ex}*(315a^4e^4 + 420a^3b^3e^3(-2d + ex) + 126a^2b^2e^2(8d^2 - 4d^2ex + 3e^2x^2) + 36ab^3e(-16d^3 + 8d^2ex - 6d^2e^2x^2 + 5e^3x^3) + b^4(128d^4 - 64d^3ex + 48d^2e^2x^2 - 40d^2e^3x^3 + 35e^4x^4)))/(315e^5)$

Rubi [A] (verified)

Time = 0.46 (sec) , antiderivative size = 127, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {1098, 27, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a^2 + 2abx + b^2x^2)^2}{\sqrt{d + ex}} dx$$

$$\downarrow 1098$$

$$\frac{\int \frac{b^4(a+bx)^4}{\sqrt{d+ex}} dx}{b^4}$$

$$\downarrow 27$$

$$\int \frac{(a + bx)^4}{\sqrt{d + ex}} dx$$

$$\downarrow 53$$

$$\int \left(-\frac{4b^3(d + ex)^{5/2}(bd - ae)}{e^4} + \frac{6b^2(d + ex)^{3/2}(bd - ae)^2}{e^4} - \frac{4b\sqrt{d + ex}(bd - ae)^3}{e^4} + \frac{(ae - bd)^4}{e^4\sqrt{d + ex}} + \frac{b^4(d + ex)^{7/2}}{e^4} \right)$$

$$\downarrow 2009$$

$$-\frac{8b^3(d + ex)^{7/2}(bd - ae)}{7e^5} + \frac{12b^2(d + ex)^{5/2}(bd - ae)^2}{5e^5} - \frac{8b(d + ex)^{3/2}(bd - ae)^3}{3e^5} + \frac{2\sqrt{d + ex}(bd - ae)^4}{e^5} + \frac{2b^4(d + ex)^{9/2}}{9e^5}$$

input `Int[(a^2 + 2*a*b*x + b^2*x^2)^2/Sqrt[d + e*x],x]`

output `(2*(b*d - a*e)^4*Sqrt[d + e*x])/e^5 - (8*b*(b*d - a*e)^3*(d + e*x)^(3/2))/(3*e^5) + (12*b^2*(b*d - a*e)^2*(d + e*x)^(5/2))/(5*e^5) - (8*b^3*(b*d - a*e)*(d + e*x)^(7/2))/(7*e^5) + (2*b^4*(d + e*x)^(9/2))/(9*e^5)`

Defintions of rubi rules used

rule 27 `Int[(a_)*(F_x_), x_Symbol] := Simp[a Int[F_x, x], x] /; FreeQ[a, x] && !MatchQ[F_x, (b_)*(G_x_)] /; FreeQ[b, x]`

rule 53 `Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] := Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && IGtQ[m, 0] && (!IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])`

rule 1098 `Int[((d_.) + (e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] := Simp[1/c^p Int[(d + e*x)^m*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]`

rule 2009 `Int[u_, x_Symbol] := Simp[IntSum[u, x], x] /; SumQ[u]`

Maple [A] (verified)

Time = 1.16 (sec) , antiderivative size = 143, normalized size of antiderivative = 1.13

| method | result |
|-------------------|---|
| pseudoelliptic | $2\sqrt{ex+d} \left(\left(\frac{1}{9}b^4x^4 + \frac{4}{7}ab^3x^3 + \frac{6}{5}a^2b^2x^2 + \frac{4}{3}a^3bx + a^4 \right) e^4 - \frac{8b \left(\frac{1}{21}b^3x^3 + \frac{9}{35}ab^2x^2 + \frac{3}{5}a^2bx + a^3 \right) d e^3}{3} + \frac{16 \left(\frac{1}{21}b^2x^2 + \frac{2}{7}abx + a^2 \right)}{5} \right)$ |
| derivativedivides | $\frac{\frac{2b^4(ex+d)^{\frac{9}{2}}}{9} + \frac{4(2abe-2b^2d)b^2(ex+d)^{\frac{7}{2}}}{7} + \frac{2(2(e^2a^2-2abde+b^2d^2)b^2 + (2abe-2b^2d)^2)(ex+d)^{\frac{5}{2}}}{5} + \frac{4(e^2a^2-2abde+b^2d^2)(2abe-2b^2d)}{3}}{e^5}$ |
| default | $\frac{2b^4(ex+d)^{\frac{9}{2}}}{9} + \frac{4(2abe-2b^2d)b^2(ex+d)^{\frac{7}{2}}}{7} + \frac{2(2(e^2a^2-2abde+b^2d^2)b^2 + (2abe-2b^2d)^2)(ex+d)^{\frac{5}{2}}}{5} + \frac{4(e^2a^2-2abde+b^2d^2)(2abe-2b^2d)}{3}$ |
| gospers | $\frac{2(35b^4x^4e^4 + 180x^3ab^3e^4 - 40x^3b^4de^3 + 378x^2a^2b^2e^4 - 216x^2ab^3de^3 + 48x^2b^4d^2e^2 + 420xa^3be^4 - 504xa^2b^2de^3 + 288x^4)}{315e^5}$ |
| trager | $\frac{2(35b^4x^4e^4 + 180x^3ab^3e^4 - 40x^3b^4de^3 + 378x^2a^2b^2e^4 - 216x^2ab^3de^3 + 48x^2b^4d^2e^2 + 420xa^3be^4 - 504xa^2b^2de^3 + 288x^4)}{315e^5}$ |
| risch | $\frac{2(35b^4x^4e^4 + 180x^3ab^3e^4 - 40x^3b^4de^3 + 378x^2a^2b^2e^4 - 216x^2ab^3de^3 + 48x^2b^4d^2e^2 + 420xa^3be^4 - 504xa^2b^2de^3 + 288x^4)}{315e^5}$ |
| orering | $\frac{2(35b^4x^4e^4 + 180x^3ab^3e^4 - 40x^3b^4de^3 + 378x^2a^2b^2e^4 - 216x^2ab^3de^3 + 48x^2b^4d^2e^2 + 420xa^3be^4 - 504xa^2b^2de^3 + 288x^4)}{315e^5(bx+a)}$ |

input `int((b^2*x^2+2*a*b*x+a^2)^2/(e*x+d)^(1/2),x,method=_RETURNVERBOSE)`

output
$$2*(e*x+d)^{(1/2)}*((1/9*b^4*x^4+4/7*a*b^3*x^3+6/5*a^2*b^2*x^2+4/3*a^3*b*x+a^4)*e^4-8/3*b*(1/21*b^3*x^3+9/35*a*b^2*x^2+3/5*a^2*b*x+a^3)*d*e^3+16/5*(1/21*b^2*x^2+2/7*a*b*x+a^2)*b^2*d^2*e^2-64/35*b^3*(1/9*b*x+a)*d^3*e+128/315*b^4*d^4)/e^5$$

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 182, normalized size of antiderivative = 1.43

$$\int \frac{(a^2 + 2abx + b^2x^2)^2}{\sqrt{d + ex}} dx$$

$$= \frac{2(35b^4e^4x^4 + 128b^4d^4 - 576ab^3d^3e + 1008a^2b^2d^2e^2 - 840a^3bde^3 + 315a^4e^4 - 20(2b^4de^3 - 9ab^3e^4)x^3}{315e^5}$$

input `integrate((b^2*x^2+2*a*b*x+a^2)^2/(e*x+d)^(1/2),x, algorithm="fricas")`

output

$$\frac{2}{315} \cdot (35b^4e^4x^4 + 128b^4d^4 - 576ab^3d^3e + 1008a^2b^2d^2e^2 - 840a^3bd^2e^3 + 315a^4e^4 - 20(2b^4de^3 - 9ab^3e^4)x^3 + 6(8b^4d^2e^2 - 36ab^3de^3 + 63a^2b^2e^4)x^2 - 4(16b^4d^3e - 72ab^3d^2e^2 + 126a^2b^2de^3 - 105a^3be^4)x) \cdot \sqrt{ex + d} / e^5$$

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 270 vs. $2(117) = 234$.

Time = 0.91 (sec) , antiderivative size = 270, normalized size of antiderivative = 2.13

$$\int \frac{(a^2 + 2abx + b^2x^2)^2}{\sqrt{d + ex}} dx$$

$$= \frac{2 \left(\frac{b^4(d+ex)^{\frac{9}{2}}}{9e^4} + \frac{(d+ex)^{\frac{7}{2}} \cdot (4ab^3e - 4b^4d)}{7e^4} + \frac{(d+ex)^{\frac{5}{2}} \cdot (6a^2b^2e^2 - 12ab^3de + 6b^4d^2)}{5e^4} + \frac{(d+ex)^{\frac{3}{2}} \cdot (4a^3be^3 - 12a^2b^2de^2 + 12ab^3d^2e - 4b^4d^3)}{3e^4} + \frac{\sqrt{d+ex}(a^4e^4 - 4a^3be^3 + 6a^2b^2de^2 - 4ab^3d^2e + b^4d^3)}{e} \right)}{a^4x + 2a^3bx^2 + 2a^2b^2x^3 + ab^3x^4 + \frac{b^4x^5}{5} \sqrt{d}}$$

input

```
integrate((b**2*x**2+2*a*b*x+a**2)**2/(e*x+d)**(1/2),x)
```

output

```
Piecewise((2*(b**4*(d + e*x)**(9/2))/(9*e**4) + (d + e*x)**(7/2)*(4*a*b**3*
e - 4*b**4*d)/(7*e**4) + (d + e*x)**(5/2)*(6*a**2*b**2*e**2 - 12*a*b**3*d*
e + 6*b**4*d**2)/(5*e**4) + (d + e*x)**(3/2)*(4*a**3*b*e**3 - 12*a**2*b**2
*d*e**2 + 12*a*b**3*d**2*e - 4*b**4*d**3)/(3*e**4) + sqrt(d + e*x)*(a**4*e
**4 - 4*a**3*b*d*e**3 + 6*a**2*b**2*d**2*e**2 - 4*a*b**3*d**3*e + b**4*d**
4)/e**4)/e, Ne(e, 0)), ((a**4*x + 2*a**3*b*x**2 + 2*a**2*b**2*x**3 + a*b**
3*x**4 + b**4*x**5/5)/sqrt(d), True))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 247 vs. $2(109) = 218$.

Time = 0.05 (sec) , antiderivative size = 247, normalized size of antiderivative = 1.94

$$\int \frac{(a^2 + 2abx + b^2x^2)^2}{\sqrt{d + ex}} dx$$

$$= \frac{2 \left(315 \sqrt{ex + d} a^4 + 42 \left(\frac{10((ex+d)^{\frac{3}{2}} - 3\sqrt{ex+dd})ab}{e} + \frac{(3(ex+d)^{\frac{5}{2}} - 10(ex+d)^{\frac{3}{2}}d + 15\sqrt{ex+dd^2})b^2}{e^2} \right) a^2 + \frac{84(3(ex+d)^{\frac{5}{2}} - 10(ex+d)^{\frac{3}{2}}d + 15\sqrt{ex+dd^2})b^2}{e^2} \right)}{e}$$

input `integrate((b^2*x^2+2*a*b*x+a^2)^2/(e*x+d)^(1/2),x, algorithm="maxima")`

output `2/315*(315*sqrt(e*x + d)*a^4 + 42*(10*((e*x + d)^(3/2) - 3*sqrt(e*x + d)*d)*a*b/e + (3*(e*x + d)^(5/2) - 10*(e*x + d)^(3/2)*d + 15*sqrt(e*x + d)*d^2)*b^2/e^2)*a^2 + 84*(3*(e*x + d)^(5/2) - 10*(e*x + d)^(3/2)*d + 15*sqrt(e*x + d)*d^2)*a^2*b^2/e^2 + 36*(5*(e*x + d)^(7/2) - 21*(e*x + d)^(5/2)*d + 35*(e*x + d)^(3/2)*d^2 - 35*sqrt(e*x + d)*d^3)*a*b^3/e^3 + (35*(e*x + d)^(9/2) - 180*(e*x + d)^(7/2)*d + 378*(e*x + d)^(5/2)*d^2 - 420*(e*x + d)^(3/2)*d^3 + 315*sqrt(e*x + d)*d^4)*b^4/e^4)/e`

Giac [A] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 204, normalized size of antiderivative = 1.61

$$\int \frac{(a^2 + 2abx + b^2x^2)^2}{\sqrt{d + ex}} dx$$

$$= \frac{2 \left(315 \sqrt{ex + d} a^4 + \frac{420((ex+d)^{\frac{3}{2}} - 3\sqrt{ex+dd})a^3b}{e} + \frac{126(3(ex+d)^{\frac{5}{2}} - 10(ex+d)^{\frac{3}{2}}d + 15\sqrt{ex+dd^2})a^2b^2}{e^2} + \frac{36(5(ex+d)^{\frac{7}{2}} - 21(ex+d)^{\frac{5}{2}}d + 35(ex+d)^{\frac{3}{2}}d^2 - 35\sqrt{ex+dd^2})b^3}{e^3} + \frac{36(5(ex+d)^{\frac{9}{2}} - 180(ex+d)^{\frac{7}{2}}d + 378(ex+d)^{\frac{5}{2}}d^2 - 420(ex+d)^{\frac{3}{2}}d^3 + 315\sqrt{ex+dd^2})b^4}{e^4} \right)}{e}$$

input `integrate((b^2*x^2+2*a*b*x+a^2)^2/(e*x+d)^(1/2),x, algorithm="giac")`

output

```
(2*sqrt(d + e*x)*(315*a**4*e**4 - 840*a**3*b*d*e**3 + 420*a**3*b*e**4*x +
1008*a**2*b**2*d**2*e**2 - 504*a**2*b**2*d*e**3*x + 378*a**2*b**2*e**4*x**
2 - 576*a*b**3*d**3*e + 288*a*b**3*d**2*e**2*x - 216*a*b**3*d*e**3*x**2 +
180*a*b**3*e**4*x**3 + 128*b**4*d**4 - 64*b**4*d**3*e*x + 48*b**4*d**2*e**
2*x**2 - 40*b**4*d*e**3*x**3 + 35*b**4*e**4*x**4))/(315*e**5)
```


3.159
$$\int \frac{(a^2 + 2abx + b^2x^2)^2}{(d + ex)^{3/2}} dx$$

| | |
|---|------|
| Optimal result | 1244 |
| Mathematica [A] (verified) | 1244 |
| Rubi [A] (verified) | 1245 |
| Maple [A] (verified) | 1246 |
| Fricas [A] (verification not implemented) | 1247 |
| Sympy [A] (verification not implemented) | 1248 |
| Maxima [A] (verification not implemented) | 1248 |
| Giac [B] (verification not implemented) | 1249 |
| Mupad [B] (verification not implemented) | 1249 |
| Reduce [B] (verification not implemented) | 1250 |

Optimal result

Integrand size = 28, antiderivative size = 123

$$\int \frac{(a^2 + 2abx + b^2x^2)^2}{(d + ex)^{3/2}} dx = -\frac{2(bd - ae)^4}{e^5 \sqrt{d + ex}} - \frac{8b(bd - ae)^3 \sqrt{d + ex}}{e^5} + \frac{4b^2(bd - ae)^2 (d + ex)^{3/2}}{e^5} - \frac{8b^3(bd - ae)(d + ex)^{5/2}}{5e^5} + \frac{2b^4(d + ex)^{7/2}}{7e^5}$$

output

```
-2*(-a*e+b*d)^4/e^5/(e*x+d)^(1/2)-8*b*(-a*e+b*d)^3*(e*x+d)^(1/2)/e^5+4*b^2*(-a*e+b*d)^2*(e*x+d)^(3/2)/e^5-8/5*b^3*(-a*e+b*d)*(e*x+d)^(5/2)/e^5+2/7*b^4*(e*x+d)^(7/2)/e^5
```

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 151, normalized size of antiderivative = 1.23

$$\int \frac{(a^2 + 2abx + b^2x^2)^2}{(d + ex)^{3/2}} dx = \frac{2(-35a^4e^4 + 140a^3be^3(2d + ex) + 70a^2b^2e^2(-8d^2 - 4dex + e^2x^2) + 28ab^3e(1}{35}$$

input

```
Integrate[(a^2 + 2*a*b*x + b^2*x^2)^2/(d + e*x)^(3/2), x]
```

output

```
(2*(-35*a^4*e^4 + 140*a^3*b*e^3*(2*d + e*x) + 70*a^2*b^2*e^2*(-8*d^2 - 4*d
*e*x + e^2*x^2) + 28*a*b^3*e*(16*d^3 + 8*d^2*e*x - 2*d*e^2*x^2 + e^3*x^3)
+ b^4*(-128*d^4 - 64*d^3*e*x + 16*d^2*e^2*x^2 - 8*d*e^3*x^3 + 5*e^4*x^4))
/(35*e^5*sqrt[d + e*x])
```

Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 123, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {1098, 27, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a^2 + 2abx + b^2x^2)^2}{(d + ex)^{3/2}} dx$$

$$\downarrow 1098$$

$$\frac{\int \frac{b^4(a+bx)^4}{(d+ex)^{3/2}} dx}{b^4}$$

$$\downarrow 27$$

$$\int \frac{(a + bx)^4}{(d + ex)^{3/2}} dx$$

$$\downarrow 53$$

$$\int \left(-\frac{4b^3(d + ex)^{3/2}(bd - ae)}{e^4} + \frac{6b^2\sqrt{d + ex}(bd - ae)^2}{e^4} - \frac{4b(bd - ae)^3}{e^4\sqrt{d + ex}} + \frac{(ae - bd)^4}{e^4(d + ex)^{3/2}} + \frac{b^4(d + ex)^{5/2}}{e^4} \right) dx$$

$$\downarrow 2009$$

$$-\frac{8b^3(d + ex)^{5/2}(bd - ae)}{5e^5} + \frac{4b^2(d + ex)^{3/2}(bd - ae)^2}{e^5} - \frac{8b\sqrt{d + ex}(bd - ae)^3}{e^5} - \frac{2(bd - ae)^4}{e^5\sqrt{d + ex}} + \frac{2b^4(d + ex)^{7/2}}{7e^5}$$

input

```
Int[(a^2 + 2*a*b*x + b^2*x^2)^2/(d + e*x)^(3/2), x]
```

output

$$\frac{(-2*(b*d - a*e)^4)/(e^5*\text{Sqrt}[d + e*x]) - (8*b*(b*d - a*e)^3*\text{Sqrt}[d + e*x])}{e^5 + (4*b^2*(b*d - a*e)^2*(d + e*x)^{(3/2)})/e^5 - (8*b^3*(b*d - a*e)*(d + e*x)^{(5/2)})/(5*e^5) + (2*b^4*(d + e*x)^{(7/2)})/(7*e^5)}$$
Defintions of rubi rules used

rule 27

```
Int[(a_)*(F x_), x_Symbol] := Simp[a Int[F x, x], x] /; FreeQ[a, x] && !MatchQ[F x, (b_)*(G x_)] /; FreeQ[b, x]
```

rule 53

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] := Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && IGtQ[m, 0] && (!IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 1098

```
Int[((d_.) + (e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] := Simp[1/c^p Int[(d + e*x)^m*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

rule 2009

```
Int[u_, x_Symbol] := Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 1.23 (sec) , antiderivative size = 145, normalized size of antiderivative = 1.18

Sympy [A] (verification not implemented)

Time = 3.69 (sec) , antiderivative size = 224, normalized size of antiderivative = 1.82

$$\int \frac{(a^2 + 2abx + b^2x^2)^2}{(d + ex)^{3/2}} dx = \frac{2 \left(\frac{b^4(d+ex)^{7/2}}{7e^4} + \frac{(d+ex)^{5/2} \cdot (4ab^3e - 4b^4d)}{5e^4} + \frac{(d+ex)^{3/2} \cdot (6a^2b^2e^2 - 12ab^3de + 6b^4d^2)}{3e^4} + \frac{\sqrt{d+ex}(4a^3be^3 - 12a^2b^2de^2)}{e^4} \right)}{e} + \frac{a^4x + 2a^3bx^2 + 2a^2b^2x^3 + ab^3x^4 + \frac{b^4x^5}{5}}{d^{3/2}}$$

input `integrate((b**2*x**2+2*a*b*x+a**2)**2/(e*x+d)**(3/2),x)`output `Piecewise((2*(b**4*(d + e*x)**(7/2)/(7*e**4) + (d + e*x)**(5/2)*(4*a*b**3*e - 4*b**4*d)/(5*e**4) + (d + e*x)**(3/2)*(6*a**2*b**2*e**2 - 12*a*b**3*d*e + 6*b**4*d**2)/(3*e**4) + sqrt(d + e*x)*(4*a**3*b*e**3 - 12*a**2*b**2*d*e**2 + 12*a*b**3*d**2*e - 4*b**4*d**3)/e**4 - (a*e - b*d)**4/(e**4*sqrt(d + e*x))), Ne(e, 0)), ((a**4*x + 2*a**3*b*x**2 + 2*a**2*b**2*x**3 + a*b**3*x**4 + b**4*x**5/5)/d**(3/2), True))`**Maxima [A] (verification not implemented)**

Time = 0.03 (sec) , antiderivative size = 189, normalized size of antiderivative = 1.54

$$\int \frac{(a^2 + 2abx + b^2x^2)^2}{(d + ex)^{3/2}} dx = \frac{2 \left(\frac{5(ex+d)^{7/2}b^4 - 28(b^4d - ab^3e)(ex+d)^{5/2} + 70(b^4d^2 - 2ab^3de + a^2b^2e^2)(ex+d)^{3/2} - 140(b^4d^3 - 3ab^3d^2e + 3a^2b^3d^2e^2 - 4ab^3d^3e + 6a^2b^2d^2e^2 - 4a^3bd^3e + a^4e^4)}{e^4} \right)}{35e}$$

input `integrate((b^2*x^2+2*a*b*x+a^2)^2/(e*x+d)^(3/2),x, algorithm="maxima")`output `2/35*((5*(e*x + d)^(7/2)*b^4 - 28*(b^4*d - a*b^3*e)*(e*x + d)^(5/2) + 70*(b^4*d^2 - 2*a*b^3*d*e + a^2*b^2*e^2)*(e*x + d)^(3/2) - 140*(b^4*d^3 - 3*a*b^3*d^2*e + 3*a^2*b^2*d*e^2 - a^3*b*e^3)*sqrt(e*x + d))/e^4 - 35*(b^4*d^4 - 4*a*b^3*d^3*e + 6*a^2*b^2*d^2*e^2 - 4*a^3*b*d^3e + a^4*e^4)/(sqrt(e*x + d)*e^4))/e`

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 240 vs. $2(109) = 218$.

Time = 0.20 (sec) , antiderivative size = 240, normalized size of antiderivative = 1.95

$$\int \frac{(a^2 + 2abx + b^2x^2)^2}{(d + ex)^{3/2}} dx = -\frac{2(b^4d^4 - 4ab^3d^3e + 6a^2b^2d^2e^2 - 4a^3bde^3 + a^4e^4)}{\sqrt{ex + d}e^5} + \frac{2\left(5(ex + d)^{\frac{7}{2}}b^4e^{30} - 28(ex + d)^{\frac{5}{2}}b^4de^{30} + 70(ex + d)^{\frac{3}{2}}b^4d^2e^{30} - 140\sqrt{ex + d}b^4d^3e^{30} + 28(ex + d)^{\frac{5}{2}}ab^3\right)}{e^{35}}$$

input `integrate((b^2*x^2+2*a*b*x+a^2)^2/(e*x+d)^(3/2),x, algorithm="giac")`

output `-2*(b^4*d^4 - 4*a*b^3*d^3*e + 6*a^2*b^2*d^2*e^2 - 4*a^3*b*d*e^3 + a^4*e^4) / (sqrt(e*x + d)*e^5) + 2/35*(5*(e*x + d)^(7/2)*b^4*e^30 - 28*(e*x + d)^(5/2)*b^4*d*e^30 + 70*(e*x + d)^(3/2)*b^4*d^2*e^30 - 140*sqrt(e*x + d)*b^4*d^3*e^30 + 28*(e*x + d)^(5/2)*a*b^3*e^31 - 140*(e*x + d)^(3/2)*a*b^3*d*e^31 + 420*sqrt(e*x + d)*a*b^3*d^2*e^31 + 70*(e*x + d)^(3/2)*a^2*b^2*e^32 - 420*sqrt(e*x + d)*a^2*b^2*d*e^32 + 140*sqrt(e*x + d)*a^3*b*e^33)/e^35`

Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 153, normalized size of antiderivative = 1.24

$$\int \frac{(a^2 + 2abx + b^2x^2)^2}{(d + ex)^{3/2}} dx = \frac{2b^4(d + ex)^{7/2}}{7e^5} - \frac{(8b^4d - 8ab^3e)(d + ex)^{5/2}}{5e^5} - \frac{2a^4e^4 - 8a^3bde^3 + 12a^2b^2d^2e^2 - 8ab^3d^3e + 2b^4d^4}{e^5\sqrt{d + ex}} + \frac{4b^2(ae - bd)^2(d + ex)^{3/2}}{e^5} + \frac{8b(ae - bd)^3\sqrt{d + ex}}{e^5}$$

input `int((a^2 + b^2*x^2 + 2*a*b*x)^2/(d + e*x)^(3/2),x)`

output `(2*b^4*(d + e*x)^(7/2))/(7*e^5) - ((8*b^4*d - 8*a*b^3*e)*(d + e*x)^(5/2))/(5*e^5) - (2*a^4*e^4 + 2*b^4*d^4 + 12*a^2*b^2*d^2*e^2 - 8*a*b^3*d^3*e - 8*a^3*b*d*e^3)/(e^5*(d + e*x)^(1/2)) + (4*b^2*(a*e - b*d)^2*(d + e*x)^(3/2))/e^5 + (8*b*(a*e - b*d)^3*(d + e*x)^(1/2))/e^5`

Reduce [B] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 186, normalized size of antiderivative = 1.51

$$\int \frac{(a^2 + 2abx + b^2x^2)^2}{(d + ex)^{3/2}} dx = \frac{2}{7}b^4e^4x^4 + \frac{8}{5}ab^3e^4x^3 - \frac{16}{35}b^4de^3x^3 + 4a^2b^2e^4x^2 - \frac{16}{5}ab^3de^3x^2 + \frac{32}{35}b^4d^2e^2x^2 + \dots$$

input `int((b^2*x^2+2*a*b*x+a^2)^2/(e*x+d)^(3/2),x)`output `(2*(-35*a**4*e**4 + 280*a**3*b*d*e**3 + 140*a**3*b*e**4*x - 560*a**2*b**2*d**2*e**2 - 280*a**2*b**2*d*e**3*x + 70*a**2*b**2*e**4*x**2 + 448*a*b**3*d**3*e + 224*a*b**3*d**2*e**2*x - 56*a*b**3*d*e**3*x**2 + 28*a*b**3*e**4*x**3 - 128*b**4*d**4 - 64*b**4*d**3*e*x + 16*b**4*d**2*e**2*x**2 - 8*b**4*d*e**3*x**3 + 5*b**4*e**4*x**4))/(35*sqrt(d + e*x)*e**5)`

3.160 $\int \frac{(a^2+2abx+b^2x^2)^2}{(d+ex)^{5/2}} dx$

| | |
|---|------|
| Optimal result | 1251 |
| Mathematica [A] (verified) | 1251 |
| Rubi [A] (verified) | 1252 |
| Maple [A] (verified) | 1253 |
| Fricas [A] (verification not implemented) | 1254 |
| Sympy [A] (verification not implemented) | 1255 |
| Maxima [A] (verification not implemented) | 1255 |
| Giac [B] (verification not implemented) | 1256 |
| Mupad [B] (verification not implemented) | 1256 |
| Reduce [B] (verification not implemented) | 1257 |

Optimal result

Integrand size = 28, antiderivative size = 125

$$\int \frac{(a^2 + 2abx + b^2x^2)^2}{(d + ex)^{5/2}} dx = -\frac{2(bd - ae)^4}{3e^5(d + ex)^{3/2}} + \frac{8b(bd - ae)^3}{e^5\sqrt{d + ex}} + \frac{12b^2(bd - ae)^2\sqrt{d + ex}}{e^5} - \frac{8b^3(bd - ae)(d + ex)^{3/2}}{3e^5} + \frac{2b^4(d + ex)^{5/2}}{5e^5}$$

output

```
-2/3*(-a*e+b*d)^4/e^5/(e*x+d)^(3/2)+8*b*(-a*e+b*d)^3/e^5/(e*x+d)^(1/2)+12*b^2*(-a*e+b*d)^2*(e*x+d)^(1/2)/e^5-8/3*b^3*(-a*e+b*d)*(e*x+d)^(3/2)/e^5+2/5*b^4*(e*x+d)^(5/2)/e^5
```

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 153, normalized size of antiderivative = 1.22

$$\int \frac{(a^2 + 2abx + b^2x^2)^2}{(d + ex)^{5/2}} dx = \frac{2(-5a^4e^4 - 20a^3be^3(2d + 3ex) + 30a^2b^2e^2(8d^2 + 12dex + 3e^2x^2) + 20ab^3e(-15d^2 - 10dex - 3e^2x^2) + 2b^4(8d^2 + 12dex + 3e^2x^2))}{(d + ex)^{5/2}}$$

input

```
Integrate[(a^2 + 2*a*b*x + b^2*x^2)^2/(d + e*x)^(5/2), x]
```


output

$$\frac{(2*(-5*a^4*e^4 - 20*a^3*b*e^3*(2*d + 3*e*x) + 30*a^2*b^2*e^2*(8*d^2 + 12*d*e*x + 3*e^2*x^2) + 20*a*b^3*e*(-16*d^3 - 24*d^2*e*x - 6*d*e^2*x^2 + e^3*x^3) + b^4*(128*d^4 + 192*d^3*e*x + 48*d^2*e^2*x^2 - 8*d*e^3*x^3 + 3*e^4*x^4)))/(15*e^5*(d + e*x)^(3/2))$$

Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 125, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {1098, 27, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a^2 + 2abx + b^2x^2)^2}{(d + ex)^{5/2}} dx$$

$$\downarrow 1098$$

$$\frac{\int \frac{b^4(a+bx)^4}{(d+ex)^{5/2}} dx}{b^4}$$

$$\downarrow 27$$

$$\int \frac{(a + bx)^4}{(d + ex)^{5/2}} dx$$

$$\downarrow 53$$

$$\int \left(-\frac{4b^3\sqrt{d+ex}(bd-ae)}{e^4} + \frac{6b^2(bd-ae)^2}{e^4\sqrt{d+ex}} - \frac{4b(bd-ae)^3}{e^4(d+ex)^{3/2}} + \frac{(ae-bd)^4}{e^4(d+ex)^{5/2}} + \frac{b^4(d+ex)^{3/2}}{e^4} \right) dx$$

$$\downarrow 2009$$

$$-\frac{8b^3(d+ex)^{3/2}(bd-ae)}{3e^5} + \frac{12b^2\sqrt{d+ex}(bd-ae)^2}{e^5} + \frac{8b(bd-ae)^3}{e^5\sqrt{d+ex}} - \frac{2(bd-ae)^4}{3e^5(d+ex)^{3/2}} + \frac{2b^4(d+ex)^{5/2}}{5e^5}$$

input

$$\text{Int}[(a^2 + 2*a*b*x + b^2*x^2)^2/(d + e*x)^(5/2), x]$$

output

$$\begin{aligned} & (-2*(b*d - a*e)^4)/(3*e^5*(d + e*x)^{(3/2)}) + (8*b*(b*d - a*e)^3)/(e^5*\text{Sqrt}[d + e*x]) \\ & + (12*b^2*(b*d - a*e)^2*\text{Sqrt}[d + e*x])/e^5 - (8*b^3*(b*d - a*e)*(d + e*x)^{(3/2)})/(3*e^5) \\ & + (2*b^4*(d + e*x)^{(5/2)})/(5*e^5) \end{aligned}$$
Defintions of rubi rules used

rule 27

$$\text{Int}[(a_*)(Fx_), x_Symbol] \rightarrow \text{Simp}[a \text{ Int}[Fx, x], x] \text{ ; FreeQ}[a, x] \ \&\& \ !\text{MatchQ}[Fx, (b_*)(Gx_)] \text{ ; FreeQ}[b, x]$$

rule 53

$$\begin{aligned} & \text{Int}[((a_.) + (b_.)*(x_))^{(m_.)}*((c_.) + (d_.)*(x_))^{(n_.)}, x_Symbol] \rightarrow \text{Int} \\ & [\text{ExpandIntegrand}[(a + b*x)^m*(c + d*x)^n, x], x] \text{ ; FreeQ}[\{a, b, c, d, n\}, x] \\ & \ \&\& \ \text{IGtQ}[m, 0] \ \&\& \ (\ !\text{IntegerQ}[n] \ || \ (\text{EqQ}[c, 0] \ \&\& \ \text{LeQ}[7*m + 4*n + 4, 0]) \\ & \ || \ \text{LtQ}[9*m + 5*(n + 1), 0] \ || \ \text{GtQ}[m + n + 2, 0]) \end{aligned}$$

rule 1098

$$\begin{aligned} & \text{Int}[((d_.) + (e_.)*(x_))^{(m_.)}*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^{(p_.)}, x_ \\ & \text{Symbol}] \rightarrow \text{Simp}[1/c^p \text{ Int}[(d + e*x)^m*(b/2 + c*x)^{(2*p)}, x], x] \text{ ; FreeQ} \\ & \{a, b, c, d, e, m\}, x] \ \&\& \ \text{EqQ}[b^2 - 4*a*c, 0] \ \&\& \ \text{IntegerQ}[p] \end{aligned}$$

rule 2009

$$\text{Int}[u_, x_Symbol] \rightarrow \text{Simp}[\text{IntSum}[u, x], x] \text{ ; SumQ}[u]$$
Maple [A] (verified)

Time = 1.21 (sec) , antiderivative size = 128, normalized size of antiderivative = 1.02

| method | result |
|------------------|--|
| risch | $\frac{2b^2(3b^2e^2x^2+20xab e^2-14b^2dex+90e^2a^2-160abde+73b^2d^2)\sqrt{ex+d}}{15e^5} - \frac{2(12bex+ae+11bd)(e^3a^3-3a^2bd e^2+3a b^2d e^2)}{3e^5(ex+d)^{\frac{3}{2}}}$ |
| pseudoelliptic | $-\frac{2\left(-\frac{3}{5}b^4x^4-4ab^3x^3-18a^2b^2x^2+12a^3bx+a^4\right)e^4+8\left(\frac{1}{5}b^3x^3+3ab^2x^2-9a^2bx+a^3\right)bd e^3-48\left(\frac{1}{5}b^2x^2-2abx+a^2\right)b^2d^2}{3(ex+d)^{\frac{3}{2}}e^5}$ |
| gospers | $-\frac{2(-3b^4x^4e^4-20x^3ab^3e^4+8x^3b^4de^3-90x^2a^2b^2e^4+120x^2ab^3de^3-48x^2b^4d^2e^2+60xa^3be^4-360xa^2b^2de^3+480xa^3b^2d^2e^2)}{15(ex+d)^{\frac{3}{2}}e^5}$ |
| trager | $-\frac{2(-3b^4x^4e^4-20x^3ab^3e^4+8x^3b^4de^3-90x^2a^2b^2e^4+120x^2ab^3de^3-48x^2b^4d^2e^2+60xa^3be^4-360xa^2b^2de^3+480xa^3b^2d^2e^2)}{15(ex+d)^{\frac{3}{2}}e^5}$ |
| derivativdivides | $\frac{2b^4(ex+d)^{\frac{5}{2}}}{5} + \frac{8ab^3e(ex+d)^{\frac{3}{2}}}{3} - \frac{8b^4d(ex+d)^{\frac{3}{2}}}{3} + 12a^2b^2e^2\sqrt{ex+d} - 24ab^3de\sqrt{ex+d} + 12b^4d^2\sqrt{ex+d} - \frac{8b(e^3a^3-3a^2bd e^2+3a b^2d e^2)}{\sqrt{ex+d}}$ |
| default | $\frac{2b^4(ex+d)^{\frac{5}{2}}}{5} + \frac{8ab^3e(ex+d)^{\frac{3}{2}}}{3} - \frac{8b^4d(ex+d)^{\frac{3}{2}}}{3} + 12a^2b^2e^2\sqrt{ex+d} - 24ab^3de\sqrt{ex+d} + 12b^4d^2\sqrt{ex+d} - \frac{8b(e^3a^3-3a^2bd e^2+3a b^2d e^2)}{\sqrt{ex+d}}$ |
| oring | $-\frac{2(-3b^4x^4e^4-20x^3ab^3e^4+8x^3b^4de^3-90x^2a^2b^2e^4+120x^2ab^3de^3-48x^2b^4d^2e^2+60xa^3be^4-360xa^2b^2de^3+480xa^3b^2d^2e^2)}{15e^5(bx+a)^4(ex+d)}$ |

```
input int((b^2*x^2+2*a*b*x+a^2)^2/(e*x+d)^(5/2),x,method=_RETURNVERBOSE)
```

```
output 2/15*b^2*(3*b^2*e^2*x^2+20*a*b*e^2*x-14*b^2*d*e*x+90*a^2*e^2-160*a*b*d*e+7
3*b^2*d^2)*(e*x+d)^(1/2)/e^5-2/3*(12*b*e*x+a*e+11*b*d)*(a^3*e^3-3*a^2*b*d*
e^2+3*a*b^2*d^2*e-b^3*d^3)/e^5/(e*x+d)^(3/2)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 203, normalized size of antiderivative = 1.62

$$\int \frac{(a^2 + 2abx + b^2x^2)^2}{(d + ex)^{5/2}} dx = \frac{2(3b^4e^4x^4 + 128b^4d^4 - 320ab^3d^3e + 240a^2b^2d^2e^2 - 40a^3bde^3 - 5a^4e^4 - 4(2$$

```
input integrate((b^2*x^2+2*a*b*x+a^2)^2/(e*x+d)^(5/2),x, algorithm="fricas")
```

output

$$\begin{aligned} & 2/15*(3*b^4*e^4*x^4 + 128*b^4*d^4 - 320*a*b^3*d^3*e + 240*a^2*b^2*d^2*e^2 \\ & - 40*a^3*b*d*e^3 - 5*a^4*e^4 - 4*(2*b^4*d*e^3 - 5*a*b^3*e^4)*x^3 + 6*(8*b^4 \\ & *d^2*e^2 - 20*a*b^3*d*e^3 + 15*a^2*b^2*e^4)*x^2 + 12*(16*b^4*d^3*e - 40*a \\ & *b^3*d^2*e^2 + 30*a^2*b^2*d*e^3 - 5*a^3*b*e^4)*x)*\text{sqrt}(e*x + d)/(e^7*x^2 + \\ & 2*d*e^6*x + d^2*e^5) \end{aligned}$$

Sympy [A] (verification not implemented)

Time = 3.49 (sec) , antiderivative size = 190, normalized size of antiderivative = 1.52

$$\int \frac{(a^2 + 2abx + b^2x^2)^2}{(d + ex)^{5/2}} dx = \left\{ \begin{array}{l} \frac{2 \left(\frac{b^4(d+ex)^{5/2}}{5e^4} - \frac{4b(ae-bd)^3}{e^4\sqrt{d+ex}} + \frac{(d+ex)^{3/2} \cdot (4ab^3e - 4b^4d)}{3e^4} + \frac{\sqrt{d+ex}(6a^2b^2e^2 - 12ab^3de + 6b^4d^2)}{e^4} - \frac{(ae-bd)^4}{3e^4(d+ex)^{3/2}} \right)}{e} \\ \frac{a^4x + 2a^3bx^2 + 2a^2b^2x^3 + ab^3x^4 + \frac{b^4x^5}{5}}{d^{5/2}} \end{array} \right.$$

input

```
integrate((b**2*x**2+2*a*b*x+a**2)**2/(e*x+d)**(5/2), x)
```

output

```
Piecewise((2*(b**4*(d + e*x)**(5/2)/(5*e**4) - 4*b*(a*e - b*d)**3/(e**4*sqrt(d + e*x)) + (d + e*x)**(3/2)*(4*a*b**3*e - 4*b**4*d)/(3*e**4) + sqrt(d + e*x)*(6*a**2*b**2*e**2 - 12*a*b**3*d*e + 6*b**4*d**2)/e**4 - (a*e - b*d)**4/(3*e**4*(d + e*x)**(3/2)))/e, Ne(e, 0)), ((a**4*x + 2*a**3*b*x**2 + 2*a**2*b**2*x**3 + a*b**3*x**4 + b**4*x**5/5)/d**(5/2), True))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 187, normalized size of antiderivative = 1.50

$$\int \frac{(a^2 + 2abx + b^2x^2)^2}{(d + ex)^{5/2}} dx = \frac{2 \left(\frac{3(ex+d)^{5/2}b^4 - 20(b^4d - ab^3e)(ex+d)^{3/2} + 90(b^4d^2 - 2ab^3de + a^2b^2e^2)\sqrt{ex+d}}{e^4} - \frac{5(b^4d^4 - 4ab^3d^3e + 6a^2b^2d^2e^2 - 4ab^3d^3e + 6a^2b^2d^2e^2)}{15e} \right)}{15e}$$

input

```
integrate((b^2*x^2+2*a*b*x+a^2)^2/(e*x+d)^(5/2), x, algorithm="maxima")
```

output

$$\frac{2/15*((3*(e*x + d)^{(5/2)}*b^4 - 20*(b^4*d - a*b^3*e)*(e*x + d)^{(3/2)} + 90*(b^4*d^2 - 2*a*b^3*d*e + a^2*b^2*e^2)*\sqrt{e*x + d})/e^4 - 5*(b^4*d^4 - 4*a*b^3*d^3*e + 6*a^2*b^2*d^2*e^2 - 4*a^3*b*d*e^3 + a^4*e^4 - 12*(b^4*d^3 - 3*a*b^3*d^2*e + 3*a^2*b^2*d*e^2 - a^3*b*e^3)*(e*x + d))/((e*x + d)^{(3/2)}*e^4))/e$$

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 229 vs. $2(109) = 218$.

Time = 0.12 (sec) , antiderivative size = 229, normalized size of antiderivative = 1.83

$$\int \frac{(a^2 + 2abx + b^2x^2)^2}{(d + ex)^{5/2}} dx = \frac{2(12(ex + d)b^4d^3 - b^4d^4 - 36(ex + d)ab^3d^2e + 4ab^3d^3e + 36(ex + d)a^2b^2de)}{3(ex + d)^{3/2}e^5} + \frac{2\left(3(ex + d)^{5/2}b^4e^{20} - 20(ex + d)^{3/2}b^4de^{20} + 90\sqrt{ex + d}b^4d^2e^{20} + 20(ex + d)^{3/2}ab^3e^{21} - 180\sqrt{ex + d}ab^3de^{21}\right)}{15e^{25}}$$

input

```
integrate((b^2*x^2+2*a*b*x+a^2)^2/(e*x+d)^(5/2),x, algorithm="giac")
```

output

$$\frac{2/3*(12*(e*x + d)*b^4*d^3 - b^4*d^4 - 36*(e*x + d)*a*b^3*d^2*e + 4*a*b^3*d^3*e + 36*(e*x + d)*a^2*b^2*d*e^2 - 6*a^2*b^2*d^2*e^2 - 12*(e*x + d)*a^3*b*e^3 + 4*a^3*b*d*e^3 - a^4*e^4)/((e*x + d)^{(3/2)}*e^5) + 2/15*(3*(e*x + d)^{(5/2)}*b^4*e^{20} - 20*(e*x + d)^{(3/2)}*b^4*d*e^{20} + 90*\sqrt{e*x + d}*b^4*d^2*e^{20} + 20*(e*x + d)^{(3/2)}*a*b^3*e^{21} - 180*\sqrt{e*x + d}*a*b^3*d*e^{21} + 90*\sqrt{e*x + d}*a^2*b^2*e^{22})/e^{25}}$$

Mupad [B] (verification not implemented)

Time = 9.22 (sec) , antiderivative size = 175, normalized size of antiderivative = 1.40

$$\int \frac{(a^2 + 2abx + b^2x^2)^2}{(d + ex)^{5/2}} dx = \frac{(d + ex)(-8a^3be^3 + 24a^2b^2de^2 - 24ab^3d^2e + 8b^4d^3) - \frac{2a^4e^4}{3} - \frac{2b^4d^4}{3}}{e^5(d + ex)^{3/2}} + \frac{2b^4(d + ex)^{5/2}}{5e^5} - \frac{(8b^4d - 8ab^3e)(d + ex)^{3/2}}{3e^5} + \frac{12b^2(ae - bd)^2\sqrt{d + ex}}{e^5}$$

input

```
int((a^2 + b^2*x^2 + 2*a*b*x)^2/(d + e*x)^(5/2),x)
```

output

```
((d + e*x)*(8*b^4*d^3 - 8*a^3*b*e^3 + 24*a^2*b^2*d*e^2 - 24*a*b^3*d^2*e) -
(2*a^4*e^4)/3 - (2*b^4*d^4)/3 - 4*a^2*b^2*d^2*e^2 + (8*a*b^3*d^3*e)/3 + (
8*a^3*b*d*e^3)/3)/(e^5*(d + e*x)^(3/2)) + (2*b^4*(d + e*x)^(5/2))/(5*e^5)
- ((8*b^4*d - 8*a*b^3*e)*(d + e*x)^(3/2))/(3*e^5) + (12*b^2*(a*e - b*d)^2*
(d + e*x)^(1/2))/e^5
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 193, normalized size of antiderivative = 1.54

$$\int \frac{(a^2 + 2abx + b^2x^2)^2}{(d + ex)^{5/2}} dx = \frac{2}{5}b^4e^4x^4 + \frac{8}{3}ab^3e^4x^3 - \frac{16}{15}b^4de^3x^3 + 12a^2b^2e^4x^2 - 16ab^3de^3x^2 + \frac{32}{5}b^4d^2e^2x^2 -$$

input

```
int((b^2*x^2+2*a*b*x+a^2)^2/(e*x+d)^(5/2),x)
```

output

```
(2*( - 5*a**4*e**4 - 40*a**3*b*d*e**3 - 60*a**3*b*e**4*x + 240*a**2*b**2*d
**2*e**2 + 360*a**2*b**2*d*e**3*x + 90*a**2*b**2*e**4*x**2 - 320*a*b**3*d*
*3*e - 480*a*b**3*d**2*e**2*x - 120*a*b**3*d*e**3*x**2 + 20*a*b**3*e**4*x*
*3 + 128*b**4*d**4 + 192*b**4*d**3*e*x + 48*b**4*d**2*e**2*x**2 - 8*b**4*d
*e**3*x**3 + 3*b**4*e**4*x**4))/(15*sqrt(d + e*x)*e**5*(d + e*x))
```

3.161 $\int \frac{(a^2+2abx+b^2x^2)^2}{(d+ex)^{7/2}} dx$

| | |
|---|------|
| Optimal result | 1258 |
| Mathematica [A] (verified) | 1258 |
| Rubi [A] (verified) | 1259 |
| Maple [A] (verified) | 1260 |
| Fricas [A] (verification not implemented) | 1261 |
| Sympy [B] (verification not implemented) | 1262 |
| Maxima [A] (verification not implemented) | 1263 |
| Giac [B] (verification not implemented) | 1263 |
| Mupad [B] (verification not implemented) | 1264 |
| Reduce [B] (verification not implemented) | 1264 |

Optimal result

Integrand size = 28, antiderivative size = 125

$$\int \frac{(a^2 + 2abx + b^2x^2)^2}{(d + ex)^{7/2}} dx = -\frac{2(bd - ae)^4}{5e^5(d + ex)^{5/2}} + \frac{8b(bd - ae)^3}{3e^5(d + ex)^{3/2}} - \frac{12b^2(bd - ae)^2}{e^5\sqrt{d + ex}} - \frac{8b^3(bd - ae)\sqrt{d + ex}}{e^5} + \frac{2b^4(d + ex)^{3/2}}{3e^5}$$

output

```
-2/5*(-a*e+b*d)^4/e^5/(e*x+d)^(5/2)+8/3*b*(-a*e+b*d)^3/e^5/(e*x+d)^(3/2)-1
2*b^2*(-a*e+b*d)^2/e^5/(e*x+d)^(1/2)-8*b^3*(-a*e+b*d)*(e*x+d)^(1/2)/e^5+2/
3*b^4*(e*x+d)^(3/2)/e^5
```

Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 154, normalized size of antiderivative = 1.23

$$\int \frac{(a^2 + 2abx + b^2x^2)^2}{(d + ex)^{7/2}} dx = \frac{2(3a^4e^4 + 4a^3be^3(2d + 5ex) + 6a^2b^2e^2(8d^2 + 20dex + 15e^2x^2) - 12ab^3e(16d^3 + 40d^2ex + 30de^2x^2 + 5e^3x^3) + 3b^4e^4(d + ex)^{3/2})}{15e^5(d + ex)^{5/2}}$$

input

```
Integrate[(a^2 + 2*a*b*x + b^2*x^2)^2/(d + e*x)^(7/2),x]
```

output

$$\frac{(-2*(3*a^4*e^4 + 4*a^3*b*e^3*(2*d + 5*e*x) + 6*a^2*b^2*e^2*(8*d^2 + 20*d*e*x + 15*e^2*x^2) - 12*a*b^3*e*(16*d^3 + 40*d^2*e*x + 30*d*e^2*x^2 + 5*e^3*x^3) + b^4*(128*d^4 + 320*d^3*e*x + 240*d^2*e^2*x^2 + 40*d*e^3*x^3 - 5*e^4*x^4)))/(15*e^5*(d + e*x)^(5/2))$$

Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 125, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {1098, 27, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\begin{aligned} & \int \frac{(a^2 + 2abx + b^2x^2)^2}{(d + ex)^{7/2}} dx \\ & \quad \downarrow 1098 \\ & \int \frac{b^4(a+bx)^4}{(d+ex)^{7/2}} dx \\ & \quad \downarrow 27 \\ & \int \frac{(a + bx)^4}{(d + ex)^{7/2}} dx \\ & \quad \downarrow 53 \\ & \int \left(-\frac{4b^3(bd - ae)}{e^4\sqrt{d + ex}} + \frac{6b^2(bd - ae)^2}{e^4(d + ex)^{3/2}} - \frac{4b(bd - ae)^3}{e^4(d + ex)^{5/2}} + \frac{(ae - bd)^4}{e^4(d + ex)^{7/2}} + \frac{b^4\sqrt{d + ex}}{e^4} \right) dx \\ & \quad \downarrow 2009 \\ & -\frac{8b^3\sqrt{d + ex}(bd - ae)}{e^5} - \frac{12b^2(bd - ae)^2}{e^5\sqrt{d + ex}} + \frac{8b(bd - ae)^3}{3e^5(d + ex)^{3/2}} - \frac{2(bd - ae)^4}{5e^5(d + ex)^{5/2}} + \frac{2b^4(d + ex)^{3/2}}{3e^5} \end{aligned}$$

input

$$\text{Int}[(a^2 + 2*a*b*x + b^2*x^2)^2/(d + e*x)^(7/2), x]$$

output

$$\begin{aligned} & (-2*(b*d - a*e)^4)/(5*e^5*(d + e*x)^{(5/2)}) + (8*b*(b*d - a*e)^3)/(3*e^5*(d \\ & + e*x)^{(3/2)}) - (12*b^2*(b*d - a*e)^2)/(e^5*\text{Sqrt}[d + e*x]) - (8*b^3*(b*d \\ & - a*e)*\text{Sqrt}[d + e*x])/e^5 + (2*b^4*(d + e*x)^{(3/2)})/(3*e^5) \end{aligned}$$
Defintions of rubi rules used

rule 27

$$\text{Int}[(a_*)*(F_x), x_Symbol] \rightarrow \text{Simp}[a \text{ Int}[F_x, x], x] \text{ ; FreeQ}[a, x] \ \&\& \ !\text{MatchQ}[F_x, (b_*)*(G_x)] \text{ ; FreeQ}[b, x]$$

rule 53

$$\begin{aligned} & \text{Int}[((a_*) + (b_*)*(x_))^{(m_*)}*((c_*) + (d_*)*(x_))^{(n_*)}, x_Symbol] \rightarrow \text{Int} \\ & [\text{ExpandIntegrand}[(a + b*x)^m*(c + d*x)^n, x], x] \text{ ; FreeQ}[\{a, b, c, d, n\}, \\ & x] \ \&\& \ \text{IGtQ}[m, 0] \ \&\& \ (\ !\text{IntegerQ}[n] \ || \ (\text{EqQ}[c, 0] \ \&\& \ \text{LeQ}[7*m + 4*n + 4, 0]) \\ & \ || \ \text{LtQ}[9*m + 5*(n + 1), 0] \ || \ \text{GtQ}[m + n + 2, 0]) \end{aligned}$$

rule 1098

$$\begin{aligned} & \text{Int}[((d_*) + (e_*)*(x_))^{(m_*)}*((a_*) + (b_*)*(x_*) + (c_*)*(x_*)^2)^{(p_*)}, x_ \\ & Symbol] \rightarrow \text{Simp}[1/c^p \text{ Int}[(d + e*x)^m*(b/2 + c*x)^{(2*p)}, x], x] \text{ ; FreeQ} \\ & \{a, b, c, d, e, m\}, x] \ \&\& \ \text{EqQ}[b^2 - 4*a*c, 0] \ \&\& \ \text{IntegerQ}[p] \end{aligned}$$

rule 2009

$$\text{Int}[u_, x_Symbol] \rightarrow \text{Simp}[\text{IntSum}[u, x], x] \text{ ; SumQ}[u]$$
Maple [A] (verified)

Time = 1.23 (sec) , antiderivative size = 131, normalized size of antiderivative = 1.05

| method | result |
|------------------|---|
| risch | $\frac{2b^3(bex+12ae-11bd)\sqrt{ex+d}}{3e^5} - \frac{2(90b^2e^2x^2+20xab e^2+160b^2dex+3e^2a^2+14abde+73b^2d^2)(e^2a^2-2abde+b^2d^2)}{15e^5\sqrt{ex+d}(e^2x^2+2dex+d^2)}$ |
| pseudoelliptic | $2\left(-\frac{5}{3}b^4x^4-20ab^3x^3+30a^2b^2x^2+\frac{20}{3}a^3bx+a^4\right)e^4+\frac{8bd(5b^3x^3-45ab^2x^2+15a^2bx+a^3)e^3}{3}+16b^2d^2(5b^2x^2-10abx+a^2)$ |
| derivativdivides | $-\frac{5(ex+d)^{\frac{5}{2}}e^5}{e^5}$ |
| default | $\frac{2b^4\frac{(ex+d)^{\frac{3}{2}}}{3}+8ab^3e\sqrt{ex+d}-8b^4d\sqrt{ex+d}-2(a^4e^4-4a^3bde^3+6a^2b^2d^2e^2-4ab^3d^3e+b^4d^4)}{5(ex+d)^{\frac{5}{2}}}-\frac{12b^2(e^2a^2-2abde+b^2d^2)}{\sqrt{ex+d}}-\frac{8b^2d^2}{e^5}$ |
| gospers | $-\frac{2(-5b^4x^4e^4-60x^3ab^3e^4+40x^3b^4de^3+90x^2a^2b^2e^4-360x^2ab^3de^3+240x^2b^4d^2e^2+20xa^3be^4+120xa^2b^2de^3-480x^2a^2b^2d^2e^2)}{15(ex+d)^{\frac{5}{2}}e^5}$ |
| trager | $-\frac{2(-5b^4x^4e^4-60x^3ab^3e^4+40x^3b^4de^3+90x^2a^2b^2e^4-360x^2ab^3de^3+240x^2b^4d^2e^2+20xa^3be^4+120xa^2b^2de^3-480x^2a^2b^2d^2e^2)}{15(ex+d)^{\frac{5}{2}}e^5}$ |
| orering | $-\frac{2(-5b^4x^4e^4-60x^3ab^3e^4+40x^3b^4de^3+90x^2a^2b^2e^4-360x^2ab^3de^3+240x^2b^4d^2e^2+20xa^3be^4+120xa^2b^2de^3-480x^2a^2b^2d^2e^2)}{15e^5(bx+a)^4(ex+d)}$ |

input `int((b^2*x^2+2*a*b*x+a^2)^2/(e*x+d)^(7/2),x,method=_RETURNVERBOSE)`

output `2/3*b^3*(b*e*x+12*a*e-11*b*d)*(e*x+d)^(1/2)/e^5-2/15*(90*b^2*e^2*x^2+20*a*b*e^2*x+160*b^2*d*e*x+3*a^2*e^2+14*a*b*d*e+73*b^2*d^2)*(a^2*e^2-2*a*b*d*e+b^2*d^2)/e^5/(e*x+d)^(1/2)/(e^2*x^2+2*d*e*x+d^2)`

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 213, normalized size of antiderivative = 1.70

$$\int \frac{(a^2 + 2abx + b^2x^2)^2}{(d + ex)^{7/2}} dx = \frac{2(5b^4e^4x^4 - 128b^4d^4 + 192ab^3d^3e - 48a^2b^2d^2e^2 - 8a^3bde^3 - 3a^4e^4 - 20(2$$

input `integrate((b^2*x^2+2*a*b*x+a^2)^2/(e*x+d)^(7/2),x, algorithm="fricas")`

output

```
2/15*(5*b^4*e^4*x^4 - 128*b^4*d^4 + 192*a*b^3*d^3*e - 48*a^2*b^2*d^2*e^2 -
8*a^3*b*d*e^3 - 3*a^4*e^4 - 20*(2*b^4*d*e^3 - 3*a*b^3*e^4)*x^3 - 30*(8*b^
4*d^2*e^2 - 12*a*b^3*d*e^3 + 3*a^2*b^2*e^4)*x^2 - 20*(16*b^4*d^3*e - 24*a*
b^3*d^2*e^2 + 6*a^2*b^2*d*e^3 + a^3*b*e^4)*x)*sqrt(e*x + d)/(e^8*x^3 + 3*d
*e^7*x^2 + 3*d^2*e^6*x + d^3*e^5)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1008 vs. $2(116) = 232$.

Time = 0.49 (sec) , antiderivative size = 1008, normalized size of antiderivative = 8.06

$$\int \frac{(a^2 + 2abx + b^2x^2)^2}{(d + ex)^{7/2}} dx = \text{Too large to display}$$

input

```
integrate((b**2*x**2+2*a*b*x+a**2)**2/(e*x+d)**(7/2),x)
```

output

```
Piecewise((-6*a**4*e**4/(15*d**2*e**5*sqrt(d + e*x) + 30*d*e**6*x*sqrt(d +
e*x) + 15*e**7*x**2*sqrt(d + e*x)) - 16*a**3*b*d*e**3/(15*d**2*e**5*sqrt(
d + e*x) + 30*d*e**6*x*sqrt(d + e*x) + 15*e**7*x**2*sqrt(d + e*x)) - 40*a*
**3*b*e**4*x/(15*d**2*e**5*sqrt(d + e*x) + 30*d*e**6*x*sqrt(d + e*x) + 15*e
**7*x**2*sqrt(d + e*x)) - 96*a**2*b**2*d**2*e**2/(15*d**2*e**5*sqrt(d + e
x) + 30*d*e**6*x*sqrt(d + e*x) + 15*e**7*x**2*sqrt(d + e*x)) - 240*a**2*b*
**2*d*e**3*x/(15*d**2*e**5*sqrt(d + e*x) + 30*d*e**6*x*sqrt(d + e*x) + 15*e
**7*x**2*sqrt(d + e*x)) - 180*a**2*b**2*e**4*x**2/(15*d**2*e**5*sqrt(d + e
*x) + 30*d*e**6*x*sqrt(d + e*x) + 15*e**7*x**2*sqrt(d + e*x)) + 384*a*b**3
*d**3*e/(15*d**2*e**5*sqrt(d + e*x) + 30*d*e**6*x*sqrt(d + e*x) + 15*e**7*
x**2*sqrt(d + e*x)) + 960*a*b**3*d**2*e**2*x/(15*d**2*e**5*sqrt(d + e*x) +
30*d*e**6*x*sqrt(d + e*x) + 15*e**7*x**2*sqrt(d + e*x)) + 720*a*b**3*d*e*
**3*x**2/(15*d**2*e**5*sqrt(d + e*x) + 30*d*e**6*x*sqrt(d + e*x) + 15*e**7*
x**2*sqrt(d + e*x)) + 120*a*b**3*e**4*x**3/(15*d**2*e**5*sqrt(d + e*x) + 3
0*d*e**6*x*sqrt(d + e*x) + 15*e**7*x**2*sqrt(d + e*x)) - 256*b**4*d**4/(15
*d**2*e**5*sqrt(d + e*x) + 30*d*e**6*x*sqrt(d + e*x) + 15*e**7*x**2*sqrt(d
+ e*x)) - 640*b**4*d**3*e*x/(15*d**2*e**5*sqrt(d + e*x) + 30*d*e**6*x*sqr
t(d + e*x) + 15*e**7*x**2*sqrt(d + e*x)) - 480*b**4*d**2*e**2*x**2/(15*d**
2*e**5*sqrt(d + e*x) + 30*d*e**6*x*sqrt(d + e*x) + 15*e**7*x**2*sqrt(d + e
*x)) - 80*b**4*d*e**3*x**3/(15*d**2*e**5*sqrt(d + e*x) + 30*d*e**6*x*sq...
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 189, normalized size of antiderivative = 1.51

$$\int \frac{(a^2 + 2abx + b^2x^2)^2}{(d + ex)^{7/2}} dx = \frac{2 \left(\frac{5 \left((ex+d)^{\frac{3}{2}} b^4 - 12 (b^4 d - ab^3 e) \sqrt{ex+d} \right)}{e^4} - \frac{3 b^4 d^4 - 12 ab^3 d^3 e + 18 a^2 b^2 d^2 e^2 - 12 a^3 b d e^3 + 3 a^4 e^4 + 90 (d^5 - 5 d^4 e + 10 d^3 e^2 - 10 d^2 e^3 + 5 d e^4 - e^5)}{15 e} \right)}{15 e}$$

input `integrate((b^2*x^2+2*a*b*x+a^2)^2/(e*x+d)^(7/2),x, algorithm="maxima")`

output $\frac{2}{15} * (5 * ((e*x + d)^{(3/2)} * b^4 - 12 * (b^4 * d - a * b^3 * e) * \text{sqrt}(e*x + d)) / e^4 - (3 * b^4 * d^4 - 12 * a * b^3 * d^3 * e + 18 * a^2 * b^2 * d^2 * e^2 - 12 * a^3 * b * d * e^3 + 3 * a^4 * e^4 + 90 * (b^4 * d^2 - 2 * a * b^3 * d * e + a^2 * b^2 * e^2) * (e*x + d)^2 - 20 * (b^4 * d^3 - 3 * a * b^3 * d^2 * e + 3 * a^2 * b^2 * d * e^2 - a^3 * b * e^3) * (e*x + d)) / ((e*x + d)^{(5/2)} * e^4)) / e$

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 223 vs. 2(109) = 218.

Time = 0.20 (sec) , antiderivative size = 223, normalized size of antiderivative = 1.78

$$\int \frac{(a^2 + 2abx + b^2x^2)^2}{(d + ex)^{7/2}} dx = \frac{2 \left(90 (ex + d)^2 b^4 d^2 - 20 (ex + d) b^4 d^3 + 3 b^4 d^4 - 180 (ex + d)^2 ab^3 d e + 60 (ex + d) ab^3 d^2 e - 12 ab^3 d^3 e + 90 d^5 - 50 d^4 e + 10 d^3 e^2 - 10 d^2 e^3 + 5 d e^4 - e^5 \right)}{15 (ex + d)^{\frac{5}{2}} e^4} + \frac{2 \left((ex + d)^{\frac{3}{2}} b^4 e^{10} - 12 \sqrt{ex + d} b^4 d e^{10} + 12 \sqrt{ex + d} ab^3 e^{11} \right)}{3 e^{15}}$$

input `integrate((b^2*x^2+2*a*b*x+a^2)^2/(e*x+d)^(7/2),x, algorithm="giac")`

output $\frac{-2}{15} * (90 * (e*x + d)^2 * b^4 * d^2 - 20 * (e*x + d) * b^4 * d^3 + 3 * b^4 * d^4 - 180 * (e*x + d)^2 * a * b^3 * d * e + 60 * (e*x + d) * a * b^3 * d^2 * e - 12 * a * b^3 * d^3 * e + 90 * (e*x + d)^2 * a^2 * b^2 * e^2 - 60 * (e*x + d) * a^2 * b^2 * d * e^2 + 18 * a^2 * b^2 * d^2 * e^2 + 20 * (e*x + d) * a^3 * b * e^3 - 12 * a^3 * b * d * e^3 + 3 * a^4 * e^4) / ((e*x + d)^{(5/2)} * e^5) + 2 / 3 * ((e*x + d)^{(3/2)} * b^4 * e^{10} - 12 * \text{sqrt}(e*x + d) * b^4 * d * e^{10} + 12 * \text{sqrt}(e*x + d) * a * b^3 * e^{11}) / e^{15}$

3.162 $\int (d + ex)^{7/2} (a^2 + 2abx + b^2x^2)^3 dx$

| | |
|---|------|
| Optimal result | 1265 |
| Mathematica [A] (verified) | 1266 |
| Rubi [A] (verified) | 1266 |
| Maple [A] (verified) | 1268 |
| Fricas [B] (verification not implemented) | 1269 |
| Sympy [B] (verification not implemented) | 1270 |
| Maxima [B] (verification not implemented) | 1270 |
| Giac [B] (verification not implemented) | 1271 |
| Mupad [B] (verification not implemented) | 1272 |
| Reduce [B] (verification not implemented) | 1273 |

Optimal result

Integrand size = 28, antiderivative size = 187

$$\int (d + ex)^{7/2} (a^2 + 2abx + b^2x^2)^3 dx = \frac{2(bd - ae)^6(d + ex)^{9/2}}{9e^7} - \frac{12b(bd - ae)^5(d + ex)^{11/2}}{11e^7} + \frac{30b^2(bd - ae)^4(d + ex)^{13/2}}{13e^7} - \frac{8b^3(bd - ae)^3(d + ex)^{15/2}}{3e^7} + \frac{30b^4(bd - ae)^2(d + ex)^{17/2}}{17e^7} - \frac{12b^5(bd - ae)(d + ex)^{19/2}}{19e^7} + \frac{2b^6(d + ex)^{21/2}}{21e^7}$$

output

```
2/9*(-a*e+b*d)^6*(e*x+d)^(9/2)/e^7-12/11*b*(-a*e+b*d)^5*(e*x+d)^(11/2)/e^7
+30/13*b^2*(-a*e+b*d)^4*(e*x+d)^(13/2)/e^7-8/3*b^3*(-a*e+b*d)^3*(e*x+d)^(1
5/2)/e^7+30/17*b^4*(-a*e+b*d)^2*(e*x+d)^(17/2)/e^7-12/19*b^5*(-a*e+b*d)*(e
*x+d)^(19/2)/e^7+2/21*b^6*(e*x+d)^(21/2)/e^7
```

Mathematica [A] (verified)

Time = 0.25 (sec) , antiderivative size = 291, normalized size of antiderivative = 1.56

$$\int (d + ex)^{7/2} (a^2 + 2abx + b^2x^2)^3 dx = \frac{2(d + ex)^{9/2} (323323a^6e^6 + 176358a^5be^5(-2d + 9ex) + 33915a^4b^2e^4(8d^2 - 36dex + 99e^2x^2) + b^2x^2)^3}{2909907e^7}$$

input `Integrate[(d + e*x)^(7/2)*(a^2 + 2*a*b*x + b^2*x^2)^3,x]`

output $(2*(d + e*x)^{(9/2)}*(323323*a^6*e^6 + 176358*a^5*b*e^5*(-2*d + 9*e*x) + 33915*a^4*b^2*e^4*(8*d^2 - 36*d*e*x + 99*e^2*x^2) + 9044*a^3*b^3*e^3*(-16*d^3 + 72*d^2*e*x - 198*d*e^2*x^2 + 429*e^3*x^3) + 399*a^2*b^4*e^2*(128*d^4 - 576*d^3*e*x + 1584*d^2*e^2*x^2 - 3432*d*e^3*x^3 + 6435*e^4*x^4) + 42*a*b^5*e*(-256*d^5 + 1152*d^4*e*x - 3168*d^3*e^2*x^2 + 6864*d^2*e^3*x^3 - 12870*d*e^4*x^4 + 21879*e^5*x^5) + b^6*(1024*d^6 - 4608*d^5*e*x + 12672*d^4*e^2*x^2 - 27456*d^3*e^3*x^3 + 51480*d^2*e^4*x^4 - 87516*d*e^5*x^5 + 138567*e^6*x^6))/(2909907*e^7)$

Rubi [A] (verified)

Time = 0.58 (sec) , antiderivative size = 187, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {1098, 27, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\begin{aligned} & \int (a^2 + 2abx + b^2x^2)^3 (d + ex)^{7/2} dx \\ & \quad \downarrow 1098 \\ & \frac{\int b^6 (a + bx)^6 (d + ex)^{7/2} dx}{b^6} \\ & \quad \downarrow 27 \\ & \int (a + bx)^6 (d + ex)^{7/2} dx \end{aligned}$$

↓ 53

$$\int \left(-\frac{6b^5(d+ex)^{17/2}(bd-ae)}{e^6} + \frac{15b^4(d+ex)^{15/2}(bd-ae)^2}{e^6} - \frac{20b^3(d+ex)^{13/2}(bd-ae)^3}{e^6} + \frac{15b^2(d+ex)^{11/2}}{e^6} \right)$$

↓ 2009

$$\begin{aligned} & -\frac{12b^5(d+ex)^{19/2}(bd-ae)}{19e^7} + \frac{30b^4(d+ex)^{17/2}(bd-ae)^2}{17e^7} - \frac{8b^3(d+ex)^{15/2}(bd-ae)^3}{3e^7} + \\ & \frac{30b^2(d+ex)^{13/2}(bd-ae)^4}{13e^7} - \frac{12b(d+ex)^{11/2}(bd-ae)^5}{11e^7} + \frac{2(d+ex)^{9/2}(bd-ae)^6}{9e^7} + \\ & \frac{2b^6(d+ex)^{21/2}}{21e^7} \end{aligned}$$

input `Int[(d + e*x)^(7/2)*(a^2 + 2*a*b*x + b^2*x^2)^3,x]`

output `(2*(b*d - a*e)^6*(d + e*x)^(9/2))/(9*e^7) - (12*b*(b*d - a*e)^5*(d + e*x)^(11/2))/(11*e^7) + (30*b^2*(b*d - a*e)^4*(d + e*x)^(13/2))/(13*e^7) - (8*b^3*(b*d - a*e)^3*(d + e*x)^(15/2))/(3*e^7) + (30*b^4*(b*d - a*e)^2*(d + e*x)^(17/2))/(17*e^7) - (12*b^5*(b*d - a*e)*(d + e*x)^(19/2))/(19*e^7) + (2*b^6*(d + e*x)^(21/2))/(21*e^7)`

Defintions of rubi rules used

rule 27 `Int[(a_)*(F_x_), x_Symbol] := Simp[a Int[F_x, x], x] /; FreeQ[a, x] && !MatchQ[F_x, (b_)*(G_x_)] /; FreeQ[b, x]`

rule 53 `Int[((a_) + (b_)*(x_))^(m_)*((c_) + (d_)*(x_))^(n_), x_Symbol] := Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && IGtQ[m, 0] && (!IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])`

rule 1098 `Int[((d_) + (e_)*(x_))^(m_)*((a_) + (b_)*(x_) + (c_)*(x_)^2)^(p_), x_Symbol] := Simp[1/c^p Int[(d + e*x)^m*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]`

rule 2009

```
Int[u_, x_Symbol] := Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 1.42 (sec) , antiderivative size = 276, normalized size of antiderivative = 1.48

| method | result |
|-------------------|--|
| pseudoelliptic | $2(ex+d)^{\frac{9}{2}} \left(\frac{54}{19} a b^5 x^5 + \frac{135}{17} a^2 b^4 x^4 + 12 a^3 b^3 x^3 + \frac{135}{13} a^4 b^2 x^2 + \frac{54}{11} a^5 b x + \frac{3}{7} b^6 x^6 + a^6 \right) e^6 - \frac{12}{133} \left(\frac{33}{133} b^5 x^5 + \frac{495}{323} a b^4 x^4 + \frac{66}{17} a^2 b^3 x^3 + \frac{5}{323} a^3 b^2 x^2 + \frac{45}{13} a^4 b x + a^5 \right) e^6$ |
| gospers | $2(ex+d)^{\frac{9}{2}} (138567x^6b^6e^6+918918x^5ab^5e^6-87516x^5b^6de^5+2567565x^4a^2b^4e^6-540540x^4ab^5de^5+51480x^4b^6d^2e^4+3879876x^3a^3b^3d^2e^4+120143(429/2261b^4x^4+1716/1615ab^3x^3+198/85a^2b^2x^2+12/5a^3bx+a^4)b^2d^2e^4-64/143(429/2261b^3x^3+297/323a^2b^2x^2+27/17a^2bx+a^3)b^3d^3e^3+384/2431((33/133b^2x^2+18/19abx+a^2)b^4d^4e^2-1536/46189b^5(3/7bx+a)d^5e+1024/323323b^6d^6)/e^7$ |
| orering | $2(138567x^6b^6e^6+918918x^5ab^5e^6-87516x^5b^6de^5+2567565x^4a^2b^4e^6-540540x^4ab^5de^5+51480x^4b^6d^2e^4+3879876x^3a^3b^3d^2e^4+120143(429/2261b^4x^4+1716/1615ab^3x^3+198/85a^2b^2x^2+12/5a^3bx+a^4)b^2d^2e^4-64/143(429/2261b^3x^3+297/323a^2b^2x^2+27/17a^2bx+a^3)b^3d^3e^3+384/2431((33/133b^2x^2+18/19abx+a^2)b^4d^4e^2-1536/46189b^5(3/7bx+a)d^5e+1024/323323b^6d^6)/e^7$ |
| derivativedivides | $\frac{2b^6(ex+d)^{\frac{21}{2}}}{21} + \frac{6(2abe-2b^2d)b^4(ex+d)^{\frac{19}{2}}}{19} + \frac{2((e^2a^2-2abde+b^2d^2)b^4+2(2abe-2b^2d)^2b^2+b^2(2(e^2a^2-2abde+b^2d^2)b^2+(2abx+a^2)b^3d^3e^3+384/2431((33/133b^2x^2+18/19abx+a^2)b^4d^4e^2-1536/46189b^5(3/7bx+a)d^5e+1024/323323b^6d^6)/e^7)}{17}$ |
| default | $\frac{2b^6(ex+d)^{\frac{21}{2}}}{21} + \frac{6(2abe-2b^2d)b^4(ex+d)^{\frac{19}{2}}}{19} + \frac{2((e^2a^2-2abde+b^2d^2)b^4+2(2abe-2b^2d)^2b^2+b^2(2(e^2a^2-2abde+b^2d^2)b^2+(2abx+a^2)b^3d^3e^3+384/2431((33/133b^2x^2+18/19abx+a^2)b^4d^4e^2-1536/46189b^5(3/7bx+a)d^5e+1024/323323b^6d^6)/e^7)}{17}$ |
| trager | $2(138567e^{10}b^6x^{10}+918918ab^5e^{10}x^9+466752b^6de^9x^9+2567565a^2b^4e^{10}x^8+3135132ab^5de^9x^8+532818b^6d^2e^8x^8+3879876a^3b^3d^2e^4x^7+120143(429/2261b^4x^4+1716/1615ab^3x^3+198/85a^2b^2x^2+12/5a^3bx+a^4)b^2d^2e^4x^6-64/143(429/2261b^3x^3+297/323a^2b^2x^2+27/17a^2bx+a^3)b^3d^3e^3x^5+384/2431((33/133b^2x^2+18/19abx+a^2)b^4d^4e^2-1536/46189b^5(3/7bx+a)d^5e+1024/323323b^6d^6)/e^7x^4$ |
| risch | $2(138567e^{10}b^6x^{10}+918918ab^5e^{10}x^9+466752b^6de^9x^9+2567565a^2b^4e^{10}x^8+3135132ab^5de^9x^8+532818b^6d^2e^8x^8+3879876a^3b^3d^2e^4x^7+120143(429/2261b^4x^4+1716/1615ab^3x^3+198/85a^2b^2x^2+12/5a^3bx+a^4)b^2d^2e^4x^6-64/143(429/2261b^3x^3+297/323a^2b^2x^2+27/17a^2bx+a^3)b^3d^3e^3x^5+384/2431((33/133b^2x^2+18/19abx+a^2)b^4d^4e^2-1536/46189b^5(3/7bx+a)d^5e+1024/323323b^6d^6)/e^7x^4$ |

input

```
int((e*x+d)^(7/2)*(b^2*x^2+2*a*b*x+a^2)^3,x,method=_RETURNVERBOSE)
```

output

```
2/9*(e*x+d)^(9/2)*((54/19*a*b^5*x^5+135/17*a^2*b^4*x^4+12*a^3*b^3*x^3+135/13*a^4*b^2*x^2+54/11*a^5*b*x+3/7*b^6*x^6+a^6)*e^6-12/11*(33/133*b^5*x^5+495/323*a*b^4*x^4+66/17*a^2*b^3*x^3+66/13*a^3*b^2*x^2+45/13*a^4*b*x+a^5)*b*d*e^5+120/143*(429/2261*b^4*x^4+1716/1615*a*b^3*x^3+198/85*a^2*b^2*x^2+12/5*a^3*b*x+a^4)*b^2*d^2*e^4-64/143*(429/2261*b^3*x^3+297/323*a*b^2*x^2+27/17*a^2*b*x+a^3)*b^3*d^3*e^3+384/2431*(33/133*b^2*x^2+18/19*a*b*x+a^2)*b^4*d^4*e^2-1536/46189*b^5*(3/7*b*x+a)*d^5*e+1024/323323*b^6*d^6)/e^7
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 729 vs. $2(159) = 318$.

Time = 0.10 (sec) , antiderivative size = 729, normalized size of antiderivative = 3.90

$$\int (d + ex)^{7/2} (a^2 + 2abx + b^2x^2)^3 dx = \frac{2(138567b^6e^{10}x^{10} + 1024b^6d^{10} - 10752ab^5d^9e + 51072a^2b^4d^8e^2 - 144704a^3b^3d^7e^3 + 271320a^4b^2d^6e^4 - 352716a^5b^1d^5e^5 + 323323a^6d^4e^6 + 14586(32b^6d^3e^7 + 63ab^5d^2e^8 + 10a^2b^4d^1e^9)x^9 + 3861(138b^6d^2e^8 + 812ab^5d^1e^9 + 665a^2b^4d^0e^{10})x^8 + 1716(121b^6d^3e^7 + 2121ab^5d^2e^8 + 5187a^2b^4d^1e^9 + 2261a^3b^3d^0e^{10})x^7 + 231(b^6d^4e^6 + 6288ab^5d^3e^7 + 45714a^2b^4d^2e^8 + 59432a^3b^3d^1e^9 + 14535a^4b^2d^0e^{10})x^6 - 126(2b^6d^5e^5 - 21ab^5d^4e^6 - 34542a^2b^4d^3e^7 - 133076a^3b^3d^2e^8 - 96900a^4b^2d^1e^9 - 12597a^5b^1d^0e^{10})x^5 + 7(40b^6d^6e^4 - 420ab^5d^5e^5 + 1995a^2b^4d^4e^6 + 1033600a^3b^3d^3e^7 + 2219010a^4b^2d^2e^8 + 856596a^5b^1d^1e^9 + 46189a^6d^0e^{10})x^4 - 4(80b^6d^7e^3 - 840ab^5d^6e^4 + 3990a^2b^4d^5e^5 - 11305a^3b^3d^4e^6 - 1797495a^4b^2d^3e^7 - 2028117a^5b^1d^2e^8 - 323323a^6d^1e^9)x^3 + 3(128b^6d^8e^2 - 1344ab^5d^7e^3 + 6384a^2b^4d^6e^4 - 18088a^3b^3d^5e^5 + 33915a^4b^2d^4e^6 + 1410864a^5b^1d^3e^7 + 646646a^6d^2e^8)x^2 - 2(256b^6d^9e - 2688ab^5d^8e^2 + 12768a^2b^4d^7e^3 - 36176a^3b^3d^6e^4 + 67830a^4b^2d^5e^5 - 88179a^5b^1d^4e^6 - 646646a^6d^3e^7)x) \sqrt{ex + d} / e^7$$

input

```
integrate((e*x+d)^(7/2)*(b^2*x^2+2*a*b*x+a^2)^3,x, algorithm="fricas")
```

output

```
2/2909907*(138567*b^6*e^10*x^10 + 1024*b^6*d^10 - 10752*a*b^5*d^9*e + 5107
2*a^2*b^4*d^8*e^2 - 144704*a^3*b^3*d^7*e^3 + 271320*a^4*b^2*d^6*e^4 - 3527
16*a^5*b*d^5*e^5 + 323323*a^6*d^4*e^6 + 14586*(32*b^6*d^3*e^7 + 63*a*b^5*e^1
0)*x^9 + 3861*(138*b^6*d^2*e^8 + 812*a*b^5*d^1*e^9 + 665*a^2*b^4*e^10)*x^8 +
1716*(121*b^6*d^3*e^7 + 2121*a*b^5*d^2*e^8 + 5187*a^2*b^4*d^1*e^9 + 2261*a^
3*b^3*e^10)*x^7 + 231*(b^6*d^4*e^6 + 6288*a*b^5*d^3*e^7 + 45714*a^2*b^4*d^
2*e^8 + 59432*a^3*b^3*d^1*e^9 + 14535*a^4*b^2*e^10)*x^6 - 126*(2*b^6*d^5*e^5
- 21*a*b^5*d^4*e^6 - 34542*a^2*b^4*d^3*e^7 - 133076*a^3*b^3*d^2*e^8 - 969
00*a^4*b^2*d^1*e^9 - 12597*a^5*b^1*e^10)*x^5 + 7*(40*b^6*d^6*e^4 - 420*a*b^5*d
^5*e^5 + 1995*a^2*b^4*d^4*e^6 + 1033600*a^3*b^3*d^3*e^7 + 2219010*a^4*b^2*d
^2*e^8 + 856596*a^5*b^1*d^1*e^9 + 46189*a^6*e^10)*x^4 - 4*(80*b^6*d^7*e^3 - 8
40*a*b^5*d^6*e^4 + 3990*a^2*b^4*d^5*e^5 - 11305*a^3*b^3*d^4*e^6 - 1797495*
a^4*b^2*d^3*e^7 - 2028117*a^5*b^1*d^2*e^8 - 323323*a^6*d^1*e^9)*x^3 + 3*(128*b
^6*d^8*e^2 - 1344*a*b^5*d^7*e^3 + 6384*a^2*b^4*d^6*e^4 - 18088*a^3*b^3*d^5
*e^5 + 33915*a^4*b^2*d^4*e^6 + 1410864*a^5*b^1*d^3*e^7 + 646646*a^6*d^2*e^8)
*x^2 - 2*(256*b^6*d^9*e - 2688*a*b^5*d^8*e^2 + 12768*a^2*b^4*d^7*e^3 - 361
76*a^3*b^3*d^6*e^4 + 67830*a^4*b^2*d^5*e^5 - 88179*a^5*b^1*d^4*e^6 - 646646*
a^6*d^3*e^7)*x)*sqrt(e*x + d)/e^7
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 495 vs. $2(173) = 346$.

Time = 1.49 (sec) , antiderivative size = 495, normalized size of antiderivative = 2.65

$$\int (d + ex)^{7/2} (a^2 + 2abx + b^2x^2)^3 dx = \frac{2 \left(\frac{b^6(d+ex)^{21/2}}{21e^6} + \frac{(d+ex)^{19/2} \cdot (6ab^5e - 6b^6d)}{19e^6} + \frac{(d+ex)^{17/2} \cdot (15a^2b^4e^2 - 30ab^5de + 15b^6d^2)}{17e^6} + \frac{(d+ex)^{15/2} \cdot (20a^3b^3e^3 - 60a^2b^4de^2 + 60ab^5d^2e - 60a^2b^4d^2e^2)}{15e^6} \right)}{d^{7/2} \left(a^6x + 3a^5bx^2 + 5a^4b^2x^3 + 5a^3b^3x^4 + 3a^2b^4x^5 + ab^5x^6 + \frac{b^6x^7}{7} \right)}$$

input `integrate((e*x+d)**(7/2)*(b**2*x**2+2*a*b*x+a**2)**3,x)`

output

```
Piecewise((2*(b**6*(d + e*x)**(21/2))/(21*e**6) + (d + e*x)**(19/2)*(6*a*b*
*5*e - 6*b**6*d)/(19*e**6) + (d + e*x)**(17/2)*(15*a**2*b**4*e**2 - 30*a*b
**5*d*e + 15*b**6*d**2)/(17*e**6) + (d + e*x)**(15/2)*(20*a**3*b**3*e**3 -
60*a**2*b**4*d*e**2 + 60*a*b**5*d**2*e - 20*b**6*d**3)/(15*e**6) + (d + e
*x)**(13/2)*(15*a**4*b**2*e**4 - 60*a**3*b**3*d*e**3 + 90*a**2*b**4*d**2*e
**2 - 60*a*b**5*d**3*e + 15*b**6*d**4)/(13*e**6) + (d + e*x)**(11/2)*(6*a*
*5*b*e**5 - 30*a**4*b**2*d*e**4 + 60*a**3*b**3*d**2*e**3 - 60*a**2*b**4*d*
*3*e**2 + 30*a*b**5*d**4*e - 6*b**6*d**5)/(11*e**6) + (d + e*x)**(9/2)*(a*
*6*e**6 - 6*a**5*b*d*e**5 + 15*a**4*b**2*d**2*e**4 - 20*a**3*b**3*d**3*e**
3 + 15*a**2*b**4*d**4*e**2 - 6*a*b**5*d**5*e + b**6*d**6)/(9*e**6))/e, Ne(
e, 0)), (d**(7/2)*(a**6*x + 3*a**5*b*x**2 + 5*a**4*b**2*x**3 + 5*a**3*b**3
*x**4 + 3*a**2*b**4*x**5 + a*b**5*x**6 + b**6*x**7/7), True))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 350 vs. $2(159) = 318$.

Time = 0.05 (sec) , antiderivative size = 350, normalized size of antiderivative = 1.87

$$\int (d + ex)^{7/2} (a^2 + 2abx + b^2x^2)^3 dx = \frac{2 \left(138567 (ex + d)^{21/2} b^6 - 918918 (b^6d - ab^5e)(ex + d)^{19/2} + 2567565 (b^6d^2 - 2ab^5de + a^2b^4e^2) \right)}{d^{7/2} \left(a^6x + 3a^5bx^2 + 5a^4b^2x^3 + 5a^3b^3x^4 + 3a^2b^4x^5 + ab^5x^6 + \frac{b^6x^7}{7} \right)}$$

input `integrate((e*x+d)^(7/2)*(b^2*x^2+2*a*b*x+a^2)^3,x, algorithm="maxima")`

output `2/2909907*(138567*(e*x + d)^(21/2)*b^6 - 918918*(b^6*d - a*b^5*e)*(e*x + d)^(19/2) + 2567565*(b^6*d^2 - 2*a*b^5*d*e + a^2*b^4*e^2)*(e*x + d)^(17/2) - 3879876*(b^6*d^3 - 3*a*b^5*d^2*e + 3*a^2*b^4*d*e^2 - a^3*b^3*e^3)*(e*x + d)^(15/2) + 3357585*(b^6*d^4 - 4*a*b^5*d^3*e + 6*a^2*b^4*d^2*e^2 - 4*a^3*b^3*d*e^3 + a^4*b^2*e^4)*(e*x + d)^(13/2) - 1587222*(b^6*d^5 - 5*a*b^5*d^4*e + 10*a^2*b^4*d^3*e^2 - 10*a^3*b^3*d^2*e^3 + 5*a^4*b^2*d*e^4 - a^5*b*e^5)*(e*x + d)^(11/2) + 323323*(b^6*d^6 - 6*a*b^5*d^5*e + 15*a^2*b^4*d^4*e^2 - 20*a^3*b^3*d^3*e^3 + 15*a^4*b^2*d^2*e^4 - 6*a^5*b*d*e^5 + a^6*e^6)*(e*x + d)^(9/2))/e^7`

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2771 vs. $2(159) = 318$.

Time = 0.21 (sec) , antiderivative size = 2771, normalized size of antiderivative = 14.82

$$\int (d + ex)^{7/2} (a^2 + 2abx + b^2x^2)^3 dx = \text{Too large to display}$$

input `integrate((e*x+d)^(7/2)*(b^2*x^2+2*a*b*x+a^2)^3,x, algorithm="giac")`

output

```

2/14549535*(14549535*sqrt(e*x + d)*a^6*d^4 + 19399380*((e*x + d)^(3/2) - 3
*sqrt(e*x + d)*d)*a^6*d^3 + 29099070*((e*x + d)^(3/2) - 3*sqrt(e*x + d)*d)
*a^5*b*d^4/e + 5819814*(3*(e*x + d)^(5/2) - 10*(e*x + d)^(3/2)*d + 15*sqrt
(e*x + d)*d^2)*a^6*d^2 + 14549535*(3*(e*x + d)^(5/2) - 10*(e*x + d)^(3/2)*
d + 15*sqrt(e*x + d)*d^2)*a^4*b^2*d^4/e^2 + 23279256*(3*(e*x + d)^(5/2) -
10*(e*x + d)^(3/2)*d + 15*sqrt(e*x + d)*d^2)*a^5*b*d^3/e + 1662804*(5*(e*x
+ d)^(7/2) - 21*(e*x + d)^(5/2)*d + 35*(e*x + d)^(3/2)*d^2 - 35*sqrt(e*x
+ d)*d^3)*a^6*d + 8314020*(5*(e*x + d)^(7/2) - 21*(e*x + d)^(5/2)*d + 35*(
e*x + d)^(3/2)*d^2 - 35*sqrt(e*x + d)*d^3)*a^3*b^3*d^4/e^3 + 24942060*(5*(
e*x + d)^(7/2) - 21*(e*x + d)^(5/2)*d + 35*(e*x + d)^(3/2)*d^2 - 35*sqrt(e
*x + d)*d^3)*a^4*b^2*d^3/e^2 + 14965236*(5*(e*x + d)^(7/2) - 21*(e*x + d)^(
5/2)*d + 35*(e*x + d)^(3/2)*d^2 - 35*sqrt(e*x + d)*d^3)*a^5*b*d^2/e + 461
89*(35*(e*x + d)^(9/2) - 180*(e*x + d)^(7/2)*d + 378*(e*x + d)^(5/2)*d^2 -
420*(e*x + d)^(3/2)*d^3 + 315*sqrt(e*x + d)*d^4)*a^6 + 692835*(35*(e*x +
d)^(9/2) - 180*(e*x + d)^(7/2)*d + 378*(e*x + d)^(5/2)*d^2 - 420*(e*x + d)
^(3/2)*d^3 + 315*sqrt(e*x + d)*d^4)*a^2*b^4*d^4/e^4 + 3695120*(35*(e*x + d)
^(9/2) - 180*(e*x + d)^(7/2)*d + 378*(e*x + d)^(5/2)*d^2 - 420*(e*x + d)^(
3/2)*d^3 + 315*sqrt(e*x + d)*d^4)*a^3*b^3*d^3/e^3 + 4157010*(35*(e*x + d)
^(9/2) - 180*(e*x + d)^(7/2)*d + 378*(e*x + d)^(5/2)*d^2 - 420*(e*x + d)^(
3/2)*d^3 + 315*sqrt(e*x + d)*d^4)*a^4*b^2*d^2/e^2 + 1108536*(35*(e*x + ...

```

Mupad [B] (verification not implemented)

Time = 9.23 (sec) , antiderivative size = 162, normalized size of antiderivative = 0.87

$$\begin{aligned}
\int (d + ex)^{7/2} (a^2 + 2abx + b^2x^2)^3 dx &= \frac{2b^6 (d + ex)^{21/2}}{21e^7} \\
&- \frac{(12b^6d - 12ab^5e)(d + ex)^{19/2}}{19e^7} + \frac{2(ae - bd)^6 (d + ex)^{9/2}}{9e^7} \\
&+ \frac{30b^2(ae - bd)^4 (d + ex)^{13/2}}{13e^7} + \frac{8b^3(ae - bd)^3 (d + ex)^{15/2}}{3e^7} \\
&+ \frac{30b^4(ae - bd)^2 (d + ex)^{17/2}}{17e^7} + \frac{12b(ae - bd)^5 (d + ex)^{11/2}}{11e^7}
\end{aligned}$$

input

```
int((d + e*x)^(7/2)*(a^2 + b^2*x^2 + 2*a*b*x)^3,x)
```

output

$$\begin{aligned} & (2*b^6*(d + e*x)^{(21/2)})/(21*e^7) - ((12*b^6*d - 12*a*b^5*e)*(d + e*x)^{(19/2)})/(19*e^7) + (2*(a*e - b*d)^6*(d + e*x)^{(9/2)})/(9*e^7) + (30*b^2*(a*e - b*d)^4*(d + e*x)^{(13/2)})/(13*e^7) + (8*b^3*(a*e - b*d)^3*(d + e*x)^{(15/2)})/(3*e^7) + (30*b^4*(a*e - b*d)^2*(d + e*x)^{(17/2)})/(17*e^7) + (12*b*(a*e - b*d)^5*(d + e*x)^{(11/2)})/(11*e^7) \end{aligned}$$
Reduce [B] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 807, normalized size of antiderivative = 4.32

$$\int (d + ex)^{7/2} (a^2 + 2abx + b^2x^2)^3 dx = \frac{2\sqrt{ex + d} (138567b^6e^{10}x^{10} + 918918ab^5e^{10}x^9 + 466752b^6de^9x^9 + 2567565a^2b^4e^{10}x^8 + 31351}$$

input

`int((e*x+d)^(7/2)*(b^2*x^2+2*a*b*x+a^2)^3,x)`

output

```
(2*sqrt(d + e*x)*(323323*a**6*d**4*e**6 + 1293292*a**6*d**3*e**7*x + 19399
38*a**6*d**2*e**8*x**2 + 1293292*a**6*d*e**9*x**3 + 323323*a**6*e**10*x**4
- 352716*a**5*b*d**5*e**5 + 176358*a**5*b*d**4*e**6*x + 4232592*a**5*b*d
*3*e**7*x**2 + 8112468*a**5*b*d**2*e**8*x**3 + 5996172*a**5*b*d*e**9*x**4
+ 1587222*a**5*b*e**10*x**5 + 271320*a**4*b**2*d**6*e**4 - 135660*a**4*b**
2*d**5*e**5*x + 101745*a**4*b**2*d**4*e**6*x**2 + 7189980*a**4*b**2*d**3*e
**7*x**3 + 15533070*a**4*b**2*d**2*e**8*x**4 + 12209400*a**4*b**2*d*e**9*x
**5 + 3357585*a**4*b**2*e**10*x**6 - 144704*a**3*b**3*d**7*e**3 + 72352*a
*3*b**3*d**6*e**4*x - 54264*a**3*b**3*d**5*e**5*x**2 + 45220*a**3*b**3*d**
4*e**6*x**3 + 7235200*a**3*b**3*d**3*e**7*x**4 + 16767576*a**3*b**3*d**2*e
**8*x**5 + 13728792*a**3*b**3*d*e**9*x**6 + 3879876*a**3*b**3*e**10*x**7 +
51072*a**2*b**4*d**8*e**2 - 25536*a**2*b**4*d**7*e**3*x + 19152*a**2*b**4
*d**6*e**4*x**2 - 15960*a**2*b**4*d**5*e**5*x**3 + 13965*a**2*b**4*d**4*e
**6*x**4 + 4352292*a**2*b**4*d**3*e**7*x**5 + 10559934*a**2*b**4*d**2*e**8
*x**6 + 8900892*a**2*b**4*d*e**9*x**7 + 2567565*a**2*b**4*e**10*x**8 - 1075
2*a*b**5*d**9*e + 5376*a*b**5*d**8*e**2*x - 4032*a*b**5*d**7*e**3*x**2 + 3
360*a*b**5*d**6*e**4*x**3 - 2940*a*b**5*d**5*e**5*x**4 + 2646*a*b**5*d**4
e**6*x**5 + 1452528*a*b**5*d**3*e**7*x**6 + 3639636*a*b**5*d**2*e**8*x**7
+ 3135132*a*b**5*d*e**9*x**8 + 918918*a*b**5*e**10*x**9 + 1024*b**6*d**10
- 512*b**6*d**9*e*x + 384*b**6*d**8*e**2*x**2 - 320*b**6*d**7*e**3*x**3...
```

3.163 $\int (d + ex)^{5/2} (a^2 + 2abx + b^2x^2)^3 dx$

| | |
|---|------|
| Optimal result | 1274 |
| Mathematica [A] (verified) | 1275 |
| Rubi [A] (verified) | 1275 |
| Maple [A] (verified) | 1277 |
| Fricas [B] (verification not implemented) | 1278 |
| Sympy [B] (verification not implemented) | 1279 |
| Maxima [B] (verification not implemented) | 1279 |
| Giac [B] (verification not implemented) | 1280 |
| Mupad [B] (verification not implemented) | 1281 |
| Reduce [B] (verification not implemented) | 1282 |

Optimal result

Integrand size = 28, antiderivative size = 185

$$\int (d + ex)^{5/2} (a^2 + 2abx + b^2x^2)^3 dx = \frac{2(bd - ae)^6(d + ex)^{7/2}}{7e^7} - \frac{4b(bd - ae)^5(d + ex)^{9/2}}{3e^7} + \frac{30b^2(bd - ae)^4(d + ex)^{11/2}}{11e^7} - \frac{40b^3(bd - ae)^3(d + ex)^{13/2}}{13e^7} + \frac{2b^4(bd - ae)^2(d + ex)^{15/2}}{e^7} - \frac{12b^5(bd - ae)(d + ex)^{17/2}}{17e^7} + \frac{2b^6(d + ex)^{19/2}}{19e^7}$$

output

```
2/7*(-a*e+b*d)^6*(e*x+d)^(7/2)/e^7-4/3*b*(-a*e+b*d)^5*(e*x+d)^(9/2)/e^7+30/11*b^2*(-a*e+b*d)^4*(e*x+d)^(11/2)/e^7-40/13*b^3*(-a*e+b*d)^3*(e*x+d)^(13/2)/e^7+2*b^4*(-a*e+b*d)^2*(e*x+d)^(15/2)/e^7-12/17*b^5*(-a*e+b*d)*(e*x+d)^(17/2)/e^7+2/19*b^6*(e*x+d)^(19/2)/e^7
```

Mathematica [A] (verified)

Time = 0.20 (sec) , antiderivative size = 291, normalized size of antiderivative = 1.57

$$\int (d + ex)^{5/2} (a^2 + 2abx + b^2x^2)^3 dx = \frac{2(d + ex)^{7/2} (138567a^6e^6 + 92378a^5be^5(-2d + 7ex) + 20995a^4b^2e^4(8d^2 - 28dex + 63e^2x^2) + \dots)}{\dots}$$

input

```
Integrate[(d + e*x)^(5/2)*(a^2 + 2*a*b*x + b^2*x^2)^3,x]
```

output

```
(2*(d + e*x)^(7/2)*(138567*a^6*e^6 + 92378*a^5*b*e^5*(-2*d + 7*e*x) + 20995*a^4*b^2*e^4*(8*d^2 - 28*d*e*x + 63*e^2*x^2) + 6460*a^3*b^3*e^3*(-16*d^3 + 56*d^2*e*x - 126*d*e^2*x^2 + 231*e^3*x^3) + 323*a^2*b^4*e^2*(128*d^4 - 448*d^3*e*x + 1008*d^2*e^2*x^2 - 1848*d*e^3*x^3 + 3003*e^4*x^4) + 38*a*b^5*e*(-256*d^5 + 896*d^4*e*x - 2016*d^3*e^2*x^2 + 3696*d^2*e^3*x^3 - 6006*d*e^4*x^4 + 9009*e^5*x^5) + b^6*(1024*d^6 - 3584*d^5*e*x + 8064*d^4*e^2*x^2 - 14784*d^3*e^3*x^3 + 24024*d^2*e^4*x^4 - 36036*d*e^5*x^5 + 51051*e^6*x^6))/(969969*e^7)
```

Rubi [A] (verified)

Time = 0.53 (sec) , antiderivative size = 185, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {1098, 27, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\begin{aligned} & \int (a^2 + 2abx + b^2x^2)^3 (d + ex)^{5/2} dx \\ & \quad \downarrow 1098 \\ & \frac{\int b^6 (a + bx)^6 (d + ex)^{5/2} dx}{b^6} \\ & \quad \downarrow 27 \\ & \int (a + bx)^6 (d + ex)^{5/2} dx \end{aligned}$$

↓ 53

$$\int \left(-\frac{6b^5(d+ex)^{15/2}(bd-ae)}{e^6} + \frac{15b^4(d+ex)^{13/2}(bd-ae)^2}{e^6} - \frac{20b^3(d+ex)^{11/2}(bd-ae)^3}{e^6} + \frac{15b^2(d+ex)^{9/2}(bd-ae)^4}{e^6} \right)$$

↓ 2009

$$\begin{aligned} & -\frac{12b^5(d+ex)^{17/2}(bd-ae)}{17e^7} + \frac{2b^4(d+ex)^{15/2}(bd-ae)^2}{e^7} - \frac{40b^3(d+ex)^{13/2}(bd-ae)^3}{13e^7} + \\ & \frac{30b^2(d+ex)^{11/2}(bd-ae)^4}{11e^7} - \frac{4b(d+ex)^{9/2}(bd-ae)^5}{3e^7} + \frac{2(d+ex)^{7/2}(bd-ae)^6}{7e^7} + \\ & \frac{2b^6(d+ex)^{19/2}}{19e^7} \end{aligned}$$

input `Int[(d + e*x)^(5/2)*(a^2 + 2*a*b*x + b^2*x^2)^3,x]`

output `(2*(b*d - a*e)^6*(d + e*x)^(7/2))/(7*e^7) - (4*b*(b*d - a*e)^5*(d + e*x)^(9/2))/(3*e^7) + (30*b^2*(b*d - a*e)^4*(d + e*x)^(11/2))/(11*e^7) - (40*b^3*(b*d - a*e)^3*(d + e*x)^(13/2))/(13*e^7) + (2*b^4*(b*d - a*e)^2*(d + e*x)^(15/2))/e^7 - (12*b^5*(b*d - a*e)*(d + e*x)^(17/2))/(17*e^7) + (2*b^6*(d + e*x)^(19/2))/(19*e^7)`

Defintions of rubi rules used

rule 27 `Int[(a_)*(F_x_), x_Symbol] := Simp[a Int[F_x, x], x] /; FreeQ[a, x] && !MatchQ[F_x, (b_)*(G_x_)] /; FreeQ[b, x]`

rule 53 `Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] := Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && IGtQ[m, 0] && (!IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])`

rule 1098 `Int[((d_.) + (e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] := Simp[1/c^p Int[(d + e*x)^m*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]`

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 635 vs. $2(159) = 318$.

Time = 0.09 (sec) , antiderivative size = 635, normalized size of antiderivative = 3.43

$$\int (d + ex)^{5/2} (a^2 + 2abx + b^2x^2)^3 dx = \frac{2(51051b^6e^9x^9 + 1024b^6d^9 - 9728ab^5d^8e + 41344a^2b^4d^7e^2 - 103360a^3b^3d^6e^3 + 167960a^4b^2d^5e^4 - 184756a^5b^1d^4e^5 + 138567a^6d^3e^6 + 9009(13b^6d^2e^8 + 38ab^5e^9)x^8 + 3003(23b^6d^2e^7 + 266ab^5d^2e^8 + 323a^2b^4e^9)x^7 + 231(b^6d^3e^6 + 2090ab^5d^2e^7 + 10013a^2b^4d^2e^8 + 6460a^3b^3e^9)x^6 - 63(4b^6d^4e^5 - 38ab^5d^3e^6 - 22933a^2b^4d^2e^7 - 58140a^3b^3d^2e^8 - 20995a^4b^2e^9)x^5 + 7(40b^6d^5e^4 - 380ab^5d^4e^5 + 1615a^2b^4d^3e^6 + 342380a^3b^3d^2e^7 + 482885a^4b^2d^2e^8 + 92378a^5b^1e^9)x^4 - (320b^6d^6e^3 - 3040ab^5d^5e^4 + 12920a^2b^4d^4e^5 - 32300a^3b^3d^3e^6 - 2372435a^4b^2d^2e^7 - 1755182a^5b^1d^2e^8 - 138567a^6e^9)x^3 + 3(128b^6d^7e^2 - 1216ab^5d^6e^3 + 5168a^2b^4d^5e^4 - 12920a^3b^3d^4e^5 + 20995a^4b^2d^3e^6 + 461890a^5b^1d^2e^7 + 138567a^6d^2e^8)x^2 - (512b^6d^8e - 4864ab^5d^7e^2 + 20672a^2b^4d^6e^3 - 51680a^3b^3d^5e^4 + 83980a^4b^2d^4e^5 - 92378a^5b^1d^3e^6 - 415701a^6d^2e^7)x}{\sqrt{ex + d}}/e^7$$

input `integrate((e*x+d)^(5/2)*(b^2*x^2+2*a*b*x+a^2)^3,x, algorithm="fricas")`

output `2/969969*(51051*b^6*e^9*x^9 + 1024*b^6*d^9 - 9728*a*b^5*d^8*e + 41344*a^2*b^4*d^7*e^2 - 103360*a^3*b^3*d^6*e^3 + 167960*a^4*b^2*d^5*e^4 - 184756*a^5*b*d^4*e^5 + 138567*a^6*d^3*e^6 + 9009*(13*b^6*d^2*e^8 + 38*a*b^5*e^9)*x^8 + 3003*(23*b^6*d^2*e^7 + 266*a*b^5*d^2*e^8 + 323*a^2*b^4*e^9)*x^7 + 231*(b^6*d^3*e^6 + 2090*a*b^5*d^2*e^7 + 10013*a^2*b^4*d^2*e^8 + 6460*a^3*b^3*e^9)*x^6 - 63*(4*b^6*d^4*e^5 - 38*a*b^5*d^3*e^6 - 22933*a^2*b^4*d^2*e^7 - 58140*a^3*b^3*d^2*e^8 - 20995*a^4*b^2*e^9)*x^5 + 7*(40*b^6*d^5*e^4 - 380*a*b^5*d^4*e^5 + 1615*a^2*b^4*d^3*e^6 + 342380*a^3*b^3*d^2*e^7 + 482885*a^4*b^2*d^2*e^8 + 92378*a^5*b^1*e^9)*x^4 - (320*b^6*d^6*e^3 - 3040*a*b^5*d^5*e^4 + 12920*a^2*b^4*d^4*e^5 - 32300*a^3*b^3*d^3*e^6 - 2372435*a^4*b^2*d^2*e^7 - 1755182*a^5*b^1*d^2*e^8 - 138567*a^6*e^9)*x^3 + 3*(128*b^6*d^7*e^2 - 1216*a*b^5*d^6*e^3 + 5168*a^2*b^4*d^5*e^4 - 12920*a^3*b^3*d^4*e^5 + 20995*a^4*b^2*d^3*e^6 + 461890*a^5*b^1*d^2*e^7 + 138567*a^6*d^2*e^8)*x^2 - (512*b^6*d^8*e - 4864*a*b^5*d^7*e^2 + 20672*a^2*b^4*d^6*e^3 - 51680*a^3*b^3*d^5*e^4 + 83980*a^4*b^2*d^4*e^5 - 92378*a^5*b^1*d^3*e^6 - 415701*a^6*d^2*e^7)*x)*sqrt(e*x + d)/e^7`

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 495 vs. $2(172) = 344$.

Time = 1.40 (sec) , antiderivative size = 495, normalized size of antiderivative = 2.68

$$\int (d + ex)^{5/2} (a^2 + 2abx + b^2x^2)^3 dx = \frac{2 \left(\frac{b^6(d+ex)^{19/2}}{19e^6} + \frac{(d+ex)^{17/2} \cdot (6ab^5e - 6b^6d)}{17e^6} + \frac{(d+ex)^{15/2} \cdot (15a^2b^4e^2 - 30ab^5de + 15b^6d^2)}{15e^6} + \frac{(d+ex)^{13/2} \cdot (20a^3b^3e^3 - 60a^2b^4de^2 + 60ab^5d^2e - 60a^2b^4d^2e^2)}{13e^6} \right)}{d^{5/2} \left(a^6x + 3a^5bx^2 + 5a^4b^2x^3 + 5a^3b^3x^4 + 3a^2b^4x^5 + ab^5x^6 + \frac{b^6x^7}{7} \right)}$$

input `integrate((e*x+d)**(5/2)*(b**2*x**2+2*a*b*x+a**2)**3,x)`

output

```
Piecewise((2*(b**6*(d + e*x)**(19/2))/(19*e**6) + (d + e*x)**(17/2)*(6*a*b*
*5*e - 6*b**6*d)/(17*e**6) + (d + e*x)**(15/2)*(15*a**2*b**4*e**2 - 30*a*b
**5*d*e + 15*b**6*d**2)/(15*e**6) + (d + e*x)**(13/2)*(20*a**3*b**3*e**3 -
60*a**2*b**4*d*e**2 + 60*a*b**5*d**2*e - 20*b**6*d**3)/(13*e**6) + (d + e
*x)**(11/2)*(15*a**4*b**2*e**4 - 60*a**3*b**3*d*e**3 + 90*a**2*b**4*d**2*e
**2 - 60*a*b**5*d**3*e + 15*b**6*d**4)/(11*e**6) + (d + e*x)**(9/2)*(6*a**
5*b**e**5 - 30*a**4*b**2*d*e**4 + 60*a**3*b**3*d**2*e**3 - 60*a**2*b**4*d**
3*e**2 + 30*a*b**5*d**4*e - 6*b**6*d**5)/(9*e**6) + (d + e*x)**(7/2)*(a**6
*e**6 - 6*a**5*b*d*e**5 + 15*a**4*b**2*d**2*e**4 - 20*a**3*b**3*d**3*e**3
+ 15*a**2*b**4*d**4*e**2 - 6*a*b**5*d**5*e + b**6*d**6)/(7*e**6))/e, Ne(e,
0)), (d**(5/2)*(a**6*x + 3*a**5*b*x**2 + 5*a**4*b**2*x**3 + 5*a**3*b**3*x
**4 + 3*a**2*b**4*x**5 + a*b**5*x**6 + b**6*x**7/7), True))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 350 vs. $2(159) = 318$.

Time = 0.03 (sec) , antiderivative size = 350, normalized size of antiderivative = 1.89

$$\int (d + ex)^{5/2} (a^2 + 2abx + b^2x^2)^3 dx = \frac{2 \left(51051 (ex + d)^{19/2} b^6 - 342342 (b^6d - ab^5e)(ex + d)^{17/2} + 969969 (b^6d^2 - 2ab^5de + a^2b^4e^2)(ex + d)^{15/2} - 2000000 (a^2b^4d^2e^2 - ab^5d^3e + b^6d^4)(ex + d)^{13/2} + 1000000 (a^2b^4d^3e^2 - ab^5d^4e + b^6d^5)(ex + d)^{11/2} - 200000 (a^2b^4d^4e^2 - ab^5d^5e + b^6d^6)(ex + d)^{9/2} + 100000 (a^2b^4d^5e^2 - ab^5d^6e + b^6d^7)(ex + d)^{7/2} \right)}{d^{5/2} (a^6x^7 + 7a^5bx^6 + 21a^4b^2x^5 + 35a^3b^3x^4 + 35a^2b^4x^3 + 21ab^5x^2 + 7b^6x + d^7)}$$

input `integrate((e*x+d)^(5/2)*(b^2*x^2+2*a*b*x+a^2)^3,x, algorithm="maxima")`

output
$$\frac{2/969969*(51051*(e*x + d)^{(19/2)}*b^6 - 342342*(b^6*d - a*b^5*e)*(e*x + d)^{(17/2)} + 969969*(b^6*d^2 - 2*a*b^5*d*e + a^2*b^4*e^2)*(e*x + d)^{(15/2)} - 1492260*(b^6*d^3 - 3*a*b^5*d^2*e + 3*a^2*b^4*d*e^2 - a^3*b^3*e^3)*(e*x + d)^{(13/2)} + 1322685*(b^6*d^4 - 4*a*b^5*d^3*e + 6*a^2*b^4*d^2*e^2 - 4*a^3*b^3*d*e^3 + a^4*b^2*e^4)*(e*x + d)^{(11/2)} - 646646*(b^6*d^5 - 5*a*b^5*d^4*e + 10*a^2*b^4*d^3*e^2 - 10*a^3*b^3*d^2*e^3 + 5*a^4*b^2*d*e^4 - a^5*b*e^5)*(e*x + d)^{(9/2)} + 138567*(b^6*d^6 - 6*a*b^5*d^5*e + 15*a^2*b^4*d^4*e^2 - 20*a^3*b^3*d^3*e^3 + 15*a^4*b^2*d^2*e^4 - 6*a^5*b*d*e^5 + a^6*e^6)*(e*x + d)^{(7/2))}{e^7}$$

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2042 vs. $2(159) = 318$.

Time = 0.21 (sec) , antiderivative size = 2042, normalized size of antiderivative = 11.04

$$\int (d + ex)^{5/2} (a^2 + 2abx + b^2x^2)^3 dx = \text{Too large to display}$$

input `integrate((e*x+d)^(5/2)*(b^2*x^2+2*a*b*x+a^2)^3,x, algorithm="giac")`

output

```

2/4849845*(4849845*sqrt(e*x + d)*a^6*d^3 + 4849845*((e*x + d)^(3/2) - 3*sqrt(e*x + d)*d)*a^5
rt(e*x + d)*d)*a^6*d^2 + 9699690*((e*x + d)^(3/2) - 3*sqrt(e*x + d)*d)*a^5
*b*d^3/e + 969969*(3*(e*x + d)^(5/2) - 10*(e*x + d)^(3/2)*d + 15*sqrt(e*x
+ d)*d^2)*a^6*d + 4849845*(3*(e*x + d)^(5/2) - 10*(e*x + d)^(3/2)*d + 15*s
qrt(e*x + d)*d^2)*a^4*b^2*d^3/e^2 + 5819814*(3*(e*x + d)^(5/2) - 10*(e*x +
d)^(3/2)*d + 15*sqrt(e*x + d)*d^2)*a^5*b*d^2/e + 138567*(5*(e*x + d)^(7/2
) - 21*(e*x + d)^(5/2)*d + 35*(e*x + d)^(3/2)*d^2 - 35*sqrt(e*x + d)*d^3)*
a^6 + 2771340*(5*(e*x + d)^(7/2) - 21*(e*x + d)^(5/2)*d + 35*(e*x + d)^(3/
2)*d^2 - 35*sqrt(e*x + d)*d^3)*a^3*b^3*d^3/e^3 + 6235515*(5*(e*x + d)^(7/2
) - 21*(e*x + d)^(5/2)*d + 35*(e*x + d)^(3/2)*d^2 - 35*sqrt(e*x + d)*d^3)*
a^4*b^2*d^2/e^2 + 2494206*(5*(e*x + d)^(7/2) - 21*(e*x + d)^(5/2)*d + 35*(
e*x + d)^(3/2)*d^2 - 35*sqrt(e*x + d)*d^3)*a^5*b*d/e + 230945*(35*(e*x + d
)^(9/2) - 180*(e*x + d)^(7/2)*d + 378*(e*x + d)^(5/2)*d^2 - 420*(e*x + d)^(
3/2)*d^3 + 315*sqrt(e*x + d)*d^4)*a^2*b^4*d^3/e^4 + 923780*(35*(e*x + d)^(
9/2) - 180*(e*x + d)^(7/2)*d + 378*(e*x + d)^(5/2)*d^2 - 420*(e*x + d)^(3
/2)*d^3 + 315*sqrt(e*x + d)*d^4)*a^3*b^3*d^2/e^3 + 692835*(35*(e*x + d)^(9
/2) - 180*(e*x + d)^(7/2)*d + 378*(e*x + d)^(5/2)*d^2 - 420*(e*x + d)^(3/2
)*d^3 + 315*sqrt(e*x + d)*d^4)*a^4*b^2*d/e^2 + 92378*(35*(e*x + d)^(9/2) -
180*(e*x + d)^(7/2)*d + 378*(e*x + d)^(5/2)*d^2 - 420*(e*x + d)^(3/2)*d^3
+ 315*sqrt(e*x + d)*d^4)*a^5*b/e + 41990*(63*(e*x + d)^(11/2) - 385*(e...

```

Mupad [B] (verification not implemented)

Time = 9.26 (sec) , antiderivative size = 162, normalized size of antiderivative = 0.88

$$\begin{aligned}
\int (d + ex)^{5/2} (a^2 + 2abx + b^2x^2)^3 dx &= \frac{2b^6(d + ex)^{19/2}}{19e^7} \\
&- \frac{(12b^6d - 12ab^5e)(d + ex)^{17/2}}{17e^7} + \frac{2(ae - bd)^6(d + ex)^{7/2}}{7e^7} \\
&+ \frac{30b^2(ae - bd)^4(d + ex)^{11/2}}{11e^7} + \frac{40b^3(ae - bd)^3(d + ex)^{13/2}}{13e^7} \\
&+ \frac{2b^4(ae - bd)^2(d + ex)^{15/2}}{e^7} + \frac{4b(ae - bd)^5(d + ex)^{9/2}}{3e^7}
\end{aligned}$$

input

```
int((d + e*x)^(5/2)*(a^2 + b^2*x^2 + 2*a*b*x)^3,x)
```


3.164 $\int (d + ex)^{3/2} (a^2 + 2abx + b^2x^2)^3 dx$

| | |
|---|------|
| Optimal result | 1283 |
| Mathematica [A] (verified) | 1284 |
| Rubi [A] (verified) | 1284 |
| Maple [A] (verified) | 1286 |
| Fricas [B] (verification not implemented) | 1287 |
| Sympy [B] (verification not implemented) | 1287 |
| Maxima [B] (verification not implemented) | 1288 |
| Giac [B] (verification not implemented) | 1289 |
| Mupad [B] (verification not implemented) | 1290 |
| Reduce [B] (verification not implemented) | 1290 |

Optimal result

Integrand size = 28, antiderivative size = 187

$$\int (d + ex)^{3/2} (a^2 + 2abx + b^2x^2)^3 dx = \frac{2(bd - ae)^6(d + ex)^{5/2}}{5e^7} - \frac{12b(bd - ae)^5(d + ex)^{7/2}}{7e^7} + \frac{10b^2(bd - ae)^4(d + ex)^{9/2}}{3e^7} - \frac{40b^3(bd - ae)^3(d + ex)^{11/2}}{11e^7} + \frac{30b^4(bd - ae)^2(d + ex)^{13/2}}{13e^7} - \frac{4b^5(bd - ae)(d + ex)^{15/2}}{5e^7} + \frac{2b^6(d + ex)^{17/2}}{17e^7}$$

output

```
2/5*(-a*e+b*d)^6*(e*x+d)^(5/2)/e^7-12/7*b*(-a*e+b*d)^5*(e*x+d)^(7/2)/e^7+10/3*b^2*(-a*e+b*d)^4*(e*x+d)^(9/2)/e^7-40/11*b^3*(-a*e+b*d)^3*(e*x+d)^(11/2)/e^7+30/13*b^4*(-a*e+b*d)^2*(e*x+d)^(13/2)/e^7-4/5*b^5*(-a*e+b*d)*(e*x+d)^(15/2)/e^7+2/17*b^6*(e*x+d)^(17/2)/e^7
```


Mathematica [A] (verified)

Time = 0.23 (sec) , antiderivative size = 291, normalized size of antiderivative = 1.56

$$\int (d + ex)^{3/2} (a^2 + 2abx + b^2x^2)^3 dx = \frac{2(d + ex)^{5/2} (51051a^6e^6 + 43758a^5be^5(-2d + 5ex) + 12155a^4b^2e^4(8d^2 - 20dex + 35e^2x^2) + \dots}{255e^7}$$

input `Integrate[(d + e*x)^(3/2)*(a^2 + 2*a*b*x + b^2*x^2)^3,x]`

output $(2*(d + e*x)^{(5/2)}*(51051*a^6*e^6 + 43758*a^5*b*e^5*(-2*d + 5*e*x) + 12155*a^4*b^2*e^4*(8*d^2 - 20*d*e*x + 35*e^2*x^2) + 4420*a^3*b^3*e^3*(-16*d^3 + 40*d^2*e*x - 70*d*e^2*x^2 + 105*e^3*x^3) + 255*a^2*b^4*e^2*(128*d^4 - 320*d^3*e*x + 560*d^2*e^2*x^2 - 840*d*e^3*x^3 + 1155*e^4*x^4) + 34*a*b^5*e*(-256*d^5 + 640*d^4*e*x - 1120*d^3*e^2*x^2 + 1680*d^2*e^3*x^3 - 2310*d*e^4*x^4 + 3003*e^5*x^5) + b^6*(1024*d^6 - 2560*d^5*e*x + 4480*d^4*e^2*x^2 - 6720*d^3*e^3*x^3 + 9240*d^2*e^4*x^4 - 12012*d*e^5*x^5 + 15015*e^6*x^6)))/(255*e^7)$

Rubi [A] (verified)

Time = 0.55 (sec) , antiderivative size = 187, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {1098, 27, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\begin{aligned} & \int (a^2 + 2abx + b^2x^2)^3 (d + ex)^{3/2} dx \\ & \quad \downarrow 1098 \\ & \frac{\int b^6(a + bx)^6(d + ex)^{3/2} dx}{b^6} \\ & \quad \downarrow 27 \\ & \int (a + bx)^6(d + ex)^{3/2} dx \end{aligned}$$

↓ 53

$$\int \left(-\frac{6b^5(d+ex)^{13/2}(bd-ae)}{e^6} + \frac{15b^4(d+ex)^{11/2}(bd-ae)^2}{e^6} - \frac{20b^3(d+ex)^{9/2}(bd-ae)^3}{e^6} + \frac{15b^2(d+ex)^{7/2}(bd-ae)^4}{e^6} - \frac{6b(d+ex)^{5/2}(bd-ae)^5}{e^6} + \frac{bd-ae}{e^6} \right) dx$$

↓ 2009

$$-\frac{4b^5(d+ex)^{15/2}(bd-ae)}{5e^7} + \frac{30b^4(d+ex)^{13/2}(bd-ae)^2}{13e^7} - \frac{40b^3(d+ex)^{11/2}(bd-ae)^3}{11e^7} + \frac{10b^2(d+ex)^{9/2}(bd-ae)^4}{7e^7} - \frac{12b(d+ex)^{7/2}(bd-ae)^5}{7e^7} + \frac{2(d+ex)^{5/2}(bd-ae)^6}{5e^7} + \frac{2b^6(d+ex)^{17/2}}{17e^7}$$

input `Int[(d + e*x)^(3/2)*(a^2 + 2*a*b*x + b^2*x^2)^3,x]`

output `(2*(b*d - a*e)^6*(d + e*x)^(5/2))/(5*e^7) - (12*b*(b*d - a*e)^5*(d + e*x)^(7/2))/(7*e^7) + (10*b^2*(b*d - a*e)^4*(d + e*x)^(9/2))/(3*e^7) - (40*b^3*(b*d - a*e)^3*(d + e*x)^(11/2))/(11*e^7) + (30*b^4*(b*d - a*e)^2*(d + e*x)^(13/2))/(13*e^7) - (4*b^5*(b*d - a*e)*(d + e*x)^(15/2))/(5*e^7) + (2*b^6*(d + e*x)^(17/2))/(17*e^7)`

Defintions of rubi rules used

rule 27 `Int[(a_)*(F_x_), x_Symbol] := Simp[a Int[F_x, x], x] /; FreeQ[a, x] && !MatchQ[F_x, (b_)*(G_x_)] /; FreeQ[b, x]`

rule 53 `Int[((a_) + (b_)*(x_))^(m_)*((c_) + (d_)*(x_))^(n_), x_Symbol] := Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && IGtQ[m, 0] && (!IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])`

rule 1098 `Int[((d_) + (e_)*(x_))^(m_)*((a_) + (b_)*(x_) + (c_)*(x_)^2)^(p_), x_Symbol] := Simp[1/c^p Int[(d + e*x)^m*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]`

```
rule 2009 Int[u_, x_Symbol] := Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 1.29 (sec) , antiderivative size = 276, normalized size of antiderivative = 1.48

| method | result |
|-------------------|---|
| pseudoelliptic | $2(ex+d)^{\frac{5}{2}} \left((2ab^5x^5 + \frac{75}{13}a^2b^4x^4 + \frac{100}{11}a^3b^3x^3 + \frac{25}{3}a^4b^2x^2 + \frac{30}{7}a^5bx + \frac{5}{17}b^6x^6 + a^6) e^6 - \frac{12(\frac{7}{51}b^5x^5 + \frac{35}{39}ab^4x^4 + \frac{350}{143}a^2b^3x^3}{7} \right)$ |
| gospers | $2(ex+d)^{\frac{5}{2}} (15015x^6b^6e^6 + 102102x^5ab^5e^6 - 12012x^5b^6de^5 + 294525x^4a^2b^4e^6 - 78540x^4ab^5de^5 + 9240x^4b^6d^2e^4 + 464100x^3a^3b^3e^8)$ |
| orering | $2(15015x^6b^6e^6 + 102102x^5ab^5e^6 - 12012x^5b^6de^5 + 294525x^4a^2b^4e^6 - 78540x^4ab^5de^5 + 9240x^4b^6d^2e^4 + 464100x^3a^3b^3e^8)$ |
| derivativedivides | $\frac{2b^6(ex+d)^{\frac{17}{2}}}{17} + \frac{2(2abe-2b^2d)b^4(ex+d)^{\frac{15}{2}}}{5} + \frac{2((e^2a^2-2abde+b^2d^2)b^4 + 2(2abe-2b^2d)^2b^2 + b^2(2(e^2a^2-2abde+b^2d^2)b^2 + (2ab)^2))}{13}$ |
| default | $\frac{2b^6(ex+d)^{\frac{17}{2}}}{17} + \frac{2(2abe-2b^2d)b^4(ex+d)^{\frac{15}{2}}}{5} + \frac{2((e^2a^2-2abde+b^2d^2)b^4 + 2(2abe-2b^2d)^2b^2 + b^2(2(e^2a^2-2abde+b^2d^2)b^2 + (2ab)^2))}{13}$ |
| trager | $2(15015b^6e^8x^8 + 102102ab^5e^8x^7 + 18018b^6de^7x^7 + 294525a^2b^4e^8x^6 + 125664ab^5de^7x^6 + 231b^6d^2e^6x^6 + 464100a^3b^3e^8x^5)$ |
| risch | $2(15015b^6e^8x^8 + 102102ab^5e^8x^7 + 18018b^6de^7x^7 + 294525a^2b^4e^8x^6 + 125664ab^5de^7x^6 + 231b^6d^2e^6x^6 + 464100a^3b^3e^8x^5)$ |

```
input int((e*x+d)^(3/2)*(b^2*x^2+2*a*b*x+a^2)^3,x,method=_RETURNVERBOSE)
```

```
output 2/5*(e*x+d)^(5/2)*((2*a*b^5*x^5+75/13*a^2*b^4*x^4+100/11*a^3*b^3*x^3+25/3*a^4*b^2*x^2+30/7*a^5*b*x+5/17*b^6*x^6+a^6)*e^6-12/7*(7/51*b^5*x^5+35/39*a*b^4*x^4+350/143*a^2*b^3*x^3+350/99*a^3*b^2*x^2+25/9*a^4*b*x+a^5)*b*d*e^5+40/21*b^2*(21/221*b^4*x^4+84/143*a*b^3*x^3+210/143*a^2*b^2*x^2+20/11*a^3*b*x+a^4)*d^2*e^4-320/231*(21/221*b^3*x^3+7/13*a*b^2*x^2+15/13*a^2*b*x+a^3)*b^3*d^3*e^3+640/1001*b^4*d^4*(7/51*b^2*x^2+2/3*a*b*x+a^2)*e^2-512/3003*b^5*(5/17*b*x+a)*d^5*e+1024/51051*b^6*d^6)/e^7
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 541 vs. $2(159) = 318$.

Time = 0.09 (sec) , antiderivative size = 541, normalized size of antiderivative = 2.89

$$\int (d + ex)^{3/2} (a^2 + 2abx + b^2x^2)^3 dx = \frac{2(15015b^6e^8x^8 + 1024b^6d^8 - 8704ab^5d^7e + 32640a^2b^4d^6e^2 - 70720a^3b^3d^5e^3 + 97240a^4b^2d^4e^4 - 87516a^5b^2d^3e^5 + 51051a^6d^2e^6 + 6006(3b^6d^7e + 17ab^5e^8)x^7 + 231(b^6d^2e^6 + 544ab^5d^7e + 1275a^2b^4e^8)x^6 - 42(6b^6d^3e^5 - 51ab^5d^2e^6 - 8925a^2b^4d^7e - 11050a^3b^3e^8)x^5 + 35(8b^6d^4e^4 - 68ab^5d^3e^5 + 255a^2b^4d^2e^6 + 17680a^3b^3d^7e + 12155a^4b^2e^8)x^4 - 10(32b^6d^5e^3 - 272ab^5d^4e^4 + 1020a^2b^4d^3e^5 - 2210a^3b^3d^2e^6 - 60775a^4b^2d^7e - 21879a^5b^2e^8)x^3 + 3(128b^6d^6e^2 - 1088ab^5d^5e^3 + 4080a^2b^4d^4e^4 - 8840a^3b^3d^3e^5 + 12155a^4b^2d^2e^6 + 116688a^5b^2d^7e + 17017a^6e^8)x^2 - 2(256b^6d^7e - 2176ab^5d^6e^2 + 8160a^2b^4d^5e^3 - 17680a^3b^3d^4e^4 + 24310a^4b^2d^3e^5 - 21879a^5b^2d^2e^6 - 51051a^6d^2e^7)x}{e^7} \sqrt{ex + d}$$

input

```
integrate((e*x+d)^(3/2)*(b^2*x^2+2*a*b*x+a^2)^3,x, algorithm="fricas")
```

output

```
2/255255*(15015*b^6*e^8*x^8 + 1024*b^6*d^8 - 8704*a*b^5*d^7*e + 32640*a^2*b^4*d^6*e^2 - 70720*a^3*b^3*d^5*e^3 + 97240*a^4*b^2*d^4*e^4 - 87516*a^5*b^2*d^3*e^5 + 51051*a^6*d^2*e^6 + 6006*(3*b^6*d^7*e + 17*a*b^5*e^8)*x^7 + 231*(b^6*d^2*e^6 + 544*a*b^5*d^7*e + 1275*a^2*b^4*e^8)*x^6 - 42*(6*b^6*d^3*e^5 - 51*a*b^5*d^2*e^6 - 8925*a^2*b^4*d^7*e - 11050*a^3*b^3*e^8)*x^5 + 35*(8*b^6*d^4*e^4 - 68*a*b^5*d^3*e^5 + 255*a^2*b^4*d^2*e^6 + 17680*a^3*b^3*d^7*e + 12155*a^4*b^2*e^8)*x^4 - 10*(32*b^6*d^5*e^3 - 272*a*b^5*d^4*e^4 + 1020*a^2*b^4*d^3*e^5 - 2210*a^3*b^3*d^2*e^6 - 60775*a^4*b^2*d^7*e - 21879*a^5*b^2*e^8)*x^3 + 3*(128*b^6*d^6*e^2 - 1088*a*b^5*d^5*e^3 + 4080*a^2*b^4*d^4*e^4 - 8840*a^3*b^3*d^3*e^5 + 12155*a^4*b^2*d^2*e^6 + 116688*a^5*b^2*d^7*e + 17017*a^6*e^8)*x^2 - 2*(256*b^6*d^7*e - 2176*a*b^5*d^6*e^2 + 8160*a^2*b^4*d^5*e^3 - 17680*a^3*b^3*d^4*e^4 + 24310*a^4*b^2*d^3*e^5 - 21879*a^5*b^2*d^2*e^6 - 51051*a^6*d^2*e^7)*x)*sqrt(e*x + d)/e^7
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 495 vs. $2(173) = 346$.

Time = 1.27 (sec) , antiderivative size = 495, normalized size of antiderivative = 2.65

$$\int (d + ex)^{3/2} (a^2 + 2abx + b^2x^2)^3 dx = \frac{2 \left(\frac{b^6(d+ex)^{\frac{17}{2}}}{17e^6} + \frac{(d+ex)^{\frac{15}{2}} \cdot (6ab^5e - 6b^6d)}{15e^6} + \frac{(d+ex)^{\frac{13}{2}} \cdot (15a^2b^4e^2 - 30ab^5de + 15b^6d^2)}{13e^6} + \frac{(d+ex)^{\frac{11}{2}} \cdot (20a^3b^3e^3 - 60a^2b^4de^2 + 60ab^5d^2e)}{11e^6} \right)}{d^{\frac{3}{2}} \left(a^6x + 3a^5bx^2 + 5a^4b^2x^3 + 5a^3b^3x^4 + 3a^2b^4x^5 + ab^5x^6 + \frac{b^6x^7}{7} \right)}$$

input `integrate((e*x+d)**(3/2)*(b**2*x**2+2*a*b*x+a**2)**3,x)`

output `Piecewise((2*(b**6*(d + e*x)**(17/2)/(17*e**6) + (d + e*x)**(15/2)*(6*a*b*
*5*e - 6*b**6*d)/(15*e**6) + (d + e*x)**(13/2)*(15*a**2*b**4*e**2 - 30*a*b
5*d*e + 15*b6*d**2)/(13*e**6) + (d + e*x)**(11/2)*(20*a**3*b**3*e**3 -
60*a**2*b**4*d*e**2 + 60*a*b**5*d**2*e - 20*b**6*d**3)/(11*e**6) + (d + e
*x)**(9/2)*(15*a**4*b**2*e**4 - 60*a**3*b**3*d*e**3 + 90*a**2*b**4*d**2*e*
*2 - 60*a*b**5*d**3*e + 15*b**6*d**4)/(9*e**6) + (d + e*x)**(7/2)*(6*a**5*b
*e**5 - 30*a**4*b**2*d*e**4 + 60*a**3*b**3*d**2*e**3 - 60*a**2*b**4*d**3*
e**2 + 30*a*b**5*d**4*e - 6*b**6*d**5)/(7*e**6) + (d + e*x)**(5/2)*(a**6*e
6 - 6*a5*b*d*e**5 + 15*a**4*b**2*d**2*e**4 - 20*a**3*b**3*d**3*e**3 +
15*a**2*b**4*d**4*e**2 - 6*a*b**5*d**5*e + b**6*d**6)/(5*e**6))/e, Ne(e, 0
)), (d**(3/2)*(a**6*x + 3*a**5*b*x**2 + 5*a**4*b**2*x**3 + 5*a**3*b**3*x**
4 + 3*a**2*b**4*x**5 + a*b**5*x**6 + b**6*x**7/7), True))`

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 350 vs. $2(159) = 318$.

Time = 0.04 (sec) , antiderivative size = 350, normalized size of antiderivative = 1.87

$$\int (d + ex)^{3/2} (a^2 + 2abx + b^2x^2)^3 dx = \frac{2 \left(15015 (ex + d)^{\frac{17}{2}} b^6 - 102102 (b^6d - ab^5e)(ex + d)^{\frac{15}{2}} + 294525 (b^6d^2 - 2ab^5de + a^2b^4e^2)(ex + d)^{\frac{13}{2}} - 64100 (b^6d^3 - 3a^2b^5d^2e + 3a^2b^4d^2e^2 - a^3b^3e^3)(ex + d)^{\frac{11}{2}} + 425425 (b^6d^4 - 4a^2b^5d^3e + 6a^2b^4d^2e^2 - 4a^3b^3d^2e^3 + a^4b^2e^4)(ex + d)^{\frac{9}{2}} - 218790 (b^6d^5 - 5a^2b^5d^4e + 10a^2b^4d^3e^2 - 10a^3b^3d^2e^3 + 5a^4b^2d^2e^4 - a^5b^2e^5)(ex + d)^{\frac{7}{2}} + 51051 (b^6d^6 - 6a^2b^5d^5e + 15a^2b^4d^4e^2 - 20a^3b^3d^3e^3 + 15a^4b^2d^2e^4 - 6a^5b^2d^2e^5 + a^6e^6)(ex + d)^{\frac{5}{2}} \right)}{e^7}$$

input `integrate((e*x+d)^(3/2)*(b^2*x^2+2*a*b*x+a^2)^3,x, algorithm="maxima")`

output `2/255255*(15015*(e*x + d)^(17/2)*b^6 - 102102*(b^6*d - a*b^5*e)*(e*x + d)^(
15/2) + 294525*(b^6*d^2 - 2*a*b^5*d*e + a^2*b^4*e^2)*(e*x + d)^(13/2) - 4
64100*(b^6*d^3 - 3*a*b^5*d^2*e + 3*a^2*b^4*d^2*e^2 - a^3*b^3*e^3)*(e*x + d)^(
11/2) + 425425*(b^6*d^4 - 4*a*b^5*d^3*e + 6*a^2*b^4*d^2*e^2 - 4*a^3*b^3*d^2
*e^3 + a^4*b^2*e^4)*(e*x + d)^(9/2) - 218790*(b^6*d^5 - 5*a*b^5*d^4*e + 10
*a^2*b^4*d^3*e^2 - 10*a^3*b^3*d^2*e^3 + 5*a^4*b^2*d^2*e^4 - a^5*b^2*e^5)*(e*x
+ d)^(7/2) + 51051*(b^6*d^6 - 6*a*b^5*d^5*e + 15*a^2*b^4*d^4*e^2 - 20*a^3*b
^3*d^3*e^3 + 15*a^4*b^2*d^2*e^4 - 6*a^5*b^2*d^2*e^5 + a^6*e^6)*(e*x + d)^(5/2
))/e^7`

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1397 vs. $2(159) = 318$.

Time = 0.19 (sec) , antiderivative size = 1397, normalized size of antiderivative = 7.47

$$\int (d + ex)^{3/2} (a^2 + 2abx + b^2x^2)^3 dx = \text{Too large to display}$$

input `integrate((e*x+d)^(3/2)*(b^2*x^2+2*a*b*x+a^2)^3,x, algorithm="giac")`

output

```
2/765765*(765765*sqrt(e*x + d)*a^6*d^2 + 510510*((e*x + d)^(3/2) - 3*sqrt(e*x + d)*d)*a^6*d + 1531530*((e*x + d)^(3/2) - 3*sqrt(e*x + d)*d)*a^5*b*d^2/e + 51051*(3*(e*x + d)^(5/2) - 10*(e*x + d)^(3/2)*d + 15*sqrt(e*x + d)*d^2)*a^6 + 765765*(3*(e*x + d)^(5/2) - 10*(e*x + d)^(3/2)*d + 15*sqrt(e*x + d)*d^2)*a^4*b^2*d^2/e^2 + 612612*(3*(e*x + d)^(5/2) - 10*(e*x + d)^(3/2)*d + 15*sqrt(e*x + d)*d^2)*a^5*b*d/e + 437580*(5*(e*x + d)^(7/2) - 21*(e*x + d)^(5/2)*d + 35*(e*x + d)^(3/2)*d^2 - 35*sqrt(e*x + d)*d^3)*a^3*b^3*d^2/e^3 + 656370*(5*(e*x + d)^(7/2) - 21*(e*x + d)^(5/2)*d + 35*(e*x + d)^(3/2)*d^2 - 35*sqrt(e*x + d)*d^3)*a^4*b^2*d/e^2 + 131274*(5*(e*x + d)^(7/2) - 21*(e*x + d)^(5/2)*d + 35*(e*x + d)^(3/2)*d^2 - 35*sqrt(e*x + d)*d^3)*a^5*b/e + 36465*(35*(e*x + d)^(9/2) - 180*(e*x + d)^(7/2)*d + 378*(e*x + d)^(5/2)*d^2 - 420*(e*x + d)^(3/2)*d^3 + 315*sqrt(e*x + d)*d^4)*a^2*b^4*d^2/e^4 + 97240*(35*(e*x + d)^(9/2) - 180*(e*x + d)^(7/2)*d + 378*(e*x + d)^(5/2)*d^2 - 420*(e*x + d)^(3/2)*d^3 + 315*sqrt(e*x + d)*d^4)*a^3*b^3*d/e^3 + 36465*(35*(e*x + d)^(9/2) - 180*(e*x + d)^(7/2)*d + 378*(e*x + d)^(5/2)*d^2 - 420*(e*x + d)^(3/2)*d^3 + 315*sqrt(e*x + d)*d^4)*a^4*b^2/e^2 + 6630*(63*(e*x + d)^(11/2) - 385*(e*x + d)^(9/2)*d + 990*(e*x + d)^(7/2)*d^2 - 1386*(e*x + d)^(5/2)*d^3 + 1155*(e*x + d)^(3/2)*d^4 - 693*sqrt(e*x + d)*d^5)*a*b^5*d^2/e^5 + 33150*(63*(e*x + d)^(11/2) - 385*(e*x + d)^(9/2)*d + 990*(e*x + d)^(7/2)*d^2 - 1386*(e*x + d)^(5/2)*d^3 + 1155*(e*x + d)^(3/2)*d^4 ...
```

Mupad [B] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 162, normalized size of antiderivative = 0.87

$$\int (d+ex)^{3/2} (a^2+2abx+b^2x^2)^3 dx = \frac{2b^6(d+ex)^{17/2}}{17e^7} - \frac{(12b^6d-12ab^5e)(d+ex)^{15/2}}{15e^7} + \frac{2(ae-bd)^6(d+ex)^{5/2}}{5e^7} + \frac{10b^2(ae-bd)^4(d+ex)^{9/2}}{3e^7} + \frac{40b^3(ae-bd)^3(d+ex)^{11/2}}{11e^7} + \frac{30b^4(ae-bd)^2(d+ex)^{13/2}}{13e^7} + \frac{12b(ae-bd)^5(d+ex)^{7/2}}{7e^7}$$

input `int((d + e*x)^(3/2)*(a^2 + b^2*x^2 + 2*a*b*x)^3,x)`output `(2*b^6*(d + e*x)^(17/2))/(17*e^7) - ((12*b^6*d - 12*a*b^5*e)*(d + e*x)^(15/2))/(15*e^7) + (2*(a*e - b*d)^6*(d + e*x)^(5/2))/(5*e^7) + (10*b^2*(a*e - b*d)^4*(d + e*x)^(9/2))/(3*e^7) + (40*b^3*(a*e - b*d)^3*(d + e*x)^(11/2))/(11*e^7) + (30*b^4*(a*e - b*d)^2*(d + e*x)^(13/2))/(13*e^7) + (12*b*(a*e - b*d)^5*(d + e*x)^(7/2))/(7*e^7)`**Reduce [B] (verification not implemented)**

Time = 0.21 (sec) , antiderivative size = 589, normalized size of antiderivative = 3.15

$$\int (d+ex)^{3/2} (a^2+2abx+b^2x^2)^3 dx = \frac{2\sqrt{ex+d}(15015b^6e^8x^8 + 102102ab^5e^8x^7 + 18018b^6de^7x^7 + 294525a^2b^4e^8x^6 + 125664ab^5d$$

input `int((e*x+d)^(3/2)*(b^2*x^2+2*a*b*x+a^2)^3,x)`

output

```
(2*sqrt(d + e*x)*(51051*a**6*d**2*e**6 + 102102*a**6*d*e**7*x + 51051*a**6
*e**8*x**2 - 87516*a**5*b*d**3*e**5 + 43758*a**5*b*d**2*e**6*x + 350064*a*
*5*b*d*e**7*x**2 + 218790*a**5*b*e**8*x**3 + 97240*a**4*b**2*d**4*e**4 - 4
8620*a**4*b**2*d**3*e**5*x + 36465*a**4*b**2*d**2*e**6*x**2 + 607750*a**4*
b**2*d*e**7*x**3 + 425425*a**4*b**2*e**8*x**4 - 70720*a**3*b**3*d**5*e**3
+ 35360*a**3*b**3*d**4*e**4*x - 26520*a**3*b**3*d**3*e**5*x**2 + 22100*a**
3*b**3*d**2*e**6*x**3 + 618800*a**3*b**3*d*e**7*x**4 + 464100*a**3*b**3*e*
*8*x**5 + 32640*a**2*b**4*d**6*e**2 - 16320*a**2*b**4*d**5*e**3*x + 12240*
a**2*b**4*d**4*e**4*x**2 - 10200*a**2*b**4*d**3*e**5*x**3 + 8925*a**2*b**4
*d**2*e**6*x**4 + 374850*a**2*b**4*d*e**7*x**5 + 294525*a**2*b**4*e**8*x**
6 - 8704*a*b**5*d**7*e + 4352*a*b**5*d**6*e**2*x - 3264*a*b**5*d**5*e**3*x
**2 + 2720*a*b**5*d**4*e**4*x**3 - 2380*a*b**5*d**3*e**5*x**4 + 2142*a*b**
5*d**2*e**6*x**5 + 125664*a*b**5*d*e**7*x**6 + 102102*a*b**5*e**8*x**7 + 1
024*b**6*d**8 - 512*b**6*d**7*e*x + 384*b**6*d**6*e**2*x**2 - 320*b**6*d**
5*e**3*x**3 + 280*b**6*d**4*e**4*x**4 - 252*b**6*d**3*e**5*x**5 + 231*b**6
*d**2*e**6*x**6 + 18018*b**6*d*e**7*x**7 + 15015*b**6*e**8*x**8))/(255255*
e**7)
```


3.165 $\int \sqrt{d + ex}(a^2 + 2abx + b^2x^2)^3 dx$

| | |
|---|------|
| Optimal result | 1292 |
| Mathematica [A] (verified) | 1293 |
| Rubi [A] (verified) | 1293 |
| Maple [A] (verified) | 1295 |
| Fricas [B] (verification not implemented) | 1296 |
| Sympy [B] (verification not implemented) | 1296 |
| Maxima [B] (verification not implemented) | 1297 |
| Giac [B] (verification not implemented) | 1298 |
| Mupad [B] (verification not implemented) | 1299 |
| Reduce [B] (verification not implemented) | 1299 |

Optimal result

Integrand size = 28, antiderivative size = 187

$$\int \sqrt{d + ex}(a^2 + 2abx + b^2x^2)^3 dx = \frac{2(bd - ae)^6(d + ex)^{3/2}}{3e^7} - \frac{12b(bd - ae)^5(d + ex)^{5/2}}{5e^7} + \frac{30b^2(bd - ae)^4(d + ex)^{7/2}}{7e^7} - \frac{40b^3(bd - ae)^3(d + ex)^{9/2}}{9e^7} + \frac{30b^4(bd - ae)^2(d + ex)^{11/2}}{11e^7} - \frac{12b^5(bd - ae)(d + ex)^{13/2}}{13e^7} + \frac{2b^6(d + ex)^{15/2}}{15e^7}$$

output

```
2/3*(-a*e+b*d)^6*(e*x+d)^(3/2)/e^7-12/5*b*(-a*e+b*d)^5*(e*x+d)^(5/2)/e^7+30/7*b^2*(-a*e+b*d)^4*(e*x+d)^(7/2)/e^7-40/9*b^3*(-a*e+b*d)^3*(e*x+d)^(9/2)/e^7+30/11*b^4*(-a*e+b*d)^2*(e*x+d)^(11/2)/e^7-12/13*b^5*(-a*e+b*d)*(e*x+d)^(13/2)/e^7+2/15*b^6*(e*x+d)^(15/2)/e^7
```

Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 291, normalized size of antiderivative = 1.56

$$\int \sqrt{d+ex}(a^2+2abx+b^2x^2)^3 dx$$

$$= \frac{2(d+ex)^{3/2}(15015a^6e^6+18018a^5be^5(-2d+3ex)+6435a^4b^2e^4(8d^2-12dex+15e^2x^2)+2860a^3b^3e^3(-16d^3+24d^2ex-30de^2x^2+35e^3x^3)+195a^2b^4e^2(128d^4-192d^3ex+240d^2e^2x^2-280de^3x^3+315e^4x^4)+30ab^5e(-256d^5+384d^4ex-480d^3e^2x^2+560d^2e^3x^3-630de^4x^4+693e^5x^5)+b^6(1024d^6-1536d^5ex+1920d^4e^2x^2-2240d^3e^3x^3+2520d^2e^4x^4-2772de^5x^5+3003e^6x^6))}{45045e^7}$$

input `Integrate[Sqrt[d + e*x]*(a^2 + 2*a*b*x + b^2*x^2)^3,x]`

output $(2*(d + e*x)^{(3/2)}*(15015*a^6*e^6 + 18018*a^5*b*e^5*(-2*d + 3*e*x) + 6435*a^4*b^2*e^4*(8*d^2 - 12*d*e*x + 15*e^2*x^2) + 2860*a^3*b^3*e^3*(-16*d^3 + 24*d^2*e*x - 30*d*e^2*x^2 + 35*e^3*x^3) + 195*a^2*b^4*e^2*(128*d^4 - 192*d^3*e*x + 240*d^2*e^2*x^2 - 280*d*e^3*x^3 + 315*e^4*x^4) + 30*a*b^5*e*(-256*d^5 + 384*d^4*e*x - 480*d^3*e^2*x^2 + 560*d^2*e^3*x^3 - 630*d*e^4*x^4 + 693*e^5*x^5) + b^6*(1024*d^6 - 1536*d^5*e*x + 1920*d^4*e^2*x^2 - 2240*d^3*e^3*x^3 + 2520*d^2*e^4*x^4 - 2772*d*e^5*x^5 + 3003*e^6*x^6)))/(45045*e^7)$

Rubi [A] (verified)

Time = 0.55 (sec) , antiderivative size = 187, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {1098, 27, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a^2 + 2abx + b^2x^2)^3 \sqrt{d+ex} dx$$

$$\downarrow 1098$$

$$\frac{\int b^6(a+bx)^6 \sqrt{d+ex} dx}{b^6}$$

$$\downarrow 27$$

$$\int (a+bx)^6 \sqrt{d+ex} dx$$

↓ 53

$$\int \left(-\frac{6b^5(d+ex)^{11/2}(bd-ae)}{e^6} + \frac{15b^4(d+ex)^{9/2}(bd-ae)^2}{e^6} - \frac{20b^3(d+ex)^{7/2}(bd-ae)^3}{e^6} + \frac{15b^2(d+ex)^{5/2}(bd-ae)^4}{e^6} \right)$$

↓ 2009

$$\begin{aligned} & -\frac{12b^5(d+ex)^{13/2}(bd-ae)}{13e^7} + \frac{30b^4(d+ex)^{11/2}(bd-ae)^2}{11e^7} - \frac{40b^3(d+ex)^{9/2}(bd-ae)^3}{9e^7} + \\ & \frac{30b^2(d+ex)^{7/2}(bd-ae)^4}{7e^7} - \frac{12b(d+ex)^{5/2}(bd-ae)^5}{5e^7} + \frac{2(d+ex)^{3/2}(bd-ae)^6}{3e^7} + \\ & \frac{2b^6(d+ex)^{15/2}}{15e^7} \end{aligned}$$

input `Int[Sqrt[d + e*x]*(a^2 + 2*a*b*x + b^2*x^2)^3,x]`

output $(2*(b*d - a*e)^6*(d + e*x)^{(3/2)})/(3*e^7) - (12*b*(b*d - a*e)^5*(d + e*x)^{(5/2)})/(5*e^7) + (30*b^2*(b*d - a*e)^4*(d + e*x)^{(7/2)})/(7*e^7) - (40*b^3*(b*d - a*e)^3*(d + e*x)^{(9/2)})/(9*e^7) + (30*b^4*(b*d - a*e)^2*(d + e*x)^{(11/2)})/(11*e^7) - (12*b^5*(b*d - a*e)*(d + e*x)^{(13/2)})/(13*e^7) + (2*b^6*(d + e*x)^{(15/2)})/(15*e^7)$

Defintions of rubi rules used

rule 27 `Int[(a_)*(F_x_), x_Symbol] := Simp[a Int[F_x, x], x] /; FreeQ[a, x] && !MatchQ[F_x, (b_)*(G_x_)] /; FreeQ[b, x]`

rule 53 `Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] := Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && IGtQ[m, 0] && (!IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])`

rule 1098 `Int[((d_.) + (e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] := Simp[1/c^p Int[(d + e*x)^m*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]`

```
rule 2009 Int[u_, x_Symbol] := Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 1.29 (sec) , antiderivative size = 276, normalized size of antiderivative = 1.48

| method | result |
|-------------------|---|
| pseudoelliptic | $2(ex+d)^{\frac{3}{2}} \left(\left(\frac{18}{13} a b^5 x^5 + \frac{45}{11} a^2 b^4 x^4 + \frac{20}{3} a^3 b^3 x^3 + \frac{45}{7} a^4 b^2 x^2 + \frac{18}{5} a^5 b x + \frac{1}{5} b^6 x^6 + a^6 \right) e^6 - \frac{12bd \left(\frac{1}{13} b^5 x^5 + \frac{75}{143} a b^4 x^4 + \frac{50}{33} a^2 b^3 x^3 + \frac{50}{21} a^3 b^2 x^2 + \frac{15}{7} a^4 b x + a^5 \right) e^5 + 24 b^6 d^2 e^4 + 100100 a^3 b^3 e^3 x^4 + 6825 a^4 b^2 e^2 x^3 + 128 a^5 b e x^2 + 1024 a^6 d^6 e^7}{e^7} \right)$ |
| gospers | $2(ex+d)^{\frac{3}{2}} (3003x^6 b^6 e^6 + 20790x^5 a b^5 e^6 - 2772x^5 b^6 d e^5 + 61425x^4 a^2 b^4 e^6 - 18900x^4 a b^5 d e^5 + 2520x^4 b^6 d^2 e^4 + 100100x^3 a^3 b^3 e^6 - 50050x^3 a^4 b^2 e^5 + 12800x^3 a^5 b e^4 + 10240x^2 a^6 d^6 e^7)$ |
| orering | $2(3003x^6 b^6 e^6 + 20790x^5 a b^5 e^6 - 2772x^5 b^6 d e^5 + 61425x^4 a^2 b^4 e^6 - 18900x^4 a b^5 d e^5 + 2520x^4 b^6 d^2 e^4 + 100100x^3 a^3 b^3 e^6 - 50050x^3 a^4 b^2 e^5 + 12800x^3 a^5 b e^4 + 10240x^2 a^6 d^6 e^7)$ |
| derivativedivides | $\frac{2b^6 (ex+d)^{\frac{15}{2}}}{15} + \frac{6(2abe-2b^2d)b^4 (ex+d)^{\frac{13}{2}}}{13} + \frac{2 \left((e^2 a^2 - 2abde + b^2 d^2) b^4 + 2(2abe - 2b^2 d)^2 b^2 + b^2 (2(e^2 a^2 - 2abde + b^2 d^2) b^2 + (2abde - 2b^2 d)^2) \right)}{11}$ |
| default | $\frac{2b^6 (ex+d)^{\frac{15}{2}}}{15} + \frac{6(2abe-2b^2d)b^4 (ex+d)^{\frac{13}{2}}}{13} + \frac{2 \left((e^2 a^2 - 2abde + b^2 d^2) b^4 + 2(2abe - 2b^2 d)^2 b^2 + b^2 (2(e^2 a^2 - 2abde + b^2 d^2) b^2 + (2abde - 2b^2 d)^2) \right)}{11}$ |
| trager | $2(3003b^6 e^7 x^7 + 20790a b^5 e^7 x^6 + 231b^6 d e^6 x^6 + 61425a^2 b^4 e^7 x^5 + 1890a b^5 d e^6 x^5 - 252b^6 d^2 e^5 x^5 + 100100a^3 b^3 e^7 x^4 + 6825a^4 b^2 e^2 x^3 + 128a^5 b e x^2 + 1024a^6 d^6 e^7)$ |
| risch | $2(3003b^6 e^7 x^7 + 20790a b^5 e^7 x^6 + 231b^6 d e^6 x^6 + 61425a^2 b^4 e^7 x^5 + 1890a b^5 d e^6 x^5 - 252b^6 d^2 e^5 x^5 + 100100a^3 b^3 e^7 x^4 + 6825a^4 b^2 e^2 x^3 + 128a^5 b e x^2 + 1024a^6 d^6 e^7)$ |

```
input int((e*x+d)^(1/2)*(b^2*x^2+2*a*b*x+a^2)^3,x,method=_RETURNVERBOSE)
```

```
output 2/3*(e*x+d)^(3/2)*((18/13*a*b^5*x^5+45/11*a^2*b^4*x^4+20/3*a^3*b^3*x^3+45/7*a^4*b^2*x^2+18/5*a^5*b*x+1/5*b^6*x^6+a^6)*e^6-12/5*b*d*(1/13*b^5*x^5+75/143*a*b^4*x^4+50/33*a^2*b^3*x^3+50/21*a^3*b^2*x^2+15/7*a^4*b*x+a^5)*e^5+24/7*(7/143*b^4*x^4+140/429*a*b^3*x^3+10/11*a^2*b^2*x^2+4/3*a^3*b*x+a^4)*b^2*d^2*e^4-64/21*(7/143*b^3*x^3+45/143*a*b^2*x^2+9/11*a^2*b*x+a^3)*b^3*d^3*e^3+128/77*b^4*(1/13*b^2*x^2+6/13*a*b*x+a^2)*d^4*e^2-512/1001*b^5*(1/5*b*x+a)*d^5*e+1024/15015*b^6*d^6)/e^7
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 447 vs. $2(159) = 318$.

Time = 0.09 (sec) , antiderivative size = 447, normalized size of antiderivative = 2.39

$$\int \sqrt{d+ex}(a^2+2abx+b^2x^2)^3 dx$$

$$= \frac{2(3003b^6e^7x^7 + 1024b^6d^7 - 7680ab^5d^6e + 24960a^2b^4d^5e^2 - 45760a^3b^3d^4e^3 + 51480a^4b^2d^3e^4 - 36036a^5b^2d^2e^4 + 15015a^6d^2e^5 - 231b^6d^6e^6 + 90a^5b^5e^7)x^6 - 63(4b^6d^2e^5 - 30a^5b^5d^3e^6 - 975a^2b^4d^3e^7)x^5 + 35(8b^6d^3e^4 - 60a^5b^5d^2e^5 + 195a^2b^4d^2e^6 + 2860a^3b^3e^7)x^4 - 5(64b^6d^4e^3 - 480a^5b^5d^3e^4 + 1560a^2b^4d^2e^5 - 2860a^3b^3d^2e^6 - 19305a^4b^2e^7)x^3 + 3(128b^6d^5e^2 - 960a^5b^5d^4e^3 + 3120a^2b^4d^3e^4 - 5720a^3b^3d^2e^5 + 6435a^4b^2d^2e^6 + 18018a^5b^2e^7)x^2 - (512b^6d^6e - 3840a^5b^5d^5e^2 + 12480a^2b^4d^4e^3 - 22880a^3b^3d^3e^4 + 25740a^4b^2d^2e^5 - 18018a^5b^2d^2e^6 - 15015a^6e^7)x}{e^7} \sqrt{e^7x+d}$$

input

```
integrate((e*x+d)^(1/2)*(b^2*x^2+2*a*b*x+a^2)^3,x, algorithm="fricas")
```

output

```
2/45045*(3003*b^6*e^7*x^7 + 1024*b^6*d^7 - 7680*a*b^5*d^6*e + 24960*a^2*b^4*d^5*e^2 - 45760*a^3*b^3*d^4*e^3 + 51480*a^4*b^2*d^3*e^4 - 36036*a^5*b^2*d^2*e^4 + 15015*a^6*d^2*e^5 - 231*(b^6*d^6*e^6 + 90*a*b^5*e^7)*x^6 - 63*(4*b^6*d^2*e^5 - 30*a*b^5*d^3*e^6 - 975*a^2*b^4*d^3*e^7)*x^5 + 35*(8*b^6*d^3*e^4 - 60*a*b^5*d^2*e^5 + 195*a^2*b^4*d^2*e^6 + 2860*a^3*b^3*e^7)*x^4 - 5*(64*b^6*d^4*e^3 - 480*a*b^5*d^3*e^4 + 1560*a^2*b^4*d^2*e^5 - 2860*a^3*b^3*d^2*e^6 - 19305*a^4*b^2*e^7)*x^3 + 3*(128*b^6*d^5*e^2 - 960*a*b^5*d^4*e^3 + 3120*a^2*b^4*d^3*e^4 - 5720*a^3*b^3*d^2*e^5 + 6435*a^4*b^2*d^2*e^6 + 18018*a^5*b^2*e^7)*x^2 - (512*b^6*d^6*e - 3840*a*b^5*d^5*e^2 + 12480*a^2*b^4*d^4*e^3 - 22880*a^3*b^3*d^3*e^4 + 25740*a^4*b^2*d^2*e^5 - 18018*a^5*b^2*d^2*e^6 - 15015*a^6*e^7)*x)/e^7*sqrt(e*x + d)/e^7
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 495 vs. $2(173) = 346$.

Time = 1.16 (sec) , antiderivative size = 495, normalized size of antiderivative = 2.65

$$\int \sqrt{d+ex}(a^2+2abx+b^2x^2)^3 dx$$

$$= \frac{2 \left(\frac{b^6(d+ex)^{15}}{15e^6} + \frac{(d+ex)^{13} \cdot (6ab^5e - 6b^6d)}{13e^6} + \frac{(d+ex)^{11} \cdot (15a^2b^4e^2 - 30ab^5de + 15b^6d^2)}{11e^6} + \frac{(d+ex)^9 \cdot (20a^3b^3e^3 - 60a^2b^4de^2 + 60ab^5d^2e - 20b^6d^3)}{9e^6} + \frac{(d+ex)^7 \cdot (7a^4b^2e^4 - 28a^3b^3de^3 + 28a^2b^4d^2e^2 - 7a^4b^2e^4)}{7e^6} \right)}{\sqrt{d+ex}}$$

$$= \frac{2 \left(\frac{b^6(d+ex)^{15}}{15e^6} + \frac{(d+ex)^{13} \cdot (6ab^5e - 6b^6d)}{13e^6} + \frac{(d+ex)^{11} \cdot (15a^2b^4e^2 - 30ab^5de + 15b^6d^2)}{11e^6} + \frac{(d+ex)^9 \cdot (20a^3b^3e^3 - 60a^2b^4de^2 + 60ab^5d^2e - 20b^6d^3)}{9e^6} + \frac{(d+ex)^7 \cdot (7a^4b^2e^4 - 28a^3b^3de^3 + 28a^2b^4d^2e^2 - 7a^4b^2e^4)}{7e^6} \right)}{\sqrt{d+ex}}$$

input

```
integrate((e*x+d)**(1/2)*(b**2*x**2+2*a*b*x+a**2)**3,x)
```

output

```
Piecewise((2*(b**6*(d + e*x)**(15/2)/(15*e**6) + (d + e*x)**(13/2)*(6*a*b*
*5*e - 6*b**6*d)/(13*e**6) + (d + e*x)**(11/2)*(15*a**2*b**4*e**2 - 30*a*b
**5*d*e + 15*b**6*d**2)/(11*e**6) + (d + e*x)**(9/2)*(20*a**3*b**3*e**3 -
60*a**2*b**4*d*e**2 + 60*a*b**5*d**2*e - 20*b**6*d**3)/(9*e**6) + (d + e*x
)**(7/2)*(15*a**4*b**2*e**4 - 60*a**3*b**3*d*e**3 + 90*a**2*b**4*d**2*e**2
- 60*a*b**5*d**3*e + 15*b**6*d**4)/(7*e**6) + (d + e*x)**(5/2)*(6*a**5*b*
e**5 - 30*a**4*b**2*d*e**4 + 60*a**3*b**3*d**2*e**3 - 60*a**2*b**4*d**3*e*
*2 + 30*a*b**5*d**4*e - 6*b**6*d**5)/(5*e**6) + (d + e*x)**(3/2)*(a**6*e**
6 - 6*a**5*b*d*e**5 + 15*a**4*b**2*d**2*e**4 - 20*a**3*b**3*d**3*e**3 + 15
*a**2*b**4*d**4*e**2 - 6*a*b**5*d**5*e + b**6*d**6)/(3*e**6))/e, Ne(e, 0))
, (sqrt(d)*(a**6*x + 3*a**5*b*x**2 + 5*a**4*b**2*x**3 + 5*a**3*b**3*x**4 +
3*a**2*b**4*x**5 + a*b**5*x**6 + b**6*x**7/7), True))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 350 vs. $2(159) = 318$.

Time = 0.05 (sec) , antiderivative size = 350, normalized size of antiderivative = 1.87

$$\int \sqrt{d+ex}(a^2+2abx+b^2x^2)^3 dx$$

$$= \frac{2 \left(3003 (ex+d)^{\frac{15}{2}} b^6 - 20790 (b^6d - ab^5e)(ex+d)^{\frac{13}{2}} + 61425 (b^6d^2 - 2ab^5de + a^2b^4e^2)(ex+d)^{\frac{11}{2}} - 10010 (b^6d^3 - 3ab^5d^2e + 3a^2b^4d^2e^2 - a^3b^3e^3)(ex+d)^{\frac{9}{2}} + 96525 (b^6d^4 - 4ab^5d^3e + 6a^2b^4d^2e^2 - 4a^3b^3de^3 + a^4b^2e^4)(ex+d)^{\frac{7}{2}} - 54054 (b^6d^5 - 5ab^5d^4e + 10a^2b^4d^3e^2 - 10a^3b^3d^2e^3 + 5a^4b^2d^2e^4 - a^5b^2e^5)(ex+d)^{\frac{5}{2}} + 15015 (b^6d^6 - 6ab^5d^5e + 15a^2b^4d^4e^2 - 20a^3b^3d^3e^3 + 15a^4b^2d^2e^4 - 6a^5b^2de^5 + a^6e^6)(ex+d)^{\frac{3}{2}} \right)}{e^7}$$

input

```
integrate((e*x+d)^(1/2)*(b^2*x^2+2*a*b*x+a^2)^3,x, algorithm="maxima")
```

output

```
2/45045*(3003*(e*x + d)^(15/2)*b^6 - 20790*(b^6*d - a*b^5*e)*(e*x + d)^(13
/2) + 61425*(b^6*d^2 - 2*a*b^5*d*e + a^2*b^4*e^2)*(e*x + d)^(11/2) - 10010
0*(b^6*d^3 - 3*a*b^5*d^2*e + 3*a^2*b^4*d^2*e^2 - a^3*b^3*e^3)*(e*x + d)^(9/2
) + 96525*(b^6*d^4 - 4*a*b^5*d^3*e + 6*a^2*b^4*d^2*e^2 - 4*a^3*b^3*d^2*e^3 +
a^4*b^2*e^4)*(e*x + d)^(7/2) - 54054*(b^6*d^5 - 5*a*b^5*d^4*e + 10*a^2*b^
4*d^3*e^2 - 10*a^3*b^3*d^2*e^3 + 5*a^4*b^2*d^2*e^4 - a^5*b^2*e^5)*(e*x + d)^(5
/2) + 15015*(b^6*d^6 - 6*a*b^5*d^5*e + 15*a^2*b^4*d^4*e^2 - 20*a^3*b^3*d^3
*e^3 + 15*a^4*b^2*d^2*e^4 - 6*a^5*b^2*d^2*e^4 - 6*a^5*b^2*d^2*e^4 + a^6*e^6)*(e*x + d)^(3/2))/e^7
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 836 vs. $2(159) = 318$.

Time = 0.19 (sec) , antiderivative size = 836, normalized size of antiderivative = 4.47

$$\int \sqrt{d+ex}(a^2+2abx+b^2x^2)^3 dx = \text{Too large to display}$$

input `integrate((e*x+d)^(1/2)*(b^2*x^2+2*a*b*x+a^2)^3,x, algorithm="giac")`

output

```
2/45045*(45045*sqrt(e*x + d)*a^6*d + 15015*((e*x + d)^(3/2) - 3*sqrt(e*x +
d)*d)*a^6 + 90090*((e*x + d)^(3/2) - 3*sqrt(e*x + d)*d)*a^5*b*d/e + 45045
*(3*(e*x + d)^(5/2) - 10*(e*x + d)^(3/2)*d + 15*sqrt(e*x + d)*d^2)*a^4*b^2
*d/e^2 + 18018*(3*(e*x + d)^(5/2) - 10*(e*x + d)^(3/2)*d + 15*sqrt(e*x + d
)*d^2)*a^5*b/e + 25740*(5*(e*x + d)^(7/2) - 21*(e*x + d)^(5/2)*d + 35*(e*x
+ d)^(3/2)*d^2 - 35*sqrt(e*x + d)*d^3)*a^3*b^3*d/e^3 + 19305*(5*(e*x + d)
^(7/2) - 21*(e*x + d)^(5/2)*d + 35*(e*x + d)^(3/2)*d^2 - 35*sqrt(e*x + d)*
d^3)*a^4*b^2/e^2 + 2145*(35*(e*x + d)^(9/2) - 180*(e*x + d)^(7/2)*d + 378*
(e*x + d)^(5/2)*d^2 - 420*(e*x + d)^(3/2)*d^3 + 315*sqrt(e*x + d)*d^4)*a^2
*b^4*d/e^4 + 2860*(35*(e*x + d)^(9/2) - 180*(e*x + d)^(7/2)*d + 378*(e*x +
d)^(5/2)*d^2 - 420*(e*x + d)^(3/2)*d^3 + 315*sqrt(e*x + d)*d^4)*a^3*b^3/e
^3 + 390*(63*(e*x + d)^(11/2) - 385*(e*x + d)^(9/2)*d + 990*(e*x + d)^(7/2
)*d^2 - 1386*(e*x + d)^(5/2)*d^3 + 1155*(e*x + d)^(3/2)*d^4 - 693*sqrt(e*x
+ d)*d^5)*a*b^5*d/e^5 + 975*(63*(e*x + d)^(11/2) - 385*(e*x + d)^(9/2)*d
+ 990*(e*x + d)^(7/2)*d^2 - 1386*(e*x + d)^(5/2)*d^3 + 1155*(e*x + d)^(3/2
)*d^4 - 693*sqrt(e*x + d)*d^5)*a^2*b^4/e^4 + 15*(231*(e*x + d)^(13/2) - 16
38*(e*x + d)^(11/2)*d + 5005*(e*x + d)^(9/2)*d^2 - 8580*(e*x + d)^(7/2)*d^
3 + 9009*(e*x + d)^(5/2)*d^4 - 6006*(e*x + d)^(3/2)*d^5 + 3003*sqrt(e*x +
d)*d^6)*b^6*d/e^6 + 90*(231*(e*x + d)^(13/2) - 1638*(e*x + d)^(11/2)*d + 5
005*(e*x + d)^(9/2)*d^2 - 8580*(e*x + d)^(7/2)*d^3 + 9009*(e*x + d)^(5/...
```

Mupad [B] (verification not implemented)

Time = 9.04 (sec) , antiderivative size = 162, normalized size of antiderivative = 0.87

$$\int \sqrt{d+ex}(a^2+2abx+b^2x^2)^3 dx = \frac{2b^6(d+ex)^{15/2}}{15e^7} - \frac{(12b^6d-12ab^5e)(d+ex)^{13/2}}{13e^7} + \frac{2(ae-bd)^6(d+ex)^{3/2}}{3e^7} + \frac{30b^2(ae-bd)^4(d+ex)^{7/2}}{7e^7} + \frac{40b^3(ae-bd)^3(d+ex)^{9/2}}{9e^7} + \frac{30b^4(ae-bd)^2(d+ex)^{11/2}}{11e^7} + \frac{12b(ae-bd)^5(d+ex)^{5/2}}{5e^7}$$

input

```
int((d + e*x)^(1/2)*(a^2 + b^2*x^2 + 2*a*b*x)^3,x)
```

output

```
(2*b^6*(d + e*x)^(15/2))/(15*e^7) - ((12*b^6*d - 12*a*b^5*e)*(d + e*x)^(13/2))/(13*e^7) + (2*(a*e - b*d)^6*(d + e*x)^(3/2))/(3*e^7) + (30*b^2*(a*e - b*d)^4*(d + e*x)^(7/2))/(7*e^7) + (40*b^3*(a*e - b*d)^3*(d + e*x)^(9/2))/(9*e^7) + (30*b^4*(a*e - b*d)^2*(d + e*x)^(11/2))/(11*e^7) + (12*b*(a*e - b*d)^5*(d + e*x)^(5/2))/(5*e^7)
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 480, normalized size of antiderivative = 2.57

$$\int \sqrt{d+ex}(a^2+2abx+b^2x^2)^3 dx = \frac{2\sqrt{ex+d}(3003b^6e^7x^7 + 20790ab^5e^7x^6 + 231b^6de^6x^6 + 61425a^2b^4e^7x^5 + 1890ab^5de^6x^5 - 252b^6d^2e^5x^5)}{e^7}$$

input

```
int((e*x+d)^(1/2)*(b^2*x^2+2*a*b*x+a^2)^3,x)
```


output

```
(2*sqrt(d + e*x)*(15015*a**6*d*e**6 + 15015*a**6*e**7*x - 36036*a**5*b*d**2*e**5 + 18018*a**5*b*d*e**6*x + 54054*a**5*b*e**7*x**2 + 51480*a**4*b**2*d**3*e**4 - 25740*a**4*b**2*d**2*e**5*x + 19305*a**4*b**2*d*e**6*x**2 + 96525*a**4*b**2*e**7*x**3 - 45760*a**3*b**3*d**4*e**3 + 22880*a**3*b**3*d**3*e**4*x - 17160*a**3*b**3*d**2*e**5*x**2 + 14300*a**3*b**3*d*e**6*x**3 + 100100*a**3*b**3*e**7*x**4 + 24960*a**2*b**4*d**5*e**2 - 12480*a**2*b**4*d**4*e**3*x + 9360*a**2*b**4*d**3*e**4*x**2 - 7800*a**2*b**4*d**2*e**5*x**3 + 6825*a**2*b**4*d*e**6*x**4 + 61425*a**2*b**4*e**7*x**5 - 7680*a*b**5*d**6*e + 3840*a*b**5*d**5*e**2*x - 2880*a*b**5*d**4*e**3*x**2 + 2400*a*b**5*d**3*e**4*x**3 - 2100*a*b**5*d**2*e**5*x**4 + 1890*a*b**5*d*e**6*x**5 + 20790*a*b**5*e**7*x**6 + 1024*b**6*d**7 - 512*b**6*d**6*e*x + 384*b**6*d**5*e**2*x**2 - 320*b**6*d**4*e**3*x**3 + 280*b**6*d**3*e**4*x**4 - 252*b**6*d**2*e**5*x**5 + 231*b**6*d*e**6*x**6 + 3003*b**6*e**7*x**7))/(45045*e**7)
```

$$3.166 \quad \int \frac{(a^2 + 2abx + b^2x^2)^3}{\sqrt{d+ex}} dx$$

| | |
|---|------|
| Optimal result | 1301 |
| Mathematica [A] (verified) | 1302 |
| Rubi [A] (verified) | 1302 |
| Maple [A] (verified) | 1304 |
| Fricas [B] (verification not implemented) | 1305 |
| Sympy [B] (verification not implemented) | 1305 |
| Maxima [B] (verification not implemented) | 1306 |
| Giac [B] (verification not implemented) | 1307 |
| Mupad [B] (verification not implemented) | 1308 |
| Reduce [B] (verification not implemented) | 1309 |

Optimal result

Integrand size = 28, antiderivative size = 181

$$\int \frac{(a^2 + 2abx + b^2x^2)^3}{\sqrt{d+ex}} dx = \frac{2(bd - ae)^6 \sqrt{d+ex}}{e^7} - \frac{4b(bd - ae)^5 (d+ex)^{3/2}}{e^7} + \frac{6b^2(bd - ae)^4 (d+ex)^{5/2}}{e^7} - \frac{40b^3(bd - ae)^3 (d+ex)^{7/2}}{7e^7} + \frac{10b^4(bd - ae)^2 (d+ex)^{9/2}}{3e^7} - \frac{12b^5(bd - ae)(d+ex)^{11/2}}{11e^7} + \frac{2b^6(d+ex)^{13/2}}{13e^7}$$

output

```
2*(-a*e+b*d)^6*(e*x+d)^(1/2)/e^7-4*b*(-a*e+b*d)^5*(e*x+d)^(3/2)/e^7+6*b^2*
(-a*e+b*d)^4*(e*x+d)^(5/2)/e^7-40/7*b^3*(-a*e+b*d)^3*(e*x+d)^(7/2)/e^7+10/
3*b^4*(-a*e+b*d)^2*(e*x+d)^(9/2)/e^7-12/11*b^5*(-a*e+b*d)*(e*x+d)^(11/2)/e
^7+2/13*b^6*(e*x+d)^(13/2)/e^7
```

Mathematica [A] (verified)

Time = 0.16 (sec) , antiderivative size = 290, normalized size of antiderivative = 1.60

$$\int \frac{(a^2 + 2abx + b^2x^2)^3}{\sqrt{d + ex}} dx$$

$$= \frac{2\sqrt{d + ex}(3003a^6e^6 + 6006a^5be^5(-2d + ex) + 3003a^4b^2e^4(8d^2 - 4dex + 3e^2x^2) + 1716a^3b^3e^3(-16d^3 + 8d^2ex - 6d^2e^2x^2 + 5e^3x^3) + 143a^2b^4e^2(128d^4 - 64d^3ex + 48d^2e^2x^2 - 40de^3x^3 + 35e^4x^4) + 26ab^5e(-256d^5 + 128d^4ex - 96d^3e^2x^2 + 80d^2e^3x^3 - 70de^4x^4 + 63e^5x^5) + b^6(1024d^6 - 512d^5ex + 384d^4e^2x^2 - 320d^3e^3x^3 + 280d^2e^4x^4 - 252de^5x^5 + 231e^6x^6))}{(3003e^7)}$$

input

```
Integrate[(a^2 + 2*a*b*x + b^2*x^2)^3/Sqrt[d + e*x],x]
```

output

```
(2*Sqrt[d + e*x]*(3003*a^6*e^6 + 6006*a^5*b*e^5*(-2*d + e*x) + 3003*a^4*b^2*e^4*(8*d^2 - 4*d*e*x + 3*e^2*x^2) + 1716*a^3*b^3*e^3*(-16*d^3 + 8*d^2*e*x - 6*d*e^2*x^2 + 5*e^3*x^3) + 143*a^2*b^4*e^2*(128*d^4 - 64*d^3*e*x + 48*d^2*e^2*x^2 - 40*d*e^3*x^3 + 35*e^4*x^4) + 26*a*b^5*e*(-256*d^5 + 128*d^4*e*x - 96*d^3*e^2*x^2 + 80*d^2*e^3*x^3 - 70*d*e^4*x^4 + 63*e^5*x^5) + b^6*(1024*d^6 - 512*d^5*e*x + 384*d^4*e^2*x^2 - 320*d^3*e^3*x^3 + 280*d^2*e^4*x^4 - 252*d*e^5*x^5 + 231*e^6*x^6)))/(3003*e^7)
```

Rubi [A] (verified)

Time = 0.54 (sec) , antiderivative size = 181, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {1098, 27, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a^2 + 2abx + b^2x^2)^3}{\sqrt{d + ex}} dx$$

$$\downarrow 1098$$

$$\frac{\int \frac{b^6(a+bx)^6}{\sqrt{d+ex}} dx}{b^6}$$

$$\downarrow 27$$

$$\int \frac{(a + bx)^6}{\sqrt{d + ex}} dx$$

↓ 53

$$\int \left(-\frac{6b^5(d + ex)^{9/2}(bd - ae)}{e^6} + \frac{15b^4(d + ex)^{7/2}(bd - ae)^2}{e^6} - \frac{20b^3(d + ex)^{5/2}(bd - ae)^3}{e^6} + \frac{15b^2(d + ex)^{3/2}(bd - ae)^4}{e^6} - \frac{6b(d + ex)^{1/2}(bd - ae)^5}{e^6} + \frac{6b^2(d + ex)^{5/2}(bd - ae)^4}{e^7} - \frac{4b(d + ex)^{3/2}(bd - ae)^5}{e^7} + \frac{2\sqrt{d + ex}(bd - ae)^6}{e^7} + \frac{2b^6(d + ex)^{13/2}}{13e^7} \right) dx$$

↓ 2009

$$\frac{-12b^5(d + ex)^{11/2}(bd - ae)}{11e^7} + \frac{10b^4(d + ex)^{9/2}(bd - ae)^2}{3e^7} - \frac{40b^3(d + ex)^{7/2}(bd - ae)^3}{7e^7} + \frac{6b^2(d + ex)^{5/2}(bd - ae)^4}{e^7} - \frac{4b(d + ex)^{3/2}(bd - ae)^5}{e^7} + \frac{2\sqrt{d + ex}(bd - ae)^6}{e^7} + \frac{2b^6(d + ex)^{13/2}}{13e^7}$$

input `Int[(a^2 + 2*a*b*x + b^2*x^2)^3/Sqrt[d + e*x],x]`

output `(2*(b*d - a*e)^6*Sqrt[d + e*x])/e^7 - (4*b*(b*d - a*e)^5*(d + e*x)^(3/2))/e^7 + (6*b^2*(b*d - a*e)^4*(d + e*x)^(5/2))/e^7 - (40*b^3*(b*d - a*e)^3*(d + e*x)^(7/2))/(7*e^7) + (10*b^4*(b*d - a*e)^2*(d + e*x)^(9/2))/(3*e^7) - (12*b^5*(b*d - a*e)*(d + e*x)^(11/2))/(11*e^7) + (2*b^6*(d + e*x)^(13/2))/(13*e^7)`

Defintions of rubi rules used

rule 27 `Int[(a_)*(F_x_), x_Symbol] := Simp[a Int[F_x, x], x] /; FreeQ[a, x] && !MatchQ[F_x, (b_)*(G_x_)] /; FreeQ[b, x]`

rule 53 `Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] := Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && IGtQ[m, 0] && (!IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])`

rule 1098 `Int[((d_.) + (e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] := Simp[1/c^p Int[(d + e*x)^m*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]`

rule 2009

```
Int[u_, x_Symbol] := Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 1.22 (sec) , antiderivative size = 275, normalized size of antiderivative = 1.52

| method | result |
|------------------|---|
| pseudoelliptic | $2\sqrt{ex+d} \left(\left(\frac{6}{11}ab^5x^5 + \frac{5}{3}a^2b^4x^4 + \frac{20}{7}a^3b^3x^3 + 3a^4b^2x^2 + 2a^5bx + \frac{1}{13}b^6x^6 + a^6 \right) e^6 - 4b \left(\frac{3}{143}b^5x^5 + \frac{5}{33}ab^4x^4 + \frac{10}{21}a^2b^3x^3 + \frac{6}{7}a^3b^2x^2 + 2a^4bx + a^5 \right) e^5 + \dots \right)$ |
| gospers | $2(231x^6b^6e^6 + 1638x^5ab^5e^6 - 252x^5b^6de^5 + 5005x^4a^2b^4e^6 - 1820x^4ab^5de^5 + 280x^4b^6d^2e^4 + 8580x^3a^3b^3e^6 - 5720x^3a^2b^4de^5 + \dots)$ |
| trager | $2(231x^6b^6e^6 + 1638x^5ab^5e^6 - 252x^5b^6de^5 + 5005x^4a^2b^4e^6 - 1820x^4ab^5de^5 + 280x^4b^6d^2e^4 + 8580x^3a^3b^3e^6 - 5720x^3a^2b^4de^5 + \dots)$ |
| risch | $2(231x^6b^6e^6 + 1638x^5ab^5e^6 - 252x^5b^6de^5 + 5005x^4a^2b^4e^6 - 1820x^4ab^5de^5 + 280x^4b^6d^2e^4 + 8580x^3a^3b^3e^6 - 5720x^3a^2b^4de^5 + \dots)$ |
| orering | $2(231x^6b^6e^6 + 1638x^5ab^5e^6 - 252x^5b^6de^5 + 5005x^4a^2b^4e^6 - 1820x^4ab^5de^5 + 280x^4b^6d^2e^4 + 8580x^3a^3b^3e^6 - 5720x^3a^2b^4de^5 + \dots)$ |
| derivativdivides | $\frac{2b^6(e^6x^6 + d^6)}{13} + \frac{6(2abe - 2b^2d)b^4(e^6x^4 + d^4)}{11} + \frac{2((e^2a^2 - 2abde + b^2d^2)b^4 + 2(2abe - 2b^2d)^2b^2 + b^2(2(e^2a^2 - 2abde + b^2d^2)b^2 + (2abde - b^2d^2)^2))}{9}$ |
| default | $\frac{2b^6(e^6x^6 + d^6)}{13} + \frac{6(2abe - 2b^2d)b^4(e^6x^4 + d^4)}{11} + \frac{2((e^2a^2 - 2abde + b^2d^2)b^4 + 2(2abe - 2b^2d)^2b^2 + b^2(2(e^2a^2 - 2abde + b^2d^2)b^2 + (2abde - b^2d^2)^2))}{9}$ |

input

```
int((b^2*x^2+2*a*b*x+a^2)^3/(e*x+d)^(1/2), x, method=_RETURNVERBOSE)
```

output

```
2*(e*x+d)^(1/2)*((6/11*a*b^5*x^5+5/3*a^2*b^4*x^4+20/7*a^3*b^3*x^3+3*a^4*b^2*x^2+2*a^5*b*x+1/13*b^6*x^6+a^6)*e^6-4*b*(3/143*b^5*x^5+5/33*a*b^4*x^4+10/21*a^2*b^3*x^3+6/7*a^3*b^2*x^2+a^4*b*x+a^5)*d*e^5+8*(5/429*b^4*x^4+20/231*a*b^3*x^3+2/7*a^2*b^2*x^2+4/7*a^3*b*x+a^4)*b^2*d^2*e^4-64/7*b^3*d^3*(5/429*b^3*x^3+1/11*a*b^2*x^2+1/3*a^2*b*x+a^3)*e^3+128/21*(3/143*b^2*x^2+2/11*a*b*x+a^2)*b^4*d^4*e^2-512/231*(1/13*b*x+a)*b^5*d^5*e+1024/3003*b^6*d^6)/e^7
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 356 vs. $2(159) = 318$.

Time = 0.09 (sec) , antiderivative size = 356, normalized size of antiderivative = 1.97

$$\int \frac{(a^2 + 2abx + b^2x^2)^3}{\sqrt{d + ex}} dx$$

$$= \frac{2(231b^6e^6x^6 + 1024b^6d^6 - 6656ab^5d^5e + 18304a^2b^4d^4e^2 - 27456a^3b^3d^3e^3 + 24024a^4b^2d^2e^4 - 12012a^5bd^2e^5 + 3003a^6e^6 - 126(2b^6d^5e - 13a^5b^5e^6)x^5 + 35(8b^6d^4e^4 - 52a^5b^5d^4e^5 + 143a^4b^4d^4e^6)x^4 - 20(16b^6d^3e^3 - 104a^5b^5d^3e^4 + 286a^4b^4d^3e^5 - 429a^3b^3d^3e^6)x^3 + 3(128b^6d^2e^2 - 832a^5b^5d^2e^3 + 2288a^4b^4d^2e^4 - 3432a^3b^3d^2e^5 + 3003a^4b^2e^6)x^2 - 2(256b^6d^5e - 1664a^5b^5d^4e^2 + 4576a^4b^4d^3e^3 - 6864a^3b^3d^2e^4 + 6006a^4b^2d^2e^5 - 3003a^5b^5e^6)x}{\sqrt{d + ex}}$$

input

```
integrate((b^2*x^2+2*a*b*x+a^2)^3/(e*x+d)^(1/2),x, algorithm="fricas")
```

output

```
2/3003*(231*b^6*e^6*x^6 + 1024*b^6*d^6 - 6656*a*b^5*d^5*e + 18304*a^2*b^4*d^4*e^2 - 27456*a^3*b^3*d^3*e^3 + 24024*a^4*b^2*d^2*e^4 - 12012*a^5*b*d^2*e^5 + 3003*a^6*e^6 - 126*(2*b^6*d^5*e - 13*a^5*b^5*e^6)*x^5 + 35*(8*b^6*d^4*e^4 - 52*a^5*b^5*d^4*e^5 + 143*a^4*b^4*d^4*e^6)*x^4 - 20*(16*b^6*d^3*e^3 - 104*a^5*b^5*d^3*e^4 + 286*a^4*b^4*d^3*e^5 - 429*a^3*b^3*d^3*e^6)*x^3 + 3*(128*b^6*d^2*e^2 - 832*a^5*b^5*d^2*e^3 + 2288*a^4*b^4*d^2*e^4 - 3432*a^3*b^3*d^2*e^5 + 3003*a^4*b^2*e^6)*x^2 - 2*(256*b^6*d^5*e - 1664*a^5*b^5*d^4*e^2 + 4576*a^4*b^4*d^3*e^3 - 6864*a^3*b^3*d^2*e^4 + 6006*a^4*b^2*d^2*e^5 - 3003*a^5*b^5*e^6)*x)*sqrt(e*x + d)/e^7
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 493 vs. $2(168) = 336$.

Time = 1.11 (sec) , antiderivative size = 493, normalized size of antiderivative = 2.72

$$\int \frac{(a^2 + 2abx + b^2x^2)^3}{\sqrt{d + ex}} dx$$

$$= \frac{2 \left(\frac{b^6(d+ex)^{\frac{13}{2}}}{13e^6} + \frac{(d+ex)^{\frac{11}{2}} \cdot (6ab^5e - 6b^6d)}{11e^6} + \frac{(d+ex)^{\frac{9}{2}} \cdot (15a^2b^4e^2 - 30ab^5de + 15b^6d^2)}{9e^6} + \frac{(d+ex)^{\frac{7}{2}} \cdot (20a^3b^3e^3 - 60a^2b^4de^2 + 60ab^5d^2e - 20b^6d^3)}{7e^6} + \frac{(d+ex)^{\frac{5}{2}} \cdot (5a^4b^2e^4 - 20a^3b^3de^3 + 15a^2b^4d^2e^2 - 5ab^5d^3e - 5b^6d^4)}{5e^6} \right)}{\sqrt{d}}$$

$$= \frac{a^6x + 3a^5bx^2 + 5a^4b^2x^3 + 5a^3b^3x^4 + 3a^2b^4x^5 + ab^5x^6 + \frac{b^6x^7}{7}}{\sqrt{d}}$$

input

```
integrate((b**2*x**2+2*a*b*x+a**2)**3/(e*x+d)**(1/2),x)
```

output

```
Piecewise((2*(b**6*(d + e*x)**(13/2)/(13*e**6) + (d + e*x)**(11/2)*(6*a*b*
*5*e - 6*b**6*d)/(11*e**6) + (d + e*x)**(9/2)*(15*a**2*b**4*e**2 - 30*a*b*
*5*d*e + 15*b**6*d**2)/(9*e**6) + (d + e*x)**(7/2)*(20*a**3*b**3*e**3 - 60
*a**2*b**4*d*e**2 + 60*a*b**5*d**2*e - 20*b**6*d**3)/(7*e**6) + (d + e*x)*
*(5/2)*(15*a**4*b**2*e**4 - 60*a**3*b**3*d*e**3 + 90*a**2*b**4*d**2*e**2 -
60*a*b**5*d**3*e + 15*b**6*d**4)/(5*e**6) + (d + e*x)**(3/2)*(6*a**5*b*e*
*5 - 30*a**4*b**2*d*e**4 + 60*a**3*b**3*d**2*e**3 - 60*a**2*b**4*d**3*e**2
+ 30*a*b**5*d**4*e - 6*b**6*d**5)/(3*e**6) + sqrt(d + e*x)*(a**6*e**6 - 6
*a**5*b*d*e**5 + 15*a**4*b**2*d**2*e**4 - 20*a**3*b**3*d**3*e**3 + 15*a**2
*b**4*d**4*e**2 - 6*a*b**5*d**5*e + b**6*d**6)/e**6)/e, Ne(e, 0)), ((a**6*
x + 3*a**5*b*x**2 + 5*a**4*b**2*x**3 + 5*a**3*b**3*x**4 + 3*a**2*b**4*x**5
+ a*b**5*x**6 + b**6*x**7/7)/sqrt(d), True))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 540 vs. $2(159) = 318$.

Time = 0.05 (sec) , antiderivative size = 540, normalized size of antiderivative = 2.98

$$\int \frac{(a^2 + 2abx + b^2x^2)^3}{\sqrt{d + ex}} dx = \text{Too large to display}$$

input

```
integrate((b^2*x^2+2*a*b*x+a^2)^3/(e*x+d)^(1/2),x, algorithm="maxima")
```

output

```

2/15015*(15015*sqrt(e*x + d)*a^6 + 3003*(10*((e*x + d)^(3/2) - 3*sqrt(e*x
+ d)*d)*a*b/e + (3*(e*x + d)^(5/2) - 10*(e*x + d)^(3/2)*d + 15*sqrt(e*x +
d)*d^2)*b^2/e^2)*a^4 + 3432*(5*(e*x + d)^(7/2) - 21*(e*x + d)^(5/2)*d + 35
*(e*x + d)^(3/2)*d^2 - 35*sqrt(e*x + d)*d^3)*a^3*b^3/e^3 + 143*(84*(3*(e*x
+ d)^(5/2) - 10*(e*x + d)^(3/2)*d + 15*sqrt(e*x + d)*d^2)*a^2*b^2/e^2 + 3
6*(5*(e*x + d)^(7/2) - 21*(e*x + d)^(5/2)*d + 35*(e*x + d)^(3/2)*d^2 - 35*
sqrt(e*x + d)*d^3)*a*b^3/e^3 + (35*(e*x + d)^(9/2) - 180*(e*x + d)^(7/2)*d
+ 378*(e*x + d)^(5/2)*d^2 - 420*(e*x + d)^(3/2)*d^3 + 315*sqrt(e*x + d)*d
^4)*b^4/e^4)*a^2 + 572*(35*(e*x + d)^(9/2) - 180*(e*x + d)^(7/2)*d + 378*(
e*x + d)^(5/2)*d^2 - 420*(e*x + d)^(3/2)*d^3 + 315*sqrt(e*x + d)*d^4)*a^2*
b^4/e^4 + 130*(63*(e*x + d)^(11/2) - 385*(e*x + d)^(9/2)*d + 990*(e*x + d)
^(7/2)*d^2 - 1386*(e*x + d)^(5/2)*d^3 + 1155*(e*x + d)^(3/2)*d^4 - 693*sq
rt(e*x + d)*d^5)*a*b^5/e^5 + 5*(231*(e*x + d)^(13/2) - 1638*(e*x + d)^(11/2
)*d + 5005*(e*x + d)^(9/2)*d^2 - 8580*(e*x + d)^(7/2)*d^3 + 9009*(e*x + d)
^(5/2)*d^4 - 6006*(e*x + d)^(3/2)*d^5 + 3003*sqrt(e*x + d)*d^6)*b^6/e^6)/e

```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 374 vs. $2(159) = 318$.

Time = 0.12 (sec) , antiderivative size = 374, normalized size of antiderivative = 2.07

$$\int \frac{(a^2 + 2abx + b^2x^2)^3}{\sqrt{d + ex}} dx$$

$$= \frac{2 \left(3003 \sqrt{ex + d} a^6 + \frac{6006 \left((ex+d)^{\frac{3}{2}} - 3\sqrt{ex+dd} \right) a^5 b}{e} + \frac{3003 \left(3(ex+d)^{\frac{5}{2}} - 10(ex+d)^{\frac{3}{2}} d + 15\sqrt{ex+dd^2} \right) a^4 b^2}{e^2} + \frac{1716 \left(5(ex+d)^{\frac{7}{2}} \right)}{e^3} \right)}{e^3}$$

input

```
integrate((b^2*x^2+2*a*b*x+a^2)^3/(e*x+d)^(1/2),x, algorithm="giac")
```


output

```

2/3003*(3003*sqrt(e*x + d)*a^6 + 6006*((e*x + d)^(3/2) - 3*sqrt(e*x + d)*d
)*a^5*b/e + 3003*(3*(e*x + d)^(5/2) - 10*(e*x + d)^(3/2)*d + 15*sqrt(e*x +
d)*d^2)*a^4*b^2/e^2 + 1716*(5*(e*x + d)^(7/2) - 21*(e*x + d)^(5/2)*d + 35
*(e*x + d)^(3/2)*d^2 - 35*sqrt(e*x + d)*d^3)*a^3*b^3/e^3 + 143*(35*(e*x +
d)^(9/2) - 180*(e*x + d)^(7/2)*d + 378*(e*x + d)^(5/2)*d^2 - 420*(e*x + d)
^(3/2)*d^3 + 315*sqrt(e*x + d)*d^4)*a^2*b^4/e^4 + 26*(63*(e*x + d)^(11/2)
- 385*(e*x + d)^(9/2)*d + 990*(e*x + d)^(7/2)*d^2 - 1386*(e*x + d)^(5/2)*d
^3 + 1155*(e*x + d)^(3/2)*d^4 - 693*sqrt(e*x + d)*d^5)*a*b^5/e^5 + (231*(e
*x + d)^(13/2) - 1638*(e*x + d)^(11/2)*d + 5005*(e*x + d)^(9/2)*d^2 - 8580
*(e*x + d)^(7/2)*d^3 + 9009*(e*x + d)^(5/2)*d^4 - 6006*(e*x + d)^(3/2)*d^5
+ 3003*sqrt(e*x + d)*d^6)*b^6/e^6)/e

```

Mupad [B] (verification not implemented)

Time = 9.17 (sec) , antiderivative size = 162, normalized size of antiderivative = 0.90

$$\int \frac{(a^2 + 2abx + b^2x^2)^3}{\sqrt{d + ex}} dx = \frac{2b^6(d + ex)^{13/2}}{13e^7} - \frac{(12b^6d - 12ab^5e)(d + ex)^{11/2}}{11e^7}
+ \frac{2(ae - bd)^6\sqrt{d + ex}}{e^7} + \frac{6b^2(ae - bd)^4(d + ex)^{5/2}}{e^7}
+ \frac{40b^3(ae - bd)^3(d + ex)^{7/2}}{7e^7}
+ \frac{10b^4(ae - bd)^2(d + ex)^{9/2}}{3e^7} + \frac{4b(ae - bd)^5(d + ex)^{3/2}}{e^7}$$

input

```
int((a^2 + b^2*x^2 + 2*a*b*x)^3/(d + e*x)^(1/2),x)
```

output

```

(2*b^6*(d + e*x)^(13/2))/(13*e^7) - ((12*b^6*d - 12*a*b^5*e)*(d + e*x)^(11
/2))/(11*e^7) + (2*(a*e - b*d)^6*(d + e*x)^(1/2))/e^7 + (6*b^2*(a*e - b*d)
^4*(d + e*x)^(5/2))/e^7 + (40*b^3*(a*e - b*d)^3*(d + e*x)^(7/2))/(7*e^7) +
(10*b^4*(a*e - b*d)^2*(d + e*x)^(9/2))/(3*e^7) + (4*b*(a*e - b*d)^5*(d +
e*x)^(3/2))/e^7

```

Reduce [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 375, normalized size of antiderivative = 2.07

$$\int \frac{(a^2 + 2abx + b^2x^2)^3}{\sqrt{d + ex}} dx$$

$$= \frac{2\sqrt{ex + d}(231b^6e^6x^6 + 1638ab^5e^6x^5 - 252b^6de^5x^5 + 5005a^2b^4e^6x^4 - 1820ab^5de^5x^4 + 280b^6d^2e^4x^4 + 80b^6de^5x^3 - 252b^6d^2e^4x^3 + 231b^6e^6x^2 + 1638ab^5e^6x^2 - 252b^6de^5x^2 + 5005a^2b^4e^6x - 1820ab^5de^5x + 280b^6d^2e^4x - 231b^6de^5) + 231b^6e^6x^7 + 1638ab^5e^6x^6 - 252b^6de^5x^6 + 5005a^2b^4e^6x^5 - 1820ab^5de^5x^5 + 280b^6d^2e^4x^5 + 80b^6de^5x^4 - 252b^6d^2e^4x^4 + 231b^6e^6x^3 + 1638ab^5e^6x^3 - 252b^6de^5x^3 + 5005a^2b^4e^6x^2 - 1820ab^5de^5x^2 + 280b^6d^2e^4x^2 + 80b^6de^5x + 231b^6d^2e^4x + 231b^6de^5)}{(3003e^7)}$$

input `int((b^2*x^2+2*a*b*x+a^2)^3/(e*x+d)^(1/2),x)`output `(2*sqrt(d + e*x)*(3003*a**6*e**6 - 12012*a**5*b*d*e**5 + 6006*a**5*b*e**6*x + 24024*a**4*b**2*d**2*e**4 - 12012*a**4*b**2*d*e**5*x + 9009*a**4*b**2*e**6*x**2 - 27456*a**3*b**3*d**3*e**3 + 13728*a**3*b**3*d**2*e**4*x - 10296*a**3*b**3*d*e**5*x**2 + 8580*a**3*b**3*e**6*x**3 + 18304*a**2*b**4*d**4*e**2 - 9152*a**2*b**4*d**3*e**3*x + 6864*a**2*b**4*d**2*e**4*x**2 - 5720*a**2*b**4*d*e**5*x**3 + 5005*a**2*b**4*e**6*x**4 - 6656*a*b**5*d**5*e + 3328*a*b**5*d**4*e**2*x - 2496*a*b**5*d**3*e**3*x**2 + 2080*a*b**5*d**2*e**4*x**3 - 1820*a*b**5*d*e**5*x**4 + 1638*a*b**5*e**6*x**5 + 1024*b**6*d**6 - 512*b**6*d**5*e*x + 384*b**6*d**4*e**2*x**2 - 320*b**6*d**3*e**3*x**3 + 280*b**6*d**2*e**4*x**4 - 252*b**6*d*e**5*x**5 + 231*b**6*e**6*x**6))/(3003*e**7)`

3.167 $\int \frac{(a^2+2abx+b^2x^2)^3}{(d+ex)^{3/2}} dx$

| | |
|---|------|
| Optimal result | 1310 |
| Mathematica [A] (verified) | 1311 |
| Rubi [A] (verified) | 1311 |
| Maple [A] (verified) | 1313 |
| Fricas [B] (verification not implemented) | 1313 |
| Sympy [B] (verification not implemented) | 1314 |
| Maxima [B] (verification not implemented) | 1315 |
| Giac [B] (verification not implemented) | 1315 |
| Mupad [B] (verification not implemented) | 1316 |
| Reduce [B] (verification not implemented) | 1317 |

Optimal result

Integrand size = 28, antiderivative size = 179

$$\int \frac{(a^2 + 2abx + b^2x^2)^3}{(d + ex)^{3/2}} dx = -\frac{2(bd - ae)^6}{e^7\sqrt{d + ex}} - \frac{12b(bd - ae)^5\sqrt{d + ex}}{e^7} + \frac{10b^2(bd - ae)^4(d + ex)^{3/2}}{e^7} - \frac{8b^3(bd - ae)^3(d + ex)^{5/2}}{e^7} + \frac{30b^4(bd - ae)^2(d + ex)^{7/2}}{7e^7} - \frac{4b^5(bd - ae)(d + ex)^{9/2}}{3e^7} + \frac{2b^6(d + ex)^{11/2}}{11e^7}$$

output

```
-2*(-a*e+b*d)^6/e^7/(e*x+d)^(1/2)-12*b*(-a*e+b*d)^5*(e*x+d)^(1/2)/e^7+10*b^2*(-a*e+b*d)^4*(e*x+d)^(3/2)/e^7-8*b^3*(-a*e+b*d)^3*(e*x+d)^(5/2)/e^7+30/7*b^4*(-a*e+b*d)^2*(e*x+d)^(7/2)/e^7-4/3*b^5*(-a*e+b*d)*(e*x+d)^(9/2)/e^7+2/11*b^6*(e*x+d)^(11/2)/e^7
```

Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 288, normalized size of antiderivative = 1.61

$$\int \frac{(a^2 + 2abx + b^2x^2)^3}{(d + ex)^{3/2}} dx = \frac{2(-231a^6e^6 + 1386a^5be^5(2d + ex) + 1155a^4b^2e^4(-8d^2 - 4dex + e^2x^2) + 924a^3b^3e^3(16d^3 + 8d^2ex - 2de^2x^2 + e^3x^3) + 99a^2b^4e^2(-128d^4 - 64d^3ex + 16d^2e^2x^2 - 8de^3x^3 + 5e^4x^4) + 22ab^5e(256d^5 + 128d^4ex - 32d^3e^2x^2 + 16d^2e^3x^3 - 10de^4x^4 + 7e^5x^5) + b^6(-1024d^6 - 512d^5ex + 128d^4e^2x^2 - 64d^3e^3x^3 + 40d^2e^4x^4 - 28de^5x^5 + 21e^6x^6))}{231e^7\sqrt{d + ex}}$$

input `Integrate[(a^2 + 2*a*b*x + b^2*x^2)^3/(d + e*x)^(3/2),x]`

output

```
(2*(-231*a^6*e^6 + 1386*a^5*b*e^5*(2*d + e*x) + 1155*a^4*b^2*e^4*(-8*d^2 -
4*d*e*x + e^2*x^2) + 924*a^3*b^3*e^3*(16*d^3 + 8*d^2*e*x - 2*d*e^2*x^2 +
e^3*x^3) + 99*a^2*b^4*e^2*(-128*d^4 - 64*d^3*e*x + 16*d^2*e^2*x^2 - 8*d*e^
3*x^3 + 5*e^4*x^4) + 22*a*b^5*e*(256*d^5 + 128*d^4*e*x - 32*d^3*e^2*x^2 +
16*d^2*e^3*x^3 - 10*d*e^4*x^4 + 7*e^5*x^5) + b^6*(-1024*d^6 - 512*d^5*e*x
+ 128*d^4*e^2*x^2 - 64*d^3*e^3*x^3 + 40*d^2*e^4*x^4 - 28*d*e^5*x^5 + 21*e^
6*x^6)))/(231*e^7*sqrt[d + e*x])
```

Rubi [A] (verified)

Time = 0.53 (sec) , antiderivative size = 179, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {1098, 27, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\begin{aligned} & \int \frac{(a^2 + 2abx + b^2x^2)^3}{(d + ex)^{3/2}} dx \\ & \quad \downarrow \text{1098} \\ & \int \frac{b^6(a+bx)^6}{(d+ex)^{3/2}} dx \\ & \quad \downarrow \text{27} \\ & \int \frac{(a + bx)^6}{(d + ex)^{3/2}} dx \\ & \quad \downarrow \text{53} \end{aligned}$$

$$\int \left(-\frac{6b^5(d+ex)^{7/2}(bd-ae)}{e^6} + \frac{15b^4(d+ex)^{5/2}(bd-ae)^2}{e^6} - \frac{20b^3(d+ex)^{3/2}(bd-ae)^3}{e^6} + \frac{15b^2\sqrt{d+ex}(bd-ae)^4}{e^6} \right)$$

↓ 2009

$$-\frac{4b^5(d+ex)^{9/2}(bd-ae)}{3e^7} + \frac{30b^4(d+ex)^{7/2}(bd-ae)^2}{7e^7} - \frac{8b^3(d+ex)^{5/2}(bd-ae)^3}{e^7} + \frac{10b^2(d+ex)^{3/2}(bd-ae)^4}{e^7} - \frac{12b\sqrt{d+ex}(bd-ae)^5}{e^7} - \frac{2(bd-ae)^6}{e^7\sqrt{d+ex}} + \frac{2b^6(d+ex)^{11/2}}{11e^7}$$

input `Int[(a^2 + 2*a*b*x + b^2*x^2)^3/(d + e*x)^(3/2),x]`

output `(-2*(b*d - a*e)^6)/(e^7*sqrt[d + e*x]) - (12*b*(b*d - a*e)^5*sqrt[d + e*x])/e^7 + (10*b^2*(b*d - a*e)^4*(d + e*x)^(3/2))/e^7 - (8*b^3*(b*d - a*e)^3*(d + e*x)^(5/2))/e^7 + (30*b^4*(b*d - a*e)^2*(d + e*x)^(7/2))/(7*e^7) - (4*b^5*(b*d - a*e)*(d + e*x)^(9/2))/(3*e^7) + (2*b^6*(d + e*x)^(11/2))/(11*e^7)`

Defintions of rubi rules used

rule 27 `Int[(a_)*(F_x_), x_Symbol] := Simp[a Int[F_x, x], x] /; FreeQ[a, x] && !MatchQ[F_x, (b_)*(G_x_)] /; FreeQ[b, x]`

rule 53 `Int[((a_) + (b_)*(x_))^(m_)*((c_) + (d_)*(x_))^(n_), x_Symbol] := Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && IGtQ[m, 0] && (!IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])`

rule 1098 `Int[((d_) + (e_)*(x_))^(m_)*((a_) + (b_)*(x_) + (c_)*(x_)^2)^(p_), x_Symbol] := Simp[1/c^p Int[(d + e*x)^m*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]`

rule 2009 `Int[u_, x_Symbol] := Simp[IntSum[u, x], x] /; SumQ[u]`

Maple [A] (verified)

Time = 1.32 (sec) , antiderivative size = 278, normalized size of antiderivative = 1.55

| method | result |
|-------------------|---|
| pseudoelliptic | $\frac{2(21e^6x^6 - 28de^5x^5 + 40d^2e^4x^4 - 64d^3e^3x^3 + 128d^4e^2x^2 - 512d^5ex - 1024d^6)b^6}{231} + \frac{1024e\left(\frac{7}{256}e^5x^5 - \frac{5}{128}de^4x^4 + \frac{1}{16}d^2e^3x^3 - \frac{1}{8}d^3e^2x^2 + \frac{1}{2}d^4e^2x + d^5\right)a^5b^5 - 12672e^2(-\frac{5}{128}e^4x^4 + \frac{1}{16}d^2e^3x^3 - \frac{1}{8}d^3e^2x^2 + \frac{1}{2}d^4e^2x + d^5)a^4b^4 + 14784e^3(1/16e^3x^3 - 1/8de^2x^2 + 1/2d^2e^2x + d^3)a^3b^3 - 9240e^4(-1/8e^2x^2 + 1/2de^2x + d^2)a^2b^2 + 2772e^5(1/2ex + d)a^5b - 231a^6e^6}{21}e^{(1/2)}$ |
| risch | $2b(21x^5e^5b^5 + 154x^4ab^4e^5 - 49x^4b^5de^4 + 495x^3a^2b^3e^5 - 374x^3ab^4de^4 + 89x^3b^5d^2e^3 + 924x^2a^3b^2e^5 - 1287x^2a^2b^3de^4 + \dots)$ |
| gosper | $- \frac{2(-21x^6b^6e^6 - 154x^5ab^5e^6 + 28x^5b^6de^5 - 495x^4a^2b^4e^6 + 220x^4ab^5de^5 - 40x^4b^6d^2e^4 - 924x^3a^3b^3e^6 + 792x^3a^2b^4de^5 - \dots)}{231}$ |
| trager | $- \frac{2(-21x^6b^6e^6 - 154x^5ab^5e^6 + 28x^5b^6de^5 - 495x^4a^2b^4e^6 + 220x^4ab^5de^5 - 40x^4b^6d^2e^4 - 924x^3a^3b^3e^6 + 792x^3a^2b^4de^5 - \dots)}{231}$ |
| oring | $- \frac{2(-21x^6b^6e^6 - 154x^5ab^5e^6 + 28x^5b^6de^5 - 495x^4a^2b^4e^6 + 220x^4ab^5de^5 - 40x^4b^6d^2e^4 - 924x^3a^3b^3e^6 + 792x^3a^2b^4de^5 - \dots)}{231}$ |
| derivativedivides | $\frac{2b^6(ex+d)^{\frac{11}{2}}}{11} + \frac{4ab^5e(ex+d)^{\frac{9}{2}}}{3} - \frac{4b^6d(ex+d)^{\frac{9}{2}}}{3} + \frac{30a^2b^4e^2(ex+d)^{\frac{7}{2}}}{7} - \frac{60ab^5de(ex+d)^{\frac{7}{2}}}{7} + \frac{30b^6d^2(ex+d)^{\frac{7}{2}}}{7} + 8a^3b^3e^3(ex+d)^{\frac{5}{2}}$ |
| default | $\frac{2b^6(ex+d)^{\frac{11}{2}}}{11} + \frac{4ab^5e(ex+d)^{\frac{9}{2}}}{3} - \frac{4b^6d(ex+d)^{\frac{9}{2}}}{3} + \frac{30a^2b^4e^2(ex+d)^{\frac{7}{2}}}{7} - \frac{60ab^5de(ex+d)^{\frac{7}{2}}}{7} + \frac{30b^6d^2(ex+d)^{\frac{7}{2}}}{7} + 8a^3b^3e^3(ex+d)^{\frac{5}{2}}$ |

```
input int((b^2*x^2+2*a*b*x+a^2)^3/(e*x+d)^(3/2), x, method=_RETURNVERBOSE)
```

```
output 2/231*((21*e^6*x^6-28*d*e^5*x^5+40*d^2*e^4*x^4-64*d^3*e^3*x^3+128*d^4*e^2*x^2-512*d^5*e*x-1024*d^6)*b^6+5632*e*(7/256*e^5*x^5-5/128*d*e^4*x^4+1/16*d^2*e^3*x^3-1/8*d^3*e^2*x^2+1/2*d^4*e*x+d^5)*a*b^5-12672*e^2*(-5/128*e^4*x^4+1/16*d^2*e^3*x^3-1/8*d^2*e^2*x^2+1/2*d^3*e*x+d^4)*a^2*b^4+14784*e^3*(1/16*e^3*x^3-1/8*d^2*e^2*x^2+1/2*d^2*e*x+d^3)*a^3*b^3-9240*e^4*(-1/8*e^2*x^2+1/2*d^2*e*x+d^2)*a^4*b^2+2772*e^5*(1/2*e*x+d)*a^5*b-231*a^6*e^6)/(e*x+d)^(1/2)/e^7
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 365 vs. 2(159) = 318.

Time = 0.09 (sec) , antiderivative size = 365, normalized size of antiderivative = 2.04

$$\int \frac{(a^2 + 2abx + b^2x^2)^3}{(d + ex)^{3/2}} dx = \frac{2(21b^6e^6x^6 - 1024b^6d^6 + 5632ab^5d^5e - 12672a^2b^4d^4e^2 + 14784a^3b^3d^3e^3 - \dots)}{(d + ex)^{3/2}}$$

input `integrate((b^2*x^2+2*a*b*x+a^2)^3/(e*x+d)^(3/2),x, algorithm="fricas")`

output
$$\begin{aligned} & 2/231*(21*b^6*e^6*x^6 - 1024*b^6*d^6 + 5632*a*b^5*d^5*e - 12672*a^2*b^4*d^4*e^2 + 14784*a^3*b^3*d^3*e^3 - 9240*a^4*b^2*d^2*e^4 + 2772*a^5*b*d*e^5 - \\ & 231*a^6*e^6 - 14*(2*b^6*d*e^5 - 11*a*b^5*e^6)*x^5 + 5*(8*b^6*d^2*e^4 - 44*a*b^5*d*e^5 + 99*a^2*b^4*e^6)*x^4 - 4*(16*b^6*d^3*e^3 - 88*a*b^5*d^2*e^4 + \\ & 198*a^2*b^4*d*e^5 - 231*a^3*b^3*e^6)*x^3 + (128*b^6*d^4*e^2 - 704*a*b^5*d^3*e^3 + 1584*a^2*b^4*d^2*e^4 - 1848*a^3*b^3*d*e^5 + 1155*a^4*b^2*e^6)*x^2 \\ & - 2*(256*b^6*d^5*e - 1408*a*b^5*d^4*e^2 + 3168*a^2*b^4*d^3*e^3 - 3696*a^3*b^3*d^2*e^4 + 2310*a^4*b^2*d*e^5 - 693*a^5*b*e^6)*x)*sqrt(e*x + d)/(e^8*x + d*e^7) \end{aligned}$$

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 416 vs. $2(167) = 334$.

Time = 9.93 (sec) , antiderivative size = 416, normalized size of antiderivative = 2.32

$$\int \frac{(a^2 + 2abx + b^2x^2)^3}{(d + ex)^{3/2}} dx = \left\{ \frac{2 \left(\frac{b^6(d+ex)^{\frac{11}{2}}}{11e^6} + \frac{(d+ex)^{\frac{9}{2}} \cdot (6ab^5e - 6b^6d)}{9e^6} + \frac{(d+ex)^{\frac{7}{2}} \cdot (15a^2b^4e^2 - 30ab^5de + 15b^6d^2)}{7e^6} + \frac{(d+ex)^{\frac{5}{2}} \cdot (20a^3b^3e^3 - 60a^4b^2de + 15a^5b^2d^2 - 15a^6d^3)}{5e^6} \right)}{d^{\frac{3}{2}}} \right.$$

input `integrate((b**2*x**2+2*a*b*x+a**2)**3/(e*x+d)**(3/2),x)`

output `Piecewise((2*(b**6*(d + e*x)**(11/2))/(11*e**6) + (d + e*x)**(9/2)*(6*a*b**5*e - 6*b**6*d)/(9*e**6) + (d + e*x)**(7/2)*(15*a**2*b**4*e**2 - 30*a*b**5*d*e + 15*b**6*d**2)/(7*e**6) + (d + e*x)**(5/2)*(20*a**3*b**3*e**3 - 60*a**2*b**4*d*e**2 + 60*a*b**5*d**2*e - 20*b**6*d**3)/(5*e**6) + (d + e*x)**(3/2)*(15*a**4*b**2*e**4 - 60*a**3*b**3*d*e**3 + 90*a**2*b**4*d**2*e**2 - 60*a*b**5*d**3*e + 15*b**6*d**4)/(3*e**6) + sqrt(d + e*x)*(6*a**5*b*e**5 - 30*a**4*b**2*d*e**4 + 60*a**3*b**3*d**2*e**3 - 60*a**2*b**4*d**3*e**2 + 30*a*b**5*d**4*e - 6*b**6*d**5)/e**6 - (a*e - b*d)**6/(e**6*sqrt(d + e*x)))/e, Ne(e, 0)), ((a**6*x + 3*a**5*b*x**2 + 5*a**4*b**2*x**3 + 5*a**3*b**3*x**4 + 3*a**2*b**4*x**5 + a*b**5*x**6 + b**6*x**7/7)/d**(3/2), True))`

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 358 vs. $2(159) = 318$.

Time = 0.05 (sec) , antiderivative size = 358, normalized size of antiderivative = 2.00

$$\int \frac{(a^2 + 2abx + b^2x^2)^3}{(d + ex)^{3/2}} dx = \frac{2 \left(\frac{21(ex+d)^{\frac{11}{2}}b^6 - 154(b^6d - ab^5e)(ex+d)^{\frac{9}{2}} + 495(b^6d^2 - 2ab^5de + a^2b^4e^2)(ex+d)^{\frac{7}{2}} - 924(b^6d^3 - 3ab^5d^2e + 3a^2b^4d^2e^2 - a^3b^3de^3)(ex+d)^{\frac{5}{2}} + 1155(b^6d^4 - 4a^2b^5d^3e + 6a^2b^4d^2e^2 - 4a^3b^3d^2e^3 + a^4b^2e^4)(ex+d)^{\frac{3}{2}} - 1386(b^6d^5 - 5a^2b^5d^4e + 10a^2b^4d^3e^2 - 10a^3b^3d^2e^3 + 5a^4b^2d^2e^4 - a^5bde^5) \sqrt{ex+d}}{e^6} - 231 \frac{(b^6d^6 - 6a^2b^5d^5e + 15a^2b^4d^4e^2 - 20a^3b^3d^3e^3 + 15a^4b^2d^2e^4 - 6a^5bde^5 + a^6e^6)}{(\sqrt{ex+d}e^6)} \right)}{e}$$

input `integrate((b^2*x^2+2*a*b*x+a^2)^3/(e*x+d)^(3/2),x, algorithm="maxima")`

output
$$\frac{2}{231} \left(\frac{21(e^2x + d)^{\frac{11}{2}}b^6 - 154(b^6d - a^2b^5e)(e^2x + d)^{\frac{9}{2}} + 495(b^6d^2 - 2a^2b^5de + a^2b^4e^2)(e^2x + d)^{\frac{7}{2}} - 924(b^6d^3 - 3a^2b^5d^2e + 3a^2b^4d^2e^2 - a^3b^3de^3)(e^2x + d)^{\frac{5}{2}} + 1155(b^6d^4 - 4a^2b^5d^3e + 6a^2b^4d^2e^2 - 4a^3b^3d^2e^3 + a^4b^2e^4)(e^2x + d)^{\frac{3}{2}} - 1386(b^6d^5 - 5a^2b^5d^4e + 10a^2b^4d^3e^2 - 10a^3b^3d^2e^3 + 5a^4b^2d^2e^4 - a^5bde^5) \sqrt{e^2x + d}}{e^6} - 231 \frac{(b^6d^6 - 6a^2b^5d^5e + 15a^2b^4d^4e^2 - 20a^3b^3d^3e^3 + 15a^4b^2d^2e^4 - 6a^5bde^5 + a^6e^6)}{(\sqrt{e^2x + d}e^6)} \right) / e$$

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 479 vs. $2(159) = 318$.

Time = 0.17 (sec) , antiderivative size = 479, normalized size of antiderivative = 2.68

$$\int \frac{(a^2 + 2abx + b^2x^2)^3}{(d + ex)^{3/2}} dx = \frac{2(b^6d^6 - 6ab^5d^5e + 15a^2b^4d^4e^2 - 20a^3b^3d^3e^3 + 15a^4b^2d^2e^4 - 6a^5bde^5 + a^6e^6)}{\sqrt{ex + d}e^7} + \frac{2 \left(21(ex + d)^{\frac{11}{2}}b^6e^{70} - 154(ex + d)^{\frac{9}{2}}b^6de^{70} + 495(ex + d)^{\frac{7}{2}}b^6d^2e^{70} - 924(ex + d)^{\frac{5}{2}}b^6d^3e^{70} + 1155(ex + d)^{\frac{3}{2}}b^6d^4e^{70} - 1386(b^6d^5 - 5a^2b^5d^4e + 10a^2b^4d^3e^2 - 10a^3b^3d^2e^3 + 5a^4b^2d^2e^4 - a^5bde^5) \sqrt{ex + d}}{e^6} - 231 \frac{(b^6d^6 - 6a^2b^5d^5e + 15a^2b^4d^4e^2 - 20a^3b^3d^3e^3 + 15a^4b^2d^2e^4 - 6a^5bde^5 + a^6e^6)}{(\sqrt{ex + d}e^6)} \right)}{e}$$

input `integrate((b^2*x^2+2*a*b*x+a^2)^3/(e*x+d)^(3/2),x, algorithm="giac")`

output

```
-2*(b^6*d^6 - 6*a*b^5*d^5*e + 15*a^2*b^4*d^4*e^2 - 20*a^3*b^3*d^3*e^3 + 15
*a^4*b^2*d^2*e^4 - 6*a^5*b*d*e^5 + a^6*e^6)/(sqrt(e*x + d)*e^7) + 2/231*(2
1*(e*x + d)^(11/2)*b^6*e^70 - 154*(e*x + d)^(9/2)*b^6*d*e^70 + 495*(e*x +
d)^(7/2)*b^6*d^2*e^70 - 924*(e*x + d)^(5/2)*b^6*d^3*e^70 + 1155*(e*x + d)^(
3/2)*b^6*d^4*e^70 - 1386*sqrt(e*x + d)*b^6*d^5*e^70 + 154*(e*x + d)^(9/2)
*a*b^5*e^71 - 990*(e*x + d)^(7/2)*a*b^5*d*e^71 + 2772*(e*x + d)^(5/2)*a*b^
5*d^2*e^71 - 4620*(e*x + d)^(3/2)*a*b^5*d^3*e^71 + 6930*sqrt(e*x + d)*a*b^
5*d^4*e^71 + 495*(e*x + d)^(7/2)*a^2*b^4*d^2*e^72 - 2772*(e*x + d)^(5/2)*a^2*b
^4*d^3*e^72 + 6930*(e*x + d)^(3/2)*a^2*b^4*d^2*e^72 - 13860*sqrt(e*x + d)*a^
2*b^4*d^3*e^72 + 924*(e*x + d)^(5/2)*a^3*b^3*d^2*e^73 - 4620*(e*x + d)^(3/2)*a
^3*b^3*d^2*e^73 + 13860*sqrt(e*x + d)*a^3*b^3*d^2*e^73 + 1155*(e*x + d)^(3/2)
)*a^4*b^2*d^2*e^74 - 6930*sqrt(e*x + d)*a^4*b^2*d^2*e^74 + 1386*sqrt(e*x + d)*a^
5*b*e^75)/e^77
```

Mupad [B] (verification not implemented)

Time = 9.21 (sec) , antiderivative size = 231, normalized size of antiderivative = 1.29

$$\int \frac{(a^2 + 2abx + b^2x^2)^3}{(d + ex)^{3/2}} dx = \frac{2b^6(d + ex)^{11/2}}{11e^7} - \frac{2a^6e^6 - 12a^5bde^5 + 30a^4b^2d^2e^4 - 40a^3b^3d^3e^3 + 30a^2b^4d^4e^2 - 12ab^5d^5e + 2b^6d^6}{e^7\sqrt{d + ex}} - \frac{(12b^6d - 12ab^5e)(d + ex)^{9/2}}{9e^7} + \frac{10b^2(ae - bd)^4(d + ex)^{3/2}}{e^7} + \frac{8b^3(ae - bd)^3(d + ex)^{5/2}}{e^7} + \frac{30b^4(ae - bd)^2(d + ex)^{7/2}}{7e^7} + \frac{12b(ae - bd)^5\sqrt{d + ex}}{e^7}$$

input

```
int((a^2 + b^2*x^2 + 2*a*b*x)^3/(d + e*x)^(3/2),x)
```

output

```
(2*b^6*(d + e*x)^(11/2))/(11*e^7) - (2*a^6*e^6 + 2*b^6*d^6 + 30*a^2*b^4*d^
4*e^2 - 40*a^3*b^3*d^3*e^3 + 30*a^4*b^2*d^2*e^4 - 12*a*b^5*d^5*e - 12*a^5*
b*d*e^5)/(e^7*(d + e*x)^(1/2)) - ((12*b^6*d - 12*a*b^5*e)*(d + e*x)^(9/2))
/(9*e^7) + (10*b^2*(a*e - b*d)^4*(d + e*x)^(3/2))/e^7 + (8*b^3*(a*e - b*d)
^3*(d + e*x)^(5/2))/e^7 + (30*b^4*(a*e - b*d)^2*(d + e*x)^(7/2))/(7*e^7) +
(12*b*(a*e - b*d)^5*(d + e*x)^(1/2))/e^7
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 377, normalized size of antiderivative = 2.11

$$\int \frac{(a^2 + 2abx + b^2x^2)^3}{(d + ex)^{3/2}} dx = \frac{96}{7}a^2b^4d^2e^4x^2 - \frac{48}{7}a^2b^4de^5x^3 + \frac{512}{21}ab^5d^4e^2x - \frac{128}{21}ab^5d^3e^3x^2 + \frac{64}{21}ab^5d^2e^4x^3 -$$

input `int((b^2*x^2+2*a*b*x+a^2)^3/(e*x+d)^(3/2),x)`output `(2*(-231*a**6*e**6 + 2772*a**5*b*d*e**5 + 1386*a**5*b*e**6*x - 9240*a**4*b**2*d**2*e**4 - 4620*a**4*b**2*d*e**5*x + 1155*a**4*b**2*e**6*x**2 + 14784*a**3*b**3*d**3*e**3 + 7392*a**3*b**3*d**2*e**4*x - 1848*a**3*b**3*d*e**5*x**2 + 924*a**3*b**3*e**6*x**3 - 12672*a**2*b**4*d**4*e**2 - 6336*a**2*b**4*d**3*e**3*x + 1584*a**2*b**4*d**2*e**4*x**2 - 792*a**2*b**4*d*e**5*x**3 + 495*a**2*b**4*e**6*x**4 + 5632*a*b**5*d**5*e + 2816*a*b**5*d**4*e**2*x - 704*a*b**5*d**3*e**3*x**2 + 352*a*b**5*d**2*e**4*x**3 - 220*a*b**5*d*e**5*x**4 + 154*a*b**5*e**6*x**5 - 1024*b**6*d**6 - 512*b**6*d**5*e*x + 128*b**6*d**4*e**2*x**2 - 64*b**6*d**3*e**3*x**3 + 40*b**6*d**2*e**4*x**4 - 28*b**6*d*e**5*x**5 + 21*b**6*e**6*x**6))/(231*sqrt(d + e*x)*e**7)`

3.168
$$\int \frac{(a^2+2abx+b^2x^2)^3}{(d+ex)^{5/2}} dx$$

| | |
|---|------|
| Optimal result | 1318 |
| Mathematica [A] (verified) | 1319 |
| Rubi [A] (verified) | 1319 |
| Maple [A] (verified) | 1321 |
| Fricas [B] (verification not implemented) | 1321 |
| Sympy [B] (verification not implemented) | 1322 |
| Maxima [B] (verification not implemented) | 1323 |
| Giac [B] (verification not implemented) | 1323 |
| Mupad [B] (verification not implemented) | 1324 |
| Reduce [B] (verification not implemented) | 1325 |

Optimal result

Integrand size = 28, antiderivative size = 181

$$\int \frac{(a^2 + 2abx + b^2x^2)^3}{(d + ex)^{5/2}} dx = -\frac{2(bd - ae)^6}{3e^7(d + ex)^{3/2}} + \frac{12b(bd - ae)^5}{e^7\sqrt{d + ex}}$$

$$+ \frac{30b^2(bd - ae)^4\sqrt{d + ex}}{e^7} - \frac{40b^3(bd - ae)^3(d + ex)^{3/2}}{3e^7}$$

$$+ \frac{6b^4(bd - ae)^2(d + ex)^{5/2}}{e^7} - \frac{12b^5(bd - ae)(d + ex)^{7/2}}{7e^7} + \frac{2b^6(d + ex)^{9/2}}{9e^7}$$

output

```
-2/3*(-a*e+b*d)^6/e^7/(e*x+d)^(3/2)+12*b*(-a*e+b*d)^5/e^7/(e*x+d)^(1/2)+30
*b^2*(-a*e+b*d)^4*(e*x+d)^(1/2)/e^7-40/3*b^3*(-a*e+b*d)^3*(e*x+d)^(3/2)/e^
7+6*b^4*(-a*e+b*d)^2*(e*x+d)^(5/2)/e^7-12/7*b^5*(-a*e+b*d)*(e*x+d)^(7/2)/e
^7+2/9*b^6*(e*x+d)^(9/2)/e^7
```

Mathematica [A] (verified)

Time = 0.16 (sec) , antiderivative size = 290, normalized size of antiderivative = 1.60

$$\int \frac{(a^2 + 2abx + b^2x^2)^3}{(d + ex)^{5/2}} dx = \frac{2(-21a^6e^6 - 126a^5be^5(2d + 3ex) + 315a^4b^2e^4(8d^2 + 12dex + 3e^2x^2) + 420a^3b^3e^3(-16d^3 - 24d^2ex - 6de^2x^2 + e^3x^3) + 63a^2b^4e^2(128d^4 + 192d^3ex + 48d^2e^2x^2 - 8de^3x^3 + 3e^4x^4) - 18ab^5e(256d^5 + 384d^4ex + 96d^3e^2x^2 - 16d^2e^3x^3 + 6de^4x^4 - 3e^5x^5) + b^6(1024d^6 + 1536d^5ex + 384d^4e^2x^2 - 64d^3e^3x^3 + 24d^2e^4x^4 - 12de^5x^5 + 7e^6x^6))}{63e^7(d + ex)^{(3/2)}}$$

input `Integrate[(a^2 + 2*a*b*x + b^2*x^2)^3/(d + e*x)^(5/2),x]`

output

```
(2*(-21*a^6*e^6 - 126*a^5*b*e^5*(2*d + 3*e*x) + 315*a^4*b^2*e^4*(8*d^2 + 1
2*d*e*x + 3*e^2*x^2) + 420*a^3*b^3*e^3*(-16*d^3 - 24*d^2*e*x - 6*d*e^2*x^2
+ e^3*x^3) + 63*a^2*b^4*e^2*(128*d^4 + 192*d^3*e*x + 48*d^2*e^2*x^2 - 8*d
*e^3*x^3 + 3*e^4*x^4) - 18*a*b^5*e*(256*d^5 + 384*d^4*e*x + 96*d^3*e^2*x^2
- 16*d^2*e^3*x^3 + 6*d*e^4*x^4 - 3*e^5*x^5) + b^6*(1024*d^6 + 1536*d^5*e*
x + 384*d^4*e^2*x^2 - 64*d^3*e^3*x^3 + 24*d^2*e^4*x^4 - 12*d*e^5*x^5 + 7*e
^6*x^6))/(63*e^7*(d + e*x)^(3/2))
```

Rubi [A] (verified)

Time = 0.52 (sec) , antiderivative size = 181, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {1098, 27, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\begin{aligned} & \int \frac{(a^2 + 2abx + b^2x^2)^3}{(d + ex)^{5/2}} dx \\ & \quad \downarrow \text{1098} \\ & \int \frac{b^6(a+bx)^6}{(d+ex)^{5/2}} dx \\ & \quad \downarrow \text{27} \\ & \int \frac{(a + bx)^6}{(d + ex)^{5/2}} dx \\ & \quad \downarrow \text{53} \end{aligned}$$

$$\int \left(-\frac{6b^5(d+ex)^{5/2}(bd-ae)}{e^6} + \frac{15b^4(d+ex)^{3/2}(bd-ae)^2}{e^6} - \frac{20b^3\sqrt{d+ex}(bd-ae)^3}{e^6} + \frac{15b^2(bd-ae)^4}{e^6\sqrt{d+ex}} - \frac{6b(bd-ae)^5}{e^6(d+ex)} \right)$$

↓ 2009

$$-\frac{12b^5(d+ex)^{7/2}(bd-ae)}{7e^7} + \frac{6b^4(d+ex)^{5/2}(bd-ae)^2}{e^7} - \frac{40b^3(d+ex)^{3/2}(bd-ae)^3}{3e^7} + \frac{30b^2\sqrt{d+ex}(bd-ae)^4}{e^7} + \frac{12b(bd-ae)^5}{e^7\sqrt{d+ex}} - \frac{2(bd-ae)^6}{3e^7(d+ex)^{3/2}} + \frac{2b^6(d+ex)^{9/2}}{9e^7}$$

input `Int[(a^2 + 2*a*b*x + b^2*x^2)^3/(d + e*x)^(5/2),x]`

output `(-2*(b*d - a*e)^6)/(3*e^7*(d + e*x)^(3/2)) + (12*b*(b*d - a*e)^5)/(e^7*sqrt[d + e*x]) + (30*b^2*(b*d - a*e)^4*sqrt[d + e*x])/e^7 - (40*b^3*(b*d - a*e)^3*(d + e*x)^(3/2))/(3*e^7) + (6*b^4*(b*d - a*e)^2*(d + e*x)^(5/2))/e^7 - (12*b^5*(b*d - a*e)*(d + e*x)^(7/2))/(7*e^7) + (2*b^6*(d + e*x)^(9/2))/(9*e^7)`

Defintions of rubi rules used

rule 27 `Int[(a_)*(F_x_), x_Symbol] := Simp[a Int[F_x, x], x] /; FreeQ[a, x] && !MatchQ[F_x, (b_)*(G_x_)] /; FreeQ[b, x]`

rule 53 `Int[((a_) + (b_)*(x_))^(m_)*((c_) + (d_)*(x_))^(n_), x_Symbol] := Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && IGtQ[m, 0] && (!IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])`

rule 1098 `Int[((d_) + (e_)*(x_))^(m_)*((a_) + (b_)*(x_) + (c_)*(x_)^2)^(p_), x_Symbol] := Simp[1/c^p Int[(d + e*x)^m*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]`

rule 2009 `Int[u_, x_Symbol] := Simp[IntSum[u, x], x] /; SumQ[u]`

input `integrate((b^2*x^2+2*a*b*x+a^2)^3/(e*x+d)^(5/2),x, algorithm="fricas")`

output
$$\frac{2}{63}(7b^6e^6x^6 + 1024b^6d^6 - 4608ab^5d^5e + 8064a^2b^4d^4e^2 - 6720a^3b^3d^3e^3 + 2520a^4b^2d^2e^4 - 252a^5b^1d^1e^5 - 21a^6e^6 - 6(2b^6d^5e^5 - 9a^2b^5e^6)x^5 + 3(8b^6d^4e^4 - 36a^2b^5d^3e^5 + 63a^3b^4e^6)x^4 - 4(16b^6d^3e^3 - 72a^2b^5d^2e^4 + 126a^3b^4d^1e^5 - 105a^4b^3e^6)x^3 + 3(128b^6d^4e^2 - 576a^2b^5d^3e^3 + 1008a^3b^4d^2e^4 - 840a^4b^3d^1e^5 + 315a^5b^2e^6)x^2 + 6(256b^6d^5e - 1152a^2b^5d^4e^2 + 2016a^3b^4d^3e^3 - 1680a^4b^3d^2e^4 + 630a^5b^2d^1e^5 - 63a^6e^6)x) \sqrt{e*x + d} / (e^9x^2 + 2*d*e^8x + d^2e^7)$$

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 352 vs. $2(168) = 336$.

Time = 10.69 (sec) , antiderivative size = 352, normalized size of antiderivative = 1.94

$$\int \frac{(a^2 + 2abx + b^2x^2)^3}{(d + ex)^{5/2}} dx = \left\{ \frac{2 \left(\frac{b^6(d+ex)^{\frac{9}{2}}}{9e^6} - \frac{6b(ae-bd)^5}{e^6\sqrt{d+ex}} + \frac{(d+ex)^{\frac{7}{2}} \cdot (6ab^5e - 6b^6d)}{7e^6} + \frac{(d+ex)^{\frac{5}{2}} \cdot (15a^2b^4e^2 - 30ab^5de + 15b^6d^2)}{5e^6} + \frac{(d+ex)^{\frac{3}{2}} \cdot (2a^6x + 3a^5bx^2 + 5a^4b^2x^3 + 5a^3b^3x^4 + 3a^2b^4x^5 + ab^5x^6 + \frac{b^6x^7}{7})}{d^{\frac{5}{2}}} \right)}{d^{\frac{5}{2}}} \right.$$

input `integrate((b**2*x**2+2*a*b*x+a**2)**3/(e*x+d)**(5/2),x)`

output `Piecewise((2*(b**6*(d + e*x)**(9/2)/(9*e**6) - 6*b*(a*e - b*d)**5/(e**6*sqrt(d + e*x)) + (d + e*x)**(7/2)*(6*a*b**5*e - 6*b**6*d)/(7*e**6) + (d + e*x)**(5/2)*(15*a**2*b**4*e**2 - 30*a*b**5*d*e + 15*b**6*d**2)/(5*e**6) + (d + e*x)**(3/2)*(20*a**3*b**3*e**3 - 60*a**2*b**4*d*e**2 + 60*a*b**5*d**2*e - 20*b**6*d**3)/(3*e**6) + sqrt(d + e*x)*(15*a**4*b**2*e**4 - 60*a**3*b**3*d*e**3 + 90*a**2*b**4*d**2*e**2 - 60*a*b**5*d**3*e + 15*b**6*d**4)/e**6 - (a*e - b*d)**6/(3*e**6*(d + e*x)**(3/2)))/e, Ne(e, 0)), ((a**6*x + 3*a**5*b*x**2 + 5*a**4*b**2*x**3 + 5*a**3*b**3*x**4 + 3*a**2*b**4*x**5 + a*b**5*x**6 + b**6*x**7/7)/d**(5/2), True))`

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 356 vs. $2(159) = 318$.

Time = 0.03 (sec) , antiderivative size = 356, normalized size of antiderivative = 1.97

$$\int \frac{(a^2 + 2abx + b^2x^2)^3}{(d + ex)^{5/2}} dx = \frac{2 \left(\frac{7(ex+d)^{\frac{9}{2}}b^6 - 54(b^6d - ab^5e)(ex+d)^{\frac{7}{2}} + 189(b^6d^2 - 2ab^5de + a^2b^4e^2)(ex+d)^{\frac{5}{2}} - 420(b^6d^3 - 3ab^5d^2e + a^2b^4d^2e^2)(ex+d)^{\frac{3}{2}} + 945(b^6d^4 - 4ab^5d^3e + 6a^2b^4d^2e^2 - 4a^3b^3d^2e^3 + a^4b^2d^2e^4) \sqrt{ex+d}}{e^6} - 21(b^6d^6 - 6a^2b^5d^5e + 15a^2b^4d^4e^2 - 20a^3b^3d^3e^3 + 15a^4b^2d^2e^4 - 6a^5bd^2e^5 + a^6e^6 - 18(b^6d^5 - 5ab^5d^4e + 10a^2b^4d^3e^2 - 10a^3b^3d^2e^3 + 5a^4b^2d^2e^4 - a^5bd^2e^5) \sqrt{ex+d}}{e^6} \right)}{e}$$

input `integrate((b^2*x^2+2*a*b*x+a^2)^3/(e*x+d)^(5/2),x, algorithm="maxima")`

output
$$\frac{2/63*((7*(ex + d)^{(9/2)}*b^6 - 54*(b^6*d - a*b^5*e)*(ex + d)^{(7/2)} + 189*(b^6*d^2 - 2*a*b^5*d*e + a^2*b^4*e^2)*(ex + d)^{(5/2)} - 420*(b^6*d^3 - 3*a*b^5*d^2*e + 3*a^2*b^4*d*e^2 - a^3*b^3*e^3)*(ex + d)^{(3/2)} + 945*(b^6*d^4 - 4*a*b^5*d^3*e + 6*a^2*b^4*d^2*e^2 - 4*a^3*b^3*d^2*e^3 + a^4*b^2*e^4)*\sqrt{ex + d})/e^6 - 21*(b^6*d^6 - 6*a*b^5*d^5*e + 15*a^2*b^4*d^4*e^2 - 20*a^3*b^3*d^3*e^3 + 15*a^4*b^2*d^2*e^4 - 6*a^5*b*d^2*e^5 + a^6*e^6 - 18*(b^6*d^5 - 5*a*b^5*d^4*e + 10*a^2*b^4*d^3*e^2 - 10*a^3*b^3*d^2*e^3 + 5*a^4*b^2*d^2*e^4 - a^5*b*d^2*e^5)*(ex + d))/((ex + d)^{(3/2)}*e^6))/e$$

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 464 vs. $2(159) = 318$.

Time = 0.20 (sec) , antiderivative size = 464, normalized size of antiderivative = 2.56

$$\int \frac{(a^2 + 2abx + b^2x^2)^3}{(d + ex)^{5/2}} dx = \frac{2(18(ex+d)b^6d^5 - b^6d^6 - 90(ex+d)ab^5d^4e + 6ab^5d^5e + 180(ex+d)a^2b^4d^4e^2 - 2(7(ex+d)^{\frac{9}{2}}b^6e^{56} - 54(ex+d)^{\frac{7}{2}}b^6de^{56} + 189(ex+d)^{\frac{5}{2}}b^6d^2e^{56} - 420(ex+d)^{\frac{3}{2}}b^6d^3e^{56} + 945\sqrt{ex+d}b^6d^4e^{56} - 21(b^6d^6 - 6a^2b^5d^5e + 15a^2b^4d^4e^2 - 20a^3b^3d^3e^3 + 15a^4b^2d^2e^4 - 6a^5bd^2e^5 + a^6e^6 - 18(b^6d^5 - 5ab^5d^4e + 10a^2b^4d^3e^2 - 10a^3b^3d^2e^3 + 5a^4b^2d^2e^4 - a^5bd^2e^5)\sqrt{ex+d}))/e^6}{e}$$

input `integrate((b^2*x^2+2*a*b*x+a^2)^3/(e*x+d)^(5/2),x, algorithm="giac")`

output

```

2/3*(18*(e*x + d)*b^6*d^5 - b^6*d^6 - 90*(e*x + d)*a*b^5*d^4*e + 6*a*b^5*d^5*e + 180*(e*x + d)*a^2*b^4*d^3*e^2 - 15*a^2*b^4*d^4*e^2 - 180*(e*x + d)*a^3*b^3*d^2*e^3 + 20*a^3*b^3*d^3*e^3 + 90*(e*x + d)*a^4*b^2*d*e^4 - 15*a^4*b^2*d^2*e^4 - 18*(e*x + d)*a^5*b*e^5 + 6*a^5*b*d*e^5 - a^6*e^6)/((e*x + d)^(3/2)*e^7) + 2/63*(7*(e*x + d)^(9/2)*b^6*e^56 - 54*(e*x + d)^(7/2)*b^6*d*e^56 + 189*(e*x + d)^(5/2)*b^6*d^2*e^56 - 420*(e*x + d)^(3/2)*b^6*d^3*e^56 + 945*sqrt(e*x + d)*b^6*d^4*e^56 + 54*(e*x + d)^(7/2)*a*b^5*e^57 - 378*(e*x + d)^(5/2)*a*b^5*d*e^57 + 1260*(e*x + d)^(3/2)*a*b^5*d^2*e^57 - 3780*sqrt(e*x + d)*a*b^5*d^3*e^57 + 189*(e*x + d)^(5/2)*a^2*b^4*e^58 - 1260*(e*x + d)^(3/2)*a^2*b^4*d*e^58 + 5670*sqrt(e*x + d)*a^2*b^4*d^2*e^58 + 420*(e*x + d)^(3/2)*a^3*b^3*e^59 - 3780*sqrt(e*x + d)*a^3*b^3*d*e^59 + 945*sqrt(e*x + d)*a^4*b^2*e^60)/e^63

```

Mupad [B] (verification not implemented)

Time = 9.25 (sec) , antiderivative size = 281, normalized size of antiderivative = 1.55

$$\begin{aligned}
& \int \frac{(a^2 + 2abx + b^2x^2)^3}{(d + ex)^{5/2}} dx = \frac{2b^6(d + ex)^{9/2}}{9e^7} - \frac{(12b^6d - 12ab^5e)(d + ex)^{7/2}}{7e^7} \\
& + \frac{(d + ex)(-12a^5be^5 + 60a^4b^2de^4 - 120a^3b^3d^2e^3 + 120a^2b^4d^3e^2 - 60ab^5d^4e + 12b^6d^5) - \frac{2a^6e^6}{3}}{e^7(d + ex)^{3/2}} \\
& + \frac{30b^2(ae - bd)^4\sqrt{d + ex}}{e^7} + \frac{40b^3(ae - bd)^3(d + ex)^{3/2}}{3e^7} + \frac{6b^4(ae - bd)^2(d + ex)^{5/2}}{e^7}
\end{aligned}$$

input

```
int((a^2 + b^2*x^2 + 2*a*b*x)^3/(d + e*x)^(5/2), x)
```

output

```

(2*b^6*(d + e*x)^(9/2))/(9*e^7) - ((12*b^6*d - 12*a*b^5*e)*(d + e*x)^(7/2))/(7*e^7) + ((d + e*x)*(12*b^6*d^5 - 12*a^5*b*e^5 + 60*a^4*b^2*d*e^4 + 120*a^2*b^4*d^3*e^2 - 120*a^3*b^3*d^2*e^3 - 60*a*b^5*d^4*e) - (2*a^6*e^6)/3 - (2*b^6*d^6)/3 - 10*a^2*b^4*d^4*e^2 + (40*a^3*b^3*d^3*e^3)/3 - 10*a^4*b^2*d^2*e^4 + 4*a*b^5*d^5*e + 4*a^5*b*d*e^5)/(e^7*(d + e*x)^(3/2)) + (30*b^2*(a*e - b*d)^4*(d + e*x)^(1/2))/e^7 + (40*b^3*(a*e - b*d)^3*(d + e*x)^(3/2))/(3*e^7) + (6*b^4*(a*e - b*d)^2*(d + e*x)^(5/2))/e^7

```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 384, normalized size of antiderivative = 2.12

$$\int \frac{(a^2 + 2abx + b^2x^2)^3}{(d + ex)^{5/2}} dx = \frac{96a^2b^4d^2e^4x^2 - 16a^2b^4de^5x^3 - \frac{1536}{7}ab^5d^4e^2x - \frac{384}{7}ab^5d^3e^3x^2 + \frac{64}{7}ab^5d^2e^4x^3 - \dots}{(d + ex)^{5/2}}$$

input `int((b^2*x^2+2*a*b*x+a^2)^3/(e*x+d)^(5/2),x)`output `(2*(- 21*a**6*e**6 - 252*a**5*b*d*e**5 - 378*a**5*b*e**6*x + 2520*a**4*b**2*d**2*e**4 + 3780*a**4*b**2*d*e**5*x + 945*a**4*b**2*e**6*x**2 - 6720*a**3*b**3*d**3*e**3 - 10080*a**3*b**3*d**2*e**4*x - 2520*a**3*b**3*d*e**5*x**2 + 420*a**3*b**3*e**6*x**3 + 8064*a**2*b**4*d**4*e**2 + 12096*a**2*b**4*d**3*e**3*x + 3024*a**2*b**4*d**2*e**4*x**2 - 504*a**2*b**4*d*e**5*x**3 + 189*a**2*b**4*e**6*x**4 - 4608*a*b**5*d**5*e - 6912*a*b**5*d**4*e**2*x - 1728*a*b**5*d**3*e**3*x**2 + 288*a*b**5*d**2*e**4*x**3 - 108*a*b**5*d*e**5*x**4 + 54*a*b**5*e**6*x**5 + 1024*b**6*d**6 + 1536*b**6*d**5*e*x + 384*b**6*d**4*e**2*x**2 - 64*b**6*d**3*e**3*x**3 + 24*b**6*d**2*e**4*x**4 - 12*b**6*d*e**5*x**5 + 7*b**6*e**6*x**6))/(63*sqrt(d + e*x)*e**7*(d + e*x))`

3.169
$$\int \frac{(a^2+2abx+b^2x^2)^3}{(d+ex)^{7/2}} dx$$

| | |
|---|------|
| Optimal result | 1326 |
| Mathematica [A] (verified) | 1327 |
| Rubi [A] (verified) | 1327 |
| Maple [A] (verified) | 1329 |
| Fricas [B] (verification not implemented) | 1330 |
| Sympy [A] (verification not implemented) | 1330 |
| Maxima [B] (verification not implemented) | 1331 |
| Giac [B] (verification not implemented) | 1332 |
| Mupad [B] (verification not implemented) | 1332 |
| Reduce [B] (verification not implemented) | 1333 |

Optimal result

Integrand size = 28, antiderivative size = 179

$$\int \frac{(a^2 + 2abx + b^2x^2)^3}{(d + ex)^{7/2}} dx = -\frac{2(bd - ae)^6}{5e^7(d + ex)^{5/2}} + \frac{4b(bd - ae)^5}{e^7(d + ex)^{3/2}} - \frac{30b^2(bd - ae)^4}{e^7\sqrt{d + ex}} - \frac{40b^3(bd - ae)^3\sqrt{d + ex}}{e^7} + \frac{10b^4(bd - ae)^2(d + ex)^{3/2}}{e^7} - \frac{12b^5(bd - ae)(d + ex)^{5/2}}{5e^7} + \frac{2b^6(d + ex)^{7/2}}{7e^7}$$

output

```
-2/5*(-a*e+b*d)^6/e^7/(e*x+d)^(5/2)+4*b*(-a*e+b*d)^5/e^7/(e*x+d)^(3/2)-30*
b^2*(-a*e+b*d)^4/e^7/(e*x+d)^(1/2)-40*b^3*(-a*e+b*d)^3*(e*x+d)^(1/2)/e^7+1
0*b^4*(-a*e+b*d)^2*(e*x+d)^(3/2)/e^7-12/5*b^5*(-a*e+b*d)*(e*x+d)^(5/2)/e^7
+2/7*b^6*(e*x+d)^(7/2)/e^7
```

Mathematica [A] (verified)

Time = 0.21 (sec) , antiderivative size = 291, normalized size of antiderivative = 1.63

$$\int \frac{(a^2 + 2abx + b^2x^2)^3}{(d + ex)^{7/2}} dx = \frac{2(7a^6e^6 + 14a^5be^5(2d + 5ex) + 35a^4b^2e^4(8d^2 + 20dex + 15e^2x^2) - 140a^3b^3e^3(16d^3 + 40d^2ex + 30de^2x^2 -$$

input

```
Integrate[(a^2 + 2*a*b*x + b^2*x^2)^3/(d + e*x)^(7/2),x]
```

output

```
(-2*(7*a^6*e^6 + 14*a^5*b*e^5*(2*d + 5*e*x) + 35*a^4*b^2*e^4*(8*d^2 + 20*d
*e*x + 15*e^2*x^2) - 140*a^3*b^3*e^3*(16*d^3 + 40*d^2*e*x + 30*d*e^2*x^2 +
5*e^3*x^3) + 35*a^2*b^4*e^2*(128*d^4 + 320*d^3*e*x + 240*d^2*e^2*x^2 + 40
*d*e^3*x^3 - 5*e^4*x^4) - 14*a*b^5*e*(256*d^5 + 640*d^4*e*x + 480*d^3*e^2*x
^2 + 80*d^2*e^3*x^3 - 10*d*e^4*x^4 + 3*e^5*x^5) + b^6*(1024*d^6 + 2560*d^
5*e*x + 1920*d^4*e^2*x^2 + 320*d^3*e^3*x^3 - 40*d^2*e^4*x^4 + 12*d*e^5*x^5
- 5*e^6*x^6)))/(35*e^7*(d + e*x)^(5/2))
```

Rubi [A] (verified)

Time = 0.53 (sec) , antiderivative size = 179, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {1098, 27, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a^2 + 2abx + b^2x^2)^3}{(d + ex)^{7/2}} dx$$

$$\downarrow 1098$$

$$\frac{\int \frac{b^6(a+bx)^6}{(d+ex)^{7/2}} dx}{b^6}$$

$$\downarrow 27$$

$$\int \frac{(a + bx)^6}{(d + ex)^{7/2}} dx$$

↓ 53

$$\int \left(-\frac{6b^5(d + ex)^{3/2}(bd - ae)}{e^6} + \frac{15b^4\sqrt{d + ex}(bd - ae)^2}{e^6} - \frac{20b^3(bd - ae)^3}{e^6\sqrt{d + ex}} + \frac{15b^2(bd - ae)^4}{e^6(d + ex)^{3/2}} - \frac{6b(bd - ae)^5}{e^6(d + ex)^{5/2}} + \dots \right)$$

↓ 2009

$$-\frac{12b^5(d + ex)^{5/2}(bd - ae)}{5e^7} + \frac{10b^4(d + ex)^{3/2}(bd - ae)^2}{e^7} - \frac{40b^3\sqrt{d + ex}(bd - ae)^3}{e^7} - \frac{30b^2(bd - ae)^4}{e^7\sqrt{d + ex}} + \frac{4b(bd - ae)^5}{e^7(d + ex)^{3/2}} - \frac{2(bd - ae)^6}{5e^7(d + ex)^{5/2}} + \frac{2b^6(d + ex)^{7/2}}{7e^7}$$

input `Int[(a^2 + 2*a*b*x + b^2*x^2)^3/(d + e*x)^(7/2),x]`

output `(-2*(b*d - a*e)^6)/(5*e^7*(d + e*x)^(5/2)) + (4*b*(b*d - a*e)^5)/(e^7*(d + e*x)^(3/2)) - (30*b^2*(b*d - a*e)^4)/(e^7*sqrt[d + e*x]) - (40*b^3*(b*d - a*e)^3*sqrt[d + e*x])/e^7 + (10*b^4*(b*d - a*e)^2*(d + e*x)^(3/2))/e^7 - (12*b^5*(b*d - a*e)*(d + e*x)^(5/2))/(5*e^7) + (2*b^6*(d + e*x)^(7/2))/(7*e^7)`

Defintions of rubi rules used

rule 27 `Int[(a_)*(Fx_), x_Symbol] := Simp[a Int[Fx, x], x] /; FreeQ[a, x] && !MatchQ[Fx, (b_)*(Gx_)] /; FreeQ[b, x]`

rule 53 `Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] := Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && IGtQ[m, 0] && (!IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])`

rule 1098 `Int[((d_.) + (e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] := Simp[1/c^p Int[(d + e*x)^m*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]`

rule 2009

```
Int[u_, x_Symbol] := Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 1.32 (sec) , antiderivative size = 248, normalized size of antiderivative = 1.39

| method | result |
|-------------------|--|
| risch | $\frac{2b^3(5e^3x^3b^3+42x^2ab^2e^3-27x^2b^3de^2+175a^2be^3x-266xab^2de^2+106b^3d^2ex+700e^3a^3-1925a^2bde^2+1792ab^2d^2e-562b^3d^3)}{35e^7}$ |
| pseudoelliptic | $-\frac{2\left((-6ab^5x^5-25a^2b^4x^4-100a^3b^3x^3+75a^4b^2x^2+10a^5bx-\frac{5}{7}b^6x^6+a^6\right)e^6+4\left(\frac{3}{7}b^5x^5+5ab^4x^4+50a^2b^3x^3-150a^3b^2x^2-100a^4b^2x+10a^5\right)e^5}{e^7}$ |
| gosper | $-\frac{2(-5x^6b^6e^6-42x^5ab^5e^6+12x^5b^6de^5-175x^4a^2b^4e^6+140x^4ab^5de^5-40x^4b^6d^2e^4-700x^3a^3b^3e^6+1400x^3a^2b^4de^5-562b^3d^3e^3)}{e^7}$ |
| trager | $-\frac{2(-5x^6b^6e^6-42x^5ab^5e^6+12x^5b^6de^5-175x^4a^2b^4e^6+140x^4ab^5de^5-40x^4b^6d^2e^4-700x^3a^3b^3e^6+1400x^3a^2b^4de^5-562b^3d^3e^3)}{e^7}$ |
| derivativedivides | $\frac{2b^6(e^3x+d)^{\frac{7}{2}}}{7} + \frac{12ab^5e(e^3x+d)^{\frac{5}{2}}}{5} - \frac{12b^6d(e^3x+d)^{\frac{5}{2}}}{5} + 10a^2b^4e^2(e^3x+d)^{\frac{3}{2}} - 20ab^5de(e^3x+d)^{\frac{3}{2}} + 10b^6d^2(e^3x+d)^{\frac{3}{2}} + 40a^3b^3e^3\sqrt{e^3x+d}$ |
| default | $\frac{2b^6(e^3x+d)^{\frac{7}{2}}}{7} + \frac{12ab^5e(e^3x+d)^{\frac{5}{2}}}{5} - \frac{12b^6d(e^3x+d)^{\frac{5}{2}}}{5} + 10a^2b^4e^2(e^3x+d)^{\frac{3}{2}} - 20ab^5de(e^3x+d)^{\frac{3}{2}} + 10b^6d^2(e^3x+d)^{\frac{3}{2}} + 40a^3b^3e^3\sqrt{e^3x+d}$ |
| orering | $-\frac{2(-5x^6b^6e^6-42x^5ab^5e^6+12x^5b^6de^5-175x^4a^2b^4e^6+140x^4ab^5de^5-40x^4b^6d^2e^4-700x^3a^3b^3e^6+1400x^3a^2b^4de^5-562b^3d^3e^3)}{e^7}$ |

```
input int((b^2*x^2+2*a*b*x+a^2)^3/(e*x+d)^(7/2), x, method=_RETURNVERBOSE)
```

```
output 2/35*b^3*(5*b^3*e^3*x^3+42*a*b^2*e^3*x^2-27*b^3*d*e^2*x^2+175*a^2*b*e^3*x-266*a*b^2*d*e^2*x+106*b^3*d^2*e*x+700*a^3*e^3-1925*a^2*b*d*e^2+1792*a*b^2*d^2*e-562*b^3*d^3)*(e*x+d)^(1/2)/e^7-2/5*(75*b^2*e^2*x^2+10*a*b*e^2*x+140*b^2*d*e*x+a^2*e^2+8*a*b*d*e+66*b^2*d^2)*(a^4*e^4-4*a^3*b*d*e^3+6*a^2*b^2*d^2*e^2-4*a*b^3*d^3*e+b^4*d^4)/e^7/(e*x+d)^(1/2)/(e^2*x^2+2*d*e*x+d^2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 388 vs. $2(159) = 318$.

Time = 0.10 (sec) , antiderivative size = 388, normalized size of antiderivative = 2.17

$$\int \frac{(a^2 + 2abx + b^2x^2)^3}{(d + ex)^{7/2}} dx = \frac{2(5b^6e^6x^6 - 1024b^6d^6 + 3584ab^5d^5e - 4480a^2b^4d^4e^2 + 2240a^3b^3d^3e^3 - 280a^4b^2d^2e^4 - 28a^5b^1d^1e^5 - 7a^6e^6)}{(d + ex)^{7/2}}$$

input `integrate((b^2*x^2+2*a*b*x+a^2)^3/(e*x+d)^(7/2),x, algorithm="fricas")`

output
$$\frac{2/35*(5*b^6*e^6*x^6 - 1024*b^6*d^6 + 3584*a*b^5*d^5*e - 4480*a^2*b^4*d^4*e^2 + 2240*a^3*b^3*d^3*e^3 - 280*a^4*b^2*d^2*e^4 - 28*a^5*b*d*e^5 - 7*a^6*e^6 - 6*(2*b^6*d*e^5 - 7*a*b^5*e^6)*x^5 + 5*(8*b^6*d^2*e^4 - 28*a*b^5*d*e^5 + 35*a^2*b^4*e^6)*x^4 - 20*(16*b^6*d^3*e^3 - 56*a*b^5*d^2*e^4 + 70*a^2*b^4*d*e^5 - 35*a^3*b^3*e^6)*x^3 - 15*(128*b^6*d^4*e^2 - 448*a*b^5*d^3*e^3 + 560*a^2*b^4*d^2*e^4 - 280*a^3*b^3*d*e^5 + 35*a^4*b^2*e^6)*x^2 - 10*(256*b^6*d^5*e - 896*a*b^5*d^4*e^2 + 1120*a^2*b^4*d^3*e^3 - 560*a^3*b^3*d^2*e^4 + 70*a^4*b^2*d*e^5 + 7*a^5*b*e^6)*x)*sqrt(e*x + d)/(e^10*x^3 + 3*d*e^9*x^2 + 3*d^2*e^8*x + d^3*e^7)}$$

Sympy [A] (verification not implemented)

Time = 10.08 (sec) , antiderivative size = 301, normalized size of antiderivative = 1.68

$$\int \frac{(a^2 + 2abx + b^2x^2)^3}{(d + ex)^{7/2}} dx = \left\{ \begin{array}{l} \frac{2 \left(\frac{b^6(d+ex)^{7/2}}{7e^6} - \frac{15b^2(ae-bd)^4}{e^6\sqrt{d+ex}} - \frac{2b(ae-bd)^5}{e^6(d+ex)^{3/2}} + \frac{(d+ex)^{5/2} \cdot (6ab^5e - 6b^6d)}{5e^6} + \frac{(d+ex)^{3/2} \cdot (15a^2b^4e^2 - 30ab^5de + 15b^6d^2)}{3e^6} \right)}{e} \\ \frac{a^6x + 3a^5bx^2 + 5a^4b^2x^3 + 5a^3b^3x^4 + 3a^2b^4x^5 + ab^5x^6 + \frac{b^6x^7}{7}}{d^{7/2}} \end{array} \right.$$

input `integrate((b**2*x**2+2*a*b*x+a**2)**3/(e*x+d)**(7/2),x)`

output

```
Piecewise((2*(b**6*(d + e*x)**(7/2))/(7*e**6) - 15*b**2*(a*e - b*d)**4/(e**6*sqrt(d + e*x)) - 2*b*(a*e - b*d)**5/(e**6*(d + e*x)**(3/2)) + (d + e*x)**(5/2)*(6*a*b**5*e - 6*b**6*d)/(5*e**6) + (d + e*x)**(3/2)*(15*a**2*b**4*e**2 - 30*a*b**5*d*e + 15*b**6*d**2)/(3*e**6) + sqrt(d + e*x)*(20*a**3*b**3*e**3 - 60*a**2*b**4*d*e**2 + 60*a*b**5*d**2*e - 20*b**6*d**3)/e**6 - (a*e - b*d)**6/(5*e**6*(d + e*x)**(5/2)))/e, Ne(e, 0)), ((a**6*x + 3*a**5*b*x**2 + 5*a**4*b**2*x**3 + 5*a**3*b**3*x**4 + 3*a**2*b**4*x**5 + a*b**5*x**6 + b**6*x**7/7)/d**(7/2), True))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 356 vs. $2(159) = 318$.

Time = 0.04 (sec) , antiderivative size = 356, normalized size of antiderivative = 1.99

$$\int \frac{(a^2 + 2abx + b^2x^2)^3}{(d + ex)^{7/2}} dx = \frac{2 \left(\frac{5(ex+d)^{7/2} b^6 - 42(b^6 d - ab^5 e)(ex+d)^{5/2} + 175(b^6 d^2 - 2ab^5 de + a^2 b^4 e^2)(ex+d)^{3/2} - 700(b^6 d^3 - 3ab^5 d^2 e + a^2 b^4 d e^2 - 3a^3 b^3 e^3) \sqrt{ex+d}}{e^6} \right)}{(d + ex)^{7/2}}$$

input

```
integrate((b^2*x^2+2*a*b*x+a^2)^3/(e*x+d)^(7/2),x, algorithm="maxima")
```

output

```
2/35*((5*(e*x + d)^(7/2)*b^6 - 42*(b^6*d - a*b^5*e)*(e*x + d)^(5/2) + 175*(b^6*d^2 - 2*a*b^5*d*e + a^2*b^4*e^2)*(e*x + d)^(3/2) - 700*(b^6*d^3 - 3*a*b^5*d^2*e + 3*a^2*b^4*d*e^2 - a^3*b^3*e^3)*sqrt(e*x + d))/e^6 - 7*(b^6*d^6 - 6*a*b^5*d^5*e + 15*a^2*b^4*d^4*e^2 - 20*a^3*b^3*d^3*e^3 + 15*a^4*b^2*d^2*e^4 - 6*a^5*b*d*e^5 + a^6*e^6 + 75*(b^6*d^4 - 4*a*b^5*d^3*e + 6*a^2*b^4*d^2*e^2 - 4*a^3*b^3*d*e^3 + a^4*b^2*e^4)*(e*x + d)^2 - 10*(b^6*d^5 - 5*a*b^5*d^4*e + 10*a^2*b^4*d^3*e^2 - 10*a^3*b^3*d^2*e^3 + 5*a^4*b^2*d*e^4 - a^5*b*e^5)*(e*x + d))/((e*x + d)^(5/2)*e^6))/e
```


Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 457 vs. $2(159) = 318$.

Time = 0.16 (sec) , antiderivative size = 457, normalized size of antiderivative = 2.55

$$\int \frac{(a^2 + 2abx + b^2x^2)^3}{(d + ex)^{7/2}} dx =$$

$$\frac{2(75(ex + d)^2b^6d^4 - 10(ex + d)b^6d^5 + b^6d^6 - 300(ex + d)^2ab^5d^3e + 50(ex + d)ab^5d^4e - 6ab^5d^5e + 450a^2b^5d^3e^2 - 100a^2b^4d^2e^2 + 15a^2b^4d^4e^2 - 300a^3b^3d^2e^3 + 100a^3b^3d^4e^3 - 20a^3b^3d^3e^3 + 75a^4b^2d^2e^4 - 50a^4b^2d^4e^4 + 15a^4b^2d^2e^4 + 10a^5bde^5 - 6a^5bde^5 + a^6e^6)/((ex + d)^{5/2}e^7) + \frac{2(5(ex + d)^{7/2}b^6e^{42} - 42(ex + d)^{5/2}b^6de^{42} + 175(ex + d)^{3/2}b^6d^2e^{42} - 700\sqrt{ex + d}b^6d^3e^{42} + 42(ex + d)^{5/2}ab^6d^3e^{42} - 42(ex + d)^{3/2}ab^6d^4e^{42} + 2100\sqrt{ex + d}ab^6d^4e^{42} - 2100\sqrt{ex + d}ab^6d^5e^{42} + 700\sqrt{ex + d}a^2b^5d^3e^{42} - 700\sqrt{ex + d}a^2b^5d^4e^{42} + 2100\sqrt{ex + d}a^2b^5d^5e^{42} - 2100\sqrt{ex + d}a^3b^4d^2e^{42} + 2100\sqrt{ex + d}a^3b^4d^3e^{42} - 2100\sqrt{ex + d}a^3b^4d^4e^{42} + 2100\sqrt{ex + d}a^4b^3d^2e^{42} - 2100\sqrt{ex + d}a^4b^3d^3e^{42} + 2100\sqrt{ex + d}a^4b^3d^4e^{42} - 2100\sqrt{ex + d}a^5b^2d^2e^{42} + 2100\sqrt{ex + d}a^5b^2d^3e^{42} - 2100\sqrt{ex + d}a^5b^2d^4e^{42} + 2100\sqrt{ex + d}a^6de^{42})/e^{49}}$$

input

```
integrate((b^2*x^2+2*a*b*x+a^2)^3/(e*x+d)^(7/2),x, algorithm="giac")
```

output

```
-2/5*(75*(e*x + d)^2*b^6*d^4 - 10*(e*x + d)*b^6*d^5 + b^6*d^6 - 300*(e*x + d)^2*a*b^5*d^3*e + 50*(e*x + d)*a*b^5*d^4*e - 6*a*b^5*d^5*e + 450*(e*x + d)^2*a^2*b^4*d^2*e^2 - 100*(e*x + d)*a^2*b^4*d^3*e^2 + 15*a^2*b^4*d^4*e^2 - 300*(e*x + d)^2*a^3*b^3*d^2*e^3 + 100*(e*x + d)*a^3*b^3*d^4*e^3 - 20*a^3*b^3*d^3*e^3 + 75*(e*x + d)^2*a^4*b^2*d^2*e^4 - 50*(e*x + d)*a^4*b^2*d^4*e^4 + 15*a^4*b^2*d^2*e^4 + 10*(e*x + d)*a^5*b*d^2*e^5 - 6*a^5*b*d^4*e^5 + a^6*e^6)/((e*x + d)^(5/2)*e^7) + 2/35*(5*(e*x + d)^(7/2)*b^6*e^42 - 42*(e*x + d)^(5/2)*b^6*d^2*e^42 + 175*(e*x + d)^(3/2)*b^6*d^3*e^42 - 700*sqrt(e*x + d)*b^6*d^4*e^42 + 42*(e*x + d)^(5/2)*a*b^5*d^3*e^42 - 42*(e*x + d)^(3/2)*a*b^5*d^4*e^42 + 2100*sqrt(e*x + d)*a*b^5*d^5*e^42 - 2100*sqrt(e*x + d)*a^2*b^4*d^2*e^42 + 2100*sqrt(e*x + d)*a^2*b^4*d^3*e^42 - 2100*sqrt(e*x + d)*a^2*b^4*d^4*e^42 + 2100*sqrt(e*x + d)*a^3*b^3*d^2*e^42 - 2100*sqrt(e*x + d)*a^3*b^3*d^3*e^42 + 2100*sqrt(e*x + d)*a^3*b^3*d^4*e^42 - 2100*sqrt(e*x + d)*a^4*b^2*d^2*e^42 + 2100*sqrt(e*x + d)*a^4*b^2*d^3*e^42 - 2100*sqrt(e*x + d)*a^4*b^2*d^4*e^42 + 2100*sqrt(e*x + d)*a^5*b*d^2*e^42 - 2100*sqrt(e*x + d)*a^5*b*d^3*e^42 - 2100*sqrt(e*x + d)*a^5*b*d^4*e^42 + 2100*sqrt(e*x + d)*a^6*d^2*e^42)/e^49
```

Mupad [B] (verification not implemented)

Time = 9.11 (sec) , antiderivative size = 322, normalized size of antiderivative = 1.80

$$\int \frac{(a^2 + 2abx + b^2x^2)^3}{(d + ex)^{7/2}} dx = \frac{2b^6(d + ex)^{7/2}}{7e^7} - \frac{(12b^6d - 12ab^5e)(d + ex)^{5/2}}{5e^7}$$

$$- \frac{(d + ex)^2(30a^4b^2e^4 - 120a^3b^3de^3 + 180a^2b^4d^2e^2 - 120ab^5d^3e + 30b^6d^4) - (d + ex)(-4a^5be^5 + 40b^3(ae - bd)^3\sqrt{d + ex} + 10b^4(ae - bd)^2(d + ex)^{3/2})}{e^7}$$

input `int((a^2 + b^2*x^2 + 2*a*b*x)^3/(d + e*x)^(7/2),x)`

output
$$\begin{aligned} & (2*b^6*(d + e*x)^(7/2))/(7*e^7) - ((12*b^6*d - 12*a*b^5*e)*(d + e*x)^(5/2)) / (5*e^7) - ((d + e*x)^2*(30*b^6*d^4 + 30*a^4*b^2*e^4 - 120*a^3*b^3*d*e^3 \\ & + 180*a^2*b^4*d^2*e^2 - 120*a*b^5*d^3*e) - (d + e*x)*(4*b^6*d^5 - 4*a^5*b*e^5 + 20*a^4*b^2*d*e^4 + 40*a^2*b^4*d^3*e^2 - 40*a^3*b^3*d^2*e^3 - 20*a*b^5*d^4*e) + (2*a^6*e^6)/5 + (2*b^6*d^6)/5 + 6*a^2*b^4*d^4*e^2 - 8*a^3*b^3*d^3*e^3 + 6*a^4*b^2*d^2*e^4 - (12*a*b^5*d^5*e)/5 - (12*a^5*b*d*e^5)/5) / (e^7 * (d + e*x)^(5/2)) + (40*b^3*(a*e - b*d)^3*(d + e*x)^(1/2))/e^7 + (10*b^4*(a*e - b*d)^2*(d + e*x)^(3/2))/e^7 \end{aligned}$$

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 395, normalized size of antiderivative = 2.21

$$\int \frac{(a^2 + 2abx + b^2x^2)^3}{(d + ex)^{7/2}} dx = \frac{-480a^2b^4d^2e^4x^2 - 80a^2b^4de^5x^3 + 512ab^5d^4e^2x + 384ab^5d^3e^3x^2 + 64ab^5d^2e^4}{(d + ex)^{7/2}}$$

input `int((b^2*x^2+2*a*b*x+a^2)^3/(e*x+d)^(7/2),x)`

output
$$\begin{aligned} & (2*(- 7*a**6*e**6 - 28*a**5*b*d*e**5 - 70*a**5*b*e**6*x - 280*a**4*b**2*d \\ & **2*e**4 - 700*a**4*b**2*d*e**5*x - 525*a**4*b**2*e**6*x**2 + 2240*a**3*b* \\ & *3*d**3*e**3 + 5600*a**3*b**3*d**2*e**4*x + 4200*a**3*b**3*d*e**5*x**2 + 7 \\ & 00*a**3*b**3*e**6*x**3 - 4480*a**2*b**4*d**4*e**2 - 11200*a**2*b**4*d**3*e \\ & **3*x - 8400*a**2*b**4*d**2*e**4*x**2 - 1400*a**2*b**4*d*e**5*x**3 + 175*a \\ & **2*b**4*e**6*x**4 + 3584*a*b**5*d**5*e + 8960*a*b**5*d**4*e**2*x + 6720*a \\ & *b**5*d**3*e**3*x**2 + 1120*a*b**5*d**2*e**4*x**3 - 140*a*b**5*d*e**5*x**4 \\ & + 42*a*b**5*e**6*x**5 - 1024*b**6*d**6 - 2560*b**6*d**5*e*x - 1920*b**6*d \\ & **4*e**2*x**2 - 320*b**6*d**3*e**3*x**3 + 40*b**6*d**2*e**4*x**4 - 12*b**6 \\ & *d*e**5*x**5 + 5*b**6*e**6*x**6))/(35*sqrt(d + e*x)*e**7*(d**2 + 2*d*e*x + \\ & e**2*x**2)) \end{aligned}$$

3.170 $\int \frac{(d+ex)^{9/2}}{a^2+2abx+b^2x^2} dx$

| | |
|---|------|
| Optimal result | 1334 |
| Mathematica [A] (verified) | 1335 |
| Rubi [A] (verified) | 1335 |
| Maple [A] (verified) | 1339 |
| Fricas [B] (verification not implemented) | 1340 |
| Sympy [F(-1)] | 1341 |
| Maxima [F(-2)] | 1342 |
| Giac [B] (verification not implemented) | 1342 |
| Mupad [B] (verification not implemented) | 1343 |
| Reduce [B] (verification not implemented) | 1344 |

Optimal result

Integrand size = 28, antiderivative size = 162

$$\int \frac{(d+ex)^{9/2}}{a^2+2abx+b^2x^2} dx = \frac{9e(bd-ae)^3\sqrt{d+ex}}{b^5} + \frac{3e(bd-ae)^2(d+ex)^{3/2}}{b^4} + \frac{9e(bd-ae)(d+ex)^{5/2}}{5b^3} + \frac{9e(d+ex)^{7/2}}{7b^2} - \frac{(d+ex)^{9/2}}{b(a+bx)} - \frac{9e(bd-ae)^{7/2}\operatorname{arctanh}\left(\frac{\sqrt{b}\sqrt{d+ex}}{\sqrt{bd-ae}}\right)}{b^{11/2}}$$

output

```
9*e*(-a*e+b*d)^3*(e*x+d)^(1/2)/b^5+3*e*(-a*e+b*d)^2*(e*x+d)^(3/2)/b^4+9/5*
e*(-a*e+b*d)*(e*x+d)^(5/2)/b^3+9/7*e*(e*x+d)^(7/2)/b^2-(e*x+d)^(9/2)/b/(b*
x+a)-9*e*(-a*e+b*d)^(7/2)*arctanh(b^(1/2)*(e*x+d)^(1/2)/(-a*e+b*d)^(1/2))/
b^(11/2)
```

Mathematica [A] (verified)

Time = 0.49 (sec) , antiderivative size = 208, normalized size of antiderivative = 1.28

$$\int \frac{(d+ex)^{9/2}}{a^2+2abx+b^2x^2} dx = \frac{\sqrt{d+ex}(315a^4e^4+210a^3be^3(-5d+ex)-42a^2b^2e^2(-29d^2+17dex+e^2x^2)+6ab^3e(-88d^3+142d^2ex-35b^5(a+bx))}{35b^5(a+bx)} + \frac{9e(-bd+ae)^{7/2} \arctan\left(\frac{\sqrt{b}\sqrt{d+ex}}{\sqrt{-bd+ae}}\right)}{b^{11/2}}$$

input

```
Integrate[(d + e*x)^(9/2)/(a^2 + 2*a*b*x + b^2*x^2), x]
```

output

```
-1/35*(Sqrt[d + e*x]*(315*a^4*e^4 + 210*a^3*b*e^3*(-5*d + e*x) - 42*a^2*b^2*e^2*(-29*d^2 + 17*d*e*x + e^2*x^2) + 6*a*b^3*e*(-88*d^3 + 142*d^2*e*x + 23*d*e^2*x^2 + 3*e^3*x^3) + b^4*(35*d^4 - 388*d^3*e*x - 156*d^2*e^2*x^2 - 58*d*e^3*x^3 - 10*e^4*x^4)))/(b^5*(a + b*x)) + (9*e*(-(b*d) + a*e)^(7/2)*ArcTan[(Sqrt[b]*Sqrt[d + e*x])/Sqrt[-(b*d) + a*e]])/b^(11/2)
```

Rubi [A] (verified)

Time = 0.51 (sec) , antiderivative size = 179, normalized size of antiderivative = 1.10, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.321$, Rules used = {1098, 27, 51, 60, 60, 60, 60, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex)^{9/2}}{a^2+2abx+b^2x^2} dx$$

↓ 1098

$$b^2 \int \frac{(d+ex)^{9/2}}{b^2(a+bx)^2} dx$$

↓ 27

$$\begin{aligned}
& \int \frac{(d+ex)^{9/2}}{(a+bx)^2} dx \\
& \quad \downarrow 51 \\
& \frac{9e \int \frac{(d+ex)^{7/2}}{a+bx} dx}{2b} - \frac{(d+ex)^{9/2}}{b(a+bx)} \\
& \quad \downarrow 60 \\
& \frac{9e \left(\frac{(bd-ae) \int \frac{(d+ex)^{5/2}}{a+bx} dx}{b} + \frac{2(d+ex)^{7/2}}{7b} \right)}{2b} - \frac{(d+ex)^{9/2}}{b(a+bx)} \\
& \quad \downarrow 60 \\
& \frac{9e \left(\frac{(bd-ae) \left(\frac{(bd-ae) \int \frac{(d+ex)^{3/2}}{a+bx} dx}{b} + \frac{2(d+ex)^{5/2}}{5b} \right)}{b} + \frac{2(d+ex)^{7/2}}{7b} \right)}{2b} - \frac{(d+ex)^{9/2}}{b(a+bx)} \\
& \quad \downarrow 60 \\
& \frac{9e \left(\frac{(bd-ae) \left(\frac{(bd-ae) \left(\frac{(bd-ae) \int \frac{\sqrt{d+ex}}{a+bx} dx}{b} + \frac{2(d+ex)^{3/2}}{3b} \right)}{b} + \frac{2(d+ex)^{5/2}}{5b} \right)}{b} + \frac{2(d+ex)^{7/2}}{7b} \right)}{2b} - \frac{(d+ex)^{9/2}}{b(a+bx)} \\
& \quad \downarrow 60
\end{aligned}$$

$$9e \left(\frac{(bd-ae) \left(\frac{(bd-ae) \int \frac{1}{(a+bx)\sqrt{d+ex}} dx + 2\sqrt{d+ex}}{b} \right) + \frac{2(d+ex)^{3/2}}{3b}}{b} + \frac{2(d+ex)^{5/2}}{5b} \right) + \frac{2(d+ex)^{7/2}}{7b}$$

$$\frac{2b}{(d+ex)^{9/2}} \cdot \frac{1}{b(a+bx)}$$

↓ 73

$$9e \left(\frac{(bd-ae) \left(\frac{2(bd-ae) \int \frac{1}{a + \frac{b(d+ex)}{e} - \frac{bd}{e}} d\sqrt{d+ex}}{b} + 2\sqrt{d+ex}}{b} \right) + \frac{2(d+ex)^{3/2}}{3b}}{b} + \frac{2(d+ex)^{5/2}}{5b} \right) + \frac{2(d+ex)^{7/2}}{7b}$$

$$\frac{2b}{(d+ex)^{9/2}} \cdot \frac{1}{b(a+bx)}$$

↓ 221

$$\frac{9e \left((bd-ae) \left(\frac{(bd-ae) \left(\frac{2\sqrt{d+ex}}{b} - \frac{2\sqrt{bd-ae} \operatorname{arctanh}\left(\frac{\sqrt{b}\sqrt{d+ex}}{\sqrt{bd-ae}}\right)}{b^{3/2}} \right)}{b} + \frac{2(d+ex)^{3/2}}{3b} \right) + \frac{2(d+ex)^{5/2}}{5b} \right) + \frac{2(d+ex)^{7/2}}{7b}}{b} + \frac{2(d+ex)^{9/2}}{7b}}{2b \frac{(d+ex)^{9/2}}{b(a+bx)}}$$

```
input Int[(d + e*x)^(9/2)/(a^2 + 2*a*b*x + b^2*x^2), x]
```

```
output -((d + e*x)^(9/2)/(b*(a + b*x))) + (9*e*((2*(d + e*x)^(7/2))/(7*b) + ((b*d - a*e)*((2*(d + e*x)^(5/2))/(5*b) + ((b*d - a*e)*((2*(d + e*x)^(3/2))/(3*b) + ((b*d - a*e)*((2*Sqrt[d + e*x])/b - (2*Sqrt[b*d - a*e]*ArcTanh[(Sqrt[b]*Sqrt[d + e*x])/Sqrt[b*d - a*e]])/b^(3/2)))/b))/b))/(2*b)
```

Defintions of rubi rules used

```
rule 27 Int[(a_)*(Fx_), x_Symbol] := Simp[a Int[Fx, x], x] /; FreeQ[a, x] && !MatchQ[Fx, (b_)*(Gx_)] /; FreeQ[b, x]
```

```
rule 51 Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] := Simp[(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Simp[d*(n/(b*(m + 1))) Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d, n}, x] && ILtQ[m, -1] && FractionQ[n] && GtQ[n, 0]
```

rule 60 `Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] := Simp[(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*(b*c - a*d)/(b*(m + n + 1)) Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] && !(IGtQ[m, 0] && (!IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) && !ILtQ[m + n + 2, 0] && IntLinearQ[a, b, c, d, m, n, x]`

rule 73 `Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] := With[{p = Denominator[m]}, Simp[p/b Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) + d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntLinearQ[a, b, c, d, m, n, x]`

rule 221 `Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[-a/b, 2]/a)*ArcTanh[x/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]`

rule 1098 `Int[((d_.) + (e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] := Simp[1/c^p Int[(d + e*x)^m*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]`

Maple [A] (verified)

Time = 3.08 (sec) , antiderivative size = 217, normalized size of antiderivative = 1.34

| method | result |
|-------------------|---|
| pseudoelliptic | $\frac{9e(ae-bd)^4(bx+a) \arctan\left(\frac{b\sqrt{ex+d}}{\sqrt{b(ae-bd)}}\right) - 9\sqrt{ex+d} \sqrt{b(ae-bd)} \left(-\frac{2}{63}e^4x^4 - \frac{58}{315}de^3x^3 - \frac{52}{105}d^2e^2x^2 - \frac{388}{315}d^3ex + \frac{1}{9}d^4 \right)}{b^5(bx+a)\sqrt{b(ae-bd)}}$ |
| risch | $-\frac{2e(-5e^3x^3b^3+14x^2ab^2e^3-29x^2b^3de^2-35a^2be^3x+98xab^2de^2-78b^3d^2ex+140e^3a^3-455a^2bde^2+504ab^2d^2e-19d^3)}{35b^5}$ |
| derivativedivides | $2e \left(-\frac{(ex+d)^{\frac{7}{2}}b^3}{7} + \frac{2ab^2e(ex+d)^{\frac{5}{2}}}{5} - \frac{2b^3d(ex+d)^{\frac{5}{2}}}{5} - a^2be^2(ex+d)^{\frac{3}{2}} + \frac{2ab^2de(ex+d)^{\frac{3}{2}}}{b^5} - \frac{b^3d^2(ex+d)^{\frac{3}{2}}}{b^5} + 4a^3e^3\sqrt{ex+d} \right)$ |
| default | $2e \left(-\frac{(ex+d)^{\frac{7}{2}}b^3}{7} + \frac{2ab^2e(ex+d)^{\frac{5}{2}}}{5} - \frac{2b^3d(ex+d)^{\frac{5}{2}}}{5} - a^2be^2(ex+d)^{\frac{3}{2}} + \frac{2ab^2de(ex+d)^{\frac{3}{2}}}{b^5} - \frac{b^3d^2(ex+d)^{\frac{3}{2}}}{b^5} + 4a^3e^3\sqrt{ex+d} \right)$ |

```
input int((e*x+d)^(9/2)/(b^2*x^2+2*a*b*x+a^2),x,method=_RETURNVERBOSE)
```

```
output 9*(e*(a*e-b*d)^4*(b*x+a)*arctan(b*(e*x+d)^(1/2)/(b*(a*e-b*d))^(1/2))-(e*x+d)^(1/2)*(b*(a*e-b*d))^(1/2)*((-2/63*e^4*x^4-58/315*d*e^3*x^3-52/105*d^2*e^2*x^2-388/315*d^3*e*x+1/9*d^4)*b^4-176/105*e*(-3/88*e^3*x^3-23/88*d*e^2*x^2-71/44*d^2*e*x+d^3)*a*b^3+58/15*e^2*a^2*(-1/29*e^2*x^2-17/29*d*e*x+d^2)*b^2-10/3*e^3*a^3*(-1/5*e*x+d)*b+a^4*e^4)/(b*(a*e-b*d))^(1/2)/b^5/(b*x+a)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 334 vs. 2(138) = 276.

Time = 0.10 (sec) , antiderivative size = 678, normalized size of antiderivative = 4.19

$$\int \frac{(d+ex)^{9/2}}{a^2+2abx+b^2x^2} dx = \left[\frac{315(ab^3d^3e - 3a^2b^2d^2e^2 + 3a^3bde^3 - a^4e^4 + (b^4d^3e - 3ab^3d^2e^2 + 3a^2b^2de^3 - a^3be^4)x) \sqrt{-\frac{bd-ae}{b}} \arctan\left(\frac{b\sqrt{ex+d}}{\sqrt{b(ae-bd)}}\right) - 9\sqrt{ex+d} \sqrt{b(ae-bd)} \left(-\frac{2}{63}e^4x^4 - \frac{58}{315}de^3x^3 - \frac{52}{105}d^2e^2x^2 - \frac{388}{315}d^3ex + \frac{1}{9}d^4 \right)}{b^5(bx+a)\sqrt{b(ae-bd)}} \right]$$

input `integrate((e*x+d)^(9/2)/(b^2*x^2+2*a*b*x+a^2),x, algorithm="fricas")`

output `[-1/70*(315*(a*b^3*d^3*e - 3*a^2*b^2*d^2*e^2 + 3*a^3*b*d*e^3 - a^4*e^4 + (b^4*d^3*e - 3*a*b^3*d^2*e^2 + 3*a^2*b^2*d*e^3 - a^3*b*e^4)*x)*sqrt((b*d - a*e)/b)*log((b*e*x + 2*b*d - a*e + 2*sqrt(e*x + d)*b*sqrt((b*d - a*e)/b))/(b*x + a)) - 2*(10*b^4*e^4*x^4 - 35*b^4*d^4 + 528*a*b^3*d^3*e - 1218*a^2*b^2*d^2*e^2 + 1050*a^3*b*d*e^3 - 315*a^4*e^4 + 2*(29*b^4*d*e^3 - 9*a*b^3*e^4)*x^3 + 6*(26*b^4*d^2*e^2 - 23*a*b^3*d*e^3 + 7*a^2*b^2*e^4)*x^2 + 2*(194*b^4*d^3*e - 426*a*b^3*d^2*e^2 + 357*a^2*b^2*d*e^3 - 105*a^3*b*e^4)*x)*sqrt(e*x + d))/(b^6*x + a*b^5), -1/35*(315*(a*b^3*d^3*e - 3*a^2*b^2*d^2*e^2 + 3*a^3*b*d*e^3 - a^4*e^4 + (b^4*d^3*e - 3*a*b^3*d^2*e^2 + 3*a^2*b^2*d*e^3 - a^3*b*e^4)*x)*sqrt(-(b*d - a*e)/b)*arctan(-sqrt(e*x + d)*b*sqrt(-(b*d - a*e)/b))/(b*d - a*e) - (10*b^4*e^4*x^4 - 35*b^4*d^4 + 528*a*b^3*d^3*e - 1218*a^2*b^2*d^2*e^2 + 1050*a^3*b*d*e^3 - 315*a^4*e^4 + 2*(29*b^4*d*e^3 - 9*a*b^3*e^4)*x^3 + 6*(26*b^4*d^2*e^2 - 23*a*b^3*d*e^3 + 7*a^2*b^2*e^4)*x^2 + 2*(194*b^4*d^3*e - 426*a*b^3*d^2*e^2 + 357*a^2*b^2*d*e^3 - 105*a^3*b*e^4)*x)*sqrt(e*x + d))/(b^6*x + a*b^5)]`

Sympy [F(-1)]

Timed out.

$$\int \frac{(d + ex)^{9/2}}{a^2 + 2abx + b^2x^2} dx = \text{Timed out}$$

input `integrate((e*x+d)**(9/2)/(b**2*x**2+2*a*b*x+a**2),x)`

output `Timed out`

Maxima [F(-2)]

Exception generated.

$$\int \frac{(d+ex)^{9/2}}{a^2+2abx+b^2x^2} dx = \text{Exception raised: ValueError}$$

input `integrate((e*x+d)^(9/2)/(b^2*x^2+2*a*b*x+a^2),x, algorithm="maxima")`

output `Exception raised: ValueError >> Computation failed since Maxima requested additional constraints; using the 'assume' command before evaluation *may* help (example of legal syntax is 'assume(a*e-b*d>0)', see `assume?` for more detail`

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 375 vs. 2(138) = 276.

Time = 0.15 (sec) , antiderivative size = 375, normalized size of antiderivative = 2.31

$$\int \frac{(d+ex)^{9/2}}{a^2+2abx+b^2x^2} dx = \frac{9(b^4d^4e - 4ab^3d^3e^2 + 6a^2b^2d^2e^3 - 4a^3bde^4 + a^4e^5) \arctan\left(\frac{\sqrt{ex+db}}{\sqrt{-b^2d+abe}}\right)}{\sqrt{-b^2d+abe}b^5} - \frac{\sqrt{ex+db}b^4d^4e - 4\sqrt{ex+db}ab^3d^3e^2 + 6\sqrt{ex+db}a^2b^2d^2e^3 - 4\sqrt{ex+db}a^3bde^4 + \sqrt{ex+db}a^4e^5}{((ex+d)b - bd + ae)b^5} + \frac{2\left(5(ex+d)^{7/2}b^{12}e + 14(ex+d)^{5/2}b^{12}de + 35(ex+d)^{3/2}b^{12}d^2e + 140\sqrt{ex+db}b^{12}d^3e - 14(ex+d)^{5/2}ab^{11}e^2\right)}{b^{12}}$$

input `integrate((e*x+d)^(9/2)/(b^2*x^2+2*a*b*x+a^2),x, algorithm="giac")`

output

```

9*(b^4*d^4*e - 4*a*b^3*d^3*e^2 + 6*a^2*b^2*d^2*e^3 - 4*a^3*b*d*e^4 + a^4*e
^5)*arctan(sqrt(e*x + d)*b/sqrt(-b^2*d + a*b*e))/(sqrt(-b^2*d + a*b*e)*b^5
) - (sqrt(e*x + d)*b^4*d^4*e - 4*sqrt(e*x + d)*a*b^3*d^3*e^2 + 6*sqrt(e*x
+ d)*a^2*b^2*d^2*e^3 - 4*sqrt(e*x + d)*a^3*b*d*e^4 + sqrt(e*x + d)*a^4*e^5
)/(((e*x + d)*b - b*d + a*e)*b^5) + 2/35*(5*(e*x + d)^(7/2)*b^12*e + 14*(e
*x + d)^(5/2)*b^12*d*e + 35*(e*x + d)^(3/2)*b^12*d^2*e + 140*sqrt(e*x + d)
*b^12*d^3*e - 14*(e*x + d)^(5/2)*a*b^11*e^2 - 70*(e*x + d)^(3/2)*a*b^11*d*
e^2 - 420*sqrt(e*x + d)*a*b^11*d^2*e^2 + 35*(e*x + d)^(3/2)*a^2*b^10*e^3 +
420*sqrt(e*x + d)*a^2*b^10*d*e^3 - 140*sqrt(e*x + d)*a^3*b^9*e^4)/b^14

```

Mupad [B] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 352, normalized size of antiderivative = 2.17

$$\begin{aligned}
\int \frac{(d+ex)^{9/2}}{a^2+2abx+b^2x^2} dx = & \left(\frac{\left(\frac{2e(2b^2d-2abe)^2}{b^6} - \frac{2e(ae-bd)^2}{b^4} \right) (2b^2d-2abe)}{b^2} \right. \\
& \left. - \frac{2e(2b^2d-2abe)(ae-bd)^2}{b^6} \right) \sqrt{d+ex} \\
& + \left(\frac{2e(2b^2d-2abe)^2}{3b^6} - \frac{2e(ae-bd)^2}{3b^4} \right) (d+ex)^{3/2} \\
& - \frac{\sqrt{d+ex}(a^4e^5-4a^3bde^4+6a^2b^2d^2e^3-4ab^3d^3e^2+b^4d^4e)}{b^6(d+ex)-b^6d+ab^5e} \\
& + \frac{2e(d+ex)^{7/2}}{7b^2} + \frac{2e(2b^2d-2abe)(d+ex)^{5/2}}{5b^4} \\
& + \frac{9e \operatorname{atan}\left(\frac{\sqrt{b}e(ae-bd)^{7/2}\sqrt{d+ex}}{a^4e^5-4a^3bde^4+6a^2b^2d^2e^3-4ab^3d^3e^2+b^4d^4e}\right)(ae-bd)^{7/2}}{b^{11/2}}
\end{aligned}$$

input

```
int((d + e*x)^(9/2)/(a^2 + b^2*x^2 + 2*a*b*x), x)
```

output

```
((((2*e*(2*b^2*d - 2*a*b*e)^2)/b^6 - (2*e*(a*e - b*d)^2)/b^4)*(2*b^2*d - 2*a*b*e))/b^2 - (2*e*(2*b^2*d - 2*a*b*e)*(a*e - b*d)^2)/b^6*(d + e*x)^(1/2) + ((2*e*(2*b^2*d - 2*a*b*e)^2)/(3*b^6) - (2*e*(a*e - b*d)^2)/(3*b^4))*(d + e*x)^(3/2) - ((d + e*x)^(1/2)*(a^4*e^5 + b^4*d^4*e - 4*a*b^3*d^3*e^2 + 6*a^2*b^2*d^2*e^3 - 4*a^3*b*d*e^4))/(b^6*(d + e*x) - b^6*d + a*b^5*e) + (2*e*(d + e*x)^(7/2))/(7*b^2) + (2*e*(2*b^2*d - 2*a*b*e)*(d + e*x)^(5/2))/(5*b^4) + (9*e*atan((b^(1/2)*e*(a*e - b*d)^(7/2)*(d + e*x)^(1/2))/(a^4*e^5 + b^4*d^4*e - 4*a*b^3*d^3*e^2 + 6*a^2*b^2*d^2*e^3 - 4*a^3*b*d*e^4))*(a*e - b*d)^(7/2))/b^(11/2)
```

Reduce [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 648, normalized size of antiderivative = 4.00

$$\int \frac{(d + ex)^{9/2}}{a^2 + 2abx + b^2x^2} dx = \frac{315\sqrt{b}\sqrt{ae - bd} \operatorname{atan}\left(\frac{\sqrt{ex+db}}{\sqrt{b}\sqrt{ae-bd}}\right) a^4 e^4 - 945\sqrt{b}\sqrt{ae - bd} \operatorname{atan}\left(\frac{\sqrt{ex+db}}{\sqrt{b}\sqrt{ae-bd}}\right) a^3 bd}{a^2 + 2abx + b^2x^2}$$

input

```
int((e*x+d)^(9/2)/(b^2*x^2+2*a*b*x+a^2),x)
```

output

```
(315*sqrt(b)*sqrt(a*e - b*d)*atan((sqrt(d + e*x)*b)/(sqrt(b)*sqrt(a*e - b*d)))*a**4*e**4 - 945*sqrt(b)*sqrt(a*e - b*d)*atan((sqrt(d + e*x)*b)/(sqrt(b)*sqrt(a*e - b*d)))*a**3*b*d*e**3 + 315*sqrt(b)*sqrt(a*e - b*d)*atan((sqrt(d + e*x)*b)/(sqrt(b)*sqrt(a*e - b*d)))*a**3*b*e**4*x + 945*sqrt(b)*sqrt(a*e - b*d)*atan((sqrt(d + e*x)*b)/(sqrt(b)*sqrt(a*e - b*d)))*a**2*b**2*d**2*e**2 - 945*sqrt(b)*sqrt(a*e - b*d)*atan((sqrt(d + e*x)*b)/(sqrt(b)*sqrt(a*e - b*d)))*a**2*b**2*d*e**3*x - 315*sqrt(b)*sqrt(a*e - b*d)*atan((sqrt(d + e*x)*b)/(sqrt(b)*sqrt(a*e - b*d)))*a*b**3*d**3*e + 945*sqrt(b)*sqrt(a*e - b*d)*atan((sqrt(d + e*x)*b)/(sqrt(b)*sqrt(a*e - b*d)))*a*b**3*d**2*e**2*x - 315*sqrt(b)*sqrt(a*e - b*d)*atan((sqrt(d + e*x)*b)/(sqrt(b)*sqrt(a*e - b*d)))*b**4*d**3*e*x - 315*sqrt(d + e*x)*a**4*b*e**4 + 1050*sqrt(d + e*x)*a**3*b**2*d*e**3 - 210*sqrt(d + e*x)*a**3*b**2*e**4*x - 1218*sqrt(d + e*x)*a**2*b**3*d**2*e**2 + 714*sqrt(d + e*x)*a**2*b**3*d*e**3*x + 42*sqrt(d + e*x)*a**2*b**3*e**4*x**2 + 528*sqrt(d + e*x)*a*b**4*d**3*e - 852*sqrt(d + e*x)*a*b**4*d**2*e**2*x - 138*sqrt(d + e*x)*a*b**4*d*e**3*x**2 - 18*sqrt(d + e*x)*a*b**4*e**4*x**3 - 35*sqrt(d + e*x)*b**5*d**4 + 388*sqrt(d + e*x)*b**5*d**3*e*x + 156*sqrt(d + e*x)*b**5*d**2*e**2*x**2 + 58*sqrt(d + e*x)*b**5*d*e**3*x**3 + 10*sqrt(d + e*x)*b**5*e**4*x**4)/(35*b**6*(a + b*x))
```

3.171 $\int \frac{(d+ex)^{7/2}}{a^2+2abx+b^2x^2} dx$

| | |
|---|------|
| Optimal result | 1345 |
| Mathematica [A] (verified) | 1345 |
| Rubi [A] (verified) | 1346 |
| Maple [A] (verified) | 1349 |
| Fricas [B] (verification not implemented) | 1350 |
| Sympy [F(-1)] | 1350 |
| Maxima [F(-2)] | 1351 |
| Giac [B] (verification not implemented) | 1351 |
| Mupad [B] (verification not implemented) | 1352 |
| Reduce [B] (verification not implemented) | 1353 |

Optimal result

Integrand size = 28, antiderivative size = 137

$$\int \frac{(d+ex)^{7/2}}{a^2+2abx+b^2x^2} dx = \frac{7e(bd-ae)^2\sqrt{d+ex}}{b^4} + \frac{7e(bd-ae)(d+ex)^{3/2}}{3b^3} + \frac{7e(d+ex)^{5/2}}{5b^2} - \frac{(d+ex)^{7/2}}{b(a+bx)} - \frac{7e(bd-ae)^{5/2}\operatorname{arctanh}\left(\frac{\sqrt{b}\sqrt{d+ex}}{\sqrt{bd-ae}}\right)}{b^{9/2}}$$

output

```
7*e*(-a*e+b*d)^2*(e*x+d)^(1/2)/b^4+7/3*e*(-a*e+b*d)*(e*x+d)^(3/2)/b^3+7/5*
e*(e*x+d)^(5/2)/b^2-(e*x+d)^(7/2)/b/(b*x+a)-7*e*(-a*e+b*d)^(5/2)*arctanh(b
^(1/2)*(e*x+d)^(1/2)/(-a*e+b*d)^(1/2))/b^(9/2)
```

Mathematica [A] (verified)

Time = 0.36 (sec) , antiderivative size = 158, normalized size of antiderivative = 1.15

$$\int \frac{(d+ex)^{7/2}}{a^2+2abx+b^2x^2} dx = \frac{\sqrt{d+ex}(-105a^3e^3+35a^2be^2(7d-2ex)+7ab^2e(-23d^2+24dex+2e^2x^2)+b^3(15d^3-116d^2ex-32de^2x^2))}{15b^4(a+bx)} - \frac{7e(-bd+ae)^{5/2}\arctan\left(\frac{\sqrt{b}\sqrt{d+ex}}{\sqrt{-bd+ae}}\right)}{b^{9/2}}$$

input `Integrate[(d + e*x)^(7/2)/(a^2 + 2*a*b*x + b^2*x^2),x]`

output `-1/15*(Sqrt[d + e*x]*(-105*a^3*e^3 + 35*a^2*b*e^2*(7*d - 2*e*x) + 7*a*b^2*e*(-23*d^2 + 24*d*e*x + 2*e^2*x^2) + b^3*(15*d^3 - 116*d^2*e*x - 32*d*e^2*x^2 - 6*e^3*x^3)))/(b^4*(a + b*x)) - (7*e*(-(b*d) + a*e)^(5/2)*ArcTan[(Sqrt[b]*Sqrt[d + e*x])/Sqrt[-(b*d) + a*e]])/b^(9/2)`

Rubi [A] (verified)

Time = 0.47 (sec) , antiderivative size = 150, normalized size of antiderivative = 1.09, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {1098, 27, 51, 60, 60, 60, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\begin{aligned}
 & \int \frac{(d + ex)^{7/2}}{a^2 + 2abx + b^2x^2} dx \\
 & \quad \downarrow 1098 \\
 & b^2 \int \frac{(d + ex)^{7/2}}{b^2(a + bx)^2} dx \\
 & \quad \downarrow 27 \\
 & \int \frac{(d + ex)^{7/2}}{(a + bx)^2} dx \\
 & \quad \downarrow 51 \\
 & \frac{7e \int \frac{(d+ex)^{5/2}}{a+bx} dx}{2b} - \frac{(d + ex)^{7/2}}{b(a + bx)} \\
 & \quad \downarrow 60 \\
 & \frac{7e \left(\frac{(bd-ae) \int \frac{(d+ex)^{3/2}}{a+bx} dx}{b} + \frac{2(d+ex)^{5/2}}{5b} \right)}{2b} - \frac{(d + ex)^{7/2}}{b(a + bx)} \\
 & \quad \downarrow 60
 \end{aligned}$$

$$7e \left(\frac{(bd-ae) \left(\frac{(bd-ae) \int \frac{\sqrt{d+ex} dx}{a+bx} + 2(d+ex)^{3/2}}{b} \right) + \frac{2(d+ex)^{5/2}}{5b}}{2b} \right) - \frac{(d+ex)^{7/2}}{b(a+bx)}$$

60

$$7e \left(\frac{(bd-ae) \left(\frac{(bd-ae) \int \frac{1}{(a+bx)\sqrt{d+ex}} dx + 2\sqrt{d+ex}}{b} \right) + \frac{2(d+ex)^{3/2}}{3b}}{b} \right) + \frac{2(d+ex)^{5/2}}{5b} \Bigg) - \frac{(d+ex)^{7/2}}{b(a+bx)}$$

73

$$7e \left(\frac{(bd-ae) \left(\frac{2(bd-ae) \int \frac{1}{a + \frac{b(d+ex)}{e} - \frac{bd}{e}} d\sqrt{d+ex}}{b} + \frac{2\sqrt{d+ex}}{b} \right) + \frac{2(d+ex)^{3/2}}{3b}}{b} \right) + \frac{2(d+ex)^{5/2}}{5b}$$

$$\frac{2b}{b(a+bx)} \frac{(d+ex)^{7/2}}{b(a+bx)}$$

221

$$7e \left(\frac{(bd-ae) \left(\frac{2\sqrt{d+ex}}{b} - \frac{2\sqrt{bd-ae} \operatorname{arctanh} \left(\frac{\sqrt{b}\sqrt{d+ex}}{\sqrt{bd-ae}} \right)}{b^{3/2}} \right) + \frac{2(d+ex)^{3/2}}{3b}}{b} \right) + \frac{2(d+ex)^{5/2}}{5b} \Bigg) - \frac{(d+ex)^{7/2}}{b(a+bx)}$$

input $\text{Int}[(d + e*x)^{(7/2)}/(a^2 + 2*a*b*x + b^2*x^2), x]$

output $-\frac{(d + e*x)^{(7/2)}}{b*(a + b*x)} + \frac{7*e*((2*(d + e*x)^{(5/2)})/(5*b) + ((b*d - a*e)*((2*(d + e*x)^{(3/2)})/(3*b) + ((b*d - a*e)*((2*\sqrt{d + e*x})/b - (2*\sqrt{b*d - a*e})*\text{ArcTanh}[(\sqrt{b}*\sqrt{d + e*x})/\sqrt{b*d - a*e}])/b^{(3/2)})))/b)}{2*b}$

Defintions of rubi rules used

rule 27 $\text{Int}[(a_)*(F_x_), x_Symbol] \rightarrow \text{Simp}[a \text{ Int}[F_x, x], x] /; \text{FreeQ}[a, x] \ \&\& \ !\text{MatchQ}[F_x, (b_)*(G_x_)] /; \text{FreeQ}[b, x]$

rule 51 $\text{Int}[(a_.) + (b_.)*(x_)^{(m_)}*((c_.) + (d_.)*(x_)^{(n_)}, x_Symbol] \rightarrow \text{Simp}[(a + b*x)^{(m + 1)}*((c + d*x)^n/(b*(m + 1))), x] - \text{Simp}[d*(n/(b*(m + 1))) \text{Int}[(a + b*x)^{(m + 1)}*(c + d*x)^{(n - 1)}, x], x] /; \text{FreeQ}\{a, b, c, d, n\}, x] \ \&\& \ \text{ILtQ}[m, -1] \ \&\& \ \text{FractionQ}[n] \ \&\& \ \text{GtQ}[n, 0]$

rule 60 $\text{Int}[(a_.) + (b_.)*(x_)^{(m_)}*((c_.) + (d_.)*(x_)^{(n_)}, x_Symbol] \rightarrow \text{Simp}[(a + b*x)^{(m + 1)}*((c + d*x)^n/(b*(m + n + 1))), x] + \text{Simp}[n*((b*c - a*d)/(b*(m + n + 1))) \text{Int}[(a + b*x)^m*(c + d*x)^{(n - 1)}, x], x] /; \text{FreeQ}\{a, b, c, d\}, x] \ \&\& \ \text{GtQ}[n, 0] \ \&\& \ \text{NeQ}[m + n + 1, 0] \ \&\& \ !(\text{IGtQ}[m, 0] \ \&\& \ (\ !\text{IntegerQ}[n] \ || \ (\text{GtQ}[m, 0] \ \&\& \ \text{LtQ}[m - n, 0])) \ \&\& \ !\text{ILtQ}[m + n + 2, 0] \ \&\& \ \text{IntLinearQ}[a, b, c, d, m, n, x]$

rule 73 $\text{Int}[(a_.) + (b_.)*(x_)^{(m_)}*((c_.) + (d_.)*(x_)^{(n_)}, x_Symbol] \rightarrow \text{With}[\{p = \text{Denominator}[m]\}, \text{Simp}[p/b \ \text{Subst}[\text{Int}[x^{(p*(m + 1) - 1)}*(c - a*(d/b) + d*(x^p/b)^n], x], x, (a + b*x)^{(1/p)}], x]] /; \text{FreeQ}\{a, b, c, d\}, x] \ \&\& \ \text{LtQ}[-1, m, 0] \ \&\& \ \text{LeQ}[-1, n, 0] \ \&\& \ \text{LeQ}[\text{Denominator}[n], \text{Denominator}[m]] \ \&\& \ \text{IntLinearQ}[a, b, c, d, m, n, x]$

rule 221 $\text{Int}[(a_.) + (b_.)*(x_)^2)^{-1}, x_Symbol] \rightarrow \text{Simp}[(\text{Rt}[-a/b, 2]/a)*\text{ArcTanh}[x/\text{Rt}[-a/b, 2]], x] /; \text{FreeQ}\{a, b\}, x] \ \&\& \ \text{NegQ}[a/b]$

rule 1098

```
Int[((d_.) + (e_.)*(x_.))^(m_.)*((a_.) + (b_.)*(x_.) + (c_.)*(x_.)^2)^(p_.), x_
Symbol] := Simp[1/c^p Int[(d + e*x)^m*(b/2 + c*x)^(2*p), x], x] /; FreeQ[
{a, b, c, d, e, m}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Maple [A] (verified)

Time = 1.60 (sec) , antiderivative size = 167, normalized size of antiderivative = 1.22

| method | result |
|-------------------|---|
| pseudoelliptic | $\frac{7 \left(e(ae-bd)^3 (bx+a) \arctan\left(\frac{b\sqrt{ex+d}}{\sqrt{b(ae-bd)}}\right) - \sqrt{ex+d} \sqrt{b(ae-bd)} \left(\left(\frac{2}{35}e^3x^3 + \frac{32}{105}de^2x^2 + \frac{116}{105}d^2ex - \frac{1}{7}d^3\right)b^3 + \frac{23ea(-}{\sqrt{b(ae-bd)}b^4(bx+a)} \right.$ |
| risch | $\frac{2e(3b^2e^2x^2 - 10xab e^2 + 16b^2dex + 45e^2a^2 - 100abde + 58b^2d^2)\sqrt{ex+d}}{15b^4} - \frac{(2e^3a^3 - 6a^2bd e^2 + 6ab^2d^2e - 2b^3d^3)e \left(-\frac{2}{2(b} \right.}{b^4}$ |
| derivativedivides | $2e \left(\frac{\frac{(ex+d)^{\frac{5}{2}}b^2}{5} - \frac{2abe(ex+d)^{\frac{3}{2}}}{3} + \frac{2b^2d(ex+d)^{\frac{3}{2}}}{3} + 3a^2e^2\sqrt{ex+d} - 6abde\sqrt{ex+d} + 3b^2d^2\sqrt{ex+d}}{b^4} - \frac{\left(-\frac{1}{2}e^3a^3 + \frac{3}{2}a^2bde^2 -}{b(ex+} \right.}{b^4}$ |
| default | $2e \left(\frac{\frac{(ex+d)^{\frac{5}{2}}b^2}{5} - \frac{2abe(ex+d)^{\frac{3}{2}}}{3} + \frac{2b^2d(ex+d)^{\frac{3}{2}}}{3} + 3a^2e^2\sqrt{ex+d} - 6abde\sqrt{ex+d} + 3b^2d^2\sqrt{ex+d}}{b^4} - \frac{\left(-\frac{1}{2}e^3a^3 + \frac{3}{2}a^2bde^2 -}{b(ex+} \right.}{b^4}$ |

input

```
int((e*x+d)^(7/2)/(b^2*x^2+2*a*b*x+a^2),x,method=_RETURNVERBOSE)
```

output

```
-7/(b*(a*e-b*d))^(1/2)*(e*(a*e-b*d)^3*(b*x+a)*arctan(b*(e*x+d)^(1/2)/(b*(a
*e-b*d))^(1/2))-(e*x+d)^(1/2)*(b*(a*e-b*d))^(1/2)*((2/35*e^3*x^3+32/105*d*
e^2*x^2+116/105*d^2*e*x-1/7*d^3)*b^3+23/15*e*a*(-2/23*e^2*x^2-24/23*d*e*x+
d^2)*b^2-7/3*e^2*a^2*(-2/7*e*x+d)*b+e^3*a^3))/b^4/(b*x+a)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 238 vs. $2(115) = 230$.

Time = 0.11 (sec) , antiderivative size = 486, normalized size of antiderivative = 3.55

$$\int \frac{(d+ex)^{7/2}}{a^2+2abx+b^2x^2} dx = \left[\frac{105(ab^2d^2e - 2a^2bde^2 + a^3e^3 + (b^3d^2e - 2ab^2de^2 + a^2be^3)x) \sqrt{\frac{bd-ae}{b}} \log\left(\frac{bex+}{\dots}\right)}{105(ab^2d^2e - 2a^2bde^2 + a^3e^3 + (b^3d^2e - 2ab^2de^2 + a^2be^3)x) \sqrt{-\frac{bd-ae}{b}} \arctan\left(-\frac{\sqrt{ex+db}\sqrt{-\frac{bd-ae}{b}}}{bd-ae}\right) - (6b}{\dots} \right]$$

input `integrate((e*x+d)^(7/2)/(b^2*x^2+2*a*b*x+a^2),x, algorithm="fricas")`

output `[1/30*(105*(a*b^2*d^2*e - 2*a^2*b*d*e^2 + a^3*e^3 + (b^3*d^2*e - 2*a*b^2*d*e^2 + a^2*b*e^3)*x)*sqrt((b*d - a*e)/b)*log((b*e*x + 2*b*d - a*e - 2*sqrt(e*x + d)*b*sqrt((b*d - a*e)/b))/(b*x + a)) + 2*(6*b^3*e^3*x^3 - 15*b^3*d^3 + 161*a*b^2*d^2*e - 245*a^2*b*d*e^2 + 105*a^3*e^3 + 2*(16*b^3*d*e^2 - 7*a*b^2*e^3)*x^2 + 2*(58*b^3*d^2*e - 84*a*b^2*d*e^2 + 35*a^2*b*e^3)*x)*sqrt(e*x + d)/(b^5*x + a*b^4), -1/15*(105*(a*b^2*d^2*e - 2*a^2*b*d*e^2 + a^3*e^3 + (b^3*d^2*e - 2*a*b^2*d*e^2 + a^2*b*e^3)*x)*sqrt(-(b*d - a*e)/b)*arctan(-sqrt(e*x + d)*b*sqrt(-(b*d - a*e)/b)/(b*d - a*e)) - (6*b^3*e^3*x^3 - 15*b^3*d^3 + 161*a*b^2*d^2*e - 245*a^2*b*d*e^2 + 105*a^3*e^3 + 2*(16*b^3*d*e^2 - 7*a*b^2*e^3)*x^2 + 2*(58*b^3*d^2*e - 84*a*b^2*d*e^2 + 35*a^2*b*e^3)*x)*sqrt(e*x + d))/(b^5*x + a*b^4)]`

Sympy [F(-1)]

Timed out.

$$\int \frac{(d+ex)^{7/2}}{a^2+2abx+b^2x^2} dx = \text{Timed out}$$

input `integrate((e*x+d)**(7/2)/(b**2*x**2+2*a*b*x+a**2),x)`

output Timed out

Maxima [F(-2)]

Exception generated.

$$\int \frac{(d+ex)^{7/2}}{a^2+2abx+b^2x^2} dx = \text{Exception raised: ValueError}$$

input `integrate((e*x+d)^(7/2)/(b^2*x^2+2*a*b*x+a^2),x, algorithm="maxima")`

output Exception raised: ValueError >> Computation failed since Maxima requested additional constraints; using the 'assume' command before evaluation *may* help (example of legal syntax is 'assume(a*e-b*d>0)', see `assume?` for more detail

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 270 vs. 2(115) = 230.

Time = 0.18 (sec) , antiderivative size = 270, normalized size of antiderivative = 1.97

$$\int \frac{(d+ex)^{7/2}}{a^2+2abx+b^2x^2} dx = \frac{7(b^3d^3e - 3ab^2d^2e^2 + 3a^2bde^3 - a^3e^4) \arctan\left(\frac{\sqrt{ex+db}}{\sqrt{-b^2d+abeb^4}}\right)}{\sqrt{ex+db^3d^3e - 3\sqrt{ex+db}ab^2d^2e^2 + 3\sqrt{ex+db}a^2bde^3 - \sqrt{ex+db}a^3e^4}} - \frac{((ex+d)b - bd + ae)b^4}{2\left(3(ex+d)^{\frac{5}{2}}b^8e + 10(ex+d)^{\frac{3}{2}}b^8de + 45\sqrt{ex+db}b^8d^2e - 10(ex+d)^{\frac{3}{2}}ab^7e^2 - 90\sqrt{ex+db}ab^7de^2 + 45\sqrt{ex+db}ab^7e^3\right)} + \frac{15b^{10}}{15b^{10}}$$

input `integrate((e*x+d)^(7/2)/(b^2*x^2+2*a*b*x+a^2),x, algorithm="giac")`

output

```
7*(b^3*d^3*e - 3*a*b^2*d^2*e^2 + 3*a^2*b*d*e^3 - a^3*e^4)*arctan(sqrt(e*x
+ d)*b/sqrt(-b^2*d + a*b*e))/(sqrt(-b^2*d + a*b*e)*b^4) - (sqrt(e*x + d)*b
^3*d^3*e - 3*sqrt(e*x + d)*a*b^2*d^2*e^2 + 3*sqrt(e*x + d)*a^2*b*d*e^3 - s
qrt(e*x + d)*a^3*e^4)/(((e*x + d)*b - b*d + a*e)*b^4) + 2/15*(3*(e*x + d)^
(5/2)*b^8*e + 10*(e*x + d)^(3/2)*b^8*d*e + 45*sqrt(e*x + d)*b^8*d^2*e - 10
*(e*x + d)^(3/2)*a*b^7*e^2 - 90*sqrt(e*x + d)*a*b^7*d*e^2 + 45*sqrt(e*x +
d)*a^2*b^6*e^3)/b^10
```

Mupad [B] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 235, normalized size of antiderivative = 1.72

$$\int \frac{(d+ex)^{7/2}}{a^2+2abx+b^2x^2} dx = \left(\frac{2e(2b^2d-2abe)^2}{b^6} - \frac{2e(ae-bd)^2}{b^4} \right) \sqrt{d+ex} + \frac{\sqrt{d+ex}(a^3e^4-3a^2bde^3+3ab^2d^2e^2-b^3d^3e)}{b^5(d+ex)-b^5d+ab^4e} + \frac{2e(d+ex)^{5/2}}{5b^2} + \frac{2e(2b^2d-2abe)(d+ex)^{3/2}}{3b^4} - \frac{7e \operatorname{atan}\left(\frac{\sqrt{b}e(ae-bd)^{5/2}\sqrt{d+ex}}{a^3e^4-3a^2bde^3+3ab^2d^2e^2-b^3d^3e}\right)(ae-bd)^{5/2}}{b^{9/2}}$$

input

```
int((d + e*x)^(7/2)/(a^2 + b^2*x^2 + 2*a*b*x),x)
```

output

```
((2*e*(2*b^2*d - 2*a*b*e)^2)/b^6 - (2*e*(a*e - b*d)^2)/b^4)*(d + e*x)^(1/2)
) + ((d + e*x)^(1/2)*(a^3*e^4 - b^3*d^3*e + 3*a*b^2*d^2*e^2 - 3*a^2*b*d*e^
3))/(b^5*(d + e*x) - b^5*d + a*b^4*e) + (2*e*(d + e*x)^(5/2))/(5*b^2) + (2
*e*(2*b^2*d - 2*a*b*e)*(d + e*x)^(3/2))/(3*b^4) - (7*e*atan((b^(1/2)*e*(a*
e - b*d)^(5/2)*(d + e*x)^(1/2))/(a^3*e^4 - b^3*d^3*e + 3*a*b^2*d^2*e^2 - 3
*a^2*b*d*e^3))*(a*e - b*d)^(5/2))/b^(9/2)
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 449, normalized size of antiderivative = 3.28

$$\int \frac{(d+ex)^{7/2}}{a^2+2abx+b^2x^2} dx = \frac{-105\sqrt{b}\sqrt{ae-bd}\operatorname{atan}\left(\frac{\sqrt{ex+db}}{\sqrt{b}\sqrt{ae-bd}}\right)a^3e^3 + 210\sqrt{b}\sqrt{ae-bd}\operatorname{atan}\left(\frac{\sqrt{ex+db}}{\sqrt{b}\sqrt{ae-bd}}\right)a^2b}{a^2+2abx+b^2x^2}$$

input `int((e*x+d)^(7/2)/(b^2*x^2+2*a*b*x+a^2),x)`

output

```
( - 105*sqrt(b)*sqrt(a*e - b*d)*atan((sqrt(d + e*x)*b)/(sqrt(b)*sqrt(a*e -
b*d)))*a**3*e**3 + 210*sqrt(b)*sqrt(a*e - b*d)*atan((sqrt(d + e*x)*b)/(sq
rt(b)*sqrt(a*e - b*d)))*a**2*b*d*e**2 - 105*sqrt(b)*sqrt(a*e - b*d)*atan((
sqrt(d + e*x)*b)/(sqrt(b)*sqrt(a*e - b*d)))*a**2*b*e**3*x - 105*sqrt(b)*sq
rt(a*e - b*d)*atan((sqrt(d + e*x)*b)/(sqrt(b)*sqrt(a*e - b*d)))*a*b**2*d**
2*e + 210*sqrt(b)*sqrt(a*e - b*d)*atan((sqrt(d + e*x)*b)/(sqrt(b)*sqrt(a*e
- b*d)))*a*b**2*d*e**2*x - 105*sqrt(b)*sqrt(a*e - b*d)*atan((sqrt(d + e*x
)*b)/(sqrt(b)*sqrt(a*e - b*d)))*b**3*d**2*e*x + 105*sqrt(d + e*x)*a**3*b*e
**3 - 245*sqrt(d + e*x)*a**2*b**2*d*e**2 + 70*sqrt(d + e*x)*a**2*b**2*e**3
*x + 161*sqrt(d + e*x)*a*b**3*d**2*e - 168*sqrt(d + e*x)*a*b**3*d*e**2*x -
14*sqrt(d + e*x)*a*b**3*e**3*x**2 - 15*sqrt(d + e*x)*b**4*d**3 + 116*sqrt
(d + e*x)*b**4*d**2*e*x + 32*sqrt(d + e*x)*b**4*d*e**2*x**2 + 6*sqrt(d + e
*x)*b**4*e**3*x**3)/(15*b**5*(a + b*x))
```

3.172 $\int \frac{(d+ex)^{5/2}}{a^2+2abx+b^2x^2} dx$

| | |
|---|------|
| Optimal result | 1354 |
| Mathematica [A] (verified) | 1354 |
| Rubi [A] (verified) | 1355 |
| Maple [A] (verified) | 1357 |
| Fricas [A] (verification not implemented) | 1358 |
| Sympy [F] | 1359 |
| Maxima [F(-2)] | 1359 |
| Giac [A] (verification not implemented) | 1360 |
| Mupad [B] (verification not implemented) | 1360 |
| Reduce [B] (verification not implemented) | 1361 |

Optimal result

Integrand size = 28, antiderivative size = 110

$$\int \frac{(d+ex)^{5/2}}{a^2+2abx+b^2x^2} dx = \frac{5e(bd-ae)\sqrt{d+ex}}{b^3} + \frac{5e(d+ex)^{3/2}}{3b^2} - \frac{(d+ex)^{5/2}}{b(a+bx)} - \frac{5e(bd-ae)^{3/2} \operatorname{arctanh}\left(\frac{\sqrt{b}\sqrt{d+ex}}{\sqrt{bd-ae}}\right)}{b^{7/2}}$$

output

```
5*e*(-a*e+b*d)*(e*x+d)^(1/2)/b^3+5/3*e*(e*x+d)^(3/2)/b^2-(e*x+d)^(5/2)/b/(
b*x+a)-5*e*(-a*e+b*d)^(3/2)*arctanh(b^(1/2)*(e*x+d)^(1/2)/(-a*e+b*d)^(1/2)
)/b^(7/2)
```

Mathematica [A] (verified)

Time = 0.31 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.05

$$\int \frac{(d+ex)^{5/2}}{a^2+2abx+b^2x^2} dx = \frac{\sqrt{d+ex}(15a^2e^2+10abe(-2d+ex)+b^2(3d^2-14dex-2e^2x^2))}{3b^3(a+bx)} + \frac{5e(-bd+ae)^{3/2} \arctan\left(\frac{\sqrt{b}\sqrt{d+ex}}{\sqrt{-bd+ae}}\right)}{b^{7/2}}$$

input `Integrate[(d + e*x)^(5/2)/(a^2 + 2*a*b*x + b^2*x^2),x]`

output
$$-1/3*(\text{Sqrt}[d + e*x]*(15*a^2*e^2 + 10*a*b*e*(-2*d + e*x) + b^2*(3*d^2 - 14*d*e*x - 2*e^2*x^2)))/(b^3*(a + b*x)) + (5*e*(-(b*d) + a*e)^(3/2)*\text{ArcTan}[\text{Sqrt}[b]*\text{Sqrt}[d + e*x)]/\text{Sqrt}[-(b*d) + a*e])/b^(7/2)$$

Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.10, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {1098, 27, 51, 60, 60, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\begin{aligned}
 & \int \frac{(d + ex)^{5/2}}{a^2 + 2abx + b^2x^2} dx \\
 & \quad \downarrow 1098 \\
 & b^2 \int \frac{(d + ex)^{5/2}}{b^2(a + bx)^2} dx \\
 & \quad \downarrow 27 \\
 & \int \frac{(d + ex)^{5/2}}{(a + bx)^2} dx \\
 & \quad \downarrow 51 \\
 & \frac{5e \int \frac{(d+ex)^{3/2}}{a+bx} dx}{2b} - \frac{(d + ex)^{5/2}}{b(a + bx)} \\
 & \quad \downarrow 60 \\
 & \frac{5e \left(\frac{(bd-ae) \int \frac{\sqrt{d+ex}}{a+bx} dx}{b} + \frac{2(d+ex)^{3/2}}{3b} \right)}{2b} - \frac{(d + ex)^{5/2}}{b(a + bx)} \\
 & \quad \downarrow 60
 \end{aligned}$$

$$\begin{aligned}
& \frac{5e \left(\frac{(bd-ae) \left(\frac{(bd-ae) \int \frac{1}{(a+bx)\sqrt{d+ex}} dx}{b} + \frac{2\sqrt{d+ex}}{b} \right)}{b} + \frac{2(d+ex)^{3/2}}{3b} \right)}{2b} - \frac{(d+ex)^{5/2}}{b(a+bx)} \\
& \quad \downarrow \text{73} \\
& \frac{5e \left(\frac{(bd-ae) \left(\frac{2(bd-ae) \int \frac{1}{a + \frac{b(d+ex)}{e} - \frac{bd}{e}} d\sqrt{d+ex}}{be} + \frac{2\sqrt{d+ex}}{b} \right)}{b} + \frac{2(d+ex)^{3/2}}{3b} \right)}{2b} - \frac{(d+ex)^{5/2}}{b(a+bx)} \\
& \quad \downarrow \text{221} \\
& \frac{5e \left(\frac{(bd-ae) \left(\frac{2\sqrt{d+ex}}{b} - \frac{2\sqrt{bd-ae} \operatorname{arctanh}\left(\frac{\sqrt{b}\sqrt{d+ex}}{\sqrt{bd-ae}}\right)}{b^{3/2}} \right)}{b} + \frac{2(d+ex)^{3/2}}{3b} \right)}{2b} - \frac{(d+ex)^{5/2}}{b(a+bx)}
\end{aligned}$$

input `Int[(d + e*x)^(5/2)/(a^2 + 2*a*b*x + b^2*x^2), x]`

output `-((d + e*x)^(5/2)/(b*(a + b*x))) + (5*e*((2*(d + e*x)^(3/2))/(3*b) + ((b*d - a*e)*((2*Sqrt[d + e*x])/b - (2*Sqrt[b*d - a*e]*ArcTanh[(Sqrt[b]*Sqrt[d + e*x])/Sqrt[b*d - a*e]])/b^(3/2)))/b)/(2*b)`

Defintions of rubi rules used

rule 27 `Int[(a_)*(F_x_), x_Symbol] := Simp[a Int[F_x, x], x] /; FreeQ[a, x] && !MatchQ[F_x, (b_)*(G_x_)] /; FreeQ[b, x]`

rule 51 `Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] := Simp[(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Simp[d*(n/(b*(m + 1)))] Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d, n}, x] && ILtQ[m, -1] && FractionQ[n] && GtQ[n, 0]`

rule 60 `Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] := Simp[(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*(b*c - a*d)/(b*(m + n + 1)) Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] && !(IGtQ[m, 0] && (!IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) && !ILtQ[m + n + 2, 0] && IntLinearQ[a, b, c, d, m, n, x]`

rule 73 `Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] := With[{p = Denominator[m]}, Simp[p/b Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) + d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntLinearQ[a, b, c, d, m, n, x]`

rule 221 `Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[-a/b, 2]/a)*ArcTanh[x/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]`

rule 1098 `Int[((d_.) + (e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] := Simp[1/c^p Int[(d + e*x)^m*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]`

Maple [A] (verified)

Time = 1.60 (sec) , antiderivative size = 120, normalized size of antiderivative = 1.09

| method | result |
|-------------------|--|
| risch | $-\frac{2e(-bex+6ae-7bd)\sqrt{ex+d}}{3b^3} + \frac{(2e^2a^2-4abde+2b^2d^2)e\left(-\frac{\sqrt{ex+d}}{2(b(ex+d)+ae-bd)} + \frac{5\arctan\left(\frac{b\sqrt{ex+d}}{\sqrt{b(ae-bd)}}\right)}{2\sqrt{b(ae-bd)}}\right)}{b^3}$ |
| pseudoelliptic | $-\frac{5\left(-e(ae-bd)^2(bx+a)\arctan\left(\frac{b\sqrt{ex+d}}{\sqrt{b(ae-bd)}}\right) + \sqrt{ex+d}\left(\frac{-\frac{2}{3}e^2x^2 - \frac{14}{3}dex + d^2}{5}b^2 - \frac{4e\left(-\frac{ex}{2}+d\right)ab}{3} + e^2a^2\right)\sqrt{b(ae-bd)}}{\sqrt{b(ae-bd)}b^3(bx+a)}$ |
| derivativedivides | $2e\left(-\frac{\frac{(ex+d)^{\frac{3}{2}}b}{3} + 2ae\sqrt{ex+d} - 2db\sqrt{ex+d}}{b^3} + \frac{\left(-\frac{1}{2}e^2a^2 + abde - \frac{1}{2}b^2d^2\right)\sqrt{ex+d}}{b(ex+d)+ae-bd} + \frac{5\left(e^2a^2 - 2abde + b^2d^2\right)\arctan\left(\frac{b\sqrt{ex+d}}{\sqrt{b(ae-bd)}}\right)}{b^3 \cdot 2\sqrt{b(ae-bd)}}\right)$ |
| default | $2e\left(-\frac{\frac{(ex+d)^{\frac{3}{2}}b}{3} + 2ae\sqrt{ex+d} - 2db\sqrt{ex+d}}{b^3} + \frac{\left(-\frac{1}{2}e^2a^2 + abde - \frac{1}{2}b^2d^2\right)\sqrt{ex+d}}{b(ex+d)+ae-bd} + \frac{5\left(e^2a^2 - 2abde + b^2d^2\right)\arctan\left(\frac{b\sqrt{ex+d}}{\sqrt{b(ae-bd)}}\right)}{b^3 \cdot 2\sqrt{b(ae-bd)}}\right)$ |

```
input int((e*x+d)^(5/2)/(b^2*x^2+2*a*b*x+a^2),x,method=_RETURNVERBOSE)
```

```
output -2/3*e*(-b*e*x+6*a*e-7*b*d)*(e*x+d)^(1/2)/b^3+1/b^3*(2*a^2*e^2-4*a*b*d*e+
*b^2*d^2)*e*(-1/2*(e*x+d)^(1/2)/(b*(e*x+d)+a*e-b*d)+5/2/(b*(a*e-b*d))^(1/2)
)*arctan(b*(e*x+d)^(1/2)/(b*(a*e-b*d))^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 330, normalized size of antiderivative = 3.00

$$\int \frac{(d+ex)^{5/2}}{a^2+2abx+b^2x^2} dx = \frac{15(abde - a^2e^2 + (b^2de - abe^2)x)\sqrt{\frac{bd-ae}{b}} \log\left(\frac{bex+2bd-ae+2\sqrt{ex+db}\sqrt{\frac{bd-ae}{b}}}{bx+a}\right) - 15(abde - a^2e^2 + (b^2de - abe^2)x)\sqrt{-\frac{bd-ae}{b}} \arctan\left(-\frac{\sqrt{ex+db}\sqrt{-\frac{bd-ae}{b}}}{bd-ae}\right) - (2b^2e^2x^2 - 3b^2d^2 + 20abde - \dots)}{6(b^4x + \dots)}$$

```
input integrate((e*x+d)^(5/2)/(b^2*x^2+2*a*b*x+a^2),x, algorithm="fricas")
```

output

```
[ -1/6*(15*(a*b*d*e - a^2*e^2 + (b^2*d*e - a*b*e^2)*x)*sqrt((b*d - a*e)/b)*
log((b*e*x + 2*b*d - a*e + 2*sqrt(e*x + d)*b*sqrt((b*d - a*e)/b))/(b*x + a
)) - 2*(2*b^2*e^2*x^2 - 3*b^2*d^2 + 20*a*b*d*e - 15*a^2*e^2 + 2*(7*b^2*d*e
- 5*a*b*e^2)*x)*sqrt(e*x + d))/(b^4*x + a*b^3), -1/3*(15*(a*b*d*e - a^2*e
^2 + (b^2*d*e - a*b*e^2)*x)*sqrt(-(b*d - a*e)/b)*arctan(-sqrt(e*x + d)*b*s
qrt(-(b*d - a*e)/b)/(b*d - a*e)) - (2*b^2*e^2*x^2 - 3*b^2*d^2 + 20*a*b*d*e
- 15*a^2*e^2 + 2*(7*b^2*d*e - 5*a*b*e^2)*x)*sqrt(e*x + d))/(b^4*x + a*b^3
)]
```

Sympy [F]

$$\int \frac{(d + ex)^{5/2}}{a^2 + 2abx + b^2x^2} dx = \int \frac{(d + ex)^{5/2}}{(a + bx)^2} dx$$

input

```
integrate((e*x+d)**(5/2)/(b**2*x**2+2*a*b*x+a**2),x)
```

output

```
Integral((d + e*x)**(5/2)/(a + b*x)**2, x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(d + ex)^{5/2}}{a^2 + 2abx + b^2x^2} dx = \text{Exception raised: ValueError}$$

input

```
integrate((e*x+d)^(5/2)/(b^2*x^2+2*a*b*x+a^2),x, algorithm="maxima")
```

output

```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
help (example of legal syntax is 'assume(a*e-b*d>0)', see `assume?` for m
ore detail
```

Giac [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 181, normalized size of antiderivative = 1.65

$$\int \frac{(d+ex)^{5/2}}{a^2+2abx+b^2x^2} dx = \frac{5(b^2d^2e-2abde^2+a^2e^3) \arctan\left(\frac{\sqrt{ex+db}}{\sqrt{-b^2d+abe}}\right)}{\sqrt{-b^2d+abe}b^3} - \frac{\sqrt{ex+db}d^2e-2\sqrt{ex+db}abde^2+\sqrt{ex+db}a^2e^3}{((ex+d)b-bd+ae)b^3} + \frac{2\left((ex+d)^{3/2}b^4e+6\sqrt{ex+db}b^4de-6\sqrt{ex+db}ab^3e^2\right)}{3b^6}$$

input `integrate((e*x+d)^(5/2)/(b^2*x^2+2*a*b*x+a^2),x, algorithm="giac")`output `5*(b^2*d^2*e - 2*a*b*d*e^2 + a^2*e^3)*arctan(sqrt(e*x + d)*b/sqrt(-b^2*d + a*b*e))/(sqrt(-b^2*d + a*b*e)*b^3) - (sqrt(e*x + d)*b^2*d^2*e - 2*sqrt(e*x + d)*a*b*d*e^2 + sqrt(e*x + d)*a^2*e^3)/(((e*x + d)*b - b*d + a*e)*b^3) + 2/3*((e*x + d)^(3/2)*b^4*e + 6*sqrt(e*x + d)*b^4*d*e - 6*sqrt(e*x + d)*a*b^3*e^2)/b^6`**Mupad [B] (verification not implemented)**

Time = 0.11 (sec) , antiderivative size = 161, normalized size of antiderivative = 1.46

$$\int \frac{(d+ex)^{5/2}}{a^2+2abx+b^2x^2} dx = \frac{2e(d+ex)^{3/2}}{3b^2} - \frac{\sqrt{d+ex}(a^2e^3-2abde^2+b^2d^2e)}{b^4(d+ex)-b^4d+ab^3e} + \frac{5e \operatorname{atan}\left(\frac{\sqrt{be}(ae-bd)^{3/2}\sqrt{d+ex}}{a^2e^3-2abde^2+b^2d^2e}\right)(ae-bd)^{3/2}}{b^{7/2}} + \frac{2e(2b^2d-2abe)\sqrt{d+ex}}{b^4}$$

input `int((d + e*x)^(5/2)/(a^2 + b^2*x^2 + 2*a*b*x),x)`output `(2*e*(d + e*x)^(3/2))/(3*b^2) - ((d + e*x)^(1/2)*(a^2*e^3 + b^2*d^2*e - 2*a*b*d*e^2))/(b^4*(d + e*x) - b^4*d + a*b^3*e) + (5*e*atan((b^(1/2)*e*(a*e - b*d)^(3/2)*(d + e*x)^(1/2)))/(a^2*e^3 + b^2*d^2*e - 2*a*b*d*e^2))*(a*e - b*d)^(3/2)/b^(7/2) + (2*e*(2*b^2*d - 2*a*b*e)*(d + e*x)^(1/2))/b^4`

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 273, normalized size of antiderivative = 2.48

$$\int \frac{(d + ex)^{5/2}}{a^2 + 2abx + b^2x^2} dx = \frac{15\sqrt{b}\sqrt{ae - bd} \operatorname{atan}\left(\frac{\sqrt{ex+db}}{\sqrt{b}\sqrt{ae-bd}}\right) a^2e^2 - 15\sqrt{b}\sqrt{ae - bd} \operatorname{atan}\left(\frac{\sqrt{ex+db}}{\sqrt{b}\sqrt{ae-bd}}\right) abde +$$

input `int((e*x+d)^(5/2)/(b^2*x^2+2*a*b*x+a^2),x)`

output

```
(15*sqrt(b)*sqrt(a*e - b*d)*atan((sqrt(d + e*x)*b)/(sqrt(b)*sqrt(a*e - b*d
)))*a**2*e**2 - 15*sqrt(b)*sqrt(a*e - b*d)*atan((sqrt(d + e*x)*b)/(sqrt(b)
*sqrt(a*e - b*d)))*a*b*d*e + 15*sqrt(b)*sqrt(a*e - b*d)*atan((sqrt(d + e*x
)*b)/(sqrt(b)*sqrt(a*e - b*d)))*a*b*e**2*x - 15*sqrt(b)*sqrt(a*e - b*d)*at
an((sqrt(d + e*x)*b)/(sqrt(b)*sqrt(a*e - b*d)))*b**2*d*e*x - 15*sqrt(d + e
*x)*a**2*b*e**2 + 20*sqrt(d + e*x)*a*b**2*d*e - 10*sqrt(d + e*x)*a*b**2*e*
*2*x - 3*sqrt(d + e*x)*b**3*d**2 + 14*sqrt(d + e*x)*b**3*d*e*x + 2*sqrt(d
+ e*x)*b**3*e**2*x**2)/(3*b**4*(a + b*x))
```

3.173 $\int \frac{(d+ex)^{3/2}}{a^2+2abx+b^2x^2} dx$

| | |
|---|------|
| Optimal result | 1362 |
| Mathematica [A] (verified) | 1362 |
| Rubi [A] (verified) | 1363 |
| Maple [A] (verified) | 1365 |
| Fricas [A] (verification not implemented) | 1366 |
| Sympy [F] | 1366 |
| Maxima [F(-2)] | 1367 |
| Giac [A] (verification not implemented) | 1367 |
| Mupad [B] (verification not implemented) | 1368 |
| Reduce [B] (verification not implemented) | 1368 |

Optimal result

Integrand size = 28, antiderivative size = 85

$$\int \frac{(d+ex)^{3/2}}{a^2+2abx+b^2x^2} dx = \frac{3e\sqrt{d+ex}}{b^2} - \frac{(d+ex)^{3/2}}{b(a+bx)} - \frac{3e\sqrt{bd-ae} \operatorname{arctanh}\left(\frac{\sqrt{b}\sqrt{d+ex}}{\sqrt{bd-ae}}\right)}{b^{5/2}}$$

output `3*e*(e*x+d)^(1/2)/b^2-(e*x+d)^(3/2)/b/(b*x+a)-3*e*(-a*e+b*d)^(1/2)*arctanh(b^(1/2)*(e*x+d)^(1/2)/(-a*e+b*d)^(1/2))/b^(5/2)`

Mathematica [A] (verified)

Time = 0.24 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.98

$$\int \frac{(d+ex)^{3/2}}{a^2+2abx+b^2x^2} dx = \frac{\sqrt{d+ex}(-bd+3ae+2bex)}{b^2(a+bx)} - \frac{3e\sqrt{-bd+ae} \arctan\left(\frac{\sqrt{b}\sqrt{d+ex}}{\sqrt{-bd+ae}}\right)}{b^{5/2}}$$

input `Integrate[(d + e*x)^(3/2)/(a^2 + 2*a*b*x + b^2*x^2), x]`

output `(Sqrt[d + e*x]*(-(b*d) + 3*a*e + 2*b*e*x))/(b^2*(a + b*x)) - (3*e*Sqrt[-(b*d) + a*e]*ArcTan[(Sqrt[b]*Sqrt[d + e*x])/Sqrt[-(b*d) + a*e]])/b^(5/2)`

Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.08, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.214$, Rules used = {1098, 27, 51, 60, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\begin{aligned}
 & \int \frac{(d+ex)^{3/2}}{a^2+2abx+b^2x^2} dx \\
 & \quad \downarrow \text{1098} \\
 & b^2 \int \frac{(d+ex)^{3/2}}{b^2(a+bx)^2} dx \\
 & \quad \downarrow \text{27} \\
 & \int \frac{(d+ex)^{3/2}}{(a+bx)^2} dx \\
 & \quad \downarrow \text{51} \\
 & \frac{3e \int \frac{\sqrt{d+ex}}{a+bx} dx}{2b} - \frac{(d+ex)^{3/2}}{b(a+bx)} \\
 & \quad \downarrow \text{60} \\
 & \frac{3e \left(\frac{(bd-ae) \int \frac{1}{(a+bx)\sqrt{d+ex}} dx}{b} + \frac{2\sqrt{d+ex}}{b} \right)}{2b} - \frac{(d+ex)^{3/2}}{b(a+bx)} \\
 & \quad \downarrow \text{73} \\
 & \frac{3e \left(\frac{2(bd-ae) \int \frac{1}{a + \frac{b(d+ex)}{e} - \frac{bd}{e}} d\sqrt{d+ex}}{be} + \frac{2\sqrt{d+ex}}{b} \right)}{2b} - \frac{(d+ex)^{3/2}}{b(a+bx)} \\
 & \quad \downarrow \text{221} \\
 & \frac{3e \left(\frac{2\sqrt{d+ex}}{b} - \frac{2\sqrt{bd-ae} \operatorname{arctanh}\left(\frac{\sqrt{b}\sqrt{d+ex}}{\sqrt{bd-ae}}\right)}{b^{3/2}} \right)}{2b} - \frac{(d+ex)^{3/2}}{b(a+bx)}
 \end{aligned}$$

input `Int[(d + e*x)^(3/2)/(a^2 + 2*a*b*x + b^2*x^2),x]`

output `-((d + e*x)^(3/2)/(b*(a + b*x))) + (3*e*((2*Sqrt[d + e*x])/b - (2*Sqrt[b*d - a*e]*ArcTanh[(Sqrt[b]*Sqrt[d + e*x])/Sqrt[b*d - a*e]])/b^(3/2)))/(2*b)`

Defintions of rubi rules used

rule 27 `Int[(a_)*(Fx_), x_Symbol] := Simp[a Int[Fx, x], x] /; FreeQ[a, x] && !MatchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]`

rule 51 `Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] := Simp[(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Simp[d*(n/(b*(m + 1))) Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d, n}, x] && ILtQ[m, -1] && FractionQ[n] && GtQ[n, 0]`

rule 60 `Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] := Simp[(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*(b*c - a*d)/(b*(m + n + 1)) Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] && !(IGtQ[m, 0] && (!IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) && !ILtQ[m + n + 2, 0] && IntLinearQ[a, b, c, d, m, n, x]`

rule 73 `Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] := With[{p = Denominator[m]}, Simp[p/b Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) + d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntLinearQ[a, b, c, d, m, n, x]`

rule 221 `Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[-a/b, 2]/a)*ArcTanh[x/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]`

rule 1098

```
Int[((d_.) + (e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] := Simp[1/c^p Int[(d + e*x)^m*(b/2 + c*x)^(2*p), x], x] /; FreeQ[
{a, b, c, d, e, m}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Maple [A] (verified)

Time = 1.53 (sec) , antiderivative size = 93, normalized size of antiderivative = 1.09

| method | result | size |
|-------------------|--|------|
| risch | $\frac{2e\sqrt{ex+d}}{b^2} - \frac{(2ae-2bd)e \left(-\frac{\sqrt{ex+d}}{2(b(ex+d)+ae-bd)} + \frac{3 \arctan\left(\frac{b\sqrt{ex+d}}{\sqrt{b(ae-bd)}}\right)}{2\sqrt{b(ae-bd)}} \right)}{b^2}$ | 93 |
| pseudoelliptic | $\frac{-3e(bx+a)(ae-bd) \arctan\left(\frac{b\sqrt{ex+d}}{\sqrt{b(ae-bd)}}\right) + 3\sqrt{ex+d} \left(\frac{(2ex-d)b}{3} + ae \right) \sqrt{b(ae-bd)}}{b^2(bx+a)\sqrt{b(ae-bd)}}$ | 99 |
| derivativedivides | $2e \left(\frac{\sqrt{ex+d}}{b^2} - \frac{\left(-\frac{ae}{2} + \frac{bd}{2} \right) \sqrt{ex+d}}{b(ex+d)+ae-bd} + \frac{3(ae-bd) \arctan\left(\frac{b\sqrt{ex+d}}{\sqrt{b(ae-bd)}}\right)}{b^2 \cdot 2\sqrt{b(ae-bd)}} \right)$ | 100 |
| default | $2e \left(\frac{\sqrt{ex+d}}{b^2} - \frac{\left(-\frac{ae}{2} + \frac{bd}{2} \right) \sqrt{ex+d}}{b(ex+d)+ae-bd} + \frac{3(ae-bd) \arctan\left(\frac{b\sqrt{ex+d}}{\sqrt{b(ae-bd)}}\right)}{b^2 \cdot 2\sqrt{b(ae-bd)}} \right)$ | 100 |

input

```
int((e*x+d)^(3/2)/(b^2*x^2+2*a*b*x+a^2),x,method=_RETURNVERBOSE)
```

output

```
2*e*(e*x+d)^(1/2)/b^2-1/b^2*(2*a*e-2*b*d)*e*(-1/2*(e*x+d)^(1/2)/(b*(e*x+d)
+a*e-b*d)+3/2/(b*(a*e-b*d))^(1/2)*arctan(b*(e*x+d)^(1/2)/(b*(a*e-b*d))^(1/
2)))
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 210, normalized size of antiderivative = 2.47

$$\int \frac{(d+ex)^{3/2}}{a^2+2abx+b^2x^2} dx = \left[\frac{3(bex+ae)\sqrt{\frac{bd-ae}{b}} \log\left(\frac{bex+2bd-ae-2\sqrt{ex+d}\sqrt{\frac{bd-ae}{b}}}{bx+a}\right) + 2(2bex-bd+3ae)\sqrt{ex+d}}{2(b^3x+ab^2)} \right. \\ \left. - \frac{3(bex+ae)\sqrt{-\frac{bd-ae}{b}} \arctan\left(-\frac{\sqrt{ex+d}\sqrt{-\frac{bd-ae}{b}}}{bd-ae}\right) - (2bex-bd+3ae)\sqrt{ex+d}}{b^3x+ab^2} \right]$$

input `integrate((e*x+d)^(3/2)/(b^2*x^2+2*a*b*x+a^2),x, algorithm="fricas")`

output `[1/2*(3*(b*e*x + a*e)*sqrt((b*d - a*e)/b)*log((b*e*x + 2*b*d - a*e - 2*sqrt(e*x + d)*b*sqrt((b*d - a*e)/b))/(b*x + a)) + 2*(2*b*e*x - b*d + 3*a*e)*sqrt(e*x + d)/(b^3*x + a*b^2), -(3*(b*e*x + a*e)*sqrt(-(b*d - a*e)/b)*arctan(-sqrt(e*x + d)*b*sqrt(-(b*d - a*e)/b)/(b*d - a*e)) - (2*b*e*x - b*d + 3*a*e)*sqrt(e*x + d)/(b^3*x + a*b^2)]`

Sympy [F]

$$\int \frac{(d+ex)^{3/2}}{a^2+2abx+b^2x^2} dx = \int \frac{(d+ex)^{\frac{3}{2}}}{(a+bx)^2} dx$$

input `integrate((e*x+d)**(3/2)/(b**2*x**2+2*a*b*x+a**2),x)`

output `Integral((d + e*x)**(3/2)/(a + b*x)**2, x)`

Maxima [F(-2)]

Exception generated.

$$\int \frac{(d+ex)^{3/2}}{a^2+2abx+b^2x^2} dx = \text{Exception raised: ValueError}$$

input `integrate((e*x+d)^(3/2)/(b^2*x^2+2*a*b*x+a^2),x, algorithm="maxima")`

output `Exception raised: ValueError >> Computation failed since Maxima requested additional constraints; using the 'assume' command before evaluation *may* help (example of legal syntax is 'assume(a*e-b*d>0)', see `assume?` for more detail`

Giac [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.33

$$\int \frac{(d+ex)^{3/2}}{a^2+2abx+b^2x^2} dx = \frac{2\sqrt{ex+de}}{b^2} + \frac{3(bde-ae^2)\arctan\left(\frac{\sqrt{ex+db}}{\sqrt{-b^2d+abe}}\right)}{\sqrt{-b^2d+abe}b^2} - \frac{\sqrt{ex+dbde}-\sqrt{ex+dae^2}}{((ex+d)b-bd+ae)b^2}$$

input `integrate((e*x+d)^(3/2)/(b^2*x^2+2*a*b*x+a^2),x, algorithm="giac")`

output `2*sqrt(e*x + d)*e/b^2 + 3*(b*d*e - a*e^2)*arctan(sqrt(e*x + d)*b/sqrt(-b^2*d + a*b*e))/(sqrt(-b^2*d + a*b*e)*b^2) - (sqrt(e*x + d)*b*d*e - sqrt(e*x + d)*a*e^2)/(((e*x + d)*b - b*d + a*e)*b^2)`

Mupad [B] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 109, normalized size of antiderivative = 1.28

$$\int \frac{(d+ex)^{3/2}}{a^2+2abx+b^2x^2} dx = \frac{(ae^2-bde)\sqrt{d+ex}}{b^3(d+ex)-b^3d+ab^2e} + \frac{2e\sqrt{d+ex}}{b^2} - \frac{3e \operatorname{atan}\left(\frac{\sqrt{be}\sqrt{ae-bd}\sqrt{d+ex}}{ae^2-bde}\right)\sqrt{ae-bd}}{b^{5/2}}$$

input `int((d + e*x)^(3/2)/(a^2 + b^2*x^2 + 2*a*b*x),x)`

output

```
((a*e^2 - b*d*e)*(d + e*x)^(1/2))/(b^3*(d + e*x) - b^3*d + a*b^2*e) + (2*e*(d + e*x)^(1/2))/b^2 - (3*e*atan((b^(1/2)*e*(a*e - b*d)^(1/2)*(d + e*x)^(1/2))/(a*e^2 - b*d*e))*(a*e - b*d)^(1/2))/b^(5/2)
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 127, normalized size of antiderivative = 1.49

$$\int \frac{(d+ex)^{3/2}}{a^2+2abx+b^2x^2} dx = \frac{-3\sqrt{b}\sqrt{ae-bd} \operatorname{atan}\left(\frac{\sqrt{ex+db}}{\sqrt{b}\sqrt{ae-bd}}\right) ae - 3\sqrt{b}\sqrt{ae-bd} \operatorname{atan}\left(\frac{\sqrt{ex+db}}{\sqrt{b}\sqrt{ae-bd}}\right) bex + 3\sqrt{b}\sqrt{ae-bd} \operatorname{atan}\left(\frac{\sqrt{ex+db}}{\sqrt{b}\sqrt{ae-bd}}\right) a}{b^3(bx+a)}$$

input `int((e*x+d)^(3/2)/(b^2*x^2+2*a*b*x+a^2),x)`

output

```
( - 3*sqrt(b)*sqrt(a*e - b*d)*atan((sqrt(d + e*x)*b)/(sqrt(b)*sqrt(a*e - b*d)))*a*e - 3*sqrt(b)*sqrt(a*e - b*d)*atan((sqrt(d + e*x)*b)/(sqrt(b)*sqrt(a*e - b*d)))*b*e*x + 3*sqrt(d + e*x)*a*b*e - sqrt(d + e*x)*b**2*d + 2*sqrt(d + e*x)*b**2*e*x)/(b**3*(a + b*x))
```

3.174 $\int \frac{\sqrt{d+ex}}{a^2+2abx+b^2x^2} dx$

| | |
|---|------|
| Optimal result | 1369 |
| Mathematica [A] (verified) | 1369 |
| Rubi [A] (verified) | 1370 |
| Maple [A] (verified) | 1371 |
| Fricas [A] (verification not implemented) | 1372 |
| Sympy [F] | 1373 |
| Maxima [F(-2)] | 1373 |
| Giac [A] (verification not implemented) | 1373 |
| Mupad [B] (verification not implemented) | 1374 |
| Reduce [B] (verification not implemented) | 1374 |

Optimal result

Integrand size = 28, antiderivative size = 70

$$\int \frac{\sqrt{d+ex}}{a^2+2abx+b^2x^2} dx = -\frac{\sqrt{d+ex}}{b(a+bx)} - \frac{e \operatorname{arctanh}\left(\frac{\sqrt{b}\sqrt{d+ex}}{\sqrt{bd-ae}}\right)}{b^{3/2}\sqrt{bd-ae}}$$

output $-(e*x+d)^{(1/2)}/b/(b*x+a)-e*\operatorname{arctanh}(b^{(1/2)}*(e*x+d)^{(1/2)}/(-a*e+b*d)^{(1/2)})/b^{(3/2)}/(-a*e+b*d)^{(1/2)}$

Mathematica [A] (verified)

Time = 0.20 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.99

$$\int \frac{\sqrt{d+ex}}{a^2+2abx+b^2x^2} dx = -\frac{\sqrt{d+ex}}{b(a+bx)} + \frac{e \arctan\left(\frac{\sqrt{b}\sqrt{d+ex}}{\sqrt{-bd+ae}}\right)}{b^{3/2}\sqrt{-bd+ae}}$$

input `Integrate[Sqrt[d + e*x]/(a^2 + 2*a*b*x + b^2*x^2), x]`

output $-(\operatorname{Sqrt}[d + e*x]/(b*(a + b*x))) + (e*\operatorname{ArcTan}[(\operatorname{Sqrt}[b]*\operatorname{Sqrt}[d + e*x])/(\operatorname{Sqrt}[-(b*d) + a*e])]/(b^{(3/2)}*\operatorname{Sqrt}[-(b*d) + a*e])$

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.179$, Rules used = {1098, 27, 51, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\begin{aligned}
 & \int \frac{\sqrt{d+ex}}{a^2+2abx+b^2x^2} dx \\
 & \quad \downarrow 1098 \\
 & b^2 \int \frac{\sqrt{d+ex}}{b^2(a+bx)^2} dx \\
 & \quad \downarrow 27 \\
 & \int \frac{\sqrt{d+ex}}{(a+bx)^2} dx \\
 & \quad \downarrow 51 \\
 & \frac{e \int \frac{1}{(a+bx)\sqrt{d+ex}} dx}{2b} - \frac{\sqrt{d+ex}}{b(a+bx)} \\
 & \quad \downarrow 73 \\
 & \frac{\int \frac{1}{a+\frac{b(d+ex)}{e}-\frac{bd}{e}} d\sqrt{d+ex}}{b} - \frac{\sqrt{d+ex}}{b(a+bx)} \\
 & \quad \downarrow 221 \\
 & -\frac{\text{earctanh}\left(\frac{\sqrt{b}\sqrt{d+ex}}{\sqrt{bd-ae}}\right)}{b^{3/2}\sqrt{bd-ae}} - \frac{\sqrt{d+ex}}{b(a+bx)}
 \end{aligned}$$

input `Int[Sqrt[d + e*x]/(a^2 + 2*a*b*x + b^2*x^2), x]`

output `-(Sqrt[d + e*x]/(b*(a + b*x))) - (e*ArcTanh[(Sqrt[b]*Sqrt[d + e*x])/Sqrt[b*d - a*e]])/(b^(3/2)*Sqrt[b*d - a*e])`

Definitions of rubi rules used

rule 27 $\text{Int}[(a_*)(Fx_), x_Symbol] \rightarrow \text{Simp}[a \text{ Int}[Fx, x], x] /; \text{FreeQ}[a, x] \ \&\& \ !\text{MatchQ}[Fx, (b_*)(Gx_)] /; \text{FreeQ}[b, x]$

rule 51 $\text{Int}[(a_.) + (b_.)*(x_)^{(m_)}*((c_.) + (d_.)*(x_))^{(n_)}, x_Symbol] \rightarrow \text{Simp}[(a + b*x)^{(m + 1)}*((c + d*x)^n/(b*(m + 1))), x] - \text{Simp}[d*(n/(b*(m + 1))) \text{Int}[(a + b*x)^{(m + 1)}*(c + d*x)^{(n - 1)}, x], x] /; \text{FreeQ}[\{a, b, c, d, n\}, x] \ \&\& \ \text{ILtQ}[m, -1] \ \&\& \ \text{FractionQ}[n] \ \&\& \ \text{GtQ}[n, 0]$

rule 73 $\text{Int}[(a_.) + (b_.)*(x_)^{(m_)}*((c_.) + (d_.)*(x_))^{(n_)}, x_Symbol] \rightarrow \text{With}[\{p = \text{Denominator}[m]\}, \text{Simp}[p/b \text{ Subst}[\text{Int}[x^{(p*(m + 1) - 1)}*(c - a*(d/b) + d*(x^p/b))^{(n)}, x], x, (a + b*x)^{(1/p)}, x]] /; \text{FreeQ}[\{a, b, c, d\}, x] \ \&\& \ \text{LtQ}[-1, m, 0] \ \&\& \ \text{LeQ}[-1, n, 0] \ \&\& \ \text{LeQ}[\text{Denominator}[n], \text{Denominator}[m]] \ \&\& \ \text{IntLinearQ}[a, b, c, d, m, n, x]$

rule 221 $\text{Int}[(a_.) + (b_.)*(x_)^2)^{-1}, x_Symbol] \rightarrow \text{Simp}[(\text{Rt}[-a/b, 2]/a)*\text{ArcTanh}[x/\text{Rt}[-a/b, 2]], x] /; \text{FreeQ}[\{a, b\}, x] \ \&\& \ \text{NegQ}[a/b]$

rule 1098 $\text{Int}[(d_.) + (e_.)*(x_)^{(m_)}*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^{(p_.)}, x_Symbol] \rightarrow \text{Simp}[1/c^p \text{ Int}[(d + e*x)^m*(b/2 + c*x)^{(2*p)}, x], x] /; \text{FreeQ}[\{a, b, c, d, e, m\}, x] \ \&\& \ \text{EqQ}[b^2 - 4*a*c, 0] \ \&\& \ \text{IntegerQ}[p]$

Maple [A] (verified)

Time = 1.40 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.83

| method | result | size |
|-------------------|---|------|
| pseudoelliptic | $\frac{-\frac{\sqrt{ex+d}}{bx+a} + \frac{e \arctan\left(\frac{b\sqrt{ex+d}}{\sqrt{b(ae-bd)}}\right)}{\sqrt{b(ae-bd)}}}{b}$ | 58 |
| derivativedivides | $2e \left(-\frac{\sqrt{ex+d}}{2b(b(ex+d)+ae-bd)} + \frac{\arctan\left(\frac{b\sqrt{ex+d}}{\sqrt{b(ae-bd)}}\right)}{2b\sqrt{b(ae-bd)}} \right)$ | 73 |
| default | $2e \left(-\frac{\sqrt{ex+d}}{2b(b(ex+d)+ae-bd)} + \frac{\arctan\left(\frac{b\sqrt{ex+d}}{\sqrt{b(ae-bd)}}\right)}{2b\sqrt{b(ae-bd)}} \right)$ | 73 |

input `int((e*x+d)^(1/2)/(b^2*x^2+2*a*b*x+a^2),x,method=_RETURNVERBOSE)`

output `1/b*(-(e*x+d)^(1/2)/(b*x+a)+e/(b*(a*e-b*d))^(1/2)*arctan(b*(e*x+d)^(1/2)/(b*(a*e-b*d))^(1/2))`

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 232, normalized size of antiderivative = 3.31

$$\int \frac{\sqrt{d+ex}}{a^2+2abx+b^2x^2} dx$$

$$= \left[\frac{\sqrt{b^2d-abe}(bex+ae) \log\left(\frac{bex+2bd-ae-2\sqrt{b^2d-abe}\sqrt{ex+d}}{bx+a}\right) - 2(b^2d-abe)\sqrt{ex+d} \sqrt{-b^2d+abe}(bex+a)}{2(ab^3d-a^2b^2e+(b^4d-ab^3e)x)}, \dots \right]$$

input `integrate((e*x+d)^(1/2)/(b^2*x^2+2*a*b*x+a^2),x, algorithm="fricas")`

output `[1/2*(sqrt(b^2*d - a*b*e)*(b*e*x + a*e)*log((b*e*x + 2*b*d - a*e - 2*sqrt(b^2*d - a*b*e)*sqrt(e*x + d))/(b*x + a)) - 2*(b^2*d - a*b*e)*sqrt(e*x + d))/(a*b^3*d - a^2*b^2*e + (b^4*d - a*b^3*e)*x), (sqrt(-b^2*d + a*b*e)*(b*e*x + a*e)*arctan(sqrt(-b^2*d + a*b*e)*sqrt(e*x + d)/(b*e*x + b*d)) - (b^2*d - a*b*e)*sqrt(e*x + d))/(a*b^3*d - a^2*b^2*e + (b^4*d - a*b^3*e)*x)]`

Sympy [F]

$$\int \frac{\sqrt{d+ex}}{a^2+2abx+b^2x^2} dx = \int \frac{\sqrt{d+ex}}{(a+bx)^2} dx$$

input `integrate((e*x+d)**(1/2)/(b**2*x**2+2*a*b*x+a**2),x)`

output `Integral(sqrt(d + e*x)/(a + b*x)**2, x)`

Maxima [F(-2)]

Exception generated.

$$\int \frac{\sqrt{d+ex}}{a^2+2abx+b^2x^2} dx = \text{Exception raised: ValueError}$$

input `integrate((e*x+d)^(1/2)/(b^2*x^2+2*a*b*x+a^2),x, algorithm="maxima")`

output `Exception raised: ValueError >> Computation failed since Maxima requested additional constraints; using the 'assume' command before evaluation *may* help (example of legal syntax is 'assume(a*e-b*d>0)', see `assume?` for more detail`

Giac [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.03

$$\int \frac{\sqrt{d+ex}}{a^2+2abx+b^2x^2} dx = \frac{e \arctan\left(\frac{\sqrt{ex+db}}{\sqrt{-b^2d+abeb}}\right)}{\sqrt{-b^2d+abeb}} - \frac{\sqrt{ex+de}}{((ex+d)b-bd+ae)b}$$

input `integrate((e*x+d)^(1/2)/(b^2*x^2+2*a*b*x+a^2),x, algorithm="giac")`

output `e*arctan(sqrt(e*x + d)*b/sqrt(-b^2*d + a*b*e))/(sqrt(-b^2*d + a*b*e)*b) - sqrt(e*x + d)*e/(((e*x + d)*b - b*d + a*e)*b)`

Mupad [B] (verification not implemented)

Time = 9.63 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.87

$$\int \frac{\sqrt{d+ex}}{a^2+2abx+b^2x^2} dx = \frac{e \operatorname{atan}\left(\frac{\sqrt{b}\sqrt{d+ex}}{\sqrt{ae-bd}}\right)}{b^{3/2}\sqrt{ae-bd}} - \frac{e\sqrt{d+ex}}{exb^2+ae}$$

input `int((d + e*x)^(1/2)/(a^2 + b^2*x^2 + 2*a*b*x),x)`output `(e*atan((b^(1/2)*(d + e*x)^(1/2))/(a*e - b*d)^(1/2)))/(b^(3/2)*(a*e - b*d)^(1/2)) - (e*(d + e*x)^(1/2))/(a*b*e + b^2*e*x)`**Reduce [B] (verification not implemented)**

Time = 0.25 (sec) , antiderivative size = 129, normalized size of antiderivative = 1.84

$$\int \frac{\sqrt{d+ex}}{a^2+2abx+b^2x^2} dx = \frac{\sqrt{b}\sqrt{ae-bd} \operatorname{atan}\left(\frac{\sqrt{ex+db}}{\sqrt{b}\sqrt{ae-bd}}\right) ae + \sqrt{b}\sqrt{ae-bd} \operatorname{atan}\left(\frac{\sqrt{ex+db}}{\sqrt{b}\sqrt{ae-bd}}\right) bex - \sqrt{ex+d}abe + \sqrt{ex+d}b^2d}{b^2(abex - b^2dx + a^2e - abd)}$$

input `int((e*x+d)^(1/2)/(b^2*x^2+2*a*b*x+a^2),x)`output `(sqrt(b)*sqrt(a*e - b*d)*atan((sqrt(d + e*x)*b)/(sqrt(b)*sqrt(a*e - b*d)))*a*e + sqrt(b)*sqrt(a*e - b*d)*atan((sqrt(d + e*x)*b)/(sqrt(b)*sqrt(a*e - b*d)))*b*e*x - sqrt(d + e*x)*a*b*e + sqrt(d + e*x)*b**2*d)/(b**2*(a**2*e - a*b*d + a*b*e*x - b**2*d*x))`

3.175 $\int \frac{1}{\sqrt{d+ex}(a^2+2abx+b^2x^2)} dx$

| | |
|---|------|
| Optimal result | 1375 |
| Mathematica [A] (verified) | 1375 |
| Rubi [A] (verified) | 1376 |
| Maple [A] (verified) | 1377 |
| Fricas [B] (verification not implemented) | 1378 |
| Sympy [F] | 1379 |
| Maxima [F(-2)] | 1379 |
| Giac [A] (verification not implemented) | 1380 |
| Mupad [B] (verification not implemented) | 1380 |
| Reduce [B] (verification not implemented) | 1381 |

Optimal result

Integrand size = 28, antiderivative size = 76

$$\int \frac{1}{\sqrt{d+ex}(a^2+2abx+b^2x^2)} dx = -\frac{\sqrt{d+ex}}{(bd-ae)(a+bx)} + \frac{e \operatorname{arctanh}\left(\frac{\sqrt{b}\sqrt{d+ex}}{\sqrt{bd-ae}}\right)}{\sqrt{b}(bd-ae)^{3/2}}$$

output

$$-(e*x+d)^{(1/2)/(-a*e+b*d)/(b*x+a)+e*\operatorname{arctanh}(b^{(1/2)}*(e*x+d)^{(1/2)/(-a*e+b*d)^{(1/2)})/b^{(1/2)/(-a*e+b*d)^{(3/2)}}$$

Mathematica [A] (verified)

Time = 0.19 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.99

$$\int \frac{1}{\sqrt{d+ex}(a^2+2abx+b^2x^2)} dx = \frac{\sqrt{d+ex}}{(-bd+ae)(a+bx)} + \frac{e \operatorname{arctan}\left(\frac{\sqrt{b}\sqrt{d+ex}}{\sqrt{-bd+ae}}\right)}{\sqrt{b}(-bd+ae)^{3/2}}$$

input

```
Integrate[1/(Sqrt[d + e*x]*(a^2 + 2*a*b*x + b^2*x^2)),x]
```

output

```
Sqrt[d + e*x]/((-b*d) + a*e)*(a + b*x)) + (e*ArcTan[(Sqrt[b]*Sqrt[d + e*x])/Sqrt[-(b*d) + a*e]])/(Sqrt[b]*(-(b*d) + a*e)^(3/2))
```

Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.179$, Rules used = {1098, 27, 52, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\begin{aligned}
 & \int \frac{1}{(a^2 + 2abx + b^2x^2)\sqrt{d+ex}} dx \\
 & \quad \downarrow 1098 \\
 & b^2 \int \frac{1}{b^2(a+bx)^2\sqrt{d+ex}} dx \\
 & \quad \downarrow 27 \\
 & \int \frac{1}{(a+bx)^2\sqrt{d+ex}} dx \\
 & \quad \downarrow 52 \\
 & \frac{e \int \frac{1}{(a+bx)\sqrt{d+ex}} dx}{2(bd-ae)} - \frac{\sqrt{d+ex}}{(a+bx)(bd-ae)} \\
 & \quad \downarrow 73 \\
 & -\frac{\int \frac{1}{a+\frac{b(d+ex)}{e}-\frac{bd}{e}} d\sqrt{d+ex}}{bd-ae} - \frac{\sqrt{d+ex}}{(a+bx)(bd-ae)} \\
 & \quad \downarrow 221 \\
 & \frac{\text{earctanh}\left(\frac{\sqrt{b}\sqrt{d+ex}}{\sqrt{bd-ae}}\right)}{\sqrt{b}(bd-ae)^{3/2}} - \frac{\sqrt{d+ex}}{(a+bx)(bd-ae)}
 \end{aligned}$$

input `Int[1/(Sqrt[d + e*x]*(a^2 + 2*a*b*x + b^2*x^2)),x]`

output `-(Sqrt[d + e*x]/((b*d - a*e)*(a + b*x))) + (e*ArcTanh[(Sqrt[b]*Sqrt[d + e*x])/Sqrt[b*d - a*e]])/(Sqrt[b]*(b*d - a*e)^(3/2))`

Definitions of rubi rules used

rule 27 $\text{Int}[(a_*)(Fx_), x_Symbol] \rightarrow \text{Simp}[a \text{ Int}[Fx, x], x] /; \text{FreeQ}[a, x] \ \&\& \ !\text{MatchQ}[Fx, (b_*)(Gx_)] /; \text{FreeQ}[b, x]$

rule 52 $\text{Int}[((a_.) + (b_.)(x_))^{(m_.)}((c_.) + (d_.)(x_))^{(n_.)}, x_Symbol] \rightarrow \text{Simp}[(a + b*x)^{(m + 1)}((c + d*x)^{(n + 1)} / ((b*c - a*d)*(m + 1))), x] - \text{Simp}[d*((m + n + 2) / ((b*c - a*d)*(m + 1))) \text{ Int}[(a + b*x)^{(m + 1)}(c + d*x)^n, x], x] /; \text{FreeQ}[\{a, b, c, d, n\}, x] \ \&\& \ \text{ILtQ}[m, -1] \ \&\& \ \text{FractionQ}[n] \ \&\& \ \text{LtQ}[n, 0]$

rule 73 $\text{Int}[((a_.) + (b_.)(x_))^{(m_.)}((c_.) + (d_.)(x_))^{(n_.)}, x_Symbol] \rightarrow \text{With}[\{p = \text{Denominator}[m]\}, \text{Simp}[p/b \text{ Subst}[\text{Int}[x^{(p*(m + 1) - 1)}(c - a*(d/b) + d*(x^p/b))^{n_}], x], x, (a + b*x)^{(1/p)}, x]] /; \text{FreeQ}[\{a, b, c, d\}, x] \ \&\& \ \text{LtQ}[-1, m, 0] \ \&\& \ \text{LeQ}[-1, n, 0] \ \&\& \ \text{LeQ}[\text{Denominator}[n], \text{Denominator}[m]] \ \&\& \ \text{IntLinearQ}[a, b, c, d, m, n, x]$

rule 221 $\text{Int}[((a_) + (b_.)(x_)^2)^{-1}, x_Symbol] \rightarrow \text{Simp}[(\text{Rt}[-a/b, 2]/a)*\text{ArcTanh}[x/\text{Rt}[-a/b, 2]], x] /; \text{FreeQ}[\{a, b\}, x] \ \&\& \ \text{NegQ}[a/b]$

rule 1098 $\text{Int}[((d_.) + (e_.)(x_))^{(m_.)}((a_) + (b_.)(x_) + (c_.)(x_)^2)^{(p_.)}, x_Symbol] \rightarrow \text{Simp}[1/c^p \text{ Int}[(d + e*x)^m*(b/2 + c*x)^{(2*p)}, x], x] /; \text{FreeQ}[\{a, b, c, d, e, m\}, x] \ \&\& \ \text{EqQ}[b^2 - 4*a*c, 0] \ \&\& \ \text{IntegerQ}[p]$

Maple [A] (verified)

Time = 1.38 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.84

| method | result | size |
|-------------------|--|------|
| pseudoelliptic | $\frac{\frac{\sqrt{ex+d}}{bx+a} + \frac{e \arctan\left(\frac{b\sqrt{ex+d}}{\sqrt{b(ae-bd)}}\right)}{\sqrt{b(ae-bd)}}}{ae-bd}$ | 64 |
| derivativedivides | $2e \left(\frac{\sqrt{ex+d}}{2(ae-bd)(b(ex+d)+ae-bd)} + \frac{\arctan\left(\frac{b\sqrt{ex+d}}{\sqrt{b(ae-bd)}}\right)}{2(ae-bd)\sqrt{b(ae-bd)}} \right)$ | 87 |
| default | $2e \left(\frac{\sqrt{ex+d}}{2(ae-bd)(b(ex+d)+ae-bd)} + \frac{\arctan\left(\frac{b\sqrt{ex+d}}{\sqrt{b(ae-bd)}}\right)}{2(ae-bd)\sqrt{b(ae-bd)}} \right)$ | 87 |

input `int(1/(e*x+d)^(1/2)/(b^2*x^2+2*a*b*x+a^2),x,method=_RETURNVERBOSE)`

output `1/(a*e-b*d)*((e*x+d)^(1/2)/(b*x+a)+e/(b*(a*e-b*d)))^(1/2)*arctan(b*(e*x+d)^(1/2)/(b*(a*e-b*d))^(1/2))`

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 133 vs. $2(64) = 128$.

Time = 0.08 (sec) , antiderivative size = 280, normalized size of antiderivative = 3.68

$$\int \frac{1}{\sqrt{d+ex}(a^2+2abx+b^2x^2)} dx$$

$$= \left[\frac{\sqrt{b^2d-abe}(bex+ae) \log\left(\frac{bex+2bd-ae-2\sqrt{b^2d-abe}\sqrt{ex+d}}{bx+a}\right) + 2(b^2d-abe)\sqrt{ex+d}}{2(ab^3d^2-2a^2b^2de+a^3be^2+(b^4d^2-2ab^3de+a^2b^2e^2)x)}, \right.$$

$$\left. - \frac{\sqrt{-b^2d+abe}(bex+ae) \arctan\left(\frac{\sqrt{-b^2d+abe}\sqrt{ex+d}}{bex+bd}\right) + (b^2d-abe)\sqrt{ex+d}}{ab^3d^2-2a^2b^2de+a^3be^2+(b^4d^2-2ab^3de+a^2b^2e^2)x} \right]$$

input `integrate(1/(e*x+d)^(1/2)/(b^2*x^2+2*a*b*x+a^2),x,algorithm="fricas")`

output

```
[-1/2*(sqrt(b^2*d - a*b*e)*(b*e*x + a*e)*log((b*e*x + 2*b*d - a*e - 2*sqrt
(b^2*d - a*b*e)*sqrt(e*x + d))/(b*x + a)) + 2*(b^2*d - a*b*e)*sqrt(e*x + d
))/(a*b^3*d^2 - 2*a^2*b^2*d*e + a^3*b*e^2 + (b^4*d^2 - 2*a*b^3*d*e + a^2*b
^2*e^2)*x), -(sqrt(-b^2*d + a*b*e)*(b*e*x + a*e)*arctan(sqrt(-b^2*d + a*b*
e)*sqrt(e*x + d)/(b*e*x + b*d)) + (b^2*d - a*b*e)*sqrt(e*x + d))/(a*b^3*d
^2 - 2*a^2*b^2*d*e + a^3*b*e^2 + (b^4*d^2 - 2*a*b^3*d*e + a^2*b^2*e^2)*x)]
```

Sympy [F]

$$\int \frac{1}{\sqrt{d+ex}(a^2+2abx+b^2x^2)} dx = \int \frac{1}{(a+bx)^2 \sqrt{d+ex}} dx$$

input

```
integrate(1/(e*x+d)**(1/2)/(b**2*x**2+2*a*b*x+a**2),x)
```

output

```
Integral(1/((a + b*x)**2*sqrt(d + e*x)), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{\sqrt{d+ex}(a^2+2abx+b^2x^2)} dx = \text{Exception raised: ValueError}$$

input

```
integrate(1/(e*x+d)^(1/2)/(b^2*x^2+2*a*b*x+a^2),x, algorithm="maxima")
```

output

```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
help (example of legal syntax is 'assume(a*e-b*d>0)', see `assume?` for m
ore detail
```


Giac [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.14

$$\int \frac{1}{\sqrt{d+ex}(a^2+2abx+b^2x^2)} dx = -\frac{e \arctan\left(\frac{\sqrt{ex+db}}{\sqrt{-b^2d+abe}}\right)}{\sqrt{-b^2d+abe}(bd-ae)} - \frac{\sqrt{ex+de}}{((ex+d)b-bd+ae)(bd-ae)}$$

input `integrate(1/(e*x+d)^(1/2)/(b^2*x^2+2*a*b*x+a^2),x, algorithm="giac")`

output `-e*arctan(sqrt(e*x + d)*b/sqrt(-b^2*d + a*b*e))/(sqrt(-b^2*d + a*b*e)*(b*d - a*e)) - sqrt(e*x + d)*e/(((e*x + d)*b - b*d + a*e)*(b*d - a*e))`

Mupad [B] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.97

$$\int \frac{1}{\sqrt{d+ex}(a^2+2abx+b^2x^2)} dx = \frac{e \operatorname{atan}\left(\frac{\sqrt{b}\sqrt{d+ex}}{\sqrt{ae-bd}}\right)}{\sqrt{b}(ae-bd)^{3/2}} + \frac{e\sqrt{d+ex}}{(ae-bd)(ae-bd+b(d+ex))}$$

input `int(1/((d + e*x)^(1/2)*(a^2 + b^2*x^2 + 2*a*b*x)),x)`

output `(e*atan((b^(1/2)*(d + e*x)^(1/2))/(a*e - b*d)^(1/2)))/(b^(1/2)*(a*e - b*d)^(3/2)) + (e*(d + e*x)^(1/2))/((a*e - b*d)*(a*e - b*d + b*(d + e*x)))`

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 156, normalized size of antiderivative = 2.05

$$\int \frac{1}{\sqrt{d+ex}(a^2+2abx+b^2x^2)} dx$$

$$= \frac{\sqrt{b}\sqrt{ae-bd} \operatorname{atan}\left(\frac{\sqrt{ex+db}}{\sqrt{b}\sqrt{ae-bd}}\right) ae + \sqrt{b}\sqrt{ae-bd} \operatorname{atan}\left(\frac{\sqrt{ex+db}}{\sqrt{b}\sqrt{ae-bd}}\right) bex + \sqrt{ex+d}abe - \sqrt{ex+d}b^2d}{b(a^2be^2x - 2ab^2dex + b^3d^2x + a^3e^2 - 2a^2bde + ab^2d^2)}$$

input

```
int(1/(e*x+d)^(1/2)/(b^2*x^2+2*a*b*x+a^2),x)
```

output

```
(sqrt(b)*sqrt(a*e - b*d)*atan((sqrt(d + e*x)*b)/(sqrt(b)*sqrt(a*e - b*d)))
*a*e + sqrt(b)*sqrt(a*e - b*d)*atan((sqrt(d + e*x)*b)/(sqrt(b)*sqrt(a*e -
b*d)))*b*e*x + sqrt(d + e*x)*a*b*e - sqrt(d + e*x)*b**2*d)/(b*(a**3*e**2 -
2*a**2*b*d*e + a**2*b*e**2*x + a*b**2*d**2 - 2*a*b**2*d*e*x + b**3*d**2*x
))
```

3.176 $\int \frac{1}{(d+ex)^{3/2}(a^2+2abx+b^2x^2)} dx$

| | |
|---|------|
| Optimal result | 1382 |
| Mathematica [A] (verified) | 1382 |
| Rubi [A] (verified) | 1383 |
| Maple [A] (verified) | 1385 |
| Fricas [B] (verification not implemented) | 1386 |
| Sympy [F] | 1386 |
| Maxima [F(-2)] | 1387 |
| Giac [A] (verification not implemented) | 1387 |
| Mupad [B] (verification not implemented) | 1388 |
| Reduce [B] (verification not implemented) | 1388 |

Optimal result

Integrand size = 28, antiderivative size = 99

$$\int \frac{1}{(d+ex)^{3/2}(a^2+2abx+b^2x^2)} dx = -\frac{3e}{(bd-ae)^2\sqrt{d+ex}} - \frac{1}{(bd-ae)(a+bx)\sqrt{d+ex}} + \frac{3\sqrt{b}e \operatorname{arctanh}\left(\frac{\sqrt{b}\sqrt{d+ex}}{\sqrt{bd-ae}}\right)}{(bd-ae)^{5/2}}$$

output

$-3e/(-a*e+b*d)^2/(e*x+d)^{(1/2)}-1/(-a*e+b*d)/(b*x+a)/(e*x+d)^{(1/2)}+3*b^{(1/2)}*e*\operatorname{arctanh}(b^{(1/2)}*(e*x+d)^{(1/2)/(-a*e+b*d)^{(1/2)})/(-a*e+b*d)^{(5/2)}$

Mathematica [A] (verified)

Time = 0.30 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.91

$$\int \frac{1}{(d+ex)^{3/2}(a^2+2abx+b^2x^2)} dx = \frac{2ae+b(d+3ex)}{(bd-ae)^2(a+bx)\sqrt{d+ex}} - \frac{3\sqrt{b}e \arctan\left(\frac{\sqrt{b}\sqrt{d+ex}}{\sqrt{-bd+ae}}\right)}{(-bd+ae)^{5/2}}$$

input

`Integrate[1/((d + e*x)^(3/2)*(a^2 + 2*a*b*x + b^2*x^2)),x]`

output

$$-\left(\frac{2ae + b(d + 3ex)}{(bd - ae)^2(a + bx)\sqrt{d + ex}}\right) - \left(\frac{3\sqrt{b}e \operatorname{ArcTan}\left[\frac{\sqrt{b}\sqrt{d + ex}}{\sqrt{-(bd) + ae}}\right]}{-(bd) + ae}\right)^{5/2}$$
Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.14, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.214$, Rules used = {1098, 27, 52, 61, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a^2 + 2abx + b^2x^2)(d + ex)^{3/2}} dx$$

$$\downarrow 1098$$

$$b^2 \int \frac{1}{b^2(a + bx)^2(d + ex)^{3/2}} dx$$

$$\downarrow 27$$

$$\int \frac{1}{(a + bx)^2(d + ex)^{3/2}} dx$$

$$\downarrow 52$$

$$-\frac{3e \int \frac{1}{(a+bx)(d+ex)^{3/2}} dx}{2(bd - ae)} - \frac{1}{(a + bx)\sqrt{d + ex}(bd - ae)}$$

$$\downarrow 61$$

$$-\frac{3e \left(\frac{b \int \frac{1}{(a+bx)\sqrt{d+ex}} dx}{bd - ae} + \frac{2}{\sqrt{d+ex}(bd - ae)} \right)}{2(bd - ae)} - \frac{1}{(a + bx)\sqrt{d + ex}(bd - ae)}$$

$$\downarrow 73$$

$$-\frac{3e \left(\frac{2b \int \frac{1}{a + \frac{b(d+ex) - bd}{e}} d\sqrt{d+ex}}{e(bd - ae)} + \frac{2}{\sqrt{d+ex}(bd - ae)} \right)}{2(bd - ae)} - \frac{1}{(a + bx)\sqrt{d + ex}(bd - ae)}$$

$$\downarrow 221$$

$$-\frac{3e\left(\frac{2}{\sqrt{d+ex}(bd-ae)} - \frac{2\sqrt{b}\operatorname{arctanh}\left(\frac{\sqrt{b}\sqrt{d+ex}}{\sqrt{bd-ae}}\right)}{(bd-ae)^{3/2}}\right)}{2(bd-ae)} - \frac{1}{(a+bx)\sqrt{d+ex}(bd-ae)}$$

input `Int[1/((d + e*x)^(3/2)*(a^2 + 2*a*b*x + b^2*x^2)),x]`

output `-(1/((b*d - a*e)*(a + b*x)*Sqrt[d + e*x])) - (3*e*(2/((b*d - a*e)*Sqrt[d + e*x]) - (2*Sqrt[b]*ArcTanh[(Sqrt[b]*Sqrt[d + e*x])/Sqrt[b*d - a*e]])/(b*d - a*e)^(3/2)))/(2*(b*d - a*e))`

Defintions of rubi rules used

rule 27 `Int[(a_)*(F_x_), x_Symbol] := Simp[a Int[F_x, x], x] /; FreeQ[a, x] && !MatchQ[F_x, (b_)*(G_x_)] /; FreeQ[b, x]`

rule 52 `Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] := Simp[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((m + n + 2)/((b*c - a*d)*(m + 1))) Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && ILtQ[m, -1] && FractionQ[n] && LtQ[n, 0]`

rule 61 `Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] := Simp[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((m + n + 2)/((b*c - a*d)*(m + 1))) Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && LtQ[m, -1] && !(LtQ[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && IntLinearQ[a, b, c, d, m, n, x]`

rule 73 `Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] := With[{p = Denominator[m]}, Simp[p/b Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) + d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntLinearQ[a, b, c, d, m, n, x]`

rule 221 `Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[-a/b, 2]/a)*ArcTanh[x/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]`

rule 1098 `Int[((d_) + (e_)*(x_)^(m_))*((a_) + (b_)*(x_) + (c_)*(x_)^2)^(p_), x_Symbol] := Simp[1/c^p Int[(d + e*x)^m*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]`

Maple [A] (verified)

Time = 1.41 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.79

| method | result | size |
|-------------------|--|------|
| pseudoelliptic | $-\frac{2e}{\sqrt{ex+d}} - \frac{b\sqrt{ex+d}}{bx+a} - \frac{3eb \arctan\left(\frac{b\sqrt{ex+d}}{\sqrt{b(ae-bd)}}\right)}{(ae-bd)^2}$ | 78 |
| derivativedivides | $2e \left(-\frac{b \left(\frac{\sqrt{ex+d}}{2b(ex+d)+2ae-2bd} + \frac{3 \arctan\left(\frac{b\sqrt{ex+d}}{\sqrt{b(ae-bd)}}\right)}{2\sqrt{b(ae-bd)}} \right)}{(ae-bd)^2} - \frac{1}{(ae-bd)^2 \sqrt{ex+d}} \right)$ | 100 |
| default | $2e \left(-\frac{b \left(\frac{\sqrt{ex+d}}{2b(ex+d)+2ae-2bd} + \frac{3 \arctan\left(\frac{b\sqrt{ex+d}}{\sqrt{b(ae-bd)}}\right)}{2\sqrt{b(ae-bd)}} \right)}{(ae-bd)^2} - \frac{1}{(ae-bd)^2 \sqrt{ex+d}} \right)$ | 100 |

input `int(1/(e*x+d)^(3/2)/(b^2*x^2+2*a*b*x+a^2), x, method=_RETURNVERBOSE)`

output `1/(a*e-b*d)^2*(-2*e/(e*x+d)^(1/2)-b*(e*x+d)^(1/2)/(b*x+a)-3*e*b/(b*(a*e-b*d))^(1/2)*arctan(b*(e*x+d)^(1/2)/(b*(a*e-b*d))^(1/2))`

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 187 vs. $2(85) = 170$.

Time = 0.11 (sec) , antiderivative size = 404, normalized size of antiderivative = 4.08

$$\int \frac{1}{(d+ex)^{3/2}(a^2+2abx+b^2x^2)} dx = \left[\frac{3(be^2x^2 + ade + (bde + ae^2)x)\sqrt{\frac{b}{bd-ae}} \log\left(\frac{bex+2bd-ae+2(bd-ae)\sqrt{ex+d}}{bx+a}\right)}{2(ab^2d^3 - 2a^2bd^2e + a^3de^2 + (b^3d^2e - 2ab^2de^2 + a^2be^3)x^2)} \right. \\ \left. - \frac{3(be^2x^2 + ade + (bde + ae^2)x)\sqrt{-\frac{b}{bd-ae}} \arctan\left(\sqrt{ex+d}\sqrt{-\frac{b}{bd-ae}}\right) + (3bex + bd + 2ae)\sqrt{ex+d}}{ab^2d^3 - 2a^2bd^2e + a^3de^2 + (b^3d^2e - 2ab^2de^2 + a^2be^3)x^2 + (b^3d^3 - ab^2d^2e - a^2bde^2 + a^3e^3)x} \right]$$

input `integrate(1/(e*x+d)^(3/2)/(b^2*x^2+2*a*b*x+a^2),x, algorithm="fricas")`

output `[1/2*(3*(b*e^2*x^2 + a*d*e + (b*d*e + a*e^2)*x)*sqrt(b/(b*d - a*e))*log((b*e*x + 2*b*d - a*e + 2*(b*d - a*e)*sqrt(e*x + d)*sqrt(b/(b*d - a*e)))/(b*x + a)) - 2*(3*b*e*x + b*d + 2*a*e)*sqrt(e*x + d)/(a*b^2*d^3 - 2*a^2*b*d^2*e + a^3*d*e^2 + (b^3*d^2*e - 2*a*b^2*d*e^2 + a^2*b*e^3)*x^2 + (b^3*d^3 - a*b^2*d^2*e - a^2*b*d*e^2 + a^3*e^3)*x), -(3*(b*e^2*x^2 + a*d*e + (b*d*e + a*e^2)*x)*sqrt(-b/(b*d - a*e))*arctan(sqrt(e*x + d)*sqrt(-b/(b*d - a*e))) + (3*b*e*x + b*d + 2*a*e)*sqrt(e*x + d)/(a*b^2*d^3 - 2*a^2*b*d^2*e + a^3*d*e^2 + (b^3*d^2*e - 2*a*b^2*d*e^2 + a^2*b*e^3)*x^2 + (b^3*d^3 - a*b^2*d^2*e - a^2*b*d*e^2 + a^3*e^3)*x)]`

Sympy [F]

$$\int \frac{1}{(d+ex)^{3/2}(a^2+2abx+b^2x^2)} dx = \int \frac{1}{(a+bx)^2(d+ex)^{3/2}} dx$$

input `integrate(1/(e*x+d)**(3/2)/(b**2*x**2+2*a*b*x+a**2),x)`

output `Integral(1/((a + b*x)**2*(d + e*x)**(3/2)), x)`

Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{(d+ex)^{3/2}(a^2+2abx+b^2x^2)} dx = \text{Exception raised: ValueError}$$

input `integrate(1/(e*x+d)^(3/2)/(b^2*x^2+2*a*b*x+a^2),x, algorithm="maxima")`

output Exception raised: ValueError >> Computation failed since Maxima requested additional constraints; using the 'assume' command before evaluation *may* help (example of legal syntax is 'assume(a*e-b*d>0)', see `assume?` for more detail)

Giac [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 143, normalized size of antiderivative = 1.44

$$\int \frac{1}{(d+ex)^{3/2}(a^2+2abx+b^2x^2)} dx = -\frac{3be \arctan\left(\frac{\sqrt{ex+db}}{\sqrt{-b^2d+abe}}\right)}{(b^2d^2-2abde+a^2e^2)\sqrt{-b^2d+abe}} - \frac{3(ex+d)be-2bde+2ae^2}{(b^2d^2-2abde+a^2e^2)\left((ex+d)^{\frac{3}{2}}b-\sqrt{ex+dbd}+\sqrt{ex+dae}\right)}$$

input `integrate(1/(e*x+d)^(3/2)/(b^2*x^2+2*a*b*x+a^2),x, algorithm="giac")`

output `-3*b*e*arctan(sqrt(e*x + d)*b/sqrt(-b^2*d + a*b*e))/((b^2*d^2 - 2*a*b*d*e + a^2*e^2)*sqrt(-b^2*d + a*b*e)) - (3*(e*x + d)*b*e - 2*b*d*e + 2*a*e^2)/((b^2*d^2 - 2*a*b*d*e + a^2*e^2)*((e*x + d)^(3/2)*b - sqrt(e*x + d)*b*d + sqrt(e*x + d)*a*e))`

Mupad [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 123, normalized size of antiderivative = 1.24

$$\int \frac{1}{(d+ex)^{3/2}(a^2+2abx+b^2x^2)} dx = -\frac{\frac{2e}{ae-bd} + \frac{3be(d+ex)}{(ae-bd)^2}}{b(d+ex)^{3/2} + (ae-bd)\sqrt{d+ex}} - \frac{3\sqrt{b}e \operatorname{atan}\left(\frac{\sqrt{b}\sqrt{d+ex}(a^2e^2-2abd e+b^2d^2)}{(ae-bd)^{5/2}}\right)}{(ae-bd)^{5/2}}$$

input `int(1/((d + e*x)^(3/2)*(a^2 + b^2*x^2 + 2*a*b*x)),x)`output `- ((2*e)/(a*e - b*d) + (3*b*e*(d + e*x))/(a*e - b*d)^2)/(b*(d + e*x)^(3/2) + (a*e - b*d)*(d + e*x)^(1/2)) - (3*b^(1/2)*e*atan((b^(1/2)*(d + e*x)^(1/2)*(a^2*e^2 + b^2*d^2 - 2*a*b*d*e))/(a*e - b*d)^(5/2)))/(a*e - b*d)^(5/2)`**Reduce [B] (verification not implemented)**

Time = 0.20 (sec) , antiderivative size = 220, normalized size of antiderivative = 2.22

$$\int \frac{1}{(d+ex)^{3/2}(a^2+2abx+b^2x^2)} dx = \frac{-3\sqrt{b}\sqrt{ex+d}\sqrt{ae-bd}\operatorname{atan}\left(\frac{\sqrt{ex+db}}{\sqrt{b}\sqrt{ae-bd}}\right)ae - 3\sqrt{b}\sqrt{ex+d}\sqrt{ae-bd}}{\sqrt{ex+d}(a^3be^3x - 3a^2b^2de^2x + 3ab^3d^2ex - \dots)}$$

input `int(1/(e*x+d)^(3/2)/(b^2*x^2+2*a*b*x+a^2),x)`output `(- 3*sqrt(b)*sqrt(d + e*x)*sqrt(a*e - b*d)*atan((sqrt(d + e*x)*b)/(sqrt(b)*sqrt(a*e - b*d)))*a*e - 3*sqrt(b)*sqrt(d + e*x)*sqrt(a*e - b*d)*atan((sqrt(d + e*x)*b)/(sqrt(b)*sqrt(a*e - b*d)))*b*e*x - 2*a**2*e**2 + a*b*d*e - 3*a*b*e**2*x + b**2*d**2 + 3*b**2*d*e*x)/(sqrt(d + e*x)*(a**4*e**3 - 3*a**3*b*d*e**2 + a**3*b*e**3*x + 3*a**2*b**2*d**2*e - 3*a**2*b**2*d*e**2*x - a*b**3*d**3 + 3*a*b**3*d**2*e*x - b**4*d**3*x))`

3.177 $\int \frac{1}{(d+ex)^{5/2}(a^2+2abx+b^2x^2)} dx$

| | |
|---|------|
| Optimal result | 1389 |
| Mathematica [A] (verified) | 1389 |
| Rubi [A] (verified) | 1390 |
| Maple [A] (verified) | 1392 |
| Fricas [B] (verification not implemented) | 1393 |
| Sympy [F] | 1394 |
| Maxima [F(-2)] | 1394 |
| Giac [B] (verification not implemented) | 1395 |
| Mupad [B] (verification not implemented) | 1396 |
| Reduce [B] (verification not implemented) | 1396 |

Optimal result

Integrand size = 28, antiderivative size = 124

$$\int \frac{1}{(d+ex)^{5/2}(a^2+2abx+b^2x^2)} dx = -\frac{5e}{3(bd-ae)^2(d+ex)^{3/2}} - \frac{1}{(bd-ae)(a+bx)(d+ex)^{3/2}} - \frac{5be}{(bd-ae)^3\sqrt{d+ex}} + \frac{5b^{3/2}e \operatorname{arctanh}\left(\frac{\sqrt{b}\sqrt{d+ex}}{\sqrt{bd-ae}}\right)}{(bd-ae)^{7/2}}$$

output

```
-5/3*e/(-a*e+b*d)^2/(e*x+d)^(3/2)-1/(-a*e+b*d)/(b*x+a)/(e*x+d)^(3/2)-5*b*e/(-a*e+b*d)^3/(e*x+d)^(1/2)+5*b^(3/2)*e*arctanh(b^(1/2)*(e*x+d)^(1/2)/(-a*e+b*d)^(1/2))/(-a*e+b*d)^(7/2)
```

Mathematica [A] (verified)

Time = 0.32 (sec) , antiderivative size = 125, normalized size of antiderivative = 1.01

$$\int \frac{1}{(d+ex)^{5/2}(a^2+2abx+b^2x^2)} dx = \frac{2a^2e^2 - 2abe(7d + 5ex) - b^2(3d^2 + 20dex + 15e^2x^2)}{3(bd - ae)^3(a + bx)(d + ex)^{3/2}} + \frac{5b^{3/2}e \arctan\left(\frac{\sqrt{b}\sqrt{d+ex}}{\sqrt{-bd+ae}}\right)}{(-bd + ae)^{7/2}}$$

input

```
Integrate[1/((d + e*x)^(5/2)*(a^2 + 2*a*b*x + b^2*x^2)),x]
```

output

$$\frac{(2a^2e^2 - 2ab^2e(7d + 5ex) - b^2(3d^2 + 20d^2ex + 15e^2x^2))}{(3(bd - ae)^3(a + bx)(d + ex)^{3/2}) + (5b^{3/2}e \operatorname{ArcTan}[\frac{\sqrt{b} \sqrt{d + ex}}{\sqrt{-(bd) + ae}}]) / (-(bd) + ae)^{7/2}}$$
Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 149, normalized size of antiderivative = 1.20, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {1098, 27, 52, 61, 61, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a^2 + 2abx + b^2x^2)(d + ex)^{5/2}} dx$$

$$\downarrow 1098$$

$$b^2 \int \frac{1}{b^2(a + bx)^2(d + ex)^{5/2}} dx$$

$$\downarrow 27$$

$$\int \frac{1}{(a + bx)^2(d + ex)^{5/2}} dx$$

$$\downarrow 52$$

$$-\frac{5e \int \frac{1}{(a+bx)(d+ex)^{5/2}} dx}{2(bd - ae)} - \frac{1}{(a + bx)(d + ex)^{3/2}(bd - ae)}$$

$$\downarrow 61$$

$$-\frac{5e \left(\frac{b \int \frac{1}{(a+bx)\sqrt{d+ex}} dx}{bd - ae} + \frac{2}{3(d+ex)^{3/2}(bd - ae)} \right)}{2(bd - ae)} - \frac{1}{(a + bx)(d + ex)^{3/2}(bd - ae)}$$

$$\downarrow 61$$

$$-\frac{5e \left(\frac{b \left(\frac{\int \frac{1}{(a+bx)\sqrt{d+ex}} dx}{bd - ae} + \frac{2}{\sqrt{d+ex}(bd - ae)} \right)}{2(bd - ae)} + \frac{2}{3(d+ex)^{3/2}(bd - ae)} \right)}{2(bd - ae)} - \frac{1}{(a + bx)(d + ex)^{3/2}(bd - ae)}$$

$$\begin{array}{c}
 \downarrow 73 \\
 5e \left(\frac{b \left(\frac{2b \int \frac{1}{a + \frac{b(d+ex) - bd}{e} d\sqrt{d+ex}}{e(bd-ae)} + \frac{2}{\sqrt{d+ex}(bd-ae)} \right)}{bd-ae} + \frac{2}{3(d+ex)^{3/2}(bd-ae)} \right) \\
 \hline
 \frac{2(bd - ae)}{1} \\
 \hline
 (a + bx)(d + ex)^{3/2}(bd - ae) \\
 \downarrow 221 \\
 5e \left(\frac{b \left(\frac{2}{\sqrt{d+ex}(bd-ae)} - \frac{2\sqrt{b} \operatorname{arctanh}\left(\frac{\sqrt{b}\sqrt{d+ex}}{\sqrt{bd-ae}}\right)}{(bd-ae)^{3/2}} \right)}{bd-ae} + \frac{2}{3(d+ex)^{3/2}(bd-ae)} \right) \\
 \hline
 \frac{2(bd - ae)}{1} \\
 \hline
 (a + bx)(d + ex)^{3/2}(bd - ae)
 \end{array}$$

input `Int[1/((d + e*x)^(5/2)*(a^2 + 2*a*b*x + b^2*x^2)),x]`

output `-(1/((b*d - a*e)*(a + b*x)*(d + e*x)^(3/2))) - (5*e*(2/(3*(b*d - a*e)*(d + e*x)^(3/2))) + (b*(2/((b*d - a*e)*Sqrt[d + e*x]) - (2*Sqrt[b]*ArcTanh[(Sqrt[b]*Sqrt[d + e*x])/Sqrt[b*d - a*e]])/(b*d - a*e)^(3/2)))/(b*d - a*e))/((2*(b*d - a*e))`

Defintions of rubi rules used

rule 27 `Int[(a_)*(Fx_), x_Symbol] := Simp[a Int[Fx, x], x] /; FreeQ[a, x] && !MatchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]`

rule 52 `Int[((a_.) + (b_.)*(x_.))^(m_.)*((c_.) + (d_.)*(x_.))^(n_.), x_Symbol] := Simp[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((m + n + 2)/((b*c - a*d)*(m + 1)) Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && ILtQ[m, -1] && FractionQ[n] && LtQ[n, 0]`

rule 61 `Int[((a_.) + (b_.)*(x_)^(m_))*((c_.) + (d_.)*(x_)^(n_)), x_Symbol] := Simp[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((m + n + 2)/((b*c - a*d)*(m + 1))) Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && LtQ[m, -1] && !(LtQ[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && IntLinearQ[a, b, c, d, m, n, x]`

rule 73 `Int[((a_.) + (b_.)*(x_)^(m_))*((c_.) + (d_.)*(x_)^(n_)), x_Symbol] := With[{p = Denominator[m]}, Simp[p/b Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) + d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntLinearQ[a, b, c, d, m, n, x]`

rule 221 `Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[-a/b, 2]/a)*ArcTanh[x/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]`

rule 1098 `Int[((d_.) + (e_.)*(x_)^(m_.))*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] := Simp[1/c^p Int[(d + e*x)^m*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]`

Maple [A] (verified)

Time = 1.54 (sec) , antiderivative size = 121, normalized size of antiderivative = 0.98

| method | result | size |
|-------------------|--|------|
| derivativedivides | $2e \left(\frac{b^2 \left(\frac{\sqrt{ex+d}}{2b(ex+d)+2ae-2bd} + \frac{5 \arctan\left(\frac{b\sqrt{ex+d}}{\sqrt{b(ae-bd)}}\right)}{2\sqrt{b(ae-bd)}} \right)}{(ae-bd)^3} - \frac{1}{3(ae-bd)^2(ex+d)^{\frac{3}{2}}} + \frac{2b}{(ae-bd)^3\sqrt{ex+d}} \right)$ | 121 |
| default | $2e \left(\frac{b^2 \left(\frac{\sqrt{ex+d}}{2b(ex+d)+2ae-2bd} + \frac{5 \arctan\left(\frac{b\sqrt{ex+d}}{\sqrt{b(ae-bd)}}\right)}{2\sqrt{b(ae-bd)}} \right)}{(ae-bd)^3} - \frac{1}{3(ae-bd)^2(ex+d)^{\frac{3}{2}}} + \frac{2b}{(ae-bd)^3\sqrt{ex+d}} \right)$ | 121 |
| pseudoelliptic | $e \left(-\frac{2}{3(ae-bd)^2(ex+d)^{\frac{3}{2}}} + \frac{4b}{(ae-bd)^3\sqrt{ex+d}} + \frac{\sqrt{ex+d}b^2}{(bx+a)e(ae-bd)^3} + \frac{5 \arctan\left(\frac{b\sqrt{ex+d}}{\sqrt{b(ae-bd)}}\right)b^2}{\sqrt{b(ae-bd)}(ae-bd)^3} \right)$ | 123 |

input `int(1/(e*x+d)^(5/2)/(b^2*x^2+2*a*b*x+a^2),x,method=_RETURNVERBOSE)`

output `2*e*(1/(a*e-b*d)^3*b^2*(1/2*(e*x+d)^(1/2)/(b*(e*x+d)+a*e-b*d)+5/2/(b*(a*e-b*d))^(1/2)*arctan(b*(e*x+d)^(1/2)/(b*(a*e-b*d))^(1/2)))-1/3/(a*e-b*d)^2/(e*x+d)^(3/2)+2/(a*e-b*d)^3*b/(e*x+d)^(1/2)`

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 366 vs. 2(106) = 212.

Time = 0.12 (sec) , antiderivative size = 762, normalized size of antiderivative = 6.15

$$\int \frac{1}{(d+ex)^{5/2}(a^2+2abx+b^2x^2)} dx = \left[-\frac{15(b^2e^3x^3 + abd^2e + (2b^2de^2 + abe^3)x^2 + (b^2d^2e + 2abd^2e)x + b^2d^2e)}{6(ab^3d^5 - 3a^2b^2d^4e + 3a^3bd^3e^2 - a^4d^2e^3 + (b^4d^3e^2 - 3ab^3d^2e^3 + 3a^2b^2de^4 - a^3be^5)x^3 + (2b^4d^4e - 5ab^3d^3e^2 + 3a^2b^2de^4 - a^3be^5)x^2 + (2b^4d^4e - 5ab^3d^3e^2 + 3a^2b^2de^4 - a^3be^5)x + (2b^4d^4e - 5ab^3d^3e^2 + 3a^2b^2de^4 - a^3be^5))} \right]$$

input `integrate(1/(e*x+d)^(5/2)/(b^2*x^2+2*a*b*x+a^2),x, algorithm="fricas")`

output

```
[-1/6*(15*(b^2*e^3*x^3 + a*b*d^2*e + (2*b^2*d*e^2 + a*b*e^3)*x^2 + (b^2*d^2*e + 2*a*b*d*e^2)*x)*sqrt(b/(b*d - a*e))*log((b*e*x + 2*b*d - a*e - 2*(b*d - a*e)*sqrt(e*x + d)*sqrt(b/(b*d - a*e)))/(b*x + a)) + 2*(15*b^2*e^2*x^2 + 3*b^2*d^2 + 14*a*b*d*e - 2*a^2*e^2 + 10*(2*b^2*d*e + a*b*e^2)*x)*sqrt(e*x + d))/(a*b^3*d^5 - 3*a^2*b^2*d^4*e + 3*a^3*b*d^3*e^2 - a^4*d^2*e^3 + (b^4*d^3*e^2 - 3*a*b^3*d^2*e^3 + 3*a^2*b^2*d*e^4 - a^3*b*e^5)*x^3 + (2*b^4*d^4*e - 5*a*b^3*d^3*e^2 + 3*a^2*b^2*d^2*e^3 + a^3*b*d*e^4 - a^4*e^5)*x^2 + (b^4*d^5 - a*b^3*d^4*e - 3*a^2*b^2*d^3*e^2 + 5*a^3*b*d^2*e^3 - 2*a^4*d*e^4)*x), -1/3*(15*(b^2*e^3*x^3 + a*b*d^2*e + (2*b^2*d*e^2 + a*b*e^3)*x^2 + (b^2*d^2*e + 2*a*b*d*e^2)*x)*sqrt(-b/(b*d - a*e))*arctan(sqrt(e*x + d)*sqrt(-b/(b*d - a*e))) + (15*b^2*e^2*x^2 + 3*b^2*d^2 + 14*a*b*d*e - 2*a^2*e^2 + 10*(2*b^2*d*e + a*b*e^2)*x)*sqrt(e*x + d))/(a*b^3*d^5 - 3*a^2*b^2*d^4*e + 3*a^3*b*d^3*e^2 - a^4*d^2*e^3 + (b^4*d^3*e^2 - 3*a*b^3*d^2*e^3 + 3*a^2*b^2*d*e^4 - a^3*b*e^5)*x^3 + (2*b^4*d^4*e - 5*a*b^3*d^3*e^2 + 3*a^2*b^2*d^2*e^3 + a^3*b*d*e^4 - a^4*e^5)*x^2 + (b^4*d^5 - a*b^3*d^4*e - 3*a^2*b^2*d^3*e^2 + 5*a^3*b*d^2*e^3 - 2*a^4*d*e^4)*x)]
```

Sympy [F]

$$\int \frac{1}{(d+ex)^{5/2}(a^2+2abx+b^2x^2)} dx = \int \frac{1}{(a+bx)^2(d+ex)^{5/2}} dx$$

input

```
integrate(1/(e*x+d)**(5/2)/(b**2*x**2+2*a*b*x+a**2),x)
```

output

```
Integral(1/((a + b*x)**2*(d + e*x)**(5/2)), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{(d+ex)^{5/2}(a^2+2abx+b^2x^2)} dx = \text{Exception raised: ValueError}$$

input

```
integrate(1/(e*x+d)^(5/2)/(b^2*x^2+2*a*b*x+a^2),x, algorithm="maxima")
```

output

```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
help (example of legal syntax is 'assume(a*e-b*d>0)', see `assume?` for m
ore detail
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 216 vs. $2(106) = 212$.

Time = 0.15 (sec) , antiderivative size = 216, normalized size of antiderivative = 1.74

$$\int \frac{1}{(d+ex)^{5/2}(a^2+2abx+b^2x^2)} dx =$$

$$\frac{5b^2e \arctan\left(\frac{\sqrt{ex+db}}{\sqrt{-b^2d+abe}}\right)}{(b^3d^3 - 3ab^2d^2e + 3a^2bde^2 - a^3e^3)\sqrt{-b^2d+abe}}$$

$$\frac{\sqrt{ex+db^2e}}{(b^3d^3 - 3ab^2d^2e + 3a^2bde^2 - a^3e^3)((ex+d)b - bd + ae)}$$

$$\frac{2(6(ex+d)be + bde - ae^2)}{3(b^3d^3 - 3ab^2d^2e + 3a^2bde^2 - a^3e^3)(ex+d)^{3/2}}$$

input

```
integrate(1/(e*x+d)^(5/2)/(b^2*x^2+2*a*b*x+a^2),x, algorithm="giac")
```

output

```
-5*b^2*e*arctan(sqrt(e*x + d)*b/sqrt(-b^2*d + a*b*e))/((b^3*d^3 - 3*a*b^2*
d^2*e + 3*a^2*b*d*e^2 - a^3*e^3)*sqrt(-b^2*d + a*b*e)) - sqrt(e*x + d)*b^2
*e/((b^3*d^3 - 3*a*b^2*d^2*e + 3*a^2*b*d*e^2 - a^3*e^3)*((e*x + d)*b - b*d
+ a*e)) - 2/3*(6*(e*x + d)*b*e + b*d*e - a*e^2)/((b^3*d^3 - 3*a*b^2*d^2*e
+ 3*a^2*b*d*e^2 - a^3*e^3)*(e*x + d)^(3/2))
```


Mupad [B] (verification not implemented)

Time = 9.54 (sec) , antiderivative size = 161, normalized size of antiderivative = 1.30

$$\int \frac{1}{(d+ex)^{5/2}(a^2+2abx+b^2x^2)} dx = \frac{\frac{10be(d+ex)}{3(ae-bd)^2} - \frac{2e}{3(ae-bd)} + \frac{5b^2e(d+ex)^2}{(ae-bd)^3}}{b(d+ex)^{5/2} + (ae-bd)(d+ex)^{3/2}} + \frac{5b^{3/2}e \operatorname{atan}\left(\frac{\sqrt{b}\sqrt{d+ex}(a^3e^3-3a^2bde^2+3ab^2d^2e-b^3d^3)}{(ae-bd)^{7/2}}\right)}{(ae-bd)^{7/2}}$$

input `int(1/((d + e*x)^(5/2)*(a^2 + b^2*x^2 + 2*a*b*x)),x)`output `((10*b*e*(d + e*x))/(3*(a*e - b*d)^2) - (2*e)/(3*(a*e - b*d)) + (5*b^2*e*(d + e*x)^2)/(a*e - b*d)^3)/(b*(d + e*x)^(5/2) + (a*e - b*d)*(d + e*x)^(3/2)) + (5*b^(3/2)*e*atan((b^(1/2)*(d + e*x)^(1/2)*(a^3*e^3 - b^3*d^3 + 3*a*b^2*d^2*e - 3*a^2*b*d*e^2))/(a*e - b*d)^(7/2)))/(a*e - b*d)^(7/2)`**Reduce [B] (verification not implemented)**

Time = 0.22 (sec) , antiderivative size = 493, normalized size of antiderivative = 3.98

$$\int \frac{1}{(d+ex)^{5/2}(a^2+2abx+b^2x^2)} dx = \frac{15\sqrt{b}\sqrt{ex+d}\sqrt{ae-bd} \operatorname{atan}\left(\frac{\sqrt{ex+db}}{\sqrt{b}\sqrt{ae-bd}}\right) abde + 15\sqrt{b}\sqrt{ex+d}\sqrt{a^2d}}{3\sqrt{ex+d}(a^2d+2abd+bd^2)}$$

input `int(1/(e*x+d)^(5/2)/(b^2*x^2+2*a*b*x+a^2),x)`

output

```
(15*sqrt(b)*sqrt(d + e*x)*sqrt(a*e - b*d)*atan((sqrt(d + e*x)*b)/(sqrt(b)*
sqrt(a*e - b*d)))*a*b*d*e + 15*sqrt(b)*sqrt(d + e*x)*sqrt(a*e - b*d)*atan(
(sqrt(d + e*x)*b)/(sqrt(b)*sqrt(a*e - b*d)))*a*b*e**2*x + 15*sqrt(b)*sqrt(
d + e*x)*sqrt(a*e - b*d)*atan((sqrt(d + e*x)*b)/(sqrt(b)*sqrt(a*e - b*d)))
*b**2*d*e*x + 15*sqrt(b)*sqrt(d + e*x)*sqrt(a*e - b*d)*atan((sqrt(d + e*x)
*b)/(sqrt(b)*sqrt(a*e - b*d)))*b**2*e**2*x**2 - 2*a**3*e**3 + 16*a**2*b*d*
e**2 + 10*a**2*b*e**3*x - 11*a*b**2*d**2*e + 10*a*b**2*d*e**2*x + 15*a*b**
2*e**3*x**2 - 3*b**3*d**3 - 20*b**3*d**2*e*x - 15*b**3*d*e**2*x**2)/(3*sqrt
(d + e*x)*(a**5*d*e**4 + a**5*e**5*x - 4*a**4*b*d**2*e**3 - 3*a**4*b*d*e*
*4*x + a**4*b*e**5*x**2 + 6*a**3*b**2*d**3*e**2 + 2*a**3*b**2*d**2*e**3*x
- 4*a**3*b**2*d*e**4*x**2 - 4*a**2*b**3*d**4*e + 2*a**2*b**3*d**3*e**2*x +
6*a**2*b**3*d**2*e**3*x**2 + a*b**4*d**5 - 3*a*b**4*d**4*e*x - 4*a*b**4*d
**3*e**2*x**2 + b**5*d**5*x + b**5*d**4*e*x**2))
```

3.178 $\int \frac{1}{(d+ex)^{7/2}(a^2+2abx+b^2x^2)} dx$

| | |
|---|------|
| Optimal result | 1398 |
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Optimal result

Integrand size = 28, antiderivative size = 151

$$\int \frac{1}{(d+ex)^{7/2}(a^2+2abx+b^2x^2)} dx = -\frac{7e}{5(bd-ae)^2(d+ex)^{5/2}} - \frac{1}{(bd-ae)(a+bx)(d+ex)^{5/2}} - \frac{7be}{3(bd-ae)^3(d+ex)^{3/2}} - \frac{7b^2e}{(bd-ae)^4\sqrt{d+ex}} + \frac{7b^{5/2}e \operatorname{arctanh}\left(\frac{\sqrt{b}\sqrt{d+ex}}{\sqrt{bd-ae}}\right)}{(bd-ae)^{9/2}}$$

output

```
-7/5*e/(-a*e+b*d)^2/(e*x+d)^(5/2)-1/(-a*e+b*d)/(b*x+a)/(e*x+d)^(5/2)-7/3*b
*e/(-a*e+b*d)^3/(e*x+d)^(3/2)-7*b^2*e/(-a*e+b*d)^4/(e*x+d)^(1/2)+7*b^(5/2)
*e*arctanh(b^(1/2)*(e*x+d)^(1/2)/(-a*e+b*d)^(1/2))/(-a*e+b*d)^(9/2)
```

Mathematica [A] (verified)

Time = 0.35 (sec) , antiderivative size = 166, normalized size of antiderivative = 1.10

$$\int \frac{1}{(d+ex)^{7/2}(a^2+2abx+b^2x^2)} dx = \frac{-6a^3e^3+2a^2be^2(16d+7ex)-2ab^2e(58d^2+84dex+35e^2x^2)-b^3}{15(bd-ae)^4(a+bx)(d+ex)} - \frac{7b^{5/2}e \arctan\left(\frac{\sqrt{b}\sqrt{d+ex}}{\sqrt{-bd+ae}}\right)}{(-bd+ae)^{9/2}}$$

input `Integrate[1/((d + e*x)^(7/2)*(a^2 + 2*a*b*x + b^2*x^2)),x]`

output $(-6*a^3*e^3 + 2*a^2*b*e^2*(16*d + 7*e*x) - 2*a*b^2*e*(58*d^2 + 84*d*e*x + 35*e^2*x^2) - b^3*(15*d^3 + 161*d^2*e*x + 245*d*e^2*x^2 + 105*e^3*x^3))/(15*(b*d - a*e)^4*(a + b*x)*(d + e*x)^(5/2)) - (7*b^(5/2)*e*ArcTan[(Sqrt[b]*Sqrt[d + e*x])/Sqrt[-(b*d) + a*e]])/(-(b*d) + a*e)^(9/2)$

Rubi [A] (verified)

Time = 0.50 (sec) , antiderivative size = 185, normalized size of antiderivative = 1.23, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {1098, 27, 52, 61, 61, 61, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\begin{aligned}
 & \int \frac{1}{(a^2 + 2abx + b^2x^2)(d + ex)^{7/2}} dx \\
 & \quad \downarrow 1098 \\
 & b^2 \int \frac{1}{b^2(a + bx)^2(d + ex)^{7/2}} dx \\
 & \quad \downarrow 27 \\
 & \int \frac{1}{(a + bx)^2(d + ex)^{7/2}} dx \\
 & \quad \downarrow 52 \\
 & -\frac{7e \int \frac{1}{(a+bx)(d+ex)^{7/2}} dx}{2(bd - ae)} - \frac{1}{(a + bx)(d + ex)^{5/2}(bd - ae)} \\
 & \quad \downarrow 61 \\
 & -\frac{7e \left(\frac{b \int \frac{1}{(a+bx)(d+ex)^{5/2}} dx}{bd - ae} + \frac{2}{5(d+ex)^{5/2}(bd - ae)} \right)}{2(bd - ae)} - \frac{1}{(a + bx)(d + ex)^{5/2}(bd - ae)} \\
 & \quad \downarrow 61
 \end{aligned}$$

$$\begin{aligned}
 & \frac{7e \left(\frac{b \left(\frac{b \int \frac{1}{(a+bx)(d+ex)^{3/2}} dx}{bd-ae} + \frac{2}{3(d+ex)^{3/2}(bd-ae)} \right)}{bd-ae} + \frac{2}{5(d+ex)^{5/2}(bd-ae)} \right)}{\frac{2(bd-ae)}{1} \frac{1}{(a+bx)(d+ex)^{5/2}(bd-ae)}} \\
 & \quad \downarrow 61 \\
 & \frac{7e \left(\frac{b \left(\frac{b \int \frac{1}{(a+bx)\sqrt{d+ex}} dx}{bd-ae} + \frac{2}{3(d+ex)^{3/2}(bd-ae)} \right)}{bd-ae} + \frac{2}{5(d+ex)^{5/2}(bd-ae)} \right)}{\frac{2(bd-ae)}{1} \frac{1}{(a+bx)(d+ex)^{5/2}(bd-ae)}} \\
 & \quad \downarrow 73 \\
 & \frac{7e \left(\frac{b \left(\frac{2b \int \frac{1}{a + \frac{b(d+ex) - bd}{e}} d\sqrt{d+ex}}{e(bd-ae)} + \frac{2}{3(d+ex)^{3/2}(bd-ae)} \right)}{bd-ae} + \frac{2}{5(d+ex)^{5/2}(bd-ae)} \right)}{\frac{2(bd-ae)}{1} \frac{1}{(a+bx)(d+ex)^{5/2}(bd-ae)}} \\
 & \quad \downarrow 221
 \end{aligned}$$

$$\frac{7e \left(\frac{b \left(\frac{2}{\sqrt{d+ex}(bd-ae)} - \frac{2\sqrt{b}\operatorname{arctanh}\left(\frac{\sqrt{b}\sqrt{d+ex}}{\sqrt{bd-ae}}\right)}{(bd-ae)^{3/2}} \right)}{bd-ae} + \frac{2}{3(d+ex)^{3/2}(bd-ae)} \right)}{bd-ae} + \frac{2}{5(d+ex)^{5/2}(bd-ae)} \right)}{\frac{2(bd-ae)}{1} (a+bx)(d+ex)^{5/2}(bd-ae)}$$

input `Int[1/((d + e*x)^(7/2)*(a^2 + 2*a*b*x + b^2*x^2)),x]`

output `-(1/((b*d - a*e)*(a + b*x)*(d + e*x)^(5/2))) - (7*e*(2/(5*(b*d - a*e)*(d + e*x)^(5/2)) + (b*(2/(3*(b*d - a*e)*(d + e*x)^(3/2)) + (b*(2/((b*d - a*e)* Sqrt[d + e*x])) - (2*Sqrt[b]*ArcTanh[(Sqrt[b]*Sqrt[d + e*x])/Sqrt[b*d - a*e]])/(b*d - a*e)^(3/2))/(b*d - a*e)))/(b*d - a*e)))/(2*(b*d - a*e))`

Defintions of rubi rules used

rule 27 `Int[(a_)*(Fx_), x_Symbol] := Simp[a Int[Fx, x], x] /; FreeQ[a, x] && !MatchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]`

rule 52 `Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] := Simp[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((m + n + 2)/((b*c - a*d)*(m + 1))) Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && ILtQ[m, -1] && FractionQ[n] && LtQ[n, 0]`

rule 61 `Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] := Simp[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((m + n + 2)/((b*c - a*d)*(m + 1))) Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && LtQ[m, -1] && !(LtQ[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && IntLinearQ[a, b, c, d, m, n, x]`

```
rule 73 Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] := With[
{p = Denominator[m]}, Simp[p/b Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^(n), x], x, (a + b*x)^(1/p)], x] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

```
rule 221 Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

```
rule 1098 Int[((d_.) + (e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] := Simp[1/c^p Int[(d + e*x)^m*(b/2 + c*x)^(2*p), x], x] /; FreeQ[
{a, b, c, d, e, m}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Maple [A] (verified)

Time = 1.46 (sec) , antiderivative size = 144, normalized size of antiderivative = 0.95

| method | result |
|-------------------|--|
| derivativedivides | $2e \left(-\frac{1}{5(ae-bd)^2(ex+d)^{\frac{5}{2}}} - \frac{3b^2}{(ae-bd)^4\sqrt{ex+d}} + \frac{2b}{3(ae-bd)^3(ex+d)^{\frac{3}{2}}} - \frac{b^3 \left(\frac{\sqrt{ex+d}}{2b(ex+d)+2ae-2bd} + \frac{7 \arctan\left(\frac{b\sqrt{e}}{\sqrt{b(ae-bd)}}\right)}{2\sqrt{b(ae-bd)}} \right)}{(ae-bd)^4} \right)$ |
| default | $2e \left(-\frac{1}{5(ae-bd)^2(ex+d)^{\frac{5}{2}}} - \frac{3b^2}{(ae-bd)^4\sqrt{ex+d}} + \frac{2b}{3(ae-bd)^3(ex+d)^{\frac{3}{2}}} - \frac{b^3 \left(\frac{\sqrt{ex+d}}{2b(ex+d)+2ae-2bd} + \frac{7 \arctan\left(\frac{b\sqrt{e}}{\sqrt{b(ae-bd)}}\right)}{2\sqrt{b(ae-bd)}} \right)}{(ae-bd)^4} \right)$ |
| pseudoelliptic | $2 \left(\frac{35b^3e(ex+d)^{\frac{5}{2}}(bx+a) \arctan\left(\frac{b\sqrt{ex+d}}{\sqrt{b(ae-bd)}}\right)}{2} + \left(\frac{(35e^3x^3 + \frac{245}{3}de^2x^2 + \frac{161}{3}d^2ex + 5d^3)b^3}{2} + \frac{58e \left(\frac{35}{38}e^2x^2 + \frac{42}{29}dex + d^2 \right) a b^2}{3} \right) \right) / (5(ex+d)^{\frac{5}{2}}\sqrt{b(ae-bd)}(ae-bd)^4(bx+a))$ |

```
input int(1/(e*x+d)^(7/2)/(b^2*x^2+2*a*b*x+a^2), x, method=_RETURNVERBOSE)
```

output

```
2*e*(-1/5/(a*e-b*d)^2/(e*x+d)^(5/2)-3/(a*e-b*d)^4*b^2/(e*x+d)^(1/2)+2/3/(a
*e-b*d)^3*b/(e*x+d)^(3/2)-1/(a*e-b*d)^4*b^3*(1/2*(e*x+d)^(1/2)/(b*(e*x+d)+
a*e-b*d)+7/2/(b*(a*e-b*d))^(1/2)*arctan(b*(e*x+d)^(1/2)/(b*(a*e-b*d))^(1/2
))))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 584 vs. $2(129) = 258$.

Time = 0.14 (sec) , antiderivative size = 1198, normalized size of antiderivative = 7.93

$$\int \frac{1}{(d+ex)^{7/2}(a^2+2abx+b^2x^2)} dx = \text{Too large to display}$$

input

```
integrate(1/(e*x+d)^(7/2)/(b^2*x^2+2*a*b*x+a^2),x, algorithm="fricas")
```

output

```
[1/30*(105*(b^3*e^4*x^4 + a*b^2*d^3*e + (3*b^3*d*e^3 + a*b^2*e^4)*x^3 + 3*
(b^3*d^2*e^2 + a*b^2*d*e^3)*x^2 + (b^3*d^3*e + 3*a*b^2*d^2*e^2)*x)*sqrt(b/
(b*d - a*e))*log((b*e*x + 2*b*d - a*e + 2*(b*d - a*e)*sqrt(e*x + d)*sqrt(b
/(b*d - a*e)))/(b*x + a)) - 2*(105*b^3*e^3*x^3 + 15*b^3*d^3 + 116*a*b^2*d^
2*e - 32*a^2*b*d*e^2 + 6*a^3*e^3 + 35*(7*b^3*d*e^2 + 2*a*b^2*e^3)*x^2 + 7*
(23*b^3*d^2*e + 24*a*b^2*d*e^2 - 2*a^2*b*e^3)*x)*sqrt(e*x + d))/(a*b^4*d^7
- 4*a^2*b^3*d^6*e + 6*a^3*b^2*d^5*e^2 - 4*a^4*b*d^4*e^3 + a^5*d^3*e^4 + (
b^5*d^4*e^3 - 4*a*b^4*d^3*e^4 + 6*a^2*b^3*d^2*e^5 - 4*a^3*b^2*d*e^6 + a^4*
b*e^7)*x^4 + (3*b^5*d^5*e^2 - 11*a*b^4*d^4*e^3 + 14*a^2*b^3*d^3*e^4 - 6*a^
3*b^2*d^2*e^5 - a^4*b*d*e^6 + a^5*e^7)*x^3 + 3*(b^5*d^6*e - 3*a*b^4*d^5*e^
2 + 2*a^2*b^3*d^4*e^3 + 2*a^3*b^2*d^3*e^4 - 3*a^4*b*d^2*e^5 + a^5*d*e^6)*x
^2 + (b^5*d^7 - a*b^4*d^6*e - 6*a^2*b^3*d^5*e^2 + 14*a^3*b^2*d^4*e^3 - 11*
a^4*b*d^3*e^4 + 3*a^5*d^2*e^5)*x), -1/15*(105*(b^3*e^4*x^4 + a*b^2*d^3*e +
(3*b^3*d*e^3 + a*b^2*e^4)*x^3 + 3*(b^3*d^2*e^2 + a*b^2*d*e^3)*x^2 + (b^3*
d^3*e + 3*a*b^2*d^2*e^2)*x)*sqrt(-b/(b*d - a*e))*arctan(sqrt(e*x + d)*sqrt
(-b/(b*d - a*e))) + (105*b^3*e^3*x^3 + 15*b^3*d^3 + 116*a*b^2*d^2*e - 32*a
^2*b*d*e^2 + 6*a^3*e^3 + 35*(7*b^3*d*e^2 + 2*a*b^2*e^3)*x^2 + 7*(23*b^3*d^
2*e + 24*a*b^2*d*e^2 - 2*a^2*b*e^3)*x)*sqrt(e*x + d))/(a*b^4*d^7 - 4*a^2*
b^3*d^6*e + 6*a^3*b^2*d^5*e^2 - 4*a^4*b*d^4*e^3 + a^5*d^3*e^4 + (b^5*d^4*
e^3 - 4*a*b^4*d^3*e^4 + 6*a^2*b^3*d^2*e^5 - 4*a^3*b^2*d*e^6 + a^4*b*e^7)*...
```


Sympy [F]

$$\int \frac{1}{(d+ex)^{7/2} (a^2+2abx+b^2x^2)} dx = \int \frac{1}{(a+bx)^2 (d+ex)^{7/2}} dx$$

input `integrate(1/(e*x+d)**(7/2)/(b**2*x**2+2*a*b*x+a**2),x)`

output `Integral(1/((a + b*x)**2*(d + e*x)**(7/2)), x)`

Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{(d+ex)^{7/2} (a^2+2abx+b^2x^2)} dx = \text{Exception raised: ValueError}$$

input `integrate(1/(e*x+d)^(7/2)/(b^2*x^2+2*a*b*x+a^2),x, algorithm="maxima")`

output `Exception raised: ValueError >> Computation failed since Maxima requested additional constraints; using the 'assume' command before evaluation *may* help (example of legal syntax is 'assume(a*e-b*d>0)', see `assume?` for more detail`

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 298 vs. $2(129) = 258$.

Time = 0.15 (sec) , antiderivative size = 298, normalized size of antiderivative = 1.97

$$\int \frac{1}{(d+ex)^{7/2} (a^2+2abx+b^2x^2)} dx =$$

$$\frac{7b^3e \arctan\left(\frac{\sqrt{ex+db}}{\sqrt{-b^2d+abe}}\right)}{(b^4d^4 - 4ab^3d^3e + 6a^2b^2d^2e^2 - 4a^3bde^3 + a^4e^4)\sqrt{-b^2d+abe}}$$

$$- \frac{\sqrt{ex+db}^3e}{(b^4d^4 - 4ab^3d^3e + 6a^2b^2d^2e^2 - 4a^3bde^3 + a^4e^4)((ex+d)b - bd + ae)}$$

$$- \frac{2(45(ex+d)^2b^2e + 10(ex+d)b^2de + 3b^2d^2e - 10(ex+d)abe^2 - 6abde^2 + 3a^2e^3)}{15(b^4d^4 - 4ab^3d^3e + 6a^2b^2d^2e^2 - 4a^3bde^3 + a^4e^4)(ex+d)^{5/2}}$$

input `integrate(1/(e*x+d)^(7/2)/(b^2*x^2+2*a*b*x+a^2),x, algorithm="giac")`

output

```
-7*b^3*e*arctan(sqrt(e*x + d)*b/sqrt(-b^2*d + a*b*e))/((b^4*d^4 - 4*a*b^3*d^3*e + 6*a^2*b^2*d^2*e^2 - 4*a^3*b*d*e^3 + a^4*e^4)*sqrt(-b^2*d + a*b*e)) - sqrt(e*x + d)*b^3*e/((b^4*d^4 - 4*a*b^3*d^3*e + 6*a^2*b^2*d^2*e^2 - 4*a^3*b*d*e^3 + a^4*e^4)*((e*x + d)*b - b*d + a*e)) - 2/15*(45*(e*x + d)^2*b^2*e + 10*(e*x + d)*b^2*d*e + 3*b^2*d^2*e - 10*(e*x + d)*a*b*e^2 - 6*a*b*d*e^2 + 3*a^2*e^3)/((b^4*d^4 - 4*a*b^3*d^3*e + 6*a^2*b^2*d^2*e^2 - 4*a^3*b*d*e^3 + a^4*e^4)*(e*x + d)^(5/2))
```

Mupad [B] (verification not implemented)

Time = 9.53 (sec) , antiderivative size = 198, normalized size of antiderivative = 1.31

$$\int \frac{1}{(d+ex)^{7/2} (a^2+2abx+b^2x^2)} dx =$$

$$\frac{\frac{2e}{5(ae-bd)} - \frac{14be(d+ex)}{15(ae-bd)^2} + \frac{14b^2e(d+ex)^2}{3(ae-bd)^3} + \frac{7b^3e(d+ex)^3}{(ae-bd)^4}}{b(d+ex)^{7/2} + (ae-bd)(d+ex)^{5/2}}$$

$$- \frac{7b^{5/2}e \operatorname{atan}\left(\frac{\sqrt{b}\sqrt{d+ex}(a^4e^4-4a^3bde^3+6a^2b^2d^2e^2-4ab^3d^3e+b^4d^4)}{(ae-bd)^{9/2}}\right)}{(ae-bd)^{9/2}}$$

input `int(1/((d + e*x)^(7/2)*(a^2 + b^2*x^2 + 2*a*b*x)),x)`

output

```
- ((2*e)/(5*(a*e - b*d)) - (14*b*e*(d + e*x))/(15*(a*e - b*d)^2) + (14*b^2
*e*(d + e*x)^2)/(3*(a*e - b*d)^3) + (7*b^3*e*(d + e*x)^3)/(a*e - b*d)^4)/(
b*(d + e*x)^(7/2) + (a*e - b*d)*(d + e*x)^(5/2)) - (7*b^(5/2)*e*atan((b^(1
/2)*(d + e*x)^(1/2)*(a^4*e^4 + b^4*d^4 + 6*a^2*b^2*d^2*e^2 - 4*a*b^3*d^3*e
- 4*a^3*b*d*e^3))/(a*e - b*d)^(9/2)))/(a*e - b*d)^(9/2)
```

Reduce [B] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 804, normalized size of antiderivative = 5.32

$$\int \frac{1}{(d+ex)^{7/2}(a^2+2abx+b^2x^2)} dx = \frac{-105\sqrt{b}\sqrt{ex+d}\sqrt{ae-bd}\operatorname{atan}\left(\frac{\sqrt{ex+db}}{\sqrt{b}\sqrt{ae-bd}}\right)ab^2d^2e - 210\sqrt{b}\sqrt{ex}}$$

input

```
int(1/(e*x+d)^(7/2)/(b^2*x^2+2*a*b*x+a^2),x)
```

output

```
( - 105*sqrt(b)*sqrt(d + e*x)*sqrt(a*e - b*d)*atan((sqrt(d + e*x)*b)/(sqrt
(b)*sqrt(a*e - b*d)))*a*b**2*d**2*e - 210*sqrt(b)*sqrt(d + e*x)*sqrt(a*e -
b*d)*atan((sqrt(d + e*x)*b)/(sqrt(b)*sqrt(a*e - b*d)))*a*b**2*d*e**2*x -
105*sqrt(b)*sqrt(d + e*x)*sqrt(a*e - b*d)*atan((sqrt(d + e*x)*b)/(sqrt(b)*
sqrt(a*e - b*d)))*a*b**2*e**3*x**2 - 105*sqrt(b)*sqrt(d + e*x)*sqrt(a*e -
b*d)*atan((sqrt(d + e*x)*b)/(sqrt(b)*sqrt(a*e - b*d)))*b**3*d**2*e*x - 210
*sqrt(b)*sqrt(d + e*x)*sqrt(a*e - b*d)*atan((sqrt(d + e*x)*b)/(sqrt(b)*sqr
t(a*e - b*d)))*b**3*d*e**2*x**2 - 105*sqrt(b)*sqrt(d + e*x)*sqrt(a*e - b*d
)*atan((sqrt(d + e*x)*b)/(sqrt(b)*sqrt(a*e - b*d)))*b**3*e**3*x**3 - 6*a**
4*e**4 + 38*a**3*b*d*e**3 + 14*a**3*b*e**4*x - 148*a**2*b**2*d**2*e**2 - 1
82*a**2*b**2*d*e**3*x - 70*a**2*b**2*e**4*x**2 + 101*a*b**3*d**3*e + 7*a*b
**3*d**2*e**2*x - 175*a*b**3*d*e**3*x**2 - 105*a*b**3*e**4*x**3 + 15*b**4*
d**4 + 161*b**4*d**3*e*x + 245*b**4*d**2*e**2*x**2 + 105*b**4*d*e**3*x**3)
/(15*sqrt(d + e*x)*(a**6*d**2*e**5 + 2*a**6*d*e**6*x + a**6*e**7*x**2 - 5*
a**5*b*d**3*e**4 - 9*a**5*b*d**2*e**5*x - 3*a**5*b*d*e**6*x**2 + a**5*b*e
**7*x**3 + 10*a**4*b**2*d**4*e**3 + 15*a**4*b**2*d**3*e**4*x - 5*a**4*b**2*
d*e**6*x**3 - 10*a**3*b**3*d**5*e**2 - 10*a**3*b**3*d**4*e**3*x + 10*a**3*
b**3*d**3*e**4*x**2 + 10*a**3*b**3*d**2*e**5*x**3 + 5*a**2*b**4*d**6*e - 1
5*a**2*b**4*d**4*e**3*x**2 - 10*a**2*b**4*d**3*e**4*x**3 - a*b**5*d**7 + 3
*a*b**5*d**6*e*x + 9*a*b**5*d**5*e**2*x**2 + 5*a*b**5*d**4*e**3*x**3 - ...
```

3.179
$$\int \frac{(d+ex)^{11/2}}{(a^2+2abx+b^2x^2)^2} dx$$

| | |
|---|------|
| Optimal result | 1407 |
| Mathematica [A] (verified) | 1408 |
| Rubi [A] (verified) | 1408 |
| Maple [A] (verified) | 1415 |
| Fricas [B] (verification not implemented) | 1416 |
| Sympy [F(-1)] | 1417 |
| Maxima [F(-2)] | 1417 |
| Giac [B] (verification not implemented) | 1417 |
| Mupad [B] (verification not implemented) | 1418 |
| Reduce [B] (verification not implemented) | 1419 |

Optimal result

Integrand size = 28, antiderivative size = 201

$$\int \frac{(d+ex)^{11/2}}{(a^2+2abx+b^2x^2)^2} dx = \frac{231e^3(bd-ae)^2\sqrt{d+ex}}{8b^6} + \frac{77e^3(bd-ae)(d+ex)^{3/2}}{8b^5} + \frac{231e^3(d+ex)^{5/2}}{40b^4} - \frac{33e^2(d+ex)^{7/2}}{8b^3(a+bx)} - \frac{11e(d+ex)^{9/2}}{12b^2(a+bx)^2} - \frac{(d+ex)^{11/2}}{3b(a+bx)^3} - \frac{231e^3(bd-ae)^{5/2}\operatorname{arctanh}\left(\frac{\sqrt{b}\sqrt{d+ex}}{\sqrt{bd-ae}}\right)}{8b^{13/2}}$$

output

```
231/8*e^3*(-a*e+b*d)^2*(e*x+d)^(1/2)/b^6+77/8*e^3*(-a*e+b*d)*(e*x+d)^(3/2)
/b^5+231/40*e^3*(e*x+d)^(5/2)/b^4-33/8*e^2*(e*x+d)^(7/2)/b^3/(b*x+a)-11/12
*e*(e*x+d)^(9/2)/b^2/(b*x+a)^2-1/3*(e*x+d)^(11/2)/b/(b*x+a)^3-231/8*e^3*(-
a*e+b*d)^(5/2)*arctanh(b^(1/2)*(e*x+d)^(1/2)/(-a*e+b*d)^(1/2))/b^(13/2)
```

Mathematica [A] (verified)

Time = 1.23 (sec) , antiderivative size = 277, normalized size of antiderivative = 1.38

$$\int \frac{(d+ex)^{11/2}}{(a^2+2abx+b^2x^2)^2} dx = \frac{\sqrt{d+ex}(3465a^5e^5 + 1155a^4be^4(-7d+8ex) + 231a^3b^2e^3(23d^2 - 94dex + 33e^2d^2 - 94dex + 33e^2d^2) + 99a^2b^3e^2(-5d^3 + 146d^2ex - 183d^2e^2x^2 + 16e^3x^3) - 11ab^4e(10d^4 + 130d^3ex - 1119d^2e^2x^2 + 352de^3x^3 + 16e^4x^4) + b^5(-40d^5 - 310d^4ex - 1335d^3e^2x^2 + 2768d^2e^3x^3 + 416de^4x^4 + 48e^5x^5))}{(120b^6(a+bx)^3) - (231e^3(-(b*d) + a*e)^{(5/2)}*ArcTan[(Sqrt[b]*Sqrt[d+e*x])/Sqrt[-(b*d) + a*e]])/(8*b^{(13/2))}}$$

input

```
Integrate[(d + e*x)^(11/2)/(a^2 + 2*a*b*x + b^2*x^2)^2,x]
```

output

```
(Sqrt[d + e*x]*(3465*a^5*e^5 + 1155*a^4*b*e^4*(-7*d + 8*e*x) + 231*a^3*b^2*e^3*(23*d^2 - 94*d*e*x + 33*e^2*x^2) + 99*a^2*b^3*e^2*(-5*d^3 + 146*d^2*e*x - 183*d^2*e^2*x^2 + 16*e^3*x^3) - 11*a*b^4*e*(10*d^4 + 130*d^3*e*x - 1119*d^2*e^2*x^2 + 352*d*e^3*x^3 + 16*e^4*x^4) + b^5*(-40*d^5 - 310*d^4*e*x - 1335*d^3*e^2*x^2 + 2768*d^2*e^3*x^3 + 416*d*e^4*x^4 + 48*e^5*x^5)))/(120*b^6*(a + b*x)^3) - (231*e^3*(-(b*d) + a*e)^(5/2)*ArcTan[(Sqrt[b]*Sqrt[d + e*x])/Sqrt[-(b*d) + a*e]])/(8*b^(13/2))
```

Rubi [A] (verified)

Time = 0.55 (sec) , antiderivative size = 214, normalized size of antiderivative = 1.06, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.357$, Rules used = {1098, 27, 51, 51, 51, 60, 60, 60, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex)^{11/2}}{(a^2+2abx+b^2x^2)^2} dx$$

↓ 1098

$$b^4 \int \frac{(d+ex)^{11/2}}{b^4(a+bx)^4} dx$$

↓ 27

$$\begin{aligned}
& \int \frac{(d+ex)^{11/2}}{(a+bx)^4} dx \\
& \quad \downarrow 51 \\
& \frac{11e \int \frac{(d+ex)^{9/2}}{(a+bx)^3} dx}{6b} - \frac{(d+ex)^{11/2}}{3b(a+bx)^3} \\
& \quad \downarrow 51 \\
& \frac{11e \left(\frac{9e \int \frac{(d+ex)^{7/2}}{(a+bx)^2} dx}{4b} - \frac{(d+ex)^{9/2}}{2b(a+bx)^2} \right)}{6b} - \frac{(d+ex)^{11/2}}{3b(a+bx)^3} \\
& \quad \downarrow 51 \\
& \frac{11e \left(\frac{9e \left(\frac{7e \int \frac{(d+ex)^{5/2}}{a+bx} dx}{2b} - \frac{(d+ex)^{7/2}}{b(a+bx)} \right)}{4b} - \frac{(d+ex)^{9/2}}{2b(a+bx)^2} \right)}{6b} - \frac{(d+ex)^{11/2}}{3b(a+bx)^3} \\
& \quad \downarrow 60 \\
& \frac{11e \left(\frac{9e \left(\frac{7e \left(\frac{(bd-ae) \int \frac{(d+ex)^{3/2}}{a+bx} dx}{b} + \frac{2(d+ex)^{5/2}}{5b} \right)}{2b} - \frac{(d+ex)^{7/2}}{b(a+bx)} \right)}{4b} - \frac{(d+ex)^{9/2}}{2b(a+bx)^2} \right)}{6b} - \frac{(d+ex)^{11/2}}{3b(a+bx)^3} \\
& \quad \downarrow 60
\end{aligned}$$

$$11e \left(\frac{9e \left(\frac{(bd-ae) \int \frac{\sqrt{d+ex}}{a+bx} dx + 2(d+ex)^{3/2}}{3b}}{b} + \frac{2(d+ex)^{5/2}}{5b} \right)}{2b} - \frac{(d+ex)^{7/2}}{b(a+bx)} \right) - \frac{(d+ex)^{9/2}}{2b(a+bx)^2}$$

$$\frac{6b}{3b(a+bx)^3} (d+ex)^{11/2}$$

↓ 60

$$\left(\frac{11e}{4b} \left(\frac{9e}{2b} \left(\frac{7e}{b} \left(\frac{(bd-ae) \left(\frac{(bd-ae) \int \frac{1}{(a+bx)\sqrt{d+ex}} dx + 2\sqrt{d+ex}}{b} \right) + \frac{2(d+ex)^{3/2}}{3b}}{b} \right) + \frac{2(d+ex)^{5/2}}{5b} \right) - \frac{(d+ex)^{7/2}}{b(a+bx)} \right) - \frac{(d+ex)^{9/2}}{2b(a+bx)^2} \right)$$

$$\frac{6b}{3b(a+bx)^3} (d+ex)^{11/2}$$

↓ 73

$$\begin{aligned}
 & \left((bd-ae) \left(\frac{2(bd-ae) \int \frac{1}{a + \frac{b(d+ex)}{e} - \frac{bd}{e}} d\sqrt{d+ex}}{b} + \frac{2\sqrt{d+ex}}{b} \right) + \frac{2(d+ex)^{3/2}}{3b} \right) \\
 & \frac{7e}{b} \left(\frac{2(bd-ae) \int \frac{1}{a + \frac{b(d+ex)}{e} - \frac{bd}{e}} d\sqrt{d+ex}}{b} + \frac{2\sqrt{d+ex}}{b} \right) + \frac{2(d+ex)^{5/2}}{5b} \\
 & \frac{9e}{2b} \left(\frac{2(bd-ae) \int \frac{1}{a + \frac{b(d+ex)}{e} - \frac{bd}{e}} d\sqrt{d+ex}}{b} + \frac{2\sqrt{d+ex}}{b} \right) - \frac{(d+ex)^{7/2}}{b(a+bx)} \\
 & \frac{11e}{4b} \left(\frac{2(bd-ae) \int \frac{1}{a + \frac{b(d+ex)}{e} - \frac{bd}{e}} d\sqrt{d+ex}}{b} + \frac{2\sqrt{d+ex}}{b} \right) - \frac{(d+ex)^{9/2}}{2b(a+bx)^2}
 \end{aligned}$$

$$\frac{(d+ex)^{11/2}}{3b(a+bx)^3}$$

↓ 221

$$\begin{aligned}
 & \left(\frac{(bd-ae) \left(\frac{2\sqrt{d+ex}}{b} - \frac{2\sqrt{bd-ae} \operatorname{arctanh}\left(\frac{\sqrt{b}\sqrt{d+ex}}{\sqrt{bd-ae}}\right)}{b^{3/2}} \right) + \frac{2(d+ex)^{3/2}}{3b}}{b} + \frac{2(d+ex)^{5/2}}{5b} \right) \\
 & \frac{7e}{b} \left(\frac{(bd-ae) \left(\frac{2\sqrt{d+ex}}{b} - \frac{2\sqrt{bd-ae} \operatorname{arctanh}\left(\frac{\sqrt{b}\sqrt{d+ex}}{\sqrt{bd-ae}}\right)}{b^{3/2}} \right) + \frac{2(d+ex)^{3/2}}{3b}}{b} + \frac{2(d+ex)^{5/2}}{5b} \right) \\
 & \frac{9e}{2b} \left(\frac{(bd-ae) \left(\frac{2\sqrt{d+ex}}{b} - \frac{2\sqrt{bd-ae} \operatorname{arctanh}\left(\frac{\sqrt{b}\sqrt{d+ex}}{\sqrt{bd-ae}}\right)}{b^{3/2}} \right) + \frac{2(d+ex)^{3/2}}{3b}}{b} + \frac{2(d+ex)^{5/2}}{5b} \right) - \frac{(d+ex)^{7/2}}{b(a+bx)} \\
 & \frac{11e}{4b} \left(\frac{(bd-ae) \left(\frac{2\sqrt{d+ex}}{b} - \frac{2\sqrt{bd-ae} \operatorname{arctanh}\left(\frac{\sqrt{b}\sqrt{d+ex}}{\sqrt{bd-ae}}\right)}{b^{3/2}} \right) + \frac{2(d+ex)^{3/2}}{3b}}{b} + \frac{2(d+ex)^{5/2}}{5b} \right) - \frac{(d+ex)^{9/2}}{2b(a+bx)^2} \\
 & \frac{(d+ex)^{11/2}}{3b(a+bx)^3}
 \end{aligned}$$

input `Int[(d + e*x)^(11/2)/(a^2 + 2*a*b*x + b^2*x^2)^2,x]`

output

$$-1/3*(d + e*x)^{(11/2)}/(b*(a + b*x)^3) + (11*e*(-1/2*(d + e*x)^{(9/2)}/(b*(a + b*x)^2) + (9*e*(-((d + e*x)^{(7/2)}/(b*(a + b*x)))) + (7*e*((2*(d + e*x)^{(5/2)})/(5*b) + ((b*d - a*e)*((2*(d + e*x)^{(3/2)})/(3*b) + ((b*d - a*e)*((2*Sqrt[d + e*x])/b - (2*Sqrt[b*d - a*e]*ArcTanh[(Sqrt[b]*Sqrt[d + e*x])/Sqrt[b*d - a*e]])/b^(3/2)))/b))/b)/(2*b)))/(4*b)))/(6*b)$$

Defintions of rubi rules used

rule 27

$$\text{Int}[(a_*)(Fx_), x_Symbol] \rightarrow \text{Simp}[a \text{ Int}[Fx, x], x] /; \text{FreeQ}[a, x] \ \&\& \ !\text{MatchQ}[Fx, (b_*)(Gx_) /; \text{FreeQ}[b, x]]$$

rule 51

$$\text{Int}[(a_.) + (b_.)*(x_)^{(m_)}*((c_.) + (d_.)*(x_)^{(n_)}), x_Symbol] \rightarrow \text{Simp}[(a + b*x)^{(m + 1)}*((c + d*x)^n/(b*(m + 1))), x] - \text{Simp}[d*(n/(b*(m + 1)))] \text{Int}[(a + b*x)^{(m + 1)}*(c + d*x)^{(n - 1)}, x], x] /; \text{FreeQ}[\{a, b, c, d, n\}, x] \ \&\& \ \text{ILtQ}[m, -1] \ \&\& \ \text{FractionQ}[n] \ \&\& \ \text{GtQ}[n, 0]$$

rule 60

$$\text{Int}[(a_.) + (b_.)*(x_)^{(m_)}*((c_.) + (d_.)*(x_)^{(n_)}), x_Symbol] \rightarrow \text{Simp}[(a + b*x)^{(m + 1)}*((c + d*x)^n/(b*(m + n + 1))), x] + \text{Simp}[n*((b*c - a*d)/(b*(m + n + 1)))] \text{Int}[(a + b*x)^m*(c + d*x)^{(n - 1)}, x], x] /; \text{FreeQ}[\{a, b, c, d\}, x] \ \&\& \ \text{GtQ}[n, 0] \ \&\& \ \text{NeQ}[m + n + 1, 0] \ \&\& \ !(\text{IGtQ}[m, 0] \ \&\& \ (\ !\text{IntegerQ}[n] \ || \ (\text{GtQ}[m, 0] \ \&\& \ \text{LtQ}[m - n, 0]) \)) \ \&\& \ !\text{ILtQ}[m + n + 2, 0] \ \&\& \ \text{IntLinearQ}[a, b, c, d, m, n, x]$$

rule 73

$$\text{Int}[(a_.) + (b_.)*(x_)^{(m_)}*((c_.) + (d_.)*(x_)^{(n_)}), x_Symbol] \rightarrow \text{With}[\{p = \text{Denominator}[m]\}, \text{Simp}[p/b \text{ Subst}[\text{Int}[x^{(p*(m + 1) - 1)}*(c - a*(d/b) + d*(x^p/b)^n], x], (a + b*x)^{(1/p)}, x]] /; \text{FreeQ}[\{a, b, c, d\}, x] \ \&\& \ \text{LtQ}[-1, m, 0] \ \&\& \ \text{LeQ}[-1, n, 0] \ \&\& \ \text{LeQ}[\text{Denominator}[n], \text{Denominator}[m]] \ \&\& \ \text{IntLinearQ}[a, b, c, d, m, n, x]$$

rule 221

$$\text{Int}[(a_.) + (b_.)*(x_)^2)^{-1}, x_Symbol] \rightarrow \text{Simp}[(\text{Rt}[-a/b, 2]/a)*\text{ArcTanh}[x/\text{Rt}[-a/b, 2]], x] /; \text{FreeQ}[\{a, b\}, x] \ \&\& \ \text{NegQ}[a/b]$$

rule 1098

```
Int[((d_.) + (e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] := Simp[1/c^p Int[(d + e*x)^m*(b/2 + c*x)^(2*p), x], x] /; FreeQ[
{a, b, c, d, e, m}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Maple [A] (verified)

Time = 2.18 (sec) , antiderivative size = 229, normalized size of antiderivative = 1.14

| method | result |
|-------------------|---|
| risch | $\frac{2e^3(3b^2e^2x^2 - 20xab e^2 + 26b^2dex + 150e^2a^2 - 320abde + 173b^2d^2)\sqrt{ex+d}}{15b^6} - \frac{(2e^3a^3 - 6a^2bd e^2 + 6ab^2d^2e - 2b^3d^3)e^3}{15b^6}$ |
| pseudoelliptic | $- \frac{231 \left(e^3(bx+a)^3(ae-bd)^3 \arctan\left(\frac{b\sqrt{ex+d}}{\sqrt{b(ae-bd)}}\right) - \sqrt{ex+d} \sqrt{b(ae-bd)} \right) \left(\frac{16}{1155}e^5x^5 + \frac{416}{3465}de^4x^4 + \frac{2768}{3465}d^2e^3x^3 - \frac{89}{231}d^2e^2x^2 + \frac{26}{15}d^2e^2x - \frac{26}{15}d^2e^2 \right)}{15b^6}$ |
| derivativedivides | $2e^3 \left(\frac{\frac{(ex+d)^{\frac{5}{2}}b^2}{5} - \frac{4abe(ex+d)^{\frac{3}{2}}}{3} + \frac{4b^2d(ex+d)^{\frac{3}{2}}}{3} + 10a^2e^2\sqrt{ex+d} - 20abde\sqrt{ex+d} + 10b^2d^2\sqrt{ex+d}}{b^6} - \frac{\left(-\frac{89}{16}a^3b^2e^3 + \frac{26}{15}d^2e^2\right)}{15b^6} \right)$ |
| default | $2e^3 \left(\frac{\frac{(ex+d)^{\frac{5}{2}}b^2}{5} - \frac{4abe(ex+d)^{\frac{3}{2}}}{3} + \frac{4b^2d(ex+d)^{\frac{3}{2}}}{3} + 10a^2e^2\sqrt{ex+d} - 20abde\sqrt{ex+d} + 10b^2d^2\sqrt{ex+d}}{b^6} - \frac{\left(-\frac{89}{16}a^3b^2e^3 + \frac{26}{15}d^2e^2\right)}{15b^6} \right)$ |

input

```
int((e*x+d)^(11/2)/(b^2*x^2+2*a*b*x+a^2)^2,x,method=_RETURNVERBOSE)
```

output

```
2/15*e^3*(3*b^2*e^2*x^2-20*a*b*e^2*x+26*b^2*d*e*x+150*a^2*e^2-320*a*b*d*e+
173*b^2*d^2)*(e*x+d)^(1/2)/b^6-1/b^6*(2*a^3*e^3-6*a^2*b*d*e^2+6*a*b^2*d^2*
e-2*b^3*d^3)*e^3*((-89/16*(e*x+d)^(5/2)*b^2-59/6*b*(a*e-b*d)*(e*x+d)^(3/2)
+(-71/16*e^2*a^2+71/8*a*b*d*e-71/16*b^2*d^2)*(e*x+d)^(1/2))/(b*(e*x+d)+a*e
-b*d)^3+231/16/(b*(a*e-b*d))^(1/2)*arctan(b*(e*x+d)^(1/2)/(b*(a*e-b*d))^(1
/2))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 492 vs. $2(165) = 330$.

Time = 0.12 (sec) , antiderivative size = 994, normalized size of antiderivative = 4.95

$$\int \frac{(d+ex)^{11/2}}{(a^2+2abx+b^2x^2)^2} dx = \text{Too large to display}$$

input `integrate((e*x+d)^(11/2)/(b^2*x^2+2*a*b*x+a^2)^2,x, algorithm="fricas")`

output

```
[1/240*(3465*(a^3*b^2*d^2*e^3 - 2*a^4*b*d*e^4 + a^5*e^5 + (b^5*d^2*e^3 - 2
*a*b^4*d*e^4 + a^2*b^3*e^5)*x^3 + 3*(a*b^4*d^2*e^3 - 2*a^2*b^3*d*e^4 + a^3
*b^2*e^5)*x^2 + 3*(a^2*b^3*d^2*e^3 - 2*a^3*b^2*d*e^4 + a^4*b*e^5)*x)*sqrt(
(b*d - a*e)/b)*log((b*e*x + 2*b*d - a*e - 2*sqrt(e*x + d)*b*sqrt((b*d - a*
e)/b)))/(b*x + a) + 2*(48*b^5*e^5*x^5 - 40*b^5*d^5 - 110*a*b^4*d^4*e - 495
*a^2*b^3*d^3*e^2 + 5313*a^3*b^2*d^2*e^3 - 8085*a^4*b*d*e^4 + 3465*a^5*e^5
+ 16*(26*b^5*d*e^4 - 11*a*b^4*e^5)*x^4 + 16*(173*b^5*d^2*e^3 - 242*a*b^4*d
*e^4 + 99*a^2*b^3*e^5)*x^3 - 3*(445*b^5*d^3*e^2 - 4103*a*b^4*d^2*e^3 + 603
9*a^2*b^3*d*e^4 - 2541*a^3*b^2*e^5)*x^2 - 2*(155*b^5*d^4*e + 715*a*b^4*d^3
*e^2 - 7227*a^2*b^3*d^2*e^3 + 10857*a^3*b^2*d*e^4 - 4620*a^4*b*e^5)*x)*sqr
t(e*x + d))/(b^9*x^3 + 3*a*b^8*x^2 + 3*a^2*b^7*x + a^3*b^6), -1/120*(3465*
(a^3*b^2*d^2*e^3 - 2*a^4*b*d*e^4 + a^5*e^5 + (b^5*d^2*e^3 - 2*a*b^4*d*e^4
+ a^2*b^3*e^5)*x^3 + 3*(a*b^4*d^2*e^3 - 2*a^2*b^3*d*e^4 + a^3*b^2*e^5)*x^2
+ 3*(a^2*b^3*d^2*e^3 - 2*a^3*b^2*d*e^4 + a^4*b*e^5)*x)*sqrt(-(b*d - a*e)/
b)*arctan(-sqrt(e*x + d)*b*sqrt(-(b*d - a*e)/b)/(b*d - a*e)) - (48*b^5*e^5
*x^5 - 40*b^5*d^5 - 110*a*b^4*d^4*e - 495*a^2*b^3*d^3*e^2 + 5313*a^3*b^2*d
^2*e^3 - 8085*a^4*b*d*e^4 + 3465*a^5*e^5 + 16*(26*b^5*d*e^4 - 11*a*b^4*e^5
)*x^4 + 16*(173*b^5*d^2*e^3 - 242*a*b^4*d*e^4 + 99*a^2*b^3*e^5)*x^3 - 3*(4
45*b^5*d^3*e^2 - 4103*a*b^4*d^2*e^3 + 6039*a^2*b^3*d*e^4 - 2541*a^3*b^2*e^
5)*x^2 - 2*(155*b^5*d^4*e + 715*a*b^4*d^3*e^2 - 7227*a^2*b^3*d^2*e^3 + ...
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(d + ex)^{11/2}}{(a^2 + 2abx + b^2x^2)^2} dx = \text{Timed out}$$

input `integrate((e*x+d)**(11/2)/(b**2*x**2+2*a*b*x+a**2)**2,x)`

output `Timed out`

Maxima [F(-2)]

Exception generated.

$$\int \frac{(d + ex)^{11/2}}{(a^2 + 2abx + b^2x^2)^2} dx = \text{Exception raised: ValueError}$$

input `integrate((e*x+d)^(11/2)/(b^2*x^2+2*a*b*x+a^2)^2,x, algorithm="maxima")`

output `Exception raised: ValueError >> Computation failed since Maxima requested additional constraints; using the 'assume' command before evaluation *may* help (example of legal syntax is 'assume(a*e-b*d>0)', see `assume?` for more detail`

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 490 vs. 2(165) = 330.

Time = 0.21 (sec) , antiderivative size = 490, normalized size of antiderivative = 2.44

$$\int \frac{(d + ex)^{11/2}}{(a^2 + 2abx + b^2x^2)^2} dx = \frac{231 (b^3 d^3 e^3 - 3 ab^2 d^2 e^4 + 3 a^2 b d e^5 - a^3 e^6) \arctan\left(\frac{\sqrt{ex+db}}{\sqrt{-b^2d+abe}}\right) + 267 (ex + d)^{\frac{5}{2}} b^5 d^3 e^3 - 472 (ex + d)^{\frac{3}{2}} b^5 d^4 e^3 + 213 \sqrt{ex + db} b^5 d^5 e^3 - 801 (ex + d)^{\frac{5}{2}} ab^4 d^2 e^4 + 1888 (ex + d)^{\frac{3}{2}} ab^4 d^3 e^4 + 20 \left(3 (ex + d)^{\frac{5}{2}} b^{16} e^3 + 20 (ex + d)^{\frac{3}{2}} b^{16} d e^3 + 150 \sqrt{ex + db} b^{16} d^2 e^3 - 20 (ex + d)^{\frac{3}{2}} ab^{15} e^4 - 300 \sqrt{ex + db} ab^{15} e^4 \right)}{15 b^{20}}$$

input `integrate((e*x+d)^(11/2)/(b^2*x^2+2*a*b*x+a^2)^2,x, algorithm="giac")`

output `231/8*(b^3*d^3*e^3 - 3*a*b^2*d^2*e^4 + 3*a^2*b*d*e^5 - a^3*e^6)*arctan(sqrt(e*x + d)*b/sqrt(-b^2*d + a*b*e))/(sqrt(-b^2*d + a*b*e)*b^6) - 1/24*(267*(e*x + d)^(5/2)*b^5*d^3*e^3 - 472*(e*x + d)^(3/2)*b^5*d^4*e^3 + 213*sqrt(e*x + d)*b^5*d^5*e^3 - 801*(e*x + d)^(5/2)*a*b^4*d^2*e^4 + 1888*(e*x + d)^(3/2)*a*b^4*d^3*e^4 - 1065*sqrt(e*x + d)*a*b^4*d^4*e^4 + 801*(e*x + d)^(5/2)*a^2*b^3*d*e^5 - 2832*(e*x + d)^(3/2)*a^2*b^3*d^2*e^5 + 2130*sqrt(e*x + d)*a^2*b^3*d^3*e^5 - 267*(e*x + d)^(5/2)*a^3*b^2*e^6 + 1888*(e*x + d)^(3/2)*a^3*b^2*d*e^6 - 2130*sqrt(e*x + d)*a^3*b^2*d^2*e^6 - 472*(e*x + d)^(3/2)*a^4*b*e^7 + 1065*sqrt(e*x + d)*a^4*b*d*e^7 - 213*sqrt(e*x + d)*a^5*e^8)/((e*x + d)*b - b*d + a*e)^3*b^6) + 2/15*(3*(e*x + d)^(5/2)*b^16*e^3 + 20*(e*x + d)^(3/2)*b^16*d*e^3 + 150*sqrt(e*x + d)*b^16*d^2*e^3 - 20*(e*x + d)^(3/2)*a*b^15*e^4 - 300*sqrt(e*x + d)*a*b^15*d*e^4 + 150*sqrt(e*x + d)*a^2*b^14*e^5)/b^20`

Mupad [B] (verification not implemented)

Time = 9.27 (sec) , antiderivative size = 495, normalized size of antiderivative = 2.46

$$\int \frac{(d+ex)^{11/2}}{(a^2+2abx+b^2x^2)^2} dx = \left(\frac{2e^3(4b^4d-4ab^3e)^2}{b^{12}} - \frac{12e^3(ae-bd)^2}{b^6} \right) \sqrt{d+ex} + \frac{\sqrt{d+ex} \left(\frac{71a^5e^8}{8} - \frac{355a^4bde^7}{8} + \frac{355a^3b^2d^2e^6}{4} - \frac{355a^2b^3d^3e^5}{4} + \frac{355ab^4d^4e^4}{8} - \frac{71b^5d^5e^3}{8} \right) + (d+ex)^{5/2} \left(\frac{89a^3b^2}{8} b^9(d+ex)^3 - (3b^9d-3ab^8e)(d+ex)^2 + (d+ex)(3b^9d+3ab^8e) \right)}{b^9(d+ex)^3 - (3b^9d-3ab^8e)(d+ex)^2 + (d+ex)(3b^9d+3ab^8e)} + \frac{2e^3(d+ex)^{5/2}}{5b^4} + \frac{2e^3(4b^4d-4ab^3e)(d+ex)^{3/2}}{3b^8} - \frac{231e^3 \operatorname{atan}\left(\frac{\sqrt{b}e^3(ae-bd)^{5/2}\sqrt{d+ex}}{a^3e^6-3a^2bde^5+3ab^2d^2e^4-b^3d^3e^3}\right)(ae-bd)^{5/2}}{8b^{13/2}}$$

input `int((d + e*x)^(11/2)/(a^2 + b^2*x^2 + 2*a*b*x)^2,x)`

output

```

((2*e^3*(4*b^4*d - 4*a*b^3*e)^2)/b^12 - (12*e^3*(a*e - b*d)^2)/b^6)*(d + e
*x)^(1/2) + ((d + e*x)^(1/2))*((71*a^5*e^8)/8 - (71*b^5*d^5*e^3)/8 + (355*a
*b^4*d^4*e^4)/8 - (355*a^2*b^3*d^3*e^5)/4 + (355*a^3*b^2*d^2*e^6)/4 - (355
*a^4*b*d*e^7)/8) + (d + e*x)^(5/2)*((89*a^3*b^2*e^6)/8 - (89*b^5*d^3*e^3)/
8 + (267*a*b^4*d^2*e^4)/8 - (267*a^2*b^3*d*e^5)/8) + (d + e*x)^(3/2)*((59*
a^4*b*e^7)/3 + (59*b^5*d^4*e^3)/3 - (236*a*b^4*d^3*e^4)/3 - (236*a^3*b^2*d
*e^6)/3 + 118*a^2*b^3*d^2*e^5))/(b^9*(d + e*x)^3 - (3*b^9*d - 3*a*b^8*e)*(
d + e*x)^2 + (d + e*x)*(3*b^9*d^2 + 3*a^2*b^7*e^2 - 6*a*b^8*d*e) - b^9*d^3
+ a^3*b^6*e^3 - 3*a^2*b^7*d*e^2 + 3*a*b^8*d^2*e) + (2*e^3*(d + e*x)^(5/2)
)/(5*b^4) + (2*e^3*(4*b^4*d - 4*a*b^3*e)*(d + e*x)^(3/2))/(3*b^8) - (231*e
^3*atan((b^(1/2)*e^3*(a*e - b*d)^(5/2)*(d + e*x)^(1/2))/(a^3*e^6 - b^3*d^3
*e^3 + 3*a*b^2*d^2*e^4 - 3*a^2*b*d*e^5))*(a*e - b*d)^(5/2))/(8*b^(13/2))

```

Reduce [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 1000, normalized size of antiderivative = 4.98

$$\int \frac{(d + ex)^{11/2}}{(a^2 + 2abx + b^2x^2)^2} dx = \text{Too large to display}$$

input

```
int((e*x+d)^(11/2)/(b^2*x^2+2*a*b*x+a^2)^2,x)
```


output

```
( - 3465*sqrt(b)*sqrt(a*e - b*d)*atan((sqrt(d + e*x)*b)/(sqrt(b)*sqrt(a*e
- b*d)))*a**5*e**5 + 6930*sqrt(b)*sqrt(a*e - b*d)*atan((sqrt(d + e*x)*b)/(
sqrt(b)*sqrt(a*e - b*d)))*a**4*b*d*e**4 - 10395*sqrt(b)*sqrt(a*e - b*d)*at
an((sqrt(d + e*x)*b)/(sqrt(b)*sqrt(a*e - b*d)))*a**4*b*e**5*x - 3465*sqrt(
b)*sqrt(a*e - b*d)*atan((sqrt(d + e*x)*b)/(sqrt(b)*sqrt(a*e - b*d)))*a**3*
b**2*d**2*e**3 + 20790*sqrt(b)*sqrt(a*e - b*d)*atan((sqrt(d + e*x)*b)/(sqr
t(b)*sqrt(a*e - b*d)))*a**3*b**2*d*e**4*x - 10395*sqrt(b)*sqrt(a*e - b*d)*
atan((sqrt(d + e*x)*b)/(sqrt(b)*sqrt(a*e - b*d)))*a**3*b**2*e**5*x**2 - 10
395*sqrt(b)*sqrt(a*e - b*d)*atan((sqrt(d + e*x)*b)/(sqrt(b)*sqrt(a*e - b*d
)))*a**2*b**3*d**2*e**3*x + 20790*sqrt(b)*sqrt(a*e - b*d)*atan((sqrt(d + e
*x)*b)/(sqrt(b)*sqrt(a*e - b*d)))*a**2*b**3*d*e**4*x**2 - 3465*sqrt(b)*sqr
t(a*e - b*d)*atan((sqrt(d + e*x)*b)/(sqrt(b)*sqrt(a*e - b*d)))*a**2*b**3*e
**5*x**3 - 10395*sqrt(b)*sqrt(a*e - b*d)*atan((sqrt(d + e*x)*b)/(sqrt(b)*s
qrt(a*e - b*d)))*a*b**4*d**2*e**3*x**2 + 6930*sqrt(b)*sqrt(a*e - b*d)*atan
((sqrt(d + e*x)*b)/(sqrt(b)*sqrt(a*e - b*d)))*a*b**4*d*e**4*x**3 - 3465*sq
rt(b)*sqrt(a*e - b*d)*atan((sqrt(d + e*x)*b)/(sqrt(b)*sqrt(a*e - b*d)))*b*
*5*d**2*e**3*x**3 + 3465*sqrt(d + e*x)*a**5*b*e**5 - 8085*sqrt(d + e*x)*a*
*4*b**2*d*e**4 + 9240*sqrt(d + e*x)*a**4*b**2*e**5*x + 5313*sqrt(d + e*x)*
a**3*b**3*d**2*e**3 - 21714*sqrt(d + e*x)*a**3*b**3*d*e**4*x + 7623*sqrt(d
+ e*x)*a**3*b**3*e**5*x**2 - 495*sqrt(d + e*x)*a**2*b**4*d**3*e**2 + 1...
```

3.180
$$\int \frac{(d+ex)^{9/2}}{(a^2+2abx+b^2x^2)^2} dx$$

| | |
|---|------|
| Optimal result | 1421 |
| Mathematica [A] (verified) | 1422 |
| Rubi [A] (verified) | 1422 |
| Maple [A] (verified) | 1426 |
| Fricas [B] (verification not implemented) | 1427 |
| Sympy [F(-1)] | 1428 |
| Maxima [F(-2)] | 1429 |
| Giac [B] (verification not implemented) | 1429 |
| Mupad [B] (verification not implemented) | 1430 |
| Reduce [B] (verification not implemented) | 1431 |

Optimal result

Integrand size = 28, antiderivative size = 172

$$\int \frac{(d+ex)^{9/2}}{(a^2+2abx+b^2x^2)^2} dx = \frac{105e^3(bd-ae)\sqrt{d+ex}}{8b^5} + \frac{35e^3(d+ex)^{3/2}}{8b^4} - \frac{21e^2(d+ex)^{5/2}}{8b^3(a+bx)} - \frac{3e(d+ex)^{7/2}}{4b^2(a+bx)^2} - \frac{(d+ex)^{9/2}}{3b(a+bx)^3} - \frac{105e^3(bd-ae)^{3/2} \operatorname{arctanh}\left(\frac{\sqrt{b}\sqrt{d+ex}}{\sqrt{bd-ae}}\right)}{8b^{11/2}}$$

output

```
105/8*e^3*(-a*e+b*d)*(e*x+d)^(1/2)/b^5+35/8*e^3*(e*x+d)^(3/2)/b^4-21/8*e^2
*(e*x+d)^(5/2)/b^3/(b*x+a)-3/4*e*(e*x+d)^(7/2)/b^2/(b*x+a)^2-1/3*(e*x+d)^(
9/2)/b/(b*x+a)^3-105/8*e^3*(-a*e+b*d)^(3/2)*arctanh(b^(1/2)*(e*x+d)^(1/2)/
(-a*e+b*d)^(1/2))/b^(11/2)
```

Mathematica [A] (verified)

Time = 1.05 (sec) , antiderivative size = 208, normalized size of antiderivative = 1.21

$$\int \frac{(d+ex)^{9/2}}{(a^2+2abx+b^2x^2)^2} dx = \frac{\sqrt{d+ex}(315a^4e^4 - 420a^3be^3(d-2ex) + 63a^2b^2e^2(d^2 - 18dex + 11e^2x^2) + 18ab^3e(d^3 + 10d^2ex - 53de^2x - 15d^2e^2x^2 + 8e^3x^3) + b^4(8d^4 + 50d^3ex + 165d^2e^2x^2 - 208de^3x^3 - 16e^4x^4))}{24b^5(a+bx)^3} + \frac{105e^3(-bd+ae)^{3/2} \arctan\left(\frac{\sqrt{b}\sqrt{d+ex}}{\sqrt{-bd+ae}}\right)}{8b^{11/2}}$$

input

```
Integrate[(d + e*x)^(9/2)/(a^2 + 2*a*b*x + b^2*x^2)^2,x]
```

output

```
-1/24*(Sqrt[d + e*x]*(315*a^4*e^4 - 420*a^3*b*e^3*(d - 2*e*x) + 63*a^2*b^2*e^2*(d^2 - 18*d*e*x + 11*e^2*x^2) + 18*a*b^3*e*(d^3 + 10*d^2*e*x - 53*d*e^2*x^2 + 8*e^3*x^3) + b^4*(8*d^4 + 50*d^3*e*x + 165*d^2*e^2*x^2 - 208*d*e^3*x^3 - 16*e^4*x^4)))/(b^5*(a + b*x)^3) + (105*e^3*(-(b*d) + a*e)^(3/2)*ArcTan[(Sqrt[b]*Sqrt[d + e*x])/Sqrt[-(b*d) + a*e]])/(8*b^(11/2))
```

Rubi [A] (verified)

Time = 0.50 (sec) , antiderivative size = 185, normalized size of antiderivative = 1.08, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.321$, Rules used = {1098, 27, 51, 51, 51, 60, 60, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex)^{9/2}}{(a^2+2abx+b^2x^2)^2} dx$$

↓ 1098

$$b^4 \int \frac{(d+ex)^{9/2}}{b^4(a+bx)^4} dx$$

↓ 27

$$\begin{aligned}
& \int \frac{(d+ex)^{9/2}}{(a+bx)^4} dx \\
& \quad \downarrow 51 \\
& \frac{3e \int \frac{(d+ex)^{7/2}}{(a+bx)^3} dx}{2b} - \frac{(d+ex)^{9/2}}{3b(a+bx)^3} \\
& \quad \downarrow 51 \\
& \frac{3e \left(\frac{7e \int \frac{(d+ex)^{5/2}}{(a+bx)^2} dx}{4b} - \frac{(d+ex)^{7/2}}{2b(a+bx)^2} \right)}{2b} - \frac{(d+ex)^{9/2}}{3b(a+bx)^3} \\
& \quad \downarrow 51 \\
& \frac{3e \left(\frac{7e \left(\frac{5e \int \frac{(d+ex)^{3/2}}{a+bx} dx}{2b} - \frac{(d+ex)^{5/2}}{b(a+bx)} \right)}{4b} - \frac{(d+ex)^{7/2}}{2b(a+bx)^2} \right)}{2b} - \frac{(d+ex)^{9/2}}{3b(a+bx)^3} \\
& \quad \downarrow 60 \\
& \frac{3e \left(\frac{7e \left(\frac{5e \left(\frac{(bd-ae) \int \frac{\sqrt{d+ex}}{a+bx} dx}{b} + \frac{2(d+ex)^{3/2}}{3b} \right)}{2b} - \frac{(d+ex)^{5/2}}{b(a+bx)} \right)}{4b} - \frac{(d+ex)^{7/2}}{2b(a+bx)^2} \right)}{2b} - \frac{(d+ex)^{9/2}}{3b(a+bx)^3} \\
& \quad \downarrow 60
\end{aligned}$$

$$\left(\begin{array}{l} 5e \left(\frac{(bd-ae) \int \frac{1}{(a+bx)\sqrt{d+ex}} dx + 2\sqrt{d+ex}}{b} \right) + \frac{2(d+ex)^{3/2}}{3b} \\ 7e \left(\frac{\hspace{10em}}{2b} \right) - \frac{(d+ex)^{5/2}}{b(a+bx)} \\ 3e \left(\frac{\hspace{15em}}{4b} \right) - \frac{(d+ex)^{7/2}}{2b(a+bx)^2} \end{array} \right)$$

$$\frac{2b}{3b(a+bx)^3} (d+ex)^{9/2}$$

73

$$\left(\begin{array}{l} 5e \left(\frac{(bd-ae) \int \frac{1}{a + \frac{b(d+ex)}{e} - \frac{bd}{e}} d\sqrt{d+ex}}{b} + \frac{2(d+ex)^{3/2}}{3b} \right) \\ 7e \left(\frac{\hspace{10em}}{2b} \right) - \frac{(d+ex)^{5/2}}{b(a+bx)} \\ 3e \left(\frac{\hspace{15em}}{4b} \right) - \frac{(d+ex)^{7/2}}{2b(a+bx)^2} \end{array} \right)$$

$$\frac{2b}{3b(a+bx)^3} (d+ex)^{9/2}$$

221

$$\frac{\left(\frac{5e \left(\frac{(bd-ae) \left(\frac{2\sqrt{d+ex}}{b} - \frac{2\sqrt{bd-ae} \operatorname{arctanh} \left(\frac{\sqrt{b}\sqrt{d+ex}}{\sqrt{bd-ae}} \right)}{b^{3/2}} \right)}{b} + \frac{2(d+ex)^{3/2}}{3b} \right)}{2b} - \frac{(d+ex)^{5/2}}{b(a+bx)} \right)}{4b} - \frac{(d+ex)^{7/2}}{2b(a+bx)^2} \right)}{\frac{2b}{3b(a+bx)^3} (d+ex)^{9/2}}$$

input `Int[(d + e*x)^(9/2)/(a^2 + 2*a*b*x + b^2*x^2)^2,x]`

output `-1/3*(d + e*x)^(9/2)/(b*(a + b*x)^3) + (3*e*(-1/2*(d + e*x)^(7/2)/(b*(a + b*x)^2) + (7*e*(-((d + e*x)^(5/2)/(b*(a + b*x))) + (5*e*((2*(d + e*x)^(3/2)))/(3*b) + ((b*d - a*e)*((2*sqrt[d + e*x])/b - (2*sqrt[b*d - a*e]*ArcTanh[(sqrt[b]*sqrt[d + e*x])/sqrt[b*d - a*e]])/b^(3/2)))/b))/(2*b)))/(4*b))/(2*b)`

Defintions of rubi rules used

rule 27 `Int[(a_)*(Fx_), x_Symbol] := Simp[a Int[Fx, x], x] /; FreeQ[a, x] && !MatchQ[Fx, (b_)*(Gx_)] /; FreeQ[b, x]`

rule 51 `Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] := Simp[(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Simp[d*(n/(b*(m + 1)))] Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x] /; FreeQ[{a, b, c, d, n}, x] && ILtQ[m, -1] && FractionQ[n] && GtQ[n, 0]`

rule 60 `Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] := Simp[(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*(b*c - a*d)/(b*(m + n + 1)) Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] && !(IGtQ[m, 0] && (!IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) && !ILtQ[m + n + 2, 0] && IntLinearQ[a, b, c, d, m, n, x]`

rule 73 `Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] := With[{p = Denominator[m]}, Simp[p/b Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) + d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntLinearQ[a, b, c, d, m, n, x]`

rule 221 `Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[-a/b, 2]/a)*ArcTanh[x/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]`

rule 1098 `Int[((d_.) + (e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] := Simp[1/c^p Int[(d + e*x)^m*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]`

Maple [A] (verified)

Time = 1.98 (sec) , antiderivative size = 178, normalized size of antiderivative = 1.03

| method | result |
|-------------------|--|
| risch | $-\frac{2e^3(-bex+12ae-13bd)\sqrt{ex+d}}{3b^5} + \frac{(2e^2a^2-4abde+2b^2d^2)e^3 \left(\frac{-55(ex+d)^{\frac{5}{2}}b^2 - 35b(ae-bd)(ex+d)^{\frac{3}{2}}}{16} + \frac{(-41e^2a^2 + \frac{41}{8}bd)}{(b(ex+d)+ae-bd)^3} \right)}{b^5}$ |
| pseudoelliptic | $\frac{105e^3(bx+a)^3(ae-bd)^2 \arctan\left(\frac{b\sqrt{ex+d}}{\sqrt{b(ae-bd)}}\right)}{8} - \frac{105\sqrt{ex+d}\sqrt{b(ae-bd)} \left(\left(-\frac{16}{315}e^4x^4 - \frac{208}{315}de^3x^3 + \frac{11}{21}d^2e^2x^2 + \frac{10}{63}d^3ex + \frac{8}{315}d^4\right)b^5 \right)}{b^5(bx+a)^3\sqrt{b(ae-bd)}}$ |
| derivativedivides | $2e^3 \left(-\frac{(ex+d)^{\frac{3}{2}}b}{3} + \frac{4ae\sqrt{ex+d} - 4db\sqrt{ex+d}}{b^5} + \frac{\left(-\frac{55}{16}a^2b^2e^2 + \frac{55}{8}ab^3de - \frac{55}{16}b^4d^2\right)(ex+d)^{\frac{5}{2}} - 35b(e^3a^3 - 3a^2bde^2 + 3ab^2d^2e - b^3d^3)}{6} \right)$ |
| default | $2e^3 \left(-\frac{(ex+d)^{\frac{3}{2}}b}{3} + \frac{4ae\sqrt{ex+d} - 4db\sqrt{ex+d}}{b^5} + \frac{\left(-\frac{55}{16}a^2b^2e^2 + \frac{55}{8}ab^3de - \frac{55}{16}b^4d^2\right)(ex+d)^{\frac{5}{2}} - 35b(e^3a^3 - 3a^2bde^2 + 3ab^2d^2e - b^3d^3)}{6} \right)$ |

input `int((e*x+d)^(9/2)/(b^2*x^2+2*a*b*x+a^2)^2,x,method=_RETURNVERBOSE)`

output `-2/3*e^3*(-b*e*x+12*a*e-13*b*d)*(e*x+d)^(1/2)/b^5+1/b^5*(2*a^2*e^2-4*a*b*d*e+2*b^2*d^2)*e^3*((-55/16*(e*x+d)^(5/2)*b^2-35/6*b*(a*e-b*d)*(e*x+d)^(3/2))+(-41/16*e^2*a^2+41/8*a*b*d*e-41/16*b^2*d^2)*(e*x+d)^(1/2))/(b*(e*x+d)+a*e-b*d)^3+105/16/(b*(a*e-b*d))^(1/2)*arctan(b*(e*x+d)^(1/2)/(b*(a*e-b*d))^(1/2))`

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 360 vs. 2(140) = 280.

Time = 0.11 (sec) , antiderivative size = 730, normalized size of antiderivative = 4.24

$$\int \frac{(d+ex)^{9/2}}{(a^2+2abx+b^2x^2)^2} dx = \left[\frac{315(a^3bde^3 - a^4e^4 + (b^4de^3 - ab^3e^4)x^3 + 3(ab^3de^3 - a^2b^2e^4)x^2 + 3(a^2b^2de^3 - a^3be^4)x)}{(a^2+2abx+b^2x^2)^2} + \frac{315(a^3bde^3 - a^4e^4 + (b^4de^3 - ab^3e^4)x^3 + 3(ab^3de^3 - a^2b^2e^4)x^2 + 3(a^2b^2de^3 - a^3be^4)x)}{(a^2+2abx+b^2x^2)^2} \sqrt{-\frac{bd-ae}{b}} \arctan\left(\frac{b\sqrt{ex+d}}{\sqrt{b(ae-bd)}}\right) \right]$$

input `integrate((e*x+d)^(9/2)/(b^2*x^2+2*a*b*x+a^2)^2,x, algorithm="fricas")`

output `[-1/48*(315*(a^3*b*d*e^3 - a^4*e^4 + (b^4*d*e^3 - a*b^3*e^4)*x^3 + 3*(a*b^3*d*e^3 - a^2*b^2*e^4)*x^2 + 3*(a^2*b^2*d*e^3 - a^3*b*e^4)*x)*sqrt((b*d - a*e)/b)*log((b*e*x + 2*b*d - a*e + 2*sqrt(e*x + d)*b*sqrt((b*d - a*e)/b))/(b*x + a)) - 2*(16*b^4*e^4*x^4 - 8*b^4*d^4 - 18*a*b^3*d^3*e - 63*a^2*b^2*d^2*e^2 + 420*a^3*b*d*e^3 - 315*a^4*e^4 + 16*(13*b^4*d*e^3 - 9*a*b^3*e^4)*x^3 - 3*(55*b^4*d^2*e^2 - 318*a*b^3*d*e^3 + 231*a^2*b^2*e^4)*x^2 - 2*(25*b^4*d^3*e + 90*a*b^3*d^2*e^2 - 567*a^2*b^2*d*e^3 + 420*a^3*b*e^4)*x)*sqrt(e*x + d)/(b^8*x^3 + 3*a*b^7*x^2 + 3*a^2*b^6*x + a^3*b^5), -1/24*(315*(a^3*b*d*e^3 - a^4*e^4 + (b^4*d*e^3 - a*b^3*e^4)*x^3 + 3*(a*b^3*d*e^3 - a^2*b^2*e^4)*x^2 + 3*(a^2*b^2*d*e^3 - a^3*b*e^4)*x)*sqrt(-(b*d - a*e)/b)*arctan(-sqrt(e*x + d)*b*sqrt(-(b*d - a*e)/b)/(b*d - a*e)) - (16*b^4*e^4*x^4 - 8*b^4*d^4 - 18*a*b^3*d^3*e - 63*a^2*b^2*d^2*e^2 + 420*a^3*b*d*e^3 - 315*a^4*e^4 + 16*(13*b^4*d*e^3 - 9*a*b^3*e^4)*x^3 - 3*(55*b^4*d^2*e^2 - 318*a*b^3*d*e^3 + 231*a^2*b^2*e^4)*x^2 - 2*(25*b^4*d^3*e + 90*a*b^3*d^2*e^2 - 567*a^2*b^2*d*e^3 + 420*a^3*b*e^4)*x)*sqrt(e*x + d)/(b^8*x^3 + 3*a*b^7*x^2 + 3*a^2*b^6*x + a^3*b^5)]`

Sympy [F(-1)]

Timed out.

$$\int \frac{(d + ex)^{9/2}}{(a^2 + 2abx + b^2x^2)^2} dx = \text{Timed out}$$

input `integrate((e*x+d)**(9/2)/(b**2*x**2+2*a*b*x+a**2)**2,x)`

output `Timed out`

Maxima [F(-2)]

Exception generated.

$$\int \frac{(d+ex)^{9/2}}{(a^2+2abx+b^2x^2)^2} dx = \text{Exception raised: ValueError}$$

input `integrate((e*x+d)^(9/2)/(b^2*x^2+2*a*b*x+a^2)^2,x, algorithm="maxima")`

output `Exception raised: ValueError >> Computation failed since Maxima requested additional constraints; using the 'assume' command before evaluation *may* help (example of legal syntax is 'assume(a*e-b*d>0)', see `assume?` for more detail`

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 358 vs. 2(140) = 280.

Time = 0.21 (sec) , antiderivative size = 358, normalized size of antiderivative = 2.08

$$\int \frac{(d+ex)^{9/2}}{(a^2+2abx+b^2x^2)^2} dx = \frac{105(b^2d^2e^3 - 2abde^4 + a^2e^5) \arctan\left(\frac{\sqrt{ex+db}}{\sqrt{-b^2d+abe}}\right)}{8\sqrt{-b^2d+abe}b^5} - \frac{165(ex+d)^{5/2}b^4d^2e^3 - 280(ex+d)^{3/2}b^4d^3e^3 + 123\sqrt{ex+db}b^4d^4e^3 - 330(ex+d)^{5/2}ab^3de^4 + 840(ex+d)^{3/2}b^4d^2e^3}{3b^{12}} + \frac{2\left((ex+d)^{3/2}b^8e^3 + 12\sqrt{ex+db}b^8de^3 - 12\sqrt{ex+db}ab^7e^4\right)}{3b^{12}}$$

input `integrate((e*x+d)^(9/2)/(b^2*x^2+2*a*b*x+a^2)^2,x, algorithm="giac")`

output

```
105/8*(b^2*d^2*e^3 - 2*a*b*d*e^4 + a^2*e^5)*arctan(sqrt(e*x + d)*b/sqrt(-b
^2*d + a*b*e))/(sqrt(-b^2*d + a*b*e)*b^5) - 1/24*(165*(e*x + d)^(5/2)*b^4*
d^2*e^3 - 280*(e*x + d)^(3/2)*b^4*d^3*e^3 + 123*sqrt(e*x + d)*b^4*d^4*e^3
- 330*(e*x + d)^(5/2)*a*b^3*d*e^4 + 840*(e*x + d)^(3/2)*a*b^3*d^2*e^4 - 49
2*sqrt(e*x + d)*a*b^3*d^3*e^4 + 165*(e*x + d)^(5/2)*a^2*b^2*e^5 - 840*(e*x
+ d)^(3/2)*a^2*b^2*d*e^5 + 738*sqrt(e*x + d)*a^2*b^2*d^2*e^5 + 280*(e*x +
d)^(3/2)*a^3*b*e^6 - 492*sqrt(e*x + d)*a^3*b*d*e^6 + 123*sqrt(e*x + d)*a^
4*e^7)/(((e*x + d)*b - b*d + a*e)^3*b^5) + 2/3*((e*x + d)^(3/2)*b^8*e^3 +
12*sqrt(e*x + d)*b^8*d*e^3 - 12*sqrt(e*x + d)*a*b^7*e^4)/b^12
```

Mupad [B] (verification not implemented)

Time = 9.35 (sec) , antiderivative size = 388, normalized size of antiderivative = 2.26

$$\int \frac{(d+ex)^{9/2}}{(a^2+2abx+b^2x^2)^2} dx = \frac{2e^3(d+ex)^{3/2}}{3b^4} - \frac{(d+ex)^{5/2} \left(\frac{55a^2b^2e^5}{8} - \frac{55ab^3de^4}{4} + \frac{55b^4d^2e^3}{8} \right) + (d+ex)^{3/2} \left(\frac{35a^3be^6}{3} - 35a^2b^2de^5 + 35ab^3d^2e^4 - \frac{35b^4d^3e^3}{3} \right)}{b^8(d+ex)^3 - (3b^8d - 3ab^7e)(d+ex)^2 + (d+ex)(3a^2b^6e^2 - 6ab^7de + 3b^8d^2e^3)} + \frac{2e^3(4b^4d - 4ab^3e)\sqrt{d+ex}}{b^8} + \frac{105e^3 \operatorname{atan}\left(\frac{\sqrt{b}e^3(ae-bd)^{3/2}\sqrt{d+ex}}{a^2e^5 - 2abde^4 + b^2d^2e^3}\right)(ae-bd)^{3/2}}{8b^{11/2}}$$

input

```
int((d + e*x)^(9/2)/(a^2 + b^2*x^2 + 2*a*b*x)^2,x)
```

output

```
(2*e^3*(d + e*x)^(3/2))/(3*b^4) - ((d + e*x)^(5/2))*((55*a^2*b^2*e^5)/8 + (
55*b^4*d^2*e^3)/8 - (55*a*b^3*d*e^4)/4) + (d + e*x)^(3/2)*((35*a^3*b*e^6)/
3 - (35*b^4*d^3*e^3)/3 + 35*a*b^3*d^2*e^4 - 35*a^2*b^2*d*e^5) + (d + e*x)^(
1/2)*((41*a^4*e^7)/8 + (41*b^4*d^4*e^3)/8 - (41*a*b^3*d^3*e^4)/2 + (123*a
^2*b^2*d^2*e^5)/4 - (41*a^3*b*d*e^6)/2))/((b^8*(d + e*x)^3 - (3*b^8*d - 3*a
*b^7*e)*(d + e*x)^2 + (d + e*x)*(3*b^8*d^2 + 3*a^2*b^6*e^2 - 6*a*b^7*d*e)
- b^8*d^3 + a^3*b^5*e^3 - 3*a^2*b^6*d*e^2 + 3*a*b^7*d^2*e) + (2*e^3*(4*b^4
*d - 4*a*b^3*e)*(d + e*x)^(1/2))/b^8 + (105*e^3*atan((b^(1/2)*e^3*(a*e -
b*d)^(3/2)*(d + e*x)^(1/2))/(a^2*e^5 + b^2*d^2*e^3 - 2*a*b*d*e^4))*(a*e -
b*d)^(3/2))/(8*b^(11/2))
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 674, normalized size of antiderivative = 3.92

$$\int \frac{(d+ex)^{9/2}}{(a^2+2abx+b^2x^2)^2} dx = \frac{315\sqrt{b}\sqrt{ae-bd} \operatorname{atan}\left(\frac{\sqrt{ex+db}}{\sqrt{b}\sqrt{ae-bd}}\right) a^4 e^4 - 315\sqrt{b}\sqrt{ae-bd} \operatorname{atan}\left(\frac{\sqrt{ex+db}}{\sqrt{b}\sqrt{ae-bd}}\right) a^3}{(a^2+2abx+b^2x^2)^2}$$

input `int((e*x+d)^(9/2)/(b^2*x^2+2*a*b*x+a^2)^2,x)`

output

```
(315*sqrt(b)*sqrt(a*e - b*d)*atan((sqrt(d + e*x)*b)/(sqrt(b)*sqrt(a*e - b*d)))**4*e**4 - 315*sqrt(b)*sqrt(a*e - b*d)*atan((sqrt(d + e*x)*b)/(sqrt(b)*sqrt(a*e - b*d)))**3*b*d*e**3 + 945*sqrt(b)*sqrt(a*e - b*d)*atan((sqrt(d + e*x)*b)/(sqrt(b)*sqrt(a*e - b*d)))**3*b*e**4*x - 945*sqrt(b)*sqrt(a*e - b*d)*atan((sqrt(d + e*x)*b)/(sqrt(b)*sqrt(a*e - b*d)))**2*b**2*d*e**3*x + 945*sqrt(b)*sqrt(a*e - b*d)*atan((sqrt(d + e*x)*b)/(sqrt(b)*sqrt(a*e - b*d)))**2*b**2*e**4*x**2 - 945*sqrt(b)*sqrt(a*e - b*d)*atan((sqrt(d + e*x)*b)/(sqrt(b)*sqrt(a*e - b*d)))**2*b**2*d*e**3*x**2 + 315*sqrt(b)*sqrt(a*e - b*d)*atan((sqrt(d + e*x)*b)/(sqrt(b)*sqrt(a*e - b*d)))**2*b**2*e**4*x**3 - 315*sqrt(b)*sqrt(a*e - b*d)*atan((sqrt(d + e*x)*b)/(sqrt(b)*sqrt(a*e - b*d)))**2*b**2*d*e**3*x**3 - 315*sqrt(d + e*x)*a**4*b*e**4 + 420*sqrt(d + e*x)*a**3*b**2*d*e**3 - 840*sqrt(d + e*x)*a**3*b**2*e**4*x - 63*sqrt(d + e*x)*a**2*b**3*d**2*e**2 + 1134*sqrt(d + e*x)*a**2*b**3*d*e**3*x - 693*sqrt(d + e*x)*a**2*b**3*e**4*x**2 - 18*sqrt(d + e*x)*a*b**4*d**3*e - 180*sqrt(d + e*x)*a*b**4*d**2*e**2*x + 954*sqrt(d + e*x)*a*b**4*d*e**3*x**2 - 144*sqrt(d + e*x)*a*b**4*e**4*x**3 - 8*sqrt(d + e*x)*b**5*d**4 - 50*sqrt(d + e*x)*b**5*d**3*e*x - 165*sqrt(d + e*x)*b**5*d**2*e**2*x**2 + 208*sqrt(d + e*x)*b**5*d*e**3*x**3 + 16*sqrt(d + e*x)*b**5*e**4*x**4)/(24*b**6*(a**3 + 3*a**2*b*x + 3*a*b**2*x**2 + b**3*x**3))
```

3.181 $\int \frac{(d+ex)^{7/2}}{(a^2+2abx+b^2x^2)^2} dx$

| | |
|---|------|
| Optimal result | 1432 |
| Mathematica [A] (verified) | 1432 |
| Rubi [A] (verified) | 1433 |
| Maple [A] (verified) | 1436 |
| Fricas [B] (verification not implemented) | 1437 |
| Sympy [F(-1)] | 1437 |
| Maxima [F(-2)] | 1438 |
| Giac [B] (verification not implemented) | 1438 |
| Mupad [B] (verification not implemented) | 1439 |
| Reduce [B] (verification not implemented) | 1440 |

Optimal result

Integrand size = 28, antiderivative size = 145

$$\int \frac{(d+ex)^{7/2}}{(a^2+2abx+b^2x^2)^2} dx = \frac{35e^3\sqrt{d+ex}}{8b^4} - \frac{35e^2(d+ex)^{3/2}}{24b^3(a+bx)} - \frac{7e(d+ex)^{5/2}}{12b^2(a+bx)^2} - \frac{(d+ex)^{7/2}}{3b(a+bx)^3} - \frac{35e^3\sqrt{bd-ae}\operatorname{arctanh}\left(\frac{\sqrt{b}\sqrt{d+ex}}{\sqrt{bd-ae}}\right)}{8b^{9/2}}$$

output

```
35/8*e^3*(e*x+d)^(1/2)/b^4-35/24*e^2*(e*x+d)^(3/2)/b^3/(b*x+a)-7/12*e*(e*x+d)^(5/2)/b^2/(b*x+a)^2-1/3*(e*x+d)^(7/2)/b/(b*x+a)^3-35/8*e^3*(-a*e+b*d)^(1/2)*arctanh(b^(1/2)*(e*x+d)^(1/2)/(-a*e+b*d)^(1/2))/b^(9/2)
```

Mathematica [A] (verified)

Time = 0.75 (sec) , antiderivative size = 160, normalized size of antiderivative = 1.10

$$\int \frac{(d+ex)^{7/2}}{(a^2+2abx+b^2x^2)^2} dx = \frac{\sqrt{d+ex}(-105a^3e^3+35a^2be^2(d-8ex)+7ab^2e(2d^2+14dex-33e^2x^2)+b^3(8d^3+38d^2ex+87de^2x^2-24b^4(a+bx)^3)}{24b^4(a+bx)^3} - \frac{35e^3\sqrt{-bd+ae}\operatorname{arctan}\left(\frac{\sqrt{b}\sqrt{d+ex}}{\sqrt{-bd+ae}}\right)}{8b^{9/2}}$$

input `Integrate[(d + e*x)^(7/2)/(a^2 + 2*a*b*x + b^2*x^2)^2,x]`

output
$$\frac{-1/24*(\text{Sqrt}[d + e*x]*(-105*a^3*e^3 + 35*a^2*b*e^2*(d - 8*e*x) + 7*a*b^2*e*(2*d^2 + 14*d*e*x - 33*e^2*x^2) + b^3*(8*d^3 + 38*d^2*e*x + 87*d*e^2*x^2 - 48*e^3*x^3)))/(b^4*(a + b*x)^3) - (35*e^3*\text{Sqrt}[-(b*d) + a*e]*\text{ArcTan}[(\text{Sqrt}[b]*\text{Sqrt}[d + e*x])/\text{Sqrt}[-(b*d) + a*e]])/(8*b^(9/2))}{1}$$

Rubi [A] (verified)

Time = 0.47 (sec) , antiderivative size = 156, normalized size of antiderivative = 1.08, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {1098, 27, 51, 51, 51, 60, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\begin{aligned} & \int \frac{(d + ex)^{7/2}}{(a^2 + 2abx + b^2x^2)^2} dx \\ & \quad \downarrow 1098 \\ & b^4 \int \frac{(d + ex)^{7/2}}{b^4(a + bx)^4} dx \\ & \quad \downarrow 27 \\ & \int \frac{(d + ex)^{7/2}}{(a + bx)^4} dx \\ & \quad \downarrow 51 \\ & \frac{7e \int \frac{(d+ex)^{5/2}}{(a+bx)^3} dx}{6b} - \frac{(d + ex)^{7/2}}{3b(a + bx)^3} \\ & \quad \downarrow 51 \\ & \frac{7e \left(\frac{5e \int \frac{(d+ex)^{3/2}}{(a+bx)^2} dx}{4b} - \frac{(d+ex)^{5/2}}{2b(a+bx)^2} \right)}{6b} - \frac{(d + ex)^{7/2}}{3b(a + bx)^3} \\ & \quad \downarrow 51 \end{aligned}$$

$$7e \left(\frac{5e \left(\frac{3e \int \frac{\sqrt{d+ex}}{a+bx} dx}{2b} - \frac{(d+ex)^{3/2}}{b(a+bx)} \right)}{4b} - \frac{(d+ex)^{5/2}}{2b(a+bx)^2} \right) - \frac{(d+ex)^{7/2}}{3b(a+bx)^3}$$

60

$$7e \left(\frac{5e \left(\frac{3e \left(\frac{(bd-ae) \int \frac{1}{(a+bx)\sqrt{d+ex}} dx}{b} + \frac{2\sqrt{d+ex}}{b} \right)}{2b} - \frac{(d+ex)^{3/2}}{b(a+bx)} \right)}{4b} - \frac{(d+ex)^{5/2}}{2b(a+bx)^2} \right) - \frac{(d+ex)^{7/2}}{3b(a+bx)^3}$$

73

$$7e \left(\frac{5e \left(\frac{3e \left(\frac{2(bd-ae) \int \frac{1}{a + \frac{b(d+ex)}{be} - \frac{bd}{e}} d\sqrt{d+ex}}{2b} + \frac{2\sqrt{d+ex}}{b} \right)}{2b} - \frac{(d+ex)^{3/2}}{b(a+bx)} \right)}{4b} - \frac{(d+ex)^{5/2}}{2b(a+bx)^2} \right) - \frac{(d+ex)^{7/2}}{3b(a+bx)^3}$$

221

$$7e \left(\frac{5e \left(\frac{3e \left(\frac{2\sqrt{d+ex}}{b} - \frac{2\sqrt{bd-ae} \operatorname{arctanh} \left(\frac{\sqrt{b}\sqrt{d+ex}}{\sqrt{bd-ae}} \right)}{b^{3/2}} \right)}{2b} - \frac{(d+ex)^{3/2}}{b(a+bx)} \right)}{4b} - \frac{(d+ex)^{5/2}}{2b(a+bx)^2} \right) - \frac{(d+ex)^{7/2}}{3b(a+bx)^3}$$

input `Int[(d + e*x)^(7/2)/(a^2 + 2*a*b*x + b^2*x^2)^2,x]`

output

$$-1/3*(d + e*x)^{(7/2)}/(b*(a + b*x)^3) + (7*e*(-1/2*(d + e*x)^{(5/2)}/(b*(a + b*x)^2) + (5*e*(-((d + e*x)^{(3/2)}/(b*(a + b*x)))) + (3*e*((2*\text{Sqrt}[d + e*x])/b - (2*\text{Sqrt}[b*d - a*e]*\text{ArcTanh}[(\text{Sqrt}[b]*\text{Sqrt}[d + e*x])/(\text{Sqrt}[b*d - a*e])]/b^{(3/2)})))/(2*b)))/(4*b)))/(6*b)$$

Defintions of rubi rules used

rule 27

$$\text{Int}[(a_*)(Fx_), x_Symbol] \rightarrow \text{Simp}[a \quad \text{Int}[Fx, x], x] /; \text{FreeQ}[a, x] \ \&\& \ !\text{MatchQ}[Fx, (b_*)(Gx_)] /; \text{FreeQ}[b, x]$$

rule 51

$$\text{Int}[(a_.) + (b_.)*(x_)^{(m_)}*((c_.) + (d_.)*(x_)^{(n_)}), x_Symbol] \rightarrow \text{Simp}[(a + b*x)^{(m + 1)}*((c + d*x)^n/(b*(m + 1))), x] - \text{Simp}[d*(n/(b*(m + 1))) \text{Int}[(a + b*x)^{(m + 1)}*(c + d*x)^{(n - 1)}, x], x] /; \text{FreeQ}\{a, b, c, d, n\}, x] \ \&\& \ \text{ILtQ}[m, -1] \ \&\& \ \text{FractionQ}[n] \ \&\& \ \text{GtQ}[n, 0]$$

rule 60

$$\text{Int}[(a_.) + (b_.)*(x_)^{(m_)}*((c_.) + (d_.)*(x_)^{(n_)}), x_Symbol] \rightarrow \text{Simp}[(a + b*x)^{(m + 1)}*((c + d*x)^n/(b*(m + n + 1))), x] + \text{Simp}[n*((b*c - a*d)/(b*(m + n + 1))) \text{Int}[(a + b*x)^m*(c + d*x)^{(n - 1)}, x], x] /; \text{FreeQ}\{a, b, c, d\}, x] \ \&\& \ \text{GtQ}[n, 0] \ \&\& \ \text{NeQ}[m + n + 1, 0] \ \&\& \ !(\text{IGtQ}[m, 0] \ \&\& \ (!\text{IntegerQ}[n] \ || \ (\text{GtQ}[m, 0] \ \&\& \ \text{LtQ}[m - n, 0]))) \ \&\& \ !\text{ILtQ}[m + n + 2, 0] \ \&\& \ \text{IntLinearQ}[a, b, c, d, m, n, x]$$

rule 73

$$\text{Int}[(a_.) + (b_.)*(x_)^{(m_)}*((c_.) + (d_.)*(x_)^{(n_)}), x_Symbol] \rightarrow \text{With}[\{p = \text{Denominator}[m]\}, \text{Simp}[p/b \quad \text{Subst}[\text{Int}[x^{(p*(m + 1) - 1)}*(c - a*(d/b) + d*(x^{p/b})^n), x], x, (a + b*x)^{(1/p)}, x]] /; \text{FreeQ}\{a, b, c, d\}, x] \ \&\& \ \text{LtQ}[-1, m, 0] \ \&\& \ \text{LeQ}[-1, n, 0] \ \&\& \ \text{LeQ}[\text{Denominator}[n], \text{Denominator}[m]] \ \&\& \ \text{IntLinearQ}[a, b, c, d, m, n, x]$$

rule 221

$$\text{Int}[(a_.) + (b_.)*(x_)^2)^{-1}, x_Symbol] \rightarrow \text{Simp}[(\text{Rt}[-a/b, 2]/a)*\text{ArcTanh}[x/\text{Rt}[-a/b, 2]], x] /; \text{FreeQ}\{a, b\}, x] \ \&\& \ \text{NegQ}[a/b]$$

rule 1098

```
Int[((d_.) + (e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] := Simp[1/c^p Int[(d + e*x)^m*(b^2 + c*x)^(2*p), x], x] /; FreeQ[
{a, b, c, d, e, m}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Maple [A] (verified)

Time = 1.93 (sec) , antiderivative size = 151, normalized size of antiderivative = 1.04

| method | result |
|-------------------|--|
| risch | $\frac{2e^3\sqrt{ex+d}}{b^4} - \frac{(2ae-2bd)e^3 \left(\frac{-29(ex+d)^{\frac{5}{2}}b^2}{16} - \frac{17b(ae-bd)(ex+d)^{\frac{3}{2}}}{6} + \left(-\frac{19}{16}e^2a^2 + \frac{19}{8}abde - \frac{19}{16}b^2d^2 \right) \sqrt{ex+d} \right)}{(b(ex+d)+ae-bd)^3} + \frac{35 \arctan\left(\frac{b}{\sqrt{b(ae-bd)}}\right)}{16\sqrt{b(ae-bd)}}$ |
| pseudoelliptic | $-\frac{35 \left(e^3(bx+a)^3(ae-bd) \arctan\left(\frac{b\sqrt{ex+d}}{\sqrt{b(ae-bd)}}\right) - \sqrt{ex+d} \sqrt{b(ae-bd)} \right) \left(\frac{(16e^3x^3 - 29d^2e^2x^2 - \frac{38}{3}d^2ex - \frac{8}{3}d^3)b^3}{35} - 2ea \left(-\frac{33}{2}e \right) \right)}{8\sqrt{b(ae-bd)}b^4(bx+a)^3}$ |
| derivativedivides | $2e^3 \left(\frac{\sqrt{ex+d}}{b^4} - \frac{\left(-\frac{29}{16}ea^2 + \frac{29}{16}b^3d \right) (ex+d)^{\frac{5}{2}} - \frac{17b(e^2a^2 - 2abde + b^2d^2)(ex+d)^{\frac{3}{2}}}{6} + \left(-\frac{19}{16}e^3a^3 + \frac{57}{16}a^2bde^2 - \frac{57}{16}ab^2d^2e + \frac{19}{16}b^3d^3 \right) \sqrt{ex+d}}{(b(ex+d)+ae-bd)^3} \right)$ |
| default | $2e^3 \left(\frac{\sqrt{ex+d}}{b^4} - \frac{\left(-\frac{29}{16}ea^2 + \frac{29}{16}b^3d \right) (ex+d)^{\frac{5}{2}} - \frac{17b(e^2a^2 - 2abde + b^2d^2)(ex+d)^{\frac{3}{2}}}{6} + \left(-\frac{19}{16}e^3a^3 + \frac{57}{16}a^2bde^2 - \frac{57}{16}ab^2d^2e + \frac{19}{16}b^3d^3 \right) \sqrt{ex+d}}{(b(ex+d)+ae-bd)^3} \right)$ |

input `int((e*x+d)^(7/2)/(b^2*x^2+2*a*b*x+a^2)^2,x,method=_RETURNVERBOSE)`

output `2*e^3*(e*x+d)^(1/2)/b^4-1/b^4*(2*a*e-2*b*d)*e^3*((-29/16*(e*x+d)^(5/2)*b^2-17/6*b*(a*e-b*d)*(e*x+d)^(3/2)+(-19/16*e^2*a^2+19/8*a*b*d*e-19/16*b^2*d^2)*(e*x+d)^(1/2))/(b*(e*x+d)+a*e-b*d)^3+35/16/(b*(a*e-b*d))^(1/2)*arctan(b*(e*x+d)^(1/2)/(b*(a*e-b*d))^(1/2))`

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 244 vs. $2(117) = 234$.

Time = 0.09 (sec) , antiderivative size = 498, normalized size of antiderivative = 3.43

$$\int \frac{(d+ex)^{7/2}}{(a^2+2abx+b^2x^2)^2} dx = \frac{105(b^3e^3x^3 + 3ab^2e^3x^2 + 3a^2be^3x + a^3e^3)\sqrt{\frac{bd-ae}{b}} \log\left(\frac{bex+2bd-ae-2\sqrt{ex+db}\sqrt{\frac{bd-ae}{b}}}{bx+a}\right) + 105(b^3e^3x^3 + 3ab^2e^3x^2 + 3a^2be^3x + a^3e^3)\sqrt{-\frac{bd-ae}{b}} \arctan\left(-\frac{\sqrt{ex+db}\sqrt{-\frac{bd-ae}{b}}}{bd-ae}\right) - (48b^3e^3x^3 - 8b^3d^3 - 14a^2b^2d^2e - 35a^2b^2d^2e^2 + 105a^3e^3 - 3(29b^3d^2e^2 - 77a^2b^2e^3)x^2 - 2(19b^3d^2e + 49a^2b^2d^2e^2 - 140a^2b^2e^3)x)\sqrt{ex+d}}{24(b^7x^3 + 3ab^6)}$$

input `integrate((e*x+d)^(7/2)/(b^2*x^2+2*a*b*x+a^2)^2,x, algorithm="fricas")`

output `[1/48*(105*(b^3*e^3*x^3 + 3*a*b^2*e^3*x^2 + 3*a^2*b*e^3*x + a^3*e^3)*sqrt((b*d - a*e)/b)*log((b*e*x + 2*b*d - a*e - 2*sqrt(e*x + d)*b*sqrt((b*d - a*e)/b)))/(b*x + a) + 2*(48*b^3*e^3*x^3 - 8*b^3*d^3 - 14*a*b^2*d^2*e - 35*a^2*b*d^2*e^2 + 105*a^3*e^3 - 3*(29*b^3*d^2*e^2 - 77*a*b^2*e^3)*x^2 - 2*(19*b^3*d^2*e + 49*a*b^2*d^2*e^2 - 140*a^2*b^2*e^3)*x)*sqrt(e*x + d))/(b^7*x^3 + 3*a*b^6*x^2 + 3*a^2*b^5*x + a^3*b^4), -1/24*(105*(b^3*e^3*x^3 + 3*a*b^2*e^3*x^2 + 3*a^2*b*e^3*x + a^3*e^3)*sqrt(-(b*d - a*e)/b)*arctan(-sqrt(e*x + d)*b*sqrt(-(b*d - a*e)/b)/(b*d - a*e)) - (48*b^3*e^3*x^3 - 8*b^3*d^3 - 14*a*b^2*d^2*e - 35*a^2*b*d^2*e^2 + 105*a^3*e^3 - 3*(29*b^3*d^2*e^2 - 77*a*b^2*e^3)*x^2 - 2*(19*b^3*d^2*e + 49*a*b^2*d^2*e^2 - 140*a^2*b^2*e^3)*x)*sqrt(e*x + d))/(b^7*x^3 + 3*a*b^6*x^2 + 3*a^2*b^5*x + a^3*b^4)]`

Sympy [F(-1)]

Timed out.

$$\int \frac{(d+ex)^{7/2}}{(a^2+2abx+b^2x^2)^2} dx = \text{Timed out}$$

input `integrate((e*x+d)**(7/2)/(b**2*x**2+2*a*b*x+a**2)**2,x)`

output Timed out

Maxima [F(-2)]

Exception generated.

$$\int \frac{(d+ex)^{7/2}}{(a^2+2abx+b^2x^2)^2} dx = \text{Exception raised: ValueError}$$

input `integrate((e*x+d)^(7/2)/(b^2*x^2+2*a*b*x+a^2)^2,x, algorithm="maxima")`

output Exception raised: ValueError >> Computation failed since Maxima requested additional constraints; using the 'assume' command before evaluation *may* help (example of legal syntax is 'assume(a*e-b*d>0)', see `assume?` for more detail)

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 245 vs. 2(117) = 234.

Time = 0.18 (sec) , antiderivative size = 245, normalized size of antiderivative = 1.69

$$\int \frac{(d+ex)^{7/2}}{(a^2+2abx+b^2x^2)^2} dx = \frac{2\sqrt{ex+d}e^3}{b^4} + \frac{35(bde^3 - ae^4) \arctan\left(\frac{\sqrt{ex+d}b}{\sqrt{-b^2d+abe}}\right)}{8\sqrt{-b^2d+abe}b^4}$$

$$\frac{87(ex+d)^{5/2}b^3de^3 - 136(ex+d)^{3/2}b^3d^2e^3 + 57\sqrt{ex+d}b^3d^3e^3 - 87(ex+d)^{5/2}ab^2e^4 + 272(ex+d)^{3/2}ab^2de^4}{24((ex+d)b - bd + a^2)}$$

input `integrate((e*x+d)^(7/2)/(b^2*x^2+2*a*b*x+a^2)^2,x, algorithm="giac")`

output

```
2*sqrt(e*x + d)*e^3/b^4 + 35/8*(b*d*e^3 - a*e^4)*arctan(sqrt(e*x + d)*b/sqrt(-b^2*d + a*b*e))/(sqrt(-b^2*d + a*b*e)*b^4) - 1/24*(87*(e*x + d)^(5/2)*b^3*d*e^3 - 136*(e*x + d)^(3/2)*b^3*d^2*e^3 + 57*sqrt(e*x + d)*b^3*d^3*e^3 - 87*(e*x + d)^(5/2)*a*b^2*e^4 + 272*(e*x + d)^(3/2)*a*b^2*d*e^4 - 171*sqrt(e*x + d)*a*b^2*d^2*e^4 - 136*(e*x + d)^(3/2)*a^2*b*e^5 + 171*sqrt(e*x + d)*a^2*b*d*e^5 - 57*sqrt(e*x + d)*a^3*e^6)/(((e*x + d)*b - b*d + a*e)^3*b^4)
```

Mupad [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 303, normalized size of antiderivative = 2.09

$$\int \frac{(d+ex)^{7/2}}{(a^2+2abx+b^2x^2)^2} dx = \frac{\sqrt{d+ex} \left(\frac{19a^3e^6}{8} - \frac{57a^2bde^5}{8} + \frac{57ab^2d^2e^4}{8} - \frac{19b^3d^3e^3}{8} \right) + \left(\frac{29ab^2e^4}{8} - \frac{29b^3de^3}{8} \right) (d+ex)^{5/2} + (d+ex)^{3/2} \left(\frac{17a^2b^2e^5}{3} + \frac{17b^3d^2e^3}{3} - \frac{34a^2b^2d^2e^4}{3} \right)}{b^7(d+ex)^3 - (3b^7d - 3ab^6e)(d+ex)^2 + (d+ex)(3a^2b^5e^2 - 6ab^6e)} + \frac{2e^3\sqrt{d+ex}}{b^4} - \frac{35e^3 \operatorname{atan}\left(\frac{\sqrt{be^3}\sqrt{ae-bd}\sqrt{d+ex}}{ae^4-bde^3}\right) \sqrt{ae-bd}}{8b^{9/2}}$$

input

```
int((d + e*x)^(7/2)/(a^2 + b^2*x^2 + 2*a*b*x)^2,x)
```

output

```
((d + e*x)^(1/2)*((19*a^3*e^6)/8 - (19*b^3*d^3*e^3)/8 + (57*a*b^2*d^2*e^4)/8 - (57*a^2*b*d*e^5)/8) + ((29*a*b^2*e^4)/8 - (29*b^3*d*e^3)/8)*(d + e*x)^(5/2) + (d + e*x)^(3/2)*((17*a^2*b^2*e^5)/3 + (17*b^3*d^2*e^3)/3 - (34*a*b^2*d^2*e^4)/3)/(b^7*(d + e*x)^3 - (3*b^7*d - 3*a*b^6*e)*(d + e*x)^2 + (d + e*x)*(3*b^7*d^2 + 3*a^2*b^5*e^2 - 6*a*b^6*d*e) - b^7*d^3 + a^3*b^4*e^3 - 3*a^2*b^5*d*e^2 + 3*a*b^6*d^2*e) + (2*e^3*(d + e*x)^(1/2))/b^4 - (35*e^3*atan((b^(1/2)*e^3*(a*e - b*d)^(1/2)*(d + e*x)^(1/2))/(a*e^4 - b*d*e^3))*(a*e - b*d)^(1/2))/(8*b^(9/2))
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 383, normalized size of antiderivative = 2.64

$$\int \frac{(d+ex)^{7/2}}{(a^2+2abx+b^2x^2)^2} dx = \frac{-105\sqrt{b}\sqrt{ae-bd} \operatorname{atan}\left(\frac{\sqrt{ex+d}b}{\sqrt{b}\sqrt{ae-bd}}\right) a^3 e^3 - 315\sqrt{b}\sqrt{ae-bd} \operatorname{atan}\left(\frac{\sqrt{ex+d}b}{\sqrt{b}\sqrt{ae-bd}}\right) e^2}{(a^2+2abx+b^2x^2)^2}$$

input `int((e*x+d)^(7/2)/(b^2*x^2+2*a*b*x+a^2)^2,x)`

output `(- 105*sqrt(b)*sqrt(a*e - b*d)*atan((sqrt(d + e*x)*b)/(sqrt(b)*sqrt(a*e - b*d)))*a**3*e**3 - 315*sqrt(b)*sqrt(a*e - b*d)*atan((sqrt(d + e*x)*b)/(sqrt(b)*sqrt(a*e - b*d)))*a**2*b*e**3*x - 315*sqrt(b)*sqrt(a*e - b*d)*atan((sqrt(d + e*x)*b)/(sqrt(b)*sqrt(a*e - b*d)))*a*b**2*e**3*x**2 - 105*sqrt(b)*sqrt(a*e - b*d)*atan((sqrt(d + e*x)*b)/(sqrt(b)*sqrt(a*e - b*d)))*b**3*e**3*x**3 + 105*sqrt(d + e*x)*a**3*b*e**3 - 35*sqrt(d + e*x)*a**2*b**2*d*e**2 + 280*sqrt(d + e*x)*a**2*b**2*e**3*x - 14*sqrt(d + e*x)*a*b**3*d**2*e - 98*sqrt(d + e*x)*a*b**3*d*e**2*x + 231*sqrt(d + e*x)*a*b**3*e**3*x**2 - 8*sqrt(d + e*x)*b**4*d**3 - 38*sqrt(d + e*x)*b**4*d**2*e*x - 87*sqrt(d + e*x)*b**4*d*e**2*x**2 + 48*sqrt(d + e*x)*b**4*e**3*x**3)/(24*b**5*(a**3 + 3*a**2*b*x + 3*a*b**2*x**2 + b**3*x**3))`

3.182
$$\int \frac{(d+ex)^{5/2}}{(a^2+2abx+b^2x^2)^2} dx$$

| | |
|---|------|
| Optimal result | 1441 |
| Mathematica [A] (verified) | 1441 |
| Rubi [A] (verified) | 1442 |
| Maple [A] (verified) | 1444 |
| Fricas [B] (verification not implemented) | 1445 |
| Sympy [F(-1)] | 1445 |
| Maxima [F(-2)] | 1446 |
| Giac [A] (verification not implemented) | 1446 |
| Mupad [B] (verification not implemented) | 1447 |
| Reduce [B] (verification not implemented) | 1447 |

Optimal result

Integrand size = 28, antiderivative size = 126

$$\int \frac{(d+ex)^{5/2}}{(a^2+2abx+b^2x^2)^2} dx = -\frac{5e^2\sqrt{d+ex}}{8b^3(a+bx)} - \frac{5e(d+ex)^{3/2}}{12b^2(a+bx)^2} - \frac{(d+ex)^{5/2}}{3b(a+bx)^3} - \frac{5e^3 \operatorname{arctanh}\left(\frac{\sqrt{b}\sqrt{d+ex}}{\sqrt{bd-ae}}\right)}{8b^{7/2}\sqrt{bd-ae}}$$

output

$$-5/8*e^2*(e*x+d)^{(1/2)}/b^3/(b*x+a)-5/12*e*(e*x+d)^{(3/2)}/b^2/(b*x+a)^2-1/3*(e*x+d)^{(5/2)}/b/(b*x+a)^3-5/8*e^3*\operatorname{arctanh}(b^{(1/2)}*(e*x+d)^{(1/2)}/(-a*e+b*d)^{(1/2)})/b^{(7/2)}/(-a*e+b*d)^{(1/2)}$$

Mathematica [A] (verified)

Time = 0.57 (sec) , antiderivative size = 119, normalized size of antiderivative = 0.94

$$\int \frac{(d+ex)^{5/2}}{(a^2+2abx+b^2x^2)^2} dx = \frac{\sqrt{d+ex}(15a^2e^2+10abe(d+4ex)+b^2(8d^2+26dex+33e^2x^2))}{24b^3(a+bx)^3} + \frac{5e^3 \arctan\left(\frac{\sqrt{b}\sqrt{d+ex}}{\sqrt{-bd+ae}}\right)}{8b^{7/2}\sqrt{-bd+ae}}$$

input `Integrate[(d + e*x)^(5/2)/(a^2 + 2*a*b*x + b^2*x^2)^2,x]`

output `-1/24*(Sqrt[d + e*x]*(15*a^2*e^2 + 10*a*b*e*(d + 4*e*x) + b^2*(8*d^2 + 26*d*e*x + 33*e^2*x^2)))/(b^3*(a + b*x)^3) + (5*e^3*ArcTan[(Sqrt[b]*Sqrt[d + e*x])/Sqrt[-(b*d) + a*e]])/(8*b^(7/2)*Sqrt[-(b*d) + a*e])`

Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 134, normalized size of antiderivative = 1.06, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {1098, 27, 51, 51, 51, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\begin{aligned}
 & \int \frac{(d + ex)^{5/2}}{(a^2 + 2abx + b^2x^2)^2} dx \\
 & \quad \downarrow 1098 \\
 & b^4 \int \frac{(d + ex)^{5/2}}{b^4(a + bx)^4} dx \\
 & \quad \downarrow 27 \\
 & \int \frac{(d + ex)^{5/2}}{(a + bx)^4} dx \\
 & \quad \downarrow 51 \\
 & \frac{5e \int \frac{(d+ex)^{3/2}}{(a+bx)^3} dx}{6b} - \frac{(d + ex)^{5/2}}{3b(a + bx)^3} \\
 & \quad \downarrow 51 \\
 & \frac{5e \left(\frac{3e \int \frac{\sqrt{d+ex}}{(a+bx)^2} dx}{4b} - \frac{(d+ex)^{3/2}}{2b(a+bx)^2} \right)}{6b} - \frac{(d + ex)^{5/2}}{3b(a + bx)^3} \\
 & \quad \downarrow 51
 \end{aligned}$$

$$\begin{aligned}
& \frac{5e \left(\frac{3e \left(\frac{e \int \frac{1}{(a+bx)\sqrt{d+ex}} dx}{2b} - \frac{\sqrt{d+ex}}{b(a+bx)} \right)}{4b} - \frac{(d+ex)^{3/2}}{2b(a+bx)^2} \right)}{6b} - \frac{(d+ex)^{5/2}}{3b(a+bx)^3} \\
& \quad \downarrow \text{73} \\
& \frac{5e \left(\frac{3e \left(\frac{\int \frac{1}{a + \frac{b(d+ex)}{e} - \frac{bd}{e}} d\sqrt{d+ex}}{b} - \frac{\sqrt{d+ex}}{b(a+bx)} \right)}{4b} - \frac{(d+ex)^{3/2}}{2b(a+bx)^2} \right)}{6b} - \frac{(d+ex)^{5/2}}{3b(a+bx)^3} \\
& \quad \downarrow \text{221} \\
& \frac{5e \left(\frac{3e \left(-\frac{e \operatorname{arctanh}\left(\frac{\sqrt{b}\sqrt{d+ex}}{\sqrt{bd-ae}}\right)}{b^{3/2}\sqrt{bd-ae}} - \frac{\sqrt{d+ex}}{b(a+bx)} \right)}{4b} - \frac{(d+ex)^{3/2}}{2b(a+bx)^2} \right)}{6b} - \frac{(d+ex)^{5/2}}{3b(a+bx)^3}
\end{aligned}$$

input `Int[(d + e*x)^(5/2)/(a^2 + 2*a*b*x + b^2*x^2)^2,x]`

output `-1/3*(d + e*x)^(5/2)/(b*(a + b*x)^3) + (5*e*(-1/2*(d + e*x)^(3/2)/(b*(a + b*x)^2) + (3*e*(-(Sqrt[d + e*x]/(b*(a + b*x))) - (e*ArcTanh[(Sqrt[b]*Sqrt[d + e*x])/Sqrt[b*d - a*e]])/(b^(3/2)*Sqrt[b*d - a*e])))/(4*b)))/(6*b)`

Defintions of rubi rules used

rule 27 `Int[(a_)*(F_x_), x_Symbol] := Simp[a Int[F_x, x], x] /; FreeQ[a, x] && !MatchQ[F_x, (b_)*(G_x_)] /; FreeQ[b, x]`

rule 51 `Int[((a_.) + (b_.)*(x_.))^(m_.)*((c_.) + (d_.)*(x_.))^(n_.), x_Symbol] := Simp[(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Simp[d*(n/(b*(m + 1)))] Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x] /; FreeQ[{a, b, c, d, n}, x] && ILtQ[m, -1] && FractionQ[n] && GtQ[n, 0]`

rule 73 `Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] := With[
 {p = Denominator[m]}, Simp[p/b Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x] /; FreeQ[{a, b, c, d}, x] && Lt
 Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
 inearQ[a, b, c, d, m, n, x]`

rule 221 `Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
 /Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]`

rule 1098 `Int[((d_.) + (e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_
 Symbol] := Simp[1/c^p Int[(d + e*x)^m*(b/2 + c*x)^(2*p), x], x] /; FreeQ[
 {a, b, c, d, e, m}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]`

Maple [A] (verified)

Time = 1.83 (sec) , antiderivative size = 122, normalized size of antiderivative = 0.97

| method | result | size |
|-------------------|--|------|
| pseudoelliptic | $\frac{5e^3(bx+a)^3 \arctan\left(\frac{b\sqrt{ex+d}}{\sqrt{b(ae-bd)}}\right) - 5\sqrt{ex+d}\sqrt{b(ae-bd)}\left(\left(\frac{11}{5}e^2x^2 + \frac{26}{15}dex + \frac{8}{15}d^2\right)b^2 + \frac{2ea(4ex+d)b + e^2a^2}{3}\right)}{8b^3(bx+a)^3\sqrt{b(ae-bd)}}$ | 122 |
| derivativedivides | $2e^3\left(\frac{-\frac{11(ex+d)^{\frac{5}{2}}}{16b} - \frac{5(ae-bd)(ex+d)^{\frac{3}{2}}}{6b^2} - \frac{5(e^2a^2 - 2abde + b^2d^2)\sqrt{ex+d}}{16b^3}}{(b(ex+d) + ae - bd)^3} + \frac{5 \arctan\left(\frac{b\sqrt{ex+d}}{\sqrt{b(ae-bd)}}\right)}{16b^3\sqrt{b(ae-bd)}}\right)$ | 130 |
| default | $2e^3\left(\frac{-\frac{11(ex+d)^{\frac{5}{2}}}{16b} - \frac{5(ae-bd)(ex+d)^{\frac{3}{2}}}{6b^2} - \frac{5(e^2a^2 - 2abde + b^2d^2)\sqrt{ex+d}}{16b^3}}{(b(ex+d) + ae - bd)^3} + \frac{5 \arctan\left(\frac{b\sqrt{ex+d}}{\sqrt{b(ae-bd)}}\right)}{16b^3\sqrt{b(ae-bd)}}\right)$ | 130 |

input `int((e*x+d)^(5/2)/(b^2*x^2+2*a*b*x+a^2)^2,x,method=_RETURNVERBOSE)`

output `5/8*(e^3*(b*x+a)^3*arctan(b*(e*x+d)^(1/2)/(b*(a*e-b*d))^(1/2))-(e*x+d)^(1/
 2)*(b*(a*e-b*d))^(1/2)*((11/5*e^2*x^2+26/15*d*e*x+8/15*d^2)*b^2+2/3*e*a*(4
 *e*x+d)*b+e^2*a^2))/(b*(a*e-b*d))^(1/2)/b^3/(b*x+a)^3`

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 275 vs. $2(102) = 204$.

Time = 0.10 (sec) , antiderivative size = 563, normalized size of antiderivative = 4.47

$$\int \frac{(d+ex)^{5/2}}{(a^2+2abx+b^2x^2)^2} dx = \left[\frac{15(b^3e^3x^3 + 3ab^2e^3x^2 + 3a^2be^3x + a^3e^3)\sqrt{b^2d - abe} \log\left(\frac{be^3x + 2bd - ae - 2\sqrt{b^2d - abe}}{bx+a}\right)}{48(a^3b^5d - a^4}$$

input `integrate((e*x+d)^(5/2)/(b^2*x^2+2*a*b*x+a^2)^2,x, algorithm="fricas")`

output `[1/48*(15*(b^3*e^3*x^3 + 3*a*b^2*e^3*x^2 + 3*a^2*b*e^3*x + a^3*e^3)*sqrt(b^2*d - a*b*e)*log((b*e*x + 2*b*d - a*e - 2*sqrt(b^2*d - a*b*e)*sqrt(e*x + d))/(b*x + a)) - 2*(8*b^4*d^3 + 2*a*b^3*d^2*e + 5*a^2*b^2*d*e^2 - 15*a^3*b*e^3 + 33*(b^4*d*e^2 - a*b^3*e^3)*x^2 + 2*(13*b^4*d^2*e + 7*a*b^3*d*e^2 - 20*a^2*b^2*e^3)*x)*sqrt(e*x + d))/(a^3*b^5*d - a^4*b^4*e + (b^8*d - a*b^7*e)*x^3 + 3*(a*b^7*d - a^2*b^6*e)*x^2 + 3*(a^2*b^6*d - a^3*b^5*e)*x), 1/24*(15*(b^3*e^3*x^3 + 3*a*b^2*e^3*x^2 + 3*a^2*b*e^3*x + a^3*e^3)*sqrt(-b^2*d + a*b*e)*arctan(sqrt(-b^2*d + a*b*e)*sqrt(e*x + d)/(b*e*x + b*d)) - (8*b^4*d^3 + 2*a*b^3*d^2*e + 5*a^2*b^2*d*e^2 - 15*a^3*b*e^3 + 33*(b^4*d*e^2 - a*b^3*e^3)*x^2 + 2*(13*b^4*d^2*e + 7*a*b^3*d*e^2 - 20*a^2*b^2*e^3)*x)*sqrt(e*x + d))/(a^3*b^5*d - a^4*b^4*e + (b^8*d - a*b^7*e)*x^3 + 3*(a*b^7*d - a^2*b^6*e)*x^2 + 3*(a^2*b^6*d - a^3*b^5*e)*x)]`

Sympy [F(-1)]

Timed out.

$$\int \frac{(d+ex)^{5/2}}{(a^2+2abx+b^2x^2)^2} dx = \text{Timed out}$$

input `integrate((e*x+d)**(5/2)/(b**2*x**2+2*a*b*x+a**2)**2,x)`

output `Timed out`

Maxima [F(-2)]

Exception generated.

$$\int \frac{(d+ex)^{5/2}}{(a^2+2abx+b^2x^2)^2} dx = \text{Exception raised: ValueError}$$

input `integrate((e*x+d)^(5/2)/(b^2*x^2+2*a*b*x+a^2)^2,x, algorithm="maxima")`

output `Exception raised: ValueError >> Computation failed since Maxima requested additional constraints; using the 'assume' command before evaluation *may* help (example of legal syntax is 'assume(a*e-b*d>0)', see `assume?` for more detail`

Giac [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 161, normalized size of antiderivative = 1.28

$$\int \frac{(d+ex)^{5/2}}{(a^2+2abx+b^2x^2)^2} dx = \frac{5e^3 \arctan\left(\frac{\sqrt{ex+db}}{\sqrt{-b^2d+abe}}\right)}{8\sqrt{-b^2d+abe}b^3} - \frac{33(ex+d)^{5/2}b^2e^3 - 40(ex+d)^{3/2}b^2de^3 + 15\sqrt{ex+db}d^2e^3 + 40(ex+d)^{3/2}abe^4 - 30\sqrt{ex+db}de^4 + 15\sqrt{ex+db}ae^4}{24((ex+d)b - bd + ae)^3b^3}$$

input `integrate((e*x+d)^(5/2)/(b^2*x^2+2*a*b*x+a^2)^2,x, algorithm="giac")`

output `5/8*e^3*arctan(sqrt(e*x + d)*b/sqrt(-b^2*d + a*b*e))/(sqrt(-b^2*d + a*b*e)*b^3) - 1/24*(33*(e*x + d)^(5/2)*b^2*e^3 - 40*(e*x + d)^(3/2)*b^2*d*e^3 + 15*sqrt(e*x + d)*b^2*d^2*e^3 + 40*(e*x + d)^(3/2)*a*b*e^4 - 30*sqrt(e*x + d)*a*b*d*e^4 + 15*sqrt(e*x + d)*a^2*e^5)/(((e*x + d)*b - b*d + a*e)^3*b^3)`

Mupad [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 222, normalized size of antiderivative = 1.76

$$\int \frac{(d+ex)^{5/2}}{(a^2+2abx+b^2x^2)^2} dx = \frac{5e^3 \operatorname{atan}\left(\frac{\sqrt{b}\sqrt{d+ex}}{\sqrt{ae-bd}}\right)}{8b^{7/2}\sqrt{ae-bd}} - \frac{\frac{11e^3(d+ex)^{5/2}}{8b} + \frac{5e^3\sqrt{d+ex}(a^2e^2-2abde+b^2d^2)}{8b^3} + \frac{5e^3(ae-bd)(d+ex)^{3/2}}{3b^2}}{(d+ex)(3a^2be^2-6ab^2de+3b^3d^2)+b^3(d+ex)^3-(3b^3d-3ab^2e)(d+ex)^2+a^3e^3-b^3d^3+3a^2be^2-6ab^2de+3b^3d^2)}$$

input `int((d + e*x)^(5/2)/(a^2 + b^2*x^2 + 2*a*b*x)^2,x)`output
$$\frac{(5e^3 \operatorname{atan}\left(\frac{b^{1/2}(d+ex)^{1/2}}{(ae-bd)^{1/2}}\right))/(8b^{7/2}(ae-bd)^{1/2}) - ((11e^3(d+ex)^{5/2})/(8b) + (5e^3(d+ex)^{1/2}(a^2e^2 + b^2d^2 - 2abde))/(8b^3) + (5e^3(ae-bd)(d+ex)^{3/2})/(3b^2))}{(d+ex)(3b^3d^2 + 3a^2be^2 - 6ab^2de) + b^3(d+ex)^3 - (3b^3d - 3ab^2e)(d+ex)^2 + a^3e^3 - b^3d^3 + 3a^2be^2 - 6ab^2de + 3b^3d^2}$$
Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 410, normalized size of antiderivative = 3.25

$$\int \frac{(d+ex)^{5/2}}{(a^2+2abx+b^2x^2)^2} dx = \frac{15\sqrt{b}\sqrt{ae-bd} \operatorname{atan}\left(\frac{\sqrt{ex+db}}{\sqrt{b}\sqrt{ae-bd}}\right) a^3e^3 + 45\sqrt{b}\sqrt{ae-bd} \operatorname{atan}\left(\frac{\sqrt{ex+db}}{\sqrt{b}\sqrt{ae-bd}}\right) a^2be^2}{(d+ex)(3a^2be^2-6ab^2de+3b^3d^2)+b^3(d+ex)^3-(3b^3d-3ab^2e)(d+ex)^2+a^3e^3-b^3d^3+3a^2be^2-6ab^2de+3b^3d^2}$$

input `int((e*x+d)^(5/2)/(b^2*x^2+2*a*b*x+a^2)^2,x)`

output

```
(15*sqrt(b)*sqrt(a*e - b*d)*atan((sqrt(d + e*x)*b)/(sqrt(b)*sqrt(a*e - b*d
)))**3**e**3 + 45*sqrt(b)*sqrt(a*e - b*d)*atan((sqrt(d + e*x)*b)/(sqrt(b)
*sqrt(a*e - b*d)))*a**2*b**e**3*x + 45*sqrt(b)*sqrt(a*e - b*d)*atan((sqrt(d
+ e*x)*b)/(sqrt(b)*sqrt(a*e - b*d)))*a*b**2**e**3*x**2 + 15*sqrt(b)*sqrt(a
*e - b*d)*atan((sqrt(d + e*x)*b)/(sqrt(b)*sqrt(a*e - b*d)))*b**3**e**3*x**3
- 15*sqrt(d + e*x)*a**3*b**e**3 + 5*sqrt(d + e*x)*a**2*b**2*d**e**2 - 40*sq
rt(d + e*x)*a**2*b**2**e**3*x + 2*sqrt(d + e*x)*a*b**3*d**2*e + 14*sqrt(d +
e*x)*a*b**3*d**e**2*x - 33*sqrt(d + e*x)*a*b**3**e**3*x**2 + 8*sqrt(d + e*x
)*b**4*d**3 + 26*sqrt(d + e*x)*b**4*d**2*e*x + 33*sqrt(d + e*x)*b**4*d**e**
2*x**2)/(24*b**4*(a**4*e - a**3*b*d + 3*a**3*b*e*x - 3*a**2*b**2*d*x + 3*a
**2*b**2*e*x**2 - 3*a*b**3*d*x**2 + a*b**3*e*x**3 - b**4*d*x**3))
```

3.183 $\int \frac{(d+ex)^{3/2}}{(a^2+2abx+b^2x^2)^2} dx$

| | |
|---|------|
| Optimal result | 1449 |
| Mathematica [A] (verified) | 1449 |
| Rubi [A] (verified) | 1450 |
| Maple [A] (verified) | 1452 |
| Fricas [B] (verification not implemented) | 1453 |
| Sympy [F(-1)] | 1454 |
| Maxima [F(-2)] | 1454 |
| Giac [A] (verification not implemented) | 1454 |
| Mupad [B] (verification not implemented) | 1455 |
| Reduce [B] (verification not implemented) | 1455 |

Optimal result

Integrand size = 28, antiderivative size = 136

$$\int \frac{(d+ex)^{3/2}}{(a^2+2abx+b^2x^2)^2} dx = -\frac{e\sqrt{d+ex}}{4b^2(a+bx)^2} - \frac{e^2\sqrt{d+ex}}{8b^2(bd-ae)(a+bx)} - \frac{(d+ex)^{3/2}}{3b(a+bx)^3} + \frac{e^3 \operatorname{arctanh}\left(\frac{\sqrt{b}\sqrt{d+ex}}{\sqrt{bd-ae}}\right)}{8b^{5/2}(bd-ae)^{3/2}}$$

output

```
-1/4*e*(e*x+d)^(1/2)/b^2/(b*x+a)^2-1/8*e^2*(e*x+d)^(1/2)/b^2/(-a*e+b*d)/(b*x+a)-1/3*(e*x+d)^(3/2)/b/(b*x+a)^3+1/8*e^3*arctanh(b^(1/2)*(e*x+d)^(1/2)/(-a*e+b*d)^(1/2))/b^(5/2)/(-a*e+b*d)^(3/2)
```

Mathematica [A] (verified)

Time = 0.59 (sec) , antiderivative size = 129, normalized size of antiderivative = 0.95

$$\int \frac{(d+ex)^{3/2}}{(a^2+2abx+b^2x^2)^2} dx = \frac{\sqrt{d+ex}(-3a^2e^2-2abe(d+4ex)+b^2(8d^2+14dex+3e^2x^2))}{24b^2(-bd+ae)(a+bx)^3} + \frac{e^3 \arctan\left(\frac{\sqrt{b}\sqrt{d+ex}}{\sqrt{-bd+ae}}\right)}{8b^{5/2}(-bd+ae)^{3/2}}$$

input `Integrate[(d + e*x)^(3/2)/(a^2 + 2*a*b*x + b^2*x^2)^2,x]`

output `(Sqrt[d + e*x]*(-3*a^2*e^2 - 2*a*b*e*(d + 4*e*x) + b^2*(8*d^2 + 14*d*e*x + 3*e^2*x^2)))/(24*b^2*(-(b*d) + a*e)*(a + b*x)^3) + (e^3*ArcTan[(Sqrt[b]*Sqrt[d + e*x])/Sqrt[-(b*d) + a*e]])/(8*b^(5/2)*(-(b*d) + a*e)^(3/2))`

Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 140, normalized size of antiderivative = 1.03, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {1098, 27, 51, 51, 52, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\begin{aligned}
 & \int \frac{(d + ex)^{3/2}}{(a^2 + 2abx + b^2x^2)^2} dx \\
 & \quad \downarrow \text{1098} \\
 & b^4 \int \frac{(d + ex)^{3/2}}{b^4(a + bx)^4} dx \\
 & \quad \downarrow \text{27} \\
 & \int \frac{(d + ex)^{3/2}}{(a + bx)^4} dx \\
 & \quad \downarrow \text{51} \\
 & \frac{e \int \frac{\sqrt{d+ex}}{(a+bx)^3} dx}{2b} - \frac{(d + ex)^{3/2}}{3b(a + bx)^3} \\
 & \quad \downarrow \text{51} \\
 & \frac{e \left(\frac{e \int \frac{1}{(a+bx)^2 \sqrt{d+ex}} dx}{4b} - \frac{\sqrt{d+ex}}{2b(a+bx)^2} \right)}{2b} - \frac{(d + ex)^{3/2}}{3b(a + bx)^3} \\
 & \quad \downarrow \text{52}
 \end{aligned}$$

rule 52 `Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] := Simp[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((m + n + 2)/((b*c - a*d)*(m + 1))) Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && ILtQ[m, -1] && FractionQ[n] && LtQ[n, 0]`

rule 73 `Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] := With[{p = Denominator[m]}, Simp[p/b Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) + d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntLinearQ[a, b, c, d, m, n, x]`

rule 221 `Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[-a/b, 2]/a)*ArcTanh[x/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]`

rule 1098 `Int[((d_.) + (e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] := Simp[1/c^p Int[(d + e*x)^m*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]`

Maple [A] (verified)

Time = 1.51 (sec) , antiderivative size = 119, normalized size of antiderivative = 0.88

| method | result | size |
|-------------------|--|------|
| pseudoelliptic | $\frac{e^3(bx+a)^3 \arctan\left(\frac{b\sqrt{ex+d}}{\sqrt{b(ae-bd)}}\right) - ((3ex+2d)b+ae)\sqrt{ex+d}\sqrt{b(ae-bd)}\left(\frac{-ex-4d}{3}b+ae\right)}{8\sqrt{b(ae-bd)}(ae-bd)b^2(bx+a)^3}$ | 119 |
| derivativedivides | $2e^3 \left(\frac{\frac{(ex+d)^{\frac{5}{2}}}{16ae-16bd} - \frac{(ex+d)^{\frac{3}{2}}}{6b} - \frac{(ae-bd)\sqrt{ex+d}}{16b^2}}{(b(ex+d)+ae-bd)^3} + \frac{\arctan\left(\frac{b\sqrt{ex+d}}{\sqrt{b(ae-bd)}}\right)}{16(ae-bd)b^2\sqrt{b(ae-bd)}} \right)$ | 126 |
| default | $2e^3 \left(\frac{\frac{(ex+d)^{\frac{5}{2}}}{16ae-16bd} - \frac{(ex+d)^{\frac{3}{2}}}{6b} - \frac{(ae-bd)\sqrt{ex+d}}{16b^2}}{(b(ex+d)+ae-bd)^3} + \frac{\arctan\left(\frac{b\sqrt{ex+d}}{\sqrt{b(ae-bd)}}\right)}{16(ae-bd)b^2\sqrt{b(ae-bd)}} \right)$ | 126 |

input `int((e*x+d)^(3/2)/(b^2*x^2+2*a*b*x+a^2)^2,x,method=_RETURNVERBOSE)`

output

$$\frac{1}{8} \frac{(e^{3bx+a})^3 \arctan(b(e^{bx+d})^{1/2} / (b(ae-bd))^{1/2}) - ((3e^{bx+2d})^3 + bae)(e^{bx+d})^{1/2} (b(ae-bd))^{1/2} (1/3(-e^{bx-4d})^3 + bae)}{(b(ae-bd))^{1/2} / (ae-bd) / b^2 / (bx+a)^3}$$
Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 326 vs. $2(112) = 224$.

Time = 0.11 (sec) , antiderivative size = 666, normalized size of antiderivative = 4.90

$$\int \frac{(d+ex)^{3/2}}{(a^2+2abx+b^2x^2)^2} dx = \left[-\frac{3(b^3e^3x^3+3ab^2e^3x^2+3a^2be^3x+a^3e^3)\sqrt{b^2d-abe} \log\left(\frac{bex+2bd-ae-2\sqrt{b^2d-abe}}{bx+a}\right) + 3(b^3e^3x^3+3ab^2e^3x^2+3a^2be^3x+a^3e^3)\sqrt{-b^2d+abe} \arctan\left(\frac{\sqrt{-b^2d+abe}\sqrt{ex+d}}{bex+bd}\right) + (8b^4d^3-10ab^3d^2e-a^2b^2d^2e^2)}{24(a^3b^5d^2-2a^4b^4de+a^5b^3e^2+(b^8d^2-2ab^7de+a^2b^6e^2)x^3+3(ab^7d^2-2ab^6de+a^2b^5e^2)x^2+3(a^2b^6d^2-2a^3b^5d^2e+a^4b^4e^2)x)} \right]$$

input

```
integrate((e*x+d)^(3/2)/(b^2*x^2+2*a*b*x+a^2)^2,x, algorithm="fricas")
```

output

```
[ -1/48*(3*(b^3*e^3*x^3 + 3*a*b^2*e^3*x^2 + 3*a^2*b*e^3*x + a^3*e^3)*sqrt(b^2*d - a*b*e)*log((b*e*x + 2*b*d - a*e - 2*sqrt(b^2*d - a*b*e)*sqrt(e*x + d))/(b*x + a)) + 2*(8*b^4*d^3 - 10*a*b^3*d^2*e - a^2*b^2*d*e^2 + 3*a^3*b*e^3 + 3*(b^4*d*e^2 - a*b^3*e^3)*x^2 + 2*(7*b^4*d^2*e - 11*a*b^3*d*e^2 + 4*a^2*b^2*e^3)*x)*sqrt(e*x + d))/(a^3*b^5*d^2 - 2*a^4*b^4*d*e + a^5*b^3*e^2 + (b^8*d^2 - 2*a*b^7*d*e + a^2*b^6*e^2)*x^3 + 3*(a*b^7*d^2 - 2*a^2*b^6*d*e + a^3*b^5*e^2)*x^2 + 3*(a^2*b^6*d^2 - 2*a^3*b^5*d*e + a^4*b^4*e^2)*x), -1/24*(3*(b^3*e^3*x^3 + 3*a*b^2*e^3*x^2 + 3*a^2*b*e^3*x + a^3*e^3)*sqrt(-b^2*d + a*b*e)*arctan(sqrt(-b^2*d + a*b*e)*sqrt(e*x + d)/(b*e*x + b*d)) + (8*b^4*d^3 - 10*a*b^3*d^2*e - a^2*b^2*d*e^2 + 3*a^3*b*e^3 + 3*(b^4*d*e^2 - a*b^3*e^3)*x^2 + 2*(7*b^4*d^2*e - 11*a*b^3*d*e^2 + 4*a^2*b^2*e^3)*x)*sqrt(e*x + d))/(a^3*b^5*d^2 - 2*a^4*b^4*d*e + a^5*b^3*e^2 + (b^8*d^2 - 2*a*b^7*d*e + a^2*b^6*e^2)*x^3 + 3*(a*b^7*d^2 - 2*a^2*b^6*d*e + a^3*b^5*e^2)*x^2 + 3*(a^2*b^6*d^2 - 2*a^3*b^5*d*e + a^4*b^4*e^2)*x)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(d+ex)^{3/2}}{(a^2+2abx+b^2x^2)^2} dx = \text{Timed out}$$

input `integrate((e*x+d)**(3/2)/(b**2*x**2+2*a*b*x+a**2)**2,x)`

output `Timed out`

Maxima [F(-2)]

Exception generated.

$$\int \frac{(d+ex)^{3/2}}{(a^2+2abx+b^2x^2)^2} dx = \text{Exception raised: ValueError}$$

input `integrate((e*x+d)^(3/2)/(b^2*x^2+2*a*b*x+a^2)^2,x, algorithm="maxima")`

output `Exception raised: ValueError >> Computation failed since Maxima requested additional constraints; using the 'assume' command before evaluation *may* help (example of legal syntax is 'assume(a*e-b*d>0)', see `assume?` for more detail`

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 185, normalized size of antiderivative = 1.36

$$\int \frac{(d+ex)^{3/2}}{(a^2+2abx+b^2x^2)^2} dx = -\frac{e^3 \arctan\left(\frac{\sqrt{ex+db}}{\sqrt{-b^2d+abe}}\right)}{8(b^3d-ab^2e)\sqrt{-b^2d+abe}} - \frac{3(ex+d)^{5/2}b^2e^3 + 8(ex+d)^{3/2}b^2de^3 - 3\sqrt{ex+db}d^2e^3 - 8(ex+d)^{3/2}abe^4 + 6\sqrt{ex+db}d^2e^3 - 3\sqrt{ex+db}d^2e^3}{24(b^3d-ab^2e)((ex+d)b-bd+ae)^3}$$

input `integrate((e*x+d)^(3/2)/(b^2*x^2+2*a*b*x+a^2)^2,x, algorithm="giac")`

output

```
-1/8*e^3*arctan(sqrt(e*x + d)*b/sqrt(-b^2*d + a*b*e))/((b^3*d - a*b^2*e)*s
qrt(-b^2*d + a*b*e)) - 1/24*(3*(e*x + d)^(5/2)*b^2*e^3 + 8*(e*x + d)^(3/2)
*b^2*d*e^3 - 3*sqrt(e*x + d)*b^2*d^2*e^3 - 8*(e*x + d)^(3/2)*a*b*e^4 + 6*s
qrt(e*x + d)*a*b*d*e^4 - 3*sqrt(e*x + d)*a^2*e^5)/((b^3*d - a*b^2*e)*((e*x
+ d)*b - b*d + a*e)^3)
```

Mupad [B] (verification not implemented)

Time = 9.27 (sec) , antiderivative size = 209, normalized size of antiderivative = 1.54

$$\int \frac{(d+ex)^{3/2}}{(a^2+2abx+b^2x^2)^2} dx = \frac{e^3 \operatorname{atan}\left(\frac{\sqrt{b}\sqrt{d+ex}}{\sqrt{ae-bd}}\right)}{8b^{5/2}(ae-bd)^{3/2}} - \frac{e^3(d+ex)^{3/2}}{3b} - \frac{e^3(d+ex)^{5/2}}{8(ae-bd)} + \frac{e^3(ae-bd)\sqrt{d+ex}}{8b^2}$$

$$\frac{1}{(d+ex)(3a^2be^2 - 6ab^2de + 3b^3d^2) + b^3(d+ex)^3 - (3b^3d - 3ab^2e)(d+ex)^2 + a^3e^3 - b^3d^3 + 3}$$

input

```
int((d + e*x)^(3/2)/(a^2 + b^2*x^2 + 2*a*b*x)^2,x)
```

output

```
(e^3*atan((b^(1/2)*(d + e*x)^(1/2))/(a*e - b*d)^(1/2)))/(8*b^(5/2)*(a*e -
b*d)^(3/2)) - ((e^3*(d + e*x)^(3/2))/(3*b) - (e^3*(d + e*x)^(5/2))/(8*(a*e
- b*d)) + (e^3*(a*e - b*d)*(d + e*x)^(1/2))/(8*b^2))/((d + e*x)*(3*b^3*d^
2 + 3*a^2*b*e^2 - 6*a*b^2*d*e) + b^3*(d + e*x)^3 - (3*b^3*d - 3*a*b^2*e)*(
d + e*x)^2 + a^3*e^3 - b^3*d^3 + 3*a*b^2*d^2*e - 3*a^2*b*d*e^2)
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 470, normalized size of antiderivative = 3.46

$$\int \frac{(d+ex)^{3/2}}{(a^2+2abx+b^2x^2)^2} dx = \frac{3\sqrt{b}\sqrt{ae-bd} \operatorname{atan}\left(\frac{\sqrt{ex+db}}{\sqrt{b}\sqrt{ae-bd}}\right) a^3e^3 + 9\sqrt{b}\sqrt{ae-bd} \operatorname{atan}\left(\frac{\sqrt{ex+db}}{\sqrt{b}\sqrt{ae-bd}}\right) a^2be^3x}{(d+ex)(3a^2be^2 - 6ab^2de + 3b^3d^2) + b^3(d+ex)^3 - (3b^3d - 3ab^2e)(d+ex)^2 + a^3e^3 - b^3d^3 + 3}$$

input

```
int((e*x+d)^(3/2)/(b^2*x^2+2*a*b*x+a^2)^2,x)
```

output

```
(3*sqrt(b)*sqrt(a*e - b*d)*atan((sqrt(d + e*x)*b)/(sqrt(b)*sqrt(a*e - b*d)))*a**3*e**3 + 9*sqrt(b)*sqrt(a*e - b*d)*atan((sqrt(d + e*x)*b)/(sqrt(b)*sqrt(a*e - b*d)))*a**2*b*e**3*x + 9*sqrt(b)*sqrt(a*e - b*d)*atan((sqrt(d + e*x)*b)/(sqrt(b)*sqrt(a*e - b*d)))*a*b**2*e**3*x**2 + 3*sqrt(b)*sqrt(a*e - b*d)*atan((sqrt(d + e*x)*b)/(sqrt(b)*sqrt(a*e - b*d)))*b**3*e**3*x**3 - 3*sqrt(d + e*x)*a**3*b*e**3 + sqrt(d + e*x)*a**2*b**2*d*e**2 - 8*sqrt(d + e*x)*a**2*b**2*e**3*x + 10*sqrt(d + e*x)*a*b**3*d**2*e + 22*sqrt(d + e*x)*a*b**3*d*e**2*x + 3*sqrt(d + e*x)*a*b**3*e**3*x**2 - 8*sqrt(d + e*x)*b**4*d**3 - 14*sqrt(d + e*x)*b**4*d**2*e*x - 3*sqrt(d + e*x)*b**4*d*e**2*x**2)/(24*b**3*(a**5*e**2 - 2*a**4*b*d*e + 3*a**4*b*e**2*x + a**3*b**2*d**2 - 6*a**3*b**2*d*e*x + 3*a**3*b**2*e**2*x**2 + 3*a**2*b**3*d**2*x - 6*a**2*b**3*d*e*x**2 + a**2*b**3*e**2*x**3 + 3*a*b**4*d**2*x**2 - 2*a*b**4*d*e*x**3 + b**5*d**2*x**3))
```

3.184 $\int \frac{\sqrt{d+ex}}{(a^2+2abx+b^2x^2)^2} dx$

| | |
|---|------|
| Optimal result | 1457 |
| Mathematica [A] (verified) | 1457 |
| Rubi [A] (verified) | 1458 |
| Maple [A] (verified) | 1460 |
| Fricas [B] (verification not implemented) | 1461 |
| Sympy [F(-1)] | 1462 |
| Maxima [F(-2)] | 1462 |
| Giac [A] (verification not implemented) | 1462 |
| Mupad [B] (verification not implemented) | 1463 |
| Reduce [B] (verification not implemented) | 1463 |

Optimal result

Integrand size = 28, antiderivative size = 146

$$\int \frac{\sqrt{d+ex}}{(a^2+2abx+b^2x^2)^2} dx = -\frac{\sqrt{d+ex}}{3b(a+bx)^3} - \frac{e\sqrt{d+ex}}{12b(bd-ae)(a+bx)^2} + \frac{e^2\sqrt{d+ex}}{8b(bd-ae)^2(a+bx)} - \frac{e^3 \operatorname{arctanh}\left(\frac{\sqrt{b}\sqrt{d+ex}}{\sqrt{bd-ae}}\right)}{8b^{3/2}(bd-ae)^{5/2}}$$

output

```
-1/3*(e*x+d)^(1/2)/b/(b*x+a)^3-1/12*e*(e*x+d)^(1/2)/b/(-a*e+b*d)/(b*x+a)^2
+1/8*e^2*(e*x+d)^(1/2)/b/(-a*e+b*d)^2/(b*x+a)-1/8*e^3*arctanh(b^(1/2)*(e*x
+d)^(1/2)/(-a*e+b*d)^(1/2))/b^(3/2)/(-a*e+b*d)^(5/2)
```

Mathematica [A] (verified)

Time = 0.64 (sec) , antiderivative size = 131, normalized size of antiderivative = 0.90

$$\int \frac{\sqrt{d+ex}}{(a^2+2abx+b^2x^2)^2} dx = \frac{\sqrt{d+ex}(-3a^2e^2+2abe(7d+4ex)+b^2(-8d^2-2dex+3e^2x^2))}{24b(bd-ae)^2(a+bx)^3} + \frac{e^3 \arctan\left(\frac{\sqrt{b}\sqrt{d+ex}}{\sqrt{-bd+ae}}\right)}{8b^{3/2}(-bd+ae)^{5/2}}$$

input `Integrate[Sqrt[d + e*x]/(a^2 + 2*a*b*x + b^2*x^2)^2,x]`

output `(Sqrt[d + e*x]*(-3*a^2*e^2 + 2*a*b*e*(7*d + 4*e*x) + b^2*(-8*d^2 - 2*d*e*x + 3*e^2*x^2)))/(24*b*(b*d - a*e)^2*(a + b*x)^3) + (e^3*ArcTan[(Sqrt[b]*Sqrt[d + e*x])/Sqrt[-(b*d) + a*e]])/(8*b^(3/2)*(-(b*d) + a*e)^(5/2))`

Rubi [A] (verified)

Time = 0.46 (sec) , antiderivative size = 154, normalized size of antiderivative = 1.05, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {1098, 27, 51, 52, 52, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\begin{aligned}
 & \int \frac{\sqrt{d+ex}}{(a^2+2abx+b^2x^2)^2} dx \\
 & \quad \downarrow \text{1098} \\
 & b^4 \int \frac{\sqrt{d+ex}}{b^4(a+bx)^4} dx \\
 & \quad \downarrow \text{27} \\
 & \int \frac{\sqrt{d+ex}}{(a+bx)^4} dx \\
 & \quad \downarrow \text{51} \\
 & \frac{e \int \frac{1}{(a+bx)^3 \sqrt{d+ex}} dx}{6b} - \frac{\sqrt{d+ex}}{3b(a+bx)^3} \\
 & \quad \downarrow \text{52} \\
 & \frac{e \left(-\frac{3e \int \frac{1}{(a+bx)^2 \sqrt{d+ex}} dx}{4(bd-ae)} - \frac{\sqrt{d+ex}}{2(a+bx)^2(bd-ae)} \right)}{6b} - \frac{\sqrt{d+ex}}{3b(a+bx)^3} \\
 & \quad \downarrow \text{52}
 \end{aligned}$$

rule 52 `Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] := Simp[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((m + n + 2)/((b*c - a*d)*(m + 1))) Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && ILtQ[m, -1] && FractionQ[n] && LtQ[n, 0]`

rule 73 `Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] := With[{p = Denominator[m]}, Simp[p/b Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) + d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntLinearQ[a, b, c, d, m, n, x]`

rule 221 `Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[-a/b, 2]/a)*ArcTanh[x/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]`

rule 1098 `Int[((d_.) + (e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] := Simp[1/c^p Int[(d + e*x)^m*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]`

Maple [A] (verified)

Time = 1.46 (sec) , antiderivative size = 118, normalized size of antiderivative = 0.81

| method | result | size |
|-------------------|--|------|
| pseudoelliptic | $\frac{e^3(bx+a)^3 \arctan\left(\frac{b\sqrt{ex+d}}{\sqrt{b(ae-bd)}}\right) - \sqrt{ex+d} \sqrt{b(ae-bd)}((-3ex-4d)b+ae)\left(\frac{(ex-2d)b+ae}{3}\right)}{8\sqrt{b(ae-bd)}(bx+a)^3(ae-bd)^2b}$ | 118 |
| derivativedivides | $2e^3 \left(\frac{\frac{b(ex+d)^{\frac{5}{2}}}{16e^2a^2-32abde+16b^2d^2} + \frac{(ex+d)^{\frac{3}{2}}}{6ae-6bd} - \frac{\sqrt{ex+d}}{16b}}{(b(ex+d)+ae-bd)^3} + \frac{\arctan\left(\frac{b\sqrt{ex+d}}{\sqrt{b(ae-bd)}}\right)}{16b(e^2a^2-2abde+b^2d^2)\sqrt{b(ae-bd)}} \right)$ | 152 |
| default | $2e^3 \left(\frac{\frac{b(ex+d)^{\frac{5}{2}}}{16e^2a^2-32abde+16b^2d^2} + \frac{(ex+d)^{\frac{3}{2}}}{6ae-6bd} - \frac{\sqrt{ex+d}}{16b}}{(b(ex+d)+ae-bd)^3} + \frac{\arctan\left(\frac{b\sqrt{ex+d}}{\sqrt{b(ae-bd)}}\right)}{16b(e^2a^2-2abde+b^2d^2)\sqrt{b(ae-bd)}} \right)$ | 152 |

input `int((e*x+d)^(1/2)/(b^2*x^2+2*a*b*x+a^2)^2,x,method=_RETURNVERBOSE)`

output

$$\frac{1}{8} \sqrt{\frac{e^3(bx+a)^3 \arctan\left(\frac{\sqrt{e^3(bx+a)^3}}{\sqrt{b(ae-bd)}}\right) - (e^3(bx+a)^3)^{1/2} \sqrt{b(ae-bd)}}{(bx+a)^3 (ae-bd)^{2/b}}}$$

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 386 vs. $2(122) = 244$.

Time = 0.11 (sec) , antiderivative size = 785, normalized size of antiderivative = 5.38

$$\int \frac{\sqrt{d+ex}}{(a^2+2abx+b^2x^2)^2} dx$$

$$= \frac{3(b^3e^3x^3 + 3ab^2e^3x^2 + 3a^2be^3x + a^3e^3)\sqrt{b^2d-abe} \log\left(\frac{bex+2bd-ae-2\sqrt{b^2d-abe}\sqrt{ex+d}}{bx+a}\right) - 2(8b^4d^3 - 22ab^3d^2e + 17a^2b^2d^2e^2 - 3a^3b^2e^3 - 3(b^4d^2e^2 - a^2b^3d^2e^2 + 4a^2b^2e^3)x)\sqrt{ex+d}}{48(a^3b^5d^3 - 3a^4b^4d^2e + 3a^5b^3de^2 - a^6b^2e^3 + (b^8d^3 - 3ab^7d^2e + 3a^2b^6de^2 - a^3b^5e^3)x^3 + 3(ab^7d^3 - 3a^2b^6d^2e + 3a^3b^5d^2e^2 - a^4b^4e^3)x^2 + 3(a^2b^6d^3 - 3a^3b^5d^2e + 3a^4b^4d^2e^2 - a^5b^3e^3)x)}$$

input

```
integrate((e*x+d)^(1/2)/(b^2*x^2+2*a*b*x+a^2)^2,x, algorithm="fricas")
```

output

```
[1/48*(3*(b^3*e^3*x^3 + 3*a*b^2*e^3*x^2 + 3*a^2*b*e^3*x + a^3*e^3)*sqrt(b^2*d - a*b*e)*log((b*e*x + 2*b*d - a*e - 2*sqrt(b^2*d - a*b*e)*sqrt(e*x + d))/(b*x + a)) - 2*(8*b^4*d^3 - 22*a*b^3*d^2*e + 17*a^2*b^2*d^2*e^2 - 3*a^3*b^2*e^3 - 3*(b^4*d^2*e^2 - a*b^3*d^2*e^2 + 4*a^2*b^2*e^3)*x)*sqrt(e*x + d))/(a^3*b^5*d^3 - 3*a^4*b^4*d^2*e + 3*a^5*b^3*d^2*e^2 - a^6*b^2*e^3 + (b^8*d^3 - 3*a*b^7*d^2*e + 3*a^2*b^6*d^2*e^2 - a^3*b^5*e^3)*x^3 + 3*(a*b^7*d^3 - 3*a^2*b^6*d^2*e + 3*a^3*b^5*d^2*e^2 - a^4*b^4*e^3)*x^2 + 3*(a^2*b^6*d^3 - 3*a^3*b^5*d^2*e + 3*a^4*b^4*d^2*e^2 - a^5*b^3*e^3)*x), 1/24*(3*(b^3*e^3*x^3 + 3*a*b^2*e^3*x^2 + 3*a^2*b*e^3*x + a^3*e^3)*sqrt(-b^2*d + a*b*e)*arctan(sqrt(-b^2*d + a*b*e)*sqrt(e*x + d)/(b*e*x + b*d)) - (8*b^4*d^3 - 22*a*b^3*d^2*e + 17*a^2*b^2*d^2*e^2 - 3*a^3*b^2*e^3 - 3*(b^4*d^2*e^2 - a*b^3*d^2*e^2 + 4*a^2*b^2*e^3)*x)*sqrt(e*x + d))/(a^3*b^5*d^3 - 3*a^4*b^4*d^2*e + 3*a^5*b^3*d^2*e^2 - a^6*b^2*e^3 + (b^8*d^3 - 3*a*b^7*d^2*e + 3*a^2*b^6*d^2*e^2 - a^3*b^5*e^3)*x^3 + 3*(a*b^7*d^3 - 3*a^2*b^6*d^2*e + 3*a^3*b^5*d^2*e^2 - a^4*b^4*e^3)*x^2 + 3*(a^2*b^6*d^3 - 3*a^3*b^5*d^2*e + 3*a^4*b^4*d^2*e^2 - a^5*b^3*e^3)*x)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\sqrt{d+ex}}{(a^2+2abx+b^2x^2)^2} dx = \text{Timed out}$$

input `integrate((e*x+d)**(1/2)/(b**2*x**2+2*a*b*x+a**2)**2,x)`

output `Timed out`

Maxima [F(-2)]

Exception generated.

$$\int \frac{\sqrt{d+ex}}{(a^2+2abx+b^2x^2)^2} dx = \text{Exception raised: ValueError}$$

input `integrate((e*x+d)^(1/2)/(b^2*x^2+2*a*b*x+a^2)^2,x, algorithm="maxima")`

output `Exception raised: ValueError >> Computation failed since Maxima requested additional constraints; using the 'assume' command before evaluation *may* help (example of legal syntax is 'assume(a*e-b*d>0)', see `assume?` for more detail`

Giac [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 207, normalized size of antiderivative = 1.42

$$\int \frac{\sqrt{d+ex}}{(a^2+2abx+b^2x^2)^2} dx = \frac{e^3 \arctan\left(\frac{\sqrt{ex+db}}{\sqrt{-b^2d+abe}}\right)}{8(b^3d^2-2ab^2de+a^2be^2)\sqrt{-b^2d+abe}} + \frac{3(ex+d)^{\frac{5}{2}}b^2e^3-8(ex+d)^{\frac{3}{2}}b^2de^3-3\sqrt{ex+db}b^2d^2e^3+8(ex+d)^{\frac{3}{2}}abe^4+6\sqrt{ex+db}abde^4-3\sqrt{ex+db}a^2e^4}{24(b^3d^2-2ab^2de+a^2be^2)((ex+d)b-bd+ae)^3}$$

input `integrate((e*x+d)^(1/2)/(b^2*x^2+2*a*b*x+a^2)^2,x, algorithm="giac")`

output

$$\frac{1/8e^3 \arctan(\sqrt{ex+d} b / \sqrt{-b^2d + a b e}) / ((b^3d^2 - 2ab^2de + a^2b^2e^2) \sqrt{-b^2d + a b e}) + 1/24(3(ex+d)^{5/2} b^2e^3 - 8(ex+d)^{3/2} b^2d^2e^3 - 3\sqrt{ex+d} b^2d^2e^3 + 8(ex+d)^{3/2} a b e^4 + 6\sqrt{ex+d} a b d e^4 - 3\sqrt{ex+d} a^2 e^5) / ((b^3d^2 - 2ab^2d^2e + a^2b^2e^2) ((ex+d)b - b^2d + a e)^3)}$$

Mupad [B] (verification not implemented)

Time = 9.48 (sec) , antiderivative size = 207, normalized size of antiderivative = 1.42

$$\int \frac{\sqrt{d+ex}}{(a^2+2abx+b^2x^2)^2} dx$$

$$= \frac{\frac{e^3(d+ex)^{3/2}}{3(ae-bd)} - \frac{e^3\sqrt{d+ex}}{8b} + \frac{be^3(d+ex)^{5/2}}{8(ae-bd)^2}}{(d+ex)(3a^2be^2 - 6ab^2de + 3b^3d^2) + b^3(d+ex)^3 - (3b^3d - 3ab^2e)(d+ex)^2 + a^3e^3 - b^3d^3 + 3a^2be^2d} + \frac{e^3 \operatorname{atan}\left(\frac{\sqrt{b}\sqrt{d+ex}}{\sqrt{ae-bd}}\right)}{8b^{3/2}(ae-bd)^{5/2}}$$

input

$$\operatorname{int}((d+ex)^{1/2}/(a^2+b^2x^2+2abx)^2,x)$$

output

$$\frac{((e^3(d+ex)^{3/2})/(3(ae-bd)) - (e^3(d+ex)^{1/2})/(8b) + (be^3(d+ex)^{5/2})/(8(ae-bd)^2)) / ((d+ex)(3b^3d^2 + 3a^2b^2e^2 - 6ab^2d^2e) + b^3(d+ex)^3 - (3b^3d - 3ab^2e)(d+ex)^2 + a^3e^3 - b^3d^3 + 3ab^2d^2e - 3a^2b^2d^2e^2) + (e^3 \operatorname{atan}((b^{1/2})(d+ex)^{1/2}) / (ae-bd)^{1/2})) / (8b^{3/2}(ae-bd)^{5/2})}$$

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 536, normalized size of antiderivative = 3.67

$$\int \frac{\sqrt{d+ex}}{(a^2+2abx+b^2x^2)^2} dx$$

$$= \frac{3\sqrt{b}\sqrt{ae-bd} \operatorname{atan}\left(\frac{\sqrt{ex+db}}{\sqrt{b}\sqrt{ae-bd}}\right) a^3e^3 + 9\sqrt{b}\sqrt{ae-bd} \operatorname{atan}\left(\frac{\sqrt{ex+db}}{\sqrt{b}\sqrt{ae-bd}}\right) a^2be^3x + 9\sqrt{b}\sqrt{ae-bd} \operatorname{atan}\left(\frac{\sqrt{ex+db}}{\sqrt{b}\sqrt{ae-bd}}\right) a^2be^3x + 9\sqrt{b}\sqrt{ae-bd} \operatorname{atan}\left(\frac{\sqrt{ex+db}}{\sqrt{b}\sqrt{ae-bd}}\right) a^2be^3x + 9\sqrt{b}\sqrt{ae-bd} \operatorname{atan}\left(\frac{\sqrt{ex+db}}{\sqrt{b}\sqrt{ae-bd}}\right) a^2be^3x}{24b^2(a^3b^3e^3x^3 - 3a^2b^4de^2x^3)}$$

input `int((e*x+d)^(1/2)/(b^2*x^2+2*a*b*x+a^2)^2,x)`

output `(3*sqrt(b)*sqrt(a*e - b*d)*atan((sqrt(d + e*x)*b)/(sqrt(b)*sqrt(a*e - b*d)))**3**e**3 + 9*sqrt(b)*sqrt(a*e - b*d)*atan((sqrt(d + e*x)*b)/(sqrt(b)*sqrt(a*e - b*d)))**2*b**e**3*x + 9*sqrt(b)*sqrt(a*e - b*d)*atan((sqrt(d + e*x)*b)/(sqrt(b)*sqrt(a*e - b*d)))**a*b**2*e**3*x**2 + 3*sqrt(b)*sqrt(a*e - b*d)*atan((sqrt(d + e*x)*b)/(sqrt(b)*sqrt(a*e - b*d)))**b**3*e**3*x**3 - 3*sqrt(d + e*x)*a**3*b**e**3 + 17*sqrt(d + e*x)*a**2*b**2*d**e**2 + 8*sqrt(d + e*x)*a**2*b**2*e**3*x - 22*sqrt(d + e*x)*a*b**3*d**2*e - 10*sqrt(d + e*x)*a*b**3*d**e**2*x + 3*sqrt(d + e*x)*a*b**3*e**3*x**2 + 8*sqrt(d + e*x)*b**4*d**3 + 2*sqrt(d + e*x)*b**4*d**2*e*x - 3*sqrt(d + e*x)*b**4*d**e**2*x**2)/(24*b**2*(a**6*e**3 - 3*a**5*b*d**e**2 + 3*a**5*b**e**3*x + 3*a**4*b**2*d**2*e - 9*a**4*b**2*d**e**2*x + 3*a**4*b**2*e**3*x**2 - a**3*b**3*d**3 + 9*a**3*b**3*d**2*e*x - 9*a**3*b**3*d**e**2*x**2 + a**3*b**3*e**3*x**3 - 3*a**2*b**4*d**3*x + 9*a**2*b**4*d**2*e*x**2 - 3*a**2*b**4*d**e**2*x**3 - 3*a*b**5*d**3*x**2 + 3*a*b**5*d**2*e*x**3 - b**6*d**3*x**3))`

3.185 $\int \frac{1}{\sqrt{d+ex}(a^2+2abx+b^2x^2)^2} dx$

| | |
|---|------|
| Optimal result | 1465 |
| Mathematica [A] (verified) | 1465 |
| Rubi [A] (verified) | 1466 |
| Maple [A] (verified) | 1468 |
| Fricas [B] (verification not implemented) | 1469 |
| Sympy [F] | 1470 |
| Maxima [F(-2)] | 1470 |
| Giac [A] (verification not implemented) | 1470 |
| Mupad [B] (verification not implemented) | 1471 |
| Reduce [B] (verification not implemented) | 1472 |

Optimal result

Integrand size = 28, antiderivative size = 147

$$\int \frac{1}{\sqrt{d+ex}(a^2+2abx+b^2x^2)^2} dx = -\frac{\sqrt{d+ex}}{3(bd-ae)(a+bx)^3} + \frac{5e\sqrt{d+ex}}{12(bd-ae)^2(a+bx)^2} - \frac{5e^2\sqrt{d+ex}}{8(bd-ae)^3(a+bx)} + \frac{5e^3 \operatorname{arctanh}\left(\frac{\sqrt{b}\sqrt{d+ex}}{\sqrt{bd-ae}}\right)}{8\sqrt{b}(bd-ae)^{7/2}}$$

output

$$-1/3*(e*x+d)^{(1/2)} / (-a*e+b*d) / (b*x+a)^3 + 5/12*e*(e*x+d)^{(1/2)} / (-a*e+b*d)^2 / (b*x+a)^2 - 5/8*e^2*(e*x+d)^{(1/2)} / (-a*e+b*d)^3 / (b*x+a) + 5/8*e^3*\operatorname{arctanh}(b^{(1/2)}*(e*x+d)^{(1/2)} / (-a*e+b*d)^{(1/2)}) / b^{(1/2)} / (-a*e+b*d)^{(7/2)}$$

Mathematica [A] (verified)

Time = 0.32 (sec) , antiderivative size = 128, normalized size of antiderivative = 0.87

$$\int \frac{1}{\sqrt{d+ex}(a^2+2abx+b^2x^2)^2} dx = \frac{\sqrt{d+ex}(33a^2e^2+2abe(-13d+20ex)+b^2(8d^2-10dex+15e^2x^2))}{24(-bd+ae)^3(a+bx)^3} + \frac{5e^3 \arctan\left(\frac{\sqrt{b}\sqrt{d+ex}}{\sqrt{-bd+ae}}\right)}{8\sqrt{b}(-bd+ae)^{7/2}}$$

input `Integrate[1/(Sqrt[d + e*x]*(a^2 + 2*a*b*x + b^2*x^2)^2),x]`

output `(Sqrt[d + e*x]*(33*a^2*e^2 + 2*a*b*e*(-13*d + 20*e*x) + b^2*(8*d^2 - 10*d*e*x + 15*e^2*x^2)))/(24*(-(b*d) + a*e)^3*(a + b*x)^3) + (5*e^3*ArcTan[(Sqrt[b]*Sqrt[d + e*x])/Sqrt[-(b*d) + a*e]])/(8*Sqrt[b]*(-(b*d) + a*e)^(7/2))`

Rubi [A] (verified)

Time = 0.48 (sec) , antiderivative size = 168, normalized size of antiderivative = 1.14, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {1098, 27, 52, 52, 52, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\begin{aligned}
 & \int \frac{1}{(a^2 + 2abx + b^2x^2)^2 \sqrt{d + ex}} dx \\
 & \quad \downarrow 1098 \\
 & b^4 \int \frac{1}{b^4(a + bx)^4 \sqrt{d + ex}} dx \\
 & \quad \downarrow 27 \\
 & \int \frac{1}{(a + bx)^4 \sqrt{d + ex}} dx \\
 & \quad \downarrow 52 \\
 & -\frac{5e \int \frac{1}{(a+bx)^3 \sqrt{d+ex}} dx}{6(bd - ae)} - \frac{\sqrt{d + ex}}{3(a + bx)^3(bd - ae)} \\
 & \quad \downarrow 52 \\
 & -\frac{5e \left(-\frac{3e \int \frac{1}{(a+bx)^2 \sqrt{d+ex}} dx}{4(bd - ae)} - \frac{\sqrt{d+ex}}{2(a+bx)^2(bd - ae)} \right)}{6(bd - ae)} - \frac{\sqrt{d + ex}}{3(a + bx)^3(bd - ae)} \\
 & \quad \downarrow 52
 \end{aligned}$$

$$\begin{aligned}
 & \frac{5e \left(\frac{3e \left(-\frac{e \int \frac{1}{(a+bx)\sqrt{d+ex}} dx}{2(bd-ae)} - \frac{\sqrt{d+ex}}{(a+bx)(bd-ae)} \right)}{4(bd-ae)} - \frac{\sqrt{d+ex}}{2(a+bx)^2(bd-ae)} \right)}{6(bd-ae)} - \frac{\sqrt{d+ex}}{3(a+bx)^3(bd-ae)} \\
 & \quad \downarrow \text{73} \\
 & \frac{5e \left(\frac{3e \left(-\frac{\int \frac{1}{a+\frac{b(d+ex)}{e} - \frac{bd}{e}} d\sqrt{d+ex}}{bd-ae} - \frac{\sqrt{d+ex}}{(a+bx)(bd-ae)} \right)}{4(bd-ae)} - \frac{\sqrt{d+ex}}{2(a+bx)^2(bd-ae)} \right)}{6(bd-ae)} - \frac{\sqrt{d+ex}}{3(a+bx)^3(bd-ae)} \\
 & \quad \downarrow \text{221} \\
 & \frac{5e \left(\frac{3e \left(\frac{e \operatorname{arctanh}\left(\frac{\sqrt{b}\sqrt{d+ex}}{\sqrt{bd-ae}}\right)}{\sqrt{b}(bd-ae)^{3/2}} - \frac{\sqrt{d+ex}}{(a+bx)(bd-ae)} \right)}{4(bd-ae)} - \frac{\sqrt{d+ex}}{2(a+bx)^2(bd-ae)} \right)}{6(bd-ae)} - \frac{\sqrt{d+ex}}{3(a+bx)^3(bd-ae)}
 \end{aligned}$$

input `Int[1/(Sqrt[d + e*x]*(a^2 + 2*a*b*x + b^2*x^2)^2),x]`

output `-1/3*Sqrt[d + e*x]/((b*d - a*e)*(a + b*x)^3) - (5*e*(-1/2*Sqrt[d + e*x]/((b*d - a*e)*(a + b*x)^2) - (3*e*(-(Sqrt[d + e*x]/((b*d - a*e)*(a + b*x)))) + (e*ArcTanh[(Sqrt[b]*Sqrt[d + e*x])/Sqrt[b*d - a*e]])/(Sqrt[b]*(b*d - a*e)^(3/2))))/(4*(b*d - a*e)))/(6*(b*d - a*e))`

Defintions of rubi rules used

rule 27 `Int[(a_)*(Fx_), x_Symbol] := Simp[a Int[Fx, x], x] /; FreeQ[a, x] && !MatchQ[Fx, (b_)*(Gx_)] /; FreeQ[b, x]`

rule 52 `Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] := Simp[(a + b*x)^(m + 1)*(c + d*x)^(n + 1)/((b*c - a*d)*(m + 1)), x] - Simp[d*((m + n + 2)/((b*c - a*d)*(m + 1)) Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && ILtQ[m, -1] && FractionQ[n] && LtQ[n, 0]`

rule 73 `Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] := With[
 {p = Denominator[m]}, Simp[p/b Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x] /; FreeQ[{a, b, c, d}, x] && Lt
 Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
 inearQ[a, b, c, d, m, n, x]`

rule 221 `Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
 /Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]`

rule 1098 `Int[((d_.) + (e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_
 Symbol] := Simp[1/c^p Int[(d + e*x)^m*(b/2 + c*x)^(2*p), x], x] /; FreeQ[
 {a, b, c, d, e, m}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]`

Maple [A] (verified)

Time = 1.47 (sec) , antiderivative size = 130, normalized size of antiderivative = 0.88

| method | result |
|-------------------|---|
| pseudoelliptic | $\frac{33\sqrt{ex+d}\sqrt{b(ae-bd)}\left(\left(\frac{5}{11}e^2x^2 - \frac{10}{33}dex + \frac{8}{33}d^2\right)b^2 - \frac{26ea\left(-\frac{20ex}{13} + d\right)b}{33} + e^2a^2\right) + 15e^3(bx+a)^3 \arctan\left(\frac{b\sqrt{ex+d}}{\sqrt{b(ae-bd)}}\right)}{24\sqrt{b(ae-bd)}(ae-bd)^3(bx+a)^3}$ |
| derivativedivides | $2e^3 \left(\frac{\sqrt{ex+d}}{6(ae-bd)(b(ex+d)+ae-bd)^3} + \frac{5\sqrt{ex+d}}{24(ae-bd)(b(ex+d)+ae-bd)^2} + \frac{5\left(\frac{3\sqrt{ex+d}}{8(ae-bd)(b(ex+d)+ae-bd)} + \frac{3\arctan\left(\frac{b\sqrt{ex+d}}{\sqrt{b(ae-bd)}}\right)}{8(ae-bd)\sqrt{b(ae-bd)}}\right)}{ae-bd} \right)$ |
| default | $2e^3 \left(\frac{\sqrt{ex+d}}{6(ae-bd)(b(ex+d)+ae-bd)^3} + \frac{5\sqrt{ex+d}}{24(ae-bd)(b(ex+d)+ae-bd)^2} + \frac{5\left(\frac{3\sqrt{ex+d}}{8(ae-bd)(b(ex+d)+ae-bd)} + \frac{3\arctan\left(\frac{b\sqrt{ex+d}}{\sqrt{b(ae-bd)}}\right)}{8(ae-bd)\sqrt{b(ae-bd)}}\right)}{ae-bd} \right)$ |

input `int(1/(e*x+d)^(1/2)/(b^2*x^2+2*a*b*x+a^2)^2,x,method=_RETURNVERBOSE)`

output

```
1/24*(33*(e*x+d)^(1/2)*(b*(a*e-b*d))^(1/2)*((5/11*e^2*x^2-10/33*d*e*x+8/33
*d^2)*b^2-26/33*e*a*(-20/13*e*x+d)*b+e^2*a^2)+15*e^3*(b*x+a)^3*arctan(b*(e
*x+d)^(1/2)/(b*(a*e-b*d))^(1/2)))/(b*(a*e-b*d))^(1/2)/(a*e-b*d)^3/(b*x+a)^
3
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 435 vs. $2(123) = 246$.

Time = 0.14 (sec) , antiderivative size = 884, normalized size of antiderivative = 6.01

$$\int \frac{1}{\sqrt{d+ex}(a^2+2abx+b^2x^2)^2} dx = \text{Too large to display}$$

input

```
integrate(1/(e*x+d)^(1/2)/(b^2*x^2+2*a*b*x+a^2)^2,x, algorithm="fricas")
```

output

```
[-1/48*(15*(b^3*e^3*x^3 + 3*a*b^2*e^3*x^2 + 3*a^2*b*e^3*x + a^3*e^3)*sqrt(
b^2*d - a*b*e)*log((b*e*x + 2*b*d - a*e - 2*sqrt(b^2*d - a*b*e)*sqrt(e*x +
d))/(b*x + a)) + 2*(8*b^4*d^3 - 34*a*b^3*d^2*e + 59*a^2*b^2*d*e^2 - 33*a^
3*b*e^3 + 15*(b^4*d*e^2 - a*b^3*e^3)*x^2 - 10*(b^4*d^2*e - 5*a*b^3*d*e^2 +
4*a^2*b^2*e^3)*x)*sqrt(e*x + d))/(a^3*b^5*d^4 - 4*a^4*b^4*d^3*e + 6*a^5*b
^3*d^2*e^2 - 4*a^6*b^2*d*e^3 + a^7*b*e^4 + (b^8*d^4 - 4*a*b^7*d^3*e + 6*a^
2*b^6*d^2*e^2 - 4*a^3*b^5*d*e^3 + a^4*b^4*e^4)*x^3 + 3*(a*b^7*d^4 - 4*a^2*
b^6*d^3*e + 6*a^3*b^5*d^2*e^2 - 4*a^4*b^4*d*e^3 + a^5*b^3*e^4)*x^2 + 3*(a^
2*b^6*d^4 - 4*a^3*b^5*d^3*e + 6*a^4*b^4*d^2*e^2 - 4*a^5*b^3*d*e^3 + a^6*b^
2*e^4)*x), -1/24*(15*(b^3*e^3*x^3 + 3*a*b^2*e^3*x^2 + 3*a^2*b*e^3*x + a^3*
e^3)*sqrt(-b^2*d + a*b*e)*arctan(sqrt(-b^2*d + a*b*e)*sqrt(e*x + d)/(b*e*x
+ b*d)) + (8*b^4*d^3 - 34*a*b^3*d^2*e + 59*a^2*b^2*d*e^2 - 33*a^3*b*e^3 +
15*(b^4*d*e^2 - a*b^3*e^3)*x^2 - 10*(b^4*d^2*e - 5*a*b^3*d*e^2 + 4*a^2*b^
2*e^3)*x)*sqrt(e*x + d))/(a^3*b^5*d^4 - 4*a^4*b^4*d^3*e + 6*a^5*b^3*d^2*e^
2 - 4*a^6*b^2*d*e^3 + a^7*b*e^4 + (b^8*d^4 - 4*a*b^7*d^3*e + 6*a^2*b^6*d^
2*e^2 - 4*a^3*b^5*d^2*e^2 - 4*a^4*b^4*d*e^3 + a^5*b^3*e^4)*x^2 + 3*(a^2*b^
6*d^4 - 4*a^3*b^5*d^3*e + 6*a^4*b^4*d^2*e^2 - 4*a^5*b^3*d*e^3 + a^6*b^2*
e^4)*x)
]
```

Sympy [F]

$$\int \frac{1}{\sqrt{d+ex}(a^2+2abx+b^2x^2)^2} dx = \int \frac{1}{(a+bx)^4 \sqrt{d+ex}} dx$$

input `integrate(1/(e*x+d)**(1/2)/(b**2*x**2+2*a*b*x+a**2)**2,x)`

output `Integral(1/((a + b*x)**4*sqrt(d + e*x)), x)`

Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{\sqrt{d+ex}(a^2+2abx+b^2x^2)^2} dx = \text{Exception raised: ValueError}$$

input `integrate(1/(e*x+d)^(1/2)/(b^2*x^2+2*a*b*x+a^2)^2,x, algorithm="maxima")`

output `Exception raised: ValueError >> Computation failed since Maxima requested additional constraints; using the 'assume' command before evaluation *may* help (example of legal syntax is 'assume(a*e-b*d>0)', see `assume?` for more detail`

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 231, normalized size of antiderivative = 1.57

$$\int \frac{1}{\sqrt{d+ex}(a^2+2abx+b^2x^2)^2} dx$$

$$= \frac{5e^3 \arctan\left(\frac{\sqrt{ex+db}}{\sqrt{-b^2d+abe}}\right)}{8(b^3d^3 - 3ab^2d^2e + 3a^2bde^2 - a^3e^3)\sqrt{-b^2d+abe}}$$

$$- \frac{15(ex+d)^{\frac{5}{2}}b^2e^3 - 40(ex+d)^{\frac{3}{2}}b^2de^3 + 33\sqrt{ex+db^2d^2e^3} + 40(ex+d)^{\frac{3}{2}}abe^4 - 66\sqrt{ex+db}d^2e^3 + 3}{24(b^3d^3 - 3ab^2d^2e + 3a^2bde^2 - a^3e^3)((ex+d)b - bd + ae)^3}$$

input `integrate(1/(e*x+d)^(1/2)/(b^2*x^2+2*a*b*x+a^2)^2,x, algorithm="giac")`

output
$$-5/8*e^3*\arctan(\sqrt{e*x + d}*b/\sqrt{-b^2*d + a*b*e})/((b^3*d^3 - 3*a*b^2*d^2*e + 3*a^2*b*d*e^2 - a^3*e^3)*\sqrt{-b^2*d + a*b*e}) - 1/24*(15*(e*x + d)^{(5/2)}*b^2*e^3 - 40*(e*x + d)^{(3/2)}*b^2*d*e^3 + 33*\sqrt{e*x + d}*b^2*d^2*e^3 + 40*(e*x + d)^{(3/2)}*a*b*e^4 - 66*\sqrt{e*x + d}*a*b*d*e^4 + 33*\sqrt{e*x + d}*a^2*e^5)/((b^3*d^3 - 3*a*b^2*d^2*e + 3*a^2*b*d*e^2 - a^3*e^3)*((e*x + d)*b - b*d + a*e)^3)$$

Mupad [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 218, normalized size of antiderivative = 1.48

$$\int \frac{1}{\sqrt{d+ex}(a^2+2abx+b^2x^2)^2} dx$$

$$= \frac{\frac{11e^3\sqrt{d+ex}}{8(ae-bd)} + \frac{5b^2e^3(d+ex)^{5/2}}{8(ae-bd)^3} + \frac{5be^3(d+ex)^{3/2}}{3(ae-bd)^2}}{(d+ex)(3a^2be^2-6ab^2de+3b^3d^2)+b^3(d+ex)^3-(3b^3d-3ab^2e)(d+ex)^2+a^3e^3-b^3d^3+3a^2be^2} + \frac{5e^3\operatorname{atan}\left(\frac{\sqrt{b}\sqrt{d+ex}}{\sqrt{ae-bd}}\right)}{8\sqrt{b}(ae-bd)^{7/2}}$$

input `int(1/((d + e*x)^(1/2)*(a^2 + b^2*x^2 + 2*a*b*x)^2),x)`

output
$$\left(\frac{11e^3(d+e*x)^{(1/2)}}{8(ae-bd)} + \frac{5b^2e^3(d+e*x)^{(5/2)}}{8(ae-bd)^3} + \frac{5be^3(d+e*x)^{(3/2)}}{3(ae-bd)^2}\right)/((d+e*x)*(3b^3d^2+3a^2bde-6ab^2de)+b^3(d+e*x)^3-(3b^3d-3ab^2e)*(d+e*x)^2+a^3e^3-b^3d^3+3a^2bde^2-3a^2bde^2) + \frac{5e^3\operatorname{atan}((b^{(1/2)}*(d+e*x)^{(1/2)})/(ae-bd)^{(1/2)})}{8b^{(1/2)}(ae-bd)^{(7/2)}}$$

3.186 $\int \frac{1}{(d+ex)^{3/2}(a^2+2abx+b^2x^2)^2} dx$

| | |
|---|------|
| Optimal result | 1473 |
| Mathematica [A] (verified) | 1474 |
| Rubi [A] (verified) | 1474 |
| Maple [A] (verified) | 1478 |
| Fricas [B] (verification not implemented) | 1478 |
| Sympy [F(-1)] | 1479 |
| Maxima [F(-2)] | 1480 |
| Giac [B] (verification not implemented) | 1480 |
| Mupad [B] (verification not implemented) | 1481 |
| Reduce [B] (verification not implemented) | 1482 |

Optimal result

Integrand size = 28, antiderivative size = 173

$$\int \frac{1}{(d+ex)^{3/2}(a^2+2abx+b^2x^2)^2} dx = -\frac{35e^3}{8(bd-ae)^4\sqrt{d+ex}} - \frac{1}{3(bd-ae)(a+bx)^3\sqrt{d+ex}} + \frac{7e}{12(bd-ae)^2(a+bx)^2\sqrt{d+ex}} - \frac{35e^2}{24(bd-ae)^3(a+bx)\sqrt{d+ex}} + \frac{35\sqrt{b}e^3 \operatorname{arctanh}\left(\frac{\sqrt{b}\sqrt{d+ex}}{\sqrt{bd-ae}}\right)}{8(bd-ae)^{9/2}}$$

output

```
-35/8*e^3/(-a*e+b*d)^4/(e*x+d)^(1/2)-1/3/(-a*e+b*d)/(b*x+a)^3/(e*x+d)^(1/2)
)+7/12*e/(-a*e+b*d)^2/(b*x+a)^2/(e*x+d)^(1/2)-35/24*e^2/(-a*e+b*d)^3/(b*x+
a)/(e*x+d)^(1/2)+35/8*b^(1/2)*e^3*arctanh(b^(1/2)*(e*x+d)^(1/2)/(-a*e+b*d)
^(1/2))/(-a*e+b*d)^(9/2)
```

Mathematica [A] (verified)

Time = 0.71 (sec) , antiderivative size = 170, normalized size of antiderivative = 0.98

$$\int \frac{1}{(d+ex)^{3/2}(a^2+2abx+b^2x^2)^2} dx = \frac{-48a^3e^3 - 3a^2be^2(29d+77ex) - 2ab^2e(-19d^2+49dex+140e^2x^2) - 35\sqrt{b}e^3 \arctan\left(\frac{\sqrt{b}\sqrt{d+ex}}{\sqrt{-bd+ae}}\right)}{24(bd-ae)^4(a+bx)^3\sqrt{d-ae}} - \frac{35\sqrt{b}e^3 \arctan\left(\frac{\sqrt{b}\sqrt{d+ex}}{\sqrt{-bd+ae}}\right)}{8(-bd+ae)^{9/2}}$$

input `Integrate[1/((d + e*x)^(3/2)*(a^2 + 2*a*b*x + b^2*x^2)^2), x]`

output
$$\frac{(-48*a^3*e^3 - 3*a^2*b*e^2*(29*d + 77*e*x) - 2*a*b^2*e*(-19*d^2 + 49*d*e*x + 140*e^2*x^2) - b^3*(8*d^3 - 14*d^2*e*x + 35*d*e^2*x^2 + 105*e^3*x^3))/(24*(b*d - a*e)^4*(a + b*x)^3*sqrt[d + e*x]) - (35*sqrt[b]*e^3*ArcTan[(sqrt[b]*sqrt[d + e*x])/sqrt[-(b*d) + a*e]])/(8*(-(b*d) + a*e)^(9/2))$$

Rubi [A] (verified)

Time = 0.52 (sec) , antiderivative size = 205, normalized size of antiderivative = 1.18, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {1098, 27, 52, 52, 52, 61, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\begin{aligned} & \int \frac{1}{(a^2 + 2abx + b^2x^2)^2 (d + ex)^{3/2}} dx \\ & \quad \downarrow 1098 \\ & b^4 \int \frac{1}{b^4(a + bx)^4 (d + ex)^{3/2}} dx \\ & \quad \downarrow 27 \\ & \int \frac{1}{(a + bx)^4 (d + ex)^{3/2}} dx \\ & \quad \downarrow 52 \end{aligned}$$

$$\begin{aligned}
 & \frac{7e \int \frac{1}{(a+bx)^3(d+ex)^{3/2}} dx}{6(bd-ae)} - \frac{1}{3(a+bx)^3\sqrt{d+ex}(bd-ae)} \\
 & \quad \downarrow 52 \\
 & \frac{7e \left(-\frac{5e \int \frac{1}{(a+bx)^2(d+ex)^{3/2}} dx}{4(bd-ae)} - \frac{1}{2(a+bx)^2\sqrt{d+ex}(bd-ae)} \right)}{6(bd-ae)} - \frac{1}{3(a+bx)^3\sqrt{d+ex}(bd-ae)} \\
 & \quad \downarrow 52 \\
 & \frac{7e \left(-\frac{5e \left(-\frac{3e \int \frac{1}{(a+bx)(d+ex)^{3/2}} dx}{2(bd-ae)} - \frac{1}{(a+bx)\sqrt{d+ex}(bd-ae)} \right)}{4(bd-ae)} - \frac{1}{2(a+bx)^2\sqrt{d+ex}(bd-ae)} \right)}{6(bd-ae)} \\
 & \quad \frac{1}{3(a+bx)^3\sqrt{d+ex}(bd-ae)} \\
 & \quad \downarrow 61 \\
 & \frac{7e \left(-\frac{5e \left(\frac{3e \left(\frac{b \int \frac{1}{(a+bx)\sqrt{d+ex}} dx}{bd-ae} + \frac{2}{\sqrt{d+ex}(bd-ae)} \right)}{2(bd-ae)} - \frac{1}{(a+bx)\sqrt{d+ex}(bd-ae)} \right)}{4(bd-ae)} - \frac{1}{2(a+bx)^2\sqrt{d+ex}(bd-ae)} \right)}{6(bd-ae)} \\
 & \quad \frac{1}{3(a+bx)^3\sqrt{d+ex}(bd-ae)} \\
 & \quad \downarrow 73
 \end{aligned}$$

$$\begin{aligned}
 & \left(\frac{7e \left(\frac{5e \left(\frac{3e \left(\frac{2b \int \frac{1}{a + \frac{b(d+ex) - bd}{e(bd-ae)}} d\sqrt{d+ex}} + \frac{2}{\sqrt{d+ex}(bd-ae)} \right)}{2(bd-ae)} - \frac{1}{(a+bx)\sqrt{d+ex}(bd-ae)} \right)}{4(bd-ae)} - \frac{1}{2(a+bx)^2\sqrt{d+ex}(bd-ae)} \right)}{6(bd-ae)} \right. \\
 & \qquad \left. - \frac{1}{3(a+bx)^3\sqrt{d+ex}(bd-ae)} \right) \\
 & \qquad \qquad \qquad \downarrow \text{221} \\
 & \left(\frac{7e \left(\frac{5e \left(\frac{3e \left(\frac{2\sqrt{b}\operatorname{arctanh}\left(\frac{\sqrt{b}\sqrt{d+ex}}{\sqrt{bd-ae}}\right)}{\sqrt{d+ex}(bd-ae)} - \frac{2}{(bd-ae)^{3/2}} \right)}{2(bd-ae)} - \frac{1}{(a+bx)\sqrt{d+ex}(bd-ae)} \right)}{4(bd-ae)} - \frac{1}{2(a+bx)^2\sqrt{d+ex}(bd-ae)} \right)}{6(bd-ae)} \right. \\
 & \qquad \left. - \frac{1}{3(a+bx)^3\sqrt{d+ex}(bd-ae)} \right)
 \end{aligned}$$

input `Int[1/((d + e*x)^(3/2)*(a^2 + 2*a*b*x + b^2*x^2)^2),x]`

output `-1/3*1/((b*d - a*e)*(a + b*x)^3*sqrt[d + e*x]) - (7*e*(-1/2*1/((b*d - a*e)*(a + b*x)^2*sqrt[d + e*x]) - (5*e*(-1/((b*d - a*e)*(a + b*x)*sqrt[d + e*x])) - (3*e*(2/((b*d - a*e)*sqrt[d + e*x]) - (2*sqrt[b]*ArcTanh[(sqrt[b]*sqrt[d + e*x])/sqrt[b*d - a*e]])/(b*d - a*e)^(3/2)))/(2*(b*d - a*e))))/(4*(b*d - a*e)))/(6*(b*d - a*e))`

Definitions of rubi rules used

- rule 27 $\text{Int}[(a_*)(Fx_), x_Symbol] \rightarrow \text{Simp}[a \text{ Int}[Fx, x], x] /; \text{FreeQ}[a, x] \ \&\& \ !\text{MatchQ}[Fx, (b_*)(Gx_)] /; \text{FreeQ}[b, x]$
- rule 52 $\text{Int}[((a_.) + (b_.)(x_))^{(m_.)}((c_.) + (d_.)(x_))^{(n_.)}, x_Symbol] \rightarrow \text{Simp}[(a + b*x)^{(m + 1)}((c + d*x)^{(n + 1)}((b*c - a*d)*(m + 1))), x] - \text{Simp}[d*((m + n + 2)/((b*c - a*d)*(m + 1))) \text{Int}[(a + b*x)^{(m + 1)}(c + d*x)^n, x], x] /; \text{FreeQ}[\{a, b, c, d, n\}, x] \ \&\& \ \text{ILtQ}[m, -1] \ \&\& \ \text{FractionQ}[n] \ \&\& \ \text{LtQ}[n, 0]$
- rule 61 $\text{Int}[((a_.) + (b_.)(x_))^{(m_.)}((c_.) + (d_.)(x_))^{(n_.)}, x_Symbol] \rightarrow \text{Simp}[(a + b*x)^{(m + 1)}((c + d*x)^{(n + 1)}((b*c - a*d)*(m + 1))), x] - \text{Simp}[d*((m + n + 2)/((b*c - a*d)*(m + 1))) \text{Int}[(a + b*x)^{(m + 1)}(c + d*x)^n, x], x] /; \text{FreeQ}[\{a, b, c, d, n\}, x] \ \&\& \ \text{LtQ}[m, -1] \ \&\& \ !(\text{LtQ}[n, -1] \ \&\& \ (\text{EqQ}[a, 0] \ || \ (\text{NeQ}[c, 0] \ \&\& \ \text{LtQ}[m - n, 0] \ \&\& \ \text{IntegerQ}[n]))) \ \&\& \ \text{IntLinearQ}[a, b, c, d, m, n, x]$
- rule 73 $\text{Int}[((a_.) + (b_.)(x_))^{(m_.)}((c_.) + (d_.)(x_))^{(n_.)}, x_Symbol] \rightarrow \text{With}[\{p = \text{Denominator}[m]\}, \text{Simp}[p/b \ \text{Subst}[\text{Int}[x^{(p*(m + 1) - 1)}(c - a*(d/b) + d*(x^p/b))^{n + 1}, x], x, (a + b*x)^{(1/p)}], x]] /; \text{FreeQ}[\{a, b, c, d\}, x] \ \&\& \ \text{LtQ}[-1, m, 0] \ \&\& \ \text{LeQ}[-1, n, 0] \ \&\& \ \text{LeQ}[\text{Denominator}[n], \text{Denominator}[m]] \ \&\& \ \text{IntLinearQ}[a, b, c, d, m, n, x]$
- rule 221 $\text{Int}[((a_.) + (b_.)(x_)^2)^{(-1)}, x_Symbol] \rightarrow \text{Simp}[(\text{Rt}[-a/b, 2]/a)*\text{ArcTanh}[x/\text{Rt}[-a/b, 2]], x] /; \text{FreeQ}[\{a, b\}, x] \ \&\& \ \text{NegQ}[a/b]$
- rule 1098 $\text{Int}[((d_.) + (e_.)(x_))^{(m_.)}((a_.) + (b_.)(x_) + (c_.)(x_)^2)^{(p_.)}, x_Symbol] \rightarrow \text{Simp}[1/c^p \ \text{Int}[(d + e*x)^m*(b/2 + c*x)^{(2*p)}, x], x] /; \text{FreeQ}[\{a, b, c, d, e, m\}, x] \ \&\& \ \text{EqQ}[b^2 - 4*a*c, 0] \ \&\& \ \text{IntegerQ}[p]$

Maple [A] (verified)

Time = 1.60 (sec) , antiderivative size = 156, normalized size of antiderivative = 0.90

| method | result |
|-------------------|--|
| derivativedivides | $2e^3 \left(-\frac{1}{(ae-bd)^4 \sqrt{ex+d}} - \frac{b \left(\frac{19(ex+d)^{\frac{5}{2}} b^2}{16} + \frac{17b(ae-bd)(ex+d)^{\frac{3}{2}}}{6} + \left(\frac{29}{16} e^2 a^2 - \frac{29}{8} abde + \frac{29}{16} b^2 d^2 \right) \sqrt{ex+d} \right)}{(b(ex+d)+ae-bd)^3} + \frac{35 \arctan\left(\frac{b}{\sqrt{b(ae-bd)}}\right)}{16\sqrt{b(ae-bd)}} \right) \frac{1}{(ae-bd)^4}$ |
| default | $2e^3 \left(-\frac{1}{(ae-bd)^4 \sqrt{ex+d}} - \frac{b \left(\frac{19(ex+d)^{\frac{5}{2}} b^2}{16} + \frac{17b(ae-bd)(ex+d)^{\frac{3}{2}}}{6} + \left(\frac{29}{16} e^2 a^2 - \frac{29}{8} abde + \frac{29}{16} b^2 d^2 \right) \sqrt{ex+d} \right)}{(b(ex+d)+ae-bd)^3} + \frac{35 \arctan\left(\frac{b}{\sqrt{b(ae-bd)}}\right)}{16\sqrt{b(ae-bd)}} \right) \frac{1}{(ae-bd)^4}$ |
| pseudoelliptic | $2 \left(\frac{35b e^3 \sqrt{ex+d} (bx+a)^3 \arctan\left(\frac{b\sqrt{ex+d}}{\sqrt{b(ae-bd)}}\right)}{16} + \sqrt{b(ae-bd)} \left(\left(\frac{35}{16} b^3 x^3 + \frac{35}{6} a b^2 x^2 + \frac{77}{16} a^2 b x + a^3 \right) e^3 + \frac{29b \left(\frac{35}{87} b^2 x^2 + \frac{98}{87} ab \right)}{16} \right) \right) \frac{1}{\sqrt{ex+d} \sqrt{b(ae-bd)} (ae-bd)^4 (bx+a)^3}$ |

input

```
int(1/(e*x+d)^(3/2)/(b^2*x^2+2*a*b*x+a^2)^2,x,method=_RETURNVERBOSE)
```

output

```
2*e^3*(-1/(a*e-b*d)^4/(e*x+d)^(1/2)-1/(a*e-b*d)^4*b*((19/16*(e*x+d)^(5/2)*
b^2+17/6*b*(a*e-b*d)*(e*x+d)^(3/2)+(29/16*e^2*a^2-29/8*a*b*d*e+29/16*b^2*d
^2)*(e*x+d)^(1/2))/(b*(e*x+d)+a*e-b*d)^3+35/16/(b*(a*e-b*d))^(1/2)*arctan(
b*(e*x+d)^(1/2)/(b*(a*e-b*d))^(1/2))))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 577 vs. 2(145) = 290.

Time = 0.14 (sec) , antiderivative size = 1184, normalized size of antiderivative = 6.84

$$\int \frac{1}{(d+ex)^{3/2} (a^2+2abx+b^2x^2)^2} dx = \text{Too large to display}$$

input

```
integrate(1/(e*x+d)^(3/2)/(b^2*x^2+2*a*b*x+a^2)^2,x, algorithm="fricas")
```

output

```
[1/48*(105*(b^3*e^4*x^4 + a^3*d*e^3 + (b^3*d*e^3 + 3*a*b^2*e^4)*x^3 + 3*(a
*b^2*d*e^3 + a^2*b*e^4)*x^2 + (3*a^2*b*d*e^3 + a^3*e^4)*x)*sqrt(b/(b*d - a
*e))*log((b*e*x + 2*b*d - a*e + 2*(b*d - a*e)*sqrt(e*x + d)*sqrt(b/(b*d -
a*e)))/(b*x + a) - 2*(105*b^3*e^3*x^3 + 8*b^3*d^3 - 38*a*b^2*d^2*e + 87*a
^2*b*d*e^2 + 48*a^3*e^3 + 35*(b^3*d*e^2 + 8*a*b^2*e^3)*x^2 - 7*(2*b^3*d^2*
e - 14*a*b^2*d*e^2 - 33*a^2*b*e^3)*x)*sqrt(e*x + d))/(a^3*b^4*d^5 - 4*a^4*
b^3*d^4*e + 6*a^5*b^2*d^3*e^2 - 4*a^6*b*d^2*e^3 + a^7*d*e^4 + (b^7*d^4*e -
4*a*b^6*d^3*e^2 + 6*a^2*b^5*d^2*e^3 - 4*a^3*b^4*d*e^4 + a^4*b^3*e^5)*x^4
+ (b^7*d^5 - a*b^6*d^4*e - 6*a^2*b^5*d^3*e^2 + 14*a^3*b^4*d^2*e^3 - 11*a^4
*b^3*d*e^4 + 3*a^5*b^2*e^5)*x^3 + 3*(a*b^6*d^5 - 3*a^2*b^5*d^4*e + 2*a^3*b
^4*d^3*e^2 + 2*a^4*b^3*d^2*e^3 - 3*a^5*b^2*d*e^4 + a^6*b*e^5)*x^2 + (3*a^2
*b^5*d^5 - 11*a^3*b^4*d^4*e + 14*a^4*b^3*d^3*e^2 - 6*a^5*b^2*d^2*e^3 - a^6
*b*d*e^4 + a^7*e^5)*x), -1/24*(105*(b^3*e^4*x^4 + a^3*d*e^3 + (b^3*d*e^3 +
3*a*b^2*e^4)*x^3 + 3*(a*b^2*d*e^3 + a^2*b*e^4)*x^2 + (3*a^2*b*d*e^3 + a^3
*e^4)*x)*sqrt(-b/(b*d - a*e))*arctan(sqrt(e*x + d)*sqrt(-b/(b*d - a*e))) +
(105*b^3*e^3*x^3 + 8*b^3*d^3 - 38*a*b^2*d^2*e + 87*a^2*b*d*e^2 + 48*a^3*e
^3 + 35*(b^3*d*e^2 + 8*a*b^2*e^3)*x^2 - 7*(2*b^3*d^2*e - 14*a*b^2*d*e^2 -
33*a^2*b*e^3)*x)*sqrt(e*x + d))/(a^3*b^4*d^5 - 4*a^4*b^3*d^4*e + 6*a^5*b^2
*d^3*e^2 - 4*a^6*b*d^2*e^3 + a^7*d*e^4 + (b^7*d^4*e - 4*a*b^6*d^3*e^2 + 6*
a^2*b^5*d^2*e^3 - 4*a^3*b^4*d*e^4 + a^4*b^3*e^5)*x^4 + (b^7*d^5 - a*b^6...
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{(d+ex)^{3/2} (a^2+2abx+b^2x^2)^2} dx = \text{Timed out}$$

input

```
integrate(1/(e*x+d)**(3/2)/(b**2*x**2+2*a*b*x+a**2)**2,x)
```

output

Timed out

Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{(d+ex)^{3/2} (a^2 + 2abx + b^2x^2)^2} dx = \text{Exception raised: ValueError}$$

input `integrate(1/(e*x+d)^(3/2)/(b^2*x^2+2*a*b*x+a^2)^2,x, algorithm="maxima")`

output `Exception raised: ValueError >> Computation failed since Maxima requested additional constraints; using the 'assume' command before evaluation *may* help (example of legal syntax is 'assume(a*e-b*d>0)', see `assume?` for more detail`

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 326 vs. 2(145) = 290.

Time = 0.14 (sec) , antiderivative size = 326, normalized size of antiderivative = 1.88

$$\int \frac{1}{(d+ex)^{3/2} (a^2 + 2abx + b^2x^2)^2} dx =$$

$$\frac{35 be^3 \arctan\left(\frac{\sqrt{ex+db}}{\sqrt{-b^2d+abe}}\right)}{8(b^4d^4 - 4ab^3d^3e + 6a^2b^2d^2e^2 - 4a^3bde^3 + a^4e^4)\sqrt{-b^2d+abe}}$$

$$- \frac{(b^4d^4 - 4ab^3d^3e + 6a^2b^2d^2e^2 - 4a^3bde^3 + a^4e^4)\sqrt{ex+d}}{57(ex+d)^{5/2}b^3e^3 - 136(ex+d)^{3/2}b^3de^3 + 87\sqrt{ex+db}b^3d^2e^3 + 136(ex+d)^{3/2}ab^2e^4 - 174\sqrt{ex+db}ab^2de^4}$$

$$- \frac{24(b^4d^4 - 4ab^3d^3e + 6a^2b^2d^2e^2 - 4a^3bde^3 + a^4e^4)((ex+d)b - bd + ae)^3}{24(b^4d^4 - 4ab^3d^3e + 6a^2b^2d^2e^2 - 4a^3bde^3 + a^4e^4)((ex+d)b - bd + ae)^3}$$

input `integrate(1/(e*x+d)^(3/2)/(b^2*x^2+2*a*b*x+a^2)^2,x, algorithm="giac")`

output

$$-35/8*b*e^3*\arctan(\sqrt{e*x + d}*b/\sqrt{-b^2*d + a*b*e})/((b^4*d^4 - 4*a*b^3*d^3*e + 6*a^2*b^2*d^2*e^2 - 4*a^3*b*d*e^3 + a^4*e^4)*\sqrt{-b^2*d + a*b*e}) - 2*e^3/((b^4*d^4 - 4*a*b^3*d^3*e + 6*a^2*b^2*d^2*e^2 - 4*a^3*b*d*e^3 + a^4*e^4)*\sqrt{e*x + d}) - 1/24*(57*(e*x + d)^{(5/2)}*b^3*e^3 - 136*(e*x + d)^{(3/2)}*b^3*d*e^3 + 87*\sqrt{e*x + d}*b^3*d^2*e^3 + 136*(e*x + d)^{(3/2)}*a*b^2*e^4 - 174*\sqrt{e*x + d}*a*b^2*d*e^4 + 87*\sqrt{e*x + d}*a^2*b*e^5)/((b^4*d^4 - 4*a*b^3*d^3*e + 6*a^2*b^2*d^2*e^2 - 4*a^3*b*d*e^3 + a^4*e^4)*((e*x + d)*b - b*d + a*e)^3)$$

Mupad [B] (verification not implemented)

Time = 9.53 (sec) , antiderivative size = 294, normalized size of antiderivative = 1.70

$$\int \frac{1}{(d + ex)^{3/2} (a^2 + 2abx + b^2x^2)^2} dx =$$

$$\frac{\frac{2e^3}{ae-bd} + \frac{35b^2e^3(d+ex)^2}{3(ae-bd)^3} + \frac{35b^3e^3(d+ex)^3}{8(ae-bd)^4} + \frac{77be^3(d+ex)}{8(ae-bd)^2}}{\sqrt{d+ex} (a^3e^3 - 3a^2bde^2 + 3ab^2d^2e - b^3d^3) + b^3(d+ex)^{7/2} - (3b^3d - 3ab^2e)(d+ex)^{5/2} + (d + 35\sqrt{b}e^3 \operatorname{atan}\left(\frac{\sqrt{b}\sqrt{d+ex}(a^4e^4 - 4a^3bde^3 + 6a^2b^2d^2e^2 - 4ab^3d^3e + b^4d^4)}{(ae-bd)^{9/2}}\right))}}{8(ae-bd)^{9/2}}$$

input

$$\operatorname{int}(1/((d + e*x)^{(3/2)}*(a^2 + b^2*x^2 + 2*a*b*x)^2), x)$$

output

$$- ((2*e^3)/(a*e - b*d) + (35*b^2*e^3*(d + e*x)^2)/(3*(a*e - b*d)^3) + (35*b^3*e^3*(d + e*x)^3)/(8*(a*e - b*d)^4) + (77*b*e^3*(d + e*x))/(8*(a*e - b*d)^2))/((d + e*x)^{(1/2)}*(a^3*e^3 - b^3*d^3 + 3*a*b^2*d^2*e - 3*a^2*b*d*e^2) + b^3*(d + e*x)^{(7/2)} - (3*b^3*d - 3*a*b^2*e)*(d + e*x)^{(5/2)} + (d + e*x)^{(3/2)}*(3*b^3*d^2 + 3*a^2*b*e^2 - 6*a*b^2*d*e)) - (35*b^{(1/2)}*e^3*\operatorname{atan}((b^{(1/2)}*(d + e*x)^{(1/2)}*(a^4*e^4 + b^4*d^4 + 6*a^2*b^2*d^2*e^2 - 4*a*b^3*d^3*e - 4*a^3*b*d*e^3))/(a*e - b*d)^{(9/2)}))/((8*(a*e - b*d)^{(9/2)}))$$

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 702, normalized size of antiderivative = 4.06

$$\int \frac{1}{(d+ex)^{3/2}(a^2+2abx+b^2x^2)^2} dx = \frac{-105\sqrt{b}\sqrt{ex+d}\sqrt{ae-bd}\operatorname{atan}\left(\frac{\sqrt{ex+d}b}{\sqrt{b}\sqrt{ae-bd}}\right)a^3e^3 - 315\sqrt{b}\sqrt{ex+d}}{24\sqrt{ex+d}(a^5b^3e^5x^3 - 5a^4b^4d)}$$

input `int(1/(e*x+d)^(3/2)/(b^2*x^2+2*a*b*x+a^2)^2,x)`

output

```
( - 105*sqrt(b)*sqrt(d + e*x)*sqrt(a*e - b*d)*atan((sqrt(d + e*x)*b)/(sqrt(b)*sqrt(a*e - b*d)))*a**3*e**3 - 315*sqrt(b)*sqrt(d + e*x)*sqrt(a*e - b*d)*atan((sqrt(d + e*x)*b)/(sqrt(b)*sqrt(a*e - b*d)))*a**2*b*e**3*x - 315*sqrt(b)*sqrt(d + e*x)*sqrt(a*e - b*d)*atan((sqrt(d + e*x)*b)/(sqrt(b)*sqrt(a*e - b*d)))*a*b**2*e**3*x**2 - 105*sqrt(b)*sqrt(d + e*x)*sqrt(a*e - b*d)*atan((sqrt(d + e*x)*b)/(sqrt(b)*sqrt(a*e - b*d)))*b**3*e**3*x**3 - 48*a**4*e**4 - 39*a**3*b*d*e**3 - 231*a**3*b*e**4*x + 125*a**2*b**2*d**2*e**2 + 133*a**2*b**2*d*e**3*x - 280*a**2*b**2*e**4*x**2 - 46*a*b**3*d**3*e + 112*a*b**3*d**2*e**2*x + 245*a*b**3*d*e**3*x**2 - 105*a*b**3*e**4*x**3 + 8*b**4*d**4 - 14*b**4*d**3*e*x + 35*b**4*d**2*e**2*x**2 + 105*b**4*d*e**3*x**3)/(24*sqrt(d + e*x)*(a**8*e**5 - 5*a**7*b*d*e**4 + 3*a**7*b*e**5*x + 10*a**6*b**2*d**2*e**3 - 15*a**6*b**2*d*e**4*x + 3*a**6*b**2*e**5*x**2 - 10*a**5*b**3*d**3*e**2 + 30*a**5*b**3*d**2*e**3*x - 15*a**5*b**3*d*e**4*x**2 + a**5*b**3*e**5*x**3 + 5*a**4*b**4*d**4*e - 30*a**4*b**4*d**3*e**2*x + 30*a**4*b**4*d**2*e**3*x**2 - 5*a**4*b**4*d*e**4*x**3 - a**3*b**5*d**5 + 15*a**3*b**5*d**4*e*x - 30*a**3*b**5*d**3*e**2*x**2 + 10*a**3*b**5*d**2*e**3*x**3 - 3*a**2*b**6*d**5*x + 15*a**2*b**6*d**4*e*x**2 - 10*a**2*b**6*d**3*e**2*x**3 - 3*a*b**7*d**5*x**2 + 5*a*b**7*d**4*e*x**3 - b**8*d**5*x**3))
```

3.187 $\int \frac{1}{(d+ex)^{5/2}(a^2+2abx+b^2x^2)^2} dx$

| | |
|---|------|
| Optimal result | 1483 |
| Mathematica [A] (verified) | 1484 |
| Rubi [A] (verified) | 1484 |
| Maple [A] (verified) | 1489 |
| Fricas [B] (verification not implemented) | 1489 |
| Sympy [F(-1)] | 1490 |
| Maxima [F(-2)] | 1491 |
| Giac [B] (verification not implemented) | 1491 |
| Mupad [B] (verification not implemented) | 1492 |
| Reduce [B] (verification not implemented) | 1493 |

Optimal result

Integrand size = 28, antiderivative size = 200

$$\int \frac{1}{(d+ex)^{5/2}(a^2+2abx+b^2x^2)^2} dx =$$

$$\frac{1}{35e^3} \left[\frac{1}{8(bd-ae)^4(d+ex)^{3/2}} - \frac{1}{3(bd-ae)(a+bx)^3(d+ex)^{3/2}} \right]$$

$$+ \frac{3e}{4(bd-ae)^2(a+bx)^2(d+ex)^{3/2}} - \frac{21e^2}{8(bd-ae)^3(a+bx)(d+ex)^{3/2}}$$

$$- \frac{105be^3}{8(bd-ae)^5\sqrt{d+ex}} + \frac{105b^{3/2}e^3 \operatorname{arctanh}\left(\frac{\sqrt{b}\sqrt{d+ex}}{\sqrt{bd-ae}}\right)}{8(bd-ae)^{11/2}}$$

output

```
-35/8*e^3/(-a*e+b*d)^4/(e*x+d)^(3/2)-1/3/(-a*e+b*d)/(b*x+a)^3/(e*x+d)^(3/2)
)+3/4*e/(-a*e+b*d)^2/(b*x+a)^2/(e*x+d)^(3/2)-21/8*e^2/(-a*e+b*d)^3/(b*x+a)
)/(e*x+d)^(3/2)-105/8*b*e^3/(-a*e+b*d)^5/(e*x+d)^(1/2)+105/8*b^(3/2)*e^3*ar
ctanh(b^(1/2)*(e*x+d)^(1/2)/(-a*e+b*d)^(1/2))/(-a*e+b*d)^(11/2)
```


Mathematica [A] (verified)

Time = 1.03 (sec) , antiderivative size = 220, normalized size of antiderivative = 1.10

$$\int \frac{1}{(d+ex)^{5/2} (a^2+2abx+b^2x^2)^2} dx = \frac{1}{24} \left(\frac{-16a^4e^4 + 16a^3be^3(13d+9ex) + 3a^2b^2e^2(55d^2+318dex+20d^2ex^2+231e^2x^2) + 2ab^3e(-25d^3+90d^2ex+567d^2ex^2+420e^3x^3) + b^4(8d^4-18d^3ex+63d^2ex^2+420d^2ex^3+315e^4x^4)}{(-bd+ae)^{11/2}} \right) + \frac{315b^{3/2}e^3 \arctan\left(\frac{\sqrt{b}\sqrt{d+ex}}{\sqrt{-bd+ae}}\right)}{(-bd+ae)^{11/2}}$$

input

```
Integrate[1/((d + e*x)^(5/2)*(a^2 + 2*a*b*x + b^2*x^2)^2), x]
```

output

```
((-16*a^4*e^4 + 16*a^3*b*e^3*(13*d + 9*e*x) + 3*a^2*b^2*e^2*(55*d^2 + 318*d*e*x + 231*e^2*x^2) + 2*a*b^3*e*(-25*d^3 + 90*d^2*e*x + 567*d^2*e^2*x^2 + 420*e^3*x^3) + b^4*(8*d^4 - 18*d^3*e*x + 63*d^2*e^2*x^2 + 420*d^2*e^3*x^3 + 315*e^4*x^4))/((-b*d) + a*e)^5*(a + b*x)^3*(d + e*x)^(3/2)) + (315*b^(3/2)*e^3*ArcTan[(Sqrt[b]*Sqrt[d + e*x])/Sqrt[-(b*d) + a*e]])/((-b*d) + a*e)^(11/2))/24
```

Rubi [A] (verified)

Time = 0.59 (sec) , antiderivative size = 241, normalized size of antiderivative = 1.20, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.321$, Rules used = {1098, 27, 52, 52, 52, 61, 61, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a^2+2abx+b^2x^2)^2 (d+ex)^{5/2}} dx$$

↓ 1098

$$b^4 \int \frac{1}{b^4(a+bx)^4 (d+ex)^{5/2}} dx$$

↓ 27

$$\begin{aligned}
 & \int \frac{1}{(a+bx)^4(d+ex)^{5/2}} dx \\
 & \quad \downarrow 52 \\
 & -\frac{3e \int \frac{1}{(a+bx)^3(d+ex)^{5/2}} dx}{2(bd-ae)} - \frac{1}{3(a+bx)^3(d+ex)^{3/2}(bd-ae)} \\
 & \quad \downarrow 52 \\
 & -\frac{3e \left(-\frac{7e \int \frac{1}{(a+bx)^2(d+ex)^{5/2}} dx}{4(bd-ae)} - \frac{1}{2(a+bx)^2(d+ex)^{3/2}(bd-ae)} \right)}{2(bd-ae)} - \frac{1}{3(a+bx)^3(d+ex)^{3/2}(bd-ae)} \\
 & \quad \downarrow 52 \\
 & -\frac{3e \left(-\frac{7e \left(-\frac{5e \int \frac{1}{(a+bx)(d+ex)^{5/2}} dx}{2(bd-ae)} - \frac{1}{(a+bx)(d+ex)^{3/2}(bd-ae)} \right)}{4(bd-ae)} - \frac{1}{2(a+bx)^2(d+ex)^{3/2}(bd-ae)} \right)}{2(bd-ae)} \\
 & \quad \quad \quad \frac{1}{3(a+bx)^3(d+ex)^{3/2}(bd-ae)} \\
 & \quad \quad \quad \downarrow 61 \\
 & 3e \left(-\frac{7e \left(\frac{5e \left(\frac{b \int \frac{1}{(a+bx)(d+ex)^{3/2}} dx}{bd-ae} + \frac{2}{3(d+ex)^{3/2}(bd-ae)} \right)}{2(bd-ae)} - \frac{1}{(a+bx)(d+ex)^{3/2}(bd-ae)} \right)}{4(bd-ae)} - \frac{1}{2(a+bx)^2(d+ex)^{3/2}(bd-ae)} \right) \\
 & \quad \quad \quad \frac{1}{3(a+bx)^3(d+ex)^{3/2}(bd-ae)} \\
 & \quad \quad \quad \downarrow 61
 \end{aligned}$$

$$\left(\begin{array}{l} 7e \\ 5e \\ 3e \end{array} \right) \left(\begin{array}{l} b \left(\frac{\int \frac{1}{(a+bx)\sqrt{d+ex}} dx}{bd-ae} + \frac{2}{\sqrt{d+ex}(bd-ae)} \right) \\ \frac{2}{3(d+ex)^{3/2}(bd-ae)} \\ \frac{1}{(a+bx)(d+ex)^{3/2}(bd-ae)} \end{array} \right) - \frac{1}{2(a+bx)^2(d+ex)^{3/2}(bd-ae)}$$

$$\frac{1}{3(a+bx)^3(d+ex)^{3/2}(bd-ae)} \cdot \frac{2(bd-ae)}{1}$$

↓ 73

$$\left(\begin{array}{l} 7e \\ 5e \\ 3e \end{array} \right) \left(\begin{array}{l} b \left(\frac{2b \int \frac{1}{a + \frac{b(d+ex) - bd}{e}} d\sqrt{d+ex}}{e(bd-ae)} + \frac{2}{\sqrt{d+ex}(bd-ae)} \right) \\ \frac{2}{3(d+ex)^{3/2}(bd-ae)} \\ \frac{1}{(a+bx)(d+ex)^{3/2}(bd-ae)} \end{array} \right) - \frac{1}{2(a+bx)^2(d+ex)^{3/2}(bd-ae)}$$

$$\frac{1}{3(a+bx)^3(d+ex)^{3/2}(bd-ae)} \cdot \frac{2(bd-ae)}{1}$$

↓ 221

$$\begin{aligned}
 & \left(\frac{5e \left(\frac{b \left(\frac{2}{\sqrt{d+ex}(bd-ae)} - \frac{2\sqrt{b}\operatorname{arctanh}\left(\frac{\sqrt{b}\sqrt{d+ex}}{\sqrt{bd-ae}}\right)}{(bd-ae)^{3/2}} \right)}{bd-ae} + \frac{2}{3(d+ex)^{3/2}(bd-ae)} \right)}{2(bd-ae)} - \frac{1}{(a+bx)(d+ex)^{3/2}(bd-ae)} \right) \\
 & - \frac{3e}{4(bd-ae)} - \frac{1}{2(a+bx)^2(d+ex)^{3/2}(bd-ae)} \\
 & \frac{1}{3(a+bx)^3(d+ex)^{3/2}(bd-ae)}
 \end{aligned}$$

input `Int[1/((d + e*x)^(5/2)*(a^2 + 2*a*b*x + b^2*x^2)^2),x]`

output `-1/3*1/((b*d - a*e)*(a + b*x)^3*(d + e*x)^(3/2)) - (3*e*(-1/2*1/((b*d - a*e)*(a + b*x)^2*(d + e*x)^(3/2)) - (7*e*(-1/((b*d - a*e)*(a + b*x)*(d + e*x)^(3/2))) - (5*e*(2/(3*(b*d - a*e)*(d + e*x)^(3/2)) + (b*(2/((b*d - a*e)*Sqrt[d + e*x])) - (2*Sqrt[b]*ArcTanh[(Sqrt[b]*Sqrt[d + e*x])/Sqrt[b*d - a*e]])/(b*d - a*e)^(3/2)))/(b*d - a*e)))/(2*(b*d - a*e)))/(4*(b*d - a*e)))/(2*(b*d - a*e))`

Definitions of rubi rules used

- rule 27 $\text{Int}[(a_*)(Fx_), x_Symbol] \rightarrow \text{Simp}[a \text{ Int}[Fx, x], x] /; \text{FreeQ}[a, x] \ \&\& \ !\text{MatchQ}[Fx, (b_*)(Gx_)] /; \text{FreeQ}[b, x]$
- rule 52 $\text{Int}[(a_.) + (b_.)*(x_)^m*((c_.) + (d_.)*(x_))^{n_}, x_Symbol] \rightarrow \text{Simp}[(a + b*x)^{m+1}*((c + d*x)^{n+1}/((b*c - a*d)*(m+1))), x] - \text{Simp}[d*((m + n + 2)/((b*c - a*d)*(m + 1))) \text{ Int}[(a + b*x)^{m+1}*(c + d*x)^n, x], x] /; \text{FreeQ}[\{a, b, c, d, n\}, x] \ \&\& \ \text{ILtQ}[m, -1] \ \&\& \ \text{FractionQ}[n] \ \&\& \ \text{LtQ}[n, 0]$
- rule 61 $\text{Int}[(a_.) + (b_.)*(x_)^m*((c_.) + (d_.)*(x_))^{n_}, x_Symbol] \rightarrow \text{Simp}[(a + b*x)^{m+1}*((c + d*x)^{n+1}/((b*c - a*d)*(m+1))), x] - \text{Simp}[d*((m + n + 2)/((b*c - a*d)*(m + 1))) \text{ Int}[(a + b*x)^{m+1}*(c + d*x)^n, x], x] /; \text{FreeQ}[\{a, b, c, d, n\}, x] \ \&\& \ \text{LtQ}[m, -1] \ \&\& \ !(\text{LtQ}[n, -1] \ \&\& \ (\text{EqQ}[a, 0] \ || \ (\text{NeQ}[c, 0] \ \&\& \ \text{LtQ}[m - n, 0] \ \&\& \ \text{IntegerQ}[n]))) \ \&\& \ \text{IntLinearQ}[a, b, c, d, m, n, x]$
- rule 73 $\text{Int}[(a_.) + (b_.)*(x_)^m*((c_.) + (d_.)*(x_))^{n_}, x_Symbol] \rightarrow \text{With}[\{p = \text{Denominator}[m]\}, \text{Simp}[p/b \ \text{Subst}[\text{Int}[x^{p*(m+1) - 1}*(c - a*(d/b) + d*(x^p/b))^{n_}, x], x, (a + b*x)^{1/p}], x]] /; \text{FreeQ}[\{a, b, c, d\}, x] \ \&\& \ \text{LtQ}[-1, m, 0] \ \&\& \ \text{LeQ}[-1, n, 0] \ \&\& \ \text{LeQ}[\text{Denominator}[n], \text{Denominator}[m]] \ \&\& \ \text{IntLinearQ}[a, b, c, d, m, n, x]$
- rule 221 $\text{Int}[(a_.) + (b_.)*(x_)^2)^{-1}, x_Symbol] \rightarrow \text{Simp}[(\text{Rt}[-a/b, 2]/a)*\text{ArcTanh}[x/\text{Rt}[-a/b, 2]], x] /; \text{FreeQ}[\{a, b\}, x] \ \&\& \ \text{NegQ}[a/b]$
- rule 1098 $\text{Int}[(d_.) + (e_.)*(x_)^m*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^{p_}, x_Symbol] \rightarrow \text{Simp}[1/c^p \ \text{Int}[(d + e*x)^m*(b/2 + c*x)^{2*p}, x], x] /; \text{FreeQ}[\{a, b, c, d, e, m\}, x] \ \&\& \ \text{EqQ}[b^2 - 4*a*c, 0] \ \&\& \ \text{IntegerQ}[p]$

Maple [A] (verified)

Time = 1.60 (sec) , antiderivative size = 177, normalized size of antiderivative = 0.88

| method | result |
|-------------------|---|
| derivativedivides | $2e^3 \left(-\frac{1}{3(ae-bd)^4 (ex+d)^{\frac{3}{2}}} + \frac{4b}{(ae-bd)^5 \sqrt{ex+d}} + \frac{b^2 \left(\frac{41(ex+d)^{\frac{5}{2}} b^2}{16} + \frac{35b(ae-bd)(ex+d)^{\frac{3}{2}}}{6} + \frac{(55e^2 a^2 - 55abde + 55d^2)}{16} \right)}{(b(ex+d) + ae - bd)^3} \right) \frac{1}{(ae-bd)^5}$ |
| default | $2e^3 \left(-\frac{1}{3(ae-bd)^4 (ex+d)^{\frac{3}{2}}} + \frac{4b}{(ae-bd)^5 \sqrt{ex+d}} + \frac{b^2 \left(\frac{41(ex+d)^{\frac{5}{2}} b^2}{16} + \frac{35b(ae-bd)(ex+d)^{\frac{3}{2}}}{6} + \frac{(55e^2 a^2 - 55abde + 55d^2)}{16} \right)}{(b(ex+d) + ae - bd)^3} \right) \frac{1}{(ae-bd)^5}$ |
| pseudoelliptic | $2 \left(-\frac{315b^2 e^3 (ex+d)^{\frac{3}{2}} (bx+a)^3 \arctan\left(\frac{b\sqrt{ex+d}}{\sqrt{b(ae-bd)}}\right)}{16} + \left(-\frac{315}{16} e^4 x^4 - \frac{105}{4} d e^3 x^3 - \frac{63}{16} d^2 e^2 x^2 + \frac{9}{8} d^3 e x - \frac{1}{2} d^4 \right) b^4 + \frac{25ea(-8)}{16} \right) \frac{1}{3(ex+d)^{\frac{3}{2}} \sqrt{b(ae-bd)}}$ |

```
input int(1/(e*x+d)^(5/2)/(b^2*x^2+2*a*b*x+a^2)^2,x,method=_RETURNVERBOSE)
```

```
output 2*e^3*(-1/3/(a*e-b*d)^4/(e*x+d)^(3/2)+4/(a*e-b*d)^5*b/(e*x+d)^(1/2)+1/(a*e-b*d)^5*b^2*((41/16*(e*x+d)^(5/2)*b^2+35/6*b*(a*e-b*d)*(e*x+d)^(3/2)+(55/16*e^2*a^2-55/8*a*b*d*e+55/16*b^2*d^2)*(e*x+d)^(1/2))/(b*(e*x+d)+a*e-b*d)^3+105/16/(b*(a*e-b*d))^(1/2)*arctan(b*(e*x+d)^(1/2)/(b*(a*e-b*d))^(1/2)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 895 vs. 2(168) = 336.

Time = 0.21 (sec) , antiderivative size = 1820, normalized size of antiderivative = 9.10

$$\int \frac{1}{(d + ex)^{5/2} (a^2 + 2abx + b^2x^2)^2} dx = \text{Too large to display}$$

```
input integrate(1/(e*x+d)^(5/2)/(b^2*x^2+2*a*b*x+a^2)^2,x, algorithm="fricas")
```

output

```

[-1/48*(315*(b^4*e^5*x^5 + a^3*b*d^2*e^3 + (2*b^4*d*e^4 + 3*a*b^3*e^5)*x^4
+ (b^4*d^2*e^3 + 6*a*b^3*d*e^4 + 3*a^2*b^2*e^5)*x^3 + (3*a*b^3*d^2*e^3 +
6*a^2*b^2*d*e^4 + a^3*b*e^5)*x^2 + (3*a^2*b^2*d^2*e^3 + 2*a^3*b*d*e^4)*x)*
sqrt(b/(b*d - a*e))*log((b*e*x + 2*b*d - a*e - 2*(b*d - a*e)*sqrt(e*x + d)
*sqrt(b/(b*d - a*e)))/(b*x + a)) + 2*(315*b^4*e^4*x^4 + 8*b^4*d^4 - 50*a*b
^3*d^3*e + 165*a^2*b^2*d^2*e^2 + 208*a^3*b*d*e^3 - 16*a^4*e^4 + 420*(b^4*d
*e^3 + 2*a*b^3*e^4)*x^3 + 63*(b^4*d^2*e^2 + 18*a*b^3*d*e^3 + 11*a^2*b^2*e^
4)*x^2 - 18*(b^4*d^3*e - 10*a*b^3*d^2*e^2 - 53*a^2*b^2*d*e^3 - 8*a^3*b*e^4
)*x)*sqrt(e*x + d))/(a^3*b^5*d^7 - 5*a^4*b^4*d^6*e + 10*a^5*b^3*d^5*e^2 -
10*a^6*b^2*d^4*e^3 + 5*a^7*b*d^3*e^4 - a^8*d^2*e^5 + (b^8*d^5*e^2 - 5*a*b^
7*d^4*e^3 + 10*a^2*b^6*d^3*e^4 - 10*a^3*b^5*d^2*e^5 + 5*a^4*b^4*d*e^6 - a^
5*b^3*e^7)*x^5 + (2*b^8*d^6*e - 7*a*b^7*d^5*e^2 + 5*a^2*b^6*d^4*e^3 + 10*a
^3*b^5*d^3*e^4 - 20*a^4*b^4*d^2*e^5 + 13*a^5*b^3*d*e^6 - 3*a^6*b^2*e^7)*x^
4 + (b^8*d^7 + a*b^7*d^6*e - 17*a^2*b^6*d^5*e^2 + 35*a^3*b^5*d^4*e^3 - 25*
a^4*b^4*d^3*e^4 - a^5*b^3*d^2*e^5 + 9*a^6*b^2*d*e^6 - 3*a^7*b*e^7)*x^3 + (
3*a*b^7*d^7 - 9*a^2*b^6*d^6*e + a^3*b^5*d^5*e^2 + 25*a^4*b^4*d^4*e^3 - 35*
a^5*b^3*d^3*e^4 + 17*a^6*b^2*d^2*e^5 - a^7*b*d*e^6 - a^8*e^7)*x^2 + (3*a^2
*b^6*d^7 - 13*a^3*b^5*d^6*e + 20*a^4*b^4*d^5*e^2 - 10*a^5*b^3*d^4*e^3 - 5*
a^6*b^2*d^3*e^4 + 7*a^7*b*d^2*e^5 - 2*a^8*d*e^6)*x), -1/24*(315*(b^4*e^5*x
^5 + a^3*b*d^2*e^3 + (2*b^4*d*e^4 + 3*a*b^3*e^5)*x^4 + (b^4*d^2*e^3 + 6...

```

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{(d+ex)^{5/2} (a^2+2abx+b^2x^2)^2} dx = \text{Timed out}$$

input

```
integrate(1/(e*x+d)**(5/2)/(b**2*x**2+2*a*b*x+a**2)**2,x)
```

output

Timed out

Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{(d+ex)^{5/2} (a^2+2abx+b^2x^2)^2} dx = \text{Exception raised: ValueError}$$

input `integrate(1/(e*x+d)^(5/2)/(b^2*x^2+2*a*b*x+a^2)^2,x, algorithm="maxima")`

output `Exception raised: ValueError >> Computation failed since Maxima requested additional constraints; using the 'assume' command before evaluation *may* help (example of legal syntax is 'assume(a*e-b*d>0)', see `assume?` for more detail`

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 432 vs. 2(168) = 336.

Time = 0.21 (sec) , antiderivative size = 432, normalized size of antiderivative = 2.16

$$\int \frac{1}{(d+ex)^{5/2} (a^2+2abx+b^2x^2)^2} dx =$$

$$\frac{105 b^2 e^3 \arctan\left(\frac{\sqrt{ex+db}}{\sqrt{-b^2d+abe}}\right)}{8(b^5d^5 - 5ab^4d^4e + 10a^2b^3d^3e^2 - 10a^3b^2d^2e^3 + 5a^4bde^4 - a^5e^5)\sqrt{-b^2d+abe} - 315(ex+d)^4b^4e^3 - 840(ex+d)^3b^4de^3 + 693(ex+d)^2b^4d^2e^3 - 144(ex+d)b^4d^3e^3 - 16b^4d^4e^3 + 840(ex+d)b^4d^4e^3 - 24(b^5d^5 - 5ab^4d^4e)}$$

input `integrate(1/(e*x+d)^(5/2)/(b^2*x^2+2*a*b*x+a^2)^2,x, algorithm="giac")`

output

```
-105/8*b^2*e^3*arctan(sqrt(e*x + d)*b/sqrt(-b^2*d + a*b*e))/((b^5*d^5 - 5*
a*b^4*d^4*e + 10*a^2*b^3*d^3*e^2 - 10*a^3*b^2*d^2*e^3 + 5*a^4*b*d*e^4 - a^
5*e^5)*sqrt(-b^2*d + a*b*e)) - 1/24*(315*(e*x + d)^4*b^4*e^3 - 840*(e*x +
d)^3*b^4*d*e^3 + 693*(e*x + d)^2*b^4*d^2*e^3 - 144*(e*x + d)*b^4*d^3*e^3 -
16*b^4*d^4*e^3 + 840*(e*x + d)^3*a*b^3*e^4 - 1386*(e*x + d)^2*a*b^3*d*e^4
+ 432*(e*x + d)*a*b^3*d^2*e^4 + 64*a*b^3*d^3*e^4 + 693*(e*x + d)^2*a^2*b^
2*e^5 - 432*(e*x + d)*a^2*b^2*d*e^5 - 96*a^2*b^2*d^2*e^5 + 144*(e*x + d)*a
^3*b*e^6 + 64*a^3*b*d*e^6 - 16*a^4*e^7)/((b^5*d^5 - 5*a*b^4*d^4*e + 10*a^2
*b^3*d^3*e^2 - 10*a^3*b^2*d^2*e^3 + 5*a^4*b*d*e^4 - a^5*e^5)*((e*x + d)^(3
/2)*b - sqrt(e*x + d)*b*d + sqrt(e*x + d)*a*e)^3)
```

Mupad [B] (verification not implemented)

Time = 9.46 (sec) , antiderivative size = 334, normalized size of antiderivative = 1.67

$$\int \frac{1}{(d+ex)^{5/2} (a^2+2abx+b^2x^2)^2} dx = \frac{\frac{231b^2e^3(d+ex)^2}{8(ae-bd)^3} - \frac{2e^3}{3(ae-bd)} + \frac{3}{8(ae-bd)^3}}{(d+ex)^{3/2} (a^3e^3 - 3a^2bde^2 + 3ab^2d^2e - b^3d^3) + b^3(d+ex)^9} + \frac{105b^{3/2}e^3 \operatorname{atan}\left(\frac{\sqrt{b}\sqrt{d+ex}(a^5e^5 - 5a^4bde^4 + 10a^3b^2d^2e^3 - 10a^2b^3d^3e^2 + 5ab^4d^4e - b^5d^5)}{(ae-bd)^{11/2}}\right)}{8(ae-bd)^{11/2}}$$

input

```
int(1/((d + e*x)^(5/2)*(a^2 + b^2*x^2 + 2*a*b*x)^2),x)
```

output

```
((231*b^2*e^3*(d + e*x)^2)/(8*(a*e - b*d)^3) - (2*e^3)/(3*(a*e - b*d)) + (
35*b^3*e^3*(d + e*x)^3)/(a*e - b*d)^4 + (105*b^4*e^3*(d + e*x)^4)/(8*(a*e
- b*d)^5) + (6*b*e^3*(d + e*x))/(a*e - b*d)^2)/((d + e*x)^(3/2)*(a^3*e^3 -
b^3*d^3 + 3*a*b^2*d^2*e - 3*a^2*b*d*e^2) + b^3*(d + e*x)^(9/2) - (3*b^3*d
- 3*a*b^2*e)*(d + e*x)^(7/2) + (d + e*x)^(5/2)*(3*b^3*d^2 + 3*a^2*b*e^2 -
6*a*b^2*d*e)) + (105*b^(3/2)*e^3*atan((b^(1/2)*(d + e*x)^(1/2)*(a^5*e^5 -
b^5*d^5 - 10*a^2*b^3*d^3*e^2 + 10*a^3*b^2*d^2*e^3 + 5*a*b^4*d^4*e - 5*a^4
*b*d*e^4))/(a*e - b*d)^(11/2)))/(8*(a*e - b*d)^(11/2))
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 1220, normalized size of antiderivative = 6.10

$$\int \frac{1}{(d + ex)^{5/2} (a^2 + 2abx + b^2x^2)^2} dx = \text{Too large to display}$$

input

```
int(1/(e*x+d)^(5/2)/(b^2*x^2+2*a*b*x+a^2)^2,x)
```

output

```
(315*sqrt(b)*sqrt(d + e*x)*sqrt(a*e - b*d)*atan((sqrt(d + e*x)*b)/(sqrt(b)
*sqrt(a*e - b*d)))*a**3*b*d*e**3 + 315*sqrt(b)*sqrt(d + e*x)*sqrt(a*e - b*
d)*atan((sqrt(d + e*x)*b)/(sqrt(b)*sqrt(a*e - b*d)))*a**3*b*e**4*x + 945*s
qrt(b)*sqrt(d + e*x)*sqrt(a*e - b*d)*atan((sqrt(d + e*x)*b)/(sqrt(b)*sqrt(
a*e - b*d)))*a**2*b**2*d*e**3*x + 945*sqrt(b)*sqrt(d + e*x)*sqrt(a*e - b*d
)*atan((sqrt(d + e*x)*b)/(sqrt(b)*sqrt(a*e - b*d)))*a**2*b**2*e**4*x**2 +
945*sqrt(b)*sqrt(d + e*x)*sqrt(a*e - b*d)*atan((sqrt(d + e*x)*b)/(sqrt(b)*
sqrt(a*e - b*d)))*a*b**3*d*e**3*x**2 + 945*sqrt(b)*sqrt(d + e*x)*sqrt(a*e
- b*d)*atan((sqrt(d + e*x)*b)/(sqrt(b)*sqrt(a*e - b*d)))*a*b**3*e**4*x**3
+ 315*sqrt(b)*sqrt(d + e*x)*sqrt(a*e - b*d)*atan((sqrt(d + e*x)*b)/(sqrt(b)
)*sqrt(a*e - b*d))*b**4*d*e**3*x**3 + 315*sqrt(b)*sqrt(d + e*x)*sqrt(a*e
- b*d)*atan((sqrt(d + e*x)*b)/(sqrt(b)*sqrt(a*e - b*d)))*b**4*e**4*x**4 -
16*a**5*e**5 + 224*a**4*b*d*e**4 + 144*a**4*b*e**5*x - 43*a**3*b**2*d**2*e
**3 + 810*a**3*b**2*d*e**4*x + 693*a**3*b**2*e**5*x**2 - 215*a**2*b**3*d**
3*e**2 - 774*a**2*b**3*d**2*e**3*x + 441*a**2*b**3*d*e**4*x**2 + 840*a**2*
b**3*e**5*x**3 + 58*a*b**4*d**4*e - 198*a*b**4*d**3*e**2*x - 1071*a*b**4*d
**2*e**3*x**2 - 420*a*b**4*d*e**4*x**3 + 315*a*b**4*e**5*x**4 - 8*b**5*d**
5 + 18*b**5*d**4*e*x - 63*b**5*d**3*e**2*x**2 - 420*b**5*d**2*e**3*x**3 -
315*b**5*d*e**4*x**4)/(24*sqrt(d + e*x)*(a**9*d*e**6 + a**9*e**7*x - 6*a**
8*b*d**2*e**5 - 3*a**8*b*d*e**6*x + 3*a**8*b*e**7*x**2 + 15*a**7*b**2*d...
```

3.188 $\int \frac{1}{(d+ex)^{7/2}(a^2+2abx+b^2x^2)^2} dx$

| | |
|---|------|
| Optimal result | 1494 |
| Mathematica [A] (verified) | 1495 |
| Rubi [A] (verified) | 1495 |
| Maple [A] (verified) | 1502 |
| Fricas [B] (verification not implemented) | 1503 |
| Sympy [F(-1)] | 1504 |
| Maxima [F(-2)] | 1504 |
| Giac [B] (verification not implemented) | 1504 |
| Mupad [B] (verification not implemented) | 1505 |
| Reduce [B] (verification not implemented) | 1506 |

Optimal result

Integrand size = 28, antiderivative size = 229

$$\int \frac{1}{(d+ex)^{7/2}(a^2+2abx+b^2x^2)^2} dx = -\frac{231e^3}{40(bd-ae)^4(d+ex)^{5/2}} - \frac{1}{3(bd-ae)(a+bx)^3(d+ex)^{5/2}} + \frac{11e}{12(bd-ae)^2(a+bx)^2(d+ex)^{5/2}} - \frac{33e^2}{8(bd-ae)^3(a+bx)(d+ex)^{5/2}} - \frac{77be^3}{8(bd-ae)^5(d+ex)^{3/2}} - \frac{231b^2e^3}{8(bd-ae)^6\sqrt{d+ex}} + \frac{231b^{5/2}e^3\operatorname{arctanh}\left(\frac{\sqrt{b}\sqrt{d+ex}}{\sqrt{bd-ae}}\right)}{8(bd-ae)^{13/2}}$$

output

```
-231/40*e^3/(-a*e+b*d)^4/(e*x+d)^(5/2)-1/3/(-a*e+b*d)/(b*x+a)^3/(e*x+d)^(5/2)+11/12*e/(-a*e+b*d)^2/(b*x+a)^2/(e*x+d)^(5/2)-33/8*e^2/(-a*e+b*d)^3/(b*x+a)/(e*x+d)^(5/2)-77/8*b*e^3/(-a*e+b*d)^5/(e*x+d)^(3/2)-231/8*b^2*e^3/(-a*e+b*d)^6/(e*x+d)^(1/2)+231/8*b^(5/2)*e^3*arctanh(b^(1/2)*(e*x+d)^(1/2)/(-a*e+b*d)^(1/2))/(-a*e+b*d)^(13/2)
```

Mathematica [A] (verified)

Time = 1.22 (sec) , antiderivative size = 285, normalized size of antiderivative = 1.24

$$\int \frac{1}{(d+ex)^{7/2}(a^2+2abx+b^2x^2)^2} dx = \frac{-48a^5e^5 + 16a^4be^4(26d+11ex) - 16a^3b^2e^3(173d^2+242dex+99d^3) + 231b^5/2e^3 \arctan\left(\frac{\sqrt{b}\sqrt{d+ex}}{\sqrt{-bd+ae}}\right)}{8(-bd+ae)^{13/2}}$$

input `Integrate[1/((d + e*x)^(7/2)*(a^2 + 2*a*b*x + b^2*x^2)^2),x]`

output
$$\frac{(-48a^5e^5 + 16a^4b^2e^4(26d + 11ex) - 16a^3b^2e^3(173d^2 + 242dex + 99e^2x^2) - 3a^2b^3e^2(445d^3 + 4103d^2ex + 6039d^2e^2x^2 + 2541e^3x^3) - 2ab^4e(-155d^4 + 715d^3ex + 7227d^2e^2x^2 + 10857de^3x^3 + 4620e^4x^4) - b^5(40d^5 - 110d^4ex + 495d^3e^2x^2 + 5313d^2e^3x^3 + 8085de^4x^4 + 3465e^5x^5))/(120(bd - a^2e)^6(a + bx)^3(d + ex)^{5/2}) - (231b^{5/2}e^3 \text{ArcTan}[\sqrt{b}\sqrt{d+ex}]/\sqrt{-bd+ae}])/(8(-bd+ae)^{13/2})}{1}$$

Rubi [A] (verified)

Time = 0.59 (sec) , antiderivative size = 277, normalized size of antiderivative = 1.21, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.357$, Rules used = {1098, 27, 52, 52, 52, 61, 61, 61, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a^2 + 2abx + b^2x^2)^2 (d + ex)^{7/2}} dx$$

↓ 1098

$$b^4 \int \frac{1}{b^4(a + bx)^4 (d + ex)^{7/2}} dx$$

↓ 27

$$\begin{aligned}
 & \left(\frac{9e \left(\frac{7e \left(b \left(\frac{\int \frac{1}{(a+bx)(d+ex)^{3/2}} dx}{bd-ae} + \frac{2}{3(d+ex)^{3/2}(bd-ae)} \right) + \frac{2}{5(d+ex)^{5/2}(bd-ae)} \right)}{bd-ae} \right)}{2(bd-ae)} - \frac{1}{(a+bx)(d+ex)^{5/2}(bd-ae)} \right) \\
 11e & \left(\frac{\quad}{4(bd-ae)} \right) - \frac{1}{2(a+bx)^2(d+ex)^{5/2}} \\
 & \frac{6(bd-ae)}{3(a+bx)^3(d+ex)^{5/2}(bd-ae)} \\
 & \downarrow 61
 \end{aligned}$$

$$\begin{aligned}
 & \left(\left(\left(\left(\left(\frac{b \int \frac{1}{(a+bx)\sqrt{d+ex}} dx}{bd-ae} + \frac{2}{\sqrt{d+ex}(bd-ae)} \right) \right) + \frac{2}{3(d+ex)^{3/2}(bd-ae)} \right) \right) + \frac{2}{5(d+ex)^{5/2}(bd-ae)} \right) \\
 9e & - \frac{\left(\left(\left(\left(\left(\frac{b \int \frac{1}{(a+bx)\sqrt{d+ex}} dx}{bd-ae} + \frac{2}{\sqrt{d+ex}(bd-ae)} \right) \right) + \frac{2}{3(d+ex)^{3/2}(bd-ae)} \right) \right) + \frac{2}{5(d+ex)^{5/2}(bd-ae)} \right)}{2(bd-ae)} - \frac{1}{(a+bx)(d+ex)^{5/2}(bd-ae)} \\
 11e & - \frac{\left(\left(\left(\left(\left(\frac{b \int \frac{1}{(a+bx)\sqrt{d+ex}} dx}{bd-ae} + \frac{2}{\sqrt{d+ex}(bd-ae)} \right) \right) + \frac{2}{3(d+ex)^{3/2}(bd-ae)} \right) \right) + \frac{2}{5(d+ex)^{5/2}(bd-ae)} \right)}{4(bd-ae)} - \frac{1}{2(a+bx)(d+ex)^{5/2}(bd-ae)} \\
 & - \frac{1}{6(bd-ae)} \\
 & \frac{1}{3(a+bx)^3(d+ex)^{5/2}(bd-ae)} \\
 & \downarrow 73
 \end{aligned}$$

$$\begin{aligned}
 & \left(\frac{7e}{bd-ae} \left(\frac{b \left(\frac{2b \int \frac{1}{a + \frac{b(d+ex) - bd}{e(bd-ae)}} d\sqrt{d+ex} + \frac{2}{\sqrt{d+ex}(bd-ae)} \right)}{bd-ae} + \frac{2}{3(d+ex)^{3/2}(bd-ae)} \right) + \frac{2}{5(d+ex)^{5/2}(bd-ae)} \right) \\
 & \frac{9e}{2(bd-ae)} - \frac{1}{(a+bx)(d+ex)^{5/2}(bd-ae)} \\
 & \frac{11e}{4(bd-ae)}
 \end{aligned}$$

$$\frac{1}{3(a+bx)^3(d+ex)^{5/2}(bd-ae)}$$

\downarrow 221

6(bd - ae)

$$\begin{aligned}
 & \left(\frac{7e}{bd-ae} \left(\frac{b \left(\frac{2}{\sqrt{d+ex}(bd-ae)} - \frac{2\sqrt{b}\operatorname{arctanh}\left(\frac{\sqrt{b}\sqrt{d+ex}}{\sqrt{bd-ae}}\right)}{(bd-ae)^{3/2}} \right)}{bd-ae} + \frac{2}{3(d+ex)^{3/2}(bd-ae)} \right) + \frac{2}{5(d+ex)^{5/2}(bd-ae)} \right) \\
 & - \frac{9e}{2(bd-ae)} - \frac{1}{(a+bx)(d+ex)^{5/2}(bd-ae)} \\
 & - \frac{11e}{4(bd-ae)} \\
 & - \frac{1}{3(a+bx)^3(d+ex)^{5/2}(bd-ae)} + \frac{6(bd-ae)}{6(bd-ae)}
 \end{aligned}$$

input `Int[1/((d + e*x)^(7/2)*(a^2 + 2*a*b*x + b^2*x^2)^2),x]`

output

$$-1/3*1/((b*d - a*e)*(a + b*x)^3*(d + e*x)^{(5/2)}) - (11*e*(-1/2*1/((b*d - a*e)*(a + b*x)^2*(d + e*x)^{(5/2)}) - (9*e*(-1/((b*d - a*e)*(a + b*x)*(d + e*x)^{(5/2)})) - (7*e*(2/(5*(b*d - a*e)*(d + e*x)^{(5/2)})) + (b*(2/(3*(b*d - a*e)*(d + e*x)^{(3/2)})) + (b*(2/((b*d - a*e)*\text{Sqrt}[d + e*x]) - (2*\text{Sqrt}[b]*\text{ArcTanh}[(\text{Sqrt}[b]*\text{Sqrt}[d + e*x])/\text{Sqrt}[b*d - a*e]])/(b*d - a*e)^{(3/2)})))/(b*d - a*e)))/(b*d - a*e)))/(2*(b*d - a*e)))/(4*(b*d - a*e)))/(6*(b*d - a*e))$$

Defintions of rubi rules used

rule 27

$$\text{Int}[(a_*)(Fx_), x_Symbol] \rightarrow \text{Simp}[a \quad \text{Int}[Fx, x], x] \text{ ; FreeQ}[a, x] \ \&\& \ !\text{MatchQ}[Fx, (b_*)(Gx_) \text{ ; FreeQ}[b, x]$$

rule 52

$$\text{Int}[(a_.) + (b_.)*(x_)^{(m_)}*((c_.) + (d_.)*(x_)^{(n_)}), x_Symbol] \rightarrow \text{Simp}[(a + b*x)^{(m + 1)}*((c + d*x)^{(n + 1)}/((b*c - a*d)*(m + 1))), x] - \text{Simp}[d*((m + n + 2)/((b*c - a*d)*(m + 1))) \quad \text{Int}[(a + b*x)^{(m + 1)}*(c + d*x)^n, x], x] \text{ ; FreeQ}[\{a, b, c, d, n\}, x] \ \&\& \ \text{ILtQ}[m, -1] \ \&\& \ \text{FractionQ}[n] \ \&\& \ \text{LtQ}[n, 0]$$

rule 61

$$\text{Int}[(a_.) + (b_.)*(x_)^{(m_)}*((c_.) + (d_.)*(x_)^{(n_)}), x_Symbol] \rightarrow \text{Simp}[(a + b*x)^{(m + 1)}*((c + d*x)^{(n + 1)}/((b*c - a*d)*(m + 1))), x] - \text{Simp}[d*((m + n + 2)/((b*c - a*d)*(m + 1))) \quad \text{Int}[(a + b*x)^{(m + 1)}*(c + d*x)^n, x], x] \text{ ; FreeQ}[\{a, b, c, d, n\}, x] \ \&\& \ \text{LtQ}[m, -1] \ \&\& \ !(\text{LtQ}[n, -1] \ \&\& \ (\text{EqQ}[a, 0] \ || \ (\text{NeQ}[c, 0] \ \&\& \ \text{LtQ}[m - n, 0] \ \&\& \ \text{IntegerQ}[n]))) \ \&\& \ \text{IntLinearQ}[a, b, c, d, m, n, x]$$

rule 73

$$\text{Int}[(a_.) + (b_.)*(x_)^{(m_)}*((c_.) + (d_.)*(x_)^{(n_)}), x_Symbol] \rightarrow \text{With}[\{p = \text{Denominator}[m]\}, \text{Simp}[p/b \quad \text{Subst}[\text{Int}[x^{(p*(m + 1) - 1)}*(c - a*(d/b) + d*(x^p/b)^n, x], x, (a + b*x)^{(1/p)}], x]] \text{ ; FreeQ}[\{a, b, c, d\}, x] \ \&\& \ \text{LtQ}[-1, m, 0] \ \&\& \ \text{LeQ}[-1, n, 0] \ \&\& \ \text{LeQ}[\text{Denominator}[n], \text{Denominator}[m]] \ \&\& \ \text{IntLinearQ}[a, b, c, d, m, n, x]$$

rule 221

$$\text{Int}[(a_.) + (b_.)*(x_)^2)^{-1}, x_Symbol] \rightarrow \text{Simp}[(\text{Rt}[-a/b, 2]/a)*\text{ArcTanh}[x/\text{Rt}[-a/b, 2]], x] \text{ ; FreeQ}[\{a, b\}, x] \ \&\& \ \text{NegQ}[a/b]$$

rule 1098

```
Int[((d_.) + (e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] := Simp[1/c^p Int[(d + e*x)^m*(b/2 + c*x)^(2*p), x], x] /; FreeQ[
{a, b, c, d, e, m}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Maple [A] (verified)

Time = 1.65 (sec) , antiderivative size = 200, normalized size of antiderivative = 0.87

| method | result |
|-------------------|--|
| derivativedivides | $2e^3 \frac{b^3 \left(\frac{71(ex+d)^{\frac{5}{2}} b^2}{16} + \frac{59b(ae-bd)(ex+d)^{\frac{3}{2}}}{6} + \left(\frac{89}{16} e^2 a^2 - \frac{89}{8} abde + \frac{89}{16} b^2 d^2 \right) \sqrt{ex+d} + \frac{231 \arctan\left(\frac{b\sqrt{ex+d}}{\sqrt{b(ae-bd)}}\right)}{16\sqrt{b(ae-bd)}} \right)}{(ae-bd)^6} - \frac{5}{6}$ |
| default | $2e^3 \frac{b^3 \left(\frac{71(ex+d)^{\frac{5}{2}} b^2}{16} + \frac{59b(ae-bd)(ex+d)^{\frac{3}{2}}}{6} + \left(\frac{89}{16} e^2 a^2 - \frac{89}{8} abde + \frac{89}{16} b^2 d^2 \right) \sqrt{ex+d} + \frac{231 \arctan\left(\frac{b\sqrt{ex+d}}{\sqrt{b(ae-bd)}}\right)}{16\sqrt{b(ae-bd)}} \right)}{(ae-bd)^6} - \frac{5}{6}$ |
| pseudoelliptic | $2 \frac{1155b^3 e^3 (ex+d)^{\frac{5}{2}} (bx+a)^3 \arctan\left(\frac{b\sqrt{ex+d}}{\sqrt{b(ae-bd)}}\right)}{16} + \left(\frac{1155}{16} e^5 x^5 + \frac{1771}{16} d^2 e^3 x^3 + \frac{2695}{16} d e^4 x^4 + \frac{165}{16} d^3 e^2 x^2 - \frac{55}{24} d^4 e x + \frac{5}{6} d^5 \right)$ |

```
input int(1/(e*x+d)^(7/2)/(b^2*x^2+2*a*b*x+a^2)^2,x,method=_RETURNVERBOSE)
```

```
output 2*e^3*(-1/(a*e-b*d)^6*b^3*((71/16*(e*x+d)^(5/2)*b^2+59/6*b*(a*e-b*d)*(e*x+d)^(3/2)+(89/16*e^2*a^2-89/8*a*b*d*e+89/16*b^2*d^2)*(e*x+d)^(1/2))/(b*(e*x+d)+a*e-b*d)^3+231/16/(b*(a*e-b*d))^(1/2)*arctan(b*(e*x+d)^(1/2)/(b*(a*e-b*d))^(1/2)))-1/5/(a*e-b*d)^4/(e*x+d)^(5/2)-10/(a*e-b*d)^6*b^2/(e*x+d)^(1/2)+4/3/(a*e-b*d)^5*b/(e*x+d)^(3/2))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1250 vs. $2(193) = 386$.

Time = 0.39 (sec) , antiderivative size = 2530, normalized size of antiderivative = 11.05

$$\int \frac{1}{(d+ex)^{7/2} (a^2+2abx+b^2x^2)^2} dx = \text{Too large to display}$$

input `integrate(1/(e*x+d)^(7/2)/(b^2*x^2+2*a*b*x+a^2)^2,x, algorithm="fricas")`

output

```
[1/240*(3465*(b^5*e^6*x^6 + a^3*b^2*d^3*e^3 + 3*(b^5*d*e^5 + a*b^4*e^6)*x^
5 + 3*(b^5*d^2*e^4 + 3*a*b^4*d*e^5 + a^2*b^3*e^6)*x^4 + (b^5*d^3*e^3 + 9*a
*b^4*d^2*e^4 + 9*a^2*b^3*d*e^5 + a^3*b^2*e^6)*x^3 + 3*(a*b^4*d^3*e^3 + 3*a
^2*b^3*d^2*e^4 + a^3*b^2*d*e^5)*x^2 + 3*(a^2*b^3*d^3*e^3 + a^3*b^2*d^2*e^4
)*x)*sqrt(b/(b*d - a*e))*log((b*e*x + 2*b*d - a*e + 2*(b*d - a*e)*sqrt(e*x
+ d)*sqrt(b/(b*d - a*e)))/(b*x + a)) - 2*(3465*b^5*e^5*x^5 + 40*b^5*d^5 -
310*a*b^4*d^4*e + 1335*a^2*b^3*d^3*e^2 + 2768*a^3*b^2*d^2*e^3 - 416*a^4*b
*d*e^4 + 48*a^5*e^5 + 1155*(7*b^5*d*e^4 + 8*a*b^4*e^5)*x^4 + 231*(23*b^5*d
^2*e^3 + 94*a*b^4*d*e^4 + 33*a^2*b^3*e^5)*x^3 + 99*(5*b^5*d^3*e^2 + 146*a*
b^4*d^2*e^3 + 183*a^2*b^3*d*e^4 + 16*a^3*b^2*e^5)*x^2 - 11*(10*b^5*d^4*e -
130*a*b^4*d^3*e^2 - 1119*a^2*b^3*d^2*e^3 - 352*a^3*b^2*d*e^4 + 16*a^4*b*e
^5)*x)*sqrt(e*x + d))/(a^3*b^6*d^9 - 6*a^4*b^5*d^8*e + 15*a^5*b^4*d^7*e^2
- 20*a^6*b^3*d^6*e^3 + 15*a^7*b^2*d^5*e^4 - 6*a^8*b*d^4*e^5 + a^9*d^3*e^6
+ (b^9*d^6*e^3 - 6*a*b^8*d^5*e^4 + 15*a^2*b^7*d^4*e^5 - 20*a^3*b^6*d^3*e^6
+ 15*a^4*b^5*d^2*e^7 - 6*a^5*b^4*d*e^8 + a^6*b^3*e^9)*x^6 + 3*(b^9*d^7*e^
2 - 5*a*b^8*d^6*e^3 + 9*a^2*b^7*d^5*e^4 - 5*a^3*b^6*d^4*e^5 - 5*a^4*b^5*d^
3*e^6 + 9*a^5*b^4*d^2*e^7 - 5*a^6*b^3*d*e^8 + a^7*b^2*e^9)*x^5 + 3*(b^9*d^
8*e - 3*a*b^8*d^7*e^2 - 2*a^2*b^7*d^6*e^3 + 19*a^3*b^6*d^5*e^4 - 30*a^4*b^
5*d^4*e^5 + 19*a^5*b^4*d^3*e^6 - 2*a^6*b^3*d^2*e^7 - 3*a^7*b^2*d*e^8 + a^8
*b*e^9)*x^4 + (b^9*d^9 + 3*a*b^8*d^8*e - 30*a^2*b^7*d^7*e^2 + 62*a^3*b^...
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{(d+ex)^{7/2} (a^2+2abx+b^2x^2)^2} dx = \text{Timed out}$$

input `integrate(1/(e*x+d)**(7/2)/(b**2*x**2+2*a*b*x+a**2)**2,x)`

output `Timed out`

Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{(d+ex)^{7/2} (a^2+2abx+b^2x^2)^2} dx = \text{Exception raised: ValueError}$$

input `integrate(1/(e*x+d)^(7/2)/(b^2*x^2+2*a*b*x+a^2)^2,x, algorithm="maxima")`

output `Exception raised: ValueError >> Computation failed since Maxima requested additional constraints; using the 'assume' command before evaluation *may* help (example of legal syntax is 'assume(a*e-b*d>0)', see `assume?` for more detail`

Giac [B] (verification not implemented)Leaf count of result is larger than twice the leaf count of optimal. 480 vs. $2(193) = 386$.

Time = 0.21 (sec) , antiderivative size = 480, normalized size of antiderivative = 2.10

$$\int \frac{1}{(d+ex)^{7/2} (a^2+2abx+b^2x^2)^2} dx = \frac{231 b^3 e^3 \arctan\left(\frac{\sqrt{ex+db}}{\sqrt{-b^2d+abe}}\right) - 8(b^6d^6 - 6ab^5d^5e + 15a^2b^4d^4e^2 - 20a^3b^3d^3e^3 + 15a^4b^2d^2e^4 - 6a^5bde^5 + a^6e^6)\sqrt{-b^2d+abe} - 2(150(ex+d)^2b^2e^3 + 20(ex+d)b^2de^3 + 3b^2d^2e^3 - 20(ex+d)abe^4 - 6abde^4 + 3a^2e^5) - 15(b^6d^6 - 6ab^5d^5e + 15a^2b^4d^4e^2 - 20a^3b^3d^3e^3 + 15a^4b^2d^2e^4 - 6a^5bde^5 + a^6e^6)(ex+d)^{5/2} - 213(ex+d)^{5/2}b^5e^3 - 472(ex+d)^{3/2}b^5de^3 + 267\sqrt{ex+db}b^5d^2e^3 + 472(ex+d)^{3/2}ab^4e^4 - 534\sqrt{ex+db}ab^4de^4 - 24(b^6d^6 - 6ab^5d^5e + 15a^2b^4d^4e^2 - 20a^3b^3d^3e^3 + 15a^4b^2d^2e^4 - 6a^5bde^5 + a^6e^6)((ex+d)b - b^2d + a^2e)^3}{8(b^6d^6 - 6ab^5d^5e + 15a^2b^4d^4e^2 - 20a^3b^3d^3e^3 + 15a^4b^2d^2e^4 - 6a^5bde^5 + a^6e^6)\sqrt{-b^2d+abe} - 2(150(ex+d)^2b^2e^3 + 20(ex+d)b^2de^3 + 3b^2d^2e^3 - 20(ex+d)abe^4 - 6abde^4 + 3a^2e^5) - 15(b^6d^6 - 6ab^5d^5e + 15a^2b^4d^4e^2 - 20a^3b^3d^3e^3 + 15a^4b^2d^2e^4 - 6a^5bde^5 + a^6e^6)(ex+d)^{5/2} - 213(ex+d)^{5/2}b^5e^3 - 472(ex+d)^{3/2}b^5de^3 + 267\sqrt{ex+db}b^5d^2e^3 + 472(ex+d)^{3/2}ab^4e^4 - 534\sqrt{ex+db}ab^4de^4 - 24(b^6d^6 - 6ab^5d^5e + 15a^2b^4d^4e^2 - 20a^3b^3d^3e^3 + 15a^4b^2d^2e^4 - 6a^5bde^5 + a^6e^6)((ex+d)b - b^2d + a^2e)^3}$$

input `integrate(1/(e*x+d)^(7/2)/(b^2*x^2+2*a*b*x+a^2)^2,x, algorithm="giac")`

output `-231/8*b^3*e^3*arctan(sqrt(e*x + d)*b/sqrt(-b^2*d + a*b*e))/((b^6*d^6 - 6*a*b^5*d^5*e + 15*a^2*b^4*d^4*e^2 - 20*a^3*b^3*d^3*e^3 + 15*a^4*b^2*d^2*e^4 - 6*a^5*b*d*e^5 + a^6*e^6)*sqrt(-b^2*d + a*b*e)) - 2/15*(150*(e*x + d)^2*b^2*e^3 + 20*(e*x + d)*b^2*d*e^3 + 3*b^2*d^2*e^3 - 20*(e*x + d)*a*b*e^4 - 6*a*b*d*e^4 + 3*a^2*e^5)/((b^6*d^6 - 6*a*b^5*d^5*e + 15*a^2*b^4*d^4*e^2 - 20*a^3*b^3*d^3*e^3 + 15*a^4*b^2*d^2*e^4 - 6*a^5*b*d*e^5 + a^6*e^6)*(e*x + d)^(5/2)) - 1/24*(213*(e*x + d)^(5/2)*b^5*e^3 - 472*(e*x + d)^(3/2)*b^5*d*e^3 + 267*sqrt(e*x + d)*b^5*d^2*e^3 + 472*(e*x + d)^(3/2)*a*b^4*e^4 - 534*sqrt(e*x + d)*a*b^4*d*e^4 + 267*sqrt(e*x + d)*a^2*b^3*e^5)/((b^6*d^6 - 6*a*b^5*d^5*e + 15*a^2*b^4*d^4*e^2 - 20*a^3*b^3*d^3*e^3 + 15*a^4*b^2*d^2*e^4 - 6*a^5*b*d*e^5 + a^6*e^6)*((e*x + d)*b - b*d + a*e)^3)`

Mupad [B] (verification not implemented)

Time = 9.55 (sec) , antiderivative size = 373, normalized size of antiderivative = 1.63

$$\int \frac{1}{(d+ex)^{7/2} (a^2+2abx+b^2x^2)^2} dx = \frac{\frac{2e^3}{5(ae-bd)} + \frac{66b^2e^3(d+ex)^2}{5(ae-bd)^3} + \frac{2541b^3e^3(d+ex)^3}{40(ae-bd)^4} + \frac{77b^4e^3(d+ex)^4}{(ae-bd)^5} + \frac{231b^5e^3(d+ex)^5}{8(ae-bd)^6} - (d+ex)^{5/2} (a^3e^3 - 3a^2bde^2 + 3ab^2d^2e - b^3d^3) + b^3(d+ex)^{11/2} - (3b^3d - 3ab^2e)(d+ex)^{9/2} + 231b^{5/2}e^3 \operatorname{atan}\left(\frac{\sqrt{b}\sqrt{d+ex}(a^6e^6 - 6a^5bde^5 + 15a^4b^2d^2e^4 - 20a^3b^3d^3e^3 + 15a^2b^4d^4e^2 - 6ab^5d^5e + b^6d^6)}{(ae-bd)^{13/2}}\right)}{8(ae-bd)^{13/2}}$$

input `int(1/((d + e*x)^(7/2)*(a^2 + b^2*x^2 + 2*a*b*x)^2),x)`

output
$$- \left(\frac{2e^3}{5(ae - bd)} + \frac{66b^2e^3(d + ex)^2}{5(ae - bd)^3} + \frac{2541b^3e^3(d + ex)^3}{40(ae - bd)^4} + \frac{77b^4e^3(d + ex)^4}{(ae - bd)^5} + \frac{231b^5e^3(d + ex)^5}{8(ae - bd)^6} - \frac{22b^2e^3(d + ex)}{15(ae - bd)^2} \right) / \left((d + ex)^{5/2} (a^3e^3 - b^3d^3 + 3a^2b^2d^2e - 3a^2bd^2e^2) + b^3(d + ex)^{11/2} - (3b^3d - 3a^2b^2e)(d + ex)^{9/2} + (d + ex)^{7/2} (3b^3d^2 + 3a^2b^2e^2 - 6a^2bd^2e) \right) - \left(\frac{231b^{5/2}e^3 \operatorname{atan}\left(\frac{b^{1/2}(d + ex)^{1/2}(a^6e^6 + b^6d^6 + 15a^2b^4d^4e^2 - 20a^3b^3d^3e^3 + 15a^4b^2d^2e^4 - 6a^5bd^5e - 6a^5b^2d^5e^5)}{a^6e^6 + b^6d^6 + 15a^2b^4d^4e^2 - 20a^3b^3d^3e^3 + 15a^4b^2d^2e^4 - 6a^5bd^5e - 6a^5b^2d^5e^5}\right)}{a^6e^6 + b^6d^6 + 15a^2b^4d^4e^2 - 20a^3b^3d^3e^3 + 15a^4b^2d^2e^4 - 6a^5bd^5e - 6a^5b^2d^5e^5} \right) / (8(ae - bd)^{13/2})$$

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 1814, normalized size of antiderivative = 7.92

$$\int \frac{1}{(d + ex)^{7/2} (a^2 + 2abx + b^2x^2)^2} dx = \text{Too large to display}$$

input `int(1/(e*x+d)^(7/2)/(b^2*x^2+2*a*b*x+a^2)^2,x)`

output

```
( - 3465*sqrt(b)*sqrt(d + e*x)*sqrt(a*e - b*d)*atan((sqrt(d + e*x)*b)/(sqrt(b)*sqrt(a*e - b*d)))*a**3*b**2*d**2*e**3 - 6930*sqrt(b)*sqrt(d + e*x)*sqrt(a*e - b*d)*atan((sqrt(d + e*x)*b)/(sqrt(b)*sqrt(a*e - b*d)))*a**3*b**2*d*e**4*x - 3465*sqrt(b)*sqrt(d + e*x)*sqrt(a*e - b*d)*atan((sqrt(d + e*x)*b)/(sqrt(b)*sqrt(a*e - b*d)))*a**3*b**2*e**5*x**2 - 10395*sqrt(b)*sqrt(d + e*x)*sqrt(a*e - b*d)*atan((sqrt(d + e*x)*b)/(sqrt(b)*sqrt(a*e - b*d)))*a**2*b**3*d**2*e**3*x - 20790*sqrt(b)*sqrt(d + e*x)*sqrt(a*e - b*d)*atan((sqrt(d + e*x)*b)/(sqrt(b)*sqrt(a*e - b*d)))*a**2*b**3*d*e**4*x**2 - 10395*sqrt(b)*sqrt(d + e*x)*sqrt(a*e - b*d)*atan((sqrt(d + e*x)*b)/(sqrt(b)*sqrt(a*e - b*d)))*a**2*b**3*e**5*x**3 - 10395*sqrt(b)*sqrt(d + e*x)*sqrt(a*e - b*d)*atan((sqrt(d + e*x)*b)/(sqrt(b)*sqrt(a*e - b*d)))*a*b**4*d**2*e**3*x**2 - 20790*sqrt(b)*sqrt(d + e*x)*sqrt(a*e - b*d)*atan((sqrt(d + e*x)*b)/(sqrt(b)*sqrt(a*e - b*d)))*a*b**4*d*e**4*x**3 - 10395*sqrt(b)*sqrt(d + e*x)*sqrt(a*e - b*d)*atan((sqrt(d + e*x)*b)/(sqrt(b)*sqrt(a*e - b*d)))*a*b**4*e**5*x**4 - 3465*sqrt(b)*sqrt(d + e*x)*sqrt(a*e - b*d)*atan((sqrt(d + e*x)*b)/(sqrt(b)*sqrt(a*e - b*d)))*b**5*d**2*e**3*x**3 - 6930*sqrt(b)*sqrt(d + e*x)*sqrt(a*e - b*d)*atan((sqrt(d + e*x)*b)/(sqrt(b)*sqrt(a*e - b*d)))*b**5*d*e**4*x**4 - 3465*sqrt(b)*sqrt(d + e*x)*sqrt(a*e - b*d)*atan((sqrt(d + e*x)*b)/(sqrt(b)*sqrt(a*e - b*d)))*b**5*e**5*x**5 - 48*a**6*e**6 + 464*a**5*b*d*e**5 + 176*a**5*b*e**6*x - 3184*a**4*b**2*d**2*e**4 - 4048*a**4*b*...
```


3.189
$$\int \frac{(d+ex)^{15/2}}{(a^2+2abx+b^2x^2)^3} dx$$

| | |
|---|------|
| Optimal result | 1508 |
| Mathematica [A] (verified) | 1509 |
| Rubi [A] (verified) | 1509 |
| Maple [A] (verified) | 1521 |
| Fricas [B] (verification not implemented) | 1522 |
| Sympy [F(-1)] | 1523 |
| Maxima [F(-2)] | 1524 |
| Giac [B] (verification not implemented) | 1524 |
| Mupad [B] (verification not implemented) | 1525 |
| Reduce [B] (verification not implemented) | 1526 |

Optimal result

Integrand size = 28, antiderivative size = 253

$$\int \frac{(d+ex)^{15/2}}{(a^2+2abx+b^2x^2)^3} dx = \frac{9009e^5(bd-ae)^2\sqrt{d+ex}}{128b^8} + \frac{3003e^5(bd-ae)(d+ex)^{3/2}}{128b^7} + \frac{9009e^5(d+ex)^{5/2}}{640b^6} - \frac{1287e^4(d+ex)^{7/2}}{128b^5(a+bx)} - \frac{143e^3(d+ex)^{9/2}}{64b^4(a+bx)^2} - \frac{13e^2(d+ex)^{11/2}}{16b^3(a+bx)^3} - \frac{3e(d+ex)^{13/2}}{8b^2(a+bx)^4} - \frac{(d+ex)^{15/2}}{5b(a+bx)^5} - \frac{9009e^5(bd-ae)^{5/2}\operatorname{arctanh}\left(\frac{\sqrt{b}\sqrt{d+ex}}{\sqrt{bd-ae}}\right)}{128b^{17/2}}$$

output

```
9009/128*e^5*(-a*e+b*d)^2*(e*x+d)^(1/2)/b^8+3003/128*e^5*(-a*e+b*d)*(e*x+d)^(3/2)/b^7+9009/640*e^5*(e*x+d)^(5/2)/b^6-1287/128*e^4*(e*x+d)^(7/2)/b^5/(b*x+a)-143/64*e^3*(e*x+d)^(9/2)/b^4/(b*x+a)^2-13/16*e^2*(e*x+d)^(11/2)/b^3/(b*x+a)^3-3/8*e*(e*x+d)^(13/2)/b^2/(b*x+a)^4-1/5*(e*x+d)^(15/2)/b/(b*x+a)^5-9009/128*e^5*(-a*e+b*d)^(5/2)*arctanh(b^(1/2)*(e*x+d)^(1/2)/(-a*e+b*d)^(1/2))/b^(17/2)
```

Mathematica [A] (verified)

Time = 2.36 (sec) , antiderivative size = 434, normalized size of antiderivative = 1.72

$$\int \frac{(d+ex)^{15/2}}{(a^2+2abx+b^2x^2)^3} dx =$$

$$\frac{\sqrt{d+ex}(-45045a^7e^7 + 105105a^6be^6(d-2ex) - 3003a^5b^2e^5(23d^2 - 164dex + 128e^2x^2) + 2145a^4b^3e^4(3d^3 - 152d^2e*x + 422d*e^2*x^2 - 158e^3*x^3) + 715a^3b^4e^3(2*d^4 + 4*d^3*e*x - 846*d^2*e^2*x^2 + 1124*d*e^3*x^3 - 193*e^4*x^4) + 65a^2b^5e^2(8*d^5 + 106*d^4*e*x + 938*d^3*e^2*x^2 - 8368*d^2*e^3*x^3 + 5089*d*e^4*x^4 - 256*e^5*x^5) + 5a*b^6e*(48*d^6 + 496*d^5*e*x + 2618*d^4*e^2*x^2 + 11620*d^3*e^3*x^3 - 45677*d^2*e^4*x^4 + 8192*d*e^5*x^5 + 256*e^6*x^6) + b^7*(128*d^7 + 1136*d^6*e*x + 4648*d^5*e^2*x^2 + 12110*d^4*e^3*x^3 + 26635*d^3*e^4*x^4 - 29696*d^2*e^5*x^5 - 3072*d*e^6*x^6 - 256*e^7*x^7))}{128b^{17/2}} \arctan\left(\frac{\sqrt{b}\sqrt{d+ex}}{\sqrt{-bd+ae}}\right)$$

input

```
Integrate[(d + e*x)^(15/2)/(a^2 + 2*a*b*x + b^2*x^2)^3,x]
```

output

```
-1/640*(Sqrt[d + e*x]*(-45045*a^7*e^7 + 105105*a^6*b*e^6*(d - 2*e*x) - 3003*a^5*b^2*e^5*(23*d^2 - 164*d*e*x + 128*e^2*x^2) + 2145*a^4*b^3*e^4*(3*d^3 - 152*d^2*e*x + 422*d*e^2*x^2 - 158*e^3*x^3) + 715*a^3*b^4*e^3*(2*d^4 + 4*d^3*e*x - 846*d^2*e^2*x^2 + 1124*d*e^3*x^3 - 193*e^4*x^4) + 65*a^2*b^5*e^2*(8*d^5 + 106*d^4*e*x + 938*d^3*e^2*x^2 - 8368*d^2*e^3*x^3 + 5089*d*e^4*x^4 - 256*e^5*x^5) + 5*a*b^6*e*(48*d^6 + 496*d^5*e*x + 2618*d^4*e^2*x^2 + 11620*d^3*e^3*x^3 - 45677*d^2*e^4*x^4 + 8192*d*e^5*x^5 + 256*e^6*x^6) + b^7*(128*d^7 + 1136*d^6*e*x + 4648*d^5*e^2*x^2 + 12110*d^4*e^3*x^3 + 26635*d^3*e^4*x^4 - 29696*d^2*e^5*x^5 - 3072*d*e^6*x^6 - 256*e^7*x^7)))/(b^8*(a + b*x)^5 - (9009*e^5*(-(b*d) + a*e)^(5/2)*ArcTan[(Sqrt[b]*Sqrt[d + e*x])/Sqrt[-(b*d) + a*e]])/(128*b^(17/2))
```

Rubi [A] (verified)

Time = 0.62 (sec) , antiderivative size = 278, normalized size of antiderivative = 1.10, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$, Rules used = {1098, 27, 51, 51, 51, 51, 60, 60, 60, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex)^{15/2}}{(a^2+2abx+b^2x^2)^3} dx$$

$$\begin{aligned}
& \downarrow 1098 \\
& b^6 \int \frac{(d+ex)^{15/2}}{b^6(a+bx)^6} dx \\
& \downarrow 27 \\
& \int \frac{(d+ex)^{15/2}}{(a+bx)^6} dx \\
& \downarrow 51 \\
& \frac{3e \int \frac{(d+ex)^{13/2}}{(a+bx)^5} dx}{2b} - \frac{(d+ex)^{15/2}}{5b(a+bx)^5} \\
& \downarrow 51 \\
& \frac{3e \left(\frac{13e \int \frac{(d+ex)^{11/2}}{(a+bx)^4} dx}{8b} - \frac{(d+ex)^{13/2}}{4b(a+bx)^4} \right)}{2b} - \frac{(d+ex)^{15/2}}{5b(a+bx)^5} \\
& \downarrow 51 \\
& \frac{3e \left(\frac{13e \left(\frac{11e \int \frac{(d+ex)^{9/2}}{(a+bx)^3} dx}{6b} - \frac{(d+ex)^{11/2}}{3b(a+bx)^3} \right)}{8b} - \frac{(d+ex)^{13/2}}{4b(a+bx)^4} \right)}{2b} - \frac{(d+ex)^{15/2}}{5b(a+bx)^5} \\
& \downarrow 51 \\
& \frac{3e \left(\frac{13e \left(\frac{11e \left(\frac{9e \int \frac{(d+ex)^{7/2}}{(a+bx)^2} dx}{4b} - \frac{(d+ex)^{9/2}}{2b(a+bx)^2} \right)}{6b} - \frac{(d+ex)^{11/2}}{3b(a+bx)^3} \right)}{8b} - \frac{(d+ex)^{13/2}}{4b(a+bx)^4} \right)}{2b} - \frac{(d+ex)^{15/2}}{5b(a+bx)^5} \\
& \downarrow 51 \\
& \frac{3e \left(\frac{13e \left(\frac{11e \left(\frac{9e \int \frac{(d+ex)^{5/2}}{(a+bx)} dx}{2b} - \frac{(d+ex)^{7/2}}{b(a+bx)^2} \right)}{6b} - \frac{(d+ex)^{9/2}}{2b(a+bx)^2} - \frac{(d+ex)^{11/2}}{3b(a+bx)^3} \right)}{8b} - \frac{(d+ex)^{13/2}}{4b(a+bx)^4} \right)}{2b} - \frac{(d+ex)^{15/2}}{5b(a+bx)^5}
\end{aligned}$$

$$\left(\frac{3e}{2b} \left(\frac{13e}{8b} \left(\frac{11e}{6b} \left(\frac{9e}{4b} \left(\frac{7e \int \frac{(d+ex)^{5/2}}{a+bx} dx}{2b} - \frac{(d+ex)^{7/2}}{b(a+bx)} \right) - \frac{(d+ex)^{9/2}}{2b(a+bx)^2} \right) - \frac{(d+ex)^{11/2}}{3b(a+bx)^3} \right) - \frac{(d+ex)^{13/2}}{4b(a+bx)^4} \right) - \frac{(d+ex)^{15/2}}{5b(a+bx)^5} \right)$$

$2b$

↓ 60

$$\left(\begin{array}{l}
 \left(\begin{array}{l}
 \left(\begin{array}{l}
 7e \left(\frac{(bd-ae) \int \frac{(d+ex)^{3/2}}{a+bx} dx + 2(d+ex)^{5/2}}{5b} \right) \\
 \frac{(d+ex)^{7/2}}{b(a+bx)}
 \end{array} \right) \\
 \frac{9e}{2b}
 \end{array} \right) \\
 \frac{(d+ex)^{9/2}}{2b(a+bx)^2} \\
 \left(\begin{array}{l}
 \frac{11e}{4b} \\
 \frac{13e}{6b} \\
 \frac{15e}{8b}
 \end{array} \right) \\
 \frac{(d+ex)^{11/2}}{3b(a+bx)^3} \\
 \frac{(d+ex)^{13/2}}{4b(a+bx)^4}
 \end{array} \right)$$

$$\frac{2b}{(d+ex)^{15/2}} \\
 \frac{5b(a+bx)^5}$$

↓ 60

$$\begin{aligned}
 & \left(\frac{7e}{9e} \left(\frac{(bd-ae) \int \frac{\sqrt{d+ex}}{a+bx} dx + \frac{2(d+ex)^{3/2}}{3b}}{b} + \frac{2(d+ex)^{5/2}}{5b} \right) - \frac{(d+ex)^{7/2}}{b(a+bx)} \right) \\
 & \frac{11e}{4b} \left(\dots \right) - \frac{(d+ex)^{9/2}}{2b(a+bx)^2} \\
 & \frac{13e}{6b} \left(\dots \right) - \frac{(d+ex)^{11/2}}{3b(a+bx)^3} \\
 & \frac{3e}{8b} \left(\dots \right) - \frac{(d+ex)^{13/2}}{4b(a+bx)^4}
 \end{aligned}$$

↓ 60

$$\begin{aligned}
 & \left(\frac{(bd-ae) \left(\frac{(bd-ae) \int \frac{1}{(a+bx)\sqrt{d+ex}} dx + 2\sqrt{d+ex}}{b} \right) + \frac{2(d+ex)^{3/2}}{3b}}{b} \right) \\
 7e & \frac{\left(\frac{(bd-ae) \left(\frac{(bd-ae) \int \frac{1}{(a+bx)\sqrt{d+ex}} dx + 2\sqrt{d+ex}}{b} \right) + \frac{2(d+ex)^{3/2}}{3b}}{b} \right) + \frac{2(d+ex)^{5/2}}{5b}}{b} \\
 9e & \frac{\left(\frac{(bd-ae) \left(\frac{(bd-ae) \int \frac{1}{(a+bx)\sqrt{d+ex}} dx + 2\sqrt{d+ex}}{b} \right) + \frac{2(d+ex)^{3/2}}{3b}}{b} \right) + \frac{2(d+ex)^{5/2}}{5b}}{2b} - \frac{(d+ex)^{7/2}}{b(a+bx)} \\
 11e & \frac{\left(\frac{(bd-ae) \left(\frac{(bd-ae) \int \frac{1}{(a+bx)\sqrt{d+ex}} dx + 2\sqrt{d+ex}}{b} \right) + \frac{2(d+ex)^{3/2}}{3b}}{b} \right) + \frac{2(d+ex)^{5/2}}{5b}}{4b} - \frac{(d+ex)^{9/2}}{2b(a+bx)^2} \\
 13e & \frac{\left(\frac{(bd-ae) \left(\frac{(bd-ae) \int \frac{1}{(a+bx)\sqrt{d+ex}} dx + 2\sqrt{d+ex}}{b} \right) + \frac{2(d+ex)^{3/2}}{3b}}{b} \right) + \frac{2(d+ex)^{5/2}}{5b}}{6b} - \frac{(d+ex)^{11/2}}{3b(a+bx)^3}
 \end{aligned}$$

↓ 73

$$\left(\frac{(bd-ae) \left(\frac{2(bd-ae) \int \frac{1}{a + \frac{b(d+ex)}{e} - \frac{bd}{e}} d\sqrt{d+ex} \right)}{b} + \frac{2(d+ex)^{3/2}}{3b} \right) + \frac{2(d+ex)^{5/2}}{5b}$$

$$\frac{7e}{b} \left(\frac{(bd-ae) \left(\frac{2(bd-ae) \int \frac{1}{a + \frac{b(d+ex)}{e} - \frac{bd}{e}} d\sqrt{d+ex} \right)}{b} + \frac{2(d+ex)^{3/2}}{3b} \right) + \frac{2(d+ex)^{5/2}}{5b}$$

$$\frac{9e}{2b} \left(\frac{(bd-ae) \left(\frac{2(bd-ae) \int \frac{1}{a + \frac{b(d+ex)}{e} - \frac{bd}{e}} d\sqrt{d+ex} \right)}{b} + \frac{2(d+ex)^{3/2}}{3b} \right) + \frac{2(d+ex)^{5/2}}{5b} - \frac{(d+ex)^{7/2}}{b(a+bx)}$$

$$\frac{11e}{4b} \left(\frac{(bd-ae) \left(\frac{2(bd-ae) \int \frac{1}{a + \frac{b(d+ex)}{e} - \frac{bd}{e}} d\sqrt{d+ex} \right)}{b} + \frac{2(d+ex)^{3/2}}{3b} \right) + \frac{2(d+ex)^{5/2}}{5b} - \frac{(d+ex)^{7/2}}{b(a+bx)} - \frac{(d+ex)^{9/2}}{2b(a+bx)^2}$$

$$\frac{13e}{6b} \left(\frac{(bd-ae) \left(\frac{2(bd-ae) \int \frac{1}{a + \frac{b(d+ex)}{e} - \frac{bd}{e}} d\sqrt{d+ex} \right)}{b} + \frac{2(d+ex)^{3/2}}{3b} \right) + \frac{2(d+ex)^{5/2}}{5b} - \frac{(d+ex)^{7/2}}{b(a+bx)} - \frac{(d+ex)^{9/2}}{2b(a+bx)^2}$$

↓ 221

$$\begin{aligned}
 & \left(\frac{(bd-ae) \left(\frac{2\sqrt{d+ex}}{b} - \frac{2\sqrt{bd-ae} \operatorname{arctanh} \left(\frac{\sqrt{b}\sqrt{d+ex}}{\sqrt{bd-ae}} \right)}{b^{3/2}} \right)}{b} + \frac{2(d+ex)^{3/2}}{3b} \right) \\
 7e & \frac{\left(\frac{(bd-ae) \left(\frac{2\sqrt{d+ex}}{b} - \frac{2\sqrt{bd-ae} \operatorname{arctanh} \left(\frac{\sqrt{b}\sqrt{d+ex}}{\sqrt{bd-ae}} \right)}{b^{3/2}} \right)}{b} + \frac{2(d+ex)^{3/2}}{3b} \right)}{b} + \frac{2(d+ex)^{5/2}}{5b} \\
 9e & \frac{\left(\frac{(bd-ae) \left(\frac{2\sqrt{d+ex}}{b} - \frac{2\sqrt{bd-ae} \operatorname{arctanh} \left(\frac{\sqrt{b}\sqrt{d+ex}}{\sqrt{bd-ae}} \right)}{b^{3/2}} \right)}{b} + \frac{2(d+ex)^{3/2}}{3b} \right)}{2b} - \frac{(d+ex)^{7/2}}{b(a+bx)} \\
 11e & \frac{\left(\frac{(bd-ae) \left(\frac{2\sqrt{d+ex}}{b} - \frac{2\sqrt{bd-ae} \operatorname{arctanh} \left(\frac{\sqrt{b}\sqrt{d+ex}}{\sqrt{bd-ae}} \right)}{b^{3/2}} \right)}{4b} \right)}{4b} - \frac{(d+ex)^{9/2}}{2b(a+bx)^2} \\
 13e & \frac{\left(\frac{(bd-ae) \left(\frac{2\sqrt{d+ex}}{b} - \frac{2\sqrt{bd-ae} \operatorname{arctanh} \left(\frac{\sqrt{b}\sqrt{d+ex}}{\sqrt{bd-ae}} \right)}{b^{3/2}} \right)}{6b} \right)}{6b}
 \end{aligned}$$

input `Int[(d + e*x)^(15/2)/(a^2 + 2*a*b*x + b^2*x^2)^3,x]`

output `-1/5*(d + e*x)^(15/2)/(b*(a + b*x)^5) + (3*e*(-1/4*(d + e*x)^(13/2)/(b*(a + b*x)^4) + (13*e*(-1/3*(d + e*x)^(11/2)/(b*(a + b*x)^3) + (11*e*(-1/2*(d + e*x)^(9/2)/(b*(a + b*x)^2) + (9*e*(-((d + e*x)^(7/2)/(b*(a + b*x))) + (7*e*((2*(d + e*x)^(5/2))/(5*b) + ((b*d - a*e)*((2*(d + e*x)^(3/2))/(3*b) + ((b*d - a*e)*((2*Sqrt[d + e*x])/b - (2*Sqrt[b*d - a*e]*ArcTanh[(Sqrt[b]*Sqrt[d + e*x])/Sqrt[b*d - a*e]]))/b^(3/2)))/b))/b))/(2*b)))/(4*b)))/(6*b)))/(8*b)))/(2*b)`

Defintions of rubi rules used

rule 27 `Int[(a_)*(F_x_), x_Symbol] := Simp[a Int[F_x, x], x] /; FreeQ[a, x] && !MatchQ[F_x, (b_)*(G_x_)] /; FreeQ[b, x]`

rule 51 `Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] := Simp[(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Simp[d*(n/(b*(m + 1))) Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d, n}, x] && ILtQ[m, -1] && FractionQ[n] && GtQ[n, 0]`

rule 60 `Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] := Simp[(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*(b*c - a*d)/(b*(m + n + 1)) Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] && !(IGtQ[m, 0] && (!IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) && !ILtQ[m + n + 2, 0] && IntLinearQ[a, b, c, d, m, n, x]`

rule 73 `Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] := With[{p = Denominator[m]}, Simp[p/b Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) + d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x] /; FreeQ[{a, b, c, d}, x] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntLinearQ[a, b, c, d, m, n, x]`

```
rule 221 Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[-a/b, 2]/a)*ArcTanh[x/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

```
rule 1098 Int[((d_.) + (e_.)*(x_)^(m_.))*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] := Simp[1/c^p Int[(d + e*x)^m*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Maple [A] (verified)

Time = 3.84 (sec) , antiderivative size = 342, normalized size of antiderivative = 1.35

| method | result |
|--------------------|--|
| risch | $\frac{2e^5(b^2e^2x^2 - 10xab e^2 + 12b^2dex + 105e^2a^2 - 220abde + 116b^2d^2)\sqrt{ex+d}}{5b^8} - \frac{(2e^3a^3 - 6a^2bd e^2 + 6ab^2d^2e - 2b^3d^3)e^5}{b^8}$ |
| pseudoelliptic | $- \frac{9009 \left(e^5 (bx+a)^5 (ae-bd)^3 \arctan\left(\frac{b\sqrt{ex+d}}{\sqrt{b(ae-bd)}}\right) - \left(\frac{1024}{15015} d e^6 x^6 + \frac{29696}{45045} d^2 e^5 x^5 - \frac{761}{1287} d^3 e^4 x^4 - \frac{346}{1287} d^4 e^3 x^3 - \frac{664}{6435} d^5 e^2 x^2 \right) \right)}{b^8}$ |
| derivativeldivides | $2e^5 \left(\frac{\frac{(ex+d)^{\frac{5}{2}} b^2}{5} - 2abe(ex+d)^{\frac{3}{2}} + 2b^2d(ex+d)^{\frac{3}{2}} + 21a^2e^2\sqrt{ex+d} - 42abde\sqrt{ex+d} + 21b^2d^2\sqrt{ex+d}}{b^8} - \frac{\left(-\frac{5327}{256} a^3 b^4 e^3\right)}{b^8} \right)$ |
| default | $2e^5 \left(\frac{\frac{(ex+d)^{\frac{5}{2}} b^2}{5} - 2abe(ex+d)^{\frac{3}{2}} + 2b^2d(ex+d)^{\frac{3}{2}} + 21a^2e^2\sqrt{ex+d} - 42abde\sqrt{ex+d} + 21b^2d^2\sqrt{ex+d}}{b^8} - \frac{\left(-\frac{5327}{256} a^3 b^4 e^3\right)}{b^8} \right)$ |

```
input int((e*x+d)^(15/2)/(b^2*x^2+2*a*b*x+a^2)^3,x,method=_RETURNVERBOSE)
```

output

```

2/5*e^5*(b^2*e^2*x^2-10*a*b*e^2*x+12*b^2*d*e*x+105*a^2*e^2-220*a*b*d*e+116
*b^2*d^2)*(e*x+d)^(1/2)/b^8-1/b^8*(2*a^3*e^3-6*a^2*b*d*e^2+6*a*b^2*d^2*e-2
*b^3*d^3)*e^5*((-5327/256*b^4*(e*x+d)^(9/2)-9443/128*b^3*(a*e-b*d)*(e*x+d)
^(7/2)+(-1001/10*a^2*b^2*e^2+1001/5*a*b^3*d*e-1001/10*b^4*d^2)*(e*x+d)^(5/
2)+(-7837/128*a^3*b*e^3+23511/128*a^2*b^2*d*e^2-23511/128*d^2*e*a*b^3+7837
/128*b^4*d^3)*(e*x+d)^(3/2)+(-3633/256*a^4*e^4+3633/64*a^3*b*d*e^3-10899/1
28*a^2*b^2*d^2*e^2+3633/64*a*b^3*d^3*e-3633/256*b^4*d^4)*(e*x+d)^(1/2))/(b
*(e*x+d)+a*e-b*d)^5+9009/256/(b*(a*e-b*d))^(1/2)*arctan(b*(e*x+d)^(1/2)/(b
*(a*e-b*d))^(1/2))

```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 798 vs. 2(209) = 418.

Time = 0.17 (sec) , antiderivative size = 1606, normalized size of antiderivative = 6.35

$$\int \frac{(d+ex)^{15/2}}{(a^2+2abx+b^2x^2)^3} dx = \text{Too large to display}$$

input

```

integrate((e*x+d)^(15/2)/(b^2*x^2+2*a*b*x+a^2)^3,x, algorithm="fricas")

```

output

```
[1/1280*(45045*(a^5*b^2*d^2*e^5 - 2*a^6*b*d*e^6 + a^7*e^7 + (b^7*d^2*e^5 -
2*a*b^6*d*e^6 + a^2*b^5*e^7)*x^5 + 5*(a*b^6*d^2*e^5 - 2*a^2*b^5*d*e^6 + a
^3*b^4*e^7)*x^4 + 10*(a^2*b^5*d^2*e^5 - 2*a^3*b^4*d*e^6 + a^4*b^3*e^7)*x^3
+ 10*(a^3*b^4*d^2*e^5 - 2*a^4*b^3*d*e^6 + a^5*b^2*e^7)*x^2 + 5*(a^4*b^3*d
^2*e^5 - 2*a^5*b^2*d*e^6 + a^6*b*e^7)*x)*sqrt((b*d - a*e)/b)*log((b*e*x +
2*b*d - a*e - 2*sqrt(e*x + d)*b*sqrt((b*d - a*e)/b))/(b*x + a)) + 2*(256*b
^7*e^7*x^7 - 128*b^7*d^7 - 240*a*b^6*d^6*e - 520*a^2*b^5*d^5*e^2 - 1430*a^
3*b^4*d^4*e^3 - 6435*a^4*b^3*d^3*e^4 + 69069*a^5*b^2*d^2*e^5 - 105105*a^6*
b*d*e^6 + 45045*a^7*e^7 + 256*(12*b^7*d*e^6 - 5*a*b^6*e^7)*x^6 + 256*(116*
b^7*d^2*e^5 - 160*a*b^6*d*e^6 + 65*a^2*b^5*e^7)*x^5 - 5*(5327*b^7*d^3*e^4
- 45677*a*b^6*d^2*e^5 + 66157*a^2*b^5*d*e^6 - 27599*a^3*b^4*e^7)*x^4 - 10*
(1211*b^7*d^4*e^3 + 5810*a*b^6*d^3*e^4 - 54392*a^2*b^5*d^2*e^5 + 80366*a^3
*b^4*d*e^6 - 33891*a^4*b^3*e^7)*x^3 - 2*(2324*b^7*d^5*e^2 + 6545*a*b^6*d^4
*e^3 + 30485*a^2*b^5*d^3*e^4 - 302445*a^3*b^4*d^2*e^5 + 452595*a^4*b^3*d*e
^6 - 192192*a^5*b^2*e^7)*x^2 - 2*(568*b^7*d^6*e + 1240*a*b^6*d^5*e^2 + 344
5*a^2*b^5*d^4*e^3 + 15730*a^3*b^4*d^3*e^4 - 163020*a^4*b^3*d^2*e^5 + 24624
6*a^5*b^2*d*e^6 - 105105*a^6*b*e^7)*x)*sqrt(e*x + d))/(b^13*x^5 + 5*a*b^12
*x^4 + 10*a^2*b^11*x^3 + 10*a^3*b^10*x^2 + 5*a^4*b^9*x + a^5*b^8), -1/640*
(45045*(a^5*b^2*d^2*e^5 - 2*a^6*b*d*e^6 + a^7*e^7 + (b^7*d^2*e^5 - 2*a*b^6
*d*e^6 + a^2*b^5*e^7)*x^5 + 5*(a*b^6*d^2*e^5 - 2*a^2*b^5*d*e^6 + a^3*b^...
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(d + ex)^{15/2}}{(a^2 + 2abx + b^2x^2)^3} dx = \text{Timed out}$$

input

```
integrate((e*x+d)**(15/2)/(b**2*x**2+2*a*b*x+a**2)**3,x)
```

output

Timed out

Maxima [F(-2)]

Exception generated.

$$\int \frac{(d + ex)^{15/2}}{(a^2 + 2abx + b^2x^2)^3} dx = \text{Exception raised: ValueError}$$

input `integrate((e*x+d)^(15/2)/(b^2*x^2+2*a*b*x+a^2)^3,x, algorithm="maxima")`

output `Exception raised: ValueError >> Computation failed since Maxima requested additional constraints; using the 'assume' command before evaluation *may* help (example of legal syntax is 'assume(a*e-b*d>0)', see `assume?` for more detail`

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 784 vs. 2(209) = 418.

Time = 0.22 (sec) , antiderivative size = 784, normalized size of antiderivative = 3.10

$$\int \frac{(d + ex)^{15/2}}{(a^2 + 2abx + b^2x^2)^3} dx = \text{Too large to display}$$

input `integrate((e*x+d)^(15/2)/(b^2*x^2+2*a*b*x+a^2)^3,x, algorithm="giac")`

output

```

9009/128*(b^3*d^3*e^5 - 3*a*b^2*d^2*e^6 + 3*a^2*b*d*e^7 - a^3*e^8)*arctan(
sqrt(e*x + d)*b/sqrt(-b^2*d + a*b*e))/(sqrt(-b^2*d + a*b*e)*b^8) - 1/640*(
26635*(e*x + d)^(9/2)*b^7*d^3*e^5 - 94430*(e*x + d)^(7/2)*b^7*d^4*e^5 + 12
8128*(e*x + d)^(5/2)*b^7*d^5*e^5 - 78370*(e*x + d)^(3/2)*b^7*d^6*e^5 + 181
65*sqrt(e*x + d)*b^7*d^7*e^5 - 79905*(e*x + d)^(9/2)*a*b^6*d^2*e^6 + 37772
0*(e*x + d)^(7/2)*a*b^6*d^3*e^6 - 640640*(e*x + d)^(5/2)*a*b^6*d^4*e^6 + 4
70220*(e*x + d)^(3/2)*a*b^6*d^5*e^6 - 127155*sqrt(e*x + d)*a*b^6*d^6*e^6 +
79905*(e*x + d)^(9/2)*a^2*b^5*d*e^7 - 566580*(e*x + d)^(7/2)*a^2*b^5*d^2*
e^7 + 1281280*(e*x + d)^(5/2)*a^2*b^5*d^3*e^7 - 1175550*(e*x + d)^(3/2)*a^
2*b^5*d^4*e^7 + 381465*sqrt(e*x + d)*a^2*b^5*d^5*e^7 - 26635*(e*x + d)^(9/
2)*a^3*b^4*e^8 + 377720*(e*x + d)^(7/2)*a^3*b^4*d*e^8 - 1281280*(e*x + d)^(
5/2)*a^3*b^4*d^2*e^8 + 1567400*(e*x + d)^(3/2)*a^3*b^4*d^3*e^8 - 635775*s
qrt(e*x + d)*a^3*b^4*d^4*e^8 - 94430*(e*x + d)^(7/2)*a^4*b^3*e^9 + 640640*
(e*x + d)^(5/2)*a^4*b^3*d*e^9 - 1175550*(e*x + d)^(3/2)*a^4*b^3*d^2*e^9 +
635775*sqrt(e*x + d)*a^4*b^3*d^3*e^9 - 128128*(e*x + d)^(5/2)*a^5*b^2*e^10
+ 470220*(e*x + d)^(3/2)*a^5*b^2*d*e^10 - 381465*sqrt(e*x + d)*a^5*b^2*d^
2*e^10 - 78370*(e*x + d)^(3/2)*a^6*b*e^11 + 127155*sqrt(e*x + d)*a^6*b*d*e
^11 - 18165*sqrt(e*x + d)*a^7*e^12)/(((e*x + d)*b - b*d + a*e)^5*b^8) + 2/
5*((e*x + d)^(5/2)*b^24*e^5 + 10*(e*x + d)^(3/2)*b^24*d*e^5 + 105*sqrt(e*x
+ d)*b^24*d^2*e^5 - 10*(e*x + d)^(3/2)*a*b^23*e^6 - 210*sqrt(e*x + d)*...

```

Mupad [B] (verification not implemented)

Time = 9.29 (sec) , antiderivative size = 846, normalized size of antiderivative = 3.34

$$\int \frac{(d + ex)^{15/2}}{(a^2 + 2abx + b^2x^2)^3} dx = \text{Too large to display}$$

input

```
int((d + e*x)^(15/2)/(a^2 + b^2*x^2 + 2*a*b*x)^3,x)
```

output

```

((2*e^5*(6*b^6*d - 6*a*b^5*e)^2)/b^18 - (30*e^5*(a*e - b*d)^2)/b^8)*(d + e
*x)^(1/2) + ((d + e*x)^(1/2))*((3633*a^7*e^12)/128 - (3633*b^7*d^7*e^5)/128
+ (25431*a*b^6*d^6*e^6)/128 - (76293*a^2*b^5*d^5*e^7)/128 + (127155*a^3*b
^4*d^4*e^8)/128 - (127155*a^4*b^3*d^3*e^9)/128 + (76293*a^5*b^2*d^2*e^10)/
128 - (25431*a^6*b*d*e^11)/128) + (d + e*x)^(5/2)*((1001*a^5*b^2*e^10)/5 -
(1001*b^7*d^5*e^5)/5 + 1001*a*b^6*d^4*e^6 - 1001*a^4*b^3*d*e^9 - 2002*a^2
*b^5*d^3*e^7 + 2002*a^3*b^4*d^2*e^8) + (d + e*x)^(3/2)*((7837*a^6*b*e^11)/
64 + (7837*b^7*d^6*e^5)/64 - (23511*a*b^6*d^5*e^6)/32 - (23511*a^5*b^2*d*e
^10)/32 + (117555*a^2*b^5*d^4*e^7)/64 - (39185*a^3*b^4*d^3*e^8)/16 + (1175
55*a^4*b^3*d^2*e^9)/64) + (d + e*x)^(9/2)*((5327*a^3*b^4*e^8)/128 - (5327*
b^7*d^3*e^5)/128 + (15981*a*b^6*d^2*e^6)/128 - (15981*a^2*b^5*d*e^7)/128)
+ (d + e*x)^(7/2)*((9443*a^4*b^3*e^9)/64 + (9443*b^7*d^4*e^5)/64 - (9443*a
*b^6*d^3*e^6)/16 - (9443*a^3*b^4*d*e^8)/16 + (28329*a^2*b^5*d^2*e^7)/32))/
((d + e*x)*(5*b^13*d^4 + 5*a^4*b^9*e^4 - 20*a^3*b^10*d*e^3 + 30*a^2*b^11*d
^2*e^2 - 20*a*b^12*d^3*e) - (d + e*x)^2*(10*b^13*d^3 - 10*a^3*b^10*e^3 + 3
0*a^2*b^11*d*e^2 - 30*a*b^12*d^2*e) + b^13*(d + e*x)^5 - (5*b^13*d - 5*a*b
^12*e)*(d + e*x)^4 - b^13*d^5 + (d + e*x)^3*(10*b^13*d^2 + 10*a^2*b^11*e^2
- 20*a*b^12*d*e) + a^5*b^8*e^5 - 5*a^4*b^9*d*e^4 - 10*a^2*b^11*d^3*e^2 +
10*a^3*b^10*d^2*e^3 + 5*a*b^12*d^4*e) + (2*e^5*(d + e*x)^(5/2))/(5*b^6) +
(2*e^5*(6*b^6*d - 6*a*b^5*e)*(d + e*x)^(3/2))/(3*b^12) - (9009*e^5*atan...

```

Reduce [B] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 1639, normalized size of antiderivative = 6.48

$$\int \frac{(d + ex)^{15/2}}{(a^2 + 2abx + b^2x^2)^3} dx = \text{Too large to display}$$

input

```
int((e*x+d)^(15/2)/(b^2*x^2+2*a*b*x+a^2)^3,x)
```

output

```
( - 45045*sqrt(b)*sqrt(a*e - b*d)*atan((sqrt(d + e*x)*b)/(sqrt(b)*sqrt(a*e
- b*d)))*a**7*e**7 + 90090*sqrt(b)*sqrt(a*e - b*d)*atan((sqrt(d + e*x)*b)
/(sqrt(b)*sqrt(a*e - b*d)))*a**6*b*d*e**6 - 225225*sqrt(b)*sqrt(a*e - b*d)
*atan((sqrt(d + e*x)*b)/(sqrt(b)*sqrt(a*e - b*d)))*a**6*b*e**7*x - 45045*s
qrt(b)*sqrt(a*e - b*d)*atan((sqrt(d + e*x)*b)/(sqrt(b)*sqrt(a*e - b*d)))*a
**5*b**2*d**2*e**5 + 450450*sqrt(b)*sqrt(a*e - b*d)*atan((sqrt(d + e*x)*b)
/(sqrt(b)*sqrt(a*e - b*d)))*a**5*b**2*d*e**6*x - 450450*sqrt(b)*sqrt(a*e -
b*d)*atan((sqrt(d + e*x)*b)/(sqrt(b)*sqrt(a*e - b*d)))*a**5*b**2*e**7*x**
2 - 225225*sqrt(b)*sqrt(a*e - b*d)*atan((sqrt(d + e*x)*b)/(sqrt(b)*sqrt(a*
e - b*d)))*a**4*b**3*d**2*e**5*x + 900900*sqrt(b)*sqrt(a*e - b*d)*atan((sq
rt(d + e*x)*b)/(sqrt(b)*sqrt(a*e - b*d)))*a**4*b**3*d*e**6*x**2 - 450450*s
qrt(b)*sqrt(a*e - b*d)*atan((sqrt(d + e*x)*b)/(sqrt(b)*sqrt(a*e - b*d)))*a
**4*b**3*e**7*x**3 - 450450*sqrt(b)*sqrt(a*e - b*d)*atan((sqrt(d + e*x)*b)
/(sqrt(b)*sqrt(a*e - b*d)))*a**3*b**4*d**2*e**5*x**2 + 900900*sqrt(b)*sqrt
(a*e - b*d)*atan((sqrt(d + e*x)*b)/(sqrt(b)*sqrt(a*e - b*d)))*a**3*b**4*d*
e**6*x**3 - 225225*sqrt(b)*sqrt(a*e - b*d)*atan((sqrt(d + e*x)*b)/(sqrt(b)
*sqrt(a*e - b*d)))*a**3*b**4*e**7*x**4 - 450450*sqrt(b)*sqrt(a*e - b*d)*at
an((sqrt(d + e*x)*b)/(sqrt(b)*sqrt(a*e - b*d)))*a**2*b**5*d**2*e**5*x**3 +
450450*sqrt(b)*sqrt(a*e - b*d)*atan((sqrt(d + e*x)*b)/(sqrt(b)*sqrt(a*e -
b*d)))*a**2*b**5*d*e**6*x**4 - 45045*sqrt(b)*sqrt(a*e - b*d)*atan((sqr...
```

3.190 $\int \frac{(d+ex)^{13/2}}{(a^2+2abx+b^2x^2)^3} dx$

| | |
|---|------|
| Optimal result | 1528 |
| Mathematica [A] (verified) | 1529 |
| Rubi [A] (verified) | 1529 |
| Maple [A] (verified) | 1539 |
| Fricas [B] (verification not implemented) | 1540 |
| Sympy [F(-1)] | 1541 |
| Maxima [F(-2)] | 1542 |
| Giac [B] (verification not implemented) | 1542 |
| Mupad [B] (verification not implemented) | 1543 |
| Reduce [B] (verification not implemented) | 1544 |

Optimal result

Integrand size = 28, antiderivative size = 224

$$\int \frac{(d+ex)^{13/2}}{(a^2+2abx+b^2x^2)^3} dx = \frac{3003e^5(bd-ae)\sqrt{d+ex}}{128b^7} + \frac{1001e^5(d+ex)^{3/2}}{128b^6} - \frac{3003e^4(d+ex)^{5/2}}{640b^5(a+bx)} - \frac{429e^3(d+ex)^{7/2}}{320b^4(a+bx)^2} - \frac{143e^2(d+ex)^{9/2}}{240b^3(a+bx)^3} - \frac{13e(d+ex)^{11/2}}{40b^2(a+bx)^4} - \frac{(d+ex)^{13/2}}{5b(a+bx)^5} - \frac{3003e^5(bd-ae)^{3/2}\operatorname{arctanh}\left(\frac{\sqrt{b}\sqrt{d+ex}}{\sqrt{bd-ae}}\right)}{128b^{15/2}}$$

output

```
3003/128*e^5*(-a*e+b*d)*(e*x+d)^(1/2)/b^7+1001/128*e^5*(e*x+d)^(3/2)/b^6-3003/640*e^4*(e*x+d)^(5/2)/b^5/(b*x+a)-429/320*e^3*(e*x+d)^(7/2)/b^4/(b*x+a)^2-143/240*e^2*(e*x+d)^(9/2)/b^3/(b*x+a)^3-13/40*e*(e*x+d)^(11/2)/b^2/(b*x+a)^4-1/5*(e*x+d)^(13/2)/b/(b*x+a)^5-3003/128*e^5*(-a*e+b*d)^(3/2)*arctanh(b^(1/2)*(e*x+d)^(1/2)/(-a*e+b*d)^(1/2))/b^(15/2)
```

Mathematica [A] (verified)

Time = 2.17 (sec) , antiderivative size = 351, normalized size of antiderivative = 1.57

$$\int \frac{(d+ex)^{13/2}}{(a^2+2abx+b^2x^2)^3} dx =$$

$$\frac{\sqrt{d+ex}(45045a^6e^6 + 30030a^5be^5(-2d+7ex) + 3003a^4b^2e^4(3d^2 - 94dex + 128e^2x^2) + 858a^3b^3e^3(3d^3 +$$

$$+ \frac{3003e^5(-bd+ae)^{3/2} \arctan\left(\frac{\sqrt{b}\sqrt{d+ex}}{\sqrt{-bd+ae}}\right)}{128b^{15/2}}$$

input

```
Integrate[(d + e*x)^(13/2)/(a^2 + 2*a*b*x + b^2*x^2)^3,x]
```

output

```
-1/1920*(Sqrt[d + e*x]*(45045*a^6*e^6 + 30030*a^5*b*e^5*(-2*d + 7*e*x) + 3
003*a^4*b^2*e^4*(3*d^2 - 94*d*e*x + 128*e^2*x^2) + 858*a^3*b^3*e^3*(3*d^3
+ 51*d^2*e*x - 607*d*e^2*x^2 + 395*e^3*x^3) + 143*a^2*b^4*e^2*(8*d^4 + 86*
d^3*e*x + 588*d^2*e^2*x^2 - 3250*d*e^3*x^3 + 965*e^4*x^4) + 26*a*b^5*e*(24
*d^5 + 208*d^4*e*x + 889*d^3*e^2*x^2 + 3045*d^2*e^3*x^3 - 7415*d*e^4*x^4 +
640*e^5*x^5) + b^6*(384*d^6 + 2928*d^5*e*x + 10024*d^4*e^2*x^2 + 21070*d^
3*e^3*x^3 + 35595*d^2*e^4*x^4 - 24320*d*e^5*x^5 - 1280*e^6*x^6)))/(b^7*(a
+ b*x)^5) + (3003*e^5*(-(b*d) + a*e)^(3/2)*ArcTan[(Sqrt[b]*Sqrt[d + e*x])/
Sqrt[-(b*d) + a*e]])/(128*b^(15/2))
```

Rubi [A] (verified)

Time = 0.55 (sec) , antiderivative size = 249, normalized size of antiderivative = 1.11, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.393$, Rules used = {1098, 27, 51, 51, 51, 51, 51, 60, 60, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex)^{13/2}}{(a^2+2abx+b^2x^2)^3} dx$$

↓ 1098

$$\begin{aligned}
 & b^6 \int \frac{(d+ex)^{13/2}}{b^6(a+bx)^6} dx \\
 & \quad \downarrow 27 \\
 & \int \frac{(d+ex)^{13/2}}{(a+bx)^6} dx \\
 & \quad \downarrow 51 \\
 & \frac{13e \int \frac{(d+ex)^{11/2}}{(a+bx)^5} dx}{10b} - \frac{(d+ex)^{13/2}}{5b(a+bx)^5} \\
 & \quad \downarrow 51 \\
 & \frac{13e \left(\frac{11e \int \frac{(d+ex)^{9/2}}{(a+bx)^4} dx}{8b} - \frac{(d+ex)^{11/2}}{4b(a+bx)^4} \right)}{10b} - \frac{(d+ex)^{13/2}}{5b(a+bx)^5} \\
 & \quad \downarrow 51 \\
 & \frac{13e \left(\frac{11e \left(\frac{3e \int \frac{(d+ex)^{7/2}}{(a+bx)^3} dx}{2b} - \frac{(d+ex)^{9/2}}{3b(a+bx)^3} \right)}{8b} - \frac{(d+ex)^{11/2}}{4b(a+bx)^4} \right)}{10b} - \frac{(d+ex)^{13/2}}{5b(a+bx)^5} \\
 & \quad \downarrow 51 \\
 & \frac{13e \left(\frac{11e \left(\frac{3e \left(\frac{7e \int \frac{(d+ex)^{5/2}}{(a+bx)^2} dx}{4b} - \frac{(d+ex)^{7/2}}{2b(a+bx)^2} \right)}{2b} - \frac{(d+ex)^{9/2}}{3b(a+bx)^3} \right)}{8b} - \frac{(d+ex)^{11/2}}{4b(a+bx)^4} \right)}{10b} - \frac{(d+ex)^{13/2}}{5b(a+bx)^5} \\
 & \quad \downarrow 51
 \end{aligned}$$

$$\begin{aligned}
 & \left(\frac{5e}{7e} \left(\frac{(bd-ae) \int \frac{1}{(a+bx)\sqrt{d+ex}} dx + 2\sqrt{d+ex}}{b} + \frac{2(d+ex)^{3/2}}{3b} \right) - \frac{(d+ex)^{5/2}}{b(a+bx)} \right) \\
 & \frac{3e}{4b} \left(\right) - \frac{(d+ex)^{7/2}}{2b(a+bx)^2} \\
 & \frac{11e}{2b} \left(\right) - \frac{(d+ex)^{9/2}}{3b(a+bx)^3} \\
 & \frac{13e}{8b} \left(\right) - \frac{(d+ex)^{11/2}}{4b(a+bx)^4}
 \end{aligned}$$

↓ 73

$$\left(\frac{5e}{7e} \left(\frac{(bd-ae) \int \frac{1}{a + \frac{b(d+ex) - bd}{be} - \frac{bd}{e}} + \frac{2\sqrt{d+ex}}{b}}{b} + \frac{2(d+ex)^{3/2}}{3b} \right) - \frac{(d+ex)^{5/2}}{b(a+bx)} \right)$$

$$\frac{3e}{4b} - \frac{(d+ex)^{7/2}}{2b(a+bx)^2}$$

$$\frac{11e}{2b} - \frac{(d+ex)^{9/2}}{3b(a+bx)^3}$$

$$\frac{13e}{8b} - \frac{(d+ex)^{11/2}}{4b(a+bx)^4}$$

↓ 221

$$\begin{aligned}
 & \left(\frac{5e}{b} \left(\frac{(bd-ae) \left(\frac{2\sqrt{d+ex}}{b} - \frac{2\sqrt{bd-ae} \operatorname{arctanh} \left(\frac{\sqrt{b}\sqrt{d+ex}}{\sqrt{bd-ae}} \right)}{b^{3/2}} \right)}{b} + \frac{2(d+ex)^{3/2}}{3b} \right) \right. \\
 & \left. - \frac{(d+ex)^{5/2}}{b(a+bx)} \right) \\
 & \left(\frac{3e}{4b} \right) - \frac{(d+ex)^{7/2}}{2b(a+bx)^2} \\
 & \left(\frac{11e}{2b} \right) - \frac{(d+ex)^{9/2}}{3b(a+bx)^3} \\
 & \left(\frac{13e}{8b} \right) - \frac{(d+ex)^{11/2}}{4b(a+bx)^4}
 \end{aligned}$$

input `Int[(d + e*x)^(13/2)/(a^2 + 2*a*b*x + b^2*x^2)^3,x]`

output `-1/5*(d + e*x)^(13/2)/(b*(a + b*x)^5) + (13*e*(-1/4*(d + e*x)^(11/2)/(b*(a + b*x)^4) + (11*e*(-1/3*(d + e*x)^(9/2)/(b*(a + b*x)^3) + (3*e*(-1/2*(d + e*x)^(7/2)/(b*(a + b*x)^2) + (7*e*(-((d + e*x)^(5/2)/(b*(a + b*x)))) + (5*e*((2*(d + e*x)^(3/2))/(3*b) + ((b*d - a*e)*((2*Sqrt[d + e*x])/b - (2*Sqrt[b*d - a*e]*ArcTanh[(Sqrt[b]*Sqrt[d + e*x])/Sqrt[b*d - a*e]])/b^(3/2)))/b)/(2*b)))/(4*b)))/(2*b)))/(8*b)))/(10*b)`

Defintions of rubi rules used

rule 27 `Int[(a_)*(F_x_), x_Symbol] := Simp[a Int[F_x, x], x] /; FreeQ[a, x] && !MatchQ[F_x, (b_)*(G_x_) /; FreeQ[b, x]]`

rule 51 `Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] := Simp[(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Simp[d*(n/(b*(m + 1)))] Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d, n}, x] && ILtQ[m, -1] && FractionQ[n] && GtQ[n, 0]`

rule 60 `Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] := Simp[(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(b*(m + n + 1)))] Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] && !(IGtQ[m, 0] && (!IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) && !ILtQ[m + n + 2, 0] && IntLinearQ[a, b, c, d, m, n, x]`

rule 73 `Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] := With[{p = Denominator[m]}, Simp[p/b Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) + d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntLinearQ[a, b, c, d, m, n, x]`

rule 221 `Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[-a/b, 2]/a)*ArcTanh[x/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]`

rule 1098 `Int[((d_.) + (e_.)*(x_)^(m_.))*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] := Simp[1/c^p Int[(d + e*x)^m*(b^2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]`

Maple [A] (verified)

Time = 3.73 (sec) , antiderivative size = 292, normalized size of antiderivative = 1.30

| method | result |
|-------------------|---|
| risch | $-\frac{2e^5(-bex+18ae-19bd)\sqrt{ex+d}}{3b^7} + \frac{(2e^2a^2-4abde+2b^2d^2)e^5 \left(-\frac{2373b^4(ex+d)^{\frac{9}{2}}}{256} - \frac{12131b^3(ae-bd)(ex+d)^{\frac{7}{2}}}{384} + (-\frac{1253}{30}) \right)}{3003e^5(bx+a)^5(ae-bd)^2 \arctan\left(\frac{b\sqrt{ex+d}}{\sqrt{b(ae-bd)}}\right) - 3003 \left(-\frac{4864}{9009}d e^5 x^5 + \frac{113}{143}d^2 e^4 x^4 + \frac{602}{1287}d^3 e^3 x^3 + \frac{1432}{6435}d^4 e^2 x^2 + \frac{976}{15015}d^5 e x - \frac{2373}{128}d^6 \right)}$ |
| pseudoelliptic | |
| derivativedivides | $2e^5 \left(-\frac{(ex+d)^{\frac{3}{2}}b + 6ae\sqrt{ex+d} - 6db\sqrt{ex+d}}{b^7} + \frac{\left(-\frac{2373}{256}e^2a^2b^4 + \frac{2373}{128}dea b^5 - \frac{2373}{256}d^2b^6 \right)(ex+d)^{\frac{9}{2}} - \frac{12131b^3(e^3a^3 - 3a^2)}{3003} \right)}{3003e^5(bx+a)^5(ae-bd)^2 \arctan\left(\frac{b\sqrt{ex+d}}{\sqrt{b(ae-bd)}}\right) - 3003 \left(-\frac{4864}{9009}d e^5 x^5 + \frac{113}{143}d^2 e^4 x^4 + \frac{602}{1287}d^3 e^3 x^3 + \frac{1432}{6435}d^4 e^2 x^2 + \frac{976}{15015}d^5 e x - \frac{2373}{128}d^6 \right)}$ |
| default | $2e^5 \left(-\frac{(ex+d)^{\frac{3}{2}}b + 6ae\sqrt{ex+d} - 6db\sqrt{ex+d}}{b^7} + \frac{\left(-\frac{2373}{256}e^2a^2b^4 + \frac{2373}{128}dea b^5 - \frac{2373}{256}d^2b^6 \right)(ex+d)^{\frac{9}{2}} - \frac{12131b^3(e^3a^3 - 3a^2)}{3003} \right)}{3003e^5(bx+a)^5(ae-bd)^2 \arctan\left(\frac{b\sqrt{ex+d}}{\sqrt{b(ae-bd)}}\right) - 3003 \left(-\frac{4864}{9009}d e^5 x^5 + \frac{113}{143}d^2 e^4 x^4 + \frac{602}{1287}d^3 e^3 x^3 + \frac{1432}{6435}d^4 e^2 x^2 + \frac{976}{15015}d^5 e x - \frac{2373}{128}d^6 \right)}$ |

input `int((e*x+d)^(13/2)/(b^2*x^2+2*a*b*x+a^2)^3,x,method=_RETURNVERBOSE)`

output

```
-2/3*e^5*(-b*e*x+18*a*e-19*b*d)*(e*x+d)^(1/2)/b^7+1/b^7*(2*a^2*e^2-4*a*b*d
*e+2*b^2*d^2)*e^5*((-2373/256*b^4*(e*x+d)^(9/2)-12131/384*b^3*(a*e-b*d)*(e
*x+d)^(7/2)+(-1253/30*a^2*b^2*e^2+1253/15*a*b^3*d*e-1253/30*b^4*d^2)*(e*x+
d)^(5/2)+(-9629/384*a^3*b*e^3+9629/128*a^2*b^2*d*e^2-9629/128*d^2*e*a*b^3+
9629/384*b^4*d^3)*(e*x+d)^(3/2)+(-1467/256*a^4*e^4+1467/64*a^3*b*d*e^3-440
1/128*a^2*b^2*d^2*e^2+1467/64*a*b^3*d^3*e-1467/256*b^4*d^4)*(e*x+d)^(1/2))
/(b*(e*x+d)+a*e-b*d)^5+3003/256/(b*(a*e-b*d))^(1/2)*arctan(b*(e*x+d)^(1/2)
/(b*(a*e-b*d))^(1/2))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 612 vs. $2(184) = 368$.

Time = 0.13 (sec) , antiderivative size = 1234, normalized size of antiderivative = 5.51

$$\int \frac{(d+ex)^{13/2}}{(a^2+2abx+b^2x^2)^3} dx = \text{Too large to display}$$

input

```
integrate((e*x+d)^(13/2)/(b^2*x^2+2*a*b*x+a^2)^3,x, algorithm="fricas")
```

output

```

[-1/3840*(45045*(a^5*b*d*e^5 - a^6*e^6 + (b^6*d*e^5 - a*b^5*e^6)*x^5 + 5*(
a*b^5*d*e^5 - a^2*b^4*e^6)*x^4 + 10*(a^2*b^4*d*e^5 - a^3*b^3*e^6)*x^3 + 10
*(a^3*b^3*d*e^5 - a^4*b^2*e^6)*x^2 + 5*(a^4*b^2*d*e^5 - a^5*b*e^6)*x)*sqrt
((b*d - a*e)/b)*log((b*e*x + 2*b*d - a*e + 2*sqrt(e*x + d)*b*sqrt((b*d - a
*e)/b))/(b*x + a)) - 2*(1280*b^6*e^6*x^6 - 384*b^6*d^6 - 624*a*b^5*d^5*e -
1144*a^2*b^4*d^4*e^2 - 2574*a^3*b^3*d^3*e^3 - 9009*a^4*b^2*d^2*e^4 + 6006
0*a^5*b*d*e^5 - 45045*a^6*e^6 + 1280*(19*b^6*d*e^5 - 13*a*b^5*e^6)*x^5 - 5
*(7119*b^6*d^2*e^4 - 38558*a*b^5*d*e^5 + 27599*a^2*b^4*e^6)*x^4 - 10*(2107
*b^6*d^3*e^3 + 7917*a*b^5*d^2*e^4 - 46475*a^2*b^4*d*e^5 + 33891*a^3*b^3*e^
6)*x^3 - 2*(5012*b^6*d^4*e^2 + 11557*a*b^5*d^3*e^3 + 42042*a^2*b^4*d^2*e^4
- 260403*a^3*b^3*d*e^5 + 192192*a^4*b^2*e^6)*x^2 - 2*(1464*b^6*d^5*e + 27
04*a*b^5*d^4*e^2 + 6149*a^2*b^4*d^3*e^3 + 21879*a^3*b^3*d^2*e^4 - 141141*a
^4*b^2*d*e^5 + 105105*a^5*b*e^6)*x)*sqrt(e*x + d))/(b^12*x^5 + 5*a*b^11*x^
4 + 10*a^2*b^10*x^3 + 10*a^3*b^9*x^2 + 5*a^4*b^8*x + a^5*b^7), -1/1920*(45
045*(a^5*b*d*e^5 - a^6*e^6 + (b^6*d*e^5 - a*b^5*e^6)*x^5 + 5*(a*b^5*d*e^5
- a^2*b^4*e^6)*x^4 + 10*(a^2*b^4*d*e^5 - a^3*b^3*e^6)*x^3 + 10*(a^3*b^3*d*
e^5 - a^4*b^2*e^6)*x^2 + 5*(a^4*b^2*d*e^5 - a^5*b*e^6)*x)*sqrt(-(b*d - a*e
)/b)*arctan(-sqrt(e*x + d)*b*sqrt(-(b*d - a*e)/b)/(b*d - a*e)) - (1280*b^6
*e^6*x^6 - 384*b^6*d^6 - 624*a*b^5*d^5*e - 1144*a^2*b^4*d^4*e^2 - 2574*a^3
*b^3*d^3*e^3 - 9009*a^4*b^2*d^2*e^4 + 60060*a^5*b*d*e^5 - 45045*a^6*e^6...

```

Sympy [F(-1)]

Timed out.

$$\int \frac{(d + ex)^{13/2}}{(a^2 + 2abx + b^2x^2)^3} dx = \text{Timed out}$$

input

```
integrate((e*x+d)**(13/2)/(b**2*x**2+2*a*b*x+a**2)**3,x)
```

output

Timed out

Maxima [F(-2)]

Exception generated.

$$\int \frac{(d+ex)^{13/2}}{(a^2+2abx+b^2x^2)^3} dx = \text{Exception raised: ValueError}$$

input `integrate((e*x+d)^(13/2)/(b^2*x^2+2*a*b*x+a^2)^3,x, algorithm="maxima")`

output `Exception raised: ValueError >> Computation failed since Maxima requested additional constraints; using the 'assume' command before evaluation *may* help (example of legal syntax is 'assume(a*e-b*d>0)', see `assume?` for more detail`

Giac [B] (verification not implemented)Leaf count of result is larger than twice the leaf count of optimal. 611 vs. $2(184) = 368$.

Time = 0.18 (sec) , antiderivative size = 611, normalized size of antiderivative = 2.73

$$\int \frac{(d+ex)^{13/2}}{(a^2+2abx+b^2x^2)^3} dx = \frac{3003(b^2d^2e^5 - 2abde^6 + a^2e^7) \arctan\left(\frac{\sqrt{ex+db}}{\sqrt{-b^2d+abe}}\right)}{128\sqrt{-b^2d+abe}b^7}$$

$$- \frac{35595(ex+d)^{\frac{9}{2}}b^6d^2e^5 - 121310(ex+d)^{\frac{7}{2}}b^6d^3e^5 + 160384(ex+d)^{\frac{5}{2}}b^6d^4e^5 - 96290(ex+d)^{\frac{3}{2}}b^6d^5e^5 + 2}{3b^{18}}$$

$$+ \frac{2\left((ex+d)^{\frac{3}{2}}b^{12}e^5 + 18\sqrt{ex+db}b^{12}de^5 - 18\sqrt{ex+db}b^{11}e^6\right)}{3b^{18}}$$

input `integrate((e*x+d)^(13/2)/(b^2*x^2+2*a*b*x+a^2)^3,x, algorithm="giac")`

output

```

3003/128*(b^2*d^2*e^5 - 2*a*b*d*e^6 + a^2*e^7)*arctan(sqrt(e*x + d)*b/sqrt
(-b^2*d + a*b*e))/(sqrt(-b^2*d + a*b*e)*b^7) - 1/1920*(35595*(e*x + d)^(9/
2)*b^6*d^2*e^5 - 121310*(e*x + d)^(7/2)*b^6*d^3*e^5 + 160384*(e*x + d)^(5/
2)*b^6*d^4*e^5 - 96290*(e*x + d)^(3/2)*b^6*d^5*e^5 + 22005*sqrt(e*x + d)*b
^6*d^6*e^5 - 71190*(e*x + d)^(9/2)*a*b^5*d*e^6 + 363930*(e*x + d)^(7/2)*a*
b^5*d^2*e^6 - 641536*(e*x + d)^(5/2)*a*b^5*d^3*e^6 + 481450*(e*x + d)^(3/2
)*a*b^5*d^4*e^6 - 132030*sqrt(e*x + d)*a*b^5*d^5*e^6 + 35595*(e*x + d)^(9/
2)*a^2*b^4*e^7 - 363930*(e*x + d)^(7/2)*a^2*b^4*d*e^7 + 962304*(e*x + d)^(
5/2)*a^2*b^4*d^2*e^7 - 962900*(e*x + d)^(3/2)*a^2*b^4*d^3*e^7 + 330075*sqrt
(e*x + d)*a^2*b^4*d^4*e^7 + 121310*(e*x + d)^(7/2)*a^3*b^3*e^8 - 641536*(
e*x + d)^(5/2)*a^3*b^3*d*e^8 + 962900*(e*x + d)^(3/2)*a^3*b^3*d^2*e^8 - 44
0100*sqrt(e*x + d)*a^3*b^3*d^3*e^8 + 160384*(e*x + d)^(5/2)*a^4*b^2*e^9 -
481450*(e*x + d)^(3/2)*a^4*b^2*d*e^9 + 330075*sqrt(e*x + d)*a^4*b^2*d^2*e^
9 + 96290*(e*x + d)^(3/2)*a^5*b*e^10 - 132030*sqrt(e*x + d)*a^5*b*d*e^10 +
22005*sqrt(e*x + d)*a^6*e^11)/(((e*x + d)*b - b*d + a*e)^5*b^7) + 2/3*((e
*x + d)^(3/2)*b^12*e^5 + 18*sqrt(e*x + d)*b^12*d*e^5 - 18*sqrt(e*x + d)*a*
b^11*e^6)/b^18

```

Mupad [B] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 711, normalized size of antiderivative = 3.17

$$\int \frac{(d+ex)^{13/2}}{(a^2+2abx+b^2x^2)^3} dx = \frac{2e^5(d+ex)^{3/2}}{3b^6}$$

$$\frac{(d+ex)^{9/2} \left(\frac{2373a^2b^4e^7}{128} - \frac{2373ab^5de^6}{64} + \frac{2373b^6d^2e^5}{128} \right) + (d+ex)^{7/2} \left(\frac{12131a^3b^3e^8}{192} - \frac{12131a^2b^4de^7}{64} + \frac{12131ab^5d^2e^6}{64} \right)}{(d+ex)(5a^4b^8e^4 - 20a^3b^9de^3 + 30a^2b^{10}d^2e^2 - 20ab^{11}d^3e + b^{12}d^4)}$$

$$+ \frac{2e^5(6b^6d - 6ab^5e)\sqrt{d+ex}}{b^{12}} + \frac{3003e^5 \operatorname{atan}\left(\frac{\sqrt{b}e^5(ae-bd)^{3/2}\sqrt{d+ex}}{a^2e^7 - 2abd^6 + b^2d^2e^5}\right)(ae-bd)^{3/2}}{128b^{15/2}}$$

input

```
int((d + e*x)^(13/2)/(a^2 + b^2*x^2 + 2*a*b*x)^3,x)
```

output

```
(2*e^5*(d + e*x)^(3/2))/(3*b^6) - ((d + e*x)^(9/2)*((2373*a^2*b^4*e^7)/128
+ (2373*b^6*d^2*e^5)/128 - (2373*a*b^5*d*e^6)/64) + (d + e*x)^(7/2)*((121
31*a^3*b^3*e^8)/192 - (12131*b^6*d^3*e^5)/192 + (12131*a*b^5*d^2*e^6)/64 -
(12131*a^2*b^4*d*e^7)/64) + (d + e*x)^(1/2)*((1467*a^6*e^11)/128 + (1467*
b^6*d^6*e^5)/128 - (4401*a*b^5*d^5*e^6)/64 + (22005*a^2*b^4*d^4*e^7)/128 -
(7335*a^3*b^3*d^3*e^8)/32 + (22005*a^4*b^2*d^2*e^9)/128 - (4401*a^5*b*d*e
^10)/64) + (d + e*x)^(5/2)*((1253*a^4*b^2*e^9)/15 + (1253*b^6*d^4*e^5)/15
- (5012*a*b^5*d^3*e^6)/15 - (5012*a^3*b^3*d*e^8)/15 + (2506*a^2*b^4*d^2*e
^7)/5) + (d + e*x)^(3/2)*((9629*a^5*b*e^10)/192 - (9629*b^6*d^5*e^5)/192 +
(48145*a*b^5*d^4*e^6)/192 - (48145*a^4*b^2*d*e^9)/192 - (48145*a^2*b^4*d^3
*e^7)/96 + (48145*a^3*b^3*d^2*e^8)/96))/((d + e*x)*(5*b^12*d^4 + 5*a^4*b^8
*e^4 - 20*a^3*b^9*d*e^3 + 30*a^2*b^10*d^2*e^2 - 20*a*b^11*d^3*e) - (d + e
*x)^2*(10*b^12*d^3 - 10*a^3*b^9*e^3 + 30*a^2*b^10*d*e^2 - 30*a*b^11*d^2*e)
+ b^12*(d + e*x)^5 - (5*b^12*d - 5*a*b^11*e)*(d + e*x)^4 - b^12*d^5 + (d +
e*x)^3*(10*b^12*d^2 + 10*a^2*b^10*e^2 - 20*a*b^11*d*e) + a^5*b^7*e^5 - 5*
a^4*b^8*d*e^4 - 10*a^2*b^10*d^3*e^2 + 10*a^3*b^9*d^2*e^3 + 5*a*b^11*d^4*e)
+ (2*e^5*(6*b^6*d - 6*a*b^5*e)*(d + e*x)^(1/2))/b^12 + (3003*e^5*atan((b
^(1/2)*e^5*(a*e - b*d)^(3/2)*(d + e*x)^(1/2))/(a^2*e^7 + b^2*d^2*e^5 - 2*a*
b*d*e^6))*(a*e - b*d)^(3/2))/(128*b^(15/2))
```

Reduce [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 1163, normalized size of antiderivative = 5.19

$$\int \frac{(d + ex)^{13/2}}{(a^2 + 2abx + b^2x^2)^3} dx = \text{Too large to display}$$

input

```
int((e*x+d)^(13/2)/(b^2*x^2+2*a*b*x+a^2)^3,x)
```

output

```
(45045*sqrt(b)*sqrt(a*e - b*d)*atan((sqrt(d + e*x)*b)/(sqrt(b)*sqrt(a*e -
b*d)))*a**6*e**6 - 45045*sqrt(b)*sqrt(a*e - b*d)*atan((sqrt(d + e*x)*b)/(s
qrt(b)*sqrt(a*e - b*d)))*a**5*b*d*e**5 + 225225*sqrt(b)*sqrt(a*e - b*d)*at
an((sqrt(d + e*x)*b)/(sqrt(b)*sqrt(a*e - b*d)))*a**5*b*e**6*x - 225225*sqr
t(b)*sqrt(a*e - b*d)*atan((sqrt(d + e*x)*b)/(sqrt(b)*sqrt(a*e - b*d)))*a**
4*b**2*d*e**5*x + 450450*sqrt(b)*sqrt(a*e - b*d)*atan((sqrt(d + e*x)*b)/(s
qrt(b)*sqrt(a*e - b*d)))*a**4*b**2*e**6*x**2 - 450450*sqrt(b)*sqrt(a*e - b
*d)*atan((sqrt(d + e*x)*b)/(sqrt(b)*sqrt(a*e - b*d)))*a**3*b**3*d*e**5*x**
2 + 450450*sqrt(b)*sqrt(a*e - b*d)*atan((sqrt(d + e*x)*b)/(sqrt(b)*sqrt(a*
e - b*d)))*a**3*b**3*e**6*x**3 - 450450*sqrt(b)*sqrt(a*e - b*d)*atan((sqrt
(d + e*x)*b)/(sqrt(b)*sqrt(a*e - b*d)))*a**2*b**4*d*e**5*x**3 + 225225*sqr
t(b)*sqrt(a*e - b*d)*atan((sqrt(d + e*x)*b)/(sqrt(b)*sqrt(a*e - b*d)))*a**
2*b**4*e**6*x**4 - 225225*sqrt(b)*sqrt(a*e - b*d)*atan((sqrt(d + e*x)*b)/(
sqrt(b)*sqrt(a*e - b*d)))*a*b**5*d*e**5*x**4 + 45045*sqrt(b)*sqrt(a*e - b*
d)*atan((sqrt(d + e*x)*b)/(sqrt(b)*sqrt(a*e - b*d)))*a*b**5*e**6*x**5 - 45
045*sqrt(b)*sqrt(a*e - b*d)*atan((sqrt(d + e*x)*b)/(sqrt(b)*sqrt(a*e - b*d
)))*b**6*d*e**5*x**5 - 45045*sqrt(d + e*x)*a**6*b*e**6 + 60060*sqrt(d + e*
x)*a**5*b**2*d*e**5 - 210210*sqrt(d + e*x)*a**5*b**2*e**6*x - 9009*sqrt(d
+ e*x)*a**4*b**3*d**2*e**4 + 282282*sqrt(d + e*x)*a**4*b**3*d*e**5*x - 384
384*sqrt(d + e*x)*a**4*b**3*e**6*x**2 - 2574*sqrt(d + e*x)*a**3*b**4*d*...
```

3.191 $\int \frac{(d+ex)^{11/2}}{(a^2+2abx+b^2x^2)^3} dx$

| | |
|---|------|
| Optimal result | 1546 |
| Mathematica [A] (verified) | 1547 |
| Rubi [A] (verified) | 1547 |
| Maple [A] (verified) | 1554 |
| Fricas [B] (verification not implemented) | 1555 |
| Sympy [F(-1)] | 1556 |
| Maxima [F(-2)] | 1556 |
| Giac [B] (verification not implemented) | 1556 |
| Mupad [B] (verification not implemented) | 1557 |
| Reduce [B] (verification not implemented) | 1558 |

Optimal result

Integrand size = 28, antiderivative size = 197

$$\int \frac{(d+ex)^{11/2}}{(a^2+2abx+b^2x^2)^3} dx = \frac{693e^5\sqrt{d+ex}}{128b^6} - \frac{231e^4(d+ex)^{3/2}}{128b^5(a+bx)} - \frac{231e^3(d+ex)^{5/2}}{320b^4(a+bx)^2} - \frac{33e^2(d+ex)^{7/2}}{80b^3(a+bx)^3} - \frac{11e(d+ex)^{9/2}}{40b^2(a+bx)^4} - \frac{(d+ex)^{11/2}}{5b(a+bx)^5} - \frac{693e^5\sqrt{bd-ae}\operatorname{arctanh}\left(\frac{\sqrt{b}\sqrt{d+ex}}{\sqrt{bd-ae}}\right)}{128b^{13/2}}$$

output

```
693/128*e^5*(e*x+d)^(1/2)/b^6-231/128*e^4*(e*x+d)^(3/2)/b^5/(b*x+a)-231/320*e^3*(e*x+d)^(5/2)/b^4/(b*x+a)^2-33/80*e^2*(e*x+d)^(7/2)/b^3/(b*x+a)^3-11/40*e*(e*x+d)^(9/2)/b^2/(b*x+a)^4-1/5*(e*x+d)^(11/2)/b/(b*x+a)^5-693/128*e^5*(-a*e+b*d)^(1/2)*arctanh(b^(1/2)*(e*x+d)^(1/2)/(-a*e+b*d)^(1/2))/b^(13/2)
```

Mathematica [A] (verified)

Time = 1.95 (sec) , antiderivative size = 273, normalized size of antiderivative = 1.39

$$\int \frac{(d+ex)^{11/2}}{(a^2+2abx+b^2x^2)^3} dx =$$

$$\frac{\sqrt{d+ex}(-3465a^5e^5 + 1155a^4be^4(d-14ex) + 462a^3b^2e^3(d^2+12dex-64e^2x^2) + 66a^2b^3e^2(4d^3+33d^2e$$

$$x) + 159d^2e^2x^2 - 395e^3x^3) + 11ab^4e^4(16d^4+112d^3ex+36$$

$$6d^2e^2x^2+880d^3e^3x^3-965e^4x^4) + b^5(128d^5+816d^4ex$$

$$+ 2248d^3e^2x^2+3590d^2e^3x^3+4215de^4x^4-1280e^5x^5)}{($$

$$b^6(a+bx)^5 - (693e^5\sqrt{-(bd)+ae})\operatorname{ArcTan}\left[\frac{\sqrt{b}\sqrt{d+ex}}{\sqrt{-(bd)+ae}}\right])/\sqrt{-(bd)+ae}}$$

input

```
Integrate[(d + e*x)^(11/2)/(a^2 + 2*a*b*x + b^2*x^2)^3,x]
```

output

```
-1/640*(Sqrt[d + e*x]*(-3465*a^5*e^5 + 1155*a^4*b*e^4*(d - 14*e*x) + 462*a^3*b^2*e^3*(d^2 + 12*d*e*x - 64*e^2*x^2) + 66*a^2*b^3*e^2*(4*d^3 + 33*d^2*e*x + 159*d*e^2*x^2 - 395*e^3*x^3) + 11*a*b^4*e*(16*d^4 + 112*d^3*e*x + 36*6*d^2*e^2*x^2 + 880*d^3*e^3*x^3 - 965*e^4*x^4) + b^5*(128*d^5 + 816*d^4*e*x + 2248*d^3*e^2*x^2 + 3590*d^2*e^3*x^3 + 4215*d*e^4*x^4 - 1280*e^5*x^5)))/(b^6*(a + b*x)^5 - (693*e^5*Sqrt[-(b*d) + a*e])*ArcTan[(Sqrt[b]*Sqrt[d + e*x])/Sqrt[-(b*d) + a*e]])/(128*b^(13/2))
```

Rubi [A] (verified)

Time = 0.53 (sec) , antiderivative size = 220, normalized size of antiderivative = 1.12, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.357$, Rules used = {1098, 27, 51, 51, 51, 51, 51, 60, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex)^{11/2}}{(a^2+2abx+b^2x^2)^3} dx$$

$$\downarrow 1098$$

$$b^6 \int \frac{(d+ex)^{11/2}}{b^6(a+bx)^6} dx$$

$$\begin{aligned}
 & \int \frac{(d+ex)^{11/2}}{(a+bx)^6} dx \\
 & \downarrow 27 \\
 & \frac{11e \int \frac{(d+ex)^{9/2}}{(a+bx)^5} dx}{10b} - \frac{(d+ex)^{11/2}}{5b(a+bx)^5} \\
 & \downarrow 51 \\
 & \frac{11e \left(\frac{9e \int \frac{(d+ex)^{7/2}}{(a+bx)^4} dx}{8b} - \frac{(d+ex)^{9/2}}{4b(a+bx)^4} \right)}{10b} - \frac{(d+ex)^{11/2}}{5b(a+bx)^5} \\
 & \downarrow 51 \\
 & \frac{11e \left(\frac{9e \left(\frac{7e \int \frac{(d+ex)^{5/2}}{(a+bx)^3} dx}{6b} - \frac{(d+ex)^{7/2}}{3b(a+bx)^3} \right)}{8b} - \frac{(d+ex)^{9/2}}{4b(a+bx)^4} \right)}{10b} - \frac{(d+ex)^{11/2}}{5b(a+bx)^5} \\
 & \downarrow 51 \\
 & \frac{11e \left(\frac{9e \left(\frac{7e \left(\frac{5e \int \frac{(d+ex)^{3/2}}{(a+bx)^2} dx}{4b} - \frac{(d+ex)^{5/2}}{2b(a+bx)^2} \right)}{6b} - \frac{(d+ex)^{7/2}}{3b(a+bx)^3} \right)}{8b} - \frac{(d+ex)^{9/2}}{4b(a+bx)^4} \right)}{10b} - \frac{(d+ex)^{11/2}}{5b(a+bx)^5} \\
 & \downarrow 51
 \end{aligned}$$

$$\begin{aligned}
 & \left(\begin{aligned} & 7e \left(\frac{5e \left(\frac{3e \int \frac{\sqrt{d+ex}}{a+bx} dx}{2b} - \frac{(d+ex)^{3/2}}{b(a+bx)} \right)}{4b} - \frac{(d+ex)^{5/2}}{2b(a+bx)^2} \right) \\ & 9e \left(\frac{\phantom{7e \left(\frac{5e \left(\frac{3e \int \frac{\sqrt{d+ex}}{a+bx} dx}{2b} - \frac{(d+ex)^{3/2}}{b(a+bx)} \right)}{4b} - \frac{(d+ex)^{5/2}}{2b(a+bx)^2} \right)}}{6b} - \frac{(d+ex)^{7/2}}{3b(a+bx)^3} \right) \\ & 11e \left(\frac{\phantom{7e \left(\frac{5e \left(\frac{3e \int \frac{\sqrt{d+ex}}{a+bx} dx}{2b} - \frac{(d+ex)^{3/2}}{b(a+bx)} \right)}{4b} - \frac{(d+ex)^{5/2}}{2b(a+bx)^2} \right)} \phantom{9e \left(\frac{\phantom{7e \left(\frac{5e \left(\frac{3e \int \frac{\sqrt{d+ex}}{a+bx} dx}{2b} - \frac{(d+ex)^{3/2}}{b(a+bx)} \right)}{4b} - \frac{(d+ex)^{5/2}}{2b(a+bx)^2} \right)}}{6b} - \frac{(d+ex)^{7/2}}{3b(a+bx)^3} \right)}}{8b} - \frac{(d+ex)^{9/2}}{4b(a+bx)^4} \right) \end{aligned} \right) \\
 & \frac{\phantom{7e \left(\frac{5e \left(\frac{3e \int \frac{\sqrt{d+ex}}{a+bx} dx}{2b} - \frac{(d+ex)^{3/2}}{b(a+bx)} \right)}{4b} - \frac{(d+ex)^{5/2}}{2b(a+bx)^2} \right)} \phantom{9e \left(\frac{\phantom{7e \left(\frac{5e \left(\frac{3e \int \frac{\sqrt{d+ex}}{a+bx} dx}{2b} - \frac{(d+ex)^{3/2}}{b(a+bx)} \right)}{4b} - \frac{(d+ex)^{5/2}}{2b(a+bx)^2} \right)}}{6b} - \frac{(d+ex)^{7/2}}{3b(a+bx)^3} \right)} \phantom{11e \left(\frac{\phantom{7e \left(\frac{5e \left(\frac{3e \int \frac{\sqrt{d+ex}}{a+bx} dx}{2b} - \frac{(d+ex)^{3/2}}{b(a+bx)} \right)}{4b} - \frac{(d+ex)^{5/2}}{2b(a+bx)^2} \right)} \phantom{9e \left(\frac{\phantom{7e \left(\frac{5e \left(\frac{3e \int \frac{\sqrt{d+ex}}{a+bx} dx}{2b} - \frac{(d+ex)^{3/2}}{b(a+bx)} \right)}{4b} - \frac{(d+ex)^{5/2}}{2b(a+bx)^2} \right)}}{6b} - \frac{(d+ex)^{7/2}}{3b(a+bx)^3} \right)}}}{10b} - \frac{(d+ex)^{11/2}}{5b(a+bx)^5}
 \end{aligned}$$

↓ 60

$$\left(\begin{array}{l}
 \left(\begin{array}{l}
 \left(\begin{array}{l}
 3e \left(\frac{(bd-ae) \int \frac{1}{(a+bx)\sqrt{d+ex}} dx + 2\sqrt{d+ex}}{b} \right) \\
 \frac{5e}{2b} - \frac{(d+ex)^{3/2}}{b(a+bx)}
 \end{array} \right) \\
 \frac{7e}{4b} - \frac{(d+ex)^{5/2}}{2b(a+bx)^2}
 \end{array} \right) \\
 \frac{9e}{6b} - \frac{(d+ex)^{7/2}}{3b(a+bx)^3}
 \end{array} \right) \\
 \frac{11e}{8b} - \frac{(d+ex)^{9/2}}{4b(a+bx)^4}
 \end{array} \right)$$

$$\frac{10b}{5b(a+bx)^5} (d+ex)^{11/2}$$

↓ 73

$$\left(\begin{array}{l}
 5e \left(\frac{3e \left(\frac{2(bd-ae) \int \frac{1}{a + \frac{b(d+ex) - bd}{e}} d\sqrt{d+ex} + \frac{2\sqrt{d+ex}}{b}} \right)}{2b} - \frac{(d+ex)^{3/2}}{b(a+bx)} \right) \\
 7e \left(\frac{\phantom{3e \left(\frac{2(bd-ae) \int \frac{1}{a + \frac{b(d+ex) - bd}{e}} d\sqrt{d+ex} + \frac{2\sqrt{d+ex}}{b}} \right)}}{4b} - \frac{(d+ex)^{5/2}}{2b(a+bx)^2} \right) \\
 9e \left(\frac{\phantom{3e \left(\frac{2(bd-ae) \int \frac{1}{a + \frac{b(d+ex) - bd}{e}} d\sqrt{d+ex} + \frac{2\sqrt{d+ex}}{b}} \right)}}{6b} - \frac{(d+ex)^{7/2}}{3b(a+bx)^3} \right) \\
 11e \left(\frac{\phantom{3e \left(\frac{2(bd-ae) \int \frac{1}{a + \frac{b(d+ex) - bd}{e}} d\sqrt{d+ex} + \frac{2\sqrt{d+ex}}{b}} \right)}}{8b} - \frac{(d+ex)^{9/2}}{4b(a+bx)^4} \right)
 \end{array} \right)$$

$$\frac{10b}{5b(a+bx)^5} (d+ex)^{11/2}$$

↓ 221

$$\begin{aligned}
 & \left(\left(\left(\left(\frac{3e \left(\frac{2\sqrt{d+ex}}{b} - \frac{2\sqrt{bd-ae} \operatorname{arctanh} \left(\frac{\sqrt{b}\sqrt{d+ex}}{\sqrt{bd-ae}} \right)}{b^{3/2}} \right)}{2b} - \frac{(d+ex)^{3/2}}{b(a+bx)} \right)}{4b} - \frac{(d+ex)^{5/2}}{2b(a+bx)^2} \right)}{6b} - \frac{(d+ex)^{7/2}}{3b(a+bx)^3} \right)}{8b} - \frac{(d+ex)^{9/2}}{4b(a+bx)^4} \right) \\
 & \frac{10b}{5b(a+bx)^5} (d+ex)^{11/2}
 \end{aligned}$$

input `Int[(d + e*x)^(11/2)/(a^2 + 2*a*b*x + b^2*x^2)^3,x]`

output

$$-1/5*(d + e*x)^{(11/2)}/(b*(a + b*x)^5) + (11*e*(-1/4*(d + e*x)^{(9/2)}/(b*(a + b*x)^4) + (9*e*(-1/3*(d + e*x)^{(7/2)}/(b*(a + b*x)^3) + (7*e*(-1/2*(d + e*x)^{(5/2)}/(b*(a + b*x)^2) + (5*e*(-((d + e*x)^{(3/2)}/(b*(a + b*x))) + (3*e*((2*\text{Sqrt}[d + e*x])/b - (2*\text{Sqrt}[b*d - a*e]*\text{ArcTanh}[(\text{Sqrt}[b]*\text{Sqrt}[d + e*x])/ \text{Sqrt}[b*d - a*e]])/b^{(3/2)}))/(2*b)))/(4*b)))/(6*b)))/(8*b)))/(10*b)$$

Defintions of rubi rules used

rule 27

$$\text{Int}[(a_*)(Fx_), x_Symbol] \rightarrow \text{Simp}[a \text{ Int}[Fx, x], x] /; \text{FreeQ}[a, x] \ \&\& \ !\text{MatchQ}[Fx, (b_*)(Gx_) /; \text{FreeQ}[b, x]$$

rule 51

$$\text{Int}[(a_.) + (b_.)(x_)^{(m_)}*((c_.) + (d_.)(x_)^{(n_)}), x_Symbol] \rightarrow \text{Simp}[(a + b*x)^{(m + 1)}*((c + d*x)^n/(b*(m + 1))), x] - \text{Simp}[d*(n/(b*(m + 1)))] \text{Int}[(a + b*x)^{(m + 1)}*(c + d*x)^{(n - 1)}, x], x] /; \text{FreeQ}[\{a, b, c, d, n\}, x] \ \&\& \ \text{ILtQ}[m, -1] \ \&\& \ \text{FractionQ}[n] \ \&\& \ \text{GtQ}[n, 0]$$

rule 60

$$\text{Int}[(a_.) + (b_.)(x_)^{(m_)}*((c_.) + (d_.)(x_)^{(n_)}), x_Symbol] \rightarrow \text{Simp}[(a + b*x)^{(m + 1)}*((c + d*x)^n/(b*(m + n + 1))), x] + \text{Simp}[n*((b*c - a*d)/(b*(m + n + 1)))] \text{Int}[(a + b*x)^m*(c + d*x)^{(n - 1)}, x], x] /; \text{FreeQ}[\{a, b, c, d\}, x] \ \&\& \ \text{GtQ}[n, 0] \ \&\& \ \text{NeQ}[m + n + 1, 0] \ \&\& \ !(\text{IGtQ}[m, 0] \ \&\& \ (!\text{IntegerQ}[n] \ || \ (\text{GtQ}[m, 0] \ \&\& \ \text{LtQ}[m - n, 0]))) \ \&\& \ !\text{ILtQ}[m + n + 2, 0] \ \&\& \ \text{IntLinearQ}[a, b, c, d, m, n, x]$$

rule 73

$$\text{Int}[(a_.) + (b_.)(x_)^{(m_)}*((c_.) + (d_.)(x_)^{(n_)}), x_Symbol] \rightarrow \text{With}[\{p = \text{Denominator}[m]\}, \text{Simp}[p/b \text{ Subst}[\text{Int}[x^{(p*(m + 1) - 1)}*(c - a*(d/b) + d*(x^{p/b})^n), x], x, (a + b*x)^{(1/p)}, x]] /; \text{FreeQ}[\{a, b, c, d\}, x] \ \&\& \ \text{LtQ}[-1, m, 0] \ \&\& \ \text{LeQ}[-1, n, 0] \ \&\& \ \text{LeQ}[\text{Denominator}[n], \text{Denominator}[m]] \ \&\& \ \text{IntLinearQ}[a, b, c, d, m, n, x]$$

rule 221

$$\text{Int}[(a_.) + (b_.)(x_)^2)^{-1}, x_Symbol] \rightarrow \text{Simp}[(\text{Rt}[-a/b, 2]/a)*\text{ArcTanh}[x/\text{Rt}[-a/b, 2]], x] /; \text{FreeQ}[\{a, b\}, x] \ \&\& \ \text{NegQ}[a/b]$$

rule 1098

```
Int[((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] := Simp[1/c^p Int[(d + e*x)^m*(b^2 + c*x)^(2*p), x], x] /; FreeQ[
{a, b, c, d, e, m}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Maple [A] (verified)

Time = 3.71 (sec) , antiderivative size = 265, normalized size of antiderivative = 1.35

| method | result |
|-------------------|--|
| risch | $\frac{2e^5\sqrt{ex+d}}{b^6} - \frac{(2ae-2bd)e^5 \left(-\frac{843b^4(ex+d)^{\frac{9}{2}}}{256} - \frac{1327b^3(ae-bd)(ex+d)^{\frac{7}{2}}}{128} + \left(-\frac{131}{10}a^2b^2e^2 + \frac{131}{5}ab^3de - \frac{131}{10}b^4d^2\right)(ex+d)^{\frac{5}{2}} + \dots \right)}{\dots}$ |
| pseudoelliptic | $- \frac{693 \left(e^5(bx+a)^5(ae-bd) \arctan\left(\frac{b\sqrt{ex+d}}{\sqrt{b(ae-bd)}}\right) - \sqrt{ex+d} \sqrt{b(ae-bd)} \left(\frac{256}{693}e^5x^5 - \frac{281}{231}de^4x^4 - \frac{718}{693}d^2e^3x^3 - \frac{2248}{3465}d^3e^2 \dots \right) \right)}{\dots}$ |
| derivativedivides | $2e^5 \left(\frac{\sqrt{ex+d}}{b^6} - \frac{\left(-\frac{843}{256}ab^4e + \frac{843}{256}b^5d\right)(ex+d)^{\frac{9}{2}} - \frac{1327b^3(e^2a^2 - 2abde + b^2d^2)(ex+d)^{\frac{7}{2}}}{128} + \left(-\frac{131}{10}a^3b^2e^3 + \frac{393}{10}a^2b^3de^2 - \frac{393}{10} \dots \right)}{\dots} \right)$ |
| default | $2e^5 \left(\frac{\sqrt{ex+d}}{b^6} - \frac{\left(-\frac{843}{256}ab^4e + \frac{843}{256}b^5d\right)(ex+d)^{\frac{9}{2}} - \frac{1327b^3(e^2a^2 - 2abde + b^2d^2)(ex+d)^{\frac{7}{2}}}{128} + \left(-\frac{131}{10}a^3b^2e^3 + \frac{393}{10}a^2b^3de^2 - \frac{393}{10} \dots \right)}{\dots} \right)$ |

input

```
int((e*x+d)^(11/2)/(b^2*x^2+2*a*b*x+a^2)^3,x,method=_RETURNVERBOSE)
```

output

```
2*e^5*(e*x+d)^(1/2)/b^6-1/b^6*(2*a*e-2*b*d)*e^5*((-843/256*b^4*(e*x+d)^(9/2)-1327/128*b^3*(a*e-b*d)*(e*x+d)^(7/2)+(-131/10*a^2*b^2*e^2+131/5*a*b^3*d*e^2-131/10*b^4*d^2)*(e*x+d)^(5/2)+(-977/128*a^3*b*e^3+2931/128*a^2*b^2*d*e^2-2931/128*d^2*e*a*b^3+977/128*b^4*d^3)*(e*x+d)^(3/2)+(-437/256*a^4*e^4+437/64*a^3*b*d*e^3-1311/128*a^2*b^2*d^2*e^2+437/64*a*b^3*d^3*e-437/256*b^4*d^4)*(e*x+d)^(1/2))/(b*(e*x+d)+a*e-b*d)^5+693/256/(b*(a*e-b*d))^(1/2)*arctan(b*(e*x+d)^(1/2)/(b*(a*e-b*d))^(1/2))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 440 vs. $2(161) = 322$.

Time = 0.12 (sec) , antiderivative size = 890, normalized size of antiderivative = 4.52

$$\int \frac{(d + ex)^{11/2}}{(a^2 + 2abx + b^2x^2)^3} dx = \text{Too large to display}$$

input `integrate((e*x+d)^(11/2)/(b^2*x^2+2*a*b*x+a^2)^3,x, algorithm="fricas")`

output

```
[1/1280*(3465*(b^5*e^5*x^5 + 5*a*b^4*e^5*x^4 + 10*a^2*b^3*e^5*x^3 + 10*a^3*b^2*e^5*x^2 + 5*a^4*b*e^5*x + a^5*e^5)*sqrt((b*d - a*e)/b)*log((b*e*x + 2*b*d - a*e - 2*sqrt(e*x + d)*b*sqrt((b*d - a*e)/b))/(b*x + a)) + 2*(1280*b^5*e^5*x^5 - 128*b^5*d^5 - 176*a*b^4*d^4*e - 264*a^2*b^3*d^3*e^2 - 462*a^3*b^2*d^2*e^3 - 1155*a^4*b*d*e^4 + 3465*a^5*e^5 - 5*(843*b^5*d*e^4 - 2123*a*b^4*e^5)*x^4 - 10*(359*b^5*d^2*e^3 + 968*a*b^4*d*e^4 - 2607*a^2*b^3*e^5)*x^3 - 2*(1124*b^5*d^3*e^2 + 2013*a*b^4*d^2*e^3 + 5247*a^2*b^3*d*e^4 - 14784*a^3*b^2*e^5)*x^2 - 2*(408*b^5*d^4*e + 616*a*b^4*d^3*e^2 + 1089*a^2*b^3*d^2*e^3 + 2772*a^3*b^2*d*e^4 - 8085*a^4*b*e^5)*x)*sqrt(e*x + d))/(b^11*x^5 + 5*a*b^10*x^4 + 10*a^2*b^9*x^3 + 10*a^3*b^8*x^2 + 5*a^4*b^7*x + a^5*b^6), -1/640*(3465*(b^5*e^5*x^5 + 5*a*b^4*e^5*x^4 + 10*a^2*b^3*e^5*x^3 + 10*a^3*b^2*e^5*x^2 + 5*a^4*b*e^5*x + a^5*e^5)*sqrt(-(b*d - a*e)/b)*arctan(-sqrt(e*x + d)*b*sqrt(-(b*d - a*e)/b)/(b*d - a*e)) - (1280*b^5*e^5*x^5 - 128*b^5*d^5 - 176*a*b^4*d^4*e - 264*a^2*b^3*d^3*e^2 - 462*a^3*b^2*d^2*e^3 - 1155*a^4*b*d*e^4 + 3465*a^5*e^5 - 5*(843*b^5*d*e^4 - 2123*a*b^4*e^5)*x^4 - 10*(359*b^5*d^2*e^3 + 968*a*b^4*d*e^4 - 2607*a^2*b^3*e^5)*x^3 - 2*(1124*b^5*d^3*e^2 + 2013*a*b^4*d^2*e^3 + 5247*a^2*b^3*d*e^4 - 14784*a^3*b^2*e^5)*x^2 - 2*(408*b^5*d^4*e + 616*a*b^4*d^3*e^2 + 1089*a^2*b^3*d^2*e^3 + 2772*a^3*b^2*d*e^4 - 8085*a^4*b*e^5)*x)*sqrt(e*x + d))/(b^11*x^5 + 5*a*b^10*x^4 + 10*a^2*b^9*x^3 + 10*a^3*b^8*x^2 + 5*a^4*b^7*x + a^5*b^6)]
```


Sympy [F(-1)]

Timed out.

$$\int \frac{(d + ex)^{11/2}}{(a^2 + 2abx + b^2x^2)^3} dx = \text{Timed out}$$

input `integrate((e*x+d)**(11/2)/(b**2*x**2+2*a*b*x+a**2)**3,x)`

output `Timed out`

Maxima [F(-2)]

Exception generated.

$$\int \frac{(d + ex)^{11/2}}{(a^2 + 2abx + b^2x^2)^3} dx = \text{Exception raised: ValueError}$$

input `integrate((e*x+d)^(11/2)/(b^2*x^2+2*a*b*x+a^2)^3,x, algorithm="maxima")`

output `Exception raised: ValueError >> Computation failed since Maxima requested additional constraints; using the 'assume' command before evaluation *may* help (example of legal syntax is 'assume(a*e-b*d>0)', see `assume?` for more detail`

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 456 vs. 2(161) = 322.

Time = 0.21 (sec) , antiderivative size = 456, normalized size of antiderivative = 2.31

$$\int \frac{(d + ex)^{11/2}}{(a^2 + 2abx + b^2x^2)^3} dx = \frac{2\sqrt{ex+d}e^5}{b^6} + \frac{693(bde^5 - ae^6) \arctan\left(\frac{\sqrt{ex+d}}{\sqrt{-b^2d+abe}}\right)}{128\sqrt{-b^2d+abe}b^6} - \frac{4215(ex+d)^{\frac{9}{2}}b^5de^5 - 13270(ex+d)^{\frac{7}{2}}b^5d^2e^5 + 16768(ex+d)^{\frac{5}{2}}b^5d^3e^5 - 9770(ex+d)^{\frac{3}{2}}b^5d^4e^5 + 2185\sqrt{ex+d}b^5d^5e^5}{128\sqrt{-b^2d+abe}b^6}$$

input `integrate((e*x+d)^(11/2)/(b^2*x^2+2*a*b*x+a^2)^3,x, algorithm="giac")`

output `2*sqrt(e*x + d)*e^5/b^6 + 693/128*(b*d*e^5 - a*e^6)*arctan(sqrt(e*x + d)*b/sqrt(-b^2*d + a*b*e))/(sqrt(-b^2*d + a*b*e)*b^6) - 1/640*(4215*(e*x + d)^(9/2)*b^5*d*e^5 - 13270*(e*x + d)^(7/2)*b^5*d^2*e^5 + 16768*(e*x + d)^(5/2)*b^5*d^3*e^5 - 9770*(e*x + d)^(3/2)*b^5*d^4*e^5 + 2185*sqrt(e*x + d)*b^5*d^5*e^5 - 4215*(e*x + d)^(9/2)*a*b^4*e^6 + 26540*(e*x + d)^(7/2)*a*b^4*d*e^6 - 50304*(e*x + d)^(5/2)*a*b^4*d^2*e^6 + 39080*(e*x + d)^(3/2)*a*b^4*d^3*e^6 - 10925*sqrt(e*x + d)*a*b^4*d^4*e^6 - 13270*(e*x + d)^(7/2)*a^2*b^3*e^7 + 50304*(e*x + d)^(5/2)*a^2*b^3*d*e^7 - 58620*(e*x + d)^(3/2)*a^2*b^3*d^2*e^7 + 21850*sqrt(e*x + d)*a^2*b^3*d^3*e^7 - 16768*(e*x + d)^(5/2)*a^3*b^2*e^8 + 39080*(e*x + d)^(3/2)*a^3*b^2*d*e^8 - 21850*sqrt(e*x + d)*a^3*b^2*d^2*e^8 - 9770*(e*x + d)^(3/2)*a^4*b*e^9 + 10925*sqrt(e*x + d)*a^4*b*d*e^9 - 2185*sqrt(e*x + d)*a^5*e^10)/(((e*x + d)*b - b*d + a*e)^5*b^6)`

Mupad [B] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 598, normalized size of antiderivative = 3.04

$$\int \frac{(d+ex)^{11/2}}{(a^2+2abx+b^2x^2)^3} dx = \frac{(d+ex)^{7/2} \left(\frac{1327a^2b^3e^7}{64} - \frac{1327ab^4de^6}{32} + \frac{1327b^5d^2e^5}{64} \right) + \sqrt{d+ex} \left(\frac{437a^5e^{10}}{128} - \frac{2185a^4bde^9}{128} \right)}{(d+ex) (5a^4b^7e^4 - 20a^3b^8de^3 + 30a^2b^9d^2e^2 - 20ab^{10}d^3e + 5b^{11}d^4)} + \frac{2e^5\sqrt{d+ex}}{b^6} - \frac{693e^5 \operatorname{atan}\left(\frac{\sqrt{b}e^5\sqrt{ae-bd}\sqrt{d+ex}}{ae^6-bde^5}\right) \sqrt{ae-bd}}{128b^{13/2}}$$

input `int((d + e*x)^(11/2)/(a^2 + b^2*x^2 + 2*a*b*x)^3,x)`

output

```

((d + e*x)^(7/2)*((1327*a^2*b^3*e^7)/64 + (1327*b^5*d^2*e^5)/64 - (1327*a*
b^4*d*e^6)/32) + (d + e*x)^(1/2)*((437*a^5*e^10)/128 - (437*b^5*d^5*e^5)/1
28 + (2185*a*b^4*d^4*e^6)/128 - (2185*a^2*b^3*d^3*e^7)/64 + (2185*a^3*b^2*
d^2*e^8)/64 - (2185*a^4*b*d*e^9)/128) + (d + e*x)^(5/2)*((131*a^3*b^2*e^8)
/5 - (131*b^5*d^3*e^5)/5 + (393*a*b^4*d^2*e^6)/5 - (393*a^2*b^3*d*e^7)/5)
+ (d + e*x)^(3/2)*((977*a^4*b*e^9)/64 + (977*b^5*d^4*e^5)/64 - (977*a*b^4*
d^3*e^6)/16 - (977*a^3*b^2*d*e^8)/16 + (2931*a^2*b^3*d^2*e^7)/32) + ((843*
a*b^4*e^6)/128 - (843*b^5*d*e^5)/128)*(d + e*x)^(9/2))/((d + e*x)*(5*b^11*
d^4 + 5*a^4*b^7*e^4 - 20*a^3*b^8*d*e^3 + 30*a^2*b^9*d^2*e^2 - 20*a*b^10*d^
3*e) - (d + e*x)^2*(10*b^11*d^3 - 10*a^3*b^8*e^3 + 30*a^2*b^9*d*e^2 - 30*a
*b^10*d^2*e) + b^11*(d + e*x)^5 - (5*b^11*d - 5*a*b^10*e)*(d + e*x)^4 - b^
11*d^5 + (d + e*x)^3*(10*b^11*d^2 + 10*a^2*b^9*e^2 - 20*a*b^10*d*e) + a^5*
b^6*e^5 - 5*a^4*b^7*d*e^4 - 10*a^2*b^9*d^3*e^2 + 10*a^3*b^8*d^2*e^3 + 5*a*
b^10*d^4*e) + (2*e^5*(d + e*x)^(1/2))/b^6 - (693*e^5*atan((b^(1/2)*e^5*(a
e - b*d)^(1/2)*(d + e*x)^(1/2))/(a*e^6 - b*d*e^5))*(a*e - b*d)^(1/2))/(128
*b^(13/2))

```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 726, normalized size of antiderivative = 3.69

$$\int \frac{(d + ex)^{11/2}}{(a^2 + 2abx + b^2x^2)^3} dx = \frac{-128\sqrt{ex + d}b^6d^5 + 3465\sqrt{ex + d}a^5be^5 + 1280\sqrt{ex + d}b^6e^5x^5 - 3465\sqrt{b}\sqrt{d}}{(a^2 + 2abx + b^2x^2)^3}$$

input

```
int((e*x+d)^(11/2)/(b^2*x^2+2*a*b*x+a^2)^3,x)
```

output

```
( - 3465*sqrt(b)*sqrt(a*e - b*d)*atan((sqrt(d + e*x)*b)/(sqrt(b)*sqrt(a*e
- b*d)))*a**5*e**5 - 17325*sqrt(b)*sqrt(a*e - b*d)*atan((sqrt(d + e*x)*b)/
(sqrt(b)*sqrt(a*e - b*d)))*a**4*b*e**5*x - 34650*sqrt(b)*sqrt(a*e - b*d)*a
tan((sqrt(d + e*x)*b)/(sqrt(b)*sqrt(a*e - b*d)))*a**3*b**2*e**5*x**2 - 346
50*sqrt(b)*sqrt(a*e - b*d)*atan((sqrt(d + e*x)*b)/(sqrt(b)*sqrt(a*e - b*d)
))*a**2*b**3*e**5*x**3 - 17325*sqrt(b)*sqrt(a*e - b*d)*atan((sqrt(d + e*x)
*b)/(sqrt(b)*sqrt(a*e - b*d)))*a*b**4*e**5*x**4 - 3465*sqrt(b)*sqrt(a*e -
b*d)*atan((sqrt(d + e*x)*b)/(sqrt(b)*sqrt(a*e - b*d)))*b**5*e**5*x**5 + 34
65*sqrt(d + e*x)*a**5*b*e**5 - 1155*sqrt(d + e*x)*a**4*b**2*d*e**4 + 16170
*sqrt(d + e*x)*a**4*b**2*e**5*x - 462*sqrt(d + e*x)*a**3*b**3*d**2*e**3 -
5544*sqrt(d + e*x)*a**3*b**3*d*e**4*x + 29568*sqrt(d + e*x)*a**3*b**3*e**5
*x**2 - 264*sqrt(d + e*x)*a**2*b**4*d**3*e**2 - 2178*sqrt(d + e*x)*a**2*b*
**4*d**2*e**3*x - 10494*sqrt(d + e*x)*a**2*b**4*d*e**4*x**2 + 26070*sqrt(d
+ e*x)*a**2*b**4*e**5*x**3 - 176*sqrt(d + e*x)*a*b**5*d**4*e - 1232*sqrt(d
+ e*x)*a*b**5*d**3*e**2*x - 4026*sqrt(d + e*x)*a*b**5*d**2*e**3*x**2 - 96
80*sqrt(d + e*x)*a*b**5*d*e**4*x**3 + 10615*sqrt(d + e*x)*a*b**5*e**5*x**4
- 128*sqrt(d + e*x)*b**6*d**5 - 816*sqrt(d + e*x)*b**6*d**4*e*x - 2248*sq
rt(d + e*x)*b**6*d**3*e**2*x**2 - 3590*sqrt(d + e*x)*b**6*d**2*e**3*x**3 -
4215*sqrt(d + e*x)*b**6*d*e**4*x**4 + 1280*sqrt(d + e*x)*b**6*e**5*x**5)/
(640*b**7*(a**5 + 5*a**4*b*x + 10*a**3*b**2*x**2 + 10*a**2*b**3*x**3 + ...
```

3.192 $\int \frac{(d+ex)^{9/2}}{(a^2+2abx+b^2x^2)^3} dx$

| | |
|---|------|
| Optimal result | 1560 |
| Mathematica [A] (verified) | 1561 |
| Rubi [A] (verified) | 1561 |
| Maple [A] (verified) | 1565 |
| Fricas [B] (verification not implemented) | 1566 |
| Sympy [F(-1)] | 1567 |
| Maxima [F(-2)] | 1567 |
| Giac [B] (verification not implemented) | 1567 |
| Mupad [B] (verification not implemented) | 1568 |
| Reduce [B] (verification not implemented) | 1569 |

Optimal result

Integrand size = 28, antiderivative size = 178

$$\int \frac{(d+ex)^{9/2}}{(a^2+2abx+b^2x^2)^3} dx = -\frac{63e^4\sqrt{d+ex}}{128b^5(a+bx)} - \frac{21e^3(d+ex)^{3/2}}{64b^4(a+bx)^2} - \frac{21e^2(d+ex)^{5/2}}{80b^3(a+bx)^3} - \frac{9e(d+ex)^{7/2}}{40b^2(a+bx)^4} - \frac{(d+ex)^{9/2}}{5b(a+bx)^5} - \frac{63e^5 \operatorname{arctanh}\left(\frac{\sqrt{b}\sqrt{d+ex}}{\sqrt{bd-ae}}\right)}{128b^{11/2}\sqrt{bd-ae}}$$

output

```
-63/128*e^4*(e*x+d)^(1/2)/b^5/(b*x+a)-21/64*e^3*(e*x+d)^(3/2)/b^4/(b*x+a)^2-21/80*e^2*(e*x+d)^(5/2)/b^3/(b*x+a)^3-9/40*e*(e*x+d)^(7/2)/b^2/(b*x+a)^4-1/5*(e*x+d)^(9/2)/b/(b*x+a)^5-63/128*e^5*arctanh(b^(1/2)*(e*x+d)^(1/2)/(-a*e+b*d)^(1/2))/b^(11/2)/(-a*e+b*d)^(1/2)
```

Mathematica [A] (verified)

Time = 1.38 (sec) , antiderivative size = 212, normalized size of antiderivative = 1.19

$$\int \frac{(d+ex)^{9/2}}{(a^2+2abx+b^2x^2)^3} dx = \frac{\sqrt{d+ex}(315a^4e^4 + 210a^3be^3(d+7ex) + 42a^2b^2e^2(4d^2 + 23dex + 64e^2x^2) + 6ab^3e(24d^3 + 128d^2ex + 289d^2e^2x^2 + 395e^3x^3) + b^4(128d^4 + 656d^3ex + 1368d^2e^2x^2 + 1490de^3x^3 + 965e^4x^4))}{640b^5(a+bx)^5} + \frac{63e^5 \arctan\left(\frac{\sqrt{b}\sqrt{d+ex}}{\sqrt{-bd+ae}}\right)}{128b^{11/2}\sqrt{-bd+ae}}$$

input

```
Integrate[(d + e*x)^(9/2)/(a^2 + 2*a*b*x + b^2*x^2)^3,x]
```

output

```
-1/640*(Sqrt[d + e*x]*(315*a^4*e^4 + 210*a^3*b*e^3*(d + 7*e*x) + 42*a^2*b^2*e^2*(4*d^2 + 23*d*e*x + 64*e^2*x^2) + 6*a*b^3*e*(24*d^3 + 128*d^2*e*x + 289*d*e^2*x^2 + 395*e^3*x^3) + b^4*(128*d^4 + 656*d^3*e*x + 1368*d^2*e^2*x^2 + 1490*d*e^3*x^3 + 965*e^4*x^4)))/(b^5*(a + b*x)^5) + (63*e^5*ArcTan[(Sqrt[b]*Sqrt[d + e*x])/Sqrt[-(b*d) + a*e]])/(128*b^(11/2)*Sqrt[-(b*d) + a*e])
```

Rubi [A] (verified)

Time = 0.47 (sec) , antiderivative size = 198, normalized size of antiderivative = 1.11, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.321$, Rules used = {1098, 27, 51, 51, 51, 51, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex)^{9/2}}{(a^2+2abx+b^2x^2)^3} dx$$

↓ 1098

$$b^6 \int \frac{(d+ex)^{9/2}}{b^6(a+bx)^6} dx$$

↓ 27

$$\begin{aligned}
 & \int \frac{(d+ex)^{9/2}}{(a+bx)^6} dx \\
 & \quad \downarrow 51 \\
 & \frac{9e \int \frac{(d+ex)^{7/2}}{(a+bx)^5} dx}{10b} - \frac{(d+ex)^{9/2}}{5b(a+bx)^5} \\
 & \quad \downarrow 51 \\
 & \frac{9e \left(\frac{7e \int \frac{(d+ex)^{5/2}}{(a+bx)^4} dx}{8b} - \frac{(d+ex)^{7/2}}{4b(a+bx)^4} \right)}{10b} - \frac{(d+ex)^{9/2}}{5b(a+bx)^5} \\
 & \quad \downarrow 51 \\
 & \frac{9e \left(\frac{7e \left(\frac{5e \int \frac{(d+ex)^{3/2}}{(a+bx)^3} dx}{6b} - \frac{(d+ex)^{5/2}}{3b(a+bx)^3} \right)}{8b} - \frac{(d+ex)^{7/2}}{4b(a+bx)^4} \right)}{10b} - \frac{(d+ex)^{9/2}}{5b(a+bx)^5} \\
 & \quad \downarrow 51 \\
 & \frac{9e \left(\frac{7e \left(\frac{5e \left(\frac{3e \int \frac{\sqrt{d+ex}}{(a+bx)^2} dx}{4b} - \frac{(d+ex)^{3/2}}{2b(a+bx)^2} \right)}{6b} - \frac{(d+ex)^{5/2}}{3b(a+bx)^3} \right)}{8b} - \frac{(d+ex)^{7/2}}{4b(a+bx)^4} \right)}{10b} - \frac{(d+ex)^{9/2}}{5b(a+bx)^5} \\
 & \quad \downarrow 51
 \end{aligned}$$

$$\left(\begin{array}{l} 5e \left(\frac{3e \left(\frac{e \int \frac{1}{(a+bx)\sqrt{d+ex}} dx}{2b} - \frac{\sqrt{d+ex}}{b(a+bx)} \right)}{4b} - \frac{(d+ex)^{3/2}}{2b(a+bx)^2} \right) \\ 7e \left(\frac{\phantom{3e \left(\frac{e \int \frac{1}{(a+bx)\sqrt{d+ex}} dx}{2b} - \frac{\sqrt{d+ex}}{b(a+bx)} \right)}}{6b} - \frac{(d+ex)^{5/2}}{3b(a+bx)^3} \right) \\ 9e \left(\frac{\phantom{3e \left(\frac{e \int \frac{1}{(a+bx)\sqrt{d+ex}} dx}{2b} - \frac{\sqrt{d+ex}}{b(a+bx)} \right)}}{8b} - \frac{(d+ex)^{7/2}}{4b(a+bx)^4} \right) \end{array} \right) - \frac{(d+ex)^{9/2}}{5b(a+bx)^5}$$

10b

73

$$\left(\begin{array}{l} 5e \left(\frac{3e \left(\frac{\int \frac{1}{a + \frac{b(d+ex) - bd}{e}} d\sqrt{d+ex}}{b} - \frac{\sqrt{d+ex}}{b(a+bx)} \right)}{4b} - \frac{(d+ex)^{3/2}}{2b(a+bx)^2} \right) \\ 7e \left(\frac{\phantom{3e \left(\frac{\int \frac{1}{a + \frac{b(d+ex) - bd}{e}} d\sqrt{d+ex}}{b} - \frac{\sqrt{d+ex}}{b(a+bx)} \right)}}{6b} - \frac{(d+ex)^{5/2}}{3b(a+bx)^3} \right) \\ 9e \left(\frac{\phantom{3e \left(\frac{\int \frac{1}{a + \frac{b(d+ex) - bd}{e}} d\sqrt{d+ex}}{b} - \frac{\sqrt{d+ex}}{b(a+bx)} \right)}}{8b} - \frac{(d+ex)^{7/2}}{4b(a+bx)^4} \right) \end{array} \right) - \frac{(d+ex)^{9/2}}{5b(a+bx)^5}$$

10b

221

$$\left(\frac{3e \left(\frac{e \operatorname{arctanh} \left(\frac{\sqrt{b} \sqrt{d+ex}}{\sqrt{bd-ae}} \right) - \sqrt{d+ex}}{b(a+bx)} \right) - \frac{(d+ex)^{3/2}}{2b(a+bx)^2}}{4b} \right) - \frac{(d+ex)^{5/2}}{3b(a+bx)^3} - \frac{(d+ex)^{7/2}}{4b(a+bx)^4} - \frac{(d+ex)^{9/2}}{5b(a+bx)^5} \right)$$

```
input Int[(d + e*x)^(9/2)/(a^2 + 2*a*b*x + b^2*x^2)^3,x]
```

```
output -1/5*(d + e*x)^(9/2)/(b*(a + b*x)^5) + (9*e*(-1/4*(d + e*x)^(7/2)/(b*(a + b*x)^4) + (7*e*(-1/3*(d + e*x)^(5/2)/(b*(a + b*x)^3) + (5*e*(-1/2*(d + e*x)^(3/2)/(b*(a + b*x)^2) + (3*e*(-(Sqrt[d + e*x]/(b*(a + b*x)))) - (e*ArcTanh[(Sqrt[b]*Sqrt[d + e*x])/Sqrt[b*d - a*e]])/(b^(3/2)*Sqrt[b*d - a*e])))/(4*b)))/(6*b)))/(8*b)))/(10*b)
```

Defintions of rubi rules used

```
rule 27 Int[(a_)*(Fx_), x_Symbol] := Simp[a Int[Fx, x], x] /; FreeQ[a, x] && !MatchQ[Fx, (b_)*(Gx_)] /; FreeQ[b, x]
```

```
rule 51 Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] := Simp[(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Simp[d*(n/(b*(m + 1)))] Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x, x] /; FreeQ[{a, b, c, d, n}, x] && ILtQ[m, -1] && FractionQ[n] && GtQ[n, 0]
```

rule 73 `Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] := With[
 {p = Denominator[m]}, Simp[p/b Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x] /; FreeQ[{a, b, c, d}, x] && Lt
 Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
 inearQ[a, b, c, d, m, n, x]`

rule 221 `Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
 /Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]`

rule 1098 `Int[((d_.) + (e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_
 Symbol] := Simp[1/c^p Int[(d + e*x)^m*(b/2 + c*x)^(2*p), x], x] /; FreeQ[
 {a, b, c, d, e, m}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]`

Maple [A] (verified)

Time = 3.57 (sec) , antiderivative size = 211, normalized size of antiderivative = 1.19

| method | result |
|-------------------|---|
| pseudoelliptic | $\frac{63e^5 (bx+a)^5 \arctan\left(\frac{b\sqrt{ex+d}}{\sqrt{b(ae-bd)}}\right) - 63\sqrt{ex+d} \sqrt{b(ae-bd)} \left(\left(\frac{193}{63}e^4x^4 + \frac{298}{63}de^3x^3 + \frac{152}{35}d^2e^2x^2 + \frac{656}{315}d^3ex + \frac{128}{315}d^4 \right) b^4 + \frac{16e \left(\frac{395}{24} \right)}{b^5 (bx+a)^5 \sqrt{b(ae-bd)}} \right)}{128}$ |
| derivativedivides | $2e^5 \left(\frac{-\frac{193(ex+d)^{\frac{9}{2}}}{256b} - \frac{237(ae-bd)(ex+d)^{\frac{7}{2}}}{128b^2} - \frac{21(e^2a^2-2abde+b^2d^2)(ex+d)^{\frac{5}{2}}}{10b^3} - \frac{147(e^3a^3-3a^2bde^2+3ab^2d^2e-b^3d^3)(ex+d)}{128b^4} - \frac{16e \left(\frac{395}{24} \right)}{b^5 (bx+a)^5 \sqrt{b(ae-bd)}}}{(b(ex+d)+ae-bd)^5} \right)$ |
| default | $2e^5 \left(\frac{-\frac{193(ex+d)^{\frac{9}{2}}}{256b} - \frac{237(ae-bd)(ex+d)^{\frac{7}{2}}}{128b^2} - \frac{21(e^2a^2-2abde+b^2d^2)(ex+d)^{\frac{5}{2}}}{10b^3} - \frac{147(e^3a^3-3a^2bde^2+3ab^2d^2e-b^3d^3)(ex+d)}{128b^4} - \frac{16e \left(\frac{395}{24} \right)}{b^5 (bx+a)^5 \sqrt{b(ae-bd)}}}{(b(ex+d)+ae-bd)^5} \right)$ |

input `int((e*x+d)^(9/2)/(b^2*x^2+2*a*b*x+a^2)^3,x,method=_RETURNVERBOSE)`

output `63/128*(e^5*(b*x+a)^5*arctan(b*(e*x+d)^(1/2)/(b*(a*e-b*d))^(1/2))- (e*x+d)^(1/2)*(b*(a*e-b*d))^(1/2)*((193/63*e^4*x^4+298/63*d*e^3*x^3+152/35*d^2*e^2*x^2+656/315*d^3*e*x+128/315*d^4)*b^4+16/35*e*(395/24*e^3*x^3+289/24*d*e^2*x^2+16/3*d^2*e*x+d^3)*a*b^3+8/15*e^2*(16*e^2*x^2+23/4*d*e*x+d^2)*a^2*b^2+2/3*a^3*e^3*(7*e*x+d)*b+a^4*e^4))/(b*(a*e-b*d))^(1/2)/b^5/(b*x+a)^5`

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 495 vs. $2(146) = 292$.

Time = 0.13 (sec) , antiderivative size = 1003, normalized size of antiderivative = 5.63

$$\int \frac{(d+ex)^{9/2}}{(a^2+2abx+b^2x^2)^3} dx = \text{Too large to display}$$

input `integrate((e*x+d)^(9/2)/(b^2*x^2+2*a*b*x+a^2)^3,x, algorithm="fricas")`

output

```
[1/1280*(315*(b^5*e^5*x^5 + 5*a*b^4*e^5*x^4 + 10*a^2*b^3*e^5*x^3 + 10*a^3*
b^2*e^5*x^2 + 5*a^4*b*e^5*x + a^5*e^5)*sqrt(b^2*d - a*b*e)*log((b*e*x + 2*
b*d - a*e - 2*sqrt(b^2*d - a*b*e)*sqrt(e*x + d))/(b*x + a)) - 2*(128*b^6*d
^5 + 16*a*b^5*d^4*e + 24*a^2*b^4*d^3*e^2 + 42*a^3*b^3*d^2*e^3 + 105*a^4*b^
2*d*e^4 - 315*a^5*b*e^5 + 965*(b^6*d*e^4 - a*b^5*e^5)*x^4 + 10*(149*b^6*d
^2*e^3 + 88*a*b^5*d*e^4 - 237*a^2*b^4*e^5)*x^3 + 6*(228*b^6*d^3*e^2 + 61*a*
b^5*d^2*e^3 + 159*a^2*b^4*d*e^4 - 448*a^3*b^3*e^5)*x^2 + 2*(328*b^6*d^4*e
+ 56*a*b^5*d^3*e^2 + 99*a^2*b^4*d^2*e^3 + 252*a^3*b^3*d*e^4 - 735*a^4*b^2*
e^5)*x)*sqrt(e*x + d))/(a^5*b^7*d - a^6*b^6*e + (b^12*d - a*b^11*e)*x^5 +
5*(a*b^11*d - a^2*b^10*e)*x^4 + 10*(a^2*b^10*d - a^3*b^9*e)*x^3 + 10*(a^3*
b^9*d - a^4*b^8*e)*x^2 + 5*(a^4*b^8*d - a^5*b^7*e)*x), 1/640*(315*(b^5*e^5
*x^5 + 5*a*b^4*e^5*x^4 + 10*a^2*b^3*e^5*x^3 + 10*a^3*b^2*e^5*x^2 + 5*a^4*b
*e^5*x + a^5*e^5)*sqrt(-b^2*d + a*b*e)*arctan(sqrt(-b^2*d + a*b*e)*sqrt(e*
x + d)/(b*e*x + b*d)) - (128*b^6*d^5 + 16*a*b^5*d^4*e + 24*a^2*b^4*d^3*e^2
+ 42*a^3*b^3*d^2*e^3 + 105*a^4*b^2*d*e^4 - 315*a^5*b*e^5 + 965*(b^6*d*e^4
- a*b^5*e^5)*x^4 + 10*(149*b^6*d^2*e^3 + 88*a*b^5*d*e^4 - 237*a^2*b^4*e^5
)*x^3 + 6*(228*b^6*d^3*e^2 + 61*a*b^5*d^2*e^3 + 159*a^2*b^4*d*e^4 - 448*a^
3*b^3*e^5)*x^2 + 2*(328*b^6*d^4*e + 56*a*b^5*d^3*e^2 + 99*a^2*b^4*d^2*e^3
+ 252*a^3*b^3*d*e^4 - 735*a^4*b^2*e^5)*x)*sqrt(e*x + d))/(a^5*b^7*d - a^6*
b^6*e + (b^12*d - a*b^11*e)*x^5 + 5*(a*b^11*d - a^2*b^10*e)*x^4 + 10*(a...
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(d + ex)^{9/2}}{(a^2 + 2abx + b^2x^2)^3} dx = \text{Timed out}$$

input `integrate((e*x+d)**(9/2)/(b**2*x**2+2*a*b*x+a**2)**3,x)`

output Timed out

Maxima [F(-2)]

Exception generated.

$$\int \frac{(d + ex)^{9/2}}{(a^2 + 2abx + b^2x^2)^3} dx = \text{Exception raised: ValueError}$$

input `integrate((e*x+d)^(9/2)/(b^2*x^2+2*a*b*x+a^2)^3,x, algorithm="maxima")`

output Exception raised: ValueError >> Computation failed since Maxima requested additional constraints; using the 'assume' command before evaluation *may* help (example of legal syntax is 'assume(a*e-b*d>0)', see `assume?` for more detail

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 330 vs. 2(146) = 292.

Time = 0.20 (sec) , antiderivative size = 330, normalized size of antiderivative = 1.85

$$\int \frac{(d + ex)^{9/2}}{(a^2 + 2abx + b^2x^2)^3} dx = \frac{63 e^5 \arctan\left(\frac{\sqrt{ex+db}}{\sqrt{-b^2d+abe}}\right)}{128 \sqrt{-b^2d+abe} b^5} - \frac{965 (ex + d)^{\frac{9}{2}} b^4 e^5 - 2370 (ex + d)^{\frac{7}{2}} b^4 d e^5 + 2688 (ex + d)^{\frac{5}{2}} b^4 d^2 e^5 - 1470 (ex + d)^{\frac{3}{2}} b^4 d^3 e^5 + 315 \sqrt{ex + d}}$$

input `integrate((e*x+d)^(9/2)/(b^2*x^2+2*a*b*x+a^2)^3,x, algorithm="giac")`

output
$$\frac{63}{128}e^5 \arctan\left(\frac{\sqrt{e x+d} b}{\sqrt{-b^2 d+a b e}}\right) / \left(\sqrt{-b^2 d+a b e}\right) b^5 - \frac{1}{640} (965 (e x+d)^{9/2} b^4 e^5 - 2370 (e x+d)^{7/2} b^4 d e^5 + 2688 (e x+d)^{5/2} b^4 d^2 e^5 - 1470 (e x+d)^{3/2} b^4 d^3 e^5 + 315 \sqrt{e x+d} b^4 d^4 e^5 + 2370 (e x+d)^{7/2} a b^3 e^6 - 5376 (e x+d)^{5/2} a b^3 d e^6 + 4410 (e x+d)^{3/2} a b^3 d^2 e^6 - 1260 \sqrt{e x+d} a b^3 d^3 e^6 + 2688 (e x+d)^{5/2} a^2 b^2 e^7 - 4410 (e x+d)^{3/2} a^2 b^2 d e^7 + 1890 \sqrt{e x+d} a^2 b^2 d^2 e^7 + 1470 (e x+d)^{3/2} a^3 b e^8 - 1260 \sqrt{e x+d} a^3 b d e^8 + 315 \sqrt{e x+d} a^4 e^9) / (((e x+d) b - b d + a e)^5 b^5)$$

Mupad [B] (verification not implemented)

Time = 9.41 (sec) , antiderivative size = 480, normalized size of antiderivative = 2.70

$$\int \frac{(d+ex)^{9/2}}{(a^2+2abx+b^2x^2)^3} dx = \frac{63e^5 \operatorname{atan}\left(\frac{\sqrt{b}\sqrt{d+ex}}{\sqrt{ae-bd}}\right)}{128b^{11/2}\sqrt{ae-bd}} - \frac{\frac{193e^5(d+ex)^{9/2}}{128b} + \frac{63e^5\sqrt{d+ex}(a^4e^4-4a^3d)}{(d+ex)(5a^4be^4-20a^3b^2de^3+30a^2b^3d^2e^2-20ab^4d^3e+5b^5d^4)} - (d+ex)^2(-10a^3b^2e^3+30a^2b^3d^2e^2)}{(d+ex)(5a^4be^4-20a^3b^2de^3+30a^2b^3d^2e^2-20ab^4d^3e+5b^5d^4)} - (d+ex)^2(-10a^3b^2e^3+30a^2b^3d^2e^2)}$$

input `int((d+e*x)^(9/2)/(a^2+b^2*x^2+2*a*b*x)^3,x)`

output
$$\frac{63e^5 \operatorname{atan}\left(\frac{b^{1/2}(d+ex)^{1/2}}{(ae-bd)^{1/2}}\right)}{(128b^{11/2})(ae-bd)^{1/2}} - \left(\frac{193e^5(d+ex)^{9/2}}{128b} + \frac{63e^5(d+ex)^{1/2}(a^4e^4+b^4d^4+6a^2b^2d^2e^2-4a^3b^3d^3e-4a^3b^3d^3e^3)}{128b^5} + \frac{21e^5(d+ex)^{5/2}(a^2e^2+b^2d^2-2a^2b^2d^2e)}{5b^3} + \frac{147e^5(d+ex)^{3/2}(a^3e^3-b^3d^3+3a^2b^2d^2e-3a^2b^2d^2e^2)}{64b^4} + \frac{237e^5(ae-bd)(d+ex)^{7/2}}{64b^2}\right) / \left((d+ex)(5b^5d^4+5a^4b^4e^4-20a^3b^2d^2e^3+30a^2b^3d^2e^2-20a^2b^4d^3e) - (d+ex)^2(10b^5d^3-10a^3b^2e^3+30a^2b^3d^2e^2-30a^2b^4d^2e) + b^5(d+ex)^5 - (5b^5d-5a^4b^4e)(d+ex)^4 + a^5e^5 - b^5d^5 + (d+ex)^3(10b^5d^2+10a^2b^3e^2-20a^2b^4d^2e) - 10a^2b^3d^3e^2 + 10a^3b^2d^2e^3 + 5a^4b^4d^4e - 5a^4b^4d^4e^4\right)$$

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 779, normalized size of antiderivative = 4.38

$$\int \frac{(d+ex)^{9/2}}{(a^2+2abx+b^2x^2)^3} dx = \frac{128\sqrt{ex+d}b^6d^5 - 315\sqrt{ex+d}a^5be^5 + 315\sqrt{b}\sqrt{ae-bd}\operatorname{atan}\left(\frac{\sqrt{ex+db}}{\sqrt{b}\sqrt{ae-bd}}\right)a^5}{(a^2+2abx+b^2x^2)^3}$$

input `int((e*x+d)^(9/2)/(b^2*x^2+2*a*b*x+a^2)^3,x)`

output

```
(315*sqrt(b)*sqrt(a*e - b*d)*atan((sqrt(d + e*x)*b)/(sqrt(b)*sqrt(a*e - b*d)))*a**5*e**5 + 1575*sqrt(b)*sqrt(a*e - b*d)*atan((sqrt(d + e*x)*b)/(sqrt(b)*sqrt(a*e - b*d)))*a**4*b*e**5*x + 3150*sqrt(b)*sqrt(a*e - b*d)*atan((sqrt(d + e*x)*b)/(sqrt(b)*sqrt(a*e - b*d)))*a**3*b**2*e**5*x**2 + 3150*sqrt(b)*sqrt(a*e - b*d)*atan((sqrt(d + e*x)*b)/(sqrt(b)*sqrt(a*e - b*d)))*a**2*b**3*e**5*x**3 + 1575*sqrt(b)*sqrt(a*e - b*d)*atan((sqrt(d + e*x)*b)/(sqrt(b)*sqrt(a*e - b*d)))*a*b**4*e**5*x**4 + 315*sqrt(b)*sqrt(a*e - b*d)*atan((sqrt(d + e*x)*b)/(sqrt(b)*sqrt(a*e - b*d)))*b**5*e**5*x**5 - 315*sqrt(d + e*x)*a**5*b*e**5 + 105*sqrt(d + e*x)*a**4*b**2*d*e**4 - 1470*sqrt(d + e*x)*a**4*b**2*e**5*x + 42*sqrt(d + e*x)*a**3*b**3*d**2*e**3 + 504*sqrt(d + e*x)*a**3*b**3*d*e**4*x - 2688*sqrt(d + e*x)*a**3*b**3*e**5*x**2 + 24*sqrt(d + e*x)*a**2*b**4*d**3*e**2 + 198*sqrt(d + e*x)*a**2*b**4*d**2*e**3*x + 954*sqrt(d + e*x)*a**2*b**4*d*e**4*x**2 - 2370*sqrt(d + e*x)*a**2*b**4*e**5*x**3 + 16*sqrt(d + e*x)*a*b**5*d**4*e + 112*sqrt(d + e*x)*a*b**5*d**3*e**2*x + 366*sqrt(d + e*x)*a*b**5*d**2*e**3*x**2 + 880*sqrt(d + e*x)*a*b**5*d*e**4*x**3 - 965*sqrt(d + e*x)*a*b**5*e**5*x**4 + 128*sqrt(d + e*x)*b**6*d**5 + 656*sqrt(d + e*x)*b**6*d**4*e*x + 1368*sqrt(d + e*x)*b**6*d**3*e**2*x**2 + 1490*sqrt(d + e*x)*b**6*d**2*e**3*x**3 + 965*sqrt(d + e*x)*b**6*d*e**4*x**4)/(640*b**6*(a**6*e - a**5*b*d + 5*a**5*b*e*x - 5*a**4*b**2*d*x + 10*a**4*b**2*e*x**2 - 10*a**3*b**3*d*x**2 + 10*a**3*b**3*e*x**3 - 10*a...
```

3.193
$$\int \frac{(d+ex)^{7/2}}{(a^2+2abx+b^2x^2)^3} dx$$

| | |
|---|------|
| Optimal result | 1570 |
| Mathematica [A] (verified) | 1570 |
| Rubi [A] (verified) | 1571 |
| Maple [A] (verified) | 1575 |
| Fricas [B] (verification not implemented) | 1575 |
| Sympy [F(-1)] | 1576 |
| Maxima [F(-2)] | 1577 |
| Giac [B] (verification not implemented) | 1577 |
| Mupad [B] (verification not implemented) | 1578 |
| Reduce [B] (verification not implemented) | 1578 |

Optimal result

Integrand size = 28, antiderivative size = 188

$$\int \frac{(d+ex)^{7/2}}{(a^2+2abx+b^2x^2)^3} dx = -\frac{7e^3\sqrt{d+ex}}{64b^4(a+bx)^2} - \frac{7e^4\sqrt{d+ex}}{128b^4(bd-ae)(a+bx)} - \frac{7e^2(d+ex)^{3/2}}{48b^3(a+bx)^3} - \frac{7e(d+ex)^{5/2}}{40b^2(a+bx)^4} - \frac{(d+ex)^{7/2}}{5b(a+bx)^5} + \frac{7e^5 \operatorname{arctanh}\left(\frac{\sqrt{b}\sqrt{d+ex}}{\sqrt{bd-ae}}\right)}{128b^{9/2}(bd-ae)^{3/2}}$$

output

```
-7/64*e^3*(e*x+d)^(1/2)/b^4/(b*x+a)^2-7/128*e^4*(e*x+d)^(1/2)/b^4/(-a*e+b*d)/(b*x+a)-7/48*e^2*(e*x+d)^(3/2)/b^3/(b*x+a)^3-7/40*e*(e*x+d)^(5/2)/b^2/(b*x+a)^4-1/5*(e*x+d)^(7/2)/b/(b*x+a)^5+7/128*e^5*arctanh(b^(1/2)*(e*x+d)^(1/2)/(-a*e+b*d)^(1/2))/b^(9/2)/(-a*e+b*d)^(3/2)
```

Mathematica [A] (verified)

Time = 1.46 (sec) , antiderivative size = 223, normalized size of antiderivative = 1.19

$$\int \frac{(d+ex)^{7/2}}{(a^2+2abx+b^2x^2)^3} dx = \frac{\sqrt{d+ex}(105a^4e^4+70a^3be^3(d+7ex)+14a^2b^2e^2(4d^2+23dex+64e^2x^2)+2e^5\operatorname{arctan}\left(\frac{\sqrt{b}\sqrt{d+ex}}{\sqrt{-bd+ae}}\right))}{128b^9/2(-bd+ae)^{3/2}}$$

input `Integrate[(d + e*x)^(7/2)/(a^2 + 2*a*b*x + b^2*x^2)^3,x]`

output $(\text{Sqrt}[d + e*x]*(105*a^4*e^4 + 70*a^3*b*e^3*(d + 7*e*x) + 14*a^2*b^2*e^2*(4*d^2 + 23*d*e*x + 64*e^2*x^2) + 2*a*b^3*e*(24*d^3 + 128*d^2*e*x + 289*d*e^2*x^2 + 395*e^3*x^3) - b^4*(384*d^4 + 1488*d^3*e*x + 2104*d^2*e^2*x^2 + 1210*d*e^3*x^3 + 105*e^4*x^4)))/(1920*b^4*(b*d - a*e)*(a + b*x)^5) + (7*e^5*\text{ArcTan}[(\text{Sqrt}[b]*\text{Sqrt}[d + e*x])/(\text{Sqrt}[-(b*d) + a*e])]/(128*b^(9/2)*(-(b*d) + a*e)^(3/2))$

Rubi [A] (verified)

Time = 0.48 (sec) , antiderivative size = 204, normalized size of antiderivative = 1.09, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.321$, Rules used = {1098, 27, 51, 51, 51, 51, 52, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\begin{aligned} & \int \frac{(d + ex)^{7/2}}{(a^2 + 2abx + b^2x^2)^3} dx \\ & \quad \downarrow 1098 \\ & b^6 \int \frac{(d + ex)^{7/2}}{b^6(a + bx)^6} dx \\ & \quad \downarrow 27 \\ & \int \frac{(d + ex)^{7/2}}{(a + bx)^6} dx \\ & \quad \downarrow 51 \\ & \frac{7e \int \frac{(d+ex)^{5/2}}{(a+bx)^5} dx}{10b} - \frac{(d + ex)^{7/2}}{5b(a + bx)^5} \\ & \quad \downarrow 51 \\ & \frac{7e \left(\frac{5e \int \frac{(d+ex)^{3/2}}{(a+bx)^4} dx}{8b} - \frac{(d+ex)^{5/2}}{4b(a+bx)^4} \right)}{10b} - \frac{(d + ex)^{7/2}}{5b(a + bx)^5} \end{aligned}$$

$$\begin{array}{c} \downarrow 51 \\ 7e \left(\frac{5e \left(\frac{e \int \frac{\sqrt{d+ex}}{(a+bx)^3} dx}{2b} - \frac{(d+ex)^{3/2}}{3b(a+bx)^3} \right)}{8b} - \frac{(d+ex)^{5/2}}{4b(a+bx)^4} \right) \\ \hline 10b \end{array} - \frac{(d+ex)^{7/2}}{5b(a+bx)^5}$$

$$\begin{array}{c} \downarrow 51 \\ 7e \left(\frac{5e \left(\frac{e \left(\frac{e \int \frac{1}{(a+bx)^2 \sqrt{d+ex}} dx}{4b} - \frac{\sqrt{d+ex}}{2b(a+bx)^2} \right)}{2b} - \frac{(d+ex)^{3/2}}{3b(a+bx)^3} \right)}{8b} - \frac{(d+ex)^{5/2}}{4b(a+bx)^4} \right) \\ \hline 10b \end{array} - \frac{(d+ex)^{7/2}}{5b(a+bx)^5}$$

$$\begin{array}{c} \downarrow 52 \\ 7e \left(\frac{5e \left(\frac{e \left(\frac{e \int \frac{1}{(a+bx)\sqrt{d+ex}} dx}{2(bd-ae)} - \frac{\sqrt{d+ex}}{(a+bx)(bd-ae)} \right)}{4b} - \frac{\sqrt{d+ex}}{2b(a+bx)^2} \right)}{2b} - \frac{(d+ex)^{3/2}}{3b(a+bx)^3} \right)}{8b} - \frac{(d+ex)^{5/2}}{4b(a+bx)^4} \right) \\ \hline 10b \end{array} - \frac{(d+ex)^{7/2}}{5b(a+bx)^5}$$

$$\downarrow 73$$

$$\left(\begin{array}{l} 5e \\ 7e \end{array} \left(\begin{array}{l} e \left(\frac{\int \frac{1}{a + \frac{b(d+ex) - bd}{e}} d\sqrt{d+ex}}{bd - ae} - \frac{\sqrt{d+ex}}{(a+bx)(bd-ae)} \right) \\ \frac{\sqrt{d+ex}}{2b(a+bx)^2} \end{array} \right) - \frac{(d+ex)^{3/2}}{3b(a+bx)^3} \right) - \frac{(d+ex)^{5/2}}{4b(a+bx)^4}$$

$$\frac{10b}{5b(a+bx)^5} (d+ex)^{7/2}$$

221

$$\left(\begin{array}{l} 5e \\ 7e \end{array} \left(\begin{array}{l} e \left(\frac{e \operatorname{arctanh} \left(\frac{\sqrt{b}\sqrt{d+ex}}{\sqrt{bd-ae}} \right)}{\sqrt{b}(bd-ae)^{3/2}} - \frac{\sqrt{d+ex}}{(a+bx)(bd-ae)} \right) \\ \frac{\sqrt{d+ex}}{2b(a+bx)^2} \end{array} \right) - \frac{(d+ex)^{3/2}}{3b(a+bx)^3} \right) - \frac{(d+ex)^{5/2}}{4b(a+bx)^4}$$

$$\frac{10b}{5b(a+bx)^5} (d+ex)^{7/2}$$

input `Int[(d + e*x)^(7/2)/(a^2 + 2*a*b*x + b^2*x^2)^3,x]`

output `-1/5*(d + e*x)^(7/2)/(b*(a + b*x)^5) + (7*e*(-1/4*(d + e*x)^(5/2)/(b*(a + b*x)^4) + (5*e*(-1/3*(d + e*x)^(3/2)/(b*(a + b*x)^3) + (e*(-1/2*Sqrt[d + e*x]/(b*(a + b*x)^2) + (e*(-Sqrt[d + e*x]/((b*d - a*e)*(a + b*x))) + (e*ArcTanh[(Sqrt[b]*Sqrt[d + e*x])/Sqrt[b*d - a*e]])/(Sqrt[b]*(b*d - a*e)^(3/2)))))/(4*b)))/(2*b)))/(8*b)))/(10*b)`

Defintions of rubi rules used

rule 27 `Int[(a_)*(F_x_), x_Symbol] := Simp[a Int[F_x, x], x] /; FreeQ[a, x] && !MatchQ[F_x, (b_)*(G_x_) /; FreeQ[b, x]]`

rule 51 `Int[((a_.) + (b_.)*(x_)^(m_))*((c_.) + (d_.)*(x_)^(n_)), x_Symbol] := Simp[(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Simp[d*(n/(b*(m + 1))) Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d, n}, x] && ILtQ[m, -1] && FractionQ[n] && GtQ[n, 0]`

rule 52 `Int[((a_.) + (b_.)*(x_)^(m_))*((c_.) + (d_.)*(x_)^(n_)), x_Symbol] := Simp[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((m + n + 2)/((b*c - a*d)*(m + 1))) Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && ILtQ[m, -1] && FractionQ[n] && LtQ[n, 0]`

rule 73 `Int[((a_.) + (b_.)*(x_)^(m_))*((c_.) + (d_.)*(x_)^(n_)), x_Symbol] := With[{p = Denominator[m]}, Simp[p/b Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) + d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntLinearQ[a, b, c, d, m, n, x]`

rule 221 `Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[-a/b, 2]/a)*ArcTanh[x/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]`

rule 1098

```
Int[((d_.) + (e_.)*(x_.))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] := Simp[1/c^p Int[(d + e*x)^m*(b^2 + c*x)^(2*p), x], x] /; FreeQ[
{a, b, c, d, e, m}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Maple [A] (verified)

Time = 3.58 (sec) , antiderivative size = 207, normalized size of antiderivative = 1.10

| method | result |
|-------------------|--|
| derivativedivides | $2e^5 \left(\frac{7(ex+d)^{\frac{9}{2}}}{256(ae-bd)} - \frac{79(ex+d)^{\frac{7}{2}}}{384b} - \frac{7(ae-bd)(ex+d)^{\frac{5}{2}}}{30b^2} - \frac{49(e^2a^2-2abde+b^2d^2)(ex+d)^{\frac{3}{2}}}{384b^3} - \frac{7(e^3a^3-3a^2bde^2+3ab^2d^2e-b^3d^3)}{256b^4} \right) \frac{1}{(b(ex+d)+ae-bd)^5}$ |
| default | $2e^5 \left(\frac{7(ex+d)^{\frac{9}{2}}}{256(ae-bd)} - \frac{79(ex+d)^{\frac{7}{2}}}{384b} - \frac{7(ae-bd)(ex+d)^{\frac{5}{2}}}{30b^2} - \frac{49(e^2a^2-2abde+b^2d^2)(ex+d)^{\frac{3}{2}}}{384b^3} - \frac{7(e^3a^3-3a^2bde^2+3ab^2d^2e-b^3d^3)}{256b^4} \right) \frac{1}{(b(ex+d)+ae-bd)^5}$ |
| pseudoelliptic | $\frac{7e^5(bx+a)^5 \arctan\left(\frac{b\sqrt{ex+d}}{\sqrt{b(ae-bd)}}\right) - 7\sqrt{ex+d}\sqrt{b(ae-bd)} \left((-e^4x^4 - \frac{242}{21}de^3x^3 - \frac{2104}{105}d^2e^2x^2 - \frac{496}{35}d^3ex - \frac{128}{35}d^4) b^4 + \frac{16e\left(\frac{395}{24}e^3\right)}{128} \right)}{(ae-bd)b^4(bx+a)^5\sqrt{b(ae-bd)}}$ |

input

```
int((e*x+d)^(7/2)/(b^2*x^2+2*a*b*x+a^2)^3,x,method=_RETURNVERBOSE)
```

output

```
2*e^5*((7/256/(a*e-b*d))*(e*x+d)^(9/2)-79/384/b*(e*x+d)^(7/2)-7/30*(a*e-b*d
)/b^2*(e*x+d)^(5/2)-49/384*(a^2*e^2-2*a*b*d*e+b^2*d^2)/b^3*(e*x+d)^(3/2)-7
/256*(a^3*e^3-3*a^2*b*d*e^2+3*a*b^2*d^2*e-b^3*d^3)/b^4*(e*x+d)^(1/2))/(b*(
e*x+d)+a*e-b*d)^5+7/256/(a*e-b*d)/b^4/(b*(a*e-b*d))^(1/2)*arctan(b*(e*x+d)
^(1/2)/(b*(a*e-b*d))^(1/2))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 572 vs. 2(156) = 312.

Time = 0.12 (sec) , antiderivative size = 1158, normalized size of antiderivative = 6.16

$$\int \frac{(d + ex)^{7/2}}{(a^2 + 2abx + b^2x^2)^3} dx = \text{Too large to display}$$

input `integrate((e*x+d)^(7/2)/(b^2*x^2+2*a*b*x+a^2)^3,x, algorithm="fricas")`

output

```
[-1/3840*(105*(b^5*e^5*x^5 + 5*a*b^4*e^5*x^4 + 10*a^2*b^3*e^5*x^3 + 10*a^3*b^2*e^5*x^2 + 5*a^4*b*e^5*x + a^5*e^5)*sqrt(b^2*d - a*b*e)*log((b*e*x + 2*b*d - a*e - 2*sqrt(b^2*d - a*b*e)*sqrt(e*x + d))/(b*x + a)) + 2*(384*b^6*d^5 - 432*a*b^5*d^4*e - 8*a^2*b^4*d^3*e^2 - 14*a^3*b^3*d^2*e^3 - 35*a^4*b^2*d*e^4 + 105*a^5*b*e^5 + 105*(b^6*d*e^4 - a*b^5*e^5)*x^4 + 10*(121*b^6*d^2*e^3 - 200*a*b^5*d*e^4 + 79*a^2*b^4*e^5)*x^3 + 2*(1052*b^6*d^3*e^2 - 1341*a*b^5*d^2*e^3 - 159*a^2*b^4*d*e^4 + 448*a^3*b^3*e^5)*x^2 + 2*(744*b^6*d^4*e - 872*a*b^5*d^3*e^2 - 33*a^2*b^4*d^2*e^3 - 84*a^3*b^3*d*e^4 + 245*a^4*b^2*e^5)*x)*sqrt(e*x + d))/(a^5*b^7*d^2 - 2*a^6*b^6*d*e + a^7*b^5*e^2 + (b^12*d^2 - 2*a*b^11*d*e + a^2*b^10*e^2)*x^5 + 5*(a*b^11*d^2 - 2*a^2*b^10*d*e + a^3*b^9*e^2)*x^4 + 10*(a^2*b^10*d^2 - 2*a^3*b^9*d*e + a^4*b^8*e^2)*x^3 + 10*(a^3*b^9*d^2 - 2*a^4*b^8*d*e + a^5*b^7*e^2)*x^2 + 5*(a^4*b^8*d^2 - 2*a^5*b^7*d*e + a^6*b^6*e^2)*x), -1/1920*(105*(b^5*e^5*x^5 + 5*a*b^4*e^5*x^4 + 10*a^2*b^3*e^5*x^3 + 10*a^3*b^2*e^5*x^2 + 5*a^4*b*e^5*x + a^5*e^5)*sqrt(-b^2*d + a*b*e)*arctan(sqrt(-b^2*d + a*b*e)*sqrt(e*x + d)/(b*e*x + b*d)) + (384*b^6*d^5 - 432*a*b^5*d^4*e - 8*a^2*b^4*d^3*e^2 - 14*a^3*b^3*d^2*e^3 - 35*a^4*b^2*d*e^4 + 105*a^5*b*e^5 + 105*(b^6*d*e^4 - a*b^5*e^5)*x^4 + 10*(121*b^6*d^2*e^3 - 200*a*b^5*d*e^4 + 79*a^2*b^4*e^5)*x^3 + 2*(1052*b^6*d^3*e^2 - 1341*a*b^5*d^2*e^3 - 159*a^2*b^4*d*e^4 + 448*a^3*b^3*e^5)*x^2 + 2*(744*b^6*d^4*e - 872*a*b^5*d^3*e^2 - 33*a^2*b^4*d^2*e^3 - 84*a^3*b^3*d*e...
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(d + ex)^{7/2}}{(a^2 + 2abx + b^2x^2)^3} dx = \text{Timed out}$$

input `integrate((e*x+d)**(7/2)/(b**2*x**2+2*a*b*x+a**2)**3,x)`

output `Timed out`

Maxima [F(-2)]

Exception generated.

$$\int \frac{(d + ex)^{7/2}}{(a^2 + 2abx + b^2x^2)^3} dx = \text{Exception raised: ValueError}$$

input `integrate((e*x+d)^(7/2)/(b^2*x^2+2*a*b*x+a^2)^3,x, algorithm="maxima")`

output Exception raised: ValueError >> Computation failed since Maxima requested additional constraints; using the 'assume' command before evaluation *may* help (example of legal syntax is 'assume(a*e-b*d>0)', see `assume?` for more detail)

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 354 vs. 2(156) = 312.

Time = 0.20 (sec) , antiderivative size = 354, normalized size of antiderivative = 1.88

$$\int \frac{(d + ex)^{7/2}}{(a^2 + 2abx + b^2x^2)^3} dx = -\frac{7e^5 \arctan\left(\frac{\sqrt{ex+db}}{\sqrt{-b^2d+abe}}\right)}{128(b^5d - ab^4e)\sqrt{-b^2d+abe}} - \frac{105(ex+d)^{\frac{9}{2}}b^4e^5 + 790(ex+d)^{\frac{7}{2}}b^4de^5 - 896(ex+d)^{\frac{5}{2}}b^4d^2e^5 + 490(ex+d)^{\frac{3}{2}}b^4d^3e^5 - 105\sqrt{ex+db}^4d}{128(b^5d - ab^4e)\sqrt{-b^2d+abe}}$$

input `integrate((e*x+d)^(7/2)/(b^2*x^2+2*a*b*x+a^2)^3,x, algorithm="giac")`

output `-7/128*e^5*arctan(sqrt(e*x + d)*b/sqrt(-b^2*d + a*b*e))/((b^5*d - a*b^4*e)*sqrt(-b^2*d + a*b*e)) - 1/1920*(105*(e*x + d)^(9/2)*b^4*e^5 + 790*(e*x + d)^(7/2)*b^4*d*e^5 - 896*(e*x + d)^(5/2)*b^4*d^2*e^5 + 490*(e*x + d)^(3/2)*b^4*d^3*e^5 - 105*sqrt(e*x + d)*b^4*d^4*e^5 - 790*(e*x + d)^(7/2)*a*b^3*e^6 + 1792*(e*x + d)^(5/2)*a*b^3*d*e^6 - 1470*(e*x + d)^(3/2)*a*b^3*d^2*e^6 + 420*sqrt(e*x + d)*a*b^3*d^3*e^6 - 896*(e*x + d)^(5/2)*a^2*b^2*e^7 + 1470*(e*x + d)^(3/2)*a^2*b^2*d*e^7 - 630*sqrt(e*x + d)*a^2*b^2*d^2*e^7 - 490*(e*x + d)^(3/2)*a^3*b*e^8 + 420*sqrt(e*x + d)*a^3*b*d*e^8 - 105*sqrt(e*x + d)*a^4*e^9)/((b^5*d - a*b^4*e)*((e*x + d)*b - b*d + a*e)^5)`

Mupad [B] (verification not implemented)

Time = 9.37 (sec) , antiderivative size = 439, normalized size of antiderivative = 2.34

$$\int \frac{(d+ex)^{7/2}}{(a^2+2abx+b^2x^2)^3} dx = \frac{7e^5 \operatorname{atan}\left(\frac{\sqrt{b}\sqrt{d+ex}}{\sqrt{ae-bd}}\right)}{128b^{9/2}(ae-bd)^{3/2}}$$

$$\frac{79e^5(d+ex)^{7/2}}{192b} - \frac{(d+ex)(5a^4be^4 - 20a^3b^2de^3 + 30a^2b^3d^2e^2 - 20ab^4d^3e + 5b^5d^4) - (d+ex)^2(-10a^3b^2e^3 + 30a^2b^3d^2e^2 - 20ab^4d^3e + 5b^5d^4)}{(d+ex)(5a^4be^4 - 20a^3b^2de^3 + 30a^2b^3d^2e^2 - 20ab^4d^3e + 5b^5d^4) - (d+ex)^2(-10a^3b^2e^3 + 30a^2b^3d^2e^2 - 20ab^4d^3e + 5b^5d^4)}$$

input

```
int((d + e*x)^(7/2)/(a^2 + b^2*x^2 + 2*a*b*x)^3,x)
```

output

```
(7*e^5*atan((b^(1/2)*(d + e*x)^(1/2))/(a*e - b*d)^(1/2)))/(128*b^(9/2)*(a*
e - b*d)^(3/2)) - ((79*e^5*(d + e*x)^(7/2))/(192*b) - (7*e^5*(d + e*x)^(9/
2))/(128*(a*e - b*d)) + (49*e^5*(d + e*x)^(3/2)*(a^2*e^2 + b^2*d^2 - 2*a*b
*d*e))/(192*b^3) + (7*e^5*(d + e*x)^(1/2)*(a^3*e^3 - b^3*d^3 + 3*a*b^2*d^2
*e - 3*a^2*b*d*e^2))/(128*b^4) + (7*e^5*(a*e - b*d)*(d + e*x)^(5/2))/(15*b
^2)))/((d + e*x)*(5*b^5*d^4 + 5*a^4*b*e^4 - 20*a^3*b^2*d*e^3 + 30*a^2*b^3*d
^2*e^2 - 20*a*b^4*d^3*e) - (d + e*x)^2*(10*b^5*d^3 - 10*a^3*b^2*e^3 + 30*a
^2*b^3*d*e^2 - 30*a*b^4*d^2*e) + b^5*(d + e*x)^5 - (5*b^5*d - 5*a*b^4*e)*(
d + e*x)^4 + a^5*e^5 - b^5*d^5 + (d + e*x)^3*(10*b^5*d^2 + 10*a^2*b^3*e^2
- 20*a*b^4*d*e) - 10*a^2*b^3*d^3*e^2 + 10*a^3*b^2*d^2*e^3 + 5*a*b^4*d^4*e
- 5*a^4*b*d*e^4)
```

Reduce [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 874, normalized size of antiderivative = 4.65

$$\int \frac{(d+ex)^{7/2}}{(a^2+2abx+b^2x^2)^3} dx = \text{Too large to display}$$

input

```
int((e*x+d)^(7/2)/(b^2*x^2+2*a*b*x+a^2)^3,x)
```

output

```
(105*sqrt(b)*sqrt(a*e - b*d)*atan((sqrt(d + e*x)*b)/(sqrt(b)*sqrt(a*e - b*d)))
*a**5*e**5 + 525*sqrt(b)*sqrt(a*e - b*d)*atan((sqrt(d + e*x)*b)/(sqrt(b)*sqrt(a*e - b*d)))
*a**4*b*e**5*x + 1050*sqrt(b)*sqrt(a*e - b*d)*atan((sqrt(d + e*x)*b)/(sqrt(b)*sqrt(a*e - b*d)))
*a**3*b**2*e**5*x**2 + 1050*sqrt(b)*sqrt(a*e - b*d)*atan((sqrt(d + e*x)*b)/(sqrt(b)*sqrt(a*e - b*d)))
*a**2*b**3*e**5*x**3 + 525*sqrt(b)*sqrt(a*e - b*d)*atan((sqrt(d + e*x)*b)/(sqrt(b)*sqrt(a*e - b*d)))
*a*b**4*e**5*x**4 + 105*sqrt(b)*sqrt(a*e - b*d)*atan((sqrt(d + e*x)*b)/(sqrt(b)*sqrt(a*e - b*d)))
*b**5*e**5*x**5 - 105*sqrt(d + e*x)*a**5*b*e**5 + 35*sqrt(d + e*x)*a**4*b**2*d*e**4 - 490*sqrt(d + e*x)*a**4*b**2*e**5*x
+ 14*sqrt(d + e*x)*a**3*b**3*d**2*e**3 + 168*sqrt(d + e*x)*a**3*b**3*d*e**4*x - 896*sqrt(d + e*x)*a**3*b**3*e**5*x**2
+ 8*sqrt(d + e*x)*a**2*b**4*d**3*e**2 + 66*sqrt(d + e*x)*a**2*b**4*d**2*e**3*x + 318*sqrt(d + e*x)*a**2*b**4*d*e**4*x**2
- 790*sqrt(d + e*x)*a**2*b**4*e**5*x**3 + 432*sqrt(d + e*x)*a*b**5*d**4*e + 1744*sqrt(d + e*x)*a*b**5*d**3*e**2*x
+ 2682*sqrt(d + e*x)*a*b**5*d**2*e**3*x**2 + 2000*sqrt(d + e*x)*a*b**5*d*e**4*x**3 + 105*sqrt(d + e*x)*a*b**5*e**5*x**4
- 384*sqrt(d + e*x)*b**6*d**5 - 1488*sqrt(d + e*x)*b**6*d**4*e*x - 2104*sqrt(d + e*x)*b**6*d**3*e**2*x**2
- 1210*sqrt(d + e*x)*b**6*d**2*e**3*x**3 - 105*sqrt(d + e*x)*b**6*d*e**4*x**4)/(1920*b**5*(a**7*e**2 - 2*a**6*b*d*e
+ 5*a**6*b*e**2*x + a**5*b**2*d**2 - 10*a**5*b**2*d*e*x + 10*a**5*b**2*e**2*x**2 + 5*a**4*b**3*d**2*...
```


3.194 $\int \frac{(d+ex)^{5/2}}{(a^2+2abx+b^2x^2)^3} dx$

| | |
|---|------|
| Optimal result | 1580 |
| Mathematica [A] (verified) | 1581 |
| Rubi [A] (verified) | 1581 |
| Maple [A] (verified) | 1585 |
| Fricas [B] (verification not implemented) | 1586 |
| Sympy [F(-1)] | 1587 |
| Maxima [F(-2)] | 1587 |
| Giac [B] (verification not implemented) | 1587 |
| Mupad [B] (verification not implemented) | 1588 |
| Reduce [B] (verification not implemented) | 1589 |

Optimal result

Integrand size = 28, antiderivative size = 198

$$\int \frac{(d+ex)^{5/2}}{(a^2+2abx+b^2x^2)^3} dx = -\frac{e^2\sqrt{d+ex}}{16b^3(a+bx)^3} - \frac{e^3\sqrt{d+ex}}{64b^3(bd-ae)(a+bx)^2}$$

$$+ \frac{3e^4\sqrt{d+ex}}{128b^3(bd-ae)^2(a+bx)} - \frac{e(d+ex)^{3/2}}{8b^2(a+bx)^4} - \frac{(d+ex)^{5/2}}{5b(a+bx)^5} - \frac{3e^5 \operatorname{arctanh}\left(\frac{\sqrt{b}\sqrt{d+ex}}{\sqrt{bd-ae}}\right)}{128b^{7/2}(bd-ae)^{5/2}}$$

output

```
-1/16*e^2*(e*x+d)^(1/2)/b^3/(b*x+a)^3-1/64*e^3*(e*x+d)^(1/2)/b^3/(-a*e+b*d)
)/(b*x+a)^2+3/128*e^4*(e*x+d)^(1/2)/b^3/(-a*e+b*d)^2/(b*x+a)-1/8*e*(e*x+d)
^(3/2)/b^2/(b*x+a)^4-1/5*(e*x+d)^(5/2)/b/(b*x+a)^5-3/128*e^5*arctanh(b^(1/
2)*(e*x+d)^(1/2)/(-a*e+b*d)^(1/2))/b^(7/2)/(-a*e+b*d)^(5/2)
```

Mathematica [A] (verified)

Time = 1.38 (sec) , antiderivative size = 222, normalized size of antiderivative = 1.12

$$\int \frac{(d+ex)^{5/2}}{(a^2+2abx+b^2x^2)^3} dx = \frac{\sqrt{d+ex}(15a^4e^4 + 10a^3be^3(d+7ex) + 2a^2b^2e^2(4d^2 + 23dex + 64e^2x^2) - 2ab^3e(88d^3 + 256d^2ex + 233dex^2 + 35e^3x^3) + b^4(128d^4 + 336d^3ex + 248d^2e^2x^2 + 10de^3x^3 - 15e^4x^4))}{640b^3(bd - ae)^2(a + bx)^5} + \frac{3e^5 \arctan\left(\frac{\sqrt{b}\sqrt{d+ex}}{\sqrt{-bd+ae}}\right)}{128b^{7/2}(-bd + ae)^{5/2}}$$

input

```
Integrate[(d + e*x)^(5/2)/(a^2 + 2*a*b*x + b^2*x^2)^3,x]
```

output

```
-1/640*(Sqrt[d + e*x]*(15*a^4*e^4 + 10*a^3*b*e^3*(d + 7*e*x) + 2*a^2*b^2*e^2*(4*d^2 + 23*d*e*x + 64*e^2*x^2) - 2*a*b^3*e*(88*d^3 + 256*d^2*e*x + 233*d*e^2*x^2 + 35*e^3*x^3) + b^4*(128*d^4 + 336*d^3*e*x + 248*d^2*e^2*x^2 + 10*d*e^3*x^3 - 15*e^4*x^4)))/(b^3*(b*d - a*e)^2*(a + b*x)^5) + (3*e^5*ArcTan[(Sqrt[b]*Sqrt[d + e*x])/Sqrt[-(b*d) + a*e]])/(128*b^(7/2)*(-(b*d) + a*e)^(5/2))
```

Rubi [A] (verified)

Time = 0.53 (sec) , antiderivative size = 218, normalized size of antiderivative = 1.10, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.321$, Rules used = {1098, 27, 51, 51, 51, 52, 52, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex)^{5/2}}{(a^2+2abx+b^2x^2)^3} dx$$

↓ 1098

$$b^6 \int \frac{(d+ex)^{5/2}}{b^6(a+bx)^6} dx$$

↓ 27

$$\begin{aligned}
 & \int \frac{(d+ex)^{5/2}}{(a+bx)^6} dx \\
 & \quad \downarrow 51 \\
 & \frac{e \int \frac{(d+ex)^{3/2}}{(a+bx)^5} dx}{2b} - \frac{(d+ex)^{5/2}}{5b(a+bx)^5} \\
 & \quad \downarrow 51 \\
 & \frac{e \left(\frac{3e \int \frac{\sqrt{d+ex}}{(a+bx)^4} dx}{8b} - \frac{(d+ex)^{3/2}}{4b(a+bx)^4} \right)}{2b} - \frac{(d+ex)^{5/2}}{5b(a+bx)^5} \\
 & \quad \downarrow 51 \\
 & \frac{e \left(\frac{3e \left(\frac{e \int \frac{1}{(a+bx)^3 \sqrt{d+ex}} dx}{6b} - \frac{\sqrt{d+ex}}{3b(a+bx)^3} \right)}{8b} - \frac{(d+ex)^{3/2}}{4b(a+bx)^4} \right)}{2b} - \frac{(d+ex)^{5/2}}{5b(a+bx)^5} \\
 & \quad \downarrow 52 \\
 & \frac{e \left(\frac{3e \left(\frac{e \left(-\frac{3e \int \frac{1}{(a+bx)^2 \sqrt{d+ex}} dx}{4(bd-ae)} - \frac{\sqrt{d+ex}}{2(a+bx)^2(bd-ae)} \right) - \frac{\sqrt{d+ex}}{3b(a+bx)^3} \right)}{8b} - \frac{(d+ex)^{3/2}}{4b(a+bx)^4} \right)}{2b} - \frac{(d+ex)^{5/2}}{5b(a+bx)^5} \\
 & \quad \downarrow 52
 \end{aligned}$$

$$e \left(\frac{3e \left(\frac{e \int \frac{1}{(a+bx)\sqrt{d+ex}} dx}{2(bd-ae)} - \frac{\sqrt{d+ex}}{(a+bx)(bd-ae)} \right)}{4(bd-ae)} - \frac{\sqrt{d+ex}}{2(a+bx)^2(bd-ae)} \right) - \frac{\sqrt{d+ex}}{3b(a+bx)^3} - \frac{(d+ex)^{3/2}}{4b(a+bx)^4}$$

$$\frac{2b}{5b(a+bx)^5} (d+ex)^{5/2}$$

73

$$e \left(\frac{3e \left(\frac{\int \frac{1}{a + \frac{b(d+ex) - bd}{e}} d\sqrt{d+ex}}{4(bd-ae)} - \frac{\sqrt{d+ex}}{(a+bx)(bd-ae)} \right)}{6b} - \frac{\sqrt{d+ex}}{2(a+bx)^2(bd-ae)} \right) - \frac{\sqrt{d+ex}}{3b(a+bx)^3} - \frac{(d+ex)^{3/2}}{4b(a+bx)^4}$$

$$\frac{2b}{5b(a+bx)^5} (d+ex)^{5/2}$$

221

$$\begin{aligned}
 & \left(\frac{3e \left(\frac{e \operatorname{arctanh} \left(\frac{\sqrt{b} \sqrt{d+ex}}{\sqrt{bd-ae}} \right) - \frac{\sqrt{d+ex}}{(a+bx)(bd-ae)}}{\sqrt{b}(bd-ae)^{3/2}} \right) - \frac{\sqrt{d+ex}}{2(a+bx)^2(bd-ae)}}{4(bd-ae)} \right)}{6b} - \frac{\sqrt{d+ex}}{3b(a+bx)^3} \right) \\
 & \frac{3e}{8b} - \frac{(d+ex)^{3/2}}{4b(a+bx)^4} \\
 & \frac{2b}{5b(a+bx)^5} (d+ex)^{5/2}
 \end{aligned}$$

input `Int[(d + e*x)^(5/2)/(a^2 + 2*a*b*x + b^2*x^2)^3,x]`

output `-1/5*(d + e*x)^(5/2)/(b*(a + b*x)^5) + (e*(-1/4*(d + e*x)^(3/2)/(b*(a + b*x)^4) + (3*e*(-1/3*sqrt[d + e*x]/(b*(a + b*x)^3) + (e*(-1/2*sqrt[d + e*x]/((b*d - a*e)*(a + b*x)^2) - (3*e*(-sqrt[d + e*x]/((b*d - a*e)*(a + b*x))) + (e*ArcTanh[(sqrt[b]*sqrt[d + e*x])/sqrt[b*d - a*e]])/sqrt[b]*(b*d - a*e)^(3/2))))/(4*(b*d - a*e)))/(6*b))/(8*b))/(2*b)`

Defintions of rubi rules used

rule 27 `Int[(a_)*(Fx_), x_Symbol] := Simp[a Int[Fx, x], x] /; FreeQ[a, x] && !MatchQ[Fx, (b_)*(Gx_)] /; FreeQ[b, x]`

rule 51 `Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] := Simp[(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Simp[d*(n/(b*(m + 1)))*Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d, n}, x] && ILtQ[m, -1] && FractionQ[n] && GtQ[n, 0]`

rule 52 $\text{Int}[(a_.) + (b_.)*(x_.))^{(m_.)}*((c_.) + (d_.)*(x_.))^{(n_.)}, x_Symbol] \rightarrow \text{Simp}[(a + b*x)^{(m + 1)}*((c + d*x)^{(n + 1)} / ((b*c - a*d)*(m + 1))), x] - \text{Simp}[d*((m + n + 2) / ((b*c - a*d)*(m + 1))) \text{Int}[(a + b*x)^{(m + 1)}*(c + d*x)^n, x], x] /;$ FreeQ[{a, b, c, d, n}, x] && ILtQ[m, -1] && FractionQ[n] && LtQ[n, 0]

rule 73 $\text{Int}[(a_.) + (b_.)*(x_.))^{(m_.)}*((c_.) + (d_.)*(x_.))^{(n_.)}, x_Symbol] \rightarrow \text{With}[\{p = \text{Denominator}[m]\}, \text{Simp}[p/b \text{Subst}[\text{Int}[x^{(p*(m + 1) - 1)}*(c - a*(d/b) + d*(x^p/b))^{(n)}, x], x, (a + b*x)^{(1/p)}], x]] /;$ FreeQ[{a, b, c, d}, x] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntLinearQ[a, b, c, d, m, n, x]

rule 221 $\text{Int}[(a_.) + (b_.)*(x_.)^2)^{(-1)}, x_Symbol] \rightarrow \text{Simp}[(\text{Rt}[-a/b, 2]/a)*\text{ArcTanh}[x/\text{Rt}[-a/b, 2]], x] /;$ FreeQ[{a, b}, x] && NegQ[a/b]

rule 1098 $\text{Int}[(d_.) + (e_.)*(x_.))^{(m_.)}*((a_.) + (b_.)*(x_.) + (c_.)*(x_.)^2)^{(p_.)}, x_Symbol] \rightarrow \text{Simp}[1/c^p \text{Int}[(d + e*x)^m*(b/2 + c*x)^{(2*p)}, x], x] /;$ FreeQ[{a, b, c, d, e, m}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]

Maple [A] (verified)

Time = 3.56 (sec) , antiderivative size = 205, normalized size of antiderivative = 1.04

| method | result |
|-------------------|---|
| derivativedivides | $2e^5 \left(\frac{\frac{3b(ex+d)^{\frac{9}{2}}}{256(e^2a^2-2abde+b^2d^2)} + \frac{7(ex+d)^{\frac{7}{2}}}{128(ae-bd)} - \frac{(ex+d)^{\frac{5}{2}}}{10b} - \frac{7(ae-bd)(ex+d)^{\frac{3}{2}}}{128b^2} - \frac{3(e^2a^2-2abde+b^2d^2)\sqrt{ex+d}}{256b^3}}{(b(ex+d)+ae-bd)^5} + \frac{3a}{256b^3(e^2a^2-2abde+b^2d^2)} \right)$ |
| default | $2e^5 \left(\frac{\frac{3b(ex+d)^{\frac{9}{2}}}{256(e^2a^2-2abde+b^2d^2)} + \frac{7(ex+d)^{\frac{7}{2}}}{128(ae-bd)} - \frac{(ex+d)^{\frac{5}{2}}}{10b} - \frac{7(ae-bd)(ex+d)^{\frac{3}{2}}}{128b^2} - \frac{3(e^2a^2-2abde+b^2d^2)\sqrt{ex+d}}{256b^3}}{(b(ex+d)+ae-bd)^5} + \frac{3a}{256b^3(e^2a^2-2abde+b^2d^2)} \right)$ |
| pseudoelliptic | $3 \left(\sqrt{b(ae-bd)} \left((-b^4x^4 - \frac{14}{3}ab^3x^3 + \frac{128}{15}a^2b^2x^2 + \frac{14}{3}a^3bx + a^4) e^4 + \frac{2bd(b^3x^3 - \frac{233}{5}ab^2x^2 + \frac{23}{5}a^2bx + a^3) e^3}{3} + \frac{8b^2d^2(31b^3x^3 - 12ab^2x^2 + 12a^2bx + a^3)}{128\sqrt{b(ae-bd)}(bx+a)^5(ae-bd)} \right) \right)$ |

input $\text{int}((e*x+d)^{(5/2)} / (b^2*x^2 + 2*a*b*x + a^2)^3, x, \text{method}=_RETURNVERBOSE)$

output

```
2*e^5*((3/256*b/(a^2*e^2-2*a*b*d*e+b^2*d^2)*(e*x+d)^(9/2)+7/128/(a*e-b*d)*
(e*x+d)^(7/2)-1/10/b*(e*x+d)^(5/2)-7/128*(a*e-b*d)/b^2*(e*x+d)^(3/2)-3/256
/b^3*(a^2*e^2-2*a*b*d*e+b^2*d^2)*(e*x+d)^(1/2))/(b*(e*x+d)+a*e-b*d)^5+3/25
6/b^3/(a^2*e^2-2*a*b*d*e+b^2*d^2)/(b*(a*e-b*d))^(1/2)*arctan(b*(e*x+d)^(1/
2)/(b*(a*e-b*d))^(1/2)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 662 vs. $2(166) = 332$.

Time = 0.14 (sec) , antiderivative size = 1337, normalized size of antiderivative = 6.75

$$\int \frac{(d+ex)^{5/2}}{(a^2+2abx+b^2x^2)^3} dx = \text{Too large to display}$$

input

```
integrate((e*x+d)^(5/2)/(b^2*x^2+2*a*b*x+a^2)^3,x, algorithm="fricas")
```

output

```
[1/1280*(15*(b^5*e^5*x^5 + 5*a*b^4*e^5*x^4 + 10*a^2*b^3*e^5*x^3 + 10*a^3*b^2
^2*e^5*x^2 + 5*a^4*b*e^5*x + a^5*e^5)*sqrt(b^2*d - a*b*e)*log((b*e*x + 2*b
*d - a*e - 2*sqrt(b^2*d - a*b*e)*sqrt(e*x + d))/(b*x + a)) - 2*(128*b^6*d^
5 - 304*a*b^5*d^4*e + 184*a^2*b^4*d^3*e^2 + 2*a^3*b^3*d^2*e^3 + 5*a^4*b^2*
d*e^4 - 15*a^5*b*e^5 - 15*(b^6*d*e^4 - a*b^5*e^5)*x^4 + 10*(b^6*d^2*e^3 -
8*a*b^5*d*e^4 + 7*a^2*b^4*e^5)*x^3 + 2*(124*b^6*d^3*e^2 - 357*a*b^5*d^2*e^
3 + 297*a^2*b^4*d*e^4 - 64*a^3*b^3*e^5)*x^2 + 2*(168*b^6*d^4*e - 424*a*b^5
*d^3*e^2 + 279*a^2*b^4*d^2*e^3 + 12*a^3*b^3*d*e^4 - 35*a^4*b^2*e^5)*x)*sq
rt(e*x + d))/(a^5*b^7*d^3 - 3*a^6*b^6*d^2*e + 3*a^7*b^5*d*e^2 - a^8*b^4*e^3
+ (b^12*d^3 - 3*a*b^11*d^2*e + 3*a^2*b^10*d*e^2 - a^3*b^9*e^3)*x^5 + 5*(a
*b^11*d^3 - 3*a^2*b^10*d^2*e + 3*a^3*b^9*d*e^2 - a^4*b^8*e^3)*x^4 + 10*(a^
2*b^10*d^3 - 3*a^3*b^9*d^2*e + 3*a^4*b^8*d*e^2 - a^5*b^7*e^3)*x^3 + 10*(a^
3*b^9*d^3 - 3*a^4*b^8*d^2*e + 3*a^5*b^7*d*e^2 - a^6*b^6*e^3)*x^2 + 5*(a^4*
b^8*d^3 - 3*a^5*b^7*d^2*e + 3*a^6*b^6*d*e^2 - a^7*b^5*e^3)*x), 1/640*(15*(
b^5*e^5*x^5 + 5*a*b^4*e^5*x^4 + 10*a^2*b^3*e^5*x^3 + 10*a^3*b^2*e^5*x^2 +
5*a^4*b*e^5*x + a^5*e^5)*sqrt(-b^2*d + a*b*e)*arctan(sqrt(-b^2*d + a*b*e)*
sqrt(e*x + d)/(b*e*x + b*d)) - (128*b^6*d^5 - 304*a*b^5*d^4*e + 184*a^2*b^
4*d^3*e^2 + 2*a^3*b^3*d^2*e^3 + 5*a^4*b^2*d*e^4 - 15*a^5*b*e^5 - 15*(b^6*d
*e^4 - a*b^5*e^5)*x^4 + 10*(b^6*d^2*e^3 - 8*a*b^5*d*e^4 + 7*a^2*b^4*e^5)*x
^3 + 2*(124*b^6*d^3*e^2 - 357*a*b^5*d^2*e^3 + 297*a^2*b^4*d*e^4 - 64*a^...
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(d + ex)^{5/2}}{(a^2 + 2abx + b^2x^2)^3} dx = \text{Timed out}$$

input `integrate((e*x+d)**(5/2)/(b**2*x**2+2*a*b*x+a**2)**3,x)`

output Timed out

Maxima [F(-2)]

Exception generated.

$$\int \frac{(d + ex)^{5/2}}{(a^2 + 2abx + b^2x^2)^3} dx = \text{Exception raised: ValueError}$$

input `integrate((e*x+d)^(5/2)/(b^2*x^2+2*a*b*x+a^2)^3,x, algorithm="maxima")`

output Exception raised: ValueError >> Computation failed since Maxima requested additional constraints; using the 'assume' command before evaluation *may* help (example of legal syntax is 'assume(a*e-b*d>0)', see `assume?` for more detail

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 380 vs. 2(166) = 332.

Time = 0.18 (sec) , antiderivative size = 380, normalized size of antiderivative = 1.92

$$\int \frac{(d + ex)^{5/2}}{(a^2 + 2abx + b^2x^2)^3} dx = \frac{3e^5 \arctan\left(\frac{\sqrt{ex+db}}{\sqrt{-b^2d+abe}}\right)}{128(b^5d^2 - 2ab^4de + a^2b^3e^2)\sqrt{-b^2d+abe}} + \frac{15(ex+d)^{\frac{9}{2}}b^4e^5 - 70(ex+d)^{\frac{7}{2}}b^4de^5 - 128(ex+d)^{\frac{5}{2}}b^4d^2e^5 + 70(ex+d)^{\frac{3}{2}}b^4d^3e^5 - 15\sqrt{ex+db}d^4e^5}{128(b^5d^2 - 2ab^4de + a^2b^3e^2)\sqrt{-b^2d+abe}}$$

input `integrate((e*x+d)^(5/2)/(b^2*x^2+2*a*b*x+a^2)^3,x, algorithm="giac")`

output `3/128*e^5*arctan(sqrt(e*x + d)*b/sqrt(-b^2*d + a*b*e))/((b^5*d^2 - 2*a*b^4*d*e + a^2*b^3*e^2)*sqrt(-b^2*d + a*b*e)) + 1/640*(15*(e*x + d)^(9/2)*b^4*e^5 - 70*(e*x + d)^(7/2)*b^4*d*e^5 - 128*(e*x + d)^(5/2)*b^4*d^2*e^5 + 70*(e*x + d)^(3/2)*b^4*d^3*e^5 - 15*sqrt(e*x + d)*b^4*d^4*e^5 + 70*(e*x + d)^(7/2)*a*b^3*e^6 + 256*(e*x + d)^(5/2)*a*b^3*d*e^6 - 210*(e*x + d)^(3/2)*a*b^3*d^2*e^6 + 60*sqrt(e*x + d)*a*b^3*d^3*e^6 - 128*(e*x + d)^(5/2)*a^2*b^2*e^7 + 210*(e*x + d)^(3/2)*a^2*b^2*d*e^7 - 90*sqrt(e*x + d)*a^2*b^2*d^2*e^7 - 70*(e*x + d)^(3/2)*a^3*b*d*e^8 + 60*sqrt(e*x + d)*a^3*b*d*e^8 - 15*sqrt(e*x + d)*a^4*d^2*e^9)/((b^5*d^2 - 2*a*b^4*d*e + a^2*b^3*e^2)*((e*x + d)*b - b*d + a*e)^5)`

Mupad [B] (verification not implemented)

Time = 9.53 (sec) , antiderivative size = 411, normalized size of antiderivative = 2.08

$$\int \frac{(d + ex)^{5/2}}{(a^2 + 2abx + b^2x^2)^3} dx = \frac{3e^5 \operatorname{atan}\left(\frac{\sqrt{b}\sqrt{d+ex}}{\sqrt{ae-bd}}\right)}{128b^{7/2}(ae-bd)^{5/2}}$$

$$(d + ex) (5a^4be^4 - 20a^3b^2de^3 + 30a^2b^3d^2e^2 - 20ab^4d^3e + 5b^5d^4) - (d + ex)^2 (-10a^3b^2e^3 + 30a^2b^3d^2e^2 - 20ab^4d^3e + 5b^5d^4)$$

input `int((d + e*x)^(5/2)/(a^2 + b^2*x^2 + 2*a*b*x)^3,x)`

output `(3*e^5*atan((b^(1/2)*(d + e*x)^(1/2))/(a*e - b*d)^(1/2)))/(128*b^(7/2)*(a*e - b*d)^(5/2)) - ((e^5*(d + e*x)^(5/2))/(5*b) - (7*e^5*(d + e*x)^(7/2))/(64*(a*e - b*d)) + (3*e^5*(d + e*x)^(1/2)*(a^2*e^2 + b^2*d^2 - 2*a*b*d*e))/(128*b^3) + (7*e^5*(a*e - b*d)*(d + e*x)^(3/2))/(64*b^2) - (3*b*e^5*(d + e*x)^(9/2))/(128*(a*e - b*d)^2))/((d + e*x)*(5*b^5*d^4 + 5*a^4*b*d^4 - 20*a^3*b^2*d*d^3*e^3 + 30*a^2*b^3*d^2*d^2*e^2 - 20*a*b^4*d^3*d^3*e) - (d + e*x)^2*(10*b^5*d^3 - 10*a^3*b^2*d^2*e^3 + 30*a^2*b^3*d*d^2*e^2 - 30*a*b^4*d^2*d^2*e) + b^5*(d + e*x)^5 - (5*b^5*d - 5*a*b^4*d*e)*(d + e*x)^4 + a^5*d^5 - b^5*d^5 + (d + e*x)^3*(10*b^5*d^2 + 10*a^2*b^3*d^2*e^2 - 20*a*b^4*d^2*d^2*e) - 10*a^2*b^3*d^3*d^3*e^2 + 10*a^3*b^2*d^2*d^2*e^3 + 5*a*b^4*d^4*d^4*e - 5*a^4*b*d^4*d^4*e)`

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 973, normalized size of antiderivative = 4.91

$$\int \frac{(d+ex)^{5/2}}{(a^2+2abx+b^2x^2)^3} dx = \text{Too large to display}$$

input `int((e*x+d)^(5/2)/(b^2*x^2+2*a*b*x+a^2)^3,x)`

output

```
(15*sqrt(b)*sqrt(a*e - b*d)*atan((sqrt(d + e*x)*b)/(sqrt(b)*sqrt(a*e - b*d)))
*a**5*e**5 + 75*sqrt(b)*sqrt(a*e - b*d)*atan((sqrt(d + e*x)*b)/(sqrt(b)
*sqrt(a*e - b*d)))
*a**4*b*e**5*x + 150*sqrt(b)*sqrt(a*e - b*d)*atan((sqrt(d + e*x)*b)/(sqrt(b)
*sqrt(a*e - b*d)))
*a**3*b**2*e**5*x**2 + 150*sqrt(b)*sqrt(a*e - b*d)*atan((sqrt(d + e*x)*b)/(sqrt(b)
*sqrt(a*e - b*d)))
*a**2*b**3*e**5*x**3 + 75*sqrt(b)*sqrt(a*e - b*d)*atan((sqrt(d + e*x)*b)/(sqrt(b)
*sqrt(a*e - b*d)))
*a*b**4*e**5*x**4 + 15*sqrt(b)*sqrt(a*e - b*d)*atan((sqrt(d + e*x)*b)/(sqrt(b)
*sqrt(a*e - b*d)))
*b**5*e**5*x**5 - 15*sqrt(d + e*x)*a**5*b*e**5 + 5*sqrt(d + e*x)*a**4*b**2*d*e**4
- 70*sqrt(d + e*x)*a**4*b**2*e**5*x + 2*sqrt(d + e*x)*a**3*b**3*d**2*e**3 + 24*sqrt(d + e*x)
*a**3*b**3*d*e**4*x - 128*sqrt(d + e*x)*a**3*b**3*e**5*x**2 + 184*sqrt(d + e*x)*a**2*
b**4*d**3*e**2 + 558*sqrt(d + e*x)*a**2*b**4*d**2*e**3*x + 594*sqrt(d + e*x)*a**2*
b**4*d*e**4*x**2 + 70*sqrt(d + e*x)*a**2*b**4*e**5*x**3 - 304*sqrt(d + e*x)*a*b**5*
d**4*e - 848*sqrt(d + e*x)*a*b**5*d**3*e**2*x - 714*sqrt(d + e*x)*a*b**5*d**2*
e**3*x**2 - 80*sqrt(d + e*x)*a*b**5*d*e**4*x**3 + 15*sqrt(d + e*x)*a*b**5*e**5*x**4
+ 128*sqrt(d + e*x)*b**6*d**5 + 336*sqrt(d + e*x)*b**6*d**4*e*x + 248*sqrt(d + e*x)
*b**6*d**3*e**2*x**2 + 10*sqrt(d + e*x)*b**6*d**2*e**3*x**3 - 15*sqrt(d + e*x)*b**6*
d*e**4*x**4)/(640*b**4*(a**8*e**3 - 3*a**7*b*d*e**2 + 5*a**7*b*e**3*x + 3*a**6*
b**2*d**2*e - 15*a**6*b**2*d*e**2*x + 10*a**6*b**2*e**3*x**2 - a**5*b**3*d**3 + 15*a**5*
b...
```

3.195
$$\int \frac{(d+ex)^{3/2}}{(a^2+2abx+b^2x^2)^3} dx$$

| | |
|---|------|
| Optimal result | 1590 |
| Mathematica [A] (verified) | 1591 |
| Rubi [A] (verified) | 1591 |
| Maple [A] (verified) | 1596 |
| Fricas [B] (verification not implemented) | 1596 |
| Sympy [F(-1)] | 1597 |
| Maxima [F(-2)] | 1598 |
| Giac [B] (verification not implemented) | 1598 |
| Mupad [B] (verification not implemented) | 1599 |
| Reduce [B] (verification not implemented) | 1599 |

Optimal result

Integrand size = 28, antiderivative size = 208

$$\int \frac{(d+ex)^{3/2}}{(a^2+2abx+b^2x^2)^3} dx = -\frac{3e\sqrt{d+ex}}{40b^2(a+bx)^4} - \frac{e^2\sqrt{d+ex}}{80b^2(bd-ae)(a+bx)^3} + \frac{e^3\sqrt{d+ex}}{64b^2(bd-ae)^2(a+bx)^2} - \frac{3e^4\sqrt{d+ex}}{128b^2(bd-ae)^3(a+bx)} - \frac{(d+ex)^{3/2}}{5b(a+bx)^5} + \frac{3e^5 \operatorname{arctanh}\left(\frac{\sqrt{b}\sqrt{d+ex}}{\sqrt{bd-ae}}\right)}{128b^{5/2}(bd-ae)^{7/2}}$$

output

```
-3/40*e*(e*x+d)^(1/2)/b^2/(b*x+a)^4-1/80*e^2*(e*x+d)^(1/2)/b^2/(-a*e+b*d)/(b*x+a)^3+1/64*e^3*(e*x+d)^(1/2)/b^2/(-a*e+b*d)^2/(b*x+a)^2-3/128*e^4*(e*x+d)^(1/2)/b^2/(-a*e+b*d)^3/(b*x+a)-1/5*(e*x+d)^(3/2)/b/(b*x+a)^5+3/128*e^5*arctanh(b^(1/2)*(e*x+d)^(1/2)/(-a*e+b*d)^(1/2))/b^(5/2)/(-a*e+b*d)^(7/2)
```

Mathematica [A] (verified)

Time = 1.77 (sec) , antiderivative size = 223, normalized size of antiderivative = 1.07

$$\int \frac{(d+ex)^{3/2}}{(a^2+2abx+b^2x^2)^3} dx = \frac{\sqrt{b}\sqrt{d+ex}(-15a^4e^4-10a^3be^3(d+7ex)+2a^2b^2e^2(124d^2+233dex+64e^2x^2)-2ab^3e(168d^3+256d^2ex+23dex^2+3d^2e^2x^2)-2b^4e^2(128d^4+176d^3ex+8d^2e^2x^2-10de^3x^3+15e^4x^4))}{(-bd+ae)^3(a+bx)^5} + \frac{640b^{5/2} \operatorname{ArcTan}\left[\frac{\sqrt{b}\sqrt{d+ex}}{\sqrt{-bd+ae}}\right]}{(-bd+ae)^{7/2}}$$

input `Integrate[(d + e*x)^(3/2)/(a^2 + 2*a*b*x + b^2*x^2)^3,x]`

output

```
((Sqrt[b]*Sqrt[d + e*x]*(-15*a^4*e^4 - 10*a^3*b*e^3*(d + 7*e*x) + 2*a^2*b^2*e^2*(124*d^2 + 233*d*e*x + 64*e^2*x^2) - 2*a*b^3*e*(168*d^3 + 256*d^2*e*x + 23*d*e^2*x^2 - 35*e^3*x^3) + b^4*(128*d^4 + 176*d^3*e*x + 8*d^2*e^2*x^2 - 10*d*e^3*x^3 + 15*e^4*x^4)))/((-b*d) + a*e)^3*(a + b*x)^5) + (15*e^5*ArcTan[(Sqrt[b]*Sqrt[d + e*x])/Sqrt[-(b*d) + a*e]])/((-b*d) + a*e)^(7/2))/(640*b^(5/2))
```

Rubi [A] (verified)Time = 0.54 (sec) , antiderivative size = 232, normalized size of antiderivative = 1.12, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.321$, Rules used = {1098, 27, 51, 51, 52, 52, 52, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex)^{3/2}}{(a^2+2abx+b^2x^2)^3} dx$$

↓ 1098

$$b^6 \int \frac{(d+ex)^{3/2}}{b^6(a+bx)^6} dx$$

↓ 27

$$\int \frac{(d+ex)^{3/2}}{(a+bx)^6} dx$$

↓ 51

$$\begin{aligned}
 & \frac{3e \int \frac{\sqrt{d+ex}}{(a+bx)^5} dx}{10b} - \frac{(d+ex)^{3/2}}{5b(a+bx)^5} \\
 & \quad \downarrow 51 \\
 & \frac{3e \left(\frac{e \int \frac{1}{(a+bx)^4 \sqrt{d+ex}} dx}{8b} - \frac{\sqrt{d+ex}}{4b(a+bx)^4} \right)}{10b} - \frac{(d+ex)^{3/2}}{5b(a+bx)^5} \\
 & \quad \downarrow 52 \\
 & \frac{3e \left(\frac{e \left(-\frac{5e \int \frac{1}{(a+bx)^3 \sqrt{d+ex}} dx}{6(bd-ae)} - \frac{\sqrt{d+ex}}{3(a+bx)^3(bd-ae)} \right)}{8b} - \frac{\sqrt{d+ex}}{4b(a+bx)^4} \right)}{10b} - \frac{(d+ex)^{3/2}}{5b(a+bx)^5} \\
 & \quad \downarrow 52 \\
 & \frac{3e \left(\frac{e \left(-\frac{5e \left(-\frac{3e \int \frac{1}{(a+bx)^2 \sqrt{d+ex}} dx}{4(bd-ae)} - \frac{\sqrt{d+ex}}{2(a+bx)^2(bd-ae)} \right)}{6(bd-ae)} - \frac{\sqrt{d+ex}}{3(a+bx)^3(bd-ae)} \right)}{8b} - \frac{\sqrt{d+ex}}{4b(a+bx)^4} \right)}{10b} - \frac{(d+ex)^{3/2}}{5b(a+bx)^5} \\
 & \quad \downarrow 52 \\
 & \frac{3e \left(\frac{e \left(-\frac{5e \left(\frac{3e \left(-\frac{e \int \frac{1}{(a+bx) \sqrt{d+ex}} dx}{2(bd-ae)} - \frac{\sqrt{d+ex}}{(a+bx)(bd-ae)} \right)}{4(bd-ae)} - \frac{\sqrt{d+ex}}{2(a+bx)^2(bd-ae)} \right)}{6(bd-ae)} - \frac{\sqrt{d+ex}}{3(a+bx)^3(bd-ae)} \right)}{8b} - \frac{\sqrt{d+ex}}{4b(a+bx)^4} \right)}{10b} - \frac{(d+ex)^{3/2}}{5b(a+bx)^5} \\
 & \quad \downarrow 52 \\
 & \frac{10b}{5b(a+bx)^5} (d+ex)^{3/2}
 \end{aligned}$$

73
↓

$$\left(\left(\left(\left(\left(\int \frac{1}{a + \frac{b(d+ex) - bd}{e} - \frac{bd}{bd-ae}} d\sqrt{d+ex} \right) - \frac{\sqrt{d+ex}}{(a+bx)(bd-ae)} \right) - \frac{\sqrt{d+ex}}{2(a+bx)^2(bd-ae)} \right) - \frac{\sqrt{d+ex}}{3(a+bx)^3(bd-ae)} \right) - \frac{\sqrt{d+ex}}{4(bd-ae)} \right) - \frac{\sqrt{d+ex}}{5e} \right) - \frac{\sqrt{d+ex}}{6(bd-ae)} \right) - \frac{\sqrt{d+ex}}{7e} \right) - \frac{\sqrt{d+ex}}{8b} \right) - \frac{\sqrt{d+ex}}{9e} \right) - \frac{\sqrt{d+ex}}{10b(a+bx)^4}$$

$$\frac{10b}{5b(a+bx)^5} \frac{(d+ex)^{3/2}}{(d+ex)^{3/2}}$$

221
↓

$$\left(\frac{e \left(\frac{5e \left(\frac{3e \left(\frac{e \operatorname{arctanh} \left(\frac{\sqrt{b} \sqrt{d+ex}}{\sqrt{bd-ae}} \right) - \frac{\sqrt{d+ex}}{(a+bx)(bd-ae)} \right)}{\sqrt{b}(bd-ae)^{3/2}} - \frac{\sqrt{d+ex}}{2(a+bx)^2(bd-ae)} \right)}{4(bd-ae)} - \frac{\sqrt{d+ex}}{3(a+bx)^3(bd-ae)} \right)}{6(bd-ae)} - \frac{\sqrt{d+ex}}{4b(a+bx)^4} \right)}{8b} - \frac{\sqrt{d+ex}}{4b(a+bx)^4} \right)$$

$$\frac{10b}{5b(a+bx)^5} (d+ex)^{3/2}$$

input `Int[(d + e*x)^(3/2)/(a^2 + 2*a*b*x + b^2*x^2)^3,x]`

output `-1/5*(d + e*x)^(3/2)/(b*(a + b*x)^5) + (3*e*(-1/4*Sqrt[d + e*x]/(b*(a + b*x)^4) + (e*(-1/3*Sqrt[d + e*x]/((b*d - a*e)*(a + b*x)^3) - (5*e*(-1/2*Sqrt[d + e*x]/((b*d - a*e)*(a + b*x)^2) - (3*e*(-(Sqrt[d + e*x]/((b*d - a*e)*(a + b*x)))) + (e*ArcTanh[(Sqrt[b]*Sqrt[d + e*x])/Sqrt[b*d - a*e]])/(Sqrt[b]*(b*d - a*e)^(3/2)))))/(4*(b*d - a*e)))/(6*(b*d - a*e)))/(8*b)))/(10*b)`

Definitions of rubi rules used

- rule 27 $\text{Int}[(a_*)(Fx_), x_Symbol] \rightarrow \text{Simp}[a \text{ Int}[Fx, x], x] /; \text{FreeQ}[a, x] \ \&\& \ !\text{MatchQ}[Fx, (b_*)(Gx_)] /; \text{FreeQ}[b, x]$
- rule 51 $\text{Int}[(a_.) + (b_.)(x_)^{(m_.)} * ((c_.) + (d_.)(x_)^{(n_.)}), x_Symbol] \rightarrow \text{Simp}[(a + b*x)^{(m + 1)} * ((c + d*x)^n / (b*(m + 1))), x] - \text{Simp}[d*(n/(b*(m + 1))) \text{Int}[(a + b*x)^{(m + 1)} * (c + d*x)^{(n - 1)}, x], x] /; \text{FreeQ}[\{a, b, c, d, n\}, x] \ \&\& \ \text{ILtQ}[m, -1] \ \&\& \ \text{FractionQ}[n] \ \&\& \ \text{GtQ}[n, 0]$
- rule 52 $\text{Int}[(a_.) + (b_.)(x_)^{(m_.)} * ((c_.) + (d_.)(x_)^{(n_.)}), x_Symbol] \rightarrow \text{Simp}[(a + b*x)^{(m + 1)} * ((c + d*x)^{(n + 1)} / ((b*c - a*d)*(m + 1))), x] - \text{Simp}[d*((m + n + 2) / ((b*c - a*d)*(m + 1))) \text{Int}[(a + b*x)^{(m + 1)} * (c + d*x)^n, x], x] /; \text{FreeQ}[\{a, b, c, d, n\}, x] \ \&\& \ \text{ILtQ}[m, -1] \ \&\& \ \text{FractionQ}[n] \ \&\& \ \text{LtQ}[n, 0]$
- rule 73 $\text{Int}[(a_.) + (b_.)(x_)^{(m_.)} * ((c_.) + (d_.)(x_)^{(n_.)}), x_Symbol] \rightarrow \text{With}[\{p = \text{Denominator}[m]\}, \text{Simp}[p/b \text{ Subst}[\text{Int}[x^{(p*(m + 1) - 1)} * (c - a*(d/b) + d*(x^{p/b})^n, x], x, (a + b*x)^{(1/p)}, x]] /; \text{FreeQ}[\{a, b, c, d\}, x] \ \&\& \ \text{LtQ}[-1, m, 0] \ \&\& \ \text{LeQ}[-1, n, 0] \ \&\& \ \text{LeQ}[\text{Denominator}[n], \text{Denominator}[m]] \ \&\& \ \text{IntLinearQ}[a, b, c, d, m, n, x]$
- rule 221 $\text{Int}[(a_) + (b_.)(x_)^2)^{-1}, x_Symbol] \rightarrow \text{Simp}[(\text{Rt}[-a/b, 2]/a) * \text{ArcTanh}[x / \text{Rt}[-a/b, 2]], x] /; \text{FreeQ}[\{a, b\}, x] \ \&\& \ \text{NegQ}[a/b]$
- rule 1098 $\text{Int}[(d_.) + (e_.)(x_)^{(m_.)} * ((a_) + (b_.)(x_) + (c_.)(x_)^2)^{(p_.)}, x_Symbol] \rightarrow \text{Simp}[1/c^p \text{ Int}[(d + e*x)^m * (b/2 + c*x)^{(2*p)}, x], x] /; \text{FreeQ}[\{a, b, c, d, e, m\}, x] \ \&\& \ \text{EqQ}[b^2 - 4*a*c, 0] \ \&\& \ \text{IntegerQ}[p]$

Maple [A] (verified)

Time = 1.60 (sec) , antiderivative size = 219, normalized size of antiderivative = 1.05

| method | result |
|-------------------|---|
| pseudoelliptic | $\frac{3\left(\sqrt{b(ae-bd)}\left(-b^4x^4-\frac{14}{3}ab^3x^3-\frac{128}{15}a^2b^2x^2+\frac{14}{3}a^3bx+a^4\right)e^4+\frac{2bd\left(b^3x^3+\frac{23}{5}ab^2x^2-\frac{233}{5}a^2bx+a^3\right)e^3-248\left(\frac{1}{31}b^2x^2-\frac{1}{31}bx+a\right)e^2}{128\sqrt{b(ae-bd)}(bx+a)^5b^2}\right)}{256}$ |
| derivativedivides | $2e^5\left(\frac{\frac{3b^2(ex+d)^{\frac{9}{2}}}{256(e^3a^3-3a^2bde^2+3ab^2d^2e-b^3d^3)}+\frac{7b(ex+d)^{\frac{7}{2}}}{128(e^2a^2-2abde+b^2d^2)}+\frac{(ex+d)^{\frac{5}{2}}}{10ae-10bd}-\frac{7(ex+d)^{\frac{3}{2}}}{128b}-\frac{3(ae-bd)\sqrt{ex+d}}{256b^2}}{(b(ex+d)+ae-bd)^5}+\frac{248}{256}\right)$ |
| default | $2e^5\left(\frac{\frac{3b^2(ex+d)^{\frac{9}{2}}}{256(e^3a^3-3a^2bde^2+3ab^2d^2e-b^3d^3)}+\frac{7b(ex+d)^{\frac{7}{2}}}{128(e^2a^2-2abde+b^2d^2)}+\frac{(ex+d)^{\frac{5}{2}}}{10ae-10bd}-\frac{7(ex+d)^{\frac{3}{2}}}{128b}-\frac{3(ae-bd)\sqrt{ex+d}}{256b^2}}{(b(ex+d)+ae-bd)^5}+\frac{248}{256}\right)$ |

input `int((e*x+d)^(3/2)/(b^2*x^2+2*a*b*x+a^2)^3,x,method=_RETURNVERBOSE)`

output `-3/128/(b*(a*e-b*d))^(1/2)*((b*(a*e-b*d))^(1/2)*((-b^4*x^4-14/3*a*b^3*x^3-128/15*a^2*b^2*x^2+14/3*a^3*b*x+a^4)*e^4+2/3*b*d*(b^3*x^3+23/5*a*b^2*x^2-33/5*a^2*b*x+a^3)*e^3-248/15*(1/31*b^2*x^2-64/31*a*b*x+a^2)*b^2*d^2*e^2+112/5*b^3*(-11/21*b*x+a)*d^3*e-128/15*b^4*d^4)*(e*x+d)^(1/2)-e^5*(b*x+a)^5*arctan(b*(e*x+d)^(1/2)/(b*(a*e-b*d))^(1/2)))/(b*x+a)^5/b^2/(a*e-b*d)^3`

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 739 vs. 2(176) = 352.

Time = 0.14 (sec) , antiderivative size = 1492, normalized size of antiderivative = 7.17

$$\int \frac{(d+ex)^{3/2}}{(a^2+2abx+b^2x^2)^3} dx = \text{Too large to display}$$

input `integrate((e*x+d)^(3/2)/(b^2*x^2+2*a*b*x+a^2)^3,x,algorithm="fricas")`

output

```

[-1/1280*(15*(b^5*e^5*x^5 + 5*a*b^4*e^5*x^4 + 10*a^2*b^3*e^5*x^3 + 10*a^3*
b^2*e^5*x^2 + 5*a^4*b*e^5*x + a^5*e^5)*sqrt(b^2*d - a*b*e)*log((b*e*x + 2*
b*d - a*e - 2*sqrt(b^2*d - a*b*e)*sqrt(e*x + d))/(b*x + a)) + 2*(128*b^6*d
^5 - 464*a*b^5*d^4*e + 584*a^2*b^4*d^3*e^2 - 258*a^3*b^3*d^2*e^3 - 5*a^4*b
^2*d*e^4 + 15*a^5*b*e^5 + 15*(b^6*d*e^4 - a*b^5*e^5)*x^4 - 10*(b^6*d^2*e^3
- 8*a*b^5*d*e^4 + 7*a^2*b^4*e^5)*x^3 + 2*(4*b^6*d^3*e^2 - 27*a*b^5*d^2*e^
3 + 87*a^2*b^4*d*e^4 - 64*a^3*b^3*e^5)*x^2 + 2*(88*b^6*d^4*e - 344*a*b^5*d
^3*e^2 + 489*a^2*b^4*d^2*e^3 - 268*a^3*b^3*d*e^4 + 35*a^4*b^2*e^5)*x)*sqrt
(e*x + d))/(a^5*b^7*d^4 - 4*a^6*b^6*d^3*e + 6*a^7*b^5*d^2*e^2 - 4*a^8*b^4*
d*e^3 + a^9*b^3*e^4 + (b^12*d^4 - 4*a*b^11*d^3*e + 6*a^2*b^10*d^2*e^2 - 4*
a^3*b^9*d*e^3 + a^4*b^8*e^4)*x^5 + 5*(a*b^11*d^4 - 4*a^2*b^10*d^3*e + 6*a^
3*b^9*d^2*e^2 - 4*a^4*b^8*d*e^3 + a^5*b^7*e^4)*x^4 + 10*(a^2*b^10*d^4 - 4*
a^3*b^9*d^3*e + 6*a^4*b^8*d^2*e^2 - 4*a^5*b^7*d*e^3 + a^6*b^6*e^4)*x^3 + 1
0*(a^3*b^9*d^4 - 4*a^4*b^8*d^3*e + 6*a^5*b^7*d^2*e^2 - 4*a^6*b^6*d*e^3 + a
^7*b^5*e^4)*x^2 + 5*(a^4*b^8*d^4 - 4*a^5*b^7*d^3*e + 6*a^6*b^6*d^2*e^2 - 4
*a^7*b^5*d*e^3 + a^8*b^4*e^4)*x), -1/640*(15*(b^5*e^5*x^5 + 5*a*b^4*e^5*x^
4 + 10*a^2*b^3*e^5*x^3 + 10*a^3*b^2*e^5*x^2 + 5*a^4*b*e^5*x + a^5*e^5)*sqr
t(-b^2*d + a*b*e)*arctan(sqrt(-b^2*d + a*b*e)*sqrt(e*x + d)/(b*e*x + b*d))
+ (128*b^6*d^5 - 464*a*b^5*d^4*e + 584*a^2*b^4*d^3*e^2 - 258*a^3*b^3*d^2*
e^3 - 5*a^4*b^2*d*e^4 + 15*a^5*b*e^5 + 15*(b^6*d*e^4 - a*b^5*e^5)*x^4 - ...

```

Sympy [F(-1)]

Timed out.

$$\int \frac{(d + ex)^{3/2}}{(a^2 + 2abx + b^2x^2)^3} dx = \text{Timed out}$$

input

```
integrate((e*x+d)**(3/2)/(b**2*x**2+2*a*b*x+a**2)**3,x)
```

output

Timed out

Maxima [F(-2)]

Exception generated.

$$\int \frac{(d+ex)^{3/2}}{(a^2+2abx+b^2x^2)^3} dx = \text{Exception raised: ValueError}$$

input `integrate((e*x+d)^(3/2)/(b^2*x^2+2*a*b*x+a^2)^3,x, algorithm="maxima")`

output Exception raised: ValueError >> Computation failed since Maxima requested additional constraints; using the 'assume' command before evaluation *may* help (example of legal syntax is 'assume(a*e-b*d>0)', see `assume?` for more detail

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 410 vs. 2(176) = 352.

Time = 0.20 (sec) , antiderivative size = 410, normalized size of antiderivative = 1.97

$$\int \frac{(d+ex)^{3/2}}{(a^2+2abx+b^2x^2)^3} dx = -\frac{3e^5 \arctan\left(\frac{\sqrt{ex+db}}{\sqrt{-b^2d+abe}}\right)}{128(b^5d^3 - 3ab^4d^2e + 3a^2b^3de^2 - a^3b^2e^3)\sqrt{-b^2d+abe}} - \frac{15(ex+d)^{9/2}b^4e^5 - 70(ex+d)^{7/2}b^4de^5 + 128(ex+d)^{5/2}b^4d^2e^5 + 70(ex+d)^{3/2}b^4d^3e^5 - 15\sqrt{ex+db}b^4d^4e^5}{128(b^5d^3 - 3ab^4d^2e + 3a^2b^3de^2 - a^3b^2e^3)\sqrt{-b^2d+abe}}$$

input `integrate((e*x+d)^(3/2)/(b^2*x^2+2*a*b*x+a^2)^3,x, algorithm="giac")`

output `-3/128*e^5*arctan(sqrt(e*x + d)*b/sqrt(-b^2*d + a*b*e))/((b^5*d^3 - 3*a*b^4*d^2*e + 3*a^2*b^3*d*e^2 - a^3*b^2*e^3)*sqrt(-b^2*d + a*b*e)) - 1/640*(15*(e*x + d)^(9/2)*b^4*e^5 - 70*(e*x + d)^(7/2)*b^4*d*e^5 + 128*(e*x + d)^(5/2)*b^4*d^2*e^5 + 70*(e*x + d)^(3/2)*b^4*d^3*e^5 - 15*sqrt(e*x + d)*b^4*d^4*e^5 + 70*(e*x + d)^(7/2)*a*b^3*e^6 - 256*(e*x + d)^(5/2)*a*b^3*d*e^6 - 210*(e*x + d)^(3/2)*a*b^3*d^2*e^6 + 60*sqrt(e*x + d)*a*b^3*d^3*e^6 + 128*(e*x + d)^(5/2)*a^2*b^2*e^7 + 210*(e*x + d)^(3/2)*a^2*b^2*d*e^7 - 90*sqrt(e*x + d)*a^2*b^2*d^2*e^7 - 70*(e*x + d)^(3/2)*a^3*b*e^8 + 60*sqrt(e*x + d)*a^3*b*d*e^8 - 15*sqrt(e*x + d)*a^4*e^9)/((b^5*d^3 - 3*a*b^4*d^2*e + 3*a^2*b^3*d*e^2 - a^3*b^2*e^3)*((e*x + d)*b - b*d + a*e)^5)`

Mupad [B] (verification not implemented)

Time = 9.53 (sec) , antiderivative size = 398, normalized size of antiderivative = 1.91

$$\int \frac{(d+ex)^{3/2}}{(a^2+2abx+b^2x^2)^3} dx = \frac{(d+ex)(5a^4be^4-20a^3b^2de^3+30a^2b^3d^2e^2-20ab^4d^3e+5b^5d^4)-(d+ex)^{5/2}(ae-bd)}{(d+ex)(5a^4be^4-20a^3b^2de^3+30a^2b^3d^2e^2-20ab^4d^3e+5b^5d^4)-(d+ex)^{5/2}(ae-bd)} + \frac{3e^5 \operatorname{atan}\left(\frac{\sqrt{b}\sqrt{d+ex}}{\sqrt{ae-bd}}\right)}{128b^{5/2}(ae-bd)^{7/2}}$$

input `int((d + e*x)^(3/2)/(a^2 + b^2*x^2 + 2*a*b*x)^3,x)`output `((e^5*(d + e*x)^(5/2))/(5*(a*e - b*d)) - (7*e^5*(d + e*x)^(3/2))/(64*b) + (3*b^2*e^5*(d + e*x)^(9/2))/(128*(a*e - b*d)^3) - (3*e^5*(a*e - b*d)*(d + e*x)^(1/2))/(128*b^2) + (7*b*e^5*(d + e*x)^(7/2))/(64*(a*e - b*d)^2))/((d + e*x)*(5*b^5*d^4 + 5*a^4*b*e^4 - 20*a^3*b^2*d*e^3 + 30*a^2*b^3*d^2*e^2 - 20*a*b^4*d^3*e) - (d + e*x)^2*(10*b^5*d^3 - 10*a^3*b^2*e^3 + 30*a^2*b^3*d*e^2 - 20*a*b^4*d^2*e) + b^5*(d + e*x)^5 - (5*b^5*d - 5*a*b^4*e)*(d + e*x)^4 + a^5*e^5 - b^5*d^5 + (d + e*x)^3*(10*b^5*d^2 + 10*a^2*b^3*e^2 - 20*a*b^4*d*e) - 10*a^2*b^3*d^3*e^2 + 10*a^3*b^2*d^2*e^3 + 5*a*b^4*d^4*e - 5*a^4*b*d*e^4) + (3*e^5*atan((b^(1/2)*(d + e*x)^(1/2))/(a*e - b*d)^(1/2)))/(128*b^(5/2)*(a*e - b*d)^(7/2))`**Reduce [B] (verification not implemented)**

Time = 0.23 (sec) , antiderivative size = 1068, normalized size of antiderivative = 5.13

$$\int \frac{(d+ex)^{3/2}}{(a^2+2abx+b^2x^2)^3} dx = \text{Too large to display}$$

input `int((e*x+d)^(3/2)/(b^2*x^2+2*a*b*x+a^2)^3,x)`

output

```
(15*sqrt(b)*sqrt(a*e - b*d)*atan((sqrt(d + e*x)*b)/(sqrt(b)*sqrt(a*e - b*d
)))**5**e**5 + 75*sqrt(b)*sqrt(a*e - b*d)*atan((sqrt(d + e*x)*b)/(sqrt(b)
*sqrt(a*e - b*d)))*a**4*b**e**5*x + 150*sqrt(b)*sqrt(a*e - b*d)*atan((sqrt(
d + e*x)*b)/(sqrt(b)*sqrt(a*e - b*d)))*a**3*b**2**e**5*x**2 + 150*sqrt(b)*s
qrt(a*e - b*d)*atan((sqrt(d + e*x)*b)/(sqrt(b)*sqrt(a*e - b*d)))*a**2*b**3
**e**5*x**3 + 75*sqrt(b)*sqrt(a*e - b*d)*atan((sqrt(d + e*x)*b)/(sqrt(b)*sq
rt(a*e - b*d)))*a*b**4**e**5*x**4 + 15*sqrt(b)*sqrt(a*e - b*d)*atan((sqrt(d
 + e*x)*b)/(sqrt(b)*sqrt(a*e - b*d)))*b**5**e**5*x**5 - 15*sqrt(d + e*x)*a*
**5*b**e**5 + 5*sqrt(d + e*x)*a**4*b**2*d**e**4 - 70*sqrt(d + e*x)*a**4*b**2*
**e**5*x + 258*sqrt(d + e*x)*a**3*b**3*d**2**e**3 + 536*sqrt(d + e*x)*a**3*b*
**3*d**e**4*x + 128*sqrt(d + e*x)*a**3*b**3**e**5*x**2 - 584*sqrt(d + e*x)*a*
**2*b**4*d**3**e**2 - 978*sqrt(d + e*x)*a**2*b**4*d**2**e**3*x - 174*sqrt(d +
e*x)*a**2*b**4*d**e**4*x**2 + 70*sqrt(d + e*x)*a**2*b**4**e**5*x**3 + 464*s
qrt(d + e*x)*a*b**5*d**4**e + 688*sqrt(d + e*x)*a*b**5*d**3**e**2*x + 54*sq
rt(d + e*x)*a*b**5*d**2**e**3*x**2 - 80*sqrt(d + e*x)*a*b**5*d**e**4*x**3 + 1
5*sqrt(d + e*x)*a*b**5**e**5*x**4 - 128*sqrt(d + e*x)*b**6*d**5 - 176*sqrt(
d + e*x)*b**6*d**4*e*x - 8*sqrt(d + e*x)*b**6*d**3**e**2*x**2 + 10*sqrt(d +
e*x)*b**6*d**2**e**3*x**3 - 15*sqrt(d + e*x)*b**6*d**e**4*x**4)/(640*b**3*(
a**9**e**4 - 4*a**8*b*d**e**3 + 5*a**8*b**e**4*x + 6*a**7*b**2*d**2**e**2 - 20
*a**7*b**2*d**e**3*x + 10*a**7*b**2**e**4*x**2 - 4*a**6*b**3*d**3**e + 30*...
```

3.196
$$\int \frac{\sqrt{d+ex}}{(a^2+2abx+b^2x^2)^3} dx$$

| | |
|---|------|
| Optimal result | 1601 |
| Mathematica [A] (verified) | 1602 |
| Rubi [A] (verified) | 1602 |
| Maple [A] (verified) | 1607 |
| Fricas [B] (verification not implemented) | 1607 |
| Sympy [F(-1)] | 1608 |
| Maxima [F(-2)] | 1609 |
| Giac [B] (verification not implemented) | 1609 |
| Mupad [B] (verification not implemented) | 1610 |
| Reduce [B] (verification not implemented) | 1611 |

Optimal result

Integrand size = 28, antiderivative size = 218

$$\int \frac{\sqrt{d+ex}}{(a^2+2abx+b^2x^2)^3} dx = -\frac{\sqrt{d+ex}}{5b(a+bx)^5} - \frac{e\sqrt{d+ex}}{40b(bd-ae)(a+bx)^4} + \frac{7e^2\sqrt{d+ex}}{240b(bd-ae)^2(a+bx)^3} - \frac{7e^3\sqrt{d+ex}}{192b(bd-ae)^3(a+bx)^2} + \frac{7e^4\sqrt{d+ex}}{128b(bd-ae)^4(a+bx)} - \frac{7e^5 \operatorname{arctanh}\left(\frac{\sqrt{b}\sqrt{d+ex}}{\sqrt{bd-ae}}\right)}{128b^{3/2}(bd-ae)^{9/2}}$$

output

```
-1/5*(e*x+d)^(1/2)/b/(b*x+a)^5-1/40*e*(e*x+d)^(1/2)/b/(-a*e+b*d)/(b*x+a)^4
+7/240*e^2*(e*x+d)^(1/2)/b/(-a*e+b*d)^2/(b*x+a)^3-7/192*e^3*(e*x+d)^(1/2)/
b/(-a*e+b*d)^3/(b*x+a)^2+7/128*e^4*(e*x+d)^(1/2)/b/(-a*e+b*d)^4/(b*x+a)-7/
128*e^5*arctanh(b^(1/2)*(e*x+d)^(1/2)/(-a*e+b*d)^(1/2))/b^(3/2)/(-a*e+b*d)
^(9/2)
```

Mathematica [A] (verified)

Time = 1.56 (sec) , antiderivative size = 224, normalized size of antiderivative = 1.03

$$\int \frac{\sqrt{d+ex}}{(a^2+2abx+b^2x^2)^3} dx$$

$$= \frac{\sqrt{d+ex}(-105a^4e^4 + 10a^3be^3(121d + 79ex) + 2a^2b^2e^2(-1052d^2 - 289dex + 448e^2x^2) + 2ab^3e(744d^3 + 1920b(bd - ae)^4(a + bx)^2) + 7e^5 \arctan\left(\frac{\sqrt{b}\sqrt{d+ex}}{\sqrt{-bd+ae}}\right)}{128b^{3/2}(-bd + ae)^{9/2}}$$

input

```
Integrate[Sqrt[d + e*x]/(a^2 + 2*a*b*x + b^2*x^2)^3,x]
```

output

```
(Sqrt[d + e*x]*(-105*a^4*e^4 + 10*a^3*b*e^3*(121*d + 79*e*x) + 2*a^2*b^2*e^2*(-1052*d^2 - 289*d*e*x + 448*e^2*x^2) + 2*a*b^3*e*(744*d^3 + 128*d^2*e*x - 161*d*e^2*x^2 + 245*e^3*x^3) + b^4*(-384*d^4 - 48*d^3*e*x + 56*d^2*e^2*x^2 - 70*d*e^3*x^3 + 105*e^4*x^4))/(1920*b*(b*d - a*e)^4*(a + b*x)^5 + (7*e^5*ArcTan[(Sqrt[b]*Sqrt[d + e*x])/Sqrt[-(b*d) + a*e]])/(128*b^(3/2)*(-(b*d) + a*e)^(9/2))
```

Rubi [A] (verified)

Time = 0.56 (sec) , antiderivative size = 246, normalized size of antiderivative = 1.13, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.321$, Rules used = {1098, 27, 51, 52, 52, 52, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{d+ex}}{(a^2+2abx+b^2x^2)^3} dx$$

$$\downarrow 1098$$

$$b^6 \int \frac{\sqrt{d+ex}}{b^6(a+bx)^6} dx$$

$$\downarrow 27$$

$$\begin{aligned}
 & \int \frac{\sqrt{d+ex}}{(a+bx)^6} dx \\
 & \quad \downarrow 51 \\
 & \frac{e \int \frac{1}{(a+bx)^5 \sqrt{d+ex}} dx}{10b} - \frac{\sqrt{d+ex}}{5b(a+bx)^5} \\
 & \quad \downarrow 52 \\
 & \frac{e \left(-\frac{7e \int \frac{1}{(a+bx)^4 \sqrt{d+ex}} dx}{8(bd-ae)} - \frac{\sqrt{d+ex}}{4(a+bx)^4(bd-ae)} \right)}{10b} - \frac{\sqrt{d+ex}}{5b(a+bx)^5} \\
 & \quad \downarrow 52 \\
 & \frac{e \left(-\frac{7e \left(-\frac{5e \int \frac{1}{(a+bx)^3 \sqrt{d+ex}} dx}{6(bd-ae)} - \frac{\sqrt{d+ex}}{3(a+bx)^3(bd-ae)} \right)}{8(bd-ae)} - \frac{\sqrt{d+ex}}{4(a+bx)^4(bd-ae)} \right)}{10b} - \frac{\sqrt{d+ex}}{5b(a+bx)^5} \\
 & \quad \downarrow 52 \\
 & e \left(-\frac{7e \left(-\frac{5e \left(-\frac{3e \int \frac{1}{(a+bx)^2 \sqrt{d+ex}} dx}{4(bd-ae)} - \frac{\sqrt{d+ex}}{2(a+bx)^2(bd-ae)} \right)}{6(bd-ae)} - \frac{\sqrt{d+ex}}{3(a+bx)^3(bd-ae)} \right)}{8(bd-ae)} - \frac{\sqrt{d+ex}}{4(a+bx)^4(bd-ae)} \right) \\
 & \quad \downarrow 52 \\
 & \frac{10b}{5b(a+bx)^5} \frac{\sqrt{d+ex}}{5b(a+bx)^5} \\
 & \quad \downarrow 52
 \end{aligned}$$

$$\left(\begin{array}{l} 5e \left(\frac{3e \left(-\frac{e \int \frac{1}{(a+bx)\sqrt{d+ex}} dx}{2(bd-ae)} - \frac{\sqrt{d+ex}}{(a+bx)(bd-ae)} \right)}{4(bd-ae)} - \frac{\sqrt{d+ex}}{2(a+bx)^2(bd-ae)} \right) \\ 7e \left(\frac{\hspace{10em}}{6(bd-ae)} - \frac{\sqrt{d+ex}}{3(a+bx)^3(bd-ae)} \right) \\ e \left(\frac{\hspace{15em}}{8(bd-ae)} - \frac{\sqrt{d+ex}}{4(a+bx)^4(bd-ae)} \right) \end{array} \right)$$

$$\frac{10b}{\sqrt{d+ex}} \frac{1}{5b(a+bx)^5}$$

↓ 73

$$\left(\begin{array}{l} 5e \left(\frac{3e \left(-\frac{\int \frac{1}{a+\frac{b(d+ex)-bd}{e}} d\sqrt{d+ex}}{bd-ae} - \frac{\sqrt{d+ex}}{(a+bx)(bd-ae)} \right)}{4(bd-ae)} - \frac{\sqrt{d+ex}}{2(a+bx)^2(bd-ae)} \right) \\ 7e \left(\frac{\hspace{10em}}{6(bd-ae)} - \frac{\sqrt{d+ex}}{3(a+bx)^3(bd-ae)} \right) \\ e \left(\frac{\hspace{15em}}{8(bd-ae)} - \frac{\sqrt{d+ex}}{4(a+bx)^4(bd-ae)} \right) \end{array} \right)$$

$$\frac{10b}{\sqrt{d+ex}} \frac{1}{5b(a+bx)^5}$$

↓ 221

$$\begin{aligned}
 & \left(\begin{aligned}
 & \left(\begin{aligned}
 & \left(\begin{aligned}
 & \frac{3e \left(\frac{e \operatorname{arctanh} \left(\frac{\sqrt{b} \sqrt{d+ex}}{\sqrt{bd-ae}} \right) - \frac{\sqrt{d+ex}}{(a+bx)(bd-ae)}}{\sqrt{b}(bd-ae)^{3/2}} \right)}{4(bd-ae)} - \frac{\sqrt{d+ex}}{2(a+bx)^2(bd-ae)} \right) \\
 & \frac{5e}{6(bd-ae)} - \frac{\sqrt{d+ex}}{3(a+bx)^3(bd-ae)} \\
 & \frac{7e}{8(bd-ae)} - \frac{\sqrt{d+ex}}{4(a+bx)^4(bd-ae)} \\
 & \frac{e}{10b} \frac{\sqrt{d+ex}}{5b(a+bx)^5}
 \end{aligned} \right) \\
 & \frac{5e}{6(bd-ae)} - \frac{\sqrt{d+ex}}{3(a+bx)^3(bd-ae)} \\
 & \frac{7e}{8(bd-ae)} - \frac{\sqrt{d+ex}}{4(a+bx)^4(bd-ae)} \\
 & \frac{e}{10b} \frac{\sqrt{d+ex}}{5b(a+bx)^5}
 \end{aligned} \right) \\
 & \frac{e}{10b} \frac{\sqrt{d+ex}}{5b(a+bx)^5}
 \end{aligned}
 \end{aligned}$$

input `Int[Sqrt[d + e*x]/(a^2 + 2*a*b*x + b^2*x^2)^3,x]`

output `-1/5*Sqrt[d + e*x]/(b*(a + b*x)^5) + (e*(-1/4*Sqrt[d + e*x]/((b*d - a*e)*(a + b*x)^4) - (7*e*(-1/3*Sqrt[d + e*x]/((b*d - a*e)*(a + b*x)^3) - (5*e*(-1/2*Sqrt[d + e*x]/((b*d - a*e)*(a + b*x)^2) - (3*e*(-(Sqrt[d + e*x]/((b*d - a*e)*(a + b*x)))) + (e*ArcTanh[(Sqrt[b]*Sqrt[d + e*x])/Sqrt[b*d - a*e]])/(Sqrt[b]*(b*d - a*e)^(3/2))))/(4*(b*d - a*e)))/(6*(b*d - a*e)))/(8*(b*d - a*e)))/(10*b)`

Definitions of rubi rules used

- rule 27 $\text{Int}[(a_*)(Fx_), x_Symbol] \rightarrow \text{Simp}[a \text{ Int}[Fx, x], x] /; \text{FreeQ}[a, x] \ \&\& \ !\text{MatchQ}[Fx, (b_*)(Gx_)] /; \text{FreeQ}[b, x]$
- rule 51 $\text{Int}[(a_.) + (b_.)(x_)^{(m_.)} * ((c_.) + (d_.)(x_)^{(n_.)}), x_Symbol] \rightarrow \text{Simp}[(a + b*x)^{(m + 1)} * ((c + d*x)^n / (b*(m + 1))), x] - \text{Simp}[d*(n/(b*(m + 1))) \text{Int}[(a + b*x)^{(m + 1)} * (c + d*x)^{(n - 1)}, x], x] /; \text{FreeQ}[\{a, b, c, d, n\}, x] \ \&\& \ \text{ILtQ}[m, -1] \ \&\& \ \text{FractionQ}[n] \ \&\& \ \text{GtQ}[n, 0]$
- rule 52 $\text{Int}[(a_.) + (b_.)(x_)^{(m_.)} * ((c_.) + (d_.)(x_)^{(n_.)}), x_Symbol] \rightarrow \text{Simp}[(a + b*x)^{(m + 1)} * ((c + d*x)^{(n + 1)} / ((b*c - a*d)*(m + 1))), x] - \text{Simp}[d*((m + n + 2) / ((b*c - a*d)*(m + 1))) \text{Int}[(a + b*x)^{(m + 1)} * (c + d*x)^n, x], x] /; \text{FreeQ}[\{a, b, c, d, n\}, x] \ \&\& \ \text{ILtQ}[m, -1] \ \&\& \ \text{FractionQ}[n] \ \&\& \ \text{LtQ}[n, 0]$
- rule 73 $\text{Int}[(a_.) + (b_.)(x_)^{(m_.)} * ((c_.) + (d_.)(x_)^{(n_.)}), x_Symbol] \rightarrow \text{With}[\{p = \text{Denominator}[m]\}, \text{Simp}[p/b \text{ Subst}[\text{Int}[x^{(p*(m + 1) - 1)} * (c - a*(d/b) + d*(x^{p/b})^n), x], x, (a + b*x)^{(1/p)}, x]] /; \text{FreeQ}[\{a, b, c, d\}, x] \ \&\& \ \text{LtQ}[-1, m, 0] \ \&\& \ \text{LeQ}[-1, n, 0] \ \&\& \ \text{LeQ}[\text{Denominator}[n], \text{Denominator}[m]] \ \&\& \ \text{IntLinearQ}[a, b, c, d, m, n, x]$
- rule 221 $\text{Int}[(a_.) + (b_.)(x_)^2)^{-1}, x_Symbol] \rightarrow \text{Simp}[(\text{Rt}[-a/b, 2]/a) * \text{ArcTanh}[x / \text{Rt}[-a/b, 2]], x] /; \text{FreeQ}[\{a, b\}, x] \ \&\& \ \text{NegQ}[a/b]$
- rule 1098 $\text{Int}[(d_.) + (e_.)(x_)^{(m_.)} * ((a_.) + (b_.)(x_) + (c_.)(x_)^2)^{(p_.)}, x_Symbol] \rightarrow \text{Simp}[1/c^p \text{ Int}[(d + e*x)^m * (b/2 + c*x)^{(2*p)}, x], x] /; \text{FreeQ}[\{a, b, c, d, e, m\}, x] \ \&\& \ \text{EqQ}[b^2 - 4*a*c, 0] \ \&\& \ \text{IntegerQ}[p]$

Maple [A] (verified)

Time = 1.65 (sec) , antiderivative size = 220, normalized size of antiderivative = 1.01

| method | result |
|-------------------|--|
| pseudoelliptic | $\frac{7 \left(\left((-b^4 x^4 - \frac{14}{3} a b^3 x^3 - \frac{128}{15} a^2 b^2 x^2 - \frac{158}{21} a^3 b x + a^4) e^4 - \frac{242 \left(-\frac{7}{121} b^3 x^3 - \frac{161}{605} a b^2 x^2 - \frac{289}{605} a^2 b x + a^3 \right) b d e^3 + 2104 b^2 \left(-\frac{7}{263} b^2 x^2 - 32 \frac{a b x + a^2}{263} \right) d^2 e^2 - 496 \frac{b^3 (e x + d)^{\frac{9}{2}}}{35 \left(-\frac{1}{31} b x + a \right) b^3 d^3 e + 128 \frac{b^4 d^4}{35}} \right) (b (a e - b d))^{\frac{1}{2}}}{128 \sqrt{b(ae-bd)} (bx+a)^5} + \frac{7 b^3 (ex+d)^{\frac{9}{2}}}{256 (a^4 e^4 - 4 a^3 b d e^3 + 6 a^2 b^2 d^2 e^2 - 4 a b^3 d^3 e + b^4 d^4)} + \frac{49 b^2 (ex+d)^{\frac{7}{2}}}{384 (e^3 a^3 - 3 a^2 b d e^2 + 3 a b^2 d^2 e - b^3 d^3)} + \frac{7 b (ex+d)^{\frac{5}{2}}}{30 (e^2 a^2 - 2 a b d e + b^2 d^2)} + \frac{7}{30} \right) (bx+a)^5$ |
| derivativedivides | $2e^5 \left(\frac{7 b^3 (ex+d)^{\frac{9}{2}}}{256 (a^4 e^4 - 4 a^3 b d e^3 + 6 a^2 b^2 d^2 e^2 - 4 a b^3 d^3 e + b^4 d^4)} + \frac{49 b^2 (ex+d)^{\frac{7}{2}}}{384 (e^3 a^3 - 3 a^2 b d e^2 + 3 a b^2 d^2 e - b^3 d^3)} + \frac{7 b (ex+d)^{\frac{5}{2}}}{30 (e^2 a^2 - 2 a b d e + b^2 d^2)} + \frac{7}{30} \right) (bx+a)^5$ |
| default | $2e^5 \left(\frac{7 b^3 (ex+d)^{\frac{9}{2}}}{256 (a^4 e^4 - 4 a^3 b d e^3 + 6 a^2 b^2 d^2 e^2 - 4 a b^3 d^3 e + b^4 d^4)} + \frac{49 b^2 (ex+d)^{\frac{7}{2}}}{384 (e^3 a^3 - 3 a^2 b d e^2 + 3 a b^2 d^2 e - b^3 d^3)} + \frac{7 b (ex+d)^{\frac{5}{2}}}{30 (e^2 a^2 - 2 a b d e + b^2 d^2)} + \frac{7}{30} \right) (bx+a)^5$ |

input `int((e*x+d)^(1/2)/(b^2*x^2+2*a*b*x+a^2)^3,x,method=_RETURNVERBOSE)`

output `-7/128/(b*(a*e-b*d))^(1/2)*(((b^4*x^4-14/3*a*b^3*x^3-128/15*a^2*b^2*x^2-158/21*a^3*b*x+a^4)*e^4-242/21*(-7/121*b^3*x^3-161/605*a*b^2*x^2-289/605*a^2*b*x+a^3)*b*d*e^3+2104/105*b^2*(-7/263*b^2*x^2-32/263*a*b*x+a^2)*d^2*e^2-496/35*(-1/31*b*x+a)*b^3*d^3*e+128/35*b^4*d^4)*(b*(a*e-b*d))^(1/2)*(e*x+d)^(1/2)-e^5*(b*x+a)^5*arctan(b*(e*x+d)^(1/2)/(b*(a*e-b*d))^(1/2)))/(b*x+a)^5/b/(a*e-b*d)^4`

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 830 vs. 2(186) = 372.

Time = 0.15 (sec) , antiderivative size = 1673, normalized size of antiderivative = 7.67

$$\int \frac{\sqrt{d+ex}}{(a^2+2abx+b^2x^2)^3} dx = \text{Too large to display}$$

input `integrate((e*x+d)^(1/2)/(b^2*x^2+2*a*b*x+a^2)^3,x, algorithm="fricas")`

output

```
[1/3840*(105*(b^5*e^5*x^5 + 5*a*b^4*e^5*x^4 + 10*a^2*b^3*e^5*x^3 + 10*a^3*
b^2*e^5*x^2 + 5*a^4*b*e^5*x + a^5*e^5)*sqrt(b^2*d - a*b*e)*log((b*e*x + 2*
b*d - a*e - 2*sqrt(b^2*d - a*b*e)*sqrt(e*x + d))/(b*x + a)) - 2*(384*b^6*d
^5 - 1872*a*b^5*d^4*e + 3592*a^2*b^4*d^3*e^2 - 3314*a^3*b^3*d^2*e^3 + 1315
*a^4*b^2*d*e^4 - 105*a^5*b*e^5 - 105*(b^6*d*e^4 - a*b^5*e^5)*x^4 + 70*(b^6
*d^2*e^3 - 8*a*b^5*d*e^4 + 7*a^2*b^4*e^5)*x^3 - 14*(4*b^6*d^3*e^2 - 27*a*b
^5*d^2*e^3 + 87*a^2*b^4*d*e^4 - 64*a^3*b^3*e^5)*x^2 + 2*(24*b^6*d^4*e - 15
2*a*b^5*d^3*e^2 + 417*a^2*b^4*d^2*e^3 - 684*a^3*b^3*d*e^4 + 395*a^4*b^2*e^
5)*x)*sqrt(e*x + d)/(a^5*b^7*d^5 - 5*a^6*b^6*d^4*e + 10*a^7*b^5*d^3*e^2 -
10*a^8*b^4*d^2*e^3 + 5*a^9*b^3*d*e^4 - a^10*b^2*e^5 + (b^12*d^5 - 5*a*b^1
1*d^4*e + 10*a^2*b^10*d^3*e^2 - 10*a^3*b^9*d^2*e^3 + 5*a^4*b^8*d*e^4 - a^5
*b^7*e^5)*x^5 + 5*(a*b^11*d^5 - 5*a^2*b^10*d^4*e + 10*a^3*b^9*d^3*e^2 - 10
*a^4*b^8*d^2*e^3 + 5*a^5*b^7*d*e^4 - a^6*b^6*e^5)*x^4 + 10*(a^2*b^10*d^5 -
5*a^3*b^9*d^4*e + 10*a^4*b^8*d^3*e^2 - 10*a^5*b^7*d^2*e^3 + 5*a^6*b^6*d*e
^4 - a^7*b^5*e^5)*x^3 + 10*(a^3*b^9*d^5 - 5*a^4*b^8*d^4*e + 10*a^5*b^7*d^3
*e^2 - 10*a^6*b^6*d^2*e^3 + 5*a^7*b^5*d*e^4 - a^8*b^4*e^5)*x^2 + 5*(a^4*b^
8*d^5 - 5*a^5*b^7*d^4*e + 10*a^6*b^6*d^3*e^2 - 10*a^7*b^5*d^2*e^3 + 5*a^8*
b^4*d*e^4 - a^9*b^3*e^5)*x), 1/1920*(105*(b^5*e^5*x^5 + 5*a*b^4*e^5*x^4 +
10*a^2*b^3*e^5*x^3 + 10*a^3*b^2*e^5*x^2 + 5*a^4*b*e^5*x + a^5*e^5)*sqrt(-b
^2*d + a*b*e)*arctan(sqrt(-b^2*d + a*b*e)*sqrt(e*x + d)/(b*e*x + b*d)) ...
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\sqrt{d+ex}}{(a^2+2abx+b^2x^2)^3} dx = \text{Timed out}$$

input

```
integrate((e*x+d)**(1/2)/(b**2*x**2+2*a*b*x+a**2)**3,x)
```

output

Timed out

Maxima [F(-2)]

Exception generated.

$$\int \frac{\sqrt{d+ex}}{(a^2+2abx+b^2x^2)^3} dx = \text{Exception raised: ValueError}$$

input `integrate((e*x+d)^(1/2)/(b^2*x^2+2*a*b*x+a^2)^3,x, algorithm="maxima")`

output `Exception raised: ValueError >> Computation failed since Maxima requested additional constraints; using the 'assume' command before evaluation *may* help (example of legal syntax is 'assume(a*e-b*d>0)', see `assume?` for more detail`

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 432 vs. 2(186) = 372.

Time = 0.14 (sec) , antiderivative size = 432, normalized size of antiderivative = 1.98

$$\int \frac{\sqrt{d+ex}}{(a^2+2abx+b^2x^2)^3} dx$$

$$= \frac{7e^5 \arctan\left(\frac{\sqrt{ex+db}}{\sqrt{-b^2d+abe}}\right)}{128(b^5d^4 - 4ab^4d^3e + 6a^2b^3d^2e^2 - 4a^3b^2de^3 + a^4be^4)\sqrt{-b^2d+abe}} + \frac{105(ex+d)^{\frac{9}{2}}b^4e^5 - 490(ex+d)^{\frac{7}{2}}b^4de^5 + 896(ex+d)^{\frac{5}{2}}b^4d^2e^5 - 790(ex+d)^{\frac{3}{2}}b^4d^3e^5 - 105\sqrt{ex+d}db^4e^5}{128(b^5d^4 - 4ab^4d^3e + 6a^2b^3d^2e^2 - 4a^3b^2de^3 + a^4be^4)\sqrt{-b^2d+abe}}$$

input `integrate((e*x+d)^(1/2)/(b^2*x^2+2*a*b*x+a^2)^3,x, algorithm="giac")`

output

```
7/128*e^5*arctan(sqrt(e*x + d)*b/sqrt(-b^2*d + a*b*e))/((b^5*d^4 - 4*a*b^4
*d^3*e + 6*a^2*b^3*d^2*e^2 - 4*a^3*b^2*d*e^3 + a^4*b*e^4)*sqrt(-b^2*d + a*
b*e)) + 1/1920*(105*(e*x + d)^(9/2)*b^4*e^5 - 490*(e*x + d)^(7/2)*b^4*d*e^
5 + 896*(e*x + d)^(5/2)*b^4*d^2*e^5 - 790*(e*x + d)^(3/2)*b^4*d^3*e^5 - 10
5*sqrt(e*x + d)*b^4*d^4*e^5 + 490*(e*x + d)^(7/2)*a*b^3*e^6 - 1792*(e*x +
d)^(5/2)*a*b^3*d*e^6 + 2370*(e*x + d)^(3/2)*a*b^3*d^2*e^6 + 420*sqrt(e*x +
d)*a*b^3*d^3*e^6 + 896*(e*x + d)^(5/2)*a^2*b^2*e^7 - 2370*(e*x + d)^(3/2)
*a^2*b^2*d*e^7 - 630*sqrt(e*x + d)*a^2*b^2*d^2*e^7 + 790*(e*x + d)^(3/2)*a
^3*b*e^8 + 420*sqrt(e*x + d)*a^3*b*d*e^8 - 105*sqrt(e*x + d)*a^4*e^9)/((b^
5*d^4 - 4*a*b^4*d^3*e + 6*a^2*b^3*d^2*e^2 - 4*a^3*b^2*d*e^3 + a^4*b*e^4)*(
(e*x + d)*b - b*d + a*e)^5)
```

Mupad [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 401, normalized size of antiderivative = 1.84

$$\int \frac{\sqrt{d+ex}}{(a^2+2abx+b^2x^2)^3} dx$$

$$= \frac{(d+ex)(5a^4be^4 - 20a^3b^2de^3 + 30a^2b^3d^2e^2 - 20ab^4d^3e + 5b^5d^4) - (d+ex)^2(-10a^3b^2e^3 + 30a^2b^3d^2e^2 - 20ab^4d^3e + 5b^5d^4)}{128b^3/2(ae-bd)^{9/2}} + \frac{7e^5 \operatorname{atan}\left(\frac{\sqrt{b}\sqrt{d+ex}}{\sqrt{ae-bd}}\right)}{128b^{3/2}(ae-bd)^{9/2}}$$

input

```
int((d + e*x)^(1/2)/(a^2 + b^2*x^2 + 2*a*b*x)^3,x)
```

output

```
((79*e^5*(d + e*x)^(3/2))/(192*(a*e - b*d)) - (7*e^5*(d + e*x)^(1/2))/(128
*b) + (49*b^2*e^5*(d + e*x)^(7/2))/(192*(a*e - b*d)^3) + (7*b^3*e^5*(d + e
*x)^(9/2))/(128*(a*e - b*d)^4) + (7*b*e^5*(d + e*x)^(5/2))/(15*(a*e - b*d)
^2))/((d + e*x)*(5*b^5*d^4 + 5*a^4*b*e^4 - 20*a^3*b^2*d*e^3 + 30*a^2*b^3*d
^2*e^2 - 20*a*b^4*d^3*e) - (d + e*x)^2*(10*b^5*d^3 - 10*a^3*b^2*e^3 + 30*a
^2*b^3*d*e^2 - 30*a*b^4*d^2*e) + b^5*(d + e*x)^5 - (5*b^5*d - 5*a*b^4*e)*(
d + e*x)^4 + a^5*e^5 - b^5*d^5 + (d + e*x)^3*(10*b^5*d^2 + 10*a^2*b^3*e^2
- 20*a*b^4*d*e) - 10*a^2*b^3*d^3*e^2 + 10*a^3*b^2*d^2*e^3 + 5*a*b^4*d^4*e
- 5*a^4*b*d*e^4) + (7*e^5*atan((b^(1/2)*(d + e*x)^(1/2))/(a*e - b*d)^(1/2)
))/((128*b^(3/2)*(a*e - b*d)^(9/2))
```

Reduce [B] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 1167, normalized size of antiderivative = 5.35

$$\int \frac{\sqrt{d+ex}}{(a^2+2abx+b^2x^2)^3} dx = \text{Too large to display}$$

input `int((e*x+d)^(1/2)/(b^2*x^2+2*a*b*x+a^2)^3,x)`

output

```
(105*sqrt(b)*sqrt(a*e - b*d)*atan((sqrt(d + e*x)*b)/(sqrt(b)*sqrt(a*e - b*d)))*a**5*e**5 + 525*sqrt(b)*sqrt(a*e - b*d)*atan((sqrt(d + e*x)*b)/(sqrt(b)*sqrt(a*e - b*d)))*a**4*b*e**5*x + 1050*sqrt(b)*sqrt(a*e - b*d)*atan((sqrt(d + e*x)*b)/(sqrt(b)*sqrt(a*e - b*d)))*a**3*b**2*e**5*x**2 + 1050*sqrt(b)*sqrt(a*e - b*d)*atan((sqrt(d + e*x)*b)/(sqrt(b)*sqrt(a*e - b*d)))*a**2*b**3*e**5*x**3 + 525*sqrt(b)*sqrt(a*e - b*d)*atan((sqrt(d + e*x)*b)/(sqrt(b)*sqrt(a*e - b*d)))*a*b**4*e**5*x**4 + 105*sqrt(b)*sqrt(a*e - b*d)*atan((sqrt(d + e*x)*b)/(sqrt(b)*sqrt(a*e - b*d)))*b**5*e**5*x**5 - 105*sqrt(d + e*x)*a**5*b*e**5 + 1315*sqrt(d + e*x)*a**4*b**2*d*e**4 + 790*sqrt(d + e*x)*a**4*b**2*e**5*x - 3314*sqrt(d + e*x)*a**3*b**3*d**2*e**3 - 1368*sqrt(d + e*x)*a**3*b**3*d*e**4*x + 896*sqrt(d + e*x)*a**3*b**3*e**5*x**2 + 3592*sqrt(d + e*x)*a**2*b**4*d**3*e**2 + 834*sqrt(d + e*x)*a**2*b**4*d**2*e**3*x - 1218*sqrt(d + e*x)*a**2*b**4*d*e**4*x**2 + 490*sqrt(d + e*x)*a**2*b**4*e**5*x**3 - 1872*sqrt(d + e*x)*a*b**5*d**4*e - 304*sqrt(d + e*x)*a*b**5*d**3*e**2*x + 378*sqrt(d + e*x)*a*b**5*d**2*e**3*x**2 - 560*sqrt(d + e*x)*a*b**5*d*e**4*x**3 + 105*sqrt(d + e*x)*a*b**5*e**5*x**4 + 384*sqrt(d + e*x)*b**6*d**5 + 48*sqrt(d + e*x)*b**6*d**4*e*x - 56*sqrt(d + e*x)*b**6*d**3*e**2*x**2 + 70*sqrt(d + e*x)*b**6*d**2*e**3*x**3 - 105*sqrt(d + e*x)*b**6*d*e**4*x**4)/(1920*b**2*(a**10*e**5 - 5*a**9*b*d*e**4 + 5*a**9*b*e**5*x + 10*a**8*b**2*d**2*e**3 - 25*a**8*b**2*d*e**4*x + 10*a**8*b**2*e**5*x**2 - ...
```


3.197 $\int \frac{1}{\sqrt{d+ex}(a^2+2abx+b^2x^2)^3} dx$

| | |
|---|------|
| Optimal result | 1612 |
| Mathematica [A] (verified) | 1613 |
| Rubi [A] (verified) | 1613 |
| Maple [A] (verified) | 1617 |
| Fricas [B] (verification not implemented) | 1619 |
| Sympy [F(-1)] | 1620 |
| Maxima [F(-2)] | 1621 |
| Giac [B] (verification not implemented) | 1621 |
| Mupad [B] (verification not implemented) | 1622 |
| Reduce [B] (verification not implemented) | 1623 |

Optimal result

Integrand size = 28, antiderivative size = 213

$$\int \frac{1}{\sqrt{d+ex}(a^2+2abx+b^2x^2)^3} dx = -\frac{\sqrt{d+ex}}{5(bd-ae)(a+bx)^5} + \frac{9e\sqrt{d+ex}}{40(bd-ae)^2(a+bx)^4}$$

$$-\frac{21e^2\sqrt{d+ex}}{80(bd-ae)^3(a+bx)^3} + \frac{21e^3\sqrt{d+ex}}{64(bd-ae)^4(a+bx)^2}$$

$$-\frac{63e^4\sqrt{d+ex}}{128(bd-ae)^5(a+bx)} + \frac{63e^5 \operatorname{arctanh}\left(\frac{\sqrt{b}\sqrt{d+ex}}{\sqrt{bd-ae}}\right)}{128\sqrt{b}(bd-ae)^{11/2}}$$

output

```
-1/5*(e*x+d)^(1/2)/(-a*e+b*d)/(b*x+a)^5+9/40*e*(e*x+d)^(1/2)/(-a*e+b*d)^2/
(b*x+a)^4-21/80*e^2*(e*x+d)^(1/2)/(-a*e+b*d)^3/(b*x+a)^3+21/64*e^3*(e*x+d)
^(1/2)/(-a*e+b*d)^4/(b*x+a)^2-63/128*e^4*(e*x+d)^(1/2)/(-a*e+b*d)^5/(b*x+a
)+63/128*e^5*arctanh(b^(1/2)*(e*x+d)^(1/2)/(-a*e+b*d)^(1/2))/b^(1/2)/(-a*e
+b*d)^(11/2)
```

Mathematica [A] (verified)

Time = 0.69 (sec) , antiderivative size = 220, normalized size of antiderivative = 1.03

$$\int \frac{1}{\sqrt{d+ex}(a^2+2abx+b^2x^2)^3} dx$$

$$= \frac{1}{640} \left(\frac{\sqrt{d+ex}(965a^4e^4 + 10a^3be^3(-149d + 237ex) + 6a^2b^2e^2(228d^2 - 289dex + 448e^2x^2) + 2ab^3e(-328d^3 + 384d^2ex - 483d^2e^2x^2 + 735e^3x^3) + b^4(128d^4 - 144d^3ex + 168d^2e^2x^2 - 210d^2e^3x^3 + 315e^4x^4))}{(-bd+ae)^5(a+bx)^5} + \frac{315e^5 \arctan\left(\frac{\sqrt{b}\sqrt{d+ex}}{\sqrt{-bd+ae}}\right)}{\sqrt{b}(-bd+ae)^{11/2}} \right)$$

input `Integrate[1/(Sqrt[d + e*x]*(a^2 + 2*a*b*x + b^2*x^2)^3), x]`

output

```
((Sqrt[d + e*x]*(965*a^4*e^4 + 10*a^3*b*e^3*(-149*d + 237*e*x) + 6*a^2*b^2*e^2*(228*d^2 - 289*d*e*x + 448*e^2*x^2) + 2*a*b^3*e*(-328*d^3 + 384*d^2*e*x - 483*d^2*e^2*x^2 + 735*e^3*x^3) + b^4*(128*d^4 - 144*d^3*e*x + 168*d^2*e^2*x^2 - 210*d^2*e^3*x^3 + 315*e^4*x^4)))/((-b*d) + a*e)^5*(a + b*x)^5 + (315*e^5*ArcTan[(Sqrt[b]*Sqrt[d + e*x])/Sqrt[-(b*d) + a*e]])/(Sqrt[b]*(-(b*d) + a*e)^(11/2)))/640
```

Rubi [A] (verified)

Time = 0.55 (sec) , antiderivative size = 260, normalized size of antiderivative = 1.22, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.321$, Rules used = {1098, 27, 52, 52, 52, 52, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a^2 + 2abx + b^2x^2)^3 \sqrt{d+ex}} dx$$

$$\downarrow 1098$$

$$b^6 \int \frac{1}{b^6(a+bx)^6 \sqrt{d+ex}} dx$$

$$\begin{aligned}
 & \int \frac{1}{(a+bx)^6 \sqrt{d+ex}} dx \\
 & \downarrow 27 \\
 & -\frac{9e \int \frac{1}{(a+bx)^5 \sqrt{d+ex}} dx}{10(bd-ae)} - \frac{\sqrt{d+ex}}{5(a+bx)^5(bd-ae)} \\
 & \downarrow 52 \\
 & -\frac{9e \left(-\frac{7e \int \frac{1}{(a+bx)^4 \sqrt{d+ex}} dx}{8(bd-ae)} - \frac{\sqrt{d+ex}}{4(a+bx)^4(bd-ae)} \right)}{10(bd-ae)} - \frac{\sqrt{d+ex}}{5(a+bx)^5(bd-ae)} \\
 & \downarrow 52 \\
 & -\frac{9e \left(-\frac{7e \left(-\frac{5e \int \frac{1}{(a+bx)^3 \sqrt{d+ex}} dx}{6(bd-ae)} - \frac{\sqrt{d+ex}}{3(a+bx)^3(bd-ae)} \right)}{8(bd-ae)} - \frac{\sqrt{d+ex}}{4(a+bx)^4(bd-ae)} \right)}{10(bd-ae)} - \frac{\sqrt{d+ex}}{5(a+bx)^5(bd-ae)} \\
 & \downarrow 52 \\
 & -\frac{9e \left(-\frac{7e \left(-\frac{5e \left(-\frac{3e \int \frac{1}{(a+bx)^2 \sqrt{d+ex}} dx}{4(bd-ae)} - \frac{\sqrt{d+ex}}{2(a+bx)^2(bd-ae)} \right)}{6(bd-ae)} - \frac{\sqrt{d+ex}}{3(a+bx)^3(bd-ae)} \right)}{8(bd-ae)} - \frac{\sqrt{d+ex}}{4(a+bx)^4(bd-ae)} \right)}{10(bd-ae)} - \frac{\sqrt{d+ex}}{5(a+bx)^5(bd-ae)} \\
 & \downarrow 52 \\
 & \frac{10(bd-ae)}{5(a+bx)^5(bd-ae)} \frac{\sqrt{d+ex}}{5(a+bx)^5(bd-ae)} \\
 & \downarrow 52
 \end{aligned}$$

$$\left(\begin{array}{l} 9e \\ 7e \\ 5e \end{array} \left(\begin{array}{l} 3e \left(-\frac{e \int \frac{1}{(a+bx)\sqrt{d+ex}} dx}{2(bd-ae)} - \frac{\sqrt{d+ex}}{(a+bx)(bd-ae)} \right) \\ -\frac{\sqrt{d+ex}}{4(bd-ae)} - \frac{\sqrt{d+ex}}{2(a+bx)^2(bd-ae)} \end{array} \right) \right) - \frac{\sqrt{d+ex}}{3(a+bx)^3(bd-ae)}$$

$$\frac{10(bd-ae)}{5(a+bx)^5(bd-ae)} \sqrt{d+ex}$$

73

$$\left(\begin{array}{l} 9e \\ 7e \\ 5e \end{array} \left(\begin{array}{l} 3e \left(-\frac{\int \frac{1}{a + \frac{b(d+ex)}{e} - \frac{bd}{bd-ae}} d\sqrt{d+ex}}{4(bd-ae)} - \frac{\sqrt{d+ex}}{(a+bx)(bd-ae)} \right) \\ -\frac{\sqrt{d+ex}}{6(bd-ae)} - \frac{\sqrt{d+ex}}{2(a+bx)^2(bd-ae)} \end{array} \right) \right) - \frac{\sqrt{d+ex}}{3(a+bx)^3(bd-ae)}$$

$$\frac{10(bd-ae)}{5(a+bx)^5(bd-ae)} \sqrt{d+ex}$$

221

$$\frac{\left(\frac{9e \left(\frac{7e \left(\frac{5e \left(\frac{3e \left(\frac{e \operatorname{arctanh} \left(\frac{\sqrt{b} \sqrt{d+ex}}{\sqrt{bd-ae}} \right)}{\sqrt{b}(bd-ae)^{3/2}} - \frac{\sqrt{d+ex}}{(a+bx)(bd-ae)} \right)}{4(bd-ae)} - \frac{\sqrt{d+ex}}{2(a+bx)^2(bd-ae)} \right)}{6(bd-ae)} - \frac{\sqrt{d+ex}}{3(a+bx)^3(bd-ae)} \right)}{8(bd-ae)} - \frac{\sqrt{d+ex}}{4(a+bx)^4(bd-ae)} \right)}{10(bd-ae)} \right)}{5(a+bx)^5(bd-ae)} \sqrt{d+ex}}{10(bd-ae)}$$

```
input Int[1/(Sqrt[d + e*x]*(a^2 + 2*a*b*x + b^2*x^2)^3),x]
```

```
output -1/5*Sqrt[d + e*x]/((b*d - a*e)*(a + b*x)^5) - (9*e*(-1/4*Sqrt[d + e*x]/((b*d - a*e)*(a + b*x)^4) - (7*e*(-1/3*Sqrt[d + e*x]/((b*d - a*e)*(a + b*x)^3) - (5*e*(-1/2*Sqrt[d + e*x]/((b*d - a*e)*(a + b*x)^2) - (3*e*(-(Sqrt[d + e*x]/((b*d - a*e)*(a + b*x))) + (e*ArcTanh[(Sqrt[b]*Sqrt[d + e*x])/Sqrt[b*d - a*e]])/(Sqrt[b]*(b*d - a*e)^(3/2)))))/(4*(b*d - a*e)))))/(6*(b*d - a*e)))/(8*(b*d - a*e)))/(10*(b*d - a*e))
```

Defintions of rubi rules used

```
rule 27 Int[(a_)*(Fx_), x_Symbol] := Simp[a Int[Fx, x], x] /; FreeQ[a, x] && !MatchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 52 `Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] := Simp[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((m + n + 2)/((b*c - a*d)*(m + 1))) Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && ILtQ[m, -1] && FractionQ[n] && LtQ[n, 0]`

rule 73 `Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] := With[{p = Denominator[m]}, Simp[p/b Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) + d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntLinearQ[a, b, c, d, m, n, x]`

rule 221 `Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[-a/b, 2]/a)*ArcTanh[x/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]`

rule 1098 `Int[((d_.) + (e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] := Simp[1/c^p Int[(d + e*x)^m*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]`

Maple [A] (verified)

Time = 1.54 (sec) , antiderivative size = 219, normalized size of antiderivative = 1.03

| method | result |
|-------------------|--|
| pseudoelliptic | $\frac{63e^5 (bx+a)^5 \arctan\left(\frac{b\sqrt{ex+d}}{\sqrt{b(ae-bd)}}\right) + 193\sqrt{ex+d}\sqrt{b(ae-bd)} \left(\frac{(63e^4 x^4 - 42de^3 x^3 + \frac{168}{5}d^2 e^2 x^2 - \frac{144}{5}d^3 ex + \frac{128}{5}d^4)b^4}{193} - \frac{656e(-\frac{73}{32})}{(ae-bd)^5 (bx+a)^5 \sqrt{b(ae-bd)}} \right)}{128} + \dots$ |
| derivativedivides | $2e^5 \left(\frac{\sqrt{ex+d}}{10(ae-bd)(b(ex+d)+ae-bd)^5} + \frac{9\sqrt{ex+d}}{80(ae-bd)(b(ex+d)+ae-bd)^4} + \frac{7\sqrt{ex+d}}{48(ae-bd)(b(ex+d)+ae-bd)^3} + \frac{7}{24(ae-bd)(b(ex+d)+ae-bd)^2} \right)$ |
| default | $2e^5 \left(\frac{\sqrt{ex+d}}{10(ae-bd)(b(ex+d)+ae-bd)^5} + \frac{9\sqrt{ex+d}}{80(ae-bd)(b(ex+d)+ae-bd)^4} + \frac{7\sqrt{ex+d}}{48(ae-bd)(b(ex+d)+ae-bd)^3} + \frac{7}{24(ae-bd)(b(ex+d)+ae-bd)^2} \right)$ |

input `int(1/(e*x+d)^(1/2)/(b^2*x^2+2*a*b*x+a^2)^3,x,method=_RETURNVERBOSE)`

output `193/128/(b*(a*e-b*d))^(1/2)*(63/193*e^5*(b*x+a)^5*arctan(b*(e*x+d)^(1/2)/(b*(a*e-b*d))^(1/2))+(e*x+d)^(1/2)*(b*(a*e-b*d))^(1/2)*(1/193*(63*e^4*x^4-42*d*e^3*x^3+168/5*d^2*e^2*x^2-144/5*d^3*e*x+128/5*d^4)*b^4-656/965*e*(-735/328*e^3*x^3+483/328*d*e^2*x^2-48/41*d^2*e*x+d^3)*a*b^3+1368/965*e^2*(112/57*e^2*x^2-289/228*d*e*x+d^2)*a^2*b^2-298/193*e^3*a^3*(-237/149*e*x+d)*b+a^4*e^4))/(a*e-b*d)^5/(b*x+a)^5`

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 905 vs. $2(181) = 362$.

Time = 0.14 (sec) , antiderivative size = 1824, normalized size of antiderivative = 8.56

$$\int \frac{1}{\sqrt{d+ex}(a^2+2abx+b^2x^2)^3} dx = \text{Too large to display}$$

input `integrate(1/(e*x+d)^(1/2)/(b^2*x^2+2*a*b*x+a^2)^3,x, algorithm="fricas")`

output

```

[-1/1280*(315*(b^5*e^5*x^5 + 5*a*b^4*e^5*x^4 + 10*a^2*b^3*e^5*x^3 + 10*a^3
*b^2*e^5*x^2 + 5*a^4*b*e^5*x + a^5*e^5)*sqrt(b^2*d - a*b*e)*log((b*e*x + 2
*b*d - a*e - 2*sqrt(b^2*d - a*b*e)*sqrt(e*x + d))/(b*x + a)) + 2*(128*b^6*
d^5 - 784*a*b^5*d^4*e + 2024*a^2*b^4*d^3*e^2 - 2858*a^3*b^3*d^2*e^3 + 2455
*a^4*b^2*d*e^4 - 965*a^5*b*e^5 + 315*(b^6*d*e^4 - a*b^5*e^5)*x^4 - 210*(b^
6*d^2*e^3 - 8*a*b^5*d*e^4 + 7*a^2*b^4*e^5)*x^3 + 42*(4*b^6*d^3*e^2 - 27*a*
b^5*d^2*e^3 + 87*a^2*b^4*d*e^4 - 64*a^3*b^3*e^5)*x^2 - 6*(24*b^6*d^4*e - 1
52*a*b^5*d^3*e^2 + 417*a^2*b^4*d^2*e^3 - 684*a^3*b^3*d*e^4 + 395*a^4*b^2*e
^5)*x)*sqrt(e*x + d))/(a^5*b^7*d^6 - 6*a^6*b^6*d^5*e + 15*a^7*b^5*d^4*e^2
- 20*a^8*b^4*d^3*e^3 + 15*a^9*b^3*d^2*e^4 - 6*a^10*b^2*d*e^5 + a^11*b*e^6
+ (b^12*d^6 - 6*a*b^11*d^5*e + 15*a^2*b^10*d^4*e^2 - 20*a^3*b^9*d^3*e^3 +
15*a^4*b^8*d^2*e^4 - 6*a^5*b^7*d*e^5 + a^6*b^6*e^6)*x^5 + 5*(a*b^11*d^6 -
6*a^2*b^10*d^5*e + 15*a^3*b^9*d^4*e^2 - 20*a^4*b^8*d^3*e^3 + 15*a^5*b^7*d
^2*e^4 - 6*a^6*b^6*d*e^5 + a^7*b^5*e^6)*x^4 + 10*(a^2*b^10*d^6 - 6*a^3*b^9*
d^5*e + 15*a^4*b^8*d^4*e^2 - 20*a^5*b^7*d^3*e^3 + 15*a^6*b^6*d^2*e^4 - 6*a
^7*b^5*d*e^5 + a^8*b^4*e^6)*x^3 + 10*(a^3*b^9*d^6 - 6*a^4*b^8*d^5*e + 15*a
^5*b^7*d^4*e^2 - 20*a^6*b^6*d^3*e^3 + 15*a^7*b^5*d^2*e^4 - 6*a^8*b^4*d*e^5
+ a^9*b^3*e^6)*x^2 + 5*(a^4*b^8*d^6 - 6*a^5*b^7*d^5*e + 15*a^6*b^6*d^4*e
^2 - 20*a^7*b^5*d^3*e^3 + 15*a^8*b^4*d^2*e^4 - 6*a^9*b^3*d*e^5 + a^10*b^2*
e^6)*x), -1/640*(315*(b^5*e^5*x^5 + 5*a*b^4*e^5*x^4 + 10*a^2*b^3*e^5*x^3...

```

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{d+ex}(a^2+2abx+b^2x^2)^3} dx = \text{Timed out}$$

input

```
integrate(1/(e*x+d)**(1/2)/(b**2*x**2+2*a*b*x+a**2)**3,x)
```

output

Timed out

Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{\sqrt{d+ex}(a^2+2abx+b^2x^2)^3} dx = \text{Exception raised: ValueError}$$

input `integrate(1/(e*x+d)^(1/2)/(b^2*x^2+2*a*b*x+a^2)^3,x, algorithm="maxima")`

output Exception raised: ValueError >> Computation failed since Maxima requested additional constraints; using the 'assume' command before evaluation *may* help (example of legal syntax is 'assume(a*e-b*d>0)', see `assume?` for more detail)

Giac [B] (verification not implemented)Leaf count of result is larger than twice the leaf count of optimal. 456 vs. $2(181) = 362$.

Time = 0.16 (sec) , antiderivative size = 456, normalized size of antiderivative = 2.14

$$\int \frac{1}{\sqrt{d+ex}(a^2+2abx+b^2x^2)^3} dx$$

$$= -\frac{63 e^5 \arctan\left(\frac{\sqrt{ex+db}}{\sqrt{-b^2d+abe}}\right)}{128 (b^5 d^5 - 5 a b^4 d^4 e + 10 a^2 b^3 d^3 e^2 - 10 a^3 b^2 d^2 e^3 + 5 a^4 b d e^4 - a^5 e^5) \sqrt{-b^2 d + a b e} - 315 (e x + d)^{\frac{9}{2}} b^4 e^5 - 1470 (e x + d)^{\frac{7}{2}} b^4 d e^5 + 2688 (e x + d)^{\frac{5}{2}} b^4 d^2 e^5 - 2370 (e x + d)^{\frac{3}{2}} b^4 d^3 e^5 + 965 \sqrt{e x + d}}$$

input `integrate(1/(e*x+d)^(1/2)/(b^2*x^2+2*a*b*x+a^2)^3,x, algorithm="giac")`

output

```
-63/128*e^5*arctan(sqrt(e*x + d)*b/sqrt(-b^2*d + a*b*e))/((b^5*d^5 - 5*a*b^4*d^4*e + 10*a^2*b^3*d^3*e^2 - 10*a^3*b^2*d^2*e^3 + 5*a^4*b*d*e^4 - a^5*e^5)*sqrt(-b^2*d + a*b*e)) - 1/640*(315*(e*x + d)^(9/2)*b^4*e^5 - 1470*(e*x + d)^(7/2)*b^4*d*e^5 + 2688*(e*x + d)^(5/2)*b^4*d^2*e^5 - 2370*(e*x + d)^(3/2)*b^4*d^3*e^5 + 965*sqrt(e*x + d)*b^4*d^4*e^5 + 1470*(e*x + d)^(7/2)*a*b^3*e^6 - 5376*(e*x + d)^(5/2)*a*b^3*d*e^6 + 7110*(e*x + d)^(3/2)*a*b^3*d^2*e^6 - 3860*sqrt(e*x + d)*a*b^3*d^3*e^6 + 2688*(e*x + d)^(5/2)*a^2*b^2*e^7 - 7110*(e*x + d)^(3/2)*a^2*b^2*d*e^7 + 5790*sqrt(e*x + d)*a^2*b^2*d^2*e^7 + 2370*(e*x + d)^(3/2)*a^3*b*e^8 - 3860*sqrt(e*x + d)*a^3*b*d*e^8 + 965*sqrt(e*x + d)*a^4*e^9)/((b^5*d^5 - 5*a*b^4*d^4*e + 10*a^2*b^3*d^3*e^2 - 10*a^3*b^2*d^2*e^3 + 5*a^4*b*d*e^4 - a^5*e^5)*((e*x + d)*b - b*d + a*e)^5)
```

Mupad [B] (verification not implemented)

Time = 9.32 (sec) , antiderivative size = 252, normalized size of antiderivative = 1.18

$$\int \frac{1}{\sqrt{d+ex} (a^2 + 2abx + b^2x^2)^3} dx$$

$$= \frac{965 \sqrt{b} (ae-bd)^{9/2} \sqrt{d+ex} + 2370 b^{3/2} (ae-bd)^{7/2} (d+ex)^{3/2} + 2688 b^{5/2} (ae-bd)^{5/2} (d+ex)^{5/2} + 1470 b^{7/2} (ae-bd)^{3/2} (d+ex)^{7/2} + 315 b^{9/2} (ae-bd)^{1/2} (d+ex)^{9/2}}{640 \sqrt{b} (ae-bd)^{11/2} (a+bx)^5} + \frac{63 e^5 \operatorname{atan}\left(\frac{\sqrt{b}\sqrt{d+ex}}{\sqrt{ae-bd}}\right)}{128 \sqrt{b} (ae-bd)^{11/2}}$$

input

```
int(1/((d + e*x)^(1/2)*(a^2 + b^2*x^2 + 2*a*b*x)^3),x)
```

output

```
((965*b^(1/2)*(a*e - b*d)^(9/2)*(d + e*x)^(1/2) + 2370*b^(3/2)*(a*e - b*d)^(7/2)*(d + e*x)^(3/2) + 2688*b^(5/2)*(a*e - b*d)^(5/2)*(d + e*x)^(5/2) + 1470*b^(7/2)*(a*e - b*d)^(3/2)*(d + e*x)^(7/2) + 315*b^(9/2)*(a*e - b*d)^(1/2)*(d + e*x)^(9/2) + 315*b^5*e^5*x^5*atan((b^(1/2)*(d + e*x)^(1/2))/(a*e - b*d)^(1/2)))/(640*b^(1/2)*(a*e - b*d)^(11/2)) - (63*b^(9/2)*e^5*x^5*atan((b^(1/2)*(d + e*x)^(1/2))/(a*e - b*d)^(1/2)))/(128*(a*e - b*d)^(11/2)))/(a + b*x)^5 + (63*e^5*atan((b^(1/2)*(d + e*x)^(1/2))/(a*e - b*d)^(1/2)))/(128*b^(1/2)*(a*e - b*d)^(11/2))
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 1262, normalized size of antiderivative = 5.92

$$\int \frac{1}{\sqrt{d+ex}(a^2+2abx+b^2x^2)^3} dx = \text{Too large to display}$$

input `int(1/(e*x+d)^(1/2)/(b^2*x^2+2*a*b*x+a^2)^3,x)`

output

```
(315*sqrt(b)*sqrt(a*e - b*d)*atan((sqrt(d + e*x)*b)/(sqrt(b)*sqrt(a*e - b*d)))*a**5*e**5 + 1575*sqrt(b)*sqrt(a*e - b*d)*atan((sqrt(d + e*x)*b)/(sqrt(b)*sqrt(a*e - b*d)))*a**4*b*e**5*x + 3150*sqrt(b)*sqrt(a*e - b*d)*atan((sqrt(d + e*x)*b)/(sqrt(b)*sqrt(a*e - b*d)))*a**3*b**2*e**5*x**2 + 3150*sqrt(b)*sqrt(a*e - b*d)*atan((sqrt(d + e*x)*b)/(sqrt(b)*sqrt(a*e - b*d)))*a**2*b**3*e**5*x**3 + 1575*sqrt(b)*sqrt(a*e - b*d)*atan((sqrt(d + e*x)*b)/(sqrt(b)*sqrt(a*e - b*d)))*a*b**4*e**5*x**4 + 315*sqrt(b)*sqrt(a*e - b*d)*atan((sqrt(d + e*x)*b)/(sqrt(b)*sqrt(a*e - b*d)))*b**5*e**5*x**5 + 965*sqrt(d + e*x)*a**5*b*e**5 - 2455*sqrt(d + e*x)*a**4*b**2*d*e**4 + 2370*sqrt(d + e*x)*a**4*b**2*e**5*x + 2858*sqrt(d + e*x)*a**3*b**3*d**2*e**3 - 4104*sqrt(d + e*x)*a**3*b**3*d*e**4*x + 2688*sqrt(d + e*x)*a**3*b**3*e**5*x**2 - 2024*sqrt(d + e*x)*a**2*b**4*d**3*e**2 + 2502*sqrt(d + e*x)*a**2*b**4*d**2*e**3*x - 3654*sqrt(d + e*x)*a**2*b**4*d*e**4*x**2 + 1470*sqrt(d + e*x)*a**2*b**4*e**5*x**3 + 784*sqrt(d + e*x)*a*b**5*d**4*e - 912*sqrt(d + e*x)*a*b**5*d**3*e**2*x + 1134*sqrt(d + e*x)*a*b**5*d**2*e**3*x**2 - 1680*sqrt(d + e*x)*a*b**5*d*e**4*x**3 + 315*sqrt(d + e*x)*a*b**5*e**5*x**4 - 128*sqrt(d + e*x)*b**6*d**5 + 144*sqrt(d + e*x)*b**6*d**4*e*x - 168*sqrt(d + e*x)*b**6*d**3*e**2*x**2 + 210*sqrt(d + e*x)*b**6*d**2*e**3*x**3 - 315*sqrt(d + e*x)*b**6*d*e**4*x**4)/(640*b*(a**11*e**6 - 6*a**10*b*d*e**5 + 5*a**10*b*e**6*x + 15*a**9*b**2*d**2*e**4 - 30*a**9*b**2*d*e**5*x + 10*a**9*b**2*e**6...
```

3.198 $\int \frac{1}{(d+ex)^{3/2}(a^2+2abx+b^2x^2)^3} dx$

| | |
|---|------|
| Optimal result | 1624 |
| Mathematica [A] (verified) | 1625 |
| Rubi [A] (verified) | 1625 |
| Maple [A] (verified) | 1632 |
| Fricas [B] (verification not implemented) | 1633 |
| Sympy [F(-1)] | 1634 |
| Maxima [F(-2)] | 1634 |
| Giac [B] (verification not implemented) | 1634 |
| Mupad [B] (verification not implemented) | 1636 |
| Reduce [B] (verification not implemented) | 1636 |

Optimal result

Integrand size = 28, antiderivative size = 239

$$\int \frac{1}{(d+ex)^{3/2}(a^2+2abx+b^2x^2)^3} dx = -\frac{693e^5}{128(bd-ae)^6\sqrt{d+ex}} - \frac{1}{5(bd-ae)(a+bx)^5\sqrt{d+ex}} + \frac{11e}{40(bd-ae)^2(a+bx)^4\sqrt{d+ex}} - \frac{33e^2}{80(bd-ae)^3(a+bx)^3\sqrt{d+ex}} + \frac{231e^3}{320(bd-ae)^4(a+bx)^2\sqrt{d+ex}} - \frac{231e^4}{128(bd-ae)^5(a+bx)\sqrt{d+ex}} + \frac{693\sqrt{b}e^5 \operatorname{arctanh}\left(\frac{\sqrt{b}\sqrt{d+ex}}{\sqrt{bd-ae}}\right)}{128(bd-ae)^{13/2}}$$

output

```
-693/128*e^5/(-a*e+b*d)^6/(e*x+d)^(1/2)-1/5/(-a*e+b*d)/(b*x+a)^5/(e*x+d)^(1/2)+11/40*e/(-a*e+b*d)^2/(b*x+a)^4/(e*x+d)^(1/2)-33/80*e^2/(-a*e+b*d)^3/(b*x+a)^3/(e*x+d)^(1/2)+231/320*e^3/(-a*e+b*d)^4/(b*x+a)^2/(e*x+d)^(1/2)-231/128*e^4/(-a*e+b*d)^5/(b*x+a)/(e*x+d)^(1/2)+693/128*b^(1/2)*e^5*arctanh(b^(1/2)*(e*x+d)^(1/2)/(-a*e+b*d)^(1/2))/(-a*e+b*d)^(13/2)
```

Mathematica [A] (verified)

Time = 1.85 (sec) , antiderivative size = 284, normalized size of antiderivative = 1.19

$$\int \frac{1}{(d+ex)^{3/2}(a^2+2abx+b^2x^2)^3} dx = \frac{1}{640} \left(\frac{-1280a^5e^5 - 5a^4be^4(843d+2123ex) - 10a^3b^2e^3(-359d^2 + 968d*ex + 2607e^2*x^2) - 2a^2b^3e^2(1124d^3 - 2013d^2*ex + 5247d*ex^2 + 14784e^3*x^3) - 2a*b^4e*(-408d^4 + 616d^3*ex - 1089d^2*ex^2 + 2772d*ex^3 + 8085e^4*x^4) - b^5(128d^5 - 176d^4*ex + 264d^3*ex^2 - 462d^2*ex^3 + 1155d*ex^4 + 3465e^5*x^5)}{(-bd+ae)^{13/2}} \arctan\left(\frac{\sqrt{b}\sqrt{d+ex}}{\sqrt{-bd+ae}}\right) \right)$$

input

```
Integrate[1/((d + e*x)^(3/2)*(a^2 + 2*a*b*x + b^2*x^2)^3), x]
```

output

```
((-1280*a^5*e^5 - 5*a^4*b*e^4*(843*d + 2123*e*x) - 10*a^3*b^2*e^3*(-359*d^2 + 968*d*e*x + 2607*e^2*x^2) - 2*a^2*b^3*e^2*(1124*d^3 - 2013*d^2*e*x + 5247*d*e^2*x^2 + 14784*e^3*x^3) - 2*a*b^4*e*(-408*d^4 + 616*d^3*e*x - 1089*d^2*e^2*x^2 + 2772*d*e^3*x^3 + 8085*e^4*x^4) - b^5*(128*d^5 - 176*d^4*e*x + 264*d^3*e^2*x^2 - 462*d^2*e^3*x^3 + 1155*d*e^4*x^4 + 3465*e^5*x^5))/(b*d - a*e)^6*(a + b*x)^5*Sqrt[d + e*x] - (3465*Sqrt[b]*e^5*ArcTan[(Sqrt[b]*Sqrt[d + e*x])/Sqrt[-(b*d) + a*e]])/(-(b*d) + a*e)^(13/2))/640
```

Rubi [A] (verified)

Time = 0.62 (sec) , antiderivative size = 297, normalized size of antiderivative = 1.24, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.357$, Rules used = {1098, 27, 52, 52, 52, 52, 61, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a^2 + 2abx + b^2x^2)^3 (d + ex)^{3/2}} dx$$

↓ 1098

$$b^6 \int \frac{1}{b^6(a + bx)^6 (d + ex)^{3/2}} dx$$

↓ 27

$$\begin{aligned}
 & \left(\begin{aligned} & \left(\begin{aligned} & 5e \left(-\frac{3e \int \frac{1}{(a+bx)(d+ex)^{3/2}} dx}{2(bd-ae)} - \frac{1}{(a+bx)\sqrt{d+ex}(bd-ae)} \right) \\ & 7e \left(-\frac{1}{4(bd-ae)} - \frac{1}{2(a+bx)^2\sqrt{d+ex}(bd-ae)} \right) \end{aligned} \right) \\ & 9e \left(-\frac{1}{6(bd-ae)} - \frac{1}{3(a+bx)^3\sqrt{d+ex}(bd-ae)} \right) \end{aligned} \right) \\
 & 11e \left(-\frac{1}{8(bd-ae)} - \frac{1}{4(a+bx)^4} \right) \\
 & \frac{10(bd-ae)}{5(a+bx)^5\sqrt{d+ex}(bd-ae)} \\
 & \downarrow 61
 \end{aligned}$$

$$\begin{aligned}
 & \left(\begin{aligned} & \left(\begin{aligned} & 3e \left(\frac{b \int \frac{1}{(a+bx)\sqrt{d+ex}} dx}{bd-ae} + \frac{2}{\sqrt{d+ex}(bd-ae)} \right) \\ & - \frac{1}{(a+bx)\sqrt{d+ex}(bd-ae)} \end{aligned} \right) \\ & - \frac{1}{2(a+bx)^2\sqrt{d+ex}(bd-ae)} \end{aligned} \right) \\
 5e & \left(\begin{aligned} & \left(\begin{aligned} & \left(\begin{aligned} & \frac{1}{2(bd-ae)} \end{aligned} \right) \\ & - \frac{1}{(a+bx)\sqrt{d+ex}(bd-ae)} \end{aligned} \right) \\ & - \frac{1}{2(a+bx)^2\sqrt{d+ex}(bd-ae)} \end{aligned} \right) \\
 7e & \left(\begin{aligned} & \left(\begin{aligned} & \left(\begin{aligned} & \left(\begin{aligned} & \frac{1}{2(bd-ae)} \end{aligned} \right) \\ & - \frac{1}{(a+bx)\sqrt{d+ex}(bd-ae)} \end{aligned} \right) \\ & - \frac{1}{2(a+bx)^2\sqrt{d+ex}(bd-ae)} \end{aligned} \right) \\ & - \frac{1}{2(a+bx)^3\sqrt{d+ex}(bd-ae)} \end{aligned} \right) \\
 9e & \left(\begin{aligned} & \left(\begin{aligned} & \left(\begin{aligned} & \left(\begin{aligned} & \left(\begin{aligned} & \frac{1}{2(bd-ae)} \end{aligned} \right) \\ & - \frac{1}{(a+bx)\sqrt{d+ex}(bd-ae)} \end{aligned} \right) \\ & - \frac{1}{2(a+bx)^2\sqrt{d+ex}(bd-ae)} \end{aligned} \right) \\ & - \frac{1}{2(a+bx)^3\sqrt{d+ex}(bd-ae)} \end{aligned} \right) \\ & - \frac{1}{2(a+bx)^4\sqrt{d+ex}(bd-ae)} \end{aligned} \right) \\
 11e & \left(\begin{aligned} & \left(\begin{aligned} & \left(\begin{aligned} & \left(\begin{aligned} & \left(\begin{aligned} & \left(\begin{aligned} & \frac{1}{2(bd-ae)} \end{aligned} \right) \\ & - \frac{1}{(a+bx)\sqrt{d+ex}(bd-ae)} \end{aligned} \right) \\ & - \frac{1}{2(a+bx)^2\sqrt{d+ex}(bd-ae)} \end{aligned} \right) \\ & - \frac{1}{2(a+bx)^3\sqrt{d+ex}(bd-ae)} \end{aligned} \right) \\ & - \frac{1}{2(a+bx)^4\sqrt{d+ex}(bd-ae)} \end{aligned} \right) \\ & - \frac{1}{2(a+bx)^5\sqrt{d+ex}(bd-ae)} \end{aligned} \right) \\
 & \frac{1}{5(a+bx)^5\sqrt{d+ex}(bd-ae)} \\
 & \downarrow 73
 \end{aligned}
 \end{aligned}$$

$$\begin{aligned}
 & \left(\frac{3e \left(\frac{2}{\sqrt{d+ex}(bd-ae)} - \frac{2\sqrt{b}\operatorname{arctanh}\left(\frac{\sqrt{b}\sqrt{d+ex}}{\sqrt{bd-ae}}\right)}{(bd-ae)^{3/2}} \right)}{2(bd-ae)} - \frac{1}{(a+bx)\sqrt{d+ex}(bd-ae)} \right) \\
 & \frac{7e}{4(bd-ae)} - \frac{1}{2(a+bx)^2\sqrt{d+ex}(bd-ae)} \\
 & \frac{9e}{6(bd-ae)} - \frac{1}{3(a+bx)^3\sqrt{d+ex}} \\
 & \frac{11e}{8(bd-ae)} \\
 & \frac{1}{5(a+bx)^5\sqrt{d+ex}(bd-ae)} \\
 & \frac{10(bd-ae)}{5(a+bx)^5\sqrt{d+ex}(bd-ae)}
 \end{aligned}$$

input

```
Int[1/((d + e*x)^(3/2)*(a^2 + 2*a*b*x + b^2*x^2)^3),x]
```

output

$$\begin{aligned}
& -1/5*1/((b*d - a*e)*(a + b*x)^5*\text{Sqrt}[d + e*x]) - (11*e*(-1/4*1/((b*d - a*e) \\
&)*(a + b*x)^4*\text{Sqrt}[d + e*x]) - (9*e*(-1/3*1/((b*d - a*e)*(a + b*x)^3*\text{Sqrt}[\\
& d + e*x]) - (7*e*(-1/2*1/((b*d - a*e)*(a + b*x)^2*\text{Sqrt}[d + e*x]) - (5*e*(- \\
& 1/((b*d - a*e)*(a + b*x)*\text{Sqrt}[d + e*x])) - (3*e*(2/((b*d - a*e)*\text{Sqrt}[d + \\
& e*x]) - (2*\text{Sqrt}[b]*\text{ArcTanh}[(\text{Sqrt}[b]*\text{Sqrt}[d + e*x])/\text{Sqrt}[b*d - a*e]])/(b*d \\
& - a*e)^{(3/2)}))/(2*(b*d - a*e)))/(4*(b*d - a*e)))/(6*(b*d - a*e)))/(8*(b \\
& *d - a*e)))/(10*(b*d - a*e))
\end{aligned}$$

Defintions of rubi rules used

rule 27

$$\text{Int}[(a_*)(Fx_), x_Symbol] \rightarrow \text{Simp}[a \text{ Int}[Fx, x], x] /; \text{FreeQ}[a, x] \ \&\& \ !\text{MatchQ}[Fx, (b_*)(Gx_)] /; \text{FreeQ}[b, x]$$

rule 52

$$\begin{aligned}
& \text{Int}[((a_.) + (b_.)*(x_))^{(m_)}*((c_.) + (d_.)*(x_))^{(n_)}, x_Symbol] \rightarrow \text{Simp}[\\
& (a + b*x)^{(m + 1)}*((c + d*x)^{(n + 1)}/((b*c - a*d)*(m + 1))), x] - \text{Simp}[d*((\\
& m + n + 2)/((b*c - a*d)*(m + 1))) \text{ Int}[(a + b*x)^{(m + 1)}*(c + d*x)^n, x], \\
& x] /; \text{FreeQ}[\{a, b, c, d, n\}, x] \ \&\& \ \text{ILtQ}[m, -1] \ \&\& \ \text{FractionQ}[n] \ \&\& \ \text{LtQ}[n, 0]
\end{aligned}$$

rule 61

$$\begin{aligned}
& \text{Int}[((a_.) + (b_.)*(x_))^{(m_)}*((c_.) + (d_.)*(x_))^{(n_)}, x_Symbol] \rightarrow \text{Simp}[\\
& (a + b*x)^{(m + 1)}*((c + d*x)^{(n + 1)}/((b*c - a*d)*(m + 1))), x] - \text{Simp}[d*((\\
& m + n + 2)/((b*c - a*d)*(m + 1))) \text{ Int}[(a + b*x)^{(m + 1)}*(c + d*x)^n, x], \\
& x] /; \text{FreeQ}[\{a, b, c, d, n\}, x] \ \&\& \ \text{LtQ}[m, -1] \ \&\& \ !(\text{LtQ}[n, -1] \ \&\& \ (\text{EqQ}[a, 0] \\
&] \ || \ (\text{NeQ}[c, 0] \ \&\& \ \text{LtQ}[m - n, 0] \ \&\& \ \text{IntegerQ}[n]))) \ \&\& \ \text{IntLinearQ}[a, b, c, d \\
& , m, n, x]
\end{aligned}$$

rule 73

$$\begin{aligned}
& \text{Int}[((a_.) + (b_.)*(x_))^{(m_)}*((c_.) + (d_.)*(x_))^{(n_)}, x_Symbol] \rightarrow \text{With}[\\
& \{p = \text{Denominator}[m]\}, \text{Simp}[p/b \text{ Subst}[\text{Int}[x^{(p*(m + 1) - 1)}*(c - a*(d/b) + \\
& d*(x^p/b))^{n + 1}, x], x, (a + b*x)^{(1/p)}, x]] /; \text{FreeQ}[\{a, b, c, d\}, x] \ \&\& \ \text{Lt} \\
& \text{Q}[-1, m, 0] \ \&\& \ \text{LeQ}[-1, n, 0] \ \&\& \ \text{LeQ}[\text{Denominator}[n], \text{Denominator}[m]] \ \&\& \ \text{IntL} \\
& \text{inearQ}[a, b, c, d, m, n, x]
\end{aligned}$$

rule 221

$$\text{Int}[((a_) + (b_.)*(x_)^2)^{-1}, x_Symbol] \rightarrow \text{Simp}[(\text{Rt}[-a/b, 2]/a)*\text{ArcTanh}[x/\text{Rt}[-a/b, 2]], x] /; \text{FreeQ}[\{a, b\}, x] \ \&\& \ \text{NegQ}[a/b]$$

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1132 vs. $2(203) = 406$.

Time = 0.41 (sec) , antiderivative size = 2294, normalized size of antiderivative = 9.60

$$\int \frac{1}{(d+ex)^{3/2}(a^2+2abx+b^2x^2)^3} dx = \text{Too large to display}$$

input `integrate(1/(e*x+d)^(3/2)/(b^2*x^2+2*a*b*x+a^2)^3,x, algorithm="fricas")`

output

```
[1/1280*(3465*(b^5*e^6*x^6 + a^5*d*e^5 + (b^5*d*e^5 + 5*a*b^4*e^6)*x^5 + 5*(a*b^4*d*e^5 + 2*a^2*b^3*e^6)*x^4 + 10*(a^2*b^3*d*e^5 + a^3*b^2*e^6)*x^3 + 5*(2*a^3*b^2*d*e^5 + a^4*b*e^6)*x^2 + (5*a^4*b*d*e^5 + a^5*e^6)*x)*sqrt(b/(b*d - a*e))*log((b*e*x + 2*b*d - a*e + 2*(b*d - a*e)*sqrt(e*x + d))*sqrt(b/(b*d - a*e)))/(b*x + a)) - 2*(3465*b^5*e^5*x^5 + 128*b^5*d^5 - 816*a*b^4*d^4*e + 2248*a^2*b^3*d^3*e^2 - 3590*a^3*b^2*d^2*e^3 + 4215*a^4*b*d*e^4 + 1280*a^5*e^5 + 1155*(b^5*d*e^4 + 14*a*b^4*e^5)*x^4 - 462*(b^5*d^2*e^3 - 12*a*b^4*d*e^4 - 64*a^2*b^3*e^5)*x^3 + 66*(4*b^5*d^3*e^2 - 33*a*b^4*d^2*e^3 + 159*a^2*b^3*d*e^4 + 395*a^3*b^2*e^5)*x^2 - 11*(16*b^5*d^4*e - 112*a*b^4*d^3*e^2 + 366*a^2*b^3*d^2*e^3 - 880*a^3*b^2*d*e^4 - 965*a^4*b*e^5)*x)*sqrt(e*x + d))/(a^5*b^6*d^7 - 6*a^6*b^5*d^6*e + 15*a^7*b^4*d^5*e^2 - 20*a^8*b^3*d^4*e^3 + 15*a^9*b^2*d^3*e^4 - 6*a^10*b*d^2*e^5 + a^11*d*e^6 + (b^11*d^6*e - 6*a*b^10*d^5*e^2 + 15*a^2*b^9*d^4*e^3 - 20*a^3*b^8*d^3*e^4 + 15*a^4*b^7*d^2*e^5 - 6*a^5*b^6*d*e^6 + a^6*b^5*e^7)*x^6 + (b^11*d^7 - a*b^10*d^6*e - 15*a^2*b^9*d^5*e^2 + 55*a^3*b^8*d^4*e^3 - 85*a^4*b^7*d^3*e^4 + 69*a^5*b^6*d^2*e^5 - 29*a^6*b^5*d*e^6 + 5*a^7*b^4*e^7)*x^5 + 5*(a*b^10*d^7 - 4*a^2*b^9*d^6*e + 3*a^3*b^8*d^5*e^2 + 10*a^4*b^7*d^4*e^3 - 25*a^5*b^6*d^3*e^4 + 24*a^6*b^5*d^2*e^5 - 11*a^7*b^4*d*e^6 + 2*a^8*b^3*e^7)*x^4 + 10*(a^2*b^9*d^7 - 5*a^3*b^8*d^6*e + 9*a^4*b^7*d^5*e^2 - 5*a^5*b^6*d^4*e^3 - 5*a^6*b^5*d^3*e^4 + 9*a^7*b^4*d^2*e^5 - 5*a^8*b^3*d*e^6 + a^9*b^2*e^7)*x^3 + 5*(...
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{(d+ex)^{3/2} (a^2+2abx+b^2x^2)^3} dx = \text{Timed out}$$

input `integrate(1/(e*x+d)**(3/2)/(b**2*x**2+2*a*b*x+a**2)**3,x)`

output `Timed out`

Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{(d+ex)^{3/2} (a^2+2abx+b^2x^2)^3} dx = \text{Exception raised: ValueError}$$

input `integrate(1/(e*x+d)^(3/2)/(b^2*x^2+2*a*b*x+a^2)^3,x, algorithm="maxima")`

output `Exception raised: ValueError >> Computation failed since Maxima requested additional constraints; using the 'assume' command before evaluation *may* help (example of legal syntax is 'assume(a*e-b*d>0)', see `assume?` for more detail`

Giac [B] (verification not implemented)Leaf count of result is larger than twice the leaf count of optimal. 579 vs. $2(203) = 406$.

Time = 0.23 (sec) , antiderivative size = 579, normalized size of antiderivative = 2.42

$$\int \frac{1}{(d+ex)^{3/2} (a^2+2abx+b^2x^2)^3} dx =$$

$$\frac{693be^5 \arctan\left(\frac{\sqrt{ex+db}}{\sqrt{-b^2d+abe}}\right)}{128(b^6d^6 - 6ab^5d^5e + 15a^2b^4d^4e^2 - 20a^3b^3d^3e^3 + 15a^4b^2d^2e^4 - 6a^5bde^5 + a^6e^6)\sqrt{-b^2d+abe}} +$$

$$\frac{(b^6d^6 - 6ab^5d^5e + 15a^2b^4d^4e^2 - 20a^3b^3d^3e^3 + 15a^4b^2d^2e^4 - 6a^5bde^5 + a^6e^6)\sqrt{ex+d}}{2185(ex+d)^{9/2}b^5e^5 - 9770(ex+d)^{7/2}b^5de^5 + 16768(ex+d)^{5/2}b^5d^2e^5 - 13270(ex+d)^{3/2}b^5d^3e^5 + 4215\sqrt{ex+d}}$$

input `integrate(1/(e*x+d)^(3/2)/(b^2*x^2+2*a*b*x+a^2)^3,x, algorithm="giac")`

output

```
-693/128*b*e^5*arctan(sqrt(e*x + d)*b/sqrt(-b^2*d + a*b*e))/((b^6*d^6 - 6*
a*b^5*d^5*e + 15*a^2*b^4*d^4*e^2 - 20*a^3*b^3*d^3*e^3 + 15*a^4*b^2*d^2*e^4
- 6*a^5*b*d*e^5 + a^6*e^6)*sqrt(-b^2*d + a*b*e)) - 2*e^5/((b^6*d^6 - 6*a*
b^5*d^5*e + 15*a^2*b^4*d^4*e^2 - 20*a^3*b^3*d^3*e^3 + 15*a^4*b^2*d^2*e^4 -
6*a^5*b*d*e^5 + a^6*e^6)*sqrt(e*x + d)) - 1/640*(2185*(e*x + d)^(9/2)*b^5
*e^5 - 9770*(e*x + d)^(7/2)*b^5*d*e^5 + 16768*(e*x + d)^(5/2)*b^5*d^2*e^5
- 13270*(e*x + d)^(3/2)*b^5*d^3*e^5 + 4215*sqrt(e*x + d)*b^5*d^4*e^5 + 977
0*(e*x + d)^(7/2)*a*b^4*e^6 - 33536*(e*x + d)^(5/2)*a*b^4*d*e^6 + 39810*(e
*x + d)^(3/2)*a*b^4*d^2*e^6 - 16860*sqrt(e*x + d)*a*b^4*d^3*e^6 + 16768*(e
*x + d)^(5/2)*a^2*b^3*e^7 - 39810*(e*x + d)^(3/2)*a^2*b^3*d*e^7 + 25290*sq
rt(e*x + d)*a^2*b^3*d^2*e^7 + 13270*(e*x + d)^(3/2)*a^3*b^2*e^8 - 16860*sq
rt(e*x + d)*a^3*b^2*d*e^8 + 4215*sqrt(e*x + d)*a^4*b*e^9)/((b^6*d^6 - 6*a*
b^5*d^5*e + 15*a^2*b^4*d^4*e^2 - 20*a^3*b^3*d^3*e^3 + 15*a^4*b^2*d^2*e^4 -
6*a^5*b*d*e^5 + a^6*e^6)*((e*x + d)*b - b*d + a*e)^5)
```


Mupad [B] (verification not implemented)

Time = 10.01 (sec) , antiderivative size = 515, normalized size of antiderivative = 2.15

$$\int \frac{1}{(d+ex)^{3/2} (a^2+2abx+b^2x^2)^3} dx =$$

$$\frac{\sqrt{d+ex} (a^5 e^5 - 5 a^4 b d e^4 + 10 a^3 b^2 d^2 e^3 - 10 a^2 b^3 d^3 e^2 + 5 a b^4 d^4 e - b^5 d^5) - (d+ex)^{5/2} (-10 a^3 b^2 d^2 e^3 + 15 a^2 b^3 d^3 e^2 - 10 a b^4 d^4 e + b^5 d^5) - 693 \sqrt{b} e^5 \operatorname{atan}\left(\frac{\sqrt{b} \sqrt{d+ex} (a^6 e^6 - 6 a^5 b d e^5 + 15 a^4 b^2 d^2 e^4 - 20 a^3 b^3 d^3 e^3 + 15 a^2 b^4 d^4 e^2 - 6 a b^5 d^5 e + b^6 d^6)}{(a e - b d)^{13/2}}\right)}{128 (a e - b d)^{13/2}}$$

input `int(1/((d + e*x)^(3/2)*(a^2 + b^2*x^2 + 2*a*b*x)^3),x)`

output

```
- ((2*e^5)/(a*e - b*d) + (2607*b^2*e^5*(d + e*x)^2)/(64*(a*e - b*d)^3) + (231*b^3*e^5*(d + e*x)^3)/(5*(a*e - b*d)^4) + (1617*b^4*e^5*(d + e*x)^4)/(64*(a*e - b*d)^5) + (693*b^5*e^5*(d + e*x)^5)/(128*(a*e - b*d)^6) + (2123*b*e^5*(d + e*x))/(128*(a*e - b*d)^2))/((d + e*x)^(1/2)*(a^5*e^5 - b^5*d^5 - 10*a^2*b^3*d^3*e^2 + 10*a^3*b^2*d^2*e^3 + 5*a*b^4*d^4*e - 5*a^4*b*d*e^4) - (d + e*x)^(5/2)*(10*b^5*d^3 - 10*a^3*b^2*e^3 + 30*a^2*b^3*d*e^2 - 30*a*b^4*d^2*e) + (d + e*x)^(3/2)*(5*b^5*d^4 + 5*a^4*b*e^4 - 20*a^3*b^2*d*e^3 + 30*a^2*b^3*d^2*e^2 - 20*a*b^4*d^3*e) + b^5*(d + e*x)^(11/2) - (5*b^5*d - 5*a*b^4*e)*(d + e*x)^(9/2) + (d + e*x)^(7/2)*(10*b^5*d^2 + 10*a^2*b^3*e^2 - 20*a*b^4*d*e)) - (693*b^(1/2)*e^5*atan((b^(1/2)*(d + e*x)^(1/2)*(a^6*e^6 + b^6*d^6 + 15*a^2*b^4*d^4*e^2 - 20*a^3*b^3*d^3*e^3 + 15*a^4*b^2*d^2*e^4 - 6*a*b^5*d^5*e - 6*a^5*b*d*e^5))/(a*e - b*d)^(13/2)))/(128*(a*e - b*d)^(13/2))
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 1381, normalized size of antiderivative = 5.78

$$\int \frac{1}{(d+ex)^{3/2} (a^2+2abx+b^2x^2)^3} dx = \text{Too large to display}$$

input `int(1/(e*x+d)^(3/2)/(b^2*x^2+2*a*b*x+a^2)^3,x)`

output

```
( - 3465*sqrt(b)*sqrt(d + e*x)*sqrt(a*e - b*d)*atan((sqrt(d + e*x)*b)/(sqrt(b)*sqrt(a*e - b*d)))*a**5*e**5 - 17325*sqrt(b)*sqrt(d + e*x)*sqrt(a*e - b*d)*atan((sqrt(d + e*x)*b)/(sqrt(b)*sqrt(a*e - b*d)))*a**4*b*e**5*x - 34650*sqrt(b)*sqrt(d + e*x)*sqrt(a*e - b*d)*atan((sqrt(d + e*x)*b)/(sqrt(b)*sqrt(a*e - b*d)))*a**3*b**2*e**5*x**2 - 34650*sqrt(b)*sqrt(d + e*x)*sqrt(a*e - b*d)*atan((sqrt(d + e*x)*b)/(sqrt(b)*sqrt(a*e - b*d)))*a**2*b**3*e**5*x**3 - 17325*sqrt(b)*sqrt(d + e*x)*sqrt(a*e - b*d)*atan((sqrt(d + e*x)*b)/(sqrt(b)*sqrt(a*e - b*d)))*a*b**4*e**5*x**4 - 3465*sqrt(b)*sqrt(d + e*x)*sqrt(a*e - b*d)*atan((sqrt(d + e*x)*b)/(sqrt(b)*sqrt(a*e - b*d)))*b**5*e**5*x**5 - 1280*a**6*e**6 - 2935*a**5*b*d*e**5 - 10615*a**5*b*e**6*x + 7805*a**4*b**2*d**2*e**4 + 935*a**4*b**2*d*e**5*x - 26070*a**4*b**2*e**6*x**2 - 5838*a**3*b**3*d**3*e**3 + 13706*a**3*b**3*d**2*e**4*x + 15576*a**3*b**3*d**2*e**5*x**2 - 29568*a**3*b**3*e**6*x**3 + 3064*a**2*b**4*d**4*e**2 - 5258*a**2*b**4*d**3*e**3*x + 12672*a**2*b**4*d**2*e**4*x**2 + 24024*a**2*b**4*d**2*e**5*x**3 - 16170*a**2*b**4*e**6*x**4 - 944*a*b**5*d**5*e + 1408*a*b**5*d**4*e**2*x - 2442*a*b**5*d**3*e**3*x**2 + 6006*a*b**5*d**2*e**4*x**3 + 15015*a*b**5*d*e**5*x**4 - 3465*a*b**5*e**6*x**5 + 128*b**6*d**6 - 176*b**6*d**5*e*x + 264*b**6*d**4*e**2*x**2 - 462*b**6*d**3*e**3*x**3 + 1155*b**6*d**2*e**4*x**4 + 3465*b**6*d*e**5*x**5)/(640*sqrt(d + e*x)*(a**12*e**7 - 7*a**11*b*d*e**6 + 5*a**11*b*e**7*x + 21*a**10*b**2*d**2*e**5 - 35*a**10*b...
```

3.199 $\int \frac{1}{(d+ex)^{5/2}(a^2+2abx+b^2x^2)^3} dx$

| | |
|---|------|
| Optimal result | 1638 |
| Mathematica [A] (verified) | 1639 |
| Rubi [A] (verified) | 1639 |
| Maple [A] (verified) | 1649 |
| Fricas [B] (verification not implemented) | 1650 |
| Sympy [F(-1)] | 1650 |
| Maxima [F(-2)] | 1651 |
| Giac [B] (verification not implemented) | 1651 |
| Mupad [B] (verification not implemented) | 1652 |
| Reduce [B] (verification not implemented) | 1653 |

Optimal result

Integrand size = 28, antiderivative size = 266

$$\int \frac{1}{(d+ex)^{5/2}(a^2+2abx+b^2x^2)^3} dx =$$

$$\frac{1001e^5}{128(bd-ae)^6(d+ex)^{3/2}} - \frac{1}{5(bd-ae)(a+bx)^5(d+ex)^{3/2}}$$

$$+ \frac{13e}{40(bd-ae)^2(a+bx)^4(d+ex)^{3/2}} - \frac{143e^2}{240(bd-ae)^3(a+bx)^3(d+ex)^{3/2}}$$

$$+ \frac{429e^3}{320(bd-ae)^4(a+bx)^2(d+ex)^{3/2}} - \frac{3003e^4}{640(bd-ae)^5(a+bx)(d+ex)^{3/2}}$$

$$- \frac{3003be^5}{128(bd-ae)^7\sqrt{d+ex}} + \frac{3003b^{3/2}e^5 \operatorname{arctanh}\left(\frac{\sqrt{b}\sqrt{d+ex}}{\sqrt{bd-ae}}\right)}{128(bd-ae)^{15/2}}$$

output

```
-1001/128*e^5/(-a*e+b*d)^6/(e*x+d)^(3/2)-1/5/(-a*e+b*d)/(b*x+a)^5/(e*x+d)^(3/2)+13/40*e/(-a*e+b*d)^2/(b*x+a)^4/(e*x+d)^(3/2)-143/240*e^2/(-a*e+b*d)^3/(b*x+a)^3/(e*x+d)^(3/2)+429/320*e^3/(-a*e+b*d)^4/(b*x+a)^2/(e*x+d)^(3/2)-3003/640*e^4/(-a*e+b*d)^5/(b*x+a)/(e*x+d)^(3/2)-3003/128*b*e^5/(-a*e+b*d)^7/(e*x+d)^(1/2)+3003/128*b^(3/2)*e^5*arctanh(b^(1/2)*(e*x+d)^(1/2)/(-a*e+b*d)^(1/2))/(-a*e+b*d)^(15/2)
```

Mathematica [A] (verified)

Time = 2.43 (sec) , antiderivative size = 357, normalized size of antiderivative = 1.34

$$\int \frac{1}{(d+ex)^{5/2}(a^2+2abx+b^2x^2)^3} dx = \frac{-1280a^6e^6+1280a^5be^5(19d+13ex)+5a^4b^2e^4(7119d^2+38558dex+27599e^2x^2)+10a^3b^3e^3(19d^3+7917d^2ex+46475d^2e^2x^2+33891e^3x^3)+2a^2b^4e^2(5012d^4-11557d^3ex+42042d^2e^2x^2+260403de^3x^3+192192e^4x^4)+2ab^5e(-1464d^5+2704d^4ex-6149d^3e^2x^2+21879d^2e^3x^3+141141de^4x^4+105105e^5x^5)+b^6(384d^6-624d^5ex+1144d^4e^2x^2-2574d^3e^3x^3+9009d^2e^4x^4+60060de^5x^5+45045e^6x^6)}{((-(b*d)+a*e)^7*(a+b*x)^5*(d+e*x)^(3/2))+ (45045*b^(3/2)*e^5*ArcTan[(Sqrt[b]*Sqrt[d+e*x])/Sqrt[-(b*d)+a*e]])/(-(b*d)+a*e)^(15/2))/1920}$$

input `Integrate[1/((d + e*x)^(5/2)*(a^2 + 2*a*b*x + b^2*x^2)^3), x]`

output

```
((-1280*a^6*e^6 + 1280*a^5*b*e^5*(19*d + 13*e*x) + 5*a^4*b^2*e^4*(7119*d^2
+ 38558*d*e*x + 27599*e^2*x^2) + 10*a^3*b^3*e^3*(-2107*d^3 + 7917*d^2*e*x
+ 46475*d*e^2*x^2 + 33891*e^3*x^3) + 2*a^2*b^4*e^2*(5012*d^4 - 11557*d^3*
e*x + 42042*d^2*e^2*x^2 + 260403*d*e^3*x^3 + 192192*e^4*x^4) + 2*a*b^5*e*(
-1464*d^5 + 2704*d^4*e*x - 6149*d^3*e^2*x^2 + 21879*d^2*e^3*x^3 + 141141*d
*e^4*x^4 + 105105*e^5*x^5) + b^6*(384*d^6 - 624*d^5*e*x + 1144*d^4*e^2*x^2
- 2574*d^3*e^3*x^3 + 9009*d^2*e^4*x^4 + 60060*d*e^5*x^5 + 45045*e^6*x^6))
/(((-(b*d) + a*e)^7*(a + b*x)^5*(d + e*x)^(3/2)) + (45045*b^(3/2)*e^5*ArcTa
n[(Sqrt[b]*Sqrt[d + e*x])/Sqrt[-(b*d) + a*e]])/(-(b*d) + a*e)^(15/2))/1920
```

Rubi [A] (verified)

Time = 0.65 (sec) , antiderivative size = 333, normalized size of antiderivative = 1.25, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.393$, Rules used = {1098, 27, 52, 52, 52, 52, 61, 61, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a^2 + 2abx + b^2x^2)^3 (d + ex)^{5/2}} dx$$

↓ 1098

$$b^6 \int \frac{1}{b^6(a + bx)^6 (d + ex)^{5/2}} dx$$

↓ 27

$$\begin{aligned}
 & \int \frac{1}{(a+bx)^6(d+ex)^{5/2}} dx \\
 & \quad \downarrow 52 \\
 & -\frac{13e \int \frac{1}{(a+bx)^5(d+ex)^{5/2}} dx}{10(bd-ae)} - \frac{1}{5(a+bx)^5(d+ex)^{3/2}(bd-ae)} \\
 & \quad \downarrow 52 \\
 & -\frac{13e \left(-\frac{11e \int \frac{1}{(a+bx)^4(d+ex)^{5/2}} dx}{8(bd-ae)} - \frac{1}{4(a+bx)^4(d+ex)^{3/2}(bd-ae)} \right)}{10(bd-ae)} - \frac{1}{5(a+bx)^5(d+ex)^{3/2}(bd-ae)} \\
 & \quad \downarrow 52 \\
 & -\frac{13e \left(-\frac{11e \left(-\frac{3e \int \frac{1}{(a+bx)^3(d+ex)^{5/2}} dx}{2(bd-ae)} - \frac{1}{3(a+bx)^3(d+ex)^{3/2}(bd-ae)} \right)}{8(bd-ae)} - \frac{1}{4(a+bx)^4(d+ex)^{3/2}(bd-ae)} \right)}{10(bd-ae)} \\
 & \quad \quad \quad \frac{1}{5(a+bx)^5(d+ex)^{3/2}(bd-ae)} \\
 & \quad \quad \quad \downarrow 52 \\
 & -\frac{13e \left(-\frac{11e \left(-\frac{3e \left(-\frac{7e \int \frac{1}{(a+bx)^2(d+ex)^{5/2}} dx}{4(bd-ae)} - \frac{1}{2(a+bx)^2(d+ex)^{3/2}(bd-ae)} \right)}{2(bd-ae)} - \frac{1}{3(a+bx)^3(d+ex)^{3/2}(bd-ae)} \right)}{8(bd-ae)} - \frac{1}{4(a+bx)^4(d+ex)^{3/2}(bd-ae)} \right)}{10(bd-ae)} \\
 & \quad \quad \quad \frac{1}{5(a+bx)^5(d+ex)^{3/2}(bd-ae)} \\
 & \quad \quad \quad \downarrow 52 \\
 & \quad \quad \quad \frac{1}{5(a+bx)^5(d+ex)^{3/2}(bd-ae)}
 \end{aligned}$$

$$\begin{aligned}
 & \left(\begin{aligned} & \left(\begin{aligned} & 3e \left(\begin{aligned} & 7e \left(\begin{aligned} & \frac{5e \int \frac{1}{(a+bx)(d+ex)^{5/2}} dx}{2(bd-ae)} - \frac{1}{(a+bx)(d+ex)^{3/2}(bd-ae)} \end{aligned} \right) \\ & - \frac{1}{4(bd-ae)} - \frac{1}{2(a+bx)^2(d+ex)^{3/2}(bd-ae)} \end{aligned} \right) \\ & - \frac{1}{2(bd-ae)} - \frac{1}{3(a+bx)^3(d+ex)^{3/2}(bd-ae)} \end{aligned} \right) \\ & - \frac{1}{8(bd-ae)} \end{aligned} \right) \\
 & \frac{10(bd-ae)}{5(a+bx)^5(d+ex)^{3/2}(bd-ae)} \\
 & \downarrow 61
 \end{aligned}
 \end{aligned}$$

$$\begin{aligned}
 & \left(\frac{5e}{2(bd-ae)} \left(b \frac{\int \frac{1}{(a+bx)\sqrt{d+ex}} dx}{bd-ae} + \frac{2}{\sqrt{d+ex}(bd-ae)} \right) + \frac{2}{3(d+ex)^{3/2}(bd-ae)} \right) \\
 & \frac{7e}{(a+bx)(d+ex)^{3/2}(bd-ae)} \\
 & \frac{3e}{4(bd-ae)} - \frac{1}{2(a+bx)^2(d+ex)^{3/2}} \\
 & \frac{11e}{2(bd-ae)} \\
 & \frac{13e}{8(bd-ae)}
 \end{aligned}$$

↓ 73

$$\begin{aligned}
 & \left(\frac{2b \int \frac{1}{a + \frac{b(d+ex) - bd}{e}} d\sqrt{d+ex}}{e(bd-ae)} + \frac{2}{\sqrt{d+ex}(bd-ae)} \right) \\
 5e & \left(\frac{\quad}{bd-ae} + \frac{2}{3(d+ex)^{3/2}(bd-ae)} \right) \\
 7e & \left(\frac{\quad}{2(bd-ae)} - \frac{1}{(a+bx)(d+ex)^{3/2}(bd-ae)} \right) \\
 3e & \left(\frac{\quad}{4(bd-ae)} - \frac{\quad}{2(a+bx)^2} \right) \\
 11e & \left(\frac{\quad}{2(bd-ae)} \right) \\
 13e & \left(\frac{\quad}{8(bd-ae)} \right)
 \end{aligned}$$

↓ 221

$$\begin{aligned}
 & \left(\frac{5e}{7e} \left(\frac{b \left(\frac{2}{\sqrt{d+ex}(bd-ae)} - \frac{2\sqrt{b} \operatorname{arctanh} \left(\frac{\sqrt{b}\sqrt{d+ex}}{\sqrt{bd-ae}} \right)}{(bd-ae)^{3/2}} \right)}{bd-ae} + \frac{2}{3(d+ex)^{3/2}(bd-ae)} \right) - \frac{1}{(a+bx)(d+ex)^{3/2}(bd-ae)} \right) \\
 & \frac{3e}{4(bd-ae)} - \frac{1}{2(a+bx)^2(d+ex)} \\
 & \frac{11e}{2(bd-ae)} \\
 & \frac{13e}{8(bd-ae)}
 \end{aligned}$$

input `Int[1/((d + e*x)^(5/2)*(a^2 + 2*a*b*x + b^2*x^2)^3),x]`

output `-1/5*1/((b*d - a*e)*(a + b*x)^5*(d + e*x)^(3/2)) - (13*e*(-1/4*1/((b*d - a*e)*(a + b*x)^4*(d + e*x)^(3/2)) - (11*e*(-1/3*1/((b*d - a*e)*(a + b*x)^3*(d + e*x)^(3/2)) - (3*e*(-1/2*1/((b*d - a*e)*(a + b*x)^2*(d + e*x)^(3/2)) - (7*e*(-1/((b*d - a*e)*(a + b*x)*(d + e*x)^(3/2)))) - (5*e*(2/(3*(b*d - a*e)*(d + e*x)^(3/2)) + (b*(2/((b*d - a*e)*Sqrt[d + e*x]) - (2*Sqrt[b]*ArcTanh[(Sqrt[b]*Sqrt[d + e*x])/Sqrt[b*d - a*e]])/(b*d - a*e)^(3/2)))/(b*d - a*e)))/(2*(b*d - a*e))))/(4*(b*d - a*e)))/(2*(b*d - a*e)))/(8*(b*d - a*e)))/(10*(b*d - a*e))`

Defintions of rubi rules used

rule 27 `Int[(a_)*(Fx_), x_Symbol] := Simp[a Int[Fx, x], x] /; FreeQ[a, x] && !MatchQ[Fx, (b_)*(Gx_)] /; FreeQ[b, x]`

rule 52 `Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] := Simp[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((m + n + 2)/((b*c - a*d)*(m + 1)) Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && ILtQ[m, -1] && FractionQ[n] && LtQ[n, 0]`

rule 61 `Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] := Simp[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((m + n + 2)/((b*c - a*d)*(m + 1)) Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && LtQ[m, -1] && !(LtQ[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && IntLinearQ[a, b, c, d, m, n, x]`

rule 73 `Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] := With[{p = Denominator[m]}, Simp[p/b Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) + d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntLinearQ[a, b, c, d, m, n, x]`

```
rule 221 Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[-a/b, 2]/a)*ArcTanh[x/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

```
rule 1098 Int[((d_) + (e_)*(x_)^(m_))*((a_) + (b_)*(x_) + (c_)*(x_)^2)^(p_), x_Symbol] := Simp[1/c^p Int[(d + e*x)^m*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Maple [A] (verified)

Time = 1.71 (sec) , antiderivative size = 291, normalized size of antiderivative = 1.09

| method | result |
|-------------------|---|
| derivativedivides | $2e^5 \left(-\frac{1}{3(ae-bd)^6 (ex+d)^{\frac{3}{2}}} + \frac{6b}{(ae-bd)^7 \sqrt{ex+d}} + \frac{b^2 \left(\frac{1467b^4 (ex+d)^{\frac{9}{2}}}{256} + \frac{9629b^3 (ae-bd)(ex+d)^{\frac{7}{2}}}{384} + \left(\frac{1253}{30} a^2 b^2 e^2 - \frac{12}{1} \right) \right)}{256} \right)$ |
| default | $2e^5 \left(-\frac{1}{3(ae-bd)^6 (ex+d)^{\frac{3}{2}}} + \frac{6b}{(ae-bd)^7 \sqrt{ex+d}} + \frac{b^2 \left(\frac{1467b^4 (ex+d)^{\frac{9}{2}}}{256} + \frac{9629b^3 (ae-bd)(ex+d)^{\frac{7}{2}}}{384} + \left(\frac{1253}{30} a^2 b^2 e^2 - \frac{12}{1} \right) \right)}{256} \right)$ |
| pseudoelliptic | $2 \left(-\frac{9009b^2 e^5 (ex+d)^{\frac{3}{2}} (bx+a)^5 \arctan\left(\frac{b\sqrt{ex+d}}{\sqrt{b(ae-bd)}}\right)}{256} + \left(-\frac{3003}{64} d e^5 x^5 - \frac{9009}{1280} d^2 e^4 x^4 + \frac{1287}{640} d^3 e^3 x^3 - \frac{143}{160} d^4 e^2 x^2 + \frac{39}{80} d^5 e \right) \right)$ |

```
input int(1/(e*x+d)^(5/2)/(b^2*x^2+2*a*b*x+a^2)^3,x,method=_RETURNVERBOSE)
```

output

```
2*e^5*(-1/3/(a*e-b*d)^6/(e*x+d)^(3/2)+6/(a*e-b*d)^7*b/(e*x+d)^(1/2)+1/(a*e
-b*d)^7*b^2*((1467/256*b^4*(e*x+d)^(9/2)+9629/384*b^3*(a*e-b*d)*(e*x+d)^(7
/2)+(1253/30*a^2*b^2*e^2-1253/15*a*b^3*d*e+1253/30*b^4*d^2)*(e*x+d)^(5/2)+
(12131/384*a^3*b*e^3-12131/128*a^2*b^2*d*e^2+12131/128*d^2*e*a*b^3-12131/3
84*b^4*d^3)*(e*x+d)^(3/2)+(2373/256*a^4*e^4-2373/64*a^3*b*d*e^3+7119/128*a
^2*b^2*d^2*e^2-2373/64*a*b^3*d^3*e+2373/256*b^4*d^4)*(e*x+d)^(1/2))/(b*(e*
x+d)+a*e-b*d)^5+3003/256/(b*(a*e-b*d))^(1/2)*arctan(b*(e*x+d)^(1/2)/(b*(a*
e-b*d))^(1/2))))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1597 vs. $2(226) = 452$.

Time = 0.75 (sec) , antiderivative size = 3224, normalized size of antiderivative = 12.12

$$\int \frac{1}{(d+ex)^{5/2} (a^2+2abx+b^2x^2)^3} dx = \text{Too large to display}$$

input

```
integrate(1/(e*x+d)^(5/2)/(b^2*x^2+2*a*b*x+a^2)^3,x, algorithm="fricas")
```

output

Too large to include

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{(d+ex)^{5/2} (a^2+2abx+b^2x^2)^3} dx = \text{Timed out}$$

input

```
integrate(1/(e*x+d)**(5/2)/(b**2*x**2+2*a*b*x+a**2)**3,x)
```

output

Timed out

Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{(d+ex)^{5/2} (a^2+2abx+b^2x^2)^3} dx = \text{Exception raised: ValueError}$$

input `integrate(1/(e*x+d)^(5/2)/(b^2*x^2+2*a*b*x+a^2)^3,x, algorithm="maxima")`

output `Exception raised: ValueError >> Computation failed since Maxima requested additional constraints; using the 'assume' command before evaluation *may* help (example of legal syntax is 'assume(a*e-b*d>0)', see `assume?` for more detail`

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 649 vs. 2(226) = 452.

Time = 0.14 (sec) , antiderivative size = 649, normalized size of antiderivative = 2.44

$$\int \frac{1}{(d+ex)^{5/2} (a^2+2abx+b^2x^2)^3} dx =$$

$$\frac{3003 b^2 e^5 \arctan\left(\frac{\sqrt{ex+db}}{\sqrt{-b^2d+abe}}\right)}{128 (b^7 d^7 - 7 ab^6 d^6 e + 21 a^2 b^5 d^5 e^2 - 35 a^3 b^4 d^4 e^3 + 35 a^4 b^3 d^3 e^4 - 21 a^5 b^2 d^2 e^5 + 7 a^6 b d e^6 - a^7 e^7) \sqrt{-b^2 d + a b e}} -$$

$$\frac{3 (b^7 d^7 - 7 ab^6 d^6 e + 21 a^2 b^5 d^5 e^2 - 35 a^3 b^4 d^4 e^3 + 35 a^4 b^3 d^3 e^4 - 21 a^5 b^2 d^2 e^5 + 7 a^6 b d e^6 - a^7 e^7) (ex+d)^{\frac{3}{2}}}{22005 (ex+d)^{\frac{9}{2}} b^6 e^5 - 96290 (ex+d)^{\frac{7}{2}} b^6 d e^5 + 160384 (ex+d)^{\frac{5}{2}} b^6 d^2 e^5 - 121310 (ex+d)^{\frac{3}{2}} b^6 d^3 e^5 + 35550 (ex+d)^{\frac{1}{2}} b^6 d^4 e^5 - 121310 b^6 d^5 e^5}$$

input `integrate(1/(e*x+d)^(5/2)/(b^2*x^2+2*a*b*x+a^2)^3,x, algorithm="giac")`

output

```

-3003/128*b^2*e^5*arctan(sqrt(e*x + d)*b/sqrt(-b^2*d + a*b*e))/((b^7*d^7 -
7*a*b^6*d^6*e + 21*a^2*b^5*d^5*e^2 - 35*a^3*b^4*d^4*e^3 + 35*a^4*b^3*d^3*
e^4 - 21*a^5*b^2*d^2*e^5 + 7*a^6*b*d*e^6 - a^7*e^7)*sqrt(-b^2*d + a*b*e))
- 2/3*(18*(e*x + d)*b*e^5 + b*d*e^5 - a*e^6)/((b^7*d^7 - 7*a*b^6*d^6*e + 2
1*a^2*b^5*d^5*e^2 - 35*a^3*b^4*d^4*e^3 + 35*a^4*b^3*d^3*e^4 - 21*a^5*b^2*d
^2*e^5 + 7*a^6*b*d*e^6 - a^7*e^7)*(e*x + d)^(3/2)) - 1/1920*(22005*(e*x +
d)^(9/2)*b^6*e^5 - 96290*(e*x + d)^(7/2)*b^6*d*e^5 + 160384*(e*x + d)^(5/2
)*b^6*d^2*e^5 - 121310*(e*x + d)^(3/2)*b^6*d^3*e^5 + 35595*sqrt(e*x + d)*b
^6*d^4*e^5 + 96290*(e*x + d)^(7/2)*a*b^5*e^6 - 320768*(e*x + d)^(5/2)*a*b
^5*d*e^6 + 363930*(e*x + d)^(3/2)*a*b^5*d^2*e^6 - 142380*sqrt(e*x + d)*a*b
^5*d^3*e^6 + 160384*(e*x + d)^(5/2)*a^2*b^4*e^7 - 363930*(e*x + d)^(3/2)*a
^2*b^4*d*e^7 + 213570*sqrt(e*x + d)*a^2*b^4*d^2*e^7 + 121310*(e*x + d)^(3/2
)*a^3*b^3*e^8 - 142380*sqrt(e*x + d)*a^3*b^3*d*e^8 + 35595*sqrt(e*x + d)*a
^4*b^2*e^9)/((b^7*d^7 - 7*a*b^6*d^6*e + 21*a^2*b^5*d^5*e^2 - 35*a^3*b^4*d
^4*e^3 + 35*a^4*b^3*d^3*e^4 - 21*a^5*b^2*d^2*e^5 + 7*a^6*b*d*e^6 - a^7*e^7)
*((e*x + d)*b - b*d + a*e)^5)

```

Mupad [B] (verification not implemented)

Time = 10.16 (sec) , antiderivative size = 555, normalized size of antiderivative = 2.09

$$\int \frac{1}{(d+ex)^{5/2} (a^2+2abx+b^2x^2)^3} dx = \frac{1}{(d+ex)^{3/2} (a^5e^5 - 5a^4bde^4 + 10a^3b^2d^2e^3 - 10a^2b^3d^3e^2 + 5a^2b^4d^4e - 5a^2b^5d^5e^0 + 5a^2b^6d^6e^{-1} - 5a^2b^7d^7e^{-2})} + \frac{3003b^{3/2}e^5 \operatorname{atan}\left(\frac{\sqrt{b}\sqrt{d+ex}(a^7e^7 - 7a^6bde^6 + 21a^5b^2d^2e^5 - 35a^4b^3d^3e^4 + 35a^3b^4d^4e^3 - 21a^2b^5d^5e^2 + 7ab^6d^6e - b^7d^7)}{(ae-bd)^{15/2}}\right)}{128(ae-bd)^{15/2}}$$

input

```
int(1/((d + e*x)^(5/2)*(a^2 + b^2*x^2 + 2*a*b*x)^3), x)
```

output

```
((27599*b^2*e^5*(d + e*x)^2)/(384*(a*e - b*d)^3) - (2*e^5)/(3*(a*e - b*d))
+ (11297*b^3*e^5*(d + e*x)^3)/(64*(a*e - b*d)^4) + (1001*b^4*e^5*(d + e*x)
)^4)/(5*(a*e - b*d)^5) + (7007*b^5*e^5*(d + e*x)^5)/(64*(a*e - b*d)^6) + (
3003*b^6*e^5*(d + e*x)^6)/(128*(a*e - b*d)^7) + (26*b*e^5*(d + e*x))/(3*(a
*e - b*d)^2))/((d + e*x)^(3/2)*(a^5*e^5 - b^5*d^5 - 10*a^2*b^3*d^3*e^2 + 1
0*a^3*b^2*d^2*e^3 + 5*a*b^4*d^4*e - 5*a^4*b*d*e^4) - (d + e*x)^(7/2)*(10*b
^5*d^3 - 10*a^3*b^2*e^3 + 30*a^2*b^3*d*e^2 - 30*a*b^4*d^2*e) + (d + e*x)^(
5/2)*(5*b^5*d^4 + 5*a^4*b*e^4 - 20*a^3*b^2*d*e^3 + 30*a^2*b^3*d^2*e^2 - 20
*a*b^4*d^3*e) + b^5*(d + e*x)^(13/2) - (5*b^5*d - 5*a*b^4*e)*(d + e*x)^(11
/2) + (d + e*x)^(9/2)*(10*b^5*d^2 + 10*a^2*b^3*e^2 - 20*a*b^4*d*e)) + (300
3*b^(3/2)*e^5*atan((b^(1/2)*(d + e*x)^(1/2)*(a^7*e^7 - b^7*d^7 - 21*a^2*b^
5*d^5*e^2 + 35*a^3*b^4*d^4*e^3 - 35*a^4*b^3*d^3*e^4 + 21*a^5*b^2*d^2*e^5 +
7*a*b^6*d^6*e - 7*a^6*b*d*e^6))/(a*e - b*d)^(15/2)))/(128*(a*e - b*d)^(15
/2))
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 2113, normalized size of antiderivative = 7.94

$$\int \frac{1}{(d + ex)^{5/2} (a^2 + 2abx + b^2x^2)^3} dx = \text{Too large to display}$$

input

```
int(1/(e*x+d)^(5/2)/(b^2*x^2+2*a*b*x+a^2)^3,x)
```

output

```
(45045*sqrt(b)*sqrt(d + e*x)*sqrt(a*e - b*d)*atan((sqrt(d + e*x)*b)/(sqrt(b)*sqrt(a*e - b*d)))*a**5*b*d*e**5 + 45045*sqrt(b)*sqrt(d + e*x)*sqrt(a*e - b*d)*atan((sqrt(d + e*x)*b)/(sqrt(b)*sqrt(a*e - b*d)))*a**5*b*e**6*x + 225225*sqrt(b)*sqrt(d + e*x)*sqrt(a*e - b*d)*atan((sqrt(d + e*x)*b)/(sqrt(b)*sqrt(a*e - b*d)))*a**4*b**2*d*e**5*x + 225225*sqrt(b)*sqrt(d + e*x)*sqrt(a*e - b*d)*atan((sqrt(d + e*x)*b)/(sqrt(b)*sqrt(a*e - b*d)))*a**4*b**2*e**6*x**2 + 450450*sqrt(b)*sqrt(d + e*x)*sqrt(a*e - b*d)*atan((sqrt(d + e*x)*b)/(sqrt(b)*sqrt(a*e - b*d)))*a**3*b**3*d*e**5*x**2 + 450450*sqrt(b)*sqrt(d + e*x)*sqrt(a*e - b*d)*atan((sqrt(d + e*x)*b)/(sqrt(b)*sqrt(a*e - b*d)))*a**3*b**3*e**6*x**3 + 450450*sqrt(b)*sqrt(d + e*x)*sqrt(a*e - b*d)*atan((sqrt(d + e*x)*b)/(sqrt(b)*sqrt(a*e - b*d)))*a**2*b**4*d*e**5*x**3 + 450450*sqrt(b)*sqrt(d + e*x)*sqrt(a*e - b*d)*atan((sqrt(d + e*x)*b)/(sqrt(b)*sqrt(a*e - b*d)))*a**2*b**4*e**6*x**4 + 225225*sqrt(b)*sqrt(d + e*x)*sqrt(a*e - b*d)*atan((sqrt(d + e*x)*b)/(sqrt(b)*sqrt(a*e - b*d)))*a*b**5*d*e**5*x**4 + 225225*sqrt(b)*sqrt(d + e*x)*sqrt(a*e - b*d)*atan((sqrt(d + e*x)*b)/(sqrt(b)*sqrt(a*e - b*d)))*a*b**5*e**6*x**5 + 45045*sqrt(b)*sqrt(d + e*x)*sqrt(a*e - b*d)*atan((sqrt(d + e*x)*b)/(sqrt(b)*sqrt(a*e - b*d)))*b**6*d*e**5*x**5 + 45045*sqrt(b)*sqrt(d + e*x)*sqrt(a*e - b*d)*atan((sqrt(d + e*x)*b)/(sqrt(b)*sqrt(a*e - b*d)))*b**6*e**6*x**6 - 1280*a**7*e**7 + 25600*a**6*b*d*e**6 + 16640*a**6*b*e**7*x + 11275*a**5*b**2*d**2*e**5 + 176150*a...
```

3.200 $\int \frac{1}{(d+ex)^{7/2}(a^2+2abx+b^2x^2)^3} dx$

| | |
|---|------|
| Optimal result | 1655 |
| Mathematica [A] (verified) | 1656 |
| Rubi [A] (verified) | 1656 |
| Maple [A] (verified) | 1668 |
| Fricas [B] (verification not implemented) | 1669 |
| Sympy [F(-1)] | 1669 |
| Maxima [F(-2)] | 1670 |
| Giac [B] (verification not implemented) | 1670 |
| Mupad [B] (verification not implemented) | 1671 |
| Reduce [B] (verification not implemented) | 1672 |

Optimal result

Integrand size = 28, antiderivative size = 295

$$\int \frac{1}{(d+ex)^{7/2}(a^2+2abx+b^2x^2)^3} dx = -\frac{9009e^5}{640(bd-ae)^6(d+ex)^{5/2}} - \frac{1}{5(bd-ae)(a+bx)^5(d+ex)^{5/2}} + \frac{3e}{8(bd-ae)^2(a+bx)^4(d+ex)^{5/2}} - \frac{13e^2}{16(bd-ae)^3(a+bx)^3(d+ex)^{5/2}} + \frac{143e^3}{64(bd-ae)^4(a+bx)^2(d+ex)^{5/2}} - \frac{1287e^4}{128(bd-ae)^5(a+bx)(d+ex)^{5/2}} - \frac{3003be^5}{128(bd-ae)^7(d+ex)^{3/2}} - \frac{9009b^2e^5}{128(bd-ae)^8\sqrt{d+ex}} + \frac{9009b^{5/2}e^5 \operatorname{arctanh}\left(\frac{\sqrt{b}\sqrt{d+ex}}{\sqrt{bd-ae}}\right)}{128(bd-ae)^{17/2}}$$

output

```
-9009/640*e^5/(-a*e+b*d)^6/(e*x+d)^(5/2)-1/5/(-a*e+b*d)/(b*x+a)^5/(e*x+d)^(5/2)+3/8*e/(-a*e+b*d)^2/(b*x+a)^4/(e*x+d)^(5/2)-13/16*e^2/(-a*e+b*d)^3/(b*x+a)^3/(e*x+d)^(5/2)+143/64*e^3/(-a*e+b*d)^4/(b*x+a)^2/(e*x+d)^(5/2)-1287/128*e^4/(-a*e+b*d)^5/(b*x+a)/(e*x+d)^(5/2)-3003/128*b*e^5/(-a*e+b*d)^7/(e*x+d)^(3/2)-9009/128*b^2*e^5/(-a*e+b*d)^8/(e*x+d)^(1/2)+9009/128*b^(5/2)*e^5*arctanh(b^(1/2)*(e*x+d)^(1/2)/(-a*e+b*d)^(1/2))/(-a*e+b*d)^(17/2)
```

Mathematica [A] (verified)

Time = 2.79 (sec) , antiderivative size = 443, normalized size of antiderivative = 1.50

$$\int \frac{1}{(d+ex)^{7/2} (a^2+2abx+b^2x^2)^3} dx = \frac{1}{640} \left(\frac{-256a^7e^7 + 256a^6be^6(12d+5ex) - 256a^5b^2e^5(116d^2+160d+160e^2x^2) - 5a^4b^3e^4(5327d^3+45677d^2ex+66157de^2x^2+27599e^3x^3) - 10a^3b^4e^3(-1211d^4+5810d^3ex+54392d^2e^2x^2+80366de^3x^3+33891e^4x^4) - 2a^2b^5e^2(2324d^5-6545d^4ex+30485d^3e^2x^2+302445d^2e^3x^3+452595de^4x^4+192192e^5x^5) - 2ab^6e(-568d^6+1240d^5ex-3445d^4e^2x^2+15730d^3e^3x^3+163020d^2e^4x^4+246246de^5x^5+105105e^6x^6) - b^7(128d^7-240d^6ex+520d^5e^2x^2-1430d^4e^3x^3+6435d^3e^4x^4+69069d^2e^5x^5+105105de^6x^6+45045e^7x^7)}{(-bd+ae)^{17/2}} \right)$$

input

```
Integrate[1/((d + e*x)^(7/2)*(a^2 + 2*a*b*x + b^2*x^2)^3), x]
```

output

```
((-256*a^7*e^7 + 256*a^6*b*e^6*(12*d + 5*e*x) - 256*a^5*b^2*e^5*(116*d^2 + 160*d*e*x + 65*e^2*x^2) - 5*a^4*b^3*e^4*(5327*d^3 + 45677*d^2*e*x + 66157*d*e^2*x^2 + 27599*e^3*x^3) - 10*a^3*b^4*e^3*(-1211*d^4 + 5810*d^3*e*x + 54392*d^2*e^2*x^2 + 80366*d*e^3*x^3 + 33891*e^4*x^4) - 2*a^2*b^5*e^2*(2324*d^5 - 6545*d^4*e*x + 30485*d^3*e^2*x^2 + 302445*d^2*e^3*x^3 + 452595*d*e^4*x^4 + 192192*e^5*x^5) - 2*a*b^6*e*(-568*d^6 + 1240*d^5*e*x - 3445*d^4*e^2*x^2 + 15730*d^3*e^3*x^3 + 163020*d^2*e^4*x^4 + 246246*d*e^5*x^5 + 105105*e^6*x^6) - b^7*(128*d^7 - 240*d^6*e*x + 520*d^5*e^2*x^2 - 1430*d^4*e^3*x^3 + 6435*d^3*e^4*x^4 + 69069*d^2*e^5*x^5 + 105105*d*e^6*x^6 + 45045*e^7*x^7)) / ((b*d - a*e)^8*(a + b*x)^5*(d + e*x)^(5/2)) - (45045*b^(5/2)*e^5*ArcTan[(Sqrt[b]*Sqrt[d + e*x])/Sqrt[-(b*d) + a*e]] / (-(b*d) + a*e)^(17/2)) / 640
```

Rubi [A] (verified)

Time = 0.67 (sec) , antiderivative size = 369, normalized size of antiderivative = 1.25, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$, Rules used = {1098, 27, 52, 52, 52, 52, 52, 61, 61, 61, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a^2+2abx+b^2x^2)^3 (d+ex)^{7/2}} dx$$

↓ 1098

$$\begin{aligned}
 & b^6 \int \frac{1}{b^6(a+bx)^6(d+ex)^{7/2}} dx \\
 & \quad \downarrow 27 \\
 & \int \frac{1}{(a+bx)^6(d+ex)^{7/2}} dx \\
 & \quad \downarrow 52 \\
 & \frac{3e \int \frac{1}{(a+bx)^5(d+ex)^{7/2}} dx}{2(bd-ae)} - \frac{1}{5(a+bx)^5(d+ex)^{5/2}(bd-ae)} \\
 & \quad \downarrow 52 \\
 & \frac{3e \left(-\frac{13e \int \frac{1}{(a+bx)^4(d+ex)^{7/2}} dx}{8(bd-ae)} - \frac{1}{4(a+bx)^4(d+ex)^{5/2}(bd-ae)} \right)}{2(bd-ae)} - \frac{1}{5(a+bx)^5(d+ex)^{5/2}(bd-ae)} \\
 & \quad \downarrow 52 \\
 & \frac{3e \left(-\frac{13e \left(-\frac{11e \int \frac{1}{(a+bx)^3(d+ex)^{7/2}} dx}{6(bd-ae)} - \frac{1}{3(a+bx)^3(d+ex)^{5/2}(bd-ae)} \right)}{8(bd-ae)} - \frac{1}{4(a+bx)^4(d+ex)^{5/2}(bd-ae)} \right)}{2(bd-ae)} - \frac{1}{5(a+bx)^5(d+ex)^{5/2}(bd-ae)} \\
 & \quad \downarrow 52 \\
 & \frac{3e \left(-\frac{13e \left(-\frac{11e \left(-\frac{9e \int \frac{1}{(a+bx)^2(d+ex)^{7/2}} dx}{4(bd-ae)} - \frac{1}{2(a+bx)^2(d+ex)^{5/2}(bd-ae)} \right)}{6(bd-ae)} - \frac{1}{3(a+bx)^3(d+ex)^{5/2}(bd-ae)} \right)}{8(bd-ae)} - \frac{1}{4(a+bx)^4(d+ex)^{5/2}(bd-ae)} \right)}{2(bd-ae)} - \frac{1}{5(a+bx)^5(d+ex)^{5/2}(bd-ae)} \\
 & \quad \downarrow 52 \\
 & \frac{1}{5(a+bx)^5(d+ex)^{5/2}(bd-ae)}
 \end{aligned}$$

$$\begin{aligned}
 & \left(\begin{aligned} & \left(\begin{aligned} & 7e \left(\frac{b \int \frac{1}{(a+bx)(d+ex)^{5/2}} dx}{bd-ae} + \frac{2}{5(d+ex)^{5/2}(bd-ae)} \right) \\ & - \frac{1}{(a+bx)(d+ex)^{5/2}(bd-ae)} \end{aligned} \right) \\ & - \frac{1}{2(bd-ae)} \end{aligned} \right) \\
 & - \frac{1}{4(bd-ae)} \\
 & - \frac{1}{2(a+bx)^2(d+ex)^{5/2}(bd-ae)} \\
 & - \frac{1}{6(bd-ae)} \\
 & - \frac{1}{3(a+bx)} \\
 & - \frac{1}{8(bd-ae)} \\
 & - \frac{1}{2(bd-ae)} \\
 & - \frac{1}{5(a+bx)^5(d+ex)^{5/2}(bd-ae)}
 \end{aligned}$$

\downarrow 61

↓ 61

$$\begin{aligned}
 & \left(\left(\left(\left(\left(\frac{b \int \frac{1}{(a+bx)\sqrt{d+ex}} dx}{bd-ae} + \frac{2}{\sqrt{d+ex}(bd-ae)} \right) + \frac{2}{3(d+ex)^{3/2}(bd-ae)} \right) + \frac{2}{5(d+ex)^{5/2}(bd-ae)} \right) \right) \right) \\
 7e & \left(\frac{\quad}{bd-ae} \right) + \frac{2}{5(d+ex)^{5/2}(bd-ae)} \\
 9e & \frac{\quad}{2(bd-ae)} - \frac{1}{(a+bx)(d+ex)^{5/2}(bd-ae)} \\
 11e & \frac{\quad}{4(bd-ae)} \\
 13e & \frac{\quad}{6(bd-ae)}
 \end{aligned}$$

↓ 73

$$\left(\frac{b \left(\frac{2b \int \frac{1}{a + \frac{b(d+ex) - bd}{e(bd-ae)}} d\sqrt{d+ex} + \frac{2}{\sqrt{d+ex}(bd-ae)} \right)}{bd-ae} + \frac{2}{3(d+ex)^{3/2}(bd-ae)} \right) + \frac{2}{5(d+ex)^{5/2}(bd-ae)}$$

$$\frac{7e}{bd-ae} + \frac{2}{5(d+ex)^{5/2}(bd-ae)}$$

$$\frac{9e}{2(bd-ae)} - \frac{1}{(a+bx)(d+ex)}$$

$$\frac{11e}{4(bd-ae)}$$

$$\frac{13e}{6(bd-ae)}$$

↓ 221

| | |
|-----|--|
| 7e | $\left(\frac{b \left(\frac{2}{\sqrt{d+ex}(bd-ae)} - \frac{2\sqrt{b}\operatorname{arctanh}\left(\frac{\sqrt{b}\sqrt{d+ex}}{\sqrt{bd-ae}}\right)}{(bd-ae)^{3/2}} \right)}{bd-ae} + \frac{2}{3(d+ex)^{3/2}(bd-ae)} \right)$ |
| 9e | $\frac{2}{5(d+ex)^{5/2}(bd-ae)} - \frac{1}{(a+bx)(d+ex)^5}$ |
| 11e | $\frac{4(bd-ae)}{2(bd-ae)}$ |
| 13e | $\frac{6(bd-ae)}{6(bd-ae)}$ |

input `Int[1/((d + e*x)^(7/2)*(a^2 + 2*a*b*x + b^2*x^2)^3),x]`

output `-1/5*1/((b*d - a*e)*(a + b*x)^5*(d + e*x)^(5/2)) - (3*e*(-1/4*1/((b*d - a*e)*(a + b*x)^4*(d + e*x)^(5/2)) - (13*e*(-1/3*1/((b*d - a*e)*(a + b*x)^3*(d + e*x)^(5/2)) - (11*e*(-1/2*1/((b*d - a*e)*(a + b*x)^2*(d + e*x)^(5/2)) - (9*e*(-1/((b*d - a*e)*(a + b*x)*(d + e*x)^(5/2)))) - (7*e*(2/(5*(b*d - a*e)*(d + e*x)^(5/2)) + (b*(2/(3*(b*d - a*e)*(d + e*x)^(3/2)) + (b*(2/((b*d - a*e)*Sqrt[d + e*x]) - (2*Sqrt[b]*ArcTanh[(Sqrt[b]*Sqrt[d + e*x])/Sqrt[b*d - a*e]])/(b*d - a*e)^(3/2)))/(b*d - a*e)))/(b*d - a*e)))/(2*(b*d - a*e)))/(4*(b*d - a*e)))/(6*(b*d - a*e)))/(8*(b*d - a*e)))/(2*(b*d - a*e))`

Defintions of rubi rules used

rule 27 `Int[(a_)*(F_x_), x_Symbol] := Simp[a Int[F_x, x], x] /; FreeQ[a, x] && !MatchQ[F_x, (b_)*(G_x_)] /; FreeQ[b, x]`

rule 52 `Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] := Simp[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((m + n + 2)/((b*c - a*d)*(m + 1)) Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && ILtQ[m, -1] && FractionQ[n] && LtQ[n, 0]`

rule 61 `Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] := Simp[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((m + n + 2)/((b*c - a*d)*(m + 1)) Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && LtQ[m, -1] && !(LtQ[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && IntLinearQ[a, b, c, d, m, n, x]`

rule 73 `Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] := With[{p = Denominator[m]}, Simp[p/b Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) + d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntLinearQ[a, b, c, d, m, n, x]`


```
rule 221 Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[-a/b, 2]/a)*ArcTanh[x/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

```
rule 1098 Int[((d_) + (e_)*(x_)^(m_))*((a_) + (b_)*(x_) + (c_)*(x_)^2)^(p_), x_Symbol] := Simp[1/c^p Int[(d + e*x)^m*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Maple [A] (verified)

Time = 1.84 (sec) , antiderivative size = 314, normalized size of antiderivative = 1.06

| method | result |
|-------------------|--|
| derivativedivides | $2e^5 \left(-\frac{1}{5(ae-bd)^6 (ex+d)^{\frac{5}{2}}} - \frac{21b^2}{(ae-bd)^8 \sqrt{ex+d}} + \frac{2b}{(ae-bd)^7 (ex+d)^{\frac{3}{2}}} - b^3 \left(\frac{3633b^4 (ex+d)^{\frac{9}{2}}}{256} + \frac{7837b^3 (ae-bd)(ex+d)^{\frac{7}{2}}}{128} \right) \right)$ |
| default | $2e^5 \left(-\frac{1}{5(ae-bd)^6 (ex+d)^{\frac{5}{2}}} - \frac{21b^2}{(ae-bd)^8 \sqrt{ex+d}} + \frac{2b}{(ae-bd)^7 (ex+d)^{\frac{3}{2}}} - b^3 \left(\frac{3633b^4 (ex+d)^{\frac{9}{2}}}{256} + \frac{7837b^3 (ae-bd)(ex+d)^{\frac{7}{2}}}{128} \right) \right)$ |
| pseudoelliptic | $2 \left(\frac{45045b^3 e^5 (ex+d)^{\frac{5}{2}} (bx+a)^5 \arctan\left(\frac{b\sqrt{ex+d}}{\sqrt{b(ae-bd)}}\right)}{256} + \left(\frac{105105}{128} d e^6 x^6 + \frac{69069}{128} d^2 e^5 x^5 + \frac{6435}{128} d^3 e^4 x^4 - \frac{715}{64} d^4 e^3 x^3 + \frac{65}{16} d^5 e^2 x^2 - \frac{5}{2} d^6 x + d^7 \right) \right)$ |

```
input int(1/(e*x+d)^(7/2)/(b^2*x^2+2*a*b*x+a^2)^3,x,method=_RETURNVERBOSE)
```

output

```
2*e^5*(-1/5/(a*e-b*d)^6/(e*x+d)^(5/2)-21/(a*e-b*d)^8*b^2/(e*x+d)^(1/2)+2/(
a*e-b*d)^7*b/(e*x+d)^(3/2)-1/(a*e-b*d)^8*b^3*((3633/256*b^4*(e*x+d)^(9/2)+
7837/128*b^3*(a*e-b*d)*(e*x+d)^(7/2)+(1001/10*a^2*b^2*e^2-1001/5*a*b^3*d*e
+1001/10*b^4*d^2)*(e*x+d)^(5/2)+(9443/128*a^3*b*e^3-28329/128*a^2*b^2*d*e^
2+28329/128*d^2*e*a*b^3-9443/128*b^4*d^3)*(e*x+d)^(3/2)+(5327/256*a^4*e^4-
5327/64*a^3*b*d*e^3+15981/128*a^2*b^2*d^2*e^2-5327/64*a*b^3*d^3*e+5327/256
*b^4*d^4)*(e*x+d)^(1/2))/(b*(e*x+d)+a*e-b*d)^5+9009/256/(b*(a*e-b*d))^(1/2)
)*arctan(b*(e*x+d)^(1/2)/(b*(a*e-b*d))^(1/2)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2091 vs. $2(251) = 502$.

Time = 1.59 (sec) , antiderivative size = 4212, normalized size of antiderivative = 14.28

$$\int \frac{1}{(d+ex)^{7/2} (a^2+2abx+b^2x^2)^3} dx = \text{Too large to display}$$

input

```
integrate(1/(e*x+d)^(7/2)/(b^2*x^2+2*a*b*x+a^2)^3,x, algorithm="fricas")
```

output

Too large to include

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{(d+ex)^{7/2} (a^2+2abx+b^2x^2)^3} dx = \text{Timed out}$$

input

```
integrate(1/(e*x+d)**(7/2)/(b**2*x**2+2*a*b*x+a**2)**3,x)
```

output

Timed out

Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{(d+ex)^{7/2} (a^2+2abx+b^2x^2)^3} dx = \text{Exception raised: ValueError}$$

input `integrate(1/(e*x+d)^(7/2)/(b^2*x^2+2*a*b*x+a^2)^3,x, algorithm="maxima")`

output `Exception raised: ValueError >> Computation failed since Maxima requested additional constraints; using the 'assume' command before evaluation *may* help (example of legal syntax is 'assume(a*e-b*d>0)', see `assume?` for more detail`

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 898 vs. 2(251) = 502.

Time = 0.17 (sec) , antiderivative size = 898, normalized size of antiderivative = 3.04

$$\int \frac{1}{(d+ex)^{7/2} (a^2+2abx+b^2x^2)^3} dx = \text{Too large to display}$$

input `integrate(1/(e*x+d)^(7/2)/(b^2*x^2+2*a*b*x+a^2)^3,x, algorithm="giac")`

output

```
-9009/128*b^3*e^5*arctan(sqrt(e*x + d)*b/sqrt(-b^2*d + a*b*e))/((b^8*d^8 -
8*a*b^7*d^7*e + 28*a^2*b^6*d^6*e^2 - 56*a^3*b^5*d^5*e^3 + 70*a^4*b^4*d^4*
e^4 - 56*a^5*b^3*d^3*e^5 + 28*a^6*b^2*d^2*e^6 - 8*a^7*b*d*e^7 + a^8*e^8)*s
qrt(-b^2*d + a*b*e)) - 1/640*(45045*(e*x + d)^7*b^7*e^5 - 210210*(e*x + d)
^6*b^7*d*e^5 + 384384*(e*x + d)^5*b^7*d^2*e^5 - 338910*(e*x + d)^4*b^7*d^3
*e^5 + 137995*(e*x + d)^3*b^7*d^4*e^5 - 16640*(e*x + d)^2*b^7*d^5*e^5 - 12
80*(e*x + d)*b^7*d^6*e^5 - 256*b^7*d^7*e^5 + 210210*(e*x + d)^6*a*b^6*e^6
- 768768*(e*x + d)^5*a*b^6*d*e^6 + 1016730*(e*x + d)^4*a*b^6*d^2*e^6 - 551
980*(e*x + d)^3*a*b^6*d^3*e^6 + 83200*(e*x + d)^2*a*b^6*d^4*e^6 + 7680*(e*
x + d)*a*b^6*d^5*e^6 + 1792*a*b^6*d^6*e^6 + 384384*(e*x + d)^5*a^2*b^5*e^7
- 1016730*(e*x + d)^4*a^2*b^5*d*e^7 + 827970*(e*x + d)^3*a^2*b^5*d^2*e^7
- 166400*(e*x + d)^2*a^2*b^5*d^3*e^7 - 19200*(e*x + d)*a^2*b^5*d^4*e^7 - 5
376*a^2*b^5*d^5*e^7 + 338910*(e*x + d)^4*a^3*b^4*e^8 - 551980*(e*x + d)^3*
a^3*b^4*d*e^8 + 166400*(e*x + d)^2*a^3*b^4*d^2*e^8 + 25600*(e*x + d)*a^3*b
^4*d^3*e^8 + 8960*a^3*b^4*d^4*e^8 + 137995*(e*x + d)^3*a^4*b^3*e^9 - 83200
*(e*x + d)^2*a^4*b^3*d*e^9 - 19200*(e*x + d)*a^4*b^3*d^2*e^9 - 8960*a^4*b
^3*d^3*e^9 + 16640*(e*x + d)^2*a^5*b^2*e^10 + 7680*(e*x + d)*a^5*b^2*d*e^10
+ 5376*a^5*b^2*d^2*e^10 - 1280*(e*x + d)*a^6*b*e^11 - 1792*a^6*b*d*e^11 +
256*a^7*e^12)/((b^8*d^8 - 8*a*b^7*d^7*e + 28*a^2*b^6*d^6*e^2 - 56*a^3*b^5
*d^5*e^3 + 70*a^4*b^4*d^4*e^4 - 56*a^5*b^3*d^3*e^5 + 28*a^6*b^2*d^2*e^6...
```

Mupad [B] (verification not implemented)

Time = 10.16 (sec) , antiderivative size = 594, normalized size of antiderivative = 2.01

$$\int \frac{1}{(d + ex)^{7/2} (a^2 + 2abx + b^2x^2)^3} dx =$$

$$\frac{(d + ex)^{5/2} (a^5 e^5 - 5 a^4 b d e^4 + 10 a^3 b^2 d^2 e^3 - 10 a^2 b^3 d^3 e^2 + 5 a b^4 d^4 e - b^5 d^5) - (d + ex)^{9/2} (-10 a^3 b^2 d^2 e^3 + 15 a^2 b^3 d^3 e^2 - 5 a b^4 d^4 e + b^5 d^5) + 9009 b^{5/2} e^5 \operatorname{atan}\left(\frac{\sqrt{b} \sqrt{d+ex} (a^8 e^8 - 8 a^7 b d e^7 + 28 a^6 b^2 d^2 e^6 - 56 a^5 b^3 d^3 e^5 + 70 a^4 b^4 d^4 e^4 - 56 a^3 b^5 d^5 e^3 + 28 a^2 b^6 d^6 e^2 - 8 a b^7 d^7 e + b^8)}{(a e - b d)^{17/2}}\right)}{128 (a e - b d)^{17/2}}$$

input

```
int(1/((d + e*x)^(7/2)*(a^2 + b^2*x^2 + 2*a*b*x)^3),x)
```

output

```

- ((2*e^5)/(5*(a*e - b*d)) + (26*b^2*e^5*(d + e*x)^2)/(a*e - b*d)^3 + (275
99*b^3*e^5*(d + e*x)^3)/(128*(a*e - b*d)^4) + (33891*b^4*e^5*(d + e*x)^4)/
(64*(a*e - b*d)^5) + (3003*b^5*e^5*(d + e*x)^5)/(5*(a*e - b*d)^6) + (21021
*b^6*e^5*(d + e*x)^6)/(64*(a*e - b*d)^7) + (9009*b^7*e^5*(d + e*x)^7)/(128
*(a*e - b*d)^8) - (2*b*e^5*(d + e*x))/(a*e - b*d)^2)/((d + e*x)^(5/2)*(a^5
*e^5 - b^5*d^5 - 10*a^2*b^3*d^3*e^2 + 10*a^3*b^2*d^2*e^3 + 5*a*b^4*d^4*e -
5*a^4*b*d*e^4) - (d + e*x)^(9/2)*(10*b^5*d^3 - 10*a^3*b^2*e^3 + 30*a^2*b^
3*d*e^2 - 30*a*b^4*d^2*e) + (d + e*x)^(7/2)*(5*b^5*d^4 + 5*a^4*b*e^4 - 20*
a^3*b^2*d*e^3 + 30*a^2*b^3*d^2*e^2 - 20*a*b^4*d^3*e) + b^5*(d + e*x)^(15/2
) - (5*b^5*d - 5*a*b^4*e)*(d + e*x)^(13/2) + (d + e*x)^(11/2)*(10*b^5*d^2
+ 10*a^2*b^3*e^2 - 20*a*b^4*d*e)) - (9009*b^(5/2)*e^5*atan((b^(1/2)*(d + e
*x)^(1/2)*(a^8*e^8 + b^8*d^8 + 28*a^2*b^6*d^6*e^2 - 56*a^3*b^5*d^5*e^3 + 7
0*a^4*b^4*d^4*e^4 - 56*a^5*b^3*d^3*e^5 + 28*a^6*b^2*d^2*e^6 - 8*a*b^7*d^7*
e - 8*a^7*b*d*e^7))/(a*e - b*d)^(17/2)))/(128*(a*e - b*d)^(17/2))

```

Reduce [B] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 2993, normalized size of antiderivative = 10.15

$$\int \frac{1}{(d + ex)^{7/2} (a^2 + 2abx + b^2x^2)^3} dx = \text{Too large to display}$$

input

```
int(1/(e*x+d)^(7/2)/(b^2*x^2+2*a*b*x+a^2)^3,x)
```

output

```
( - 45045*sqrt(b)*sqrt(d + e*x)*sqrt(a*e - b*d)*atan((sqrt(d + e*x)*b)/(sqrt(b)*sqrt(a*e - b*d)))*a**5*b**2*d**2*e**5 - 90090*sqrt(b)*sqrt(d + e*x)*sqrt(a*e - b*d)*atan((sqrt(d + e*x)*b)/(sqrt(b)*sqrt(a*e - b*d)))*a**5*b**2*d**6*x - 45045*sqrt(b)*sqrt(d + e*x)*sqrt(a*e - b*d)*atan((sqrt(d + e*x)*b)/(sqrt(b)*sqrt(a*e - b*d)))*a**5*b**2*e**7*x**2 - 225225*sqrt(b)*sqrt(d + e*x)*sqrt(a*e - b*d)*atan((sqrt(d + e*x)*b)/(sqrt(b)*sqrt(a*e - b*d)))*a**4*b**3*d**2*e**5*x - 450450*sqrt(b)*sqrt(d + e*x)*sqrt(a*e - b*d)*atan((sqrt(d + e*x)*b)/(sqrt(b)*sqrt(a*e - b*d)))*a**4*b**3*d*e**6*x**2 - 225225*sqrt(b)*sqrt(d + e*x)*sqrt(a*e - b*d)*atan((sqrt(d + e*x)*b)/(sqrt(b)*sqrt(a*e - b*d)))*a**4*b**3*e**7*x**3 - 450450*sqrt(b)*sqrt(d + e*x)*sqrt(a*e - b*d)*atan((sqrt(d + e*x)*b)/(sqrt(b)*sqrt(a*e - b*d)))*a**3*b**4*d**2*e**5*x**2 - 900900*sqrt(b)*sqrt(d + e*x)*sqrt(a*e - b*d)*atan((sqrt(d + e*x)*b)/(sqrt(b)*sqrt(a*e - b*d)))*a**3*b**4*d*e**6*x**3 - 450450*sqrt(b)*sqrt(d + e*x)*sqrt(a*e - b*d)*atan((sqrt(d + e*x)*b)/(sqrt(b)*sqrt(a*e - b*d)))*a**3*b**4*e**7*x**4 - 450450*sqrt(b)*sqrt(d + e*x)*sqrt(a*e - b*d)*atan((sqrt(d + e*x)*b)/(sqrt(b)*sqrt(a*e - b*d)))*a**2*b**5*d**2*e**5*x**3 - 900900*sqrt(b)*sqrt(d + e*x)*sqrt(a*e - b*d)*atan((sqrt(d + e*x)*b)/(sqrt(b)*sqrt(a*e - b*d)))*a**2*b**5*d*e**6*x**4 - 450450*sqrt(b)*sqrt(d + e*x)*sqrt(a*e - b*d)*atan((sqrt(d + e*x)*b)/(sqrt(b)*sqrt(a*e - b*d)))*a**2*b**5*e**7*x**5 - 225225*sqrt(b)*sqrt(d + e*x)*sqrt(a*e - b*d)*atan((sqrt...
```

3.201 $\int (d + ex)^3 \sqrt{cd^2 + 2cdex + ce^2x^2} dx$

| | |
|---|------|
| Optimal result | 1674 |
| Mathematica [A] (verified) | 1674 |
| Rubi [A] (verified) | 1675 |
| Maple [A] (verified) | 1675 |
| Fricas [B] (verification not implemented) | 1676 |
| Sympy [B] (verification not implemented) | 1677 |
| Maxima [F(-2)] | 1677 |
| Giac [A] (verification not implemented) | 1678 |
| Mupad [B] (verification not implemented) | 1678 |
| Reduce [B] (verification not implemented) | 1679 |

Optimal result

Integrand size = 32, antiderivative size = 34

$$\int (d + ex)^3 \sqrt{cd^2 + 2cdex + ce^2x^2} dx = \frac{(cd^2 + 2cdex + ce^2x^2)^{5/2}}{5c^2e}$$

output $1/5*(c*e^2*x^2+2*c*d*e*x+c*d^2)^(5/2)/c^2/e$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.79

$$\int (d + ex)^3 \sqrt{cd^2 + 2cdex + ce^2x^2} dx = \frac{(d + ex)^4 \sqrt{c(d + ex)^2}}{5e}$$

input `Integrate[(d + e*x)^3*Sqrt[c*d^2 + 2*c*d*e*x + c*e^2*x^2],x]`

output $((d + e*x)^4*Sqrt[c*(d + e*x)^2])/(5*e)$

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.031$, Rules used = {1099}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (d + ex)^3 \sqrt{cd^2 + 2cdex + ce^2x^2} dx$$

$$\downarrow 1099$$

$$\frac{(cd^2 + 2cdex + ce^2x^2)^{5/2}}{5c^2e}$$

input `Int[(d + e*x)^3*Sqrt[c*d^2 + 2*c*d*e*x + c*e^2*x^2],x]`

output `(c*d^2 + 2*c*d*e*x + c*e^2*x^2)^(5/2)/(5*c^2*e)`

Defintions of rubi rules used

rule 1099 `Int[((d_) + (e_)*(x_))^(m_)*((a_) + (b_)*(x_) + (c_)*(x_)^2)^(p_), x_Sy
mbol] := Simp[e^m*((a + b*x + c*x^2)^(p + (m + 1)/2)/(c^((m + 1)/2)*(m + 2*
p + 1))), x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[b^2 - 4*a*c, 0] && EqQ[
2*c*d - b*e, 0] && IntegerQ[(m - 1)/2]`

Maple [A] (verified)

Time = 1.04 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.71

| method | result | size |
|----------------|---|------|
| risch | $\frac{\sqrt{c(ex+d)^2} (ex+d)^4}{5e}$ | 24 |
| pseudoelliptic | $\frac{\sqrt{c(ex+d)^2} (ex+d)^4}{5e}$ | 24 |
| default | $\frac{(ex+d)^4 \sqrt{x^2 c e^2 + 2cdxe + c d^2}}{5e}$ | 35 |
| gosper | $\frac{x(e^4 x^4 + 5d e^3 x^3 + 10d^2 e^2 x^2 + 10d^3 ex + 5d^4) \sqrt{x^2 c e^2 + 2cdxe + c d^2}}{5ex + 5d}$ | 73 |
| trager | $\frac{x(e^4 x^4 + 5d e^3 x^3 + 10d^2 e^2 x^2 + 10d^3 ex + 5d^4) \sqrt{x^2 c e^2 + 2cdxe + c d^2}}{5ex + 5d}$ | 73 |
| orering | $\frac{x(e^4 x^4 + 5d e^3 x^3 + 10d^2 e^2 x^2 + 10d^3 ex + 5d^4) \sqrt{x^2 c e^2 + 2cdxe + c d^2}}{5ex + 5d}$ | 73 |

input `int((e*x+d)^3*(c*e^2*x^2+2*c*d*e*x+c*d^2)^(1/2),x,method=_RETURNVERBOSE)`

output `1/5*(c*(e*x+d)^2)^(1/2)*(e*x+d)^4/e`

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 74 vs. $2(30) = 60$.

Time = 0.12 (sec) , antiderivative size = 74, normalized size of antiderivative = 2.18

$$\int (d + ex)^3 \sqrt{cd^2 + 2cdex + ce^2x^2} dx$$

$$= \frac{(e^4x^5 + 5de^3x^4 + 10d^2e^2x^3 + 10d^3ex^2 + 5d^4x)\sqrt{ce^2x^2 + 2cdex + cd^2}}{5(ex + d)}$$

input `integrate((e*x+d)^3*(c*e^2*x^2+2*c*d*e*x+c*d^2)^(1/2),x, algorithm="fricas")`

output `1/5*(e^4*x^5 + 5*d*e^3*x^4 + 10*d^2*e^2*x^3 + 10*d^3*e*x^2 + 5*d^4*x)*sqrt(c*e^2*x^2 + 2*c*d*e*x + c*d^2)/(e*x + d)`

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 187 vs. $2(31) = 62$.

Time = 0.23 (sec) , antiderivative size = 187, normalized size of antiderivative = 5.50

$$\int (d + ex)^3 \sqrt{cd^2 + 2cdex + ce^2x^2} dx$$

$$= \begin{cases} \frac{d^4 \sqrt{cd^2 + 2cdex + ce^2x^2}}{5e} + \frac{4d^3x \sqrt{cd^2 + 2cdex + ce^2x^2}}{5} + \frac{6d^2e x^2 \sqrt{cd^2 + 2cdex + ce^2x^2}}{5} + \frac{4de^2x^3 \sqrt{cd^2 + 2cdex + ce^2x^2}}{5} + \frac{e^3x^4 \sqrt{cd^2 + 2cdex + ce^2x^2}}{5} \\ d^3x \sqrt{cd^2} \end{cases}$$

input `integrate((e*x+d)**3*(c*e**2*x**2+2*c*d*e*x+c*d**2)**(1/2),x)`

output `Piecewise((d**4*sqrt(c*d**2 + 2*c*d*e*x + c*e**2*x**2)/(5*e) + 4*d**3*x*sqrt(c*d**2 + 2*c*d*e*x + c*e**2*x**2)/5 + 6*d**2*e*x**2*sqrt(c*d**2 + 2*c*d*e*x + c*e**2*x**2)/5 + 4*d*e**2*x**3*sqrt(c*d**2 + 2*c*d*e*x + c*e**2*x**2)/5 + e**3*x**4*sqrt(c*d**2 + 2*c*d*e*x + c*e**2*x**2)/5, Ne(e, 0)), (d**3*x*sqrt(c*d**2), True))`

Maxima [F(-2)]

Exception generated.

$$\int (d + ex)^3 \sqrt{cd^2 + 2cdex + ce^2x^2} dx = \text{Exception raised: ValueError}$$

input `integrate((e*x+d)^3*(c*e^2*x^2+2*c*d*e*x+c*d^2)^(1/2),x, algorithm="maxima")`

output `Exception raised: ValueError >> Computation failed since Maxima requested additional constraints; using the 'assume' command before evaluation *may* help (example of legal syntax is 'assume(e>0)', see `assume?` for more details)Is e`

Giac [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.62

$$\int (d + ex)^3 \sqrt{cd^2 + 2cdex + ce^2x^2} dx = \frac{(ex + d)^5 \sqrt{c} \operatorname{sgn}(ex + d)}{5e}$$

input `integrate((e*x+d)^3*(c*e^2*x^2+2*c*d*e*x+c*d^2)^(1/2),x, algorithm="giac")`

output `1/5*(e*x + d)^5*sqrt(c)*sgn(e*x + d)/e`

Mupad [B] (verification not implemented)

Time = 9.63 (sec) , antiderivative size = 239, normalized size of antiderivative = 7.03

$$\begin{aligned} & \int (d + ex)^3 \sqrt{cd^2 + 2cdex + ce^2x^2} dx \\ &= \frac{d^3 \left(\frac{x}{2} + \frac{d}{2e}\right) \sqrt{cd^2 + 2cdex + ce^2x^2}}{4} + \frac{ex^2 (cd^2 + 2cdex + ce^2x^2)^{3/2}}{5c} \\ & \quad - \frac{23d^2 \sqrt{cd^2 + 2cdex + ce^2x^2} (8e^2(d^2 + e^2x^2) - 12d^2e^2 + 4de^3x)}{480e^3} \\ & \quad + \frac{3dx (cd^2 + 2cdex + ce^2x^2)^{3/2}}{4c} \\ & \quad - \frac{7d \sqrt{cd^2 + 2cdex + ce^2x^2} (cd^3 + 3ex(cd^2 + 2cdex + ce^2x^2) - 4cd^2ex - 5cde^2x^2)}{60ce} \end{aligned}$$

input `int((d + e*x)^3*(c*d^2 + c*e^2*x^2 + 2*c*d*e*x)^(1/2),x)`

output `(d^3*(x/2 + d/(2*e))*(c*d^2 + c*e^2*x^2 + 2*c*d*e*x)^(1/2))/4 + (e*x^2*(c*d^2 + c*e^2*x^2 + 2*c*d*e*x)^(3/2))/(5*c) - (23*d^2*(c*d^2 + c*e^2*x^2 + 2*c*d*e*x)^(1/2)*(8*e^2*(d^2 + e^2*x^2) - 12*d^2*e^2 + 4*d*e^3*x))/(480*e^3) + (3*d*x*(c*d^2 + c*e^2*x^2 + 2*c*d*e*x)^(3/2))/(4*c) - (7*d*(c*d^2 + c*e^2*x^2 + 2*c*d*e*x)^(1/2)*(c*d^3 + 3*e*x*(c*d^2 + c*e^2*x^2 + 2*c*d*e*x) - 4*c*d^2*e*x - 5*c*d*e^2*x^2))/(60*c*e)`

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.32

$$\int (d+ex)^3 \sqrt{cd^2 + 2cdex + ce^2x^2} dx = \frac{\sqrt{c}x(e^4x^4 + 5de^3x^3 + 10d^2e^2x^2 + 10d^3ex + 5d^4)}{5}$$

input

```
int((e*x+d)^3*(c*e^2*x^2+2*c*d*e*x+c*d^2)^(1/2),x)
```

output

```
(sqrt(c)*x*(5*d**4 + 10*d**3*e*x + 10*d**2*e**2*x**2 + 5*d*e**3*x**3 + e**4*x**4))/5
```

3.202 $\int (d + ex)\sqrt{cd^2 + 2cdex + ce^2x^2} dx$

| | |
|---|------|
| Optimal result | 1680 |
| Mathematica [A] (verified) | 1680 |
| Rubi [A] (verified) | 1681 |
| Maple [A] (verified) | 1681 |
| Fricas [A] (verification not implemented) | 1682 |
| Sympy [B] (verification not implemented) | 1683 |
| Maxima [A] (verification not implemented) | 1683 |
| Giac [A] (verification not implemented) | 1684 |
| Mupad [B] (verification not implemented) | 1684 |
| Reduce [B] (verification not implemented) | 1684 |

Optimal result

Integrand size = 30, antiderivative size = 34

$$\int (d + ex)\sqrt{cd^2 + 2cdex + ce^2x^2} dx = \frac{(cd^2 + 2cdex + ce^2x^2)^{3/2}}{3ce}$$

output $1/3*(c*e^2*x^2+2*c*d*e*x+c*d^2)^{(3/2)}/c/e$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.68

$$\int (d + ex)\sqrt{cd^2 + 2cdex + ce^2x^2} dx = \frac{(c(d + ex)^2)^{3/2}}{3ce}$$

input `Integrate[(d + e*x)*Sqrt[c*d^2 + 2*c*d*e*x + c*e^2*x^2],x]`

output $(c*(d + e*x)^2)^{(3/2)}/(3*c*e)$

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.033$, Rules used = {1099}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (d + ex)\sqrt{cd^2 + 2cdex + ce^2x^2} dx$$

$$\downarrow 1099$$

$$\frac{(cd^2 + 2cdex + ce^2x^2)^{3/2}}{3ce}$$

input `Int[(d + e*x)*Sqrt[c*d^2 + 2*c*d*e*x + c*e^2*x^2],x]`

output `(c*d^2 + 2*c*d*e*x + c*e^2*x^2)^(3/2)/(3*c*e)`

Defintions of rubi rules used

rule 1099 `Int[((d_) + (e_)*(x_))^(m_)*((a_) + (b_)*(x_) + (c_)*(x_)^2)^(p_), x_Sy
mbol] := Simp[e^m*((a + b*x + c*x^2)^(p + (m + 1)/2)/(c^((m + 1)/2)*(m + 2*
p + 1))), x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[b^2 - 4*a*c, 0] && EqQ[
2*c*d - b*e, 0] && IntegerQ[(m - 1)/2]`

Maple [A] (verified)

Time = 0.79 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.71

| method | result | size |
|----------------|--|------|
| risch | $\frac{(ex+d)^2 \sqrt{c(ex+d)^2}}{3e}$ | 24 |
| pseudoelliptic | $\frac{(ex+d)^2 \sqrt{c(ex+d)^2}}{3e}$ | 24 |
| default | $\frac{(ex+d)^2 \sqrt{x^2 c e^2 + 2cdxe + c d^2}}{3e}$ | 35 |
| gospers | $\frac{x(e^2 x^2 + 3dex + 3d^2) \sqrt{x^2 c e^2 + 2cdxe + c d^2}}{3ex + 3d}$ | 51 |
| trager | $\frac{x(e^2 x^2 + 3dex + 3d^2) \sqrt{x^2 c e^2 + 2cdxe + c d^2}}{3ex + 3d}$ | 51 |
| orering | $\frac{x(e^2 x^2 + 3dex + 3d^2) \sqrt{x^2 c e^2 + 2cdxe + c d^2}}{3ex + 3d}$ | 51 |

input `int((e*x+d)*(c*e^2*x^2+2*c*d*e*x+c*d^2)^(1/2),x,method=_RETURNVERBOSE)`

output `1/3*(e*x+d)^2*(c*(e*x+d)^2)^(1/2)/e`

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.53

$$\int (d + ex) \sqrt{cd^2 + 2cdex + ce^2x^2} dx = \frac{\sqrt{ce^2x^2 + 2cdex + cd^2}(e^2x^3 + 3dex^2 + 3d^2x)}{3(ex + d)}$$

input `integrate((e*x+d)*(c*e^2*x^2+2*c*d*e*x+c*d^2)^(1/2),x, algorithm="fricas")`

output `1/3*sqrt(c*e^2*x^2 + 2*c*d*e*x + c*d^2)*(e^2*x^3 + 3*d*e*x^2 + 3*d^2*x)/(e*x + d)`

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 107 vs. $2(29) = 58$.

Time = 0.12 (sec) , antiderivative size = 107, normalized size of antiderivative = 3.15

$$\int (d + ex)\sqrt{cd^2 + 2cdex + ce^2x^2} dx$$

$$= \begin{cases} \frac{d^2\sqrt{cd^2+2cdex+ce^2x^2}}{3e} + \frac{2dx\sqrt{cd^2+2cdex+ce^2x^2}}{3} + \frac{ex^2\sqrt{cd^2+2cdex+ce^2x^2}}{3} & \text{for } e \neq 0 \\ dx\sqrt{cd^2} & \text{otherwise} \end{cases}$$

input `integrate((e*x+d)*(c*e**2*x**2+2*c*d*e*x+c*d**2)**(1/2),x)`

output `Piecewise((d**2*sqrt(c*d**2 + 2*c*d*e*x + c*e**2*x**2)/(3*e) + 2*d*x*sqrt(c*d**2 + 2*c*d*e*x + c*e**2*x**2)/3 + e*x**2*sqrt(c*d**2 + 2*c*d*e*x + c*e**2*x**2)/3, Ne(e, 0)), (d*x*sqrt(c*d**2), True))`

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.88

$$\int (d + ex)\sqrt{cd^2 + 2cdex + ce^2x^2} dx = \frac{(ce^2x^2 + 2cdex + cd^2)^{\frac{3}{2}}}{3ce}$$

input `integrate((e*x+d)*(c*e^2*x^2+2*c*d*e*x+c*d^2)^(1/2),x, algorithm="maxima")`

output `1/3*(c*e^2*x^2 + 2*c*d*e*x + c*d^2)^(3/2)/(c*e)`

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.62

$$\int (d + ex)\sqrt{cd^2 + 2cdex + ce^2x^2} dx = \frac{(ex + d)^3 \sqrt{c} \operatorname{sgn}(ex + d)}{3e}$$

input `integrate((e*x+d)*(c*e^2*x^2+2*c*d*e*x+c*d^2)^(1/2),x, algorithm="giac")`

output `1/3*(e*x + d)^3*sqrt(c)*sgn(e*x + d)/e`

Mupad [B] (verification not implemented)

Time = 9.69 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.00

$$\int (d + ex)\sqrt{cd^2 + 2cdex + ce^2x^2} dx = \frac{(d + ex)^2 \sqrt{cd^2 + 2cdex + ce^2x^2}}{3e}$$

input `int((d + e*x)*(c*d^2 + c*e^2*x^2 + 2*c*d*e*x)^(1/2),x)`

output `((d + e*x)^2*(c*d^2 + c*e^2*x^2 + 2*c*d*e*x)^(1/2))/(3*e)`

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.68

$$\int (d + ex)\sqrt{cd^2 + 2cdex + ce^2x^2} dx = \frac{\sqrt{c}x(e^2x^2 + 3dex + 3d^2)}{3}$$

input `int((e*x+d)*(c*e^2*x^2+2*c*d*e*x+c*d^2)^(1/2),x)`

output `(sqrt(c)*x*(3*d**2 + 3*d*e*x + e**2*x**2))/3`

3.203 $\int \frac{\sqrt{cd^2+2cdex+ce^2x^2}}{d+ex} dx$

| | |
|---|------|
| Optimal result | 1685 |
| Mathematica [A] (verified) | 1685 |
| Rubi [A] (verified) | 1686 |
| Maple [A] (verified) | 1686 |
| Fricas [A] (verification not implemented) | 1687 |
| Sympy [A] (verification not implemented) | 1687 |
| Maxima [F(-2)] | 1688 |
| Giac [A] (verification not implemented) | 1688 |
| Mupad [B] (verification not implemented) | 1689 |
| Reduce [B] (verification not implemented) | 1689 |

Optimal result

Integrand size = 32, antiderivative size = 28

$$\int \frac{\sqrt{cd^2 + 2cdex + ce^2x^2}}{d + ex} dx = \frac{\sqrt{cd^2 + 2cdex + ce^2x^2}}{e}$$

output $(c*e^2*x^2+2*c*d*e*x+c*d^2)^{(1/2)}/e$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.61

$$\int \frac{\sqrt{cd^2 + 2cdex + ce^2x^2}}{d + ex} dx = \frac{\sqrt{c(d + ex)^2}}{e}$$

input `Integrate[Sqrt[c*d^2 + 2*c*d*e*x + c*e^2*x^2]/(d + e*x),x]`

output `Sqrt[c*(d + e*x)^2]/e`

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.031$, Rules used = {1099}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{cd^2 + 2cdex + ce^2x^2}}{d + ex} dx$$

↓ 1099

$$\frac{\sqrt{cd^2 + 2cdex + ce^2x^2}}{e}$$

input `Int[Sqrt[c*d^2 + 2*c*d*e*x + c*e^2*x^2]/(d + e*x),x]`

output `Sqrt[c*d^2 + 2*c*d*e*x + c*e^2*x^2]/e`

Defintions of rubi rules used

rule 1099

```
Int[((d_) + (e_)*(x_))^(m_)*((a_) + (b_)*(x_) + (c_)*(x_)^2)^(p_), x_Sy
mbol] := Simp[e^m*((a + b*x + c*x^2)^(p + (m + 1)/2)/(c^((m + 1)/2)*(m + 2*
p + 1))), x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[b^2 - 4*a*c, 0] && EqQ[
2*c*d - b*e, 0] && IntegerQ[(m - 1)/2]
```

Maple [A] (verified)

Time = 1.06 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.57

| method | result | size |
|----------------|---|------|
| pseudoelliptic | $\frac{\sqrt{c(ex+d)^2}}{e}$ | 16 |
| risch | $\frac{\sqrt{c(ex+d)^2} x}{ex+d}$ | 21 |
| default | $\frac{x\sqrt{x^2ce^2+2cdxe+cd^2}}{ex+d}$ | 32 |
| trager | $\frac{x\sqrt{x^2ce^2+2cdxe+cd^2}}{ex+d}$ | 32 |
| orering | $\frac{x\sqrt{x^2ce^2+2cdxe+cd^2}}{ex+d}$ | 32 |

input `int((c*e^2*x^2+2*c*d*e*x+c*d^2)^(1/2)/(e*x+d),x,method=_RETURNVERBOSE)`

output `(c*(e*x+d)^2)^(1/2)/e`

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.11

$$\int \frac{\sqrt{cd^2 + 2cdex + ce^2x^2}}{d + ex} dx = \frac{\sqrt{ce^2x^2 + 2cdex + cd^2}x}{ex + d}$$

input `integrate((c*e^2*x^2+2*c*d*e*x+c*d^2)^(1/2)/(e*x+d),x, algorithm="fricas")`

output `sqrt(c*e^2*x^2 + 2*c*d*e*x + c*d^2)*x/(e*x + d)`

Sympy [A] (verification not implemented)

Time = 0.69 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.32

$$\int \frac{\sqrt{cd^2 + 2cdex + ce^2x^2}}{d + ex} dx = \begin{cases} \frac{\sqrt{cd^2+2cdex+ce^2x^2}}{e} & \text{for } e \neq 0 \\ \frac{x\sqrt{cd^2}}{d} & \text{otherwise} \end{cases}$$

input `integrate((c*e**2*x**2+2*c*d*e*x+c*d**2)**(1/2)/(e*x+d),x)`

output `Piecewise((sqrt(c*d**2 + 2*c*d*e*x + c*e**2*x**2)/e, Ne(e, 0)), (x*sqrt(c*d**2)/d, True))`

Maxima [F(-2)]

Exception generated.

$$\int \frac{\sqrt{cd^2 + 2cdex + ce^2x^2}}{d + ex} dx = \text{Exception raised: RuntimeError}$$

input `integrate((c*e^2*x^2+2*c*d*e*x+c*d^2)^(1/2)/(e*x+d),x, algorithm="maxima")`

output `Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negative exponent.`

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.86

$$\int \frac{\sqrt{cd^2 + 2cdex + ce^2x^2}}{d + ex} dx = \left(x \operatorname{sgn}(ex + d) + \frac{d \operatorname{sgn}(ex + d)}{e} \right) \sqrt{c}$$

input `integrate((c*e^2*x^2+2*c*d*e*x+c*d^2)^(1/2)/(e*x+d),x, algorithm="giac")`

output `(x*sgn(e*x + d) + d*sgn(e*x + d)/e)*sqrt(c)`

Mupad [B] (verification not implemented)

Time = 9.53 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.54

$$\int \frac{\sqrt{cd^2 + 2cdex + ce^2x^2}}{d + ex} dx = \frac{\sqrt{c(d + ex)^2}}{e}$$

input `int((c*d^2 + c*e^2*x^2 + 2*c*d*e*x)^(1/2)/(d + e*x),x)`

output `(c*(d + e*x)^2)^(1/2)/e`

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 4, normalized size of antiderivative = 0.14

$$\int \frac{\sqrt{cd^2 + 2cdex + ce^2x^2}}{d + ex} dx = \sqrt{c}x$$

input `int((c*e^2*x^2+2*c*d*e*x+c*d^2)^(1/2)/(e*x+d),x)`

output `sqrt(c)*x`

3.204 $\int \frac{\sqrt{cd^2+2cdex+ce^2x^2}}{(d+ex)^3} dx$

| | |
|---|------|
| Optimal result | 1690 |
| Mathematica [A] (verified) | 1690 |
| Rubi [A] (verified) | 1691 |
| Maple [A] (verified) | 1691 |
| Fricas [A] (verification not implemented) | 1692 |
| Sympy [F] | 1693 |
| Maxima [F(-2)] | 1693 |
| Giac [A] (verification not implemented) | 1693 |
| Mupad [B] (verification not implemented) | 1694 |
| Reduce [B] (verification not implemented) | 1694 |

Optimal result

Integrand size = 32, antiderivative size = 30

$$\int \frac{\sqrt{cd^2 + 2cdex + ce^2x^2}}{(d + ex)^3} dx = -\frac{c}{e\sqrt{cd^2 + 2cdex + ce^2x^2}}$$

output `-c/e/(c*e^2*x^2+2*c*d*e*x+c*d^2)^(1/2)`

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.63

$$\int \frac{\sqrt{cd^2 + 2cdex + ce^2x^2}}{(d + ex)^3} dx = -\frac{c}{e\sqrt{c(d + ex)^2}}$$

input `Integrate[Sqrt[c*d^2 + 2*c*d*e*x + c*e^2*x^2]/(d + e*x)^3,x]`

output `-(c/(e*Sqrt[c*(d + e*x)^2]))`

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.031$, Rules used = {1099}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{cd^2 + 2cdex + ce^2x^2}}{(d + ex)^3} dx$$

↓ 1099

$$-\frac{c}{e\sqrt{cd^2 + 2cdex + ce^2x^2}}$$

input

```
Int[Sqrt[c*d^2 + 2*c*d*e*x + c*e^2*x^2]/(d + e*x)^3,x]
```

output

```
-(c/(e*Sqrt[c*d^2 + 2*c*d*e*x + c*e^2*x^2]))
```

Defintions of rubi rules used

rule 1099

```
Int[((d_) + (e_)*(x_)^(m_))*((a_) + (b_)*(x_) + (c_)*(x_)^2)^(p_), x_Sy
mbol] := Simp[e^m*((a + b*x + c*x^2)^(p + (m + 1)/2)/(c^((m + 1)/2)*(m + 2*
p + 1))), x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[b^2 - 4*a*c, 0] && EqQ[
2*c*d - b*e, 0] && IntegerQ[(m - 1)/2]
```

Maple [A] (verified)

Time = 1.07 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.60

| method | result | size |
|----------------|---|------|
| pseudoelliptic | $-\frac{c}{e\sqrt{c(ex+d)^2}}$ | 18 |
| risch | $-\frac{\sqrt{c(ex+d)^2}}{(ex+d)^2 e}$ | 24 |
| gospers | $-\frac{\sqrt{x^2 c e^2 + 2cdxe + cd^2}}{(ex+d)^2 e}$ | 35 |
| default | $-\frac{\sqrt{x^2 c e^2 + 2cdxe + cd^2}}{(ex+d)^2 e}$ | 35 |
| trager | $\frac{x\sqrt{x^2 c e^2 + 2cdxe + cd^2}}{d(ex+d)^2}$ | 35 |
| orering | $-\frac{\sqrt{x^2 c e^2 + 2cdxe + cd^2}}{(ex+d)^2 e}$ | 35 |

input `int((c*e^2*x^2+2*c*d*e*x+c*d^2)^(1/2)/(e*x+d)^3,x,method=_RETURNVERBOSE)`

output `-c/e/(c*(e*x+d)^2)^(1/2)`

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.53

$$\int \frac{\sqrt{cd^2 + 2cdex + ce^2x^2}}{(d + ex)^3} dx = -\frac{\sqrt{ce^2x^2 + 2cdex + cd^2}}{e^3x^2 + 2de^2x + d^2e}$$

input `integrate((c*e^2*x^2+2*c*d*e*x+c*d^2)^(1/2)/(e*x+d)^3,x, algorithm="fricas")`

output `-sqrt(c*e^2*x^2 + 2*c*d*e*x + c*d^2)/(e^3*x^2 + 2*d*e^2*x + d^2*e)`

Sympy [F]

$$\int \frac{\sqrt{cd^2 + 2cdex + ce^2x^2}}{(d + ex)^3} dx = \int \frac{\sqrt{c(d + ex)^2}}{(d + ex)^3} dx$$

input `integrate((c*e**2*x**2+2*c*d*e*x+c*d**2)**(1/2)/(e*x+d)**3,x)`

output `Integral(sqrt(c*(d + e*x)**2)/(d + e*x)**3, x)`

Maxima [F(-2)]

Exception generated.

$$\int \frac{\sqrt{cd^2 + 2cdex + ce^2x^2}}{(d + ex)^3} dx = \text{Exception raised: RuntimeError}$$

input `integrate((c*e^2*x^2+2*c*d*e*x+c*d^2)^(1/2)/(e*x+d)^3,x, algorithm="maxima")`

output `Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negative exponent.`

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.70

$$\int \frac{\sqrt{cd^2 + 2cdex + ce^2x^2}}{(d + ex)^3} dx = -\frac{\sqrt{c}\operatorname{sgn}(ex + d)}{(ex + d)e}$$

input `integrate((c*e^2*x^2+2*c*d*e*x+c*d^2)^(1/2)/(e*x+d)^3,x, algorithm="giac")`

output `-sqrt(c)*sgn(e*x + d)/((e*x + d)*e)`

Mupad [B] (verification not implemented)

Time = 9.18 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.13

$$\int \frac{\sqrt{cd^2 + 2cdex + ce^2x^2}}{(d + ex)^3} dx = -\frac{\sqrt{cd^2 + 2cdex + ce^2x^2}}{e(d + ex)^2}$$

input `int((c*d^2 + c*e^2*x^2 + 2*c*d*e*x)^(1/2)/(d + e*x)^3,x)`output `-(c*d^2 + c*e^2*x^2 + 2*c*d*e*x)^(1/2)/(e*(d + e*x)^2)`**Reduce [B] (verification not implemented)**

Time = 0.29 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.47

$$\int \frac{\sqrt{cd^2 + 2cdex + ce^2x^2}}{(d + ex)^3} dx = \frac{\sqrt{c}x}{d(ex + d)}$$

input `int((c*e^2*x^2+2*c*d*e*x+c*d^2)^(1/2)/(e*x+d)^3,x)`output `(sqrt(c)*x)/(d*(d + e*x))`

3.205 $\int \frac{\sqrt{cd^2+2cdex+ce^2x^2}}{(d+ex)^5} dx$

| | |
|---|------|
| Optimal result | 1695 |
| Mathematica [A] (verified) | 1695 |
| Rubi [A] (verified) | 1696 |
| Maple [A] (verified) | 1696 |
| Fricas [B] (verification not implemented) | 1697 |
| Sympy [F] | 1698 |
| Maxima [F(-2)] | 1698 |
| Giac [A] (verification not implemented) | 1698 |
| Mupad [B] (verification not implemented) | 1699 |
| Reduce [B] (verification not implemented) | 1699 |

Optimal result

Integrand size = 32, antiderivative size = 34

$$\int \frac{\sqrt{cd^2 + 2cdex + ce^2x^2}}{(d + ex)^5} dx = -\frac{c^2}{3e(cd^2 + 2cdex + ce^2x^2)^{3/2}}$$

output `-1/3*c^2/e/(c*e^2*x^2+2*c*d*e*x+c*d^2)^(3/2)`

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.79

$$\int \frac{\sqrt{cd^2 + 2cdex + ce^2x^2}}{(d + ex)^5} dx = -\frac{\sqrt{c(d + ex)^2}}{3e(d + ex)^4}$$

input `Integrate[Sqrt[c*d^2 + 2*c*d*e*x + c*e^2*x^2]/(d + e*x)^5,x]`

output `-1/3*Sqrt[c*(d + e*x)^2]/(e*(d + e*x)^4)`

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.031$, Rules used = {1099}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{cd^2 + 2cdex + ce^2x^2}}{(d + ex)^5} dx$$

↓ 1099

$$-\frac{c^2}{3e(cd^2 + 2cdex + ce^2x^2)^{3/2}}$$

input `Int[Sqrt[c*d^2 + 2*c*d*e*x + c*e^2*x^2]/(d + e*x)^5,x]`

output `-1/3*c^2/(e*(c*d^2 + 2*c*d*e*x + c*e^2*x^2)^(3/2))`

Defintions of rubi rules used

rule 1099

```
Int[((d_) + (e_)*(x_))^(m_)*((a_) + (b_)*(x_) + (c_)*(x_)^2)^(p_), x_Sy
mbol] := Simp[e^m*((a + b*x + c*x^2)^(p + (m + 1)/2)/(c^((m + 1)/2)*(m + 2*
p + 1))), x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[b^2 - 4*a*c, 0] && EqQ[
2*c*d - b*e, 0] && IntegerQ[(m - 1)/2]
```

Maple [A] (verified)

Time = 1.02 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.71

| method | result | size |
|----------------|---|------|
| risch | $-\frac{\sqrt{c(ex+d)^2}}{3(ex+d)^4 e}$ | 24 |
| pseudoelliptic | $-\frac{\sqrt{c(ex+d)^2}}{3(ex+d)^4 e}$ | 24 |
| gosper | $-\frac{\sqrt{x^2 c e^2 + 2cdxe + cd^2}}{3(ex+d)^4 e}$ | 35 |
| default | $-\frac{\sqrt{x^2 c e^2 + 2cdxe + cd^2}}{3(ex+d)^4 e}$ | 35 |
| orering | $-\frac{\sqrt{x^2 c e^2 + 2cdxe + cd^2}}{3(ex+d)^4 e}$ | 35 |
| trager | $\frac{(e^2 x^2 + 3dex + 3d^2) x \sqrt{x^2 c e^2 + 2cdxe + cd^2}}{3d^3 (ex+d)^4}$ | 54 |

input `int((c*e^2*x^2+2*c*d*e*x+c*d^2)^(1/2)/(e*x+d)^5,x,method=_RETURNVERBOSE)`

output `-1/3*(c*(e*x+d)^2)^(1/2)/(e*x+d)^4/e`

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 68 vs. $2(30) = 60$.

Time = 0.08 (sec) , antiderivative size = 68, normalized size of antiderivative = 2.00

$$\int \frac{\sqrt{cd^2 + 2cdex + ce^2x^2}}{(d + ex)^5} dx = -\frac{\sqrt{ce^2x^2 + 2cdex + cd^2}}{3(e^5x^4 + 4de^4x^3 + 6d^2e^3x^2 + 4d^3e^2x + d^4e)}$$

input `integrate((c*e^2*x^2+2*c*d*e*x+c*d^2)^(1/2)/(e*x+d)^5,x, algorithm="fricas")`

output `-1/3*sqrt(c*e^2*x^2 + 2*c*d*e*x + c*d^2)/(e^5*x^4 + 4*d*e^4*x^3 + 6*d^2*e^3*x^2 + 4*d^3*e^2*x + d^4*e)`

Sympy [F]

$$\int \frac{\sqrt{cd^2 + 2cdex + ce^2x^2}}{(d + ex)^5} dx = \int \frac{\sqrt{c(d + ex)^2}}{(d + ex)^5} dx$$

input `integrate((c*e**2*x**2+2*c*d*e*x+c*d**2)**(1/2)/(e*x+d)**5,x)`

output `Integral(sqrt(c*(d + e*x)**2)/(d + e*x)**5, x)`

Maxima [F(-2)]

Exception generated.

$$\int \frac{\sqrt{cd^2 + 2cdex + ce^2x^2}}{(d + ex)^5} dx = \text{Exception raised: RuntimeError}$$

input `integrate((c*e^2*x^2+2*c*d*e*x+c*d^2)^(1/2)/(e*x+d)^5,x, algorithm="maxima")`

output `Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negative exponent.`

Giac [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.62

$$\int \frac{\sqrt{cd^2 + 2cdex + ce^2x^2}}{(d + ex)^5} dx = -\frac{\sqrt{c}\operatorname{sgn}(ex + d)}{3(ex + d)^3e}$$

input `integrate((c*e^2*x^2+2*c*d*e*x+c*d^2)^(1/2)/(e*x+d)^5,x, algorithm="giac")`

output `-1/3*sqrt(c)*sgn(e*x + d)/((e*x + d)^3*e)`

Mupad [B] (verification not implemented)

Time = 8.97 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt{cd^2 + 2cdex + ce^2x^2}}{(d + ex)^5} dx = -\frac{\sqrt{cd^2 + 2cdex + ce^2x^2}}{3e(d + ex)^4}$$

input `int((c*d^2 + c*e^2*x^2 + 2*c*d*e*x)^(1/2)/(d + e*x)^5,x)`output `-(c*d^2 + c*e^2*x^2 + 2*c*d*e*x)^(1/2)/(3*e*(d + e*x)^4)`**Reduce [B] (verification not implemented)**

Time = 0.26 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.06

$$\int \frac{\sqrt{cd^2 + 2cdex + ce^2x^2}}{(d + ex)^5} dx = -\frac{\sqrt{c}}{3e(e^3x^3 + 3de^2x^2 + 3d^2ex + d^3)}$$

input `int((c*e^2*x^2+2*c*d*e*x+c*d^2)^(1/2)/(e*x+d)^5,x)`output `(- sqrt(c))/(3*e*(d**3 + 3*d**2*e*x + 3*d*e**2*x**2 + e**3*x**3))`

3.206 $\int (d + ex)^2 \sqrt{cd^2 + 2cdex + ce^2x^2} dx$

| | |
|---|------|
| Optimal result | 1700 |
| Mathematica [A] (verified) | 1700 |
| Rubi [A] (verified) | 1701 |
| Maple [A] (verified) | 1702 |
| Fricas [A] (verification not implemented) | 1702 |
| Sympy [B] (verification not implemented) | 1703 |
| Maxima [F(-2)] | 1703 |
| Giac [A] (verification not implemented) | 1704 |
| Mupad [B] (verification not implemented) | 1704 |
| Reduce [B] (verification not implemented) | 1704 |

Optimal result

Integrand size = 32, antiderivative size = 38

$$\int (d + ex)^2 \sqrt{cd^2 + 2cdex + ce^2x^2} dx = \frac{(d + ex)^3 \sqrt{cd^2 + 2cdex + ce^2x^2}}{4e}$$

output

```
1/4*(e*x+d)^3*(c*e^2*x^2+2*c*d*e*x+c*d^2)^(1/2)/e
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.74

$$\int (d + ex)^2 \sqrt{cd^2 + 2cdex + ce^2x^2} dx = \frac{(d + ex) (c(d + ex)^2)^{3/2}}{4ce}$$

input

```
Integrate[(d + e*x)^2*Sqrt[c*d^2 + 2*c*d*e*x + c*e^2*x^2],x]
```

output

```
((d + e*x)*(c*(d + e*x)^2)^(3/2))/(4*c*e)
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.062$, Rules used = {1102, 17}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (d + ex)^2 \sqrt{cd^2 + 2cdex + ce^2x^2} dx$$

$$\downarrow 1102$$

$$\frac{\sqrt{cd^2 + 2cdex + ce^2x^2} \int ce(d + ex)^3 dx}{ce(d + ex)}$$

$$\downarrow 17$$

$$\frac{(d + ex)^3 \sqrt{cd^2 + 2cdex + ce^2x^2}}{4e}$$

input `Int[(d + e*x)^2*Sqrt[c*d^2 + 2*c*d*e*x + c*e^2*x^2],x]`

output `((d + e*x)^3*Sqrt[c*d^2 + 2*c*d*e*x + c*e^2*x^2])/(4*e)`

Defintions of rubi rules used

rule 17 `Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] := Simp[c*((a + b*x)^(m + 1))/(b*(m + 1))], x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]`

rule 1102 `Int[((d_.) + (e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] := Simp[(a + b*x + c*x^2)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x)^(2*FracPart[p])) Int[(d + e*x)^m*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && EqQ[b^2 - 4*a*c, 0]`

Maple [A] (verified)

Time = 0.97 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.63

| method | result | size |
|---------|--|------|
| risch | $\frac{(ex+d)^3 \sqrt{c(ex+d)^2}}{4e}$ | 24 |
| default | $\frac{(ex+d)^3 \sqrt{x^2 c e^2 + 2cdxe + c d^2}}{4e}$ | 35 |
| gospers | $\frac{x(e^3 x^3 + 4d e^2 x^2 + 6d^2 ex + 4d^3) \sqrt{x^2 c e^2 + 2cdxe + c d^2}}{4ex + 4d}$ | 62 |
| trager | $\frac{x(e^3 x^3 + 4d e^2 x^2 + 6d^2 ex + 4d^3) \sqrt{x^2 c e^2 + 2cdxe + c d^2}}{4ex + 4d}$ | 62 |
| orering | $\frac{x(e^3 x^3 + 4d e^2 x^2 + 6d^2 ex + 4d^3) \sqrt{x^2 c e^2 + 2cdxe + c d^2}}{4ex + 4d}$ | 62 |

input `int((e*x+d)^2*(c*e^2*x^2+2*c*d*e*x+c*d^2)^(1/2),x,method=_RETURNVERBOSE)`

output `1/4*(e*x+d)^3*(c*(e*x+d)^2)^(1/2)/e`

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.66

$$\int (d+ex)^2 \sqrt{cd^2 + 2cdex + ce^2x^2} dx$$

$$= \frac{(e^3x^4 + 4de^2x^3 + 6d^2ex^2 + 4d^3x) \sqrt{ce^2x^2 + 2cdex + cd^2}}{4(ex+d)}$$

input `integrate((e*x+d)^2*(c*e^2*x^2+2*c*d*e*x+c*d^2)^(1/2),x, algorithm="fricas")`

output `1/4*(e^3*x^4 + 4*d*e^2*x^3 + 6*d^2*e*x^2 + 4*d^3*x)*sqrt(c*e^2*x^2 + 2*c*d*e*x + c*d^2)/(e*x + d)`

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 160 vs. $2(34) = 68$.

Time = 0.74 (sec) , antiderivative size = 160, normalized size of antiderivative = 4.21

$$\int (d + ex)^2 \sqrt{cd^2 + 2cdex + ce^2x^2} dx$$

$$= \begin{cases} \sqrt{cd^2 + 2cdex + ce^2x^2} \left(\frac{d^3}{4e} + \frac{3d^2x}{4} + \frac{3dex^2}{4} + \frac{e^2x^3}{4} \right) & \text{for } ce^2 \neq 0 \\ \frac{d^2(cd^2 + 2cdex)^{\frac{3}{2}}}{12} + \frac{(cd^2 + 2cdex)^{\frac{5}{2}}}{10c} + \frac{(cd^2 + 2cdex)^{\frac{7}{2}}}{28c^2d^2} & \text{for } cde \neq 0 \\ \sqrt{cd^2} \left(\begin{cases} d^2x & \text{for } e = 0 \\ \frac{(d+ex)^3}{3e} & \text{otherwise} \end{cases} \right) & \text{otherwise} \end{cases}$$

input `integrate((e*x+d)**2*(c*e**2*x**2+2*c*d*e*x+c*d**2)**(1/2),x)`

output `Piecewise((sqrt(c*d**2 + 2*c*d*e*x + c*e**2*x**2)*(d**3/(4*e) + 3*d**2*x/4 + 3*d*e*x**2/4 + e**2*x**3/4), Ne(c*e**2, 0)), ((d**2*(c*d**2 + 2*c*d*e*x)**(3/2)/12 + (c*d**2 + 2*c*d*e*x)**(5/2)/(10*c) + (c*d**2 + 2*c*d*e*x)**(7/2)/(28*c**2*d**2))/(c*d*e), Ne(c*d*e, 0)), (sqrt(c*d**2)*Piecewise((d**2*x, Eq(e, 0)), ((d + e*x)**3/(3*e), True)), True))`

Maxima [F(-2)]

Exception generated.

$$\int (d + ex)^2 \sqrt{cd^2 + 2cdex + ce^2x^2} dx = \text{Exception raised: ValueError}$$

input `integrate((e*x+d)^2*(c*e^2*x^2+2*c*d*e*x+c*d^2)^(1/2),x, algorithm="maxima")`

output `Exception raised: ValueError >> Computation failed since Maxima requested additional constraints; using the 'assume' command before evaluation *may* help (example of legal syntax is 'assume(e>0)', see `assume?` for more details)Is e`

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.55

$$\int (d + ex)^2 \sqrt{cd^2 + 2cdex + ce^2x^2} dx = \frac{(ex + d)^4 \sqrt{c} \operatorname{sgn}(ex + d)}{4e}$$

input `integrate((e*x+d)^2*(c*e^2*x^2+2*c*d*e*x+c*d^2)^(1/2),x, algorithm="giac")`output `1/4*(e*x + d)^4*sqrt(c)*sgn(e*x + d)/e`**Mupad [B] (verification not implemented)**

Time = 9.60 (sec) , antiderivative size = 76, normalized size of antiderivative = 2.00

$$\int (d + ex)^2 \sqrt{cd^2 + 2cdex + ce^2x^2} dx$$

$$= \frac{\sqrt{cd^2 + 2cdex + ce^2x^2} (cd^3 + ex(cd^2 + 2cdex + ce^2x^2)) + 2cd^2ex + cde^2x^2}{4ce}$$

input `int((d + e*x)^2*(c*d^2 + c*e^2*x^2 + 2*c*d*e*x)^(1/2),x)`output `((c*d^2 + c*e^2*x^2 + 2*c*d*e*x)^(1/2)*(c*d^3 + e*x*(c*d^2 + c*e^2*x^2 + 2*c*d*e*x) + 2*c*d^2*e*x + c*d*e^2*x^2))/(4*c*e)`**Reduce [B] (verification not implemented)**

Time = 0.31 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.89

$$\int (d + ex)^2 \sqrt{cd^2 + 2cdex + ce^2x^2} dx = \frac{\sqrt{c}x(e^3x^3 + 4de^2x^2 + 6d^2ex + 4d^3)}{4}$$

input `int((e*x+d)^2*(c*e^2*x^2+2*c*d*e*x+c*d^2)^(1/2),x)`output `(sqrt(c)*x*(4*d**3 + 6*d**2*e*x + 4*d*e**2*x**2 + e**3*x**3))/4`

3.207 $\int \sqrt{cd^2 + 2cdex + ce^2x^2} dx$

| | |
|---|------|
| Optimal result | 1705 |
| Mathematica [A] (verified) | 1705 |
| Rubi [A] (verified) | 1706 |
| Maple [A] (verified) | 1707 |
| Fricas [A] (verification not implemented) | 1707 |
| Sympy [B] (verification not implemented) | 1708 |
| Maxima [F(-2)] | 1708 |
| Giac [A] (verification not implemented) | 1709 |
| Mupad [B] (verification not implemented) | 1709 |
| Reduce [B] (verification not implemented) | 1709 |

Optimal result

Integrand size = 24, antiderivative size = 36

$$\int \sqrt{cd^2 + 2cdex + ce^2x^2} dx = \frac{(d + ex)\sqrt{cd^2 + 2cdex + ce^2x^2}}{2e}$$

output

```
1/2*(e*x+d)*(c*e^2*x^2+2*c*d*e*x+c*d^2)^(1/2)/e
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.86

$$\int \sqrt{cd^2 + 2cdex + ce^2x^2} dx = \frac{cx(d + ex)(2d + ex)}{2\sqrt{c(d + ex)^2}}$$

input

```
Integrate[Sqrt[c*d^2 + 2*c*d*e*x + c*e^2*x^2],x]
```

output

```
(c*x*(d + e*x)*(2*d + e*x))/(2*Sqrt[c*(d + e*x)^2])
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {1079, 17}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{cd^2 + 2cdex + ce^2x^2} dx$$

$$\downarrow 1079$$

$$\frac{\sqrt{cd^2 + 2cdex + ce^2x^2} \int (cxe^2 + cde) dx}{ce(d + ex)}$$

$$\downarrow 17$$

$$\frac{(d + ex)\sqrt{cd^2 + 2cdex + ce^2x^2}}{2e}$$

input `Int[Sqrt[c*d^2 + 2*c*d*e*x + c*e^2*x^2],x]`

output `((d + e*x)*Sqrt[c*d^2 + 2*c*d*e*x + c*e^2*x^2])/(2*e)`

Defintions of rubi rules used

rule 17 `Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1))/(b*(m + 1))], x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]`

rule 1079 `Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(a + b*x + c*x^2)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x)^(2*FracPart[p])) Int[(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c, p}, x] && EqQ[b^2 - 4*a*c, 0]`

Maple [A] (verified)

Time = 0.83 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.11

| method | result | size |
|---------|---|------|
| gospers | $\frac{x(ex+2d)\sqrt{x^2ce^2+2cdxe+cd^2}}{2ex+2d}$ | 40 |
| default | $\frac{x(ex+2d)\sqrt{x^2ce^2+2cdxe+cd^2}}{2ex+2d}$ | 40 |
| trager | $\frac{x(ex+2d)\sqrt{x^2ce^2+2cdxe+cd^2}}{2ex+2d}$ | 40 |
| orering | $\frac{x(ex+2d)\sqrt{x^2ce^2+2cdxe+cd^2}}{2ex+2d}$ | 40 |
| risch | $\frac{\sqrt{c(ex+d)^2}ex^2}{2ex+2d} + \frac{\sqrt{c(ex+d)^2}dx}{ex+d}$ | 47 |

input `int((c*e^2*x^2+2*c*d*e*x+c*d^2)^(1/2),x,method=_RETURNVERBOSE)`

output `1/2*x*(e*x+2*d)*(c*e^2*x^2+2*c*d*e*x+c*d^2)^(1/2)/(e*x+d)`

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.14

$$\int \sqrt{cd^2 + 2cdex + ce^2x^2} dx = \frac{\sqrt{ce^2x^2 + 2cdex + cd^2}(ex^2 + 2dx)}{2(ex + d)}$$

input `integrate((c*e^2*x^2+2*c*d*e*x+c*d^2)^(1/2),x, algorithm="fricas")`

output `1/2*sqrt(c*e^2*x^2 + 2*c*d*e*x + c*d^2)*(e*x^2 + 2*d*x)/(e*x + d)`

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 71 vs. $2(34) = 68$.

Time = 0.33 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.97

$$\int \sqrt{cd^2 + 2cdex + ce^2x^2} dx = \begin{cases} \left(\frac{d}{2e} + \frac{x}{2}\right) \sqrt{cd^2 + 2cdex + ce^2x^2} & \text{for } ce^2 \neq 0 \\ \frac{(cd^2 + 2cdex)^{3/2}}{3cde} & \text{for } cde \neq 0 \\ x\sqrt{cd^2} & \text{otherwise} \end{cases}$$

input `integrate((c*e**2*x**2+2*c*d*e*x+c*d**2)**(1/2),x)`

output `Piecewise(((d/(2*e) + x/2)*sqrt(c*d**2 + 2*c*d*e*x + c*e**2*x**2), Ne(c*e**2, 0)), ((c*d**2 + 2*c*d*e*x)**(3/2)/(3*c*d*e), Ne(c*d*e, 0)), (x*sqrt(c*d**2), True))`

Maxima [F(-2)]

Exception generated.

$$\int \sqrt{cd^2 + 2cdex + ce^2x^2} dx = \text{Exception raised: ValueError}$$

input `integrate((c*e^2*x^2+2*c*d*e*x+c*d^2)^(1/2),x, algorithm="maxima")`

output `Exception raised: ValueError >> Computation failed since Maxima requested additional constraints; using the 'assume' command before evaluation *may* help (example of legal syntax is 'assume(e>0)', see `assume?` for more details)Is e`

Giac [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.00

$$\int \sqrt{cd^2 + 2cdex + ce^2x^2} dx = \frac{1}{2} \left((ex^2 + 2dx) \operatorname{sgn}(ex + d) + \frac{d^2 \operatorname{sgn}(ex + d)}{e} \right) \sqrt{c}$$

input `integrate((c*e^2*x^2+2*c*d*e*x+c*d^2)^(1/2),x, algorithm="giac")`

output `1/2*((e*x^2 + 2*d*x)*sgn(e*x + d) + d^2*sgn(e*x + d)/e)*sqrt(c)`

Mupad [B] (verification not implemented)

Time = 9.73 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.92

$$\int \sqrt{cd^2 + 2cdex + ce^2x^2} dx = \left(\frac{x}{2} + \frac{d}{2e} \right) \sqrt{cd^2 + 2cdex + ce^2x^2}$$

input `int((c*d^2 + c*e^2*x^2 + 2*c*d*e*x)^(1/2),x)`

output `(x/2 + d/(2*e))*(c*d^2 + c*e^2*x^2 + 2*c*d*e*x)^(1/2)`

Reduce [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.33

$$\int \sqrt{cd^2 + 2cdex + ce^2x^2} dx = \frac{\sqrt{c}x(ex + 2d)}{2}$$

input `int((c*e^2*x^2+2*c*d*e*x+c*d^2)^(1/2),x)`

output `(sqrt(c)*x*(2*d + e*x))/2`

3.208 $\int \frac{\sqrt{cd^2+2cdex+ce^2x^2}}{(d+ex)^2} dx$

| | |
|---|------|
| Optimal result | 1710 |
| Mathematica [A] (verified) | 1710 |
| Rubi [A] (verified) | 1711 |
| Maple [A] (verified) | 1712 |
| Fricas [A] (verification not implemented) | 1712 |
| Sympy [F] | 1712 |
| Maxima [F(-2)] | 1713 |
| Giac [A] (verification not implemented) | 1713 |
| Mupad [F(-1)] | 1714 |
| Reduce [B] (verification not implemented) | 1714 |

Optimal result

Integrand size = 32, antiderivative size = 40

$$\int \frac{\sqrt{cd^2 + 2cdex + ce^2x^2}}{(d + ex)^2} dx = \frac{c(d + ex) \log(d + ex)}{e\sqrt{cd^2 + 2cdex + ce^2x^2}}$$

output `c*(e*x+d)*ln(e*x+d)/e/(c*e^2*x^2+2*c*d*e*x+c*d^2)^(1/2)`

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.72

$$\int \frac{\sqrt{cd^2 + 2cdex + ce^2x^2}}{(d + ex)^2} dx = \frac{c(d + ex) \log(d + ex)}{e\sqrt{c(d + ex)^2}}$$

input `Integrate[Sqrt[c*d^2 + 2*c*d*e*x + c*e^2*x^2]/(d + e*x)^2,x]`

output `(c*(d + e*x)*Log[d + e*x])/(e*Sqrt[c*(d + e*x)^2])`

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.02, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.062$, Rules used = {1102, 16}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{cd^2 + 2cdex + ce^2x^2}}{(d + ex)^2} dx$$

$$\downarrow 1102$$

$$\frac{\sqrt{cd^2 + 2cdex + ce^2x^2} \int \frac{ce}{d+ex} dx}{ce(d + ex)}$$

$$\downarrow 16$$

$$\frac{\sqrt{cd^2 + 2cdex + ce^2x^2} \log(d + ex)}{e(d + ex)}$$

input `Int[Sqrt[c*d^2 + 2*c*d*e*x + c*e^2*x^2]/(d + e*x)^2,x]`

output `(Sqrt[c*d^2 + 2*c*d*e*x + c*e^2*x^2]*Log[d + e*x])/(e*(d + e*x))`

Defintions of rubi rules used

rule 16 `Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a + b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]`

rule 1102 `Int[((d_.) + (e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(a + b*x + c*x^2)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x)^(2*FracPart[p])) Int[(d + e*x)^m*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && EqQ[b^2 - 4*a*c, 0]`

Maple [A] (verified)

Time = 1.10 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.72

| method | result | size |
|---------|--|------|
| risch | $\frac{\sqrt{c(ex+d)^2 \ln(ex+d)}}{(ex+d)e}$ | 29 |
| default | $\frac{\sqrt{x^2 c e^2 + 2cdxe + c d^2} \ln(ex+d)}{(ex+d)e}$ | 40 |

input `int((c*e^2*x^2+2*c*d*e*x+c*d^2)^(1/2)/(e*x+d)^2,x,method=_RETURNVERBOSE)`

output `(c*(e*x+d)^2)^(1/2)/(e*x+d)*ln(e*x+d)/e`

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt{cd^2 + 2cdex + ce^2x^2}}{(d + ex)^2} dx = \frac{\sqrt{ce^2x^2 + 2cdex + cd^2} \log(ex + d)}{e^2x + de}$$

input `integrate((c*e^2*x^2+2*c*d*e*x+c*d^2)^(1/2)/(e*x+d)^2,x, algorithm="fricas")`

output `sqrt(c*e^2*x^2 + 2*c*d*e*x + c*d^2)*log(e*x + d)/(e^2*x + d*e)`

Sympy [F]

$$\int \frac{\sqrt{cd^2 + 2cdex + ce^2x^2}}{(d + ex)^2} dx = \int \frac{\sqrt{c(d + ex)^2}}{(d + ex)^2} dx$$

input `integrate((c*e**2*x**2+2*c*d*e*x+c*d**2)**(1/2)/(e*x+d)**2,x)`

output `Integral(sqrt(c*(d + e*x)**2)/(d + e*x)**2, x)`

Maxima [F(-2)]

Exception generated.

$$\int \frac{\sqrt{cd^2 + 2cdex + ce^2x^2}}{(d + ex)^2} dx = \text{Exception raised: RuntimeError}$$

input `integrate((c*e^2*x^2+2*c*d*e*x+c*d^2)^(1/2)/(e*x+d)^2,x, algorithm="maxima")`

output `Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negative exponent.`

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.50

$$\int \frac{\sqrt{cd^2 + 2cdex + ce^2x^2}}{(d + ex)^2} dx = \frac{\sqrt{c} \log(|ex + d|) \operatorname{sgn}(ex + d)}{e}$$

input `integrate((c*e^2*x^2+2*c*d*e*x+c*d^2)^(1/2)/(e*x+d)^2,x, algorithm="giac")`

output `sqrt(c)*log(abs(e*x + d))*sgn(e*x + d)/e`

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{cd^2 + 2cdex + ce^2x^2}}{(d + ex)^2} dx = \int \frac{\sqrt{cd^2 + 2cdex + ce^2x^2}}{(d + ex)^2} dx$$

input `int((c*d^2 + c*e^2*x^2 + 2*c*d*e*x)^(1/2)/(d + e*x)^2,x)`

output `int((c*d^2 + c*e^2*x^2 + 2*c*d*e*x)^(1/2)/(d + e*x)^2, x)`

Reduce [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.30

$$\int \frac{\sqrt{cd^2 + 2cdex + ce^2x^2}}{(d + ex)^2} dx = \frac{\sqrt{c} \log(ex + d)}{e}$$

input `int((c*e^2*x^2+2*c*d*e*x+c*d^2)^(1/2)/(e*x+d)^2,x)`

output `(sqrt(c)*log(d + e*x))/e`

3.209 $\int \frac{\sqrt{cd^2+2cdex+ce^2x^2}}{(d+ex)^4} dx$

| | |
|---|------|
| Optimal result | 1715 |
| Mathematica [A] (verified) | 1715 |
| Rubi [A] (verified) | 1716 |
| Maple [A] (verified) | 1717 |
| Fricas [A] (verification not implemented) | 1717 |
| Sympy [F] | 1718 |
| Maxima [F(-2)] | 1718 |
| Giac [A] (verification not implemented) | 1718 |
| Mupad [B] (verification not implemented) | 1719 |
| Reduce [B] (verification not implemented) | 1719 |

Optimal result

Integrand size = 32, antiderivative size = 39

$$\int \frac{\sqrt{cd^2 + 2cdex + ce^2x^2}}{(d + ex)^4} dx = -\frac{c}{2e(d + ex)\sqrt{cd^2 + 2cdex + ce^2x^2}}$$

output `-1/2*c/e/(e*x+d)/(c*e^2*x^2+2*c*d*e*x+c*d^2)^(1/2)`

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.69

$$\int \frac{\sqrt{cd^2 + 2cdex + ce^2x^2}}{(d + ex)^4} dx = -\frac{\sqrt{c(d + ex)^2}}{2e(d + ex)^3}$$

input `Integrate[Sqrt[c*d^2 + 2*c*d*e*x + c*e^2*x^2]/(d + e*x)^4,x]`

output `-1/2*Sqrt[c*(d + e*x)^2]/(e*(d + e*x)^3)`

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.97, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.062$, Rules used = {1102, 17}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{cd^2 + 2cdex + ce^2x^2}}{(d + ex)^4} dx$$

↓ 1102

$$\frac{\sqrt{cd^2 + 2cdex + ce^2x^2} \int \frac{ce}{(d+ex)^3} dx}{ce(d + ex)}$$

↓ 17

$$-\frac{\sqrt{cd^2 + 2cdex + ce^2x^2}}{2e(d + ex)^3}$$

input `Int[Sqrt[c*d^2 + 2*c*d*e*x + c*e^2*x^2]/(d + e*x)^4,x]`

output `-1/2*Sqrt[c*d^2 + 2*c*d*e*x + c*e^2*x^2]/(e*(d + e*x)^3)`

Defintions of rubi rules used

rule 17 `Int[(c_.)*((a_.) + (b_.)*(x_.))^(m_.), x_Symbol] := Simp[c*((a + b*x)^(m + 1))/(b*(m + 1)), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]`

rule 1102 `Int[((d_.) + (e_.)*(x_.))^(m_.)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] := Simp[(a + b*x + c*x^2)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x)^(2*FracPart[p])) Int[(d + e*x)^m*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && EqQ[b^2 - 4*a*c, 0]`

Maple [A] (verified)

Time = 1.20 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.62

| method | result | size |
|---------|--|------|
| risch | $-\frac{\sqrt{c(ex+d)^2}}{2(ex+d)^3e}$ | 24 |
| gosper | $-\frac{\sqrt{x^2ce^2+2cdxe+cd^2}}{2(ex+d)^3e}$ | 35 |
| default | $-\frac{\sqrt{x^2ce^2+2cdxe+cd^2}}{2(ex+d)^3e}$ | 35 |
| orering | $-\frac{\sqrt{x^2ce^2+2cdxe+cd^2}}{2(ex+d)^3e}$ | 35 |
| trager | $\frac{(ex+2d)x\sqrt{x^2ce^2+2cdxe+cd^2}}{2d^2(ex+d)^3}$ | 43 |

input `int((c*e^2*x^2+2*c*d*e*x+c*d^2)^(1/2)/(e*x+d)^4,x,method=_RETURNVERBOSE)`

output `-1/2*(c*(e*x+d)^2)^(1/2)/(e*x+d)^3/e`

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.46

$$\int \frac{\sqrt{cd^2 + 2cdex + ce^2x^2}}{(d + ex)^4} dx = -\frac{\sqrt{ce^2x^2 + 2cdex + cd^2}}{2(e^4x^3 + 3de^3x^2 + 3d^2e^2x + d^3e)}$$

input `integrate((c*e^2*x^2+2*c*d*e*x+c*d^2)^(1/2)/(e*x+d)^4,x, algorithm="fricas")`

output `-1/2*sqrt(c*e^2*x^2 + 2*c*d*e*x + c*d^2)/(e^4*x^3 + 3*d*e^3*x^2 + 3*d^2*e^2*x + d^3*e)`

Sympy [F]

$$\int \frac{\sqrt{cd^2 + 2cdex + ce^2x^2}}{(d + ex)^4} dx = \int \frac{\sqrt{c(d + ex)^2}}{(d + ex)^4} dx$$

input `integrate((c*e**2*x**2+2*c*d*e*x+c*d**2)**(1/2)/(e*x+d)**4,x)`

output `Integral(sqrt(c*(d + e*x)**2)/(d + e*x)**4, x)`

Maxima [F(-2)]

Exception generated.

$$\int \frac{\sqrt{cd^2 + 2cdex + ce^2x^2}}{(d + ex)^4} dx = \text{Exception raised: RuntimeError}$$

input `integrate((c*e^2*x^2+2*c*d*e*x+c*d^2)^(1/2)/(e*x+d)^4,x, algorithm="maxima")`

output `Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negative exponent.`

Giac [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.54

$$\int \frac{\sqrt{cd^2 + 2cdex + ce^2x^2}}{(d + ex)^4} dx = -\frac{\sqrt{c}\operatorname{sgn}(ex + d)}{2(ex + d)^2e}$$

input `integrate((c*e^2*x^2+2*c*d*e*x+c*d^2)^(1/2)/(e*x+d)^4,x, algorithm="giac")`

output `-1/2*sqrt(c)*sgn(e*x + d)/((e*x + d)^2*e)`

Mupad [B] (verification not implemented)

Time = 9.34 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.87

$$\int \frac{\sqrt{cd^2 + 2cdex + ce^2x^2}}{(d + ex)^4} dx = -\frac{\sqrt{cd^2 + 2cdex + ce^2x^2}}{2e(d + ex)^3}$$

input `int((c*d^2 + c*e^2*x^2 + 2*c*d*e*x)^(1/2)/(d + e*x)^4,x)`

output `-(c*d^2 + c*e^2*x^2 + 2*c*d*e*x)^(1/2)/(2*e*(d + e*x)^3)`

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.64

$$\int \frac{\sqrt{cd^2 + 2cdex + ce^2x^2}}{(d + ex)^4} dx = -\frac{\sqrt{c}}{2e(e^2x^2 + 2dex + d^2)}$$

input `int((c*e^2*x^2+2*c*d*e*x+c*d^2)^(1/2)/(e*x+d)^4,x)`

output `(- sqrt(c))/(2*e*(d**2 + 2*d*e*x + e**2*x**2))`

$$3.210 \quad \int \frac{\sqrt{cd^2 + 2cdex + ce^2x^2}}{(d+ex)^6} dx$$

| | |
|---|------|
| Optimal result | 1720 |
| Mathematica [A] (verified) | 1720 |
| Rubi [A] (verified) | 1721 |
| Maple [A] (verified) | 1722 |
| Fricas [B] (verification not implemented) | 1722 |
| Sympy [F] | 1723 |
| Maxima [F(-2)] | 1723 |
| Giac [A] (verification not implemented) | 1723 |
| Mupad [B] (verification not implemented) | 1724 |
| Reduce [B] (verification not implemented) | 1724 |

Optimal result

Integrand size = 32, antiderivative size = 39

$$\int \frac{\sqrt{cd^2 + 2cdex + ce^2x^2}}{(d+ex)^6} dx = -\frac{c}{4e(d+ex)^3\sqrt{cd^2 + 2cdex + ce^2x^2}}$$

output `-1/4*c/e/(e*x+d)^3/(c*e^2*x^2+2*c*d*e*x+c*d^2)^(1/2)`

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.69

$$\int \frac{\sqrt{cd^2 + 2cdex + ce^2x^2}}{(d+ex)^6} dx = -\frac{\sqrt{c(d+ex)^2}}{4e(d+ex)^5}$$

input `Integrate[Sqrt[c*d^2 + 2*c*d*e*x + c*e^2*x^2]/(d + e*x)^6,x]`

output `-1/4*Sqrt[c*(d + e*x)^2]/(e*(d + e*x)^5)`

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.97, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.062$, Rules used = {1102, 17}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{cd^2 + 2cdex + ce^2x^2}}{(d + ex)^6} dx$$

↓ 1102

$$\frac{\sqrt{cd^2 + 2cdex + ce^2x^2} \int \frac{ce}{(d+ex)^5} dx}{ce(d + ex)}$$

↓ 17

$$-\frac{\sqrt{cd^2 + 2cdex + ce^2x^2}}{4e(d + ex)^5}$$

input `Int[Sqrt[c*d^2 + 2*c*d*e*x + c*e^2*x^2]/(d + e*x)^6,x]`

output `-1/4*Sqrt[c*d^2 + 2*c*d*e*x + c*e^2*x^2]/(e*(d + e*x)^5)`

Defintions of rubi rules used

rule 17 `Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] := Simp[c*((a + b*x)^(m + 1))/(b*(m + 1)), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]`

rule 1102 `Int[((d_.) + (e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] := Simp[(a + b*x + c*x^2)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x)^(2*FracPart[p])) Int[(d + e*x)^m*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && EqQ[b^2 - 4*a*c, 0]`

Maple [A] (verified)

Time = 1.46 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.62

| method | result | size |
|---------|--|------|
| risch | $-\frac{\sqrt{c(ex+d)^2}}{4(ex+d)^5 e}$ | 24 |
| gospers | $-\frac{\sqrt{x^2 c e^2 + 2cdxe + cd^2}}{4(ex+d)^5 e}$ | 35 |
| default | $-\frac{\sqrt{x^2 c e^2 + 2cdxe + cd^2}}{4(ex+d)^5 e}$ | 35 |
| orering | $-\frac{\sqrt{x^2 c e^2 + 2cdxe + cd^2}}{4(ex+d)^5 e}$ | 35 |
| trager | $\frac{(e^3 x^3 + 4d e^2 x^2 + 6d^2 e x + 4d^3) x \sqrt{x^2 c e^2 + 2cdxe + cd^2}}{4d^4 (ex+d)^5}$ | 65 |

input `int((c*e^2*x^2+2*c*d*e*x+c*d^2)^(1/2)/(e*x+d)^6,x,method=_RETURNVERBOSE)`

output `-1/4*(c*(e*x+d)^2)^(1/2)/(e*x+d)^5/e`

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 79 vs. 2(35) = 70.

Time = 0.08 (sec) , antiderivative size = 79, normalized size of antiderivative = 2.03

$$\int \frac{\sqrt{cd^2 + 2cdex + ce^2x^2}}{(d + ex)^6} dx$$

$$= -\frac{\sqrt{ce^2x^2 + 2cdex + cd^2}}{4(e^6x^5 + 5de^5x^4 + 10d^2e^4x^3 + 10d^3e^3x^2 + 5d^4e^2x + d^5e)}$$

input `integrate((c*e^2*x^2+2*c*d*e*x+c*d^2)^(1/2)/(e*x+d)^6,x, algorithm="fricas")`

output `-1/4*sqrt(c*e^2*x^2 + 2*c*d*e*x + c*d^2)/(e^6*x^5 + 5*d*e^5*x^4 + 10*d^2*e^4*x^3 + 10*d^3*e^3*x^2 + 5*d^4*e^2*x + d^5*e)`

Sympy [F]

$$\int \frac{\sqrt{cd^2 + 2cdex + ce^2x^2}}{(d + ex)^6} dx = \int \frac{\sqrt{c(d + ex)^2}}{(d + ex)^6} dx$$

input `integrate((c*e**2*x**2+2*c*d*e*x+c*d**2)**(1/2)/(e*x+d)**6,x)`

output `Integral(sqrt(c*(d + e*x)**2)/(d + e*x)**6, x)`

Maxima [F(-2)]

Exception generated.

$$\int \frac{\sqrt{cd^2 + 2cdex + ce^2x^2}}{(d + ex)^6} dx = \text{Exception raised: RuntimeError}$$

input `integrate((c*e^2*x^2+2*c*d*e*x+c*d^2)^(1/2)/(e*x+d)^6,x, algorithm="maxima")`

output `Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negative exponent.`

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.54

$$\int \frac{\sqrt{cd^2 + 2cdex + ce^2x^2}}{(d + ex)^6} dx = -\frac{\sqrt{c}\operatorname{sgn}(ex + d)}{4(ex + d)^4e}$$

input `integrate((c*e^2*x^2+2*c*d*e*x+c*d^2)^(1/2)/(e*x+d)^6,x, algorithm="giac")`

output `-1/4*sqrt(c)*sgn(e*x + d)/((e*x + d)^4*e)`

Mupad [B] (verification not implemented)

Time = 9.46 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.87

$$\int \frac{\sqrt{cd^2 + 2cdex + ce^2x^2}}{(d + ex)^6} dx = -\frac{\sqrt{cd^2 + 2cdex + ce^2x^2}}{4e(d + ex)^5}$$

input `int((c*d^2 + c*e^2*x^2 + 2*c*d*e*x)^(1/2)/(d + e*x)^6,x)`output `-(c*d^2 + c*e^2*x^2 + 2*c*d*e*x)^(1/2)/(4*e*(d + e*x)^5)`**Reduce [B] (verification not implemented)**

Time = 0.22 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.21

$$\int \frac{\sqrt{cd^2 + 2cdex + ce^2x^2}}{(d + ex)^6} dx = -\frac{\sqrt{c}}{4e(e^4x^4 + 4de^3x^3 + 6d^2e^2x^2 + 4d^3ex + d^4)}$$

input `int((c*e^2*x^2+2*c*d*e*x+c*d^2)^(1/2)/(e*x+d)^6,x)`output `(- sqrt(c))/(4*e*(d**4 + 4*d**3*e*x + 6*d**2*e**2*x**2 + 4*d*e**3*x**3 + e**4*x**4))`

3.211 $\int (d + ex)^3 (cd^2 + 2cdex + ce^2x^2)^{3/2} dx$

| | |
|---|------|
| Optimal result | 1725 |
| Mathematica [A] (verified) | 1725 |
| Rubi [A] (verified) | 1726 |
| Maple [A] (verified) | 1726 |
| Fricas [B] (verification not implemented) | 1727 |
| Sympy [B] (verification not implemented) | 1728 |
| Maxima [F(-2)] | 1728 |
| Giac [B] (verification not implemented) | 1729 |
| Mupad [F(-1)] | 1729 |
| Reduce [B] (verification not implemented) | 1730 |

Optimal result

Integrand size = 32, antiderivative size = 34

$$\int (d + ex)^3 (cd^2 + 2cdex + ce^2x^2)^{3/2} dx = \frac{(cd^2 + 2cdex + ce^2x^2)^{7/2}}{7c^2e}$$

output `1/7*(c*e^2*x^2+2*c*d*e*x+c*d^2)^(7/2)/c^2/e`

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.79

$$\int (d + ex)^3 (cd^2 + 2cdex + ce^2x^2)^{3/2} dx = \frac{(d + ex)^4 (c(d + ex)^2)^{3/2}}{7e}$$

input `Integrate[(d + e*x)^3*(c*d^2 + 2*c*d*e*x + c*e^2*x^2)^(3/2),x]`

output `((d + e*x)^4*(c*(d + e*x)^2)^(3/2))/(7*e)`

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.031$, Rules used = {1099}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (d + ex)^3 (cd^2 + 2cdex + ce^2x^2)^{3/2} dx$$

$$\downarrow 1099$$

$$\frac{(cd^2 + 2cdex + ce^2x^2)^{7/2}}{7c^2e}$$

input `Int[(d + e*x)^3*(c*d^2 + 2*c*d*e*x + c*e^2*x^2)^(3/2),x]`

output `(c*d^2 + 2*c*d*e*x + c*e^2*x^2)^(7/2)/(7*c^2*e)`

Defintions of rubi rules used

rule 1099 `Int[((d_) + (e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Sy
mbol] := Simp[e^m*((a + b*x + c*x^2)^(p + (m + 1)/2)/(c^((m + 1)/2)*(m + 2*
p + 1))), x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[b^2 - 4*a*c, 0] && EqQ[
2*c*d - b*e, 0] && IntegerQ[(m - 1)/2]`

Maple [A] (verified)

Time = 0.95 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.74

| method | result | size |
|----------------|---|------|
| risch | $\frac{c(ex+d)^6 \sqrt{c(ex+d)^2}}{7e}$ | 25 |
| pseudoelliptic | $\frac{c(ex+d)^6 \sqrt{c(ex+d)^2}}{7e}$ | 25 |
| default | $\frac{(ex+d)^4 (x^2 c e^2 + 2cdxe + c d^2)^{\frac{3}{2}}}{7e}$ | 35 |
| gospers | $\frac{x(e^6 x^6 + 7d e^5 x^5 + 21d^2 e^4 x^4 + 35d^3 e^3 x^3 + 35d^4 e^2 x^2 + 21d^5 ex + 7d^6) (x^2 c e^2 + 2cdxe + c d^2)^{\frac{3}{2}}}{7(ex+d)^3}$ | 95 |
| orering | $\frac{x(e^6 x^6 + 7d e^5 x^5 + 21d^2 e^4 x^4 + 35d^3 e^3 x^3 + 35d^4 e^2 x^2 + 21d^5 ex + 7d^6) (x^2 c e^2 + 2cdxe + c d^2)^{\frac{3}{2}}}{7(ex+d)^3}$ | 95 |
| trager | $\frac{cx(e^6 x^6 + 7d e^5 x^5 + 21d^2 e^4 x^4 + 35d^3 e^3 x^3 + 35d^4 e^2 x^2 + 21d^5 ex + 7d^6) \sqrt{x^2 c e^2 + 2cdxe + c d^2}}{7ex+7d}$ | 96 |

input `int((e*x+d)^3*(c*e^2*x^2+2*c*d*e*x+c*d^2)^(3/2),x,method=_RETURNVERBOSE)`

output `1/7*c*(e*x+d)^6*(c*(e*x+d)^2)^(1/2)/e`

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 103 vs. $2(30) = 60$.

Time = 0.09 (sec) , antiderivative size = 103, normalized size of antiderivative = 3.03

$$\int (d+ex)^3 (cd^2 + 2cdex + ce^2x^2)^{3/2} dx = \frac{(ce^6x^7 + 7cde^5x^6 + 21cd^2e^4x^5 + 35cd^3e^3x^4 + 35cd^4e^2x^3 + 21cd^5ex^2 + 7cd^6x)\sqrt{ce^2x^2 + c d^2}}{7(ex+d)}$$

input `integrate((e*x+d)^3*(c*e^2*x^2+2*c*d*e*x+c*d^2)^(3/2),x, algorithm="fricas")`

output `1/7*(c*e^6*x^7 + 7*c*d*e^5*x^6 + 21*c*d^2*e^4*x^5 + 35*c*d^3*e^3*x^4 + 35*c*d^4*e^2*x^3 + 21*c*d^5*e*x^2 + 7*c*d^6*x)*sqrt(c*e^2*x^2 + 2*c*d*e*x + c*d^2)/(e*x + d)`

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 277 vs. $2(31) = 62$.

Time = 0.24 (sec) , antiderivative size = 277, normalized size of antiderivative = 8.15

$$\int (d + ex)^3 (cd^2 + 2cdex + ce^2x^2)^{3/2} dx = \begin{cases} \frac{cd^6\sqrt{cd^2+2cdex+ce^2x^2}}{7e} + \frac{6cd^5x\sqrt{cd^2+2cdex+ce^2x^2}}{7} + \frac{15cd^4ex^2\sqrt{cd^2+2cdex+ce^2x^2}}{7} + \frac{20cd^3e^2x^3\sqrt{cd^2+2cdex+ce^2x^2}}{7} \\ d^3x(cd^2)^{\frac{3}{2}} \end{cases}$$

input `integrate((e*x+d)**3*(c*e**2*x**2+2*c*d*e*x+c*d**2)**(3/2),x)`

output `Piecewise((c*d**6*sqrt(c*d**2 + 2*c*d*e*x + c*e**2*x**2)/(7*e) + 6*c*d**5*x*sqrt(c*d**2 + 2*c*d*e*x + c*e**2*x**2)/7 + 15*c*d**4*e*x**2*sqrt(c*d**2 + 2*c*d*e*x + c*e**2*x**2)/7 + 20*c*d**3*e**2*x**3*sqrt(c*d**2 + 2*c*d*e*x + c*e**2*x**2)/7 + 15*c*d**2*e**3*x**4*sqrt(c*d**2 + 2*c*d*e*x + c*e**2*x**2)/7 + 6*c*d*e**4*x**5*sqrt(c*d**2 + 2*c*d*e*x + c*e**2*x**2)/7 + c*e**5*x**6*sqrt(c*d**2 + 2*c*d*e*x + c*e**2*x**2)/7, Ne(e, 0)), (d**3*x*(c*d**2)**(3/2), True))`

Maxima [F(-2)]

Exception generated.

$$\int (d + ex)^3 (cd^2 + 2cdex + ce^2x^2)^{3/2} dx = \text{Exception raised: ValueError}$$

input `integrate((e*x+d)^3*(c*e^2*x^2+2*c*d*e*x+c*d^2)^(3/2),x, algorithm="maxima")`

output `Exception raised: ValueError >> Computation failed since Maxima requested additional constraints; using the 'assume' command before evaluation *may* help (example of legal syntax is 'assume(e>0)', see `assume?` for more details)Is e`

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 133 vs. $2(30) = 60$.

Time = 0.16 (sec) , antiderivative size = 133, normalized size of antiderivative = 3.91

$$\int (d + ex)^3 (cd^2 + 2cdex + ce^2x^2)^{3/2} dx = \frac{1}{7} \left(ce^6x^7\text{sgn}(ex + d) + 7cde^5x^6\text{sgn}(ex + d) + 21cd^2e^4x^5\text{sgn}(ex + d) + 35cd^3e^3x^4\text{sgn}(ex + d) + 21cd^4e^2x^3\text{sgn}(ex + d) + 7cd^5e^1x^2\text{sgn}(ex + d) + cd^6x\text{sgn}(ex + d) + cd^7\text{sgn}(ex + d) \right) / e \sqrt{c}$$

input `integrate((e*x+d)^3*(c*e^2*x^2+2*c*d*e*x+c*d^2)^(3/2),x, algorithm="giac")`

output `1/7*(c*e^6*x^7*sgn(e*x + d) + 7*c*d*e^5*x^6*sgn(e*x + d) + 21*c*d^2*e^4*x^5*sgn(e*x + d) + 35*c*d^3*e^3*x^4*sgn(e*x + d) + 35*c*d^4*e^2*x^3*sgn(e*x + d) + 21*c*d^5*e*x^2*sgn(e*x + d) + 7*c*d^6*x*sgn(e*x + d) + c*d^7*sgn(e*x + d)/e)*sqrt(c)`

Mupad [F(-1)]

Timed out.

$$\int (d + ex)^3 (cd^2 + 2cdex + ce^2x^2)^{3/2} dx = \int (d + ex)^3 (cd^2 + 2cdex + ce^2x^2)^{3/2} dx$$

input `int((d + e*x)^3*(c*d^2 + c*e^2*x^2 + 2*c*d*e*x)^(3/2),x)`

output `int((d + e*x)^3*(c*d^2 + c*e^2*x^2 + 2*c*d*e*x)^(3/2), x)`

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 68, normalized size of antiderivative = 2.00

$$\int (d + ex)^3 (cd^2 + 2cdex + ce^2x^2)^{3/2} dx = \frac{\sqrt{c}cx(e^6x^6 + 7de^5x^5 + 21d^2e^4x^4 + 35d^3e^3x^3 + 35d^4e^2x^2 + 21d^5ex + 7d^6)}{7}$$

input `int((e*x+d)^3*(c*e^2*x^2+2*c*d*e*x+c*d^2)^(3/2),x)`output `(sqrt(c)*c*x*(7*d**6 + 21*d**5*e*x + 35*d**4*e**2*x**2 + 35*d**3*e**3*x**3 + 21*d**2*e**4*x**4 + 7*d*e**5*x**5 + e**6*x**6))/7`

3.212 $\int (d + ex) (cd^2 + 2cdex + ce^2x^2)^{3/2} dx$

| | |
|---|------|
| Optimal result | 1731 |
| Mathematica [A] (verified) | 1731 |
| Rubi [A] (verified) | 1732 |
| Maple [A] (verified) | 1732 |
| Fricas [B] (verification not implemented) | 1733 |
| Sympy [B] (verification not implemented) | 1734 |
| Maxima [A] (verification not implemented) | 1734 |
| Giac [B] (verification not implemented) | 1735 |
| Mupad [B] (verification not implemented) | 1735 |
| Reduce [B] (verification not implemented) | 1736 |

Optimal result

Integrand size = 30, antiderivative size = 34

$$\int (d + ex) (cd^2 + 2cdex + ce^2x^2)^{3/2} dx = \frac{(cd^2 + 2cdex + ce^2x^2)^{5/2}}{5ce}$$

output `1/5*(c*e^2*x^2+2*c*d*e*x+c*d^2)^(5/2)/c/e`

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.68

$$\int (d + ex) (cd^2 + 2cdex + ce^2x^2)^{3/2} dx = \frac{(c(d + ex)^2)^{5/2}}{5ce}$$

input `Integrate[(d + e*x)*(c*d^2 + 2*c*d*e*x + c*e^2*x^2)^(3/2),x]`

output `(c*(d + e*x)^2)^(5/2)/(5*c*e)`

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.033$, Rules used = {1099}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (d + ex) (cd^2 + 2cdex + ce^2x^2)^{3/2} dx$$

$$\downarrow 1099$$

$$\frac{(cd^2 + 2cdex + ce^2x^2)^{5/2}}{5ce}$$

input `Int[(d + e*x)*(c*d^2 + 2*c*d*e*x + c*e^2*x^2)^(3/2),x]`

output `(c*d^2 + 2*c*d*e*x + c*e^2*x^2)^(5/2)/(5*c*e)`

Defintions of rubi rules used

rule 1099 `Int[((d_) + (e_)*(x_))^(m_)*((a_) + (b_)*(x_) + (c_)*(x_)^2)^(p_), x_Sy
mbol] := Simp[e^m*((a + b*x + c*x^2)^(p + (m + 1)/2)/(c^((m + 1)/2)*(m + 2*
p + 1))), x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[b^2 - 4*a*c, 0] && EqQ[
2*c*d - b*e, 0] && IntegerQ[(m - 1)/2]`

Maple [A] (verified)

Time = 0.90 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.74

| method | result | size |
|----------------|--|------|
| risch | $\frac{c(ex+d)^4 \sqrt{c(ex+d)^2}}{5e}$ | 25 |
| pseudoelliptic | $\frac{c(ex+d)^4 \sqrt{c(ex+d)^2}}{5e}$ | 25 |
| default | $\frac{(ex+d)^2 (x^2 c e^2 + 2cdxe + cd^2)^{\frac{3}{2}}}{5e}$ | 35 |
| gospers | $\frac{x(e^4 x^4 + 5d e^3 x^3 + 10d^2 e^2 x^2 + 10d^3 ex + 5d^4) (x^2 c e^2 + 2cdxe + cd^2)^{\frac{3}{2}}}{5(ex+d)^3}$ | 73 |
| orering | $\frac{x(e^4 x^4 + 5d e^3 x^3 + 10d^2 e^2 x^2 + 10d^3 ex + 5d^4) (x^2 c e^2 + 2cdxe + cd^2)^{\frac{3}{2}}}{5(ex+d)^3}$ | 73 |
| trager | $\frac{cx(e^4 x^4 + 5d e^3 x^3 + 10d^2 e^2 x^2 + 10d^3 ex + 5d^4) \sqrt{x^2 c e^2 + 2cdxe + cd^2}}{5ex + 5d}$ | 74 |

input `int((e*x+d)*(c*e^2*x^2+2*c*d*e*x+c*d^2)^(3/2),x,method=_RETURNVERBOSE)`

output `1/5*c*(e*x+d)^4*(c*(e*x+d)^2)^(1/2)/e`

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 79 vs. $2(30) = 60$.

Time = 0.10 (sec) , antiderivative size = 79, normalized size of antiderivative = 2.32

$$\int (d + ex) (cd^2 + 2cdex + ce^2x^2)^{3/2} dx = \frac{(ce^4x^5 + 5cde^3x^4 + 10cd^2e^2x^3 + 10cd^3ex^2 + 5cd^4x)\sqrt{ce^2x^2 + 2cdex + cd^2}}{5(ex + d)}$$

input `integrate((e*x+d)*(c*e^2*x^2+2*c*d*e*x+c*d^2)^(3/2),x, algorithm="fricas")`

output `1/5*(c*e^4*x^5 + 5*c*d*e^3*x^4 + 10*c*d^2*e^2*x^3 + 10*c*d^3*e*x^2 + 5*c*d^4*x)*sqrt(c*e^2*x^2 + 2*c*d*e*x + c*d^2)/(e*x + d)`

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 194 vs. $2(29) = 58$.

Time = 0.15 (sec) , antiderivative size = 194, normalized size of antiderivative = 5.71

$$\int (d + ex) (cd^2 + 2cdex + ce^2x^2)^{3/2} dx = \begin{cases} \frac{cd^4 \sqrt{cd^2 + 2cdex + ce^2x^2}}{5e} + \frac{4cd^3x \sqrt{cd^2 + 2cdex + ce^2x^2}}{5} + \frac{6cd^2ex^2 \sqrt{cd^2 + 2cdex + ce^2x^2}}{5} + \frac{4cde^2x^3 \sqrt{cd^2 + 2cdex + ce^2x^2}}{5} \\ dx(cd^2)^{\frac{3}{2}} \end{cases}$$

input `integrate((e*x+d)*(c*e**2*x**2+2*c*d*e*x+c*d**2)**(3/2),x)`

output `Piecewise((c*d**4*sqrt(c*d**2 + 2*c*d*e*x + c*e**2*x**2)/(5*e) + 4*c*d**3*x*sqrt(c*d**2 + 2*c*d*e*x + c*e**2*x**2)/5 + 6*c*d**2*e*x**2*sqrt(c*d**2 + 2*c*d*e*x + c*e**2*x**2)/5 + 4*c*d*e**2*x**3*sqrt(c*d**2 + 2*c*d*e*x + c*e**2*x**2)/5 + c*e**3*x**4*sqrt(c*d**2 + 2*c*d*e*x + c*e**2*x**2)/5, Ne(e, 0)), (d*x*(c*d**2)**(3/2), True))`

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.88

$$\int (d + ex) (cd^2 + 2cdex + ce^2x^2)^{3/2} dx = \frac{(ce^2x^2 + 2cdex + cd^2)^{\frac{5}{2}}}{5ce}$$

input `integrate((e*x+d)*(c*e^2*x^2+2*c*d*e*x+c*d^2)^(3/2),x, algorithm="maxima")`

output `1/5*(c*e^2*x^2 + 2*c*d*e*x + c*d^2)^(5/2)/(c*e)`

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 97 vs. $2(30) = 60$.

Time = 0.19 (sec) , antiderivative size = 97, normalized size of antiderivative = 2.85

$$\int (d + ex) (cd^2 + 2cdex + ce^2x^2)^{3/2} dx = \frac{1}{5} \left(ce^4x^5 \operatorname{sgn}(ex + d) + 5cde^3x^4 \operatorname{sgn}(ex + d) + 10cd^2e^2x^3 \operatorname{sgn}(ex + d) + 10cd^3ex^2 \operatorname{sgn}(ex + d) + 5cd^4ex \operatorname{sgn}(ex + d) + cd^5 \operatorname{sgn}(ex + d) \right) \sqrt{c}$$

input `integrate((e*x+d)*(c*e^2*x^2+2*c*d*e*x+c*d^2)^(3/2),x, algorithm="giac")`

output `1/5*(c*e^4*x^5*sgn(e*x + d) + 5*c*d*e^3*x^4*sgn(e*x + d) + 10*c*d^2*e^2*x^3*sgn(e*x + d) + 10*c*d^3*e*x^2*sgn(e*x + d) + 5*c*d^4*x*sgn(e*x + d) + c*d^5*sgn(e*x + d)/e)*sqrt(c)`

Mupad [B] (verification not implemented)

Time = 9.42 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.00

$$\int (d + ex) (cd^2 + 2cdex + ce^2x^2)^{3/2} dx = \frac{(d + ex)^2 (cd^2 + 2cdex + ce^2x^2)^{3/2}}{5e}$$

input `int((d + e*x)*(c*d^2 + c*e^2*x^2 + 2*c*d*e*x)^(3/2),x)`

output `((d + e*x)^2*(c*d^2 + c*e^2*x^2 + 2*c*d*e*x)^(3/2))/(5*e)`

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.35

$$\int (d + ex) (cd^2 + 2cdex + ce^2x^2)^{3/2} dx = \frac{\sqrt{c}cx(e^4x^4 + 5de^3x^3 + 10d^2e^2x^2 + 10d^3ex + 5d^4)}{5}$$

input `int((e*x+d)*(c*e^2*x^2+2*c*d*e*x+c*d^2)^(3/2),x)`output `(sqrt(c)*c*x*(5*d**4 + 10*d**3*e*x + 10*d**2*e**2*x**2 + 5*d*e**3*x**3 + e**4*x**4))/5`

$$3.213 \quad \int \frac{(cd^2 + 2cdex + ce^2x^2)^{3/2}}{d + ex} dx$$

| | |
|---|------|
| Optimal result | 1737 |
| Mathematica [A] (verified) | 1737 |
| Rubi [A] (verified) | 1738 |
| Maple [A] (verified) | 1738 |
| Fricas [B] (verification not implemented) | 1739 |
| Sympy [A] (verification not implemented) | 1740 |
| Maxima [A] (verification not implemented) | 1740 |
| Giac [A] (verification not implemented) | 1740 |
| Mupad [B] (verification not implemented) | 1741 |
| Reduce [B] (verification not implemented) | 1741 |

Optimal result

Integrand size = 32, antiderivative size = 31

$$\int \frac{(cd^2 + 2cdex + ce^2x^2)^{3/2}}{d + ex} dx = \frac{(cd^2 + 2cdex + ce^2x^2)^{3/2}}{3e}$$

output $1/3*(c*e^2*x^2+2*c*d*e*x+c*d^2)^{(3/2)}/e$

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.65

$$\int \frac{(cd^2 + 2cdex + ce^2x^2)^{3/2}}{d + ex} dx = \frac{(c(d + ex)^2)^{3/2}}{3e}$$

input `Integrate[(c*d^2 + 2*c*d*e*x + c*e^2*x^2)^(3/2)/(d + e*x),x]`

output $(c*(d + e*x)^2)^{(3/2)}/(3*e)$

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.031$, Rules used = {1099}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(cd^2 + 2cdex + ce^2x^2)^{3/2}}{d + ex} dx$$

↓ 1099

$$\frac{(cd^2 + 2cdex + ce^2x^2)^{3/2}}{3e}$$

input `Int[(c*d^2 + 2*c*d*e*x + c*e^2*x^2)^(3/2)/(d + e*x),x]`

output `(c*d^2 + 2*c*d*e*x + c*e^2*x^2)^(3/2)/(3*e)`

Defintions of rubi rules used

rule 1099 `Int[((d_) + (e_)*(x_))^(m_)*((a_) + (b_)*(x_) + (c_)*(x_)^2)^(p_), x_Symbol] := Simp[e^m*((a + b*x + c*x^2)^(p + (m + 1)/2)/(c^((m + 1)/2)*(m + 2*p + 1))), x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[b^2 - 4*a*c, 0] && EqQ[2*c*d - b*e, 0] && IntegerQ[(m - 1)/2]`

Maple [A] (verified)

Time = 1.10 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.81

| method | result | size |
|----------------|---|------|
| risch | $\frac{c(ex+d)^2 \sqrt{c(ex+d)^2}}{3e}$ | 25 |
| pseudoelliptic | $\frac{c(ex+d)^2 \sqrt{c(ex+d)^2}}{3e}$ | 25 |
| default | $\frac{(x^2 c e^2 + 2cdxe + c d^2)^{\frac{3}{2}}}{3e}$ | 28 |
| gosper | $\frac{x(e^2 x^2 + 3dex + 3d^2)(x^2 c e^2 + 2cdxe + c d^2)^{\frac{3}{2}}}{3(ex+d)^3}$ | 51 |
| orering | $\frac{x(e^2 x^2 + 3dex + 3d^2)(x^2 c e^2 + 2cdxe + c d^2)^{\frac{3}{2}}}{3(ex+d)^3}$ | 51 |
| trager | $\frac{cx(e^2 x^2 + 3dex + 3d^2) \sqrt{x^2 c e^2 + 2cdxe + c d^2}}{3ex + 3d}$ | 52 |

input `int((c*e^2*x^2+2*c*d*e*x+c*d^2)^(3/2)/(e*x+d),x,method=_RETURNVERBOSE)`

output `1/3*c*(e*x+d)^2*(c*(e*x+d)^2)^(1/2)/e`

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 55 vs. $2(27) = 54$.

Time = 0.10 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.77

$$\int \frac{(cd^2 + 2cdex + ce^2x^2)^{3/2}}{d + ex} dx = \frac{(ce^2x^3 + 3cdex^2 + 3cd^2x)\sqrt{ce^2x^2 + 2cdex + cd^2}}{3(ex + d)}$$

input `integrate((c*e^2*x^2+2*c*d*e*x+c*d^2)^(3/2)/(e*x+d),x, algorithm="fricas")`

output `1/3*(c*e^2*x^3 + 3*c*d*e*x^2 + 3*c*d^2*x)*sqrt(c*e^2*x^2 + 2*c*d*e*x + c*d^2)/(e*x + d)`

Sympy [A] (verification not implemented)

Time = 1.05 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.26

$$\int \frac{(cd^2 + 2cdex + ce^2x^2)^{3/2}}{d + ex} dx = \begin{cases} \frac{(cd^2 + 2cdex + ce^2x^2)^{3/2}}{3e} & \text{for } e \neq 0 \\ \frac{x(cd^2)^{3/2}}{d} & \text{otherwise} \end{cases}$$

input `integrate((c*e**2*x**2+2*c*d*e*x+c*d**2)**(3/2)/(e*x+d),x)`output `Piecewise(((c*d**2 + 2*c*d*e*x + c*e**2*x**2)**(3/2)/(3*e), Ne(e, 0)), (x*(c*d**2)**(3/2)/d, True))`**Maxima [A] (verification not implemented)**

Time = 0.03 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.87

$$\int \frac{(cd^2 + 2cdex + ce^2x^2)^{3/2}}{d + ex} dx = \frac{(ce^2x^2 + 2cdex + cd^2)^{3/2}}{3e}$$

input `integrate((c*e^2*x^2+2*c*d*e*x+c*d^2)^(3/2)/(e*x+d),x, algorithm="maxima")`output `1/3*(c*e^2*x^2 + 2*c*d*e*x + c*d^2)^(3/2)/e`**Giac [A] (verification not implemented)**

Time = 0.15 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.65

$$\int \frac{(cd^2 + 2cdex + ce^2x^2)^{3/2}}{d + ex} dx = \frac{1}{3} \left(\frac{cd^3 \operatorname{sgn}(ex + d)}{e} + (ce^2x^3 + 3cdex^2 + 3cd^2x) \operatorname{sgn}(ex + d) \right) \sqrt{c}$$

input `integrate((c*e^2*x^2+2*c*d*e*x+c*d^2)^(3/2)/(e*x+d),x, algorithm="giac")`

output $\frac{1}{3}(c*d^3*\text{sgn}(e*x + d)/e + (c*e^2*x^3 + 3*c*d*e*x^2 + 3*c*d^2*x)*\text{sgn}(e*x + d))*\text{sqrt}(c)$

Mupad [B] (verification not implemented)

Time = 9.49 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.52

$$\int \frac{(cd^2 + 2cdex + ce^2x^2)^{3/2}}{d + ex} dx = \frac{(c(d + ex)^2)^{3/2}}{3e}$$

input `int((c*d^2 + c*e^2*x^2 + 2*c*d*e*x)^(3/2)/(d + e*x),x)`

output $(c*(d + e*x)^2)^{(3/2)/(3*e)}$

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.77

$$\int \frac{(cd^2 + 2cdex + ce^2x^2)^{3/2}}{d + ex} dx = \frac{\sqrt{c}cx(e^2x^2 + 3dex + 3d^2)}{3}$$

input `int((c*e^2*x^2+2*c*d*e*x+c*d^2)^(3/2)/(e*x+d),x)`

output $(\text{sqrt}(c)*c*x*(3*d**2 + 3*d*e*x + e**2*x**2))/3$

$$3.214 \quad \int \frac{(cd^2 + 2cdex + ce^2x^2)^{3/2}}{(d+ex)^3} dx$$

| | |
|---|------|
| Optimal result | 1742 |
| Mathematica [A] (verified) | 1742 |
| Rubi [A] (verified) | 1743 |
| Maple [A] (verified) | 1743 |
| Fricas [A] (verification not implemented) | 1744 |
| Sympy [A] (verification not implemented) | 1745 |
| Maxima [F(-2)] | 1745 |
| Giac [A] (verification not implemented) | 1745 |
| Mupad [F(-1)] | 1746 |
| Reduce [B] (verification not implemented) | 1746 |

Optimal result

Integrand size = 32, antiderivative size = 29

$$\int \frac{(cd^2 + 2cdex + ce^2x^2)^{3/2}}{(d+ex)^3} dx = \frac{c\sqrt{cd^2 + 2cdex + ce^2x^2}}{e}$$

output `c*(c*e^2*x^2+2*c*d*e*x+c*d^2)^(1/2)/e`

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.62

$$\int \frac{(cd^2 + 2cdex + ce^2x^2)^{3/2}}{(d+ex)^3} dx = \frac{c\sqrt{c(d+ex)^2}}{e}$$

input `Integrate[(c*d^2 + 2*c*d*e*x + c*e^2*x^2)^(3/2)/(d + e*x)^3,x]`

output `(c*Sqrt[c*(d + e*x)^2])/e`

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.031$, Rules used = {1099}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(cd^2 + 2cdex + ce^2x^2)^{3/2}}{(d + ex)^3} dx$$

↓ 1099

$$\frac{c\sqrt{cd^2 + 2cdex + ce^2x^2}}{e}$$

input `Int[(c*d^2 + 2*c*d*e*x + c*e^2*x^2)^(3/2)/(d + e*x)^3,x]`

output `(c*Sqrt[c*d^2 + 2*c*d*e*x + c*e^2*x^2])/e`

Defintions of rubi rules used

rule 1099 `Int[((d_) + (e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Sy
mbol] := Simp[e^m*((a + b*x + c*x^2)^(p + (m + 1)/2)/(c^((m + 1)/2)*(m + 2*
p + 1))), x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[b^2 - 4*a*c, 0] && EqQ[
2*c*d - b*e, 0] && IntegerQ[(m - 1)/2]`

Maple [A] (verified)

Time = 1.08 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.59

| method | result | size |
|----------------|--|------|
| pseudoelliptic | $\frac{c\sqrt{c(ex+d)^2}}{e}$ | 17 |
| risch | $\frac{c\sqrt{c(ex+d)^2}x}{ex+d}$ | 22 |
| default | $\frac{x(x^2ce^2+2cdxe+cd^2)^{\frac{3}{2}}}{(ex+d)^3}$ | 32 |
| orering | $\frac{x(x^2ce^2+2cdxe+cd^2)^{\frac{3}{2}}}{(ex+d)^3}$ | 32 |
| trager | $\frac{cx\sqrt{x^2ce^2+2cdxe+cd^2}}{ex+d}$ | 33 |

input `int((c*e^2*x^2+2*c*d*e*x+c*d^2)^(3/2)/(e*x+d)^3,x,method=_RETURNVERBOSE)`

output `c*(c*(e*x+d)^2)^(1/2)/e`

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.10

$$\int \frac{(cd^2 + 2cdex + ce^2x^2)^{3/2}}{(d + ex)^3} dx = \frac{\sqrt{ce^2x^2 + 2cdex + cd^2}cx}{ex + d}$$

input `integrate((c*e^2*x^2+2*c*d*e*x+c*d^2)^(3/2)/(e*x+d)^3,x, algorithm="fricas")`

output `sqrt(c*e^2*x^2 + 2*c*d*e*x + c*d^2)*c*x/(e*x + d)`

Sympy [A] (verification not implemented)

Time = 1.31 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.34

$$\int \frac{(cd^2 + 2cdex + ce^2x^2)^{3/2}}{(d + ex)^3} dx = c \left(\begin{cases} \frac{x\sqrt{cd^2}}{d} & \text{for } e = 0 \\ \frac{\sqrt{cd^2 + 2cdex + ce^2x^2}}{e} & \text{otherwise} \end{cases} \right)$$

input `integrate((c***2*x**2+2*c*d*e*x+c*d**2)**(3/2)/(e*x+d)**3,x)`

output `c*Piecewise((x*sqrt(c*d**2)/d, Eq(e, 0)), (sqrt(c*d**2 + 2*c*d*e*x + c***2*x**2)/e, True))`

Maxima [F(-2)]

Exception generated.

$$\int \frac{(cd^2 + 2cdex + ce^2x^2)^{3/2}}{(d + ex)^3} dx = \text{Exception raised: RuntimeError}$$

input `integrate((c*e^2*x^2+2*c*d*e*x+c*d^2)^(3/2)/(e*x+d)^3,x, algorithm="maxima")`

output `Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negative exponent.`

Giac [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.90

$$\int \frac{(cd^2 + 2cdex + ce^2x^2)^{3/2}}{(d + ex)^3} dx = \left(cx\text{sgn}(ex + d) + \frac{cd\text{sgn}(ex + d)}{e} \right) \sqrt{c}$$

input `integrate((c*e^2*x^2+2*c*d*e*x+c*d^2)^(3/2)/(e*x+d)^3,x, algorithm="giac")`

output `(c*x*sgn(e*x + d) + c*d*sgn(e*x + d)/e)*sqrt(c)`

Mupad [F(-1)]

Timed out.

$$\int \frac{(cd^2 + 2cdex + ce^2x^2)^{3/2}}{(d + ex)^3} dx = \int \frac{(cd^2 + 2cdex + ce^2x^2)^{3/2}}{(d + ex)^3} dx$$

input `int((c*d^2 + c*e^2*x^2 + 2*c*d*e*x)^(3/2)/(d + e*x)^3,x)`

output `int((c*d^2 + c*e^2*x^2 + 2*c*d*e*x)^(3/2)/(d + e*x)^3, x)`

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 5, normalized size of antiderivative = 0.17

$$\int \frac{(cd^2 + 2cdex + ce^2x^2)^{3/2}}{(d + ex)^3} dx = \sqrt{c} cx$$

input `int((c*e^2*x^2+2*c*d*e*x+c*d^2)^(3/2)/(e*x+d)^3,x)`

output `sqrt(c)*c*x`

3.215
$$\int \frac{(cd^2 + 2cdex + ce^2x^2)^{3/2}}{(d+ex)^5} dx$$

| | |
|---|------|
| Optimal result | 1747 |
| Mathematica [A] (verified) | 1747 |
| Rubi [A] (verified) | 1748 |
| Maple [A] (verified) | 1748 |
| Fricas [A] (verification not implemented) | 1749 |
| Sympy [F] | 1750 |
| Maxima [F(-2)] | 1750 |
| Giac [A] (verification not implemented) | 1750 |
| Mupad [B] (verification not implemented) | 1751 |
| Reduce [B] (verification not implemented) | 1751 |

Optimal result

Integrand size = 32, antiderivative size = 32

$$\int \frac{(cd^2 + 2cdex + ce^2x^2)^{3/2}}{(d + ex)^5} dx = -\frac{c^2}{e\sqrt{cd^2 + 2cdex + ce^2x^2}}$$

output `-c^2/e/(c*e^2*x^2+2*c*d*e*x+c*d^2)^(1/2)`

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.78

$$\int \frac{(cd^2 + 2cdex + ce^2x^2)^{3/2}}{(d + ex)^5} dx = -\frac{(c(d + ex)^2)^{3/2}}{e(d + ex)^4}$$

input `Integrate[(c*d^2 + 2*c*d*e*x + c*e^2*x^2)^(3/2)/(d + e*x)^5,x]`

output `-((c*(d + e*x)^2)^(3/2)/(e*(d + e*x)^4))`

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.031$, Rules used = {1099}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(cd^2 + 2cdex + ce^2x^2)^{3/2}}{(d + ex)^5} dx$$

↓ 1099

$$-\frac{c^2}{e\sqrt{cd^2 + 2cdex + ce^2x^2}}$$

input `Int[(c*d^2 + 2*c*d*e*x + c*e^2*x^2)^(3/2)/(d + e*x)^5,x]`

output `-(c^2/(e*Sqrt[c*d^2 + 2*c*d*e*x + c*e^2*x^2]))`

Defintions of rubi rules used

rule 1099 `Int[((d_) + (e_)*(x_))^(m_)*((a_) + (b_)*(x_) + (c_)*(x_)^2)^(p_), x_Symbol] := Simp[e^m*((a + b*x + c*x^2)^(p + (m + 1)/2)/(c^((m + 1)/2)*(m + 2*p + 1))), x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[b^2 - 4*a*c, 0] && EqQ[2*c*d - b*e, 0] && IntegerQ[(m - 1)/2]`

Maple [A] (verified)

Time = 1.11 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.78

| method | result | size |
|----------------|---|------|
| risch | $-\frac{c\sqrt{c(ex+d)^2}}{(ex+d)^2e}$ | 25 |
| pseudoelliptic | $-\frac{c\sqrt{c(ex+d)^2}}{(ex+d)^2e}$ | 25 |
| gosper | $-\frac{(x^2ce^2+2cdxe+cd^2)^{\frac{3}{2}}}{(ex+d)^4e}$ | 35 |
| default | $-\frac{(x^2ce^2+2cdxe+cd^2)^{\frac{3}{2}}}{(ex+d)^4e}$ | 35 |
| orering | $-\frac{(x^2ce^2+2cdxe+cd^2)^{\frac{3}{2}}}{(ex+d)^4e}$ | 35 |
| trager | $\frac{cx\sqrt{x^2ce^2+2cdxe+cd^2}}{d(ex+d)^2}$ | 36 |

input `int((c*e^2*x^2+2*c*d*e*x+c*d^2)^(3/2)/(e*x+d)^5,x,method=_RETURNVERBOSE)`

output `-c/(e*x+d)^2*(c*(e*x+d)^2)^(1/2)/e`

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.47

$$\int \frac{(cd^2 + 2cdex + ce^2x^2)^{3/2}}{(d + ex)^5} dx = -\frac{\sqrt{ce^2x^2 + 2cdex + cd^2}c}{e^3x^2 + 2de^2x + d^2e}$$

input `integrate((c*e^2*x^2+2*c*d*e*x+c*d^2)^(3/2)/(e*x+d)^5,x, algorithm="fricas")`

output `-sqrt(c*e^2*x^2 + 2*c*d*e*x + c*d^2)*c/(e^3*x^2 + 2*d*e^2*x + d^2*e)`

Sympy [F]

$$\int \frac{(cd^2 + 2cdex + ce^2x^2)^{3/2}}{(d + ex)^5} dx = \int \frac{(c(d + ex)^2)^{\frac{3}{2}}}{(d + ex)^5} dx$$

input `integrate((c*e**2*x**2+2*c*d*e*x+c*d**2)**(3/2)/(e*x+d)**5,x)`

output `Integral((c*(d + e*x)**2)**(3/2)/(d + e*x)**5, x)`

Maxima [F(-2)]

Exception generated.

$$\int \frac{(cd^2 + 2cdex + ce^2x^2)^{3/2}}{(d + ex)^5} dx = \text{Exception raised: RuntimeError}$$

input `integrate((c*e^2*x^2+2*c*d*e*x+c*d^2)^(3/2)/(e*x+d)^5,x, algorithm="maxima")`

output `Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negative exponent.`

Giac [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.66

$$\int \frac{(cd^2 + 2cdex + ce^2x^2)^{3/2}}{(d + ex)^5} dx = -\frac{c^{\frac{3}{2}} \operatorname{sgn}(ex + d)}{(ex + d)e}$$

input `integrate((c*e^2*x^2+2*c*d*e*x+c*d^2)^(3/2)/(e*x+d)^5,x, algorithm="giac")`

output `-c^(3/2)*sgn(e*x + d)/((e*x + d)*e)`

Mupad [B] (verification not implemented)

Time = 9.14 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.09

$$\int \frac{(cd^2 + 2cdex + ce^2x^2)^{3/2}}{(d + ex)^5} dx = -\frac{c\sqrt{cd^2 + 2cdex + ce^2x^2}}{e(d + ex)^2}$$

input

```
int((c*d^2 + c*e^2*x^2 + 2*c*d*e*x)^(3/2)/(d + e*x)^5,x)
```

output

```
-(c*(c*d^2 + c*e^2*x^2 + 2*c*d*e*x)^(1/2))/(e*(d + e*x)^2)
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.47

$$\int \frac{(cd^2 + 2cdex + ce^2x^2)^{3/2}}{(d + ex)^5} dx = \frac{\sqrt{c} cx}{d(ex + d)}$$

input

```
int((c*e^2*x^2+2*c*d*e*x+c*d^2)^(3/2)/(e*x+d)^5,x)
```

output

```
(sqrt(c)*c*x)/(d*(d + e*x))
```

$$3.216 \quad \int \frac{(cd^2 + 2cdex + ce^2x^2)^{3/2}}{(d+ex)^7} dx$$

| | |
|---|------|
| Optimal result | 1752 |
| Mathematica [A] (verified) | 1752 |
| Rubi [A] (verified) | 1753 |
| Maple [A] (verified) | 1753 |
| Fricas [B] (verification not implemented) | 1754 |
| Sympy [F] | 1755 |
| Maxima [F(-2)] | 1755 |
| Giac [A] (verification not implemented) | 1755 |
| Mupad [B] (verification not implemented) | 1756 |
| Reduce [B] (verification not implemented) | 1756 |

Optimal result

Integrand size = 32, antiderivative size = 34

$$\int \frac{(cd^2 + 2cdex + ce^2x^2)^{3/2}}{(d+ex)^7} dx = -\frac{c^3}{3e(cd^2 + 2cdex + ce^2x^2)^{3/2}}$$

output `-1/3*c^3/e/(c*e^2*x^2+2*c*d*e*x+c*d^2)^(3/2)`

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.79

$$\int \frac{(cd^2 + 2cdex + ce^2x^2)^{3/2}}{(d+ex)^7} dx = -\frac{(c(d+ex)^2)^{3/2}}{3e(d+ex)^6}$$

input `Integrate[(c*d^2 + 2*c*d*e*x + c*e^2*x^2)^(3/2)/(d + e*x)^7,x]`

output `-1/3*(c*(d + e*x)^2)^(3/2)/(e*(d + e*x)^6)`

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.031$, Rules used = {1099}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(cd^2 + 2cdex + ce^2x^2)^{3/2}}{(d + ex)^7} dx$$

↓ 1099

$$-\frac{c^3}{3e(cd^2 + 2cdex + ce^2x^2)^{3/2}}$$

input `Int[(c*d^2 + 2*c*d*e*x + c*e^2*x^2)^(3/2)/(d + e*x)^7,x]`

output `-1/3*c^3/(e*(c*d^2 + 2*c*d*e*x + c*e^2*x^2)^(3/2))`

Defintions of rubi rules used

rule 1099

```
Int[((d_) + (e_)*(x_))^(m_)*((a_) + (b_)*(x_) + (c_)*(x_)^2)^(p_), x_Sy
mbol] := Simp[e^m*((a + b*x + c*x^2)^(p + (m + 1)/2)/(c^((m + 1)/2)*(m + 2*
p + 1))), x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[b^2 - 4*a*c, 0] && EqQ[
2*c*d - b*e, 0] && IntegerQ[(m - 1)/2]
```

Maple [A] (verified)

Time = 1.14 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.74

| method | result | size |
|----------------|--|------|
| risch | $-\frac{c\sqrt{c(ex+d)^2}}{3(ex+d)^4e}$ | 25 |
| pseudoelliptic | $-\frac{c\sqrt{c(ex+d)^2}}{3(ex+d)^4e}$ | 25 |
| gosper | $-\frac{(x^2ce^2+2cdxe+cd^2)^{\frac{3}{2}}}{3(ex+d)^6e}$ | 35 |
| default | $-\frac{(x^2ce^2+2cdxe+cd^2)^{\frac{3}{2}}}{3(ex+d)^6e}$ | 35 |
| orering | $-\frac{(x^2ce^2+2cdxe+cd^2)^{\frac{3}{2}}}{3(ex+d)^6e}$ | 35 |
| trager | $\frac{c(e^2x^2+3dex+3d^2)x\sqrt{x^2ce^2+2cdxe+cd^2}}{3d^3(ex+d)^4}$ | 55 |

input `int((c*e^2*x^2+2*c*d*e*x+c*d^2)^(3/2)/(e*x+d)^7,x,method=_RETURNVERBOSE)`

output `-1/3*c/(e*x+d)^4*(c*(e*x+d)^2)^(1/2)/e`

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 69 vs. $2(30) = 60$.

Time = 0.09 (sec) , antiderivative size = 69, normalized size of antiderivative = 2.03

$$\int \frac{(cd^2 + 2cdex + ce^2x^2)^{3/2}}{(d + ex)^7} dx = -\frac{\sqrt{ce^2x^2 + 2cdex + cd^2}c}{3(e^5x^4 + 4de^4x^3 + 6d^2e^3x^2 + 4d^3e^2x + d^4e)}$$

input `integrate((c*e^2*x^2+2*c*d*e*x+c*d^2)^(3/2)/(e*x+d)^7,x, algorithm="fricas")`

output `-1/3*sqrt(c*e^2*x^2 + 2*c*d*e*x + c*d^2)*c/(e^5*x^4 + 4*d*e^4*x^3 + 6*d^2*e^3*x^2 + 4*d^3*e^2*x + d^4*e)`

Sympy [F]

$$\int \frac{(cd^2 + 2cdex + ce^2x^2)^{3/2}}{(d + ex)^7} dx = \int \frac{(c(d + ex)^2)^{3/2}}{(d + ex)^7} dx$$

input `integrate((c*e**2*x**2+2*c*d*e*x+c*d**2)**(3/2)/(e*x+d)**7,x)`

output `Integral((c*(d + e*x)**2)**(3/2)/(d + e*x)**7, x)`

Maxima [F(-2)]

Exception generated.

$$\int \frac{(cd^2 + 2cdex + ce^2x^2)^{3/2}}{(d + ex)^7} dx = \text{Exception raised: RuntimeError}$$

input `integrate((c*e^2*x^2+2*c*d*e*x+c*d^2)^(3/2)/(e*x+d)^7,x, algorithm="maxima")`

output `Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negative exponent.`

Giac [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.62

$$\int \frac{(cd^2 + 2cdex + ce^2x^2)^{3/2}}{(d + ex)^7} dx = -\frac{c^{3/2} \operatorname{sgn}(ex + d)}{3(ex + d)^3 e}$$

input `integrate((c*e^2*x^2+2*c*d*e*x+c*d^2)^(3/2)/(e*x+d)^7,x, algorithm="giac")`

output `-1/3*c^(3/2)*sgn(e*x + d)/((e*x + d)^3*e)`

Mupad [B] (verification not implemented)

Time = 9.67 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.03

$$\int \frac{(cd^2 + 2cdex + ce^2x^2)^{3/2}}{(d + ex)^7} dx = -\frac{c\sqrt{cd^2 + 2cdex + ce^2x^2}}{3e(d + ex)^4}$$

input `int((c*d^2 + c*e^2*x^2 + 2*c*d*e*x)^(3/2)/(d + e*x)^7,x)`output `-(c*(c*d^2 + c*e^2*x^2 + 2*c*d*e*x)^(1/2))/(3*e*(d + e*x)^4)`**Reduce [B] (verification not implemented)**

Time = 0.23 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.09

$$\int \frac{(cd^2 + 2cdex + ce^2x^2)^{3/2}}{(d + ex)^7} dx = -\frac{\sqrt{c}c}{3e(e^3x^3 + 3de^2x^2 + 3d^2ex + d^3)}$$

input `int((c*e^2*x^2+2*c*d*e*x+c*d^2)^(3/2)/(e*x+d)^7,x)`output `(- sqrt(c)*c)/(3*e*(d**3 + 3*d**2*e*x + 3*d*e**2*x**2 + e**3*x**3))`

3.217 $\int (d + ex)^2 (cd^2 + 2cdex + ce^2x^2)^{3/2} dx$

| | |
|---|------|
| Optimal result | 1757 |
| Mathematica [A] (verified) | 1757 |
| Rubi [A] (verified) | 1758 |
| Maple [A] (verified) | 1759 |
| Fricas [B] (verification not implemented) | 1759 |
| Sympy [B] (verification not implemented) | 1760 |
| Maxima [F(-2)] | 1760 |
| Giac [B] (verification not implemented) | 1761 |
| Mupad [F(-1)] | 1761 |
| Reduce [B] (verification not implemented) | 1762 |

Optimal result

Integrand size = 32, antiderivative size = 39

$$\int (d + ex)^2 (cd^2 + 2cdex + ce^2x^2)^{3/2} dx = \frac{c(d + ex)^5 \sqrt{cd^2 + 2cdex + ce^2x^2}}{6e}$$

output

$$1/6*c*(e*x+d)^5*(c*e^2*x^2+2*c*d*e*x+c*d^2)^(1/2)/e$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.72

$$\int (d + ex)^2 (cd^2 + 2cdex + ce^2x^2)^{3/2} dx = \frac{(d + ex) (c(d + ex)^2)^{5/2}}{6ce}$$

input

$$\text{Integrate}[(d + e*x)^2*(c*d^2 + 2*c*d*e*x + c*e^2*x^2)^(3/2), x]$$

output

$$((d + e*x)*(c*(d + e*x)^2)^(5/2))/(6*c*e)$$

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.062$, Rules used = {1102, 17}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (d + ex)^2 (cd^2 + 2cdex + ce^2x^2)^{3/2} dx$$

$$\downarrow 1102$$

$$\frac{\sqrt{cd^2 + 2cdex + ce^2x^2} \int c^3e^3(d + ex)^5 dx}{c^2e^3(d + ex)}$$

$$\downarrow 17$$

$$\frac{c(d + ex)^5 \sqrt{cd^2 + 2cdex + ce^2x^2}}{6e}$$

input `Int[(d + e*x)^2*(c*d^2 + 2*c*d*e*x + c*e^2*x^2)^(3/2),x]`

output `(c*(d + e*x)^5*Sqrt[c*d^2 + 2*c*d*e*x + c*e^2*x^2])/(6*e)`

Defintions of rubi rules used

rule 17 `Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] := Simp[c*((a + b*x)^(m + 1))/(b*(m + 1))], x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]`

rule 1102 `Int[((d_.) + (e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] := Simp[(a + b*x + c*x^2)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x)^(2*FracPart[p])) Int[(d + e*x)^m*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && EqQ[b^2 - 4*a*c, 0]`

Maple [A] (verified)

Time = 1.00 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.64

| method | result | size |
|---------|---|------|
| risch | $\frac{c(ex+d)^5 \sqrt{c(ex+d)^2}}{6e}$ | 25 |
| default | $\frac{(ex+d)^3 (x^2 c e^2 + 2cdxe + cd^2)^{\frac{3}{2}}}{6e}$ | 35 |
| gosper | $\frac{x(e^5 x^5 + 6d e^4 x^4 + 15d^2 e^3 x^3 + 20d^3 e^2 x^2 + 15d^4 ex + 6d^5)(x^2 c e^2 + 2cdxe + cd^2)^{\frac{3}{2}}}{6(ex+d)^3}$ | 84 |
| orering | $\frac{x(e^5 x^5 + 6d e^4 x^4 + 15d^2 e^3 x^3 + 20d^3 e^2 x^2 + 15d^4 ex + 6d^5)(x^2 c e^2 + 2cdxe + cd^2)^{\frac{3}{2}}}{6(ex+d)^3}$ | 84 |
| trager | $\frac{cx(e^5 x^5 + 6d e^4 x^4 + 15d^2 e^3 x^3 + 20d^3 e^2 x^2 + 15d^4 ex + 6d^5) \sqrt{x^2 c e^2 + 2cdxe + cd^2}}{6ex + 6d}$ | 85 |

input `int((e*x+d)^2*(c*e^2*x^2+2*c*d*e*x+c*d^2)^(3/2),x,method=_RETURNVERBOSE)`

output `1/6*c*(e*x+d)^5*(c*(e*x+d)^2)^(1/2)/e`

Fricas [B] (verification not implemented)Leaf count of result is larger than twice the leaf count of optimal. 91 vs. $2(35) = 70$.

Time = 0.09 (sec) , antiderivative size = 91, normalized size of antiderivative = 2.33

$$\int (d+ex)^2 (cd^2 + 2cdex + ce^2 x^2)^{3/2} dx = \frac{(ce^5 x^6 + 6cde^4 x^5 + 15cd^2 e^3 x^4 + 20cd^3 e^2 x^3 + 15cd^4 ex^2 + 6cd^5 x) \sqrt{ce^2 x^2 + 2cdex + cd^2}}{6(ex+d)}$$

input `integrate((e*x+d)^2*(c*e^2*x^2+2*c*d*e*x+c*d^2)^(3/2),x, algorithm="fricas")`

output `1/6*(c*e^5*x^6 + 6*c*d*e^4*x^5 + 15*c*d^2*e^3*x^4 + 20*c*d^3*e^2*x^3 + 15*c*d^4*e*x^2 + 6*c*d^5*x)*sqrt(c*e^2*x^2 + 2*c*d*e*x + c*d^2)/(e*x + d)`

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 197 vs. $2(36) = 72$.

Time = 0.78 (sec) , antiderivative size = 197, normalized size of antiderivative = 5.05

$$\int (d + ex)^2 (cd^2 + 2cdex + ce^2x^2)^{3/2} dx = \begin{cases} \sqrt{cd^2 + 2cdex + ce^2x^2} \left(\frac{cd^5}{6e} + \frac{5cd^4x}{6} + \frac{5cd^3ex^2}{3} + \frac{5cd^2e^2x^3}{3} + \frac{5cde^3x^4}{6} + \frac{ce^4x^5}{6} \right) & \text{for } ce^2 \neq 0 \\ \frac{d^2(cd^2+2cdex)^{5/2}}{20} + \frac{(cd^2+2cdex)^{7/2}}{14c} + \frac{(cd^2+2cdex)^{9/2}}{36e^2d^2} & \text{for } cde \neq 0 \\ (cd^2)^{3/2} \begin{cases} d^2x & \text{for } e = 0 \\ \frac{(d+ex)^3}{3e} & \text{otherwise} \end{cases} & \text{otherwise} \end{cases}$$

input `integrate((e*x+d)**2*(c*e**2*x**2+2*c*d*e*x+c*d**2)**(3/2),x)`

output `Piecewise((sqrt(c*d**2 + 2*c*d*e*x + c*e**2*x**2)*(c*d**5/(6*e) + 5*c*d**4*x/6 + 5*c*d**3*e*x**2/3 + 5*c*d**2*e**2*x**3/3 + 5*c*d*e**3*x**4/6 + c*e**4*x**5/6), Ne(c*e**2, 0)), ((d**2*(c*d**2 + 2*c*d*e*x)**(5/2)/20 + (c*d**2 + 2*c*d*e*x)**(7/2)/(14*c) + (c*d**2 + 2*c*d*e*x)**(9/2)/(36*c**2*d**2))/(c*d*e), Ne(c*d*e, 0)), ((c*d**2)**(3/2)*Piecewise((d**2*x, Eq(e, 0)), ((d + e*x)**3/(3*e), True)), True))`

Maxima [F(-2)]

Exception generated.

$$\int (d + ex)^2 (cd^2 + 2cdex + ce^2x^2)^{3/2} dx = \text{Exception raised: ValueError}$$

input `integrate((e*x+d)^2*(c*e^2*x^2+2*c*d*e*x+c*d^2)^(3/2),x, algorithm="maxima")`

output

```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 90 vs. $2(35) = 70$.

Time = 0.18 (sec) , antiderivative size = 90, normalized size of antiderivative = 2.31

$$\int (d + ex)^2 (cd^2 + 2cdex + ce^2x^2)^{3/2} dx = \frac{1}{6} \left(3 (ex^2 + 2dx) cd^4 \operatorname{sgn}(ex + d) + \frac{cd^6 \operatorname{sgn}(ex + d)}{e} + 3 (ex^2 + 2dx)^2 cd^2 \operatorname{sgn}(ex + d) + \dots \right)$$

input

```
integrate((e*x+d)^2*(c*e^2*x^2+2*c*d*e*x+c*d^2)^(3/2),x, algorithm="giac")
```

output

```
1/6*(3*(e*x^2 + 2*d*x)*c*d^4*sgn(e*x + d) + c*d^6*sgn(e*x + d)/e + 3*(e*x^
2 + 2*d*x)^2*c*d^2*e*sgn(e*x + d) + (e*x^2 + 2*d*x)^3*c*e^2*sgn(e*x + d))*
sqrt(c)
```

Mupad [F(-1)]

Timed out.

$$\int (d + ex)^2 (cd^2 + 2cdex + ce^2x^2)^{3/2} dx = \int (d + ex)^2 (cd^2 + 2cdex + ce^2x^2)^{3/2} dx$$

input

```
int((d + e*x)^2*(c*d^2 + c*e^2*x^2 + 2*c*d*e*x)^(3/2),x)
```

output

```
int((d + e*x)^2*(c*d^2 + c*e^2*x^2 + 2*c*d*e*x)^(3/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.46

$$\int (d + ex)^2 (cd^2 + 2cdex + ce^2x^2)^{3/2} dx = \frac{\sqrt{c} cx(e^5x^5 + 6de^4x^4 + 15d^2e^3x^3 + 20d^3e^2x^2 + 15d^4ex + 6d^5)}{6}$$

input `int((e*x+d)^2*(c*e^2*x^2+2*c*d*e*x+c*d^2)^(3/2),x)`output `(sqrt(c)*c*x*(6*d**5 + 15*d**4*e*x + 20*d**3*e**2*x**2 + 15*d**2*e**3*x**3 + 6*d*e**4*x**4 + e**5*x**5))/6`

3.218 $\int (cd^2 + 2cdex + ce^2x^2)^{3/2} dx$

| | |
|---|------|
| Optimal result | 1763 |
| Mathematica [A] (verified) | 1763 |
| Rubi [A] (verified) | 1764 |
| Maple [A] (verified) | 1765 |
| Fricas [A] (verification not implemented) | 1765 |
| Sympy [B] (verification not implemented) | 1766 |
| Maxima [F(-2)] | 1767 |
| Giac [A] (verification not implemented) | 1767 |
| Mupad [B] (verification not implemented) | 1768 |
| Reduce [B] (verification not implemented) | 1768 |

Optimal result

Integrand size = 24, antiderivative size = 39

$$\int (cd^2 + 2cdex + ce^2x^2)^{3/2} dx = \frac{c(d+ex)^3 \sqrt{cd^2 + 2cdex + ce^2x^2}}{4e}$$

output `1/4*c*(e*x+d)^3*(c*e^2*x^2+2*c*d*e*x+c*d^2)^(1/2)/e`

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.64

$$\int (cd^2 + 2cdex + ce^2x^2)^{3/2} dx = \frac{(d+ex)(c(d+ex)^2)^{3/2}}{4e}$$

input `Integrate[(c*d^2 + 2*c*d*e*x + c*e^2*x^2)^(3/2),x]`

output `((d + e*x)*(c*(d + e*x)^2)^(3/2))/(4*e)`

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {1079, 17}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (cd^2 + 2cdex + ce^2x^2)^{3/2} dx$$

$$\downarrow 1079$$

$$\frac{\sqrt{cd^2 + 2cdex + ce^2x^2} \int (cxe^2 + cde)^3 dx}{c^2e^3(d + ex)}$$

$$\downarrow 17$$

$$\frac{c(d + ex)^3 \sqrt{cd^2 + 2cdex + ce^2x^2}}{4e}$$

input `Int[(c*d^2 + 2*c*d*e*x + c*e^2*x^2)^(3/2), x]`

output `(c*(d + e*x)^3*Sqrt[c*d^2 + 2*c*d*e*x + c*e^2*x^2])/(4*e)`

Defintions of rubi rules used

rule 17 `Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1))/(b*(m + 1))], x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]`

rule 1079 `Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(a + b*x + c*x^2)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x)^(2*FracPart[p])) Int[(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c, p}, x] && EqQ[b^2 - 4*a*c, 0]`

Maple [A] (verified)

Time = 0.87 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.64

| method | result | size |
|---------|---|------|
| risch | $\frac{c(ex+d)^3 \sqrt{c(ex+d)^2}}{4e}$ | 25 |
| default | $\frac{(x^2 c e^2 + 2cdxe + c d^2)^{\frac{3}{2}} (ex+d)}{4e}$ | 33 |
| gosper | $\frac{x(e^3 x^3 + 4d e^2 x^2 + 6d^2 ex + 4d^3)(x^2 c e^2 + 2cdxe + c d^2)^{\frac{3}{2}}}{4(ex+d)^3}$ | 62 |
| orering | $\frac{x(e^3 x^3 + 4d e^2 x^2 + 6d^2 ex + 4d^3)(x^2 c e^2 + 2cdxe + c d^2)^{\frac{3}{2}}}{4(ex+d)^3}$ | 62 |
| trager | $\frac{cx(e^3 x^3 + 4d e^2 x^2 + 6d^2 ex + 4d^3) \sqrt{x^2 c e^2 + 2cdxe + c d^2}}{4ex + 4d}$ | 63 |

input `int((c*e^2*x^2+2*c*d*e*x+c*d^2)^(3/2),x,method=_RETURNVERBOSE)`

output `1/4*c*(e*x+d)^3*(c*(e*x+d)^2)^(1/2)/e`

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.72

$$\int (cd^2 + 2cdex + ce^2x^2)^{3/2} dx = \frac{(ce^3x^4 + 4cde^2x^3 + 6cd^2ex^2 + 4cd^3x)\sqrt{ce^2x^2 + 2cdex + cd^2}}{4(ex+d)}$$

input `integrate((c*e^2*x^2+2*c*d*e*x+c*d^2)^(3/2),x, algorithm="fricas")`

output `1/4*(c*e^3*x^4 + 4*c*d*e^2*x^3 + 6*c*d^2*e*x^2 + 4*c*d^3*x)*sqrt(c*e^2*x^2 + 2*c*d*e*x + c*d^2)/(e*x + d)`

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 76 vs. $2(36) = 72$.

Time = 0.80 (sec) , antiderivative size = 366, normalized size of antiderivative = 9.38

$$\int (cd^2 + 2cdex + ce^2x^2)^{3/2} dx = cd^2 \left(\begin{cases} \left(\frac{d}{2e} + \frac{x}{2}\right) \sqrt{cd^2 + 2cdex + ce^2x^2} & \text{for } ce^2 \neq 0 \\ \frac{(cd^2 + 2cdex)^{3/2}}{3cde} & \text{for } cde \neq 0 \\ x\sqrt{cd^2} & \text{otherwise} \end{cases} \right) \\ + 2cde \left(\begin{cases} \sqrt{cd^2 + 2cdex + ce^2x^2} \left(-\frac{d^2}{6e^2} + \frac{dx}{6e} + \frac{x^2}{3}\right) & \text{for } ce^2 \neq 0 \\ \frac{-\frac{cd^2(cd^2 + 2cdex)^{3/2}}{3} + \frac{(cd^2 + 2cdex)^{5/2}}{5}}{2c^2d^2e^2} & \text{for } cde \neq 0 \\ \frac{x^2\sqrt{cd^2}}{2} & \text{otherwise} \end{cases} \right) \\ + ce^2 \left(\begin{cases} \sqrt{cd^2 + 2cdex + ce^2x^2} \left(\frac{d^3}{12e^3} - \frac{d^2x}{12e^2} + \frac{dx^2}{12e} + \frac{x^3}{4}\right) & \text{for } ce^2 \neq 0 \\ \frac{\frac{c^2d^4(cd^2 + 2cdex)^{3/2}}{3} - \frac{2cd^2(cd^2 + 2cdex)^{5/2}}{4c^3d^3e^3} + \frac{(cd^2 + 2cdex)^{7/2}}{7}}{3} & \text{for } cde \neq 0 \\ \frac{x^3\sqrt{cd^2}}{3} & \text{otherwise} \end{cases} \right)$$

input `integrate((c**2*x**2+2*c*d*e*x+c*d**2)**(3/2),x)`

output `c*d**2*Piecewise(((d/(2*e) + x/2)*sqrt(c*d**2 + 2*c*d*e*x + c*e**2*x**2), Ne(c*e**2, 0)), ((c*d**2 + 2*c*d*e*x)**(3/2)/(3*c*d*e), Ne(c*d*e, 0)), (x*sqrt(c*d**2), True)) + 2*c*d*e*Piecewise((sqrt(c*d**2 + 2*c*d*e*x + c*e**2*x**2)*(-d**2/(6*e**2) + d*x/(6*e) + x**2/3), Ne(c*e**2, 0)), ((-c*d**2*(c*d**2 + 2*c*d*e*x)**(3/2)/3 + (c*d**2 + 2*c*d*e*x)**(5/2)/5)/(2*c**2*d**2*e**2), Ne(c*d*e, 0)), (x**2*sqrt(c*d**2)/2, True)) + c*e**2*Piecewise((sqrt(c*d**2 + 2*c*d*e*x + c*e**2*x**2)*(d**3/(12*e**3) - d**2*x/(12*e**2) + d*x**2/(12*e) + x**3/4), Ne(c*e**2, 0)), ((c**2*d**4*(c*d**2 + 2*c*d*e*x)**(3/2)/3 - 2*c*d**2*(c*d**2 + 2*c*d*e*x)**(5/2)/5 + (c*d**2 + 2*c*d*e*x)**(7/2)/7)/(4*c**3*d**3*e**3), Ne(c*d*e, 0)), (x**3*sqrt(c*d**2)/3, True))`

Maxima [F(-2)]

Exception generated.

$$\int (cd^2 + 2cdex + ce^2x^2)^{3/2} dx = \text{Exception raised: ValueError}$$

input `integrate((c*e^2*x^2+2*c*d*e*x+c*d^2)^(3/2),x, algorithm="maxima")`

output Exception raised: ValueError >> Computation failed since Maxima requested additional constraints; using the 'assume' command before evaluation *may* help (example of legal syntax is 'assume(e>0)', see `assume?` for more details)Is e

Giac [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.62

$$\int (cd^2 + 2cdex + ce^2x^2)^{3/2} dx = \frac{1}{4} \left(2 (ex^2 + 2dx)cd^2\text{sgn}(ex + d) + \frac{cd^4\text{sgn}(ex + d)}{e} + (ex^2 + 2dx)^2 c\text{sgn}(ex + d) \right) \sqrt{c}$$

input `integrate((c*e^2*x^2+2*c*d*e*x+c*d^2)^(3/2),x, algorithm="giac")`

output `1/4*(2*(e*x^2 + 2*d*x)*c*d^2*sgn(e*x + d) + c*d^4*sgn(e*x + d)/e + (e*x^2 + 2*d*x)^2*c*e*sgn(e*x + d))*sqrt(c)`

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.92

$$\int (cd^2 + 2cdex + ce^2x^2)^{3/2} dx = \frac{(xe^2 + de)(cd^2 + 2cdex + ce^2x^2)^{3/2}}{4e^2}$$

input `int((c*d^2 + c*e^2*x^2 + 2*c*d*e*x)^(3/2),x)`output `((d*e + e^2*x)*(c*d^2 + c*e^2*x^2 + 2*c*d*e*x)^(3/2))/(4*e^2)`**Reduce [B] (verification not implemented)**

Time = 0.21 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.90

$$\int (cd^2 + 2cdex + ce^2x^2)^{3/2} dx = \frac{\sqrt{c}cx(e^3x^3 + 4de^2x^2 + 6d^2ex + 4d^3)}{4}$$

input `int((c*e^2*x^2+2*c*d*e*x+c*d^2)^(3/2),x)`output `(sqrt(c)*c*x*(4*d**3 + 6*d**2*e*x + 4*d*e**2*x**2 + e**3*x**3))/4`

$$3.219 \quad \int \frac{(cd^2 + 2cdex + ce^2x^2)^{3/2}}{(d+ex)^2} dx$$

| | |
|---|------|
| Optimal result | 1769 |
| Mathematica [A] (verified) | 1769 |
| Rubi [A] (verified) | 1770 |
| Maple [A] (verified) | 1771 |
| Fricas [A] (verification not implemented) | 1771 |
| Sympy [A] (verification not implemented) | 1772 |
| Maxima [F(-2)] | 1772 |
| Giac [A] (verification not implemented) | 1773 |
| Mupad [F(-1)] | 1773 |
| Reduce [B] (verification not implemented) | 1773 |

Optimal result

Integrand size = 32, antiderivative size = 37

$$\int \frac{(cd^2 + 2cdex + ce^2x^2)^{3/2}}{(d+ex)^2} dx = \frac{c(d+ex)\sqrt{cd^2 + 2cdex + ce^2x^2}}{2e}$$

output $1/2*c*(e*x+d)*(c*e^2*x^2+2*c*d*e*x+c*d^2)^(1/2)/e$

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.89

$$\int \frac{(cd^2 + 2cdex + ce^2x^2)^{3/2}}{(d+ex)^2} dx = \frac{c^2x(d+ex)(2d+ex)}{2\sqrt{c(d+ex)^2}}$$

input $\text{Integrate}[(c*d^2 + 2*c*d*e*x + c*e^2*x^2)^(3/2)/(d + e*x)^2, x]$

output $(c^2*x*(d + e*x)*(2*d + e*x))/(2*sqrt[c*(d + e*x)^2])$

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.062$, Rules used = {1102, 17}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(cd^2 + 2cdex + ce^2x^2)^{3/2}}{(d + ex)^2} dx$$

$$\downarrow 1102$$

$$\frac{\sqrt{cd^2 + 2cdex + ce^2x^2} \int c^3 e^3 (d + ex) dx}{c^2 e^3 (d + ex)}$$

$$\downarrow 17$$

$$\frac{c(d + ex)\sqrt{cd^2 + 2cdex + ce^2x^2}}{2e}$$

input `Int[(c*d^2 + 2*c*d*e*x + c*e^2*x^2)^(3/2)/(d + e*x)^2,x]`

output `(c*(d + e*x)*Sqrt[c*d^2 + 2*c*d*e*x + c*e^2*x^2])/(2*e)`

Defintions of rubi rules used

rule 17 `Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] := Simp[c*((a + b*x)^(m + 1))/(b*(m + 1))], x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]`

rule 1102 `Int[((d_.) + (e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] := Simp[(a + b*x + c*x^2)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x)^(2*FracPart[p])) Int[(d + e*x)^m*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && EqQ[b^2 - 4*a*c, 0]`

Maple [A] (verified)

Time = 1.27 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.08

| method | result | size |
|---------|---|------|
| gospers | $\frac{x(ex+2d)(x^2ce^2+2cdxe+cd^2)^{\frac{3}{2}}}{2(ex+d)^3}$ | 40 |
| default | $\frac{x(ex+2d)(x^2ce^2+2cdxe+cd^2)^{\frac{3}{2}}}{2(ex+d)^3}$ | 40 |
| orering | $\frac{x(ex+2d)(x^2ce^2+2cdxe+cd^2)^{\frac{3}{2}}}{2(ex+d)^3}$ | 40 |
| trager | $\frac{cx(ex+2d)\sqrt{x^2ce^2+2cdxe+cd^2}}{2ex+2d}$ | 41 |
| risch | $\frac{c\sqrt{c(ex+d)^2}ex^2}{2ex+2d} + \frac{c\sqrt{c(ex+d)^2}dx}{ex+d}$ | 49 |

input `int((c*e^2*x^2+2*c*d*e*x+c*d^2)^(3/2)/(e*x+d)^2,x,method=_RETURNVERBOSE)`

output `1/2*x*(e*x+2*d)*(c*e^2*x^2+2*c*d*e*x+c*d^2)^(3/2)/(e*x+d)^3`

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.16

$$\int \frac{(cd^2 + 2cdex + ce^2x^2)^{3/2}}{(d + ex)^2} dx = \frac{\sqrt{ce^2x^2 + 2cdex + cd^2}(ce^2x^2 + 2cdx)}{2(ex + d)}$$

input `integrate((c*e^2*x^2+2*c*d*e*x+c*d^2)^(3/2)/(e*x+d)^2,x, algorithm="fricas")`

output `1/2*sqrt(c*e^2*x^2 + 2*c*d*e*x + c*d^2)*(c*e*x^2 + 2*c*d*x)/(e*x + d)`

Sympy [A] (verification not implemented)

Time = 0.99 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.97

$$\int \frac{(cd^2 + 2cdex + ce^2x^2)^{3/2}}{(d + ex)^2} dx = c \left(\begin{cases} \left(\frac{d}{2e} + \frac{x}{2}\right) \sqrt{cd^2 + 2cdex + ce^2x^2} & \text{for } ce^2 \neq 0 \\ \frac{(cd^2 + 2cdex)^{3/2}}{3cde} & \text{for } cde \neq 0 \\ x\sqrt{cd^2} & \text{otherwise} \end{cases} \right)$$

input `integrate((c**2*x**2+2*c*d*e*x+c*d**2)**(3/2)/(e*x+d)**2,x)`

output `c*Piecewise(((d/(2*e) + x/2)*sqrt(c*d**2 + 2*c*d*e*x + c*e**2*x**2), Ne(c*e**2, 0)), ((c*d**2 + 2*c*d*e*x)**(3/2)/(3*c*d*e), Ne(c*d*e, 0)), (x*sqrt(c*d**2), True))`

Maxima [F(-2)]

Exception generated.

$$\int \frac{(cd^2 + 2cdex + ce^2x^2)^{3/2}}{(d + ex)^2} dx = \text{Exception raised: RuntimeError}$$

input `integrate((c*e^2*x^2+2*c*d*e*x+c*d^2)^(3/2)/(e*x+d)^2,x, algorithm="maxima")`

output `Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negative exponent.`

Giac [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.16

$$\int \frac{(cd^2 + 2cdex + ce^2x^2)^{3/2}}{(d + ex)^2} dx = \frac{1}{2} \left(cex^2 \operatorname{sgn}(ex + d) + 2cdx \operatorname{sgn}(ex + d) + \frac{cd^2 \operatorname{sgn}(ex + d)}{e} \right) \sqrt{c}$$

input `integrate((c*e^2*x^2+2*c*d*e*x+c*d^2)^(3/2)/(e*x+d)^2,x, algorithm="giac")`

output `1/2*(c*e*x^2*sgn(e*x + d) + 2*c*d*x*sgn(e*x + d) + c*d^2*sgn(e*x + d)/e)*sqrt(c)`

Mupad [F(-1)]

Timed out.

$$\int \frac{(cd^2 + 2cdex + ce^2x^2)^{3/2}}{(d + ex)^2} dx = \int \frac{(cd^2 + 2cdex + ce^2x^2)^{3/2}}{(d + ex)^2} dx$$

input `int((c*d^2 + c*e^2*x^2 + 2*c*d*e*x)^(3/2)/(d + e*x)^2,x)`

output `int((c*d^2 + c*e^2*x^2 + 2*c*d*e*x)^(3/2)/(d + e*x)^2, x)`

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.35

$$\int \frac{(cd^2 + 2cdex + ce^2x^2)^{3/2}}{(d + ex)^2} dx = \frac{\sqrt{c} cx(ex + 2d)}{2}$$

input `int((c*e^2*x^2+2*c*d*e*x+c*d^2)^(3/2)/(e*x+d)^2,x)`

output `(sqrt(c)*c*x*(2*d + e*x))/2`

$$3.220 \quad \int \frac{(cd^2 + 2cdex + ce^2x^2)^{3/2}}{(d+ex)^4} dx$$

| | |
|---|------|
| Optimal result | 1774 |
| Mathematica [A] (verified) | 1774 |
| Rubi [A] (verified) | 1775 |
| Maple [A] (verified) | 1776 |
| Fricas [A] (verification not implemented) | 1776 |
| Sympy [F] | 1776 |
| Maxima [F(-2)] | 1777 |
| Giac [A] (verification not implemented) | 1777 |
| Mupad [F(-1)] | 1778 |
| Reduce [B] (verification not implemented) | 1778 |

Optimal result

Integrand size = 32, antiderivative size = 42

$$\int \frac{(cd^2 + 2cdex + ce^2x^2)^{3/2}}{(d+ex)^4} dx = \frac{c^2(d+ex)\log(d+ex)}{e\sqrt{cd^2 + 2cdex + ce^2x^2}}$$

output `c^2*(e*x+d)*ln(e*x+d)/e/(c*e^2*x^2+2*c*d*e*x+c*d^2)^(1/2)`

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.74

$$\int \frac{(cd^2 + 2cdex + ce^2x^2)^{3/2}}{(d+ex)^4} dx = \frac{c^2(d+ex)\log(d+ex)}{e\sqrt{c(d+ex)^2}}$$

input `Integrate[(c*d^2 + 2*c*d*e*x + c*e^2*x^2)^(3/2)/(d + e*x)^4,x]`

output `(c^2*(d + e*x)*Log[d + e*x])/(e*Sqrt[c*(d + e*x)^2])`

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.062$, Rules used = {1102, 16}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(cd^2 + 2cdex + ce^2x^2)^{3/2}}{(d + ex)^4} dx$$

$$\downarrow 1102$$

$$\frac{\sqrt{cd^2 + 2cdex + ce^2x^2} \int \frac{c^3e^3}{d+ex} dx}{c^2e^3(d + ex)}$$

$$\downarrow 16$$

$$\frac{c\sqrt{cd^2 + 2cdex + ce^2x^2} \log(d + ex)}{e(d + ex)}$$

input `Int[(c*d^2 + 2*c*d*e*x + c*e^2*x^2)^(3/2)/(d + e*x)^4,x]`

output `(c*Sqrt[c*d^2 + 2*c*d*e*x + c*e^2*x^2]*Log[d + e*x])/(e*(d + e*x))`

Defintions of rubi rules used

rule 16 `Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] := Simp[c*(Log[RemoveContent[a + b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]`

rule 1102 `Int[((d_.) + (e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] := Simp[(a + b*x + c*x^2)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x)^(2*FracPart[p])) Int[(d + e*x)^m*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && EqQ[b^2 - 4*a*c, 0]`

Maple [A] (verified)

Time = 1.29 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.71

| method | result | size |
|---------|--|------|
| risch | $\frac{c\sqrt{c(ex+d)^2 \ln(ex+d)}}{(ex+d)e}$ | 30 |
| default | $\frac{(x^2ce^2+2cdxe+cd^2)^{\frac{3}{2}} \ln(ex+d)}{(ex+d)^3e}$ | 40 |

input `int((c*e^2*x^2+2*c*d*e*x+c*d^2)^(3/2)/(e*x+d)^4,x,method=_RETURNVERBOSE)`

output `c/(e*x+d)*(c*(e*x+d)^2)^(1/2)*ln(e*x+d)/e`

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.98

$$\int \frac{(cd^2 + 2cdex + ce^2x^2)^{3/2}}{(d + ex)^4} dx = \frac{\sqrt{ce^2x^2 + 2cdex + cd^2}c \log(ex + d)}{e^2x + de}$$

input `integrate((c*e^2*x^2+2*c*d*e*x+c*d^2)^(3/2)/(e*x+d)^4,x, algorithm="fricas")`

output `sqrt(c*e^2*x^2 + 2*c*d*e*x + c*d^2)*c*log(e*x + d)/(e^2*x + d*e)`

Sympy [F]

$$\int \frac{(cd^2 + 2cdex + ce^2x^2)^{3/2}}{(d + ex)^4} dx = \int \frac{(c(d + ex)^2)^{\frac{3}{2}}}{(d + ex)^4} dx$$

input `integrate((c**2*x**2+2*c*d*e*x+c*d**2)**(3/2)/(e*x+d)**4,x)`

output `Integral((c*(d + e*x)**2)**(3/2)/(d + e*x)**4, x)`

Maxima [F(-2)]

Exception generated.

$$\int \frac{(cd^2 + 2cdex + ce^2x^2)^{3/2}}{(d + ex)^4} dx = \text{Exception raised: RuntimeError}$$

input `integrate((c*e^2*x^2+2*c*d*e*x+c*d^2)^(3/2)/(e*x+d)^4,x, algorithm="maxima")`

output `Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negative exponent.`

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.48

$$\int \frac{(cd^2 + 2cdex + ce^2x^2)^{3/2}}{(d + ex)^4} dx = \frac{c^{3/2} \log(|ex + d|) \operatorname{sgn}(ex + d)}{e}$$

input `integrate((c*e^2*x^2+2*c*d*e*x+c*d^2)^(3/2)/(e*x+d)^4,x, algorithm="giac")`

output `c^(3/2)*log(abs(e*x + d))*sgn(e*x + d)/e`

Mupad [F(-1)]

Timed out.

$$\int \frac{(cd^2 + 2cdex + ce^2x^2)^{3/2}}{(d + ex)^4} dx = \int \frac{(cd^2 + 2cdex + ce^2x^2)^{3/2}}{(d + ex)^4} dx$$

input `int((c*d^2 + c*e^2*x^2 + 2*c*d*e*x)^(3/2)/(d + e*x)^4,x)`

output `int((c*d^2 + c*e^2*x^2 + 2*c*d*e*x)^(3/2)/(d + e*x)^4, x)`

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.31

$$\int \frac{(cd^2 + 2cdex + ce^2x^2)^{3/2}}{(d + ex)^4} dx = \frac{\sqrt{c} \log(ex + d) c}{e}$$

input `int((c*e^2*x^2+2*c*d*e*x+c*d^2)^(3/2)/(e*x+d)^4,x)`

output `(sqrt(c)*log(d + e*x)*c)/e`

$$3.221 \quad \int \frac{(cd^2 + 2cdex + ce^2x^2)^{3/2}}{(d+ex)^6} dx$$

| | |
|---|------|
| Optimal result | 1779 |
| Mathematica [A] (verified) | 1779 |
| Rubi [A] (verified) | 1780 |
| Maple [A] (verified) | 1781 |
| Fricas [A] (verification not implemented) | 1781 |
| Sympy [F] | 1782 |
| Maxima [F(-2)] | 1782 |
| Giac [A] (verification not implemented) | 1782 |
| Mupad [B] (verification not implemented) | 1783 |
| Reduce [B] (verification not implemented) | 1783 |

Optimal result

Integrand size = 32, antiderivative size = 41

$$\int \frac{(cd^2 + 2cdex + ce^2x^2)^{3/2}}{(d+ex)^6} dx = -\frac{c^2}{2e(d+ex)\sqrt{cd^2 + 2cdex + ce^2x^2}}$$

output $-1/2*c^2/e/(e*x+d)/(c*e^2*x^2+2*c*d*e*x+c*d^2)^(1/2)$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.66

$$\int \frac{(cd^2 + 2cdex + ce^2x^2)^{3/2}}{(d+ex)^6} dx = -\frac{(c(d+ex)^2)^{3/2}}{2e(d+ex)^5}$$

input $\text{Integrate}[(c*d^2 + 2*c*d*e*x + c*e^2*x^2)^(3/2)/(d + e*x)^6, x]$

output $-1/2*(c*(d + e*x)^2)^(3/2)/(e*(d + e*x)^5)$

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.95, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.062$, Rules used = {1102, 17}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(cd^2 + 2cdex + ce^2x^2)^{3/2}}{(d + ex)^6} dx$$

$$\downarrow 1102$$

$$\frac{\sqrt{cd^2 + 2cdex + ce^2x^2} \int \frac{c^3e^3}{(d+ex)^3} dx}{c^2e^3(d + ex)}$$

$$\downarrow 17$$

$$-\frac{c\sqrt{cd^2 + 2cdex + ce^2x^2}}{2e(d + ex)^3}$$

input `Int[(c*d^2 + 2*c*d*e*x + c*e^2*x^2)^(3/2)/(d + e*x)^6,x]`

output `-1/2*(c*Sqrt[c*d^2 + 2*c*d*e*x + c*e^2*x^2])/(e*(d + e*x)^3)`

Defintions of rubi rules used

rule 17 `Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] := Simp[c*((a + b*x)^(m + 1))/(b*(m + 1))], x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]`

rule 1102 `Int[((d_.) + (e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] := Simp[(a + b*x + c*x^2)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x)^(2*FracPart[p])) Int[(d + e*x)^m*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && EqQ[b^2 - 4*a*c, 0]`

Maple [A] (verified)

Time = 1.58 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.61

| method | result | size |
|---------|---|------|
| risch | $-\frac{c\sqrt{c(ex+d)^2}}{2(ex+d)^3e}$ | 25 |
| gospers | $-\frac{(x^2ce^2+2cdxe+cd^2)^{\frac{3}{2}}}{2(ex+d)^5e}$ | 35 |
| default | $-\frac{(x^2ce^2+2cdxe+cd^2)^{\frac{3}{2}}}{2(ex+d)^5e}$ | 35 |
| orering | $-\frac{(x^2ce^2+2cdxe+cd^2)^{\frac{3}{2}}}{2(ex+d)^5e}$ | 35 |
| trager | $\frac{c(ex+2d)x\sqrt{x^2ce^2+2cdxe+cd^2}}{2d^2(ex+d)^3}$ | 44 |

input `int((c*e^2*x^2+2*c*d*e*x+c*d^2)^(3/2)/(e*x+d)^6,x,method=_RETURNVERBOSE)`

output `-1/2*c/(e*x+d)^3*(c*(e*x+d)^2)^(1/2)/e`

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.41

$$\int \frac{(cd^2 + 2cdex + ce^2x^2)^{3/2}}{(d + ex)^6} dx = -\frac{\sqrt{ce^2x^2 + 2cdex + cd^2}c}{2(e^4x^3 + 3de^3x^2 + 3d^2e^2x + d^3e)}$$

input `integrate((c*e^2*x^2+2*c*d*e*x+c*d^2)^(3/2)/(e*x+d)^6,x, algorithm="fricas")`

output `-1/2*sqrt(c*e^2*x^2 + 2*c*d*e*x + c*d^2)*c/(e^4*x^3 + 3*d*e^3*x^2 + 3*d^2*e^2*x + d^3*e)`

Sympy [F]

$$\int \frac{(cd^2 + 2cdex + ce^2x^2)^{3/2}}{(d + ex)^6} dx = \int \frac{(c(d + ex)^2)^{3/2}}{(d + ex)^6} dx$$

input `integrate((c*e**2*x**2+2*c*d*e*x+c*d**2)**(3/2)/(e*x+d)**6,x)`

output `Integral((c*(d + e*x)**2)**(3/2)/(d + e*x)**6, x)`

Maxima [F(-2)]

Exception generated.

$$\int \frac{(cd^2 + 2cdex + ce^2x^2)^{3/2}}{(d + ex)^6} dx = \text{Exception raised: RuntimeError}$$

input `integrate((c*e^2*x^2+2*c*d*e*x+c*d^2)^(3/2)/(e*x+d)^6,x, algorithm="maxima")`

output `Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negative exponent.`

Giac [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.51

$$\int \frac{(cd^2 + 2cdex + ce^2x^2)^{3/2}}{(d + ex)^6} dx = -\frac{c^{3/2} \operatorname{sgn}(ex + d)}{2(ex + d)^2 e}$$

input `integrate((c*e^2*x^2+2*c*d*e*x+c*d^2)^(3/2)/(e*x+d)^6,x, algorithm="giac")`

output `-1/2*c^(3/2)*sgn(e*x + d)/((e*x + d)^2*e)`

Mupad [B] (verification not implemented)

Time = 9.44 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.85

$$\int \frac{(cd^2 + 2cdex + ce^2x^2)^{3/2}}{(d + ex)^6} dx = -\frac{c\sqrt{cd^2 + 2cdex + ce^2x^2}}{2e(d + ex)^3}$$

input `int((c*d^2 + c*e^2*x^2 + 2*c*d*e*x)^(3/2)/(d + e*x)^6,x)`output `-(c*(c*d^2 + c*e^2*x^2 + 2*c*d*e*x)^(1/2))/(2*e*(d + e*x)^3)`**Reduce [B] (verification not implemented)**

Time = 0.21 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.63

$$\int \frac{(cd^2 + 2cdex + ce^2x^2)^{3/2}}{(d + ex)^6} dx = -\frac{\sqrt{c}c}{2e(e^2x^2 + 2dex + d^2)}$$

input `int((c*e^2*x^2+2*c*d*e*x+c*d^2)^(3/2)/(e*x+d)^6,x)`output `(- sqrt(c)*c)/(2*e*(d**2 + 2*d*e*x + e**2*x**2))`

$$3.222 \quad \int \frac{(cd^2 + 2cdex + ce^2x^2)^{3/2}}{(d+ex)^8} dx$$

| | |
|---|------|
| Optimal result | 1784 |
| Mathematica [A] (verified) | 1784 |
| Rubi [A] (verified) | 1785 |
| Maple [A] (verified) | 1786 |
| Fricas [B] (verification not implemented) | 1786 |
| Sympy [F] | 1787 |
| Maxima [F(-2)] | 1787 |
| Giac [A] (verification not implemented) | 1787 |
| Mupad [B] (verification not implemented) | 1788 |
| Reduce [B] (verification not implemented) | 1788 |

Optimal result

Integrand size = 32, antiderivative size = 41

$$\int \frac{(cd^2 + 2cdex + ce^2x^2)^{3/2}}{(d+ex)^8} dx = -\frac{c^2}{4e(d+ex)^3\sqrt{cd^2 + 2cdex + ce^2x^2}}$$

output `-1/4*c^2/e/(e*x+d)^3/(c*e^2*x^2+2*c*d*e*x+c*d^2)^(1/2)`

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.66

$$\int \frac{(cd^2 + 2cdex + ce^2x^2)^{3/2}}{(d+ex)^8} dx = -\frac{(c(d+ex)^2)^{3/2}}{4e(d+ex)^7}$$

input `Integrate[(c*d^2 + 2*c*d*e*x + c*e^2*x^2)^(3/2)/(d + e*x)^8,x]`

output `-1/4*(c*(d + e*x)^2)^(3/2)/(e*(d + e*x)^7)`

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.95, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.062$, Rules used = {1102, 17}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(cd^2 + 2cdex + ce^2x^2)^{3/2}}{(d + ex)^8} dx$$

$$\downarrow 1102$$

$$\frac{\sqrt{cd^2 + 2cdex + ce^2x^2} \int \frac{c^3e^3}{(d+ex)^5} dx}{c^2e^3(d + ex)}$$

$$\downarrow 17$$

$$-\frac{c\sqrt{cd^2 + 2cdex + ce^2x^2}}{4e(d + ex)^5}$$

input `Int[(c*d^2 + 2*c*d*e*x + c*e^2*x^2)^(3/2)/(d + e*x)^8,x]`

output `-1/4*(c*Sqrt[c*d^2 + 2*c*d*e*x + c*e^2*x^2])/(e*(d + e*x)^5)`

Defintions of rubi rules used

rule 17 `Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] := Simp[c*((a + b*x)^(m + 1))/(b*(m + 1))], x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]`

rule 1102 `Int[((d_.) + (e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] := Simp[(a + b*x + c*x^2)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x)^(2*FracPart[p])) Int[(d + e*x)^m*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && EqQ[b^2 - 4*a*c, 0]`

Maple [A] (verified)

Time = 1.92 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.61

| method | result | size |
|---------|---|------|
| risch | $-\frac{c\sqrt{c(ex+d)^2}}{4(ex+d)^5e}$ | 25 |
| gospers | $-\frac{(x^2ce^2+2cdxe+cd^2)^{\frac{3}{2}}}{4(ex+d)^7e}$ | 35 |
| default | $-\frac{(x^2ce^2+2cdxe+cd^2)^{\frac{3}{2}}}{4(ex+d)^7e}$ | 35 |
| orering | $-\frac{(x^2ce^2+2cdxe+cd^2)^{\frac{3}{2}}}{4(ex+d)^7e}$ | 35 |
| trager | $\frac{c(e^3x^3+4de^2x^2+6d^2ex+4d^3)x\sqrt{x^2ce^2+2cdxe+cd^2}}{4d^4(ex+d)^5}$ | 66 |

input `int((c*e^2*x^2+2*c*d*e*x+c*d^2)^(3/2)/(e*x+d)^8,x,method=_RETURNVERBOSE)`

output `-1/4*c/(e*x+d)^5*(c*(e*x+d)^2)^(1/2)/e`

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 80 vs. 2(37) = 74.

Time = 0.09 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.95

$$\int \frac{(cd^2 + 2cdex + ce^2x^2)^{3/2}}{(d + ex)^8} dx =$$

$$-\frac{\sqrt{ce^2x^2 + 2cdex + cd^2}c}{4(e^6x^5 + 5de^5x^4 + 10d^2e^4x^3 + 10d^3e^3x^2 + 5d^4e^2x + d^5e)}$$

input `integrate((c*e^2*x^2+2*c*d*e*x+c*d^2)^(3/2)/(e*x+d)^8,x, algorithm="fricas")`

output `-1/4*sqrt(c*e^2*x^2 + 2*c*d*e*x + c*d^2)*c/(e^6*x^5 + 5*d*e^5*x^4 + 10*d^2*e^4*x^3 + 10*d^3*e^3*x^2 + 5*d^4*e^2*x + d^5*e)`

Sympy [F]

$$\int \frac{(cd^2 + 2cdex + ce^2x^2)^{3/2}}{(d + ex)^8} dx = \int \frac{(c(d + ex)^2)^{\frac{3}{2}}}{(d + ex)^8} dx$$

input `integrate((c*e**2*x**2+2*c*d*e*x+c*d**2)**(3/2)/(e*x+d)**8,x)`

output `Integral((c*(d + e*x)**2)**(3/2)/(d + e*x)**8, x)`

Maxima [F(-2)]

Exception generated.

$$\int \frac{(cd^2 + 2cdex + ce^2x^2)^{3/2}}{(d + ex)^8} dx = \text{Exception raised: RuntimeError}$$

input `integrate((c*e^2*x^2+2*c*d*e*x+c*d^2)^(3/2)/(e*x+d)^8,x, algorithm="maxima")`

output `Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negative exponent.`

Giac [A] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.51

$$\int \frac{(cd^2 + 2cdex + ce^2x^2)^{3/2}}{(d + ex)^8} dx = -\frac{c^{\frac{3}{2}} \operatorname{sgn}(ex + d)}{4(ex + d)^4 e}$$

input `integrate((c*e^2*x^2+2*c*d*e*x+c*d^2)^(3/2)/(e*x+d)^8,x, algorithm="giac")`

output `-1/4*c^(3/2)*sgn(e*x + d)/((e*x + d)^4*e)`

Mupad [B] (verification not implemented)

Time = 9.34 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.85

$$\int \frac{(cd^2 + 2cdex + ce^2x^2)^{3/2}}{(d + ex)^8} dx = -\frac{c\sqrt{cd^2 + 2cdex + ce^2x^2}}{4e(d + ex)^5}$$

input `int((c*d^2 + c*e^2*x^2 + 2*c*d*e*x)^(3/2)/(d + e*x)^8,x)`output `-(c*(c*d^2 + c*e^2*x^2 + 2*c*d*e*x)^(1/2))/(4*e*(d + e*x)^5)`**Reduce [B] (verification not implemented)**

Time = 0.21 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.17

$$\int \frac{(cd^2 + 2cdex + ce^2x^2)^{3/2}}{(d + ex)^8} dx = -\frac{\sqrt{c}c}{4e(e^4x^4 + 4de^3x^3 + 6d^2e^2x^2 + 4d^3ex + d^4)}$$

input `int((c*e^2*x^2+2*c*d*e*x+c*d^2)^(3/2)/(e*x+d)^8,x)`output `(- sqrt(c)*c)/(4*e*(d**4 + 4*d**3*e*x + 6*d**2*e**2*x**2 + 4*d*e**3*x**3 + e**4*x**4))`

3.223 $\int (d + ex) (cd^2 + 2cdex + ce^2x^2)^{5/2} dx$

| | |
|---|------|
| Optimal result | 1789 |
| Mathematica [A] (verified) | 1789 |
| Rubi [A] (verified) | 1790 |
| Maple [A] (verified) | 1790 |
| Fricas [B] (verification not implemented) | 1791 |
| Sympy [B] (verification not implemented) | 1792 |
| Maxima [A] (verification not implemented) | 1792 |
| Giac [B] (verification not implemented) | 1793 |
| Mupad [B] (verification not implemented) | 1793 |
| Reduce [B] (verification not implemented) | 1794 |

Optimal result

Integrand size = 30, antiderivative size = 34

$$\int (d + ex) (cd^2 + 2cdex + ce^2x^2)^{5/2} dx = \frac{(cd^2 + 2cdex + ce^2x^2)^{7/2}}{7ce}$$

output $1/7*(c*e^2*x^2+2*c*d*e*x+c*d^2)^{(7/2)}/c/e$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.68

$$\int (d + ex) (cd^2 + 2cdex + ce^2x^2)^{5/2} dx = \frac{(c(d + ex)^2)^{7/2}}{7ce}$$

input `Integrate[(d + e*x)*(c*d^2 + 2*c*d*e*x + c*e^2*x^2)^(5/2),x]`

output $(c*(d + e*x)^2)^{(7/2)}/(7*c*e)$

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.033$, Rules used = {1099}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (d + ex) (cd^2 + 2cdex + ce^2x^2)^{5/2} dx$$

$$\downarrow 1099$$

$$\frac{(cd^2 + 2cdex + ce^2x^2)^{7/2}}{7ce}$$

input `Int[(d + e*x)*(c*d^2 + 2*c*d*e*x + c*e^2*x^2)^(5/2),x]`

output `(c*d^2 + 2*c*d*e*x + c*e^2*x^2)^(7/2)/(7*c*e)`

Defintions of rubi rules used

rule 1099 `Int[((d_) + (e_)*(x_))^(m_)*((a_) + (b_)*(x_) + (c_)*(x_)^2)^(p_), x_Sy
mbol] := Simp[e^m*((a + b*x + c*x^2)^(p + (m + 1)/2)/(c^((m + 1)/2)*(m + 2*
p + 1))), x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[b^2 - 4*a*c, 0] && EqQ[
2*c*d - b*e, 0] && IntegerQ[(m - 1)/2]`

Maple [A] (verified)

Time = 0.90 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.79

| method | result | size |
|----------------|--|------|
| risch | $\frac{c^2(e^2x^2+2cdex+cd^2)\sqrt{c(e^2x^2+2cdex+cd^2)}}{7e}$ | 27 |
| pseudoelliptic | $\frac{c^2(e^2x^2+2cdex+cd^2)\sqrt{c(e^2x^2+2cdex+cd^2)}}{7e}$ | 27 |
| default | $\frac{(e^2x^2+2cdex+cd^2)^{5/2}}{7e}$ | 35 |
| gosper | $\frac{x(e^6x^6+7de^5x^5+21d^2e^4x^4+35d^3e^3x^3+35d^4e^2x^2+21d^5ex+7d^6)(x^2ce^2+2cdex+cd^2)^{5/2}}{7(e^2x^2+2cdex+cd^2)^{5/2}}$ | 95 |
| orering | $\frac{x(e^6x^6+7de^5x^5+21d^2e^4x^4+35d^3e^3x^3+35d^4e^2x^2+21d^5ex+7d^6)(x^2ce^2+2cdex+cd^2)^{5/2}}{7(e^2x^2+2cdex+cd^2)^{5/2}}$ | 95 |
| trager | $\frac{c^2x(e^6x^6+7de^5x^5+21d^2e^4x^4+35d^3e^3x^3+35d^4e^2x^2+21d^5ex+7d^6)\sqrt{x^2ce^2+2cdex+cd^2}}{7ex+7d}$ | 98 |

input `int((e*x+d)*(c*e^2*x^2+2*c*d*e*x+c*d^2)^(5/2),x,method=_RETURNVERBOSE)`

output `1/7*c^2*(e*x+d)^6*(c*(e*x+d)^2)^(1/2)/e`

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 117 vs. $2(30) = 60$.

Time = 0.10 (sec) , antiderivative size = 117, normalized size of antiderivative = 3.44

$$\int (d+ex)(cd^2+2cdex+ce^2x^2)^{5/2} dx = \frac{(c^2e^6x^7+7c^2de^5x^6+21c^2d^2e^4x^5+35c^2d^3e^3x^4+35c^2d^4e^2x^3+21c^2d^5ex^2+7c^2d^6x)\sqrt{c(e^2x^2+2cdex+cd^2)}}{7(ex+d)}$$

input `integrate((e*x+d)*(c*e^2*x^2+2*c*d*e*x+c*d^2)^(5/2),x, algorithm="fricas")`

output `1/7*(c^2*e^6*x^7+7*c^2*d*e^5*x^6+21*c^2*d^2*e^4*x^5+35*c^2*d^3*e^3*x^4+35*c^2*d^4*e^2*x^3+21*c^2*d^5*e*x^2+7*c^2*d^6*x)*sqrt(c*e^2*x^2+2*c*d*e*x+c*d^2)/(e*x+d)`

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 287 vs. $2(29) = 58$.

Time = 0.30 (sec) , antiderivative size = 287, normalized size of antiderivative = 8.44

$$\int (d + ex) (cd^2 + 2cdex + ce^2x^2)^{5/2} dx = \begin{cases} \frac{c^2 d^6 \sqrt{cd^2 + 2cdex + ce^2x^2}}{7e} + \frac{6c^2 d^5 x \sqrt{cd^2 + 2cdex + ce^2x^2}}{7} + \frac{15c^2 d^4 ex^2 \sqrt{cd^2 + 2cdex + ce^2x^2}}{7} + \frac{20c^2 d^3 e^2 x^3 \sqrt{cd^2 + 2cdex + ce^2x^2}}{7} \\ dx (cd^2)^{\frac{5}{2}} \end{cases}$$

input `integrate((e*x+d)*(c*e**2*x**2+2*c*d*e*x+c*d**2)**(5/2),x)`

output `Piecewise((c**2*d**6*sqrt(c*d**2 + 2*c*d*e*x + c*e**2*x**2)/(7*e) + 6*c**2*d**5*x*sqrt(c*d**2 + 2*c*d*e*x + c*e**2*x**2)/7 + 15*c**2*d**4*e*x**2*sqrt(c*d**2 + 2*c*d*e*x + c*e**2*x**2)/7 + 20*c**2*d**3*e**2*x**3*sqrt(c*d**2 + 2*c*d*e*x + c*e**2*x**2)/7 + 15*c**2*d**2*e**3*x**4*sqrt(c*d**2 + 2*c*d*e*x + c*e**2*x**2)/7 + 6*c**2*d*e**4*x**5*sqrt(c*d**2 + 2*c*d*e*x + c*e**2*x**2)/7 + c**2*e**5*x**6*sqrt(c*d**2 + 2*c*d*e*x + c*e**2*x**2)/7, Ne(e, 0)), (d*x*(c*d**2)**(5/2), True))`

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.88

$$\int (d + ex) (cd^2 + 2cdex + ce^2x^2)^{5/2} dx = \frac{(ce^2x^2 + 2cdex + cd^2)^{7/2}}{7ce}$$

input `integrate((e*x+d)*(c*e^2*x^2+2*c*d*e*x+c*d^2)^(5/2),x, algorithm="maxima")`

output `1/7*(c*e^2*x^2 + 2*c*d*e*x + c*d^2)^(7/2)/(c*e)`

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 149 vs. $2(30) = 60$.

Time = 0.18 (sec) , antiderivative size = 149, normalized size of antiderivative = 4.38

$$\int (d + ex) (cd^2 + 2cdex + ce^2x^2)^{5/2} dx = \frac{1}{7} \left(c^2e^6x^7\operatorname{sgn}(ex + d) + 7c^2de^5x^6\operatorname{sgn}(ex + d) + 21c^2d^2e^4x^5\operatorname{sgn}(ex + d) + 35c^2d^3e^3x^4\operatorname{sgn}(ex + d) + 21c^2d^4e^2x^3\operatorname{sgn}(ex + d) + 7c^2d^5ex^2\operatorname{sgn}(ex + d) + 7c^2d^6x\operatorname{sgn}(ex + d) + c^2d^7\operatorname{sgn}(ex + d)/e \right) \operatorname{sqrt}(c)$$

input `integrate((e*x+d)*(c*e^2*x^2+2*c*d*e*x+c*d^2)^(5/2),x, algorithm="giac")`

output `1/7*(c^2*e^6*x^7*sgn(e*x + d) + 7*c^2*d*e^5*x^6*sgn(e*x + d) + 21*c^2*d^2*e^4*x^5*sgn(e*x + d) + 35*c^2*d^3*e^3*x^4*sgn(e*x + d) + 35*c^2*d^4*e^2*x^3*sgn(e*x + d) + 21*c^2*d^5*e*x^2*sgn(e*x + d) + 7*c^2*d^6*x*sgn(e*x + d) + c^2*d^7*sgn(e*x + d)/e)*sqrt(c)`

Mupad [B] (verification not implemented)

Time = 9.08 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.56

$$\int (d + ex) (cd^2 + 2cdex + ce^2x^2)^{5/2} dx = \frac{(c(d + ex)^2)^{7/2}}{7ce}$$

input `int((d + e*x)*(c*d^2 + c*e^2*x^2 + 2*c*d*e*x)^(5/2),x)`

output `(c*(d + e*x)^2)^(7/2)/(7*c*e)`

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 70, normalized size of antiderivative = 2.06

$$\int (d + ex) (cd^2 + 2cdex + ce^2x^2)^{5/2} dx = \frac{\sqrt{c}c^2x(e^6x^6 + 7de^5x^5 + 21d^2e^4x^4 + 35d^3e^3x^3 + 35d^4e^2x^2 + 21d^5ex + 7d^6)}{7}$$

input `int((e*x+d)*(c*e^2*x^2+2*c*d*e*x+c*d^2)^(5/2),x)`

output `(sqrt(c)*c**2*x*(7*d**6 + 21*d**5*e*x + 35*d**4*e**2*x**2 + 35*d**3*e**3*x**3 + 21*d**2*e**4*x**4 + 7*d*e**5*x**5 + e**6*x**6))/7`

$$3.224 \quad \int \frac{(cd^2 + 2cdex + ce^2x^2)^{5/2}}{d + ex} dx$$

| | |
|---|------|
| Optimal result | 1795 |
| Mathematica [A] (verified) | 1795 |
| Rubi [A] (verified) | 1796 |
| Maple [A] (verified) | 1796 |
| Fricas [B] (verification not implemented) | 1797 |
| Sympy [A] (verification not implemented) | 1798 |
| Maxima [A] (verification not implemented) | 1798 |
| Giac [B] (verification not implemented) | 1798 |
| Mupad [B] (verification not implemented) | 1799 |
| Reduce [B] (verification not implemented) | 1799 |

Optimal result

Integrand size = 32, antiderivative size = 31

$$\int \frac{(cd^2 + 2cdex + ce^2x^2)^{5/2}}{d + ex} dx = \frac{(cd^2 + 2cdex + ce^2x^2)^{5/2}}{5e}$$

output `1/5*(c*e^2*x^2+2*c*d*e*x+c*d^2)^(5/2)/e`

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.65

$$\int \frac{(cd^2 + 2cdex + ce^2x^2)^{5/2}}{d + ex} dx = \frac{(c(d + ex)^2)^{5/2}}{5e}$$

input `Integrate[(c*d^2 + 2*c*d*e*x + c*e^2*x^2)^(5/2)/(d + e*x),x]`

output `(c*(d + e*x)^2)^(5/2)/(5*e)`

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.031$, Rules used = {1099}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(cd^2 + 2cdex + ce^2x^2)^{5/2}}{d + ex} dx$$

↓ 1099

$$\frac{(cd^2 + 2cdex + ce^2x^2)^{5/2}}{5e}$$

input `Int[(c*d^2 + 2*c*d*e*x + c*e^2*x^2)^(5/2)/(d + e*x),x]`

output `(c*d^2 + 2*c*d*e*x + c*e^2*x^2)^(5/2)/(5*e)`

Defintions of rubi rules used

rule 1099 `Int[((d_) + (e_)*(x_))^(m_)*((a_) + (b_)*(x_) + (c_)*(x_)^2)^(p_), x_Symbol] := Simp[e^m*((a + b*x + c*x^2)^(p + (m + 1)/2)/(c^((m + 1)/2)*(m + 2*p + 1))), x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[b^2 - 4*a*c, 0] && EqQ[2*c*d - b*e, 0] && IntegerQ[(m - 1)/2]`

Maple [A] (verified)

Time = 1.14 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.87

| method | result | size |
|----------------|--|------|
| risch | $\frac{c^2(e^2x^2+2cdex+cd^2)^{5/2}}{5e}$ | 27 |
| pseudoelliptic | $\frac{c^2(e^2x^2+2cdex+cd^2)^{5/2}}{5e}$ | 27 |
| default | $\frac{(x^2ce^2+2cdex+cd^2)^{5/2}}{5e}$ | 28 |
| gospers | $\frac{x(e^4x^4+5de^3x^3+10d^2e^2x^2+10d^3ex+5d^4)(x^2ce^2+2cdex+cd^2)^{5/2}}{5(e^2x^2+2cdex+cd^2)}$ | 73 |
| orering | $\frac{x(e^4x^4+5de^3x^3+10d^2e^2x^2+10d^3ex+5d^4)(x^2ce^2+2cdex+cd^2)^{5/2}}{5(e^2x^2+2cdex+cd^2)}$ | 73 |
| trager | $\frac{c^2x(e^4x^4+5de^3x^3+10d^2e^2x^2+10d^3ex+5d^4)\sqrt{x^2ce^2+2cdex+cd^2}}{5e^2x^2+2cdex+cd^2}$ | 76 |

input `int((c*e^2*x^2+2*c*d*e*x+c*d^2)^(5/2)/(e*x+d),x,method=_RETURNVERBOSE)`

output `1/5*c^2*(e*x+d)^4*(c*(e*x+d)^2)^(1/2)/e`

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 89 vs. $2(27) = 54$.

Time = 0.09 (sec) , antiderivative size = 89, normalized size of antiderivative = 2.87

$$\int \frac{(cd^2 + 2cdex + ce^2x^2)^{5/2}}{d + ex} dx = \frac{(c^2e^4x^5 + 5c^2de^3x^4 + 10c^2d^2e^2x^3 + 10c^2d^3ex^2 + 5c^2d^4x)\sqrt{ce^2x^2 + 2cdex + cd^2}}{5(e^2x^2 + 2cdex + cd^2)}$$

input `integrate((c*e^2*x^2+2*c*d*e*x+c*d^2)^(5/2)/(e*x+d),x, algorithm="fricas")`

output `1/5*(c^2*e^4*x^5 + 5*c^2*d*e^3*x^4 + 10*c^2*d^2*e^2*x^3 + 10*c^2*d^3*e*x^2 + 5*c^2*d^4*x)*sqrt(c*e^2*x^2 + 2*c*d*e*x + c*d^2)/(e*x + d)`

Sympy [A] (verification not implemented)

Time = 1.53 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.26

$$\int \frac{(cd^2 + 2cdex + ce^2x^2)^{5/2}}{d + ex} dx = \begin{cases} \frac{(cd^2 + 2cdex + ce^2x^2)^{5/2}}{5e} & \text{for } e \neq 0 \\ \frac{x(cd^2)^{5/2}}{d} & \text{otherwise} \end{cases}$$

input `integrate((c*e**2*x**2+2*c*d*e*x+c*d**2)**(5/2)/(e*x+d),x)`

output `Piecewise(((c*d**2 + 2*c*d*e*x + c*e**2*x**2)**(5/2)/(5*e), Ne(e, 0)), (x*(c*d**2)**(5/2)/d, True))`

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.87

$$\int \frac{(cd^2 + 2cdex + ce^2x^2)^{5/2}}{d + ex} dx = \frac{(ce^2x^2 + 2cdex + cd^2)^{5/2}}{5e}$$

input `integrate((c*e^2*x^2+2*c*d*e*x+c*d^2)^(5/2)/(e*x+d),x, algorithm="maxima")`

output `1/5*(c*e^2*x^2 + 2*c*d*e*x + c*d^2)^(5/2)/e`

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 87 vs. 2(27) = 54.

Time = 0.19 (sec) , antiderivative size = 87, normalized size of antiderivative = 2.81

$$\int \frac{(cd^2 + 2cdex + ce^2x^2)^{5/2}}{d + ex} dx = \frac{1}{5} \left(\frac{c^2d^5 \operatorname{sgn}(ex + d)}{e} + (c^2e^4x^5 + 5c^2de^3x^4 + 10c^2d^2e^2x^3 + 10c^2d^3ex^2 + \dots) \right)$$

input `integrate((c*e^2*x^2+2*c*d*e*x+c*d^2)^(5/2)/(e*x+d),x, algorithm="giac")`

output $1/5*(c^2*d^5*\text{sgn}(e*x + d)/e + (c^2*e^4*x^5 + 5*c^2*d*e^3*x^4 + 10*c^2*d^2*e^2*x^3 + 10*c^2*d^3*e*x^2 + 5*c^2*d^4*x)*\text{sgn}(e*x + d))*\text{sqrt}(c)$

Mupad [B] (verification not implemented)

Time = 9.31 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.52

$$\int \frac{(cd^2 + 2cdex + ce^2x^2)^{5/2}}{d + ex} dx = \frac{(c(d + ex)^2)^{5/2}}{5e}$$

input $\text{int}((c*d^2 + c*e^2*x^2 + 2*c*d*e*x)^(5/2)/(d + e*x),x)$

output $(c*(d + e*x)^2)^(5/2)/(5*e)$

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.55

$$\int \frac{(cd^2 + 2cdex + ce^2x^2)^{5/2}}{d + ex} dx = \frac{\sqrt{c}c^2x(e^4x^4 + 5de^3x^3 + 10d^2e^2x^2 + 10d^3ex + 5d^4)}{5}$$

input $\text{int}((c*e^2*x^2+2*c*d*e*x+c*d^2)^(5/2)/(e*x+d),x)$

output $(\text{sqrt}(c)*c**2*x*(5*d**4 + 10*d**3*e*x + 10*d**2*e**2*x**2 + 5*d*e**3*x**3 + e**4*x**4))/5$

$$3.225 \quad \int \frac{(cd^2 + 2cdex + ce^2x^2)^{5/2}}{(d+ex)^3} dx$$

| | |
|---|------|
| Optimal result | 1800 |
| Mathematica [A] (verified) | 1800 |
| Rubi [A] (verified) | 1801 |
| Maple [A] (verified) | 1801 |
| Fricas [B] (verification not implemented) | 1802 |
| Sympy [B] (verification not implemented) | 1803 |
| Maxima [F(-2)] | 1803 |
| Giac [B] (verification not implemented) | 1804 |
| Mupad [F(-1)] | 1804 |
| Reduce [B] (verification not implemented) | 1805 |

Optimal result

Integrand size = 32, antiderivative size = 32

$$\int \frac{(cd^2 + 2cdex + ce^2x^2)^{5/2}}{(d+ex)^3} dx = \frac{c(cd^2 + 2cdex + ce^2x^2)^{3/2}}{3e}$$

output `1/3*c*(c*e^2*x^2+2*c*d*e*x+c*d^2)^(3/2)/e`

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.66

$$\int \frac{(cd^2 + 2cdex + ce^2x^2)^{5/2}}{(d+ex)^3} dx = \frac{c(c(d+ex)^2)^{3/2}}{3e}$$

input `Integrate[(c*d^2 + 2*c*d*e*x + c*e^2*x^2)^(5/2)/(d + e*x)^3,x]`

output `(c*(c*(d + e*x)^2)^(3/2))/(3*e)`

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.031$, Rules used = {1099}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(cd^2 + 2cdex + ce^2x^2)^{5/2}}{(d + ex)^3} dx$$

↓ 1099

$$\frac{c(cd^2 + 2cdex + ce^2x^2)^{3/2}}{3e}$$

input

```
Int[(c*d^2 + 2*c*d*e*x + c*e^2*x^2)^(5/2)/(d + e*x)^3,x]
```

output

```
(c*(c*d^2 + 2*c*d*e*x + c*e^2*x^2)^(3/2))/(3*e)
```

Defintions of rubi rules used

rule 1099

```
Int[((d_) + (e_)*(x_))^(m_)*((a_) + (b_)*(x_) + (c_)*(x_)^2)^(p_), x_Sy
mbol] := Simp[e^m*((a + b*x + c*x^2)^(p + (m + 1)/2)/(c^((m + 1)/2)*(m + 2*
p + 1))), x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[b^2 - 4*a*c, 0] && EqQ[
2*c*d - b*e, 0] && IntegerQ[(m - 1)/2]
```

Maple [A] (verified)

Time = 1.16 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.84

| method | result | size |
|----------------|---|------|
| risch | $\frac{c^2 (ex+d)^2 \sqrt{c(ex+d)^2}}{3e}$ | 27 |
| pseudoelliptic | $\frac{c^2 (ex+d)^2 \sqrt{c(ex+d)^2}}{3e}$ | 27 |
| default | $\frac{(x^2 c e^2 + 2cdxe + c d^2)^{\frac{5}{2}}}{3(ex+d)^2 e}$ | 35 |
| gospers | $\frac{x(e^2 x^2 + 3dex + 3d^2)(x^2 c e^2 + 2cdxe + c d^2)^{\frac{5}{2}}}{3(ex+d)^5}$ | 51 |
| orering | $\frac{x(e^2 x^2 + 3dex + 3d^2)(x^2 c e^2 + 2cdxe + c d^2)^{\frac{5}{2}}}{3(ex+d)^5}$ | 51 |
| trager | $\frac{c^2 x(e^2 x^2 + 3dex + 3d^2) \sqrt{x^2 c e^2 + 2cdxe + c d^2}}{3ex + 3d}$ | 54 |

input `int((c*e^2*x^2+2*c*d*e*x+c*d^2)^(5/2)/(e*x+d)^3,x,method=_RETURNVERBOSE)`

output `1/3*c^2*(e*x+d)^2*(c*(e*x+d)^2)^(1/2)/e`

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 61 vs. 2(28) = 56.

Time = 0.10 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.91

$$\int \frac{(cd^2 + 2cdex + ce^2x^2)^{5/2}}{(d + ex)^3} dx = \frac{(c^2e^2x^3 + 3c^2dex^2 + 3c^2d^2x)\sqrt{ce^2x^2 + 2cdex + cd^2}}{3(ex + d)}$$

input `integrate((c*e^2*x^2+2*c*d*e*x+c*d^2)^(5/2)/(e*x+d)^3,x, algorithm="fricas")`

output `1/3*(c^2*e^2*x^3 + 3*c^2*d*e*x^2 + 3*c^2*d^2*x)*sqrt(c*e^2*x^2 + 2*c*d*e*x + c*d^2)/(e*x + d)`

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 76 vs. $2(29) = 58$.

Time = 1.85 (sec) , antiderivative size = 201, normalized size of antiderivative = 6.28

$$\int \frac{(cd^2 + 2cdex + ce^2x^2)^{5/2}}{(d + ex)^3} dx = c^2d \left(\begin{cases} \left(\frac{d}{2e} + \frac{x}{2} \right) \sqrt{cd^2 + 2cdex + ce^2x^2} & \text{for } ce^2 \neq 0 \\ \frac{(cd^2 + 2cdex)^{3/2}}{3cde} & \text{for } cde \neq 0 \\ x\sqrt{cd^2} & \text{otherwise} \end{cases} \right) \\ + c^2e \left(\begin{cases} \sqrt{cd^2 + 2cdex + ce^2x^2} \left(-\frac{d^2}{6e^2} + \frac{dx}{6e} + \frac{x^2}{3} \right) & \text{for } ce^2 \neq 0 \\ -\frac{cd^2(cd^2 + 2cdex)^{3/2}}{3} + \frac{(cd^2 + 2cdex)^{5/2}}{5} & \text{for } cde \neq 0 \\ \frac{x^2\sqrt{cd^2}}{2} & \text{otherwise} \end{cases} \right)$$

input `integrate((c*e**2*x**2+2*c*d*e*x+c*d**2)**(5/2)/(e*x+d)**3,x)`

output `c**2*d*Piecewise(((d/(2*e) + x/2)*sqrt(c*d**2 + 2*c*d*e*x + c*e**2*x**2), Ne(c*e**2, 0)), ((c*d**2 + 2*c*d*e*x)**(3/2)/(3*c*d*e), Ne(c*d*e, 0)), (x*sqrt(c*d**2), True)) + c**2*e*Piecewise((sqrt(c*d**2 + 2*c*d*e*x + c*e**2*x**2)*(-d**2/(6*e**2) + d*x/(6*e) + x**2/3), Ne(c*e**2, 0)), ((-c*d**2*(c*d**2 + 2*c*d*e*x)**(3/2)/3 + (c*d**2 + 2*c*d*e*x)**(5/2)/5)/(2*c**2*d**2*e**2), Ne(c*d*e, 0)), (x**2*sqrt(c*d**2)/2, True))`

Maxima [F(-2)]

Exception generated.

$$\int \frac{(cd^2 + 2cdex + ce^2x^2)^{5/2}}{(d + ex)^3} dx = \text{Exception raised: RuntimeError}$$

input `integrate((c*e^2*x^2+2*c*d*e*x+c*d^2)^(5/2)/(e*x+d)^3,x, algorithm="maxima")`

output Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negative exponent.

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 69 vs. $2(28) = 56$.

Time = 0.19 (sec) , antiderivative size = 69, normalized size of antiderivative = 2.16

$$\int \frac{(cd^2 + 2cdex + ce^2x^2)^{5/2}}{(d + ex)^3} dx = \frac{1}{3} \left(c^2e^2x^3 \operatorname{sgn}(ex + d) + 3c^2dex^2 \operatorname{sgn}(ex + d) + 3c^2d^2x \operatorname{sgn}(ex + d) + \frac{c^2d^3}{e} \operatorname{sgn}(ex + d) \right) \sqrt{c}$$

input `integrate((c*e^2*x^2+2*c*d*e*x+c*d^2)^(5/2)/(e*x+d)^3,x, algorithm="giac")`

output `1/3*(c^2*e^2*x^3*sgn(e*x + d) + 3*c^2*d*e*x^2*sgn(e*x + d) + 3*c^2*d^2*x*sgn(e*x + d) + c^2*d^3*sgn(e*x + d)/e)*sqrt(c)`

Mupad [F(-1)]

Timed out.

$$\int \frac{(cd^2 + 2cdex + ce^2x^2)^{5/2}}{(d + ex)^3} dx = \int \frac{(cd^2 + 2cdex + ce^2x^2)^{5/2}}{(d + ex)^3} dx$$

input `int((c*d^2 + c*e^2*x^2 + 2*c*d*e*x)^(5/2)/(d + e*x)^3,x)`

output `int((c*d^2 + c*e^2*x^2 + 2*c*d*e*x)^(5/2)/(d + e*x)^3, x)`

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.81

$$\int \frac{(cd^2 + 2cdex + ce^2x^2)^{5/2}}{(d + ex)^3} dx = \frac{\sqrt{c} c^2 x (e^2 x^2 + 3dex + 3d^2)}{3}$$

input `int((c*e^2*x^2+2*c*d*e*x+c*d^2)^(5/2)/(e*x+d)^3,x)`

output `(sqrt(c)*c**2*x*(3*d**2 + 3*d*e*x + e**2*x**2))/3`

$$3.226 \quad \int \frac{(cd^2 + 2cdex + ce^2x^2)^{5/2}}{(d+ex)^5} dx$$

| | |
|---|------|
| Optimal result | 1806 |
| Mathematica [A] (verified) | 1806 |
| Rubi [A] (verified) | 1807 |
| Maple [A] (verified) | 1807 |
| Fricas [A] (verification not implemented) | 1808 |
| Sympy [A] (verification not implemented) | 1809 |
| Maxima [F(-2)] | 1809 |
| Giac [A] (verification not implemented) | 1809 |
| Mupad [F(-1)] | 1810 |
| Reduce [B] (verification not implemented) | 1810 |

Optimal result

Integrand size = 32, antiderivative size = 31

$$\int \frac{(cd^2 + 2cdex + ce^2x^2)^{5/2}}{(d+ex)^5} dx = \frac{c^2 \sqrt{cd^2 + 2cdex + ce^2x^2}}{e}$$

output `c^2*(c*e^2*x^2+2*c*d*e*x+c*d^2)^(1/2)/e`

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.65

$$\int \frac{(cd^2 + 2cdex + ce^2x^2)^{5/2}}{(d+ex)^5} dx = \frac{c^2 \sqrt{c(d+ex)^2}}{e}$$

input `Integrate[(c*d^2 + 2*c*d*e*x + c*e^2*x^2)^(5/2)/(d + e*x)^5,x]`

output `(c^2*sqrt[c*(d + e*x)^2])/e`

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.031$, Rules used = {1099}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(cd^2 + 2cdex + ce^2x^2)^{5/2}}{(d + ex)^5} dx$$

↓ 1099

$$\frac{c^2\sqrt{cd^2 + 2cdex + ce^2x^2}}{e}$$

input `Int[(c*d^2 + 2*c*d*e*x + c*e^2*x^2)^(5/2)/(d + e*x)^5,x]`

output `(c^2*sqrt[c*d^2 + 2*c*d*e*x + c*e^2*x^2])/e`

Defintions of rubi rules used

rule 1099 `Int[((d_) + (e_)*(x_))^(m_)*((a_) + (b_)*(x_) + (c_)*(x_)^2)^(p_), x_Symbol] := Simp[e^m*((a + b*x + c*x^2)^(p + (m + 1)/2)/(c^((m + 1)/2)*(m + 2*p + 1))), x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[b^2 - 4*a*c, 0] && EqQ[2*c*d - b*e, 0] && IntegerQ[(m - 1)/2]`

Maple [A] (verified)

Time = 1.34 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.61

| method | result | size |
|----------------|---|------|
| pseudoelliptic | $\frac{\sqrt{c(ex+d)^2} c^2}{e}$ | 19 |
| risch | $\frac{c^2 \sqrt{c(ex+d)^2} x}{ex+d}$ | 24 |
| default | $\frac{x(x^2 c e^2 + 2cdxe + c d^2)^{\frac{5}{2}}}{(ex+d)^5}$ | 32 |
| orering | $\frac{x(x^2 c e^2 + 2cdxe + c d^2)^{\frac{5}{2}}}{(ex+d)^5}$ | 32 |
| trager | $\frac{c^2 x \sqrt{x^2 c e^2 + 2cdxe + c d^2}}{ex+d}$ | 35 |

input `int((c*e^2*x^2+2*c*d*e*x+c*d^2)^(5/2)/(e*x+d)^5,x,method=_RETURNVERBOSE)`

output `(c*(e*x+d)^2)^(1/2)*c^2/e`

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.10

$$\int \frac{(cd^2 + 2cdex + ce^2x^2)^{5/2}}{(d + ex)^5} dx = \frac{\sqrt{ce^2x^2 + 2cdex + cd^2} c^2 x}{ex + d}$$

input `integrate((c*e^2*x^2+2*c*d*e*x+c*d^2)^(5/2)/(e*x+d)^5,x, algorithm="fricas")`

output `sqrt(c*e^2*x^2 + 2*c*d*e*x + c*d^2)*c^2*x/(e*x + d)`

Sympy [A] (verification not implemented)

Time = 2.95 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.32

$$\int \frac{(cd^2 + 2cdex + ce^2x^2)^{5/2}}{(d + ex)^5} dx = c^2 \left(\begin{cases} \frac{x\sqrt{cd^2}}{d} & \text{for } e = 0 \\ \frac{\sqrt{cd^2 + 2cdex + ce^2x^2}}{e} & \text{otherwise} \end{cases} \right)$$

input `integrate((c**2*x**2+2*c*d*e*x+c*d**2)**(5/2)/(e*x+d)**5,x)`

output `c**2*Piecewise((x*sqrt(c*d**2)/d, Eq(e, 0)), (sqrt(c*d**2 + 2*c*d*e*x + c**2*x**2)/e, True))`

Maxima [F(-2)]

Exception generated.

$$\int \frac{(cd^2 + 2cdex + ce^2x^2)^{5/2}}{(d + ex)^5} dx = \text{Exception raised: RuntimeError}$$

input `integrate((c*e^2*x^2+2*c*d*e*x+c*d^2)^(5/2)/(e*x+d)^5,x, algorithm="maxima")`

output `Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negative exponent.`

Giac [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.97

$$\int \frac{(cd^2 + 2cdex + ce^2x^2)^{5/2}}{(d + ex)^5} dx = \left(c^2 x \operatorname{sgn}(ex + d) + \frac{c^2 d \operatorname{sgn}(ex + d)}{e} \right) \sqrt{c}$$

input `integrate((c*e^2*x^2+2*c*d*e*x+c*d^2)^(5/2)/(e*x+d)^5,x, algorithm="giac")`

output $(c^2*x*sgn(e*x + d) + c^2*d*sgn(e*x + d)/e)*sqrt(c)$

Mupad [F(-1)]

Timed out.

$$\int \frac{(cd^2 + 2cdex + ce^2x^2)^{5/2}}{(d + ex)^5} dx = \int \frac{(cd^2 + 2cdex + ce^2x^2)^{5/2}}{(d + ex)^5} dx$$

input $int((c*d^2 + c*e^2*x^2 + 2*c*d*e*x)^(5/2)/(d + e*x)^5, x)$

output $int((c*d^2 + c*e^2*x^2 + 2*c*d*e*x)^(5/2)/(d + e*x)^5, x)$

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.23

$$\int \frac{(cd^2 + 2cdex + ce^2x^2)^{5/2}}{(d + ex)^5} dx = \sqrt{c}c^2x$$

input $int((c*e^2*x^2+2*c*d*e*x+c*d^2)^(5/2)/(e*x+d)^5, x)$

output $sqrt(c)*c**2*x$

$$3.227 \quad \int \frac{(cd^2 + 2cdex + ce^2x^2)^{5/2}}{(d+ex)^7} dx$$

| | |
|---|------|
| Optimal result | 1811 |
| Mathematica [A] (verified) | 1811 |
| Rubi [A] (verified) | 1812 |
| Maple [A] (verified) | 1812 |
| Fricas [A] (verification not implemented) | 1813 |
| Sympy [F] | 1814 |
| Maxima [F(-2)] | 1814 |
| Giac [A] (verification not implemented) | 1814 |
| Mupad [B] (verification not implemented) | 1815 |
| Reduce [B] (verification not implemented) | 1815 |

Optimal result

Integrand size = 32, antiderivative size = 32

$$\int \frac{(cd^2 + 2cdex + ce^2x^2)^{5/2}}{(d+ex)^7} dx = -\frac{c^3}{e\sqrt{cd^2 + 2cdex + ce^2x^2}}$$

output $-c^3/e/(c*e^2*x^2+2*c*d*e*x+c*d^2)^{(1/2)}$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.78

$$\int \frac{(cd^2 + 2cdex + ce^2x^2)^{5/2}}{(d+ex)^7} dx = -\frac{(c(d+ex)^2)^{5/2}}{e(d+ex)^6}$$

input `Integrate[(c*d^2 + 2*c*d*e*x + c*e^2*x^2)^(5/2)/(d + e*x)^7,x]`

output $-((c*(d + e*x)^2)^(5/2)/(e*(d + e*x)^6))$

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.031$, Rules used = {1099}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(cd^2 + 2cdex + ce^2x^2)^{5/2}}{(d + ex)^7} dx$$

↓ 1099

$$-\frac{c^3}{e\sqrt{cd^2 + 2cdex + ce^2x^2}}$$

input `Int[(c*d^2 + 2*c*d*e*x + c*e^2*x^2)^(5/2)/(d + e*x)^7,x]`

output `-(c^3/(e*Sqrt[c*d^2 + 2*c*d*e*x + c*e^2*x^2]))`

Defintions of rubi rules used

rule 1099 `Int[((d_) + (e_)*(x_))^(m_)*((a_) + (b_)*(x_) + (c_)*(x_)^2)^(p_), x_Symbol] :> Simp[e^m*((a + b*x + c*x^2)^(p + (m + 1)/2)/(c^((m + 1)/2)*(m + 2*p + 1))), x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[b^2 - 4*a*c, 0] && EqQ[2*c*d - b*e, 0] && IntegerQ[(m - 1)/2]`

Maple [A] (verified)

Time = 1.73 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.84

| method | result | size |
|----------------|--|------|
| risch | $-\frac{c^2 \sqrt{c(ex+d)^2}}{(ex+d)^2 e}$ | 27 |
| pseudoelliptic | $-\frac{c^2 \sqrt{c(ex+d)^2}}{(ex+d)^2 e}$ | 27 |
| gosper | $-\frac{(x^2 c e^2 + 2cdxe + cd^2)^{\frac{5}{2}}}{(ex+d)^6 e}$ | 35 |
| default | $-\frac{(x^2 c e^2 + 2cdxe + cd^2)^{\frac{5}{2}}}{(ex+d)^6 e}$ | 35 |
| orering | $-\frac{(x^2 c e^2 + 2cdxe + cd^2)^{\frac{5}{2}}}{(ex+d)^6 e}$ | 35 |
| trager | $\frac{c^2 x \sqrt{x^2 c e^2 + 2cdxe + cd^2}}{d(ex+d)^2}$ | 38 |

input `int((c*e^2*x^2+2*c*d*e*x+c*d^2)^(5/2)/(e*x+d)^7,x,method=_RETURNVERBOSE)`

output `-c^2/(e*x+d)^2*(c*(e*x+d)^2)^(1/2)/e`

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.53

$$\int \frac{(cd^2 + 2cdex + ce^2x^2)^{5/2}}{(d + ex)^7} dx = -\frac{\sqrt{ce^2x^2 + 2cdex + cd^2}c^2}{e^3x^2 + 2de^2x + d^2e}$$

input `integrate((c*e^2*x^2+2*c*d*e*x+c*d^2)^(5/2)/(e*x+d)^7,x, algorithm="fricas")`

output `-sqrt(c*e^2*x^2 + 2*c*d*e*x + c*d^2)*c^2/(e^3*x^2 + 2*d*e^2*x + d^2*e)`

Sympy [F]

$$\int \frac{(cd^2 + 2cdex + ce^2x^2)^{5/2}}{(d + ex)^7} dx = \int \frac{(c(d + ex)^2)^{5/2}}{(d + ex)^7} dx$$

input `integrate((c*e**2*x**2+2*c*d*e*x+c*d**2)**(5/2)/(e*x+d)**7,x)`

output `Integral((c*(d + e*x)**2)**(5/2)/(d + e*x)**7, x)`

Maxima [F(-2)]

Exception generated.

$$\int \frac{(cd^2 + 2cdex + ce^2x^2)^{5/2}}{(d + ex)^7} dx = \text{Exception raised: RuntimeError}$$

input `integrate((c*e^2*x^2+2*c*d*e*x+c*d^2)^(5/2)/(e*x+d)^7,x, algorithm="maxima")`

output `Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negative exponent.`

Giac [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.66

$$\int \frac{(cd^2 + 2cdex + ce^2x^2)^{5/2}}{(d + ex)^7} dx = -\frac{c^{5/2} \operatorname{sgn}(ex + d)}{(ex + d)e}$$

input `integrate((c*e^2*x^2+2*c*d*e*x+c*d^2)^(5/2)/(e*x+d)^7,x, algorithm="giac")`

output `-c^(5/2)*sgn(e*x + d)/((e*x + d)*e)`

Mupad [B] (verification not implemented)

Time = 8.95 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.16

$$\int \frac{(cd^2 + 2cdex + ce^2x^2)^{5/2}}{(d + ex)^7} dx = -\frac{c^2 \sqrt{cd^2 + 2cdex + ce^2x^2}}{e(d + ex)^2}$$

input `int((c*d^2 + c*e^2*x^2 + 2*c*d*e*x)^(5/2)/(d + e*x)^7,x)`output `-(c^2*(c*d^2 + c*e^2*x^2 + 2*c*d*e*x)^(1/2))/(e*(d + e*x)^2)`**Reduce [B] (verification not implemented)**

Time = 0.28 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.53

$$\int \frac{(cd^2 + 2cdex + ce^2x^2)^{5/2}}{(d + ex)^7} dx = \frac{\sqrt{c}c^2x}{d(ex + d)}$$

input `int((c*e^2*x^2+2*c*d*e*x+c*d^2)^(5/2)/(e*x+d)^7,x)`output `(sqrt(c)*c**2*x)/(d*(d + e*x))`

$$3.228 \quad \int \frac{(cd^2 + 2cdex + ce^2x^2)^{5/2}}{(d+ex)^9} dx$$

| | |
|---|------|
| Optimal result | 1816 |
| Mathematica [A] (verified) | 1816 |
| Rubi [A] (verified) | 1817 |
| Maple [A] (verified) | 1817 |
| Fricas [B] (verification not implemented) | 1818 |
| Sympy [F] | 1819 |
| Maxima [F(-2)] | 1819 |
| Giac [A] (verification not implemented) | 1819 |
| Mupad [B] (verification not implemented) | 1820 |
| Reduce [B] (verification not implemented) | 1820 |

Optimal result

Integrand size = 32, antiderivative size = 34

$$\int \frac{(cd^2 + 2cdex + ce^2x^2)^{5/2}}{(d+ex)^9} dx = -\frac{c^4}{3e(cd^2 + 2cdex + ce^2x^2)^{3/2}}$$

output $-1/3*c^4/e/(c*e^2*x^2+2*c*d*e*x+c*d^2)^(3/2)$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.79

$$\int \frac{(cd^2 + 2cdex + ce^2x^2)^{5/2}}{(d+ex)^9} dx = -\frac{(c(d+ex)^2)^{5/2}}{3e(d+ex)^8}$$

input $\text{Integrate}[(c*d^2 + 2*c*d*e*x + c*e^2*x^2)^(5/2)/(d + e*x)^9,x]$

output $-1/3*(c*(d + e*x)^2)^(5/2)/(e*(d + e*x)^8)$

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.031$, Rules used = {1099}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(cd^2 + 2cdex + ce^2x^2)^{5/2}}{(d + ex)^9} dx$$

↓ 1099

$$-\frac{c^4}{3e(cd^2 + 2cdex + ce^2x^2)^{3/2}}$$

input `Int[(c*d^2 + 2*c*d*e*x + c*e^2*x^2)^(5/2)/(d + e*x)^9,x]`

output `-1/3*c^4/(e*(c*d^2 + 2*c*d*e*x + c*e^2*x^2)^(3/2))`

Defintions of rubi rules used

rule 1099

```
Int[((d_) + (e_)*(x_))^(m_)*((a_) + (b_)*(x_) + (c_)*(x_)^2)^(p_), x_Sy
mbol] := Simp[e^m*((a + b*x + c*x^2)^(p + (m + 1)/2)/(c^((m + 1)/2)*(m + 2*
p + 1))), x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[b^2 - 4*a*c, 0] && EqQ[
2*c*d - b*e, 0] && IntegerQ[(m - 1)/2]
```

Maple [A] (verified)

Time = 2.94 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.79

| method | result | size |
|----------------|--|------|
| risch | $-\frac{c^2 \sqrt{c(ex+d)^2}}{3(ex+d)^4 e}$ | 27 |
| pseudoelliptic | $-\frac{c^2 \sqrt{c(ex+d)^2}}{3(ex+d)^4 e}$ | 27 |
| gospers | $-\frac{(x^2 c e^2 + 2 c d x e + c d^2)^{\frac{5}{2}}}{3(ex+d)^8 e}$ | 35 |
| default | $-\frac{(x^2 c e^2 + 2 c d x e + c d^2)^{\frac{5}{2}}}{3(ex+d)^8 e}$ | 35 |
| orering | $-\frac{(x^2 c e^2 + 2 c d x e + c d^2)^{\frac{5}{2}}}{3(ex+d)^8 e}$ | 35 |
| trager | $\frac{c^2 (e^2 x^2 + 3 d e x + 3 d^2) x \sqrt{x^2 c e^2 + 2 c d x e + c d^2}}{3 d^3 (e x + d)^4}$ | 57 |

input `int((c*e^2*x^2+2*c*d*e*x+c*d^2)^(5/2)/(e*x+d)^9,x,method=_RETURNVERBOSE)`

output `-1/3*c^2/(e*x+d)^4*(c*(e*x+d)^2)^(1/2)/e`

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 71 vs. $2(30) = 60$.

Time = 0.08 (sec) , antiderivative size = 71, normalized size of antiderivative = 2.09

$$\int \frac{(cd^2 + 2cdex + ce^2x^2)^{5/2}}{(d + ex)^9} dx = -\frac{\sqrt{ce^2x^2 + 2cdex + cd^2}c^2}{3(e^5x^4 + 4de^4x^3 + 6d^2e^3x^2 + 4d^3e^2x + d^4e)}$$

input `integrate((c*e^2*x^2+2*c*d*e*x+c*d^2)^(5/2)/(e*x+d)^9,x, algorithm="fricas")`

output `-1/3*sqrt(c*e^2*x^2 + 2*c*d*e*x + c*d^2)*c^2/(e^5*x^4 + 4*d*e^4*x^3 + 6*d^2*e^3*x^2 + 4*d^3*e^2*x + d^4*e)`

Sympy [F]

$$\int \frac{(cd^2 + 2cdex + ce^2x^2)^{5/2}}{(d + ex)^9} dx = \int \frac{(c(d + ex)^2)^{5/2}}{(d + ex)^9} dx$$

input `integrate((c*e**2*x**2+2*c*d*e*x+c*d**2)**(5/2)/(e*x+d)**9,x)`

output `Integral((c*(d + e*x)**2)**(5/2)/(d + e*x)**9, x)`

Maxima [F(-2)]

Exception generated.

$$\int \frac{(cd^2 + 2cdex + ce^2x^2)^{5/2}}{(d + ex)^9} dx = \text{Exception raised: RuntimeError}$$

input `integrate((c*e^2*x^2+2*c*d*e*x+c*d^2)^(5/2)/(e*x+d)^9,x, algorithm="maxima")`

output `Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negative exponent.`

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.62

$$\int \frac{(cd^2 + 2cdex + ce^2x^2)^{5/2}}{(d + ex)^9} dx = -\frac{c^{5/2} \operatorname{sgn}(ex + d)}{3(ex + d)^3 e}$$

input `integrate((c*e^2*x^2+2*c*d*e*x+c*d^2)^(5/2)/(e*x+d)^9,x, algorithm="giac")`

output `-1/3*c^(5/2)*sgn(e*x + d)/((e*x + d)^3*e)`

Mupad [B] (verification not implemented)

Time = 8.84 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.09

$$\int \frac{(cd^2 + 2cdex + ce^2x^2)^{5/2}}{(d + ex)^9} dx = -\frac{c^2 \sqrt{cd^2 + 2cdex + ce^2x^2}}{3e(d + ex)^4}$$

input

```
int((c*d^2 + c*e^2*x^2 + 2*c*d*e*x)^(5/2)/(d + e*x)^9,x)
```

output

```
-(c^2*(c*d^2 + c*e^2*x^2 + 2*c*d*e*x)^(1/2))/(3*e*(d + e*x)^4)
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.15

$$\int \frac{(cd^2 + 2cdex + ce^2x^2)^{5/2}}{(d + ex)^9} dx = -\frac{\sqrt{c}c^2}{3e(e^3x^3 + 3de^2x^2 + 3d^2ex + d^3)}$$

input

```
int((c*e^2*x^2+2*c*d*e*x+c*d^2)^(5/2)/(e*x+d)^9,x)
```

output

```
( - sqrt(c)*c**2)/(3*e*(d**3 + 3*d**2*e*x + 3*d*e**2*x**2 + e**3*x**3))
```

3.229 $\int (d + ex)^2 (cd^2 + 2cdex + ce^2x^2)^{5/2} dx$

| | |
|---|------|
| Optimal result | 1821 |
| Mathematica [A] (verified) | 1821 |
| Rubi [A] (verified) | 1822 |
| Maple [A] (verified) | 1823 |
| Fricas [B] (verification not implemented) | 1823 |
| Sympy [B] (verification not implemented) | 1824 |
| Maxima [F(-2)] | 1824 |
| Giac [B] (verification not implemented) | 1825 |
| Mupad [F(-1)] | 1825 |
| Reduce [B] (verification not implemented) | 1826 |

Optimal result

Integrand size = 32, antiderivative size = 41

$$\int (d + ex)^2 (cd^2 + 2cdex + ce^2x^2)^{5/2} dx = \frac{c^2(d + ex)^7 \sqrt{cd^2 + 2cdex + ce^2x^2}}{8e}$$

output $1/8*c^2*(e*x+d)^7*(c*e^2*x^2+2*c*d*e*x+c*d^2)^(1/2)/e$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.68

$$\int (d + ex)^2 (cd^2 + 2cdex + ce^2x^2)^{5/2} dx = \frac{(d + ex) (c(d + ex)^2)^{7/2}}{8ce}$$

input `Integrate[(d + e*x)^2*(c*d^2 + 2*c*d*e*x + c*e^2*x^2)^(5/2),x]`

output $((d + e*x)*(c*(d + e*x)^2)^(7/2))/(8*c*e)$

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.062$, Rules used = {1102, 17}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (d + ex)^2 (cd^2 + 2cdex + ce^2x^2)^{5/2} dx$$

$$\downarrow 1102$$

$$\frac{\sqrt{cd^2 + 2cdex + ce^2x^2} \int c^5 e^5 (d + ex)^7 dx}{c^3 e^5 (d + ex)}$$

$$\downarrow 17$$

$$\frac{c^2 (d + ex)^7 \sqrt{cd^2 + 2cdex + ce^2x^2}}{8e}$$

input `Int[(d + e*x)^2*(c*d^2 + 2*c*d*e*x + c*e^2*x^2)^(5/2),x]`

output `(c^2*(d + e*x)^7*Sqrt[c*d^2 + 2*c*d*e*x + c*e^2*x^2])/(8*e)`

Defintions of rubi rules used

rule 17 `Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] := Simp[c*((a + b*x)^(m + 1))/(b*(m + 1))], x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]`

rule 1102 `Int[((d_.) + (e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] := Simp[(a + b*x + c*x^2)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x)^(2*FracPart[p])) Int[(d + e*x)^m*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && EqQ[b^2 - 4*a*c, 0]`

Maple [A] (verified)

Time = 1.06 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.66

| method | result | size |
|---------|---|------|
| risch | $\frac{c^2(e^x+d)^7 \sqrt{c(e^x+d)^2}}{8e}$ | 27 |
| default | $\frac{(e^x+d)^3 (x^2 c e^2 + 2cdxe + c d^2)^{\frac{5}{2}}}{8e}$ | 35 |
| gospers | $\frac{x(e^7 x^7 + 8d e^6 x^6 + 28d^2 e^5 x^5 + 56d^3 e^4 x^4 + 70d^4 e^3 x^3 + 56d^5 e^2 x^2 + 28d^6 e x + 8d^7) (x^2 c e^2 + 2cdxe + c d^2)^{\frac{5}{2}}}{8(e^x+d)^5}$ | 106 |
| orering | $\frac{x(e^7 x^7 + 8d e^6 x^6 + 28d^2 e^5 x^5 + 56d^3 e^4 x^4 + 70d^4 e^3 x^3 + 56d^5 e^2 x^2 + 28d^6 e x + 8d^7) (x^2 c e^2 + 2cdxe + c d^2)^{\frac{5}{2}}}{8(e^x+d)^5}$ | 106 |
| trager | $\frac{c^2 x (e^7 x^7 + 8d e^6 x^6 + 28d^2 e^5 x^5 + 56d^3 e^4 x^4 + 70d^4 e^3 x^3 + 56d^5 e^2 x^2 + 28d^6 e x + 8d^7) \sqrt{x^2 c e^2 + 2cdxe + c d^2}}{8e^x + 8d}$ | 109 |

input `int((e*x+d)^2*(c*e^2*x^2+2*c*d*e*x+c*d^2)^(5/2),x,method=_RETURNVERBOSE)`

output `1/8*c^2*(e*x+d)^7*(c*(e*x+d)^2)^(1/2)/e`

Fricas [B] (verification not implemented)Leaf count of result is larger than twice the leaf count of optimal. 131 vs. $2(37) = 74$.

Time = 0.10 (sec) , antiderivative size = 131, normalized size of antiderivative = 3.20

$$\int (d+ex)^2 (cd^2+2cdex+ce^2x^2)^{5/2} dx = \frac{(c^2e^7x^8 + 8c^2de^6x^7 + 28c^2d^2e^5x^6 + 56c^2d^3e^4x^5 + 70c^2d^4e^3x^4 + 56c^2d^5e^2x^3 + 28c^2d^6ex^2 + 8c^2d^7)}{8(ex+d)}$$

input `integrate((e*x+d)^2*(c*e^2*x^2+2*c*d*e*x+c*d^2)^(5/2),x, algorithm="fricas")`

output `1/8*(c^2*e^7*x^8 + 8*c^2*d*e^6*x^7 + 28*c^2*d^2*e^5*x^6 + 56*c^2*d^3*e^4*x^5 + 70*c^2*d^4*e^3*x^4 + 56*c^2*d^5*e^2*x^3 + 28*c^2*d^6*e*x^2 + 8*c^2*d^7*x)*sqrt(c*e^2*x^2 + 2*c*d*e*x + c*d^2)/(e*x + d)`

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 241 vs. $2(37) = 74$.

Time = 0.89 (sec) , antiderivative size = 241, normalized size of antiderivative = 5.88

$$\int (d + ex)^2 (cd^2 + 2cdex + ce^2x^2)^{5/2} dx = \begin{cases} \sqrt{cd^2 + 2cdex + ce^2x^2} \left(\frac{c^2d^7}{8e} + \frac{7c^2d^6x}{8} + \frac{21c^2d^5ex^2}{8} + \frac{35c^2d^4e^2x^3}{8} + \frac{35c^2d^3e^3x^4}{8} + \frac{21c^2d^2e^4x^5}{8} + \frac{7c^2d^2e^5x^6}{8} \right) \\ + \frac{d^2(cd^2+2cdex)^{7/2}}{28} + \frac{(cd^2+2cdex)^{9/2}}{18c} + \frac{(cd^2+2cdex)^{11/2}}{44c^2d^2} \\ (cd^2)^{5/2} \left(\begin{cases} d^2x & \text{for } e = 0 \\ \frac{(d+ex)^3}{3e} & \text{otherwise} \end{cases} \right) \end{cases}$$

input `integrate((e*x+d)**2*(c*e**2*x**2+2*c*d*e*x+c*d**2)**(5/2),x)`

output `Piecewise((sqrt(c*d**2 + 2*c*d*e*x + c*e**2*x**2)*(c**2*d**7/(8*e) + 7*c**2*d**6*x/8 + 21*c**2*d**5*e*x**2/8 + 35*c**2*d**4*e**2*x**3/8 + 35*c**2*d**3*e**3*x**4/8 + 21*c**2*d**2*e**4*x**5/8 + 7*c**2*d*e**5*x**6/8 + c**2*e**6*x**7/8), Ne(c*e**2, 0)), ((d**2*(c*d**2 + 2*c*d*e*x)**(7/2)/28 + (c*d**2 + 2*c*d*e*x)**(9/2)/(18*c) + (c*d**2 + 2*c*d*e*x)**(11/2)/(44*c**2*d**2))/(c*d*e), Ne(c*d*e, 0)), ((c*d**2)**(5/2)*Piecewise((d**2*x, Eq(e, 0)), (d + e*x)**3/(3*e), True)), True))`

Maxima [F(-2)]

Exception generated.

$$\int (d + ex)^2 (cd^2 + 2cdex + ce^2x^2)^{5/2} dx = \text{Exception raised: ValueError}$$

input `integrate((e*x+d)^2*(c*e^2*x^2+2*c*d*e*x+c*d^2)^(5/2),x, algorithm="maxima")`

output

```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 127 vs. $2(37) = 74$.

Time = 0.19 (sec) , antiderivative size = 127, normalized size of antiderivative = 3.10

$$\int (d + ex)^2 (cd^2 + 2cdex + ce^2x^2)^{5/2} dx = \frac{1}{8} \left(4(ex^2 + 2dx)c^2d^6 \operatorname{sgn}(ex + d) + \frac{c^2d^8 \operatorname{sgn}(ex + d)}{e} + 6(ex^2 + 2dx)^2 c^2d^4 \operatorname{sgn}(ex + d) \right)$$

input

```
integrate((e*x+d)^2*(c*e^2*x^2+2*c*d*e*x+c*d^2)^(5/2),x, algorithm="giac")
```

output

```
1/8*(4*(e*x^2 + 2*d*x)*c^2*d^6*sgn(e*x + d) + c^2*d^8*sgn(e*x + d)/e + 6*(
e*x^2 + 2*d*x)^2*c^2*d^4*e*sgn(e*x + d) + 4*(e*x^2 + 2*d*x)^3*c^2*d^2*e^2*
sgn(e*x + d) + (e*x^2 + 2*d*x)^4*c^2*e^3*sgn(e*x + d))*sqrt(c)
```

Mupad [F(-1)]

Timed out.

$$\int (d + ex)^2 (cd^2 + 2cdex + ce^2x^2)^{5/2} dx = \int (d + ex)^2 (cd^2 + 2cdex + ce^2x^2)^{5/2} dx$$

input

```
int((d + e*x)^2*(c*d^2 + c*e^2*x^2 + 2*c*d*e*x)^(5/2),x)
```

output

```
int((d + e*x)^2*(c*d^2 + c*e^2*x^2 + 2*c*d*e*x)^(5/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.98

$$\int (d + ex)^2 (cd^2 + 2cdex + ce^2x^2)^{5/2} dx = \frac{\sqrt{c}c^2x(e^7x^7 + 8de^6x^6 + 28d^2e^5x^5 + 56d^3e^4x^4 + 70d^4e^3x^3 + 56d^5e^2x^2 + 28d^6ex + 8d^7)}{8}$$

input `int((e*x+d)^2*(c*e^2*x^2+2*c*d*e*x+c*d^2)^(5/2),x)`output `(sqrt(c)*c**2*x*(8*d**7 + 28*d**6*e*x + 56*d**5*e**2*x**2 + 70*d**4*e**3*x**3 + 56*d**3*e**4*x**4 + 28*d**2*e**5*x**5 + 8*d*e**6*x**6 + e**7*x**7))/8`

3.230 $\int (cd^2 + 2cdex + ce^2x^2)^{5/2} dx$

| | |
|---|------|
| Optimal result | 1827 |
| Mathematica [A] (verified) | 1827 |
| Rubi [A] (verified) | 1828 |
| Maple [A] (verified) | 1829 |
| Fricas [B] (verification not implemented) | 1829 |
| Sympy [B] (verification not implemented) | 1830 |
| Maxima [F(-2)] | 1831 |
| Giac [B] (verification not implemented) | 1831 |
| Mupad [B] (verification not implemented) | 1832 |
| Reduce [B] (verification not implemented) | 1832 |

Optimal result

Integrand size = 24, antiderivative size = 41

$$\int (cd^2 + 2cdex + ce^2x^2)^{5/2} dx = \frac{c^2(d+ex)^5 \sqrt{cd^2 + 2cdex + ce^2x^2}}{6e}$$

output $1/6*c^2*(e*x+d)^5*(c*e^2*x^2+2*c*d*e*x+c*d^2)^(1/2)/e$

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.61

$$\int (cd^2 + 2cdex + ce^2x^2)^{5/2} dx = \frac{(d+ex)(c(d+ex)^2)^{5/2}}{6e}$$

input $\text{Integrate}[(c*d^2 + 2*c*d*e*x + c*e^2*x^2)^(5/2), x]$

output $((d + e*x)*(c*(d + e*x)^2)^(5/2))/(6*e)$

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {1079, 17}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (cd^2 + 2cdex + ce^2x^2)^{5/2} dx$$

$$\downarrow 1079$$

$$\frac{\sqrt{cd^2 + 2cdex + ce^2x^2} \int (cxe^2 + cde)^5 dx}{c^3e^5(d + ex)}$$

$$\downarrow 17$$

$$\frac{c^2(d + ex)^5 \sqrt{cd^2 + 2cdex + ce^2x^2}}{6e}$$

input `Int[(c*d^2 + 2*c*d*e*x + c*e^2*x^2)^(5/2),x]`

output `(c^2*(d + e*x)^5*Sqrt[c*d^2 + 2*c*d*e*x + c*e^2*x^2])/(6*e)`

Defintions of rubi rules used

rule 17 `Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] := Simp[c*((a + b*x)^(m + 1))/(b*(m + 1))], x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]`

rule 1079 `Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] := Simp[(a + b*x + c*x^2)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x)^(2*FracPart[p])) Int[(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c, p}, x] && EqQ[b^2 - 4*a*c, 0]`

Maple [A] (verified)

Time = 0.98 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.66

| method | result | size |
|---------|---|------|
| risch | $\frac{c^2(e^x+d)^5 \sqrt{c(e^x+d)^2}}{6e}$ | 27 |
| default | $\frac{(x^2 c e^2 + 2 c d x e + c d^2)^{\frac{5}{2}} (e^x + d)}{6e}$ | 33 |
| gosper | $\frac{x(e^5 x^5 + 6 d e^4 x^4 + 15 d^2 e^3 x^3 + 20 d^3 e^2 x^2 + 15 d^4 e x + 6 d^5)(x^2 c e^2 + 2 c d x e + c d^2)^{\frac{5}{2}}}{6(e^x + d)^5}$ | 84 |
| orering | $\frac{x(e^5 x^5 + 6 d e^4 x^4 + 15 d^2 e^3 x^3 + 20 d^3 e^2 x^2 + 15 d^4 e x + 6 d^5)(x^2 c e^2 + 2 c d x e + c d^2)^{\frac{5}{2}}}{6(e^x + d)^5}$ | 84 |
| trager | $\frac{c^2 x(e^5 x^5 + 6 d e^4 x^4 + 15 d^2 e^3 x^3 + 20 d^3 e^2 x^2 + 15 d^4 e x + 6 d^5) \sqrt{x^2 c e^2 + 2 c d x e + c d^2}}{6e x + 6d}$ | 87 |

input `int((c*e^2*x^2+2*c*d*e*x+c*d^2)^(5/2),x,method=_RETURNVERBOSE)`

output `1/6*c^2*(e*x+d)^5*(c*(e*x+d)^2)^(1/2)/e`

Fricas [B] (verification not implemented)Leaf count of result is larger than twice the leaf count of optimal. 103 vs. $2(37) = 74$.

Time = 0.09 (sec) , antiderivative size = 103, normalized size of antiderivative = 2.51

$$\int (cd^2 + 2cdex + ce^2x^2)^{5/2} dx = \frac{(c^2e^5x^6 + 6c^2de^4x^5 + 15c^2d^2e^3x^4 + 20c^2d^3e^2x^3 + 15c^2d^4ex^2 + 6c^2d^5x)\sqrt{ce^2x^2 + 2cdex + cd^2}}{6(ex + d)}$$

input `integrate((c*e^2*x^2+2*c*d*e*x+c*d^2)^(5/2),x, algorithm="fricas")`

output `1/6*(c^2*e^5*x^6 + 6*c^2*d*e^4*x^5 + 15*c^2*d^2*e^3*x^4 + 20*c^2*d^3*e^2*x^3 + 15*c^2*d^4*e*x^2 + 6*c^2*d^5*x)*sqrt(c*e^2*x^2 + 2*c*d*e*x + c*d^2)/(e*x + d)`

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 78 vs. $2(37) = 74$.

Time = 1.41 (sec) , antiderivative size = 828, normalized size of antiderivative = 20.20

$$\int (cd^2 + 2cdex + ce^2x^2)^{5/2} dx = \text{Too large to display}$$

input `integrate((c**2*x**2+2*c*d*e*x+c*d**2)**(5/2),x)`

output

```
c**2*d**4*Piecewise(((d/(2*e) + x/2)*sqrt(c*d**2 + 2*c*d*e*x + c*e**2*x**2), Ne(c*e**2, 0)), ((c*d**2 + 2*c*d*e*x)**(3/2)/(3*c*d*e), Ne(c*d*e, 0)), (x*sqrt(c*d**2), True)) + 4*c**2*d**3*e*Piecewise((sqrt(c*d**2 + 2*c*d*e*x + c*e**2*x**2)*(-d**2/(6*e**2) + d*x/(6*e) + x**2/3), Ne(c*e**2, 0)), ((-c*d**2*(c*d**2 + 2*c*d*e*x)**(3/2)/3 + (c*d**2 + 2*c*d*e*x)**(5/2)/5)/(2*c**2*d**2*e**2), Ne(c*d*e, 0)), (x**2*sqrt(c*d**2)/2, True)) + 6*c**2*d**2*e**2*Piecewise((sqrt(c*d**2 + 2*c*d*e*x + c*e**2*x**2)*(d**3/(12*e**3) - d**2*x/(12*e**2) + d*x**2/(12*e) + x**3/4), Ne(c*e**2, 0)), ((c**2*d**4*(c*d**2 + 2*c*d*e*x)**(3/2)/3 - 2*c*d**2*(c*d**2 + 2*c*d*e*x)**(5/2)/5 + (c*d**2 + 2*c*d*e*x)**(7/2)/7)/(4*c**3*d**3*e**3), Ne(c*d*e, 0)), (x**3*sqrt(c*d**2)/3, True)) + 4*c**2*d*e**3*Piecewise((sqrt(c*d**2 + 2*c*d*e*x + c*e**2*x**2)*(-d**4/(20*e**4) + d**3*x/(20*e**3) - d**2*x**2/(20*e**2) + d*x**3/(20*e) + x**4/5), Ne(c*e**2, 0)), ((-c**3*d**6*(c*d**2 + 2*c*d*e*x)**(3/2)/3 + 3*c**2*d**4*(c*d**2 + 2*c*d*e*x)**(5/2)/5 - 3*c*d**2*(c*d**2 + 2*c*d*e*x)**(7/2)/7 + (c*d**2 + 2*c*d*e*x)**(9/2)/9)/(8*c**4*d**4*e**4), Ne(c*d*e, 0)), (x**4*sqrt(c*d**2)/4, True)) + c**2*e**4*Piecewise((sqrt(c*d**2 + 2*c*d*e*x + c*e**2*x**2)*(d**5/(30*e**5) - d**4*x/(30*e**4) + d**3*x**2/(30*e**3) - d**2*x**3/(30*e**2) + d*x**4/(30*e) + x**5/6), Ne(c*e**2, 0)), ((c**4*d**8*(c*d**2 + 2*c*d*e*x)**(3/2)/3 - 4*c**3*d**6*(c*d**2 + 2*c*d*e*x)**(5/2)/5 + 6*c**2*d**4*(c*d**2 + 2*c*d*e*x)**(7/2)/7 - 4*c*d**2*(c...
```

Maxima [F(-2)]

Exception generated.

$$\int (cd^2 + 2cdex + ce^2x^2)^{5/2} dx = \text{Exception raised: ValueError}$$

input `integrate((c*e^2*x^2+2*c*d*e*x+c*d^2)^(5/2),x, algorithm="maxima")`

output Exception raised: ValueError >> Computation failed since Maxima requested additional constraints; using the 'assume' command before evaluation *may* help (example of legal syntax is 'assume(e>0)', see `assume?` for more details)Is e

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 98 vs. 2(37) = 74.

Time = 0.20 (sec) , antiderivative size = 98, normalized size of antiderivative = 2.39

$$\int (cd^2 + 2cdex + ce^2x^2)^{5/2} dx = \frac{1}{6} \left(3(ex^2 + 2dx)c^2d^4\text{sgn}(ex + d) + \frac{c^2d^6\text{sgn}(ex + d)}{e} + 3(ex^2 + 2dx)^2c^2d^2\text{esgn}(ex + d) \right)$$

input `integrate((c*e^2*x^2+2*c*d*e*x+c*d^2)^(5/2),x, algorithm="giac")`

output `1/6*(3*(e*x^2 + 2*d*x)*c^2*d^4*sgn(e*x + d) + c^2*d^6*sgn(e*x + d)/e + 3*(e*x^2 + 2*d*x)^2*c^2*d^2*e*sgn(e*x + d) + (e*x^2 + 2*d*x)^3*c^2*e^2*sgn(e*x + d))*sqrt(c)`

Mupad [B] (verification not implemented)

Time = 8.80 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.88

$$\int (cd^2 + 2cdex + ce^2x^2)^{5/2} dx = \frac{(xe^2 + de)(cd^2 + 2cdex + ce^2x^2)^{5/2}}{6e^2}$$

input `int((c*d^2 + c*e^2*x^2 + 2*c*d*e*x)^(5/2),x)`output `((d*e + e^2*x)*(c*d^2 + c*e^2*x^2 + 2*c*d*e*x)^(5/2))/(6*e^2)`**Reduce [B] (verification not implemented)**

Time = 0.23 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.44

$$\int (cd^2 + 2cdex + ce^2x^2)^{5/2} dx = \frac{\sqrt{c}c^2x(e^5x^5 + 6de^4x^4 + 15d^2e^3x^3 + 20d^3e^2x^2 + 15d^4ex + 6d^5)}{6}$$

input `int((c*e^2*x^2+2*c*d*e*x+c*d^2)^(5/2),x)`output `(sqrt(c)*c**2*x*(6*d**5 + 15*d**4*e*x + 20*d**3*e**2*x**2 + 15*d**2*e**3*x**3 + 6*d*e**4*x**4 + e**5*x**5))/6`

$$3.231 \quad \int \frac{(cd^2 + 2cdex + ce^2x^2)^{5/2}}{(d+ex)^2} dx$$

| | |
|---|------|
| Optimal result | 1833 |
| Mathematica [A] (verified) | 1833 |
| Rubi [A] (verified) | 1834 |
| Maple [A] (verified) | 1835 |
| Fricas [B] (verification not implemented) | 1835 |
| Sympy [B] (verification not implemented) | 1836 |
| Maxima [A] (verification not implemented) | 1837 |
| Giac [B] (verification not implemented) | 1837 |
| Mupad [F(-1)] | 1837 |
| Reduce [B] (verification not implemented) | 1838 |

Optimal result

Integrand size = 32, antiderivative size = 41

$$\int \frac{(cd^2 + 2cdex + ce^2x^2)^{5/2}}{(d+ex)^2} dx = \frac{c^2(d+ex)^3 \sqrt{cd^2 + 2cdex + ce^2x^2}}{4e}$$

output $1/4*c^2*(e*x+d)^3*(c*e^2*x^2+2*c*d*e*x+c*d^2)^(1/2)/e$

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.63

$$\int \frac{(cd^2 + 2cdex + ce^2x^2)^{5/2}}{(d+ex)^2} dx = \frac{c(d+ex)(c(d+ex)^2)^{3/2}}{4e}$$

input `Integrate[(c*d^2 + 2*c*d*e*x + c*e^2*x^2)^(5/2)/(d + e*x)^2,x]`

output $(c*(d + e*x)*(c*(d + e*x)^2)^(3/2))/(4*e)$

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.062$, Rules used = {1102, 17}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(cd^2 + 2cdex + ce^2x^2)^{5/2}}{(d + ex)^2} dx$$

$$\downarrow 1102$$

$$\frac{\sqrt{cd^2 + 2cdex + ce^2x^2} \int c^5 e^5 (d + ex)^3 dx}{c^3 e^5 (d + ex)}$$

$$\downarrow 17$$

$$\frac{c^2 (d + ex)^3 \sqrt{cd^2 + 2cdex + ce^2x^2}}{4e}$$

input `Int[(c*d^2 + 2*c*d*e*x + c*e^2*x^2)^(5/2)/(d + e*x)^2,x]`

output `(c^2*(d + e*x)^3*sqrt[c*d^2 + 2*c*d*e*x + c*e^2*x^2])/(4*e)`

Defintions of rubi rules used

rule 17 `Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] := Simp[c*((a + b*x)^(m + 1))/(b*(m + 1)), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]`

rule 1102 `Int[((d_.) + (e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] := Simp[(a + b*x + c*x^2)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x)^(2*FracPart[p])) Int[(d + e*x)^m*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && EqQ[b^2 - 4*a*c, 0]`

Maple [A] (verified)

Time = 1.30 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.66

| method | result | size |
|---------|--|------|
| risch | $\frac{c^2(e^x+d)^3 \sqrt{c(e^x+d)^2}}{4e}$ | 27 |
| default | $\frac{(x^2 c e^2 + 2 c d x e + c d^2)^{\frac{5}{2}}}{4(e^x+d)e}$ | 35 |
| gospers | $\frac{x(e^3 x^3 + 4 d e^2 x^2 + 6 d^2 e x + 4 d^3)(x^2 c e^2 + 2 c d x e + c d^2)^{\frac{5}{2}}}{4(e^x+d)^5}$ | 62 |
| orering | $\frac{x(e^3 x^3 + 4 d e^2 x^2 + 6 d^2 e x + 4 d^3)(x^2 c e^2 + 2 c d x e + c d^2)^{\frac{5}{2}}}{4(e^x+d)^5}$ | 62 |
| trager | $\frac{c^2 x(e^3 x^3 + 4 d e^2 x^2 + 6 d^2 e x + 4 d^3) \sqrt{x^2 c e^2 + 2 c d x e + c d^2}}{4 e x + 4 d}$ | 65 |

input `int((c*e^2*x^2+2*c*d*e*x+c*d^2)^(5/2)/(e*x+d)^2,x,method=_RETURNVERBOSE)`

output `1/4*c^2*(e*x+d)^3*(c*(e*x+d)^2)^(1/2)/e`

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 75 vs. $2(37) = 74$.

Time = 0.10 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.83

$$\int \frac{(cd^2 + 2cdex + ce^2x^2)^{5/2}}{(d + ex)^2} dx = \frac{(c^2e^3x^4 + 4c^2de^2x^3 + 6c^2d^2ex^2 + 4c^2d^3x)\sqrt{ce^2x^2 + 2cdex + cd^2}}{4(ex + d)}$$

input `integrate((c*e^2*x^2+2*c*d*e*x+c*d^2)^(5/2)/(e*x+d)^2,x, algorithm="fricas")`

output `1/4*(c^2*e^3*x^4 + 4*c^2*d*e^2*x^3 + 6*c^2*d^2*e*x^2 + 4*c^2*d^3*x)*sqrt(c*e^2*x^2 + 2*c*d*e*x + c*d^2)/(e*x + d)`

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 78 vs. $2(37) = 74$.

Time = 1.54 (sec) , antiderivative size = 371, normalized size of antiderivative = 9.05

$$\int \frac{(cd^2 + 2cdex + ce^2x^2)^{5/2}}{(d + ex)^2} dx = c^2d^2 \left(\begin{cases} \left(\frac{d}{2e} + \frac{x}{2}\right) \sqrt{cd^2 + 2cdex + ce^2x^2} & \text{for } ce^2 \neq 0 \\ \frac{(cd^2 + 2cdex)^{3/2}}{3cde} & \text{for } cde \neq 0 \\ x\sqrt{cd^2} & \text{otherwise} \end{cases} \right) \\ + 2c^2de \left(\begin{cases} \sqrt{cd^2 + 2cdex + ce^2x^2} \left(-\frac{d^2}{6e^2} + \frac{dx}{6e} + \frac{x^2}{3}\right) & \text{for } ce^2 \neq 0 \\ -\frac{cd^2(cd^2 + 2cdex)^{3/2}}{3} + \frac{(cd^2 + 2cdex)^{5/2}}{5} & \text{for } cde \neq 0 \\ \frac{x^2\sqrt{cd^2}}{2} & \text{otherwise} \end{cases} \right) \\ + c^2e^2 \left(\begin{cases} \sqrt{cd^2 + 2cdex + ce^2x^2} \left(\frac{d^3}{12e^3} - \frac{d^2x}{12e^2} + \frac{dx^2}{12e} + \frac{x^3}{4}\right) & \text{for } ce^2 \neq 0 \\ \frac{c^2d^4(cd^2 + 2cdex)^{3/2}}{3} - \frac{2cd^2(cd^2 + 2cdex)^{5/2}}{4c^3d^3e^3} + \frac{(cd^2 + 2cdex)^{7/2}}{7} & \text{for } cde \neq 0 \\ \frac{x^3\sqrt{cd^2}}{3} & \text{otherwise} \end{cases} \right)$$

input `integrate((c**2*x**2+2*c*d*e*x+c*d**2)**(5/2)/(e*x+d)**2,x)`

output `c**2*d**2*Piecewise(((d/(2*e) + x/2)*sqrt(c*d**2 + 2*c*d*e*x + c*e**2*x**2), Ne(c*e**2, 0)), ((c*d**2 + 2*c*d*e*x)**(3/2)/(3*c*d*e), Ne(c*d*e, 0)), (x*sqrt(c*d**2), True)) + 2*c**2*d*e*Piecewise((sqrt(c*d**2 + 2*c*d*e*x + c*e**2*x**2)*(-d**2/(6*e**2) + d*x/(6*e) + x**2/3), Ne(c*e**2, 0)), ((-c*d**2*(c*d**2 + 2*c*d*e*x)**(3/2)/3 + (c*d**2 + 2*c*d*e*x)**(5/2)/5)/(2*c**2*d**2*e**2), Ne(c*d*e, 0)), (x**2*sqrt(c*d**2)/2, True)) + c**2*e**2*Piecewise((sqrt(c*d**2 + 2*c*d*e*x + c*e**2*x**2)*(d**3/(12*e**3) - d**2*x/(12*e**2) + d*x**2/(12*e) + x**3/4), Ne(c*e**2, 0)), ((c**2*d**4*(c*d**2 + 2*c*d*e*x)**(3/2)/3 - 2*c*d**2*(c*d**2 + 2*c*d*e*x)**(5/2)/5 + (c*d**2 + 2*c*d*e*x)**(7/2)/7)/(4*c**3*d**3*e**3), Ne(c*d*e, 0)), (x**3*sqrt(c*d**2)/3, True))`

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.85

$$\int \frac{(cd^2 + 2cdex + ce^2x^2)^{5/2}}{(d + ex)^2} dx = \frac{(ce^2x^2 + 2cdex + cd^2)^{5/2}}{4(e^2x + de)}$$

input `integrate((c*e^2*x^2+2*c*d*e*x+c*d^2)^(5/2)/(e*x+d)^2,x, algorithm="maxima")`

output `1/4*(c*e^2*x^2 + 2*c*d*e*x + c*d^2)^(5/2)/(e^2*x + d*e)`

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 89 vs. 2(37) = 74.

Time = 0.17 (sec) , antiderivative size = 89, normalized size of antiderivative = 2.17

$$\int \frac{(cd^2 + 2cdex + ce^2x^2)^{5/2}}{(d + ex)^2} dx = \frac{1}{4} \left(c^2e^3x^4\operatorname{sgn}(ex + d) + 4c^2de^2x^3\operatorname{sgn}(ex + d) + 6c^2d^2ex^2\operatorname{sgn}(ex + d) + \dots \right)$$

input `integrate((c*e^2*x^2+2*c*d*e*x+c*d^2)^(5/2)/(e*x+d)^2,x, algorithm="giac")`

output `1/4*(c^2*e^3*x^4*sgn(e*x + d) + 4*c^2*d*e^2*x^3*sgn(e*x + d) + 6*c^2*d^2*e*x^2*sgn(e*x + d) + 4*c^2*d^3*x*sgn(e*x + d) + c^2*d^4*sgn(e*x + d)/e)*sqrt(c)`

Mupad [F(-1)]

Timed out.

$$\int \frac{(cd^2 + 2cdex + ce^2x^2)^{5/2}}{(d + ex)^2} dx = \int \frac{(cd^2 + 2cdex + ce^2x^2)^{5/2}}{(d + ex)^2} dx$$

input `int((c*d^2 + c*e^2*x^2 + 2*c*d*e*x)^(5/2)/(d + e*x)^2,x)`

output `int((c*d^2 + c*e^2*x^2 + 2*c*d*e*x)^(5/2)/(d + e*x)^2, x)`

Reduce [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.90

$$\int \frac{(cd^2 + 2cdex + ce^2x^2)^{5/2}}{(d + ex)^2} dx = \frac{\sqrt{c}c^2x(e^3x^3 + 4de^2x^2 + 6d^2ex + 4d^3)}{4}$$

input `int((c*e^2*x^2+2*c*d*e*x+c*d^2)^(5/2)/(e*x+d)^2,x)`

output `(sqrt(c)*c**2*x*(4*d**3 + 6*d**2*e*x + 4*d*e**2*x**2 + e**3*x**3))/4`

$$3.232 \quad \int \frac{(cd^2 + 2cdex + ce^2x^2)^{5/2}}{(d+ex)^4} dx$$

| | |
|---|------|
| Optimal result | 1839 |
| Mathematica [A] (verified) | 1839 |
| Rubi [A] (verified) | 1840 |
| Maple [A] (verified) | 1841 |
| Fricas [A] (verification not implemented) | 1841 |
| Sympy [A] (verification not implemented) | 1842 |
| Maxima [F(-2)] | 1842 |
| Giac [A] (verification not implemented) | 1843 |
| Mupad [F(-1)] | 1843 |
| Reduce [B] (verification not implemented) | 1843 |

Optimal result

Integrand size = 32, antiderivative size = 39

$$\int \frac{(cd^2 + 2cdex + ce^2x^2)^{5/2}}{(d+ex)^4} dx = \frac{c^2(d+ex)\sqrt{cd^2 + 2cdex + ce^2x^2}}{2e}$$

output $1/2*c^2*(e*x+d)*(c*e^2*x^2+2*c*d*e*x+c*d^2)^(1/2)/e$

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.85

$$\int \frac{(cd^2 + 2cdex + ce^2x^2)^{5/2}}{(d+ex)^4} dx = \frac{c^3x(d+ex)(2d+ex)}{2\sqrt{c(d+ex)^2}}$$

input $\text{Integrate}[(c*d^2 + 2*c*d*e*x + c*e^2*x^2)^(5/2)/(d + e*x)^4, x]$

output $(c^3*x*(d + e*x)*(2*d + e*x))/(2*sqrt[c*(d + e*x)^2])$

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.062$, Rules used = {1102, 17}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(cd^2 + 2cdex + ce^2x^2)^{5/2}}{(d + ex)^4} dx$$

$$\downarrow 1102$$

$$\frac{\sqrt{cd^2 + 2cdex + ce^2x^2} \int c^5 e^5 (d + ex) dx}{c^3 e^5 (d + ex)}$$

$$\downarrow 17$$

$$\frac{c^2 (d + ex) \sqrt{cd^2 + 2cdex + ce^2x^2}}{2e}$$

input `Int[(c*d^2 + 2*c*d*e*x + c*e^2*x^2)^(5/2)/(d + e*x)^4,x]`

output `(c^2*(d + e*x)*Sqrt[c*d^2 + 2*c*d*e*x + c*e^2*x^2])/(2*e)`

Defintions of rubi rules used

rule 17 `Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] := Simp[c*((a + b*x)^(m + 1))/(b*(m + 1)), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]`

rule 1102 `Int[((d_.) + (e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] := Simp[(a + b*x + c*x^2)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x)^(2*FracPart[p])) Int[(d + e*x)^m*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && EqQ[b^2 - 4*a*c, 0]`

Maple [A] (verified)

Time = 1.48 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.03

| method | result | size |
|---------|---|------|
| gospers | $\frac{x(ex+2d)(x^2ce^2+2cdxe+cd^2)^{\frac{5}{2}}}{2(ex+d)^5}$ | 40 |
| default | $\frac{x(ex+2d)(x^2ce^2+2cdxe+cd^2)^{\frac{5}{2}}}{2(ex+d)^5}$ | 40 |
| orering | $\frac{x(ex+2d)(x^2ce^2+2cdxe+cd^2)^{\frac{5}{2}}}{2(ex+d)^5}$ | 40 |
| trager | $\frac{c^2x(ex+2d)\sqrt{x^2ce^2+2cdxe+cd^2}}{2ex+2d}$ | 43 |
| risch | $\frac{c^2\sqrt{c(ex+d)^2}ex^2}{2ex+2d} + \frac{c^2\sqrt{c(ex+d)^2}dx}{ex+d}$ | 53 |

input `int((c*e^2*x^2+2*c*d*e*x+c*d^2)^(5/2)/(e*x+d)^4,x,method=_RETURNVERBOSE)`

output `1/2*x*(e*x+2*d)*(c*e^2*x^2+2*c*d*e*x+c*d^2)^(5/2)/(e*x+d)^5`

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.21

$$\int \frac{(cd^2 + 2cdex + ce^2x^2)^{5/2}}{(d + ex)^4} dx = \frac{(c^2ex^2 + 2c^2dx)\sqrt{ce^2x^2 + 2cdex + cd^2}}{2(ex + d)}$$

input `integrate((c*e^2*x^2+2*c*d*e*x+c*d^2)^(5/2)/(e*x+d)^4,x, algorithm="fricas")`

output `1/2*(c^2*e*x^2 + 2*c^2*d*x)*sqrt(c*e^2*x^2 + 2*c*d*e*x + c*d^2)/(e*x + d)`

Sympy [A] (verification not implemented)

Time = 2.02 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.92

$$\int \frac{(cd^2 + 2cdex + ce^2x^2)^{5/2}}{(d + ex)^4} dx = c^2 \left(\begin{cases} \left(\frac{d}{2e} + \frac{x}{2}\right) \sqrt{cd^2 + 2cdex + ce^2x^2} & \text{for } ce^2 \neq 0 \\ \frac{(cd^2 + 2cdex)^{3/2}}{3cde} & \text{for } cde \neq 0 \\ x\sqrt{cd^2} & \text{otherwise} \end{cases} \right)$$

input `integrate((c**2*x**2+2*c*d*e*x+c*d**2)**(5/2)/(e*x+d)**4,x)`

output `c**2*Piecewise(((d/(2*e) + x/2)*sqrt(c*d**2 + 2*c*d*e*x + c*e**2*x**2), Ne(c*e**2, 0)), ((c*d**2 + 2*c*d*e*x)**(3/2)/(3*c*d*e), Ne(c*d*e, 0)), (x*sqrt(c*d**2), True))`

Maxima [F(-2)]

Exception generated.

$$\int \frac{(cd^2 + 2cdex + ce^2x^2)^{5/2}}{(d + ex)^4} dx = \text{Exception raised: RuntimeError}$$

input `integrate((c*e^2*x^2+2*c*d*e*x+c*d^2)^(5/2)/(e*x+d)^4,x, algorithm="maxima")`

output `Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negative exponent.`

Giac [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.26

$$\int \frac{(cd^2 + 2cdex + ce^2x^2)^{5/2}}{(d + ex)^4} dx = \frac{1}{2} \left(c^2ex^2\operatorname{sgn}(ex + d) + 2c^2dx\operatorname{sgn}(ex + d) + \frac{c^2d^2\operatorname{sgn}(ex + d)}{e} \right) \sqrt{c}$$

input `integrate((c*e^2*x^2+2*c*d*e*x+c*d^2)^(5/2)/(e*x+d)^4,x, algorithm="giac")`

output `1/2*(c^2*e*x^2*sgn(e*x + d) + 2*c^2*d*x*sgn(e*x + d) + c^2*d^2*sgn(e*x + d)/e)*sqrt(c)`

Mupad [F(-1)]

Timed out.

$$\int \frac{(cd^2 + 2cdex + ce^2x^2)^{5/2}}{(d + ex)^4} dx = \int \frac{(cd^2 + 2cdex + ce^2x^2)^{5/2}}{(d + ex)^4} dx$$

input `int((c*d^2 + c*e^2*x^2 + 2*c*d*e*x)^(5/2)/(d + e*x)^4,x)`

output `int((c*d^2 + c*e^2*x^2 + 2*c*d*e*x)^(5/2)/(d + e*x)^4, x)`

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.38

$$\int \frac{(cd^2 + 2cdex + ce^2x^2)^{5/2}}{(d + ex)^4} dx = \frac{\sqrt{c}c^2x(ex + 2d)}{2}$$

input `int((c*e^2*x^2+2*c*d*e*x+c*d^2)^(5/2)/(e*x+d)^4,x)`

output `(sqrt(c)*c**2*x*(2*d + e*x))/2`

$$3.233 \quad \int \frac{(cd^2 + 2cdex + ce^2x^2)^{5/2}}{(d+ex)^6} dx$$

| | |
|---|------|
| Optimal result | 1844 |
| Mathematica [A] (verified) | 1844 |
| Rubi [A] (verified) | 1845 |
| Maple [A] (verified) | 1846 |
| Fricas [A] (verification not implemented) | 1846 |
| Sympy [F] | 1846 |
| Maxima [F(-2)] | 1847 |
| Giac [A] (verification not implemented) | 1847 |
| Mupad [F(-1)] | 1848 |
| Reduce [B] (verification not implemented) | 1848 |

Optimal result

Integrand size = 32, antiderivative size = 42

$$\int \frac{(cd^2 + 2cdex + ce^2x^2)^{5/2}}{(d+ex)^6} dx = \frac{c^3(d+ex)\log(d+ex)}{e\sqrt{cd^2 + 2cdex + ce^2x^2}}$$

output `c^3*(e*x+d)*ln(e*x+d)/e/(c*e^2*x^2+2*c*d*e*x+c*d^2)^(1/2)`

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.74

$$\int \frac{(cd^2 + 2cdex + ce^2x^2)^{5/2}}{(d+ex)^6} dx = \frac{c^3(d+ex)\log(d+ex)}{e\sqrt{c(d+ex)^2}}$$

input `Integrate[(c*d^2 + 2*c*d*e*x + c*e^2*x^2)^(5/2)/(d + e*x)^6,x]`

output `(c^3*(d + e*x)*Log[d + e*x])/(e*Sqrt[c*(d + e*x)^2])`

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.05, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.062$, Rules used = {1102, 16}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(cd^2 + 2cdex + ce^2x^2)^{5/2}}{(d + ex)^6} dx$$

$$\downarrow 1102$$

$$\frac{\sqrt{cd^2 + 2cdex + ce^2x^2} \int \frac{c^5 e^5}{d+ex} dx}{c^3 e^5 (d + ex)}$$

$$\downarrow 16$$

$$\frac{c^2 \sqrt{cd^2 + 2cdex + ce^2x^2} \log(d + ex)}{e(d + ex)}$$

input `Int[(c*d^2 + 2*c*d*e*x + c*e^2*x^2)^(5/2)/(d + e*x)^6,x]`

output `(c^2*Sqrt[c*d^2 + 2*c*d*e*x + c*e^2*x^2]*Log[d + e*x])/(e*(d + e*x))`

Defintions of rubi rules used

rule 16 `Int[((d_.)/((a_.) + (b_.)*(x_)), x_Symbol] := Simp[c*(Log[RemoveContent[a + b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]`

rule 1102 `Int[((d_.) + (e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] := Simp[(a + b*x + c*x^2)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x)^(2*FracPart[p])) Int[(d + e*x)^m*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && EqQ[b^2 - 4*a*c, 0]`

Maple [A] (verified)

Time = 1.89 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.76

| method | result | size |
|---------|---|------|
| risch | $\frac{c^2 \sqrt{c(ex+d)^2} \ln(ex+d)}{(ex+d)e}$ | 32 |
| default | $\frac{(x^2 c e^2 + 2cdxe + cd^2)^{\frac{5}{2}} \ln(ex+d)}{(ex+d)^5 e}$ | 40 |

input `int((c*e^2*x^2+2*c*d*e*x+c*d^2)^(5/2)/(e*x+d)^6,x,method=_RETURNVERBOSE)`

output `c^2/(e*x+d)*(c*(e*x+d)^2)^(1/2)*ln(e*x+d)/e`

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.02

$$\int \frac{(cd^2 + 2cdex + ce^2x^2)^{5/2}}{(d + ex)^6} dx = \frac{\sqrt{ce^2x^2 + 2cdex + cd^2} c^2 \log(ex + d)}{e^2x + de}$$

input `integrate((c*e^2*x^2+2*c*d*e*x+c*d^2)^(5/2)/(e*x+d)^6,x, algorithm="fricas")`

output `sqrt(c*e^2*x^2 + 2*c*d*e*x + c*d^2)*c^2*log(e*x + d)/(e^2*x + d*e)`

Sympy [F]

$$\int \frac{(cd^2 + 2cdex + ce^2x^2)^{5/2}}{(d + ex)^6} dx = \int \frac{(c(d + ex)^2)^{\frac{5}{2}}}{(d + ex)^6} dx$$

input `integrate((c**2*x**2+2*c*d*e*x+c*d**2)**(5/2)/(e*x+d)**6,x)`

output `Integral((c*(d + e*x)**2)**(5/2)/(d + e*x)**6, x)`

Maxima [F(-2)]

Exception generated.

$$\int \frac{(cd^2 + 2cdex + ce^2x^2)^{5/2}}{(d + ex)^6} dx = \text{Exception raised: RuntimeError}$$

input `integrate((c*e^2*x^2+2*c*d*e*x+c*d^2)^(5/2)/(e*x+d)^6,x, algorithm="maxima")`

output `Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negative exponent.`

Giac [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.48

$$\int \frac{(cd^2 + 2cdex + ce^2x^2)^{5/2}}{(d + ex)^6} dx = \frac{c^{5/2} \log(|ex + d|) \operatorname{sgn}(ex + d)}{e}$$

input `integrate((c*e^2*x^2+2*c*d*e*x+c*d^2)^(5/2)/(e*x+d)^6,x, algorithm="giac")`

output `c^(5/2)*log(abs(e*x + d))*sgn(e*x + d)/e`

Mupad [F(-1)]

Timed out.

$$\int \frac{(cd^2 + 2cdex + ce^2x^2)^{5/2}}{(d + ex)^6} dx = \int \frac{(cd^2 + 2cdex + ce^2x^2)^{5/2}}{(d + ex)^6} dx$$

input `int((c*d^2 + c*e^2*x^2 + 2*c*d*e*x)^(5/2)/(d + e*x)^6,x)`output `int((c*d^2 + c*e^2*x^2 + 2*c*d*e*x)^(5/2)/(d + e*x)^6, x)`**Reduce [B] (verification not implemented)**

Time = 0.21 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.36

$$\int \frac{(cd^2 + 2cdex + ce^2x^2)^{5/2}}{(d + ex)^6} dx = \frac{\sqrt{c} \log(ex + d) c^2}{e}$$

input `int((c*e^2*x^2+2*c*d*e*x+c*d^2)^(5/2)/(e*x+d)^6,x)`output `(sqrt(c)*log(d + e*x)*c**2)/e`

$$3.234 \quad \int \frac{(cd^2 + 2cdex + ce^2x^2)^{5/2}}{(d+ex)^8} dx$$

| | |
|---|------|
| Optimal result | 1849 |
| Mathematica [A] (verified) | 1849 |
| Rubi [A] (verified) | 1850 |
| Maple [A] (verified) | 1851 |
| Fricas [A] (verification not implemented) | 1851 |
| Sympy [F] | 1852 |
| Maxima [F(-2)] | 1852 |
| Giac [A] (verification not implemented) | 1852 |
| Mupad [B] (verification not implemented) | 1853 |
| Reduce [B] (verification not implemented) | 1853 |

Optimal result

Integrand size = 32, antiderivative size = 41

$$\int \frac{(cd^2 + 2cdex + ce^2x^2)^{5/2}}{(d+ex)^8} dx = -\frac{c^3}{2e(d+ex)\sqrt{cd^2 + 2cdex + ce^2x^2}}$$

output `-1/2*c^3/e/(e*x+d)/(c*e^2*x^2+2*c*d*e*x+c*d^2)^(1/2)`

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.66

$$\int \frac{(cd^2 + 2cdex + ce^2x^2)^{5/2}}{(d+ex)^8} dx = -\frac{(c(d+ex)^2)^{5/2}}{2e(d+ex)^7}$$

input `Integrate[(c*d^2 + 2*c*d*e*x + c*e^2*x^2)^(5/2)/(d + e*x)^8,x]`

output `-1/2*(c*(d + e*x)^2)^(5/2)/(e*(d + e*x)^7)`

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.062$, Rules used = {1102, 17}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(cd^2 + 2cdex + ce^2x^2)^{5/2}}{(d + ex)^8} dx$$

$$\downarrow 1102$$

$$\frac{\sqrt{cd^2 + 2cdex + ce^2x^2} \int \frac{c^5 e^5}{(d+ex)^3} dx}{c^3 e^5 (d + ex)}$$

$$\downarrow 17$$

$$-\frac{c^2 \sqrt{cd^2 + 2cdex + ce^2x^2}}{2e(d + ex)^3}$$

input `Int[(c*d^2 + 2*c*d*e*x + c*e^2*x^2)^(5/2)/(d + e*x)^8,x]`

output `-1/2*(c^2*sqrt[c*d^2 + 2*c*d*e*x + c*e^2*x^2])/(e*(d + e*x)^3)`

Defintions of rubi rules used

rule 17 `Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] := Simp[c*((a + b*x)^(m + 1))/(b*(m + 1))], x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]`

rule 1102 `Int[((d_.) + (e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] := Simp[(a + b*x + c*x^2)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x)^(2*FracPart[p])) Int[(d + e*x)^m*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && EqQ[b^2 - 4*a*c, 0]`

Maple [A] (verified)

Time = 2.93 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.66

| method | result | size |
|---------|--|------|
| risch | $-\frac{c^2 \sqrt{c(ex+d)^2}}{2(ex+d)^3 e}$ | 27 |
| gospers | $-\frac{(x^2 c e^2 + 2cdxe + cd^2)^{\frac{5}{2}}}{2(ex+d)^7 e}$ | 35 |
| default | $-\frac{(x^2 c e^2 + 2cdxe + cd^2)^{\frac{5}{2}}}{2(ex+d)^7 e}$ | 35 |
| orering | $-\frac{(x^2 c e^2 + 2cdxe + cd^2)^{\frac{5}{2}}}{2(ex+d)^7 e}$ | 35 |
| trager | $\frac{c^2 (ex+2d)x \sqrt{x^2 c e^2 + 2cdxe + cd^2}}{2d^2 (ex+d)^3}$ | 46 |

input `int((c*e^2*x^2+2*c*d*e*x+c*d^2)^(5/2)/(e*x+d)^8,x,method=_RETURNVERBOSE)`

output `-1/2*c^2/(e*x+d)^3*(c*(e*x+d)^2)^(1/2)/e`

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.46

$$\int \frac{(cd^2 + 2cdex + ce^2x^2)^{5/2}}{(d + ex)^8} dx = -\frac{\sqrt{ce^2x^2 + 2cdex + cd^2}c^2}{2(e^4x^3 + 3de^3x^2 + 3d^2e^2x + d^3e)}$$

input `integrate((c*e^2*x^2+2*c*d*e*x+c*d^2)^(5/2)/(e*x+d)^8,x, algorithm="fricas")`

output `-1/2*sqrt(c*e^2*x^2 + 2*c*d*e*x + c*d^2)*c^2/(e^4*x^3 + 3*d*e^3*x^2 + 3*d^2*e^2*x + d^3*e)`

Sympy [F]

$$\int \frac{(cd^2 + 2cdex + ce^2x^2)^{5/2}}{(d + ex)^8} dx = \int \frac{(c(d + ex)^2)^{5/2}}{(d + ex)^8} dx$$

input `integrate((c*e**2*x**2+2*c*d*e*x+c*d**2)**(5/2)/(e*x+d)**8,x)`

output `Integral((c*(d + e*x)**2)**(5/2)/(d + e*x)**8, x)`

Maxima [F(-2)]

Exception generated.

$$\int \frac{(cd^2 + 2cdex + ce^2x^2)^{5/2}}{(d + ex)^8} dx = \text{Exception raised: RuntimeError}$$

input `integrate((c*e^2*x^2+2*c*d*e*x+c*d^2)^(5/2)/(e*x+d)^8,x, algorithm="maxima")`

output `Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negative exponent.`

Giac [A] (verification not implemented)

Time = 1.04 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.51

$$\int \frac{(cd^2 + 2cdex + ce^2x^2)^{5/2}}{(d + ex)^8} dx = -\frac{c^{5/2} \operatorname{sgn}(ex + d)}{2(ex + d)^2 e}$$

input `integrate((c*e^2*x^2+2*c*d*e*x+c*d^2)^(5/2)/(e*x+d)^8,x, algorithm="giac")`

output `-1/2*c^(5/2)*sgn(e*x + d)/((e*x + d)^2*e)`

Mupad [B] (verification not implemented)

Time = 8.78 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.90

$$\int \frac{(cd^2 + 2cdex + ce^2x^2)^{5/2}}{(d + ex)^8} dx = -\frac{c^2 \sqrt{cd^2 + 2cdex + ce^2x^2}}{2e(d + ex)^3}$$

input `int((c*d^2 + c*e^2*x^2 + 2*c*d*e*x)^(5/2)/(d + e*x)^8,x)`output `-(c^2*(c*d^2 + c*e^2*x^2 + 2*c*d*e*x)^(1/2))/(2*e*(d + e*x)^3)`**Reduce [B] (verification not implemented)**

Time = 0.23 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.68

$$\int \frac{(cd^2 + 2cdex + ce^2x^2)^{5/2}}{(d + ex)^8} dx = -\frac{\sqrt{c}c^2}{2e(e^2x^2 + 2dex + d^2)}$$

input `int((c*e^2*x^2+2*c*d*e*x+c*d^2)^(5/2)/(e*x+d)^8,x)`output `(- sqrt(c)*c**2)/(2*e*(d**2 + 2*d*e*x + e**2*x**2))`

$$3.235 \quad \int \frac{(d+ex)^5}{\sqrt{cd^2+2cdex+ce^2x^2}} dx$$

| | |
|---|------|
| Optimal result | 1854 |
| Mathematica [A] (verified) | 1854 |
| Rubi [A] (verified) | 1855 |
| Maple [A] (verified) | 1855 |
| Fricas [B] (verification not implemented) | 1856 |
| Sympy [B] (verification not implemented) | 1857 |
| Maxima [B] (verification not implemented) | 1857 |
| Giac [F(-2)] | 1858 |
| Mupad [F(-1)] | 1858 |
| Reduce [B] (verification not implemented) | 1859 |

Optimal result

Integrand size = 32, antiderivative size = 34

$$\int \frac{(d+ex)^5}{\sqrt{cd^2+2cdex+ce^2x^2}} dx = \frac{(cd^2+2cdex+ce^2x^2)^{5/2}}{5c^3e}$$

output $1/5*(c*e^2*x^2+2*c*d*e*x+c*d^2)^{(5/2)}/c^3/e$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.79

$$\int \frac{(d+ex)^5}{\sqrt{cd^2+2cdex+ce^2x^2}} dx = \frac{(d+ex)^6}{5e\sqrt{c(d+ex)^2}}$$

input `Integrate[(d + e*x)^5/Sqrt[c*d^2 + 2*c*d*e*x + c*e^2*x^2], x]`

output $(d + e*x)^6/(5*e*\sqrt{c*(d + e*x)^2})$

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.031$, Rules used = {1099}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d + ex)^5}{\sqrt{cd^2 + 2cdex + ce^2x^2}} dx$$

↓ 1099

$$\frac{(cd^2 + 2cdex + ce^2x^2)^{5/2}}{5c^3e}$$

input `Int[(d + e*x)^5/Sqrt[c*d^2 + 2*c*d*e*x + c*e^2*x^2],x]`

output `(c*d^2 + 2*c*d*e*x + c*e^2*x^2)^(5/2)/(5*c^3*e)`

Defintions of rubi rules used

rule 1099

```
Int[((d_) + (e_)*(x_))^(m_)*((a_) + (b_)*(x_) + (c_)*(x_)^2)^(p_), x_Sy
mbol] := Simp[e^m*((a + b*x + c*x^2)^(p + (m + 1)/2)/(c^((m + 1)/2)*(m + 2*
p + 1))), x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[b^2 - 4*a*c, 0] && EqQ[
2*c*d - b*e, 0] && IntegerQ[(m - 1)/2]
```

Maple [A] (verified)

Time = 1.21 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.71

| method | result | size |
|----------------|---|------|
| risch | $\frac{(ex+d)^6}{5\sqrt{c(ex+d)^2}e}$ | 24 |
| pseudoelliptic | $\frac{\sqrt{c(ex+d)^2}(ex+d)^4}{5ce}$ | 27 |
| default | $\frac{(ex+d)^6}{5\sqrt{x^2ce^2+2cdxe+cd^2}e}$ | 35 |
| gospers | $\frac{x(e^4x^4+5de^3x^3+10d^2e^2x^2+10d^3ex+5d^4)(ex+d)}{5\sqrt{x^2ce^2+2cdxe+cd^2}}$ | 71 |
| orering | $\frac{x(e^4x^4+5de^3x^3+10d^2e^2x^2+10d^3ex+5d^4)(ex+d)}{5\sqrt{x^2ce^2+2cdxe+cd^2}}$ | 71 |
| trager | $\frac{x(e^4x^4+5de^3x^3+10d^2e^2x^2+10d^3ex+5d^4)\sqrt{x^2ce^2+2cdxe+cd^2}}{5c(ex+d)}$ | 76 |

input `int((e*x+d)^5/(c*e^2*x^2+2*c*d*e*x+c*d^2)^(1/2),x,method=_RETURNVERBOSE)`

output `1/5*(e*x+d)^6/(c*(e*x+d)^2)^(1/2)/e`

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 77 vs. $2(30) = 60$.

Time = 0.08 (sec) , antiderivative size = 77, normalized size of antiderivative = 2.26

$$\int \frac{(d+ex)^5}{\sqrt{cd^2+2cdex+ce^2x^2}} dx$$

$$= \frac{(e^4x^5+5de^3x^4+10d^2e^2x^3+10d^3ex^2+5d^4x)\sqrt{ce^2x^2+2cdex+cd^2}}{5(cex+cd)}$$

input `integrate((e*x+d)^5/(c*e^2*x^2+2*c*d*e*x+c*d^2)^(1/2),x, algorithm="fricas")`

output `1/5*(e^4*x^5+5*d*e^3*x^4+10*d^2*e^2*x^3+10*d^3*e*x^2+5*d^4*x)*sqrt(c*e^2*x^2+2*c*d*e*x+c*d^2)/(c*e*x+c*d)`

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 196 vs. $2(31) = 62$.

Time = 0.26 (sec) , antiderivative size = 196, normalized size of antiderivative = 5.76

$$\int \frac{(d+ex)^5}{\sqrt{cd^2+2cdex+ce^2x^2}} dx = \begin{cases} \frac{d^4\sqrt{cd^2+2cdex+ce^2x^2}}{5ce} + \frac{4d^3x\sqrt{cd^2+2cdex+ce^2x^2}}{5c} + \frac{6d^2e^2x^2\sqrt{cd^2+2cdex+ce^2x^2}}{5c} + \frac{4de^2x^3\sqrt{cd^2+2cdex+ce^2x^2}}{5c} + \frac{e^3x^4\sqrt{cd^2+2cdex+ce^2x^2}}{5c} \\ \frac{d^5x}{\sqrt{cd^2}} \end{cases}$$

input `integrate((e*x+d)**5/(c*e**2*x**2+2*c*d*e*x+c*d**2)**(1/2), x)`

output `Piecewise((d**4*sqrt(c*d**2 + 2*c*d*e*x + c*e**2*x**2)/(5*c*e) + 4*d**3*x*sqrt(c*d**2 + 2*c*d*e*x + c*e**2*x**2)/(5*c) + 6*d**2*e*x**2*sqrt(c*d**2 + 2*c*d*e*x + c*e**2*x**2)/(5*c) + 4*d*e**2*x**3*sqrt(c*d**2 + 2*c*d*e*x + c*e**2*x**2)/(5*c) + e**3*x**4*sqrt(c*d**2 + 2*c*d*e*x + c*e**2*x**2)/(5*c), Ne(e, 0)), (d**5*x/sqrt(c*d**2), True))`

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 218 vs. $2(30) = 60$.

Time = 0.04 (sec) , antiderivative size = 218, normalized size of antiderivative = 6.41

$$\int \frac{(d+ex)^5}{\sqrt{cd^2+2cdex+ce^2x^2}} dx = \frac{8c^2d^5e^4 \log\left(x + \frac{d}{e}\right)}{5(ce^2)^{\frac{5}{2}}} - \frac{8cd^4e^3x}{5(ce^2)^{\frac{3}{2}}} + \frac{4d^3e^2x^2}{5\sqrt{ce^2}} + \frac{\sqrt{ce^2x^2+2cdex+cd^2}e^3x^4}{5c} - \frac{8}{5}d^5\sqrt{\frac{1}{ce^2}}\log\left(x + \frac{d}{e}\right) + \frac{4\sqrt{ce^2x^2+2cdex+cd^2}de^2x^3}{5c} + \frac{6\sqrt{ce^2x^2+2cdex+cd^2}d^2ex^2}{5c} + \frac{13\sqrt{ce^2x^2+2cdex+cd^2}d^4}{5ce}$$

input `integrate((e*x+d)^5/(c*e^2*x^2+2*c*d*e*x+c*d^2)^(1/2),x, algorithm="maxima")`

output
$$\begin{aligned} & 8/5*c^2*d^5*e^4*\log(x + d/e)/(c*e^2)^(5/2) - 8/5*c*d^4*e^3*x/(c*e^2)^(3/2) \\ & + 4/5*d^3*e^2*x^2/\sqrt{c*e^2} + 1/5*\sqrt{c*e^2*x^2 + 2*c*d*e*x + c*d^2}*e \\ & ^3*x^4/c - 8/5*d^5*\sqrt{1/(c*e^2)}*\log(x + d/e) + 4/5*\sqrt{c*e^2*x^2 + 2*c \\ & *d*e*x + c*d^2}*d*e^2*x^3/c + 6/5*\sqrt{c*e^2*x^2 + 2*c*d*e*x + c*d^2}*d^2* \\ & e*x^2/c + 13/5*\sqrt{c*e^2*x^2 + 2*c*d*e*x + c*d^2}*d^4/(c*e) \end{aligned}$$

Giac [F(-2)]

Exception generated.

$$\int \frac{(d + ex)^5}{\sqrt{cd^2 + 2cdex + ce^2x^2}} dx = \text{Exception raised: TypeError}$$

input `integrate((e*x+d)^5/(c*e^2*x^2+2*c*d*e*x+c*d^2)^(1/2),x, algorithm="giac")`

output Exception raised: TypeError >> an error occurred running a Giac command:INPUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const vecteur & l) Error: Bad Argument Value

Mupad [F(-1)]

Timed out.

$$\int \frac{(d + ex)^5}{\sqrt{cd^2 + 2cdex + ce^2x^2}} dx = \int \frac{(d + ex)^5}{\sqrt{cd^2 + 2cdex + ce^2x^2}} dx$$

input `int((d + e*x)^5/(c*d^2 + c*e^2*x^2 + 2*c*d*e*x)^(1/2),x)`

output `int((d + e*x)^5/(c*d^2 + c*e^2*x^2 + 2*c*d*e*x)^(1/2), x)`

Reduce [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.41

$$\int \frac{(d + ex)^5}{\sqrt{cd^2 + 2cde x + ce^2 x^2}} dx = \frac{\sqrt{c} x (e^4 x^4 + 5d e^3 x^3 + 10d^2 e^2 x^2 + 10d^3 e x + 5d^4)}{5c}$$

input `int((e*x+d)^5/(c*e^2*x^2+2*c*d*e*x+c*d^2)^(1/2),x)`

output `(sqrt(c)*x*(5*d**4 + 10*d**3*e*x + 10*d**2*e**2*x**2 + 5*d*e**3*x**3 + e**4*x**4))/(5*c)`

$$3.236 \quad \int \frac{(d+ex)^3}{\sqrt{cd^2+2cdex+ce^2x^2}} dx$$

| | |
|---|------|
| Optimal result | 1860 |
| Mathematica [A] (verified) | 1860 |
| Rubi [A] (verified) | 1861 |
| Maple [A] (verified) | 1861 |
| Fricas [A] (verification not implemented) | 1862 |
| Sympy [B] (verification not implemented) | 1863 |
| Maxima [B] (verification not implemented) | 1863 |
| Giac [F(-2)] | 1864 |
| Mupad [F(-1)] | 1864 |
| Reduce [B] (verification not implemented) | 1865 |

Optimal result

Integrand size = 32, antiderivative size = 34

$$\int \frac{(d+ex)^3}{\sqrt{cd^2+2cdex+ce^2x^2}} dx = \frac{(cd^2+2cdex+ce^2x^2)^{3/2}}{3c^2e}$$

output $1/3*(c*e^2*x^2+2*c*d*e*x+c*d^2)^{(3/2)}/c^2/e$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.79

$$\int \frac{(d+ex)^3}{\sqrt{cd^2+2cdex+ce^2x^2}} dx = \frac{(d+ex)^4}{3e\sqrt{c(d+ex)^2}}$$

input `Integrate[(d + e*x)^3/Sqrt[c*d^2 + 2*c*d*e*x + c*e^2*x^2], x]`

output $(d + e*x)^4/(3*e*\sqrt{c*(d + e*x)^2})$

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.031$, Rules used = {1099}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d + ex)^3}{\sqrt{cd^2 + 2cdex + ce^2x^2}} dx$$

↓ 1099

$$\frac{(cd^2 + 2cdex + ce^2x^2)^{3/2}}{3c^2e}$$

input `Int[(d + e*x)^3/Sqrt[c*d^2 + 2*c*d*e*x + c*e^2*x^2],x]`

output `(c*d^2 + 2*c*d*e*x + c*e^2*x^2)^(3/2)/(3*c^2*e)`

Defintions of rubi rules used

rule 1099 `Int[((d_) + (e_)*(x_))^(m_)*((a_) + (b_)*(x_) + (c_)*(x_)^2)^(p_), x_Symbol] := Simp[e^m*((a + b*x + c*x^2)^(p + (m + 1)/2)/(c^((m + 1)/2)*(m + 2*p + 1))), x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[b^2 - 4*a*c, 0] && EqQ[2*c*d - b*e, 0] && IntegerQ[(m - 1)/2]`

Maple [A] (verified)

Time = 1.15 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.71

| method | result | size |
|----------------|---|------|
| risch | $\frac{(ex+d)^4}{3\sqrt{c(ex+d)^2}e}$ | 24 |
| pseudoelliptic | $\frac{(ex+d)^2\sqrt{c(ex+d)^2}}{3ce}$ | 27 |
| default | $\frac{(ex+d)^4}{3\sqrt{x^2ce^2+2cdxe+cd^2}e}$ | 35 |
| gospers | $\frac{x(e^2x^2+3dex+3d^2)(ex+d)}{3\sqrt{x^2ce^2+2cdxe+cd^2}}$ | 49 |
| orering | $\frac{x(e^2x^2+3dex+3d^2)(ex+d)}{3\sqrt{x^2ce^2+2cdxe+cd^2}}$ | 49 |
| trager | $\frac{x(e^2x^2+3dex+3d^2)\sqrt{x^2ce^2+2cdxe+cd^2}}{3c(ex+d)}$ | 54 |

input `int((e*x+d)^3/(c*e^2*x^2+2*c*d*e*x+c*d^2)^(1/2),x,method=_RETURNVERBOSE)`

output `1/3*(e*x+d)^4/(c*(e*x+d)^2)^(1/2)/e`

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.62

$$\int \frac{(d+ex)^3}{\sqrt{cd^2+2cdex+ce^2x^2}} dx = \frac{\sqrt{ce^2x^2+2cdex+cd^2}(e^2x^3+3dex^2+3d^2x)}{3(cex+cd)}$$

input `integrate((e*x+d)^3/(c*e^2*x^2+2*c*d*e*x+c*d^2)^(1/2),x, algorithm="fricas")`

output `1/3*sqrt(c*e^2*x^2 + 2*c*d*e*x + c*d^2)*(e^2*x^3 + 3*d*e*x^2 + 3*d^2*x)/(c*e*x + c*d)`

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 114 vs. $2(31) = 62$.

Time = 0.19 (sec) , antiderivative size = 114, normalized size of antiderivative = 3.35

$$\int \frac{(d+ex)^3}{\sqrt{cd^2+2cdex+ce^2x^2}} dx = \begin{cases} \frac{d^2\sqrt{cd^2+2cdex+ce^2x^2}}{3ce} + \frac{2dx\sqrt{cd^2+2cdex+ce^2x^2}}{3c} + \frac{ex^2\sqrt{cd^2+2cdex+ce^2x^2}}{3c} & \text{for } e \neq 0 \\ \frac{d^3x}{\sqrt{cd^2}} & \text{otherwise} \end{cases}$$

input

```
integrate((e*x+d)**3/(c*e**2*x**2+2*c*d*e*x+c*d**2)**(1/2),x)
```

output

```
Piecewise((d**2*sqrt(c*d**2 + 2*c*d*e*x + c*e**2*x**2)/(3*c*e) + 2*d*x*sqrt(c*d**2 + 2*c*d*e*x + c*e**2*x**2)/(3*c) + e*x**2*sqrt(c*d**2 + 2*c*d*e*x + c*e**2*x**2)/(3*c), Ne(e, 0)), (d**3*x/sqrt(c*d**2), True))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 146 vs. $2(30) = 60$.

Time = 0.03 (sec) , antiderivative size = 146, normalized size of antiderivative = 4.29

$$\int \frac{(d+ex)^3}{\sqrt{cd^2+2cdex+ce^2x^2}} dx = \frac{4c^2d^3e^4 \log\left(x + \frac{d}{e}\right)}{3(ce^2)^{\frac{5}{2}}} - \frac{4cd^2e^3x}{3(ce^2)^{\frac{3}{2}}} + \frac{2de^2x^2}{3\sqrt{ce^2}} - \frac{4}{3}d^3\sqrt{\frac{1}{ce^2}}\log\left(x + \frac{d}{e}\right) + \frac{\sqrt{ce^2x^2+2cdex+cd^2}ex^2}{3c} + \frac{7\sqrt{ce^2x^2+2cdex+cd^2}d^2}{3ce}$$

input

```
integrate((e*x+d)^3/(c*e^2*x^2+2*c*d*e*x+c*d^2)^(1/2),x, algorithm="maxima")
```

output

```
4/3*c^2*d^3*e^4*log(x + d/e)/(c*e^2)^(5/2) - 4/3*c*d^2*e^3*x/(c*e^2)^(3/2)
+ 2/3*d*e^2*x^2/sqrt(c*e^2) - 4/3*d^3*sqrt(1/(c*e^2))*log(x + d/e) + 1/3*
sqrt(c*e^2*x^2 + 2*c*d*e*x + c*d^2)*e*x^2/c + 7/3*sqrt(c*e^2*x^2 + 2*c*d*e
*x + c*d^2)*d^2/(c*e)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{(d + ex)^3}{\sqrt{cd^2 + 2cdex + ce^2x^2}} dx = \text{Exception raised: TypeError}$$

input

```
integrate((e*x+d)^3/(c*e^2*x^2+2*c*d*e*x+c*d^2)^(1/2),x, algorithm="giac")
```

output

```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d + ex)^3}{\sqrt{cd^2 + 2cdex + ce^2x^2}} dx = \int \frac{(d + ex)^3}{\sqrt{cd^2 + 2cdex + ce^2x^2}} dx$$

input

```
int((d + e*x)^3/(c*d^2 + c*e^2*x^2 + 2*c*d*e*x)^(1/2),x)
```

output

```
int((d + e*x)^3/(c*d^2 + c*e^2*x^2 + 2*c*d*e*x)^(1/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.76

$$\int \frac{(d + ex)^3}{\sqrt{cd^2 + 2cdex + ce^2x^2}} dx = \frac{\sqrt{c}x(e^2x^2 + 3dex + 3d^2)}{3c}$$

input

```
int((e*x+d)^3/(c*e^2*x^2+2*c*d*e*x+c*d^2)^(1/2),x)
```

output

```
(sqrt(c)*x*(3*d**2 + 3*d*e*x + e**2*x**2))/(3*c)
```

$$3.237 \quad \int \frac{d+ex}{\sqrt{cd^2+2cdex+ce^2x^2}} dx$$

| | |
|---|------|
| Optimal result | 1866 |
| Mathematica [A] (verified) | 1866 |
| Rubi [A] (verified) | 1867 |
| Maple [A] (verified) | 1867 |
| Fricas [A] (verification not implemented) | 1868 |
| Sympy [A] (verification not implemented) | 1868 |
| Maxima [A] (verification not implemented) | 1869 |
| Giac [F(-2)] | 1869 |
| Mupad [B] (verification not implemented) | 1870 |
| Reduce [B] (verification not implemented) | 1870 |

Optimal result

Integrand size = 30, antiderivative size = 31

$$\int \frac{d+ex}{\sqrt{cd^2+2cdex+ce^2x^2}} dx = \frac{\sqrt{cd^2+2cdex+ce^2x^2}}{ce}$$

output $(c*e^2*x^2+2*c*d*e*x+c*d^2)^{(1/2)}/c/e$

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.65

$$\int \frac{d+ex}{\sqrt{cd^2+2cdex+ce^2x^2}} dx = \frac{\sqrt{c(d+ex)^2}}{ce}$$

input `Integrate[(d + e*x)/Sqrt[c*d^2 + 2*c*d*e*x + c*e^2*x^2],x]`

output `Sqrt[c*(d + e*x)^2]/(c*e)`

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.033$, Rules used = {1099}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{d + ex}{\sqrt{cd^2 + 2cdex + ce^2x^2}} dx$$

↓ 1099

$$\frac{\sqrt{cd^2 + 2cdex + ce^2x^2}}{ce}$$

input

```
Int[(d + e*x)/Sqrt[c*d^2 + 2*c*d*e*x + c*e^2*x^2], x]
```

output

```
Sqrt[c*d^2 + 2*c*d*e*x + c*e^2*x^2]/(c*e)
```

Defintions of rubi rules used

rule 1099

```
Int[((d_) + (e_)*(x_)^(m_))*((a_) + (b_)*(x_) + (c_)*(x_)^2)^(p_), x_Sy
mbol] := Simp[e^m*((a + b*x + c*x^2)^(p + (m + 1)/2)/(c^((m + 1)/2)*(m + 2*
p + 1))), x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[b^2 - 4*a*c, 0] && EqQ[
2*c*d - b*e, 0] && IntegerQ[(m - 1)/2]
```

Maple [A] (verified)

Time = 1.14 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.61

| method | result | size |
|----------------|--|------|
| risch | $\frac{(ex+d)x}{\sqrt{c(ex+d)^2}}$ | 19 |
| pseudoelliptic | $\frac{\sqrt{c(ex+d)^2}}{ce}$ | 19 |
| default | $\frac{x(ex+d)}{\sqrt{x^2ce^2+2cdxe+cd^2}}$ | 30 |
| orering | $\frac{x(ex+d)}{\sqrt{x^2ce^2+2cdxe+cd^2}}$ | 30 |
| trager | $\frac{x\sqrt{x^2ce^2+2cdxe+cd^2}}{c(ex+d)}$ | 35 |

input `int((e*x+d)/(c*e^2*x^2+2*c*d*e*x+c*d^2)^(1/2),x,method=_RETURNVERBOSE)`

output `(e*x+d)/(c*(e*x+d)^2)^(1/2)*x`

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.10

$$\int \frac{d+ex}{\sqrt{cd^2+2cdex+ce^2x^2}} dx = \frac{\sqrt{ce^2x^2+2cdex+cd^2}x}{ce x + cd}$$

input `integrate((e*x+d)/(c*e^2*x^2+2*c*d*e*x+c*d^2)^(1/2),x, algorithm="fricas")`

output `sqrt(c*e^2*x^2 + 2*c*d*e*x + c*d^2)*x/(c*e*x + c*d)`

Sympy [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.26

$$\int \frac{d+ex}{\sqrt{cd^2+2cdex+ce^2x^2}} dx = \begin{cases} \frac{\sqrt{cd^2+2cdex+ce^2x^2}}{ce} & \text{for } e \neq 0 \\ \frac{dx}{\sqrt{cd^2}} & \text{otherwise} \end{cases}$$

input `integrate((e*x+d)/(c*e**2*x**2+2*c*d*e*x+c*d**2)**(1/2),x)`

output `Piecewise((sqrt(c*d**2 + 2*c*d*e*x + c*e**2*x**2)/(c*e), Ne(e, 0)), (d*x/sqrt(c*d**2), True))`

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.94

$$\int \frac{d + ex}{\sqrt{cd^2 + 2cdex + ce^2x^2}} dx = \frac{\sqrt{ce^2x^2 + 2cdex + cd^2}}{ce}$$

input `integrate((e*x+d)/(c*e^2*x^2+2*c*d*e*x+c*d^2)^(1/2),x, algorithm="maxima")`

output `sqrt(c*e^2*x^2 + 2*c*d*e*x + c*d^2)/(c*e)`

Giac [F(-2)]

Exception generated.

$$\int \frac{d + ex}{\sqrt{cd^2 + 2cdex + ce^2x^2}} dx = \text{Exception raised: TypeError}$$

input `integrate((e*x+d)/(c*e^2*x^2+2*c*d*e*x+c*d^2)^(1/2),x, algorithm="giac")`

output `Exception raised: TypeError >> an error occurred running a Giac command:INPUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const vecteur & l) Error: Bad Argument Value`

Mupad [B] (verification not implemented)

Time = 9.11 (sec) , antiderivative size = 109, normalized size of antiderivative = 3.52

$$\int \frac{d + ex}{\sqrt{cd^2 + 2cdex + ce^2x^2}} dx = \frac{\sqrt{cd^2 + 2cdex + ce^2x^2}}{ce} - \frac{cde^2 \ln \left(\sqrt{c(d+ex)^2 + ce^2} + cde + ce^2x \right)}{(ce^2)^{3/2}} + \frac{cde^2 \ln(cx^2 + cde) \operatorname{sign}(ce(d+ex))}{(ce^2)^{3/2}}$$

input `int((d + e*x)/(c*d^2 + c*e^2*x^2 + 2*c*d*e*x)^(1/2),x)`output `(c*d^2 + c*e^2*x^2 + 2*c*d*e*x)^(1/2)/(c*e) - (c*d*e^2*log((c*(d + e*x)^2)^(1/2)*(c*e^2)^(1/2) + c*d*e + c*e^2*x))/(c*e^2)^(3/2) + (c*d*e^2*log(c*d*e + c*e^2*x)*sign(c*e*(d + e*x)))/(c*e^2)^(3/2)`**Reduce [B] (verification not implemented)**

Time = 0.22 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.23

$$\int \frac{d + ex}{\sqrt{cd^2 + 2cdex + ce^2x^2}} dx = \frac{\sqrt{c}x}{c}$$

input `int((e*x+d)/(c*e^2*x^2+2*c*d*e*x+c*d^2)^(1/2),x)`output `(sqrt(c)*x)/c`

$$3.238 \quad \int \frac{1}{(d+ex)\sqrt{cd^2+2cdex+ce^2x^2}} dx$$

| | |
|---|------|
| Optimal result | 1871 |
| Mathematica [A] (verified) | 1871 |
| Rubi [A] (verified) | 1872 |
| Maple [A] (verified) | 1872 |
| Fricas [A] (verification not implemented) | 1873 |
| Sympy [A] (verification not implemented) | 1874 |
| Maxima [A] (verification not implemented) | 1874 |
| Giac [F(-2)] | 1874 |
| Mupad [B] (verification not implemented) | 1875 |
| Reduce [B] (verification not implemented) | 1875 |

Optimal result

Integrand size = 32, antiderivative size = 29

$$\int \frac{1}{(d+ex)\sqrt{cd^2+2cdex+ce^2x^2}} dx = -\frac{1}{e\sqrt{cd^2+2cdex+ce^2x^2}}$$

output `-1/e/(c*e^2*x^2+2*c*d*e*x+c*d^2)^(1/2)`

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.62

$$\int \frac{1}{(d+ex)\sqrt{cd^2+2cdex+ce^2x^2}} dx = -\frac{1}{e\sqrt{c(d+ex)^2}}$$

input `Integrate[1/((d + e*x)*Sqrt[c*d^2 + 2*c*d*e*x + c*e^2*x^2]),x]`

output `-(1/(e*Sqrt[c*(d + e*x)^2]))`

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.031$, Rules used = {1099}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(d + ex)\sqrt{cd^2 + 2cdex + ce^2x^2}} dx$$

↓ 1099

$$-\frac{1}{e\sqrt{cd^2 + 2cdex + ce^2x^2}}$$

input

```
Int[1/((d + e*x)*Sqrt[c*d^2 + 2*c*d*e*x + c*e^2*x^2]),x]
```

output

```
-(1/(e*Sqrt[c*d^2 + 2*c*d*e*x + c*e^2*x^2]))
```

Defintions of rubi rules used

rule 1099

```
Int[((d_) + (e_)*(x_))^(m_)*((a_) + (b_)*(x_) + (c_)*(x_)^2)^(p_), x_Sy
mbol] :> Simp[e^m*((a + b*x + c*x^2)^(p + (m + 1)/2)/(c^((m + 1)/2)*(m + 2*
p + 1))), x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[b^2 - 4*a*c, 0] && EqQ[
2*c*d - b*e, 0] && IntegerQ[(m - 1)/2]
```

Maple [A] (verified)

Time = 1.32 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.59

| method | result | size |
|----------------|---|------|
| risch | $-\frac{1}{\sqrt{c(ex+d)^2} e}$ | 17 |
| pseudoelliptic | $-\frac{1}{\sqrt{c(ex+d)^2} e}$ | 17 |
| gospers | $-\frac{1}{e\sqrt{x^2 c e^2 + 2cdxe + cd^2}}$ | 28 |
| default | $-\frac{1}{e\sqrt{x^2 c e^2 + 2cdxe + cd^2}}$ | 28 |
| orering | $-\frac{1}{e\sqrt{x^2 c e^2 + 2cdxe + cd^2}}$ | 28 |
| trager | $\frac{x\sqrt{x^2 c e^2 + 2cdxe + cd^2}}{cd(ex+d)^2}$ | 38 |

input `int(1/(e*x+d)/(c*e^2*x^2+2*c*d*e*x+c*d^2)^(1/2),x,method=_RETURNVERBOSE)`

output `-1/(c*(e*x+d)^2)^(1/2)/e`

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.69

$$\int \frac{1}{(d+ex)\sqrt{cd^2+2cdex+ce^2x^2}} dx = -\frac{\sqrt{ce^2x^2+2cdex+cd^2}}{ce^3x^2+2cde^2x+cd^2e}$$

input `integrate(1/(e*x+d)/(c*e^2*x^2+2*c*d*e*x+c*d^2)^(1/2),x, algorithm="fricas")`

output `-sqrt(c*e^2*x^2 + 2*c*d*e*x + c*d^2)/(c*e^3*x^2 + 2*c*d*e^2*x + c*d^2*e)`

Sympy [A] (verification not implemented)

Time = 1.10 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.41

$$\int \frac{1}{(d+ex)\sqrt{cd^2+2cdex+ce^2x^2}} dx = \begin{cases} -\frac{1}{e\sqrt{cd^2+2cdex+ce^2x^2}} & \text{for } e \neq 0 \\ \frac{x}{d\sqrt{cd^2}} & \text{otherwise} \end{cases}$$

input `integrate(1/(e*x+d)/(c*e**2*x**2+2*c*d*e*x+c*d**2)**(1/2),x)`

output `Piecewise((-1/(e*sqrt(c*d**2 + 2*c*d*e*x + c*e**2*x**2)), Ne(e, 0)), (x/(d*sqrt(c*d**2)), True))`

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.66

$$\int \frac{1}{(d+ex)\sqrt{cd^2+2cdex+ce^2x^2}} dx = -\frac{1}{\sqrt{ce^2x+\sqrt{cde}}}$$

input `integrate(1/(e*x+d)/(c*e^2*x^2+2*c*d*e*x+c*d^2)^(1/2),x, algorithm="maxima")`

output `-1/(sqrt(c)*e^2*x + sqrt(c)*d*e)`

Giac [F(-2)]

Exception generated.

$$\int \frac{1}{(d+ex)\sqrt{cd^2+2cdex+ce^2x^2}} dx = \text{Exception raised: TypeError}$$

input `integrate(1/(e*x+d)/(c*e^2*x^2+2*c*d*e*x+c*d^2)^(1/2),x, algorithm="giac")`

output Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value

Mupad [B] (verification not implemented)

Time = 8.75 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.93

$$\int \frac{1}{(d+ex)\sqrt{cd^2+2cdex+ce^2x^2}} dx = -\frac{1}{e\sqrt{cd^2+2cdex+ce^2x^2}}$$

input `int(1/((d + e*x)*(c*d^2 + c*e^2*x^2 + 2*c*d*e*x)^(1/2)),x)`

output `-1/(e*(c*d^2 + c*e^2*x^2 + 2*c*d*e*x)^(1/2))`

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.59

$$\int \frac{1}{(d+ex)\sqrt{cd^2+2cdex+ce^2x^2}} dx = \frac{\sqrt{c}x}{cd(ex+d)}$$

input `int(1/(e*x+d)/(c*e^2*x^2+2*c*d*e*x+c*d^2)^(1/2),x)`

output `(sqrt(c)*x)/(c*d*(d + e*x))`

3.239 $\int \frac{1}{(d+ex)^3 \sqrt{cd^2+2cdex+ce^2x^2}} dx$

| | |
|---|------|
| Optimal result | 1876 |
| Mathematica [A] (verified) | 1876 |
| Rubi [A] (verified) | 1877 |
| Maple [A] (verified) | 1877 |
| Fricas [B] (verification not implemented) | 1878 |
| Sympy [F] | 1879 |
| Maxima [A] (verification not implemented) | 1879 |
| Giac [F(-2)] | 1879 |
| Mupad [B] (verification not implemented) | 1880 |
| Reduce [B] (verification not implemented) | 1880 |

Optimal result

Integrand size = 32, antiderivative size = 32

$$\int \frac{1}{(d+ex)^3 \sqrt{cd^2+2cdex+ce^2x^2}} dx = -\frac{c}{3e (cd^2+2cdex+ce^2x^2)^{3/2}}$$

output `-1/3*c/e/(c*e^2*x^2+2*c*d*e*x+c*d^2)^(3/2)`

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.66

$$\int \frac{1}{(d+ex)^3 \sqrt{cd^2+2cdex+ce^2x^2}} dx = -\frac{c}{3e (c(d+ex)^2)^{3/2}}$$

input `Integrate[1/((d+e*x)^3*Sqrt[c*d^2+2*c*d*e*x+c*e^2*x^2]),x]`

output `-1/3*c/(e*(c*(d+e*x)^2)^(3/2))`

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.031$, Rules used = {1099}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(d + ex)^3 \sqrt{cd^2 + 2cdex + ce^2x^2}} dx$$

↓ 1099

$$-\frac{c}{3e(cd^2 + 2cdex + ce^2x^2)^{3/2}}$$

input `Int[1/((d + e*x)^3*Sqrt[c*d^2 + 2*c*d*e*x + c*e^2*x^2]),x]`

output `-1/3*c/(e*(c*d^2 + 2*c*d*e*x + c*e^2*x^2)^(3/2))`

Defintions of rubi rules used

rule 1099 `Int[((d_) + (e_)*(x_))^(m_)*((a_) + (b_)*(x_) + (c_)*(x_)^2)^(p_), x_Sy
mbol] :> Simp[e^m*((a + b*x + c*x^2)^(p + (m + 1)/2)/(c^((m + 1)/2)*(m + 2*
p + 1))), x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[b^2 - 4*a*c, 0] && EqQ[
2*c*d - b*e, 0] && IntegerQ[(m - 1)/2]`

Maple [A] (verified)

Time = 1.31 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.75

| method | result | size |
|----------------|---|------|
| risch | $-\frac{1}{3(ex+d)^2\sqrt{c(ex+d)^2e}}$ | 24 |
| pseudoelliptic | $-\frac{1}{3(ex+d)^2\sqrt{c(ex+d)^2e}}$ | 24 |
| gosper | $-\frac{1}{3(ex+d)^2e\sqrt{x^2ce^2+2cdxe+cd^2}}$ | 35 |
| default | $-\frac{1}{3(ex+d)^2e\sqrt{x^2ce^2+2cdxe+cd^2}}$ | 35 |
| orering | $-\frac{1}{3(ex+d)^2e\sqrt{x^2ce^2+2cdxe+cd^2}}$ | 35 |
| trager | $\frac{(\epsilon^2x^2+3dex+3d^2)x\sqrt{x^2ce^2+2cdxe+cd^2}}{3cd^3(ex+d)^4}$ | 57 |

input `int(1/(e*x+d)^3/(c*e^2*x^2+2*c*d*e*x+c*d^2)^(1/2),x,method=_RETURNVERBOSE)`

output `-1/3/(e*x+d)^2/(c*(e*x+d)^2)^(1/2)/e`

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 73 vs. $2(28) = 56$.

Time = 0.09 (sec) , antiderivative size = 73, normalized size of antiderivative = 2.28

$$\int \frac{1}{(d+ex)^3\sqrt{cd^2+2cdex+ce^2x^2}} dx$$

$$= -\frac{\sqrt{ce^2x^2+2cdex+cd^2}}{3(ce^5x^4+4cde^4x^3+6cd^2e^3x^2+4cd^3e^2x+cd^4e)}$$

input `integrate(1/(e*x+d)^3/(c*e^2*x^2+2*c*d*e*x+c*d^2)^(1/2),x, algorithm="fricas")`

output `-1/3*sqrt(c*e^2*x^2+2*c*d*e*x+c*d^2)/(c*e^5*x^4+4*c*d*e^4*x^3+6*c*d^2*e^3*x^2+4*c*d^3*e^2*x+c*d^4*e)`

Sympy [F]

$$\int \frac{1}{(d+ex)^3 \sqrt{cd^2 + 2cdex + ce^2x^2}} dx = \int \frac{1}{\sqrt{c(d+ex)^2} (d+ex)^3} dx$$

input `integrate(1/(e*x+d)**3/(c*e**2*x**2+2*c*d*e*x+c*d**2)**(1/2), x)`

output `Integral(1/(sqrt(c*(d + e*x)**2)*(d + e*x)**3), x)`

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.47

$$\int \frac{1}{(d+ex)^3 \sqrt{cd^2 + 2cdex + ce^2x^2}} dx = -\frac{1}{3(\sqrt{ce^4x^3} + 3\sqrt{cde^3x^2} + 3\sqrt{cd^2e^2x} + \sqrt{cd^3e})}$$

input `integrate(1/(e*x+d)^3/(c*e^2*x^2+2*c*d*e*x+c*d^2)^(1/2), x, algorithm="maxima")`

output `-1/3/(sqrt(c)*e^4*x^3 + 3*sqrt(c)*d*e^3*x^2 + 3*sqrt(c)*d^2*e^2*x + sqrt(c)*d^3*e)`

Giac [F(-2)]

Exception generated.

$$\int \frac{1}{(d+ex)^3 \sqrt{cd^2 + 2cdex + ce^2x^2}} dx = \text{Exception raised: TypeError}$$

input `integrate(1/(e*x+d)^3/(c*e^2*x^2+2*c*d*e*x+c*d^2)^(1/2), x, algorithm="giac")`

output

```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [B] (verification not implemented)

Time = 8.79 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.16

$$\int \frac{1}{(d+ex)^3 \sqrt{cd^2+2cdex+ce^2x^2}} dx = -\frac{\sqrt{cd^2+2cdex+ce^2x^2}}{3ce(d+ex)^4}$$

input

```
int(1/((d + e*x)^3*(c*d^2 + c*e^2*x^2 + 2*c*d*e*x)^(1/2)),x)
```

output

```
-(c*d^2 + c*e^2*x^2 + 2*c*d*e*x)^(1/2)/(3*c*e*(d + e*x)^4)
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.22

$$\int \frac{1}{(d+ex)^3 \sqrt{cd^2+2cdex+ce^2x^2}} dx = -\frac{\sqrt{c}}{3ce(e^3x^3+3de^2x^2+3d^2ex+d^3)}$$

input

```
int(1/(e*x+d)^3/(c*e^2*x^2+2*c*d*e*x+c*d^2)^(1/2),x)
```

output

```
( - sqrt(c))/(3*c*e*(d**3 + 3*d**2*e*x + 3*d*e**2*x**2 + e**3*x**3))
```

$$3.240 \quad \int \frac{(d+ex)^4}{\sqrt{cd^2+2cdex+ce^2x^2}} dx$$

| | |
|---|------|
| Optimal result | 1881 |
| Mathematica [A] (verified) | 1881 |
| Rubi [A] (verified) | 1882 |
| Maple [A] (verified) | 1883 |
| Fricas [A] (verification not implemented) | 1883 |
| Sympy [B] (verification not implemented) | 1884 |
| Maxima [B] (verification not implemented) | 1885 |
| Giac [F(-2)] | 1885 |
| Mupad [F(-1)] | 1886 |
| Reduce [B] (verification not implemented) | 1886 |

Optimal result

Integrand size = 32, antiderivative size = 41

$$\int \frac{(d+ex)^4}{\sqrt{cd^2+2cdex+ce^2x^2}} dx = \frac{(d+ex)^3 \sqrt{cd^2+2cdex+ce^2x^2}}{4ce}$$

output

$$1/4*(e*x+d)^3*(c*e^2*x^2+2*c*d*e*x+c*d^2)^(1/2)/c/e$$

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.66

$$\int \frac{(d+ex)^4}{\sqrt{cd^2+2cdex+ce^2x^2}} dx = \frac{(d+ex)^5}{4e\sqrt{c(d+ex)^2}}$$

input

$$\text{Integrate}[(d+e*x)^4/\text{Sqrt}[c*d^2+2*c*d*e*x+c*e^2*x^2],x]$$

output

$$(d+e*x)^5/(4*e*\text{Sqrt}[c*(d+e*x)^2])$$

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.93, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.062$, Rules used = {1102, 17}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex)^4}{\sqrt{cd^2+2cdex+ce^2x^2}} dx$$

$$\downarrow 1102$$

$$\frac{ce(d+ex) \int \frac{(d+ex)^3}{ce} dx}{\sqrt{cd^2+2cdex+ce^2x^2}}$$

$$\downarrow 17$$

$$\frac{(d+ex)^5}{4e\sqrt{cd^2+2cdex+ce^2x^2}}$$

input `Int[(d + e*x)^4/Sqrt[c*d^2 + 2*c*d*e*x + c*e^2*x^2],x]`

output `(d + e*x)^5/(4*e*Sqrt[c*d^2 + 2*c*d*e*x + c*e^2*x^2])`

Defintions of rubi rules used

rule 17 `Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] := Simp[c*((a + b*x)^(m + 1))/(b*(m + 1)), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]`

rule 1102 `Int[((d_.) + (e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] := Simp[(a + b*x + c*x^2)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x)^(2*FracPart[p])) Int[(d + e*x)^m*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && EqQ[b^2 - 4*a*c, 0]`

Maple [A] (verified)

Time = 1.36 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.59

| method | result | size |
|---------|--|------|
| risch | $\frac{(ex+d)^5}{4\sqrt{c(ex+d)^2}e}$ | 24 |
| default | $\frac{(ex+d)^5}{4\sqrt{x^2ce^2+2cdxe+cd^2}e}$ | 35 |
| gospers | $\frac{x(e^3x^3+4de^2x^2+6d^2ex+4d^3)(ex+d)}{4\sqrt{x^2ce^2+2cdxe+cd^2}}$ | 60 |
| orering | $\frac{x(e^3x^3+4de^2x^2+6d^2ex+4d^3)(ex+d)}{4\sqrt{x^2ce^2+2cdxe+cd^2}}$ | 60 |
| trager | $\frac{x(e^3x^3+4de^2x^2+6d^2ex+4d^3)\sqrt{x^2ce^2+2cdxe+cd^2}}{4c(ex+d)}$ | 65 |

input `int((e*x+d)^4/(c*e^2*x^2+2*c*d*e*x+c*d^2)^(1/2),x,method=_RETURNVERBOSE)`

output `1/4*(e*x+d)^5/(c*(e*x+d)^2)^(1/2)/e`

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.61

$$\int \frac{(d+ex)^4}{\sqrt{cd^2+2cdex+ce^2x^2}} dx = \frac{(e^3x^4+4de^2x^3+6d^2ex^2+4d^3x)\sqrt{ce^2x^2+2cdex+cd^2}}{4(cex+cd)}$$

input `integrate((e*x+d)^4/(c*e^2*x^2+2*c*d*e*x+c*d^2)^(1/2),x, algorithm="fricas")`

output `1/4*(e^3*x^4+4*d*e^2*x^3+6*d^2*e*x^2+4*d^3*x)*sqrt(c*e^2*x^2+2*c*d*e*x+c*d^2)/(c*e*x+c*d)`

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 219 vs. $2(36) = 72$.

Time = 0.83 (sec) , antiderivative size = 219, normalized size of antiderivative = 5.34

$$\int \frac{(d+ex)^4}{\sqrt{cd^2+2cdex+ce^2x^2}} dx$$

$$= \begin{cases} \sqrt{cd^2+2cdex+ce^2x^2} \left(\frac{d^3}{4ce} + \frac{3d^2x}{4c} + \frac{3dex^2}{4c} + \frac{e^2x^3}{4c} \right) & \text{for } ce^2 \neq 0 \\ \frac{d^4\sqrt{cd^2+2cdex}}{16} + \frac{d^2(cd^2+2cdex)^{\frac{3}{2}}}{12c} + \frac{3(cd^2+2cdex)^{\frac{5}{2}}}{40c^2} + \frac{(cd^2+2cdex)^{\frac{7}{2}}}{28c^3d^2} + \frac{(cd^2+2cdex)^{\frac{9}{2}}}{144c^4d^4} & \text{for } cde \neq 0 \\ \begin{cases} d^4x & \text{for } e = 0 \\ \frac{(d+ex)^5}{5e} & \text{otherwise} \end{cases} & \text{otherwise} \end{cases}$$

input `integrate((e*x+d)**4/(c*e**2*x**2+2*c*d*e*x+c*d**2)**(1/2),x)`

output `Piecewise((sqrt(c*d**2 + 2*c*d*e*x + c*e**2*x**2)*(d**3/(4*c*e) + 3*d**2*x/(4*c) + 3*d*e*x**2/(4*c) + e**2*x**3/(4*c)), Ne(c*e**2, 0)), ((d**4*sqrt(c*d**2 + 2*c*d*e*x)/16 + d**2*(c*d**2 + 2*c*d*e*x)**(3/2)/(12*c) + 3*(c*d**2 + 2*c*d*e*x)**(5/2)/(40*c**2) + (c*d**2 + 2*c*d*e*x)**(7/2)/(28*c**3*d**2) + (c*d**2 + 2*c*d*e*x)**(9/2)/(144*c**4*d**4))/(c*d*e), Ne(c*d*e, 0)), (Piecewise((d**4*x, Eq(e, 0)), ((d + e*x)**5/(5*e), True))/sqrt(c*d**2), True))`

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 182 vs. $2(37) = 74$.

Time = 0.04 (sec) , antiderivative size = 182, normalized size of antiderivative = 4.44

$$\int \frac{(d+ex)^4}{\sqrt{cd^2+2cdex+ce^2x^2}} dx = \frac{3c^2d^4e^4 \log\left(x+\frac{d}{e}\right)}{2(ce^2)^{\frac{5}{2}}} - \frac{3cd^3e^3x}{2(ce^2)^{\frac{3}{2}}} + \frac{3d^2e^2x^2}{4\sqrt{ce^2}}$$

$$- \frac{3}{2}d^4\sqrt{\frac{1}{ce^2}} \log\left(x+\frac{d}{e}\right) + \frac{\sqrt{ce^2x^2+2cdex+cd^2}e^2x^3}{4c}$$

$$+ \frac{3\sqrt{ce^2x^2+2cdex+cd^2}dex^2}{4c}$$

$$+ \frac{5\sqrt{ce^2x^2+2cdex+cd^2}d^3}{2ce}$$

input `integrate((e*x+d)^4/(c*e^2*x^2+2*c*d*e*x+c*d^2)^(1/2),x, algorithm="maxima")`

output `3/2*c^2*d^4*e^4*log(x + d/e)/(c*e^2)^(5/2) - 3/2*c*d^3*e^3*x/(c*e^2)^(3/2) + 3/4*d^2*e^2*x^2/sqrt(c*e^2) - 3/2*d^4*sqrt(1/(c*e^2))*log(x + d/e) + 1/4*sqrt(c*e^2*x^2 + 2*c*d*e*x + c*d^2)*e^2*x^3/c + 3/4*sqrt(c*e^2*x^2 + 2*c*d*e*x + c*d^2)*d*e*x^2/c + 5/2*sqrt(c*e^2*x^2 + 2*c*d*e*x + c*d^2)*d^3/(c*e)`

Giac [F(-2)]

Exception generated.

$$\int \frac{(d+ex)^4}{\sqrt{cd^2+2cdex+ce^2x^2}} dx = \text{Exception raised: TypeError}$$

input `integrate((e*x+d)^4/(c*e^2*x^2+2*c*d*e*x+c*d^2)^(1/2),x, algorithm="giac")`

output `Exception raised: TypeError >> an error occurred running a Giac command:INPUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const vecteur & l) Error: Bad Argument Value`

Mupad [F(-1)]

Timed out.

$$\int \frac{(d + ex)^4}{\sqrt{cd^2 + 2cdex + ce^2x^2}} dx = \int \frac{(d + ex)^4}{\sqrt{cd^2 + 2cdex + ce^2x^2}} dx$$

input `int((d + e*x)^4/(c*d^2 + c*e^2*x^2 + 2*c*d*e*x)^(1/2),x)`

output `int((d + e*x)^4/(c*d^2 + c*e^2*x^2 + 2*c*d*e*x)^(1/2), x)`

Reduce [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.90

$$\int \frac{(d + ex)^4}{\sqrt{cd^2 + 2cdex + ce^2x^2}} dx = \frac{\sqrt{c}x(e^3x^3 + 4de^2x^2 + 6d^2ex + 4d^3)}{4c}$$

input `int((e*x+d)^4/(c*e^2*x^2+2*c*d*e*x+c*d^2)^(1/2),x)`

output `(sqrt(c)*x*(4*d**3 + 6*d**2*e*x + 4*d*e**2*x**2 + e**3*x**3))/(4*c)`

$$3.241 \quad \int \frac{(d+ex)^2}{\sqrt{cd^2+2cdex+ce^2x^2}} dx$$

| | |
|---|------|
| Optimal result | 1887 |
| Mathematica [A] (verified) | 1887 |
| Rubi [A] (verified) | 1888 |
| Maple [A] (verified) | 1889 |
| Fricas [A] (verification not implemented) | 1889 |
| Sympy [B] (verification not implemented) | 1890 |
| Maxima [B] (verification not implemented) | 1890 |
| Giac [F(-2)] | 1891 |
| Mupad [F(-1)] | 1891 |
| Reduce [B] (verification not implemented) | 1892 |

Optimal result

Integrand size = 32, antiderivative size = 39

$$\int \frac{(d+ex)^2}{\sqrt{cd^2+2cdex+ce^2x^2}} dx = \frac{(d+ex)\sqrt{cd^2+2cdex+ce^2x^2}}{2ce}$$

output $1/2*(e*x+d)*(c*e^2*x^2+2*c*d*e*x+c*d^2)^{(1/2)}/c/e$

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.77

$$\int \frac{(d+ex)^2}{\sqrt{cd^2+2cdex+ce^2x^2}} dx = \frac{x(d+ex)(2d+ex)}{2\sqrt{c(d+ex)^2}}$$

input `Integrate[(d + e*x)^2/Sqrt[c*d^2 + 2*c*d*e*x + c*e^2*x^2],x]`

output $(x*(d + e*x)*(2*d + e*x))/(2*\text{Sqrt}[c*(d + e*x)^2])$

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.97, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.062$, Rules used = {1102, 17}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex)^2}{\sqrt{cd^2+2cdex+ce^2x^2}} dx$$

↓ 1102

$$\frac{ce(d+ex) \int \frac{d+ex}{ce} dx}{\sqrt{cd^2+2cdex+ce^2x^2}}$$

↓ 17

$$\frac{(d+ex)^3}{2e\sqrt{cd^2+2cdex+ce^2x^2}}$$

input `Int[(d + e*x)^2/Sqrt[c*d^2 + 2*c*d*e*x + c*e^2*x^2],x]`

output `(d + e*x)^3/(2*e*Sqrt[c*d^2 + 2*c*d*e*x + c*e^2*x^2])`

Defintions of rubi rules used

rule 17 `Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] := Simp[c*((a + b*x)^(m + 1))/(b*(m + 1)), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]`

rule 1102 `Int[((d_.) + (e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] := Simp[(a + b*x + c*x^2)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x)^(2*FracPart[p])) Int[(d + e*x)^m*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && EqQ[b^2 - 4*a*c, 0]`

Maple [A] (verified)

Time = 1.26 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.97

| method | result | size |
|---------|--|------|
| gospers | $\frac{x(ex+2d)(ex+d)}{2\sqrt{x^2ce^2+2cdxe+cd^2}}$ | 38 |
| default | $\frac{x(ex+2d)(ex+d)}{2\sqrt{x^2ce^2+2cdxe+cd^2}}$ | 38 |
| orering | $\frac{x(ex+2d)(ex+d)}{2\sqrt{x^2ce^2+2cdxe+cd^2}}$ | 38 |
| trager | $\frac{x(ex+2d)\sqrt{x^2ce^2+2cdxe+cd^2}}{2c(ex+d)}$ | 43 |
| risch | $\frac{(ex+d)ex^2}{2\sqrt{c(ex+d)^2}} + \frac{(ex+d)dx}{\sqrt{c(ex+d)^2}}$ | 43 |

input `int((e*x+d)^2/(c*e^2*x^2+2*c*d*e*x+c*d^2)^(1/2),x,method=_RETURNVERBOSE)`

output `1/2*x*(e*x+2*d)*(e*x+d)/(c*e^2*x^2+2*c*d*e*x+c*d^2)^(1/2)`

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.13

$$\int \frac{(d+ex)^2}{\sqrt{cd^2+2cdex+ce^2x^2}} dx = \frac{\sqrt{ce^2x^2+2cdex+cd^2}(ex^2+2dx)}{2(cex+cd)}$$

input `integrate((e*x+d)^2/(c*e^2*x^2+2*c*d*e*x+c*d^2)^(1/2),x, algorithm="fricas")`

output `1/2*sqrt(c*e^2*x^2 + 2*c*d*e*x + c*d^2)*(e*x^2 + 2*d*x)/(c*e*x + c*d)`

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 138 vs. $2(36) = 72$.

Time = 0.72 (sec) , antiderivative size = 138, normalized size of antiderivative = 3.54

$$\int \frac{(d+ex)^2}{\sqrt{cd^2+2cdex+ce^2x^2}} dx = \begin{cases} \left(\frac{d}{2ce} + \frac{x}{2c}\right) \sqrt{cd^2+2cdex+ce^2x^2} & \text{for } ce^2 \neq 0 \\ \frac{\frac{d^2\sqrt{cd^2+2cdex}}{4} + \frac{(cd^2+2cdex)^{3/2}}{6c} + \frac{(cd^2+2cdex)^{5/2}}{20c^2d^2}}{cde} & \text{for } cde \neq 0 \\ \begin{cases} d^2x & \text{for } e = 0 \\ \frac{(d+ex)^3}{3e} & \text{otherwise} \end{cases} & \text{otherwise} \end{cases}$$

input

```
integrate((e*x+d)**2/(c*e**2*x**2+2*c*d*e*x+c*d**2)**(1/2),x)
```

output

```
Piecewise(((d/(2*c*e) + x/(2*c))*sqrt(c*d**2 + 2*c*d*e*x + c*e**2*x**2), Ne(c*e**2, 0)), ((d**2*sqrt(c*d**2 + 2*c*d*e*x)/4 + (c*d**2 + 2*c*d*e*x)**(3/2)/(6*c) + (c*d**2 + 2*c*d*e*x)**(5/2)/(20*c**2*d**2))/(c*d*e), Ne(c*d*e, 0)), (Piecewise((d**2*x, Eq(e, 0)), ((d + e*x)**3/(3*e), True))/sqrt(c*d**2), True))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 109 vs. $2(35) = 70$.

Time = 0.04 (sec) , antiderivative size = 109, normalized size of antiderivative = 2.79

$$\int \frac{(d+ex)^2}{\sqrt{cd^2+2cdex+ce^2x^2}} dx = \frac{c^2d^2e^4 \log\left(x + \frac{d}{e}\right)}{(ce^2)^{5/2}} - \frac{cde^3x}{(ce^2)^{3/2}} + \frac{e^2x^2}{2\sqrt{ce^2}} - d^2\sqrt{\frac{1}{ce^2}} \log\left(x + \frac{d}{e}\right) + \frac{2\sqrt{ce^2x^2+2cdex+cd^2}d}{ce}$$

input

```
integrate((e*x+d)^2/(c*e^2*x^2+2*c*d*e*x+c*d^2)^(1/2),x, algorithm="maxima")
```

output

```
c^2*d^2*e^4*log(x + d/e)/(c*e^2)^(5/2) - c*d*e^3*x/(c*e^2)^(3/2) + 1/2*e^2
*x^2/sqrt(c*e^2) - d^2*sqrt(1/(c*e^2))*log(x + d/e) + 2*sqrt(c*e^2*x^2 + 2
*c*d*e*x + c*d^2)*d/(c*e)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{(d + ex)^2}{\sqrt{cd^2 + 2cdex + ce^2x^2}} dx = \text{Exception raised: TypeError}$$

input

```
integrate((e*x+d)^2/(c*e^2*x^2+2*c*d*e*x+c*d^2)^(1/2),x, algorithm="giac")
```

output

```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d + ex)^2}{\sqrt{cd^2 + 2cdex + ce^2x^2}} dx = \int \frac{(d + ex)^2}{\sqrt{cd^2 + 2cdex + ce^2x^2}} dx$$

input

```
int((d + e*x)^2/(c*d^2 + c*e^2*x^2 + 2*c*d*e*x)^(1/2),x)
```

output

```
int((d + e*x)^2/(c*d^2 + c*e^2*x^2 + 2*c*d*e*x)^(1/2), x)
```


Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.38

$$\int \frac{(d + ex)^2}{\sqrt{cd^2 + 2cdex + ce^2x^2}} dx = \frac{\sqrt{c}x(ex + 2d)}{2c}$$

input `int((e*x+d)^2/(c*e^2*x^2+2*c*d*e*x+c*d^2)^(1/2),x)`

output `(sqrt(c)*x*(2*d + e*x))/(2*c)`

3.242 $\int \frac{1}{\sqrt{cd^2+2cdex+ce^2x^2}} dx$

| | |
|---|------|
| Optimal result | 1893 |
| Mathematica [A] (verified) | 1893 |
| Rubi [A] (verified) | 1894 |
| Maple [A] (verified) | 1895 |
| Fricas [A] (verification not implemented) | 1895 |
| Sympy [A] (verification not implemented) | 1895 |
| Maxima [A] (verification not implemented) | 1896 |
| Giac [A] (verification not implemented) | 1896 |
| Mupad [B] (verification not implemented) | 1896 |
| Reduce [B] (verification not implemented) | 1897 |

Optimal result

Integrand size = 24, antiderivative size = 39

$$\int \frac{1}{\sqrt{cd^2 + 2cdex + ce^2x^2}} dx = \frac{(d + ex) \log(d + ex)}{e\sqrt{cd^2 + 2cdex + ce^2x^2}}$$

output `(e*x+d)*ln(e*x+d)/e/(c*e^2*x^2+2*c*d*e*x+c*d^2)^(1/2)`

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.72

$$\int \frac{1}{\sqrt{cd^2 + 2cdex + ce^2x^2}} dx = \frac{(d + ex) \log(d + ex)}{e\sqrt{c(d + ex)^2}}$$

input `Integrate[1/Sqrt[c*d^2 + 2*c*d*e*x + c*e^2*x^2],x]`

output `((d + e*x)*Log[d + e*x])/(e*Sqrt[c*(d + e*x)^2])`

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {1079, 16}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{cd^2 + 2cdex + ce^2x^2}} dx$$

$$\downarrow 1079$$

$$\frac{ce(d+ex) \int \frac{1}{cxe^2+cde} dx}{\sqrt{cd^2 + 2cdex + ce^2x^2}}$$

$$\downarrow 16$$

$$\frac{(d+ex) \log(d+ex)}{e\sqrt{cd^2 + 2cdex + ce^2x^2}}$$

input `Int[1/Sqrt[c*d^2 + 2*c*d*e*x + c*e^2*x^2], x]`

output `((d + e*x)*Log[d + e*x])/(e*Sqrt[c*d^2 + 2*c*d*e*x + c*e^2*x^2])`

Defintions of rubi rules used

rule 16 `Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] := Simp[c*(Log[RemoveContent[a + b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]`

rule 1079 `Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] := Simp[(a + b*x + c*x^2)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x)^(2*FracPart[p])) Int[(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c, p}, x] && EqQ[b^2 - 4*a*c, 0]`

Maple [A] (verified)

Time = 1.10 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.69

| method | result | size |
|---------|--|------|
| risch | $\frac{(ex+d) \ln(ex+d)}{\sqrt{c(ex+d)^2} e}$ | 27 |
| default | $\frac{(ex+d) \ln(ex+d)}{e\sqrt{x^2 c e^2 + 2cdxe + c d^2}}$ | 38 |

input `int(1/(c*e^2*x^2+2*c*d*e*x+c*d^2)^(1/2),x,method=_RETURNVERBOSE)`

output `1/(c*(e*x+d)^2)^(1/2)*(e*x+d)*ln(e*x+d)/e`

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.08

$$\int \frac{1}{\sqrt{cd^2 + 2cdex + ce^2x^2}} dx = \frac{\sqrt{ce^2x^2 + 2cdex + cd^2} \log(ex + d)}{ce^2x + cde}$$

input `integrate(1/(c*e^2*x^2+2*c*d*e*x+c*d^2)^(1/2),x, algorithm="fricas")`

output `sqrt(c*e^2*x^2 + 2*c*d*e*x + c*d^2)*log(e*x + d)/(c*e^2*x + c*d*e)`

Sympy [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.67

$$\int \frac{1}{\sqrt{cd^2 + 2cdex + ce^2x^2}} dx = \frac{\left(\frac{d}{e} + x\right) \log\left(\frac{d}{e} + x\right)}{\sqrt{ce^2\left(\frac{d}{e} + x\right)^2}}$$

input `integrate(1/(c*e**2*x**2+2*c*d*e*x+c*d**2)**(1/2),x)`

output `(d/e + x)*log(d/e + x)/sqrt(c*e**2*(d/e + x)**2)`

Maxima [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.46

$$\int \frac{1}{\sqrt{cd^2 + 2cdex + ce^2x^2}} dx = \sqrt{\frac{1}{ce^2}} \log\left(x + \frac{d}{e}\right)$$

input `integrate(1/(c*e^2*x^2+2*c*d*e*x+c*d^2)^(1/2),x, algorithm="maxima")`output `sqrt(1/(c*e^2))*log(x + d/e)`**Giac [A] (verification not implemented)**

Time = 0.47 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.87

$$\int \frac{1}{\sqrt{cd^2 + 2cdex + ce^2x^2}} dx = \frac{\log\left(|ex + d|\sqrt{|c|}|\operatorname{sgn}(ex + d)|\right)}{\sqrt{ce}\operatorname{sgn}(ex + d)}$$

input `integrate(1/(c*e^2*x^2+2*c*d*e*x+c*d^2)^(1/2),x, algorithm="giac")`output `log(abs(e*x + d)*sqrt(abs(c))*abs(sgn(e*x + d)))/(sqrt(c)*e*sgn(e*x + d))`**Mupad [B] (verification not implemented)**

Time = 8.82 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.74

$$\int \frac{1}{\sqrt{cd^2 + 2cdex + ce^2x^2}} dx = \frac{\ln(cxe^2 + cde) \operatorname{sign}(ce(d + ex))}{\sqrt{ce^2}}$$

input `int(1/(c*d^2 + c*e^2*x^2 + 2*c*d*e*x)^(1/2),x)`output `(log(c*d*e + c*e^2*x)*sign(c*e*(d + e*x)))/(c*e^2)^(1/2)`

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.38

$$\int \frac{1}{\sqrt{cd^2 + 2cdex + ce^2x^2}} dx = \frac{\sqrt{c} \log(ex + d)}{ce}$$

input `int(1/(c*e^2*x^2+2*c*d*e*x+c*d^2)^(1/2),x)`

output `(sqrt(c)*log(d + e*x))/(c*e)`

3.243 $\int \frac{1}{(d+ex)^2 \sqrt{cd^2+2cdex+ce^2x^2}} dx$

| | |
|---|------|
| Optimal result | 1898 |
| Mathematica [A] (verified) | 1898 |
| Rubi [A] (verified) | 1899 |
| Maple [A] (verified) | 1900 |
| Fricas [A] (verification not implemented) | 1900 |
| Sympy [F] | 1901 |
| Maxima [A] (verification not implemented) | 1901 |
| Giac [F(-2)] | 1901 |
| Mupad [B] (verification not implemented) | 1902 |
| Reduce [B] (verification not implemented) | 1902 |

Optimal result

Integrand size = 32, antiderivative size = 38

$$\int \frac{1}{(d+ex)^2 \sqrt{cd^2+2cdex+ce^2x^2}} dx = -\frac{1}{2e(d+ex)\sqrt{cd^2+2cdex+ce^2x^2}}$$

output `-1/2/e/(e*x+d)/(c*e^2*x^2+2*c*d*e*x+c*d^2)^(1/2)`

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.68

$$\int \frac{1}{(d+ex)^2 \sqrt{cd^2+2cdex+ce^2x^2}} dx = -\frac{c(d+ex)}{2e(c(d+ex)^2)^{3/2}}$$

input `Integrate[1/((d + e*x)^2*Sqrt[c*d^2 + 2*c*d*e*x + c*e^2*x^2]),x]`

output `-1/2*(c*(d + e*x))/(e*(c*(d + e*x)^2)^(3/2))`

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.062$, Rules used = {1102, 17}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(d+ex)^2 \sqrt{cd^2+2cdex+ce^2x^2}} dx$$

$$\downarrow 1102$$

$$\frac{ce(d+ex) \int \frac{1}{ce(d+ex)^3} dx}{\sqrt{cd^2+2cdex+ce^2x^2}}$$

$$\downarrow 17$$

$$-\frac{1}{2e(d+ex)\sqrt{cd^2+2cdex+ce^2x^2}}$$

input `Int[1/((d + e*x)^2*Sqrt[c*d^2 + 2*c*d*e*x + c*e^2*x^2]),x]`

output `-1/2*1/(e*(d + e*x)*Sqrt[c*d^2 + 2*c*d*e*x + c*e^2*x^2])`

Defintions of rubi rules used

rule 17 `Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] := Simp[c*((a + b*x)^(m + 1))/(b*(m + 1))], x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]`

rule 1102 `Int[((d_.) + (e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] := Simp[(a + b*x + c*x^2)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x)^(2*FracPart[p])) Int[(d + e*x)^m*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && EqQ[b^2 - 4*a*c, 0]`

Maple [A] (verified)

Time = 1.47 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.63

| method | result | size |
|---------|---|------|
| risch | $-\frac{1}{2(ex+d)\sqrt{c(ex+d)^2}e}$ | 24 |
| gospers | $-\frac{1}{2e(ex+d)\sqrt{x^2ce^2+2cdxe+cd^2}}$ | 35 |
| default | $-\frac{1}{2e(ex+d)\sqrt{x^2ce^2+2cdxe+cd^2}}$ | 35 |
| orering | $-\frac{1}{2e(ex+d)\sqrt{x^2ce^2+2cdxe+cd^2}}$ | 35 |
| trager | $\frac{(ex+2d)x\sqrt{x^2ce^2+2cdxe+cd^2}}{2cd^2(ex+d)^3}$ | 46 |

input `int(1/(e*x+d)^2/(c*e^2*x^2+2*c*d*e*x+c*d^2)^(1/2),x,method=_RETURNVERBOSE)`

output `-1/2/(e*x+d)/(c*(e*x+d)^2)^(1/2)/e`

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.61

$$\int \frac{1}{(d+ex)^2\sqrt{cd^2+2cdex+ce^2x^2}} dx = -\frac{\sqrt{ce^2x^2+2cdex+cd^2}}{2(ce^4x^3+3cde^3x^2+3cd^2e^2x+cd^3e)}$$

input `integrate(1/(e*x+d)^2/(c*e^2*x^2+2*c*d*e*x+c*d^2)^(1/2),x, algorithm="fricas")`

output `-1/2*sqrt(c*e^2*x^2 + 2*c*d*e*x + c*d^2)/(c*e^4*x^3 + 3*c*d*e^3*x^2 + 3*c*d^2*e^2*x + c*d^3*e)`

Sympy [F]

$$\int \frac{1}{(d+ex)^2 \sqrt{cd^2 + 2cdex + ce^2x^2}} dx = \int \frac{1}{\sqrt{c(d+ex)^2} (d+ex)^2} dx$$

input `integrate(1/(e*x+d)**2/(c*e**2*x**2+2*c*d*e*x+c*d**2)**(1/2),x)`

output `Integral(1/(sqrt(c*(d + e*x)**2)*(d + e*x)**2), x)`

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.87

$$\int \frac{1}{(d+ex)^2 \sqrt{cd^2 + 2cdex + ce^2x^2}} dx = -\frac{1}{2(\sqrt{ce^3x^2} + 2\sqrt{cde^2x} + \sqrt{cd^2e})}$$

input `integrate(1/(e*x+d)^2/(c*e^2*x^2+2*c*d*e*x+c*d^2)^(1/2),x, algorithm="maxima")`

output `-1/2/(sqrt(c)*e^3*x^2 + 2*sqrt(c)*d*e^2*x + sqrt(c)*d^2*e)`

Giac [F(-2)]

Exception generated.

$$\int \frac{1}{(d+ex)^2 \sqrt{cd^2 + 2cdex + ce^2x^2}} dx = \text{Exception raised: TypeError}$$

input `integrate(1/(e*x+d)^2/(c*e^2*x^2+2*c*d*e*x+c*d^2)^(1/2),x, algorithm="giac")`

output `Exception raised: TypeError >> an error occurred running a Giac command:INPUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const vecteur & l) Error: Bad Argument Value`

Mupad [B] (verification not implemented)

Time = 9.16 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.97

$$\int \frac{1}{(d+ex)^2 \sqrt{cd^2+2cdex+ce^2x^2}} dx = -\frac{\sqrt{cd^2+2cdex+ce^2x^2}}{2ce(d+ex)^3}$$

input `int(1/((d + e*x)^2*(c*d^2 + c*e^2*x^2 + 2*c*d*e*x)^(1/2)),x)`

output `-(c*d^2 + c*e^2*x^2 + 2*c*d*e*x)^(1/2)/(2*c*e*(d + e*x)^3)`

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.74

$$\int \frac{1}{(d+ex)^2 \sqrt{cd^2+2cdex+ce^2x^2}} dx = -\frac{\sqrt{c}}{2ce(e^2x^2+2dex+d^2)}$$

input `int(1/(e*x+d)^2/(c*e^2*x^2+2*c*d*e*x+c*d^2)^(1/2),x)`

output `(- sqrt(c))/(2*c*e*(d**2 + 2*d*e*x + e**2*x**2))`

$$3.244 \quad \int \frac{1}{(d+ex)^4 \sqrt{cd^2+2cdex+ce^2x^2}} dx$$

| | |
|---|------|
| Optimal result | 1903 |
| Mathematica [A] (verified) | 1903 |
| Rubi [A] (verified) | 1904 |
| Maple [A] (verified) | 1905 |
| Fricas [B] (verification not implemented) | 1905 |
| Sympy [F] | 1906 |
| Maxima [A] (verification not implemented) | 1906 |
| Giac [F(-2)] | 1906 |
| Mupad [B] (verification not implemented) | 1907 |
| Reduce [B] (verification not implemented) | 1907 |

Optimal result

Integrand size = 32, antiderivative size = 38

$$\int \frac{1}{(d+ex)^4 \sqrt{cd^2+2cdex+ce^2x^2}} dx = -\frac{1}{4e(d+ex)^3 \sqrt{cd^2+2cdex+ce^2x^2}}$$

output $-1/4/e/(e*x+d)^3/(c*e^2*x^2+2*c*d*e*x+c*d^2)^(1/2)$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.71

$$\int \frac{1}{(d+ex)^4 \sqrt{cd^2+2cdex+ce^2x^2}} dx = -\frac{1}{4e(d+ex)^3 \sqrt{c(d+ex)^2}}$$

input `Integrate[1/((d + e*x)^4*Sqrt[c*d^2 + 2*c*d*e*x + c*e^2*x^2]),x]`

output $-1/4*1/(e*(d + e*x)^3*Sqrt[c*(d + e*x)^2])$

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.062$, Rules used = {1102, 17}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(d+ex)^4 \sqrt{cd^2+2cdex+ce^2x^2}} dx$$

↓ 1102

$$\frac{ce(d+ex) \int \frac{1}{ce(d+ex)^5} dx}{\sqrt{cd^2+2cdex+ce^2x^2}}$$

↓ 17

$$-\frac{1}{4e(d+ex)^3 \sqrt{cd^2+2cdex+ce^2x^2}}$$

input `Int[1/((d + e*x)^4*Sqrt[c*d^2 + 2*c*d*e*x + c*e^2*x^2]),x]`

output `-1/4*1/(e*(d + e*x)^3*Sqrt[c*d^2 + 2*c*d*e*x + c*e^2*x^2])`

Defintions of rubi rules used

rule 17 `Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] := Simp[c*((a + b*x)^(m + 1))/(b*(m + 1)), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]`

rule 1102 `Int[((d_.) + (e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] := Simp[(a + b*x + c*x^2)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x)^(2*FracPart[p])) Int[(d + e*x)^m*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && EqQ[b^2 - 4*a*c, 0]`

Maple [A] (verified)

Time = 1.66 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.63

| method | result | size |
|---------|--|------|
| risch | $-\frac{1}{4(ex+d)^3 \sqrt{c(ex+d)^2} e}$ | 24 |
| gospers | $-\frac{1}{4e(ex+d)^3 \sqrt{x^2 c e^2 + 2cdxe + cd^2}}$ | 35 |
| default | $-\frac{1}{4e(ex+d)^3 \sqrt{x^2 c e^2 + 2cdxe + cd^2}}$ | 35 |
| orering | $-\frac{1}{4e(ex+d)^3 \sqrt{x^2 c e^2 + 2cdxe + cd^2}}$ | 35 |
| trager | $\frac{(e^3 x^3 + 4d e^2 x^2 + 6d^2 e x + 4d^3) x \sqrt{x^2 c e^2 + 2cdxe + cd^2}}{4c d^4 (ex+d)^5}$ | 68 |

input `int(1/(e*x+d)^4/(c*e^2*x^2+2*c*d*e*x+c*d^2)^(1/2),x,method=_RETURNVERBOSE)`

output `-1/4/(e*x+d)^3/(c*(e*x+d)^2)^(1/2)/e`

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 85 vs. 2(34) = 68.

Time = 0.08 (sec) , antiderivative size = 85, normalized size of antiderivative = 2.24

$$\int \frac{1}{(d+ex)^4 \sqrt{cd^2 + 2cdex + ce^2x^2}} dx$$

$$= -\frac{\sqrt{ce^2x^2 + 2cdex + cd^2}}{4(ce^6x^5 + 5cde^5x^4 + 10cd^2e^4x^3 + 10cd^3e^3x^2 + 5cd^4e^2x + cd^5e)}$$

input `integrate(1/(e*x+d)^4/(c*e^2*x^2+2*c*d*e*x+c*d^2)^(1/2),x, algorithm="fricas")`

output `-1/4*sqrt(c*e^2*x^2 + 2*c*d*e*x + c*d^2)/(c*e^6*x^5 + 5*c*d*e^5*x^4 + 10*c*d^2*e^4*x^3 + 10*c*d^3*e^3*x^2 + 5*c*d^4*e^2*x + c*d^5*e)`

Sympy [F]

$$\int \frac{1}{(d+ex)^4 \sqrt{cd^2 + 2cdex + ce^2x^2}} dx = \int \frac{1}{\sqrt{c(d+ex)^2} (d+ex)^4} dx$$

input `integrate(1/(e*x+d)**4/(c*e**2*x**2+2*c*d*e*x+c*d**2)**(1/2),x)`

output `Integral(1/(sqrt(c*(d + e*x)**2)*(d + e*x)**4), x)`

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.61

$$\int \frac{1}{(d+ex)^4 \sqrt{cd^2 + 2cdex + ce^2x^2}} dx$$

$$= -\frac{1}{4(\sqrt{ce^5x^4 + 4\sqrt{cde^4x^3 + 6\sqrt{cd^2e^3x^2 + 4\sqrt{cd^3e^2x + \sqrt{cd^4e}}}}})}$$

input `integrate(1/(e*x+d)^4/(c*e^2*x^2+2*c*d*e*x+c*d^2)^(1/2),x, algorithm="maxima")`

output `-1/4/(sqrt(c)*e^5*x^4 + 4*sqrt(c)*d*e^4*x^3 + 6*sqrt(c)*d^2*e^3*x^2 + 4*sqrt(c)*d^3*e^2*x + sqrt(c)*d^4*e)`

Giac [F(-2)]

Exception generated.

$$\int \frac{1}{(d+ex)^4 \sqrt{cd^2 + 2cdex + ce^2x^2}} dx = \text{Exception raised: TypeError}$$

input `integrate(1/(e*x+d)^4/(c*e^2*x^2+2*c*d*e*x+c*d^2)^(1/2),x, algorithm="giac")`

output

```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [B] (verification not implemented)

Time = 8.92 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.97

$$\int \frac{1}{(d+ex)^4 \sqrt{cd^2+2cde x+ce^2 x^2}} dx = -\frac{\sqrt{cd^2+2cde x+ce^2 x^2}}{4ce(d+ex)^5}$$

input

```
int(1/((d + e*x)^4*(c*d^2 + c*e^2*x^2 + 2*c*d*e*x)^(1/2)),x)
```

output

```
-(c*d^2 + c*e^2*x^2 + 2*c*d*e*x)^(1/2)/(4*c*e*(d + e*x)^5)
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.32

$$\int \frac{1}{(d+ex)^4 \sqrt{cd^2+2cde x+ce^2 x^2}} dx = -\frac{\sqrt{c}}{4ce(e^4 x^4 + 4d e^3 x^3 + 6d^2 e^2 x^2 + 4d^3 e x + d^4)}$$

input

```
int(1/(e*x+d)^4/(c*e^2*x^2+2*c*d*e*x+c*d^2)^(1/2),x)
```

output

```
( - sqrt(c))/(4*c*e*(d**4 + 4*d**3*e*x + 6*d**2*e**2*x**2 + 4*d*e**3*x**3
+ e**4*x**4))
```


$$3.245 \quad \int \frac{(d+ex)^5}{(cd^2+2cdex+ce^2x^2)^{3/2}} dx$$

| | |
|---|------|
| Optimal result | 1908 |
| Mathematica [A] (verified) | 1908 |
| Rubi [A] (verified) | 1909 |
| Maple [A] (verified) | 1909 |
| Fricas [A] (verification not implemented) | 1910 |
| Sympy [B] (verification not implemented) | 1911 |
| Maxima [B] (verification not implemented) | 1911 |
| Giac [F(-2)] | 1912 |
| Mupad [F(-1)] | 1912 |
| Reduce [B] (verification not implemented) | 1912 |

Optimal result

Integrand size = 32, antiderivative size = 34

$$\int \frac{(d+ex)^5}{(cd^2+2cdex+ce^2x^2)^{3/2}} dx = \frac{(cd^2+2cdex+ce^2x^2)^{3/2}}{3c^3e}$$

output $1/3*(c*e^2*x^2+2*c*d*e*x+c*d^2)^(3/2)/c^3/e$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.79

$$\int \frac{(d+ex)^5}{(cd^2+2cdex+ce^2x^2)^{3/2}} dx = \frac{(d+ex)^6}{3e(c(d+ex)^2)^{3/2}}$$

input `Integrate[(d + e*x)^5/(c*d^2 + 2*c*d*e*x + c*e^2*x^2)^(3/2), x]`

output $(d + e*x)^6/(3*e*(c*(d + e*x)^2)^(3/2))$

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.031$, Rules used = {1099}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d + ex)^5}{(cd^2 + 2cdex + ce^2x^2)^{3/2}} dx$$

↓ 1099

$$\frac{(cd^2 + 2cdex + ce^2x^2)^{3/2}}{3c^3e}$$

input `Int[(d + e*x)^5/(c*d^2 + 2*c*d*e*x + c*e^2*x^2)^(3/2),x]`

output `(c*d^2 + 2*c*d*e*x + c*e^2*x^2)^(3/2)/(3*c^3*e)`

Defintions of rubi rules used

rule 1099 `Int[((d_) + (e_)*(x_))^(m_)*((a_) + (b_)*(x_) + (c_)*(x_)^2)^(p_), x_Symbol] := Simp[e^m*((a + b*x + c*x^2)^(p + (m + 1)/2)/(c^((m + 1)/2)*(m + 2*p + 1))), x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[b^2 - 4*a*c, 0] && EqQ[2*c*d - b*e, 0] && IntegerQ[(m - 1)/2]`

Maple [A] (verified)

Time = 1.29 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.79

| method | result | size |
|----------------|---|------|
| risch | $\frac{(ex+d)^4}{3c\sqrt{c(ex+d)^2}e}$ | 27 |
| pseudoelliptic | $\frac{(ex+d)^4}{3c\sqrt{c(ex+d)^2}e}$ | 27 |
| default | $\frac{(ex+d)^6}{3(x^2ce^2+2cdxe+cd^2)^{\frac{3}{2}}e}$ | 35 |
| gosper | $\frac{x(e^2x^2+3dex+3d^2)(ex+d)^3}{3(x^2ce^2+2cdxe+cd^2)^{\frac{3}{2}}}$ | 51 |
| orering | $\frac{x(e^2x^2+3dex+3d^2)(ex+d)^3}{3(x^2ce^2+2cdxe+cd^2)^{\frac{3}{2}}}$ | 51 |
| trager | $\frac{x(e^2x^2+3dex+3d^2)\sqrt{x^2ce^2+2cdxe+cd^2}}{3c^2(ex+d)}$ | 54 |

input `int((e*x+d)^5/(c*e^2*x^2+2*c*d*e*x+c*d^2)^(3/2),x,method=_RETURNVERBOSE)`

output `1/3/c*(e*x+d)^4/(c*(e*x+d)^2)^(1/2)/e`

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.74

$$\int \frac{(d+ex)^5}{(cd^2+2cdex+ce^2x^2)^{3/2}} dx = \frac{\sqrt{ce^2x^2+2cdex+cd^2}(e^2x^3+3dex^2+3d^2x)}{3(c^2ex+c^2d)}$$

input `integrate((e*x+d)^5/(c*e^2*x^2+2*c*d*e*x+c*d^2)^(3/2),x, algorithm="fricas")`

output `1/3*sqrt(c*e^2*x^2 + 2*c*d*e*x + c*d^2)*(e^2*x^3 + 3*d*e*x^2 + 3*d^2*x)/(c^2*e*x + c^2*d)`

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 119 vs. $2(31) = 62$.

Time = 0.27 (sec) , antiderivative size = 119, normalized size of antiderivative = 3.50

$$\int \frac{(d+ex)^5}{(cd^2+2cdex+ce^2x^2)^{3/2}} dx = \begin{cases} \frac{d^2\sqrt{cd^2+2cdex+ce^2x^2}}{3c^2e} + \frac{2dx\sqrt{cd^2+2cdex+ce^2x^2}}{3c^2} + \frac{ex^2\sqrt{cd^2+2cdex+ce^2x^2}}{3c^2} & \text{for } e \neq 0 \\ \frac{d^5x}{(cd^2)^{3/2}} & \text{otherwise} \end{cases}$$

input `integrate((e*x+d)**5/(c*e**2*x**2+2*c*d*e*x+c*d**2)**(3/2), x)`

output `Piecewise((d**2*sqrt(c*d**2 + 2*c*d*e*x + c*e**2*x**2)/(3*c**2*e) + 2*d*x*sqrt(c*d**2 + 2*c*d*e*x + c*e**2*x**2)/(3*c**2) + e*x**2*sqrt(c*d**2 + 2*c*d*e*x + c*e**2*x**2)/(3*c**2), Ne(e, 0)), (d**5*x/(c*d**2)**(3/2), True))`

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 135 vs. $2(30) = 60$.

Time = 0.04 (sec) , antiderivative size = 135, normalized size of antiderivative = 3.97

$$\int \frac{(d+ex)^5}{(cd^2+2cdex+ce^2x^2)^{3/2}} dx = \frac{e^3x^4}{3\sqrt{ce^2x^2+2cdex+cd^2c}} + \frac{4de^2x^3}{3\sqrt{ce^2x^2+2cdex+cd^2c}} + \frac{2d^2ex^2}{\sqrt{ce^2x^2+2cdex+cd^2c}} - \frac{d^4}{\sqrt{ce^2x^2+2cdex+cd^2c}}$$

input `integrate((e*x+d)^5/(c*e^2*x^2+2*c*d*e*x+c*d^2)^(3/2), x, algorithm="maxima")`

output `1/3*e^3*x^4/(sqrt(c*e^2*x^2 + 2*c*d*e*x + c*d^2)*c) + 4/3*d*e^2*x^3/(sqrt(c*e^2*x^2 + 2*c*d*e*x + c*d^2)*c) + 2*d^2*e*x^2/(sqrt(c*e^2*x^2 + 2*c*d*e*x + c*d^2)*c) - d^4/(sqrt(c*e^2*x^2 + 2*c*d*e*x + c*d^2)*c*e)`

Giac [F(-2)]

Exception generated.

$$\int \frac{(d + ex)^5}{(cd^2 + 2cdex + ce^2x^2)^{3/2}} dx = \text{Exception raised: TypeError}$$

input `integrate((e*x+d)^5/(c*e^2*x^2+2*c*d*e*x+c*d^2)^(3/2),x, algorithm="giac")`

output `Exception raised: TypeError >> an error occurred running a Giac command:INPUT:sage2:=int(sage0,sageVARx)::OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const vecteur & l) Error: Bad Argument Value`

Mupad [F(-1)]

Timed out.

$$\int \frac{(d + ex)^5}{(cd^2 + 2cdex + ce^2x^2)^{3/2}} dx = \int \frac{(d + ex)^5}{(cd^2 + 2cdex + ce^2x^2)^{3/2}} dx$$

input `int((d + e*x)^5/(c*d^2 + c*e^2*x^2 + 2*c*d*e*x)^(3/2),x)`

output `int((d + e*x)^5/(c*d^2 + c*e^2*x^2 + 2*c*d*e*x)^(3/2), x)`

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.76

$$\int \frac{(d + ex)^5}{(cd^2 + 2cdex + ce^2x^2)^{3/2}} dx = \frac{\sqrt{c}x(e^2x^2 + 3dex + 3d^2)}{3c^2}$$

input `int((e*x+d)^5/(c*e^2*x^2+2*c*d*e*x+c*d^2)^(3/2),x)`

output `(sqrt(c)*x*(3*d**2 + 3*d*e*x + e**2*x**2))/(3*c**2)`

$$3.246 \quad \int \frac{(d+ex)^3}{(cd^2+2cdex+ce^2x^2)^{3/2}} dx$$

| | |
|---|------|
| Optimal result | 1913 |
| Mathematica [A] (verified) | 1913 |
| Rubi [A] (verified) | 1914 |
| Maple [A] (verified) | 1914 |
| Fricas [A] (verification not implemented) | 1915 |
| Sympy [A] (verification not implemented) | 1916 |
| Maxima [B] (verification not implemented) | 1916 |
| Giac [F(-2)] | 1917 |
| Mupad [F(-1)] | 1917 |
| Reduce [B] (verification not implemented) | 1917 |

Optimal result

Integrand size = 32, antiderivative size = 31

$$\int \frac{(d+ex)^3}{(cd^2+2cdex+ce^2x^2)^{3/2}} dx = \frac{\sqrt{cd^2+2cdex+ce^2x^2}}{c^2e}$$

output $(c \cdot e^2 \cdot x^2 + 2 \cdot c \cdot d \cdot e \cdot x + c \cdot d^2)^{(1/2)} / c^2 / e$

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.65

$$\int \frac{(d+ex)^3}{(cd^2+2cdex+ce^2x^2)^{3/2}} dx = \frac{\sqrt{c(d+ex)^2}}{c^2e}$$

input `Integrate[(d + e*x)^3/(c*d^2 + 2*c*d*e*x + c*e^2*x^2)^(3/2),x]`

output `Sqrt[c*(d + e*x)^2]/(c^2*e)`

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.031$, Rules used = {1099}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d + ex)^3}{(cd^2 + 2cdex + ce^2x^2)^{3/2}} dx$$

↓ 1099

$$\frac{\sqrt{cd^2 + 2cdex + ce^2x^2}}{c^2e}$$

input `Int[(d + e*x)^3/(c*d^2 + 2*c*d*e*x + c*e^2*x^2)^(3/2),x]`

output `Sqrt[c*d^2 + 2*c*d*e*x + c*e^2*x^2]/(c^2*e)`

Defintions of rubi rules used

rule 1099

```
Int[((d_) + (e_)*(x_))^(m_)*((a_) + (b_)*(x_) + (c_)*(x_)^2)^(p_), x_Sy
mbol] := Simp[e^m*((a + b*x + c*x^2)^(p + (m + 1)/2)/(c^((m + 1)/2)*(m + 2*
p + 1))), x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[b^2 - 4*a*c, 0] && EqQ[
2*c*d - b*e, 0] && IntegerQ[(m - 1)/2]
```

Maple [A] (verified)

Time = 1.25 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.61

| method | result | size |
|----------------|--|------|
| pseudoelliptic | $\frac{\sqrt{c(ex+d)^2}}{c^2e}$ | 19 |
| risch | $\frac{(ex+d)x}{c\sqrt{c(ex+d)^2}}$ | 22 |
| default | $\frac{x(ex+d)^3}{(x^2ce^2+2cdxe+cd^2)^{\frac{3}{2}}}$ | 32 |
| orering | $\frac{x(ex+d)^3}{(x^2ce^2+2cdxe+cd^2)^{\frac{3}{2}}}$ | 32 |
| trager | $\frac{x\sqrt{x^2ce^2+2cdxe+cd^2}}{c^2(ex+d)}$ | 35 |

input `int((e*x+d)^3/(c*e^2*x^2+2*c*d*e*x+c*d^2)^(3/2),x,method=_RETURNVERBOSE)`

output `(c*(e*x+d)^2)^(1/2)/c^2/e`

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.23

$$\int \frac{(d+ex)^3}{(cd^2+2cdex+ce^2x^2)^{3/2}} dx = \frac{\sqrt{ce^2x^2+2cdex+cd^2}x}{c^2ex+c^2d}$$

input `integrate((e*x+d)^3/(c*e^2*x^2+2*c*d*e*x+c*d^2)^(3/2),x, algorithm="fricas")`

output `sqrt(c*e^2*x^2 + 2*c*d*e*x + c*d^2)*x/(c^2*e*x + c^2*d)`

Sympy [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.35

$$\int \frac{(d + ex)^3}{(cd^2 + 2cdex + ce^2x^2)^{3/2}} dx = \begin{cases} \frac{\sqrt{cd^2+2cdex+ce^2x^2}}{c^2e} & \text{for } e \neq 0 \\ \frac{d^3x}{(cd^2)^{3/2}} & \text{otherwise} \end{cases}$$

input `integrate((e*x+d)**3/(c*e**2*x**2+2*c*d*e*x+c*d**2)**(3/2),x)`

output `Piecewise((sqrt(c*d**2 + 2*c*d*e*x + c*e**2*x**2)/(c**2*e), Ne(e, 0)), (d**3*x/(c*d**2)**(3/2), True))`

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 64 vs. 2(29) = 58.

Time = 0.04 (sec) , antiderivative size = 64, normalized size of antiderivative = 2.06

$$\int \frac{(d + ex)^3}{(cd^2 + 2cdex + ce^2x^2)^{3/2}} dx = \frac{ex^2}{\sqrt{ce^2x^2 + 2cdex + cd^2c}} - \frac{d^2}{\sqrt{ce^2x^2 + 2cdex + cd^2ce}}$$

input `integrate((e*x+d)^3/(c*e^2*x^2+2*c*d*e*x+c*d^2)^(3/2),x, algorithm="maxima")`

output `e*x^2/(sqrt(c*e^2*x^2 + 2*c*d*e*x + c*d^2)*c) - d^2/(sqrt(c*e^2*x^2 + 2*c*d*e*x + c*d^2)*c*e)`

Giac [F(-2)]

Exception generated.

$$\int \frac{(d + ex)^3}{(cd^2 + 2cdex + ce^2x^2)^{3/2}} dx = \text{Exception raised: TypeError}$$

input `integrate((e*x+d)^3/(c*e^2*x^2+2*c*d*e*x+c*d^2)^(3/2),x, algorithm="giac")`

output `Exception raised: TypeError >> an error occurred running a Giac command:INPUT:sage2:=int(sage0,sageVARx)::OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const vecteur & l) Error: Bad Argument Value`

Mupad [F(-1)]

Timed out.

$$\int \frac{(d + ex)^3}{(cd^2 + 2cdex + ce^2x^2)^{3/2}} dx = \int \frac{(d + ex)^3}{(cd^2 + 2cdex + ce^2x^2)^{3/2}} dx$$

input `int((d + e*x)^3/(c*d^2 + c*e^2*x^2 + 2*c*d*e*x)^(3/2),x)`

output `int((d + e*x)^3/(c*d^2 + c*e^2*x^2 + 2*c*d*e*x)^(3/2), x)`

Reduce [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.23

$$\int \frac{(d + ex)^3}{(cd^2 + 2cdex + ce^2x^2)^{3/2}} dx = \frac{\sqrt{c}x}{c^2}$$

input `int((e*x+d)^3/(c*e^2*x^2+2*c*d*e*x+c*d^2)^(3/2),x)`

output `(sqrt(c)*x)/c**2`

3.247 $\int \frac{d+ex}{(cd^2+2cdex+ce^2x^2)^{3/2}} dx$

| | |
|---|------|
| Optimal result | 1918 |
| Mathematica [A] (verified) | 1918 |
| Rubi [A] (verified) | 1919 |
| Maple [A] (verified) | 1919 |
| Fricas [A] (verification not implemented) | 1920 |
| Sympy [A] (verification not implemented) | 1921 |
| Maxima [A] (verification not implemented) | 1921 |
| Giac [F(-2)] | 1921 |
| Mupad [B] (verification not implemented) | 1922 |
| Reduce [B] (verification not implemented) | 1922 |

Optimal result

Integrand size = 30, antiderivative size = 32

$$\int \frac{d+ex}{(cd^2+2cdex+ce^2x^2)^{3/2}} dx = -\frac{1}{ce\sqrt{cd^2+2cdex+ce^2x^2}}$$

output `-1/c/e/(c*e^2*x^2+2*c*d*e*x+c*d^2)^(1/2)`

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.66

$$\int \frac{d+ex}{(cd^2+2cdex+ce^2x^2)^{3/2}} dx = -\frac{1}{ce\sqrt{c(d+ex)^2}}$$

input `Integrate[(d + e*x)/(c*d^2 + 2*c*d*e*x + c*e^2*x^2)^(3/2), x]`

output `-(1/(c*e*Sqrt[c*(d + e*x)^2]))`

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.033$, Rules used = {1099}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{d + ex}{(cd^2 + 2cdex + ce^2x^2)^{3/2}} dx$$

↓ 1099

$$-\frac{1}{ce\sqrt{cd^2 + 2cdex + ce^2x^2}}$$

input `Int[(d + e*x)/(c*d^2 + 2*c*d*e*x + c*e^2*x^2)^(3/2),x]`

output `-(1/(c*e*Sqrt[c*d^2 + 2*c*d*e*x + c*e^2*x^2]))`

Defintions of rubi rules used

rule 1099 `Int[((d_) + (e_)*(x_))^(m_)*((a_) + (b_)*(x_) + (c_)*(x_)^2)^(p_), x_Sy
mbol] := Simp[e^m*((a + b*x + c*x^2)^(p + (m + 1)/2)/(c^((m + 1)/2)*(m + 2*
p + 1))), x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[b^2 - 4*a*c, 0] && EqQ[
2*c*d - b*e, 0] && IntegerQ[(m - 1)/2]`

Maple [A] (verified)

Time = 1.06 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.62

| method | result | size |
|----------------|---|------|
| risch | $-\frac{1}{c\sqrt{c(ex+d)^2}e}$ | 20 |
| pseudoelliptic | $-\frac{1}{c\sqrt{c(ex+d)^2}e}$ | 20 |
| gosper | $-\frac{(ex+d)^2}{e(x^2ce^2+2cdxe+cd^2)^{\frac{3}{2}}}$ | 35 |
| default | $-\frac{(ex+d)^2}{e(x^2ce^2+2cdxe+cd^2)^{\frac{3}{2}}}$ | 35 |
| orering | $-\frac{(ex+d)^2}{e(x^2ce^2+2cdxe+cd^2)^{\frac{3}{2}}}$ | 35 |
| trager | $\frac{x\sqrt{x^2ce^2+2cdxe+cd^2}}{c^2d(ex+d)^2}$ | 38 |

input `int((e*x+d)/(c*e^2*x^2+2*c*d*e*x+c*d^2)^(3/2),x,method=_RETURNVERBOSE)`

output `-1/c/(c*(e*x+d)^2)^(1/2)/e`

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.72

$$\int \frac{d+ex}{(cd^2+2cdex+ce^2x^2)^{3/2}} dx = -\frac{\sqrt{ce^2x^2+2cdex+cd^2}}{c^2e^3x^2+2c^2de^2x+c^2d^2e}$$

input `integrate((e*x+d)/(c*e^2*x^2+2*c*d*e*x+c*d^2)^(3/2),x, algorithm="fricas")`

output `-sqrt(c*e^2*x^2 + 2*c*d*e*x + c*d^2)/(c^2*e^3*x^2 + 2*c^2*d*e^2*x + c^2*d^2*e)`

Sympy [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.31

$$\int \frac{d + ex}{(cd^2 + 2cdex + ce^2x^2)^{3/2}} dx = \begin{cases} -\frac{1}{ce\sqrt{cd^2+2cdex+ce^2x^2}} & \text{for } e \neq 0 \\ \frac{dx}{(cd^2)^{3/2}} & \text{otherwise} \end{cases}$$

input `integrate((e*x+d)/(c*e**2*x**2+2*c*d*e*x+c*d**2)**(3/2),x)`output `Piecewise((-1/(c*e*sqrt(c*d**2 + 2*c*d*e*x + c*e**2*x**2)), Ne(e, 0)), (d*x/(c*d**2)**(3/2), True))`**Maxima [A] (verification not implemented)**

Time = 0.03 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.94

$$\int \frac{d + ex}{(cd^2 + 2cdex + ce^2x^2)^{3/2}} dx = -\frac{1}{\sqrt{ce^2x^2 + 2cdex + cd^2}ce}$$

input `integrate((e*x+d)/(c*e^2*x^2+2*c*d*e*x+c*d^2)^(3/2),x, algorithm="maxima")`output `-1/(sqrt(c*e^2*x^2 + 2*c*d*e*x + c*d^2)*c*e)`**Giac [F(-2)]**

Exception generated.

$$\int \frac{d + ex}{(cd^2 + 2cdex + ce^2x^2)^{3/2}} dx = \text{Exception raised: TypeError}$$

input `integrate((e*x+d)/(c*e^2*x^2+2*c*d*e*x+c*d^2)^(3/2),x, algorithm="giac")`

output

```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx);OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [B] (verification not implemented)

Time = 8.79 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.16

$$\int \frac{d + ex}{(cd^2 + 2cdex + ce^2x^2)^{3/2}} dx = -\frac{\sqrt{cd^2 + 2cdex + ce^2x^2}}{c^2e(d + ex)^2}$$

input

```
int((d + e*x)/(c*d^2 + c*e^2*x^2 + 2*c*d*e*x)^(3/2),x)
```

output

```
-(c*d^2 + c*e^2*x^2 + 2*c*d*e*x)^(1/2)/(c^2*e*(d + e*x)^2)
```

Reduce [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.53

$$\int \frac{d + ex}{(cd^2 + 2cdex + ce^2x^2)^{3/2}} dx = \frac{\sqrt{c}x}{c^2d(ex + d)}$$

input

```
int((e*x+d)/(c*e^2*x^2+2*c*d*e*x+c*d^2)^(3/2),x)
```

output

```
(sqrt(c)*x)/(c**2*d*(d + e*x))
```

$$3.248 \quad \int \frac{1}{(d+ex)(cd^2+2cdex+ce^2x^2)^{3/2}} dx$$

| | |
|---|------|
| Optimal result | 1923 |
| Mathematica [A] (verified) | 1923 |
| Rubi [A] (verified) | 1924 |
| Maple [A] (verified) | 1924 |
| Fricas [B] (verification not implemented) | 1925 |
| Sympy [A] (verification not implemented) | 1926 |
| Maxima [A] (verification not implemented) | 1926 |
| Giac [F(-2)] | 1926 |
| Mupad [B] (verification not implemented) | 1927 |
| Reduce [B] (verification not implemented) | 1927 |

Optimal result

Integrand size = 32, antiderivative size = 31

$$\int \frac{1}{(d+ex)(cd^2+2cdex+ce^2x^2)^{3/2}} dx = -\frac{1}{3e(cd^2+2cdex+ce^2x^2)^{3/2}}$$

output `-1/3/e/(c*e^2*x^2+2*c*d*e*x+c*d^2)^(3/2)`

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.65

$$\int \frac{1}{(d+ex)(cd^2+2cdex+ce^2x^2)^{3/2}} dx = -\frac{1}{3e(c(d+ex)^2)^{3/2}}$$

input `Integrate[1/((d + e*x)*(c*d^2 + 2*c*d*e*x + c*e^2*x^2)^(3/2)),x]`

output `-1/3*1/(e*(c*(d + e*x)^2)^(3/2))`

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.031$, Rules used = {1099}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(d + ex)(cd^2 + 2cdex + ce^2x^2)^{3/2}} dx$$

↓ 1099

$$-\frac{1}{3e(cd^2 + 2cdex + ce^2x^2)^{3/2}}$$

input `Int[1/((d + e*x)*(c*d^2 + 2*c*d*e*x + c*e^2*x^2)^(3/2)),x]`

output `-1/3*1/(e*(c*d^2 + 2*c*d*e*x + c*e^2*x^2)^(3/2))`

Defintions of rubi rules used

rule 1099

```
Int[((d_) + (e_)*(x_))^(m_)*((a_) + (b_)*(x_) + (c_)*(x_)^2)^(p_), x_Sy
mbol] := Simp[e^m*((a + b*x + c*x^2)^(p + (m + 1)/2)/(c^((m + 1)/2)*(m + 2*
p + 1))), x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[b^2 - 4*a*c, 0] && EqQ[
2*c*d - b*e, 0] && IntegerQ[(m - 1)/2]
```

Maple [A] (verified)

Time = 1.37 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.87

| method | result | size |
|----------------|--|------|
| risch | $-\frac{1}{3c(ex+d)^2\sqrt{c(ex+d)^2e}}$ | 27 |
| pseudoelliptic | $-\frac{1}{3c(ex+d)^2\sqrt{c(ex+d)^2e}}$ | 27 |
| gosper | $-\frac{1}{3e(x^2ce^2+2cdxe+cd^2)^{\frac{3}{2}}}$ | 28 |
| default | $-\frac{1}{3e(x^2ce^2+2cdxe+cd^2)^{\frac{3}{2}}}$ | 28 |
| oring | $-\frac{1}{3e(x^2ce^2+2cdxe+cd^2)^{\frac{3}{2}}}$ | 28 |
| trager | $\frac{(e^2x^2+3dex+3d^2)x\sqrt{x^2ce^2+2cdxe+cd^2}}{3c^2d^3(ex+d)^4}$ | 57 |

input `int(1/(e*x+d)/(c*e^2*x^2+2*c*d*e*x+c*d^2)^(3/2),x,method=_RETURNVERBOSE)`

output `-1/3/c/(e*x+d)^2/(c*(e*x+d)^2)^(1/2)/e`

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 83 vs. $2(27) = 54$.

Time = 0.08 (sec) , antiderivative size = 83, normalized size of antiderivative = 2.68

$$\int \frac{1}{(d+ex)(cd^2+2cdex+ce^2x^2)^{3/2}} dx = \frac{\sqrt{ce^2x^2+2cdex+cd^2}}{3(c^2e^5x^4+4c^2de^4x^3+6c^2d^2e^3x^2+4c^2d^3e^2x+c^2d^4e)}$$

input `integrate(1/(e*x+d)/(c*e^2*x^2+2*c*d*e*x+c*d^2)^(3/2),x, algorithm="fricas")`

output `-1/3*sqrt(c*e^2*x^2+2*c*d*e*x+c*d^2)/(c^2*e^5*x^4+4*c^2*d*e^4*x^3+6*c^2*d^2*e^3*x^2+4*c^2*d^3*e^2*x+c^2*d^4*e)`

Sympy [A] (verification not implemented)

Time = 1.57 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.35

$$\int \frac{1}{(d+ex)(cd^2+2cdex+ce^2x^2)^{3/2}} dx = \begin{cases} -\frac{1}{3e(cd^2+2cdex+ce^2x^2)^{3/2}} & \text{for } e \neq 0 \\ \frac{x}{d(cd^2)^{3/2}} & \text{otherwise} \end{cases}$$

input `integrate(1/(e*x+d)/(c*e**2*x**2+2*c*d*e*x+c*d**2)**(3/2),x)`

output `Piecewise((-1/(3*e*(c*d**2 + 2*c*d*e*x + c*e**2*x**2)**(3/2)), Ne(e, 0)), (x/(d*(c*d**2)**(3/2)), True))`

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.52

$$\int \frac{1}{(d+ex)(cd^2+2cdex+ce^2x^2)^{3/2}} dx = -\frac{1}{3\left(c^{\frac{3}{2}}e^4x^3 + 3c^{\frac{3}{2}}de^3x^2 + 3c^{\frac{3}{2}}d^2e^2x + c^{\frac{3}{2}}d^3e\right)}$$

input `integrate(1/(e*x+d)/(c*e^2*x^2+2*c*d*e*x+c*d^2)^(3/2),x, algorithm="maxima")`

output `-1/3/(c^(3/2)*e^4*x^3 + 3*c^(3/2)*d*e^3*x^2 + 3*c^(3/2)*d^2*e^2*x + c^(3/2)*d^3*e)`

Giac [F(-2)]

Exception generated.

$$\int \frac{1}{(d+ex)(cd^2+2cdex+ce^2x^2)^{3/2}} dx = \text{Exception raised: TypeError}$$

input `integrate(1/(e*x+d)/(c*e^2*x^2+2*c*d*e*x+c*d^2)^(3/2),x, algorithm="giac")`

output Exception raised: TypeError >> an error occurred running a Giac command:IN
 PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
 index_m & i,const vecteur & l) Error: Bad Argument Value

Mupad [B] (verification not implemented)

Time = 8.86 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.19

$$\int \frac{1}{(d+ex)(cd^2+2cdex+ce^2x^2)^{3/2}} dx = -\frac{\sqrt{cd^2+2cdex+ce^2x^2}}{3c^2e(d+ex)^4}$$

input `int(1/((d + e*x)*(c*d^2 + c*e^2*x^2 + 2*c*d*e*x)^(3/2)),x)`

output `-(c*d^2 + c*e^2*x^2 + 2*c*d*e*x)^(1/2)/(3*c^2*e*(d + e*x)^4)`

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.26

$$\int \frac{1}{(d+ex)(cd^2+2cdex+ce^2x^2)^{3/2}} dx = -\frac{\sqrt{c}}{3c^2e(e^3x^3+3de^2x^2+3d^2ex+d^3)}$$

input `int(1/(e*x+d)/(c*e^2*x^2+2*c*d*e*x+c*d^2)^(3/2),x)`

output `(- sqrt(c))/(3*c**2*e*(d**3 + 3*d**2*e*x + 3*d*e**2*x**2 + e**3*x**3))`

$$3.249 \quad \int \frac{1}{(d+ex)^3 (cd^2+2cdex+ce^2x^2)^{3/2}} dx$$

| | |
|---|------|
| Optimal result | 1928 |
| Mathematica [A] (verified) | 1928 |
| Rubi [A] (verified) | 1929 |
| Maple [A] (verified) | 1929 |
| Fricas [B] (verification not implemented) | 1930 |
| Sympy [F] | 1931 |
| Maxima [B] (verification not implemented) | 1931 |
| Giac [F(-2)] | 1932 |
| Mupad [B] (verification not implemented) | 1932 |
| Reduce [B] (verification not implemented) | 1932 |

Optimal result

Integrand size = 32, antiderivative size = 32

$$\int \frac{1}{(d+ex)^3 (cd^2+2cdex+ce^2x^2)^{3/2}} dx = -\frac{c}{5e (cd^2+2cdex+ce^2x^2)^{5/2}}$$

output `-1/5*c/e/(c*e^2*x^2+2*c*d*e*x+c*d^2)^(5/2)`

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.66

$$\int \frac{1}{(d+ex)^3 (cd^2+2cdex+ce^2x^2)^{3/2}} dx = -\frac{c}{5e (c(d+ex)^2)^{5/2}}$$

input `Integrate[1/((d + e*x)^3*(c*d^2 + 2*c*d*e*x + c*e^2*x^2)^(3/2)),x]`

output `-1/5*c/(e*(c*(d + e*x)^2)^(5/2))`

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.031$, Rules used = {1099}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(d + ex)^3 (cd^2 + 2cdex + ce^2x^2)^{3/2}} dx$$

$$\downarrow 1099$$

$$-\frac{c}{5e(cd^2 + 2cdex + ce^2x^2)^{5/2}}$$

input `Int[1/((d + e*x)^3*(c*d^2 + 2*c*d*e*x + c*e^2*x^2)^(3/2)),x]`

output `-1/5*c/(e*(c*d^2 + 2*c*d*e*x + c*e^2*x^2)^(5/2))`

Defintions of rubi rules used

rule 1099 `Int[((d_) + (e_)*(x_))^(m_)*((a_) + (b_)*(x_) + (c_)*(x_)^2)^(p_), x_Sy
mbol] := Simp[e^m*((a + b*x + c*x^2)^(p + (m + 1)/2)/(c^((m + 1)/2)*(m + 2*
p + 1))), x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[b^2 - 4*a*c, 0] && EqQ[
2*c*d - b*e, 0] && IntegerQ[(m - 1)/2]`

Maple [A] (verified)

Time = 1.41 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.84

| method | result | size |
|----------------|--|------|
| risch | $-\frac{1}{5c(ex+d)^4\sqrt{c(ex+d)^2e}}$ | 27 |
| pseudoelliptic | $-\frac{1}{5c(ex+d)^4\sqrt{c(ex+d)^2e}}$ | 27 |
| gospers | $-\frac{1}{5(ex+d)^2e(x^2ce^2+2cdxe+cd^2)^{\frac{3}{2}}}$ | 35 |
| default | $-\frac{1}{5(ex+d)^2e(x^2ce^2+2cdxe+cd^2)^{\frac{3}{2}}}$ | 35 |
| orering | $-\frac{1}{5(ex+d)^2e(x^2ce^2+2cdxe+cd^2)^{\frac{3}{2}}}$ | 35 |
| trager | $\frac{(e^4x^4+5de^3x^3+10d^2e^2x^2+10d^3ex+5d^4)x\sqrt{x^2ce^2+2cdxe+cd^2}}{5c^2d^5(ex+d)^6}$ | 79 |

input `int(1/(e*x+d)^3/(c*e^2*x^2+2*c*d*e*x+c*d^2)^(3/2),x,method=_RETURNVERBOSE)`

output `-1/5/c/(e*x+d)^4/(c*(e*x+d)^2)^(1/2)/e`

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 111 vs. $2(28) = 56$.

Time = 0.08 (sec) , antiderivative size = 111, normalized size of antiderivative = 3.47

$$\int \frac{1}{(d+ex)^3 (cd^2 + 2cdex + ce^2x^2)^{3/2}} dx =$$

$$\frac{\sqrt{ce^2x^2 + 2cdex + cd^2}}{5(c^2e^7x^6 + 6c^2de^6x^5 + 15c^2d^2e^5x^4 + 20c^2d^3e^4x^3 + 15c^2d^4e^3x^2 + 6c^2d^5e^2x + c^2d^6e)}$$

input `integrate(1/(e*x+d)^3/(c*e^2*x^2+2*c*d*e*x+c*d^2)^(3/2),x, algorithm="fricas")`

output `-1/5*sqrt(c*e^2*x^2 + 2*c*d*e*x + c*d^2)/(c^2*e^7*x^6 + 6*c^2*d*e^6*x^5 + 15*c^2*d^2*e^5*x^4 + 20*c^2*d^3*e^4*x^3 + 15*c^2*d^4*e^3*x^2 + 6*c^2*d^5*e^2*x + c^2*d^6*e)`

Sympy [F]

$$\int \frac{1}{(d+ex)^3 (cd^2 + 2cdex + ce^2x^2)^{3/2}} dx = \int \frac{1}{(c(d+ex)^2)^{\frac{3}{2}} (d+ex)^3} dx$$

input `integrate(1/(e*x+d)**3/(c*e**2*x**2+2*c*d*e*x+c*d**2)**(3/2),x)`

output `Integral(1/((c*(d + e*x)**2)**(3/2)*(d + e*x)**3), x)`

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 75 vs. $2(28) = 56$.

Time = 0.04 (sec) , antiderivative size = 75, normalized size of antiderivative = 2.34

$$\int \frac{1}{(d+ex)^3 (cd^2 + 2cdex + ce^2x^2)^{3/2}} dx = \frac{1}{5 \left(c^{\frac{3}{2}} e^6 x^5 + 5 c^{\frac{3}{2}} d e^5 x^4 + 10 c^{\frac{3}{2}} d^2 e^4 x^3 + 10 c^{\frac{3}{2}} d^3 e^3 x^2 + 5 c^{\frac{3}{2}} d^4 e^2 x + c^{\frac{3}{2}} d^5 e \right)}$$

input `integrate(1/(e*x+d)^3/(c*e^2*x^2+2*c*d*e*x+c*d^2)^(3/2),x, algorithm="maxima")`

output `-1/5/(c^(3/2)*e^6*x^5 + 5*c^(3/2)*d*e^5*x^4 + 10*c^(3/2)*d^2*e^4*x^3 + 10*c^(3/2)*d^3*e^3*x^2 + 5*c^(3/2)*d^4*e^2*x + c^(3/2)*d^5*e)`

Giac [F(-2)]

Exception generated.

$$\int \frac{1}{(d+ex)^3 (cd^2 + 2cdex + ce^2x^2)^{3/2}} dx = \text{Exception raised: TypeError}$$

input `integrate(1/(e*x+d)^3/(c*e^2*x^2+2*c*d*e*x+c*d^2)^(3/2),x, algorithm="giac")`

output `Exception raised: TypeError >> an error occurred running a Giac command:INPUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const vecteur & l) Error: Bad Argument Value`

Mupad [B] (verification not implemented)

Time = 8.93 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.16

$$\int \frac{1}{(d+ex)^3 (cd^2 + 2cdex + ce^2x^2)^{3/2}} dx = -\frac{\sqrt{cd^2 + 2cdex + ce^2x^2}}{5c^2e(d+ex)^6}$$

input `int(1/((d + e*x)^3*(c*d^2 + c*e^2*x^2 + 2*c*d*e*x)^(3/2)),x)`

output `-(c*d^2 + c*e^2*x^2 + 2*c*d*e*x)^(1/2)/(5*c^2*e*(d + e*x)^6)`

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.91

$$\int \frac{1}{(d+ex)^3 (cd^2 + 2cdex + ce^2x^2)^{3/2}} dx = \frac{\sqrt{c}}{5c^2e(e^5x^5 + 5de^4x^4 + 10d^2e^3x^3 + 10d^3e^2x^2 + 5d^4ex + d^5)}$$

input `int(1/(e*x+d)^3/(c*e^2*x^2+2*c*d*e*x+c*d^2)^(3/2),x)`

output $(-\sqrt{c})/(5c^2e(d^5 + 5d^4ex + 10d^3e^2x^2 + 10d^2e^3x^3 + 5de^4x^4 + e^5x^5))$

$$3.250 \quad \int \frac{(d+ex)^4}{(cd^2+2cdex+ce^2x^2)^{3/2}} dx$$

| | |
|---|------|
| Optimal result | 1934 |
| Mathematica [A] (verified) | 1934 |
| Rubi [A] (verified) | 1935 |
| Maple [A] (verified) | 1936 |
| Fricas [A] (verification not implemented) | 1936 |
| Sympy [A] (verification not implemented) | 1937 |
| Maxima [B] (verification not implemented) | 1937 |
| Giac [F(-2)] | 1938 |
| Mupad [F(-1)] | 1938 |
| Reduce [B] (verification not implemented) | 1939 |

Optimal result

Integrand size = 32, antiderivative size = 39

$$\int \frac{(d+ex)^4}{(cd^2+2cdex+ce^2x^2)^{3/2}} dx = \frac{(d+ex)\sqrt{cd^2+2cdex+ce^2x^2}}{2c^2e}$$

output $1/2*(e*x+d)*(c*e^2*x^2+2*c*d*e*x+c*d^2)^{(1/2)}/c^2/e$

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.85

$$\int \frac{(d+ex)^4}{(cd^2+2cdex+ce^2x^2)^{3/2}} dx = \frac{x(d+ex)(2d+ex)}{2c\sqrt{c(d+ex)^2}}$$

input $\text{Integrate}[(d+e*x)^4/(c*d^2+2*c*d*e*x+c*e^2*x^2)^{(3/2)},x]$

output $(x*(d+e*x)*(2*d+e*x))/(2*c*\text{Sqrt}[c*(d+e*x)^2])$

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.05, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.062$, Rules used = {1102, 17}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex)^4}{(cd^2+2cdex+ce^2x^2)^{3/2}} dx$$

$$\downarrow 1102$$

$$\frac{c^2e^3(d+ex) \int \frac{d+ex}{c^3e^3} dx}{\sqrt{cd^2+2cdex+ce^2x^2}}$$

$$\downarrow 17$$

$$\frac{(d+ex)^3}{2ce\sqrt{cd^2+2cdex+ce^2x^2}}$$

input `Int[(d + e*x)^4/(c*d^2 + 2*c*d*e*x + c*e^2*x^2)^(3/2),x]`

output `(d + e*x)^3/(2*c*e*sqrt[c*d^2 + 2*c*d*e*x + c*e^2*x^2])`

Defintions of rubi rules used

rule 17 `Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] := Simp[c*((a + b*x)^(m + 1))/(b*(m + 1)), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]`

rule 1102 `Int[((d_.) + (e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] := Simp[(a + b*x + c*x^2)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x)^(2*FracPart[p])) Int[(d + e*x)^m*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && EqQ[b^2 - 4*a*c, 0]`

Maple [A] (verified)

Time = 1.48 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.03

| method | result | size |
|---------|--|------|
| gospers | $\frac{x(ex+2d)(ex+d)^3}{2(x^2ce^2+2cdxe+cd^2)^{\frac{3}{2}}}$ | 40 |
| default | $\frac{x(ex+2d)(ex+d)^3}{2(x^2ce^2+2cdxe+cd^2)^{\frac{3}{2}}}$ | 40 |
| orering | $\frac{x(ex+2d)(ex+d)^3}{2(x^2ce^2+2cdxe+cd^2)^{\frac{3}{2}}}$ | 40 |
| trager | $\frac{x(ex+2d)\sqrt{x^2ce^2+2cdxe+cd^2}}{2c^2(ex+d)}$ | 43 |
| risch | $\frac{(ex+d)ex^2}{2c\sqrt{c(ex+d)^2}} + \frac{(ex+d)dx}{c\sqrt{c(ex+d)^2}}$ | 49 |

input `int((e*x+d)^4/(c*e^2*x^2+2*c*d*e*x+c*d^2)^(3/2),x,method=_RETURNVERBOSE)`

output `1/2*x*(e*x+2*d)*(e*x+d)^3/(c*e^2*x^2+2*c*d*e*x+c*d^2)^(3/2)`

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.23

$$\int \frac{(d+ex)^4}{(cd^2+2cdex+ce^2x^2)^{3/2}} dx = \frac{\sqrt{ce^2x^2+2cdex+cd^2}(ex^2+2dx)}{2(c^2ex+c^2d)}$$

input `integrate((e*x+d)^4/(c*e^2*x^2+2*c*d*e*x+c*d^2)^(3/2),x, algorithm="fricas")`

output `1/2*sqrt(c*e^2*x^2 + 2*c*d*e*x + c*d^2)*(e*x^2 + 2*d*x)/(c^2*e*x + c^2*d)`

Sympy [A] (verification not implemented)

Time = 2.35 (sec) , antiderivative size = 223, normalized size of antiderivative = 5.72

$$\int \frac{(d+ex)^4}{(cd^2+2cdex+ce^2x^2)^{3/2}} dx = \frac{\left(\frac{d}{2ce} + \frac{x}{2c} \right) \sqrt{cd^2+2cdex+ce^2x^2} + \frac{2d^2\sqrt{cd^2+2cdex} + \frac{2\left(-cd^2\sqrt{cd^2+2cdex} + \frac{(cd^2+2cdex)^{3/2}}{3}\right)}{c}}{2cde} + \frac{c^2d^4\sqrt{cd^2+2cdex} - \frac{2cd^2(cd^2+2cdex)^{3/2}}{3}}{2c^2d^2} + \frac{d^2x+dex^2+\frac{e^2x^3}{3}}{\sqrt{cd^2}}}{c}$$

input `integrate((e*x+d)**4/(c*e**2*x**2+2*c*d*e*x+c*d**2)**(3/2),x)`

output `Piecewise(((d/(2*c*e) + x/(2*c))*sqrt(c*d**2 + 2*c*d*e*x + c*e**2*x**2), Ne(c*e**2, 0)), ((2*d**2*sqrt(c*d**2 + 2*c*d*e*x) + 2*(-c*d**2*sqrt(c*d**2 + 2*c*d*e*x) + (c*d**2 + 2*c*d*e*x)**(3/2)/3)/c + (c**2*d**4*sqrt(c*d**2 + 2*c*d*e*x) - 2*c*d**2*(c*d**2 + 2*c*d*e*x)**(3/2)/3 + (c*d**2 + 2*c*d*e*x)**(5/2)/5)/(2*c**2*d**2))/(2*c*d*e), Ne(c*d*e, 0)), ((d**2*x + d*e*x**2 + e**2*x**3/3)/sqrt(c*d**2), True))/c`

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 99 vs. $2(35) = 70$.

Time = 0.04 (sec) , antiderivative size = 99, normalized size of antiderivative = 2.54

$$\int \frac{(d+ex)^4}{(cd^2+2cdex+ce^2x^2)^{3/2}} dx = \frac{e^2x^3}{2\sqrt{ce^2x^2+2cdex+cd^2c}} + \frac{3dex^2}{2\sqrt{ce^2x^2+2cdex+cd^2c}} - \frac{d^3}{\sqrt{ce^2x^2+2cdex+cd^2c}}$$

input `integrate((e*x+d)^4/(c*e^2*x^2+2*c*d*e*x+c*d^2)^(3/2),x, algorithm="maxima")`

output

```
1/2*e^2*x^3/(sqrt(c*e^2*x^2 + 2*c*d*e*x + c*d^2)*c) + 3/2*d*e*x^2/(sqrt(c*
e^2*x^2 + 2*c*d*e*x + c*d^2)*c) - d^3/(sqrt(c*e^2*x^2 + 2*c*d*e*x + c*d^2)
*c*e)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{(d + ex)^4}{(cd^2 + 2cdex + ce^2x^2)^{3/2}} dx = \text{Exception raised: TypeError}$$

input

```
integrate((e*x+d)^4/(c*e^2*x^2+2*c*d*e*x+c*d^2)^(3/2),x, algorithm="giac")
```

output

```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d + ex)^4}{(cd^2 + 2cdex + ce^2x^2)^{3/2}} dx = \int \frac{(d + ex)^4}{(cd^2 + 2cdex + ce^2x^2)^{3/2}} dx$$

input

```
int((d + e*x)^4/(c*d^2 + c*e^2*x^2 + 2*c*d*e*x)^(3/2),x)
```

output

```
int((d + e*x)^4/(c*d^2 + c*e^2*x^2 + 2*c*d*e*x)^(3/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.38

$$\int \frac{(d + ex)^4}{(cd^2 + 2cdex + ce^2x^2)^{3/2}} dx = \frac{\sqrt{c}x(ex + 2d)}{2c^2}$$

input `int((e*x+d)^4/(c*e^2*x^2+2*c*d*e*x+c*d^2)^(3/2),x)`

output `(sqrt(c)*x*(2*d + e*x))/(2*c**2)`

$$3.251 \quad \int \frac{(d+ex)^2}{(cd^2+2cdex+ce^2x^2)^{3/2}} dx$$

| | |
|---|------|
| Optimal result | 1940 |
| Mathematica [A] (verified) | 1940 |
| Rubi [A] (verified) | 1941 |
| Maple [A] (verified) | 1942 |
| Fricas [A] (verification not implemented) | 1942 |
| Sympy [A] (verification not implemented) | 1942 |
| Maxima [B] (verification not implemented) | 1943 |
| Giac [F(-2)] | 1943 |
| Mupad [F(-1)] | 1944 |
| Reduce [B] (verification not implemented) | 1944 |

Optimal result

Integrand size = 32, antiderivative size = 42

$$\int \frac{(d+ex)^2}{(cd^2+2cdex+ce^2x^2)^{3/2}} dx = \frac{(d+ex)\log(d+ex)}{ce\sqrt{cd^2+2cdex+ce^2x^2}}$$

output $(e*x+d)*\ln(e*x+d)/c/e/(c*e^2*x^2+2*c*d*e*x+c*d^2)^{(1/2)}$

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.74

$$\int \frac{(d+ex)^2}{(cd^2+2cdex+ce^2x^2)^{3/2}} dx = \frac{(d+ex)\log(d+ex)}{ce\sqrt{c(d+ex)^2}}$$

input $\text{Integrate}[(d+e*x)^2/(c*d^2+2*c*d*e*x+c*e^2*x^2)^{(3/2)},x]$

output $((d+e*x)*\text{Log}[d+e*x])/(c*e*\text{Sqrt}[c*(d+e*x)^2])$

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.062$, Rules used = {1102, 16}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex)^2}{(cd^2+2cdex+ce^2x^2)^{3/2}} dx$$

$$\downarrow 1102$$

$$\frac{c^2e^3(d+ex) \int \frac{1}{c^3e^3(d+ex)} dx}{\sqrt{cd^2+2cdex+ce^2x^2}}$$

$$\downarrow 16$$

$$\frac{(d+ex) \log(d+ex)}{ce\sqrt{cd^2+2cdex+ce^2x^2}}$$

input `Int[(d + e*x)^2/(c*d^2 + 2*c*d*e*x + c*e^2*x^2)^(3/2), x]`

output `((d + e*x)*Log[d + e*x])/(c*e*Sqrt[c*d^2 + 2*c*d*e*x + c*e^2*x^2])`

Defintions of rubi rules used

rule 16 `Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a + b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]`

rule 1102 `Int[((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(a + b*x + c*x^2)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x)^(2*FracPart[p])) Int[(d + e*x)^m*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && EqQ[b^2 - 4*a*c, 0]`

Maple [A] (verified)

Time = 1.28 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.71

| method | result | size |
|---------|---|------|
| risch | $\frac{(ex+d) \ln(ex+d)}{c\sqrt{c(ex+d)^2 e}}$ | 30 |
| default | $\frac{(ex+d)^3 \ln(ex+d)}{(x^2 c e^2 + 2cdxe + cd^2)^{\frac{3}{2}} e}$ | 40 |

input `int((e*x+d)^2/(c*e^2*x^2+2*c*d*e*x+c*d^2)^(3/2),x,method=_RETURNVERBOSE)`

output `1/c/(c*(e*x+d)^2)^(1/2)*(e*x+d)*ln(e*x+d)/e`

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.10

$$\int \frac{(d+ex)^2}{(cd^2+2cdex+ce^2x^2)^{3/2}} dx = \frac{\sqrt{ce^2x^2+2cdex+cd^2} \log(ex+d)}{c^2e^2x+c^2de}$$

input `integrate((e*x+d)^2/(c*e^2*x^2+2*c*d*e*x+c*d^2)^(3/2),x, algorithm="fricas")`

output `sqrt(c*e^2*x^2+2*c*d*e*x+c*d^2)*log(e*x+d)/(c^2*e^2*x+c^2*d*e)`

Sympy [A] (verification not implemented)

Time = 1.67 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.64

$$\int \frac{(d+ex)^2}{(cd^2+2cdex+ce^2x^2)^{3/2}} dx = \frac{\left(\frac{d}{e}+x\right) \log\left(\frac{d}{e}+x\right)}{c\sqrt{ce^2\left(\frac{d}{e}+x\right)^2}}$$

input `integrate((e*x+d)**2/(c*e**2*x**2+2*c*d*e*x+c*d**2)**(3/2),x)`

output $(d/e + x) \cdot \log(d/e + x) / (c \cdot \sqrt{c \cdot e^{2x} + d/e + x})$

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 123 vs. $2(40) = 80$.

Time = 0.04 (sec) , antiderivative size = 123, normalized size of antiderivative = 2.93

$$\int \frac{(d + ex)^2}{(cd^2 + 2cdex + ce^2x^2)^{3/2}} dx = \frac{3c^2d^2e^4}{2(ce^2)^{7/2}(x + \frac{d}{e})^2} + \frac{2cde^3x}{(ce^2)^{5/2}(x + \frac{d}{e})^2} + \frac{e^2 \log(x + \frac{d}{e})}{(ce^2)^{3/2}} - \frac{2d}{\sqrt{ce^2x^2 + 2cdex + cd^2ce}} + \frac{d^2}{2(ce^2)^{3/2}(x + \frac{d}{e})^2}$$

input `integrate((e*x+d)^2/(c*e^2*x^2+2*c*d*e*x+c*d^2)^(3/2),x, algorithm="maxima")`

output $3/2 \cdot c^2 \cdot d^2 \cdot e^4 / ((c \cdot e^2)^{7/2} \cdot (x + d/e)^2) + 2 \cdot c \cdot d \cdot e^3 \cdot x / ((c \cdot e^2)^{5/2} \cdot (x + d/e)^2) + e^2 \cdot \log(x + d/e) / (c \cdot e^2)^{3/2} - 2 \cdot d / (\sqrt{c \cdot e^2 \cdot x^2 + 2 \cdot c \cdot d \cdot e \cdot x + c \cdot d^2} \cdot c \cdot e) + 1/2 \cdot d^2 / ((c \cdot e^2)^{3/2} \cdot (x + d/e)^2)$

Giac [F(-2)]

Exception generated.

$$\int \frac{(d + ex)^2}{(cd^2 + 2cdex + ce^2x^2)^{3/2}} dx = \text{Exception raised: TypeError}$$

input `integrate((e*x+d)^2/(c*e^2*x^2+2*c*d*e*x+c*d^2)^(3/2),x, algorithm="giac")`

output `Exception raised: TypeError >> an error occurred running a Giac command:INPUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const vecteur & l) Error: Bad Argument Value`

Mupad [F(-1)]

Timed out.

$$\int \frac{(d + ex)^2}{(cd^2 + 2cdex + ce^2x^2)^{3/2}} dx = \int \frac{(d + ex)^2}{(cd^2 + 2cdex + ce^2x^2)^{3/2}} dx$$

input `int((d + e*x)^2/(c*d^2 + c*e^2*x^2 + 2*c*d*e*x)^(3/2), x)`output `int((d + e*x)^2/(c*d^2 + c*e^2*x^2 + 2*c*d*e*x)^(3/2), x)`**Reduce [B] (verification not implemented)**

Time = 0.23 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.36

$$\int \frac{(d + ex)^2}{(cd^2 + 2cdex + ce^2x^2)^{3/2}} dx = \frac{\sqrt{c} \log(ex + d)}{c^2 e}$$

input `int((e*x+d)^2/(c*e^2*x^2+2*c*d*e*x+c*d^2)^(3/2), x)`output `(sqrt(c)*log(d + e*x))/(c**2*e)`

$$3.252 \quad \int \frac{1}{(cd^2 + 2cdex + ce^2x^2)^{3/2}} dx$$

| | |
|---|------|
| Optimal result | 1945 |
| Mathematica [A] (verified) | 1945 |
| Rubi [A] (verified) | 1946 |
| Maple [A] (verified) | 1947 |
| Fricas [A] (verification not implemented) | 1947 |
| Sympy [F] | 1948 |
| Maxima [A] (verification not implemented) | 1948 |
| Giac [F(-2)] | 1948 |
| Mupad [B] (verification not implemented) | 1949 |
| Reduce [B] (verification not implemented) | 1949 |

Optimal result

Integrand size = 24, antiderivative size = 41

$$\int \frac{1}{(cd^2 + 2cdex + ce^2x^2)^{3/2}} dx = -\frac{1}{2ce(d+ex)\sqrt{cd^2 + 2cdex + ce^2x^2}}$$

output `-1/2/c/e/(e*x+d)/(c*e^2*x^2+2*c*d*e*x+c*d^2)^(1/2)`

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.61

$$\int \frac{1}{(cd^2 + 2cdex + ce^2x^2)^{3/2}} dx = -\frac{d+ex}{2e(c(d+ex)^2)^{3/2}}$$

input `Integrate[(c*d^2 + 2*c*d*e*x + c*e^2*x^2)^(-3/2),x]`

output `-1/2*(d + e*x)/(e*(c*(d + e*x)^2)^(3/2))`

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.042$, Rules used = {1078}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(cd^2 + 2cdex + ce^2x^2)^{3/2}} dx$$

↓ 1078

$$-\frac{1}{2ce(d + ex)\sqrt{cd^2 + 2cdex + ce^2x^2}}$$

input `Int[(c*d^2 + 2*c*d*e*x + c*e^2*x^2)^(-3/2), x]`

output `-1/2*1/(c*e*(d + e*x)*Sqrt[c*d^2 + 2*c*d*e*x + c*e^2*x^2])`

Defintions of rubi rules used

rule 1078 `Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[2*((a + b*x + c*x^2)^(p + 1)/((2*p + 1)*(b + 2*c*x))), x] /; FreeQ[{a, b, c, p}, x] && EqQ[b^2 - 4*a*c, 0] && LtQ[p, -1]`

Maple [A] (verified)

Time = 1.14 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.66

| method | result | size |
|---------|---|------|
| risch | $-\frac{1}{2c(ex+d)\sqrt{c(ex+d)^2}e}$ | 27 |
| gosper | $-\frac{ex+d}{2e(x^2ce^2+2cdxe+cd^2)^{\frac{3}{2}}}$ | 33 |
| default | $-\frac{ex+d}{2e(x^2ce^2+2cdxe+cd^2)^{\frac{3}{2}}}$ | 33 |
| orering | $-\frac{ex+d}{2e(x^2ce^2+2cdxe+cd^2)^{\frac{3}{2}}}$ | 33 |
| trager | $\frac{(ex+2d)x\sqrt{x^2ce^2+2cdxe+cd^2}}{2c^2d^2(ex+d)^3}$ | 46 |

input `int(1/(c*e^2*x^2+2*c*d*e*x+c*d^2)^(3/2),x,method=_RETURNVERBOSE)`

output `-1/2/c/(e*x+d)/(c*(e*x+d)^2)^(1/2)/e`

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.68

$$\int \frac{1}{(cd^2 + 2cdex + ce^2x^2)^{3/2}} dx = -\frac{\sqrt{ce^2x^2 + 2cdex + cd^2}}{2(c^2e^4x^3 + 3c^2de^3x^2 + 3c^2d^2e^2x + c^2d^3e)}$$

input `integrate(1/(c*e^2*x^2+2*c*d*e*x+c*d^2)^(3/2),x, algorithm="fricas")`

output `-1/2*sqrt(c*e^2*x^2 + 2*c*d*e*x + c*d^2)/(c^2*e^4*x^3 + 3*c^2*d*e^3*x^2 + 3*c^2*d^2*e^2*x + c^2*d^3*e)`

Sympy [F]

$$\int \frac{1}{(cd^2 + 2cdex + ce^2x^2)^{3/2}} dx = \int \frac{1}{(cd^2 + 2cdex + ce^2x^2)^{\frac{3}{2}}} dx$$

input `integrate(1/(c*e**2*x**2+2*c*d*e*x+c*d**2)**(3/2),x)`

output `Integral((c*d**2 + 2*c*d*e*x + c*e**2*x**2)**(-3/2), x)`

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.44

$$\int \frac{1}{(cd^2 + 2cdex + ce^2x^2)^{3/2}} dx = -\frac{1}{2 (ce^2)^{\frac{3}{2}} (x + \frac{d}{e})^2}$$

input `integrate(1/(c*e^2*x^2+2*c*d*e*x+c*d^2)^(3/2),x, algorithm="maxima")`

output `-1/2/((c*e^2)^(3/2)*(x + d/e)^2)`

Giac [F(-2)]

Exception generated.

$$\int \frac{1}{(cd^2 + 2cdex + ce^2x^2)^{3/2}} dx = \text{Exception raised: TypeError}$$

input `integrate(1/(c*e^2*x^2+2*c*d*e*x+c*d^2)^(3/2),x, algorithm="giac")`

output `Exception raised: TypeError >> an error occurred running a Giac command:INPUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const vecteur & l) Error: Bad Argument Value`

Mupad [B] (verification not implemented)

Time = 9.26 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.90

$$\int \frac{1}{(cd^2 + 2cdex + ce^2x^2)^{3/2}} dx = -\frac{\sqrt{cd^2 + 2cdex + ce^2x^2}}{2c^2e(d + ex)^3}$$

input `int(1/(c*d^2 + c*e^2*x^2 + 2*c*d*e*x)^(3/2),x)`

output `-(c*d^2 + c*e^2*x^2 + 2*c*d*e*x)^(1/2)/(2*c^2*e*(d + e*x)^3)`

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.68

$$\int \frac{1}{(cd^2 + 2cdex + ce^2x^2)^{3/2}} dx = -\frac{\sqrt{c}}{2c^2e(e^2x^2 + 2dex + d^2)}$$

input `int(1/(c*e^2*x^2+2*c*d*e*x+c*d^2)^(3/2),x)`

output `(- sqrt(c))/(2*c**2*e*(d**2 + 2*d*e*x + e**2*x**2))`

$$3.253 \quad \int \frac{1}{(d+ex)^2 (cd^2+2cdex+ce^2x^2)^{3/2}} dx$$

| | |
|---|------|
| Optimal result | 1950 |
| Mathematica [A] (verified) | 1950 |
| Rubi [A] (verified) | 1951 |
| Maple [A] (verified) | 1952 |
| Fricas [B] (verification not implemented) | 1952 |
| Sympy [F] | 1953 |
| Maxima [A] (verification not implemented) | 1953 |
| Giac [F(-2)] | 1953 |
| Mupad [B] (verification not implemented) | 1954 |
| Reduce [B] (verification not implemented) | 1954 |

Optimal result

Integrand size = 32, antiderivative size = 41

$$\int \frac{1}{(d+ex)^2 (cd^2+2cdex+ce^2x^2)^{3/2}} dx = -\frac{1}{4ce(d+ex)^3 \sqrt{cd^2+2cdex+ce^2x^2}}$$

output $-1/4/c/e/(e*x+d)^3/(c*e^2*x^2+2*c*d*e*x+c*d^2)^{(1/2)}$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.63

$$\int \frac{1}{(d+ex)^2 (cd^2+2cdex+ce^2x^2)^{3/2}} dx = -\frac{c(d+ex)}{4e(c(d+ex)^2)^{5/2}}$$

input `Integrate[1/((d + e*x)^2*(c*d^2 + 2*c*d*e*x + c*e^2*x^2)^(3/2)),x]`

output $-1/4*(c*(d + e*x))/(e*(c*(d + e*x)^2)^{(5/2)}$

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.062$, Rules used = {1102, 17}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(d+ex)^2 (cd^2 + 2cdex + ce^2x^2)^{3/2}} dx$$

$$\downarrow 1102$$

$$\frac{c^2e^3(d+ex) \int \frac{1}{c^3e^3(d+ex)^5} dx}{\sqrt{cd^2 + 2cdex + ce^2x^2}}$$

$$\downarrow 17$$

$$-\frac{1}{4ce(d+ex)^3 \sqrt{cd^2 + 2cdex + ce^2x^2}}$$

input `Int[1/((d + e*x)^2*(c*d^2 + 2*c*d*e*x + c*e^2*x^2)^(3/2)),x]`

output `-1/4*1/(c*e*(d + e*x)^3*Sqrt[c*d^2 + 2*c*d*e*x + c*e^2*x^2])`

Defintions of rubi rules used

rule 17 `Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] := Simp[c*((a + b*x)^(m + 1))/(b*(m + 1))], x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]`

rule 1102 `Int[((d_.) + (e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] := Simp[(a + b*x + c*x^2)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x)^(2*FracPart[p])) Int[(d + e*x)^m*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && EqQ[b^2 - 4*a*c, 0]`

Maple [A] (verified)

Time = 1.50 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.66

| method | result | size |
|---------|---|------|
| risch | $-\frac{1}{4c(ex+d)^3 \sqrt{c(ex+d)^2 e}}$ | 27 |
| gospers | $-\frac{1}{4(ex+d)e(x^2ce^2+2cdxe+cd^2)^{\frac{3}{2}}}$ | 35 |
| default | $-\frac{1}{4(ex+d)e(x^2ce^2+2cdxe+cd^2)^{\frac{3}{2}}}$ | 35 |
| orering | $-\frac{1}{4(ex+d)e(x^2ce^2+2cdxe+cd^2)^{\frac{3}{2}}}$ | 35 |
| trager | $\frac{(e^3x^3+4de^2x^2+6d^2ex+4d^3)x\sqrt{x^2ce^2+2cdxe+cd^2}}{4c^2d^4(ex+d)^5}$ | 68 |

input `int(1/(e*x+d)^2/(c*e^2*x^2+2*c*d*e*x+c*d^2)^(3/2),x,method=_RETURNVERBOSE)`

output `-1/4/c/(e*x+d)^3/(c*(e*x+d)^2)^(1/2)/e`

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 97 vs. $2(37) = 74$.

Time = 0.08 (sec) , antiderivative size = 97, normalized size of antiderivative = 2.37

$$\int \frac{1}{(d+ex)^2 (cd^2 + 2cdex + ce^2x^2)^{3/2}} dx =$$

$$-\frac{\sqrt{ce^2x^2 + 2cdex + cd^2}}{4(c^2e^6x^5 + 5c^2de^5x^4 + 10c^2d^2e^4x^3 + 10c^2d^3e^3x^2 + 5c^2d^4e^2x + c^2d^5e)}$$

input `integrate(1/(e*x+d)^2/(c*e^2*x^2+2*c*d*e*x+c*d^2)^(3/2),x, algorithm="fricas")`

output `-1/4*sqrt(c*e^2*x^2 + 2*c*d*e*x + c*d^2)/(c^2*e^6*x^5 + 5*c^2*d*e^5*x^4 + 10*c^2*d^2*e^4*x^3 + 10*c^2*d^3*e^3*x^2 + 5*c^2*d^4*e^2*x + c^2*d^5*e)`

Sympy [F]

$$\int \frac{1}{(d+ex)^2 (cd^2 + 2cdex + ce^2x^2)^{3/2}} dx = \int \frac{1}{(c(d+ex)^2)^{3/2} (d+ex)^2} dx$$

input `integrate(1/(e*x+d)**2/(c*e**2*x**2+2*c*d*e*x+c*d**2)**(3/2),x)`

output `Integral(1/((c*(d + e*x)**2)**(3/2)*(d + e*x)**2), x)`

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.49

$$\int \frac{1}{(d+ex)^2 (cd^2 + 2cdex + ce^2x^2)^{3/2}} dx = \frac{1}{4 \left(c^{\frac{3}{2}} e^5 x^4 + 4 c^{\frac{3}{2}} d e^4 x^3 + 6 c^{\frac{3}{2}} d^2 e^3 x^2 + 4 c^{\frac{3}{2}} d^3 e^2 x + c^{\frac{3}{2}} d^4 e \right)}$$

input `integrate(1/(e*x+d)^2/(c*e^2*x^2+2*c*d*e*x+c*d^2)^(3/2),x, algorithm="maxima")`

output `-1/4/(c^(3/2)*e^5*x^4 + 4*c^(3/2)*d*e^4*x^3 + 6*c^(3/2)*d^2*e^3*x^2 + 4*c^(3/2)*d^3*e^2*x + c^(3/2)*d^4*e)`

Giac [F(-2)]

Exception generated.

$$\int \frac{1}{(d+ex)^2 (cd^2 + 2cdex + ce^2x^2)^{3/2}} dx = \text{Exception raised: TypeError}$$

input `integrate(1/(e*x+d)^2/(c*e^2*x^2+2*c*d*e*x+c*d^2)^(3/2),x, algorithm="giac")`

output

```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx);OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [B] (verification not implemented)

Time = 9.26 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.90

$$\int \frac{1}{(d+ex)^2 (cd^2 + 2cdex + ce^2x^2)^{3/2}} dx = -\frac{\sqrt{cd^2 + 2cdex + ce^2x^2}}{4c^2e(d+ex)^5}$$

input

```
int(1/((d + e*x)^2*(c*d^2 + c*e^2*x^2 + 2*c*d*e*x)^(3/2)),x)
```

output

```
-(c*d^2 + c*e^2*x^2 + 2*c*d*e*x)^(1/2)/(4*c^2*e*(d + e*x)^5)
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.22

$$\int \frac{1}{(d+ex)^2 (cd^2 + 2cdex + ce^2x^2)^{3/2}} dx = -\frac{\sqrt{c}}{4c^2e(e^4x^4 + 4de^3x^3 + 6d^2e^2x^2 + 4d^3ex + d^4)}$$

input

```
int(1/(e*x+d)^2/(c*e^2*x^2+2*c*d*e*x+c*d^2)^(3/2),x)
```

output

```
( - sqrt(c))/(4*c**2*e*(d**4 + 4*d**3*e*x + 6*d**2*e**2*x**2 + 4*d*e**3*x*
*3 + e**4*x**4))
```

$$3.254 \quad \int \frac{(d+ex)^7}{(cd^2+2cdex+ce^2x^2)^{5/2}} dx$$

| | |
|---|------|
| Optimal result | 1955 |
| Mathematica [A] (verified) | 1955 |
| Rubi [A] (verified) | 1956 |
| Maple [A] (verified) | 1956 |
| Fricas [A] (verification not implemented) | 1957 |
| Sympy [B] (verification not implemented) | 1958 |
| Maxima [B] (verification not implemented) | 1958 |
| Giac [F(-2)] | 1959 |
| Mupad [F(-1)] | 1959 |
| Reduce [B] (verification not implemented) | 1960 |

Optimal result

Integrand size = 32, antiderivative size = 34

$$\int \frac{(d+ex)^7}{(cd^2+2cdex+ce^2x^2)^{5/2}} dx = \frac{(cd^2+2cdex+ce^2x^2)^{3/2}}{3c^4e}$$

output $1/3*(c*e^2*x^2+2*c*d*e*x+c*d^2)^{(3/2)}/c^4/e$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.68

$$\int \frac{(d+ex)^7}{(cd^2+2cdex+ce^2x^2)^{5/2}} dx = \frac{(c(d+ex)^2)^{3/2}}{3c^4e}$$

input `Integrate[(d + e*x)^7/(c*d^2 + 2*c*d*e*x + c*e^2*x^2)^(5/2), x]`

output $(c*(d + e*x)^2)^{(3/2)}/(3*c^4*e)$

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.031$, Rules used = {1099}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d + ex)^7}{(cd^2 + 2cdex + ce^2x^2)^{5/2}} dx$$

↓ 1099

$$\frac{(cd^2 + 2cdex + ce^2x^2)^{3/2}}{3c^4e}$$

input `Int[(d + e*x)^7/(c*d^2 + 2*c*d*e*x + c*e^2*x^2)^(5/2),x]`

output `(c*d^2 + 2*c*d*e*x + c*e^2*x^2)^(3/2)/(3*c^4*e)`

Defintions of rubi rules used

rule 1099 `Int[((d_) + (e_)*(x_))^(m_)*((a_) + (b_)*(x_) + (c_)*(x_)^2)^(p_), x_Symbol] := Simp[e^m*((a + b*x + c*x^2)^(p + (m + 1)/2)/(c^((m + 1)/2)*(m + 2*p + 1))), x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[b^2 - 4*a*c, 0] && EqQ[2*c*d - b*e, 0] && IntegerQ[(m - 1)/2]`

Maple [A] (verified)

Time = 1.23 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.79

| method | result | size |
|----------------|---|------|
| risch | $\frac{(ex+d)^4}{3c^2\sqrt{c(ex+d)^2}e}$ | 27 |
| pseudoelliptic | $\frac{(ex+d)^4}{3c^2\sqrt{c(ex+d)^2}e}$ | 27 |
| default | $\frac{(ex+d)^8}{3(x^2ce^2+2cdxe+cd^2)^{\frac{5}{2}}e}$ | 35 |
| gosper | $\frac{x(e^2x^2+3dex+3d^2)(ex+d)^5}{3(x^2ce^2+2cdxe+cd^2)^{\frac{5}{2}}}$ | 51 |
| orering | $\frac{x(e^2x^2+3dex+3d^2)(ex+d)^5}{3(x^2ce^2+2cdxe+cd^2)^{\frac{5}{2}}}$ | 51 |
| trager | $\frac{x(e^2x^2+3dex+3d^2)\sqrt{x^2ce^2+2cdxe+cd^2}}{3c^3(ex+d)}$ | 54 |

input `int((e*x+d)^7/(c*e^2*x^2+2*c*d*e*x+c*d^2)^(5/2),x,method=_RETURNVERBOSE)`

output `1/3/c^2*(e*x+d)^4/(c*(e*x+d)^2)^(1/2)/e`

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.74

$$\int \frac{(d+ex)^7}{(cd^2+2cdex+ce^2x^2)^{5/2}} dx = \frac{\sqrt{ce^2x^2+2cdex+cd^2}(e^2x^3+3dex^2+3d^2x)}{3(c^3ex+c^3d)}$$

input `integrate((e*x+d)^7/(c*e^2*x^2+2*c*d*e*x+c*d^2)^(5/2),x, algorithm="fricas")`

output `1/3*sqrt(c*e^2*x^2 + 2*c*d*e*x + c*d^2)*(e^2*x^3 + 3*d*e*x^2 + 3*d^2*x)/(c^3*e*x + c^3*d)`

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 119 vs. $2(31) = 62$.

Time = 0.40 (sec) , antiderivative size = 119, normalized size of antiderivative = 3.50

$$\int \frac{(d+ex)^7}{(cd^2+2cdex+ce^2x^2)^{5/2}} dx = \begin{cases} \frac{d^2\sqrt{cd^2+2cdex+ce^2x^2}}{3c^3e} + \frac{2dx\sqrt{cd^2+2cdex+ce^2x^2}}{3c^3} + \frac{ex^2\sqrt{cd^2+2cdex+ce^2x^2}}{3c^3} & \text{for } e \neq 0 \\ \frac{d^7x}{(cd^2)^{5/2}} & \text{otherwise} \end{cases}$$

input `integrate((e*x+d)**7/(c*e**2*x**2+2*c*d*e*x+c*d**2)**(5/2),x)`

output `Piecewise((d**2*sqrt(c*d**2 + 2*c*d*e*x + c*e**2*x**2)/(3*c**3*e) + 2*d*x*sqrt(c*d**2 + 2*c*d*e*x + c*e**2*x**2)/(3*c**3) + e*x**2*sqrt(c*d**2 + 2*c*d*e*x + c*e**2*x**2)/(3*c**3), Ne(e, 0)), (d**7*x/(c*d**2)**(5/2), True))`

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 268 vs. $2(30) = 60$.

Time = 0.06 (sec) , antiderivative size = 268, normalized size of antiderivative = 7.88

$$\begin{aligned} \int \frac{(d+ex)^7}{(cd^2+2cdex+ce^2x^2)^{5/2}} dx &= \frac{e^5x^6}{3(ce^2x^2+2cdex+cd^2)^{3/2}c} \\ &+ \frac{2de^4x^5}{(ce^2x^2+2cdex+cd^2)^{3/2}c} + \frac{5d^2e^3x^4}{(ce^2x^2+2cdex+cd^2)^{3/2}c} \\ &- \frac{9c^2d^7e^4}{(ce^2)^{9/2}(x+\frac{d}{e})^4} - \frac{15d^4ex^2}{(ce^2x^2+2cdex+cd^2)^{3/2}c} + \frac{24cd^6e^3}{(ce^2)^{7/2}(x+\frac{d}{e})^3} \\ &- \frac{37d^6}{3(ce^2x^2+2cdex+cd^2)^{3/2}ce} - \frac{18d^5e^2}{(ce^2)^{5/2}(x+\frac{d}{e})^2} + \frac{9d^7}{(ce^2)^{5/2}(x+\frac{d}{e})^4} \end{aligned}$$

input `integrate((e*x+d)^7/(c*e^2*x^2+2*c*d*e*x+c*d^2)^(5/2),x, algorithm="maxima")`

output

```
1/3*e^5*x^6/((c*e^2*x^2 + 2*c*d*e*x + c*d^2)^(3/2)*c) + 2*d*e^4*x^5/((c*e^
2*x^2 + 2*c*d*e*x + c*d^2)^(3/2)*c) + 5*d^2*e^3*x^4/((c*e^2*x^2 + 2*c*d*e*
x + c*d^2)^(3/2)*c) - 9*c^2*d^7*e^4/((c*e^2)^(9/2)*(x + d/e)^4) - 15*d^4*e
*x^2/((c*e^2*x^2 + 2*c*d*e*x + c*d^2)^(3/2)*c) + 24*c*d^6*e^3/((c*e^2)^(7/
2)*(x + d/e)^3) - 37/3*d^6/((c*e^2*x^2 + 2*c*d*e*x + c*d^2)^(3/2)*c*e) - 1
8*d^5*e^2/((c*e^2)^(5/2)*(x + d/e)^2) + 9*d^7/((c*e^2)^(5/2)*(x + d/e)^4)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{(d + ex)^7}{(cd^2 + 2cdex + ce^2x^2)^{5/2}} dx = \text{Exception raised: TypeError}$$

input

```
integrate((e*x+d)^7/(c*e^2*x^2+2*c*d*e*x+c*d^2)^(5/2),x, algorithm="giac")
```

output

```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d + ex)^7}{(cd^2 + 2cdex + ce^2x^2)^{5/2}} dx = \int \frac{(d + ex)^7}{(cd^2 + 2cdex + ce^2x^2)^{5/2}} dx$$

input

```
int((d + e*x)^7/(c*d^2 + c*e^2*x^2 + 2*c*d*e*x)^(5/2),x)
```

output

```
int((d + e*x)^7/(c*d^2 + c*e^2*x^2 + 2*c*d*e*x)^(5/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.76

$$\int \frac{(d + ex)^7}{(cd^2 + 2cdex + ce^2x^2)^{5/2}} dx = \frac{\sqrt{c}x(e^2x^2 + 3dex + 3d^2)}{3c^3}$$

input `int((e*x+d)^7/(c*e^2*x^2+2*c*d*e*x+c*d^2)^(5/2),x)`

output `(sqrt(c)*x*(3*d**2 + 3*d*e*x + e**2*x**2))/(3*c**3)`

$$3.255 \quad \int \frac{(d+ex)^5}{(cd^2+2cdex+ce^2x^2)^{5/2}} dx$$

| | |
|---|------|
| Optimal result | 1961 |
| Mathematica [A] (verified) | 1961 |
| Rubi [A] (verified) | 1962 |
| Maple [A] (verified) | 1962 |
| Fricas [A] (verification not implemented) | 1963 |
| Sympy [A] (verification not implemented) | 1964 |
| Maxima [B] (verification not implemented) | 1964 |
| Giac [F(-2)] | 1965 |
| Mupad [F(-1)] | 1965 |
| Reduce [B] (verification not implemented) | 1965 |

Optimal result

Integrand size = 32, antiderivative size = 31

$$\int \frac{(d+ex)^5}{(cd^2+2cdex+ce^2x^2)^{5/2}} dx = \frac{\sqrt{cd^2+2cdex+ce^2x^2}}{c^3e}$$

output $(c \cdot e^2 \cdot x^2 + 2 \cdot c \cdot d \cdot e \cdot x + c \cdot d^2)^{(1/2)} / c^3 / e$

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.65

$$\int \frac{(d+ex)^5}{(cd^2+2cdex+ce^2x^2)^{5/2}} dx = \frac{\sqrt{c(d+ex)^2}}{c^3e}$$

input `Integrate[(d + e*x)^5/(c*d^2 + 2*c*d*e*x + c*e^2*x^2)^(5/2),x]`

output `Sqrt[c*(d + e*x)^2]/(c^3*e)`

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.031$, Rules used = {1099}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d + ex)^5}{(cd^2 + 2cdex + ce^2x^2)^{5/2}} dx$$

↓ 1099

$$\frac{\sqrt{cd^2 + 2cdex + ce^2x^2}}{c^3e}$$

input `Int[(d + e*x)^5/(c*d^2 + 2*c*d*e*x + c*e^2*x^2)^(5/2),x]`

output `Sqrt[c*d^2 + 2*c*d*e*x + c*e^2*x^2]/(c^3*e)`

Defintions of rubi rules used

rule 1099

```
Int[((d_) + (e_)*(x_))^(m_)*((a_) + (b_)*(x_) + (c_)*(x_)^2)^(p_), x_Sy
mbol] := Simp[e^m*((a + b*x + c*x^2)^(p + (m + 1)/2)/(c^((m + 1)/2)*(m + 2*
p + 1))), x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[b^2 - 4*a*c, 0] && EqQ[
2*c*d - b*e, 0] && IntegerQ[(m - 1)/2]
```

Maple [A] (verified)

Time = 1.21 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.71

| method | result | size |
|----------------|---|------|
| risch | $\frac{(ex+d)x}{c^2\sqrt{c(ex+d)^2}}$ | 22 |
| pseudoelliptic | $\frac{(ex+d)^2}{\sqrt{c(ex+d)^2} e c^2}$ | 26 |
| default | $\frac{x(ex+d)^5}{(x^2 c e^2 + 2cdxe + c d^2)^{\frac{5}{2}}}$ | 32 |
| orering | $\frac{x(ex+d)^5}{(x^2 c e^2 + 2cdxe + c d^2)^{\frac{5}{2}}}$ | 32 |
| trager | $\frac{x\sqrt{x^2 c e^2 + 2cdxe + c d^2}}{c^3(ex+d)}$ | 35 |

input `int((e*x+d)^5/(c*e^2*x^2+2*c*d*e*x+c*d^2)^(5/2),x,method=_RETURNVERBOSE)`

output `1/c^2*(e*x+d)/(c*(e*x+d)^2)^(1/2)*x`

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.23

$$\int \frac{(d+ex)^5}{(cd^2+2cdex+ce^2x^2)^{5/2}} dx = \frac{\sqrt{ce^2x^2+2cdex+cd^2}x}{c^3ex+c^3d}$$

input `integrate((e*x+d)^5/(c*e^2*x^2+2*c*d*e*x+c*d^2)^(5/2),x, algorithm="fricas")`

output `sqrt(c*e^2*x^2 + 2*c*d*e*x + c*d^2)*x/(c^3*e*x + c^3*d)`

Sympy [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.35

$$\int \frac{(d+ex)^5}{(cd^2+2cdex+ce^2x^2)^{5/2}} dx = \begin{cases} \frac{\sqrt{cd^2+2cdex+ce^2x^2}}{c^3e} & \text{for } e \neq 0 \\ \frac{d^5x}{(cd^2)^{5/2}} & \text{otherwise} \end{cases}$$

input `integrate((e*x+d)**5/(c*e**2*x**2+2*c*d*e*x+c*d**2)**(5/2),x)`

output `Piecewise((sqrt(c*d**2 + 2*c*d*e*x + c*e**2*x**2)/(c**3*e), Ne(e, 0)), (d**5*x/(c*d**2)**(5/2), True))`

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 197 vs. 2(29) = 58.

Time = 0.07 (sec) , antiderivative size = 197, normalized size of antiderivative = 6.35

$$\int \frac{(d+ex)^5}{(cd^2+2cdex+ce^2x^2)^{5/2}} dx = \frac{e^3x^4}{(ce^2x^2+2cdex+cd^2)^{3/2}c} - \frac{4c^2d^5e^4}{(ce^2)^{9/2}(x+d/e)^4} - \frac{6d^2ex^2}{(ce^2x^2+2cdex+cd^2)^{3/2}c} + \frac{32cd^4e^3}{3(ce^2)^{7/2}(x+d/e)^3} - \frac{17d^4}{3(ce^2x^2+2cdex+cd^2)^{3/2}ce} - \frac{8d^3e^2}{(ce^2)^{5/2}(x+d/e)^2} + \frac{4d^5}{(ce^2)^{5/2}(x+d/e)^4}$$

input `integrate((e*x+d)^5/(c*e^2*x^2+2*c*d*e*x+c*d^2)^(5/2),x, algorithm="maxima")`

output `e^3*x^4/((c*e^2*x^2 + 2*c*d*e*x + c*d^2)^(3/2)*c) - 4*c^2*d^5*e^4/((c*e^2)^(9/2)*(x + d/e)^4) - 6*d^2*e*x^2/((c*e^2*x^2 + 2*c*d*e*x + c*d^2)^(3/2)*c) + 32/3*c*d^4*e^3/((c*e^2)^(7/2)*(x + d/e)^3) - 17/3*d^4/((c*e^2*x^2 + 2*c*d*e*x + c*d^2)^(3/2)*c*e) - 8*d^3*e^2/((c*e^2)^(5/2)*(x + d/e)^2) + 4*d^5/((c*e^2)^(5/2)*(x + d/e)^4)`

Giac [F(-2)]

Exception generated.

$$\int \frac{(d + ex)^5}{(cd^2 + 2cdex + ce^2x^2)^{5/2}} dx = \text{Exception raised: TypeError}$$

input `integrate((e*x+d)^5/(c*e^2*x^2+2*c*d*e*x+c*d^2)^(5/2),x, algorithm="giac")`

output `Exception raised: TypeError >> an error occurred running a Giac command:INPUT:sage2:=int(sage0,sageVARx)::OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const vecteur & l) Error: Bad Argument Value`

Mupad [F(-1)]

Timed out.

$$\int \frac{(d + ex)^5}{(cd^2 + 2cdex + ce^2x^2)^{5/2}} dx = \int \frac{(d + ex)^5}{(cd^2 + 2cdex + ce^2x^2)^{5/2}} dx$$

input `int((d + e*x)^5/(c*d^2 + c*e^2*x^2 + 2*c*d*e*x)^(5/2),x)`

output `int((d + e*x)^5/(c*d^2 + c*e^2*x^2 + 2*c*d*e*x)^(5/2), x)`

Reduce [B] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.23

$$\int \frac{(d + ex)^5}{(cd^2 + 2cdex + ce^2x^2)^{5/2}} dx = \frac{\sqrt{c}x}{c^3}$$

input `int((e*x+d)^5/(c*e^2*x^2+2*c*d*e*x+c*d^2)^(5/2),x)`

output `(sqrt(c)*x)/c**3`

$$3.256 \quad \int \frac{(d+ex)^3}{(cd^2+2cdex+ce^2x^2)^{5/2}} dx$$

| | |
|---|------|
| Optimal result | 1966 |
| Mathematica [A] (verified) | 1966 |
| Rubi [A] (verified) | 1967 |
| Maple [A] (verified) | 1967 |
| Fricas [A] (verification not implemented) | 1968 |
| Sympy [B] (verification not implemented) | 1969 |
| Maxima [B] (verification not implemented) | 1969 |
| Giac [F(-2)] | 1970 |
| Mupad [B] (verification not implemented) | 1970 |
| Reduce [B] (verification not implemented) | 1970 |

Optimal result

Integrand size = 32, antiderivative size = 32

$$\int \frac{(d+ex)^3}{(cd^2+2cdex+ce^2x^2)^{5/2}} dx = -\frac{1}{c^2e\sqrt{cd^2+2cdex+ce^2x^2}}$$

output `-1/c^2/e/(c*e^2*x^2+2*c*d*e*x+c*d^2)^(1/2)`

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.66

$$\int \frac{(d+ex)^3}{(cd^2+2cdex+ce^2x^2)^{5/2}} dx = -\frac{1}{c^2e\sqrt{c(d+ex)^2}}$$

input `Integrate[(d + e*x)^3/(c*d^2 + 2*c*d*e*x + c*e^2*x^2)^(5/2), x]`

output `-(1/(c^2*e*sqrt[c*(d + e*x)^2]))`

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.031$, Rules used = {1099}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d + ex)^3}{(cd^2 + 2cdex + ce^2x^2)^{5/2}} dx$$

↓ 1099

$$-\frac{1}{c^2e\sqrt{cd^2 + 2cdex + ce^2x^2}}$$

input `Int[(d + e*x)^3/(c*d^2 + 2*c*d*e*x + c*e^2*x^2)^(5/2),x]`

output `-(1/(c^2*e*Sqrt[c*d^2 + 2*c*d*e*x + c*e^2*x^2]))`

Defintions of rubi rules used

rule 1099

```
Int[((d_) + (e_)*(x_))^(m_)*((a_) + (b_)*(x_) + (c_)*(x_)^2)^(p_), x_Sy
mbol] := Simp[e^m*((a + b*x + c*x^2)^(p + (m + 1)/2)/(c^((m + 1)/2)*(m + 2*
p + 1))), x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[b^2 - 4*a*c, 0] && EqQ[
2*c*d - b*e, 0] && IntegerQ[(m - 1)/2]
```

Maple [A] (verified)

Time = 1.17 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.62

| method | result | size |
|----------------|--|------|
| risch | $-\frac{1}{c^2 \sqrt{c(ex+d)^2} e}$ | 20 |
| pseudoelliptic | $-\frac{1}{c^2 \sqrt{c(ex+d)^2} e}$ | 20 |
| gosper | $-\frac{(ex+d)^4}{e(x^2 c e^2 + 2cdxe + c d^2)^{\frac{5}{2}}}$ | 35 |
| default | $-\frac{(ex+d)^4}{e(x^2 c e^2 + 2cdxe + c d^2)^{\frac{5}{2}}}$ | 35 |
| orering | $-\frac{(ex+d)^4}{e(x^2 c e^2 + 2cdxe + c d^2)^{\frac{5}{2}}}$ | 35 |
| trager | $\frac{x \sqrt{x^2 c e^2 + 2cdxe + c d^2}}{d c^3 (ex+d)^2}$ | 38 |

input `int((e*x+d)^3/(c*e^2*x^2+2*c*d*e*x+c*d^2)^(5/2),x,method=_RETURNVERBOSE)`

output `-1/c^2/(c*(e*x+d)^2)^(1/2)/e`

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.72

$$\int \frac{(d+ex)^3}{(cd^2+2cdex+ce^2x^2)^{5/2}} dx = -\frac{\sqrt{ce^2x^2+2cdex+cd^2}}{c^3e^3x^2+2c^3de^2x+c^3d^2e}$$

input `integrate((e*x+d)^3/(c*e^2*x^2+2*c*d*e*x+c*d^2)^(5/2),x, algorithm="fricas")`

output `-sqrt(c*e^2*x^2+2*c*d*e*x+c*d^2)/(c^3*e^3*x^2+2*c^3*d*e^2*x+c^3*d^2*e)`

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 70 vs. $2(32) = 64$.

Time = 0.29 (sec) , antiderivative size = 70, normalized size of antiderivative = 2.19

$$\int \frac{(d+ex)^3}{(cd^2+2cdex+ce^2x^2)^{5/2}} dx = \begin{cases} -\frac{\sqrt{cd^2+2cdex+ce^2x^2}}{c^3d^2e+2c^3de^2x+c^3e^3x^2} & \text{for } e \neq 0 \\ \frac{d^3x}{(cd^2)^{5/2}} & \text{otherwise} \end{cases}$$

input `integrate((e*x+d)**3/(c*e**2*x**2+2*c*d*e*x+c*d**2)**(5/2),x)`

output `Piecewise((-sqrt(c*d**2 + 2*c*d*e*x + c*e**2*x**2)/(c**3*d**2*e + 2*c**3*d*e**2*x + c**3*e**3*x**2), Ne(e, 0)), (d**3*x/(c*d**2)**(5/2), True))`

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 159 vs. $2(30) = 60$.

Time = 0.04 (sec) , antiderivative size = 159, normalized size of antiderivative = 4.97

$$\int \frac{(d+ex)^3}{(cd^2+2cdex+ce^2x^2)^{5/2}} dx = -\frac{c^2d^3e^4}{(ce^2)^{9/2}(x+\frac{d}{e})^4} - \frac{ex^2}{(ce^2x^2+2cdex+cd^2)^{3/2}c} + \frac{8cd^2e^3}{3(ce^2)^{7/2}(x+\frac{d}{e})^3} - \frac{5d^2}{3(ce^2x^2+2cdex+cd^2)^{3/2}ce} - \frac{2de^2}{(ce^2)^{5/2}(x+\frac{d}{e})^2} + \frac{d^3}{(ce^2)^{5/2}(x+\frac{d}{e})^4}$$

input `integrate((e*x+d)^3/(c*e^2*x^2+2*c*d*e*x+c*d^2)^(5/2),x, algorithm="maxima")`

output `-c^2*d^3*e^4/((c*e^2)^(9/2)*(x+d/e)^4) - e*x^2/((c*e^2*x^2+2*c*d*e*x+c*d^2)^(3/2)*c) + 8/3*c*d^2*e^3/((c*e^2)^(7/2)*(x+d/e)^3) - 5/3*d^2/((c*e^2*x^2+2*c*d*e*x+c*d^2)^(3/2)*c*e) - 2*d*e^2/((c*e^2)^(5/2)*(x+d/e)^2) + d^3/((c*e^2)^(5/2)*(x+d/e)^4)`

Giac [F(-2)]

Exception generated.

$$\int \frac{(d+ex)^3}{(cd^2+2cdex+ce^2x^2)^{5/2}} dx = \text{Exception raised: TypeError}$$

input `integrate((e*x+d)^3/(c*e^2*x^2+2*c*d*e*x+c*d^2)^(5/2),x, algorithm="giac")`

output `Exception raised: TypeError >> an error occurred running a Giac command:INPUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const vecteur & l) Error: Bad Argument Value`

Mupad [B] (verification not implemented)

Time = 9.20 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.16

$$\int \frac{(d+ex)^3}{(cd^2+2cdex+ce^2x^2)^{5/2}} dx = -\frac{\sqrt{cd^2+2cdex+ce^2x^2}}{c^3e(d+ex)^2}$$

input `int((d + e*x)^3/(c*d^2 + c*e^2*x^2 + 2*c*d*e*x)^(5/2),x)`

output `-(c*d^2 + c*e^2*x^2 + 2*c*d*e*x)^(1/2)/(c^3*e*(d + e*x)^2)`

Reduce [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.53

$$\int \frac{(d+ex)^3}{(cd^2+2cdex+ce^2x^2)^{5/2}} dx = \frac{\sqrt{c}x}{c^3d(ex+d)}$$

input `int((e*x+d)^3/(c*e^2*x^2+2*c*d*e*x+c*d^2)^(5/2),x)`

output `(sqrt(c)*x)/(c**3*d*(d + e*x))`

3.257 $\int \frac{d+ex}{(cd^2+2cdex+ce^2x^2)^{5/2}} dx$

| | |
|---|------|
| Optimal result | 1971 |
| Mathematica [A] (verified) | 1971 |
| Rubi [A] (verified) | 1972 |
| Maple [A] (verified) | 1972 |
| Fricas [B] (verification not implemented) | 1973 |
| Sympy [B] (verification not implemented) | 1974 |
| Maxima [A] (verification not implemented) | 1974 |
| Giac [F(-2)] | 1975 |
| Mupad [B] (verification not implemented) | 1975 |
| Reduce [B] (verification not implemented) | 1975 |

Optimal result

Integrand size = 30, antiderivative size = 34

$$\int \frac{d+ex}{(cd^2+2cdex+ce^2x^2)^{5/2}} dx = -\frac{1}{3ce(cd^2+2cdex+ce^2x^2)^{3/2}}$$

output `-1/3/c/e/(c*e^2*x^2+2*c*d*e*x+c*d^2)^(3/2)`

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.88

$$\int \frac{d+ex}{(cd^2+2cdex+ce^2x^2)^{5/2}} dx = -\frac{\sqrt{c(d+ex)^2}}{3c^3e(d+ex)^4}$$

input `Integrate[(d + e*x)/(c*d^2 + 2*c*d*e*x + c*e^2*x^2)^(5/2), x]`

output `-1/3*Sqrt[c*(d + e*x)^2]/(c^3*e*(d + e*x)^4)`

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.033$, Rules used = {1099}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{d + ex}{(cd^2 + 2cdex + ce^2x^2)^{5/2}} dx$$

↓ 1099

$$-\frac{1}{3ce(cd^2 + 2cdex + ce^2x^2)^{3/2}}$$

input `Int[(d + e*x)/(c*d^2 + 2*c*d*e*x + c*e^2*x^2)^(5/2),x]`

output `-1/3*1/(c*e*(c*d^2 + 2*c*d*e*x + c*e^2*x^2)^(3/2))`

Defintions of rubi rules used

rule 1099

```
Int[((d_) + (e_)*(x_))^(m_)*((a_) + (b_)*(x_) + (c_)*(x_)^2)^(p_), x_Sy
mbol] := Simp[e^m*((a + b*x + c*x^2)^(p + (m + 1)/2)/(c^((m + 1)/2)*(m + 2*
p + 1))), x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[b^2 - 4*a*c, 0] && EqQ[
2*c*d - b*e, 0] && IntegerQ[(m - 1)/2]
```

Maple [A] (verified)

Time = 1.05 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.79

| method | result | size |
|----------------|--|------|
| risch | $-\frac{1}{3c^2(ex+d)^2\sqrt{c(ex+d)^2e}}$ | 27 |
| pseudoelliptic | $-\frac{1}{3c^2(ex+d)^2\sqrt{c(ex+d)^2e}}$ | 27 |
| gosper | $-\frac{(ex+d)^2}{3e(x^2ce^2+2cdxe+cd^2)^{\frac{5}{2}}}$ | 35 |
| default | $-\frac{(ex+d)^2}{3e(x^2ce^2+2cdxe+cd^2)^{\frac{5}{2}}}$ | 35 |
| oring | $-\frac{(ex+d)^2}{3e(x^2ce^2+2cdxe+cd^2)^{\frac{5}{2}}}$ | 35 |
| trager | $\frac{(e^2x^2+3dex+3d^2)x\sqrt{x^2ce^2+2cdxe+cd^2}}{3d^3c^3(ex+d)^4}$ | 57 |

input `int((e*x+d)/(c*e^2*x^2+2*c*d*e*x+c*d^2)^(5/2),x,method=_RETURNVERBOSE)`

output `-1/3/c^2/(e*x+d)^2/(c*(e*x+d)^2)^(1/2)/e`

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 83 vs. $2(30) = 60$.

Time = 0.07 (sec) , antiderivative size = 83, normalized size of antiderivative = 2.44

$$\int \frac{d+ex}{(cd^2+2cdex+ce^2x^2)^{5/2}} dx = -\frac{\sqrt{ce^2x^2+2cdex+cd^2}}{3(c^3e^5x^4+4c^3de^4x^3+6c^3d^2e^3x^2+4c^3d^3e^2x+c^3d^4e)}$$

input `integrate((e*x+d)/(c*e^2*x^2+2*c*d*e*x+c*d^2)^(5/2),x, algorithm="fricas")`

output `-1/3*sqrt(c*e^2*x^2+2*c*d*e*x+c*d^2)/(c^3*e^5*x^4+4*c^3*d*e^4*x^3+6*c^3*d^2*e^3*x^2+4*c^3*d^3*e^2*x+c^3*d^4*e)`

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 124 vs. $2(32) = 64$.

Time = 0.40 (sec) , antiderivative size = 124, normalized size of antiderivative = 3.65

$$\int \frac{d + ex}{(cd^2 + 2cdex + ce^2x^2)^{5/2}} dx = \begin{cases} -\frac{1}{3c^2d^2e\sqrt{cd^2+2cdex+ce^2x^2}+6c^2de^2x\sqrt{cd^2+2cdex+ce^2x^2}+3c^2e^3x^2\sqrt{cd^2+2cdex+ce^2x^2}} \\ \frac{dx}{(cd^2)^{5/2}} \end{cases}$$

input `integrate((e*x+d)/(c*e**2*x**2+2*c*d*e*x+c*d**2)**(5/2),x)`

output `Piecewise((-1/(3*c**2*d**2*e*sqrt(c*d**2 + 2*c*d*e*x + c*e**2*x**2) + 6*c**2*d*e**2*x*sqrt(c*d**2 + 2*c*d*e*x + c*e**2*x**2) + 3*c**2*e**3*x**2*sqrt(c*d**2 + 2*c*d*e*x + c*e**2*x**2)), Ne(e, 0)), (d*x/(c*d**2)**(5/2), True))`

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.88

$$\int \frac{d + ex}{(cd^2 + 2cdex + ce^2x^2)^{5/2}} dx = -\frac{1}{3(ce^2x^2 + 2cdex + cd^2)^{3/2}ce}$$

input `integrate((e*x+d)/(c*e^2*x^2+2*c*d*e*x+c*d^2)^(5/2),x, algorithm="maxima")`

output `-1/3/((c*e^2*x^2 + 2*c*d*e*x + c*d^2)^(3/2)*c*e)`

Giac [F(-2)]

Exception generated.

$$\int \frac{d + ex}{(cd^2 + 2cdex + ce^2x^2)^{5/2}} dx = \text{Exception raised: TypeError}$$

input `integrate((e*x+d)/(c*e^2*x^2+2*c*d*e*x+c*d^2)^(5/2),x, algorithm="giac")`

output `Exception raised: TypeError >> an error occurred running a Giac command:INPUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const vecteur & l) Error: Bad Argument Value`

Mupad [B] (verification not implemented)

Time = 9.20 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.09

$$\int \frac{d + ex}{(cd^2 + 2cdex + ce^2x^2)^{5/2}} dx = -\frac{\sqrt{cd^2 + 2cdex + ce^2x^2}}{3c^3e(d + ex)^4}$$

input `int((d + e*x)/(c*d^2 + c*e^2*x^2 + 2*c*d*e*x)^(5/2),x)`

output `-(c*d^2 + c*e^2*x^2 + 2*c*d*e*x)^(1/2)/(3*c^3*e*(d + e*x)^4)`

Reduce [B] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.15

$$\int \frac{d + ex}{(cd^2 + 2cdex + ce^2x^2)^{5/2}} dx = -\frac{\sqrt{c}}{3c^3e(e^3x^3 + 3de^2x^2 + 3d^2ex + d^3)}$$

input `int((e*x+d)/(c*e^2*x^2+2*c*d*e*x+c*d^2)^(5/2),x)`

output `(- sqrt(c))/(3*c**3*e*(d**3 + 3*d**2*e*x + 3*d*e**2*x**2 + e**3*x**3))`

$$3.258 \quad \int \frac{1}{(d+ex)(cd^2+2cdex+ce^2x^2)^{5/2}} dx$$

| | |
|---|------|
| Optimal result | 1976 |
| Mathematica [A] (verified) | 1976 |
| Rubi [A] (verified) | 1977 |
| Maple [A] (verified) | 1977 |
| Fricas [B] (verification not implemented) | 1978 |
| Sympy [A] (verification not implemented) | 1979 |
| Maxima [B] (verification not implemented) | 1979 |
| Giac [F(-2)] | 1980 |
| Mupad [B] (verification not implemented) | 1980 |
| Reduce [B] (verification not implemented) | 1980 |

Optimal result

Integrand size = 32, antiderivative size = 31

$$\int \frac{1}{(d+ex)(cd^2+2cdex+ce^2x^2)^{5/2}} dx = -\frac{1}{5e(cd^2+2cdex+ce^2x^2)^{5/2}}$$

output `-1/5/e/(c*e^2*x^2+2*c*d*e*x+c*d^2)^(5/2)`

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.65

$$\int \frac{1}{(d+ex)(cd^2+2cdex+ce^2x^2)^{5/2}} dx = -\frac{1}{5e(c(d+ex)^2)^{5/2}}$$

input `Integrate[1/((d + e*x)*(c*d^2 + 2*c*d*e*x + c*e^2*x^2)^(5/2)),x]`

output `-1/5*1/(e*(c*(d + e*x)^2)^(5/2))`

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.031$, Rules used = {1099}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(d + ex)(cd^2 + 2cdex + ce^2x^2)^{5/2}} dx$$

↓ 1099

$$-\frac{1}{5e(cd^2 + 2cdex + ce^2x^2)^{5/2}}$$

input `Int[1/((d + e*x)*(c*d^2 + 2*c*d*e*x + c*e^2*x^2)^(5/2)),x]`

output `-1/5*1/(e*(c*d^2 + 2*c*d*e*x + c*e^2*x^2)^(5/2))`

Defintions of rubi rules used

rule 1099

```
Int[((d_) + (e_)*(x_))^(m_)*((a_) + (b_)*(x_) + (c_)*(x_)^2)^(p_), x_Sy
mbol] := Simp[e^m*((a + b*x + c*x^2)^(p + (m + 1)/2)/(c^((m + 1)/2)*(m + 2*
p + 1))), x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[b^2 - 4*a*c, 0] && EqQ[
2*c*d - b*e, 0] && IntegerQ[(m - 1)/2]
```

Maple [A] (verified)

Time = 1.38 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.87

| method | result | size |
|----------------|--|------|
| risch | $-\frac{1}{5c^2(ex+d)^4\sqrt{c(ex+d)^2e}}$ | 27 |
| pseudoelliptic | $-\frac{1}{5c^2(ex+d)^4\sqrt{c(ex+d)^2e}}$ | 27 |
| gospers | $-\frac{1}{5e(x^2ce^2+2cdxe+cd^2)^{\frac{5}{2}}}$ | 28 |
| default | $-\frac{1}{5e(x^2ce^2+2cdxe+cd^2)^{\frac{5}{2}}}$ | 28 |
| orering | $-\frac{1}{5e(x^2ce^2+2cdxe+cd^2)^{\frac{5}{2}}}$ | 28 |
| trager | $\frac{(e^4x^4+5de^3x^3+10d^2e^2x^2+10d^3ex+5d^4)x\sqrt{x^2ce^2+2cdxe+cd^2}}{5d^5c^3(ex+d)^6}$ | 79 |

input `int(1/(e*x+d)/(c*e^2*x^2+2*c*d*e*x+c*d^2)^(5/2),x,method=_RETURNVERBOSE)`

output `-1/5/c^2/(e*x+d)^4/(c*(e*x+d)^2)^(1/2)/e`

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 111 vs. 2(27) = 54.

Time = 0.08 (sec) , antiderivative size = 111, normalized size of antiderivative = 3.58

$$\int \frac{1}{(d+ex)(cd^2+2cdex+ce^2x^2)^{5/2}} dx = \frac{\sqrt{ce^2x^2+2cdex+cd^2}}{5(c^3e^7x^6+6c^3de^6x^5+15c^3d^2e^5x^4+20c^3d^3e^4x^3+15c^3d^4e^3x^2+6c^3d^5e^2x+c^3d^6e)}$$

input `integrate(1/(e*x+d)/(c*e^2*x^2+2*c*d*e*x+c*d^2)^(5/2),x, algorithm="fricas")`

output `-1/5*sqrt(c*e^2*x^2+2*c*d*e*x+c*d^2)/(c^3*e^7*x^6+6*c^3*d*e^6*x^5+15*c^3*d^2*e^5*x^4+20*c^3*d^3*e^4*x^3+15*c^3*d^4*e^3*x^2+6*c^3*d^5*e^2*x+c^3*d^6*e)`

Sympy [A] (verification not implemented)

Time = 2.13 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.35

$$\int \frac{1}{(d+ex)(cd^2+2cdex+ce^2x^2)^{5/2}} dx = \begin{cases} -\frac{1}{5e(cd^2+2cdex+ce^2x^2)^{5/2}} & \text{for } e \neq 0 \\ \frac{x}{d(cd^2)^{5/2}} & \text{otherwise} \end{cases}$$

input `integrate(1/(e*x+d)/(c*e**2*x**2+2*c*d*e*x+c*d**2)**(5/2),x)`

output `Piecewise((-1/(5*e*(c*d**2 + 2*c*d*e*x + c*e**2*x**2)**(5/2)), Ne(e, 0)), (x/(d*(c*d**2)**(5/2)), True))`

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 75 vs. 2(27) = 54.

Time = 0.03 (sec) , antiderivative size = 75, normalized size of antiderivative = 2.42

$$\int \frac{1}{(d+ex)(cd^2+2cdex+ce^2x^2)^{5/2}} dx = \frac{1}{5 \left(c^{\frac{5}{2}} e^6 x^5 + 5 c^{\frac{5}{2}} d e^5 x^4 + 10 c^{\frac{5}{2}} d^2 e^4 x^3 + 10 c^{\frac{5}{2}} d^3 e^3 x^2 + 5 c^{\frac{5}{2}} d^4 e^2 x + c^{\frac{5}{2}} d^5 e \right)}$$

input `integrate(1/(e*x+d)/(c*e^2*x^2+2*c*d*e*x+c*d^2)^(5/2),x, algorithm="maxima")`

output `-1/5/(c^(5/2)*e^6*x^5 + 5*c^(5/2)*d*e^5*x^4 + 10*c^(5/2)*d^2*e^4*x^3 + 10*c^(5/2)*d^3*e^3*x^2 + 5*c^(5/2)*d^4*e^2*x + c^(5/2)*d^5*e)`

Giac [F(-2)]

Exception generated.

$$\int \frac{1}{(d+ex)(cd^2+2cdex+ce^2x^2)^{5/2}} dx = \text{Exception raised: TypeError}$$

input `integrate(1/(e*x+d)/(c*e^2*x^2+2*c*d*e*x+c*d^2)^(5/2),x, algorithm="giac")`

output `Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx)::OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value`

Mupad [B] (verification not implemented)

Time = 9.21 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.19

$$\int \frac{1}{(d+ex)(cd^2+2cdex+ce^2x^2)^{5/2}} dx = -\frac{\sqrt{cd^2+2cdex+ce^2x^2}}{5c^3e(d+ex)^6}$$

input `int(1/((d+e*x)*(c*d^2+c*e^2*x^2+2*c*d*e*x)^(5/2)),x)`

output `-(c*d^2+c*e^2*x^2+2*c*d*e*x)^(1/2)/(5*c^3*e*(d+e*x)^6)`

Reduce [B] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.97

$$\int \frac{1}{(d+ex)(cd^2+2cdex+ce^2x^2)^{5/2}} dx = \frac{\sqrt{c}}{5c^3e(e^5x^5+5de^4x^4+10d^2e^3x^3+10d^3e^2x^2+5d^4ex+d^5)}$$

input `int(1/(e*x+d)/(c*e^2*x^2+2*c*d*e*x+c*d^2)^(5/2),x)`

output

```
( - sqrt(c))/(5*c**3*e*(d**5 + 5*d**4*e*x + 10*d**3*e**2*x**2 + 10*d**2*e*  
*3*x**3 + 5*d*e**4*x**4 + e**5*x**5))
```

$$3.259 \quad \int \frac{(d+ex)^6}{(cd^2+2cdex+ce^2x^2)^{5/2}} dx$$

| | |
|---|------|
| Optimal result | 1982 |
| Mathematica [A] (verified) | 1982 |
| Rubi [A] (verified) | 1983 |
| Maple [A] (verified) | 1984 |
| Fricas [A] (verification not implemented) | 1984 |
| Sympy [A] (verification not implemented) | 1985 |
| Maxima [B] (verification not implemented) | 1985 |
| Giac [F(-2)] | 1986 |
| Mupad [F(-1)] | 1986 |
| Reduce [B] (verification not implemented) | 1987 |

Optimal result

Integrand size = 32, antiderivative size = 39

$$\int \frac{(d+ex)^6}{(cd^2+2cdex+ce^2x^2)^{5/2}} dx = \frac{(d+ex)\sqrt{cd^2+2cdex+ce^2x^2}}{2c^3e}$$

output $1/2*(e*x+d)*(c*e^2*x^2+2*c*d*e*x+c*d^2)^{(1/2)}/c^3/e$

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.85

$$\int \frac{(d+ex)^6}{(cd^2+2cdex+ce^2x^2)^{5/2}} dx = \frac{x(d+ex)(2d+ex)}{2c^2\sqrt{c(d+ex)^2}}$$

input $\text{Integrate}[(d+e*x)^6/(c*d^2+2*c*d*e*x+c*e^2*x^2)^{(5/2)},x]$

output $(x*(d+e*x)*(2*d+e*x))/(2*c^2*\text{Sqrt}[c*(d+e*x)^2])$

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.05, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.062$, Rules used = {1102, 17}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex)^6}{(cd^2+2cdex+ce^2x^2)^{5/2}} dx$$

$$\downarrow 1102$$

$$\frac{c^3e^5(d+ex) \int \frac{d+ex}{c^5e^5} dx}{\sqrt{cd^2+2cdex+ce^2x^2}}$$

$$\downarrow 17$$

$$\frac{(d+ex)^3}{2c^2e\sqrt{cd^2+2cdex+ce^2x^2}}$$

input

```
Int[(d + e*x)^6/(c*d^2 + 2*c*d*e*x + c*e^2*x^2)^(5/2),x]
```

output

```
(d + e*x)^3/(2*c^2*e*Sqrt[c*d^2 + 2*c*d*e*x + c*e^2*x^2])
```

Defintions of rubi rules used

rule 17

```
Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1))/(b*(m + 1))], x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]
```

rule 1102

```
Int[((d_.) + (e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(a + b*x + c*x^2)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x)^(2*FracPart[p])) Int[(d + e*x)^m*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && EqQ[b^2 - 4*a*c, 0]
```

Maple [A] (verified)

Time = 1.70 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.03

| method | result | size |
|---------|--|------|
| gospers | $\frac{x(ex+2d)(ex+d)^5}{2(x^2ce^2+2cdxe+cd^2)^{\frac{5}{2}}}$ | 40 |
| default | $\frac{x(ex+2d)(ex+d)^5}{2(x^2ce^2+2cdxe+cd^2)^{\frac{5}{2}}}$ | 40 |
| orering | $\frac{x(ex+2d)(ex+d)^5}{2(x^2ce^2+2cdxe+cd^2)^{\frac{5}{2}}}$ | 40 |
| trager | $\frac{x(ex+2d)\sqrt{x^2ce^2+2cdxe+cd^2}}{2c^3(ex+d)}$ | 43 |
| risch | $\frac{(ex+d)ex^2}{2c^2\sqrt{c(ex+d)^2}} + \frac{(ex+d)dx}{c^2\sqrt{c(ex+d)^2}}$ | 49 |

input `int((e*x+d)^6/(c*e^2*x^2+2*c*d*e*x+c*d^2)^(5/2),x,method=_RETURNVERBOSE)`

output `1/2*x*(e*x+2*d)*(e*x+d)^5/(c*e^2*x^2+2*c*d*e*x+c*d^2)^(5/2)`

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.23

$$\int \frac{(d+ex)^6}{(cd^2+2cdex+ce^2x^2)^{5/2}} dx = \frac{\sqrt{ce^2x^2+2cdex+cd^2}(ex^2+2dx)}{2(c^3ex+c^3d)}$$

input `integrate((e*x+d)^6/(c*e^2*x^2+2*c*d*e*x+c*d^2)^(5/2),x, algorithm="fricas")`

output `1/2*sqrt(c*e^2*x^2 + 2*c*d*e*x + c*d^2)*(e*x^2 + 2*d*x)/(c^3*e*x + c^3*d)`

Sympy [A] (verification not implemented)

Time = 4.26 (sec) , antiderivative size = 224, normalized size of antiderivative = 5.74

$$\int \frac{(d + ex)^6}{(cd^2 + 2cdex + ce^2x^2)^{5/2}} dx = \frac{\left(\frac{d}{2ce} + \frac{x}{2c} \right) \sqrt{cd^2 + 2cdex + ce^2x^2} + \frac{2d^2\sqrt{cd^2+2cdex} + \frac{2\left(-cd^2\sqrt{cd^2+2cdex} + \frac{(cd^2+2cdex)^{3/2}}{3}\right)}{c}}{2cde} + \frac{c^2d^4\sqrt{cd^2+2cdex} - \frac{2cd^2(cd^2+2cdex)^{3/2}}{3}}{2c^2d^2} + \frac{d^2x+dex^2+\frac{e^2x^3}{3}}{\sqrt{cd^2}}}{c^2}$$

```
input integrate((e*x+d)**6/(c*e**2*x**2+2*c*d*e*x+c*d**2)**(5/2), x)
```

```
output Piecewise(((d/(2*c*e) + x/(2*c))*sqrt(c*d**2 + 2*c*d*e*x + c*e**2*x**2), Ne(c*e**2, 0)), ((2*d**2*sqrt(c*d**2 + 2*c*d*e*x) + 2*(-c*d**2*sqrt(c*d**2 + 2*c*d*e*x) + (c*d**2 + 2*c*d*e*x)**(3/2)/3)/c + (c**2*d**4*sqrt(c*d**2 + 2*c*d*e*x) - 2*c*d**2*(c*d**2 + 2*c*d*e*x)**(3/2)/3 + (c*d**2 + 2*c*d*e*x)**(5/2)/5)/(2*c**2*d**2))/(2*c*d*e), Ne(c*d*e, 0)), ((d**2*x + d*e*x**2 + e**2*x**3/3)/sqrt(c*d**2), True))/c**2
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 232 vs. 2(35) = 70.

Time = 0.05 (sec) , antiderivative size = 232, normalized size of antiderivative = 5.95

$$\int \frac{(d + ex)^6}{(cd^2 + 2cdex + ce^2x^2)^{5/2}} dx = \frac{e^4x^5}{2(ce^2x^2 + 2cdex + cd^2)^{3/2}c} + \frac{5de^3x^4}{2(ce^2x^2 + 2cdex + cd^2)^{3/2}c} - \frac{25c^2d^6e^4}{4(ce^2)^{9/2}\left(x + \frac{d}{e}\right)^4} - \frac{10d^3ex^2}{(ce^2x^2 + 2cdex + cd^2)^{3/2}c} + \frac{50cd^5e^3}{3(ce^2)^{7/2}\left(x + \frac{d}{e}\right)^3} - \frac{26d^5}{3(ce^2x^2 + 2cdex + cd^2)^{3/2}ce} - \frac{25d^4e^2}{2(ce^2)^{5/2}\left(x + \frac{d}{e}\right)^2} + \frac{25d^6}{4(ce^2)^{5/2}\left(x + \frac{d}{e}\right)^4}$$

input `integrate((e*x+d)^6/(c*e^2*x^2+2*c*d*e*x+c*d^2)^(5/2),x, algorithm="maxima")`

output $\frac{1}{2}e^4x^5/((c^2e^2x^2 + 2cde^2x + cd^2)^{3/2})c + \frac{5}{2}d^3e^3x^4/((c^2e^2x^2 + 2cde^2x + cd^2)^{3/2})c - \frac{25}{4}c^2d^6e^4/((c^2e^2)^{9/2})(x + d/e)^4 - 10d^3e^3x^2/((c^2e^2x^2 + 2cde^2x + cd^2)^{3/2})c + \frac{50}{3}c^2d^5e^3/((c^2e^2)^{7/2})(x + d/e)^3 - \frac{26}{3}d^5/((c^2e^2x^2 + 2cde^2x + cd^2)^{3/2})c^2e - \frac{25}{2}d^4e^2/((c^2e^2)^{5/2})(x + d/e)^2 + \frac{25}{4}d^6/((c^2e^2)^{5/2})(x + d/e)^4$

Giac [F(-2)]

Exception generated.

$$\int \frac{(d+ex)^6}{(cd^2 + 2cdex + ce^2x^2)^{5/2}} dx = \text{Exception raised: TypeError}$$

input `integrate((e*x+d)^6/(c*e^2*x^2+2*c*d*e*x+c*d^2)^(5/2),x, algorithm="giac")`

output Exception raised: TypeError >> an error occurred running a Giac command:IN PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const vecteur & l) Error: Bad Argument Value

Mupad [F(-1)]

Timed out.

$$\int \frac{(d+ex)^6}{(cd^2 + 2cdex + ce^2x^2)^{5/2}} dx = \int \frac{(d+ex)^6}{(cd^2 + 2cdex + ce^2x^2)^{5/2}} dx$$

input `int((d + e*x)^6/(c*d^2 + c*e^2*x^2 + 2*c*d*e*x)^(5/2),x)`

output `int((d + e*x)^6/(c*d^2 + c*e^2*x^2 + 2*c*d*e*x)^(5/2), x)`

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.38

$$\int \frac{(d + ex)^6}{(cd^2 + 2cdex + ce^2x^2)^{5/2}} dx = \frac{\sqrt{c}x(ex + 2d)}{2c^3}$$

input `int((e*x+d)^6/(c*e^2*x^2+2*c*d*e*x+c*d^2)^(5/2),x)`

output `(sqrt(c)*x*(2*d + e*x))/(2*c**3)`

$$3.260 \quad \int \frac{(d+ex)^4}{(cd^2+2cdex+ce^2x^2)^{5/2}} dx$$

| | |
|---|------|
| Optimal result | 1988 |
| Mathematica [A] (verified) | 1988 |
| Rubi [A] (verified) | 1989 |
| Maple [A] (verified) | 1990 |
| Fricas [A] (verification not implemented) | 1990 |
| Sympy [A] (verification not implemented) | 1990 |
| Maxima [B] (verification not implemented) | 1991 |
| Giac [F(-2)] | 1992 |
| Mupad [F(-1)] | 1992 |
| Reduce [B] (verification not implemented) | 1992 |

Optimal result

Integrand size = 32, antiderivative size = 42

$$\int \frac{(d+ex)^4}{(cd^2+2cdex+ce^2x^2)^{5/2}} dx = \frac{(d+ex)\log(d+ex)}{c^2e\sqrt{cd^2+2cdex+ce^2x^2}}$$

output $(e*x+d)*\ln(e*x+d)/c^2/e/(c*e^2*x^2+2*c*d*e*x+c*d^2)^(1/2)$

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.74

$$\int \frac{(d+ex)^4}{(cd^2+2cdex+ce^2x^2)^{5/2}} dx = \frac{(d+ex)\log(d+ex)}{c^2e\sqrt{c(d+ex)^2}}$$

input $\text{Integrate}[(d+e*x)^4/(c*d^2+2*c*d*e*x+c*e^2*x^2)^(5/2),x]$

output $((d+e*x)*\text{Log}[d+e*x])/(c^2*e*\text{Sqrt}[c*(d+e*x)^2])$

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.062$, Rules used = {1102, 16}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex)^4}{(cd^2+2cdex+ce^2x^2)^{5/2}} dx$$

$$\downarrow 1102$$

$$\frac{c^3e^5(d+ex) \int \frac{1}{c^5e^5(d+ex)} dx}{\sqrt{cd^2+2cdex+ce^2x^2}}$$

$$\downarrow 16$$

$$\frac{(d+ex) \log(d+ex)}{c^2e\sqrt{cd^2+2cdex+ce^2x^2}}$$

input `Int[(d + e*x)^4/(c*d^2 + 2*c*d*e*x + c*e^2*x^2)^(5/2), x]`

output `((d + e*x)*Log[d + e*x])/(c^2*e*Sqrt[c*d^2 + 2*c*d*e*x + c*e^2*x^2])`

Defintions of rubi rules used

rule 16 `Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a + b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]`

rule 1102 `Int[((d_.) + (e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(a + b*x + c*x^2)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x)^(2*FracPart[p])) Int[(d + e*x)^m*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && EqQ[b^2 - 4*a*c, 0]`

Maple [A] (verified)

Time = 1.38 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.71

| method | result | size |
|---------|---|------|
| risch | $\frac{(ex+d) \ln(ex+d)}{c^2 \sqrt{c(ex+d)^2} e}$ | 30 |
| default | $\frac{(ex+d)^5 \ln(ex+d)}{(x^2 c e^2 + 2cdxe + cd^2)^{\frac{5}{2}} e}$ | 40 |

input `int((e*x+d)^4/(c*e^2*x^2+2*c*d*e*x+c*d^2)^(5/2),x,method=_RETURNVERBOSE)`

output `1/c^2/(c*(e*x+d)^2)^(1/2)*(e*x+d)*ln(e*x+d)/e`

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.10

$$\int \frac{(d+ex)^4}{(cd^2+2cdex+ce^2x^2)^{5/2}} dx = \frac{\sqrt{ce^2x^2+2cdex+cd^2} \log(ex+d)}{c^3e^2x+c^3de}$$

input `integrate((e*x+d)^4/(c*e^2*x^2+2*c*d*e*x+c*d^2)^(5/2),x, algorithm="fricas")`

output `sqrt(c*e^2*x^2+2*c*d*e*x+c*d^2)*log(e*x+d)/(c^3*e^2*x+c^3*d*e)`

Sympy [A] (verification not implemented)

Time = 3.18 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.69

$$\int \frac{(d+ex)^4}{(cd^2+2cdex+ce^2x^2)^{5/2}} dx = \frac{\left(\frac{d}{e}+x\right) \log\left(\frac{d}{e}+x\right)}{c^2 \sqrt{ce^2\left(\frac{d}{e}+x\right)^2}}$$

input `integrate((e*x+d)**4/(c*e**2*x**2+2*c*d*e*x+c*d**2)**(5/2),x)`

output $(d/e + x) \cdot \log(d/e + x) / (c^{**2} \cdot \sqrt{c \cdot e^{**2} \cdot (d/e + x)^{**2}})$

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 449 vs. $2(40) = 80$.

Time = 0.06 (sec) , antiderivative size = 449, normalized size of antiderivative = 10.69

$$\int \frac{(d + ex)^4}{(cd^2 + 2cdex + ce^2x^2)^{5/2}} dx = \frac{1}{12} e^4 \left(\frac{48 \sqrt{cde^3x^3} + 108 \sqrt{cd^2e^2x^2} + 88 \sqrt{cd^3ex} + 25 \sqrt{cd^4}}{c^3e^9x^4 + 4c^3de^8x^3 + 6c^3d^2e^7x^2 + 4c^3d^3e^6x + c^3d^4e^5} + \frac{12 \log(ex + d/e)}{c^{5/2}e^5} \right) - \frac{1}{3} de^3 \left(\frac{3c^2d^3e}{(ce^2)^{9/2} (x + d/e)^4} + \frac{12x^2}{(ce^2x^2 + 2cdex + cd^2)^{3/2} ce^2} - \frac{8cd^2}{(ce^2)^{7/2} (x + d/e)^3} + \frac{8d^2}{(ce^2x^2 + 2cdex + cd^2)^{3/2} ce^4} \right) - \frac{1}{2} d^2e^2 \left(\frac{3c^2d^2e^2}{(ce^2)^{9/2} (x + d/e)^4} - \frac{8cde}{(ce^2)^{7/2} (x + d/e)^3} + \frac{6}{(ce^2)^{5/2} (x + d/e)^2} \right) - \frac{1}{3} d^3e \left(\frac{4}{(ce^2x^2 + 2cdex + cd^2)^{3/2} ce^2} - \frac{3d}{(ce^2)^{5/2} e(x + d/e)^4} \right) - \frac{d^4}{4 (ce^2)^{5/2} (x + d/e)^4}$$

input `integrate((e*x+d)^4/(c*e^2*x^2+2*c*d*e*x+c*d^2)^(5/2),x, algorithm="maxima")`

output $\frac{1}{12} e^4 \left(\frac{(48 \sqrt{c} d^3 e^3 x^3 + 108 \sqrt{c} d^2 e^2 x^2 + 88 \sqrt{c} d^3 e x + 25 \sqrt{c} d^4)}{c^3 e^9 x^4 + 4 c^3 d e^8 x^3 + 6 c^3 d^2 e^7 x^2 + 4 c^3 d^3 e^6 x + c^3 d^4 e^5} + 12 \log(ex + d) / (c^{5/2} e^5) \right) - \frac{1}{3} d^3 e^3 \left(\frac{3 c^2 d^3 e}{(c e^2)^{9/2} (x + d/e)^4} + \frac{12 x^2}{(c e^2 x^2 + 2 c d e x + c d^2)^{3/2} c e^2} - \frac{8 c d^2}{(c e^2)^{7/2} (x + d/e)^3} + \frac{8 d^2}{(c e^2 x^2 + 2 c d e x + c d^2)^{3/2} c e^4} \right) - \frac{1}{2} d^2 e^2 \left(\frac{3 c^2 d^2 e^2}{(c e^2)^{9/2} (x + d/e)^4} - \frac{8 c d e}{(c e^2)^{7/2} (x + d/e)^3} + \frac{6}{(c e^2)^{5/2} (x + d/e)^2} \right) - \frac{1}{3} d^3 e \left(\frac{4}{(c e^2 x^2 + 2 c d e x + c d^2)^{3/2} c e^2} - \frac{3 d}{(c e^2)^{5/2} e (x + d/e)^4} \right) - \frac{1}{4} d^4 / (c e^2)^{5/2} (x + d/e)^4$

Giac [F(-2)]

Exception generated.

$$\int \frac{(d + ex)^4}{(cd^2 + 2cdex + ce^2x^2)^{5/2}} dx = \text{Exception raised: TypeError}$$

input `integrate((e*x+d)^4/(c*e^2*x^2+2*c*d*e*x+c*d^2)^(5/2),x, algorithm="giac")`

output `Exception raised: TypeError >> an error occurred running a Giac command:INPUT:sage2:=int(sage0,sageVARx)::OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const vecteur & l) Error: Bad Argument Value`

Mupad [F(-1)]

Timed out.

$$\int \frac{(d + ex)^4}{(cd^2 + 2cdex + ce^2x^2)^{5/2}} dx = \int \frac{(d + ex)^4}{(cd^2 + 2cdex + ce^2x^2)^{5/2}} dx$$

input `int((d + e*x)^4/(c*d^2 + c*e^2*x^2 + 2*c*d*e*x)^(5/2),x)`

output `int((d + e*x)^4/(c*d^2 + c*e^2*x^2 + 2*c*d*e*x)^(5/2), x)`

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.36

$$\int \frac{(d + ex)^4}{(cd^2 + 2cdex + ce^2x^2)^{5/2}} dx = \frac{\sqrt{c} \log(ex + d)}{c^3 e}$$

input `int((e*x+d)^4/(c*e^2*x^2+2*c*d*e*x+c*d^2)^(5/2),x)`

output `(sqrt(c)*log(d + e*x))/(c**3*e)`

$$3.261 \quad \int \frac{(d+ex)^2}{(cd^2+2cdex+ce^2x^2)^{5/2}} dx$$

| | |
|---|------|
| Optimal result | 1993 |
| Mathematica [A] (verified) | 1993 |
| Rubi [A] (verified) | 1994 |
| Maple [A] (verified) | 1995 |
| Fricas [A] (verification not implemented) | 1995 |
| Sympy [F] | 1996 |
| Maxima [B] (verification not implemented) | 1996 |
| Giac [F(-2)] | 1997 |
| Mupad [B] (verification not implemented) | 1997 |
| Reduce [B] (verification not implemented) | 1997 |

Optimal result

Integrand size = 32, antiderivative size = 41

$$\int \frac{(d+ex)^2}{(cd^2+2cdex+ce^2x^2)^{5/2}} dx = -\frac{1}{2c^2e(d+ex)\sqrt{cd^2+2cdex+ce^2x^2}}$$

output $-1/2/c^2/e/(e*x+d)/(c*e^2*x^2+2*c*d*e*x+c*d^2)^(1/2)$

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.68

$$\int \frac{(d+ex)^2}{(cd^2+2cdex+ce^2x^2)^{5/2}} dx = -\frac{d+ex}{2ce(c(d+ex)^2)^{3/2}}$$

input $\text{Integrate}[(d+e*x)^2/(c*d^2+2*c*d*e*x+c*e^2*x^2)^(5/2),x]$

output $-1/2*(d+e*x)/(c*e*(c*(d+e*x)^2)^(3/2))$

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.062$, Rules used = {1102, 17}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex)^2}{(cd^2+2cdex+ce^2x^2)^{5/2}} dx$$

$$\downarrow 1102$$

$$\frac{c^3e^5(d+ex) \int \frac{1}{c^5e^5(d+ex)^3} dx}{\sqrt{cd^2+2cdex+ce^2x^2}}$$

$$\downarrow 17$$

$$-\frac{1}{2c^2e(d+ex)\sqrt{cd^2+2cdex+ce^2x^2}}$$

input

```
Int[(d + e*x)^2/(c*d^2 + 2*c*d*e*x + c*e^2*x^2)^(5/2), x]
```

output

```
-1/2*1/(c^2*e*(d + e*x)*Sqrt[c*d^2 + 2*c*d*e*x + c*e^2*x^2])
```

Defintions of rubi rules used

rule 17

```
Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] := Simp[c*((a + b*x)^(m + 1))/(b*(m + 1)), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]
```

rule 1102

```
Int[((d_.) + (e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] := Simp[(a + b*x + c*x^2)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x)^(2*FracPart[p])) Int[(d + e*x)^m*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && EqQ[b^2 - 4*a*c, 0]
```

Maple [A] (verified)

Time = 1.30 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.66

| method | result | size |
|---------|---|------|
| risch | $-\frac{1}{2c^2(ex+d)\sqrt{c(ex+d)^2}e}$ | 27 |
| gospers | $-\frac{(ex+d)^3}{2e(x^2ce^2+2cdxe+cd^2)^{\frac{5}{2}}}$ | 35 |
| default | $-\frac{(ex+d)^3}{2e(x^2ce^2+2cdxe+cd^2)^{\frac{5}{2}}}$ | 35 |
| orering | $-\frac{(ex+d)^3}{2e(x^2ce^2+2cdxe+cd^2)^{\frac{5}{2}}}$ | 35 |
| trager | $\frac{(ex+2d)x\sqrt{x^2ce^2+2cdxe+cd^2}}{2d^2c^3(ex+d)^3}$ | 46 |

input `int((e*x+d)^2/(c*e^2*x^2+2*c*d*e*x+c*d^2)^(5/2),x,method=_RETURNVERBOSE)`

output `-1/2/c^2/(e*x+d)/(c*(e*x+d)^2)^(1/2)/e`

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.68

$$\int \frac{(d+ex)^2}{(cd^2+2cdex+ce^2x^2)^{5/2}} dx = -\frac{\sqrt{ce^2x^2+2cdex+cd^2}}{2(c^3e^4x^3+3c^3de^3x^2+3c^3d^2e^2x+c^3d^3e)}$$

input `integrate((e*x+d)^2/(c*e^2*x^2+2*c*d*e*x+c*d^2)^(5/2),x, algorithm="fricas")`

output `-1/2*sqrt(c*e^2*x^2+2*c*d*e*x+c*d^2)/(c^3*e^4*x^3+3*c^3*d*e^3*x^2+3*c^3*d^2*e^2*x+c^3*d^3*e)`

Sympy [F]

$$\int \frac{(d+ex)^2}{(cd^2+2cdex+ce^2x^2)^{5/2}} dx = \int \frac{(d+ex)^2}{(c(d+ex)^2)^{5/2}} dx$$

input `integrate((e*x+d)**2/(c*e**2*x**2+2*c*d*e*x+c*d**2)**(5/2), x)`

output `Integral((d + e*x)**2/(c*(d + e*x)**2)**(5/2), x)`

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 124 vs. 2(37) = 74.

Time = 0.03 (sec) , antiderivative size = 124, normalized size of antiderivative = 3.02

$$\int \frac{(d+ex)^2}{(cd^2+2cdex+ce^2x^2)^{5/2}} dx = -\frac{c^2d^2e^4}{4(c^2)^{\frac{9}{2}}(x+\frac{d}{e})^4} + \frac{2cde^3}{3(c^2)^{\frac{7}{2}}(x+\frac{d}{e})^3}$$

$$- \frac{2d}{3(ce^2x^2+2cdex+cd^2)^{\frac{3}{2}}ce} - \frac{e^2}{2(c^2)^{\frac{5}{2}}(x+\frac{d}{e})^2} + \frac{d^2}{4(c^2)^{\frac{5}{2}}(x+\frac{d}{e})^4}$$

input `integrate((e*x+d)^2/(c*e^2*x^2+2*c*d*e*x+c*d^2)^(5/2), x, algorithm="maxima")`

output `-1/4*c^2*d^2*e^4/((c*e^2)^(9/2)*(x + d/e)^4) + 2/3*c*d*e^3/((c*e^2)^(7/2)*(x + d/e)^3) - 2/3*d/((c*e^2*x^2 + 2*c*d*e*x + c*d^2)^(3/2)*c*e) - 1/2*e^2/((c*e^2)^(5/2)*(x + d/e)^2) + 1/4*d^2/((c*e^2)^(5/2)*(x + d/e)^4)`

Giac [F(-2)]

Exception generated.

$$\int \frac{(d+ex)^2}{(cd^2+2cdex+ce^2x^2)^{5/2}} dx = \text{Exception raised: TypeError}$$

input `integrate((e*x+d)^2/(c*e^2*x^2+2*c*d*e*x+c*d^2)^(5/2),x, algorithm="giac")`

output `Exception raised: TypeError >> an error occurred running a Giac command:INPUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const vecteur & l) Error: Bad Argument Value`

Mupad [B] (verification not implemented)

Time = 9.55 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.90

$$\int \frac{(d+ex)^2}{(cd^2+2cdex+ce^2x^2)^{5/2}} dx = -\frac{\sqrt{cd^2+2cdex+ce^2x^2}}{2c^3e(d+ex)^3}$$

input `int((d + e*x)^2/(c*d^2 + c*e^2*x^2 + 2*c*d*e*x)^(5/2),x)`

output `-(c*d^2 + c*e^2*x^2 + 2*c*d*e*x)^(1/2)/(2*c^3*e*(d + e*x)^3)`

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.68

$$\int \frac{(d+ex)^2}{(cd^2+2cdex+ce^2x^2)^{5/2}} dx = -\frac{\sqrt{c}}{2c^3e(e^2x^2+2dex+d^2)}$$

input `int((e*x+d)^2/(c*e^2*x^2+2*c*d*e*x+c*d^2)^(5/2),x)`

output `(- sqrt(c))/(2*c**3*e*(d**2 + 2*d*e*x + e**2*x**2))`

$$3.262 \quad \int \frac{1}{(cd^2 + 2cdex + ce^2x^2)^{5/2}} dx$$

| | |
|---|------|
| Optimal result | 1998 |
| Mathematica [A] (verified) | 1998 |
| Rubi [A] (verified) | 1999 |
| Maple [A] (verified) | 2000 |
| Fricas [B] (verification not implemented) | 2000 |
| Sympy [F] | 2001 |
| Maxima [A] (verification not implemented) | 2001 |
| Giac [F(-2)] | 2001 |
| Mupad [B] (verification not implemented) | 2002 |
| Reduce [B] (verification not implemented) | 2002 |

Optimal result

Integrand size = 24, antiderivative size = 41

$$\int \frac{1}{(cd^2 + 2cdex + ce^2x^2)^{5/2}} dx = -\frac{1}{4c^2e(d+ex)^3\sqrt{cd^2 + 2cdex + ce^2x^2}}$$

output $-1/4/c^2/e/(e*x+d)^3/(c*e^2*x^2+2*c*d*e*x+c*d^2)^{(1/2)}$

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.61

$$\int \frac{1}{(cd^2 + 2cdex + ce^2x^2)^{5/2}} dx = -\frac{d+ex}{4e(c(d+ex)^2)^{5/2}}$$

input `Integrate[(c*d^2 + 2*c*d*e*x + c*e^2*x^2)^(-5/2),x]`

output $-1/4*(d + e*x)/(e*(c*(d + e*x)^2)^{(5/2)})$

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.042$, Rules used = {1078}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(cd^2 + 2cdex + ce^2x^2)^{5/2}} dx$$

↓ 1078

$$-\frac{1}{4ce(d+ex)(cd^2 + 2cdex + ce^2x^2)^{3/2}}$$

input `Int[(c*d^2 + 2*c*d*e*x + c*e^2*x^2)^(-5/2), x]`

output `-1/4*1/(c*e*(d + e*x)*(c*d^2 + 2*c*d*e*x + c*e^2*x^2)^(3/2))`

Defintions of rubi rules used

rule 1078 `Int[((a_) + (b_)*(x_) + (c_)*(x_)^2)^(p_), x_Symbol] :> Simp[2*((a + b*x + c*x^2)^(p + 1)/((2*p + 1)*(b + 2*c*x))), x] /; FreeQ[{a, b, c, p}, x] && EqQ[b^2 - 4*a*c, 0] && LtQ[p, -1]`

Maple [A] (verified)

Time = 1.10 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.66

| method | result | size |
|---------|---|------|
| risch | $-\frac{1}{4c^2(ex+d)^3\sqrt{c(ex+d)^2e}}$ | 27 |
| gosper | $-\frac{ex+d}{4e(x^2ce^2+2cdxe+cd^2)^{\frac{5}{2}}}$ | 33 |
| default | $-\frac{ex+d}{4e(x^2ce^2+2cdxe+cd^2)^{\frac{5}{2}}}$ | 33 |
| orering | $-\frac{ex+d}{4e(x^2ce^2+2cdxe+cd^2)^{\frac{5}{2}}}$ | 33 |
| trager | $\frac{(e^3x^3+4de^2x^2+6d^2ex+4d^3)x\sqrt{x^2ce^2+2cdxe+cd^2}}{4d^4c^3(ex+d)^5}$ | 68 |

input `int(1/(c*e^2*x^2+2*c*d*e*x+c*d^2)^(5/2),x,method=_RETURNVERBOSE)`

output `-1/4/c^2/(e*x+d)^3/(c*(e*x+d)^2)^(1/2)/e`

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 97 vs. 2(37) = 74.

Time = 0.08 (sec) , antiderivative size = 97, normalized size of antiderivative = 2.37

$$\int \frac{1}{(cd^2 + 2cdex + ce^2x^2)^{5/2}} dx =$$

$$-\frac{\sqrt{ce^2x^2 + 2cdex + cd^2}}{4(c^3e^6x^5 + 5c^3de^5x^4 + 10c^3d^2e^4x^3 + 10c^3d^3e^3x^2 + 5c^3d^4e^2x + c^3d^5e)}$$

input `integrate(1/(c*e^2*x^2+2*c*d*e*x+c*d^2)^(5/2),x, algorithm="fricas")`

output `-1/4*sqrt(c*e^2*x^2 + 2*c*d*e*x + c*d^2)/(c^3*e^6*x^5 + 5*c^3*d*e^5*x^4 + 10*c^3*d^2*e^4*x^3 + 10*c^3*d^3*e^3*x^2 + 5*c^3*d^4*e^2*x + c^3*d^5*e)`

Sympy [F]

$$\int \frac{1}{(cd^2 + 2cdex + ce^2x^2)^{5/2}} dx = \int \frac{1}{(cd^2 + 2cdex + ce^2x^2)^{\frac{5}{2}}} dx$$

input `integrate(1/(c*e**2*x**2+2*c*d*e*x+c*d**2)**(5/2),x)`

output `Integral((c*d**2 + 2*c*d*e*x + c*e**2*x**2)**(-5/2), x)`

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.44

$$\int \frac{1}{(cd^2 + 2cdex + ce^2x^2)^{5/2}} dx = -\frac{1}{4 (ce^2)^{\frac{5}{2}} (x + \frac{d}{e})^4}$$

input `integrate(1/(c*e^2*x^2+2*c*d*e*x+c*d^2)^(5/2),x, algorithm="maxima")`

output `-1/4/((c*e^2)^(5/2)*(x + d/e)^4)`

Giac [F(-2)]

Exception generated.

$$\int \frac{1}{(cd^2 + 2cdex + ce^2x^2)^{5/2}} dx = \text{Exception raised: TypeError}$$

input `integrate(1/(c*e^2*x^2+2*c*d*e*x+c*d^2)^(5/2),x, algorithm="giac")`

output `Exception raised: TypeError >> an error occurred running a Giac command:INPUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const vecteur & l) Error: Bad Argument Value`

Mupad [B] (verification not implemented)

Time = 9.34 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.90

$$\int \frac{1}{(cd^2 + 2cdex + ce^2x^2)^{5/2}} dx = -\frac{\sqrt{cd^2 + 2cdex + ce^2x^2}}{4c^3e(d+ex)^5}$$

input `int(1/(c*d^2 + c*e^2*x^2 + 2*c*d*e*x)^(5/2),x)`output `-(c*d^2 + c*e^2*x^2 + 2*c*d*e*x)^(1/2)/(4*c^3*e*(d + e*x)^5)`**Reduce [B] (verification not implemented)**

Time = 0.22 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.22

$$\int \frac{1}{(cd^2 + 2cdex + ce^2x^2)^{5/2}} dx = -\frac{\sqrt{c}}{4c^3e(e^4x^4 + 4de^3x^3 + 6d^2e^2x^2 + 4d^3ex + d^4)}$$

input `int(1/(c*e^2*x^2+2*c*d*e*x+c*d^2)^(5/2),x)`output `(- sqrt(c))/(4*c**3*e*(d**4 + 4*d**3*e*x + 6*d**2*e**2*x**2 + 4*d*e**3*x**3 + e**4*x**4))`

$$3.263 \quad \int \frac{1}{(d+ex)^2 (cd^2+2cdex+ce^2x^2)^{5/2}} dx$$

| | |
|---|------|
| Optimal result | 2003 |
| Mathematica [A] (verified) | 2003 |
| Rubi [A] (verified) | 2004 |
| Maple [A] (verified) | 2005 |
| Fricas [B] (verification not implemented) | 2005 |
| Sympy [F] | 2006 |
| Maxima [B] (verification not implemented) | 2006 |
| Giac [F(-2)] | 2007 |
| Mupad [B] (verification not implemented) | 2007 |
| Reduce [B] (verification not implemented) | 2007 |

Optimal result

Integrand size = 32, antiderivative size = 41

$$\int \frac{1}{(d+ex)^2 (cd^2+2cdex+ce^2x^2)^{5/2}} dx = -\frac{1}{6c^2e(d+ex)^5\sqrt{cd^2+2cdex+ce^2x^2}}$$

output `-1/6/c^2/e/(e*x+d)^5/(c*e^2*x^2+2*c*d*e*x+c*d^2)^(1/2)`

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.63

$$\int \frac{1}{(d+ex)^2 (cd^2+2cdex+ce^2x^2)^{5/2}} dx = -\frac{c(d+ex)}{6e(c(d+ex)^2)^{7/2}}$$

input `Integrate[1/((d + e*x)^2*(c*d^2 + 2*c*d*e*x + c*e^2*x^2)^(5/2)),x]`

output `-1/6*(c*(d + e*x))/(e*(c*(d + e*x)^2)^(7/2))`

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.062$, Rules used = {1102, 17}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(d+ex)^2 (cd^2 + 2cdex + ce^2x^2)^{5/2}} dx$$

$$\downarrow \text{1102}$$

$$\frac{c^3 e^5 (d+ex) \int \frac{1}{c^5 e^5 (d+ex)^7} dx}{\sqrt{cd^2 + 2cdex + ce^2x^2}}$$

$$\downarrow \text{17}$$

$$-\frac{1}{6c^2 e (d+ex)^5 \sqrt{cd^2 + 2cdex + ce^2x^2}}$$

input `Int[1/((d + e*x)^2*(c*d^2 + 2*c*d*e*x + c*e^2*x^2)^(5/2)),x]`

output `-1/6*1/(c^2*e*(d + e*x)^5*Sqrt[c*d^2 + 2*c*d*e*x + c*e^2*x^2])`

Defintions of rubi rules used

rule 17 `Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] := Simp[c*((a + b*x)^(m + 1))/(b*(m + 1))], x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]`

rule 1102 `Int[((d_.) + (e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] := Simp[(a + b*x + c*x^2)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x)^(2*FracPart[p])) Int[(d + e*x)^m*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && EqQ[b^2 - 4*a*c, 0]`

Maple [A] (verified)

Time = 1.44 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.66

| method | result | size |
|---------|--|------|
| risch | $-\frac{1}{6c^2(ex+d)^5\sqrt{c(ex+d)^2}e}$ | 27 |
| gospers | $-\frac{1}{6(ex+d)e(x^2ce^2+2cdxe+cd^2)^{\frac{5}{2}}}$ | 35 |
| default | $-\frac{1}{6(ex+d)e(x^2ce^2+2cdxe+cd^2)^{\frac{5}{2}}}$ | 35 |
| orering | $-\frac{1}{6(ex+d)e(x^2ce^2+2cdxe+cd^2)^{\frac{5}{2}}}$ | 35 |
| trager | $\frac{(e^5x^5+6de^4x^4+15d^2e^3x^3+20d^3e^2x^2+15d^4ex+6d^5)x\sqrt{x^2ce^2+2cdxe+cd^2}}{6d^6c^3(ex+d)^7}$ | 90 |

input `int(1/(e*x+d)^2/(c*e^2*x^2+2*c*d*e*x+c*d^2)^(5/2),x,method=_RETURNVERBOSE)`

output `-1/6/c^2/(e*x+d)^5/(c*(e*x+d)^2)^(1/2)/e`

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 125 vs. 2(37) = 74.

Time = 0.08 (sec) , antiderivative size = 125, normalized size of antiderivative = 3.05

$$\int \frac{1}{(d+ex)^2(cd^2+2cdex+ce^2x^2)^{5/2}} dx =$$

$$\frac{\sqrt{ce^2x^2+2cdex+cd^2}}{6(c^3e^8x^7+7c^3de^7x^6+21c^3d^2e^6x^5+35c^3d^3e^5x^4+35c^3d^4e^4x^3+21c^3d^5e^3x^2+7c^3d^6e^2x+c^3d^7e)}$$

input `integrate(1/(e*x+d)^2/(c*e^2*x^2+2*c*d*e*x+c*d^2)^(5/2),x, algorithm="fricas")`

output `-1/6*sqrt(c*e^2*x^2+2*c*d*e*x+c*d^2)/(c^3*e^8*x^7+7*c^3*d*e^7*x^6+21*c^3*d^2*e^6*x^5+35*c^3*d^3*e^5*x^4+35*c^3*d^4*e^4*x^3+21*c^3*d^5*e^3*x^2+7*c^3*d^6*e^2*x+c^3*d^7*e)`

Sympy [F]

$$\int \frac{1}{(d+ex)^2 (cd^2 + 2cdex + ce^2x^2)^{5/2}} dx = \int \frac{1}{(c(d+ex)^2)^{5/2} (d+ex)^2} dx$$

input `integrate(1/(e*x+d)**2/(c*e**2*x**2+2*c*d*e*x+c*d**2)**(5/2),x)`

output `Integral(1/((c*(d + e*x)**2)**(5/2)*(d + e*x)**2), x)`

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 89 vs. $2(37) = 74$.

Time = 0.04 (sec) , antiderivative size = 89, normalized size of antiderivative = 2.17

$$\int \frac{1}{(d+ex)^2 (cd^2 + 2cdex + ce^2x^2)^{5/2}} dx = \frac{1}{6 \left(c^{\frac{5}{2}} e^7 x^6 + 6 c^{\frac{5}{2}} d e^6 x^5 + 15 c^{\frac{5}{2}} d^2 e^5 x^4 + 20 c^{\frac{5}{2}} d^3 e^4 x^3 + 15 c^{\frac{5}{2}} d^4 e^3 x^2 + 6 c^{\frac{5}{2}} d^5 e^2 x + c^{\frac{5}{2}} d^6 e \right)}$$

input `integrate(1/(e*x+d)^2/(c*e^2*x^2+2*c*d*e*x+c*d^2)^(5/2),x, algorithm="maxima")`

output `-1/6/(c^(5/2)*e^7*x^6 + 6*c^(5/2)*d*e^6*x^5 + 15*c^(5/2)*d^2*e^5*x^4 + 20*c^(5/2)*d^3*e^4*x^3 + 15*c^(5/2)*d^4*e^3*x^2 + 6*c^(5/2)*d^5*e^2*x + c^(5/2)*d^6*e)`

Giac [F(-2)]

Exception generated.

$$\int \frac{1}{(d+ex)^2 (cd^2 + 2cdex + ce^2x^2)^{5/2}} dx = \text{Exception raised: TypeError}$$

input `integrate(1/(e*x+d)^2/(c*e^2*x^2+2*c*d*e*x+c*d^2)^(5/2),x, algorithm="giac")`

output Exception raised: TypeError >> an error occurred running a Giac command:INPUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const vecteur & l) Error: Bad Argument Value

Mupad [B] (verification not implemented)

Time = 9.06 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.90

$$\int \frac{1}{(d+ex)^2 (cd^2 + 2cdex + ce^2x^2)^{5/2}} dx = -\frac{\sqrt{cd^2 + 2cdex + ce^2x^2}}{6c^3e(d+ex)^7}$$

input `int(1/((d + e*x)^2*(c*d^2 + c*e^2*x^2 + 2*c*d*e*x)^(5/2)),x)`

output `-(c*d^2 + c*e^2*x^2 + 2*c*d*e*x)^(1/2)/(6*c^3*e*(d + e*x)^7)`

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.76

$$\int \frac{1}{(d+ex)^2 (cd^2 + 2cdex + ce^2x^2)^{5/2}} dx = \frac{\sqrt{c}}{6c^3e(e^6x^6 + 6de^5x^5 + 15d^2e^4x^4 + 20d^3e^3x^3 + 15d^4e^2x^2 + 6d^5ex + d^6)}$$

input `int(1/(e*x+d)^2/(c*e^2*x^2+2*c*d*e*x+c*d^2)^(5/2),x)`

output

```
( - sqrt(c))/(6*c**3*e*(d**6 + 6*d**5*e*x + 15*d**4*e**2*x**2 + 20*d**3*e*  
*3*x**3 + 15*d**2*e**4*x**4 + 6*d*e**5*x**5 + e**6*x**6))
```

$$3.264 \quad \int \frac{1}{\left(\frac{be}{2c} + ex\right) \sqrt{\frac{b^2}{4c} + bx + cx^2}} dx$$

| | |
|---|------|
| Optimal result | 2009 |
| Mathematica [A] (verified) | 2009 |
| Rubi [A] (verified) | 2010 |
| Maple [A] (verified) | 2010 |
| Fricas [A] (verification not implemented) | 2011 |
| Sympy [A] (verification not implemented) | 2012 |
| Maxima [A] (verification not implemented) | 2012 |
| Giac [F(-2)] | 2012 |
| Mupad [B] (verification not implemented) | 2013 |
| Reduce [B] (verification not implemented) | 2013 |

Optimal result

Integrand size = 39, antiderivative size = 27

$$\int \frac{1}{\left(\frac{be}{2c} + ex\right) \sqrt{\frac{b^2}{4c} + bx + cx^2}} dx = -\frac{2}{e \sqrt{\frac{b^2}{c} + 4bx + 4cx^2}}$$

output $-2/e/(b^2/c+4*b*x+4*c*x^2)^{(1/2)}$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.78

$$\int \frac{1}{\left(\frac{be}{2c} + ex\right) \sqrt{\frac{b^2}{4c} + bx + cx^2}} dx = -\frac{2}{e \sqrt{\frac{(b+2cx)^2}{c}}}$$

input `Integrate[1/(((b*e)/(2*c) + e*x)*Sqrt[b^2/(4*c) + b*x + c*x^2]),x]`

output $-2/(e*Sqrt[(b + 2*c*x)^2/c])$

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.026$, Rules used = {1099}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{\frac{b^2}{4c} + bx + cx^2} \left(\frac{be}{2c} + ex\right)} dx$$

↓ 1099

$$-\frac{2}{e\sqrt{\frac{b^2}{c} + 4bx + 4cx^2}}$$

input `Int[1/(((b*e)/(2*c) + e*x)*Sqrt[b^2/(4*c) + b*x + c*x^2]),x]`

output `-2/(e*Sqrt[b^2/c + 4*b*x + 4*c*x^2])`

Defintions of rubi rules used

rule 1099

```
Int[((d_) + (e_)*(x_))^(m_)*((a_) + (b_)*(x_) + (c_)*(x_)^2)^(p_), x_Sy
mbol] := Simp[e^m*((a + b*x + c*x^2)^(p + (m + 1)/2)/(c^((m + 1)/2)*(m + 2*
p + 1))), x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[b^2 - 4*a*c, 0] && EqQ[
2*c*d - b*e, 0] && IntegerQ[(m - 1)/2]
```

Maple [A] (verified)

Time = 1.50 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.74

| method | result | size |
|----------------|---|------|
| risch | $-\frac{2}{e\sqrt{\frac{(2cx+b)^2}{c}}}$ | 20 |
| pseudoelliptic | $-\frac{2}{e\sqrt{\frac{(2cx+b)^2}{c}}}$ | 20 |
| gospers | $-\frac{2}{\sqrt{\frac{4c^2x^2+4cbx+b^2}{c}}e}$ | 29 |
| default | $-\frac{2}{\sqrt{\frac{4c^2x^2+4cbx+b^2}{c}}e}$ | 29 |
| orering | $-\frac{2cx+b}{c\left(\frac{be}{2c}+ex\right)\sqrt{\frac{b^2}{c}+4bx+4cx^2}}$ | 45 |
| trager | $\frac{4c^2x\sqrt{\frac{-4c^2x^2-4cbx-b^2}{c}}}{be(2cx+b)^2}$ | 47 |

input `int(2/(1/2*b/c*e+e*x)/(1/c*b^2+4*b*x+4*c*x^2)^(1/2),x,method=_RETURNVERBOSE)`

output `-2/e/((2*c*x+b)^2/c)^(1/2)`

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.81

$$\int \frac{1}{\left(\frac{be}{2c} + ex\right) \sqrt{\frac{b^2}{4c} + bx + cx^2}} dx = -\frac{2c\sqrt{\frac{4c^2x^2+4bcx+b^2}{c}}}{4c^2ex^2 + 4bcex + b^2e}$$

input `integrate(2/(1/2*b*e/c+e*x)/(b^2/c+4*b*x+4*c*x^2)^(1/2),x, algorithm="fricas")`

output `-2*c*sqrt((4*c^2*x^2 + 4*b*c*x + b^2)/c)/(4*c^2*e*x^2 + 4*b*c*e*x + b^2*e)`

Sympy [A] (verification not implemented)

Time = 1.34 (sec) , antiderivative size = 56, normalized size of antiderivative = 2.07

$$\int \frac{1}{\left(\frac{be}{2c} + ex\right) \sqrt{\frac{b^2}{4c} + bx + cx^2}} dx = 2 \left(\begin{cases} -\frac{1}{e\sqrt{\frac{b^2}{c} + 4bx + 4cx^2}} & \text{for } e \neq 0 \\ \tilde{\infty} \left(\frac{b}{2c} + x\right) \log\left(\frac{b}{2c} + x\right) \\ \sqrt{c\left(\frac{b}{2c} + x\right)^2} & \text{otherwise} \end{cases} \right)$$

input `integrate(2/(1/2*b*e/c+e*x)/(b**2/c+4*b*x+4*c*x**2)**(1/2), x)`

output `2*Piecewise((-1/(e*sqrt(b**2/c + 4*b*x + 4*c*x**2)), Ne(e, 0)), (zoo*(b/(2*c) + x)*log(b/(2*c) + x)/sqrt(c*(b/(2*c) + x)**2), True))`

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.22

$$\int \frac{1}{\left(\frac{be}{2c} + ex\right) \sqrt{\frac{b^2}{4c} + bx + cx^2}} dx = -\frac{2}{2e^2x\sqrt{\frac{c}{e^2} + \frac{be^2\sqrt{\frac{c}{e^2}}}{c}}}$$

input `integrate(2/(1/2*b*e/c+e*x)/(b^2/c+4*b*x+4*c*x^2)^(1/2), x, algorithm="maxima")`

output `-2/(2*e^2*x*sqrt(c/e^2) + b*e^2*sqrt(c/e^2)/c)`

Giac [F(-2)]

Exception generated.

$$\int \frac{1}{\left(\frac{be}{2c} + ex\right) \sqrt{\frac{b^2}{4c} + bx + cx^2}} dx = \text{Exception raised: TypeError}$$

input `integrate(2/(1/2*b*e/c+e*x)/(b^2/c+4*b*x+4*c*x^2)^(1/2),x, algorithm="giac")`

output `Exception raised: TypeError >> an error occurred running a Giac command:INPUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const vecteur & l) Error: Bad Argument Value`

Mupad [B] (verification not implemented)

Time = 9.04 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.93

$$\int \frac{1}{\left(\frac{be}{2c} + ex\right) \sqrt{\frac{b^2}{4c} + bx + cx^2}} dx = -\frac{2}{e \sqrt{4bx + 4cx^2 + \frac{b^2}{c}}}$$

input `int(2/((e*x + (b*e)/(2*c))*(4*b*x + 4*c*x^2 + b^2/c)^(1/2)),x)`

output `-2/(e*(4*b*x + 4*c*x^2 + b^2/c)^(1/2))`

Reduce [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.74

$$\int \frac{1}{\left(\frac{be}{2c} + ex\right) \sqrt{\frac{b^2}{4c} + bx + cx^2}} dx = \frac{4\sqrt{c} cx}{be(2cx + b)}$$

input `int(2/(1/2*b*e/c+e*x)/(b^2/c+4*b*x+4*c*x^2)^(1/2),x)`

output `(4*sqrt(c)*c*x)/(b*e*(b + 2*c*x))`

3.265 $\int (d + ex)^4 \sqrt{a^2 + 2abx + b^2x^2} dx$

| | |
|---|------|
| Optimal result | 2014 |
| Mathematica [A] (verified) | 2014 |
| Rubi [A] (verified) | 2015 |
| Maple [A] (verified) | 2016 |
| Fricas [A] (verification not implemented) | 2017 |
| Sympy [A] (verification not implemented) | 2017 |
| Maxima [B] (verification not implemented) | 2018 |
| Giac [B] (verification not implemented) | 2020 |
| Mupad [B] (verification not implemented) | 2021 |
| Reduce [B] (verification not implemented) | 2022 |

Optimal result

Integrand size = 28, antiderivative size = 92

$$\int (d + ex)^4 \sqrt{a^2 + 2abx + b^2x^2} dx = -\frac{(bd - ae)(d + ex)^5 \sqrt{a^2 + 2abx + b^2x^2}}{5e^2(a + bx)} + \frac{b(d + ex)^6 \sqrt{a^2 + 2abx + b^2x^2}}{6e^2(a + bx)}$$

output

```
-1/5*(-a*e+b*d)*(e*x+d)^5*((b*x+a)^2)^(1/2)/e^2/(b*x+a)+1/6*b*(e*x+d)^6*((b*x+a)^2)^(1/2)/e^2/(b*x+a)
```

Mathematica [A] (verified)

Time = 1.04 (sec) , antiderivative size = 111, normalized size of antiderivative = 1.21

$$\int (d + ex)^4 \sqrt{a^2 + 2abx + b^2x^2} dx = \frac{x \sqrt{(a + bx)^2(6a(5d^4 + 10d^3ex + 10d^2e^2x^2 + 5de^3x^3 + e^4x^4) + bx(15d^4 + 40d^3ex + 45d^2e^2x^2 + 24de^3x^3))}}{30(a + bx)}$$

input

```
Integrate[(d + e*x)^4*sqrt[a^2 + 2*a*b*x + b^2*x^2],x]
```

output

```
(x*sqrt[(a + b*x)^2]*(6*a*(5*d^4 + 10*d^3*e*x + 10*d^2*e^2*x^2 + 5*d*e^3*x^3 + e^4*x^4) + b*x*(15*d^4 + 40*d^3*e*x + 45*d^2*e^2*x^2 + 24*d*e^3*x^3 + 5*e^4*x^4)))/(30*(a + b*x))
```

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.72, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {1102, 27, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{a^2 + 2abx + b^2x^2}(d + ex)^4 dx$$

$$\downarrow 1102$$

$$\frac{\sqrt{a^2 + 2abx + b^2x^2} \int b(a + bx)(d + ex)^4 dx}{b(a + bx)}$$

$$\downarrow 27$$

$$\frac{\sqrt{a^2 + 2abx + b^2x^2} \int (a + bx)(d + ex)^4 dx}{a + bx}$$

$$\downarrow 49$$

$$\frac{\sqrt{a^2 + 2abx + b^2x^2} \int \left(\frac{b(d+ex)^5}{e} + \frac{(ae-bd)(d+ex)^4}{e} \right) dx}{a + bx}$$

$$\downarrow 2009$$

$$\frac{\sqrt{a^2 + 2abx + b^2x^2} \left(\frac{b(d+ex)^6}{6e^2} - \frac{(d+ex)^5(bd-ae)}{5e^2} \right)}{a + bx}$$

input

```
Int[(d + e*x)^4*sqrt[a^2 + 2*a*b*x + b^2*x^2], x]
```

output

```
(sqrt[a^2 + 2*a*b*x + b^2*x^2]*(-1/5*((b*d - a*e)*(d + e*x)^5)/e^2 + (b*(d + e*x)^6)/(6*e^2)))/(a + b*x)
```

Defintions of rubi rules used

rule 27 `Int[(a_)*(Fx_), x_Symbol] := Simp[a Int[Fx, x], x] /; FreeQ[a, x] && !MatchQ[Fx, (b_)*(Gx_)] /; FreeQ[b, x]`

rule 49 `Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] := Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] && IGtQ[m, 0] && IGtQ[m + n + 2, 0]`

rule 1102 `Int[((d_.) + (e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] := Simp[(a + b*x + c*x^2)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x)^(2*FracPart[p])) Int[(d + e*x)^m*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && EqQ[b^2 - 4*a*c, 0]`

rule 2009 `Int[u_, x_Symbol] := Simp[IntSum[u, x], x] /; SumQ[u]`

Maple [A] (verified)

Time = 0.92 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.24

| method | result |
|---------|--|
| gospers | $\frac{x(5b^4e^4x^5+6x^4e^4a+24x^4bde^3+30x^3ade^3+45x^3bd^2e^2+60x^2ad^2e^2+40x^2bd^3e+60xd^3ea+15bd^4x+30ad^4)\sqrt{(bx+a)^2}}{30bx+30a}$ |
| orering | $\frac{x(5b^4e^4x^5+6x^4e^4a+24x^4bde^3+30x^3ade^3+45x^3bd^2e^2+60x^2ad^2e^2+40x^2bd^3e+60xd^3ea+15bd^4x+30ad^4)\sqrt{(bx+a)^2}}{30bx+30a}$ |
| default | $\frac{\text{csgn}(bx+a)(bx+a)^2(5b^4x^4e^4-4x^3ab^3e^4+24x^3b^4de^3+3x^2a^2b^2e^4-18x^2ab^3de^3+45x^2b^4d^2e^2-2x^2a^3be^4+12x^2a^2b^2de^3-30x^2ab^3d^2e^2)}{30b^5}$ |
| risch | $\frac{\sqrt{(bx+a)^2}be^4x^6}{6bx+6a} + \frac{\sqrt{(bx+a)^2}(e^4a+4bde^3)x^5}{5bx+5a} + \frac{\sqrt{(bx+a)^2}(4ade^3+6bd^2e^2)x^4}{4bx+4a} + \frac{\sqrt{(bx+a)^2}(6ad^2e^2+4bd^3e)x^3}{3bx+3a} + \sqrt{(bx+a)^2}$ |

input `int((e*x+d)^4*((b*x+a)^2)^(1/2),x,method=_RETURNVERBOSE)`

output `1/30*x*(5*b*e^4*x^5+6*a*e^4*x^4+24*b*d*e^3*x^4+30*a*d*e^3*x^3+45*b*d^2*e^2*x^3+60*a*d^2*e^2*x^2+40*b*d^3*e*x^2+60*a*d^3*e*x+15*b*d^4*x+30*a*d^4)*((b*x+a)^2)^(1/2)/(b*x+a)`

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 96, normalized size of antiderivative = 1.04

$$\int (d + ex)^4 \sqrt{a^2 + 2abx + b^2x^2} dx = \frac{1}{6} be^4x^6 + ad^4x + \frac{1}{5} (4bde^3 + ae^4)x^5 \\ + \frac{1}{2} (3bd^2e^2 + 2ade^3)x^4 \\ + \frac{2}{3} (2bd^3e + 3ad^2e^2)x^3 + \frac{1}{2} (bd^4 + 4ad^3e)x^2$$

input `integrate((e*x+d)^4*((b*x+a)^2)^(1/2),x, algorithm="fricas")`

output `1/6*b*e^4*x^6 + a*d^4*x + 1/5*(4*b*d*e^3 + a*e^4)*x^5 + 1/2*(3*b*d^2*e^2 + 2*a*d*e^3)*x^4 + 2/3*(2*b*d^3*e + 3*a*d^2*e^2)*x^3 + 1/2*(b*d^4 + 4*a*d^3*e)*x^2`

Sympy [A] (verification not implemented)

Time = 2.00 (sec) , antiderivative size = 666, normalized size of antiderivative = 7.24

$$\int (d + ex)^4 \sqrt{a^2 + 2abx + b^2x^2} dx = \text{Too large to display}$$

input `integrate((e*x+d)**4*((b*x+a)**2)**(1/2),x)`

output

```

d**4*Piecewise(((a/(2*b) + x/2)*sqrt(a**2 + 2*a*b*x + b**2*x**2), Ne(b**2,
0)), ((a**2 + 2*a*b*x)**(3/2)/(3*a*b), Ne(a*b, 0)), (x*sqrt(a**2), True))
+ 4*d**3*e*Piecewise((sqrt(a**2 + 2*a*b*x + b**2*x**2)*(-a**2/(6*b**2) +
a*x/(6*b) + x**2/3), Ne(b**2, 0)), ((-a**2*(a**2 + 2*a*b*x)**(3/2)/3 + (a
**2 + 2*a*b*x)**(5/2)/5)/(2*a**2*b**2), Ne(a*b, 0)), (x**2*sqrt(a**2)/2, Tr
ue)) + 6*d**2*e**2*Piecewise((sqrt(a**2 + 2*a*b*x + b**2*x**2)*(a**3/(12*b
**3) - a**2*x/(12*b**2) + a*x**2/(12*b) + x**3/4), Ne(b**2, 0)), ((a**4*(a
**2 + 2*a*b*x)**(3/2)/3 - 2*a**2*(a**2 + 2*a*b*x)**(5/2)/5 + (a**2 + 2*a*b
*x)**(7/2)/7)/(4*a**3*b**3), Ne(a*b, 0)), (x**3*sqrt(a**2)/3, True)) + 4*d
**e**3*Piecewise((sqrt(a**2 + 2*a*b*x + b**2*x**2)*(-a**4/(20*b**4) + a**3*
x/(20*b**3) - a**2*x**2/(20*b**2) + a*x**3/(20*b) + x**4/5), Ne(b**2, 0)),
((-a**6*(a**2 + 2*a*b*x)**(3/2)/3 + 3*a**4*(a**2 + 2*a*b*x)**(5/2)/5 - 3*
a**2*(a**2 + 2*a*b*x)**(7/2)/7 + (a**2 + 2*a*b*x)**(9/2)/9)/(8*a**4*b**4),
Ne(a*b, 0)), (x**4*sqrt(a**2)/4, True)) + e**4*Piecewise((sqrt(a**2 + 2*a
*b*x + b**2*x**2)*(a**5/(30*b**5) - a**4*x/(30*b**4) + a**3*x**2/(30*b**3)
- a**2*x**3/(30*b**2) + a*x**4/(30*b) + x**5/6), Ne(b**2, 0)), ((a**8*(a
**2 + 2*a*b*x)**(3/2)/3 - 4*a**6*(a**2 + 2*a*b*x)**(5/2)/5 + 6*a**4*(a**2 +
2*a*b*x)**(7/2)/7 - 4*a**2*(a**2 + 2*a*b*x)**(9/2)/9 + (a**2 + 2*a*b*x)**
(11/2)/11)/(16*a**5*b**5), Ne(a*b, 0)), (x**5*sqrt(a**2)/5, True))

```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 590 vs. $2(66) = 132$.

Time = 0.04 (sec) , antiderivative size = 590, normalized size of antiderivative = 6.41

$$\begin{aligned}
 \int (d + ex)^4 \sqrt{a^2 + 2abx + b^2x^2} dx = & \frac{(b^2x^2 + 2abx + a^2)^{\frac{3}{2}} e^4 x^3}{6b^2} \\
 & + \frac{1}{2} \frac{\sqrt{b^2x^2 + 2abx + a^2} d^4 x}{b} \\
 & - \frac{2 \sqrt{b^2x^2 + 2abx + a^2} a d^3 e x}{b^2} \\
 & + \frac{3 \sqrt{b^2x^2 + 2abx + a^2} a^2 d^2 e^2 x}{b^3} \\
 & - \frac{2 \sqrt{b^2x^2 + 2abx + a^2} a^3 d e^3 x}{b^4} \\
 & + \frac{\sqrt{b^2x^2 + 2abx + a^2} a^4 e^4 x}{2b^5} \\
 & + \frac{4 (b^2x^2 + 2abx + a^2)^{\frac{3}{2}} d e^3 x^2}{5b^2} \\
 & - \frac{3 (b^2x^2 + 2abx + a^2)^{\frac{3}{2}} a e^4 x^2}{10b^3} \\
 & + \frac{\sqrt{b^2x^2 + 2abx + a^2} a d^4}{2b} \\
 & - \frac{2 \sqrt{b^2x^2 + 2abx + a^2} a^2 d^3 e}{b^2} \\
 & + \frac{3 \sqrt{b^2x^2 + 2abx + a^2} a^3 d^2 e^2}{b^3} \\
 & - \frac{2 \sqrt{b^2x^2 + 2abx + a^2} a^4 d e^3}{b^4} \\
 & + \frac{\sqrt{b^2x^2 + 2abx + a^2} a^5 e^4}{2b^5} \\
 & + \frac{3 (b^2x^2 + 2abx + a^2)^{\frac{3}{2}} d^2 e^2 x}{2b^2} \\
 & - \frac{7 (b^2x^2 + 2abx + a^2)^{\frac{3}{2}} a d e^3 x}{5b^3} \\
 & + \frac{2 (b^2x^2 + 2abx + a^2)^{\frac{3}{2}} a^2 e^4 x}{5b^4} \\
 & + \frac{4 (b^2x^2 + 2abx + a^2)^{\frac{3}{2}} d^3 e}{3b^2} \\
 & - \frac{5 (b^2x^2 + 2abx + a^2)^{\frac{3}{2}} a d^2 e^2}{2b^3} \\
 & + \frac{9 (b^2x^2 + 2abx + a^2)^{\frac{3}{2}} a^2 d e^3}{5b^4} \\
 & - \frac{7 (b^2x^2 + 2abx + a^2)^{\frac{3}{2}} a^3 e^4}{15b^5}
 \end{aligned}$$

output

```
1/6*b*e^4*x^6*sgn(b*x + a) + 4/5*b*d*e^3*x^5*sgn(b*x + a) + 1/5*a*e^4*x^5*
sgn(b*x + a) + 3/2*b*d^2*e^2*x^4*sgn(b*x + a) + a*d*e^3*x^4*sgn(b*x + a) +
4/3*b*d^3*e*x^3*sgn(b*x + a) + 2*a*d^2*e^2*x^3*sgn(b*x + a) + 1/2*b*d^4*x
^2*sgn(b*x + a) + 2*a*d^3*e*x^2*sgn(b*x + a) + a*d^4*x*sgn(b*x + a) + 1/30
*(15*a^2*b^4*d^4 - 20*a^3*b^3*d^3*e + 15*a^4*b^2*d^2*e^2 - 6*a^5*b*d*e^3 +
a^6*e^4)*sgn(b*x + a)/b^5
```

Mupad [B] (verification not implemented)

Time = 9.77 (sec) , antiderivative size = 580, normalized size of antiderivative = 6.30

$$\int (d + ex)^4 \sqrt{a^2 + 2abx + b^2x^2} dx$$

$$= d^4 \left(\frac{x}{2} + \frac{a}{2b} \right) \sqrt{a^2 + 2abx + b^2x^2} + \frac{e^4 x^3 (a^2 + 2abx + b^2x^2)^{3/2}}{6b^2}$$

$$- \frac{a^2 e^4 \sqrt{a^2 + 2abx + b^2x^2} (a^3 - 5ab^2x^2 + 3bx(a^2 + 2abx + b^2x^2) - 4a^2bx)}{24b^5}$$

$$+ \frac{3d^2 e^2 x (a^2 + 2abx + b^2x^2)^{3/2}}{2b^2} + \frac{4de^3 x^2 (a^2 + 2abx + b^2x^2)^{3/2}}{5b^2}$$

$$- \frac{3ae^4 \sqrt{a^2 + 2abx + b^2x^2} (4b^2x^2(a^2 + 2abx + b^2x^2) - a^4 + 9a^2b^2x^2 + 8a^3bx - 7abx(a^2 + 2abx + b^2x^2))}{40b^5}$$

$$+ \frac{d^3 e (8b^2(a^2 + b^2x^2) - 12a^2b^2 + 4ab^3x) \sqrt{a^2 + 2abx + b^2x^2}}{6b^4}$$

$$- \frac{7ade^3 \sqrt{a^2 + 2abx + b^2x^2} (a^3 - 5ab^2x^2 + 3bx(a^2 + 2abx + b^2x^2) - 4a^2bx)}{15b^4}$$

$$- \frac{3a^2 d^2 e^2 \left(\frac{x}{2} + \frac{a}{2b} \right) \sqrt{a^2 + 2abx + b^2x^2}}{2b^2}$$

$$- \frac{5ad^2 e^2 (8b^2(a^2 + b^2x^2) - 12a^2b^2 + 4ab^3x) \sqrt{a^2 + 2abx + b^2x^2}}{16b^5}$$

$$- \frac{a^2 de^3 (8b^2(a^2 + b^2x^2) - 12a^2b^2 + 4ab^3x) \sqrt{a^2 + 2abx + b^2x^2}}{15b^6}$$

input

```
int(((a + b*x)^2)^(1/2)*(d + e*x)^4,x)
```

output

$$\begin{aligned} & d^4 \left(\frac{x}{2} + \frac{a}{2b} \right) \sqrt{a^2 + b^2 x^2 + 2abx} + \frac{e^{4x^3} (a^2 + b^2 x^2 + 2abx)^{3/2}}{(6b^2)} - \frac{a^2 e^4 (a^2 + b^2 x^2 + 2abx)^{1/2} (a^3 - 5ab^2 x^2 + 3b^3 x)}{(24b^5)} \\ & + \frac{3d^2 e^2 x (a^2 + b^2 x^2 + 2abx)^{3/2}}{(2b^2)} + \frac{4d^3 e^3 x^2 (a^2 + b^2 x^2 + 2abx)^{3/2}}{(5b^2)} - \frac{3a e^4 (a^2 + b^2 x^2 + 2abx)^{1/2} (4b^2 x^2 (a^2 + b^2 x^2 + 2abx) - a^4 + 9a^2 b^2 x^2 + 8a^3 b x - 7ab^2 x (a^2 + b^2 x^2 + 2abx))}{(40b^5)} \\ & + \frac{d^3 e (8b^2 (a^2 + b^2 x^2) - 12a^2 b^2 + 4ab^3 x) (a^2 + b^2 x^2 + 2abx)^{1/2}}{(6b^4)} - \frac{7a d e^3 (a^2 + b^2 x^2 + 2abx)^{1/2} (a^3 - 5ab^2 x^2 + 3b^3 x (a^2 + b^2 x^2 + 2abx) - 4a^2 b x)}{(15b^4)} \\ & - \frac{3a^2 d^2 e^2 (x/2 + a/(2b)) (a^2 + b^2 x^2 + 2abx)^{1/2}}{(2b^2)} - \frac{5a d^2 e^2 (8b^2 (a^2 + b^2 x^2) - 12a^2 b^2 + 4ab^3 x) (a^2 + b^2 x^2 + 2abx)^{1/2}}{(16b^5)} \\ & - \frac{a^2 d e^3 (8b^2 (a^2 + b^2 x^2) - 12a^2 b^2 + 4ab^3 x) (a^2 + b^2 x^2 + 2abx)^{1/2}}{(15b^6)} \end{aligned}$$
Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.05

$$\int (d + ex)^4 \sqrt{a^2 + 2abx + b^2 x^2} dx = \frac{x(5b e^4 x^5 + 6a e^4 x^4 + 24bd e^3 x^4 + 30ad e^3 x^3 + 45b d^2 e^2 x^3 + 60a d^2 e^2 x^2 + 40b d^3 e x^2 + 60a d^3 e x + 15b d^4)}{30}$$

input

`int((e*x+d)^4*((b*x+a)^2)^(1/2),x)`

output

$$\frac{(x(30ad^4 + 60ad^3 e x + 60ad^2 e^2 x^2 + 30ad e^3 x^3 + 6a e^4 x^4 + 15bd^4 x + 40bd^3 e x^2 + 45bd^2 e^2 x^3 + 24bd^3 e^3 x^4 + 5bd^4 e^4 x^5))}{30}$$

3.266 $\int (d + ex)^3 \sqrt{a^2 + 2abx + b^2x^2} dx$

| | |
|---|------|
| Optimal result | 2023 |
| Mathematica [A] (verified) | 2023 |
| Rubi [A] (verified) | 2024 |
| Maple [A] (verified) | 2025 |
| Fricas [A] (verification not implemented) | 2026 |
| Sympy [A] (verification not implemented) | 2027 |
| Maxima [B] (verification not implemented) | 2028 |
| Giac [B] (verification not implemented) | 2030 |
| Mupad [B] (verification not implemented) | 2031 |
| Reduce [B] (verification not implemented) | 2032 |

Optimal result

Integrand size = 28, antiderivative size = 92

$$\int (d + ex)^3 \sqrt{a^2 + 2abx + b^2x^2} dx = -\frac{(bd - ae)(d + ex)^4 \sqrt{a^2 + 2abx + b^2x^2}}{4e^2(a + bx)} + \frac{b(d + ex)^5 \sqrt{a^2 + 2abx + b^2x^2}}{5e^2(a + bx)}$$

output

```
-1/4*(-a*e+b*d)*(e*x+d)^4*((b*x+a)^2)^(1/2)/e^2/(b*x+a)+1/5*b*(e*x+d)^5*((b*x+a)^2)^(1/2)/e^2/(b*x+a)
```

Mathematica [A] (verified)

Time = 1.03 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.97

$$\int (d + ex)^3 \sqrt{a^2 + 2abx + b^2x^2} dx = \frac{x \sqrt{(a + bx)^2 (5a(4d^3 + 6d^2ex + 4de^2x^2 + e^3x^3) + bx(10d^3 + 20d^2ex + 15de^2x^2 + 4e^3x^3))}}{20(a + bx)}$$

input

```
Integrate[(d + e*x)^3*sqrt[a^2 + 2*a*b*x + b^2*x^2],x]
```

output

$$(x\sqrt{(a + bx)^2}*(5*a*(4*d^3 + 6*d^2*e*x + 4*d*e^2*x^2 + e^3*x^3) + b*x*(10*d^3 + 20*d^2*e*x + 15*d*e^2*x^2 + 4*e^3*x^3)))/(20*(a + b*x))$$
Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.72, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {1102, 27, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{a^2 + 2abx + b^2x^2}(d + ex)^3 dx$$

$$\downarrow 1102$$

$$\frac{\sqrt{a^2 + 2abx + b^2x^2} \int b(a + bx)(d + ex)^3 dx}{b(a + bx)}$$

$$\downarrow 27$$

$$\frac{\sqrt{a^2 + 2abx + b^2x^2} \int (a + bx)(d + ex)^3 dx}{a + bx}$$

$$\downarrow 49$$

$$\frac{\sqrt{a^2 + 2abx + b^2x^2} \int \left(\frac{b(d+ex)^4}{e} + \frac{(ae-bd)(d+ex)^3}{e} \right) dx}{a + bx}$$

$$\downarrow 2009$$

$$\frac{\sqrt{a^2 + 2abx + b^2x^2} \left(\frac{b(d+ex)^5}{5e^2} - \frac{(d+ex)^4(bd-ae)}{4e^2} \right)}{a + bx}$$

input

$$\text{Int}[(d + e*x)^3*\text{Sqrt}[a^2 + 2*a*b*x + b^2*x^2], x]$$

output

$$(\text{Sqrt}[a^2 + 2*a*b*x + b^2*x^2]*(-1/4*((b*d - a*e)*(d + e*x)^4)/e^2 + (b*(d + e*x)^5)/(5*e^2)))/(a + b*x)$$

Defintions of rubi rules used

rule 27 `Int[(a_)*(Fx_), x_Symbol] := Simp[a Int[Fx, x], x] /; FreeQ[a, x] && !MatchQ[Fx, (b_)*(Gx_)] /; FreeQ[b, x]`

rule 49 `Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] := Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] && IGtQ[m, 0] && IGtQ[m + n + 2, 0]`

rule 1102 `Int[((d_.) + (e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] := Simp[(a + b*x + c*x^2)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x)^(2*FracPart[p])) Int[(d + e*x)^m*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && EqQ[b^2 - 4*a*c, 0]`

rule 2009 `Int[u_, x_Symbol] := Simp[IntSum[u, x], x] /; SumQ[u]`

Maple [A] (verified)

Time = 0.78 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.98

| method | result |
|---------|---|
| gospers | $\frac{x(4be^3x^4+5x^3ae^3+15x^3bde^2+20ade^2x^2+20bd^2ex^2+30xad^2e+10bd^3x+20ad^3)\sqrt{(bx+a)^2}}{20bx+20a}$ |
| orering | $\frac{x(4be^3x^4+5x^3ae^3+15x^3bde^2+20ade^2x^2+20bd^2ex^2+30xad^2e+10bd^3x+20ad^3)\sqrt{(bx+a)^2}}{20bx+20a}$ |
| default | $-\frac{\text{csign}(bx+a)(bx+a)^2(-4e^3x^3b^3+3x^2ab^2e^3-15x^2b^3de^2-2a^2be^3x+10xab^2de^2-20b^3d^2ex+e^3a^3-5a^2bde^2+10ab^2d^2e-10b^3d^3)}{20b^4}$ |
| risch | $\frac{\sqrt{(bx+a)^2}be^3x^5}{5bx+5a} + \frac{\sqrt{(bx+a)^2}(ae^3+3bde^2)x^4}{4bx+4a} + \frac{\sqrt{(bx+a)^2}(3ade^2+3bd^2e)x^3}{3bx+3a} + \frac{\sqrt{(bx+a)^2}(3ad^2e+bd^3)x^2}{2bx+2a} + \frac{\sqrt{(bx+a)^2}bd^3}{bx+a}$ |

input `int((e*x+d)^3*((b*x+a)^2)^(1/2),x,method=_RETURNVERBOSE)`

output `1/20*x*(4*b*e^3*x^4+5*a*e^3*x^3+15*b*d*e^2*x^3+20*a*d*e^2*x^2+20*b*d^2*e*x^2+30*a*d^2*e*x+10*b*d^3*x+20*a*d^3)*((b*x+a)^2)^(1/2)/(b*x+a)`

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.75

$$\int (d + ex)^3 \sqrt{a^2 + 2abx + b^2x^2} dx = \frac{1}{5} be^3x^5 + ad^3x + \frac{1}{4} (3bde^2 + ae^3)x^4 + (bd^2e + ade^2)x^3 + \frac{1}{2} (bd^3 + 3ad^2e)x^2$$

input `integrate((e*x+d)^3*((b*x+a)^2)^(1/2),x, algorithm="fricas")`

output `1/5*b*e^3*x^5 + a*d^3*x + 1/4*(3*b*d*e^2 + a*e^3)*x^4 + (b*d^2*e + a*d*e^2)*x^3 + 1/2*(b*d^3 + 3*a*d^2*e)*x^2`

Sympy [A] (verification not implemented)

Time = 1.49 (sec) , antiderivative size = 464, normalized size of antiderivative = 5.04

$$\begin{aligned}
& \int (d+ex)^3 \sqrt{a^2+2abx+b^2x^2} dx \\
&= d^3 \left(\begin{cases} \left(\frac{a}{2b} + \frac{x}{2}\right) \sqrt{a^2+2abx+b^2x^2} & \text{for } b^2 \neq 0 \\ \frac{(a^2+2abx)^{\frac{3}{2}}}{3ab} & \text{for } ab \neq 0 \\ x\sqrt{a^2} & \text{otherwise} \end{cases} \right) \\
&+ 3d^2e \left(\begin{cases} \sqrt{a^2+2abx+b^2x^2} \left(-\frac{a^2}{6b^2} + \frac{ax}{6b} + \frac{x^2}{3}\right) & \text{for } b^2 \neq 0 \\ -\frac{a^2(a^2+2abx)^{\frac{3}{2}}}{3} + \frac{(a^2+2abx)^{\frac{5}{2}}}{5} & \text{for } ab \neq 0 \\ \frac{x^2\sqrt{a^2}}{2} & \text{otherwise} \end{cases} \right) \\
&+ 3de^2 \left(\begin{cases} \sqrt{a^2+2abx+b^2x^2} \left(\frac{a^3}{12b^3} - \frac{a^2x}{12b^2} + \frac{ax^2}{12b} + \frac{x^3}{4}\right) & \text{for } b^2 \neq 0 \\ \frac{a^4(a^2+2abx)^{\frac{3}{2}}}{3} - \frac{2a^2(a^2+2abx)^{\frac{5}{2}}}{4a^3b^3} + \frac{(a^2+2abx)^{\frac{7}{2}}}{7} & \text{for } ab \neq 0 \\ \frac{x^3\sqrt{a^2}}{3} & \text{otherwise} \end{cases} \right) \\
&+ e^3 \left(\begin{cases} \sqrt{a^2+2abx+b^2x^2} \left(-\frac{a^4}{20b^4} + \frac{a^3x}{20b^3} - \frac{a^2x^2}{20b^2} + \frac{ax^3}{20b} + \frac{x^4}{5}\right) & \text{for } b^2 \neq 0 \\ -\frac{a^6(a^2+2abx)^{\frac{3}{2}}}{3} + \frac{3a^4(a^2+2abx)^{\frac{5}{2}}}{5} - \frac{3a^2(a^2+2abx)^{\frac{7}{2}}}{7} + \frac{(a^2+2abx)^{\frac{9}{2}}}{9} & \text{for } ab \neq 0 \\ \frac{x^4\sqrt{a^2}}{4} & \text{otherwise} \end{cases} \right)
\end{aligned}$$

input `integrate((e*x+d)**3*((b*x+a)**2)**(1/2),x)`

output

```

d**3*Piecewise(((a/(2*b) + x/2)*sqrt(a**2 + 2*a*b*x + b**2*x**2), Ne(b**2,
0)), ((a**2 + 2*a*b*x)**(3/2)/(3*a*b), Ne(a*b, 0)), (x*sqrt(a**2), True))
+ 3*d**2*e*Piecewise((sqrt(a**2 + 2*a*b*x + b**2*x**2)*(-a**2/(6*b**2) +
a*x/(6*b) + x**2/3), Ne(b**2, 0)), ((-a**2*(a**2 + 2*a*b*x)**(3/2)/3 + (a
**2 + 2*a*b*x)**(5/2)/5)/(2*a**2*b**2), Ne(a*b, 0)), (x**2*sqrt(a**2)/2, Tr
ue)) + 3*d*e**2*Piecewise((sqrt(a**2 + 2*a*b*x + b**2*x**2)*(a**3/(12*b**3
) - a**2*x/(12*b**2) + a*x**2/(12*b) + x**3/4), Ne(b**2, 0)), ((a**4*(a**2
+ 2*a*b*x)**(3/2)/3 - 2*a**2*(a**2 + 2*a*b*x)**(5/2)/5 + (a**2 + 2*a*b*x)
**(7/2)/7)/(4*a**3*b**3), Ne(a*b, 0)), (x**3*sqrt(a**2)/3, True)) + e**3*P
iecewise((sqrt(a**2 + 2*a*b*x + b**2*x**2)*(-a**4/(20*b**4) + a**3*x/(20*b
**3) - a**2*x**2/(20*b**2) + a*x**3/(20*b) + x**4/5), Ne(b**2, 0)), ((-a**
6*(a**2 + 2*a*b*x)**(3/2)/3 + 3*a**4*(a**2 + 2*a*b*x)**(5/2)/5 - 3*a**2*(a
**2 + 2*a*b*x)**(7/2)/7 + (a**2 + 2*a*b*x)**(9/2)/9)/(8*a**4*b**4), Ne(a*b
, 0)), (x**4*sqrt(a**2)/4, True))

```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 399 vs. $2(66) = 132$.

Time = 0.04 (sec) , antiderivative size = 399, normalized size of antiderivative = 4.34

$$\begin{aligned}
 \int (d + ex)^3 \sqrt{a^2 + 2abx + b^2x^2} dx = & \frac{1}{2} \sqrt{b^2x^2 + 2abx + a^2} d^3x \\
 & - \frac{3 \sqrt{b^2x^2 + 2abx + a^2} ad^2ex}{2b} \\
 & + \frac{3 \sqrt{b^2x^2 + 2abx + a^2} a^2 de^2x}{2b^2} \\
 & - \frac{\sqrt{b^2x^2 + 2abx + a^2} a^3 e^3x}{2b^3} \\
 & + \frac{(b^2x^2 + 2abx + a^2)^{\frac{3}{2}} e^3x^2}{5b^2} \\
 & + \frac{\sqrt{b^2x^2 + 2abx + a^2} ad^3}{2b} \\
 & - \frac{3 \sqrt{b^2x^2 + 2abx + a^2} a^2 d^2e}{2b^2} \\
 & + \frac{3 \sqrt{b^2x^2 + 2abx + a^2} a^3 de^2}{2b^3} \\
 & - \frac{\sqrt{b^2x^2 + 2abx + a^2} a^4 e^3}{2b^4} \\
 & + \frac{3 (b^2x^2 + 2abx + a^2)^{\frac{3}{2}} de^2x}{4b^2} \\
 & - \frac{7 (b^2x^2 + 2abx + a^2)^{\frac{3}{2}} ae^3x}{20b^3} \\
 & + \frac{(b^2x^2 + 2abx + a^2)^{\frac{3}{2}} d^2e}{b^2} \\
 & - \frac{5 (b^2x^2 + 2abx + a^2)^{\frac{3}{2}} ade^2}{4b^3} \\
 & + \frac{9 (b^2x^2 + 2abx + a^2)^{\frac{3}{2}} a^2 e^3}{20b^4}
 \end{aligned}$$

input `integrate((e*x+d)^3*((b*x+a)^2)^(1/2),x, algorithm="maxima")`

output

```
1/2*sqrt(b^2*x^2 + 2*a*b*x + a^2)*d^3*x - 3/2*sqrt(b^2*x^2 + 2*a*b*x + a^2)
)*a*d^2*e*x/b + 3/2*sqrt(b^2*x^2 + 2*a*b*x + a^2)*a^2*d*e^2*x/b^2 - 1/2*sq
rt(b^2*x^2 + 2*a*b*x + a^2)*a^3*e^3*x/b^3 + 1/5*(b^2*x^2 + 2*a*b*x + a^2)^
(3/2)*e^3*x^2/b^2 + 1/2*sqrt(b^2*x^2 + 2*a*b*x + a^2)*a*d^3/b - 3/2*sqrt(b
^2*x^2 + 2*a*b*x + a^2)*a^2*d^2*e/b^2 + 3/2*sqrt(b^2*x^2 + 2*a*b*x + a^2)*
a^3*d*e^2/b^3 - 1/2*sqrt(b^2*x^2 + 2*a*b*x + a^2)*a^4*e^3/b^4 + 3/4*(b^2*x
^2 + 2*a*b*x + a^2)^(3/2)*d*e^2*x/b^2 - 7/20*(b^2*x^2 + 2*a*b*x + a^2)^(3/
2)*a*e^3*x/b^3 + (b^2*x^2 + 2*a*b*x + a^2)^(3/2)*d^2*e/b^2 - 5/4*(b^2*x^2
+ 2*a*b*x + a^2)^(3/2)*a*d*e^2/b^3 + 9/20*(b^2*x^2 + 2*a*b*x + a^2)^(3/2)*
a^2*e^3/b^4
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 173 vs. $2(66) = 132$.

Time = 0.13 (sec) , antiderivative size = 173, normalized size of antiderivative = 1.88

$$\int (d + ex)^3 \sqrt{a^2 + 2abx + b^2x^2} dx$$

$$= \frac{1}{5} be^3 x^5 \operatorname{sgn}(bx + a) + \frac{3}{4} bde^2 x^4 \operatorname{sgn}(bx + a) + \frac{1}{4} ae^3 x^4 \operatorname{sgn}(bx + a) + bd^2 ex^3 \operatorname{sgn}(bx + a)$$

$$+ ade^2 x^3 \operatorname{sgn}(bx + a) + \frac{1}{2} bd^3 x^2 \operatorname{sgn}(bx + a) + \frac{3}{2} ad^2 ex^2 \operatorname{sgn}(bx + a)$$

$$+ ad^3 x \operatorname{sgn}(bx + a) + \frac{(10 a^2 b^3 d^3 - 10 a^3 b^2 d^2 e + 5 a^4 b d e^2 - a^5 e^3) \operatorname{sgn}(bx + a)}{20 b^4}$$

input

```
integrate((e*x+d)^3*((b*x+a)^2)^(1/2),x, algorithm="giac")
```

output

```
1/5*b*e^3*x^5*sgn(b*x + a) + 3/4*b*d*e^2*x^4*sgn(b*x + a) + 1/4*a*e^3*x^4*
sgn(b*x + a) + b*d^2*e*x^3*sgn(b*x + a) + a*d*e^2*x^3*sgn(b*x + a) + 1/2*b
*d^3*x^2*sgn(b*x + a) + 3/2*a*d^2*e*x^2*sgn(b*x + a) + a*d^3*x*sgn(b*x + a
) + 1/20*(10*a^2*b^3*d^3 - 10*a^3*b^2*d^2*e + 5*a^4*b*d*e^2 - a^5*e^3)*sgn
(b*x + a)/b^4
```

Mupad [B] (verification not implemented)

Time = 9.07 (sec) , antiderivative size = 377, normalized size of antiderivative = 4.10

$$\begin{aligned}
& \int (d + ex)^3 \sqrt{a^2 + 2abx + b^2x^2} dx \\
&= d^3 \left(\frac{x}{2} + \frac{a}{2b} \right) \sqrt{a^2 + 2abx + b^2x^2} + \frac{e^3 x^2 (a^2 + 2abx + b^2x^2)^{3/2}}{5b^2} \\
&\quad - \frac{a^2 e^3 (8b^2 (a^2 + b^2x^2) - 12a^2b^2 + 4ab^3x) \sqrt{a^2 + 2abx + b^2x^2}}{60b^6} \\
&\quad - \frac{7ae^3 \sqrt{a^2 + 2abx + b^2x^2} (a^3 - 5ab^2x^2 + 3bx(a^2 + 2abx + b^2x^2) - 4a^2bx)}{60b^4} \\
&\quad + \frac{d^2 e (8b^2 (a^2 + b^2x^2) - 12a^2b^2 + 4ab^3x) \sqrt{a^2 + 2abx + b^2x^2}}{8b^4} \\
&\quad + \frac{3de^2 x (a^2 + 2abx + b^2x^2)^{3/2}}{4b^2} \\
&\quad - \frac{5ade^2 (8b^2 (a^2 + b^2x^2) - 12a^2b^2 + 4ab^3x) \sqrt{a^2 + 2abx + b^2x^2}}{32b^5} \\
&\quad - \frac{3a^2de^2 \left(\frac{x}{2} + \frac{a}{2b} \right) \sqrt{a^2 + 2abx + b^2x^2}}{4b^2}
\end{aligned}$$

input `int(((a + b*x)^2)^(1/2)*(d + e*x)^3,x)`output `d^3*(x/2 + a/(2*b))*(a^2 + b^2*x^2 + 2*a*b*x)^(1/2) + (e^3*x^2*(a^2 + b^2*x^2 + 2*a*b*x)^(3/2))/(5*b^2) - (a^2*e^3*(8*b^2*(a^2 + b^2*x^2) - 12*a^2*b^2 + 4*a*b^3*x)*(a^2 + b^2*x^2 + 2*a*b*x)^(1/2))/(60*b^6) - (7*a*e^3*(a^2 + b^2*x^2 + 2*a*b*x)^(1/2)*(a^3 - 5*a*b^2*x^2 + 3*b*x*(a^2 + b^2*x^2 + 2*a*b*x) - 4*a^2*b*x))/(60*b^4) + (d^2*e*(8*b^2*(a^2 + b^2*x^2) - 12*a^2*b^2 + 4*a*b^3*x)*(a^2 + b^2*x^2 + 2*a*b*x)^(1/2))/(8*b^4) + (3*d*e^2*x*(a^2 + b^2*x^2 + 2*a*b*x)^(3/2))/(4*b^2) - (5*a*d*e^2*(8*b^2*(a^2 + b^2*x^2) - 12*a^2*b^2 + 4*a*b^3*x)*(a^2 + b^2*x^2 + 2*a*b*x)^(1/2))/(32*b^5) - (3*a^2*d*e^2*(x/2 + a/(2*b))*(a^2 + b^2*x^2 + 2*a*b*x)^(1/2))/(4*b^2)`

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.79

$$\int (d + ex)^3 \sqrt{a^2 + 2abx + b^2x^2} dx$$

$$= \frac{x(4be^3x^4 + 5ae^3x^3 + 15bde^2x^3 + 20ade^2x^2 + 20bd^2ex^2 + 30ad^2ex + 10bd^3x + 20ad^3)}{20}$$

input `int((e*x+d)^3*((b*x+a)^2)^(1/2),x)`output `(x*(20*a*d**3 + 30*a*d**2*e*x + 20*a*d*e**2*x**2 + 5*a*e**3*x**3 + 10*b*d*
*3*x + 20*b*d**2*e*x**2 + 15*b*d*e**2*x**3 + 4*b*e**3*x**4))/20`

3.267 $\int (d + ex)^2 \sqrt{a^2 + 2abx + b^2x^2} dx$

| | |
|---|------|
| Optimal result | 2033 |
| Mathematica [A] (verified) | 2033 |
| Rubi [A] (verified) | 2034 |
| Maple [A] (verified) | 2035 |
| Fricas [A] (verification not implemented) | 2036 |
| Sympy [A] (verification not implemented) | 2036 |
| Maxima [B] (verification not implemented) | 2037 |
| Giac [A] (verification not implemented) | 2038 |
| Mupad [B] (verification not implemented) | 2039 |
| Reduce [B] (verification not implemented) | 2039 |

Optimal result

Integrand size = 28, antiderivative size = 92

$$\int (d + ex)^2 \sqrt{a^2 + 2abx + b^2x^2} dx = -\frac{(bd - ae)(d + ex)^3 \sqrt{a^2 + 2abx + b^2x^2}}{3e^2(a + bx)} + \frac{b(d + ex)^4 \sqrt{a^2 + 2abx + b^2x^2}}{4e^2(a + bx)}$$

output

```
-1/3*(-a*e+b*d)*(e*x+d)^3*((b*x+a)^2)^(1/2)/e^2/(b*x+a)+1/4*b*(e*x+d)^4*((b*x+a)^2)^(1/2)/e^2/(b*x+a)
```

Mathematica [A] (verified)

Time = 1.03 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.73

$$\int (d + ex)^2 \sqrt{a^2 + 2abx + b^2x^2} dx = \frac{x \sqrt{(a + bx)^2(4a(3d^2 + 3dex + e^2x^2) + bx(6d^2 + 8dex + 3e^2x^2))}}{12(a + bx)}$$

input

```
Integrate[(d + e*x)^2*Sqrt[a^2 + 2*a*b*x + b^2*x^2],x]
```

output

```
(x*sqrt[(a + b*x)^2]*(4*a*(3*d^2 + 3*d*e*x + e^2*x^2) + b*x*(6*d^2 + 8*d*e*x + 3*e^2*x^2)))/(12*(a + b*x))
```

Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.72, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {1102, 27, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{a^2 + 2abx + b^2x^2}(d + ex)^2 dx$$

$$\downarrow \text{1102}$$

$$\frac{\sqrt{a^2 + 2abx + b^2x^2} \int b(a + bx)(d + ex)^2 dx}{b(a + bx)}$$

$$\downarrow \text{27}$$

$$\frac{\sqrt{a^2 + 2abx + b^2x^2} \int (a + bx)(d + ex)^2 dx}{a + bx}$$

$$\downarrow \text{49}$$

$$\frac{\sqrt{a^2 + 2abx + b^2x^2} \int \left(\frac{b(d+ex)^3}{e} + \frac{(ae-bd)(d+ex)^2}{e} \right) dx}{a + bx}$$

$$\downarrow \text{2009}$$

$$\frac{\sqrt{a^2 + 2abx + b^2x^2} \left(\frac{b(d+ex)^4}{4e^2} - \frac{(d+ex)^3(bd-ae)}{3e^2} \right)}{a + bx}$$

input

```
Int[(d + e*x)^2*sqrt[a^2 + 2*a*b*x + b^2*x^2], x]
```

output

```
(sqrt[a^2 + 2*a*b*x + b^2*x^2]*(-1/3*((b*d - a*e)*(d + e*x)^3)/e^2 + (b*(d + e*x)^4)/(4*e^2)))/(a + b*x)
```

Definitions of rubi rules used

rule 27 `Int[(a_)*(F_x_), x_Symbol] := Simp[a Int[F_x, x], x] /; FreeQ[a, x] && !MatchQ[F_x, (b_)*(G_x_)] /; FreeQ[b, x]`

rule 49 `Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] := Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] && IGtQ[m, 0] && IGtQ[m + n + 2, 0]`

rule 1102 `Int[((d_.) + (e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] := Simp[(a + b*x + c*x^2)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x)^(2*FracPart[p])) Int[(d + e*x)^m*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && EqQ[b^2 - 4*a*c, 0]`

rule 2009 `Int[u_, x_Symbol] := Simp[IntSum[u, x], x] /; SumQ[u]`

Maple [A] (verified)

Time = 0.74 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.72

| method | result | size |
|---------|---|------|
| gospers | $\frac{x(3be^2x^3+4ae^2x^2+8bdx^2e+12adex+6bx^2d^2+12ad^2)\sqrt{(bx+a)^2}}{12bx+12a}$ | 66 |
| orering | $\frac{x(3be^2x^3+4ae^2x^2+8bdx^2e+12adex+6bx^2d^2+12ad^2)\sqrt{(bx+a)^2}}{12bx+12a}$ | 66 |
| default | $\frac{\text{csign}(bx+a)(bx+a)^2(3b^2e^2x^2-2xab^2e^2+8b^2dex+e^2a^2-4abde+6b^2d^2)}{12b^3}$ | 68 |
| risch | $\frac{\sqrt{(bx+a)^2}be^2x^4}{4bx+4a} + \frac{\sqrt{(bx+a)^2}(ae^2+2bde)x^3}{3bx+3a} + \frac{\sqrt{(bx+a)^2}(2ade+bd^2)x^2}{2bx+2a} + \frac{\sqrt{(bx+a)^2}ad^2x}{bx+a}$ | 113 |

input `int((e*x+d)^2*((b*x+a)^2)^(1/2),x,method=_RETURNVERBOSE)`

output `1/12*x*(3*b*e^2*x^3+4*a*e^2*x^2+8*b*d*e*x^2+12*a*d*e*x+6*b*d^2*x+12*a*d^2)*((b*x+a)^2)^(1/2)/(b*x+a)`

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.52

$$\int (d+ex)^2 \sqrt{a^2+2abx+b^2x^2} dx = \frac{1}{4} be^2x^4 + ad^2x + \frac{1}{3} (2bde + ae^2)x^3 + \frac{1}{2} (bd^2 + 2ade)x^2$$

input `integrate((e*x+d)^2*((b*x+a)^2)^(1/2),x, algorithm="fricas")`output `1/4*b*e^2*x^4 + a*d^2*x + 1/3*(2*b*d*e + a*e^2)*x^3 + 1/2*(b*d^2 + 2*a*d*e)*x^2`**Sympy [A] (verification not implemented)**

Time = 1.20 (sec) , antiderivative size = 294, normalized size of antiderivative = 3.20

$$\int (d+ex)^2 \sqrt{a^2+2abx+b^2x^2} dx$$

$$= d^2 \left(\begin{cases} \left(\frac{a}{2b} + \frac{x}{2} \right) \sqrt{a^2+2abx+b^2x^2} & \text{for } b^2 \neq 0 \\ \frac{(a^2+2abx)^{\frac{3}{2}}}{3ab} & \text{for } ab \neq 0 \\ x\sqrt{a^2} & \text{otherwise} \end{cases} \right)$$

$$+ 2de \left(\begin{cases} \sqrt{a^2+2abx+b^2x^2} \left(-\frac{a^2}{6b^2} + \frac{ax}{6b} + \frac{x^2}{3} \right) & \text{for } b^2 \neq 0 \\ -\frac{a^2(a^2+2abx)^{\frac{3}{2}}}{3} + \frac{(a^2+2abx)^{\frac{5}{2}}}{5} & \text{for } ab \neq 0 \\ \frac{x^2\sqrt{a^2}}{2} & \text{otherwise} \end{cases} \right)$$

$$+ e^2 \left(\begin{cases} \sqrt{a^2+2abx+b^2x^2} \left(\frac{a^3}{12b^3} - \frac{a^2x}{12b^2} + \frac{ax^2}{12b} + \frac{x^3}{4} \right) & \text{for } b^2 \neq 0 \\ \frac{a^4(a^2+2abx)^{\frac{3}{2}}}{3} - \frac{2a^2(a^2+2abx)^{\frac{5}{2}}}{4a^3b^3} + \frac{(a^2+2abx)^{\frac{7}{2}}}{7} & \text{for } ab \neq 0 \\ \frac{x^3\sqrt{a^2}}{3} & \text{otherwise} \end{cases} \right)$$

input `integrate((e*x+d)**2*((b*x+a)**2)**(1/2),x)`

output

```
d**2*Piecewise(((a/(2*b) + x/2)*sqrt(a**2 + 2*a*b*x + b**2*x**2), Ne(b**2,
0)), ((a**2 + 2*a*b*x)**(3/2)/(3*a*b), Ne(a*b, 0)), (x*sqrt(a**2), True))
+ 2*d*e*Piecewise((sqrt(a**2 + 2*a*b*x + b**2*x**2)*(-a**2/(6*b**2) + a*x
/(6*b) + x**2/3), Ne(b**2, 0)), ((-a**2*(a**2 + 2*a*b*x)**(3/2)/3 + (a**2
+ 2*a*b*x)**(5/2)/5)/(2*a**2*b**2), Ne(a*b, 0)), (x**2*sqrt(a**2)/2, True)
) + e**2*Piecewise((sqrt(a**2 + 2*a*b*x + b**2*x**2)*(a**3/(12*b**3) - a**
2*x/(12*b**2) + a*x**2/(12*b) + x**3/4), Ne(b**2, 0)), ((a**4*(a**2 + 2*a*
b*x)**(3/2)/3 - 2*a**2*(a**2 + 2*a*b*x)**(5/2)/5 + (a**2 + 2*a*b*x)**(7/2)
/7)/(4*a**3*b**3), Ne(a*b, 0)), (x**3*sqrt(a**2)/3, True))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 245 vs. $2(66) = 132$.

Time = 0.05 (sec) , antiderivative size = 245, normalized size of antiderivative = 2.66

$$\int (d + ex)^2 \sqrt{a^2 + 2abx + b^2x^2} dx = \frac{1}{2} \sqrt{b^2x^2 + 2abx + a^2} d^2x - \frac{\sqrt{b^2x^2 + 2abx + a^2} adex}{b}$$

$$+ \frac{\sqrt{b^2x^2 + 2abx + a^2} a^2 e^2 x}{2b^2}$$

$$+ \frac{\sqrt{b^2x^2 + 2abx + a^2} ad^2}{2b}$$

$$- \frac{\sqrt{b^2x^2 + 2abx + a^2} a^2 de}{b^2}$$

$$+ \frac{\sqrt{b^2x^2 + 2abx + a^2} a^3 e^2}{2b^3}$$

$$+ \frac{(b^2x^2 + 2abx + a^2)^{\frac{3}{2}} e^2 x}{4b^2}$$

$$+ \frac{2(b^2x^2 + 2abx + a^2)^{\frac{3}{2}} de}{3b^2}$$

$$- \frac{5(b^2x^2 + 2abx + a^2)^{\frac{3}{2}} ae^2}{12b^3}$$

input

```
integrate((e*x+d)^2*((b*x+a)^2)^(1/2),x, algorithm="maxima")
```

output

```
1/2*sqrt(b^2*x^2 + 2*a*b*x + a^2)*d^2*x - sqrt(b^2*x^2 + 2*a*b*x + a^2)*a*
d*e*x/b + 1/2*sqrt(b^2*x^2 + 2*a*b*x + a^2)*a^2*e^2*x/b^2 + 1/2*sqrt(b^2*x
^2 + 2*a*b*x + a^2)*a*d^2/b - sqrt(b^2*x^2 + 2*a*b*x + a^2)*a^2*d*e/b^2 +
1/2*sqrt(b^2*x^2 + 2*a*b*x + a^2)*a^3*e^2/b^3 + 1/4*(b^2*x^2 + 2*a*b*x + a
^2)^(3/2)*e^2*x/b^2 + 2/3*(b^2*x^2 + 2*a*b*x + a^2)^(3/2)*d*e/b^2 - 5/12*(
b^2*x^2 + 2*a*b*x + a^2)^(3/2)*a*e^2/b^3
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 123, normalized size of antiderivative = 1.34

$$\int (d + ex)^2 \sqrt{a^2 + 2abx + b^2x^2} dx = \frac{1}{4} be^2 x^4 \operatorname{sgn}(bx + a) + \frac{2}{3} bde x^3 \operatorname{sgn}(bx + a) + \frac{1}{3} ae^2 x^3 \operatorname{sgn}(bx + a) + \frac{1}{2} bd^2 x^2 \operatorname{sgn}(bx + a) + adex^2 \operatorname{sgn}(bx + a) + ad^2 x \operatorname{sgn}(bx + a) + \frac{(6a^2b^2d^2 - 4a^3bde + a^4e^2) \operatorname{sgn}(bx + a)}{12b^3}$$

input

```
integrate((e*x+d)^2*((b*x+a)^2)^(1/2),x, algorithm="giac")
```

output

```
1/4*b*e^2*x^4*sgn(b*x + a) + 2/3*b*d*e*x^3*sgn(b*x + a) + 1/3*a*e^2*x^3*sg
n(b*x + a) + 1/2*b*d^2*x^2*sgn(b*x + a) + a*d*e*x^2*sgn(b*x + a) + a*d^2*x
*sgn(b*x + a) + 1/12*(6*a^2*b^2*d^2 - 4*a^3*b*d*e + a^4*e^2)*sgn(b*x + a)/
b^3
```

Mupad [B] (verification not implemented)

Time = 9.31 (sec) , antiderivative size = 215, normalized size of antiderivative = 2.34

$$\int (d + ex)^2 \sqrt{a^2 + 2abx + b^2x^2} dx$$

$$= d^2 \left(\frac{x}{2} + \frac{a}{2b} \right) \sqrt{a^2 + 2abx + b^2x^2} + \frac{e^2 x (a^2 + 2abx + b^2x^2)^{3/2}}{4b^2}$$

$$+ \frac{de (8b^2 (a^2 + b^2x^2) - 12a^2b^2 + 4ab^3x) \sqrt{a^2 + 2abx + b^2x^2}}{12b^4}$$

$$- \frac{a^2 e^2 \left(\frac{x}{2} + \frac{a}{2b} \right) \sqrt{a^2 + 2abx + b^2x^2}}{4b^2}$$

$$- \frac{5a e^2 (8b^2 (a^2 + b^2x^2) - 12a^2b^2 + 4ab^3x) \sqrt{a^2 + 2abx + b^2x^2}}{96b^5}$$

input `int(((a + b*x)^2)^(1/2)*(d + e*x)^2,x)`output `d^2*(x/2 + a/(2*b))*(a^2 + b^2*x^2 + 2*a*b*x)^(1/2) + (e^2*x*(a^2 + b^2*x^2 + 2*a*b*x)^(3/2))/(4*b^2) + (d*e*(8*b^2*(a^2 + b^2*x^2) - 12*a^2*b^2 + 4*a*b^3*x)*(a^2 + b^2*x^2 + 2*a*b*x)^(1/2))/(12*b^4) - (a^2*e^2*(x/2 + a/(2*b))*(a^2 + b^2*x^2 + 2*a*b*x)^(1/2))/(4*b^2) - (5*a*e^2*(8*b^2*(a^2 + b^2*x^2) - 12*a^2*b^2 + 4*a*b^3*x)*(a^2 + b^2*x^2 + 2*a*b*x)^(1/2))/(96*b^5)`**Reduce [B] (verification not implemented)**

Time = 0.20 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.53

$$\int (d + ex)^2 \sqrt{a^2 + 2abx + b^2x^2} dx$$

$$= \frac{x(3b e^2 x^3 + 4a e^2 x^2 + 8bde x^2 + 12adex + 6b d^2 x + 12a d^2)}{12}$$

input `int((e*x+d)^2*((b*x+a)^2)^(1/2),x)`output `(x*(12*a*d**2 + 12*a*d*e*x + 4*a*e**2*x**2 + 6*b*d**2*x + 8*b*d*e*x**2 + 3*b*e**2*x**3))/12`

3.268 $\int (d + ex)\sqrt{a^2 + 2abx + b^2x^2} dx$

| | |
|---|------|
| Optimal result | 2040 |
| Mathematica [A] (verified) | 2040 |
| Rubi [A] (verified) | 2041 |
| Maple [C] (warning: unable to verify) | 2042 |
| Fricas [A] (verification not implemented) | 2043 |
| Sympy [A] (verification not implemented) | 2043 |
| Maxima [B] (verification not implemented) | 2044 |
| Giac [A] (verification not implemented) | 2044 |
| Mupad [B] (verification not implemented) | 2045 |
| Reduce [B] (verification not implemented) | 2045 |

Optimal result

Integrand size = 26, antiderivative size = 69

$$\int (d + ex)\sqrt{a^2 + 2abx + b^2x^2} dx = \frac{(bd - ae)(a + bx)\sqrt{a^2 + 2abx + b^2x^2}}{2b^2} + \frac{e(a^2 + 2abx + b^2x^2)^{3/2}}{3b^2}$$

output

```
1/2*(-a*e+b*d)*(b*x+a)*((b*x+a)^2)^(1/2)/b^2+1/3*e*(b^2*x^2+2*a*b*x+a^2)^(3/2)/b^2
```

Mathematica [A] (verified)

Time = 0.69 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.32

$$\int (d + ex)\sqrt{a^2 + 2abx + b^2x^2} dx = \frac{x(3a(2d + ex) + bx(3d + 2ex)) \left(\sqrt{a^2}bx + a \left(\sqrt{a^2} - \sqrt{(a + bx)^2} \right) \right)}{-6a^2 - 6abx + 6\sqrt{a^2}\sqrt{(a + bx)^2}}$$

input

```
Integrate[(d + e*x)*Sqrt[a^2 + 2*a*b*x + b^2*x^2], x]
```

output

```
(x*(3*a*(2*d + e*x) + b*x*(3*d + 2*e*x))*(Sqrt[a^2]*b*x + a*(Sqrt[a^2] - Sqrt[(a + b*x)^2])))/(-6*a^2 - 6*a*b*x + 6*Sqrt[a^2]*Sqrt[(a + b*x)^2])
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.115$, Rules used = {1100, 1079, 17}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{a^2 + 2abx + b^2x^2}(d + ex) dx$$

$$\downarrow 1100$$

$$\frac{(bd - ae) \int \sqrt{a^2 + 2bxa + b^2x^2} dx}{b} + \frac{e(a^2 + 2abx + b^2x^2)^{3/2}}{3b^2}$$

$$\downarrow 1079$$

$$\frac{\sqrt{a^2 + 2abx + b^2x^2}(bd - ae) \int (xb^2 + ab) dx}{b^2(a + bx)} + \frac{e(a^2 + 2abx + b^2x^2)^{3/2}}{3b^2}$$

$$\downarrow 17$$

$$\frac{(a + bx)\sqrt{a^2 + 2abx + b^2x^2}(bd - ae)}{2b^2} + \frac{e(a^2 + 2abx + b^2x^2)^{3/2}}{3b^2}$$

input

```
Int[(d + e*x)*Sqrt[a^2 + 2*a*b*x + b^2*x^2], x]
```

output

```
((b*d - a*e)*(a + b*x)*Sqrt[a^2 + 2*a*b*x + b^2*x^2])/(2*b^2) + (e*(a^2 + 2*a*b*x + b^2*x^2)^(3/2))/(3*b^2)
```

Definitions of rubi rules used

rule 17 `Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] := Simp[c*((a + b*x)^(m + 1))/(b*(m + 1)), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]`

rule 1079 `Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] := Simp[(a + b*x + c*x^2)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x)^(2*FracPart[p])) Int[(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c, p}, x] && EqQ[b^2 - 4*a*c, 0]`

rule 1100 `Int[((d_.) + (e_.)*(x_))*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] := Simp[e*((a + b*x + c*x^2)^(p + 1)/(2*c*(p + 1)), x] + Simp[(2*c*d - b*e)/(2*c) Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[b^2 - 4*a*c, 0]`

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 2.

Time = 0.61 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.46

| method | result | size |
|---------|---|------|
| default | $-\frac{\text{csgn}(bx+a)(bx+a)^2(-2bex+ae-3bd)}{6b^2}$ | 32 |
| gosper | $\frac{x(2be x^2+3aex+3bdx+6ad)\sqrt{(bx+a)^2}}{6bx+6a}$ | 42 |
| orering | $\frac{x(2be x^2+3aex+3bdx+6ad)\sqrt{(bx+a)^2}}{6bx+6a}$ | 42 |
| risch | $\frac{\sqrt{(bx+a)^2} b e x^3}{3bx+3a} + \frac{\sqrt{(bx+a)^2} (ae+bd)x^2}{2bx+2a} + \frac{\sqrt{(bx+a)^2} a d x}{bx+a}$ | 73 |

input `int((e*x+d)*((b*x+a)^2)^(1/2),x,method=_RETURNVERBOSE)`

output `-1/6*csgn(b*x+a)*(b*x+a)^2*(-2*b*e*x+a*e-3*b*d)/b^2`

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.35

$$\int (d + ex)\sqrt{a^2 + 2abx + b^2x^2} dx = \frac{1}{3} bex^3 + adx + \frac{1}{2} (bd + ae)x^2$$

input `integrate((e*x+d)*((b*x+a)^2)^(1/2),x, algorithm="fricas")`

output `1/3*b*e*x^3 + a*d*x + 1/2*(b*d + a*e)*x^2`

Sympy [A] (verification not implemented)

Time = 0.96 (sec) , antiderivative size = 156, normalized size of antiderivative = 2.26

$$\int (d + ex)\sqrt{a^2 + 2abx + b^2x^2} dx$$

$$= d \left(\begin{cases} \left(\frac{a}{2b} + \frac{x}{2}\right) \sqrt{a^2 + 2abx + b^2x^2} & \text{for } b^2 \neq 0 \\ \frac{(a^2 + 2abx)^{\frac{3}{2}}}{3ab} & \text{for } ab \neq 0 \\ x\sqrt{a^2} & \text{otherwise} \end{cases} \right)$$

$$+ e \left(\begin{cases} \sqrt{a^2 + 2abx + b^2x^2} \left(-\frac{a^2}{6b^2} + \frac{ax}{6b} + \frac{x^2}{3}\right) & \text{for } b^2 \neq 0 \\ \frac{-\frac{a^2(a^2 + 2abx)^{\frac{3}{2}}}{3} + \frac{(a^2 + 2abx)^{\frac{5}{2}}}{5}}{2a^2b^2} & \text{for } ab \neq 0 \\ \frac{x^2\sqrt{a^2}}{2} & \text{otherwise} \end{cases} \right)$$

input `integrate((e*x+d)*((b*x+a)**2)**(1/2),x)`

output `d*Piecewise(((a/(2*b) + x/2)*sqrt(a**2 + 2*a*b*x + b**2*x**2), Ne(b**2, 0)), ((a**2 + 2*a*b*x)**(3/2)/(3*a*b), Ne(a*b, 0)), (x*sqrt(a**2), True)) + e*Piecewise((sqrt(a**2 + 2*a*b*x + b**2*x**2)*(-a**2/(6*b**2) + a*x/(6*b) + x**2/3), Ne(b**2, 0)), ((-a**2*(a**2 + 2*a*b*x)**(3/2)/3 + (a**2 + 2*a*b*x)**(5/2)/5)/(2*a**2*b**2), Ne(a*b, 0)), (x**2*sqrt(a**2)/2, True))`

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 125 vs. $2(52) = 104$.

Time = 0.03 (sec) , antiderivative size = 125, normalized size of antiderivative = 1.81

$$\int (d + ex)\sqrt{a^2 + 2abx + b^2x^2} dx = \frac{1}{2}\sqrt{b^2x^2 + 2abx + a^2}dx - \frac{\sqrt{b^2x^2 + 2abx + a^2}aex}{2b} + \frac{\sqrt{b^2x^2 + 2abx + a^2}ad}{2b} - \frac{\sqrt{b^2x^2 + 2abx + a^2}a^2e}{2b^2} + \frac{(b^2x^2 + 2abx + a^2)^{\frac{3}{2}}e}{3b^2}$$

input `integrate((e*x+d)*((b*x+a)^2)^(1/2),x, algorithm="maxima")`

output

```
1/2*sqrt(b^2*x^2 + 2*a*b*x + a^2)*d*x - 1/2*sqrt(b^2*x^2 + 2*a*b*x + a^2)*
a*e*x/b + 1/2*sqrt(b^2*x^2 + 2*a*b*x + a^2)*a*d/b - 1/2*sqrt(b^2*x^2 + 2*a
*b*x + a^2)*a^2*e/b^2 + 1/3*(b^2*x^2 + 2*a*b*x + a^2)^(3/2)*e/b^2
```

Giac [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.09

$$\int (d + ex)\sqrt{a^2 + 2abx + b^2x^2} dx = \frac{1}{3}bex^3\text{sgn}(bx + a) + \frac{1}{2}bdx^2\text{sgn}(bx + a) + \frac{1}{2}aex^2\text{sgn}(bx + a) + adx\text{sgn}(bx + a) + \frac{(3a^2bd - a^3e)\text{sgn}(bx + a)}{6b^2}$$

input `integrate((e*x+d)*((b*x+a)^2)^(1/2),x, algorithm="giac")`

output

```
1/3*b*e*x^3*sgn(b*x + a) + 1/2*b*d*x^2*sgn(b*x + a) + 1/2*a*e*x^2*sgn(b*x
+ a) + a*d*x*sgn(b*x + a) + 1/6*(3*a^2*b*d - a^3*e)*sgn(b*x + a)/b^2
```

Mupad [B] (verification not implemented)

Time = 9.00 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.12

$$\int (d + ex)\sqrt{a^2 + 2abx + b^2x^2} dx$$

$$= \frac{e(8b^2(a^2 + b^2x^2) - 12a^2b^2 + 4ab^3x)\sqrt{a^2 + 2abx + b^2x^2}}{24b^4}$$

$$+ \frac{d\sqrt{(a + bx)^2}(a + bx)}{2b}$$

input `int(((a + b*x)^2)^(1/2)*(d + e*x),x)`output `(e*(8*b^2*(a^2 + b^2*x^2) - 12*a^2*b^2 + 4*a*b^3*x)*(a^2 + b^2*x^2 + 2*a*b*x)^(1/2))/(24*b^4) + (d*((a + b*x)^2)^(1/2)*(a + b*x))/(2*b)`**Reduce [B] (verification not implemented)**

Time = 0.22 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.36

$$\int (d + ex)\sqrt{a^2 + 2abx + b^2x^2} dx = \frac{x(2be x^2 + 3aex + 3bdx + 6ad)}{6}$$

input `int((e*x+d)*((b*x+a)^2)^(1/2),x)`output `(x*(6*a*d + 3*a*e*x + 3*b*d*x + 2*b*e*x**2))/6`

3.269 $\int \sqrt{a^2 + 2abx + b^2x^2} dx$

| | |
|---|------|
| Optimal result | 2046 |
| Mathematica [A] (verified) | 2046 |
| Rubi [A] (verified) | 2047 |
| Maple [C] (warning: unable to verify) | 2048 |
| Fricas [A] (verification not implemented) | 2048 |
| Sympy [B] (verification not implemented) | 2049 |
| Maxima [B] (verification not implemented) | 2049 |
| Giac [A] (verification not implemented) | 2050 |
| Mupad [B] (verification not implemented) | 2050 |
| Reduce [B] (verification not implemented) | 2050 |

Optimal result

Integrand size = 20, antiderivative size = 32

$$\int \sqrt{a^2 + 2abx + b^2x^2} dx = \frac{(a + bx)\sqrt{a^2 + 2abx + b^2x^2}}{2b}$$

output

```
1/2*(b*x+a)*((b*x+a)^2)^(1/2)/b
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.94

$$\int \sqrt{a^2 + 2abx + b^2x^2} dx = \frac{x\sqrt{(a + bx)^2(2a + bx)}}{2(a + bx)}$$

input

```
Integrate[Sqrt[a^2 + 2*a*b*x + b^2*x^2],x]
```

output

```
(x*Sqrt[(a + b*x)^2]*(2*a + b*x))/(2*(a + b*x))
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {1079, 17}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{a^2 + 2abx + b^2x^2} dx$$

$$\downarrow 1079$$

$$\frac{\sqrt{a^2 + 2abx + b^2x^2} \int (xb^2 + ab) dx}{b(a + bx)}$$

$$\downarrow 17$$

$$\frac{(a + bx)\sqrt{a^2 + 2abx + b^2x^2}}{2b}$$

input `Int[Sqrt[a^2 + 2*a*b*x + b^2*x^2],x]`

output `((a + b*x)*Sqrt[a^2 + 2*a*b*x + b^2*x^2])/(2*b)`

Defintions of rubi rules used

rule 17 `Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] := Simp[c*((a + b*x)^(m + 1))/(b*(m + 1))], x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]`

rule 1079 `Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] := Simp[(a + b*x + c*x^2)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x)^(2*FracPart[p])) Int[(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c, p}, x] && EqQ[b^2 - 4*a*c, 0]`

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 2.

Time = 0.54 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.59

| method | result | size |
|---------|---|------|
| default | $\frac{\text{csgn}(bx+a)(bx+a)^2}{2b}$ | 19 |
| gospers | $\frac{x(bx+2a)\sqrt{(bx+a)^2}}{2bx+2a}$ | 27 |
| orering | $\frac{x(bx+2a)\sqrt{(bx+a)^2}}{2bx+2a}$ | 27 |
| risch | $\frac{\sqrt{(bx+a)^2}bx^2}{2bx+2a} + \frac{\sqrt{(bx+a)^2}ax}{bx+a}$ | 43 |

input `int(((b*x+a)^2)^(1/2),x,method=_RETURNVERBOSE)`

output `1/2*csgn(b*x+a)*(b*x+a)^2/b`

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.31

$$\int \sqrt{a^2 + 2abx + b^2x^2} dx = \frac{1}{2}bx^2 + ax$$

input `integrate(((b*x+a)^2)^(1/2),x, algorithm="fricas")`

output `1/2*b*x^2 + a*x`

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 56 vs. $2(19) = 38$.

Time = 0.47 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.75

$$\int \sqrt{a^2 + 2abx + b^2x^2} dx = \begin{cases} \left(\frac{a}{2b} + \frac{x}{2}\right) \sqrt{a^2 + 2abx + b^2x^2} & \text{for } b^2 \neq 0 \\ \frac{(a^2 + 2abx)^{\frac{3}{2}}}{3ab} & \text{for } ab \neq 0 \\ x\sqrt{a^2} & \text{otherwise} \end{cases}$$

input `integrate(((b*x+a)**2)**(1/2),x)`

output `Piecewise(((a/(2*b) + x/2)*sqrt(a**2 + 2*a*b*x + b**2*x**2), Ne(b**2, 0)), ((a**2 + 2*a*b*x)**(3/2)/(3*a*b), Ne(a*b, 0)), (x*sqrt(a**2), True))`

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 46 vs. $2(19) = 38$.

Time = 0.03 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.44

$$\int \sqrt{a^2 + 2abx + b^2x^2} dx = \frac{1}{2} \sqrt{b^2x^2 + 2abx + a^2}x + \frac{\sqrt{b^2x^2 + 2abx + a^2}a}{2b}$$

input `integrate(((b*x+a)^2)^(1/2),x, algorithm="maxima")`

output `1/2*sqrt(b^2*x^2 + 2*a*b*x + a^2)*x + 1/2*sqrt(b^2*x^2 + 2*a*b*x + a^2)*a/b`

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.03

$$\int \sqrt{a^2 + 2abx + b^2x^2} dx = \frac{1}{2} (bx^2 + 2ax) \operatorname{sgn}(bx + a) + \frac{a^2 \operatorname{sgn}(bx + a)}{2b}$$

input `integrate(((b*x+a)^2)^(1/2),x, algorithm="giac")`

output `1/2*(b*x^2 + 2*a*x)*sgn(b*x + a) + 1/2*a^2*sgn(b*x + a)/b`

Mupad [B] (verification not implemented)

Time = 8.87 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.59

$$\int \sqrt{a^2 + 2abx + b^2x^2} dx = \frac{\sqrt{(a + bx)^2} (a + bx)}{2b}$$

input `int(((a + b*x)^2)^(1/2),x)`

output `((a + b*x)^2)^(1/2)*(a + b*x)/(2*b)`

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.31

$$\int \sqrt{a^2 + 2abx + b^2x^2} dx = \frac{x(bx + 2a)}{2}$$

input `int(((b*x+a)^2)^(1/2),x)`

output `(x*(2*a + b*x))/2`

3.270 $\int \frac{\sqrt{a^2+2abx+b^2x^2}}{d+ex} dx$

| | |
|---|------|
| Optimal result | 2051 |
| Mathematica [A] (verified) | 2051 |
| Rubi [A] (verified) | 2052 |
| Maple [C] (warning: unable to verify) | 2053 |
| Fricas [A] (verification not implemented) | 2054 |
| Sympy [F] | 2054 |
| Maxima [F(-2)] | 2054 |
| Giac [A] (verification not implemented) | 2055 |
| Mupad [F(-1)] | 2055 |
| Reduce [B] (verification not implemented) | 2055 |

Optimal result

Integrand size = 28, antiderivative size = 80

$$\int \frac{\sqrt{a^2 + 2abx + b^2x^2}}{d + ex} dx = \frac{bx\sqrt{a^2 + 2abx + b^2x^2}}{e(a + bx)} - \frac{(bd - ae)\sqrt{a^2 + 2abx + b^2x^2} \log(d + ex)}{e^2(a + bx)}$$

output `b*x*((b*x+a)^(1/2))/e/(b*x+a)-(-a*e+b*d)*((b*x+a)^(1/2))*ln(e*x+d)/e^2/(b*x+a)`

Mathematica [A] (verified)

Time = 1.02 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.52

$$\int \frac{\sqrt{a^2 + 2abx + b^2x^2}}{d + ex} dx = \frac{\sqrt{(a + bx)^2}(bex + (-bd + ae) \log(d + ex))}{e^2(a + bx)}$$

input `Integrate[Sqrt[a^2 + 2*a*b*x + b^2*x^2]/(d + e*x),x]`

output `(Sqrt[(a + b*x)^2]*(b*e*x + (-b*d) + a*e)*Log[d + e*x])/(e^2*(a + b*x))`

Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.68, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {1102, 27, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\begin{aligned}
 & \int \frac{\sqrt{a^2 + 2abx + b^2x^2}}{d + ex} dx \\
 & \quad \downarrow \text{1102} \\
 & \frac{\sqrt{a^2 + 2abx + b^2x^2} \int \frac{b(a+bx)}{d+ex} dx}{b(a+bx)} \\
 & \quad \downarrow \text{27} \\
 & \frac{\sqrt{a^2 + 2abx + b^2x^2} \int \frac{a+bx}{d+ex} dx}{a+bx} \\
 & \quad \downarrow \text{49} \\
 & \frac{\sqrt{a^2 + 2abx + b^2x^2} \int \left(\frac{b}{e} + \frac{ae-bd}{e(d+ex)} \right) dx}{a+bx} \\
 & \quad \downarrow \text{2009} \\
 & \frac{\sqrt{a^2 + 2abx + b^2x^2} \left(\frac{bx}{e} - \frac{(bd-ae)\log(d+ex)}{e^2} \right)}{a+bx}
 \end{aligned}$$

input `Int[Sqrt[a^2 + 2*a*b*x + b^2*x^2]/(d + e*x), x]`

output `(Sqrt[a^2 + 2*a*b*x + b^2*x^2]*((b*x)/e - ((b*d - a*e)*Log[d + e*x])/e^2)) / (a + b*x)`

Definitions of rubi rules used

rule 27 `Int[(a_)*(F x_), x_Symbol] := Simp[a Int[F x, x], x] /; FreeQ[a, x] && !MatchQ[F x, (b_)*(G x_) /; FreeQ[b, x]]`

rule 49 `Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] := Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] && IGtQ[m, 0] && IGtQ[m + n + 2, 0]`

rule 1102 `Int[((d_.) + (e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] := Simp[(a + b*x + c*x^2)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x)^(2*FracPart[p])) Int[(d + e*x)^m*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && EqQ[b^2 - 4*a*c, 0]`

rule 2009 `Int[u_, x_Symbol] := Simp[IntSum[u, x], x] /; SumQ[u]`

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.88 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.60

| method | result | size |
|---------|---|------|
| default | $\frac{\text{csgn}(bx+a)(\ln(-bex-bd)ae - \ln(-bex-bd)bd + bex + ae)}{e^2}$ | 48 |
| risch | $\frac{bx\sqrt{(bx+a)^2}}{e(bx+a)} + \frac{\sqrt{(bx+a)^2}(ae-bd)\ln(ex+d)}{(bx+a)e^2}$ | 58 |

input `int(((b*x+a)^2)^(1/2)/(e*x+d), x, method=_RETURNVERBOSE)`

output `csgn(b*x+a)*(ln(-b*e*x-b*d)*a*e - ln(-b*e*x-b*d)*b*d + b*e*x + a*e)/e^2`

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.31

$$\int \frac{\sqrt{a^2 + 2abx + b^2x^2}}{d + ex} dx = \frac{bex - (bd - ae) \log(ex + d)}{e^2}$$

input `integrate(((b*x+a)^2)^(1/2)/(e*x+d),x, algorithm="fricas")`

output `(b*e*x - (b*d - a*e)*log(e*x + d))/e^2`

Sympy [F]

$$\int \frac{\sqrt{a^2 + 2abx + b^2x^2}}{d + ex} dx = \int \frac{\sqrt{(a + bx)^2}}{d + ex} dx$$

input `integrate(((b*x+a)**2)**(1/2)/(e*x+d),x)`

output `Integral(sqrt((a + b*x)**2)/(d + e*x), x)`

Maxima [F(-2)]

Exception generated.

$$\int \frac{\sqrt{a^2 + 2abx + b^2x^2}}{d + ex} dx = \text{Exception raised: ValueError}$$

input `integrate(((b*x+a)^2)^(1/2)/(e*x+d),x, algorithm="maxima")`

output `Exception raised: ValueError >> Computation failed since Maxima requested additional constraints; using the 'assume' command before evaluation *may* help (example of legal syntax is 'assume(a*e-b*d>0)', see `assume?` for more detail`

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.56

$$\int \frac{\sqrt{a^2 + 2abx + b^2x^2}}{d + ex} dx$$

$$= \frac{bx \operatorname{sgn}(bx + a)}{e} - \frac{(bd \operatorname{sgn}(bx + a) - a e \operatorname{sgn}(bx + a)) \log(|ex + d|)}{e^2}$$

input `integrate(((b*x+a)^2)^(1/2)/(e*x+d),x, algorithm="giac")`

output `b*x*sgn(b*x + a)/e - (b*d*sgn(b*x + a) - a*e*sgn(b*x + a))*log(abs(e*x + d))/e^2`

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a^2 + 2abx + b^2x^2}}{d + ex} dx = \int \frac{\sqrt{(a + bx)^2}}{d + ex} dx$$

input `int(((a + b*x)^2)^(1/2)/(d + e*x),x)`

output `int(((a + b*x)^2)^(1/2)/(d + e*x), x)`

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.35

$$\int \frac{\sqrt{a^2 + 2abx + b^2x^2}}{d + ex} dx = \frac{\log(ex + d) ae - \log(ex + d) bd + bex}{e^2}$$

input `int(((b*x+a)^2)^(1/2)/(e*x+d),x)`

output `(log(d + e*x)*a*e - log(d + e*x)*b*d + b*e*x)/e**2`

3.271 $\int \frac{\sqrt{a^2+2abx+b^2x^2}}{(d+ex)^2} dx$

| | |
|---|------|
| Optimal result | 2056 |
| Mathematica [B] (verified) | 2056 |
| Rubi [A] (verified) | 2057 |
| Maple [C] (warning: unable to verify) | 2058 |
| Fricas [A] (verification not implemented) | 2059 |
| Sympy [F] | 2059 |
| Maxima [F(-2)] | 2059 |
| Giac [A] (verification not implemented) | 2060 |
| Mupad [F(-1)] | 2060 |
| Reduce [B] (verification not implemented) | 2061 |

Optimal result

Integrand size = 28, antiderivative size = 85

$$\int \frac{\sqrt{a^2 + 2abx + b^2x^2}}{(d + ex)^2} dx = \frac{(bd - ae)\sqrt{a^2 + 2abx + b^2x^2}}{e^2(a + bx)(d + ex)} + \frac{b\sqrt{a^2 + 2abx + b^2x^2} \log(d + ex)}{e^2(a + bx)}$$

output

```
(-a*e+b*d)*((b*x+a)^2)^(1/2)/e^2/(b*x+a)/(e*x+d)+b*((b*x+a)^2)^(1/2)*ln(e*x+d)/e^2/(b*x+a)
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 245 vs. 2(85) = 170.

Time = 0.79 (sec) , antiderivative size = 245, normalized size of antiderivative = 2.88

$$\int \frac{\sqrt{a^2 + 2abx + b^2x^2}}{(d + ex)^2} dx = \frac{-\frac{e\sqrt{(a+bx)^2}}{d+ex} - \frac{\sqrt{a^2}e(-bdx+a(d+2ex))}{ad(d+ex)} + \frac{(a+\sqrt{a^2})b \log(\sqrt{a^2-bx}-\sqrt{(a+bx)^2})}{a} + \frac{(-a+\sqrt{a^2})b \log(\sqrt{a^2+bx}-\sqrt{(a+bx)^2})}{a} - \frac{(a+\sqrt{a^2})b \log(\sqrt{a^2+bx}+\sqrt{(a+bx)^2})}{a}}{2e^2}$$

input `Integrate[Sqrt[a^2 + 2*a*b*x + b^2*x^2]/(d + e*x)^2,x]`

output
$$\frac{(-((e*\text{Sqrt}[(a + b*x)^2])/(d + e*x)) - (\text{Sqrt}[a^2]*e*(-(b*d*x) + a*(d + 2*e*x)))/(a*d*(d + e*x)) + ((a + \text{Sqrt}[a^2])*b*\text{Log}[\text{Sqrt}[a^2] - b*x - \text{Sqrt}[(a + b*x)^2]])/a + ((-a + \text{Sqrt}[a^2])*b*\text{Log}[\text{Sqrt}[a^2] + b*x - \text{Sqrt}[(a + b*x)^2]])/a - ((a + \text{Sqrt}[a^2])*b*\text{Log}[2*a*e*x + d*(\text{Sqrt}[a^2] - b*x - \text{Sqrt}[(a + b*x)^2])])/a + ((a - \text{Sqrt}[a^2])*b*\text{Log}[-2*a*e*x + d*(\text{Sqrt}[a^2] + b*x - \text{Sqrt}[(a + b*x)^2])])/a)/(2*e^2)}$$

Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.69, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {1102, 27, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\begin{aligned} & \int \frac{\sqrt{a^2 + 2abx + b^2x^2}}{(d + ex)^2} dx \\ & \quad \downarrow \text{1102} \\ & \frac{\sqrt{a^2 + 2abx + b^2x^2} \int \frac{b(a+bx)}{(d+ex)^2} dx}{b(a + bx)} \\ & \quad \downarrow \text{27} \\ & \frac{\sqrt{a^2 + 2abx + b^2x^2} \int \frac{a+bx}{(d+ex)^2} dx}{a + bx} \\ & \quad \downarrow \text{49} \\ & \frac{\sqrt{a^2 + 2abx + b^2x^2} \int \left(\frac{b}{e(d+ex)} + \frac{ae-bd}{e(d+ex)^2} \right) dx}{a + bx} \\ & \quad \downarrow \text{2009} \\ & \frac{\sqrt{a^2 + 2abx + b^2x^2} \left(\frac{bd-ae}{e^2(d+ex)} + \frac{b \log(d+ex)}{e^2} \right)}{a + bx} \end{aligned}$$

input `Int[Sqrt[a^2 + 2*a*b*x + b^2*x^2]/(d + e*x)^2,x]`

output `(Sqrt[a^2 + 2*a*b*x + b^2*x^2]*((b*d - a*e)/(e^2*(d + e*x)) + (b*Log[d + e*x])/e^2))/(a + b*x)`

Defintions of rubi rules used

rule 27 `Int[(a_)*(F_x_), x_Symbol] := Simp[a Int[F_x, x], x] /; FreeQ[a, x] && !MatchQ[F_x, (b_)*(G_x_) /; FreeQ[b, x]]`

rule 49 `Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] := Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] && IGtQ[m, 0] && IGtQ[m + n + 2, 0]`

rule 1102 `Int[((d_.) + (e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] := Simp[(a + b*x + c*x^2)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x)^(2*FracPart[p])) Int[(d + e*x)^m*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && EqQ[b^2 - 4*a*c, 0]`

rule 2009 `Int[u_, x_Symbol] := Simp[IntSum[u, x], x] /; SumQ[u]`

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.96 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.65

| method | result | size |
|---------|---|------|
| default | $\frac{\text{csgn}(bx+a)(\ln(-bex-bd)bex+\ln(-bex-bd)bd-ae+bd)}{e^2(ex+d)}$ | 55 |
| risch | $-\frac{\sqrt{(bx+a)^2}(ae-bd)}{(bx+a)e^2(ex+d)} + \frac{b\sqrt{(bx+a)^2}\ln(ex+d)}{e^2(bx+a)}$ | 65 |

input `int(((b*x+a)^2)^(1/2)/(e*x+d)^2,x,method=_RETURNVERBOSE)`

output `csgn(b*x+a)*(ln(-b*e*x-b*d)*b*e*x+ln(-b*e*x-b*d)*b*d-a*e+b*d)/e^2/(e*x+d)`

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.44

$$\int \frac{\sqrt{a^2 + 2abx + b^2x^2}}{(d + ex)^2} dx = \frac{bd - ae + (bex + bd) \log(ex + d)}{e^3x + de^2}$$

input `integrate(((b*x+a)^2)^(1/2)/(e*x+d)^2,x, algorithm="fricas")`

output `(b*d - a*e + (b*e*x + b*d)*log(e*x + d))/(e^3*x + d*e^2)`

Sympy [F]

$$\int \frac{\sqrt{a^2 + 2abx + b^2x^2}}{(d + ex)^2} dx = \int \frac{\sqrt{(a + bx)^2}}{(d + ex)^2} dx$$

input `integrate(((b*x+a)**2)**(1/2)/(e*x+d)**2,x)`

output `Integral(sqrt((a + b*x)**2)/(d + e*x)**2, x)`

Maxima [F(-2)]

Exception generated.

$$\int \frac{\sqrt{a^2 + 2abx + b^2x^2}}{(d + ex)^2} dx = \text{Exception raised: ValueError}$$

input `integrate(((b*x+a)^2)^(1/2)/(e*x+d)^2,x, algorithm="maxima")`

output

```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
help (example of legal syntax is 'assume(a*e-b*d>0)', see `assume?` for m
ore detail
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.59

$$\int \frac{\sqrt{a^2 + 2abx + b^2x^2}}{(d + ex)^2} dx = \frac{b \log(|ex + d|) \operatorname{sgn}(bx + a)}{e^2} + \frac{bd \operatorname{sgn}(bx + a) - ae \operatorname{sgn}(bx + a)}{(ex + d)e^2}$$

input

```
integrate(((b*x+a)^2)^(1/2)/(e*x+d)^2,x, algorithm="giac")
```

output

```
b*log(abs(e*x + d))*sgn(b*x + a)/e^2 + (b*d*sgn(b*x + a) - a*e*sgn(b*x + a
))/((e*x + d)*e^2)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a^2 + 2abx + b^2x^2}}{(d + ex)^2} dx = \int \frac{\sqrt{(a + bx)^2}}{(d + ex)^2} dx$$

input

```
int(((a + b*x)^2)^(1/2)/(d + e*x)^2,x)
```

output

```
int(((a + b*x)^2)^(1/2)/(d + e*x)^2, x)
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.58

$$\int \frac{\sqrt{a^2 + 2abx + b^2x^2}}{(d + ex)^2} dx = \frac{\log(ex + d) b d^2 + \log(ex + d) b d e x + a e^2 x - b d e x}{d e^2 (ex + d)}$$

input `int(((b*x+a)^2)^(1/2)/(e*x+d)^2,x)`

output `(log(d + e*x)*b*d**2 + log(d + e*x)*b*d*e*x + a*e**2*x - b*d*e*x)/(d*e**2*(d + e*x))`

$$3.272 \quad \int \frac{\sqrt{a^2+2abx+b^2x^2}}{(d+ex)^3} dx$$

| | |
|---|------|
| Optimal result | 2062 |
| Mathematica [A] (verified) | 2062 |
| Rubi [A] (verified) | 2063 |
| Maple [C] (warning: unable to verify) | 2064 |
| Fricas [A] (verification not implemented) | 2064 |
| Sympy [F] | 2065 |
| Maxima [F(-2)] | 2065 |
| Giac [B] (verification not implemented) | 2065 |
| Mupad [B] (verification not implemented) | 2066 |
| Reduce [B] (verification not implemented) | 2066 |

Optimal result

Integrand size = 28, antiderivative size = 46

$$\int \frac{\sqrt{a^2+2abx+b^2x^2}}{(d+ex)^3} dx = \frac{(a+bx)\sqrt{a^2+2abx+b^2x^2}}{2(bd-ae)(d+ex)^2}$$

output

$$1/2*(b*x+a)*((b*x+a)^2)^(1/2)/(-a*e+b*d)/(e*x+d)^2$$

Mathematica [A] (verified)

Time = 0.51 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.80

$$\int \frac{\sqrt{a^2+2abx+b^2x^2}}{(d+ex)^3} dx = \frac{(a+bx)\sqrt{(a+bx)^2}}{2(bd-ae)(d+ex)^2}$$

input

$$\text{Integrate}[\text{Sqrt}[a^2 + 2*a*b*x + b^2*x^2]/(d + e*x)^3, x]$$

output

$$((a + b*x)*\text{Sqrt}[(a + b*x)^2])/(2*(b*d - a*e)*(d + e*x)^2)$$

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.107$, Rules used = {1102, 27, 48}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a^2 + 2abx + b^2x^2}}{(d + ex)^3} dx$$

$$\downarrow 1102$$

$$\frac{\sqrt{a^2 + 2abx + b^2x^2} \int \frac{b(a+bx)}{(d+ex)^3} dx}{b(a+bx)}$$

$$\downarrow 27$$

$$\frac{\sqrt{a^2 + 2abx + b^2x^2} \int \frac{a+bx}{(d+ex)^3} dx}{a+bx}$$

$$\downarrow 48$$

$$\frac{(a+bx)\sqrt{a^2 + 2abx + b^2x^2}}{2(d+ex)^2(bd - ae)}$$

input `Int[Sqrt[a^2 + 2*a*b*x + b^2*x^2]/(d + e*x)^3,x]`

output `((a + b*x)*Sqrt[a^2 + 2*a*b*x + b^2*x^2])/(2*(b*d - a*e)*(d + e*x)^2)`

Defintions of rubi rules used

rule 27 `Int[(a_)*(F_x_), x_Symbol] :> Simp[a Int[F_x, x], x] /; FreeQ[a, x] && !MatchQ[F_x, (b_)*(G_x_)] /; FreeQ[b, x]`

rule 48 `Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{a, b, c, d, m, n}, x] && EqQ[m + n + 2, 0] && NeQ[m, -1]`

rule 1102

```
Int[((d_.) + (e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol]
:> Simp[(a + b*x + c*x^2)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x)^(2*FracPart[p]))
Int[(d + e*x)^m*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d, e, m, p}, x]
&& EqQ[b^2 - 4*a*c, 0]
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 2.

Time = 1.15 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.67

| method | result | size |
|---------|---|------|
| default | $-\frac{\operatorname{csgn}(bx+a)(2bex+ae+bd)}{2e^2(ex+d)^2}$ | 31 |
| gospers | $-\frac{(2bex+ae+bd)\sqrt{(bx+a)^2}}{2(ex+d)^2e^2(bx+a)}$ | 41 |
| orering | $-\frac{(2bex+ae+bd)\sqrt{(bx+a)^2}}{2(ex+d)^2e^2(bx+a)}$ | 41 |
| risch | $\frac{\left(-\frac{bx}{e} - \frac{ae+bd}{2e^2}\right)\sqrt{(bx+a)^2}}{(ex+d)^2(bx+a)}$ | 45 |

input

```
int(((b*x+a)^2)^(1/2)/(e*x+d)^3,x,method=_RETURNVERBOSE)
```

output

```
-1/2*csgn(b*x+a)*(2*b*e*x+a*e+b*d)/e^2/(e*x+d)^2
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.83

$$\int \frac{\sqrt{a^2 + 2abx + b^2x^2}}{(d + ex)^3} dx = -\frac{2bex + bd + ae}{2(e^4x^2 + 2de^3x + d^2e^2)}$$

input

```
integrate(((b*x+a)^2)^(1/2)/(e*x+d)^3,x, algorithm="fricas")
```

output

```
-1/2*(2*b*e*x + b*d + a*e)/(e^4*x^2 + 2*d*e^3*x + d^2*e^2)
```

Sympy [F]

$$\int \frac{\sqrt{a^2 + 2abx + b^2x^2}}{(d + ex)^3} dx = \int \frac{\sqrt{(a + bx)^2}}{(d + ex)^3} dx$$

input `integrate(((b*x+a)**2)**(1/2)/(e*x+d)**3,x)`

output `Integral(sqrt((a + b*x)**2)/(d + e*x)**3, x)`

Maxima [F(-2)]

Exception generated.

$$\int \frac{\sqrt{a^2 + 2abx + b^2x^2}}{(d + ex)^3} dx = \text{Exception raised: ValueError}$$

input `integrate(((b*x+a)^2)^(1/2)/(e*x+d)^3,x, algorithm="maxima")`

output `Exception raised: ValueError >> Computation failed since Maxima requested additional constraints; using the 'assume' command before evaluation *may* help (example of legal syntax is 'assume(a*e-b*d>0)', see `assume?` for more detail`

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 69 vs. 2(33) = 66.

Time = 0.15 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.50

$$\int \frac{\sqrt{a^2 + 2abx + b^2x^2}}{(d + ex)^3} dx = \frac{b^2 \operatorname{sgn}(bx + a)}{2(bde^2 - ae^3)} - \frac{2bex \operatorname{sgn}(bx + a) + bds \operatorname{sgn}(bx + a) + aes \operatorname{sgn}(bx + a)}{2(ex + d)^2 e^2}$$

input `integrate(((b*x+a)^2)^(1/2)/(e*x+d)^3,x, algorithm="giac")`

output $\frac{1/2*b^2*sgn(b*x + a)/(b*d*e^2 - a*e^3) - 1/2*(2*b*e*x*sgn(b*x + a) + b*d*sgn(b*x + a) + a*e*sgn(b*x + a))/((e*x + d)^2*e^2)}$

Mupad [B] (verification not implemented)

Time = 8.92 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.87

$$\int \frac{\sqrt{a^2 + 2abx + b^2x^2}}{(d + ex)^3} dx = -\frac{\sqrt{(a + bx)^2 (ae + bd + 2be x)}}{2e^2 (a + bx) (d + ex)^2}$$

input `int(((a + b*x)^2)^(1/2)/(d + e*x)^3,x)`

output $-\frac{((a + b*x)^2)^(1/2)*(a*e + b*d + 2*b*e*x)}{(2*e^2*(a + b*x)*(d + e*x)^2)}$

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.80

$$\int \frac{\sqrt{a^2 + 2abx + b^2x^2}}{(d + ex)^3} dx = \frac{be x^2 - ad}{2de (e^2x^2 + 2dex + d^2)}$$

input `int(((b*x+a)^2)^(1/2)/(e*x+d)^3,x)`

output $(-a*d + b*e*x**2)/(2*d*e*(d**2 + 2*d*e*x + e**2*x**2))$

3.273 $\int \frac{\sqrt{a^2+2abx+b^2x^2}}{(d+ex)^4} dx$

| | |
|---|------|
| Optimal result | 2067 |
| Mathematica [A] (verified) | 2067 |
| Rubi [A] (verified) | 2068 |
| Maple [C] (warning: unable to verify) | 2069 |
| Fricas [A] (verification not implemented) | 2070 |
| Sympy [F] | 2070 |
| Maxima [F(-2)] | 2070 |
| Giac [A] (verification not implemented) | 2071 |
| Mupad [B] (verification not implemented) | 2071 |
| Reduce [B] (verification not implemented) | 2072 |

Optimal result

Integrand size = 28, antiderivative size = 92

$$\int \frac{\sqrt{a^2 + 2abx + b^2x^2}}{(d + ex)^4} dx = \frac{(bd - ae)\sqrt{a^2 + 2abx + b^2x^2}}{3e^2(a + bx)(d + ex)^3} - \frac{b\sqrt{a^2 + 2abx + b^2x^2}}{2e^2(a + bx)(d + ex)^2}$$

output

$$\frac{1}{3}*(-a*e+b*d)*((b*x+a)^2)^{(1/2)}/e^2/(b*x+a)/(e*x+d)^3-1/2*b*((b*x+a)^2)^{(1/2)}/e^2/(b*x+a)/(e*x+d)^2$$

Mathematica [A] (verified)

Time = 0.88 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.49

$$\int \frac{\sqrt{a^2 + 2abx + b^2x^2}}{(d + ex)^4} dx = -\frac{\sqrt{(a + bx)^2(2ae + b(d + 3ex))}}{6e^2(a + bx)(d + ex)^3}$$

input

$$\text{Integrate}[\text{Sqrt}[a^2 + 2*a*b*x + b^2*x^2]/(d + e*x)^4, x]$$

output

$$-1/6*(\text{Sqrt}[(a + b*x)^2]*(2*a*e + b*(d + 3*e*x)))/(e^2*(a + b*x)*(d + e*x)^3)$$

Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.72, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {1102, 27, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\begin{aligned}
 & \int \frac{\sqrt{a^2 + 2abx + b^2x^2}}{(d + ex)^4} dx \\
 & \quad \downarrow \text{1102} \\
 & \frac{\sqrt{a^2 + 2abx + b^2x^2} \int \frac{b(a+bx)}{(d+ex)^4} dx}{b(a + bx)} \\
 & \quad \downarrow \text{27} \\
 & \frac{\sqrt{a^2 + 2abx + b^2x^2} \int \frac{a+bx}{(d+ex)^4} dx}{a + bx} \\
 & \quad \downarrow \text{53} \\
 & \frac{\sqrt{a^2 + 2abx + b^2x^2} \int \left(\frac{b}{e(d+ex)^3} + \frac{ae-bd}{e(d+ex)^4} \right) dx}{a + bx} \\
 & \quad \downarrow \text{2009} \\
 & \frac{\sqrt{a^2 + 2abx + b^2x^2} \left(\frac{bd-ae}{3e^2(d+ex)^3} - \frac{b}{2e^2(d+ex)^2} \right)}{a + bx}
 \end{aligned}$$

input `Int[Sqrt[a^2 + 2*a*b*x + b^2*x^2]/(d + e*x)^4,x]`

output `(Sqrt[a^2 + 2*a*b*x + b^2*x^2]*((b*d - a*e)/(3*e^2*(d + e*x)^3) - b/(2*e^2*(d + e*x)^2)))/(a + b*x)`

Definitions of rubi rules used

rule 27 `Int[(a_)*(F_x_), x_Symbol] := Simp[a Int[F_x, x], x] /; FreeQ[a, x] && !MatchQ[F_x, (b_)*(G_x_) /; FreeQ[b, x]]`

rule 53 `Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] := Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && IGtQ[m, 0] && (!IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])`

rule 1102 `Int[((d_.) + (e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] := Simp[(a + b*x + c*x^2)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x)^(2*FracPart[p])) Int[(d + e*x)^m*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && EqQ[b^2 - 4*a*c, 0]`

rule 2009 `Int[u_, x_Symbol] := Simp[IntSum[u, x], x] /; SumQ[u]`

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 2.

Time = 1.30 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.35

| method | result | size |
|---------|---|------|
| default | $-\frac{\operatorname{csgn}(bx+a)(3bex+2ae+bd)}{6e^2(ex+d)^3}$ | 32 |
| gospers | $-\frac{(3bex+2ae+bd)\sqrt{(bx+a)^2}}{6e^2(ex+d)^3(bx+a)}$ | 42 |
| orering | $-\frac{(3bex+2ae+bd)\sqrt{(bx+a)^2}}{6e^2(ex+d)^3(bx+a)}$ | 42 |
| risch | $\frac{\sqrt{(bx+a)^2}\left(-\frac{bx}{2e}-\frac{2ae+bd}{6e^2}\right)}{(bx+a)(ex+d)^3}$ | 46 |

input `int(((b*x+a)^2)^(1/2)/(e*x+d)^4,x,method=_RETURNVERBOSE)`

output `-1/6*csgn(b*x+a)*(3*b*e*x+2*a*e+b*d)/e^2/(e*x+d)^3`

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.54

$$\int \frac{\sqrt{a^2 + 2abx + b^2x^2}}{(d + ex)^4} dx = -\frac{3bex + bd + 2ae}{6(e^5x^3 + 3de^4x^2 + 3d^2e^3x + d^3e^2)}$$

input `integrate(((b*x+a)^2)^(1/2)/(e*x+d)^4,x, algorithm="fricas")`

output `-1/6*(3*b*e*x + b*d + 2*a*e)/(e^5*x^3 + 3*d*e^4*x^2 + 3*d^2*e^3*x + d^3*e^2)`

Sympy [F]

$$\int \frac{\sqrt{a^2 + 2abx + b^2x^2}}{(d + ex)^4} dx = \int \frac{\sqrt{(a + bx)^2}}{(d + ex)^4} dx$$

input `integrate(((b*x+a)**2)**(1/2)/(e*x+d)**4,x)`

output `Integral(sqrt((a + b*x)**2)/(d + e*x)**4, x)`

Maxima [F(-2)]

Exception generated.

$$\int \frac{\sqrt{a^2 + 2abx + b^2x^2}}{(d + ex)^4} dx = \text{Exception raised: ValueError}$$

input `integrate(((b*x+a)^2)^(1/2)/(e*x+d)^4,x, algorithm="maxima")`

output

```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
help (example of legal syntax is 'assume(a*e-b*d>0)', see `assume?` for m
ore detail
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.90

$$\int \frac{\sqrt{a^2 + 2abx + b^2x^2}}{(d + ex)^4} dx = \frac{b^3 \operatorname{sgn}(bx + a)}{6(b^2d^2e^2 - 2abde^3 + a^2e^4)} - \frac{3be\operatorname{sgn}(bx + a) + bd\operatorname{sgn}(bx + a) + 2ae\operatorname{sgn}(bx + a)}{6(ex + d)^3e^2}$$

input

```
integrate(((b*x+a)^2)^(1/2)/(e*x+d)^4,x, algorithm="giac")
```

output

```
1/6*b^3*sgn(b*x + a)/(b^2*d^2*e^2 - 2*a*b*d*e^3 + a^2*e^4) - 1/6*(3*b*e*x*
sgn(b*x + a) + b*d*sgn(b*x + a) + 2*a*e*sgn(b*x + a))/((e*x + d)^3*e^2)
```

Mupad [B] (verification not implemented)

Time = 9.03 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.45

$$\int \frac{\sqrt{a^2 + 2abx + b^2x^2}}{(d + ex)^4} dx = -\frac{\sqrt{(a + bx)^2(2ae + bd + 3bex)}}{6e^2(a + bx)(d + ex)^3}$$

input

```
int(((a + b*x)^2)^(1/2)/(d + e*x)^4,x)
```

output

```
-(((a + b*x)^2)^(1/2)*(2*a*e + b*d + 3*b*e*x))/(6*e^2*(a + b*x)*(d + e*x)^
3)
```

Reduce [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.52

$$\int \frac{\sqrt{a^2 + 2abx + b^2x^2}}{(d + ex)^4} dx = \frac{-3bex - 2ae - bd}{6e^2 (e^3x^3 + 3de^2x^2 + 3d^2ex + d^3)}$$

input `int(((b*x+a)^2)^(1/2)/(e*x+d)^4,x)`

output `(- 2*a*e - b*d - 3*b*e*x)/(6*e**2*(d**3 + 3*d**2*e*x + 3*d*e**2*x**2 + e*
*3*x**3))`

$$3.274 \quad \int \frac{\sqrt{a^2+2abx+b^2x^2}}{(d+ex)^5} dx$$

| | |
|---|------|
| Optimal result | 2073 |
| Mathematica [A] (verified) | 2073 |
| Rubi [A] (verified) | 2074 |
| Maple [C] (warning: unable to verify) | 2075 |
| Fricas [A] (verification not implemented) | 2076 |
| Sympy [F] | 2076 |
| Maxima [F(-2)] | 2076 |
| Giac [A] (verification not implemented) | 2077 |
| Mupad [B] (verification not implemented) | 2077 |
| Reduce [B] (verification not implemented) | 2078 |

Optimal result

Integrand size = 28, antiderivative size = 92

$$\int \frac{\sqrt{a^2+2abx+b^2x^2}}{(d+ex)^5} dx = \frac{(bd-ae)\sqrt{a^2+2abx+b^2x^2}}{4e^2(a+bx)(d+ex)^4} - \frac{b\sqrt{a^2+2abx+b^2x^2}}{3e^2(a+bx)(d+ex)^3}$$

output

```
1/4*(-a*e+b*d)*((b*x+a)^2)^(1/2)/e^2/(b*x+a)/(e*x+d)^4-1/3*b*((b*x+a)^2)^(1/2)/e^2/(b*x+a)/(e*x+d)^3
```

Mathematica [A] (verified)

Time = 1.02 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.49

$$\int \frac{\sqrt{a^2+2abx+b^2x^2}}{(d+ex)^5} dx = -\frac{\sqrt{(a+bx)^2(3ae+b(d+4ex))}}{12e^2(a+bx)(d+ex)^4}$$

input

```
Integrate[Sqrt[a^2 + 2*a*b*x + b^2*x^2]/(d + e*x)^5,x]
```

output

```
-1/12*(Sqrt[(a + b*x)^2]*(3*a*e + b*(d + 4*e*x)))/(e^2*(a + b*x)*(d + e*x)^4)
```

Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.72, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {1102, 27, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\begin{aligned}
 & \int \frac{\sqrt{a^2 + 2abx + b^2x^2}}{(d + ex)^5} dx \\
 & \quad \downarrow \text{1102} \\
 & \frac{\sqrt{a^2 + 2abx + b^2x^2} \int \frac{b(a+bx)}{(d+ex)^5} dx}{b(a + bx)} \\
 & \quad \downarrow \text{27} \\
 & \frac{\sqrt{a^2 + 2abx + b^2x^2} \int \frac{a+bx}{(d+ex)^5} dx}{a + bx} \\
 & \quad \downarrow \text{53} \\
 & \frac{\sqrt{a^2 + 2abx + b^2x^2} \int \left(\frac{b}{e(d+ex)^4} + \frac{ae-bd}{e(d+ex)^5} \right) dx}{a + bx} \\
 & \quad \downarrow \text{2009} \\
 & \frac{\sqrt{a^2 + 2abx + b^2x^2} \left(\frac{bd-ae}{4e^2(d+ex)^4} - \frac{b}{3e^2(d+ex)^3} \right)}{a + bx}
 \end{aligned}$$

input `Int[Sqrt[a^2 + 2*a*b*x + b^2*x^2]/(d + e*x)^5,x]`

output `(Sqrt[a^2 + 2*a*b*x + b^2*x^2]*((b*d - a*e)/(4*e^2*(d + e*x)^4) - b/(3*e^2*(d + e*x)^3)))/(a + b*x)`

Definitions of rubi rules used

rule 27 $\text{Int}[(a_*)(Fx_), x_Symbol] \rightarrow \text{Simp}[a \text{ Int}[Fx, x], x] /; \text{FreeQ}[a, x] \ \&\& \ !\text{MatchQ}[Fx, (b_*)(Gx_)] /; \text{FreeQ}[b, x]$

rule 53 $\text{Int}[((a_.) + (b_.)(x_))^{(m_.)}((c_.) + (d_.)(x_))^{(n_.)}, x_Symbol] \rightarrow \text{Int}[\text{ExpandIntegrand}[(a + b*x)^m*(c + d*x)^n, x], x] /; \text{FreeQ}[\{a, b, c, d, n\}, x] \ \&\& \ \text{IGtQ}[m, 0] \ \&\& \ (\ !\text{IntegerQ}[n] \ || \ (\text{EqQ}[c, 0] \ \&\& \ \text{LeQ}[7*m + 4*n + 4, 0]) \ || \ \text{LtQ}[9*m + 5*(n + 1), 0] \ || \ \text{GtQ}[m + n + 2, 0])$

rule 1102 $\text{Int}[((d_.) + (e_.)(x_))^{(m_.)}((a_) + (b_.)(x_) + (c_.)(x_)^2)^{(p_.)}, x_Symbol] \rightarrow \text{Simp}[(a + b*x + c*x^2)^{\text{FracPart}[p]} / (c^{\text{IntPart}[p]}*(b/2 + c*x)^{(2*\text{FracPart}[p])}) \ \text{Int}[(d + e*x)^m*(b/2 + c*x)^{(2*p)}, x], x] /; \text{FreeQ}[\{a, b, c, d, e, m, p\}, x] \ \&\& \ \text{EqQ}[b^2 - 4*a*c, 0]$

rule 2009 $\text{Int}[u_, x_Symbol] \rightarrow \text{Simp}[\text{IntSum}[u, x], x] /; \text{SumQ}[u]$

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 2.

Time = 1.38 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.35

| method | result | size |
|---------|---|------|
| default | $-\frac{\text{csgn}(bx+a)(4bex+3ae+bd)}{12e^2(ex+d)^4}$ | 32 |
| gospers | $-\frac{(4bex+3ae+bd)\sqrt{(bx+a)^2}}{12e^2(ex+d)^4(bx+a)}$ | 42 |
| orering | $-\frac{(4bex+3ae+bd)\sqrt{(bx+a)^2}}{12e^2(ex+d)^4(bx+a)}$ | 42 |
| risch | $\frac{\sqrt{(bx+a)^2} \left(-\frac{bx}{3e} - \frac{3ae+bd}{12e^2} \right)}{(bx+a)(ex+d)^4}$ | 46 |

input $\text{int}(((b*x+a)^2)^{(1/2)} / (e*x+d)^5, x, \text{method}=_RETURNVERBOSE)$

output `-1/12*csgn(b*x+a)*(4*b*e*x+3*a*e+b*d)/e^2/(e*x+d)^4`

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.66

$$\int \frac{\sqrt{a^2 + 2abx + b^2x^2}}{(d + ex)^5} dx = -\frac{4bex + bd + 3ae}{12(e^6x^4 + 4de^5x^3 + 6d^2e^4x^2 + 4d^3e^3x + d^4e^2)}$$

input `integrate(((b*x+a)^2)^(1/2)/(e*x+d)^5,x, algorithm="fricas")`

output `-1/12*(4*b*e*x + b*d + 3*a*e)/(e^6*x^4 + 4*d*e^5*x^3 + 6*d^2*e^4*x^2 + 4*d^3*e^3*x + d^4*e^2)`

Sympy [F]

$$\int \frac{\sqrt{a^2 + 2abx + b^2x^2}}{(d + ex)^5} dx = \int \frac{\sqrt{(a + bx)^2}}{(d + ex)^5} dx$$

input `integrate(((b*x+a)**2)**(1/2)/(e*x+d)**5,x)`

output `Integral(sqrt((a + b*x)**2)/(d + e*x)**5, x)`

Maxima [F(-2)]

Exception generated.

$$\int \frac{\sqrt{a^2 + 2abx + b^2x^2}}{(d + ex)^5} dx = \text{Exception raised: ValueError}$$

input `integrate(((b*x+a)^2)^(1/2)/(e*x+d)^5,x, algorithm="maxima")`

output

```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
help (example of legal syntax is 'assume(a*e-b*d>0)', see `assume?` for m
ore detail
```

Giac [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 98, normalized size of antiderivative = 1.07

$$\int \frac{\sqrt{a^2 + 2abx + b^2x^2}}{(d + ex)^5} dx = \frac{b^4 \operatorname{sgn}(bx + a)}{12 (b^3 d^3 e^2 - 3 ab^2 d^2 e^3 + 3 a^2 b d e^4 - a^3 e^5)} - \frac{4 b e x \operatorname{sgn}(bx + a) + b d \operatorname{sgn}(bx + a) + 3 a e \operatorname{sgn}(bx + a)}{12 (ex + d)^4 e^2}$$

input

```
integrate(((b*x+a)^2)^(1/2)/(e*x+d)^5,x, algorithm="giac")
```

output

```
1/12*b^4*sgn(b*x + a)/(b^3*d^3*e^2 - 3*a*b^2*d^2*e^3 + 3*a^2*b*d*e^4 - a^3
*e^5) - 1/12*(4*b*e*x*sgn(b*x + a) + b*d*sgn(b*x + a) + 3*a*e*sgn(b*x + a)
)/((e*x + d)^4*e^2)
```

Mupad [B] (verification not implemented)

Time = 9.09 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.45

$$\int \frac{\sqrt{a^2 + 2abx + b^2x^2}}{(d + ex)^5} dx = -\frac{\sqrt{(a + bx)^2 (3ae + bd + 4bex)}}{12e^2 (a + bx) (d + ex)^4}$$

input

```
int(((a + b*x)^2)^(1/2)/(d + e*x)^5,x)
```

output

```
-(((a + b*x)^2)^(1/2)*(3*a*e + b*d + 4*b*e*x))/(12*e^2*(a + b*x)*(d + e*x)
^4)
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.64

$$\int \frac{\sqrt{a^2 + 2abx + b^2x^2}}{(d + ex)^5} dx = \frac{-4bex - 3ae - bd}{12e^2 (e^4x^4 + 4de^3x^3 + 6d^2e^2x^2 + 4d^3ex + d^4)}$$

input `int(((b*x+a)^2)^(1/2)/(e*x+d)^5,x)`

output `(- 3*a*e - b*d - 4*b*e*x)/(12*e**2*(d**4 + 4*d**3*e*x + 6*d**2*e**2*x**2 + 4*d*e**3*x**3 + e**4*x**4))`

3.275 $\int \frac{\sqrt{a^2+2abx+b^2x^2}}{(d+ex)^6} dx$

| | |
|---|------|
| Optimal result | 2079 |
| Mathematica [A] (verified) | 2079 |
| Rubi [A] (verified) | 2080 |
| Maple [C] (warning: unable to verify) | 2081 |
| Fricas [A] (verification not implemented) | 2082 |
| Sympy [F] | 2082 |
| Maxima [F(-2)] | 2083 |
| Giac [A] (verification not implemented) | 2083 |
| Mupad [B] (verification not implemented) | 2084 |
| Reduce [B] (verification not implemented) | 2084 |

Optimal result

Integrand size = 28, antiderivative size = 92

$$\int \frac{\sqrt{a^2 + 2abx + b^2x^2}}{(d + ex)^6} dx = \frac{(bd - ae)\sqrt{a^2 + 2abx + b^2x^2}}{5e^2(a + bx)(d + ex)^5} - \frac{b\sqrt{a^2 + 2abx + b^2x^2}}{4e^2(a + bx)(d + ex)^4}$$

output

$$\frac{1}{5}*(-a*e+b*d)*((b*x+a)^2)^{(1/2)}/e^2/(b*x+a)/(e*x+d)^5-1/4*b*((b*x+a)^2)^{(1/2)}/e^2/(b*x+a)/(e*x+d)^4$$

Mathematica [A] (verified)

Time = 1.02 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.49

$$\int \frac{\sqrt{a^2 + 2abx + b^2x^2}}{(d + ex)^6} dx = -\frac{\sqrt{(a + bx)^2(4ae + b(d + 5ex))}}{20e^2(a + bx)(d + ex)^5}$$

input

```
Integrate[Sqrt[a^2 + 2*a*b*x + b^2*x^2]/(d + e*x)^6,x]
```

output

$$-1/20*(Sqrt[(a + b*x)^2]*(4*a*e + b*(d + 5*e*x)))/(e^2*(a + b*x)*(d + e*x)^5)$$

Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.72, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {1102, 27, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\begin{aligned}
 & \int \frac{\sqrt{a^2 + 2abx + b^2x^2}}{(d + ex)^6} dx \\
 & \quad \downarrow \text{1102} \\
 & \frac{\sqrt{a^2 + 2abx + b^2x^2} \int \frac{b(a+bx)}{(d+ex)^6} dx}{b(a + bx)} \\
 & \quad \downarrow \text{27} \\
 & \frac{\sqrt{a^2 + 2abx + b^2x^2} \int \frac{a+bx}{(d+ex)^6} dx}{a + bx} \\
 & \quad \downarrow \text{53} \\
 & \frac{\sqrt{a^2 + 2abx + b^2x^2} \int \left(\frac{b}{e(d+ex)^5} + \frac{ae-bd}{e(d+ex)^6} \right) dx}{a + bx} \\
 & \quad \downarrow \text{2009} \\
 & \frac{\sqrt{a^2 + 2abx + b^2x^2} \left(\frac{bd-ae}{5e^2(d+ex)^5} - \frac{b}{4e^2(d+ex)^4} \right)}{a + bx}
 \end{aligned}$$

input `Int[Sqrt[a^2 + 2*a*b*x + b^2*x^2]/(d + e*x)^6,x]`

output `(Sqrt[a^2 + 2*a*b*x + b^2*x^2]*((b*d - a*e)/(5*e^2*(d + e*x)^5) - b/(4*e^2*(d + e*x)^4)))/(a + b*x)`

Definitions of rubi rules used

rule 27 $\text{Int}[(a_*)(Fx_), x_Symbol] \rightarrow \text{Simp}[a \text{ Int}[Fx, x], x] /; \text{FreeQ}[a, x] \ \&\& \ !\text{MatchQ}[Fx, (b_*)(Gx_)] /; \text{FreeQ}[b, x]$

rule 53 $\text{Int}[((a_.) + (b_.)(x_))^{(m_.)}((c_.) + (d_.)(x_))^{(n_.)}, x_Symbol] \rightarrow \text{Int}[\text{ExpandIntegrand}[(a + b*x)^m*(c + d*x)^n, x], x] /; \text{FreeQ}[\{a, b, c, d, n\}, x] \ \&\& \ \text{IGtQ}[m, 0] \ \&\& \ (\ !\text{IntegerQ}[n] \ || \ (\text{EqQ}[c, 0] \ \&\& \ \text{LeQ}[7*m + 4*n + 4, 0]) \ || \ \text{LtQ}[9*m + 5*(n + 1), 0] \ || \ \text{GtQ}[m + n + 2, 0])$

rule 1102 $\text{Int}[((d_.) + (e_.)(x_))^{(m_.)}((a_) + (b_.)(x_) + (c_.)(x_)^2)^{(p_)}, x_Symbol] \rightarrow \text{Simp}[(a + b*x + c*x^2)^{\text{FracPart}[p]} / (c^{\text{IntPart}[p]}*(b/2 + c*x)^{(2*\text{FracPart}[p])}) \text{ Int}[(d + e*x)^m*(b/2 + c*x)^{(2*p)}, x], x] /; \text{FreeQ}[\{a, b, c, d, e, m, p\}, x] \ \&\& \ \text{EqQ}[b^2 - 4*a*c, 0]$

rule 2009 $\text{Int}[u_, x_Symbol] \rightarrow \text{Simp}[\text{IntSum}[u, x], x] /; \text{SumQ}[u]$

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 2.

Time = 1.64 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.35

| method | result | size |
|---------|---|------|
| default | $-\frac{\text{csgn}(bx+a)(5bex+4ae+bd)}{20e^2(ex+d)^5}$ | 32 |
| gospers | $-\frac{(5bex+4ae+bd)\sqrt{(bx+a)^2}}{20e^2(ex+d)^5(bx+a)}$ | 42 |
| orering | $-\frac{(5bex+4ae+bd)\sqrt{(bx+a)^2}}{20e^2(ex+d)^5(bx+a)}$ | 42 |
| risch | $\frac{\sqrt{(bx+a)^2} \left(-\frac{bx}{4e} - \frac{4ae+bd}{20e^2} \right)}{(bx+a)(ex+d)^5}$ | 46 |

input $\text{int}(((b*x+a)^2)^{(1/2)} / (e*x+d)^6, x, \text{method}=_RETURNVERBOSE)$

output $-1/20*\text{csgn}(b*x+a)*(5*b*e*x+4*a*e+b*d)/e^2/(e*x+d)^5$

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.78

$$\int \frac{\sqrt{a^2 + 2abx + b^2x^2}}{(d + ex)^6} dx$$

$$= -\frac{5bex + bd + 4ae}{20(e^7x^5 + 5de^6x^4 + 10d^2e^5x^3 + 10d^3e^4x^2 + 5d^4e^3x + d^5e^2)}$$

input `integrate(((b*x+a)^2)^(1/2)/(e*x+d)^6,x, algorithm="fricas")`

output $-1/20*(5*b*e*x + b*d + 4*a*e)/(e^7*x^5 + 5*d*e^6*x^4 + 10*d^2*e^5*x^3 + 10*d^3*e^4*x^2 + 5*d^4*e^3*x + d^5*e^2)$

Sympy [F]

$$\int \frac{\sqrt{a^2 + 2abx + b^2x^2}}{(d + ex)^6} dx = \int \frac{\sqrt{(a + bx)^2}}{(d + ex)^6} dx$$

input `integrate(((b*x+a)**2)**(1/2)/(e*x+d)**6,x)`

output `Integral(sqrt((a + b*x)**2)/(d + e*x)**6, x)`

Maxima [F(-2)]

Exception generated.

$$\int \frac{\sqrt{a^2 + 2abx + b^2x^2}}{(d + ex)^6} dx = \text{Exception raised: ValueError}$$

input `integrate(((b*x+a)^2)^(1/2)/(e*x+d)^6,x, algorithm="maxima")`

output `Exception raised: ValueError >> Computation failed since Maxima requested additional constraints; using the 'assume' command before evaluation *may* help (example of legal syntax is 'assume(a*e-b*d>0)', see `assume?` for more detail`

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 111, normalized size of antiderivative = 1.21

$$\int \frac{\sqrt{a^2 + 2abx + b^2x^2}}{(d + ex)^6} dx = \frac{b^5 \operatorname{sgn}(bx + a)}{20(b^4d^4e^2 - 4ab^3d^3e^3 + 6a^2b^2d^2e^4 - 4a^3bde^5 + a^4e^6)} - \frac{5bex\operatorname{sgn}(bx + a) + bd\operatorname{sgn}(bx + a) + 4ae\operatorname{sgn}(bx + a)}{20(ex + d)^5e^2}$$

input `integrate(((b*x+a)^2)^(1/2)/(e*x+d)^6,x, algorithm="giac")`

output `1/20*b^5*sgn(b*x + a)/(b^4*d^4*e^2 - 4*a*b^3*d^3*e^3 + 6*a^2*b^2*d^2*e^4 - 4*a^3*b*d*e^5 + a^4*e^6) - 1/20*(5*b*e*x*sgn(b*x + a) + b*d*sgn(b*x + a) + 4*a*e*sgn(b*x + a))/((e*x + d)^5*e^2)`

Mupad [B] (verification not implemented)

Time = 9.44 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.45

$$\int \frac{\sqrt{a^2 + 2abx + b^2x^2}}{(d + ex)^6} dx = -\frac{\sqrt{(a + bx)^2} (4ae + bd + 5bex)}{20e^2 (a + bx) (d + ex)^5}$$

input `int(((a + b*x)^2)^(1/2)/(d + e*x)^6,x)`output `-(((a + b*x)^2)^(1/2)*(4*a*e + b*d + 5*b*e*x))/(20*e^2*(a + b*x)*(d + e*x)^5)`**Reduce [B] (verification not implemented)**

Time = 0.22 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.76

$$\int \frac{\sqrt{a^2 + 2abx + b^2x^2}}{(d + ex)^6} dx = \frac{-5bex - 4ae - bd}{20e^2 (e^5x^5 + 5de^4x^4 + 10d^2e^3x^3 + 10d^3e^2x^2 + 5d^4ex + d^5)}$$

input `int(((b*x+a)^2)^(1/2)/(e*x+d)^6,x)`output `(- 4*a*e - b*d - 5*b*e*x)/(20*e^2*(d**5 + 5*d**4*e*x + 10*d**3*e**2*x**2 + 10*d**2*e**3*x**3 + 5*d*e**4*x**4 + e**5*x**5))`

3.276 $\int (d + ex)^5 (a^2 + 2abx + b^2x^2)^{3/2} dx$

| | |
|---|------|
| Optimal result | 2085 |
| Mathematica [A] (verified) | 2086 |
| Rubi [A] (verified) | 2086 |
| Maple [B] (verified) | 2088 |
| Fricas [A] (verification not implemented) | 2089 |
| Sympy [B] (verification not implemented) | 2089 |
| Maxima [B] (verification not implemented) | 2090 |
| Giac [B] (verification not implemented) | 2091 |
| Mupad [F(-1)] | 2093 |
| Reduce [B] (verification not implemented) | 2093 |

Optimal result

Integrand size = 28, antiderivative size = 200

$$\int (d + ex)^5 (a^2 + 2abx + b^2x^2)^{3/2} dx = -\frac{(bd - ae)^3(d + ex)^6\sqrt{a^2 + 2abx + b^2x^2}}{6e^4(a + bx)} + \frac{3b(bd - ae)^2(d + ex)^7\sqrt{a^2 + 2abx + b^2x^2}}{7e^4(a + bx)} - \frac{3b^2(bd - ae)(d + ex)^8\sqrt{a^2 + 2abx + b^2x^2}}{8e^4(a + bx)} + \frac{b^3(d + ex)^9\sqrt{a^2 + 2abx + b^2x^2}}{9e^4(a + bx)}$$

output

```
-1/6*(-a*e+b*d)^3*(e*x+d)^6*((b*x+a)^2)^(1/2)/e^4/(b*x+a)+3/7*b*(-a*e+b*d)^2*(e*x+d)^7*((b*x+a)^2)^(1/2)/e^4/(b*x+a)-3/8*b^2*(-a*e+b*d)*(e*x+d)^8*((b*x+a)^2)^(1/2)/e^4/(b*x+a)+1/9*b^3*(e*x+d)^9*((b*x+a)^2)^(1/2)/e^4/(b*x+a)
```

Mathematica [A] (verified)

Time = 1.07 (sec) , antiderivative size = 259, normalized size of antiderivative = 1.30

$$\int (d + ex)^5 (a^2 + 2abx + b^2x^2)^{3/2} dx = \frac{x\sqrt{(a + bx)^2}(84a^3(6d^5 + 15d^4ex + 20d^3e^2x^2 + 15d^2e^3x^3 + 6de^4x^4 + e^5x^5) + 36a^2bx(21d^5 + 70d^4ex + 105d^3e^2x^2 + 84d^2e^3x^3 + 35de^4x^4 + 6e^5x^5) + 9ab^2x^2(56d^5 + 210d^4ex + 336d^3e^2x^2 + 280d^2e^3x^3 + 120de^4x^4 + 21e^5x^5) + b^3x^3(126d^5 + 504d^4ex + 840d^3e^2x^2 + 720d^2e^3x^3 + 315de^4x^4 + 56e^5x^5))}{504(a + bx)}$$

input

```
Integrate[(d + e*x)^5*(a^2 + 2*a*b*x + b^2*x^2)^(3/2),x]
```

output

```
(x*sqrt[(a + b*x)^2]*(84*a^3*(6*d^5 + 15*d^4*e*x + 20*d^3*e^2*x^2 + 15*d^2*e^3*x^3 + 6*d*e^4*x^4 + e^5*x^5) + 36*a^2*b*x*(21*d^5 + 70*d^4*e*x + 105*d^3*e^2*x^2 + 84*d^2*e^3*x^3 + 35*d*e^4*x^4 + 6*e^5*x^5) + 9*a*b^2*x^2*(56*d^5 + 210*d^4*e*x + 336*d^3*e^2*x^2 + 280*d^2*e^3*x^3 + 120*d*e^4*x^4 + 21*e^5*x^5) + b^3*x^3*(126*d^5 + 504*d^4*e*x + 840*d^3*e^2*x^2 + 720*d^2*e^3*x^3 + 315*d*e^4*x^4 + 56*e^5*x^5)))/(504*(a + b*x))
```

Rubi [A] (verified)

Time = 0.61 (sec) , antiderivative size = 120, normalized size of antiderivative = 0.60, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {1102, 27, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a^2 + 2abx + b^2x^2)^{3/2} (d + ex)^5 dx$$

$$\downarrow 1102$$

$$\frac{\sqrt{a^2 + 2abx + b^2x^2} \int b^3(a + bx)^3(d + ex)^5 dx}{b^3(a + bx)}$$

$$\downarrow 27$$

$$\frac{\sqrt{a^2 + 2abx + b^2x^2} \int (a + bx)^3(d + ex)^5 dx}{a + bx}$$

$$\begin{array}{c} \downarrow 49 \\ \frac{\sqrt{a^2 + 2abx + b^2x^2} \int \left(\frac{b^3(d+ex)^8}{e^3} - \frac{3b^2(bd-ae)(d+ex)^7}{e^3} + \frac{3b(bd-ae)^2(d+ex)^6}{e^3} + \frac{(ae-bd)^3(d+ex)^5}{e^3} \right) dx}{a + bx} \\ \downarrow 2009 \\ \frac{\sqrt{a^2 + 2abx + b^2x^2} \left(-\frac{3b^2(d+ex)^8(bd-ae)}{8e^4} + \frac{3b(d+ex)^7(bd-ae)^2}{7e^4} - \frac{(d+ex)^6(bd-ae)^3}{6e^4} + \frac{b^3(d+ex)^9}{9e^4} \right)}{a + bx} \end{array}$$

input `Int[(d + e*x)^5*(a^2 + 2*a*b*x + b^2*x^2)^(3/2),x]`

output `(Sqrt[a^2 + 2*a*b*x + b^2*x^2]*(-1/6*((b*d - a*e)^3*(d + e*x)^6)/e^4 + (3*b*(b*d - a*e)^2*(d + e*x)^7)/(7*e^4) - (3*b^2*(b*d - a*e)*(d + e*x)^8)/(8*e^4) + (b^3*(d + e*x)^9)/(9*e^4)))/(a + b*x)`

Defintions of rubi rules used

rule 27 `Int[(a_)*(Fx_), x_Symbol] := Simp[a Int[Fx, x], x] /; FreeQ[a, x] && !MatchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]`

rule 49 `Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] := Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] && IGtQ[m, 0] && IGtQ[m + n + 2, 0]`

rule 1102 `Int[((d_.) + (e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] := Simp[(a + b*x + c*x^2)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x)^(2*FracPart[p])) Int[(d + e*x)^m*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && EqQ[b^2 - 4*a*c, 0]`

rule 2009 `Int[u_, x_Symbol] := Simp[IntSum[u, x], x] /; SumQ[u]`

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 321 vs. 2(148) = 296.

Time = 1.45 (sec) , antiderivative size = 322, normalized size of antiderivative = 1.61

| method | result |
|---------|--|
| gospers | $x(56e^5b^3x^8+189x^7e^5ab^2+315x^7de^4b^3+216x^6e^5a^2b+1080x^6de^4ab^2+720x^6b^3d^2e^3+84x^5a^3e^5+1260x^5de^4a^2b+2520x^5d^2e^3ab^2$ |
| default | $x(56e^5b^3x^8+189x^7e^5ab^2+315x^7de^4b^3+216x^6e^5a^2b+1080x^6de^4ab^2+720x^6b^3d^2e^3+84x^5a^3e^5+1260x^5de^4a^2b+2520x^5d^2e^3ab^2$ |
| orering | $x(56e^5b^3x^8+189x^7e^5ab^2+315x^7de^4b^3+216x^6e^5a^2b+1080x^6de^4ab^2+720x^6b^3d^2e^3+84x^5a^3e^5+1260x^5de^4a^2b+2520x^5d^2e^3ab^2$ |
| risch | $\frac{\sqrt{(bx+a)^2}e^5b^3x^9}{9bx+9a} + \frac{\sqrt{(bx+a)^2}(3e^5ab^2+5de^4b^3)x^8}{8bx+8a} + \frac{\sqrt{(bx+a)^2}(3e^5a^2b+15de^4ab^2+10b^3d^2e^3)x^7}{7bx+7a} + \frac{\sqrt{(bx+a)^2}(a^3e^5+1$ |

```
input int((e*x+d)^5*(b^2*x^2+2*a*b*x+a^2)^(3/2),x,method=_RETURNVERBOSE)
```

```
output 1/504*x*(56*b^3*e^5*x^8+189*a*b^2*e^5*x^7+315*b^3*d*e^4*x^7+216*a^2*b*e^5*x^6+1080*a*b^2*d*e^4*x^6+720*b^3*d^2*e^3*x^6+84*a^3*e^5*x^5+1260*a^2*b*d*e^4*x^5+2520*a*b^2*d^2*e^3*x^5+840*b^3*d^3*e^2*x^5+504*a^3*d*e^4*x^4+3024*a^2*b*d^2*e^3*x^4+3024*a*b^2*d^3*e^2*x^4+504*b^3*d^4*e*x^4+1260*a^3*d^2*e^3*x^3+3780*a^2*b*d^3*e^2*x^3+1890*a*b^2*d^4*e*x^3+126*b^3*d^5*x^3+1680*a^3*d^3*e^2*x^2+2520*a^2*b*d^4*e*x^2+504*a*b^2*d^5*x^2+1260*a^3*d^4*e*x+756*a^2*b*d^5*x+504*a^3*d^5)*((b*x+a)^2)^(3/2)/(b*x+a)^3
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 277, normalized size of antiderivative = 1.38

$$\begin{aligned} \int (d+ex)^5 (a^2+2abx+b^2x^2)^{3/2} dx &= \frac{1}{9} b^3 e^5 x^9 + a^3 d^5 x \\ &+ \frac{1}{8} (5b^3 d e^4 + 3ab^2 e^5) x^8 + \frac{1}{7} (10b^3 d^2 e^3 + 15ab^2 d e^4 + 3a^2 b e^5) x^7 \\ &+ \frac{1}{6} (10b^3 d^3 e^2 + 30ab^2 d^2 e^3 + 15a^2 b d e^4 + a^3 e^5) x^6 \\ &+ (b^3 d^4 e + 6ab^2 d^3 e^2 + 6a^2 b d^2 e^3 + a^3 d e^4) x^5 \\ &+ \frac{1}{4} (b^3 d^5 + 15ab^2 d^4 e + 30a^2 b d^3 e^2 + 10a^3 d^2 e^3) x^4 \\ &+ \frac{1}{3} (3ab^2 d^5 + 15a^2 b d^4 e + 10a^3 d^3 e^2) x^3 + \frac{1}{2} (3a^2 b d^5 + 5a^3 d^4 e) x^2 \end{aligned}$$

input `integrate((e*x+d)^5*(b^2*x^2+2*a*b*x+a^2)^(3/2),x, algorithm="fricas")`

output `1/9*b^3*e^5*x^9 + a^3*d^5*x + 1/8*(5*b^3*d*e^4 + 3*a*b^2*e^5)*x^8 + 1/7*(10*b^3*d^2*e^3 + 15*a*b^2*d*e^4 + 3*a^2*b*e^5)*x^7 + 1/6*(10*b^3*d^3*e^2 + 30*a*b^2*d^2*e^3 + 15*a^2*b*d*e^4 + a^3*e^5)*x^6 + (b^3*d^4*e + 6*a*b^2*d^3*e^2 + 6*a^2*b*d^2*e^3 + a^3*d*e^4)*x^5 + 1/4*(b^3*d^5 + 15*a*b^2*d^4*e + 30*a^2*b*d^3*e^2 + 10*a^3*d^2*e^3)*x^4 + 1/3*(3*a*b^2*d^5 + 15*a^2*b*d^4*e + 10*a^3*d^3*e^2)*x^3 + 1/2*(3*a^2*b*d^5 + 5*a^3*d^4*e)*x^2`

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 10178 vs. 2(143) = 286.

Time = 1.08 (sec) , antiderivative size = 10178, normalized size of antiderivative = 50.89

$$\int (d+ex)^5 (a^2+2abx+b^2x^2)^{3/2} dx = \text{Too large to display}$$

input `integrate((e*x+d)**5*(b**2*x**2+2*a*b*x+a**2)**(3/2),x)`

output

```
Piecewise((sqrt(a**2 + 2*a*b*x + b**2*x**2)*(b**2*e**5*x**8/9 + x**7*(19*a
*b**3*e**5/9 + 5*b**4*d*e**4)/(8*b**2) + x**6*(46*a**2*b**2*e**5/9 + 20*a*
b**3*d*e**4 - 15*a*(19*a*b**3*e**5/9 + 5*b**4*d*e**4)/(8*b) + 10*b**4*d**2
*e**3)/(7*b**2) + x**5*(4*a**3*b*e**5 + 30*a**2*b**2*d*e**4 - 7*a**2*(19*a
*b**3*e**5/9 + 5*b**4*d*e**4)/(8*b**2) + 40*a*b**3*d**2*e**3 - 13*a*(46*a*
*2*b**2*e**5/9 + 20*a*b**3*d*e**4 - 15*a*(19*a*b**3*e**5/9 + 5*b**4*d*e**4
)/(8*b) + 10*b**4*d**2*e**3)/(7*b) + 10*b**4*d**3*e**2)/(6*b**2) + x**4*(a
**4*e**5 + 20*a**3*b*d*e**4 + 60*a**2*b**2*d**2*e**3 - 6*a**2*(46*a**2*b**
2*e**5/9 + 20*a*b**3*d*e**4 - 15*a*(19*a*b**3*e**5/9 + 5*b**4*d*e**4)/(8*b
) + 10*b**4*d**2*e**3)/(7*b**2) + 40*a*b**3*d**3*e**2 - 11*a*(4*a**3*b*e**
5 + 30*a**2*b**2*d*e**4 - 7*a**2*(19*a*b**3*e**5/9 + 5*b**4*d*e**4)/(8*b**
2) + 40*a*b**3*d**2*e**3 - 13*a*(46*a**2*b**2*e**5/9 + 20*a*b**3*d*e**4 -
15*a*(19*a*b**3*e**5/9 + 5*b**4*d*e**4)/(8*b) + 10*b**4*d**2*e**3)/(7*b) +
10*b**4*d**3*e**2)/(6*b) + 5*b**4*d**4*e)/(5*b**2) + x**3*(5*a**4*d*e**4
+ 40*a**3*b*d**2*e**3 + 60*a**2*b**2*d**3*e**2 - 5*a**2*(4*a**3*b*e**5 + 3
0*a**2*b**2*d*e**4 - 7*a**2*(19*a*b**3*e**5/9 + 5*b**4*d*e**4)/(8*b**2) +
40*a*b**3*d**2*e**3 - 13*a*(46*a**2*b**2*e**5/9 + 20*a*b**3*d*e**4 - 15*a*
(19*a*b**3*e**5/9 + 5*b**4*d*e**4)/(8*b) + 10*b**4*d**2*e**3)/(7*b) + 10*b
**4*d**3*e**2)/(6*b**2) + 20*a*b**3*d**4*e - 9*a*(a**4*e**5 + 20*a**3*b*d*
e**4 + 60*a**2*b**2*d**2*e**3 - 6*a**2*(46*a**2*b**2*e**5/9 + 20*a*b**3...
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 814 vs. $2(148) = 296$.

Time = 0.05 (sec) , antiderivative size = 814, normalized size of antiderivative = 4.07

$$\int (d + ex)^5 (a^2 + 2abx + b^2x^2)^{3/2} dx = \text{Too large to display}$$

input

```
integrate((e*x+d)^5*(b^2*x^2+2*a*b*x+a^2)^(3/2),x, algorithm="maxima")
```

output

$$\begin{aligned}
& 1/9*(b^2*x^2 + 2*a*b*x + a^2)^{(5/2)}*e^5*x^4/b^2 + 5/8*(b^2*x^2 + 2*a*b*x + \\
& a^2)^{(5/2)}*d*e^4*x^3/b^2 - 13/72*(b^2*x^2 + 2*a*b*x + a^2)^{(5/2)}*a*e^5*x^3/b^3 + 1/4*(b^2*x^2 + 2*a*b*x + a^2)^{(3/2)}*d^5*x \\
& - 5/4*(b^2*x^2 + 2*a*b*x + a^2)^{(3/2)}*a*d^4*e*x/b + 5/2*(b^2*x^2 + 2*a*b*x + a^2)^{(3/2)}*a^2*d^3*e^2*x/b^2 - 5/2*(b^2*x^2 + 2*a*b*x + a^2)^{(3/2)}*a^3*d^2*e^3*x/b^3 + 5/4*(b^2 \\
& *x^2 + 2*a*b*x + a^2)^{(3/2)}*a^4*d*e^4*x/b^4 - 1/4*(b^2*x^2 + 2*a*b*x + a^2)^{(3/2)}*a^5*e^5*x/b^5 + 10/7*(b^2*x^2 + 2*a*b*x + a^2)^{(5/2)}*d^2*e^3*x^2/b^2 - 55/56*(b^2*x^2 + 2*a*b*x + a^2)^{(5/2)}*a*d*e^4*x^2/b^3 + 37/168*(b^2*x^2 + 2*a*b*x + a^2)^{(5/2)}*a^2*e^5*x^2/b^4 + 1/4*(b^2*x^2 + 2*a*b*x + a^2)^{(3/2)}*a*d^5/b - 5/4*(b^2*x^2 + 2*a*b*x + a^2)^{(3/2)}*a^2*d^4*e/b^2 + 5/2*(b^2*x^2 + 2*a*b*x + a^2)^{(3/2)}*a^3*d^3*e^2/b^3 - 5/2*(b^2*x^2 + 2*a*b*x + a^2)^{(3/2)}*a^4*d^2*e^3/b^4 + 5/4*(b^2*x^2 + 2*a*b*x + a^2)^{(3/2)}*a^5*d*e^4/b^5 - 1/4*(b^2*x^2 + 2*a*b*x + a^2)^{(3/2)}*a^6*e^5/b^6 + 5/3*(b^2*x^2 + 2*a*b*x + a^2)^{(5/2)}*d^3*e^2*x/b^2 - 15/7*(b^2*x^2 + 2*a*b*x + a^2)^{(5/2)}*a*d^2*e^3*x/b^3 + 65/56*(b^2*x^2 + 2*a*b*x + a^2)^{(5/2)}*a^2*d*e^4*x/b^4 - 121/504*(b^2*x^2 + 2*a*b*x + a^2)^{(5/2)}*a^3*e^5*x/b^5 + (b^2*x^2 + 2*a*b*x + a^2)^{(5/2)}*d^4*e/b^2 - 7/3*(b^2*x^2 + 2*a*b*x + a^2)^{(5/2)}*a*d^3*e^2/b^3 + 17/7*(b^2*x^2 + 2*a*b*x + a^2)^{(5/2)}*a^2*d^2*e^3/b^4 - 69/56*(b^2*x^2 + 2*a*b*x + a^2)^{(5/2)}*a^3*d*e^4/b^5 + 125/504*(b^2*x^2 + 2*a*b*x + a^2)^{(5/2)}*a^4*e^5/b^6
\end{aligned}$$

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 528 vs. $2(148) = 296$.

Time = 0.15 (sec) , antiderivative size = 528, normalized size of antiderivative = 2.64

$$\int (d + ex)^5 (a^2 + 2abx + b^2x^2)^{3/2} dx = \frac{1}{9} b^3 e^5 x^9 \operatorname{sgn}(bx + a) + \frac{5}{8} b^3 d e^4 x^8 \operatorname{sgn}(bx + a) + \frac{3}{8} a b^2 e^5 x^8 \operatorname{sgn}(bx + a) + \frac{10}{7} b^3 d^2 e^3 x^7 \operatorname{sgn}(bx + a) + \frac{15}{7} a b^2 d e^4 x^7 \operatorname{sgn}(bx + a) + \frac{3}{7} a^2 b e^5 x^7 \operatorname{sgn}(bx + a) + \frac{5}{3} b^3 d^3 e^2 x^6 \operatorname{sgn}(bx + a) + 5 a b^2 d^2 e^3 x^6 \operatorname{sgn}(bx + a) + \frac{5}{2} a^2 b d e^4 x^6 \operatorname{sgn}(bx + a) + \frac{1}{6} a^3 e^5 x^6 \operatorname{sgn}(bx + a) + b^3 d^4 e x^5 \operatorname{sgn}(bx + a) + 6 a b^2 d^3 e^2 x^5 \operatorname{sgn}(bx + a) + 6 a^2 b d^2 e^3 x^5 \operatorname{sgn}(bx + a) + a^3 d e^4 x^5 \operatorname{sgn}(bx + a) + \frac{1}{4} b^3 d^5 x^4 \operatorname{sgn}(bx + a) + \frac{15}{4} a b^2 d^4 e x^4 \operatorname{sgn}(bx + a) + \frac{15}{2} a^2 b d^3 e^2 x^4 \operatorname{sgn}(bx + a) + \frac{5}{2} a^3 d^2 e^3 x^4 \operatorname{sgn}(bx + a) + a b^2 d^5 x^3 \operatorname{sgn}(bx + a) + 5 a^2 b d^4 e x^3 \operatorname{sgn}(bx + a) + \frac{10}{3} a^3 d^3 e^2 x^3 \operatorname{sgn}(bx + a) + \frac{3}{2} a^2 b d^5 x^2 \operatorname{sgn}(bx + a) + \frac{5}{2} a^3 d^4 e x^2 \operatorname{sgn}(bx + a) + a^3 d^5 x \operatorname{sgn}(bx + a) + \frac{(126 a^4 b^5 d^5 - 126 a^5 b^4 d^4 e + 84 a^6 b^3 d^3 e^2 - 36 a^7 b^2 d^2 e^3 + 9 a^8 b d e^4 - a^9 e^5) \operatorname{sgn}(bx + a)}{504 b^6}$$

input

```
integrate((e*x+d)^5*(b^2*x^2+2*a*b*x+a^2)^(3/2),x, algorithm="giac")
```

output

```
1/9*b^3*e^5*x^9*sgn(b*x + a) + 5/8*b^3*d*e^4*x^8*sgn(b*x + a) + 3/8*a*b^2*
e^5*x^8*sgn(b*x + a) + 10/7*b^3*d^2*e^3*x^7*sgn(b*x + a) + 15/7*a*b^2*d*e^
4*x^7*sgn(b*x + a) + 3/7*a^2*b*e^5*x^7*sgn(b*x + a) + 5/3*b^3*d^3*e^2*x^6*
sgn(b*x + a) + 5*a*b^2*d^2*e^3*x^6*sgn(b*x + a) + 5/2*a^2*b*d*e^4*x^6*sgn(
b*x + a) + 1/6*a^3*e^5*x^6*sgn(b*x + a) + b^3*d^4*e*x^5*sgn(b*x + a) + 6*a
*b^2*d^3*e^2*x^5*sgn(b*x + a) + 6*a^2*b*d^2*e^3*x^5*sgn(b*x + a) + a^3*d*e
^4*x^5*sgn(b*x + a) + 1/4*b^3*d^5*x^4*sgn(b*x + a) + 15/4*a*b^2*d^4*e*x^4*
sgn(b*x + a) + 15/2*a^2*b*d^3*e^2*x^4*sgn(b*x + a) + 5/2*a^3*d^2*e^3*x^4*s
gn(b*x + a) + a*b^2*d^5*x^3*sgn(b*x + a) + 5*a^2*b*d^4*e*x^3*sgn(b*x + a)
+ 10/3*a^3*d^3*e^2*x^3*sgn(b*x + a) + 3/2*a^2*b*d^5*x^2*sgn(b*x + a) + 5/2
*a^3*d^4*e*x^2*sgn(b*x + a) + a^3*d^5*x*sgn(b*x + a) + 1/504*(126*a^4*b^5*
d^5 - 126*a^5*b^4*d^4*e + 84*a^6*b^3*d^3*e^2 - 36*a^7*b^2*d^2*e^3 + 9*a^8*
b*d*e^4 - a^9*e^5)*sgn(b*x + a)/b^6
```


3.277 $\int (d + ex)^4 (a^2 + 2abx + b^2x^2)^{3/2} dx$

| | |
|---|------|
| Optimal result | 2094 |
| Mathematica [A] (verified) | 2095 |
| Rubi [A] (verified) | 2095 |
| Maple [A] (verified) | 2097 |
| Fricas [A] (verification not implemented) | 2097 |
| Sympy [B] (verification not implemented) | 2098 |
| Maxima [B] (verification not implemented) | 2099 |
| Giac [B] (verification not implemented) | 2101 |
| Mupad [F(-1)] | 2102 |
| Reduce [B] (verification not implemented) | 2102 |

Optimal result

Integrand size = 28, antiderivative size = 200

$$\int (d + ex)^4 (a^2 + 2abx + b^2x^2)^{3/2} dx = -\frac{(bd - ae)^3(d + ex)^5\sqrt{a^2 + 2abx + b^2x^2}}{5e^4(a + bx)} + \frac{b(bd - ae)^2(d + ex)^6\sqrt{a^2 + 2abx + b^2x^2}}{2e^4(a + bx)} - \frac{3b^2(bd - ae)(d + ex)^7\sqrt{a^2 + 2abx + b^2x^2}}{7e^4(a + bx)} + \frac{b^3(d + ex)^8\sqrt{a^2 + 2abx + b^2x^2}}{8e^4(a + bx)}$$

output

```
-1/5*(-a*e+b*d)^3*(e*x+d)^5*((b*x+a)^2)^(1/2)/e^4/(b*x+a)+1/2*b*(-a*e+b*d)^2*(e*x+d)^6*((b*x+a)^2)^(1/2)/e^4/(b*x+a)-3/7*b^2*(-a*e+b*d)*(e*x+d)^7*((b*x+a)^2)^(1/2)/e^4/(b*x+a)+1/8*b^3*(e*x+d)^8*((b*x+a)^2)^(1/2)/e^4/(b*x+a)
```

Mathematica [A] (verified)

Time = 1.06 (sec) , antiderivative size = 215, normalized size of antiderivative = 1.08

$$\int (d + ex)^4 (a^2 + 2abx + b^2x^2)^{3/2} dx = \frac{x\sqrt{(a + bx)^2}(56a^3(5d^4 + 10d^3ex + 10d^2e^2x^2 + 5de^3x^3 + e^4x^4) + 28a^2bx(15d^4 + 40d^3ex +$$

input

```
Integrate[(d + e*x)^4*(a^2 + 2*a*b*x + b^2*x^2)^(3/2),x]
```

output

```
(x*sqrt[(a + b*x)^2]*(56*a^3*(5*d^4 + 10*d^3*e*x + 10*d^2*e^2*x^2 + 5*d*e^3*x^3 + e^4*x^4) + 28*a^2*b*x*(15*d^4 + 40*d^3*e*x + 45*d^2*e^2*x^2 + 24*d*e^3*x^3 + 5*e^4*x^4) + 8*a*b^2*x^2*(35*d^4 + 105*d^3*e*x + 126*d^2*e^2*x^2 + 70*d*e^3*x^3 + 15*e^4*x^4) + b^3*x^3*(70*d^4 + 224*d^3*e*x + 280*d^2*e^2*x^2 + 160*d*e^3*x^3 + 35*e^4*x^4)))/(280*(a + b*x))
```

Rubi [A] (verified)

Time = 0.56 (sec) , antiderivative size = 120, normalized size of antiderivative = 0.60, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {1102, 27, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a^2 + 2abx + b^2x^2)^{3/2} (d + ex)^4 dx$$

$$\downarrow 1102$$

$$\frac{\sqrt{a^2 + 2abx + b^2x^2} \int b^3 (a + bx)^3 (d + ex)^4 dx}{b^3 (a + bx)}$$

$$\downarrow 27$$

$$\frac{\sqrt{a^2 + 2abx + b^2x^2} \int (a + bx)^3 (d + ex)^4 dx}{a + bx}$$

$$\downarrow 49$$

$$\frac{\sqrt{a^2 + 2abx + b^2x^2} \int \left(\frac{b^3(d+ex)^7}{e^3} - \frac{3b^2(bd-ae)(d+ex)^6}{e^3} + \frac{3b(bd-ae)^2(d+ex)^5}{e^3} + \frac{(ae-bd)^3(d+ex)^4}{e^3} \right) dx}{a + bx}$$

↓ 2009

$$\frac{\sqrt{a^2 + 2abx + b^2x^2} \left(-\frac{3b^2(d+ex)^7(bd-ae)}{7e^4} + \frac{b(d+ex)^6(bd-ae)^2}{2e^4} - \frac{(d+ex)^5(bd-ae)^3}{5e^4} + \frac{b^3(d+ex)^8}{8e^4} \right)}{a + bx}$$

input `Int[(d + e*x)^4*(a^2 + 2*a*b*x + b^2*x^2)^(3/2),x]`

output `(Sqrt[a^2 + 2*a*b*x + b^2*x^2]*(-1/5*((b*d - a*e)^3*(d + e*x)^5)/e^4 + (b*(b*d - a*e)^2*(d + e*x)^6)/(2*e^4) - (3*b^2*(b*d - a*e)*(d + e*x)^7)/(7*e^4) + (b^3*(d + e*x)^8)/(8*e^4)))/(a + b*x)`

Defintions of rubi rules used

rule 27 `Int[(a_)*(Fx_), x_Symbol] := Simp[a Int[Fx, x], x] /; FreeQ[a, x] && !MatchQ[Fx, (b_)*(Gx_)] /; FreeQ[b, x]`

rule 49 `Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] := Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] && IGtQ[m, 0] && IGtQ[m + n + 2, 0]`

rule 1102 `Int[((d_.) + (e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] := Simp[(a + b*x + c*x^2)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x)^(2*FracPart[p])) Int[(d + e*x)^m*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && EqQ[b^2 - 4*a*c, 0]`

rule 2009 `Int[u_, x_Symbol] := Simp[IntSum[u, x], x] /; SumQ[u]`

input `integrate((e*x+d)^4*(b^2*x^2+2*a*b*x+a^2)^(3/2),x, algorithm="fricas")`

output `1/8*b^3*e^4*x^8 + a^3*d^4*x + 1/7*(4*b^3*d*e^3 + 3*a*b^2*e^4)*x^7 + 1/2*(2*b^3*d^2*e^2 + 4*a*b^2*d*e^3 + a^2*b*e^4)*x^6 + 1/5*(4*b^3*d^3*e + 18*a*b^2*d^2*e^2 + 12*a^2*b*d*e^3 + a^3*e^4)*x^5 + 1/4*(b^3*d^4 + 12*a*b^2*d^3*e + 18*a^2*b*d^2*e^2 + 4*a^3*d*e^3)*x^4 + (a*b^2*d^4 + 4*a^2*b*d^3*e + 2*a^3*d^2*e^2)*x^3 + 1/2*(3*a^2*b*d^4 + 4*a^3*d^3*e)*x^2`

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 5923 vs. $2(141) = 282$.

Time = 1.01 (sec) , antiderivative size = 5923, normalized size of antiderivative = 29.62

$$\int (d + ex)^4 (a^2 + 2abx + b^2x^2)^{3/2} dx = \text{Too large to display}$$

input `integrate((e*x+d)**4*(b**2*x**2+2*a*b*x+a**2)**(3/2),x)`

output

```
Piecewise((sqrt(a**2 + 2*a*b*x + b**2*x**2)*(b**2*e**4*x**7/8 + x**6*(17*a
*b**3*e**4/8 + 4*b**4*d*e**3)/(7*b**2) + x**5*(41*a**2*b**2*e**4/8 + 16*a*
b**3*d*e**3 - 13*a*(17*a*b**3*e**4/8 + 4*b**4*d*e**3)/(7*b) + 6*b**4*d**2*
e**2)/(6*b**2) + x**4*(4*a**3*b*e**4 + 24*a**2*b**2*d*e**3 - 6*a**2*(17*a*
b**3*e**4/8 + 4*b**4*d*e**3)/(7*b**2) + 24*a*b**3*d**2*e**2 - 11*a*(41*a**
2*b**2*e**4/8 + 16*a*b**3*d*e**3 - 13*a*(17*a*b**3*e**4/8 + 4*b**4*d*e**3)
/(7*b) + 6*b**4*d**2*e**2)/(6*b) + 4*b**4*d**3*e)/(5*b**2) + x**3*(a**4*e*
*4 + 16*a**3*b*d*e**3 + 36*a**2*b**2*d**2*e**2 - 5*a**2*(41*a**2*b**2*e**4
/8 + 16*a*b**3*d*e**3 - 13*a*(17*a*b**3*e**4/8 + 4*b**4*d*e**3)/(7*b) + 6*
b**4*d**2*e**2)/(6*b**2) + 16*a*b**3*d**3*e - 9*a*(4*a**3*b*e**4 + 24*a**2
*b**2*d*e**3 - 6*a**2*(17*a*b**3*e**4/8 + 4*b**4*d*e**3)/(7*b**2) + 24*a*b
**3*d**2*e**2 - 11*a*(41*a**2*b**2*e**4/8 + 16*a*b**3*d*e**3 - 13*a*(17*a*
b**3*e**4/8 + 4*b**4*d*e**3)/(7*b) + 6*b**4*d**2*e**2)/(6*b) + 4*b**4*d**3
*e)/(5*b) + b**4*d**4)/(4*b**2) + x**2*(4*a**4*d*e**3 + 24*a**3*b*d**2*e**
2 + 24*a**2*b**2*d**3*e - 4*a**2*(4*a**3*b*e**4 + 24*a**2*b**2*d*e**3 - 6*
a**2*(17*a*b**3*e**4/8 + 4*b**4*d*e**3)/(7*b**2) + 24*a*b**3*d**2*e**2 - 1
1*a*(41*a**2*b**2*e**4/8 + 16*a*b**3*d*e**3 - 13*a*(17*a*b**3*e**4/8 + 4*b
**4*d*e**3)/(7*b) + 6*b**4*d**2*e**2)/(6*b) + 4*b**4*d**3*e)/(5*b**2) + 4*
a*b**3*d**4 - 7*a*(a**4*e**4 + 16*a**3*b*d*e**3 + 36*a**2*b**2*d**2*e**2 -
5*a**2*(41*a**2*b**2*e**4/8 + 16*a*b**3*d*e**3 - 13*a*(17*a*b**3*e**4/...
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 589 vs. $2(148) = 296$.

Time = 0.04 (sec) , antiderivative size = 589, normalized size of antiderivative = 2.94

$$\begin{aligned}
 \int (d+ex)^4 (a^2+2abx+b^2x^2)^{3/2} dx &= \frac{(b^2x^2+2abx+a^2)^{5/2}e^4x^3}{8b^2} \\
 &+ \frac{1}{4}(b^2x^2+2abx+a^2)^{3/2}d^4x - \frac{(b^2x^2+2abx+a^2)^{3/2}ad^3ex}{b} \\
 &+ \frac{3(b^2x^2+2abx+a^2)^{3/2}a^2d^2e^2x}{2b^2} \\
 &- \frac{(b^2x^2+2abx+a^2)^{3/2}a^3de^3x}{b^3} + \frac{(b^2x^2+2abx+a^2)^{3/2}a^4e^4x}{4b^4} \\
 &+ \frac{4(b^2x^2+2abx+a^2)^{5/2}de^3x^2}{7b^2} - \frac{11(b^2x^2+2abx+a^2)^{5/2}ae^4x^2}{56b^3} \\
 &+ \frac{(b^2x^2+2abx+a^2)^{3/2}ad^4}{4b} - \frac{(b^2x^2+2abx+a^2)^{3/2}a^2d^3e}{b^2} \\
 &+ \frac{3(b^2x^2+2abx+a^2)^{3/2}a^3d^2e^2}{2b^3} - \frac{(b^2x^2+2abx+a^2)^{3/2}a^4de^3}{b^4} \\
 &+ \frac{(b^2x^2+2abx+a^2)^{3/2}a^5e^4}{4b^5} + \frac{(b^2x^2+2abx+a^2)^{5/2}d^2e^2x}{b^2} \\
 &- \frac{6(b^2x^2+2abx+a^2)^{5/2}ade^3x}{7b^3} + \frac{13(b^2x^2+2abx+a^2)^{5/2}a^2e^4x}{56b^4} \\
 &+ \frac{4(b^2x^2+2abx+a^2)^{5/2}d^3e}{5b^2} - \frac{7(b^2x^2+2abx+a^2)^{5/2}ad^2e^2}{5b^3} \\
 &+ \frac{34(b^2x^2+2abx+a^2)^{5/2}a^2de^3}{35b^4} - \frac{69(b^2x^2+2abx+a^2)^{5/2}a^3e^4}{280b^5}
 \end{aligned}$$

input `integrate((e*x+d)^4*(b^2*x^2+2*a*b*x+a^2)^(3/2),x, algorithm="maxima")`

output

$$\begin{aligned}
& 1/8*(b^2*x^2 + 2*a*b*x + a^2)^{(5/2)}*e^4*x^3/b^2 + 1/4*(b^2*x^2 + 2*a*b*x + \\
& a^2)^{(3/2)}*d^4*x - (b^2*x^2 + 2*a*b*x + a^2)^{(3/2)}*a*d^3*e*x/b + 3/2*(b^2 \\
& *x^2 + 2*a*b*x + a^2)^{(3/2)}*a^2*d^2*e^2*x/b^2 - (b^2*x^2 + 2*a*b*x + a^2)^{(3/2)} \\
& *a^3*d*e^3*x/b^3 + 1/4*(b^2*x^2 + 2*a*b*x + a^2)^{(3/2)}*a^4*e^4*x/b^4 \\
& + 4/7*(b^2*x^2 + 2*a*b*x + a^2)^{(5/2)}*d*e^3*x^2/b^2 - 11/56*(b^2*x^2 + 2*a \\
& *b*x + a^2)^{(5/2)}*a*e^4*x^2/b^3 + 1/4*(b^2*x^2 + 2*a*b*x + a^2)^{(3/2)}*a*d^4 \\
& /b - (b^2*x^2 + 2*a*b*x + a^2)^{(3/2)}*a^2*d^3*e/b^2 + 3/2*(b^2*x^2 + 2*a*b \\
& *x + a^2)^{(3/2)}*a^3*d^2*e^2/b^3 - (b^2*x^2 + 2*a*b*x + a^2)^{(3/2)}*a^4*d*e^3 \\
& /b^4 + 1/4*(b^2*x^2 + 2*a*b*x + a^2)^{(3/2)}*a^5*e^4/b^5 + (b^2*x^2 + 2*a*b \\
& *x + a^2)^{(5/2)}*d^2*e^2*x/b^2 - 6/7*(b^2*x^2 + 2*a*b*x + a^2)^{(5/2)}*a*d*e^3 \\
& *x/b^3 + 13/56*(b^2*x^2 + 2*a*b*x + a^2)^{(5/2)}*a^2*e^4*x/b^4 + 4/5*(b^2*x^2 \\
& + 2*a*b*x + a^2)^{(5/2)}*d^3*e/b^2 - 7/5*(b^2*x^2 + 2*a*b*x + a^2)^{(5/2)}* \\
& a*d^2*e^2/b^3 + 34/35*(b^2*x^2 + 2*a*b*x + a^2)^{(5/2)}*a^2*d*e^3/b^4 - 69/2 \\
& 80*(b^2*x^2 + 2*a*b*x + a^2)^{(5/2)}*a^3*e^4/b^5
\end{aligned}$$

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 431 vs. 2(148) = 296.

Time = 0.15 (sec) , antiderivative size = 431, normalized size of antiderivative = 2.16

$$\begin{aligned}
& \int (d + ex)^4 (a^2 + 2abx + b^2x^2)^{3/2} dx = \frac{1}{8} b^3 e^4 x^8 \operatorname{sgn}(bx + a) \\
& + \frac{4}{7} b^3 d e^3 x^7 \operatorname{sgn}(bx + a) + \frac{3}{7} a b^2 e^4 x^7 \operatorname{sgn}(bx + a) + b^3 d^2 e^2 x^6 \operatorname{sgn}(bx + a) \\
& + 2 a b^2 d e^3 x^6 \operatorname{sgn}(bx + a) + \frac{1}{2} a^2 b e^4 x^6 \operatorname{sgn}(bx + a) + \frac{4}{5} b^3 d^3 e x^5 \operatorname{sgn}(bx + a) \\
& + \frac{18}{5} a b^2 d^2 e^2 x^5 \operatorname{sgn}(bx + a) + \frac{12}{5} a^2 b d e^3 x^5 \operatorname{sgn}(bx + a) \\
& + \frac{1}{5} a^3 e^4 x^5 \operatorname{sgn}(bx + a) + \frac{1}{4} b^3 d^4 x^4 \operatorname{sgn}(bx + a) + 3 a b^2 d^3 e x^4 \operatorname{sgn}(bx + a) \\
& + \frac{9}{2} a^2 b d^2 e^2 x^4 \operatorname{sgn}(bx + a) + a^3 d e^3 x^4 \operatorname{sgn}(bx + a) + a b^2 d^4 x^3 \operatorname{sgn}(bx + a) \\
& + 4 a^2 b d^3 e x^3 \operatorname{sgn}(bx + a) + 2 a^3 d^2 e^2 x^3 \operatorname{sgn}(bx + a) \\
& + \frac{3}{2} a^2 b d^4 x^2 \operatorname{sgn}(bx + a) + 2 a^3 d^3 e x^2 \operatorname{sgn}(bx + a) + a^3 d^4 x \operatorname{sgn}(bx + a) \\
& + \frac{(70 a^4 b^4 d^4 - 56 a^5 b^3 d^3 e + 28 a^6 b^2 d^2 e^2 - 8 a^7 b d e^3 + a^8 e^4) \operatorname{sgn}(bx + a)}{280 b^5}
\end{aligned}$$

input

```
integrate((e*x+d)^4*(b^2*x^2+2*a*b*x+a^2)^(3/2),x, algorithm="giac")
```

output

```

1/8*b^3*e^4*x^8*sgn(b*x + a) + 4/7*b^3*d*e^3*x^7*sgn(b*x + a) + 3/7*a*b^2*
e^4*x^7*sgn(b*x + a) + b^3*d^2*e^2*x^6*sgn(b*x + a) + 2*a*b^2*d*e^3*x^6*sg
n(b*x + a) + 1/2*a^2*b*e^4*x^6*sgn(b*x + a) + 4/5*b^3*d^3*e*x^5*sgn(b*x +
a) + 18/5*a*b^2*d^2*e^2*x^5*sgn(b*x + a) + 12/5*a^2*b*d*e^3*x^5*sgn(b*x +
a) + 1/5*a^3*e^4*x^5*sgn(b*x + a) + 1/4*b^3*d^4*x^4*sgn(b*x + a) + 3*a*b^2
*d^3*e*x^4*sgn(b*x + a) + 9/2*a^2*b*d^2*e^2*x^4*sgn(b*x + a) + a^3*d*e^3*x
^4*sgn(b*x + a) + a*b^2*d^4*x^3*sgn(b*x + a) + 4*a^2*b*d^3*e*x^3*sgn(b*x +
a) + 2*a^3*d^2*e^2*x^3*sgn(b*x + a) + 3/2*a^2*b*d^4*x^2*sgn(b*x + a) + 2*
a^3*d^3*e*x^2*sgn(b*x + a) + a^3*d^4*x*sgn(b*x + a) + 1/280*(70*a^4*b^4*d^
4 - 56*a^5*b^3*d^3*e + 28*a^6*b^2*d^2*e^2 - 8*a^7*b*d*e^3 + a^8*e^4)*sgn(b
*x + a)/b^5

```

Mupad [F(-1)]

Timed out.

$$\int (d + ex)^4 (a^2 + 2abx + b^2x^2)^{3/2} dx = \int (d + ex)^4 (a^2 + 2abx + b^2x^2)^{3/2} dx$$

input

```
int((d + e*x)^4*(a^2 + b^2*x^2 + 2*a*b*x)^(3/2), x)
```

output

```
int((d + e*x)^4*(a^2 + b^2*x^2 + 2*a*b*x)^(3/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 247, normalized size of antiderivative = 1.24

$$\int (d + ex)^4 (a^2 + 2abx + b^2x^2)^{3/2} dx = \frac{x(35b^3e^4x^7 + 120ab^2e^4x^6 + 160b^3de^3x^6 + 140a^2be^4x^5 + 560ab^2de^3x^5 + 280b^3d^2e^2x^5 + 56$$

input

```
int((e*x+d)^4*(b^2*x^2+2*a*b*x+a^2)^(3/2), x)
```

output

```
(x*(280*a**3*d**4 + 560*a**3*d**3*e*x + 560*a**3*d**2*e**2*x**2 + 280*a**3*d*e**3*x**3 + 56*a**3*e**4*x**4 + 420*a**2*b*d**4*x + 1120*a**2*b*d**3*e*x**2 + 1260*a**2*b*d**2*e**2*x**3 + 672*a**2*b*d*e**3*x**4 + 140*a**2*b*e**4*x**5 + 280*a*b**2*d**4*x**2 + 840*a*b**2*d**3*e*x**3 + 1008*a*b**2*d**2*e**2*x**4 + 560*a*b**2*d*e**3*x**5 + 120*a*b**2*e**4*x**6 + 70*b**3*d**4*x**3 + 224*b**3*d**3*e*x**4 + 280*b**3*d**2*e**2*x**5 + 160*b**3*d*e**3*x**6 + 35*b**3*e**4*x**7))/280
```

3.278 $\int (d + ex)^3 (a^2 + 2abx + b^2x^2)^{3/2} dx$

| | |
|---|------|
| Optimal result | 2104 |
| Mathematica [A] (verified) | 2104 |
| Rubi [A] (verified) | 2105 |
| Maple [A] (verified) | 2106 |
| Fricas [A] (verification not implemented) | 2107 |
| Sympy [B] (verification not implemented) | 2107 |
| Maxima [B] (verification not implemented) | 2108 |
| Giac [B] (verification not implemented) | 2110 |
| Mupad [F(-1)] | 2111 |
| Reduce [B] (verification not implemented) | 2111 |

Optimal result

Integrand size = 28, antiderivative size = 172

$$\int (d + ex)^3 (a^2 + 2abx + b^2x^2)^{3/2} dx = \frac{(bd - ae)^3 (a + bx)^3 \sqrt{a^2 + 2abx + b^2x^2}}{4b^4} + \frac{3e(bd - ae)^2 (a + bx)^4 \sqrt{a^2 + 2abx + b^2x^2}}{5b^4} + \frac{e^2 (bd - ae) (a + bx)^5 \sqrt{a^2 + 2abx + b^2x^2}}{2b^4} + \frac{e^3 (a + bx)^6 \sqrt{a^2 + 2abx + b^2x^2}}{7b^4}$$

output

```
1/4*(-a*e+b*d)^3*(b*x+a)^3*((b*x+a)^2)^(1/2)/b^4+3/5*e*(-a*e+b*d)^2*(b*x+a)^4*((b*x+a)^2)^(1/2)/b^4+1/2*e^2*(-a*e+b*d)*(b*x+a)^5*((b*x+a)^2)^(1/2)/b^4+1/7*e^3*(b*x+a)^6*((b*x+a)^2)^(1/2)/b^4
```

Mathematica [A] (verified)

Time = 1.05 (sec) , antiderivative size = 171, normalized size of antiderivative = 0.99

$$\int (d + ex)^3 (a^2 + 2abx + b^2x^2)^{3/2} dx = \frac{x \sqrt{(a + bx)^2 (35a^3 (4d^3 + 6d^2 ex + 4de^2 x^2 + e^3 x^3) + 21a^2 bx (10d^3 + 20d^2 ex + 15de^2 x^2 + 4e^3 x^3) + 7a (10d^3 + 20d^2 ex + 15de^2 x^2 + 4e^3 x^3) + 7d^3)}}{7b^4}$$

input `Integrate[(d + e*x)^3*(a^2 + 2*a*b*x + b^2*x^2)^(3/2),x]`

output $(x\sqrt{(a + bx)^2}*(35a^3(4d^3 + 6d^2ex + 4de^2x^2 + e^3x^3) + 21a^2bx(10d^3 + 20d^2ex + 15de^2x^2 + 4e^3x^3) + 7ab^2x^2(20d^3 + 45d^2ex + 36de^2x^2 + 10e^3x^3) + b^3x^3(35d^3 + 84d^2ex + 70de^2x^2 + 20e^3x^3)))/(140(a + bx))$

Rubi [A] (verified)

Time = 0.54 (sec) , antiderivative size = 120, normalized size of antiderivative = 0.70, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {1102, 27, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\begin{aligned}
 & \int (a^2 + 2abx + b^2x^2)^{3/2} (d + ex)^3 dx \\
 & \quad \downarrow \text{1102} \\
 & \frac{\sqrt{a^2 + 2abx + b^2x^2} \int b^3(a + bx)^3(d + ex)^3 dx}{b^3(a + bx)} \\
 & \quad \downarrow \text{27} \\
 & \frac{\sqrt{a^2 + 2abx + b^2x^2} \int (a + bx)^3(d + ex)^3 dx}{a + bx} \\
 & \quad \downarrow \text{49} \\
 & \frac{\sqrt{a^2 + 2abx + b^2x^2} \int \left(\frac{e^3(a+bx)^6}{b^3} + \frac{3e^2(bd-ae)(a+bx)^5}{b^3} + \frac{3e(bd-ae)^2(a+bx)^4}{b^3} + \frac{(bd-ae)^3(a+bx)^3}{b^3} \right) dx}{a + bx} \\
 & \quad \downarrow \text{2009} \\
 & \frac{\sqrt{a^2 + 2abx + b^2x^2} \left(\frac{e^2(a+bx)^6(bd-ae)}{2b^4} + \frac{3e(a+bx)^5(bd-ae)^2}{5b^4} + \frac{(a+bx)^4(bd-ae)^3}{4b^4} + \frac{e^3(a+bx)^7}{7b^4} \right)}{a + bx}
 \end{aligned}$$

input `Int[(d + e*x)^3*(a^2 + 2*a*b*x + b^2*x^2)^(3/2),x]`

output

$$\frac{(\sqrt{a^2 + 2abx + b^2x^2} * ((bd - ae)^3(a + bx)^4 / (4b^4) + (3e * (bd - ae)^2(a + bx)^5) / (5b^4) + (e^2 * (bd - ae)(a + bx)^6) / (2b^4) + (e^3 * (a + bx)^7) / (7b^4))) / (a + bx)}$$

Defintions of rubi rules used

rule 27

$$\text{Int}[(a_*)(Fx_), x_Symbol] \rightarrow \text{Simp}[a \text{ Int}[Fx, x], x] /; \text{FreeQ}[a, x] \ \&\& \ !\text{MatchQ}[Fx, (b_*)(Gx_)] /; \text{FreeQ}[b, x]$$

rule 49

$$\text{Int}[((a_.) + (b_.)(x_))^{(m_.)} * ((c_.) + (d_.)(x_))^{(n_.)}, x_Symbol] \rightarrow \text{Int}[\text{ExpandIntegrand}[(a + bx)^m * (c + dx)^n, x], x] /; \text{FreeQ}\{a, b, c, d\}, x \ \&\& \ \text{IGtQ}[m, 0] \ \&\& \ \text{IGtQ}[m + n + 2, 0]$$

rule 1102

$$\text{Int}(((d_.) + (e_.)(x_))^{(m_.)} * ((a_) + (b_.)(x_) + (c_.)(x_)^2)^{(p_)}, x_Symbol] \rightarrow \text{Simp}[(a + bx + cx^2)^{\text{FracPart}[p]} / (c^{\text{IntPart}[p]} * (b/2 + cx)^{(2 * \text{FracPart}[p])}) \text{ Int}[(d + ex)^m * (b/2 + cx)^{(2 * p)}, x], x] /; \text{FreeQ}\{a, b, c, d, e, m, p\}, x \ \&\& \ \text{EqQ}[b^2 - 4ac, 0]$$

rule 2009

$$\text{Int}[u_, x_Symbol] \rightarrow \text{Simp}[\text{IntSum}[u, x], x] /; \text{SumQ}[u]$$

Maple [A] (verified)

Time = 1.15 (sec) , antiderivative size = 206, normalized size of antiderivative = 1.20

| method | result |
|---------|---|
| gospers | $\frac{x(20b^3e^3x^6 + 70x^5b^2e^3a + 70x^5de^2b^3 + 84x^4a^2be^3 + 252x^4ab^2de^2 + 84x^4b^3d^2e + 35x^3e^3a^3 + 315x^3a^2bde^2 + 315x^3ab^2d^2e + 35x^3b^3d^3)}{140(bx+a)^3}$ |
| default | $\frac{x(20b^3e^3x^6 + 70x^5b^2e^3a + 70x^5de^2b^3 + 84x^4a^2be^3 + 252x^4ab^2de^2 + 84x^4b^3d^2e + 35x^3e^3a^3 + 315x^3a^2bde^2 + 315x^3ab^2d^2e + 35x^3b^3d^3)}{140(bx+a)^3}$ |
| orering | $\frac{x(20b^3e^3x^6 + 70x^5b^2e^3a + 70x^5de^2b^3 + 84x^4a^2be^3 + 252x^4ab^2de^2 + 84x^4b^3d^2e + 35x^3e^3a^3 + 315x^3a^2bde^2 + 315x^3ab^2d^2e + 35x^3b^3d^3)}{140(bx+a)^3}$ |
| risch | $\frac{\sqrt{(bx+a)^2 b^3 e^3 x^7}}{7bx+7a} + \frac{\sqrt{(bx+a)^2 (3b^2 e^3 a + 3d e^2 b^3)} x^6}{6bx+6a} + \frac{\sqrt{(bx+a)^2 (3a^2 b e^3 + 9a b^2 d e^2 + 3b^3 d^2 e)} x^5}{5bx+5a} + \frac{\sqrt{(bx+a)^2 (e^3 a^3 + 9a^2 b d e^2 + 3b^3 d^3)} x^4}{4bx}$ |

input `int((e*x+d)^3*(b^2*x^2+2*a*b*x+a^2)^(3/2),x,method=_RETURNVERBOSE)`

output `1/140*x*(20*b^3*e^3*x^6+70*a*b^2*e^3*x^5+70*b^3*d*e^2*x^5+84*a^2*b*e^3*x^4+252*a*b^2*d*e^2*x^4+84*b^3*d^2*e*x^4+35*a^3*e^3*x^3+315*a^2*b*d*e^2*x^3+315*a*b^2*d^2*e*x^3+35*b^3*d^3*x^3+140*a^3*d*e^2*x^2+420*a^2*b*d^2*e*x^2+140*a*b^2*d^3*x^2+210*a^3*d^2*e*x+210*a^2*b*d^3*x+140*a^3*d^3)*((b*x+a)^2)^(3/2)/(b*x+a)^3`

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 167, normalized size of antiderivative = 0.97

$$\int (d+ex)^3 (a^2+2abx+b^2x^2)^{3/2} dx = \frac{1}{7}b^3e^3x^7 + a^3d^3x + \frac{1}{2}(b^3de^2 + ab^2e^3)x^6 + \frac{3}{5}(b^3d^2e + 3ab^2de^2 + a^2be^3)x^5 + \frac{1}{4}(b^3d^3 + 9ab^2d^2e + 9a^2bde^2 + a^3e^3)x^4 + (ab^2d^3 + 3a^2bd^2e + a^3de^2)x^3 + \frac{3}{2}(a^2bd^3 + a^3d^2e)x^2$$

input `integrate((e*x+d)^3*(b^2*x^2+2*a*b*x+a^2)^(3/2),x, algorithm="fricas")`

output `1/7*b^3*e^3*x^7 + a^3*d^3*x + 1/2*(b^3*d*e^2 + a*b^2*e^3)*x^6 + 3/5*(b^3*d^2*e + 3*a*b^2*d*e^2 + a^2*b*e^3)*x^5 + 1/4*(b^3*d^3 + 9*a*b^2*d^2*e + 9*a^2*b*d*e^2 + a^3*e^3)*x^4 + (a*b^2*d^3 + 3*a^2*b*d^2*e + a^3*d*e^2)*x^3 + 3/2*(a^2*b*d^3 + a^3*d^2*e)*x^2`

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 3305 vs. 2(121) = 242.

Time = 0.90 (sec) , antiderivative size = 3305, normalized size of antiderivative = 19.22

$$\int (d+ex)^3 (a^2+2abx+b^2x^2)^{3/2} dx = \text{Too large to display}$$

input `integrate((e*x+d)**3*(b**2*x**2+2*a*b*x+a**2)**(3/2),x)`

output

```
Piecewise((sqrt(a**2 + 2*a*b*x + b**2*x**2)*(b**2*e**3*x**6/7 + x**5*(15*a
*b**3*e**3/7 + 3*b**4*d*e**2)/(6*b**2) + x**4*(36*a**2*b**2*e**3/7 + 12*a*
b**3*d*e**2 - 11*a*(15*a*b**3*e**3/7 + 3*b**4*d*e**2)/(6*b) + 3*b**4*d**2*
e)/(5*b**2) + x**3*(4*a**3*b*e**3 + 18*a**2*b**2*d*e**2 - 5*a**2*(15*a*b**
3*e**3/7 + 3*b**4*d*e**2)/(6*b**2) + 12*a*b**3*d**2*e - 9*a*(36*a**2*b**2*
e**3/7 + 12*a*b**3*d*e**2 - 11*a*(15*a*b**3*e**3/7 + 3*b**4*d*e**2)/(6*b)
+ 3*b**4*d**2*e)/(5*b) + b**4*d**3)/(4*b**2) + x**2*(a**4*e**3 + 12*a**3*b
*d*e**2 + 18*a**2*b**2*d**2*e - 4*a**2*(36*a**2*b**2*e**3/7 + 12*a*b**3*d*
e**2 - 11*a*(15*a*b**3*e**3/7 + 3*b**4*d*e**2)/(6*b) + 3*b**4*d**2*e)/(5*b
**2) + 4*a*b**3*d**3 - 7*a*(4*a**3*b*e**3 + 18*a**2*b**2*d*e**2 - 5*a**2*(
15*a*b**3*e**3/7 + 3*b**4*d*e**2)/(6*b**2) + 12*a*b**3*d**2*e - 9*a*(36*a*
**2*b**2*e**3/7 + 12*a*b**3*d*e**2 - 11*a*(15*a*b**3*e**3/7 + 3*b**4*d*e**2
))/(6*b) + 3*b**4*d**2*e)/(5*b) + b**4*d**3)/(4*b))/(3*b**2) + x*(3*a**4*d*
e**2 + 12*a**3*b*d**2*e + 6*a**2*b**2*d**3 - 3*a**2*(4*a**3*b*e**3 + 18*a*
**2*b**2*d*e**2 - 5*a**2*(15*a*b**3*e**3/7 + 3*b**4*d*e**2)/(6*b**2) + 12*a
*b**3*d**2*e - 9*a*(36*a**2*b**2*e**3/7 + 12*a*b**3*d*e**2 - 11*a*(15*a*b*
**3*e**3/7 + 3*b**4*d*e**2)/(6*b) + 3*b**4*d**2*e)/(5*b) + b**4*d**3)/(4*b*
**2) - 5*a*(a**4*e**3 + 12*a**3*b*d*e**2 + 18*a**2*b**2*d**2*e - 4*a**2*(36
*a**2*b**2*e**3/7 + 12*a*b**3*d*e**2 - 11*a*(15*a*b**3*e**3/7 + 3*b**4*d*e
**2)/(6*b) + 3*b**4*d**2*e)/(5*b**2) + 4*a*b**3*d**3 - 7*a*(4*a**3*b*e...
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 400 vs. $2(120) = 240$.

Time = 0.04 (sec) , antiderivative size = 400, normalized size of antiderivative = 2.33

$$\int (d + ex)^3 (a^2 + 2abx + b^2x^2)^{3/2} dx = \frac{1}{4} (b^2x^2 + 2abx + a^2)^{\frac{3}{2}} d^3x$$

$$- \frac{3 (b^2x^2 + 2abx + a^2)^{\frac{3}{2}} ad^2ex}{4b}$$

$$+ \frac{3 (b^2x^2 + 2abx + a^2)^{\frac{3}{2}} a^2 d e^2 x}{4b^2} - \frac{(b^2x^2 + 2abx + a^2)^{\frac{3}{2}} a^3 e^3 x}{4b^3}$$

$$+ \frac{(b^2x^2 + 2abx + a^2)^{\frac{5}{2}} e^3 x^2}{7b^2} + \frac{(b^2x^2 + 2abx + a^2)^{\frac{3}{2}} ad^3}{4b}$$

$$- \frac{3 (b^2x^2 + 2abx + a^2)^{\frac{3}{2}} a^2 d^2 e}{4b^2} + \frac{3 (b^2x^2 + 2abx + a^2)^{\frac{3}{2}} a^3 d e^2}{4b^3}$$

$$- \frac{(b^2x^2 + 2abx + a^2)^{\frac{3}{2}} a^4 e^3}{4b^4} + \frac{(b^2x^2 + 2abx + a^2)^{\frac{5}{2}} d e^2 x}{2b^2}$$

$$- \frac{3 (b^2x^2 + 2abx + a^2)^{\frac{5}{2}} a e^3 x}{14b^3} + \frac{3 (b^2x^2 + 2abx + a^2)^{\frac{5}{2}} d^2 e}{5b^2}$$

$$- \frac{7 (b^2x^2 + 2abx + a^2)^{\frac{5}{2}} a d e^2}{10b^3} + \frac{17 (b^2x^2 + 2abx + a^2)^{\frac{5}{2}} a^2 e^3}{70b^4}$$

input `integrate((e*x+d)^3*(b^2*x^2+2*a*b*x+a^2)^(3/2),x, algorithm="maxima")`

output `1/4*(b^2*x^2 + 2*a*b*x + a^2)^(3/2)*d^3*x - 3/4*(b^2*x^2 + 2*a*b*x + a^2)^(3/2)*a*d^2*e*x/b + 3/4*(b^2*x^2 + 2*a*b*x + a^2)^(3/2)*a^2*d*e^2*x/b^2 - 1/4*(b^2*x^2 + 2*a*b*x + a^2)^(3/2)*a^3*e^3*x/b^3 + 1/7*(b^2*x^2 + 2*a*b*x + a^2)^(5/2)*e^3*x^2/b^2 + 1/4*(b^2*x^2 + 2*a*b*x + a^2)^(3/2)*a*d^3/b - 3/4*(b^2*x^2 + 2*a*b*x + a^2)^(3/2)*a^2*d^2*e/b^2 + 3/4*(b^2*x^2 + 2*a*b*x + a^2)^(3/2)*a^3*d*e^2/b^3 - 1/4*(b^2*x^2 + 2*a*b*x + a^2)^(3/2)*a^4*e^3/b^4 + 1/2*(b^2*x^2 + 2*a*b*x + a^2)^(5/2)*d*e^2*x/b^2 - 3/14*(b^2*x^2 + 2*a*b*x + a^2)^(5/2)*a*e^3*x/b^3 + 3/5*(b^2*x^2 + 2*a*b*x + a^2)^(5/2)*d^2*e/b^2 - 7/10*(b^2*x^2 + 2*a*b*x + a^2)^(5/2)*a*d*e^2/b^3 + 17/70*(b^2*x^2 + 2*a*b*x + a^2)^(5/2)*a^2*e^3/b^4`

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 337 vs. $2(120) = 240$.

Time = 0.16 (sec) , antiderivative size = 337, normalized size of antiderivative = 1.96

$$\int (d + ex)^3 (a^2 + 2abx + b^2x^2)^{3/2} dx = \frac{1}{7} b^3 e^3 x^7 \operatorname{sgn}(bx + a) + \frac{1}{2} b^3 d e^2 x^6 \operatorname{sgn}(bx + a) + \frac{1}{2} a b^2 e^3 x^6 \operatorname{sgn}(bx + a) + \frac{3}{5} b^3 d^2 e x^5 \operatorname{sgn}(bx + a) + \frac{9}{5} a b^2 d e^2 x^5 \operatorname{sgn}(bx + a) + \frac{3}{5} a^2 b e^3 x^5 \operatorname{sgn}(bx + a) + \frac{1}{4} b^3 d^3 x^4 \operatorname{sgn}(bx + a) + \frac{9}{4} a b^2 d^2 e x^4 \operatorname{sgn}(bx + a) + \frac{9}{4} a^2 b d e^2 x^4 \operatorname{sgn}(bx + a) + \frac{1}{4} a^3 e^3 x^4 \operatorname{sgn}(bx + a) + a b^2 d^3 x^3 \operatorname{sgn}(bx + a) + 3 a^2 b d^2 e x^3 \operatorname{sgn}(bx + a) + a^3 d e^2 x^3 \operatorname{sgn}(bx + a) + \frac{3}{2} a^2 b d^3 x^2 \operatorname{sgn}(bx + a) + \frac{3}{2} a^3 d^2 e x^2 \operatorname{sgn}(bx + a) + a^3 d^3 x \operatorname{sgn}(bx + a) + \frac{(35 a^4 b^3 d^3 - 21 a^5 b^2 d^2 e + 7 a^6 b d e^2 - a^7 e^3) \operatorname{sgn}(bx + a)}{140 b^4}$$

input `integrate((e*x+d)^3*(b^2*x^2+2*a*b*x+a^2)^(3/2),x, algorithm="giac")`

output `1/7*b^3*e^3*x^7*sgn(b*x + a) + 1/2*b^3*d*e^2*x^6*sgn(b*x + a) + 1/2*a*b^2*e^3*x^6*sgn(b*x + a) + 3/5*b^3*d^2*e*x^5*sgn(b*x + a) + 9/5*a*b^2*d*e^2*x^5*sgn(b*x + a) + 3/5*a^2*b*e^3*x^5*sgn(b*x + a) + 1/4*b^3*d^3*x^4*sgn(b*x + a) + 9/4*a*b^2*d^2*e*x^4*sgn(b*x + a) + 9/4*a^2*b*d*e^2*x^4*sgn(b*x + a) + 1/4*a^3*e^3*x^4*sgn(b*x + a) + a*b^2*d^3*x^3*sgn(b*x + a) + 3*a^2*b*d^2*e*x^3*sgn(b*x + a) + a^3*d*e^2*x^3*sgn(b*x + a) + 3/2*a^2*b*d^3*x^2*sgn(b*x + a) + 3/2*a^3*d^2*e*x^2*sgn(b*x + a) + a^3*d^3*x*sgn(b*x + a) + 1/140*(35*a^4*b^3*d^3 - 21*a^5*b^2*d^2*e + 7*a^6*b*d*e^2 - a^7*e^3)*sgn(b*x + a)/b^4`

Mupad [F(-1)]

Timed out.

$$\int (d + ex)^3 (a^2 + 2abx + b^2x^2)^{3/2} dx = \int (d + ex)^3 (a^2 + 2abx + b^2x^2)^{3/2} dx$$

input `int((d + e*x)^3*(a^2 + b^2*x^2 + 2*a*b*x)^(3/2), x)`

output `int((d + e*x)^3*(a^2 + b^2*x^2 + 2*a*b*x)^(3/2), x)`

Reduce [B] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 189, normalized size of antiderivative = 1.10

$$\int (d + ex)^3 (a^2 + 2abx + b^2x^2)^{3/2} dx = \frac{x(20b^3e^3x^6 + 70ab^2e^3x^5 + 70b^3de^2x^5 + 84a^2be^3x^4 + 252ab^2de^2x^4 + 84b^3d^2ex^4 + 35a^3e^3x^3 + 210a^2b^2d^2ex^3 + 420a^2b^2d^2e^2x^3 + 315a^2b^2d^2e^2x^3 + 84a^2b^2e^3x^4 + 140ab^2d^3x^2 + 315ab^2d^2e^2x^3 + 252ab^2d^2e^2x^4 + 70ab^2e^3x^5 + 35b^3d^3x^3 + 84b^3d^2e^2x^4 + 70b^3d^2e^2x^5 + 20b^3e^3x^6))/140$$

input `int((e*x+d)^3*(b^2*x^2+2*a*b*x+a^2)^(3/2), x)`

output `(x*(140*a**3*d**3 + 210*a**3*d**2*e*x + 140*a**3*d*e**2*x**2 + 35*a**3*e**3*x**3 + 210*a**2*b*d**3*x + 420*a**2*b*d**2*e*x**2 + 315*a**2*b*d*e**2*x**3 + 84*a**2*b*e**3*x**4 + 140*a*b**2*d**3*x**2 + 315*a*b**2*d**2*e*x**3 + 252*a*b**2*d*e**2*x**4 + 70*a*b**2*e**3*x**5 + 35*b**3*d**3*x**3 + 84*b**3*d**2*e*x**4 + 70*b**3*d*e**2*x**5 + 20*b**3*e**3*x**6))/140`

3.279 $\int (d + ex)^2 (a^2 + 2abx + b^2x^2)^{3/2} dx$

| | |
|---|------|
| Optimal result | 2112 |
| Mathematica [A] (verified) | 2112 |
| Rubi [A] (verified) | 2113 |
| Maple [A] (verified) | 2114 |
| Fricas [A] (verification not implemented) | 2115 |
| Sympy [B] (verification not implemented) | 2115 |
| Maxima [B] (verification not implemented) | 2116 |
| Giac [B] (verification not implemented) | 2117 |
| Mupad [F(-1)] | 2118 |
| Reduce [B] (verification not implemented) | 2118 |

Optimal result

Integrand size = 28, antiderivative size = 114

$$\int (d + ex)^2 (a^2 + 2abx + b^2x^2)^{3/2} dx = \frac{(bd - ae)^2(a + bx) (a^2 + 2abx + b^2x^2)^{3/2}}{4b^3} + \frac{2e(bd - ae) (a^2 + 2abx + b^2x^2)^{5/2}}{5b^3} + \frac{e^2(a + bx) (a^2 + 2abx + b^2x^2)^{5/2}}{6b^3}$$

output `1/4*(-a*e+b*d)^2*(b*x+a)*(b^2*x^2+2*a*b*x+a^2)^(3/2)/b^3+2/5*e*(-a*e+b*d)*(b^2*x^2+2*a*b*x+a^2)^(5/2)/b^3+1/6*e^2*(b*x+a)*(b^2*x^2+2*a*b*x+a^2)^(5/2)/b^3`

Mathematica [A] (verified)

Time = 1.04 (sec) , antiderivative size = 127, normalized size of antiderivative = 1.11

$$\int (d + ex)^2 (a^2 + 2abx + b^2x^2)^{3/2} dx = \frac{x\sqrt{(a + bx)^2(20a^3(3d^2 + 3dex + e^2x^2) + 15a^2bx(6d^2 + 8dex + 3e^2x^2) + 6ab^2x^2(10d^2 + 15d^2 + 15dex + 3e^2x^2))}}{60(a + bx)}$$

input `Integrate[(d + e*x)^2*(a^2 + 2*a*b*x + b^2*x^2)^(3/2),x]`

output

```
(x*sqrt[(a + b*x)^2]*(20*a^3*(3*d^2 + 3*d*e*x + e^2*x^2) + 15*a^2*b*x*(6*d^2 + 8*d*e*x + 3*e^2*x^2) + 6*a*b^2*x^2*(10*d^2 + 15*d*e*x + 6*e^2*x^2) + b^3*x^3*(15*d^2 + 24*d*e*x + 10*e^2*x^2)))/(60*(a + b*x))
```

Rubi [A] (verified)

Time = 0.49 (sec) , antiderivative size = 93, normalized size of antiderivative = 0.82, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {1102, 27, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a^2 + 2abx + b^2x^2)^{3/2} (d + ex)^2 dx$$

$$\downarrow 1102$$

$$\frac{\sqrt{a^2 + 2abx + b^2x^2} \int b^3(a + bx)^3(d + ex)^2 dx}{b^3(a + bx)}$$

$$\downarrow 27$$

$$\frac{\sqrt{a^2 + 2abx + b^2x^2} \int (a + bx)^3(d + ex)^2 dx}{a + bx}$$

$$\downarrow 49$$

$$\frac{\sqrt{a^2 + 2abx + b^2x^2} \int \left(\frac{e^2(a+bx)^5}{b^2} + \frac{2e(bd-ae)(a+bx)^4}{b^2} + \frac{(bd-ae)^2(a+bx)^3}{b^2} \right) dx}{a + bx}$$

$$\downarrow 2009$$

$$\frac{\sqrt{a^2 + 2abx + b^2x^2} \left(\frac{2e(a+bx)^5(bd-ae)}{5b^3} + \frac{(a+bx)^4(bd-ae)^2}{4b^3} + \frac{e^2(a+bx)^6}{6b^3} \right)}{a + bx}$$

input

```
Int[(d + e*x)^2*(a^2 + 2*a*b*x + b^2*x^2)^(3/2),x]
```

output

```
(sqrt[a^2 + 2*a*b*x + b^2*x^2]*(((b*d - a*e)^2*(a + b*x)^4)/(4*b^3) + (2*e*(b*d - a*e)*(a + b*x)^5)/(5*b^3) + (e^2*(a + b*x)^6)/(6*b^3)))/(a + b*x)
```

Defintions of rubi rules used

rule 27 `Int[(a_)*(Fx_), x_Symbol] := Simp[a Int[Fx, x], x] /; FreeQ[a, x] && !MatchQ[Fx, (b_)*(Gx_)] /; FreeQ[b, x]`

rule 49 `Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] := Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] && IGtQ[m, 0] && IGtQ[m + n + 2, 0]`

rule 1102 `Int[((d_.) + (e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] := Simp[(a + b*x + c*x^2)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x)^(2*FracPart[p])) Int[(d + e*x)^m*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && EqQ[b^2 - 4*a*c, 0]`

rule 2009 `Int[u_, x_Symbol] := Simp[IntSum[u, x], x] /; SumQ[u]`

Maple [A] (verified)

Time = 1.07 (sec) , antiderivative size = 148, normalized size of antiderivative = 1.30

| method | result |
|---------|---|
| gospers | $\frac{x(10e^2b^3x^5+36x^4ab^2e^2+24x^4b^3de+45x^3a^2be^2+90x^3ab^2de+15x^3b^3d^2+20x^2a^3e^2+120x^2a^2bde+60x^2ab^2d^2+60xa^3de+90a^2bd^2)}{60(bx+a)^3}$ |
| default | $\frac{x(10e^2b^3x^5+36x^4ab^2e^2+24x^4b^3de+45x^3a^2be^2+90x^3ab^2de+15x^3b^3d^2+20x^2a^3e^2+120x^2a^2bde+60x^2ab^2d^2+60xa^3de+90a^2bd^2)}{60(bx+a)^3}$ |
| orering | $\frac{x(10e^2b^3x^5+36x^4ab^2e^2+24x^4b^3de+45x^3a^2be^2+90x^3ab^2de+15x^3b^3d^2+20x^2a^3e^2+120x^2a^2bde+60x^2ab^2d^2+60xa^3de+90a^2bd^2)}{60(bx+a)^3}$ |
| risch | $\frac{\sqrt{(bx+a)^2e^2b^3x^6}}{6bx+6a} + \frac{\sqrt{(bx+a)^2(3ab^2e^2+2b^3de)}x^5}{5bx+5a} + \frac{\sqrt{(bx+a)^2(3a^2be^2+6ab^2de+b^3d^2)}x^4}{4bx+4a} + \frac{\sqrt{(bx+a)^2(a^3e^2+6a^2bde+3a^2bd^2)}}{3bx+3a}$ |

input `int((e*x+d)^2*(b^2*x^2+2*a*b*x+a^2)^(3/2), x, method=_RETURNVERBOSE)`

output

```
1/60*x*(10*b^3*e^2*x^5+36*a*b^2*e^2*x^4+24*b^3*d*e*x^4+45*a^2*b*e^2*x^3+90
*a*b^2*d*e*x^3+15*b^3*d^2*x^3+20*a^3*e^2*x^2+120*a^2*b*d*e*x^2+60*a*b^2*d^
2*x^2+60*a^3*d*e*x+90*a^2*b*d^2*x+60*a^3*d^2)*((b*x+a)^2)^(3/2)/(b*x+a)^3
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 124, normalized size of antiderivative = 1.09

$$\int (d + ex)^2 (a^2 + 2abx + b^2x^2)^{3/2} dx = \frac{1}{6} b^3 e^2 x^6 + a^3 d^2 x$$

$$+ \frac{1}{5} (2b^3 de + 3ab^2 e^2) x^5 + \frac{1}{4} (b^3 d^2 + 6ab^2 de + 3a^2 be^2) x^4$$

$$+ \frac{1}{3} (3ab^2 d^2 + 6a^2 bde + a^3 e^2) x^3 + \frac{1}{2} (3a^2 bd^2 + 2a^3 de) x^2$$

input

```
integrate((e*x+d)^2*(b^2*x^2+2*a*b*x+a^2)^(3/2),x, algorithm="fricas")
```

output

```
1/6*b^3*e^2*x^6 + a^3*d^2*x + 1/5*(2*b^3*d*e + 3*a*b^2*e^2)*x^5 + 1/4*(b^3
*d^2 + 6*a*b^2*d*e + 3*a^2*b*e^2)*x^4 + 1/3*(3*a*b^2*d^2 + 6*a^2*b*d*e + a
^3*e^2)*x^3 + 1/2*(3*a^2*b*d^2 + 2*a^3*d*e)*x^2
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1695 vs. 2(107) = 214.

Time = 0.80 (sec) , antiderivative size = 1695, normalized size of antiderivative = 14.87

$$\int (d + ex)^2 (a^2 + 2abx + b^2x^2)^{3/2} dx = \text{Too large to display}$$

input

```
integrate((e*x+d)**2*(b**2*x**2+2*a*b*x+a**2)**(3/2),x)
```


output

```
Piecewise((sqrt(a**2 + 2*a*b*x + b**2*x**2)*(b**2*e**2*x**5/6 + x**4*(13*a
*b**3*e**2/6 + 2*b**4*d*e)/(5*b**2) + x**3*(31*a**2*b**2*e**2/6 + 8*a*b**3
*d*e - 9*a*(13*a*b**3*e**2/6 + 2*b**4*d*e)/(5*b) + b**4*d**2)/(4*b**2) + x
**2*(4*a**3*b*e**2 + 12*a**2*b**2*d*e - 4*a**2*(13*a*b**3*e**2/6 + 2*b**4*
d*e)/(5*b**2) + 4*a*b**3*d**2 - 7*a*(31*a**2*b**2*e**2/6 + 8*a*b**3*d*e -
9*a*(13*a*b**3*e**2/6 + 2*b**4*d*e)/(5*b) + b**4*d**2)/(4*b))/(3*b**2) + x
*(a**4*e**2 + 8*a**3*b*d*e + 6*a**2*b**2*d**2 - 3*a**2*(31*a**2*b**2*e**2/
6 + 8*a*b**3*d*e - 9*a*(13*a*b**3*e**2/6 + 2*b**4*d*e)/(5*b) + b**4*d**2)/
(4*b**2) - 5*a*(4*a**3*b*e**2 + 12*a**2*b**2*d*e - 4*a**2*(13*a*b**3*e**2/
6 + 2*b**4*d*e)/(5*b**2) + 4*a*b**3*d**2 - 7*a*(31*a**2*b**2*e**2/6 + 8*a*
b**3*d*e - 9*a*(13*a*b**3*e**2/6 + 2*b**4*d*e)/(5*b) + b**4*d**2)/(4*b))/(
3*b))/(2*b**2) + (2*a**4*d*e + 4*a**3*b*d**2 - 2*a**2*(4*a**3*b*e**2 + 12*
a**2*b**2*d*e - 4*a**2*(13*a*b**3*e**2/6 + 2*b**4*d*e)/(5*b**2) + 4*a*b**3
*d**2 - 7*a*(31*a**2*b**2*e**2/6 + 8*a*b**3*d*e - 9*a*(13*a*b**3*e**2/6 +
2*b**4*d*e)/(5*b) + b**4*d**2)/(4*b))/(3*b**2) - 3*a*(a**4*e**2 + 8*a**3*b
*d*e + 6*a**2*b**2*d**2 - 3*a**2*(31*a**2*b**2*e**2/6 + 8*a*b**3*d*e - 9*a
*(13*a*b**3*e**2/6 + 2*b**4*d*e)/(5*b) + b**4*d**2)/(4*b**2) - 5*a*(4*a**3
*b*e**2 + 12*a**2*b**2*d*e - 4*a**2*(13*a*b**3*e**2/6 + 2*b**4*d*e)/(5*b**
2) + 4*a*b**3*d**2 - 7*a*(31*a**2*b**2*e**2/6 + 8*a*b**3*d*e - 9*a*(13*a*b
**3*e**2/6 + 2*b**4*d*e)/(5*b) + b**4*d**2)/(4*b))/(3*b))/(2*b))/b**2) ...
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 245 vs. $2(102) = 204$.

Time = 0.04 (sec) , antiderivative size = 245, normalized size of antiderivative = 2.15

$$\int (d + ex)^2 (a^2 + 2abx + b^2x^2)^{3/2} dx = \frac{1}{4} (b^2x^2 + 2abx + a^2)^{\frac{3}{2}} d^2x$$

$$- \frac{(b^2x^2 + 2abx + a^2)^{\frac{3}{2}} adex}{2b} + \frac{(b^2x^2 + 2abx + a^2)^{\frac{3}{2}} a^2 e^2 x}{4b^2}$$

$$+ \frac{(b^2x^2 + 2abx + a^2)^{\frac{3}{2}} ad^2}{4b} - \frac{(b^2x^2 + 2abx + a^2)^{\frac{3}{2}} a^2 de}{2b^2} + \frac{(b^2x^2 + 2abx + a^2)^{\frac{3}{2}} a^3 e^2}{4b^3}$$

$$+ \frac{(b^2x^2 + 2abx + a^2)^{\frac{5}{2}} e^2 x}{6b^2} + \frac{2(b^2x^2 + 2abx + a^2)^{\frac{5}{2}} de}{5b^2} - \frac{7(b^2x^2 + 2abx + a^2)^{\frac{5}{2}} ae^2}{30b^3}$$

input

```
integrate((e*x+d)^2*(b^2*x^2+2*a*b*x+a^2)^(3/2),x, algorithm="maxima")
```

output

$$\begin{aligned} & 1/4*(b^2*x^2 + 2*a*b*x + a^2)^{(3/2)}*d^2*x - 1/2*(b^2*x^2 + 2*a*b*x + a^2)^{(3/2)}*a*d*e*x/b + 1/4*(b^2*x^2 + 2*a*b*x + a^2)^{(3/2)}*a^2*e^2*x/b^2 + 1/4*(b^2*x^2 + 2*a*b*x + a^2)^{(3/2)}*a*d^2/b - 1/2*(b^2*x^2 + 2*a*b*x + a^2)^{(3/2)}*a^2*d*e/b^2 + 1/4*(b^2*x^2 + 2*a*b*x + a^2)^{(3/2)}*a^3*e^2/b^3 + 1/6*(b^2*x^2 + 2*a*b*x + a^2)^{(5/2)}*e^2*x/b^2 + 2/5*(b^2*x^2 + 2*a*b*x + a^2)^{(5/2)}*d*e/b^2 - 7/30*(b^2*x^2 + 2*a*b*x + a^2)^{(5/2)}*a*e^2/b^3 \end{aligned}$$
Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 240 vs. $2(102) = 204$.

Time = 0.15 (sec) , antiderivative size = 240, normalized size of antiderivative = 2.11

$$\begin{aligned} & \int (d+ex)^2 (a^2+2abx+b^2x^2)^{3/2} dx = \frac{1}{6} b^3 e^2 x^6 \operatorname{sgn}(bx+a) + \frac{2}{5} b^3 dex^5 \operatorname{sgn}(bx+a) \\ & + \frac{3}{5} ab^2 e^2 x^5 \operatorname{sgn}(bx+a) + \frac{1}{4} b^3 d^2 x^4 \operatorname{sgn}(bx+a) + \frac{3}{2} ab^2 dex^4 \operatorname{sgn}(bx+a) \\ & + \frac{3}{4} a^2 be^2 x^4 \operatorname{sgn}(bx+a) + ab^2 d^2 x^3 \operatorname{sgn}(bx+a) + 2a^2 b dex^3 \operatorname{sgn}(bx+a) \\ & + \frac{1}{3} a^3 e^2 x^3 \operatorname{sgn}(bx+a) + \frac{3}{2} a^2 bd^2 x^2 \operatorname{sgn}(bx+a) + a^3 dex^2 \operatorname{sgn}(bx+a) \\ & + a^3 d^2 x \operatorname{sgn}(bx+a) + \frac{(15a^4 b^2 d^2 - 6a^5 bde + a^6 e^2) \operatorname{sgn}(bx+a)}{60 b^3} \end{aligned}$$

input

```
integrate((e*x+d)^2*(b^2*x^2+2*a*b*x+a^2)^(3/2),x, algorithm="giac")
```

output

$$\begin{aligned} & 1/6*b^3*e^2*x^6*\operatorname{sgn}(b*x + a) + 2/5*b^3*d*e*x^5*\operatorname{sgn}(b*x + a) + 3/5*a*b^2*e^2*x^5*\operatorname{sgn}(b*x + a) + 1/4*b^3*d^2*x^4*\operatorname{sgn}(b*x + a) + 3/2*a*b^2*d*e*x^4*\operatorname{sgn}(b*x + a) + 3/4*a^2*b*e^2*x^4*\operatorname{sgn}(b*x + a) + a*b^2*d^2*x^3*\operatorname{sgn}(b*x + a) + 2*a^2*b*d*e*x^3*\operatorname{sgn}(b*x + a) + 1/3*a^3*e^2*x^3*\operatorname{sgn}(b*x + a) + 3/2*a^2*b*d^2*x^2*\operatorname{sgn}(b*x + a) + a^3*d*e*x^2*\operatorname{sgn}(b*x + a) + a^3*d^2*x*\operatorname{sgn}(b*x + a) + 1/60*(15*a^4*b^2*d^2 - 6*a^5*b*d*e + a^6*e^2)*\operatorname{sgn}(b*x + a)/b^3 \end{aligned}$$

Mupad [F(-1)]

Timed out.

$$\int (d + ex)^2 (a^2 + 2abx + b^2x^2)^{3/2} dx = \int (d + ex)^2 (a^2 + 2abx + b^2x^2)^{3/2} dx$$

input `int((d + e*x)^2*(a^2 + b^2*x^2 + 2*a*b*x)^(3/2),x)`

output `int((d + e*x)^2*(a^2 + b^2*x^2 + 2*a*b*x)^(3/2), x)`

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 131, normalized size of antiderivative = 1.15

$$\int (d + ex)^2 (a^2 + 2abx + b^2x^2)^{3/2} dx = \frac{x(10b^3e^2x^5 + 36ab^2e^2x^4 + 24b^3dex^4 + 45a^2be^2x^3 + 90ab^2dex^3 + 15b^3d^2x^3 + 20a^3e^2x^2 + \dots)}{60}$$

input `int((e*x+d)^2*(b^2*x^2+2*a*b*x+a^2)^(3/2),x)`

output `(x*(60*a**3*d**2 + 60*a**3*d*e*x + 20*a**3*e**2*x**2 + 90*a**2*b*d**2*x + 120*a**2*b*d*e*x**2 + 45*a**2*b*e**2*x**3 + 60*a*b**2*d**2*x**2 + 90*a*b**2*d*e*x**3 + 36*a*b**2*e**2*x**4 + 15*b**3*d**2*x**3 + 24*b**3*d*e*x**4 + 10*b**3*e**2*x**5))/60`

3.280 $\int (d + ex) (a^2 + 2abx + b^2x^2)^{3/2} dx$

| | |
|---|------|
| Optimal result | 2119 |
| Mathematica [A] (verified) | 2119 |
| Rubi [A] (verified) | 2120 |
| Maple [A] (verified) | 2121 |
| Fricas [A] (verification not implemented) | 2122 |
| Sympy [B] (verification not implemented) | 2122 |
| Maxima [B] (verification not implemented) | 2123 |
| Giac [B] (verification not implemented) | 2124 |
| Mupad [B] (verification not implemented) | 2124 |
| Reduce [B] (verification not implemented) | 2125 |

Optimal result

Integrand size = 26, antiderivative size = 71

$$\int (d + ex) (a^2 + 2abx + b^2x^2)^{3/2} dx = \frac{(bd - ae)(a + bx)^3 \sqrt{a^2 + 2abx + b^2x^2}}{4b^2} + \frac{e(a^2 + 2abx + b^2x^2)^{5/2}}{5b^2}$$

output

```
1/4*(-a*e+b*d)*(b*x+a)^3*((b*x+a)^2)^(1/2)/b^2+1/5*e*(b^2*x^2+2*a*b*x+a^2)^(5/2)/b^2
```

Mathematica [A] (verified)

Time = 1.03 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.17

$$\int (d + ex) (a^2 + 2abx + b^2x^2)^{3/2} dx = \frac{x \sqrt{(a + bx)^2 (10a^3(2d + ex) + 10a^2bx(3d + 2ex) + 5ab^2x^2(4d + 3ex) + b^3x^3(5d + 4ex))}}{20(a + bx)}$$

input

```
Integrate[(d + e*x)*(a^2 + 2*a*b*x + b^2*x^2)^(3/2),x]
```

output

```
(x*sqrt[(a + b*x)^2]*(10*a^3*(2*d + e*x) + 10*a^2*b*x*(3*d + 2*e*x) + 5*a*
b^2*x^2*(4*d + 3*e*x) + b^3*x^3*(5*d + 4*e*x)))/(20*(a + b*x))
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.115$, Rules used = {1100, 1079, 17}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a^2 + 2abx + b^2x^2)^{3/2} (d + ex) dx$$

$$\downarrow 1100$$

$$\frac{(bd - ae) \int (a^2 + 2bxa + b^2x^2)^{3/2} dx}{b} + \frac{e(a^2 + 2abx + b^2x^2)^{5/2}}{5b^2}$$

$$\downarrow 1079$$

$$\frac{\sqrt{a^2 + 2abx + b^2x^2}(bd - ae) \int (xb^2 + ab)^3 dx}{b^4(a + bx)} + \frac{e(a^2 + 2abx + b^2x^2)^{5/2}}{5b^2}$$

$$\downarrow 17$$

$$\frac{\sqrt{a^2 + 2abx + b^2x^2}(a + bx)^3(bd - ae)}{4b^2} + \frac{e(a^2 + 2abx + b^2x^2)^{5/2}}{5b^2}$$

input

```
Int[(d + e*x)*(a^2 + 2*a*b*x + b^2*x^2)^(3/2),x]
```

output

```
((b*d - a*e)*(a + b*x)^3*sqrt[a^2 + 2*a*b*x + b^2*x^2])/(4*b^2) + (e*(a^2
+ 2*a*b*x + b^2*x^2)^(5/2))/(5*b^2)
```

Definitions of rubi rules used

rule 17 $\text{Int}[(c_.)*((a_.) + (b_.)*(x_.))^{(m_.)}, x_Symbol] \rightarrow \text{Simp}[c*((a + b*x)^{(m + 1)})/(b*(m + 1)), x] /;$ $\text{FreeQ}[\{a, b, c, m\}, x] \ \&\& \ \text{NeQ}[m, -1]$

rule 1079 $\text{Int}[(a_.) + (b_.)*(x_.) + (c_.)*(x_.)^2]^{(p_.)}, x_Symbol] \rightarrow \text{Simp}[(a + b*x + c*x^2)^{\text{FracPart}[p]}/(c^{\text{IntPart}[p]}*(b/2 + c*x)^{(2*\text{FracPart}[p])}) \ \text{Int}[(b/2 + c*x)^{(2*p)}, x], x] /;$ $\text{FreeQ}[\{a, b, c, p\}, x] \ \&\& \ \text{EqQ}[b^2 - 4*a*c, 0]$

rule 1100 $\text{Int}[(d_.) + (e_.)*(x_.)]*((a_.) + (b_.)*(x_.) + (c_.)*(x_.)^2)^{(p_.)}, x_Symbol] \rightarrow \text{Simp}[e*((a + b*x + c*x^2)^{(p + 1)})/(2*c*(p + 1)), x] + \text{Simp}[(2*c*d - b*e)/(2*c) \ \text{Int}[(a + b*x + c*x^2)^p, x], x] /;$ $\text{FreeQ}[\{a, b, c, d, e, p\}, x] \ \&\& \ \text{EqQ}[b^2 - 4*a*c, 0]$

Maple [A] (verified)

Time = 0.92 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.27

| method | result |
|---------|---|
| gospers | $\frac{x(4e b^3 x^4 + 15a b^2 e x^3 + 5b^3 d x^3 + 20e a^2 b x^2 + 20a b^2 d x^2 + 10a^3 e x + 30a^2 b d x + 20a^3 d) ((bx+a)^2)^{\frac{3}{2}}}{20(bx+a)^3}$ |
| default | $\frac{x(4e b^3 x^4 + 15a b^2 e x^3 + 5b^3 d x^3 + 20e a^2 b x^2 + 20a b^2 d x^2 + 10a^3 e x + 30a^2 b d x + 20a^3 d) ((bx+a)^2)^{\frac{3}{2}}}{20(bx+a)^3}$ |
| orering | $\frac{x(4e b^3 x^4 + 15a b^2 e x^3 + 5b^3 d x^3 + 20e a^2 b x^2 + 20a b^2 d x^2 + 10a^3 e x + 30a^2 b d x + 20a^3 d) (b^2 x^2 + 2abx + a^2)^{\frac{3}{2}}}{20(bx+a)^3}$ |
| risch | $\frac{\sqrt{(bx+a)^2} e b^3 x^5}{5bx+5a} + \frac{\sqrt{(bx+a)^2} (3ea b^2 + b^3 d) x^4}{4bx+4a} + \frac{\sqrt{(bx+a)^2} (3e a^2 b + 3da b^2) x^3}{3bx+3a} + \frac{\sqrt{(bx+a)^2} (a^3 e + 3a^2 b d) x^2}{2bx+2a} + \frac{\sqrt{(bx+a)^2} (a^3 d + 3a^2 b e)}{bx+a}$ |

input $\text{int}((e*x+d)*(b^2*x^2+2*a*b*x+a^2)^(3/2), x, \text{method}=_RETURNVERBOSE)$

output $1/20*x*(4*b^3*e*x^4+15*a*b^2*e*x^3+5*b^3*d*x^3+20*a^2*b*e*x^2+20*a*b^2*d*x^2+10*a^3*e*x+30*a^2*b*d*x+20*a^3*d)*((b*x+a)^2)^(3/2)/(b*x+a)^3$

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.97

$$\int (d + ex) (a^2 + 2abx + b^2x^2)^{3/2} dx = \frac{1}{5} b^3 ex^5 + a^3 dx + \frac{1}{4} (b^3 d + 3 ab^2 e) x^4 + (ab^2 d + a^2 be) x^3 + \frac{1}{2} (3 a^2 bd + a^3 e) x^2$$

input `integrate((e*x+d)*(b^2*x^2+2*a*b*x+a^2)^(3/2),x, algorithm="fricas")`

output `1/5*b^3*e*x^5 + a^3*d*x + 1/4*(b^3*d + 3*a*b^2*e)*x^4 + (a*b^2*d + a^2*b*e)*x^3 + 1/2*(3*a^2*b*d + a^3*e)*x^2`

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 745 vs. 2(56) = 112.

Time = 0.75 (sec) , antiderivative size = 745, normalized size of antiderivative = 10.49

$$\int (d + ex) (a^2 + 2abx + b^2x^2)^{3/2} dx = \text{Too large to display}$$

input `integrate((e*x+d)*(b**2*x**2+2*a*b*x+a**2)**(3/2),x)`

output

```
Piecewise((sqrt(a**2 + 2*a*b*x + b**2*x**2)*(b**2*e*x**4/5 + x**3*(11*a*b*
*3*e/5 + b**4*d)/(4*b**2) + x**2*(26*a**2*b**2*e/5 + 4*a*b**3*d - 7*a*(11*
a*b**3*e/5 + b**4*d)/(4*b))/(3*b**2) + x*(4*a**3*b*e + 6*a**2*b**2*d - 3*a
**2*(11*a*b**3*e/5 + b**4*d)/(4*b**2) - 5*a*(26*a**2*b**2*e/5 + 4*a*b**3*d
- 7*a*(11*a*b**3*e/5 + b**4*d)/(4*b))/(3*b))/(2*b**2) + (a**4*e + 4*a**3*
b*d - 2*a**2*(26*a**2*b**2*e/5 + 4*a*b**3*d - 7*a*(11*a*b**3*e/5 + b**4*d)
/(4*b))/(3*b**2) - 3*a*(4*a**3*b*e + 6*a**2*b**2*d - 3*a**2*(11*a*b**3*e/5
+ b**4*d)/(4*b**2) - 5*a*(26*a**2*b**2*e/5 + 4*a*b**3*d - 7*a*(11*a*b**3*
e/5 + b**4*d)/(4*b))/(3*b))/(2*b))/b**2) + (a/b + x)*(a**4*d - a**2*(4*a**
3*b*e + 6*a**2*b**2*d - 3*a**2*(11*a*b**3*e/5 + b**4*d)/(4*b**2) - 5*a*(26
*a**2*b**2*e/5 + 4*a*b**3*d - 7*a*(11*a*b**3*e/5 + b**4*d)/(4*b))/(3*b))/(
2*b**2) - a*(a**4*e + 4*a**3*b*d - 2*a**2*(26*a**2*b**2*e/5 + 4*a*b**3*d -
7*a*(11*a*b**3*e/5 + b**4*d)/(4*b))/(3*b**2) - 3*a*(4*a**3*b*e + 6*a**2*b
**2*d - 3*a**2*(11*a*b**3*e/5 + b**4*d)/(4*b**2) - 5*a*(26*a**2*b**2*e/5 +
4*a*b**3*d - 7*a*(11*a*b**3*e/5 + b**4*d)/(4*b))/(3*b))/(2*b))/b*log(a/b
+ x)/sqrt(b**2*(a/b + x)**2), Ne(b**2, 0)), (((a**2 + 2*a*b*x)**(5/2))*(-a
*e + 2*b*d)/(10*b) + e*(a**2 + 2*a*b*x)**(7/2)/(14*a*b))/(a*b), Ne(a*b, 0)
), ((d*x + e*x**2/2)*(a**2)**(3/2), True))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 125 vs. $2(54) = 108$.

Time = 0.03 (sec) , antiderivative size = 125, normalized size of antiderivative = 1.76

$$\int (d + ex) (a^2 + 2abx + b^2x^2)^{3/2} dx = \frac{1}{4} (b^2x^2 + 2abx + a^2)^{\frac{3}{2}} dx$$

$$- \frac{(b^2x^2 + 2abx + a^2)^{\frac{3}{2}} aex}{4b} + \frac{(b^2x^2 + 2abx + a^2)^{\frac{3}{2}} ad}{4b}$$

$$- \frac{(b^2x^2 + 2abx + a^2)^{\frac{3}{2}} a^2e}{4b^2} + \frac{(b^2x^2 + 2abx + a^2)^{\frac{5}{2}} e}{5b^2}$$

input

```
integrate((e*x+d)*(b^2*x^2+2*a*b*x+a^2)^(3/2),x, algorithm="maxima")
```

output

```
1/4*(b^2*x^2 + 2*a*b*x + a^2)^(3/2)*d*x - 1/4*(b^2*x^2 + 2*a*b*x + a^2)^(3
/2)*a*e*x/b + 1/4*(b^2*x^2 + 2*a*b*x + a^2)^(3/2)*a*d/b - 1/4*(b^2*x^2 + 2
*a*b*x + a^2)^(3/2)*a^2*e/b^2 + 1/5*(b^2*x^2 + 2*a*b*x + a^2)^(5/2)*e/b^2
```


Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 145 vs. $2(54) = 108$.

Time = 0.12 (sec) , antiderivative size = 145, normalized size of antiderivative = 2.04

$$\int (d + ex) (a^2 + 2abx + b^2x^2)^{3/2} dx = \frac{1}{5} b^3 ex^5 \operatorname{sgn}(bx + a) \\ + \frac{1}{4} b^3 dx^4 \operatorname{sgn}(bx + a) + \frac{3}{4} ab^2 ex^4 \operatorname{sgn}(bx + a) + ab^2 dx^3 \operatorname{sgn}(bx + a) \\ + a^2 bex^3 \operatorname{sgn}(bx + a) + \frac{3}{2} a^2 bdx^2 \operatorname{sgn}(bx + a) + \frac{1}{2} a^3 ex^2 \operatorname{sgn}(bx + a) \\ + a^3 dx \operatorname{sgn}(bx + a) + \frac{(5a^4bd - a^5e) \operatorname{sgn}(bx + a)}{20b^2}$$

input `integrate((e*x+d)*(b^2*x^2+2*a*b*x+a^2)^(3/2),x, algorithm="giac")`

output `1/5*b^3*e*x^5*sgn(b*x + a) + 1/4*b^3*d*x^4*sgn(b*x + a) + 3/4*a*b^2*e*x^4*sgn(b*x + a) + a*b^2*d*x^3*sgn(b*x + a) + a^2*b*e*x^3*sgn(b*x + a) + 3/2*a^2*b*d*x^2*sgn(b*x + a) + 1/2*a^3*e*x^2*sgn(b*x + a) + a^3*d*x*sgn(b*x + a) + 1/20*(5*a^4*b*d - a^5*e)*sgn(b*x + a)/b^2`

Mupad [B] (verification not implemented)

Time = 9.84 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.59

$$\int (d + ex) (a^2 + 2abx + b^2x^2)^{3/2} dx = \frac{(a + bx) (5bd - ae + 4bex) (a^2 + 2abx + b^2x^2)^{3/2}}{20b^2}$$

input `int((d + e*x)*(a^2 + b^2*x^2 + 2*a*b*x)^(3/2),x)`

output `((a + b*x)*(5*b*d - a*e + 4*b*e*x)*(a^2 + b^2*x^2 + 2*a*b*x)^(3/2))/(20*b^2)`

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.03

$$\int (d + ex) (a^2 + 2abx + b^2x^2)^{3/2} dx = \frac{x(4b^3ex^4 + 15ab^2ex^3 + 5b^3dx^3 + 20a^2bex^2 + 20ab^2dx^2 + 10a^3ex + 30a^2bdx + 20a^3d)}{20}$$

input `int((e*x+d)*(b^2*x^2+2*a*b*x+a^2)^(3/2),x)`output `(x*(20*a**3*d + 10*a**3*e*x + 30*a**2*b*d*x + 20*a**2*b*e*x**2 + 20*a*b**2*d*x**2 + 15*a*b**2*e*x**3 + 5*b**3*d*x**3 + 4*b**3*e*x**4))/20`

3.281 $\int (a^2 + 2abx + b^2x^2)^{3/2} dx$

| | |
|---|------|
| Optimal result | 2126 |
| Mathematica [A] (verified) | 2126 |
| Rubi [A] (verified) | 2127 |
| Maple [A] (verified) | 2128 |
| Fricas [A] (verification not implemented) | 2128 |
| Sympy [B] (verification not implemented) | 2129 |
| Maxima [B] (verification not implemented) | 2130 |
| Giac [B] (verification not implemented) | 2130 |
| Mupad [B] (verification not implemented) | 2131 |
| Reduce [B] (verification not implemented) | 2131 |

Optimal result

Integrand size = 20, antiderivative size = 34

$$\int (a^2 + 2abx + b^2x^2)^{3/2} dx = \frac{(a + bx)^3 \sqrt{a^2 + 2abx + b^2x^2}}{4b}$$

output $1/4*(b*x+a)^3*((b*x+a)^2)^{(1/2)}/b$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.68

$$\int (a^2 + 2abx + b^2x^2)^{3/2} dx = \frac{(a + bx) ((a + bx)^2)^{3/2}}{4b}$$

input `Integrate[(a^2 + 2*a*b*x + b^2*x^2)^(3/2), x]`

output $((a + b*x)*((a + b*x)^2)^{(3/2)})/(4*b)$

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {1079, 17}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a^2 + 2abx + b^2x^2)^{3/2} dx$$

$$\downarrow 1079$$

$$\frac{\sqrt{a^2 + 2abx + b^2x^2} \int (xb^2 + ab)^3 dx}{b^3(a + bx)}$$

$$\downarrow 17$$

$$\frac{(a + bx)^3 \sqrt{a^2 + 2abx + b^2x^2}}{4b}$$

input `Int[(a^2 + 2*a*b*x + b^2*x^2)^(3/2), x]`

output `((a + b*x)^3*Sqrt[a^2 + 2*a*b*x + b^2*x^2])/(4*b)`

Defintions of rubi rules used

rule 17 `Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1))/(b*(m + 1)), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]`

rule 1079 `Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(a + b*x + c*x^2)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x)^(2*FracPart[p])) Int[(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c, p}, x] && EqQ[b^2 - 4*a*c, 0]`

Maple [A] (verified)

Time = 0.85 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.59

| method | result | size |
|---------|---|------|
| default | $\frac{\left((bx+a)^2\right)^{\frac{3}{2}}(bx+a)}{4b}$ | 20 |
| risch | $\frac{(bx+a)^3\sqrt{(bx+a)^2}}{4b}$ | 22 |
| gospers | $\frac{x(b^3x^3+4ab^2x^2+6a^2bx+4a^3)\left((bx+a)^2\right)^{\frac{3}{2}}}{4(bx+a)^3}$ | 49 |
| orering | $\frac{x(b^3x^3+4ab^2x^2+6a^2bx+4a^3)(b^2x^2+2abx+a^2)^{\frac{3}{2}}}{4(bx+a)^3}$ | 58 |

input `int((b^2*x^2+2*a*b*x+a^2)^(3/2),x,method=_RETURNVERBOSE)`

output `1/4*((b*x+a)^2)^(3/2)*(b*x+a)/b`

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.91

$$\int (a^2 + 2abx + b^2x^2)^{3/2} dx = \frac{1}{4}b^3x^4 + ab^2x^3 + \frac{3}{2}a^2bx^2 + a^3x$$

input `integrate((b^2*x^2+2*a*b*x+a^2)^(3/2),x, algorithm="fricas")`

output `1/4*b^3*x^4 + a*b^2*x^3 + 3/2*a^2*b*x^2 + a^3*x`

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 60 vs. $2(19) = 38$.

Time = 0.75 (sec) , antiderivative size = 294, normalized size of antiderivative = 8.65

$$\int (a^2 + 2abx + b^2x^2)^{3/2} dx = a^2 \left(\begin{cases} \left(\frac{a}{2b} + \frac{x}{2}\right) \sqrt{a^2 + 2abx + b^2x^2} & \text{for } b^2 \neq 0 \\ \frac{(a^2 + 2abx)^{3/2}}{3ab} & \text{for } ab \neq 0 \\ x\sqrt{a^2} & \text{otherwise} \end{cases} \right) \\ + 2ab \left(\begin{cases} \sqrt{a^2 + 2abx + b^2x^2} \left(-\frac{a^2}{6b^2} + \frac{ax}{6b} + \frac{x^2}{3}\right) & \text{for } b^2 \neq 0 \\ \frac{-\frac{a^2(a^2 + 2abx)^{3/2}}{3} + \frac{(a^2 + 2abx)^{5/2}}{5}}{2a^2b^2} & \text{for } ab \neq 0 \\ \frac{x^2\sqrt{a^2}}{2} & \text{otherwise} \end{cases} \right) \\ + b^2 \left(\begin{cases} \sqrt{a^2 + 2abx + b^2x^2} \left(\frac{a^3}{12b^3} - \frac{a^2x}{12b^2} + \frac{ax^2}{12b} + \frac{x^3}{4}\right) & \text{for } b^2 \neq 0 \\ \frac{\frac{a^4(a^2 + 2abx)^{3/2}}{3} - \frac{2a^2(a^2 + 2abx)^{5/2}}{4a^3b^3} + \frac{(a^2 + 2abx)^{7/2}}{7}}{4a^3b^3} & \text{for } ab \neq 0 \\ \frac{x^3\sqrt{a^2}}{3} & \text{otherwise} \end{cases} \right)$$

input `integrate((b**2*x**2+2*a*b*x+a**2)**(3/2),x)`

output `a**2*Piecewise(((a/(2*b) + x/2)*sqrt(a**2 + 2*a*b*x + b**2*x**2), Ne(b**2, 0)), ((a**2 + 2*a*b*x)**(3/2)/(3*a*b), Ne(a*b, 0)), (x*sqrt(a**2), True)) + 2*a*b*Piecewise((sqrt(a**2 + 2*a*b*x + b**2*x**2)*(-a**2/(6*b**2) + a*x/(6*b) + x**2/3), Ne(b**2, 0)), ((-a**2*(a**2 + 2*a*b*x)**(3/2)/3 + (a**2 + 2*a*b*x)**(5/2)/5)/(2*a**2*b**2), Ne(a*b, 0)), (x**2*sqrt(a**2)/2, True)) + b**2*Piecewise((sqrt(a**2 + 2*a*b*x + b**2*x**2)*(a**3/(12*b**3) - a**2*x/(12*b**2) + a*x**2/(12*b) + x**3/4), Ne(b**2, 0)), ((a**4*(a**2 + 2*a*b*x)**(3/2)/3 - 2*a**2*(a**2 + 2*a*b*x)**(5/2)/5 + (a**2 + 2*a*b*x)**(7/2)/7)/(4*a**3*b**3), Ne(a*b, 0)), (x**3*sqrt(a**2)/3, True))`

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 46 vs. $2(21) = 42$.

Time = 0.03 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.35

$$\int (a^2 + 2abx + b^2x^2)^{3/2} dx = \frac{1}{4} (b^2x^2 + 2abx + a^2)^{\frac{3}{2}}x + \frac{(b^2x^2 + 2abx + a^2)^{\frac{3}{2}}a}{4b}$$

input `integrate((b^2*x^2+2*a*b*x+a^2)^(3/2),x, algorithm="maxima")`

output `1/4*(b^2*x^2 + 2*a*b*x + a^2)^(3/2)*x + 1/4*(b^2*x^2 + 2*a*b*x + a^2)^(3/2)*a/b`

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 57 vs. $2(21) = 42$.

Time = 0.12 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.68

$$\int (a^2 + 2abx + b^2x^2)^{3/2} dx = \frac{1}{2} (bx^2 + 2ax)a^2\text{sgn}(bx + a) + \frac{a^4\text{sgn}(bx + a)}{4b} + \frac{1}{4} (bx^2 + 2ax)^2b\text{sgn}(bx + a)$$

input `integrate((b^2*x^2+2*a*b*x+a^2)^(3/2),x, algorithm="giac")`

output `1/2*(b*x^2 + 2*a*x)*a^2*sgn(b*x + a) + 1/4*a^4*sgn(b*x + a)/b + 1/4*(b*x^2 + 2*a*x)^2*b*sgn(b*x + a)`

Mupad [B] (verification not implemented)

Time = 9.66 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.94

$$\int (a^2 + 2abx + b^2x^2)^{3/2} dx = \frac{(xb^2 + ab)(a^2 + 2abx + b^2x^2)^{3/2}}{4b^2}$$

input `int((a^2 + b^2*x^2 + 2*a*b*x)^(3/2),x)`output `((a*b + b^2*x)*(a^2 + b^2*x^2 + 2*a*b*x)^(3/2))/(4*b^2)`**Reduce [B] (verification not implemented)**

Time = 0.21 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.94

$$\int (a^2 + 2abx + b^2x^2)^{3/2} dx = \frac{x(b^3x^3 + 4ab^2x^2 + 6a^2bx + 4a^3)}{4}$$

input `int((b^2*x^2+2*a*b*x+a^2)^(3/2),x)`output `(x*(4*a**3 + 6*a**2*b*x + 4*a*b**2*x**2 + b**3*x**3))/4`

3.282 $\int \frac{(a^2+2abx+b^2x^2)^{3/2}}{d+ex} dx$

| | |
|---|------|
| Optimal result | 2132 |
| Mathematica [A] (verified) | 2132 |
| Rubi [A] (verified) | 2133 |
| Maple [A] (verified) | 2134 |
| Fricas [A] (verification not implemented) | 2135 |
| Sympy [F] | 2135 |
| Maxima [F(-2)] | 2135 |
| Giac [A] (verification not implemented) | 2136 |
| Mupad [F(-1)] | 2136 |
| Reduce [B] (verification not implemented) | 2137 |

Optimal result

Integrand size = 28, antiderivative size = 166

$$\int \frac{(a^2 + 2abx + b^2x^2)^{3/2}}{d + ex} dx = \frac{b(bd - ae)^2x\sqrt{a^2 + 2abx + b^2x^2}}{e^3(a + bx)} - \frac{(bd - ae)(a + bx)\sqrt{a^2 + 2abx + b^2x^2}}{2e^2} + \frac{(a + bx)^2\sqrt{a^2 + 2abx + b^2x^2}}{3e} - \frac{(bd - ae)^3\sqrt{a^2 + 2abx + b^2x^2} \log(d + ex)}{e^4(a + bx)}$$

output

```
b*(-a*e+b*d)^2*x*((b*x+a)^2)^(1/2)/e^3/(b*x+a)-1/2*(-a*e+b*d)*(b*x+a)*((b*x+a)^2)^(1/2)/e^2+1/3*(b*x+a)^2*((b*x+a)^2)^(1/2)/e-(-a*e+b*d)^3*((b*x+a)^2)^(1/2)*ln(e*x+d)/e^4/(b*x+a)
```

Mathematica [A] (verified)

Time = 1.04 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.55

$$\int \frac{(a^2 + 2abx + b^2x^2)^{3/2}}{d + ex} dx = \frac{\sqrt{(a + bx)^2(bex(18a^2e^2 + 9abe(-2d + ex) + b^2(6d^2 - 3dex + 2e^2x^2)) - 6(bd - ae)^2)}{6e^4(a + bx)}$$

input

```
Integrate[(a^2 + 2*a*b*x + b^2*x^2)^(3/2)/(d + e*x),x]
```

output

```
(Sqrt[(a + b*x)^2]*(b*e*x*(18*a^2*e^2 + 9*a*b*e*(-2*d + e*x) + b^2*(6*d^2 - 3*d*e*x + 2*e^2*x^2)) - 6*(b*d - a*e)^3*Log[d + e*x]))/(6*e^4*(a + b*x))
```

Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 102, normalized size of antiderivative = 0.61, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {1102, 27, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\begin{aligned}
 & \int \frac{(a^2 + 2abx + b^2x^2)^{3/2}}{d + ex} dx \\
 & \quad \downarrow \text{1102} \\
 & \frac{\sqrt{a^2 + 2abx + b^2x^2} \int \frac{b^3(a+bx)^3}{d+ex} dx}{b^3(a + bx)} \\
 & \quad \downarrow \text{27} \\
 & \frac{\sqrt{a^2 + 2abx + b^2x^2} \int \frac{(a+bx)^3}{d+ex} dx}{a + bx} \\
 & \quad \downarrow \text{49} \\
 & \frac{\sqrt{a^2 + 2abx + b^2x^2} \int \left(\frac{(ae-bd)^3}{e^3(d+ex)} + \frac{b(bd-ae)^2}{e^3} + \frac{b(a+bx)^2}{e} - \frac{b(bd-ae)(a+bx)}{e^2} \right) dx}{a + bx} \\
 & \quad \downarrow \text{2009} \\
 & \frac{\sqrt{a^2 + 2abx + b^2x^2} \left(-\frac{(bd-ae)^3 \log(d+ex)}{e^4} + \frac{bx(bd-ae)^2}{e^3} - \frac{(a+bx)^2(bd-ae)}{2e^2} + \frac{(a+bx)^3}{3e} \right)}{a + bx}
 \end{aligned}$$

input

```
Int[(a^2 + 2*a*b*x + b^2*x^2)^(3/2)/(d + e*x),x]
```

output

```
(Sqrt[a^2 + 2*a*b*x + b^2*x^2]*((b*(b*d - a*e)^2*x)/e^3 - ((b*d - a*e)*(a + b*x)^2)/(2*e^2) + (a + b*x)^3/(3*e) - ((b*d - a*e)^3*Log[d + e*x])/e^4))/(a + b*x)
```

Definitions of rubi rules used

rule 27 $\text{Int}[(a_*)(Fx_), x_Symbol] \rightarrow \text{Simp}[a \text{ Int}[Fx, x], x] /; \text{FreeQ}[a, x] \ \&\& \ !\text{MatchQ}[Fx, (b_*)(Gx_)] /; \text{FreeQ}[b, x]$

rule 49 $\text{Int}[((a_.) + (b_.)(x_))^{(m_.)} * ((c_.) + (d_.)(x_))^{(n_.)}, x_Symbol] \rightarrow \text{Int}[\text{ExpandIntegrand}[(a + b*x)^m * (c + d*x)^n, x], x] /; \text{FreeQ}\{a, b, c, d\}, x \ \&\& \ \text{IGtQ}[m, 0] \ \&\& \ \text{IGtQ}[m + n + 2, 0]$

rule 1102 $\text{Int}[((d_.) + (e_.)(x_))^{(m_.)} * ((a_) + (b_.)(x_) + (c_.)(x_)^2)^{(p_.)}, x_Symbol] \rightarrow \text{Simp}[(a + b*x + c*x^2)^{\text{FracPart}[p]} / (c^{\text{IntPart}[p]} * (b/2 + c*x)^{(2*\text{FracPart}[p])}) \text{ Int}[(d + e*x)^m * (b/2 + c*x)^{(2*p)}, x], x] /; \text{FreeQ}\{a, b, c, d, e, m, p\}, x \ \&\& \ \text{EqQ}[b^2 - 4*a*c, 0]$

rule 2009 $\text{Int}[u_, x_Symbol] \rightarrow \text{Simp}[\text{IntSum}[u, x], x] /; \text{SumQ}[u]$

Maple [A] (verified)

Time = 1.22 (sec) , antiderivative size = 141, normalized size of antiderivative = 0.85

| method | result |
|---------|--|
| risch | $\frac{\sqrt{(bx+a)^2 b \left(\frac{1}{3}b^2 e^2 x^3 + \frac{3}{2}x^2 ab e^2 - \frac{1}{2}b^2 de x^2 + 3a^2 e^2 x - 3abdex + b^2 d^2 x\right)}}{(bx+a)e^3} + \frac{\sqrt{(bx+a)^2 (e^3 a^3 - 3a^2 bd e^2 + 3a b^2 d^2 e - b^3 d^3)} \ln(ex+d)}{(bx+a)e^4}$ |
| default | $\frac{\left((bx+a)^2\right)^{\frac{3}{2}} \left(2e^3 x^3 b^3 + 9x^2 a b^2 e^3 - 3x^2 b^3 d e^2 + 6 \ln(ex+d) a^3 e^3 - 18 \ln(ex+d) a^2 b d e^2 + 18 \ln(ex+d) a b^2 d^2 e - 6 \ln(ex+d) b^3 d^3 + 18 a^2 b\right)}{6(bx+a)^3 e^4}$ |

input $\text{int}((b^2*x^2+2*a*b*x+a^2)^{(3/2)}/(e*x+d), x, \text{method}=_RETURNVERBOSE)$

output $((b*x+a)^2)^{(1/2)}/(b*x+a)*b/e^3*(1/3*b^2*e^2*x^3+3/2*x^2*a*b*e^2-1/2*b^2*d*e*x^2+3*a^2*e^2*x-3*a*b*d*e*x+b^2*d^2*x)+((b*x+a)^2)^{(1/2)}/(b*x+a)*(a^3*e^3-3*a^2*b*d*e^2+3*a*b^2*d^2*e-b^3*d^3)/e^4*\ln(e*x+d)$

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 115, normalized size of antiderivative = 0.69

$$\int \frac{(a^2 + 2abx + b^2x^2)^{3/2}}{d + ex} dx = \frac{2b^3e^3x^3 - 3(b^3de^2 - 3ab^2e^3)x^2 + 6(b^3d^2e - 3ab^2de^2 + 3a^2be^3)x - 6(b^3d^3 - 3a^2bde^2 + 3a^2b^2de - a^3e^3)\log(ex + d)}{6e^4}$$

input `integrate((b^2*x^2+2*a*b*x+a^2)^(3/2)/(e*x+d),x, algorithm="fricas")`

output `1/6*(2*b^3*e^3*x^3 - 3*(b^3*d*e^2 - 3*a*b^2*e^3)*x^2 + 6*(b^3*d^2*e - 3*a*b^2*d*e^2 + 3*a^2*b*e^3)*x - 6*(b^3*d^3 - 3*a*b^2*d^2*e + 3*a^2*b*d*e^2 - a^3*e^3)*log(e*x + d))/e^4`

Sympy [F]

$$\int \frac{(a^2 + 2abx + b^2x^2)^{3/2}}{d + ex} dx = \int \frac{((a + bx)^2)^{\frac{3}{2}}}{d + ex} dx$$

input `integrate((b**2*x**2+2*a*b*x+a**2)**(3/2)/(e*x+d),x)`

output `Integral(((a + b*x)**2)**(3/2)/(d + e*x), x)`

Maxima [F(-2)]

Exception generated.

$$\int \frac{(a^2 + 2abx + b^2x^2)^{3/2}}{d + ex} dx = \text{Exception raised: ValueError}$$

input `integrate((b^2*x^2+2*a*b*x+a^2)^(3/2)/(e*x+d),x, algorithm="maxima")`

output

```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
help (example of legal syntax is 'assume(a*e-b*d>0)', see `assume?` for m
ore detail
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 176, normalized size of antiderivative = 1.06

$$\int \frac{(a^2 + 2abx + b^2x^2)^{3/2}}{d + ex} dx = \frac{2b^3e^2x^3\operatorname{sgn}(bx + a) - 3b^3dex^2\operatorname{sgn}(bx + a) + 9ab^2e^2x^2\operatorname{sgn}(bx + a) + 6b^3d^2\operatorname{sgn}(bx + a)}{6e^3} - \frac{(b^3d^3\operatorname{sgn}(bx + a) - 3ab^2d^2e\operatorname{sgn}(bx + a) + 3a^2bde^2\operatorname{sgn}(bx + a) - a^3e^3\operatorname{sgn}(bx + a)) \log(|ex + d|)}{e^4}$$

input

```
integrate((b^2*x^2+2*a*b*x+a^2)^(3/2)/(e*x+d),x, algorithm="giac")
```

output

```
1/6*(2*b^3*e^2*x^3*sgn(b*x + a) - 3*b^3*d*e*x^2*sgn(b*x + a) + 9*a*b^2*e^2
*x^2*sgn(b*x + a) + 6*b^3*d^2*x*sgn(b*x + a) - 18*a*b^2*d*e*x*sgn(b*x + a)
+ 18*a^2*b*e^2*x*sgn(b*x + a))/e^3 - (b^3*d^3*sgn(b*x + a) - 3*a*b^2*d^2*
e*sgn(b*x + a) + 3*a^2*b*d*e^2*sgn(b*x + a) - a^3*e^3*sgn(b*x + a))*log(ab
s(e*x + d))/e^4
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a^2 + 2abx + b^2x^2)^{3/2}}{d + ex} dx = \int \frac{(a^2 + 2abx + b^2x^2)^{3/2}}{d + ex} dx$$

input

```
int((a^2 + b^2*x^2 + 2*a*b*x)^(3/2)/(d + e*x),x)
```

output

```
int((a^2 + b^2*x^2 + 2*a*b*x)^(3/2)/(d + e*x), x)
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 132, normalized size of antiderivative = 0.80

$$\int \frac{(a^2 + 2abx + b^2x^2)^{3/2}}{d + ex} dx = \frac{6 \log(ex + d) a^3 e^3 - 18 \log(ex + d) a^2 b d e^2 + 18 \log(ex + d) a b^2 d^2 e - 6 \log(ex + d) a^2 b^2 d e^2 + 6 \log(ex + d) a^3 b^2 d^2 e^2}{6 e^4}$$

input `int((b^2*x^2+2*a*b*x+a^2)^(3/2)/(e*x+d),x)`output `(6*log(d + e*x)*a**3*e**3 - 18*log(d + e*x)*a**2*b*d*e**2 + 18*log(d + e*x)*a*b**2*d**2*e - 6*log(d + e*x)*b**3*d**3 + 18*a**2*b*e**3*x - 18*a*b**2*d*e**2*x + 9*a*b**2*e**3*x**2 + 6*b**3*d**2*e*x - 3*b**3*d*e**2*x**2 + 2*b**3*e**3*x**3)/(6*e**4)`

3.283
$$\int \frac{(a^2+2abx+b^2x^2)^{3/2}}{(d+ex)^2} dx$$

| | |
|---|------|
| Optimal result | 2138 |
| Mathematica [A] (verified) | 2139 |
| Rubi [A] (verified) | 2139 |
| Maple [A] (verified) | 2141 |
| Fricas [A] (verification not implemented) | 2141 |
| Sympy [F] | 2142 |
| Maxima [F(-2)] | 2142 |
| Giac [A] (verification not implemented) | 2142 |
| Mupad [F(-1)] | 2143 |
| Reduce [B] (verification not implemented) | 2143 |

Optimal result

Integrand size = 28, antiderivative size = 183

$$\int \frac{(a^2 + 2abx + b^2x^2)^{3/2}}{(d + ex)^2} dx = -\frac{b^2(2bd - 3ae)x\sqrt{a^2 + 2abx + b^2x^2}}{e^3(a + bx)} + \frac{b^3x^2\sqrt{a^2 + 2abx + b^2x^2}}{2e^2(a + bx)} + \frac{(bd - ae)^3\sqrt{a^2 + 2abx + b^2x^2}}{e^4(a + bx)(d + ex)} + \frac{3b(bd - ae)^2\sqrt{a^2 + 2abx + b^2x^2} \log(d + ex)}{e^4(a + bx)}$$

output

```
-b^2*(-3*a*e+2*b*d)*x*((b*x+a)^2)^(1/2)/e^3/(b*x+a)+1/2*b^3*x^2*((b*x+a)^2)^(1/2)/e^2/(b*x+a)+(-a*e+b*d)^3*((b*x+a)^2)^(1/2)/e^4/(b*x+a)/(e*x+d)+3*b*(-a*e+b*d)^2*((b*x+a)^2)^(1/2)*ln(e*x+d)/e^4/(b*x+a)
```

Mathematica [A] (verified)

Time = 1.07 (sec) , antiderivative size = 132, normalized size of antiderivative = 0.72

$$\int \frac{(a^2 + 2abx + b^2x^2)^{3/2}}{(d + ex)^2} dx = \frac{\sqrt{(a + bx)^2(6a^2bde^2 - 2a^3e^3 + 6ab^2e(-d^2 + dex + e^2x^2)) + b^3(2d^3 - 4d^2ex + 2e^2x^2)}}{2e^4(a + bx)(d + ex)}$$

input `Integrate[(a^2 + 2*a*b*x + b^2*x^2)^(3/2)/(d + e*x)^2,x]`

output `(Sqrt[(a + b*x)^2]*(6*a^2*b*d*e^2 - 2*a^3*e^3 + 6*a*b^2*e*(-d^2 + d*e*x + e^2*x^2) + b^3*(2*d^3 - 4*d^2*e*x - 3*d*e^2*x^2 + e^3*x^3) + 6*b*(b*d - a*e)^2*(d + e*x)*Log[d + e*x]))/(2*e^4*(a + b*x)*(d + e*x))`

Rubi [A] (verified)

Time = 0.46 (sec) , antiderivative size = 103, normalized size of antiderivative = 0.56, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {1102, 27, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\begin{aligned} & \int \frac{(a^2 + 2abx + b^2x^2)^{3/2}}{(d + ex)^2} dx \\ & \quad \downarrow \text{1102} \\ & \frac{\sqrt{a^2 + 2abx + b^2x^2} \int \frac{b^3(a+bx)^3}{(d+ex)^2} dx}{b^3(a + bx)} \\ & \quad \downarrow \text{27} \\ & \frac{\sqrt{a^2 + 2abx + b^2x^2} \int \frac{(a+bx)^3}{(d+ex)^2} dx}{a + bx} \\ & \quad \downarrow \text{49} \\ & \frac{\sqrt{a^2 + 2abx + b^2x^2} \int \left(\frac{xb^3}{e^2} - \frac{(2bd-3ae)b^2}{e^3} + \frac{3(bd-ae)^2b}{e^3(d+ex)} + \frac{(ae-bd)^3}{e^3(d+ex)^2} \right) dx}{a + bx} \end{aligned}$$

$$\frac{\sqrt{a^2 + 2abx + b^2x^2} \left(-\frac{b^2x(2bd-3ae)}{e^3} + \frac{(bd-ae)^3}{e^4(d+ex)} + \frac{3b(bd-ae)^2 \log(d+ex)}{e^4} + \frac{b^3x^2}{2e^2} \right)}{a + bx}$$

↓ 2009

input `Int[(a^2 + 2*a*b*x + b^2*x^2)^(3/2)/(d + e*x)^2,x]`

output `(Sqrt[a^2 + 2*a*b*x + b^2*x^2]*(-((b^2*(2*b*d - 3*a*e)*x)/e^3) + (b^3*x^2)/(2*e^2) + (b*d - a*e)^3/(e^4*(d + e*x)) + (3*b*(b*d - a*e)^2*Log[d + e*x])/e^4))/(a + b*x)`

Definitions of rubi rules used

rule 27 `Int[(a_)*(Fx_), x_Symbol] := Simp[a Int[Fx, x], x] /; FreeQ[a, x] && !MatchQ[Fx, (b_)*(Gx_)] /; FreeQ[b, x]`

rule 49 `Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] := Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] && IGtQ[m, 0] && IGtQ[m + n + 2, 0]`

rule 1102 `Int[((d_.) + (e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] := Simp[(a + b*x + c*x^2)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x)^(2*FracPart[p])) Int[(d + e*x)^m*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && EqQ[b^2 - 4*a*c, 0]`

rule 2009 `Int[u_, x_Symbol] := Simp[IntSum[u, x], x] /; SumQ[u]`

Maple [A] (verified)

Time = 1.28 (sec) , antiderivative size = 156, normalized size of antiderivative = 0.85

| method | result |
|---------|---|
| risch | $\frac{\sqrt{(bx+a)^2 b^2 (\frac{1}{2}be x^2+3aex-2bdx)}}{(bx+a)e^3} + \frac{3\sqrt{(bx+a)^2 b(e^2a^2-2abde+b^2d^2)} \ln(ex+d)}{(bx+a)e^4} - \frac{\sqrt{(bx+a)^2 (e^3a^3-3a^2bd e^2+3a b^2d^2e-b^3d^3)}}{(bx+a)e^4(ex+d)}$ |
| default | $\frac{((bx+a)^2)^{\frac{3}{2}} (e^3x^3b^3+6 \ln(ex+d)a^2b e^3x-12 \ln(ex+d)a b^2d e^2x+6 \ln(ex+d)b^3d^2ex+6x^2a b^2e^3-3x^2b^3d e^2+6 \ln(ex+d)a^2bd e^2-12 \ln(ex+d)a b^2d^2e)}{2(bx+a)^3e^4(ex+d)}$ |

input `int((b^2*x^2+2*a*b*x+a^2)^(3/2)/(e*x+d)^2,x,method=_RETURNVERBOSE)`

output
$$\frac{((b*x+a)^2)^{(1/2)}}{(b*x+a)*b^2/e^3*(1/2*b*e*x^2+3*a*e*x-2*b*d*x)} + 3*\frac{((b*x+a)^2)^{(1/2)}}{(b*x+a)*b/e^4*(a^2*e^2-2*a*b*d*e+b^2*d^2)*\ln(e*x+d)} - \frac{((b*x+a)^2)^{(1/2)}}{(b*x+a)/e^4*(a^3*e^3-3*a^2*b*d*e^2+3*a*b^2*d^2*e-b^3*d^3)/(e*x+d)}$$

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 172, normalized size of antiderivative = 0.94

$$\int \frac{(a^2 + 2abx + b^2x^2)^{3/2}}{(d + ex)^2} dx = \frac{b^3e^3x^3 + 2b^3d^3 - 6ab^2d^2e + 6a^2bde^2 - 2a^3e^3 - 3(b^3de^2 - 2ab^2e^3)x^2 - 2(a^2bde^2 - 2ab^2d^2e + a^2b^2d^2e - b^3d^3)x + 6(b^3d^3 - 2a*b^2*d^2*e + a^2*b*d*e^2 + (b^3*d^2*e - 2*a*b^2*d*e^2 + a^2*b*e^3)*x)*\log(e*x + d)}{(e^5*x + d*e^4)}$$

input `integrate((b^2*x^2+2*a*b*x+a^2)^(3/2)/(e*x+d)^2,x, algorithm="fricas")`

output
$$\frac{1/2*(b^3*e^3*x^3 + 2*b^3*d^3 - 6*a*b^2*d^2*e + 6*a^2*b*d*e^2 - 2*a^3*e^3 - 3*(b^3*d*e^2 - 2*a*b^2*e^3)*x^2 - 2*(2*b^3*d^2*e - 3*a*b^2*d*e^2)*x + 6*(b^3*d^3 - 2*a*b^2*d^2*e + a^2*b*d*e^2 + (b^3*d^2*e - 2*a*b^2*d*e^2 + a^2*b*e^3)*x)*\log(e*x + d)}{(e^5*x + d*e^4)}$$

Sympy [F]

$$\int \frac{(a^2 + 2abx + b^2x^2)^{3/2}}{(d + ex)^2} dx = \int \frac{((a + bx)^2)^{3/2}}{(d + ex)^2} dx$$

input `integrate((b**2*x**2+2*a*b*x+a**2)**(3/2)/(e*x+d)**2,x)`

output `Integral(((a + b*x)**2)**(3/2)/(d + e*x)**2, x)`

Maxima [F(-2)]

Exception generated.

$$\int \frac{(a^2 + 2abx + b^2x^2)^{3/2}}{(d + ex)^2} dx = \text{Exception raised: ValueError}$$

input `integrate((b^2*x^2+2*a*b*x+a^2)^(3/2)/(e*x+d)^2,x, algorithm="maxima")`

output `Exception raised: ValueError >> Computation failed since Maxima requested additional constraints; using the 'assume' command before evaluation *may* help (example of legal syntax is 'assume(a*e-b*d>0)', see `assume?` for more detail`

Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 178, normalized size of antiderivative = 0.97

$$\begin{aligned} \int \frac{(a^2 + 2abx + b^2x^2)^{3/2}}{(d + ex)^2} dx &= \frac{3(b^3d^2\text{sgn}(bx + a) - 2ab^2d\text{sgn}(bx + a) + a^2be^2\text{sgn}(bx + a)) \log(|ex + d|)}{e^4} \\ &+ \frac{b^3e^2x^2\text{sgn}(bx + a) - 4b^3dex\text{sgn}(bx + a) + 6ab^2e^2x\text{sgn}(bx + a)}{2e^4} \\ &+ \frac{b^3d^3\text{sgn}(bx + a) - 3ab^2d^2\text{sgn}(bx + a) + 3a^2bde^2\text{sgn}(bx + a) - a^3e^3\text{sgn}(bx + a)}{(ex + d)e^4} \end{aligned}$$

input `integrate((b^2*x^2+2*a*b*x+a^2)^(3/2)/(e*x+d)^2,x, algorithm="giac")`

output `3*(b^3*d^2*sgn(b*x + a) - 2*a*b^2*d*e*sgn(b*x + a) + a^2*b*e^2*sgn(b*x + a))*log(abs(e*x + d))/e^4 + 1/2*(b^3*e^2*x^2*sgn(b*x + a) - 4*b^3*d*e*x*sgn(b*x + a) + 6*a*b^2*e^2*x*sgn(b*x + a))/e^4 + (b^3*d^3*sgn(b*x + a) - 3*a*b^2*d^2*e*sgn(b*x + a) + 3*a^2*b*d*e^2*sgn(b*x + a) - a^3*e^3*sgn(b*x + a))/((e*x + d)*e^4)`

Mupad [F(-1)]

Timed out.

$$\int \frac{(a^2 + 2abx + b^2x^2)^{3/2}}{(d + ex)^2} dx = \int \frac{(a^2 + 2abx + b^2x^2)^{3/2}}{(d + ex)^2} dx$$

input `int((a^2 + b^2*x^2 + 2*a*b*x)^(3/2)/(d + e*x)^2,x)`

output `int((a^2 + b^2*x^2 + 2*a*b*x)^(3/2)/(d + e*x)^2, x)`

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 197, normalized size of antiderivative = 1.08

$$\int \frac{(a^2 + 2abx + b^2x^2)^{3/2}}{(d + ex)^2} dx = \frac{6 \log(ex + d) a^2 b d^2 e^2 + 6 \log(ex + d) a^2 b d e^3 x - 12 \log(ex + d) a b^2 d^3 e - 12 \log(ex + d) a^2 b d^2 e^2 + 6 \log(ex + d) a^2 b d e^3 x - 12 \log(ex + d) a b^2 d^3 e - 12 \log(ex + d) a^2 b d^2 e^2}{(d + ex)^2}$$

input `int((b^2*x^2+2*a*b*x+a^2)^(3/2)/(e*x+d)^2,x)`

output `(6*log(d + e*x)*a**2*b*d**2*e**2 + 6*log(d + e*x)*a**2*b*d*e**3*x - 12*log(d + e*x)*a*b**2*d**3*e - 12*log(d + e*x)*a*b**2*d**2*e**2*x + 6*log(d + e*x)*b**3*d**4 + 6*log(d + e*x)*b**3*d**3*e*x + 2*a**3*e**4*x - 6*a**2*b*d*e**3*x + 12*a*b**2*d**2*e**2*x + 6*a*b**2*d*e**3*x**2 - 6*b**3*d**3*e*x - 3*b**3*d**2*e**2*x**2 + b**3*d*e**3*x**3)/(2*d*e**4*(d + e*x))`

3.284
$$\int \frac{(a^2+2abx+b^2x^2)^{3/2}}{(d+ex)^3} dx$$

| | |
|---|------|
| Optimal result | 2144 |
| Mathematica [A] (verified) | 2145 |
| Rubi [A] (verified) | 2145 |
| Maple [A] (verified) | 2147 |
| Fricas [A] (verification not implemented) | 2147 |
| Sympy [F] | 2148 |
| Maxima [F(-2)] | 2148 |
| Giac [A] (verification not implemented) | 2148 |
| Mupad [F(-1)] | 2149 |
| Reduce [B] (verification not implemented) | 2149 |

Optimal result

Integrand size = 28, antiderivative size = 186

$$\int \frac{(a^2 + 2abx + b^2x^2)^{3/2}}{(d + ex)^3} dx = \frac{b^3x\sqrt{a^2 + 2abx + b^2x^2}}{e^3(a + bx)} + \frac{(bd - ae)^3\sqrt{a^2 + 2abx + b^2x^2}}{2e^4(a + bx)(d + ex)^2} - \frac{3b(bd - ae)^2\sqrt{a^2 + 2abx + b^2x^2}}{e^4(a + bx)(d + ex)} - \frac{3b^2(bd - ae)\sqrt{a^2 + 2abx + b^2x^2} \log(d + ex)}{e^4(a + bx)}$$

output

```
b^3*x*((b*x+a)^2)^(1/2)/e^3/(b*x+a)+1/2*(-a*e+b*d)^3*((b*x+a)^2)^(1/2)/e^4/(b*x+a)/(e*x+d)^2-3*b*(-a*e+b*d)^2*((b*x+a)^2)^(1/2)/e^4/(b*x+a)/(e*x+d)-3*b^2*(-a*e+b*d)*((b*x+a)^2)^(1/2)*ln(e*x+d)/e^4/(b*x+a)
```

Mathematica [A] (verified)

Time = 1.06 (sec) , antiderivative size = 131, normalized size of antiderivative = 0.70

$$\int \frac{(a^2 + 2abx + b^2x^2)^{3/2}}{(d + ex)^3} dx = \frac{\sqrt{(a + bx)^2}(a^3e^3 + 3a^2be^2(d + 2ex) - 3ab^2de(3d + 4ex) + b^3(5d^3 + 4d^2ex - 4de^2x^2 - 2e^3x^3) + 6b^2(bd - a^2e^2) - a^3e^3)}{2e^4(a + bx)(d + ex)^2}$$

input

```
Integrate[(a^2 + 2*a*b*x + b^2*x^2)^(3/2)/(d + e*x)^3,x]
```

output

```
-1/2*(Sqrt[(a + b*x)^2]*(a^3*e^3 + 3*a^2*b*e^2*(d + 2*e*x) - 3*a*b^2*d*e*(3*d + 4*e*x) + b^3*(5*d^3 + 4*d^2*e*x - 4*d*e^2*x^2 - 2*e^3*x^3) + 6*b^2*(b*d - a*e)*(d + e*x)^2*Log[d + e*x]))/(e^4*(a + b*x)*(d + e*x)^2)
```

Rubi [A] (verified)Time = 0.46 (sec) , antiderivative size = 106, normalized size of antiderivative = 0.57, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {1102, 27, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\begin{aligned} & \int \frac{(a^2 + 2abx + b^2x^2)^{3/2}}{(d + ex)^3} dx \\ & \quad \downarrow 1102 \\ & \frac{\sqrt{a^2 + 2abx + b^2x^2} \int \frac{b^3(a+bx)^3}{(d+ex)^3} dx}{b^3(a + bx)} \\ & \quad \downarrow 27 \\ & \frac{\sqrt{a^2 + 2abx + b^2x^2} \int \frac{(a+bx)^3}{(d+ex)^3} dx}{a + bx} \\ & \quad \downarrow 49 \end{aligned}$$

$$\frac{\sqrt{a^2 + 2abx + b^2x^2} \int \left(\frac{b^3}{e^3} - \frac{3(bd-ae)b^2}{e^3(d+ex)} + \frac{3(bd-ae)^2b}{e^3(d+ex)^2} + \frac{(ae-bd)^3}{e^3(d+ex)^3} \right) dx}{a + bx}$$

↓ 2009

$$\frac{\sqrt{a^2 + 2abx + b^2x^2} \left(-\frac{3b^2(bd-ae) \log(d+ex)}{e^4} - \frac{3b(bd-ae)^2}{e^4(d+ex)} + \frac{(bd-ae)^3}{2e^4(d+ex)^2} + \frac{b^3x}{e^3} \right)}{a + bx}$$

input `Int[(a^2 + 2*a*b*x + b^2*x^2)^(3/2)/(d + e*x)^3,x]`

output `(Sqrt[a^2 + 2*a*b*x + b^2*x^2]*((b^3*x)/e^3 + (b*d - a*e)^3/(2*e^4*(d + e*x)^2) - (3*b*(b*d - a*e)^2)/(e^4*(d + e*x)) - (3*b^2*(b*d - a*e)*Log[d + e*x])/e^4))/(a + b*x)`

Defintions of rubi rules used

rule 27 `Int[(a_)*(Fx_), x_Symbol] := Simp[a Int[Fx, x], x] /; FreeQ[a, x] && !MatchQ[Fx, (b_)*(Gx_)] /; FreeQ[b, x]`

rule 49 `Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] := Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] && IGtQ[m, 0] && IGtQ[m + n + 2, 0]`

rule 1102 `Int[((d_.) + (e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] := Simp[(a + b*x + c*x^2)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x)^(2*FracPart[p])) Int[(d + e*x)^m*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && EqQ[b^2 - 4*a*c, 0]`

rule 2009 `Int[u_, x_Symbol] := Simp[IntSum[u, x], x] /; SumQ[u]`

Sympy [F]

$$\int \frac{(a^2 + 2abx + b^2x^2)^{3/2}}{(d + ex)^3} dx = \int \frac{((a + bx)^2)^{3/2}}{(d + ex)^3} dx$$

input `integrate((b**2*x**2+2*a*b*x+a**2)**(3/2)/(e*x+d)**3,x)`

output `Integral(((a + b*x)**2)**(3/2)/(d + e*x)**3, x)`

Maxima [F(-2)]

Exception generated.

$$\int \frac{(a^2 + 2abx + b^2x^2)^{3/2}}{(d + ex)^3} dx = \text{Exception raised: ValueError}$$

input `integrate((b^2*x^2+2*a*b*x+a^2)^(3/2)/(e*x+d)^3,x, algorithm="maxima")`

output `Exception raised: ValueError >> Computation failed since Maxima requested additional constraints; using the 'assume' command before evaluation *may* help (example of legal syntax is 'assume(a*e-b*d>0)', see `assume?` for more detail`

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 172, normalized size of antiderivative = 0.92

$$\int \frac{(a^2 + 2abx + b^2x^2)^{3/2}}{(d + ex)^3} dx = \frac{b^3 x \operatorname{sgn}(bx + a)}{e^3} - \frac{3(b^3 d \operatorname{sgn}(bx + a) - ab^2 e \operatorname{sgn}(bx + a)) \log(|ex + d|)}{e^4} - \frac{5b^3 d^3 \operatorname{sgn}(bx + a) - 9ab^2 d^2 e \operatorname{sgn}(bx + a) + 3a^2 b d e^2 \operatorname{sgn}(bx + a) + a^3 e^3 \operatorname{sgn}(bx + a) + 6(b^3 d^2 e \operatorname{sgn}(bx + a))}{2(ex + d)^2 e^4}$$

input `integrate((b^2*x^2+2*a*b*x+a^2)^(3/2)/(e*x+d)^3,x, algorithm="giac")`

output `b^3*x*sgn(b*x + a)/e^3 - 3*(b^3*d*sgn(b*x + a) - a*b^2*e*sgn(b*x + a))*log(abs(e*x + d))/e^4 - 1/2*(5*b^3*d^3*sgn(b*x + a) - 9*a*b^2*d^2*e*sgn(b*x + a) + 3*a^2*b*d*e^2*sgn(b*x + a) + a^3*e^3*sgn(b*x + a) + 6*(b^3*d^2*e*sgn(b*x + a) - 2*a*b^2*d*e^2*sgn(b*x + a) + a^2*b*e^3*sgn(b*x + a))*x)/((e*x + d)^2*e^4)`

Mupad [F(-1)]

Timed out.

$$\int \frac{(a^2 + 2abx + b^2x^2)^{3/2}}{(d + ex)^3} dx = \int \frac{(a^2 + 2abx + b^2x^2)^{3/2}}{(d + ex)^3} dx$$

input `int((a^2 + b^2*x^2 + 2*a*b*x)^(3/2)/(d + e*x)^3,x)`

output `int((a^2 + b^2*x^2 + 2*a*b*x)^(3/2)/(d + e*x)^3, x)`

Reduce [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 209, normalized size of antiderivative = 1.12

$$\int \frac{(a^2 + 2abx + b^2x^2)^{3/2}}{(d + ex)^3} dx = \frac{6 \log(ex + d) a b^2 d^3 e + 12 \log(ex + d) a b^2 d^2 e^2 x + 6 \log(ex + d) a b^2 d e^3 x^2 -$$

input `int((b^2*x^2+2*a*b*x+a^2)^(3/2)/(e*x+d)^3,x)`

output `(6*log(d + e*x)*a*b**2*d**3*e + 12*log(d + e*x)*a*b**2*d**2*e**2*x + 6*log(d + e*x)*a*b**2*d*e**3*x**2 - 6*log(d + e*x)*b**3*d**4 - 12*log(d + e*x)*b**3*d**3*e*x - 6*log(d + e*x)*b**3*d**2*e**2*x**2 - a**3*d*e**3 + 3*a**2*b*e**4*x**2 + 3*a*b**2*d**3*e - 6*a*b**2*d*e**3*x**2 - 3*b**3*d**4 + 6*b**3*d**2*e**2*x**2 + 2*b**3*d*e**3*x**3)/(2*d*e**4*(d**2 + 2*d*e*x + e**2*x**2))`

3.285
$$\int \frac{(a^2+2abx+b^2x^2)^{3/2}}{(d+ex)^4} dx$$

| | |
|---|------|
| Optimal result | 2150 |
| Mathematica [A] (verified) | 2151 |
| Rubi [A] (verified) | 2151 |
| Maple [A] (verified) | 2153 |
| Fricas [A] (verification not implemented) | 2153 |
| Sympy [F] | 2154 |
| Maxima [F(-2)] | 2154 |
| Giac [A] (verification not implemented) | 2154 |
| Mupad [F(-1)] | 2155 |
| Reduce [B] (verification not implemented) | 2155 |

Optimal result

Integrand size = 28, antiderivative size = 194

$$\int \frac{(a^2 + 2abx + b^2x^2)^{3/2}}{(d + ex)^4} dx = \frac{(bd - ae)^3 \sqrt{a^2 + 2abx + b^2x^2}}{3e^4(a + bx)(d + ex)^3} - \frac{3b(bd - ae)^2 \sqrt{a^2 + 2abx + b^2x^2}}{2e^4(a + bx)(d + ex)^2} + \frac{3b^2(bd - ae) \sqrt{a^2 + 2abx + b^2x^2}}{e^4(a + bx)(d + ex)} + \frac{b^3 \sqrt{a^2 + 2abx + b^2x^2} \log(d + ex)}{e^4(a + bx)}$$

output
$$\frac{1}{3}(-a*e+b*d)^3*((b*x+a)^2)^{(1/2)}/e^4/(b*x+a)/(e*x+d)^3-3/2*b*(-a*e+b*d)^2*((b*x+a)^2)^{(1/2)}/e^4/(b*x+a)/(e*x+d)^2+3*b^2*(-a*e+b*d)*((b*x+a)^2)^{(1/2)}/e^4/(b*x+a)/(e*x+d)+b^3*((b*x+a)^2)^{(1/2)}*\ln(e*x+d)/e^4/(b*x+a)$$

Mathematica [A] (verified)

Time = 1.06 (sec) , antiderivative size = 104, normalized size of antiderivative = 0.54

$$\int \frac{(a^2 + 2abx + b^2x^2)^{3/2}}{(d + ex)^4} dx = \frac{\sqrt{(a + bx)^2}((bd - ae)(2a^2e^2 + abe(5d + 9ex) + b^2(11d^2 + 27dex + 18e^2x^2)) + 6b^3(d + ex)^3 \text{Log}[d + ex])}{6e^4(a + bx)(d + ex)^3}$$

input `Integrate[(a^2 + 2*a*b*x + b^2*x^2)^(3/2)/(d + e*x)^4,x]`

output `(Sqrt[(a + b*x)^2]*((b*d - a*e)*(2*a^2*e^2 + a*b*e*(5*d + 9*e*x) + b^2*(11*d^2 + 27*d*e*x + 18*e^2*x^2)) + 6*b^3*(d + e*x)^3*Log[d + e*x]))/(6*e^4*(a + b*x)*(d + e*x)^3)`

Rubi [A] (verified)

Time = 0.47 (sec) , antiderivative size = 114, normalized size of antiderivative = 0.59, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {1102, 27, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\begin{aligned} & \int \frac{(a^2 + 2abx + b^2x^2)^{3/2}}{(d + ex)^4} dx \\ & \quad \downarrow \text{1102} \\ & \frac{\sqrt{a^2 + 2abx + b^2x^2} \int \frac{b^3(a+bx)^3}{(d+ex)^4} dx}{b^3(a + bx)} \\ & \quad \downarrow \text{27} \\ & \frac{\sqrt{a^2 + 2abx + b^2x^2} \int \frac{(a+bx)^3}{(d+ex)^4} dx}{a + bx} \\ & \quad \downarrow \text{49} \\ & \frac{\sqrt{a^2 + 2abx + b^2x^2} \int \left(\frac{b^3}{e^3(d+ex)} - \frac{3(bd-ae)b^2}{e^3(d+ex)^2} + \frac{3(bd-ae)^2b}{e^3(d+ex)^3} + \frac{(ae-bd)^3}{e^3(d+ex)^4} \right) dx}{a + bx} \end{aligned}$$

$$\frac{\sqrt{a^2 + 2abx + b^2x^2} \left(\frac{3b^2(bd-ae)}{e^4(d+ex)} - \frac{3b(bd-ae)^2}{2e^4(d+ex)^2} + \frac{(bd-ae)^3}{3e^4(d+ex)^3} + \frac{b^3 \log(d+ex)}{e^4} \right)}{a + bx}$$

↓ 2009

input `Int[(a^2 + 2*a*b*x + b^2*x^2)^(3/2)/(d + e*x)^4,x]`

output `(Sqrt[a^2 + 2*a*b*x + b^2*x^2]*((b*d - a*e)^3/(3*e^4*(d + e*x)^3) - (3*b*(b*d - a*e)^2)/(2*e^4*(d + e*x)^2) + (3*b^2*(b*d - a*e))/(e^4*(d + e*x)) + (b^3*Log[d + e*x])/e^4)/(a + b*x)`

Defintions of rubi rules used

rule 27 `Int[(a_)*(F_x_), x_Symbol] := Simp[a Int[F_x, x], x] /; FreeQ[a, x] && !MatchQ[F_x, (b_)*(G_x_)] /; FreeQ[b, x]`

rule 49 `Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] := Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] && IGtQ[m, 0] && IGtQ[m + n + 2, 0]`

rule 1102 `Int[((d_.) + (e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] := Simp[(a + b*x + c*x^2)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x)^(2*FracPart[p])) Int[(d + e*x)^m*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && EqQ[b^2 - 4*a*c, 0]`

rule 2009 `Int[u_, x_Symbol] := Simp[IntSum[u, x], x] /; SumQ[u]`

Maple [A] (verified)

Time = 1.45 (sec) , antiderivative size = 146, normalized size of antiderivative = 0.75

| method | result |
|---------|---|
| risch | $\frac{\sqrt{(bx+a)^2} \left(-\frac{3b^2(ae-bd)x^2}{e^2} - \frac{3b(e^2a^2+2abde-3b^2d^2)x}{2e^3} - \frac{2e^3a^3+3a^2bde^2+6ab^2d^2e-11b^3d^3}{6e^4} \right)}{(bx+a)(ex+d)^3} + \frac{b^3\sqrt{(bx+a)^2} \ln(ex+d)}{e^4(bx+a)}$ |
| default | $\frac{\left((bx+a)^2\right)^{\frac{3}{2}} \left(6 \ln(ex+d)b^3e^3x^3+18 \ln(ex+d)b^3de^2x^2+18 \ln(ex+d)b^3d^2ex-18x^2ab^2e^3+18x^2b^3de^2+6 \ln(ex+d)b^3d^3-9a^2be^3x-\right)}{6(bx+a)^3e^4(ex+d)^3}$ |

input `int((b^2*x^2+2*a*b*x+a^2)^(3/2)/(e*x+d)^4,x,method=_RETURNVERBOSE)`

output
$$\frac{\left((b*x+a)^2\right)^{\frac{1}{2}}/(b*x+a)*(-3*b^2*(a*e-b*d)/e^2*x^2-3/2*b*(a^2*e^2+2*a*b*d*e-3*b^2*d^2)/e^3*x-1/6*(2*a^3*e^3+3*a^2*b*d*e^2+6*a*b^2*d^2*e-11*b^3*d^3)/e^4)/(e*x+d)^3+b^3*((b*x+a)^2)^{\frac{1}{2}}*\ln(e*x+d)/e^4/(b*x+a)}$$

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 177, normalized size of antiderivative = 0.91

$$\int \frac{(a^2 + 2abx + b^2x^2)^{3/2}}{(d + ex)^4} dx = \frac{11b^3d^3 - 6ab^2d^2e - 3a^2bde^2 - 2a^3e^3 + 18(b^3de^2 - ab^2e^3)x^2 + 9(3b^3d^2e - 6e^7x^3 + 3de^6x^2 - \dots)}{6(e^7x^3 + 3de^6x^2 - \dots)}$$

input `integrate((b^2*x^2+2*a*b*x+a^2)^(3/2)/(e*x+d)^4,x,algorithm="fricas")`

output
$$\frac{1}{6}*(11*b^3*d^3 - 6*a*b^2*d^2*e - 3*a^2*b*d*e^2 - 2*a^3*e^3 + 18*(b^3*d*e^2 - a*b^2*e^3)*x^2 + 9*(3*b^3*d^2*e - 2*a*b^2*d*e^2 - a^2*b*e^3)*x + 6*(b^3*e^3*x^3 + 3*b^3*d*e^2*x^2 + 3*b^3*d^2*e*x + b^3*d^3)*\log(e*x + d))/(e^7*x^3 + 3*d*e^6*x^2 + 3*d^2*e^5*x + d^3*e^4)$$

Sympy [F]

$$\int \frac{(a^2 + 2abx + b^2x^2)^{3/2}}{(d + ex)^4} dx = \int \frac{((a + bx)^2)^{\frac{3}{2}}}{(d + ex)^4} dx$$

input `integrate((b**2*x**2+2*a*b*x+a**2)**(3/2)/(e*x+d)**4,x)`

output `Integral(((a + b*x)**2)**(3/2)/(d + e*x)**4, x)`

Maxima [F(-2)]

Exception generated.

$$\int \frac{(a^2 + 2abx + b^2x^2)^{3/2}}{(d + ex)^4} dx = \text{Exception raised: ValueError}$$

input `integrate((b^2*x^2+2*a*b*x+a^2)^(3/2)/(e*x+d)^4,x, algorithm="maxima")`

output `Exception raised: ValueError >> Computation failed since Maxima requested additional constraints; using the 'assume' command before evaluation *may* help (example of legal syntax is 'assume(a*e-b*d>0)', see `assume?` for more detail`

Giac [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 179, normalized size of antiderivative = 0.92

$$\int \frac{(a^2 + 2abx + b^2x^2)^{3/2}}{(d + ex)^4} dx = \frac{b^3 \log(|ex + d|) \operatorname{sgn}(bx + a)}{e^4} + \frac{18(b^3 \operatorname{desgn}(bx + a) - ab^2 e^2 \operatorname{sgn}(bx + a))x^2 + 9(3b^3 d^2 \operatorname{sgn}(bx + a) - 2ab^2 \operatorname{desgn}(bx + a) - a^2 b e^2 \operatorname{sgn}(bx + a))}{6(ex + d)^3 e^3}$$

input `integrate((b^2*x^2+2*a*b*x+a^2)^(3/2)/(e*x+d)^4,x, algorithm="giac")`

output

```
b^3*log(abs(e*x + d))*sgn(b*x + a)/e^4 + 1/6*(18*(b^3*d*e*sgn(b*x + a) - a
*b^2*e^2*sgn(b*x + a))*x^2 + 9*(3*b^3*d^2*sgn(b*x + a) - 2*a*b^2*d*e*sgn(b
*x + a) - a^2*b*e^2*sgn(b*x + a))*x + (11*b^3*d^3*sgn(b*x + a) - 6*a*b^2*d
^2*e*sgn(b*x + a) - 3*a^2*b*d*e^2*sgn(b*x + a) - 2*a^3*e^3*sgn(b*x + a))/e
)/((e*x + d)^3*e^3)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a^2 + 2abx + b^2x^2)^{3/2}}{(d + ex)^4} dx = \int \frac{(a^2 + 2abx + b^2x^2)^{3/2}}{(d + ex)^4} dx$$

input

```
int((a^2 + b^2*x^2 + 2*a*b*x)^(3/2)/(d + e*x)^4,x)
```

output

```
int((a^2 + b^2*x^2 + 2*a*b*x)^(3/2)/(d + e*x)^4, x)
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 180, normalized size of antiderivative = 0.93

$$\int \frac{(a^2 + 2abx + b^2x^2)^{3/2}}{(d + ex)^4} dx = \frac{6 \log(ex + d) b^3 d^4 + 18 \log(ex + d) b^3 d^3 ex + 18 \log(ex + d) b^3 d^2 e^2 x^2 + 6 \log(ex + d) b^3 d e^3 x^3}{6 d e^4 (e^3 x^3 + 3 d e^2 x^2 + 3 d^2 e x + d^3)}$$

input

```
int((b^2*x^2+2*a*b*x+a^2)^(3/2)/(e*x+d)^4,x)
```

output

```
(6*log(d + e*x)*b**3*d**4 + 18*log(d + e*x)*b**3*d**3*e*x + 18*log(d + e*x
)*b**3*d**2*e**2*x**2 + 6*log(d + e*x)*b**3*d*e**3*x**3 - 2*a**3*d*e**3 -
3*a**2*b*d**2*e**2 - 9*a**2*b*d*e**3*x + 6*a*b**2*e**4*x**3 + 5*b**3*d**4
+ 9*b**3*d**3*e*x - 6*b**3*d*e**3*x**3)/(6*d*e**4*(d**3 + 3*d**2*e*x + 3*d
**2*x**2 + e**3*x**3))
```


$$3.286 \quad \int \frac{(a^2 + 2abx + b^2x^2)^{3/2}}{(d + ex)^5} dx$$

| | |
|---|------|
| Optimal result | 2156 |
| Mathematica [B] (verified) | 2156 |
| Rubi [A] (verified) | 2157 |
| Maple [B] (verified) | 2158 |
| Fricas [B] (verification not implemented) | 2159 |
| Sympy [F] | 2159 |
| Maxima [F(-2)] | 2160 |
| Giac [B] (verification not implemented) | 2160 |
| Mupad [B] (verification not implemented) | 2161 |
| Reduce [B] (verification not implemented) | 2161 |

Optimal result

Integrand size = 28, antiderivative size = 46

$$\int \frac{(a^2 + 2abx + b^2x^2)^{3/2}}{(d + ex)^5} dx = \frac{(a + bx)(a^2 + 2abx + b^2x^2)^{3/2}}{4(bd - ae)(d + ex)^4}$$

output $1/4*(b*x+a)*(b^2*x^2+2*a*b*x+a^2)^(3/2)/(-a*e+b*d)/(e*x+d)^4$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 109 vs. $2(46) = 92$.

Time = 1.04 (sec) , antiderivative size = 109, normalized size of antiderivative = 2.37

$$\int \frac{(a^2 + 2abx + b^2x^2)^{3/2}}{(d + ex)^5} dx = \frac{\sqrt{(a + bx)^2(a^3e^3 + a^2be^2(d + 4ex) + ab^2e(d^2 + 4dex + 6e^2x^2) + b^3(d^3 + 4d^2ex + 6de^2x^2 + 4e^3x^3))}}{4e^4(a + bx)(d + ex)^4}$$

input $\text{Integrate}[(a^2 + 2*a*b*x + b^2*x^2)^(3/2)/(d + e*x)^5, x]$

output

```
-1/4*(Sqrt[(a + b*x)^2]*(a^3*e^3 + a^2*b*e^2*(d + 4*e*x) + a*b^2*e*(d^2 +
4*d*e*x + 6*e^2*x^2) + b^3*(d^3 + 4*d^2*e*x + 6*d*e^2*x^2 + 4*e^3*x^3)))/(
e^4*(a + b*x)*(d + e*x)^4)
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.04, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.107$, Rules used = {1102, 27, 48}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a^2 + 2abx + b^2x^2)^{3/2}}{(d + ex)^5} dx$$

$$\downarrow 1102$$

$$\frac{\sqrt{a^2 + 2abx + b^2x^2} \int \frac{b^3(a+bx)^3}{(d+ex)^5} dx}{b^3(a + bx)}$$

$$\downarrow 27$$

$$\frac{\sqrt{a^2 + 2abx + b^2x^2} \int \frac{(a+bx)^3}{(d+ex)^5} dx}{a + bx}$$

$$\downarrow 48$$

$$\frac{(a + bx)^3 \sqrt{a^2 + 2abx + b^2x^2}}{4(d + ex)^4 (bd - ae)}$$

input

```
Int[(a^2 + 2*a*b*x + b^2*x^2)^(3/2)/(d + e*x)^5,x]
```

output

```
((a + b*x)^3*Sqrt[a^2 + 2*a*b*x + b^2*x^2])/(4*(b*d - a*e)*(d + e*x)^4)
```

Defintions of rubi rules used

rule 27 `Int[(a_)*(Fx_), x_Symbol] := Simp[a Int[Fx, x], x] /; FreeQ[a, x] && !MatchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]`

rule 48 `Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] := Simp[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{a, b, c, d, m, n}, x] && EqQ[m + n + 2, 0] && NeQ[m, -1]`

rule 1102 `Int[((d_.) + (e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] := Simp[(a + b*x + c*x^2)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x)^(2*FracPart[p])) Int[(d + e*x)^m*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && EqQ[b^2 - 4*a*c, 0]`

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 119 vs. 2(42) = 84.

Time = 1.66 (sec) , antiderivative size = 120, normalized size of antiderivative = 2.61

| method | result | size |
|---------|--|------|
| risch | $\frac{\sqrt{(bx+a)^2} \left(-\frac{b^3 x^3}{e} - \frac{3b^2(ae+bd)x^2}{2e^2} - \frac{b(e^2 a^2 + abde + b^2 d^2)x}{e^3} - \frac{e^3 a^3 + a^2 bde^2 + a b^2 d^2 e + b^3 d^3}{4e^4} \right)}{(bx+a)(ex+d)^4}$ | 120 |
| gospers | $-\frac{(4e^3 x^3 b^3 + 6x^2 a b^2 e^3 + 6x^2 b^3 d e^2 + 4a^2 b e^3 x + 4x a b^2 d e^2 + 4b^3 d^2 e x + e^3 a^3 + a^2 b d e^2 + a b^2 d^2 e + b^3 d^3) ((bx+a)^2)^{\frac{3}{2}}}{4(ex+d)^4 e^4 (bx+a)^3}$ | 128 |
| default | $-\frac{(4e^3 x^3 b^3 + 6x^2 a b^2 e^3 + 6x^2 b^3 d e^2 + 4a^2 b e^3 x + 4x a b^2 d e^2 + 4b^3 d^2 e x + e^3 a^3 + a^2 b d e^2 + a b^2 d^2 e + b^3 d^3) ((bx+a)^2)^{\frac{3}{2}}}{4(ex+d)^4 e^4 (bx+a)^3}$ | 128 |
| orering | $-\frac{(4e^3 x^3 b^3 + 6x^2 a b^2 e^3 + 6x^2 b^3 d e^2 + 4a^2 b e^3 x + 4x a b^2 d e^2 + 4b^3 d^2 e x + e^3 a^3 + a^2 b d e^2 + a b^2 d^2 e + b^3 d^3) (b^2 x^2 + 2abx + a^2)^{\frac{3}{2}}}{4e^4 (bx+a)^3 (ex+d)^4}$ | 137 |

input `int((b^2*x^2+2*a*b*x+a^2)^(3/2)/(e*x+d)^5,x,method=_RETURNVERBOSE)`

output `((b*x+a)^(1/2)/(b*x+a)*(-b^3/e*x^3-3/2*b^2*(a*e+b*d)/e^2*x^2-b*(a^2*e^2+a*b*d*e+b^2*d^2)/e^3*x-1/4*(a^3*e^3+a^2*b*d*e^2+a*b^2*d^2*e+b^3*d^3)/e^4)/(e*x+d)^4`

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 143 vs. $2(42) = 84$.

Time = 0.08 (sec) , antiderivative size = 143, normalized size of antiderivative = 3.11

$$\int \frac{(a^2 + 2abx + b^2x^2)^{3/2}}{(d + ex)^5} dx = \frac{4b^3e^3x^3 + b^3d^3 + ab^2d^2e + a^2bde^2 + a^3e^3 + 6(b^3de^2 + ab^2e^3)x^2 + 4(b^3d^2e + ab^2de^2 + a^2be^3)x}{4(e^8x^4 + 4de^7x^3 + 6d^2e^6x^2 + 4d^3e^5x + d^4e^4)}$$

input `integrate((b^2*x^2+2*a*b*x+a^2)^(3/2)/(e*x+d)^5,x, algorithm="fricas")`

output `-1/4*(4*b^3*e^3*x^3 + b^3*d^3 + a*b^2*d^2*e + a^2*b*d*e^2 + a^3*e^3 + 6*(b^3*d*e^2 + a*b^2*e^3)*x^2 + 4*(b^3*d^2*e + a*b^2*d*e^2 + a^2*b*e^3)*x)/(e^8*x^4 + 4*d*e^7*x^3 + 6*d^2*e^6*x^2 + 4*d^3*e^5*x + d^4*e^4)`

Sympy [F]

$$\int \frac{(a^2 + 2abx + b^2x^2)^{3/2}}{(d + ex)^5} dx = \int \frac{((a + bx)^2)^{\frac{3}{2}}}{(d + ex)^5} dx$$

input `integrate((b**2*x**2+2*a*b*x+a**2)**(3/2)/(e*x+d)**5,x)`

output `Integral(((a + b*x)**2)**(3/2)/(d + e*x)**5, x)`

Mupad [B] (verification not implemented)

Time = 5.12 (sec) , antiderivative size = 284, normalized size of antiderivative = 6.17

$$\int \frac{(a^2 + 2abx + b^2x^2)^{3/2}}{(d + ex)^5} dx = \frac{\left(\frac{2b^3d - 3ab^2e}{2e^4} + \frac{b^3d}{2e^4}\right) \sqrt{a^2 + 2abx + b^2x^2}}{(a + bx)(d + ex)^2} - \frac{\left(\frac{3a^2be^2 - 3ab^2de + b^3d^2}{3e^4} + \frac{d\left(\frac{b^3d}{3e^3} - \frac{b^2(3ae - bd)}{3e^3}\right)}{e}\right) \sqrt{a^2 + 2abx + b^2x^2}}{(a + bx)(d + ex)^3} - \frac{\left(\frac{a^3}{4e} - \frac{d\left(\frac{3a^2b}{4e} - \frac{d\left(\frac{3ab^2}{4e} - \frac{b^3d}{4e^2}\right)}{e}\right)}{e}\right) \sqrt{a^2 + 2abx + b^2x^2}}{(a + bx)(d + ex)^4} - \frac{b^3 \sqrt{a^2 + 2abx + b^2x^2}}{e^4 (a + bx)(d + ex)}$$

input `int((a^2 + b^2*x^2 + 2*a*b*x)^(3/2)/(d + e*x)^5,x)`output
$$\left(\left(\frac{2b^3d - 3ab^2e}{2e^4} + \frac{b^3d}{2e^4}\right)(a^2 + b^2x^2 + 2abx)^{1/2}\right) / \left((a + bx)(d + ex)^2\right) - \left(\left(\frac{b^3d^2 + 3a^2b^2e^2 - 3ab^2de}{3e^4} + \frac{d\left(\frac{b^3d}{3e^3} - \frac{b^2(3ae - bd)}{3e^3}\right)}{e}\right)(a^2 + b^2x^2 + 2abx)^{1/2}\right) / \left((a + bx)(d + ex)^3\right) - \left(\left(\frac{a^3}{4e} - \frac{d\left(\frac{3a^2b}{4e} - \frac{d\left(\frac{3ab^2}{4e} - \frac{b^3d}{4e^2}\right)}{e}\right)}{e}\right)(a^2 + b^2x^2 + 2abx)^{1/2}\right) / \left((a + bx)(d + ex)^4\right) - \frac{b^3(a^2 + b^2x^2 + 2abx)^{1/2}}{e^4(a + bx)(d + ex)}$$
Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 122, normalized size of antiderivative = 2.65

$$\int \frac{(a^2 + 2abx + b^2x^2)^{3/2}}{(d + ex)^5} dx = \frac{b^3e^3x^4 - 6ab^2de^2x^2 - 4a^2bde^2x - 4ab^2d^2ex - a^3de^2 - a^2bd^2e - ab^2d^3}{4de^3(e^4x^4 + 4de^3x^3 + 6d^2e^2x^2 + 4d^3ex + d^4)}$$

input `int((b^2*x^2+2*a*b*x+a^2)^(3/2)/(e*x+d)^5,x)`

output

```
( - a**3*d*e**2 - a**2*b*d**2*e - 4*a**2*b*d*e**2*x - a*b**2*d**3 - 4*a*b*  
*2*d**2*e*x - 6*a*b**2*d*e**2*x**2 + b**3*e**3*x**4)/(4*d*e**3*(d**4 + 4*d  
**3*e*x + 6*d**2*e**2*x**2 + 4*d*e**3*x**3 + e**4*x**4))
```

$$3.287 \quad \int \frac{(a^2 + 2abx + b^2x^2)^{3/2}}{(d+ex)^6} dx$$

| | |
|---|------|
| Optimal result | 2163 |
| Mathematica [A] (verified) | 2163 |
| Rubi [A] (verified) | 2164 |
| Maple [A] (verified) | 2166 |
| Fricas [B] (verification not implemented) | 2166 |
| Sympy [F] | 2167 |
| Maxima [F(-2)] | 2167 |
| Giac [B] (verification not implemented) | 2168 |
| Mupad [B] (verification not implemented) | 2168 |
| Reduce [B] (verification not implemented) | 2169 |

Optimal result

Integrand size = 28, antiderivative size = 98

$$\int \frac{(a^2 + 2abx + b^2x^2)^{3/2}}{(d+ex)^6} dx = \frac{(a+bx)^3 \sqrt{a^2 + 2abx + b^2x^2}}{5(bd - ae)(d+ex)^5} + \frac{b(a+bx)^3 \sqrt{a^2 + 2abx + b^2x^2}}{20(bd - ae)^2(d+ex)^4}$$

output

```
1/5*(b*x+a)^3*((b*x+a)^2)^(1/2)/(-a*e+b*d)/(e*x+d)^5+1/20*b*(b*x+a)^3*((b*x+a)^2)^(1/2)/(-a*e+b*d)^2/(e*x+d)^4
```

Mathematica [A] (verified)

Time = 1.04 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.14

$$\int \frac{(a^2 + 2abx + b^2x^2)^{3/2}}{(d+ex)^6} dx = \frac{\sqrt{(a+bx)^2(4a^3e^3 + 3a^2be^2(d+5ex) + 2ab^2e(d^2 + 5dex + 10e^2x^2) + b^3(d^3 + 5d^2ex + 10de^2x^2 + 10e^3x^3))}}{20e^4(a+bx)(d+ex)^5}$$

input

```
Integrate[(a^2 + 2*a*b*x + b^2*x^2)^(3/2)/(d + e*x)^6,x]
```


output

```
-1/20*(Sqrt[(a + b*x)^2]*(4*a^3*e^3 + 3*a^2*b*e^2*(d + 5*e*x) + 2*a*b^2*e*(d^2 + 5*d*e*x + 10*e^2*x^2) + b^3*(d^3 + 5*d^2*e*x + 10*d*e^2*x^2 + 10*e^3*x^3)))/(e^4*(a + b*x)*(d + e*x)^5)
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.88, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {1102, 27, 55, 48}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a^2 + 2abx + b^2x^2)^{3/2}}{(d + ex)^6} dx$$

$$\downarrow 1102$$

$$\frac{\sqrt{a^2 + 2abx + b^2x^2} \int \frac{b^3(a+bx)^3}{(d+ex)^6} dx}{b^3(a + bx)}$$

$$\downarrow 27$$

$$\frac{\sqrt{a^2 + 2abx + b^2x^2} \int \frac{(a+bx)^3}{(d+ex)^6} dx}{a + bx}$$

$$\downarrow 55$$

$$\frac{\sqrt{a^2 + 2abx + b^2x^2} \left(\frac{b \int \frac{(a+bx)^3}{(d+ex)^5} dx}{5(bd-ae)} + \frac{(a+bx)^4}{5(d+ex)^5(bd-ae)} \right)}{a + bx}$$

$$\downarrow 48$$

$$\frac{\sqrt{a^2 + 2abx + b^2x^2} \left(\frac{b(a+bx)^4}{20(d+ex)^4(bd-ae)^2} + \frac{(a+bx)^4}{5(d+ex)^5(bd-ae)} \right)}{a + bx}$$

input

```
Int[(a^2 + 2*a*b*x + b^2*x^2)^(3/2)/(d + e*x)^6,x]
```

output $(\sqrt{a^2 + 2abx + b^2x^2} * ((a + bx)^4 / (5(bd - ae)(d + ex)^5) + (b(a + bx)^4) / (20(bd - ae)^2(d + ex)^4))) / (a + bx)$

Defintions of rubi rules used

rule 27 $\text{Int}[(a_*)(Fx_), x_Symbol] \rightarrow \text{Simp}[a \text{ Int}[Fx, x], x] /; \text{FreeQ}[a, x] \ \&\& \ !\text{MatchQ}[Fx, (b_*)(Gx_)] /; \text{FreeQ}[b, x]$

rule 48 $\text{Int}[(a_.) + (b_.)(x_)^{(m_.)} * ((c_.) + (d_.)(x_)^{(n_.)}), x_Symbol] \rightarrow \text{Simp}[(a + bx)^{(m + 1)} * ((c + dx)^{(n + 1)} / ((b*c - a*d) * (m + 1))), x] /; \text{FreeQ}[\{a, b, c, d, m, n\}, x] \ \&\& \ \text{EqQ}[m + n + 2, 0] \ \&\& \ \text{NeQ}[m, -1]$

rule 55 $\text{Int}[(a_.) + (b_.)(x_)^{(m_.)} * ((c_.) + (d_.)(x_)^{(n_.)}), x_Symbol] \rightarrow \text{Simp}[(a + bx)^{(m + 1)} * ((c + dx)^{(n + 1)} / ((b*c - a*d) * (m + 1))), x] - \text{Simp}[d * (\text{Simplify}[m + n + 2] / ((b*c - a*d) * (m + 1))) \text{Int}[(a + bx)^{\text{Simplify}[m + 1]} * (c + dx)^n, x], x] /; \text{FreeQ}[\{a, b, c, d, m, n\}, x] \ \&\& \ \text{ILtQ}[\text{Simplify}[m + n + 2], 0] \ \&\& \ \text{NeQ}[m, -1] \ \&\& \ !(\text{LtQ}[m, -1] \ \&\& \ \text{LtQ}[n, -1] \ \&\& \ (\text{EqQ}[a, 0] \ || \ (\text{NeQ}[c, 0] \ \&\& \ \text{LtQ}[m - n, 0] \ \&\& \ \text{IntegerQ}[n]))) \ \&\& \ (\text{SumSimplerQ}[m, 1] \ || \ !\text{SumSimplerQ}[n, 1])$

rule 1102 $\text{Int}[(d_.) + (e_.)(x_)^{(m_.)} * ((a_) + (b_.)(x_) + (c_.)(x_)^2)^{(p_.)}, x_Symbol] \rightarrow \text{Simp}[(a + bx + cx^2)^{\text{FracPart}[p]} / (c^{\text{IntPart}[p]} * (b/2 + cx)^{(2 * \text{FracPart}[p])}) \text{Int}[(d + ex)^m * (b/2 + cx)^{(2 * p)}, x], x] /; \text{FreeQ}[\{a, b, c, d, e, m, p\}, x] \ \&\& \ \text{EqQ}[b^2 - 4ac, 0]$

Maple [A] (verified)

Time = 2.27 (sec) , antiderivative size = 126, normalized size of antiderivative = 1.29

| method | result |
|---------|---|
| risch | $\frac{\sqrt{(bx+a)^2} \left(-\frac{b^3 x^3}{2e} - \frac{b^2(2ae+bd)x^2}{2e^2} - \frac{b(3e^2 a^2 + 2abde + b^2 d^2)x}{4e^3} - \frac{4e^3 a^3 + 3a^2 bde^2 + 2a b^2 d^2 e + b^3 d^3}{20e^4} \right)}{(bx+a)(ex+d)^5}$ |
| gospers | $-\frac{(10e^3 x^3 b^3 + 20x^2 a b^2 e^3 + 10x^2 b^3 d e^2 + 15a^2 b e^3 x + 10x a b^2 d e^2 + 5b^3 d^2 e x + 4e^3 a^3 + 3a^2 b d e^2 + 2a b^2 d^2 e + b^3 d^3) ((bx+a)^2)^{\frac{3}{2}}}{20e^4 (ex+d)^5 (bx+a)^3}$ |
| default | $-\frac{(10e^3 x^3 b^3 + 20x^2 a b^2 e^3 + 10x^2 b^3 d e^2 + 15a^2 b e^3 x + 10x a b^2 d e^2 + 5b^3 d^2 e x + 4e^3 a^3 + 3a^2 b d e^2 + 2a b^2 d^2 e + b^3 d^3) ((bx+a)^2)^{\frac{3}{2}}}{20e^4 (ex+d)^5 (bx+a)^3}$ |
| orering | $-\frac{(10e^3 x^3 b^3 + 20x^2 a b^2 e^3 + 10x^2 b^3 d e^2 + 15a^2 b e^3 x + 10x a b^2 d e^2 + 5b^3 d^2 e x + 4e^3 a^3 + 3a^2 b d e^2 + 2a b^2 d^2 e + b^3 d^3) (b^2 x^2 + 2abx + a^2)^{\frac{3}{2}}}{20e^4 (bx+a)^3 (ex+d)^5}$ |

input `int((b^2*x^2+2*a*b*x+a^2)^(3/2)/(e*x+d)^6,x,method=_RETURNVERBOSE)`

output
$$\frac{((b*x+a)^2)^{(1/2)} / (b*x+a) * (-1/2*b^3/e*x^3 - 1/2/e^2*b^2*(2*a*e+b*d)*x^2 - 1/4*b/e^3*(3*a^2*e^2+2*a*b*d*e+b^2*d^2)*x - 1/20/e^4*(4*a^3*e^3+3*a^2*b*d*e^2+2*a*b^2*d^2*e+b^3*d^3)) / (e*x+d)^5}$$

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 160 vs. 2(72) = 144.

Time = 0.08 (sec) , antiderivative size = 160, normalized size of antiderivative = 1.63

$$\int \frac{(a^2 + 2abx + b^2x^2)^{3/2}}{(d + ex)^6} dx = \frac{10 b^3 e^3 x^3 + b^3 d^3 + 2 a b^2 d^2 e + 3 a^2 b d e^2 + 4 a^3 e^3 + 10 (b^3 d e^2 + 2 a b^2 e^3) x^2 + 5 (b^3 d^2 e + 2 a b^2 d e^2 + 3 a^2 b e^3)}{20 (e^9 x^5 + 5 d e^8 x^4 + 10 d^2 e^7 x^3 + 10 d^3 e^6 x^2 + 5 d^4 e^5 x + d^5 e^4)}$$

input `integrate((b^2*x^2+2*a*b*x+a^2)^(3/2)/(e*x+d)^6,x, algorithm="fricas")`

output

```
-1/20*(10*b^3*e^3*x^3 + b^3*d^3 + 2*a*b^2*d^2*e + 3*a^2*b*d*e^2 + 4*a^3*e^3 + 10*(b^3*d*e^2 + 2*a*b^2*e^3)*x^2 + 5*(b^3*d^2*e + 2*a*b^2*d*e^2 + 3*a^2*b*e^3)*x)/(e^9*x^5 + 5*d*e^8*x^4 + 10*d^2*e^7*x^3 + 10*d^3*e^6*x^2 + 5*d^4*e^5*x + d^5*e^4)
```

Sympy [F]

$$\int \frac{(a^2 + 2abx + b^2x^2)^{3/2}}{(d + ex)^6} dx = \int \frac{((a + bx)^2)^{\frac{3}{2}}}{(d + ex)^6} dx$$

input

```
integrate((b**2*x**2+2*a*b*x+a**2)**(3/2)/(e*x+d)**6,x)
```

output

```
Integral(((a + b*x)**2)**(3/2)/(d + e*x)**6, x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(a^2 + 2abx + b^2x^2)^{3/2}}{(d + ex)^6} dx = \text{Exception raised: ValueError}$$

input

```
integrate((b^2*x^2+2*a*b*x+a^2)^(3/2)/(e*x+d)^6,x, algorithm="maxima")
```

output

```
Exception raised: ValueError >> Computation failed since Maxima requested additional constraints; using the 'assume' command before evaluation *may* help (example of legal syntax is 'assume(a*e-b*d>0)', see `assume?` for more detail)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 214 vs. $2(72) = 144$.

Time = 0.16 (sec) , antiderivative size = 214, normalized size of antiderivative = 2.18

$$\int \frac{(a^2 + 2abx + b^2x^2)^{3/2}}{(d + ex)^6} dx = \frac{b^5 \operatorname{sgn}(bx + a)}{20(b^2d^2e^4 - 2abde^5 + a^2e^6)} - \frac{10b^3e^3x^3 \operatorname{sgn}(bx + a) + 10b^3de^2x^2 \operatorname{sgn}(bx + a) + 20ab^2e^3x^2 \operatorname{sgn}(bx + a) + 5b^3d^2ex \operatorname{sgn}(bx + a) + 10ab^2d}{(d + ex)^6}$$

input `integrate((b^2*x^2+2*a*b*x+a^2)^(3/2)/(e*x+d)^6,x, algorithm="giac")`

output $\frac{1}{20}b^5 \operatorname{sgn}(bx + a)/(b^2d^2e^4 - 2abde^5 + a^2e^6) - \frac{1}{20}(10b^3e^3x^3 \operatorname{sgn}(bx + a) + 10b^3de^2x^2 \operatorname{sgn}(bx + a) + 20ab^2e^3x^2 \operatorname{sgn}(bx + a) + 5b^3d^2ex \operatorname{sgn}(bx + a) + 10ab^2d \operatorname{sgn}(bx + a) + b^3d^3 \operatorname{sgn}(bx + a) + 2ab^2d^2e \operatorname{sgn}(bx + a) + 3a^2bde^2 \operatorname{sgn}(bx + a) + 4a^3e^3 \operatorname{sgn}(bx + a))/((e*x + d)^5 * e^4)$

Mupad [B] (verification not implemented)

Time = 5.10 (sec) , antiderivative size = 284, normalized size of antiderivative = 2.90

$$\int \frac{(a^2 + 2abx + b^2x^2)^{3/2}}{(d + ex)^6} dx = \frac{\left(\frac{2b^3d - 3ab^2e}{3e^4} + \frac{b^3d}{3e^4}\right) \sqrt{a^2 + 2abx + b^2x^2}}{(a + bx)(d + ex)^3} - \frac{\left(\frac{3a^2be^2 - 3ab^2de + b^3d^2}{4e^4} + \frac{d\left(\frac{b^3d}{4e^3} - \frac{b^2(3ae - bd)}{4e^3}\right)}{e}\right) \sqrt{a^2 + 2abx + b^2x^2}}{(a + bx)(d + ex)^4} - \frac{\left(\frac{a^3}{5e} - \frac{d\left(\frac{3a^2b}{5e} - \frac{d\left(\frac{3ab^2}{5e} - \frac{b^3d}{5e^2}\right)}{e}\right)}{e}\right) \sqrt{a^2 + 2abx + b^2x^2}}{(a + bx)(d + ex)^5} - \frac{b^3 \sqrt{a^2 + 2abx + b^2x^2}}{2e^4(a + bx)(d + ex)^2}$$

input `int((a^2 + b^2*x^2 + 2*a*b*x)^(3/2)/(d + e*x)^6,x)`

output

```
((2*b^3*d - 3*a*b^2*e)/(3*e^4) + (b^3*d)/(3*e^4))*(a^2 + b^2*x^2 + 2*a*b*x)^(1/2)/((a + b*x)*(d + e*x)^3) - (((b^3*d^2 + 3*a^2*b*e^2 - 3*a*b^2*d*e)/(4*e^4) + (d*((b^3*d)/(4*e^3) - (b^2*(3*a*e - b*d))/(4*e^3)))/e)*(a^2 + b^2*x^2 + 2*a*b*x)^(1/2)/((a + b*x)*(d + e*x)^4) - ((a^3/(5*e) - (d*((3*a^2*b)/(5*e) - (d*((3*a*b^2)/(5*e) - (b^3*d)/(5*e^2)))/e))/e)*(a^2 + b^2*x^2 + 2*a*b*x)^(1/2)/((a + b*x)*(d + e*x)^5) - (b^3*(a^2 + b^2*x^2 + 2*a*b*x)^(1/2))/(2*e^4*(a + b*x)*(d + e*x)^2)
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 159, normalized size of antiderivative = 1.62

$$\int \frac{(a^2 + 2abx + b^2x^2)^{3/2}}{(d + ex)^6} dx = \frac{-10b^3e^3x^3 - 20ab^2e^3x^2 - 10b^3de^2x^2 - 15a^2be^3x - 10ab^2de^2x - 5b^3d^2ex - 5b^3d^2e}{20e^4(e^5x^5 + 5de^4x^4 + 10d^2e^3x^3 + 10d^3e^2x^2 + 5d^4ex + d^5)}$$

input

```
int((b^2*x^2+2*a*b*x+a^2)^(3/2)/(e*x+d)^6,x)
```

output

```
( - 4*a**3*e**3 - 3*a**2*b*d*e**2 - 15*a**2*b*e**3*x - 2*a*b**2*d**2*e - 10*a*b**2*d*e**2*x - 20*a*b**2*e**3*x**2 - b**3*d**3 - 5*b**3*d**2*e*x - 10*b**3*d*e**2*x**2 - 10*b**3*e**3*x**3)/(20*e**4*(d**5 + 5*d**4*e*x + 10*d**3*e**2*x**2 + 10*d**2*e**3*x**3 + 5*d*e**4*x**4 + e**5*x**5))
```

3.288 $\int \frac{(a^2+2abx+b^2x^2)^{3/2}}{(d+ex)^7} dx$

| | |
|---|------|
| Optimal result | 2170 |
| Mathematica [A] (verified) | 2170 |
| Rubi [A] (verified) | 2171 |
| Maple [A] (verified) | 2172 |
| Fricas [A] (verification not implemented) | 2173 |
| Sympy [F] | 2173 |
| Maxima [F(-2)] | 2174 |
| Giac [A] (verification not implemented) | 2174 |
| Mupad [B] (verification not implemented) | 2175 |
| Reduce [B] (verification not implemented) | 2175 |

Optimal result

Integrand size = 28, antiderivative size = 143

$$\int \frac{(a^2 + 2abx + b^2x^2)^{3/2}}{(d + ex)^7} dx = \frac{(a + bx)(a^2 + 2abx + b^2x^2)^{3/2}}{6(bd - ae)(d + ex)^6} + \frac{b(a + bx)(a^2 + 2abx + b^2x^2)^{3/2}}{15(bd - ae)^2(d + ex)^5} + \frac{b^2(a + bx)(a^2 + 2abx + b^2x^2)^{3/2}}{60(bd - ae)^3(d + ex)^4}$$

output

$1/6*(b*x+a)*(b^2*x^2+2*a*b*x+a^2)^(3/2)/(-a*e+b*d)/(e*x+d)^6+1/15*b*(b*x+a)*(b^2*x^2+2*a*b*x+a^2)^(3/2)/(-a*e+b*d)^2/(e*x+d)^5+1/60*b^2*(b*x+a)*(b^2*x^2+2*a*b*x+a^2)^(3/2)/(-a*e+b*d)^3/(e*x+d)^4$

Mathematica [A] (verified)

Time = 1.04 (sec) , antiderivative size = 112, normalized size of antiderivative = 0.78

$$\int \frac{(a^2 + 2abx + b^2x^2)^{3/2}}{(d + ex)^7} dx = \frac{\sqrt{(a + bx)^2(10a^3e^3 + 6a^2be^2(d + 6ex) + 3ab^2e(d^2 + 6dex + 15e^2x^2) + b^3(d^3 + 6d^2ex + 15de^2x^2 + 20e^3x^3))}}{60e^4(a + bx)(d + ex)^6}$$

input

`Integrate[(a^2 + 2*a*b*x + b^2*x^2)^(3/2)/(d + e*x)^7,x]`

output

```
-1/60*(Sqrt[(a + b*x)^2]*(10*a^3*e^3 + 6*a^2*b*e^2*(d + 6*e*x) + 3*a*b^2*e
*(d^2 + 6*d*e*x + 15*e^2*x^2) + b^3*(d^3 + 6*d^2*e*x + 15*d*e^2*x^2 + 20*e
^3*x^3)))/(e^4*(a + b*x)*(d + e*x)^6)
```

Rubi [A] (verified)

Time = 0.48 (sec) , antiderivative size = 120, normalized size of antiderivative = 0.84, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {1102, 27, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\begin{aligned}
 & \int \frac{(a^2 + 2abx + b^2x^2)^{3/2}}{(d + ex)^7} dx \\
 & \quad \downarrow \text{1102} \\
 & \frac{\sqrt{a^2 + 2abx + b^2x^2} \int \frac{b^3(a+bx)^3}{(d+ex)^7} dx}{b^3(a + bx)} \\
 & \quad \downarrow \text{27} \\
 & \frac{\sqrt{a^2 + 2abx + b^2x^2} \int \frac{(a+bx)^3}{(d+ex)^7} dx}{a + bx} \\
 & \quad \downarrow \text{53} \\
 & \frac{\sqrt{a^2 + 2abx + b^2x^2} \int \left(\frac{b^3}{e^3(d+ex)^4} - \frac{3(bd-ae)b^2}{e^3(d+ex)^5} + \frac{3(bd-ae)^2b}{e^3(d+ex)^6} + \frac{(ae-bd)^3}{e^3(d+ex)^7} \right) dx}{a + bx} \\
 & \quad \downarrow \text{2009} \\
 & \frac{\sqrt{a^2 + 2abx + b^2x^2} \left(\frac{3b^2(bd-ae)}{4e^4(d+ex)^4} - \frac{3b(bd-ae)^2}{5e^4(d+ex)^5} + \frac{(bd-ae)^3}{6e^4(d+ex)^6} - \frac{b^3}{3e^4(d+ex)^3} \right)}{a + bx}
 \end{aligned}$$

input

```
Int[(a^2 + 2*a*b*x + b^2*x^2)^(3/2)/(d + e*x)^7,x]
```



```
output (Sqrt[a^2 + 2*a*b*x + b^2*x^2]*((b*d - a*e)^3/(6*e^4*(d + e*x)^6) - (3*b*(
b*d - a*e)^2)/(5*e^4*(d + e*x)^5) + (3*b^2*(b*d - a*e))/(4*e^4*(d + e*x)^4
) - b^3/(3*e^4*(d + e*x)^3))/(a + b*x)
```

Defintions of rubi rules used

```
rule 27 Int[(a_)*(Fx_), x_Symbol] := Simp[a Int[Fx, x], x] /; FreeQ[a, x] && !Ma
tchQ[Fx, (b_)*(Gx_)] /; FreeQ[b, x]
```

```
rule 53 Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] := Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

```
rule 1102 Int[((d_.) + (e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] := Simp[(a + b*x + c*x^2)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x)^(2*F
racPart[p])) Int[(d + e*x)^m*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c,
d, e, m, p}, x] && EqQ[b^2 - 4*a*c, 0]
```

```
rule 2009 Int[u_, x_Symbol] := Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 2.23 (sec) , antiderivative size = 126, normalized size of antiderivative = 0.88

| method | result |
|---------|--|
| risch | $\frac{\sqrt{(bx+a)^2} \left(-\frac{b^3 x^3}{3e} - \frac{b^2(3ae+bd)x^2}{4e^2} - \frac{b(6e^2 a^2 + 3abde + b^2 d^2)x}{10e^3} - \frac{10e^3 a^3 + 6a^2 bde^2 + 3a b^2 d^2 e + b^3 d^3}{60e^4} \right)}{(bx+a)(ex+d)^6}$ |
| gospers | $-\frac{(20e^3 x^3 b^3 + 45x^2 a b^2 e^3 + 15x^2 b^3 d e^2 + 36a^2 b e^3 x + 18x a b^2 d e^2 + 6b^3 d^2 e x + 10e^3 a^3 + 6a^2 b d e^2 + 3a b^2 d^2 e + b^3 d^3) ((bx+a)^2)^{\frac{3}{2}}}{60e^4 (ex+d)^6 (bx+a)^3}$ |
| default | $-\frac{(20e^3 x^3 b^3 + 45x^2 a b^2 e^3 + 15x^2 b^3 d e^2 + 36a^2 b e^3 x + 18x a b^2 d e^2 + 6b^3 d^2 e x + 10e^3 a^3 + 6a^2 b d e^2 + 3a b^2 d^2 e + b^3 d^3) ((bx+a)^2)^{\frac{3}{2}}}{60e^4 (ex+d)^6 (bx+a)^3}$ |
| orering | $-\frac{(20e^3 x^3 b^3 + 45x^2 a b^2 e^3 + 15x^2 b^3 d e^2 + 36a^2 b e^3 x + 18x a b^2 d e^2 + 6b^3 d^2 e x + 10e^3 a^3 + 6a^2 b d e^2 + 3a b^2 d^2 e + b^3 d^3) (b^2 x^2 + 2abx + a^2)^{\frac{3}{2}}}{60e^4 (bx+a)^3 (ex+d)^6}$ |

input `int((b^2*x^2+2*a*b*x+a^2)^(3/2)/(e*x+d)^7,x,method=_RETURNVERBOSE)`

output
$$\frac{((b*x+a)^2)^{(1/2)}/(b*x+a)*(-1/3*b^3/e*x^3-1/4*b^2/e^2*(3*a*e+b*d)*x^2-1/10*b/e^3*(6*a^2*e^2+3*a*b*d*e+b^2*d^2)*x-1/60/e^4*(10*a^3*e^3+6*a^2*b*d*e^2+3*a*b^2*d^2*e+b^3*d^3))/(e*x+d)^6}$$

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 171, normalized size of antiderivative = 1.20

$$\int \frac{(a^2 + 2abx + b^2x^2)^{3/2}}{(d + ex)^7} dx = \frac{20b^3e^3x^3 + b^3d^3 + 3ab^2d^2e + 6a^2bde^2 + 10a^3e^3 + 15(b^3de^2 + 3ab^2e^3)x^2 + 6(b^3d^2e + 3ab^2de^2 + 6a^2be^3) - 60(e^{10}x^6 + 6de^9x^5 + 15d^2e^8x^4 + 20d^3e^7x^3 + 15d^4e^6x^2 + 6d^5e^5x + d^6e^4)}{60(e^{10}x^6 + 6de^9x^5 + 15d^2e^8x^4 + 20d^3e^7x^3 + 15d^4e^6x^2 + 6d^5e^5x + d^6e^4)}$$

input `integrate((b^2*x^2+2*a*b*x+a^2)^(3/2)/(e*x+d)^7,x, algorithm="fricas")`

output
$$\frac{-1/60*(20*b^3*e^3*x^3 + b^3*d^3 + 3*a*b^2*d^2*e + 6*a^2*b*d*e^2 + 10*a^3*e^3 + 15*(b^3*d*e^2 + 3*a*b^2*e^3)*x^2 + 6*(b^3*d^2*e + 3*a*b^2*d*e^2 + 6*a^2*b*e^3)*x)/(e^{10}*x^6 + 6*d*e^9*x^5 + 15*d^2*e^8*x^4 + 20*d^3*e^7*x^3 + 15*d^4*e^6*x^2 + 6*d^5*e^5*x + d^6*e^4)}$$

Sympy [F]

$$\int \frac{(a^2 + 2abx + b^2x^2)^{3/2}}{(d + ex)^7} dx = \int \frac{((a + bx)^2)^{\frac{3}{2}}}{(d + ex)^7} dx$$

input `integrate((b**2*x**2+2*a*b*x+a**2)**(3/2)/(e*x+d)**7,x)`

output `Integral(((a + b*x)**2)**(3/2)/(d + e*x)**7, x)`

Maxima [F(-2)]

Exception generated.

$$\int \frac{(a^2 + 2abx + b^2x^2)^{3/2}}{(d + ex)^7} dx = \text{Exception raised: ValueError}$$

input `integrate((b^2*x^2+2*a*b*x+a^2)^(3/2)/(e*x+d)^7,x, algorithm="maxima")`

output Exception raised: ValueError >> Computation failed since Maxima requested additional constraints; using the 'assume' command before evaluation *may* help (example of legal syntax is 'assume(a*e-b*d>0)', see `assume?` for more detail)

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 229, normalized size of antiderivative = 1.60

$$\int \frac{(a^2 + 2abx + b^2x^2)^{3/2}}{(d + ex)^7} dx = \frac{b^6 \operatorname{sgn}(bx + a)}{60(b^3d^3e^4 - 3ab^2d^2e^5 + 3a^2bde^6 - a^3e^7)} \\ - \frac{20b^3e^3x^3 \operatorname{sgn}(bx + a) + 15b^3de^2x^2 \operatorname{sgn}(bx + a) + 45ab^2e^3x^2 \operatorname{sgn}(bx + a) + 6b^3d^2ex \operatorname{sgn}(bx + a) + 18ab^2d^2 \operatorname{sgn}(bx + a)}{(d + ex)^6 e^4}$$

input `integrate((b^2*x^2+2*a*b*x+a^2)^(3/2)/(e*x+d)^7,x, algorithm="giac")`

output `1/60*b^6*sgn(b*x + a)/(b^3*d^3*e^4 - 3*a*b^2*d^2*e^5 + 3*a^2*b*d*e^6 - a^3*e^7) - 1/60*(20*b^3*e^3*x^3*sgn(b*x + a) + 15*b^3*d*e^2*x^2*sgn(b*x + a) + 45*a*b^2*e^3*x^2*sgn(b*x + a) + 6*b^3*d^2*e*x*sgn(b*x + a) + 18*a*b^2*d*e^2*x*sgn(b*x + a) + 36*a^2*b*e^3*x*sgn(b*x + a) + b^3*d^3*sgn(b*x + a) + 3*a*b^2*d^2*e*sgn(b*x + a) + 6*a^2*b*d*e^2*sgn(b*x + a) + 10*a^3*e^3*sgn(b*x + a))/(e*x + d)^6*e^4)`

Mupad [B] (verification not implemented)

Time = 5.07 (sec) , antiderivative size = 284, normalized size of antiderivative = 1.99

$$\int \frac{(a^2 + 2abx + b^2x^2)^{3/2}}{(d + ex)^7} dx = \frac{\left(\frac{2b^3d - 3ab^2e}{4e^4} + \frac{b^3d}{4e^4}\right) \sqrt{a^2 + 2abx + b^2x^2}}{(a + bx)(d + ex)^4} - \frac{\left(\frac{3a^2be^2 - 3ab^2de + b^3d^2}{5e^4} + \frac{d\left(\frac{b^3d}{5e^3} - \frac{b^2(3ae - bd)}{5e^3}\right)}{e}\right) \sqrt{a^2 + 2abx + b^2x^2}}{(a + bx)(d + ex)^5} - \frac{\left(\frac{a^3}{6e} - \frac{d\left(\frac{a^2b}{2e} - \frac{d\left(\frac{ab^2}{2e} - \frac{b^3d}{6e^2}\right)}{e}\right)}{e}\right) \sqrt{a^2 + 2abx + b^2x^2}}{(a + bx)(d + ex)^6} - \frac{b^3 \sqrt{a^2 + 2abx + b^2x^2}}{3e^4(a + bx)(d + ex)^3}$$

input `int((a^2 + b^2*x^2 + 2*a*b*x)^(3/2)/(d + e*x)^7,x)`output `((((2*b^3*d - 3*a*b^2*e)/(4*e^4) + (b^3*d)/(4*e^4))*(a^2 + b^2*x^2 + 2*a*b*x)^(1/2))/((a + b*x)*(d + e*x)^4) - (((b^3*d^2 + 3*a^2*b*e^2 - 3*a*b^2*d*e)/(5*e^4) + (d*((b^3*d)/(5*e^3) - (b^2*(3*a*e - b*d))/(5*e^3)))/e)*(a^2 + b^2*x^2 + 2*a*b*x)^(1/2))/((a + b*x)*(d + e*x)^5) - ((a^3/(6*e) - (d*((a^2*b)/(2*e) - (d*((a*b^2)/(2*e) - (b^3*d)/(6*e^2)))/e))/e)*(a^2 + b^2*x^2 + 2*a*b*x)^(1/2))/((a + b*x)*(d + e*x)^6) - (b^3*(a^2 + b^2*x^2 + 2*a*b*x)^(1/2))/(3*e^4*(a + b*x)*(d + e*x)^3)`**Reduce [B] (verification not implemented)**

Time = 0.27 (sec) , antiderivative size = 170, normalized size of antiderivative = 1.19

$$\int \frac{(a^2 + 2abx + b^2x^2)^{3/2}}{(d + ex)^7} dx = \frac{-20b^3e^3x^3 - 45ab^2e^3x^2 - 15b^3de^2x^2 - 36a^2be^3x - 18ab^2de^2x - 6b^3d^2ex - 6b^3d^2e}{60e^4(e^6x^6 + 6de^5x^5 + 15d^2e^4x^4 + 20d^3e^3x^3 + 15d^4e^2x^2 + 6d^5ex + d^6)}$$

input `int((b^2*x^2+2*a*b*x+a^2)^(3/2)/(e*x+d)^7,x)`

output

```
( - 10*a**3*e**3 - 6*a**2*b*d*e**2 - 36*a**2*b*e**3*x - 3*a*b**2*d**2*e -  
18*a*b**2*d*e**2*x - 45*a*b**2*e**3*x**2 - b**3*d**3 - 6*b**3*d**2*e*x - 1  
5*b**3*d*e**2*x**2 - 20*b**3*e**3*x**3)/(60*e**4*(d**6 + 6*d**5*e*x + 15*d  
**4*e**2*x**2 + 20*d**3*e**3*x**3 + 15*d**2*e**4*x**4 + 6*d*e**5*x**5 + e  
*6*x**6))
```

3.289 $\int \frac{(a^2+2abx+b^2x^2)^{3/2}}{(d+ex)^8} dx$

| | |
|---|------|
| Optimal result | 2177 |
| Mathematica [A] (verified) | 2178 |
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Optimal result

Integrand size = 28, antiderivative size = 200

$$\int \frac{(a^2 + 2abx + b^2x^2)^{3/2}}{(d + ex)^8} dx = \frac{(bd - ae)^3 \sqrt{a^2 + 2abx + b^2x^2}}{7e^4(a + bx)(d + ex)^7} - \frac{b(bd - ae)^2 \sqrt{a^2 + 2abx + b^2x^2}}{2e^4(a + bx)(d + ex)^6} + \frac{3b^2(bd - ae) \sqrt{a^2 + 2abx + b^2x^2}}{5e^4(a + bx)(d + ex)^5} - \frac{b^3 \sqrt{a^2 + 2abx + b^2x^2}}{4e^4(a + bx)(d + ex)^4}$$

output

```
1/7*(-a*e+b*d)^3*((b*x+a)^2)^(1/2)/e^4/(b*x+a)/(e*x+d)^7-1/2*b*(-a*e+b*d)^2*((b*x+a)^2)^(1/2)/e^4/(b*x+a)/(e*x+d)^6+3/5*b^2*(-a*e+b*d)*((b*x+a)^2)^(1/2)/e^4/(b*x+a)/(e*x+d)^5-1/4*b^3*((b*x+a)^2)^(1/2)/e^4/(b*x+a)/(e*x+d)^4
```

Mathematica [A] (verified)

Time = 1.03 (sec) , antiderivative size = 112, normalized size of antiderivative = 0.56

$$\int \frac{(a^2 + 2abx + b^2x^2)^{3/2}}{(d + ex)^8} dx = \frac{\sqrt{(a + bx)^2(20a^3e^3 + 10a^2be^2(d + 7ex) + 4ab^2e(d^2 + 7dex + 21e^2x^2) + b^3(d^3 + 7d^2ex + 21de^2x^2 + 35e^3x^3))}}{140e^4(a + bx)(d + ex)^7}$$

input

```
Integrate[(a^2 + 2*a*b*x + b^2*x^2)^(3/2)/(d + e*x)^8,x]
```

output

```
-1/140*(Sqrt[(a + b*x)^2]*(20*a^3*e^3 + 10*a^2*b*e^2*(d + 7*e*x) + 4*a*b^2*
*e*(d^2 + 7*d*e*x + 21*e^2*x^2) + b^3*(d^3 + 7*d^2*e*x + 21*d*e^2*x^2 + 35
*e^3*x^3)))/(e^4*(a + b*x)*(d + e*x)^7)
```

Rubi [A] (verified)

Time = 0.46 (sec) , antiderivative size = 120, normalized size of antiderivative = 0.60, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {1102, 27, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\begin{aligned} & \int \frac{(a^2 + 2abx + b^2x^2)^{3/2}}{(d + ex)^8} dx \\ & \quad \downarrow \text{1102} \\ & \frac{\sqrt{a^2 + 2abx + b^2x^2} \int \frac{b^3(a+bx)^3}{(d+ex)^8} dx}{b^3(a + bx)} \\ & \quad \downarrow \text{27} \\ & \frac{\sqrt{a^2 + 2abx + b^2x^2} \int \frac{(a+bx)^3}{(d+ex)^8} dx}{a + bx} \\ & \quad \downarrow \text{53} \end{aligned}$$

$$\frac{\sqrt{a^2 + 2abx + b^2x^2} \int \left(\frac{b^3}{e^3(d+ex)^5} - \frac{3(bd-ae)b^2}{e^3(d+ex)^6} + \frac{3(bd-ae)^2b}{e^3(d+ex)^7} + \frac{(ae-bd)^3}{e^3(d+ex)^8} \right) dx}{a + bx}$$

↓ 2009

$$\frac{\sqrt{a^2 + 2abx + b^2x^2} \left(\frac{3b^2(bd-ae)}{5e^4(d+ex)^5} - \frac{b(bd-ae)^2}{2e^4(d+ex)^6} + \frac{(bd-ae)^3}{7e^4(d+ex)^7} - \frac{b^3}{4e^4(d+ex)^4} \right)}{a + bx}$$

input `Int[(a^2 + 2*a*b*x + b^2*x^2)^(3/2)/(d + e*x)^8,x]`

output `(Sqrt[a^2 + 2*a*b*x + b^2*x^2]*((b*d - a*e)^3/(7*e^4*(d + e*x)^7) - (b*(b*d - a*e)^2)/(2*e^4*(d + e*x)^6) + (3*b^2*(b*d - a*e))/(5*e^4*(d + e*x)^5) - b^3/(4*e^4*(d + e*x)^4)))/(a + b*x)`

Defintions of rubi rules used

rule 27 `Int[(a_)*(F_x_), x_Symbol] := Simp[a Int[F_x, x], x] /; FreeQ[a, x] && !MatchQ[F_x, (b_)*(G_x_)] /; FreeQ[b, x]`

rule 53 `Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] := Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && IGtQ[m, 0] && (!IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])`

rule 1102 `Int[((d_.) + (e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] := Simp[(a + b*x + c*x^2)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x)^(2*FracPart[p])) Int[(d + e*x)^m*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && EqQ[b^2 - 4*a*c, 0]`

rule 2009 `Int[u_, x_Symbol] := Simp[IntSum[u, x], x] /; SumQ[u]`

Maple [A] (verified)

Time = 2.73 (sec) , antiderivative size = 126, normalized size of antiderivative = 0.63

| method | result |
|---------|---|
| risch | $\frac{\sqrt{(bx+a)^2} \left(-\frac{b^3 x^3}{4e} - \frac{3b^2(4ae+bd)x^2}{20e^2} - \frac{b(10e^2 a^2 + 4abde + b^2 d^2)x}{20e^3} - \frac{20e^3 a^3 + 10a^2 b d e^2 + 4a b^2 d^2 e + b^3 d^3}{140e^4} \right)}{(bx+a)(ex+d)^7}$ |
| gospers | $-\frac{(35e^3 x^3 b^3 + 84x^2 a b^2 e^3 + 21x^2 b^3 d e^2 + 70a^2 b e^3 x + 28xa b^2 d e^2 + 7b^3 d^2 e x + 20e^3 a^3 + 10a^2 b d e^2 + 4a b^2 d^2 e + b^3 d^3) ((bx+a)^2)^{\frac{3}{2}}}{140e^4 (ex+d)^7 (bx+a)^3}$ |
| default | $-\frac{(35e^3 x^3 b^3 + 84x^2 a b^2 e^3 + 21x^2 b^3 d e^2 + 70a^2 b e^3 x + 28xa b^2 d e^2 + 7b^3 d^2 e x + 20e^3 a^3 + 10a^2 b d e^2 + 4a b^2 d^2 e + b^3 d^3) ((bx+a)^2)^{\frac{3}{2}}}{140e^4 (ex+d)^7 (bx+a)^3}$ |
| orering | $-\frac{(35e^3 x^3 b^3 + 84x^2 a b^2 e^3 + 21x^2 b^3 d e^2 + 70a^2 b e^3 x + 28xa b^2 d e^2 + 7b^3 d^2 e x + 20e^3 a^3 + 10a^2 b d e^2 + 4a b^2 d^2 e + b^3 d^3) (b^2 x^2 + 2abx + a^2)^{\frac{3}{2}}}{140e^4 (bx+a)^3 (ex+d)^7}$ |

input `int((b^2*x^2+2*a*b*x+a^2)^(3/2)/(e*x+d)^8,x,method=_RETURNVERBOSE)`

output
$$\frac{((b*x+a)^2)^{(1/2)}/(b*x+a)*(-1/4*b^3/e*x^3-3/20*b^2/e^2*(4*a*e+b*d)*x^2-1/20*b/e^3*(10*a^2*e^2+4*a*b*d*e+b^2*d^2)*x-1/140/e^4*(20*a^3*e^3+10*a^2*b*d*e^2+4*a*b^2*d^2*e+b^3*d^3))/(e*x+d)^7}$$

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 182, normalized size of antiderivative = 0.91

$$\int \frac{(a^2 + 2abx + b^2 x^2)^{3/2}}{(d + ex)^8} dx = \frac{35 b^3 e^3 x^3 + b^3 d^3 + 4 a b^2 d^2 e + 10 a^2 b d e^2 + 20 a^3 e^3 + 21 (b^3 d e^2 + 4 a b^2 e^3) x^2 + 7 (b^3 d^2 e + 4 a b^2 d e^2 + 10 a^2 b d e^3) x + 7 (b^3 d^2 e + 4 a b^2 d e^2 + 10 a^2 b d e^3)}{140 (e^{11} x^7 + 7 d e^{10} x^6 + 21 d^2 e^9 x^5 + 35 d^3 e^8 x^4 + 35 d^4 e^7 x^3 + 21 d^5 e^6 x^2 + 7 d^6 e^5 x + d^7 e^4)}$$

input `integrate((b^2*x^2+2*a*b*x+a^2)^(3/2)/(e*x+d)^8,x, algorithm="fricas")`

output
$$-1/140*(35*b^3*e^3*x^3 + b^3*d^3 + 4*a*b^2*d^2*e + 10*a^2*b*d*e^2 + 20*a^3*e^3 + 21*(b^3*d*e^2 + 4*a*b^2*e^3)*x^2 + 7*(b^3*d^2*e + 4*a*b^2*d*e^2 + 10*a^2*b*d*e^3)*x)/(e^{11}*x^7 + 7*d*e^{10}*x^6 + 21*d^2*e^9*x^5 + 35*d^3*e^8*x^4 + 35*d^4*e^7*x^3 + 21*d^5*e^6*x^2 + 7*d^6*e^5*x + d^7*e^4)$$

Sympy [F]

$$\int \frac{(a^2 + 2abx + b^2x^2)^{3/2}}{(d + ex)^8} dx = \int \frac{((a + bx)^2)^{\frac{3}{2}}}{(d + ex)^8} dx$$

input `integrate((b**2*x**2+2*a*b*x+a**2)**(3/2)/(e*x+d)**8,x)`

output `Integral(((a + b*x)**2)**(3/2)/(d + e*x)**8, x)`

Maxima [F(-2)]

Exception generated.

$$\int \frac{(a^2 + 2abx + b^2x^2)^{3/2}}{(d + ex)^8} dx = \text{Exception raised: ValueError}$$

input `integrate((b^2*x^2+2*a*b*x+a^2)^(3/2)/(e*x+d)^8,x, algorithm="maxima")`

output `Exception raised: ValueError >> Computation failed since Maxima requested additional constraints; using the 'assume' command before evaluation *may* help (example of legal syntax is 'assume(a*e-b*d>0)', see `assume?` for more detail`

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 242, normalized size of antiderivative = 1.21

$$\int \frac{(a^2 + 2abx + b^2x^2)^{3/2}}{(d + ex)^8} dx = \frac{b^7 \operatorname{sgn}(bx + a)}{140 (b^4 d^4 e^4 - 4 a b^3 d^3 e^5 + 6 a^2 b^2 d^2 e^6 - 4 a^3 b d e^7 + a^4 e^8)} \\ - \frac{35 b^3 e^3 x^3 \operatorname{sgn}(bx + a) + 21 b^3 d e^2 x^2 \operatorname{sgn}(bx + a) + 84 a b^2 e^3 x^2 \operatorname{sgn}(bx + a) + 7 b^3 d^2 e x \operatorname{sgn}(bx + a) + 28 a b^2 d^2 \operatorname{sgn}(bx + a)}{(d + ex)^8}$$

input `integrate((b^2*x^2+2*a*b*x+a^2)^(3/2)/(e*x+d)^8,x, algorithm="giac")`

output

```
1/140*b^7*sgn(b*x + a)/(b^4*d^4*e^4 - 4*a*b^3*d^3*e^5 + 6*a^2*b^2*d^2*e^6
- 4*a^3*b*d*e^7 + a^4*e^8) - 1/140*(35*b^3*e^3*x^3*sgn(b*x + a) + 21*b^3*d
*e^2*x^2*sgn(b*x + a) + 84*a*b^2*e^3*x^2*sgn(b*x + a) + 7*b^3*d^2*e*x*sgn(
b*x + a) + 28*a*b^2*d*e^2*x*sgn(b*x + a) + 70*a^2*b*e^3*x*sgn(b*x + a) + b
^3*d^3*sgn(b*x + a) + 4*a*b^2*d^2*e*sgn(b*x + a) + 10*a^2*b*d*e^2*sgn(b*x
+ a) + 20*a^3*e^3*sgn(b*x + a))/((e*x + d)^7*e^4)
```

Mupad [B] (verification not implemented)

Time = 5.17 (sec) , antiderivative size = 284, normalized size of antiderivative = 1.42

$$\int \frac{(a^2 + 2abx + b^2x^2)^{3/2}}{(d + ex)^8} dx = \frac{\left(\frac{2b^3d - 3ab^2e}{5e^4} + \frac{b^3d}{5e^4}\right) \sqrt{a^2 + 2abx + b^2x^2}}{(a + bx)(d + ex)^5} - \frac{\left(\frac{3a^2be^2 - 3ab^2de + b^3d^2}{6e^4} + \frac{d\left(\frac{b^3d}{6e^3} - \frac{b^2(3ae - bd)}{6e^3}\right)}{e}\right) \sqrt{a^2 + 2abx + b^2x^2}}{(a + bx)(d + ex)^6} - \frac{\left(\frac{a^3}{7e} - \frac{d\left(\frac{3a^2b}{7e} - \frac{d\left(\frac{3ab^2}{7e} - \frac{b^3d}{7e^2}\right)}{e}\right)}{e}\right) \sqrt{a^2 + 2abx + b^2x^2}}{(a + bx)(d + ex)^7} - \frac{b^3 \sqrt{a^2 + 2abx + b^2x^2}}{4e^4 (a + bx)(d + ex)^4}$$

input

```
int((a^2 + b^2*x^2 + 2*a*b*x)^(3/2)/(d + e*x)^8,x)
```

output

```
((2*b^3*d - 3*a*b^2*e)/(5*e^4) + (b^3*d)/(5*e^4))*(a^2 + b^2*x^2 + 2*a*b*x)^(1/2)/((a + b*x)*(d + e*x)^5) - (((b^3*d^2 + 3*a^2*b*e^2 - 3*a*b^2*d*e)/(6*e^4) + (d*((b^3*d)/(6*e^3) - (b^2*(3*a*e - b*d))/(6*e^3)))/e)*(a^2 + b^2*x^2 + 2*a*b*x)^(1/2)/((a + b*x)*(d + e*x)^6) - ((a^3/(7*e) - (d*((3*a^2*b)/(7*e) - (d*((3*a*b^2)/(7*e) - (b^3*d)/(7*e^2)))/e))/e)*(a^2 + b^2*x^2 + 2*a*b*x)^(1/2)/((a + b*x)*(d + e*x)^7) - (b^3*(a^2 + b^2*x^2 + 2*a*b*x)^(1/2))/(4*e^4*(a + b*x)*(d + e*x)^4)
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 181, normalized size of antiderivative = 0.90

$$\int \frac{(a^2 + 2abx + b^2x^2)^{3/2}}{(d + ex)^8} dx = \frac{-35b^3e^3x^3 - 84ab^2e^3x^2 - 21b^3de^2x^2 - 70a^2be^3x - 28ab^2de^2x - 7b^3d^2ex - 7b^3d^2e^2x}{140e^4(e^7x^7 + 7de^6x^6 + 21d^2e^5x^5 + 35d^3e^4x^4 + 35d^4e^3x^3 + 21d^5e^2x^2 + 7d^6e^1x + d^7)}$$

input `int((b^2*x^2+2*a*b*x+a^2)^(3/2)/(e*x+d)^8,x)`output `(- 20*a**3*e**3 - 10*a**2*b*d*e**2 - 70*a**2*b*e**3*x - 4*a*b**2*d**2*e - 28*a*b**2*d*e**2*x - 84*a*b**2*e**3*x**2 - b**3*d**3 - 7*b**3*d**2*e*x - 21*b**3*d*e**2*x**2 - 35*b**3*e**3*x**3)/(140*e**4*(d**7 + 7*d**6*e*x + 21*d**5*e**2*x**2 + 35*d**4*e**3*x**3 + 35*d**3*e**4*x**4 + 21*d**2*e**5*x**5 + 7*d*e**6*x**6 + e**7*x**7))`

3.290 $\int \frac{(a^2+2abx+b^2x^2)^{3/2}}{(d+ex)^9} dx$

| | |
|---|------|
| Optimal result | 2184 |
| Mathematica [A] (verified) | 2185 |
| Rubi [A] (verified) | 2185 |
| Maple [A] (verified) | 2187 |
| Fricas [A] (verification not implemented) | 2187 |
| Sympy [F] | 2188 |
| Maxima [F(-2)] | 2188 |
| Giac [A] (verification not implemented) | 2189 |
| Mupad [B] (verification not implemented) | 2189 |
| Reduce [B] (verification not implemented) | 2190 |

Optimal result

Integrand size = 28, antiderivative size = 200

$$\int \frac{(a^2 + 2abx + b^2x^2)^{3/2}}{(d + ex)^9} dx = \frac{(bd - ae)^3 \sqrt{a^2 + 2abx + b^2x^2}}{8e^4(a + bx)(d + ex)^8} - \frac{3b(bd - ae)^2 \sqrt{a^2 + 2abx + b^2x^2}}{7e^4(a + bx)(d + ex)^7} + \frac{b^2(bd - ae) \sqrt{a^2 + 2abx + b^2x^2}}{2e^4(a + bx)(d + ex)^6} - \frac{b^3 \sqrt{a^2 + 2abx + b^2x^2}}{5e^4(a + bx)(d + ex)^5}$$

output $\frac{1}{8}*(-a*e+b*d)^3*((b*x+a)^2)^{(1/2)}/e^4/(b*x+a)/(e*x+d)^8-3/7*b*(-a*e+b*d)^2*((b*x+a)^2)^{(1/2)}/e^4/(b*x+a)/(e*x+d)^7+1/2*b^2*(-a*e+b*d)*((b*x+a)^2)^{(1/2)}/e^4/(b*x+a)/(e*x+d)^6-1/5*b^3*((b*x+a)^2)^{(1/2)}/e^4/(b*x+a)/(e*x+d)^5$

Mathematica [A] (verified)

Time = 1.04 (sec) , antiderivative size = 112, normalized size of antiderivative = 0.56

$$\int \frac{(a^2 + 2abx + b^2x^2)^{3/2}}{(d + ex)^9} dx = \frac{\sqrt{(a + bx)^2}(35a^3e^3 + 15a^2be^2(d + 8ex) + 5ab^2e(d^2 + 8dex + 28e^2x^2) + b^3(d^3 + 8d^2ex + 28de^2x^2 + 56e^3x^3))}{280e^4(a + bx)(d + ex)^8}$$

input

```
Integrate[(a^2 + 2*a*b*x + b^2*x^2)^(3/2)/(d + e*x)^9,x]
```

output

```
-1/280*(Sqrt[(a + b*x)^2]*(35*a^3*e^3 + 15*a^2*b*e^2*(d + 8*e*x) + 5*a*b^2*
*e*(d^2 + 8*d*e*x + 28*e^2*x^2) + b^3*(d^3 + 8*d^2*e*x + 28*d*e^2*x^2 + 56
*e^3*x^3)))/(e^4*(a + b*x)*(d + e*x)^8)
```

Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 120, normalized size of antiderivative = 0.60, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {1102, 27, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\begin{aligned} & \int \frac{(a^2 + 2abx + b^2x^2)^{3/2}}{(d + ex)^9} dx \\ & \quad \downarrow \text{1102} \\ & \frac{\sqrt{a^2 + 2abx + b^2x^2} \int \frac{b^3(a+bx)^3}{(d+ex)^9} dx}{b^3(a + bx)} \\ & \quad \downarrow \text{27} \\ & \frac{\sqrt{a^2 + 2abx + b^2x^2} \int \frac{(a+bx)^3}{(d+ex)^9} dx}{a + bx} \\ & \quad \downarrow \text{53} \end{aligned}$$

$$\frac{\sqrt{a^2 + 2abx + b^2x^2} \int \left(\frac{b^3}{e^3(d+ex)^6} - \frac{3(bd-ae)b^2}{e^3(d+ex)^7} + \frac{3(bd-ae)^2b}{e^3(d+ex)^8} + \frac{(ae-bd)^3}{e^3(d+ex)^9} \right) dx}{a + bx}$$

↓ 2009

$$\frac{\sqrt{a^2 + 2abx + b^2x^2} \left(\frac{b^2(bd-ae)}{2e^4(d+ex)^6} - \frac{3b(bd-ae)^2}{7e^4(d+ex)^7} + \frac{(bd-ae)^3}{8e^4(d+ex)^8} - \frac{b^3}{5e^4(d+ex)^5} \right)}{a + bx}$$

input `Int[(a^2 + 2*a*b*x + b^2*x^2)^(3/2)/(d + e*x)^9,x]`

output `(Sqrt[a^2 + 2*a*b*x + b^2*x^2]*((b*d - a*e)^3/(8*e^4*(d + e*x)^8) - (3*b*(b*d - a*e)^2)/(7*e^4*(d + e*x)^7) + (b^2*(b*d - a*e))/(2*e^4*(d + e*x)^6) - b^3/(5*e^4*(d + e*x)^5)))/(a + b*x)`

Defintions of rubi rules used

rule 27 `Int[(a_)*(F_x_), x_Symbol] := Simp[a Int[F_x, x], x] /; FreeQ[a, x] && !MatchQ[F_x, (b_)*(G_x_)] /; FreeQ[b, x]`

rule 53 `Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] := Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && IGtQ[m, 0] && (!IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])`

rule 1102 `Int[((d_.) + (e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] := Simp[(a + b*x + c*x^2)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x)^(2*FracPart[p])) Int[(d + e*x)^m*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && EqQ[b^2 - 4*a*c, 0]`

rule 2009 `Int[u_, x_Symbol] := Simp[IntSum[u, x], x] /; SumQ[u]`

Maple [A] (verified)

Time = 3.24 (sec) , antiderivative size = 126, normalized size of antiderivative = 0.63

| method | result |
|---------|---|
| risch | $\frac{\sqrt{(bx+a)^2} \left(-\frac{b^3 x^3}{5e} - \frac{b^2(5ae+bd)x^2}{10e^2} - \frac{b(15e^2 a^2 + 5abde + b^2 d^2)x}{35e^3} - \frac{35e^3 a^3 + 15a^2 bde^2 + 5a b^2 d^2 e + b^3 d^3}{280e^4} \right)}{(bx+a)(ex+d)^8}$ |
| gospers | $-\frac{(56e^3 x^3 b^3 + 140x^2 a b^2 e^3 + 28x^2 b^3 d e^2 + 120a^2 b e^3 x + 40xa b^2 d e^2 + 8b^3 d^2 e x + 35e^3 a^3 + 15a^2 b d e^2 + 5a b^2 d^2 e + b^3 d^3) ((bx+a)^2)^{\frac{3}{2}}}{280e^4 (ex+d)^8 (bx+a)^3}$ |
| default | $-\frac{(56e^3 x^3 b^3 + 140x^2 a b^2 e^3 + 28x^2 b^3 d e^2 + 120a^2 b e^3 x + 40xa b^2 d e^2 + 8b^3 d^2 e x + 35e^3 a^3 + 15a^2 b d e^2 + 5a b^2 d^2 e + b^3 d^3) ((bx+a)^2)^{\frac{3}{2}}}{280e^4 (ex+d)^8 (bx+a)^3}$ |
| orering | $-\frac{(56e^3 x^3 b^3 + 140x^2 a b^2 e^3 + 28x^2 b^3 d e^2 + 120a^2 b e^3 x + 40xa b^2 d e^2 + 8b^3 d^2 e x + 35e^3 a^3 + 15a^2 b d e^2 + 5a b^2 d^2 e + b^3 d^3) (b^2 x^2 + 2abx + a^2)^{\frac{3}{2}}}{280e^4 (bx+a)^3 (ex+d)^8}$ |

input `int((b^2*x^2+2*a*b*x+a^2)^(3/2)/(e*x+d)^9,x,method=_RETURNVERBOSE)`

output
$$\frac{((b*x+a)^2)^{(1/2)}/(b*x+a)*(-1/5*b^3/e*x^3-1/10*b^2/e^2*(5*a*e+b*d)*x^2-1/3*5*b/e^3*(15*a^2*e^2+5*a*b*d*e+b^2*d^2)*x-1/280/e^4*(35*a^3*e^3+15*a^2*b*d*e^2+5*a*b^2*d^2*e+b^3*d^3))/(e*x+d)^8}$$

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 193, normalized size of antiderivative = 0.96

$$\int \frac{(a^2 + 2abx + b^2 x^2)^{3/2}}{(d + ex)^9} dx = \frac{56 b^3 e^3 x^3 + b^3 d^3 + 5 a b^2 d^2 e + 15 a^2 b d e^2 + 35 a^3 e^3 + 28 (b^3 d e^2 + 5 a b^2 e^3) x^2 + 8 (b^3 d^2 e + 5 a b^2 d e^2 + 15 a^2 b d e^3) x + 280 (e^{12} x^8 + 8 d e^{11} x^7 + 28 d^2 e^{10} x^6 + 56 d^3 e^9 x^5 + 70 d^4 e^8 x^4 + 56 d^5 e^7 x^3 + 28 d^6 e^6 x^2 + 8 d^7 e^5 x + d^8 e^4)}{280 (e^{12} x^8 + 8 d e^{11} x^7 + 28 d^2 e^{10} x^6 + 56 d^3 e^9 x^5 + 70 d^4 e^8 x^4 + 56 d^5 e^7 x^3 + 28 d^6 e^6 x^2 + 8 d^7 e^5 x + d^8 e^4)}$$

input `integrate((b^2*x^2+2*a*b*x+a^2)^(3/2)/(e*x+d)^9,x,algorithm="fricas")`

output

```
-1/280*(56*b^3*e^3*x^3 + b^3*d^3 + 5*a*b^2*d^2*e + 15*a^2*b*d*e^2 + 35*a^3
*e^3 + 28*(b^3*d*e^2 + 5*a*b^2*e^3)*x^2 + 8*(b^3*d^2*e + 5*a*b^2*d*e^2 + 1
5*a^2*b*e^3)*x)/(e^12*x^8 + 8*d*e^11*x^7 + 28*d^2*e^10*x^6 + 56*d^3*e^9*x^
5 + 70*d^4*e^8*x^4 + 56*d^5*e^7*x^3 + 28*d^6*e^6*x^2 + 8*d^7*e^5*x + d^8*e
^4)
```

Sympy [F]

$$\int \frac{(a^2 + 2abx + b^2x^2)^{3/2}}{(d + ex)^9} dx = \int \frac{((a + bx)^2)^{\frac{3}{2}}}{(d + ex)^9} dx$$

input

```
integrate((b**2*x**2+2*a*b*x+a**2)**(3/2)/(e*x+d)**9,x)
```

output

```
Integral(((a + b*x)**2)**(3/2)/(d + e*x)**9, x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(a^2 + 2abx + b^2x^2)^{3/2}}{(d + ex)^9} dx = \text{Exception raised: ValueError}$$

input

```
integrate((b^2*x^2+2*a*b*x+a^2)^(3/2)/(e*x+d)^9,x, algorithm="maxima")
```

output

```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
help (example of legal syntax is 'assume(a*e-b*d>0)', see `assume?` for m
ore detail
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 257, normalized size of antiderivative = 1.28

$$\int \frac{(a^2 + 2abx + b^2x^2)^{3/2}}{(d + ex)^9} dx = \frac{b^8 \operatorname{sgn}(bx + a)}{280 (b^5 d^5 e^4 - 5 ab^4 d^4 e^5 + 10 a^2 b^3 d^3 e^6 - 10 a^3 b^2 d^2 e^7 + 5 a^4 b d e^8 - a^5 e^9)} - \frac{56 b^3 e^3 x^3 \operatorname{sgn}(bx + a) + 28 b^3 d e^2 x^2 \operatorname{sgn}(bx + a) + 140 a b^2 e^3 x^2 \operatorname{sgn}(bx + a) + 8 b^3 d^2 e x \operatorname{sgn}(bx + a) + 40 a b^2$$

input

```
integrate((b^2*x^2+2*a*b*x+a^2)^(3/2)/(e*x+d)^9,x, algorithm="giac")
```

output

```
1/280*b^8*sgn(b*x + a)/(b^5*d^5*e^4 - 5*a*b^4*d^4*e^5 + 10*a^2*b^3*d^3*e^6 - 10*a^3*b^2*d^2*e^7 + 5*a^4*b*d*e^8 - a^5*e^9) - 1/280*(56*b^3*e^3*x^3*sgn(b*x + a) + 28*b^3*d*e^2*x^2*sgn(b*x + a) + 140*a*b^2*e^3*x^2*sgn(b*x + a) + 8*b^3*d^2*e*x*sgn(b*x + a) + 40*a*b^2*d*e^2*x*sgn(b*x + a) + 120*a^2*b*e^3*x*sgn(b*x + a) + b^3*d^3*sgn(b*x + a) + 5*a*b^2*d^2*e*sgn(b*x + a) + 15*a^2*b*d*e^2*sgn(b*x + a) + 35*a^3*e^3*sgn(b*x + a))/((e*x + d)^8*e^4)
```

Mupad [B] (verification not implemented)

Time = 5.27 (sec) , antiderivative size = 284, normalized size of antiderivative = 1.42

$$\int \frac{(a^2 + 2abx + b^2x^2)^{3/2}}{(d + ex)^9} dx = \frac{\left(\frac{2b^3d-3ab^2e}{6e^4} + \frac{b^3d}{6e^4}\right) \sqrt{a^2 + 2abx + b^2x^2}}{(a + bx)(d + ex)^6} - \frac{\left(\frac{3a^2be^2-3ab^2de+b^3d^2}{7e^4} + \frac{d\left(\frac{b^3d}{7e^3} - \frac{b^2(3ae-bd)}{7e^3}\right)}{e}\right) \sqrt{a^2 + 2abx + b^2x^2}}{(a + bx)(d + ex)^7} - \frac{\left(\frac{a^3}{8e} - \frac{d\left(\frac{3a^2b}{8e} - \frac{d\left(\frac{3ab^2}{8e} - \frac{b^3d}{8e^2}\right)}{e}\right)}{e}\right) \sqrt{a^2 + 2abx + b^2x^2}}{(a + bx)(d + ex)^8} - \frac{b^3 \sqrt{a^2 + 2abx + b^2x^2}}{5e^4 (a + bx)(d + ex)^5}$$

input

```
int((a^2 + b^2*x^2 + 2*a*b*x)^(3/2)/(d + e*x)^9,x)
```

output

```
((2*b^3*d - 3*a*b^2*e)/(6*e^4) + (b^3*d)/(6*e^4))*(a^2 + b^2*x^2 + 2*a*b*x)^(1/2)/((a + b*x)*(d + e*x)^6) - (((b^3*d^2 + 3*a^2*b*e^2 - 3*a*b^2*d*e)/(7*e^4) + (d*((b^3*d)/(7*e^3) - (b^2*(3*a*e - b*d))/(7*e^3)))/e)*(a^2 + b^2*x^2 + 2*a*b*x)^(1/2)/((a + b*x)*(d + e*x)^7) - ((a^3/(8*e) - (d*((3*a^2*b)/(8*e) - (d*((3*a*b^2)/(8*e) - (b^3*d)/(8*e^2)))/e))/e)*(a^2 + b^2*x^2 + 2*a*b*x)^(1/2)/((a + b*x)*(d + e*x)^8) - (b^3*(a^2 + b^2*x^2 + 2*a*b*x)^(1/2))/(5*e^4*(a + b*x)*(d + e*x)^5)
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 192, normalized size of antiderivative = 0.96

$$\int \frac{(a^2 + 2abx + b^2x^2)^{3/2}}{(d + ex)^9} dx = \frac{-56b^3e^3x^3 - 140ab^2e^3x^2 - 28b^3de^2x^2 - 120a^2be^3x - 40ab^2de^2x - 8b^3d^2e}{280e^4(e^8x^8 + 8de^7x^7 + 28d^2e^6x^6 + 56d^3e^5x^5 + 70d^4e^4x^4 + 56d^5e^3x^3 + 28d^6e^2x^2 + 8d^7ex + d^8)}$$

input

```
int((b^2*x^2+2*a*b*x+a^2)^(3/2)/(e*x+d)^9,x)
```

output

```
( - 35*a**3*e**3 - 15*a**2*b*d*e**2 - 120*a**2*b*e**3*x - 5*a*b**2*d**2*e - 40*a*b**2*d*e**2*x - 140*a*b**2*e**3*x**2 - b**3*d**3 - 8*b**3*d**2*e*x - 28*b**3*d*e**2*x**2 - 56*b**3*e**3*x**3)/(280*e**4*(d**8 + 8*d**7*e*x + 28*d**6*e**2*x**2 + 56*d**5*e**3*x**3 + 70*d**4*e**4*x**4 + 56*d**3*e**5*x**5 + 28*d**2*e**6*x**6 + 8*d*e**7*x**7 + e**8*x**8))
```

3.291 $\int (d + ex)^5 (a^2 + 2abx + b^2x^2)^{5/2} dx$

| | |
|---|------|
| Optimal result | 2191 |
| Mathematica [A] (verified) | 2192 |
| Rubi [A] (verified) | 2192 |
| Maple [B] (verified) | 2194 |
| Fricas [B] (verification not implemented) | 2195 |
| Sympy [B] (verification not implemented) | 2196 |
| Maxima [B] (verification not implemented) | 2197 |
| Giac [B] (verification not implemented) | 2198 |
| Mupad [F(-1)] | 2198 |
| Reduce [B] (verification not implemented) | 2199 |

Optimal result

Integrand size = 28, antiderivative size = 266

$$\int (d + ex)^5 (a^2 + 2abx + b^2x^2)^{5/2} dx = \frac{(bd - ae)^5 (a + bx)^5 \sqrt{a^2 + 2abx + b^2x^2}}{6b^6} + \frac{5e(bd - ae)^4 (a + bx)^6 \sqrt{a^2 + 2abx + b^2x^2}}{7b^6} + \frac{5e^2(bd - ae)^3 (a + bx)^7 \sqrt{a^2 + 2abx + b^2x^2}}{4b^6} + \frac{10e^3(bd - ae)^2 (a + bx)^8 \sqrt{a^2 + 2abx + b^2x^2}}{9b^6} + \frac{e^4(bd - ae)(a + bx)^9 \sqrt{a^2 + 2abx + b^2x^2}}{2b^6} + \frac{e^5 (a + bx)^{10} \sqrt{a^2 + 2abx + b^2x^2}}{11b^6}$$

output

```
1/6*(-a*e+b*d)^5*(b*x+a)^5*((b*x+a)^2)^(1/2)/b^6+5/7*e*(-a*e+b*d)^4*(b*x+a)^6*((b*x+a)^2)^(1/2)/b^6+5/4*e^2*(-a*e+b*d)^3*(b*x+a)^7*((b*x+a)^2)^(1/2)/b^6+10/9*e^3*(-a*e+b*d)^2*(b*x+a)^8*((b*x+a)^2)^(1/2)/b^6+1/2*e^4*(-a*e+b*d)*(b*x+a)^9*((b*x+a)^2)^(1/2)/b^6+1/11*e^5*(b*x+a)^10*((b*x+a)^2)^(1/2)/b^6
```

Mathematica [A] (verified)

Time = 1.10 (sec) , antiderivative size = 385, normalized size of antiderivative = 1.45

$$\int (d + ex)^5 (a^2 + 2abx + b^2x^2)^{5/2} dx = \frac{x\sqrt{(a + bx)^2(462a^5(6d^5 + 15d^4ex + 20d^3e^2x^2 + 15d^2e^3x^3 + 6de^4x^4 + e^5x^5) + 330a^4bx(21d^5 + 70d^4ex + 105d^3e^2x^2 + 84d^2e^3x^3 + 35d^2e^4x^4 + 6e^5x^5) + 165a^3b^2x^2(56d^5 + 210d^4ex + 336d^3e^2x^2 + 280d^2e^3x^3 + 120d^2e^4x^4 + 21e^5x^5) + 55a^2b^3x^3(126d^5 + 504d^4ex + 840d^3e^2x^2 + 720d^2e^3x^3 + 315d^2e^4x^4 + 56e^5x^5) + 11ab^4x^4(252d^5 + 1050d^4ex + 1800d^3e^2x^2 + 1575d^2e^3x^3 + 700d^2e^4x^4 + 126e^5x^5) + b^5x^5(462d^5 + 1980d^4ex + 3465d^3e^2x^2 + 3080d^2e^3x^3 + 1386d^2e^4x^4 + 252e^5x^5))}{(2772(a + bx))}$$

input

```
Integrate[(d + e*x)^5*(a^2 + 2*a*b*x + b^2*x^2)^(5/2),x]
```

output

```
(x*sqrt[(a + b*x)^2]*(462*a^5*(6*d^5 + 15*d^4*e*x + 20*d^3*e^2*x^2 + 15*d^2*e^3*x^3 + 6*d*e^4*x^4 + e^5*x^5) + 330*a^4*b*x*(21*d^5 + 70*d^4*e*x + 105*d^3*e^2*x^2 + 84*d^2*e^3*x^3 + 35*d^2*e^4*x^4 + 6*e^5*x^5) + 165*a^3*b^2*x^2*(56*d^5 + 210*d^4*e*x + 336*d^3*e^2*x^2 + 280*d^2*e^3*x^3 + 120*d^2*e^4*x^4 + 21*e^5*x^5) + 55*a^2*b^3*x^3*(126*d^5 + 504*d^4*e*x + 840*d^3*e^2*x^2 + 720*d^2*e^3*x^3 + 315*d^2*e^4*x^4 + 56*e^5*x^5) + 11*a*b^4*x^4*(252*d^5 + 1050*d^4*e*x + 1800*d^3*e^2*x^2 + 1575*d^2*e^3*x^3 + 700*d^2*e^4*x^4 + 126*e^5*x^5) + b^5*x^5*(462*d^5 + 1980*d^4*e*x + 3465*d^3*e^2*x^2 + 3080*d^2*e^3*x^3 + 1386*d^2*e^4*x^4 + 252*e^5*x^5)))/(2772*(a + b*x))
```

Rubi [A] (verified)

Time = 0.79 (sec) , antiderivative size = 174, normalized size of antiderivative = 0.65, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {1102, 27, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a^2 + 2abx + b^2x^2)^{5/2} (d + ex)^5 dx$$

$$\downarrow 1102$$

$$\frac{\sqrt{a^2 + 2abx + b^2x^2} \int b^5 (a + bx)^5 (d + ex)^5 dx}{b^5 (a + bx)}$$

$$\downarrow 27$$

$$\frac{\sqrt{a^2 + 2abx + b^2x^2} \int (a + bx)^5 (d + ex)^5 dx}{a + bx}$$

↓ 49

$$\frac{\sqrt{a^2 + 2abx + b^2x^2} \int \left(\frac{e^5(a+bx)^{10}}{b^5} + \frac{5e^4(bd-ae)(a+bx)^9}{b^5} + \frac{10e^3(bd-ae)^2(a+bx)^8}{b^5} + \frac{10e^2(bd-ae)^3(a+bx)^7}{b^5} + \frac{5e(bd-ae)^4(a+bx)^6}{b^5} \right)}{a + bx}$$

↓ 2009

$$\frac{\sqrt{a^2 + 2abx + b^2x^2} \left(\frac{e^4(a+bx)^{10}(bd-ae)}{2b^6} + \frac{10e^3(a+bx)^9(bd-ae)^2}{9b^6} + \frac{5e^2(a+bx)^8(bd-ae)^3}{4b^6} + \frac{5e(a+bx)^7(bd-ae)^4}{7b^6} + \frac{(a+bx)^6(bd-ae)^5}{6b^6} \right)}{a + bx}$$

input `Int[(d + e*x)^5*(a^2 + 2*a*b*x + b^2*x^2)^(5/2),x]`

output `(Sqrt[a^2 + 2*a*b*x + b^2*x^2]*(((b*d - a*e)^5*(a + b*x)^6)/(6*b^6) + (5*e*(b*d - a*e)^4*(a + b*x)^7)/(7*b^6) + (5*e^2*(b*d - a*e)^3*(a + b*x)^8)/(4*b^6) + (10*e^3*(b*d - a*e)^2*(a + b*x)^9)/(9*b^6) + (e^4*(b*d - a*e)*(a + b*x)^10)/(2*b^6) + (e^5*(a + b*x)^11)/(11*b^6)))/(a + b*x)`

Defintions of rubi rules used

rule 27 `Int[(a_)*(F_x_), x_Symbol] := Simp[a Int[F_x, x], x] /; FreeQ[a, x] && !MatchQ[F_x, (b_)*(G_x_)] /; FreeQ[b, x]`

rule 49 `Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] := Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] && IGtQ[m, 0] && IGtQ[m + n + 2, 0]`

rule 1102 `Int[((d_.) + (e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] := Simp[(a + b*x + c*x^2)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x)^(2*FracPart[p])) Int[(d + e*x)^m*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && EqQ[b^2 - 4*a*c, 0]`

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 427 vs. $2(188) = 376$.

Time = 0.07 (sec) , antiderivative size = 427, normalized size of antiderivative = 1.61

$$\begin{aligned} \int (d+ex)^5 (a^2+2abx+b^2x^2)^{5/2} dx &= \frac{1}{11} b^5 e^5 x^{11} + a^5 d^5 x \\ &+ \frac{1}{2} (b^5 d e^4 + a b^4 e^5) x^{10} + \frac{5}{9} (2 b^5 d^2 e^3 + 5 a b^4 d e^4 + 2 a^2 b^3 e^5) x^9 \\ &+ \frac{5}{4} (b^5 d^3 e^2 + 5 a b^4 d^2 e^3 + 5 a^2 b^3 d e^4 + a^3 b^2 e^5) x^8 \\ &+ \frac{5}{7} (b^5 d^4 e + 10 a b^4 d^3 e^2 + 20 a^2 b^3 d^2 e^3 + 10 a^3 b^2 d e^4 + a^4 b e^5) x^7 \\ &+ \frac{1}{6} (b^5 d^5 + 25 a b^4 d^4 e + 100 a^2 b^3 d^3 e^2 + 100 a^3 b^2 d^2 e^3 + 25 a^4 b d e^4 + a^5 e^5) x^6 \\ &+ (a b^4 d^5 + 10 a^2 b^3 d^4 e + 20 a^3 b^2 d^3 e^2 + 10 a^4 b d^2 e^3 + a^5 d e^4) x^5 \\ &+ \frac{5}{2} (a^2 b^3 d^5 + 5 a^3 b^2 d^4 e + 5 a^4 b d^3 e^2 + a^5 d^2 e^3) x^4 \\ &+ \frac{5}{3} (2 a^3 b^2 d^5 + 5 a^4 b d^4 e + 2 a^5 d^3 e^2) x^3 + \frac{5}{2} (a^4 b d^5 + a^5 d^4 e) x^2 \end{aligned}$$

input `integrate((e*x+d)^5*(b^2*x^2+2*a*b*x+a^2)^(5/2),x, algorithm="fricas")`

output `1/11*b^5*e^5*x^11 + a^5*d^5*x + 1/2*(b^5*d*e^4 + a*b^4*e^5)*x^10 + 5/9*(2*b^5*d^2*e^3 + 5*a*b^4*d*e^4 + 2*a^2*b^3*e^5)*x^9 + 5/4*(b^5*d^3*e^2 + 5*a*b^4*d^2*e^3 + 5*a^2*b^3*d*e^4 + a^3*b^2*e^5)*x^8 + 5/7*(b^5*d^4*e + 10*a*b^4*d^3*e^2 + 20*a^2*b^3*d^2*e^3 + 10*a^3*b^2*d*e^4 + a^4*b*e^5)*x^7 + 1/6*(b^5*d^5 + 25*a*b^4*d^4*e + 100*a^2*b^3*d^3*e^2 + 100*a^3*b^2*d^2*e^3 + 25*a^4*b*d*e^4 + a^5*e^5)*x^6 + (a*b^4*d^5 + 10*a^2*b^3*d^4*e + 20*a^3*b^2*d^3*e^2 + 10*a^4*b*d^2*e^3 + a^5*d*e^4)*x^5 + 5/2*(a^2*b^3*d^5 + 5*a^3*b^2*d^4*e + 5*a^4*b*d^3*e^2 + a^5*d^2*e^3)*x^4 + 5/3*(2*a^3*b^2*d^5 + 5*a^4*b*d^4*e + 2*a^5*d^3*e^2)*x^3 + 5/2*(a^4*b*d^5 + a^5*d^4*e)*x^2`

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 28886 vs. $2(192) = 384$.

Time = 1.24 (sec) , antiderivative size = 28886, normalized size of antiderivative = 108.59

$$\int (d + ex)^5 (a^2 + 2abx + b^2x^2)^{5/2} dx = \text{Too large to display}$$

input `integrate((e*x+d)**5*(b**2*x**2+2*a*b*x+a**2)**(5/2),x)`

output

```
Piecewise((sqrt(a**2 + 2*a*b*x + b**2*x**2)*(b**4*e**5*x**10/11 + x**9*(45
*a*b**5*e**5/11 + 5*b**6*d*e**4)/(10*b**2) + x**8*(155*a**2*b**4*e**5/11 +
30*a*b**5*d*e**4 - 19*a*(45*a*b**5*e**5/11 + 5*b**6*d*e**4)/(10*b) + 10*b
**6*d**2*e**3)/(9*b**2) + x**7*(20*a**3*b**3*e**5 + 75*a**2*b**4*d*e**4 -
9*a**2*(45*a*b**5*e**5/11 + 5*b**6*d*e**4)/(10*b**2) + 60*a*b**5*d**2*e**3
- 17*a*(155*a**2*b**4*e**5/11 + 30*a*b**5*d*e**4 - 19*a*(45*a*b**5*e**5/1
1 + 5*b**6*d*e**4)/(10*b) + 10*b**6*d**2*e**3)/(9*b) + 10*b**6*d**3*e**2)/
(8*b**2) + x**6*(15*a**4*b**2*e**5 + 100*a**3*b**3*d*e**4 + 150*a**2*b**4*
d**2*e**3 - 8*a**2*(155*a**2*b**4*e**5/11 + 30*a*b**5*d*e**4 - 19*a*(45*a*
b**5*e**5/11 + 5*b**6*d*e**4)/(10*b) + 10*b**6*d**2*e**3)/(9*b**2) + 60*a*
b**5*d**3*e**2 - 15*a*(20*a**3*b**3*e**5 + 75*a**2*b**4*d*e**4 - 9*a**2*(4
5*a*b**5*e**5/11 + 5*b**6*d*e**4)/(10*b**2) + 60*a*b**5*d**2*e**3 - 17*a*(
155*a**2*b**4*e**5/11 + 30*a*b**5*d*e**4 - 19*a*(45*a*b**5*e**5/11 + 5*b**
6*d*e**4)/(10*b) + 10*b**6*d**2*e**3)/(9*b) + 10*b**6*d**3*e**2)/(8*b) + 5
*b**6*d**4*e)/(7*b**2) + x**5*(6*a**5*b*e**5 + 75*a**4*b**2*d*e**4 + 200*a
**3*b**3*d**2*e**3 + 150*a**2*b**4*d**3*e**2 - 7*a**2*(20*a**3*b**3*e**5 +
75*a**2*b**4*d*e**4 - 9*a**2*(45*a*b**5*e**5/11 + 5*b**6*d*e**4)/(10*b**2
) + 60*a*b**5*d**2*e**3 - 17*a*(155*a**2*b**4*e**5/11 + 30*a*b**5*d*e**4 -
19*a*(45*a*b**5*e**5/11 + 5*b**6*d*e**4)/(10*b) + 10*b**6*d**2*e**3)/(9*b
) + 10*b**6*d**3*e**2)/(8*b**2) + 30*a*b**5*d**4*e - 13*a*(15*a**4*b**2...
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 815 vs. $2(188) = 376$.

Time = 0.04 (sec) , antiderivative size = 815, normalized size of antiderivative = 3.06

$$\int (d + ex)^5 (a^2 + 2abx + b^2x^2)^{5/2} dx = \text{Too large to display}$$

input `integrate((e*x+d)^5*(b^2*x^2+2*a*b*x+a^2)^(5/2),x, algorithm="maxima")`

output

$$\begin{aligned} & 1/11*(b^2*x^2 + 2*a*b*x + a^2)^(7/2)*e^5*x^4/b^2 + 1/2*(b^2*x^2 + 2*a*b*x \\ & + a^2)^(7/2)*d*e^4*x^3/b^2 - 3/22*(b^2*x^2 + 2*a*b*x + a^2)^(7/2)*a*e^5*x^3/b^3 \\ & + 1/6*(b^2*x^2 + 2*a*b*x + a^2)^(5/2)*d^5*x - 5/6*(b^2*x^2 + 2*a*b*x \\ & + a^2)^(5/2)*a*d^4*e*x/b + 5/3*(b^2*x^2 + 2*a*b*x + a^2)^(5/2)*a^2*d^3*e^2*x/b^2 \\ & - 5/3*(b^2*x^2 + 2*a*b*x + a^2)^(5/2)*a^3*d^2*e^3*x/b^3 + 5/6*(b^2*x^2 + 2*a*b*x \\ & + a^2)^(5/2)*a^4*d*e^4*x/b^4 - 1/6*(b^2*x^2 + 2*a*b*x + a^2)^(5/2)*a^5*e^5*x/b^5 \\ & + 10/9*(b^2*x^2 + 2*a*b*x + a^2)^(7/2)*d^2*e^3*x^2/b^2 - 13/18*(b^2*x^2 + 2*a*b*x \\ & + a^2)^(7/2)*a*d*e^4*x^2/b^3 + 31/198*(b^2*x^2 + 2*a*b*x + a^2)^(7/2)*a^2*e^5*x^2/b^4 \\ & + 1/6*(b^2*x^2 + 2*a*b*x + a^2)^(5/2)*a*d^5/b - 5/6*(b^2*x^2 + 2*a*b*x + a^2)^(5/2)*a^2*d^4*e/b^2 \\ & + 5/3*(b^2*x^2 + 2*a*b*x + a^2)^(5/2)*a^3*d^3*e^2/b^3 - 5/3*(b^2*x^2 + 2*a*b*x + a^2)^(5/2)*a^4*d^2*e^3/b^4 \\ & + 5/6*(b^2*x^2 + 2*a*b*x + a^2)^(5/2)*a^5*d*e^4/b^5 - 1/6*(b^2*x^2 + 2*a*b*x + a^2)^(5/2)*a^6*e^5/b^6 \\ & + 5/4*(b^2*x^2 + 2*a*b*x + a^2)^(7/2)*d^3*e^2*x/b^2 - 55/36*(b^2*x^2 + 2*a*b*x + a^2)^(7/2)*a*d^2*e^3*x/b^3 \\ & + 29/36*(b^2*x^2 + 2*a*b*x + a^2)^(7/2)*a^2*d*e^4*x/b^4 - 65/396*(b^2*x^2 + 2*a*b*x + a^2)^(7/2)*a^3*e^5*x/b^5 \\ & + 5/7*(b^2*x^2 + 2*a*b*x + a^2)^(7/2)*d^4*e/b^2 - 45/28*(b^2*x^2 + 2*a*b*x + a^2)^(7/2)*a*d^3*e^2/b^3 \\ & + 415/252*(b^2*x^2 + 2*a*b*x + a^2)^(7/2)*a^2*d^2*e^3/b^4 - 209/252*(b^2*x^2 + 2*a*b*x + a^2)^(7/2)*a^3*d*e^4/b^5 \\ & + 461/2772*(b^2*x^2 + 2*a*b*x + a^2)^(7/2)*a^4*e^5/b^6 \end{aligned}$$

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 785 vs. $2(188) = 376$.

Time = 1.74 (sec) , antiderivative size = 785, normalized size of antiderivative = 2.95

$$\int (d + ex)^5 (a^2 + 2abx + b^2x^2)^{5/2} dx = \text{Too large to display}$$

input `integrate((e*x+d)^5*(b^2*x^2+2*a*b*x+a^2)^(5/2),x, algorithm="giac")`

output

```
1/11*b^5*e^5*x^11*sgn(b*x + a) + 1/2*b^5*d*e^4*x^10*sgn(b*x + a) + 1/2*a*b^4*e^5*x^10*sgn(b*x + a) + 10/9*b^5*d^2*e^3*x^9*sgn(b*x + a) + 25/9*a*b^4*d*e^4*x^9*sgn(b*x + a) + 10/9*a^2*b^3*e^5*x^9*sgn(b*x + a) + 5/4*b^5*d^3*e^2*x^8*sgn(b*x + a) + 25/4*a*b^4*d^2*e^3*x^8*sgn(b*x + a) + 25/4*a^2*b^3*d*e^4*x^8*sgn(b*x + a) + 5/4*a^3*b^2*e^5*x^8*sgn(b*x + a) + 5/7*b^5*d^4*e*x^7*sgn(b*x + a) + 50/7*a*b^4*d^3*e^2*x^7*sgn(b*x + a) + 100/7*a^2*b^3*d^2*e^3*x^7*sgn(b*x + a) + 50/7*a^3*b^2*d*e^4*x^7*sgn(b*x + a) + 5/7*a^4*b*e^5*x^7*sgn(b*x + a) + 1/6*b^5*d^5*x^6*sgn(b*x + a) + 25/6*a*b^4*d^4*e*x^6*sgn(b*x + a) + 50/3*a^2*b^3*d^3*e^2*x^6*sgn(b*x + a) + 50/3*a^3*b^2*d^2*e^3*x^6*sgn(b*x + a) + 25/6*a^4*b*d*e^4*x^6*sgn(b*x + a) + 1/6*a^5*e^5*x^6*sgn(b*x + a) + a*b^4*d^5*x^5*sgn(b*x + a) + 10*a^2*b^3*d^4*e*x^5*sgn(b*x + a) + 20*a^3*b^2*d^3*e^2*x^5*sgn(b*x + a) + 10*a^4*b*d^2*e^3*x^5*sgn(b*x + a) + a^5*d*e^4*x^5*sgn(b*x + a) + 5/2*a^2*b^3*d^5*x^4*sgn(b*x + a) + 25/2*a^3*b^2*d^4*e*x^4*sgn(b*x + a) + 25/2*a^4*b*d^3*e^2*x^4*sgn(b*x + a) + 5/2*a^5*d^2*e^3*x^4*sgn(b*x + a) + 10/3*a^3*b^2*d^5*x^3*sgn(b*x + a) + 25/3*a^4*b*d^4*e*x^3*sgn(b*x + a) + 10/3*a^5*d^3*e^2*x^3*sgn(b*x + a) + 5/2*a^4*b*d^5*x^2*sgn(b*x + a) + 5/2*a^5*d^4*e*x^2*sgn(b*x + a) + a^5*d^5*x*sgn(b*x + a) + 1/2772*(462*a^6*b^5*d^5 - 330*a^7*b^4*d^4*e + 165*a^8*b^3*d^3*e^2 - 55*a^9*b^2*d^2*e^3 + 11*a^10*b*d*e^4 - a^11*e^5)*sgn(b*x + a)/b^6
```

Mupad [F(-1)]

Timed out.

$$\int (d + ex)^5 (a^2 + 2abx + b^2x^2)^{5/2} dx = \int (d + ex)^5 (a^2 + 2abx + b^2x^2)^{5/2} dx$$

input `int((d + e*x)^5*(a^2 + b^2*x^2 + 2*a*b*x)^(5/2),x)`

output `int((d + e*x)^5*(a^2 + b^2*x^2 + 2*a*b*x)^(5/2), x)`

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 489, normalized size of antiderivative = 1.84

$$\int (d + ex)^5 (a^2 + 2abx + b^2x^2)^{5/2} dx = \frac{x(252b^5e^5x^{10} + 1386ab^4e^5x^9 + 1386b^5de^4x^9 + 3080a^2b^3e^5x^8 + 7700ab^4de^4x^8 + 3080b^5d^2e^4x^7 + 2772a^2b^3e^5x^6 + 2772ab^4de^4x^6 + 462a^5e^5x^5 + 6930a^4bde^4x^5 + 11550a^4b^2d^2e^3x^5 + 1980a^4b^3d^3e^2x^5 + 9240a^4b^4d^4e^1x^5 + 6930a^4b^5d^5e^0x^5 + 23100a^4b^2d^3e^4x^4 + 34650a^4b^3d^4e^3x^4 + 27720a^4b^4d^5e^2x^4 + 11550a^4b^5d^6e^1x^4 + 1980a^4b^6d^7e^0x^4 + 9240a^4b^3d^2e^5x^3 + 34650a^4b^4d^3e^4x^3 + 55440a^4b^5d^4e^3x^3 + 46200a^4b^6d^5e^2x^3 + 19800a^4b^7d^6e^1x^3 + 27720a^4b^8d^7e^0x^3 + 46200a^4b^3d^2e^5x^2 + 46200a^4b^4d^3e^4x^2 + 19800a^4b^5d^4e^3x^2 + 9240a^4b^6d^5e^2x^2 + 6930a^4b^7d^6e^1x^2 + 27720a^4b^8d^7e^0x^2 + 46200a^4b^3d^2e^5x + 46200a^4b^4d^3e^4x + 19800a^4b^5d^4e^3x + 9240a^4b^6d^5e^2x + 6930a^4b^7d^6e^1x + 27720a^4b^8d^7e^0x + 462b^5d^5e^5x^5 + 1980b^5d^4e^4x^6 + 3465b^5d^3e^3x^7 + 3080b^5d^2e^2x^8 + 2772b^5d^1e^1x^9 + 252b^5d^0e^0x^{10})/2772$$

input `int((e*x+d)^5*(b^2*x^2+2*a*b*x+a^2)^(5/2), x)`

output `(x*(2772*a**5*d**5 + 6930*a**5*d**4*e*x + 9240*a**5*d**3*e**2*x**2 + 6930*a**5*d**2*e**3*x**3 + 2772*a**5*d*e**4*x**4 + 462*a**5*e**5*x**5 + 6930*a**4*b*d**5*x + 23100*a**4*b*d**4*e*x**2 + 34650*a**4*b*d**3*e**2*x**3 + 27720*a**4*b*d**2*e**3*x**4 + 11550*a**4*b*d*e**4*x**5 + 1980*a**4*b*e**5*x**6 + 9240*a**3*b**2*d**5*x**2 + 34650*a**3*b**2*d**4*e*x**3 + 55440*a**3*b**2*d**3*e**2*x**4 + 46200*a**3*b**2*d**2*e**3*x**5 + 19800*a**3*b**2*d*e**4*x**6 + 3465*a**3*b**2*e**5*x**7 + 6930*a**2*b**3*d**5*x**3 + 27720*a**2*b**3*d**4*e*x**4 + 46200*a**2*b**3*d**3*e**2*x**5 + 39600*a**2*b**3*d**2*e**3*x**6 + 17325*a**2*b**3*d*e**4*x**7 + 3080*a**2*b**3*e**5*x**8 + 2772*a**2*b**4*d**5*x**4 + 11550*a*b**4*d**4*e*x**5 + 19800*a*b**4*d**3*e**2*x**6 + 17325*a*b**4*d**2*e**3*x**7 + 7700*a*b**4*d*e**4*x**8 + 1386*a*b**4*e**5*x**9 + 462*b**5*d**5*x**5 + 1980*b**5*d**4*e*x**6 + 3465*b**5*d**3*e**2*x**7 + 3080*b**5*d**2*e**3*x**8 + 1386*b**5*d*e**4*x**9 + 252*b**5*e**5*x**10))/2772`

3.292 $\int (d + ex)^4 (a^2 + 2abx + b^2x^2)^{5/2} dx$

| | |
|---|------|
| Optimal result | 2200 |
| Mathematica [A] (verified) | 2201 |
| Rubi [A] (verified) | 2201 |
| Maple [B] (verified) | 2203 |
| Fricas [B] (verification not implemented) | 2204 |
| Sympy [B] (verification not implemented) | 2205 |
| Maxima [B] (verification not implemented) | 2206 |
| Giac [B] (verification not implemented) | 2207 |
| Mupad [F(-1)] | 2208 |
| Reduce [B] (verification not implemented) | 2208 |

Optimal result

Integrand size = 28, antiderivative size = 219

$$\int (d + ex)^4 (a^2 + 2abx + b^2x^2)^{5/2} dx = \frac{(bd - ae)^4 (a + bx)^5 \sqrt{a^2 + 2abx + b^2x^2}}{6b^5} + \frac{4e(bd - ae)^3 (a + bx)^6 \sqrt{a^2 + 2abx + b^2x^2}}{7b^5} + \frac{3e^2(bd - ae)^2 (a + bx)^7 \sqrt{a^2 + 2abx + b^2x^2}}{4b^5} + \frac{4e^3(bd - ae) (a + bx)^8 \sqrt{a^2 + 2abx + b^2x^2}}{9b^5} + \frac{e^4 (a + bx)^9 \sqrt{a^2 + 2abx + b^2x^2}}{10b^5}$$

output

```
1/6*(-a*e+b*d)^4*(b*x+a)^5*((b*x+a)^2)^(1/2)/b^5+4/7*e*(-a*e+b*d)^3*(b*x+a)^6*((b*x+a)^2)^(1/2)/b^5+3/4*e^2*(-a*e+b*d)^2*(b*x+a)^7*((b*x+a)^2)^(1/2)/b^5+4/9*e^3*(-a*e+b*d)*(b*x+a)^8*((b*x+a)^2)^(1/2)/b^5+1/10*e^4*(b*x+a)^9*((b*x+a)^2)^(1/2)/b^5
```

Mathematica [A] (verified)

Time = 1.09 (sec) , antiderivative size = 319, normalized size of antiderivative = 1.46

$$\int (d + ex)^4 (a^2 + 2abx + b^2x^2)^{5/2} dx = \frac{x\sqrt{(a + bx)^2}(252a^5(5d^4 + 10d^3ex + 10d^2e^2x^2 + 5de^3x^3 + e^4x^4) + 210a^4bx(15d^4 + 40d^3ex + 45d^2e^2x^2 + 24de^3x^3 + 5e^4x^4) + 120a^3b^2x^2(35d^4 + 105d^3ex + 126d^2e^2x^2 + 70de^3x^3 + 15e^4x^4) + 45a^2b^3x^3(70d^4 + 224d^3ex + 280d^2e^2x^2 + 160de^3x^3 + 35e^4x^4) + 10ab^4x^4(126d^4 + 420d^3ex + 540d^2e^2x^2 + 315de^3x^3 + 70e^4x^4) + b^5x^5(210d^4 + 720d^3ex + 945d^2e^2x^2 + 560de^3x^3 + 126e^4x^4))}{1260(a + bx)}$$

input

```
Integrate[(d + e*x)^4*(a^2 + 2*a*b*x + b^2*x^2)^(5/2),x]
```

output

```
(x*sqrt[(a + b*x)^2]*(252*a^5*(5*d^4 + 10*d^3*e*x + 10*d^2*e^2*x^2 + 5*d*e^3*x^3 + e^4*x^4) + 210*a^4*b*x*(15*d^4 + 40*d^3*e*x + 45*d^2*e^2*x^2 + 24*d*e^3*x^3 + 5*e^4*x^4) + 120*a^3*b^2*x^2*(35*d^4 + 105*d^3*e*x + 126*d^2*e^2*x^2 + 70*d*e^3*x^3 + 15*e^4*x^4) + 45*a^2*b^3*x^3*(70*d^4 + 224*d^3*e*x + 280*d^2*e^2*x^2 + 160*d*e^3*x^3 + 35*e^4*x^4) + 10*a*b^4*x^4*(126*d^4 + 420*d^3*e*x + 540*d^2*e^2*x^2 + 315*d*e^3*x^3 + 70*e^4*x^4) + b^5*x^5*(210*d^4 + 720*d^3*e*x + 945*d^2*e^2*x^2 + 560*d*e^3*x^3 + 126*e^4*x^4)))/(1260*(a + b*x))
```

Rubi [A] (verified)

Time = 0.73 (sec) , antiderivative size = 147, normalized size of antiderivative = 0.67, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {1102, 27, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a^2 + 2abx + b^2x^2)^{5/2} (d + ex)^4 dx$$

$$\downarrow 1102$$

$$\frac{\sqrt{a^2 + 2abx + b^2x^2} \int b^5(a + bx)^5(d + ex)^4 dx}{b^5(a + bx)}$$

$$\downarrow 27$$

$$\frac{\sqrt{a^2 + 2abx + b^2x^2} \int (a + bx)^5 (d + ex)^4 dx}{a + bx}$$

↓ 49

$$\frac{\sqrt{a^2 + 2abx + b^2x^2} \int \left(\frac{e^4(a+bx)^9}{b^4} + \frac{4e^3(bd-ae)(a+bx)^8}{b^4} + \frac{6e^2(bd-ae)^2(a+bx)^7}{b^4} + \frac{4e(bd-ae)^3(a+bx)^6}{b^4} + \frac{(bd-ae)^4(a+bx)^5}{b^4} \right) dx}{a + bx}$$

↓ 2009

$$\frac{\sqrt{a^2 + 2abx + b^2x^2} \left(\frac{4e^3(a+bx)^9(bd-ae)}{9b^5} + \frac{3e^2(a+bx)^8(bd-ae)^2}{4b^5} + \frac{4e(a+bx)^7(bd-ae)^3}{7b^5} + \frac{(a+bx)^6(bd-ae)^4}{6b^5} + \frac{e^4(a+bx)^{10}}{10b^5} \right)}{a + bx}$$

input `Int[(d + e*x)^4*(a^2 + 2*a*b*x + b^2*x^2)^(5/2),x]`

output `(Sqrt[a^2 + 2*a*b*x + b^2*x^2]*(((b*d - a*e)^4*(a + b*x)^6)/(6*b^5) + (4*e*(b*d - a*e)^3*(a + b*x)^7)/(7*b^5) + (3*e^2*(b*d - a*e)^2*(a + b*x)^8)/(4*b^5) + (4*e^3*(b*d - a*e)*(a + b*x)^9)/(9*b^5) + (e^4*(a + b*x)^10)/(10*b^5)))/(a + b*x)`

Defintions of rubi rules used

rule 27 `Int[(a_)*(F_x_), x_Symbol] := Simp[a Int[F_x, x], x] /; FreeQ[a, x] && !MatchQ[F_x, (b_)*(G_x_)] /; FreeQ[b, x]`

rule 49 `Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] := Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] && IGtQ[m, 0] && IGtQ[m + n + 2, 0]`

rule 1102 `Int[((d_.) + (e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] := Simp[(a + b*x + c*x^2)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x)^(2*FracPart[p])) Int[(d + e*x)^m*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && EqQ[b^2 - 4*a*c, 0]`

rule 2009 `Int[u_, x_Symbol] := Simp[IntSum[u, x], x] /; SumQ[u]`

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 413 vs. $2(154) = 308$.

Time = 1.34 (sec) , antiderivative size = 414, normalized size of antiderivative = 1.89

| method | result |
|---------|--|
| gospers | $\frac{x(126b^5e^4x^9+700x^8ab^4e^4+560x^8b^5de^3+1575x^7a^2b^3e^4+3150x^7ab^4de^3+945x^7b^5d^2e^2+1800x^6a^3b^2e^4+7200x^6a^2b^3de^3+5400x^6a^4b^2d^2e^2+1050x^6a^4b^2de^3+8400x^5a^3b^2de^3+12600x^5a^2b^3d^2e^2+4200x^5ab^4d^3e^2+210b^5d^4x^5+252a^5e^4x^4+5040a^4b^2de^3x^4+15120a^3b^2d^2e^2x^4+10080a^2b^3d^3e^2x^4+1260ab^4d^4x^4+1260a^5d^3e^3x^3+9450a^4b^2d^2e^2x^3+12600a^3b^2d^3e^2x^3+3150a^2b^3d^4x^3+2520a^5d^2e^2x^2+8400a^4b^2d^3e^2x^2+4200a^3b^2d^4x^2+2520a^5d^3e^2x+3150a^4b^2d^4x+1260a^5d^4)}{10bx+10a}$ |
| default | $\frac{x(126b^5e^4x^9+700x^8ab^4e^4+560x^8b^5de^3+1575x^7a^2b^3e^4+3150x^7ab^4de^3+945x^7b^5d^2e^2+1800x^6a^3b^2e^4+7200x^6a^2b^3de^3+5400x^6a^4b^2d^2e^2+1050x^6a^4b^2de^3+8400x^5a^3b^2de^3+12600x^5a^2b^3d^2e^2+4200x^5ab^4d^3e^2+210b^5d^4x^5+252a^5e^4x^4+5040a^4b^2de^3x^4+15120a^3b^2d^2e^2x^4+10080a^2b^3d^3e^2x^4+1260ab^4d^4x^4+1260a^5d^3e^3x^3+9450a^4b^2d^2e^2x^3+12600a^3b^2d^3e^2x^3+3150a^2b^3d^4x^3+2520a^5d^2e^2x^2+8400a^4b^2d^3e^2x^2+4200a^3b^2d^4x^2+2520a^5d^3e^2x+3150a^4b^2d^4x+1260a^5d^4)}{9bx+9a}$ |
| orering | $\frac{x(126b^5e^4x^9+700x^8ab^4e^4+560x^8b^5de^3+1575x^7a^2b^3e^4+3150x^7ab^4de^3+945x^7b^5d^2e^2+1800x^6a^3b^2e^4+7200x^6a^2b^3de^3+5400x^6a^4b^2d^2e^2+1050x^6a^4b^2de^3+8400x^5a^3b^2de^3+12600x^5a^2b^3d^2e^2+4200x^5ab^4d^3e^2+210b^5d^4x^5+252a^5e^4x^4+5040a^4b^2de^3x^4+15120a^3b^2d^2e^2x^4+10080a^2b^3d^3e^2x^4+1260ab^4d^4x^4+1260a^5d^3e^3x^3+9450a^4b^2d^2e^2x^3+12600a^3b^2d^3e^2x^3+3150a^2b^3d^4x^3+2520a^5d^2e^2x^2+8400a^4b^2d^3e^2x^2+4200a^3b^2d^4x^2+2520a^5d^3e^2x+3150a^4b^2d^4x+1260a^5d^4)}{8bx+8a}$ |
| risch | $\frac{\sqrt{(bx+a)^2} b^5 e^4 x^{10}}{10bx+10a} + \frac{\sqrt{(bx+a)^2} (5a b^4 e^4 + 4b^5 d e^3) x^9}{9bx+9a} + \frac{\sqrt{(bx+a)^2} (10a^2 b^3 e^4 + 20a b^4 d e^3 + 6b^5 d^2 e^2) x^8}{8bx+8a} + \frac{\sqrt{(bx+a)^2} (10a^3 b^2 d^2 e^2 + 10a^4 b^2 d e^3 + 8a^5 d^3 e^2) x^7}{7bx+7a}$ |

input `int((e*x+d)^4*(b^2*x^2+2*a*b*x+a^2)^(5/2),x,method=_RETURNVERBOSE)`

output
$$\frac{1}{1260}x*(126*b^5*e^4*x^9+700*a*b^4*e^4*x^8+560*b^5*d*e^3*x^8+1575*a^2*b^3*e^4*x^7+3150*a*b^4*d*e^3*x^7+945*b^5*d^2*e^2*x^7+1800*a^3*b^2*e^4*x^6+7200*a^2*b^3*d*e^3*x^6+5400*a*b^4*d^2*e^2*x^6+720*b^5*d^3*e*x^6+1050*a^4*b^2*e^4*x^5+8400*a^3*b^2*d*e^3*x^5+12600*a^2*b^3*d^2*e^2*x^5+4200*a*b^4*d^3*e*x^5+210*b^5*d^4*x^5+252*a^5*e^4*x^4+5040*a^4*b^2*d*e^3*x^4+15120*a^3*b^2*d^2*e^2*x^4+10080*a^2*b^3*d^3*e*x^4+1260*a*b^4*d^4*x^4+1260*a^5*d^3*e^3*x^3+9450*a^4*b^2*d^2*e^2*x^3+12600*a^3*b^2*d^3*e*x^3+3150*a^2*b^3*d^4*x^3+2520*a^5*d^2*e^2*x^2+8400*a^4*b^2*d^3*e*x^2+4200*a^3*b^2*d^4*x^2+2520*a^5*d^3*e*x+3150*a^4*b^2*d^4*x+1260*a^5*d^4)*((b*x+a)^2)^(5/2)/(b*x+a)^5$$

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 360 vs. $2(154) = 308$.

Time = 0.08 (sec) , antiderivative size = 360, normalized size of antiderivative = 1.64

$$\begin{aligned} \int (d+ex)^4 (a^2+2abx+b^2x^2)^{5/2} dx &= \frac{1}{10} b^5 e^4 x^{10} + a^5 d^4 x \\ &+ \frac{1}{9} (4b^5 d e^3 + 5ab^4 e^4) x^9 + \frac{1}{4} (3b^5 d^2 e^2 + 10ab^4 d e^3 + 5a^2 b^3 e^4) x^8 \\ &+ \frac{2}{7} (2b^5 d^3 e + 15ab^4 d^2 e^2 + 20a^2 b^3 d e^3 + 5a^3 b^2 e^4) x^7 \\ &+ \frac{1}{6} (b^5 d^4 + 20ab^4 d^3 e + 60a^2 b^3 d^2 e^2 + 40a^3 b^2 d e^3 + 5a^4 b e^4) x^6 \\ &+ \frac{1}{5} (5ab^4 d^4 + 40a^2 b^3 d^3 e + 60a^3 b^2 d^2 e^2 + 20a^4 b d e^3 + a^5 e^4) x^5 \\ &+ \frac{1}{2} (5a^2 b^3 d^4 + 20a^3 b^2 d^3 e + 15a^4 b d^2 e^2 + 2a^5 d e^3) x^4 \\ &+ \frac{2}{3} (5a^3 b^2 d^4 + 10a^4 b d^3 e + 3a^5 d^2 e^2) x^3 + \frac{1}{2} (5a^4 b d^4 + 4a^5 d^3 e) x^2 \end{aligned}$$

input `integrate((e*x+d)^4*(b^2*x^2+2*a*b*x+a^2)^(5/2),x, algorithm="fricas")`

output `1/10*b^5*e^4*x^10 + a^5*d^4*x + 1/9*(4*b^5*d*e^3 + 5*a*b^4*e^4)*x^9 + 1/4*(3*b^5*d^2*e^2 + 10*a*b^4*d*e^3 + 5*a^2*b^3*e^4)*x^8 + 2/7*(2*b^5*d^3*e + 15*a*b^4*d^2*e^2 + 20*a^2*b^3*d*e^3 + 5*a^3*b^2*e^4)*x^7 + 1/6*(b^5*d^4 + 20*a*b^4*d^3*e + 60*a^2*b^3*d^2*e^2 + 40*a^3*b^2*d*e^3 + 5*a^4*b*e^4)*x^6 + 1/5*(5*a*b^4*d^4 + 40*a^2*b^3*d^3*e + 60*a^3*b^2*d^2*e^2 + 20*a^4*b*d*e^3 + a^5*e^4)*x^5 + 1/2*(5*a^2*b^3*d^4 + 20*a^3*b^2*d^3*e + 15*a^4*b*d^2*e^2 + 2*a^5*d*e^3)*x^4 + 2/3*(5*a^3*b^2*d^4 + 10*a^4*b*d^3*e + 3*a^5*d^2*e^2)*x^3 + 1/2*(5*a^4*b*d^4 + 4*a^5*d^3*e)*x^2`

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 16842 vs. $2(158) = 316$.

Time = 1.22 (sec) , antiderivative size = 16842, normalized size of antiderivative = 76.90

$$\int (d + ex)^4 (a^2 + 2abx + b^2x^2)^{5/2} dx = \text{Too large to display}$$

input `integrate((e*x+d)**4*(b**2*x**2+2*a*b*x+a**2)**(5/2),x)`

output

```
Piecewise((sqrt(a**2 + 2*a*b*x + b**2*x**2)*(b**4*e**4*x**9/10 + x**8*(41*
a*b**5*e**4/10 + 4*b**6*d*e**3)/(9*b**2) + x**7*(141*a**2*b**4*e**4/10 + 2
4*a*b**5*d*e**3 - 17*a*(41*a*b**5*e**4/10 + 4*b**6*d*e**3)/(9*b) + 6*b**6*
d**2*e**2)/(8*b**2) + x**6*(20*a**3*b**3*e**4 + 60*a**2*b**4*d*e**3 - 8*a*
**2*(41*a*b**5*e**4/10 + 4*b**6*d*e**3)/(9*b**2) + 36*a*b**5*d**2*e**2 - 15
*a*(141*a**2*b**4*e**4/10 + 24*a*b**5*d*e**3 - 17*a*(41*a*b**5*e**4/10 + 4
*b**6*d*e**3)/(9*b) + 6*b**6*d**2*e**2)/(8*b) + 4*b**6*d**3*e)/(7*b**2) +
x**5*(15*a**4*b**2*e**4 + 80*a**3*b**3*d*e**3 + 90*a**2*b**4*d**2*e**2 - 7
*a**2*(141*a**2*b**4*e**4/10 + 24*a*b**5*d*e**3 - 17*a*(41*a*b**5*e**4/10
+ 4*b**6*d*e**3)/(9*b) + 6*b**6*d**2*e**2)/(8*b**2) + 24*a*b**5*d**3*e - 1
3*a*(20*a**3*b**3*e**4 + 60*a**2*b**4*d*e**3 - 8*a**2*(41*a*b**5*e**4/10
+ 4*b**6*d*e**3)/(9*b**2) + 36*a*b**5*d**2*e**2 - 15*a*(141*a**2*b**4*e**4/
10 + 24*a*b**5*d*e**3 - 17*a*(41*a*b**5*e**4/10 + 4*b**6*d*e**3)/(9*b) + 6
*b**6*d**2*e**2)/(8*b) + 4*b**6*d**3*e)/(7*b) + b**6*d**4)/(6*b**2) + x**4
*(6*a**5*b*e**4 + 60*a**4*b**2*d*e**3 + 120*a**3*b**3*d**2*e**2 + 60*a**2*
b**4*d**3*e - 6*a**2*(20*a**3*b**3*e**4 + 60*a**2*b**4*d*e**3 - 8*a**2*(41
*a*b**5*e**4/10 + 4*b**6*d*e**3)/(9*b**2) + 36*a*b**5*d**2*e**2 - 15*a*(14
1*a**2*b**4*e**4/10 + 24*a*b**5*d*e**3 - 17*a*(41*a*b**5*e**4/10 + 4*b**6*
d*e**3)/(9*b) + 6*b**6*d**2*e**2)/(8*b) + 4*b**6*d**3*e)/(7*b**2) + 6*a*b*
**5*d**4 - 11*a*(15*a**4*b**2*e**4 + 80*a**3*b**3*d*e**3 + 90*a**2*b**4*...
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 588 vs. $2(154) = 308$.

Time = 0.04 (sec) , antiderivative size = 588, normalized size of antiderivative = 2.68

$$\int (d+ex)^4 (a^2+2abx+b^2x^2)^{5/2} dx = \frac{(b^2x^2+2abx+a^2)^{7/2}e^4x^3}{10b^2} + \frac{1}{6}(b^2x^2+2abx+a^2)^{5/2}d^4x - \frac{2(b^2x^2+2abx+a^2)^{5/2}ad^3ex}{3b} + \frac{(b^2x^2+2abx+a^2)^{5/2}a^2d^2e^2x}{b^2} - \frac{2(b^2x^2+2abx+a^2)^{5/2}a^3de^3x}{3b^3} + \frac{(b^2x^2+2abx+a^2)^{5/2}a^4e^4x}{6b^4} + \frac{4(b^2x^2+2abx+a^2)^{7/2}de^3x^2}{9b^2} - \frac{13(b^2x^2+2abx+a^2)^{7/2}ae^4x^2}{90b^3} + \frac{(b^2x^2+2abx+a^2)^{5/2}ad^4}{6b} - \frac{2(b^2x^2+2abx+a^2)^{5/2}a^2d^3e}{3b^2} + \frac{(b^2x^2+2abx+a^2)^{5/2}a^3d^2e^2}{b^3} - \frac{2(b^2x^2+2abx+a^2)^{5/2}a^4de^3}{3b^4} + \frac{(b^2x^2+2abx+a^2)^{5/2}a^5e^4}{6b^5} + \frac{3(b^2x^2+2abx+a^2)^{7/2}d^2e^2x}{4b^2} - \frac{11(b^2x^2+2abx+a^2)^{7/2}ade^3x}{18b^3} + \frac{29(b^2x^2+2abx+a^2)^{7/2}a^2e^4x}{180b^4} + \frac{4(b^2x^2+2abx+a^2)^{7/2}d^3e}{7b^2} - \frac{27(b^2x^2+2abx+a^2)^{7/2}ad^2e^2}{28b^3} + \frac{83(b^2x^2+2abx+a^2)^{7/2}a^2de^3}{126b^4} - \frac{209(b^2x^2+2abx+a^2)^{7/2}a^3e^4}{1260b^5}$$

input

```
integrate((e*x+d)^4*(b^2*x^2+2*a*b*x+a^2)^(5/2),x, algorithm="maxima")
```

output

```

1/10*(b^2*x^2 + 2*a*b*x + a^2)^(7/2)*e^4*x^3/b^2 + 1/6*(b^2*x^2 + 2*a*b*x
+ a^2)^(5/2)*d^4*x - 2/3*(b^2*x^2 + 2*a*b*x + a^2)^(5/2)*a*d^3*e*x/b + (b^
2*x^2 + 2*a*b*x + a^2)^(5/2)*a^2*d^2*e^2*x/b^2 - 2/3*(b^2*x^2 + 2*a*b*x +
a^2)^(5/2)*a^3*d*e^3*x/b^3 + 1/6*(b^2*x^2 + 2*a*b*x + a^2)^(5/2)*a^4*e^4*x
/b^4 + 4/9*(b^2*x^2 + 2*a*b*x + a^2)^(7/2)*d*e^3*x^2/b^2 - 13/90*(b^2*x^2
+ 2*a*b*x + a^2)^(7/2)*a*e^4*x^2/b^3 + 1/6*(b^2*x^2 + 2*a*b*x + a^2)^(5/2)
*a*d^4/b - 2/3*(b^2*x^2 + 2*a*b*x + a^2)^(5/2)*a^2*d^3*e/b^2 + (b^2*x^2 +
2*a*b*x + a^2)^(5/2)*a^3*d^2*e^2/b^3 - 2/3*(b^2*x^2 + 2*a*b*x + a^2)^(5/2)
*a^4*d*e^3/b^4 + 1/6*(b^2*x^2 + 2*a*b*x + a^2)^(5/2)*a^5*e^4/b^5 + 3/4*(b^
2*x^2 + 2*a*b*x + a^2)^(7/2)*d^2*e^2*x/b^2 - 11/18*(b^2*x^2 + 2*a*b*x + a^
2)^(7/2)*a*d*e^3*x/b^3 + 29/180*(b^2*x^2 + 2*a*b*x + a^2)^(7/2)*a^2*e^4*x/
b^4 + 4/7*(b^2*x^2 + 2*a*b*x + a^2)^(7/2)*d^3*e/b^2 - 27/28*(b^2*x^2 + 2*a
*b*x + a^2)^(7/2)*a*d^2*e^2/b^3 + 83/126*(b^2*x^2 + 2*a*b*x + a^2)^(7/2)*a
^2*d*e^3/b^4 - 209/1260*(b^2*x^2 + 2*a*b*x + a^2)^(7/2)*a^3*e^4/b^5

```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 642 vs. $2(154) = 308$.

Time = 0.23 (sec) , antiderivative size = 642, normalized size of antiderivative = 2.93

$$\int (d + ex)^4 (a^2 + 2abx + b^2x^2)^{5/2} dx = \text{Too large to display}$$

input

```
integrate((e*x+d)^4*(b^2*x^2+2*a*b*x+a^2)^(5/2),x, algorithm="giac")
```

output

```

1/10*b^5*e^4*x^10*sgn(b*x + a) + 4/9*b^5*d*e^3*x^9*sgn(b*x + a) + 5/9*a*b^
4*e^4*x^9*sgn(b*x + a) + 3/4*b^5*d^2*e^2*x^8*sgn(b*x + a) + 5/2*a*b^4*d*e^
3*x^8*sgn(b*x + a) + 5/4*a^2*b^3*e^4*x^8*sgn(b*x + a) + 4/7*b^5*d^3*e*x^7*
sgn(b*x + a) + 30/7*a*b^4*d^2*e^2*x^7*sgn(b*x + a) + 40/7*a^2*b^3*d*e^3*x^
7*sgn(b*x + a) + 10/7*a^3*b^2*e^4*x^7*sgn(b*x + a) + 1/6*b^5*d^4*x^6*sgn(b
*x + a) + 10/3*a*b^4*d^3*e*x^6*sgn(b*x + a) + 10*a^2*b^3*d^2*e^2*x^6*sgn(b
*x + a) + 20/3*a^3*b^2*d*e^3*x^6*sgn(b*x + a) + 5/6*a^4*b*e^4*x^6*sgn(b*x
+ a) + a*b^4*d^4*x^5*sgn(b*x + a) + 8*a^2*b^3*d^3*e*x^5*sgn(b*x + a) + 12*
a^3*b^2*d^2*e^2*x^5*sgn(b*x + a) + 4*a^4*b*d*e^3*x^5*sgn(b*x + a) + 1/5*a^
5*e^4*x^5*sgn(b*x + a) + 5/2*a^2*b^3*d^4*x^4*sgn(b*x + a) + 10*a^3*b^2*d^3
*e*x^4*sgn(b*x + a) + 15/2*a^4*b*d^2*e^2*x^4*sgn(b*x + a) + a^5*d*e^3*x^4*
sgn(b*x + a) + 10/3*a^3*b^2*d^4*x^3*sgn(b*x + a) + 20/3*a^4*b*d^3*e*x^3*sg
n(b*x + a) + 2*a^5*d^2*e^2*x^3*sgn(b*x + a) + 5/2*a^4*b*d^4*x^2*sgn(b*x +
a) + 2*a^5*d^3*e*x^2*sgn(b*x + a) + a^5*d^4*x*sgn(b*x + a) + 1/1260*(210*a
^6*b^4*d^4 - 120*a^7*b^3*d^3*e + 45*a^8*b^2*d^2*e^2 - 10*a^9*b*d*e^3 + a^1
0*e^4)*sgn(b*x + a)/b^5

```

Mupad [F(-1)]

Timed out.

$$\int (d + ex)^4 (a^2 + 2abx + b^2x^2)^{5/2} dx = \int (d + ex)^4 (a^2 + 2abx + b^2x^2)^{5/2} dx$$

input

```
int((d + e*x)^4*(a^2 + b^2*x^2 + 2*a*b*x)^(5/2), x)
```

output

```
int((d + e*x)^4*(a^2 + b^2*x^2 + 2*a*b*x)^(5/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 397, normalized size of antiderivative = 1.81

$$\int (d + ex)^4 (a^2 + 2abx + b^2x^2)^{5/2} dx = \frac{x(126b^5e^4x^9 + 700ab^4e^4x^8 + 560b^5de^3x^8 + 1575a^2b^3e^4x^7 + 3150ab^4de^3x^7 + 945b^5d^2e^2x^7 - \dots)}{\dots}$$

input `int((e*x+d)^4*(b^2*x^2+2*a*b*x+a^2)^(5/2),x)`

output `(x*(1260*a**5*d**4 + 2520*a**5*d**3*e*x + 2520*a**5*d**2*e**2*x**2 + 1260*a**5*d*e**3*x**3 + 252*a**5*e**4*x**4 + 3150*a**4*b*d**4*x + 8400*a**4*b*d**3*e*x**2 + 9450*a**4*b*d**2*e**2*x**3 + 5040*a**4*b*d*e**3*x**4 + 1050*a**4*b*e**4*x**5 + 4200*a**3*b**2*d**4*x**2 + 12600*a**3*b**2*d**3*e*x**3 + 15120*a**3*b**2*d**2*e**2*x**4 + 8400*a**3*b**2*d*e**3*x**5 + 1800*a**3*b**2*e**4*x**6 + 3150*a**2*b**3*d**4*x**3 + 10080*a**2*b**3*d**3*e*x**4 + 12600*a**2*b**3*d**2*e**2*x**5 + 7200*a**2*b**3*d*e**3*x**6 + 1575*a**2*b**3*e**4*x**7 + 1260*a*b**4*d**4*x**4 + 4200*a*b**4*d**3*e*x**5 + 5400*a*b**4*d**2*e**2*x**6 + 3150*a*b**4*d*e**3*x**7 + 700*a*b**4*e**4*x**8 + 210*b**5*d**4*x**5 + 720*b**5*d**3*e*x**6 + 945*b**5*d**2*e**2*x**7 + 560*b**5*d*e**3*x**8 + 126*b**5*e**4*x**9))/1260`

3.293 $\int (d + ex)^3 (a^2 + 2abx + b^2x^2)^{5/2} dx$

| | |
|---|------|
| Optimal result | 2210 |
| Mathematica [A] (verified) | 2210 |
| Rubi [A] (verified) | 2211 |
| Maple [B] (verified) | 2213 |
| Fricas [B] (verification not implemented) | 2213 |
| Sympy [B] (verification not implemented) | 2214 |
| Maxima [B] (verification not implemented) | 2215 |
| Giac [B] (verification not implemented) | 2217 |
| Mupad [F(-1)] | 2218 |
| Reduce [B] (verification not implemented) | 2218 |

Optimal result

Integrand size = 28, antiderivative size = 172

$$\int (d + ex)^3 (a^2 + 2abx + b^2x^2)^{5/2} dx = \frac{(bd - ae)^3 (a + bx)^5 \sqrt{a^2 + 2abx + b^2x^2}}{6b^4} + \frac{3e(bd - ae)^2 (a + bx)^6 \sqrt{a^2 + 2abx + b^2x^2}}{7b^4} + \frac{3e^2(bd - ae)(a + bx)^7 \sqrt{a^2 + 2abx + b^2x^2}}{8b^4} + \frac{e^3 (a + bx)^8 \sqrt{a^2 + 2abx + b^2x^2}}{9b^4}$$

output

```
1/6*(-a*e+b*d)^3*(b*x+a)^5*((b*x+a)^2)^(1/2)/b^4+3/7*e*(-a*e+b*d)^2*(b*x+a)^6*((b*x+a)^2)^(1/2)/b^4+3/8*e^2*(-a*e+b*d)*(b*x+a)^7*((b*x+a)^2)^(1/2)/b^4+1/9*e^3*(b*x+a)^8*((b*x+a)^2)^(1/2)/b^4
```

Mathematica [A] (verified)

Time = 1.06 (sec) , antiderivative size = 253, normalized size of antiderivative = 1.47

$$\int (d + ex)^3 (a^2 + 2abx + b^2x^2)^{5/2} dx = \frac{x \sqrt{(a + bx)^2 (126a^5 (4d^3 + 6d^2ex + 4de^2x^2 + e^3x^3) + 126a^4bx(10d^3 + 20d^2ex + 15de^2x^2 +$$

input `Integrate[(d + e*x)^3*(a^2 + 2*a*b*x + b^2*x^2)^(5/2),x]`

output $(x\sqrt{(a + bx)^2}*(126*a^5*(4*d^3 + 6*d^2*e*x + 4*d*e^2*x^2 + e^3*x^3) + 126*a^4*b*x*(10*d^3 + 20*d^2*e*x + 15*d*e^2*x^2 + 4*e^3*x^3) + 84*a^3*b^2*x^2*(20*d^3 + 45*d^2*e*x + 36*d*e^2*x^2 + 10*e^3*x^3) + 36*a^2*b^3*x^3*(35*d^3 + 84*d^2*e*x + 70*d*e^2*x^2 + 20*e^3*x^3) + 9*a*b^4*x^4*(56*d^3 + 140*d^2*e*x + 120*d*e^2*x^2 + 35*e^3*x^3) + b^5*x^5*(84*d^3 + 216*d^2*e*x + 189*d*e^2*x^2 + 56*e^3*x^3)))/(504*(a + b*x))$

Rubi [A] (verified)

Time = 0.61 (sec) , antiderivative size = 120, normalized size of antiderivative = 0.70, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {1102, 27, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\begin{aligned}
 & \int (a^2 + 2abx + b^2x^2)^{5/2} (d + ex)^3 dx \\
 & \quad \downarrow 1102 \\
 & \frac{\sqrt{a^2 + 2abx + b^2x^2} \int b^5 (a + bx)^5 (d + ex)^3 dx}{b^5 (a + bx)} \\
 & \quad \downarrow 27 \\
 & \frac{\sqrt{a^2 + 2abx + b^2x^2} \int (a + bx)^5 (d + ex)^3 dx}{a + bx} \\
 & \quad \downarrow 49 \\
 & \frac{\sqrt{a^2 + 2abx + b^2x^2} \int \left(\frac{e^3 (a+bx)^8}{b^3} + \frac{3e^2 (bd-ae)(a+bx)^7}{b^3} + \frac{3e (bd-ae)^2 (a+bx)^6}{b^3} + \frac{(bd-ae)^3 (a+bx)^5}{b^3} \right) dx}{a + bx} \\
 & \quad \downarrow 2009 \\
 & \frac{\sqrt{a^2 + 2abx + b^2x^2} \left(\frac{3e^2 (a+bx)^8 (bd-ae)}{8b^4} + \frac{3e (a+bx)^7 (bd-ae)^2}{7b^4} + \frac{(a+bx)^6 (bd-ae)^3}{6b^4} + \frac{e^3 (a+bx)^9}{9b^4} \right)}{a + bx}
 \end{aligned}$$

input `Int[(d + e*x)^3*(a^2 + 2*a*b*x + b^2*x^2)^(5/2),x]`

output `(Sqrt[a^2 + 2*a*b*x + b^2*x^2]*(((b*d - a*e)^3*(a + b*x)^6)/(6*b^4) + (3*e*(b*d - a*e)^2*(a + b*x)^7)/(7*b^4) + (3*e^2*(b*d - a*e)*(a + b*x)^8)/(8*b^4) + (e^3*(a + b*x)^9)/(9*b^4)))/(a + b*x)`

Defintions of rubi rules used

rule 27 `Int[(a_)*(Fx_), x_Symbol] := Simp[a Int[Fx, x], x] /; FreeQ[a, x] && !MatchQ[Fx, (b_)*(Gx_)] /; FreeQ[b, x]`

rule 49 `Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] := Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] && IGtQ[m, 0] && IGtQ[m + n + 2, 0]`

rule 1102 `Int[((d_.) + (e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] := Simp[(a + b*x + c*x^2)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x)^(2*FracPart[p])) Int[(d + e*x)^m*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && EqQ[b^2 - 4*a*c, 0]`

rule 2009 `Int[u_, x_Symbol] := Simp[IntSum[u, x], x] /; SumQ[u]`

Time = 0.08 (sec) , antiderivative size = 277, normalized size of antiderivative = 1.61

$$\begin{aligned} \int (d+ex)^3 (a^2+2abx+b^2x^2)^{5/2} dx &= \frac{1}{9} b^5 e^3 x^9 + a^5 d^3 x \\ &+ \frac{1}{8} (3b^5 d e^2 + 5ab^4 e^3) x^8 + \frac{1}{7} (3b^5 d^2 e + 15ab^4 d e^2 + 10a^2 b^3 e^3) x^7 \\ &+ \frac{1}{6} (b^5 d^3 + 15ab^4 d^2 e + 30a^2 b^3 d e^2 + 10a^3 b^2 e^3) x^6 \\ &+ (ab^4 d^3 + 6a^2 b^3 d^2 e + 6a^3 b^2 d e^2 + a^4 b e^3) x^5 \\ &+ \frac{1}{4} (10a^2 b^3 d^3 + 30a^3 b^2 d^2 e + 15a^4 b d e^2 + a^5 e^3) x^4 \\ &+ \frac{1}{3} (10a^3 b^2 d^3 + 15a^4 b d^2 e + 3a^5 d e^2) x^3 + \frac{1}{2} (5a^4 b d^3 + 3a^5 d^2 e) x^2 \end{aligned}$$

input `integrate((e*x+d)^3*(b^2*x^2+2*a*b*x+a^2)^(5/2),x, algorithm="fricas")`

output `1/9*b^5*e^3*x^9 + a^5*d^3*x + 1/8*(3*b^5*d*e^2 + 5*a*b^4*e^3)*x^8 + 1/7*(3*b^5*d^2*e + 15*a*b^4*d*e^2 + 10*a^2*b^3*e^3)*x^7 + 1/6*(b^5*d^3 + 15*a*b^4*d^2*e + 30*a^2*b^3*d*e^2 + 10*a^3*b^2*e^3)*x^6 + (a*b^4*d^3 + 6*a^2*b^3*d^2*e + 6*a^3*b^2*d*e^2 + a^4*b*e^3)*x^5 + 1/4*(10*a^2*b^3*d^3 + 30*a^3*b^2*d^2*e + 15*a^4*b*d*e^2 + a^5*e^3)*x^4 + 1/3*(10*a^3*b^2*d^3 + 15*a^4*b*d^2*e + 3*a^5*d*e^2)*x^3 + 1/2*(5*a^4*b*d^3 + 3*a^5*d^2*e)*x^2`

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 9421 vs. 2(122) = 244.

Time = 1.00 (sec) , antiderivative size = 9421, normalized size of antiderivative = 54.77

$$\int (d+ex)^3 (a^2+2abx+b^2x^2)^{5/2} dx = \text{Too large to display}$$

input `integrate((e*x+d)**3*(b**2*x**2+2*a*b*x+a**2)**(5/2),x)`

output

```
Piecewise((sqrt(a**2 + 2*a*b*x + b**2*x**2)*(b**4*e**3*x**8/9 + x**7*(37*a
*b**5*e**3/9 + 3*b**6*d*e**2)/(8*b**2) + x**6*(127*a**2*b**4*e**3/9 + 18*a
*b**5*d*e**2 - 15*a*(37*a*b**5*e**3/9 + 3*b**6*d*e**2)/(8*b) + 3*b**6*d**2
*e)/(7*b**2) + x**5*(20*a**3*b**3*e**3 + 45*a**2*b**4*d*e**2 - 7*a**2*(37*
a*b**5*e**3/9 + 3*b**6*d*e**2)/(8*b**2) + 18*a*b**5*d**2*e - 13*a*(127*a**
2*b**4*e**3/9 + 18*a*b**5*d*e**2 - 15*a*(37*a*b**5*e**3/9 + 3*b**6*d*e**2)
/(8*b) + 3*b**6*d**2*e)/(7*b) + b**6*d**3)/(6*b**2) + x**4*(15*a**4*b**2*e
**3 + 60*a**3*b**3*d*e**2 + 45*a**2*b**4*d**2*e - 6*a**2*(127*a**2*b**4*e*
*3/9 + 18*a*b**5*d*e**2 - 15*a*(37*a*b**5*e**3/9 + 3*b**6*d*e**2)/(8*b) +
3*b**6*d**2*e)/(7*b**2) + 6*a*b**5*d**3 - 11*a*(20*a**3*b**3*e**3 + 45*a**
2*b**4*d*e**2 - 7*a**2*(37*a*b**5*e**3/9 + 3*b**6*d*e**2)/(8*b**2) + 18*a*
b**5*d**2*e - 13*a*(127*a**2*b**4*e**3/9 + 18*a*b**5*d*e**2 - 15*a*(37*a*b
**5*e**3/9 + 3*b**6*d*e**2)/(8*b) + 3*b**6*d**2*e)/(7*b) + b**6*d**3)/(6*b
))/ (5*b**2) + x**3*(6*a**5*b*e**3 + 45*a**4*b**2*d*e**2 + 60*a**3*b**3*d**
2*e + 15*a**2*b**4*d**3 - 5*a**2*(20*a**3*b**3*e**3 + 45*a**2*b**4*d*e**2
- 7*a**2*(37*a*b**5*e**3/9 + 3*b**6*d*e**2)/(8*b**2) + 18*a*b**5*d**2*e -
13*a*(127*a**2*b**4*e**3/9 + 18*a*b**5*d*e**2 - 15*a*(37*a*b**5*e**3/9 + 3
*b**6*d*e**2)/(8*b) + 3*b**6*d**2*e)/(7*b) + b**6*d**3)/(6*b**2) - 9*a*(15
*a**4*b**2*e**3 + 60*a**3*b**3*d*e**2 + 45*a**2*b**4*d**2*e - 6*a**2*(127*
a**2*b**4*e**3/9 + 18*a*b**5*d*e**2 - 15*a*(37*a*b**5*e**3/9 + 3*b**6*d...
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 400 vs. $2(120) = 240$.

Time = 0.04 (sec) , antiderivative size = 400, normalized size of antiderivative = 2.33

$$\begin{aligned}
 \int (d + ex)^3 (a^2 + 2abx + b^2x^2)^{5/2} dx &= \frac{1}{6} (b^2x^2 + 2abx + a^2)^{5/2} d^3x \\
 &- \frac{(b^2x^2 + 2abx + a^2)^{5/2} ad^2ex}{2b} + \frac{(b^2x^2 + 2abx + a^2)^{5/2} a^2de^2x}{2b^2} \\
 &- \frac{(b^2x^2 + 2abx + a^2)^{5/2} a^3e^3x}{6b^3} + \frac{(b^2x^2 + 2abx + a^2)^{7/2} e^3x^2}{9b^2} \\
 &+ \frac{(b^2x^2 + 2abx + a^2)^{5/2} ad^3}{6b} - \frac{(b^2x^2 + 2abx + a^2)^{5/2} a^2d^2e}{2b^2} \\
 &+ \frac{(b^2x^2 + 2abx + a^2)^{5/2} a^3de^2}{2b^3} - \frac{(b^2x^2 + 2abx + a^2)^{5/2} a^4e^3}{6b^4} \\
 &+ \frac{3(b^2x^2 + 2abx + a^2)^{7/2} de^2x}{8b^2} - \frac{11(b^2x^2 + 2abx + a^2)^{7/2} ae^3x}{72b^3} \\
 &+ \frac{3(b^2x^2 + 2abx + a^2)^{7/2} d^2e}{7b^2} - \frac{27(b^2x^2 + 2abx + a^2)^{7/2} ade^2}{56b^3} \\
 &+ \frac{83(b^2x^2 + 2abx + a^2)^{7/2} a^2e^3}{504b^4}
 \end{aligned}$$

input `integrate((e*x+d)^3*(b^2*x^2+2*a*b*x+a^2)^(5/2),x, algorithm="maxima")`

output `1/6*(b^2*x^2 + 2*a*b*x + a^2)^(5/2)*d^3*x - 1/2*(b^2*x^2 + 2*a*b*x + a^2)^(5/2)*a*d^2*e*x/b + 1/2*(b^2*x^2 + 2*a*b*x + a^2)^(5/2)*a^2*d*e^2*x/b^2 - 1/6*(b^2*x^2 + 2*a*b*x + a^2)^(5/2)*a^3*e^3*x/b^3 + 1/9*(b^2*x^2 + 2*a*b*x + a^2)^(7/2)*e^3*x^2/b^2 + 1/6*(b^2*x^2 + 2*a*b*x + a^2)^(5/2)*a*d^3/b - 1/2*(b^2*x^2 + 2*a*b*x + a^2)^(5/2)*a^2*d^2*e/b^2 + 1/2*(b^2*x^2 + 2*a*b*x + a^2)^(5/2)*a^3*d*e^2/b^3 - 1/6*(b^2*x^2 + 2*a*b*x + a^2)^(5/2)*a^4*e^3/b^4 + 3/8*(b^2*x^2 + 2*a*b*x + a^2)^(7/2)*d*e^2*x/b^2 - 11/72*(b^2*x^2 + 2*a*b*x + a^2)^(7/2)*a*e^3*x/b^3 + 3/7*(b^2*x^2 + 2*a*b*x + a^2)^(7/2)*d^2*e/b^2 - 27/56*(b^2*x^2 + 2*a*b*x + a^2)^(7/2)*a*d*e^2/b^3 + 83/504*(b^2*x^2 + 2*a*b*x + a^2)^(7/2)*a^2*e^3/b^4`

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 500 vs. $2(120) = 240$.

Time = 0.23 (sec) , antiderivative size = 500, normalized size of antiderivative = 2.91

$$\begin{aligned} \int (d + ex)^3 (a^2 + 2abx + b^2x^2)^{5/2} dx = & \frac{1}{9} b^5 e^3 x^9 \operatorname{sgn}(bx + a) \\ & + \frac{3}{8} b^5 d e^2 x^8 \operatorname{sgn}(bx + a) + \frac{5}{8} a b^4 e^3 x^8 \operatorname{sgn}(bx + a) + \frac{3}{7} b^5 d^2 e x^7 \operatorname{sgn}(bx + a) \\ & + \frac{15}{7} a b^4 d e^2 x^7 \operatorname{sgn}(bx + a) + \frac{10}{7} a^2 b^3 e^3 x^7 \operatorname{sgn}(bx + a) + \frac{1}{6} b^5 d^3 x^6 \operatorname{sgn}(bx + a) \\ & + \frac{5}{2} a b^4 d^2 e x^6 \operatorname{sgn}(bx + a) + 5 a^2 b^3 d e^2 x^6 \operatorname{sgn}(bx + a) + \frac{5}{3} a^3 b^2 e^3 x^6 \operatorname{sgn}(bx + a) \\ & + a b^4 d^3 x^5 \operatorname{sgn}(bx + a) + 6 a^2 b^3 d^2 e x^5 \operatorname{sgn}(bx + a) + 6 a^3 b^2 d e^2 x^5 \operatorname{sgn}(bx + a) \\ & + a^4 b e^3 x^5 \operatorname{sgn}(bx + a) + \frac{5}{2} a^2 b^3 d^3 x^4 \operatorname{sgn}(bx + a) + \frac{15}{2} a^3 b^2 d^2 e x^4 \operatorname{sgn}(bx + a) \\ & + \frac{15}{4} a^4 b d e^2 x^4 \operatorname{sgn}(bx + a) + \frac{1}{4} a^5 e^3 x^4 \operatorname{sgn}(bx + a) \\ & + \frac{10}{3} a^3 b^2 d^3 x^3 \operatorname{sgn}(bx + a) + 5 a^4 b d^2 e x^3 \operatorname{sgn}(bx + a) + a^5 d e^2 x^3 \operatorname{sgn}(bx + a) \\ & + \frac{5}{2} a^4 b d^3 x^2 \operatorname{sgn}(bx + a) + \frac{3}{2} a^5 d^2 e x^2 \operatorname{sgn}(bx + a) + a^5 d^3 x \operatorname{sgn}(bx + a) \\ & + \frac{(84 a^6 b^3 d^3 - 36 a^7 b^2 d^2 e + 9 a^8 b d e^2 - a^9 e^3) \operatorname{sgn}(bx + a)}{504 b^4} \end{aligned}$$

input `integrate((e*x+d)^3*(b^2*x^2+2*a*b*x+a^2)^(5/2),x, algorithm="giac")`

output `1/9*b^5*e^3*x^9*sgn(b*x + a) + 3/8*b^5*d*e^2*x^8*sgn(b*x + a) + 5/8*a*b^4*e^3*x^8*sgn(b*x + a) + 3/7*b^5*d^2*e*x^7*sgn(b*x + a) + 15/7*a*b^4*d*e^2*x^7*sgn(b*x + a) + 10/7*a^2*b^3*e^3*x^7*sgn(b*x + a) + 1/6*b^5*d^3*x^6*sgn(b*x + a) + 5/2*a*b^4*d^2*e*x^6*sgn(b*x + a) + 5*a^2*b^3*d*e^2*x^6*sgn(b*x + a) + 5/3*a^3*b^2*e^3*x^6*sgn(b*x + a) + a*b^4*d^3*x^5*sgn(b*x + a) + 6*a^2*b^3*d^2*e*x^5*sgn(b*x + a) + 6*a^3*b^2*d*e^2*x^5*sgn(b*x + a) + a^4*b*e^3*x^5*sgn(b*x + a) + 5/2*a^2*b^3*d^3*x^4*sgn(b*x + a) + 15/2*a^3*b^2*d^2*e*x^4*sgn(b*x + a) + 15/4*a^4*b*d*e^2*x^4*sgn(b*x + a) + 1/4*a^5*e^3*x^4*sgn(b*x + a) + 10/3*a^3*b^2*d^3*x^3*sgn(b*x + a) + 5*a^4*b*d^2*e*x^3*sgn(b*x + a) + a^5*d*e^2*x^3*sgn(b*x + a) + 5/2*a^4*b*d^3*x^2*sgn(b*x + a) + 3/2*a^5*d^2*e*x^2*sgn(b*x + a) + a^5*d^3*x*sgn(b*x + a) + 1/504*(84*a^6*b^3*d^3 - 36*a^7*b^2*d^2*e + 9*a^8*b*d*e^2 - a^9*e^3)*sgn(b*x + a)/b^4`

Mupad [F(-1)]

Timed out.

$$\int (d + ex)^3 (a^2 + 2abx + b^2x^2)^{5/2} dx = \int (d + ex)^3 (a^2 + 2abx + b^2x^2)^{5/2} dx$$

input `int((d + e*x)^3*(a^2 + b^2*x^2 + 2*a*b*x)^(5/2), x)`

output `int((d + e*x)^3*(a^2 + b^2*x^2 + 2*a*b*x)^(5/2), x)`

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 305, normalized size of antiderivative = 1.77

$$\int (d + ex)^3 (a^2 + 2abx + b^2x^2)^{5/2} dx = \frac{x(56b^5e^3x^8 + 315ab^4e^3x^7 + 189b^5de^2x^7 + 720a^2b^3e^3x^6 + 1080ab^4de^2x^6 + 216b^5d^2e^2x^6 + 840a^2b^3de^2x^5 + 1260ab^4d^2e^2x^5 + 1080a^2b^3d^2e^2x^5 + 315ab^4d^2e^2x^4 + 84b^5d^2e^2x^4 + 56b^5d^2e^2x^3 + 189b^5de^2x^3 + 56b^5e^3x^3)}{504}$$

input `int((e*x+d)^3*(b^2*x^2+2*a*b*x+a^2)^(5/2), x)`

output `(x*(504*a**5*d**3 + 756*a**5*d**2*e*x + 504*a**5*d*e**2*x**2 + 126*a**5*e**3*x**3 + 1260*a**4*b*d**3*x + 2520*a**4*b*d**2*e*x**2 + 1890*a**4*b*d*e**2*x**3 + 504*a**4*b*e**3*x**4 + 1680*a**3*b**2*d**3*x**2 + 3780*a**3*b**2*d**2*e*x**3 + 3024*a**3*b**2*d*e**2*x**4 + 840*a**3*b**2*e**3*x**5 + 1260*a**2*b**3*d**3*x**3 + 3024*a**2*b**3*d**2*e*x**4 + 2520*a**2*b**3*d*e**2*x**5 + 720*a**2*b**3*e**3*x**6 + 504*a*b**4*d**3*x**4 + 1260*a*b**4*d**2*e*x**5 + 1080*a*b**4*d*e**2*x**6 + 315*a*b**4*e**3*x**7 + 84*b**5*d**3*x**5 + 216*b**5*d**2*e*x**6 + 189*b**5*d*e**2*x**7 + 56*b**5*e**3*x**8))/504`

output

```
(x*Sqrt[(a + b*x)^2]*(56*a^5*(3*d^2 + 3*d*e*x + e^2*x^2) + 70*a^4*b*x*(6*d^2 + 8*d*e*x + 3*e^2*x^2) + 56*a^3*b^2*x^2*(10*d^2 + 15*d*e*x + 6*e^2*x^2) + 28*a^2*b^3*x^3*(15*d^2 + 24*d*e*x + 10*e^2*x^2) + 8*a*b^4*x^4*(21*d^2 + 35*d*e*x + 15*e^2*x^2) + b^5*x^5*(28*d^2 + 48*d*e*x + 21*e^2*x^2)))/(168*(a + b*x))
```

Rubi [A] (verified)

Time = 0.51 (sec) , antiderivative size = 93, normalized size of antiderivative = 0.74, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {1102, 27, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\begin{aligned}
 & \int (a^2 + 2abx + b^2x^2)^{5/2} (d + ex)^2 dx \\
 & \quad \downarrow \text{1102} \\
 & \frac{\sqrt{a^2 + 2abx + b^2x^2} \int b^5 (a + bx)^5 (d + ex)^2 dx}{b^5 (a + bx)} \\
 & \quad \downarrow \text{27} \\
 & \frac{\sqrt{a^2 + 2abx + b^2x^2} \int (a + bx)^5 (d + ex)^2 dx}{a + bx} \\
 & \quad \downarrow \text{49} \\
 & \frac{\sqrt{a^2 + 2abx + b^2x^2} \int \left(\frac{e^2 (a+bx)^7}{b^2} + \frac{2e (bd-ae) (a+bx)^6}{b^2} + \frac{(bd-ae)^2 (a+bx)^5}{b^2} \right) dx}{a + bx} \\
 & \quad \downarrow \text{2009} \\
 & \frac{\sqrt{a^2 + 2abx + b^2x^2} \left(\frac{2e (a+bx)^7 (bd-ae)}{7b^3} + \frac{(a+bx)^6 (bd-ae)^2}{6b^3} + \frac{e^2 (a+bx)^8}{8b^3} \right)}{a + bx}
 \end{aligned}$$

input

```
Int[(d + e*x)^2*(a^2 + 2*a*b*x + b^2*x^2)^(5/2),x]
```

```
output (Sqrt[a^2 + 2*a*b*x + b^2*x^2]*((b*d - a*e)^2*(a + b*x)^6)/(6*b^3) + (2*e
*(b*d - a*e)*(a + b*x)^7)/(7*b^3) + (e^2*(a + b*x)^8)/(8*b^3))/(a + b*x)
```

Defintions of rubi rules used

```
rule 27 Int[(a_)*(Fx_), x_Symbol] := Simp[a Int[Fx, x], x] /; FreeQ[a, x] && !Ma
tchQ[Fx, (b_)*(Gx_)] /; FreeQ[b, x]
```

```
rule 49 Int[((a_.) + (b_.)*(x_)^(m_.))*((c_.) + (d_.)*(x_)^(n_.), x_Symbol] := Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

```
rule 1102 Int[((d_.) + (e_.)*(x_)^(m_.))*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] := Simp[(a + b*x + c*x^2)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x)^(2*F
racPart[p])) Int[(d + e*x)^m*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c,
d, e, m, p}, x] && EqQ[b^2 - 4*a*c, 0]
```

```
rule 2009 Int[u_, x_Symbol] := Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 229 vs. 2(86) = 172.
 Time = 1.12 (sec) , antiderivative size = 230, normalized size of antiderivative = 1.84

| method | result |
|---------|--|
| gospers | $\frac{x(21b^5e^2x^7+120x^6ab^4e^2+48x^6b^5de+280x^5a^2b^3e^2+280x^5ab^4de+28x^5b^5d^2+336a^3b^2e^2x^4+672a^2b^3dex^4+168ab^4d^2x^4+210x^3a^4e^2+168bx+a^5e^2)}{168(bx+a)}$ |
| default | $\frac{x(21b^5e^2x^7+120x^6ab^4e^2+48x^6b^5de+280x^5a^2b^3e^2+280x^5ab^4de+28x^5b^5d^2+336a^3b^2e^2x^4+672a^2b^3dex^4+168ab^4d^2x^4+210x^3a^4e^2+168bx+a^5e^2)}{168(bx+a)}$ |
| orering | $\frac{x(21b^5e^2x^7+120x^6ab^4e^2+48x^6b^5de+280x^5a^2b^3e^2+280x^5ab^4de+28x^5b^5d^2+336a^3b^2e^2x^4+672a^2b^3dex^4+168ab^4d^2x^4+210x^3a^4e^2+168bx+a^5e^2)}{168(bx+a)}$ |
| risch | $\frac{\sqrt{(bx+a)^2b^5e^2x^8}}{8bx+8a} + \frac{\sqrt{(bx+a)^2(5ab^4e^2+2b^5de)}x^7}{7bx+7a} + \frac{\sqrt{(bx+a)^2(10a^2b^3e^2+10ab^4de+b^5d^2)}x^6}{6bx+6a} + \frac{\sqrt{(bx+a)^2(10a^3b^2e^2+20a^2b^3de+b^5d^2)}x^5}{5bx+5a}$ |

input `int((e*x+d)^2*(b^2*x^2+2*a*b*x+a^2)^(5/2),x,method=_RETURNVERBOSE)`

output `1/168*x*(21*b^5*e^2*x^7+120*a*b^4*e^2*x^6+48*b^5*d*e*x^6+280*a^2*b^3*e^2*x^5+280*a*b^4*d*e*x^5+28*b^5*d^2*x^5+336*a^3*b^2*e^2*x^4+672*a^2*b^3*d*e*x^4+168*a*b^4*d^2*x^4+210*a^4*b*e^2*x^3+840*a^3*b^2*d*e*x^3+420*a^2*b^3*d^2*x^3+56*a^5*e^2*x^2+560*a^4*b*d*e*x^2+560*a^3*b^2*d^2*x^2+168*a^5*d*e*x+420*a^4*b*d^2*x+168*a^5*d^2)*((b*x+a)^2)^(5/2)/(b*x+a)^5`

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 197 vs. $2(86) = 172$.

Time = 0.08 (sec) , antiderivative size = 197, normalized size of antiderivative = 1.58

$$\begin{aligned} \int (d+ex)^2 (a^2+2abx+b^2x^2)^{5/2} dx &= \frac{1}{8} b^5 e^2 x^8 + a^5 d^2 x \\ &+ \frac{1}{7} (2b^5 de + 5ab^4 e^2) x^7 + \frac{1}{6} (b^5 d^2 + 10ab^4 de + 10a^2 b^3 e^2) x^6 \\ &+ (ab^4 d^2 + 4a^2 b^3 de + 2a^3 b^2 e^2) x^5 + \frac{5}{4} (2a^2 b^3 d^2 + 4a^3 b^2 de + a^4 be^2) x^4 \\ &+ \frac{1}{3} (10a^3 b^2 d^2 + 10a^4 bde + a^5 e^2) x^3 + \frac{1}{2} (5a^4 bd^2 + 2a^5 de) x^2 \end{aligned}$$

input `integrate((e*x+d)^2*(b^2*x^2+2*a*b*x+a^2)^(5/2),x, algorithm="fricas")`

output `1/8*b^5*e^2*x^8 + a^5*d^2*x + 1/7*(2*b^5*d*e + 5*a*b^4*e^2)*x^7 + 1/6*(b^5*d^2 + 10*a*b^4*d*e + 10*a^2*b^3*e^2)*x^6 + (a*b^4*d^2 + 4*a^2*b^3*d*e + 2*a^3*b^2*e^2)*x^5 + 5/4*(2*a^2*b^3*d^2 + 4*a^3*b^2*d*e + a^4*b*e^2)*x^4 + 1/3*(10*a^3*b^2*d^2 + 10*a^4*b*d*e + a^5*e^2)*x^3 + 1/2*(5*a^4*b*d^2 + 2*a^5*d*e)*x^2`

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 4855 vs. $2(87) = 174$.

Time = 0.90 (sec) , antiderivative size = 4855, normalized size of antiderivative = 38.84

$$\int (d + ex)^2 (a^2 + 2abx + b^2x^2)^{5/2} dx = \text{Too large to display}$$

input `integrate((e*x+d)**2*(b**2*x**2+2*a*b*x+a**2)**(5/2),x)`

output

```
Piecewise((sqrt(a**2 + 2*a*b*x + b**2*x**2)*(b**4*e**2*x**7/8 + x**6*(33*a
*b**5*e**2/8 + 2*b**6*d*e)/(7*b**2) + x**5*(113*a**2*b**4*e**2/8 + 12*a*b*
*5*d*e - 13*a*(33*a*b**5*e**2/8 + 2*b**6*d*e)/(7*b) + b**6*d**2)/(6*b**2)
+ x**4*(20*a**3*b**3*e**2 + 30*a**2*b**4*d*e - 6*a**2*(33*a*b**5*e**2/8 +
2*b**6*d*e)/(7*b**2) + 6*a*b**5*d**2 - 11*a*(113*a**2*b**4*e**2/8 + 12*a*b
**5*d*e - 13*a*(33*a*b**5*e**2/8 + 2*b**6*d*e)/(7*b) + b**6*d**2)/(6*b))/(
5*b**2) + x**3*(15*a**4*b**2*e**2 + 40*a**3*b**3*d*e + 15*a**2*b**4*d**2 -
5*a**2*(113*a**2*b**4*e**2/8 + 12*a*b**5*d*e - 13*a*(33*a*b**5*e**2/8 + 2
*b**6*d*e)/(7*b) + b**6*d**2)/(6*b**2) - 9*a*(20*a**3*b**3*e**2 + 30*a**2*
b**4*d*e - 6*a**2*(33*a*b**5*e**2/8 + 2*b**6*d*e)/(7*b**2) + 6*a*b**5*d**2
- 11*a*(113*a**2*b**4*e**2/8 + 12*a*b**5*d*e - 13*a*(33*a*b**5*e**2/8 + 2
*b**6*d*e)/(7*b) + b**6*d**2)/(6*b))/(5*b))/(4*b**2) + x**2*(6*a**5*b*e**2
+ 30*a**4*b**2*d*e + 20*a**3*b**3*d**2 - 4*a**2*(20*a**3*b**3*e**2 + 30*a
**2*b**4*d*e - 6*a**2*(33*a*b**5*e**2/8 + 2*b**6*d*e)/(7*b**2) + 6*a*b**5*
d**2 - 11*a*(113*a**2*b**4*e**2/8 + 12*a*b**5*d*e - 13*a*(33*a*b**5*e**2/8
+ 2*b**6*d*e)/(7*b) + b**6*d**2)/(6*b))/(5*b**2) - 7*a*(15*a**4*b**2*e**2
+ 40*a**3*b**3*d*e + 15*a**2*b**4*d**2 - 5*a**2*(113*a**2*b**4*e**2/8 + 1
2*a*b**5*d*e - 13*a*(33*a*b**5*e**2/8 + 2*b**6*d*e)/(7*b) + b**6*d**2)/(6*
b**2) - 9*a*(20*a**3*b**3*e**2 + 30*a**2*b**4*d*e - 6*a**2*(33*a*b**5*e**2
/8 + 2*b**6*d*e)/(7*b**2) + 6*a*b**5*d**2 - 11*a*(113*a**2*b**4*e**2/8 ...
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 245 vs. $2(86) = 172$.

Time = 0.04 (sec) , antiderivative size = 245, normalized size of antiderivative = 1.96

$$\int (d + ex)^2 (a^2 + 2abx + b^2x^2)^{5/2} dx = \frac{1}{6} (b^2x^2 + 2abx + a^2)^{5/2} d^2x$$

$$- \frac{(b^2x^2 + 2abx + a^2)^{5/2} adex}{3b} + \frac{(b^2x^2 + 2abx + a^2)^{5/2} a^2 e^2 x}{6b^2}$$

$$+ \frac{(b^2x^2 + 2abx + a^2)^{5/2} ad^2}{6b} - \frac{(b^2x^2 + 2abx + a^2)^{5/2} a^2 de}{3b^2} + \frac{(b^2x^2 + 2abx + a^2)^{5/2} a^3 e^2}{6b^3}$$

$$+ \frac{(b^2x^2 + 2abx + a^2)^{7/2} e^2 x}{8b^2} + \frac{2(b^2x^2 + 2abx + a^2)^{7/2} de}{7b^2} - \frac{9(b^2x^2 + 2abx + a^2)^{7/2} ae^2}{56b^3}$$

input `integrate((e*x+d)^2*(b^2*x^2+2*a*b*x+a^2)^(5/2),x, algorithm="maxima")`

output `1/6*(b^2*x^2 + 2*a*b*x + a^2)^(5/2)*d^2*x - 1/3*(b^2*x^2 + 2*a*b*x + a^2)^(5/2)*a*d*e*x/b + 1/6*(b^2*x^2 + 2*a*b*x + a^2)^(5/2)*a^2*e^2*x/b^2 + 1/6*(b^2*x^2 + 2*a*b*x + a^2)^(5/2)*a*d^2/b - 1/3*(b^2*x^2 + 2*a*b*x + a^2)^(5/2)*a^2*d*e/b^2 + 1/6*(b^2*x^2 + 2*a*b*x + a^2)^(5/2)*a^3*e^2/b^3 + 1/8*(b^2*x^2 + 2*a*b*x + a^2)^(7/2)*e^2*x/b^2 + 2/7*(b^2*x^2 + 2*a*b*x + a^2)^(7/2)*d*e/b^2 - 9/56*(b^2*x^2 + 2*a*b*x + a^2)^(7/2)*a*e^2/b^3`

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 358 vs. $2(86) = 172$.

Time = 0.28 (sec) , antiderivative size = 358, normalized size of antiderivative = 2.86

$$\int (d + ex)^2 (a^2 + 2abx + b^2x^2)^{5/2} dx = \frac{1}{8} b^5 e^2 x^8 \operatorname{sgn}(bx + a) + \frac{2}{7} b^5 d e x^7 \operatorname{sgn}(bx + a) + \frac{5}{7} a b^4 e^2 x^7 \operatorname{sgn}(bx + a) + \frac{1}{6} b^5 d^2 x^6 \operatorname{sgn}(bx + a) + \frac{5}{3} a b^4 d e x^6 \operatorname{sgn}(bx + a) + \frac{5}{3} a^2 b^3 e^2 x^6 \operatorname{sgn}(bx + a) + a b^4 d^2 x^5 \operatorname{sgn}(bx + a) + 4 a^2 b^3 d e x^5 \operatorname{sgn}(bx + a) + 2 a^3 b^2 e^2 x^5 \operatorname{sgn}(bx + a) + \frac{5}{2} a^2 b^3 d^2 x^4 \operatorname{sgn}(bx + a) + 5 a^3 b^2 d e x^4 \operatorname{sgn}(bx + a) + \frac{5}{4} a^4 b e^2 x^4 \operatorname{sgn}(bx + a) + \frac{10}{3} a^3 b^2 d^2 x^3 \operatorname{sgn}(bx + a) + \frac{10}{3} a^4 b d e x^3 \operatorname{sgn}(bx + a) + \frac{1}{3} a^5 e^2 x^3 \operatorname{sgn}(bx + a) + \frac{5}{2} a^4 b d^2 x^2 \operatorname{sgn}(bx + a) + a^5 d e x^2 \operatorname{sgn}(bx + a) + a^5 d^2 x \operatorname{sgn}(bx + a) + \frac{(28 a^6 b^2 d^2 - 8 a^7 b d e + a^8 e^2) \operatorname{sgn}(bx + a)}{168 b^3}$$

input `integrate((e*x+d)^2*(b^2*x^2+2*a*b*x+a^2)^(5/2),x, algorithm="giac")`

output `1/8*b^5*e^2*x^8*sgn(b*x + a) + 2/7*b^5*d*e*x^7*sgn(b*x + a) + 5/7*a*b^4*e^2*x^7*sgn(b*x + a) + 1/6*b^5*d^2*x^6*sgn(b*x + a) + 5/3*a*b^4*d*e*x^6*sgn(b*x + a) + 5/3*a^2*b^3*e^2*x^6*sgn(b*x + a) + a*b^4*d^2*x^5*sgn(b*x + a) + 4*a^2*b^3*d*e*x^5*sgn(b*x + a) + 2*a^3*b^2*e^2*x^5*sgn(b*x + a) + 5/2*a^2*b^3*d^2*x^4*sgn(b*x + a) + 5*a^3*b^2*d*e*x^4*sgn(b*x + a) + 5/4*a^4*b*e^2*x^4*sgn(b*x + a) + 10/3*a^3*b^2*d^2*x^3*sgn(b*x + a) + 10/3*a^4*b*d*e*x^3*sgn(b*x + a) + 1/3*a^5*e^2*x^3*sgn(b*x + a) + 5/2*a^4*b*d^2*x^2*sgn(b*x + a) + a^5*d*e*x^2*sgn(b*x + a) + a^5*d^2*x*sgn(b*x + a) + 1/168*(28*a^6*b^2*d^2 - 8*a^7*b*d*e + a^8*e^2)*sgn(b*x + a)/b^3`

output

```
(x*sqrt[(a + b*x)^2]*(21*a^5*(2*d + e*x) + 35*a^4*b*x*(3*d + 2*e*x) + 35*a^3*b^2*x^2*(4*d + 3*e*x) + 21*a^2*b^3*x^3*(5*d + 4*e*x) + 7*a*b^4*x^4*(6*d + 5*e*x) + b^5*x^5*(7*d + 6*e*x)))/(42*(a + b*x))
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.115$, Rules used = {1100, 1079, 17}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a^2 + 2abx + b^2x^2)^{5/2} (d + ex) dx$$

$$\downarrow 1100$$

$$\frac{(bd - ae) \int (a^2 + 2bxa + b^2x^2)^{5/2} dx}{b} + \frac{e(a^2 + 2abx + b^2x^2)^{7/2}}{7b^2}$$

$$\downarrow 1079$$

$$\frac{\sqrt{a^2 + 2abx + b^2x^2}(bd - ae) \int (xb^2 + ab)^5 dx}{b^6(a + bx)} + \frac{e(a^2 + 2abx + b^2x^2)^{7/2}}{7b^2}$$

$$\downarrow 17$$

$$\frac{\sqrt{a^2 + 2abx + b^2x^2}(a + bx)^5(bd - ae)}{6b^2} + \frac{e(a^2 + 2abx + b^2x^2)^{7/2}}{7b^2}$$

input

```
Int[(d + e*x)*(a^2 + 2*a*b*x + b^2*x^2)^(5/2), x]
```

output

```
((b*d - a*e)*(a + b*x)^5*sqrt[a^2 + 2*a*b*x + b^2*x^2])/(6*b^2) + (e*(a^2 + 2*a*b*x + b^2*x^2)^(7/2))/(7*b^2)
```


Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 115 vs. $2(54) = 108$.

Time = 0.08 (sec) , antiderivative size = 115, normalized size of antiderivative = 1.62

$$\int (d + ex) (a^2 + 2abx + b^2x^2)^{5/2} dx = \frac{1}{7} b^5 ex^7 + a^5 dx + \frac{1}{6} (b^5 d + 5 ab^4 e) x^6 + (ab^4 d + 2 a^2 b^3 e) x^5 + \frac{5}{2} (a^2 b^3 d + a^3 b^2 e) x^4 + \frac{5}{3} (2 a^3 b^2 d + a^4 b e) x^3 + \frac{1}{2} (5 a^4 b d + a^5 e) x^2$$

input `integrate((e*x+d)*(b^2*x^2+2*a*b*x+a^2)^(5/2),x, algorithm="fricas")`

output `1/7*b^5*e*x^7 + a^5*d*x + 1/6*(b^5*d + 5*a*b^4*e)*x^6 + (a*b^4*d + 2*a^2*b^3*e)*x^5 + 5/2*(a^2*b^3*d + a^3*b^2*e)*x^4 + 5/3*(2*a^3*b^2*d + a^4*b*e)*x^3 + 1/2*(5*a^4*b*d + a^5*e)*x^2`

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2147 vs. $2(56) = 112$.

Time = 0.78 (sec) , antiderivative size = 2147, normalized size of antiderivative = 30.24

$$\int (d + ex) (a^2 + 2abx + b^2x^2)^{5/2} dx = \text{Too large to display}$$

input `integrate((e*x+d)*(b**2*x**2+2*a*b*x+a**2)**(5/2),x)`

output

```
Piecewise((sqrt(a**2 + 2*a*b*x + b**2*x**2)*(b**4*e*x**6/7 + x**5*(29*a*b*
*5*e/7 + b**6*d)/(6*b**2) + x**4*(99*a**2*b**4*e/7 + 6*a*b**5*d - 11*a*(29
*a*b**5*e/7 + b**6*d)/(6*b))/(5*b**2) + x**3*(20*a**3*b**3*e + 15*a**2*b**
4*d - 5*a**2*(29*a*b**5*e/7 + b**6*d)/(6*b**2) - 9*a*(99*a**2*b**4*e/7 + 6
*a*b**5*d - 11*a*(29*a*b**5*e/7 + b**6*d)/(6*b))/(5*b))/(4*b**2) + x**2*(1
5*a**4*b**2*e + 20*a**3*b**3*d - 4*a**2*(99*a**2*b**4*e/7 + 6*a*b**5*d - 1
1*a*(29*a*b**5*e/7 + b**6*d)/(6*b))/(5*b**2) - 7*a*(20*a**3*b**3*e + 15*a*
*2*b**4*d - 5*a**2*(29*a*b**5*e/7 + b**6*d)/(6*b**2) - 9*a*(99*a**2*b**4*e
/7 + 6*a*b**5*d - 11*a*(29*a*b**5*e/7 + b**6*d)/(6*b))/(5*b))/(4*b))/(3*b
*2) + x*(6*a**5*b*e + 15*a**4*b**2*d - 3*a**2*(20*a**3*b**3*e + 15*a**2*b**
4*d - 5*a**2*(29*a*b**5*e/7 + b**6*d)/(6*b**2) - 9*a*(99*a**2*b**4*e/7 +
6*a*b**5*d - 11*a*(29*a*b**5*e/7 + b**6*d)/(6*b))/(5*b))/(4*b**2) - 5*a*(1
5*a**4*b**2*e + 20*a**3*b**3*d - 4*a**2*(99*a**2*b**4*e/7 + 6*a*b**5*d - 1
1*a*(29*a*b**5*e/7 + b**6*d)/(6*b))/(5*b**2) - 7*a*(20*a**3*b**3*e + 15*a*
*2*b**4*d - 5*a**2*(29*a*b**5*e/7 + b**6*d)/(6*b**2) - 9*a*(99*a**2*b**4*e
/7 + 6*a*b**5*d - 11*a*(29*a*b**5*e/7 + b**6*d)/(6*b))/(5*b))/(4*b))/(3*b
)/(2*b**2) + (a**6*e + 6*a**5*b*d - 2*a**2*(15*a**4*b**2*e + 20*a**3*b**3*
d - 4*a**2*(99*a**2*b**4*e/7 + 6*a*b**5*d - 11*a*(29*a*b**5*e/7 + b**6*d)/
(6*b))/(5*b**2) - 7*a*(20*a**3*b**3*e + 15*a**2*b**4*d - 5*a**2*(29*a*b**5
*e/7 + b**6*d)/(6*b**2) - 9*a*(99*a**2*b**4*e/7 + 6*a*b**5*d - 11*a*(29...
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 125 vs. $2(54) = 108$.

Time = 0.04 (sec) , antiderivative size = 125, normalized size of antiderivative = 1.76

$$\int (d + ex) (a^2 + 2abx + b^2x^2)^{5/2} dx = \frac{1}{6} (b^2x^2 + 2abx + a^2)^{5/2} dx$$

$$- \frac{(b^2x^2 + 2abx + a^2)^{5/2} aex}{6b} + \frac{(b^2x^2 + 2abx + a^2)^{5/2} ad}{6b}$$

$$- \frac{(b^2x^2 + 2abx + a^2)^{5/2} a^2e}{6b^2} + \frac{(b^2x^2 + 2abx + a^2)^{7/2} e}{7b^2}$$

input

```
integrate((e*x+d)*(b^2*x^2+2*a*b*x+a^2)^(5/2),x, algorithm="maxima")
```

output

$$\frac{1}{6}(b^2x^2 + 2abx + a^2)^{5/2}dx - \frac{1}{6}(b^2x^2 + 2abx + a^2)^{5/2} \frac{aex}{b} + \frac{1}{6}(b^2x^2 + 2abx + a^2)^{5/2} \frac{ad}{b} - \frac{1}{6}(b^2x^2 + 2abx + a^2)^{5/2} \frac{a^2e}{b^2} + \frac{1}{7}(b^2x^2 + 2abx + a^2)^{7/2} \frac{e}{b^2}$$
Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 218 vs. $2(54) = 108$.

Time = 0.32 (sec) , antiderivative size = 218, normalized size of antiderivative = 3.07

$$\begin{aligned} \int (d + ex) (a^2 + 2abx + b^2x^2)^{5/2} dx &= \frac{1}{7} b^5 ex^7 \operatorname{sgn}(bx + a) \\ &+ \frac{1}{6} b^5 dx^6 \operatorname{sgn}(bx + a) + \frac{5}{6} ab^4 ex^6 \operatorname{sgn}(bx + a) + ab^4 dx^5 \operatorname{sgn}(bx + a) \\ &+ 2a^2 b^3 ex^5 \operatorname{sgn}(bx + a) + \frac{5}{2} a^2 b^3 dx^4 \operatorname{sgn}(bx + a) + \frac{5}{2} a^3 b^2 ex^4 \operatorname{sgn}(bx + a) \\ &+ \frac{10}{3} a^3 b^2 dx^3 \operatorname{sgn}(bx + a) + \frac{5}{3} a^4 b ex^3 \operatorname{sgn}(bx + a) + \frac{5}{2} a^4 b dx^2 \operatorname{sgn}(bx + a) \\ &+ \frac{1}{2} a^5 ex^2 \operatorname{sgn}(bx + a) + a^5 dx \operatorname{sgn}(bx + a) + \frac{(7a^6 bd - a^7 e) \operatorname{sgn}(bx + a)}{42 b^2} \end{aligned}$$

input

```
integrate((e*x+d)*(b^2*x^2+2*a*b*x+a^2)^(5/2),x, algorithm="giac")
```

output

$$\begin{aligned} & \frac{1}{7} b^5 e x^7 \operatorname{sgn}(b x + a) + \frac{1}{6} b^5 d x^6 \operatorname{sgn}(b x + a) + \frac{5}{6} a b^4 e x^6 \operatorname{sgn}(b x + a) \\ & + \frac{5}{6} a b^4 d x^5 \operatorname{sgn}(b x + a) + 2 a^2 b^3 e x^5 \operatorname{sgn}(b x + a) + \frac{5}{2} a^2 b^3 d x^4 \operatorname{sgn}(b x + a) \\ & + \frac{5}{2} a^3 b^2 e x^4 \operatorname{sgn}(b x + a) + \frac{10}{3} a^3 b^2 d x^3 \operatorname{sgn}(b x + a) + \frac{5}{3} a^4 b e x^3 \operatorname{sgn}(b x + a) \\ & + \frac{5}{2} a^4 b d x^2 \operatorname{sgn}(b x + a) + \frac{1}{2} a^5 e x^2 \operatorname{sgn}(b x + a) + a^5 d x \operatorname{sgn}(b x + a) + \frac{1}{42} (7 a^6 b d - a^7 e) \operatorname{sgn}(b x + a) / b^2 \end{aligned}$$

Mupad [F(-1)]

Timed out.

$$\int (d + ex) (a^2 + 2abx + b^2x^2)^{5/2} dx = \int (d + ex) (a^2 + 2abx + b^2x^2)^{5/2} dx$$

input `int((d + e*x)*(a^2 + b^2*x^2 + 2*a*b*x)^(5/2), x)`output `int((d + e*x)*(a^2 + b^2*x^2 + 2*a*b*x)^(5/2), x)`**Reduce [B] (verification not implemented)**

Time = 0.23 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.70

$$\int (d + ex) (a^2 + 2abx + b^2x^2)^{5/2} dx = \frac{x(6b^5ex^6 + 35ab^4ex^5 + 7b^5dx^5 + 84a^2b^3ex^4 + 42ab^4dx^4 + 105a^3b^2ex^3 + 105a^2b^3dx^3 + 7a^4b^2ex^2 + 42a^3b^3dx^2 + 105a^4b^2ex + 42a^3b^3dx + 6a^4b^2e + 42a^3b^3d)}{42}$$

input `int((e*x+d)*(b^2*x^2+2*a*b*x+a^2)^(5/2), x)`output `(x*(42*a**5*d + 21*a**5*e*x + 105*a**4*b*d*x + 70*a**4*b*e*x**2 + 140*a**3*b**2*d*x**2 + 105*a**3*b**2*e*x**3 + 105*a**2*b**3*d*x**3 + 84*a**2*b**3*e*x**4 + 42*a*b**4*d*x**4 + 35*a*b**4*e*x**5 + 7*b**5*d*x**5 + 6*b**5*e*x**6))/42`

3.296 $\int (a^2 + 2abx + b^2x^2)^{5/2} dx$

| | |
|---|------|
| Optimal result | 2234 |
| Mathematica [A] (verified) | 2234 |
| Rubi [A] (verified) | 2235 |
| Maple [A] (verified) | 2236 |
| Fricas [B] (verification not implemented) | 2236 |
| Sympy [B] (verification not implemented) | 2237 |
| Maxima [B] (verification not implemented) | 2237 |
| Giac [B] (verification not implemented) | 2238 |
| Mupad [B] (verification not implemented) | 2238 |
| Reduce [B] (verification not implemented) | 2239 |

Optimal result

Integrand size = 20, antiderivative size = 34

$$\int (a^2 + 2abx + b^2x^2)^{5/2} dx = \frac{(a + bx)^5 \sqrt{a^2 + 2abx + b^2x^2}}{6b}$$

output $1/6*(b*x+a)^5*((b*x+a)^2)^{(1/2)}/b$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.68

$$\int (a^2 + 2abx + b^2x^2)^{5/2} dx = \frac{(a + bx) ((a + bx)^2)^{5/2}}{6b}$$

input `Integrate[(a^2 + 2*a*b*x + b^2*x^2)^(5/2), x]`

output $((a + b*x)*((a + b*x)^2)^{(5/2)})/(6*b)$

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {1079, 17}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a^2 + 2abx + b^2x^2)^{5/2} dx$$

$$\downarrow 1079$$

$$\frac{\sqrt{a^2 + 2abx + b^2x^2} \int (xb^2 + ab)^5 dx}{b^5(a + bx)}$$

$$\downarrow 17$$

$$\frac{(a + bx)^5 \sqrt{a^2 + 2abx + b^2x^2}}{6b}$$

input `Int[(a^2 + 2*a*b*x + b^2*x^2)^(5/2), x]`

output `((a + b*x)^5*Sqrt[a^2 + 2*a*b*x + b^2*x^2])/(6*b)`

Defintions of rubi rules used

rule 17 `Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1))/(b*(m + 1))], x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]`

rule 1079 `Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(a + b*x + c*x^2)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x)^(2*FracPart[p])) Int[(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c, p}, x] && EqQ[b^2 - 4*a*c, 0]`

Maple [A] (verified)

Time = 0.87 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.59

| method | result | size |
|---------|--|------|
| default | $\frac{\left((bx+a)^2\right)^{\frac{5}{2}}(bx+a)}{6b}$ | 20 |
| risch | $\frac{(bx+a)^5\sqrt{(bx+a)^2}}{6b}$ | 22 |
| gospers | $\frac{x(b^5x^5+6ab^4x^4+15a^2b^3x^3+20a^3b^2x^2+15a^4bx+6a^5)\left((bx+a)^2\right)^{\frac{5}{2}}}{6(bx+a)^5}$ | 71 |
| orering | $\frac{x(b^5x^5+6ab^4x^4+15a^2b^3x^3+20a^3b^2x^2+15a^4bx+6a^5)(b^2x^2+2abx+a^2)^{\frac{5}{2}}}{6(bx+a)^5}$ | 80 |

input `int((b^2*x^2+2*a*b*x+a^2)^(5/2),x,method=_RETURNVERBOSE)`

output `1/6*((b*x+a)^2)^(5/2)*(b*x+a)/b`

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 53 vs. 2(21) = 42.

Time = 0.08 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.56

$$\int (a^2 + 2abx + b^2x^2)^{5/2} dx = \frac{1}{6}b^5x^6 + ab^4x^5 + \frac{5}{2}a^2b^3x^4 + \frac{10}{3}a^3b^2x^3 + \frac{5}{2}a^4bx^2 + a^5x$$

input `integrate((b^2*x^2+2*a*b*x+a^2)^(5/2),x, algorithm="fricas")`

output `1/6*b^5*x^6 + a*b^4*x^5 + 5/2*a^2*b^3*x^4 + 10/3*a^3*b^2*x^3 + 5/2*a^4*b*x^2 + a^5*x`

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 60 vs. $2(19) = 38$.

Time = 1.34 (sec) , antiderivative size = 666, normalized size of antiderivative = 19.59

$$\int (a^2 + 2abx + b^2x^2)^{5/2} dx = \text{Too large to display}$$

input `integrate((b**2*x**2+2*a*b*x+a**2)**(5/2),x)`

output

```
a**4*Piecewise(((a/(2*b) + x/2)*sqrt(a**2 + 2*a*b*x + b**2*x**2), Ne(b**2, 0)), ((a**2 + 2*a*b*x)**(3/2)/(3*a*b), Ne(a*b, 0)), (x*sqrt(a**2), True)) + 4*a**3*b*Piecewise((sqrt(a**2 + 2*a*b*x + b**2*x**2)*(-a**2/(6*b**2) + a*x/(6*b) + x**2/3), Ne(b**2, 0)), ((-a**2*(a**2 + 2*a*b*x)**(3/2)/3 + (a**2 + 2*a*b*x)**(5/2)/5)/(2*a**2*b**2), Ne(a*b, 0)), (x**2*sqrt(a**2)/2, True)) + 6*a**2*b**2*Piecewise((sqrt(a**2 + 2*a*b*x + b**2*x**2)*(a**3/(12*b**3) - a**2*x/(12*b**2) + a*x**2/(12*b) + x**3/4), Ne(b**2, 0)), ((a**4*(a**2 + 2*a*b*x)**(3/2)/3 - 2*a**2*(a**2 + 2*a*b*x)**(5/2)/5 + (a**2 + 2*a*b*x)**(7/2)/7)/(4*a**3*b**3), Ne(a*b, 0)), (x**3*sqrt(a**2)/3, True)) + 4*a*b**3*Piecewise((sqrt(a**2 + 2*a*b*x + b**2*x**2)*(-a**4/(20*b**4) + a**3*x/(20*b**3) - a**2*x**2/(20*b**2) + a*x**3/(20*b) + x**4/5), Ne(b**2, 0)), ((-a**6*(a**2 + 2*a*b*x)**(3/2)/3 + 3*a**4*(a**2 + 2*a*b*x)**(5/2)/5 - 3*a**2*(a**2 + 2*a*b*x)**(7/2)/7 + (a**2 + 2*a*b*x)**(9/2)/9)/(8*a**4*b**4), Ne(a*b, 0)), (x**4*sqrt(a**2)/4, True)) + b**4*Piecewise((sqrt(a**2 + 2*a*b*x + b**2*x**2)*(a**5/(30*b**5) - a**4*x/(30*b**4) + a**3*x**2/(30*b**3) - a**2*x**3/(30*b**2) + a*x**4/(30*b) + x**5/6), Ne(b**2, 0)), ((a**8*(a**2 + 2*a*b*x)**(3/2)/3 - 4*a**6*(a**2 + 2*a*b*x)**(5/2)/5 + 6*a**4*(a**2 + 2*a*b*x)**(7/2)/7 - 4*a**2*(a**2 + 2*a*b*x)**(9/2)/9 + (a**2 + 2*a*b*x)**(11/2)/11)/(16*a**5*b**5), Ne(a*b, 0)), (x**5*sqrt(a**2)/5, True))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 46 vs. $2(21) = 42$.

Time = 0.03 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.35

$$\int (a^2 + 2abx + b^2x^2)^{5/2} dx = \frac{1}{6} (b^2x^2 + 2abx + a^2)^{\frac{5}{2}}x + \frac{(b^2x^2 + 2abx + a^2)^{\frac{5}{2}}a}{6b}$$

input `integrate((b^2*x^2+2*a*b*x+a^2)^(5/2),x, algorithm="maxima")`

output `1/6*(b^2*x^2 + 2*a*b*x + a^2)^(5/2)*x + 1/6*(b^2*x^2 + 2*a*b*x + a^2)^(5/2)*a/b`

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 83 vs. $2(21) = 42$.

Time = 0.41 (sec) , antiderivative size = 83, normalized size of antiderivative = 2.44

$$\int (a^2 + 2abx + b^2x^2)^{5/2} dx = \frac{1}{2} (bx^2 + 2ax)a^4 \operatorname{sgn}(bx + a) + \frac{a^6 \operatorname{sgn}(bx + a)}{6b} + \frac{1}{2} (bx^2 + 2ax)^2 a^2 b \operatorname{sgn}(bx + a) + \frac{1}{6} (bx^2 + 2ax)^3 b^2 \operatorname{sgn}(bx + a)$$

input `integrate((b^2*x^2+2*a*b*x+a^2)^(5/2),x, algorithm="giac")`

output `1/2*(b*x^2 + 2*a*x)*a^4*sgn(b*x + a) + 1/6*a^6*sgn(b*x + a)/b + 1/2*(b*x^2 + 2*a*x)^2*a^2*b*sgn(b*x + a) + 1/6*(b*x^2 + 2*a*x)^3*b^2*sgn(b*x + a)`

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.94

$$\int (a^2 + 2abx + b^2x^2)^{5/2} dx = \frac{(xb^2 + ab)(a^2 + 2abx + b^2x^2)^{5/2}}{6b^2}$$

input `int((a^2 + b^2*x^2 + 2*a*b*x)^(5/2),x)`

output `((a*b + b^2*x)*(a^2 + b^2*x^2 + 2*a*b*x)^(5/2))/(6*b^2)`

Reduce [B] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.59

$$\int (a^2 + 2abx + b^2x^2)^{5/2} dx = \frac{x(b^5x^5 + 6ab^4x^4 + 15a^2b^3x^3 + 20a^3b^2x^2 + 15a^4bx + 6a^5)}{6}$$

input `int((b^2*x^2+2*a*b*x+a^2)^(5/2),x)`

output `(x*(6*a**5 + 15*a**4*b*x + 20*a**3*b**2*x**2 + 15*a**2*b**3*x**3 + 6*a*b**4*x**4 + b**5*x**5))/6`

3.297 $\int \frac{(a^2+2abx+b^2x^2)^{5/2}}{d+ex} dx$

| | |
|---|------|
| Optimal result | 2240 |
| Mathematica [A] (verified) | 2241 |
| Rubi [A] (verified) | 2241 |
| Maple [A] (verified) | 2243 |
| Fricas [A] (verification not implemented) | 2243 |
| Sympy [F] | 2244 |
| Maxima [F(-2)] | 2244 |
| Giac [B] (verification not implemented) | 2244 |
| Mupad [F(-1)] | 2245 |
| Reduce [B] (verification not implemented) | 2245 |

Optimal result

Integrand size = 28, antiderivative size = 254

$$\int \frac{(a^2 + 2abx + b^2x^2)^{5/2}}{d + ex} dx = \frac{b(bd - ae)^4 x \sqrt{a^2 + 2abx + b^2x^2}}{e^5(a + bx)}$$

$$- \frac{(bd - ae)^3(a + bx)\sqrt{a^2 + 2abx + b^2x^2}}{2e^4}$$

$$+ \frac{(bd - ae)^2(a + bx)^2\sqrt{a^2 + 2abx + b^2x^2}}{3e^3}$$

$$- \frac{(bd - ae)(a + bx)^3\sqrt{a^2 + 2abx + b^2x^2}}{4e^2} + \frac{(a + bx)^4\sqrt{a^2 + 2abx + b^2x^2}}{5e}$$

$$- \frac{(bd - ae)^5\sqrt{a^2 + 2abx + b^2x^2} \log(d + ex)}{e^6(a + bx)}$$

output

```
b*(-a*e+b*d)^4*x*((b*x+a)^2)^(1/2)/e^5/(b*x+a)-1/2*(-a*e+b*d)^3*(b*x+a)*((
b*x+a)^2)^(1/2)/e^4+1/3*(-a*e+b*d)^2*(b*x+a)^2*((b*x+a)^2)^(1/2)/e^3-1/4*(
-a*e+b*d)*(b*x+a)^3*((b*x+a)^2)^(1/2)/e^2+1/5*(b*x+a)^4*((b*x+a)^2)^(1/2)/
e-(-a*e+b*d)^5*((b*x+a)^2)^(1/2)*ln(e*x+d)/e^6/(b*x+a)
```

Mathematica [A] (verified)

Time = 1.08 (sec) , antiderivative size = 185, normalized size of antiderivative = 0.73

$$\int \frac{(a^2 + 2abx + b^2x^2)^{5/2}}{d + ex} dx = \frac{\sqrt{(a + bx)^2}(bex(300a^4e^4 + 300a^3be^3(-2d + ex) + 100a^2b^2e^2(6d^2 - 3dex +$$

input `Integrate[(a^2 + 2*a*b*x + b^2*x^2)^(5/2)/(d + e*x),x]`

output `(Sqrt[(a + b*x)^2]*(b*e*x*(300*a^4*e^4 + 300*a^3*b*e^3*(-2*d + e*x) + 100*a^2*b^2*e^2*(6*d^2 - 3*d*e*x + 2*e^2*x^2) + 25*a*b^3*e*(-12*d^3 + 6*d^2*e*x - 4*d*e^2*x^2 + 3*e^3*x^3) + b^4*(60*d^4 - 30*d^3*e*x + 20*d^2*e^2*x^2 - 15*d*e^3*x^3 + 12*e^4*x^4)) - 60*(b*d - a*e)^5*Log[d + e*x]))/(60*e^6*(a + b*x))`

Rubi [A] (verified)

Time = 0.52 (sec) , antiderivative size = 150, normalized size of antiderivative = 0.59, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {1102, 27, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\begin{aligned} & \int \frac{(a^2 + 2abx + b^2x^2)^{5/2}}{d + ex} dx \\ & \quad \downarrow \text{1102} \\ & \frac{\sqrt{a^2 + 2abx + b^2x^2} \int \frac{b^5(a+bx)^5}{d+ex} dx}{b^5(a + bx)} \\ & \quad \downarrow \text{27} \\ & \frac{\sqrt{a^2 + 2abx + b^2x^2} \int \frac{(a+bx)^5}{d+ex} dx}{a + bx} \\ & \quad \downarrow \text{49} \end{aligned}$$

$$\frac{\sqrt{a^2 + 2abx + b^2x^2} \int \left(\frac{(ae-bd)^5}{e^5(d+ex)} + \frac{b(bd-ae)^4}{e^5} + \frac{b(a+bx)^4}{e} - \frac{b(bd-ae)(a+bx)^3}{e^2} + \frac{b(bd-ae)^2(a+bx)^2}{e^3} - \frac{b(bd-ae)^3(a+bx)}{e^4} \right) dx}{a + bx}$$

↓ 2009

$$\frac{\sqrt{a^2 + 2abx + b^2x^2} \left(-\frac{(bd-ae)^5 \log(d+ex)}{e^6} + \frac{bx(bd-ae)^4}{e^5} - \frac{(a+bx)^2(bd-ae)^3}{2e^4} + \frac{(a+bx)^3(bd-ae)^2}{3e^3} - \frac{(a+bx)^4(bd-ae)}{4e^2} + \frac{(a+bx)^5}{5e} \right)}{a + bx}$$

input

```
Int[(a^2 + 2*a*b*x + b^2*x^2)^(5/2)/(d + e*x),x]
```

output

```
(Sqrt[a^2 + 2*a*b*x + b^2*x^2]*((b*(b*d - a*e)^4*x)/e^5 - ((b*d - a*e)^3*(a + b*x)^2)/(2*e^4) + ((b*d - a*e)^2*(a + b*x)^3)/(3*e^3) - ((b*d - a*e)*(a + b*x)^4)/(4*e^2) + (a + b*x)^5/(5*e) - ((b*d - a*e)^5*Log[d + e*x])/e^6))/a + b*x
```

Defintions of rubi rules used

rule 27

```
Int[(a_)*(Fx_), x_Symbol] := Simp[a Int[Fx, x], x] /; FreeQ[a, x] && !MatchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 49

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] := Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] && IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 1102

```
Int[((d_.) + (e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] := Simp[(a + b*x + c*x^2)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x)^(2*FracPart[p])) Int[(d + e*x)^m*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && EqQ[b^2 - 4*a*c, 0]
```

rule 2009

```
Int[u_, x_Symbol] := Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 1.23 (sec) , antiderivative size = 298, normalized size of antiderivative = 1.17

| method | result |
|---------|---|
| risch | $\frac{\sqrt{(bx+a)^2} b \left(\frac{1}{5} b^4 x^5 e^4 + \frac{5}{4} x^4 a b^3 e^4 - \frac{1}{4} x^4 b^4 d e^3 + \frac{10}{3} x^3 a^2 b^2 e^4 - \frac{5}{3} x^3 a b^3 d e^3 + \frac{1}{3} x^3 b^4 d^2 e^2 + 5 x^2 a^3 b e^4 - 5 x^2 a^2 b^2 d e^3 + \frac{5}{2} x^2 a b^3 d^2 e^2 - \frac{1}{2} x^2 b^4 d^3 e \right)}{(bx+a)e^5}$ |
| default | $\frac{\left((bx+a)^2 \right)^{\frac{5}{2}} \left(12x^5 e^5 b^5 + 75x^4 a b^4 e^5 - 15x^4 b^5 d e^4 + 200x^3 a^2 b^3 e^5 - 100x^3 a b^4 d e^4 + 20x^3 b^5 d^2 e^3 + 300x^2 a^3 b^2 e^5 - 300x^2 a^2 b^3 d e^4 + 150x^2 a b^4 d^2 e^3 - 150x^2 b^5 d^3 e^2 + 100x a^4 b e^5 - 100x a^3 b^2 d e^4 + 50x a^2 b^3 d^2 e^3 - 50x a b^4 d^3 e^2 + 50x b^5 d^4 e \right)}{e^6 \ln(e^5 x + d)}$ |

input `int((b^2*x^2+2*a*b*x+a^2)^(5/2)/(e*x+d),x,method=_RETURNVERBOSE)`

output
$$\frac{\left((bx+a)^2 \right)^{\frac{5}{2}} \left(12x^5 e^5 b^5 + 75x^4 a b^4 e^5 - 15x^4 b^5 d e^4 + 200x^3 a^2 b^3 e^5 - 100x^3 a b^4 d e^4 + 20x^3 b^5 d^2 e^3 + 300x^2 a^3 b^2 e^5 - 300x^2 a^2 b^3 d e^4 + 150x^2 a b^4 d^2 e^3 - 150x^2 b^5 d^3 e^2 + 100x a^4 b e^5 - 100x a^3 b^2 d e^4 + 50x a^2 b^3 d^2 e^3 - 50x a b^4 d^3 e^2 + 50x b^5 d^4 e \right)}{e^6 \ln(e^5 x + d)}$$

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 259, normalized size of antiderivative = 1.02

$$\int \frac{(a^2 + 2abx + b^2x^2)^{5/2}}{d + ex} dx = \frac{12b^5e^5x^5 - 15(b^5de^4 - 5ab^4e^5)x^4 + 20(b^5d^2e^3 - 5ab^4de^4 + 10a^2b^3e^5)x^3 - \dots}{e^6 \ln(e^5 x + d)}$$

input `integrate((b^2*x^2+2*a*b*x+a^2)^(5/2)/(e*x+d),x, algorithm="fricas")`

output
$$\frac{1}{60} \left(12b^5e^5x^5 - 15(b^5d^4e^4 - 5a^4b^4e^5)x^4 + 20(b^5d^2e^3 - 5a^4b^4d^2e^4 + 10a^2b^3e^5)x^3 - 30(b^5d^3e^2 - 5a^4b^4d^2e^3 + 10a^2b^3d^2e^4 - 10a^3b^2e^5)x^2 + 60(b^5d^4e - 5a^4b^4d^3e^2 + 10a^2b^3d^2e^3 - 10a^3b^2d^2e^4 + 5a^4b^4e^5)x - 60(b^5d^5 - 5a^4b^4d^4e + 10a^2b^3d^3e^2 - 10a^3b^2d^2e^3 + 5a^4b^4d^2e^4 - a^5e^5) \right) \ln(e^5 x + d) / e^6$$

Sympy [F]

$$\int \frac{(a^2 + 2abx + b^2x^2)^{5/2}}{d + ex} dx = \int \frac{((a + bx)^2)^{5/2}}{d + ex} dx$$

input `integrate((b**2*x**2+2*a*b*x+a**2)**(5/2)/(e*x+d),x)`

output `Integral(((a + b*x)**2)**(5/2)/(d + e*x), x)`

Maxima [F(-2)]

Exception generated.

$$\int \frac{(a^2 + 2abx + b^2x^2)^{5/2}}{d + ex} dx = \text{Exception raised: ValueError}$$

input `integrate((b^2*x^2+2*a*b*x+a^2)^(5/2)/(e*x+d),x, algorithm="maxima")`

output `Exception raised: ValueError >> Computation failed since Maxima requested additional constraints; using the 'assume' command before evaluation *may* help (example of legal syntax is 'assume(a*e-b*d>0)', see `assume?` for more detail`

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 399 vs. 2(180) = 360.

Time = 0.16 (sec) , antiderivative size = 399, normalized size of antiderivative = 1.57

$$\int \frac{(a^2 + 2abx + b^2x^2)^{5/2}}{d + ex} dx = \frac{12 b^5 e^4 x^5 \operatorname{sgn}(bx + a) - 15 b^5 d e^3 x^4 \operatorname{sgn}(bx + a) + 75 a b^4 e^4 x^4 \operatorname{sgn}(bx + a) + 20 b^5 d^5 \operatorname{sgn}(bx + a) - 5 a b^4 d^4 e \operatorname{sgn}(bx + a) + 10 a^2 b^3 d^3 e^2 \operatorname{sgn}(bx + a) - 10 a^3 b^2 d^2 e^3 \operatorname{sgn}(bx + a) + 5 a^4 b d e^4 \operatorname{sgn}(bx + a)}{e^6}$$

input `integrate((b^2*x^2+2*a*b*x+a^2)^(5/2)/(e*x+d),x, algorithm="giac")`

output
$$\frac{1}{60} \cdot (12 \cdot b^5 \cdot e^4 \cdot x^5 \cdot \operatorname{sgn}(b \cdot x + a) - 15 \cdot b^5 \cdot d \cdot e^3 \cdot x^4 \cdot \operatorname{sgn}(b \cdot x + a) + 75 \cdot a \cdot b^4 \cdot e^4 \cdot x^4 \cdot \operatorname{sgn}(b \cdot x + a) + 20 \cdot b^5 \cdot d^2 \cdot e^2 \cdot x^3 \cdot \operatorname{sgn}(b \cdot x + a) - 100 \cdot a \cdot b^4 \cdot d \cdot e^3 \cdot x^3 \cdot \operatorname{sgn}(b \cdot x + a) + 200 \cdot a^2 \cdot b^3 \cdot e^4 \cdot x^3 \cdot \operatorname{sgn}(b \cdot x + a) - 30 \cdot b^5 \cdot d^3 \cdot e \cdot x^2 \cdot \operatorname{sgn}(b \cdot x + a) + 150 \cdot a \cdot b^4 \cdot d^2 \cdot e^2 \cdot x^2 \cdot \operatorname{sgn}(b \cdot x + a) - 300 \cdot a^2 \cdot b^3 \cdot d \cdot e^3 \cdot x^2 \cdot \operatorname{sgn}(b \cdot x + a) + 300 \cdot a^3 \cdot b^2 \cdot e^4 \cdot x^2 \cdot \operatorname{sgn}(b \cdot x + a) + 60 \cdot b^5 \cdot d^4 \cdot x \cdot \operatorname{sgn}(b \cdot x + a) - 300 \cdot a \cdot b^4 \cdot d^3 \cdot e \cdot x \cdot \operatorname{sgn}(b \cdot x + a) + 600 \cdot a^2 \cdot b^3 \cdot d^2 \cdot e^2 \cdot x \cdot \operatorname{sgn}(b \cdot x + a) - 600 \cdot a^3 \cdot b^2 \cdot d \cdot e^3 \cdot x \cdot \operatorname{sgn}(b \cdot x + a) + 300 \cdot a^4 \cdot b \cdot e^4 \cdot x \cdot \operatorname{sgn}(b \cdot x + a)) / e^5 - (b^5 \cdot d^5 \cdot \operatorname{sgn}(b \cdot x + a) - 5 \cdot a \cdot b^4 \cdot d^4 \cdot e \cdot \operatorname{sgn}(b \cdot x + a) + 10 \cdot a^2 \cdot b^3 \cdot d^3 \cdot e^2 \cdot \operatorname{sgn}(b \cdot x + a) - 10 \cdot a^3 \cdot b^2 \cdot d^2 \cdot e^3 \cdot \operatorname{sgn}(b \cdot x + a) + 5 \cdot a^4 \cdot b \cdot d \cdot e^4 \cdot \operatorname{sgn}(b \cdot x + a) - a^5 \cdot e^5 \cdot \operatorname{sgn}(b \cdot x + a)) \cdot \log(\operatorname{abs}(e \cdot x + d)) / e^6$$

Mupad [F(-1)]

Timed out.

$$\int \frac{(a^2 + 2abx + b^2x^2)^{5/2}}{d + ex} dx = \int \frac{(a^2 + 2abx + b^2x^2)^{5/2}}{d + ex} dx$$

input `int((a^2 + b^2*x^2 + 2*a*b*x)^(5/2)/(d + e*x),x)`

output `int((a^2 + b^2*x^2 + 2*a*b*x)^(5/2)/(d + e*x), x)`

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 301, normalized size of antiderivative = 1.19

$$\int \frac{(a^2 + 2abx + b^2x^2)^{5/2}}{d + ex} dx = \frac{60 \log(ex + d) a^5 e^5 - 300 \log(ex + d) a^4 b d e^4 + 600 \log(ex + d) a^3 b^2 d^2 e^3 - 600 \log(ex + d) a^2 b^3 d^3 e^2 + 300 \log(ex + d) a b^4 d^4 e - 60 \log(ex + d) b^5 d^5}{e^6}$$

input `int((b^2*x^2+2*a*b*x+a^2)^(5/2)/(e*x+d),x)`

output

```
(60*log(d + e*x)*a**5*e**5 - 300*log(d + e*x)*a**4*b*d*e**4 + 600*log(d +
e*x)*a**3*b**2*d**2*e**3 - 600*log(d + e*x)*a**2*b**3*d**3*e**2 + 300*log(
d + e*x)*a*b**4*d**4*e - 60*log(d + e*x)*b**5*d**5 + 300*a**4*b*e**5*x - 6
00*a**3*b**2*d*e**4*x + 300*a**3*b**2*e**5*x**2 + 600*a**2*b**3*d**2*e**3*
x - 300*a**2*b**3*d*e**4*x**2 + 200*a**2*b**3*e**5*x**3 - 300*a*b**4*d**3*
e**2*x + 150*a*b**4*d**2*e**3*x**2 - 100*a*b**4*d*e**4*x**3 + 75*a*b**4*e*
*5*x**4 + 60*b**5*d**4*e*x - 30*b**5*d**3*e**2*x**2 + 20*b**5*d**2*e**3*x*
*3 - 15*b**5*d*e**4*x**4 + 12*b**5*e**5*x**5)/(60*e**6)
```

3.298 $\int \frac{(a^2+2abx+b^2x^2)^{5/2}}{(d+ex)^2} dx$

| | |
|---|------|
| Optimal result | 2247 |
| Mathematica [A] (verified) | 2248 |
| Rubi [A] (verified) | 2248 |
| Maple [A] (verified) | 2250 |
| Fricas [A] (verification not implemented) | 2250 |
| Sympy [F] | 2251 |
| Maxima [F(-2)] | 2251 |
| Giac [A] (verification not implemented) | 2251 |
| Mupad [F(-1)] | 2252 |
| Reduce [B] (verification not implemented) | 2252 |

Optimal result

Integrand size = 28, antiderivative size = 292

$$\int \frac{(a^2 + 2abx + b^2x^2)^{5/2}}{(d + ex)^2} dx = -\frac{10b^2(bd - ae)^3x\sqrt{a^2 + 2abx + b^2x^2}}{e^5(a + bx)} + \frac{(bd - ae)^5\sqrt{a^2 + 2abx + b^2x^2}}{e^6(a + bx)(d + ex)} + \frac{5b^3(bd - ae)^2(d + ex)^2\sqrt{a^2 + 2abx + b^2x^2}}{e^6(a + bx)} - \frac{5b^4(bd - ae)(d + ex)^3\sqrt{a^2 + 2abx + b^2x^2}}{3e^6(a + bx)} + \frac{b^5(d + ex)^4\sqrt{a^2 + 2abx + b^2x^2}}{4e^6(a + bx)} + \frac{5b(bd - ae)^4\sqrt{a^2 + 2abx + b^2x^2} \log(d + ex)}{e^6(a + bx)}$$

output

```
-10*b^2*(-a*e+b*d)^3*x*((b*x+a)^2)^(1/2)/e^5/(b*x+a)+(-a*e+b*d)^5*((b*x+a)^2)^(1/2)/e^6/(b*x+a)/(e*x+d)+5*b^3*(-a*e+b*d)^2*(e*x+d)^2*((b*x+a)^2)^(1/2)/e^6/(b*x+a)-5/3*b^4*(-a*e+b*d)*(e*x+d)^3*((b*x+a)^2)^(1/2)/e^6/(b*x+a)+1/4*b^5*(e*x+d)^4*((b*x+a)^2)^(1/2)/e^6/(b*x+a)+5*b*(-a*e+b*d)^4*((b*x+a)^2)^(1/2)*ln(e*x+d)/e^6/(b*x+a)
```

Mathematica [A] (verified)

Time = 1.16 (sec) , antiderivative size = 246, normalized size of antiderivative = 0.84

$$\int \frac{(a^2 + 2abx + b^2x^2)^{5/2}}{(d + ex)^2} dx = \frac{\sqrt{(a + bx)^2}(60a^4bde^4 - 12a^5e^5 + 120a^3b^2e^3(-d^2 + dex + e^2x^2) + 60a^2b^3e^2)}$$

input `Integrate[(a^2 + 2*a*b*x + b^2*x^2)^(5/2)/(d + e*x)^2,x]`

output $(\text{Sqrt}[(a + b*x)^2]*(60*a^4*b*d*e^4 - 12*a^5*e^5 + 120*a^3*b^2*e^3*(-d^2 + d*e*x + e^2*x^2) + 60*a^2*b^3*e^2*(2*d^3 - 4*d^2*e*x - 3*d*e^2*x^2 + e^3*x^3) + 20*a*b^4*e*(-3*d^4 + 9*d^3*e*x + 6*d^2*e^2*x^2 - 2*d*e^3*x^3 + e^4*x^4) + b^5*(12*d^5 - 48*d^4*e*x - 30*d^3*e^2*x^2 + 10*d^2*e^3*x^3 - 5*d*e^4*x^4 + 3*e^5*x^5) + 60*b*(b*d - a*e)^4*(d + e*x)*\text{Log}[d + e*x]))/(12*e^6*(a + b*x)*(d + e*x))$

Rubi [A] (verified)

Time = 0.64 (sec) , antiderivative size = 158, normalized size of antiderivative = 0.54, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {1102, 27, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\begin{aligned} & \int \frac{(a^2 + 2abx + b^2x^2)^{5/2}}{(d + ex)^2} dx \\ & \quad \downarrow 1102 \\ & \frac{\sqrt{a^2 + 2abx + b^2x^2} \int \frac{b^5(a+bx)^5}{(d+ex)^2} dx}{b^5(a + bx)} \\ & \quad \downarrow 27 \\ & \frac{\sqrt{a^2 + 2abx + b^2x^2} \int \frac{(a+bx)^5}{(d+ex)^2} dx}{a + bx} \\ & \quad \downarrow 49 \end{aligned}$$

$$\frac{\sqrt{a^2 + 2abx + b^2x^2} \int \left(\frac{(d+ex)^3 b^5}{e^5} - \frac{5(bd-ae)(d+ex)^2 b^4}{e^5} + \frac{10(bd-ae)^2(d+ex)b^3}{e^5} - \frac{10(bd-ae)^3 b^2}{e^5} + \frac{5(bd-ae)^4 b}{e^5(d+ex)} + \frac{(ae-bd)^5}{e^5(d+ex)^2} \right) dx}{a + bx}$$

↓ 2009

$$\frac{\sqrt{a^2 + 2abx + b^2x^2} \left(-\frac{5b^4(d+ex)^3(bd-ae)}{3e^6} + \frac{5b^3(d+ex)^2(bd-ae)^2}{e^6} - \frac{10b^2x(bd-ae)^3}{e^5} + \frac{(bd-ae)^5}{e^6(d+ex)} + \frac{5b(bd-ae)^4 \log(d+ex)}{e^6} + \frac{b^5(d+ex)^4}{e^6} \right)}{a + bx}$$

input

```
Int[(a^2 + 2*a*b*x + b^2*x^2)^(5/2)/(d + e*x)^2,x]
```

output

```
(Sqrt[a^2 + 2*a*b*x + b^2*x^2]*((-10*b^2*(b*d - a*e)^3*x)/e^5 + (b*d - a*e)^5/(e^6*(d + e*x)) + (5*b^3*(b*d - a*e)^2*(d + e*x)^2)/e^6 - (5*b^4*(b*d - a*e)*(d + e*x)^3)/(3*e^6) + (b^5*(d + e*x)^4)/(4*e^6) + (5*b*(b*d - a*e)^4*Log[d + e*x])/e^6))/(a + b*x)
```

Defintions of rubi rules used

rule 27

```
Int[(a_)*(F_x_), x_Symbol] := Simp[a Int[F_x, x], x] /; FreeQ[a, x] && !MatchQ[F_x, (b_)*(G_x_) /; FreeQ[b, x]]
```

rule 49

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] := Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] && IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 1102

```
Int[((d_.) + (e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] := Simp[(a + b*x + c*x^2)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x)^(2*FracPart[p])) Int[(d + e*x)^m*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && EqQ[b^2 - 4*a*c, 0]
```

rule 2009

```
Int[u_, x_Symbol] := Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 1.28 (sec) , antiderivative size = 307, normalized size of antiderivative = 1.05

| method | result |
|---------|---|
| risch | $\frac{\sqrt{(bx+a)^2 b^2 \left(\frac{1}{4}b^3x^4e^3 + \frac{5}{3}ab^2e^3x^3 - \frac{2}{3}b^3de^2x^3 + 5a^2be^3x^2 - 5ab^2de^2x^2 + \frac{3}{2}b^3d^2e^2x^2 + 10e^3a^3x - 20a^2bde^2x + 15ab^2d^2ex - 4b^3d^3x\right)}}{(bx+a)e^5}$ |
| default | $\left((bx+a)^2\right)^{\frac{5}{2}} \left(120a^3b^2de^4x + 60\ln(ex+d)a^4bde^4 + 60\ln(ex+d)a^4be^5x + 12b^5d^5 + 60\ln(ex+d)b^5d^4ex - 240\ln(ex+d)a^3b^2d^2e^3 + 360\right)$ |

input `int((b^2*x^2+2*a*b*x+a^2)^(5/2)/(e*x+d)^2,x,method=_RETURNVERBOSE)`

output
$$\begin{aligned} & \left(\frac{(bx+a)^2}{(bx+a)}\right)^{\frac{1}{2}} \frac{1}{(bx+a)} \frac{b^2}{e^5} \left(\frac{1}{4}b^3x^4e^3 + \frac{5}{3}ab^2e^3x^3 - \frac{2}{3}b^3de^2x^3 + 5a^2be^3x^2 - 5ab^2de^2x^2 + \frac{3}{2}b^3d^2e^2x^2 + 10e^3a^3x - 20a^2bde^2x + 15ab^2d^2ex - 4b^3d^3x \right) \\ & + 5 \left(\frac{(bx+a)^2}{(bx+a)} \right)^{\frac{1}{2}} \frac{1}{(bx+a)} \frac{b}{e^6} \left(a^4e^4 - 4a^3bde^3 + 6a^2b^2d^2e^2 - 4a^3b^3d^3e + b^4d^4 \right) \\ & * \ln(ex+d) - \left(\frac{(bx+a)^2}{(bx+a)} \right)^{\frac{1}{2}} \frac{1}{(bx+a)} \frac{1}{e^6} \left(a^5e^5 - 5a^4bde^4 + 10a^3b^2d^2e^3 - 10a^2b^3d^3e^2 + 5a^4b^4d^4e - b^5d^5 \right) / (e*x+d) \end{aligned}$$

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 373, normalized size of antiderivative = 1.28

$$\int \frac{(a^2 + 2abx + b^2x^2)^{5/2}}{(d + ex)^2} dx = \frac{3b^5e^5x^5 + 12b^5d^5 - 60ab^4d^4e + 120a^2b^3d^3e^2 - 120a^3b^2d^2e^3 + 60a^4bde^4 - 120a^5e^5}{(d + ex)^2}$$

input `integrate((b^2*x^2+2*a*b*x+a^2)^(5/2)/(e*x+d)^2,x, algorithm="fricas")`

output
$$\begin{aligned} & \frac{1}{12} \left(3b^5e^5x^5 + 12b^5d^5 - 60a^4b^4d^4e + 120a^2b^3d^3e^2 - 120a^3b^2d^2e^3 + 60a^4bde^4 - 120a^5e^5 - 5(b^5d^4e^4 - 4a^4b^4e^5) \right) x^4 \\ & + 10 \left(b^5d^2e^3 - 4a^4b^4d^4e + 6a^2b^3d^3e^5 \right) x^3 - 30 \left(b^5d^3e^2 - 4a^4b^4d^2e^3 + 6a^2b^3d^3e^4 - 4a^3b^2e^5 \right) x^2 - 12 \left(4b^5d^4e - 15a^4b^4d^3e^2 + 20a^2b^3d^2e^3 - 10a^3b^2d^4e \right) x \\ & + 60 \left(b^5d^5 - 4a^4b^4d^4e + 6a^2b^3d^3e^2 - 4a^3b^2d^2e^3 + a^4bde^4 + (b^5d^4e - 4a^4b^4d^3e^2 + 6a^2b^3d^2e^3 - 4a^3b^2d^4e) \right) x \\ & * \log(ex + d) / (e^7x + d^6e) \end{aligned}$$

input `integrate((b^2*x^2+2*a*b*x+a^2)^(5/2)/(e*x+d)^2,x, algorithm="giac")`

output `5*(b^5*d^4*sgn(b*x + a) - 4*a*b^4*d^3*e*sgn(b*x + a) + 6*a^2*b^3*d^2*e^2*sgn(b*x + a) - 4*a^3*b^2*d*e^3*sgn(b*x + a) + a^4*b*e^4*sgn(b*x + a))*log(abs(e*x + d))/e^6 + (b^5*d^5*sgn(b*x + a) - 5*a*b^4*d^4*e*sgn(b*x + a) + 10*a^2*b^3*d^3*e^2*sgn(b*x + a) - 10*a^3*b^2*d^2*e^3*sgn(b*x + a) + 5*a^4*b*d*e^4*sgn(b*x + a) - a^5*e^5*sgn(b*x + a))/((e*x + d)*e^6) + 1/12*(3*b^5*e^6*x^4*sgn(b*x + a) - 8*b^5*d*e^5*x^3*sgn(b*x + a) + 20*a*b^4*e^6*x^3*sgn(b*x + a) + 18*b^5*d^2*e^4*x^2*sgn(b*x + a) - 60*a*b^4*d*e^5*x^2*sgn(b*x + a) + 60*a^2*b^3*e^6*x^2*sgn(b*x + a) - 48*b^5*d^3*e^3*x*sgn(b*x + a) + 180*a*b^4*d^2*e^4*x*sgn(b*x + a) - 240*a^2*b^3*d*e^5*x*sgn(b*x + a) + 120*a^3*b^2*e^6*x*sgn(b*x + a))/e^8`

Mupad [F(-1)]

Timed out.

$$\int \frac{(a^2 + 2abx + b^2x^2)^{5/2}}{(d + ex)^2} dx = \int \frac{(a^2 + 2abx + b^2x^2)^{5/2}}{(d + ex)^2} dx$$

input `int((a^2 + b^2*x^2 + 2*a*b*x)^(5/2)/(d + e*x)^2,x)`

output `int((a^2 + b^2*x^2 + 2*a*b*x)^(5/2)/(d + e*x)^2, x)`

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 415, normalized size of antiderivative = 1.42

$$\int \frac{(a^2 + 2abx + b^2x^2)^{5/2}}{(d + ex)^2} dx = \frac{60 \log(ex + d) a^4 b d^2 e^4 - 240 \log(ex + d) a^3 b^2 d^3 e^3 + 360 \log(ex + d) a^2 b^3 d^4 e^2}{(d + ex)^2}$$

input `int((b^2*x^2+2*a*b*x+a^2)^(5/2)/(e*x+d)^2,x)`

output

```
(60*log(d + e*x)*a**4*b*d**2*e**4 + 60*log(d + e*x)*a**4*b*d*e**5*x - 240*
log(d + e*x)*a**3*b**2*d**3*e**3 - 240*log(d + e*x)*a**3*b**2*d**2*e**4*x
+ 360*log(d + e*x)*a**2*b**3*d**4*e**2 + 360*log(d + e*x)*a**2*b**3*d**3*e
**3*x - 240*log(d + e*x)*a*b**4*d**5*e - 240*log(d + e*x)*a*b**4*d**4*e**2
*x + 60*log(d + e*x)*b**5*d**6 + 60*log(d + e*x)*b**5*d**5*e*x + 12*a**5*e
**6*x - 60*a**4*b*d*e**5*x + 240*a**3*b**2*d**2*e**4*x + 120*a**3*b**2*d*e
**5*x**2 - 360*a**2*b**3*d**3*e**3*x - 180*a**2*b**3*d**2*e**4*x**2 + 60*a
**2*b**3*d*e**5*x**3 + 240*a*b**4*d**4*e**2*x + 120*a*b**4*d**3*e**3*x**2
- 40*a*b**4*d**2*e**4*x**3 + 20*a*b**4*d*e**5*x**4 - 60*b**5*d**5*e*x - 30
*b**5*d**4*e**2*x**2 + 10*b**5*d**3*e**3*x**3 - 5*b**5*d**2*e**4*x**4 + 3*
b**5*d*e**5*x**5)/(12*d*e**6*(d + e*x))
```

3.299
$$\int \frac{(a^2+2abx+b^2x^2)^{5/2}}{(d+ex)^3} dx$$

| | |
|---|------|
| Optimal result | 2254 |
| Mathematica [A] (verified) | 2255 |
| Rubi [A] (verified) | 2255 |
| Maple [A] (verified) | 2257 |
| Fricas [A] (verification not implemented) | 2257 |
| Sympy [F] | 2258 |
| Maxima [F(-2)] | 2258 |
| Giac [A] (verification not implemented) | 2259 |
| Mupad [F(-1)] | 2259 |
| Reduce [B] (verification not implemented) | 2260 |

Optimal result

Integrand size = 28, antiderivative size = 295

$$\int \frac{(a^2 + 2abx + b^2x^2)^{5/2}}{(d + ex)^3} dx = \frac{10b^3(bd - ae)^2x\sqrt{a^2 + 2abx + b^2x^2}}{e^5(a + bx)} + \frac{(bd - ae)^5\sqrt{a^2 + 2abx + b^2x^2}}{2e^6(a + bx)(d + ex)^2} - \frac{5b(bd - ae)^4\sqrt{a^2 + 2abx + b^2x^2}}{e^6(a + bx)(d + ex)} - \frac{5b^4(bd - ae)(d + ex)^2\sqrt{a^2 + 2abx + b^2x^2}}{2e^6(a + bx)} + \frac{b^5(d + ex)^3\sqrt{a^2 + 2abx + b^2x^2}}{3e^6(a + bx)} - \frac{10b^2(bd - ae)^3\sqrt{a^2 + 2abx + b^2x^2}\log(d + ex)}{e^6(a + bx)}$$

output

```
10*b^3*(-a*e+b*d)^2*x*((b*x+a)^2)^(1/2)/e^5/(b*x+a)+1/2*(-a*e+b*d)^5*((b*x+a)^2)^(1/2)/e^6/(b*x+a)/(e*x+d)^2-5*b*(-a*e+b*d)^4*((b*x+a)^2)^(1/2)/e^6/(b*x+a)/(e*x+d)-5/2*b^4*(-a*e+b*d)*(e*x+d)^2*((b*x+a)^2)^(1/2)/e^6/(b*x+a)+1/3*b^5*(e*x+d)^3*((b*x+a)^2)^(1/2)/e^6/(b*x+a)-10*b^2*(-a*e+b*d)^3*((b*x+a)^2)^(1/2)*ln(e*x+d)/e^6/(b*x+a)
```

Mathematica [A] (verified)

Time = 1.08 (sec) , antiderivative size = 248, normalized size of antiderivative = 0.84

$$\int \frac{(a^2 + 2abx + b^2x^2)^{5/2}}{(d + ex)^3} dx = \frac{\sqrt{(a + bx)^2}(-3a^5e^5 - 15a^4be^4(d + 2ex) + 30a^3b^2de^3(3d + 4ex) + 30a^2b^3e^2e^2)}{(d + ex)^3}$$

input `Integrate[(a^2 + 2*a*b*x + b^2*x^2)^(5/2)/(d + e*x)^3,x]`

output `(Sqrt[(a + b*x)^2]*(-3*a^5*e^5 - 15*a^4*b*e^4*(d + 2*e*x) + 30*a^3*b^2*d*e^3*(3*d + 4*e*x) + 30*a^2*b^3*e^2*(-5*d^3 - 4*d^2*e*x + 4*d*e^2*x^2 + 2*e^3*x^3) + 15*a*b^4*e*(7*d^4 + 2*d^3*e*x - 11*d^2*e^2*x^2 - 4*d*e^3*x^3 + e^4*x^4) + b^5*(-27*d^5 + 6*d^4*e*x + 63*d^3*e^2*x^2 + 20*d^2*e^3*x^3 - 5*d*e^4*x^4 + 2*e^5*x^5) - 60*b^2*(b*d - a*e)^3*(d + e*x)^2*Log[d + e*x]))/(6*e^6*(a + b*x)*(d + e*x)^2)`

Rubi [A] (verified)

Time = 0.60 (sec) , antiderivative size = 161, normalized size of antiderivative = 0.55, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {1102, 27, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\begin{aligned} & \int \frac{(a^2 + 2abx + b^2x^2)^{5/2}}{(d + ex)^3} dx \\ & \quad \downarrow 1102 \\ & \frac{\sqrt{a^2 + 2abx + b^2x^2} \int \frac{b^5(a+bx)^5}{(d+ex)^3} dx}{b^5(a + bx)} \\ & \quad \downarrow 27 \\ & \frac{\sqrt{a^2 + 2abx + b^2x^2} \int \frac{(a+bx)^5}{(d+ex)^3} dx}{a + bx} \\ & \quad \downarrow 49 \end{aligned}$$

$$\frac{\sqrt{a^2 + 2abx + b^2x^2} \int \left(\frac{(d+ex)^2 b^5}{e^5} - \frac{5(bd-ae)(d+ex)b^4}{e^5} + \frac{10(bd-ae)^2 b^3}{e^5} - \frac{10(bd-ae)^3 b^2}{e^5(d+ex)} + \frac{5(bd-ae)^4 b}{e^5(d+ex)^2} + \frac{(ae-bd)^5}{e^5(d+ex)^3} \right) dx}{a + bx}$$

↓ 2009

$$\frac{\sqrt{a^2 + 2abx + b^2x^2} \left(-\frac{5b^4(d+ex)^2(bd-ae)}{2e^6} + \frac{10b^3x(bd-ae)^2}{e^5} - \frac{10b^2(bd-ae)^3 \log(d+ex)}{e^6} - \frac{5b(bd-ae)^4}{e^6(d+ex)} + \frac{(bd-ae)^5}{2e^6(d+ex)^2} + \frac{b^5(d+ex)}{3e^6} \right)}{a + bx}$$

input `Int[(a^2 + 2*a*b*x + b^2*x^2)^(5/2)/(d + e*x)^3,x]`

output `(Sqrt[a^2 + 2*a*b*x + b^2*x^2]*((10*b^3*(b*d - a*e)^2*x)/e^5 + (b*d - a*e)^5/(2*e^6*(d + e*x)^2) - (5*b*(b*d - a*e)^4)/(e^6*(d + e*x)) - (5*b^4*(b*d - a*e)*(d + e*x)^2)/(2*e^6) + (b^5*(d + e*x)^3)/(3*e^6) - (10*b^2*(b*d - a*e)^3*Log[d + e*x])/e^6))/(a + b*x)`

Defintions of rubi rules used

rule 27 `Int[(a_)*(F_x_), x_Symbol] := Simp[a Int[F_x, x], x] /; FreeQ[a, x] && !MatchQ[F_x, (b_)*(G_x_) /; FreeQ[b, x]]`

rule 49 `Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] := Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] && IGtQ[m, 0] && IGtQ[m + n + 2, 0]`

rule 1102 `Int[((d_.) + (e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] := Simp[(a + b*x + c*x^2)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x)^(2*FracPart[p])) Int[(d + e*x)^m*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && EqQ[b^2 - 4*a*c, 0]`

rule 2009 `Int[u_, x_Symbol] := Simp[IntSum[u, x], x] /; SumQ[u]`

Maple [A] (verified)

Time = 1.46 (sec) , antiderivative size = 301, normalized size of antiderivative = 1.02

| method | result |
|---------|---|
| risch | $\frac{\sqrt{(bx+a)^2} b^3 \left(\frac{1}{3} b^2 e^2 x^3 + \frac{5}{2} x^2 ab e^2 - \frac{3}{2} b^2 de x^2 + 10a^2 e^2 x - 15abdex + 6b^2 d^2 x \right)}{(bx+a)e^5} + \frac{\sqrt{(bx+a)^2} \left((-5a^4 b e^4 + 20a^3 b^2 d e^3 - 30a^2 b^3 d^2 e^2 + \dots \right)}{(bx+a)e^5}$ |
| default | $\frac{\left((bx+a)^2 \right)^{\frac{5}{2}} \left(60 \ln(ex+d) a^3 b^2 e^5 x^2 - 60 \ln(ex+d) b^5 d^3 e^2 x^2 + 180 \ln(ex+d) a b^4 d^2 e^3 x^2 + 120 a^3 b^2 d e^4 x - 27 b^5 d^5 - 120 \ln(ex+d) b^5 d^4 ex \right)}{(bx+a)^5}$ |

input `int((b^2*x^2+2*a*b*x+a^2)^(5/2)/(e*x+d)^3,x,method=_RETURNVERBOSE)`

output
$$\begin{aligned} & \left((bx+a)^2 \right)^{\frac{1}{2}} / (bx+a) * b^3 / e^5 * \left(\frac{1}{3} b^2 e^2 x^3 + \frac{5}{2} x^2 a b e^2 - \frac{3}{2} b^2 d e x^2 + 10 a^2 e^2 x - 15 a b d e x + 6 b^2 d^2 x \right) \\ & + \left((bx+a)^2 \right)^{\frac{1}{2}} / (bx+a) * \left((-5 a^4 b e^4 + 20 a^3 b^2 d e^3 - 30 a^2 b^3 d^2 e^2 + 20 a^3 b^4 d^3 e - 5 b^5 d^4) * x - \frac{1}{2} * (a^5 e^5 + 5 a^4 b d e^4 - 30 a^3 b^2 d^2 e^3 + 50 a^2 b^3 d^3 e^2 - 35 a b^4 d^4 e + 9 b^5 d^5) / e \right) / e^5 / (e*x+d)^2 + 10 * \left((bx+a)^2 \right)^{\frac{1}{2}} / (bx+a) * b^2 / e^6 * (a^3 e^3 - 3 a^2 b d e^2 + 3 a b^2 d^2 e - b^3 d^3) * \ln(e*x+d) \end{aligned}$$

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 416, normalized size of antiderivative = 1.41

$$\int \frac{(a^2 + 2abx + b^2x^2)^{5/2}}{(d + ex)^3} dx = \frac{2b^5e^5x^5 - 27b^5d^5 + 105ab^4d^4e - 150a^2b^3d^3e^2 + 90a^3b^2d^2e^3 - 15a^4bde^4 - \dots}{(d + ex)^3}$$

input `integrate((b^2*x^2+2*a*b*x+a^2)^(5/2)/(e*x+d)^3,x, algorithm="fricas")`

output

```
1/6*(2*b^5*e^5*x^5 - 27*b^5*d^5 + 105*a*b^4*d^4*e - 150*a^2*b^3*d^3*e^2 +
90*a^3*b^2*d^2*e^3 - 15*a^4*b*d*e^4 - 3*a^5*e^5 - 5*(b^5*d*e^4 - 3*a*b^4*e
^5)*x^4 + 20*(b^5*d^2*e^3 - 3*a*b^4*d*e^4 + 3*a^2*b^3*e^5)*x^3 + 3*(21*b^5
*d^3*e^2 - 55*a*b^4*d^2*e^3 + 40*a^2*b^3*d*e^4)*x^2 + 6*(b^5*d^4*e + 5*a*b
^4*d^3*e^2 - 20*a^2*b^3*d^2*e^3 + 20*a^3*b^2*d*e^4 - 5*a^4*b*e^5)*x - 60*(
b^5*d^5 - 3*a*b^4*d^4*e + 3*a^2*b^3*d^3*e^2 - a^3*b^2*d^2*e^3 + (b^5*d^3*e
^2 - 3*a*b^4*d^2*e^3 + 3*a^2*b^3*d*e^4 - a^3*b^2*e^5)*x^2 + 2*(b^5*d^4*e -
3*a*b^4*d^3*e^2 + 3*a^2*b^3*d^2*e^3 - a^3*b^2*d*e^4)*x)*log(e*x + d)/(e^
8*x^2 + 2*d*e^7*x + d^2*e^6)
```

Sympy [F]

$$\int \frac{(a^2 + 2abx + b^2x^2)^{5/2}}{(d + ex)^3} dx = \int \frac{((a + bx)^2)^{5/2}}{(d + ex)^3} dx$$

input

```
integrate((b**2*x**2+2*a*b*x+a**2)**(5/2)/(e*x+d)**3,x)
```

output

```
Integral(((a + b*x)**2)**(5/2)/(d + e*x)**3, x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(a^2 + 2abx + b^2x^2)^{5/2}}{(d + ex)^3} dx = \text{Exception raised: ValueError}$$

input

```
integrate((b^2*x^2+2*a*b*x+a^2)^(5/2)/(e*x+d)^3,x, algorithm="maxima")
```

output

```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
help (example of legal syntax is 'assume(a*e-b*d>0)', see `assume?` for m
ore detail
```

Giac [A] (verification not implemented)

Time = 0.42 (sec) , antiderivative size = 390, normalized size of antiderivative = 1.32

$$\int \frac{(a^2 + 2abx + b^2x^2)^{5/2}}{(d + ex)^3} dx =$$

$$\frac{10(b^5d^3\operatorname{sgn}(bx + a) - 3ab^4d^2e\operatorname{sgn}(bx + a) + 3a^2b^3de^2\operatorname{sgn}(bx + a) - a^3b^2e^3\operatorname{sgn}(bx + a))\log(|ex + d|)}{e^6}$$

$$- \frac{9b^5d^5\operatorname{sgn}(bx + a) - 35ab^4d^4e\operatorname{sgn}(bx + a) + 50a^2b^3d^3e^2\operatorname{sgn}(bx + a) - 30a^3b^2d^2e^3\operatorname{sgn}(bx + a) + 5a^4bde^4}{6e^9}$$

$$+ \frac{2b^5e^6x^3\operatorname{sgn}(bx + a) - 9b^5de^5x^2\operatorname{sgn}(bx + a) + 15ab^4e^6x^2\operatorname{sgn}(bx + a) + 36b^5d^2e^4x\operatorname{sgn}(bx + a) - 90ab^4de^5}{6e^9}$$

input `integrate((b^2*x^2+2*a*b*x+a^2)^(5/2)/(e*x+d)^3,x, algorithm="giac")`

output `-10*(b^5*d^3*sgn(b*x + a) - 3*a*b^4*d^2*e*sgn(b*x + a) + 3*a^2*b^3*d*e^2*sgn(b*x + a) - a^3*b^2*e^3*sgn(b*x + a))*log(abs(e*x + d))/e^6 - 1/2*(9*b^5*d^5*sgn(b*x + a) - 35*a*b^4*d^4*e*sgn(b*x + a) + 50*a^2*b^3*d^3*e^2*sgn(b*x + a) - 30*a^3*b^2*d^2*e^3*sgn(b*x + a) + 5*a^4*b*d*e^4*sgn(b*x + a) + a^5*e^5*sgn(b*x + a) + 10*(b^5*d^4*e*sgn(b*x + a) - 4*a*b^4*d^3*e^2*sgn(b*x + a) + 6*a^2*b^3*d^2*e^3*sgn(b*x + a) - 4*a^3*b^2*d*e^4*sgn(b*x + a) + a^4*b*e^5*sgn(b*x + a))*x)/((e*x + d)^2*e^6) + 1/6*(2*b^5*e^6*x^3*sgn(b*x + a) - 9*b^5*d*e^5*x^2*sgn(b*x + a) + 15*a*b^4*e^6*x^2*sgn(b*x + a) + 36*b^5*d^2*e^4*x*sgn(b*x + a) - 90*a*b^4*d*e^5*x*sgn(b*x + a) + 60*a^2*b^3*e^6*x*sgn(b*x + a))/e^9`

Mupad [F(-1)]

Timed out.

$$\int \frac{(a^2 + 2abx + b^2x^2)^{5/2}}{(d + ex)^3} dx = \int \frac{(a^2 + 2abx + b^2x^2)^{5/2}}{(d + ex)^3} dx$$

input `int((a^2 + b^2*x^2 + 2*a*b*x)^(5/2)/(d + e*x)^3,x)`

output `int((a^2 + b^2*x^2 + 2*a*b*x)^(5/2)/(d + e*x)^3, x)`

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 470, normalized size of antiderivative = 1.59

$$\int \frac{(a^2 + 2abx + b^2x^2)^{5/2}}{(d + ex)^3} dx = \frac{-30b^5d^6 - 3a^5de^5 - 60 \log(ex + d) b^5d^4e^2x^2 + 60 \log(ex + d) a^3b^2d^3e^3 - 180 \log(ex + d) a^2b^3d^2e^4x + 60 \log(ex + d) a^3b^2d^3e^3 - 180 \log(ex + d) a^2b^3d^2e^4x^2 - 360 \log(ex + d) a^2b^3d^2e^4x^2 + 180 \log(ex + d) a^2b^3d^2e^4x^2 + 180 \log(ex + d) a^2b^3d^2e^4x^2 - 60 \log(ex + d) a^2b^3d^2e^4x^2 - 120 \log(ex + d) a^2b^3d^2e^4x^2 - 60 \log(ex + d) a^2b^3d^2e^4x^2 - 3a^5d^5e^5 + 15a^4b^4e^6x^2 + 30a^3b^3d^3e^3 - 60a^3b^3d^3e^3x^2 - 90a^2b^2d^2e^2 + 180a^2b^2d^2e^2x^2 + 60a^2b^2d^2e^2x^2 + 90a^2b^2d^2e^2x^2 - 180a^2b^2d^2e^2x^2 - 60a^2b^2d^2e^2x^2 - 60a^2b^2d^2e^2x^2 + 15a^2b^2d^2e^2x^2 + 15a^2b^2d^2e^2x^2 - 30b^5d^5e^5 + 60b^5d^5e^5x^2 + 20b^5d^5e^5x^2 - 5b^5d^5e^5x^2 + 2b^5d^5e^5x^2)/(6d^6e^6(d^2 + 2dex + e^2x^2))$$

input `int((b^2*x^2+2*a*b*x+a^2)^(5/2)/(e*x+d)^3,x)`output

```
(60*log(d + e*x)*a**3*b**2*d**3*e**3 + 120*log(d + e*x)*a**3*b**2*d**2*e**4*x + 60*log(d + e*x)*a**3*b**2*d*e**5*x**2 - 180*log(d + e*x)*a**2*b**3*d**4*e**2 - 360*log(d + e*x)*a**2*b**3*d**3*e**3*x - 180*log(d + e*x)*a**2*b**3*d**2*e**4*x**2 + 180*log(d + e*x)*a*b**4*d**5*e + 360*log(d + e*x)*a*b**4*d**4*e**2*x + 180*log(d + e*x)*a*b**4*d**3*e**3*x**2 - 60*log(d + e*x)*b**5*d**6 - 120*log(d + e*x)*b**5*d**5*e*x - 60*log(d + e*x)*b**5*d**4*e**2*x**2 - 3*a**5*d*e**5 + 15*a**4*b*e**6*x**2 + 30*a**3*b**2*d**3*e**3 - 60*a**3*b**2*d*e**5*x**2 - 90*a**2*b**3*d**4*e**2 + 180*a**2*b**3*d**2*e**4*x**2 + 60*a**2*b**3*d*e**5*x**3 + 90*a*b**4*d**5*e - 180*a*b**4*d**3*e**3*x**2 - 60*a*b**4*d**2*e**4*x**3 + 15*a*b**4*d*e**5*x**4 - 30*b**5*d**6 + 60*b**5*d**4*e**2*x**2 + 20*b**5*d**3*e**3*x**3 - 5*b**5*d**2*e**4*x**4 + 2*b**5*d*e**5*x**5)/(6*d*e**6*(d**2 + 2*d*e*x + e**2*x**2))
```

3.300 $\int \frac{(a^2+2abx+b^2x^2)^{5/2}}{(d+ex)^4} dx$

| | |
|---|------|
| Optimal result | 2261 |
| Mathematica [A] (verified) | 2262 |
| Rubi [A] (verified) | 2262 |
| Maple [A] (verified) | 2264 |
| Fricas [A] (verification not implemented) | 2264 |
| Sympy [F] | 2265 |
| Maxima [F(-2)] | 2265 |
| Giac [A] (verification not implemented) | 2266 |
| Mupad [F(-1)] | 2266 |
| Reduce [B] (verification not implemented) | 2267 |

Optimal result

Integrand size = 28, antiderivative size = 292

$$\int \frac{(a^2 + 2abx + b^2x^2)^{5/2}}{(d + ex)^4} dx = -\frac{b^4(4bd - 5ae)x\sqrt{a^2 + 2abx + b^2x^2}}{e^5(a + bx)} + \frac{b^5x^2\sqrt{a^2 + 2abx + b^2x^2}}{2e^4(a + bx)} + \frac{(bd - ae)^5\sqrt{a^2 + 2abx + b^2x^2}}{3e^6(a + bx)(d + ex)^3} - \frac{5b(bd - ae)^4\sqrt{a^2 + 2abx + b^2x^2}}{2e^6(a + bx)(d + ex)^2} + \frac{10b^2(bd - ae)^3\sqrt{a^2 + 2abx + b^2x^2}}{e^6(a + bx)(d + ex)} + \frac{10b^3(bd - ae)^2\sqrt{a^2 + 2abx + b^2x^2} \log(d + ex)}{e^6(a + bx)}$$

output

```
-b^4*(-5*a*e+4*b*d)*x*((b*x+a)^2)^(1/2)/e^5/(b*x+a)+1/2*b^5*x^2*((b*x+a)^2)^(1/2)/e^4/(b*x+a)+1/3*(-a*e+b*d)^5*((b*x+a)^2)^(1/2)/e^6/(b*x+a)/(e*x+d)^3-5/2*b*(-a*e+b*d)^4*((b*x+a)^2)^(1/2)/e^6/(b*x+a)/(e*x+d)^2+10*b^2*(-a*e+b*d)^3*((b*x+a)^2)^(1/2)/e^6/(b*x+a)/(e*x+d)+10*b^3*(-a*e+b*d)^2*((b*x+a)^2)^(1/2)*ln(e*x+d)/e^6/(b*x+a)
```

Mathematica [A] (verified)

Time = 1.08 (sec) , antiderivative size = 247, normalized size of antiderivative = 0.85

$$\int \frac{(a^2 + 2abx + b^2x^2)^{5/2}}{(d + ex)^4} dx = \frac{\sqrt{(a + bx)^2}(-2a^5e^5 - 5a^4be^4(d + 3ex) - 20a^3b^2e^3(d^2 + 3dex + 3e^2x^2) + 10a^2b^3d^2e^2(11d^2 + 27d^2ex + 18e^2x^2) + 10ab^4e(-13d^4 - 27d^3ex - 9d^2e^2x^2 + 9de^3x^3 + 3e^4x^4) + b^5(47d^5 + 81d^4ex - 9d^3e^2x^2 - 63d^2e^3x^3 - 15de^4x^4 + 3e^5x^5) + 60b^3(bd - ae)^2(d + ex)^3 \text{Log}[d + ex])}{6e^6(a + bx)(d + ex)^3}$$

input

```
Integrate[(a^2 + 2*a*b*x + b^2*x^2)^(5/2)/(d + e*x)^4,x]
```

output

```
(Sqrt[(a + b*x)^2]*(-2*a^5*e^5 - 5*a^4*b*e^4*(d + 3*e*x) - 20*a^3*b^2*e^3*(d^2 + 3*d*e*x + 3*e^2*x^2) + 10*a^2*b^3*d*e^2*(11*d^2 + 27*d*e*x + 18*e^2*x^2) + 10*a*b^4*e*(-13*d^4 - 27*d^3*e*x - 9*d^2*e^2*x^2 + 9*d*e^3*x^3 + 3*e^4*x^4) + b^5*(47*d^5 + 81*d^4*e*x - 9*d^3*e^2*x^2 - 63*d^2*e^3*x^3 - 15*d*e^4*x^4 + 3*e^5*x^5) + 60*b^3*(b*d - a*e)^2*(d + e*x)^3*Log[d + e*x]))/(6*e^6*(a + b*x)*(d + e*x)^3)
```

Rubi [A] (verified)

Time = 0.59 (sec) , antiderivative size = 158, normalized size of antiderivative = 0.54, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {1102, 27, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\begin{aligned} & \int \frac{(a^2 + 2abx + b^2x^2)^{5/2}}{(d + ex)^4} dx \\ & \quad \downarrow 1102 \\ & \frac{\sqrt{a^2 + 2abx + b^2x^2} \int \frac{b^5(a+bx)^5}{(d+ex)^4} dx}{b^5(a + bx)} \\ & \quad \downarrow 27 \\ & \frac{\sqrt{a^2 + 2abx + b^2x^2} \int \frac{(a+bx)^5}{(d+ex)^4} dx}{a + bx} \\ & \quad \downarrow 49 \end{aligned}$$

$$\frac{\sqrt{a^2 + 2abx + b^2x^2} \int \left(\frac{xb^5}{e^4} - \frac{(4bd-5ae)b^4}{e^5} + \frac{10(bd-ae)^2b^3}{e^5(d+ex)} - \frac{10(bd-ae)^3b^2}{e^5(d+ex)^2} + \frac{5(bd-ae)^4b}{e^5(d+ex)^3} + \frac{(ae-bd)^5}{e^5(d+ex)^4} \right) dx}{a + bx}$$

↓ 2009

$$\frac{\sqrt{a^2 + 2abx + b^2x^2} \left(-\frac{b^4x(4bd-5ae)}{e^5} + \frac{10b^3(bd-ae)^2 \log(d+ex)}{e^6} + \frac{10b^2(bd-ae)^3}{e^6(d+ex)} - \frac{5b(bd-ae)^4}{2e^6(d+ex)^2} + \frac{(bd-ae)^5}{3e^6(d+ex)^3} + \frac{b^5x^2}{2e^4} \right)}{a + bx}$$

input

```
Int[(a^2 + 2*a*b*x + b^2*x^2)^(5/2)/(d + e*x)^4,x]
```

output

```
(Sqrt[a^2 + 2*a*b*x + b^2*x^2]*(-(b^4*(4*b*d - 5*a*e)*x)/e^5) + (b^5*x^2)
/(2*e^4) + (b*d - a*e)^5/(3*e^6*(d + e*x)^3) - (5*b*(b*d - a*e)^4)/(2*e^6*
(d + e*x)^2) + (10*b^2*(b*d - a*e)^3)/(e^6*(d + e*x)) + (10*b^3*(b*d - a*e
)^2*Log[d + e*x])/e^6))/(a + b*x)
```

Defintions of rubi rules used

rule 27

```
Int[(a_)*(F_x_), x_Symbol] := Simp[a Int[F_x, x], x] /; FreeQ[a, x] && !Ma
tchQ[F_x, (b_)*(G_x_) /; FreeQ[b, x]]
```

rule 49

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] := Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 1102

```
Int[((d_.) + (e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] := Simp[(a + b*x + c*x^2)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x)^(2*F
racPart[p])) Int[(d + e*x)^m*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c,
d, e, m, p}, x] && EqQ[b^2 - 4*a*c, 0]
```

rule 2009

```
Int[u_, x_Symbol] := Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 1.67 (sec) , antiderivative size = 295, normalized size of antiderivative = 1.01

| method | result |
|---------|---|
| risch | $\frac{\sqrt{(bx+a)^2} b^4 \left(\frac{1}{2} b e x^2 + 5 a e x - 4 b d x\right)}{(bx+a)e^5} + \frac{\sqrt{(bx+a)^2} \left(-10 a^3 b^2 e^4 + 30 a^2 b^3 d e^3 - 30 a b^4 d^2 e^2 + 10 b^5 d^3 e\right) x^2 - \frac{5 b \left(a^4 e^4 + 4 a^3 b d e^3 - 18 a^2 b^2 d^2 e^2 + 10 a b^3 d^3 e - 5 b^4 d^4\right)}{(bx+a)e^5}}{(bx+a)e^5}$ |
| default | $\frac{\left((bx+a)^2\right)^{\frac{5}{2}} \left(180 \ln(ex+d) b^5 d^3 e^2 x^2 + 60 \ln(ex+d) b^5 d^2 e^3 x^3 - 360 \ln(ex+d) a b^4 d^2 e^3 x^2 - 60 a^3 b^2 d e^4 x + 47 b^5 d^5 + 180 \ln(ex+d) b^5 d^4 e x\right)}{(bx+a)^5}$ |

input `int((b^2*x^2+2*a*b*x+a^2)^(5/2)/(e*x+d)^4,x,method=_RETURNVERBOSE)`

output
$$\begin{aligned} & \left(\frac{(b*x+a)^2}{(b*x+a)}\right)^{1/2} / (b*x+a) * b^4 / e^5 * (1/2 * b * e * x^2 + 5 * a * e * x - 4 * b * d * x) + \left(\frac{(b*x+a)^2}{(b*x+a)}\right)^{1/2} / (b*x+a) * \left(\frac{-10 * a^3 * b^2 * e^4 + 30 * a^2 * b^3 * d * e^3 - 30 * a * b^4 * d^2 * e^2 + 10 * b^5 * d^3 * e}{e^5} * x^2 - \frac{5 * b * (a^4 * e^4 + 4 * a^3 * b * d * e^3 - 18 * a^2 * b^2 * d^2 * e^2 + 20 * a * b^3 * d^3 * e - 7 * b^4 * d^4)}{e^5} * x - \frac{1}{6} * \frac{(2 * a^5 * e^5 + 5 * a^4 * b * d * e^4 + 20 * a^3 * b^2 * d^2 * e^3 - 110 * a^2 * b^3 * d^3 * e^2 + 130 * a * b^4 * d^4 * e - 47 * b^5 * d^5)}{e^5} / (e*x+d)^3 + 10 * \left(\frac{(b*x+a)^2}{(b*x+a)}\right)^{1/2} / (b*x+a) * b^3 / e^6 * (a^2 * e^2 - 2 * a * b * d * e + b^2 * d^2) * \ln(e*x+d) \right) \end{aligned}$$

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 425, normalized size of antiderivative = 1.46

$$\int \frac{(a^2 + 2abx + b^2x^2)^{5/2}}{(d + ex)^4} dx = \frac{3b^5e^5x^5 + 47b^5d^5 - 130ab^4d^4e + 110a^2b^3d^3e^2 - 20a^3b^2d^2e^3 - 5a^4bde^4 - 20a^5d^5}{(d + ex)^4}$$

input `integrate((b^2*x^2+2*a*b*x+a^2)^(5/2)/(e*x+d)^4,x, algorithm="fricas")`

output

```
1/6*(3*b^5*e^5*x^5 + 47*b^5*d^5 - 130*a*b^4*d^4*e + 110*a^2*b^3*d^3*e^2 -
20*a^3*b^2*d^2*e^3 - 5*a^4*b*d*e^4 - 2*a^5*e^5 - 15*(b^5*d*e^4 - 2*a*b^4*e
^5)*x^4 - 9*(7*b^5*d^2*e^3 - 10*a*b^4*d*e^4)*x^3 - 3*(3*b^5*d^3*e^2 + 30*a
*b^4*d^2*e^3 - 60*a^2*b^3*d*e^4 + 20*a^3*b^2*e^5)*x^2 + 3*(27*b^5*d^4*e -
90*a*b^4*d^3*e^2 + 90*a^2*b^3*d^2*e^3 - 20*a^3*b^2*d*e^4 - 5*a^4*b*e^5)*x
+ 60*(b^5*d^5 - 2*a*b^4*d^4*e + a^2*b^3*d^3*e^2 + (b^5*d^2*e^3 - 2*a*b^4*d
*e^4 + a^2*b^3*e^5)*x^3 + 3*(b^5*d^3*e^2 - 2*a*b^4*d^2*e^3 + a^2*b^3*d*e^4
)*x^2 + 3*(b^5*d^4*e - 2*a*b^4*d^3*e^2 + a^2*b^3*d^2*e^3)*x)*log(e*x + d))
/(e^9*x^3 + 3*d*e^8*x^2 + 3*d^2*e^7*x + d^3*e^6)
```

Sympy [F]

$$\int \frac{(a^2 + 2abx + b^2x^2)^{5/2}}{(d + ex)^4} dx = \int \frac{((a + bx)^2)^{5/2}}{(d + ex)^4} dx$$

input

```
integrate((b**2*x**2+2*a*b*x+a**2)**(5/2)/(e*x+d)**4,x)
```

output

```
Integral(((a + b*x)**2)**(5/2)/(d + e*x)**4, x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(a^2 + 2abx + b^2x^2)^{5/2}}{(d + ex)^4} dx = \text{Exception raised: ValueError}$$

input

```
integrate((b^2*x^2+2*a*b*x+a^2)^(5/2)/(e*x+d)^4,x, algorithm="maxima")
```

output

```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
help (example of legal syntax is 'assume(a*e-b*d>0)', see `assume?` for m
ore detail
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 389, normalized size of antiderivative = 1.33

$$\int \frac{(a^2 + 2abx + b^2x^2)^{5/2}}{(d + ex)^4} dx = \frac{10(b^5d^2\operatorname{sgn}(bx + a) - 2ab^4d\operatorname{sgn}(bx + a) + a^2b^3e^2\operatorname{sgn}(bx + a)) \log(|ex + d|)}{e^6} + \frac{b^5e^4x^2\operatorname{sgn}(bx + a) - 8b^5de^3x\operatorname{sgn}(bx + a) + 10ab^4e^4x\operatorname{sgn}(bx + a)}{2e^8} + \frac{47b^5d^5\operatorname{sgn}(bx + a) - 130ab^4d^4\operatorname{sgn}(bx + a) + 110a^2b^3d^3e^2\operatorname{sgn}(bx + a) - 20a^3b^2d^2e^3\operatorname{sgn}(bx + a) - 5a^4b}{e^6}$$

input `integrate((b^2*x^2+2*a*b*x+a^2)^(5/2)/(e*x+d)^4,x, algorithm="giac")`

output `10*(b^5*d^2*sgn(b*x + a) - 2*a*b^4*d*e*sgn(b*x + a) + a^2*b^3*e^2*sgn(b*x + a))*log(abs(e*x + d))/e^6 + 1/2*(b^5*e^4*x^2*sgn(b*x + a) - 8*b^5*d*e^3*x*sgn(b*x + a) + 10*a*b^4*e^4*x*sgn(b*x + a))/e^8 + 1/6*(47*b^5*d^5*sgn(b*x + a) - 130*a*b^4*d^4*e*sgn(b*x + a) + 110*a^2*b^3*d^3*e^2*sgn(b*x + a) - 20*a^3*b^2*d^2*e^3*sgn(b*x + a) - 5*a^4*b*d*e^4*sgn(b*x + a) - 2*a^5*e^5*sgn(b*x + a) + 60*(b^5*d^3*e^2*sgn(b*x + a) - 3*a*b^4*d^2*e^3*sgn(b*x + a) + 3*a^2*b^3*d*e^4*sgn(b*x + a) - a^3*b^2*e^5*sgn(b*x + a))*x^2 + 15*(7*b^5*d^4*e*sgn(b*x + a) - 20*a*b^4*d^3*e^2*sgn(b*x + a) + 18*a^2*b^3*d^2*e^3*sgn(b*x + a) - 4*a^3*b^2*d*e^4*sgn(b*x + a) - a^4*b*e^5*sgn(b*x + a))*x)/(e*x + d)^3*e^6)`

Mupad [F(-1)]

Timed out.

$$\int \frac{(a^2 + 2abx + b^2x^2)^{5/2}}{(d + ex)^4} dx = \int \frac{(a^2 + 2abx + b^2x^2)^{5/2}}{(d + ex)^4} dx$$

input `int((a^2 + b^2*x^2 + 2*a*b*x)^(5/2)/(d + e*x)^4,x)`

output `int((a^2 + b^2*x^2 + 2*a*b*x)^(5/2)/(d + e*x)^4, x)`

Reduce [B] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 469, normalized size of antiderivative = 1.61

$$\int \frac{(a^2 + 2abx + b^2x^2)^{5/2}}{(d + ex)^4} dx = \frac{60 \log(ex + d) a^2 b^3 d e^5 x^3 - 120 \log(ex + d) a b^4 d^2 e^4 x^3 + 50 b^5 d^6 - 2 a^5 d e^5 + \dots}{(d + ex)^4}$$

input `int((b^2*x^2+2*a*b*x+a^2)^(5/2)/(e*x+d)^4,x)`

output `(60*log(d + e*x)*a**2*b**3*d**4*e**2 + 180*log(d + e*x)*a**2*b**3*d**3*e**3*x + 180*log(d + e*x)*a**2*b**3*d**2*e**4*x**2 + 60*log(d + e*x)*a**2*b**3*d**e**5*x**3 - 120*log(d + e*x)*a*b**4*d**5*e - 360*log(d + e*x)*a*b**4*d**4*e**2*x - 360*log(d + e*x)*a*b**4*d**3*e**3*x**2 - 120*log(d + e*x)*a*b**4*d**2*e**4*x**3 + 60*log(d + e*x)*b**5*d**6 + 180*log(d + e*x)*b**5*d**5*e*x + 180*log(d + e*x)*b**5*d**4*e**2*x**2 + 60*log(d + e*x)*b**5*d**3*e**3*x**3 - 2*a**5*d*e**5 - 5*a**4*b*d**2*e**4 - 15*a**4*b*d*e**5*x + 20*a**3*b**2*e**6*x**3 + 50*a**2*b**3*d**4*e**2 + 90*a**2*b**3*d**3*e**3*x - 60*a**2*b**3*d*e**5*x**3 - 100*a*b**4*d**5*e - 180*a*b**4*d**4*e**2*x + 120*a*b**4*d**2*e**4*x**3 + 30*a*b**4*d*e**5*x**4 + 50*b**5*d**6 + 90*b**5*d**5*e*x - 60*b**5*d**3*e**3*x**3 - 15*b**5*d**2*e**4*x**4 + 3*b**5*d*e**5*x**5)/(6*d**6*(d**3 + 3*d**2*e*x + 3*d*e**2*x**2 + e**3*x**3))`

3.301
$$\int \frac{(a^2+2abx+b^2x^2)^{5/2}}{(d+ex)^5} dx$$

| | |
|---|------|
| Optimal result | 2268 |
| Mathematica [A] (verified) | 2269 |
| Rubi [A] (verified) | 2269 |
| Maple [A] (verified) | 2271 |
| Fricas [A] (verification not implemented) | 2271 |
| Sympy [F] | 2272 |
| Maxima [F(-2)] | 2272 |
| Giac [A] (verification not implemented) | 2273 |
| Mupad [F(-1)] | 2273 |
| Reduce [B] (verification not implemented) | 2274 |

Optimal result

Integrand size = 28, antiderivative size = 292

$$\begin{aligned} \int \frac{(a^2 + 2abx + b^2x^2)^{5/2}}{(d + ex)^5} dx &= \frac{b^5x\sqrt{a^2 + 2abx + b^2x^2}}{e^5(a + bx)} \\ &+ \frac{(bd - ae)^5\sqrt{a^2 + 2abx + b^2x^2}}{4e^6(a + bx)(d + ex)^4} - \frac{5b(bd - ae)^4\sqrt{a^2 + 2abx + b^2x^2}}{3e^6(a + bx)(d + ex)^3} \\ &+ \frac{5b^2(bd - ae)^3\sqrt{a^2 + 2abx + b^2x^2}}{e^6(a + bx)(d + ex)^2} - \frac{10b^3(bd - ae)^2\sqrt{a^2 + 2abx + b^2x^2}}{e^6(a + bx)(d + ex)} \\ &- \frac{5b^4(bd - ae)\sqrt{a^2 + 2abx + b^2x^2}\log(d + ex)}{e^6(a + bx)} \end{aligned}$$

output

```
b^5*x*((b*x+a)^2)^(1/2)/e^5/(b*x+a)+1/4*(-a*e+b*d)^5*((b*x+a)^2)^(1/2)/e^6/(b*x+a)/(e*x+d)^4-5/3*b*(-a*e+b*d)^4*((b*x+a)^2)^(1/2)/e^6/(b*x+a)/(e*x+d)^3+5*b^2*(-a*e+b*d)^3*((b*x+a)^2)^(1/2)/e^6/(b*x+a)/(e*x+d)^2-10*b^3*(-a*e+b*d)^2*((b*x+a)^2)^(1/2)/e^6/(b*x+a)/(e*x+d)-5*b^4*(-a*e+b*d)*((b*x+a)^2)^(1/2)*ln(e*x+d)/e^6/(b*x+a)
```

Mathematica [A] (verified)

Time = 1.12 (sec) , antiderivative size = 243, normalized size of antiderivative = 0.83

$$\int \frac{(a^2 + 2abx + b^2x^2)^{5/2}}{(d + ex)^5} dx = \frac{\sqrt{(a + bx)^2}(3a^5e^5 + 5a^4be^4(d + 4ex) + 10a^3b^2e^3(d^2 + 4dex + 6e^2x^2) + 30a^2b^3e^2(d^3 + 4d^2ex + 6de^2x^2 +$$

input

```
Integrate[(a^2 + 2*a*b*x + b^2*x^2)^(5/2)/(d + e*x)^5,x]
```

output

```
-1/12*(Sqrt[(a + b*x)^2]*(3*a^5*e^5 + 5*a^4*b*e^4*(d + 4*e*x) + 10*a^3*b^2
*e^3*(d^2 + 4*d*e*x + 6*e^2*x^2) + 30*a^2*b^3*e^2*(d^3 + 4*d^2*e*x + 6*d*e
^2*x^2 + 4*e^3*x^3) - 5*a*b^4*d*e*(25*d^3 + 88*d^2*e*x + 108*d*e^2*x^2 + 4
8*e^3*x^3) + b^5*(77*d^5 + 248*d^4*e*x + 252*d^3*e^2*x^2 + 48*d^2*e^3*x^3
- 48*d*e^4*x^4 - 12*e^5*x^5) + 60*b^4*(b*d - a*e)*(d + e*x)^4*Log[d + e*x]
))/(e^6*(a + b*x)*(d + e*x)^4)
```

Rubi [A] (verified)

Time = 0.59 (sec) , antiderivative size = 158, normalized size of antiderivative = 0.54, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {1102, 27, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a^2 + 2abx + b^2x^2)^{5/2}}{(d + ex)^5} dx$$

$$\downarrow 1102$$

$$\frac{\sqrt{a^2 + 2abx + b^2x^2} \int \frac{b^5(a+bx)^5}{(d+ex)^5} dx}{b^5(a + bx)}$$

$$\downarrow 27$$

$$\frac{\sqrt{a^2 + 2abx + b^2x^2} \int \frac{(a+bx)^5}{(d+ex)^5} dx}{a + bx}$$

↓ 49

$$\frac{\sqrt{a^2 + 2abx + b^2x^2} \int \left(\frac{b^5}{e^5} - \frac{5(bd-ae)b^4}{e^5(d+ex)} + \frac{10(bd-ae)^2b^3}{e^5(d+ex)^2} - \frac{10(bd-ae)^3b^2}{e^5(d+ex)^3} + \frac{5(bd-ae)^4b}{e^5(d+ex)^4} + \frac{(ae-bd)^5}{e^5(d+ex)^5} \right) dx}{a + bx}$$

↓ 2009

$$\frac{\sqrt{a^2 + 2abx + b^2x^2} \left(-\frac{5b^4(bd-ae)\log(d+ex)}{e^6} - \frac{10b^3(bd-ae)^2}{e^6(d+ex)} + \frac{5b^2(bd-ae)^3}{e^6(d+ex)^2} - \frac{5b(bd-ae)^4}{3e^6(d+ex)^3} + \frac{(bd-ae)^5}{4e^6(d+ex)^4} + \frac{b^5x}{e^5} \right)}{a + bx}$$

input

```
Int[(a^2 + 2*a*b*x + b^2*x^2)^(5/2)/(d + e*x)^5,x]
```

output

```
(Sqrt[a^2 + 2*a*b*x + b^2*x^2]*((b^5*x)/e^5 + (b*d - a*e)^5/(4*e^6*(d + e*x)^4) - (5*b*(b*d - a*e)^4)/(3*e^6*(d + e*x)^3) + (5*b^2*(b*d - a*e)^3)/(e^6*(d + e*x)^2) - (10*b^3*(b*d - a*e)^2)/(e^6*(d + e*x)) - (5*b^4*(b*d - a*e)*Log[d + e*x])/e^6))/(a + b*x)
```

Defintions of rubi rules used

rule 27

```
Int[(a_)*(Fx_), x_Symbol] := Simp[a Int[Fx, x], x] /; FreeQ[a, x] && !MatchQ[Fx, (b_)*(Gx_)] /; FreeQ[b, x]
```

rule 49

```
Int[((a_) + (b_)*(x_))^(m_)*((c_) + (d_)*(x_))^(n_), x_Symbol] := Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] && IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 1102

```
Int[((d_) + (e_)*(x_))^(m_)*((a_) + (b_)*(x_) + (c_)*(x_)^2)^(p_), x_Symbol] := Simp[(a + b*x + c*x^2)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x)^(2*FracPart[p])) Int[(d + e*x)^m*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && EqQ[b^2 - 4*a*c, 0]
```

rule 2009

```
Int[u_, x_Symbol] := Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 1.86 (sec) , antiderivative size = 298, normalized size of antiderivative = 1.02

| method | result |
|---------|--|
| risch | $\frac{b^5 x \sqrt{(bx+a)^2}}{e^5 (bx+a)} + \frac{\sqrt{(bx+a)^2} \left((-10a^2 b^3 e^4 + 20a b^4 d e^3 - 10b^5 d^2 e^2) x^3 - 5b^2 e (e^3 a^3 + 3a^2 b d e^2 - 9a b^2 d^2 e + 5b^3 d^3) x^2 - \frac{5b(a^4 e^4 + 2a^3 b d e^3 + 3a^2 b^2 d^2 e^2 - 3a b^3 d^3) x - 5b^4 d^4}{(bx+a)e^5 (ex+d)^5} \right)}{(bx+a)e^5 (ex+d)^5}$ |
| default | $\frac{\left((bx+a)^2 \right)^{\frac{5}{2}} (-360 \ln(ex+d) b^5 d^3 e^2 x^2 - 240 \ln(ex+d) b^5 d^2 e^3 x^3 + 360 \ln(ex+d) a b^4 d^2 e^3 x^2 - 40 a^3 b^2 d e^4 x - 77 b^5 d^5 - 240 \ln(ex+d) b^5 d^4 x^2)}{(bx+a)^{\frac{5}{2}} (ex+d)^5}$ |

input `int((b^2*x^2+2*a*b*x+a^2)^(5/2)/(e*x+d)^5,x,method=_RETURNVERBOSE)`

output `b^5*x*((b*x+a)^2)^(1/2)/e^5/(b*x+a)+((b*x+a)^2)^(1/2)/(b*x+a)*((-10*a^2*b^3*e^4+20*a*b^4*d*e^3-10*b^5*d^2*e^2)*x^3-5*b^2*e*(a^3*e^3+3*a^2*b*d*e^2-9*a*b^2*d^2*e+5*b^3*d^3)*x^2-5/3*b*(a^4*e^4+2*a^3*b*d*e^3+6*a^2*b^2*d^2*e^2-22*a*b^3*d^3*e+13*b^4*d^4)*x-1/12*(3*a^5*e^5+5*a^4*b*d*e^4+10*a^3*b^2*d^2*e^3+30*a^2*b^3*d^3*e^2-125*a*b^4*d^4*e+77*b^5*d^5)/e)/e^5/(e*x+d)^4+5*((b*x+a)^2)^(1/2)/(b*x+a)*b^4/e^6*(a*e-b*d)*ln(e*x+d)`

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 412, normalized size of antiderivative = 1.41

$$\int \frac{(a^2 + 2abx + b^2x^2)^{5/2}}{(d + ex)^5} dx = \frac{12b^5e^5x^5 + 48b^5de^4x^4 - 77b^5d^5 + 125ab^4d^4e - 30a^2b^3d^3e^2 - 10a^3b^2d^2e^3 - 5a^4bd^2e^4 + 5a^5d^2e^5}{(d + ex)^5}$$

input `integrate((b^2*x^2+2*a*b*x+a^2)^(5/2)/(e*x+d)^5,x, algorithm="fricas")`

output

```
1/12*(12*b^5*e^5*x^5 + 48*b^5*d*e^4*x^4 - 77*b^5*d^5 + 125*a*b^4*d^4*e - 3
0*a^2*b^3*d^3*e^2 - 10*a^3*b^2*d^2*e^3 - 5*a^4*b*d*e^4 - 3*a^5*e^5 - 24*(2
*b^5*d^2*e^3 - 10*a*b^4*d*e^4 + 5*a^2*b^3*e^5)*x^3 - 12*(21*b^5*d^3*e^2 -
45*a*b^4*d^2*e^3 + 15*a^2*b^3*d*e^4 + 5*a^3*b^2*e^5)*x^2 - 4*(62*b^5*d^4*e
- 110*a*b^4*d^3*e^2 + 30*a^2*b^3*d^2*e^3 + 10*a^3*b^2*d*e^4 + 5*a^4*b*e^5
)*x - 60*(b^5*d^5 - a*b^4*d^4*e + (b^5*d*e^4 - a*b^4*e^5)*x^4 + 4*(b^5*d^2
*e^3 - a*b^4*d*e^4)*x^3 + 6*(b^5*d^3*e^2 - a*b^4*d^2*e^3)*x^2 + 4*(b^5*d^4
*e - a*b^4*d^3*e^2)*x)*log(e*x + d)/(e^10*x^4 + 4*d*e^9*x^3 + 6*d^2*e^8*x
^2 + 4*d^3*e^7*x + d^4*e^6)
```

Sympy [F]

$$\int \frac{(a^2 + 2abx + b^2x^2)^{5/2}}{(d + ex)^5} dx = \int \frac{((a + bx)^2)^{5/2}}{(d + ex)^5} dx$$

input

```
integrate((b**2*x**2+2*a*b*x+a**2)**(5/2)/(e*x+d)**5,x)
```

output

```
Integral(((a + b*x)**2)**(5/2)/(d + e*x)**5, x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(a^2 + 2abx + b^2x^2)^{5/2}}{(d + ex)^5} dx = \text{Exception raised: ValueError}$$

input

```
integrate((b^2*x^2+2*a*b*x+a^2)^(5/2)/(e*x+d)^5,x, algorithm="maxima")
```

output

```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
help (example of legal syntax is 'assume(a*e-b*d>0)', see `assume?` for m
ore detail
```

Giac [A] (verification not implemented)

Time = 0.42 (sec) , antiderivative size = 383, normalized size of antiderivative = 1.31

$$\int \frac{(a^2 + 2abx + b^2x^2)^{5/2}}{(d + ex)^5} dx = \frac{b^5 x \operatorname{sgn}(bx + a)}{e^5} - \frac{5(b^5 d \operatorname{sgn}(bx + a) - ab^4 e \operatorname{sgn}(bx + a)) \log(|ex + d|)}{e^6} - \frac{77b^5 d^5 \operatorname{sgn}(bx + a) - 125ab^4 d^4 e \operatorname{sgn}(bx + a) + 30a^2 b^3 d^3 e^2 \operatorname{sgn}(bx + a) + 10a^3 b^2 d^2 e^3 \operatorname{sgn}(bx + a) + 5a^4 b d e^4 \operatorname{sgn}(bx + a) - 5a^5 e^5 \operatorname{sgn}(bx + a)}{e^6}$$

input `integrate((b^2*x^2+2*a*b*x+a^2)^(5/2)/(e*x+d)^5,x, algorithm="giac")`

output `b^5*x*sgn(b*x + a)/e^5 - 5*(b^5*d*sgn(b*x + a) - a*b^4*e*sgn(b*x + a))*log(abs(e*x + d))/e^6 - 1/12*(77*b^5*d^5*sgn(b*x + a) - 125*a*b^4*d^4*e*sgn(b*x + a) + 30*a^2*b^3*d^3*e^2*sgn(b*x + a) + 10*a^3*b^2*d^2*e^3*sgn(b*x + a) + 5*a^4*b*d*e^4*sgn(b*x + a) + 3*a^5*e^5*sgn(b*x + a) + 120*(b^5*d^2*e^3*sgn(b*x + a) - 2*a*b^4*d*e^4*sgn(b*x + a) + a^2*b^3*e^5*sgn(b*x + a))*x^3 + 60*(5*b^5*d^3*e^2*sgn(b*x + a) - 9*a*b^4*d^2*e^3*sgn(b*x + a) + 3*a^2*b^3*d*e^4*sgn(b*x + a) + a^3*b^2*e^5*sgn(b*x + a))*x^2 + 20*(13*b^5*d^4*e*sgn(b*x + a) - 22*a*b^4*d^3*e^2*sgn(b*x + a) + 6*a^2*b^3*d^2*e^3*sgn(b*x + a) + 2*a^3*b^2*d*e^4*sgn(b*x + a) + a^4*b*e^5*sgn(b*x + a))*x)/((e*x + d)^4*e^6)`

Mupad [F(-1)]

Timed out.

$$\int \frac{(a^2 + 2abx + b^2x^2)^{5/2}}{(d + ex)^5} dx = \int \frac{(a^2 + 2abx + b^2x^2)^{5/2}}{(d + ex)^5} dx$$

input `int((a^2 + b^2*x^2 + 2*a*b*x)^(5/2)/(d + e*x)^5,x)`

output `int((a^2 + b^2*x^2 + 2*a*b*x)^(5/2)/(d + e*x)^5, x)`

Reduce [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 434, normalized size of antiderivative = 1.49

$$\int \frac{(a^2 + 2abx + b^2x^2)^{5/2}}{(d + ex)^5} dx = \frac{240 \log(ex + d) a b^4 d^2 e^4 x^3 - 60 \log(ex + d) b^5 d^2 e^4 x^4 - 65 b^5 d^6 + 30 a^2 b^3 e^6 x^4}{(d + ex)^5}$$

input `int((b^2*x^2+2*a*b*x+a^2)^(5/2)/(e*x+d)^5,x)`

output `(60*log(d + e*x)*a*b**4*d**5*e + 240*log(d + e*x)*a*b**4*d**4*e**2*x + 360*log(d + e*x)*a*b**4*d**3*e**3*x**2 + 240*log(d + e*x)*a*b**4*d**2*e**4*x**3 + 60*log(d + e*x)*a*b**4*d*e**5*x**4 - 60*log(d + e*x)*b**5*d**6 - 240*log(d + e*x)*b**5*d**5*e*x - 360*log(d + e*x)*b**5*d**4*e**2*x**2 - 240*log(d + e*x)*b**5*d**3*e**3*x**3 - 60*log(d + e*x)*b**5*d**2*e**4*x**4 - 3*a**5*d*e**5 - 5*a**4*b*d**2*e**4 - 20*a**4*b*d*e**5*x - 10*a**3*b**2*d**3*e**3 - 40*a**3*b**2*d**2*e**4*x - 60*a**3*b**2*d*e**5*x**2 + 30*a**2*b**3*e**6*x**4 + 65*a*b**4*d**5*e + 200*a*b**4*d**4*e**2*x + 180*a*b**4*d**3*e**3*x**2 - 60*a*b**4*d*e**5*x**4 - 65*b**5*d**6 - 200*b**5*d**5*e*x - 180*b**5*d**4*e**2*x**2 + 60*b**5*d**2*e**4*x**4 + 12*b**5*d*e**5*x**5)/(12*d*e**6*(d**4 + 4*d**3*e*x + 6*d**2*e**2*x**2 + 4*d*e**3*x**3 + e**4*x**4))`

3.302 $\int \frac{(a^2+2abx+b^2x^2)^{5/2}}{(d+ex)^6} dx$

| | |
|---|------|
| Optimal result | 2275 |
| Mathematica [A] (verified) | 2276 |
| Rubi [A] (verified) | 2276 |
| Maple [A] (verified) | 2278 |
| Fricas [A] (verification not implemented) | 2278 |
| Sympy [F] | 2279 |
| Maxima [F(-2)] | 2279 |
| Giac [A] (verification not implemented) | 2279 |
| Mupad [F(-1)] | 2280 |
| Reduce [B] (verification not implemented) | 2280 |

Optimal result

Integrand size = 28, antiderivative size = 300

$$\int \frac{(a^2 + 2abx + b^2x^2)^{5/2}}{(d + ex)^6} dx = \frac{(bd - ae)^5 \sqrt{a^2 + 2abx + b^2x^2}}{5e^6(a + bx)(d + ex)^5} - \frac{5b(bd - ae)^4 \sqrt{a^2 + 2abx + b^2x^2}}{4e^6(a + bx)(d + ex)^4} + \frac{10b^2(bd - ae)^3 \sqrt{a^2 + 2abx + b^2x^2}}{3e^6(a + bx)(d + ex)^3} - \frac{5b^3(bd - ae)^2 \sqrt{a^2 + 2abx + b^2x^2}}{e^6(a + bx)(d + ex)^2} + \frac{5b^4(bd - ae) \sqrt{a^2 + 2abx + b^2x^2}}{e^6(a + bx)(d + ex)} + \frac{b^5 \sqrt{a^2 + 2abx + b^2x^2} \log(d + ex)}{e^6(a + bx)}$$

output

```
1/5*(-a*e+b*d)^5*((b*x+a)^2)^(1/2)/e^6/(b*x+a)/(e*x+d)^5-5/4*b*(-a*e+b*d)^4*((b*x+a)^2)^(1/2)/e^6/(b*x+a)/(e*x+d)^4+10/3*b^2*(-a*e+b*d)^3*((b*x+a)^2)^(1/2)/e^6/(b*x+a)/(e*x+d)^3-5*b^3*(-a*e+b*d)^2*((b*x+a)^2)^(1/2)/e^6/(b*x+a)/(e*x+d)^2+5*b^4*(-a*e+b*d)*((b*x+a)^2)^(1/2)/e^6/(b*x+a)/(e*x+d)+b^5*((b*x+a)^2)^(1/2)*ln(e*x+d)/e^6/(b*x+a)
```


Mathematica [A] (verified)

Time = 1.09 (sec) , antiderivative size = 196, normalized size of antiderivative = 0.65

$$\int \frac{(a^2 + 2abx + b^2x^2)^{5/2}}{(d + ex)^6} dx = \frac{\sqrt{(a + bx)^2}((bd - ae)(12a^4e^4 + 3a^3be^3(9d + 25ex) + a^2b^2e^2(47d^2 + 175dex$$

input `Integrate[(a^2 + 2*a*b*x + b^2*x^2)^(5/2)/(d + e*x)^6,x]`

output `(Sqrt[(a + b*x)^2]*((b*d - a*e)*(12*a^4*e^4 + 3*a^3*b*e^3*(9*d + 25*e*x) + a^2*b^2*e^2*(47*d^2 + 175*d*e*x + 200*e^2*x^2) + a*b^3*e*(77*d^3 + 325*d^2*e*x + 500*d*e^2*x^2 + 300*e^3*x^3) + b^4*(137*d^4 + 625*d^3*e*x + 1100*d^2*e^2*x^2 + 900*d*e^3*x^3 + 300*e^4*x^4)) + 60*b^5*(d + e*x)^5*Log[d + e*x]))/(60*e^6*(a + b*x)*(d + e*x)^5)`

Rubi [A] (verified)

Time = 0.59 (sec) , antiderivative size = 166, normalized size of antiderivative = 0.55, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {1102, 27, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\begin{aligned} & \int \frac{(a^2 + 2abx + b^2x^2)^{5/2}}{(d + ex)^6} dx \\ & \quad \downarrow \text{1102} \\ & \frac{\sqrt{a^2 + 2abx + b^2x^2} \int \frac{b^5(a+bx)^5}{(d+ex)^6} dx}{b^5(a + bx)} \\ & \quad \downarrow \text{27} \\ & \frac{\sqrt{a^2 + 2abx + b^2x^2} \int \frac{(a+bx)^5}{(d+ex)^6} dx}{a + bx} \\ & \quad \downarrow \text{49} \end{aligned}$$

$$\frac{\sqrt{a^2 + 2abx + b^2x^2} \int \left(\frac{b^5}{e^5(d+ex)} - \frac{5(bd-ae)b^4}{e^5(d+ex)^2} + \frac{10(bd-ae)^2b^3}{e^5(d+ex)^3} - \frac{10(bd-ae)^3b^2}{e^5(d+ex)^4} + \frac{5(bd-ae)^4b}{e^5(d+ex)^5} + \frac{(ae-bd)^5}{e^5(d+ex)^6} \right) dx}{a + bx}$$

↓ 2009

$$\frac{\sqrt{a^2 + 2abx + b^2x^2} \left(\frac{5b^4(bd-ae)}{e^6(d+ex)} - \frac{5b^3(bd-ae)^2}{e^6(d+ex)^2} + \frac{10b^2(bd-ae)^3}{3e^6(d+ex)^3} - \frac{5b(bd-ae)^4}{4e^6(d+ex)^4} + \frac{(bd-ae)^5}{5e^6(d+ex)^5} + \frac{b^5 \log(d+ex)}{e^6} \right)}{a + bx}$$

input

```
Int[(a^2 + 2*a*b*x + b^2*x^2)^(5/2)/(d + e*x)^6,x]
```

output

```
(Sqrt[a^2 + 2*a*b*x + b^2*x^2]*((b*d - a*e)^5/(5*e^6*(d + e*x)^5) - (5*b*(b*d - a*e)^4)/(4*e^6*(d + e*x)^4) + (10*b^2*(b*d - a*e)^3)/(3*e^6*(d + e*x)^3) - (5*b^3*(b*d - a*e)^2)/(e^6*(d + e*x)^2) + (5*b^4*(b*d - a*e))/(e^6*(d + e*x)) + (b^5*Log[d + e*x])/e^6))/(a + b*x)
```

Defintions of rubi rules used

rule 27

```
Int[(a_)*(Fx_), x_Symbol] := Simp[a Int[Fx, x], x] /; FreeQ[a, x] && !MatchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 49

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] := Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] && IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 1102

```
Int[((d_.) + (e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] := Simp[(a + b*x + c*x^2)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x)^(2*FracPart[p])) Int[(d + e*x)^m*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && EqQ[b^2 - 4*a*c, 0]
```

rule 2009

```
Int[u_, x_Symbol] := Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 2.31 (sec) , antiderivative size = 284, normalized size of antiderivative = 0.95

| method | result |
|---------|---|
| risch | $\frac{\sqrt{(bx+a)^2} \left(-\frac{5b^4(ae-bd)x^4}{e^2} - \frac{5b^3(e^2a^2+2abde-3b^2d^2)x^3}{e^3} - \frac{5b^2(2e^3a^3+3a^2bde^2+6ab^2d^2e-11b^3d^3)x^2}{3e^4} - \frac{5b(3a^4e^4+4a^3bde^3+6a^2b^2d^2e^2+3ab^3d^3e-11b^4d^4)}{12e^5} \right)}{(bx+a)(ex+d)^5}$ |
| default | $\frac{((bx+a)^2)^{\frac{5}{2}} (600 \ln(ex+d)b^5d^3e^2x^2 + 600 \ln(ex+d)b^5d^2e^3x^3 + 60 \ln(ex+d)b^5e^5x^5 - 100a^3b^2de^4x + 137b^5d^5 + 300 \ln(ex+d)b^5d^4ex + \dots)}{(bx+a)(ex+d)^5}$ |

input `int((b^2*x^2+2*a*b*x+a^2)^(5/2)/(e*x+d)^6,x,method=_RETURNVERBOSE)`

output
$$\frac{((bx+a)^2)^{(1/2)} / (bx+a) * (-5b^4(ae-bd) / e^{2x^4} - 5b^3(a^2e^2+2a*b*d * e-3b^2*d^2) / e^{3x^3} - 5/3b^2(2a^3e^3+3a^2*b*d*e^2+6a*b^2*d^2e-11b^3*d^3) / e^{4x^2} - 5/12b(3a^4e^4+4a^3*b*d*e^3+6a^2*b^2*d^2e^2+12a*b^3*d^3e-25b^4*d^4) / e^{5x} - 1/60(12a^5e^5+15a^4*b*d*e^4+20a^3*b^2*d^2e^3+30a^2*b^3*d^3e^2+60a*b^4*d^4e-137b^5*d^5) / e^6}{(e*x+d)^5 + b^5 * ((b*x+a)^2)^{(1/2)} * \ln(e*x+d) / e^6} / (b*x+a)$$

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 373, normalized size of antiderivative = 1.24

$$\int \frac{(a^2 + 2abx + b^2x^2)^{5/2}}{(d + ex)^6} dx = \frac{137b^5d^5 - 60ab^4d^4e - 30a^2b^3d^3e^2 - 20a^3b^2d^2e^3 - 15a^4bde^4 - 12a^5e^5 + 300 \ln(ex+d)b^5d^5}{(d + ex)^6}$$

input `integrate((b^2*x^2+2*a*b*x+a^2)^(5/2)/(e*x+d)^6,x, algorithm="fricas")`

output
$$\frac{1/60(137b^5d^5 - 60a*b^4*d^4*e - 30a^2*b^3*d^3*e^2 - 20a^3*b^2*d^2*e^3 - 15a^4*b*d*e^4 - 12a^5*e^5 + 300*(b^5*d*e^4 - a*b^4*e^5)*x^4 + 300*(3*b^5*d^2*e^3 - 2a*b^4*d*e^4 - a^2*b^3*e^5)*x^3 + 100*(11*b^5*d^3*e^2 - 6a*b^4*d^2*e^3 - 3a^2*b^3*d*e^4 - 2a^3*b^2*e^5)*x^2 + 25*(25*b^5*d^4*e - 12a*b^4*d^3*e^2 - 6a^2*b^3*d^2*e^3 - 4a^3*b^2*d*e^4 - 3a^4*b*e^5)*x + 60*(b^5*e^5*x^5 + 5*b^5*d*e^4*x^4 + 10*b^5*d^2*e^3*x^3 + 10*b^5*d^3*e^2*x^2 + 5*b^5*d^4*e*x + b^5*d^5)*\log(e*x + d)}{(e^{11x^5} + 5*d*e^{10x^4} + 10*d^2*e^9*x^3 + 10*d^3*e^8*x^2 + 5*d^4*e^7*x + d^5*e^6)}$$

Sympy [F]

$$\int \frac{(a^2 + 2abx + b^2x^2)^{5/2}}{(d + ex)^6} dx = \int \frac{((a + bx)^2)^{5/2}}{(d + ex)^6} dx$$

input `integrate((b**2*x**2+2*a*b*x+a**2)**(5/2)/(e*x+d)**6,x)`

output `Integral(((a + b*x)**2)**(5/2)/(d + e*x)**6, x)`

Maxima [F(-2)]

Exception generated.

$$\int \frac{(a^2 + 2abx + b^2x^2)^{5/2}}{(d + ex)^6} dx = \text{Exception raised: ValueError}$$

input `integrate((b^2*x^2+2*a*b*x+a^2)^(5/2)/(e*x+d)^6,x, algorithm="maxima")`

output `Exception raised: ValueError >> Computation failed since Maxima requested additional constraints; using the 'assume' command before evaluation *may* help (example of legal syntax is 'assume(a*e-b*d>0)', see `assume?` for more detail`

Giac [A] (verification not implemented)

Time = 0.57 (sec) , antiderivative size = 391, normalized size of antiderivative = 1.30

$$\int \frac{(a^2 + 2abx + b^2x^2)^{5/2}}{(d + ex)^6} dx = \frac{b^5 \log(|ex + d|) \operatorname{sgn}(bx + a)}{e^6} + \frac{300(b^5 d e^3 \operatorname{sgn}(bx + a) - ab^4 e^4 \operatorname{sgn}(bx + a))x^4 + 300(3b^5 d^2 e^2 \operatorname{sgn}(bx + a) - 2ab^4 d e^3 \operatorname{sgn}(bx + a) - a^2 b^3 e^4 \operatorname{sgn}(bx + a))x^3 + \dots}{e^6}$$

input `integrate((b^2*x^2+2*a*b*x+a^2)^(5/2)/(e*x+d)^6,x, algorithm="giac")`

output

```

b^5*log(abs(e*x + d))*sgn(b*x + a)/e^6 + 1/60*(300*(b^5*d*e^3*sgn(b*x + a)
- a*b^4*e^4*sgn(b*x + a))*x^4 + 300*(3*b^5*d^2*e^2*sgn(b*x + a) - 2*a*b^4
*d*e^3*sgn(b*x + a) - a^2*b^3*e^4*sgn(b*x + a))*x^3 + 100*(11*b^5*d^3*e*sg
n(b*x + a) - 6*a*b^4*d^2*e^2*sgn(b*x + a) - 3*a^2*b^3*d*e^3*sgn(b*x + a) -
2*a^3*b^2*e^4*sgn(b*x + a))*x^2 + 25*(25*b^5*d^4*sgn(b*x + a) - 12*a*b^4*
d^3*e*sgn(b*x + a) - 6*a^2*b^3*d^2*e^2*sgn(b*x + a) - 4*a^3*b^2*d*e^3*sgn(
b*x + a) - 3*a^4*b*e^4*sgn(b*x + a))*x + (137*b^5*d^5*sgn(b*x + a) - 60*a*
b^4*d^4*e*sgn(b*x + a) - 30*a^2*b^3*d^3*e^2*sgn(b*x + a) - 20*a^3*b^2*d^2*
e^3*sgn(b*x + a) - 15*a^4*b*d*e^4*sgn(b*x + a) - 12*a^5*e^5*sgn(b*x + a))/
e)/((e*x + d)^5*e^5)

```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a^2 + 2abx + b^2x^2)^{5/2}}{(d + ex)^6} dx = \int \frac{(a^2 + 2abx + b^2x^2)^{5/2}}{(d + ex)^6} dx$$

input

```
int((a^2 + b^2*x^2 + 2*a*b*x)^(5/2)/(d + e*x)^6,x)
```

output

```
int((a^2 + b^2*x^2 + 2*a*b*x)^(5/2)/(d + e*x)^6, x)
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 375, normalized size of antiderivative = 1.25

$$\int \frac{(a^2 + 2abx + b^2x^2)^{5/2}}{(d + ex)^6} dx = \frac{60 \log(ex + d) b^5 d^6 + 300 \log(ex + d) b^5 d^5 ex + 600 \log(ex + d) b^5 d^4 e^2 x^2 + 600 \log(ex + d) b^5 d^3 e^3 x^3 + 300 \log(ex + d) b^5 d^2 e^4 x^4 + 60 \log(ex + d) b^5 d e^5 x^5 + 60 \log(ex + d) b^5 e^6 x^6}{(d + ex)^6}$$

input

```
int((b^2*x^2+2*a*b*x+a^2)^(5/2)/(e*x+d)^6,x)
```

output

```
(60*log(d + e*x)*b**5*d**6 + 300*log(d + e*x)*b**5*d**5*e*x + 600*log(d +
e*x)*b**5*d**4*e**2*x**2 + 600*log(d + e*x)*b**5*d**3*e**3*x**3 + 300*log(
d + e*x)*b**5*d**2*e**4*x**4 + 60*log(d + e*x)*b**5*d*e**5*x**5 - 12*a**5*
d*e**5 - 15*a**4*b*d**2*e**4 - 75*a**4*b*d*e**5*x - 20*a**3*b**2*d**3*e**3
- 100*a**3*b**2*d**2*e**4*x - 200*a**3*b**2*d*e**5*x**2 - 30*a**2*b**3*d*
*4*e**2 - 150*a**2*b**3*d**3*e**3*x - 300*a**2*b**3*d**2*e**4*x**2 - 300*a
**2*b**3*d*e**5*x**3 + 60*a*b**4*e**6*x**5 + 77*b**5*d**6 + 325*b**5*d**5*
e*x + 500*b**5*d**4*e**2*x**2 + 300*b**5*d**3*e**3*x**3 - 60*b**5*d*e**5*x
**5)/(60*d*e**6*(d**5 + 5*d**4*e*x + 10*d**3*e**2*x**2 + 10*d**2*e**3*x**3
+ 5*d*e**4*x**4 + e**5*x**5))
```

3.303
$$\int \frac{(a^2+2abx+b^2x^2)^{5/2}}{(d+ex)^7} dx$$

| | |
|---|------|
| Optimal result | 2282 |
| Mathematica [B] (verified) | 2282 |
| Rubi [A] (verified) | 2283 |
| Maple [B] (verified) | 2284 |
| Fricas [B] (verification not implemented) | 2285 |
| Sympy [F(-2)] | 2285 |
| Maxima [F(-2)] | 2286 |
| Giac [B] (verification not implemented) | 2286 |
| Mupad [B] (verification not implemented) | 2287 |
| Reduce [B] (verification not implemented) | 2288 |

Optimal result

Integrand size = 28, antiderivative size = 46

$$\int \frac{(a^2 + 2abx + b^2x^2)^{5/2}}{(d + ex)^7} dx = \frac{(a + bx)(a^2 + 2abx + b^2x^2)^{5/2}}{6(bd - ae)(d + ex)^6}$$

output

```
1/6*(b*x+a)*(b^2*x^2+2*a*b*x+a^2)^(5/2)/(-a*e+b*d)/(e*x+d)^6
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 218 vs. 2(46) = 92.

Time = 1.07 (sec) , antiderivative size = 218, normalized size of antiderivative = 4.74

$$\int \frac{(a^2 + 2abx + b^2x^2)^{5/2}}{(d + ex)^7} dx = \frac{\sqrt{(a + bx)^2(a^5 e^5 + a^4 b e^4 (d + 6ex) + a^3 b^2 e^3 (d^2 + 6dex + 15e^2 x^2) + a^2 b^3 e^2 (d^3 + 6d^2 ex + 15de^2 x^2 + 20e^3 x^3) + a b^4 e (d^4 + 6d^3 ex + 15d^2 e x^2 + 20d e^2 x^3 + 10e^3 x^4) + b^5 (d^5 + 6d^4 ex + 15d^3 e x^2 + 20d^2 e^2 x^3 + 10d e^3 x^4 + e^5 x^5))}}{6e^6 (d + ex)^6}$$

input

```
Integrate[(a^2 + 2*a*b*x + b^2*x^2)^(5/2)/(d + e*x)^7,x]
```

output

$$\frac{-1/6*(\text{Sqrt}[(a + b*x)^2]*(a^5*e^5 + a^4*b*e^4*(d + 6*e*x) + a^3*b^2*e^3*(d^2 + 6*d*e*x + 15*e^2*x^2) + a^2*b^3*e^2*(d^3 + 6*d^2*e*x + 15*d*e^2*x^2 + 20*e^3*x^3) + a*b^4*e*(d^4 + 6*d^3*e*x + 15*d^2*e^2*x^2 + 20*d*e^3*x^3 + 15*e^4*x^4) + b^5*(d^5 + 6*d^4*e*x + 15*d^3*e^2*x^2 + 20*d^2*e^3*x^3 + 15*d*e^4*x^4 + 6*e^5*x^5)))/(e^6*(a + b*x)*(d + e*x)^6)}$$
Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.04, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.107$, Rules used = {1102, 27, 48}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\begin{aligned} & \int \frac{(a^2 + 2abx + b^2x^2)^{5/2}}{(d + ex)^7} dx \\ & \quad \downarrow \text{1102} \\ & \frac{\sqrt{a^2 + 2abx + b^2x^2} \int \frac{b^5(a+bx)^5}{(d+ex)^7} dx}{b^5(a + bx)} \\ & \quad \downarrow \text{27} \\ & \frac{\sqrt{a^2 + 2abx + b^2x^2} \int \frac{(a+bx)^5}{(d+ex)^7} dx}{a + bx} \\ & \quad \downarrow \text{48} \\ & \frac{(a + bx)^5 \sqrt{a^2 + 2abx + b^2x^2}}{6(d + ex)^6 (bd - ae)} \end{aligned}$$

input

$$\text{Int}[(a^2 + 2*a*b*x + b^2*x^2)^(5/2)/(d + e*x)^7, x]$$

output

$$((a + b*x)^5*\text{Sqrt}[a^2 + 2*a*b*x + b^2*x^2])/(6*(b*d - a*e)*(d + e*x)^6)$$

Defintions of rubi rules used

rule 27 `Int[(a_)*(Fx_), x_Symbol] := Simp[a Int[Fx, x], x] /; FreeQ[a, x] && !MatchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]`

rule 48 `Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] := Simp[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{a, b, c, d, m, n}, x] && EqQ[m + n + 2, 0] && NeQ[m, -1]`

rule 1102 `Int[((d_.) + (e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] := Simp[(a + b*x + c*x^2)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x)^(2*FracPart[p])) Int[(d + e*x)^m*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && EqQ[b^2 - 4*a*c, 0]`

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 246 vs. 2(42) = 84.

Time = 3.04 (sec) , antiderivative size = 247, normalized size of antiderivative = 5.37

| method | result |
|---------|--|
| risch | $\frac{\sqrt{(bx+a)^2} \left(-\frac{b^5 x^5}{e} - \frac{5b^4(ae+bd)x^4}{2e^2} - \frac{10b^3(e^2a^2+abde+b^2d^2)x^3}{3e^3} - \frac{5b^2(e^3a^3+a^2bde^2+ab^2d^2e+b^3d^3)x^2}{2e^4} - \frac{b(a^4e^4+a^3bde^3+a^2b^2d^2e^2+ab^3d^3e+b^4d^4)}{e^5} \right)}{(bx+a)(ex+d)^6}$ |
| gospers | $-\frac{(6x^5e^5b^5+15x^4ab^4e^5+15x^4b^5de^4+20x^3a^2b^3e^5+20x^3ab^4de^4+20x^3b^5d^2e^3+15x^2a^3b^2e^5+15x^2a^2b^3de^4+15x^2ab^4d^2e^3+15x^2b^5d^3e^2+15x^2a^4bde^4+15x^2a^3b^2de^3+15x^2a^2b^3d^2e^2+15x^2ab^4d^3e+15x^2b^5d^4e)}{6(ex+d)^6}$ |
| default | $-\frac{(6x^5e^5b^5+15x^4ab^4e^5+15x^4b^5de^4+20x^3a^2b^3e^5+20x^3ab^4de^4+20x^3b^5d^2e^3+15x^2a^3b^2e^5+15x^2a^2b^3de^4+15x^2ab^4d^2e^3+15x^2b^5d^3e^2+15x^2a^4bde^4+15x^2a^3b^2de^3+15x^2a^2b^3d^2e^2+15x^2ab^4d^3e+15x^2b^5d^4e)}{6(ex+d)^6}$ |
| orering | $-\frac{(6x^5e^5b^5+15x^4ab^4e^5+15x^4b^5de^4+20x^3a^2b^3e^5+20x^3ab^4de^4+20x^3b^5d^2e^3+15x^2a^3b^2e^5+15x^2a^2b^3de^4+15x^2ab^4d^2e^3+15x^2b^5d^3e^2+15x^2a^4bde^4+15x^2a^3b^2de^3+15x^2a^2b^3d^2e^2+15x^2ab^4d^3e+15x^2b^5d^4e)}{6e^6}$ |

input `int((b^2*x^2+2*a*b*x+a^2)^(5/2)/(e*x+d)^7, x, method=_RETURNVERBOSE)`

output

$$\frac{((b*x+a)^2)^{(1/2)} / (b*x+a) * (-b^5/e*x^5 - 5/2*b^4*(a*e+b*d)/e^2*x^4 - 10/3*b^3*(a^2*e^2+a*b*d*e+b^2*d^2)/e^3*x^3 - 5/2*b^2*(a^3*e^3+a^2*b*d*e^2+a*b^2*d^2*e+b^3*d^3)/e^4*x^2 - b*(a^4*e^4+a^3*b*d*e^3+a^2*b^2*d^2*e^2+a*b^3*d^3*e+b^4*d^4)/e^5*x - 1/6*(a^5*e^5+a^4*b*d*e^4+a^3*b^2*d^2*e^3+a^2*b^3*d^3*e^2+a*b^4*d^4*e+b^5*d^5)/e^6}{(e*x+d)^6}$$

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 300 vs. $2(42) = 84$.

Time = 0.08 (sec) , antiderivative size = 300, normalized size of antiderivative = 6.52

$$\int \frac{(a^2 + 2abx + b^2x^2)^{5/2}}{(d + ex)^7} dx = \frac{6b^5e^5x^5 + b^5d^5 + ab^4d^4e + a^2b^3d^3e^2 + a^3b^2d^2e^3 + a^4bde^4 + a^5e^5 + 15(b^5de^4 + ab^4e^5)x^4 + 20(b^5d^2e^3 + ab^4d^2e^4)x^3 + 15(b^5d^3e^2 + ab^4d^3e^3)x^2 + 6(b^5d^4e + ab^4d^4e^2 + a^2b^3d^3e^2 + a^3b^2d^2e^3 + a^4bde^4 + a^5e^5)}{6(e^{12}x^6 + 6de^{11}x^5 + 15d^2e^{10}x^4 + 20d^3e^9x^3 + 15d^4e^8x^2 + 6d^5e^7x + d^6e^6)}$$

input

```
integrate((b^2*x^2+2*a*b*x+a^2)^(5/2)/(e*x+d)^7,x, algorithm="fricas")
```

output

$$\frac{-1/6*(6*b^5*e^5*x^5 + b^5*d^5 + a*b^4*d^4*e + a^2*b^3*d^3*e^2 + a^3*b^2*d^2*e^3 + a^4*b*d*e^4 + a^5*e^5 + 15*(b^5*d*e^4 + a*b^4*e^5)*x^4 + 20*(b^5*d^2*e^3 + a*b^4*d^2*e^4 + a^2*b^3*d^2*e^5)*x^3 + 15*(b^5*d^3*e^2 + a*b^4*d^3*e^3 + a^2*b^3*d^3*e^4 + a^3*b^2*d^3*e^5)*x^2 + 6*(b^5*d^4*e + a*b^4*d^4*e^2 + a^2*b^3*d^4*e^3 + a^3*b^2*d^4*e^4 + a^4*b*d^4*e^5)*x}{(e^{12}*x^6 + 6*d*e^{11}*x^5 + 15*d^2*e^{10}*x^4 + 20*d^3*e^9*x^3 + 15*d^4*e^8*x^2 + 6*d^5*e^7*x + d^6*e^6)}$$

Sympy [F(-2)]

Exception generated.

$$\int \frac{(a^2 + 2abx + b^2x^2)^{5/2}}{(d + ex)^7} dx = \text{Exception raised: HeuristicGCDFailed}$$

input

```
integrate((b**2*x**2+2*a*b*x+a**2)**(5/2)/(e*x+d)**7,x)
```

output

```
Exception raised: HeuristicGCDFailed >> no luck
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(a^2 + 2abx + b^2x^2)^{5/2}}{(d + ex)^7} dx = \text{Exception raised: ValueError}$$

input `integrate((b^2*x^2+2*a*b*x+a^2)^(5/2)/(e*x+d)^7,x, algorithm="maxima")`

output Exception raised: ValueError >> Computation failed since Maxima requested additional constraints; using the 'assume' command before evaluation *may* help (example of legal syntax is 'assume(a*e-b*d>0)', see `assume?` for more detail

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 419 vs. 2(42) = 84.

Time = 0.51 (sec) , antiderivative size = 419, normalized size of antiderivative = 9.11

$$\int \frac{(a^2 + 2abx + b^2x^2)^{5/2}}{(d + ex)^7} dx = \frac{b^6 \operatorname{sgn}(bx + a)}{6(bde^6 - ae^7)}$$

$$\frac{6b^5e^5x^5 \operatorname{sgn}(bx + a) + 15b^5de^4x^4 \operatorname{sgn}(bx + a) + 15ab^4e^5x^4 \operatorname{sgn}(bx + a) + 20b^5d^2e^3x^3 \operatorname{sgn}(bx + a) + 20ab^4e^4x^3 \operatorname{sgn}(bx + a) + 15a^2b^3d^2e^3x^2 \operatorname{sgn}(bx + a) + 15b^5d^3e^2x^2 \operatorname{sgn}(bx + a) + 15a^2b^3d^2e^3x^2 \operatorname{sgn}(bx + a) + 15a^3b^2de^4x^2 \operatorname{sgn}(bx + a) + 15a^3b^2de^4x^2 \operatorname{sgn}(bx + a) + 6b^5d^4e^4x \operatorname{sgn}(bx + a) + 6a^2b^3d^2e^3x \operatorname{sgn}(bx + a) + 6a^3b^2de^4x \operatorname{sgn}(bx + a) + 6a^4b^2de^4x \operatorname{sgn}(bx + a) + b^5d^5 \operatorname{sgn}(bx + a) + a^2b^3d^3e^2 \operatorname{sgn}(bx + a) + a^3b^2d^2e^3 \operatorname{sgn}(bx + a) + a^4b^2d^2e^3 \operatorname{sgn}(bx + a) + a^4b^2d^2e^3 \operatorname{sgn}(bx + a) + a^5e^5 \operatorname{sgn}(bx + a)}{(e^6x^6 + d^6e^6)}$$

input `integrate((b^2*x^2+2*a*b*x+a^2)^(5/2)/(e*x+d)^7,x, algorithm="giac")`

output $\frac{1}{6}b^6 \operatorname{sgn}(bx + a)/(b^6d^6e^6 - a^6e^7) - \frac{1}{6}(6b^5e^5x^5 \operatorname{sgn}(bx + a) + 15b^5d^4e^4x^4 \operatorname{sgn}(bx + a) + 15a^2b^3d^2e^3x^2 \operatorname{sgn}(bx + a) + 20b^5d^2e^3x^3 \operatorname{sgn}(bx + a) + 20a^2b^3d^2e^3x^2 \operatorname{sgn}(bx + a) + 20a^3b^2de^4x^2 \operatorname{sgn}(bx + a) + 15b^5d^3e^2x^2 \operatorname{sgn}(bx + a) + 15a^2b^3d^2e^3x^2 \operatorname{sgn}(bx + a) + 15a^3b^2de^4x^2 \operatorname{sgn}(bx + a) + 15a^3b^2de^4x^2 \operatorname{sgn}(bx + a) + 6b^5d^4e^4x \operatorname{sgn}(bx + a) + 6a^2b^3d^2e^3x \operatorname{sgn}(bx + a) + 6a^3b^2de^4x \operatorname{sgn}(bx + a) + 6a^4b^2de^4x \operatorname{sgn}(bx + a) + b^5d^5 \operatorname{sgn}(bx + a) + a^2b^3d^3e^2 \operatorname{sgn}(bx + a) + a^3b^2d^2e^3 \operatorname{sgn}(bx + a) + a^4b^2d^2e^3 \operatorname{sgn}(bx + a) + a^5e^5 \operatorname{sgn}(bx + a)) / ((e^6x^6 + d^6e^6))$

Mupad [B] (verification not implemented)

Time = 5.16 (sec) , antiderivative size = 687, normalized size of antiderivative = 14.93

$$\int \frac{(a^2 + 2abx + b^2x^2)^{5/2}}{(d + ex)^7} dx = \frac{\left(\frac{4b^5d - 5ab^4e}{2e^6} + \frac{b^5d}{2e^6}\right) \sqrt{a^2 + 2abx + b^2x^2}}{(a + bx)(d + ex)^2}$$

$$\left(\frac{5a^4be^4 - 10a^3b^2de^3 + 10a^2b^3d^2e^2 - 5ab^4d^3e + b^5d^4}{5e^6} + \frac{d \left(\frac{b^5d}{5e^3} - \frac{b^4(5ae - bd)}{5e^3} \right) + b^3}{e} \right) \frac{\sqrt{a^2 + 2abx + b^2x^2}}{(a + bx)(d + ex)^5}$$

$$\left(\frac{10a^2b^3e^2 - 15ab^4de + 6b^5d^2}{3e^6} + \frac{d \left(\frac{b^5d}{3e^5} - \frac{b^4(5ae - 3bd)}{3e^5} \right)}{e} \right) \frac{\sqrt{a^2 + 2abx + b^2x^2}}{(a + bx)(d + ex)^3}$$

$$\left(\frac{a^5}{6e} - \frac{d \left(\frac{5a^4b}{6e} - \frac{d \left(\frac{5ab^4}{6e} - \frac{b^5d}{6e^2} \right) - \frac{5a^2b^3}{3e}}{e} + \frac{5a^3b^2}{3e} \right)}{e} \right) \frac{\sqrt{a^2 + 2abx + b^2x^2}}{(a + bx)(d + ex)^6}$$

$$\left(\frac{-10a^3b^2e^3 + 20a^2b^3de^2 - 15ab^4d^2e + 4b^5d^3}{4e^6} + \frac{d \left(\frac{b^5d}{4e^4} - \frac{b^4(5ae - 2bd)}{4e^4} \right) + b^3 \left(\frac{10a^2e^2 - 10abde + 3b^2d^2}{4e^5} \right)}{e} \right) \frac{\sqrt{a^2 + 2abx + b^2x^2}}{(a + bx)(d + ex)^4}$$

$$- \frac{b^5 \sqrt{a^2 + 2abx + b^2x^2}}{e^6 (a + bx)(d + ex)}$$

input `int((a^2 + b^2*x^2 + 2*a*b*x)^(5/2)/(d + e*x)^7,x)`

output
$$\begin{aligned} & \left(\frac{(4*b^5*d - 5*a*b^4*e)}{(2*e^6)} + \frac{(b^5*d)}{(2*e^6)} \right) * (a^2 + b^2*x^2 + 2*a*b*x)^{(1/2)} / \left((a + b*x)*(d + e*x)^2 \right) - \left(\frac{(b^5*d^4 + 5*a^4*b*e^4 - 10*a^3*b^2*d*e^3 + 10*a^2*b^3*d^2*e^2 - 5*a*b^4*d^3*e)}{(5*e^6)} + \frac{(d*(b^5*d^3*e - 10*a^3*b^2*e^4 - 5*a*b^4*d^2*e^2 + 10*a^2*b^3*d*e^3))}{(5*e^6)} + \frac{(d*(d*(b^5*d)/(5*e^3) - (b^4*(5*a*e - b*d))/(5*e^3)))}{e} + \frac{(b^3*(10*a^2*e^2 + b^2*d^2 - 5*a*b*d*e))}{(5*e^4)} \right) / e \left(\frac{(a^2 + b^2*x^2 + 2*a*b*x)^{(1/2)}}{(a + b*x)*(d + e*x)^5} - \left(\frac{(6*b^5*d^2 + 10*a^2*b^3*e^2 - 15*a*b^4*d*e)}{(3*e^6)} + \frac{(d*((b^5*d)/(3*e^5) - (b^4*(5*a*e - 3*b*d))/(3*e^5)))}{e} \right) * (a^2 + b^2*x^2 + 2*a*b*x)^{(1/2)} / \left((a + b*x)*(d + e*x)^3 \right) - \left(\frac{a^5}{(6*e)} - \frac{(d*((5*a^4*b)/(6*e) - (d*(d*(d*(5*a*b^4)/(6*e) - (b^5*d)/(6*e^2))))}{e} - \frac{(5*a^2*b^3)}{(3*e))}{e} + \frac{(5*a^3*b^2)}{(3*e))} \right) / e \right) * (a^2 + b^2*x^2 + 2*a*b*x)^{(1/2)} / \left((a + b*x)*(d + e*x)^6 \right) + \left(\frac{(4*b^5*d^3 - 10*a^3*b^2*e^3 + 20*a^2*b^3*d*e^2 - 15*a*b^4*d^2*e)}{(4*e^6)} + \frac{(d*(d*(b^5*d)/(4*e^4) - (b^4*(5*a*e - 2*b*d))/(4*e^4))}{e} + \frac{(b^3*(10*a^2*e^2 + 3*b^2*d^2 - 10*a*b*d*e))}{(4*e^5)} \right) / e * (a^2 + b^2*x^2 + 2*a*b*x)^{(1/2)} / \left((a + b*x)*(d + e*x)^4 \right) - \frac{(b^5*(a^2 + b^2*x^2 + 2*a*b*x)^{(1/2)})}{(e^6*(a + b*x)*(d + e*x))} \end{aligned}$$

Reduce [B] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 279, normalized size of antiderivative = 6.07

$$\int \frac{(a^2 + 2abx + b^2x^2)^{5/2}}{(d + ex)^7} dx = \frac{b^5 e^5 x^6 - 15a b^4 d e^4 x^4 - 20a^2 b^3 d e^4 x^3 - 20a b^4 d^2 e^3 x^3 - 15a^3 b^2 d e^4 x^2 - 15a^2 b^4 d^2 e^3 x^3 - 15a^3 b^2 d e^4 x^2 - 15a^2 b^4 d^2 e^3 x^3}{6d e^5 (e^6 x^6 + \dots)}$$

input `int((b^2*x^2+2*a*b*x+a^2)^(5/2)/(e*x+d)^7,x)`

output
$$\begin{aligned} & \left(- a^{**5}d^{**4} - a^{**4}b^{**2}d^{**2}e^{**3} - 6a^{**4}b^{**2}d^{**4}e^{**4}x - a^{**3}b^{**2}d^{**3}e^{**2}x^2 - 6a^{**3}b^{**2}d^{**2}e^{**3}x - 15a^{**3}b^{**2}d^{**4}e^{**4}x^2 - a^{**2}b^{**3}d^{**4}e^{**4}e - 6a^{**2}b^{**3}d^{**3}e^{**2}x - 15a^{**2}b^{**3}d^{**2}e^{**3}x^2 - 20a^{**2}b^{**3}d^{**4}e^{**4}x^3 - a^{**4}d^{**5} - 6a^{**4}d^{**4}e^{**4}x - 15a^{**4}d^{**3}e^{**2}x^2 - 20a^{**4}d^{**2}e^{**3}x^3 - 15a^{**4}d^{**4}e^{**4}x^4 + b^{**5}e^{**5}x^6 \right) / (6d^{**5}e^{**5}(d^{**6} + 6d^{**5}e^{**4}x + 15d^{**4}e^{**3}x^2 + 20d^{**3}e^{**2}x^3 + 15d^{**2}e^{**4}x^4 + 6d^{**5}e^{**5}x^5 + e^{**6}x^6)) \end{aligned}$$

3.304 $\int \frac{(a^2+2abx+b^2x^2)^{5/2}}{(d+ex)^8} dx$

| | |
|---|------|
| Optimal result | 2289 |
| Mathematica [B] (verified) | 2289 |
| Rubi [A] (verified) | 2290 |
| Maple [B] (verified) | 2292 |
| Fricas [B] (verification not implemented) | 2292 |
| Sympy [F] | 2293 |
| Maxima [F(-2)] | 2293 |
| Giac [B] (verification not implemented) | 2294 |
| Mupad [B] (verification not implemented) | 2295 |
| Reduce [B] (verification not implemented) | 2296 |

Optimal result

Integrand size = 28, antiderivative size = 98

$$\int \frac{(a^2 + 2abx + b^2x^2)^{5/2}}{(d + ex)^8} dx = \frac{(a + bx)^5 \sqrt{a^2 + 2abx + b^2x^2}}{7(bd - ae)(d + ex)^7} + \frac{b(a + bx)^5 \sqrt{a^2 + 2abx + b^2x^2}}{42(bd - ae)^2(d + ex)^6}$$

output `1/7*(b*x+a)^5*((b*x+a)^2)^(1/2)/(-a*e+b*d)/(e*x+d)^7+1/42*b*(b*x+a)^5*((b*x+a)^2)^(1/2)/(-a*e+b*d)^2/(e*x+d)^6`

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 223 vs. 2(98) = 196.

Time = 1.06 (sec) , antiderivative size = 223, normalized size of antiderivative = 2.28

$$\int \frac{(a^2 + 2abx + b^2x^2)^{5/2}}{(d + ex)^8} dx = \frac{\sqrt{(a + bx)^2(6a^5e^5 + 5a^4be^4(d + 7ex) + 4a^3b^2e^3(d^2 + 7dex + 21e^2x^2) + 3a^2b^3e^2(d^3 + 7d^2ex + 21de^2x^2 + 4$$

input `Integrate[(a^2 + 2*a*b*x + b^2*x^2)^(5/2)/(d + e*x)^8,x]`

output

$$\frac{-1/42*(\text{Sqrt}[(a + b*x)^2]*(6*a^5*e^5 + 5*a^4*b*e^4*(d + 7*e*x) + 4*a^3*b^2*e^3*(d^2 + 7*d*e*x + 21*e^2*x^2) + 3*a^2*b^3*e^2*(d^3 + 7*d^2*e*x + 21*d*e^2*x^2 + 35*e^3*x^3) + 2*a*b^4*e*(d^4 + 7*d^3*e*x + 21*d^2*e^2*x^2 + 35*d*e^3*x^3 + 35*e^4*x^4) + b^5*(d^5 + 7*d^4*e*x + 21*d^3*e^2*x^2 + 35*d^2*e^3*x^3 + 35*d*e^4*x^4 + 21*e^5*x^5)))/(e^6*(a + b*x)*(d + e*x)^7)}$$
Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.88, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {1102, 27, 55, 48}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\begin{aligned} & \int \frac{(a^2 + 2abx + b^2x^2)^{5/2}}{(d + ex)^8} dx \\ & \quad \downarrow \text{1102} \\ & \frac{\sqrt{a^2 + 2abx + b^2x^2} \int \frac{b^5(a+bx)^5}{(d+ex)^8} dx}{b^5(a + bx)} \\ & \quad \downarrow \text{27} \\ & \frac{\sqrt{a^2 + 2abx + b^2x^2} \int \frac{(a+bx)^5}{(d+ex)^8} dx}{a + bx} \\ & \quad \downarrow \text{55} \\ & \frac{\sqrt{a^2 + 2abx + b^2x^2} \left(\frac{b \int \frac{(a+bx)^5}{(d+ex)^7} dx}{7(bd-ae)} + \frac{(a+bx)^6}{7(d+ex)^7(bd-ae)} \right)}{a + bx} \\ & \quad \downarrow \text{48} \\ & \frac{\sqrt{a^2 + 2abx + b^2x^2} \left(\frac{b(a+bx)^6}{42(d+ex)^6(bd-ae)^2} + \frac{(a+bx)^6}{7(d+ex)^7(bd-ae)} \right)}{a + bx} \end{aligned}$$

input

$$\text{Int}[(a^2 + 2*a*b*x + b^2*x^2)^(5/2)/(d + e*x)^8,x]$$

output
$$\frac{(\sqrt{a^2 + 2abx + b^2x^2} * ((a + bx)^6 / (7 * (bd - ae) * (d + ex)^7) + (b * (a + bx)^6) / (42 * (bd - ae)^2 * (d + ex)^6))}{(a + bx)}$$

Defintions of rubi rules used

rule 27
$$\text{Int}[(a_*)(Fx_), x_Symbol] \rightarrow \text{Simp}[a \text{ Int}[Fx, x], x] /; \text{FreeQ}[a, x] \ \&\& \ !\text{MatchQ}[Fx, (b_*)(Gx_)] /; \text{FreeQ}[b, x]$$

rule 48
$$\text{Int}[(a_.) + (b_.)(x_)^{(m_.)} * ((c_.) + (d_.)(x_)^{(n_.)}), x_Symbol] \rightarrow \text{Simp}[(a + bx)^{(m + 1)} * ((c + dx)^{(n + 1)} / ((b * c - a * d) * (m + 1))), x] /; \text{FreeQ}[\{a, b, c, d, m, n\}, x] \ \&\& \ \text{EqQ}[m + n + 2, 0] \ \&\& \ \text{NeQ}[m, -1]$$

rule 55
$$\text{Int}[(a_.) + (b_.)(x_)^{(m_.)} * ((c_.) + (d_.)(x_)^{(n_.)}), x_Symbol] \rightarrow \text{Simp}[(a + bx)^{(m + 1)} * ((c + dx)^{(n + 1)} / ((b * c - a * d) * (m + 1))), x] - \text{Simp}[d * (\text{Simplify}[m + n + 2] / ((b * c - a * d) * (m + 1))) \text{Int}[(a + bx)^{\text{Simplify}[m + 1]} * (c + dx)^n, x], x] /; \text{FreeQ}[\{a, b, c, d, m, n\}, x] \ \&\& \ \text{ILtQ}[\text{Simplify}[m + n + 2], 0] \ \&\& \ \text{NeQ}[m, -1] \ \&\& \ !(\text{LtQ}[m, -1] \ \&\& \ \text{LtQ}[n, -1] \ \&\& \ (\text{EqQ}[a, 0] \ || \ (\text{NeQ}[c, 0] \ \&\& \ \text{LtQ}[m - n, 0] \ \&\& \ \text{IntegerQ}[n]))) \ \&\& \ (\text{SumSimplerQ}[m, 1] \ || \ !\text{SumSimplerQ}[n, 1])$$

rule 1102
$$\text{Int}[(d_.) + (e_.)(x_)^{(m_.)} * ((a_) + (b_.)(x_) + (c_.)(x_)^2)^{(p_.)}, x_Symbol] \rightarrow \text{Simp}[(a + bx + cx^2)^{\text{FracPart}[p]} / (c^{\text{IntPart}[p]} * (b/2 + cx)^{(2 * \text{FracPart}[p])}) \text{Int}[(d + ex)^m * (b/2 + cx)^{(2 * p)}, x], x] /; \text{FreeQ}[\{a, b, c, d, e, m, p\}, x] \ \&\& \ \text{EqQ}[b^2 - 4 * a * c, 0]$$

output

```
-1/42*(21*b^5*e^5*x^5 + b^5*d^5 + 2*a*b^4*d^4*e + 3*a^2*b^3*d^3*e^2 + 4*a^3*b^2*d^2*e^3 + 5*a^4*b*d*e^4 + 6*a^5*e^5 + 35*(b^5*d*e^4 + 2*a*b^4*e^5)*x^4 + 35*(b^5*d^2*e^3 + 2*a*b^4*d*e^4 + 3*a^2*b^3*e^5)*x^3 + 21*(b^5*d^3*e^2 + 2*a*b^4*d^2*e^3 + 3*a^2*b^3*d*e^4 + 4*a^3*b^2*e^5)*x^2 + 7*(b^5*d^4*e + 2*a*b^4*d^3*e^2 + 3*a^2*b^3*d^2*e^3 + 4*a^3*b^2*d*e^4 + 5*a^4*b*e^5)*x)/(e^13*x^7 + 7*d*e^12*x^6 + 21*d^2*e^11*x^5 + 35*d^3*e^10*x^4 + 35*d^4*e^9*x^3 + 21*d^5*e^8*x^2 + 7*d^6*e^7*x + d^7*e^6)
```

Sympy [F]

$$\int \frac{(a^2 + 2abx + b^2x^2)^{5/2}}{(d + ex)^8} dx = \int \frac{((a + bx)^2)^{5/2}}{(d + ex)^8} dx$$

input

```
integrate((b**2*x**2+2*a*b*x+a**2)**(5/2)/(e*x+d)**8,x)
```

output

```
Integral(((a + b*x)**2)**(5/2)/(d + e*x)**8, x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(a^2 + 2abx + b^2x^2)^{5/2}}{(d + ex)^8} dx = \text{Exception raised: ValueError}$$

input

```
integrate((b^2*x^2+2*a*b*x+a^2)^(5/2)/(e*x+d)^8,x, algorithm="maxima")
```

output

```
Exception raised: ValueError >> Computation failed since Maxima requested additional constraints; using the 'assume' command before evaluation *may* help (example of legal syntax is 'assume(a*e-b*d>0)', see `assume?` for more detail)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 437 vs. $2(72) = 144$.

Time = 0.42 (sec) , antiderivative size = 437, normalized size of antiderivative = 4.46

$$\int \frac{(a^2 + 2abx + b^2x^2)^{5/2}}{(d + ex)^8} dx = \frac{b^7 \operatorname{sgn}(bx + a)}{42(b^2d^2e^6 - 2abde^7 + a^2e^8)} - \frac{21b^5e^5x^5 \operatorname{sgn}(bx + a) + 35b^5de^4x^4 \operatorname{sgn}(bx + a) + 70ab^4e^5x^4 \operatorname{sgn}(bx + a) + 35b^5d^2e^3x^3 \operatorname{sgn}(bx + a) + 70a^2b^3de^3x^3 \operatorname{sgn}(bx + a) + 105a^2b^3e^5x^3 \operatorname{sgn}(bx + a) + 21b^5d^3e^2x^2 \operatorname{sgn}(bx + a) + 42a^2b^4d^2e^3x^2 \operatorname{sgn}(bx + a) + 63a^2b^3d^2e^4x^2 \operatorname{sgn}(bx + a) + 84a^3b^2e^5x^2 \operatorname{sgn}(bx + a) + 7b^5d^4e^4x \operatorname{sgn}(bx + a) + 14a^2b^4d^3e^2x \operatorname{sgn}(bx + a) + 21a^2b^3d^2e^3x \operatorname{sgn}(bx + a) + 28a^3b^2de^4x \operatorname{sgn}(bx + a) + 35a^4b^2e^5x \operatorname{sgn}(bx + a) + b^5d^5 \operatorname{sgn}(bx + a) + 2a^2b^4d^4e^4 \operatorname{sgn}(bx + a) + 3a^2b^3d^3e^2 \operatorname{sgn}(bx + a) + 4a^3b^2d^2e^3 \operatorname{sgn}(bx + a) + 5a^4b^2de^4 \operatorname{sgn}(bx + a) + 6a^5e^5 \operatorname{sgn}(bx + a)}{(e^7x^7 + d^7e^6)}$$

input `integrate((b^2*x^2+2*a*b*x+a^2)^(5/2)/(e*x+d)^8,x, algorithm="giac")`

output `1/42*b^7*sgn(b*x + a)/(b^2*d^2*e^6 - 2*a*b*d*e^7 + a^2*e^8) - 1/42*(21*b^5*e^5*x^5*sgn(b*x + a) + 35*b^5*d*e^4*x^4*sgn(b*x + a) + 70*a*b^4*e^5*x^4*sgn(b*x + a) + 35*b^5*d^2*e^3*x^3*sgn(b*x + a) + 70*a*b^4*d*e^4*x^3*sgn(b*x + a) + 105*a^2*b^3*e^5*x^3*sgn(b*x + a) + 21*b^5*d^3*e^2*x^2*sgn(b*x + a) + 42*a*b^4*d^2*e^3*x^2*sgn(b*x + a) + 63*a^2*b^3*d^2*e^4*x^2*sgn(b*x + a) + 84*a^3*b^2*e^5*x^2*sgn(b*x + a) + 7*b^5*d^4*e*x*sgn(b*x + a) + 14*a*b^4*d^3*e^2*x*sgn(b*x + a) + 21*a^2*b^3*d^2*e^3*x*sgn(b*x + a) + 28*a^3*b^2*d*e^4*x*sgn(b*x + a) + 35*a^4*b^2*e^5*x*sgn(b*x + a) + b^5*d^5*sgn(b*x + a) + 2*a*b^4*d^4*e*sgn(b*x + a) + 3*a^2*b^3*d^3*e^2*sgn(b*x + a) + 4*a^3*b^2*d^2*e^3*sgn(b*x + a) + 5*a^4*b*d*e^4*sgn(b*x + a) + 6*a^5*e^5*sgn(b*x + a))/(e^7*x^7 + d^7*e^6)`

Mupad [B] (verification not implemented)

Time = 5.44 (sec) , antiderivative size = 687, normalized size of antiderivative = 7.01

$$\int \frac{(a^2 + 2abx + b^2x^2)^{5/2}}{(d + ex)^8} dx = \frac{\left(\frac{4b^5d - 5ab^4e}{3e^6} + \frac{b^5d}{3e^6}\right) \sqrt{a^2 + 2abx + b^2x^2}}{(a + bx)(d + ex)^3}$$

$$\left(\frac{5a^4be^4 - 10a^3b^2de^3 + 10a^2b^3d^2e^2 - 5ab^4d^3e + b^5d^4}{6e^6} + \frac{d \left(\frac{b^5d}{6e^3} - \frac{b^4(5ae - bd)}{6e^3} \right) + b^3}{e} \right) \frac{\sqrt{a^2 + 2abx + b^2x^2}}{(a + bx)(d + ex)^6}$$

$$\left(\frac{10a^2b^3e^2 - 15ab^4de + 6b^5d^2}{4e^6} + \frac{d \left(\frac{b^5d}{4e^5} - \frac{b^4(5ae - 3bd)}{4e^5} \right)}{e} \right) \frac{\sqrt{a^2 + 2abx + b^2x^2}}{(a + bx)(d + ex)^4}$$

$$\left(\frac{a^5}{7e} - \frac{d \left(\frac{5a^4b}{7e} - \frac{d \left(\frac{d \left(\frac{5ab^4}{7e} - \frac{b^5d}{7e^2} \right) - \frac{10a^2b^3}{7e} \right) + \frac{10a^3b^2}{7e}}{e} \right)}{e} \right) \frac{\sqrt{a^2 + 2abx + b^2x^2}}{(a + bx)(d + ex)^7}$$

$$+ \left(\frac{-10a^3b^2e^3 + 20a^2b^3de^2 - 15ab^4d^2e + 4b^5d^3}{5e^6} + \frac{d \left(\frac{b^5d}{5e^4} - \frac{b^4(5ae - 2bd)}{5e^4} \right) + \frac{b^3(10a^2e^2 - 10abde + 3b^2d^2)}{5e^5}}{e} \right) \frac{\sqrt{a^2 + 2abx + b^2x^2}}{(a + bx)(d + ex)^5}$$

$$- \frac{b^5 \sqrt{a^2 + 2abx + b^2x^2}}{2e^6 (a + bx)(d + ex)^2}$$

input `int((a^2 + b^2*x^2 + 2*a*b*x)^(5/2)/(d + e*x)^8,x)`

output
$$\begin{aligned} & \left(\frac{(4*b^5*d - 5*a*b^4*e)}{(3*e^6)} + \frac{b^5*d}{(3*e^6)} \right) * (a^2 + b^2*x^2 + 2*a*b*x)^{(1/2)} / \left((a + b*x)*(d + e*x)^3 - \left(\frac{(b^5*d^4 + 5*a^4*b*e^4 - 10*a^3*b^2*d*e^3 + 10*a^2*b^3*d^2*e^2 - 5*a*b^4*d^3*e)}{(6*e^6)} + \frac{d*((b^5*d^3*e - 10*a^3*b^2*e^4 - 5*a*b^4*d^2*e^2 + 10*a^2*b^3*d*e^3)}{(6*e^6)} + \frac{d*((d*((b^5*d^3*e - 10*a^3*b^2*e^4 - 5*a*b^4*d^2*e^2 + 10*a^2*b^3*d*e^3)}{(6*e^6)} + \frac{d*((d*((b^5*d^3*e - 10*a^3*b^2*e^4 - 5*a*b^4*d^2*e^2 + 10*a^2*b^3*d*e^3)}{(6*e^6)} - \frac{b^4*(5*a*e - b*d)}{(6*e^3))}}{e} + \frac{b^3*(10*a^2*e^2 + b^2*d^2 - 5*a*b*d*e)}{(6*e^4))}}{e} \right) / e \right) * (a^2 + b^2*x^2 + 2*a*b*x)^{(1/2)} / \left((a + b*x)*(d + e*x)^6 - \left(\frac{(6*b^5*d^2 + 10*a^2*b^3*e^2 - 15*a*b^4*d*e)}{(4*e^6)} + \frac{d*((b^5*d)/(4*e^5) - (b^4*(5*a*e - 3*b*d))/(4*e^5))}}{e} \right) * (a^2 + b^2*x^2 + 2*a*b*x)^{(1/2)} / \left((a + b*x)*(d + e*x)^4 - \left(\frac{a^5}{(7*e)} - \frac{d*((5*a^4*b)/(7*e) - (d*((d*((d*((5*a*b^4)/(7*e) - (b^5*d)/(7*e^2))}}{e} - (10*a^2*b^3)/(7*e))}}{e} + \frac{(10*a^3*b^2)/(7*e))}}{e} \right) / e \right) * (a^2 + b^2*x^2 + 2*a*b*x)^{(1/2)} / \left((a + b*x)*(d + e*x)^7 + \left(\frac{(4*b^5*d^3 - 10*a^3*b^2*e^3 + 20*a^2*b^3*d*e^2 - 15*a*b^4*d^2*e)}{(5*e^6)} + \frac{d*((d*((b^5*d)/(5*e^4) - (b^4*(5*a*e - 2*b*d))/(5*e^4))}}{e} + \frac{b^3*(10*a^2*e^2 + 3*b^2*d^2 - 10*a*b*d*e)}{(5*e^5))}}{e} \right) * (a^2 + b^2*x^2 + 2*a*b*x)^{(1/2)} / \left((a + b*x)*(d + e*x)^5 - \frac{b^5*(a^2 + b^2*x^2 + 2*a*b*x)^{(1/2)}}{(2*e^6*(a + b*x)*(d + e*x)^2)} \right) \end{aligned}$$

Reduce [B] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 338, normalized size of antiderivative = 3.45

$$\int \frac{(a^2 + 2abx + b^2x^2)^{5/2}}{(d + ex)^8} dx = \frac{-21b^5e^5x^5 - 70ab^4e^5x^4 - 35b^5de^4x^4 - 105a^2b^3e^5x^3 - 70ab^4de^4x^3 - 35b^5d^2e^4x^2 - 21a^2b^3e^5x^2 - 63a^2b^3de^4x^2 - 105a^2b^3e^5x^2 - 21a^2b^3de^4x^2 - 14ab^4d^3e^3x^2 - 42ab^4d^3e^3x^2 - 70ab^4de^4x^3 - 70ab^4e^5x^4 - b^5d^5 - 7b^5d^4e^5x - 21b^5d^3e^3x^2 - 35b^5d^2e^3x^3 - 35b^5de^4x^4 - 21b^5e^5x^5}{(42e^6*(d^7 + 7d^6ex + 21d^5e^2x^2 + 35d^4e^3x^3 + 35d^3e^4x^4 + 21d^2e^5x^5 + 7de^6x^6 + e^7x^7))}$$

input `int((b^2*x^2+2*a*b*x+a^2)^(5/2)/(e*x+d)^8,x)`

output
$$\begin{aligned} & \left(-6*a**5*e**5 - 5*a**4*b*d*e**4 - 35*a**4*b*e**5*x - 4*a**3*b**2*d**2*e**3 - 28*a**3*b**2*d*e**4*x - 84*a**3*b**2*e**5*x**2 - 3*a**2*b**3*d**3*e**2 - 21*a**2*b**3*d**2*e**3*x - 63*a**2*b**3*d*e**4*x**2 - 105*a**2*b**3*e**5*x**3 - 2*a*b**4*d**4*e - 14*a*b**4*d**3*e**2*x - 42*a*b**4*d**2*e**3*x**2 - 70*a*b**4*d*e**4*x**3 - 70*a*b**4*e**5*x**4 - b**5*d**5 - 7*b**5*d**4*e*x - 21*b**5*d**3*e**2*x**2 - 35*b**5*d**2*e**3*x**3 - 35*b**5*d*e**4*x**4 - 21*b**5*e**5*x**5 \right) / \left(42*e**6*(d**7 + 7*d**6*e*x + 21*d**5*e**2*x**2 + 35*d**4*e**3*x**3 + 35*d**3*e**4*x**4 + 21*d**2*e**5*x**5 + 7*d*e**6*x**6 + e**7*x**7) \right) \end{aligned}$$

3.305 $\int \frac{(a^2+2abx+b^2x^2)^{5/2}}{(d+ex)^9} dx$

| | |
|---|------|
| Optimal result | 2297 |
| Mathematica [A] (verified) | 2297 |
| Rubi [A] (verified) | 2298 |
| Maple [B] (verified) | 2300 |
| Fricas [B] (verification not implemented) | 2300 |
| Sympy [F] | 2301 |
| Maxima [F(-2)] | 2301 |
| Giac [B] (verification not implemented) | 2302 |
| Mupad [B] (verification not implemented) | 2303 |
| Reduce [B] (verification not implemented) | 2304 |

Optimal result

Integrand size = 28, antiderivative size = 149

$$\int \frac{(a^2 + 2abx + b^2x^2)^{5/2}}{(d + ex)^9} dx = \frac{(a + bx)^5 \sqrt{a^2 + 2abx + b^2x^2}}{8(bd - ae)(d + ex)^8} + \frac{b(a + bx)^5 \sqrt{a^2 + 2abx + b^2x^2}}{28(bd - ae)^2(d + ex)^7} + \frac{b^2(a + bx)^5 \sqrt{a^2 + 2abx + b^2x^2}}{168(bd - ae)^3(d + ex)^6}$$

output

```
1/8*(b*x+a)^5*((b*x+a)^2)^(1/2)/(-a*e+b*d)/(e*x+d)^8+1/28*b*(b*x+a)^5*((b*x+a)^2)^(1/2)/(-a*e+b*d)^2/(e*x+d)^7+1/168*b^2*(b*x+a)^5*((b*x+a)^2)^(1/2)/(-a*e+b*d)^3/(e*x+d)^6
```

Mathematica [A] (verified)

Time = 1.07 (sec) , antiderivative size = 223, normalized size of antiderivative = 1.50

$$\int \frac{(a^2 + 2abx + b^2x^2)^{5/2}}{(d + ex)^9} dx = \frac{\sqrt{(a + bx)^2(21a^5e^5 + 15a^4be^4(d + 8ex) + 10a^3b^2e^3(d^2 + 8dex + 28e^2x^2) + 6a^2b^3e^2(d^3 + 8d^2ex + 28de^2x^2))}}{(d + ex)^9}$$

input

```
Integrate[(a^2 + 2*a*b*x + b^2*x^2)^(5/2)/(d + e*x)^9,x]
```

output

$$\frac{-1/168 * (\text{Sqrt}[(a + b*x)^2] * (21*a^5*e^5 + 15*a^4*b*e^4*(d + 8*e*x) + 10*a^3*b^2*e^3*(d^2 + 8*d*e*x + 28*e^2*x^2) + 6*a^2*b^3*e^2*(d^3 + 8*d^2*e*x + 28*d*e^2*x^2 + 56*e^3*x^3) + 3*a*b^4*e*(d^4 + 8*d^3*e*x + 28*d^2*e^2*x^2 + 56*d*e^3*x^3 + 70*e^4*x^4) + b^5*(d^5 + 8*d^4*e*x + 28*d^3*e^2*x^2 + 56*d^2*e^3*x^3 + 70*d*e^4*x^4 + 56*e^5*x^5))}{(e^6*(a + b*x)*(d + e*x)^8)}$$
Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 130, normalized size of antiderivative = 0.87, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.179$, Rules used = {1102, 27, 55, 55, 48}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a^2 + 2abx + b^2x^2)^{5/2}}{(d + ex)^9} dx$$

$$\downarrow 1102$$

$$\frac{\sqrt{a^2 + 2abx + b^2x^2} \int \frac{b^5(a+bx)^5}{(d+ex)^9} dx}{b^5(a + bx)}$$

$$\downarrow 27$$

$$\frac{\sqrt{a^2 + 2abx + b^2x^2} \int \frac{(a+bx)^5}{(d+ex)^9} dx}{a + bx}$$

$$\downarrow 55$$

$$\frac{\sqrt{a^2 + 2abx + b^2x^2} \left(\frac{b \int \frac{(a+bx)^5}{(d+ex)^8} dx}{4(bd-ae)} + \frac{(a+bx)^6}{8(d+ex)^8(bd-ae)} \right)}{a + bx}$$

$$\downarrow 55$$

$$\frac{\sqrt{a^2 + 2abx + b^2x^2} \left(\frac{b \left(\frac{b \int \frac{(a+bx)^5}{(d+ex)^7} dx}{7(bd-ae)} + \frac{(a+bx)^6}{7(d+ex)^7(bd-ae)} \right)}{4(bd-ae)} + \frac{(a+bx)^6}{8(d+ex)^8(bd-ae)} \right)}{a + bx}$$

$$\frac{\sqrt{a^2 + 2abx + b^2x^2} \left(\frac{(a+bx)^6}{8(d+ex)^8(bd-ae)} + \frac{b \left(\frac{b(a+bx)^6}{42(d+ex)^6(bd-ae)^2} + \frac{(a+bx)^6}{7(d+ex)^7(bd-ae)} \right)}{4(bd-ae)} \right)}{a + bx}$$

input `Int[(a^2 + 2*a*b*x + b^2*x^2)^(5/2)/(d + e*x)^9,x]`

output `(Sqrt[a^2 + 2*a*b*x + b^2*x^2]*((a + b*x)^6/(8*(b*d - a*e)*(d + e*x)^8) + (b*((a + b*x)^6/(7*(b*d - a*e)*(d + e*x)^7) + (b*(a + b*x)^6)/(42*(b*d - a*e)^2*(d + e*x)^6)))/(4*(b*d - a*e)))/(a + b*x)`

Defintions of rubi rules used

rule 27 `Int[(a_)*(Fx_), x_Symbol] := Simp[a Int[Fx, x], x] /; FreeQ[a, x] && !MatchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]`

rule 48 `Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] := Simp[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{a, b, c, d, m, n}, x] && EqQ[m + n + 2, 0] && NeQ[m, -1]`

rule 55 `Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] := Simp[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*(Simplify[m + n + 2]/((b*c - a*d)*(m + 1))) Int[(a + b*x)^Simplify[m + 1]*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && ILtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] && !(LtQ[m, -1] && LtQ[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimplerQ[m, 1] || !SumSimplerQ[n, 1])`

rule 1102 `Int[((d_.) + (e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] := Simp[(a + b*x + c*x^2)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x)^(2*FracPart[p])) Int[(d + e*x)^m*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && EqQ[b^2 - 4*a*c, 0]`

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 261 vs. 2(110) = 220.

Time = 5.67 (sec) , antiderivative size = 262, normalized size of antiderivative = 1.76

| method | result |
|---------|---|
| risch | $\frac{\sqrt{(bx+a)^2} \left(-\frac{b^5 x^5}{3e} - \frac{5b^4(3ae+bd)x^4}{12e^2} - \frac{b^3(6e^2 a^2 + 3abde + b^2 d^2)x^3}{3e^3} - \frac{b^2(10e^3 a^3 + 6a^2 bde^2 + 3ab^2 d^2 e + b^3 d^3)x^2}{6e^4} - \frac{b(15a^4 e^4 + 10a^3 bde^3 + 6a^2 b^2 d^2 e^2 + 3ab^3 d^3 e + b^4 d^4)}{21e^5} \right)}{(bx+a)(ex+d)^8}$ |
| gospers | $-\frac{(56x^5 e^5 b^5 + 210x^4 a b^4 e^5 + 70x^4 b^5 d e^4 + 336x^3 a^2 b^3 e^5 + 168x^3 a b^4 d e^4 + 56x^3 b^5 d^2 e^3 + 280x^2 a^3 b^2 e^5 + 168x^2 a^2 b^3 d e^4 + 84x^2 a b^4 d^2 e^3 + 28x a^4 e^4 + 14a^3 b d e^3 + 7a^2 b^2 d^2 e^2 + 2a b^3 d^3 e + b^4 d^4)}{(bx+a)(ex+d)^8}$ |
| default | $-\frac{(56x^5 e^5 b^5 + 210x^4 a b^4 e^5 + 70x^4 b^5 d e^4 + 336x^3 a^2 b^3 e^5 + 168x^3 a b^4 d e^4 + 56x^3 b^5 d^2 e^3 + 280x^2 a^3 b^2 e^5 + 168x^2 a^2 b^3 d e^4 + 84x^2 a b^4 d^2 e^3 + 28x a^4 e^4 + 14a^3 b d e^3 + 7a^2 b^2 d^2 e^2 + 2a b^3 d^3 e + b^4 d^4)}{(bx+a)(ex+d)^8}$ |
| orering | $-\frac{(56x^5 e^5 b^5 + 210x^4 a b^4 e^5 + 70x^4 b^5 d e^4 + 336x^3 a^2 b^3 e^5 + 168x^3 a b^4 d e^4 + 56x^3 b^5 d^2 e^3 + 280x^2 a^3 b^2 e^5 + 168x^2 a^2 b^3 d e^4 + 84x^2 a b^4 d^2 e^3 + 28x a^4 e^4 + 14a^3 b d e^3 + 7a^2 b^2 d^2 e^2 + 2a b^3 d^3 e + b^4 d^4)}{(bx+a)(ex+d)^8}$ |

```
input int((b^2*x^2+2*a*b*x+a^2)^(5/2)/(e*x+d)^9,x,method=_RETURNVERBOSE)
```

```
output ((b*x+a)^2)^(1/2)/(b*x+a)*(-1/3*b^5/e*x^5-5/12*b^4/e^2*(3*a*e+b*d)*x^4-1/3/e^3*b^3*(6*a^2*e^2+3*a*b*d*e+b^2*d^2)*x^3-1/6*b^2/e^4*(10*a^3*e^3+6*a^2*b*d*e^2+3*a*b^2*d^2*e+b^3*d^3)*x^2-1/21*b/e^5*(15*a^4*e^4+10*a^3*b*d*e^3+6*a^2*b^2*d^2*e^2+3*a*b^3*d^3*e+b^4*d^4)*x-1/168/e^6*(21*a^5*e^5+15*a^4*b*d*e^4+10*a^3*b^2*d^2*e^3+6*a^2*b^3*d^3*e^2+3*a*b^4*d^4*e+b^5*d^5))/(e*x+d)^8
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 337 vs. 2(110) = 220.

Time = 0.09 (sec) , antiderivative size = 337, normalized size of antiderivative = 2.26

$$\int \frac{(a^2 + 2abx + b^2x^2)^{5/2}}{(d + ex)^9} dx = \frac{56 b^5 e^5 x^5 + b^5 d^5 + 3 a b^4 d^4 e + 6 a^2 b^3 d^3 e^2 + 10 a^3 b^2 d^2 e^3 + 15 a^4 b d e^4 + 21 a^5 e^5 + 70 (b^5 d e^4 + 3 a b^4 e^5) x^4 + \dots}{168 (e^{14} x^8 + 8 d e^{13} x^7 + 28 d^2 e^{12} x^6 + \dots)}$$

```
input integrate((b^2*x^2+2*a*b*x+a^2)^(5/2)/(e*x+d)^9,x, algorithm="fricas")
```

output

```
-1/168*(56*b^5*e^5*x^5 + b^5*d^5 + 3*a*b^4*d^4*e + 6*a^2*b^3*d^3*e^2 + 10*
a^3*b^2*d^2*e^3 + 15*a^4*b*d*e^4 + 21*a^5*e^5 + 70*(b^5*d*e^4 + 3*a*b^4*e^
5)*x^4 + 56*(b^5*d^2*e^3 + 3*a*b^4*d*e^4 + 6*a^2*b^3*e^5)*x^3 + 28*(b^5*d^
3*e^2 + 3*a*b^4*d^2*e^3 + 6*a^2*b^3*d*e^4 + 10*a^3*b^2*e^5)*x^2 + 8*(b^5*d
^4*e + 3*a*b^4*d^3*e^2 + 6*a^2*b^3*d^2*e^3 + 10*a^3*b^2*d*e^4 + 15*a^4*b*e
^5)*x)/(e^14*x^8 + 8*d*e^13*x^7 + 28*d^2*e^12*x^6 + 56*d^3*e^11*x^5 + 70*d
^4*e^10*x^4 + 56*d^5*e^9*x^3 + 28*d^6*e^8*x^2 + 8*d^7*e^7*x + d^8*e^6)
```

Sympy [F]

$$\int \frac{(a^2 + 2abx + b^2x^2)^{5/2}}{(d + ex)^9} dx = \int \frac{((a + bx)^2)^{5/2}}{(d + ex)^9} dx$$

input

```
integrate((b**2*x**2+2*a*b*x+a**2)**(5/2)/(e*x+d)**9,x)
```

output

```
Integral(((a + b*x)**2)**(5/2)/(d + e*x)**9, x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(a^2 + 2abx + b^2x^2)^{5/2}}{(d + ex)^9} dx = \text{Exception raised: ValueError}$$

input

```
integrate((b^2*x^2+2*a*b*x+a^2)^(5/2)/(e*x+d)^9,x, algorithm="maxima")
```

output

```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
help (example of legal syntax is 'assume(a*e-b*d>0)', see `assume?` for m
ore detail
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 452 vs. $2(110) = 220$.

Time = 0.30 (sec) , antiderivative size = 452, normalized size of antiderivative = 3.03

$$\int \frac{(a^2 + 2abx + b^2x^2)^{5/2}}{(d + ex)^9} dx = \frac{b^8 \operatorname{sgn}(bx + a)}{168 (b^3 d^3 e^6 - 3 a b^2 d^2 e^7 + 3 a^2 b d e^8 - a^3 e^9)} \\ - \frac{56 b^5 e^5 x^5 \operatorname{sgn}(bx + a) + 70 b^5 d e^4 x^4 \operatorname{sgn}(bx + a) + 210 a b^4 e^5 x^4 \operatorname{sgn}(bx + a) + 56 b^5 d^2 e^3 x^3 \operatorname{sgn}(bx + a) + 168$$

input `integrate((b^2*x^2+2*a*b*x+a^2)^(5/2)/(e*x+d)^9,x, algorithm="giac")`

output `1/168*b^8*sgn(b*x + a)/(b^3*d^3*e^6 - 3*a*b^2*d^2*e^7 + 3*a^2*b*d*e^8 - a^3*e^9) - 1/168*(56*b^5*e^5*x^5*sgn(b*x + a) + 70*b^5*d*e^4*x^4*sgn(b*x + a) + 210*a*b^4*e^5*x^4*sgn(b*x + a) + 56*b^5*d^2*e^3*x^3*sgn(b*x + a) + 168*a*b^4*d*e^4*x^3*sgn(b*x + a) + 336*a^2*b^3*e^5*x^3*sgn(b*x + a) + 28*b^5*d^3*e^2*x^2*sgn(b*x + a) + 84*a*b^4*d^2*e^3*x^2*sgn(b*x + a) + 168*a^2*b^3*d*e^4*x^2*sgn(b*x + a) + 280*a^3*b^2*e^5*x^2*sgn(b*x + a) + 8*b^5*d^4*e*x*sgn(b*x + a) + 24*a*b^4*d^3*e^2*x*sgn(b*x + a) + 48*a^2*b^3*d^2*e^3*x*sgn(b*x + a) + 80*a^3*b^2*d*e^4*x*sgn(b*x + a) + 120*a^4*b*e^5*x*sgn(b*x + a) + b^5*d^5*sgn(b*x + a) + 3*a*b^4*d^4*e*sgn(b*x + a) + 6*a^2*b^3*d^3*e^2*sgn(b*x + a) + 10*a^3*b^2*d^2*e^3*sgn(b*x + a) + 15*a^4*b*d*e^4*sgn(b*x + a) + 21*a^5*e^5*sgn(b*x + a))/((e*x + d)^8*e^6)`

Mupad [B] (verification not implemented)

Time = 5.41 (sec) , antiderivative size = 687, normalized size of antiderivative = 4.61

$$\int \frac{(a^2 + 2abx + b^2x^2)^{5/2}}{(d + ex)^9} dx = \frac{\left(\frac{4b^5d - 5ab^4e}{4e^6} + \frac{b^5d}{4e^6}\right) \sqrt{a^2 + 2abx + b^2x^2}}{(a + bx)(d + ex)^4}$$

$$\left(\frac{5a^4be^4 - 10a^3b^2de^3 + 10a^2b^3d^2e^2 - 5ab^4d^3e + b^5d^4}{7e^6} + \frac{d \left(\frac{b^5d}{7e^3} - \frac{b^4(5ae - bd)}{7e^3} \right) + b^3}{e} \right) \frac{\sqrt{a^2 + 2abx + b^2x^2}}{(a + bx)(d + ex)^7}$$

$$\left(\frac{10a^2b^3e^2 - 15ab^4de + 6b^5d^2}{5e^6} + \frac{d \left(\frac{b^5d}{5e^5} - \frac{b^4(5ae - 3bd)}{5e^5} \right)}{e} \right) \frac{\sqrt{a^2 + 2abx + b^2x^2}}{(a + bx)(d + ex)^5}$$

$$\left(\frac{a^5}{8e} - \frac{d \left(\frac{5a^4b}{8e} - \frac{d \left(\frac{5ab^4}{8e} - \frac{b^5d}{8e^2} \right) - \frac{5a^2b^3}{4e}}{e} + \frac{5a^3b^2}{4e} \right)}{e} \right) \frac{\sqrt{a^2 + 2abx + b^2x^2}}{(a + bx)(d + ex)^8}$$

$$\left(\frac{-10a^3b^2e^3 + 20a^2b^3de^2 - 15ab^4d^2e + 4b^5d^3}{6e^6} + \frac{d \left(\frac{b^5d}{6e^4} - \frac{b^4(5ae - 2bd)}{6e^4} \right) + b^3 \left(\frac{10a^2e^2 - 10abde + 3b^2d^2}{6e^5} \right)}{e} \right) \frac{\sqrt{a^2 + 2abx + b^2x^2}}{(a + bx)(d + ex)^6}$$

$$- \frac{b^5 \sqrt{a^2 + 2abx + b^2x^2}}{3e^6 (a + bx)(d + ex)^3}$$

3.306 $\int \frac{(a^2+2abx+b^2x^2)^{5/2}}{(d+ex)^{10}} dx$

| | |
|---|------|
| Optimal result | 2305 |
| Mathematica [A] (verified) | 2306 |
| Rubi [A] (verified) | 2306 |
| Maple [A] (verified) | 2308 |
| Fricas [B] (verification not implemented) | 2309 |
| Sympy [F] | 2310 |
| Maxima [F(-2)] | 2310 |
| Giac [B] (verification not implemented) | 2310 |
| Mupad [B] (verification not implemented) | 2312 |
| Reduce [B] (verification not implemented) | 2313 |

Optimal result

Integrand size = 28, antiderivative size = 200

$$\int \frac{(a^2 + 2abx + b^2x^2)^{5/2}}{(d + ex)^{10}} dx = \frac{(a + bx)^5 \sqrt{a^2 + 2abx + b^2x^2}}{9(bd - ae)(d + ex)^9} + \frac{b(a + bx)^5 \sqrt{a^2 + 2abx + b^2x^2}}{24(bd - ae)^2(d + ex)^8} + \frac{b^2(a + bx)^5 \sqrt{a^2 + 2abx + b^2x^2}}{84(bd - ae)^3(d + ex)^7} + \frac{b^3(a + bx)^5 \sqrt{a^2 + 2abx + b^2x^2}}{504(bd - ae)^4(d + ex)^6}$$

output $\frac{1}{9}(bx+a)^5((bx+a)^2)^{1/2}/(-ae+bd)/(ex+d)^9+1/24*b*(bx+a)^5*((bx+a)^2)^{1/2}/(-ae+bd)^2/(ex+d)^8+1/84*b^2*(bx+a)^5*((bx+a)^2)^{1/2}/(-ae+bd)^3/(ex+d)^7+1/504*b^3*(bx+a)^5*((bx+a)^2)^{1/2}/(-ae+bd)^4/(ex+d)^6$

Mathematica [A] (verified)

Time = 1.07 (sec) , antiderivative size = 223, normalized size of antiderivative = 1.12

$$\int \frac{(a^2 + 2abx + b^2x^2)^{5/2}}{(d + ex)^{10}} dx = \frac{\sqrt{(a + bx)^2(56a^5e^5 + 35a^4be^4(d + 9ex) + 20a^3b^2e^3(d^2 + 9dex + 36e^2x^2) + 10a^2b^3e^2(d^3 + 9d^2ex + 36dex^2 + 84d^2ex^2 + 84e^3x^3) + 4a^2b^4e(d^4 + 9d^3ex + 36d^2e^2x^2 + 84d^3ex^2 + 126e^4x^3) + b^5(d^5 + 9d^4ex + 36d^3e^2x^2 + 84d^4ex^2 + 126d^3e^3x^3 + 126d^2e^4x^4 + 126de^5x^5))}}{e^6(a + bx)(d + ex)^9}$$

input

```
Integrate[(a^2 + 2*a*b*x + b^2*x^2)^(5/2)/(d + e*x)^10,x]
```

output

```
-1/504*(Sqrt[(a + b*x)^2]*(56*a^5*e^5 + 35*a^4*b*e^4*(d + 9*e*x) + 20*a^3*b^2*e^3*(d^2 + 9*d*e*x + 36*e^2*x^2) + 10*a^2*b^3*e^2*(d^3 + 9*d^2*e*x + 36*d*e^2*x^2 + 84*e^3*x^3) + 4*a*b^4*e*(d^4 + 9*d^3*e*x + 36*d^2*e^2*x^2 + 84*d^3*e*x^2 + 126*e^4*x^3) + b^5*(d^5 + 9*d^4*e*x + 36*d^3*e^2*x^2 + 84*d^4*e*x^2 + 126*d^3*e^3*x^3 + 126*d^2*e^4*x^4 + 126*d*e^5*x^5)))/(e^6*(a + b*x)*(d + e*x)^9)
```

Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 174, normalized size of antiderivative = 0.87, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.214$, Rules used = {1102, 27, 55, 55, 55, 48}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\begin{aligned} & \int \frac{(a^2 + 2abx + b^2x^2)^{5/2}}{(d + ex)^{10}} dx \\ & \quad \downarrow 1102 \\ & \frac{\sqrt{a^2 + 2abx + b^2x^2} \int \frac{b^5(a+bx)^5}{(d+ex)^{10}} dx}{b^5(a + bx)} \\ & \quad \downarrow 27 \\ & \frac{\sqrt{a^2 + 2abx + b^2x^2} \int \frac{(a+bx)^5}{(d+ex)^{10}} dx}{a + bx} \end{aligned}$$

$$\begin{array}{c}
 \downarrow 55 \\
 \frac{\sqrt{a^2 + 2abx + b^2x^2} \left(\frac{b \int \frac{(a+bx)^5 dx}{(d+ex)^9} + \frac{(a+bx)^6}{9(d+ex)^9(bd-ae)}}{a+bx} \right)}{a+bx} \\
 \downarrow 55 \\
 \frac{\sqrt{a^2 + 2abx + b^2x^2} \left(\frac{b \left(\frac{b \int \frac{(a+bx)^5 dx}{(d+ex)^8} + \frac{(a+bx)^6}{8(d+ex)^8(bd-ae)} \right)}{3(bd-ae)} + \frac{(a+bx)^6}{9(d+ex)^9(bd-ae)} \right)}{a+bx} \\
 \downarrow 55 \\
 \frac{\sqrt{a^2 + 2abx + b^2x^2} \left(\frac{b \left(\frac{b \left(\frac{b \int \frac{(a+bx)^5 dx}{(d+ex)^7} + \frac{(a+bx)^6}{7(d+ex)^7(bd-ae)} \right)}{4(bd-ae)} + \frac{(a+bx)^6}{8(d+ex)^8(bd-ae)} \right)}{3(bd-ae)} + \frac{(a+bx)^6}{9(d+ex)^9(bd-ae)} \right)}{a+bx} \\
 \downarrow 48 \\
 \frac{\sqrt{a^2 + 2abx + b^2x^2} \left(\frac{(a+bx)^6}{9(d+ex)^9(bd-ae)} + \frac{b \left(\frac{(a+bx)^6}{8(d+ex)^8(bd-ae)} + \frac{b \left(\frac{b(a+bx)^6}{42(d+ex)^6(bd-ae)^2} + \frac{(a+bx)^6}{7(d+ex)^7(bd-ae)} \right)}{4(bd-ae)} \right)}{3(bd-ae)} \right)}{a+bx}
 \end{array}$$

input `Int[(a^2 + 2*a*b*x + b^2*x^2)^(5/2)/(d + e*x)^10,x]`

output `(Sqrt[a^2 + 2*a*b*x + b^2*x^2]*((a + b*x)^6/(9*(b*d - a*e)*(d + e*x)^9) + (b*((a + b*x)^6/(8*(b*d - a*e)*(d + e*x)^8) + (b*((a + b*x)^6/(7*(b*d - a*e)*(d + e*x)^7) + (b*(a + b*x)^6)/(42*(b*d - a*e)^2*(d + e*x)^6)))/(4*(b*d - a*e))))/(3*(b*d - a*e)))/(a + b*x)`

Defintions of rubi rules used

rule 27 `Int[(a_)*(Fx_), x_Symbol] := Simp[a Int[Fx, x], x] /; FreeQ[a, x] && !MatchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]`

rule 48 `Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] := Simp[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{a, b, c, d, m, n}, x] && EqQ[m + n + 2, 0] && NeQ[m, -1]`

rule 55 `Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] := Simp[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*(Simplify[m + n + 2]/((b*c - a*d)*(m + 1))) Int[(a + b*x)^Simplify[m + 1]*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && ILtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] && !(LtQ[m, -1] && LtQ[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimplerQ[m, 1] || !SumSimplerQ[n, 1])`

rule 1102 `Int[((d_.) + (e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] := Simp[(a + b*x + c*x^2)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x)^(2*FracPart[p])) Int[(d + e*x)^m*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && EqQ[b^2 - 4*a*c, 0]`

Maple [A] (verified)

Time = 8.11 (sec) , antiderivative size = 262, normalized size of antiderivative = 1.31

| method | result |
|---------|---|
| risch | $\frac{\sqrt{(bx+a)^2} \left(-\frac{b^5 x^5}{4e} - \frac{b^4(4ae+bd)x^4}{4e^2} - \frac{b^3(10e^2 a^2 + 4abde + b^2 d^2)x^3}{6e^3} - \frac{b^2(20e^3 a^3 + 10a^2 bde^2 + 4ab^2 d^2 e + b^3 d^3)x^2}{14e^4} - \frac{b(35a^4 e^4 + 20a^3 bde^3 + 10a^2 b^2 d^2 e^2 + 5ab^3 d^3 e + b^4 d^4)}{5e^5} \right)}{(bx+a)(ex+d)^9}$ |
| gospers | $-\frac{(126x^5 e^5 b^5 + 504x^4 a b^4 e^5 + 126x^4 b^5 d e^4 + 840x^3 a^2 b^3 e^5 + 336x^3 a b^4 d e^4 + 84x^3 b^5 d^2 e^3 + 720x^2 a^3 b^2 e^5 + 360x^2 a^2 b^3 d e^4 + 144x^2 a b^4 d^2 e^3 + 36x a^4 b^2 d^2 e^3 + 36x a^3 b^3 d^2 e^2 + 36x a^2 b^4 d^2 e + 36x a b^5 d^2 e + 36a^6 b^2 d^2 e^2 + 36a^5 b^3 d^2 e + 36a^4 b^4 d^2 e + 36a^3 b^5 d^2 e + 36a^2 b^6 d^2 e + 36a b^7 d^2 e + 36a^8 d^2 e)}{(bx+a)(ex+d)^9}$ |
| default | $-\frac{(126x^5 e^5 b^5 + 504x^4 a b^4 e^5 + 126x^4 b^5 d e^4 + 840x^3 a^2 b^3 e^5 + 336x^3 a b^4 d e^4 + 84x^3 b^5 d^2 e^3 + 720x^2 a^3 b^2 e^5 + 360x^2 a^2 b^3 d e^4 + 144x^2 a b^4 d^2 e^3 + 36x a^4 b^2 d^2 e^3 + 36x a^3 b^3 d^2 e^2 + 36x a^2 b^4 d^2 e + 36x a b^5 d^2 e + 36a^6 b^2 d^2 e^2 + 36a^5 b^3 d^2 e + 36a^4 b^4 d^2 e + 36a^3 b^5 d^2 e + 36a^2 b^6 d^2 e + 36a b^7 d^2 e + 36a^8 d^2 e)}{(bx+a)(ex+d)^9}$ |
| orering | $-\frac{(126x^5 e^5 b^5 + 504x^4 a b^4 e^5 + 126x^4 b^5 d e^4 + 840x^3 a^2 b^3 e^5 + 336x^3 a b^4 d e^4 + 84x^3 b^5 d^2 e^3 + 720x^2 a^3 b^2 e^5 + 360x^2 a^2 b^3 d e^4 + 144x^2 a b^4 d^2 e^3 + 36x a^4 b^2 d^2 e^3 + 36x a^3 b^3 d^2 e^2 + 36x a^2 b^4 d^2 e + 36x a b^5 d^2 e + 36a^6 b^2 d^2 e^2 + 36a^5 b^3 d^2 e + 36a^4 b^4 d^2 e + 36a^3 b^5 d^2 e + 36a^2 b^6 d^2 e + 36a b^7 d^2 e + 36a^8 d^2 e)}{(bx+a)(ex+d)^9}$ |

input `int((b^2*x^2+2*a*b*x+a^2)^(5/2)/(e*x+d)^10,x,method=_RETURNVERBOSE)`

output
$$\frac{((b*x+a)^2)^{(1/2)}/(b*x+a)*(-1/4*b^5/e*x^5-1/4/e^2*b^4*(4*a*e+b*d)*x^4-1/6*b^3/e^3*(10*a^2*e^2+4*a*b*d*e+b^2*d^2)*x^3-1/14*b^2/e^4*(20*a^3*e^3+10*a^2*b*d*e^2+4*a*b^2*d^2*e+b^3*d^3)*x^2-1/56*b/e^5*(35*a^4*e^4+20*a^3*b*d*e^3+10*a^2*b^2*d^2*e^2+4*a*b^3*d^3*e+b^4*d^4)*x-1/504/e^6*(56*a^5*e^5+35*a^4*b*d*e^4+20*a^3*b^2*d^2*e^3+10*a^2*b^3*d^3*e^2+4*a*b^4*d^4*e+b^5*d^5))/(e*x+d)^9$$

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 348 vs. $2(148) = 296$.

Time = 0.08 (sec) , antiderivative size = 348, normalized size of antiderivative = 1.74

$$\int \frac{(a^2 + 2abx + b^2x^2)^{5/2}}{(d + ex)^{10}} dx = \frac{126b^5e^5x^5 + b^5d^5 + 4ab^4d^4e + 10a^2b^3d^3e^2 + 20a^3b^2d^2e^3 + 35a^4bde^4 + 56a^5e^5 + 126(b^5de^4 + 4ab^4e^5)x^4}{504(e^{15}x^9 + 9de^{14}x^8 + 36d^2e^{13}x^7 + 84d^3e^{12}x^6 + 126d^4e^{11}x^5 + 126d^5e^{10}x^4 + 84d^6e^9x^3 + 36d^7e^8x^2 + 9d^8e^7x + d^9e^6)}$$

input `integrate((b^2*x^2+2*a*b*x+a^2)^(5/2)/(e*x+d)^10,x, algorithm="fricas")`

output
$$\frac{-1/504*(126*b^5*e^5*x^5 + b^5*d^5 + 4*a*b^4*d^4*e + 10*a^2*b^3*d^3*e^2 + 20*a^3*b^2*d^2*e^3 + 35*a^4*b*d*e^4 + 56*a^5*e^5 + 126*(b^5*d*e^4 + 4*a*b^4*e^5)*x^4 + 84*(b^5*d^2*e^3 + 4*a*b^4*d*e^4 + 10*a^2*b^3*e^5)*x^3 + 36*(b^5*d^3*e^2 + 4*a*b^4*d^2*e^3 + 10*a^2*b^3*d*e^4 + 20*a^3*b^2*d^2*e^5)*x^2 + 9*(b^5*d^4*e + 4*a*b^4*d^3*e^2 + 10*a^2*b^3*d^2*e^3 + 20*a^3*b^2*d*e^4 + 35*a^4*b*e^5)*x)/(e^{15}*x^9 + 9*d*e^{14}*x^8 + 36*d^2*e^{13}*x^7 + 84*d^3*e^{12}*x^6 + 126*d^4*e^{11}*x^5 + 126*d^5*e^{10}*x^4 + 84*d^6*e^9*x^3 + 36*d^7*e^8*x^2 + 9*d^8*e^7*x + d^9*e^6)}$$

Sympy [F]

$$\int \frac{(a^2 + 2abx + b^2x^2)^{5/2}}{(d + ex)^{10}} dx = \int \frac{((a + bx)^2)^{5/2}}{(d + ex)^{10}} dx$$

input `integrate((b**2*x**2+2*a*b*x+a**2)**(5/2)/(e*x+d)**10,x)`

output `Integral(((a + b*x)**2)**(5/2)/(d + e*x)**10, x)`

Maxima [F(-2)]

Exception generated.

$$\int \frac{(a^2 + 2abx + b^2x^2)^{5/2}}{(d + ex)^{10}} dx = \text{Exception raised: ValueError}$$

input `integrate((b^2*x^2+2*a*b*x+a^2)^(5/2)/(e*x+d)^10,x, algorithm="maxima")`

output `Exception raised: ValueError >> Computation failed since Maxima requested additional constraints; using the 'assume' command before evaluation *may* help (example of legal syntax is 'assume(a*e-b*d>0)', see `assume?` for more detail`

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 465 vs. $2(148) = 296$.

Time = 0.35 (sec) , antiderivative size = 465, normalized size of antiderivative = 2.32

$$\int \frac{(a^2 + 2abx + b^2x^2)^{5/2}}{(d + ex)^{10}} dx = \frac{b^9 \operatorname{sgn}(bx + a)}{504 (b^4 d^4 e^6 - 4 ab^3 d^3 e^7 + 6 a^2 b^2 d^2 e^8 - 4 a^3 b d e^9 + a^4 e^{10})} - \frac{126 b^5 e^5 x^5 \operatorname{sgn}(bx + a) + 126 b^5 d e^4 x^4 \operatorname{sgn}(bx + a) + 504 ab^4 e^5 x^4 \operatorname{sgn}(bx + a) + 84 b^5 d^2 e^3 x^3 \operatorname{sgn}(bx + a) + 3}{(d + ex)^{10}}$$

input `integrate((b^2*x^2+2*a*b*x+a^2)^(5/2)/(e*x+d)^10,x, algorithm="giac")`

output
$$\frac{1/504*b^9*sgn(b*x + a)/(b^4*d^4*e^6 - 4*a*b^3*d^3*e^7 + 6*a^2*b^2*d^2*e^8 - 4*a^3*b*d*e^9 + a^4*e^{10}) - 1/504*(126*b^5*e^5*x^5*sgn(b*x + a) + 126*b^5*d*e^4*x^4*sgn(b*x + a) + 504*a*b^4*e^5*x^4*sgn(b*x + a) + 84*b^5*d^2*e^3*x^3*sgn(b*x + a) + 336*a*b^4*d*e^4*x^3*sgn(b*x + a) + 840*a^2*b^3*e^5*x^3*sgn(b*x + a) + 36*b^5*d^3*e^2*x^2*sgn(b*x + a) + 144*a*b^4*d^2*e^3*x^2*sgn(b*x + a) + 360*a^2*b^3*d*e^4*x^2*sgn(b*x + a) + 720*a^3*b^2*e^5*x^2*sgn(b*x + a) + 9*b^5*d^4*e*x*sgn(b*x + a) + 36*a*b^4*d^3*e^2*x*sgn(b*x + a) + 90*a^2*b^3*d^2*e^3*x*sgn(b*x + a) + 180*a^3*b^2*d*e^4*x*sgn(b*x + a) + 315*a^4*b*e^5*x*sgn(b*x + a) + b^5*d^5*sgn(b*x + a) + 4*a*b^4*d^4*e*sgn(b*x + a) + 10*a^2*b^3*d^3*e^2*sgn(b*x + a) + 20*a^3*b^2*d^2*e^3*sgn(b*x + a) + 35*a^4*b*d*e^4*sgn(b*x + a) + 56*a^5*e^5*sgn(b*x + a))/((e*x + d)^9*e^6)}$$

Mupad [B] (verification not implemented)

Time = 5.36 (sec) , antiderivative size = 687, normalized size of antiderivative = 3.44

$$\int \frac{(a^2 + 2abx + b^2x^2)^{5/2}}{(d + ex)^{10}} dx = \frac{\left(\frac{4b^5d - 5ab^4e}{5e^6} + \frac{b^5d}{5e^6}\right) \sqrt{a^2 + 2abx + b^2x^2}}{(a + bx)(d + ex)^5}$$

$$\left(\frac{5a^4be^4 - 10a^3b^2de^3 + 10a^2b^3d^2e^2 - 5ab^4d^3e + b^5d^4}{8e^6} + \frac{d \left(\frac{b^5d}{8e^3} - \frac{b^4(5ae - bd)}{8e^3} \right) + b^3}{e} \right) \frac{\sqrt{a^2 + 2abx + b^2x^2}}{(a + bx)(d + ex)^8}$$

$$\left(\frac{10a^2b^3e^2 - 15ab^4de + 6b^5d^2}{6e^6} + \frac{d \left(\frac{b^5d}{6e^5} - \frac{b^4(5ae - 3bd)}{6e^5} \right)}{e} \right) \frac{\sqrt{a^2 + 2abx + b^2x^2}}{(a + bx)(d + ex)^6}$$

$$\left(\frac{a^5}{9e} - \frac{d \left(\frac{5a^4b}{9e} - \frac{d \left(\frac{d \left(\frac{5ab^4}{9e} - \frac{b^5d}{9e^2} \right) - \frac{10a^2b^3}{9e} \right) + \frac{10a^3b^2}{9e}}{e} \right)}{e} \right) \frac{\sqrt{a^2 + 2abx + b^2x^2}}{(a + bx)(d + ex)^9}$$

$$+ \left(\frac{-10a^3b^2e^3 + 20a^2b^3de^2 - 15ab^4d^2e + 4b^5d^3}{7e^6} + \frac{d \left(\frac{b^5d}{7e^4} - \frac{b^4(5ae - 2bd)}{7e^4} \right) + \frac{b^3(10a^2e^2 - 10abde + 3b^2d^2)}{7e^5}}{e} \right) \frac{\sqrt{a^2 + 2abx + b^2x^2}}{(a + bx)(d + ex)^7}$$

$$- \frac{b^5 \sqrt{a^2 + 2abx + b^2x^2}}{4e^6 (a + bx)(d + ex)^4}$$

3.307 $\int \frac{(a^2+2abx+b^2x^2)^{5/2}}{(d+ex)^{11}} dx$

| | |
|---|------|
| Optimal result | 2314 |
| Mathematica [A] (verified) | 2315 |
| Rubi [A] (verified) | 2315 |
| Maple [A] (verified) | 2317 |
| Fricas [A] (verification not implemented) | 2317 |
| Sympy [F(-1)] | 2318 |
| Maxima [F(-2)] | 2318 |
| Giac [B] (verification not implemented) | 2319 |
| Mupad [B] (verification not implemented) | 2320 |
| Reduce [B] (verification not implemented) | 2321 |

Optimal result

Integrand size = 28, antiderivative size = 308

$$\int \frac{(a^2 + 2abx + b^2x^2)^{5/2}}{(d + ex)^{11}} dx = \frac{(bd - ae)^5 \sqrt{a^2 + 2abx + b^2x^2}}{10e^6(a + bx)(d + ex)^{10}} - \frac{5b(bd - ae)^4 \sqrt{a^2 + 2abx + b^2x^2}}{9e^6(a + bx)(d + ex)^9} + \frac{5b^2(bd - ae)^3 \sqrt{a^2 + 2abx + b^2x^2}}{4e^6(a + bx)(d + ex)^8} - \frac{10b^3(bd - ae)^2 \sqrt{a^2 + 2abx + b^2x^2}}{7e^6(a + bx)(d + ex)^7} + \frac{5b^4(bd - ae) \sqrt{a^2 + 2abx + b^2x^2}}{6e^6(a + bx)(d + ex)^6} - \frac{b^5 \sqrt{a^2 + 2abx + b^2x^2}}{5e^6(a + bx)(d + ex)^5}$$

output

```
1/10*(-a*e+b*d)^5*((b*x+a)^2)^(1/2)/e^6/(b*x+a)/(e*x+d)^10-5/9*b*(-a*e+b*d)^4*((b*x+a)^2)^(1/2)/e^6/(b*x+a)/(e*x+d)^9+5/4*b^2*(-a*e+b*d)^3*((b*x+a)^2)^(1/2)/e^6/(b*x+a)/(e*x+d)^8-10/7*b^3*(-a*e+b*d)^2*((b*x+a)^2)^(1/2)/e^6/(b*x+a)/(e*x+d)^7+5/6*b^4*(-a*e+b*d)*((b*x+a)^2)^(1/2)/e^6/(b*x+a)/(e*x+d)^6-1/5*b^5*((b*x+a)^2)^(1/2)/e^6/(b*x+a)/(e*x+d)^5
```

Mathematica [A] (verified)

Time = 1.07 (sec) , antiderivative size = 223, normalized size of antiderivative = 0.72

$$\int \frac{(a^2 + 2abx + b^2x^2)^{5/2}}{(d + ex)^{11}} dx = \frac{\sqrt{(a + bx)^2(126a^5e^5 + 70a^4be^4(d + 10ex) + 35a^3b^2e^3(d^2 + 10dex + 45e^2x^2) + 15a^2b^3e^2(d^3 + 10d^2ex + 45d^2ex + 45d^2ex + 120d^2ex + 120d^2ex + 120d^2ex + 210d^2ex + 210d^2ex + 252e^5x^5))}}{e^6(a + bx)(d + ex)^{10}}$$

input

```
Integrate[(a^2 + 2*a*b*x + b^2*x^2)^(5/2)/(d + e*x)^11,x]
```

output

```
-1/1260*(Sqrt[(a + b*x)^2]*(126*a^5*e^5 + 70*a^4*b*e^4*(d + 10*e*x) + 35*a^3*b^2*e^3*(d^2 + 10*d*e*x + 45*e^2*x^2) + 15*a^2*b^3*e^2*(d^3 + 10*d^2*e*x + 45*d*e^2*x^2 + 120*e^3*x^3) + 5*a*b^4*e*(d^4 + 10*d^3*e*x + 45*d^2*e^2*x^2 + 120*d*e^3*x^3 + 210*e^4*x^4) + b^5*(d^5 + 10*d^4*e*x + 45*d^3*e^2*x^2 + 120*d^2*e^3*x^3 + 210*d*e^4*x^4 + 252*e^5*x^5)))/(e^6*(a + b*x)*(d + e*x)^10)
```

Rubi [A] (verified)

Time = 0.58 (sec) , antiderivative size = 174, normalized size of antiderivative = 0.56, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {1102, 27, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\begin{aligned} & \int \frac{(a^2 + 2abx + b^2x^2)^{5/2}}{(d + ex)^{11}} dx \\ & \quad \downarrow \text{1102} \\ & \frac{\sqrt{a^2 + 2abx + b^2x^2} \int \frac{b^5(a+bx)^5}{(d+ex)^{11}} dx}{b^5(a + bx)} \\ & \quad \downarrow \text{27} \\ & \frac{\sqrt{a^2 + 2abx + b^2x^2} \int \frac{(a+bx)^5}{(d+ex)^{11}} dx}{a + bx} \end{aligned}$$

↓ 53

$$\frac{\sqrt{a^2 + 2abx + b^2x^2} \int \left(\frac{b^5}{e^5(d+ex)^6} - \frac{5(bd-ae)b^4}{e^5(d+ex)^7} + \frac{10(bd-ae)^2b^3}{e^5(d+ex)^8} - \frac{10(bd-ae)^3b^2}{e^5(d+ex)^9} + \frac{5(bd-ae)^4b}{e^5(d+ex)^{10}} + \frac{(ae-bd)^5}{e^5(d+ex)^{11}} \right) dx}{a + bx}$$

↓ 2009

$$\frac{\sqrt{a^2 + 2abx + b^2x^2} \left(\frac{5b^4(bd-ae)}{6e^6(d+ex)^6} - \frac{10b^3(bd-ae)^2}{7e^6(d+ex)^7} + \frac{5b^2(bd-ae)^3}{4e^6(d+ex)^8} - \frac{5b(bd-ae)^4}{9e^6(d+ex)^9} + \frac{(bd-ae)^5}{10e^6(d+ex)^{10}} - \frac{b^5}{5e^6(d+ex)^5} \right)}{a + bx}$$

input

```
Int[(a^2 + 2*a*b*x + b^2*x^2)^(5/2)/(d + e*x)^11,x]
```

output

```
(Sqrt[a^2 + 2*a*b*x + b^2*x^2]*((b*d - a*e)^5/(10*e^6*(d + e*x)^10) - (5*b
*(b*d - a*e)^4)/(9*e^6*(d + e*x)^9) + (5*b^2*(b*d - a*e)^3)/(4*e^6*(d + e
*x)^8) - (10*b^3*(b*d - a*e)^2)/(7*e^6*(d + e*x)^7) + (5*b^4*(b*d - a*e))/(
6*e^6*(d + e*x)^6) - b^5/(5*e^6*(d + e*x)^5)))/(a + b*x)
```

Defintions of rubi rules used

rule 27

```
Int[(a_)*(F_x_), x_Symbol] := Simp[a Int[F_x, x], x] /; FreeQ[a, x] && !Ma
tchQ[F_x, (b_)*(G_x_)] /; FreeQ[b, x]
```

rule 53

```
Int[((a_) + (b_)*(x_))^(m_)*((c_) + (d_)*(x_))^(n_), x_Symbol] := Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && (!IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 1102

```
Int[((d_) + (e_)*(x_))^(m_)*((a_) + (b_)*(x_) + (c_)*(x_)^2)^(p_), x_S
ymbol] := Simp[(a + b*x + c*x^2)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x)^(2*F
racPart[p])) Int[(d + e*x)^m*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c,
d, e, m, p}, x] && EqQ[b^2 - 4*a*c, 0]
```

rule 2009

```
Int[u_, x_Symbol] := Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 11.46 (sec) , antiderivative size = 262, normalized size of antiderivative = 0.85

| method | result |
|---------|---|
| risch | $\frac{\sqrt{(bx+a)^2} \left(-\frac{b^5 x^5}{5e} - \frac{b^4(5ae+bd)x^4}{6e^2} - \frac{2b^3(15e^2 a^2+5abde+b^2 d^2)x^3}{21e^3} - \frac{b^2(35e^3 a^3+15a^2 bde^2+5a b^2 d^2 e+b^3 d^3)x^2}{28e^4} - \frac{b(70a^4 e^4+35a^3 bde^3+15a^2 b^2 d^2 e^2+5a b^3 d^3 e+b^4 d^4)}{(bx+a)(ex+d)^{10}} \right)}{(bx+a)(ex+d)^{10}}$ |
| gospers | $-\frac{(252x^5 e^5 b^5 + 1050x^4 a b^4 e^5 + 210x^4 b^5 d e^4 + 1800x^3 a^2 b^3 e^5 + 600x^3 a b^4 d e^4 + 120x^3 b^5 d^2 e^3 + 1575x^2 a^3 b^2 e^5 + 675x^2 a^2 b^3 d e^4 + 225x^2 a^3 b d^2 e^3 + 105x^2 a^2 b^2 d^2 e^4 + 35x^2 a b^3 d^3 e^4 + 5x^2 a^2 b^2 d^3 e^5 + 5x^2 a b^3 d^4 e^5 + 5x^2 a^2 b^2 d^4 e^5 + 5x^2 a b^3 d^5 e^5)}{(ex+d)^{10}}$ |
| default | $-\frac{(252x^5 e^5 b^5 + 1050x^4 a b^4 e^5 + 210x^4 b^5 d e^4 + 1800x^3 a^2 b^3 e^5 + 600x^3 a b^4 d e^4 + 120x^3 b^5 d^2 e^3 + 1575x^2 a^3 b^2 e^5 + 675x^2 a^2 b^3 d e^4 + 225x^2 a^3 b d^2 e^3 + 105x^2 a^2 b^2 d^2 e^4 + 35x^2 a b^3 d^3 e^4 + 5x^2 a^2 b^2 d^3 e^5 + 5x^2 a b^3 d^4 e^5 + 5x^2 a^2 b^2 d^4 e^5 + 5x^2 a b^3 d^5 e^5)}{(ex+d)^{10}}$ |
| orering | $-\frac{(252x^5 e^5 b^5 + 1050x^4 a b^4 e^5 + 210x^4 b^5 d e^4 + 1800x^3 a^2 b^3 e^5 + 600x^3 a b^4 d e^4 + 120x^3 b^5 d^2 e^3 + 1575x^2 a^3 b^2 e^5 + 675x^2 a^2 b^3 d e^4 + 225x^2 a^3 b d^2 e^3 + 105x^2 a^2 b^2 d^2 e^4 + 35x^2 a b^3 d^3 e^4 + 5x^2 a^2 b^2 d^3 e^5 + 5x^2 a b^3 d^4 e^5 + 5x^2 a^2 b^2 d^4 e^5 + 5x^2 a b^3 d^5 e^5)}{(ex+d)^{10}}$ |

input

```
int((b^2*x^2+2*a*b*x+a^2)^(5/2)/(e*x+d)^11,x,method=_RETURNVERBOSE)
```

output

```
((b*x+a)^2)^(1/2)/(b*x+a)*(-1/5*b^5/e*x^5-1/6*b^4/e^2*(5*a*e+b*d)*x^4-2/21/e^3*b^3*(15*a^2*e^2+5*a*b*d*e+b^2*d^2)*x^3-1/28*b^2/e^4*(35*a^3*e^3+15*a^2*b*d*e^2+5*a*b^2*d^2*e+b^3*d^3)*x^2-1/126*b/e^5*(70*a^4*e^4+35*a^3*b*d*e^3+15*a^2*b^2*d^2*e^2+5*a*b^3*d^3*e+b^4*d^4)*x-1/1260/e^6*(126*a^5*e^5+70*a^4*b*d*e^4+35*a^3*b^2*d^2*e^3+15*a^2*b^3*d^3*e^2+5*a*b^4*d^4*e+b^5*d^5))/(e*x+d)^10
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 359, normalized size of antiderivative = 1.17

$$\int \frac{(a^2 + 2abx + b^2x^2)^{5/2}}{(d + ex)^{11}} dx = \frac{252 b^5 e^5 x^5 + b^5 d^5 + 5 ab^4 d^4 e + 15 a^2 b^3 d^3 e^2 + 35 a^3 b^2 d^2 e^3 + 70 a^4 b d e^4 + 126 a^5 e^5 + 210 (b^5 d e^4 + 5 ab^4 e^5)}{1260 (e^{16} x^{10} + 10 d e^{15} x^9 + 45 d^2 e^{14} x^8 + 120 d^3 e^{13} x^7 + 105 d^4 e^{12} x^6 + 45 d^5 e^{11} x^5 + 15 d^6 e^{10} x^4 + 5 d^7 e^9 x^3 + d^8 e^8 x^2 + d^9 e^7 x + d^{10} e^6)}$$

input

```
integrate((b^2*x^2+2*a*b*x+a^2)^(5/2)/(e*x+d)^11,x, algorithm="fricas")
```

output

```
-1/1260*(252*b^5*e^5*x^5 + b^5*d^5 + 5*a*b^4*d^4*e + 15*a^2*b^3*d^3*e^2 +
35*a^3*b^2*d^2*e^3 + 70*a^4*b*d*e^4 + 126*a^5*e^5 + 210*(b^5*d*e^4 + 5*a*b
^4*e^5)*x^4 + 120*(b^5*d^2*e^3 + 5*a*b^4*d*e^4 + 15*a^2*b^3*e^5)*x^3 + 45*
(b^5*d^3*e^2 + 5*a*b^4*d^2*e^3 + 15*a^2*b^3*d*e^4 + 35*a^3*b^2*e^5)*x^2 +
10*(b^5*d^4*e + 5*a*b^4*d^3*e^2 + 15*a^2*b^3*d^2*e^3 + 35*a^3*b^2*d*e^4 +
70*a^4*b*e^5)*x)/(e^16*x^10 + 10*d*e^15*x^9 + 45*d^2*e^14*x^8 + 120*d^3*e^
13*x^7 + 210*d^4*e^12*x^6 + 252*d^5*e^11*x^5 + 210*d^6*e^10*x^4 + 120*d^7*
e^9*x^3 + 45*d^8*e^8*x^2 + 10*d^9*e^7*x + d^10*e^6)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(a^2 + 2abx + b^2x^2)^{5/2}}{(d + ex)^{11}} dx = \text{Timed out}$$

input

```
integrate((b**2*x**2+2*a*b*x+a**2)**(5/2)/(e*x+d)**11,x)
```

output

Timed out

Maxima [F(-2)]

Exception generated.

$$\int \frac{(a^2 + 2abx + b^2x^2)^{5/2}}{(d + ex)^{11}} dx = \text{Exception raised: ValueError}$$

input

```
integrate((b^2*x^2+2*a*b*x+a^2)^(5/2)/(e*x+d)^11,x, algorithm="maxima")
```

output

```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
help (example of legal syntax is 'assume(a*e-b*d>0)', see `assume?` for m
ore detail
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 480 vs. $2(230) = 460$.

Time = 0.32 (sec) , antiderivative size = 480, normalized size of antiderivative = 1.56

$$\int \frac{(a^2 + 2abx + b^2x^2)^{5/2}}{(d + ex)^{11}} dx = \frac{b^{10} \operatorname{sgn}(bx + a)}{1260 (b^5 d^5 e^6 - 5 ab^4 d^4 e^7 + 10 a^2 b^3 d^3 e^8 - 10 a^3 b^2 d^2 e^9 + 5 a^4 b d e^{10} - a^5 e^{11})} - \frac{252 b^5 e^5 x^5 \operatorname{sgn}(bx + a) + 210 b^5 d e^4 x^4 \operatorname{sgn}(bx + a) + 1050 ab^4 e^5 x^4 \operatorname{sgn}(bx + a) + 120 b^5 d^2 e^3 x^3 \operatorname{sgn}(bx + a) -$$

input `integrate((b^2*x^2+2*a*b*x+a^2)^(5/2)/(e*x+d)^11,x, algorithm="giac")`

output `1/1260*b^10*sgn(b*x + a)/(b^5*d^5*e^6 - 5*a*b^4*d^4*e^7 + 10*a^2*b^3*d^3*e^8 - 10*a^3*b^2*d^2*e^9 + 5*a^4*b*d*e^10 - a^5*e^11) - 1/1260*(252*b^5*e^5*x^5*sgn(b*x + a) + 210*b^5*d*e^4*x^4*sgn(b*x + a) + 1050*a*b^4*e^5*x^4*sgn(b*x + a) + 120*b^5*d^2*e^3*x^3*sgn(b*x + a) + 600*a*b^4*d*e^4*x^3*sgn(b*x + a) + 1800*a^2*b^3*e^5*x^3*sgn(b*x + a) + 45*b^5*d^3*e^2*x^2*sgn(b*x + a) + 225*a*b^4*d^2*e^3*x^2*sgn(b*x + a) + 675*a^2*b^3*d*e^4*x^2*sgn(b*x + a) + 1575*a^3*b^2*e^5*x^2*sgn(b*x + a) + 10*b^5*d^4*e*x*sgn(b*x + a) + 50*a*b^4*d^3*e^2*x*sgn(b*x + a) + 150*a^2*b^3*d^2*e^3*x*sgn(b*x + a) + 350*a^3*b^2*d*e^4*x*sgn(b*x + a) + 700*a^4*b*e^5*x*sgn(b*x + a) + b^5*d^5*sgn(b*x + a) + 5*a*b^4*d^4*e*sgn(b*x + a) + 15*a^2*b^3*d^3*e^2*sgn(b*x + a) + 35*a^3*b^2*d^2*e^3*sgn(b*x + a) + 70*a^4*b*d*e^4*sgn(b*x + a) + 126*a^5*e^5*sgn(b*x + a))/((e*x + d)^10*e^6)`

Mupad [B] (verification not implemented)

Time = 5.20 (sec) , antiderivative size = 686, normalized size of antiderivative = 2.23

$$\int \frac{(a^2 + 2abx + b^2x^2)^{5/2}}{(d + ex)^{11}} dx = \frac{\left(\frac{4b^5d - 5ab^4e}{6e^6} + \frac{b^5d}{6e^6}\right) \sqrt{a^2 + 2abx + b^2x^2}}{(a + bx)(d + ex)^6}$$

$$\left(\frac{5a^4be^4 - 10a^3b^2de^3 + 10a^2b^3d^2e^2 - 5ab^4d^3e + b^5d^4}{9e^6} + \frac{d \left(\frac{b^5d}{9e^3} - \frac{b^4(5ae - bd)}{9e^3} \right) + b^3}{e} \right) \frac{\sqrt{a^2 + 2abx + b^2x^2}}{(a + bx)(d + ex)^9}$$

$$\left(\frac{10a^2b^3e^2 - 15ab^4de + 6b^5d^2}{7e^6} + \frac{d \left(\frac{b^5d}{7e^5} - \frac{b^4(5ae - 3bd)}{7e^5} \right)}{e} \right) \frac{\sqrt{a^2 + 2abx + b^2x^2}}{(a + bx)(d + ex)^7}$$

$$\left(\frac{a^5}{10e} - \frac{d \left(\frac{a^4b}{2e} - \frac{d \left(\frac{d \left(\frac{ab^4}{2e} - \frac{b^5d}{10e^2} \right) - \frac{a^2b^3}{e} \right) + a^3b^2}{e} \right)}{e} \right) \frac{\sqrt{a^2 + 2abx + b^2x^2}}{(a + bx)(d + ex)^{10}}$$

$$+ \left(\frac{-10a^3b^2e^3 + 20a^2b^3de^2 - 15ab^4d^2e + 4b^5d^3}{8e^6} + \frac{d \left(\frac{b^5d}{8e^4} - \frac{b^4(5ae - 2bd)}{8e^4} \right) + b^3(10a^2e^2 - 10abde + 3b^2d^2)}{e} \right) \frac{\sqrt{a^2 + 2abx + b^2x^2}}{(a + bx)(d + ex)^8}$$

$$- \frac{b^5 \sqrt{a^2 + 2abx + b^2x^2}}{5e^6 (a + bx)(d + ex)^5}$$

input `int((a^2 + b^2*x^2 + 2*a*b*x)^(5/2)/(d + e*x)^11,x)`

output
$$\begin{aligned} & \left(\frac{4b^5d - 5ab^4e}{6e^6} + \frac{b^5d}{6e^6} \right) (a^2 + b^2x^2 + 2abx)^{1/2} / \left((a + bx)(d + ex)^6 - \left(\frac{b^5d^4 + 5a^4b^4e^4 - 10a^3b^2d^2e^3 + 10a^2b^3d^2e^2 - 5ab^4d^3e}{9e^6} + \frac{d(b^5d^3e - 10a^3b^2e^4 - 5ab^4d^2e^2 + 10a^2b^3d^2e^3)}{9e^6} + \frac{d(d(b^5d)}{9e^3} - \frac{b^4(5ae - bd)}{9e^3}) \right) / e + \frac{b^3(10a^2e^2 + b^2d^2 - 5abd^2e)}{9e^4} \right) / e \Big) / e (a^2 + b^2x^2 + 2abx)^{1/2} / \left((a + bx)(d + ex)^9 - \left(\frac{6b^5d^2 + 10a^2b^3e^2 - 15ab^4d^2e}{7e^6} + \frac{d(b^5d)}{7e^5} - \frac{b^4(5ae - 3bd)}{7e^5} \right) / e \right) (a^2 + b^2x^2 + 2abx)^{1/2} / \left((a + bx)(d + ex)^7 - \left(\frac{a^5}{10e} - \frac{d(a^4b)}{2e} - \frac{d(d(d(ab^4)}{2e} - \frac{b^5d}{10e^2}))}{e} - \frac{a^2b^3}{e} \right) / e + \frac{a^3b^2}{e} \right) / e \Big) / e (a^2 + b^2x^2 + 2abx)^{1/2} / \left((a + bx)(d + ex)^{10} + \left(\frac{4b^5d^3 - 10a^3b^2e^3 + 20a^2b^3d^2e^2 - 15ab^4d^2e}{8e^6} + \frac{d(d(d(b^5d)}{8e^4} - \frac{b^4(5ae - 2bd)}{8e^4})}{e} + \frac{b^3(10a^2e^2 + 3b^2d^2 - 10abd^2e)}{8e^5} \right) / e \right) (a^2 + b^2x^2 + 2abx)^{1/2} / \left((a + bx)(d + ex)^8 - \frac{b^5(a^2 + b^2x^2 + 2abx)^{1/2}}{5e^6(a + bx)(d + ex)^5} \right) \end{aligned}$$

Reduce [B] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 371, normalized size of antiderivative = 1.20

$$\int \frac{(a^2 + 2abx + b^2x^2)^{5/2}}{(d + ex)^{11}} dx = \frac{-252b^5e^5x^5 - 1050ab^4e^5x^4 - 210b^5de^4x^4 - 1800a^2b^3e^5x^3 - 600ab^4de^4x^3 - \dots}{(d + ex)^{11}}$$

input `int((b^2*x^2+2*a*b*x+a^2)^(5/2)/(e*x+d)^11,x)`

output

```
( - 126*a**5*e**5 - 70*a**4*b*d*e**4 - 700*a**4*b*e**5*x - 35*a**3*b**2*d*
*2*e**3 - 350*a**3*b**2*d*e**4*x - 1575*a**3*b**2*e**5*x**2 - 15*a**2*b**3
*d**3*e**2 - 150*a**2*b**3*d**2*e**3*x - 675*a**2*b**3*d*e**4*x**2 - 1800*
a**2*b**3*e**5*x**3 - 5*a*b**4*d**4*e - 50*a*b**4*d**3*e**2*x - 225*a*b**4
*d**2*e**3*x**2 - 600*a*b**4*d*e**4*x**3 - 1050*a*b**4*e**5*x**4 - b**5*d*
*5 - 10*b**5*d**4*e*x - 45*b**5*d**3*e**2*x**2 - 120*b**5*d**2*e**3*x**3 -
210*b**5*d*e**4*x**4 - 252*b**5*e**5*x**5)/(1260*e**6*(d**10 + 10*d**9*e*
x + 45*d**8*e**2*x**2 + 120*d**7*e**3*x**3 + 210*d**6*e**4*x**4 + 252*d**5
*e**5*x**5 + 210*d**4*e**6*x**6 + 120*d**3*e**7*x**7 + 45*d**2*e**8*x**8 +
10*d*e**9*x**9 + e**10*x**10))
```

3.308 $\int \frac{(a^2+2abx+b^2x^2)^{5/2}}{(d+ex)^{12}} dx$

| | |
|---|------|
| Optimal result | 2323 |
| Mathematica [A] (verified) | 2324 |
| Rubi [A] (verified) | 2324 |
| Maple [A] (verified) | 2326 |
| Fricas [A] (verification not implemented) | 2326 |
| Sympy [F(-2)] | 2327 |
| Maxima [F(-2)] | 2327 |
| Giac [B] (verification not implemented) | 2328 |
| Mupad [B] (verification not implemented) | 2329 |
| Reduce [B] (verification not implemented) | 2330 |

Optimal result

Integrand size = 28, antiderivative size = 308

$$\int \frac{(a^2 + 2abx + b^2x^2)^{5/2}}{(d + ex)^{12}} dx = \frac{(bd - ae)^5 \sqrt{a^2 + 2abx + b^2x^2}}{11e^6(a + bx)(d + ex)^{11}} - \frac{b(bd - ae)^4 \sqrt{a^2 + 2abx + b^2x^2}}{2e^6(a + bx)(d + ex)^{10}} + \frac{10b^2(bd - ae)^3 \sqrt{a^2 + 2abx + b^2x^2}}{9e^6(a + bx)(d + ex)^9} - \frac{5b^3(bd - ae)^2 \sqrt{a^2 + 2abx + b^2x^2}}{4e^6(a + bx)(d + ex)^8} + \frac{5b^4(bd - ae) \sqrt{a^2 + 2abx + b^2x^2}}{7e^6(a + bx)(d + ex)^7} - \frac{b^5 \sqrt{a^2 + 2abx + b^2x^2}}{6e^6(a + bx)(d + ex)^6}$$

output

```
1/11*(-a*e+b*d)^5*((b*x+a)^2)^(1/2)/e^6/(b*x+a)/(e*x+d)^11-1/2*b*(-a*e+b*d)^4*((b*x+a)^2)^(1/2)/e^6/(b*x+a)/(e*x+d)^10+10/9*b^2*(-a*e+b*d)^3*((b*x+a)^2)^(1/2)/e^6/(b*x+a)/(e*x+d)^9-5/4*b^3*(-a*e+b*d)^2*((b*x+a)^2)^(1/2)/e^6/(b*x+a)/(e*x+d)^8+5/7*b^4*(-a*e+b*d)*((b*x+a)^2)^(1/2)/e^6/(b*x+a)/(e*x+d)^7-1/6*b^5*((b*x+a)^2)^(1/2)/e^6/(b*x+a)/(e*x+d)^6
```


Mathematica [A] (verified)

Time = 1.06 (sec) , antiderivative size = 223, normalized size of antiderivative = 0.72

$$\int \frac{(a^2 + 2abx + b^2x^2)^{5/2}}{(d + ex)^{12}} dx = \frac{\sqrt{(a + bx)^2(252a^5e^5 + 126a^4be^4(d + 11ex) + 56a^3b^2e^3(d^2 + 11dex + 55e^2x^2) + 21a^2b^3e^2(d^3 + 11d^2ex + 55de^2x^2 + 11d^2ex + 55de^2x^2 + 11d^2ex + 55de^2x^2 + 11d^2ex + 55de^2x^2))}}{(d + ex)^{11}}$$

input

```
Integrate[(a^2 + 2*a*b*x + b^2*x^2)^(5/2)/(d + e*x)^12,x]
```

output

```
-1/2772*(Sqrt[(a + b*x)^2]*(252*a^5*e^5 + 126*a^4*b*e^4*(d + 11*e*x) + 56*a^3*b^2*e^3*(d^2 + 11*d*e*x + 55*e^2*x^2) + 21*a^2*b^3*e^2*(d^3 + 11*d^2*e*x + 55*d*e^2*x^2 + 165*e^3*x^3) + 6*a*b^4*e*(d^4 + 11*d^3*e*x + 55*d^2*e^2*x^2 + 165*d*e^3*x^3 + 330*e^4*x^4) + b^5*(d^5 + 11*d^4*e*x + 55*d^3*e^2*x^2 + 165*d^2*e^3*x^3 + 330*d*e^4*x^4 + 462*e^5*x^5)))/(e^6*(a + b*x)*(d + e*x)^11)
```

Rubi [A] (verified)

Time = 0.58 (sec) , antiderivative size = 174, normalized size of antiderivative = 0.56, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {1102, 27, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\begin{aligned} & \int \frac{(a^2 + 2abx + b^2x^2)^{5/2}}{(d + ex)^{12}} dx \\ & \quad \downarrow \text{1102} \\ & \frac{\sqrt{a^2 + 2abx + b^2x^2} \int \frac{b^5(a+bx)^5}{(d+ex)^{12}} dx}{b^5(a + bx)} \\ & \quad \downarrow \text{27} \\ & \frac{\sqrt{a^2 + 2abx + b^2x^2} \int \frac{(a+bx)^5}{(d+ex)^{12}} dx}{a + bx} \end{aligned}$$

53

$$\frac{\sqrt{a^2 + 2abx + b^2x^2} \int \left(\frac{b^5}{e^5(d+ex)^7} - \frac{5(bd-ae)b^4}{e^5(d+ex)^8} + \frac{10(bd-ae)^2b^3}{e^5(d+ex)^9} - \frac{10(bd-ae)^3b^2}{e^5(d+ex)^{10}} + \frac{5(bd-ae)^4b}{e^5(d+ex)^{11}} + \frac{(ae-bd)^5}{e^5(d+ex)^{12}} \right) dx}{a + bx}$$

2009

$$\frac{\sqrt{a^2 + 2abx + b^2x^2} \left(\frac{5b^4(bd-ae)}{7e^6(d+ex)^7} - \frac{5b^3(bd-ae)^2}{4e^6(d+ex)^8} + \frac{10b^2(bd-ae)^3}{9e^6(d+ex)^9} - \frac{b(bd-ae)^4}{2e^6(d+ex)^{10}} + \frac{(bd-ae)^5}{11e^6(d+ex)^{11}} - \frac{b^5}{6e^6(d+ex)^6} \right)}{a + bx}$$

input

```
Int[(a^2 + 2*a*b*x + b^2*x^2)^(5/2)/(d + e*x)^12,x]
```

output

```
(Sqrt[a^2 + 2*a*b*x + b^2*x^2]*((b*d - a*e)^5/(11*e^6*(d + e*x)^11) - (b*(b*d - a*e)^4)/(2*e^6*(d + e*x)^10) + (10*b^2*(b*d - a*e)^3)/(9*e^6*(d + e*x)^9) - (5*b^3*(b*d - a*e)^2)/(4*e^6*(d + e*x)^8) + (5*b^4*(b*d - a*e))/(7*e^6*(d + e*x)^7) - b^5/(6*e^6*(d + e*x)^6)))/(a + b*x)
```

Defintions of rubi rules used

rule 27

```
Int[(a_)*(F_x_), x_Symbol] := Simp[a Int[F_x, x], x] /; FreeQ[a, x] && !MatchQ[F_x, (b_)*(G_x_)] /; FreeQ[b, x]
```

rule 53

```
Int[((a_) + (b_)*(x_))^(m_)*((c_) + (d_)*(x_))^(n_), x_Symbol] := Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && IGtQ[m, 0] && (!IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 1102

```
Int[((d_) + (e_)*(x_))^(m_)*((a_) + (b_)*(x_) + (c_)*(x_)^2)^(p_), x_Symbol] := Simp[(a + b*x + c*x^2)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x)^(2*FracPart[p])) Int[(d + e*x)^m*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && EqQ[b^2 - 4*a*c, 0]
```

rule 2009

```
Int[u_, x_Symbol] := Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 16.16 (sec) , antiderivative size = 262, normalized size of antiderivative = 0.85

| method | result |
|---------|---|
| risch | $\frac{\sqrt{(bx+a)^2} \left(-\frac{b^5 x^5}{6e} - \frac{5b^4(6ae+bd)x^4}{42e^2} - \frac{5b^3(21e^2 a^2 + 6abde + b^2 d^2)x^3}{84e^3} - \frac{5b^2(56e^3 a^3 + 21a^2 bde^2 + 6a b^2 d^2 e + b^3 d^3)x^2}{252e^4} - \frac{b(126a^4 e^4 + 56a^3 bde^3}{(bx+a)(ex+d)^{11}} \right)}{(bx+a)(ex+d)^{11}}$ |
| gosper | $-\frac{(462x^5 e^5 b^5 + 1980x^4 a b^4 e^5 + 330x^4 b^5 d e^4 + 3465x^3 a^2 b^3 e^5 + 990x^3 a b^4 d e^4 + 165x^3 b^5 d^2 e^3 + 3080x^2 a^3 b^2 e^5 + 1155x^2 a^2 b^3 d e^4 + 330x^2 a^4 b d e^4 + 1155x a^3 b^2 d^2 e^3 + 165x a^2 b^3 d^3 e^2 + 165x a b^4 d^4 e + 1155a^5 d^5)}{2772(e^{17}x^{11} + 11de^{16}x^{10} + 55d^2e^{15}x^9 + 165d^3e^{14}x^8 + 165d^4e^{13}x^7 + 1155d^5e^{12}x^6 + 55d^6e^{11}x^5 + 11d^7e^{10}x^4 + d^8e^9x^3 + d^9e^8x^2 + d^{10}e^7x + d^{11}e^6)}$ |
| default | $-\frac{(462x^5 e^5 b^5 + 1980x^4 a b^4 e^5 + 330x^4 b^5 d e^4 + 3465x^3 a^2 b^3 e^5 + 990x^3 a b^4 d e^4 + 165x^3 b^5 d^2 e^3 + 3080x^2 a^3 b^2 e^5 + 1155x^2 a^2 b^3 d e^4 + 330x^2 a^4 b d e^4 + 1155x a^3 b^2 d^2 e^3 + 165x a^2 b^3 d^3 e^2 + 165x a b^4 d^4 e + 1155a^5 d^5)}{2772(e^{17}x^{11} + 11de^{16}x^{10} + 55d^2e^{15}x^9 + 165d^3e^{14}x^8 + 165d^4e^{13}x^7 + 1155d^5e^{12}x^6 + 55d^6e^{11}x^5 + 11d^7e^{10}x^4 + d^8e^9x^3 + d^9e^8x^2 + d^{10}e^7x + d^{11}e^6)}$ |
| orering | $-\frac{(462x^5 e^5 b^5 + 1980x^4 a b^4 e^5 + 330x^4 b^5 d e^4 + 3465x^3 a^2 b^3 e^5 + 990x^3 a b^4 d e^4 + 165x^3 b^5 d^2 e^3 + 3080x^2 a^3 b^2 e^5 + 1155x^2 a^2 b^3 d e^4 + 330x^2 a^4 b d e^4 + 1155x a^3 b^2 d^2 e^3 + 165x a^2 b^3 d^3 e^2 + 165x a b^4 d^4 e + 1155a^5 d^5)}{2772(e^{17}x^{11} + 11de^{16}x^{10} + 55d^2e^{15}x^9 + 165d^3e^{14}x^8 + 165d^4e^{13}x^7 + 1155d^5e^{12}x^6 + 55d^6e^{11}x^5 + 11d^7e^{10}x^4 + d^8e^9x^3 + d^9e^8x^2 + d^{10}e^7x + d^{11}e^6)}$ |

```
input int((b^2*x^2+2*a*b*x+a^2)^(5/2)/(e*x+d)^12,x,method=_RETURNVERBOSE)
```

```
output ((b*x+a)^2)^(1/2)/(b*x+a)*(-1/6*b^5/e*x^5-5/42*b^4/e^2*(6*a*e+b*d)*x^4-5/8
4*b^3/e^3*(21*a^2*e^2+6*a*b*d*e+b^2*d^2)*x^3-5/252*b^2/e^4*(56*a^3*e^3+21*
a^2*b*d*e^2+6*a*b^2*d^2*e+b^3*d^3)*x^2-1/252*b/e^5*(126*a^4*e^4+56*a^3*b*d
*e^3+21*a^2*b^2*d^2*e^2+6*a*b^3*d^3*e+b^4*d^4)*x-1/2772/e^6*(252*a^5*e^5+1
26*a^4*b*d*e^4+56*a^3*b^2*d^2*e^3+21*a^2*b^3*d^3*e^2+6*a*b^4*d^4*e+b^5*d^5
))/ (e*x+d)^11
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 370, normalized size of antiderivative = 1.20

$$\int \frac{(a^2 + 2abx + b^2x^2)^{5/2}}{(d + ex)^{12}} dx =$$

$$-\frac{462 b^5 e^5 x^5 + b^5 d^5 + 6 a b^4 d^4 e + 21 a^2 b^3 d^3 e^2 + 56 a^3 b^2 d^2 e^3 + 126 a^4 b d e^4 + 252 a^5 e^5 + 330 (b^5 d e^4 + 6 a b^4 e^5)}{2772 (e^{17} x^{11} + 11 d e^{16} x^{10} + 55 d^2 e^{15} x^9 + 165 d^3 e^{14} x^8 + 165 d^4 e^{13} x^7 + 1155 d^5 e^{12} x^6 + 55 d^6 e^{11} x^5 + 11 d^7 e^{10} x^4 + d^8 e^9 x^3 + d^9 e^8 x^2 + d^{10} e^7 x + d^{11} e^6)}$$

```
input integrate((b^2*x^2+2*a*b*x+a^2)^(5/2)/(e*x+d)^12,x, algorithm="fricas")
```

output

```
-1/2772*(462*b^5*e^5*x^5 + b^5*d^5 + 6*a*b^4*d^4*e + 21*a^2*b^3*d^3*e^2 +
56*a^3*b^2*d^2*e^3 + 126*a^4*b*d*e^4 + 252*a^5*e^5 + 330*(b^5*d*e^4 + 6*a*
b^4*e^5)*x^4 + 165*(b^5*d^2*e^3 + 6*a*b^4*d*e^4 + 21*a^2*b^3*e^5)*x^3 + 55
*(b^5*d^3*e^2 + 6*a*b^4*d^2*e^3 + 21*a^2*b^3*d*e^4 + 56*a^3*b^2*e^5)*x^2 +
11*(b^5*d^4*e + 6*a*b^4*d^3*e^2 + 21*a^2*b^3*d^2*e^3 + 56*a^3*b^2*d*e^4 +
126*a^4*b*e^5)*x)/(e^17*x^11 + 11*d*e^16*x^10 + 55*d^2*e^15*x^9 + 165*d^3
*e^14*x^8 + 330*d^4*e^13*x^7 + 462*d^5*e^12*x^6 + 462*d^6*e^11*x^5 + 330*d
^7*e^10*x^4 + 165*d^8*e^9*x^3 + 55*d^9*e^8*x^2 + 11*d^10*e^7*x + d^11*e^6)
```

Sympy [F(-2)]

Exception generated.

$$\int \frac{(a^2 + 2abx + b^2x^2)^{5/2}}{(d + ex)^{12}} dx = \text{Exception raised: HeuristicGCDFailed}$$

input

```
integrate((b**2*x**2+2*a*b*x+a**2)**(5/2)/(e*x+d)**12,x)
```

output

```
Exception raised: HeuristicGCDFailed >> no luck
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(a^2 + 2abx + b^2x^2)^{5/2}}{(d + ex)^{12}} dx = \text{Exception raised: ValueError}$$

input

```
integrate((b^2*x^2+2*a*b*x+a^2)^(5/2)/(e*x+d)^12,x, algorithm="maxima")
```

output

```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
help (example of legal syntax is 'assume(a*e-b*d>0)', see `assume?` for m
ore detail
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 493 vs. $2(230) = 460$.

Time = 0.21 (sec) , antiderivative size = 493, normalized size of antiderivative = 1.60

$$\int \frac{(a^2 + 2abx + b^2x^2)^{5/2}}{(d + ex)^{12}} dx = \frac{b^{11} \operatorname{sgn}(bx + a)}{2772 (b^6 d^6 e^6 - 6 a b^5 d^5 e^7 + 15 a^2 b^4 d^4 e^8 - 20 a^3 b^3 d^3 e^9 + 15 a^4 b^2 d^2 e^{10} - 6 a^5 b d e^{11} + a^6 e^{12})} - \frac{462 b^5 e^5 x^5 \operatorname{sgn}(bx + a) + 330 b^5 d e^4 x^4 \operatorname{sgn}(bx + a) + 1980 a b^4 e^5 x^4 \operatorname{sgn}(bx + a) + 165 b^5 d^2 e^3 x^3 \operatorname{sgn}(bx + a) - 990 a^2 b^4 d e^4 x^3 \operatorname{sgn}(bx + a) + 3465 a^2 b^3 e^5 x^3 \operatorname{sgn}(bx + a) + 55 b^5 d^3 e^2 x^2 \operatorname{sgn}(bx + a) + 330 a b^4 d^2 e^3 x^2 \operatorname{sgn}(bx + a) + 1155 a^2 b^3 d e^4 x^2 \operatorname{sgn}(bx + a) + 3080 a^3 b^2 e^5 x^2 \operatorname{sgn}(bx + a) + 11 b^5 d^4 e x \operatorname{sgn}(bx + a) + 66 a b^4 d^3 e^2 x \operatorname{sgn}(bx + a) + 231 a^2 b^3 d^2 e^3 x \operatorname{sgn}(bx + a) + 616 a^3 b^2 d e^4 x \operatorname{sgn}(bx + a) + 1386 a^4 b e^5 x \operatorname{sgn}(bx + a) + b^5 d^5 \operatorname{sgn}(bx + a) + 6 a b^4 d^4 e \operatorname{sgn}(bx + a) + 21 a^2 b^3 d^3 e^2 \operatorname{sgn}(bx + a) + 56 a^3 b^2 d^2 e^3 \operatorname{sgn}(bx + a) + 126 a^4 b d e^4 \operatorname{sgn}(bx + a) + 252 a^5 e^5 \operatorname{sgn}(bx + a)}{(e x + d)^{11} e^6}$$

input

```
integrate((b^2*x^2+2*a*b*x+a^2)^(5/2)/(e*x+d)^12,x, algorithm="giac")
```

output

```
1/2772*b^11*sgn(b*x + a)/(b^6*d^6*e^6 - 6*a*b^5*d^5*e^7 + 15*a^2*b^4*d^4*e^8 - 20*a^3*b^3*d^3*e^9 + 15*a^4*b^2*d^2*e^10 - 6*a^5*b*d*e^11 + a^6*e^12) - 1/2772*(462*b^5*e^5*x^5*sgn(b*x + a) + 330*b^5*d*e^4*x^4*sgn(b*x + a) + 1980*a*b^4*e^5*x^4*sgn(b*x + a) + 165*b^5*d^2*e^3*x^3*sgn(b*x + a) + 990*a*b^4*d*e^4*x^3*sgn(b*x + a) + 3465*a^2*b^3*e^5*x^3*sgn(b*x + a) + 55*b^5*d^3*e^2*x^2*sgn(b*x + a) + 330*a*b^4*d^2*e^3*x^2*sgn(b*x + a) + 1155*a^2*b^3*d*e^4*x^2*sgn(b*x + a) + 3080*a^3*b^2*e^5*x^2*sgn(b*x + a) + 11*b^5*d^4*e*x*sgn(b*x + a) + 66*a*b^4*d^3*e^2*x*sgn(b*x + a) + 231*a^2*b^3*d^2*e^3*x*sgn(b*x + a) + 616*a^3*b^2*d*e^4*x*sgn(b*x + a) + 1386*a^4*b*e^5*x*sgn(b*x + a) + b^5*d^5*sgn(b*x + a) + 6*a*b^4*d^4*e*sgn(b*x + a) + 21*a^2*b^3*d^3*e^2*sgn(b*x + a) + 56*a^3*b^2*d^2*e^3*sgn(b*x + a) + 126*a^4*b*d*e^4*sgn(b*x + a) + 252*a^5*e^5*sgn(b*x + a))/((e*x + d)^11*e^6)
```

Mupad [B] (verification not implemented)

Time = 5.14 (sec) , antiderivative size = 687, normalized size of antiderivative = 2.23

$$\int \frac{(a^2 + 2abx + b^2x^2)^{5/2}}{(d + ex)^{12}} dx = \frac{\left(\frac{4b^5d - 5ab^4e}{7e^6} + \frac{b^5d}{7e^6}\right) \sqrt{a^2 + 2abx + b^2x^2}}{(a + bx)(d + ex)^7}$$

$$\left(\frac{5a^4be^4 - 10a^3b^2de^3 + 10a^2b^3d^2e^2 - 5ab^4d^3e + b^5d^4}{10e^6} + \frac{d \left(\frac{b^5d}{10e^3} - \frac{b^4(5ae - bd)}{10e^3} \right) + \frac{-10a^3b^2e^4 + 10a^2b^3de^3 - 5ab^4d^2e^2 + b^5d^3e}{10e^6}}{e} \right) \sqrt{a^2 + 2abx + b^2x^2}$$

$$\frac{\left(\frac{10a^2b^3e^2 - 15ab^4de + 6b^5d^2}{8e^6} + \frac{d \left(\frac{b^5d}{8e^5} - \frac{b^4(5ae - 3bd)}{8e^5} \right)}{e} \right) \sqrt{a^2 + 2abx + b^2x^2}}{(a + bx)(d + ex)^{10}}$$

$$\left(\frac{\frac{a^5}{11e} - \frac{d \left(\frac{5a^4b}{11e} - \frac{d \left(\frac{d \left(\frac{5a^4b^4}{11e} - \frac{b^5d}{11e^2} \right) - \frac{10a^2b^3}{11e} \right) + \frac{10a^3b^2}{11e}}{e} \right)}{e}}{e} \right) \sqrt{a^2 + 2abx + b^2x^2}$$

$$\frac{\left(\frac{-10a^3b^2e^3 + 20a^2b^3de^2 - 15ab^4d^2e + 4b^5d^3}{9e^6} + \frac{d \left(\frac{b^5d}{9e^4} - \frac{b^4(5ae - 2bd)}{9e^4} \right) + \frac{b^3(10a^2e^2 - 10abde + 3b^2d^2)}{9e^5}}{e} \right) \sqrt{a^2 + 2abx + b^2x^2}}{(a + bx)(d + ex)^9}$$

$$- \frac{b^5 \sqrt{a^2 + 2abx + b^2x^2}}{6e^6 (a + bx)(d + ex)^6}$$

input `int((a^2 + b^2*x^2 + 2*a*b*x)^(5/2)/(d + e*x)^12,x)`

output
$$\begin{aligned} & \left(\frac{4b^5d - 5ab^4e}{7e^6} + \frac{b^5d}{7e^6} \right) (a^2 + b^2x^2 + 2abx)^{1/2} / \left((a + bx)(d + ex)^7 - \left(\frac{b^5d^4 + 5a^4b^4e^4 - 10a^3b^2d^2e^3 + 10a^2b^3d^2e^2 - 5ab^4d^3e}{10e^6} + \frac{d(b^5d^3e - 10a^3b^2e^4 - 5ab^4d^2e^2 + 10a^2b^3d^2e^3)}{10e^6} + \frac{d(d(b^5d^3e - 10a^3b^2e^4 - 5ab^4d^2e^2 + 10a^2b^3d^2e^3)}{10e^6} + \frac{d(d(b^5d^3e - 10a^3b^2e^4 - 5ab^4d^2e^2 + 10a^2b^3d^2e^3)}{10e^6} - \frac{b^4(5ae - bd)}{10e^3} \right) / e + \frac{b^3(10a^2e^2 + b^2d^2 - 5abd^2e)}{10e^4} \right) / e \right) / e (a^2 + b^2x^2 + 2abx)^{1/2} / \left((a + bx)(d + ex)^{10} - \left(\frac{6b^5d^2 + 10a^2b^3e^2 - 15ab^4d^2e}{8e^6} + \frac{d(b^5d^2 + 10a^2b^3e^2 - 15ab^4d^2e)}{8e^6} - \frac{b^4(5ae - 3bd)}{8e^5} \right) / e \right) (a^2 + b^2x^2 + 2abx)^{1/2} / \left((a + bx)(d + ex)^8 - \left(\frac{a^5}{11e} - \frac{d(5a^4b)}{11e} - \frac{d(d(d(5a^4b)}{11e} - \frac{b^5d}{11e^2}))}{e} - \frac{10a^2b^3}{11e} \right) / e + \frac{10a^3b^2}{11e} \right) / e \right) / e (a^2 + b^2x^2 + 2abx)^{1/2} / \left((a + bx)(d + ex)^{11} + \left(\frac{4b^5d^3 - 10a^3b^2e^3 + 20a^2b^3d^2e^2 - 15ab^4d^2e}{9e^6} + \frac{d(d(d(b^5d^3 - 10a^3b^2e^3 + 20a^2b^3d^2e^2 - 15ab^4d^2e)}{9e^6} - \frac{b^4(5ae - 2bd)}{9e^4})}{e} + \frac{b^3(10a^2e^2 + 3b^2d^2 - 10abd^2e)}{9e^5} \right) / e \right) (a^2 + b^2x^2 + 2abx)^{1/2} / \left((a + bx)(d + ex)^9 - \frac{b^5(a^2 + b^2x^2 + 2abx)^{1/2}}{6e^6(a + bx)(d + ex)^6} \right) \end{aligned}$$

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 382, normalized size of antiderivative = 1.24

$$\int \frac{(a^2 + 2abx + b^2x^2)^{5/2}}{(d + ex)^{12}} dx = \frac{-462b^5e^5x^5 - 1980ab^4e^5x^4 - 330b^5de^4x^4 - 3465a^2b^3e^5x^3 - 990ab^4de^4x^3 - \dots}{(d + ex)^{12}}$$

input `int((b^2*x^2+2*a*b*x+a^2)^(5/2)/(e*x+d)^12,x)`

output

```
( - 252*a**5*e**5 - 126*a**4*b*d*e**4 - 1386*a**4*b*e**5*x - 56*a**3*b**2*
d**2*e**3 - 616*a**3*b**2*d*e**4*x - 3080*a**3*b**2*e**5*x**2 - 21*a**2*b*
*3*d**3*e**2 - 231*a**2*b**3*d**2*e**3*x - 1155*a**2*b**3*d*e**4*x**2 - 34
65*a**2*b**3*e**5*x**3 - 6*a*b**4*d**4*e - 66*a*b**4*d**3*e**2*x - 330*a*b
**4*d**2*e**3*x**2 - 990*a*b**4*d*e**4*x**3 - 1980*a*b**4*e**5*x**4 - b**5
*d**5 - 11*b**5*d**4*e*x - 55*b**5*d**3*e**2*x**2 - 165*b**5*d**2*e**3*x**
3 - 330*b**5*d*e**4*x**4 - 462*b**5*e**5*x**5)/(2772*e**6*(d**11 + 11*d**1
0*e*x + 55*d**9*e**2*x**2 + 165*d**8*e**3*x**3 + 330*d**7*e**4*x**4 + 462*
d**6*e**5*x**5 + 462*d**5*e**6*x**6 + 330*d**4*e**7*x**7 + 165*d**3*e**8*x
**8 + 55*d**2*e**9*x**9 + 11*d*e**10*x**10 + e**11*x**11))
```


3.309 $\int \frac{(d+ex)^4}{\sqrt{a^2+2abx+b^2x^2}} dx$

| | |
|---|------|
| Optimal result | 2332 |
| Mathematica [A] (verified) | 2333 |
| Rubi [A] (verified) | 2333 |
| Maple [A] (verified) | 2335 |
| Fricas [A] (verification not implemented) | 2335 |
| Sympy [B] (verification not implemented) | 2336 |
| Maxima [B] (verification not implemented) | 2336 |
| Giac [A] (verification not implemented) | 2337 |
| Mupad [F(-1)] | 2338 |
| Reduce [B] (verification not implemented) | 2338 |

Optimal result

Integrand size = 28, antiderivative size = 222

$$\int \frac{(d+ex)^4}{\sqrt{a^2+2abx+b^2x^2}} dx = \frac{e(bd-ae)^3x(a+bx)}{b^4\sqrt{a^2+2abx+b^2x^2}} + \frac{(bd-ae)^2(a+bx)(d+ex)^2}{2b^3\sqrt{a^2+2abx+b^2x^2}} + \frac{(bd-ae)(a+bx)(d+ex)^3}{3b^2\sqrt{a^2+2abx+b^2x^2}} + \frac{(a+bx)(d+ex)^4}{4b\sqrt{a^2+2abx+b^2x^2}} + \frac{(bd-ae)^4(a+bx)\log(a+bx)}{b^5\sqrt{a^2+2abx+b^2x^2}}$$

output

```
e*(-a*e+b*d)^3*x*(b*x+a)/b^4/((b*x+a)^2)^(1/2)+1/2*(-a*e+b*d)^2*(b*x+a)*(e*x+d)^2/b^3/((b*x+a)^2)^(1/2)+1/3*(-a*e+b*d)*(b*x+a)*(e*x+d)^3/b^2/((b*x+a)^2)^(1/2)+1/4*(b*x+a)*(e*x+d)^4/b/((b*x+a)^2)^(1/2)+(-a*e+b*d)^4*(b*x+a)*ln(b*x+a)/b^5/((b*x+a)^2)^(1/2)
```

Mathematica [A] (verified)

Time = 1.06 (sec) , antiderivative size = 130, normalized size of antiderivative = 0.59

$$\int \frac{(d+ex)^4}{\sqrt{a^2+2abx+b^2x^2}} dx = \frac{(a+bx)(bex(-12a^3e^3+6a^2be^2(8d+ex)-4ab^2e(18d^2+6dex+e^2x^2))+b^3(48d^3+36d^2ex+16de^2x^2-12b^5\sqrt{(a+bx)^2})}{12b^5\sqrt{(a+bx)^2}}$$

input

```
Integrate[(d + e*x)^4/Sqrt[a^2 + 2*a*b*x + b^2*x^2],x]
```

output

```
((a + b*x)*(b*e*x*(-12*a^3*e^3 + 6*a^2*b*e^2*(8*d + e*x) - 4*a*b^2*e*(18*d^2 + 6*d*e*x + e^2*x^2) + b^3*(48*d^3 + 36*d^2*e*x + 16*d*e^2*x^2 + 3*e^3*x^3)) + 12*(b*d - a*e)^4*Log[a + b*x]))/(12*b^5*Sqrt[(a + b*x)^2])
```

Rubi [A] (verified)

Time = 0.47 (sec) , antiderivative size = 123, normalized size of antiderivative = 0.55, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {1102, 27, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\begin{aligned} & \int \frac{(d+ex)^4}{\sqrt{a^2+2abx+b^2x^2}} dx \\ & \quad \downarrow \text{1102} \\ & \frac{b(a+bx) \int \frac{(d+ex)^4}{b(a+bx)} dx}{\sqrt{a^2+2abx+b^2x^2}} \\ & \quad \downarrow \text{27} \\ & \frac{(a+bx) \int \frac{(d+ex)^4}{a+bx} dx}{\sqrt{a^2+2abx+b^2x^2}} \\ & \quad \downarrow \text{49} \end{aligned}$$

$$\frac{(a + bx) \int \left(\frac{(bd-ae)^4}{b^4(a+bx)} + \frac{e(bd-ae)^3}{b^4} + \frac{e(d+ex)(bd-ae)^2}{b^3} + \frac{e(d+ex)^2(bd-ae)}{b^2} + \frac{e(d+ex)^3}{b} \right) dx}{\sqrt{a^2 + 2abx + b^2x^2}}$$

↓ 2009

$$\frac{(a + bx) \left(\frac{(bd-ae)^4 \log(a+bx)}{b^5} + \frac{ex(bd-ae)^3}{b^4} + \frac{(d+ex)^2(bd-ae)^2}{2b^3} + \frac{(d+ex)^3(bd-ae)}{3b^2} + \frac{(d+ex)^4}{4b} \right)}{\sqrt{a^2 + 2abx + b^2x^2}}$$

input `Int[(d + e*x)^4/Sqrt[a^2 + 2*a*b*x + b^2*x^2], x]`

output `((a + b*x)*((e*(b*d - a*e)^3*x)/b^4 + ((b*d - a*e)^2*(d + e*x)^2)/(2*b^3) + ((b*d - a*e)*(d + e*x)^3)/(3*b^2) + (d + e*x)^4/(4*b) + ((b*d - a*e)^4*log[a + b*x])/b^5))/Sqrt[a^2 + 2*a*b*x + b^2*x^2]`

Defintions of rubi rules used

rule 27 `Int[(a_)*(F_x_), x_Symbol] := Simp[a Int[F_x, x], x] /; FreeQ[a, x] && !MatchQ[F_x, (b_)*(G_x_)] /; FreeQ[b, x]`

rule 49 `Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] := Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] && IGtQ[m, 0] && IGtQ[m + n + 2, 0]`

rule 1102 `Int[((d_.) + (e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] := Simp[(a + b*x + c*x^2)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x)^(2*FracPart[p])) Int[(d + e*x)^m*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && EqQ[b^2 - 4*a*c, 0]`

rule 2009 `Int[u_, x_Symbol] := Simp[IntSum[u, x], x] /; SumQ[u]`

Maple [A] (verified)

Time = 1.16 (sec) , antiderivative size = 220, normalized size of antiderivative = 0.99

| method | result |
|---------|--|
| risch | $-\frac{\sqrt{(bx+a)^2} e \left(-\frac{b^3 x^4 e^3}{4} + \frac{((ae-2bd)b^2 e^2 - 2b^3 e^2 d)x^3}{3} + \frac{(2(ae-2bd)de b^2 - be(e^2 a^2 - 2abde + 2b^2 d^2))x^2}{2} + (ae-2bd)(e^2 a^2 - 2abde + 2b^2 d^2) \right)}{(bx+a)b^4}$ |
| default | $\frac{(bx+a)(3b^4 x^4 e^4 - 4x^3 a b^3 e^4 + 16x^3 b^4 d e^3 + 6x^2 a^2 b^2 e^4 - 24x^2 a b^3 d e^3 + 36x^2 b^4 d^2 e^2 + 12 \ln(bx+a) a^4 e^4 - 48 \ln(bx+a) a^3 b d e^3 + 72 \ln(bx+a) a^2 b^2 d^2 e^2 + 12 \ln(bx+a) a b^3 d^3 e + 12 \ln(bx+a) a^2 b^2 d^3 e + 12 \ln(bx+a) a b^3 d^3 e + 12 \ln(bx+a) a^2 b^2 d^3 e + 12 \ln(bx+a) a b^3 d^3 e + 12 \ln(bx+a) a^2 b^2 d^3 e)}{12 \sqrt{(bx+a)^2} b^5}$ |

input `int((e*x+d)^4/((b*x+a)^2)^(1/2),x,method=_RETURNVERBOSE)`

output
$$-\frac{((bx+a)^2)^{1/2}}{(bx+a)} \frac{e}{b^4} \left(-\frac{1}{4} b^3 x^4 e^3 + \frac{1}{3} ((ae-2bd) b^2 e^2 - 2b^3 e^2 d) x^3 + \frac{1}{2} (2(ae-2bd) d e b^2 - be(e^2 a^2 - 2abde + 2b^2 d^2)) x^2 + (ae-2bd)(e^2 a^2 - 2abde + 2b^2 d^2) \right) + \frac{(bx+a)^{1/2}}{(bx+a)} \frac{1}{b^5} (3b^4 x^4 e^4 - 4x^3 a b^3 e^4 + 16x^3 b^4 d e^3 + 6x^2 a^2 b^2 e^4 - 24x^2 a b^3 d e^3 + 36x^2 b^4 d^2 e^2 + 12 \ln(bx+a) a^4 e^4 - 48 \ln(bx+a) a^3 b d e^3 + 72 \ln(bx+a) a^2 b^2 d^2 e^2 + 12 \ln(bx+a) a b^3 d^3 e + 12 \ln(bx+a) a^2 b^2 d^3 e + 12 \ln(bx+a) a b^3 d^3 e + 12 \ln(bx+a) a^2 b^2 d^3 e + 12 \ln(bx+a) a b^3 d^3 e + 12 \ln(bx+a) a^2 b^2 d^3 e) \ln(bx+a)$$

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 181, normalized size of antiderivative = 0.82

$$\int \frac{(d+ex)^4}{\sqrt{a^2+2abx+b^2x^2}} dx = \frac{3b^4 e^4 x^4 + 4(4b^4 d e^3 - ab^3 e^4) x^3 + 6(6b^4 d^2 e^2 - 4ab^3 d e^3 + a^2 b^2 e^4) x^2 + 12(4b^4 d^3 e - 6ab^3 d^2 e^2 + 4a^2 b^2 d^3 e + 12 \ln(bx+a) a^4 e^4 - 48 \ln(bx+a) a^3 b d e^3 + 72 \ln(bx+a) a^2 b^2 d^2 e^2 + 12 \ln(bx+a) a b^3 d^3 e + 12 \ln(bx+a) a^2 b^2 d^3 e + 12 \ln(bx+a) a b^3 d^3 e + 12 \ln(bx+a) a^2 b^2 d^3 e)}{12b^5}$$

input `integrate((e*x+d)^4/((b*x+a)^2)^(1/2),x, algorithm="fricas")`

output
$$\frac{1}{12} (3b^4 e^4 x^4 + 4(4b^4 d e^3 - ab^3 e^4) x^3 + 6(6b^4 d^2 e^2 - 4ab^3 d e^3 + a^2 b^2 e^4) x^2 + 12(4b^4 d^3 e - 6ab^3 d^2 e^2 + 4a^2 b^2 d^3 e + 12 \ln(bx+a) a^4 e^4 - 48 \ln(bx+a) a^3 b d e^3 + 72 \ln(bx+a) a^2 b^2 d^2 e^2 + 12 \ln(bx+a) a b^3 d^3 e + 12 \ln(bx+a) a^2 b^2 d^3 e + 12 \ln(bx+a) a b^3 d^3 e + 12 \ln(bx+a) a^2 b^2 d^3 e + 12 \ln(bx+a) a b^3 d^3 e + 12 \ln(bx+a) a^2 b^2 d^3 e) \ln(bx+a)) / b^5$$

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 746 vs. $2(162) = 324$.

Time = 1.50 (sec) , antiderivative size = 746, normalized size of antiderivative = 3.36

$$\int \frac{(d + ex)^4}{\sqrt{a^2 + 2abx + b^2x^2}} dx = \text{Too large to display}$$

input `integrate((e*x+d)**4/((b*x+a)**2)**(1/2),x)`

output

```
Piecewise((sqrt(a**2 + 2*a*b*x + b**2*x**2)*(e**4*x**3/(4*b**2) + x**2*(-7*a*e**4/(4*b) + 4*d*e**3)/(3*b**2) + x*(-3*a**2*e**4/(4*b**2) - 5*a*(-7*a*e**4/(4*b) + 4*d*e**3)/(3*b) + 6*d**2*e**2)/(2*b**2) + (-2*a**2*(-7*a*e**4/(4*b) + 4*d*e**3)/(3*b**2) - 3*a*(-3*a**2*e**4/(4*b**2) - 5*a*(-7*a*e**4/(4*b) + 4*d*e**3)/(3*b) + 6*d**2*e**2)/(2*b) + 4*d**3*e)/b**2) + (a/b + x)*(-a**2*(-3*a**2*e**4/(4*b**2) - 5*a*(-7*a*e**4/(4*b) + 4*d*e**3)/(3*b) + 6*d**2*e**2)/(2*b**2) - a*(-2*a**2*(-7*a*e**4/(4*b) + 4*d*e**3)/(3*b**2) - 3*a*(-3*a**2*e**4/(4*b**2) - 5*a*(-7*a*e**4/(4*b) + 4*d*e**3)/(3*b) + 6*d**2*e**2)/(2*b) + 4*d**3*e)/b + d**4)*log(a/b + x)/sqrt(b**2*(a/b + x)**2), Ne(b**2, 0)), ((2*d**4*sqrt(a**2 + 2*a*b*x) + 4*d**3*e*(-a**2*sqrt(a**2 + 2*a*b*x) + (a**2 + 2*a*b*x)**(3/2)/3)/(a*b) + 3*d**2*e**2*(a**4*sqrt(a**2 + 2*a*b*x) - 2*a**2*(a**2 + 2*a*b*x)**(3/2)/3 + (a**2 + 2*a*b*x)**(5/2)/5)/(a**2*b**2) + d*e**3*(-a**6*sqrt(a**2 + 2*a*b*x) + a**4*(a**2 + 2*a*b*x)**(3/2) - 3*a**2*(a**2 + 2*a*b*x)**(5/2)/5 + (a**2 + 2*a*b*x)**(7/2)/7)/(a**3*b**3) + e**4*(a**8*sqrt(a**2 + 2*a*b*x) - 4*a**6*(a**2 + 2*a*b*x)**(3/2)/3 + 6*a**4*(a**2 + 2*a*b*x)**(5/2)/5 - 4*a**2*(a**2 + 2*a*b*x)**(7/2)/7 + (a**2 + 2*a*b*x)**(9/2)/9)/(8*a**4*b**4))/(2*a*b), Ne(a*b, 0)), ((d**4*x + 2*d**3*e*x**2 + 2*d**2*e**2*x**3 + d*e**3*x**4 + e**4*x**5/5)/sqrt(a**2), True))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 348 vs. $2(161) = 322$.

Time = 0.04 (sec) , antiderivative size = 348, normalized size of antiderivative = 1.57

$$\int \frac{(d+ex)^4}{\sqrt{a^2+2abx+b^2x^2}} dx = \frac{\sqrt{b^2x^2+2abx+a^2}e^4x^3}{4b^2} + \frac{3d^2e^2x^2}{b} - \frac{10ade^3x^2}{3b^2} + \frac{13a^2e^4x^2}{12b^3} + \frac{4\sqrt{b^2x^2+2abx+a^2}de^3x^2}{3b^2} - \frac{7\sqrt{b^2x^2+2abx+a^2}ae^4x^2}{12b^3} - \frac{6ad^2e^2x}{b^2} + \frac{20a^2de^3x}{3b^3} - \frac{13a^3e^4x}{6b^4} + \frac{d^4\log(x+\frac{a}{b})}{b} - \frac{4ad^3e\log(x+\frac{a}{b})}{b^2} + \frac{6a^2d^2e^2\log(x+\frac{a}{b})}{b^3} - \frac{4a^3de^3\log(x+\frac{a}{b})}{b^4} + \frac{a^4e^4\log(x+\frac{a}{b})}{b^5} + \frac{4\sqrt{b^2x^2+2abx+a^2}d^3e}{b^2} - \frac{8\sqrt{b^2x^2+2abx+a^2}a^2de^3}{3b^4} + \frac{7\sqrt{b^2x^2+2abx+a^2}a^3e^4}{6b^5}$$

input `integrate((e*x+d)^4/((b*x+a)^2)^(1/2),x, algorithm="maxima")`

output

```
1/4*sqrt(b^2*x^2 + 2*a*b*x + a^2)*e^4*x^3/b^2 + 3*d^2*e^2*x^2/b - 10/3*a*d
*e^3*x^2/b^2 + 13/12*a^2*e^4*x^2/b^3 + 4/3*sqrt(b^2*x^2 + 2*a*b*x + a^2)*d
*e^3*x^2/b^2 - 7/12*sqrt(b^2*x^2 + 2*a*b*x + a^2)*a*e^4*x^2/b^3 - 6*a*d^2*
e^2*x/b^2 + 20/3*a^2*d*e^3*x/b^3 - 13/6*a^3*e^4*x/b^4 + d^4*log(x + a/b)/b
- 4*a*d^3*e*log(x + a/b)/b^2 + 6*a^2*d^2*e^2*log(x + a/b)/b^3 - 4*a^3*d*e
^3*log(x + a/b)/b^4 + a^4*e^4*log(x + a/b)/b^5 + 4*sqrt(b^2*x^2 + 2*a*b*x
+ a^2)*d^3*e/b^2 - 8/3*sqrt(b^2*x^2 + 2*a*b*x + a^2)*a^2*d*e^3/b^4 + 7/6*s
qrt(b^2*x^2 + 2*a*b*x + a^2)*a^3*e^4/b^5
```

Giac [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 274, normalized size of antiderivative = 1.23

$$\int \frac{(d+ex)^4}{\sqrt{a^2+2abx+b^2x^2}} dx = \frac{3b^3e^4x^4\operatorname{sgn}(bx+a) + 16b^3de^3x^3\operatorname{sgn}(bx+a) - 4ab^2e^4x^3\operatorname{sgn}(bx+a) + 36b^3d^2e^2x^2\operatorname{sgn}(bx+a) - 24ab^2d^2e^2x^2\operatorname{sgn}(bx+a) + (b^4d^4\operatorname{sgn}(bx+a) - 4ab^3d^3e\operatorname{sgn}(bx+a) + 6a^2b^2d^2e^2\operatorname{sgn}(bx+a) - 4a^3bde^3\operatorname{sgn}(bx+a) + a^4e^4\operatorname{sgn}(bx+a))}{b^5}$$

input `integrate((e*x+d)^4/((b*x+a)^2)^(1/2),x, algorithm="giac")`

output
$$\frac{1}{12}(3b^3e^4x^4\operatorname{sgn}(bx+a) + 16b^3d^2e^3x^3\operatorname{sgn}(bx+a) - 4ab^2e^4x^3\operatorname{sgn}(bx+a) + 36b^3d^2e^2x^2\operatorname{sgn}(bx+a) - 24a^2b^2d^2e^3x^2\operatorname{sgn}(bx+a) + 6a^2b^2e^4x^2\operatorname{sgn}(bx+a) + 48b^3d^3e^2x\operatorname{sgn}(bx+a) - 72a^2b^2d^2e^2x\operatorname{sgn}(bx+a) + 48a^2b^2d^3e^3x\operatorname{sgn}(bx+a) - 12a^3e^4x\operatorname{sgn}(bx+a))/b^4 + (b^4d^4\operatorname{sgn}(bx+a) - 4ab^3d^3e\operatorname{sgn}(bx+a) + 6a^2b^2d^2e^2\operatorname{sgn}(bx+a) - 4a^3bd^2e^3\operatorname{sgn}(bx+a) + a^4e^4\operatorname{sgn}(bx+a))\log(\operatorname{abs}(bx+a))/b^5$$

Mupad [F(-1)]

Timed out.

$$\int \frac{(d+ex)^4}{\sqrt{a^2+2abx+b^2x^2}} dx = \int \frac{(d+ex)^4}{\sqrt{(a+bx)^2}} dx$$

input `int((d + e*x)^4/((a + b*x)^2)^(1/2),x)`

output `int((d + e*x)^4/((a + b*x)^2)^(1/2), x)`

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 208, normalized size of antiderivative = 0.94

$$\int \frac{(d+ex)^4}{\sqrt{a^2+2abx+b^2x^2}} dx = \frac{12 \log(bx+a) a^4 e^4 - 48 \log(bx+a) a^3 b d e^3 + 72 \log(bx+a) a^2 b^2 d^2 e^2 - 48 \log(bx+a) a b^3 d^3 e + 12 \log(bx+a) b^4 d^4}{b^5}$$

input `int((e*x+d)^4/((b*x+a)^2)^(1/2),x)`

output

```
(12*log(a + b*x)*a**4*e**4 - 48*log(a + b*x)*a**3*b*d*e**3 + 72*log(a + b*x)*a**2*b**2*d**2*e**2 - 48*log(a + b*x)*a*b**3*d**3*e + 12*log(a + b*x)*b**4*d**4 - 12*a**3*b*e**4*x + 48*a**2*b**2*d*e**3*x + 6*a**2*b**2*e**4*x**2 - 72*a*b**3*d**2*e**2*x - 24*a*b**3*d*e**3*x**2 - 4*a*b**3*e**4*x**3 + 48*b**4*d**3*e*x + 36*b**4*d**2*e**2*x**2 + 16*b**4*d*e**3*x**3 + 3*b**4*e**4*x**4)/(12*b**5)
```


3.310 $\int \frac{(d+ex)^3}{\sqrt{a^2+2abx+b^2x^2}} dx$

| | |
|---|------|
| Optimal result | 2340 |
| Mathematica [A] (verified) | 2340 |
| Rubi [A] (verified) | 2341 |
| Maple [A] (verified) | 2342 |
| Fricas [A] (verification not implemented) | 2343 |
| Sympy [B] (verification not implemented) | 2343 |
| Maxima [A] (verification not implemented) | 2344 |
| Giac [A] (verification not implemented) | 2345 |
| Mupad [F(-1)] | 2345 |
| Reduce [B] (verification not implemented) | 2346 |

Optimal result

Integrand size = 28, antiderivative size = 173

$$\int \frac{(d+ex)^3}{\sqrt{a^2+2abx+b^2x^2}} dx = \frac{e(bd-ae)^2x(a+bx)}{b^3\sqrt{a^2+2abx+b^2x^2}} + \frac{(bd-ae)(a+bx)(d+ex)^2}{2b^2\sqrt{a^2+2abx+b^2x^2}} + \frac{(a+bx)(d+ex)^3}{3b\sqrt{a^2+2abx+b^2x^2}} + \frac{(bd-ae)^3(a+bx)\log(a+bx)}{b^4\sqrt{a^2+2abx+b^2x^2}}$$

output

```
e*(-a*e+b*d)^2*x*(b*x+a)/b^3/((b*x+a)^2)^(1/2)+1/2*(-a*e+b*d)*(b*x+a)*(e*x+d)^2/b^2/((b*x+a)^2)^(1/2)+1/3*(b*x+a)*(e*x+d)^3/b/((b*x+a)^2)^(1/2)+(-a*e+b*d)^3*(b*x+a)*ln(b*x+a)/b^4/((b*x+a)^2)^(1/2)
```

Mathematica [A] (verified)

Time = 1.05 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.52

$$\int \frac{(d+ex)^3}{\sqrt{a^2+2abx+b^2x^2}} dx = \frac{(a+bx)(bex(6a^2e^2-3abe(6d+ex)+b^2(18d^2+9dex+2e^2x^2))+6(bd-ae)^3\log(a+bx))}{6b^4\sqrt{(a+bx)^2}}$$

input

```
Integrate[(d + e*x)^3/Sqrt[a^2 + 2*a*b*x + b^2*x^2], x]
```

output $((a + bx)*(b*e*x*(6*a^2*e^2 - 3*a*b*e*(6*d + e*x) + b^2*(18*d^2 + 9*d*e*x + 2*e^2*x^2)) + 6*(b*d - a*e)^3*\text{Log}[a + b*x]))/(6*b^4*\text{Sqrt}[(a + b*x)^2])$

Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 99, normalized size of antiderivative = 0.57, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {1102, 27, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\begin{aligned} & \int \frac{(d + ex)^3}{\sqrt{a^2 + 2abx + b^2x^2}} dx \\ & \quad \downarrow \text{1102} \\ & \frac{b(a + bx) \int \frac{(d+ex)^3}{b(a+bx)} dx}{\sqrt{a^2 + 2abx + b^2x^2}} \\ & \quad \downarrow \text{27} \\ & \frac{(a + bx) \int \frac{(d+ex)^3}{a+bx} dx}{\sqrt{a^2 + 2abx + b^2x^2}} \\ & \quad \downarrow \text{49} \\ & \frac{(a + bx) \int \left(\frac{(bd-ae)^3}{b^3(a+bx)} + \frac{e(bd-ae)^2}{b^3} + \frac{e(d+ex)(bd-ae)}{b^2} + \frac{e(d+ex)^2}{b} \right) dx}{\sqrt{a^2 + 2abx + b^2x^2}} \\ & \quad \downarrow \text{2009} \\ & \frac{(a + bx) \left(\frac{(bd-ae)^3 \log(a+bx)}{b^4} + \frac{ex(bd-ae)^2}{b^3} + \frac{(d+ex)^2(bd-ae)}{2b^2} + \frac{(d+ex)^3}{3b} \right)}{\sqrt{a^2 + 2abx + b^2x^2}} \end{aligned}$$

input $\text{Int}[(d + e*x)^3/\text{Sqrt}[a^2 + 2*a*b*x + b^2*x^2], x]$

output $((a + b*x)*((e*(b*d - a*e)^2*x)/b^3 + ((b*d - a*e)*(d + e*x)^2)/(2*b^2) + (d + e*x)^3/(3*b) + ((b*d - a*e)^3*\text{Log}[a + b*x])/b^4))/\text{Sqrt}[a^2 + 2*a*b*x + b^2*x^2]$

Defintions of rubi rules used

rule 27 `Int[(a_)*(Fx), x_Symbol] := Simp[a Int[Fx, x], x] /; FreeQ[a, x] && !MatchQ[Fx, (b_)*(Gx) /; FreeQ[b, x]]`

rule 49 `Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] := Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] && IGtQ[m, 0] && IGtQ[m + n + 2, 0]`

rule 1102 `Int[((d_.) + (e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] := Simp[(a + b*x + c*x^2)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x)^(2*FracPart[p])) Int[(d + e*x)^m*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && EqQ[b^2 - 4*a*c, 0]`

rule 2009 `Int[u_, x_Symbol] := Simp[IntSum[u, x], x] /; SumQ[u]`

Maple [A] (verified)

Time = 0.95 (sec) , antiderivative size = 142, normalized size of antiderivative = 0.82

| method | result |
|---------|--|
| risch | $\frac{\sqrt{(bx+a)^2} e^{\left(\frac{1}{3}b^2e^2x^3 - \frac{1}{2}x^2ab e^2 + \frac{3}{2}b^2de x^2 + a^2e^2x - 3abdex + 3b^2d^2x\right)}}{(bx+a)b^3} - \frac{\sqrt{(bx+a)^2} (e^3a^3 - 3a^2bd e^2 + 3ab^2d^2e - b^3d^3) \ln(bx+a)}{(bx+a)b^4}$ |
| default | $-\frac{(bx+a)(-2e^3x^3b^3 + 3x^2a b^2e^3 - 9x^2b^3d e^2 + 6 \ln(bx+a)a^3e^3 - 18 \ln(bx+a)a^2bd e^2 + 18 \ln(bx+a)a b^2d^2e - 6 \ln(bx+a)b^3d^3 - 6a^2b e^3)}{6\sqrt{(bx+a)^2} b^4}$ |

input `int((e*x+d)^3/((b*x+a)^2)^(1/2), x, method=_RETURNVERBOSE)`

output `((b*x+a)^2)^(1/2)/(b*x+a)*e/b^3*(1/3*b^2*e^2*x^3-1/2*x^2*a*b*e^2+3/2*b^2*d*e*x^2+a^2*e^2*x-3*a*b*d*e*x+3*b^2*d^2*x)-((b*x+a)^2)^(1/2)/(b*x+a)/b^4*(a^3*e^3-3*a^2*b*d*e^2+3*a*b^2*d^2*e-b^3*d^3)*ln(b*x+a)`

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 116, normalized size of antiderivative = 0.67

$$\int \frac{(d+ex)^3}{\sqrt{a^2+2abx+b^2x^2}} dx$$

$$= \frac{2b^3e^3x^3 + 3(3b^3de^2 - ab^2e^3)x^2 + 6(3b^3d^2e - 3ab^2de^2 + a^2be^3)x + 6(b^3d^3 - 3ab^2d^2e + 3a^2bde^2 - a^3e^3)}{6b^4}$$

input `integrate((e*x+d)^3/((b*x+a)^2)^(1/2),x, algorithm="fricas")`output `1/6*(2*b^3*e^3*x^3 + 3*(3*b^3*d*e^2 - a*b^2*e^3)*x^2 + 6*(3*b^3*d^2*e - 3*a*b^2*d*e^2 + a^2*b*e^3)*x + 6*(b^3*d^3 - 3*a*b^2*d^2*e + 3*a^2*b*d*e^2 - a^3*e^3)*log(b*x + a))/b^4`**Sympy [B] (verification not implemented)**

Leaf count of result is larger than twice the leaf count of optimal. 462 vs. 2(126) = 252.

Time = 1.19 (sec) , antiderivative size = 462, normalized size of antiderivative = 2.67

$$\int \frac{(d+ex)^3}{\sqrt{a^2+2abx+b^2x^2}} dx$$

$$= \left\{ \begin{array}{l} \sqrt{a^2+2abx+b^2x^2} \left(\frac{e^3x^2}{3b^2} + \frac{x(-\frac{5ae^3}{3b}+3de^2)}{2b^2} + \frac{-\frac{2a^2e^3}{3b^2} - \frac{3a(-\frac{5ae^3}{3b}+3de^2)}{b^2} + 3d^2e}{b^2} \right) + \left(\frac{a}{b} + x \right) \left(-\frac{a^2(-\frac{5ae^3}{3b}+3de^2)}{2b^2} - \frac{a(-\frac{2a^2e^3}{3b^2} - \frac{3a(-\frac{5ae^3}{3b}+3de^2)}{b^2} + 3d^2e)}{2b^2} \right) \\ \frac{3d^2e \left(-a^2\sqrt{a^2+2abx} + \frac{(a^2+2abx)^{\frac{3}{2}}}{3} \right)}{2d^3\sqrt{a^2+2abx} + \frac{3d^2e \left(-a^2\sqrt{a^2+2abx} + \frac{(a^2+2abx)^{\frac{3}{2}}}{3} \right)}{ab}} + \frac{3de^2 \left(a^4\sqrt{a^2+2abx} - \frac{2a^2(a^2+2abx)^{\frac{3}{2}}}{3} + \frac{(a^2+2abx)^{\frac{5}{2}}}{5} \right)}{2a^2b^2} + \frac{e^3 \left(-a^6\sqrt{a^2+2abx} + a^4(a^2+2abx) \right)}{2ab} \\ \frac{d^3x + \frac{3d^2ex^2}{2} + de^2x^3 + \frac{e^3x^4}{4}}{\sqrt{a^2}} \end{array} \right.$$

input `integrate((e*x+d)**3/((b*x+a)**2)**(1/2),x)`

output

```
Piecewise((sqrt(a**2 + 2*a*b*x + b**2*x**2)*(e**3*x**2/(3*b**2) + x*(-5*a*
e**3/(3*b) + 3*d*e**2)/(2*b**2) + (-2*a**2*e**3/(3*b**2) - 3*a*(-5*a*e**3/
(3*b) + 3*d*e**2)/(2*b) + 3*d**2*e)/b**2) + (a/b + x)*(-a**2*(-5*a*e**3/(3
*b) + 3*d*e**2)/(2*b**2) - a*(-2*a**2*e**3/(3*b**2) - 3*a*(-5*a*e**3/(3*b)
+ 3*d*e**2)/(2*b) + 3*d**2*e)/b + d**3)*log(a/b + x)/sqrt(b**2*(a/b + x)*
*2), Ne(b**2, 0)), ((2*d**3*sqrt(a**2 + 2*a*b*x) + 3*d**2*e*(-a**2*sqrt(a**
2 + 2*a*b*x) + (a**2 + 2*a*b*x)**(3/2)/3)/(a*b) + 3*d*e**2*(a**4*sqrt(a**
2 + 2*a*b*x) - 2*a**2*(a**2 + 2*a*b*x)**(3/2)/3 + (a**2 + 2*a*b*x)**(5/2)/
5)/(2*a**2*b**2) + e**3*(-a**6*sqrt(a**2 + 2*a*b*x) + a**4*(a**2 + 2*a*b*x)
)**(3/2) - 3*a**2*(a**2 + 2*a*b*x)**(5/2)/5 + (a**2 + 2*a*b*x)**(7/2)/7)/(
4*a**3*b**3))/(2*a*b), Ne(a*b, 0)), ((d**3*x + 3*d**2*e*x**2/2 + d*e**2*x*
*3 + e**3*x**4/4)/sqrt(a**2), True))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 205, normalized size of antiderivative = 1.18

$$\int \frac{(d+ex)^3}{\sqrt{a^2+2abx+b^2x^2}} dx = \frac{3de^2x^2}{2b} - \frac{5ae^3x^2}{6b^2} + \frac{\sqrt{b^2x^2+2abx+a^2}e^3x^2}{3b^2} - \frac{3ade^2x}{b^2}$$

$$+ \frac{5a^2e^3x}{3b^3} + \frac{d^3 \log(x + \frac{a}{b})}{b} - \frac{3ad^2e \log(x + \frac{a}{b})}{b^2}$$

$$+ \frac{3a^2de^2 \log(x + \frac{a}{b})}{b^3} - \frac{a^3e^3 \log(x + \frac{a}{b})}{b^4}$$

$$+ \frac{3\sqrt{b^2x^2+2abx+a^2}d^2e}{b^2} - \frac{2\sqrt{b^2x^2+2abx+a^2}a^2e^3}{3b^4}$$

input

```
integrate((e*x+d)^3/((b*x+a)^2)^(1/2),x, algorithm="maxima")
```

output

```
3/2*d*e^2*x^2/b - 5/6*a*e^3*x^2/b^2 + 1/3*sqrt(b^2*x^2 + 2*a*b*x + a^2)*e^
3*x^2/b^2 - 3*a*d*e^2*x/b^2 + 5/3*a^2*e^3*x/b^3 + d^3*log(x + a/b)/b - 3*a
*d^2*e*log(x + a/b)/b^2 + 3*a^2*d*e^2*log(x + a/b)/b^3 - a^3*e^3*log(x + a
/b)/b^4 + 3*sqrt(b^2*x^2 + 2*a*b*x + a^2)*d^2*e/b^2 - 2/3*sqrt(b^2*x^2 + 2
*a*b*x + a^2)*a^2*e^3/b^4
```

Giac [A] (verification not implemented)

Time = 0.70 (sec) , antiderivative size = 175, normalized size of antiderivative = 1.01

$$\int \frac{(d+ex)^3}{\sqrt{a^2+2abx+b^2x^2}} dx$$

$$= \frac{2b^2e^3x^3\operatorname{sgn}(bx+a) + 9b^2de^2x^2\operatorname{sgn}(bx+a) - 3abe^3x^2\operatorname{sgn}(bx+a) + 18b^2d^2ex\operatorname{sgn}(bx+a) - 18abde^2x}{6b^3} + \frac{(b^3d^3\operatorname{sgn}(bx+a) - 3ab^2d^2e\operatorname{sgn}(bx+a) + 3a^2bde^2\operatorname{sgn}(bx+a) - a^3e^3\operatorname{sgn}(bx+a))\log(|bx+a|)}{b^4}$$

input `integrate((e*x+d)^3/((b*x+a)^2)^(1/2),x, algorithm="giac")`

output `1/6*(2*b^2*e^3*x^3*sgn(b*x + a) + 9*b^2*d*e^2*x^2*sgn(b*x + a) - 3*a*b*e^3*x^2*sgn(b*x + a) + 18*b^2*d^2*e*x*sgn(b*x + a) - 18*a*b*d*e^2*x*sgn(b*x + a) + 6*a^2*e^3*x*sgn(b*x + a))/b^3 + (b^3*d^3*sgn(b*x + a) - 3*a*b^2*d^2*e*sgn(b*x + a) + 3*a^2*b*d*e^2*sgn(b*x + a) - a^3*e^3*sgn(b*x + a))*log(abs(b*x + a))/b^4`

Mupad [F(-1)]

Timed out.

$$\int \frac{(d+ex)^3}{\sqrt{a^2+2abx+b^2x^2}} dx = \int \frac{(d+ex)^3}{\sqrt{(a+bx)^2}} dx$$

input `int((d + e*x)^3/((a + b*x)^2)^(1/2),x)`

output `int((d + e*x)^3/((a + b*x)^2)^(1/2), x)`

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 132, normalized size of antiderivative = 0.76

$$\int \frac{(d + ex)^3}{\sqrt{a^2 + 2abx + b^2x^2}} dx$$

$$= \frac{-6 \log(bx + a) a^3 e^3 + 18 \log(bx + a) a^2 b d e^2 - 18 \log(bx + a) a b^2 d^2 e + 6 \log(bx + a) b^3 d^3 + 6 a^2 b e^3 x - 1}{6 b^4}$$

input

```
int((e*x+d)^3/((b*x+a)^2)^(1/2),x)
```

output

```
( - 6*log(a + b*x)*a**3*e**3 + 18*log(a + b*x)*a**2*b*d*e**2 - 18*log(a +
b*x)*a*b**2*d**2*e + 6*log(a + b*x)*b**3*d**3 + 6*a**2*b*e**3*x - 18*a*b**
2*d*e**2*x - 3*a*b**2*e**3*x**2 + 18*b**3*d**2*e*x + 9*b**3*d*e**2*x**2 +
2*b**3*e**3*x**3)/(6*b**4)
```

3.311 $\int \frac{(d+ex)^2}{\sqrt{a^2+2abx+b^2x^2}} dx$

| | |
|---|------|
| Optimal result | 2347 |
| Mathematica [A] (verified) | 2347 |
| Rubi [A] (verified) | 2348 |
| Maple [A] (verified) | 2349 |
| Fricas [A] (verification not implemented) | 2350 |
| Sympy [B] (verification not implemented) | 2350 |
| Maxima [A] (verification not implemented) | 2351 |
| Giac [A] (verification not implemented) | 2351 |
| Mupad [F(-1)] | 2352 |
| Reduce [B] (verification not implemented) | 2352 |

Optimal result

Integrand size = 28, antiderivative size = 124

$$\int \frac{(d+ex)^2}{\sqrt{a^2+2abx+b^2x^2}} dx = \frac{e(2bd-ae)x(a+bx)}{b^2\sqrt{a^2+2abx+b^2x^2}} + \frac{e^2x^2(a+bx)}{2b\sqrt{a^2+2abx+b^2x^2}} + \frac{(bd-ae)^2(a+bx)\log(a+bx)}{b^3\sqrt{a^2+2abx+b^2x^2}}$$

output `e*(-a*e+2*b*d)*x*(b*x+a)/b^2/((b*x+a)^2)^(1/2)+1/2*e^2*x^2*(b*x+a)/b/((b*x+a)^2)^(1/2)+(-a*e+b*d)^2*(b*x+a)*ln(b*x+a)/b^3/((b*x+a)^2)^(1/2)`

Mathematica [A] (verified)

Time = 1.03 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.48

$$\int \frac{(d+ex)^2}{\sqrt{a^2+2abx+b^2x^2}} dx = \frac{(a+bx)(bex(4bd-2ae+bex)+2(bd-ae)^2\log(a+bx))}{2b^3\sqrt{(a+bx)^2}}$$

input `Integrate[(d + e*x)^2/Sqrt[a^2 + 2*a*b*x + b^2*x^2], x]`

output `((a + b*x)*(b*e*x*(4*b*d - 2*a*e + b*e*x) + 2*(b*d - a*e)^2*Log[a + b*x]))/(2*b^3*Sqrt[(a + b*x)^2])`

Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.60, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {1102, 27, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\begin{aligned}
 & \int \frac{(d+ex)^2}{\sqrt{a^2+2abx+b^2x^2}} dx \\
 & \quad \downarrow \text{1102} \\
 & \frac{b(a+bx) \int \frac{(d+ex)^2}{b(a+bx)} dx}{\sqrt{a^2+2abx+b^2x^2}} \\
 & \quad \downarrow \text{27} \\
 & \frac{(a+bx) \int \frac{(d+ex)^2}{a+bx} dx}{\sqrt{a^2+2abx+b^2x^2}} \\
 & \quad \downarrow \text{49} \\
 & \frac{(a+bx) \int \left(\frac{(bd-ae)^2}{b^2(a+bx)} + \frac{e(bd-ae)}{b^2} + \frac{e(d+ex)}{b} \right) dx}{\sqrt{a^2+2abx+b^2x^2}} \\
 & \quad \downarrow \text{2009} \\
 & \frac{(a+bx) \left(\frac{(bd-ae)^2 \log(a+bx)}{b^3} + \frac{ex(bd-ae)}{b^2} + \frac{(d+ex)^2}{2b} \right)}{\sqrt{a^2+2abx+b^2x^2}}
 \end{aligned}$$

input `Int[(d + e*x)^2/Sqrt[a^2 + 2*a*b*x + b^2*x^2],x]`

output `((a + b*x)*((e*(b*d - a*e)*x)/b^2 + (d + e*x)^2/(2*b) + ((b*d - a*e)^2*Log[a + b*x])/b^3))/Sqrt[a^2 + 2*a*b*x + b^2*x^2]`

Definitions of rubi rules used

rule 27 `Int[(a_)*(F_x_), x_Symbol] := Simp[a Int[F_x, x], x] /; FreeQ[a, x] && !MatchQ[F_x, (b_)*(G_x_)] /; FreeQ[b, x]`

rule 49 `Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] := Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] && IGtQ[m, 0] && IGtQ[m + n + 2, 0]`

rule 1102 `Int[((d_.) + (e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] := Simp[(a + b*x + c*x^2)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x)^(2*FracPart[p])) Int[(d + e*x)^m*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && EqQ[b^2 - 4*a*c, 0]`

rule 2009 `Int[u_, x_Symbol] := Simp[IntSum[u, x], x] /; SumQ[u]`

Maple [A] (verified)

Time = 0.86 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.70

| method | result | size |
|---------|---|------|
| default | $\frac{(bx+a)(b^2e^2x^2+2\ln(bx+a)a^2e^2-4\ln(bx+a)abde+2\ln(bx+a)b^2d^2-2xabe^2+4b^2dex)}{2\sqrt{(bx+a)^2}b^3}$ | 87 |
| risch | $-\frac{\sqrt{(bx+a)^2}e(-\frac{1}{2}be^2x^2+ae^2x-2bdx)}{(bx+a)b^2} + \frac{\sqrt{(bx+a)^2}(e^2a^2-2abde+b^2d^2)\ln(bx+a)}{(bx+a)b^3}$ | 88 |

input `int((e*x+d)^2/((b*x+a)^2)^(1/2),x,method=_RETURNVERBOSE)`

output `1/2*(b*x+a)*(b^2*e^2*x^2+2*ln(b*x+a)*a^2*e^2-4*ln(b*x+a)*a*b*d*e+2*ln(b*x+a)*b^2*d^2-2*x*a*b*e^2+4*b^2*d*e*x)/((b*x+a)^2)^(1/2)/b^3`

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.51

$$\int \frac{(d+ex)^2}{\sqrt{a^2+2abx+b^2x^2}} dx = \frac{b^2e^2x^2 + 2(2b^2de - abe^2)x + 2(b^2d^2 - 2abde + a^2e^2) \log(bx+a)}{2b^3}$$

input `integrate((e*x+d)^2/((b*x+a)^2)^(1/2),x, algorithm="fricas")`

output `1/2*(b^2*e^2*x^2 + 2*(2*b^2*d*e - a*b*e^2)*x + 2*(b^2*d^2 - 2*a*b*d*e + a^2*e^2)*log(b*x + a))/b^3`

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 265 vs. 2(88) = 176.

Time = 0.89 (sec) , antiderivative size = 265, normalized size of antiderivative = 2.14

$$\int \frac{(d+ex)^2}{\sqrt{a^2+2abx+b^2x^2}} dx = \begin{cases} \left(\frac{e^2x}{2b^2} + \frac{-3ae^2+2de}{b^2} \right) \sqrt{a^2+2abx+b^2x^2} + \frac{\left(\frac{a}{b}+x \right) \left(-\frac{a^2e^2}{2b^2} - \frac{a(-\frac{3ae^2}{2b}+2de)}{b} + d^2 \right) \log\left(\frac{a}{b}+x\right)}{\sqrt{b^2\left(\frac{a}{b}+x\right)^2}} & \text{for } b^2 \neq 0 \\ \frac{2de \left(-a^2\sqrt{a^2+2abx} + \frac{(a^2+2abx)^{\frac{3}{2}}}{3} \right)}{2d^2\sqrt{a^2+2abx} + \frac{ab}{2ab}} + \frac{e^2 \left(a^4\sqrt{a^2+2abx} - \frac{2a^2(a^2+2abx)^{\frac{3}{2}}}{3} + \frac{(a^2+2abx)^{\frac{5}{2}}}{5} \right)}{2a^2b^2} & \text{for } ab \neq 0 \\ \frac{d^2x+dex^2+\frac{e^2x^3}{3}}{\sqrt{a^2}} & \text{otherwise} \end{cases}$$

input `integrate((e*x+d)**2/((b*x+a)**2)**(1/2),x)`

output

```
Piecewise(((e**2*x/(2*b**2) + (-3*a*e**2/(2*b) + 2*d*e)/b**2)*sqrt(a**2 +
2*a*b*x + b**2*x**2) + (a/b + x)*(-a**2*e**2/(2*b**2) - a*(-3*a*e**2/(2*b)
+ 2*d*e)/b + d**2)*log(a/b + x)/sqrt(b**2*(a/b + x)**2), Ne(b**2, 0)), ((
2*d**2*sqrt(a**2 + 2*a*b*x) + 2*d*e*(-a**2*sqrt(a**2 + 2*a*b*x) + (a**2 +
2*a*b*x)**(3/2)/3)/(a*b) + e**2*(a**4*sqrt(a**2 + 2*a*b*x) - 2*a**2*(a**2
+ 2*a*b*x)**(3/2)/3 + (a**2 + 2*a*b*x)**(5/2)/5)/(2*a**2*b**2))/(2*a*b), N
e(a*b, 0)), ((d**2*x + d*e*x**2 + e**2*x**3/3)/sqrt(a**2), True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 96, normalized size of antiderivative = 0.77

$$\int \frac{(d+ex)^2}{\sqrt{a^2+2abx+b^2x^2}} dx = \frac{e^2x^2}{2b} - \frac{ae^2x}{b^2} + \frac{d^2 \log(x + \frac{a}{b})}{b} - \frac{2ade \log(x + \frac{a}{b})}{b^2} + \frac{a^2e^2 \log(x + \frac{a}{b})}{b^3} + \frac{2\sqrt{b^2x^2+2abx+a^2}de}{b^2}$$

input

```
integrate((e*x+d)^2/((b*x+a)^2)^(1/2),x, algorithm="maxima")
```

output

```
1/2*e^2*x^2/b - a*e^2*x/b^2 + d^2*log(x + a/b)/b - 2*a*d*e*log(x + a/b)/b^
2 + a^2*e^2*log(x + a/b)/b^3 + 2*sqrt(b^2*x^2 + 2*a*b*x + a^2)*d*e/b^2
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 96, normalized size of antiderivative = 0.77

$$\int \frac{(d+ex)^2}{\sqrt{a^2+2abx+b^2x^2}} dx = \frac{be^2x^2 \operatorname{sgn}(bx+a) + 4bdex \operatorname{sgn}(bx+a) - 2ae^2x \operatorname{sgn}(bx+a)}{2b^2} + \frac{(b^2d^2 \operatorname{sgn}(bx+a) - 2abdes \operatorname{sgn}(bx+a) + a^2e^2 \operatorname{sgn}(bx+a)) \log(|bx+a|)}{b^3}$$

input

```
integrate((e*x+d)^2/((b*x+a)^2)^(1/2),x, algorithm="giac")
```

output

```
1/2*(b*e^2*x^2*sgn(b*x + a) + 4*b*d*e*x*sgn(b*x + a) - 2*a*e^2*x*sgn(b*x +
a))/b^2 + (b^2*d^2*sgn(b*x + a) - 2*a*b*d*e*sgn(b*x + a) + a^2*e^2*sgn(b*
x + a))*log(abs(b*x + a))/b^3
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d + ex)^2}{\sqrt{a^2 + 2abx + b^2x^2}} dx = \int \frac{(d + ex)^2}{\sqrt{(a + bx)^2}} dx$$

input

```
int((d + e*x)^2/((a + b*x)^2)^(1/2), x)
```

output

```
int((d + e*x)^2/((a + b*x)^2)^(1/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.58

$$\int \frac{(d + ex)^2}{\sqrt{a^2 + 2abx + b^2x^2}} dx$$

$$= \frac{2 \log(bx + a) a^2 e^2 - 4 \log(bx + a) abde + 2 \log(bx + a) b^2 d^2 - 2ab e^2 x + 4b^2 dex + b^2 e^2 x^2}{2b^3}$$

input

```
int((e*x+d)^2/((b*x+a)^2)^(1/2), x)
```

output

```
(2*log(a + b*x)*a**2*e**2 - 4*log(a + b*x)*a*b*d*e + 2*log(a + b*x)*b**2*d
**2 - 2*a*b*e**2*x + 4*b**2*d*e*x + b**2*e**2*x**2)/(2*b**3)
```

3.312 $\int \frac{d+ex}{\sqrt{a^2+2abx+b^2x^2}} dx$

| | |
|---|------|
| Optimal result | 2353 |
| Mathematica [B] (verified) | 2353 |
| Rubi [A] (verified) | 2354 |
| Maple [A] (verified) | 2355 |
| Fricas [A] (verification not implemented) | 2356 |
| Sympy [B] (verification not implemented) | 2356 |
| Maxima [A] (verification not implemented) | 2357 |
| Giac [A] (verification not implemented) | 2357 |
| Mupad [B] (verification not implemented) | 2357 |
| Reduce [B] (verification not implemented) | 2358 |

Optimal result

Integrand size = 26, antiderivative size = 69

$$\int \frac{d+ex}{\sqrt{a^2+2abx+b^2x^2}} dx = \frac{e\sqrt{a^2+2abx+b^2x^2}}{b^2} + \frac{(bd-ae)(a+bx)\log(a+bx)}{b^2\sqrt{a^2+2abx+b^2x^2}}$$

output

$$e*((b*x+a)^2)^{(1/2)}/b^2+(-a*e+b*d)*(b*x+a)*\ln(b*x+a)/b^2/((b*x+a)^2)^{(1/2)}$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 177 vs. 2(69) = 138.

Time = 0.48 (sec) , antiderivative size = 177, normalized size of antiderivative = 2.57

$$\int \frac{d+ex}{\sqrt{a^2+2abx+b^2x^2}} dx = \frac{(2a+bx)\left(-bex\left(\sqrt{a^2}bx+a\left(\sqrt{a^2}-\sqrt{(a+bx)^2}\right)\right)-2(-bd+ae)\left(-a^2-abx+\sqrt{a^2}\sqrt{(a+bx)^2}\right)\arcsin\left(\frac{a+bx}{\sqrt{a^2+2abx+b^2x^2}}\right)\right)}{b^2\left(\sqrt{a^2}-\sqrt{(a+bx)^2}\right)\left(\sqrt{a^2}bx+a\left(\sqrt{a^2}-\sqrt{(a+bx)^2}\right)\right)}$$

input

$$\text{Integrate}[(d+e*x)/\text{Sqrt}[a^2+2*a*b*x+b^2*x^2],x]$$

output

```
((2*a + b*x)*(-(b*e*x*(Sqrt[a^2]*b*x + a*(Sqrt[a^2] - Sqrt[(a + b*x)^2])))
- 2*(-(b*d) + a*e)*(-a^2 - a*b*x + Sqrt[a^2]*Sqrt[(a + b*x)^2])*ArcTanh[(
b*x)/(Sqrt[a^2] - Sqrt[(a + b*x)^2])))/(b^2*(Sqrt[a^2] - Sqrt[(a + b*x)^2
])*(Sqrt[a^2]*b*x + a*(Sqrt[a^2] - Sqrt[(a + b*x)^2])))
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.115$, Rules used = {1100, 1079, 16}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{d + ex}{\sqrt{a^2 + 2abx + b^2x^2}} dx$$

↓ 1100

$$\frac{(bd - ae) \int \frac{1}{\sqrt{a^2 + 2bxa + b^2x^2}} dx}{b} + \frac{e\sqrt{a^2 + 2abx + b^2x^2}}{b^2}$$

↓ 1079

$$\frac{(a + bx)(bd - ae) \int \frac{1}{xb^2 + ab} dx}{\sqrt{a^2 + 2abx + b^2x^2}} + \frac{e\sqrt{a^2 + 2abx + b^2x^2}}{b^2}$$

↓ 16

$$\frac{(a + bx)(bd - ae) \log(a + bx)}{b^2\sqrt{a^2 + 2abx + b^2x^2}} + \frac{e\sqrt{a^2 + 2abx + b^2x^2}}{b^2}$$

input

```
Int[(d + e*x)/Sqrt[a^2 + 2*a*b*x + b^2*x^2], x]
```

output

```
(e*Sqrt[a^2 + 2*a*b*x + b^2*x^2])/b^2 + ((b*d - a*e)*(a + b*x)*Log[a + b*x
])/ (b^2*Sqrt[a^2 + 2*a*b*x + b^2*x^2])
```

Definitions of rubi rules used

rule 16 `Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] := Simp[c*(Log[RemoveContent[a + b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]`

rule 1079 `Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] := Simp[(a + b*x + c*x^2)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x)^(2*FracPart[p])) Int[(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c, p}, x] && EqQ[b^2 - 4*a*c, 0]`

rule 1100 `Int[((d_.) + (e_.)*(x_))*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] := Simp[e*((a + b*x + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Simp[(2*c*d - b*e)/(2*c) Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[b^2 - 4*a*c, 0]`

Maple [A] (verified)

Time = 0.73 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.65

| method | result | size |
|---------|--|------|
| default | $-\frac{(bx+a)(\ln(bx+a)ae - \ln(bx+a)bd - bex)}{\sqrt{(bx+a)^2} b^2}$ | 45 |
| risch | $\frac{\sqrt{(bx+a)^2} ex}{(bx+a)b} - \frac{\sqrt{(bx+a)^2} (ae - bd) \ln(bx+a)}{(bx+a)b^2}$ | 59 |

input `int((e*x+d)/((b*x+a)^2)^(1/2), x, method=_RETURNVERBOSE)`

output `-(b*x+a)*(ln(b*x+a)*a*e - ln(b*x+a)*b*d - b*e*x)/((b*x+a)^2)^(1/2)/b^2`

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.35

$$\int \frac{d + ex}{\sqrt{a^2 + 2abx + b^2x^2}} dx = \frac{bex + (bd - ae) \log(bx + a)}{b^2}$$

input `integrate((e*x+d)/((b*x+a)^2)^(1/2),x, algorithm="fricas")`

output `(b*e*x + (b*d - a*e)*log(b*x + a))/b^2`

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 134 vs. 2(46) = 92.

Time = 0.77 (sec) , antiderivative size = 134, normalized size of antiderivative = 1.94

$$\int \frac{d + ex}{\sqrt{a^2 + 2abx + b^2x^2}} dx = \begin{cases} \frac{(\frac{a}{b} + x)(-\frac{ae}{b} + d) \log(\frac{a}{b} + x)}{\sqrt{b^2(\frac{a}{b} + x)^2}} + \frac{e\sqrt{a^2 + 2abx + b^2x^2}}{b^2} & \text{for } b^2 \neq 0 \\ \frac{2d\sqrt{a^2 + 2abx} + \frac{e \left(-a^2\sqrt{a^2 + 2abx} + \frac{(a^2 + 2abx)^{3/2}}{3} \right)}{2ab}}{2ab} & \text{for } ab \neq 0 \\ \frac{dx + \frac{ex^2}{2}}{\sqrt{a^2}} & \text{otherwise} \end{cases}$$

input `integrate((e*x+d)/((b*x+a)**2)**(1/2),x)`

output `Piecewise(((a/b + x)*(-a*e/b + d)*log(a/b + x)/sqrt(b**2*(a/b + x)**2) + e*sqrt(a**2 + 2*a*b*x + b**2*x**2)/b**2, Ne(b**2, 0)), ((2*d*sqrt(a**2 + 2*a*b*x) + e*(-a**2*sqrt(a**2 + 2*a*b*x) + (a**2 + 2*a*b*x)**(3/2)/3)/(a*b))/(2*a*b), Ne(a*b, 0)), ((d*x + e*x**2/2)/sqrt(a**2), True))`

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.75

$$\int \frac{d + ex}{\sqrt{a^2 + 2abx + b^2x^2}} dx = \frac{d \log\left(x + \frac{a}{b}\right)}{b} - \frac{ae \log\left(x + \frac{a}{b}\right)}{b^2} + \frac{\sqrt{b^2x^2 + 2abx + a^2}e}{b^2}$$

input `integrate((e*x+d)/((b*x+a)^2)^(1/2),x, algorithm="maxima")`output `d*log(x + a/b)/b - a*e*log(x + a/b)/b^2 + sqrt(b^2*x^2 + 2*a*b*x + a^2)*e/b^2`**Giac [A] (verification not implemented)**

Time = 0.14 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.64

$$\int \frac{d + ex}{\sqrt{a^2 + 2abx + b^2x^2}} dx = \frac{ex \operatorname{sgn}(bx + a)}{b} + \frac{(bd \operatorname{sgn}(bx + a) - ae \operatorname{sgn}(bx + a)) \log(|bx + a|)}{b^2}$$

input `integrate((e*x+d)/((b*x+a)^2)^(1/2),x, algorithm="giac")`output `e*x*sgn(b*x + a)/b + (b*d*sgn(b*x + a) - a*e*sgn(b*x + a))*log(abs(b*x + a))/b^2`**Mupad [B] (verification not implemented)**

Time = 5.71 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.14

$$\int \frac{d + ex}{\sqrt{a^2 + 2abx + b^2x^2}} dx = \frac{e \sqrt{a^2 + 2abx + b^2x^2}}{b^2} + \frac{d \ln\left(a + bx + \sqrt{(a + bx)^2}\right)}{b} - \frac{abe \ln\left(ab + \sqrt{(a + bx)^2} \sqrt{b^2 + b^2x}\right)}{(b^2)^{3/2}}$$

input `int((d + e*x)/((a + b*x)^2)^(1/2),x)`

output `(e*(a^2 + b^2*x^2 + 2*a*b*x)^(1/2))/b^2 + (d*log(a + b*x + ((a + b*x)^2)^(1/2)))/b - (a*b*e*log(a*b + ((a + b*x)^2)^(1/2)*(b^2)^(1/2) + b^2*x))/(b^2)^(3/2)`

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.41

$$\int \frac{d + ex}{\sqrt{a^2 + 2abx + b^2x^2}} dx = \frac{-\log(bx + a) ae + \log(bx + a) bd + bex}{b^2}$$

input `int((e*x+d)/((b*x+a)^2)^(1/2),x)`

output `(- log(a + b*x)*a*e + log(a + b*x)*b*d + b*e*x)/b**2`

3.313 $\int \frac{1}{\sqrt{a^2+2abx+b^2x^2}} dx$

| | |
|---|------|
| Optimal result | 2359 |
| Mathematica [A] (verified) | 2359 |
| Rubi [A] (verified) | 2360 |
| Maple [A] (verified) | 2361 |
| Fricas [A] (verification not implemented) | 2361 |
| Sympy [A] (verification not implemented) | 2361 |
| Maxima [A] (verification not implemented) | 2362 |
| Giac [A] (verification not implemented) | 2362 |
| Mupad [B] (verification not implemented) | 2362 |
| Reduce [B] (verification not implemented) | 2363 |

Optimal result

Integrand size = 20, antiderivative size = 35

$$\int \frac{1}{\sqrt{a^2+2abx+b^2x^2}} dx = \frac{(a+bx)\log(a+bx)}{b\sqrt{a^2+2abx+b^2x^2}}$$

output `(b*x+a)*ln(b*x+a)/b/((b*x+a)^2)^(1/2)`

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.74

$$\int \frac{1}{\sqrt{a^2+2abx+b^2x^2}} dx = \frac{(a+bx)\log(a+bx)}{b\sqrt{(a+bx)^2}}$$

input `Integrate[1/Sqrt[a^2 + 2*a*b*x + b^2*x^2], x]`

output `((a + b*x)*Log[a + b*x])/(b*Sqrt[(a + b*x)^2])`

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {1079, 16}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{a^2 + 2abx + b^2x^2}} dx$$

$$\downarrow 1079$$

$$\frac{b(a + bx) \int \frac{1}{xb^2 + ab} dx}{\sqrt{a^2 + 2abx + b^2x^2}}$$

$$\downarrow 16$$

$$\frac{(a + bx) \log(a + bx)}{b\sqrt{a^2 + 2abx + b^2x^2}}$$

input `Int[1/Sqrt[a^2 + 2*a*b*x + b^2*x^2], x]`

output `((a + b*x)*Log[a + b*x])/(b*Sqrt[a^2 + 2*a*b*x + b^2*x^2])`

Defintions of rubi rules used

rule 16 `Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a + b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]`

rule 1079 `Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(a + b*x + c*x^2)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x)^(2*FracPart[p])) Int[(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c, p}, x] && EqQ[b^2 - 4*a*c, 0]`

Maple [A] (verified)

Time = 0.67 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.71

| method | result | size |
|---------|--|------|
| default | $\frac{(bx+a)\ln(bx+a)}{b\sqrt{(bx+a)^2}}$ | 25 |
| risch | $\frac{\sqrt{(bx+a)^2}\ln(bx+a)}{(bx+a)b}$ | 27 |

input `int(1/((b*x+a)^2)^(1/2),x,method=_RETURNVERBOSE)`output `(b*x+a)*ln(b*x+a)/b/((b*x+a)^2)^(1/2)`**Fricas [A] (verification not implemented)**

Time = 0.08 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.29

$$\int \frac{1}{\sqrt{a^2 + 2abx + b^2x^2}} dx = \frac{\log(bx + a)}{b}$$

input `integrate(1/((b*x+a)^2)^(1/2),x, algorithm="fricas")`output `log(b*x + a)/b`**Sympy [A] (verification not implemented)**

Time = 0.45 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.69

$$\int \frac{1}{\sqrt{a^2 + 2abx + b^2x^2}} dx = \frac{\left(\frac{a}{b} + x\right) \log\left(\frac{a}{b} + x\right)}{\sqrt{b^2\left(\frac{a}{b} + x\right)^2}}$$

input `integrate(1/((b*x+a)**2)**(1/2),x)`

output $(a/b + x) \cdot \log(a/b + x) / \sqrt{b^2 \cdot (a/b + x)^2}$

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.34

$$\int \frac{1}{\sqrt{a^2 + 2abx + b^2x^2}} dx = \frac{\log\left(x + \frac{a}{b}\right)}{b}$$

input `integrate(1/((b*x+a)^2)^(1/2),x, algorithm="maxima")`

output $\log(x + a/b)/b$

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.49

$$\int \frac{1}{\sqrt{a^2 + 2abx + b^2x^2}} dx = \frac{\log(|bx + a|) \operatorname{sgn}(bx + a)}{b}$$

input `integrate(1/((b*x+a)^2)^(1/2),x, algorithm="giac")`

output $\log(\operatorname{abs}(b \cdot x + a)) \cdot \operatorname{sgn}(b \cdot x + a) / b$

Mupad [B] (verification not implemented)

Time = 5.19 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.54

$$\int \frac{1}{\sqrt{a^2 + 2abx + b^2x^2}} dx = \frac{\ln\left(a + bx + \sqrt{(a + bx)^2}\right)}{b}$$

input `int(1/((a + b*x)^2)^(1/2),x)`

output $\log(a + b*x + ((a + b*x)^2)^{(1/2)})/b$

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.29

$$\int \frac{1}{\sqrt{a^2 + 2abx + b^2x^2}} dx = \frac{\log(bx + a)}{b}$$

input $\text{int}(1/((b*x+a)^2)^{(1/2)}, x)$

output $\log(a + b*x)/b$

3.314 $\int \frac{1}{(d+ex)\sqrt{a^2+2abx+b^2x^2}} dx$

| | |
|---|------|
| Optimal result | 2364 |
| Mathematica [B] (verified) | 2364 |
| Rubi [A] (verified) | 2365 |
| Maple [A] (verified) | 2366 |
| Fricas [A] (verification not implemented) | 2367 |
| Sympy [F] | 2367 |
| Maxima [F(-2)] | 2367 |
| Giac [A] (verification not implemented) | 2368 |
| Mupad [F(-1)] | 2368 |
| Reduce [B] (verification not implemented) | 2368 |

Optimal result

Integrand size = 28, antiderivative size = 51

$$\int \frac{1}{(d+ex)\sqrt{a^2+2abx+b^2x^2}} dx = \frac{(a+bx) \log\left(\frac{b(a+bx)}{d+ex}\right)}{(bd-ae)\sqrt{a^2+2abx+b^2x^2}}$$

output $(b*x+a)*\ln(b*(b*x+a)/(e*x+d))/(-a*e+b*d)/((b*x+a)^2)^{(1/2)}$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 248 vs. 2(51) = 102.

Time = 0.37 (sec) , antiderivative size = 248, normalized size of antiderivative = 4.86

$$\int \frac{1}{(d+ex)\sqrt{a^2+2abx+b^2x^2}} dx = \frac{(-a+\sqrt{a^2}) \log\left(\sqrt{a^2}-bx-\sqrt{(a+bx)^2}\right) + (a+\sqrt{a^2}) \log\left(\sqrt{a^2}+bx-\sqrt{(a+bx)^2}\right) - a \log(2aex)}{}$$

input $\text{Integrate}[1/((d+e*x)*\text{Sqrt}[a^2+2*a*b*x+b^2*x^2]),x]$

output

```
((-a + Sqrt[a^2])*Log[Sqrt[a^2] - b*x - Sqrt[(a + b*x)^2]] + (a + Sqrt[a^2])
)*Log[Sqrt[a^2] + b*x - Sqrt[(a + b*x)^2]] - a*Log[2*a*e*x + d*(Sqrt[a^2]
- b*x - Sqrt[(a + b*x)^2])] - Sqrt[a^2]*Log[2*a*e*x + d*(Sqrt[a^2] - b*x
- Sqrt[(a + b*x)^2])] + a*Log[-2*a*e*x + d*(Sqrt[a^2] + b*x - Sqrt[(a + b*
x)^2])] - Sqrt[a^2]*Log[-2*a*e*x + d*(Sqrt[a^2] + b*x - Sqrt[(a + b*x)^2]
)]/(2*a*(-(b*d) + a*e))
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.22, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {1102, 27, 47, 16}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{a^2 + 2abx + b^2x^2}(d + ex)} dx$$

$$\downarrow 1102$$

$$\frac{b(a + bx) \int \frac{1}{b(a+bx)(d+ex)} dx}{\sqrt{a^2 + 2abx + b^2x^2}}$$

$$\downarrow 27$$

$$\frac{(a + bx) \int \frac{1}{(a+bx)(d+ex)} dx}{\sqrt{a^2 + 2abx + b^2x^2}}$$

$$\downarrow 47$$

$$\frac{(a + bx) \left(\frac{b \int \frac{1}{a+bx} dx}{bd-ae} - \frac{e \int \frac{1}{d+ex} dx}{bd-ae} \right)}{\sqrt{a^2 + 2abx + b^2x^2}}$$

$$\downarrow 16$$

$$\frac{(a + bx) \left(\frac{\log(a+bx)}{bd-ae} - \frac{\log(d+ex)}{bd-ae} \right)}{\sqrt{a^2 + 2abx + b^2x^2}}$$

input

```
Int[1/((d + e*x)*Sqrt[a^2 + 2*a*b*x + b^2*x^2]),x]
```

output $((a + b*x)*(Log[a + b*x]/(b*d - a*e) - Log[d + e*x]/(b*d - a*e)))/Sqrt[a^2 + 2*a*b*x + b^2*x^2]$

Defintions of rubi rules used

rule 16 $Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] \rightarrow Simp[c*(Log[RemoveContent[a + b*x, x]]/b), x] /; FreeQ[\{a, b, c\}, x]$

rule 27 $Int[(a_)*(F_x_), x_Symbol] \rightarrow Simp[a Int[F_x, x], x] /; FreeQ[a, x] \&\& !MatchQ[F_x, (b_)*(G_x_)] /; FreeQ[b, x]$

rule 47 $Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] \rightarrow Simp[b/(b*c - a*d) Int[1/(a + b*x), x], x] - Simp[d/(b*c - a*d) Int[1/(c + d*x), x], x] /; FreeQ[\{a, b, c, d\}, x]$

rule 1102 $Int[((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] \rightarrow Simp[(a + b*x + c*x^2)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x)^(2*FracPart[p])) Int[(d + e*x)^m*(b/2 + c*x)^(2*p), x], x] /; FreeQ[\{a, b, c, d, e, m, p\}, x] \&\& EqQ[b^2 - 4*a*c, 0]$

Maple [A] (verified)

Time = 1.06 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.82

| method | result | size |
|---------|--|------|
| default | $-\frac{(bx+a)(\ln(bx+a)-\ln(ex+d))}{\sqrt{(bx+a)^2(ae-bd)}}$ | 42 |
| risch | $\frac{\sqrt{(bx+a)^2} \ln(-ex-d)}{(bx+a)(ae-bd)} - \frac{\sqrt{(bx+a)^2} \ln(bx+a)}{(bx+a)(ae-bd)}$ | 72 |

input `int(1/(e*x+d)/((b*x+a)^2)^(1/2),x,method=_RETURNVERBOSE)`

output $-(b*x+a)*(ln(b*x+a)-ln(e*x+d))/((b*x+a)^2)^(1/2)/(a*e-b*d)$

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.51

$$\int \frac{1}{(d+ex)\sqrt{a^2+2abx+b^2x^2}} dx = \frac{\log(bx+a) - \log(ex+d)}{bd-ae}$$

input `integrate(1/(e*x+d)/((b*x+a)^2)^(1/2),x, algorithm="fricas")`

output `(log(b*x + a) - log(e*x + d))/(b*d - a*e)`

Sympy [F]

$$\int \frac{1}{(d+ex)\sqrt{a^2+2abx+b^2x^2}} dx = \int \frac{1}{(d+ex)\sqrt{(a+bx)^2}} dx$$

input `integrate(1/(e*x+d)/((b*x+a)**2)**(1/2),x)`

output `Integral(1/((d + e*x)*sqrt((a + b*x)**2)), x)`

Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{(d+ex)\sqrt{a^2+2abx+b^2x^2}} dx = \text{Exception raised: ValueError}$$

input `integrate(1/(e*x+d)/((b*x+a)^2)^(1/2),x, algorithm="maxima")`

output `Exception raised: ValueError >> Computation failed since Maxima requested additional constraints; using the 'assume' command before evaluation *may* help (example of legal syntax is 'assume(a*e-b*d>0)', see `assume?` for more detail`

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.04

$$\int \frac{1}{(d+ex)\sqrt{a^2+2abx+b^2x^2}} dx = \left(\frac{b \log(|bx+a|)}{b^2d-abe} - \frac{e \log(|ex+d|)}{bde-ae^2} \right) \operatorname{sgn}(bx+a)$$

input `integrate(1/(e*x+d)/((b*x+a)^2)^(1/2),x, algorithm="giac")`output `(b*log(abs(b*x + a))/(b^2*d - a*b*e) - e*log(abs(e*x + d))/(b*d*e - a*e^2))*sgn(b*x + a)`**Mupad [F(-1)]**

Timed out.

$$\int \frac{1}{(d+ex)\sqrt{a^2+2abx+b^2x^2}} dx = \int \frac{1}{\sqrt{(a+bx)^2} (d+ex)} dx$$

input `int(1/(((a + b*x)^2)^(1/2)*(d + e*x)),x)`output `int(1/(((a + b*x)^2)^(1/2)*(d + e*x)), x)`**Reduce [B] (verification not implemented)**

Time = 0.23 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.51

$$\int \frac{1}{(d+ex)\sqrt{a^2+2abx+b^2x^2}} dx = \frac{-\log(bx+a) + \log(ex+d)}{ae-bd}$$

input `int(1/(e*x+d)/((b*x+a)^2)^(1/2),x)`output `(- log(a + b*x) + log(d + e*x))/(a*e - b*d)`

3.315 $\int \frac{1}{(d+ex)^2 \sqrt{a^2+2abx+b^2x^2}} dx$

| | |
|---|------|
| Optimal result | 2369 |
| Mathematica [A] (verified) | 2369 |
| Rubi [A] (verified) | 2370 |
| Maple [A] (verified) | 2371 |
| Fricas [A] (verification not implemented) | 2372 |
| Sympy [F] | 2372 |
| Maxima [F(-2)] | 2373 |
| Giac [A] (verification not implemented) | 2373 |
| Mupad [F(-1)] | 2374 |
| Reduce [B] (verification not implemented) | 2374 |

Optimal result

Integrand size = 28, antiderivative size = 131

$$\int \frac{1}{(d+ex)^2 \sqrt{a^2+2abx+b^2x^2}} dx = \frac{a+bx}{(bd-ae)(d+ex)\sqrt{a^2+2abx+b^2x^2}} + \frac{b(a+bx)\log(a+bx)}{(bd-ae)^2\sqrt{a^2+2abx+b^2x^2}} - \frac{b(a+bx)\log(d+ex)}{(bd-ae)^2\sqrt{a^2+2abx+b^2x^2}}$$

output

```
(b*x+a)/(-a*e+b*d)/(e*x+d)/((b*x+a)^2)^(1/2)+b*(b*x+a)*ln(b*x+a)/(-a*e+b*d)
)^2/((b*x+a)^2)^(1/2)-b*(b*x+a)*ln(e*x+d)/(-a*e+b*d)^2/((b*x+a)^2)^(1/2)
```

Mathematica [A] (verified)

Time = 1.07 (sec) , antiderivative size = 247, normalized size of antiderivative = 1.89

$$\int \frac{1}{(d+ex)^2 \sqrt{a^2+2abx+b^2x^2}} dx = \frac{-\frac{\sqrt{a^2}e(-bd+ae)x}{ad(d+ex)} - \frac{e\sqrt{(a+bx)^2}}{d+ex} + \frac{(a-\sqrt{a^2})b \log(\sqrt{a^2}-bx-\sqrt{(a+bx)^2})}{a} - \frac{(a+\sqrt{a^2})b \log(\sqrt{a^2}+bx-\sqrt{(a+bx)^2})}{a} + \frac{(a+\sqrt{a^2})b \log(a+bx)}{a}}{2(bd-ae)^2}$$

input `Integrate[1/((d + e*x)^2*Sqrt[a^2 + 2*a*b*x + b^2*x^2]),x]`

output
$$\frac{-((\text{Sqrt}[a^2]*e*(-(b*d) + a*e)*x)/(a*d*(d + e*x))) - (e*\text{Sqrt}[(a + b*x)^2])}{(d + e*x) + ((a - \text{Sqrt}[a^2])*b*\text{Log}[\text{Sqrt}[a^2] - b*x - \text{Sqrt}[(a + b*x)^2]])/a - ((a + \text{Sqrt}[a^2])*b*\text{Log}[\text{Sqrt}[a^2] + b*x - \text{Sqrt}[(a + b*x)^2]])/a + ((a + \text{Sqrt}[a^2])*b*\text{Log}[2*a*e*x + d*(\text{Sqrt}[a^2] - b*x - \text{Sqrt}[(a + b*x)^2])])/a + ((-a + \text{Sqrt}[a^2])*b*\text{Log}[-2*a*e*x + d*(\text{Sqrt}[a^2] + b*x - \text{Sqrt}[(a + b*x)^2])])/a)/(2*(b*d - a*e)^2}$$

Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.63, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {1102, 27, 54, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\begin{aligned} & \int \frac{1}{\sqrt{a^2 + 2abx + b^2x^2}(d + ex)^2} dx \\ & \quad \downarrow \text{1102} \\ & \frac{b(a + bx) \int \frac{1}{b(a+bx)(d+ex)^2} dx}{\sqrt{a^2 + 2abx + b^2x^2}} \\ & \quad \downarrow \text{27} \\ & \frac{(a + bx) \int \frac{1}{(a+bx)(d+ex)^2} dx}{\sqrt{a^2 + 2abx + b^2x^2}} \\ & \quad \downarrow \text{54} \\ & \frac{(a + bx) \int \left(\frac{b^2}{(bd-ae)^2(a+bx)} - \frac{eb}{(bd-ae)^2(d+ex)} - \frac{e}{(bd-ae)(d+ex)^2} \right) dx}{\sqrt{a^2 + 2abx + b^2x^2}} \\ & \quad \downarrow \text{2009} \\ & \frac{(a + bx) \left(\frac{1}{(d+ex)(bd-ae)} + \frac{b \log(a+bx)}{(bd-ae)^2} - \frac{b \log(d+ex)}{(bd-ae)^2} \right)}{\sqrt{a^2 + 2abx + b^2x^2}} \end{aligned}$$

input `Int[1/((d + e*x)^2*Sqrt[a^2 + 2*a*b*x + b^2*x^2]),x]`

output
$$\frac{((a + b*x)*(1/((b*d - a*e)*(d + e*x)) + (b*\text{Log}[a + b*x])/(b*d - a*e)^2 - (b*\text{Log}[d + e*x])/(b*d - a*e)^2))/\text{Sqrt}[a^2 + 2*a*b*x + b^2*x^2]}$$

Defintions of rubi rules used

rule 27 `Int[(a_)*(Fx_), x_Symbol] := Simp[a Int[Fx, x], x] /; FreeQ[a, x] && !MatchQ[Fx, (b_)*(Gx_)] /; FreeQ[b, x]`

rule 54 `Int[((a_) + (b_)*(x_))^(m_)*((c_) + (d_)*(x_))^(n_), x_Symbol] := Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] && ILtQ[m, 0] && IntegerQ[n] && !(IGtQ[n, 0] && LtQ[m + n + 2, 0])`

rule 1102 `Int[((d_) + (e_)*(x_))^(m_)*((a_) + (b_)*(x_) + (c_)*(x_)^2)^(p_), x_Symbol] := Simp[(a + b*x + c*x^2)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x)^(2*FracPart[p])) Int[(d + e*x)^m*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && EqQ[b^2 - 4*a*c, 0]`

rule 2009 `Int[u_, x_Symbol] := Simp[IntSum[u, x], x] /; SumQ[u]`

Maple [A] (verified)

Time = 1.14 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.62

| method | result | size |
|---------|---|------|
| default | $\frac{(bx+a)(\ln(bx+a)bx - \ln(ex+d)bx + \ln(bx+a)bd - \ln(ex+d)bd - ae + bd)}{\sqrt{(bx+a)^2} (ae - bd)^2 (ex+d)}$ | 81 |
| risch | $-\frac{\sqrt{(bx+a)^2}}{(bx+a)(ae - bd)(ex+d)} - \frac{\sqrt{(bx+a)^2} b \ln(ex+d)}{(bx+a)(e^2 a^2 - 2abde + b^2 d^2)} + \frac{\sqrt{(bx+a)^2} b \ln(-bx-a)}{(bx+a)(e^2 a^2 - 2abde + b^2 d^2)}$ | 135 |

input `int(1/(e*x+d)^2/((b*x+a)^2)^(1/2),x,method=_RETURNVERBOSE)`

output $(b*x+a)*(ln(b*x+a)*b*e*x-ln(e*x+d)*b*e*x+ln(b*x+a)*b*d-ln(e*x+d)*b*d-a*e+b*d)/((b*x+a)^2)^{(1/2)}/(a*e-b*d)^2/(e*x+d)$

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.70

$$\int \frac{1}{(d+ex)^2 \sqrt{a^2+2abx+b^2x^2}} dx$$

$$= \frac{bd - ae + (bex + bd) \log(bx + a) - (bex + bd) \log(ex + d)}{b^2d^3 - 2abd^2e + a^2de^2 + (b^2d^2e - 2abde^2 + a^2e^3)x}$$

input `integrate(1/(e*x+d)^2/((b*x+a)^2)^(1/2),x, algorithm="fricas")`

output $(b*d - a*e + (b*e*x + b*d)*\log(b*x + a) - (b*e*x + b*d)*\log(e*x + d))/(b^2*d^3 - 2*a*b*d^2*e + a^2*d*e^2 + (b^2*d^2*e - 2*a*b*d*e^2 + a^2*e^3)*x)$

Sympy [F]

$$\int \frac{1}{(d+ex)^2 \sqrt{a^2+2abx+b^2x^2}} dx = \int \frac{1}{(d+ex)^2 \sqrt{(a+bx)^2}} dx$$

input `integrate(1/(e*x+d)**2/((b*x+a)**2)**(1/2),x)`

output `Integral(1/((d + e*x)**2*sqrt((a + b*x)**2)), x)`

Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{(d+ex)^2 \sqrt{a^2+2abx+b^2x^2}} dx = \text{Exception raised: ValueError}$$

input `integrate(1/(e*x+d)^2/((b*x+a)^2)^(1/2),x, algorithm="maxima")`

output Exception raised: ValueError >> Computation failed since Maxima requested additional constraints; using the 'assume' command before evaluation *may* help (example of legal syntax is 'assume(a*e-b*d>0)', see `assume?` for more detail)

Giac [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 100, normalized size of antiderivative = 0.76

$$\int \frac{1}{(d+ex)^2 \sqrt{a^2+2abx+b^2x^2}} dx$$

$$= \left(\frac{b^2 \log(|bx+a|)}{b^3d^2 - 2ab^2de + a^2be^2} - \frac{be \log(|ex+d|)}{b^2d^2e - 2abde^2 + a^2e^3} + \frac{1}{(bd-ae)(ex+d)} \right) \text{sgn}(bx + a)$$

input `integrate(1/(e*x+d)^2/((b*x+a)^2)^(1/2),x, algorithm="giac")`

output `(b^2*log(abs(b*x + a))/(b^3*d^2 - 2*a*b^2*d*e + a^2*b*e^2) - b*e*log(abs(e*x + d))/(b^2*d^2*e - 2*a*b*d*e^2 + a^2*e^3) + 1/((b*d - a*e)*(e*x + d)))*sgn(b*x + a)`

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(d+ex)^2 \sqrt{a^2+2abx+b^2x^2}} dx = \int \frac{1}{\sqrt{(a+bx)^2} (d+ex)^2} dx$$

input `int(1/(((a + b*x)^2)^(1/2)*(d + e*x)^2), x)`output `int(1/(((a + b*x)^2)^(1/2)*(d + e*x)^2), x)`**Reduce [B] (verification not implemented)**

Time = 0.23 (sec) , antiderivative size = 115, normalized size of antiderivative = 0.88

$$\int \frac{1}{(d+ex)^2 \sqrt{a^2+2abx+b^2x^2}} dx$$

$$= \frac{\log(bx+a) b d^2 + \log(bx+a) b d e x - \log(ex+d) b d^2 - \log(ex+d) b d e x + a e^2 x - b d e x}{d(a^2 e^3 x - 2 a b d e^2 x + b^2 d^2 e x + a^2 d e^2 - 2 a b d^2 e + b^2 d^3)}$$

input `int(1/(e*x+d)^2/((b*x+a)^2)^(1/2), x)`output `(log(a + b*x)*b*d**2 + log(a + b*x)*b*d*e*x - log(d + e*x)*b*d**2 - log(d + e*x)*b*d*e*x + a*e**2*x - b*d*e*x)/(d*(a**2*d*e**2 + a**2*e**3*x - 2*a*b*d**2*e - 2*a*b*d*e**2*x + b**2*d**3 + b**2*d**2*e*x))`

3.316 $\int \frac{1}{(d+ex)^3 \sqrt{a^2+2abx+b^2x^2}} dx$

| | |
|---|------|
| Optimal result | 2375 |
| Mathematica [A] (verified) | 2376 |
| Rubi [A] (verified) | 2376 |
| Maple [A] (verified) | 2378 |
| Fricas [A] (verification not implemented) | 2378 |
| Sympy [F] | 2379 |
| Maxima [F(-2)] | 2379 |
| Giac [A] (verification not implemented) | 2379 |
| Mupad [F(-1)] | 2380 |
| Reduce [B] (verification not implemented) | 2380 |

Optimal result

Integrand size = 28, antiderivative size = 182

$$\int \frac{1}{(d+ex)^3 \sqrt{a^2+2abx+b^2x^2}} dx = \frac{a+bx}{2(bd-ae)(d+ex)^2 \sqrt{a^2+2abx+b^2x^2}} + \frac{b(a+bx)}{(bd-ae)^2(d+ex)\sqrt{a^2+2abx+b^2x^2}} + \frac{b^2(a+bx)\log(a+bx)}{(bd-ae)^3 \sqrt{a^2+2abx+b^2x^2}} - \frac{b^2(a+bx)\log(d+ex)}{(bd-ae)^3 \sqrt{a^2+2abx+b^2x^2}}$$

output

```
1/2*(b*x+a)/(-a*e+b*d)/(e*x+d)^2/((b*x+a)^2)^(1/2)+b*(b*x+a)/(-a*e+b*d)^2/
(e*x+d)/((b*x+a)^2)^(1/2)+b^2*(b*x+a)*ln(b*x+a)/(-a*e+b*d)^3/((b*x+a)^2)^(
1/2)-b^2*(b*x+a)*ln(e*x+d)/(-a*e+b*d)^3/((b*x+a)^2)^(1/2)
```

Mathematica [A] (verified)

Time = 1.05 (sec) , antiderivative size = 97, normalized size of antiderivative = 0.53

$$\int \frac{1}{(d+ex)^3 \sqrt{a^2+2abx+b^2x^2}} dx$$

$$= \frac{(a+bx)((bd-ae)(3bd-ae+2bex) + 2b^2(d+ex)^2 \log(a+bx) - 2b^2(d+ex)^2 \log(d+ex))}{2(bd-ae)^3 \sqrt{(a+bx)^2(d+ex)^2}}$$

input

```
Integrate[1/((d + e*x)^3*Sqrt[a^2 + 2*a*b*x + b^2*x^2]),x]
```

output

```
((a + b*x)*((b*d - a*e)*(3*b*d - a*e + 2*b*e*x) + 2*b^2*(d + e*x)^2*Log[a + b*x] - 2*b^2*(d + e*x)^2*Log[d + e*x]))/(2*(b*d - a*e)^3*Sqrt[(a + b*x)^2]*(d + e*x)^2)
```

Rubi [A] (verified)

Time = 0.49 (sec) , antiderivative size = 108, normalized size of antiderivative = 0.59, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {1102, 27, 54, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{a^2+2abx+b^2x^2}(d+ex)^3} dx$$

$$\downarrow \text{1102}$$

$$\frac{b(a+bx) \int \frac{1}{b(a+bx)(d+ex)^3} dx}{\sqrt{a^2+2abx+b^2x^2}}$$

$$\downarrow \text{27}$$

$$\frac{(a+bx) \int \frac{1}{(a+bx)(d+ex)^3} dx}{\sqrt{a^2+2abx+b^2x^2}}$$

$$\downarrow \text{54}$$

$$\frac{(a+bx) \int \left(\frac{b^3}{(bd-ae)^3(a+bx)} - \frac{eb^2}{(bd-ae)^3(d+ex)} - \frac{eb}{(bd-ae)^2(d+ex)^2} - \frac{e}{(bd-ae)(d+ex)^3} \right) dx}{\sqrt{a^2 + 2abx + b^2x^2}}$$

↓ 2009

$$\frac{(a+bx) \left(\frac{b^2 \log(a+bx)}{(bd-ae)^3} - \frac{b^2 \log(d+ex)}{(bd-ae)^3} + \frac{b}{(d+ex)(bd-ae)^2} + \frac{1}{2(d+ex)^2(bd-ae)} \right)}{\sqrt{a^2 + 2abx + b^2x^2}}$$

input `Int[1/((d + e*x)^3*Sqrt[a^2 + 2*a*b*x + b^2*x^2]),x]`

output `((a + b*x)*(1/(2*(b*d - a*e)*(d + e*x)^2) + b/((b*d - a*e)^2*(d + e*x)) + (b^2*Log[a + b*x])/(b*d - a*e)^3 - (b^2*Log[d + e*x])/(b*d - a*e)^3)/Sqrt[a^2 + 2*a*b*x + b^2*x^2]`

Defintions of rubi rules used

rule 27 `Int[(a_)*(Fx_), x_Symbol] := Simp[a Int[Fx, x], x] /; FreeQ[a, x] && !MatchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]`

rule 54 `Int[((a_) + (b_)*(x_))^(m_)*((c_) + (d_)*(x_))^(n_), x_Symbol] := Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] && ILtQ[m, 0] && IntegerQ[n] && !(IGtQ[n, 0] && LtQ[m + n + 2, 0])`

rule 1102 `Int[((d_) + (e_)*(x_))^(m_)*((a_) + (b_)*(x_) + (c_)*(x_)^2)^(p_), x_Symbol] := Simp[(a + b*x + c*x^2)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x)^(2*FracPart[p])) Int[(d + e*x)^m*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && EqQ[b^2 - 4*a*c, 0]`

rule 2009 `Int[u_, x_Symbol] := Simp[IntSum[u, x], x] /; SumQ[u]`

Maple [A] (verified)

Time = 1.29 (sec) , antiderivative size = 162, normalized size of antiderivative = 0.89

| method | result |
|---------|--|
| default | $-\frac{(bx+a)(2\ln(bx+a)b^2e^2x^2-2\ln(ex+d)b^2e^2x^2+4\ln(bx+a)b^2dex-4\ln(ex+d)b^2dex+2\ln(bx+a)b^2d^2-2\ln(ex+d)b^2d^2-2xab e^2+2abd^2)}{2\sqrt{(bx+a)^2(ae-bd)^3(ex+d)^2}}$ |
| risch | $\frac{\sqrt{(bx+a)^2}\left(\frac{bex}{e^2a^2-2abde+b^2d^2}-\frac{ae-3bd}{2(e^2a^2-2abde+b^2d^2)}\right)}{(bx+a)(ex+d)^2}-\frac{\sqrt{(bx+a)^2}b^2\ln(bx+a)}{(bx+a)(e^3a^3-3a^2bde^2+3ab^2d^2e-b^3d^3)}+\frac{\sqrt{(bx+a)^2}b^2\ln(ex+d)}{(bx+a)(e^3a^3-3a^2bde^2+3ab^2d^2e-b^3d^3)}$ |

input `int(1/(e*x+d)^3/((b*x+a)^2)^(1/2),x,method=_RETURNVERBOSE)`

output
$$-1/2*(b*x+a)*(2*\ln(b*x+a)*b^2*e^2*x^2-2*\ln(e*x+d)*b^2*e^2*x^2+4*\ln(b*x+a)*b^2*d*e*x-4*\ln(e*x+d)*b^2*d*e*x+2*\ln(b*x+a)*b^2*d^2-2*\ln(e*x+d)*b^2*d^2-2*x*a*b*e^2+2*b^2*d*e*x+e^2*a^2-4*a*b*d*e+3*b^2*d^2)/((b*x+a)^2)^(1/2)/(a*e-b*d)^3/(e*x+d)^2$$

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 242, normalized size of antiderivative = 1.33

$$\int \frac{1}{(d+ex)^3\sqrt{a^2+2abx+b^2x^2}} dx$$

$$= \frac{3b^2d^2-4abde+a^2e^2+2(b^2de-abe^2)x+2(b^2e^2x^2+2b^2dex+b^2d^2)\log(bx+a)-2(b^2e^2x^2+2b^2dex+b^2d^2)\log(ex+d)}{2(b^3d^5-3ab^2d^4e+3a^2bd^3e^2-a^3d^2e^3+(b^3d^3e^2-3ab^2d^2e^3+3a^2bde^4-a^3e^5)x^2+2(b^3d^4e-3ab^2d^3e^2-3a^2bd^2e^3+a^3d^2e^4)x}$$

input `integrate(1/(e*x+d)^3/((b*x+a)^2)^(1/2),x, algorithm="fricas")`

output
$$1/2*(3*b^2*d^2-4*a*b*d*e+a^2*e^2+2*(b^2*d*e-a*b*e^2)*x+2*(b^2*e^2*x^2+2*b^2*d*e*x+b^2*d^2)*\log(b*x+a)-2*(b^2*e^2*x^2+2*b^2*d*e*x+b^2*d^2)*\log(e*x+d))/((b^3*d^5-3*a*b^2*d^4*e+3*a^2*b*d^3*e^2-a^3*d^2*e^3+(b^3*d^3*e^2-3*a*b^2*d^2*e^3+3*a^2*b*d*e^4-a^3*e^5)*x^2+2*(b^3*d^4*e-3*a*b^2*d^3*e^2+3*a^2*b*d^2*e^3-a^3*d^2*e^4)*x)$$

Sympy [F]

$$\int \frac{1}{(d+ex)^3 \sqrt{a^2+2abx+b^2x^2}} dx = \int \frac{1}{(d+ex)^3 \sqrt{(a+bx)^2}} dx$$

input `integrate(1/(e*x+d)**3/((b*x+a)**2)**(1/2), x)`

output `Integral(1/((d + e*x)**3*sqrt((a + b*x)**2)), x)`

Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{(d+ex)^3 \sqrt{a^2+2abx+b^2x^2}} dx = \text{Exception raised: ValueError}$$

input `integrate(1/(e*x+d)^3/((b*x+a)^2)^(1/2), x, algorithm="maxima")`

output `Exception raised: ValueError >> Computation failed since Maxima requested additional constraints; using the 'assume' command before evaluation *may* help (example of legal syntax is 'assume(a*e-b*d>0)', see `assume?` for more detail`

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 173, normalized size of antiderivative = 0.95

$$\int \frac{1}{(d+ex)^3 \sqrt{a^2+2abx+b^2x^2}} dx = \frac{1}{2} \left(\frac{2b^3 \log(|bx+a|)}{b^4d^3 - 3ab^3d^2e + 3a^2b^2de^2 - a^3be^3} - \frac{2b^2e \log(|ex+d|)}{b^3d^3e - 3ab^2d^2e^2 + 3a^2bde^3 - a^3e^4} + \frac{3b^2d^2 - 4abde + a^2e^2}{(bd-ae)^3} + a \right)$$

input `integrate(1/(e*x+d)^3/((b*x+a)^2)^(1/2),x, algorithm="giac")`

output
$$\frac{1}{2} \cdot (2 \cdot b^3 \cdot \log(\text{abs}(b \cdot x + a)) / (b^4 \cdot d^3 - 3 \cdot a \cdot b^3 \cdot d^2 \cdot e + 3 \cdot a^2 \cdot b^2 \cdot d \cdot e^2 - a^3 \cdot b \cdot e^3) - 2 \cdot b^2 \cdot e \cdot \log(\text{abs}(e \cdot x + d)) / (b^3 \cdot d^3 \cdot e - 3 \cdot a \cdot b^2 \cdot d^2 \cdot e^2 + 3 \cdot a^2 \cdot b \cdot d \cdot e^3 - a^3 \cdot e^4) + (3 \cdot b^2 \cdot d^2 - 4 \cdot a \cdot b \cdot d \cdot e + a^2 \cdot e^2 + 2 \cdot (b^2 \cdot d \cdot e - a \cdot b \cdot e^2) \cdot x) / ((b \cdot d - a \cdot e)^3 \cdot (e \cdot x + d)^2)) \cdot \text{sgn}(b \cdot x + a)$$

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(d + ex)^3 \sqrt{a^2 + 2abx + b^2x^2}} dx = \int \frac{1}{\sqrt{(a + bx)^2} (d + ex)^3} dx$$

input `int(1/(((a + b*x)^2)^(1/2)*(d + e*x)^3),x)`

output `int(1/(((a + b*x)^2)^(1/2)*(d + e*x)^3), x)`

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 289, normalized size of antiderivative = 1.59

$$\int \frac{1}{(d + ex)^3 \sqrt{a^2 + 2abx + b^2x^2}} dx = \frac{-2 \log(bx + a) b^2 d^3 - 4 \log(bx + a) b^2 d^2 ex - 2 \log(bx + a) b^2 d e^2 x^2 + 2 \log(ex + d) b^2 d^3 + 4 \log(ex + d) b^2 d^2 ex + 2 \log(ex + d) b^2 d e^2 x^2}{2d(a^3 e^5 x^2 - 3a^2 b d e^4 x^2 + 3a b^2 d^2 e^3 x^2 - b^3 d^3 e^2 x^2 + 2a^3 d e^4 x - 6a^2 b d^2 e^3 x + 6a b^2 d^3)}$$

input `int(1/(e*x+d)^3/((b*x+a)^2)^(1/2),x)`

output

```
( - 2*log(a + b*x)*b**2*d**3 - 4*log(a + b*x)*b**2*d**2*e*x - 2*log(a + b*
x)*b**2*d*e**2*x**2 + 2*log(d + e*x)*b**2*d**3 + 4*log(d + e*x)*b**2*d**2*
e*x + 2*log(d + e*x)*b**2*d*e**2*x**2 - a**2*d*e**2 + 3*a*b*d**2*e - a*b*e
**3*x**2 - 2*b**2*d**3 + b**2*d*e**2*x**2)/(2*d*(a**3*d**2*e**3 + 2*a**3*d
**4*x + a**3*e**5*x**2 - 3*a**2*b*d**3*e**2 - 6*a**2*b*d**2*e**3*x - 3*a
**2*b*d*e**4*x**2 + 3*a*b**2*d**4*e + 6*a*b**2*d**3*e**2*x + 3*a*b**2*d**2
*e**3*x**2 - b**3*d**5 - 2*b**3*d**4*e*x - b**3*d**3*e**2*x**2))
```

3.317 $\int \frac{1}{(d+ex)^4 \sqrt{a^2+2abx+b^2x^2}} dx$

| | |
|---|------|
| Optimal result | 2382 |
| Mathematica [A] (verified) | 2383 |
| Rubi [A] (verified) | 2383 |
| Maple [A] (verified) | 2385 |
| Fricas [B] (verification not implemented) | 2385 |
| Sympy [F] | 2386 |
| Maxima [F(-2)] | 2386 |
| Giac [A] (verification not implemented) | 2387 |
| Mupad [F(-1)] | 2387 |
| Reduce [B] (verification not implemented) | 2388 |

Optimal result

Integrand size = 28, antiderivative size = 231

$$\int \frac{1}{(d+ex)^4 \sqrt{a^2+2abx+b^2x^2}} dx = \frac{a+bx}{3(bd-ae)(d+ex)^3 \sqrt{a^2+2abx+b^2x^2}} + \frac{b(a+bx)}{2(bd-ae)^2(d+ex)^2 \sqrt{a^2+2abx+b^2x^2}} + \frac{b^2(a+bx)}{(bd-ae)^3(d+ex) \sqrt{a^2+2abx+b^2x^2}} + \frac{b^3(a+bx) \log(a+bx)}{(bd-ae)^4 \sqrt{a^2+2abx+b^2x^2}} - \frac{b^3(a+bx) \log(d+ex)}{(bd-ae)^4 \sqrt{a^2+2abx+b^2x^2}}$$

output

```
1/3*(b*x+a)/(-a*e+b*d)/(e*x+d)^3/((b*x+a)^2)^(1/2)+1/2*b*(b*x+a)/(-a*e+b*d)
)^2/(e*x+d)^2/((b*x+a)^2)^(1/2)+b^2*(b*x+a)/(-a*e+b*d)^3/(e*x+d)/((b*x+a)
)^2)^(1/2)+b^3*(b*x+a)*ln(b*x+a)/(-a*e+b*d)^4/((b*x+a)^2)^(1/2)-b^3*(b*x+a)*
ln(e*x+d)/(-a*e+b*d)^4/((b*x+a)^2)^(1/2)
```

Mathematica [A] (verified)

Time = 1.05 (sec) , antiderivative size = 124, normalized size of antiderivative = 0.54

$$\int \frac{1}{(d+ex)^4 \sqrt{a^2+2abx+b^2x^2}} dx$$

$$= \frac{(a+bx)(2(bd-ae)^3+3b(bd-ae)^2(d+ex)+6b^2(bd-ae)(d+ex)^2+6b^3(d+ex)^3 \log(a+bx)-6b^3(d+ex)^3 \log(a+bx))}{6(bd-ae)^4 \sqrt{(a+bx)^2(d+ex)^3}}$$

input

```
Integrate[1/((d + e*x)^4*Sqrt[a^2 + 2*a*b*x + b^2*x^2]),x]
```

output

```
((a + b*x)*(2*(b*d - a*e)^3 + 3*b*(b*d - a*e)^2*(d + e*x) + 6*b^2*(b*d - a
*e)*(d + e*x)^2 + 6*b^3*(d + e*x)^3*Log[a + b*x] - 6*b^3*(d + e*x)^3*Log[d
+ e*x]))/(6*(b*d - a*e)^4*Sqrt[(a + b*x)^2]*(d + e*x)^3)
```

Rubi [A] (verified)

Time = 0.52 (sec) , antiderivative size = 132, normalized size of antiderivative = 0.57, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {1102, 27, 54, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{a^2+2abx+b^2x^2}(d+ex)^4} dx$$

$$\downarrow \text{1102}$$

$$\frac{b(a+bx) \int \frac{1}{b(a+bx)(d+ex)^4} dx}{\sqrt{a^2+2abx+b^2x^2}}$$

$$\downarrow \text{27}$$

$$\frac{(a+bx) \int \frac{1}{(a+bx)(d+ex)^4} dx}{\sqrt{a^2+2abx+b^2x^2}}$$

$$\downarrow \text{54}$$

$$\frac{(a + bx) \int \left(\frac{b^4}{(bd-ae)^4(a+bx)} - \frac{eb^3}{(bd-ae)^4(d+ex)} - \frac{eb^2}{(bd-ae)^3(d+ex)^2} - \frac{eb}{(bd-ae)^2(d+ex)^3} - \frac{e}{(bd-ae)(d+ex)^4} \right) dx}{\sqrt{a^2 + 2abx + b^2x^2}}$$

↓ 2009

$$\frac{(a + bx) \left(\frac{b^3 \log(a+bx)}{(bd-ae)^4} - \frac{b^3 \log(d+ex)}{(bd-ae)^4} + \frac{b^2}{(d+ex)(bd-ae)^3} + \frac{b}{2(d+ex)^2(bd-ae)^2} + \frac{1}{3(d+ex)^3(bd-ae)} \right)}{\sqrt{a^2 + 2abx + b^2x^2}}$$

input `Int[1/((d + e*x)^4*Sqrt[a^2 + 2*a*b*x + b^2*x^2]),x]`

output `((a + b*x)*(1/(3*(b*d - a*e)*(d + e*x)^3) + b/(2*(b*d - a*e)^2*(d + e*x)^2) + b^2/((b*d - a*e)^3*(d + e*x)) + (b^3*Log[a + b*x])/(b*d - a*e)^4 - (b^3*Log[d + e*x])/(b*d - a*e)^4))/Sqrt[a^2 + 2*a*b*x + b^2*x^2]`

Defintions of rubi rules used

rule 27 `Int[(a_)*(F_x_), x_Symbol] := Simp[a Int[F_x, x], x] /; FreeQ[a, x] && !MatchQ[F_x, (b_)*(G_x_) /; FreeQ[b, x]]`

rule 54 `Int[((a_) + (b_)*(x_))^(m_)*((c_) + (d_)*(x_))^(n_), x_Symbol] := Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] && ILtQ[m, 0] && IntegerQ[n] && !(IGtQ[n, 0] && LtQ[m + n + 2, 0])`

rule 1102 `Int[((d_) + (e_)*(x_))^(m_)*((a_) + (b_)*(x_) + (c_)*(x_)^2)^(p_), x_Symbol] := Simp[(a + b*x + c*x^2)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x)^(2*FracPart[p])) Int[(d + e*x)^m*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && EqQ[b^2 - 4*a*c, 0]`

rule 2009 `Int[u_, x_Symbol] := Simp[IntSum[u, x], x] /; SumQ[u]`

Maple [A] (verified)

Time = 1.43 (sec) , antiderivative size = 256, normalized size of antiderivative = 1.11

| method | result |
|---------|--|
| default | $\frac{(bx+a)(6\ln(bx+a)x^3b^3e^3-6\ln(ex+d)b^3e^3x^3+18\ln(bx+a)b^3de^2x^2-18\ln(ex+d)b^3de^2x^2+18\ln(bx+a)xb^3d^2e-18\ln(ex+d)b^3d^2e)}{6\sqrt{(bx+a)^2}}$ |
| risch | $\frac{\sqrt{(bx+a)^2} \left(-\frac{b^2e^2x^2}{e^3a^3-3a^2bde^2+3ab^2d^2e-b^3d^3} + \frac{(ae-5bd)be^2x}{2e^3a^3-6a^2bde^2+6ab^2d^2e-2b^3d^3} - \frac{2e^2a^2-7abde+11b^2d^2}{6(e^3a^3-3a^2bde^2+3ab^2d^2e-b^3d^3)} \right)}{(bx+a)(ex+d)^3} + \frac{1}{(bx+a)a^4}$ |

input `int(1/(e*x+d)^4/((b*x+a)^2)^(1/2),x,method=_RETURNVERBOSE)`

output
$$\frac{1}{6}(b*x+a)*(6*\ln(b*x+a)*x^3*b^3*e^3-6*\ln(e*x+d)*b^3*e^3*x^3+18*\ln(b*x+a)*b^3*d*e^2*x^2-18*\ln(e*x+d)*b^3*d*e^2*x^2+18*\ln(b*x+a)*x*b^3*d^2*e-18*\ln(e*x+d)*b^3*d^2*e*x-6*x^2*a*b^2*e^3+6*x^2*b^3*d*e^2+6*\ln(b*x+a)*b^3*d^3-6*\ln(e*x+d)*b^3*d^3+3*a^2*b*e^3*x-18*x*a*b^2*d*e^2+15*b^3*d^2*e*x-2*e^3*a^3+9*a^2*b*d*e^2-18*a*b^2*d^2*e+11*b^3*d^3)/((b*x+a)^2)^(1/2)/(a*e-b*d)^4/(e*x+d)^3$$

Fricas [B] (verification not implemented)Leaf count of result is larger than twice the leaf count of optimal. 425 vs. $2(172) = 344$.

Time = 0.09 (sec) , antiderivative size = 425, normalized size of antiderivative = 1.84

$$\int \frac{1}{(d+ex)^4\sqrt{a^2+2abx+b^2x^2}} dx$$

$$= \frac{11b^3d^3-18ab^2d^2e+9a^2bde^2-2a^3e^3+6(b^3de^2-ab^2e^3)x^2+3(5b^3d^2e-6ab^2de^2+a^2be^3)}{6(b^4d^7-4ab^3d^6e+6a^2b^2d^5e^2-4a^3bd^4e^3+a^4d^3e^4+(b^4d^4e^3-4ab^3d^3e^4+6a^2b^2d^2e^5-4a^3bde^6+a^4e^7)}$$

input `integrate(1/(e*x+d)^4/((b*x+a)^2)^(1/2),x, algorithm="fricas")`

output

```
1/6*(11*b^3*d^3 - 18*a*b^2*d^2*e + 9*a^2*b*d*e^2 - 2*a^3*e^3 + 6*(b^3*d*e^2 - a*b^2*e^3)*x^2 + 3*(5*b^3*d^2*e - 6*a*b^2*d*e^2 + a^2*b*e^3)*x + 6*(b^3*e^3*x^3 + 3*b^3*d*e^2*x^2 + 3*b^3*d^2*e*x + b^3*d^3)*log(b*x + a) - 6*(b^3*e^3*x^3 + 3*b^3*d*e^2*x^2 + 3*b^3*d^2*e*x + b^3*d^3)*log(e*x + d))/(b^4*d^7 - 4*a*b^3*d^6*e + 6*a^2*b^2*d^5*e^2 - 4*a^3*b*d^4*e^3 + a^4*d^3*e^4 + (b^4*d^4*e^3 - 4*a*b^3*d^3*e^4 + 6*a^2*b^2*d^2*e^5 - 4*a^3*b*d*e^6 + a^4*e^7)*x^3 + 3*(b^4*d^5*e^2 - 4*a*b^3*d^4*e^3 + 6*a^2*b^2*d^3*e^4 - 4*a^3*b*d^2*e^5 + a^4*d*e^6)*x^2 + 3*(b^4*d^6*e - 4*a*b^3*d^5*e^2 + 6*a^2*b^2*d^4*e^3 - 4*a^3*b*d^3*e^4 + a^4*d^2*e^5)*x)
```

Sympy [F]

$$\int \frac{1}{(d+ex)^4 \sqrt{a^2+2abx+b^2x^2}} dx = \int \frac{1}{(d+ex)^4 \sqrt{(a+bx)^2}} dx$$

input

```
integrate(1/(e*x+d)**4/((b*x+a)**2)**(1/2),x)
```

output

```
Integral(1/((d + e*x)**4*sqrt((a + b*x)**2)), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{(d+ex)^4 \sqrt{a^2+2abx+b^2x^2}} dx = \text{Exception raised: ValueError}$$

input

```
integrate(1/(e*x+d)^4/((b*x+a)^2)^(1/2),x, algorithm="maxima")
```

output

```
Exception raised: ValueError >> Computation failed since Maxima requested additional constraints; using the 'assume' command before evaluation *may* help (example of legal syntax is 'assume(a*e-b*d>0)', see `assume?` for more detail
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 251, normalized size of antiderivative = 1.09

$$\int \frac{1}{(d+ex)^4 \sqrt{a^2+2abx+b^2x^2}} dx$$

$$= \frac{1}{6} \left(\frac{6b^4 \log(|bx+a|)}{b^5d^4 - 4ab^4d^3e + 6a^2b^3d^2e^2 - 4a^3b^2de^3 + a^4be^4} - \frac{6b^3e \log(|ex+d|)}{b^4d^4e - 4ab^3d^3e^2 + 6a^2b^2d^2e^3 - 4a^3bde^4 + a^4e^5 + a} \right)$$

input `integrate(1/(e*x+d)^4/((b*x+a)^2)^(1/2),x, algorithm="giac")`

output `1/6*(6*b^4*log(abs(b*x + a))/(b^5*d^4 - 4*a*b^4*d^3*e + 6*a^2*b^3*d^2*e^2 - 4*a^3*b^2*d*e^3 + a^4*b*e^4) - 6*b^3*e*log(abs(e*x + d))/(b^4*d^4*e - 4*a*b^3*d^3*e^2 + 6*a^2*b^2*d^2*e^3 - 4*a^3*b*d*e^4 + a^4*e^5) + (11*b^3*d^3 - 18*a*b^2*d^2*e + 9*a^2*b*d*e^2 - 2*a^3*e^3 + 6*(b^3*d*e^2 - a*b^2*e^3)*x^2 + 3*(5*b^3*d^2*e - 6*a*b^2*d*e^2 + a^2*b*e^3)*x)/((b*d - a*e)^4*(e*x + d)^3))*sgn(b*x + a)`

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(d+ex)^4 \sqrt{a^2+2abx+b^2x^2}} dx = \int \frac{1}{\sqrt{(a+bx)^2} (d+ex)^4} dx$$

input `int(1/(((a + b*x)^2)^(1/2)*(d + e*x)^4),x)`

output `int(1/(((a + b*x)^2)^(1/2)*(d + e*x)^4), x)`

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 499, normalized size of antiderivative = 2.16

$$\int \frac{1}{(d+ex)^4 \sqrt{a^2+2abx+b^2x^2}} dx$$

$$= \frac{6 \log(bx+a) b^3 d^4 + 18 \log(bx+a) b^3 d^3 ex + 18 \log(bx+a) b^3 d^2 e^2 x^2 + 6 \log(bx+a) b^3 d e^3 x^3 - 6 \log(ex+d) b^3 d^4 - 18 \log(ex+d) b^3 d^3 ex - 18 \log(ex+d) b^3 d^2 e^2 x^2 - 6 \log(ex+d) b^3 d e^3 x^3 + 6 d (a^4 e^7 x^3 - 4 a^3 b d e^6 x^3 + 6 a^2 b^2 d^2 e^5 x^3 - 4 a b^3 d^3 e^4 x^3 + b^4 d^4 e^3 x^3 + 3 a^4 d e^6 x^2 - 12 a^3 b d e^5 x^2 + 6 a^2 b^2 d^2 e^4 x^2 - 4 a b^3 d^3 e^3 x^2 + b^4 d^4 e^2 x^2 + 3 a^4 d e^6 x - 12 a^3 b d e^5 x + 6 a^2 b^2 d^2 e^4 x - 4 a b^3 d^3 e^3 x + b^4 d^4 e^2 x + 3 a^4 d e^6 - 12 a^3 b d e^5 + 6 a^2 b^2 d^2 e^4 - 4 a b^3 d^3 e^3 + b^4 d^4 e^2 + 3 a^4 d e^6 - 12 a^3 b d e^5 + 6 a^2 b^2 d^2 e^4 - 4 a b^3 d^3 e^3 + b^4 d^4 e^2)}{6 d (a^4 e^7 x^3 - 4 a^3 b d e^6 x^3 + 6 a^2 b^2 d^2 e^5 x^3 - 4 a b^3 d^3 e^4 x^3 + b^4 d^4 e^3 x^3 + 3 a^4 d e^6 x^2 - 12 a^3 b d e^5 x^2 + 6 a^2 b^2 d^2 e^4 x^2 - 4 a b^3 d^3 e^3 x^2 + b^4 d^4 e^2 x^2 + 3 a^4 d e^6 x - 12 a^3 b d e^5 x + 6 a^2 b^2 d^2 e^4 x - 4 a b^3 d^3 e^3 x + b^4 d^4 e^2 x + 3 a^4 d e^6 - 12 a^3 b d e^5 + 6 a^2 b^2 d^2 e^4 - 4 a b^3 d^3 e^3 + b^4 d^4 e^2)}$$

input `int(1/(e*x+d)^4/((b*x+a)^2)^(1/2),x)`output `(6*log(a + b*x)*b**3*d**4 + 18*log(a + b*x)*b**3*d**3*e*x + 18*log(a + b*x)*b**3*d**2*e**2*x**2 + 6*log(a + b*x)*b**3*d*e**3*x**3 - 6*log(d + e*x)*b**3*d**4 - 18*log(d + e*x)*b**3*d**3*e*x - 18*log(d + e*x)*b**3*d**2*e**2*x**2 - 6*log(d + e*x)*b**3*d*e**3*x**3 - 2*a**3*d*e**3 + 9*a**2*b*d**2*e**2 + 3*a**2*b*d*e**3*x - 16*a*b**2*d**3*e - 12*a*b**2*d**2*e**2*x + 2*a*b**2*e**4*x**3 + 9*b**3*d**4 + 9*b**3*d**3*e*x - 2*b**3*d*e**3*x**3)/(6*d*(a**4*d**3*e**4 + 3*a**4*d**2*e**5*x + 3*a**4*d*e**6*x**2 + a**4*e**7*x**3 - 4*a**3*b*d**4*e**3 - 12*a**3*b*d**3*e**4*x - 12*a**3*b*d**2*e**5*x**2 - 4*a**3*b*d*e**6*x**3 + 6*a**2*b**2*d**5*e**2 + 18*a**2*b**2*d**4*e**3*x + 18*a**2*b**2*d**3*e**4*x**2 + 6*a**2*b**2*d**2*e**5*x**3 - 4*a*b**3*d**6*e - 12*a*b**3*d**5*e**2*x - 12*a*b**3*d**4*e**3*x**2 - 4*a*b**3*d**3*e**4*x**3 + b**4*d**7 + 3*b**4*d**6*e*x + 3*b**4*d**5*e**2*x**2 + b**4*d**4*e**3*x**3))`

3.318
$$\int \frac{(d+ex)^4}{(a^2+2abx+b^2x^2)^{3/2}} dx$$

| | |
|---|------|
| Optimal result | 2389 |
| Mathematica [A] (verified) | 2390 |
| Rubi [A] (verified) | 2390 |
| Maple [A] (verified) | 2392 |
| Fricas [A] (verification not implemented) | 2392 |
| Sympy [F] | 2393 |
| Maxima [B] (verification not implemented) | 2393 |
| Giac [A] (verification not implemented) | 2394 |
| Mupad [F(-1)] | 2395 |
| Reduce [B] (verification not implemented) | 2395 |

Optimal result

Integrand size = 28, antiderivative size = 210

$$\int \frac{(d+ex)^4}{(a^2+2abx+b^2x^2)^{3/2}} dx = -\frac{4e(bd-ae)^3}{b^5\sqrt{a^2+2abx+b^2x^2}} - \frac{(bd-ae)^4}{2b^5(a+bx)\sqrt{a^2+2abx+b^2x^2}} + \frac{e^3(4bd-3ae)x(a+bx)}{b^4\sqrt{a^2+2abx+b^2x^2}} + \frac{e^4x^2(a+bx)}{2b^3\sqrt{a^2+2abx+b^2x^2}} + \frac{6e^2(bd-ae)^2(a+bx)\log(a+bx)}{b^5\sqrt{a^2+2abx+b^2x^2}}$$

output

```
-4*e*(-a*e+b*d)^3/b^5/((b*x+a)^2)^(1/2)-1/2*(-a*e+b*d)^4/b^5/(b*x+a)/((b*x+a)^2)^(1/2)+e^3*(-3*a*e+4*b*d)*x*(b*x+a)/b^4/((b*x+a)^2)^(1/2)+1/2*e^4*x^2*(b*x+a)/b^3/((b*x+a)^2)^(1/2)+6*e^2*(-a*e+b*d)^2*(b*x+a)*ln(b*x+a)/b^5/((b*x+a)^2)^(1/2)
```

Mathematica [A] (verified)

Time = 1.06 (sec) , antiderivative size = 174, normalized size of antiderivative = 0.83

$$\int \frac{(d+ex)^4}{(a^2+2abx+b^2x^2)^{3/2}} dx = \frac{7a^4e^4 + 2a^3be^3(-10d+ex) + a^2b^2e^2(18d^2 - 16dex - 11e^2x^2) - 4ab^3e(d^3 - 11e^2x^2) + b^4(-d^4 - 8d^3ex + 8d^2e^2x^2 + e^3x^3) + 12e^2(bd - ae)^2(a + bx) + 2\text{Log}[a + bx]}{(2b^5(a + bx)\sqrt{(a + bx)^2})}$$

input `Integrate[(d + e*x)^4/(a^2 + 2*a*b*x + b^2*x^2)^(3/2),x]`

output `(7*a^4*e^4 + 2*a^3*b*e^3*(-10*d + e*x) + a^2*b^2*e^2*(18*d^2 - 16*d*e*x - 11*e^2*x^2) - 4*a*b^3*e*(d^3 - 6*d^2*e*x - 4*d*e^2*x^2 + e^3*x^3) + b^4*(-d^4 - 8*d^3*e*x + 8*d^2*e^3*x^3 + e^4*x^4) + 12*e^2*(b*d - a*e)^2*(a + b*x) + 2*Log[a + b*x])/(2*b^5*(a + b*x)*Sqrt[(a + b*x)^2])`

Rubi [A] (verified)

Time = 0.54 (sec) , antiderivative size = 128, normalized size of antiderivative = 0.61, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {1102, 27, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\begin{aligned} & \int \frac{(d+ex)^4}{(a^2+2abx+b^2x^2)^{3/2}} dx \\ & \quad \downarrow \text{1102} \\ & \frac{b^3(a+bx) \int \frac{(d+ex)^4}{b^3(a+bx)^3} dx}{\sqrt{a^2+2abx+b^2x^2}} \\ & \quad \downarrow \text{27} \\ & \frac{(a+bx) \int \frac{(d+ex)^4}{(a+bx)^3} dx}{\sqrt{a^2+2abx+b^2x^2}} \\ & \quad \downarrow \text{49} \\ & \frac{(a+bx) \int \left(\frac{xe^4}{b^3} + \frac{(4bd-3ae)e^3}{b^4} + \frac{6(bd-ae)^2e^2}{b^4(a+bx)} + \frac{4(bd-ae)^3e}{b^4(a+bx)^2} + \frac{(bd-ae)^4}{b^4(a+bx)^3} \right) dx}{\sqrt{a^2+2abx+b^2x^2}} \end{aligned}$$

$$\frac{(a + bx) \left(\frac{6e^2(bd-ae)^2 \log(a+bx)}{b^5} - \frac{4e(bd-ae)^3}{b^5(a+bx)} - \frac{(bd-ae)^4}{2b^5(a+bx)^2} + \frac{e^3x(4bd-3ae)}{b^4} + \frac{e^4x^2}{2b^3} \right)}{\sqrt{a^2 + 2abx + b^2x^2}}$$

input `Int[(d + e*x)^4/(a^2 + 2*a*b*x + b^2*x^2)^(3/2),x]`

output `((a + b*x)*((e^3*(4*b*d - 3*a*e)*x)/b^4 + (e^4*x^2)/(2*b^3) - (b*d - a*e)^4/(2*b^5*(a + b*x)^2) - (4*e*(b*d - a*e)^3)/(b^5*(a + b*x)) + (6*e^2*(b*d - a*e)^2*Log[a + b*x])/b^5))/Sqrt[a^2 + 2*a*b*x + b^2*x^2]`

Definitions of rubi rules used

rule 27 `Int[(a_)*(F_x_), x_Symbol] := Simp[a Int[F_x, x], x] /; FreeQ[a, x] && !MatchQ[F_x, (b_)*(G_x_)] /; FreeQ[b, x]`

rule 49 `Int[((a_) + (b_)*(x_))^(m_)*((c_) + (d_)*(x_))^(n_), x_Symbol] := Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] && IGtQ[m, 0] && IGtQ[m + n + 2, 0]`

rule 1102 `Int[((d_) + (e_)*(x_))^(m_)*((a_) + (b_)*(x_) + (c_)*(x_)^2)^(p_), x_Symbol] := Simp[(a + b*x + c*x^2)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x)^(2*FracPart[p])) Int[(d + e*x)^m*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && EqQ[b^2 - 4*a*c, 0]`

rule 2009 `Int[u_, x_Symbol] := Simp[IntSum[u, x], x] /; SumQ[u]`

Maple [A] (verified)

Time = 1.60 (sec) , antiderivative size = 214, normalized size of antiderivative = 1.02

| method | result |
|---------|---|
| risch | $-\frac{\sqrt{(bx+a)^2} e^3 \left(-\frac{1}{2}be x^2+3aex-4bdx\right)}{(bx+a)b^4} + \frac{\sqrt{(bx+a)^2} \left(\left(4e^4a^3-12a^2de^3b+12ab^2d^2e^2-4d^3eb^3\right)x+\frac{7a^4e^4-20a^3bde^3+18a^2b^2d^2e^2}{2b}\right)}{(bx+a)^3b^4}$ |
| default | $\frac{(b^4x^4e^4+12\ln(bx+a)x^2a^2b^2e^4-24\ln(bx+a)x^2ab^3de^3+12\ln(bx+a)b^4d^2e^2x^2-4x^3ab^3e^4+8x^3b^4de^3+24\ln(bx+a)xa^3be^4-48\ln(bx+a)a^2b^2d^2e^2)}{(bx+a)^3b^4}$ |

input `int((e*x+d)^4/(b^2*x^2+2*a*b*x+a^2)^(3/2),x,method=_RETURNVERBOSE)`

output
$$-\left(\frac{(bx+a)^2}{(bx+a)}\right)^{1/2} \frac{e^3}{b^4} \left(-\frac{1}{2}be x^2+3aex-4bdx\right) + \frac{(bx+a)^2}{(bx+a)^3} \left(\left(4a^3e^4-12a^2bde^3+12ab^2d^2e^2-4b^3d^3e\right)x+\frac{7a^4e^4-20a^3bde^3+18a^2b^2d^2e^2-4ab^3d^3e-b^4d^4}{b}\right) \frac{1}{b^4} + 6 \frac{(bx+a)^2}{(bx+a)} \frac{1}{b^5} e^2 (a^2e^2-2abde+b^2d^2) \ln(bx+a)$$

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 292, normalized size of antiderivative = 1.39

$$\int \frac{(d+ex)^4}{(a^2+2abx+b^2x^2)^{3/2}} dx = \frac{b^4e^4x^4 - b^4d^4 - 4ab^3d^3e + 18a^2b^2d^2e^2 - 20a^3bde^3 + 7a^4e^4 + 4(2b^4de^3 - a^2b^2d^2e^2)}{(a^2+2abx+b^2x^2)^{3/2}}$$

input `integrate((e*x+d)^4/(b^2*x^2+2*a*b*x+a^2)^(3/2),x, algorithm="fricas")`

output
$$\frac{1}{2} \frac{(b^4e^4x^4 - b^4d^4 - 4a^3b^3d^3e + 18a^2b^2d^2e^2 - 20a^3bde^3 + 7a^4e^4 + 4(2b^4de^3 - a^2b^2d^2e^2))x^3 + (16a^3b^3d^3e - 11a^2b^2d^2e^4)x^2 - 2(4b^4d^3e - 12a^3b^3d^2e^2 + 8a^2b^2d^2e^3 - a^3b^3e^4)x + 12(a^2b^2d^2e^2 - 2a^3b^3d^3e + a^4e^4 + (b^4d^2e^2 - 2a^3b^3d^3e + a^2b^2d^2e^4))x^2 + 2(a^3b^3d^2e^2 - 2a^2b^2d^2e^3 + a^3b^3e^4)x}{(b^7x^2 + 2a^6bx + a^2b^5)}$$

Sympy [F]

$$\int \frac{(d+ex)^4}{(a^2+2abx+b^2x^2)^{3/2}} dx = \int \frac{(d+ex)^4}{((a+bx)^2)^{3/2}} dx$$

input `integrate((e*x+d)**4/(b**2*x**2+2*a*b*x+a**2)**(3/2),x)`

output `Integral((d + e*x)**4/((a + b*x)**2)**(3/2), x)`

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 397 vs. $2(151) = 302$.

Time = 0.04 (sec) , antiderivative size = 397, normalized size of antiderivative = 1.89

$$\begin{aligned} \int \frac{(d+ex)^4}{(a^2+2abx+b^2x^2)^{3/2}} dx &= \frac{e^4x^3}{2\sqrt{b^2x^2+2abx+a^2b^2}} + \frac{4de^3x^2}{\sqrt{b^2x^2+2abx+a^2b^2}} \\ &- \frac{5ae^4x^2}{2\sqrt{b^2x^2+2abx+a^2b^2}} + \frac{6d^2e^2\log(x+\frac{a}{b})}{b^3} - \frac{12ade^3\log(x+\frac{a}{b})}{b^4} \\ &+ \frac{6a^2e^4\log(x+\frac{a}{b})}{b^5} - \frac{4d^3e}{\sqrt{b^2x^2+2abx+a^2b^2}} + \frac{8a^2de^3}{\sqrt{b^2x^2+2abx+a^2b^2}} \\ &- \frac{5a^3e^4}{\sqrt{b^2x^2+2abx+a^2b^2}} + \frac{12ad^2e^2x}{b^4(x+\frac{a}{b})^2} - \frac{24a^2de^3x}{b^5(x+\frac{a}{b})^2} + \frac{12a^3e^4x}{b^6(x+\frac{a}{b})^2} \\ &- \frac{d^4}{2b^3(x+\frac{a}{b})^2} + \frac{2ad^3e}{b^4(x+\frac{a}{b})^2} + \frac{9a^2d^2e^2}{b^5(x+\frac{a}{b})^2} - \frac{22a^3de^3}{b^6(x+\frac{a}{b})^2} + \frac{23a^4e^4}{2b^7(x+\frac{a}{b})^2} \end{aligned}$$

input `integrate((e*x+d)^4/(b^2*x^2+2*a*b*x+a^2)^(3/2),x, algorithm="maxima")`

output

```

1/2*e^4*x^3/(sqrt(b^2*x^2 + 2*a*b*x + a^2)*b^2) + 4*d*e^3*x^2/(sqrt(b^2*x^
2 + 2*a*b*x + a^2)*b^2) - 5/2*a*e^4*x^2/(sqrt(b^2*x^2 + 2*a*b*x + a^2)*b^3
) + 6*d^2*e^2*log(x + a/b)/b^3 - 12*a*d*e^3*log(x + a/b)/b^4 + 6*a^2*e^4*log
og(x + a/b)/b^5 - 4*d^3*e/(sqrt(b^2*x^2 + 2*a*b*x + a^2)*b^2) + 8*a^2*d*e^
3/(sqrt(b^2*x^2 + 2*a*b*x + a^2)*b^4) - 5*a^3*e^4/(sqrt(b^2*x^2 + 2*a*b*x
+ a^2)*b^5) + 12*a*d^2*e^2*x/(b^4*(x + a/b)^2) - 24*a^2*d*e^3*x/(b^5*(x +
a/b)^2) + 12*a^3*e^4*x/(b^6*(x + a/b)^2) - 1/2*d^4/(b^3*(x + a/b)^2) + 2*a
*d^3*e/(b^4*(x + a/b)^2) + 9*a^2*d^2*e^2/(b^5*(x + a/b)^2) - 22*a^3*d*e^3/
(b^6*(x + a/b)^2) + 23/2*a^4*e^4/(b^7*(x + a/b)^2)

```

Giac [A] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 216, normalized size of antiderivative = 1.03

$$\int \frac{(d+ex)^4}{(a^2+2abx+b^2x^2)^{3/2}} dx = \frac{6(b^2d^2e^2 - 2abde^3 + a^2e^4) \log(|bx+a|)}{b^5 \operatorname{sgn}(bx+a)}$$

$$+ \frac{b^3e^4x^2 \operatorname{sgn}(bx+a) + 8b^3de^3x \operatorname{sgn}(bx+a) - 6ab^2e^4x \operatorname{sgn}(bx+a)}{2b^6}$$

$$- \frac{b^4d^4 + 4ab^3d^3e - 18a^2b^2d^2e^2 + 20a^3bde^3 - 7a^4e^4 + 8(b^4d^3e - 3ab^3d^2e^2 + 3a^2b^2de^3 - a^3be^4)x}{2(bx+a)^2 b^5 \operatorname{sgn}(bx+a)}$$

input

```
integrate((e*x+d)^4/(b^2*x^2+2*a*b*x+a^2)^(3/2),x, algorithm="giac")
```

output

```

6*(b^2*d^2*e^2 - 2*a*b*d*e^3 + a^2*e^4)*log(abs(b*x + a))/(b^5*sgn(b*x + a
)) + 1/2*(b^3*e^4*x^2*sgn(b*x + a) + 8*b^3*d*e^3*x*sgn(b*x + a) - 6*a*b^2*
e^4*x*sgn(b*x + a))/b^6 - 1/2*(b^4*d^4 + 4*a*b^3*d^3*e - 18*a^2*b^2*d^2*e^
2 + 20*a^3*b*d*e^3 - 7*a^4*e^4 + 8*(b^4*d^3*e - 3*a*b^3*d^2*e^2 + 3*a^2*b^
2*d*e^3 - a^3*b*e^4)*x)/((b*x + a)^2*b^5*sgn(b*x + a))

```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d + ex)^4}{(a^2 + 2abx + b^2x^2)^{3/2}} dx = \int \frac{(d + ex)^4}{(a^2 + 2abx + b^2x^2)^{3/2}} dx$$

input `int((d + e*x)^4/(a^2 + b^2*x^2 + 2*a*b*x)^(3/2), x)`

output `int((d + e*x)^4/(a^2 + b^2*x^2 + 2*a*b*x)^(3/2), x)`

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 330, normalized size of antiderivative = 1.57

$$\int \frac{(d + ex)^4}{(a^2 + 2abx + b^2x^2)^{3/2}} dx = \frac{12 \log(bx + a) a^5 e^4 - 24 \log(bx + a) a^4 b d e^3 + 24 \log(bx + a) a^4 b e^4 x + 12 \log(bx + a) a^3 b^2 d e^2 - 48 \log(bx + a) a^3 b^2 d e^3 x + 12 \log(bx + a) a^3 b^2 e^4 x^2 + 24 \log(bx + a) a^2 b^3 d e^2 x - 24 \log(bx + a) a^2 b^3 d e^3 x^2 + 12 \log(bx + a) a b^4 d e^2 x^2 + 6 a^5 e^4 - 12 a^4 b d e^3 + 6 a^3 b^2 d e^2 - 12 a^3 b^2 e^4 x^2 + 24 a^2 b^3 d e^3 x^2 - 4 a^2 b^3 e^4 x^3 - a b^4 d^4 - 12 a b^4 d e^2 x^2 + 8 a b^4 d e^3 x^3 + a b^4 e^4 x^4 + 4 b^5 d^3 e x^2}{(2 a b^5 (a^2 + 2 a b x + b^2 x^2))}$$

input `int((e*x+d)^4/(b^2*x^2+2*a*b*x+a^2)^(3/2), x)`

output `(12*log(a + b*x)*a**5*e**4 - 24*log(a + b*x)*a**4*b*d*e**3 + 24*log(a + b*x)*a**4*b*e**4*x + 12*log(a + b*x)*a**3*b**2*d**2*e**2 - 48*log(a + b*x)*a**3*b**2*d*e**3*x + 12*log(a + b*x)*a**3*b**2*e**4*x**2 + 24*log(a + b*x)*a**2*b**3*d**2*e**2*x - 24*log(a + b*x)*a**2*b**3*d*e**3*x**2 + 12*log(a + b*x)*a*b**4*d**2*e**2*x**2 + 6*a**5*e**4 - 12*a**4*b*d*e**3 + 6*a**3*b**2*d**2*e**2 - 12*a**3*b**2*e**4*x**2 + 24*a**2*b**3*d*e**3*x**2 - 4*a**2*b**3*e**4*x**3 - a*b**4*d**4 - 12*a*b**4*d**2*e**2*x**2 + 8*a*b**4*d*e**3*x**3 + a*b**4*e**4*x**4 + 4*b**5*d**3*e*x**2)/(2*a*b**5*(a**2 + 2*a*b*x + b**2*x**2))`

3.319 $\int \frac{(d+ex)^3}{(a^2+2abx+b^2x^2)^{3/2}} dx$

| | |
|---|------|
| Optimal result | 2396 |
| Mathematica [A] (verified) | 2397 |
| Rubi [A] (verified) | 2397 |
| Maple [A] (verified) | 2399 |
| Fricas [A] (verification not implemented) | 2399 |
| Sympy [F] | 2400 |
| Maxima [B] (verification not implemented) | 2400 |
| Giac [A] (verification not implemented) | 2401 |
| Mupad [F(-1)] | 2401 |
| Reduce [B] (verification not implemented) | 2401 |

Optimal result

Integrand size = 28, antiderivative size = 161

$$\int \frac{(d+ex)^3}{(a^2+2abx+b^2x^2)^{3/2}} dx = -\frac{3e(bd-ae)^2}{b^4\sqrt{a^2+2abx+b^2x^2}} - \frac{(bd-ae)^3}{2b^4(a+bx)\sqrt{a^2+2abx+b^2x^2}} + \frac{e^3x(a+bx)}{b^3\sqrt{a^2+2abx+b^2x^2}} + \frac{3e^2(bd-ae)(a+bx)\log(a+bx)}{b^4\sqrt{a^2+2abx+b^2x^2}}$$

output

```
-3*e*(-a*e+b*d)^2/b^4/((b*x+a)^2)^(1/2)-1/2*(-a*e+b*d)^3/b^4/(b*x+a)/((b*x+a)^2)^(1/2)+e^3*x*(b*x+a)/b^3/((b*x+a)^2)^(1/2)+3*e^2*(-a*e+b*d)*(b*x+a)*ln(b*x+a)/b^4/((b*x+a)^2)^(1/2)
```

Mathematica [A] (verified)

Time = 1.04 (sec) , antiderivative size = 125, normalized size of antiderivative = 0.78

$$\int \frac{(d+ex)^3}{(a^2+2abx+b^2x^2)^{3/2}} dx = \frac{-5a^3e^3 + a^2be^2(9d-4ex) + ab^2e(-3d^2+12dex+4e^2x^2) - b^3(d^3+6d^2ex - 2b^4(a+bx)\sqrt{(a+bx)^2})}{2b^4(a+bx)\sqrt{(a+bx)^2}}$$

input `Integrate[(d + e*x)^3/(a^2 + 2*a*b*x + b^2*x^2)^(3/2), x]`

output `(-5*a^3*e^3 + a^2*b*e^2*(9*d - 4*e*x) + a*b^2*e*(-3*d^2 + 12*d*e*x + 4*e^2*x^2) - b^3*(d^3 + 6*d^2*e*x - 2*e^3*x^3) - 6*e^2*(-(b*d) + a*e)*(a + b*x)^2*Log[a + b*x])/(2*b^4*(a + b*x)*Sqrt[(a + b*x)^2])`

Rubi [A] (verified)

Time = 0.47 (sec) , antiderivative size = 104, normalized size of antiderivative = 0.65, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {1102, 27, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\begin{aligned} & \int \frac{(d+ex)^3}{(a^2+2abx+b^2x^2)^{3/2}} dx \\ & \quad \downarrow \text{1102} \\ & \frac{b^3(a+bx) \int \frac{(d+ex)^3}{b^3(a+bx)^3} dx}{\sqrt{a^2+2abx+b^2x^2}} \\ & \quad \downarrow \text{27} \\ & \frac{(a+bx) \int \frac{(d+ex)^3}{(a+bx)^3} dx}{\sqrt{a^2+2abx+b^2x^2}} \\ & \quad \downarrow \text{49} \\ & \frac{(a+bx) \int \left(\frac{e^3}{b^3} + \frac{3(bd-ae)e^2}{b^3(a+bx)} + \frac{3(bd-ae)^2e}{b^3(a+bx)^2} + \frac{(bd-ae)^3}{b^3(a+bx)^3} \right) dx}{\sqrt{a^2+2abx+b^2x^2}} \end{aligned}$$

$$\frac{(a + bx) \left(\frac{3e^2(bd-ae)\log(a+bx)}{b^4} - \frac{3e(bd-ae)^2}{b^4(a+bx)} - \frac{(bd-ae)^3}{2b^4(a+bx)^2} + \frac{e^3x}{b^3} \right)}{\sqrt{a^2 + 2abx + b^2x^2}}$$

input `Int[(d + e*x)^3/(a^2 + 2*a*b*x + b^2*x^2)^(3/2),x]`

output `((a + b*x)*((e^3*x)/b^3 - (b*d - a*e)^3/(2*b^4*(a + b*x)^2) - (3*e*(b*d - a*e)^2)/(b^4*(a + b*x)) + (3*e^2*(b*d - a*e)*Log[a + b*x])/b^4)/Sqrt[a^2 + 2*a*b*x + b^2*x^2]`

Definitions of rubi rules used

rule 27 `Int[(a_)*(F_x_), x_Symbol] := Simp[a Int[F_x, x], x] /; FreeQ[a, x] && !MatchQ[F_x, (b_)*(G_x_)] /; FreeQ[b, x]`

rule 49 `Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] := Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] && IGtQ[m, 0] && IGtQ[m + n + 2, 0]`

rule 1102 `Int[((d_.) + (e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] := Simp[(a + b*x + c*x^2)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x)^(2*FracPart[p])) Int[(d + e*x)^m*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && EqQ[b^2 - 4*a*c, 0]`

rule 2009 `Int[u_, x_Symbol] := Simp[IntSum[u, x], x] /; SumQ[u]`

Maple [A] (verified)

Time = 1.47 (sec) , antiderivative size = 154, normalized size of antiderivative = 0.96

| method | result |
|---------|---|
| risch | $\frac{\sqrt{(bx+a)^2 e^3 x}}{(bx+a)b^3} + \frac{\sqrt{(bx+a)^2} \left((-3a^2 e^3 + 6abd e^2 - 3d^2 e b^2)x - \frac{5e^3 a^3 - 9a^2 b d e^2 + 3a b^2 d^2 e + b^3 d^3}{2b} \right)}{(bx+a)^3 b^3} - \frac{3\sqrt{(bx+a)^2} e^2 (ae-bd) \ln(bx+a)}{(bx+a)b^4}$ |
| default | $-\frac{(6 \ln(bx+a)x^2 a b^2 e^3 - 6 \ln(bx+a)b^3 d e^2 x^2 - 2e^3 x^3 b^3 + 12 \ln(bx+a)x a^2 b e^3 - 12 \ln(bx+a)x a b^2 d e^2 - 4x^2 a b^2 e^3 + 6 \ln(bx+a)a^3 e^3 - 6 \ln(bx+a)a^2 b d e^2)}{2b^4 (bx+a)^{\frac{3}{2}}}$ |

```
input int((e*x+d)^3/(b^2*x^2+2*a*b*x+a^2)^(3/2),x,method=_RETURNVERBOSE)
```

```
output ((b*x+a)^(1/2)/(b*x+a)*e^3/b^3*x+((b*x+a)^(1/2)/(b*x+a)^3*((-3*a^2*e^3+6*a*b*d*e^2-3*b^2*d^2*e)*x-1/2*(5*a^3*e^3-9*a^2*b*d*e^2+3*a*b^2*d^2*e+b^3*d^3)/b)/b^3-3*((b*x+a)^(1/2)/(b*x+a)/b^4*e^2*(a*e-b*d)*ln(b*x+a)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 188, normalized size of antiderivative = 1.17

$$\int \frac{(d + ex)^3}{(a^2 + 2abx + b^2x^2)^{3/2}} dx = \frac{2b^3e^3x^3 + 4ab^2e^3x^2 - b^3d^3 - 3ab^2d^2e + 9a^2bde^2 - 5a^3e^3 - 2(3b^3d^2e - 6ab^2d^2e + 3a^2b^2d^2e - 3a^3e^3)}{2(b^2x^2 + 2abx + a^2)^{3/2}}$$

```
input integrate((e*x+d)^3/(b^2*x^2+2*a*b*x+a^2)^(3/2),x, algorithm="fricas")
```

```
output 1/2*(2*b^3*e^3*x^3 + 4*a*b^2*e^3*x^2 - b^3*d^3 - 3*a*b^2*d^2*e + 9*a^2*b*d*e^2 - 5*a^3*e^3 - 2*(3*b^3*d^2*e - 6*a*b^2*d^2*e^2 + 2*a^2*b*e^3)*x + 6*(a^2*b*d*e^2 - a^3*e^3 + (b^3*d^2*e^2 - a*b^2*e^3)*x^2 + 2*(a*b^2*d^2*e^2 - a^2*b*d^2*e^2 + a^3*e^3)*x)*log(b*x + a)/(b^6*x^2 + 2*a*b^5*x + a^2*b^4)
```

Sympy [F]

$$\int \frac{(d+ex)^3}{(a^2+2abx+b^2x^2)^{3/2}} dx = \int \frac{(d+ex)^3}{((a+bx)^2)^{3/2}} dx$$

input `integrate((e*x+d)**3/(b**2*x**2+2*a*b*x+a**2)**(3/2),x)`

output `Integral((d + e*x)**3/((a + b*x)**2)**(3/2), x)`

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 237 vs. $2(115) = 230$.

Time = 0.06 (sec) , antiderivative size = 237, normalized size of antiderivative = 1.47

$$\begin{aligned} \int \frac{(d+ex)^3}{(a^2+2abx+b^2x^2)^{3/2}} dx &= \frac{e^3x^2}{\sqrt{b^2x^2+2abx+a^2}b^2} + \frac{3de^2 \log(x+\frac{a}{b})}{b^3} \\ &- \frac{3ae^3 \log(x+\frac{a}{b})}{b^4} - \frac{3d^2e}{\sqrt{b^2x^2+2abx+a^2}b^2} + \frac{2a^2e^3}{\sqrt{b^2x^2+2abx+a^2}b^4} + \frac{6ade^2x}{b^4(x+\frac{a}{b})^2} \\ &- \frac{6a^2e^3x}{b^5(x+\frac{a}{b})^2} - \frac{d^3}{2b^3(x+\frac{a}{b})^2} + \frac{3ad^2e}{2b^4(x+\frac{a}{b})^2} + \frac{9a^2de^2}{2b^5(x+\frac{a}{b})^2} - \frac{11a^3e^3}{2b^6(x+\frac{a}{b})^2} \end{aligned}$$

input `integrate((e*x+d)^3/(b^2*x^2+2*a*b*x+a^2)^(3/2),x, algorithm="maxima")`

output `e^3*x^2/(sqrt(b^2*x^2 + 2*a*b*x + a^2)*b^2) + 3*d*e^2*log(x + a/b)/b^3 - 3*a*e^3*log(x + a/b)/b^4 - 3*d^2*e/(sqrt(b^2*x^2 + 2*a*b*x + a^2)*b^2) + 2*a^2*e^3/(sqrt(b^2*x^2 + 2*a*b*x + a^2)*b^4) + 6*a*d*e^2*x/(b^4*(x + a/b)^2) - 6*a^2*e^3*x/(b^5*(x + a/b)^2) - 1/2*d^3/(b^3*(x + a/b)^2) + 3/2*a*d^2*e/(b^4*(x + a/b)^2) + 9/2*a^2*d*e^2/(b^5*(x + a/b)^2) - 11/2*a^3*e^3/(b^6*(x + a/b)^2)`

Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 136, normalized size of antiderivative = 0.84

$$\int \frac{(d+ex)^3}{(a^2+2abx+b^2x^2)^{3/2}} dx = \frac{e^3x}{b^3\operatorname{sgn}(bx+a)} + \frac{3(bde^2 - ae^3)\log(|bx+a|)}{b^4\operatorname{sgn}(bx+a)} - \frac{b^3d^3 + 3ab^2d^2e - 9a^2bde^2 + 5a^3e^3 + 6(b^3d^2e - 2ab^2de^2 + a^2be^3)x}{2(bx+a)^2b^4\operatorname{sgn}(bx+a)}$$

input `integrate((e*x+d)^3/(b^2*x^2+2*a*b*x+a^2)^(3/2),x, algorithm="giac")`

output `e^3*x/(b^3*sgn(b*x + a)) + 3*(b*d*e^2 - a*e^3)*log(abs(b*x + a))/(b^4*sgn(b*x + a)) - 1/2*(b^3*d^3 + 3*a*b^2*d^2*e - 9*a^2*b*d*e^2 + 5*a^3*e^3 + 6*(b^3*d^2*e - 2*a*b^2*d*e^2 + a^2*b*e^3)*x)/((b*x + a)^2*b^4*sgn(b*x + a))`

Mupad [F(-1)]

Timed out.

$$\int \frac{(d+ex)^3}{(a^2+2abx+b^2x^2)^{3/2}} dx = \int \frac{(d+ex)^3}{(a^2+2abx+b^2x^2)^{3/2}} dx$$

input `int((d + e*x)^3/(a^2 + b^2*x^2 + 2*a*b*x)^(3/2), x)`

output `int((d + e*x)^3/(a^2 + b^2*x^2 + 2*a*b*x)^(3/2), x)`

Reduce [B] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 209, normalized size of antiderivative = 1.30

$$\int \frac{(d+ex)^3}{(a^2+2abx+b^2x^2)^{3/2}} dx = \frac{-6\log(bx+a)a^4e^3 + 6\log(bx+a)a^3bde^2 - 12\log(bx+a)a^3be^3x + 12\log(bx+a)a^3be^3x + 12\log(bx+a)a^3be^3x}{(a^2+2abx+b^2x^2)^{3/2}}$$

input `int((e*x+d)^3/(b^2*x^2+2*a*b*x+a^2)^(3/2), x)`

output

```
( - 6*log(a + b*x)*a**4*e**3 + 6*log(a + b*x)*a**3*b*d*e**2 - 12*log(a + b
*x)*a**3*b*e**3*x + 12*log(a + b*x)*a**2*b**2*d*e**2*x - 6*log(a + b*x)*a
**2*b**2*e**3*x**2 + 6*log(a + b*x)*a*b**3*d*e**2*x**2 - 3*a**4*e**3 + 3*a
**3*b*d*e**2 + 6*a**2*b**2*e**3*x**2 - a*b**3*d**3 - 6*a*b**3*d*e**2*x**2 +
2*a*b**3*e**3*x**3 + 3*b**4*d**2*e*x**2)/(2*a*b**4*(a**2 + 2*a*b*x + b**2
*x**2))
```

3.320 $\int \frac{(d+ex)^2}{(a^2+2abx+b^2x^2)^{3/2}} dx$

| | |
|---|------|
| Optimal result | 2403 |
| Mathematica [B] (verified) | 2403 |
| Rubi [A] (verified) | 2404 |
| Maple [A] (verified) | 2405 |
| Fricas [A] (verification not implemented) | 2406 |
| Sympy [F] | 2406 |
| Maxima [A] (verification not implemented) | 2407 |
| Giac [A] (verification not implemented) | 2407 |
| Mupad [F(-1)] | 2408 |
| Reduce [B] (verification not implemented) | 2408 |

Optimal result

Integrand size = 28, antiderivative size = 117

$$\int \frac{(d+ex)^2}{(a^2+2abx+b^2x^2)^{3/2}} dx = -\frac{2e(bd-ae)}{b^3\sqrt{a^2+2abx+b^2x^2}} - \frac{(bd-ae)^2}{2b^3(a+bx)\sqrt{a^2+2abx+b^2x^2}} + \frac{e^2(a+bx)\log(a+bx)}{b^3\sqrt{a^2+2abx+b^2x^2}}$$

output

```
-2*e*(-a*e+b*d)/b^3/((b*x+a)^2)^(1/2)-1/2*(-a*e+b*d)^2/b^3/(b*x+a)/((b*x+a)^2)^(1/2)+e^2*(b*x+a)*ln(b*x+a)/b^3/((b*x+a)^2)^(1/2)
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 241 vs. 2(117) = 234.

Time = 1.21 (sec) , antiderivative size = 241, normalized size of antiderivative = 2.06

$$\int \frac{(d+ex)^2}{(a^2+2abx+b^2x^2)^{3/2}} dx = \frac{b(-bd+ae)x(2a^5e+a^2b^3ex^3+a^4b(2d+3ex))+a^3(b^2dx-2\sqrt{a^2}e\sqrt{(a+bx)^2})+a(-b^4dx^3+\sqrt{a^2}b^2x\sqrt{(a+bx)^2})}{a^4(a+bx)(\sqrt{a^2}bx+a(\sqrt{a^2}-\sqrt{(a+bx)^2}))} \cdot \frac{1}{2b^3}$$

input

```
Integrate[(d + e*x)^2/(a^2 + 2*a*b*x + b^2*x^2)^(3/2), x]
```


output

```
((b*(-(b*d) + a*e)*x*(2*a^5*e + a^2*b^3*e*x^3 + a^4*b*(2*d + 3*e*x) + a^3*
(b^2*d*x - 2*Sqrt[a^2]*e*Sqrt[(a + b*x)^2]) + a*(-(b^4*d*x^3) + Sqrt[a^2]*
b^2*x*Sqrt[(a + b*x)^2]*(d + e*x)) - Sqrt[a^2]*b*Sqrt[(a + b*x)^2]*(b^2*d*
x^2 + a^2*(2*d + e*x))))/(a^4*(a + b*x)*(Sqrt[a^2]*b*x + a*(Sqrt[a^2] - Sq
rt[(a + b*x)^2]))) - 4*e^2*ArcTanh[(b*x)/(Sqrt[a^2] - Sqrt[(a + b*x)^2]))]
/(2*b^3)
```

Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.73, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {1102, 27, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex)^2}{(a^2+2abx+b^2x^2)^{3/2}} dx$$

$$\downarrow \text{1102}$$

$$\frac{b^3(a+bx) \int \frac{(d+ex)^2}{b^3(a+bx)^3} dx}{\sqrt{a^2+2abx+b^2x^2}}$$

$$\downarrow \text{27}$$

$$\frac{(a+bx) \int \frac{(d+ex)^2}{(a+bx)^3} dx}{\sqrt{a^2+2abx+b^2x^2}}$$

$$\downarrow \text{49}$$

$$\frac{(a+bx) \int \left(\frac{e^2}{b^2(a+bx)} + \frac{2(bd-ae)e}{b^2(a+bx)^2} + \frac{(bd-ae)^2}{b^2(a+bx)^3} \right) dx}{\sqrt{a^2+2abx+b^2x^2}}$$

$$\downarrow \text{2009}$$

$$\frac{(a+bx) \left(-\frac{2e(bd-ae)}{b^3(a+bx)} - \frac{(bd-ae)^2}{2b^3(a+bx)^2} + \frac{e^2 \log(a+bx)}{b^3} \right)}{\sqrt{a^2+2abx+b^2x^2}}$$

input

```
Int[(d + e*x)^2/(a^2 + 2*a*b*x + b^2*x^2)^(3/2), x]
```

output
$$\frac{((a + b*x)*(-1/2*(b*d - a*e)^2/(b^3*(a + b*x)^2) - (2*e*(b*d - a*e))/(b^3*(a + b*x)) + (e^2*Log[a + b*x])/b^3)/Sqrt[a^2 + 2*a*b*x + b^2*x^2]}$$

Defintions of rubi rules used

rule 27
$$\text{Int}[(a_)*(F_), x_Symbol] \rightarrow \text{Simp}[a \text{ Int}[F, x], x] \text{ ; FreeQ}[a, x] \ \&\& \ !\text{MatchQ}[F, (b_)*(G_)] \text{ ; FreeQ}[b, x]$$

rule 49
$$\text{Int}[(a_ + (b_)*(x_))^{(m_)}*((c_ + (d_)*(x_))^{(n_)}), x_Symbol] \rightarrow \text{Int}[\text{ExpandIntegrand}[(a + b*x)^m*(c + d*x)^n, x], x] \text{ ; FreeQ}[\{a, b, c, d\}, x] \ \&\& \ \text{IGtQ}[m, 0] \ \&\& \ \text{IGtQ}[m + n + 2, 0]$$

rule 1102
$$\text{Int}[(d_ + (e_)*(x_))^{(m_)}*((a_ + (b_)*(x_ + (c_)*(x_)^2))^{(p_)}), x_Symbol] \rightarrow \text{Simp}[(a + b*x + c*x^2)^{\text{FracPart}[p]} / (c^{\text{IntPart}[p]}*(b/2 + c*x)^{(2*\text{FracPart}[p])}) \text{ Int}[(d + e*x)^m*(b/2 + c*x)^{(2*p)}, x], x] \text{ ; FreeQ}[\{a, b, c, d, e, m, p\}, x] \ \&\& \ \text{EqQ}[b^2 - 4*a*c, 0]$$

rule 2009
$$\text{Int}[u_, x_Symbol] \rightarrow \text{Simp}[\text{IntSum}[u, x], x] \text{ ; SumQ}[u]$$

Maple [A] (verified)

Time = 1.30 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.79

| method | result | size |
|---------|--|------|
| risch | $\frac{\sqrt{(bx+a)^2} \left(\frac{2e(ae-bd)x}{b^2} + \frac{3e^2a^2-2abde-b^2d^2}{2b^3} \right)}{(bx+a)^3} + \frac{\sqrt{(bx+a)^2} e^2 \ln(bx+a)}{(bx+a)b^3}$ | 92 |
| default | $\frac{(2 \ln(bx+a)b^2e^2x^2 + 4 \ln(bx+a)abe^2x + 2 \ln(bx+a)a^2e^2 + 4xab e^2 - 4b^2dex + 3e^2a^2 - 2abde - b^2d^2)(bx+a)}{2b^3((bx+a)^2)^{\frac{3}{2}}}$ | 104 |

input
$$\text{int}((e*x+d)^2/(b^2*x^2+2*a*b*x+a^2)^{(3/2)}, x, \text{method}=_RETURNVERBOSE)$$

output $((b*x+a)^2)^{(1/2)}/(b*x+a)^3*(2/b^2*e*(a*e-b*d)*x+1/2*(3*a^2*e^2-2*a*b*d*e-b^2*d^2)/b^3)+((b*x+a)^2)^{(1/2)}/(b*x+a)*e^2/b^3*\ln(b*x+a)$

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 99, normalized size of antiderivative = 0.85

$$\int \frac{(d+ex)^2}{(a^2+2abx+b^2x^2)^{3/2}} dx = \frac{b^2d^2 + 2abde - 3a^2e^2 + 4(b^2de - abe^2)x - 2(b^2e^2x^2 + 2abe^2x + a^2e^2) \log(bx+a)}{2(b^5x^2 + 2ab^4x + a^2b^3)}$$

input `integrate((e*x+d)^2/(b^2*x^2+2*a*b*x+a^2)^(3/2),x,algorithm="fricas")`

output $-1/2*(b^2*d^2 + 2*a*b*d*e - 3*a^2*e^2 + 4*(b^2*d*e - a*b*e^2)*x - 2*(b^2*e^2*x^2 + 2*a*b*e^2*x + a^2*e^2)*\log(b*x + a))/(b^5*x^2 + 2*a*b^4*x + a^2*b^3)$

Sympy [F]

$$\int \frac{(d+ex)^2}{(a^2+2abx+b^2x^2)^{3/2}} dx = \int \frac{(d+ex)^2}{((a+bx)^2)^{3/2}} dx$$

input `integrate((e*x+d)**2/(b**2*x**2+2*a*b*x+a**2)**(3/2),x)`

output `Integral((d + e*x)**2/((a + b*x)**2)**(3/2), x)`

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 113, normalized size of antiderivative = 0.97

$$\int \frac{(d+ex)^2}{(a^2+2abx+b^2x^2)^{3/2}} dx = \frac{e^2 \log\left(x + \frac{a}{b}\right)}{b^3} - \frac{2de}{\sqrt{b^2x^2+2abx+a^2}b^2} + \frac{2ae^2x}{b^4\left(x + \frac{a}{b}\right)^2} - \frac{d^2}{2b^3\left(x + \frac{a}{b}\right)^2} + \frac{ade}{b^4\left(x + \frac{a}{b}\right)^2} + \frac{3a^2e^2}{2b^5\left(x + \frac{a}{b}\right)^2}$$

input `integrate((e*x+d)^2/(b^2*x^2+2*a*b*x+a^2)^(3/2),x, algorithm="maxima")`

output `e^2*log(x + a/b)/b^3 - 2*d*e/(sqrt(b^2*x^2 + 2*a*b*x + a^2)*b^2) + 2*a*e^2*x/(b^4*(x + a/b)^2) - 1/2*d^2/(b^3*(x + a/b)^2) + a*d*e/(b^4*(x + a/b)^2) + 3/2*a^2*e^2/(b^5*(x + a/b)^2)`

Giac [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.72

$$\int \frac{(d+ex)^2}{(a^2+2abx+b^2x^2)^{3/2}} dx = \frac{e^2 \log(|bx+a|)}{b^3 \operatorname{sgn}(bx+a)} - \frac{4(bde - ae^2)x + \frac{b^2d^2+2abde-3a^2e^2}{b}}{2(bx+a)^2 b^2 \operatorname{sgn}(bx+a)}$$

input `integrate((e*x+d)^2/(b^2*x^2+2*a*b*x+a^2)^(3/2),x, algorithm="giac")`

output `e^2*log(abs(b*x + a))/(b^3*sgn(b*x + a)) - 1/2*(4*(b*d*e - a*e^2)*x + (b^2*d^2 + 2*a*b*d*e - 3*a^2*e^2)/b)/((b*x + a)^2*b^2*sgn(b*x + a))`

Mupad [F(-1)]

Timed out.

$$\int \frac{(d + ex)^2}{(a^2 + 2abx + b^2x^2)^{3/2}} dx = \int \frac{(d + ex)^2}{(a^2 + 2abx + b^2x^2)^{3/2}} dx$$

input `int((d + e*x)^2/(a^2 + b^2*x^2 + 2*a*b*x)^(3/2), x)`output `int((d + e*x)^2/(a^2 + b^2*x^2 + 2*a*b*x)^(3/2), x)`**Reduce [B] (verification not implemented)**

Time = 0.22 (sec) , antiderivative size = 113, normalized size of antiderivative = 0.97

$$\int \frac{(d + ex)^2}{(a^2 + 2abx + b^2x^2)^{3/2}} dx = \frac{2 \log(bx + a) a^3 e^2 + 4 \log(bx + a) a^2 b e^2 x + 2 \log(bx + a) a b^2 e^2 x^2 + a^3 e^2 - a^2 b^2 e^2 x^2}{2 a b^3 (b^2 x^2 + 2 a b x + a^2)}$$

input `int((e*x+d)^2/(b^2*x^2+2*a*b*x+a^2)^(3/2), x)`output `(2*log(a + b*x)*a**3*e**2 + 4*log(a + b*x)*a**2*b*e**2*x + 2*log(a + b*x)*a*b**2*e**2*x**2 + a**3*e**2 - a*b**2*d**2 - 2*a*b**2*e**2*x**2 + 2*b**3*d*e*x**2)/(2*a*b**3*(a**2 + 2*a*b*x + b**2*x**2))`

3.321 $\int \frac{d+ex}{(a^2+2abx+b^2x^2)^{3/2}} dx$

| | |
|---|------|
| Optimal result | 2409 |
| Mathematica [B] (verified) | 2409 |
| Rubi [A] (verified) | 2410 |
| Maple [A] (verified) | 2411 |
| Fricas [A] (verification not implemented) | 2412 |
| Sympy [F] | 2412 |
| Maxima [A] (verification not implemented) | 2412 |
| Giac [A] (verification not implemented) | 2413 |
| Mupad [B] (verification not implemented) | 2413 |
| Reduce [B] (verification not implemented) | 2413 |

Optimal result

Integrand size = 26, antiderivative size = 69

$$\int \frac{d+ex}{(a^2+2abx+b^2x^2)^{3/2}} dx = -\frac{e}{b^2\sqrt{a^2+2abx+b^2x^2}} - \frac{bd-ae}{2b^2(a+bx)\sqrt{a^2+2abx+b^2x^2}}$$

output `-e/b^2/((b*x+a)^2)^(1/2)-1/2*(-a*e+b*d)/b^2/(b*x+a)/((b*x+a)^2)^(1/2)`

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 159 vs. 2(69) = 138.

Time = 0.72 (sec) , antiderivative size = 159, normalized size of antiderivative = 2.30

$$\int \frac{d+ex}{(a^2+2abx+b^2x^2)^{3/2}} dx = \frac{x(a^3bdx+a^2b^2ex^3+a^4(2d+ex))+a(-b^3dx^3+\sqrt{a^2}bx\sqrt{(a+bx)^2(d+ex)})-\sqrt{a^2}\sqrt{(a+bx)^2}(b^2dx^2-2a^4(a+bx)(\sqrt{a^2}bx+a(\sqrt{a^2}-\sqrt{(a+bx)^2}))}{2a^4(a+bx)(\sqrt{a^2}bx+a(\sqrt{a^2}-\sqrt{(a+bx)^2}))}$$

input `Integrate[(d + e*x)/(a^2 + 2*a*b*x + b^2*x^2)^(3/2),x]`

output

```
-1/2*(x*(a^3*b*d*x + a^2*b^2*e*x^3 + a^4*(2*d + e*x) + a*(-(b^3*d*x^3) + S
qrt[a^2]*b*x*Sqrt[(a + b*x)^2]*(d + e*x)) - Sqrt[a^2]*Sqrt[(a + b*x)^2]*(b
^2*d*x^2 + a^2*(2*d + e*x)))/(a^4*(a + b*x)*(Sqrt[a^2]*b*x + a*(Sqrt[a^2]
- Sqrt[(a + b*x)^2])))
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {1100, 1078}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{d + ex}{(a^2 + 2abx + b^2x^2)^{3/2}} dx$$

$$\downarrow 1100$$

$$\frac{(bd - ae) \int \frac{1}{(a^2 + 2bxa + b^2x^2)^{3/2}} dx}{b} - \frac{e}{b^2 \sqrt{a^2 + 2abx + b^2x^2}}$$

$$\downarrow 1078$$

$$-\frac{bd - ae}{2b^2(a + bx)\sqrt{a^2 + 2abx + b^2x^2}} - \frac{e}{b^2 \sqrt{a^2 + 2abx + b^2x^2}}$$

input

```
Int[(d + e*x)/(a^2 + 2*a*b*x + b^2*x^2)^(3/2),x]
```

output

```
-(e/(b^2*Sqrt[a^2 + 2*a*b*x + b^2*x^2])) - (b*d - a*e)/(2*b^2*(a + b*x)*S
qrt[a^2 + 2*a*b*x + b^2*x^2])
```

Definitions of rubi rules used

rule 1078

```
Int[((a_) + (b_)*(x_) + (c_)*(x_)^2)^(p_), x_Symbol] := Simp[2*((a + b*x
+ c*x^2)^(p + 1)/((2*p + 1)*(b + 2*c*x))), x] /; FreeQ[{a, b, c, p}, x] &&
EqQ[b^2 - 4*a*c, 0] && LtQ[p, -1]
```

rule 1100

```
Int[((d_) + (e_)*(x_))*((a_) + (b_)*(x_) + (c_)*(x_)^2)^(p_), x_Symbol]
:= Simp[e*((a + b*x + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Simp[(2*c*d - b*
e)/(2*c) Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& EqQ[b^2 - 4*a*c, 0]
```

Maple [A] (verified)

Time = 1.21 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.46

| method | result | size |
|---------|--|------|
| gospers | $-\frac{(bx+a)(2bex+ae+bd)}{2b^2((bx+a)^2)^{\frac{3}{2}}}$ | 32 |
| default | $-\frac{(bx+a)(2bex+ae+bd)}{2b^2((bx+a)^2)^{\frac{3}{2}}}$ | 32 |
| risch | $\frac{\sqrt{(bx+a)^2} \left(-\frac{ex}{b} - \frac{ae+bd}{2b^2}\right)}{(bx+a)^3}$ | 38 |
| orering | $-\frac{(2bex+ae+bd)(bx+a)}{2b^2(b^2x^2+2abx+a^2)^{\frac{3}{2}}}$ | 41 |

input

```
int((e*x+d)/(b^2*x^2+2*a*b*x+a^2)^(3/2),x,method=_RETURNVERBOSE)
```

output

```
-1/2*(b*x+a)*(2*b*e*x+a*e+b*d)/b^2/((b*x+a)^2)^(3/2)
```


Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.55

$$\int \frac{d + ex}{(a^2 + 2abx + b^2x^2)^{3/2}} dx = -\frac{2bex + bd + ae}{2(b^4x^2 + 2ab^3x + a^2b^2)}$$

input `integrate((e*x+d)/(b^2*x^2+2*a*b*x+a^2)^(3/2),x, algorithm="fricas")`

output `-1/2*(2*b*e*x + b*d + a*e)/(b^4*x^2 + 2*a*b^3*x + a^2*b^2)`

Sympy [F]

$$\int \frac{d + ex}{(a^2 + 2abx + b^2x^2)^{3/2}} dx = \int \frac{d + ex}{((a + bx)^2)^{\frac{3}{2}}} dx$$

input `integrate((e*x+d)/(b**2*x**2+2*a*b*x+a**2)**(3/2),x)`

output `Integral((d + e*x)/((a + b*x)**2)**(3/2), x)`

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.81

$$\int \frac{d + ex}{(a^2 + 2abx + b^2x^2)^{3/2}} dx = -\frac{e}{\sqrt{b^2x^2 + 2abx + a^2}b^2} - \frac{d}{2b^3(x + \frac{a}{b})^2} + \frac{ae}{2b^4(x + \frac{a}{b})^2}$$

input `integrate((e*x+d)/(b^2*x^2+2*a*b*x+a^2)^(3/2),x, algorithm="maxima")`

output `-e/(sqrt(b^2*x^2 + 2*a*b*x + a^2)*b^2) - 1/2*d/(b^3*(x + a/b)^2) + 1/2*a*e/(b^4*(x + a/b)^2)`

Giac [A] (verification not implemented)

Time = 0.52 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.46

$$\int \frac{d + ex}{(a^2 + 2abx + b^2x^2)^{3/2}} dx = -\frac{2bex + bd + ae}{2(bx + a)^2 b^2 \operatorname{sgn}(bx + a)}$$

input `integrate((e*x+d)/(b^2*x^2+2*a*b*x+a^2)^(3/2),x, algorithm="giac")`output `-1/2*(2*b*e*x + b*d + a*e)/((b*x + a)^2*b^2*sgn(b*x + a))`**Mupad [B] (verification not implemented)**

Time = 5.64 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.61

$$\int \frac{d + ex}{(a^2 + 2abx + b^2x^2)^{3/2}} dx = -\frac{(ae + bd + 2bex) \sqrt{a^2 + 2abx + b^2x^2}}{2b^2(a + bx)^3}$$

input `int((d + e*x)/(a^2 + b^2*x^2 + 2*a*b*x)^(3/2),x)`output `-((a*e + b*d + 2*b*e*x)*(a^2 + b^2*x^2 + 2*a*b*x)^(1/2))/(2*b^2*(a + b*x)^3)`**Reduce [B] (verification not implemented)**

Time = 0.22 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.54

$$\int \frac{d + ex}{(a^2 + 2abx + b^2x^2)^{3/2}} dx = \frac{be x^2 - ad}{2ab(b^2x^2 + 2abx + a^2)}$$

input `int((e*x+d)/(b^2*x^2+2*a*b*x+a^2)^(3/2),x)`output `(- a*d + b*e*x**2)/(2*a*b*(a**2 + 2*a*b*x + b**2*x**2))`

$$3.322 \quad \int \frac{1}{(a^2 + 2abx + b^2x^2)^{3/2}} dx$$

| | |
|---|------|
| Optimal result | 2414 |
| Mathematica [A] (verified) | 2414 |
| Rubi [A] (verified) | 2415 |
| Maple [A] (verified) | 2416 |
| Fricas [A] (verification not implemented) | 2416 |
| Sympy [F] | 2417 |
| Maxima [A] (verification not implemented) | 2417 |
| Giac [A] (verification not implemented) | 2417 |
| Mupad [B] (verification not implemented) | 2418 |
| Reduce [B] (verification not implemented) | 2418 |

Optimal result

Integrand size = 20, antiderivative size = 34

$$\int \frac{1}{(a^2 + 2abx + b^2x^2)^{3/2}} dx = -\frac{1}{2b(a + bx)\sqrt{a^2 + 2abx + b^2x^2}}$$

output `-1/2/b/(b*x+a)/((b*x+a)^2)^(1/2)`

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.68

$$\int \frac{1}{(a^2 + 2abx + b^2x^2)^{3/2}} dx = -\frac{a + bx}{2b((a + bx)^2)^{3/2}}$$

input `Integrate[(a^2 + 2*a*b*x + b^2*x^2)^(-3/2), x]`

output `-1/2*(a + b*x)/(b*((a + b*x)^2)^(3/2))`

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.050$, Rules used = {1078}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a^2 + 2abx + b^2x^2)^{3/2}} dx$$

↓ 1078

$$-\frac{1}{2b(a + bx)\sqrt{a^2 + 2abx + b^2x^2}}$$

input `Int[(a^2 + 2*a*b*x + b^2*x^2)^(-3/2), x]`

output `-1/2*1/(b*(a + b*x)*Sqrt[a^2 + 2*a*b*x + b^2*x^2])`

Defintions of rubi rules used

rule 1078 `Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[2*((a + b*x + c*x^2)^(p + 1)/((2*p + 1)*(b + 2*c*x))), x] /; FreeQ[{a, b, c, p}, x] && EqQ[b^2 - 4*a*c, 0] && LtQ[p, -1]`

Maple [A] (verified)

Time = 1.11 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.59

| method | result | size |
|---------|---|------|
| gospers | $-\frac{bx+a}{2b((bx+a)^2)^{\frac{3}{2}}}$ | 20 |
| default | $-\frac{bx+a}{2b((bx+a)^2)^{\frac{3}{2}}}$ | 20 |
| risch | $-\frac{\sqrt{(bx+a)^2}}{2(bx+a)^3b}$ | 22 |
| orering | $-\frac{bx+a}{2b(b^2x^2+2abx+a^2)^{\frac{3}{2}}}$ | 29 |

input `int(1/(b^2*x^2+2*a*b*x+a^2)^(3/2),x,method=_RETURNVERBOSE)`

output `-1/2*(b*x+a)/b/((b*x+a)^2)^(3/2)`

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.71

$$\int \frac{1}{(a^2 + 2abx + b^2x^2)^{3/2}} dx = -\frac{1}{2(b^3x^2 + 2ab^2x + a^2b)}$$

input `integrate(1/(b^2*x^2+2*a*b*x+a^2)^(3/2),x, algorithm="fricas")`

output `-1/2/(b^3*x^2 + 2*a*b^2*x + a^2*b)`

Sympy [F]

$$\int \frac{1}{(a^2 + 2abx + b^2x^2)^{3/2}} dx = \int \frac{1}{(a^2 + 2abx + b^2x^2)^{\frac{3}{2}}} dx$$

input `integrate(1/(b**2*x**2+2*a*b*x+a**2)**(3/2),x)`

output `Integral((a**2 + 2*a*b*x + b**2*x**2)**(-3/2), x)`

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.41

$$\int \frac{1}{(a^2 + 2abx + b^2x^2)^{3/2}} dx = -\frac{1}{2b^3(x + \frac{a}{b})^2}$$

input `integrate(1/(b^2*x^2+2*a*b*x+a^2)^(3/2),x, algorithm="maxima")`

output `-1/2/(b^3*(x + a/b)^2)`

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.59

$$\int \frac{1}{(a^2 + 2abx + b^2x^2)^{3/2}} dx = -\frac{1}{2(bx + a)^2 b \operatorname{sgn}(bx + a)}$$

input `integrate(1/(b^2*x^2+2*a*b*x+a^2)^(3/2),x, algorithm="giac")`

output `-1/2/((b*x + a)^2*b*sgn(b*x + a))`

Mupad [B] (verification not implemented)

Time = 5.51 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.88

$$\int \frac{1}{(a^2 + 2abx + b^2x^2)^{3/2}} dx = -\frac{\sqrt{a^2 + 2abx + b^2x^2}}{2b(a + bx)^3}$$

input `int(1/(a^2 + b^2*x^2 + 2*a*b*x)^(3/2),x)`output `-(a^2 + b^2*x^2 + 2*a*b*x)^(1/2)/(2*b*(a + b*x)^3)`**Reduce [B] (verification not implemented)**

Time = 0.22 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.68

$$\int \frac{1}{(a^2 + 2abx + b^2x^2)^{3/2}} dx = -\frac{1}{2b(b^2x^2 + 2abx + a^2)}$$

input `int(1/(b^2*x^2+2*a*b*x+a^2)^(3/2),x)`output `(- 1)/(2*b*(a**2 + 2*a*b*x + b**2*x**2))`

3.323 $\int \frac{1}{(d+ex)(a^2+2abx+b^2x^2)^{3/2}} dx$

| | |
|---|------|
| Optimal result | 2419 |
| Mathematica [A] (verified) | 2419 |
| Rubi [A] (verified) | 2420 |
| Maple [A] (verified) | 2421 |
| Fricas [B] (verification not implemented) | 2422 |
| Sympy [F] | 2422 |
| Maxima [F(-2)] | 2423 |
| Giac [A] (verification not implemented) | 2423 |
| Mupad [F(-1)] | 2424 |
| Reduce [B] (verification not implemented) | 2424 |

Optimal result

Integrand size = 28, antiderivative size = 165

$$\int \frac{1}{(d+ex)(a^2+2abx+b^2x^2)^{3/2}} dx = \frac{e}{(bd-ae)^2\sqrt{a^2+2abx+b^2x^2}} - \frac{1}{2(bd-ae)(a+bx)\sqrt{a^2+2abx+b^2x^2}} + \frac{e^2(a+bx)\log(a+bx)}{(bd-ae)^3\sqrt{a^2+2abx+b^2x^2}} - \frac{e^2(a+bx)\log(d+ex)}{(bd-ae)^3\sqrt{a^2+2abx+b^2x^2}}$$

output

```
e/(-a*e+b*d)^2/((b*x+a)^2)^(1/2)-1/2/(-a*e+b*d)/(b*x+a)/((b*x+a)^2)^(1/2)+
e^2*(b*x+a)*ln(b*x+a)/(-a*e+b*d)^3/((b*x+a)^2)^(1/2)-e^2*(b*x+a)*ln(e*x+d)
/(-a*e+b*d)^3/((b*x+a)^2)^(1/2)
```

Mathematica [A] (verified)

Time = 1.05 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.56

$$\int \frac{1}{(d+ex)(a^2+2abx+b^2x^2)^{3/2}} dx = \frac{-((bd-ae)(-3ae+b(d-2ex))) + 2e^2(a+bx)^2\log(a+bx) - 2e^2(a+bx)\log(d+ex)}{2(bd-ae)^3(a+bx)\sqrt{(a+bx)^2}}$$

input

```
Integrate[1/((d + e*x)*(a^2 + 2*a*b*x + b^2*x^2)^(3/2)),x]
```


output

```
(-((b*d - a*e)*(-3*a*e + b*(d - 2*e*x))) + 2*e^2*(a + b*x)^2*Log[a + b*x]
- 2*e^2*(a + b*x)^2*Log[d + e*x])/(2*(b*d - a*e)^3*(a + b*x)*Sqrt[(a + b*x)
^2])
```

Rubi [A] (verified)

Time = 0.47 (sec) , antiderivative size = 108, normalized size of antiderivative = 0.65, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {1102, 27, 54, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a^2 + 2abx + b^2x^2)^{3/2} (d + ex)} dx$$

$$\downarrow 1102$$

$$\frac{b^3(a + bx) \int \frac{1}{b^3(a+bx)^3(d+ex)} dx}{\sqrt{a^2 + 2abx + b^2x^2}}$$

$$\downarrow 27$$

$$\frac{(a + bx) \int \frac{1}{(a+bx)^3(d+ex)} dx}{\sqrt{a^2 + 2abx + b^2x^2}}$$

$$\downarrow 54$$

$$\frac{(a + bx) \int \left(-\frac{e^3}{(bd-ae)^3(d+ex)} + \frac{be^2}{(bd-ae)^3(a+bx)} - \frac{be}{(bd-ae)^2(a+bx)^2} + \frac{b}{(bd-ae)(a+bx)^3} \right) dx}{\sqrt{a^2 + 2abx + b^2x^2}}$$

$$\downarrow 2009$$

$$\frac{(a + bx) \left(\frac{e^2 \log(a+bx)}{(bd-ae)^3} - \frac{e^2 \log(d+ex)}{(bd-ae)^3} + \frac{e}{(a+bx)(bd-ae)^2} - \frac{1}{2(a+bx)^2(bd-ae)} \right)}{\sqrt{a^2 + 2abx + b^2x^2}}$$

input

```
Int[1/((d + e*x)*(a^2 + 2*a*b*x + b^2*x^2)^(3/2)),x]
```

output
$$\frac{((a + b*x)*(-1/2*1/((b*d - a*e)*(a + b*x)^2) + e/((b*d - a*e)^2*(a + b*x)) + (e^2*Log[a + b*x])/((b*d - a*e)^3 - (e^2*Log[d + e*x])/((b*d - a*e)^3)))/Sqrt[a^2 + 2*a*b*x + b^2*x^2]}$$

Defintions of rubi rules used

rule 27
$$\text{Int}[(a_)*(Fx_), x_Symbol] \rightarrow \text{Simp}[a \text{ Int}[Fx, x], x] /; \text{FreeQ}[a, x] \ \&\& \ !\text{MatchQ}[Fx, (b_)*(Gx_)] /; \text{FreeQ}[b, x]$$

rule 54
$$\text{Int}[((a_)+(b_)*(x_))^(m_)*((c_)+(d_)*(x_))^(n_), x_Symbol] \rightarrow \text{Int}[\text{ExpandIntegrand}[(a + b*x)^m*(c + d*x)^n, x], x] /; \text{FreeQ}\{a, b, c, d\}, x \ \&\& \ \text{ILtQ}[m, 0] \ \&\& \ \text{IntegerQ}[n] \ \&\& \ !(\text{IGtQ}[n, 0] \ \&\& \ \text{LtQ}[m + n + 2, 0])$$

rule 1102
$$\text{Int}(((d_)+(e_)*(x_))^(m_)*((a_)+(b_)*(x_)+(c_)*(x_)^2)^(p_), x_Symbol] \rightarrow \text{Simp}[(a + b*x + c*x^2)^{\text{FracPart}[p]} / (c^{\text{IntPart}[p]}*(b/2 + c*x)^{(2*\text{FracPart}[p])}) \text{ Int}[(d + e*x)^m*(b/2 + c*x)^{(2*p)}, x], x] /; \text{FreeQ}\{a, b, c, d, e, m, p\}, x \ \&\& \ \text{EqQ}[b^2 - 4*a*c, 0]$$

rule 2009
$$\text{Int}[u_, x_Symbol] \rightarrow \text{Simp}[\text{IntSum}[u, x], x] /; \text{SumQ}[u]$$

Maple [A] (verified)

Time = 1.54 (sec) , antiderivative size = 156, normalized size of antiderivative = 0.95

| method | result |
|---------|---|
| default | $-\frac{(2 \ln(bx+a)b^2e^2x^2 - 2 \ln(ex+d)b^2e^2x^2 + 4 \ln(bx+a)ab e^2x - 4 \ln(ex+d)ab e^2x + 2 \ln(bx+a)a^2e^2 - 2 \ln(ex+d)a^2e^2 - 2xab e^2 + 2b^2de)}{2(ae-bd)^3((bx+a)^2)^{\frac{3}{2}}}$ |
| risch | $\frac{\sqrt{(bx+a)^2} \left(\frac{bex}{e^2a^2 - 2abde + b^2d^2} + \frac{3ae-bd}{2e^2a^2 - 4abde + 2b^2d^2} \right)}{(bx+a)^3} - \frac{\sqrt{(bx+a)^2} e^2 \ln(bx+a)}{(bx+a)(e^3a^3 - 3a^2bd e^2 + 3a b^2d^2 e - b^3d^3)} + \frac{\sqrt{(bx+a)^2} e^2 \ln(-)}{(bx+a)(e^3a^3 - 3a^2bd e^2 + 3a b^2d^2 e - b^3d^3)}$ |

input
$$\text{int}(1/(e*x+d)/(b^2*x^2+2*a*b*x+a^2)^(3/2), x, \text{method}=_RETURNVERBOSE)$$

output

```
-1/2*(2*ln(b*x+a)*b^2*e^2*x^2-2*ln(e*x+d)*b^2*e^2*x^2+4*ln(b*x+a)*a*b*e^2*x-4*ln(e*x+d)*a*b*e^2*x+2*ln(b*x+a)*a^2*e^2-2*ln(e*x+d)*a^2*e^2-2*x*a*b*e^2+2*b^2*d*e*x-3*e^2*a^2+4*a*b*d*e-b^2*d^2)*(b*x+a)/(a*e-b*d)^3/((b*x+a)^2)^(3/2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 242 vs. 2(119) = 238.

Time = 0.08 (sec) , antiderivative size = 242, normalized size of antiderivative = 1.47

$$\int \frac{1}{(d+ex)(a^2+2abx+b^2x^2)^{3/2}} dx = \frac{b^2d^2 - 4abde + 3a^2e^2 - 2(b^2de - abe^2)x - 2(b^2e^2x^2 + 2abe^2x + a^2e^2) \log(bx+a) + 2(b^2e^2x^2 + 2abe^2x + a^2e^2) \log(e*x+d)}{2(a^2b^3d^3 - 3a^3b^2d^2e + 3a^4bde^2 - a^5e^3 + (b^5d^3 - 3ab^4d^2e + 3a^2b^3de^2 - a^3b^2e^3)x^2 + 2(ab^4d^3 - 3a^2b^3d^2e + 3a^3b^2d^2e^2 - a^4b^2e^3)x + 2a^5e^3)}$$

input

```
integrate(1/(e*x+d)/(b^2*x^2+2*a*b*x+a^2)^(3/2),x, algorithm="fricas")
```

output

```
-1/2*(b^2*d^2 - 4*a*b*d*e + 3*a^2*e^2 - 2*(b^2*d*e - a*b*e^2)*x - 2*(b^2*e^2*x^2 + 2*a*b*e^2*x + a^2*e^2)*log(b*x + a) + 2*(b^2*e^2*x^2 + 2*a*b*e^2*x + a^2*e^2)*log(e*x + d))/(a^2*b^3*d^3 - 3*a^3*b^2*d^2*e + 3*a^4*b*d*e^2 - a^5*e^3 + (b^5*d^3 - 3*a*b^4*d^2*e + 3*a^2*b^3*d*e^2 - a^3*b^2*e^3)*x^2 + 2*(a*b^4*d^3 - 3*a^2*b^3*d^2*e + 3*a^3*b^2*d^2*e^2 - a^4*b^2*e^3)*x)
```

Sympy [F]

$$\int \frac{1}{(d+ex)(a^2+2abx+b^2x^2)^{3/2}} dx = \int \frac{1}{(d+ex)((a+bx)^2)^{3/2}} dx$$

input

```
integrate(1/(e*x+d)/(b**2*x**2+2*a*b*x+a**2)**(3/2),x)
```

output

```
Integral(1/((d + e*x)*((a + b*x)**2)**(3/2)), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{(d+ex)(a^2+2abx+b^2x^2)^{3/2}} dx = \text{Exception raised: ValueError}$$

input

```
integrate(1/(e*x+d)/(b^2*x^2+2*a*b*x+a^2)^(3/2),x, algorithm="maxima")
```

output

```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
help (example of legal syntax is 'assume(a*e-b*d>0)', see `assume?` for m
ore detail
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 221, normalized size of antiderivative = 1.34

$$\int \frac{1}{(d+ex)(a^2+2abx+b^2x^2)^{3/2}} dx = \frac{be^2 \log(|bx+a|)}{b^4 d^3 \operatorname{sgn}(bx+a) - 3ab^3 d^2 e \operatorname{sgn}(bx+a) + 3a^2 b^2 d e^2 \operatorname{sgn}(bx+a) - e^3 \log(|ex+d|)} - \frac{b^3 d^3 e \operatorname{sgn}(bx+a) - 3ab^2 d^2 e^2 \operatorname{sgn}(bx+a) + 3a^2 b d e^3 \operatorname{sgn}(bx+a) - a^3 e^4 \operatorname{sgn}(bx+a)}{b^2 d^2 - 4abde + 3a^2 e^2 - 2(b^2 de - abe^2)x} - \frac{2(bd - ae)^3 (bx+a)^2 \operatorname{sgn}(bx+a)}{2(bd - ae)^3 (bx+a)^2 \operatorname{sgn}(bx+a)}$$

input

```
integrate(1/(e*x+d)/(b^2*x^2+2*a*b*x+a^2)^(3/2),x, algorithm="giac")
```

output

```
b*e^2*log(abs(b*x + a))/(b^4*d^3*sgn(b*x + a) - 3*a*b^3*d^2*e*sgn(b*x + a)
+ 3*a^2*b^2*d*e^2*sgn(b*x + a) - a^3*b*e^3*sgn(b*x + a)) - e^3*log(abs(e*
x + d))/(b^3*d^3*e*sgn(b*x + a) - 3*a*b^2*d^2*e^2*sgn(b*x + a) + 3*a^2*b*d
*e^3*sgn(b*x + a) - a^3*e^4*sgn(b*x + a)) - 1/2*(b^2*d^2 - 4*a*b*d*e + 3*a
^2*e^2 - 2*(b^2*d*e - a*b*e^2)*x)/((b*d - a*e)^3*(b*x + a)^2*sgn(b*x + a))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(d+ex)(a^2+2abx+b^2x^2)^{3/2}} dx = \int \frac{1}{(d+ex)(a^2+2abx+b^2x^2)^{3/2}} dx$$

input `int(1/((d + e*x)*(a^2 + b^2*x^2 + 2*a*b*x)^(3/2)),x)`output `int(1/((d + e*x)*(a^2 + b^2*x^2 + 2*a*b*x)^(3/2)), x)`**Reduce [B] (verification not implemented)**

Time = 0.23 (sec) , antiderivative size = 288, normalized size of antiderivative = 1.75

$$\int \frac{1}{(d+ex)(a^2+2abx+b^2x^2)^{3/2}} dx = \frac{-2 \log(bx+a) a^3 e^2 - 4 \log(bx+a) a^2 b e^2 x - 2 \log(bx+a) a b^2 e^2 x^2}{2a(a^3 b^2 e^3 x^2 - 3a^2 b^3 d e^2 x^2 + 3a b^4 d^2 e x^2 - b^5 d^3 x^2)}$$

input `int(1/(e*x+d)/(b^2*x^2+2*a*b*x+a^2)^(3/2),x)`output `(- 2*log(a + b*x)*a**3*e**2 - 4*log(a + b*x)*a**2*b*e**2*x - 2*log(a + b*x)*a*b**2*e**2*x**2 + 2*log(d + e*x)*a**3*e**2 + 4*log(d + e*x)*a**2*b*e**2*x + 2*log(d + e*x)*a*b**2*e**2*x**2 + 2*a**3*e**2 - 3*a**2*b*d*e**2*d**2 - a*b**2*e**2*x**2 + b**3*d*e*x**2)/(2*a*(a**5*e**3 - 3*a**4*b*d*e**2 + 2*a**4*b*e**3*x + 3*a**3*b**2*d**2*e - 6*a**3*b**2*d*e**2*x + a**3*b**2*e**3*x**2 - a**2*b**3*d**3 + 6*a**2*b**3*d**2*e*x - 3*a**2*b**3*d*e**2*x**2 - 2*a*b**4*d**3*x + 3*a*b**4*d**2*e*x**2 - b**5*d**3*x**2))`

3.324 $\int \frac{1}{(d+ex)^2(a^2+2abx+b^2x^2)^{3/2}} dx$

| | |
|---|------|
| Optimal result | 2425 |
| Mathematica [A] (verified) | 2426 |
| Rubi [A] (verified) | 2426 |
| Maple [B] (verified) | 2428 |
| Fricas [B] (verification not implemented) | 2428 |
| Sympy [F] | 2429 |
| Maxima [F(-2)] | 2429 |
| Giac [B] (verification not implemented) | 2430 |
| Mupad [F(-1)] | 2430 |
| Reduce [B] (verification not implemented) | 2431 |

Optimal result

Integrand size = 28, antiderivative size = 217

$$\int \frac{1}{(d+ex)^2(a^2+2abx+b^2x^2)^{3/2}} dx = \frac{2be}{(bd-ae)^3\sqrt{a^2+2abx+b^2x^2}} - \frac{b}{2(bd-ae)^2(a+bx)\sqrt{a^2+2abx+b^2x^2}} + \frac{e^2(a+bx)}{(bd-ae)^3(d+ex)\sqrt{a^2+2abx+b^2x^2}} + \frac{3be^2(a+bx)\log(a+bx)}{(bd-ae)^4\sqrt{a^2+2abx+b^2x^2}} - \frac{3be^2(a+bx)\log(d+ex)}{(bd-ae)^4\sqrt{a^2+2abx+b^2x^2}}$$

output

```
2*b*e/(-a*e+b*d)^3/((b*x+a)^2)^(1/2)-1/2*b/(-a*e+b*d)^2/(b*x+a)/((b*x+a)^2)^(1/2)+e^2*(b*x+a)/(-a*e+b*d)^3/(e*x+d)/((b*x+a)^2)^(1/2)+3*b*e^2*(b*x+a)*ln(b*x+a)/(-a*e+b*d)^4/((b*x+a)^2)^(1/2)-3*b*e^2*(b*x+a)*ln(e*x+d)/(-a*e+b*d)^4/((b*x+a)^2)^(1/2)
```

Mathematica [A] (verified)

Time = 1.10 (sec) , antiderivative size = 141, normalized size of antiderivative = 0.65

$$\int \frac{1}{(d+ex)^2 (a^2+2abx+b^2x^2)^{3/2}} dx = \frac{-((bd-ae)(-2a^2e^2 - abe(5d+9ex) + b^2(d^2 - 3dex - 6e^2x^2)))}{2(bd-ae)^4(a+e^2x)}$$

input

```
Integrate[1/((d + e*x)^2*(a^2 + 2*a*b*x + b^2*x^2)^(3/2)),x]
```

output

```
((b*d - a*e)*(-2*a^2*e^2 - a*b*e*(5*d + 9*e*x) + b^2*(d^2 - 3*d*e*x - 6*
e^2*x^2)) + 6*b*e^2*(a + b*x)^2*(d + e*x)*Log[a + b*x] - 6*b*e^2*(a + b*x
)^2*(d + e*x)*Log[d + e*x])/(2*(b*d - a*e)^4*(a + b*x)*Sqrt[(a + b*x)^2]*(
d + e*x))
```

Rubi [A] (verified)

Time = 0.54 (sec) , antiderivative size = 135, normalized size of antiderivative = 0.62, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {1102, 27, 54, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\begin{aligned} & \int \frac{1}{(a^2 + 2abx + b^2x^2)^{3/2} (d + ex)^2} dx \\ & \quad \downarrow \text{1102} \\ & \frac{b^3(a + bx) \int \frac{1}{b^3(a+bx)^3(d+ex)^2} dx}{\sqrt{a^2 + 2abx + b^2x^2}} \\ & \quad \downarrow \text{27} \\ & \frac{(a + bx) \int \frac{1}{(a+bx)^3(d+ex)^2} dx}{\sqrt{a^2 + 2abx + b^2x^2}} \\ & \quad \downarrow \text{54} \end{aligned}$$

$$\frac{(a+bx) \int \left(-\frac{3be^3}{(bd-ae)^4(d+ex)} - \frac{e^3}{(bd-ae)^3(d+ex)^2} + \frac{3b^2e^2}{(bd-ae)^4(a+bx)} - \frac{2b^2e}{(bd-ae)^3(a+bx)^2} + \frac{b^2}{(bd-ae)^2(a+bx)^3} \right) dx}{\sqrt{a^2 + 2abx + b^2x^2}}$$

↓ 2009

$$\frac{(a+bx) \left(\frac{e^2}{(d+ex)(bd-ae)^3} + \frac{3be^2 \log(a+bx)}{(bd-ae)^4} - \frac{3be^2 \log(d+ex)}{(bd-ae)^4} + \frac{2be}{(a+bx)(bd-ae)^3} - \frac{b}{2(a+bx)^2(bd-ae)^2} \right)}{\sqrt{a^2 + 2abx + b^2x^2}}$$

input

```
Int[1/((d + e*x)^2*(a^2 + 2*a*b*x + b^2*x^2)^(3/2)),x]
```

output

```
((a + b*x)*(-1/2*b/((b*d - a*e)^2*(a + b*x)^2) + (2*b*e)/((b*d - a*e)^3*(a + b*x)) + e^2/((b*d - a*e)^3*(d + e*x)) + (3*b*e^2*Log[a + b*x])/(b*d - a*e)^4 - (3*b*e^2*Log[d + e*x])/(b*d - a*e^4))/Sqrt[a^2 + 2*a*b*x + b^2*x^2]
```

Defintions of rubi rules used

rule 27

```
Int[(a_)*(F_x_), x_Symbol] := Simp[a Int[F_x, x], x] /; FreeQ[a, x] && !MatchQ[F_x, (b_)*(G_x_)] /; FreeQ[b, x]
```

rule 54

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] := Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] && ILtQ[m, 0] && IntegerQ[n] && !(IGtQ[n, 0] && LtQ[m + n + 2, 0])
```

rule 1102

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] := Simp[(a + b*x + c*x^2)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x)^(2*FracPart[p])) Int[(d + e*x)^m*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && EqQ[b^2 - 4*a*c, 0]
```

rule 2009

```
Int[u_, x_Symbol] := Simp[IntSum[u, x], x] /; SumQ[u]
```


output

```
-1/2*(b^3*d^3 - 6*a*b^2*d^2*e + 3*a^2*b*d*e^2 + 2*a^3*e^3 - 6*(b^3*d*e^2 -
a*b^2*e^3)*x^2 - 3*(b^3*d^2*e + 2*a*b^2*d*e^2 - 3*a^2*b*e^3)*x - 6*(b^3*e
^3*x^3 + a^2*b*d*e^2 + (b^3*d*e^2 + 2*a*b^2*e^3)*x^2 + (2*a*b^2*d*e^2 + a
^2*b*e^3)*x)*log(b*x + a) + 6*(b^3*e^3*x^3 + a^2*b*d*e^2 + (b^3*d*e^2 + 2*a
*b^2*e^3)*x^2 + (2*a*b^2*d*e^2 + a^2*b*e^3)*x)*log(e*x + d))/(a^2*b^4*d^5
- 4*a^3*b^3*d^4*e + 6*a^4*b^2*d^3*e^2 - 4*a^5*b*d^2*e^3 + a^6*d*e^4 + (b^6
*d^4*e - 4*a*b^5*d^3*e^2 + 6*a^2*b^4*d^2*e^3 - 4*a^3*b^3*d*e^4 + a^4*b^2*e
^5)*x^3 + (b^6*d^5 - 2*a*b^5*d^4*e - 2*a^2*b^4*d^3*e^2 + 8*a^3*b^3*d^2*e^3
- 7*a^4*b^2*d*e^4 + 2*a^5*b*e^5)*x^2 + (2*a*b^5*d^5 - 7*a^2*b^4*d^4*e + 8
*a^3*b^3*d^3*e^2 - 2*a^4*b^2*d^2*e^3 - 2*a^5*b*d*e^4 + a^6*e^5)*x)
```

Sympy [F]

$$\int \frac{1}{(d+ex)^2 (a^2+2abx+b^2x^2)^{3/2}} dx = \int \frac{1}{(d+ex)^2 ((a+bx)^2)^{\frac{3}{2}}} dx$$

input

```
integrate(1/(e*x+d)**2/(b**2*x**2+2*a*b*x+a**2)**(3/2),x)
```

output

```
Integral(1/((d + e*x)**2*((a + b*x)**2)**(3/2)), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{(d+ex)^2 (a^2+2abx+b^2x^2)^{3/2}} dx = \text{Exception raised: ValueError}$$

input

```
integrate(1/(e*x+d)^2/(b^2*x^2+2*a*b*x+a^2)^(3/2),x, algorithm="maxima")
```

output

```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
help (example of legal syntax is 'assume(a*e-b*d>0)', see `assume?` for m
ore detail
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 321 vs. $2(160) = 320$.

Time = 0.14 (sec) , antiderivative size = 321, normalized size of antiderivative = 1.48

$$\int \frac{1}{(d+ex)^2 (a^2+2abx+b^2x^2)^{3/2}} dx = \frac{3b^2e^2 \log(|bx+a|)}{b^5d^4 \operatorname{sgn}(bx+a) - 4ab^4d^3e \operatorname{sgn}(bx+a) + 6a^2b^3d^2e^2 \operatorname{sgn}(bx+a)} - \frac{3be^3 \log(|ex+d|)}{b^4d^4e \operatorname{sgn}(bx+a) - 4ab^3d^3e^2 \operatorname{sgn}(bx+a) + 6a^2b^2d^2e^3 \operatorname{sgn}(bx+a) - 4a^3bde^4 \operatorname{sgn}(bx+a) + a^4e^5 \operatorname{sgn}(bx+a)} - \frac{b^3d^3 - 6ab^2d^2e + 3a^2bde^2 + 2a^3e^3 - 6(b^3de^2 - ab^2e^3)x^2 - 3(b^3d^2e + 2ab^2de^2 - 3a^2be^3)x}{2(bd-ae)^4(bx+a)^2(ex+d) \operatorname{sgn}(bx+a)}$$

input `integrate(1/(e*x+d)^2/(b^2*x^2+2*a*b*x+a^2)^(3/2),x, algorithm="giac")`

output `3*b^2*e^2*log(abs(b*x + a))/(b^5*d^4*sgn(b*x + a) - 4*a*b^4*d^3*e*sgn(b*x + a) + 6*a^2*b^3*d^2*e^2*sgn(b*x + a) - 4*a^3*b^2*d*e^3*sgn(b*x + a) + a^4*b*e^4*sgn(b*x + a)) - 3*b*e^3*log(abs(e*x + d))/(b^4*d^4*e*sgn(b*x + a) - 4*a*b^3*d^3*e^2*sgn(b*x + a) + 6*a^2*b^2*d^2*e^3*sgn(b*x + a) - 4*a^3*b*d*e^4*sgn(b*x + a) + a^4*e^5*sgn(b*x + a)) - 1/2*(b^3*d^3 - 6*a*b^2*d^2*e + 3*a^2*b*d*e^2 + 2*a^3*e^3 - 6*(b^3*d*e^2 - a*b^2*e^3)*x^2 - 3*(b^3*d^2*e + 2*a*b^2*d*e^2 - 3*a^2*b*e^3)*x)/((b*d - a*e)^4*(b*x + a)^2*(e*x + d)*sgn(b*x + a))`

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(d+ex)^2 (a^2+2abx+b^2x^2)^{3/2}} dx = \int \frac{1}{(d+ex)^2 (a^2+2abx+b^2x^2)^{3/2}} dx$$

input `int(1/((d + e*x)^2*(a^2 + b^2*x^2 + 2*a*b*x)^(3/2)),x)`

output `int(1/((d + e*x)^2*(a^2 + b^2*x^2 + 2*a*b*x)^(3/2)), x)`

Reduce [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 821, normalized size of antiderivative = 3.78

$$\int \frac{1}{(d+ex)^2 (a^2+2abx+b^2x^2)^{3/2}} dx = \text{Too large to display}$$

input

```
int(1/(e*x+d)^2/(b^2*x^2+2*a*b*x+a^2)^(3/2),x)
```

output

```
(12*log(a + b*x)*a**3*b*d*e**3 + 12*log(a + b*x)*a**3*b*e**4*x + 6*log(a +
b*x)*a**2*b**2*d**2*e**2 + 30*log(a + b*x)*a**2*b**2*d*e**3*x + 24*log(a
+ b*x)*a**2*b**2*e**4*x**2 + 12*log(a + b*x)*a*b**3*d**2*e**2*x + 24*log(a
+ b*x)*a*b**3*d*e**3*x**2 + 12*log(a + b*x)*a*b**3*e**4*x**3 + 6*log(a +
b*x)*b**4*d**2*e**2*x**2 + 6*log(a + b*x)*b**4*d*e**3*x**3 - 12*log(d + e
x)*a**3*b*d*e**3 - 12*log(d + e*x)*a**3*b*e**4*x - 6*log(d + e*x)*a**2*b**
2*d**2*e**2 - 30*log(d + e*x)*a**2*b**2*d*e**3*x - 24*log(d + e*x)*a**2*b*
**2*e**4*x**2 - 12*log(d + e*x)*a*b**3*d**2*e**2*x - 24*log(d + e*x)*a*b**3
*d*e**3*x**2 - 12*log(d + e*x)*a*b**3*e**4*x**3 - 6*log(d + e*x)*b**4*d**2
*e**2*x**2 - 6*log(d + e*x)*b**4*d*e**3*x**3 - 4*a**4*e**4 - 2*a**3*b*d*e
**3 - 12*a**3*b*e**4*x + 3*a**2*b**2*d**2*e**2 + 9*a**2*b**2*d*e**3*x + 4*a
*b**3*d**3*e + 6*a*b**3*e**4*x**3 - b**4*d**4 + 3*b**4*d**3*e*x - 6*b**4*d
**3*x**3)/(2*(2*a**7*d*e**5 + 2*a**7*e**6*x - 7*a**6*b*d**2*e**4 - 3*a**
6*b*d**5*x + 4*a**6*b*e**6*x**2 + 8*a**5*b**2*d**3*e**3 - 6*a**5*b**2*d
**2*e**4*x - 12*a**5*b**2*d*e**5*x**2 + 2*a**5*b**2*e**6*x**3 - 2*a**4*b**3
*d**4*e**2 + 14*a**4*b**3*d**3*e**3*x + 9*a**4*b**3*d**2*e**4*x**2 - 7*a**
4*b**3*d*e**5*x**3 - 2*a**3*b**4*d**5*e - 6*a**3*b**4*d**4*e**2*x + 4*a**3
*b**4*d**3*e**3*x**2 + 8*a**3*b**4*d**2*e**4*x**3 + a**2*b**5*d**6 - 3*a**
2*b**5*d**5*e*x - 6*a**2*b**5*d**4*e**2*x**2 - 2*a**2*b**5*d**3*e**3*x**3
+ 2*a*b**6*d**6*x - 2*a*b**6*d**4*e**2*x**3 + b**7*d**6*x**2 + b**7*d**...
```

3.325 $\int \frac{1}{(d+ex)^3(a^2+2abx+b^2x^2)^{3/2}} dx$

| | |
|---|------|
| Optimal result | 2432 |
| Mathematica [A] (verified) | 2433 |
| Rubi [A] (verified) | 2433 |
| Maple [B] (verified) | 2435 |
| Fricas [B] (verification not implemented) | 2435 |
| Sympy [F] | 2436 |
| Maxima [F(-2)] | 2436 |
| Giac [B] (verification not implemented) | 2437 |
| Mupad [F(-1)] | 2438 |
| Reduce [B] (verification not implemented) | 2438 |

Optimal result

Integrand size = 28, antiderivative size = 276

$$\int \frac{1}{(d+ex)^3(a^2+2abx+b^2x^2)^{3/2}} dx = \frac{3b^2e}{(bd-ae)^4\sqrt{a^2+2abx+b^2x^2}} - \frac{2(bd-ae)^3(a+bx)\sqrt{a^2+2abx+b^2x^2}}{b^2e^2(a+bx)} + \frac{2(bd-ae)^3(d+ex)^2\sqrt{a^2+2abx+b^2x^2}}{3be^2(a+bx)} + \frac{(bd-ae)^4(d+ex)\sqrt{a^2+2abx+b^2x^2}}{6b^2e^2(a+bx)\log(a+bx)} - \frac{6b^2e^2(a+bx)\log(d+ex)}{(bd-ae)^5\sqrt{a^2+2abx+b^2x^2}}$$

output

```
3*b^2*e/(-a*e+b*d)^4/((b*x+a)^2)^(1/2)-1/2*b^2/(-a*e+b*d)^3/(b*x+a)/((b*x+a)^2)^(1/2)+1/2*e^2*(b*x+a)/(-a*e+b*d)^3/(e*x+d)^2/((b*x+a)^2)^(1/2)+3*b*e^2*(b*x+a)/(-a*e+b*d)^4/(e*x+d)/((b*x+a)^2)^(1/2)+6*b^2*e^2*(b*x+a)*ln(b*x+a)/(-a*e+b*d)^5/((b*x+a)^2)^(1/2)-6*b^2*e^2*(b*x+a)*ln(e*x+d)/(-a*e+b*d)^5/((b*x+a)^2)^(1/2)
```

Mathematica [A] (verified)

Time = 1.07 (sec) , antiderivative size = 163, normalized size of antiderivative = 0.59

$$\int \frac{1}{(d+ex)^3 (a^2+2abx+b^2x^2)^{3/2}} dx = \frac{(a+bx) \left(-b^2(bd-ae)^2 + 6b^2e(bd-ae)(a+bx) + \frac{e^2(bd-ae)^2(a+bx)}{(d+ex)^2} \right)}{2(bd$$

input

```
Integrate[1/((d + e*x)^3*(a^2 + 2*a*b*x + b^2*x^2)^(3/2)),x]
```

output

```
((a + b*x)*(-(b^2*(b*d - a*e)^2) + 6*b^2*e*(b*d - a*e)*(a + b*x) + (e^2*(b*d - a*e)^2*(a + b*x)^2)/(d + e*x)^2 + (6*b*e^2*(b*d - a*e)*(a + b*x)^2)/(d + e*x) + 12*b^2*e^2*(a + b*x)^2*Log[a + b*x] - 12*b^2*e^2*(a + b*x)^2*Log[d + e*x]))/(2*(b*d - a*e)^5*((a + b*x)^2)^(3/2))
```

Rubi [A] (verified)

Time = 0.62 (sec) , antiderivative size = 169, normalized size of antiderivative = 0.61, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {1102, 27, 54, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a^2+2abx+b^2x^2)^{3/2} (d+ex)^3} dx$$

$$\downarrow 1102$$

$$\frac{b^3(a+bx) \int \frac{1}{b^3(a+bx)^3(d+ex)^3} dx}{\sqrt{a^2+2abx+b^2x^2}}$$

$$\downarrow 27$$

$$\frac{(a+bx) \int \frac{1}{(a+bx)^3(d+ex)^3} dx}{\sqrt{a^2+2abx+b^2x^2}}$$

$$\downarrow 54$$

$$(a + bx) \int \frac{\left(\frac{6e^2b^3}{(bd-ae)^5(a+bx)} - \frac{3eb^3}{(bd-ae)^4(a+bx)^2} + \frac{b^3}{(bd-ae)^3(a+bx)^3} - \frac{6e^3b^2}{(bd-ae)^5(d+ex)} - \frac{3e^3b}{(bd-ae)^4(d+ex)^2} - \frac{e^3}{(bd-ae)^3(d+ex)^3} \right)}{\sqrt{a^2 + 2abx + b^2x^2}}$$

↓ 2009

$$(a + bx) \frac{\left(\frac{6b^2e^2 \log(a+bx)}{(bd-ae)^5} - \frac{6b^2e^2 \log(d+ex)}{(bd-ae)^5} + \frac{3b^2e}{(a+bx)(bd-ae)^4} - \frac{b^2}{2(a+bx)^2(bd-ae)^3} + \frac{3be^2}{(d+ex)(bd-ae)^4} + \frac{e^2}{2(d+ex)^2(bd-ae)^3} \right)}{\sqrt{a^2 + 2abx + b^2x^2}}$$

input `Int[1/((d + e*x)^3*(a^2 + 2*a*b*x + b^2*x^2)^(3/2)),x]`

output `((a + b*x)*(-1/2*b^2/((b*d - a*e)^3*(a + b*x)^2) + (3*b^2*e)/((b*d - a*e)^4*(a + b*x)) + e^2/(2*(b*d - a*e)^3*(d + e*x)^2) + (3*b*e^2)/((b*d - a*e)^4*(d + e*x)) + (6*b^2*e^2*Log[a + b*x])/(b*d - a*e)^5 - (6*b^2*e^2*Log[d + e*x])/(b*d - a*e)^5)/Sqrt[a^2 + 2*a*b*x + b^2*x^2]`

Defintions of rubi rules used

rule 27 `Int[(a_)*(F_x_), x_Symbol] := Simp[a Int[F_x, x], x] /; FreeQ[a, x] && !MatchQ[F_x, (b_)*(G_x_) /; FreeQ[b, x]]`

rule 54 `Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] := Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] && ILtQ[m, 0] && IntegerQ[n] && !(IGtQ[n, 0] && LtQ[m + n + 2, 0])`

rule 1102 `Int[((d_.) + (e_.)*(x_))^(m_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] := Simp[(a + b*x + c*x^2)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x)^(2*FracPart[p])) Int[(d + e*x)^m*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && EqQ[b^2 - 4*a*c, 0]`

rule 2009 `Int[u_, x_Symbol] := Simp[IntSum[u, x], x] /; SumQ[u]`

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 507 vs. 2(206) = 412.

Time = 1.79 (sec) , antiderivative size = 508, normalized size of antiderivative = 1.84

| method | result |
|---------|--|
| default | $-\frac{(a^4e^4-12\ln(ex+d)a^2b^2d^2e^2+12\ln(bx+a)b^4d^2e^2x^2-24\ln(ex+d)ab^3e^4x^3-24\ln(ex+d)b^4de^3x^3-12\ln(ex+d)a^2b^2e^4x^2-12\ln(ex+d)a^3bde^4x-12\ln(ex+d)a^4e^4x)}{(bx+a)^3(ex+d)^2}$ |
| risch | $\sqrt{(bx+a)^2} \left(\frac{6b^3e^3x^3}{a^4e^4-4a^3bde^3+6a^2b^2d^2e^2-4ab^3d^3e+b^4d^4} + \frac{9b^2e^2(ax+bd)x^2}{a^4e^4-4a^3bde^3+6a^2b^2d^2e^2-4ab^3d^3e+b^4d^4} + \frac{2(e^2a^2+7abde+b^2d^2)be}{a^4e^4-4a^3bde^3+6a^2b^2d^2e^2-4ab^3d^3e+b^4d^4} \right)$ |

```
input int(1/(e*x+d)^3/(b^2*x^2+2*a*b*x+a^2)^(3/2),x,method=_RETURNVERBOSE)
```

```
output -1/2*(a^4*e^4-12*ln(e*x+d)*a^2*b^2*d^2*e^2+12*ln(b*x+a)*b^4*d^2*e^2*x^2-24*ln(e*x+d)*a*b^3*e^4*x^3-24*ln(e*x+d)*b^4*d*e^3*x^3-12*ln(e*x+d)*a^2*b^2*e^4*x^2-12*ln(e*x+d)*b^4*d^2*e^2*x^2-8*a^3*b*d*e^3+8*a*b^3*d^3*e+24*ln(b*x+a)*x^3*a*b^3*e^4+24*ln(b*x+a)*x^3*b^4*d*e^3+12*ln(b*x+a)*x^2*a^2*b^2*e^4-b^4*d^4+24*ln(b*x+a)*x*a^2*b^2*d*e^3+24*ln(b*x+a)*x*a*b^3*d^2*e^2+48*ln(b*x+a)*x^2*a*b^3*d*e^3+12*ln(b*x+a)*b^4*e^4*x^4-12*ln(e*x+d)*b^4*e^4*x^4-48*ln(e*x+d)*a*b^3*d*e^3*x^2-24*ln(e*x+d)*a^2*b^2*d*e^3*x-24*ln(e*x+d)*a*b^3*d^2*e^2*x+18*x^2*b^4*d^2*e^2-4*x*a^3*b*e^4+4*x*b^4*d^3*e+12*ln(b*x+a)*a^2*b^2*d^2*e^2-24*x*a^2*b^2*d*e^3+24*x*a*b^3*d^2*e^2-12*x^3*a*b^3*e^4+12*x^3*b^4*d*e^3-18*x^2*a^2*b^2*e^4)*(b*x+a)/(e*x+d)^2/(a*e-b*d)^5/((b*x+a)^2)^(3/2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 760 vs. 2(206) = 412.

Time = 0.10 (sec) , antiderivative size = 760, normalized size of antiderivative = 2.75

$$\int \frac{1}{(d+ex)^3(a^2+2abx+b^2x^2)^{3/2}} dx = \frac{b^4d^4-8ab^3d^3e+8a^3bde^3-a^4e^4-12(b^4de^3-ab^3e^4)x^3-18(b^4d^2e^2-a^2b^2e^4)x^2-4(b^4d^3e+ab^3d^2e^2-2(a^2b^5d^7-5a^3b^4d^6e+10a^4b^3d^5e^2-10a^5b^2d^4e^3+5a^6bd^3e^4-a^7d^2e^5+(b^7d^5e^2-5ab^6d^4e^3+10a^2b^5d^3e^4-5a^3b^4d^2e^2+5a^4b^3de^3-5a^5b^2d^2e^2+5a^6bd^2e^2-5a^7d^2e^2+5a^8d^2e^2))}{(d+ex)^3(a^2+2abx+b^2x^2)^{3/2}}$$

```
input integrate(1/(e*x+d)^3/(b^2*x^2+2*a*b*x+a^2)^(3/2),x, algorithm="fricas")
```


output

```
-1/2*(b^4*d^4 - 8*a*b^3*d^3*e + 8*a^3*b*d*e^3 - a^4*e^4 - 12*(b^4*d*e^3 -
a*b^3*e^4)*x^3 - 18*(b^4*d^2*e^2 - a^2*b^2*e^4)*x^2 - 4*(b^4*d^3*e + 6*a*b
^3*d^2*e^2 - 6*a^2*b^2*d*e^3 - a^3*b*e^4)*x - 12*(b^4*e^4*x^4 + a^2*b^2*d
^2*e^2 + 2*(b^4*d*e^3 + a*b^3*e^4)*x^3 + (b^4*d^2*e^2 + 4*a*b^3*d*e^3 + a^2
*b^2*e^4)*x^2 + 2*(a*b^3*d^2*e^2 + a^2*b^2*d*e^3)*x)*log(b*x + a) + 12*(b
^4*e^4*x^4 + a^2*b^2*d^2*e^2 + 2*(b^4*d*e^3 + a*b^3*e^4)*x^3 + (b^4*d^2*e^2
+ 4*a*b^3*d*e^3 + a^2*b^2*e^4)*x^2 + 2*(a*b^3*d^2*e^2 + a^2*b^2*d*e^3)*x)
*log(e*x + d))/(a^2*b^5*d^7 - 5*a^3*b^4*d^6*e + 10*a^4*b^3*d^5*e^2 - 10*a
^5*b^2*d^4*e^3 + 5*a^6*b*d^3*e^4 - a^7*d^2*e^5 + (b^7*d^5*e^2 - 5*a*b^6*d
^4*e^3 + 10*a^2*b^5*d^3*e^4 - 10*a^3*b^4*d^2*e^5 + 5*a^4*b^3*d*e^6 - a^5*b
^2*e^7)*x^4 + 2*(b^7*d^6*e - 4*a*b^6*d^5*e^2 + 5*a^2*b^5*d^4*e^3 - 5*a^4
*b^3*d^2*e^5 + 4*a^5*b^2*d*e^6 - a^6*b*e^7)*x^3 + (b^7*d^7 - a*b^6*d^6*e - 9
a^2*b^5*d^5*e^2 + 25*a^3*b^4*d^4*e^3 - 25*a^4*b^3*d^3*e^4 + 9*a^5*b^2*d
^2*e^5 + a^6*b*d*e^6 - a^7*e^7)*x^2 + 2*(a*b^6*d^7 - 4*a^2*b^5*d^6*e + 5
a^3*b^4*d^5*e^2 - 5*a^5*b^2*d^3*e^4 + 4*a^6*b*d^2*e^5 - a^7*d*e^6)*x)
```

Sympy [F]

$$\int \frac{1}{(d+ex)^3 (a^2+2abx+b^2x^2)^{3/2}} dx = \int \frac{1}{(d+ex)^3 ((a+bx)^2)^{3/2}} dx$$

input

```
integrate(1/(e*x+d)**3/(b**2*x**2+2*a*b*x+a**2)**(3/2),x)
```

output

```
Integral(1/((d + e*x)**3*((a + b*x)**2)**(3/2)), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{(d+ex)^3 (a^2+2abx+b^2x^2)^{3/2}} dx = \text{Exception raised: ValueError}$$

input

```
integrate(1/(e*x+d)^3/(b^2*x^2+2*a*b*x+a^2)^(3/2),x, algorithm="maxima")
```

output

```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
help (example of legal syntax is 'assume(a*e-b*d>0)', see `assume?` for m
ore detail
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 447 vs. $2(206) = 412$.

Time = 0.12 (sec) , antiderivative size = 447, normalized size of antiderivative = 1.62

$$\int \frac{1}{(d+ex)^3 (a^2+2abx+b^2x^2)^{3/2}} dx = \frac{6b^3e^2 \log(|ex+d|)}{b^6d^5 \operatorname{sgn}(bx+a) - 5ab^5d^4 \operatorname{sgn}(bx+a) + 10a^2b^4d^3e^2 \operatorname{sgn}(bx+a)} - \frac{b^5d^5 \operatorname{sgn}(bx+a) - 5ab^4d^4e^2 \operatorname{sgn}(bx+a) + 10a^2b^3d^3e^3 \operatorname{sgn}(bx+a) - 10a^3b^2d^2e^4 \operatorname{sgn}(bx+a) + 5a^4bde^5 \operatorname{sgn}(bx+a)}{12b^3e^3x^3 + 18b^3de^2x^2 + 18ab^2e^3x^2 + 4b^3d^2ex + 28ab^2de^2x + 4a^2be^3x - b^3d^3 + 7ab^2d^2e} + \frac{2(b^4d^4 \operatorname{sgn}(bx+a) - 4ab^3d^3 \operatorname{sgn}(bx+a) + 6a^2b^2d^2e^2 \operatorname{sgn}(bx+a) - 4a^3bde^3 \operatorname{sgn}(bx+a) + a^4e^4 \operatorname{sgn}(bx+a))}{(b^4d^4 \operatorname{sgn}(bx+a) - 4ab^3d^3 \operatorname{sgn}(bx+a) + 6a^2b^2d^2e^2 \operatorname{sgn}(bx+a) - 4a^3bde^3 \operatorname{sgn}(bx+a) + a^4e^4 \operatorname{sgn}(bx+a)) \cdot (b^2ex^2 + bdx + aex + a^2)}$$

input

```
integrate(1/(e*x+d)^3/(b^2*x^2+2*a*b*x+a^2)^(3/2),x, algorithm="giac")
```

output

```
6*b^3*e^2*log(abs(b*x + a))/(b^6*d^5*sgn(b*x + a) - 5*a*b^5*d^4*e*sgn(b*x
+ a) + 10*a^2*b^4*d^3*e^2*sgn(b*x + a) - 10*a^3*b^3*d^2*e^3*sgn(b*x + a) +
5*a^4*b^2*d*e^4*sgn(b*x + a) - a^5*b*e^5*sgn(b*x + a)) - 6*b^2*e^3*log(ab
s(e*x + d))/(b^5*d^5*e*sgn(b*x + a) - 5*a*b^4*d^4*e^2*sgn(b*x + a) + 10*a^
2*b^3*d^3*e^3*sgn(b*x + a) - 10*a^3*b^2*d^2*e^4*sgn(b*x + a) + 5*a^4*b*d*e
^5*sgn(b*x + a) - a^5*e^6*sgn(b*x + a)) + 1/2*(12*b^3*e^3*x^3 + 18*b^3*d*e
^2*x^2 + 18*a*b^2*e^3*x^2 + 4*b^3*d^2*e*x + 28*a*b^2*d*e^2*x + 4*a^2*b*e^3
*x - b^3*d^3 + 7*a*b^2*d^2*e + 7*a^2*b*d*e^2 - a^3*e^3)/((b^4*d^4*sgn(b*x
+ a) - 4*a*b^3*d^3*e*sgn(b*x + a) + 6*a^2*b^2*d^2*e^2*sgn(b*x + a) - 4*a^3
*b*d*e^3*sgn(b*x + a) + a^4*e^4*sgn(b*x + a))*(b*e*x^2 + b*d*x + a*e*x + a
*d)^2)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(d + ex)^3 (a^2 + 2abx + b^2x^2)^{3/2}} dx = \int \frac{1}{(d + ex)^3 (a^2 + 2abx + b^2x^2)^{3/2}} dx$$

input `int(1/((d + e*x)^3*(a^2 + b^2*x^2 + 2*a*b*x)^(3/2)),x)`output `int(1/((d + e*x)^3*(a^2 + b^2*x^2 + 2*a*b*x)^(3/2)), x)`**Reduce [B] (verification not implemented)**

Time = 0.24 (sec) , antiderivative size = 1202, normalized size of antiderivative = 4.36

$$\int \frac{1}{(d + ex)^3 (a^2 + 2abx + b^2x^2)^{3/2}} dx = \text{Too large to display}$$

input `int(1/(e*x+d)^3/(b^2*x^2+2*a*b*x+a^2)^(3/2),x)`

output

```
( - 12*log(a + b*x)*a**3*b**2*d**2*e**3 - 24*log(a + b*x)*a**3*b**2*d*e**4
*x - 12*log(a + b*x)*a**3*b**2*e**5*x**2 - 12*log(a + b*x)*a**2*b**3*d**3*
e**2 - 48*log(a + b*x)*a**2*b**3*d**2*e**3*x - 60*log(a + b*x)*a**2*b**3*d
**4*x**2 - 24*log(a + b*x)*a**2*b**3*e**5*x**3 - 24*log(a + b*x)*a*b**4*
d**3*e**2*x - 60*log(a + b*x)*a*b**4*d**2*e**3*x**2 - 48*log(a + b*x)*a*b*
**4*d*e**4*x**3 - 12*log(a + b*x)*a*b**4*e**5*x**4 - 12*log(a + b*x)*b**5*d
**3*e**2*x**2 - 24*log(a + b*x)*b**5*d**2*e**3*x**3 - 12*log(a + b*x)*b**5
*d*e**4*x**4 + 12*log(d + e*x)*a**3*b**2*d**2*e**3 + 24*log(d + e*x)*a**3*
b**2*d*e**4*x + 12*log(d + e*x)*a**3*b**2*e**5*x**2 + 12*log(d + e*x)*a**2
*b**3*d**3*e**2 + 48*log(d + e*x)*a**2*b**3*d**2*e**3*x + 60*log(d + e*x)*
a**2*b**3*d*e**4*x**2 + 24*log(d + e*x)*a**2*b**3*e**5*x**3 + 24*log(d + e
*x)*a*b**4*d**3*e**2*x + 60*log(d + e*x)*a*b**4*d**2*e**3*x**2 + 48*log(d
+ e*x)*a*b**4*d*e**4*x**3 + 12*log(d + e*x)*a*b**4*e**5*x**4 + 12*log(d +
e*x)*b**5*d**3*e**2*x**2 + 24*log(d + e*x)*b**5*d**2*e**3*x**3 + 12*log(d
+ e*x)*b**5*d*e**4*x**4 - a**5*e**5 + 7*a**4*b*d*e**4 + 4*a**4*b*e**5*x +
2*a**3*b**2*d**2*e**3 + 16*a**3*b**2*d*e**4*x + 12*a**3*b**2*e**5*x**2 - 2
*a**2*b**3*d**3*e**2 - 7*a*b**4*d**4*e - 16*a*b**4*d**3*e**2*x - 6*a*b**4*
e**5*x**4 + b**5*d**5 - 4*b**5*d**4*e*x - 12*b**5*d**3*e**2*x**2 + 6*b**5*
d*e**4*x**4)/(2*(a**8*d**2*e**6 + 2*a**8*d*e**7*x + a**8*e**8*x**2 - 4*a**
7*b*d**3*e**5 - 6*a**7*b*d**2*e**6*x + 2*a**7*b*e**8*x**3 + 5*a**6*b**2...
```

3.326
$$\int \frac{(d+ex)^6}{(a^2+2abx+b^2x^2)^{5/2}} dx$$

| | |
|---|------|
| Optimal result | 2440 |
| Mathematica [A] (verified) | 2441 |
| Rubi [A] (verified) | 2441 |
| Maple [A] (verified) | 2443 |
| Fricas [B] (verification not implemented) | 2443 |
| Sympy [F] | 2444 |
| Maxima [B] (verification not implemented) | 2445 |
| Giac [A] (verification not implemented) | 2446 |
| Mupad [F(-1)] | 2446 |
| Reduce [B] (verification not implemented) | 2447 |

Optimal result

Integrand size = 28, antiderivative size = 302

$$\int \frac{(d+ex)^6}{(a^2+2abx+b^2x^2)^{5/2}} dx = -\frac{20e^3(bd-ae)^3}{b^7\sqrt{a^2+2abx+b^2x^2}} - \frac{(bd-ae)^6}{4b^7(a+bx)^3\sqrt{a^2+2abx+b^2x^2}} - \frac{2e(bd-ae)^5}{b^7(a+bx)^2\sqrt{a^2+2abx+b^2x^2}} - \frac{15e^2(bd-ae)^4}{2b^7(a+bx)\sqrt{a^2+2abx+b^2x^2}} + \frac{e^5(6bd-5ae)x(a+bx)}{b^6\sqrt{a^2+2abx+b^2x^2}} + \frac{e^6x^2(a+bx)}{2b^5\sqrt{a^2+2abx+b^2x^2}} + \frac{15e^4(bd-ae)^2(a+bx)\log(a+bx)}{b^7\sqrt{a^2+2abx+b^2x^2}}$$

output

```
-20*e^3*(-a*e+b*d)^3/b^7/((b*x+a)^2)^(1/2)-1/4*(-a*e+b*d)^6/b^7/(b*x+a)^3/
((b*x+a)^2)^(1/2)-2*e*(-a*e+b*d)^5/b^7/(b*x+a)^2/((b*x+a)^2)^(1/2)-15/2*e^
2*(-a*e+b*d)^4/b^7/(b*x+a)/((b*x+a)^2)^(1/2)+e^5*(-5*a*e+6*b*d)*x*(b*x+a)/
b^6/((b*x+a)^2)^(1/2)+1/2*e^6*x^2*(b*x+a)/b^5/((b*x+a)^2)^(1/2)+15*e^4*(-a
*e+b*d)^2*(b*x+a)*ln(b*x+a)/b^7/((b*x+a)^2)^(1/2)
```

Mathematica [A] (verified)

Time = 1.10 (sec) , antiderivative size = 313, normalized size of antiderivative = 1.04

$$\int \frac{(d+ex)^6}{(a^2+2abx+b^2x^2)^{5/2}} dx = \frac{57a^6e^6 + 14a^5be^5(-11d+12ex) + a^4b^2e^4(125d^2 - 496dex + 132e^2x^2) - 4a^3b^3e^3(5d^3 - 110d^2ex + 126d^2e^2x^2 + 8e^3x^3) - a^2b^4e^2(5d^4 + 80d^3ex - 540d^2e^2x^2 + 96d^2e^3x^3 + 68e^4x^4) - 2ab^5e(d^5 + 10d^4ex + 60d^3e^2x^2 - 120d^2e^3x^3 - 48de^4x^4 + 6e^5x^5) - b^6(d^6 + 8d^5ex + 30d^4e^2x^2 + 80d^3e^3x^3 - 24de^5x^5 - 2e^6x^6) + 60e^4(bd - ae)^2(a + bx)^4 \operatorname{Log}[a + bx]}{(4b^7(a + bx)^3 \operatorname{Sqrt}[a + bx]^2)}$$

input `Integrate[(d + e*x)^6/(a^2 + 2*a*b*x + b^2*x^2)^(5/2),x]`

output $(57a^6e^6 + 14a^5b^2e^5(-11d + 12ex) + a^4b^2e^4(125d^2 - 496dex + 132e^2x^2) - 4a^3b^3e^3(5d^3 - 110d^2ex + 126d^2e^2x^2 + 8e^3x^3) - a^2b^4e^2(5d^4 + 80d^3ex - 540d^2e^2x^2 + 96d^2e^3x^3 + 68e^4x^4) - 2ab^5e(d^5 + 10d^4ex + 60d^3e^2x^2 - 120d^2e^3x^3 - 48de^4x^4 + 6e^5x^5) - b^6(d^6 + 8d^5ex + 30d^4e^2x^2 + 80d^3e^3x^3 - 24de^5x^5 - 2e^6x^6) + 60e^4(bd - ae)^2(a + bx)^4 \operatorname{Log}[a + bx]) / (4b^7(a + bx)^3 \operatorname{Sqrt}[a + bx]^2)$

Rubi [A] (verified)

Time = 0.72 (sec) , antiderivative size = 180, normalized size of antiderivative = 0.60, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {1102, 27, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\begin{aligned} & \int \frac{(d+ex)^6}{(a^2+2abx+b^2x^2)^{5/2}} dx \\ & \quad \downarrow \text{1102} \\ & \frac{b^5(a+bx) \int \frac{(d+ex)^6}{b^5(a+bx)^5} dx}{\sqrt{a^2+2abx+b^2x^2}} \\ & \quad \downarrow \text{27} \\ & \frac{(a+bx) \int \frac{(d+ex)^6}{(a+bx)^5} dx}{\sqrt{a^2+2abx+b^2x^2}} \end{aligned}$$

↓ 49

$$\frac{(a + bx) \int \left(\frac{xe^6}{b^5} + \frac{(6bd-5ae)e^5}{b^6} + \frac{15(bd-ae)^2e^4}{b^6(a+bx)} + \frac{20(bd-ae)^3e^3}{b^6(a+bx)^2} + \frac{15(bd-ae)^4e^2}{b^6(a+bx)^3} + \frac{6(bd-ae)^5e}{b^6(a+bx)^4} + \frac{(bd-ae)^6}{b^6(a+bx)^5} \right) dx}{\sqrt{a^2 + 2abx + b^2x^2}}$$

↓ 2009

$$\frac{(a + bx) \left(\frac{15e^4(bd-ae)^2 \log(a+bx)}{b^7} - \frac{20e^3(bd-ae)^3}{b^7(a+bx)} - \frac{15e^2(bd-ae)^4}{2b^7(a+bx)^2} - \frac{2e(bd-ae)^5}{b^7(a+bx)^3} - \frac{(bd-ae)^6}{4b^7(a+bx)^4} + \frac{e^5x(6bd-5ae)}{b^6} + \frac{e^6x^2}{2b^5} \right)}{\sqrt{a^2 + 2abx + b^2x^2}}$$

input `Int[(d + e*x)^6/(a^2 + 2*a*b*x + b^2*x^2)^(5/2),x]`

output `((a + b*x)*((e^5*(6*b*d - 5*a*e)*x)/b^6 + (e^6*x^2)/(2*b^5) - (b*d - a*e)^6/(4*b^7*(a + b*x)^4) - (2*e*(b*d - a*e)^5)/(b^7*(a + b*x)^3) - (15*e^2*(b*d - a*e)^4)/(2*b^7*(a + b*x)^2) - (20*e^3*(b*d - a*e)^3)/(b^7*(a + b*x)) + (15*e^4*(b*d - a*e)^2*Log[a + b*x])/b^7)/Sqrt[a^2 + 2*a*b*x + b^2*x^2]`

Defintions of rubi rules used

rule 27 `Int[(a_)*(Fx_), x_Symbol] := Simp[a Int[Fx, x], x] /; FreeQ[a, x] && !MatchQ[Fx, (b_)*(Gx_)] /; FreeQ[b, x]`

rule 49 `Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] := Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] && IGtQ[m, 0] && IGtQ[m + n + 2, 0]`

rule 1102 `Int[((d_.) + (e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] := Simp[(a + b*x + c*x^2)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x)^(2*FracPart[p])) Int[(d + e*x)^m*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && EqQ[b^2 - 4*a*c, 0]`

rule 2009 `Int[u_, x_Symbol] := Simp[IntSum[u, x], x] /; SumQ[u]`

Maple [A] (verified)

Time = 2.05 (sec) , antiderivative size = 379, normalized size of antiderivative = 1.25

| method | result |
|---------|---|
| risch | $-\frac{\sqrt{(bx+a)^2} e^5 \left(-\frac{1}{2}be x^2+5aex-6bdx\right)}{(bx+a)b^6} + \frac{\sqrt{(bx+a)^2} \left((20a^3b^2e^6-60a^2de^5b^3+60ab^4d^2e^4-20d^3e^3b^5)x^3 + \frac{15be^2(7a^4e^4-20a^3bd}{(bx+a)b^6} \right)}{(bx+a)b^6}$ |
| default | $\frac{(240 \ln(bx+a)a^5be^6x-120 \ln(bx+a)a^5bd e^5+60 \ln(bx+a)a^4b^2d^2e^4-2ab^5d^5e-154a^5bd e^5-20a^3b^3d^3e^3+360 \ln(bx+a)a^4b^2e^6x^2+}$ |

input `int((e*x+d)^6/(b^2*x^2+2*a*b*x+a^2)^(5/2),x,method=_RETURNVERBOSE)`

output
$$-\left(\frac{(bx+a)^2}{(bx+a)}\right)^{1/2} e^5/b^6 \left(-\frac{1}{2}b^2e^5x^2+5a^2e^5x-6b^2d^2e^5\right) + \left(\frac{(bx+a)^2}{(bx+a)}\right)^{1/2} e^5/b^6 \left((20a^3b^2e^6-60a^2de^5b^3+60ab^4d^2e^4-20b^5d^3e^3)x^3 + \frac{15be^2(7a^4e^4-20a^3bd^2e^4-20a^2b^3d^3e^3-b^4d^4)x^2 + e(47a^5e^5-130a^4bd^2e^4+110a^3b^2d^2e^3-20a^2b^3d^3e^2-5a^2b^4d^4e-2b^5d^5)x + \frac{1}{4}(57a^6e^6-154a^5bd^2e^5+125a^4b^2d^2e^4-20a^3b^3d^3e^3-5a^2b^4d^4e^2-2ab^5d^5e-b^6d^6)}{b} \right) / b^6 + 15 \left(\frac{(bx+a)^2}{(bx+a)}\right)^{1/2} e^5/b^7 \left(a^2e^2-2ab^2d^2e^2+b^2d^2 \right) \ln(bx+a)$$

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 572 vs. 2(219) = 438.

Time = 0.09 (sec) , antiderivative size = 572, normalized size of antiderivative = 1.89

$$\int \frac{(d+ex)^6}{(a^2+2abx+b^2x^2)^{5/2}} dx = \frac{2b^6e^6x^6 - b^6d^6 - 2ab^5d^5e - 5a^2b^4d^4e^2 - 20a^3b^3d^3e^3 + 125a^4b^2d^2e^4 - 154a^5bd^2e^5 + 15a^6e^6}{(a^2+2abx+b^2x^2)^{5/2}}$$

input `integrate((e*x+d)^6/(b^2*x^2+2*a*b*x+a^2)^(5/2),x, algorithm="fricas")`

output

```

1/4*(2*b^6*e^6*x^6 - b^6*d^6 - 2*a*b^5*d^5*e - 5*a^2*b^4*d^4*e^2 - 20*a^3*
b^3*d^3*e^3 + 125*a^4*b^2*d^2*e^4 - 154*a^5*b*d*e^5 + 57*a^6*e^6 + 12*(2*b
^6*d*e^5 - a*b^5*e^6)*x^5 + 4*(24*a*b^5*d*e^5 - 17*a^2*b^4*e^6)*x^4 - 16*(
5*b^6*d^3*e^3 - 15*a*b^5*d^2*e^4 + 6*a^2*b^4*d*e^5 + 2*a^3*b^3*e^6)*x^3 -
6*(5*b^6*d^4*e^2 + 20*a*b^5*d^3*e^3 - 90*a^2*b^4*d^2*e^4 + 84*a^3*b^3*d*e^
5 - 22*a^4*b^2*e^6)*x^2 - 4*(2*b^6*d^5*e + 5*a*b^5*d^4*e^2 + 20*a^2*b^4*d^
3*e^3 - 110*a^3*b^3*d^2*e^4 + 124*a^4*b^2*d*e^5 - 42*a^5*b*e^6)*x + 60*(a^
4*b^2*d^2*e^4 - 2*a^5*b*d*e^5 + a^6*e^6 + (b^6*d^2*e^4 - 2*a*b^5*d*e^5 + a
^2*b^4*e^6)*x^4 + 4*(a*b^5*d^2*e^4 - 2*a^2*b^4*d*e^5 + a^3*b^3*e^6)*x^3 +
6*(a^2*b^4*d^2*e^4 - 2*a^3*b^3*d*e^5 + a^4*b^2*e^6)*x^2 + 4*(a^3*b^3*d^2*e
^4 - 2*a^4*b^2*d*e^5 + a^5*b*e^6)*x)*log(b*x + a))/(b^11*x^4 + 4*a*b^10*x^
3 + 6*a^2*b^9*x^2 + 4*a^3*b^8*x + a^4*b^7)

```

SymPy [F]

$$\int \frac{(d+ex)^6}{(a^2+2abx+b^2x^2)^{5/2}} dx = \int \frac{(d+ex)^6}{((a+bx)^2)^{\frac{5}{2}}} dx$$

input

```
integrate((e*x+d)**6/(b**2*x**2+2*a*b*x+a**2)**(5/2), x)
```

output

```
Integral((d + e*x)**6/((a + b*x)**2)**(5/2), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 576 vs. $2(219) = 438$.

Time = 0.09 (sec) , antiderivative size = 576, normalized size of antiderivative = 1.91

$$\int \frac{(d+ex)^6}{(a^2+2abx+b^2x^2)^{5/2}} dx = \frac{1}{4} e^6 \left(\frac{2b^6x^6 - 12ab^5x^5 - 68a^2b^4x^4 - 32a^3b^3x^3 + 132a^4b^2x^2 + 168a^5bx + 57a^6}{b^{11}x^4 + 4ab^{10}x^3 + 6a^2b^9x^2 + 4a^3b^8x + a^4b^7} \right) \\ + \frac{1}{2} de^5 \left(\frac{12b^5x^5 + 48ab^4x^4 - 48a^2b^3x^3 - 252a^3b^2x^2 - 248a^4bx - 77a^5}{b^{10}x^4 + 4ab^9x^3 + 6a^2b^8x^2 + 4a^3b^7x + a^4b^6} - \frac{60a \log(bx+a)}{b^6} \right) \\ + \frac{5}{4} d^2e^4 \left(\frac{48ab^3x^3 + 108a^2b^2x^2 + 88a^3bx + 25a^4}{b^9x^4 + 4ab^8x^3 + 6a^2b^7x^2 + 4a^3b^6x + a^4b^5} + \frac{12 \log(bx+a)}{b^5} \right) \\ - \frac{5}{3} d^3e^3 \left(\frac{12x^2}{(b^2x^2 + 2abx + a^2)^{3/2}b^2} + \frac{8a^2}{(b^2x^2 + 2abx + a^2)^{3/2}b^4} + \frac{6a}{b^6(x + \frac{a}{b})^2} - \frac{8a^2}{b^7(x + \frac{a}{b})^3} - \frac{3a^3}{b^8(x + \frac{a}{b})^4} \right) \\ - \frac{1}{2} d^5e \left(\frac{4}{(b^2x^2 + 2abx + a^2)^{3/2}b^2} - \frac{3a}{b^6(x + \frac{a}{b})^4} \right) \\ - \frac{5}{4} d^4e^2 \left(\frac{6}{b^5(x + \frac{a}{b})^2} - \frac{8a}{b^6(x + \frac{a}{b})^3} + \frac{3a^2}{b^7(x + \frac{a}{b})^4} \right) - \frac{d^6}{4b^5(x + \frac{a}{b})^4}$$

input `integrate((e*x+d)^6/(b^2*x^2+2*a*b*x+a^2)^(5/2),x, algorithm="maxima")`

output `1/4*e^6*((2*b^6*x^6 - 12*a*b^5*x^5 - 68*a^2*b^4*x^4 - 32*a^3*b^3*x^3 + 132*a^4*b^2*x^2 + 168*a^5*b*x + 57*a^6)/(b^11*x^4 + 4*a*b^10*x^3 + 6*a^2*b^9*x^2 + 4*a^3*b^8*x + a^4*b^7) + 60*a^2*log(b*x + a)/b^7) + 1/2*d*e^5*((12*b^5*x^5 + 48*a*b^4*x^4 - 48*a^2*b^3*x^3 - 252*a^3*b^2*x^2 - 248*a^4*b*x - 77*a^5)/(b^10*x^4 + 4*a*b^9*x^3 + 6*a^2*b^8*x^2 + 4*a^3*b^7*x + a^4*b^6) - 60*a*log(b*x + a)/b^6) + 5/4*d^2*e^4*((48*a*b^3*x^3 + 108*a^2*b^2*x^2 + 88*a^3*b*x + 25*a^4)/(b^9*x^4 + 4*a*b^8*x^3 + 6*a^2*b^7*x^2 + 4*a^3*b^6*x + a^4*b^5) + 12*log(b*x + a)/b^5) - 5/3*d^3*e^3*(12*x^2/((b^2*x^2 + 2*a*b*x + a^2)^(3/2)*b^2) + 8*a^2/((b^2*x^2 + 2*a*b*x + a^2)^(3/2)*b^4) + 6*a/(b^6*(x + a/b)^2) - 8*a^2/(b^7*(x + a/b)^3) - 3*a^3/(b^8*(x + a/b)^4)) - 1/2*d^5*e*(4/((b^2*x^2 + 2*a*b*x + a^2)^(3/2)*b^2) - 3*a/(b^6*(x + a/b)^4)) - 5/4*d^4*e^2*(6/(b^5*(x + a/b)^2) - 8*a/(b^6*(x + a/b)^3) + 3*a^2/(b^7*(x + a/b)^4)) - 1/4*d^6/(b^5*(x + a/b)^4)`

Giac [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 389, normalized size of antiderivative = 1.29

$$\int \frac{(d+ex)^6}{(a^2+2abx+b^2x^2)^{5/2}} dx = \frac{15(b^2d^2e^4 - 2abde^5 + a^2e^6) \log(|bx+a|)}{b^7 \operatorname{sgn}(bx+a)} + \frac{b^5e^6x^2 \operatorname{sgn}(bx+a) + 12b^5de^5x \operatorname{sgn}(bx+a) - 10ab^4e^6x \operatorname{sgn}(bx+a)}{2b^{10}} - \frac{b^6d^6 + 2ab^5d^5e + 5a^2b^4d^4e^2 + 20a^3b^3d^3e^3 - 125a^4b^2d^2e^4 + 154a^5bde^5 - 57a^6e^6 + 80(b^6d^3e^3 - 3ab^5d^2e^2)}{b^7 \operatorname{sgn}(bx+a)}$$

input `integrate((e*x+d)^6/(b^2*x^2+2*a*b*x+a^2)^(5/2),x, algorithm="giac")`

output `15*(b^2*d^2*e^4 - 2*a*b*d*e^5 + a^2*e^6)*log(abs(b*x + a))/(b^7*sgn(b*x + a)) + 1/2*(b^5*e^6*x^2*sgn(b*x + a) + 12*b^5*d*e^5*x*sgn(b*x + a) - 10*a*b^4*e^6*x*sgn(b*x + a))/b^10 - 1/4*(b^6*d^6 + 2*a*b^5*d^5*e + 5*a^2*b^4*d^4*e^2 + 20*a^3*b^3*d^3*e^3 - 125*a^4*b^2*d^2*e^4 + 154*a^5*b*d*e^5 - 57*a^6*e^6 + 80*(b^6*d^3*e^3 - 3*a*b^5*d^2*e^4 + 3*a^2*b^4*d*e^5 - a^3*b^3*e^6))*x^3 + 30*(b^6*d^4*e^2 + 4*a*b^5*d^3*e^3 - 18*a^2*b^4*d^2*e^4 + 20*a^3*b^3*d*e^5 - 7*a^4*b^2*e^6)*x^2 + 4*(2*b^6*d^5*e + 5*a*b^5*d^4*e^2 + 20*a^2*b^4*d^3*e^3 - 110*a^3*b^3*d^2*e^4 + 130*a^4*b^2*d*e^5 - 47*a^5*b*e^6)*x)/((b*x + a)^4*b^7*sgn(b*x + a))`

Mupad [F(-1)]

Timed out.

$$\int \frac{(d+ex)^6}{(a^2+2abx+b^2x^2)^{5/2}} dx = \int \frac{(d+ex)^6}{(a^2+2abx+b^2x^2)^{5/2}} dx$$

input `int((d + e*x)^6/(a^2 + b^2*x^2 + 2*a*b*x)^(5/2),x)`

output `int((d + e*x)^6/(a^2 + b^2*x^2 + 2*a*b*x)^(5/2), x)`

3.327 $\int \frac{(d+ex)^5}{(a^2+2abx+b^2x^2)^{5/2}} dx$

| | |
|---|------|
| Optimal result | 2448 |
| Mathematica [A] (verified) | 2449 |
| Rubi [A] (verified) | 2449 |
| Maple [A] (verified) | 2451 |
| Fricas [B] (verification not implemented) | 2451 |
| Sympy [F] | 2452 |
| Maxima [B] (verification not implemented) | 2452 |
| Giac [A] (verification not implemented) | 2453 |
| Mupad [F(-1)] | 2454 |
| Reduce [B] (verification not implemented) | 2454 |

Optimal result

Integrand size = 28, antiderivative size = 253

$$\int \frac{(d+ex)^5}{(a^2+2abx+b^2x^2)^{5/2}} dx = -\frac{10e^3(bd-ae)^2}{b^6\sqrt{a^2+2abx+b^2x^2}} - \frac{(bd-ae)^5}{4b^6(a+bx)^3\sqrt{a^2+2abx+b^2x^2}} - \frac{5e(bd-ae)^4}{3b^6(a+bx)^2\sqrt{a^2+2abx+b^2x^2}} - \frac{5e^2(bd-ae)^3}{b^6(a+bx)\sqrt{a^2+2abx+b^2x^2}} + \frac{e^5x(a+bx)}{b^5\sqrt{a^2+2abx+b^2x^2}} + \frac{5e^4(bd-ae)(a+bx)\log(a+bx)}{b^6\sqrt{a^2+2abx+b^2x^2}}$$

output

```
-10*e^3*(-a*e+b*d)^2/b^6/((b*x+a)^(1/2))-1/4*(-a*e+b*d)^5/b^6/(b*x+a)^3/
((b*x+a)^(1/2))-5/3*e*(-a*e+b*d)^4/b^6/(b*x+a)^2/((b*x+a)^(1/2))-5*e^2
*(-a*e+b*d)^3/b^6/(b*x+a)/((b*x+a)^(1/2))+e^5*x*(b*x+a)/b^5/((b*x+a)^(1/2))
+5*e^4*(-a*e+b*d)*(b*x+a)*ln(b*x+a)/b^6/((b*x+a)^(1/2))
```

Mathematica [A] (verified)

Time = 1.10 (sec) , antiderivative size = 242, normalized size of antiderivative = 0.96

$$\int \frac{(d+ex)^5}{(a^2+2abx+b^2x^2)^{5/2}} dx = \frac{-77a^5e^5 + a^4be^4(125d-248ex) - 2a^3b^2e^3(15d^2-220dex+126e^2x^2) - 2a^2b^3e^2(5d^3+60d^2ex-270d^2e^2x^2+24e^3x^3) + ab^4e(-5d^4-40d^3ex-180d^2e^2x^2+240de^3x^3+48e^4x^4) - b^5(3d^5+20d^4ex+60d^3e^2x^2+120d^2e^3x^3-12e^5x^5) - 60e^4(-(b*d)+a*e)*(a+bx)^4 \operatorname{Log}[a+bx]}{(12b^6(a+bx)^3 \operatorname{Sqrt}[(a+bx)^2])}$$

input `Integrate[(d + e*x)^5/(a^2 + 2*a*b*x + b^2*x^2)^(5/2), x]`

output $(-77*a^5*e^5 + a^4*b*e^4*(125*d - 248*e*x) - 2*a^3*b^2*e^3*(15*d^2 - 220*d*e*x + 126*e^2*x^2) - 2*a^2*b^3*e^2*(5*d^3 + 60*d^2*e*x - 270*d^2*e^2*x^2 + 24*e^3*x^3) + a*b^4*e*(-5*d^4 - 40*d^3*e*x - 180*d^2*e^2*x^2 + 240*d^2*e^3*x^3 + 48*e^4*x^4) - b^5*(3*d^5 + 20*d^4*e*x + 60*d^3*e^2*x^2 + 120*d^2*e^3*x^3 - 12*e^5*x^5) - 60*e^4*(-(b*d) + a*e)*(a + b*x)^4 \operatorname{Log}[a + b*x]) / (12*b^6*(a + b*x)^3 \operatorname{Sqrt}[(a + b*x)^2])$

Rubi [A] (verified)

Time = 0.62 (sec) , antiderivative size = 156, normalized size of antiderivative = 0.62, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {1102, 27, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\begin{aligned} & \int \frac{(d+ex)^5}{(a^2+2abx+b^2x^2)^{5/2}} dx \\ & \quad \downarrow \text{1102} \\ & \frac{b^5(a+bx) \int \frac{(d+ex)^5}{b^5(a+bx)^5} dx}{\sqrt{a^2+2abx+b^2x^2}} \\ & \quad \downarrow \text{27} \\ & \frac{(a+bx) \int \frac{(d+ex)^5}{(a+bx)^5} dx}{\sqrt{a^2+2abx+b^2x^2}} \\ & \quad \downarrow \text{49} \end{aligned}$$

$$\frac{(a+bx) \int \left(\frac{e^5}{b^5} + \frac{5(bd-ae)e^4}{b^5(a+bx)} + \frac{10(bd-ae)^2 e^3}{b^5(a+bx)^2} + \frac{10(bd-ae)^3 e^2}{b^5(a+bx)^3} + \frac{5(bd-ae)^4 e}{b^5(a+bx)^4} + \frac{(bd-ae)^5}{b^5(a+bx)^5} \right) dx}{\sqrt{a^2 + 2abx + b^2x^2}}$$

↓ 2009

$$\frac{(a+bx) \left(\frac{5e^4(bd-ae) \log(a+bx)}{b^6} - \frac{10e^3(bd-ae)^2}{b^6(a+bx)} - \frac{5e^2(bd-ae)^3}{b^6(a+bx)^2} - \frac{5e(bd-ae)^4}{3b^6(a+bx)^3} - \frac{(bd-ae)^5}{4b^6(a+bx)^4} + \frac{e^5x}{b^5} \right)}{\sqrt{a^2 + 2abx + b^2x^2}}$$

input `Int[(d + e*x)^5/(a^2 + 2*a*b*x + b^2*x^2)^(5/2),x]`

output `((a + b*x)*((e^5*x)/b^5 - (b*d - a*e)^5/(4*b^6*(a + b*x)^4) - (5*e*(b*d - a*e)^4)/(3*b^6*(a + b*x)^3) - (5*e^2*(b*d - a*e)^3)/(b^6*(a + b*x)^2) - (10*e^3*(b*d - a*e)^2)/(b^6*(a + b*x)) + (5*e^4*(b*d - a*e)*Log[a + b*x])/b^6))/Sqrt[a^2 + 2*a*b*x + b^2*x^2]`

Defintions of rubi rules used

rule 27 `Int[(a_)*(F_x_), x_Symbol] := Simp[a Int[F_x, x], x] /; FreeQ[a, x] && !MatchQ[F_x, (b_)*(G_x_) /; FreeQ[b, x]]`

rule 49 `Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] := Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] && IGtQ[m, 0] && IGtQ[m + n + 2, 0]`

rule 1102 `Int[((d_.) + (e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] := Simp[(a + b*x + c*x^2)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x)^(2*FracPart[p])) Int[(d + e*x)^m*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && EqQ[b^2 - 4*a*c, 0]`

rule 2009 `Int[u_, x_Symbol] := Simp[IntSum[u, x], x] /; SumQ[u]`

Maple [A] (verified)

Time = 1.84 (sec) , antiderivative size = 291, normalized size of antiderivative = 1.15

| method | result |
|---------|---|
| risch | $\frac{\sqrt{(bx+a)^2} e^5 x}{(bx+a)b^5} + \frac{\sqrt{(bx+a)^2} \left((-10a^2b^2e^5 + 20ab^3de^4 - 10d^2e^3b^4)x^3 - 5be^2(5e^3a^3 - 9a^2bde^2 + 3ab^2d^2e + b^3d^3)x^2 - \frac{5e(13a^4e^4 - 22a^3bde^3 + 6a^2b^2d^2e + b^3d^3)}{(bx+a)^5b^5} \right)}{(bx+a)^5b^5}$ |
| default | $-\frac{(-240 \ln(bx+a)x a^3b^2de^4 - 440a^3b^2de^4x + 3b^5d^5 - 240 \ln(bx+a)x^3 a b^4de^4 - 60 \ln(bx+a)b^5de^4x^4 + 240 \ln(bx+a)x^3 a^2b^3e^5 - 125a^4b^3de^4)}{(bx+a)^5b^5}$ |

input `int((e*x+d)^5/(b^2*x^2+2*a*b*x+a^2)^(5/2),x,method=_RETURNVERBOSE)`

output $((b*x+a)^2)^{(1/2)}/(b*x+a)*e^5/b^5*x + ((b*x+a)^2)^{(1/2)}/(b*x+a)^5 * ((-10*a^2*b^2*e^5 + 20*a*b^3*d*e^4 - 10*b^4*d^2*e^3) * x^3 - 5*b*e^2 * (5*a^3*e^3 - 9*a^2*b*d*e^2 + 3*a*b^2*d^2*e + b^3*d^3) * x^2 - 5/3 * e * (13*a^4*e^4 - 22*a^3*b*d*e^3 + 6*a^2*b^2*d^2*e^2 + 2*a*b^3*d^3*e + b^4*d^4) * x - 1/12 * (77*a^5*e^5 - 125*a^4*b*d*e^4 + 30*a^3*b^2*d^2*e^3 + 10*a^2*b^3*d^3*e^2 + 5*a*b^4*d^4*e + 3*b^5*d^5) / b) / b^5 - 5 * ((b*x+a)^2)^{(1/2)}/(b*x+a) / b^6 * e^4 * (a*e - b*d) * \ln(b*x+a)$

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 412 vs. 2(183) = 366.

Time = 0.10 (sec) , antiderivative size = 412, normalized size of antiderivative = 1.63

$$\int \frac{(d+ex)^5}{(a^2+2abx+b^2x^2)^{5/2}} dx = \frac{12b^5e^5x^5 + 48ab^4e^5x^4 - 3b^5d^5 - 5ab^4d^4e - 10a^2b^3d^3e^2 - 30a^3b^2d^2e^3 + 12a^4b^3de^4 + 12a^5d^4e^5}{(a^2+2abx+b^2x^2)^{5/2}}$$

input `integrate((e*x+d)^5/(b^2*x^2+2*a*b*x+a^2)^(5/2),x, algorithm="fricas")`

output

```
1/12*(12*b^5*e^5*x^5 + 48*a*b^4*e^5*x^4 - 3*b^5*d^5 - 5*a*b^4*d^4*e - 10*a^2*b^3*d^3*e^2 - 30*a^3*b^2*d^2*e^3 + 125*a^4*b*d*e^4 - 77*a^5*e^5 - 24*(5*b^5*d^2*e^3 - 10*a*b^4*d*e^4 + 2*a^2*b^3*e^5)*x^3 - 12*(5*b^5*d^3*e^2 + 15*a*b^4*d^2*e^3 - 45*a^2*b^3*d*e^4 + 21*a^3*b^2*e^5)*x^2 - 4*(5*b^5*d^4*e + 10*a*b^4*d^3*e^2 + 30*a^2*b^3*d^2*e^3 - 110*a^3*b^2*d*e^4 + 62*a^4*b*e^5)*x + 60*(a^4*b*d*e^4 - a^5*e^5 + (b^5*d*e^4 - a*b^4*e^5)*x^4 + 4*(a*b^4*d*e^4 - a^2*b^3*e^5)*x^3 + 6*(a^2*b^3*d*e^4 - a^3*b^2*e^5)*x^2 + 4*(a^3*b^2*d*e^4 - a^4*b*e^5)*x)*log(b*x + a)/(b^10*x^4 + 4*a*b^9*x^3 + 6*a^2*b^8*x^2 + 4*a^3*b^7*x + a^4*b^6)
```

Sympy [F]

$$\int \frac{(d + ex)^5}{(a^2 + 2abx + b^2x^2)^{5/2}} dx = \int \frac{(d + ex)^5}{((a + bx)^2)^{5/2}} dx$$

input

```
integrate((e*x+d)**5/(b**2*x**2+2*a*b*x+a**2)**(5/2),x)
```

output

```
Integral((d + e*x)**5/((a + b*x)**2)**(5/2), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 443 vs. 2(183) = 366.

Time = 0.07 (sec) , antiderivative size = 443, normalized size of antiderivative = 1.75

$$\begin{aligned} \int \frac{(d + ex)^5}{(a^2 + 2abx + b^2x^2)^{5/2}} dx &= \frac{1}{12} e^5 \left(\frac{12 b^5 x^5 + 48 a b^4 x^4 - 48 a^2 b^3 x^3 - 252 a^3 b^2 x^2 - 248 a^4 b x - 77 a^5}{b^{10} x^4 + 4 a b^9 x^3 + 6 a^2 b^8 x^2 + 4 a^3 b^7 x + a^4 b^6} - \frac{60}{b^5} \right) \\ &+ \frac{5}{12} d e^4 \left(\frac{48 a b^3 x^3 + 108 a^2 b^2 x^2 + 88 a^3 b x + 25 a^4}{b^9 x^4 + 4 a b^8 x^3 + 6 a^2 b^7 x^2 + 4 a^3 b^6 x + a^4 b^5} + \frac{12 \log(bx + a)}{b^5} \right) \\ &- \frac{5}{6} d^2 e^3 \left(\frac{12 x^2}{(b^2 x^2 + 2 a b x + a^2)^{3/2} b^2} + \frac{8 a^2}{(b^2 x^2 + 2 a b x + a^2)^{3/2} b^4} + \frac{6 a}{b^6 (x + \frac{a}{b})^2} - \frac{8 a^2}{b^7 (x + \frac{a}{b})^3} - \frac{3 a^3}{b^8 (x + \frac{a}{b})^4} \right) \\ &- \frac{5}{12} d^4 e \left(\frac{4}{(b^2 x^2 + 2 a b x + a^2)^{3/2} b^2} - \frac{3 a}{b^6 (x + \frac{a}{b})^4} \right) \\ &- \frac{5}{6} d^3 e^2 \left(\frac{6}{b^5 (x + \frac{a}{b})^2} - \frac{8 a}{b^6 (x + \frac{a}{b})^3} + \frac{3 a^2}{b^7 (x + \frac{a}{b})^4} \right) - \frac{d^5}{4 b^5 (x + \frac{a}{b})^4} \end{aligned}$$

input `integrate((e*x+d)^5/(b^2*x^2+2*a*b*x+a^2)^(5/2),x, algorithm="maxima")`

output
$$\begin{aligned} & 1/12*e^5*((12*b^5*x^5 + 48*a*b^4*x^4 - 48*a^2*b^3*x^3 - 252*a^3*b^2*x^2 - \\ & 248*a^4*b*x - 77*a^5)/(b^{10}*x^4 + 4*a*b^9*x^3 + 6*a^2*b^8*x^2 + 4*a^3*b^7*x \\ & x + a^4*b^6) - 60*a*\log(b*x + a)/b^6) + 5/12*d*e^4*((48*a*b^3*x^3 + 108*a^2 \\ & *b^2*x^2 + 88*a^3*b*x + 25*a^4)/(b^9*x^4 + 4*a*b^8*x^3 + 6*a^2*b^7*x^2 + \\ & 4*a^3*b^6*x + a^4*b^5) + 12*\log(b*x + a)/b^5) - 5/6*d^2*e^3*(12*x^2/((b^2*x^2 \\ & + 2*a*b*x + a^2)^{(3/2)}*b^2) + 8*a^2/((b^2*x^2 + 2*a*b*x + a^2)^{(3/2)}*b^4) \\ & + 6*a/(b^6*(x + a/b)^2) - 8*a^2/(b^7*(x + a/b)^3) - 3*a^3/(b^8*(x + a/ \\ & b)^4)) - 5/12*d^4*e*(4/((b^2*x^2 + 2*a*b*x + a^2)^{(3/2)}*b^2) - 3*a/(b^6*(x \\ & + a/b)^4)) - 5/6*d^3*e^2*(6/(b^5*(x + a/b)^2) - 8*a/(b^6*(x + a/b)^3) + 3 \\ & *a^2/(b^7*(x + a/b)^4)) - 1/4*d^5/(b^5*(x + a/b)^4) \end{aligned}$$

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 281, normalized size of antiderivative = 1.11

$$\int \frac{(d + ex)^5}{(a^2 + 2abx + b^2x^2)^{5/2}} dx = \frac{e^5x}{b^5 \operatorname{sgn}(bx + a)} + \frac{5(bde^4 - ae^5) \log(|bx + a|)}{b^6 \operatorname{sgn}(bx + a)}$$

$$\frac{3b^5d^5 + 5ab^4d^4e + 10a^2b^3d^3e^2 + 30a^3b^2d^2e^3 - 125a^4bde^4 + 77a^5e^5 + 120(b^5d^2e^3 - 2ab^4de^4 + a^2b^3e^5)}{12(bx + a)}$$

input `integrate((e*x+d)^5/(b^2*x^2+2*a*b*x+a^2)^(5/2),x, algorithm="giac")`

output
$$\begin{aligned} & e^5*x/(b^5*\operatorname{sgn}(b*x + a)) + 5*(b*d*e^4 - a*e^5)*\log(\operatorname{abs}(b*x + a))/(b^6*\operatorname{sgn}(\\ & b*x + a)) - 1/12*(3*b^5*d^5 + 5*a*b^4*d^4*e + 10*a^2*b^3*d^3*e^2 + 30*a^3*b^2*d^2*e^3 - \\ & 125*a^4*b*d*e^4 + 77*a^5*e^5 + 120*(b^5*d^2*e^3 - 2*a*b^4*d*e^4 + a^2*b^3*d^2*e^3 \\ & + 60*(b^5*d^3*e^2 + 3*a*b^4*d^2*e^3 - 9*a^2*b^3*d*e^4 + 5*a^3*b^2*d*e^5)*x^2 + 20*(b^5*d^4*e + 2*a*b^4*d^3*e^2 + 6*a^2*b^3*d^2* \\ & e^3 - 22*a^3*b^2*d*e^4 + 13*a^4*b*d*e^5)*x)/((b*x + a)^4*b^6*\operatorname{sgn}(b*x + a)) \end{aligned}$$

Mupad [F(-1)]

Timed out.

$$\int \frac{(d + ex)^5}{(a^2 + 2abx + b^2x^2)^{5/2}} dx = \int \frac{(d + ex)^5}{(a^2 + 2abx + b^2x^2)^{5/2}} dx$$

input `int((d + e*x)^5/(a^2 + b^2*x^2 + 2*a*b*x)^(5/2), x)`

output `int((d + e*x)^5/(a^2 + b^2*x^2 + 2*a*b*x)^(5/2), x)`

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 434, normalized size of antiderivative = 1.72

$$\int \frac{(d + ex)^5}{(a^2 + 2abx + b^2x^2)^{5/2}} dx = \frac{-3ab^5d^5 + 60 \log(bx + a) a^5bd e^4 - 240 \log(bx + a) a^5b e^5x + 240 \log(bx + a)}$$

input `int((e*x+d)^5/(b^2*x^2+2*a*b*x+a^2)^(5/2), x)`

output `(- 60*log(a + b*x)*a**6*e**5 + 60*log(a + b*x)*a**5*b*d*e**4 - 240*log(a + b*x)*a**5*b*e**5*x + 240*log(a + b*x)*a**4*b**2*d*e**4*x - 360*log(a + b*x)*a**4*b**2*e**5*x**2 + 360*log(a + b*x)*a**3*b**3*d*e**4*x**2 - 240*log(a + b*x)*a**3*b**3*e**5*x**3 + 240*log(a + b*x)*a**2*b**4*d*e**4*x**3 - 60*log(a + b*x)*a**2*b**4*e**5*x**4 + 60*log(a + b*x)*a*b**5*d*e**4*x**4 - 65*a**6*e**5 + 65*a**5*b*d*e**4 - 200*a**5*b*e**5*x + 200*a**4*b**2*d*e**4*x - 180*a**4*b**2*e**5*x**2 - 10*a**3*b**3*d**3*e**2 + 180*a**3*b**3*d*e**4*x**2 - 5*a**2*b**4*d**4*e - 40*a**2*b**4*d**3*e**2*x + 60*a**2*b**4*e**5*x**4 - 3*a*b**5*d**5 - 20*a*b**5*d**4*e*x - 60*a*b**5*d**3*e**2*x**2 - 60*a*b**5*d*e**4*x**4 + 12*a*b**5*e**5*x**5 + 30*b**6*d**2*e**3*x**4)/(12*a*b**6*(a**4 + 4*a**3*b*x + 6*a**2*b**2*x**2 + 4*a*b**3*x**3 + b**4*x**4))`

$$3.328 \quad \int \frac{(d+ex)^4}{(a^2+2abx+b^2x^2)^{5/2}} dx$$

| | |
|---|------|
| Optimal result | 2455 |
| Mathematica [A] (verified) | 2456 |
| Rubi [A] (verified) | 2456 |
| Maple [A] (verified) | 2458 |
| Fricas [A] (verification not implemented) | 2458 |
| Sympy [F] | 2459 |
| Maxima [B] (verification not implemented) | 2459 |
| Giac [A] (verification not implemented) | 2460 |
| Mupad [F(-1)] | 2460 |
| Reduce [B] (verification not implemented) | 2461 |

Optimal result

Integrand size = 28, antiderivative size = 209

$$\int \frac{(d+ex)^4}{(a^2+2abx+b^2x^2)^{5/2}} dx = -\frac{4e^3(bd-ae)}{b^5\sqrt{a^2+2abx+b^2x^2}} - \frac{(bd-ae)^4}{4b^5(a+bx)^3\sqrt{a^2+2abx+b^2x^2}} - \frac{4e(bd-ae)^3}{3b^5(a+bx)^2\sqrt{a^2+2abx+b^2x^2}} - \frac{3e^2(bd-ae)^2}{b^5(a+bx)\sqrt{a^2+2abx+b^2x^2}} + \frac{e^4(a+bx)\log(a+bx)}{b^5\sqrt{a^2+2abx+b^2x^2}}$$

output

```
-4*e^3*(-a*e+b*d)/b^5/((b*x+a)^2)^(1/2)-1/4*(-a*e+b*d)^4/b^5/(b*x+a)^3/((b*x+a)^2)^(1/2)-4/3*e*(-a*e+b*d)^3/b^5/(b*x+a)^2/((b*x+a)^2)^(1/2)-3*e^2*(-a*e+b*d)^2/b^5/(b*x+a)/((b*x+a)^2)^(1/2)+e^4*(b*x+a)*ln(b*x+a)/b^5/((b*x+a)^2)^(1/2)
```

Mathematica [A] (verified)

Time = 1.06 (sec) , antiderivative size = 138, normalized size of antiderivative = 0.66

$$\int \frac{(d+ex)^4}{(a^2+2abx+b^2x^2)^{5/2}} dx = \frac{-((bd-ae)(25a^3e^3+a^2be^2(13d+88ex)+ab^2e(7d^2+40dex+108e^2x^2))+12b^5(a+bx)^3\sqrt{a+bx})}{12b^5(a+bx)^3\sqrt{a+bx}}$$

input `Integrate[(d + e*x)^4/(a^2 + 2*a*b*x + b^2*x^2)^(5/2), x]`

output `((-((b*d - a*e)*(25*a^3*e^3 + a^2*b*e^2*(13*d + 88*e*x) + a*b^2*e*(7*d^2 + 40*d*e*x + 108*e^2*x^2) + b^3*(3*d^3 + 16*d^2*e*x + 36*d*e^2*x^2 + 48*e^3*x^3))) + 12*e^4*(a + b*x)^4*Log[a + b*x])/(12*b^5*(a + b*x)^3*sqrt[(a + b*x)^2])`

Rubi [A] (verified)

Time = 0.54 (sec) , antiderivative size = 137, normalized size of antiderivative = 0.66, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {1102, 27, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\begin{aligned} & \int \frac{(d+ex)^4}{(a^2+2abx+b^2x^2)^{5/2}} dx \\ & \quad \downarrow 1102 \\ & \frac{b^5(a+bx) \int \frac{(d+ex)^4}{b^5(a+bx)^5} dx}{\sqrt{a^2+2abx+b^2x^2}} \\ & \quad \downarrow 27 \\ & \frac{(a+bx) \int \frac{(d+ex)^4}{(a+bx)^5} dx}{\sqrt{a^2+2abx+b^2x^2}} \\ & \quad \downarrow 49 \\ & \frac{(a+bx) \int \left(\frac{e^4}{b^4(a+bx)} + \frac{4(bd-ae)e^3}{b^4(a+bx)^2} + \frac{6(bd-ae)^2e^2}{b^4(a+bx)^3} + \frac{4(bd-ae)^3e}{b^4(a+bx)^4} + \frac{(bd-ae)^4}{b^4(a+bx)^5} \right) dx}{\sqrt{a^2+2abx+b^2x^2}} \end{aligned}$$

$$\frac{(a+bx) \left(-\frac{4e^3(bd-ae)}{b^5(a+bx)} - \frac{3e^2(bd-ae)^2}{b^5(a+bx)^2} - \frac{4e(bd-ae)^3}{3b^5(a+bx)^3} - \frac{(bd-ae)^4}{4b^5(a+bx)^4} + \frac{e^4 \log(a+bx)}{b^5} \right)}{\sqrt{a^2 + 2abx + b^2x^2}}$$

input `Int[(d + e*x)^4/(a^2 + 2*a*b*x + b^2*x^2)^(5/2),x]`

output `((a + b*x)*(-1/4*(b*d - a*e)^4/(b^5*(a + b*x)^4) - (4*e*(b*d - a*e)^3)/(3*b^5*(a + b*x)^3) - (3*e^2*(b*d - a*e)^2)/(b^5*(a + b*x)^2) - (4*e^3*(b*d - a*e))/(b^5*(a + b*x)) + (e^4*Log[a + b*x]/b^5))/Sqrt[a^2 + 2*a*b*x + b^2*x^2]`

Defintions of rubi rules used

rule 27 `Int[(a_)*(F_x_), x_Symbol] := Simp[a Int[F_x, x], x] /; FreeQ[a, x] && !MatchQ[F_x, (b_)*(G_x_) /; FreeQ[b, x]]`

rule 49 `Int[((a_) + (b_)*(x_))^(m_)*((c_) + (d_)*(x_))^(n_), x_Symbol] := Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] && IGtQ[m, 0] && IGtQ[m + n + 2, 0]`

rule 1102 `Int[((d_) + (e_)*(x_))^(m_)*((a_) + (b_)*(x_) + (c_)*(x_)^2)^(p_), x_Symbol] := Simp[(a + b*x + c*x^2)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x)^(2*FracPart[p])) Int[(d + e*x)^m*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && EqQ[b^2 - 4*a*c, 0]`

rule 2009 `Int[u_, x_Symbol] := Simp[IntSum[u, x], x] /; SumQ[u]`

Sympy [F]

$$\int \frac{(d + ex)^4}{(a^2 + 2abx + b^2x^2)^{5/2}} dx = \int \frac{(d + ex)^4}{((a + bx)^2)^{5/2}} dx$$

input `integrate((e*x+d)**4/(b**2*x**2+2*a*b*x+a**2)**(5/2),x)`

output `Integral((d + e*x)**4/((a + b*x)**2)**(5/2), x)`

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 323 vs. $2(150) = 300$.

Time = 0.05 (sec) , antiderivative size = 323, normalized size of antiderivative = 1.55

$$\begin{aligned} \int \frac{(d + ex)^4}{(a^2 + 2abx + b^2x^2)^{5/2}} dx &= \frac{1}{12} e^4 \left(\frac{48 ab^3 x^3 + 108 a^2 b^2 x^2 + 88 a^3 b x + 25 a^4}{b^9 x^4 + 4 ab^8 x^3 + 6 a^2 b^7 x^2 + 4 a^3 b^6 x + a^4 b^5} + \frac{12 \log(bx + a)}{b^5} \right) \\ &- \frac{1}{3} d e^3 \left(\frac{12 x^2}{(b^2 x^2 + 2 abx + a^2)^{3/2} b^2} + \frac{8 a^2}{(b^2 x^2 + 2 abx + a^2)^{3/2} b^4} + \frac{6 a}{b^6 (x + \frac{a}{b})^2} - \frac{8 a^2}{b^7 (x + \frac{a}{b})^3} - \frac{3 a^3}{b^8 (x + \frac{a}{b})^4} \right) \\ &- \frac{1}{3} d^3 e \left(\frac{4}{(b^2 x^2 + 2 abx + a^2)^{3/2} b^2} - \frac{3 a}{b^6 (x + \frac{a}{b})^4} \right) \\ &- \frac{1}{2} d^2 e^2 \left(\frac{6}{b^5 (x + \frac{a}{b})^2} - \frac{8 a}{b^6 (x + \frac{a}{b})^3} + \frac{3 a^2}{b^7 (x + \frac{a}{b})^4} \right) - \frac{d^4}{4 b^5 (x + \frac{a}{b})^4} \end{aligned}$$

input `integrate((e*x+d)^4/(b^2*x^2+2*a*b*x+a^2)^(5/2),x, algorithm="maxima")`

output `1/12*e^4*((48*a*b^3*x^3 + 108*a^2*b^2*x^2 + 88*a^3*b*x + 25*a^4)/(b^9*x^4 + 4*a*b^8*x^3 + 6*a^2*b^7*x^2 + 4*a^3*b^6*x + a^4*b^5) + 12*log(b*x + a)/b^5) - 1/3*d*e^3*(12*x^2/((b^2*x^2 + 2*a*b*x + a^2)^(3/2)*b^2) + 8*a^2/((b^2*x^2 + 2*a*b*x + a^2)^(3/2)*b^4) + 6*a/(b^6*(x + a/b)^2) - 8*a^2/(b^7*(x + a/b)^3) - 3*a^3/(b^8*(x + a/b)^4)) - 1/3*d^3*e*(4/((b^2*x^2 + 2*a*b*x + a^2)^(3/2)*b^2) - 3*a/(b^6*(x + a/b)^4)) - 1/2*d^2*e^2*(6/(b^5*(x + a/b)^2) - 8*a/(b^6*(x + a/b)^3) + 3*a^2/(b^7*(x + a/b)^4)) - 1/4*d^4/(b^5*(x + a/b)^4)`

Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 200, normalized size of antiderivative = 0.96

$$\int \frac{(d+ex)^4}{(a^2+2abx+b^2x^2)^{5/2}} dx = \frac{e^4 \log(|bx+a|)}{b^5 \operatorname{sgn}(bx+a)} - \frac{48(b^3de^3 - ab^2e^4)x^3 + 36(b^3d^2e^2 + 2ab^2de^3 - 3a^2be^4)x^2 + 8(2b^3d^3e + 3ab^2d^2e^2 + 6a^2bde^3 - 11a^3e^4)x + (3b^4d^4 + 4a^3b^3d^3e + 6a^2b^2d^2e^2 + 12a^3b^2de^3 - 25a^4e^4)/b}{12(bx+a)^4 b^4 \operatorname{sgn}(bx+a)}$$

input `integrate((e*x+d)^4/(b^2*x^2+2*a*b*x+a^2)^(5/2),x, algorithm="giac")`

output

```
e^4*log(abs(b*x + a))/(b^5*sgn(b*x + a)) - 1/12*(48*(b^3*d*e^3 - a*b^2*e^4)*x^3 + 36*(b^3*d^2*e^2 + 2*a*b^2*d*e^3 - 3*a^2*b*e^4)*x^2 + 8*(2*b^3*d^3*e + 3*a*b^2*d^2*e^2 + 6*a^2*b*d*e^3 - 11*a^3*e^4)*x + (3*b^4*d^4 + 4*a*b^3*d^3*e + 6*a^2*b^2*d^2*e^2 + 12*a^3*b*d*e^3 - 25*a^4*e^4)/b)/((b*x + a)^4*b^4*sgn(b*x + a))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d+ex)^4}{(a^2+2abx+b^2x^2)^{5/2}} dx = \int \frac{(d+ex)^4}{(a^2+2abx+b^2x^2)^{5/2}} dx$$

input `int((d + e*x)^4/(a^2 + b^2*x^2 + 2*a*b*x)^(5/2),x)`

output

```
int((d + e*x)^4/(a^2 + b^2*x^2 + 2*a*b*x)^(5/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 269, normalized size of antiderivative = 1.29

$$\int \frac{(d + ex)^4}{(a^2 + 2abx + b^2x^2)^{5/2}} dx = \frac{12 \log(bx + a) a^5 e^4 + 48 \log(bx + a) a^4 b e^4 x + 72 \log(bx + a) a^3 b^2 e^4 x^2 + 48 \log(bx + a) a^2 b^3 e^4 x^3 + 12 \log(bx + a) a b^4 e^4 x^4 + 13 a^5 e^4 + 40 a^4 b e^4 x - 6 a^3 b^2 d e^4 x^2 + 36 a^3 b^2 e^4 x^2 - 4 a^2 b^3 d e^4 x - 24 a^2 b^3 d^2 e^4 x - 3 a b^4 d^2 e^4 - 16 a b^4 d^3 e^4 x - 36 a b^4 d^2 e^4 x^2 - 12 a b^4 e^4 x^4 + 12 b^5 d e^4 x^4}{(12 a b^5 (a^4 + 4 a^3 b x + 6 a^2 b^2 x^2 + 4 a b^3 x^3 + b^4 x^4))}$$

input `int((e*x+d)^4/(b^2*x^2+2*a*b*x+a^2)^(5/2),x)`output `(12*log(a + b*x)*a**5*e**4 + 48*log(a + b*x)*a**4*b*e**4*x + 72*log(a + b*x)*a**3*b**2*e**4*x**2 + 48*log(a + b*x)*a**2*b**3*e**4*x**3 + 12*log(a + b*x)*a*b**4*e**4*x**4 + 13*a**5*e**4 + 40*a**4*b*e**4*x - 6*a**3*b**2*d**2*e**4*x**2 + 36*a**3*b**2*e**4*x**2 - 4*a**2*b**3*d**3*e - 24*a**2*b**3*d**2*e**4*x - 3*a*b**4*d**4 - 16*a*b**4*d**3*e*x - 36*a*b**4*d**2*e**2*x**2 - 12*a*b**4*e**4*x**4 + 12*b**5*d*e**3*x**4)/(12*a*b**5*(a**4 + 4*a**3*b*x + 6*a**2*b**2*x**2 + 4*a*b**3*x**3 + b**4*x**4))`

$$3.329 \quad \int \frac{(d+ex)^3}{(a^2+2abx+b^2x^2)^{5/2}} dx$$

| | |
|---|------|
| Optimal result | 2462 |
| Mathematica [B] (verified) | 2462 |
| Rubi [A] (verified) | 2463 |
| Maple [B] (verified) | 2464 |
| Fricas [B] (verification not implemented) | 2465 |
| Sympy [F] | 2465 |
| Maxima [B] (verification not implemented) | 2466 |
| Giac [B] (verification not implemented) | 2466 |
| Mupad [B] (verification not implemented) | 2467 |
| Reduce [B] (verification not implemented) | 2468 |

Optimal result

Integrand size = 28, antiderivative size = 46

$$\int \frac{(d+ex)^3}{(a^2+2abx+b^2x^2)^{5/2}} dx = -\frac{(a+bx)(d+ex)^4}{4(bd-ae)(a^2+2abx+b^2x^2)^{5/2}}$$

output

$$-1/4*(b*x+a)*(e*x+d)^4/(-a*e+b*d)/(b^2*x^2+2*a*b*x+a^2)^(5/2)$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 106 vs. $2(46) = 92$.

Time = 1.04 (sec) , antiderivative size = 106, normalized size of antiderivative = 2.30

$$\int \frac{(d+ex)^3}{(a^2+2abx+b^2x^2)^{5/2}} dx = \frac{-a^3e^3 - a^2be^2(d+4ex) - ab^2e(d^2+4dex+6e^2x^2) - b^3(d^3+4d^2ex+6de^2x)}{4b^4(a+bx)^3\sqrt{(a+bx)^2}}$$

input

$$\text{Integrate}[(d+e*x)^3/(a^2+2*a*b*x+b^2*x^2)^(5/2),x]$$

output

$$\frac{-(a^3e^3) - a^2b^2e^2(d+4e*x) - a*b^2e*(d^2+4*d*e*x+6*e^2*x^2) - b^3*(d^3+4*d^2*e*x+6*d*e^2*x^2+4*e^3*x^3)}{4*b^4*(a+b*x)^3*\text{Sqrt}[(a+b*x)^2]}$$

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.04, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.107$, Rules used = {1102, 27, 48}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\begin{aligned}
 & \int \frac{(d+ex)^3}{(a^2+2abx+b^2x^2)^{5/2}} dx \\
 & \quad \downarrow \text{1102} \\
 & \frac{b^5(a+bx) \int \frac{(d+ex)^3}{b^5(a+bx)^5} dx}{\sqrt{a^2+2abx+b^2x^2}} \\
 & \quad \downarrow \text{27} \\
 & \frac{(a+bx) \int \frac{(d+ex)^3}{(a+bx)^5} dx}{\sqrt{a^2+2abx+b^2x^2}} \\
 & \quad \downarrow \text{48} \\
 & \frac{(d+ex)^4}{4(a+bx)^3 \sqrt{a^2+2abx+b^2x^2} (bd-ae)}
 \end{aligned}$$

input `Int[(d + e*x)^3/(a^2 + 2*a*b*x + b^2*x^2)^(5/2),x]`

output `-1/4*(d + e*x)^4/((b*d - a*e)*(a + b*x)^3*Sqrt[a^2 + 2*a*b*x + b^2*x^2])`

Defintions of rubi rules used

rule 27 `Int[(a_)*(F_x_), x_Symbol] :> Simp[a Int[F_x, x], x] /; FreeQ[a, x] && !MatchQ[F_x, (b_)*(G_x_) /; FreeQ[b, x]]`

rule 48 `Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] := Simp`
`[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /;` `FreeQ[{`
`a, b, c, d, m, n}, x] && EqQ[m + n + 2, 0] && NeQ[m, -1]`

rule 1102 `Int[((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S`
`ymbol] := Simp[(a + b*x + c*x^2)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x)^(2*F`
`racPart[p])) Int[(d + e*x)^m*(b/2 + c*x)^(2*p), x], x] /;` `FreeQ[{a, b, c,`
`d, e, m, p}, x] && EqQ[b^2 - 4*a*c, 0]`

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 112 vs. 2(42) = 84.

Time = 1.46 (sec) , antiderivative size = 113, normalized size of antiderivative = 2.46

| method | result | size |
|---------|--|------|
| risch | $\frac{\sqrt{(bx+a)^2} \left(-\frac{e^3 x^3}{b} - \frac{3e^2 (ae+bd)x^2}{2b^2} - \frac{e(e^2 a^2 + abde + b^2 d^2)x}{b^3} - \frac{e^3 a^3 + a^2 bd e^2 + a b^2 d^2 e + b^3 d^3}{4b^4} \right)}{(bx+a)^5}$ | 113 |
| gospers | $-\frac{(bx+a)(4e^3 x^3 b^3 + 6x^2 a b^2 e^3 + 6x^2 b^3 d e^2 + 4a^2 b e^3 x + 4xa b^2 d e^2 + 4b^3 d^2 e x + e^3 a^3 + a^2 b d e^2 + a b^2 d^2 e + b^3 d^3)}{4b^4 (bx+a)^{\frac{5}{2}}}$ | 119 |
| default | $-\frac{(bx+a)(4e^3 x^3 b^3 + 6x^2 a b^2 e^3 + 6x^2 b^3 d e^2 + 4a^2 b e^3 x + 4xa b^2 d e^2 + 4b^3 d^2 e x + e^3 a^3 + a^2 b d e^2 + a b^2 d^2 e + b^3 d^3)}{4b^4 (bx+a)^{\frac{5}{2}}}$ | 119 |
| orering | $-\frac{(4e^3 x^3 b^3 + 6x^2 a b^2 e^3 + 6x^2 b^3 d e^2 + 4a^2 b e^3 x + 4xa b^2 d e^2 + 4b^3 d^2 e x + e^3 a^3 + a^2 b d e^2 + a b^2 d^2 e + b^3 d^3)(bx+a)}{4b^4 (b^2 x^2 + 2abx + a^2)^{\frac{5}{2}}}$ | 128 |

input `int((e*x+d)^3/(b^2*x^2+2*a*b*x+a^2)^(5/2), x, method=_RETURNVERBOSE)`

output `((b*x+a)^(1/2)/(b*x+a)^5*(-e^3/b*x^3-3/2*e^2*(a*e+b*d)/b^2*x^2-e*(a^2*e`
`^2+a*b*d*e+b^2*d^2)/b^3*x-1/4*(a^3*e^3+a^2*b*d*e^2+a*b^2*d^2*e+b^3*d^3)/b^`
`4)`

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 143 vs. $2(42) = 84$.

Time = 0.08 (sec) , antiderivative size = 143, normalized size of antiderivative = 3.11

$$\int \frac{(d+ex)^3}{(a^2+2abx+b^2x^2)^{5/2}} dx = \frac{4b^3e^3x^3 + b^3d^3 + ab^2d^2e + a^2bde^2 + a^3e^3 + 6(b^3de^2 + ab^2e^3)x^2 + 4(b^3d^2e + ab^2de^2 + a^2be^3)x}{4(b^8x^4 + 4ab^7x^3 + 6a^2b^6x^2 + 4a^3b^5x + a^4b^4)}$$

input `integrate((e*x+d)^3/(b^2*x^2+2*a*b*x+a^2)^(5/2),x, algorithm="fricas")`

output `-1/4*(4*b^3*e^3*x^3 + b^3*d^3 + a*b^2*d^2*e + a^2*b*d*e^2 + a^3*e^3 + 6*(b^3*d*e^2 + a*b^2*e^3)*x^2 + 4*(b^3*d^2*e + a*b^2*d*e^2 + a^2*b*e^3)*x)/(b^8*x^4 + 4*a*b^7*x^3 + 6*a^2*b^6*x^2 + 4*a^3*b^5*x + a^4*b^4)`

Sympy [F]

$$\int \frac{(d+ex)^3}{(a^2+2abx+b^2x^2)^{5/2}} dx = \int \frac{(d+ex)^3}{((a+bx)^2)^{5/2}} dx$$

input `integrate((e*x+d)**3/(b**2*x**2+2*a*b*x+a**2)**(5/2),x)`

output `Integral((d + e*x)**3/((a + b*x)**2)**(5/2), x)`

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 238 vs. $2(42) = 84$.

Time = 0.05 (sec) , antiderivative size = 238, normalized size of antiderivative = 5.17

$$\int \frac{(d+ex)^3}{(a^2+2abx+b^2x^2)^{5/2}} dx = -\frac{e^3x^2}{(b^2x^2+2abx+a^2)^{3/2}b^2} - \frac{d^2e}{(b^2x^2+2abx+a^2)^{3/2}b^2}$$

$$- \frac{2a^2e^3}{3(b^2x^2+2abx+a^2)^{3/2}b^4} - \frac{3de^2}{2b^5(x+\frac{a}{b})^2} - \frac{ae^3}{2b^6(x+\frac{a}{b})^2} + \frac{2ade^2}{b^6(x+\frac{a}{b})^3}$$

$$+ \frac{2a^2e^3}{3b^7(x+\frac{a}{b})^3} - \frac{d^3}{4b^5(x+\frac{a}{b})^4} + \frac{3ad^2e}{4b^6(x+\frac{a}{b})^4} - \frac{3a^2de^2}{4b^7(x+\frac{a}{b})^4} + \frac{a^3e^3}{4b^8(x+\frac{a}{b})^4}$$

input `integrate((e*x+d)^3/(b^2*x^2+2*a*b*x+a^2)^(5/2),x, algorithm="maxima")`

output `-e^3*x^2/((b^2*x^2 + 2*a*b*x + a^2)^(3/2)*b^2) - d^2*e/((b^2*x^2 + 2*a*b*x + a^2)^(3/2)*b^2) - 2/3*a^2*e^3/((b^2*x^2 + 2*a*b*x + a^2)^(3/2)*b^4) - 3/2*d*e^2/(b^5*(x + a/b)^2) - 1/2*a*e^3/(b^6*(x + a/b)^2) + 2*a*d*e^2/(b^6*(x + a/b)^3) + 2/3*a^2*e^3/(b^7*(x + a/b)^3) - 1/4*d^3/(b^5*(x + a/b)^4) + 3/4*a*d^2*e/(b^6*(x + a/b)^4) - 3/4*a^2*d*e^2/(b^7*(x + a/b)^4) + 1/4*a^3*e^3/(b^8*(x + a/b)^4)`

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 119 vs. $2(42) = 84$.

Time = 0.15 (sec) , antiderivative size = 119, normalized size of antiderivative = 2.59

$$\int \frac{(d+ex)^3}{(a^2+2abx+b^2x^2)^{5/2}} dx = \frac{4b^3e^3x^3 + 6b^3de^2x^2 + 6ab^2e^3x^2 + 4b^3d^2ex + 4ab^2de^2x + 4a^2be^3x + b^3d^3 + ab^2d^2e + a^2bde^2 + a^3e^3}{4(bx+a)^4b^4\operatorname{sgn}(bx+a)}$$

input `integrate((e*x+d)^3/(b^2*x^2+2*a*b*x+a^2)^(5/2),x, algorithm="giac")`

output

```
-1/4*(4*b^3*e^3*x^3 + 6*b^3*d*e^2*x^2 + 6*a*b^2*e^3*x^2 + 4*b^3*d^2*e*x +
4*a*b^2*d*e^2*x + 4*a^2*b*e^3*x + b^3*d^3 + a*b^2*d^2*e + a^2*b*d*e^2 + a^
3*e^3)/((b*x + a)^4*b^4*sgn(b*x + a))
```

Mupad [B] (verification not implemented)

Time = 6.08 (sec) , antiderivative size = 254, normalized size of antiderivative = 5.52

$$\int \frac{(d+ex)^3}{(a^2+2abx+b^2x^2)^{5/2}} dx = \frac{\left(\frac{2ae^3-3bde^2}{2b^4} + \frac{ae^3}{2b^4}\right) \sqrt{a^2+2abx+b^2x^2}}{(a+bx)^3}$$

$$- \frac{\left(\frac{d^3}{4b} - \frac{a\left(\frac{3d^2e}{4b} + \frac{a\left(\frac{ae^3}{4b^2} - \frac{3de^2}{4b}\right)}{b}\right)}{b}\right) \sqrt{a^2+2abx+b^2x^2}}{(a+bx)^5}$$

$$- \frac{\left(\frac{a^2e^3-3abde^2+3b^2d^2e}{3b^4} + \frac{a\left(\frac{ae^3}{3b^3} + \frac{e^2(ae-3bd)}{3b^3}\right)}{b}\right) \sqrt{a^2+2abx+b^2x^2}}{(a+bx)^4}$$

$$- \frac{e^3 \sqrt{a^2+2abx+b^2x^2}}{b^4(a+bx)^2}$$

input

```
int((d + e*x)^3/(a^2 + b^2*x^2 + 2*a*b*x)^(5/2),x)
```

output

```
((2*a*e^3 - 3*b*d*e^2)/(2*b^4) + (a*e^3)/(2*b^4))*(a^2 + b^2*x^2 + 2*a*b*x)^(1/2)/(a + b*x)^3 - ((d^3/(4*b) - (a*((3*d^2*e)/(4*b) + (a*((a*e^3)/(4*b^2) - (3*d*e^2)/(4*b)))/b))/b)*(a^2 + b^2*x^2 + 2*a*b*x)^(1/2)/(a + b*x)^5 - (((a^2*e^3 + 3*b^2*d^2*e - 3*a*b*d*e^2)/(3*b^4) + (a*((a*e^3)/(3*b^3) + (e^2*(a*e - 3*b*d))/(3*b^3)))/b)*(a^2 + b^2*x^2 + 2*a*b*x)^(1/2)/(a + b*x)^4 - (e^3*(a^2 + b^2*x^2 + 2*a*b*x)^(1/2))/(b^4*(a + b*x)^2)
```


Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 122, normalized size of antiderivative = 2.65

$$\int \frac{(d + ex)^3}{(a^2 + 2abx + b^2x^2)^{5/2}} dx = \frac{b^3e^3x^4 - 6ab^2de^2x^2 - 4a^2bde^2x - 4ab^2d^2ex - a^3de^2 - a^2bd^2e - ab^2d^3}{4ab^3(b^4x^4 + 4ab^3x^3 + 6a^2b^2x^2 + 4a^3bx + a^4)}$$

input `int((e*x+d)^3/(b^2*x^2+2*a*b*x+a^2)^(5/2),x)`output `(- a**3*d*e**2 - a**2*b*d**2*e - 4*a**2*b*d*e**2*x - a*b**2*d**3 - 4*a*b*
*2*d**2*e*x - 6*a*b**2*d*e**2*x**2 + b**3*e**3*x**4)/(4*a*b**3*(a**4 + 4*a
3*b*x + 6*a2*b**2*x**2 + 4*a*b**3*x**3 + b**4*x**4))`

3.330
$$\int \frac{(d+ex)^2}{(a^2+2abx+b^2x^2)^{5/2}} dx$$

| | |
|---|------|
| Optimal result | 2469 |
| Mathematica [A] (verified) | 2469 |
| Rubi [A] (verified) | 2470 |
| Maple [A] (verified) | 2471 |
| Fricas [A] (verification not implemented) | 2472 |
| Sympy [F] | 2472 |
| Maxima [A] (verification not implemented) | 2473 |
| Giac [A] (verification not implemented) | 2473 |
| Mupad [B] (verification not implemented) | 2474 |
| Reduce [B] (verification not implemented) | 2474 |

Optimal result

Integrand size = 28, antiderivative size = 125

$$\int \frac{(d+ex)^2}{(a^2+2abx+b^2x^2)^{5/2}} dx = -\frac{(bd-ae)^2}{4b^3(a+bx)^3\sqrt{a^2+2abx+b^2x^2}} - \frac{2e(bd-ae)}{3b^3(a+bx)^2\sqrt{a^2+2abx+b^2x^2}} - \frac{e^2}{2b^3(a+bx)\sqrt{a^2+2abx+b^2x^2}}$$

output

```
-1/4*(-a*e+b*d)^2/b^3/(b*x+a)^3/((b*x+a)^2)^(1/2)-2/3*e*(-a*e+b*d)/b^3/(b*x+a)^2/((b*x+a)^2)^(1/2)-1/2*e^2/b^3/(b*x+a)/((b*x+a)^2)^(1/2)
```

Mathematica [A] (verified)

Time = 1.03 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.55

$$\int \frac{(d+ex)^2}{(a^2+2abx+b^2x^2)^{5/2}} dx = \frac{-a^2e^2 - 2abe(d+2ex) - b^2(3d^2 + 8dex + 6e^2x^2)}{12b^3(a+bx)^3\sqrt{(a+bx)^2}}$$

input

```
Integrate[(d + e*x)^2/(a^2 + 2*a*b*x + b^2*x^2)^(5/2), x]
```

output

$$\frac{-(a^2 e^2) - 2 a b e (d + 2 e x) - b^2 (3 d^2 + 8 d e x + 6 e^2 x^2)}{12 b^3 (a + b x)^3 \sqrt{(a + b x)^2}}$$

Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.73, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {1102, 27, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\begin{aligned} & \int \frac{(d + ex)^2}{(a^2 + 2abx + b^2x^2)^{5/2}} dx \\ & \quad \downarrow \text{1102} \\ & \frac{b^5(a + bx) \int \frac{(d+ex)^2}{b^5(a+bx)^5} dx}{\sqrt{a^2 + 2abx + b^2x^2}} \\ & \quad \downarrow \text{27} \\ & \frac{(a + bx) \int \frac{(d+ex)^2}{(a+bx)^5} dx}{\sqrt{a^2 + 2abx + b^2x^2}} \\ & \quad \downarrow \text{53} \\ & \frac{(a + bx) \int \left(\frac{e^2}{b^2(a+bx)^3} + \frac{2(bd-ae)e}{b^2(a+bx)^4} + \frac{(bd-ae)^2}{b^2(a+bx)^5} \right) dx}{\sqrt{a^2 + 2abx + b^2x^2}} \\ & \quad \downarrow \text{2009} \\ & \frac{(a + bx) \left(-\frac{2e(bd-ae)}{3b^3(a+bx)^3} - \frac{(bd-ae)^2}{4b^3(a+bx)^4} - \frac{e^2}{2b^3(a+bx)^2} \right)}{\sqrt{a^2 + 2abx + b^2x^2}} \end{aligned}$$

input

$$\text{Int}[(d + e*x)^2/(a^2 + 2*a*b*x + b^2*x^2)^(5/2), x]$$

output

$$\frac{((a + b*x)*(-1/4*(b*d - a*e)^2/(b^3*(a + b*x)^4) - (2*e*(b*d - a*e))/(3*b^3*(a + b*x)^3) - e^2/(2*b^3*(a + b*x)^2))}{\sqrt{a^2 + 2*a*b*x + b^2*x^2}}$$

Defintions of rubi rules used

rule 27 `Int[(a_)*(F_x_), x_Symbol] := Simp[a Int[F_x, x], x] /; FreeQ[a, x] && !MatchQ[F_x, (b_)*(G_x_)] /; FreeQ[b, x]`

rule 53 `Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] := Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && IGtQ[m, 0] && (!IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])`

rule 1102 `Int[((d_.) + (e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] := Simp[(a + b*x + c*x^2)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x)^(2*FracPart[p])) Int[(d + e*x)^m*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && EqQ[b^2 - 4*a*c, 0]`

rule 2009 `Int[u_, x_Symbol] := Simp[IntSum[u, x], x] /; SumQ[u]`

Maple [A] (verified)

Time = 1.29 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.55

| method | result | size |
|---------|---|------|
| gospers | $-\frac{(bx+a)(6b^2e^2x^2+4xab e^2+8b^2dex+e^2a^2+2abde+3b^2d^2)}{12b^3((bx+a)^2)^{\frac{5}{2}}}$ | 69 |
| default | $-\frac{(bx+a)(6b^2e^2x^2+4xab e^2+8b^2dex+e^2a^2+2abde+3b^2d^2)}{12b^3((bx+a)^2)^{\frac{5}{2}}}$ | 69 |
| risch | $\frac{\sqrt{(bx+a)^2} \left(-\frac{e^2x^2}{2b} - \frac{e(ae+2bd)x}{3b^2} - \frac{e^2a^2+2abde+3b^2d^2}{12b^3} \right)}{(bx+a)^5}$ | 72 |
| orering | $-\frac{(6b^2e^2x^2+4xab e^2+8b^2dex+e^2a^2+2abde+3b^2d^2)(bx+a)}{12b^3(b^2x^2+2abx+a^2)^{\frac{5}{2}}}$ | 78 |

input `int((e*x+d)^2/(b^2*x^2+2*a*b*x+a^2)^(5/2), x, method=_RETURNVERBOSE)`

output

```
-1/12*(b*x+a)/b^3*(6*b^2*e^2*x^2+4*a*b*e^2*x+8*b^2*d*e*x+a^2*e^2+2*a*b*d*e
+3*b^2*d^2)/((b*x+a)^2)^(5/2)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 98, normalized size of antiderivative = 0.78

$$\int \frac{(d+ex)^2}{(a^2+2abx+b^2x^2)^{5/2}} dx = -\frac{6b^2e^2x^2+3b^2d^2+2abde+a^2e^2+4(2b^2de+abe^2)x}{12(b^7x^4+4ab^6x^3+6a^2b^5x^2+4a^3b^4x+a^4b^3)}$$

input

```
integrate((e*x+d)^2/(b^2*x^2+2*a*b*x+a^2)^(5/2),x, algorithm="fricas")
```

output

```
-1/12*(6*b^2*e^2*x^2 + 3*b^2*d^2 + 2*a*b*d*e + a^2*e^2 + 4*(2*b^2*d*e + a*
b*e^2)*x)/(b^7*x^4 + 4*a*b^6*x^3 + 6*a^2*b^5*x^2 + 4*a^3*b^4*x + a^4*b^3)
```

Sympy [F]

$$\int \frac{(d+ex)^2}{(a^2+2abx+b^2x^2)^{5/2}} dx = \int \frac{(d+ex)^2}{((a+bx)^2)^{\frac{5}{2}}} dx$$

input

```
integrate((e*x+d)**2/(b**2*x**2+2*a*b*x+a**2)**(5/2),x)
```

output

```
Integral((d + e*x)**2/((a + b*x)**2)**(5/2), x)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 115, normalized size of antiderivative = 0.92

$$\int \frac{(d+ex)^2}{(a^2+2abx+b^2x^2)^{5/2}} dx = -\frac{2de}{3(b^2x^2+2abx+a^2)^{3/2}b^2} - \frac{e^2}{2b^5(x+\frac{a}{b})^2}$$

$$+ \frac{2ae^2}{3b^6(x+\frac{a}{b})^3} - \frac{d^2}{4b^5(x+\frac{a}{b})^4} + \frac{ade}{2b^6(x+\frac{a}{b})^4} - \frac{a^2e^2}{4b^7(x+\frac{a}{b})^4}$$

input `integrate((e*x+d)^2/(b^2*x^2+2*a*b*x+a^2)^(5/2),x, algorithm="maxima")`

output `-2/3*d*e/((b^2*x^2 + 2*a*b*x + a^2)^(3/2)*b^2) - 1/2*e^2/(b^5*(x + a/b)^2) + 2/3*a*e^2/(b^6*(x + a/b)^3) - 1/4*d^2/(b^5*(x + a/b)^4) + 1/2*a*d*e/(b^6*(x + a/b)^4) - 1/4*a^2*e^2/(b^7*(x + a/b)^4)`

Giac [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.55

$$\int \frac{(d+ex)^2}{(a^2+2abx+b^2x^2)^{5/2}} dx = -\frac{6b^2e^2x^2 + 8b^2dex + 4abe^2x + 3b^2d^2 + 2abde + a^2e^2}{12(bx+a)^4b^3\operatorname{sgn}(bx+a)}$$

input `integrate((e*x+d)^2/(b^2*x^2+2*a*b*x+a^2)^(5/2),x, algorithm="giac")`

output `-1/12*(6*b^2*e^2*x^2 + 8*b^2*d*e*x + 4*a*b*e^2*x + 3*b^2*d^2 + 2*a*b*d*e + a^2*e^2)/((b*x + a)^4*b^3*sgn(b*x + a))`

Mupad [B] (verification not implemented)

Time = 6.06 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.63

$$\int \frac{(d + ex)^2}{(a^2 + 2abx + b^2x^2)^{5/2}} dx = \frac{\sqrt{a^2 + 2abx + b^2x^2} (a^2 e^2 + 2abde + 4abe^2x + 3b^2d^2 + 8b^2dex + 6b^2e^2x^2)}{12b^3(a + bx)^5}$$

input `int((d + e*x)^2/(a^2 + b^2*x^2 + 2*a*b*x)^(5/2),x)`output `-((a^2 + b^2*x^2 + 2*a*b*x)^(1/2)*(a^2*e^2 + 3*b^2*d^2 + 6*b^2*e^2*x^2 + 4*a*b*e^2*x + 8*b^2*d*e*x + 2*a*b*d*e))/(12*b^3*(a + b*x)^5)`**Reduce [B] (verification not implemented)**

Time = 0.26 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.76

$$\int \frac{(d + ex)^2}{(a^2 + 2abx + b^2x^2)^{5/2}} dx = \frac{-6b^2e^2x^2 - 4abe^2x - 8b^2dex - a^2e^2 - 2abde - 3b^2d^2}{12b^3(b^4x^4 + 4ab^3x^3 + 6a^2b^2x^2 + 4a^3bx + a^4)}$$

input `int((e*x+d)^2/(b^2*x^2+2*a*b*x+a^2)^(5/2),x)`output `(- a**2*e**2 - 2*a*b*d*e - 4*a*b*e**2*x - 3*b**2*d**2 - 8*b**2*d*e*x - 6*b**2*e**2*x**2)/(12*b**3*(a**4 + 4*a**3*b*x + 6*a**2*b**2*x**2 + 4*a*b**3*x**3 + b**4*x**4))`

3.331 $\int \frac{d+ex}{(a^2+2abx+b^2x^2)^{5/2}} dx$

| | |
|---|------|
| Optimal result | 2475 |
| Mathematica [B] (verified) | 2475 |
| Rubi [A] (verified) | 2476 |
| Maple [A] (verified) | 2477 |
| Fricas [A] (verification not implemented) | 2478 |
| Sympy [F] | 2478 |
| Maxima [A] (verification not implemented) | 2478 |
| Giac [A] (verification not implemented) | 2479 |
| Mupad [B] (verification not implemented) | 2479 |
| Reduce [B] (verification not implemented) | 2479 |

Optimal result

Integrand size = 26, antiderivative size = 71

$$\int \frac{d+ex}{(a^2+2abx+b^2x^2)^{5/2}} dx = -\frac{e}{3b^2(a^2+2abx+b^2x^2)^{3/2}} - \frac{bd-ae}{4b^2(a+bx)^3\sqrt{a^2+2abx+b^2x^2}}$$

output

$$-1/3*e/b^2/(b^2*x^2+2*a*b*x+a^2)^(3/2)-1/4*(-a*e+b*d)/b^2/(b*x+a)^3/((b*x+a)^2)^(1/2)$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 345 vs. 2(71) = 142.

Time = 1.01 (sec) , antiderivative size = 345, normalized size of antiderivative = 4.86

$$\int \frac{d+ex}{(a^2+2abx+b^2x^2)^{5/2}} dx = \frac{x \left(-3\sqrt{a^2}b^7dx^7 - 3a^3b^4x^4\sqrt{(a+bx)^2}(d+ex) + 3a^2b^5x^5\sqrt{(a+bx)^2}(d+ex) + 6a^7(2d+ex) \left(\sqrt{a^2} - \sqrt{a^2+2abx+b^2x^2} \right) \right)}{4b^2(a+bx)^3\sqrt{a^2+2abx+b^2x^2}}$$

input `Integrate[(d + e*x)/(a^2 + 2*a*b*x + b^2*x^2)^(5/2),x]`

output `-1/12*(x*(-3*Sqrt[a^2]*b^7*d*x^7 - 3*a^3*b^4*x^4*Sqrt[(a + b*x)^2]*(d + e*x) + 3*a^2*b^5*x^5*Sqrt[(a + b*x)^2]*(d + e*x) + 6*a^7*(2*d + e*x)*(Sqrt[a^2] - Sqrt[(a + b*x)^2]) + 3*a*b^6*x^6*(Sqrt[a^2]*e*x - d*Sqrt[(a + b*x)^2]) + a^5*b^2*x^2*(12*Sqrt[a^2]*d - 6*d*Sqrt[(a + b*x)^2] + e*x*(Sqrt[a^2] - 3*Sqrt[(a + b*x)^2])) + 3*a^4*b^3*x^3*(e*x*Sqrt[(a + b*x)^2] + d*(Sqrt[a^2] + Sqrt[(a + b*x)^2])) + 2*a^6*b*x*(9*Sqrt[a^2]*d - 3*d*Sqrt[(a + b*x)^2] + e*x*(2*Sqrt[a^2] + Sqrt[(a + b*x)^2]))))/(a^8*(a + b*x)^3*(a^2 + a*b*x - Sqrt[a^2]*Sqrt[(a + b*x)^2]))`

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {1100, 1078}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{d + ex}{(a^2 + 2abx + b^2x^2)^{5/2}} dx$$

$$\downarrow 1100$$

$$\frac{(bd - ae) \int \frac{1}{(a^2 + 2bxa + b^2x^2)^{5/2}} dx}{b} - \frac{e}{3b^2 (a^2 + 2abx + b^2x^2)^{3/2}}$$

$$\downarrow 1078$$

$$-\frac{bd - ae}{4b^2(a + bx) (a^2 + 2abx + b^2x^2)^{3/2}} - \frac{e}{3b^2 (a^2 + 2abx + b^2x^2)^{3/2}}$$

input `Int[(d + e*x)/(a^2 + 2*a*b*x + b^2*x^2)^(5/2),x]`

output `-1/3*e/(b^2*(a^2 + 2*a*b*x + b^2*x^2)^(3/2)) - (b*d - a*e)/(4*b^2*(a + b*x)*(a^2 + 2*a*b*x + b^2*x^2)^(3/2))`

Definitions of rubi rules used

rule 1078

```
Int[((a_) + (b_)*(x_) + (c_)*(x_)^2)^(p_), x_Symbol] := Simp[2*((a + b*x
+ c*x^2)^(p + 1)/((2*p + 1)*(b + 2*c*x))), x] /; FreeQ[{a, b, c, p}, x] &&
EqQ[b^2 - 4*a*c, 0] && LtQ[p, -1]
```

rule 1100

```
Int[((d_) + (e_)*(x_))*((a_) + (b_)*(x_) + (c_)*(x_)^2)^(p_), x_Symbol]
:= Simp[e*((a + b*x + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Simp[(2*c*d - b*
e)/(2*c) Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& EqQ[b^2 - 4*a*c, 0]
```

Maple [A] (verified)

Time = 1.12 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.46

| method | result | size |
|---------|---|------|
| gospers | $-\frac{(bx+a)(4bex+ae+3bd)}{12b^2((bx+a)^2)^{\frac{5}{2}}}$ | 33 |
| default | $-\frac{(bx+a)(4bex+ae+3bd)}{12b^2((bx+a)^2)^{\frac{5}{2}}}$ | 33 |
| risch | $\frac{\sqrt{(bx+a)^2} \left(-\frac{ex}{3b} - \frac{ae+3bd}{12b^2} \right)}{(bx+a)^5}$ | 39 |
| orering | $-\frac{(4bex+ae+3bd)(bx+a)}{12b^2(b^2x^2+2abx+a^2)^{\frac{5}{2}}}$ | 42 |

input

```
int((e*x+d)/(b^2*x^2+2*a*b*x+a^2)^(5/2),x,method=_RETURNVERBOSE)
```

output

```
-1/12*(b*x+a)/b^2*(4*b*e*x+a*e+3*b*d)/((b*x+a)^2)^(5/2)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.86

$$\int \frac{d + ex}{(a^2 + 2abx + b^2x^2)^{5/2}} dx = -\frac{4bex + 3bd + ae}{12(b^6x^4 + 4ab^5x^3 + 6a^2b^4x^2 + 4a^3b^3x + a^4b^2)}$$

input `integrate((e*x+d)/(b^2*x^2+2*a*b*x+a^2)^(5/2),x, algorithm="fricas")`

output `-1/12*(4*b*e*x + 3*b*d + a*e)/(b^6*x^4 + 4*a*b^5*x^3 + 6*a^2*b^4*x^2 + 4*a^3*b^3*x + a^4*b^2)`

Sympy [F]

$$\int \frac{d + ex}{(a^2 + 2abx + b^2x^2)^{5/2}} dx = \int \frac{d + ex}{((a + bx)^2)^{\frac{5}{2}}} dx$$

input `integrate((e*x+d)/(b**2*x**2+2*a*b*x+a**2)**(5/2),x)`

output `Integral((d + e*x)/((a + b*x)**2)**(5/2), x)`

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.79

$$\int \frac{d + ex}{(a^2 + 2abx + b^2x^2)^{5/2}} dx = -\frac{e}{3(b^2x^2 + 2abx + a^2)^{\frac{3}{2}}b^2} - \frac{d}{4b^5(x + \frac{a}{b})^4} + \frac{ae}{4b^6(x + \frac{a}{b})^4}$$

input `integrate((e*x+d)/(b^2*x^2+2*a*b*x+a^2)^(5/2),x, algorithm="maxima")`

output `-1/3*e/((b^2*x^2 + 2*a*b*x + a^2)^(3/2)*b^2) - 1/4*d/(b^5*(x + a/b)^4) + 1/4*a*e/(b^6*(x + a/b)^4)`

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.46

$$\int \frac{d + ex}{(a^2 + 2abx + b^2x^2)^{5/2}} dx = -\frac{4bex + 3bd + ae}{12(bx + a)^4 b^2 \operatorname{sgn}(bx + a)}$$

input `integrate((e*x+d)/(b^2*x^2+2*a*b*x+a^2)^(5/2),x, algorithm="giac")`output `-1/12*(4*b*e*x + 3*b*d + a*e)/((b*x + a)^4*b^2*sgn(b*x + a))`**Mupad [B] (verification not implemented)**

Time = 6.30 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.61

$$\int \frac{d + ex}{(a^2 + 2abx + b^2x^2)^{5/2}} dx = -\frac{(ae + 3bd + 4bex) \sqrt{a^2 + 2abx + b^2x^2}}{12b^2(a + bx)^5}$$

input `int((d + e*x)/(a^2 + b^2*x^2 + 2*a*b*x)^(5/2),x)`output `-((a*e + 3*b*d + 4*b*e*x)*(a^2 + b^2*x^2 + 2*a*b*x)^(1/2))/(12*b^2*(a + b*x)^5)`**Reduce [B] (verification not implemented)**

Time = 0.25 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.83

$$\int \frac{d + ex}{(a^2 + 2abx + b^2x^2)^{5/2}} dx = \frac{-4bex - ae - 3bd}{12b^2(b^4x^4 + 4ab^3x^3 + 6a^2b^2x^2 + 4a^3bx + a^4)}$$

input `int((e*x+d)/(b^2*x^2+2*a*b*x+a^2)^(5/2),x)`output `(- a*e - 3*b*d - 4*b*e*x)/(12*b**2*(a**4 + 4*a**3*b*x + 6*a**2*b**2*x**2 + 4*a*b**3*x**3 + b**4*x**4))`

$$3.332 \quad \int \frac{1}{(a^2 + 2abx + b^2x^2)^{5/2}} dx$$

| | |
|---|------|
| Optimal result | 2480 |
| Mathematica [A] (verified) | 2480 |
| Rubi [A] (verified) | 2481 |
| Maple [A] (verified) | 2482 |
| Fricas [B] (verification not implemented) | 2482 |
| Sympy [F] | 2483 |
| Maxima [A] (verification not implemented) | 2483 |
| Giac [A] (verification not implemented) | 2483 |
| Mupad [B] (verification not implemented) | 2484 |
| Reduce [B] (verification not implemented) | 2484 |

Optimal result

Integrand size = 20, antiderivative size = 34

$$\int \frac{1}{(a^2 + 2abx + b^2x^2)^{5/2}} dx = -\frac{1}{4b(a + bx)^3 \sqrt{a^2 + 2abx + b^2x^2}}$$

output `-1/4/b/(b*x+a)^3/((b*x+a)^2)^(1/2)`

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.68

$$\int \frac{1}{(a^2 + 2abx + b^2x^2)^{5/2}} dx = -\frac{a + bx}{4b((a + bx)^2)^{5/2}}$$

input `Integrate[(a^2 + 2*a*b*x + b^2*x^2)^(-5/2), x]`

output `-1/4*(a + b*x)/(b*((a + b*x)^2)^(5/2))`

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.050$, Rules used = {1078}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a^2 + 2abx + b^2x^2)^{5/2}} dx$$

↓ 1078

$$-\frac{1}{4b(a + bx)(a^2 + 2abx + b^2x^2)^{3/2}}$$

input `Int[(a^2 + 2*a*b*x + b^2*x^2)^(-5/2), x]`

output `-1/4*1/(b*(a + b*x)*(a^2 + 2*a*b*x + b^2*x^2)^(3/2))`

Defintions of rubi rules used

rule 1078

```
Int[((a_) + (b_)*(x_) + (c_)*(x_)^2)^(p_), x_Symbol] :> Simp[2*((a + b*x + c*x^2)^(p + 1)/((2*p + 1)*(b + 2*c*x))), x] /; FreeQ[{a, b, c, p}, x] && EqQ[b^2 - 4*a*c, 0] && LtQ[p, -1]
```

Maple [A] (verified)

Time = 1.06 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.59

| method | result | size |
|---------|---|------|
| gospers | $-\frac{bx+a}{4b((bx+a)^2)^{\frac{5}{2}}}$ | 20 |
| default | $-\frac{bx+a}{4b((bx+a)^2)^{\frac{5}{2}}}$ | 20 |
| risch | $-\frac{\sqrt{(bx+a)^2}}{4(bx+a)^5b}$ | 22 |
| orering | $-\frac{bx+a}{4b(b^2x^2+2abx+a^2)^{\frac{5}{2}}}$ | 29 |

input `int(1/(b^2*x^2+2*a*b*x+a^2)^(5/2),x,method=_RETURNVERBOSE)`

output `-1/4*(b*x+a)/b/((b*x+a)^2)^(5/2)`

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 46 vs. 2(21) = 42.

Time = 0.08 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.35

$$\int \frac{1}{(a^2 + 2abx + b^2x^2)^{5/2}} dx = -\frac{1}{4(b^5x^4 + 4ab^4x^3 + 6a^2b^3x^2 + 4a^3b^2x + a^4b)}$$

input `integrate(1/(b^2*x^2+2*a*b*x+a^2)^(5/2),x, algorithm="fricas")`

output `-1/4/(b^5*x^4 + 4*a*b^4*x^3 + 6*a^2*b^3*x^2 + 4*a^3*b^2*x + a^4*b)`

Sympy [F]

$$\int \frac{1}{(a^2 + 2abx + b^2x^2)^{5/2}} dx = \int \frac{1}{(a^2 + 2abx + b^2x^2)^{5/2}} dx$$

input `integrate(1/(b**2*x**2+2*a*b*x+a**2)**(5/2),x)`

output `Integral((a**2 + 2*a*b*x + b**2*x**2)**(-5/2), x)`

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.41

$$\int \frac{1}{(a^2 + 2abx + b^2x^2)^{5/2}} dx = -\frac{1}{4b^5(x + \frac{a}{b})^4}$$

input `integrate(1/(b^2*x^2+2*a*b*x+a^2)^(5/2),x, algorithm="maxima")`

output `-1/4/(b^5*(x + a/b)^4)`

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.59

$$\int \frac{1}{(a^2 + 2abx + b^2x^2)^{5/2}} dx = -\frac{1}{4(bx + a)^4 b \operatorname{sgn}(bx + a)}$$

input `integrate(1/(b^2*x^2+2*a*b*x+a^2)^(5/2),x, algorithm="giac")`

output `-1/4/((b*x + a)^4*b*sgn(b*x + a))`

Mupad [B] (verification not implemented)

Time = 6.22 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.88

$$\int \frac{1}{(a^2 + 2abx + b^2x^2)^{5/2}} dx = -\frac{\sqrt{a^2 + 2abx + b^2x^2}}{4b(a + bx)^5}$$

input `int(1/(a^2 + b^2*x^2 + 2*a*b*x)^(5/2),x)`output `-(a^2 + b^2*x^2 + 2*a*b*x)^(1/2)/(4*b*(a + b*x)^5)`**Reduce [B] (verification not implemented)**

Time = 0.23 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.32

$$\int \frac{1}{(a^2 + 2abx + b^2x^2)^{5/2}} dx = -\frac{1}{4b(b^4x^4 + 4ab^3x^3 + 6a^2b^2x^2 + 4a^3bx + a^4)}$$

input `int(1/(b^2*x^2+2*a*b*x+a^2)^(5/2),x)`output `(- 1)/(4*b*(a**4 + 4*a**3*b*x + 6*a**2*b**2*x**2 + 4*a*b**3*x**3 + b**4*x**4))`

3.333 $\int \frac{1}{(d+ex)(a^2+2abx+b^2x^2)^{5/2}} dx$

| | |
|---|------|
| Optimal result | 2485 |
| Mathematica [A] (verified) | 2486 |
| Rubi [A] (verified) | 2486 |
| Maple [A] (verified) | 2488 |
| Fricas [B] (verification not implemented) | 2488 |
| Sympy [F] | 2489 |
| Maxima [F(-2)] | 2489 |
| Giac [B] (verification not implemented) | 2490 |
| Mupad [F(-1)] | 2491 |
| Reduce [B] (verification not implemented) | 2491 |

Optimal result

Integrand size = 28, antiderivative size = 253

$$\int \frac{1}{(d+ex)(a^2+2abx+b^2x^2)^{5/2}} dx = \frac{e^3}{(bd-ae)^4\sqrt{a^2+2abx+b^2x^2}} - \frac{1}{4(bd-ae)(a+bx)^3\sqrt{a^2+2abx+b^2x^2}} + \frac{e}{3(bd-ae)^2(a+bx)^2\sqrt{a^2+2abx+b^2x^2}} - \frac{2(bd-ae)^3(a+bx)\sqrt{a^2+2abx+b^2x^2}}{e^4(a+bx)\log(a+bx)} + \frac{e^4(a+bx)\log(a+bx)}{(bd-ae)^5\sqrt{a^2+2abx+b^2x^2}} - \frac{e^4(a+bx)\log(d+ex)}{(bd-ae)^5\sqrt{a^2+2abx+b^2x^2}}$$

output

```
e^3/(-a*e+b*d)^4/((b*x+a)^2)^(1/2)-1/4/(-a*e+b*d)/(b*x+a)^3/((b*x+a)^2)^(1/2)+1/3*e/(-a*e+b*d)^2/(b*x+a)^2/((b*x+a)^2)^(1/2)-1/2*e^2/(-a*e+b*d)^3/(b*x+a)/((b*x+a)^2)^(1/2)+e^4*(b*x+a)*ln(b*x+a)/(-a*e+b*d)^5/((b*x+a)^2)^(1/2)-e^4*(b*x+a)*ln(e*x+d)/(-a*e+b*d)^5/((b*x+a)^2)^(1/2)
```

Mathematica [A] (verified)

Time = 1.09 (sec) , antiderivative size = 163, normalized size of antiderivative = 0.64

$$\int \frac{1}{(d+ex)(a^2+2abx+b^2x^2)^{5/2}} dx = \frac{-((bd-ae)(-25a^3e^3+a^2be^2(23d-52ex))+ab^2e(-13d^2+20dex$$

input `Integrate[1/((d + e*x)*(a^2 + 2*a*b*x + b^2*x^2)^(5/2)),x]`

output `(-((b*d - a*e)*(-25*a^3*e^3 + a^2*b*e^2*(23*d - 52*e*x) + a*b^2*e*(-13*d^2 + 20*d*e*x - 42*e^2*x^2) + b^3*(3*d^3 - 4*d^2*e*x + 6*d*e^2*x^2 - 12*e^3*x^3))) + 12*e^4*(a + b*x)^4*Log[a + b*x] - 12*e^4*(a + b*x)^4*Log[d + e*x])/(12*(b*d - a*e)^5*(a + b*x)^3*Sqrt[(a + b*x)^2])`

Rubi [A] (verified)

Time = 0.56 (sec) , antiderivative size = 156, normalized size of antiderivative = 0.62, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {1102, 27, 54, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\begin{aligned} & \int \frac{1}{(a^2 + 2abx + b^2x^2)^{5/2} (d + ex)} dx \\ & \quad \downarrow \text{1102} \\ & \frac{b^5(a + bx) \int \frac{1}{b^5(a+bx)^5(d+ex)} dx}{\sqrt{a^2 + 2abx + b^2x^2}} \\ & \quad \downarrow \text{27} \\ & \frac{(a + bx) \int \frac{1}{(a+bx)^5(d+ex)} dx}{\sqrt{a^2 + 2abx + b^2x^2}} \\ & \quad \downarrow \text{54} \end{aligned}$$

$$(a + bx) \int \frac{\left(-\frac{e^5}{(bd-ae)^5(d+ex)} + \frac{be^4}{(bd-ae)^5(a+bx)} - \frac{be^3}{(bd-ae)^4(a+bx)^2} + \frac{be^2}{(bd-ae)^3(a+bx)^3} - \frac{be}{(bd-ae)^2(a+bx)^4} + \frac{b}{(bd-ae)(a+bx)^5} \right)}{\sqrt{a^2 + 2abx + b^2x^2}}$$

↓ 2009

$$(a + bx) \frac{\left(\frac{e^4 \log(a+bx)}{(bd-ae)^5} - \frac{e^4 \log(d+ex)}{(bd-ae)^5} + \frac{e^3}{(a+bx)(bd-ae)^4} - \frac{e^2}{2(a+bx)^2(bd-ae)^3} + \frac{e}{3(a+bx)^3(bd-ae)^2} - \frac{1}{4(a+bx)^4(bd-ae)} \right)}{\sqrt{a^2 + 2abx + b^2x^2}}$$

input `Int[1/((d + e*x)*(a^2 + 2*a*b*x + b^2*x^2)^(5/2)),x]`

output `((a + b*x)*(-1/4*1/((b*d - a*e)*(a + b*x)^4) + e/(3*(b*d - a*e)^2*(a + b*x)^3) - e^2/(2*(b*d - a*e)^3*(a + b*x)^2) + e^3/((b*d - a*e)^4*(a + b*x)) + (e^4*Log[a + b*x])/(b*d - a*e)^5 - (e^4*Log[d + e*x])/(b*d - a*e)^5)/Sqrt[a^2 + 2*a*b*x + b^2*x^2]`

Defintions of rubi rules used

rule 27 `Int[(a_)*(F_x_), x_Symbol] := Simp[a Int[F_x, x], x] /; FreeQ[a, x] && !MatchQ[F_x, (b_)*(G_x_) /; FreeQ[b, x]]`

rule 54 `Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] := Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] && ILtQ[m, 0] && IntegerQ[n] && !(IGtQ[n, 0] && LtQ[m + n + 2, 0])`

rule 1102 `Int[((d_.) + (e_.)*(x_))^(m_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] := Simp[(a + b*x + c*x^2)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x)^(2*FracPart[p])) Int[(d + e*x)^m*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && EqQ[b^2 - 4*a*c, 0]`

rule 2009 `Int[u_, x_Symbol] := Simp[IntSum[u, x], x] /; SumQ[u]`

Maple [A] (verified)

Time = 1.50 (sec) , antiderivative size = 359, normalized size of antiderivative = 1.42

| method | result |
|---------|--|
| default | $-\frac{(12 \ln(bx+a)b^4e^4x^4 - 12 \ln(ex+d)b^4e^4x^4 + 48 \ln(bx+a)x^3ab^3e^4 - 48 \ln(ex+d)ab^3e^4x^3 + 72 \ln(bx+a)x^2a^2b^2e^4 - 72 \ln(ex+d)a^2b^2e^4x^2 + 48 \ln(bx+a)ab^2e^4x^2 - 48 \ln(ex+d)ab^2e^4x^2 + 24 \ln(bx+a)a^2b^2e^4x^2 - 24 \ln(ex+d)a^2b^2e^4x^2 + 12 \ln(bx+a)a^2b^2e^4x^2 - 12 \ln(ex+d)a^2b^2e^4x^2 + 6 \ln(bx+a)a^2b^2e^4x^2 - 6 \ln(ex+d)a^2b^2e^4x^2 + 3 \ln(bx+a)a^2b^2e^4x^2 - 3 \ln(ex+d)a^2b^2e^4x^2 + 1.5 \ln(bx+a)a^2b^2e^4x^2 - 1.5 \ln(ex+d)a^2b^2e^4x^2 + 0.5 \ln(bx+a)a^2b^2e^4x^2 - 0.5 \ln(ex+d)a^2b^2e^4x^2 + 0.25 \ln(bx+a)a^2b^2e^4x^2 - 0.25 \ln(ex+d)a^2b^2e^4x^2 + 0.125 \ln(bx+a)a^2b^2e^4x^2 - 0.125 \ln(ex+d)a^2b^2e^4x^2 + 0.0625 \ln(bx+a)a^2b^2e^4x^2 - 0.0625 \ln(ex+d)a^2b^2e^4x^2 + 0.03125 \ln(bx+a)a^2b^2e^4x^2 - 0.03125 \ln(ex+d)a^2b^2e^4x^2 + 0.015625 \ln(bx+a)a^2b^2e^4x^2 - 0.015625 \ln(ex+d)a^2b^2e^4x^2 + 0.0078125 \ln(bx+a)a^2b^2e^4x^2 - 0.0078125 \ln(ex+d)a^2b^2e^4x^2 + 0.00390625 \ln(bx+a)a^2b^2e^4x^2 - 0.00390625 \ln(ex+d)a^2b^2e^4x^2 + 0.001953125 \ln(bx+a)a^2b^2e^4x^2 - 0.001953125 \ln(ex+d)a^2b^2e^4x^2 + 0.0009765625 \ln(bx+a)a^2b^2e^4x^2 - 0.0009765625 \ln(ex+d)a^2b^2e^4x^2 + 0.00048828125 \ln(bx+a)a^2b^2e^4x^2 - 0.00048828125 \ln(ex+d)a^2b^2e^4x^2 + 0.000244140625 \ln(bx+a)a^2b^2e^4x^2 - 0.000244140625 \ln(ex+d)a^2b^2e^4x^2 + 0.0001220703125 \ln(bx+a)a^2b^2e^4x^2 - 0.0001220703125 \ln(ex+d)a^2b^2e^4x^2 + 6.103515625e-05 \ln(bx+a)a^2b^2e^4x^2 - 6.103515625e-05 \ln(ex+d)a^2b^2e^4x^2 + 3.0517578125e-05 \ln(bx+a)a^2b^2e^4x^2 - 3.0517578125e-05 \ln(ex+d)a^2b^2e^4x^2 + 1.52587890625e-05 \ln(bx+a)a^2b^2e^4x^2 - 1.52587890625e-05 \ln(ex+d)a^2b^2e^4x^2 + 7.62939453125e-06 \ln(bx+a)a^2b^2e^4x^2 - 7.62939453125e-06 \ln(ex+d)a^2b^2e^4x^2 + 3.814697265625e-06 \ln(bx+a)a^2b^2e^4x^2 - 3.814697265625e-06 \ln(ex+d)a^2b^2e^4x^2 + 1.9073486328125e-06 \ln(bx+a)a^2b^2e^4x^2 - 1.9073486328125e-06 \ln(ex+d)a^2b^2e^4x^2 + 9.5367431640625e-07 \ln(bx+a)a^2b^2e^4x^2 - 9.5367431640625e-07 \ln(ex+d)a^2b^2e^4x^2 + 4.76837158203125e-07 \ln(bx+a)a^2b^2e^4x^2 - 4.76837158203125e-07 \ln(ex+d)a^2b^2e^4x^2 + 2.384185791015625e-07 \ln(bx+a)a^2b^2e^4x^2 - 2.384185791015625e-07 \ln(ex+d)a^2b^2e^4x^2 + 1.1920928955078125e-07 \ln(bx+a)a^2b^2e^4x^2 - 1.1920928955078125e-07 \ln(ex+d)a^2b^2e^4x^2 + 5.9604644775390625e-08 \ln(bx+a)a^2b^2e^4x^2 - 5.9604644775390625e-08 \ln(ex+d)a^2b^2e^4x^2 + 2.98023223876953125e-08 \ln(bx+a)a^2b^2e^4x^2 - 2.98023223876953125e-08 \ln(ex+d)a^2b^2e^4x^2 + 1.490116119384765625e-08 \ln(bx+a)a^2b^2e^4x^2 - 1.490116119384765625e-08 \ln(ex+d)a^2b^2e^4x^2 + 7.450580596923828125e-09 \ln(bx+a)a^2b^2e^4x^2 - 7.450580596923828125e-09 \ln(ex+d)a^2b^2e^4x^2 + 3.7252902984619140625e-09 \ln(bx+a)a^2b^2e^4x^2 - 3.7252902984619140625e-09 \ln(ex+d)a^2b^2e^4x^2 + 1.86264514923095703125e-09 \ln(bx+a)a^2b^2e^4x^2 - 1.86264514923095703125e-09 \ln(ex+d)a^2b^2e^4x^2 + 9.31322574615478515625e-10 \ln(bx+a)a^2b^2e^4x^2 - 9.31322574615478515625e-10 \ln(ex+d)a^2b^2e^4x^2 + 4.656612873077392578125e-10 \ln(bx+a)a^2b^2e^4x^2 - 4.656612873077392578125e-10 \ln(ex+d)a^2b^2e^4x^2 + 2.3283064365386962890625e-10 \ln(bx+a)a^2b^2e^4x^2 - 2.3283064365386962890625e-10 \ln(ex+d)a^2b^2e^4x^2 + 1.16415321826934814453125e-10 \ln(bx+a)a^2b^2e^4x^2 - 1.16415321826934814453125e-10 \ln(ex+d)a^2b^2e^4x^2 + 5.82076609134674072265625e-11 \ln(bx+a)a^2b^2e^4x^2 - 5.82076609134674072265625e-11 \ln(ex+d)a^2b^2e^4x^2 + 2.910383045673370361328125e-11 \ln(bx+a)a^2b^2e^4x^2 - 2.910383045673370361328125e-11 \ln(ex+d)a^2b^2e^4x^2 + 1.4551915228366851806640625e-11 \ln(bx+a)a^2b^2e^4x^2 - 1.4551915228366851806640625e-11 \ln(ex+d)a^2b^2e^4x^2 + 7.2759576141834259033203125e-12 \ln(bx+a)a^2b^2e^4x^2 - 7.2759576141834259033203125e-12 \ln(ex+d)a^2b^2e^4x^2 + 3.63797880709171295166015625e-12 \ln(bx+a)a^2b^2e^4x^2 - 3.63797880709171295166015625e-12 \ln(ex+d)a^2b^2e^4x^2 + 1.818989403545856475830078125e-12 \ln(bx+a)a^2b^2e^4x^2 - 1.818989403545856475830078125e-12 \ln(ex+d)a^2b^2e^4x^2 + 9.094947017729282379150390625e-13 \ln(bx+a)a^2b^2e^4x^2 - 9.094947017729282379150390625e-13 \ln(ex+d)a^2b^2e^4x^2 + 4.5474735088646411895751953125e-13 \ln(bx+a)a^2b^2e^4x^2 - 4.5474735088646411895751953125e-13 \ln(ex+d)a^2b^2e^4x^2 + 2.27373675443232059478759765625e-13 \ln(bx+a)a^2b^2e^4x^2 - 2.27373675443232059478759765625e-13 \ln(ex+d)a^2b^2e^4x^2 + 1.136868377216160297393798828125e-13 \ln(bx+a)a^2b^2e^4x^2 - 1.136868377216160297393798828125e-13 \ln(ex+d)a^2b^2e^4x^2 + 5.684341886080801486968994140625e-14 \ln(bx+a)a^2b^2e^4x^2 - 5.684341886080801486968994140625e-14 \ln(ex+d)a^2b^2e^4x^2 + 2.8421709430404007434844970703125e-14 \ln(bx+a)a^2b^2e^4x^2 - 2.8421709430404007434844970703125e-14 \ln(ex+d)a^2b^2e^4x^2 + 1.42108547152020037174224853515625e-14 \ln(bx+a)a^2b^2e^4x^2 - 1.42108547152020037174224853515625e-14 \ln(ex+d)a^2b^2e^4x^2 + 7.10542735760100185871124267578125e-15 \ln(bx+a)a^2b^2e^4x^2 - 7.10542735760100185871124267578125e-15 \ln(ex+d)a^2b^2e^4x^2 + 3.552713678800500929355621337890625e-15 \ln(bx+a)a^2b^2e^4x^2 - 3.552713678800500929355621337890625e-15 \ln(ex+d)a^2b^2e^4x^2 + 1.7763568394002504646778106689453125e-15 \ln(bx+a)a^2b^2e^4x^2 - 1.7763568394002504646778106689453125e-15 \ln(ex+d)a^2b^2e^4x^2 + 8.8817841970012523233890533447265625e-16 \ln(bx+a)a^2b^2e^4x^2 - 8.8817841970012523233890533447265625e-16 \ln(ex+d)a^2b^2e^4x^2 + 4.44089209850062616169452667236328125e-16 \ln(bx+a)a^2b^2e^4x^2 - 4.44089209850062616169452667236328125e-16 \ln(ex+d)a^2b^2e^4x^2 + 2.220446049250313080847263336181640625e-16 \ln(bx+a)a^2b^2e^4x^2 - 2.220446049250313080847263336181640625e-16 \ln(ex+d)a^2b^2e^4x^2 + 1.1102230246251565404236316680908203125e-16 \ln(bx+a)a^2b^2e^4x^2 - 1.1102230246251565404236316680908203125e-16 \ln(ex+d)a^2b^2e^4x^2 + 5.5511151231257827021181583440541015625e-17 \ln(bx+a)a^2b^2e^4x^2 - 5.5511151231257827021181583440541015625e-17 \ln(ex+d)a^2b^2e^4x^2 + 2.77555756156289135105907917202705078125e-17 \ln(bx+a)a^2b^2e^4x^2 - 2.77555756156289135105907917202705078125e-17 \ln(ex+d)a^2b^2e^4x^2 + 1.387778780781445675529539586013525390625e-17 \ln(bx+a)a^2b^2e^4x^2 - 1.387778780781445675529539586013525390625e-17 \ln(ex+d)a^2b^2e^4x^2 + 6.938893903907228377647697930067626953125e-18 \ln(bx+a)a^2b^2e^4x^2 - 6.938893903907228377647697930067626953125e-18 \ln(ex+d)a^2b^2e^4x^2 + 3.4694469519536141888238489650338134765625e-18 \ln(bx+a)a^2b^2e^4x^2 - 3.4694469519536141888238489650338134765625e-18 \ln(ex+d)a^2b^2e^4x^2 + 1.73472347597680709441192448251690673828125e-18 \ln(bx+a)a^2b^2e^4x^2 - 1.73472347597680709441192448251690673828125e-18 \ln(ex+d)a^2b^2e^4x^2 + 8.67361737988403547205962241258453369140625e-19 \ln(bx+a)a^2b^2e^4x^2 - 8.67361737988403547205962241258453369140625e-19 \ln(ex+d)a^2b^2e^4x^2 + 4.336808689942017736029811206292266845703125e-19 \ln(bx+a)a^2b^2e^4x^2 - 4.336808689942017736029811206292266845703125e-19 \ln(ex+d)a^2b^2e^4x^2 + 2.1684043449710088680149056031461333228515625e-19 \ln(bx+a)a^2b^2e^4x^2 - 2.1684043449710088680149056031461333228515625e-19 \ln(ex+d)a^2b^2e^4x^2 + 1.08420217248550443400745280157306666142578125e-19 \ln(bx+a)a^2b^2e^4x^2 - 1.08420217248550443400745280157306666142578125e-19 \ln(ex+d)a^2b^2e^4x^2 + 5.42101086242752217003726400786533330712890625e-20 \ln(bx+a)a^2b^2e^4x^2 - 5.42101086242752217003726400786533330712890625e-20 \ln(ex+d)a^2b^2e^4x^2 + 2.710505431213761085018632003932666653564453125e-20 \ln(bx+a)a^2b^2e^4x^2 - 2.710505431213761085018632003932666653564453125e-20 \ln(ex+d)a^2b^2e^4x^2 + 1.3552527156068805425093160019663333267822265625e-20 \ln(bx+a)a^2b^2e^4x^2 - 1.3552527156068805425093160019663333267822265625e-20 \ln(ex+d)a^2b^2e^4x^2 + 6.7762635780344027125475800098316666339111328125e-21 \ln(bx+a)a^2b^2e^4x^2 - 6.7762635780344027125475800098316666339111328125e-21 \ln(ex+d)a^2b^2e^4x^2 + 3.38813178901720135627379000491583331695556640625e-21 \ln(bx+a)a^2b^2e^4x^2 - 3.38813178901720135627379000491583331695556640625e-21 \ln(ex+d)a^2b^2e^4x^2 + 1.694065894508600678136895002457916658477783203125e-21 \ln(bx+a)a^2b^2e^4x^2 - 1.694065894508600678136895002457916658477783203125e-21 \ln(ex+d)a^2b^2e^4x^2 + 8.470329472543003390684475001239583292388916015625e-22 \ln(bx+a)a^2b^2e^4x^2 - 8.470329472543003390684475001239583292388916015625e-22 \ln(ex+d)a^2b^2e^4x^2 + 4.2351647362715016953422375006197916461944580078125e-22 \ln(bx+a)a^2b^2e^4x^2 - 4.2351647362715016953422375006197916461944580078125e-22 \ln(ex+d)a^2b^2e^4x^2 + 2.11758236813575084767111875030989582309722900390625e-22 \ln(bx+a)a^2b^2e^4x^2 - 2.11758236813575084767111875030989582309722900390625e-22 \ln(ex+d)a^2b^2e^4x^2 + 1.058791184067875423835559375154947911548614501953125e-22 \ln(bx+a)a^2b^2e^4x^2 - 1.058791184067875423835559375154947911548614501953125e-22 \ln(ex+d)a^2b^2e^4x^2 + 5.293955920339377119177796875774739557743072509765625e-23 \ln(bx+a)a^2b^2e^4x^2 - 5.293955920339377119177796875774739557743072509765625e-23 \ln(ex+d)a^2b^2e^4x^2 + 2.6469779601696885595888984378873697788715362548828125e-23 \ln(bx+a)a^2b^2e^4x^2 - 2.6469779601696885595888984378873697788715362548828125e-23 \ln(ex+d)a^2b^2e^4x^2 + 1.32348898008484427979444921894368488943576812744140625e-23 \ln(bx+a)a^2b^2e^4x^2 - 1.32348898008484427979444921894368488943576812744140625e-23 \ln(ex+d)a^2b^2e^4x^2 + 6.61744490042422139897224609471842444717884063720703125e-24 \ln(bx+a)a^2b^2e^4x^2 - 6.61744490042422139897224609471842444717884063720703125e-24 \ln(ex+d)a^2b^2e^4x^2 + 3.308722450212110699486123047359212223589420318603515625e-24 \ln(bx+a)a^2b^2e^4x^2 - 3.308722450212110699486123047359212223589420318603515625e-24 \ln(ex+d)a^2b^2e^4x^2 + 1.6543612251060553497430615236796061117947101593017578125e-24 \ln(bx+a)a^2b^2e^4x^2 - 1.6543612251060553497430615236796061117947101593017578125e-24 \ln(ex+d)a^2b^2e^4x^2 + 8.2718061255302767487153076183980305889735507965087890625e-25 \ln(bx+a)a^2b^2e^4x^2 - 8.2718061255302767487153076183980305889735507965087890625e-25 \ln(ex+d)a^2b^2e^4x^2 + 4.13590306276513837435765380919901544448677539825439453125e-25 \ln(bx+a)a^2b^2e^4x^2 - 4.13590306276513837435765380919901544448677539825439453125e-25 \ln(ex+d)a^2b^2e^4x^2 + 2.067951531382569187178776904599507722243387699127197265625e-25 \ln(bx+a)a^2b^2e^4x^2 - 2.067951531382569187178776904599507722243387699127197265625e-25 \ln(ex+d)a^2b^2e^4x^2 + 1.0339757656912845935893884522997538611216688495635986328125e-25 \ln(bx+a)a^2b^2e^4x^2 - 1.0339757656912845935893884522997538611216688495635986328125e-25 \ln(ex+d)a^2b^2e^4x^2 + 5.1698788284564229679469422614987693056083344477829931640625e-26 \ln(bx+a)a^2b^2e^4x^2 - 5.1698788284564229679469422614987693056083344477829931640625e-26 \ln(ex+d)a^2b^2e^4x^2 + 2.58493941422821148397347113074938465280416722389149658203125e-26 \ln(bx+a)a^2b^2e^4x^2 - 2.58493941422821148397347113074938465280416722389149658203125e-26 \ln(ex+d)a^2b^2e^4x^2 + 1.292469707114105741986735565374692326402083611945748291015625e-26 \ln(bx+a)a^2b^2e^4x^2 - 1.292469707114105741986735565374692326402083611945748291015625e-26 \ln(ex+d)a^2b^2e^4x^2 + 6.462348535570528709933677826873461632010418059728741455078125e-27 \ln(bx+a)a^2b^2e^4x^2 - 6.462348535570528709933677826873461632010418059728741455078125e-27 \ln(ex+d)a^2b^2e^4x^2 + 3.2311742677852643549668389134367308160052090298643707275390625e-27 \ln(bx+a)a^2b^2e^4x^2 - 3.2311742677852643549668389134367308160052090298643707275390625e-27 \ln(ex+d)a^2b^2e^4x^2 + 1.61558713389263217748341945671836540800260451493218536376953125e-27 \ln(bx+a)a^2b^2e^4x^2 - 1.61558713389263217748341945671836540800260451493218536376953125e-27 \ln(ex+d)a^2b^2e^4x^2 + 8.07793566946316088741709728359182704001302257466092681884765625e-28 \ln(bx+a)a^2b^2e^4x^2 - 8.07793566946316088741709728359182704001302257466092681884765625e-28 \ln(ex+d)a^2b^2e^4x^2 + 4.038967834731580443708548641795913520006511287330463409423828125e-28 \ln(bx+a)a^2b^2e^4x^2 - 4.038967834731580443708548641795913520006511287330463409423828125e-28 \ln(ex+d)a^2b^2e^4x^2 + 2.0194839173657902218542743208979567600032556436652317047119140625e-28 \ln(bx+a)a^2b^2e^4x^2 - 2.0194839173657902218542743208979567600032556436652317047119140625e-28 \ln(ex+d)a^2b^2e^4x^2 + 1.00974195868289511092713716044897838000162782183261585235595703125e-28 \ln(bx+a)a^2b^2e^4x^2 - 1.00974195868289511092713716044897838000162782183261585235595703125e-28 \ln(ex+d)a^2b^2e^4x^2 + 5.04870979341447555463568580224489190000813910916307926177978515625e-29 \ln(bx+a)a^2b^2e^4x^2 - 5.04870979341447555463568580224489190000813910916307926177978515625e-29 \ln(ex+d)a^2b^2e^4x^2 + 2.524354896707237777317842901122445950004069554581539630889892578125e-29 \ln(bx+a)a^2b^2e^4x^2 - 2.524354896707237777317842901122445950004069554581539630889892578125e-29 \ln(ex+d)a^2b^2e^4x^2 + 1.2621774483536188886589214505612229750020347772907698154449462890625e-29 \ln(bx+a)a^2b^2e^4x^2 - 1.2621774483536188886589214505612229750020347772907698154449462890625e-29 \ln(ex+d)a^2b^2e^4x^2 + 6.3108872417680944432946072502806114750101738864538490772247314453125e-30 \ln(bx+a)a^2b^2e^4x^2 - 6.3108872417680944432946072502806114750101738864538490772247314453125e-30 \ln(ex+d)a^2b^2e^4x^2 + 3.15544362088404722164730362514030573750508694322692453861236722265625e-30 \ln(bx+a)a^2b^2e^4x^2 - 3.15544362088404722164730362514030573750508694322692453861236722265625e-30 \ln(ex+d)a^2b^2e^4x^2 + 1.577721810442023610823651812570152868752543471613462269306183611328125e-30 \ln(bx+a)a^2b^2e^4x^2 - 1.577721810442023610823651812570152868752543471613462269306183611328125e-30 \ln(ex+d)a^2b^2e^4x^2 + 7.888609052210118054118259062850764343762717358067311346530918056640625e-31 \ln(bx+a)a^2b^2e^4x^2 - 7.8886090522101180541182590628507643$ |

output

```
-1/12*(3*b^4*d^4 - 16*a*b^3*d^3*e + 36*a^2*b^2*d^2*e^2 - 48*a^3*b*d*e^3 +
25*a^4*e^4 - 12*(b^4*d*e^3 - a*b^3*e^4)*x^3 + 6*(b^4*d^2*e^2 - 8*a*b^3*d*e
^3 + 7*a^2*b^2*e^4)*x^2 - 4*(b^4*d^3*e - 6*a*b^3*d^2*e^2 + 18*a^2*b^2*d*e^
3 - 13*a^3*b*e^4)*x - 12*(b^4*e^4*x^4 + 4*a*b^3*e^4*x^3 + 6*a^2*b^2*e^4*x^
2 + 4*a^3*b*e^4*x + a^4*e^4)*log(b*x + a) + 12*(b^4*e^4*x^4 + 4*a*b^3*e^4*
x^3 + 6*a^2*b^2*e^4*x^2 + 4*a^3*b*e^4*x + a^4*e^4)*log(e*x + d))/(a^4*b^5*
d^5 - 5*a^5*b^4*d^4*e + 10*a^6*b^3*d^3*e^2 - 10*a^7*b^2*d^2*e^3 + 5*a^8*b*
d*e^4 - a^9*e^5 + (b^9*d^5 - 5*a*b^8*d^4*e + 10*a^2*b^7*d^3*e^2 - 10*a^3*b
^6*d^2*e^3 + 5*a^4*b^5*d*e^4 - a^5*b^4*e^5)*x^4 + 4*(a*b^8*d^5 - 5*a^2*b^7
*d^4*e + 10*a^3*b^6*d^3*e^2 - 10*a^4*b^5*d^2*e^3 + 5*a^5*b^4*d*e^4 - a^6*b
^3*e^5)*x^3 + 6*(a^2*b^7*d^5 - 5*a^3*b^6*d^4*e + 10*a^4*b^5*d^3*e^2 - 10*a
^5*b^4*d^2*e^3 + 5*a^6*b^3*d*e^4 - a^7*b^2*e^5)*x^2 + 4*(a^3*b^6*d^5 - 5*a
^4*b^5*d^4*e + 10*a^5*b^4*d^3*e^2 - 10*a^6*b^3*d^2*e^3 + 5*a^7*b^2*d*e^4 -
a^8*b*e^5)*x)
```

Sympy [F]

$$\int \frac{1}{(d+ex)(a^2+2abx+b^2x^2)^{5/2}} dx = \int \frac{1}{(d+ex)((a+bx)^2)^{5/2}} dx$$

input

```
integrate(1/(e*x+d)/(b**2*x**2+2*a*b*x+a**2)**(5/2),x)
```

output

```
Integral(1/((d + e*x)*((a + b*x)**2)**(5/2)), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{(d+ex)(a^2+2abx+b^2x^2)^{5/2}} dx = \text{Exception raised: ValueError}$$

input

```
integrate(1/(e*x+d)/(b^2*x^2+2*a*b*x+a^2)^(5/2),x, algorithm="maxima")
```

output

```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
help (example of legal syntax is 'assume(a*e-b*d>0)', see `assume?` for m
ore detail
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 418 vs. $2(181) = 362$.

Time = 0.15 (sec) , antiderivative size = 418, normalized size of antiderivative = 1.65

$$\int \frac{1}{(d+ex)(a^2+2abx+b^2x^2)^{5/2}} dx = \frac{be^4 \log\left(\frac{b^5 d^5 \operatorname{sgn}(bx+a) - 5ab^5 d^4 e \operatorname{sgn}(bx+a) + 10a^2 b^4 d^3 e^2 \operatorname{sgn}(bx+a) - e^5 \log(|ex+d|)}{b^5 d^5 \operatorname{sgn}(bx+a) - 5ab^4 d^4 e^2 \operatorname{sgn}(bx+a) + 10a^2 b^3 d^3 e^3 \operatorname{sgn}(bx+a) - 10a^3 b^2 d^2 e^4 \operatorname{sgn}(bx+a) + 5a^4 b d e^5 \operatorname{sgn}(bx+a) - a^5 e^6 \operatorname{sgn}(bx+a)}\right) - e^5 \log(\operatorname{abs}(ex+d))}{12(bd-ae)^5(bx+a)^4 \operatorname{sgn}(bx+a)}$$

input

```
integrate(1/(e*x+d)/(b^2*x^2+2*a*b*x+a^2)^(5/2),x, algorithm="giac")
```

output

```
b*e^4*log(abs(b*x + a))/(b^6*d^5*sgn(b*x + a) - 5*a*b^5*d^4*e*sgn(b*x + a)
+ 10*a^2*b^4*d^3*e^2*sgn(b*x + a) - 10*a^3*b^3*d^2*e^3*sgn(b*x + a) + 5*a
^4*b^2*d*e^4*sgn(b*x + a) - a^5*b*e^5*sgn(b*x + a)) - e^5*log(abs(e*x + d)
)/(b^5*d^5*e*sgn(b*x + a) - 5*a*b^4*d^4*e^2*sgn(b*x + a) + 10*a^2*b^3*d^3*
e^3*sgn(b*x + a) - 10*a^3*b^2*d^2*e^4*sgn(b*x + a) + 5*a^4*b*d*e^5*sgn(b*x
+ a) - a^5*e^6*sgn(b*x + a)) - 1/12*(3*b^4*d^4 - 16*a*b^3*d^3*e + 36*a^2*
b^2*d^2*e^2 - 48*a^3*b*d*e^3 + 25*a^4*e^4 - 12*(b^4*d*e^3 - a*b^3*e^4)*x^3
+ 6*(b^4*d^2*e^2 - 8*a*b^3*d*e^3 + 7*a^2*b^2*e^4)*x^2 - 4*(b^4*d^3*e - 6*
a*b^3*d^2*e^2 + 18*a^2*b^2*d*e^3 - 13*a^3*b*e^4)*x)/((b*d - a*e)^5*(b*x +
a)^4*sgn(b*x + a))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(d+ex)(a^2+2abx+b^2x^2)^{5/2}} dx = \int \frac{1}{(d+ex)(a^2+2abx+b^2x^2)^{5/2}} dx$$

input `int(1/((d + e*x)*(a^2 + b^2*x^2 + 2*a*b*x)^(5/2)),x)`output `int(1/((d + e*x)*(a^2 + b^2*x^2 + 2*a*b*x)^(5/2)), x)`**Reduce [B] (verification not implemented)**

Time = 0.23 (sec) , antiderivative size = 769, normalized size of antiderivative = 3.04

$$\int \frac{1}{(d+ex)(a^2+2abx+b^2x^2)^{5/2}} dx = \frac{-12 \log(bx+a) a^5 e^4 - 48 \log(bx+a) a^4 b e^4 x - 72 \log(bx+a) a^3 b^2 e^4 x^2 + 12 a^5 b^4 e^5 x^4 - 5 a^4 b^5 d e^4 x^4 + 10 a^3 b^6 d^2 e^3 x^4 - 10 a^2 b^7 d^3 e^2 x^4}{12 a (a^5 b^4 e^5 x^4 - 5 a^4 b^5 d e^4 x^4 + 10 a^3 b^6 d^2 e^3 x^4 - 10 a^2 b^7 d^3 e^2 x^4)}$$

input `int(1/(e*x+d)/(b^2*x^2+2*a*b*x+a^2)^(5/2),x)`

output

```
( - 12*log(a + b*x)*a**5*e**4 - 48*log(a + b*x)*a**4*b*e**4*x - 72*log(a +
b*x)*a**3*b**2*e**4*x**2 - 48*log(a + b*x)*a**2*b**3*e**4*x**3 - 12*log(a
+ b*x)*a*b**4*e**4*x**4 + 12*log(d + e*x)*a**5*e**4 + 48*log(d + e*x)*a**
4*b*e**4*x + 72*log(d + e*x)*a**3*b**2*e**4*x**2 + 48*log(d + e*x)*a**2*b*
**3*e**4*x**3 + 12*log(d + e*x)*a*b**4*e**4*x**4 + 22*a**5*e**4 - 45*a**4*b
*d*e**3 + 40*a**4*b*e**4*x + 36*a**3*b**2*d**2*e**2 - 60*a**3*b**2*d*e**3*
x + 24*a**3*b**2*e**4*x**2 - 16*a**2*b**3*d**3*e + 24*a**2*b**3*d**2*e**2*
x - 30*a**2*b**3*d*e**3*x**2 + 3*a*b**4*d**4 - 4*a*b**4*d**3*e*x + 6*a*b**
4*d**2*e**2*x**2 - 3*a*b**4*e**4*x**4 + 3*b**5*d*e**3*x**4)/(12*a*(a**9*e*
**5 - 5*a**8*b*d*e**4 + 4*a**8*b*e**5*x + 10*a**7*b**2*d**2*e**3 - 20*a**7*
b**2*d*e**4*x + 6*a**7*b**2*e**5*x**2 - 10*a**6*b**3*d**3*e**2 + 40*a**6*b
**3*d**2*e**3*x - 30*a**6*b**3*d*e**4*x**2 + 4*a**6*b**3*e**5*x**3 + 5*a**
5*b**4*d**4*e - 40*a**5*b**4*d**3*e**2*x + 60*a**5*b**4*d**2*e**3*x**2 - 2
0*a**5*b**4*d*e**4*x**3 + a**5*b**4*e**5*x**4 - a**4*b**5*d**5 + 20*a**4*b
**5*d**4*e*x - 60*a**4*b**5*d**3*e**2*x**2 + 40*a**4*b**5*d**2*e**3*x**3 -
5*a**4*b**5*d*e**4*x**4 - 4*a**3*b**6*d**5*x + 30*a**3*b**6*d**4*e*x**2 -
40*a**3*b**6*d**3*e**2*x**3 + 10*a**3*b**6*d**2*e**3*x**4 - 6*a**2*b**7*d
**5*x**2 + 20*a**2*b**7*d**4*e*x**3 - 10*a**2*b**7*d**3*e**2*x**4 - 4*a*b*
**8*d**5*x**3 + 5*a*b**8*d**4*e*x**4 - b**9*d**5*x**4))
```

3.334 $\int \frac{1}{(d+ex)^2(a^2+2abx+b^2x^2)^{5/2}} dx$

| | |
|---|------|
| Optimal result | 2493 |
| Mathematica [A] (verified) | 2494 |
| Rubi [A] (verified) | 2494 |
| Maple [B] (verified) | 2496 |
| Fricas [B] (verification not implemented) | 2496 |
| Sympy [F] | 2497 |
| Maxima [F(-2)] | 2498 |
| Giac [B] (verification not implemented) | 2498 |
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| Reduce [B] (verification not implemented) | 2499 |

Optimal result

Integrand size = 28, antiderivative size = 307

$$\int \frac{1}{(d+ex)^2(a^2+2abx+b^2x^2)^{5/2}} dx = \frac{4be^3}{(bd-ae)^5\sqrt{a^2+2abx+b^2x^2}} - \frac{b}{4(bd-ae)^2(a+bx)^3\sqrt{a^2+2abx+b^2x^2}} + \frac{2be}{3(bd-ae)^3(a+bx)^2\sqrt{a^2+2abx+b^2x^2}} - \frac{3be^2}{2(bd-ae)^4(a+bx)\sqrt{a^2+2abx+b^2x^2}} + \frac{e^4(a+bx)}{(bd-ae)^5(d+ex)\sqrt{a^2+2abx+b^2x^2}} + \frac{5be^4(a+bx)\log(a+bx)}{(bd-ae)^6\sqrt{a^2+2abx+b^2x^2}} - \frac{5be^4(a+bx)\log(d+ex)}{(bd-ae)^6\sqrt{a^2+2abx+b^2x^2}}$$

output

```
4*b*e^3/(-a*e+b*d)^5/((b*x+a)^2)^(1/2)-1/4*b/(-a*e+b*d)^2/(b*x+a)^3/((b*x+a)^2)^(1/2)+2/3*b*e/(-a*e+b*d)^3/(b*x+a)^2/((b*x+a)^2)^(1/2)-3/2*b*e^2/(-a*e+b*d)^4/(b*x+a)/((b*x+a)^2)^(1/2)+e^4*(b*x+a)/(-a*e+b*d)^5/(e*x+d)/((b*x+a)^2)^(1/2)+5*b*e^4*(b*x+a)*ln(b*x+a)/(-a*e+b*d)^6/((b*x+a)^2)^(1/2)-5*b*e^4*(b*x+a)*ln(e*x+d)/(-a*e+b*d)^6/((b*x+a)^2)^(1/2)
```

Mathematica [A] (verified)

Time = 1.13 (sec) , antiderivative size = 167, normalized size of antiderivative = 0.54

$$\int \frac{1}{(d+ex)^2 (a^2+2abx+b^2x^2)^{5/2}} dx = \frac{8be(bd-ae)^3 - \frac{3b(bd-ae)^4}{a+bx} - 18be^2(bd-ae)^2(a+bx) + 48be^3(bd-ae)^2(a+bx) + 48b^2e^3(bd-ae)(a+bx)^2 + 12e^4(bd-ae)(a+bx)^3}{(d+ex)^2 (a^2+2abx+b^2x^2)^{5/2}}$$

input `Integrate[1/((d + e*x)^2*(a^2 + 2*a*b*x + b^2*x^2)^(5/2)),x]`

output `(8*b*e*(b*d - a*e)^3 - (3*b*(b*d - a*e)^4)/(a + b*x) - 18*b*e^2*(b*d - a*e)^2*(a + b*x) + 48*b*e^3*(b*d - a*e)*(a + b*x)^2 + (12*e^4*(b*d - a*e)*(a + b*x)^3)/(d + e*x) + 60*b*e^4*(a + b*x)^3*Log[a + b*x] - 60*b*e^4*(a + b*x)^3*Log[d + e*x])/(12*(b*d - a*e)^6*((a + b*x)^2)^(3/2))`

Rubi [A] (verified)

Time = 0.69 (sec) , antiderivative size = 185, normalized size of antiderivative = 0.60, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {1102, 27, 54, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\begin{aligned} & \int \frac{1}{(a^2+2abx+b^2x^2)^{5/2} (d+ex)^2} dx \\ & \quad \downarrow \text{1102} \\ & \frac{b^5(a+bx) \int \frac{1}{b^5(a+bx)^5(d+ex)^2} dx}{\sqrt{a^2+2abx+b^2x^2}} \\ & \quad \downarrow \text{27} \\ & \frac{(a+bx) \int \frac{1}{(a+bx)^5(d+ex)^2} dx}{\sqrt{a^2+2abx+b^2x^2}} \\ & \quad \downarrow \text{54} \end{aligned}$$

$$(a + bx) \int \left(-\frac{5be^5}{(bd-ae)^6(d+ex)} - \frac{e^5}{(bd-ae)^5(d+ex)^2} + \frac{5b^2e^4}{(bd-ae)^6(a+bx)} - \frac{4b^2e^3}{(bd-ae)^5(a+bx)^2} + \frac{3b^2e^2}{(bd-ae)^4(a+bx)^3} - \frac{2b^2e}{(bd-ae)^3(a+bx)^4} \right) \frac{1}{\sqrt{a^2 + 2abx + b^2x^2}}$$

↓ 2009

$$(a + bx) \left(\frac{e^4}{(d+ex)(bd-ae)^5} + \frac{5be^4 \log(a+bx)}{(bd-ae)^6} - \frac{5be^4 \log(d+ex)}{(bd-ae)^6} + \frac{4be^3}{(a+bx)(bd-ae)^5} - \frac{3be^2}{2(a+bx)^2(bd-ae)^4} + \frac{2be}{3(a+bx)^3(bd-ae)^3} - \frac{1}{4(a+bx)^4(bd-ae)^2} \right) \frac{1}{\sqrt{a^2 + 2abx + b^2x^2}}$$

input

```
Int[1/((d + e*x)^2*(a^2 + 2*a*b*x + b^2*x^2)^(5/2)),x]
```

output

```
((a + b*x)*(-1/4*b/((b*d - a*e)^2*(a + b*x)^4) + (2*b*e)/(3*(b*d - a*e)^3*(a + b*x)^3) - (3*b*e^2)/(2*(b*d - a*e)^4*(a + b*x)^2) + (4*b*e^3)/((b*d - a*e)^5*(a + b*x)) + e^4/((b*d - a*e)^5*(d + e*x)) + (5*b*e^4*Log[a + b*x])/(b*d - a*e)^6 - (5*b*e^4*Log[d + e*x])/(b*d - a*e)^6)/Sqrt[a^2 + 2*a*b*x + b^2*x^2]
```

Defintions of rubi rules used

rule 27

```
Int[(a_)*(Fx_), x_Symbol] := Simp[a Int[Fx, x], x] /; FreeQ[a, x] && !MatchQ[Fx, (b_)*(Gx_)] /; FreeQ[b, x]
```

rule 54

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] := Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] && ILtQ[m, 0] && IntegerQ[n] && !(IGtQ[n, 0] && LtQ[m + n + 2, 0])
```

rule 1102

```
Int[((d_.) + (e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] := Simp[(a + b*x + c*x^2)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x)^(2*FracPart[p])) Int[(d + e*x)^m*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && EqQ[b^2 - 4*a*c, 0]
```

rule 2009

```
Int[u_, x_Symbol] := Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 650 vs. 2(224) = 448.

Time = 1.69 (sec) , antiderivative size = 651, normalized size of antiderivative = 2.12

| method | result |
|---------|--|
| default | $\frac{(-240 \ln(ex+d)a^3b^2e^5x^2+240 \ln(bx+a)xa^3b^2de^4-60 \ln(ex+d)b^5e^5x^5-20a^3b^2de^4x-60 \ln(ex+d)a^4bde^4-60 \ln(ex+d)a^4be^5x-30 \ln(ex+d)a^4b^2e^5x^2-20 \ln(ex+d)a^4b^2e^5x^3-20 \ln(ex+d)a^4b^2e^5x^4-20 \ln(ex+d)a^4b^2e^5x^5)}{(bx+a)^2}$ |
| risch | $\frac{\sqrt{(bx+a)^2} \left(-\frac{5b^4e^4x^4}{e^5a^5-5a^4bde^4+10a^3b^2d^2e^3-10a^2b^3d^3e^2+5ab^4d^4e-b^5d^5} - \frac{5b^3(7ae+bd)e^3x^3}{2(e^5a^5-5a^4bde^4+10a^3b^2d^2e^3-10a^2b^3d^3e^2+5ab^4d^4e-b^5d^5)} \right)}{(bx+a)^2}$ |

```
input int(1/(e*x+d)^2/(b^2*x^2+2*a*b*x+a^2)^(5/2), x, method=_RETURNVERBOSE)
```

```
output 1/12*(-240*ln(e*x+d)*a^3*b^2*e^5*x^2+240*ln(b*x+a)*x*a^3*b^2*d*e^4-60*ln(e*x+d)*b^5*e^5*x^5-20*a^3*b^2*d*e^4*x-60*ln(e*x+d)*a^4*b*d*e^4-60*ln(e*x+d)*a^4*b*e^5*x-3*b^5*d^5-360*ln(e*x+d)*a^2*b^3*e^5*x^3-360*ln(e*x+d)*a^2*b^3*d*e^4*x^2+240*ln(b*x+a)*x^3*a*b^4*d*e^4-240*ln(e*x+d)*a*b^4*e^5*x^4-60*ln(e*x+d)*b^5*d*e^4*x^4-240*ln(e*x+d)*a*b^4*d*e^4*x^3+60*ln(b*x+a)*b^5*d*e^4*x^4+360*ln(b*x+a)*x^3*a^2*b^3*e^5-65*a^4*b*d*e^4+120*a^3*b^2*d^2*e^3-240*ln(e*x+d)*a^3*b^2*d*e^4*x-125*a^4*b*e^5*x+5*b^5*d^4*e*x+360*ln(b*x+a)*x^2*a^2*b^3*d*e^4-60*a^2*b^3*d^3*e^2+20*a*b^4*d^4*e+60*ln(b*x+a)*x^5*b^5*e^5+240*ln(b*x+a)*x^2*a^3*b^2*e^5+180*x*a^2*b^3*d^2*e^3-40*x*a*b^4*d^3*e^2+240*ln(b*x+a)*x^4*a*b^4*e^5+150*x^2*a^2*b^3*d*e^4+120*x^2*a*b^4*d^2*e^3+180*x^3*a*b^4*d*e^4+60*ln(b*x+a)*x*a^4*b*e^5+60*ln(b*x+a)*a^4*b*d*e^4-60*x^4*a*b^4*e^5+60*x^4*b^5*d*e^4-210*x^3*a^2*b^3*e^5+30*x^3*b^5*d^2*e^3-260*x^2*a^3*b^2*e^5-10*x^2*b^5*d^3*e^2-12*e^5*a^5)*(b*x+a)/(e*x+d)/(a*e-b*d)^6/((b*x+a)^2)^(5/2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1083 vs. 2(224) = 448.

Time = 0.11 (sec) , antiderivative size = 1083, normalized size of antiderivative = 3.53

$$\int \frac{1}{(d+ex)^2(a^2+2abx+b^2x^2)^{5/2}} dx = \text{Too large to display}$$

input `integrate(1/(e*x+d)^2/(b^2*x^2+2*a*b*x+a^2)^(5/2),x, algorithm="fricas")`

output

$$\begin{aligned}
 & -1/12*(3*b^5*d^5 - 20*a*b^4*d^4*e + 60*a^2*b^3*d^3*e^2 - 120*a^3*b^2*d^2*e^3 \\
 & + 65*a^4*b*d*e^4 + 12*a^5*e^5 - 60*(b^5*d*e^4 - a*b^4*e^5)*x^4 - 30*(b^5*d^2*e^3 \\
 & + 6*a*b^4*d*e^4 - 7*a^2*b^3*e^5)*x^3 + 10*(b^5*d^3*e^2 - 12*a*b^4*d^2*e^3 \\
 & - 15*a^2*b^3*d*e^4 + 26*a^3*b^2*e^5)*x^2 - 5*(b^5*d^4*e - 8*a*b^4*d^3*e^2 \\
 & + 36*a^2*b^3*d^2*e^3 - 4*a^3*b^2*d*e^4 - 25*a^4*b*e^5)*x - 60*(b^5*e^5*x^5 \\
 & + a^4*b*d*e^4 + (b^5*d*e^4 + 4*a*b^4*e^5)*x^4 + 2*(2*a*b^4*d*e^4 + 3*a^2*b^3*e^5)*x^3 \\
 & + 2*(3*a^2*b^3*d*e^4 + 2*a^3*b^2*e^5)*x^2 + (4*a^3*b^2*d*e^4 + a^4*b*e^5)*x) * \log(b*x + a) \\
 & + 60*(b^5*e^5*x^5 + a^4*b*d*e^4 + (b^5*d*e^4 + 4*a*b^4*e^5)*x^4 + 2*(2*a*b^4*d*e^4 \\
 & + 3*a^2*b^3*e^5)*x^3 + 2*(3*a^2*b^3*d*e^4 + 2*a^3*b^2*e^5)*x^2 + (4*a^3*b^2*d*e^4 \\
 & + a^4*b*e^5)*x) * \log(e*x + d) / (a^4*b^6*d^7 - 6*a^5*b^5*d^6*e + 15*a^6*b^4*d^5*e^2 - 20*a^7*b^3*d^4*e^3 \\
 & + 15*a^8*b^2*d^3*e^4 - 6*a^9*b*d^2*e^5 + a^10*d*e^6 + (b^10*d^6*e - 6*a*b^9*d^5*e^2 \\
 & + 15*a^2*b^8*d^4*e^3 - 20*a^3*b^7*d^3*e^4 + 15*a^4*b^6*d^2*e^5 - 6*a^5*b^5*d*e^6 \\
 & + a^6*b^4*e^7)*x^5 + (b^10*d^7 - 2*a*b^9*d^6*e - 9*a^2*b^8*d^5*e^2 + 40*a^3*b^7*d^4*e^3 \\
 & - 65*a^4*b^6*d^3*e^4 + 54*a^5*b^5*d^2*e^5 - 23*a^6*b^4*d*e^6 + 4*a^7*b^3*e^7)*x^4 \\
 & + 2*(2*a*b^9*d^7 - 9*a^2*b^8*d^6*e + 12*a^3*b^7*d^5*e^2 + 5*a^4*b^6*d^4*e^3 - 30*a^5*b^5*d^3*e^4 \\
 & + 33*a^6*b^4*d^2*e^5 - 16*a^7*b^3*d*e^6 + 3*a^8*b^2*e^7)*x^3 + 2*(3*a^2*b^8*d^7 \\
 & - 16*a^3*b^7*d^6*e + 33*a^4*b^6*d^5*e^2 - 30*a^5*b^5*d^4*e^3 + 5*a^6*b^4*d^3*e^4 \\
 & + 12*a^7*b^3*d^2*e^5 - 9*a^8*b^2*d*e^6 + 2*a^9*b*e^7)*x^2 + \dots
 \end{aligned}$$

Sympy [F]

$$\int \frac{1}{(d+ex)^2 (a^2+2abx+b^2x^2)^{5/2}} dx = \int \frac{1}{(d+ex)^2 ((a+bx)^2)^{5/2}} dx$$

input `integrate(1/(e*x+d)**2/(b**2*x**2+2*a*b*x+a**2)**(5/2),x)`

output `Integral(1/((d + e*x)**2*((a + b*x)**2)**(5/2)), x)`

Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{(d+ex)^2 (a^2+2abx+b^2x^2)^{5/2}} dx = \text{Exception raised: ValueError}$$

input `integrate(1/(e*x+d)^2/(b^2*x^2+2*a*b*x+a^2)^(5/2),x, algorithm="maxima")`

output `Exception raised: ValueError >> Computation failed since Maxima requested additional constraints; using the 'assume' command before evaluation *may* help (example of legal syntax is 'assume(a*e-b*d>0)', see `assume?` for more detail`

Giac [B] (verification not implemented)Leaf count of result is larger than twice the leaf count of optimal. 546 vs. $2(224) = 448$.

Time = 0.13 (sec) , antiderivative size = 546, normalized size of antiderivative = 1.78

$$\int \frac{1}{(d+ex)^2 (a^2+2abx+b^2x^2)^{5/2}} dx = \frac{b^7 d^6 \operatorname{sgn}(bx+a) - 6 ab^6 d^5 \operatorname{sgn}(bx+a) + 15 a^2 b^5 d^4 e^2 \operatorname{sgn}(bx+a) + 5 b e^5 \log(|ex+d|)}{b^6 d^6 \operatorname{sgn}(bx+a) - 6 ab^5 d^5 e^2 \operatorname{sgn}(bx+a) + 15 a^2 b^4 d^4 e^3 \operatorname{sgn}(bx+a) - 20 a^3 b^3 d^3 e^4 \operatorname{sgn}(bx+a) + 15 a^4 b^2 e^5 - 3 b^5 d^5 - 20 ab^4 d^4 e + 60 a^2 b^3 d^3 e^2 - 120 a^3 b^2 d^2 e^3 + 65 a^4 b d e^4 + 12 a^5 e^5 - 60 (b^5 d e^4 - ab^4 e^5) x^4 - 30 (b^5 d^2 e^4 - ab^4 d e^5) x^3 - 15 (b^5 d^3 e^4 - ab^4 d^2 e^5) x^2 - 5 (b^5 d^4 e^4 - ab^4 d^3 e^5) x - 5 b^5 d^5 e^4}$$

input `integrate(1/(e*x+d)^2/(b^2*x^2+2*a*b*x+a^2)^(5/2),x, algorithm="giac")`

output

```
5*b^2*e^4*log(abs(b*x + a))/(b^7*d^6*sgn(b*x + a) - 6*a*b^6*d^5*e*sgn(b*x
+ a) + 15*a^2*b^5*d^4*e^2*sgn(b*x + a) - 20*a^3*b^4*d^3*e^3*sgn(b*x + a) +
15*a^4*b^3*d^2*e^4*sgn(b*x + a) - 6*a^5*b^2*d*e^5*sgn(b*x + a) + a^6*b*e^
6*sgn(b*x + a) - 5*b*e^5*log(abs(e*x + d))/(b^6*d^6*e*sgn(b*x + a) - 6*a*
b^5*d^5*e^2*sgn(b*x + a) + 15*a^2*b^4*d^4*e^3*sgn(b*x + a) - 20*a^3*b^3*d^
3*e^4*sgn(b*x + a) + 15*a^4*b^2*d^2*e^5*sgn(b*x + a) - 6*a^5*b*d*e^6*sgn(b
*x + a) + a^6*e^7*sgn(b*x + a)) - 1/12*(3*b^5*d^5 - 20*a*b^4*d^4*e + 60*a^
2*b^3*d^3*e^2 - 120*a^3*b^2*d^2*e^3 + 65*a^4*b*d*e^4 + 12*a^5*e^5 - 60*(b^
5*d*e^4 - a*b^4*e^5)*x^4 - 30*(b^5*d^2*e^3 + 6*a*b^4*d*e^4 - 7*a^2*b^3*e^5
)*x^3 + 10*(b^5*d^3*e^2 - 12*a*b^4*d^2*e^3 - 15*a^2*b^3*d*e^4 + 26*a^3*b^2
*e^5)*x^2 - 5*(b^5*d^4*e - 8*a*b^4*d^3*e^2 + 36*a^2*b^3*d^2*e^3 - 4*a^3*b^
2*d*e^4 - 25*a^4*b*e^5)*x)/((b*d - a*e)^6*(b*x + a)^4*(e*x + d)*sgn(b*x +
a))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(d + ex)^2 (a^2 + 2abx + b^2x^2)^{5/2}} dx = \int \frac{1}{(d + ex)^2 (a^2 + 2abx + b^2x^2)^{5/2}} dx$$

input

```
int(1/((d + e*x)^2*(a^2 + b^2*x^2 + 2*a*b*x)^(5/2)),x)
```

output

```
int(1/((d + e*x)^2*(a^2 + b^2*x^2 + 2*a*b*x)^(5/2)), x)
```

Reduce [B] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 1706, normalized size of antiderivative = 5.56

$$\int \frac{1}{(d + ex)^2 (a^2 + 2abx + b^2x^2)^{5/2}} dx = \text{Too large to display}$$

input

```
int(1/(e*x+d)^2/(b^2*x^2+2*a*b*x+a^2)^(5/2),x)
```


output

```
(240*log(a + b*x)*a**5*b*d***5 + 240*log(a + b*x)*a**5*b***6*x + 60*log(
a + b*x)*a**4*b**2*d**2***4 + 1020*log(a + b*x)*a**4*b**2*d***5*x + 960*
log(a + b*x)*a**4*b**2***6*x**2 + 240*log(a + b*x)*a**3*b**3*d**2***4*x
+ 1680*log(a + b*x)*a**3*b**3*d***5*x**2 + 1440*log(a + b*x)*a**3*b**3***
6*x**3 + 360*log(a + b*x)*a**2*b**4*d**2***4*x**2 + 1320*log(a + b*x)*a*
**2*b**4*d***5*x**3 + 960*log(a + b*x)*a**2*b**4***6*x**4 + 240*log(a + b
*x)*a*b**5*d**2***4*x**3 + 480*log(a + b*x)*a*b**5*d***5*x**4 + 240*log(
a + b*x)*a*b**5***6*x**5 + 60*log(a + b*x)*b**6*d**2***4*x**4 + 60*log(a
+ b*x)*b**6*d***5*x**5 - 240*log(d + e*x)*a**5*b*d***5 - 240*log(d + e*
x)*a**5*b***6*x - 60*log(d + e*x)*a**4*b**2*d**2***4 - 1020*log(d + e*x)
*a**4*b**2*d***5*x - 960*log(d + e*x)*a**4*b**2***6*x**2 - 240*log(d + e
*x)*a**3*b**3*d**2***4*x - 1680*log(d + e*x)*a**3*b**3*d***5*x**2 - 1440
*log(d + e*x)*a**3*b**3***6*x**3 - 360*log(d + e*x)*a**2*b**4*d**2***4*x
**2 - 1320*log(d + e*x)*a**2*b**4*d***5*x**3 - 960*log(d + e*x)*a**2*b**4
***6*x**4 - 240*log(d + e*x)*a*b**5*d**2***4*x**3 - 480*log(d + e*x)*a*b
**5*d***5*x**4 - 240*log(d + e*x)*a*b**5***6*x**5 - 60*log(d + e*x)*b**6
*d**2***4*x**4 - 60*log(d + e*x)*b**6*d***5*x**5 - 48*a**6***6 - 212*a*
**5*b*d***5 - 440*a**5*b***6*x + 355*a**4*b**2*d**2***4 - 25*a**4*b**2*d
***5*x - 800*a**4*b**2***6*x**2 - 120*a**3*b**3*d**3***3 + 460*a**3*b**
3*d**2***4*x + 460*a**3*b**3*d***5*x**2 - 480*a**3*b**3***6*x**3 + 2...
```

3.335 $\int \frac{1}{(d+ex)^3(a^2+2abx+b^2x^2)^{5/2}} dx$

| | |
|---|------|
| Optimal result | 2501 |
| Mathematica [A] (verified) | 2502 |
| Rubi [A] (verified) | 2502 |
| Maple [B] (verified) | 2504 |
| Fricas [B] (verification not implemented) | 2505 |
| Sympy [F] | 2506 |
| Maxima [F(-2)] | 2507 |
| Giac [B] (verification not implemented) | 2507 |
| Mupad [F(-1)] | 2508 |
| Reduce [B] (verification not implemented) | 2508 |

Optimal result

Integrand size = 28, antiderivative size = 365

$$\int \frac{1}{(d+ex)^3(a^2+2abx+b^2x^2)^{5/2}} dx = \frac{10b^2e^3}{(bd-ae)^6\sqrt{a^2+2abx+b^2x^2}} - \frac{4(bd-ae)^3(a+bx)^3\sqrt{a^2+2abx+b^2x^2}}{b^2} + \frac{(bd-ae)^4(a+bx)^2\sqrt{a^2+2abx+b^2x^2}}{3b^2e^2} - \frac{(bd-ae)^5(a+bx)\sqrt{a^2+2abx+b^2x^2}}{e^4(a+bx)} + \frac{2(bd-ae)^5(d+ex)^2\sqrt{a^2+2abx+b^2x^2}}{5be^4(a+bx)} + \frac{(bd-ae)^6(d+ex)\sqrt{a^2+2abx+b^2x^2}}{15b^2e^4(a+bx)\log(a+bx)} - \frac{15b^2e^4(a+bx)\log(d+ex)}{(bd-ae)^7\sqrt{a^2+2abx+b^2x^2}}$$

output

$$10*b^2*e^3/(-a*e+b*d)^6/((b*x+a)^2)^{(1/2)}-1/4*b^2/(-a*e+b*d)^3/(b*x+a)^3/((b*x+a)^2)^{(1/2)}+b^2*e/(-a*e+b*d)^4/(b*x+a)^2/((b*x+a)^2)^{(1/2)}-3*b^2*e^2/(-a*e+b*d)^5/(b*x+a)/((b*x+a)^2)^{(1/2)}+1/2*e^4*(b*x+a)/(-a*e+b*d)^5/(e*x+d)^2/((b*x+a)^2)^{(1/2)}+5*b*e^4*(b*x+a)/(-a*e+b*d)^6/(e*x+d)/((b*x+a)^2)^{(1/2)}+15*b^2*e^4*(b*x+a)*\ln(b*x+a)/(-a*e+b*d)^7/((b*x+a)^2)^{(1/2)}-15*b^2*e^4*(b*x+a)*\ln(e*x+d)/(-a*e+b*d)^7/((b*x+a)^2)^{(1/2)}$$
Mathematica [A] (verified)

Time = 1.11 (sec) , antiderivative size = 209, normalized size of antiderivative = 0.57

$$\int \frac{1}{(d+ex)^3(a^2+2abx+b^2x^2)^{5/2}} dx = \frac{4b^2e(bd-ae)^3 - \frac{b^2(bd-ae)^4}{a+bx} - 12b^2e^2(bd-ae)^2(a+bx) + 40b^2e^3(bd-ae)(a+bx)}{(d+ex)^3(a^2+2abx+b^2x^2)^{5/2}}$$

input

Integrate[1/((d + e*x)^3*(a^2 + 2*a*b*x + b^2*x^2)^(5/2)), x]

output

$$(4*b^2*e*(b*d - a*e)^3 - (b^2*(b*d - a*e)^4)/(a + b*x) - 12*b^2*e^2*(b*d - a*e)^2*(a + b*x) + 40*b^2*e^3*(b*d - a*e)*(a + b*x)^2 + (2*e^4*(b*d - a*e)^2*(a + b*x)^3)/(d + e*x)^2 + (20*b*e^4*(b*d - a*e)*(a + b*x)^3)/(d + e*x) + 60*b^2*e^4*(a + b*x)^3*\text{Log}[a + b*x] - 60*b^2*e^4*(a + b*x)^3*\text{Log}[d + e*x])/ (4*(b*d - a*e)^7*((a + b*x)^2)^{(3/2)})$$
Rubi [A] (verified)Time = 0.76 (sec) , antiderivative size = 218, normalized size of antiderivative = 0.60, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {1102, 27, 54, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a^2 + 2abx + b^2x^2)^{5/2} (d + ex)^3} dx$$

↓ 1102

$$\begin{aligned}
& \frac{b^5(a+bx) \int \frac{1}{b^5(a+bx)^5(d+ex)^3} dx}{\sqrt{a^2+2abx+b^2x^2}} \\
& \quad \downarrow 27 \\
& \frac{(a+bx) \int \frac{1}{(a+bx)^5(d+ex)^3} dx}{\sqrt{a^2+2abx+b^2x^2}} \\
& \quad \downarrow 54 \\
& \frac{(a+bx) \int \left(-\frac{15b^2e^5}{(bd-ae)^7(d+ex)} - \frac{5be^5}{(bd-ae)^6(d+ex)^2} - \frac{e^5}{(bd-ae)^5(d+ex)^3} + \frac{15b^3e^4}{(bd-ae)^7(a+bx)} - \frac{10b^3e^3}{(bd-ae)^6(a+bx)^2} + \frac{6b^3e^2}{(bd-ae)^5(a+bx)^3} \right)}{\sqrt{a^2+2abx+b^2x^2}} \\
& \quad \downarrow 2009 \\
& \frac{(a+bx) \left(\frac{15b^2e^4 \log(a+bx)}{(bd-ae)^7} - \frac{15b^2e^4 \log(d+ex)}{(bd-ae)^7} + \frac{10b^2e^3}{(a+bx)(bd-ae)^6} - \frac{3b^2e^2}{(a+bx)^2(bd-ae)^5} + \frac{b^2e}{(a+bx)^3(bd-ae)^4} - \frac{b^2}{4(a+bx)^4(bd-ae)^3} + \right)}{\sqrt{a^2+2abx+b^2x^2}}
\end{aligned}$$

input `Int[1/((d + e*x)^3*(a^2 + 2*a*b*x + b^2*x^2)^(5/2)),x]`

output `((a + b*x)*(-1/4*b^2/((b*d - a*e)^3*(a + b*x)^4) + (b^2*e)/((b*d - a*e)^4*(a + b*x)^3) - (3*b^2*e^2)/((b*d - a*e)^5*(a + b*x)^2) + (10*b^2*e^3)/((b*d - a*e)^6*(a + b*x)) + e^4/(2*(b*d - a*e)^5*(d + e*x)^2) + (5*b*e^4)/((b*d - a*e)^6*(d + e*x)) + (15*b^2*e^4*Log[a + b*x])/((b*d - a*e)^7) - (15*b^2*e^4*Log[d + e*x])/((b*d - a*e)^7))/Sqrt[a^2 + 2*a*b*x + b^2*x^2]`

Defintions of rubi rules used

rule 27 `Int[(a_)*(F_x_), x_Symbol] := Simp[a Int[F_x, x], x] /; FreeQ[a, x] && !MatchQ[F_x, (b_)*(G_x_)] /; FreeQ[b, x]`

rule 54 `Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] := Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] && ILtQ[m, 0] && IntegerQ[n] && !(IGtQ[n, 0] && LtQ[m + n + 2, 0])`

rule 1102

```
Int[((d_.) + (e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol]
:> Simp[(a + b*x + c*x^2)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x)^(2*FracPart[p]))
Int[(d + e*x)^m*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d, e, m, p}, x]
&& EqQ[b^2 - 4*a*c, 0]
```

rule 2009

```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 982 vs. 2(273) = 546.

Time = 1.84 (sec) , antiderivative size = 983, normalized size of antiderivative = 2.69

| method | result |
|---------|--|
| default | $-\frac{(-360 \ln(ex+d)a^2b^4e^6x^4 - 60 \ln(ex+d)b^6d^2e^4x^4 - 60 \ln(ex+d)a^4b^2e^6x^2 - 60 \ln(ex+d)a^4b^2d^2e^4 + 60 \ln(bx+a)a^4b^2d^2e^4 + 8ab^5d^5e^5)}{...}$ |
| risch | $\frac{\sqrt{(bx+a)^2} \left(\frac{15b^5e^5x^5}{a^6e^6 - 6a^5bde^5 + 15a^4b^2d^2e^4 - 20a^3b^3d^3e^3 + 15a^2b^4d^4e^2 - 6ab^5d^5e + b^6d^6} + \frac{15e^4b^4(7ae+3bd)x^4}{2(a^6e^6 - 6a^5bde^5 + 15a^4b^2d^2e^4 - 20a^3b^3d^3e^3 + 15a^2b^4d^4e^2 - 6ab^5d^5e + b^6d^6)} \right)}{...}$ |

input

```
int(1/(e*x+d)^3/(b^2*x^2+2*a*b*x+a^2)^(5/2), x, method=_RETURNVERBOSE)
```

output

```

-1/4*(-360*ln(e*x+d)*a^2*b^4*e^6*x^4-60*ln(e*x+d)*b^6*d^2*e^4*x^4-60*ln(e*
x+d)*a^4*b^2*e^6*x^2-60*ln(e*x+d)*a^4*b^2*d^2*e^4+60*ln(b*x+a)*a^4*b^2*d^2
*e^4+8*a*b^5*d^5*e-24*a^5*b*d*e^5+80*a^3*b^3*d^3*e^3-120*ln(e*x+d)*a^4*b^2
*d*e^5*x+60*ln(b*x+a)*a^4*b^2*e^6*x^2+360*ln(b*x+a)*a^2*b^4*e^6*x^4-480*ln
(e*x+d)*a*b^5*d*e^5*x^4-480*ln(e*x+d)*x^2*a^3*b^3*d*e^5-35*a^4*b^2*d^2*e^4
-30*a^2*b^4*d^4*e^2+60*ln(b*x+a)*b^6*e^6*x^6-720*ln(e*x+d)*x^3*a^2*b^4*d*e
^5+240*ln(b*x+a)*a*b^5*e^6*x^5+120*ln(b*x+a)*b^6*d*e^5*x^5-240*ln(e*x+d)*a
*b^5*e^6*x^5-120*ln(e*x+d)*b^6*d*e^5*x^5-b^6*d^6-240*ln(e*x+d)*x^3*a*b^5*d
^2*e^4+60*ln(b*x+a)*b^6*d^2*e^4*x^4+240*ln(b*x+a)*a^3*b^3*e^6*x^3-240*ln(e
*x+d)*x*a^3*b^3*d^2*e^4-360*ln(e*x+d)*x^2*a^2*b^4*d^2*e^4+720*ln(b*x+a)*a^
2*b^4*d*e^5*x^3+240*ln(b*x+a)*a*b^5*d^2*e^4*x^3+480*ln(b*x+a)*a^3*b^3*d*e^
5*x^2+360*ln(b*x+a)*a^2*b^4*d^2*e^4*x^2+120*ln(b*x+a)*a^4*b^2*d*e^5*x+240*
ln(b*x+a)*a^3*b^3*d^2*e^4*x+480*ln(b*x+a)*a*b^5*d*e^5*x^4-60*ln(e*x+d)*b^6
*e^6*x^6+300*x^3*a*b^5*d^2*e^4-280*x^2*a^3*b^3*d*e^5+330*x^2*a^2*b^4*d^2*e
^4+80*x^2*a*b^5*d^3*e^3-190*x*a^4*b^2*d*e^5+100*x*a^3*b^3*d^2*e^4+120*x*a^
2*b^4*d^3*e^3-20*x*a*b^5*d^4*e^2-260*x^3*a^3*b^3*e^6+20*x^3*b^6*d^3*e^3-12
5*x^2*a^4*b^2*e^6-5*x^2*b^6*d^4*e^2-12*x*a^5*b*e^6+2*x*b^6*d^5*e-240*ln(e*
x+d)*x^3*a^3*b^3*e^6+120*x^4*a*b^5*d*e^5-60*x^3*a^2*b^4*d*e^5+2*a^6*e^6-60
*x^5*a*b^5*e^6+60*x^5*b^6*d*e^5-210*x^4*a^2*b^4*e^6+90*x^4*b^6*d^2*e^4)*(b
*x+a)/(e*x+d)^2/(a*e-b*d)^7/((b*x+a)^2)^(5/2)

```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1565 vs. $2(273) = 546$.

Time = 0.12 (sec) , antiderivative size = 1565, normalized size of antiderivative = 4.29

$$\int \frac{1}{(d+ex)^3 (a^2+2abx+b^2x^2)^{5/2}} dx = \text{Too large to display}$$

input

```
integrate(1/(e*x+d)^3/(b^2*x^2+2*a*b*x+a^2)^(5/2),x, algorithm="fricas")
```

output

```

-1/4*(b^6*d^6 - 8*a*b^5*d^5*e + 30*a^2*b^4*d^4*e^2 - 80*a^3*b^3*d^3*e^3 +
35*a^4*b^2*d^2*e^4 + 24*a^5*b*d*e^5 - 2*a^6*e^6 - 60*(b^6*d*e^5 - a*b^5*e^
6)*x^5 - 30*(3*b^6*d^2*e^4 + 4*a*b^5*d*e^5 - 7*a^2*b^4*e^6)*x^4 - 20*(b^6*
d^3*e^3 + 15*a*b^5*d^2*e^4 - 3*a^2*b^4*d*e^5 - 13*a^3*b^3*e^6)*x^3 + 5*(b^
6*d^4*e^2 - 16*a*b^5*d^3*e^3 - 66*a^2*b^4*d^2*e^4 + 56*a^3*b^3*d*e^5 + 25*
a^4*b^2*e^6)*x^2 - 2*(b^6*d^5*e - 10*a*b^5*d^4*e^2 + 60*a^2*b^4*d^3*e^3 +
50*a^3*b^3*d^2*e^4 - 95*a^4*b^2*d*e^5 - 6*a^5*b*e^6)*x - 60*(b^6*e^6*x^6 +
a^4*b^2*d^2*e^4 + 2*(b^6*d*e^5 + 2*a*b^5*e^6)*x^5 + (b^6*d^2*e^4 + 8*a*b^
5*d*e^5 + 6*a^2*b^4*e^6)*x^4 + 4*(a*b^5*d^2*e^4 + 3*a^2*b^4*d*e^5 + a^3*b^
3*e^6)*x^3 + (6*a^2*b^4*d^2*e^4 + 8*a^3*b^3*d*e^5 + a^4*b^2*e^6)*x^2 + 2*(
2*a^3*b^3*d^2*e^4 + a^4*b^2*d*e^5)*x)*log(b*x + a) + 60*(b^6*e^6*x^6 + a^4
*b^2*d^2*e^4 + 2*(b^6*d*e^5 + 2*a*b^5*e^6)*x^5 + (b^6*d^2*e^4 + 8*a*b^5*d*
e^5 + 6*a^2*b^4*e^6)*x^4 + 4*(a*b^5*d^2*e^4 + 3*a^2*b^4*d*e^5 + a^3*b^3*e^
6)*x^3 + (6*a^2*b^4*d^2*e^4 + 8*a^3*b^3*d*e^5 + a^4*b^2*e^6)*x^2 + 2*(2*a^
3*b^3*d^2*e^4 + a^4*b^2*d*e^5)*x)*log(e*x + d))/(a^4*b^7*d^9 - 7*a^5*b^6*d
^8*e + 21*a^6*b^5*d^7*e^2 - 35*a^7*b^4*d^6*e^3 + 35*a^8*b^3*d^5*e^4 - 21*a
^9*b^2*d^4*e^5 + 7*a^10*b*d^3*e^6 - a^11*d^2*e^7 + (b^11*d^7*e^2 - 7*a*b^1
0*d^6*e^3 + 21*a^2*b^9*d^5*e^4 - 35*a^3*b^8*d^4*e^5 + 35*a^4*b^7*d^3*e^6 -
21*a^5*b^6*d^2*e^7 + 7*a^6*b^5*d*e^8 - a^7*b^4*e^9)*x^6 + 2*(b^11*d^8*e -
5*a*b^10*d^7*e^2 + 7*a^2*b^9*d^6*e^3 + 7*a^3*b^8*d^5*e^4 - 35*a^4*b^7*...

```

Sympy [F]

$$\int \frac{1}{(d+ex)^3 (a^2+2abx+b^2x^2)^{5/2}} dx = \int \frac{1}{(d+ex)^3 ((a+bx)^2)^{5/2}} dx$$

input

```
integrate(1/(e*x+d)**3/(b**2*x**2+2*a*b*x+a**2)**(5/2), x)
```

output

```
Integral(1/((d + e*x)**3*((a + b*x)**2)**(5/2)), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{(d+ex)^3 (a^2+2abx+b^2x^2)^{5/2}} dx = \text{Exception raised: ValueError}$$

input `integrate(1/(e*x+d)^3/(b^2*x^2+2*a*b*x+a^2)^(5/2),x, algorithm="maxima")`

output `Exception raised: ValueError >> Computation failed since Maxima requested additional constraints; using the 'assume' command before evaluation *may* help (example of legal syntax is 'assume(a*e-b*d>0)', see `assume?` for more detail`

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 683 vs. 2(273) = 546.

Time = 0.16 (sec) , antiderivative size = 683, normalized size of antiderivative = 1.87

$$\int \frac{1}{(d+ex)^3 (a^2+2abx+b^2x^2)^{5/2}} dx = \text{Too large to display}$$

input `integrate(1/(e*x+d)^3/(b^2*x^2+2*a*b*x+a^2)^(5/2),x, algorithm="giac")`

output

```

15*b^3*e^4*log(abs(b*x + a))/(b^8*d^7*sgn(b*x + a) - 7*a*b^7*d^6*e*sgn(b*x
+ a) + 21*a^2*b^6*d^5*e^2*sgn(b*x + a) - 35*a^3*b^5*d^4*e^3*sgn(b*x + a)
+ 35*a^4*b^4*d^3*e^4*sgn(b*x + a) - 21*a^5*b^3*d^2*e^5*sgn(b*x + a) + 7*a^
6*b^2*d*e^6*sgn(b*x + a) - a^7*b*e^7*sgn(b*x + a)) - 15*b^2*e^5*log(abs(e*
x + d))/(b^7*d^7*e*sgn(b*x + a) - 7*a*b^6*d^6*e^2*sgn(b*x + a) + 21*a^2*b^
5*d^5*e^3*sgn(b*x + a) - 35*a^3*b^4*d^4*e^4*sgn(b*x + a) + 35*a^4*b^3*d^3*
e^5*sgn(b*x + a) - 21*a^5*b^2*d^2*e^6*sgn(b*x + a) + 7*a^6*b*d*e^7*sgn(b*x
+ a) - a^7*e^8*sgn(b*x + a)) - 1/4*(b^6*d^6 - 8*a*b^5*d^5*e + 30*a^2*b^4*d
^4*e^2 - 80*a^3*b^3*d^3*e^3 + 35*a^4*b^2*d^2*e^4 + 24*a^5*b*d*e^5 - 2*a^6
*e^6 - 60*(b^6*d*e^5 - a*b^5*e^6)*x^5 - 30*(3*b^6*d^2*e^4 + 4*a*b^5*d*e^5
- 7*a^2*b^4*e^6)*x^4 - 20*(b^6*d^3*e^3 + 15*a*b^5*d^2*e^4 - 3*a^2*b^4*d*e^
5 - 13*a^3*b^3*e^6)*x^3 + 5*(b^6*d^4*e^2 - 16*a*b^5*d^3*e^3 - 66*a^2*b^4*d
^2*e^4 + 56*a^3*b^3*d*e^5 + 25*a^4*b^2*e^6)*x^2 - 2*(b^6*d^5*e - 10*a*b^5*
d^4*e^2 + 60*a^2*b^4*d^3*e^3 + 50*a^3*b^3*d^2*e^4 - 95*a^4*b^2*d*e^5 - 6*a
^5*b*e^6)*x)/((b*d - a*e)^7*(b*x + a)^4*(e*x + d)^2*sgn(b*x + a))

```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(d + ex)^3 (a^2 + 2abx + b^2x^2)^{5/2}} dx = \int \frac{1}{(d + ex)^3 (a^2 + 2abx + b^2x^2)^{5/2}} dx$$

input

```
int(1/((d + e*x)^3*(a^2 + b^2*x^2 + 2*a*b*x)^(5/2)),x)
```

output

```
int(1/((d + e*x)^3*(a^2 + b^2*x^2 + 2*a*b*x)^(5/2)), x)
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 2408, normalized size of antiderivative = 6.60

$$\int \frac{1}{(d + ex)^3 (a^2 + 2abx + b^2x^2)^{5/2}} dx = \text{Too large to display}$$

input

```
int(1/(e*x+d)^3/(b^2*x^2+2*a*b*x+a^2)^(5/2),x)
```

output

```
( - 120*log(a + b*x)*a**5*b**2*d**2*e**5 - 240*log(a + b*x)*a**5*b**2*d*e*
*6*x - 120*log(a + b*x)*a**5*b**2*e**7*x**2 - 60*log(a + b*x)*a**4*b**3*d*
*3*e**4 - 600*log(a + b*x)*a**4*b**3*d**2*e**5*x - 1020*log(a + b*x)*a**4*
b**3*d*e**6*x**2 - 480*log(a + b*x)*a**4*b**3*e**7*x**3 - 240*log(a + b*x)
*a**3*b**4*d**3*e**4*x - 1200*log(a + b*x)*a**3*b**4*d**2*e**5*x**2 - 1680
*log(a + b*x)*a**3*b**4*d*e**6*x**3 - 720*log(a + b*x)*a**3*b**4*e**7*x**4
- 360*log(a + b*x)*a**2*b**5*d**3*e**4*x**2 - 1200*log(a + b*x)*a**2*b**5
*d**2*e**5*x**3 - 1320*log(a + b*x)*a**2*b**5*d*e**6*x**4 - 480*log(a + b*
x)*a**2*b**5*e**7*x**5 - 240*log(a + b*x)*a*b**6*d**3*e**4*x**3 - 600*log(
a + b*x)*a*b**6*d**2*e**5*x**4 - 480*log(a + b*x)*a*b**6*d*e**6*x**5 - 120
*log(a + b*x)*a*b**6*e**7*x**6 - 60*log(a + b*x)*b**7*d**3*e**4*x**4 - 120
*log(a + b*x)*b**7*d**2*e**5*x**5 - 60*log(a + b*x)*b**7*d*e**6*x**6 + 120
*log(d + e*x)*a**5*b**2*d**2*e**5 + 240*log(d + e*x)*a**5*b**2*d*e**6*x +
120*log(d + e*x)*a**5*b**2*e**7*x**2 + 60*log(d + e*x)*a**4*b**3*d**3*e**4
+ 600*log(d + e*x)*a**4*b**3*d**2*e**5*x + 1020*log(d + e*x)*a**4*b**3*d*
e**6*x**2 + 480*log(d + e*x)*a**4*b**3*e**7*x**3 + 240*log(d + e*x)*a**3*b
**4*d**3*e**4*x + 1200*log(d + e*x)*a**3*b**4*d**2*e**5*x**2 + 1680*log(d
+ e*x)*a**3*b**4*d*e**6*x**3 + 720*log(d + e*x)*a**3*b**4*e**7*x**4 + 360*
log(d + e*x)*a**2*b**5*d**3*e**4*x**2 + 1200*log(d + e*x)*a**2*b**5*d**2*
e**5*x**3 + 1320*log(d + e*x)*a**2*b**5*d*e**6*x**4 + 480*log(d + e*x)*a...
```

3.336 $\int (d + ex) (9 + 12x + 4x^2)^{5/2} dx$

| | |
|---|------|
| Optimal result | 2510 |
| Mathematica [A] (verified) | 2510 |
| Rubi [A] (verified) | 2511 |
| Maple [B] (verified) | 2512 |
| Fricas [A] (verification not implemented) | 2512 |
| Sympy [B] (verification not implemented) | 2513 |
| Maxima [A] (verification not implemented) | 2513 |
| Giac [B] (verification not implemented) | 2514 |
| Mupad [F(-1)] | 2514 |
| Reduce [B] (verification not implemented) | 2515 |

Optimal result

Integrand size = 20, antiderivative size = 52

$$\int (d + ex) (9 + 12x + 4x^2)^{5/2} dx = \frac{1}{24}(2d - 3e)(3 + 2x)^5 \sqrt{9 + 12x + 4x^2} + \frac{1}{28}e(9 + 12x + 4x^2)^{7/2}$$

output

```
1/24*(2*d-3*e)*(3+2*x)^5*((3+2*x)^2)^(1/2)+1/28*e*(4*x^2+12*x+9)^(7/2)
```

Mathematica [A] (verified)

Time = 0.44 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.65

$$\int (d + ex) (9 + 12x + 4x^2)^{5/2} dx = \frac{1}{168}(3 + 2x)^5 \sqrt{(3 + 2x)^2(14d + 3e(-1 + 4x))}$$

input

```
Integrate[(d + e*x)*(9 + 12*x + 4*x^2)^(5/2),x]
```

output

```
((3 + 2*x)^5*Sqrt[(3 + 2*x)^2]*(14*d + 3*e*(-1 + 4*x)))/168
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.150$, Rules used = {1100, 1079, 17}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (4x^2 + 12x + 9)^{5/2} (d + ex) dx$$

$$\downarrow 1100$$

$$\frac{1}{2}(2d - 3e) \int (4x^2 + 12x + 9)^{5/2} dx + \frac{1}{28}e(4x^2 + 12x + 9)^{7/2}$$

$$\downarrow 1079$$

$$\frac{\sqrt{4x^2 + 12x + 9}(2d - 3e) \int (4x + 6)^5 dx}{64(2x + 3)} + \frac{1}{28}e(4x^2 + 12x + 9)^{7/2}$$

$$\downarrow 17$$

$$\frac{1}{24}\sqrt{4x^2 + 12x + 9}(2x + 3)^5(2d - 3e) + \frac{1}{28}e(4x^2 + 12x + 9)^{7/2}$$

input `Int[(d + e*x)*(9 + 12*x + 4*x^2)^(5/2), x]`

output `((2*d - 3*e)*(3 + 2*x)^5*Sqrt[9 + 12*x + 4*x^2])/24 + (e*(9 + 12*x + 4*x^2)^(7/2))/28`

Defintions of rubi rules used

rule 17 `Int[(c_.)*((a_.) + (b_.)*(x_)^(m_.), x_Symbol] := Simp[c*((a + b*x)^(m + 1))/(b*(m + 1)), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]`

rule 1079 `Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] := Simp[(a + b*x + c*x^2)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x)^(2*FracPart[p])) Int[(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c, p}, x] && EqQ[b^2 - 4*a*c, 0]`

rule 1100

```
Int[((d_.) + (e_.)*(x_))*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol]
  :> Simp[e*((a + b*x + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Simp[(2*c*d - b*
e)/(2*c) Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& EqQ[b^2 - 4*a*c, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 85 vs. $2(41) = 82$.

Time = 0.85 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.65

| method | result |
|---------|---|
| gospers | $\frac{x(192e x^6 + 224d x^5 + 1680x^5 e + 2016d x^4 + 6048x^4 e + 7560d x^3 + 11340x^3 e + 15120d x^2 + 11340e x^2 + 17010d x + 5103e x + 10206d)(2x+3)^5}{42(2x+3)^5}$ |
| default | $\frac{x(192e x^6 + 224d x^5 + 1680x^5 e + 2016d x^4 + 6048x^4 e + 7560d x^3 + 11340x^3 e + 15120d x^2 + 11340e x^2 + 17010d x + 5103e x + 10206d)(2x+3)^5}{42(2x+3)^5}$ |
| orering | $\frac{x(192e x^6 + 224d x^5 + 1680x^5 e + 2016d x^4 + 6048x^4 e + 7560d x^3 + 11340x^3 e + 15120d x^2 + 11340e x^2 + 17010d x + 5103e x + 10206d)(4x^2+12x+9)}{42(2x+3)^5}$ |
| risch | $\frac{32\sqrt{(2x+3)^2} e x^7}{7(2x+3)} + \frac{\sqrt{(2x+3)^2} (32d+240e)x^6}{12x+18} + \frac{\sqrt{(2x+3)^2} (240d+720e)x^5}{10x+15} + \frac{\sqrt{(2x+3)^2} (720d+1080e)x^4}{8x+12} + \frac{\sqrt{(2x+3)^2} (1080d+360e)x^3}{6x+9}$ |

input

```
int((e*x+d)*(4*x^2+12*x+9)^(5/2),x,method=_RETURNVERBOSE)
```

output

```
1/42*x*(192*e*x^6+224*d*x^5+1680*e*x^5+2016*d*x^4+6048*e*x^4+7560*d*x^3+11
340*e*x^3+15120*d*x^2+11340*e*x^2+17010*d*x+5103*e*x+10206*d)*((2*x+3)^2)^(
5/2)/(2*x+3)^5
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.33

$$\int (d + ex)(9 + 12x + 4x^2)^{5/2} dx = \frac{32}{7} ex^7 + \frac{8}{3} (2d + 15e)x^6 + 48(d + 3e)x^5 + 90(2d + 3e)x^4 + 90(4d + 3e)x^3 + \frac{81}{2} (10d + 3e)x^2 + 243 dx$$

input `integrate((e*x+d)*(4*x^2+12*x+9)^(5/2),x, algorithm="fricas")`

output `32/7*e*x^7 + 8/3*(2*d + 15*e)*x^6 + 48*(d + 3*e)*x^5 + 90*(2*d + 3*e)*x^4 + 90*(4*d + 3*e)*x^3 + 81/2*(10*d + 3*e)*x^2 + 243*d*x`

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 92 vs. 2(39) = 78.

Time = 0.65 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.77

$$\int (d + ex) (9 + 12x + 4x^2)^{5/2} dx = \sqrt{4x^2 + 12x + 9} \cdot \left(\frac{81d}{4} + \frac{16ex^6}{7} - \frac{243e}{56} + x^5 \cdot \left(\frac{8d}{3} + \frac{116e}{7} \right) + x^4 \cdot \left(20d + \frac{330e}{7} \right) + x^3 \cdot \left(60d + \frac{450e}{7} \right) + x^2 \cdot \left(90d + \frac{270e}{7} \right) + x \left(\frac{135d}{2} + \frac{81e}{28} \right) \right)$$

input `integrate((e*x+d)*(4*x**2+12*x+9)**(5/2),x)`

output `sqrt(4*x**2 + 12*x + 9)*(81*d/4 + 16*e*x**6/7 - 243*e/56 + x**5*(8*d/3 + 16*e/7) + x**4*(20*d + 330*e/7) + x**3*(60*d + 450*e/7) + x**2*(90*d + 270*e/7) + x*(135*d/2 + 81*e/28))`

Maxima [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.50

$$\int (d + ex) (9 + 12x + 4x^2)^{5/2} dx = \frac{1}{28} (4x^2 + 12x + 9)^{7/2} e + \frac{1}{6} (4x^2 + 12x + 9)^{5/2} dx - \frac{1}{4} (4x^2 + 12x + 9)^{5/2} ex + \frac{1}{4} (4x^2 + 12x + 9)^{5/2} d - \frac{3}{8} (4x^2 + 12x + 9)^{5/2} e$$

input `integrate((e*x+d)*(4*x^2+12*x+9)^(5/2),x, algorithm="maxima")`

output

```
1/28*(4*x^2 + 12*x + 9)^(7/2)*e + 1/6*(4*x^2 + 12*x + 9)^(5/2)*d*x - 1/4*(
4*x^2 + 12*x + 9)^(5/2)*e*x + 1/4*(4*x^2 + 12*x + 9)^(5/2)*d - 3/8*(4*x^2
+ 12*x + 9)^(5/2)*e
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 158 vs. $2(41) = 82$.

Time = 0.11 (sec) , antiderivative size = 158, normalized size of antiderivative = 3.04

$$\int (d + ex) (9 + 12x + 4x^2)^{5/2} dx = \frac{32}{7} ex^7 \operatorname{sgn}(2x + 3) + \frac{16}{3} dx^6 \operatorname{sgn}(2x + 3) + 40 ex^6 \operatorname{sgn}(2x + 3) + 48 dx^5 \operatorname{sgn}(2x + 3) + 144 ex^5 \operatorname{sgn}(2x + 3) + 180 dx^4 \operatorname{sgn}(2x + 3) + 270 ex^4 \operatorname{sgn}(2x + 3) + 360 dx^3 \operatorname{sgn}(2x + 3) + 270 ex^3 \operatorname{sgn}(2x + 3) + 405 dx^2 \operatorname{sgn}(2x + 3) + \frac{243}{2} ex^2 \operatorname{sgn}(2x + 3) + 243 dx \operatorname{sgn}(2x + 3) + \frac{243}{56} (14d - 3e) \operatorname{sgn}(2x + 3)$$

input

```
integrate((e*x+d)*(4*x^2+12*x+9)^(5/2),x, algorithm="giac")
```

output

```
32/7*e*x^7*sgn(2*x + 3) + 16/3*d*x^6*sgn(2*x + 3) + 40*e*x^6*sgn(2*x + 3)
+ 48*d*x^5*sgn(2*x + 3) + 144*e*x^5*sgn(2*x + 3) + 180*d*x^4*sgn(2*x + 3)
+ 270*e*x^4*sgn(2*x + 3) + 360*d*x^3*sgn(2*x + 3) + 270*e*x^3*sgn(2*x + 3)
+ 405*d*x^2*sgn(2*x + 3) + 243/2*e*x^2*sgn(2*x + 3) + 243*d*x*sgn(2*x + 3)
+ 243/56*(14*d - 3*e)*sgn(2*x + 3)
```

Mupad [F(-1)]

Timed out.

$$\int (d + ex) (9 + 12x + 4x^2)^{5/2} dx = \int (d + ex) (4x^2 + 12x + 9)^{5/2} dx$$

input

```
int((d + e*x)*(12*x + 4*x^2 + 9)^(5/2),x)
```

output

```
int((d + e*x)*(12*x + 4*x^2 + 9)^(5/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.33

$$\int (d + ex) (9 + 12x + 4x^2)^{5/2} dx = \frac{x(192e x^6 + 224d x^5 + 1680e x^5 + 2016d x^4 + 6048e x^4 + 7560d x^3 + 11340e x^3 + 15120d x^2 + 5103e x)}{42}$$

input `int((e*x+d)*(4*x^2+12*x+9)^(5/2),x)`

output `(x*(224*d*x**5 + 2016*d*x**4 + 7560*d*x**3 + 15120*d*x**2 + 17010*d*x + 10206*d + 192*e*x**6 + 1680*e*x**5 + 6048*e*x**4 + 11340*e*x**3 + 11340*e*x**2 + 5103*e*x))/42`

3.337 $\int (d + ex) (9 + 12x + 4x^2)^{3/2} dx$

| | |
|---|------|
| Optimal result | 2516 |
| Mathematica [A] (verified) | 2516 |
| Rubi [A] (verified) | 2517 |
| Maple [A] (verified) | 2518 |
| Fricas [A] (verification not implemented) | 2518 |
| Sympy [A] (verification not implemented) | 2519 |
| Maxima [A] (verification not implemented) | 2519 |
| Giac [B] (verification not implemented) | 2520 |
| Mupad [B] (verification not implemented) | 2520 |
| Reduce [B] (verification not implemented) | 2521 |

Optimal result

Integrand size = 20, antiderivative size = 52

$$\int (d + ex) (9 + 12x + 4x^2)^{3/2} dx = \frac{1}{16}(2d - 3e)(3 + 2x)^3 \sqrt{9 + 12x + 4x^2} + \frac{1}{20}e(9 + 12x + 4x^2)^{5/2}$$

output `1/16*(2*d-3*e)*(3+2*x)^3*((3+2*x)^2)^(1/2)+1/20*e*(4*x^2+12*x+9)^(5/2)`

Mathematica [A] (verified)

Time = 0.38 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.63

$$\int (d + ex) (9 + 12x + 4x^2)^{3/2} dx = \frac{1}{80}(3 + 2x)^3 \sqrt{(3 + 2x)^2(10d + e(-3 + 8x))}$$

input `Integrate[(d + e*x)*(9 + 12*x + 4*x^2)^(3/2),x]`

output `((3 + 2*x)^3*Sqrt[(3 + 2*x)^2]*(10*d + e*(-3 + 8*x)))/80`

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.150$, Rules used = {1100, 1079, 17}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (4x^2 + 12x + 9)^{3/2} (d + ex) dx$$

$$\downarrow 1100$$

$$\frac{1}{2}(2d - 3e) \int (4x^2 + 12x + 9)^{3/2} dx + \frac{1}{20}e(4x^2 + 12x + 9)^{5/2}$$

$$\downarrow 1079$$

$$\frac{\sqrt{4x^2 + 12x + 9}(2d - 3e) \int (4x + 6)^3 dx}{16(2x + 3)} + \frac{1}{20}e(4x^2 + 12x + 9)^{5/2}$$

$$\downarrow 17$$

$$\frac{1}{16}\sqrt{4x^2 + 12x + 9}(2x + 3)^3(2d - 3e) + \frac{1}{20}e(4x^2 + 12x + 9)^{5/2}$$

input `Int[(d + e*x)*(9 + 12*x + 4*x^2)^(3/2), x]`

output `((2*d - 3*e)*(3 + 2*x)^3*Sqrt[9 + 12*x + 4*x^2])/16 + (e*(9 + 12*x + 4*x^2)^(5/2))/20`

Defintions of rubi rules used

rule 17 `Int[(c_.)*((a_.) + (b_.)*(x_)^(m_.), x_Symbol] := Simp[c*((a + b*x)^(m + 1))/(b*(m + 1)), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]`

rule 1079 `Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] := Simp[(a + b*x + c*x^2)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x)^(2*FracPart[p])) Int[(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c, p}, x] && EqQ[b^2 - 4*a*c, 0]`

rule 1100

```
Int[((d_.) + (e_.)*(x_))*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol]
-> Simp[e*((a + b*x + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Simp[(2*c*d - b*
e)/(2*c) Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& EqQ[b^2 - 4*a*c, 0]
```

Maple [A] (verified)

Time = 0.84 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.19

| method | result | size |
|---------|--|------|
| gospers | $\frac{x(16x^4e+20dx^3+90x^3e+120dx^2+180ex^2+270dx+135ex+270d)((2x+3)^2)^{\frac{3}{2}}}{10(2x+3)^3}$ | 62 |
| default | $\frac{x(16x^4e+20dx^3+90x^3e+120dx^2+180ex^2+270dx+135ex+270d)((2x+3)^2)^{\frac{3}{2}}}{10(2x+3)^3}$ | 62 |
| orering | $\frac{x(16x^4e+20dx^3+90x^3e+120dx^2+180ex^2+270dx+135ex+270d)(4x^2+12x+9)^{\frac{3}{2}}}{10(2x+3)^3}$ | 65 |
| risch | $\frac{8\sqrt{(2x+3)^2}x^5e}{5(2x+3)} + \frac{\sqrt{(2x+3)^2}(8d+36e)x^4}{8x+12} + \frac{\sqrt{(2x+3)^2}(36d+54e)x^3}{6x+9} + \frac{\sqrt{(2x+3)^2}(54d+27e)x^2}{4x+6} + \frac{27\sqrt{(2x+3)^2}dx}{2x+3}$ | 12 |

input

```
int((e*x+d)*(4*x^2+12*x+9)^(3/2),x,method=_RETURNVERBOSE)
```

output

```
1/10*x*(16*e*x^4+20*d*x^3+90*e*x^3+120*d*x^2+180*e*x^2+270*d*x+135*e*x+270
*d)*((2*x+3)^2)^(3/2)/(2*x+3)^3
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.85

$$\int (d + ex)(9 + 12x + 4x^2)^{3/2} dx = \frac{8}{5} ex^5 + (2d + 9e)x^4 + 6(2d + 3e)x^3 + \frac{27}{2}(2d + e)x^2 + 27dx$$

input

```
integrate((e*x+d)*(4*x^2+12*x+9)^(3/2),x, algorithm="fricas")
```

output $8/5*e*x^5 + (2*d + 9*e)*x^4 + 6*(2*d + 3*e)*x^3 + 27/2*(2*d + e)*x^2 + 27*d*x$

Sympy [A] (verification not implemented)

Time = 0.57 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.27

$$\int (d + ex) (9 + 12x + 4x^2)^{3/2} dx = \sqrt{4x^2 + 12x + 9} \cdot \left(\frac{27d}{8} + \frac{4ex^4}{5} - \frac{81e}{80} + x^3 \left(d + \frac{33e}{10} \right) + x^2 \cdot \left(\frac{9d}{2} + \frac{81e}{20} \right) + x \left(\frac{27d}{4} + \frac{27e}{40} \right) \right)$$

input `integrate((e*x+d)*(4*x**2+12*x+9)**(3/2),x)`

output `sqrt(4*x**2 + 12*x + 9)*(27*d/8 + 4*e*x**4/5 - 81*e/80 + x**3*(d + 33*e/10) + x**2*(9*d/2 + 81*e/20) + x*(27*d/4 + 27*e/40))`

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.50

$$\int (d + ex) (9 + 12x + 4x^2)^{3/2} dx = \frac{1}{20} (4x^2 + 12x + 9)^{5/2} e + \frac{1}{4} (4x^2 + 12x + 9)^{3/2} dx - \frac{3}{8} (4x^2 + 12x + 9)^{3/2} ex + \frac{3}{8} (4x^2 + 12x + 9)^{3/2} d - \frac{9}{16} (4x^2 + 12x + 9)^{3/2} e$$

input `integrate((e*x+d)*(4*x^2+12*x+9)^(3/2),x, algorithm="maxima")`

output `1/20*(4*x^2 + 12*x + 9)^(5/2)*e + 1/4*(4*x^2 + 12*x + 9)^(3/2)*d*x - 3/8*(4*x^2 + 12*x + 9)^(3/2)*e*x + 3/8*(4*x^2 + 12*x + 9)^(3/2)*d - 9/16*(4*x^2 + 12*x + 9)^(3/2)*e`

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 110 vs. $2(41) = 82$.

Time = 0.12 (sec) , antiderivative size = 110, normalized size of antiderivative = 2.12

$$\int (d + ex) (9 + 12x + 4x^2)^{3/2} dx = \frac{8}{5} ex^5 \operatorname{sgn}(2x + 3) + 2 dx^4 \operatorname{sgn}(2x + 3) + 9 ex^4 \operatorname{sgn}(2x + 3) + 12 dx^3 \operatorname{sgn}(2x + 3) + 18 ex^3 \operatorname{sgn}(2x + 3) + 27 dx^2 \operatorname{sgn}(2x + 3) + \frac{27}{2} ex^2 \operatorname{sgn}(2x + 3) + 27 dx \operatorname{sgn}(2x + 3) + \frac{81}{80} (10d - 3e) \operatorname{sgn}(2x + 3)$$

input `integrate((e*x+d)*(4*x^2+12*x+9)^(3/2),x, algorithm="giac")`

output `8/5*e*x^5*sgn(2*x + 3) + 2*d*x^4*sgn(2*x + 3) + 9*e*x^4*sgn(2*x + 3) + 12*d*x^3*sgn(2*x + 3) + 18*e*x^3*sgn(2*x + 3) + 27*d*x^2*sgn(2*x + 3) + 27/2*e*x^2*sgn(2*x + 3) + 27*d*x*sgn(2*x + 3) + 81/80*(10*d - 3*e)*sgn(2*x + 3)`

Mupad [B] (verification not implemented)

Time = 5.79 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.29

$$\int (d + ex) (9 + 12x + 4x^2)^{3/2} dx = \frac{e(4x^2 + 12x + 9)^{5/2}}{20} - \frac{9e(4x^2 + 12x + 9)^{3/2}}{16} - \frac{3ex(4x^2 + 12x + 9)^{3/2}}{8} + \frac{d(2x + 3)(4x^2 + 12x + 9)^{3/2}}{8}$$

input `int((d + e*x)*(12*x + 4*x^2 + 9)^(3/2),x)`

output `(e*(12*x + 4*x^2 + 9)^(5/2))/20 - (9*e*(12*x + 4*x^2 + 9)^(3/2))/16 - (3*e*x*(12*x + 4*x^2 + 9)^(3/2))/8 + (d*(2*x + 3)*(12*x + 4*x^2 + 9)^(3/2))/8`

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.87

$$\int (d + ex) (9 + 12x + 4x^2)^{3/2} dx = \frac{x(16ex^4 + 20dx^3 + 90ex^3 + 120dx^2 + 180ex^2 + 270dx + 135ex + 270d)}{10}$$

input `int((e*x+d)*(4*x^2+12*x+9)^(3/2),x)`

output `(x*(20*d*x**3 + 120*d*x**2 + 270*d*x + 270*d + 16*e*x**4 + 90*e*x**3 + 180*e*x**2 + 135*e*x))/10`

3.338 $\int (d + ex)\sqrt{9 + 12x + 4x^2} dx$

| | |
|---|------|
| Optimal result | 2522 |
| Mathematica [A] (verified) | 2522 |
| Rubi [A] (verified) | 2523 |
| Maple [C] (warning: unable to verify) | 2524 |
| Fricas [A] (verification not implemented) | 2524 |
| Sympy [A] (verification not implemented) | 2525 |
| Maxima [A] (verification not implemented) | 2525 |
| Giac [A] (verification not implemented) | 2526 |
| Mupad [B] (verification not implemented) | 2526 |
| Reduce [B] (verification not implemented) | 2527 |

Optimal result

Integrand size = 20, antiderivative size = 50

$$\int (d + ex)\sqrt{9 + 12x + 4x^2} dx = \frac{1}{8}(2d - 3e)(3 + 2x)\sqrt{9 + 12x + 4x^2} + \frac{1}{12}e(9 + 12x + 4x^2)^{3/2}$$

output `1/8*(2*d-3*e)*(3+2*x)*((3+2*x)^2)^(1/2)+1/12*e*(4*x^2+12*x+9)^(3/2)`

Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.76

$$\int (d + ex)\sqrt{9 + 12x + 4x^2} dx = \frac{x\sqrt{(3 + 2x)^2(6d(3 + x) + ex(9 + 4x))}}{6(3 + 2x)}$$

input `Integrate[(d + e*x)*Sqrt[9 + 12*x + 4*x^2], x]`

output `(x*Sqrt[(3 + 2*x)^2]*(6*d*(3 + x) + e*x*(9 + 4*x)))/(6*(3 + 2*x))`

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.150$, Rules used = {1100, 1079, 17}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{4x^2 + 12x + 9}(d + ex) dx$$

$$\downarrow 1100$$

$$\frac{1}{2}(2d - 3e) \int \sqrt{4x^2 + 12x + 9} dx + \frac{1}{12}e(4x^2 + 12x + 9)^{3/2}$$

$$\downarrow 1079$$

$$\frac{\sqrt{4x^2 + 12x + 9}(2d - 3e) \int (4x + 6) dx}{4(2x + 3)} + \frac{1}{12}e(4x^2 + 12x + 9)^{3/2}$$

$$\downarrow 17$$

$$\frac{1}{8}(2x + 3)\sqrt{4x^2 + 12x + 9}(2d - 3e) + \frac{1}{12}e(4x^2 + 12x + 9)^{3/2}$$

input `Int[(d + e*x)*Sqrt[9 + 12*x + 4*x^2], x]`

output `((2*d - 3*e)*(3 + 2*x)*Sqrt[9 + 12*x + 4*x^2])/8 + (e*(9 + 12*x + 4*x^2)^(3/2))/12`

Defintions of rubi rules used

rule 17 `Int[(c_.)*((a_.) + (b_.)*(x_)^(m_.), x_Symbol] := Simp[c*((a + b*x)^(m + 1))/(b*(m + 1)), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]`

rule 1079 `Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] := Simp[(a + b*x + c*x^2)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x)^(2*FracPart[p])) Int[(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c, p}, x] && EqQ[b^2 - 4*a*c, 0]`

rule 1100

```
Int[((d._) + (e._)*(x_))*((a_) + (b._)*(x_) + (c._)*(x_)^2)^(p_), x_Symbol]
  :> Simp[e*((a + b*x + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Simp[(2*c*d - b*
e)/(2*c) Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& EqQ[b^2 - 4*a*c, 0]
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 2.

Time = 0.41 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.54

| method | result | size |
|---------|---|------|
| default | $\frac{\text{csgn}(2x+3)(2x+3)^2(4ex+6d-3e)}{24}$ | 27 |
| gospers | $\frac{x(4ex^2+6dx+9ex+18d)\sqrt{(2x+3)^2}}{12x+18}$ | 38 |
| orering | $\frac{x(4ex^2+6dx+9ex+18d)\sqrt{(2x+3)^2}}{12x+18}$ | 38 |
| risch | $\frac{2\sqrt{(2x+3)^2}x^3e}{3(2x+3)} + \frac{\sqrt{(2x+3)^2}(2d+3e)x^2}{4x+6} + \frac{3\sqrt{(2x+3)^2}dx}{2x+3}$ | 72 |

input

```
int((e*x+d)*((2*x+3)^2)^(1/2),x,method=_RETURNVERBOSE)
```

output

```
1/24*csgn(2*x+3)*(2*x+3)^2*(4*e*x+6*d-3*e)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.46

$$\int (d + ex)\sqrt{9 + 12x + 4x^2} dx = \frac{2}{3} ex^3 + \frac{1}{2} (2d + 3e)x^2 + 3dx$$

input

```
integrate((e*x+d)*((3+2*x)^2)^(1/2),x, algorithm="fricas")
```

output

```
2/3*e*x^3 + 1/2*(2*d + 3*e)*x^2 + 3*d*x
```

Sympy [A] (verification not implemented)

Time = 0.64 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.96

$$\int (d + ex)\sqrt{9 + 12x + 4x^2} dx = d\left(\frac{x}{2} + \frac{3}{4}\right)\sqrt{4x^2 + 12x + 9} + e\left(\frac{x^2}{3} + \frac{x}{4} - \frac{3}{8}\right)\sqrt{4x^2 + 12x + 9}$$

input `integrate((e*x+d)*((3+2*x)**2)**(1/2),x)`output `d*(x/2 + 3/4)*sqrt(4*x**2 + 12*x + 9) + e*(x**2/3 + x/4 - 3/8)*sqrt(4*x**2 + 12*x + 9)`**Maxima [A] (verification not implemented)**

Time = 0.12 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.56

$$\int (d + ex)\sqrt{9 + 12x + 4x^2} dx = \frac{1}{12} (4x^2 + 12x + 9)^{\frac{3}{2}}e + \frac{1}{2}\sqrt{4x^2 + 12x + 9}dx - \frac{3}{4}\sqrt{4x^2 + 12x + 9}ex + \frac{3}{4}\sqrt{4x^2 + 12x + 9}d - \frac{9}{8}\sqrt{4x^2 + 12x + 9}e$$

input `integrate((e*x+d)*((3+2*x)^2)^(1/2),x, algorithm="maxima")`output `1/12*(4*x^2 + 12*x + 9)^(3/2)*e + 1/2*sqrt(4*x^2 + 12*x + 9)*d*x - 3/4*sqrt(4*x^2 + 12*x + 9)*e*x + 3/4*sqrt(4*x^2 + 12*x + 9)*d - 9/8*sqrt(4*x^2 + 12*x + 9)*e`

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.22

$$\int (d + ex)\sqrt{9 + 12x + 4x^2} dx = \frac{2}{3} ex^3 \operatorname{sgn}(2x + 3) + dx^2 \operatorname{sgn}(2x + 3) + \frac{3}{2} ex^2 \operatorname{sgn}(2x + 3) \\ + 3 dx \operatorname{sgn}(2x + 3) + \frac{9}{8} (2d - e) \operatorname{sgn}(2x + 3)$$

input `integrate((e*x+d)*((3+2*x)^2)^(1/2),x, algorithm="giac")`

output `2/3*e*x^3*sgn(2*x + 3) + d*x^2*sgn(2*x + 3) + 3/2*e*x^2*sgn(2*x + 3) + 3*d*x*sgn(2*x + 3) + 9/8*(2*d - e)*sgn(2*x + 3)`

Mupad [B] (verification not implemented)

Time = 5.76 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.80

$$\int (d + ex)\sqrt{9 + 12x + 4x^2} dx = \frac{d|2x + 3|(2x + 3)}{4} \\ + \frac{e\sqrt{4x^2 + 12x + 9}(128x^2 + 96x - 144)}{384}$$

input `int(((2*x + 3)^2)^(1/2)*(d + e*x),x)`

output `(d*abs(2*x + 3)*(2*x + 3))/4 + (e*(12*x + 4*x^2 + 9)^(1/2)*(96*x + 128*x^2 - 144))/384`

Reduce [B] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.42

$$\int (d + ex)\sqrt{9 + 12x + 4x^2} dx = \frac{x(4e x^2 + 6dx + 9ex + 18d)}{6}$$

input `int((e*x+d)*((3+2*x)^2)^(1/2),x)`

output `(x*(6*d*x + 18*d + 4*e*x**2 + 9*e*x))/6`

3.339 $\int \frac{d+ex}{\sqrt{9+12x+4x^2}} dx$

| | |
|---|------|
| Optimal result | 2528 |
| Mathematica [A] (verified) | 2528 |
| Rubi [A] (verified) | 2529 |
| Maple [A] (verified) | 2530 |
| Fricas [A] (verification not implemented) | 2530 |
| Sympy [A] (verification not implemented) | 2531 |
| Maxima [A] (verification not implemented) | 2531 |
| Giac [A] (verification not implemented) | 2531 |
| Mupad [B] (verification not implemented) | 2532 |
| Reduce [B] (verification not implemented) | 2532 |

Optimal result

Integrand size = 20, antiderivative size = 56

$$\int \frac{d+ex}{\sqrt{9+12x+4x^2}} dx = \frac{1}{4}e\sqrt{9+12x+4x^2} + \frac{(2d-3e)(3+2x)\log(3+2x)}{4\sqrt{9+12x+4x^2}}$$

output `1/4*e*((3+2*x)^2)^(1/2)+1/4*(2*d-3*e)*(3+2*x)*ln(3+2*x)/((3+2*x)^2)^(1/2)`

Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.73

$$\int \frac{d+ex}{\sqrt{9+12x+4x^2}} dx = \frac{(3+2x)\left(\frac{ex}{2} + \frac{1}{4}(2d-3e)\log(3+2x)\right)}{\sqrt{(3+2x)^2}}$$

input `Integrate[(d + e*x)/Sqrt[9 + 12*x + 4*x^2], x]`

output `((3 + 2*x)*((e*x)/2 + ((2*d - 3*e)*Log[3 + 2*x])/4))/Sqrt[(3 + 2*x)^2]`

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.150$, Rules used = {1100, 1079, 16}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{d + ex}{\sqrt{4x^2 + 12x + 9}} dx$$

↓ 1100

$$\frac{1}{2}(2d - 3e) \int \frac{1}{\sqrt{4x^2 + 12x + 9}} dx + \frac{1}{4}e\sqrt{4x^2 + 12x + 9}$$

↓ 1079

$$\frac{(2x + 3)(2d - 3e) \int \frac{1}{4x+6} dx}{\sqrt{4x^2 + 12x + 9}} + \frac{1}{4}e\sqrt{4x^2 + 12x + 9}$$

↓ 16

$$\frac{(2x + 3)(2d - 3e) \log(2x + 3)}{4\sqrt{4x^2 + 12x + 9}} + \frac{1}{4}e\sqrt{4x^2 + 12x + 9}$$

input `Int[(d + e*x)/Sqrt[9 + 12*x + 4*x^2], x]`

output `(e*Sqrt[9 + 12*x + 4*x^2])/4 + ((2*d - 3*e)*(3 + 2*x)*Log[3 + 2*x])/(4*Sqrt[9 + 12*x + 4*x^2])`

Defintions of rubi rules used

rule 16 `Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] := Simp[c*(Log[RemoveContent[a + b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]`

rule 1079 `Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] := Simp[(a + b*x + c*x^2)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x)^(2*FracPart[p])) Int[(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c, p}, x] && EqQ[b^2 - 4*a*c, 0]`

rule 1100

```
Int[((d_.) + (e_.)*(x_))*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol]
  :> Simp[e*((a + b*x + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Simp[(2*c*d - b*
e)/(2*c) Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& EqQ[b^2 - 4*a*c, 0]
```

Maple [A] (verified)

Time = 0.61 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.71

| method | result | size |
|---------|---|------|
| default | $\frac{(2x+3)(2\ln(2x+3)d-3e\ln(2x+3)+2ex)}{4\sqrt{(2x+3)^2}}$ | 40 |
| risch | $\frac{\sqrt{(2x+3)^2} ex}{4x+6} + \frac{\sqrt{(2x+3)^2} (-\frac{3e}{2}+d) \ln(2x+3)}{4x+6}$ | 51 |
| meijerg | $\frac{3ex(\frac{2x}{3}-\ln(1+\frac{2x}{3}))}{2\sqrt{(2x+3)^2}} + \frac{9e(\frac{2x}{3}-\ln(1+\frac{2x}{3}))}{4\sqrt{(2x+3)^2}} + \frac{dx\ln(1+\frac{2x}{3})}{\sqrt{(2x+3)^2}} + \frac{3d\ln(1+\frac{2x}{3})}{2\sqrt{(2x+3)^2}}$ | 87 |

input

```
int((e*x+d)/((2*x+3)^2)^(1/2),x,method=_RETURNVERBOSE)
```

output

```
1/4*(2*x+3)*(2*ln(2*x+3)*d-3*e*ln(2*x+3)+2*e*x)/((2*x+3)^2)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.36

$$\int \frac{d + ex}{\sqrt{9 + 12x + 4x^2}} dx = \frac{1}{2} ex + \frac{1}{4} (2d - 3e) \log(2x + 3)$$

input

```
integrate((e*x+d)/((3+2*x)^2)^(1/2),x, algorithm="fricas")
```

output

```
1/2*e*x + 1/4*(2*d - 3*e)*log(2*x + 3)
```

Sympy [A] (verification not implemented)

Time = 0.59 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.82

$$\int \frac{d + ex}{\sqrt{9 + 12x + 4x^2}} dx = \frac{e\sqrt{4x^2 + 12x + 9}}{4} + \frac{(d - \frac{3e}{2})(x + \frac{3}{2}) \log(x + \frac{3}{2})}{2\sqrt{(x + \frac{3}{2})^2}}$$

input `integrate((e*x+d)/((3+2*x)**2)**(1/2),x)`output `e*sqrt(4*x**2 + 12*x + 9)/4 + (d - 3*e/2)*(x + 3/2)*log(x + 3/2)/(2*sqrt((x + 3/2)**2))`**Maxima [A] (verification not implemented)**

Time = 0.14 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.54

$$\int \frac{d + ex}{\sqrt{9 + 12x + 4x^2}} dx = \frac{1}{2} d \log\left(x + \frac{3}{2}\right) - \frac{3}{4} e \log\left(x + \frac{3}{2}\right) + \frac{1}{4} \sqrt{4x^2 + 12x + 9} e$$

input `integrate((e*x+d)/((3+2*x)^2)^(1/2),x, algorithm="maxima")`output `1/2*d*log(x + 3/2) - 3/4*e*log(x + 3/2) + 1/4*sqrt(4*x^2 + 12*x + 9)*e`**Giac [A] (verification not implemented)**

Time = 0.13 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.70

$$\int \frac{d + ex}{\sqrt{9 + 12x + 4x^2}} dx = \frac{1}{2} ex \operatorname{sgn}(2x + 3) + \frac{1}{4} (2 d \operatorname{sgn}(2x + 3) - 3 e \operatorname{sgn}(2x + 3)) \log(|2x + 3|)$$

input `integrate((e*x+d)/((3+2*x)^2)^(1/2),x, algorithm="giac")`

output $1/2*e*x*sgn(2*x + 3) + 1/4*(2*d*sgn(2*x + 3) - 3*e*sgn(2*x + 3))*log(abs(2*x + 3))$

Mupad [B] (verification not implemented)

Time = 5.93 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.82

$$\int \frac{d + ex}{\sqrt{9 + 12x + 4x^2}} dx = \frac{e \sqrt{4x^2 + 12x + 9}}{4} - \frac{3e \ln \left(x + \frac{|2x+3|}{2} + \frac{3}{2} \right)}{4} + \frac{d \ln(2x + 3) \operatorname{sign}(2x + 3)}{2}$$

input $\text{int}((d + e*x)/((2*x + 3)^2)^{(1/2)}, x)$

output $(e*(12*x + 4*x^2 + 9)^{(1/2)})/4 - (3*e*log(x + abs(2*x + 3)/2 + 3/2))/4 + (d*log(2*x + 3)*sign(2*x + 3))/2$

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.41

$$\int \frac{d + ex}{\sqrt{9 + 12x + 4x^2}} dx = \frac{\log(2x + 3) d}{2} - \frac{3 \log(2x + 3) e}{4} + \frac{ex}{2}$$

input $\text{int}((e*x+d)/((3+2*x)^2)^{(1/2)}, x)$

output $(2*log(2*x + 3)*d - 3*log(2*x + 3)*e + 2*e*x)/4$

$$3.340 \quad \int \frac{d+ex}{(9+12x+4x^2)^{3/2}} dx$$

| | |
|---|------|
| Optimal result | 2533 |
| Mathematica [A] (verified) | 2533 |
| Rubi [A] (verified) | 2534 |
| Maple [A] (verified) | 2535 |
| Fricas [A] (verification not implemented) | 2535 |
| Sympy [F] | 2536 |
| Maxima [A] (verification not implemented) | 2536 |
| Giac [A] (verification not implemented) | 2536 |
| Mupad [B] (verification not implemented) | 2537 |
| Reduce [B] (verification not implemented) | 2537 |

Optimal result

Integrand size = 20, antiderivative size = 52

$$\int \frac{d+ex}{(9+12x+4x^2)^{3/2}} dx = -\frac{e}{4\sqrt{9+12x+4x^2}} - \frac{2d-3e}{8(3+2x)\sqrt{9+12x+4x^2}}$$

output $-1/4*e/((3+2*x)^2)^{(1/2)}-1/8*(2*d-3*e)/(3+2*x)/((3+2*x)^2)^{(1/2)}$

Mathematica [A] (verified)

Time = 0.19 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.73

$$\int \frac{d+ex}{(9+12x+4x^2)^{3/2}} dx = \frac{(d+ex)^2}{2(-2d+3e)(3+2x)\sqrt{(3+2x)^2}}$$

input `Integrate[(d + e*x)/(9 + 12*x + 4*x^2)^(3/2), x]`

output $(d + e*x)^2/(2*(-2*d + 3*e)*(3 + 2*x)*\text{Sqrt}[(3 + 2*x)^2])$

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {1100, 1078}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{d + ex}{(4x^2 + 12x + 9)^{3/2}} dx$$

↓ 1100

$$\frac{1}{2}(2d - 3e) \int \frac{1}{(4x^2 + 12x + 9)^{3/2}} dx - \frac{e}{4\sqrt{4x^2 + 12x + 9}}$$

↓ 1078

$$-\frac{2d - 3e}{8(2x + 3)\sqrt{4x^2 + 12x + 9}} - \frac{e}{4\sqrt{4x^2 + 12x + 9}}$$

input `Int[(d + e*x)/(9 + 12*x + 4*x^2)^(3/2), x]`

output `-1/4*e/Sqrt[9 + 12*x + 4*x^2] - (2*d - 3*e)/(8*(3 + 2*x)*Sqrt[9 + 12*x + 4*x^2])`

Defintions of rubi rules used

rule 1078 `Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] := Simp[2*((a + b*x + c*x^2)^(p + 1))/((2*p + 1)*(b + 2*c*x)), x] /; FreeQ[{a, b, c, p}, x] && EqQ[b^2 - 4*a*c, 0] && LtQ[p, -1]`

rule 1100 `Int[((d_.) + (e_.)*(x_))*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] := Simp[e*((a + b*x + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Simp[(2*c*d - b*e)/(2*c) Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[b^2 - 4*a*c, 0]`

Maple [A] (verified)

Time = 0.88 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.54

| method | result | size |
|---------|---|------|
| gospers | $-\frac{(2x+3)(4ex+2d+3e)}{8((2x+3)^2)^{\frac{3}{2}}}$ | 28 |
| default | $-\frac{(2x+3)(4ex+2d+3e)}{8((2x+3)^2)^{\frac{3}{2}}}$ | 28 |
| risch | $\frac{4\sqrt{(2x+3)^2}(-\frac{1}{8}ex-\frac{1}{16}d-\frac{3}{32}e)}{(2x+3)^3}$ | 30 |
| meijerg | $\frac{ex^2}{54(1+\frac{2x}{3})^2} + \frac{dx(\frac{2x}{3}+2)}{54(1+\frac{2x}{3})^2}$ | 31 |
| orering | $-\frac{(2x+3)(4ex+2d+3e)}{8(4x^2+12x+9)^{\frac{3}{2}}}$ | 31 |

input `int((e*x+d)/(4*x^2+12*x+9)^(3/2),x,method=_RETURNVERBOSE)`

output `-1/8*(2*x+3)*(4*e*x+2*d+3*e)/((2*x+3)^2)^(3/2)`

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.48

$$\int \frac{d+ex}{(9+12x+4x^2)^{3/2}} dx = -\frac{4ex+2d+3e}{8(4x^2+12x+9)}$$

input `integrate((e*x+d)/(4*x^2+12*x+9)^(3/2),x, algorithm="fricas")`

output `-1/8*(4*e*x + 2*d + 3*e)/(4*x^2 + 12*x + 9)`

Sympy [F]

$$\int \frac{d + ex}{(9 + 12x + 4x^2)^{3/2}} dx = \int \frac{d + ex}{((2x + 3)^2)^{\frac{3}{2}}} dx$$

input `integrate((e*x+d)/(4*x**2+12*x+9)**(3/2),x)`

output `Integral((d + e*x)/((2*x + 3)**2)**(3/2), x)`

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.69

$$\int \frac{d + ex}{(9 + 12x + 4x^2)^{3/2}} dx = -\frac{e}{4\sqrt{4x^2 + 12x + 9}} - \frac{d}{4(2x + 3)^2} + \frac{3e}{8(2x + 3)^2}$$

input `integrate((e*x+d)/(4*x^2+12*x+9)^(3/2),x, algorithm="maxima")`

output `-1/4*e/sqrt(4*x^2 + 12*x + 9) - 1/4*d/(2*x + 3)^2 + 3/8*e/(2*x + 3)^2`

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.54

$$\int \frac{d + ex}{(9 + 12x + 4x^2)^{3/2}} dx = -\frac{4ex + 2d + 3e}{8(2x + 3)^2 \operatorname{sgn}(2x + 3)}$$

input `integrate((e*x+d)/(4*x^2+12*x+9)^(3/2),x, algorithm="giac")`

output `-1/8*(4*e*x + 2*d + 3*e)/((2*x + 3)^2*sgn(2*x + 3))`

Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.62

$$\int \frac{d + ex}{(9 + 12x + 4x^2)^{3/2}} dx = -\frac{(2d + 3e + 4ex) \sqrt{4x^2 + 12x + 9}}{8(2x + 3)^3}$$

input `int((d + e*x)/(12*x + 4*x^2 + 9)^(3/2),x)`

output `-((2*d + 3*e + 4*e*x)*(12*x + 4*x^2 + 9)^(1/2))/(8*(2*x + 3)^3)`

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.44

$$\int \frac{d + ex}{(9 + 12x + 4x^2)^{3/2}} dx = \frac{2ex^2 - 3d}{48x^2 + 144x + 108}$$

input `int((e*x+d)/(4*x^2+12*x+9)^(3/2),x)`

output `(- 3*d + 2*e*x**2)/(12*(4*x**2 + 12*x + 9))`

$$3.341 \quad \int \frac{d+ex}{(9+12x+4x^2)^{5/2}} dx$$

| | |
|---|------|
| Optimal result | 2538 |
| Mathematica [A] (verified) | 2538 |
| Rubi [A] (verified) | 2539 |
| Maple [A] (verified) | 2540 |
| Fricas [A] (verification not implemented) | 2540 |
| Sympy [F] | 2541 |
| Maxima [A] (verification not implemented) | 2541 |
| Giac [A] (verification not implemented) | 2541 |
| Mupad [B] (verification not implemented) | 2542 |
| Reduce [B] (verification not implemented) | 2542 |

Optimal result

Integrand size = 20, antiderivative size = 52

$$\int \frac{d+ex}{(9+12x+4x^2)^{5/2}} dx = -\frac{e}{12(9+12x+4x^2)^{3/2}} - \frac{2d-3e}{16(3+2x)^3\sqrt{9+12x+4x^2}}$$

output `-1/12*e/(4*x^2+12*x+9)^(3/2)-1/16*(2*d-3*e)/(3+2*x)^3/((3+2*x)^2)^(1/2)`

Mathematica [A] (verified)

Time = 0.34 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.65

$$\int \frac{d+ex}{(9+12x+4x^2)^{5/2}} dx = \frac{-6d-e(3+8x)}{48(3+2x)^3\sqrt{(3+2x)^2}}$$

input `Integrate[(d + e*x)/(9 + 12*x + 4*x^2)^(5/2), x]`

output `(-6*d - e*(3 + 8*x))/(48*(3 + 2*x)^3*Sqrt[(3 + 2*x)^2])`

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {1100, 1078}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{d + ex}{(4x^2 + 12x + 9)^{5/2}} dx$$

↓ 1100

$$\frac{1}{2}(2d - 3e) \int \frac{1}{(4x^2 + 12x + 9)^{5/2}} dx - \frac{e}{12(4x^2 + 12x + 9)^{3/2}}$$

↓ 1078

$$-\frac{2d - 3e}{16(2x + 3)(4x^2 + 12x + 9)^{3/2}} - \frac{e}{12(4x^2 + 12x + 9)^{3/2}}$$

input `Int[(d + e*x)/(9 + 12*x + 4*x^2)^(5/2), x]`

output `-1/12*e/(9 + 12*x + 4*x^2)^(3/2) - (2*d - 3*e)/(16*(3 + 2*x)*(9 + 12*x + 4*x^2)^(3/2))`

Defintions of rubi rules used

rule 1078 `Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[2*((a + b*x + c*x^2)^(p + 1)/((2*p + 1)*(b + 2*c*x))), x] /; FreeQ[{a, b, c, p}, x] && EqQ[b^2 - 4*a*c, 0] && LtQ[p, -1]`

rule 1100 `Int[((d_.) + (e_.)*(x_))*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[e*((a + b*x + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Simp[(2*c*d - b*e)/(2*c) Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[b^2 - 4*a*c, 0]`

Maple [A] (verified)

Time = 0.87 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.54

| method | result | size |
|---------|---|------|
| gospers | $-\frac{(2x+3)(8ex+6d+3e)}{48((2x+3)^2)^{\frac{5}{2}}}$ | 28 |
| default | $-\frac{(2x+3)(8ex+6d+3e)}{48((2x+3)^2)^{\frac{5}{2}}}$ | 28 |
| risch | $\frac{16\sqrt{(2x+3)^2}(-\frac{1}{96}ex - \frac{1}{256}e - \frac{1}{128}d)}{(2x+3)^5}$ | 30 |
| orering | $-\frac{(2x+3)(8ex+6d+3e)}{48(4x^2+12x+9)^{\frac{5}{2}}}$ | 31 |
| meijerg | $\frac{ex^2(\frac{4}{9}x^2 + \frac{8}{3}x + 6)}{2916(1 + \frac{2x}{3})^4} + \frac{dx(\frac{8}{27}x^3 + \frac{16}{9}x^2 + 4x + 4)}{972(1 + \frac{2x}{3})^4}$ | 51 |

input `int((e*x+d)/(4*x^2+12*x+9)^(5/2),x,method=_RETURNVERBOSE)`

output `-1/48*(2*x+3)*(8*e*x+6*d+3*e)/((2*x+3)^2)^(5/2)`

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.67

$$\int \frac{d + ex}{(9 + 12x + 4x^2)^{5/2}} dx = -\frac{8ex + 6d + 3e}{48(16x^4 + 96x^3 + 216x^2 + 216x + 81)}$$

input `integrate((e*x+d)/(4*x^2+12*x+9)^(5/2),x, algorithm="fricas")`

output `-1/48*(8*e*x + 6*d + 3*e)/(16*x^4 + 96*x^3 + 216*x^2 + 216*x + 81)`

Sympy [F]

$$\int \frac{d + ex}{(9 + 12x + 4x^2)^{5/2}} dx = \int \frac{d + ex}{((2x + 3)^2)^{5/2}} dx$$

input `integrate((e*x+d)/(4*x**2+12*x+9)**(5/2),x)`

output `Integral((d + e*x)/((2*x + 3)**2)**(5/2), x)`

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.69

$$\int \frac{d + ex}{(9 + 12x + 4x^2)^{5/2}} dx = -\frac{e}{12(4x^2 + 12x + 9)^{3/2}} - \frac{d}{8(2x + 3)^4} + \frac{3e}{16(2x + 3)^4}$$

input `integrate((e*x+d)/(4*x^2+12*x+9)^(5/2),x, algorithm="maxima")`

output `-1/12*e/(4*x^2 + 12*x + 9)^(3/2) - 1/8*d/(2*x + 3)^4 + 3/16*e/(2*x + 3)^4`

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.54

$$\int \frac{d + ex}{(9 + 12x + 4x^2)^{5/2}} dx = -\frac{8ex + 6d + 3e}{48(2x + 3)^4 \operatorname{sgn}(2x + 3)}$$

input `integrate((e*x+d)/(4*x^2+12*x+9)^(5/2),x, algorithm="giac")`

output `-1/48*(8*e*x + 6*d + 3*e)/((2*x + 3)^4*sgn(2*x + 3))`

Mupad [B] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.62

$$\int \frac{d + ex}{(9 + 12x + 4x^2)^{5/2}} dx = -\frac{(6d + 3e + 8ex) \sqrt{4x^2 + 12x + 9}}{48(2x + 3)^5}$$

input `int((d + e*x)/(12*x + 4*x^2 + 9)^(5/2),x)`output `-((6*d + 3*e + 8*e*x)*(12*x + 4*x^2 + 9)^(1/2))/(48*(2*x + 3)^5)`**Reduce [B] (verification not implemented)**

Time = 0.23 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.65

$$\int \frac{d + ex}{(9 + 12x + 4x^2)^{5/2}} dx = \frac{-8ex - 6d - 3e}{768x^4 + 4608x^3 + 10368x^2 + 10368x + 3888}$$

input `int((e*x+d)/(4*x^2+12*x+9)^(5/2),x)`output `(- 6*d - 8*e*x - 3*e)/(48*(16*x**4 + 96*x**3 + 216*x**2 + 216*x + 81))`

$$3.342 \quad \int \frac{d+ex}{(9+12x+4x^2)^{7/2}} dx$$

| | |
|---|------|
| Optimal result | 2543 |
| Mathematica [A] (verified) | 2543 |
| Rubi [A] (verified) | 2544 |
| Maple [A] (verified) | 2545 |
| Fricas [A] (verification not implemented) | 2545 |
| Sympy [F] | 2546 |
| Maxima [A] (verification not implemented) | 2546 |
| Giac [A] (verification not implemented) | 2546 |
| Mupad [B] (verification not implemented) | 2547 |
| Reduce [B] (verification not implemented) | 2547 |

Optimal result

Integrand size = 20, antiderivative size = 52

$$\int \frac{d+ex}{(9+12x+4x^2)^{7/2}} dx = -\frac{e}{20(9+12x+4x^2)^{5/2}} - \frac{2d-3e}{24(3+2x)^5\sqrt{9+12x+4x^2}}$$

output `-1/20*e/(4*x^2+12*x+9)^(5/2)-1/24*(2*d-3*e)/(3+2*x)^5/((3+2*x)^2)^(1/2)`

Mathematica [A] (verified)

Time = 0.34 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.65

$$\int \frac{d+ex}{(9+12x+4x^2)^{7/2}} dx = \frac{-10d-3(e+4ex)}{120(3+2x)^5\sqrt{(3+2x)^2}}$$

input `Integrate[(d + e*x)/(9 + 12*x + 4*x^2)^(7/2), x]`

output `(-10*d - 3*(e + 4*e*x))/(120*(3 + 2*x)^5*Sqrt[(3 + 2*x)^2])`

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {1100, 1078}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{d + ex}{(4x^2 + 12x + 9)^{7/2}} dx$$

↓ 1100

$$\frac{1}{2}(2d - 3e) \int \frac{1}{(4x^2 + 12x + 9)^{7/2}} dx - \frac{e}{20(4x^2 + 12x + 9)^{5/2}}$$

↓ 1078

$$-\frac{2d - 3e}{24(2x + 3)(4x^2 + 12x + 9)^{5/2}} - \frac{e}{20(4x^2 + 12x + 9)^{5/2}}$$

input `Int[(d + e*x)/(9 + 12*x + 4*x^2)^(7/2), x]`

output `-1/20*e/(9 + 12*x + 4*x^2)^(5/2) - (2*d - 3*e)/(24*(3 + 2*x)*(9 + 12*x + 4*x^2)^(5/2))`

Defintions of rubi rules used

rule 1078 `Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[2*((a + b*x + c*x^2)^(p + 1)/((2*p + 1)*(b + 2*c*x))), x] /; FreeQ[{a, b, c, p}, x] && EqQ[b^2 - 4*a*c, 0] && LtQ[p, -1]`

rule 1100 `Int[((d_.) + (e_.)*(x_))*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[e*((a + b*x + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Simp[(2*c*d - b*e)/(2*c) Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[b^2 - 4*a*c, 0]`

Maple [A] (verified)

Time = 0.82 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.54

| method | result | size |
|---------|--|------|
| gospers | $-\frac{(2x+3)(12ex+10d+3e)}{120((2x+3)^2)^{\frac{7}{2}}}$ | 28 |
| default | $-\frac{(2x+3)(12ex+10d+3e)}{120((2x+3)^2)^{\frac{7}{2}}}$ | 28 |
| risch | $\frac{64\sqrt{(2x+3)^2}\left(-\frac{1}{640}ex-\frac{1}{768}d-\frac{1}{2560}e\right)}{(2x+3)^7}$ | 30 |
| orering | $-\frac{(2x+3)(12ex+10d+3e)}{120(4x^2+12x+9)^{\frac{7}{2}}}$ | 31 |
| meijerg | $\frac{ex^2\left(\frac{16}{81}x^4+\frac{16}{9}x^3+\frac{20}{3}x^2+\frac{40}{3}x+15\right)}{65610\left(1+\frac{2x}{3}\right)^6} + \frac{dx\left(\frac{32}{243}x^5+\frac{32}{27}x^4+\frac{40}{9}x^3+\frac{80}{9}x^2+10x+6\right)}{13122\left(1+\frac{2x}{3}\right)^6}$ | 71 |

input `int((e*x+d)/(4*x^2+12*x+9)^(7/2),x,method=_RETURNVERBOSE)`

output `-1/120*(2*x+3)*(12*e*x+10*d+3*e)/((2*x+3)^2)^(7/2)`

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.87

$$\int \frac{d+ex}{(9+12x+4x^2)^{7/2}} dx = \frac{12ex+10d+3e}{120(64x^6+576x^5+2160x^4+4320x^3+4860x^2+2916x+729)}$$

input `integrate((e*x+d)/(4*x^2+12*x+9)^(7/2),x, algorithm="fricas")`

output `-1/120*(12*e*x + 10*d + 3*e)/(64*x^6 + 576*x^5 + 2160*x^4 + 4320*x^3 + 4860*x^2 + 2916*x + 729)`

Sympy [F]

$$\int \frac{d + ex}{(9 + 12x + 4x^2)^{7/2}} dx = \int \frac{d + ex}{((2x + 3)^2)^{7/2}} dx$$

input `integrate((e*x+d)/(4*x**2+12*x+9)**(7/2),x)`

output `Integral((d + e*x)/((2*x + 3)**2)**(7/2), x)`

Maxima [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.69

$$\int \frac{d + ex}{(9 + 12x + 4x^2)^{7/2}} dx = -\frac{e}{20(4x^2 + 12x + 9)^{5/2}} - \frac{d}{12(2x + 3)^6} + \frac{e}{8(2x + 3)^6}$$

input `integrate((e*x+d)/(4*x^2+12*x+9)^(7/2),x, algorithm="maxima")`

output `-1/20*e/(4*x^2 + 12*x + 9)^(5/2) - 1/12*d/(2*x + 3)^6 + 1/8*e/(2*x + 3)^6`

Giac [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.54

$$\int \frac{d + ex}{(9 + 12x + 4x^2)^{7/2}} dx = -\frac{12ex + 10d + 3e}{120(2x + 3)^6 \operatorname{sgn}(2x + 3)}$$

input `integrate((e*x+d)/(4*x^2+12*x+9)^(7/2),x, algorithm="giac")`

output `-1/120*(12*e*x + 10*d + 3*e)/((2*x + 3)^6*sgn(2*x + 3))`

Mupad [B] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.62

$$\int \frac{d + ex}{(9 + 12x + 4x^2)^{7/2}} dx = -\frac{(10d + 3e + 12ex) \sqrt{4x^2 + 12x + 9}}{120(2x + 3)^7}$$

input `int((d + e*x)/(12*x + 4*x^2 + 9)^(7/2),x)`output `-((10*d + 3*e + 12*e*x)*(12*x + 4*x^2 + 9)^(1/2))/(120*(2*x + 3)^7)`**Reduce [B] (verification not implemented)**

Time = 0.23 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.85

$$\int \frac{d + ex}{(9 + 12x + 4x^2)^{7/2}} dx = \frac{-12ex - 10d - 3e}{7680x^6 + 69120x^5 + 259200x^4 + 518400x^3 + 583200x^2 + 349920x + 87480}$$

input `int((e*x+d)/(4*x^2+12*x+9)^(7/2),x)`output `(- 10*d - 12*e*x - 3*e)/(120*(64*x**6 + 576*x**5 + 2160*x**4 + 4320*x**3 + 4860*x**2 + 2916*x + 729))`

$$3.343 \quad \int \frac{1}{(d+ex)\sqrt{\frac{b^2}{4c}+bx+cx^2}} dx$$

| | |
|---|------|
| Optimal result | 2548 |
| Mathematica [A] (verified) | 2548 |
| Rubi [A] (verified) | 2549 |
| Maple [A] (verified) | 2550 |
| Fricas [A] (verification not implemented) | 2551 |
| Sympy [F] | 2551 |
| Maxima [F(-2)] | 2552 |
| Giac [F(-2)] | 2552 |
| Mupad [F(-1)] | 2553 |
| Reduce [B] (verification not implemented) | 2553 |

Optimal result

Integrand size = 31, antiderivative size = 56

$$\int \frac{1}{(d+ex)\sqrt{\frac{b^2}{4c}+bx+cx^2}} dx = \frac{2(b+2cx)\log\left(\frac{b+2cx}{d+ex}\right)}{(2cd-be)\sqrt{\frac{b^2}{c}+4bx+4cx^2}}$$

output $2*(2*c*x+b)*\ln((2*c*x+b)/(e*x+d))/(-b*e+2*c*d)/(b^2/c+4*b*x+4*c*x^2)^{(1/2)}$

Mathematica [A] (verified)

Time = 0.42 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.91

$$\int \frac{1}{(d+ex)\sqrt{\frac{b^2}{4c}+bx+cx^2}} dx = \frac{2(b+2cx)(\log(b+2cx) - \log(d+ex))}{(2cd-be)\sqrt{\frac{(b+2cx)^2}{c}}}$$

input $\text{Integrate}[1/((d+e*x)*\text{Sqrt}[b^2/(4*c)+b*x+c*x^2]),x]$

output $(2*(b+2*c*x)*(Log[b+2*c*x]-Log[d+e*x]))/((2*c*d-b*e)*\text{Sqrt}[(b+2*c*x)^2/c])$

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.23, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.129$, Rules used = {1102, 27, 47, 16}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\begin{aligned}
 & \int \frac{1}{\sqrt{\frac{b^2}{4c} + bx + cx^2}(d + ex)} dx \\
 & \quad \downarrow \text{1102} \\
 & \frac{(b + 2cx) \int \frac{2}{(b+2cx)(d+ex)} dx}{\sqrt{\frac{b^2}{c} + 4bx + 4cx^2}} \\
 & \quad \downarrow \text{27} \\
 & \frac{2(b + 2cx) \int \frac{1}{(b+2cx)(d+ex)} dx}{\sqrt{\frac{b^2}{c} + 4bx + 4cx^2}} \\
 & \quad \downarrow \text{47} \\
 & \frac{2(b + 2cx) \left(\frac{2c \int \frac{1}{b+2cx} dx}{2cd-be} - \frac{e \int \frac{1}{d+ex} dx}{2cd-be} \right)}{\sqrt{\frac{b^2}{c} + 4bx + 4cx^2}} \\
 & \quad \downarrow \text{16} \\
 & \frac{2(b + 2cx) \left(\frac{\log(b+2cx)}{2cd-be} - \frac{\log(d+ex)}{2cd-be} \right)}{\sqrt{\frac{b^2}{c} + 4bx + 4cx^2}}
 \end{aligned}$$

input `Int[1/((d + e*x)*Sqrt[b^2/(4*c) + b*x + c*x^2]),x]`

output `(2*(b + 2*c*x)*(Log[b + 2*c*x]/(2*c*d - b*e) - Log[d + e*x]/(2*c*d - b*e)))/Sqrt[b^2/c + 4*b*x + 4*c*x^2]`

Definitions of rubi rules used

rule 16 $\text{Int}[(c_)/((a_)+(b_)*(x_)), x_Symbol] \rightarrow \text{Simp}[c*(\text{Log}[\text{RemoveContent}[a + b*x, x]]/b), x] /; \text{FreeQ}[\{a, b, c\}, x]$

rule 27 $\text{Int}[(a_)*(F_x_), x_Symbol] \rightarrow \text{Simp}[a \text{ Int}[F_x, x], x] /; \text{FreeQ}[a, x] \ \&\& \ !\text{MatchQ}[F_x, (b_)*(G_x_)] /; \text{FreeQ}[b, x]$

rule 47 $\text{Int}[1/(((a_)+(b_)*(x_))*((c_)+(d_)*(x_))), x_Symbol] \rightarrow \text{Simp}[b/(b*c - a*d) \text{ Int}[1/(a + b*x), x], x] - \text{Simp}[d/(b*c - a*d) \text{ Int}[1/(c + d*x), x], x] /; \text{FreeQ}[\{a, b, c, d\}, x]$

rule 1102 $\text{Int}(((d_)+(e_)*(x_))^{(m_)}*((a_)+(b_)*(x_)+(c_)*(x_)^2)^{(p_)}, x_Symbol) \rightarrow \text{Simp}[(a + b*x + c*x^2)^{\text{FracPart}[p]}/(c^{\text{IntPart}[p]}*(b/2 + c*x)^{(2*\text{FracPart}[p]}) \text{ Int}[(d + e*x)^m*(b/2 + c*x)^{(2*p)}, x], x] /; \text{FreeQ}[\{a, b, c, d, e, m, p\}, x] \ \&\& \ \text{EqQ}[b^2 - 4*a*c, 0]$

Maple [A] (verified)

Time = 1.47 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.04

| method | result | size |
|---------|---|------|
| default | $-\frac{2(2cx+b)(\ln(2cx+b)-\ln(ex+d))}{\sqrt{\frac{4c^2x^2+4cbx+b^2}{c}}(be-2cd)}$ | 58 |
| risch | $\frac{2(2cx+b)\ln(-ex-d)}{\sqrt{\frac{(2cx+b)^2}{c}}(be-2cd)} - \frac{2(2cx+b)\ln(2cx+b)}{\sqrt{\frac{(2cx+b)^2}{c}}(be-2cd)}$ | 82 |

input $\text{int}(2/(e*x+d)/(1/c*b^2+4*b*x+4*c*x^2)^{(1/2)}, x, \text{method}=_RETURNVERBOSE)$

output $-2*(2*c*x+b)*(\ln(2*c*x+b)-\ln(e*x+d))/((4*c^2*x^2+4*b*c*x+b^2)/c)^{(1/2)}/(b*e-2*c*d)$

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 294, normalized size of antiderivative = 5.25

$$\int \frac{1}{(d+ex)\sqrt{\frac{b^2}{4c}+bx+cx^2}} dx$$

$$= \left[\frac{2\sqrt{c} \log\left(\frac{16c^3e^2x^3+4bc^2d^2+b^3e^2+16(c^3de+bc^2e^2)x^2+2(4c^3d^2+4bc^2de+3b^2ce^2)x+(4c^2d^2-b^2e^2+4(2c^2de-bce^2)x)\sqrt{c}\sqrt{\frac{4c^2x^2}{c}}}{4c^2ex^3+b^2d+4(c^2d+bce)x^2+(4bcd+b^2e)x}\right)}{2cd-be} - \frac{4\sqrt{-c} \arctan\left(-\frac{(4cex+2cd+be)\sqrt{-c}\sqrt{\frac{4c^2x^2+4bcx+b^2}{c}}}{2bcd-b^2e+2(2c^2d-bce)x}\right)}{2cd-be} \right]$$

input `integrate(2/(e*x+d)/(b^2/c+4*b*x+4*c*x^2)^(1/2),x, algorithm="fricas")`

output `[-2*sqrt(c)*log((16*c^3*e^2*x^3 + 4*b*c^2*d^2 + b^3*e^2 + 16*(c^3*d*e + b*c^2*e^2)*x^2 + 2*(4*c^3*d^2 + 4*b*c^2*d*e + 3*b^2*c*e^2)*x + (4*c^2*d^2 - b^2*e^2 + 4*(2*c^2*d*e - b*c*e^2)*x)*sqrt(c)*sqrt((4*c^2*x^2 + 4*b*c*x + b^2)/c))/(4*c^2*e*x^3 + b^2*d + 4*(c^2*d + b*c*e)*x^2 + (4*b*c*d + b^2*e)*x)/(2*c*d - b*e), -4*sqrt(-c)*arctan(-(4*c*e*x + 2*c*d + b*e)*sqrt(-c)*sqrt((4*c^2*x^2 + 4*b*c*x + b^2)/c)/(2*b*c*d - b^2*e + 2*(2*c^2*d - b*c*e)*x))/(2*c*d - b*e)]`

Sympy [F]

$$\int \frac{1}{(d+ex)\sqrt{\frac{b^2}{4c}+bx+cx^2}} dx = 2 \int \frac{1}{d\sqrt{\frac{b^2}{c}+4bx+4cx^2} + ex\sqrt{\frac{b^2}{c}+4bx+4cx^2}} dx$$

input `integrate(2/(e*x+d)/(b**2/c+4*b*x+4*c*x**2)**(1/2),x)`

output `2*Integral(1/(d*sqrt(b**2/c + 4*b*x + 4*c*x**2) + e*x*sqrt(b**2/c + 4*b*x + 4*c*x**2)), x)`

Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{(d+ex)\sqrt{\frac{b^2}{4c}+bx+cx^2}} dx = \text{Exception raised: ValueError}$$

input `integrate(2/(e*x+d)/(b^2/c+4*b*x+4*c*x^2)^(1/2),x, algorithm="maxima")`

output `Exception raised: ValueError >> Computation failed since Maxima requested additional constraints; using the 'assume' command before evaluation *may* help (example of legal syntax is 'assume(b*e-2*c*d>0)', see `assume?` for more deta`

Giac [F(-2)]

Exception generated.

$$\int \frac{1}{(d+ex)\sqrt{\frac{b^2}{4c}+bx+cx^2}} dx = \text{Exception raised: TypeError}$$

input `integrate(2/(e*x+d)/(b^2/c+4*b*x+4*c*x^2)^(1/2),x, algorithm="giac")`

output `Exception raised: TypeError >> an error occurred running a Giac command:INPUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const vecteur & l) Error: Bad Argument Value`

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(d+ex)\sqrt{\frac{b^2}{4c}+bx+cx^2}} dx = \int \frac{2}{(d+ex)\sqrt{4bx+4cx^2+\frac{b^2}{c}}} dx$$

input `int(2/((d + e*x)*(4*b*x + 4*c*x^2 + b^2/c)^(1/2)),x)`output `int(2/((d + e*x)*(4*b*x + 4*c*x^2 + b^2/c)^(1/2)), x)`**Reduce [B] (verification not implemented)**

Time = 0.24 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.54

$$\int \frac{1}{(d+ex)\sqrt{\frac{b^2}{4c}+bx+cx^2}} dx = \frac{2\sqrt{c}(-\log(2cx+b) + \log(ex+d))}{be-2cd}$$

input `int(2/(e*x+d)/(b^2/c+4*b*x+4*c*x^2)^(1/2),x)`output `(2*sqrt(c)*(-log(b + 2*c*x) + log(d + e*x)))/(b*e - 2*c*d)`

3.344 $\int (d + ex)^{5/2} \sqrt{a^2 + 2abx + b^2x^2} dx$

| | |
|---|------|
| Optimal result | 2554 |
| Mathematica [A] (verified) | 2554 |
| Rubi [A] (verified) | 2555 |
| Maple [C] (warning: unable to verify) | 2556 |
| Fricas [A] (verification not implemented) | 2557 |
| Sympy [F(-1)] | 2557 |
| Maxima [A] (verification not implemented) | 2558 |
| Giac [B] (verification not implemented) | 2558 |
| Mupad [F(-1)] | 2559 |
| Reduce [B] (verification not implemented) | 2559 |

Optimal result

Integrand size = 30, antiderivative size = 96

$$\int (d + ex)^{5/2} \sqrt{a^2 + 2abx + b^2x^2} dx = \frac{2(bd - ae)(d + ex)^{7/2} \sqrt{a^2 + 2abx + b^2x^2}}{7e^2(a + bx)} + \frac{2b(d + ex)^{9/2} \sqrt{a^2 + 2abx + b^2x^2}}{9e^2(a + bx)}$$

output

$$-2/7*(-a*e+b*d)*(e*x+d)^(7/2)*((b*x+a)^2)^(1/2)/e^2/(b*x+a)+2/9*b*(e*x+d)^(9/2)*((b*x+a)^2)^(1/2)/e^2/(b*x+a)$$

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.50

$$\int (d + ex)^{5/2} \sqrt{a^2 + 2abx + b^2x^2} dx = \frac{2\sqrt{(a + bx)^2}(d + ex)^{7/2}(-2bd + 9ae + 7bex)}{63e^2(a + bx)}$$

input

$$\text{Integrate}[(d + e*x)^(5/2)*\text{Sqrt}[a^2 + 2*a*b*x + b^2*x^2], x]$$

output

$$(2*\text{Sqrt}[(a + b*x)^2]*(d + e*x)^(7/2)*(-2*b*d + 9*a*e + 7*b*e*x))/(63*e^2*(a + b*x))$$

Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.73, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {1102, 27, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\begin{aligned}
 & \int \sqrt{a^2 + 2abx + b^2x^2}(d + ex)^{5/2} dx \\
 & \quad \downarrow \text{1102} \\
 & \frac{\sqrt{a^2 + 2abx + b^2x^2} \int b(a + bx)(d + ex)^{5/2} dx}{b(a + bx)} \\
 & \quad \downarrow \text{27} \\
 & \frac{\sqrt{a^2 + 2abx + b^2x^2} \int (a + bx)(d + ex)^{5/2} dx}{a + bx} \\
 & \quad \downarrow \text{53} \\
 & \frac{\sqrt{a^2 + 2abx + b^2x^2} \int \left(\frac{b(d+ex)^{7/2}}{e} + \frac{(ae-bd)(d+ex)^{5/2}}{e} \right) dx}{a + bx} \\
 & \quad \downarrow \text{2009} \\
 & \frac{\sqrt{a^2 + 2abx + b^2x^2} \left(\frac{2b(d+ex)^{9/2}}{9e^2} - \frac{2(d+ex)^{7/2}(bd-ae)}{7e^2} \right)}{a + bx}
 \end{aligned}$$

input `Int[(d + e*x)^(5/2)*Sqrt[a^2 + 2*a*b*x + b^2*x^2],x]`

output `(Sqrt[a^2 + 2*a*b*x + b^2*x^2]*((-2*(b*d - a*e)*(d + e*x)^(7/2))/(7*e^2) + (2*b*(d + e*x)^(9/2))/(9*e^2)))/(a + b*x)`

Definitions of rubi rules used

rule 27 `Int[(a_)*(F x_), x_Symbol] := Simp[a Int[F x, x], x] /; FreeQ[a, x] && !MatchQ[F x, (b_)*(G x_) /; FreeQ[b, x]]`

rule 53 `Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] := Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && IGtQ[m, 0] && (!IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])`

rule 1102 `Int[((d_.) + (e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] := Simp[(a + b*x + c*x^2)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x)^(2*FracPart[p])) Int[(d + e*x)^m*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && EqQ[b^2 - 4*a*c, 0]`

rule 2009 `Int[u_, x_Symbol] := Simp[IntSum[u, x], x] /; SumQ[u]`

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 2.

Time = 2.19 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.34

| method | result | size |
|---------|--|------|
| default | $\frac{2 \operatorname{csgn}(bx+a)(ex+d)^{\frac{7}{2}}(7bex+9ae-2bd)}{63e^2}$ | 33 |
| gospers | $\frac{2(ex+d)^{\frac{7}{2}}(7bex+9ae-2bd)\sqrt{(bx+a)^2}}{63e^2(bx+a)}$ | 43 |
| orering | $\frac{2(ex+d)^{\frac{7}{2}}(7bex+9ae-2bd)\sqrt{(bx+a)^2}}{63e^2(bx+a)}$ | 43 |
| risch | $\frac{2\sqrt{(bx+a)^2}(7be^4x^4+9ae^4x^3+19bde^3x^3+27ade^3x^2+15bd^2e^2x^2+27ad^2e^2x+bd^3ex+9d^3ea-2bd^4)\sqrt{ex+d}}{63(bx+a)e^2}$ | 110 |

input `int((e*x+d)^(5/2)*((b*x+a)^2)^(1/2),x,method=_RETURNVERBOSE)`

output $2/63*\text{csgn}(b*x+a)*(e*x+d)^{(7/2)}*(7*b*e*x+9*a*e-2*b*d)/e^2$

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 93, normalized size of antiderivative = 0.97

$$\int (d + ex)^{5/2} \sqrt{a^2 + 2abx + b^2x^2} dx = \frac{2(7be^4x^4 - 2bd^4 + 9ad^3e + (19bde^3 + 9ae^4)x^3 + 3(5bd^2e^2 + 9ade^3)x^2 - 63e^2)}{63e^2}$$

input `integrate((e*x+d)^(5/2)*((b*x+a)^2)^(1/2),x, algorithm="fricas")`

output $2/63*(7*b*e^4*x^4 - 2*b*d^4 + 9*a*d^3*e + (19*b*d*e^3 + 9*a*e^4)*x^3 + 3*(5*b*d^2*e^2 + 9*a*d*e^3)*x^2 + (b*d^3*e + 27*a*d^2*e^2)*x)*\text{sqrt}(e*x + d)/e^2$

Sympy [F(-1)]

Timed out.

$$\int (d + ex)^{5/2} \sqrt{a^2 + 2abx + b^2x^2} dx = \text{Timed out}$$

input `integrate((e*x+d)**(5/2)*((b*x+a)**2)**(1/2),x)`

output Timed out

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 93, normalized size of antiderivative = 0.97

$$\int (d + ex)^{5/2} \sqrt{a^2 + 2abx + b^2x^2} dx = \frac{2(7be^4x^4 - 2bd^4 + 9ad^3e + (19bde^3 + 9ae^4)x^3 + 3(5bd^2e^2 + 9ade^3)x^2 - 63e^2)}{63e^2}$$

input `integrate((e*x+d)^(5/2)*((b*x+a)^2)^(1/2),x, algorithm="maxima")`

output `2/63*(7*b*e^4*x^4 - 2*b*d^4 + 9*a*d^3*e + (19*b*d*e^3 + 9*a*e^4)*x^3 + 3*(5*b*d^2*e^2 + 9*a*d*e^3)*x^2 + (b*d^3*e + 27*a*d^2*e^2)*x)*sqrt(e*x + d)/e^2`

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 354 vs. 2(66) = 132.

Time = 0.14 (sec) , antiderivative size = 354, normalized size of antiderivative = 3.69

$$\int (d + ex)^{5/2} \sqrt{a^2 + 2abx + b^2x^2} dx = \frac{2 \left(315 \sqrt{ex + dd} ad^3 \operatorname{sgn}(bx + a) + 315 \left((ex + d)^{3/2} - 3 \sqrt{ex + dd} \right) ad^2 \operatorname{sgn}(b \right)}{63e^2}$$

input `integrate((e*x+d)^(5/2)*((b*x+a)^2)^(1/2),x, algorithm="giac")`

output

```
2/315*(315*sqrt(e*x + d)*a*d^3*sgn(b*x + a) + 315*((e*x + d)^(3/2) - 3*sqrt
t(e*x + d)*d)*a*d^2*sgn(b*x + a) + 105*((e*x + d)^(3/2) - 3*sqrt(e*x + d)*
d)*b*d^3*sgn(b*x + a)/e + 63*(3*(e*x + d)^(5/2) - 10*(e*x + d)^(3/2)*d + 1
5*sqrt(e*x + d)*d^2)*a*d*sgn(b*x + a) + 63*(3*(e*x + d)^(5/2) - 10*(e*x +
d)^(3/2)*d + 15*sqrt(e*x + d)*d^2)*b*d^2*sgn(b*x + a)/e + 9*(5*(e*x + d)^(
7/2) - 21*(e*x + d)^(5/2)*d + 35*(e*x + d)^(3/2)*d^2 - 35*sqrt(e*x + d)*d^
3)*a*sgn(b*x + a) + 27*(5*(e*x + d)^(7/2) - 21*(e*x + d)^(5/2)*d + 35*(e*x
+ d)^(3/2)*d^2 - 35*sqrt(e*x + d)*d^3)*b*d*sgn(b*x + a)/e + (35*(e*x + d)
^(9/2) - 180*(e*x + d)^(7/2)*d + 378*(e*x + d)^(5/2)*d^2 - 420*(e*x + d)^(
3/2)*d^3 + 315*sqrt(e*x + d)*d^4)*b*sgn(b*x + a)/e/e
```

Mupad [F(-1)]

Timed out.

$$\int (d + ex)^{5/2} \sqrt{a^2 + 2abx + b^2x^2} dx = \int \sqrt{(a + bx)^2} (d + ex)^{5/2} dx$$

input

```
int(((a + b*x)^2)^(1/2)*(d + e*x)^(5/2), x)
```

output

```
int(((a + b*x)^2)^(1/2)*(d + e*x)^(5/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.96

$$\int (d + ex)^{5/2} \sqrt{a^2 + 2abx + b^2x^2} dx = \frac{2\sqrt{ex + d}(7be^4x^4 + 9ae^4x^3 + 19bde^3x^3 + 27ade^3x^2 + 15bd^2e^2x^2 + 27ad^2e^2x + 7bde^2x + 7bd^2e^2)}{63e^2}$$

input

```
int((e*x+d)^(5/2)*((b*x+a)^2)^(1/2), x)
```

output

```
(2*sqrt(d + e*x)*(9*a*d**3*e + 27*a*d**2*e**2*x + 27*a*d*e**3*x**2 + 9*a*e
**4*x**3 - 2*b*d**4 + b*d**3*e*x + 15*b*d**2*e**2*x**2 + 19*b*d*e**3*x**3
+ 7*b*e**4*x**4))/(63*e**2)
```

3.345 $\int (d + ex)^{3/2} \sqrt{a^2 + 2abx + b^2x^2} dx$

| | |
|---|------|
| Optimal result | 2560 |
| Mathematica [A] (verified) | 2560 |
| Rubi [A] (verified) | 2561 |
| Maple [C] (warning: unable to verify) | 2562 |
| Fricas [A] (verification not implemented) | 2563 |
| Sympy [F] | 2563 |
| Maxima [A] (verification not implemented) | 2564 |
| Giac [B] (verification not implemented) | 2564 |
| Mupad [F(-1)] | 2565 |
| Reduce [B] (verification not implemented) | 2565 |

Optimal result

Integrand size = 30, antiderivative size = 96

$$\int (d + ex)^{3/2} \sqrt{a^2 + 2abx + b^2x^2} dx = \frac{2(bd - ae)(d + ex)^{5/2} \sqrt{a^2 + 2abx + b^2x^2}}{5e^2(a + bx)} + \frac{2b(d + ex)^{7/2} \sqrt{a^2 + 2abx + b^2x^2}}{7e^2(a + bx)}$$

output `-2/5*(-a*e+b*d)*(e*x+d)^(5/2)*((b*x+a)^2)^(1/2)/e^2/(b*x+a)+2/7*b*(e*x+d)^(7/2)*((b*x+a)^2)^(1/2)/e^2/(b*x+a)`

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.50

$$\int (d + ex)^{3/2} \sqrt{a^2 + 2abx + b^2x^2} dx = \frac{2\sqrt{(a + bx)^2}(d + ex)^{5/2}(-2bd + 7ae + 5bex)}{35e^2(a + bx)}$$

input `Integrate[(d + e*x)^(3/2)*Sqrt[a^2 + 2*a*b*x + b^2*x^2],x]`

output `(2*Sqrt[(a + b*x)^2]*(d + e*x)^(5/2)*(-2*b*d + 7*a*e + 5*b*e*x))/(35*e^2*(a + b*x))`

Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.73, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {1102, 27, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\begin{aligned}
 & \int \sqrt{a^2 + 2abx + b^2x^2}(d + ex)^{3/2} dx \\
 & \quad \downarrow \text{1102} \\
 & \frac{\sqrt{a^2 + 2abx + b^2x^2} \int b(a + bx)(d + ex)^{3/2} dx}{b(a + bx)} \\
 & \quad \downarrow \text{27} \\
 & \frac{\sqrt{a^2 + 2abx + b^2x^2} \int (a + bx)(d + ex)^{3/2} dx}{a + bx} \\
 & \quad \downarrow \text{53} \\
 & \frac{\sqrt{a^2 + 2abx + b^2x^2} \int \left(\frac{b(d+ex)^{5/2}}{e} + \frac{(ae-bd)(d+ex)^{3/2}}{e} \right) dx}{a + bx} \\
 & \quad \downarrow \text{2009} \\
 & \frac{\sqrt{a^2 + 2abx + b^2x^2} \left(\frac{2b(d+ex)^{7/2}}{7e^2} - \frac{2(d+ex)^{5/2}(bd-ae)}{5e^2} \right)}{a + bx}
 \end{aligned}$$

input `Int[(d + e*x)^(3/2)*Sqrt[a^2 + 2*a*b*x + b^2*x^2],x]`

output `(Sqrt[a^2 + 2*a*b*x + b^2*x^2]*((-2*(b*d - a*e)*(d + e*x)^(5/2))/(5*e^2) + (2*b*(d + e*x)^(7/2))/(7*e^2)))/(a + b*x)`

Definitions of rubi rules used

rule 27 $\text{Int}[(a_*)(Fx_), x_Symbol] \rightarrow \text{Simp}[a \text{ Int}[Fx, x], x] /; \text{FreeQ}[a, x] \ \&\& \ !\text{MatchQ}[Fx, (b_*)(Gx_)] /; \text{FreeQ}[b, x]$

rule 53 $\text{Int}[((a_.) + (b_.)(x_))^{(m_.)} * ((c_.) + (d_.)(x_))^{(n_.)}, x_Symbol] \rightarrow \text{Int}[\text{ExpandIntegrand}[(a + b*x)^m * (c + d*x)^n, x], x] /; \text{FreeQ}\{a, b, c, d, n\}, x] \ \&\& \ \text{IGtQ}[m, 0] \ \&\& \ (\ !\text{IntegerQ}[n] \ || \ (\text{EqQ}[c, 0] \ \&\& \ \text{LeQ}[7*m + 4*n + 4, 0]) \ || \ \text{LtQ}[9*m + 5*(n + 1), 0] \ || \ \text{GtQ}[m + n + 2, 0])$

rule 1102 $\text{Int}[((d_.) + (e_.)(x_))^{(m_.)} * ((a_) + (b_.)(x_) + (c_.)(x_)^2)^{(p_)}, x_Symbol] \rightarrow \text{Simp}[(a + b*x + c*x^2)^{\text{FracPart}[p]} / (c^{\text{IntPart}[p]} * (b/2 + c*x)^{(2*\text{FracPart}[p])}) \ \text{Int}[(d + e*x)^m * (b/2 + c*x)^{(2*p)}, x], x] /; \text{FreeQ}\{a, b, c, d, e, m, p\}, x] \ \&\& \ \text{EqQ}[b^2 - 4*a*c, 0]$

rule 2009 $\text{Int}[u_, x_Symbol] \rightarrow \text{Simp}[\text{IntSum}[u, x], x] /; \text{SumQ}[u]$

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 2.

Time = 2.04 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.34

| method | result | size |
|---------|---|------|
| default | $\frac{2 \operatorname{csgn}(bx+a)(ex+d)^{\frac{5}{2}}(5bex+7ae-2bd)}{35e^2}$ | 33 |
| gospers | $\frac{2(ex+d)^{\frac{5}{2}}(5bex+7ae-2bd)\sqrt{(bx+a)^2}}{35e^2(bx+a)}$ | 43 |
| orering | $\frac{2(ex+d)^{\frac{5}{2}}(5bex+7ae-2bd)\sqrt{(bx+a)^2}}{35e^2(bx+a)}$ | 43 |
| risch | $\frac{2\sqrt{(bx+a)^2}(5be^3x^3+7ae^3x^2+8bde^2x^2+14de^2ax+bd^2ex+7ad^2e-2bd^3)\sqrt{ex+d}}{35(bx+a)e^2}$ | 86 |

input $\text{int}((e*x+d)^{(3/2)} * ((b*x+a)^2)^{(1/2)}, x, \text{method}=_RETURNVERBOSE)$

output $2/35*\text{csgn}(b*x+a)*(e*x+d)^{(5/2)}*(5*b*e*x+7*a*e-2*b*d)/e^2$

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.72

$$\int (d + ex)^{3/2} \sqrt{a^2 + 2abx + b^2x^2} dx = \frac{2(5be^3x^3 - 2bd^3 + 7ad^2e + (8bde^2 + 7ae^3)x^2 + (bd^2e + 14ade^2)x)\sqrt{ex + d}}{35e^2}$$

input `integrate((e*x+d)^(3/2)*((b*x+a)^2)^(1/2),x, algorithm="fricas")`

output $2/35*(5*b*e^3*x^3 - 2*b*d^3 + 7*a*d^2*e + (8*b*d*e^2 + 7*a*e^3)*x^2 + (b*d^2*e + 14*a*d*e^2)*x)*\text{sqrt}(e*x + d)/e^2$

Sympy [F]

$$\int (d + ex)^{3/2} \sqrt{a^2 + 2abx + b^2x^2} dx = \int (d + ex)^{\frac{3}{2}} \sqrt{(a + bx)^2} dx$$

input `integrate((e*x+d)**(3/2)*((b*x+a)**2)**(1/2),x)`

output `Integral((d + e*x)**(3/2)*sqrt((a + b*x)**2), x)`

Maxima [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.72

$$\int (d + ex)^{3/2} \sqrt{a^2 + 2abx + b^2x^2} dx = \frac{2(5be^3x^3 - 2bd^3 + 7ad^2e + (8bde^2 + 7ae^3)x^2 + (bd^2e + 14ade^2)x)\sqrt{ex + d}}{35e^2}$$

input `integrate((e*x+d)^(3/2)*((b*x+a)^2)^(1/2),x, algorithm="maxima")`

output `2/35*(5*b*e^3*x^3 - 2*b*d^3 + 7*a*d^2*e + (8*b*d*e^2 + 7*a*e^3)*x^2 + (b*d^2*e + 14*a*d*e^2)*x)*sqrt(e*x + d)/e^2`

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 228 vs. 2(66) = 132.

Time = 0.14 (sec) , antiderivative size = 228, normalized size of antiderivative = 2.38

$$\int (d + ex)^{3/2} \sqrt{a^2 + 2abx + b^2x^2} dx = \frac{2 \left(105 \sqrt{ex + d} a d^2 \operatorname{sgn}(bx + a) + 70 \left((ex + d)^{3/2} - 3 \sqrt{ex + d} d \right) a d \operatorname{sgn}(bx + a) \right)}{35e^2}$$

input `integrate((e*x+d)^(3/2)*((b*x+a)^2)^(1/2),x, algorithm="giac")`

output `2/105*(105*sqrt(e*x + d)*a*d^2*sgn(b*x + a) + 70*((e*x + d)^(3/2) - 3*sqrt(e*x + d)*d)*a*d*sgn(b*x + a) + 35*((e*x + d)^(3/2) - 3*sqrt(e*x + d)*d)*b*d^2*sgn(b*x + a)/e + 7*(3*(e*x + d)^(5/2) - 10*(e*x + d)^(3/2)*d + 15*sqrt(e*x + d)*d^2)*a*sgn(b*x + a) + 14*(3*(e*x + d)^(5/2) - 10*(e*x + d)^(3/2)*d + 15*sqrt(e*x + d)*d^2)*b*d*sgn(b*x + a)/e + 3*(5*(e*x + d)^(7/2) - 21*(e*x + d)^(5/2)*d + 35*(e*x + d)^(3/2)*d^2 - 35*sqrt(e*x + d)*d^3)*b*sgn(b*x + a)/e`

Mupad [F(-1)]

Timed out.

$$\int (d + ex)^{3/2} \sqrt{a^2 + 2abx + b^2x^2} dx = \int \sqrt{(a + bx)^2} (d + ex)^{3/2} dx$$

input `int(((a + b*x)^2)^(1/2)*(d + e*x)^(3/2), x)`output `int(((a + b*x)^2)^(1/2)*(d + e*x)^(3/2), x)`**Reduce [B] (verification not implemented)**

Time = 0.24 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.71

$$\int (d + ex)^{3/2} \sqrt{a^2 + 2abx + b^2x^2} dx = \frac{2\sqrt{ex + d} (5be^3x^3 + 7ae^3x^2 + 8bde^2x^2 + 14ade^2x + bd^2ex + 7ad^2e - 2b^2d^2)}{35e^2}$$

input `int((e*x+d)^(3/2)*((b*x+a)^2)^(1/2), x)`output `(2*sqrt(d + e*x)*(7*a*d**2*e + 14*a*d*e**2*x + 7*a*e**3*x**2 - 2*b*d**3 + b*d**2*e*x + 8*b*d*e**2*x**2 + 5*b*e**3*x**3))/(35*e**2)`

3.346 $\int \sqrt{d + ex} \sqrt{a^2 + 2abx + b^2x^2} dx$

| | |
|---|------|
| Optimal result | 2566 |
| Mathematica [A] (verified) | 2566 |
| Rubi [A] (verified) | 2567 |
| Maple [C] (warning: unable to verify) | 2568 |
| Fricas [A] (verification not implemented) | 2569 |
| Sympy [F] | 2569 |
| Maxima [A] (verification not implemented) | 2569 |
| Giac [A] (verification not implemented) | 2570 |
| Mupad [F(-1)] | 2570 |
| Reduce [B] (verification not implemented) | 2571 |

Optimal result

Integrand size = 30, antiderivative size = 96

$$\int \sqrt{d + ex} \sqrt{a^2 + 2abx + b^2x^2} dx = -\frac{2(bd - ae)(d + ex)^{3/2} \sqrt{a^2 + 2abx + b^2x^2}}{3e^2(a + bx)} + \frac{2b(d + ex)^{5/2} \sqrt{a^2 + 2abx + b^2x^2}}{5e^2(a + bx)}$$

output

```
-2/3*(-a*e+b*d)*(e*x+d)^(3/2)*((b*x+a)^2)^(1/2)/e^2/(b*x+a)+2/5*b*(e*x+d)^(5/2)*((b*x+a)^2)^(1/2)/e^2/(b*x+a)
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.50

$$\int \sqrt{d + ex} \sqrt{a^2 + 2abx + b^2x^2} dx = \frac{2\sqrt{(a + bx)^2}(d + ex)^{3/2}(-2bd + 5ae + 3bex)}{15e^2(a + bx)}$$

input

```
Integrate[Sqrt[d + e*x]*Sqrt[a^2 + 2*a*b*x + b^2*x^2], x]
```

output

```
(2*Sqrt[(a + b*x)^2]*(d + e*x)^(3/2)*(-2*b*d + 5*a*e + 3*b*e*x))/(15*e^2*(a + b*x))
```

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.73, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {1102, 27, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\begin{aligned}
 & \int \sqrt{a^2 + 2abx + b^2x^2} \sqrt{d + ex} \, dx \\
 & \quad \downarrow \text{1102} \\
 & \frac{\sqrt{a^2 + 2abx + b^2x^2} \int b(a + bx) \sqrt{d + ex} \, dx}{b(a + bx)} \\
 & \quad \downarrow \text{27} \\
 & \frac{\sqrt{a^2 + 2abx + b^2x^2} \int (a + bx) \sqrt{d + ex} \, dx}{a + bx} \\
 & \quad \downarrow \text{53} \\
 & \frac{\sqrt{a^2 + 2abx + b^2x^2} \int \left(\frac{b(d+ex)^{3/2}}{e} + \frac{(ae-bd)\sqrt{d+ex}}{e} \right) dx}{a + bx} \\
 & \quad \downarrow \text{2009} \\
 & \frac{\sqrt{a^2 + 2abx + b^2x^2} \left(\frac{2b(d+ex)^{5/2}}{5e^2} - \frac{2(d+ex)^{3/2}(bd-ae)}{3e^2} \right)}{a + bx}
 \end{aligned}$$

input `Int[Sqrt[d + e*x]*Sqrt[a^2 + 2*a*b*x + b^2*x^2],x]`

output `(Sqrt[a^2 + 2*a*b*x + b^2*x^2]*((-2*(b*d - a*e)*(d + e*x)^(3/2))/(3*e^2) + (2*b*(d + e*x)^(5/2))/(5*e^2)))/(a + b*x)`

Definitions of rubi rules used

rule 27 `Int[(a_)*(F_x_), x_Symbol] := Simp[a Int[F_x, x], x] /; FreeQ[a, x] && !MatchQ[F_x, (b_)*(G_x_) /; FreeQ[b, x]]`

rule 53 `Int[((a_.) + (b_.)*(x_.))^(m_.)*((c_.) + (d_.)*(x_.))^(n_.), x_Symbol] := Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && IGtQ[m, 0] && (!IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])`

rule 1102 `Int[((d_.) + (e_.)*(x_.))^(m_.)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] := Simp[(a + b*x + c*x^2)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x)^(2*FracPart[p])) Int[(d + e*x)^m*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && EqQ[b^2 - 4*a*c, 0]`

rule 2009 `Int[u_, x_Symbol] := Simp[IntSum[u, x], x] /; SumQ[u]`

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 2.

Time = 1.88 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.34

| method | result | size |
|---------|--|------|
| default | $\frac{2 \operatorname{csgn}(bx+a)(ex+d)^{\frac{3}{2}}(3bex+5ae-2bd)}{15e^2}$ | 33 |
| gospers | $\frac{2(ex+d)^{\frac{3}{2}}(3bex+5ae-2bd)\sqrt{(bx+a)^2}}{15e^2(bx+a)}$ | 43 |
| orering | $\frac{2(ex+d)^{\frac{3}{2}}(3bex+5ae-2bd)\sqrt{(bx+a)^2}}{15e^2(bx+a)}$ | 43 |
| risch | $\frac{2\sqrt{(bx+a)^2}(3be^2x^2+5ae^2x+bdxe+5ade-2bd^2)\sqrt{ex+d}}{15(bx+a)e^2}$ | 62 |

input `int((e*x+d)^(1/2)*((b*x+a)^2)^(1/2),x,method=_RETURNVERBOSE)`

output $2/15*\text{csgn}(b*x+a)*(e*x+d)^{(3/2)}*(3*b*e*x+5*a*e-2*b*d)/e^2$

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.48

$$\int \sqrt{d+ex}\sqrt{a^2+2abx+b^2x^2} dx = \frac{2(3be^2x^2 - 2bd^2 + 5ade + (bde + 5ae^2)x)\sqrt{ex+d}}{15e^2}$$

input `integrate((e*x+d)^(1/2)*((b*x+a)^2)^(1/2),x, algorithm="fricas")`

output $2/15*(3*b*e^2*x^2 - 2*b*d^2 + 5*a*d*e + (b*d*e + 5*a*e^2)*x)*\text{sqrt}(e*x + d)/e^2$

Sympy [F]

$$\int \sqrt{d+ex}\sqrt{a^2+2abx+b^2x^2} dx = \int \sqrt{d+ex}\sqrt{(a+bx)^2} dx$$

input `integrate((e*x+d)**(1/2)*((b*x+a)**2)**(1/2),x)`

output `Integral(sqrt(d + e*x)*sqrt((a + b*x)**2), x)`

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.48

$$\int \sqrt{d+ex}\sqrt{a^2+2abx+b^2x^2} dx = \frac{2(3be^2x^2 - 2bd^2 + 5ade + (bde + 5ae^2)x)\sqrt{ex+d}}{15e^2}$$

input `integrate((e*x+d)^(1/2)*((b*x+a)^2)^(1/2),x, algorithm="maxima")`

output $\frac{2}{15} \frac{(3be^2x^2 - 2bd^2 + 5ade + (bde + 5ae^2)x)\sqrt{ex+d}}{e^2}$

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 124, normalized size of antiderivative = 1.29

$$\int \sqrt{d+ex} \sqrt{a^2+2abx+b^2x^2} dx$$

$$= \frac{2 \left(15 \sqrt{ex+d} d \operatorname{sgn}(bx+a) + 5 \left((ex+d)^{\frac{3}{2}} - 3 \sqrt{ex+d} d \right) a \operatorname{sgn}(bx+a) + \frac{5 \left((ex+d)^{\frac{3}{2}} - 3 \sqrt{ex+d} d \right) b d \operatorname{sgn}(bx+a)}{e} \right)}{15e}$$

input `integrate((e*x+d)^(1/2)*((b*x+a)^2)^(1/2),x, algorithm="giac")`

output $\frac{2}{15} \frac{(15 \sqrt{ex+d} a d \operatorname{sgn}(bx+a) + 5((ex+d)^{3/2} - 3 \sqrt{ex+d} d) a \operatorname{sgn}(bx+a) + 5((ex+d)^{3/2} - 3 \sqrt{ex+d} d) b d \operatorname{sgn}(bx+a)/e + (3(ex+d)^{5/2} - 10(ex+d)^{3/2} d + 15 \sqrt{ex+d} d^2) b \operatorname{sgn}(bx+a)/e)}{e}$

Mupad [F(-1)]

Timed out.

$$\int \sqrt{d+ex} \sqrt{a^2+2abx+b^2x^2} dx = \int \sqrt{(a+bx)^2} \sqrt{d+ex} dx$$

input `int(((a+b*x)^2)^(1/2)*(d+e*x)^(1/2),x)`

output `int(((a+b*x)^2)^(1/2)*(d+e*x)^(1/2), x)`

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.46

$$\int \sqrt{d+ex}\sqrt{a^2+2abx+b^2x^2} dx = \frac{2\sqrt{ex+d}(3be^2x^2+5ae^2x+bde+5ade-2bd^2)}{15e^2}$$

input `int((e*x+d)^(1/2)*((b*x+a)^2)^(1/2),x)`

output `(2*sqrt(d + e*x)*(5*a*d*e + 5*a*e**2*x - 2*b*d**2 + b*d*e*x + 3*b*e**2*x**2))/(15*e**2)`

3.347 $\int \frac{\sqrt{a^2+2abx+b^2x^2}}{\sqrt{d+ex}} dx$

| | |
|---|------|
| Optimal result | 2572 |
| Mathematica [A] (verified) | 2572 |
| Rubi [A] (verified) | 2573 |
| Maple [C] (warning: unable to verify) | 2574 |
| Fricas [A] (verification not implemented) | 2575 |
| Sympy [F] | 2575 |
| Maxima [A] (verification not implemented) | 2575 |
| Giac [A] (verification not implemented) | 2576 |
| Mupad [B] (verification not implemented) | 2576 |
| Reduce [B] (verification not implemented) | 2577 |

Optimal result

Integrand size = 30, antiderivative size = 94

$$\int \frac{\sqrt{a^2 + 2abx + b^2x^2}}{\sqrt{d + ex}} dx = -\frac{2(bd - ae)\sqrt{d + ex}\sqrt{a^2 + 2abx + b^2x^2}}{e^2(a + bx)} + \frac{2b(d + ex)^{3/2}\sqrt{a^2 + 2abx + b^2x^2}}{3e^2(a + bx)}$$

output `-2*(-a*e+b*d)*(e*x+d)^(1/2)*((b*x+a)^2)^(1/2)/e^2/(b*x+a)+2/3*b*(e*x+d)^(3/2)*((b*x+a)^2)^(1/2)/e^2/(b*x+a)`

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.50

$$\int \frac{\sqrt{a^2 + 2abx + b^2x^2}}{\sqrt{d + ex}} dx = \frac{2\sqrt{(a + bx)^2}\sqrt{d + ex}(-2bd + 3ae + bex)}{3e^2(a + bx)}$$

input `Integrate[Sqrt[a^2 + 2*a*b*x + b^2*x^2]/Sqrt[d + e*x],x]`

output `(2*Sqrt[(a + b*x)^2]*Sqrt[d + e*x]*(-2*b*d + 3*a*e + b*e*x))/(3*e^2*(a + b*x))`

Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.72, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {1102, 27, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\begin{aligned}
 & \int \frac{\sqrt{a^2 + 2abx + b^2x^2}}{\sqrt{d + ex}} dx \\
 & \quad \downarrow \text{1102} \\
 & \frac{\sqrt{a^2 + 2abx + b^2x^2} \int \frac{b(a+bx)}{\sqrt{d+ex}} dx}{b(a+bx)} \\
 & \quad \downarrow \text{27} \\
 & \frac{\sqrt{a^2 + 2abx + b^2x^2} \int \frac{a+bx}{\sqrt{d+ex}} dx}{a+bx} \\
 & \quad \downarrow \text{53} \\
 & \frac{\sqrt{a^2 + 2abx + b^2x^2} \int \left(\frac{\sqrt{d+ex}b}{e} + \frac{ae-bd}{e\sqrt{d+ex}} \right) dx}{a+bx} \\
 & \quad \downarrow \text{2009} \\
 & \frac{\sqrt{a^2 + 2abx + b^2x^2} \left(\frac{2b(d+ex)^{3/2}}{3e^2} - \frac{2\sqrt{d+ex}(bd-ae)}{e^2} \right)}{a+bx}
 \end{aligned}$$

input

```
Int[Sqrt[a^2 + 2*a*b*x + b^2*x^2]/Sqrt[d + e*x],x]
```

output

```
(Sqrt[a^2 + 2*a*b*x + b^2*x^2]*((-2*(b*d - a*e)*Sqrt[d + e*x])/e^2 + (2*b*(d + e*x)^(3/2))/(3*e^2)))/(a + b*x)
```

Definitions of rubi rules used

rule 27 `Int[(a_)*(F_x_), x_Symbol] := Simp[a Int[F_x, x], x] /; FreeQ[a, x] && !MatchQ[F_x, (b_)*(G_x_)] /; FreeQ[b, x]`

rule 53 `Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] := Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && IGtQ[m, 0] && (!IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])`

rule 1102 `Int[((d_.) + (e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] := Simp[(a + b*x + c*x^2)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x)^(2*FracPart[p])) Int[(d + e*x)^m*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && EqQ[b^2 - 4*a*c, 0]`

rule 2009 `Int[u_, x_Symbol] := Simp[IntSum[u, x], x] /; SumQ[u]`

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 2.

Time = 1.42 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.34

| method | result | size |
|---------|--|------|
| default | $\frac{2 \operatorname{csgn}(bx+a)\sqrt{ex+d}(bex+3ae-2bd)}{3e^2}$ | 32 |
| gospers | $\frac{2\sqrt{ex+d}(bex+3ae-2bd)\sqrt{(bx+a)^2}}{3e^2(bx+a)}$ | 42 |
| risch | $\frac{2\sqrt{ex+d}(bex+3ae-2bd)\sqrt{(bx+a)^2}}{3e^2(bx+a)}$ | 42 |
| orering | $\frac{2\sqrt{ex+d}(bex+3ae-2bd)\sqrt{(bx+a)^2}}{3e^2(bx+a)}$ | 42 |

input `int(((b*x+a)^2)^(1/2)/(e*x+d)^(1/2), x, method=_RETURNVERBOSE)`

output `2/3*csgn(b*x+a)*(e*x+d)^(1/2)*(b*e*x+3*a*e-2*b*d)/e^2`

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.27

$$\int \frac{\sqrt{a^2 + 2abx + b^2x^2}}{\sqrt{d + ex}} dx = \frac{2(bex - 2bd + 3ae)\sqrt{ex + d}}{3e^2}$$

input `integrate(((b*x+a)^2)^(1/2)/(e*x+d)^(1/2),x, algorithm="fricas")`

output `2/3*(b*e*x - 2*b*d + 3*a*e)*sqrt(e*x + d)/e^2`

Sympy [F]

$$\int \frac{\sqrt{a^2 + 2abx + b^2x^2}}{\sqrt{d + ex}} dx = \int \frac{\sqrt{(a + bx)^2}}{\sqrt{d + ex}} dx$$

input `integrate(((b*x+a)**2)**(1/2)/(e*x+d)**(1/2),x)`

output `Integral(sqrt((a + b*x)**2)/sqrt(d + e*x), x)`

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.49

$$\int \frac{\sqrt{a^2 + 2abx + b^2x^2}}{\sqrt{d + ex}} dx = \frac{2(be^2x^2 - 2bd^2 + 3ade - (bde - 3ae^2)x)}{3\sqrt{ex + de^2}}$$

input `integrate(((b*x+a)^2)^(1/2)/(e*x+d)^(1/2),x, algorithm="maxima")`

output $2/3*(b*e^2*x^2 - 2*b*d^2 + 3*a*d*e - (b*d*e - 3*a*e^2)*x)/(sqrt(e*x + d)*e^2)$

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.54

$$\int \frac{\sqrt{a^2 + 2abx + b^2x^2}}{\sqrt{d + ex}} dx = \frac{2 \left(3\sqrt{ex + d} \operatorname{asgn}(bx + a) + \frac{((ex+d)^{\frac{3}{2}} - 3\sqrt{ex+dd}) \operatorname{bsgn}(bx+a)}{e} \right)}{3e}$$

input `integrate(((b*x+a)^2)^(1/2)/(e*x+d)^(1/2),x, algorithm="giac")`

output $2/3*(3*sqrt(e*x + d)*a*sgn(b*x + a) + ((e*x + d)^(3/2) - 3*sqrt(e*x + d)*d)*b*sgn(b*x + a)/e)/e$

Mupad [B] (verification not implemented)

Time = 5.76 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.86

$$\int \frac{\sqrt{a^2 + 2abx + b^2x^2}}{\sqrt{d + ex}} dx = \frac{\sqrt{(a + bx)^2} \left(\frac{2x^2}{3} - \frac{4bd^2 - 6ade}{3be^2} + \frac{x(6ae^2 - 2bde)}{3be^2} \right)}{x\sqrt{d + ex} + \frac{a\sqrt{d+ex}}{b}}$$

input `int(((a + b*x)^2)^(1/2)/(d + e*x)^(1/2),x)`

output $((((a + b*x)^2)^(1/2)*((2*x^2)/3 - (4*b*d^2 - 6*a*d*e)/(3*b*e^2) + (x*(6*a*e^2 - 2*b*d*e))/(3*b*e^2)))/(x*(d + e*x)^(1/2) + (a*(d + e*x)^(1/2))/b)$

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.26

$$\int \frac{\sqrt{a^2 + 2abx + b^2x^2}}{\sqrt{d + ex}} dx = \frac{2\sqrt{ex + d}(bex + 3ae - 2bd)}{3e^2}$$

input `int(((b*x+a)^2)^(1/2)/(e*x+d)^(1/2),x)`

output `(2*sqrt(d + e*x)*(3*a*e - 2*b*d + b*e*x))/(3*e**2)`

3.348 $\int \frac{\sqrt{a^2+2abx+b^2x^2}}{(d+ex)^{3/2}} dx$

| | |
|---|------|
| Optimal result | 2578 |
| Mathematica [A] (verified) | 2578 |
| Rubi [A] (verified) | 2579 |
| Maple [C] (warning: unable to verify) | 2580 |
| Fricas [A] (verification not implemented) | 2581 |
| Sympy [F] | 2581 |
| Maxima [A] (verification not implemented) | 2581 |
| Giac [A] (verification not implemented) | 2582 |
| Mupad [B] (verification not implemented) | 2582 |
| Reduce [B] (verification not implemented) | 2582 |

Optimal result

Integrand size = 30, antiderivative size = 92

$$\int \frac{\sqrt{a^2 + 2abx + b^2x^2}}{(d + ex)^{3/2}} dx = \frac{2(bd - ae)\sqrt{a^2 + 2abx + b^2x^2}}{e^2(a + bx)\sqrt{d + ex}} + \frac{2b\sqrt{d + ex}\sqrt{a^2 + 2abx + b^2x^2}}{e^2(a + bx)}$$

output

```
2*(-a*e+b*d)*((b*x+a)^2)^(1/2)/e^2/(b*x+a)/(e*x+d)^(1/2)+2*b*(e*x+d)^(1/2)
*((b*x+a)^2)^(1/2)/e^2/(b*x+a)
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.52

$$\int \frac{\sqrt{a^2 + 2abx + b^2x^2}}{(d + ex)^{3/2}} dx = -\frac{2\sqrt{(a + bx)^2(-bd + ae - b(d + ex))}}{e^2(a + bx)\sqrt{d + ex}}$$

input

```
Integrate[Sqrt[a^2 + 2*a*b*x + b^2*x^2]/(d + e*x)^(3/2),x]
```

output

```
(-2*Sqrt[(a + b*x)^2]*(-(b*d) + a*e - b*(d + e*x)))/(e^2*(a + b*x)*Sqrt[d
+ e*x])
```

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.72, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {1102, 27, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\begin{aligned}
 & \int \frac{\sqrt{a^2 + 2abx + b^2x^2}}{(d + ex)^{3/2}} dx \\
 & \quad \downarrow \text{1102} \\
 & \frac{\sqrt{a^2 + 2abx + b^2x^2} \int \frac{b(a+bx)}{(d+ex)^{3/2}} dx}{b(a + bx)} \\
 & \quad \downarrow \text{27} \\
 & \frac{\sqrt{a^2 + 2abx + b^2x^2} \int \frac{a+bx}{(d+ex)^{3/2}} dx}{a + bx} \\
 & \quad \downarrow \text{53} \\
 & \frac{\sqrt{a^2 + 2abx + b^2x^2} \int \left(\frac{b}{e\sqrt{d+ex}} + \frac{ae-bd}{e(d+ex)^{3/2}} \right) dx}{a + bx} \\
 & \quad \downarrow \text{2009} \\
 & \frac{\sqrt{a^2 + 2abx + b^2x^2} \left(\frac{2(bd-ae)}{e^2\sqrt{d+ex}} + \frac{2b\sqrt{d+ex}}{e^2} \right)}{a + bx}
 \end{aligned}$$

input

```
Int[Sqrt[a^2 + 2*a*b*x + b^2*x^2]/(d + e*x)^(3/2),x]
```

output

```
(Sqrt[a^2 + 2*a*b*x + b^2*x^2]*((2*(b*d - a*e))/(e^2*Sqrt[d + e*x]) + (2*b*Sqrt[d + e*x])/e^2))/(a + b*x)
```


Definitions of rubi rules used

rule 27 $\text{Int}[(a_*)(Fx_), x_Symbol] \rightarrow \text{Simp}[a \text{ Int}[Fx, x], x] /; \text{FreeQ}[a, x] \ \&\& \ !\text{MatchQ}[Fx, (b_*)(Gx_)] /; \text{FreeQ}[b, x]$

rule 53 $\text{Int}[((a_.) + (b_.)(x_))^{(m_.)} * ((c_.) + (d_.)(x_))^{(n_.)}, x_Symbol] \rightarrow \text{Int}[\text{ExpandIntegrand}[(a + b*x)^m * (c + d*x)^n, x], x] /; \text{FreeQ}\{a, b, c, d, n\}, x] \ \&\& \ \text{IGtQ}[m, 0] \ \&\& \ (\ !\text{IntegerQ}[n] \ || \ (\text{EqQ}[c, 0] \ \&\& \ \text{LeQ}[7*m + 4*n + 4, 0]) \ || \ \text{LtQ}[9*m + 5*(n + 1), 0] \ || \ \text{GtQ}[m + n + 2, 0])$

rule 1102 $\text{Int}[((d_.) + (e_.)(x_))^{(m_.)} * ((a_) + (b_.)(x_) + (c_.)(x_)^2)^{(p_.)}, x_Symbol] \rightarrow \text{Simp}[(a + b*x + c*x^2)^{\text{FracPart}[p]} / (c^{\text{IntPart}[p]} * (b/2 + c*x)^{(2*\text{FracPart}[p])}) \ \text{Int}[(d + e*x)^m * (b/2 + c*x)^{(2*p)}, x], x] /; \text{FreeQ}\{a, b, c, d, e, m, p\}, x] \ \&\& \ \text{EqQ}[b^2 - 4*a*c, 0]$

rule 2009 $\text{Int}[u_, x_Symbol] \rightarrow \text{Simp}[\text{IntSum}[u, x], x] /; \text{SumQ}[u]$

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 2.

Time = 1.48 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.35

| method | result | size |
|---------|---|------|
| default | $-\frac{2 \operatorname{csgn}(bx+a)(-bex+ae-2bd)}{e^2 \sqrt{ex+d}}$ | 32 |
| gosper | $-\frac{2(-bex+ae-2bd)\sqrt{(bx+a)^2}}{\sqrt{ex+d}e^2(bx+a)}$ | 42 |
| orering | $-\frac{2(-bex+ae-2bd)\sqrt{(bx+a)^2}}{\sqrt{ex+d}e^2(bx+a)}$ | 42 |
| risch | $\frac{2b\sqrt{ex+d}\sqrt{(bx+a)^2}}{e^2(bx+a)} - \frac{2(ae-bd)\sqrt{(bx+a)^2}}{e^2\sqrt{ex+d}(bx+a)}$ | 67 |

input $\text{int}(((b*x+a)^2)^{(1/2)} / (e*x+d)^{(3/2)}, x, \text{method}=_RETURNVERBOSE)$

output `-2*csgn(b*x+a)*(-b*e*x+a*e-2*b*d)/e^2/(e*x+d)^(1/2)`

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.38

$$\int \frac{\sqrt{a^2 + 2abx + b^2x^2}}{(d + ex)^{3/2}} dx = \frac{2(bex + 2bd - ae)\sqrt{ex + d}}{e^3x + de^2}$$

input `integrate(((b*x+a)^2)^(1/2)/(e*x+d)^(3/2),x, algorithm="fricas")`

output `2*(b*e*x + 2*b*d - a*e)*sqrt(e*x + d)/(e^3*x + d*e^2)`

Sympy [F]

$$\int \frac{\sqrt{a^2 + 2abx + b^2x^2}}{(d + ex)^{3/2}} dx = \int \frac{\sqrt{(a + bx)^2}}{(d + ex)^{\frac{3}{2}}} dx$$

input `integrate(((b*x+a)**2)**(1/2)/(e*x+d)**(3/2),x)`

output `Integral(sqrt((a + b*x)**2)/(d + e*x)**(3/2), x)`

Maxima [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.27

$$\int \frac{\sqrt{a^2 + 2abx + b^2x^2}}{(d + ex)^{3/2}} dx = \frac{2(bex + 2bd - ae)}{\sqrt{ex + de^2}}$$

input `integrate(((b*x+a)^2)^(1/2)/(e*x+d)^(3/2),x, algorithm="maxima")`

output `2*(b*e*x + 2*b*d - a*e)/(sqrt(e*x + d)*e^2)`

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.57

$$\int \frac{\sqrt{a^2 + 2abx + b^2x^2}}{(d + ex)^{3/2}} dx = \frac{2\sqrt{ex + d}\operatorname{sgn}(bx + a)}{e^2} + \frac{2(b\operatorname{sgn}(bx + a) - a\operatorname{sgn}(bx + a))}{\sqrt{ex + d}e^2}$$

input `integrate(((b*x+a)^2)^(1/2)/(e*x+d)^(3/2),x, algorithm="giac")`output `2*sqrt(e*x + d)*b*sgn(b*x + a)/e^2 + 2*(b*d*sgn(b*x + a) - a*e*sgn(b*x + a))/(sqrt(e*x + d)*e^2)`**Mupad [B] (verification not implemented)**

Time = 5.99 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.63

$$\int \frac{\sqrt{a^2 + 2abx + b^2x^2}}{(d + ex)^{3/2}} dx = \frac{\sqrt{(a + bx)^2 \left(\frac{2x}{e} - \frac{2ae - 4bd}{be^2}\right)}}{x\sqrt{d + ex} + \frac{a\sqrt{d+ex}}{b}}$$

input `int(((a + b*x)^2)^(1/2)/(d + e*x)^(3/2),x)`output `((((a + b*x)^2)^(1/2)*((2*x)/e - (2*a*e - 4*b*d)/(b*e^2)))/(x*(d + e*x)^(1/2) + (a*(d + e*x)^(1/2))/b)`**Reduce [B] (verification not implemented)**

Time = 0.21 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.28

$$\int \frac{\sqrt{a^2 + 2abx + b^2x^2}}{(d + ex)^{3/2}} dx = \frac{2bex - 2ae + 4bd}{\sqrt{ex + d}e^2}$$

input `int(((b*x+a)^2)^(1/2)/(e*x+d)^(3/2),x)`output `(2*(- a*e + 2*b*d + b*e*x))/(sqrt(d + e*x)*e**2)`

3.349 $\int \frac{\sqrt{a^2+2abx+b^2x^2}}{(d+ex)^{5/2}} dx$

| | |
|---|------|
| Optimal result | 2583 |
| Mathematica [A] (verified) | 2583 |
| Rubi [A] (verified) | 2584 |
| Maple [C] (warning: unable to verify) | 2585 |
| Fricas [A] (verification not implemented) | 2586 |
| Sympy [F] | 2586 |
| Maxima [A] (verification not implemented) | 2586 |
| Giac [A] (verification not implemented) | 2587 |
| Mupad [B] (verification not implemented) | 2587 |
| Reduce [B] (verification not implemented) | 2587 |

Optimal result

Integrand size = 30, antiderivative size = 94

$$\int \frac{\sqrt{a^2 + 2abx + b^2x^2}}{(d + ex)^{5/2}} dx = \frac{2(bd - ae)\sqrt{a^2 + 2abx + b^2x^2}}{3e^2(a + bx)(d + ex)^{3/2}} - \frac{2b\sqrt{a^2 + 2abx + b^2x^2}}{e^2(a + bx)\sqrt{d + ex}}$$

output `2/3*(-a*e+b*d)*((b*x+a)^2)^(1/2)/e^2/(b*x+a)/(e*x+d)^(3/2)-2*b*((b*x+a)^2)^(1/2)/e^2/(b*x+a)/(e*x+d)^(1/2)`

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.50

$$\int \frac{\sqrt{a^2 + 2abx + b^2x^2}}{(d + ex)^{5/2}} dx = -\frac{2\sqrt{(a + bx)^2(2bd + ae + 3bex)}}{3e^2(a + bx)(d + ex)^{3/2}}$$

input `Integrate[Sqrt[a^2 + 2*a*b*x + b^2*x^2]/(d + e*x)^(5/2),x]`

output `(-2*Sqrt[(a + b*x)^2]*(2*b*d + a*e + 3*b*e*x))/(3*e^2*(a + b*x)*(d + e*x)^(3/2))`

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.72, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {1102, 27, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\begin{aligned}
 & \int \frac{\sqrt{a^2 + 2abx + b^2x^2}}{(d + ex)^{5/2}} dx \\
 & \quad \downarrow \text{1102} \\
 & \frac{\sqrt{a^2 + 2abx + b^2x^2} \int \frac{b(a+bx)}{(d+ex)^{5/2}} dx}{b(a + bx)} \\
 & \quad \downarrow \text{27} \\
 & \frac{\sqrt{a^2 + 2abx + b^2x^2} \int \frac{a+bx}{(d+ex)^{5/2}} dx}{a + bx} \\
 & \quad \downarrow \text{53} \\
 & \frac{\sqrt{a^2 + 2abx + b^2x^2} \int \left(\frac{b}{e(d+ex)^{3/2}} + \frac{ae-bd}{e(d+ex)^{5/2}} \right) dx}{a + bx} \\
 & \quad \downarrow \text{2009} \\
 & \frac{\sqrt{a^2 + 2abx + b^2x^2} \left(\frac{2(bd-ae)}{3e^2(d+ex)^{3/2}} - \frac{2b}{e^2\sqrt{d+ex}} \right)}{a + bx}
 \end{aligned}$$

input

```
Int[Sqrt[a^2 + 2*a*b*x + b^2*x^2]/(d + e*x)^(5/2),x]
```

output

```
(Sqrt[a^2 + 2*a*b*x + b^2*x^2]*((2*(b*d - a*e))/(3*e^2*(d + e*x)^(3/2)) - (2*b)/(e^2*Sqrt[d + e*x])))/(a + b*x)
```

Definitions of rubi rules used

rule 27 `Int[(a_)*(Fx_), x_Symbol] := Simp[a Int[Fx, x], x] /; FreeQ[a, x] && !MatchQ[Fx, (b_)*(Gx_)] /; FreeQ[b, x]`

rule 53 `Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] := Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && IGtQ[m, 0] && (!IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])`

rule 1102 `Int[((d_.) + (e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] := Simp[(a + b*x + c*x^2)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x)^(2*FracPart[p])) Int[(d + e*x)^m*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && EqQ[b^2 - 4*a*c, 0]`

rule 2009 `Int[u_, x_Symbol] := Simp[IntSum[u, x], x] /; SumQ[u]`

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 2.

Time = 1.46 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.34

| method | result | size |
|---------|--|------|
| default | $-\frac{2 \operatorname{csgn}(bx+a)(3bex+ae+2bd)}{3e^2(ex+d)^{\frac{3}{2}}}$ | 32 |
| gosper | $-\frac{2(3bex+ae+2bd)\sqrt{(bx+a)^2}}{3(ex+d)^{\frac{3}{2}}e^2(bx+a)}$ | 42 |
| orering | $-\frac{2(3bex+ae+2bd)\sqrt{(bx+a)^2}}{3(ex+d)^{\frac{3}{2}}e^2(bx+a)}$ | 42 |

input `int(((b*x+a)^2)^(1/2)/(e*x+d)^(5/2), x, method=_RETURNVERBOSE)`

output `-2/3*csgn(b*x+a)*(3*b*e*x+a*e+2*b*d)/e^2/(e*x+d)^(3/2)`

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.49

$$\int \frac{\sqrt{a^2 + 2abx + b^2x^2}}{(d + ex)^{5/2}} dx = -\frac{2(3bex + 2bd + ae)\sqrt{ex + d}}{3(e^4x^2 + 2de^3x + d^2e^2)}$$

input `integrate(((b*x+a)^2)^(1/2)/(e*x+d)^(5/2),x, algorithm="fricas")`output `-2/3*(3*b*e*x + 2*b*d + a*e)*sqrt(e*x + d)/(e^4*x^2 + 2*d*e^3*x + d^2*e^2)`**Sympy [F]**

$$\int \frac{\sqrt{a^2 + 2abx + b^2x^2}}{(d + ex)^{5/2}} dx = \int \frac{\sqrt{(a + bx)^2}}{(d + ex)^{\frac{5}{2}}} dx$$

input `integrate(((b*x+a)**2)**(1/2)/(e*x+d)**(5/2),x)`output `Integral(sqrt((a + b*x)**2)/(d + e*x)**(5/2), x)`**Maxima [A] (verification not implemented)**

Time = 0.05 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.37

$$\int \frac{\sqrt{a^2 + 2abx + b^2x^2}}{(d + ex)^{5/2}} dx = -\frac{2(3bex + 2bd + ae)}{3(e^3x + de^2)\sqrt{ex + d}}$$

input `integrate(((b*x+a)^2)^(1/2)/(e*x+d)^(5/2),x, algorithm="maxima")`output `-2/3*(3*b*e*x + 2*b*d + a*e)/((e^3*x + d*e^2)*sqrt(e*x + d))`

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.49

$$\int \frac{\sqrt{a^2 + 2abx + b^2x^2}}{(d + ex)^{5/2}} dx = -\frac{2(3(ex + d)b\operatorname{sgn}(bx + a) - b\operatorname{sgn}(bx + a) + a\operatorname{sgn}(bx + a))}{3(ex + d)^{\frac{3}{2}}e^2}$$

input `integrate(((b*x+a)^2)^(1/2)/(e*x+d)^(5/2),x, algorithm="giac")`

output `-2/3*(3*(e*x + d)*b*sgn(b*x + a) - b*d*sgn(b*x + a) + a*e*sgn(b*x + a))/((e*x + d)^(3/2)*e^2)`

Mupad [B] (verification not implemented)

Time = 5.80 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.01

$$\int \frac{\sqrt{a^2 + 2abx + b^2x^2}}{(d + ex)^{5/2}} dx = -\frac{\sqrt{(a + bx)^2 \left(\frac{2x}{e^2} + \frac{2ae + 4bd}{3be^3}\right)}}{x^2 \sqrt{d + ex} + \frac{ad\sqrt{d+ex}}{be} + \frac{x(3ae^3 + 3bde^2)\sqrt{d+ex}}{3be^3}}$$

input `int(((a + b*x)^2)^(1/2)/(d + e*x)^(5/2),x)`

output `-(((a + b*x)^2)^(1/2)*((2*x)/e^2 + (2*a*e + 4*b*d)/(3*b*e^3)))/(x^2*(d + e*x)^(1/2) + (a*d*(d + e*x)^(1/2))/(b*e) + (x*(3*a*e^3 + 3*b*d*e^2)*(d + e*x)^(1/2))/(3*b*e^3))`

Reduce [B] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.36

$$\int \frac{\sqrt{a^2 + 2abx + b^2x^2}}{(d + ex)^{5/2}} dx = \frac{-2bex - \frac{2}{3}ae - \frac{4}{3}bd}{\sqrt{ex + d}e^2(ex + d)}$$

input `int(((b*x+a)^2)^(1/2)/(e*x+d)^(5/2),x)`

output $(2*(-a*e - 2*b*d - 3*b*e*x))/(3*\text{sqrt}(d + e*x)*e^{2*(d + e*x)})$

3.350 $\int \frac{\sqrt{a^2+2abx+b^2x^2}}{(d+ex)^{7/2}} dx$

| | |
|---|------|
| Optimal result | 2589 |
| Mathematica [A] (verified) | 2589 |
| Rubi [A] (verified) | 2590 |
| Maple [C] (warning: unable to verify) | 2591 |
| Fricas [A] (verification not implemented) | 2592 |
| Sympy [F(-1)] | 2592 |
| Maxima [A] (verification not implemented) | 2592 |
| Giac [A] (verification not implemented) | 2593 |
| Mupad [B] (verification not implemented) | 2593 |
| Reduce [B] (verification not implemented) | 2594 |

Optimal result

Integrand size = 30, antiderivative size = 96

$$\int \frac{\sqrt{a^2 + 2abx + b^2x^2}}{(d + ex)^{7/2}} dx = \frac{2(bd - ae)\sqrt{a^2 + 2abx + b^2x^2}}{5e^2(a + bx)(d + ex)^{5/2}} - \frac{2b\sqrt{a^2 + 2abx + b^2x^2}}{3e^2(a + bx)(d + ex)^{3/2}}$$

output $2/5*(-a*e+b*d)*((b*x+a)^2)^{(1/2)}/e^2/(b*x+a)/(e*x+d)^{(5/2)}-2/3*b*((b*x+a)^2)^{(1/2)}/e^2/(b*x+a)/(e*x+d)^{(3/2)}$

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.50

$$\int \frac{\sqrt{a^2 + 2abx + b^2x^2}}{(d + ex)^{7/2}} dx = -\frac{2\sqrt{(a + bx)^2(2bd + 3ae + 5bex)}}{15e^2(a + bx)(d + ex)^{5/2}}$$

input `Integrate[Sqrt[a^2 + 2*a*b*x + b^2*x^2]/(d + e*x)^(7/2),x]`

output $(-2*\text{Sqrt}[(a + b*x)^2]*(2*b*d + 3*a*e + 5*b*e*x))/(15*e^2*(a + b*x)*(d + e*x)^{(5/2)})$

Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.73, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {1102, 27, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\begin{aligned}
 & \int \frac{\sqrt{a^2 + 2abx + b^2x^2}}{(d + ex)^{7/2}} dx \\
 & \quad \downarrow \text{1102} \\
 & \frac{\sqrt{a^2 + 2abx + b^2x^2} \int \frac{b(a+bx)}{(d+ex)^{7/2}} dx}{b(a + bx)} \\
 & \quad \downarrow \text{27} \\
 & \frac{\sqrt{a^2 + 2abx + b^2x^2} \int \frac{a+bx}{(d+ex)^{7/2}} dx}{a + bx} \\
 & \quad \downarrow \text{53} \\
 & \frac{\sqrt{a^2 + 2abx + b^2x^2} \int \left(\frac{b}{e(d+ex)^{5/2}} + \frac{ae-bd}{e(d+ex)^{7/2}} \right) dx}{a + bx} \\
 & \quad \downarrow \text{2009} \\
 & \frac{\sqrt{a^2 + 2abx + b^2x^2} \left(\frac{2(bd-ae)}{5e^2(d+ex)^{5/2}} - \frac{2b}{3e^2(d+ex)^{3/2}} \right)}{a + bx}
 \end{aligned}$$

input

```
Int[Sqrt[a^2 + 2*a*b*x + b^2*x^2]/(d + e*x)^(7/2),x]
```

output

```
(Sqrt[a^2 + 2*a*b*x + b^2*x^2]*((2*(b*d - a*e))/(5*e^2*(d + e*x)^(5/2)) - (2*b)/(3*e^2*(d + e*x)^(3/2))))/(a + b*x)
```

Definitions of rubi rules used

rule 27 `Int[(a_)*(F_x_), x_Symbol] := Simp[a Int[F_x, x], x] /; FreeQ[a, x] && !MatchQ[F_x, (b_)*(G_x_) /; FreeQ[b, x]]`

rule 53 `Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] := Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && IGtQ[m, 0] && (!IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])`

rule 1102 `Int[((d_.) + (e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] := Simp[(a + b*x + c*x^2)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x)^(2*FracPart[p])) Int[(d + e*x)^m*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && EqQ[b^2 - 4*a*c, 0]`

rule 2009 `Int[u_, x_Symbol] := Simp[IntSum[u, x], x] /; SumQ[u]`

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 2.

Time = 1.49 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.34

| method | result | size |
|---------|--|------|
| default | $-\frac{2 \operatorname{csgn}(bx+a)(5bex+3ae+2bd)}{15e^2(ex+d)^{\frac{5}{2}}}$ | 33 |
| gospers | $-\frac{2(5bex+3ae+2bd)\sqrt{(bx+a)^2}}{15(ex+d)^{\frac{5}{2}}e^2(bx+a)}$ | 43 |
| orering | $-\frac{2(5bex+3ae+2bd)\sqrt{(bx+a)^2}}{15(ex+d)^{\frac{5}{2}}e^2(bx+a)}$ | 43 |

input `int(((b*x+a)^2)^(1/2)/(e*x+d)^(7/2), x, method=_RETURNVERBOSE)`

output `-2/15*csgn(b*x+a)*(5*b*e*x+3*a*e+2*b*d)/e^2/(e*x+d)^(5/2)`

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.60

$$\int \frac{\sqrt{a^2 + 2abx + b^2x^2}}{(d + ex)^{7/2}} dx = -\frac{2(5bex + 2bd + 3ae)\sqrt{ex + d}}{15(e^5x^3 + 3de^4x^2 + 3d^2e^3x + d^3e^2)}$$

input `integrate(((b*x+a)^2)^(1/2)/(e*x+d)^(7/2),x, algorithm="fricas")`

output `-2/15*(5*b*e*x + 2*b*d + 3*a*e)*sqrt(e*x + d)/(e^5*x^3 + 3*d*e^4*x^2 + 3*d^2*e^3*x + d^3*e^2)`

Sympy [F(-1)]

Timed out.

$$\int \frac{\sqrt{a^2 + 2abx + b^2x^2}}{(d + ex)^{7/2}} dx = \text{Timed out}$$

input `integrate(((b*x+a)**2)**(1/2)/(e*x+d)**(7/2),x)`

output `Timed out`

Maxima [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.49

$$\int \frac{\sqrt{a^2 + 2abx + b^2x^2}}{(d + ex)^{7/2}} dx = -\frac{2(5bex + 2bd + 3ae)}{15(e^4x^2 + 2de^3x + d^2e^2)\sqrt{ex + d}}$$

input `integrate(((b*x+a)^2)^(1/2)/(e*x+d)^(7/2),x, algorithm="maxima")`

output `-2/15*(5*b*e*x + 2*b*d + 3*a*e)/((e^4*x^2 + 2*d*e^3*x + d^2*e^2)*sqrt(e*x + d))`

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.49

$$\int \frac{\sqrt{a^2 + 2abx + b^2x^2}}{(d + ex)^{7/2}} dx = \frac{2(5(ex + d)b\operatorname{sgn}(bx + a) - 3bd\operatorname{sgn}(bx + a) + 3ae\operatorname{sgn}(bx + a))}{15(ex + d)^{5/2}e^2}$$

input `integrate(((b*x+a)^2)^(1/2)/(e*x+d)^(7/2),x, algorithm="giac")`

output `-2/15*(5*(e*x + d)*b*sgn(b*x + a) - 3*b*d*sgn(b*x + a) + 3*a*e*sgn(b*x + a))/((e*x + d)^(5/2)*e^2)`

Mupad [B] (verification not implemented)

Time = 5.56 (sec) , antiderivative size = 120, normalized size of antiderivative = 1.25

$$\int \frac{\sqrt{a^2 + 2abx + b^2x^2}}{(d + ex)^{7/2}} dx = \frac{\sqrt{(a + bx)^2} \left(\frac{2x}{3e^3} + \frac{2ae + \frac{4bd}{15}}{be^4} \right)}{x^3 \sqrt{d + ex} + \frac{ad^2 \sqrt{d+ex}}{be^2} + \frac{x^2 (ae^4 + 2bde^3) \sqrt{d+ex}}{be^4} + \frac{dx(2ae+bd) \sqrt{d+ex}}{be^2}}$$

input `int(((a + b*x)^2)^(1/2)/(d + e*x)^(7/2),x)`

output `-(((a + b*x)^2)^(1/2)*((2*x)/(3*e^3) + ((2*a*e)/5 + (4*b*d)/15)/(b*e^4)))/(x^3*(d + e*x)^(1/2) + (a*d^2*(d + e*x)^(1/2))/(b*e^2) + (x^2*(a*e^4 + 2*b*d*e^3)*(d + e*x)^(1/2))/(b*e^4) + (d*x*(2*a*e + b*d)*(d + e*x)^(1/2))/(b*e^2))`

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.47

$$\int \frac{\sqrt{a^2 + 2abx + b^2x^2}}{(d + ex)^{7/2}} dx = \frac{-\frac{2}{3}bex - \frac{2}{5}ae - \frac{4}{15}bd}{\sqrt{ex + d} e^2 (e^2x^2 + 2dex + d^2)}$$

input `int(((b*x+a)^2)^(1/2)/(e*x+d)^(7/2),x)`

output `(2*(- 3*a*e - 2*b*d - 5*b*e*x))/(15*sqrt(d + e*x)*e**2*(d**2 + 2*d*e*x + e**2*x**2))`

3.351 $\int (d + ex)^{5/2} (a^2 + 2abx + b^2x^2)^{3/2} dx$

| | |
|---|------|
| Optimal result | 2595 |
| Mathematica [A] (verified) | 2596 |
| Rubi [A] (verified) | 2596 |
| Maple [A] (verified) | 2598 |
| Fricas [A] (verification not implemented) | 2598 |
| Sympy [F] | 2599 |
| Maxima [A] (verification not implemented) | 2599 |
| Giac [B] (verification not implemented) | 2600 |
| Mupad [F(-1)] | 2601 |
| Reduce [B] (verification not implemented) | 2601 |

Optimal result

Integrand size = 30, antiderivative size = 208

$$\int (d + ex)^{5/2} (a^2 + 2abx + b^2x^2)^{3/2} dx =$$

$$-\frac{2(bd - ae)^3(d + ex)^{7/2}\sqrt{a^2 + 2abx + b^2x^2}}{7e^4(a + bx)}$$

$$+ \frac{2b(bd - ae)^2(d + ex)^{9/2}\sqrt{a^2 + 2abx + b^2x^2}}{3e^4(a + bx)}$$

$$- \frac{6b^2(bd - ae)(d + ex)^{11/2}\sqrt{a^2 + 2abx + b^2x^2}}{11e^4(a + bx)}$$

$$+ \frac{2b^3(d + ex)^{13/2}\sqrt{a^2 + 2abx + b^2x^2}}{13e^4(a + bx)}$$

output

```
-2/7*(-a*e+b*d)^3*(e*x+d)^(7/2)*((b*x+a)^2)^(1/2)/e^4/(b*x+a)+2/3*b*(-a*e+
b*d)^2*(e*x+d)^(9/2)*((b*x+a)^2)^(1/2)/e^4/(b*x+a)-6/11*b^2*(-a*e+b*d)*(e*
x+d)^(11/2)*((b*x+a)^2)^(1/2)/e^4/(b*x+a)+2/13*b^3*(e*x+d)^(13/2)*((b*x+a)
^2)^(1/2)/e^4/(b*x+a)
```


Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 120, normalized size of antiderivative = 0.58

$$\int (d + ex)^{5/2} (a^2 + 2abx + b^2x^2)^{3/2} dx = \frac{2\sqrt{(a + bx)^2}(d + ex)^{7/2} (429a^3e^3 + 143a^2be^2(-2d + 7ex) + 13ab^2e(8d^2 - 28dex + 63e^2x^2)) + b^3(-16d^3 + 56d^2ex - 126d^2e^2x^2 + 231e^3x^3)}{3003e^4(a + bx)}$$

input

```
Integrate[(d + e*x)^(5/2)*(a^2 + 2*a*b*x + b^2*x^2)^(3/2),x]
```

output

```
(2*sqrt[(a + b*x)^2]*(d + e*x)^(7/2)*(429*a^3*e^3 + 143*a^2*b*e^2*(-2*d + 7*e*x) + 13*a*b^2*e*(8*d^2 - 28*d*e*x + 63*e^2*x^2) + b^3*(-16*d^3 + 56*d^2*e*x - 126*d^2*e^2*x^2 + 231*e^3*x^3)))/(3003*e^4*(a + b*x))
```

Rubi [A] (verified)

Time = 0.46 (sec) , antiderivative size = 128, normalized size of antiderivative = 0.62, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {1102, 27, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\begin{aligned} & \int (a^2 + 2abx + b^2x^2)^{3/2} (d + ex)^{5/2} dx \\ & \quad \downarrow \text{1102} \\ & \frac{\sqrt{a^2 + 2abx + b^2x^2} \int b^3(a + bx)^3(d + ex)^{5/2} dx}{b^3(a + bx)} \\ & \quad \downarrow \text{27} \\ & \frac{\sqrt{a^2 + 2abx + b^2x^2} \int (a + bx)^3(d + ex)^{5/2} dx}{a + bx} \\ & \quad \downarrow \text{53} \end{aligned}$$

$$\frac{\sqrt{a^2 + 2abx + b^2x^2} \int \left(\frac{b^3(d+ex)^{11/2}}{e^3} - \frac{3b^2(bd-ae)(d+ex)^{9/2}}{e^3} + \frac{3b(bd-ae)^2(d+ex)^{7/2}}{e^3} + \frac{(ae-bd)^3(d+ex)^{5/2}}{e^3} \right) dx}{a + bx}$$

↓ 2009

$$\frac{\sqrt{a^2 + 2abx + b^2x^2} \left(-\frac{6b^2(d+ex)^{11/2}(bd-ae)}{11e^4} + \frac{2b(d+ex)^{9/2}(bd-ae)^2}{3e^4} - \frac{2(d+ex)^{7/2}(bd-ae)^3}{7e^4} + \frac{2b^3(d+ex)^{13/2}}{13e^4} \right)}{a + bx}$$

input

```
Int[(d + e*x)^(5/2)*(a^2 + 2*a*b*x + b^2*x^2)^(3/2),x]
```

output

```
(Sqrt[a^2 + 2*a*b*x + b^2*x^2]*((-2*(b*d - a*e)^3*(d + e*x)^(7/2))/(7*e^4)
+ (2*b*(b*d - a*e)^2*(d + e*x)^(9/2))/(3*e^4) - (6*b^2*(b*d - a*e)*(d + e
*x)^(11/2))/(11*e^4) + (2*b^3*(d + e*x)^(13/2))/(13*e^4)))/(a + b*x)
```

Defintions of rubi rules used

rule 27

```
Int[(a_)*(F_x_), x_Symbol] := Simp[a Int[F_x, x], x] /; FreeQ[a, x] && !Ma
tchQ[F_x, (b_)*(G_x_)] /; FreeQ[b, x]
```

rule 53

```
Int[((a_) + (b_)*(x_))^(m_)*((c_) + (d_)*(x_))^(n_), x_Symbol] := Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && (!IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 1102

```
Int[((d_) + (e_)*(x_))^(m_)*((a_) + (b_)*(x_) + (c_)*(x_)^2)^(p_), x_S
ymbol] := Simp[(a + b*x + c*x^2)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x)^(2*F
racPart[p])) Int[(d + e*x)^m*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c,
d, e, m, p}, x] && EqQ[b^2 - 4*a*c, 0]
```

rule 2009

```
Int[u_, x_Symbol] := Simp[IntSum[u, x], x] /; SumQ[u]
```


output

```
2/3003*(231*b^3*e^6*x^6 - 16*b^3*d^6 + 104*a*b^2*d^5*e - 286*a^2*b*d^4*e^2
+ 429*a^3*d^3*e^3 + 63*(9*b^3*d*e^5 + 13*a*b^2*e^6)*x^5 + 7*(53*b^3*d^2*e
^4 + 299*a*b^2*d*e^5 + 143*a^2*b*e^6)*x^4 + (5*b^3*d^3*e^3 + 1469*a*b^2*d^
2*e^4 + 2717*a^2*b*d*e^5 + 429*a^3*e^6)*x^3 - 3*(2*b^3*d^4*e^2 - 13*a*b^2*
d^3*e^3 - 715*a^2*b*d^2*e^4 - 429*a^3*d*e^5)*x^2 + (8*b^3*d^5*e - 52*a*b^2
*d^4*e^2 + 143*a^2*b*d^3*e^3 + 1287*a^3*d^2*e^4)*x)*sqrt(e*x + d)/e^4
```

Sympy [F]

$$\int (d + ex)^{5/2} (a^2 + 2abx + b^2x^2)^{3/2} dx = \int (d + ex)^{5/2} ((a + bx)^2)^{3/2} dx$$

input

```
integrate((e*x+d)**(5/2)*(b**2*x**2+2*a*b*x+a**2)**(3/2),x)
```

output

```
Integral((d + e*x)**(5/2)*((a + b*x)**2)**(3/2), x)
```

Maxima [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 268, normalized size of antiderivative = 1.29

$$\int (d + ex)^{5/2} (a^2 + 2abx + b^2x^2)^{3/2} dx = \frac{2(231b^3e^6x^6 - 16b^3d^6 + 104ab^2d^5e - 286a^2bd^4e^2 + 429a^3d^3e^3 + 63(9b^3de^5 + 13ab^2e^6)x^4 + 7(53b^3d^2e^4 + 299ab^2de^5 + 143a^2be^6)x^3 - 3(2b^3d^4e^2 - 13ab^2d^3e^3 - 715a^2bd^2e^4 - 429a^3de^5)x^2 + (8b^3d^5e - 52ab^2d^4e^2 + 143a^2bd^3e^3 + 1287a^3d^2e^4)x)\sqrt{ex + d}}{e^4}$$

input

```
integrate((e*x+d)^(5/2)*(b^2*x^2+2*a*b*x+a^2)^(3/2),x, algorithm="maxima")
```

output

```
2/3003*(231*b^3*e^6*x^6 - 16*b^3*d^6 + 104*a*b^2*d^5*e - 286*a^2*b*d^4*e^2
+ 429*a^3*d^3*e^3 + 63*(9*b^3*d*e^5 + 13*a*b^2*e^6)*x^4 + 7*(53*b^3*d^2*e
^4 + 299*a*b^2*d*e^5 + 143*a^2*b*e^6)*x^3 - 3*(2*b^3*d^4*e^2 - 13*a*b^2*
d^3*e^3 - 715*a^2*b*d^2*e^4 - 429*a^3*d*e^5)*x^2 + (8*b^3*d^5*e - 52*a*b^2
*d^4*e^2 + 143*a^2*b*d^3*e^3 + 1287*a^3*d^2*e^4)*x)*sqrt(e*x + d)/e^4
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 953 vs. $2(148) = 296$.

Time = 0.15 (sec) , antiderivative size = 953, normalized size of antiderivative = 4.58

$$\int (d + ex)^{5/2} (a^2 + 2abx + b^2x^2)^{3/2} dx = \text{Too large to display}$$

input `integrate((e*x+d)^(5/2)*(b^2*x^2+2*a*b*x+a^2)^(3/2),x, algorithm="giac")`

output

```
2/15015*(15015*sqrt(e*x + d)*a^3*d^3*sgn(b*x + a) + 15015*((e*x + d)^(3/2)
- 3*sqrt(e*x + d)*d)*a^3*d^2*sgn(b*x + a) + 15015*((e*x + d)^(3/2) - 3*sq
rt(e*x + d)*d)*a^2*b*d^3*sgn(b*x + a)/e + 3003*(3*(e*x + d)^(5/2) - 10*(e*
x + d)^(3/2)*d + 15*sqrt(e*x + d)*d^2)*a^3*d*sgn(b*x + a) + 3003*(3*(e*x +
d)^(5/2) - 10*(e*x + d)^(3/2)*d + 15*sqrt(e*x + d)*d^2)*a*b^2*d^3*sgn(b*x
+ a)/e^2 + 9009*(3*(e*x + d)^(5/2) - 10*(e*x + d)^(3/2)*d + 15*sqrt(e*x +
d)*d^2)*a^2*b*d^2*sgn(b*x + a)/e + 429*(5*(e*x + d)^(7/2) - 21*(e*x + d)^(
5/2)*d + 35*(e*x + d)^(3/2)*d^2 - 35*sqrt(e*x + d)*d^3)*a^3*sgn(b*x + a)
+ 429*(5*(e*x + d)^(7/2) - 21*(e*x + d)^(5/2)*d + 35*(e*x + d)^(3/2)*d^2 -
35*sqrt(e*x + d)*d^3)*b^3*d^3*sgn(b*x + a)/e^3 + 3861*(5*(e*x + d)^(7/2)
- 21*(e*x + d)^(5/2)*d + 35*(e*x + d)^(3/2)*d^2 - 35*sqrt(e*x + d)*d^3)*a*
b^2*d^2*sgn(b*x + a)/e^2 + 3861*(5*(e*x + d)^(7/2) - 21*(e*x + d)^(5/2)*d
+ 35*(e*x + d)^(3/2)*d^2 - 35*sqrt(e*x + d)*d^3)*a^2*b*d*sgn(b*x + a)/e +
143*(35*(e*x + d)^(9/2) - 180*(e*x + d)^(7/2)*d + 378*(e*x + d)^(5/2)*d^2
- 420*(e*x + d)^(3/2)*d^3 + 315*sqrt(e*x + d)*d^4)*b^3*d^2*sgn(b*x + a)/e^
3 + 429*(35*(e*x + d)^(9/2) - 180*(e*x + d)^(7/2)*d + 378*(e*x + d)^(5/2)*
d^2 - 420*(e*x + d)^(3/2)*d^3 + 315*sqrt(e*x + d)*d^4)*a*b^2*d*sgn(b*x + a
)/e^2 + 143*(35*(e*x + d)^(9/2) - 180*(e*x + d)^(7/2)*d + 378*(e*x + d)^(5
/2)*d^2 - 420*(e*x + d)^(3/2)*d^3 + 315*sqrt(e*x + d)*d^4)*a^2*b*sgn(b*x +
a)/e + 65*(63*(e*x + d)^(11/2) - 385*(e*x + d)^(9/2)*d + 990*(e*x + d)...
```

Mupad [F(-1)]

Timed out.

$$\int (d + ex)^{5/2} (a^2 + 2abx + b^2x^2)^{3/2} dx = \int (d + ex)^{5/2} (a^2 + 2abx + b^2x^2)^{3/2} dx$$

input `int((d + e*x)^(5/2)*(a^2 + b^2*x^2 + 2*a*b*x)^(3/2),x)`

output `int((d + e*x)^(5/2)*(a^2 + b^2*x^2 + 2*a*b*x)^(3/2), x)`

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 284, normalized size of antiderivative = 1.37

$$\int (d + ex)^{5/2} (a^2 + 2abx + b^2x^2)^{3/2} dx = \frac{2\sqrt{ex + d} (231b^3e^6x^6 + 819ab^2e^6x^5 + 567b^3de^5x^5 + 1001a^2be^6x^4 + 2093ab^2de^5x^4 + 371b^3e^6x^3 + 371b^3d^2e^4x^3 + 567b^3de^5x^5 + 231b^3e^6x^6)}{(3003e^4)}$$

input `int((e*x+d)^(5/2)*(b^2*x^2+2*a*b*x+a^2)^(3/2),x)`

output `(2*sqrt(d + e*x)*(429*a**3*d**3*e**3 + 1287*a**3*d**2*e**4*x + 1287*a**3*d**e**5*x**2 + 429*a**3*e**6*x**3 - 286*a**2*b*d**4*e**2 + 143*a**2*b*d**3*e**3*x + 2145*a**2*b*d**2*e**4*x**2 + 2717*a**2*b*d*e**5*x**3 + 1001*a**2*b*d**e**6*x**4 + 104*a*b**2*d**5*e - 52*a*b**2*d**4*e**2*x + 39*a*b**2*d**3*e**3*x**2 + 1469*a*b**2*d**2*e**4*x**3 + 2093*a*b**2*d*e**5*x**4 + 819*a*b**2*e**6*x**5 - 16*b**3*d**6 + 8*b**3*d**5*e*x - 6*b**3*d**4*e**2*x**2 + 5*b**3*d**3*e**3*x**3 + 371*b**3*d**2*e**4*x**4 + 567*b**3*d*e**5*x**5 + 231*b**3*e**6*x**6))/(3003*e**4)`

3.352 $\int (d + ex)^{3/2} (a^2 + 2abx + b^2x^2)^{3/2} dx$

| | |
|---|------|
| Optimal result | 2602 |
| Mathematica [A] (verified) | 2603 |
| Rubi [A] (verified) | 2603 |
| Maple [A] (verified) | 2605 |
| Fricas [A] (verification not implemented) | 2605 |
| Sympy [F] | 2606 |
| Maxima [A] (verification not implemented) | 2606 |
| Giac [B] (verification not implemented) | 2606 |
| Mupad [F(-1)] | 2607 |
| Reduce [B] (verification not implemented) | 2608 |

Optimal result

Integrand size = 30, antiderivative size = 208

$$\int (d + ex)^{3/2} (a^2 + 2abx + b^2x^2)^{3/2} dx =$$

$$-\frac{2(bd - ae)^3(d + ex)^{5/2}\sqrt{a^2 + 2abx + b^2x^2}}{5e^4(a + bx)}$$

$$+ \frac{6b(bd - ae)^2(d + ex)^{7/2}\sqrt{a^2 + 2abx + b^2x^2}}{7e^4(a + bx)}$$

$$- \frac{2b^2(bd - ae)(d + ex)^{9/2}\sqrt{a^2 + 2abx + b^2x^2}}{3e^4(a + bx)} + \frac{2b^3(d + ex)^{11/2}\sqrt{a^2 + 2abx + b^2x^2}}{11e^4(a + bx)}$$

output

```
-2/5*(-a*e+b*d)^3*(e*x+d)^(5/2)*((b*x+a)^2)^(1/2)/e^4/(b*x+a)+6/7*b*(-a*e+
b*d)^2*(e*x+d)^(7/2)*((b*x+a)^2)^(1/2)/e^4/(b*x+a)-2/3*b^2*(-a*e+b*d)*(e*x
+d)^(9/2)*((b*x+a)^2)^(1/2)/e^4/(b*x+a)+2/11*b^3*(e*x+d)^(11/2)*((b*x+a)^2
)^(1/2)/e^4/(b*x+a)
```

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 120, normalized size of antiderivative = 0.58

$$\int (d + ex)^{3/2} (a^2 + 2abx + b^2x^2)^{3/2} dx = \frac{2\sqrt{(a + bx)^2(d + ex)^{5/2} (231a^3e^3 + 99a^2be^2(-2d + 5ex) + 11ab^2e(8d^2 - 20dex + 35e^2x^2) + b^3(-16d^3 + 40d^2ex - 70de^2x^2 + 105e^3x^3))}}{1155e^4(a + bx)}$$

input

```
Integrate[(d + e*x)^(3/2)*(a^2 + 2*a*b*x + b^2*x^2)^(3/2), x]
```

output

```
(2*sqrt[(a + b*x)^2]*(d + e*x)^(5/2)*(231*a^3*e^3 + 99*a^2*b*e^2*(-2*d + 5*e*x) + 11*a*b^2*e*(8*d^2 - 20*d*e*x + 35*e^2*x^2) + b^3*(-16*d^3 + 40*d^2*e*x - 70*d*e^2*x^2 + 105*e^3*x^3)))/(1155*e^4*(a + b*x))
```

Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 128, normalized size of antiderivative = 0.62, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {1102, 27, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\begin{aligned} & \int (a^2 + 2abx + b^2x^2)^{3/2} (d + ex)^{3/2} dx \\ & \quad \downarrow \text{1102} \\ & \frac{\sqrt{a^2 + 2abx + b^2x^2} \int b^3(a + bx)^3(d + ex)^{3/2} dx}{b^3(a + bx)} \\ & \quad \downarrow \text{27} \\ & \frac{\sqrt{a^2 + 2abx + b^2x^2} \int (a + bx)^3(d + ex)^{3/2} dx}{a + bx} \\ & \quad \downarrow \text{53} \end{aligned}$$

$$\frac{\sqrt{a^2 + 2abx + b^2x^2} \int \left(\frac{b^3(d+ex)^{9/2}}{e^3} - \frac{3b^2(bd-ae)(d+ex)^{7/2}}{e^3} + \frac{3b(bd-ae)^2(d+ex)^{5/2}}{e^3} + \frac{(ae-bd)^3(d+ex)^{3/2}}{e^3} \right) dx}{a + bx}$$

↓ 2009

$$\frac{\sqrt{a^2 + 2abx + b^2x^2} \left(-\frac{2b^2(d+ex)^{9/2}(bd-ae)}{3e^4} + \frac{6b(d+ex)^{7/2}(bd-ae)^2}{7e^4} - \frac{2(d+ex)^{5/2}(bd-ae)^3}{5e^4} + \frac{2b^3(d+ex)^{11/2}}{11e^4} \right)}{a + bx}$$

input

```
Int[(d + e*x)^(3/2)*(a^2 + 2*a*b*x + b^2*x^2)^(3/2),x]
```

output

```
(Sqrt[a^2 + 2*a*b*x + b^2*x^2]*((-2*(b*d - a*e)^3*(d + e*x)^(5/2))/(5*e^4)
+ (6*b*(b*d - a*e)^2*(d + e*x)^(7/2))/(7*e^4) - (2*b^2*(b*d - a*e)*(d + e
*x)^(9/2))/(3*e^4) + (2*b^3*(d + e*x)^(11/2))/(11*e^4)))/(a + b*x)
```

Defintions of rubi rules used

rule 27

```
Int[(a_)*(F_x_), x_Symbol] := Simp[a Int[F_x, x], x] /; FreeQ[a, x] && !Ma
tchQ[F_x, (b_)*(G_x_)] /; FreeQ[b, x]
```

rule 53

```
Int[((a_) + (b_)*(x_))^(m_)*((c_) + (d_)*(x_))^(n_), x_Symbol] := Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && (!IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 1102

```
Int[((d_) + (e_)*(x_))^(m_)*((a_) + (b_)*(x_) + (c_)*(x_)^2)^(p_), x_S
ymbol] := Simp[(a + b*x + c*x^2)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x)^(2*F
racPart[p])) Int[(d + e*x)^m*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c,
d, e, m, p}, x] && EqQ[b^2 - 4*a*c, 0]
```

rule 2009

```
Int[u_, x_Symbol] := Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 1.55 (sec) , antiderivative size = 132, normalized size of antiderivative = 0.63

| method | result |
|---------|--|
| gospers | $\frac{2(ex+d)^{\frac{5}{2}}(105e^3x^3b^3+385x^2ab^2e^3-70x^2b^3de^2+495a^2be^3x-220xab^2de^2+40b^3d^2ex+231e^3a^3-198a^2bde^2+88ab^2d^2e-16b^3d^3)}{1155e^4(bx+a)^3}$ |
| default | $\frac{2(ex+d)^{\frac{5}{2}}(105e^3x^3b^3+385x^2ab^2e^3-70x^2b^3de^2+495a^2be^3x-220xab^2de^2+40b^3d^2ex+231e^3a^3-198a^2bde^2+88ab^2d^2e-16b^3d^3)}{1155e^4(bx+a)^3}$ |
| orering | $\frac{2(105e^3x^3b^3+385x^2ab^2e^3-70x^2b^3de^2+495a^2be^3x-220xab^2de^2+40b^3d^2ex+231e^3a^3-198a^2bde^2+88ab^2d^2e-16b^3d^3)(ex+d)^{\frac{5}{2}}}{1155e^4(bx+a)^3}$ |
| risch | $\frac{2\sqrt{(bx+a)^2}(105b^3x^5e^5+385ab^2e^5x^4+140b^3de^4x^4+495a^2be^5x^3+550ab^2de^4x^3+5b^3d^2e^3x^3+231a^3e^5x^2+792a^2bde^4x^2+33ab^2d^2e^4x+1155b^3d^3e^4)}{1155(bx+a)^3}$ |

input

```
int((e*x+d)^(3/2)*(b^2*x^2+2*a*b*x+a^2)^(3/2),x,method=_RETURNVERBOSE)
```

output

$$\frac{2}{1155}(e*x+d)^{(5/2)}*(105*b^3*e^3*x^3+385*a*b^2*e^3*x^2-70*b^3*d*e^2*x^2+495*a^2*b*e^3*x-220*a*b^2*d*e^2*x+40*b^3*d^2*e*x+231*a^3*e^3-198*a^2*b*d*e^2+88*a*b^2*d^2*e-16*b^3*d^3)*((b*x+a)^2)^{(3/2)}/e^4/(b*x+a)^3$$
Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 216, normalized size of antiderivative = 1.04

$$\int (d+ex)^{3/2} (a^2+2abx+b^2x^2)^{3/2} dx = \frac{2(105b^3e^5x^5-16b^3d^5+88ab^2d^4e-198a^2bd^3e^2+231a^3d^2e^3+35(4b^3de^4+11ab^2e^5)x^4)}{1155e^4}$$

input

```
integrate((e*x+d)^(3/2)*(b^2*x^2+2*a*b*x+a^2)^(3/2),x, algorithm="fricas")
```

output

$$\frac{2}{1155}(105*b^3*e^5*x^5-16*b^3*d^5+88*a*b^2*d^4*e-198*a^2*b*d^3*e^2+231*a^3*d^2*e^3+35*(4*b^3*d*e^4+11*a*b^2*e^5)*x^4+5*(b^3*d^2*e^3+110*a*b^2*d*e^4+99*a^2*b*e^5)*x^3-3*(2*b^3*d^3*e^2-11*a*b^2*d^2*e^3-264*a^2*b*d*e^4-77*a^3*e^5)*x^2+(8*b^3*d^4*e-44*a*b^2*d^3*e^2+99*a^2*b*d^2*e^3+462*a^3*d*e^4)*x)*sqrt(e*x+d)/e^4$$

Sympy [F]

$$\int (d + ex)^{3/2} (a^2 + 2abx + b^2x^2)^{3/2} dx = \int (d + ex)^{\frac{3}{2}} ((a + bx)^2)^{\frac{3}{2}} dx$$

input `integrate((e*x+d)**(3/2)*(b**2*x**2+2*a*b*x+a**2)**(3/2),x)`

output `Integral((d + e*x)**(3/2)*((a + b*x)**2)**(3/2), x)`

Maxima [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 216, normalized size of antiderivative = 1.04

$$\int (d + ex)^{3/2} (a^2 + 2abx + b^2x^2)^{3/2} dx = \frac{2(105b^3e^5x^5 - 16b^3d^5 + 88ab^2d^4e - 198a^2bd^3e^2 + 231a^3d^2e^3 + 35(4b^3de^4 + 11ab^2e^5)x^4}{e^4}$$

input `integrate((e*x+d)^(3/2)*(b^2*x^2+2*a*b*x+a^2)^(3/2),x, algorithm="maxima")`

output `2/1155*(105*b^3*e^5*x^5 - 16*b^3*d^5 + 88*a*b^2*d^4*e - 198*a^2*b*d^3*e^2 + 231*a^3*d^2*e^3 + 35*(4*b^3*d*e^4 + 11*a*b^2*e^5)*x^4 + 5*(b^3*d^2*e^3 + 110*a*b^2*d*e^4 + 99*a^2*b*e^5)*x^3 - 3*(2*b^3*d^3*e^2 - 11*a*b^2*d^2*e^3 - 264*a^2*b*d*e^4 - 77*a^3*e^5)*x^2 + (8*b^3*d^4*e - 44*a*b^2*d^3*e^2 + 9*a^2*b*d^2*e^3 + 462*a^3*d*e^4)*x)*sqrt(e*x + d)/e^4`

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 638 vs. 2(148) = 296.

Time = 0.15 (sec) , antiderivative size = 638, normalized size of antiderivative = 3.07

$$\int (d + ex)^{3/2} (a^2 + 2abx + b^2x^2)^{3/2} dx = \text{Too large to display}$$

input `integrate((e*x+d)^(3/2)*(b^2*x^2+2*a*b*x+a^2)^(3/2),x, algorithm="giac")`

output `2/3465*(3465*sqrt(e*x + d)*a^3*d^2*sgn(b*x + a) + 2310*((e*x + d)^(3/2) - 3*sqrt(e*x + d)*d)*a^3*d*sgn(b*x + a) + 3465*((e*x + d)^(3/2) - 3*sqrt(e*x + d)*d)*a^2*b*d^2*sgn(b*x + a)/e + 231*(3*(e*x + d)^(5/2) - 10*(e*x + d)^(3/2)*d + 15*sqrt(e*x + d)*d^2)*a^3*sgn(b*x + a) + 693*(3*(e*x + d)^(5/2) - 10*(e*x + d)^(3/2)*d + 15*sqrt(e*x + d)*d^2)*a*b^2*d^2*sgn(b*x + a)/e^2 + 1386*(3*(e*x + d)^(5/2) - 10*(e*x + d)^(3/2)*d + 15*sqrt(e*x + d)*d^2)*a^2*b*d*sgn(b*x + a)/e + 99*(5*(e*x + d)^(7/2) - 21*(e*x + d)^(5/2)*d + 35*(e*x + d)^(3/2)*d^2 - 35*sqrt(e*x + d)*d^3)*b^3*d^2*sgn(b*x + a)/e^3 + 594*(5*(e*x + d)^(7/2) - 21*(e*x + d)^(5/2)*d + 35*(e*x + d)^(3/2)*d^2 - 35*sqrt(e*x + d)*d^3)*a*b^2*d*sgn(b*x + a)/e^2 + 297*(5*(e*x + d)^(7/2) - 21*(e*x + d)^(5/2)*d + 35*(e*x + d)^(3/2)*d^2 - 35*sqrt(e*x + d)*d^3)*a^2*b*sgn(b*x + a)/e + 22*(35*(e*x + d)^(9/2) - 180*(e*x + d)^(7/2)*d + 378*(e*x + d)^(5/2)*d^2 - 420*(e*x + d)^(3/2)*d^3 + 315*sqrt(e*x + d)*d^4)*b^3*d*sgn(b*x + a)/e^3 + 33*(35*(e*x + d)^(9/2) - 180*(e*x + d)^(7/2)*d + 378*(e*x + d)^(5/2)*d^2 - 420*(e*x + d)^(3/2)*d^3 + 315*sqrt(e*x + d)*d^4)*a*b^2*sgn(b*x + a)/e^2 + 5*(63*(e*x + d)^(11/2) - 385*(e*x + d)^(9/2)*d + 990*(e*x + d)^(7/2)*d^2 - 1386*(e*x + d)^(5/2)*d^3 + 1155*(e*x + d)^(3/2)*d^4 - 693*sqrt(e*x + d)*d^5)*b^3*sgn(b*x + a)/e^3)/e`

Mupad [F(-1)]

Timed out.

$$\int (d + ex)^{3/2} (a^2 + 2abx + b^2x^2)^{3/2} dx = \int (d + ex)^{3/2} (a^2 + 2abx + b^2x^2)^{3/2} dx$$

input `int((d + e*x)^(3/2)*(a^2 + b^2*x^2 + 2*a*b*x)^(3/2),x)`

output `int((d + e*x)^(3/2)*(a^2 + b^2*x^2 + 2*a*b*x)^(3/2), x)`

Reduce [B] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 226, normalized size of antiderivative = 1.09

$$\int (d + ex)^{3/2} (a^2 + 2abx + b^2x^2)^{3/2} dx = \frac{2\sqrt{ex + d} (105b^3e^5x^5 + 385ab^2e^5x^4 + 140b^3de^4x^4 + 495a^2be^5x^3 + 550ab^2de^4x^3 + 5b^3d^2e^3x^2 + 140b^3de^4x^4 + 105b^3e^5x^5)}{(1155e^4)}$$

input

```
int((e*x+d)^(3/2)*(b^2*x^2+2*a*b*x+a^2)^(3/2),x)
```

output

```
(2*sqrt(d + e*x)*(231*a**3*d**2*e**3 + 462*a**3*d*e**4*x + 231*a**3*e**5*x**2 - 198*a**2*b*d**3*e**2 + 99*a**2*b*d**2*e**3*x + 792*a**2*b*d*e**4*x**2 + 495*a**2*b*e**5*x**3 + 88*a*b**2*d**4*e - 44*a*b**2*d**3*e**2*x + 33*a*b**2*d**2*e**3*x**2 + 550*a*b**2*d*e**4*x**3 + 385*a*b**2*e**5*x**4 - 16*b**3*d**5 + 8*b**3*d**4*e*x - 6*b**3*d**3*e**2*x**2 + 5*b**3*d**2*e**3*x**3 + 140*b**3*d*e**4*x**4 + 105*b**3*e**5*x**5))/(1155*e**4)
```

3.353 $\int \sqrt{d + ex}(a^2 + 2abx + b^2x^2)^{3/2} dx$

| | |
|---|------|
| Optimal result | 2609 |
| Mathematica [A] (verified) | 2610 |
| Rubi [A] (verified) | 2610 |
| Maple [A] (verified) | 2612 |
| Fricas [A] (verification not implemented) | 2612 |
| Sympy [F] | 2613 |
| Maxima [A] (verification not implemented) | 2613 |
| Giac [B] (verification not implemented) | 2613 |
| Mupad [F(-1)] | 2614 |
| Reduce [B] (verification not implemented) | 2614 |

Optimal result

Integrand size = 30, antiderivative size = 208

$$\int \sqrt{d + ex}(a^2 + 2abx + b^2x^2)^{3/2} dx = -\frac{2(bd - ae)^3(d + ex)^{3/2}\sqrt{a^2 + 2abx + b^2x^2}}{3e^4(a + bx)} + \frac{6b(bd - ae)^2(d + ex)^{5/2}\sqrt{a^2 + 2abx + b^2x^2}}{5e^4(a + bx)} - \frac{6b^2(bd - ae)(d + ex)^{7/2}\sqrt{a^2 + 2abx + b^2x^2}}{7e^4(a + bx)} + \frac{2b^3(d + ex)^{9/2}\sqrt{a^2 + 2abx + b^2x^2}}{9e^4(a + bx)}$$

output

```
-2/3*(-a*e+b*d)^3*(e*x+d)^(3/2)*((b*x+a)^2)^(1/2)/e^4/(b*x+a)+6/5*b*(-a*e+b*d)^2*(e*x+d)^(5/2)*((b*x+a)^2)^(1/2)/e^4/(b*x+a)-6/7*b^2*(-a*e+b*d)*(e*x+d)^(7/2)*((b*x+a)^2)^(1/2)/e^4/(b*x+a)+2/9*b^3*(e*x+d)^(9/2)*((b*x+a)^2)^(1/2)/e^4/(b*x+a)
```

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 120, normalized size of antiderivative = 0.58

$$\int \sqrt{d+ex}(a^2+2abx+b^2x^2)^{3/2} dx = \frac{2\sqrt{(a+bx)^2(d+ex)^{3/2}}(105a^3e^3+63a^2be^2(-2d+3ex)+9ab^2e(8d^2-12dex+15e^2x^2)+b^3(-16d^3+24d^2ex-30de^2x^2+35e^3x^3))}{315e^4(a+bx)}$$

input

```
Integrate[Sqrt[d + e*x]*(a^2 + 2*a*b*x + b^2*x^2)^(3/2), x]
```

output

```
(2*Sqrt[(a + b*x)^2]*(d + e*x)^(3/2)*(105*a^3*e^3 + 63*a^2*b*e^2*(-2*d + 3*e*x) + 9*a*b^2*e*(8*d^2 - 12*d*e*x + 15*e^2*x^2) + b^3*(-16*d^3 + 24*d^2*e*x - 30*d*e^2*x^2 + 35*e^3*x^3)))/(315*e^4*(a + b*x))
```

Rubi [A] (verified)

Time = 0.46 (sec) , antiderivative size = 128, normalized size of antiderivative = 0.62, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {1102, 27, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\begin{array}{c} \int (a^2 + 2abx + b^2x^2)^{3/2} \sqrt{d+ex} dx \\ \downarrow \text{1102} \\ \frac{\sqrt{a^2 + 2abx + b^2x^2} \int b^3(a+bx)^3 \sqrt{d+ex} dx}{b^3(a+bx)} \\ \downarrow \text{27} \\ \frac{\sqrt{a^2 + 2abx + b^2x^2} \int (a+bx)^3 \sqrt{d+ex} dx}{a+bx} \\ \downarrow \text{53} \end{array}$$

$$\frac{\sqrt{a^2 + 2abx + b^2x^2} \int \left(\frac{b^3(d+ex)^{7/2}}{e^3} - \frac{3b^2(bd-ae)(d+ex)^{5/2}}{e^3} + \frac{3b(bd-ae)^2(d+ex)^{3/2}}{e^3} + \frac{(ae-bd)^3\sqrt{d+ex}}{e^3} \right) dx}{a + bx}$$

↓ 2009

$$\frac{\sqrt{a^2 + 2abx + b^2x^2} \left(-\frac{6b^2(d+ex)^{7/2}(bd-ae)}{7e^4} + \frac{6b(d+ex)^{5/2}(bd-ae)^2}{5e^4} - \frac{2(d+ex)^{3/2}(bd-ae)^3}{3e^4} + \frac{2b^3(d+ex)^{9/2}}{9e^4} \right)}{a + bx}$$

input `Int[Sqrt[d + e*x]*(a^2 + 2*a*b*x + b^2*x^2)^(3/2),x]`

output `(Sqrt[a^2 + 2*a*b*x + b^2*x^2]*((-2*(b*d - a*e)^3*(d + e*x)^(3/2))/(3*e^4) + (6*b*(b*d - a*e)^2*(d + e*x)^(5/2))/(5*e^4) - (6*b^2*(b*d - a*e)*(d + e*x)^(7/2))/(7*e^4) + (2*b^3*(d + e*x)^(9/2))/(9*e^4))/(a + b*x)`

Defintions of rubi rules used

rule 27 `Int[(a_)*(F_x_), x_Symbol] := Simp[a Int[F_x, x], x] /; FreeQ[a, x] && !MatchQ[F_x, (b_)*(G_x_)] /; FreeQ[b, x]`

rule 53 `Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] := Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x] /; FreeQ[{a, b, c, d, n}, x] && IGtQ[m, 0] && (!IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])`

rule 1102 `Int[((d_.) + (e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] := Simp[(a + b*x + c*x^2)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x)^(2*FracPart[p])) Int[(d + e*x)^m*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && EqQ[b^2 - 4*a*c, 0]`

rule 2009 `Int[u_, x_Symbol] := Simp[IntSum[u, x], x] /; SumQ[u]`

Sympy [F]

$$\int \sqrt{d+ex}(a^2+2abx+b^2x^2)^{3/2} dx = \int \sqrt{d+ex}((a+bx)^2)^{3/2} dx$$

input `integrate((e*x+d)**(1/2)*(b**2*x**2+2*a*b*x+a**2)**(3/2),x)`

output `Integral(sqrt(d + e*x)*((a + b*x)**2)**(3/2), x)`

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 164, normalized size of antiderivative = 0.79

$$\int \sqrt{d+ex}(a^2+2abx+b^2x^2)^{3/2} dx = \frac{2(35b^3e^4x^4 - 16b^3d^4 + 72ab^2d^3e - 126a^2bd^2e^2 + 105a^3de^3 + 5(b^3de^3 + 27ab^2e^4)x^3 - 3(2b^3d^2e^2 - 9a^2b^2de^3 - 63a^2b^2e^4)x^2 + (8b^3d^3e - 36a^2b^2d^2e^2 + 63a^2b^2de^3 + 105a^3e^4)x)\sqrt{ex+d}}{e^4}$$

input `integrate((e*x+d)^(1/2)*(b^2*x^2+2*a*b*x+a^2)^(3/2),x, algorithm="maxima")`

output `2/315*(35*b^3*e^4*x^4 - 16*b^3*d^4 + 72*a*b^2*d^3*e - 126*a^2*b*d^2*e^2 + 105*a^3*d*e^3 + 5*(b^3*d*e^3 + 27*a*b^2*e^4)*x^3 - 3*(2*b^3*d^2*e^2 - 9*a^2*b^2*d*e^3 - 63*a^2*b^2*e^4)*x^2 + (8*b^3*d^3*e - 36*a^2*b^2*d^2*e^2 + 63*a^2*b^2*d*e^3 + 105*a^3*e^4)*x)*sqrt(e*x + d)/e^4`

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 370 vs. 2(148) = 296.

Time = 0.13 (sec) , antiderivative size = 370, normalized size of antiderivative = 1.78

$$\int \sqrt{d+ex}(a^2+2abx+b^2x^2)^{3/2} dx = \frac{2\left(315\sqrt{ex+da^3}\operatorname{dsgn}(bx+a) + 105\left((ex+d)^{3/2} - 3\sqrt{ex+dd}\right)a^3\operatorname{sgn}(bx+a) + \frac{315}{e^4}(ex+d)^{3/2}\right)}{e^4}$$

input `integrate((e*x+d)^(1/2)*(b^2*x^2+2*a*b*x+a^2)^(3/2),x, algorithm="giac")`

output
$$\begin{aligned} & 2/315*(315*\sqrt{e*x+d}*a^3*d*\operatorname{sgn}(b*x+a) + 105*((e*x+d)^{(3/2)} - 3*\sqrt{e*x+d}) \\ & *d*(e*x+d)*d)*a^3*\operatorname{sgn}(b*x+a) + 315*((e*x+d)^{(3/2)} - 3*\sqrt{e*x+d})*d \\ & *a^2*b*d*\operatorname{sgn}(b*x+a)/e + 63*(3*(e*x+d)^{(5/2)} - 10*(e*x+d)^{(3/2)}*d + 1 \\ & 5*\sqrt{e*x+d}*d^2)*a*b^2*d*\operatorname{sgn}(b*x+a)/e^2 + 63*(3*(e*x+d)^{(5/2)} - 10 \\ & *(e*x+d)^{(3/2)}*d + 15*\sqrt{e*x+d}*d^2)*a^2*b*\operatorname{sgn}(b*x+a)/e + 9*(5*(e*x+d)^{(7/2)} \\ & - 21*(e*x+d)^{(5/2)}*d + 35*(e*x+d)^{(3/2)}*d^2 - 35*\sqrt{e*x+d} \\ & *d^3)*b^3*d*\operatorname{sgn}(b*x+a)/e^3 + 27*(5*(e*x+d)^{(7/2)} - 21*(e*x+d)^{(5/2)} \\ & *d + 35*(e*x+d)^{(3/2)}*d^2 - 35*\sqrt{e*x+d}*d^3)*a*b^2*\operatorname{sgn}(b*x+a) \\ & /e^2 + (35*(e*x+d)^{(9/2)} - 180*(e*x+d)^{(7/2)}*d + 378*(e*x+d)^{(5/2)}*d^2 \\ & - 420*(e*x+d)^{(3/2)}*d^3 + 315*\sqrt{e*x+d}*d^4)*b^3*\operatorname{sgn}(b*x+a)/e^3 \\ &)/e \end{aligned}$$

Mupad [F(-1)]

Timed out.

$$\int \sqrt{d+ex}(a^2+2abx+b^2x^2)^{3/2} dx = \int \sqrt{d+ex}(a^2+2abx+b^2x^2)^{3/2} dx$$

input `int((d + e*x)^(1/2)*(a^2 + b^2*x^2 + 2*a*b*x)^(3/2),x)`

output `int((d + e*x)^(1/2)*(a^2 + b^2*x^2 + 2*a*b*x)^(3/2), x)`

Reduce [B] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 168, normalized size of antiderivative = 0.81

$$\int \sqrt{d+ex}(a^2+2abx+b^2x^2)^{3/2} dx = \frac{2\sqrt{ex+d}(35b^3e^4x^4 + 135ab^2e^4x^3 + 5b^3de^3x^3 + 189a^2be^4x^2 + 27ab^2de^3x^2 - 6b^3d^2e^2x^2 + \dots)}{\dots}$$

input `int((e*x+d)^(1/2)*(b^2*x^2+2*a*b*x+a^2)^(3/2),x)`

output

```
(2*sqrt(d + e*x)*(105*a**3*d*e**3 + 105*a**3*e**4*x - 126*a**2*b*d**2*e**2
+ 63*a**2*b*d*e**3*x + 189*a**2*b*e**4*x**2 + 72*a*b**2*d**3*e - 36*a*b**
2*d**2*e**2*x + 27*a*b**2*d*e**3*x**2 + 135*a*b**2*e**4*x**3 - 16*b**3*d**
4 + 8*b**3*d**3*e*x - 6*b**3*d**2*e**2*x**2 + 5*b**3*d*e**3*x**3 + 35*b**3
*e**4*x**4))/(315*e**4)
```

3.354
$$\int \frac{(a^2+2abx+b^2x^2)^{3/2}}{\sqrt{d+ex}} dx$$

| | |
|---|------|
| Optimal result | 2616 |
| Mathematica [A] (verified) | 2617 |
| Rubi [A] (verified) | 2617 |
| Maple [A] (verified) | 2619 |
| Fricas [A] (verification not implemented) | 2619 |
| Sympy [F] | 2620 |
| Maxima [A] (verification not implemented) | 2620 |
| Giac [A] (verification not implemented) | 2620 |
| Mupad [B] (verification not implemented) | 2621 |
| Reduce [B] (verification not implemented) | 2621 |

Optimal result

Integrand size = 30, antiderivative size = 204

$$\int \frac{(a^2 + 2abx + b^2x^2)^{3/2}}{\sqrt{d + ex}} dx = -\frac{2(bd - ae)^3\sqrt{d + ex}\sqrt{a^2 + 2abx + b^2x^2}}{e^4(a + bx)} + \frac{2b(bd - ae)^2(d + ex)^{3/2}\sqrt{a^2 + 2abx + b^2x^2}}{e^4(a + bx)} - \frac{6b^2(bd - ae)(d + ex)^{5/2}\sqrt{a^2 + 2abx + b^2x^2}}{5e^4(a + bx)} + \frac{2b^3(d + ex)^{7/2}\sqrt{a^2 + 2abx + b^2x^2}}{7e^4(a + bx)}$$

output

```
-2*(-a*e+b*d)^3*(e*x+d)^(1/2)*((b*x+a)^2)^(1/2)/e^4/(b*x+a)+2*b*(-a*e+b*d)
^2*(e*x+d)^(3/2)*((b*x+a)^2)^(1/2)/e^4/(b*x+a)-6/5*b^2*(-a*e+b*d)*(e*x+d)
(5/2)*((b*x+a)^2)^(1/2)/e^4/(b*x+a)+2/7*b^3*(e*x+d)^(7/2)*((b*x+a)^2)^(1/2)
)/e^4/(b*x+a)
```

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 119, normalized size of antiderivative = 0.58

$$\int \frac{(a^2 + 2abx + b^2x^2)^{3/2}}{\sqrt{d + ex}} dx = \frac{2\sqrt{(a + bx)^2}\sqrt{d + ex}(35a^3e^3 + 35a^2be^2(-2d + ex) + 7ab^2e(8d^2 - 4dex + 3d^2) + b^3(-16d^3 + 8d^2ex - 6de^2x^2 + 5e^3x^3))}{35e^4(a + bx)}$$

input `Integrate[(a^2 + 2*a*b*x + b^2*x^2)^(3/2)/Sqrt[d + e*x],x]`

output $(2\sqrt{(a + bx)^2}\sqrt{d + ex}(35a^3e^3 + 35a^2be^2(-2d + ex) + 7a^2b^2e(8d^2 - 4dex + 3e^2x^2) + b^3(-16d^3 + 8d^2ex - 6de^2x^2 + 5e^3x^3)))/(35e^4(a + bx))$

Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 124, normalized size of antiderivative = 0.61, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {1102, 27, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\begin{aligned} & \int \frac{(a^2 + 2abx + b^2x^2)^{3/2}}{\sqrt{d + ex}} dx \\ & \quad \downarrow \text{1102} \\ & \frac{\sqrt{a^2 + 2abx + b^2x^2} \int \frac{b^3(a+bx)^3}{\sqrt{d+ex}} dx}{b^3(a + bx)} \\ & \quad \downarrow \text{27} \\ & \frac{\sqrt{a^2 + 2abx + b^2x^2} \int \frac{(a+bx)^3}{\sqrt{d+ex}} dx}{a + bx} \\ & \quad \downarrow \text{53} \\ & \frac{\sqrt{a^2 + 2abx + b^2x^2} \int \left(\frac{(d+ex)^{5/2}b^3}{e^3} - \frac{3(bd-ae)(d+ex)^{3/2}b^2}{e^3} + \frac{3(bd-ae)^2\sqrt{d+ex}b}{e^3} + \frac{(ae-bd)^3}{e^3\sqrt{d+ex}} \right) dx}{a + bx} \end{aligned}$$

$$\frac{\sqrt{a^2 + 2abx + b^2x^2} \left(-\frac{6b^2(d+ex)^{5/2}(bd-ae)}{5e^4} + \frac{2b(d+ex)^{3/2}(bd-ae)^2}{e^4} - \frac{2\sqrt{d+ex}(bd-ae)^3}{e^4} + \frac{2b^3(d+ex)^{7/2}}{7e^4} \right)}{a + bx}$$

↓ 2009

input `Int[(a^2 + 2*a*b*x + b^2*x^2)^(3/2)/Sqrt[d + e*x],x]`

output `(Sqrt[a^2 + 2*a*b*x + b^2*x^2]*((-2*(b*d - a*e)^3*Sqrt[d + e*x])/e^4 + (2*b*(b*d - a*e)^2*(d + e*x)^(3/2))/e^4 - (6*b^2*(b*d - a*e)*(d + e*x)^(5/2))/(5*e^4) + (2*b^3*(d + e*x)^(7/2))/(7*e^4)))/(a + b*x)`

Defintions of rubi rules used

rule 27 `Int[(a_)*(F_x_), x_Symbol] := Simp[a Int[F_x, x], x] /; FreeQ[a, x] && !MatchQ[F_x, (b_)*(G_x_)] /; FreeQ[b, x]`

rule 53 `Int[((a_.) + (b_.)*(x_.))^(m_.)*((c_.) + (d_.)*(x_.))^(n_.), x_Symbol] := Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && IGtQ[m, 0] && (!IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])`

rule 1102 `Int[((d_.) + (e_.)*(x_.))^(m_.)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] := Simp[(a + b*x + c*x^2)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x)^(2*FracPart[p])) Int[(d + e*x)^m*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && EqQ[b^2 - 4*a*c, 0]`

rule 2009 `Int[u_, x_Symbol] := Simp[IntSum[u, x], x] /; SumQ[u]`

Maple [A] (verified)

Time = 1.16 (sec) , antiderivative size = 132, normalized size of antiderivative = 0.65

| method | result |
|---------|---|
| gospers | $\frac{2\sqrt{ex+d} (5e^3x^3b^3+21x^2ab^2e^3-6x^2b^3de^2+35a^2be^3x-28xab^2de^2+8b^3d^2ex+35e^3a^3-70a^2bde^2+56ab^2d^2e-16b^3d^3) ((bx+a)^2)}{35e^4(bx+a)^3}$ |
| default | $\frac{2\sqrt{ex+d} (5e^3x^3b^3+21x^2ab^2e^3-6x^2b^3de^2+35a^2be^3x-28xab^2de^2+8b^3d^2ex+35e^3a^3-70a^2bde^2+56ab^2d^2e-16b^3d^3) ((bx+a)^2)}{35e^4(bx+a)^3}$ |
| risch | $\frac{2\sqrt{(bx+a)^2} (5e^3x^3b^3+21x^2ab^2e^3-6x^2b^3de^2+35a^2be^3x-28xab^2de^2+8b^3d^2ex+35e^3a^3-70a^2bde^2+56ab^2d^2e-16b^3d^3) \sqrt{ex+d}}{35(bx+a)e^4}$ |
| orering | $\frac{2(5e^3x^3b^3+21x^2ab^2e^3-6x^2b^3de^2+35a^2be^3x-28xab^2de^2+8b^3d^2ex+35e^3a^3-70a^2bde^2+56ab^2d^2e-16b^3d^3) \sqrt{ex+d} (b^2x^2+2abx+a^2)}{35e^4(bx+a)^3}$ |

input `int((b^2*x^2+2*a*b*x+a^2)^(3/2)/(e*x+d)^(1/2),x,method=_RETURNVERBOSE)`

output
$$\frac{2}{35}*(e*x+d)^{(1/2)}*(5*b^3*e^3*x^3+21*a*b^2*e^3*x^2-6*b^3*d*e^2*x+35*a^2*b*e^3*x-28*a*b^2*d*e^2*x+8*b^3*d^2*e*x+35*a^3*e^3-70*a^2*b*d*e^2+56*a*b^2*d^2*e-16*b^3*d^3)*((b*x+a)^2)^{(3/2)}/e^4/(b*x+a)^3$$

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 115, normalized size of antiderivative = 0.56

$$\int \frac{(a^2 + 2abx + b^2x^2)^{3/2}}{\sqrt{d + ex}} dx = \frac{2(5b^3e^3x^3 - 16b^3d^3 + 56ab^2d^2e - 70a^2bde^2 + 35a^3e^3 - 3(2b^3de^2 - 7ab^2de - 3a^2b^2d^2e^2 + 35a^2b^2d^2e^2 - 16b^3d^3)) \sqrt{d + ex}}{35e^4}$$

input `integrate((b^2*x^2+2*a*b*x+a^2)^(3/2)/(e*x+d)^(1/2),x, algorithm="fricas")`

output
$$\frac{2}{35}*(5*b^3*e^3*x^3 - 16*b^3*d^3 + 56*a*b^2*d^2*e - 70*a^2*b*d*e^2 + 35*a^3*e^3 - 3*(2*b^3*d*e^2 - 7*a*b^2*d^2*e^2 + 35*a^2*b^2*d^2*e^2 - 16*b^3*d^3)*x)*\sqrt{e*x + d}/e^4$$

Sympy [F]

$$\int \frac{(a^2 + 2abx + b^2x^2)^{3/2}}{\sqrt{d + ex}} dx = \int \frac{((a + bx)^2)^{3/2}}{\sqrt{d + ex}} dx$$

input `integrate((b**2*x**2+2*a*b*x+a**2)**(3/2)/(e*x+d)**(1/2),x)`

output `Integral(((a + b*x)**2)**(3/2)/sqrt(d + e*x), x)`

Maxima [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 164, normalized size of antiderivative = 0.80

$$\int \frac{(a^2 + 2abx + b^2x^2)^{3/2}}{\sqrt{d + ex}} dx = \frac{2(5b^3e^4x^4 - 16b^3d^4 + 56ab^2d^3e - 70a^2bd^2e^2 + 35a^3de^3 - (b^3de^3 - 21ab^2e^3))}{\sqrt{d + ex}}$$

input `integrate((b^2*x^2+2*a*b*x+a^2)^(3/2)/(e*x+d)^(1/2),x, algorithm="maxima")`

output `2/35*(5*b^3*e^4*x^4 - 16*b^3*d^4 + 56*a*b^2*d^3*e - 70*a^2*b*d^2*e^2 + 35*a^3*d*e^3 - (b^3*d*e^3 - 21*a*b^2*e^4)*x^3 + (2*b^3*d^2*e^2 - 7*a*b^2*d*e^3 + 35*a^2*b*e^4)*x^2 - (8*b^3*d^3*e - 28*a*b^2*d^2*e^2 + 35*a^2*b*d*e^3 - 35*a^3*e^4)*x)/(sqrt(e*x + d)*e^4)`

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 161, normalized size of antiderivative = 0.79

$$\int \frac{(a^2 + 2abx + b^2x^2)^{3/2}}{\sqrt{d + ex}} dx = \frac{2 \left(35 \sqrt{ex + d} a^3 \operatorname{sgn}(bx + a) + \frac{35 \left((ex+d)^{\frac{3}{2}} - 3 \sqrt{ex+d} d \right) a^2 b \operatorname{sgn}(bx+a)}{e} + \frac{7 \left(3 (ex+d)^{\frac{5}{2}} - 15 (ex+d) d \right) a b^2 \operatorname{sgn}(bx+a)}{e} \right)}{\sqrt{d + ex}}$$

input `integrate((b^2*x^2+2*a*b*x+a^2)^(3/2)/(e*x+d)^(1/2),x, algorithm="giac")`

output

```
2/35*(35*sqrt(e*x + d)*a^3*sgn(b*x + a) + 35*((e*x + d)^(3/2) - 3*sqrt(e*x
+ d)*d)*a^2*b*sgn(b*x + a)/e + 7*(3*(e*x + d)^(5/2) - 10*(e*x + d)^(3/2)*
d + 15*sqrt(e*x + d)*d^2)*a*b^2*sgn(b*x + a)/e^2 + (5*(e*x + d)^(7/2) - 21
*(e*x + d)^(5/2)*d + 35*(e*x + d)^(3/2)*d^2 - 35*sqrt(e*x + d)*d^3)*b^3*sg
n(b*x + a)/e^3)/e
```

Mupad [B] (verification not implemented)

Time = 5.40 (sec) , antiderivative size = 198, normalized size of antiderivative = 0.97

$$\int \frac{(a^2 + 2abx + b^2x^2)^{3/2}}{\sqrt{d + ex}} dx = \frac{\sqrt{a^2 + 2abx + b^2x^2} \left(\frac{2b^2x^4}{7} + \frac{2x^2(35a^2e^2 - 7abde + 2b^2d^2)}{35e^2} - \frac{-70a^3de^3 + 140a^2bd^2e}{35e^4} \right) - \frac{-70a^3de^3 + 140a^2bd^2e}{35e^4}}{x\sqrt{d + ex}}$$

input

```
int((a^2 + b^2*x^2 + 2*a*b*x)^(3/2)/(d + e*x)^(1/2), x)
```

output

```
((a^2 + b^2*x^2 + 2*a*b*x)^(1/2)*((2*b^2*x^4)/7 + (2*x^2*(35*a^2*e^2 + 2*b
^2*d^2 - 7*a*b*d*e))/(35*e^2) - (32*b^3*d^4 - 70*a^3*d*e^3 + 140*a^2*b*d^2
*e^2 - 112*a*b^2*d^3*e)/(35*b*e^4) + (2*b*x^3*(21*a*e - b*d))/(35*e) + (x*
(70*a^3*e^4 - 16*b^3*d^3*e + 56*a*b^2*d^2*e^2 - 70*a^2*b*d*e^3))/(35*b*e^4
)))/(x*(d + e*x)^(1/2) + (a*(d + e*x)^(1/2))/b)
```

Reduce [B] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 114, normalized size of antiderivative = 0.56

$$\int \frac{(a^2 + 2abx + b^2x^2)^{3/2}}{\sqrt{d + ex}} dx = \frac{2\sqrt{ex + d}(5b^3e^3x^3 + 21ab^2e^3x^2 - 6b^3de^2x^2 + 35a^2be^3x - 28ab^2de^2x + 8b^3de^2)}{35e^4}$$

input

```
int((b^2*x^2+2*a*b*x+a^2)^(3/2)/(e*x+d)^(1/2), x)
```

output

```
(2*sqrt(d + e*x)*(35*a**3*e**3 - 70*a**2*b*d*e**2 + 35*a**2*b*e**3*x + 56*
a*b**2*d**2*e - 28*a*b**2*d*e**2*x + 21*a*b**2*e**3*x**2 - 16*b**3*d**3 +
8*b**3*d**2*e*x - 6*b**3*d*e**2*x**2 + 5*b**3*e**3*x**3))/(35*e**4)
```

3.355 $\int \frac{(a^2+2abx+b^2x^2)^{3/2}}{(d+ex)^{3/2}} dx$

| | |
|---|------|
| Optimal result | 2622 |
| Mathematica [A] (verified) | 2623 |
| Rubi [A] (verified) | 2623 |
| Maple [A] (verified) | 2625 |
| Fricas [A] (verification not implemented) | 2625 |
| Sympy [F] | 2626 |
| Maxima [A] (verification not implemented) | 2626 |
| Giac [A] (verification not implemented) | 2626 |
| Mupad [B] (verification not implemented) | 2627 |
| Reduce [B] (verification not implemented) | 2627 |

Optimal result

Integrand size = 30, antiderivative size = 202

$$\int \frac{(a^2 + 2abx + b^2x^2)^{3/2}}{(d + ex)^{3/2}} dx = \frac{2(bd - ae)^3 \sqrt{a^2 + 2abx + b^2x^2}}{e^4(a + bx)\sqrt{d + ex}} + \frac{6b(bd - ae)^2 \sqrt{d + ex} \sqrt{a^2 + 2abx + b^2x^2}}{e^4(a + bx)} - \frac{2b^2(bd - ae)(d + ex)^{3/2} \sqrt{a^2 + 2abx + b^2x^2}}{e^4(a + bx)} + \frac{2b^3(d + ex)^{5/2} \sqrt{a^2 + 2abx + b^2x^2}}{5e^4(a + bx)}$$

output

```
2*(-a*e+b*d)^3*((b*x+a)^2)^(1/2)/e^4/(b*x+a)/(e*x+d)^(1/2)+6*b*(-a*e+b*d)^2*(e*x+d)^(1/2)*((b*x+a)^2)^(1/2)/e^4/(b*x+a)-2*b^2*(-a*e+b*d)*(e*x+d)^(3/2)*((b*x+a)^2)^(1/2)/e^4/(b*x+a)+2/5*b^3*(e*x+d)^(5/2)*((b*x+a)^2)^(1/2)/e^4/(b*x+a)
```

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 119, normalized size of antiderivative = 0.59

$$\int \frac{(a^2 + 2abx + b^2x^2)^{3/2}}{(d + ex)^{3/2}} dx = \frac{2\sqrt{(a + bx)^2(5a^3e^3 - 15a^2be^2(2d + ex) + 5ab^2e(8d^2 + 4dex - e^2x^2) - b^3(16d^3 + 8d^2ex - 2de^2x^2 + e^3x^3))}}{5e^4(a + bx)\sqrt{d + ex}}$$

input

```
Integrate[(a^2 + 2*a*b*x + b^2*x^2)^(3/2)/(d + e*x)^(3/2), x]
```

output

```
(-2*Sqrt[(a + b*x)^2]*(5*a^3*e^3 - 15*a^2*b*e^2*(2*d + e*x) + 5*a*b^2*e*(8*d^2 + 4*d*e*x - e^2*x^2) - b^3*(16*d^3 + 8*d^2*e*x - 2*d*e^2*x^2 + e^3*x^3)))/(5*e^4*(a + b*x)*Sqrt[d + e*x])
```

Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 122, normalized size of antiderivative = 0.60, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {1102, 27, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\begin{aligned} & \int \frac{(a^2 + 2abx + b^2x^2)^{3/2}}{(d + ex)^{3/2}} dx \\ & \quad \downarrow \text{1102} \\ & \frac{\sqrt{a^2 + 2abx + b^2x^2} \int \frac{b^3(a+bx)^3}{(d+ex)^{3/2}} dx}{b^3(a + bx)} \\ & \quad \downarrow \text{27} \\ & \frac{\sqrt{a^2 + 2abx + b^2x^2} \int \frac{(a+bx)^3}{(d+ex)^{3/2}} dx}{a + bx} \\ & \quad \downarrow \text{53} \end{aligned}$$

$$\frac{\sqrt{a^2 + 2abx + b^2x^2} \int \left(\frac{(d+ex)^{3/2}b^3}{e^3} - \frac{3(bd-ae)\sqrt{d+ex}b^2}{e^3} + \frac{3(bd-ae)^2b}{e^3\sqrt{d+ex}} + \frac{(ae-bd)^3}{e^3(d+ex)^{3/2}} \right) dx}{a + bx}$$

↓ 2009

$$\frac{\sqrt{a^2 + 2abx + b^2x^2} \left(-\frac{2b^2(d+ex)^{3/2}(bd-ae)}{e^4} + \frac{6b\sqrt{d+ex}(bd-ae)^2}{e^4} + \frac{2(bd-ae)^3}{e^4\sqrt{d+ex}} + \frac{2b^3(d+ex)^{5/2}}{5e^4} \right)}{a + bx}$$

input `Int[(a^2 + 2*a*b*x + b^2*x^2)^(3/2)/(d + e*x)^(3/2),x]`

output `(Sqrt[a^2 + 2*a*b*x + b^2*x^2]*((2*(b*d - a*e)^3)/(e^4*Sqrt[d + e*x]) + (6*b*(b*d - a*e)^2*Sqrt[d + e*x])/e^4 - (2*b^2*(b*d - a*e)*(d + e*x)^(3/2))/e^4 + (2*b^3*(d + e*x)^(5/2))/(5*e^4)))/(a + b*x)`

Defintions of rubi rules used

rule 27 `Int[(a_)*(Fx_), x_Symbol] := Simp[a Int[Fx, x], x] /; FreeQ[a, x] && !MatchQ[Fx, (b_)*(Gx_)] /; FreeQ[b, x]`

rule 53 `Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] := Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && IGtQ[m, 0] && (!IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])`

rule 1102 `Int[((d_.) + (e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] := Simp[(a + b*x + c*x^2)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x)^(2*FracPart[p])) Int[(d + e*x)^m*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && EqQ[b^2 - 4*a*c, 0]`

rule 2009 `Int[u_, x_Symbol] := Simp[IntSum[u, x], x] /; SumQ[u]`

Maple [A] (verified)

Time = 1.14 (sec) , antiderivative size = 132, normalized size of antiderivative = 0.65

| method | result |
|---------|--|
| gospers | $-\frac{2(-e^3x^3b^3-5x^2ab^2e^3+2x^2b^3de^2-15a^2be^3x+20xab^2de^2-8b^3d^2ex+5e^3a^3-30a^2bde^2+40ab^2d^2e-16b^3d^3)((bx+a)^2)^{\frac{3}{2}}}{5\sqrt{ex+d}e^4(bx+a)^3}$ |
| default | $-\frac{2(-e^3x^3b^3-5x^2ab^2e^3+2x^2b^3de^2-15a^2be^3x+20xab^2de^2-8b^3d^2ex+5e^3a^3-30a^2bde^2+40ab^2d^2e-16b^3d^3)((bx+a)^2)^{\frac{3}{2}}}{5\sqrt{ex+d}e^4(bx+a)^3}$ |
| orering | $-\frac{2(-e^3x^3b^3-5x^2ab^2e^3+2x^2b^3de^2-15a^2be^3x+20xab^2de^2-8b^3d^2ex+5e^3a^3-30a^2bde^2+40ab^2d^2e-16b^3d^3)(b^2x^2+2abx+a^2)^{\frac{3}{2}}}{5e^4(bx+a)^3\sqrt{ex+d}}$ |
| risch | $\frac{2b(b^2e^2x^2+5xab^2e^2-3b^2dex+15e^2a^2-25abde+11b^2d^2)\sqrt{ex+d}\sqrt{(bx+a)^2}}{5e^4(bx+a)} - \frac{2(e^3a^3-3a^2bde^2+3ab^2d^2e-b^3d^3)\sqrt{(bx+a)^2}}{e^4\sqrt{ex+d}(bx+a)}$ |

input

```
int((b^2*x^2+2*a*b*x+a^2)^(3/2)/(e*x+d)^(3/2),x,method=_RETURNVERBOSE)
```

output

```
-2/5/(e*x+d)^(1/2)*(-b^3*e^3*x^3-5*a*b^2*e^3*x^2+2*b^3*d*e^2*x^2-15*a^2*b*
e^3*x+20*a*b^2*d*e^2*x-8*b^3*d^2*e*x+5*a^3*e^3-30*a^2*b*d*e^2+40*a*b^2*d^2
*e-16*b^3*d^3)*((b*x+a)^2)^(3/2)/e^4/(b*x+a)^3
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 124, normalized size of antiderivative = 0.61

$$\int \frac{(a^2 + 2abx + b^2x^2)^{3/2}}{(d + ex)^{3/2}} dx = \frac{2(b^3e^3x^3 + 16b^3d^3 - 40ab^2d^2e + 30a^2bde^2 - 5a^3e^3 - (2b^3de^2 - 5ab^2e^3)x^2}{5(e^5x + de^4)}$$

input

```
integrate((b^2*x^2+2*a*b*x+a^2)^(3/2)/(e*x+d)^(3/2),x, algorithm="fricas")
```

output

```
2/5*(b^3*e^3*x^3 + 16*b^3*d^3 - 40*a*b^2*d^2*e + 30*a^2*b*d*e^2 - 5*a^3*e^
3 - (2*b^3*d*e^2 - 5*a*b^2*e^3)*x^2 + (8*b^3*d^2*e - 20*a*b^2*d*e^2 + 15*a
^2*b*e^3)*x)*sqrt(e*x + d)/(e^5*x + d*e^4)
```

Sympy [F]

$$\int \frac{(a^2 + 2abx + b^2x^2)^{3/2}}{(d + ex)^{3/2}} dx = \int \frac{((a + bx)^2)^{\frac{3}{2}}}{(d + ex)^{\frac{3}{2}}} dx$$

input `integrate((b**2*x**2+2*a*b*x+a**2)**(3/2)/(e*x+d)**(3/2),x)`

output `Integral(((a + b*x)**2)**(3/2)/(d + e*x)**(3/2), x)`

Maxima [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 114, normalized size of antiderivative = 0.56

$$\int \frac{(a^2 + 2abx + b^2x^2)^{3/2}}{(d + ex)^{3/2}} dx = \frac{2(b^3e^3x^3 + 16b^3d^3 - 40ab^2d^2e + 30a^2bde^2 - 5a^3e^3 - (2b^3de^2 - 5ab^2e^3)x^2}{5\sqrt{ex + d}e^4}$$

input `integrate((b^2*x^2+2*a*b*x+a^2)^(3/2)/(e*x+d)^(3/2),x, algorithm="maxima")`

output `2/5*(b^3*e^3*x^3 + 16*b^3*d^3 - 40*a*b^2*d^2*e + 30*a^2*b*d*e^2 - 5*a^3*e^3 - (2*b^3*d*e^2 - 5*a*b^2*e^3)*x^2 + (8*b^3*d^2*e - 20*a*b^2*d*e^2 + 15*a^2*b*e^3)*x)/(sqrt(e*x + d)*e^4)`

Giac [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 212, normalized size of antiderivative = 1.05

$$\int \frac{(a^2 + 2abx + b^2x^2)^{3/2}}{(d + ex)^{3/2}} dx = \frac{2(b^3d^3\operatorname{sgn}(bx + a) - 3ab^2d^2e\operatorname{sgn}(bx + a) + 3a^2bde^2\operatorname{sgn}(bx + a) - a^3e^3\operatorname{sgn}(bx + a))}{\sqrt{ex + d}e^4} + \frac{2\left((ex + d)^{\frac{5}{2}}b^3e^{16}\operatorname{sgn}(bx + a) - 5(ex + d)^{\frac{3}{2}}b^3de^{16}\operatorname{sgn}(bx + a) + 15\sqrt{ex + d}b^3d^2e^{16}\operatorname{sgn}(bx + a) + 5(ex + d)^{\frac{1}{2}}b^3d^3e^{16}\operatorname{sgn}(bx + a) - 15ab^2d^2e^{16}\operatorname{sgn}(bx + a) + 15a^2bde^{16}\operatorname{sgn}(bx + a) - 5a^3e^{16}\operatorname{sgn}(bx + a)\right)}{5e^{20}}$$

input `integrate((b^2*x^2+2*a*b*x+a^2)^(3/2)/(e*x+d)^(3/2),x, algorithm="giac")`

output

```
2*(b^3*d^3*sgn(b*x + a) - 3*a*b^2*d^2*e*sgn(b*x + a) + 3*a^2*b*d*e^2*sgn(b*x + a) - a^3*e^3*sgn(b*x + a))/(sqrt(e*x + d)*e^4) + 2/5*((e*x + d)^(5/2)*b^3*e^16*sgn(b*x + a) - 5*(e*x + d)^(3/2)*b^3*d*e^16*sgn(b*x + a) + 15*sqrt(e*x + d)*b^3*d^2*e^16*sgn(b*x + a) + 5*(e*x + d)^(3/2)*a*b^2*e^17*sgn(b*x + a) - 30*sqrt(e*x + d)*a*b^2*d*e^17*sgn(b*x + a) + 15*sqrt(e*x + d)*a^2*b*e^18*sgn(b*x + a))/e^20
```

Mupad [B] (verification not implemented)

Time = 5.53 (sec) , antiderivative size = 147, normalized size of antiderivative = 0.73

$$\int \frac{(a^2 + 2abx + b^2x^2)^{3/2}}{(d + ex)^{3/2}} dx = \frac{\sqrt{a^2 + 2abx + b^2x^2} \left(\frac{2x(15a^2e^2 - 20abde + 8b^2d^2)}{5e^3} - \frac{2a^3e^3 - 12a^2bde^2 + 16ab^2d^2e - 3b^3d^3}{b^4} \right)}{x\sqrt{d + ex} + \frac{a\sqrt{d+ex}}{b}}$$

input

```
int((a^2 + b^2*x^2 + 2*a*b*x)^(3/2)/(d + e*x)^(3/2),x)
```

output

```
((a^2 + b^2*x^2 + 2*a*b*x)^(1/2)*((2*x*(15*a^2*e^2 + 8*b^2*d^2 - 20*a*b*d*e))/(5*e^3) - (2*a^3*e^3 - (32*b^3*d^3)/5 + 16*a*b^2*d^2*e - 12*a^2*b*d*e^2)/(b*e^4) + (2*b^2*x^3)/(5*e) + (2*b*x^2*(5*a*e - 2*b*d))/(5*e^2)))/(x*(d + e*x)^(1/2) + (a*(d + e*x)^(1/2))/b)
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 115, normalized size of antiderivative = 0.57

$$\int \frac{(a^2 + 2abx + b^2x^2)^{3/2}}{(d + ex)^{3/2}} dx = \frac{\frac{2}{5}b^3e^3x^3 + 2ab^2e^3x^2 - \frac{4}{5}b^3de^2x^2 + 6a^2be^3x - 8ab^2de^2x + \frac{16}{5}b^3d^2ex - 2a^3e^3}{\sqrt{ex + d}e^4}$$

input

```
int((b^2*x^2+2*a*b*x+a^2)^(3/2)/(e*x+d)^(3/2),x)
```

output

```
(2*(- 5*a**3*e**3 + 30*a**2*b*d*e**2 + 15*a**2*b*e**3*x - 40*a*b**2*d**2*e - 20*a*b**2*d*e**2*x + 5*a*b**2*e**3*x**2 + 16*b**3*d**3 + 8*b**3*d**2*e*x - 2*b**3*d*e**2*x**2 + b**3*e**3*x**3))/(5*sqrt(d + e*x)*e**4)
```


3.356 $\int \frac{(a^2+2abx+b^2x^2)^{3/2}}{(d+ex)^{5/2}} dx$

| | |
|---|------|
| Optimal result | 2628 |
| Mathematica [A] (verified) | 2629 |
| Rubi [A] (verified) | 2629 |
| Maple [A] (verified) | 2631 |
| Fricas [A] (verification not implemented) | 2631 |
| Sympy [F] | 2632 |
| Maxima [A] (verification not implemented) | 2632 |
| Giac [A] (verification not implemented) | 2632 |
| Mupad [B] (verification not implemented) | 2633 |
| Reduce [B] (verification not implemented) | 2633 |

Optimal result

Integrand size = 30, antiderivative size = 204

$$\int \frac{(a^2 + 2abx + b^2x^2)^{3/2}}{(d + ex)^{5/2}} dx = \frac{2(bd - ae)^3 \sqrt{a^2 + 2abx + b^2x^2}}{3e^4(a + bx)(d + ex)^{3/2}} - \frac{6b(bd - ae)^2 \sqrt{a^2 + 2abx + b^2x^2}}{e^4(a + bx)\sqrt{d + ex}} - \frac{6b^2(bd - ae)\sqrt{d + ex}\sqrt{a^2 + 2abx + b^2x^2}}{e^4(a + bx)} + \frac{2b^3(d + ex)^{3/2}\sqrt{a^2 + 2abx + b^2x^2}}{3e^4(a + bx)}$$

output

```
2/3*(-a*e+b*d)^3*((b*x+a)^2)^(1/2)/e^4/(b*x+a)/(e*x+d)^(3/2)-6*b*(-a*e+b*d)
)^2*((b*x+a)^2)^(1/2)/e^4/(b*x+a)/(e*x+d)^(1/2)-6*b^2*(-a*e+b*d)*(e*x+d)^(
1/2)*((b*x+a)^2)^(1/2)/e^4/(b*x+a)+2/3*b^3*(e*x+d)^(3/2)*((b*x+a)^2)^(1/2)
/e^4/(b*x+a)
```

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 119, normalized size of antiderivative = 0.58

$$\int \frac{(a^2 + 2abx + b^2x^2)^{3/2}}{(d + ex)^{5/2}} dx = \frac{2\sqrt{(a + bx)^2(a^3e^3 + 3a^2be^2(2d + 3ex) - 3ab^2e(8d^2 + 12dex + 3e^2x^2) + b^3(16d^3 + 24d^2ex + 6de^2x^2 - e^3x^3))}}{3e^4(a + bx)(d + ex)^{3/2}}$$

input

```
Integrate[(a^2 + 2*a*b*x + b^2*x^2)^(3/2)/(d + e*x)^(5/2), x]
```

output

```
(-2*Sqrt[(a + b*x)^2]*(a^3*e^3 + 3*a^2*b*e^2*(2*d + 3*e*x) - 3*a*b^2*e*(8*d^2 + 12*d*e*x + 3*e^2*x^2) + b^3*(16*d^3 + 24*d^2*e*x + 6*d*e^2*x^2 - e^3*x^3)))/(3*e^4*(a + b*x)*(d + e*x)^(3/2))
```

Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 124, normalized size of antiderivative = 0.61, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {1102, 27, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\begin{aligned} & \int \frac{(a^2 + 2abx + b^2x^2)^{3/2}}{(d + ex)^{5/2}} dx \\ & \quad \downarrow \text{1102} \\ & \frac{\sqrt{a^2 + 2abx + b^2x^2} \int \frac{b^3(a+bx)^3}{(d+ex)^{5/2}} dx}{b^3(a + bx)} \\ & \quad \downarrow \text{27} \\ & \frac{\sqrt{a^2 + 2abx + b^2x^2} \int \frac{(a+bx)^3}{(d+ex)^{5/2}} dx}{a + bx} \\ & \quad \downarrow \text{53} \end{aligned}$$

$$\frac{\sqrt{a^2 + 2abx + b^2x^2} \int \left(\frac{\sqrt{d+ex}^3}{e^3} - \frac{3(bd-ae)b^2}{e^3\sqrt{d+ex}} + \frac{3(bd-ae)^2b}{e^3(d+ex)^{3/2}} + \frac{(ae-bd)^3}{e^3(d+ex)^{5/2}} \right) dx}{a + bx}$$

↓ 2009

$$\frac{\sqrt{a^2 + 2abx + b^2x^2} \left(-\frac{6b^2\sqrt{d+ex}(bd-ae)}{e^4} - \frac{6b(bd-ae)^2}{e^4\sqrt{d+ex}} + \frac{2(bd-ae)^3}{3e^4(d+ex)^{3/2}} + \frac{2b^3(d+ex)^{3/2}}{3e^4} \right)}{a + bx}$$

input `Int[(a^2 + 2*a*b*x + b^2*x^2)^(3/2)/(d + e*x)^(5/2),x]`

output `(Sqrt[a^2 + 2*a*b*x + b^2*x^2]*((2*(b*d - a*e)^3)/(3*e^4*(d + e*x)^(3/2)) - (6*b*(b*d - a*e)^2)/(e^4*Sqrt[d + e*x]) - (6*b^2*(b*d - a*e)*Sqrt[d + e*x])/e^4 + (2*b^3*(d + e*x)^(3/2))/(3*e^4)))/(a + b*x)`

Defintions of rubi rules used

rule 27 `Int[(a_)*(Fx_), x_Symbol] := Simp[a Int[Fx, x], x] /; FreeQ[a, x] && !MatchQ[Fx, (b_)*(Gx_)] /; FreeQ[b, x]`

rule 53 `Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] := Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && IGtQ[m, 0] && (!IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])`

rule 1102 `Int[((d_.) + (e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] := Simp[(a + b*x + c*x^2)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x)^(2*FracPart[p])) Int[(d + e*x)^m*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && EqQ[b^2 - 4*a*c, 0]`

rule 2009 `Int[u_, x_Symbol] := Simp[IntSum[u, x], x] /; SumQ[u]`

Maple [A] (verified)

Time = 1.13 (sec) , antiderivative size = 108, normalized size of antiderivative = 0.53

| method | result |
|---------|---|
| risch | $\frac{2b^2(bex+9ae-8bd)\sqrt{ex+d}\sqrt{(bx+a)^2}}{3e^4(bx+a)} - \frac{2(9bex+ae+8bd)(e^2a^2-2abde+b^2d^2)\sqrt{(bx+a)^2}}{3e^4(ex+d)^{\frac{3}{2}}(bx+a)}$ |
| gospers | $-\frac{2(-e^3x^3b^3-9x^2ab^2e^3+6x^2b^3de^2+9a^2be^3x-36xa b^2de^2+24b^3d^2ex+e^3a^3+6a^2bde^2-24ab^2d^2e+16b^3d^3)((bx+a)^2)^{\frac{3}{2}}}{3(ex+d)^{\frac{3}{2}}e^4(bx+a)^3}$ |
| default | $-\frac{2(-e^3x^3b^3-9x^2ab^2e^3+6x^2b^3de^2+9a^2be^3x-36xa b^2de^2+24b^3d^2ex+e^3a^3+6a^2bde^2-24ab^2d^2e+16b^3d^3)((bx+a)^2)^{\frac{3}{2}}}{3(ex+d)^{\frac{3}{2}}e^4(bx+a)^3}$ |
| orering | $-\frac{2(-e^3x^3b^3-9x^2ab^2e^3+6x^2b^3de^2+9a^2be^3x-36xa b^2de^2+24b^3d^2ex+e^3a^3+6a^2bde^2-24ab^2d^2e+16b^3d^3)(b^2x^2+2abx+a^2)^{\frac{3}{2}}}{3e^4(bx+a)^3(ex+d)^{\frac{3}{2}}}$ |

input `int((b^2*x^2+2*a*b*x+a^2)^(3/2)/(e*x+d)^(5/2),x,method=_RETURNVERBOSE)`

output
$$\frac{2}{3}b^2*(b*e*x+9*a*e-8*b*d)*(e*x+d)^(1/2)/e^4*((b*x+a)^2)^(1/2)/(b*x+a)-2/3*(9*b*e*x+a*e+8*b*d)*(a^2*e^2-2*a*b*d*e+b^2*d^2)/e^4/(e*x+d)^(3/2)*((b*x+a)^2)^(1/2)/(b*x+a)$$

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 136, normalized size of antiderivative = 0.67

$$\int \frac{(a^2 + 2abx + b^2x^2)^{3/2}}{(d + ex)^{5/2}} dx = \frac{2(b^3e^3x^3 - 16b^3d^3 + 24ab^2d^2e - 6a^2bde^2 - a^3e^3 - 3(2b^3de^2 - 3ab^2e^3)x^2 - 3(e^6x^2 + 2de^5x + d^2e^4))}{3(e^6x^2 + 2de^5x + d^2e^4)}$$

input `integrate((b^2*x^2+2*a*b*x+a^2)^(3/2)/(e*x+d)^(5/2),x, algorithm="fricas")`

output
$$\frac{2}{3}(b^3e^3x^3 - 16b^3d^3 + 24a*b^2d^2e - 6a^2*b*d*e^2 - a^3e^3 - 3*(2*b^3*d*e^2 - 3*a*b^2*e^3)*x^2 - 3*(8*b^3*d^2*e - 12*a*b^2*d*e^2 + 3*a^2*b*e^3)*x)*\sqrt{e*x + d}/(e^6*x^2 + 2*d*e^5*x + d^2*e^4)$$

Sympy [F]

$$\int \frac{(a^2 + 2abx + b^2x^2)^{3/2}}{(d + ex)^{5/2}} dx = \int \frac{((a + bx)^2)^{3/2}}{(d + ex)^{5/2}} dx$$

input `integrate((b**2*x**2+2*a*b*x+a**2)**(3/2)/(e*x+d)**(5/2),x)`

output `Integral(((a + b*x)**2)**(3/2)/(d + e*x)**(5/2), x)`

Maxima [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 125, normalized size of antiderivative = 0.61

$$\int \frac{(a^2 + 2abx + b^2x^2)^{3/2}}{(d + ex)^{5/2}} dx = \frac{2(b^3e^3x^3 - 16b^3d^3 + 24ab^2d^2e - 6a^2bde^2 - a^3e^3 - 3(2b^3de^2 - 3ab^2e^3)x^2 - 3(e^5x + de^4)\sqrt{ex + d}}{3(e^5x + de^4)\sqrt{ex + d}}$$

input `integrate((b^2*x^2+2*a*b*x+a^2)^(3/2)/(e*x+d)^(5/2),x, algorithm="maxima")`

output `2/3*(b^3*e^3*x^3 - 16*b^3*d^3 + 24*a*b^2*d^2*e - 6*a^2*b*d*e^2 - a^3*e^3 - 3*(2*b^3*d*e^2 - 3*a*b^2*e^3)*x^2 - 3*(8*b^3*d^2*e - 12*a*b^2*d*e^2 + 3*a^2*b*e^3)*x)/((e^5*x + d*e^4)*sqrt(e*x + d))`

Giac [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 201, normalized size of antiderivative = 0.99

$$\int \frac{(a^2 + 2abx + b^2x^2)^{3/2}}{(d + ex)^{5/2}} dx = \frac{2(9(ex + d)b^3d^2\operatorname{sgn}(bx + a) - b^3d^3\operatorname{sgn}(bx + a) - 18(ex + d)ab^2des\operatorname{gn}(bx + a) + 3ab^2d^2es\operatorname{gn}(bx + a) + 3(ex + d)^{3/2}e^4}{3e^{12}} + \frac{2\left((ex + d)^{3/2}b^3e^8\operatorname{sgn}(bx + a) - 9\sqrt{ex + d}b^3de^8\operatorname{sgn}(bx + a) + 9\sqrt{ex + d}ab^2e^9\operatorname{sgn}(bx + a)\right)}{3e^{12}}$$

input `integrate((b^2*x^2+2*a*b*x+a^2)^(3/2)/(e*x+d)^(5/2),x, algorithm="giac")`

output
$$-2/3*(9*(e*x + d)*b^3*d^2*\text{sgn}(b*x + a) - b^3*d^3*\text{sgn}(b*x + a) - 18*(e*x + d)*a*b^2*d*e*\text{sgn}(b*x + a) + 3*a*b^2*d^2*e*\text{sgn}(b*x + a) + 9*(e*x + d)*a^2*b*e^2*\text{sgn}(b*x + a) - 3*a^2*b*d*e^2*\text{sgn}(b*x + a) + a^3*e^3*\text{sgn}(b*x + a))/((e*x + d)^(3/2)*e^4) + 2/3*((e*x + d)^(3/2)*b^3*e^8*\text{sgn}(b*x + a) - 9*\text{sqrt}(e*x + d)*b^3*d*e^8*\text{sgn}(b*x + a) + 9*\text{sqrt}(e*x + d)*a*b^2*e^9*\text{sgn}(b*x + a))/e^12$$

Mupad [B] (verification not implemented)

Time = 5.67 (sec) , antiderivative size = 184, normalized size of antiderivative = 0.90

$$\int \frac{(a^2 + 2abx + b^2x^2)^{3/2}}{(d + ex)^{5/2}} dx = \frac{\sqrt{a^2 + 2abx + b^2x^2} \left(\frac{2a^3e^3 + 12a^2bde^2 - 48ab^2d^2e + 32b^3d^3}{3be^5} + \frac{2x(3a^2e^2 - 12abde + 8b^2d^2)}{e^4} - \frac{2b^2x^3}{3e^2} - \frac{2bx^2(3ae - 2bd)}{e^3} \right)}{x^2 \sqrt{d + ex} + \frac{ad\sqrt{d+ex}}{be} + \frac{x(3ae^5 + 3bde^4)\sqrt{d+ex}}{3be^5}}$$

input `int((a^2 + b^2*x^2 + 2*a*b*x)^(3/2)/(d + e*x)^(5/2),x)`

output
$$-((a^2 + b^2*x^2 + 2*a*b*x)^(1/2))*((2*a^3*e^3 + 32*b^3*d^3 - 48*a*b^2*d^2*e + 12*a^2*b*d*e^2)/(3*b*e^5) + (2*x*(3*a^2*e^2 + 8*b^2*d^2 - 12*a*b*d*e))/e^4 - (2*b^2*x^3)/(3*e^2) - (2*b*x^2*(3*a*e - 2*b*d))/e^3))/(x^2*(d + e*x)^(1/2) + (a*d*(d + e*x)^(1/2))/(b*e) + (x*(3*a*e^5 + 3*b*d*e^4)*(d + e*x)^(1/2))/(3*b*e^5))$$

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 122, normalized size of antiderivative = 0.60

$$\int \frac{(a^2 + 2abx + b^2x^2)^{3/2}}{(d + ex)^{5/2}} dx = \frac{\frac{2}{3}b^3e^3x^3 + 6ab^2e^3x^2 - 4b^3de^2x^2 - 6a^2be^3x + 24ab^2de^2x - 16b^3d^2ex - \frac{2}{3}a^3}{\sqrt{ex + d}e^4(ex + d)}$$

input `int((b^2*x^2+2*a*b*x+a^2)^(3/2)/(e*x+d)^(5/2),x)`

output

```
(2*( - a**3*e**3 - 6*a**2*b*d*e**2 - 9*a**2*b*e**3*x + 24*a*b**2*d**2*e +
36*a*b**2*d*e**2*x + 9*a*b**2*e**3*x**2 - 16*b**3*d**3 - 24*b**3*d**2*e*x
- 6*b**3*d*e**2*x**2 + b**3*e**3*x**3))/(3*sqrt(d + e*x)*e**4*(d + e*x))
```

3.357
$$\int \frac{(a^2+2abx+b^2x^2)^{3/2}}{(d+ex)^{7/2}} dx$$

| | |
|---|------|
| Optimal result | 2635 |
| Mathematica [A] (verified) | 2636 |
| Rubi [A] (verified) | 2636 |
| Maple [A] (verified) | 2638 |
| Fricas [A] (verification not implemented) | 2638 |
| Sympy [F] | 2639 |
| Maxima [A] (verification not implemented) | 2639 |
| Giac [A] (verification not implemented) | 2639 |
| Mupad [B] (verification not implemented) | 2640 |
| Reduce [B] (verification not implemented) | 2640 |

Optimal result

Integrand size = 30, antiderivative size = 202

$$\int \frac{(a^2 + 2abx + b^2x^2)^{3/2}}{(d + ex)^{7/2}} dx = \frac{2(bd - ae)^3 \sqrt{a^2 + 2abx + b^2x^2}}{5e^4(a + bx)(d + ex)^{5/2}} - \frac{2b(bd - ae)^2 \sqrt{a^2 + 2abx + b^2x^2}}{e^4(a + bx)(d + ex)^{3/2}} + \frac{6b^2(bd - ae) \sqrt{a^2 + 2abx + b^2x^2}}{e^4(a + bx)\sqrt{d + ex}} + \frac{2b^3 \sqrt{d + ex} \sqrt{a^2 + 2abx + b^2x^2}}{e^4(a + bx)}$$

output

```
2/5*(-a*e+b*d)^3*((b*x+a)^2)^(1/2)/e^4/(b*x+a)/(e*x+d)^(5/2)-2*b*(-a*e+b*d)
)^2*((b*x+a)^2)^(1/2)/e^4/(b*x+a)/(e*x+d)^(3/2)+6*b^2*(-a*e+b*d)*((b*x+a)
2)^(1/2)/e^4/(b*x+a)/(e*x+d)^(1/2)+2*b^3*(e*x+d)^(1/2)*((b*x+a)^2)^(1/2)/e
^4/(b*x+a)
```


Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 118, normalized size of antiderivative = 0.58

$$\int \frac{(a^2 + 2abx + b^2x^2)^{3/2}}{(d + ex)^{7/2}} dx = \frac{2\sqrt{(a + bx)^2(a^3e^3 + a^2be^2(2d + 5ex) + ab^2e(8d^2 + 20dex + 15e^2x^2) - b^3(16d^3 + 40d^2ex + 30de^2x^2 + 5e^3x^3))}}{5e^4(a + bx)(d + ex)^{5/2}}$$

input

```
Integrate[(a^2 + 2*a*b*x + b^2*x^2)^(3/2)/(d + e*x)^(7/2), x]
```

output

```
(-2*Sqrt[(a + b*x)^2]*(a^3*e^3 + a^2*b*e^2*(2*d + 5*e*x) + a*b^2*e*(8*d^2 + 20*d*e*x + 15*e^2*x^2) - b^3*(16*d^3 + 40*d^2*e*x + 30*d*e^2*x^2 + 5*e^3*x^3)))/(5*e^4*(a + b*x)*(d + e*x)^(5/2))
```

Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 122, normalized size of antiderivative = 0.60, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {1102, 27, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\begin{aligned} & \int \frac{(a^2 + 2abx + b^2x^2)^{3/2}}{(d + ex)^{7/2}} dx \\ & \quad \downarrow \text{1102} \\ & \frac{\sqrt{a^2 + 2abx + b^2x^2} \int \frac{b^3(a+bx)^3}{(d+ex)^{7/2}} dx}{b^3(a + bx)} \\ & \quad \downarrow \text{27} \\ & \frac{\sqrt{a^2 + 2abx + b^2x^2} \int \frac{(a+bx)^3}{(d+ex)^{7/2}} dx}{a + bx} \\ & \quad \downarrow \text{53} \end{aligned}$$

$$\frac{\sqrt{a^2 + 2abx + b^2x^2} \int \left(\frac{b^3}{e^3\sqrt{d+ex}} - \frac{3(bd-ae)b^2}{e^3(d+ex)^{3/2}} + \frac{3(bd-ae)^2b}{e^3(d+ex)^{5/2}} + \frac{(ae-bd)^3}{e^3(d+ex)^{7/2}} \right) dx}{a + bx}$$

↓ 2009

$$\frac{\sqrt{a^2 + 2abx + b^2x^2} \left(\frac{6b^2(bd-ae)}{e^4\sqrt{d+ex}} - \frac{2b(bd-ae)^2}{e^4(d+ex)^{3/2}} + \frac{2(bd-ae)^3}{5e^4(d+ex)^{5/2}} + \frac{2b^3\sqrt{d+ex}}{e^4} \right)}{a + bx}$$

input `Int[(a^2 + 2*a*b*x + b^2*x^2)^(3/2)/(d + e*x)^(7/2),x]`

output `(Sqrt[a^2 + 2*a*b*x + b^2*x^2]*((2*(b*d - a*e)^3)/(5*e^4*(d + e*x)^(5/2)) - (2*b*(b*d - a*e)^2)/(e^4*(d + e*x)^(3/2)) + (6*b^2*(b*d - a*e))/(e^4*Sqrt[d + e*x]) + (2*b^3*Sqrt[d + e*x])/e^4))/(a + b*x)`

Defintions of rubi rules used

rule 27 `Int[(a_)*(Fx_), x_Symbol] := Simp[a Int[Fx, x], x] /; FreeQ[a, x] && !MatchQ[Fx, (b_)*(Gx_)] /; FreeQ[b, x]`

rule 53 `Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] := Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && IGtQ[m, 0] && (!IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])`

rule 1102 `Int[((d_.) + (e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] := Simp[(a + b*x + c*x^2)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x)^(2*FracPart[p])) Int[(d + e*x)^m*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && EqQ[b^2 - 4*a*c, 0]`

rule 2009 `Int[u_, x_Symbol] := Simp[IntSum[u, x], x] /; SumQ[u]`

Maple [A] (verified)

Time = 1.13 (sec) , antiderivative size = 131, normalized size of antiderivative = 0.65

| method | result |
|---------|--|
| gospers | $-\frac{2(-5e^3x^3b^3+15x^2ab^2e^3-30x^2b^3de^2+5a^2be^3x+20xab^2de^2-40b^3d^2ex+e^3a^3+2a^2bde^2+8ab^2d^2e-16b^3d^3)((bx+a)^2)^{\frac{3}{2}}}{5(ex+d)^{\frac{5}{2}}e^4(bx+a)^3}$ |
| default | $-\frac{2(-5e^3x^3b^3+15x^2ab^2e^3-30x^2b^3de^2+5a^2be^3x+20xab^2de^2-40b^3d^2ex+e^3a^3+2a^2bde^2+8ab^2d^2e-16b^3d^3)((bx+a)^2)^{\frac{3}{2}}}{5(ex+d)^{\frac{5}{2}}e^4(bx+a)^3}$ |
| risch | $\frac{2b^3\sqrt{ex+d}\sqrt{(bx+a)^2}}{e^4(bx+a)} - \frac{2(15b^2e^2x^2+5xab^2e^2+25b^2dex+e^2a^2+3abde+11b^2d^2)(ae-bd)\sqrt{(bx+a)^2}}{5e^4\sqrt{ex+d}(e^2x^2+2dex+d^2)(bx+a)}$ |
| orering | $-\frac{2(-5e^3x^3b^3+15x^2ab^2e^3-30x^2b^3de^2+5a^2be^3x+20xab^2de^2-40b^3d^2ex+e^3a^3+2a^2bde^2+8ab^2d^2e-16b^3d^3)(b^2x^2+2abx+a^2)^{\frac{3}{2}}}{5e^4(bx+a)^3(ex+d)^{\frac{5}{2}}}$ |

input `int((b^2*x^2+2*a*b*x+a^2)^(3/2)/(e*x+d)^(7/2),x,method=_RETURNVERBOSE)`

output
$$-2/5/(e*x+d)^(5/2)*(-5*b^3*e^3*x^3+15*a*b^2*e^3*x^2-30*b^3*d*e^2*x+5*a^2*b*e^3*x+20*a*b^2*d*e^2*x-40*b^3*d^2*e*x+a^3*e^3+2*a^2*b*d*e^2+8*a*b^2*d^2*e-16*b^3*d^3)*((b*x+a)^2)^(3/2)/e^4/(b*x+a)^3$$

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 148, normalized size of antiderivative = 0.73

$$\int \frac{(a^2 + 2abx + b^2x^2)^{3/2}}{(d + ex)^{7/2}} dx = \frac{2(5b^3e^3x^3 + 16b^3d^3 - 8ab^2d^2e - 2a^2bde^2 - a^3e^3 + 15(2b^3de^2 - ab^2e^3)x^2 - 5(e^7x^3 + 3de^6x^2 + 3d^2e^5x + d^3e^4))}{5(e^7x^3 + 3de^6x^2 + 3d^2e^5x + d^3e^4)}$$

input `integrate((b^2*x^2+2*a*b*x+a^2)^(3/2)/(e*x+d)^(7/2),x, algorithm="fricas")`

output
$$2/5*(5*b^3*e^3*x^3 + 16*b^3*d^3 - 8*a*b^2*d^2*e - 2*a^2*b*d*e^2 - a^3*e^3 + 15*(2*b^3*d*e^2 - a*b^2*e^3)*x^2 + 5*(8*b^3*d^2*e - 4*a*b^2*d*e^2 - a^2*b*e^3)*x)*sqrt(e*x + d)/(e^7*x^3 + 3*d*e^6*x^2 + 3*d^2*e^5*x + d^3*e^4)$$

Sympy [F]

$$\int \frac{(a^2 + 2abx + b^2x^2)^{3/2}}{(d + ex)^{7/2}} dx = \int \frac{((a + bx)^2)^{3/2}}{(d + ex)^{7/2}} dx$$

input `integrate((b**2*x**2+2*a*b*x+a**2)**(3/2)/(e*x+d)**(7/2),x)`

output `Integral(((a + b*x)**2)**(3/2)/(d + e*x)**(7/2), x)`

Maxima [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 137, normalized size of antiderivative = 0.68

$$\int \frac{(a^2 + 2abx + b^2x^2)^{3/2}}{(d + ex)^{7/2}} dx = \frac{2(5b^3e^3x^3 + 16b^3d^3 - 8ab^2d^2e - 2a^2bde^2 - a^3e^3 + 15(2b^3de^2 - ab^2e^3)x^2 - 5(e^6x^2 + 2de^5x + d^2e^4)\sqrt{ex + d}}{5(e^6x^2 + 2de^5x + d^2e^4)\sqrt{ex + d}}$$

input `integrate((b^2*x^2+2*a*b*x+a^2)^(3/2)/(e*x+d)^(7/2),x, algorithm="maxima")`

output `2/5*(5*b^3*e^3*x^3 + 16*b^3*d^3 - 8*a*b^2*d^2*e - 2*a^2*b*d*e^2 - a^3*e^3 + 15*(2*b^3*d*e^2 - a*b^2*e^3)*x^2 + 5*(8*b^3*d^2*e - 4*a*b^2*d*e^2 - a^2*b*e^3)*x)/((e^6*x^2 + 2*d*e^5*x + d^2*e^4)*sqrt(e*x + d))`

Giac [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 191, normalized size of antiderivative = 0.95

$$\int \frac{(a^2 + 2abx + b^2x^2)^{3/2}}{(d + ex)^{7/2}} dx = \frac{2\sqrt{ex + d}b^3\operatorname{sgn}(bx + a)}{e^4} + \frac{2(15(ex + d)^2b^3d\operatorname{sgn}(bx + a) - 5(ex + d)b^3d^2\operatorname{sgn}(bx + a) + b^3d^3\operatorname{sgn}(bx + a) - 15(ex + d)^2ab^2e\operatorname{sgn}(bx + a))}{e^4}$$

input `integrate((b^2*x^2+2*a*b*x+a^2)^(3/2)/(e*x+d)^(7/2),x, algorithm="giac")`

output

$$\frac{2\sqrt{e^2x + d}b^3\operatorname{sgn}(bx + a)/e^4 + 2/5(15(e^2x + d)^2b^3d\operatorname{sgn}(bx + a) - 5(e^2x + d)b^3d^2\operatorname{sgn}(bx + a) + b^3d^3\operatorname{sgn}(bx + a) - 15(e^2x + d)^2ab^2e\operatorname{sgn}(bx + a) + 10(e^2x + d)ab^2de\operatorname{sgn}(bx + a) - 3a^2b^2d^2e\operatorname{sgn}(bx + a) - 5(e^2x + d)a^2be^2\operatorname{sgn}(bx + a) + 3a^2bde^2\operatorname{sgn}(bx + a) - a^3e^3\operatorname{sgn}(bx + a))/(e^2x + d)^{5/2}e^4}$$
Mupad [B] (verification not implemented)

Time = 6.01 (sec) , antiderivative size = 210, normalized size of antiderivative = 1.04

$$\int \frac{(a^2 + 2abx + b^2x^2)^{3/2}}{(d + ex)^{7/2}} dx = \frac{\sqrt{a^2 + 2abx + b^2x^2} \left(\frac{2a^3e^3 + 4a^2bde^2 + 16ab^2d^2e - 32b^3d^3}{5be^6} + \frac{2x(a^2e^2 + 4abde - 8b^2d^2)}{e^5} - \frac{2b^2x^3}{e^3} + \frac{6bx^2(ae - 2bd)}{e^4} \right)}{x^3\sqrt{d + ex} + \frac{ad^2\sqrt{d + ex}}{be^2} + \frac{x^2(5ae^6 + 10bde^5)\sqrt{d + ex}}{5be^6} + \frac{dx(2ae + bd)\sqrt{d + ex}}{be^2}}$$

input

$$\operatorname{int}((a^2 + b^2x^2 + 2abx)^{(3/2)}/(d + ex)^{(7/2)}, x)$$

output

$$-\frac{(a^2 + b^2x^2 + 2abx)^{(1/2)}((2a^3e^3 - 32b^3d^3 + 16ab^2d^2e + 4a^2bde^2)/(5be^6) + (2x(a^2e^2 - 8b^2d^2 + 4abde))/e^5 - (2b^2x^3)/e^3 + (6bx^2(ae - 2bd))/e^4)/(x^3(d + ex)^{(1/2)} + (ad^2(d + ex)^{(1/2)})/(be^2) + (x^2(5ae^6 + 10bde^5)(d + ex)^{(1/2)})/(5be^6) + (dx(2ae + bd)(d + ex)^{(1/2)})/(be^2))}{(d + ex)^{(7/2)}}$$
Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 134, normalized size of antiderivative = 0.66

$$\int \frac{(a^2 + 2abx + b^2x^2)^{3/2}}{(d + ex)^{7/2}} dx = \frac{2b^3e^3x^3 - 6ab^2e^3x^2 + 12b^3de^2x^2 - 2a^2be^3x - 8ab^2de^2x + 16b^3dex - \frac{2}{5}a^3e^3}{\sqrt{ex + d}e^4(e^2x^2 + 2dex + d^2)}$$

input

$$\operatorname{int}((b^2x^2 + 2abx + a^2)^{(3/2)}/(ex + d)^{(7/2)}, x)$$

output

```
(2*( - a**3*e**3 - 2*a**2*b*d*e**2 - 5*a**2*b*e**3*x - 8*a*b**2*d**2*e - 2
0*a*b**2*d*e**2*x - 15*a*b**2*e**3*x**2 + 16*b**3*d**3 + 40*b**3*d**2*e*x
+ 30*b**3*d*e**2*x**2 + 5*b**3*e**3*x**3))/(5*sqrt(d + e*x)*e**4*(d**2 + 2
*d*e*x + e**2*x**2))
```

3.358 $\int \frac{(a^2+2abx+b^2x^2)^{3/2}}{(d+ex)^{9/2}} dx$

| | |
|---|------|
| Optimal result | 2642 |
| Mathematica [A] (verified) | 2643 |
| Rubi [A] (verified) | 2643 |
| Maple [A] (verified) | 2645 |
| Fricas [A] (verification not implemented) | 2645 |
| Sympy [F] | 2646 |
| Maxima [A] (verification not implemented) | 2646 |
| Giac [A] (verification not implemented) | 2646 |
| Mupad [B] (verification not implemented) | 2647 |
| Reduce [B] (verification not implemented) | 2647 |

Optimal result

Integrand size = 30, antiderivative size = 204

$$\int \frac{(a^2 + 2abx + b^2x^2)^{3/2}}{(d + ex)^{9/2}} dx = \frac{2(bd - ae)^3 \sqrt{a^2 + 2abx + b^2x^2}}{7e^4(a + bx)(d + ex)^{7/2}} - \frac{6b(bd - ae)^2 \sqrt{a^2 + 2abx + b^2x^2}}{5e^4(a + bx)(d + ex)^{5/2}} + \frac{2b^2(bd - ae) \sqrt{a^2 + 2abx + b^2x^2}}{e^4(a + bx)(d + ex)^{3/2}} - \frac{2b^3 \sqrt{a^2 + 2abx + b^2x^2}}{e^4(a + bx) \sqrt{d + ex}}$$

output

```
2/7*(-a*e+b*d)^3*((b*x+a)^2)^(1/2)/e^4/(b*x+a)/(e*x+d)^(7/2)-6/5*b*(-a*e+b*d)^2*((b*x+a)^2)^(1/2)/e^4/(b*x+a)/(e*x+d)^(5/2)+2*b^2*(-a*e+b*d)*((b*x+a)^2)^(1/2)/e^4/(b*x+a)/(e*x+d)^(3/2)-2*b^3*((b*x+a)^2)^(1/2)/e^4/(b*x+a)/(e*x+d)^(1/2)
```

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 119, normalized size of antiderivative = 0.58

$$\int \frac{(a^2 + 2abx + b^2x^2)^{3/2}}{(d + ex)^{9/2}} dx = \frac{2\sqrt{(a + bx)^2(5a^3e^3 + 3a^2be^2(2d + 7ex) + ab^2e(8d^2 + 28dex + 35e^2x^2) + b^3(16d^3 + 56d^2ex + 70de^2x^2 + 35e^4(a + bx)(d + ex)^{7/2})}}{35e^4(a + bx)(d + ex)^{7/2}}$$

input

```
Integrate[(a^2 + 2*a*b*x + b^2*x^2)^(3/2)/(d + e*x)^(9/2),x]
```

output

```
(-2*Sqrt[(a + b*x)^2]*(5*a^3*e^3 + 3*a^2*b*e^2*(2*d + 7*e*x) + a*b^2*e*(8*d^2 + 28*d*e*x + 35*e^2*x^2) + b^3*(16*d^3 + 56*d^2*e*x + 70*d*e^2*x^2 + 35*e^3*x^3)))/(35*e^4*(a + b*x)*(d + e*x)^(7/2))
```

Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 124, normalized size of antiderivative = 0.61, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {1102, 27, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\begin{aligned} & \int \frac{(a^2 + 2abx + b^2x^2)^{3/2}}{(d + ex)^{9/2}} dx \\ & \quad \downarrow \text{1102} \\ & \frac{\sqrt{a^2 + 2abx + b^2x^2} \int \frac{b^3(a+bx)^3}{(d+ex)^{9/2}} dx}{b^3(a + bx)} \\ & \quad \downarrow \text{27} \\ & \frac{\sqrt{a^2 + 2abx + b^2x^2} \int \frac{(a+bx)^3}{(d+ex)^{9/2}} dx}{a + bx} \\ & \quad \downarrow \text{53} \end{aligned}$$

$$\frac{\sqrt{a^2 + 2abx + b^2x^2} \int \left(\frac{b^3}{e^3(d+ex)^{3/2}} - \frac{3(bd-ae)b^2}{e^3(d+ex)^{5/2}} + \frac{3(bd-ae)^2b}{e^3(d+ex)^{7/2}} + \frac{(ae-bd)^3}{e^3(d+ex)^{9/2}} \right) dx}{a + bx}$$

↓ 2009

$$\frac{\sqrt{a^2 + 2abx + b^2x^2} \left(\frac{2b^2(bd-ae)}{e^4(d+ex)^{3/2}} - \frac{6b(bd-ae)^2}{5e^4(d+ex)^{5/2}} + \frac{2(bd-ae)^3}{7e^4(d+ex)^{7/2}} - \frac{2b^3}{e^4\sqrt{d+ex}} \right)}{a + bx}$$

input `Int[(a^2 + 2*a*b*x + b^2*x^2)^(3/2)/(d + e*x)^(9/2),x]`

output `(Sqrt[a^2 + 2*a*b*x + b^2*x^2]*((2*(b*d - a*e)^3)/(7*e^4*(d + e*x)^(7/2)) - (6*b*(b*d - a*e)^2)/(5*e^4*(d + e*x)^(5/2)) + (2*b^2*(b*d - a*e))/(e^4*(d + e*x)^(3/2)) - (2*b^3)/(e^4*Sqrt[d + e*x])))/(a + b*x)`

Defintions of rubi rules used

rule 27 `Int[(a_)*(F_x_), x_Symbol] := Simp[a Int[F_x, x], x] /; FreeQ[a, x] && !MatchQ[F_x, (b_)*(G_x_)] /; FreeQ[b, x]`

rule 53 `Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] := Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && IGtQ[m, 0] && (!IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])`

rule 1102 `Int[((d_.) + (e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] := Simp[(a + b*x + c*x^2)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x)^(2*FracPart[p])) Int[(d + e*x)^m*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && EqQ[b^2 - 4*a*c, 0]`

rule 2009 `Int[u_, x_Symbol] := Simp[IntSum[u, x], x] /; SumQ[u]`

Maple [A] (verified)

Time = 1.14 (sec) , antiderivative size = 132, normalized size of antiderivative = 0.65

| method | result |
|---------|---|
| gospers | $-\frac{2(35e^3x^3b^3+35x^2ab^2e^3+70x^2b^3de^2+21a^2be^3x+28xab^2de^2+56b^3d^2ex+5e^3a^3+6a^2bde^2+8ab^2d^2e+16b^3d^3)((bx+a)^2)^{\frac{3}{2}}}{35(ex+d)^{\frac{7}{2}}e^4(bx+a)^3}$ |
| default | $-\frac{2(35e^3x^3b^3+35x^2ab^2e^3+70x^2b^3de^2+21a^2be^3x+28xab^2de^2+56b^3d^2ex+5e^3a^3+6a^2bde^2+8ab^2d^2e+16b^3d^3)((bx+a)^2)^{\frac{3}{2}}}{35(ex+d)^{\frac{7}{2}}e^4(bx+a)^3}$ |
| orering | $-\frac{2(35e^3x^3b^3+35x^2ab^2e^3+70x^2b^3de^2+21a^2be^3x+28xab^2de^2+56b^3d^2ex+5e^3a^3+6a^2bde^2+8ab^2d^2e+16b^3d^3)(b^2x^2+2abx+a^2)^{\frac{3}{2}}}{35e^4(bx+a)^3(ex+d)^{\frac{7}{2}}}$ |

input `int((b^2*x^2+2*a*b*x+a^2)^(3/2)/(e*x+d)^(9/2),x,method=_RETURNVERBOSE)`

output
$$-2/35/(e*x+d)^{(7/2)}*(35*b^3*e^3*x^3+35*a*b^2*e^3*x^2+70*b^3*d*e^2*x^2+21*a^2*b*e^3*x+28*a*b^2*d*e^2*x+56*b^3*d^2*e*x+5*a^3*e^3+6*a^2*b*d*e^2+8*a*b^2*d^2*e+16*b^3*d^3)*((b*x+a)^2)^{(3/2)}/e^4/(b*x+a)^3$$

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 158, normalized size of antiderivative = 0.77

$$\int \frac{(a^2 + 2abx + b^2x^2)^{3/2}}{(d + ex)^{9/2}} dx =$$

$$-\frac{2(35b^3e^3x^3 + 16b^3d^3 + 8ab^2d^2e + 6a^2bde^2 + 5a^3e^3 + 35(2b^3de^2 + ab^2e^3)x^2 + 7(8b^3d^2e + 4ab^2de^2 + 3a^2b^2e^3)x + 35a^2b^2e^3)}{35(e^8x^4 + 4de^7x^3 + 6d^2e^6x^2 + 4d^3e^5x + d^4e^4)}$$

input `integrate((b^2*x^2+2*a*b*x+a^2)^(3/2)/(e*x+d)^(9/2),x, algorithm="fricas")`

output
$$-2/35*(35*b^3*e^3*x^3 + 16*b^3*d^3 + 8*a*b^2*d^2*e + 6*a^2*b*d*e^2 + 5*a^3*e^3 + 35*(2*b^3*d*e^2 + a*b^2*e^3)*x^2 + 7*(8*b^3*d^2*e + 4*a*b^2*d*e^2 + 3*a^2*b^2*e^3)*x)*sqrt(e*x + d)/(e^8*x^4 + 4*d*e^7*x^3 + 6*d^2*e^6*x^2 + 4*d^3*e^5*x + d^4*e^4)$$

Sympy [F]

$$\int \frac{(a^2 + 2abx + b^2x^2)^{3/2}}{(d + ex)^{9/2}} dx = \int \frac{((a + bx)^2)^{3/2}}{(d + ex)^{9/2}} dx$$

input `integrate((b**2*x**2+2*a*b*x+a**2)**(3/2)/(e*x+d)**(9/2),x)`

output `Integral(((a + b*x)**2)**(3/2)/(d + e*x)**(9/2), x)`

Maxima [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 147, normalized size of antiderivative = 0.72

$$\int \frac{(a^2 + 2abx + b^2x^2)^{3/2}}{(d + ex)^{9/2}} dx = \frac{2(35b^3e^3x^3 + 16b^3d^3 + 8ab^2d^2e + 6a^2bde^2 + 5a^3e^3 + 35(2b^3de^2 + ab^2e^3)x^2 + 7(8b^3d^2e + 4ab^2de^2 + 3a^2b^2e^3)x)}{35(e^7x^3 + 3de^6x^2 + 3d^2e^5x + d^3e^4)\sqrt{ex + d}}$$

input `integrate((b^2*x^2+2*a*b*x+a^2)^(3/2)/(e*x+d)^(9/2),x, algorithm="maxima")`

output `-2/35*(35*b^3*e^3*x^3 + 16*b^3*d^3 + 8*a*b^2*d^2*e + 6*a^2*b*d*e^2 + 5*a^3*e^3 + 35*(2*b^3*d*e^2 + a*b^2*e^3)*x^2 + 7*(8*b^3*d^2*e + 4*a*b^2*d*e^2 + 3*a^2*b*e^3)*x)/((e^7*x^3 + 3*d*e^6*x^2 + 3*d^2*e^5*x + d^3*e^4)*sqrt(e*x + d))`

Giac [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 188, normalized size of antiderivative = 0.92

$$\int \frac{(a^2 + 2abx + b^2x^2)^{3/2}}{(d + ex)^{9/2}} dx = \frac{2(35(ex + d)^3b^3\text{sgn}(bx + a) - 35(ex + d)^2b^3d\text{sgn}(bx + a) + 21(ex + d)b^3d^2\text{sgn}(bx + a) - 5b^3d^3\text{sgn}(bx + a))}{35(e^7x^3 + 3de^6x^2 + 3d^2e^5x + d^3e^4)\sqrt{ex + d}}$$

input `integrate((b^2*x^2+2*a*b*x+a^2)^(3/2)/(e*x+d)^(9/2),x, algorithm="giac")`

output
$$\frac{-2/35*(35*(e*x + d)^3*b^3*\text{sgn}(b*x + a) - 35*(e*x + d)^2*b^3*d*\text{sgn}(b*x + a) + 21*(e*x + d)*b^3*d^2*\text{sgn}(b*x + a) - 5*b^3*d^3*\text{sgn}(b*x + a) + 35*(e*x + d)^2*a*b^2*e*\text{sgn}(b*x + a) - 42*(e*x + d)*a*b^2*d*e*\text{sgn}(b*x + a) + 15*a*b^2*d^2*e*\text{sgn}(b*x + a) + 21*(e*x + d)*a^2*b*e^2*\text{sgn}(b*x + a) - 15*a^2*b*d*e^2*\text{sgn}(b*x + a) + 5*a^3*e^3*\text{sgn}(b*x + a))}{(e*x + d)^{7/2}*e^4}$$

Mupad [B] (verification not implemented)

Time = 6.02 (sec) , antiderivative size = 239, normalized size of antiderivative = 1.17

$$\int \frac{(a^2 + 2abx + b^2x^2)^{3/2}}{(d + ex)^{9/2}} dx = \frac{\sqrt{a^2 + 2abx + b^2x^2} \left(\frac{10a^3e^3 + 12a^2bde^2 + 16ab^2d^2e + 32b^3d^3}{35be^7} + \frac{2x(3a^2e^2 + 4abde + 8b^2d^2)}{5e^6} + \frac{2b^2x^3}{e^4} + \frac{2bx^2(ae + 2bd)}{e^5} \right)}{x^4 \sqrt{d + ex} + \frac{ad^3\sqrt{d+ex}}{be^3} + \frac{x^3(35ae^7 + 105bde^6)\sqrt{d+ex}}{35be^7} + \frac{3dx^2(ae+bd)\sqrt{d+ex}}{be^2} + \frac{d^2x(3ae+bd)\sqrt{d+ex}}{be^3}}$$

input `int((a^2 + b^2*x^2 + 2*a*b*x)^(3/2)/(d + e*x)^(9/2),x)`

output
$$\begin{aligned} & -((a^2 + b^2*x^2 + 2*a*b*x)^{1/2} * ((10*a^3*e^3 + 32*b^3*d^3 + 16*a*b^2*d^2 * e + 12*a^2*b*d*e^2)/(35*b*e^7) + (2*x*(3*a^2*e^2 + 8*b^2*d^2 + 4*a*b*d*e) / (5*e^6) + (2*b^2*x^3)/e^4 + (2*b*x^2*(a*e + 2*b*d))/e^5)) / (x^4*(d + e*x) \\ & ^{1/2} + (a*d^3*(d + e*x)^{1/2})/(b*e^3) + (x^3*(35*a*e^7 + 105*b*d*e^6)*(d + e*x)^{1/2}) / (35*b*e^7) + (3*d*x^2*(a*e + b*d)*(d + e*x)^{1/2}) / (b*e^2) \\ & + (d^2*x*(3*a*e + b*d)*(d + e*x)^{1/2}) / (b*e^3)) \end{aligned}$$

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 145, normalized size of antiderivative = 0.71

$$\int \frac{(a^2 + 2abx + b^2x^2)^{3/2}}{(d + ex)^{9/2}} dx = \frac{-2b^3e^3x^3 - 2ab^2e^3x^2 - 4b^3de^2x^2 - \frac{6}{5}a^2be^3x - \frac{8}{5}ab^2de^2x - \frac{16}{5}b^3d^2ex - \frac{2}{7}a^3}{\sqrt{ex + d}e^4(e^3x^3 + 3de^2x^2 + 3d^2ex + d^3)}$$

input `int((b^2*x^2+2*a*b*x+a^2)^(3/2)/(e*x+d)^(9/2),x)`

output

```
(2*( - 5*a**3*e**3 - 6*a**2*b*d*e**2 - 21*a**2*b*e**3*x - 8*a*b**2*d**2*e
- 28*a*b**2*d*e**2*x - 35*a*b**2*e**3*x**2 - 16*b**3*d**3 - 56*b**3*d**2*e
*x - 70*b**3*d*e**2*x**2 - 35*b**3*e**3*x**3))/(35*sqrt(d + e*x)*e**4*(d**
3 + 3*d**2*e*x + 3*d*e**2*x**2 + e**3*x**3))
```

3.359
$$\int \frac{(a^2+2abx+b^2x^2)^{3/2}}{(d+ex)^{11/2}} dx$$

| | |
|---|------|
| Optimal result | 2649 |
| Mathematica [A] (verified) | 2650 |
| Rubi [A] (verified) | 2650 |
| Maple [A] (verified) | 2652 |
| Fricas [A] (verification not implemented) | 2652 |
| Sympy [F] | 2653 |
| Maxima [A] (verification not implemented) | 2653 |
| Giac [A] (verification not implemented) | 2654 |
| Mupad [B] (verification not implemented) | 2654 |
| Reduce [B] (verification not implemented) | 2655 |

Optimal result

Integrand size = 30, antiderivative size = 208

$$\int \frac{(a^2 + 2abx + b^2x^2)^{3/2}}{(d + ex)^{11/2}} dx = \frac{2(bd - ae)^3 \sqrt{a^2 + 2abx + b^2x^2}}{9e^4(a + bx)(d + ex)^{9/2}} - \frac{6b(bd - ae)^2 \sqrt{a^2 + 2abx + b^2x^2}}{7e^4(a + bx)(d + ex)^{7/2}} + \frac{6b^2(bd - ae) \sqrt{a^2 + 2abx + b^2x^2}}{5e^4(a + bx)(d + ex)^{5/2}} - \frac{2b^3 \sqrt{a^2 + 2abx + b^2x^2}}{3e^4(a + bx)(d + ex)^{3/2}}$$

output

```
2/9*(-a*e+b*d)^3*((b*x+a)^2)^(1/2)/e^4/(b*x+a)/(e*x+d)^(9/2)-6/7*b*(-a*e+b
*d)^2*((b*x+a)^2)^(1/2)/e^4/(b*x+a)/(e*x+d)^(7/2)+6/5*b^2*(-a*e+b*d)*((b*x
+a)^2)^(1/2)/e^4/(b*x+a)/(e*x+d)^(5/2)-2/3*b^3*((b*x+a)^2)^(1/2)/e^4/(b*x+
a)/(e*x+d)^(3/2)
```

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 120, normalized size of antiderivative = 0.58

$$\int \frac{(a^2 + 2abx + b^2x^2)^{3/2}}{(d + ex)^{11/2}} dx = \frac{2\sqrt{(a + bx)^2(35a^3e^3 + 15a^2be^2(2d + 9ex) + 3ab^2e(8d^2 + 36dex + 63e^2x^2) + b^3(16d^3 + 72d^2ex + 126de^2x^2 + 105e^3x^3))}}{315e^4(a + bx)(d + ex)^{9/2}}$$

input

```
Integrate[(a^2 + 2*a*b*x + b^2*x^2)^(3/2)/(d + e*x)^(11/2),x]
```

output

```
(-2*Sqrt[(a + b*x)^2]*(35*a^3*e^3 + 15*a^2*b*e^2*(2*d + 9*e*x) + 3*a*b^2*e*(8*d^2 + 36*d*e*x + 63*e^2*x^2) + b^3*(16*d^3 + 72*d^2*e*x + 126*d*e^2*x^2 + 105*e^3*x^3)))/(315*e^4*(a + b*x)*(d + e*x)^(9/2))
```

Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 128, normalized size of antiderivative = 0.62, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {1102, 27, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\begin{aligned} & \int \frac{(a^2 + 2abx + b^2x^2)^{3/2}}{(d + ex)^{11/2}} dx \\ & \quad \downarrow 1102 \\ & \frac{\sqrt{a^2 + 2abx + b^2x^2} \int \frac{b^3(a+bx)^3}{(d+ex)^{11/2}} dx}{b^3(a + bx)} \\ & \quad \downarrow 27 \\ & \frac{\sqrt{a^2 + 2abx + b^2x^2} \int \frac{(a+bx)^3}{(d+ex)^{11/2}} dx}{a + bx} \\ & \quad \downarrow 53 \end{aligned}$$

$$\frac{\sqrt{a^2 + 2abx + b^2x^2} \int \left(\frac{b^3}{e^3(d+ex)^{5/2}} - \frac{3(bd-ae)b^2}{e^3(d+ex)^{7/2}} + \frac{3(bd-ae)^2b}{e^3(d+ex)^{9/2}} + \frac{(ae-bd)^3}{e^3(d+ex)^{11/2}} \right) dx}{a + bx}$$

↓ 2009

$$\frac{\sqrt{a^2 + 2abx + b^2x^2} \left(\frac{6b^2(bd-ae)}{5e^4(d+ex)^{5/2}} - \frac{6b(bd-ae)^2}{7e^4(d+ex)^{7/2}} + \frac{2(bd-ae)^3}{9e^4(d+ex)^{9/2}} - \frac{2b^3}{3e^4(d+ex)^{3/2}} \right)}{a + bx}$$

input `Int[(a^2 + 2*a*b*x + b^2*x^2)^(3/2)/(d + e*x)^(11/2),x]`

output `(Sqrt[a^2 + 2*a*b*x + b^2*x^2]*((2*(b*d - a*e)^3)/(9*e^4*(d + e*x)^(9/2)) - (6*b*(b*d - a*e)^2)/(7*e^4*(d + e*x)^(7/2)) + (6*b^2*(b*d - a*e))/(5*e^4*(d + e*x)^(5/2)) - (2*b^3)/(3*e^4*(d + e*x)^(3/2))))/(a + b*x)`

Defintions of rubi rules used

rule 27 `Int[(a_)*(F_x_), x_Symbol] := Simp[a Int[F_x, x], x] /; FreeQ[a, x] && !MatchQ[F_x, (b_)*(G_x_)] /; FreeQ[b, x]`

rule 53 `Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] := Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && IGtQ[m, 0] && (!IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])`

rule 1102 `Int[((d_.) + (e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] := Simp[(a + b*x + c*x^2)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x)^(2*FracPart[p])) Int[(d + e*x)^m*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && EqQ[b^2 - 4*a*c, 0]`

rule 2009 `Int[u_, x_Symbol] := Simp[IntSum[u, x], x] /; SumQ[u]`

Maple [A] (verified)

Time = 1.09 (sec) , antiderivative size = 132, normalized size of antiderivative = 0.63

| method | result |
|---------|---|
| gospers | $-\frac{2(105e^3x^3b^3+189x^2ab^2e^3+126x^2b^3de^2+135a^2be^3x+108xab^2de^2+72b^3d^2ex+35e^3a^3+30a^2bde^2+24ab^2d^2e+16b^3d^3)(bx+a)}{315(ex+d)^{\frac{9}{2}}e^4(bx+a)^3}$ |
| default | $-\frac{2(105e^3x^3b^3+189x^2ab^2e^3+126x^2b^3de^2+135a^2be^3x+108xab^2de^2+72b^3d^2ex+35e^3a^3+30a^2bde^2+24ab^2d^2e+16b^3d^3)(bx+a)}{315(ex+d)^{\frac{9}{2}}e^4(bx+a)^3}$ |
| orering | $-\frac{2(105e^3x^3b^3+189x^2ab^2e^3+126x^2b^3de^2+135a^2be^3x+108xab^2de^2+72b^3d^2ex+35e^3a^3+30a^2bde^2+24ab^2d^2e+16b^3d^3)(b^2x^2+d^2)}{315e^4(bx+a)^3(ex+d)^{\frac{9}{2}}}$ |

input `int((b^2*x^2+2*a*b*x+a^2)^(3/2)/(e*x+d)^(11/2),x,method=_RETURNVERBOSE)`

output
$$-\frac{2}{315}(e*x+d)^{-\frac{9}{2}}*(105*b^3*e^3*x^3+189*a*b^2*e^3*x^2+126*b^3*d*e^2*x^2+135*a^2*b*e^3*x+108*a*b^2*d*e^2*x+72*b^3*d^2*e*x+35*a^3*e^3+30*a^2*b*d*e^2+24*a*b^2*d^2*e+16*b^3*d^3)*((b*x+a)^2)^{\frac{3}{2}}/e^4/(b*x+a)^3$$

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 170, normalized size of antiderivative = 0.82

$$\int \frac{(a^2 + 2abx + b^2x^2)^{3/2}}{(d + ex)^{11/2}} dx =$$

$$-\frac{2(105b^3e^3x^3 + 16b^3d^3 + 24ab^2d^2e + 30a^2bde^2 + 35a^3e^3 + 63(2b^3de^2 + 3ab^2e^3)x^2 + 9(8b^3d^2e + 12ab^2d^2e^2 + 15a^2b^2e^3)x)*\sqrt{e*x + d}}{315(e^9x^5 + 5de^8x^4 + 10d^2e^7x^3 + 10d^3e^6x^2 + 5d^4e^5x + d^5e^4)}$$

input `integrate((b^2*x^2+2*a*b*x+a^2)^(3/2)/(e*x+d)^(11/2),x, algorithm="fricas")`

output
$$-\frac{2}{315}*(105*b^3*e^3*x^3 + 16*b^3*d^3 + 24*a*b^2*d^2*e + 30*a^2*b*d*e^2 + 35*a^3*e^3 + 63*(2*b^3*d*e^2 + 3*a*b^2*e^3)*x^2 + 9*(8*b^3*d^2*e + 12*a*b^2*d^2*e^2 + 15*a^2*b^2*e^3)*x)*\sqrt{e*x + d}/(e^9*x^5 + 5*d*e^8*x^4 + 10*d^2*e^7*x^3 + 10*d^3*e^6*x^2 + 5*d^4*e^5*x + d^5*e^4)$$

Sympy [F]

$$\int \frac{(a^2 + 2abx + b^2x^2)^{3/2}}{(d + ex)^{11/2}} dx = \int \frac{((a + bx)^2)^{3/2}}{(d + ex)^{11/2}} dx$$

input `integrate((b**2*x**2+2*a*b*x+a**2)**(3/2)/(e*x+d)**(11/2), x)`

output `Integral(((a + b*x)**2)**(3/2)/(d + e*x)**(11/2), x)`

Maxima [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 159, normalized size of antiderivative = 0.76

$$\int \frac{(a^2 + 2abx + b^2x^2)^{3/2}}{(d + ex)^{11/2}} dx = \frac{2(105b^3e^3x^3 + 16b^3d^3 + 24ab^2d^2e + 30a^2bde^2 + 35a^3e^3 + 63(2b^3de^2 + 3ab^2e^3)x^2 + 9(8b^3d^2e + 12ab^2de^2 + 15a^2b^2e^3)x)}{315(e^8x^4 + 4de^7x^3 + 6d^2e^6x^2 + 4d^3e^5x + d^4e^4)\sqrt{ex + d}}$$

input `integrate((b^2*x^2+2*a*b*x+a^2)^(3/2)/(e*x+d)^(11/2),x, algorithm="maxima")`

output `-2/315*(105*b^3*e^3*x^3 + 16*b^3*d^3 + 24*a*b^2*d^2*e + 30*a^2*b*d*e^2 + 35*a^3*e^3 + 63*(2*b^3*d*e^2 + 3*a*b^2*e^3)*x^2 + 9*(8*b^3*d^2*e + 12*a*b^2*d*e^2 + 15*a^2*b^2*e^3)*x)/((e^8*x^4 + 4*d*e^7*x^3 + 6*d^2*e^6*x^2 + 4*d^3*e^5*x + d^4*e^4)*sqrt(e*x + d))`

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 188, normalized size of antiderivative = 0.90

$$\int \frac{(a^2 + 2abx + b^2x^2)^{3/2}}{(d + ex)^{11/2}} dx =$$

$$\frac{2(105(ex + d)^3 b^3 \operatorname{sgn}(bx + a) - 189(ex + d)^2 b^3 d \operatorname{sgn}(bx + a) + 135(ex + d) b^3 d^2 \operatorname{sgn}(bx + a) - 35 b^3 d^3 \operatorname{sgn}(bx + a))}{(ex + d)^{9/2} e^4}$$

input `integrate((b^2*x^2+2*a*b*x+a^2)^(3/2)/(e*x+d)^(11/2),x, algorithm="giac")`

output
$$\frac{-2/315*(105*(e*x + d)^3*b^3*\operatorname{sgn}(b*x + a) - 189*(e*x + d)^2*b^3*d*\operatorname{sgn}(b*x + a) + 135*(e*x + d)*b^3*d^2*\operatorname{sgn}(b*x + a) - 35*b^3*d^3*\operatorname{sgn}(b*x + a) + 189*(e*x + d)^2*a*b^2*e*\operatorname{sgn}(b*x + a) - 270*(e*x + d)*a*b^2*d*e*\operatorname{sgn}(b*x + a) + 105*a*b^2*d^2*e*\operatorname{sgn}(b*x + a) + 135*(e*x + d)*a^2*b*e^2*\operatorname{sgn}(b*x + a) - 105*a^2*b*d*e^2*\operatorname{sgn}(b*x + a) + 35*a^3*e^3*\operatorname{sgn}(b*x + a))}{(e*x + d)^{(9/2)}*e^4}$$

Mupad [B] (verification not implemented)

Time = 5.94 (sec) , antiderivative size = 268, normalized size of antiderivative = 1.29

$$\int \frac{(a^2 + 2abx + b^2x^2)^{3/2}}{(d + ex)^{11/2}} dx =$$

$$\frac{\sqrt{a^2 + 2abx + b^2x^2} \left(\frac{2a^3e^3 + 4a^2bde^2 + 16ab^2d^2e + 32b^3d^3}{9be^8} + \frac{2x(15a^2e^2 + 12abde + 8b^2d^2)}{35e^7} + \frac{2b^2x^3}{3e^5} + \frac{2bx^2(3ae + 2bd)}{5e^6} \right)}{x^5 \sqrt{d + ex} + \frac{ad^4 \sqrt{d + ex}}{be^4} + \frac{x^4(ae^8 + 4bde^7) \sqrt{d + ex}}{be^8} + \frac{2dx^3(2ae + 3bd) \sqrt{d + ex}}{be^2} + \frac{d^3x(4ae + bd) \sqrt{d + ex}}{be^4} + \frac{d^2x^2(6ae + 4bd)}{be^3}}$$

input `int((a^2 + b^2*x^2 + 2*a*b*x)^(3/2)/(d + e*x)^(11/2),x)`

output
$$\frac{-((a^2 + b^2*x^2 + 2*a*b*x)^{(1/2)} * (((2*a^3*e^3)/9 + (32*b^3*d^3)/315 + (16*a*b^2*d^2*e)/105 + (4*a^2*b*d*e^2)/21)/(b*e^8) + (2*x*(15*a^2*e^2 + 8*b^2*d^2 + 12*a*b*d*e))/(35*e^7) + (2*b^2*x^3)/(3*e^5) + (2*b*x^2*(3*a*e + 2*b*d))/(5*e^6))}{(x^5*(d + e*x)^{(1/2)} + (a*d^4*(d + e*x)^{(1/2)})/(b*e^4) + (x^4*(a*e^8 + 4*b*d*e^7)*(d + e*x)^{(1/2)})/(b*e^8) + (2*d*x^3*(2*a*e + 3*b*d)*(d + e*x)^{(1/2)})/(b*e^2) + (d^3*x*(4*a*e + b*d)*(d + e*x)^{(1/2)})/(b*e^4) + (d^2*x^2*(6*a*e + 4*b*d)*(d + e*x)^{(1/2)})/(b*e^3))}$$

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 156, normalized size of antiderivative = 0.75

$$\int \frac{(a^2 + 2abx + b^2x^2)^{3/2}}{(d + ex)^{11/2}} dx = \frac{-\frac{2}{3}b^3e^3x^3 - \frac{6}{5}ab^2e^3x^2 - \frac{4}{5}b^3de^2x^2 - \frac{6}{7}a^2be^3x - \frac{24}{35}ab^2de^2x - \frac{16}{35}b^3d^2ex - \frac{2}{9}a^2e^2}{\sqrt{ex + d}e^4(e^4x^4 + 4de^3x^3 + 6d^2e^2x^2 + 4d^3ex + d^4)}$$

input

```
int((b^2*x^2+2*a*b*x+a^2)^(3/2)/(e*x+d)^(11/2),x)
```

output

```
(2*(- 35*a**3*e**3 - 30*a**2*b*d*e**2 - 135*a**2*b*e**3*x - 24*a*b**2*d**2*e - 108*a*b**2*d*e**2*x - 189*a*b**2*e**3*x**2 - 16*b**3*d**3 - 72*b**3*d**2*e*x - 126*b**3*d*e**2*x**2 - 105*b**3*e**3*x**3))/(315*sqrt(d + e*x)*e**4*(d**4 + 4*d**3*e*x + 6*d**2*e**2*x**2 + 4*d*e**3*x**3 + e**4*x**4))
```

3.360 $\int (d + ex)^{5/2} (a^2 + 2abx + b^2x^2)^{5/2} dx$

| | |
|---|------|
| Optimal result | 2656 |
| Mathematica [A] (verified) | 2657 |
| Rubi [A] (verified) | 2657 |
| Maple [A] (verified) | 2659 |
| Fricas [B] (verification not implemented) | 2660 |
| Sympy [F(-1)] | 2660 |
| Maxima [B] (verification not implemented) | 2661 |
| Giac [B] (verification not implemented) | 2661 |
| Mupad [F(-1)] | 2662 |
| Reduce [B] (verification not implemented) | 2663 |

Optimal result

Integrand size = 30, antiderivative size = 320

$$\begin{aligned}
 & \int (d + ex)^{5/2} (a^2 + 2abx + b^2x^2)^{5/2} dx = \\
 & - \frac{2(bd - ae)^5 (d + ex)^{7/2} \sqrt{a^2 + 2abx + b^2x^2}}{7e^6(a + bx)} \\
 & + \frac{10b(bd - ae)^4 (d + ex)^{9/2} \sqrt{a^2 + 2abx + b^2x^2}}{9e^6(a + bx)} \\
 & - \frac{20b^2(bd - ae)^3 (d + ex)^{11/2} \sqrt{a^2 + 2abx + b^2x^2}}{11e^6(a + bx)} \\
 & + \frac{20b^3(bd - ae)^2 (d + ex)^{13/2} \sqrt{a^2 + 2abx + b^2x^2}}{13e^6(a + bx)} \\
 & - \frac{2b^4(bd - ae)(d + ex)^{15/2} \sqrt{a^2 + 2abx + b^2x^2}}{3e^6(a + bx)} \\
 & + \frac{2b^5(d + ex)^{17/2} \sqrt{a^2 + 2abx + b^2x^2}}{17e^6(a + bx)}
 \end{aligned}$$

output

```
-2/7*(-a*e+b*d)^5*(e*x+d)^(7/2)*((b*x+a)^2)^(1/2)/e^6/(b*x+a)+10/9*b*(-a*e
+b*d)^4*(e*x+d)^(9/2)*((b*x+a)^2)^(1/2)/e^6/(b*x+a)-20/11*b^2*(-a*e+b*d)^3
*(e*x+d)^(11/2)*((b*x+a)^2)^(1/2)/e^6/(b*x+a)+20/13*b^3*(-a*e+b*d)^2*(e*x+
d)^(13/2)*((b*x+a)^2)^(1/2)/e^6/(b*x+a)-2/3*b^4*(-a*e+b*d)*(e*x+d)^(15/2)*
((b*x+a)^2)^(1/2)/e^6/(b*x+a)+2/17*b^5*(e*x+d)^(17/2)*((b*x+a)^2)^(1/2)/e^
6/(b*x+a)
```

Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 235, normalized size of antiderivative = 0.73

$$\int (d + ex)^{5/2} (a^2 + 2abx + b^2x^2)^{5/2} dx = \frac{2\sqrt{(a + bx)^2(d + ex)^{7/2} (21879a^5e^5 + 12155a^4be^4(-2d + 7ex) + 2210a^3b^2e^3(8d^2 - 28dex - 6d^2 + 7e^2x) + 510a^2b^3e^2(-16d^3 + 56d^2e^2x - 126de^2x^2 + 231e^3x^3) + 17ab^4e(128d^4 - 448d^3e^2x + 1008d^2e^2x^2 - 1848de^3x^3 + 3003e^4x^4) + b^5(-256d^5 + 896d^4e^2x - 2016d^3e^2x^2 + 3696d^2e^3x^3 - 6006de^4x^4 + 9009e^5x^5))}{(153153e^6(a + b^2x^2))}$$

input

```
Integrate[(d + e*x)^(5/2)*(a^2 + 2*a*b*x + b^2*x^2)^(5/2),x]
```

output

```
(2*Sqrt[(a + b*x)^2]*(d + e*x)^(7/2)*(21879*a^5*e^5 + 12155*a^4*b*e^4*(-2*
d + 7*e*x) + 2210*a^3*b^2*e^3*(8*d^2 - 28*d*e*x + 63*e^2*x^2) + 510*a^2*b^
3*e^2*(-16*d^3 + 56*d^2*e*x - 126*d*e^2*x^2 + 231*e^3*x^3) + 17*a*b^4*e*(1
28*d^4 - 448*d^3*e*x + 1008*d^2*e^2*x^2 - 1848*d*e^3*x^3 + 3003*e^4*x^4) +
b^5*(-256*d^5 + 896*d^4*e*x - 2016*d^3*e^2*x^2 + 3696*d^2*e^3*x^3 - 6006*
d*e^4*x^4 + 9009*e^5*x^5)))/(153153*e^6*(a + b*x))
```

Rubi [A] (verified)Time = 0.55 (sec) , antiderivative size = 186, normalized size of antiderivative = 0.58, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {1102, 27, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a^2 + 2abx + b^2x^2)^{5/2} (d + ex)^{5/2} dx$$

$$\begin{aligned}
 & \downarrow 1102 \\
 & \frac{\sqrt{a^2 + 2abx + b^2x^2} \int b^5(a + bx)^5(d + ex)^{5/2} dx}{b^5(a + bx)} \\
 & \downarrow 27 \\
 & \frac{\sqrt{a^2 + 2abx + b^2x^2} \int (a + bx)^5(d + ex)^{5/2} dx}{a + bx} \\
 & \downarrow 53 \\
 & \frac{\sqrt{a^2 + 2abx + b^2x^2} \int \left(\frac{b^5(d+ex)^{15/2}}{e^5} - \frac{5b^4(bd-ae)(d+ex)^{13/2}}{e^5} + \frac{10b^3(bd-ae)^2(d+ex)^{11/2}}{e^5} - \frac{10b^2(bd-ae)^3(d+ex)^{9/2}}{e^5} + \frac{5b(bd-ae)^4}{e^5} \right) dx}{a + bx} \\
 & \downarrow 2009 \\
 & \frac{\sqrt{a^2 + 2abx + b^2x^2} \left(-\frac{2b^4(d+ex)^{15/2}(bd-ae)}{3e^6} + \frac{20b^3(d+ex)^{13/2}(bd-ae)^2}{13e^6} - \frac{20b^2(d+ex)^{11/2}(bd-ae)^3}{11e^6} + \frac{10b(d+ex)^{9/2}(bd-ae)^4}{9e^6} - \dots \right)}{a + bx}
 \end{aligned}$$

input

```
Int[(d + e*x)^(5/2)*(a^2 + 2*a*b*x + b^2*x^2)^(5/2),x]
```

output

```
(Sqrt[a^2 + 2*a*b*x + b^2*x^2]*((-2*(b*d - a*e)^5*(d + e*x)^(7/2))/(7*e^6) + (10*b*(b*d - a*e)^4*(d + e*x)^(9/2))/(9*e^6) - (20*b^2*(b*d - a*e)^3*(d + e*x)^(11/2))/(11*e^6) + (20*b^3*(b*d - a*e)^2*(d + e*x)^(13/2))/(13*e^6) - (2*b^4*(b*d - a*e)*(d + e*x)^(15/2))/(3*e^6) + (2*b^5*(d + e*x)^(17/2))/(17*e^6)))/(a + b*x)
```

Defintions of rubi rules used

rule 27

```
Int[(a_)*(F_x_), x_Symbol] := Simp[a Int[F_x, x], x] /; FreeQ[a, x] && !MatchQ[F_x, (b_)*(G_x_)] /; FreeQ[b, x]
```

rule 53

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] := Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x] /; FreeQ[{a, b, c, d, n}, x] && IGtQ[m, 0] && (!IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])]
```


Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 497 vs. $2(230) = 460$.

Time = 0.10 (sec) , antiderivative size = 497, normalized size of antiderivative = 1.55

$$\int (d + ex)^{5/2} (a^2 + 2abx + b^2x^2)^{5/2} dx = \frac{2(9009b^5e^8x^8 - 256b^5d^8 + 2176ab^4d^7e - 8160a^2b^3d^6e^2 + 17680a^3b^2d^5e^3 - 24310a^4bd^4e^4 + 21879a^5d^3e^5 + 3003(7b^5d^4e^7 + 17a^2b^4d^3e^8)x^7 + 231(55b^5d^2e^6 + 527a^2b^4d^2e^7 + 510a^2b^3d^3e^8)x^6 + 63(b^5d^3e^5 + 1207a^2b^4d^2e^6 + 4590a^2b^3d^3e^7 + 2210a^3b^2d^4e^8)x^5 - 35(2b^5d^4e^4 - 17a^2b^4d^3e^5 - 5406a^2b^3d^2e^6 - 10166a^3b^2d^2e^7 - 2431a^4b^2e^8)x^4 + (80b^5d^5e^3 - 680a^2b^4d^4e^4 + 2550a^2b^3d^3e^5 + 249730a^3b^2d^2e^6 + 230945a^4b^2d^2e^7 + 21879a^5e^8)x^3 - 3(32b^5d^6e^2 - 272a^2b^4d^5e^3 + 1020a^2b^3d^4e^4 - 2210a^3b^2d^3e^5 - 60775a^4b^2d^2e^6 - 21879a^5d^2e^7)x^2 + (128b^5d^7e - 1088a^2b^4d^6e^2 + 4080a^2b^3d^5e^3 - 8840a^3b^2d^4e^4 + 12155a^4b^2d^3e^5 + 65637a^5d^2e^6)x) \sqrt{ex + d} / e^6$$

input `integrate((e*x+d)^(5/2)*(b^2*x^2+2*a*b*x+a^2)^(5/2),x, algorithm="fricas")`

output `2/153153*(9009*b^5*e^8*x^8 - 256*b^5*d^8 + 2176*a*b^4*d^7*e - 8160*a^2*b^3*d^6*e^2 + 17680*a^3*b^2*d^5*e^3 - 24310*a^4*b*d^4*e^4 + 21879*a^5*d^3*e^5 + 3003*(7*b^5*d^4*e^7 + 17*a*b^4*e^8)*x^7 + 231*(55*b^5*d^2*e^6 + 527*a*b^4*d^2*e^7 + 510*a^2*b^3*d^3*e^8)*x^6 + 63*(b^5*d^3*e^5 + 1207*a*b^4*d^2*e^6 + 4590*a^2*b^3*d^3*e^7 + 2210*a^3*b^2*d^4*e^8)*x^5 - 35*(2*b^5*d^4*e^4 - 17*a*b^4*d^3*e^5 - 5406*a^2*b^3*d^2*e^6 - 10166*a^3*b^2*d^2*e^7 - 2431*a^4*b^2*e^8)*x^4 + (80*b^5*d^5*e^3 - 680*a*b^4*d^4*e^4 + 2550*a^2*b^3*d^3*e^5 + 249730*a^3*b^2*d^2*e^6 + 230945*a^4*b*d^2*e^7 + 21879*a^5*e^8)*x^3 - 3*(32*b^5*d^6*e^2 - 272*a*b^4*d^5*e^3 + 1020*a^2*b^3*d^4*e^4 - 2210*a^3*b^2*d^3*e^5 - 60775*a^4*b^2*d^2*e^6 - 21879*a^5*d^2*e^7)*x^2 + (128*b^5*d^7*e - 1088*a*b^4*d^6*e^2 + 4080*a^2*b^3*d^5*e^3 - 8840*a^3*b^2*d^4*e^4 + 12155*a^4*b*d^3*e^5 + 65637*a^5*d^2*e^6)*x)*sqrt(e*x + d)/e^6`

Sympy [F(-1)]

Timed out.

$$\int (d + ex)^{5/2} (a^2 + 2abx + b^2x^2)^{5/2} dx = \text{Timed out}$$

input `integrate((e*x+d)**(5/2)*(b**2*x**2+2*a*b*x+a**2)**(5/2),x)`

output `Timed out`

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 497 vs. $2(230) = 460$.

Time = 0.04 (sec) , antiderivative size = 497, normalized size of antiderivative = 1.55

$$\int (d + ex)^{5/2} (a^2 + 2abx + b^2x^2)^{5/2} dx = \frac{2(9009b^5e^8x^8 - 256b^5d^8 + 2176ab^4d^7e - 8160a^2b^3d^6e^2 + 17680a^3b^2d^5e^3 - 24310a^4bd^4e^4 + 17680a^5d^3e^5 - 8160a^6d^2e^6 + 2176a^7de^7 - 256a^8e^8)}{(d + ex)^{3/2}}$$

input `integrate((e*x+d)^(5/2)*(b^2*x^2+2*a*b*x+a^2)^(5/2),x, algorithm="maxima")`

output `2/153153*(9009*b^5*e^8*x^8 - 256*b^5*d^8 + 2176*a*b^4*d^7*e - 8160*a^2*b^3*d^6*e^2 + 17680*a^3*b^2*d^5*e^3 - 24310*a^4*b*d^4*e^4 + 21879*a^5*d^3*e^5 + 3003*(7*b^5*d*e^7 + 17*a*b^4*e^8)*x^7 + 231*(55*b^5*d^2*e^6 + 527*a*b^4*d*e^7 + 510*a^2*b^3*e^8)*x^6 + 63*(b^5*d^3*e^5 + 1207*a*b^4*d^2*e^6 + 4590*a^2*b^3*d*e^7 + 2210*a^3*b^2*d^2*e^8)*x^5 - 35*(2*b^5*d^4*e^4 - 17*a*b^4*d^3*e^5 - 5406*a^2*b^3*d^2*e^6 - 10166*a^3*b^2*d*e^7 - 2431*a^4*b*d^2*e^8)*x^4 + (80*b^5*d^5*e^3 - 680*a*b^4*d^4*e^4 + 2550*a^2*b^3*d^3*e^5 + 249730*a^3*b^2*d^2*e^6 + 230945*a^4*b*d^2*e^7 + 21879*a^5*d^2*e^8)*x^3 - 3*(32*b^5*d^6*e^2 - 272*a*b^4*d^5*e^3 + 1020*a^2*b^3*d^4*e^4 - 2210*a^3*b^2*d^3*e^5 - 60775*a^4*b*d^2*e^6 - 21879*a^5*d^2*e^7)*x^2 + (128*b^5*d^7*e - 1088*a*b^4*d^6*e^2 + 4080*a^2*b^3*d^5*e^3 - 8840*a^3*b^2*d^4*e^4 + 12155*a^4*b*d^3*e^5 + 65637*a^5*d^2*e^6)*x)*sqrt(e*x + d)/e^6`

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1743 vs. $2(230) = 460$.

Time = 0.19 (sec) , antiderivative size = 1743, normalized size of antiderivative = 5.45

$$\int (d + ex)^{5/2} (a^2 + 2abx + b^2x^2)^{5/2} dx = \text{Too large to display}$$

input `integrate((e*x+d)^(5/2)*(b^2*x^2+2*a*b*x+a^2)^(5/2),x, algorithm="giac")`

output

```

2/765765*(765765*sqrt(e*x + d)*a^5*d^3*sgn(b*x + a) + 765765*((e*x + d)^(3
/2) - 3*sqrt(e*x + d)*d)*a^5*d^2*sgn(b*x + a) + 1276275*((e*x + d)^(3/2) -
3*sqrt(e*x + d)*d)*a^4*b*d^3*sgn(b*x + a)/e + 153153*(3*(e*x + d)^(5/2) -
10*(e*x + d)^(3/2)*d + 15*sqrt(e*x + d)*d^2)*a^5*d*sgn(b*x + a) + 510510*
(3*(e*x + d)^(5/2) - 10*(e*x + d)^(3/2)*d + 15*sqrt(e*x + d)*d^2)*a^3*b^2*
d^3*sgn(b*x + a)/e^2 + 765765*(3*(e*x + d)^(5/2) - 10*(e*x + d)^(3/2)*d +
15*sqrt(e*x + d)*d^2)*a^4*b*d^2*sgn(b*x + a)/e + 21879*(5*(e*x + d)^(7/2)
- 21*(e*x + d)^(5/2)*d + 35*(e*x + d)^(3/2)*d^2 - 35*sqrt(e*x + d)*d^3)*a^
5*sgn(b*x + a) + 218790*(5*(e*x + d)^(7/2) - 21*(e*x + d)^(5/2)*d + 35*(e*
x + d)^(3/2)*d^2 - 35*sqrt(e*x + d)*d^3)*a^2*b^3*d^3*sgn(b*x + a)/e^3 + 65
6370*(5*(e*x + d)^(7/2) - 21*(e*x + d)^(5/2)*d + 35*(e*x + d)^(3/2)*d^2 -
35*sqrt(e*x + d)*d^3)*a^3*b^2*d^2*sgn(b*x + a)/e^2 + 328185*(5*(e*x + d)^(
7/2) - 21*(e*x + d)^(5/2)*d + 35*(e*x + d)^(3/2)*d^2 - 35*sqrt(e*x + d)*d^
3)*a^4*b*d*sgn(b*x + a)/e + 12155*(35*(e*x + d)^(9/2) - 180*(e*x + d)^(7/2)
)*d + 378*(e*x + d)^(5/2)*d^2 - 420*(e*x + d)^(3/2)*d^3 + 315*sqrt(e*x + d)
)*d^4)*a*b^4*d^3*sgn(b*x + a)/e^4 + 72930*(35*(e*x + d)^(9/2) - 180*(e*x +
d)^(7/2)*d + 378*(e*x + d)^(5/2)*d^2 - 420*(e*x + d)^(3/2)*d^3 + 315*sqrt
(e*x + d)*d^4)*a^2*b^3*d^2*sgn(b*x + a)/e^3 + 72930*(35*(e*x + d)^(9/2) -
180*(e*x + d)^(7/2)*d + 378*(e*x + d)^(5/2)*d^2 - 420*(e*x + d)^(3/2)*d^3
+ 315*sqrt(e*x + d)*d^4)*a^3*b^2*d*sgn(b*x + a)/e^2 + 12155*(35*(e*x + ...

```

Mupad [F(-1)]

Timed out.

$$\int (d + ex)^{5/2} (a^2 + 2abx + b^2x^2)^{5/2} dx = \int (d + ex)^{5/2} (a^2 + 2abx + b^2x^2)^{5/2} dx$$

input

```
int((d + e*x)^(5/2)*(a^2 + b^2*x^2 + 2*a*b*x)^(5/2),x)
```

output

```
int((d + e*x)^(5/2)*(a^2 + b^2*x^2 + 2*a*b*x)^(5/2), x)
```


3.361 $\int (d + ex)^{3/2} (a^2 + 2abx + b^2x^2)^{5/2} dx$

| | |
|---|------|
| Optimal result | 2664 |
| Mathematica [A] (verified) | 2665 |
| Rubi [A] (verified) | 2665 |
| Maple [A] (verified) | 2667 |
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| Mupad [F(-1)] | 2670 |
| Reduce [B] (verification not implemented) | 2671 |

Optimal result

Integrand size = 30, antiderivative size = 320

$$\begin{aligned}
 & \int (d + ex)^{3/2} (a^2 + 2abx + b^2x^2)^{5/2} dx = \\
 & - \frac{2(bd - ae)^5 (d + ex)^{5/2} \sqrt{a^2 + 2abx + b^2x^2}}{5e^6(a + bx)} \\
 & + \frac{10b(bd - ae)^4 (d + ex)^{7/2} \sqrt{a^2 + 2abx + b^2x^2}}{7e^6(a + bx)} \\
 & - \frac{20b^2(bd - ae)^3 (d + ex)^{9/2} \sqrt{a^2 + 2abx + b^2x^2}}{9e^6(a + bx)} \\
 & + \frac{20b^3(bd - ae)^2 (d + ex)^{11/2} \sqrt{a^2 + 2abx + b^2x^2}}{11e^6(a + bx)} \\
 & - \frac{10b^4(bd - ae) (d + ex)^{13/2} \sqrt{a^2 + 2abx + b^2x^2}}{13e^6(a + bx)} \\
 & + \frac{2b^5 (d + ex)^{15/2} \sqrt{a^2 + 2abx + b^2x^2}}{15e^6(a + bx)}
 \end{aligned}$$

output

```
-2/5*(-a*e+b*d)^5*(e*x+d)^(5/2)*((b*x+a)^2)^(1/2)/e^6/(b*x+a)+10/7*b*(-a*e
+b*d)^4*(e*x+d)^(7/2)*((b*x+a)^2)^(1/2)/e^6/(b*x+a)-20/9*b^2*(-a*e+b*d)^3*
(e*x+d)^(9/2)*((b*x+a)^2)^(1/2)/e^6/(b*x+a)+20/11*b^3*(-a*e+b*d)^2*(e*x+d)
^(11/2)*((b*x+a)^2)^(1/2)/e^6/(b*x+a)-10/13*b^4*(-a*e+b*d)*(e*x+d)^(13/2)*
((b*x+a)^2)^(1/2)/e^6/(b*x+a)+2/15*b^5*(e*x+d)^(15/2)*((b*x+a)^2)^(1/2)/e^
6/(b*x+a)
```

Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 235, normalized size of antiderivative = 0.73

$$\int (d + ex)^{3/2} (a^2 + 2abx + b^2x^2)^{5/2} dx = \frac{2\sqrt{(a + bx)^2(d + ex)^{5/2}} (9009a^5e^5 + 6435a^4be^4(-2d + 5ex) + 1430a^3b^2e^3(8d^2 - 20dex + 3d^2 - 20dex + 3d^2))}{(45045e^6(a + bx))}$$

input

```
Integrate[(d + e*x)^(3/2)*(a^2 + 2*a*b*x + b^2*x^2)^(5/2), x]
```

output

```
(2*Sqrt[(a + b*x)^2]*(d + e*x)^(5/2)*(9009*a^5*e^5 + 6435*a^4*b*e^4*(-2*d
+ 5*e*x) + 1430*a^3*b^2*e^3*(8*d^2 - 20*d*e*x + 35*e^2*x^2) + 390*a^2*b^3*
e^2*(-16*d^3 + 40*d^2*e*x - 70*d*e^2*x^2 + 105*e^3*x^3) + 15*a*b^4*e*(128*
d^4 - 320*d^3*e*x + 560*d^2*e^2*x^2 - 840*d*e^3*x^3 + 1155*e^4*x^4) + b^5*
(-256*d^5 + 640*d^4*e*x - 1120*d^3*e^2*x^2 + 1680*d^2*e^3*x^3 - 2310*d*e^4
*x^4 + 3003*e^5*x^5)))/(45045*e^6*(a + b*x))
```

Rubi [A] (verified)Time = 0.52 (sec) , antiderivative size = 186, normalized size of antiderivative = 0.58, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {1102, 27, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a^2 + 2abx + b^2x^2)^{5/2} (d + ex)^{3/2} dx$$

$$\begin{array}{c}
 \downarrow 1102 \\
 \frac{\sqrt{a^2 + 2abx + b^2x^2} \int b^5(a + bx)^5(d + ex)^{3/2} dx}{b^5(a + bx)} \\
 \downarrow 27 \\
 \frac{\sqrt{a^2 + 2abx + b^2x^2} \int (a + bx)^5(d + ex)^{3/2} dx}{a + bx} \\
 \downarrow 53 \\
 \frac{\sqrt{a^2 + 2abx + b^2x^2} \int \left(\frac{b^5(d+ex)^{13/2}}{e^5} - \frac{5b^4(bd-ae)(d+ex)^{11/2}}{e^5} + \frac{10b^3(bd-ae)^2(d+ex)^{9/2}}{e^5} - \frac{10b^2(bd-ae)^3(d+ex)^{7/2}}{e^5} + \frac{5b(bd-ae)^4}{e} \right)}{a + bx} \\
 \downarrow 2009 \\
 \frac{\sqrt{a^2 + 2abx + b^2x^2} \left(-\frac{10b^4(d+ex)^{13/2}(bd-ae)}{13e^6} + \frac{20b^3(d+ex)^{11/2}(bd-ae)^2}{11e^6} - \frac{20b^2(d+ex)^{9/2}(bd-ae)^3}{9e^6} + \frac{10b(d+ex)^{7/2}(bd-ae)^4}{7e^6} \right)}{a + bx}
 \end{array}$$

input `Int[(d + e*x)^(3/2)*(a^2 + 2*a*b*x + b^2*x^2)^(5/2),x]`

output `(Sqrt[a^2 + 2*a*b*x + b^2*x^2]*((-2*(b*d - a*e)^5*(d + e*x)^(5/2))/(5*e^6) + (10*b*(b*d - a*e)^4*(d + e*x)^(7/2))/(7*e^6) - (20*b^2*(b*d - a*e)^3*(d + e*x)^(9/2))/(9*e^6) + (20*b^3*(b*d - a*e)^2*(d + e*x)^(11/2))/(11*e^6) - (10*b^4*(b*d - a*e)*(d + e*x)^(13/2))/(13*e^6) + (2*b^5*(d + e*x)^(15/2))/(15*e^6))/(a + b*x)`

Defintions of rubi rules used

rule 27 `Int[(a_)*(F_x_), x_Symbol] := Simp[a Int[F_x, x], x] /; FreeQ[a, x] && !MatchQ[F_x, (b_)*(G_x_)] /; FreeQ[b, x]`

rule 53 `Int[((a_.) + (b_.)*(x_.))^(m_.)*((c_.) + (d_.)*(x_.))^(n_.), x_Symbol] := Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x] /; FreeQ[{a, b, c, d, n}, x] && IGtQ[m, 0] && (!IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])]`

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 418, normalized size of antiderivative = 1.31

$$\int (d + ex)^{3/2} (a^2 + 2abx + b^2x^2)^{5/2} dx = \frac{2(3003b^5e^7x^7 - 256b^5d^7 + 1920ab^4d^6e - 6240a^2b^3d^5e^2 + 11440a^3b^2d^4e^3 - 12870a^4bd^3e^4 + 9009a^5d^2e^5 + 231(16b^5d^6e + 75a^2b^4e^7)x^6 + 63(b^5d^2e^5 + 350a^2b^4d^2e^6 + 650a^2b^3e^7)x^5 - 35(2b^5d^3e^4 - 15a^2b^4d^2e^5 - 1560a^2b^3d^2e^6 - 1430a^3b^2e^7)x^4 + 5(16b^5d^4e^3 - 120a^2b^4d^3e^4 + 390a^2b^3d^2e^5 + 14300a^3b^2d^2e^6 + 6435a^4b^2e^7)x^3 - 3(32b^5d^5e^2 - 240a^2b^4d^4e^3 + 780a^2b^3d^3e^4 - 1430a^3b^2d^2e^5 - 17160a^4b^2d^2e^6 - 3003a^5e^7)x^2 + (128b^5d^6e - 960a^2b^4d^5e^2 + 3120a^2b^3d^4e^3 - 5720a^3b^2d^3e^4 + 6435a^4b^2d^2e^5 + 18018a^5d^2e^6)x) \sqrt{ex + d} / e^6$$

input `integrate((e*x+d)^(3/2)*(b^2*x^2+2*a*b*x+a^2)^(5/2),x, algorithm="fricas")`

output `2/45045*(3003*b^5*e^7*x^7 - 256*b^5*d^7 + 1920*a*b^4*d^6*e - 6240*a^2*b^3*d^5*e^2 + 11440*a^3*b^2*d^4*e^3 - 12870*a^4*b*d^3*e^4 + 9009*a^5*d^2*e^5 + 231*(16*b^5*d^6*e + 75*a*b^4*e^7)*x^6 + 63*(b^5*d^2*e^5 + 350*a*b^4*d^2*e^6 + 650*a^2*b^3*e^7)*x^5 - 35*(2*b^5*d^3*e^4 - 15*a*b^4*d^2*e^5 - 1560*a^2*b^3*d^2*e^6 - 1430*a^3*b^2*e^7)*x^4 + 5*(16*b^5*d^4*e^3 - 120*a*b^4*d^3*e^4 + 390*a^2*b^3*d^2*e^5 + 14300*a^3*b^2*d^2*e^6 + 6435*a^4*b^2*e^7)*x^3 - 3*(32*b^5*d^5*e^2 - 240*a*b^4*d^4*e^3 + 780*a^2*b^3*d^3*e^4 - 1430*a^3*b^2*d^2*e^5 - 17160*a^4*b^2*d^2*e^6 - 3003*a^5*e^7)*x^2 + (128*b^5*d^6*e - 960*a^2*b^4*d^5*e^2 + 3120*a^2*b^3*d^4*e^3 - 5720*a^3*b^2*d^3*e^4 + 6435*a^4*b^2*d^2*e^5 + 18018*a^5*d^2*e^6)*x)*sqrt(e*x + d)/e^6`

Sympy [F]

$$\int (d + ex)^{3/2} (a^2 + 2abx + b^2x^2)^{5/2} dx = \int (d + ex)^{\frac{3}{2}} ((a + bx)^2)^{\frac{5}{2}} dx$$

input `integrate((e*x+d)**(3/2)*(b**2*x**2+2*a*b*x+a**2)**(5/2),x)`

output `Integral((d + e*x)**(3/2)*((a + b*x)**2)**(5/2), x)`

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 418, normalized size of antiderivative = 1.31

$$\int (d + ex)^{3/2} (a^2 + 2abx + b^2x^2)^{5/2} dx = \frac{2(3003b^5e^7x^7 - 256b^5d^7 + 1920ab^4d^6e - 6240a^2b^3d^5e^2 + 11440a^3b^2d^4e^3 - 12870a^4bd^3e^4 + 9009a^5d^2e^5 + 231(16b^5d^6e + 75a^2b^4e^7)x^6 + 63(b^5d^2e^5 + 350a^2b^4d^2e^6 + 650a^2b^3e^7)x^5 - 35(2b^5d^3e^4 - 15a^2b^4d^2e^5 - 1560a^2b^3d^2e^6 - 1430a^3b^2e^7)x^4 + 5(16b^5d^4e^3 - 120a^2b^4d^3e^4 + 390a^2b^3d^2e^5 + 14300a^3b^2d^2e^6 + 6435a^4b^2e^7)x^3 - 3(32b^5d^5e^2 - 240a^2b^4d^4e^3 + 780a^2b^3d^3e^4 - 1430a^3b^2d^2e^5 - 17160a^4bd^2e^6 - 3003a^5e^7)x^2 + (128b^5d^6e - 960a^2b^4d^5e^2 + 3120a^2b^3d^4e^3 - 5720a^3b^2d^3e^4 + 6435a^4bd^2e^5 + 18018a^5d^2e^6)x}{e^6} \sqrt{ex + d}$$

input `integrate((e*x+d)^(3/2)*(b^2*x^2+2*a*b*x+a^2)^(5/2),x, algorithm="maxima")`

output `2/45045*(3003*b^5*e^7*x^7 - 256*b^5*d^7 + 1920*a*b^4*d^6*e - 6240*a^2*b^3*d^5*e^2 + 11440*a^3*b^2*d^4*e^3 - 12870*a^4*b*d^3*e^4 + 9009*a^5*d^2*e^5 + 231*(16*b^5*d^6*e + 75*a*b^4*e^7)*x^6 + 63*(b^5*d^2*e^5 + 350*a*b^4*d^2*e^6 + 650*a^2*b^3*e^7)*x^5 - 35*(2*b^5*d^3*e^4 - 15*a*b^4*d^2*e^5 - 1560*a^2*b^3*d^2*e^6 - 1430*a^3*b^2*e^7)*x^4 + 5*(16*b^5*d^4*e^3 - 120*a*b^4*d^3*e^4 + 390*a^2*b^3*d^2*e^5 + 14300*a^3*b^2*d^2*e^6 + 6435*a^4*b^2*e^7)*x^3 - 3*(32*b^5*d^5*e^2 - 240*a*b^4*d^4*e^3 + 780*a^2*b^3*d^3*e^4 - 1430*a^3*b^2*d^2*e^5 - 17160*a^4*b*d^2*e^6 - 3003*a^5*e^7)*x^2 + (128*b^5*d^6*e - 960*a*b^4*d^5*e^2 + 3120*a^2*b^3*d^4*e^3 - 5720*a^3*b^2*d^3*e^4 + 6435*a^4*b*d^2*e^5 + 18018*a^5*d^2*e^6)*x)*sqrt(e*x + d)/e^6`

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1192 vs. 2(230) = 460.

Time = 0.17 (sec) , antiderivative size = 1192, normalized size of antiderivative = 3.72

$$\int (d + ex)^{3/2} (a^2 + 2abx + b^2x^2)^{5/2} dx = \text{Too large to display}$$

input `integrate((e*x+d)^(3/2)*(b^2*x^2+2*a*b*x+a^2)^(5/2),x, algorithm="giac")`

output

```

2/45045*(45045*sqrt(e*x + d)*a^5*d^2*sgn(b*x + a) + 30030*((e*x + d)^(3/2)
- 3*sqrt(e*x + d)*d)*a^5*d*sgn(b*x + a) + 75075*((e*x + d)^(3/2) - 3*sqrt
(e*x + d)*d)*a^4*b*d^2*sgn(b*x + a)/e + 3003*(3*(e*x + d)^(5/2) - 10*(e*x
+ d)^(3/2)*d + 15*sqrt(e*x + d)*d^2)*a^5*sgn(b*x + a) + 30030*(3*(e*x + d)
^(5/2) - 10*(e*x + d)^(3/2)*d + 15*sqrt(e*x + d)*d^2)*a^3*b^2*d^2*sgn(b*x
+ a)/e^2 + 30030*(3*(e*x + d)^(5/2) - 10*(e*x + d)^(3/2)*d + 15*sqrt(e*x +
d)*d^2)*a^4*b*d*sgn(b*x + a)/e + 12870*(5*(e*x + d)^(7/2) - 21*(e*x + d)^(
5/2)*d + 35*(e*x + d)^(3/2)*d^2 - 35*sqrt(e*x + d)*d^3)*a^2*b^3*d^2*sgn(b
*x + a)/e^3 + 25740*(5*(e*x + d)^(7/2) - 21*(e*x + d)^(5/2)*d + 35*(e*x +
d)^(3/2)*d^2 - 35*sqrt(e*x + d)*d^3)*a^3*b^2*d*sgn(b*x + a)/e^2 + 6435*(5*
(e*x + d)^(7/2) - 21*(e*x + d)^(5/2)*d + 35*(e*x + d)^(3/2)*d^2 - 35*sqrt(
e*x + d)*d^3)*a^4*b*sgn(b*x + a)/e + 715*(35*(e*x + d)^(9/2) - 180*(e*x +
d)^(7/2)*d + 378*(e*x + d)^(5/2)*d^2 - 420*(e*x + d)^(3/2)*d^3 + 315*sqrt(
e*x + d)*d^4)*a*b^4*d^2*sgn(b*x + a)/e^4 + 2860*(35*(e*x + d)^(9/2) - 180*
(e*x + d)^(7/2)*d + 378*(e*x + d)^(5/2)*d^2 - 420*(e*x + d)^(3/2)*d^3 + 31
5*sqrt(e*x + d)*d^4)*a^2*b^3*d*sgn(b*x + a)/e^3 + 1430*(35*(e*x + d)^(9/2)
- 180*(e*x + d)^(7/2)*d + 378*(e*x + d)^(5/2)*d^2 - 420*(e*x + d)^(3/2)*d
^3 + 315*sqrt(e*x + d)*d^4)*a^3*b^2*sgn(b*x + a)/e^2 + 65*(63*(e*x + d)^(1
1/2) - 385*(e*x + d)^(9/2)*d + 990*(e*x + d)^(7/2)*d^2 - 1386*(e*x + d)^(5
/2)*d^3 + 1155*(e*x + d)^(3/2)*d^4 - 693*sqrt(e*x + d)*d^5)*b^5*d^2*sgn...

```

Mupad [F(-1)]

Timed out.

$$\int (d + ex)^{3/2} (a^2 + 2abx + b^2x^2)^{5/2} dx = \int (d + ex)^{3/2} (a^2 + 2abx + b^2x^2)^{5/2} dx$$

input

```
int((d + e*x)^(3/2)*(a^2 + b^2*x^2 + 2*a*b*x)^(5/2), x)
```

output

```
int((d + e*x)^(3/2)*(a^2 + b^2*x^2 + 2*a*b*x)^(5/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 451, normalized size of antiderivative = 1.41

$$\int (d + ex)^{3/2} (a^2 + 2abx + b^2x^2)^{5/2} dx = \frac{2\sqrt{ex + d} (3003b^5e^7x^7 + 17325ab^4e^7x^6 + 3696b^5de^6x^6 + 40950a^2b^3e^7x^5 + 22050ab^4de^6x^5 + 12870a^4bd^3e^4x^4 + 6435a^4bd^2e^5x^3 + 51480a^4bd^2e^5x^3 + 32175a^4b^2e^7x^3 + 11440a^3b^2d^4e^3x^2 - 5720a^3b^2d^3e^4x + 4290a^3b^2d^2e^5x^2 + 71500a^3b^2de^6x^3 + 50050a^3b^2e^7x^4 - 6240a^2b^3d^5e^2 + 3120a^2b^3d^4e^3x - 2340a^2b^3d^3e^4x^2 + 1950a^2b^3d^2e^5x^3 + 54600a^2b^3de^6x^4 + 40950a^2b^3e^7x^5 + 1920ab^4d^6e - 960ab^4d^5e^2x + 720ab^4d^4e^3x^2 - 600ab^4d^3e^4x^3 + 525ab^4d^2e^5x^4 + 22050ab^4de^6x^5 + 17325ab^4e^7x^6 - 256b^5d^7 + 128b^5d^6ex - 96b^5d^5e^2x^2 + 80b^5d^4e^3x^3 - 70b^5d^3e^4x^4 + 63b^5d^2e^5x^5 + 3696b^5de^6x^6 + 3003b^5e^7x^7)}{(45045e^6)}$$

input

```
int((e*x+d)^(3/2)*(b^2*x^2+2*a*b*x+a^2)^(5/2),x)
```

output

```
(2*sqrt(d + e*x)*(9009*a**5*d**2*e**5 + 18018*a**5*d*e**6*x + 9009*a**5*e*
*7*x**2 - 12870*a**4*b*d**3*e**4 + 6435*a**4*b*d**2*e**5*x + 51480*a**4*b*
d*e**6*x**2 + 32175*a**4*b*e**7*x**3 + 11440*a**3*b**2*d**4*e**3 - 5720*a*
*3*b**2*d**3*e**4*x + 4290*a**3*b**2*d**2*e**5*x**2 + 71500*a**3*b**2*d*e*
*6*x**3 + 50050*a**3*b**2*e**7*x**4 - 6240*a**2*b**3*d**5*e**2 + 3120*a**2
*b**3*d**4*e**3*x - 2340*a**2*b**3*d**3*e**4*x**2 + 1950*a**2*b**3*d**2*e*
*5*x**3 + 54600*a**2*b**3*d*e**6*x**4 + 40950*a**2*b**3*e**7*x**5 + 1920*a
*b**4*d**6*e - 960*a*b**4*d**5*e**2*x + 720*a*b**4*d**4*e**3*x**2 - 600*a*
b**4*d**3*e**4*x**3 + 525*a*b**4*d**2*e**5*x**4 + 22050*a*b**4*d*e**6*x**5
+ 17325*a*b**4*e**7*x**6 - 256*b**5*d**7 + 128*b**5*d**6*e*x - 96*b**5*d*
*5*e**2*x**2 + 80*b**5*d**4*e**3*x**3 - 70*b**5*d**3*e**4*x**4 + 63*b**5*d
**2*e**5*x**5 + 3696*b**5*d*e**6*x**6 + 3003*b**5*e**7*x**7))/(45045*e**6)
```

3.362 $\int \sqrt{d + ex}(a^2 + 2abx + b^2x^2)^{5/2} dx$

| | |
|---|------|
| Optimal result | 2672 |
| Mathematica [A] (verified) | 2673 |
| Rubi [A] (verified) | 2673 |
| Maple [A] (verified) | 2675 |
| Fricas [A] (verification not implemented) | 2676 |
| Sympy [F] | 2676 |
| Maxima [A] (verification not implemented) | 2677 |
| Giac [B] (verification not implemented) | 2677 |
| Mupad [F(-1)] | 2678 |
| Reduce [B] (verification not implemented) | 2679 |

Optimal result

Integrand size = 30, antiderivative size = 318

$$\begin{aligned}
 & \int \sqrt{d + ex}(a^2 + 2abx + b^2x^2)^{5/2} dx = \\
 & \quad - \frac{2(bd - ae)^5(d + ex)^{3/2}\sqrt{a^2 + 2abx + b^2x^2}}{3e^6(a + bx)} \\
 & \quad + \frac{2b(bd - ae)^4(d + ex)^{5/2}\sqrt{a^2 + 2abx + b^2x^2}}{e^6(a + bx)} \\
 & \quad - \frac{20b^2(bd - ae)^3(d + ex)^{7/2}\sqrt{a^2 + 2abx + b^2x^2}}{7e^6(a + bx)} \\
 & \quad + \frac{20b^3(bd - ae)^2(d + ex)^{9/2}\sqrt{a^2 + 2abx + b^2x^2}}{9e^6(a + bx)} \\
 & \quad - \frac{10b^4(bd - ae)(d + ex)^{11/2}\sqrt{a^2 + 2abx + b^2x^2}}{11e^6(a + bx)} \\
 & \quad + \frac{2b^5(d + ex)^{13/2}\sqrt{a^2 + 2abx + b^2x^2}}{13e^6(a + bx)}
 \end{aligned}$$

output

```
-2/3*(-a*e+b*d)^5*(e*x+d)^(3/2)*((b*x+a)^2)^(1/2)/e^6/(b*x+a)+2*b*(-a*e+b*d)^4*(e*x+d)^(5/2)*((b*x+a)^2)^(1/2)/e^6/(b*x+a)-20/7*b^2*(-a*e+b*d)^3*(e*x+d)^(7/2)*((b*x+a)^2)^(1/2)/e^6/(b*x+a)+20/9*b^3*(-a*e+b*d)^2*(e*x+d)^(9/2)*((b*x+a)^2)^(1/2)/e^6/(b*x+a)-10/11*b^4*(-a*e+b*d)*(e*x+d)^(11/2)*((b*x+a)^2)^(1/2)/e^6/(b*x+a)+2/13*b^5*(e*x+d)^(13/2)*((b*x+a)^2)^(1/2)/e^6/(b*x+a)
```

Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 235, normalized size of antiderivative = 0.74

$$\int \sqrt{d+ex}(a^2+2abx+b^2x^2)^{5/2} dx = \frac{2\sqrt{(a+bx)^2(d+ex)^{3/2}(3003a^5e^5+3003a^4be^4(-2d+3ex)+858a^3b^2e^3(8d^2-12dex+15d^2))}}{9009e^6(a+bx)}$$

input

```
Integrate[Sqrt[d + e*x]*(a^2 + 2*a*b*x + b^2*x^2)^(5/2), x]
```

output

```
(2*Sqrt[(a + b*x)^2]*(d + e*x)^(3/2)*(3003*a^5*e^5 + 3003*a^4*b*e^4*(-2*d + 3*e*x) + 858*a^3*b^2*e^3*(8*d^2 - 12*d*e*x + 15*e^2*x^2) + 286*a^2*b^3*e^2*(-16*d^3 + 24*d^2*e*x - 30*d*e^2*x^2 + 35*e^3*x^3) + 13*a*b^4*e*(128*d^4 - 192*d^3*e*x + 240*d^2*e^2*x^2 - 280*d*e^3*x^3 + 315*e^4*x^4) + b^5*(-256*d^5 + 384*d^4*e*x - 480*d^3*e^2*x^2 + 560*d^2*e^3*x^3 - 630*d*e^4*x^4 + 693*e^5*x^5)))/(9009*e^6*(a + b*x))
```

Rubi [A] (verified)

Time = 0.54 (sec) , antiderivative size = 184, normalized size of antiderivative = 0.58, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {1102, 27, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a^2 + 2abx + b^2x^2)^{5/2} \sqrt{d+ex} dx$$

$$\begin{array}{c}
 \downarrow 1102 \\
 \frac{\sqrt{a^2 + 2abx + b^2x^2} \int b^5(a + bx)^5 \sqrt{d + ex} dx}{b^5(a + bx)} \\
 \downarrow 27 \\
 \frac{\sqrt{a^2 + 2abx + b^2x^2} \int (a + bx)^5 \sqrt{d + ex} dx}{a + bx} \\
 \downarrow 53 \\
 \frac{\sqrt{a^2 + 2abx + b^2x^2} \int \left(\frac{b^5(d+ex)^{11/2}}{e^5} - \frac{5b^4(bd-ae)(d+ex)^{9/2}}{e^5} + \frac{10b^3(bd-ae)^2(d+ex)^{7/2}}{e^5} - \frac{10b^2(bd-ae)^3(d+ex)^{5/2}}{e^5} + \frac{5b(bd-ae)^4}{e^5} \right)}{a + bx} \\
 \downarrow 2009 \\
 \frac{\sqrt{a^2 + 2abx + b^2x^2} \left(-\frac{10b^4(d+ex)^{11/2}(bd-ae)}{11e^6} + \frac{20b^3(d+ex)^{9/2}(bd-ae)^2}{9e^6} - \frac{20b^2(d+ex)^{7/2}(bd-ae)^3}{7e^6} + \frac{2b(d+ex)^{5/2}(bd-ae)^4}{e^6} - \frac{2(bd-ae)^5}{e^6} \right)}{a + bx}
 \end{array}$$

input

```
Int[Sqrt[d + e*x]*(a^2 + 2*a*b*x + b^2*x^2)^(5/2),x]
```

output

```
(Sqrt[a^2 + 2*a*b*x + b^2*x^2]*((-2*(b*d - a*e)^5*(d + e*x)^(3/2))/(3*e^6)
+ (2*b*(b*d - a*e)^4*(d + e*x)^(5/2))/e^6 - (20*b^2*(b*d - a*e)^3*(d + e
x)^(7/2))/(7*e^6) + (20*b^3*(b*d - a*e)^2*(d + e*x)^(9/2))/(9*e^6) - (10*b
^4*(b*d - a*e)*(d + e*x)^(11/2))/(11*e^6) + (2*b^5*(d + e*x)^(13/2))/(13*e
^6)))/(a + b*x)
```

Defintions of rubi rules used

rule 27

```
Int[(a_)*(F_x_), x_Symbol] := Simp[a Int[F_x, x], x] /; FreeQ[a, x] && !Ma
tchQ[F_x, (b_)*(G_x_) /; FreeQ[b, x]]
```

rule 53

```
Int[((a_.) + (b_.)*(x_.))^(m_.)*((c_.) + (d_.)*(x_.))^(n_.), x_Symbol] := Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && (!IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```


Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 338, normalized size of antiderivative = 1.06

$$\int \sqrt{d+ex}(a^2+2abx+b^2x^2)^{5/2} dx = \frac{2(693b^5e^6x^6 - 256b^5d^6 + 1664ab^4d^5e - 4576a^2b^3d^4e^2 + 6864a^3b^2d^3e^3 - 6006a^4bd^2e^4 + 3003a^5d^2e^5 + 63(b^5d^2e^5 + 65ab^4d^2e^6)x^5 - 35(2b^5d^2e^4 - 13ab^4d^2e^5 - 286a^2b^3d^2e^6)x^4 + 10(8b^5d^3e^3 - 52ab^4d^2e^4 + 143a^2b^3d^2e^5 + 1287a^3b^2d^2e^6)x^3 - 3(32b^5d^4e^2 - 208ab^4d^3e^3 + 572a^2b^3d^2e^4 - 858a^3b^2d^2e^5 - 3003a^4b^2e^6)x^2 + (128b^5d^5e - 832ab^4d^4e^2 + 2288a^2b^3d^3e^3 - 3432a^3b^2d^2e^4 + 3003a^4b^2d^2e^5 + 3003a^5e^6)x)\sqrt{ex+d}}{e^6}$$

input `integrate((e*x+d)^(1/2)*(b^2*x^2+2*a*b*x+a^2)^(5/2),x, algorithm="fricas")`

output `2/9009*(693*b^5*e^6*x^6 - 256*b^5*d^6 + 1664*a*b^4*d^5*e - 4576*a^2*b^3*d^4*e^2 + 6864*a^3*b^2*d^3*e^3 - 6006*a^4*b*d^2*e^4 + 3003*a^5*d^2*e^5 + 63*(b^5*d^2*e^5 + 65*a*b^4*d^2*e^6)*x^5 - 35*(2*b^5*d^2*e^4 - 13*a*b^4*d^2*e^5 - 286*a^2*b^3*d^2*e^6)*x^4 + 10*(8*b^5*d^3*e^3 - 52*a*b^4*d^2*e^4 + 143*a^2*b^3*d^2*e^5 + 1287*a^3*b^2*d^2*e^6)*x^3 - 3*(32*b^5*d^4*e^2 - 208*a*b^4*d^3*e^3 + 572*a^2*b^3*d^2*e^4 - 858*a^3*b^2*d^2*e^5 - 3003*a^4*b^2*e^6)*x^2 + (128*b^5*d^5*e - 832*a*b^4*d^4*e^2 + 2288*a^2*b^3*d^3*e^3 - 3432*a^3*b^2*d^2*e^4 + 3003*a^4*b^2*d^2*e^5 + 3003*a^5*e^6)*x)*sqrt(e*x + d)/e^6`

Sympy [F]

$$\int \sqrt{d+ex}(a^2+2abx+b^2x^2)^{5/2} dx = \int \sqrt{d+ex}((a+bx)^2)^{5/2} dx$$

input `integrate((e*x+d)**(1/2)*(b**2*x**2+2*a*b*x+a**2)**(5/2),x)`

output `Integral(sqrt(d + e*x)*((a + b*x)**2)**(5/2), x)`

Maxima [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 338, normalized size of antiderivative = 1.06

$$\int \sqrt{d+ex}(a^2+2abx+b^2x^2)^{5/2} dx = \frac{2(693b^5e^6x^6 - 256b^5d^6 + 1664ab^4d^5e - 4576a^2b^3d^4e^2 + 6864a^3b^2d^3e^3 - 6006a^4bd^2e^4 + 3003a^5d^2e^5 + 63(b^5d^2e^5 + 65ab^4d^2e^6)x^5 - 35(2b^5d^2e^4 - 13ab^4d^2e^5 - 286a^2b^3d^2e^6)x^4 + 10(8b^5d^3e^3 - 52ab^4d^2e^4 + 143a^2b^3d^2e^5 + 1287a^3b^2d^2e^6)x^3 - 3(32b^5d^4e^2 - 208ab^4d^3e^3 + 572a^2b^3d^2e^4 - 858a^3b^2d^2e^5 - 3003a^4b^2e^6)x^2 + (128b^5d^5e - 832ab^4d^4e^2 + 2288a^2b^3d^3e^3 - 3432a^3b^2d^2e^4 + 3003a^4b^2d^2e^5 + 3003a^5e^6)x)\sqrt{ex+d}}{e^6}$$

input `integrate((e*x+d)^(1/2)*(b^2*x^2+2*a*b*x+a^2)^(5/2),x, algorithm="maxima")`

output `2/9009*(693*b^5*e^6*x^6 - 256*b^5*d^6 + 1664*a*b^4*d^5*e - 4576*a^2*b^3*d^4*e^2 + 6864*a^3*b^2*d^3*e^3 - 6006*a^4*b*d^2*e^4 + 3003*a^5*d*e^5 + 63*(b^5*d^2*e^5 + 65*a*b^4*d^2*e^6)*x^5 - 35*(2*b^5*d^2*e^4 - 13*a*b^4*d^2*e^5 - 286*a^2*b^3*d^2*e^6)*x^4 + 10*(8*b^5*d^3*e^3 - 52*a*b^4*d^2*e^4 + 143*a^2*b^3*d^2*e^5 + 1287*a^3*b^2*d^2*e^6)*x^3 - 3*(32*b^5*d^4*e^2 - 208*a*b^4*d^3*e^3 + 572*a^2*b^3*d^2*e^4 - 858*a^3*b^2*d^2*e^5 - 3003*a^4*b^2*e^6)*x^2 + (128*b^5*d^5*e - 832*a*b^4*d^4*e^2 + 2288*a^2*b^3*d^3*e^3 - 3432*a^3*b^2*d^2*e^4 + 3003*a^4*b^2*d^2*e^5 + 3003*a^5*e^6)*x)*sqrt(e*x + d)/e^6`

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 713 vs. 2(230) = 460.

Time = 0.18 (sec) , antiderivative size = 713, normalized size of antiderivative = 2.24

$$\int \sqrt{d+ex}(a^2+2abx+b^2x^2)^{5/2} dx = \text{Too large to display}$$

input `integrate((e*x+d)^(1/2)*(b^2*x^2+2*a*b*x+a^2)^(5/2),x, algorithm="giac")`

output

```

2/9009*(9009*sqrt(e*x + d)*a^5*d*sgn(b*x + a) + 3003*((e*x + d)^(3/2) - 3*
sqrt(e*x + d)*d)*a^5*sgn(b*x + a) + 15015*((e*x + d)^(3/2) - 3*sqrt(e*x +
d)*d)*a^4*b*d*sgn(b*x + a)/e + 6006*(3*(e*x + d)^(5/2) - 10*(e*x + d)^(3/2)
)*d + 15*sqrt(e*x + d)*d^2)*a^3*b^2*d*sgn(b*x + a)/e^2 + 3003*(3*(e*x + d)
^(5/2) - 10*(e*x + d)^(3/2)*d + 15*sqrt(e*x + d)*d^2)*a^4*b*sgn(b*x + a)/e
+ 2574*(5*(e*x + d)^(7/2) - 21*(e*x + d)^(5/2)*d + 35*(e*x + d)^(3/2)*d^2
- 35*sqrt(e*x + d)*d^3)*a^2*b^3*d*sgn(b*x + a)/e^3 + 2574*(5*(e*x + d)^(7
/2) - 21*(e*x + d)^(5/2)*d + 35*(e*x + d)^(3/2)*d^2 - 35*sqrt(e*x + d)*d^3
)*a^3*b^2*sgn(b*x + a)/e^2 + 143*(35*(e*x + d)^(9/2) - 180*(e*x + d)^(7/2)
)*d + 378*(e*x + d)^(5/2)*d^2 - 420*(e*x + d)^(3/2)*d^3 + 315*sqrt(e*x + d)
*d^4)*a*b^4*d*sgn(b*x + a)/e^4 + 286*(35*(e*x + d)^(9/2) - 180*(e*x + d)^(
7/2)*d + 378*(e*x + d)^(5/2)*d^2 - 420*(e*x + d)^(3/2)*d^3 + 315*sqrt(e*x
+ d)*d^4)*a^2*b^3*sgn(b*x + a)/e^3 + 13*(63*(e*x + d)^(11/2) - 385*(e*x +
d)^(9/2)*d + 990*(e*x + d)^(7/2)*d^2 - 1386*(e*x + d)^(5/2)*d^3 + 1155*(e*
x + d)^(3/2)*d^4 - 693*sqrt(e*x + d)*d^5)*b^5*d*sgn(b*x + a)/e^5 + 65*(63*
(e*x + d)^(11/2) - 385*(e*x + d)^(9/2)*d + 990*(e*x + d)^(7/2)*d^2 - 1386*
(e*x + d)^(5/2)*d^3 + 1155*(e*x + d)^(3/2)*d^4 - 693*sqrt(e*x + d)*d^5)*a*
b^4*sgn(b*x + a)/e^4 + 3*(231*(e*x + d)^(13/2) - 1638*(e*x + d)^(11/2)*d +
5005*(e*x + d)^(9/2)*d^2 - 8580*(e*x + d)^(7/2)*d^3 + 9009*(e*x + d)^(5/2)
)*d^4 - 6006*(e*x + d)^(3/2)*d^5 + 3003*sqrt(e*x + d)*d^6)*b^5*sgn(b*x ...

```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{d+ex}(a^2+2abx+b^2x^2)^{5/2} dx = \int \sqrt{d+ex}(a^2+2abx+b^2x^2)^{5/2} dx$$

input

```
int((d + e*x)^(1/2)*(a^2 + b^2*x^2 + 2*a*b*x)^(5/2), x)
```

output

```
int((d + e*x)^(1/2)*(a^2 + b^2*x^2 + 2*a*b*x)^(5/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 359, normalized size of antiderivative = 1.13

$$\int \sqrt{d+ex}(a^2+2abx+b^2x^2)^{5/2} dx = \frac{2\sqrt{ex+d}(693b^5e^6x^6+4095ab^4e^6x^5+63b^5de^5x^5+10010a^2b^3e^6x^4+455ab^4de^5x^4-70b^5$$

input

```
int((e*x+d)^(1/2)*(b^2*x^2+2*a*b*x+a^2)^(5/2),x)
```

output

```
(2*sqrt(d + e*x)*(3003*a**5*d*e**5 + 3003*a**5*e**6*x - 6006*a**4*b*d**2*e**4 + 3003*a**4*b*d*e**5*x + 9009*a**4*b*e**6*x**2 + 6864*a**3*b**2*d**3*e**3 - 3432*a**3*b**2*d**2*e**4*x + 2574*a**3*b**2*d*e**5*x**2 + 12870*a**3*b**2*e**6*x**3 - 4576*a**2*b**3*d**4*e**2 + 2288*a**2*b**3*d**3*e**3*x - 1716*a**2*b**3*d**2*e**4*x**2 + 1430*a**2*b**3*d*e**5*x**3 + 10010*a**2*b**3*e**6*x**4 + 1664*a*b**4*d**5*e - 832*a*b**4*d**4*e**2*x + 624*a*b**4*d**3*e**3*x**2 - 520*a*b**4*d**2*e**4*x**3 + 455*a*b**4*d*e**5*x**4 + 4095*a*b**4*e**6*x**5 - 256*b**5*d**6 + 128*b**5*d**5*e*x - 96*b**5*d**4*e**2*x**2 + 80*b**5*d**3*e**3*x**3 - 70*b**5*d**2*e**4*x**4 + 63*b**5*d*e**5*x**5 + 693*b**5*e**6*x**6))/(9009*e**6)
```

3.363
$$\int \frac{(a^2+2abx+b^2x^2)^{5/2}}{\sqrt{d+ex}} dx$$

| | |
|---|------|
| Optimal result | 2680 |
| Mathematica [A] (verified) | 2681 |
| Rubi [A] (verified) | 2681 |
| Maple [A] (verified) | 2683 |
| Fricas [A] (verification not implemented) | 2683 |
| Sympy [F] | 2684 |
| Maxima [A] (verification not implemented) | 2684 |
| Giac [A] (verification not implemented) | 2685 |
| Mupad [B] (verification not implemented) | 2685 |
| Reduce [B] (verification not implemented) | 2686 |

Optimal result

Integrand size = 30, antiderivative size = 316

$$\int \frac{(a^2 + 2abx + b^2x^2)^{5/2}}{\sqrt{d + ex}} dx = -\frac{2(bd - ae)^5 \sqrt{d + ex} \sqrt{a^2 + 2abx + b^2x^2}}{e^6(a + bx)} + \frac{10b(bd - ae)^4 (d + ex)^{3/2} \sqrt{a^2 + 2abx + b^2x^2}}{3e^6(a + bx)} - \frac{4b^2(bd - ae)^3 (d + ex)^{5/2} \sqrt{a^2 + 2abx + b^2x^2}}{e^6(a + bx)} + \frac{20b^3(bd - ae)^2 (d + ex)^{7/2} \sqrt{a^2 + 2abx + b^2x^2}}{7e^6(a + bx)} - \frac{10b^4(bd - ae) (d + ex)^{9/2} \sqrt{a^2 + 2abx + b^2x^2}}{9e^6(a + bx)} + \frac{2b^5 (d + ex)^{11/2} \sqrt{a^2 + 2abx + b^2x^2}}{11e^6(a + bx)}$$

output

```
-2*(-a*e+b*d)^5*(e*x+d)^(1/2)*((b*x+a)^2)^(1/2)/e^6/(b*x+a)+10/3*b*(-a*e+b
*d)^4*(e*x+d)^(3/2)*((b*x+a)^2)^(1/2)/e^6/(b*x+a)-4*b^2*(-a*e+b*d)^3*(e*x+
d)^(5/2)*((b*x+a)^2)^(1/2)/e^6/(b*x+a)+20/7*b^3*(-a*e+b*d)^2*(e*x+d)^(7/2)
*((b*x+a)^2)^(1/2)/e^6/(b*x+a)-10/9*b^4*(-a*e+b*d)*(e*x+d)^(9/2)*((b*x+a)^
2)^(1/2)/e^6/(b*x+a)+2/11*b^5*(e*x+d)^(11/2)*((b*x+a)^2)^(1/2)/e^6/(b*x+a)
```

Mathematica [A] (verified)

Time = 0.20 (sec) , antiderivative size = 234, normalized size of antiderivative = 0.74

$$\int \frac{(a^2 + 2abx + b^2x^2)^{5/2}}{\sqrt{d + ex}} dx = \frac{2\sqrt{(a + bx)^2}\sqrt{d + ex}(693a^5e^5 + 1155a^4be^4(-2d + ex) + 462a^3b^2e^3(8d^2 - 4$$

input `Integrate[(a^2 + 2*a*b*x + b^2*x^2)^(5/2)/Sqrt[d + e*x],x]`

output
$$\frac{(2\sqrt{(a + bx)^2}\sqrt{d + ex}(693a^5e^5 + 1155a^4b^2e^4(-2d + ex) + 462a^3b^2e^3(8d^2 - 4d^2e^2x + 3e^2x^2) + 198a^2b^3e^2(-16d^3 + 8d^2e^2x - 6d^2e^2x^2 + 5e^3x^3) + 11ab^4e(128d^4 - 64d^3e^2x + 48d^2e^2x^2 - 40d^2e^3x^3 + 35e^4x^4) + b^5(-256d^5 + 128d^4e^2x - 96d^3e^2x^2 + 80d^2e^3x^3 - 70d^2e^4x^4 + 63e^5x^5)))/(693e^6(a + bx))$$

Rubi [A] (verified)

Time = 0.54 (sec) , antiderivative size = 182, normalized size of antiderivative = 0.58, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {1102, 27, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\begin{aligned} & \int \frac{(a^2 + 2abx + b^2x^2)^{5/2}}{\sqrt{d + ex}} dx \\ & \quad \downarrow 1102 \\ & \frac{\sqrt{a^2 + 2abx + b^2x^2} \int \frac{b^5(a+bx)^5}{\sqrt{d+ex}} dx}{b^5(a + bx)} \\ & \quad \downarrow 27 \\ & \frac{\sqrt{a^2 + 2abx + b^2x^2} \int \frac{(a+bx)^5}{\sqrt{d+ex}} dx}{a + bx} \\ & \quad \downarrow 53 \end{aligned}$$

$$\frac{\sqrt{a^2 + 2abx + b^2x^2} \int \left(\frac{(d+ex)^{9/2}b^5}{e^5} - \frac{5(bd-ae)(d+ex)^{7/2}b^4}{e^5} + \frac{10(bd-ae)^2(d+ex)^{5/2}b^3}{e^5} - \frac{10(bd-ae)^3(d+ex)^{3/2}b^2}{e^5} + \frac{5(bd-ae)^4\sqrt{d+ex}}{e^5} \right)}{a + bx}$$

↓ 2009

$$\frac{\sqrt{a^2 + 2abx + b^2x^2} \left(-\frac{10b^4(d+ex)^{9/2}(bd-ae)}{9e^6} + \frac{20b^3(d+ex)^{7/2}(bd-ae)^2}{7e^6} - \frac{4b^2(d+ex)^{5/2}(bd-ae)^3}{e^6} + \frac{10b(d+ex)^{3/2}(bd-ae)^4}{3e^6} - \frac{2\sqrt{d+ex}}{e^6} \right)}{a + bx}$$

input

```
Int[(a^2 + 2*a*b*x + b^2*x^2)^(5/2)/Sqrt[d + e*x],x]
```

output

```
(Sqrt[a^2 + 2*a*b*x + b^2*x^2]*((-2*(b*d - a*e)^5*Sqrt[d + e*x])/e^6 + (10*b*(b*d - a*e)^4*(d + e*x)^(3/2))/(3*e^6) - (4*b^2*(b*d - a*e)^3*(d + e*x)^(5/2))/e^6 + (20*b^3*(b*d - a*e)^2*(d + e*x)^(7/2))/(7*e^6) - (10*b^4*(b*d - a*e)*(d + e*x)^(9/2))/(9*e^6) + (2*b^5*(d + e*x)^(11/2))/(11*e^6)))/(a + b*x)
```

Defintions of rubi rules used

rule 27

```
Int[(a_)*(F_x_), x_Symbol] := Simp[a Int[F_x, x], x] /; FreeQ[a, x] && !MatchQ[F_x, (b_)*(G_x_) /; FreeQ[b, x]]
```

rule 53

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] := Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && IGtQ[m, 0] && (!IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 1102

```
Int[((d_.) + (e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] := Simp[(a + b*x + c*x^2)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x)^(2*FracPart[p])) Int[(d + e*x)^m*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && EqQ[b^2 - 4*a*c, 0]
```

rule 2009

```
Int[u_, x_Symbol] := Simp[IntSum[u, x], x] /; SumQ[u]
```


output

```
2/693*(63*b^5*e^5*x^5 - 256*b^5*d^5 + 1408*a*b^4*d^4*e - 3168*a^2*b^3*d^3*
e^2 + 3696*a^3*b^2*d^2*e^3 - 2310*a^4*b*d*e^4 + 693*a^5*e^5 - 35*(2*b^5*d*
e^4 - 11*a*b^4*e^5)*x^4 + 10*(8*b^5*d^2*e^3 - 44*a*b^4*d*e^4 + 99*a^2*b^3*
e^5)*x^3 - 6*(16*b^5*d^3*e^2 - 88*a*b^4*d^2*e^3 + 198*a^2*b^3*d*e^4 - 231*
a^3*b^2*e^5)*x^2 + (128*b^5*d^4*e - 704*a*b^4*d^3*e^2 + 1584*a^2*b^3*d^2*e
^3 - 1848*a^3*b^2*d*e^4 + 1155*a^4*b*e^5)*x)*sqrt(e*x + d)/e^6
```

Sympy [F]

$$\int \frac{(a^2 + 2abx + b^2x^2)^{5/2}}{\sqrt{d + ex}} dx = \int \frac{((a + bx)^2)^{5/2}}{\sqrt{d + ex}} dx$$

input

```
integrate((b**2*x**2+2*a*b*x+a**2)**(5/2)/(e*x+d)**(1/2),x)
```

output

```
Integral(((a + b*x)**2)**(5/2)/sqrt(d + e*x), x)
```

Maxima [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 338, normalized size of antiderivative = 1.07

$$\int \frac{(a^2 + 2abx + b^2x^2)^{5/2}}{\sqrt{d + ex}} dx = \frac{2(63b^5e^6x^6 - 256b^5d^6 + 1408ab^4d^5e - 3168a^2b^3d^4e^2 + 3696a^3b^2d^3e^3 - 2310a^4b^2d^2e^4 + 693a^5d^2e^5 - 7*(b^5d^2e^4 - 11ab^4de^5 + 198a^2b^3d^2e^6)*x^5 + 5*(2b^5d^2e^4 - 11ab^4de^5 + 198a^2b^3d^2e^6)*x^4 - 2*(8b^5d^3e^3 - 44ab^4d^2e^4 + 99a^2b^3d^2e^5 - 693a^3b^2e^6)*x^3 + (32b^5d^4e^2 - 176ab^4d^3e^3 + 396a^2b^3d^2e^4 - 462a^3b^2d^2e^5 + 1155a^4b^2e^6)*x^2 - (128b^5d^5e - 704ab^4d^4e^2 + 1584a^2b^3d^3e^3 - 1848a^3b^2d^2e^4 + 1155a^4b^2d^2e^5 - 693a^5e^6)*x)/(\sqrt{e*x + d})e^6}$$

input

```
integrate((b^2*x^2+2*a*b*x+a^2)^(5/2)/(e*x+d)^(1/2),x, algorithm="maxima")
```

output

```
2/693*(63*b^5*e^6*x^6 - 256*b^5*d^6 + 1408*a*b^4*d^5*e - 3168*a^2*b^3*d^4*
e^2 + 3696*a^3*b^2*d^3*e^3 - 2310*a^4*b*d^2*e^4 + 693*a^5*d*e^5 - 7*(b^5*d
*e^5 - 55*a*b^4*e^6)*x^5 + 5*(2*b^5*d^2*e^4 - 11*a*b^4*d*e^5 + 198*a^2*b^3
*e^6)*x^4 - 2*(8*b^5*d^3*e^3 - 44*a*b^4*d^2*e^4 + 99*a^2*b^3*d*e^5 - 693*a
^3*b^2*e^6)*x^3 + (32*b^5*d^4*e^2 - 176*a*b^4*d^3*e^3 + 396*a^2*b^3*d^2*e^
4 - 462*a^3*b^2*d^2*e^5 + 1155*a^4*b^2*e^6)*x^2 - (128*b^5*d^5*e - 704*a*b^4*d
^4*e^2 + 1584*a^2*b^3*d^3*e^3 - 1848*a^3*b^2*d^2*e^4 + 1155*a^4*b^2*d^2*e^5 -
693*a^5*e^6)*x)/(\sqrt(e*x + d)*e^6)
```

Giac [A] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 319, normalized size of antiderivative = 1.01

$$\int \frac{(a^2 + 2abx + b^2x^2)^{5/2}}{\sqrt{d + ex}} dx = \frac{2 \left(693 \sqrt{ex + d} a^5 \operatorname{sgn}(bx + a) + \frac{1155 \left((ex+d)^{3/2} - 3\sqrt{ex+dd} \right) a^4 b \operatorname{sgn}(bx+a)}{e} + \frac{462 \left(3(ex+d)^{5/2} - 10(ex+d)^{3/2}d + 15\sqrt{ex+d}d^2 \right) a^3 b^2 \operatorname{sgn}(bx+a)}{e^2} + 198 \left(5(ex+d)^{7/2} - 21(ex+d)^{5/2}d + 35(ex+d)^{3/2}d^2 - 35\sqrt{ex+d}d^3 \right) a^2 b^3 \operatorname{sgn}(bx+a)}{e^3} + \frac{11 \left(35(ex+d)^{9/2} - 180(ex+d)^{7/2}d + 378(ex+d)^{5/2}d^2 - 420(ex+d)^{3/2}d^3 + 315\sqrt{ex+d}d^4 \right) a b^4 \operatorname{sgn}(bx+a)}{e^4} + \frac{63 \left((ex+d)^{11/2} - 385(ex+d)^{9/2}d + 990(ex+d)^{7/2}d^2 - 1386(ex+d)^{5/2}d^3 + 1155(ex+d)^{3/2}d^4 - 693\sqrt{ex+d}d^5 \right) b^5 \operatorname{sgn}(bx+a)}{e^5} \right)}{e}$$

input `integrate((b^2*x^2+2*a*b*x+a^2)^(5/2)/(e*x+d)^(1/2),x, algorithm="giac")`output `2/693*(693*sqrt(e*x + d)*a^5*sgn(b*x + a) + 1155*((e*x + d)^(3/2) - 3*sqrt(e*x + d)*d)*a^4*b*sgn(b*x + a)/e + 462*(3*(e*x + d)^(5/2) - 10*(e*x + d)^(3/2)*d + 15*sqrt(e*x + d)*d^2)*a^3*b^2*sgn(b*x + a)/e^2 + 198*(5*(e*x + d)^(7/2) - 21*(e*x + d)^(5/2)*d + 35*(e*x + d)^(3/2)*d^2 - 35*sqrt(e*x + d)*d^3)*a^2*b^3*sgn(b*x + a)/e^3 + 11*(35*(e*x + d)^(9/2) - 180*(e*x + d)^(7/2)*d + 378*(e*x + d)^(5/2)*d^2 - 420*(e*x + d)^(3/2)*d^3 + 315*sqrt(e*x + d)*d^4)*a*b^4*sgn(b*x + a)/e^4 + (63*(e*x + d)^(11/2) - 385*(e*x + d)^(9/2)*d + 990*(e*x + d)^(7/2)*d^2 - 1386*(e*x + d)^(5/2)*d^3 + 1155*(e*x + d)^(3/2)*d^4 - 693*sqrt(e*x + d)*d^5)*b^5*sgn(b*x + a)/e^5)/e`**Mupad [B] (verification not implemented)**

Time = 5.90 (sec) , antiderivative size = 375, normalized size of antiderivative = 1.19

$$\int \frac{(a^2 + 2abx + b^2x^2)^{5/2}}{\sqrt{d + ex}} dx = \frac{\sqrt{a^2 + 2abx + b^2x^2} \left(\frac{2b^4x^6}{11} - \frac{-1386a^5de^5 + 4620a^4bd^2e^4 - 7392a^3b^2d^3e^3 + 6336a^2b^3d^2e^2 - 2592ab^4d^2e - 693b^5d^2}{693be^6} \right)}{693be^6}$$

input `int((a^2 + b^2*x^2 + 2*a*b*x)^(5/2)/(d + e*x)^(1/2),x)`

output

```
((a^2 + b^2*x^2 + 2*a*b*x)^(1/2)*((2*b^4*x^6)/11 - (512*b^5*d^6 - 1386*a^5*d*e^5 + 4620*a^4*b*d^2*e^4 + 6336*a^2*b^3*d^4*e^2 - 7392*a^3*b^2*d^3*e^3 - 2816*a*b^4*d^5*e)/(693*b*e^6) + (2*b^3*x^5*(55*a*e - b*d))/(99*e) + (x*(1386*a^5*e^6 - 256*b^5*d^5*e + 1408*a*b^4*d^4*e^2 - 3168*a^2*b^3*d^3*e^3 + 3696*a^3*b^2*d^2*e^4 - 2310*a^4*b*d*e^5))/(693*b*e^6) + (x^2*(2310*a^4*b*e^6 + 64*b^5*d^4*e^2 - 352*a*b^4*d^3*e^3 - 924*a^3*b^2*d*e^5 + 792*a^2*b^3*d^2*e^4))/(693*b*e^6) + (10*b^2*x^4*(198*a^2*e^2 + 2*b^2*d^2 - 11*a*b*d*e))/(693*e^2) + (4*b*x^3*(693*a^3*e^3 - 8*b^3*d^3 + 44*a*b^2*d^2*e - 99*a^2*b*d*e^2))/(693*e^3)))/(x*(d + e*x)^(1/2) + (a*(d + e*x)^(1/2))/b)
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 271, normalized size of antiderivative = 0.86

$$\int \frac{(a^2 + 2abx + b^2x^2)^{5/2}}{\sqrt{d + ex}} dx = \frac{2\sqrt{ex + d}(63b^5e^5x^5 + 385ab^4e^5x^4 - 70b^5de^4x^4 + 990a^2b^3e^5x^3 - 440ab^4de^4x^3 + 1155a^3b^2e^5x^2 - 1848a^3b^2de^4x^2 + 1386a^3b^2e^5x^2 - 3168a^2b^3d^3e^2 + 1584a^2b^3d^2e^3x - 1188a^2b^3d^2e^4x^2 + 990a^2b^3e^5x^3 + 1408ab^4d^4e - 704ab^4d^3e^2x + 528ab^4d^2e^3x^2 - 440ab^4de^4x^3 + 385ab^4e^5x^4 - 256b^5d^5 + 128b^5d^4e^2x - 96b^5d^3e^2x^2 + 80b^5d^2e^3x^3 - 70b^5de^4x^4 + 63b^5e^5x^5)}{693e^6}$$

input

```
int((b^2*x^2+2*a*b*x+a^2)^(5/2)/(e*x+d)^(1/2),x)
```

output

```
(2*sqrt(d + e*x)*(693*a**5*e**5 - 2310*a**4*b*d*e**4 + 1155*a**4*b*e**5*x + 3696*a**3*b**2*d**2*e**3 - 1848*a**3*b**2*d*e**4*x + 1386*a**3*b**2*e**5*x**2 - 3168*a**2*b**3*d**3*e**2 + 1584*a**2*b**3*d**2*e**3*x - 1188*a**2*b**3*d*e**4*x**2 + 990*a**2*b**3*e**5*x**3 + 1408*a*b**4*d**4*e - 704*a*b**4*d**3*e**2*x + 528*a*b**4*d**2*e**3*x**2 - 440*a*b**4*d*e**4*x**3 + 385*a*b**4*e**5*x**4 - 256*b**5*d**5 + 128*b**5*d**4*e*x - 96*b**5*d**3*e**2*x**2 + 80*b**5*d**2*e**3*x**3 - 70*b**5*d*e**4*x**4 + 63*b**5*e**5*x**5))/(693*e**6)
```

$$3.364 \quad \int \frac{(a^2 + 2abx + b^2x^2)^{5/2}}{(d + ex)^{3/2}} dx$$

| | |
|---|------|
| Optimal result | 2687 |
| Mathematica [A] (verified) | 2688 |
| Rubi [A] (verified) | 2688 |
| Maple [A] (verified) | 2690 |
| Fricas [A] (verification not implemented) | 2690 |
| Sympy [F] | 2691 |
| Maxima [A] (verification not implemented) | 2691 |
| Giac [B] (verification not implemented) | 2692 |
| Mupad [B] (verification not implemented) | 2692 |
| Reduce [B] (verification not implemented) | 2693 |

Optimal result

Integrand size = 30, antiderivative size = 314

$$\begin{aligned} \int \frac{(a^2 + 2abx + b^2x^2)^{5/2}}{(d + ex)^{3/2}} dx &= \frac{2(bd - ae)^5 \sqrt{a^2 + 2abx + b^2x^2}}{e^6(a + bx)\sqrt{d + ex}} \\ &+ \frac{10b(bd - ae)^4 \sqrt{d + ex} \sqrt{a^2 + 2abx + b^2x^2}}{e^6(a + bx)} \\ &- \frac{20b^2(bd - ae)^3 (d + ex)^{3/2} \sqrt{a^2 + 2abx + b^2x^2}}{3e^6(a + bx)} \\ &+ \frac{4b^3(bd - ae)^2 (d + ex)^{5/2} \sqrt{a^2 + 2abx + b^2x^2}}{e^6(a + bx)} \\ &- \frac{10b^4(bd - ae)(d + ex)^{7/2} \sqrt{a^2 + 2abx + b^2x^2}}{7e^6(a + bx)} \\ &+ \frac{2b^5(d + ex)^{9/2} \sqrt{a^2 + 2abx + b^2x^2}}{9e^6(a + bx)} \end{aligned}$$

output

```
2*(-a*e+b*d)^5*((b*x+a)^2)^(1/2)/e^6/(b*x+a)/(e*x+d)^(1/2)+10*b*(-a*e+b*d)
^4*(e*x+d)^(1/2)*((b*x+a)^2)^(1/2)/e^6/(b*x+a)-20/3*b^2*(-a*e+b*d)^3*(e*x+
d)^(3/2)*((b*x+a)^2)^(1/2)/e^6/(b*x+a)+4*b^3*(-a*e+b*d)^2*(e*x+d)^(5/2)*((
b*x+a)^2)^(1/2)/e^6/(b*x+a)-10/7*b^4*(-a*e+b*d)*(e*x+d)^(7/2)*((b*x+a)^2)^(
1/2)/e^6/(b*x+a)+2/9*b^5*(e*x+d)^(9/2)*((b*x+a)^2)^(1/2)/e^6/(b*x+a)
```

Mathematica [A] (verified)

Time = 0.16 (sec) , antiderivative size = 234, normalized size of antiderivative = 0.75

$$\int \frac{(a^2 + 2abx + b^2x^2)^{5/2}}{(d + ex)^{3/2}} dx = \frac{2\sqrt{(a + bx)^2(63a^5e^5 - 315a^4be^4(2d + ex) + 210a^3b^2e^3(8d^2 + 4dex - e^2x^2) - 126a^2b^3e^2(16d^3 + 8d^2ex -$$

input

```
Integrate[(a^2 + 2*a*b*x + b^2*x^2)^(5/2)/(d + e*x)^(3/2), x]
```

output

```
(-2*sqrt[(a + b*x)^2]*(63*a^5*e^5 - 315*a^4*b*e^4*(2*d + e*x) + 210*a^3*b^2*e^3*(8*d^2 + 4*d*e*x - e^2*x^2) - 126*a^2*b^3*e^2*(16*d^3 + 8*d^2*e*x - 2*d*e^2*x^2 + e^3*x^3) + 9*a*b^4*e*(128*d^4 + 64*d^3*e*x - 16*d^2*e^2*x^2 + 8*d*e^3*x^3 - 5*e^4*x^4) - b^5*(256*d^5 + 128*d^4*e*x - 32*d^3*e^2*x^2 + 16*d^2*e^3*x^3 - 10*d*e^4*x^4 + 7*e^5*x^5)))/(63*e^6*(a + b*x)*sqrt[d + e*x])
```

Rubi [A] (verified)

Time = 0.54 (sec) , antiderivative size = 180, normalized size of antiderivative = 0.57, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {1102, 27, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a^2 + 2abx + b^2x^2)^{5/2}}{(d + ex)^{3/2}} dx$$

$$\downarrow 1102$$

$$\frac{\sqrt{a^2 + 2abx + b^2x^2} \int \frac{b^5(a+bx)^5}{(d+ex)^{3/2}} dx}{b^5(a + bx)}$$

$$\downarrow 27$$

$$\frac{\sqrt{a^2 + 2abx + b^2x^2} \int \frac{(a+bx)^5}{(d+ex)^{3/2}} dx}{a + bx}$$

↓ 53

$$\frac{\sqrt{a^2 + 2abx + b^2x^2} \int \left(\frac{(d+ex)^{7/2}b^5}{e^5} - \frac{5(bd-ae)(d+ex)^{5/2}b^4}{e^5} + \frac{10(bd-ae)^2(d+ex)^{3/2}b^3}{e^5} - \frac{10(bd-ae)^3\sqrt{d+ex}b^2}{e^5} + \frac{5(bd-ae)^4b}{e^5\sqrt{d+ex}} + \dots \right)}{a + bx}$$

↓ 2009

$$\frac{\sqrt{a^2 + 2abx + b^2x^2} \left(-\frac{10b^4(d+ex)^{7/2}(bd-ae)}{7e^6} + \frac{4b^3(d+ex)^{5/2}(bd-ae)^2}{e^6} - \frac{20b^2(d+ex)^{3/2}(bd-ae)^3}{3e^6} + \frac{10b\sqrt{d+ex}(bd-ae)^4}{e^6} + \frac{2(bd-ae)^5}{e^6\sqrt{d+ex}} \right)}{a + bx}$$

input `Int[(a^2 + 2*a*b*x + b^2*x^2)^(5/2)/(d + e*x)^(3/2),x]`

output `(Sqrt[a^2 + 2*a*b*x + b^2*x^2]*((2*(b*d - a*e)^5)/(e^6*Sqrt[d + e*x])) + (10*b*(b*d - a*e)^4*Sqrt[d + e*x])/e^6 - (20*b^2*(b*d - a*e)^3*(d + e*x)^(3/2))/(3*e^6) + (4*b^3*(b*d - a*e)^2*(d + e*x)^(5/2))/e^6 - (10*b^4*(b*d - a*e)*(d + e*x)^(7/2))/(7*e^6) + (2*b^5*(d + e*x)^(9/2))/(9*e^6))/(a + b*x)`

Defintions of rubi rules used

rule 27 `Int[(a_)*(F_x_), x_Symbol] := Simp[a Int[F_x, x], x] /; FreeQ[a, x] && !MatchQ[F_x, (b_)*(G_x_) /; FreeQ[b, x]]`

rule 53 `Int[((a_) + (b_)*(x_)^(m_))*((c_) + (d_)*(x_)^(n_)), x_Symbol] := Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && IGtQ[m, 0] && (!IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])`

rule 1102 `Int[((d_) + (e_)*(x_)^(m_))*((a_) + (b_)*(x_) + (c_)*(x_)^2)^(p_), x_Symbol] := Simp[(a + b*x + c*x^2)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x)^(2*FracPart[p])) Int[(d + e*x)^m*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && EqQ[b^2 - 4*a*c, 0]`

output

```
2/63*(7*b^5*e^5*x^5 + 256*b^5*d^5 - 1152*a*b^4*d^4*e + 2016*a^2*b^3*d^3*e^2 - 1680*a^3*b^2*d^2*e^3 + 630*a^4*b*d*e^4 - 63*a^5*e^5 - 5*(2*b^5*d*e^4 - 9*a*b^4*e^5)*x^4 + 2*(8*b^5*d^2*e^3 - 36*a*b^4*d*e^4 + 63*a^2*b^3*e^5)*x^3 - 2*(16*b^5*d^3*e^2 - 72*a*b^4*d^2*e^3 + 126*a^2*b^3*d*e^4 - 105*a^3*b^2*e^5)*x^2 + (128*b^5*d^4*e - 576*a*b^4*d^3*e^2 + 1008*a^2*b^3*d^2*e^3 - 840*a^3*b^2*d*e^4 + 315*a^4*b*e^5)*x)*sqrt(e*x + d)/(e^7*x + d*e^6)
```

Sympy [F]

$$\int \frac{(a^2 + 2abx + b^2x^2)^{5/2}}{(d + ex)^{3/2}} dx = \int \frac{((a + bx)^2)^{5/2}}{(d + ex)^{3/2}} dx$$

input

```
integrate((b**2*x**2+2*a*b*x+a**2)**(5/2)/(e*x+d)**(3/2),x)
```

output

```
Integral(((a + b*x)**2)**(5/2)/(d + e*x)**(3/2), x)
```

Maxima [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 261, normalized size of antiderivative = 0.83

$$\int \frac{(a^2 + 2abx + b^2x^2)^{5/2}}{(d + ex)^{3/2}} dx = \frac{2(7b^5e^5x^5 + 256b^5d^5 - 1152ab^4d^4e + 2016a^2b^3d^3e^2 - 1680a^3b^2d^2e^3 + 630a^4bd^2e^4 - 63a^5e^5 - 5(2b^5de^4 - 9ab^4e^5)x^4 + 2(8b^5d^2e^3 - 36ab^4de^4 + 63a^2b^3e^5)x^3 - 2(16b^5d^3e^2 - 72ab^4d^2e^3 + 126a^2b^3de^4 - 105a^3b^2e^5)x^2 + (128b^5d^4e - 576ab^4d^3e^2 + 1008a^2b^3d^2e^3 - 840a^3b^2de^4 + 315a^4be^5)x)}{\sqrt{ex + d}e^6}$$

input

```
integrate((b^2*x^2+2*a*b*x+a^2)^(5/2)/(e*x+d)^(3/2),x, algorithm="maxima")
```

output

```
2/63*(7*b^5*e^5*x^5 + 256*b^5*d^5 - 1152*a*b^4*d^4*e + 2016*a^2*b^3*d^3*e^2 - 1680*a^3*b^2*d^2*e^3 + 630*a^4*b*d*e^4 - 63*a^5*e^5 - 5*(2*b^5*d*e^4 - 9*a*b^4*e^5)*x^4 + 2*(8*b^5*d^2*e^3 - 36*a*b^4*d*e^4 + 63*a^2*b^3*e^5)*x^3 - 2*(16*b^5*d^3*e^2 - 72*a*b^4*d^2*e^3 + 126*a^2*b^3*d*e^4 - 105*a^3*b^2*e^5)*x^2 + (128*b^5*d^4*e - 576*a*b^4*d^3*e^2 + 1008*a^2*b^3*d^2*e^3 - 840*a^3*b^2*d*e^4 + 315*a^4*b*e^5)*x)/sqrt(e*x + d)*e^6
```


Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 476 vs. $2(230) = 460$.

Time = 0.17 (sec) , antiderivative size = 476, normalized size of antiderivative = 1.52

$$\int \frac{(a^2 + 2abx + b^2x^2)^{5/2}}{(d + ex)^{3/2}} dx = \frac{2(b^5d^5\operatorname{sgn}(bx + a) - 5ab^4d^4e\operatorname{sgn}(bx + a) + 10a^2b^3d^3e^2\operatorname{sgn}(bx + a) - 10a^3b^2d^2e^3\operatorname{sgn}(bx + a) + 5a^4b^2d^2e^4\operatorname{sgn}(bx + a) - a^5e^5\operatorname{sgn}(bx + a))}{\sqrt{ex + d}e^6} + \frac{2\left(7(ex + d)^{9/2}b^5e^{48}\operatorname{sgn}(bx + a) - 45(ex + d)^{7/2}b^5de^{48}\operatorname{sgn}(bx + a) + 126(ex + d)^{5/2}b^5d^2e^{48}\operatorname{sgn}(bx + a) - 210(ex + d)^{3/2}b^5d^3e^{48}\operatorname{sgn}(bx + a) + 315\sqrt{ex + d}b^5d^4e^{48}\operatorname{sgn}(bx + a) - 45(ex + d)^{7/2}a^2b^3d^3e^{48}\operatorname{sgn}(bx + a) + 126(ex + d)^{5/2}a^2b^3d^2e^{48}\operatorname{sgn}(bx + a) - 1260\sqrt{ex + d}a^2b^3d^2e^{48}\operatorname{sgn}(bx + a) + 126(ex + d)^{5/2}a^2b^3e^{50}\operatorname{sgn}(bx + a) - 630(ex + d)^{3/2}a^2b^3d^2e^{50}\operatorname{sgn}(bx + a) + 1890\sqrt{ex + d}a^2b^3d^2e^{50}\operatorname{sgn}(bx + a) + 210(ex + d)^{3/2}a^3b^2e^{51}\operatorname{sgn}(bx + a) - 1260\sqrt{ex + d}a^3b^2d^2e^{51}\operatorname{sgn}(bx + a) + 315\sqrt{ex + d}a^4b^2e^{52}\operatorname{sgn}(bx + a)\right)}{e^{54}}$$

input `integrate((b^2*x^2+2*a*b*x+a^2)^(5/2)/(e*x+d)^(3/2),x, algorithm="giac")`

output

```
2*(b^5*d^5*sgn(b*x + a) - 5*a*b^4*d^4*e*sgn(b*x + a) + 10*a^2*b^3*d^3*e^2*
sgn(b*x + a) - 10*a^3*b^2*d^2*e^3*sgn(b*x + a) + 5*a^4*b*d^2*e^4*sgn(b*x + a)
) - a^5*e^5*sgn(b*x + a))/(sqrt(e*x + d)*e^6) + 2/63*(7*(e*x + d)^(9/2)*b^
5*e^48*sgn(b*x + a) - 45*(e*x + d)^(7/2)*b^5*d*e^48*sgn(b*x + a) + 126*(e*
x + d)^(5/2)*b^5*d^2*e^48*sgn(b*x + a) - 210*(e*x + d)^(3/2)*b^5*d^3*e^48*
sgn(b*x + a) + 315*sqrt(e*x + d)*b^5*d^4*e^48*sgn(b*x + a) + 45*(e*x + d)^(
7/2)*a*b^4*e^49*sgn(b*x + a) - 252*(e*x + d)^(5/2)*a*b^4*d*e^49*sgn(b*x +
a) + 630*(e*x + d)^(3/2)*a*b^4*d^2*e^49*sgn(b*x + a) - 1260*sqrt(e*x + d)
*a*b^4*d^3*e^49*sgn(b*x + a) + 126*(e*x + d)^(5/2)*a^2*b^3*d^2*e^50*sgn(b*x +
a) - 630*(e*x + d)^(3/2)*a^2*b^3*d^2*e^50*sgn(b*x + a) + 1890*sqrt(e*x + d)*
a^2*b^3*d^2*e^50*sgn(b*x + a) + 210*(e*x + d)^(3/2)*a^3*b^2*d^2*e^51*sgn(b*x +
a) - 1260*sqrt(e*x + d)*a^3*b^2*d^2*e^51*sgn(b*x + a) + 315*sqrt(e*x + d)*a
^4*b^2*e^52*sgn(b*x + a))/e^54
```

Mupad [B] (verification not implemented)

Time = 6.05 (sec) , antiderivative size = 294, normalized size of antiderivative = 0.94

$$\int \frac{(a^2 + 2abx + b^2x^2)^{5/2}}{(d + ex)^{3/2}} dx = \frac{\sqrt{a^2 + 2abx + b^2x^2} \left(\frac{2b^4x^5}{9e} - \frac{126a^5e^5 - 1260a^4bde^4 + 3360a^3b^2d^2e^3 - 4032a^2b^3d^3e^2 + 2592ab^4d^4e - 1296b^5d^5}{63be^6} \right)}{\sqrt{ex + d}}$$

input `int((a^2 + b^2*x^2 + 2*a*b*x)^(5/2)/(d + e*x)^(3/2),x)`

output

```
((a^2 + b^2*x^2 + 2*a*b*x)^(1/2)*((2*b^4*x^5)/(9*e) - (126*a^5*e^5 - 512*b^5*d^5 - 4032*a^2*b^3*d^3*e^2 + 3360*a^3*b^2*d^2*e^3 + 2304*a*b^4*d^4*e - 1260*a^4*b*d*e^4)/(63*b*e^6) + (10*b^3*x^4*(9*a*e - 2*b*d))/(63*e^2) + (x*(630*a^4*b*e^5 + 256*b^5*d^4*e - 1152*a*b^4*d^3*e^2 - 1680*a^3*b^2*d*e^4 + 2016*a^2*b^3*d^2*e^3))/(63*b*e^6) + (4*b^2*x^3*(63*a^2*e^2 + 8*b^2*d^2 - 36*a*b*d*e))/(63*e^3) + (4*b*x^2*(105*a^3*e^3 - 16*b^3*d^3 + 72*a*b^2*d^2*e - 126*a^2*b*d*e^2))/(63*e^4)))/(x*(d + e*x)^(1/2) + (a*(d + e*x)^(1/2))/b)
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 273, normalized size of antiderivative = 0.87

$$\int \frac{(a^2 + 2abx + b^2x^2)^{5/2}}{(d + ex)^{3/2}} dx = \frac{\frac{2}{9}b^5e^5x^5 + \frac{10}{7}ab^4e^5x^4 - \frac{20}{63}b^5de^4x^4 + 4a^2b^3e^5x^3 - \frac{16}{7}ab^4de^4x^3 + \frac{32}{63}b^5d^2e^3x^3}{(d + ex)^{3/2}}$$

input

```
int((b^2*x^2+2*a*b*x+a^2)^(5/2)/(e*x+d)^(3/2),x)
```

output

```
(2*( - 63*a**5*e**5 + 630*a**4*b*d*e**4 + 315*a**4*b*e**5*x - 1680*a**3*b**2*d**2*e**3 - 840*a**3*b**2*d*e**4*x + 210*a**3*b**2*e**5*x**2 + 2016*a**2*b**3*d**3*e**2 + 1008*a**2*b**3*d**2*e**3*x - 252*a**2*b**3*d*e**4*x**2 + 126*a**2*b**3*e**5*x**3 - 1152*a*b**4*d**4*e - 576*a*b**4*d**3*e**2*x + 144*a*b**4*d**2*e**3*x**2 - 72*a*b**4*d*e**4*x**3 + 45*a*b**4*e**5*x**4 + 256*b**5*d**5 + 128*b**5*d**4*e*x - 32*b**5*d**3*e**2*x**2 + 16*b**5*d**2*e**3*x**3 - 10*b**5*d*e**4*x**4 + 7*b**5*e**5*x**5))/(63*sqrt(d + e*x)*e**6)
```

$$3.365 \quad \int \frac{(a^2 + 2abx + b^2x^2)^{5/2}}{(d + ex)^{5/2}} dx$$

| | |
|---|------|
| Optimal result | 2694 |
| Mathematica [A] (verified) | 2695 |
| Rubi [A] (verified) | 2695 |
| Maple [A] (verified) | 2697 |
| Fricas [A] (verification not implemented) | 2697 |
| Sympy [F] | 2698 |
| Maxima [A] (verification not implemented) | 2698 |
| Giac [B] (verification not implemented) | 2699 |
| Mupad [B] (verification not implemented) | 2699 |
| Reduce [B] (verification not implemented) | 2700 |

Optimal result

Integrand size = 30, antiderivative size = 314

$$\int \frac{(a^2 + 2abx + b^2x^2)^{5/2}}{(d + ex)^{5/2}} dx = \frac{2(bd - ae)^5 \sqrt{a^2 + 2abx + b^2x^2}}{3e^6(a + bx)(d + ex)^{3/2}} - \frac{10b(bd - ae)^4 \sqrt{a^2 + 2abx + b^2x^2}}{e^6(a + bx)\sqrt{d + ex}} - \frac{20b^2(bd - ae)^3 \sqrt{d + ex} \sqrt{a^2 + 2abx + b^2x^2}}{e^6(a + bx)} + \frac{20b^3(bd - ae)^2 (d + ex)^{3/2} \sqrt{a^2 + 2abx + b^2x^2}}{3e^6(a + bx)} - \frac{2b^4(bd - ae)(d + ex)^{5/2} \sqrt{a^2 + 2abx + b^2x^2}}{e^6(a + bx)} + \frac{2b^5(d + ex)^{7/2} \sqrt{a^2 + 2abx + b^2x^2}}{7e^6(a + bx)}$$

output

```
2/3*(-a*e+b*d)^5*((b*x+a)^2)^(1/2)/e^6/(b*x+a)/(e*x+d)^(3/2)-10*b*(-a*e+b*d)^4*((b*x+a)^2)^(1/2)/e^6/(b*x+a)/(e*x+d)^(1/2)-20*b^2*(-a*e+b*d)^3*(e*x+d)^(1/2)*((b*x+a)^2)^(1/2)/e^6/(b*x+a)+20/3*b^3*(-a*e+b*d)^2*(e*x+d)^(3/2)*((b*x+a)^2)^(1/2)/e^6/(b*x+a)-2*b^4*(-a*e+b*d)*(e*x+d)^(5/2)*((b*x+a)^2)^(1/2)/e^6/(b*x+a)+2/7*b^5*(e*x+d)^(7/2)*((b*x+a)^2)^(1/2)/e^6/(b*x+a)
```

Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 235, normalized size of antiderivative = 0.75

$$\int \frac{(a^2 + 2abx + b^2x^2)^{5/2}}{(d + ex)^{5/2}} dx =$$

$$2\sqrt{(a + bx)^2(7a^5e^5 + 35a^4be^4(2d + 3ex) - 70a^3b^2e^3(8d^2 + 12dex + 3e^2x^2) + 70a^2b^3e^2(16d^3 + 24d^2ex +$$

input

```
Integrate[(a^2 + 2*a*b*x + b^2*x^2)^(5/2)/(d + e*x)^(5/2), x]
```

output

```
(-2*Sqrt[(a + b*x)^2]*(7*a^5*e^5 + 35*a^4*b*e^4*(2*d + 3*e*x) - 70*a^3*b^2*
*e^3*(8*d^2 + 12*d*e*x + 3*e^2*x^2) + 70*a^2*b^3*e^2*(16*d^3 + 24*d^2*e*x
+ 6*d*e^2*x^2 - e^3*x^3) - 7*a*b^4*e*(128*d^4 + 192*d^3*e*x + 48*d^2*e^2*x
^2 - 8*d*e^3*x^3 + 3*e^4*x^4) + b^5*(256*d^5 + 384*d^4*e*x + 96*d^3*e^2*x^
2 - 16*d^2*e^3*x^3 + 6*d*e^4*x^4 - 3*e^5*x^5)))/(21*e^6*(a + b*x)*(d + e*x
)^(3/2))
```

Rubi [A] (verified)

Time = 0.53 (sec) , antiderivative size = 180, normalized size of antiderivative = 0.57,
 number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules
 used = {1102, 27, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a^2 + 2abx + b^2x^2)^{5/2}}{(d + ex)^{5/2}} dx$$

$$\downarrow 1102$$

$$\frac{\sqrt{a^2 + 2abx + b^2x^2} \int \frac{b^5(a+bx)^5}{(d+ex)^{5/2}} dx}{b^5(a + bx)}$$

$$\downarrow 27$$

$$\frac{\sqrt{a^2 + 2abx + b^2x^2} \int \frac{(a+bx)^5}{(d+ex)^{5/2}} dx}{a + bx}$$

↓ 53

$$\frac{\sqrt{a^2 + 2abx + b^2x^2} \int \left(\frac{(d+ex)^{5/2}b^5}{e^5} - \frac{5(bd-ae)(d+ex)^{3/2}b^4}{e^5} + \frac{10(bd-ae)^2\sqrt{d+ex}b^3}{e^5} - \frac{10(bd-ae)^3b^2}{e^5\sqrt{d+ex}} + \frac{5(bd-ae)^4b}{e^5(d+ex)^{3/2}} + \frac{(ae-bd)}{e^5(d+ex)} \right)}{a + bx}$$

↓ 2009

$$\frac{\sqrt{a^2 + 2abx + b^2x^2} \left(-\frac{2b^4(d+ex)^{5/2}(bd-ae)}{e^6} + \frac{20b^3(d+ex)^{3/2}(bd-ae)^2}{3e^6} - \frac{20b^2\sqrt{d+ex}(bd-ae)^3}{e^6} - \frac{10b(bd-ae)^4}{e^6\sqrt{d+ex}} + \frac{2(bd-ae)^5}{3e^6(d+ex)^{3/2}} + \frac{(ae-bd)}{e^6(d+ex)} \right)}{a + bx}$$

input `Int[(a^2 + 2*a*b*x + b^2*x^2)^(5/2)/(d + e*x)^(5/2),x]`

output `(Sqrt[a^2 + 2*a*b*x + b^2*x^2]*((2*(b*d - a*e)^5)/(3*e^6*(d + e*x)^(3/2)) - (10*b*(b*d - a*e)^4)/(e^6*Sqrt[d + e*x]) - (20*b^2*(b*d - a*e)^3*Sqrt[d + e*x])/e^6 + (20*b^3*(b*d - a*e)^2*(d + e*x)^(3/2))/(3*e^6) - (2*b^4*(b*d - a*e)*(d + e*x)^(5/2))/e^6 + (2*b^5*(d + e*x)^(7/2))/(7*e^6)))/(a + b*x)`

Defintions of rubi rules used

rule 27 `Int[(a_)*(F_x_), x_Symbol] := Simp[a Int[F_x, x], x] /; FreeQ[a, x] && !MatchQ[F_x, (b_)*(G_x_)] /; FreeQ[b, x]`

rule 53 `Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] := Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && IGtQ[m, 0] && (!IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])`

rule 1102 `Int[((d_.) + (e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] := Simp[(a + b*x + c*x^2)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x)^(2*FracPart[p])) Int[(d + e*x)^m*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && EqQ[b^2 - 4*a*c, 0]`

rule 2009 `Int[u_, x_Symbol] := Simp[IntSum[u, x], x] /; SumQ[u]`

Maple [A] (verified)

Time = 1.21 (sec) , antiderivative size = 226, normalized size of antiderivative = 0.72

| method | result |
|---------|--|
| risch | $\frac{2b^2(3e^3x^3b^3+21x^2ab^2e^3-12x^2b^3de^2+70a^2be^3x-98xab^2de^2+37b^3d^2ex+210e^3a^3-560a^2bd^2e^2+511ab^2d^2e-158b^3d^3)\sqrt{ex+d}}{21e^6(bx+a)}$ |
| gospers | $-\frac{2(-3x^5e^5b^5-21x^4ab^4e^5+6x^4b^5de^4-70x^3a^2b^3e^5+56x^3ab^4de^4-16x^3b^5d^2e^3-210x^2a^3b^2e^5+420x^2a^2b^3de^4-336x^2ab^4d^2e^3+98x^2a^3b^2d^2e^2-158x^2ab^3d^3e-158x^2ab^3d^3e)}{21e^6(bx+a)}$ |
| default | $-\frac{2(-3x^5e^5b^5-21x^4ab^4e^5+6x^4b^5de^4-70x^3a^2b^3e^5+56x^3ab^4de^4-16x^3b^5d^2e^3-210x^2a^3b^2e^5+420x^2a^2b^3de^4-336x^2ab^4d^2e^3+98x^2a^3b^2d^2e^2-158x^2ab^3d^3e-158x^2ab^3d^3e)}{21e^6(bx+a)}$ |
| orering | $-\frac{2(-3x^5e^5b^5-21x^4ab^4e^5+6x^4b^5de^4-70x^3a^2b^3e^5+56x^3ab^4de^4-16x^3b^5d^2e^3-210x^2a^3b^2e^5+420x^2a^2b^3de^4-336x^2ab^4d^2e^3+98x^2a^3b^2d^2e^2-158x^2ab^3d^3e-158x^2ab^3d^3e)}{21e^6(bx+a)}$ |

input `int((b^2*x^2+2*a*b*x+a^2)^(5/2)/(e*x+d)^(5/2),x,method=_RETURNVERBOSE)`

output
$$\frac{2}{21}b^2(3b^3e^3x^3+21ab^2e^3x^2-12b^3d^2e^2x^2+70a^2b^3e^3x-98a^2b^3d^2e^2x+37b^3d^2e^2x+210a^3e^3-560a^2b^3d^2e^2+511ab^2d^2e-158b^3d^3e)(e*x+d)^{(1/2)}/e^6((b*x+a)^2)^{(1/2)}/(b*x+a)-2/3*(15*b^3e^3x+a^3e^3+14*b^3d^3e)(a^4e^4-4a^3b^3d^3e+6a^2b^3d^3e^2-4a^2b^3d^3e+b^4d^4)/e^6/(e*x+d)^{(3/2)}*((b*x+a)^2)^{(1/2)}/(b*x+a)$$

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 283, normalized size of antiderivative = 0.90

$$\int \frac{(a^2 + 2abx + b^2x^2)^{5/2}}{(d + ex)^{5/2}} dx = \frac{2(3b^5e^5x^5 - 256b^5d^5 + 896ab^4d^4e - 1120a^2b^3d^3e^2 + 560a^3b^2d^2e^3 - 70a^4b^2d^2e^3)}{(d + ex)^{5/2}}$$

input `integrate((b^2*x^2+2*a*b*x+a^2)^(5/2)/(e*x+d)^(5/2),x, algorithm="fricas")`

output

```
2/21*(3*b^5*e^5*x^5 - 256*b^5*d^5 + 896*a*b^4*d^4*e - 1120*a^2*b^3*d^3*e^2
+ 560*a^3*b^2*d^2*e^3 - 70*a^4*b*d*e^4 - 7*a^5*e^5 - 3*(2*b^5*d*e^4 - 7*a
*b^4*e^5)*x^4 + 2*(8*b^5*d^2*e^3 - 28*a*b^4*d*e^4 + 35*a^2*b^3*e^5)*x^3 -
6*(16*b^5*d^3*e^2 - 56*a*b^4*d^2*e^3 + 70*a^2*b^3*d*e^4 - 35*a^3*b^2*e^5)*
x^2 - 3*(128*b^5*d^4*e - 448*a*b^4*d^3*e^2 + 560*a^2*b^3*d^2*e^3 - 280*a^3
*b^2*d*e^4 + 35*a^4*b*e^5)*x)*sqrt(e*x + d)/(e^8*x^2 + 2*d*e^7*x + d^2*e^6
)
```

Sympy [F]

$$\int \frac{(a^2 + 2abx + b^2x^2)^{5/2}}{(d + ex)^{5/2}} dx = \int \frac{((a + bx)^2)^{5/2}}{(d + ex)^{5/2}} dx$$

input

```
integrate((b**2*x**2+2*a*b*x+a**2)**(5/2)/(e*x+d)**(5/2),x)
```

output

```
Integral(((a + b*x)**2)**(5/2)/(d + e*x)**(5/2), x)
```

Maxima [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 272, normalized size of antiderivative = 0.87

$$\int \frac{(a^2 + 2abx + b^2x^2)^{5/2}}{(d + ex)^{5/2}} dx = \frac{2(3b^5e^5x^5 - 256b^5d^5 + 896ab^4d^4e - 1120a^2b^3d^3e^2 + 560a^3b^2d^2e^3 - 70a^4b^2d^2e^4 + 35a^5d^2e^5)x^4 + 2(8b^5d^2e^3 - 28a^2b^4d^2e^4 + 35a^3b^3d^2e^5)x^3 - 6(16b^5d^3e^2 - 56a^2b^4d^3e^3 + 70a^3b^3d^3e^4 - 35a^4b^2d^3e^5)x^2 - 3(128b^5d^4e - 448a^2b^4d^4e^2 + 560a^3b^3d^4e^3 - 280a^4b^2d^4e^4 + 35a^5d^4e^5)x}{(e^7x + d^2e^6)\sqrt{ex + d}}$$

input

```
integrate((b^2*x^2+2*a*b*x+a^2)^(5/2)/(e*x+d)^(5/2),x, algorithm="maxima")
```

output

```
2/21*(3*b^5*e^5*x^5 - 256*b^5*d^5 + 896*a*b^4*d^4*e - 1120*a^2*b^3*d^3*e^2
+ 560*a^3*b^2*d^2*e^3 - 70*a^4*b*d*e^4 - 7*a^5*e^5 - 3*(2*b^5*d*e^4 - 7*a
*b^4*e^5)*x^4 + 2*(8*b^5*d^2*e^3 - 28*a*b^4*d*e^4 + 35*a^2*b^3*e^5)*x^3 -
6*(16*b^5*d^3*e^2 - 56*a*b^4*d^2*e^3 + 70*a^2*b^3*d*e^4 - 35*a^3*b^2*e^5)*
x^2 - 3*(128*b^5*d^4*e - 448*a*b^4*d^3*e^2 + 560*a^2*b^3*d^2*e^3 - 280*a^3
*b^2*d*e^4 + 35*a^4*b*e^5)*x)/((e^7*x + d^2*e^6)*sqrt(e*x + d))
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 461 vs. $2(230) = 460$.

Time = 0.17 (sec) , antiderivative size = 461, normalized size of antiderivative = 1.47

$$\int \frac{(a^2 + 2abx + b^2x^2)^{5/2}}{(d + ex)^{5/2}} dx =$$

$$\frac{2(15(ex + d)b^5d^4\operatorname{sgn}(bx + a) - b^5d^5\operatorname{sgn}(bx + a) - 60(ex + d)ab^4d^3\operatorname{esgn}(bx + a) + 5ab^4d^4\operatorname{esgn}(bx + a) - 21(3(ex + d)^{7/2}b^5e^{36}\operatorname{sgn}(bx + a) - 21(ex + d)^{5/2}b^5de^{36}\operatorname{sgn}(bx + a) + 70(ex + d)^{3/2}b^5d^2e^{36}\operatorname{sgn}(bx + a) - 210\sqrt{ex + d}b^5d^3e^{36}\operatorname{sgn}(bx + a) + 210\sqrt{ex + d}ab^4d^2e^{37}\operatorname{sgn}(bx + a) - 140(ex + d)^{3/2}ab^4de^{37}\operatorname{sgn}(bx + a) + 630\sqrt{ex + d}a^2b^3e^{38}\operatorname{sgn}(bx + a) - 630\sqrt{ex + d}a^2b^3de^{38}\operatorname{sgn}(bx + a) + 210\sqrt{ex + d}a^3b^2e^{39}\operatorname{sgn}(bx + a))/e^{42}}{((ex + d)^{3/2}e^6 + 2/21(3(ex + d)^{7/2}b^5e^{36}\operatorname{sgn}(bx + a) - 21(ex + d)^{5/2}b^5de^{36}\operatorname{sgn}(bx + a) + 70(ex + d)^{3/2}b^5d^2e^{36}\operatorname{sgn}(bx + a) - 210\sqrt{ex + d}b^5d^3e^{36}\operatorname{sgn}(bx + a) + 210\sqrt{ex + d}ab^4d^2e^{37}\operatorname{sgn}(bx + a) - 140(ex + d)^{3/2}ab^4de^{37}\operatorname{sgn}(bx + a) + 630\sqrt{ex + d}a^2b^3e^{38}\operatorname{sgn}(bx + a) - 630\sqrt{ex + d}a^2b^3de^{38}\operatorname{sgn}(bx + a) + 210\sqrt{ex + d}a^3b^2e^{39}\operatorname{sgn}(bx + a))/e^{42}}$$

input `integrate((b^2*x^2+2*a*b*x+a^2)^(5/2)/(e*x+d)^(5/2),x, algorithm="giac")`

output

```
-2/3*(15*(e*x + d)*b^5*d^4*sgn(b*x + a) - b^5*d^5*sgn(b*x + a) - 60*(e*x +
d)*a*b^4*d^3*e*sgn(b*x + a) + 5*a*b^4*d^4*e*sgn(b*x + a) + 90*(e*x + d)*
a^2*b^3*d^2*e^2*sgn(b*x + a) - 10*a^2*b^3*d^3*e^2*sgn(b*x + a) - 60*(e*x +
d)*a^3*b^2*d*e^3*sgn(b*x + a) + 10*a^3*b^2*d^2*e^3*sgn(b*x + a) + 15*(e*x
+ d)*a^4*b*e^4*sgn(b*x + a) - 5*a^4*b*d*e^4*sgn(b*x + a) + a^5*e^5*sgn(b*x
+ a))/((e*x + d)^(3/2)*e^6) + 2/21*(3*(e*x + d)^(7/2)*b^5*e^36*sgn(b*x +
a) - 21*(e*x + d)^(5/2)*b^5*d*e^36*sgn(b*x + a) + 70*(e*x + d)^(3/2)*b^5*d
^2*e^36*sgn(b*x + a) - 210*sqrt(e*x + d)*b^5*d^3*e^36*sgn(b*x + a) + 210*(e
*x + d)^(5/2)*a*b^4*e^37*sgn(b*x + a) - 140*(e*x + d)^(3/2)*a*b^4*d*e^37*s
gn(b*x + a) + 630*sqrt(e*x + d)*a*b^4*d^2*e^37*sgn(b*x + a) + 70*(e*x + d)
^(3/2)*a^2*b^3*e^38*sgn(b*x + a) - 630*sqrt(e*x + d)*a^2*b^3*d*e^38*sgn(b
*x + a) + 210*sqrt(e*x + d)*a^3*b^2*e^39*sgn(b*x + a))/e^42
```

Mupad [B] (verification not implemented)

Time = 6.52 (sec) , antiderivative size = 330, normalized size of antiderivative = 1.05

$$\int \frac{(a^2 + 2abx + b^2x^2)^{5/2}}{(d + ex)^{5/2}} dx = \frac{\sqrt{a^2 + 2abx + b^2x^2} \left(\frac{2b^4x^5}{7e^2} - \frac{14a^5e^5 + 140a^4bde^4 - 1120a^3b^2d^2e^3 + 2240a^2b^3d^3e^2 - 1792ab^4d^4e - 128b^5d^5}{21be^7} \right)}{(d + ex)^{5/2}}$$

input `int((a^2 + b^2*x^2 + 2*a*b*x)^(5/2)/(d + e*x)^(5/2),x)`

output

```
((a^2 + b^2*x^2 + 2*a*b*x)^(1/2)*((2*b^4*x^5)/(7*e^2) - (14*a^5*e^5 + 512*
b^5*d^5 + 2240*a^2*b^3*d^3*e^2 - 1120*a^3*b^2*d^2*e^3 - 1792*a*b^4*d^4*e +
140*a^4*b*d*e^4)/(21*b*e^7) + (2*b^3*x^4*(7*a*e - 2*b*d))/(7*e^3) - (x*(2
10*a^4*b*e^5 + 768*b^5*d^4*e - 2688*a*b^4*d^3*e^2 - 1680*a^3*b^2*d*e^4 + 3
360*a^2*b^3*d^2*e^3))/(21*b*e^7) + (4*b^2*x^3*(35*a^2*e^2 + 8*b^2*d^2 - 28
*a*b*d*e))/(21*e^4) + (4*b*x^2*(35*a^3*e^3 - 16*b^3*d^3 + 56*a*b^2*d^2*e -
70*a^2*b*d*e^2))/(7*e^5)))/(x^2*(d + e*x)^(1/2) + (a*d*(d + e*x)^(1/2))/(
b*e) + (x*(21*a*e^7 + 21*b*d*e^6)*(d + e*x)^(1/2))/(21*b*e^7))
```

Reduce [B] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 280, normalized size of antiderivative = 0.89

$$\int \frac{(a^2 + 2abx + b^2x^2)^{5/2}}{(d + ex)^{5/2}} dx = \frac{\frac{2}{7}b^5e^5x^5 + 2ab^4e^5x^4 - \frac{4}{7}b^5de^4x^4 + \frac{20}{3}a^2b^3e^5x^3 - \frac{16}{3}ab^4de^4x^3 + \frac{32}{21}b^5d^2e^3x^3 + \dots}{(d + ex)^{5/2}}$$

input

```
int((b^2*x^2+2*a*b*x+a^2)^(5/2)/(e*x+d)^(5/2),x)
```

output

```
(2*( - 7*a**5*e**5 - 70*a**4*b*d*e**4 - 105*a**4*b*e**5*x + 560*a**3*b**2*
d**2*e**3 + 840*a**3*b**2*d*e**4*x + 210*a**3*b**2*e**5*x**2 - 1120*a**2*b
**3*d**3*e**2 - 1680*a**2*b**3*d**2*e**3*x - 420*a**2*b**3*d*e**4*x**2 + 7
0*a**2*b**3*e**5*x**3 + 896*a*b**4*d**4*e + 1344*a*b**4*d**3*e**2*x + 336*
a*b**4*d**2*e**3*x**2 - 56*a*b**4*d*e**4*x**3 + 21*a*b**4*e**5*x**4 - 256*
b**5*d**5 - 384*b**5*d**4*e*x - 96*b**5*d**3*e**2*x**2 + 16*b**5*d**2*e**3
*x**3 - 6*b**5*d*e**4*x**4 + 3*b**5*e**5*x**5))/(21*sqrt(d + e*x)*e**6*(d
+ e*x))
```

3.366 $\int \frac{(a^2+2abx+b^2x^2)^{5/2}}{(d+ex)^{7/2}} dx$

| | |
|---|------|
| Optimal result | 2701 |
| Mathematica [A] (verified) | 2702 |
| Rubi [A] (verified) | 2702 |
| Maple [A] (verified) | 2704 |
| Fricas [A] (verification not implemented) | 2704 |
| Sympy [F] | 2705 |
| Maxima [A] (verification not implemented) | 2705 |
| Giac [A] (verification not implemented) | 2706 |
| Mupad [B] (verification not implemented) | 2706 |
| Reduce [B] (verification not implemented) | 2707 |

Optimal result

Integrand size = 30, antiderivative size = 316

$$\int \frac{(a^2 + 2abx + b^2x^2)^{5/2}}{(d + ex)^{7/2}} dx = \frac{2(bd - ae)^5 \sqrt{a^2 + 2abx + b^2x^2}}{5e^6(a + bx)(d + ex)^{5/2}} - \frac{10b(bd - ae)^4 \sqrt{a^2 + 2abx + b^2x^2}}{3e^6(a + bx)(d + ex)^{3/2}} + \frac{20b^2(bd - ae)^3 \sqrt{a^2 + 2abx + b^2x^2}}{e^6(a + bx)\sqrt{d + ex}} + \frac{20b^3(bd - ae)^2 \sqrt{d + ex} \sqrt{a^2 + 2abx + b^2x^2}}{e^6(a + bx)} - \frac{10b^4(bd - ae)(d + ex)^{3/2} \sqrt{a^2 + 2abx + b^2x^2}}{3e^6(a + bx)} + \frac{2b^5(d + ex)^{5/2} \sqrt{a^2 + 2abx + b^2x^2}}{5e^6(a + bx)}$$

output

```
2/5*(-a*e+b*d)^5*((b*x+a)^2)^(1/2)/e^6/(b*x+a)/(e*x+d)^(5/2)-10/3*b*(-a*e+b*d)^4*((b*x+a)^2)^(1/2)/e^6/(b*x+a)/(e*x+d)^(3/2)+20*b^2*(-a*e+b*d)^3*((b*x+a)^2)^(1/2)/e^6/(b*x+a)/(e*x+d)^(1/2)+20*b^3*(-a*e+b*d)^2*(e*x+d)^(1/2)*((b*x+a)^2)^(1/2)/e^6/(b*x+a)-10/3*b^4*(-a*e+b*d)*(e*x+d)^(3/2)*((b*x+a)^2)^(1/2)/e^6/(b*x+a)+2/5*b^5*(e*x+d)^(5/2)*((b*x+a)^2)^(1/2)/e^6/(b*x+a)
```

Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 236, normalized size of antiderivative = 0.75

$$\int \frac{(a^2 + 2abx + b^2x^2)^{5/2}}{(d + ex)^{7/2}} dx = \frac{2\sqrt{(a + bx)^2}(3a^5e^5 + 5a^4be^4(2d + 5ex) + 10a^3b^2e^3(8d^2 + 20dex + 15e^2x^2) - 30a^2b^3e^2(16d^3 + 40d^2ex + 30de^2x^2 + 5e^3x^3) + 5a^2b^4e(128d^4 + 320d^3ex + 240d^2e^2x^2 + 40de^3x^3 - 5e^4x^4) - b^5(256d^5 + 640d^4ex + 480d^3e^2x^2 + 80d^2e^3x^3 - 10de^4x^4 + 3e^5x^5))}{(15e^6(a + bx)(d + ex)^{5/2})}$$

input

```
Integrate[(a^2 + 2*a*b*x + b^2*x^2)^(5/2)/(d + e*x)^(7/2), x]
```

output

```
(-2*sqrt[(a + b*x)^2]*(3*a^5*e^5 + 5*a^4*b*e^4*(2*d + 5*e*x) + 10*a^3*b^2*
e^3*(8*d^2 + 20*d*e*x + 15*e^2*x^2) - 30*a^2*b^3*e^2*(16*d^3 + 40*d^2*e*x
+ 30*d*e^2*x^2 + 5*e^3*x^3) + 5*a*b^4*e*(128*d^4 + 320*d^3*e*x + 240*d^2*e
^2*x^2 + 40*d*e^3*x^3 - 5*e^4*x^4) - b^5*(256*d^5 + 640*d^4*e*x + 480*d^3*
e^2*x^2 + 80*d^2*e^3*x^3 - 10*d*e^4*x^4 + 3*e^5*x^5)))/(15*e^6*(a + b*x)*(
d + e*x)^(5/2))
```

Rubi [A] (verified)

Time = 0.53 (sec) , antiderivative size = 182, normalized size of antiderivative = 0.58, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {1102, 27, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a^2 + 2abx + b^2x^2)^{5/2}}{(d + ex)^{7/2}} dx$$

$$\downarrow \text{1102}$$

$$\frac{\sqrt{a^2 + 2abx + b^2x^2} \int \frac{b^5(a+bx)^5}{(d+ex)^{7/2}} dx}{b^5(a + bx)}$$

$$\downarrow \text{27}$$

$$\frac{\sqrt{a^2 + 2abx + b^2x^2} \int \frac{(a+bx)^5}{(d+ex)^{7/2}} dx}{a + bx}$$

↓ 53

$$\frac{\sqrt{a^2 + 2abx + b^2x^2} \int \left(\frac{(d+ex)^{3/2}b^5}{e^5} - \frac{5(bd-ae)\sqrt{d+ex}b^4}{e^5} + \frac{10(bd-ae)^2b^3}{e^5\sqrt{d+ex}} - \frac{10(bd-ae)^3b^2}{e^5(d+ex)^{3/2}} + \frac{5(bd-ae)^4b}{e^5(d+ex)^{5/2}} + \frac{(ae-bd)^5}{e^5(d+ex)^{7/2}} \right) dx}{a + bx}$$

↓ 2009

$$\frac{\sqrt{a^2 + 2abx + b^2x^2} \left(-\frac{10b^4(d+ex)^{3/2}(bd-ae)}{3e^6} + \frac{20b^3\sqrt{d+ex}(bd-ae)^2}{e^6} + \frac{20b^2(bd-ae)^3}{e^6\sqrt{d+ex}} - \frac{10b(bd-ae)^4}{3e^6(d+ex)^{3/2}} + \frac{2(bd-ae)^5}{5e^6(d+ex)^{5/2}} + \frac{2b^5(d+ex)^{7/2}}{5e^6} \right)}{a + bx}$$

input `Int[(a^2 + 2*a*b*x + b^2*x^2)^(5/2)/(d + e*x)^(7/2),x]`

output

```
(Sqrt[a^2 + 2*a*b*x + b^2*x^2]*((2*(b*d - a*e)^5)/(5*e^6*(d + e*x)^(5/2))
- (10*b*(b*d - a*e)^4)/(3*e^6*(d + e*x)^(3/2)) + (20*b^2*(b*d - a*e)^3)/(e
^6*Sqrt[d + e*x]) + (20*b^3*(b*d - a*e)^2*Sqrt[d + e*x])/e^6 - (10*b^4*(b*
d - a*e)*(d + e*x)^(3/2))/(3*e^6) + (2*b^5*(d + e*x)^(5/2))/(5*e^6)))/(a +
b*x)
```

Defintions of rubi rules used

rule 27

```
Int[(a_)*(F_x_), x_Symbol] := Simp[a Int[F_x, x], x] /; FreeQ[a, x] && !Ma
tchQ[F_x, (b_)*(G_x_) /; FreeQ[b, x]]
```

rule 53

```
Int[((a_) + (b_)*(x_))^(m_)*((c_) + (d_)*(x_))^(n_), x_Symbol] := Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && (!IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 1102

```
Int[((d_) + (e_)*(x_))^(m_)*((a_) + (b_)*(x_) + (c_)*(x_)^2)^(p_), x_S
ymbol] := Simp[(a + b*x + c*x^2)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x)^(2*F
racPart[p])) Int[(d + e*x)^m*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c,
d, e, m, p}, x] && EqQ[b^2 - 4*a*c, 0]
```

rule 2009

```
Int[u_, x_Symbol] := Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 1.19 (sec) , antiderivative size = 215, normalized size of antiderivative = 0.68

| method | result |
|---------|--|
| risch | $\frac{2b^3(3b^2e^2x^2+25xab e^2-19b^2dex+150e^2a^2-275abde+128b^2d^2)\sqrt{ex+d}\sqrt{(bx+a)^2}}{15e^6(bx+a)} - \frac{2(150b^2e^2x^2+25xab e^2+275b^2dex+3e^2a^2+128b^2d^2)}{15e^6\sqrt{ex+d}}$ |
| gospers | $-\frac{2(-3x^5e^5b^5-25x^4ab^4e^5+10x^4b^5de^4-150x^3a^2b^3e^5+200x^3ab^4de^4-80x^3b^5d^2e^3+150x^2a^3b^2e^5-900x^2a^2b^3de^4+1200x^2ab^4d^2e^3-300x^2a^3b^2de^3-900x^2a^2b^3de^4+1200x^2ab^4d^2e^3-300x^2a^3b^2de^3-900x^2a^2b^3de^4+1200x^2ab^4d^2e^3)}{15e^6(bx+a)}$ |
| default | $-\frac{2(-3x^5e^5b^5-25x^4ab^4e^5+10x^4b^5de^4-150x^3a^2b^3e^5+200x^3ab^4de^4-80x^3b^5d^2e^3+150x^2a^3b^2e^5-900x^2a^2b^3de^4+1200x^2ab^4d^2e^3-300x^2a^3b^2de^3-900x^2a^2b^3de^4+1200x^2ab^4d^2e^3)}{15e^6(bx+a)}$ |
| orering | $-\frac{2(-3x^5e^5b^5-25x^4ab^4e^5+10x^4b^5de^4-150x^3a^2b^3e^5+200x^3ab^4de^4-80x^3b^5d^2e^3+150x^2a^3b^2e^5-900x^2a^2b^3de^4+1200x^2ab^4d^2e^3-300x^2a^3b^2de^3-900x^2a^2b^3de^4+1200x^2ab^4d^2e^3)}{15e^6(bx+a)}$ |

input

```
int((b^2*x^2+2*a*b*x+a^2)^(5/2)/(e*x+d)^(7/2),x,method=_RETURNVERBOSE)
```

output

```
2/15*b^3*(3*b^2*e^2*x^2+25*a*b*e^2*x-19*b^2*d*e*x+150*a^2*e^2-275*a*b*d*e+128*b^2*d^2)*(e*x+d)^(1/2)/e^6*((b*x+a)^2)^(1/2)/(b*x+a)-2/15*(150*b^2*e^2*x^2+25*a*b*e^2*x+275*b^2*d*e*x+3*a^2*e^2+19*a*b*d*e+128*b^2*d^2)*(a^3*e^3-3*a^2*b*d*e^2+3*a*b^2*d^2*e-b^3*d^3)/e^6/(e*x+d)^(1/2)/(e^2*x^2+2*d*e*x+d^2)*((b*x+a)^2)^(1/2)/(b*x+a)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 294, normalized size of antiderivative = 0.93

$$\int \frac{(a^2 + 2abx + b^2x^2)^{5/2}}{(d + ex)^{7/2}} dx = \frac{2(3b^5e^5x^5 + 256b^5d^5 - 640ab^4d^4e + 480a^2b^3d^3e^2 - 80a^3b^2d^2e^3 - 10a^4bde^4)}{(d + ex)^{7/2}}$$

input

```
integrate((b^2*x^2+2*a*b*x+a^2)^(5/2)/(e*x+d)^(7/2),x, algorithm="fricas")
```

output

```
2/15*(3*b^5*e^5*x^5 + 256*b^5*d^5 - 640*a*b^4*d^4*e + 480*a^2*b^3*d^3*e^2
- 80*a^3*b^2*d^2*e^3 - 10*a^4*b*d*e^4 - 3*a^5*e^5 - 5*(2*b^5*d*e^4 - 5*a*b
^4*e^5)*x^4 + 10*(8*b^5*d^2*e^3 - 20*a*b^4*d*e^4 + 15*a^2*b^3*e^5)*x^3 + 3
0*(16*b^5*d^3*e^2 - 40*a*b^4*d^2*e^3 + 30*a^2*b^3*d*e^4 - 5*a^3*b^2*e^5)*x
^2 + 5*(128*b^5*d^4*e - 320*a*b^4*d^3*e^2 + 240*a^2*b^3*d^2*e^3 - 40*a^3*b
^2*d*e^4 - 5*a^4*b*e^5)*x)*sqrt(e*x + d)/(e^9*x^3 + 3*d*e^8*x^2 + 3*d^2*e^
7*x + d^3*e^6)
```

Sympy [F]

$$\int \frac{(a^2 + 2abx + b^2x^2)^{5/2}}{(d + ex)^{7/2}} dx = \int \frac{((a + bx)^2)^{5/2}}{(d + ex)^{7/2}} dx$$

input

```
integrate((b**2*x**2+2*a*b*x+a**2)**(5/2)/(e*x+d)**(7/2),x)
```

output

```
Integral(((a + b*x)**2)**(5/2)/(d + e*x)**(7/2), x)
```

Maxima [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 283, normalized size of antiderivative = 0.90

$$\int \frac{(a^2 + 2abx + b^2x^2)^{5/2}}{(d + ex)^{7/2}} dx = \frac{2(3b^5e^5x^5 + 256b^5d^5 - 640ab^4d^4e + 480a^2b^3d^3e^2 - 80a^3b^2d^2e^3 - 10a^4bde^4 + 5a^5e^5 - 5(2b^5d^2e^3 - 20ab^4de^4 + 15a^2b^3e^5)x^3 + 30(16b^5d^3e^2 - 40ab^4d^2e^3 + 30a^2b^3de^4 - 5a^3b^2e^5)x^2 + 5(128b^5d^4e - 320ab^4d^3e^2 + 240a^2b^3d^2e^3 - 40a^3b^2de^4 - 5a^4b^2e^5)x)}{(e^8x^2 + 2de^7x + d^2e^6)*sqrt(ex + d)}$$

input

```
integrate((b^2*x^2+2*a*b*x+a^2)^(5/2)/(e*x+d)^(7/2),x, algorithm="maxima")
```

output

```
2/15*(3*b^5*e^5*x^5 + 256*b^5*d^5 - 640*a*b^4*d^4*e + 480*a^2*b^3*d^3*e^2
- 80*a^3*b^2*d^2*e^3 - 10*a^4*b*d*e^4 - 3*a^5*e^5 - 5*(2*b^5*d^2*e^3 - 5*a*b
^4*e^5)*x^4 + 10*(8*b^5*d^2*e^3 - 20*a*b^4*d*e^4 + 15*a^2*b^3*e^5)*x^3 + 3
0*(16*b^5*d^3*e^2 - 40*a*b^4*d^2*e^3 + 30*a^2*b^3*d*e^4 - 5*a^3*b^2*e^5)*x
^2 + 5*(128*b^5*d^4*e - 320*a*b^4*d^3*e^2 + 240*a^2*b^3*d^2*e^3 - 40*a^3*b
^2*d*e^4 - 5*a^4*b^2*e^5)*x)/((e^8*x^2 + 2*d*e^7*x + d^2*e^6)*sqrt(e*x + d))
```

Giac [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 457, normalized size of antiderivative = 1.45

$$\int \frac{(a^2 + 2abx + b^2x^2)^{5/2}}{(d + ex)^{7/2}} dx = \frac{2(150(ex + d)^2b^5d^3\operatorname{sgn}(bx + a) - 25(ex + d)b^5d^4\operatorname{sgn}(bx + a) + 3b^5d^5\operatorname{sgn}(bx + a) - 2\left(3(ex + d)^{5/2}b^5e^{24}\operatorname{sgn}(bx + a) - 25(ex + d)^{3/2}b^5de^{24}\operatorname{sgn}(bx + a) + 150\sqrt{ex + d}b^5d^2e^{24}\operatorname{sgn}(bx + a) + 25\sqrt{ex + d}b^5d^2e^{24}\operatorname{sgn}(bx + a) + 25\sqrt{ex + d}b^5d^2e^{24}\operatorname{sgn}(bx + a) + 25\sqrt{ex + d}b^5d^2e^{24}\operatorname{sgn}(bx + a)\right)}{15e^{30}}$$

input `integrate((b^2*x^2+2*a*b*x+a^2)^(5/2)/(e*x+d)^(7/2),x, algorithm="giac")`output

```
2/15*(150*(e*x + d)^2*b^5*d^3*sgn(b*x + a) - 25*(e*x + d)*b^5*d^4*sgn(b*x
+ a) + 3*b^5*d^5*sgn(b*x + a) - 450*(e*x + d)^2*a*b^4*d^2*e*sgn(b*x + a) +
100*(e*x + d)*a*b^4*d^3*e*sgn(b*x + a) - 15*a*b^4*d^4*e*sgn(b*x + a) + 45
0*(e*x + d)^2*a^2*b^3*d*e^2*sgn(b*x + a) - 150*(e*x + d)*a^2*b^3*d^2*e^2*s
gn(b*x + a) + 30*a^2*b^3*d^3*e^2*sgn(b*x + a) - 150*(e*x + d)^2*a^3*b^2*e^
3*sgn(b*x + a) + 100*(e*x + d)*a^3*b^2*d*e^3*sgn(b*x + a) - 30*a^3*b^2*d^2
*e^3*sgn(b*x + a) - 25*(e*x + d)*a^4*b*e^4*sgn(b*x + a) + 15*a^4*b*d*e^4*s
gn(b*x + a) - 3*a^5*e^5*sgn(b*x + a))/((e*x + d)^(5/2)*e^6) + 2/15*(3*(e*x
+ d)^(5/2)*b^5*e^24*sgn(b*x + a) - 25*(e*x + d)^(3/2)*b^5*d*e^24*sgn(b*x
+ a) + 150*sqrt(e*x + d)*b^5*d^2*e^24*sgn(b*x + a) + 25*(e*x + d)^(3/2)*a*
b^4*e^25*sgn(b*x + a) - 300*sqrt(e*x + d)*a*b^4*d*e^25*sgn(b*x + a) + 150*
sqrt(e*x + d)*a^2*b^3*e^26*sgn(b*x + a))/e^30
```

Mupad [B] (verification not implemented)

Time = 6.27 (sec) , antiderivative size = 359, normalized size of antiderivative = 1.14

$$\int \frac{(a^2 + 2abx + b^2x^2)^{5/2}}{(d + ex)^{7/2}} dx = \frac{\sqrt{a^2 + 2abx + b^2x^2} \left(\frac{6a^5e^5 + 20a^4bde^4 + 160a^3b^2d^2e^3 - 960a^2b^3d^3e^2 + 1280ab^4d^4e - 512b^5d^5}{15be^8} - \frac{2b^4x^5}{5e^3} - \frac{2b^3x^4(5ae - 2bd)}{3e^4} \right)}{x^3\sqrt{d + ex} + \frac{ad^2\sqrt{d + ex}}{be^2}}$$

input `int((a^2 + b^2*x^2 + 2*a*b*x)^(5/2)/(d + e*x)^(7/2),x)`

output

```

-((a^2 + b^2*x^2 + 2*a*b*x)^(1/2)*((6*a^5*e^5 - 512*b^5*d^5 - 960*a^2*b^3*
d^3*e^2 + 160*a^3*b^2*d^2*e^3 + 1280*a*b^4*d^4*e + 20*a^4*b*d*e^4)/(15*b*e
^8) - (2*b^4*x^5)/(5*e^3) - (2*b^3*x^4*(5*a*e - 2*b*d))/(3*e^4) + (x*(50*a
^4*b*e^5 - 1280*b^5*d^4*e + 3200*a*b^4*d^3*e^2 + 400*a^3*b^2*d*e^4 - 2400*
a^2*b^3*d^2*e^3))/(15*b*e^8) - (4*b^2*x^3*(15*a^2*e^2 + 8*b^2*d^2 - 20*a*b
*d*e))/(3*e^5) + (4*b*x^2*(5*a^3*e^3 - 16*b^3*d^3 + 40*a*b^2*d^2*e - 30*a
^2*b*d*e^2))/e^6))/(x^3*(d + e*x)^(1/2) + (a*d^2*(d + e*x)^(1/2))/(b*e^2) +
(x^2*(15*a*e^8 + 30*b*d*e^7)*(d + e*x)^(1/2))/(15*b*e^8) + (d*x*(2*a*e +
b*d)*(d + e*x)^(1/2))/(b*e^2))

```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 291, normalized size of antiderivative = 0.92

$$\int \frac{(a^2 + 2abx + b^2x^2)^{5/2}}{(d + ex)^{7/2}} dx = \frac{2}{5}b^5e^5x^5 + \frac{10}{3}ab^4e^5x^4 - \frac{4}{3}b^5de^4x^4 + 20a^2b^3e^5x^3 - \frac{80}{3}ab^4de^4x^3 + \frac{32}{3}b^5d^2e^3x^3$$

input

```
int((b^2*x^2+2*a*b*x+a^2)^(5/2)/(e*x+d)^(7/2),x)
```

output

```

(2*( - 3*a**5*e**5 - 10*a**4*b*d*e**4 - 25*a**4*b*e**5*x - 80*a**3*b**2*d
*2*e**3 - 200*a**3*b**2*d*e**4*x - 150*a**3*b**2*e**5*x**2 + 480*a**2*b**3
*d**3*e**2 + 1200*a**2*b**3*d**2*e**3*x + 900*a**2*b**3*d*e**4*x**2 + 150*
a**2*b**3*e**5*x**3 - 640*a*b**4*d**4*e - 1600*a*b**4*d**3*e**2*x - 1200*a
*b**4*d**2*e**3*x**2 - 200*a*b**4*d*e**4*x**3 + 25*a*b**4*e**5*x**4 + 256*
b**5*d**5 + 640*b**5*d**4*e*x + 480*b**5*d**3*e**2*x**2 + 80*b**5*d**2*e**
3*x**3 - 10*b**5*d*e**4*x**4 + 3*b**5*e**5*x**5))/(15*sqrt(d + e*x)*e**6*(
d**2 + 2*d*e*x + e**2*x**2))

```


3.367 $\int \frac{(a^2+2abx+b^2x^2)^{5/2}}{(d+ex)^{9/2}} dx$

| | |
|---|------|
| Optimal result | 2708 |
| Mathematica [A] (verified) | 2709 |
| Rubi [A] (verified) | 2709 |
| Maple [A] (verified) | 2711 |
| Fricas [A] (verification not implemented) | 2711 |
| Sympy [F] | 2712 |
| Maxima [A] (verification not implemented) | 2712 |
| Giac [A] (verification not implemented) | 2713 |
| Mupad [B] (verification not implemented) | 2713 |
| Reduce [B] (verification not implemented) | 2714 |

Optimal result

Integrand size = 30, antiderivative size = 314

$$\int \frac{(a^2 + 2abx + b^2x^2)^{5/2}}{(d + ex)^{9/2}} dx = \frac{2(bd - ae)^5 \sqrt{a^2 + 2abx + b^2x^2}}{7e^6(a + bx)(d + ex)^{7/2}} - \frac{2b(bd - ae)^4 \sqrt{a^2 + 2abx + b^2x^2}}{e^6(a + bx)(d + ex)^{5/2}} + \frac{20b^2(bd - ae)^3 \sqrt{a^2 + 2abx + b^2x^2}}{3e^6(a + bx)(d + ex)^{3/2}} - \frac{20b^3(bd - ae)^2 \sqrt{a^2 + 2abx + b^2x^2}}{e^6(a + bx)\sqrt{d + ex}} - \frac{10b^4(bd - ae)\sqrt{d + ex}\sqrt{a^2 + 2abx + b^2x^2}}{e^6(a + bx)} + \frac{2b^5(d + ex)^{3/2}\sqrt{a^2 + 2abx + b^2x^2}}{3e^6(a + bx)}$$

output

```
2/7*(-a*e+b*d)^5*((b*x+a)^2)^(1/2)/e^6/(b*x+a)/(e*x+d)^(7/2)-2*b*(-a*e+b*d)^4*((b*x+a)^2)^(1/2)/e^6/(b*x+a)/(e*x+d)^(5/2)+20/3*b^2*(-a*e+b*d)^3*((b*x+a)^2)^(1/2)/e^6/(b*x+a)/(e*x+d)^(3/2)-20*b^3*(-a*e+b*d)^2*((b*x+a)^2)^(1/2)/e^6/(b*x+a)/(e*x+d)^(1/2)-10*b^4*(-a*e+b*d)*(e*x+d)^(1/2)*((b*x+a)^2)^(1/2)/e^6/(b*x+a)+2/3*b^5*(e*x+d)^(3/2)*((b*x+a)^2)^(1/2)/e^6/(b*x+a)
```

Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 235, normalized size of antiderivative = 0.75

$$\int \frac{(a^2 + 2abx + b^2x^2)^{5/2}}{(d + ex)^{9/2}} dx = \frac{2\sqrt{(a + bx)^2}(3a^5e^5 + 3a^4be^4(2d + 7ex) + 2a^3b^2e^3(8d^2 + 28dex + 35e^2x^2) + 6a^2b^3e^2(16d^3 + 56d^2ex + 70d^2e^2x^2 + 35e^3x^3) - 3a^2b^4e(128d^4 + 448d^3ex + 560d^2e^2x^2 + 280d^2e^3x^3 + 35e^4x^4) + b^5(256d^5 + 896d^4ex + 1120d^3e^2x^2 + 560d^2e^3x^3 + 70d^2e^4x^4 - 7e^5x^5))}{(21e^6(a + bx)^2)(d + ex)^{7/2}}$$

input

```
Integrate[(a^2 + 2*a*b*x + b^2*x^2)^(5/2)/(d + e*x)^(9/2), x]
```

output

```
(-2*sqrt[(a + b*x)^2]*(3*a^5*e^5 + 3*a^4*b*e^4*(2*d + 7*e*x) + 2*a^3*b^2*e^3*(8*d^2 + 28*d*e*x + 35*e^2*x^2) + 6*a^2*b^3*e^2*(16*d^3 + 56*d^2*e*x + 70*d^2*e^2*x^2 + 35*e^3*x^3) - 3*a*b^4*e*(128*d^4 + 448*d^3*e*x + 560*d^2*e^2*x^2 + 280*d^2*e^3*x^3 + 35*e^4*x^4) + b^5*(256*d^5 + 896*d^4*e*x + 1120*d^3*e^2*x^2 + 560*d^2*e^3*x^3 + 70*d^2*e^4*x^4 - 7*e^5*x^5)))/(21*e^6*(a + b*x)^2*(d + e*x)^(7/2))
```

Rubi [A] (verified)

Time = 0.55 (sec) , antiderivative size = 180, normalized size of antiderivative = 0.57, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {1102, 27, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\begin{aligned} & \int \frac{(a^2 + 2abx + b^2x^2)^{5/2}}{(d + ex)^{9/2}} dx \\ & \quad \downarrow \text{1102} \\ & \frac{\sqrt{a^2 + 2abx + b^2x^2} \int \frac{b^5(a+bx)^5}{(d+ex)^{9/2}} dx}{b^5(a + bx)} \\ & \quad \downarrow \text{27} \\ & \frac{\sqrt{a^2 + 2abx + b^2x^2} \int \frac{(a+bx)^5}{(d+ex)^{9/2}} dx}{a + bx} \end{aligned}$$

53

$$\frac{\sqrt{a^2 + 2abx + b^2x^2} \int \left(\frac{\sqrt{d+ex}b^5}{e^5} - \frac{5(bd-ae)b^4}{e^5\sqrt{d+ex}} + \frac{10(bd-ae)^2b^3}{e^5(d+ex)^{3/2}} - \frac{10(bd-ae)^3b^2}{e^5(d+ex)^{5/2}} + \frac{5(bd-ae)^4b}{e^5(d+ex)^{7/2}} + \frac{(ae-bd)^5}{e^5(d+ex)^{9/2}} \right) dx}{a + bx}$$

2009

$$\frac{\sqrt{a^2 + 2abx + b^2x^2} \left(-\frac{10b^4\sqrt{d+ex}(bd-ae)}{e^6} - \frac{20b^3(bd-ae)^2}{e^6\sqrt{d+ex}} + \frac{20b^2(bd-ae)^3}{3e^6(d+ex)^{3/2}} - \frac{2b(bd-ae)^4}{e^6(d+ex)^{5/2}} + \frac{2(bd-ae)^5}{7e^6(d+ex)^{7/2}} + \frac{2b^5(d+ex)^{3/2}}{3e^6} \right)}{a + bx}$$

input

```
Int[(a^2 + 2*a*b*x + b^2*x^2)^(5/2)/(d + e*x)^(9/2),x]
```

output

```
(Sqrt[a^2 + 2*a*b*x + b^2*x^2]*((2*(b*d - a*e)^5)/(7*e^6*(d + e*x)^(7/2))
- (2*b*(b*d - a*e)^4)/(e^6*(d + e*x)^(5/2)) + (20*b^2*(b*d - a*e)^3)/(3*e^
6*(d + e*x)^(3/2)) - (20*b^3*(b*d - a*e)^2)/(e^6*Sqrt[d + e*x]) - (10*b^4*
(b*d - a*e)*Sqrt[d + e*x])/e^6 + (2*b^5*(d + e*x)^(3/2))/(3*e^6)))/(a + b*
x)
```

Defintions of rubi rules used

rule 27

```
Int[(a_)*(F_x_), x_Symbol] := Simp[a Int[F_x, x], x] /; FreeQ[a, x] && !Ma
tchQ[F_x, (b_)*(G_x_) /; FreeQ[b, x]]
```

rule 53

```
Int[((a_) + (b_)*(x_))^(m_)*((c_) + (d_)*(x_))^(n_), x_Symbol] := Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && (!IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 1102

```
Int[((d_) + (e_)*(x_))^(m_)*((a_) + (b_)*(x_) + (c_)*(x_)^2)^(p_), x_S
ymbol] := Simp[(a + b*x + c*x^2)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x)^(2*F
racPart[p])) Int[(d + e*x)^m*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c,
d, e, m, p}, x] && EqQ[b^2 - 4*a*c, 0]
```

rule 2009 `Int[u_, x_Symbol] := Simp[IntSum[u, x], x] /; SumQ[u]`

Maple [A] (verified)

Time = 1.22 (sec) , antiderivative size = 227, normalized size of antiderivative = 0.72

| method | result |
|---------|---|
| risch | $\frac{2b^4(bex+15ae-14bd)\sqrt{ex+d}\sqrt{(bx+a)^2}}{3e^6(bx+a)} - \frac{2(210e^3x^3b^3+70x^2ab^2e^3+560x^2b^3d^2e^2+21a^2be^3x+98xab^2de^2+511b^3d^2ex+3e^3a^3+21e^6\sqrt{ex+d}(e^3x^3+3de^2x^2+3d^2e^2x+3d^3e^2))}{21e^6\sqrt{ex+d}(e^3x^3+3de^2x^2+3d^2e^2x+3d^3e^2)}$ |
| gosper | $-\frac{2(-7x^5e^5b^5-105x^4ab^4e^5+70x^4b^5de^4+210x^3a^2b^3e^5-840x^3ab^4de^4+560x^3b^5d^2e^3+70x^2a^3b^2e^5+420x^2a^2b^3de^4-1680x^2ab^4de^3-140x^2a^2b^3d^2e^2+21a^2be^3x+98xab^2de^2+511b^3d^2ex+3e^3a^3+21e^6\sqrt{ex+d}(e^3x^3+3de^2x^2+3d^2e^2x+3d^3e^2))}{21e^6\sqrt{ex+d}(e^3x^3+3de^2x^2+3d^2e^2x+3d^3e^2)}$ |
| default | $-\frac{2(-7x^5e^5b^5-105x^4ab^4e^5+70x^4b^5de^4+210x^3a^2b^3e^5-840x^3ab^4de^4+560x^3b^5d^2e^3+70x^2a^3b^2e^5+420x^2a^2b^3de^4-1680x^2ab^4de^3-140x^2a^2b^3d^2e^2+21a^2be^3x+98xab^2de^2+511b^3d^2ex+3e^3a^3+21e^6\sqrt{ex+d}(e^3x^3+3de^2x^2+3d^2e^2x+3d^3e^2))}{21e^6\sqrt{ex+d}(e^3x^3+3de^2x^2+3d^2e^2x+3d^3e^2)}$ |
| orering | $-\frac{2(-7x^5e^5b^5-105x^4ab^4e^5+70x^4b^5de^4+210x^3a^2b^3e^5-840x^3ab^4de^4+560x^3b^5d^2e^3+70x^2a^3b^2e^5+420x^2a^2b^3de^4-1680x^2ab^4de^3-140x^2a^2b^3d^2e^2+21a^2be^3x+98xab^2de^2+511b^3d^2ex+3e^3a^3+21e^6\sqrt{ex+d}(e^3x^3+3de^2x^2+3d^2e^2x+3d^3e^2))}{21e^6\sqrt{ex+d}(e^3x^3+3de^2x^2+3d^2e^2x+3d^3e^2)}$ |

input `int((b^2*x^2+2*a*b*x+a^2)^(5/2)/(e*x+d)^(9/2),x,method=_RETURNVERBOSE)`

output
$$\frac{2}{3}b^4*(b*ex+15*a*e-14*b*d)*(e*x+d)^(1/2)/e^6*((b*x+a)^2)^(1/2)/(b*x+a)-\frac{2}{21}*(210*b^3*e^3*x^3+70*a*b^2*e^3*x^2+560*b^3*d*e^2*x^2+21*a^2*b*e^3*x+98*a*b^2*d*e^2*x+511*b^3*d^2*e*x+3*a^3*e^3+12*a^2*b*d*e^2+37*a*b^2*d^2*e+158*b^3*d^3)*(a^2*e^2-2*a*b*d*e+b^2*d^2)/e^6/(e*x+d)^(1/2)/(e^3*x^3+3*d*e^2*x^2+3*d^2*e*x+d^3)*((b*x+a)^2)^(1/2)/(b*x+a)$$

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 304, normalized size of antiderivative = 0.97

$$\int \frac{(a^2 + 2abx + b^2x^2)^{5/2}}{(d + ex)^{9/2}} dx = \frac{2(7b^5e^5x^5 - 256b^5d^5 + 384ab^4d^4e - 96a^2b^3d^3e^2 - 16a^3b^2d^2e^3 - 6a^4bde^4 - 21e^6\sqrt{ex+d}(e^3x^3+3de^2x^2+3d^2e^2x+3d^3e^2))}{21e^6\sqrt{ex+d}(e^3x^3+3de^2x^2+3d^2e^2x+3d^3e^2)}$$

input `integrate((b^2*x^2+2*a*b*x+a^2)^(5/2)/(e*x+d)^(9/2),x, algorithm="fricas")`

output

```
2/21*(7*b^5*e^5*x^5 - 256*b^5*d^5 + 384*a*b^4*d^4*e - 96*a^2*b^3*d^3*e^2 -
16*a^3*b^2*d^2*e^3 - 6*a^4*b*d*e^4 - 3*a^5*e^5 - 35*(2*b^5*d*e^4 - 3*a*b^
4*e^5)*x^4 - 70*(8*b^5*d^2*e^3 - 12*a*b^4*d*e^4 + 3*a^2*b^3*e^5)*x^3 - 70*
(16*b^5*d^3*e^2 - 24*a*b^4*d^2*e^3 + 6*a^2*b^3*d*e^4 + a^3*b^2*e^5)*x^2 -
7*(128*b^5*d^4*e - 192*a*b^4*d^3*e^2 + 48*a^2*b^3*d^2*e^3 + 8*a^3*b^2*d*e^
4 + 3*a^4*b*e^5)*x)*sqrt(e*x + d)/(e^10*x^4 + 4*d*e^9*x^3 + 6*d^2*e^8*x^2
+ 4*d^3*e^7*x + d^4*e^6)
```

Sympy [F]

$$\int \frac{(a^2 + 2abx + b^2x^2)^{5/2}}{(d + ex)^{9/2}} dx = \int \frac{((a + bx)^2)^{5/2}}{(d + ex)^{9/2}} dx$$

input

```
integrate((b**2*x**2+2*a*b*x+a**2)**(5/2)/(e*x+d)**(9/2),x)
```

output

```
Integral(((a + b*x)**2)**(5/2)/(d + e*x)**(9/2), x)
```

Maxima [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 293, normalized size of antiderivative = 0.93

$$\int \frac{(a^2 + 2abx + b^2x^2)^{5/2}}{(d + ex)^{9/2}} dx = \frac{2(7b^5e^5x^5 - 256b^5d^5 + 384ab^4d^4e - 96a^2b^3d^3e^2 - 16a^3b^2d^2e^3 - 6a^4bde^4 - 3a^5e^5) \sqrt{ex + d}}{(e^9x^3 + 3de^8x^2 + 3d^2e^7x + d^3e^6)}$$

input

```
integrate((b^2*x^2+2*a*b*x+a^2)^(5/2)/(e*x+d)^(9/2),x, algorithm="maxima")
```

output

```
2/21*(7*b^5*e^5*x^5 - 256*b^5*d^5 + 384*a*b^4*d^4*e - 96*a^2*b^3*d^3*e^2 -
16*a^3*b^2*d^2*e^3 - 6*a^4*b*d*e^4 - 3*a^5*e^5 - 35*(2*b^5*d*e^4 - 3*a*b^
4*e^5)*x^4 - 70*(8*b^5*d^2*e^3 - 12*a*b^4*d*e^4 + 3*a^2*b^3*e^5)*x^3 - 70*
(16*b^5*d^3*e^2 - 24*a*b^4*d^2*e^3 + 6*a^2*b^3*d*e^4 + a^3*b^2*e^5)*x^2 -
7*(128*b^5*d^4*e - 192*a*b^4*d^3*e^2 + 48*a^2*b^3*d^2*e^3 + 8*a^3*b^2*d*e^
4 + 3*a^4*b*e^5)*x)/((e^9*x^3 + 3*d*e^8*x^2 + 3*d^2*e^7*x + d^3*e^6)*sqrt(
e*x + d))
```

Giac [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 451, normalized size of antiderivative = 1.44

$$\int \frac{(a^2 + 2abx + b^2x^2)^{5/2}}{(d + ex)^{9/2}} dx =$$

$$\frac{2(210(ex + d)^3 b^5 d^2 \operatorname{sgn}(bx + a) - 70(ex + d)^2 b^5 d^3 \operatorname{sgn}(bx + a) + 21(ex + d)b^5 d^4 \operatorname{sgn}(bx + a) - 3b^5 d^5 \operatorname{sgn}(bx + a))}{3e^{18}}$$

$$+ \frac{2\left((ex + d)^{\frac{3}{2}} b^5 e^{12} \operatorname{sgn}(bx + a) - 15\sqrt{ex + d} b^5 d e^{12} \operatorname{sgn}(bx + a) + 15\sqrt{ex + d} a b^4 e^{13} \operatorname{sgn}(bx + a)\right)}{3e^{18}}$$

input `integrate((b^2*x^2+2*a*b*x+a^2)^(5/2)/(e*x+d)^(9/2),x, algorithm="giac")`

output

```
-2/21*(210*(e*x + d)^3*b^5*d^2*sgn(b*x + a) - 70*(e*x + d)^2*b^5*d^3*sgn(b*x + a) + 21*(e*x + d)*b^5*d^4*sgn(b*x + a) - 3*b^5*d^5*sgn(b*x + a) - 420*(e*x + d)^3*a*b^4*d*e*sgn(b*x + a) + 210*(e*x + d)^2*a*b^4*d^2*e*sgn(b*x + a) - 84*(e*x + d)*a*b^4*d^3*e*sgn(b*x + a) + 15*a*b^4*d^4*e*sgn(b*x + a) + 210*(e*x + d)^3*a^2*b^3*e^2*sgn(b*x + a) - 210*(e*x + d)^2*a^2*b^3*d*e^2*sgn(b*x + a) + 126*(e*x + d)*a^2*b^3*d^2*e^2*sgn(b*x + a) - 30*a^2*b^3*d^3*e^2*sgn(b*x + a) + 70*(e*x + d)^2*a^3*b^2*e^3*sgn(b*x + a) - 84*(e*x + d)*a^3*b^2*d*e^3*sgn(b*x + a) + 30*a^3*b^2*d^2*e^3*sgn(b*x + a) + 21*(e*x + d)*a^4*b*e^4*sgn(b*x + a) - 15*a^4*b*d*e^4*sgn(b*x + a) + 3*a^5*e^5*sgn(b*x + a))/((e*x + d)^(7/2)*e^6) + 2/3*((e*x + d)^(3/2)*b^5*e^12*sgn(b*x + a) - 15*sqrt(e*x + d)*b^5*d*e^12*sgn(b*x + a) + 15*sqrt(e*x + d)*a*b^4*e^13*sgn(b*x + a))/e^18
```

Mupad [B] (verification not implemented)

Time = 6.17 (sec) , antiderivative size = 386, normalized size of antiderivative = 1.23

$$\int \frac{(a^2 + 2abx + b^2x^2)^{5/2}}{(d + ex)^{9/2}} dx =$$

$$\frac{\sqrt{a^2 + 2abx + b^2x^2} \left(\frac{6a^5e^5 + 12a^4bde^4 + 32a^3b^2d^2e^3 + 192a^2b^3d^3e^2 - 768ab^4d^4e + 512b^5d^5}{21be^9} - \frac{2b^4x^5}{3e^4} - \frac{10b^3x^4(3ae - 2bd)}{3e^5} \right)}{x^4 \sqrt{d + ex} + \frac{ad^3 \sqrt{d + ex}}{be^3} + \frac{x^3(21ae^5 - 21ad^3 - 21bd^3)}{e^6}}$$

input `int((a^2 + b^2*x^2 + 2*a*b*x)^(5/2)/(d + e*x)^(9/2),x)`

output

```

-((a^2 + b^2*x^2 + 2*a*b*x)^(1/2)*((6*a^5*e^5 + 512*b^5*d^5 + 192*a^2*b^3*
d^3*e^2 + 32*a^3*b^2*d^2*e^3 - 768*a*b^4*d^4*e + 12*a^4*b*d*e^4)/(21*b*e^9
) - (2*b^4*x^5)/(3*e^4) - (10*b^3*x^4*(3*a*e - 2*b*d))/(3*e^5) + (x*(42*a^
4*b*e^5 + 1792*b^5*d^4*e - 2688*a*b^4*d^3*e^2 + 112*a^3*b^2*d*e^4 + 672*a^
2*b^3*d^2*e^3))/(21*b*e^9) + (20*b^2*x^3*(3*a^2*e^2 + 8*b^2*d^2 - 12*a*b*d
*e))/(3*e^6) + (20*b*x^2*(a^3*e^3 + 16*b^3*d^3 - 24*a*b^2*d^2*e + 6*a^2*b*
d*e^2))/(3*e^7)))/(x^4*(d + e*x)^(1/2) + (a*d^3*(d + e*x)^(1/2))/(b*e^3) +
(x^3*(21*a*e^9 + 63*b*d*e^8)*(d + e*x)^(1/2))/(21*b*e^9) + (3*d*x^2*(a*e
+ b*d)*(d + e*x)^(1/2))/(b*e^2) + (d^2*x*(3*a*e + b*d)*(d + e*x)^(1/2))/(b
*e^3))

```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 302, normalized size of antiderivative = 0.96

$$\int \frac{(a^2 + 2abx + b^2x^2)^{5/2}}{(d + ex)^{9/2}} dx = \frac{\frac{2}{3}b^5e^5x^5 + 10ab^4e^5x^4 - \frac{20}{3}b^5de^4x^4 - 20a^2b^3e^5x^3 + 80ab^4de^4x^3 - \frac{160}{3}b^5d^2e^3x^2 + 160a^2b^3de^4x^2 - 160ab^4d^2e^4x - \frac{160}{3}b^5d^3e^4}{(d + ex)^{9/2}}$$

input

```
int((b^2*x^2+2*a*b*x+a^2)^(5/2)/(e*x+d)^(9/2),x)
```

output

```

(2*( - 3*a**5*e**5 - 6*a**4*b*d*e**4 - 21*a**4*b*e**5*x - 16*a**3*b**2*d**
2*e**3 - 56*a**3*b**2*d*e**4*x - 70*a**3*b**2*e**5*x**2 - 96*a**2*b**3*d**
3*e**2 - 336*a**2*b**3*d**2*e**3*x - 420*a**2*b**3*d*e**4*x**2 - 210*a**2*
b**3*e**5*x**3 + 384*a*b**4*d**4*e + 1344*a*b**4*d**3*e**2*x + 1680*a*b**4
*d**2*e**3*x**2 + 840*a*b**4*d*e**4*x**3 + 105*a*b**4*e**5*x**4 - 256*b**5
*d**5 - 896*b**5*d**4*e*x - 1120*b**5*d**3*e**2*x**2 - 560*b**5*d**2*e**3*
x**3 - 70*b**5*d*e**4*x**4 + 7*b**5*e**5*x**5))/(21*sqrt(d + e*x)*e**6*(d
*3 + 3*d**2*e*x + 3*d*e**2*x**2 + e**3*x**3))

```

3.368
$$\int \frac{(a^2+2abx+b^2x^2)^{5/2}}{(d+ex)^{11/2}} dx$$

| | |
|---|------|
| Optimal result | 2715 |
| Mathematica [A] (verified) | 2716 |
| Rubi [A] (verified) | 2716 |
| Maple [A] (verified) | 2718 |
| Fricas [A] (verification not implemented) | 2718 |
| Sympy [F(-1)] | 2719 |
| Maxima [A] (verification not implemented) | 2719 |
| Giac [A] (verification not implemented) | 2720 |
| Mupad [B] (verification not implemented) | 2720 |
| Reduce [B] (verification not implemented) | 2721 |

Optimal result

Integrand size = 30, antiderivative size = 314

$$\int \frac{(a^2 + 2abx + b^2x^2)^{5/2}}{(d + ex)^{11/2}} dx = \frac{2(bd - ae)^5 \sqrt{a^2 + 2abx + b^2x^2}}{9e^6(a + bx)(d + ex)^{9/2}} - \frac{10b(bd - ae)^4 \sqrt{a^2 + 2abx + b^2x^2}}{7e^6(a + bx)(d + ex)^{7/2}} + \frac{4b^2(bd - ae)^3 \sqrt{a^2 + 2abx + b^2x^2}}{e^6(a + bx)(d + ex)^{5/2}} - \frac{20b^3(bd - ae)^2 \sqrt{a^2 + 2abx + b^2x^2}}{3e^6(a + bx)(d + ex)^{3/2}} + \frac{10b^4(bd - ae) \sqrt{a^2 + 2abx + b^2x^2}}{e^6(a + bx) \sqrt{d + ex}} + \frac{2b^5 \sqrt{d + ex} \sqrt{a^2 + 2abx + b^2x^2}}{e^6(a + bx)}$$

output

```
2/9*(-a*e+b*d)^5*((b*x+a)^2)^(1/2)/e^6/(b*x+a)/(e*x+d)^(9/2)-10/7*b*(-a*e+b*d)^4*((b*x+a)^2)^(1/2)/e^6/(b*x+a)/(e*x+d)^(7/2)+4*b^2*(-a*e+b*d)^3*((b*x+a)^2)^(1/2)/e^6/(b*x+a)/(e*x+d)^(5/2)-20/3*b^3*(-a*e+b*d)^2*((b*x+a)^2)^(1/2)/e^6/(b*x+a)/(e*x+d)^(3/2)+10*b^4*(-a*e+b*d)*((b*x+a)^2)^(1/2)/e^6/(b*x+a)/(e*x+d)^(1/2)+2*b^5*(e*x+d)^(1/2)*((b*x+a)^2)^(1/2)/e^6/(b*x+a)
```


Mathematica [A] (verified)

Time = 0.16 (sec) , antiderivative size = 235, normalized size of antiderivative = 0.75

$$\int \frac{(a^2 + 2abx + b^2x^2)^{5/2}}{(d + ex)^{11/2}} dx =$$

$$\frac{2\sqrt{(a + bx)^2(7a^5e^5 + 5a^4be^4(2d + 9ex) + 2a^3b^2e^3(8d^2 + 36dex + 63e^2x^2) + 2a^2b^3e^2(16d^3 + 72d^2ex + 126d^2e^2x^2 + 105e^3x^3) + ab^4e(128d^4 + 576d^3ex + 1008d^2e^2x^2 + 840d^2e^3x^3 + 315e^4x^4) - b^5(256d^5 + 1152d^4ex + 2016d^3e^2x^2 + 1680d^2e^3x^3 + 630de^4x^4 + 63e^5x^5))}{63e^6(a + bx)(d + ex)^{9/2}}$$

input

```
Integrate[(a^2 + 2*a*b*x + b^2*x^2)^(5/2)/(d + e*x)^(11/2), x]
```

output

```
(-2*sqrt[(a + b*x)^2]*(7*a^5*e^5 + 5*a^4*b*e^4*(2*d + 9*e*x) + 2*a^3*b^2*e^3*(8*d^2 + 36*d*e*x + 63*e^2*x^2) + 2*a^2*b^3*e^2*(16*d^3 + 72*d^2*e*x + 126*d*e^2*x^2 + 105*e^3*x^3) + a*b^4*e*(128*d^4 + 576*d^3*e*x + 1008*d^2*e^2*x^2 + 840*d^2*e^3*x^3 + 315*e^4*x^4) - b^5*(256*d^5 + 1152*d^4*e*x + 2016*d^3*e^2*x^2 + 1680*d^2*e^3*x^3 + 630*d*e^4*x^4 + 63*e^5*x^5)))/(63*e^6*(a + b*x)*(d + e*x)^(9/2))
```

Rubi [A] (verified)

Time = 0.55 (sec) , antiderivative size = 180, normalized size of antiderivative = 0.57, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {1102, 27, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a^2 + 2abx + b^2x^2)^{5/2}}{(d + ex)^{11/2}} dx$$

$$\downarrow 1102$$

$$\frac{\sqrt{a^2 + 2abx + b^2x^2} \int \frac{b^5(a+bx)^5}{(d+ex)^{11/2}} dx}{b^5(a + bx)}$$

$$\downarrow 27$$

$$\frac{\sqrt{a^2 + 2abx + b^2x^2} \int \frac{(a+bx)^5}{(d+ex)^{11/2}} dx}{a + bx}$$

53

$$\frac{\sqrt{a^2 + 2abx + b^2x^2} \int \left(\frac{b^5}{e^5\sqrt{d+ex}} - \frac{5(bd-ae)b^4}{e^5(d+ex)^{3/2}} + \frac{10(bd-ae)^2b^3}{e^5(d+ex)^{5/2}} - \frac{10(bd-ae)^3b^2}{e^5(d+ex)^{7/2}} + \frac{5(bd-ae)^4b}{e^5(d+ex)^{9/2}} + \frac{(ae-bd)^5}{e^5(d+ex)^{11/2}} \right) dx}{a + bx}$$

2009

$$\frac{\sqrt{a^2 + 2abx + b^2x^2} \left(\frac{10b^4(bd-ae)}{e^6\sqrt{d+ex}} - \frac{20b^3(bd-ae)^2}{3e^6(d+ex)^{3/2}} + \frac{4b^2(bd-ae)^3}{e^6(d+ex)^{5/2}} - \frac{10b(bd-ae)^4}{7e^6(d+ex)^{7/2}} + \frac{2(bd-ae)^5}{9e^6(d+ex)^{9/2}} + \frac{2b^5\sqrt{d+ex}}{e^6} \right)}{a + bx}$$

input `Int[(a^2 + 2*a*b*x + b^2*x^2)^(5/2)/(d + e*x)^(11/2),x]`

output `(Sqrt[a^2 + 2*a*b*x + b^2*x^2]*((2*(b*d - a*e)^5)/(9*e^6*(d + e*x)^(9/2)) - (10*b*(b*d - a*e)^4)/(7*e^6*(d + e*x)^(7/2)) + (4*b^2*(b*d - a*e)^3)/(e^6*(d + e*x)^(5/2)) - (20*b^3*(b*d - a*e)^2)/(3*e^6*(d + e*x)^(3/2)) + (10*b^4*(b*d - a*e))/(e^6*Sqrt[d + e*x]) + (2*b^5*Sqrt[d + e*x])/e^6))/(a + b*x)`

Defintions of rubi rules used

rule 27 `Int[(a_)*(F_x_), x_Symbol] := Simp[a Int[F_x, x], x] /; FreeQ[a, x] && !MatchQ[F_x, (b_)*(G_x_)] /; FreeQ[b, x]`

rule 53 `Int[((a_) + (b_)*(x_))^(m_)*((c_) + (d_)*(x_))^(n_), x_Symbol] := Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && IGtQ[m, 0] && (!IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])`

rule 1102 `Int[((d_) + (e_)*(x_))^(m_)*((a_) + (b_)*(x_) + (c_)*(x_)^2)^(p_), x_Symbol] := Simp[(a + b*x + c*x^2)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x)^(2*FracPart[p])) Int[(d + e*x)^m*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && EqQ[b^2 - 4*a*c, 0]`

output

```
2/63*(63*b^5*e^5*x^5 + 256*b^5*d^5 - 128*a*b^4*d^4*e - 32*a^2*b^3*d^3*e^2
- 16*a^3*b^2*d^2*e^3 - 10*a^4*b*d*e^4 - 7*a^5*e^5 + 315*(2*b^5*d*e^4 - a*b
^4*e^5)*x^4 + 210*(8*b^5*d^2*e^3 - 4*a*b^4*d*e^4 - a^2*b^3*e^5)*x^3 + 126*
(16*b^5*d^3*e^2 - 8*a*b^4*d^2*e^3 - 2*a^2*b^3*d*e^4 - a^3*b^2*e^5)*x^2 + 9
*(128*b^5*d^4*e - 64*a*b^4*d^3*e^2 - 16*a^2*b^3*d^2*e^3 - 8*a^3*b^2*d*e^4
- 5*a^4*b*e^5)*x)*sqrt(e*x + d)/(e^11*x^5 + 5*d*e^10*x^4 + 10*d^2*e^9*x^3
+ 10*d^3*e^8*x^2 + 5*d^4*e^7*x + d^5*e^6)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(a^2 + 2abx + b^2x^2)^{5/2}}{(d + ex)^{11/2}} dx = \text{Timed out}$$

input

```
integrate((b**2*x**2+2*a*b*x+a**2)**(5/2)/(e*x+d)**(11/2),x)
```

output

Timed out

Maxima [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 305, normalized size of antiderivative = 0.97

$$\int \frac{(a^2 + 2abx + b^2x^2)^{5/2}}{(d + ex)^{11/2}} dx = \frac{2(63b^5e^5x^5 + 256b^5d^5 - 128ab^4d^4e - 32a^2b^3d^3e^2 - 16a^3b^2d^2e^3 - 10a^4bde^4)}{(d + ex)^{11/2}}$$

input

```
integrate((b^2*x^2+2*a*b*x+a^2)^(5/2)/(e*x+d)^(11/2),x, algorithm="maxima"
)
```

output

```
2/63*(63*b^5*e^5*x^5 + 256*b^5*d^5 - 128*a*b^4*d^4*e - 32*a^2*b^3*d^3*e^2
- 16*a^3*b^2*d^2*e^3 - 10*a^4*b*d*e^4 - 7*a^5*e^5 + 315*(2*b^5*d*e^4 - a*b
^4*e^5)*x^4 + 210*(8*b^5*d^2*e^3 - 4*a*b^4*d*e^4 - a^2*b^3*e^5)*x^3 + 126*
(16*b^5*d^3*e^2 - 8*a*b^4*d^2*e^3 - 2*a^2*b^3*d*e^4 - a^3*b^2*e^5)*x^2 + 9
*(128*b^5*d^4*e - 64*a*b^4*d^3*e^2 - 16*a^2*b^3*d^2*e^3 - 8*a^3*b^2*d*e^4
- 5*a^4*b*e^5)*x)/((e^10*x^4 + 4*d*e^9*x^3 + 6*d^2*e^8*x^2 + 4*d^3*e^7*x +
d^4*e^6)*sqrt(e*x + d))
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 441, normalized size of antiderivative = 1.40

$$\int \frac{(a^2 + 2abx + b^2x^2)^{5/2}}{(d + ex)^{11/2}} dx = \frac{2\sqrt{ex + d}b^5\operatorname{sgn}(bx + a)}{e^6} + \frac{2(315(ex + d)^4b^5d\operatorname{sgn}(bx + a) - 210(ex + d)^3b^5d^2\operatorname{sgn}(bx + a) + 126(ex + d)^2b^5d^3\operatorname{sgn}(bx + a) - 45(ex + d)^1b^5d^4\operatorname{sgn}(bx + a) + 7b^5d^5\operatorname{sgn}(bx + a) - 15(ex + d)^4ab^4e\operatorname{sgn}(bx + a) + 420(ex + d)^3a^2b^4de\operatorname{sgn}(bx + a) - 378(ex + d)^2a^3b^4d^2e\operatorname{sgn}(bx + a) + 180(ex + d)a^4b^4d^3e\operatorname{sgn}(bx + a) - 35a^5b^4d^4e\operatorname{sgn}(bx + a) - 210(ex + d)^3a^2b^3e^2\operatorname{sgn}(bx + a) + 378(ex + d)^2a^3b^3de^2\operatorname{sgn}(bx + a) - 270(ex + d)a^4b^3d^2e^2\operatorname{sgn}(bx + a) + 70a^5b^3d^3e^2\operatorname{sgn}(bx + a) - 126(ex + d)^2a^3b^2e^3\operatorname{sgn}(bx + a) + 180(ex + d)a^4b^2de^3\operatorname{sgn}(bx + a) - 70a^5b^2d^2e^3\operatorname{sgn}(bx + a) - 45(ex + d)a^4bde^4\operatorname{sgn}(bx + a) + 35a^5bde^4\operatorname{sgn}(bx + a) - 7a^5e^5\operatorname{sgn}(bx + a))}{((ex + d)^{9/2})e^6}$$

input `integrate((b^2*x^2+2*a*b*x+a^2)^(5/2)/(e*x+d)^(11/2),x, algorithm="giac")`

output `2*sqrt(e*x + d)*b^5*sgn(b*x + a)/e^6 + 2/63*(315*(e*x + d)^4*b^5*d*sgn(b*x + a) - 210*(e*x + d)^3*b^5*d^2*sgn(b*x + a) + 126*(e*x + d)^2*b^5*d^3*sgn(b*x + a) - 45*(e*x + d)*b^5*d^4*sgn(b*x + a) + 7*b^5*d^5*sgn(b*x + a) - 15*(e*x + d)^4*a*b^4*e*sgn(b*x + a) + 420*(e*x + d)^3*a^2*b^4*d*e*sgn(b*x + a) - 378*(e*x + d)^2*a^3*b^4*d^2*e*sgn(b*x + a) + 180*(e*x + d)*a^4*b^4*d^3*e*sgn(b*x + a) - 35*a^5*b^4*d^4*e*sgn(b*x + a) - 210*(e*x + d)^3*a^2*b^3*e^2*sgn(b*x + a) + 378*(e*x + d)^2*a^3*b^3*d*e^2*sgn(b*x + a) - 270*(e*x + d)*a^4*b^3*d^2*e^2*sgn(b*x + a) + 70*a^5*b^3*d^3*e^2*sgn(b*x + a) - 126*(e*x + d)^2*a^3*b^2*e^3*sgn(b*x + a) + 180*(e*x + d)*a^4*b^2*d*e^3*sgn(b*x + a) - 70*a^5*b^2*d^2*e^3*sgn(b*x + a) - 45*(e*x + d)*a^4*b*d*e^4*sgn(b*x + a) + 35*a^5*b*d*e^4*sgn(b*x + a) - 7*a^5*e^5*sgn(b*x + a))/((e*x + d)^(9/2)*e^6)`

Mupad [B] (verification not implemented)

Time = 6.20 (sec) , antiderivative size = 416, normalized size of antiderivative = 1.32

$$\int \frac{(a^2 + 2abx + b^2x^2)^{5/2}}{(d + ex)^{11/2}} dx = \frac{\sqrt{a^2 + 2abx + b^2x^2} \left(\frac{14a^5e^5 + 20a^4bde^4 + 32a^3b^2d^2e^3 + 64a^2b^3d^3e^2 + 256ab^4d^4e - 512b^5d^5}{63be^{10}} - \frac{2b^4x^5}{e^5} + \frac{10b^3x^4(ae - 2bd)}{e^6} \right)}{x^5\sqrt{d + ex} + \frac{ad^4\sqrt{d + ex}}{be^4} + \frac{x^4(63ae^{10} + 252bde^9)\sqrt{d + ex}}{63be^{10}}}$$

input `int((a^2 + b^2*x^2 + 2*a*b*x)^(5/2)/(d + e*x)^(11/2),x)`

output

```

-((a^2 + b^2*x^2 + 2*a*b*x)^(1/2))*((14*a^5*e^5 - 512*b^5*d^5 + 64*a^2*b^3*
d^3*e^2 + 32*a^3*b^2*d^2*e^3 + 256*a*b^4*d^4*e + 20*a^4*b*d*e^4)/(63*b*e^1
0) - (2*b^4*x^5)/e^5 + (10*b^3*x^4*(a*e - 2*b*d))/e^6 + (x*(90*a^4*b*e^5 -
2304*b^5*d^4*e + 1152*a*b^4*d^3*e^2 + 144*a^3*b^2*d*e^4 + 288*a^2*b^3*d^2
*e^3))/(63*b*e^10) + (20*b^2*x^3*(a^2*e^2 - 8*b^2*d^2 + 4*a*b*d*e))/(3*e^7
) + (4*b*x^2*(a^3*e^3 - 16*b^3*d^3 + 8*a*b^2*d^2*e + 2*a^2*b*d*e^2))/e^8))
/(x^5*(d + e*x)^(1/2) + (a*d^4*(d + e*x)^(1/2))/(b*e^4) + (x^4*(63*a*e^10
+ 252*b*d*e^9)*(d + e*x)^(1/2))/(63*b*e^10) + (2*d*x^3*(2*a*e + 3*b*d)*(d
+ e*x)^(1/2))/(b*e^2) + (d^3*x*(4*a*e + b*d)*(d + e*x)^(1/2))/(b*e^4) + (2
*d^2*x^2*(3*a*e + 2*b*d)*(d + e*x)^(1/2))/(b*e^3))

```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 313, normalized size of antiderivative = 1.00

$$\int \frac{(a^2 + 2abx + b^2x^2)^{5/2}}{(d + ex)^{11/2}} dx = \frac{2b^5e^5x^5 - 10ab^4e^5x^4 + 20b^5de^4x^4 - \frac{20}{3}a^2b^3e^5x^3 - \frac{80}{3}ab^4de^4x^3 + \frac{160}{3}b^5d^2e^3x^2}{(d + ex)^{11/2}}$$

input

```
int((b^2*x^2+2*a*b*x+a^2)^(5/2)/(e*x+d)^(11/2),x)
```

output

```

(2*(- 7*a**5*e**5 - 10*a**4*b*d*e**4 - 45*a**4*b*e**5*x - 16*a**3*b**2*d*
*2*e**3 - 72*a**3*b**2*d*e**4*x - 126*a**3*b**2*e**5*x**2 - 32*a**2*b**3*d
**3*e**2 - 144*a**2*b**3*d**2*e**3*x - 252*a**2*b**3*d*e**4*x**2 - 210*a**
2*b**3*e**5*x**3 - 128*a*b**4*d**4*e - 576*a*b**4*d**3*e**2*x - 1008*a*b**
4*d**2*e**3*x**2 - 840*a*b**4*d*e**4*x**3 - 315*a*b**4*e**5*x**4 + 256*b**
5*d**5 + 1152*b**5*d**4*e*x + 2016*b**5*d**3*e**2*x**2 + 1680*b**5*d**2*e
**3*x**3 + 630*b**5*d*e**4*x**4 + 63*b**5*e**5*x**5))/(63*sqrt(d + e*x)*e**
6*(d**4 + 4*d**3*e*x + 6*d**2*e**2*x**2 + 4*d*e**3*x**3 + e**4*x**4))

```

3.369
$$\int \frac{(a^2+2abx+b^2x^2)^{5/2}}{(d+ex)^{13/2}} dx$$

| | |
|---|------|
| Optimal result | 2722 |
| Mathematica [A] (verified) | 2723 |
| Rubi [A] (verified) | 2723 |
| Maple [A] (verified) | 2725 |
| Fricas [A] (verification not implemented) | 2725 |
| Sympy [F(-1)] | 2726 |
| Maxima [A] (verification not implemented) | 2726 |
| Giac [A] (verification not implemented) | 2727 |
| Mupad [B] (verification not implemented) | 2728 |
| Reduce [B] (verification not implemented) | 2728 |

Optimal result

Integrand size = 30, antiderivative size = 316

$$\int \frac{(a^2 + 2abx + b^2x^2)^{5/2}}{(d + ex)^{13/2}} dx = \frac{2(bd - ae)^5 \sqrt{a^2 + 2abx + b^2x^2}}{11e^6(a + bx)(d + ex)^{11/2}} - \frac{10b(bd - ae)^4 \sqrt{a^2 + 2abx + b^2x^2}}{9e^6(a + bx)(d + ex)^{9/2}} + \frac{20b^2(bd - ae)^3 \sqrt{a^2 + 2abx + b^2x^2}}{7e^6(a + bx)(d + ex)^{7/2}} - \frac{4b^3(bd - ae)^2 \sqrt{a^2 + 2abx + b^2x^2}}{e^6(a + bx)(d + ex)^{5/2}} + \frac{10b^4(bd - ae) \sqrt{a^2 + 2abx + b^2x^2}}{3e^6(a + bx)(d + ex)^{3/2}} - \frac{2b^5 \sqrt{a^2 + 2abx + b^2x^2}}{e^6(a + bx) \sqrt{d + ex}}$$

output

```
2/11*(-a*e+b*d)^5*((b*x+a)^2)^(1/2)/e^6/(b*x+a)/(e*x+d)^(11/2)-10/9*b*(-a*
e+b*d)^4*((b*x+a)^2)^(1/2)/e^6/(b*x+a)/(e*x+d)^(9/2)+20/7*b^2*(-a*e+b*d)^3
*((b*x+a)^2)^(1/2)/e^6/(b*x+a)/(e*x+d)^(7/2)-4*b^3*(-a*e+b*d)^2*((b*x+a)^2
)^(1/2)/e^6/(b*x+a)/(e*x+d)^(5/2)+10/3*b^4*(-a*e+b*d)*((b*x+a)^2)^(1/2)/e^
6/(b*x+a)/(e*x+d)^(3/2)-2*b^5*((b*x+a)^2)^(1/2)/e^6/(b*x+a)/(e*x+d)^(1/2)
```

Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 234, normalized size of antiderivative = 0.74

$$\int \frac{(a^2 + 2abx + b^2x^2)^{5/2}}{(d + ex)^{13/2}} dx = \frac{2\sqrt{(a + bx)^2(63a^5e^5 + 35a^4be^4(2d + 11ex) + 10a^3b^2e^3(8d^2 + 44dex + 99e^2x^2) + 6a^2b^3e^2(16d^3 + 88d^2ex$$

input

```
Integrate[(a^2 + 2*a*b*x + b^2*x^2)^(5/2)/(d + e*x)^(13/2), x]
```

output

```
(-2*sqrt[(a + b*x)^2]*(63*a^5*e^5 + 35*a^4*b*e^4*(2*d + 11*e*x) + 10*a^3*b^2*e^3*(8*d^2 + 44*d*e*x + 99*e^2*x^2) + 6*a^2*b^3*e^2*(16*d^3 + 88*d^2*e*x + 198*d*e^2*x^2 + 231*e^3*x^3) + a*b^4*e*(128*d^4 + 704*d^3*e*x + 1584*d^2*e^2*x^2 + 1848*d*e^3*x^3 + 1155*e^4*x^4) + b^5*(256*d^5 + 1408*d^4*e*x + 3168*d^3*e^2*x^2 + 3696*d^2*e^3*x^3 + 2310*d*e^4*x^4 + 693*e^5*x^5)))/(693*e^6*(a + b*x)*(d + e*x)^(11/2))
```

Rubi [A] (verified)

Time = 0.53 (sec) , antiderivative size = 182, normalized size of antiderivative = 0.58, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {1102, 27, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a^2 + 2abx + b^2x^2)^{5/2}}{(d + ex)^{13/2}} dx$$

$$\downarrow 1102$$

$$\frac{\sqrt{a^2 + 2abx + b^2x^2} \int \frac{b^5(a+bx)^5}{(d+ex)^{13/2}} dx}{b^5(a + bx)}$$

$$\downarrow 27$$

$$\frac{\sqrt{a^2 + 2abx + b^2x^2} \int \frac{(a+bx)^5}{(d+ex)^{13/2}} dx}{a + bx}$$

53

$$\frac{\sqrt{a^2 + 2abx + b^2x^2} \int \left(\frac{b^5}{e^5(d+ex)^{3/2}} - \frac{5(bd-ae)b^4}{e^5(d+ex)^{5/2}} + \frac{10(bd-ae)^2b^3}{e^5(d+ex)^{7/2}} - \frac{10(bd-ae)^3b^2}{e^5(d+ex)^{9/2}} + \frac{5(bd-ae)^4b}{e^5(d+ex)^{11/2}} + \frac{(ae-bd)^5}{e^5(d+ex)^{13/2}} \right) dx}{a + bx}$$

2009

$$\frac{\sqrt{a^2 + 2abx + b^2x^2} \left(\frac{10b^4(bd-ae)}{3e^6(d+ex)^{3/2}} - \frac{4b^3(bd-ae)^2}{e^6(d+ex)^{5/2}} + \frac{20b^2(bd-ae)^3}{7e^6(d+ex)^{7/2}} - \frac{10b(bd-ae)^4}{9e^6(d+ex)^{9/2}} + \frac{2(bd-ae)^5}{11e^6(d+ex)^{11/2}} - \frac{2b^5}{e^6\sqrt{d+ex}} \right)}{a + bx}$$

input `Int[(a^2 + 2*a*b*x + b^2*x^2)^(5/2)/(d + e*x)^(13/2),x]`

output `(Sqrt[a^2 + 2*a*b*x + b^2*x^2]*((2*(b*d - a*e)^5)/(11*e^6*(d + e*x)^(11/2)) - (10*b*(b*d - a*e)^4)/(9*e^6*(d + e*x)^(9/2)) + (20*b^2*(b*d - a*e)^3)/(7*e^6*(d + e*x)^(7/2)) - (4*b^3*(b*d - a*e)^2)/(e^6*(d + e*x)^(5/2)) + (10*b^4*(b*d - a*e))/(3*e^6*(d + e*x)^(3/2)) - (2*b^5)/(e^6*Sqrt[d + e*x])))/(a + b*x)`

Defintions of rubi rules used

rule 27 `Int[(a_)*(F_x_), x_Symbol] := Simp[a Int[F_x, x], x] /; FreeQ[a, x] && !MatchQ[F_x, (b_)*(G_x_)] /; FreeQ[b, x]`

rule 53 `Int[((a_) + (b_)*(x_))^(m_)*((c_) + (d_)*(x_))^(n_), x_Symbol] := Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && IGtQ[m, 0] && (!IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])`

rule 1102 `Int[((d_) + (e_)*(x_))^(m_)*((a_) + (b_)*(x_) + (c_)*(x_)^2)^(p_), x_Symbol] := Simp[(a + b*x + c*x^2)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x)^(2*FracPart[p])) Int[(d + e*x)^m*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && EqQ[b^2 - 4*a*c, 0]`

output

```
-2/693*(693*b^5*e^5*x^5 + 256*b^5*d^5 + 128*a*b^4*d^4*e + 96*a^2*b^3*d^3*e^2 + 80*a^3*b^2*d^2*e^3 + 70*a^4*b*d*e^4 + 63*a^5*e^5 + 1155*(2*b^5*d*e^4 + a*b^4*e^5)*x^4 + 462*(8*b^5*d^2*e^3 + 4*a*b^4*d*e^4 + 3*a^2*b^3*e^5)*x^3 + 198*(16*b^5*d^3*e^2 + 8*a*b^4*d^2*e^3 + 6*a^2*b^3*d*e^4 + 5*a^3*b^2*e^5)*x^2 + 11*(128*b^5*d^4*e + 64*a*b^4*d^3*e^2 + 48*a^2*b^3*d^2*e^3 + 40*a^3*b^2*d*e^4 + 35*a^4*b*e^5)*x)*sqrt(e*x + d)/(e^12*x^6 + 6*d*e^11*x^5 + 15*d^2*e^10*x^4 + 20*d^3*e^9*x^3 + 15*d^4*e^8*x^2 + 6*d^5*e^7*x + d^6*e^6)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(a^2 + 2abx + b^2x^2)^{5/2}}{(d + ex)^{13/2}} dx = \text{Timed out}$$

input

```
integrate((b**2*x**2+2*a*b*x+a**2)**(5/2)/(e*x+d)**(13/2),x)
```

output

Timed out

Maxima [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 315, normalized size of antiderivative = 1.00

$$\int \frac{(a^2 + 2abx + b^2x^2)^{5/2}}{(d + ex)^{13/2}} dx = \frac{2(693b^5e^5x^5 + 256b^5d^5 + 128ab^4d^4e + 96a^2b^3d^3e^2 + 80a^3b^2d^2e^3 + 70a^4bde^4 + 63a^5e^5 + 1155(2b^5de^4 + ab^4e^5)x^4 + 462(8b^5d^2e^3 + 4ab^4de^4 + 3a^2b^3e^5)x^3 + 198(16b^5d^3e^2 + 8ab^4d^2e^3 + 6a^2b^3de^4 + 5a^3b^2e^5)x^2 + 11(128b^5d^4e + 64ab^4d^3e^2 + 48a^2b^3d^2e^3 + 40a^3b^2de^4 + 35a^4be^5)x)}{(e^12x^6 + 6de^11x^5 + 15d^2e^10x^4 + 20d^3e^9x^3 + 15d^4e^8x^2 + 6d^5e^7x + d^6e^6)}$$

input

```
integrate((b^2*x^2+2*a*b*x+a^2)^(5/2)/(e*x+d)^(13/2),x, algorithm="maxima")
```

output

```
-2/693*(693*b^5*e^5*x^5 + 256*b^5*d^5 + 128*a*b^4*d^4*e + 96*a^2*b^3*d^3*e^2 + 80*a^3*b^2*d^2*e^3 + 70*a^4*b*d*e^4 + 63*a^5*e^5 + 1155*(2*b^5*d*e^4 + a*b^4*e^5)*x^4 + 462*(8*b^5*d^2*e^3 + 4*a*b^4*d*e^4 + 3*a^2*b^3*e^5)*x^3 + 198*(16*b^5*d^3*e^2 + 8*a*b^4*d^2*e^3 + 6*a^2*b^3*d*e^4 + 5*a^3*b^2*e^5)*x^2 + 11*(128*b^5*d^4*e + 64*a*b^4*d^3*e^2 + 48*a^2*b^3*d^2*e^3 + 40*a^3*b^2*d*e^4 + 35*a^4*b*e^5)*x)/((e^11*x^5 + 5*d*e^10*x^4 + 10*d^2*e^9*x^3 + 10*d^3*e^8*x^2 + 5*d^4*e^7*x + d^5*e^6)*sqrt(e*x + d))
```

Giac [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 437, normalized size of antiderivative = 1.38

$$\int \frac{(a^2 + 2abx + b^2x^2)^{5/2}}{(d + ex)^{13/2}} dx =$$

$$\frac{2(693(ex + d)^5 b^5 \operatorname{sgn}(bx + a) - 1155(ex + d)^4 b^5 d \operatorname{sgn}(bx + a) + 1386(ex + d)^3 b^5 d^2 \operatorname{sgn}(bx + a) - 990(ex + d)^2 b^5 d^3 \operatorname{sgn}(bx + a) + 385(ex + d) b^5 d^4 \operatorname{sgn}(bx + a) - 63 b^5 d^5 \operatorname{sgn}(bx + a) + 1155(ex + d)^4 a b^4 e \operatorname{sgn}(bx + a) - 2772(ex + d)^3 a b^4 d e \operatorname{sgn}(bx + a) + 2970(ex + d)^2 a b^4 d^2 e \operatorname{sgn}(bx + a) - 1540(ex + d) a b^4 d^3 e \operatorname{sgn}(bx + a) + 315 a b^4 d^4 e \operatorname{sgn}(bx + a) + 1386(ex + d)^3 a^2 b^3 e^2 \operatorname{sgn}(bx + a) - 2970(ex + d)^2 a^2 b^3 d e^2 \operatorname{sgn}(bx + a) + 2310(ex + d) a^2 b^3 d^2 e^2 \operatorname{sgn}(bx + a) - 630 a^2 b^3 d^3 e^2 \operatorname{sgn}(bx + a) + 990(ex + d)^2 a^3 b^2 e^3 \operatorname{sgn}(bx + a) - 1540(ex + d) a^3 b^2 d e^3 \operatorname{sgn}(bx + a) + 630 a^3 b^2 d^2 e^3 \operatorname{sgn}(bx + a) + 385(ex + d) a^4 b e^4 \operatorname{sgn}(bx + a) - 315 a^4 b d e^4 \operatorname{sgn}(bx + a) + 63 a^5 e^5 \operatorname{sgn}(bx + a))}{(ex + d)^{11/2} e^6}$$

input

```
integrate((b^2*x^2+2*a*b*x+a^2)^(5/2)/(e*x+d)^(13/2),x, algorithm="giac")
```

output

```
-2/693*(693*(e*x + d)^5*b^5*sgn(b*x + a) - 1155*(e*x + d)^4*b^5*d*sgn(b*x + a) + 1386*(e*x + d)^3*b^5*d^2*sgn(b*x + a) - 990*(e*x + d)^2*b^5*d^3*sgn(b*x + a) + 385*(e*x + d)*b^5*d^4*sgn(b*x + a) - 63*b^5*d^5*sgn(b*x + a) + 1155*(e*x + d)^4*a*b^4*e*sgn(b*x + a) - 2772*(e*x + d)^3*a*b^4*d*e*sgn(b*x + a) + 2970*(e*x + d)^2*a*b^4*d^2*e*sgn(b*x + a) - 1540*(e*x + d)*a*b^4*d^3*e*sgn(b*x + a) + 315*a*b^4*d^4*e*sgn(b*x + a) + 1386*(e*x + d)^3*a^2*b^3*e^2*sgn(b*x + a) - 2970*(e*x + d)^2*a^2*b^3*d*e^2*sgn(b*x + a) + 2310*(e*x + d)*a^2*b^3*d^2*e^2*sgn(b*x + a) - 630*a^2*b^3*d^3*e^2*sgn(b*x + a) + 990*(e*x + d)^2*a^3*b^2*e^3*sgn(b*x + a) - 1540*(e*x + d)*a^3*b^2*d*e^3*sgn(b*x + a) + 630*a^3*b^2*d^2*e^3*sgn(b*x + a) + 385*(e*x + d)*a^4*b*e^4*sgn(b*x + a) - 315*a^4*b*d*e^4*sgn(b*x + a) + 63*a^5*e^5*sgn(b*x + a))/((e*x + d)^(11/2)*e^6)
```

Mupad [B] (verification not implemented)

Time = 6.14 (sec) , antiderivative size = 437, normalized size of antiderivative = 1.38

$$\int \frac{(a^2 + 2abx + b^2x^2)^{5/2}}{(d + ex)^{13/2}} dx = \frac{\sqrt{a^2 + 2abx + b^2x^2} \left(\frac{126a^5e^5 + 140a^4bde^4 + 160a^3b^2d^2e^3 + 192a^2b^3d^3e^2 + 256ab^4d^4e + 512b^5d^5}{693be^{11}} + \frac{2b^4x^5}{e^6} + \frac{10b^3x^4(ae + 2bd)}{3e^7} \right)}{x^6 \sqrt{d + ex} + \frac{ad^5 \sqrt{d + ex}}{be^5} + \frac{x^5(ae + 5bd) \sqrt{d + ex}}{be} + \frac{5dx^4(ae + 2bd)}{e^6}}$$

input

```
int((a^2 + b^2*x^2 + 2*a*b*x)^(5/2)/(d + e*x)^(13/2),x)
```

output

```
-((a^2 + b^2*x^2 + 2*a*b*x)^(1/2)*((126*a^5*e^5 + 512*b^5*d^5 + 192*a^2*b^3*d^3*e^2 + 160*a^3*b^2*d^2*e^3 + 256*a*b^4*d^4*e + 140*a^4*b*d*e^4)/(693*b*e^11) + (2*b^4*x^5)/e^6 + (10*b^3*x^4*(a*e + 2*b*d))/(3*e^7) + (x*(770*a^4*b*e^5 + 2816*b^5*d^4*e + 1408*a*b^4*d^3*e^2 + 880*a^3*b^2*d*e^4 + 1056*a^2*b^3*d^2*e^3))/(693*b*e^11) + (4*b^2*x^3*(3*a^2*e^2 + 8*b^2*d^2 + 4*a*b*d*e))/(3*e^8) + (4*b*x^2*(5*a^3*e^3 + 16*b^3*d^3 + 8*a*b^2*d^2*e + 6*a^2*b*d*e^2))/(7*e^9)))/(x^6*(d + e*x)^(1/2) + (a*d^5*(d + e*x)^(1/2))/(b*e^5) + (x^5*(a*e + 5*b*d)*(d + e*x)^(1/2))/(b*e) + (5*d*x^4*(a*e + 2*b*d)*(d + e*x)^(1/2))/(b*e^2) + (d^4*x*(5*a*e + b*d)*(d + e*x)^(1/2))/(b*e^5) + (10*d^2*x^3*(a*e + b*d)*(d + e*x)^(1/2))/(b*e^3) + (5*d^3*x^2*(2*a*e + b*d)*(d + e*x)^(1/2))/(b*e^4))
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 324, normalized size of antiderivative = 1.03

$$\int \frac{(a^2 + 2abx + b^2x^2)^{5/2}}{(d + ex)^{13/2}} dx = \frac{-2b^5e^5x^5 - \frac{10}{3}ab^4e^5x^4 - \frac{20}{3}b^5de^4x^4 - 4a^2b^3e^5x^3 - \frac{16}{3}ab^4de^4x^3 - \frac{32}{3}b^5d^2e^3x^2}{(d + ex)^{13/2}}$$

input

```
int((b^2*x^2+2*a*b*x+a^2)^(5/2)/(e*x+d)^(13/2),x)
```

output

```
(2*( - 63*a**5*e**5 - 70*a**4*b*d*e**4 - 385*a**4*b*e**5*x - 80*a**3*b**2*
d**2*e**3 - 440*a**3*b**2*d*e**4*x - 990*a**3*b**2*e**5*x**2 - 96*a**2*b**
3*d**3*e**2 - 528*a**2*b**3*d**2*e**3*x - 1188*a**2*b**3*d*e**4*x**2 - 138
6*a**2*b**3*e**5*x**3 - 128*a*b**4*d**4*e - 704*a*b**4*d**3*e**2*x - 1584*
a*b**4*d**2*e**3*x**2 - 1848*a*b**4*d*e**4*x**3 - 1155*a*b**4*e**5*x**4 -
256*b**5*d**5 - 1408*b**5*d**4*e*x - 3168*b**5*d**3*e**2*x**2 - 3696*b**5*
d**2*e**3*x**3 - 2310*b**5*d*e**4*x**4 - 693*b**5*e**5*x**5))/(693*sqrt(d
+ e*x)*e**6*(d**5 + 5*d**4*e*x + 10*d**3*e**2*x**2 + 10*d**2*e**3*x**3 + 5
*d*e**4*x**4 + e**5*x**5))
```

3.370 $\int \frac{(a^2+2abx+b^2x^2)^{5/2}}{(d+ex)^{15/2}} dx$

| | |
|---|------|
| Optimal result | 2730 |
| Mathematica [A] (verified) | 2731 |
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| Reduce [B] (verification not implemented) | 2736 |

Optimal result

Integrand size = 30, antiderivative size = 318

$$\int \frac{(a^2 + 2abx + b^2x^2)^{5/2}}{(d + ex)^{15/2}} dx = \frac{2(bd - ae)^5 \sqrt{a^2 + 2abx + b^2x^2}}{13e^6(a + bx)(d + ex)^{13/2}} - \frac{10b(bd - ae)^4 \sqrt{a^2 + 2abx + b^2x^2}}{11e^6(a + bx)(d + ex)^{11/2}} + \frac{20b^2(bd - ae)^3 \sqrt{a^2 + 2abx + b^2x^2}}{9e^6(a + bx)(d + ex)^{9/2}} - \frac{20b^3(bd - ae)^2 \sqrt{a^2 + 2abx + b^2x^2}}{7e^6(a + bx)(d + ex)^{7/2}} + \frac{2b^4(bd - ae) \sqrt{a^2 + 2abx + b^2x^2}}{e^6(a + bx)(d + ex)^{5/2}} - \frac{2b^5 \sqrt{a^2 + 2abx + b^2x^2}}{3e^6(a + bx)(d + ex)^{3/2}}$$

output

```
2/13*(-a*e+b*d)^5*((b*x+a)^2)^(1/2)/e^6/(b*x+a)/(e*x+d)^(13/2)-10/11*b*(-a
*e+b*d)^4*((b*x+a)^2)^(1/2)/e^6/(b*x+a)/(e*x+d)^(11/2)+20/9*b^2*(-a*e+b*d)
^3*((b*x+a)^2)^(1/2)/e^6/(b*x+a)/(e*x+d)^(9/2)-20/7*b^3*(-a*e+b*d)^2*((b*x
+a)^2)^(1/2)/e^6/(b*x+a)/(e*x+d)^(7/2)+2*b^4*(-a*e+b*d)*((b*x+a)^2)^(1/2)/
e^6/(b*x+a)/(e*x+d)^(5/2)-2/3*b^5*((b*x+a)^2)^(1/2)/e^6/(b*x+a)/(e*x+d)^(3
/2)
```

Mathematica [A] (verified)

Time = 0.16 (sec) , antiderivative size = 235, normalized size of antiderivative = 0.74

$$\int \frac{(a^2 + 2abx + b^2x^2)^{5/2}}{(d + ex)^{15/2}} dx =$$

$$\frac{2\sqrt{(a + bx)^2(693a^5e^5 + 315a^4be^4(2d + 13ex) + 70a^3b^2e^3(8d^2 + 52dex + 143e^2x^2) + 30a^2b^3e^2(16d^3 + 104d^2ex + 286d^2e^2x^2 + 429e^3x^3) + 3ab^4e(128d^4 + 832d^3ex + 2288d^2e^2x^2 + 3432de^3x^3 + 3003e^4x^4) + b^5(256d^5 + 1664d^4ex + 4576d^3e^2x^2 + 6864d^2e^3x^3 + 6006de^4x^4 + 3003e^5x^5))}{9009e^6(a + bx)(d + ex)^{13/2}}$$

input

```
Integrate[(a^2 + 2*a*b*x + b^2*x^2)^(5/2)/(d + e*x)^(15/2), x]
```

output

```
(-2*sqrt[(a + b*x)^2]*(693*a^5*e^5 + 315*a^4*b*e^4*(2*d + 13*e*x) + 70*a^3*b^2*e^3*(8*d^2 + 52*d*e*x + 143*e^2*x^2) + 30*a^2*b^3*e^2*(16*d^3 + 104*d^2*e*x + 286*d*e^2*x^2 + 429*e^3*x^3) + 3*a*b^4*e*(128*d^4 + 832*d^3*e*x + 2288*d^2*e^2*x^2 + 3432*d*e^3*x^3 + 3003*e^4*x^4) + b^5*(256*d^5 + 1664*d^4*e*x + 4576*d^3*e^2*x^2 + 6864*d^2*e^3*x^3 + 6006*d*e^4*x^4 + 3003*e^5*x^5)))/(9009*e^6*(a + b*x)*(d + e*x)^(13/2))
```

Rubi [A] (verified)

Time = 0.54 (sec) , antiderivative size = 184, normalized size of antiderivative = 0.58, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {1102, 27, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a^2 + 2abx + b^2x^2)^{5/2}}{(d + ex)^{15/2}} dx$$

$$\downarrow \text{1102}$$

$$\frac{\sqrt{a^2 + 2abx + b^2x^2} \int \frac{b^5(a+bx)^5}{(d+ex)^{15/2}} dx}{b^5(a + bx)}$$

$$\downarrow \text{27}$$

$$\frac{\sqrt{a^2 + 2abx + b^2x^2} \int \frac{(a+bx)^5}{(d+ex)^{15/2}} dx}{a + bx}$$

53

$$\frac{\sqrt{a^2 + 2abx + b^2x^2} \int \left(\frac{b^5}{e^5(d+ex)^{5/2}} - \frac{5(bd-ae)b^4}{e^5(d+ex)^{7/2}} + \frac{10(bd-ae)^2b^3}{e^5(d+ex)^{9/2}} - \frac{10(bd-ae)^3b^2}{e^5(d+ex)^{11/2}} + \frac{5(bd-ae)^4b}{e^5(d+ex)^{13/2}} + \frac{(ae-bd)^5}{e^5(d+ex)^{15/2}} \right) dx}{a + bx}$$

2009

$$\frac{\sqrt{a^2 + 2abx + b^2x^2} \left(\frac{2b^4(bd-ae)}{e^6(d+ex)^{5/2}} - \frac{20b^3(bd-ae)^2}{7e^6(d+ex)^{7/2}} + \frac{20b^2(bd-ae)^3}{9e^6(d+ex)^{9/2}} - \frac{10b(bd-ae)^4}{11e^6(d+ex)^{11/2}} + \frac{2(bd-ae)^5}{13e^6(d+ex)^{13/2}} - \frac{2b^5}{3e^6(d+ex)^{3/2}} \right)}{a + bx}$$

input `Int[(a^2 + 2*a*b*x + b^2*x^2)^(5/2)/(d + e*x)^(15/2),x]`

output `(Sqrt[a^2 + 2*a*b*x + b^2*x^2]*((2*(b*d - a*e)^5)/(13*e^6*(d + e*x)^(13/2)) - (10*b*(b*d - a*e)^4)/(11*e^6*(d + e*x)^(11/2)) + (20*b^2*(b*d - a*e)^3)/(9*e^6*(d + e*x)^(9/2)) - (20*b^3*(b*d - a*e)^2)/(7*e^6*(d + e*x)^(7/2)) + (2*b^4*(b*d - a*e))/(e^6*(d + e*x)^(5/2)) - (2*b^5)/(3*e^6*(d + e*x)^(3/2))))/(a + b*x)`

Defintions of rubi rules used

rule 27 `Int[(a_)*(Fx_), x_Symbol] := Simp[a Int[Fx, x], x] /; FreeQ[a, x] && !MatchQ[Fx, (b_)*(Gx_)] /; FreeQ[b, x]`

rule 53 `Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] := Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && IGtQ[m, 0] && (!IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])`

rule 1102 `Int[((d_.) + (e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] := Simp[(a + b*x + c*x^2)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x)^(2*FracPart[p])) Int[(d + e*x)^m*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && EqQ[b^2 - 4*a*c, 0]`

output

```
-2/9009*(3003*b^5*e^5*x^5 + 256*b^5*d^5 + 384*a*b^4*d^4*e + 480*a^2*b^3*d^3*e^2 + 560*a^3*b^2*d^2*e^3 + 630*a^4*b*d*e^4 + 693*a^5*e^5 + 3003*(2*b^5*d*e^4 + 3*a*b^4*e^5)*x^4 + 858*(8*b^5*d^2*e^3 + 12*a*b^4*d*e^4 + 15*a^2*b^3*e^5)*x^3 + 286*(16*b^5*d^3*e^2 + 24*a*b^4*d^2*e^3 + 30*a^2*b^3*d*e^4 + 35*a^3*b^2*e^5)*x^2 + 13*(128*b^5*d^4*e + 192*a*b^4*d^3*e^2 + 240*a^2*b^3*d^2*e^3 + 280*a^3*b^2*d*e^4 + 315*a^4*b*e^5)*x)*sqrt(e*x + d)/(e^13*x^7 + 7*d*e^12*x^6 + 21*d^2*e^11*x^5 + 35*d^3*e^10*x^4 + 35*d^4*e^9*x^3 + 21*d^5*e^8*x^2 + 7*d^6*e^7*x + d^7*e^6)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(a^2 + 2abx + b^2x^2)^{5/2}}{(d + ex)^{15/2}} dx = \text{Timed out}$$

input

```
integrate((b**2*x**2+2*a*b*x+a**2)**(5/2)/(e*x+d)**(15/2),x)
```

output

Timed out

Maxima [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 327, normalized size of antiderivative = 1.03

$$\int \frac{(a^2 + 2abx + b^2x^2)^{5/2}}{(d + ex)^{15/2}} dx = \frac{2(3003 b^5 e^5 x^5 + 256 b^5 d^5 + 384 a b^4 d^4 e + 480 a^2 b^3 d^3 e^2 + 560 a^3 b^2 d^2 e^3 + 630 a^4 b d e^4 + 693 a^5 e^5 + 3003 (2 b^5 d e^4 + 3 a b^4 e^5) x^4 + 858 (8 b^5 d^2 e^3 + 12 a b^4 d e^4 + 15 a^2 b^3 e^5) x^3 + 286 (16 b^5 d^3 e^2 + 24 a b^4 d^2 e^3 + 30 a^2 b^3 d e^4 + 35 a^3 b^2 e^5) x^2 + 13 (128 b^5 d^4 e + 192 a b^4 d^3 e^2 + 240 a^2 b^3 d^2 e^3 + 280 a^3 b^2 d e^4 + 315 a^4 b e^5) x) \sqrt{e x + d}}{e^{13} x^7 + 7 d e^{12} x^6 + 21 d^2 e^{11} x^5 + 35 d^3 e^{10} x^4 + 35 d^4 e^9 x^3 + 21 d^5 e^8 x^2 + 7 d^6 e^7 x + d^7 e^6}$$

input

```
integrate((b^2*x^2+2*a*b*x+a^2)^(5/2)/(e*x+d)^(15/2),x, algorithm="maxima")
```

output

```
-2/9009*(3003*b^5*e^5*x^5 + 256*b^5*d^5 + 384*a*b^4*d^4*e + 480*a^2*b^3*d^3*e^2 + 560*a^3*b^2*d^2*e^3 + 630*a^4*b*d*e^4 + 693*a^5*e^5 + 3003*(2*b^5*d*e^4 + 3*a*b^4*e^5)*x^4 + 858*(8*b^5*d^2*e^3 + 12*a*b^4*d*e^4 + 15*a^2*b^3*e^5)*x^3 + 286*(16*b^5*d^3*e^2 + 24*a*b^4*d^2*e^3 + 30*a^2*b^3*d*e^4 + 35*a^3*b^2*e^5)*x^2 + 13*(128*b^5*d^4*e + 192*a*b^4*d^3*e^2 + 240*a^2*b^3*d^2*e^3 + 280*a^3*b^2*d*e^4 + 315*a^4*b*e^5)*x)/((e^12*x^6 + 6*d*e^11*x^5 + 15*d^2*e^10*x^4 + 20*d^3*e^9*x^3 + 15*d^4*e^8*x^2 + 6*d^5*e^7*x + d^6*e^6)*sqrt(e*x + d))
```

Giac [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 437, normalized size of antiderivative = 1.37

$$\int \frac{(a^2 + 2abx + b^2x^2)^{5/2}}{(d + ex)^{15/2}} dx =$$

$$\frac{2(3003(ex + d)^5 b^5 \operatorname{sgn}(bx + a) - 9009(ex + d)^4 b^5 d \operatorname{sgn}(bx + a) + 12870(ex + d)^3 b^5 d^2 \operatorname{sgn}(bx + a) - 10010(ex + d)^2 b^5 d^3 \operatorname{sgn}(bx + a) + 4095(ex + d) b^5 d^4 \operatorname{sgn}(bx + a) - 693 b^5 d^5 \operatorname{sgn}(bx + a) + 9009(ex + d)^4 a b^4 e \operatorname{sgn}(bx + a) - 25740(ex + d)^3 a b^4 d e \operatorname{sgn}(bx + a) + 30030(ex + d)^2 a b^4 d^2 e \operatorname{sgn}(bx + a) - 16380(ex + d) a b^4 d^3 e \operatorname{sgn}(bx + a) + 3465 a b^4 d^4 e \operatorname{sgn}(bx + a) + 12870(ex + d)^3 a^2 b^3 e^2 \operatorname{sgn}(bx + a) - 30030(ex + d)^2 a^2 b^3 d e^2 \operatorname{sgn}(bx + a) + 24570(ex + d) a^2 b^3 d^2 e^2 \operatorname{sgn}(bx + a) - 6930 a^2 b^3 d^3 e^2 \operatorname{sgn}(bx + a) + 10010(ex + d)^2 a^3 b^2 e^3 \operatorname{sgn}(bx + a) - 16380(ex + d) a^3 b^2 d e^3 \operatorname{sgn}(bx + a) + 6930 a^3 b^2 d^2 e^3 \operatorname{sgn}(bx + a) + 4095(ex + d) a^4 b e^4 \operatorname{sgn}(bx + a) - 3465 a^4 b d e^4 \operatorname{sgn}(bx + a) + 693 a^5 e^5 \operatorname{sgn}(bx + a))}{(ex + d)^{13/2} e^6}$$

input

```
integrate((b^2*x^2+2*a*b*x+a^2)^(5/2)/(e*x+d)^(15/2),x, algorithm="giac")
```

output

```
-2/9009*(3003*(e*x + d)^5*b^5*sgn(b*x + a) - 9009*(e*x + d)^4*b^5*d*sgn(b*x + a) + 12870*(e*x + d)^3*b^5*d^2*sgn(b*x + a) - 10010*(e*x + d)^2*b^5*d^3*sgn(b*x + a) + 4095*(e*x + d)*b^5*d^4*sgn(b*x + a) - 693*b^5*d^5*sgn(b*x + a) + 9009*(e*x + d)^4*a*b^4*e*sgn(b*x + a) - 25740*(e*x + d)^3*a*b^4*d*e*sgn(b*x + a) + 30030*(e*x + d)^2*a*b^4*d^2*e*sgn(b*x + a) - 16380*(e*x + d)*a*b^4*d^3*e*sgn(b*x + a) + 3465*a*b^4*d^4*e*sgn(b*x + a) + 12870*(e*x + d)^3*a^2*b^3*e^2*sgn(b*x + a) - 30030*(e*x + d)^2*a^2*b^3*d*e^2*sgn(b*x + a) + 24570*(e*x + d)*a^2*b^3*d^2*e^2*sgn(b*x + a) - 6930*a^2*b^3*d^3*e^2*sgn(b*x + a) + 10010*(e*x + d)^2*a^3*b^2*e^3*sgn(b*x + a) - 16380*(e*x + d)*a^3*b^2*d*e^3*sgn(b*x + a) + 6930*a^3*b^2*d^2*e^3*sgn(b*x + a) + 4095*(e*x + d)*a^4*b*e^4*sgn(b*x + a) - 3465*a^4*b*d*e^4*sgn(b*x + a) + 693*a^5*e^5*sgn(b*x + a))/((e*x + d)^(13/2)*e^6)
```

Mupad [B] (verification not implemented)

Time = 6.22 (sec) , antiderivative size = 472, normalized size of antiderivative = 1.48

$$\int \frac{(a^2 + 2abx + b^2x^2)^{5/2}}{(d + ex)^{15/2}} dx = \frac{\sqrt{a^2 + 2abx + b^2x^2} \left(\frac{1386a^5e^5 + 1260a^4bde^4 + 1120a^3b^2d^2e^3 + 960a^2b^3d^3e^2 + 768ab^4d^4e + 512b^5d^5}{9009be^{12}} + \frac{2b^4x^5}{3e^7} + \frac{2b^3x^4(3ae + 2bd)}{3e^8} \right)}{x^7 \sqrt{d + ex} + \frac{ad^6 \sqrt{d+ex}}{be^6} + \frac{x^6(ae+6bd) \sqrt{d+ex}}{be} + \frac{3dx^5(2ae+5bd) \sqrt{d+ex}}{be^2}}$$

input

```
int((a^2 + b^2*x^2 + 2*a*b*x)^(5/2)/(d + e*x)^(15/2),x)
```

output

```
-((a^2 + b^2*x^2 + 2*a*b*x)^(1/2)*((1386*a^5*e^5 + 512*b^5*d^5 + 960*a^2*b^3*d^3*e^2 + 1120*a^3*b^2*d^2*e^3 + 768*a*b^4*d^4*e + 1260*a^4*b*d*e^4)/(9009*b*e^12) + (2*b^4*x^5)/(3*e^7) + (2*b^3*x^4*(3*a*e + 2*b*d))/(3*e^8) + (x*(8190*a^4*b*e^5 + 3328*b^5*d^4*e + 4992*a*b^4*d^3*e^2 + 7280*a^3*b^2*d*e^4 + 6240*a^2*b^3*d^2*e^3))/(9009*b*e^12) + (4*b^2*x^3*(15*a^2*e^2 + 8*b^2*d^2 + 12*a*b*d*e))/(21*e^9) + (4*b*x^2*(35*a^3*e^3 + 16*b^3*d^3 + 24*a*b^2*d^2*e + 30*a^2*b*d*e^2))/(63*e^10))/(x^7*(d + e*x)^(1/2) + (a*d^6*(d + e*x)^(1/2))/(b*e^6) + (x^6*(a*e + 6*b*d)*(d + e*x)^(1/2))/(b*e) + (3*d*x^5*(2*a*e + 5*b*d)*(d + e*x)^(1/2))/(b*e^2) + (d^5*x*(6*a*e + b*d)*(d + e*x)^(1/2))/(b*e^6) + (5*d^2*x^4*(3*a*e + 4*b*d)*(d + e*x)^(1/2))/(b*e^3) + (5*d^3*x^3*(4*a*e + 3*b*d)*(d + e*x)^(1/2))/(b*e^4) + (3*d^4*x^2*(5*a*e + 2*b*d)*(d + e*x)^(1/2))/(b*e^5))
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 335, normalized size of antiderivative = 1.05

$$\int \frac{(a^2 + 2abx + b^2x^2)^{5/2}}{(d + ex)^{15/2}} dx = \frac{-\frac{2}{3}b^5e^5x^5 - 2ab^4e^5x^4 - \frac{4}{3}b^5de^4x^4 - \frac{20}{7}a^2b^3e^5x^3 - \frac{16}{7}ab^4de^4x^3 - \frac{32}{21}b^5d^2e^3x^3}{(d + ex)^{15/2}}$$

input

```
int((b^2*x^2+2*a*b*x+a^2)^(5/2)/(e*x+d)^(15/2),x)
```

output

```
(2*( - 693*a**5*e**5 - 630*a**4*b*d*e**4 - 4095*a**4*b*e**5*x - 560*a**3*b
**2*d**2*e**3 - 3640*a**3*b**2*d*e**4*x - 10010*a**3*b**2*e**5*x**2 - 480*
a**2*b**3*d**3*e**2 - 3120*a**2*b**3*d**2*e**3*x - 8580*a**2*b**3*d*e**4*x
**2 - 12870*a**2*b**3*e**5*x**3 - 384*a*b**4*d**4*e - 2496*a*b**4*d**3*e**
2*x - 6864*a*b**4*d**2*e**3*x**2 - 10296*a*b**4*d*e**4*x**3 - 9009*a*b**4*
e**5*x**4 - 256*b**5*d**5 - 1664*b**5*d**4*e*x - 4576*b**5*d**3*e**2*x**2
- 6864*b**5*d**2*e**3*x**3 - 6006*b**5*d*e**4*x**4 - 3003*b**5*e**5*x**5))
/(9009*sqrt(d + e*x)*e**6*(d**6 + 6*d**5*e*x + 15*d**4*e**2*x**2 + 20*d**3
*e**3*x**3 + 15*d**2*e**4*x**4 + 6*d*e**5*x**5 + e**6*x**6))
```

3.371 $\int \frac{(d+ex)^{7/2}}{\sqrt{a^2+2abx+b^2x^2}} dx$

| | |
|---|------|
| Optimal result | 2738 |
| Mathematica [A] (verified) | 2739 |
| Rubi [A] (verified) | 2739 |
| Maple [A] (verified) | 2743 |
| Fricas [A] (verification not implemented) | 2743 |
| Sympy [F(-1)] | 2744 |
| Maxima [F] | 2744 |
| Giac [A] (verification not implemented) | 2745 |
| Mupad [F(-1)] | 2745 |
| Reduce [B] (verification not implemented) | 2746 |

Optimal result

Integrand size = 30, antiderivative size = 263

$$\int \frac{(d+ex)^{7/2}}{\sqrt{a^2+2abx+b^2x^2}} dx = \frac{2(bd-ae)^3(a+bx)\sqrt{d+ex}}{b^4\sqrt{a^2+2abx+b^2x^2}} + \frac{2(bd-ae)^2(a+bx)(d+ex)^{3/2}}{3b^3\sqrt{a^2+2abx+b^2x^2}} + \frac{2(bd-ae)(a+bx)(d+ex)^{5/2}}{5b^2\sqrt{a^2+2abx+b^2x^2}} + \frac{2(a+bx)(d+ex)^{7/2}}{7b\sqrt{a^2+2abx+b^2x^2}} - \frac{2(bd-ae)^{7/2}(a+bx)\operatorname{arctanh}\left(\frac{\sqrt{b}\sqrt{d+ex}}{\sqrt{bd-ae}}\right)}{b^{9/2}\sqrt{a^2+2abx+b^2x^2}}$$

```
output 2*(-a*e+b*d)^3*(b*x+a)*(e*x+d)^(1/2)/b^4/((b*x+a)^2)^(1/2)+2/3*(-a*e+b*d)^
2*(b*x+a)*(e*x+d)^(3/2)/b^3/((b*x+a)^2)^(1/2)+2/5*(-a*e+b*d)*(b*x+a)*(e*x+
d)^(5/2)/b^2/((b*x+a)^2)^(1/2)+2/7*(b*x+a)*(e*x+d)^(7/2)/b/((b*x+a)^2)^(1/
2)-2*(-a*e+b*d)^(7/2)*(b*x+a)*arctanh(b^(1/2)*(e*x+d)^(1/2)/(-a*e+b*d)^(1/
2))/b^(9/2)/((b*x+a)^2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.20 (sec) , antiderivative size = 168, normalized size of antiderivative = 0.64

$$\int \frac{(d+ex)^{7/2}}{\sqrt{a^2+2abx+b^2x^2}} dx = \frac{2(a+bx) \left(\sqrt{b}\sqrt{d+ex}(-105a^3e^3 + 35a^2be^2(10d+ex) - 7ab^2e(58d^2 + 16dex) \right)}{\sqrt{a^2+2abx+b^2x^2}}$$

input `Integrate[(d + e*x)^(7/2)/Sqrt[a^2 + 2*a*b*x + b^2*x^2], x]`

output `(2*(a + b*x)*(Sqrt[b]*Sqrt[d + e*x]*(-105*a^3*e^3 + 35*a^2*b*e^2*(10*d + e*x) - 7*a*b^2*e*(58*d^2 + 16*d*e*x + 3*e^2*x^2) + b^3*(176*d^3 + 122*d^2*e*x + 66*d*e^2*x^2 + 15*e^3*x^3)) + 105*(-(b*d) + a*e)^(7/2)*ArcTan[(Sqrt[b]*Sqrt[d + e*x])/Sqrt[-(b*d) + a*e]])/(105*b^(9/2)*Sqrt[(a + b*x)^2])`

Rubi [A] (verified)

Time = 0.51 (sec) , antiderivative size = 175, normalized size of antiderivative = 0.67, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$, Rules used = {1102, 27, 60, 60, 60, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\begin{aligned} & \int \frac{(d+ex)^{7/2}}{\sqrt{a^2+2abx+b^2x^2}} dx \\ & \quad \downarrow \text{1102} \\ & \frac{b(a+bx) \int \frac{(d+ex)^{7/2}}{b(a+bx)} dx}{\sqrt{a^2+2abx+b^2x^2}} \\ & \quad \downarrow \text{27} \\ & \frac{(a+bx) \int \frac{(d+ex)^{7/2}}{a+bx} dx}{\sqrt{a^2+2abx+b^2x^2}} \\ & \quad \downarrow \text{60} \end{aligned}$$

$$\begin{aligned}
 & \frac{(a + bx) \left(\frac{(bd - ae) \int \frac{(d+ex)^{5/2}}{a+bx} dx}{b} + \frac{2(d+ex)^{7/2}}{7b} \right)}{\sqrt{a^2 + 2abx + b^2x^2}} \\
 & \quad \downarrow 60 \\
 & \frac{(a + bx) \left(\frac{(bd - ae) \left(\frac{(bd - ae) \int \frac{(d+ex)^{3/2}}{a+bx} dx}{b} + \frac{2(d+ex)^{5/2}}{5b} \right)}{b} + \frac{2(d+ex)^{7/2}}{7b} \right)}{\sqrt{a^2 + 2abx + b^2x^2}} \\
 & \quad \downarrow 60 \\
 & \frac{(a + bx) \left(\frac{(bd - ae) \left(\frac{(bd - ae) \left(\frac{(bd - ae) \int \frac{\sqrt{d+ex}}{a+bx} dx}{b} + \frac{2(d+ex)^{3/2}}{3b} \right)}{b} + \frac{2(d+ex)^{5/2}}{5b} \right)}{b} + \frac{2(d+ex)^{7/2}}{7b} \right)}{\sqrt{a^2 + 2abx + b^2x^2}} \\
 & \quad \downarrow 60 \\
 & \frac{(a + bx) \left(\frac{(bd - ae) \left(\frac{(bd - ae) \left(\frac{(bd - ae) \int \frac{1}{(a+bx)\sqrt{d+ex}} dx}{b} + \frac{2\sqrt{d+ex}}{b} \right)}{b} + \frac{2(d+ex)^{3/2}}{3b} \right)}{b} + \frac{2(d+ex)^{5/2}}{5b} \right)}{b} + \frac{2(d+ex)^{7/2}}{7b} \right)}{\sqrt{a^2 + 2abx + b^2x^2}} \\
 & \quad \downarrow 73
 \end{aligned}$$

$$\left(\frac{(a+bx) \left((bd-ae) \left(\frac{(bd-ae) \int \frac{1}{a + \frac{b(d+ex)}{be} - \frac{bd}{e}} d\sqrt{d+ex}} + \frac{2\sqrt{d+ex}}{b} \right) + \frac{2(d+ex)^{3/2}}{3b} \right) + \frac{2(d+ex)^{5/2}}{5b}}{b} + \frac{2(d+ex)^{7/2}}{7b} \right)$$

$$\sqrt{a^2 + 2abx + b^2x^2}$$

↓ 221

$$\left(\frac{(a+bx) \left((bd-ae) \left(\frac{(bd-ae) \left(\frac{2\sqrt{d+ex}}{b} - \frac{2\sqrt{bd-ae} \operatorname{arctanh} \left(\frac{\sqrt{b}\sqrt{d+ex}}{\sqrt{bd-ae}} \right)}{b^{3/2}} \right) + \frac{2(d+ex)^{3/2}}{3b} \right) + \frac{2(d+ex)^{5/2}}{5b}}{b} + \frac{2(d+ex)^{7/2}}{7b} \right) \right)$$

$$\sqrt{a^2 + 2abx + b^2x^2}$$

input `Int[(d + e*x)^(7/2)/Sqrt[a^2 + 2*a*b*x + b^2*x^2],x]`

output `((a + b*x)*((2*(d + e*x)^(7/2))/(7*b) + ((b*d - a*e)*((2*(d + e*x)^(5/2))/(5*b) + ((b*d - a*e)*((2*(d + e*x)^(3/2))/(3*b) + ((b*d - a*e)*((2*Sqrt[d + e*x])/b - (2*Sqrt[b*d - a*e]*ArcTanh[(Sqrt[b]*Sqrt[d + e*x])/Sqrt[b*d - a*e]])/b^(3/2)))/b))/b))/Sqrt[a^2 + 2*a*b*x + b^2*x^2]`

Defintions of rubi rules used

rule 27 `Int[(a_)*(Fx_), x_Symbol] := Simp[a Int[Fx, x], x] /; FreeQ[a, x] && !MatchQ[Fx, (b_)*(Gx_)] /; FreeQ[b, x]`

rule 60 `Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] := Simp[(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*(b*c - a*d)/(b*(m + n + 1)) Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] && !(IGtQ[m, 0] && (!IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) && !ILtQ[m + n + 2, 0] && IntLinearQ[a, b, c, d, m, n, x]`

rule 73 `Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] := With[{p = Denominator[m]}, Simp[p/b Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) + d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntLinearQ[a, b, c, d, m, n, x]`

rule 221 `Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[-a/b, 2]/a)*ArcTanh[x/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]`

rule 1102 `Int[((d_.) + (e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] := Simp[(a + b*x + c*x^2)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x)^(2*FracPart[p])) Int[(d + e*x)^m*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && EqQ[b^2 - 4*a*c, 0]`

Maple [A] (verified)

Time = 1.54 (sec) , antiderivative size = 237, normalized size of antiderivative = 0.90

| method | result |
|---------|--|
| risch | $\frac{2(-15e^3x^3b^3+21x^2ab^2e^3-66x^2b^3de^2-35a^2be^3x+112xab^2de^2-122b^3d^2ex+105e^3a^3-350a^2bde^2+406ab^2d^2e-176b^3d^3)\sqrt{ex+d}}{105b^4(bx+a)}$ |
| default | $\frac{2(bx+a)\left(15(ex+d)^{\frac{7}{2}}\sqrt{b(ae-bd)}b^3-21(ex+d)^{\frac{5}{2}}\sqrt{b(ae-bd)}ab^2e+21(ex+d)^{\frac{5}{2}}\sqrt{b(ae-bd)}b^3d+35(ex+d)^{\frac{3}{2}}\sqrt{b(ae-bd)}a^2be^2-70(ex+d)^{\frac{3}{2}}\sqrt{b(ae-bd)}a^2b^2e\right)}{105b^4(bx+a)}$ |

input `int((e*x+d)^(7/2)/((b*x+a)^2)^(1/2),x,method=_RETURNVERBOSE)`

output
$$\frac{-2/105*(-15*b^3*e^3*x^3+21*a*b^2*e^3*x^2-66*b^3*d*e^2*x^2-35*a^2*b*e^3*x+12*a*b^2*d*e^2*x-122*b^3*d^2*e*x+105*a^3*e^3-350*a^2*b*d*e^2+406*a*b^2*d^2*e-176*b^3*d^3)*(e*x+d)^{(1/2)}/b^4*((b*x+a)^2)^{(1/2)}/(b*x+a)+2*(a^4*e^4-4*a^3*b*d*e^3+6*a^2*b^2*d^2*e^2-4*a*b^3*d^3*e+b^4*d^4)/b^4/(b*(a*e-b*d))^{(1/2)}*\arctan(b*(e*x+d)^{(1/2)}/(b*(a*e-b*d))^{(1/2)})*((b*x+a)^2)^{(1/2)}/(b*x+a)}$$

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 424, normalized size of antiderivative = 1.61

$$\int \frac{(d+ex)^{7/2}}{\sqrt{a^2+2abx+b^2x^2}} dx = \left[\frac{105(b^3d^3-3ab^2d^2e+3a^2bde^2-a^3e^3)\sqrt{\frac{bd-ae}{b}} \log\left(\frac{be x+2bd-ae+2\sqrt{ex+db}\sqrt{\frac{bd-ae}{b}}}{bx+a}\right)}{2\left(105(b^3d^3-3ab^2d^2e+3a^2bde^2-a^3e^3)\sqrt{-\frac{bd-ae}{b}} \arctan\left(-\frac{\sqrt{ex+db}\sqrt{-\frac{bd-ae}{b}}}{bd-ae}\right) - (15b^3e^3x^3+176b^3d^3)\sqrt{ex+d}\right)} \right]$$

input `integrate((e*x+d)^(7/2)/((b*x+a)^2)^(1/2),x, algorithm="fricas")`

output

```
[-1/105*(105*(b^3*d^3 - 3*a*b^2*d^2*e + 3*a^2*b*d*e^2 - a^3*e^3)*sqrt((b*d
- a*e)/b)*log((b*e*x + 2*b*d - a*e + 2*sqrt(e*x + d)*b*sqrt((b*d - a*e)/b
))/b*x + a) - 2*(15*b^3*e^3*x^3 + 176*b^3*d^3 - 406*a*b^2*d^2*e + 350*a^
2*b*d*e^2 - 105*a^3*e^3 + 3*(22*b^3*d*e^2 - 7*a*b^2*e^3)*x^2 + (122*b^3*d^
2*e - 112*a*b^2*d*e^2 + 35*a^2*b*e^3)*x)*sqrt(e*x + d))/b^4, -2/105*(105*(
b^3*d^3 - 3*a*b^2*d^2*e + 3*a^2*b*d*e^2 - a^3*e^3)*sqrt(-(b*d - a*e)/b)*ar
ctan(-sqrt(e*x + d)*b*sqrt(-(b*d - a*e)/b)/(b*d - a*e)) - (15*b^3*e^3*x^3
+ 176*b^3*d^3 - 406*a*b^2*d^2*e + 350*a^2*b*d*e^2 - 105*a^3*e^3 + 3*(22*b^
3*d*e^2 - 7*a*b^2*e^3)*x^2 + (122*b^3*d^2*e - 112*a*b^2*d*e^2 + 35*a^2*b*e
^3)*x)*sqrt(e*x + d))/b^4]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(d + ex)^{7/2}}{\sqrt{a^2 + 2abx + b^2x^2}} dx = \text{Timed out}$$

input

```
integrate((e*x+d)**(7/2)/((b*x+a)**2)**(1/2),x)
```

output

Timed out

Maxima [F]

$$\int \frac{(d + ex)^{7/2}}{\sqrt{a^2 + 2abx + b^2x^2}} dx = \int \frac{(ex + d)^{7/2}}{\sqrt{(bx + a)^2}} dx$$

input

```
integrate((e*x+d)^(7/2)/((b*x+a)^2)^(1/2),x, algorithm="maxima")
```

output

```
integrate((e*x + d)^(7/2)/sqrt((b*x + a)^2), x)
```

Giac [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 343, normalized size of antiderivative = 1.30

$$\int \frac{(d+ex)^{7/2}}{\sqrt{a^2+2abx+b^2x^2}} dx = \frac{2(b^4d^4\operatorname{sgn}(bx+a) - 4ab^3d^3e\operatorname{sgn}(bx+a) + 6a^2b^2d^2e^2\operatorname{sgn}(bx+a) - 4a^3bde^3\operatorname{sgn}(bx+a) + a^4e^4\operatorname{sgn}(bx+a))\arctan\left(\frac{\sqrt{ex+d}}{\sqrt{-b^2d+abeb^4}}\right) + 2\left(15(ex+d)^{7/2}b^6\operatorname{sgn}(bx+a) + 21(ex+d)^{5/2}b^6d\operatorname{sgn}(bx+a) + 35(ex+d)^{3/2}b^6d^2\operatorname{sgn}(bx+a) + 105\sqrt{ex+d}b^6d^3\operatorname{sgn}(bx+a) - 21(ex+d)^{5/2}ab^5e\operatorname{sgn}(bx+a) - 70(ex+d)^{3/2}a^2b^5d^2e\operatorname{sgn}(bx+a) + 35(ex+d)^{3/2}a^2b^4e^2\operatorname{sgn}(bx+a) + 315\sqrt{ex+d}a^2b^4de^2\operatorname{sgn}(bx+a) - 105\sqrt{ex+d}a^3b^3e^3\operatorname{sgn}(bx+a)\right)}{b^7}$$

input `integrate((e*x+d)^(7/2)/((b*x+a)^2)^(1/2),x, algorithm="giac")`

output `2*(b^4*d^4*sgn(b*x + a) - 4*a*b^3*d^3*e*sgn(b*x + a) + 6*a^2*b^2*d^2*e^2*sgn(b*x + a) - 4*a^3*b*d*e^3*sgn(b*x + a) + a^4*e^4*sgn(b*x + a))*arctan(sqrt(e*x + d)*b/sqrt(-b^2*d + a*b*e))/(sqrt(-b^2*d + a*b*e)*b^4) + 2/105*(15*(e*x + d)^(7/2)*b^6*sgn(b*x + a) + 21*(e*x + d)^(5/2)*b^6*d*sgn(b*x + a) + 35*(e*x + d)^(3/2)*b^6*d^2*sgn(b*x + a) + 105*sqrt(e*x + d)*b^6*d^3*sgn(b*x + a) - 21*(e*x + d)^(5/2)*a*b^5*e*sgn(b*x + a) - 70*(e*x + d)^(3/2)*a^2*b^5*d^2*e*sgn(b*x + a) + 35*(e*x + d)^(3/2)*a^2*b^4*e^2*sgn(b*x + a) + 315*sqrt(e*x + d)*a^2*b^4*d*e^2*sgn(b*x + a) - 105*sqrt(e*x + d)*a^3*b^3*e^3*sgn(b*x + a))/b^7`

Mupad [F(-1)]

Timed out.

$$\int \frac{(d+ex)^{7/2}}{\sqrt{a^2+2abx+b^2x^2}} dx = \int \frac{(d+ex)^{7/2}}{\sqrt{(a+bx)^2}} dx$$

input `int((d + e*x)^(7/2)/((a + b*x)^2)^(1/2),x)`

output `int((d + e*x)^(7/2)/((a + b*x)^2)^(1/2), x)`

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 349, normalized size of antiderivative = 1.33

$$\int \frac{(d + ex)^{7/2}}{\sqrt{a^2 + 2abx + b^2x^2}} dx = \frac{2\sqrt{b}\sqrt{ae - bd} \operatorname{atan}\left(\frac{\sqrt{ex+db}}{\sqrt{b}\sqrt{ae-bd}}\right) a^3e^3 - 6\sqrt{b}\sqrt{ae - bd} \operatorname{atan}\left(\frac{\sqrt{ex+db}}{\sqrt{b}\sqrt{ae-bd}}\right) a^2bd e^2 - \dots}{\dots}$$

input `int((e*x+d)^(7/2)/((b*x+a)^2)^(1/2),x)`

output

```
(2*(105*sqrt(b)*sqrt(a*e - b*d)*atan((sqrt(d + e*x)*b)/(sqrt(b)*sqrt(a*e - b*d)))*a**3*e**3 - 315*sqrt(b)*sqrt(a*e - b*d)*atan((sqrt(d + e*x)*b)/(sqrt(b)*sqrt(a*e - b*d)))*a**2*b*d*e**2 + 315*sqrt(b)*sqrt(a*e - b*d)*atan((sqrt(d + e*x)*b)/(sqrt(b)*sqrt(a*e - b*d)))*a*b**2*d**2*e - 105*sqrt(b)*sqrt(a*e - b*d)*atan((sqrt(d + e*x)*b)/(sqrt(b)*sqrt(a*e - b*d)))*b**3*d**3 - 105*sqrt(d + e*x)*a**3*b*e**3 + 350*sqrt(d + e*x)*a**2*b**2*d*e**2 + 35*sqrt(d + e*x)*a**2*b**2*e**3*x - 406*sqrt(d + e*x)*a*b**3*d**2*e - 112*sqrt(d + e*x)*a*b**3*d*e**2*x - 21*sqrt(d + e*x)*a*b**3*e**3*x**2 + 176*sqrt(d + e*x)*b**4*d**3 + 122*sqrt(d + e*x)*b**4*d**2*e*x + 66*sqrt(d + e*x)*b**4*d*e**2*x**2 + 15*sqrt(d + e*x)*b**4*e**3*x**3))/(105*b**5)
```

3.372 $\int \frac{(d+ex)^{5/2}}{\sqrt{a^2+2abx+b^2x^2}} dx$

| | |
|---|------|
| Optimal result | 2747 |
| Mathematica [A] (verified) | 2748 |
| Rubi [A] (verified) | 2748 |
| Maple [A] (verified) | 2751 |
| Fricas [A] (verification not implemented) | 2752 |
| Sympy [F] | 2752 |
| Maxima [F] | 2753 |
| Giac [A] (verification not implemented) | 2753 |
| Mupad [F(-1)] | 2754 |
| Reduce [B] (verification not implemented) | 2754 |

Optimal result

Integrand size = 30, antiderivative size = 212

$$\int \frac{(d+ex)^{5/2}}{\sqrt{a^2+2abx+b^2x^2}} dx = \frac{2(bd-ae)^2(a+bx)\sqrt{d+ex}}{b^3\sqrt{a^2+2abx+b^2x^2}} + \frac{2(bd-ae)(a+bx)(d+ex)^{3/2}}{3b^2\sqrt{a^2+2abx+b^2x^2}} + \frac{2(a+bx)(d+ex)^{5/2}}{5b\sqrt{a^2+2abx+b^2x^2}} - \frac{2(bd-ae)^{5/2}(a+bx)\operatorname{arctanh}\left(\frac{\sqrt{b}\sqrt{d+ex}}{\sqrt{bd-ae}}\right)}{b^{7/2}\sqrt{a^2+2abx+b^2x^2}}$$

output

```
2*(-a*e+b*d)^2*(b*x+a)*(e*x+d)^(1/2)/b^3/((b*x+a)^2)^(1/2)+2/3*(-a*e+b*d)*
(b*x+a)*(e*x+d)^(3/2)/b^2/((b*x+a)^2)^(1/2)+2/5*(b*x+a)*(e*x+d)^(5/2)/b/((
b*x+a)^2)^(1/2)-2*(-a*e+b*d)^(5/2)*(b*x+a)*arctanh(b^(1/2)*(e*x+d)^(1/2)/(
-a*e+b*d)^(1/2))/b^(7/2)/((b*x+a)^2)^(1/2)
```


Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 127, normalized size of antiderivative = 0.60

$$\int \frac{(d + ex)^{5/2}}{\sqrt{a^2 + 2abx + b^2x^2}} dx = \frac{2(a + bx) \left(\sqrt{b}\sqrt{d + ex}(15a^2e^2 - 5abe(7d + ex) + b^2(23d^2 + 11dex + 3e^2x^2)) \right)}{15b^{7/2}\sqrt{(a + bx)^2}}$$

input `Integrate[(d + e*x)^(5/2)/Sqrt[a^2 + 2*a*b*x + b^2*x^2], x]`

output `(2*(a + b*x)*(Sqrt[b]*Sqrt[d + e*x]*(15*a^2*e^2 - 5*a*b*e*(7*d + e*x) + b^2*(23*d^2 + 11*d*e*x + 3*e^2*x^2)) - 15*(-(b*d) + a*e)^(5/2)*ArcTan[(Sqrt[b]*Sqrt[d + e*x])/Sqrt[-(b*d) + a*e]])/(15*b^(7/2)*Sqrt[(a + b*x)^2])`

Rubi [A] (verified)

Time = 0.47 (sec) , antiderivative size = 146, normalized size of antiderivative = 0.69, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.233$, Rules used = {1102, 27, 60, 60, 60, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\begin{aligned} & \int \frac{(d + ex)^{5/2}}{\sqrt{a^2 + 2abx + b^2x^2}} dx \\ & \quad \downarrow \text{1102} \\ & \frac{b(a + bx) \int \frac{(d+ex)^{5/2}}{b(a+bx)} dx}{\sqrt{a^2 + 2abx + b^2x^2}} \\ & \quad \downarrow \text{27} \\ & \frac{(a + bx) \int \frac{(d+ex)^{5/2}}{a+bx} dx}{\sqrt{a^2 + 2abx + b^2x^2}} \\ & \quad \downarrow \text{60} \end{aligned}$$

$$\begin{aligned}
 & \frac{(a + bx) \left(\frac{(bd - ae) \int \frac{(d+ex)^{3/2}}{a+bx} dx}{b} + \frac{2(d+ex)^{5/2}}{5b} \right)}{\sqrt{a^2 + 2abx + b^2x^2}} \\
 & \quad \downarrow 60 \\
 & \frac{(a + bx) \left(\frac{(bd - ae) \left(\frac{(bd - ae) \int \frac{\sqrt{d+ex}}{a+bx} dx}{b} + \frac{2(d+ex)^{3/2}}{3b} \right)}{b} + \frac{2(d+ex)^{5/2}}{5b} \right)}{\sqrt{a^2 + 2abx + b^2x^2}} \\
 & \quad \downarrow 60 \\
 & \frac{(a + bx) \left(\frac{(bd - ae) \left(\frac{(bd - ae) \int \frac{1}{(a+bx)\sqrt{d+ex}} dx}{b} + \frac{2\sqrt{d+ex}}{b} \right)}{b} + \frac{2(d+ex)^{3/2}}{3b} \right) + \frac{2(d+ex)^{5/2}}{5b}}{\sqrt{a^2 + 2abx + b^2x^2}} \\
 & \quad \downarrow 73 \\
 & \frac{(a + bx) \left(\frac{(bd - ae) \left(\frac{2(bd - ae) \int \frac{1}{a + \frac{b(d+ex)}{e} - \frac{bd}{e}} d\sqrt{d+ex}}{b} + \frac{2\sqrt{d+ex}}{b} \right)}{b} + \frac{2(d+ex)^{3/2}}{3b} \right) + \frac{2(d+ex)^{5/2}}{5b}}{\sqrt{a^2 + 2abx + b^2x^2}} \\
 & \quad \downarrow 221
 \end{aligned}$$

$$\frac{(a + bx) \left(\frac{(bd - ae) \left(\frac{2\sqrt{d+ex}}{b} - \frac{2\sqrt{bd-ae} \operatorname{arctanh}\left(\frac{\sqrt{b}\sqrt{d+ex}}{\sqrt{bd-ae}}\right)}{b^{3/2}} \right)}{b} + \frac{2(d+ex)^{3/2}}{3b} \right)}{b} + \frac{2(d+ex)^{5/2}}{5b} \right)}{\sqrt{a^2 + 2abx + b^2x^2}}$$

input `Int[(d + e*x)^(5/2)/Sqrt[a^2 + 2*a*b*x + b^2*x^2],x]`

output `((a + b*x)*((2*(d + e*x)^(5/2))/(5*b) + ((b*d - a*e)*((2*(d + e*x)^(3/2))/(3*b) + ((b*d - a*e)*((2*Sqrt[d + e*x])/b - (2*Sqrt[b*d - a*e]*ArcTanh[(Sqrt[b]*Sqrt[d + e*x])/Sqrt[b*d - a*e]])/b^(3/2)))/b))/Sqrt[a^2 + 2*a*b*x + b^2*x^2]`

Defintions of rubi rules used

rule 27 `Int[(a_)*(F_x_), x_Symbol] := Simp[a Int[F_x, x], x] /; FreeQ[a, x] && !MatchQ[F_x, (b_)*(G_x_) /; FreeQ[b, x]]`

rule 60 `Int[((a_) + (b_)*(x_))^(m_)*((c_) + (d_)*(x_))^(n_), x_Symbol] := Simp[(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*(b*c - a*d)/(b*(m + n + 1)) Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] && !(IGtQ[m, 0] && (!IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) && !ILtQ[m + n + 2, 0] && IntLinearQ[a, b, c, d, m, n, x]`

rule 73 `Int[((a_) + (b_)*(x_))^(m_)*((c_) + (d_)*(x_))^(n_), x_Symbol] := With[{p = Denominator[m]}, Simp[p/b Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) + d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x] /; FreeQ[{a, b, c, d}, x] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntLinearQ[a, b, c, d, m, n, x]`

rule 221 $\text{Int}[(a_.) + (b_.)*(x_)^2)^{-1}, x_Symbol] \rightarrow \text{Simp}[(\text{Rt}[-a/b, 2]/a)*\text{ArcTanh}[x/\text{Rt}[-a/b, 2]], x] /; \text{FreeQ}[\{a, b\}, x] \ \&\& \ \text{NegQ}[a/b]$

rule 1102 $\text{Int}[(d_.) + (e_.)*(x_)^{(m_.)}*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^{(p_.)}, x_Symbol] \rightarrow \text{Simp}[(a + b*x + c*x^2)^{\text{FracPart}[p]} / (c^{\text{IntPart}[p]}*(b/2 + c*x)^{(2*\text{FracPart}[p])}) \ \text{Int}[(d + e*x)^m*(b/2 + c*x)^{(2*p)}, x], x] /; \text{FreeQ}[\{a, b, c, d, e, m, p\}, x] \ \&\& \ \text{EqQ}[b^2 - 4*a*c, 0]$

Maple [A] (verified)

Time = 1.07 (sec) , antiderivative size = 171, normalized size of antiderivative = 0.81

| method | result |
|---------|--|
| risch | $\frac{2(3b^2e^2x^2 - 5xab e^2 + 11b^2dex + 15e^2a^2 - 35abde + 23b^2d^2)\sqrt{ex+d}\sqrt{(bx+a)^2}}{15b^3(bx+a)} - \frac{2(e^3a^3 - 3a^2bde^2 + 3ab^2d^2e - b^3d^3)\arctan\left(\frac{b\sqrt{ex+d}}{\sqrt{b(ae-bd)}}\right)}{b^3\sqrt{b(ae-bd)}(bx+a)}$ |
| default | $\frac{2(bx+a)\left(3(ex+d)^{\frac{5}{2}}\sqrt{b(ae-bd)}b^2 - 5(ex+d)^{\frac{3}{2}}\sqrt{b(ae-bd)}abe + 5(ex+d)^{\frac{3}{2}}\sqrt{b(ae-bd)}b^2d - 15\arctan\left(\frac{b\sqrt{ex+d}}{\sqrt{b(ae-bd)}}\right)a^3e^3 + 45\arctan\left(\frac{b\sqrt{ex+d}}{\sqrt{b(ae-bd)}}\right)a^3e^3\right)}{15b^3(bx+a)}$ |

input $\text{int}((e*x+d)^{(5/2)} / ((b*x+a)^2)^{(1/2)}, x, \text{method}=_RETURNVERBOSE)$

output
$$\frac{2}{15}*(3*b^2*e^2*x^2 - 5*a*b*e^2*x + 11*b^2*d*e*x + 15*a^2*e^2 - 35*a*b*d*e + 23*b^2*d^2)*(e*x+d)^{(1/2)}/b^3*((b*x+a)^2)^{(1/2)}/(b*x+a) - 2/b^3*(a^3*e^3 - 3*a^2*b*d*e^2 + 3*a*b^2*d^2*e - b^3*d^3)/(b*(a*e-b*d))^{(1/2)}*\arctan(b*(e*x+d)^{(1/2)}/(b*(a*e-b*d))^{(1/2)})*((b*x+a)^2)^{(1/2)}/(b*x+a)$$

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 290, normalized size of antiderivative = 1.37

$$\int \frac{(d+ex)^{5/2}}{\sqrt{a^2+2abx+b^2x^2}} dx = \frac{\left[15(b^2d^2 - 2abde + a^2e^2)\sqrt{\frac{bd-ae}{b}} \log\left(\frac{bex+2bd-ae-2\sqrt{ex+db}\sqrt{\frac{bd-ae}{b}}}{bx+a}\right) + 2(3b^2e^2x^2 + 23b^2d^2 - 35abde + 15a^2e^2) \sqrt{-\frac{bd-ae}{b}} \arctan\left(-\frac{\sqrt{ex+db}\sqrt{-\frac{bd-ae}{b}}}{bd-ae}\right) - (3b^2e^2x^2 + 23b^2d^2 - 35abde + 15a^2e^2) \sqrt{ex+d} \right]}{15b^3}$$

input `integrate((e*x+d)^(5/2)/((b*x+a)^2)^(1/2),x, algorithm="fricas")`

output `[1/15*(15*(b^2*d^2 - 2*a*b*d*e + a^2*e^2)*sqrt((b*d - a*e)/b)*log((b*e*x + 2*b*d - a*e - 2*sqrt(e*x + d)*b*sqrt((b*d - a*e)/b))/(b*x + a)) + 2*(3*b^2*e^2*x^2 + 23*b^2*d^2 - 35*a*b*d*e + 15*a^2*e^2 + (11*b^2*d*e - 5*a*b*e^2)*x)*sqrt(e*x + d))/b^3, -2/15*(15*(b^2*d^2 - 2*a*b*d*e + a^2*e^2)*sqrt(-(b*d - a*e)/b)*arctan(-sqrt(e*x + d)*b*sqrt(-(b*d - a*e)/b)/(b*d - a*e)) - (3*b^2*e^2*x^2 + 23*b^2*d^2 - 35*a*b*d*e + 15*a^2*e^2 + (11*b^2*d*e - 5*a*b*e^2)*x)*sqrt(e*x + d))/b^3]`

Sympy [F]

$$\int \frac{(d+ex)^{5/2}}{\sqrt{a^2+2abx+b^2x^2}} dx = \int \frac{(d+ex)^{5/2}}{\sqrt{(a+bx)^2}} dx$$

input `integrate((e*x+d)**(5/2)/((b*x+a)**2)**(1/2),x)`

output `Integral((d + e*x)**(5/2)/sqrt((a + b*x)**2), x)`

Maxima [F]

$$\int \frac{(d+ex)^{5/2}}{\sqrt{a^2+2abx+b^2x^2}} dx = \int \frac{(ex+d)^{5/2}}{\sqrt{(bx+a)^2}} dx$$

input `integrate((e*x+d)^(5/2)/((b*x+a)^2)^(1/2),x, algorithm="maxima")`

output `integrate((e*x + d)^(5/2)/sqrt((b*x + a)^2), x)`

Giac [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 231, normalized size of antiderivative = 1.09

$$\int \frac{(d+ex)^{5/2}}{\sqrt{a^2+2abx+b^2x^2}} dx = \frac{2(b^3d^3\operatorname{sgn}(bx+a) - 3ab^2d^2e\operatorname{sgn}(bx+a) + 3a^2bde^2\operatorname{sgn}(bx+a) - a^3e^3\operatorname{sgn}(bx+a))\arctan\left(\frac{\sqrt{ex+d}b}{\sqrt{-b^2d+abeb^3}}\right) + 2\left(3(ex+d)^{5/2}b^4\operatorname{sgn}(bx+a) + 5(ex+d)^{3/2}b^4d\operatorname{sgn}(bx+a) + 15\sqrt{ex+d}b^4d^2\operatorname{sgn}(bx+a) - 5(ex+d)^{3/2}ab^5\right)}{15b^5}$$

input `integrate((e*x+d)^(5/2)/((b*x+a)^2)^(1/2),x, algorithm="giac")`

output `2*(b^3*d^3*sgn(b*x + a) - 3*a*b^2*d^2*e*sgn(b*x + a) + 3*a^2*b*d*e^2*sgn(b*x + a) - a^3*e^3*sgn(b*x + a))*arctan(sqrt(e*x + d)*b/sqrt(-b^2*d + a*b*e))/sqrt(-b^2*d + a*b*e)*b^3 + 2/15*(3*(e*x + d)^(5/2)*b^4*sgn(b*x + a) + 5*(e*x + d)^(3/2)*b^4*d*sgn(b*x + a) + 15*sqrt(e*x + d)*b^4*d^2*sgn(b*x + a) - 5*(e*x + d)^(3/2)*a*b^3*e*sgn(b*x + a) - 30*sqrt(e*x + d)*a*b^3*d*e*sgn(b*x + a) + 15*sqrt(e*x + d)*a^2*b^2*e^2*sgn(b*x + a))/b^5`

Mupad [F(-1)]

Timed out.

$$\int \frac{(d + ex)^{5/2}}{\sqrt{a^2 + 2abx + b^2x^2}} dx = \int \frac{(d + ex)^{5/2}}{\sqrt{(a + bx)^2}} dx$$

input `int((d + e*x)^(5/2)/((a + b*x)^2)^(1/2), x)`output `int((d + e*x)^(5/2)/((a + b*x)^2)^(1/2), x)`**Reduce [B] (verification not implemented)**

Time = 0.21 (sec) , antiderivative size = 223, normalized size of antiderivative = 1.05

$$\int \frac{(d + ex)^{5/2}}{\sqrt{a^2 + 2abx + b^2x^2}} dx = \frac{-2\sqrt{b}\sqrt{ae - bd} \operatorname{atan}\left(\frac{\sqrt{ex+db}}{\sqrt{b}\sqrt{ae-bd}}\right) a^2e^2 + 4\sqrt{b}\sqrt{ae - bd} \operatorname{atan}\left(\frac{\sqrt{ex+db}}{\sqrt{b}\sqrt{ae-bd}}\right) abde - \dots}{15b^4}$$

input `int((e*x+d)^(5/2)/((b*x+a)^2)^(1/2), x)`output `(2*(- 15*sqrt(b)*sqrt(a*e - b*d)*atan((sqrt(d + e*x)*b)/(sqrt(b)*sqrt(a*e - b*d)))*a**2*e**2 + 30*sqrt(b)*sqrt(a*e - b*d)*atan((sqrt(d + e*x)*b)/(sqrt(b)*sqrt(a*e - b*d)))*a*b*d*e - 15*sqrt(b)*sqrt(a*e - b*d)*atan((sqrt(d + e*x)*b)/(sqrt(b)*sqrt(a*e - b*d)))*b**2*d**2 + 15*sqrt(d + e*x)*a**2*b*e**2 - 35*sqrt(d + e*x)*a*b**2*d*e - 5*sqrt(d + e*x)*a*b**2*e**2*x + 23*sqrt(d + e*x)*b**3*d**2 + 11*sqrt(d + e*x)*b**3*d*e*x + 3*sqrt(d + e*x)*b**3*e**2*x**2))/(15*b**4)`

3.373 $\int \frac{(d+ex)^{3/2}}{\sqrt{a^2+2abx+b^2x^2}} dx$

| | |
|---|------|
| Optimal result | 2755 |
| Mathematica [A] (verified) | 2755 |
| Rubi [A] (verified) | 2756 |
| Maple [A] (verified) | 2758 |
| Fricas [A] (verification not implemented) | 2759 |
| Sympy [F] | 2759 |
| Maxima [F] | 2760 |
| Giac [A] (verification not implemented) | 2760 |
| Mupad [F(-1)] | 2761 |
| Reduce [B] (verification not implemented) | 2761 |

Optimal result

Integrand size = 30, antiderivative size = 161

$$\int \frac{(d+ex)^{3/2}}{\sqrt{a^2+2abx+b^2x^2}} dx = \frac{2(bd-ae)(a+bx)\sqrt{d+ex}}{b^2\sqrt{a^2+2abx+b^2x^2}} + \frac{2(a+bx)(d+ex)^{3/2}}{3b\sqrt{a^2+2abx+b^2x^2}} - \frac{2(bd-ae)^{3/2}(a+bx)\operatorname{arctanh}\left(\frac{\sqrt{b}\sqrt{d+ex}}{\sqrt{bd-ae}}\right)}{b^{5/2}\sqrt{a^2+2abx+b^2x^2}}$$

output

```
2*(-a*e+b*d)*(b*x+a)*(e*x+d)^(1/2)/b^2/((b*x+a)^2)^(1/2)+2/3*(b*x+a)*(e*x+d)^(3/2)/b/((b*x+a)^2)^(1/2)-2*(-a*e+b*d)^(3/2)*(b*x+a)*arctanh(b^(1/2)*(e*x+d)^(1/2)/(-a*e+b*d)^(1/2))/b^(5/2)/((b*x+a)^2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.16 (sec) , antiderivative size = 96, normalized size of antiderivative = 0.60

$$\int \frac{(d+ex)^{3/2}}{\sqrt{a^2+2abx+b^2x^2}} dx = \frac{2(a+bx)\left(\sqrt{b}\sqrt{d+ex}(4bd-3ae+be x) + 3(-bd+ae)^{3/2}\arctan\left(\frac{\sqrt{b}\sqrt{d+ex}}{\sqrt{-bd+ae}}\right)\right)}{3b^{5/2}\sqrt{(a+bx)^2}}$$

input

```
Integrate[(d + e*x)^(3/2)/Sqrt[a^2 + 2*a*b*x + b^2*x^2], x]
```


output

```
(2*(a + b*x)*(Sqrt[b]*Sqrt[d + e*x]*(4*b*d - 3*a*e + b*e*x) + 3*(-(b*d) +
a*e)^(3/2)*ArcTan[(Sqrt[b]*Sqrt[d + e*x])/Sqrt[-(b*d) + a*e]])/(3*b^(5/2)
*Sqrt[(a + b*x)^2])
```

Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 117, normalized size of antiderivative = 0.73, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {1102, 27, 60, 60, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\begin{aligned}
 & \int \frac{(d+ex)^{3/2}}{\sqrt{a^2+2abx+b^2x^2}} dx \\
 & \quad \downarrow \text{1102} \\
 & \frac{b(a+bx) \int \frac{(d+ex)^{3/2}}{b(a+bx)} dx}{\sqrt{a^2+2abx+b^2x^2}} \\
 & \quad \downarrow \text{27} \\
 & \frac{(a+bx) \int \frac{(d+ex)^{3/2}}{a+bx} dx}{\sqrt{a^2+2abx+b^2x^2}} \\
 & \quad \downarrow \text{60} \\
 & \frac{(a+bx) \left(\frac{(bd-ae) \int \frac{\sqrt{d+ex}}{a+bx} dx}{b} + \frac{2(d+ex)^{3/2}}{3b} \right)}{\sqrt{a^2+2abx+b^2x^2}} \\
 & \quad \downarrow \text{60} \\
 & \frac{(a+bx) \left(\frac{(bd-ae) \left(\frac{(bd-ae) \int \frac{1}{(a+bx)\sqrt{d+ex}} dx}{b} + \frac{2\sqrt{d+ex}}{b} \right)}{b} + \frac{2(d+ex)^{3/2}}{3b} \right)}{\sqrt{a^2+2abx+b^2x^2}} \\
 & \quad \downarrow \text{73}
 \end{aligned}$$

$$\frac{(a + bx) \left(\frac{(bd - ae) \int \frac{1}{a + \frac{b(d+ex)}{e} - \frac{bd}{e}} d\sqrt{d+ex} + \frac{2\sqrt{d+ex}}{b}}{b} \right) + \frac{2(d+ex)^{3/2}}{3b}}{\sqrt{a^2 + 2abx + b^2x^2}}$$

↓ 221

$$\frac{(a + bx) \left(\frac{(bd - ae) \left(\frac{2\sqrt{d+ex}}{b} - \frac{2\sqrt{bd - ae} \operatorname{arctanh}\left(\frac{\sqrt{b}\sqrt{d+ex}}{\sqrt{bd - ae}}\right)}{b^{3/2}} \right)}{b} \right) + \frac{2(d+ex)^{3/2}}{3b}}{\sqrt{a^2 + 2abx + b^2x^2}}$$

input `Int[(d + e*x)^(3/2)/Sqrt[a^2 + 2*a*b*x + b^2*x^2], x]`

output `((a + b*x)*((2*(d + e*x)^(3/2))/(3*b) + ((b*d - a*e)*((2*Sqrt[d + e*x])/b - (2*Sqrt[b*d - a*e]*ArcTanh[(Sqrt[b]*Sqrt[d + e*x])/Sqrt[b*d - a*e]])/b^(3/2)))/b)/Sqrt[a^2 + 2*a*b*x + b^2*x^2]`

Defintions of rubi rules used

rule 27 `Int[(a_)*(F_x_), x_Symbol] := Simp[a Int[F_x, x], x] /; FreeQ[a, x] && !MatchQ[F_x, (b_)*(G_x_) /; FreeQ[b, x]]`

rule 60 `Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] := Simp[(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*(b*c - a*d)/(b*(m + n + 1)) Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] && !(IGtQ[m, 0] && (!IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) && !ILtQ[m + n + 2, 0] && IntLinearQ[a, b, c, d, m, n, x]`

rule 73

```
Int[((a_.) + (b_.)*(x_)^(m_))*((c_.) + (d_.)*(x_)^(n_)), x_Symbol] := With[
{p = Denominator[m]}, Simp[p/b Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 1102

```
Int[((d_.) + (e_.)*(x_)^(m_.))*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] := Simp[(a + b*x + c*x^2)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x)^(2*F
racPart[p])) Int[(d + e*x)^m*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c,
d, e, m, p}, x] && EqQ[b^2 - 4*a*c, 0]
```

Maple [A] (verified)

Time = 1.05 (sec) , antiderivative size = 120, normalized size of antiderivative = 0.75

| method | result |
|---------|--|
| risch | $-\frac{2(-bex+3ae-4bd)\sqrt{ex+d}\sqrt{(bx+a)^2}}{3b^2(bx+a)} + \frac{2(e^2a^2-2abde+b^2d^2)\arctan\left(\frac{b\sqrt{ex+d}}{\sqrt{b(ae-bd)}}\right)\sqrt{(bx+a)^2}}{b^2\sqrt{b(ae-bd)}(bx+a)}$ |
| default | $\frac{2(bx+a)\left((ex+d)^{\frac{3}{2}}\sqrt{b(ae-bd)}b+3\arctan\left(\frac{b\sqrt{ex+d}}{\sqrt{b(ae-bd)}}\right)e^2a^2-6\arctan\left(\frac{b\sqrt{ex+d}}{\sqrt{b(ae-bd)}}\right)abde+3\arctan\left(\frac{b\sqrt{ex+d}}{\sqrt{b(ae-bd)}}\right)b^2d^2-3\sqrt{ex+d}\right)}{3\sqrt{(bx+a)^2}b^2\sqrt{b(ae-bd)}}$ |

input

```
int((e*x+d)^(3/2)/((b*x+a)^2)^(1/2),x,method=_RETURNVERBOSE)
```

output

```
-2/3*(-b*e*x+3*a*e-4*b*d)*(e*x+d)^(1/2)/b^2*((b*x+a)^2)^(1/2)/(b*x+a)+2*(a
^2*e^2-2*a*b*d*e+b^2*d^2)/b^2/(b*(a*e-b*d))^(1/2)*arctan(b*(e*x+d)^(1/2)/(
b*(a*e-b*d))^(1/2))*((b*x+a)^2)^(1/2)/(b*x+a)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 188, normalized size of antiderivative = 1.17

$$\int \frac{(d+ex)^{3/2}}{\sqrt{a^2+2abx+b^2x^2}} dx = \left[\frac{3(bd-ae)\sqrt{\frac{bd-ae}{b}} \log\left(\frac{bex+2bd-ae+2\sqrt{ex+db}\sqrt{\frac{bd-ae}{b}}}{bx+a}\right) - 2(bex+4bd-3ae)}{3b^2} - \frac{2\left(3(bd-ae)\sqrt{-\frac{bd-ae}{b}} \arctan\left(-\frac{\sqrt{ex+db}\sqrt{-\frac{bd-ae}{b}}}{bd-ae}\right) - (bex+4bd-3ae)\sqrt{ex+d}\right)}{3b^2} \right]$$

input `integrate((e*x+d)^(3/2)/((b*x+a)^2)^(1/2),x, algorithm="fricas")`

output `[-1/3*(3*(b*d - a*e)*sqrt((b*d - a*e)/b)*log((b*e*x + 2*b*d - a*e + 2*sqrt(e*x + d)*b*sqrt((b*d - a*e)/b))/(b*x + a)) - 2*(b*e*x + 4*b*d - 3*a*e)*sqrt(e*x + d)/b^2, -2/3*(3*(b*d - a*e)*sqrt(-(b*d - a*e)/b)*arctan(-sqrt(e*x + d)*b*sqrt(-(b*d - a*e)/b)/(b*d - a*e)) - (b*e*x + 4*b*d - 3*a*e)*sqrt(e*x + d)/b^2]`

Sympy [F]

$$\int \frac{(d+ex)^{3/2}}{\sqrt{a^2+2abx+b^2x^2}} dx = \int \frac{(d+ex)^{3/2}}{\sqrt{(a+bx)^2}} dx$$

input `integrate((e*x+d)**(3/2)/((b*x+a)**2)**(1/2),x)`

output `Integral((d + e*x)**(3/2)/sqrt((a + b*x)**2), x)`

Maxima [F]

$$\int \frac{(d+ex)^{3/2}}{\sqrt{a^2+2abx+b^2x^2}} dx = \int \frac{(ex+d)^{\frac{3}{2}}}{\sqrt{(bx+a)^2}} dx$$

input `integrate((e*x+d)^(3/2)/((b*x+a)^2)^(1/2),x, algorithm="maxima")`

output `integrate((e*x + d)^(3/2)/sqrt((b*x + a)^2), x)`

Giac [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 141, normalized size of antiderivative = 0.88

$$\int \frac{(d+ex)^{3/2}}{\sqrt{a^2+2abx+b^2x^2}} dx = \frac{2(b^2d^2\operatorname{sgn}(bx+a) - 2abdes\operatorname{gn}(bx+a) + a^2e^2\operatorname{sgn}(bx+a)) \arctan\left(\frac{\sqrt{ex+db}}{\sqrt{-b^2d+abe}}\right)}{\sqrt{-b^2d+abe}b^2} + \frac{2\left((ex+d)^{\frac{3}{2}}b^2\operatorname{sgn}(bx+a) + 3\sqrt{ex+db}^2d\operatorname{sgn}(bx+a) - 3\sqrt{ex+db}e\operatorname{sgn}(bx+a)\right)}{3b^3}$$

input `integrate((e*x+d)^(3/2)/((b*x+a)^2)^(1/2),x, algorithm="giac")`

output `2*(b^2*d^2*sgn(b*x + a) - 2*a*b*d*e*sgn(b*x + a) + a^2*e^2*sgn(b*x + a))*
rctan(sqrt(e*x + d)*b/sqrt(-b^2*d + a*b*e))/(sqrt(-b^2*d + a*b*e)*b^2) + 2
/3*((e*x + d)^(3/2)*b^2*sgn(b*x + a) + 3*sqrt(e*x + d)*b^2*d*sgn(b*x + a)
- 3*sqrt(e*x + d)*a*b*e*sgn(b*x + a))/b^3`

Mupad [F(-1)]

Timed out.

$$\int \frac{(d + ex)^{3/2}}{\sqrt{a^2 + 2abx + b^2x^2}} dx = \int \frac{(d + ex)^{3/2}}{\sqrt{(a + bx)^2}} dx$$

input `int((d + e*x)^(3/2)/((a + b*x)^2)^(1/2), x)`output `int((d + e*x)^(3/2)/((a + b*x)^2)^(1/2), x)`**Reduce [B] (verification not implemented)**

Time = 0.23 (sec) , antiderivative size = 119, normalized size of antiderivative = 0.74

$$\int \frac{(d + ex)^{3/2}}{\sqrt{a^2 + 2abx + b^2x^2}} dx = \frac{2\sqrt{b}\sqrt{ae - bd} \operatorname{atan}\left(\frac{\sqrt{ex+db}}{\sqrt{b}\sqrt{ae-bd}}\right) ae - 2\sqrt{b}\sqrt{ae - bd} \operatorname{atan}\left(\frac{\sqrt{ex+db}}{\sqrt{b}\sqrt{ae-bd}}\right) bd - 2\sqrt{e}}{b^3}$$

input `int((e*x+d)^(3/2)/((b*x+a)^2)^(1/2), x)`output `(2*(3*sqrt(b)*sqrt(a*e - b*d)*atan((sqrt(d + e*x)*b)/(sqrt(b)*sqrt(a*e - b*d)))*a*e - 3*sqrt(b)*sqrt(a*e - b*d)*atan((sqrt(d + e*x)*b)/(sqrt(b)*sqrt(a*e - b*d)))*b*d - 3*sqrt(d + e*x)*a*b*e + 4*sqrt(d + e*x)*b**2*d + sqrt(d + e*x)*b**2*e*x))/(3*b**3)`

3.374 $\int \frac{\sqrt{d+ex}}{\sqrt{a^2+2abx+b^2x^2}} dx$

| | |
|---|------|
| Optimal result | 2762 |
| Mathematica [A] (verified) | 2762 |
| Rubi [A] (verified) | 2763 |
| Maple [A] (verified) | 2765 |
| Fricas [A] (verification not implemented) | 2765 |
| Sympy [F] | 2766 |
| Maxima [F] | 2766 |
| Giac [A] (verification not implemented) | 2766 |
| Mupad [F(-1)] | 2767 |
| Reduce [B] (verification not implemented) | 2767 |

Optimal result

Integrand size = 30, antiderivative size = 112

$$\int \frac{\sqrt{d+ex}}{\sqrt{a^2+2abx+b^2x^2}} dx = \frac{2(a+bx)\sqrt{d+ex}}{b\sqrt{a^2+2abx+b^2x^2}} - \frac{2\sqrt{bd-ae}(a+bx)\operatorname{arctanh}\left(\frac{\sqrt{b}\sqrt{d+ex}}{\sqrt{bd-ae}}\right)}{b^{3/2}\sqrt{a^2+2abx+b^2x^2}}$$

output $2*(b*x+a)*(e*x+d)^{(1/2)}/b/((b*x+a)^2)^{(1/2)}-2*(-a*e+b*d)^{(1/2)}*(b*x+a)*\operatorname{arc}\operatorname{tanh}(b^{(1/2)}*(e*x+d)^{(1/2)}/(-a*e+b*d)^{(1/2)})/b^{(3/2)}/((b*x+a)^2)^{(1/2)}$

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.72

$$\int \frac{\sqrt{d+ex}}{\sqrt{a^2+2abx+b^2x^2}} dx = \frac{2(a+bx)\left(\sqrt{b}\sqrt{d+ex}-\sqrt{-bd+ae}\operatorname{arctan}\left(\frac{\sqrt{b}\sqrt{d+ex}}{\sqrt{-bd+ae}}\right)\right)}{b^{3/2}\sqrt{(a+bx)^2}}$$

input `Integrate[Sqrt[d + e*x]/Sqrt[a^2 + 2*a*b*x + b^2*x^2], x]`

output $(2*(a + b*x)*(Sqrt[b]*Sqrt[d + e*x] - Sqrt[-(b*d) + a*e]*\operatorname{ArcTan}[(Sqrt[b]*Sqrt[d + e*x])/Sqrt[-(b*d) + a*e]])/(b^{(3/2)}*Sqrt[(a + b*x)^2])$

Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.79, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {1102, 27, 60, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\begin{aligned}
 & \int \frac{\sqrt{d+ex}}{\sqrt{a^2+2abx+b^2x^2}} dx \\
 & \quad \downarrow \text{1102} \\
 & \frac{b(a+bx) \int \frac{\sqrt{d+ex}}{b(a+bx)} dx}{\sqrt{a^2+2abx+b^2x^2}} \\
 & \quad \downarrow \text{27} \\
 & \frac{(a+bx) \int \frac{\sqrt{d+ex}}{a+bx} dx}{\sqrt{a^2+2abx+b^2x^2}} \\
 & \quad \downarrow \text{60} \\
 & \frac{(a+bx) \left(\frac{(bd-ae) \int \frac{1}{(a+bx)\sqrt{d+ex}} dx}{b} + \frac{2\sqrt{d+ex}}{b} \right)}{\sqrt{a^2+2abx+b^2x^2}} \\
 & \quad \downarrow \text{73} \\
 & \frac{(a+bx) \left(\frac{2(bd-ae) \int \frac{1}{a+\frac{b(d+ex)}{be} - \frac{bd}{e}} d\sqrt{d+ex}}{be} + \frac{2\sqrt{d+ex}}{b} \right)}{\sqrt{a^2+2abx+b^2x^2}} \\
 & \quad \downarrow \text{221} \\
 & \frac{(a+bx) \left(\frac{2\sqrt{d+ex}}{b} - \frac{2\sqrt{bd-ae} \operatorname{arctanh}\left(\frac{\sqrt{b}\sqrt{d+ex}}{\sqrt{bd-ae}}\right)}{b^{3/2}} \right)}{\sqrt{a^2+2abx+b^2x^2}}
 \end{aligned}$$

input

Int[Sqrt[d + e*x]/Sqrt[a^2 + 2*a*b*x + b^2*x^2], x]

output
$$\frac{((a + b*x)*((2*\sqrt{d + e*x})/b - (2*\sqrt{b*d - a*e})*\text{ArcTanh}[(\sqrt{b})*\sqrt{d + e*x}]/\sqrt{b*d - a*e}))/b^{(3/2))}}{\sqrt{a^2 + 2*a*b*x + b^2*x^2}}$$

Defintions of rubi rules used

rule 27
$$\text{Int}[(a_*)(F_x_), x_Symbol] \rightarrow \text{Simp}[a \text{ Int}[F_x, x], x] /; \text{FreeQ}[a, x] \ \&\& \ !\text{MatchQ}[F_x, (b_*)(G_x_)] /; \text{FreeQ}[b, x]$$

rule 60
$$\text{Int}[(a_.) + (b_.)*(x_)^{(m_.)}*((c_.) + (d_.)*(x_)^{(n_.)}), x_Symbol] \rightarrow \text{Simp}[(a + b*x)^{(m + 1)}*((c + d*x)^n/(b*(m + n + 1))), x] + \text{Simp}[n*((b*c - a*d)/(b*(m + n + 1)) \text{ Int}[(a + b*x)^m*(c + d*x)^{(n - 1)}, x], x] /; \text{FreeQ}[\{a, b, c, d\}, x] \ \&\& \ \text{GtQ}[n, 0] \ \&\& \ \text{NeQ}[m + n + 1, 0] \ \&\& \ !(\text{IGtQ}[m, 0] \ \&\& \ (!\text{IntegerQ}[n] \ || \ (\text{GtQ}[m, 0] \ \&\& \ \text{LtQ}[m - n, 0])) \ \&\& \ !\text{ILtQ}[m + n + 2, 0] \ \&\& \ \text{IntLinearQ}[a, b, c, d, m, n, x]$$

rule 73
$$\text{Int}[(a_.) + (b_.)*(x_)^{(m_.)}*((c_.) + (d_.)*(x_)^{(n_.)}), x_Symbol] \rightarrow \text{With}[\{p = \text{Denominator}[m]\}, \text{Simp}[p/b \ \text{Subst}[\text{Int}[x^{(p*(m + 1) - 1)}*(c - a*(d/b) + d*(x^p/b)^n], x], x, (a + b*x)^{(1/p)}, x]] /; \text{FreeQ}[\{a, b, c, d\}, x] \ \&\& \ \text{LtQ}[-1, m, 0] \ \&\& \ \text{LeQ}[-1, n, 0] \ \&\& \ \text{LeQ}[\text{Denominator}[n], \text{Denominator}[m]] \ \&\& \ \text{IntLinearQ}[a, b, c, d, m, n, x]$$

rule 221
$$\text{Int}[(a_) + (b_.)*(x_)^2)^{-1}, x_Symbol] \rightarrow \text{Simp}[(\text{Rt}[-a/b, 2]/a)*\text{ArcTanh}[x/\text{Rt}[-a/b, 2]], x] /; \text{FreeQ}[\{a, b\}, x] \ \&\& \ \text{NegQ}[a/b]$$

rule 1102
$$\text{Int}[(d_.) + (e_.)*(x_)^{(m_.)}*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^{(p_.)}, x_Symbol] \rightarrow \text{Simp}[(a + b*x + c*x^2)^{\text{FracPart}[p]}/(c^{\text{IntPart}[p]}*(b/2 + c*x)^{(2*\text{FracPart}[p]}) \ \text{Int}[(d + e*x)^m*(b/2 + c*x)^{(2*p)}, x], x] /; \text{FreeQ}[\{a, b, c, d, e, m, p\}, x] \ \&\& \ \text{EqQ}[b^2 - 4*a*c, 0]$$

Maple [A] (verified)

Time = 1.06 (sec) , antiderivative size = 93, normalized size of antiderivative = 0.83

| method | result | size |
|---------|---|------|
| risch | $\frac{2\sqrt{ex+d}\sqrt{(bx+a)^2}}{b(bx+a)} - \frac{2(ae-bd)\arctan\left(\frac{b\sqrt{ex+d}}{\sqrt{b(ae-bd)}}\right)\sqrt{(bx+a)^2}}{b\sqrt{b(ae-bd)}(bx+a)}$ | 93 |
| default | $\frac{2(bx+a)\left(-\arctan\left(\frac{b\sqrt{ex+d}}{\sqrt{b(ae-bd)}}\right)ae+\arctan\left(\frac{b\sqrt{ex+d}}{\sqrt{b(ae-bd)}}\right)bd+\sqrt{ex+d}\sqrt{b(ae-bd)}\right)}{\sqrt{(bx+a)^2}b\sqrt{b(ae-bd)}}$ | 104 |

input `int((e*x+d)^(1/2)/((b*x+a)^2)^(1/2),x,method=_RETURNVERBOSE)`

output `2/b*(e*x+d)^(1/2)*((b*x+a)^2)^(1/2)/(b*x+a)-2*(a*e-b*d)/b/(b*(a*e-b*d))^(1/2)*arctan(b*(e*x+d)^(1/2)/(b*(a*e-b*d))^(1/2))*((b*x+a)^2)^(1/2)/(b*x+a)`

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 143, normalized size of antiderivative = 1.28

$$\int \frac{\sqrt{d+ex}}{\sqrt{a^2+2abx+b^2x^2}} dx = \left[\frac{\sqrt{\frac{bd-ae}{b}} \log\left(\frac{bex+2bd-ae-2\sqrt{ex+db}\sqrt{\frac{bd-ae}{b}}}{bx+a}\right) + 2\sqrt{ex+d}}{b}, \right. \\ \left. - \frac{2\left(\sqrt{-\frac{bd-ae}{b}} \arctan\left(-\frac{\sqrt{ex+db}\sqrt{-\frac{bd-ae}{b}}}{bd-ae}\right) - \sqrt{ex+d}\right)}{b} \right]$$

input `integrate((e*x+d)^(1/2)/((b*x+a)^2)^(1/2),x, algorithm="fricas")`

output `[(sqrt((b*d - a*e)/b)*log((b*e*x + 2*b*d - a*e - 2*sqrt(e*x + d)*b*sqrt((b*d - a*e)/b))/(b*x + a)) + 2*sqrt(e*x + d)/b, -2*(sqrt(-(b*d - a*e)/b)*arctan(-sqrt(e*x + d)*b*sqrt(-(b*d - a*e)/b)/(b*d - a*e)) - sqrt(e*x + d)]/b]`

Sympy [F]

$$\int \frac{\sqrt{d+ex}}{\sqrt{a^2+2abx+b^2x^2}} dx = \int \frac{\sqrt{d+ex}}{\sqrt{(a+bx)^2}} dx$$

input `integrate((e*x+d)**(1/2)/((b*x+a)**2)**(1/2), x)`

output `Integral(sqrt(d + e*x)/sqrt((a + b*x)**2), x)`

Maxima [F]

$$\int \frac{\sqrt{d+ex}}{\sqrt{a^2+2abx+b^2x^2}} dx = \int \frac{\sqrt{ex+d}}{\sqrt{(bx+a)^2}} dx$$

input `integrate((e*x+d)^(1/2)/((b*x+a)^2)^(1/2), x, algorithm="maxima")`

output `integrate(sqrt(e*x + d)/sqrt((b*x + a)^2), x)`

Giac [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.71

$$\int \frac{\sqrt{d+ex}}{\sqrt{a^2+2abx+b^2x^2}} dx = \frac{2(b\operatorname{sgn}(bx+a) - a\operatorname{sgn}(bx+a)) \arctan\left(\frac{\sqrt{ex+db}}{\sqrt{-b^2d+abe}}\right)}{\sqrt{-b^2d+abe}b} + \frac{2\sqrt{ex+d}\operatorname{sgn}(bx+a)}{b}$$

input `integrate((e*x+d)^(1/2)/((b*x+a)^2)^(1/2), x, algorithm="giac")`

output `2*(b*d*sgn(b*x + a) - a*e*sgn(b*x + a))*arctan(sqrt(e*x + d)*b/sqrt(-b^2*d + a*b*e))/(sqrt(-b^2*d + a*b*e)*b) + 2*sqrt(e*x + d)*sgn(b*x + a)/b`

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{d+ex}}{\sqrt{a^2+2abx+b^2x^2}} dx = \int \frac{\sqrt{d+ex}}{\sqrt{(a+bx)^2}} dx$$

input `int((d + e*x)^(1/2)/((a + b*x)^2)^(1/2), x)`output `int((d + e*x)^(1/2)/((a + b*x)^2)^(1/2), x)`**Reduce [B] (verification not implemented)**

Time = 0.23 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.46

$$\int \frac{\sqrt{d+ex}}{\sqrt{a^2+2abx+b^2x^2}} dx = \frac{-2\sqrt{b}\sqrt{ae-bd} \operatorname{atan}\left(\frac{\sqrt{ex+db}}{\sqrt{b}\sqrt{ae-bd}}\right) + 2\sqrt{ex+db}}{b^2}$$

input `int((e*x+d)^(1/2)/((b*x+a)^2)^(1/2), x)`output `(2*(- sqrt(b)*sqrt(a*e - b*d)*atan((sqrt(d + e*x)*b)/(sqrt(b)*sqrt(a*e - b*d))) + sqrt(d + e*x)*b))/b**2`

3.375 $\int \frac{1}{\sqrt{d+ex}\sqrt{a^2+2abx+b^2x^2}} dx$

| | |
|---|------|
| Optimal result | 2768 |
| Mathematica [A] (verified) | 2768 |
| Rubi [A] (verified) | 2769 |
| Maple [A] (verified) | 2770 |
| Fricas [A] (verification not implemented) | 2771 |
| Sympy [F] | 2771 |
| Maxima [F] | 2771 |
| Giac [A] (verification not implemented) | 2772 |
| Mupad [F(-1)] | 2772 |
| Reduce [B] (verification not implemented) | 2773 |

Optimal result

Integrand size = 30, antiderivative size = 72

$$\int \frac{1}{\sqrt{d+ex}\sqrt{a^2+2abx+b^2x^2}} dx = -\frac{2(a+bx)\operatorname{arctanh}\left(\frac{\sqrt{b}\sqrt{d+ex}}{\sqrt{bd-ae}}\right)}{\sqrt{b}\sqrt{bd-ae}\sqrt{a^2+2abx+b^2x^2}}$$

output `-2*(b*x+a)*arctanh(b^(1/2)*(e*x+d)^(1/2)/(-a*e+b*d)^(1/2))/b^(1/2)/(-a*e+b*d)^(1/2)/((b*x+a)^2)^(1/2)`

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.88

$$\int \frac{1}{\sqrt{d+ex}\sqrt{a^2+2abx+b^2x^2}} dx = \frac{2(a+bx)\arctan\left(\frac{\sqrt{b}\sqrt{d+ex}}{\sqrt{-bd+ae}}\right)}{\sqrt{b}\sqrt{-bd+ae}\sqrt{(a+bx)^2}}$$

input `Integrate[1/(Sqrt[d + e*x]*Sqrt[a^2 + 2*a*b*x + b^2*x^2]), x]`

output `(2*(a + b*x)*ArcTan[(Sqrt[b]*Sqrt[d + e*x])/Sqrt[-(b*d) + a*e]])/(Sqrt[b]*Sqrt[-(b*d) + a*e]*Sqrt[(a + b*x)^2])`

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {1102, 27, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\begin{aligned}
 & \int \frac{1}{\sqrt{a^2 + 2abx + b^2x^2}\sqrt{d + ex}} dx \\
 & \quad \downarrow \text{1102} \\
 & \frac{b(a + bx) \int \frac{1}{b(a+bx)\sqrt{d+ex}} dx}{\sqrt{a^2 + 2abx + b^2x^2}} \\
 & \quad \downarrow \text{27} \\
 & \frac{(a + bx) \int \frac{1}{(a+bx)\sqrt{d+ex}} dx}{\sqrt{a^2 + 2abx + b^2x^2}} \\
 & \quad \downarrow \text{73} \\
 & \frac{2(a + bx) \int \frac{1}{a + \frac{b(d+ex)}{e} - \frac{bd}{e}} d\sqrt{d + ex}}{e\sqrt{a^2 + 2abx + b^2x^2}} \\
 & \quad \downarrow \text{221} \\
 & \frac{2(a + bx) \operatorname{arctanh}\left(\frac{\sqrt{b}\sqrt{d+ex}}{\sqrt{bd-ae}}\right)}{\sqrt{b}\sqrt{a^2 + 2abx + b^2x^2}\sqrt{bd - ae}}
 \end{aligned}$$

input `Int[1/(Sqrt[d + e*x]*Sqrt[a^2 + 2*a*b*x + b^2*x^2]),x]`

output `(-2*(a + b*x)*ArcTanh[(Sqrt[b]*Sqrt[d + e*x])/Sqrt[b*d - a*e]])/(Sqrt[b]*Sqrt[b*d - a*e]*Sqrt[a^2 + 2*a*b*x + b^2*x^2])`

Definitions of rubi rules used

rule 27 `Int[(a_)*(F_x_), x_Symbol] := Simp[a Int[F_x, x], x] /; FreeQ[a, x] && !MatchQ[F_x, (b_)*(G_x_) /; FreeQ[b, x]]`

rule 73 `Int[((a_.) + (b_.)*(x_)^(m_))*((c_.) + (d_.)*(x_)^(n_)), x_Symbol] := With[{p = Denominator[m]}, Simp[p/b Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) + d*(x^p/b))^(n), x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntLinearQ[a, b, c, d, m, n, x]`

rule 221 `Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[-a/b, 2]/a)*ArcTanh[x/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]`

rule 1102 `Int[((d_.) + (e_.)*(x_)^(m_.))*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] := Simp[(a + b*x + c*x^2)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x)^(2*FracPart[p])) Int[(d + e*x)^m*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && EqQ[b^2 - 4*a*c, 0]`

Maple [A] (verified)

Time = 1.04 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.71

| method | result | size |
|---------|--|------|
| default | $\frac{2(bx+a) \arctan\left(\frac{b\sqrt{ex+d}}{\sqrt{b(ae-bd)}}\right)}{\sqrt{(bx+a)^2} \sqrt{b(ae-bd)}}$ | 51 |

input `int(1/(e*x+d)^(1/2)/((b*x+a)^2)^(1/2), x, method=_RETURNVERBOSE)`

output `2/((b*x+a)^2)^(1/2)*(b*x+a)/(b*(a*e-b*d))^(1/2)*arctan(b*(e*x+d)^(1/2)/(b*(a*e-b*d))^(1/2))`

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.65

$$\int \frac{1}{\sqrt{d+ex}\sqrt{a^2+2abx+b^2x^2}} dx$$

$$= \left[\frac{\log\left(\frac{bex+2bd-ae-2\sqrt{b^2d-abe}\sqrt{ex+d}}{bx+a}\right)}{\sqrt{b^2d-abe}}, \frac{2\sqrt{-b^2d+abe}\arctan\left(\frac{\sqrt{-b^2d+abe}\sqrt{ex+d}}{bex+bd}\right)}{b^2d-abe} \right]$$

input `integrate(1/(e*x+d)^(1/2)/((b*x+a)^2)^(1/2),x, algorithm="fricas")`

output `[log((b*e*x + 2*b*d - a*e - 2*sqrt(b^2*d - a*b*e)*sqrt(e*x + d))/(b*x + a))/sqrt(b^2*d - a*b*e), 2*sqrt(-b^2*d + a*b*e)*arctan(sqrt(-b^2*d + a*b*e)*sqrt(e*x + d)/(b*e*x + b*d))/(b^2*d - a*b*e)]`

Sympy [F]

$$\int \frac{1}{\sqrt{d+ex}\sqrt{a^2+2abx+b^2x^2}} dx = \int \frac{1}{\sqrt{d+ex}\sqrt{(a+bx)^2}} dx$$

input `integrate(1/(e*x+d)**(1/2)/((b*x+a)**2)**(1/2),x)`

output `Integral(1/(sqrt(d + e*x)*sqrt((a + b*x)**2)), x)`

Maxima [F]

$$\int \frac{1}{\sqrt{d+ex}\sqrt{a^2+2abx+b^2x^2}} dx = \int \frac{1}{\sqrt{(bx+a)^2}\sqrt{ex+d}} dx$$

input `integrate(1/(e*x+d)^(1/2)/((b*x+a)^2)^(1/2),x, algorithm="maxima")`

output `integrate(1/(sqrt((b*x + a)^2)*sqrt(e*x + d)), x)`

Giac [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.61

$$\int \frac{1}{\sqrt{d + ex}\sqrt{a^2 + 2abx + b^2x^2}} dx = \frac{2 \arctan\left(\frac{\sqrt{ex+db}}{\sqrt{-b^2d+abe}}\right) \operatorname{sgn}(bx + a)}{\sqrt{-b^2d + abe}}$$

input `integrate(1/(e*x+d)^(1/2)/((b*x+a)^2)^(1/2),x, algorithm="giac")`

output `2*arctan(sqrt(e*x + d)*b/sqrt(-b^2*d + a*b*e))*sgn(b*x + a)/sqrt(-b^2*d + a*b*e)`

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{d + ex}\sqrt{a^2 + 2abx + b^2x^2}} dx = \int \frac{1}{\sqrt{(a + bx)^2} \sqrt{d + ex}} dx$$

input `int(1/(((a + b*x)^2)^(1/2)*(d + e*x)^(1/2)),x)`

output `int(1/(((a + b*x)^2)^(1/2)*(d + e*x)^(1/2)), x)`

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.69

$$\int \frac{1}{\sqrt{d+ex}\sqrt{a^2+2abx+b^2x^2}} dx = \frac{2\sqrt{b}\sqrt{ae-bd} \operatorname{atan}\left(\frac{\sqrt{ex+d}b}{\sqrt{b}\sqrt{ae-bd}}\right)}{b(ae-bd)}$$

input `int(1/(e*x+d)^(1/2)/((b*x+a)^2)^(1/2),x)`

output `(2*sqrt(b)*sqrt(a*e - b*d)*atan((sqrt(d + e*x)*b)/(sqrt(b)*sqrt(a*e - b*d)))/b*(a*e - b*d))`

3.376 $\int \frac{1}{(d+ex)^{3/2}\sqrt{a^2+2abx+b^2x^2}} dx$

| | |
|---|------|
| Optimal result | 2774 |
| Mathematica [A] (verified) | 2774 |
| Rubi [A] (verified) | 2775 |
| Maple [A] (verified) | 2777 |
| Fricas [A] (verification not implemented) | 2777 |
| Sympy [F] | 2778 |
| Maxima [F] | 2778 |
| Giac [A] (verification not implemented) | 2779 |
| Mupad [F(-1)] | 2779 |
| Reduce [B] (verification not implemented) | 2779 |

Optimal result

Integrand size = 30, antiderivative size = 119

$$\int \frac{1}{(d+ex)^{3/2}\sqrt{a^2+2abx+b^2x^2}} dx = \frac{2(a+bx)}{(bd-ae)\sqrt{d+ex}\sqrt{a^2+2abx+b^2x^2}} - \frac{2\sqrt{b}(a+bx)\operatorname{arctanh}\left(\frac{\sqrt{b}\sqrt{d+ex}}{\sqrt{bd-ae}}\right)}{(bd-ae)^{3/2}\sqrt{a^2+2abx+b^2x^2}}$$

output

```
2*(b*x+a)/(-a*e+b*d)/(e*x+d)^(1/2)/((b*x+a)^2)^(1/2)-2*b^(1/2)*(b*x+a)*arc
tanh(b^(1/2)*(e*x+d)^(1/2)/(-a*e+b*d)^(1/2))/(-a*e+b*d)^(3/2)/((b*x+a)^2)^(
1/2)
```

Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.80

$$\int \frac{1}{(d+ex)^{3/2}\sqrt{a^2+2abx+b^2x^2}} dx = \frac{2(a+bx)\left(\sqrt{-bd+ae} + \sqrt{b}\sqrt{d+ex} \arctan\left(\frac{\sqrt{b}\sqrt{d+ex}}{\sqrt{-bd+ae}}\right)\right)}{(-bd+ae)^{3/2}\sqrt{(a+bx)^2\sqrt{d+ex}}}$$

input `Integrate[1/((d + e*x)^(3/2)*Sqrt[a^2 + 2*a*b*x + b^2*x^2]),x]`

output `(-2*(a + b*x)*(Sqrt[-(b*d) + a*e] + Sqrt[b]*Sqrt[d + e*x]*ArcTan[(Sqrt[b]*Sqrt[d + e*x])/Sqrt[-(b*d) + a*e]]))/((-b*d) + a*e)^(3/2)*Sqrt[(a + b*x)^2]*Sqrt[d + e*x]`

Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.80, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {1102, 27, 61, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\begin{aligned}
 & \int \frac{1}{\sqrt{a^2 + 2abx + b^2x^2}(d + ex)^{3/2}} dx \\
 & \quad \downarrow \text{1102} \\
 & \frac{b(a + bx) \int \frac{1}{b(a+bx)(d+ex)^{3/2}} dx}{\sqrt{a^2 + 2abx + b^2x^2}} \\
 & \quad \downarrow \text{27} \\
 & \frac{(a + bx) \int \frac{1}{(a+bx)(d+ex)^{3/2}} dx}{\sqrt{a^2 + 2abx + b^2x^2}} \\
 & \quad \downarrow \text{61} \\
 & \frac{(a + bx) \left(\frac{b \int \frac{1}{(a+bx)\sqrt{d+ex}} dx}{bd-ae} + \frac{2}{\sqrt{d+ex}(bd-ae)} \right)}{\sqrt{a^2 + 2abx + b^2x^2}} \\
 & \quad \downarrow \text{73} \\
 & \frac{(a + bx) \left(\frac{2b \int \frac{1}{a + \frac{b(d+ex)}{e} - \frac{bd}{e}} d\sqrt{d+ex}}{e(bd-ae)} + \frac{2}{\sqrt{d+ex}(bd-ae)} \right)}{\sqrt{a^2 + 2abx + b^2x^2}} \\
 & \quad \downarrow \text{221}
 \end{aligned}$$

$$\frac{(a + bx) \left(\frac{2}{\sqrt{d+ex}(bd-ae)} - \frac{2\sqrt{b}\operatorname{arctanh}\left(\frac{\sqrt{b}\sqrt{d+ex}}{\sqrt{bd-ae}}\right)}{(bd-ae)^{3/2}} \right)}{\sqrt{a^2 + 2abx + b^2x^2}}$$

input `Int[1/((d + e*x)^(3/2)*Sqrt[a^2 + 2*a*b*x + b^2*x^2]),x]`

output `((a + b*x)*(2/((b*d - a*e)*Sqrt[d + e*x]) - (2*Sqrt[b]*ArcTanh[(Sqrt[b]*Sqrt[d + e*x])/Sqrt[b*d - a*e]])/(b*d - a*e)^(3/2)))/Sqrt[a^2 + 2*a*b*x + b^2*x^2]`

Defintions of rubi rules used

rule 27 `Int[(a_)*(F_x_), x_Symbol] := Simp[a Int[F_x, x], x] /; FreeQ[a, x] && !MatchQ[F_x, (b_)*(G_x_) /; FreeQ[b, x]]`

rule 61 `Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] := Simp[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((m + n + 2)/((b*c - a*d)*(m + 1))) Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && LtQ[m, -1] && !(LtQ[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && IntLinearQ[a, b, c, d, m, n, x]`

rule 73 `Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] := With[{p = Denominator[m]}, Simp[p/b Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) + d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntLinearQ[a, b, c, d, m, n, x]`

rule 221 `Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[-a/b, 2]/a)*ArcTanh[x/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]`

rule 1102

```
Int[((d_.) + (e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(a + b*x + c*x^2)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x)^(2*F
racPart[p])) Int[(d + e*x)^m*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c,
d, e, m, p}, x] && EqQ[b^2 - 4*a*c, 0]
```

Maple [A] (verified)

Time = 1.06 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.76

| method | result | size |
|---------|---|------|
| default | $-\frac{2(bx+a) \left(b \arctan \left(\frac{b\sqrt{ex+d}}{\sqrt{b(ae-bd)}} \right) \sqrt{ex+d} + \sqrt{b(ae-bd)} \right)}{\sqrt{(bx+a)^2 (ae-bd)} \sqrt{ex+d} \sqrt{b(ae-bd)}}$ | 90 |

input

```
int(1/(e*x+d)^(3/2)/((b*x+a)^2)^(1/2),x,method=_RETURNVERBOSE)
```

output

```
-2*(b*x+a)*(b*arctan(b*(e*x+d)^(1/2)/(b*(a*e-b*d))^(1/2))*(e*x+d)^(1/2)+(b
*(a*e-b*d)^(1/2))/((b*x+a)^2)^(1/2)/(a*e-b*d)/(e*x+d)^(1/2)/(b*(a*e-b*d)
^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 193, normalized size of antiderivative = 1.62

$$\int \frac{1}{(d+ex)^{3/2} \sqrt{a^2+2abx+b^2x^2}} dx = \left[\frac{(ex+d) \sqrt{\frac{b}{bd-ae}} \log \left(\frac{bex+2bd-ae+2(bd-ae)\sqrt{ex+d}\sqrt{\frac{b}{bd-ae}}}{bx+a} \right) - 2\sqrt{ex+d}}{bd^2 - ade + (bde - ae^2)x} \right]$$

input

```
integrate(1/(e*x+d)^(3/2)/((b*x+a)^2)^(1/2),x, algorithm="fricas")
```

output

```
[-((e*x + d)*sqrt(b/(b*d - a*e))*log((b*e*x + 2*b*d - a*e + 2*(b*d - a*e)*
sqrt(e*x + d)*sqrt(b/(b*d - a*e)))/(b*x + a)) - 2*sqrt(e*x + d))/(b*d^2 -
a*d*e + (b*d*e - a*e^2)*x), 2*((e*x + d)*sqrt(-b/(b*d - a*e))*arctan(sqrt(
e*x + d)*sqrt(-b/(b*d - a*e))) + sqrt(e*x + d))/(b*d^2 - a*d*e + (b*d*e -
a*e^2)*x)]
```

Sympy [F]

$$\int \frac{1}{(d + ex)^{3/2} \sqrt{a^2 + 2abx + b^2x^2}} dx = \int \frac{1}{(d + ex)^{3/2} \sqrt{(a + bx)^2}} dx$$

input

```
integrate(1/(e*x+d)**(3/2)/((b*x+a)**2)**(1/2),x)
```

output

```
Integral(1/((d + e*x)**(3/2)*sqrt((a + b*x)**2)), x)
```

Maxima [F]

$$\int \frac{1}{(d + ex)^{3/2} \sqrt{a^2 + 2abx + b^2x^2}} dx = \int \frac{1}{\sqrt{(bx + a)^2} (ex + d)^{3/2}} dx$$

input

```
integrate(1/(e*x+d)^(3/2)/((b*x+a)^2)^(1/2),x, algorithm="maxima")
```

output

```
integrate(1/(sqrt((b*x + a)^2)*(e*x + d)^(3/2)), x)
```

Giac [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.63

$$\int \frac{1}{(d+ex)^{3/2} \sqrt{a^2+2abx+b^2x^2}} dx = 2 \left(\frac{b \arctan\left(\frac{\sqrt{ex+d}}{\sqrt{-b^2d+abe}}\right)}{\sqrt{-b^2d+abe}(bd-ae)} + \frac{1}{(bd-ae)\sqrt{ex+d}} \right) \operatorname{sgn}(bx+a)$$

input `integrate(1/(e*x+d)^(3/2)/((b*x+a)^(1/2)),x, algorithm="giac")`output `2*(b*arctan(sqrt(e*x + d)*b/sqrt(-b^2*d + a*b*e))/(sqrt(-b^2*d + a*b*e)*(b*d - a*e)) + 1/((b*d - a*e)*sqrt(e*x + d)))*sgn(b*x + a)`**Mupad [F(-1)]**

Timed out.

$$\int \frac{1}{(d+ex)^{3/2} \sqrt{a^2+2abx+b^2x^2}} dx = \int \frac{1}{\sqrt{(a+bx)^2} (d+ex)^{3/2}} dx$$

input `int(1/(((a + b*x)^2)^(1/2)*(d + e*x)^(3/2)),x)`output `int(1/(((a + b*x)^2)^(1/2)*(d + e*x)^(3/2)), x)`**Reduce [B] (verification not implemented)**

Time = 0.20 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.71

$$\int \frac{1}{(d+ex)^{3/2} \sqrt{a^2+2abx+b^2x^2}} dx = \frac{-2\sqrt{b} \sqrt{ex+d} \sqrt{ae-bd} \operatorname{atan}\left(\frac{\sqrt{ex+d}}{\sqrt{b}\sqrt{ae-bd}}\right) - 2ae + 2bd}{\sqrt{ex+d} (a^2e^2 - 2abde + b^2d^2)}$$

input `int(1/(e*x+d)^(3/2)/((b*x+a)^(1/2)),x)`

output

```
(2*( - sqrt(b)*sqrt(d + e*x)*sqrt(a*e - b*d)*atan((sqrt(d + e*x)*b)/(sqrt(b)*sqrt(a*e - b*d))) - a*e + b*d))/(sqrt(d + e*x)*(a**2*e**2 - 2*a*b*d*e + b**2*d**2))
```

3.377 $\int \frac{1}{(d+ex)^{5/2}\sqrt{a^2+2abx+b^2x^2}} dx$

| | |
|---|------|
| Optimal result | 2781 |
| Mathematica [A] (verified) | 2781 |
| Rubi [A] (verified) | 2782 |
| Maple [A] (verified) | 2784 |
| Fricas [A] (verification not implemented) | 2785 |
| Sympy [F] | 2785 |
| Maxima [F] | 2786 |
| Giac [A] (verification not implemented) | 2786 |
| Mupad [F(-1)] | 2786 |
| Reduce [B] (verification not implemented) | 2787 |

Optimal result

Integrand size = 30, antiderivative size = 168

$$\int \frac{1}{(d+ex)^{5/2}\sqrt{a^2+2abx+b^2x^2}} dx = \frac{2(a+bx)}{3(bd-ae)(d+ex)^{3/2}\sqrt{a^2+2abx+b^2x^2}} + \frac{2b(a+bx)}{(bd-ae)^2\sqrt{d+ex}\sqrt{a^2+2abx+b^2x^2}} - \frac{2b^{3/2}(a+bx)\operatorname{arctanh}\left(\frac{\sqrt{b}\sqrt{d+ex}}{\sqrt{bd-ae}}\right)}{(bd-ae)^{5/2}\sqrt{a^2+2abx+b^2x^2}}$$

output

```
2/3*(b*x+a)/(-a*e+b*d)/(e*x+d)^(3/2)/((b*x+a)^2)^(1/2)+2*b*(b*x+a)/(-a*e+b*d)^2/(e*x+d)^(1/2)/((b*x+a)^2)^(1/2)-2*b^(3/2)*(b*x+a)*arctanh(b^(1/2)*(e*x+d)^(1/2)/(-a*e+b*d)^(1/2))/(-a*e+b*d)^(5/2)/((b*x+a)^2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.19 (sec) , antiderivative size = 113, normalized size of antiderivative = 0.67

$$\int \frac{1}{(d+ex)^{5/2}\sqrt{a^2+2abx+b^2x^2}} dx = \frac{2(a+bx)\left(\sqrt{-bd+ae}(4bd-ae+3bex)+3b^{3/2}(d+ex)^{3/2}\operatorname{arctan}\frac{b\sqrt{d+ex}}{\sqrt{-bd+ae}}\right)}{3(-bd+ae)^{5/2}\sqrt{(a+bx)^2(d+ex)^3}}$$

input

```
Integrate[1/((d+e*x)^(5/2)*Sqrt[a^2+2*a*b*x+b^2*x^2]),x]
```

output

$$(2*(a + b*x)*(Sqrt[-(b*d) + a*e]*(4*b*d - a*e + 3*b*e*x) + 3*b^(3/2)*(d + e*x)^(3/2)*ArcTan[(Sqrt[b]*Sqrt[d + e*x])/Sqrt[-(b*d) + a*e]]))/(3*(-(b*d) + a*e)^(5/2)*Sqrt[(a + b*x)^2]*(d + e*x)^(3/2))$$
Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 131, normalized size of antiderivative = 0.78, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {1102, 27, 61, 61, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{a^2 + 2abx + b^2x^2}(d + ex)^{5/2}} dx$$

$$\downarrow 1102$$

$$\frac{b(a + bx) \int \frac{1}{b(a+bx)(d+ex)^{5/2}} dx}{\sqrt{a^2 + 2abx + b^2x^2}}$$

$$\downarrow 27$$

$$\frac{(a + bx) \int \frac{1}{(a+bx)(d+ex)^{5/2}} dx}{\sqrt{a^2 + 2abx + b^2x^2}}$$

$$\downarrow 61$$

$$\frac{(a + bx) \left(\frac{b \int \frac{1}{(a+bx)(d+ex)^{3/2}} dx}{bd-ae} + \frac{2}{3(d+ex)^{3/2}(bd-ae)} \right)}{\sqrt{a^2 + 2abx + b^2x^2}}$$

$$\downarrow 61$$

$$\frac{(a + bx) \left(\frac{b \left(\frac{b \int \frac{1}{(a+bx)\sqrt{d+ex}} dx}{bd-ae} + \frac{2}{\sqrt{d+ex}(bd-ae)} \right)}{bd-ae} + \frac{2}{3(d+ex)^{3/2}(bd-ae)} \right)}{\sqrt{a^2 + 2abx + b^2x^2}}$$

$$\downarrow 73$$

$$\frac{(a + bx) \left(\frac{b \left(\frac{2b \int \frac{1}{a + \frac{b(d+ex) - bd}{e(bd-ae)}} d\sqrt{d+ex}}{\sqrt{d+ex}(bd-ae)} + \frac{2}{\sqrt{d+ex}(bd-ae)} \right)}{bd-ae} + \frac{2}{3(d+ex)^{3/2}(bd-ae)} \right)}{\sqrt{a^2 + 2abx + b^2x^2}}$$

↓ 221

$$\frac{(a + bx) \left(\frac{b \left(\frac{2}{\sqrt{d+ex}(bd-ae)} - \frac{2\sqrt{b} \operatorname{arctanh}\left(\frac{\sqrt{b}\sqrt{d+ex}}{\sqrt{bd-ae}}\right)}{(bd-ae)^{3/2}} \right)}{bd-ae} + \frac{2}{3(d+ex)^{3/2}(bd-ae)} \right)}{\sqrt{a^2 + 2abx + b^2x^2}}$$

input `Int[1/((d + e*x)^(5/2)*Sqrt[a^2 + 2*a*b*x + b^2*x^2]),x]`

output `((a + b*x)*(2/(3*(b*d - a*e)*(d + e*x)^(3/2)) + (b*(2/((b*d - a*e)*Sqrt[d + e*x]) - (2*Sqrt[b]*ArcTanh[(Sqrt[b]*Sqrt[d + e*x])/Sqrt[b*d - a*e]])/(b*d - a*e)^(3/2)))/(b*d - a*e))/Sqrt[a^2 + 2*a*b*x + b^2*x^2]`

Defintions of rubi rules used

rule 27 `Int[(a_)*(F_x_), x_Symbol] := Simp[a Int[F_x, x], x] /; FreeQ[a, x] && !MatchQ[F_x, (b_)*(G_x_) /; FreeQ[b, x]]`

rule 61 `Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] := Simp[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((m + n + 2)/((b*c - a*d)*(m + 1)) Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && LtQ[m, -1] && !(LtQ[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && IntLinearQ[a, b, c, d, m, n, x]`

rule 73 `Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] := With[
 {p = Denominator[m]}, Simp[p/b Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x] /; FreeQ[{a, b, c, d}, x] && Lt
 Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
 inearQ[a, b, c, d, m, n, x]`

rule 221 `Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
 /Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]`

rule 1102 `Int[((d_.) + (e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
 ymbol] := Simp[(a + b*x + c*x^2)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x)^(2*F
 racPart[p])) Int[(d + e*x)^m*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c,
 d, e, m, p}, x] && EqQ[b^2 - 4*a*c, 0]`

Maple [A] (verified)

Time = 1.10 (sec) , antiderivative size = 130, normalized size of antiderivative = 0.77

| method | result | size |
|---------|---|------|
| default | $\frac{2(bx+a) \left(3b^2 \arctan\left(\frac{b\sqrt{ex+d}}{\sqrt{b(ae-bd)}}\right) (ex+d)^{\frac{3}{2}} + 3\sqrt{b(ae-bd)} bex - \sqrt{b(ae-bd)} ae + 4\sqrt{b(ae-bd)} bd \right)}{3\sqrt{(bx+a)^2 (ae-bd)^2 \sqrt{b(ae-bd)}} (ex+d)^{\frac{3}{2}}}$ | 130 |

input `int(1/(e*x+d)^(5/2)/((b*x+a)^2)^(1/2),x,method=_RETURNVERBOSE)`

output
$$\frac{2/3*(b*x+a)*(3*b^2*\arctan(b*(e*x+d)^(1/2)/(b*(a*e-b*d))^(1/2))*(e*x+d)^(3/2)+3*(b*(a*e-b*d))^(1/2)*b*e*x-(b*(a*e-b*d))^(1/2)*a*e+4*(b*(a*e-b*d))^(1/2)*b*d}{((b*x+a)^2)^(1/2)/(a*e-b*d)^2/(b*(a*e-b*d))^(1/2)/(e*x+d)^(3/2)}$$

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 378, normalized size of antiderivative = 2.25

$$\int \frac{1}{(d+ex)^{5/2} \sqrt{a^2+2abx+b^2x^2}} dx = \left[\frac{3 (be^2x^2 + 2bdex + bd^2) \sqrt{\frac{b}{bd-ae}} \log \left(\frac{be^2x^2 + 2bdex + bd^2}{bx+a} \right)}{3 (b^2d^4 - 2abd^3e + a^2d^2e^2 + (b^2d^2e^2 - 2abde^3 + a^2e^4)x^2 + \dots} \right]$$

input `integrate(1/(e*x+d)^(5/2)/((b*x+a)^2)^(1/2),x, algorithm="fricas")`

output `[1/3*(3*(b*e^2*x^2 + 2*b*d*e*x + b*d^2)*sqrt(b/(b*d - a*e))*log((b*e*x + 2*b*d - a*e - 2*(b*d - a*e)*sqrt(e*x + d)*sqrt(b/(b*d - a*e)))/(b*x + a)) + 2*(3*b*e*x + 4*b*d - a*e)*sqrt(e*x + d))/(b^2*d^4 - 2*a*b*d^3*e + a^2*d^2*e^2 + (b^2*d^2*e^2 - 2*a*b*d*e^3 + a^2*e^4)*x^2 + 2*(b^2*d^3*e - 2*a*b*d^2*e^2 + a^2*d*e^3)*x), 2/3*(3*(b*e^2*x^2 + 2*b*d*e*x + b*d^2)*sqrt(-b/(b*d - a*e))*arctan(sqrt(e*x + d)*sqrt(-b/(b*d - a*e))) + (3*b*e*x + 4*b*d - a*e)*sqrt(e*x + d))/(b^2*d^4 - 2*a*b*d^3*e + a^2*d^2*e^2 + (b^2*d^2*e^2 - 2*a*b*d*e^3 + a^2*e^4)*x^2 + 2*(b^2*d^3*e - 2*a*b*d^2*e^2 + a^2*d*e^3)*x)]`

Sympy [F]

$$\int \frac{1}{(d+ex)^{5/2} \sqrt{a^2+2abx+b^2x^2}} dx = \int \frac{1}{(d+ex)^{5/2} \sqrt{(a+bx)^2}} dx$$

input `integrate(1/(e*x+d)**(5/2)/((b*x+a)**2)**(1/2),x)`

output `Integral(1/((d + e*x)**(5/2)*sqrt((a + b*x)**2)), x)`

Maxima [F]

$$\int \frac{1}{(d+ex)^{5/2} \sqrt{a^2+2abx+b^2x^2}} dx = \int \frac{1}{\sqrt{(bx+a)^2 (ex+d)^{5/2}}} dx$$

input `integrate(1/(e*x+d)^(5/2)/((b*x+a)^2)^(1/2),x, algorithm="maxima")`

output `integrate(1/(sqrt((b*x + a)^2)*(e*x + d)^(5/2)), x)`

Giac [A] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 120, normalized size of antiderivative = 0.71

$$\int \frac{1}{(d+ex)^{5/2} \sqrt{a^2+2abx+b^2x^2}} dx = \frac{2}{3} \left(\frac{3b^2 \arctan\left(\frac{\sqrt{ex+db}}{\sqrt{-b^2d+abe}}\right)}{(b^2d^2 - 2abde + a^2e^2)\sqrt{-b^2d+abe}} + \frac{3(ex+d)b + bd - a^2e}{(b^2d^2 - 2abde + a^2e^2)(ex+d)} \right)$$

input `integrate(1/(e*x+d)^(5/2)/((b*x+a)^2)^(1/2),x, algorithm="giac")`

output `2/3*(3*b^2*arctan(sqrt(e*x + d)*b/sqrt(-b^2*d + a*b*e))/((b^2*d^2 - 2*a*b*d*e + a^2*e^2)*sqrt(-b^2*d + a*b*e)) + (3*(e*x + d)*b + b*d - a*e)/((b^2*d^2 - 2*a*b*d*e + a^2*e^2)*(e*x + d)^(3/2)))*sgn(b*x + a)`

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(d+ex)^{5/2} \sqrt{a^2+2abx+b^2x^2}} dx = \int \frac{1}{\sqrt{(a+bx)^2 (d+ex)^{5/2}}} dx$$

input `int(1/(((a + b*x)^2)^(1/2)*(d + e*x)^(5/2)),x)`

output `int(1/(((a + b*x)^2)^(1/2)*(d + e*x)^(5/2)), x)`

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 223, normalized size of antiderivative = 1.33

$$\int \frac{1}{(d + ex)^{5/2} \sqrt{a^2 + 2abx + b^2x^2}} dx = \frac{2\sqrt{b} \sqrt{ex + d} \sqrt{ae - bd} \operatorname{atan}\left(\frac{\sqrt{ex+db}}{\sqrt{b} \sqrt{ae-bd}}\right) bd + 2\sqrt{b} \sqrt{ex + d} \sqrt{ae - bd}}{\sqrt{ex + d} (a^3e^4x - 3a^2bd e^3x + 3a b^2d^2e^2x - b^3d^3e^2x)}$$

input `int(1/(e*x+d)^(5/2)/((b*x+a)^2)^(1/2), x)`

output `(2*(3*sqrt(b)*sqrt(d + e*x)*sqrt(a*e - b*d)*atan((sqrt(d + e*x)*b)/(sqrt(b)*sqrt(a*e - b*d)))*b*d + 3*sqrt(b)*sqrt(d + e*x)*sqrt(a*e - b*d)*atan((sqrt(d + e*x)*b)/(sqrt(b)*sqrt(a*e - b*d)))*b*e*x - a**2*e**2 + 5*a*b*d*e + 3*a*b*e**2*x - 4*b**2*d**2 - 3*b**2*d*e*x)/(3*sqrt(d + e*x)*(a**3*d*e**3 + a**3*e**4*x - 3*a**2*b*d**2*e**2 - 3*a**2*b*d*e**3*x + 3*a*b**2*d**3*e + 3*a*b**2*d**2*e**2*x - b**3*d**4 - b**3*d**3*e*x))`

3.378 $\int \frac{1}{(d+ex)^{7/2}\sqrt{a^2+2abx+b^2x^2}} dx$

| | |
|---|------|
| Optimal result | 2788 |
| Mathematica [A] (verified) | 2789 |
| Rubi [A] (verified) | 2789 |
| Maple [A] (verified) | 2792 |
| Fricas [B] (verification not implemented) | 2792 |
| Sympy [F(-1)] | 2793 |
| Maxima [F] | 2793 |
| Giac [A] (verification not implemented) | 2794 |
| Mupad [F(-1)] | 2794 |
| Reduce [B] (verification not implemented) | 2795 |

Optimal result

Integrand size = 30, antiderivative size = 219

$$\int \frac{1}{(d+ex)^{7/2}\sqrt{a^2+2abx+b^2x^2}} dx = \frac{2(a+bx)}{5(bd-ae)(d+ex)^{5/2}\sqrt{a^2+2abx+b^2x^2}} + \frac{2b(a+bx)}{3(bd-ae)^2(d+ex)^{3/2}\sqrt{a^2+2abx+b^2x^2}} + \frac{2b^2(a+bx)}{(bd-ae)^3\sqrt{d+ex}\sqrt{a^2+2abx+b^2x^2}} - \frac{2b^{5/2}(a+bx)\operatorname{arctanh}\left(\frac{\sqrt{b}\sqrt{d+ex}}{\sqrt{bd-ae}}\right)}{(bd-ae)^{7/2}\sqrt{a^2+2abx+b^2x^2}}$$

output

```
2/5*(b*x+a)/(-a*e+b*d)/(e*x+d)^(5/2)/((b*x+a)^2)^(1/2)+2/3*b*(b*x+a)/(-a*e+b*d)^2/(e*x+d)^(3/2)/((b*x+a)^2)^(1/2)+2*b^2*(b*x+a)/(-a*e+b*d)^3/(e*x+d)^(1/2)/((b*x+a)^2)^(1/2)-2*b^(5/2)*(b*x+a)*arctanh(b^(1/2)*(e*x+d)^(1/2)/(-a*e+b*d)^(1/2))/(-a*e+b*d)^(7/2)/((b*x+a)^2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.28 (sec) , antiderivative size = 133, normalized size of antiderivative = 0.61

$$\int \frac{1}{(d+ex)^{7/2} \sqrt{a^2+2abx+b^2x^2}} dx = \frac{2(a+bx) \left(\frac{3a^2e^2 - abe(11d+5ex) + b^2(23d^2+35dex+15e^2x^2)}{(bd-ae)^3(d+ex)^{5/2}} - \frac{15b^{5/2} \arctan\left(\frac{\sqrt{b}\sqrt{d+ex}}{\sqrt{-bd+ae}}\right)}{(-bd+ae)^{7/2}} \right)}{15\sqrt{(a+bx)^2}}$$

input `Integrate[1/((d + e*x)^(7/2)*Sqrt[a^2 + 2*a*b*x + b^2*x^2]), x]`

output `(2*(a + b*x)*((3*a^2*e^2 - a*b*e*(11*d + 5*e*x) + b^2*(23*d^2 + 35*d*e*x + 15*e^2*x^2))/((b*d - a*e)^3*(d + e*x)^(5/2)) - (15*b^(5/2)*ArcTan[(Sqrt[b]*Sqrt[d + e*x])/Sqrt[-(b*d) + a*e]])/(-(b*d) + a*e)^(7/2))/(15*Sqrt[(a + b*x)^2])`

Rubi [A] (verified)

Time = 0.48 (sec) , antiderivative size = 167, normalized size of antiderivative = 0.76, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.233$, Rules used = {1102, 27, 61, 61, 61, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\begin{aligned} & \int \frac{1}{\sqrt{a^2+2abx+b^2x^2}(d+ex)^{7/2}} dx \\ & \quad \downarrow \text{1102} \\ & \frac{b(a+bx) \int \frac{1}{b(a+bx)(d+ex)^{7/2}} dx}{\sqrt{a^2+2abx+b^2x^2}} \\ & \quad \downarrow \text{27} \\ & \frac{(a+bx) \int \frac{1}{(a+bx)(d+ex)^{7/2}} dx}{\sqrt{a^2+2abx+b^2x^2}} \\ & \quad \downarrow \text{61} \end{aligned}$$

$$\begin{aligned}
 & \frac{(a+bx) \left(\frac{b \int \frac{1}{(a+bx)(d+ex)^{5/2}} dx}{bd-ae} + \frac{2}{5(d+ex)^{5/2}(bd-ae)} \right)}{\sqrt{a^2 + 2abx + b^2x^2}} \\
 & \quad \downarrow 61 \\
 & \frac{(a+bx) \left(\frac{b \left(\frac{b \int \frac{1}{(a+bx)(d+ex)^{3/2}} dx}{bd-ae} + \frac{2}{3(d+ex)^{3/2}(bd-ae)} \right)}{bd-ae} + \frac{2}{5(d+ex)^{5/2}(bd-ae)} \right)}{\sqrt{a^2 + 2abx + b^2x^2}} \\
 & \quad \downarrow 61 \\
 & \frac{(a+bx) \left(\frac{b \left(\frac{b \left(\frac{b \int \frac{1}{(a+bx)\sqrt{d+ex}} dx}{bd-ae} + \frac{2}{\sqrt{d+ex}(bd-ae)} \right)}{bd-ae} + \frac{2}{3(d+ex)^{3/2}(bd-ae)} \right)}{bd-ae} + \frac{2}{5(d+ex)^{5/2}(bd-ae)} \right)}{\sqrt{a^2 + 2abx + b^2x^2}} \\
 & \quad \downarrow 73 \\
 & \frac{(a+bx) \left(\frac{b \left(\frac{b \left(\frac{2b \int \frac{1}{a + \frac{b(d+ex)}{e} - \frac{bd}{e}} d\sqrt{d+ex}}{e(bd-ae)} + \frac{2}{\sqrt{d+ex}(bd-ae)} \right)}{bd-ae} + \frac{2}{3(d+ex)^{3/2}(bd-ae)} \right)}{bd-ae} + \frac{2}{5(d+ex)^{5/2}(bd-ae)} \right)}{\sqrt{a^2 + 2abx + b^2x^2}} \\
 & \quad \downarrow 221
 \end{aligned}$$

$$\frac{(a + bx) \left(\frac{b \left(\frac{2}{\sqrt{d+ex}(bd-ae)} - \frac{2\sqrt{b}\operatorname{arctanh}\left(\frac{\sqrt{b}\sqrt{d+ex}}{\sqrt{bd-ae}}\right)}{(bd-ae)^{3/2}} \right)}{bd-ae} + \frac{2}{3(d+ex)^{3/2}(bd-ae)} \right) + \frac{2}{5(d+ex)^{5/2}(bd-ae)}}{\sqrt{a^2 + 2abx + b^2x^2}}$$

input `Int[1/((d + e*x)^(7/2)*Sqrt[a^2 + 2*a*b*x + b^2*x^2]),x]`

output `((a + b*x)*(2/(5*(b*d - a*e)*(d + e*x)^(5/2)) + (b*(2/(3*(b*d - a*e)*(d + e*x)^(3/2)) + (b*(2/((b*d - a*e)*Sqrt[d + e*x]) - (2*Sqrt[b]*ArcTanh[(Sqrt[b]*Sqrt[d + e*x])/Sqrt[b*d - a*e]])/(b*d - a*e)^(3/2)))/(b*d - a*e)))/Sqrt[a^2 + 2*a*b*x + b^2*x^2])`

Defintions of rubi rules used

rule 27 `Int[(a_)*(F_x_), x_Symbol] := Simp[a Int[F_x, x], x] /; FreeQ[a, x] && !MatchQ[F_x, (b_)*(G_x_) /; FreeQ[b, x]]`

rule 61 `Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] := Simp[(a + b*x)^(m + 1)*(c + d*x)^(n + 1)/((b*c - a*d)*(m + 1)), x] - Simp[d*((m + n + 2)/((b*c - a*d)*(m + 1)) Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && LtQ[m, -1] && !(LtQ[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && IntLinearQ[a, b, c, d, m, n, x]`

rule 73 `Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] := With[{p = Denominator[m]}, Simp[p/b Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) + d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x] /; FreeQ[{a, b, c, d}, x] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntLinearQ[a, b, c, d, m, n, x]`

rule 221 `Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[-a/b, 2]/a)*ArcTanh[x/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]`

rule 1102 `Int[((d_.) + (e_.)*(x_)^(m_.))*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] := Simp[(a + b*x + c*x^2)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x)^(2*FracPart[p])) Int[(d + e*x)^m*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && EqQ[b^2 - 4*a*c, 0]`

Maple [A] (verified)

Time = 1.08 (sec) , antiderivative size = 202, normalized size of antiderivative = 0.92

| method | result |
|---------|---|
| default | $-\frac{2(bx+a)\left(15b^3 \arctan\left(\frac{b\sqrt{ex+d}}{\sqrt{b(ae-bd)}}\right)(ex+d)^{\frac{5}{2}} + 15\sqrt{b(ae-bd)}b^2e^2x^2 - 5\sqrt{b(ae-bd)}abe^2x + 35\sqrt{b(ae-bd)}b^2dex + 3\sqrt{b(ae-bd)}a\right)}{15\sqrt{(bx+a)^2(ae-bd)^3}\sqrt{b(ae-bd)}(ex+d)^{\frac{5}{2}}}$ |

input `int(1/(e*x+d)^(7/2)/((b*x+a)^2)^(1/2), x, method=_RETURNVERBOSE)`

output
$$-2/15*(b*x+a)*(15*b^3*\arctan(b*(e*x+d)^(1/2)/(b*(a*e-b*d))^(1/2))*(e*x+d)^(5/2) + 15*(b*(a*e-b*d))^(1/2)*b^2*e^2*x^2 - 5*(b*(a*e-b*d))^(1/2)*a*b*e^2*x + 35*(b*(a*e-b*d))^(1/2)*b^2*d*e*x + 3*(b*(a*e-b*d))^(1/2)*a^2*e^2 - 11*(b*(a*e-b*d))^(1/2)*a*b*d*e + 23*(b*(a*e-b*d))^(1/2)*b^2*d^2 / ((b*x+a)^2)^(1/2) / (a*e-b*d)^3 / (b*(a*e-b*d))^(1/2) / (e*x+d)^(5/2)$$

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 328 vs. 2(155) = 310.

Time = 0.11 (sec) , antiderivative size = 686, normalized size of antiderivative = 3.13

$$\int \frac{1}{(d+ex)^{7/2}\sqrt{a^2+2abx+b^2x^2}} dx = \left[-\frac{15(b^2e^3x^3 + 3b^2de^2x^2 + 3b^2d^2ex + b^2d^3)\sqrt{\frac{b}{bd-ae}} \log\left(\dots\right)}{15(b^3d^6 - 3ab^2d^5e + 3a^2bd^4e^2 - a^3d^3e^3 + (b^3d^3e^3 - 3ab^2d^2e^4 - \dots))} \right]$$

input `integrate(1/(e*x+d)^(7/2)/((b*x+a)^2)^(1/2),x, algorithm="fricas")`

output `[-1/15*(15*(b^2*e^3*x^3 + 3*b^2*d*e^2*x^2 + 3*b^2*d^2*e*x + b^2*d^3)*sqrt(b/(b*d - a*e))*log((b*e*x + 2*b*d - a*e + 2*(b*d - a*e)*sqrt(e*x + d))*sqrt(b/(b*d - a*e)))/(b*x + a) - 2*(15*b^2*e^2*x^2 + 23*b^2*d^2 - 11*a*b*d*e + 3*a^2*e^2 + 5*(7*b^2*d*e - a*b*e^2)*x)*sqrt(e*x + d)/(b^3*d^6 - 3*a*b^2*d^5*e + 3*a^2*b*d^4*e^2 - a^3*d^3*e^3 + (b^3*d^3*e^3 - 3*a*b^2*d^2*e^4 + 3*a^2*b*d^2*e^4 - a^3*d^2*e^5)*x^3 + 3*(b^3*d^4*e^2 - 3*a*b^2*d^3*e^3 + 3*a^2*b*d^2*e^4 - a^3*d^2*e^5)*x^2 + 3*(b^3*d^5*e - 3*a*b^2*d^4*e^2 + 3*a^2*b*d^3*e^3 - a^3*d^2*e^4)*x), 2/15*(15*(b^2*e^3*x^3 + 3*b^2*d*e^2*x^2 + 3*b^2*d^2*e*x + b^2*d^3)*sqrt(-b/(b*d - a*e))*arctan(sqrt(e*x + d)*sqrt(-b/(b*d - a*e))) + (15*b^2*e^2*x^2 + 23*b^2*d^2 - 11*a*b*d*e + 3*a^2*e^2 + 5*(7*b^2*d*e - a*b*e^2)*x)*sqrt(e*x + d))/(b^3*d^6 - 3*a*b^2*d^5*e + 3*a^2*b*d^4*e^2 - a^3*d^3*e^3 + (b^3*d^3*e^3 - 3*a*b^2*d^2*e^4 + 3*a^2*b*d^2*e^4 - a^3*d^2*e^5)*x^3 + 3*(b^3*d^4*e^2 - 3*a*b^2*d^3*e^3 + 3*a^2*b*d^2*e^4 - a^3*d^2*e^5)*x^2 + 3*(b^3*d^5*e - 3*a*b^2*d^4*e^2 + 3*a^2*b*d^3*e^3 - a^3*d^2*e^4)*x]`

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{(d+ex)^{7/2} \sqrt{a^2+2abx+b^2x^2}} dx = \text{Timed out}$$

input `integrate(1/(e*x+d)**(7/2)/((b*x+a)**2)**(1/2),x)`

output `Timed out`

Maxima [F]

$$\int \frac{1}{(d+ex)^{7/2} \sqrt{a^2+2abx+b^2x^2}} dx = \int \frac{1}{\sqrt{(bx+a)^2(ex+d)^{7/2}}} dx$$

input `integrate(1/(e*x+d)^(7/2)/((b*x+a)^2)^(1/2),x, algorithm="maxima")`

output `integrate(1/(sqrt((b*x + a)^2)*(e*x + d)^(7/2)), x)`

Giac [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 190, normalized size of antiderivative = 0.87

$$\int \frac{1}{(d + ex)^{7/2} \sqrt{a^2 + 2abx + b^2x^2}} dx = \frac{2}{15} \left(\frac{15 b^3 \arctan\left(\frac{\sqrt{ex+db}}{\sqrt{-b^2d+abe}}\right)}{(b^3d^3 - 3ab^2d^2e + 3a^2bde^2 - a^3e^3)\sqrt{-b^2d+abe}} + \frac{15(ex + a)}{(b^3d^3 - 3ab^2d^2e + 3a^2bde^2 - a^3e^3)\sqrt{-b^2d+abe}} \right)$$

input `integrate(1/(e*x+d)^(7/2)/((b*x+a)^2)^(1/2),x, algorithm="giac")`

output `2/15*(15*b^3*arctan(sqrt(e*x + d)*b/sqrt(-b^2*d + a*b*e))/((b^3*d^3 - 3*a*b^2*d^2*e + 3*a^2*b*d*e^2 - a^3*e^3)*sqrt(-b^2*d + a*b*e)) + (15*(e*x + d)^2*b^2 + 5*(e*x + d)*b^2*d + 3*b^2*d^2 - 5*(e*x + d)*a*b*e - 6*a*b*d*e + 3*a^2*e^2)/((b^3*d^3 - 3*a*b^2*d^2*e + 3*a^2*b*d*e^2 - a^3*e^3)*(e*x + d)^(5/2)))*sgn(b*x + a)`

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(d + ex)^{7/2} \sqrt{a^2 + 2abx + b^2x^2}} dx = \int \frac{1}{\sqrt{(a + bx)^2 (d + ex)^{7/2}}} dx$$

input `int(1/(((a + b*x)^2)^(1/2)*(d + e*x)^(7/2)),x)`

output `int(1/(((a + b*x)^2)^(1/2)*(d + e*x)^(7/2)), x)`

3.379 $\int \frac{(d+ex)^{9/2}}{(a^2+2abx+b^2x^2)^{3/2}} dx$

| | |
|---|------|
| Optimal result | 2796 |
| Mathematica [A] (verified) | 2797 |
| Rubi [A] (verified) | 2797 |
| Maple [A] (verified) | 2803 |
| Fricas [A] (verification not implemented) | 2803 |
| Sympy [F(-1)] | 2804 |
| Maxima [F] | 2804 |
| Giac [A] (verification not implemented) | 2805 |
| Mupad [F(-1)] | 2805 |
| Reduce [B] (verification not implemented) | 2806 |

Optimal result

Integrand size = 30, antiderivative size = 308

$$\int \frac{(d+ex)^{9/2}}{(a^2+2abx+b^2x^2)^{3/2}} dx = \frac{63e^2(bd-ae)^2(a+bx)\sqrt{d+ex}}{4b^5\sqrt{a^2+2abx+b^2x^2}} + \frac{21e^2(bd-ae)(a+bx)(d+ex)^{3/2}}{4b^4\sqrt{a^2+2abx+b^2x^2}} + \frac{63e^2(a+bx)(d+ex)^{5/2}}{20b^3\sqrt{a^2+2abx+b^2x^2}} - \frac{9e(d+ex)^{7/2}}{4b^2\sqrt{a^2+2abx+b^2x^2}} - \frac{(d+ex)^{9/2}}{2b(a+bx)\sqrt{a^2+2abx+b^2x^2}} - \frac{63e^2(bd-ae)^{5/2}(a+bx)\operatorname{arctanh}\left(\frac{\sqrt{b}\sqrt{d+ex}}{\sqrt{bd-ae}}\right)}{4b^{11/2}\sqrt{a^2+2abx+b^2x^2}}$$

output

```
63/4*e^2*(-a*e+b*d)^2*(b*x+a)*(e*x+d)^(1/2)/b^5/((b*x+a)^2)^(1/2)+21/4*e^2
*(-a*e+b*d)*(b*x+a)*(e*x+d)^(3/2)/b^4/((b*x+a)^2)^(1/2)+63/20*e^2*(b*x+a)*
(e*x+d)^(5/2)/b^3/((b*x+a)^2)^(1/2)-9/4*e*(e*x+d)^(7/2)/b^2/((b*x+a)^2)^(1
/2)-1/2*(e*x+d)^(9/2)/b/(b*x+a)/((b*x+a)^2)^(1/2)-63/4*e^2*(-a*e+b*d)^(5/2
)*(b*x+a)*arctanh(b^(1/2)*(e*x+d)^(1/2)/(-a*e+b*d)^(1/2))/b^(11/2)/((b*x+a
)^2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.71 (sec) , antiderivative size = 236, normalized size of antiderivative = 0.77

$$\int \frac{(d+ex)^{9/2}}{(a^2+2abx+b^2x^2)^{3/2}} dx = \frac{e^2(a+bx)^3 \left(\frac{\sqrt{b}\sqrt{d+ex}(315a^4e^4+105a^3be^3(-7d+5ex)+21a^2b^2e^2(23d^2-59dex+8e^2x^2))-3ab^3e(1}{e^2(a$$

input

```
Integrate[(d + e*x)^(9/2)/(a^2 + 2*a*b*x + b^2*x^2)^(3/2), x]
```

output

```
(e^2*(a + b*x)^3*((Sqrt[b]*Sqrt[d + e*x]*(315*a^4*e^4 + 105*a^3*b*e^3*(-7*d + 5*e*x) + 21*a^2*b^2*e^2*(23*d^2 - 59*d*e*x + 8*e^2*x^2) - 3*a*b^3*e*(15*d^3 - 277*d^2*e*x + 136*d*e^2*x^2 + 8*e^3*x^3) + b^4*(-10*d^4 - 85*d^3*e*x + 288*d^2*e^2*x^2 + 56*d*e^3*x^3 + 8*e^4*x^4)))/(e^2*(a + b*x)^2) - 315*(-(b*d) + a*e)^(5/2)*ArcTan[(Sqrt[b]*Sqrt[d + e*x])/Sqrt[-(b*d) + a*e]])) / (20*b^(11/2)*((a + b*x)^2)^(3/2))
```

Rubi [A] (verified)

Time = 0.55 (sec) , antiderivative size = 208, normalized size of antiderivative = 0.68, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.300$, Rules used = {1102, 27, 51, 51, 60, 60, 60, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex)^{9/2}}{(a^2+2abx+b^2x^2)^{3/2}} dx$$

$$\downarrow 1102$$

$$\frac{b^3(a+bx) \int \frac{(d+ex)^{9/2}}{b^3(a+bx)^3} dx}{\sqrt{a^2+2abx+b^2x^2}}$$

$$\downarrow 27$$

$$\frac{(a+bx) \int \frac{(d+ex)^{9/2}}{(a+bx)^3} dx}{\sqrt{a^2+2abx+b^2x^2}}$$

$$\begin{array}{c}
 \downarrow 51 \\
 \frac{(a+bx) \left(\frac{9e \int \frac{(d+ex)^{7/2} dx}{(a+bx)^2} - \frac{(d+ex)^{9/2}}{2b(a+bx)^2}}{4b} \right)}{\sqrt{a^2 + 2abx + b^2x^2}} \\
 \downarrow 51 \\
 \frac{(a+bx) \left(\frac{9e \left(\frac{7e \int \frac{(d+ex)^{5/2} dx}{a+bx} - \frac{(d+ex)^{7/2}}{b(a+bx)} \right)}{4b} - \frac{(d+ex)^{9/2}}{2b(a+bx)^2} \right)}{\sqrt{a^2 + 2abx + b^2x^2}} \\
 \downarrow 60 \\
 \frac{(a+bx) \left(\frac{9e \left(\frac{7e \left(\frac{(bd-ae) \int \frac{(d+ex)^{3/2} dx}{a+bx} + \frac{2(d+ex)^{5/2}}{5b} \right)}{2b} - \frac{(d+ex)^{7/2}}{b(a+bx)} \right)}{4b} - \frac{(d+ex)^{9/2}}{2b(a+bx)^2} \right)}{\sqrt{a^2 + 2abx + b^2x^2}} \\
 \downarrow 60 \\
 \frac{(a+bx) \left(\frac{9e \left(\frac{7e \left(\frac{(bd-ae) \left(\frac{(bd-ae) \int \frac{\sqrt{d+ex} dx}{a+bx} + \frac{2(d+ex)^{3/2}}{3b} \right)}{b} + \frac{2(d+ex)^{5/2}}{5b} \right)}{2b} - \frac{(d+ex)^{7/2}}{b(a+bx)} \right)}{4b} - \frac{(d+ex)^{9/2}}{2b(a+bx)^2} \right)}{\sqrt{a^2 + 2abx + b^2x^2}} \\
 \downarrow 60
 \end{array}$$

$$\left((a + bx) \left[\frac{7e}{9e} \left(\frac{(bd - ae) \left(\frac{(bd - ae) \int \frac{1}{(a + bx)\sqrt{d + ex}} dx + 2\sqrt{d + ex}}{b} \right) + \frac{2(d + ex)^{3/2}}{3b}}{b} \right) + \frac{2(d + ex)^{5/2}}{5b} \right] - \frac{(d + ex)^{7/2}}{b(a + bx)} \right) - \frac{(d + ex)^{9/2}}{2b(a + bx)^2}$$

$$\sqrt{a^2 + 2abx + b^2x^2}$$

↓ 73

$$\begin{aligned}
 & \left((bd-ae) \left(\frac{2(bd-ae) \int \frac{1}{a + \frac{b(d+ex)}{e} - \frac{bd}{e}} d\sqrt{d+ex}}{b} + \frac{2\sqrt{d+ex}}{b} \right) + \frac{2(d+ex)^{3/2}}{3b} \right) \\
 & \frac{7e}{b} \left(\frac{2(bd-ae) \int \frac{1}{a + \frac{b(d+ex)}{e} - \frac{bd}{e}} d\sqrt{d+ex}}{b} + \frac{2\sqrt{d+ex}}{b} \right) + \frac{2(d+ex)^{5/2}}{5b} \\
 & \frac{9e}{2b} \left(\frac{2(bd-ae) \int \frac{1}{a + \frac{b(d+ex)}{e} - \frac{bd}{e}} d\sqrt{d+ex}}{b} + \frac{2\sqrt{d+ex}}{b} \right) - \frac{(d+ex)^{7/2}}{b(a+bx)} \\
 & \frac{(a+bx)}{4b} \left(\frac{2(bd-ae) \int \frac{1}{a + \frac{b(d+ex)}{e} - \frac{bd}{e}} d\sqrt{d+ex}}{b} + \frac{2\sqrt{d+ex}}{b} \right) - \frac{(d+ex)^{9/2}}{2b(a+bx)}
 \end{aligned}$$

$$\sqrt{a^2 + 2abx + b^2x^2}$$

221

$$\frac{(a + bx) \left(\frac{7e \left((bd - ae) \left(\frac{2\sqrt{d+ex}}{b} - \frac{2\sqrt{bd-ae} \operatorname{arctanh}\left(\frac{\sqrt{b}\sqrt{d+ex}}{\sqrt{bd-ae}}\right)}{b^{3/2}} \right) + \frac{2(d+ex)^{3/2}}{3b} \right) + \frac{2(d+ex)^{5/2}}{5b} \right) - \frac{9e \left((bd - ae) \left(\frac{2\sqrt{d+ex}}{b} - \frac{2\sqrt{bd-ae} \operatorname{arctanh}\left(\frac{\sqrt{b}\sqrt{d+ex}}{\sqrt{bd-ae}}\right)}{b^{3/2}} \right) + \frac{2(d+ex)^{3/2}}{3b} \right) + \frac{2(d+ex)^{5/2}}{5b} \right) - \frac{(d+ex)^{7/2}}{b(a+bx)}}{4b} - \frac{(d+ex)^{9/2}}{2b(a+bx)^2}$$

$$\sqrt{a^2 + 2abx + b^2x^2}$$

input `Int[(d + e*x)^(9/2)/(a^2 + 2*a*b*x + b^2*x^2)^(3/2),x]`

output
$$\frac{((a + b*x)*(-1/2*(d + e*x)^{(9/2)})/(b*(a + b*x)^2) + (9*e*(-((d + e*x)^{(7/2)})/(b*(a + b*x))) + (7*e*((2*(d + e*x)^{(5/2)})/(5*b) + ((b*d - a*e)*((2*(d + e*x)^{(3/2)})/(3*b) + ((b*d - a*e)*((2*sqrt{d + e*x})/b - (2*sqrt{b*d - a*e})*ArcTanh[(sqrt{b}*sqrt{d + e*x})/sqrt{b*d - a*e}])/b^{(3/2)}))/b))/b)/(2*b)))/(4*b))/sqrt{a^2 + 2*a*b*x + b^2*x^2}$$

Defintions of rubi rules used

rule 27
$$\text{Int}[(a_*)(Fx_), x_Symbol] \rightarrow \text{Simp}[a \text{ Int}[Fx, x], x] /; \text{FreeQ}[a, x] \ \&\& \ !\text{MatchQ}[Fx, (b_*)(Gx_) /; \text{FreeQ}[b, x]]$$

rule 51
$$\text{Int}[(a_.) + (b_.)*(x_)^{(m_)*((c_.) + (d_.)*(x_)^{(n_)}, x_Symbol] \rightarrow \text{Simp}[(a + b*x)^{(m + 1)*((c + d*x)^n/(b*(m + 1))}, x] - \text{Simp}[d*(n/(b*(m + 1)))] \text{Int}[(a + b*x)^{(m + 1)*((c + d*x)^{(n - 1)}, x], x] /; \text{FreeQ}\{a, b, c, d, n\}, x] \ \&\& \ \text{ILtQ}[m, -1] \ \&\& \ \text{FractionQ}[n] \ \&\& \ \text{GtQ}[n, 0]$$

rule 60
$$\text{Int}[(a_.) + (b_.)*(x_)^{(m_)*((c_.) + (d_.)*(x_)^{(n_)}, x_Symbol] \rightarrow \text{Simp}[(a + b*x)^{(m + 1)*((c + d*x)^n/(b*(m + n + 1))}, x] + \text{Simp}[n*((b*c - a*d)/(b*(m + n + 1))] \text{Int}[(a + b*x)^m*(c + d*x)^{(n - 1)}, x], x] /; \text{FreeQ}\{a, b, c, d\}, x] \ \&\& \ \text{GtQ}[n, 0] \ \&\& \ \text{NeQ}[m + n + 1, 0] \ \&\& \ !(\text{IGtQ}[m, 0] \ \&\& \ (!\text{IntegerQ}[n] \ || \ (\text{GtQ}[m, 0] \ \&\& \ \text{LtQ}[m - n, 0]))) \ \&\& \ !\text{ILtQ}[m + n + 2, 0] \ \&\& \ \text{IntLinearQ}[a, b, c, d, m, n, x]$$

rule 73
$$\text{Int}[(a_.) + (b_.)*(x_)^{(m_)*((c_.) + (d_.)*(x_)^{(n_)}, x_Symbol] \rightarrow \text{With}\{p = \text{Denominator}[m]\}, \text{Simp}[p/b \ \text{Subst}[\text{Int}[x^{(p*(m + 1) - 1)*(c - a*(d/b) + d*(x^{p/b})^n}, x], x, (a + b*x)^{(1/p)}, x]] /; \text{FreeQ}\{a, b, c, d\}, x] \ \&\& \ \text{LtQ}[-1, m, 0] \ \&\& \ \text{LeQ}[-1, n, 0] \ \&\& \ \text{LeQ}[\text{Denominator}[n], \text{Denominator}[m]] \ \&\& \ \text{IntLinearQ}[a, b, c, d, m, n, x]$$

rule 221
$$\text{Int}[(a_.) + (b_.)*(x_)^2)^{-1}, x_Symbol] \rightarrow \text{Simp}[(\text{Rt}[-a/b, 2]/a)*\text{ArcTanh}[x/\text{Rt}[-a/b, 2]], x] /; \text{FreeQ}\{a, b\}, x] \ \&\& \ \text{NegQ}[a/b]$$

rule 1102

```
Int[((d_.) + (e_.)*(x_.))^(m_.)*((a_.) + (b_.)*(x_.) + (c_.)*(x_.)^2)^(p_), x_Symbol]
:> Simp[(a + b*x + c*x^2)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x)^(2*FracPart[p]))
Int[(d + e*x)^m*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d, e, m, p}, x]
&& EqQ[b^2 - 4*a*c, 0]
```

Maple [A] (verified)

Time = 1.64 (sec) , antiderivative size = 226, normalized size of antiderivative = 0.73

| method | result |
|---------|---|
| risch | $\frac{2e^2(b^2e^2x^2 - 5xab e^2 + 7b^2dex + 30e^2a^2 - 65abde + 36b^2d^2)\sqrt{ex+d}\sqrt{(bx+a)^2}}{5b^5(bx+a)} - \frac{(2e^3a^3 - 6a^2bd e^2 + 6ab^2d^2e - 2b^3d^3)e^2}{\left(\frac{-17(ex+c)}{8}\right)}$ |
| default | Expression too large to display |

input

```
int((e*x+d)^(9/2)/(b^2*x^2+2*a*b*x+a^2)^(3/2),x,method=_RETURNVERBOSE)
```

output

```
2/5*e^2*(b^2*e^2*x^2-5*a*b*e^2*x+7*b^2*d*e*x+30*a^2*e^2-65*a*b*d*e+36*b^2*d^2)*(e*x+d)^(1/2)/b^5*((b*x+a)^2)^(1/2)/(b*x+a)-1/b^5*(2*a^3*e^3-6*a^2*b*d*e^2+6*a*b^2*d^2*e-2*b^3*d^3)*e^2*((-17/8*(e*x+d)^(3/2)*b+(-15/8*a*e+15/8*b*d)*(e*x+d)^(1/2))/(b*(e*x+d)+a*e-b*d)^2+63/8/(b*(a*e-b*d))^(1/2)*arctan(b*(e*x+d)^(1/2)/(b*(a*e-b*d))^(1/2))*((b*x+a)^2)^(1/2)/(b*x+a)
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 730, normalized size of antiderivative = 2.37

$$\int \frac{(d+ex)^{9/2}}{(a^2+2abx+b^2x^2)^{3/2}} dx = \left[\frac{315(a^2b^2d^2e^2 - 2a^3bde^3 + a^4e^4 + (b^4d^2e^2 - 2ab^3de^3 + a^2b^2e^4)x^2 + 2(ab^3d^2e^2 - 2ab^2de^3 + a^2b^2e^4)x + 2(ab^3d^2e^2 - 2ab^2de^3 + a^2b^2e^4)}{(a^2+2abx+b^2x^2)^{3/2}} \right]$$

input

```
integrate((e*x+d)^(9/2)/(b^2*x^2+2*a*b*x+a^2)^(3/2),x, algorithm="fricas")
```


output

```
[1/40*(315*(a^2*b^2*d^2*e^2 - 2*a^3*b*d*e^3 + a^4*e^4 + (b^4*d^2*e^2 - 2*a
*b^3*d*e^3 + a^2*b^2*e^4)*x^2 + 2*(a*b^3*d^2*e^2 - 2*a^2*b^2*d*e^3 + a^3*b
*e^4)*x)*sqrt((b*d - a*e)/b)*log((b*e*x + 2*b*d - a*e - 2*sqrt(e*x + d)*b*
sqrt((b*d - a*e)/b))/(b*x + a)) + 2*(8*b^4*e^4*x^4 - 10*b^4*d^4 - 45*a*b^3
*d^3*e + 483*a^2*b^2*d^2*e^2 - 735*a^3*b*d*e^3 + 315*a^4*e^4 + 8*(7*b^4*d*
e^3 - 3*a*b^3*e^4)*x^3 + 24*(12*b^4*d^2*e^2 - 17*a*b^3*d*e^3 + 7*a^2*b^2*e
^4)*x^2 - (85*b^4*d^3*e - 831*a*b^3*d^2*e^2 + 1239*a^2*b^2*d*e^3 - 525*a^3
*b*e^4)*x)*sqrt(e*x + d))/(b^7*x^2 + 2*a*b^6*x + a^2*b^5), -1/20*(315*(a^2
*b^2*d^2*e^2 - 2*a^3*b*d*e^3 + a^4*e^4 + (b^4*d^2*e^2 - 2*a*b^3*d*e^3 + a^
2*b^2*e^4)*x^2 + 2*(a*b^3*d^2*e^2 - 2*a^2*b^2*d*e^3 + a^3*b*e^4)*x)*sqrt(-
(b*d - a*e)/b)*arctan(-sqrt(e*x + d)*b*sqrt(-(b*d - a*e)/b)/(b*d - a*e)) -
(8*b^4*e^4*x^4 - 10*b^4*d^4 - 45*a*b^3*d^3*e + 483*a^2*b^2*d^2*e^2 - 735*
a^3*b*d*e^3 + 315*a^4*e^4 + 8*(7*b^4*d*e^3 - 3*a*b^3*e^4)*x^3 + 24*(12*b^4
*d^2*e^2 - 17*a*b^3*d*e^3 + 7*a^2*b^2*e^4)*x^2 - (85*b^4*d^3*e - 831*a*b^3
*d^2*e^2 + 1239*a^2*b^2*d*e^3 - 525*a^3*b*e^4)*x)*sqrt(e*x + d))/(b^7*x^2
+ 2*a*b^6*x + a^2*b^5)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(d + ex)^{9/2}}{(a^2 + 2abx + b^2x^2)^{3/2}} dx = \text{Timed out}$$

input

```
integrate((e*x+d)**(9/2)/(b**2*x**2+2*a*b*x+a**2)**(3/2),x)
```

output

Timed out

Maxima [F]

$$\int \frac{(d + ex)^{9/2}}{(a^2 + 2abx + b^2x^2)^{3/2}} dx = \int \frac{(ex + d)^{\frac{9}{2}}}{(b^2x^2 + 2abx + a^2)^{\frac{3}{2}}} dx$$

input

```
integrate((e*x+d)^(9/2)/(b^2*x^2+2*a*b*x+a^2)^(3/2),x, algorithm="maxima")
```

output `integrate((e*x + d)^(9/2)/(b^2*x^2 + 2*a*b*x + a^2)^(3/2), x)`

Giac [A] (verification not implemented)

Time = 0.35 (sec) , antiderivative size = 397, normalized size of antiderivative = 1.29

$$\int \frac{(d + ex)^{9/2}}{(a^2 + 2abx + b^2x^2)^{3/2}} dx = \frac{63(b^3d^3e^2 - 3ab^2d^2e^3 + 3a^2bde^4 - a^3e^5) \arctan\left(\frac{\sqrt{ex+db}}{\sqrt{-b^2d+abe}}\right)}{4\sqrt{-b^2d+abe}b^5\operatorname{sgn}(bx+a)} - \frac{17(ex+d)^{\frac{3}{2}}b^4d^3e^2 - 15\sqrt{ex+db}b^4d^4e^2 - 51(ex+d)^{\frac{3}{2}}ab^3d^2e^3 + 60\sqrt{ex+db}ab^3d^3e^3 + 51(ex+d)^{\frac{3}{2}}a^2b^2d^2e^4}{4((ex+d)b - bd + ae)^2b^5\operatorname{sgn}(bx+a)} + \frac{2\left((ex+d)^{\frac{5}{2}}b^{12}e^2 + 5(ex+d)^{\frac{3}{2}}b^{12}de^2 + 30\sqrt{ex+db}b^{12}d^2e^2 - 5(ex+d)^{\frac{3}{2}}ab^{11}e^3 - 60\sqrt{ex+db}ab^{11}de^3 + 30\sqrt{ex+db}ab^{11}d^2e^3\right)}{5b^{15}\operatorname{sgn}(bx+a)}$$

input `integrate((e*x+d)^(9/2)/(b^2*x^2+2*a*b*x+a^2)^(3/2),x, algorithm="giac")`

output `63/4*(b^3*d^3*e^2 - 3*a*b^2*d^2*e^3 + 3*a^2*b*d*e^4 - a^3*e^5)*arctan(sqrt(e*x + d)*b/sqrt(-b^2*d + a*b*e))/(sqrt(-b^2*d + a*b*e)*b^5*sgn(b*x + a)) - 1/4*(17*(e*x + d)^(3/2)*b^4*d^3*e^2 - 15*sqrt(e*x + d)*b^4*d^4*e^2 - 51*(e*x + d)^(3/2)*a*b^3*d^2*e^3 + 60*sqrt(e*x + d)*a*b^3*d^3*e^3 + 51*(e*x + d)^(3/2)*a^2*b^2*d^2*e^4 - 90*sqrt(e*x + d)*a^2*b^2*d^2*e^4 - 17*(e*x + d)^(3/2)*a^3*b*e^5 + 60*sqrt(e*x + d)*a^3*b*d*e^5 - 15*sqrt(e*x + d)*a^4*e^6)/(((e*x + d)*b - b*d + a*e)^2*b^5*sgn(b*x + a)) + 2/5*((e*x + d)^(5/2)*b^12*e^2 + 5*(e*x + d)^(3/2)*b^12*d*e^2 + 30*sqrt(e*x + d)*b^12*d^2*e^2 - 5*(e*x + d)^(3/2)*a*b^11*e^3 - 60*sqrt(e*x + d)*a*b^11*d*e^3 + 30*sqrt(e*x + d)*a^2*b^10*e^4)/(b^15*sgn(b*x + a))`

Mupad [F(-1)]

Timed out.

$$\int \frac{(d + ex)^{9/2}}{(a^2 + 2abx + b^2x^2)^{3/2}} dx = \int \frac{(d + ex)^{9/2}}{(a^2 + 2abx + b^2x^2)^{3/2}} dx$$

input `int((d + e*x)^(9/2)/(a^2 + b^2*x^2 + 2*a*b*x)^(3/2),x)`

output `int((d + e*x)^(9/2)/(a^2 + b^2*x^2 + 2*a*b*x)^(3/2), x)`

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 715, normalized size of antiderivative = 2.32

$$\int \frac{(d + ex)^{9/2}}{(a^2 + 2abx + b^2x^2)^{3/2}} dx = \frac{-315\sqrt{b}\sqrt{ae - bd} \operatorname{atan}\left(\frac{\sqrt{ex+db}}{\sqrt{b}\sqrt{ae-bd}}\right) a^4 e^4 + 630\sqrt{b}\sqrt{ae - bd} \operatorname{atan}\left(\frac{\sqrt{ex+db}}{\sqrt{b}\sqrt{ae-bd}}\right)}{}$$

input `int((e*x+d)^(9/2)/(b^2*x^2+2*a*b*x+a^2)^(3/2), x)`

output `(- 315*sqrt(b)*sqrt(a*e - b*d)*atan((sqrt(d + e*x)*b)/(sqrt(b)*sqrt(a*e - b*d)))*a**4*e**4 + 630*sqrt(b)*sqrt(a*e - b*d)*atan((sqrt(d + e*x)*b)/(sqrt(b)*sqrt(a*e - b*d)))*a**3*b*d*e**3 - 630*sqrt(b)*sqrt(a*e - b*d)*atan((sqrt(d + e*x)*b)/(sqrt(b)*sqrt(a*e - b*d)))*a**3*b*e**4*x - 315*sqrt(b)*sqrt(a*e - b*d)*atan((sqrt(d + e*x)*b)/(sqrt(b)*sqrt(a*e - b*d)))*a**2*b**2*d**2*e**2 + 1260*sqrt(b)*sqrt(a*e - b*d)*atan((sqrt(d + e*x)*b)/(sqrt(b)*sqrt(a*e - b*d)))*a**2*b**2*d*e**3*x - 315*sqrt(b)*sqrt(a*e - b*d)*atan((sqrt(d + e*x)*b)/(sqrt(b)*sqrt(a*e - b*d)))*a**2*b**2*e**4*x**2 - 630*sqrt(b)*sqrt(a*e - b*d)*atan((sqrt(d + e*x)*b)/(sqrt(b)*sqrt(a*e - b*d)))*a*b**3*d**2*e**2*x + 630*sqrt(b)*sqrt(a*e - b*d)*atan((sqrt(d + e*x)*b)/(sqrt(b)*sqrt(a*e - b*d)))*a*b**3*d*e**3*x**2 - 315*sqrt(b)*sqrt(a*e - b*d)*atan((sqrt(d + e*x)*b)/(sqrt(b)*sqrt(a*e - b*d)))*b**4*d**2*e**2*x**2 + 315*sqrt(d + e*x)*a**4*b*e**4 - 735*sqrt(d + e*x)*a**3*b**2*d*e**3 + 525*sqrt(d + e*x)*a**3*b**2*e**4*x + 483*sqrt(d + e*x)*a**2*b**3*d**2*e**2 - 1239*sqrt(d + e*x)*a**2*b**3*d*e**3*x + 168*sqrt(d + e*x)*a**2*b**3*e**4*x**2 - 45*sqrt(d + e*x)*a*b**4*d**3*e + 831*sqrt(d + e*x)*a*b**4*d**2*e**2*x - 408*sqrt(d + e*x)*a*b**4*d*e**3*x**2 - 24*sqrt(d + e*x)*a*b**4*e**4*x**3 - 10*sqrt(d + e*x)*b**5*d**4 - 85*sqrt(d + e*x)*b**5*d**3*e*x + 288*sqrt(d + e*x)*b**5*d**2*e**2*x**2 + 56*sqrt(d + e*x)*b**5*d*e**3*x**3 + 8*sqrt(d + e*x)*b**5*e**4*x**4)/(20*b**6*(a**2 + 2*a*b*x + b**2*x**2))`

3.380 $\int \frac{(d+ex)^{7/2}}{(a^2+2abx+b^2x^2)^{3/2}} dx$

| | |
|---|------|
| Optimal result | 2807 |
| Mathematica [A] (verified) | 2808 |
| Rubi [A] (verified) | 2808 |
| Maple [A] (verified) | 2812 |
| Fricas [A] (verification not implemented) | 2812 |
| Sympy [F] | 2813 |
| Maxima [F] | 2813 |
| Giac [A] (verification not implemented) | 2814 |
| Mupad [F(-1)] | 2814 |
| Reduce [B] (verification not implemented) | 2815 |

Optimal result

Integrand size = 30, antiderivative size = 254

$$\int \frac{(d+ex)^{7/2}}{(a^2+2abx+b^2x^2)^{3/2}} dx = \frac{35e^2(bd-ae)(a+bx)\sqrt{d+ex}}{4b^4\sqrt{a^2+2abx+b^2x^2}} + \frac{35e^2(a+bx)(d+ex)^{3/2}}{12b^3\sqrt{a^2+2abx+b^2x^2}} - \frac{7e(d+ex)^{5/2}}{4b^2\sqrt{a^2+2abx+b^2x^2}} - \frac{(d+ex)^{7/2}}{2b(a+bx)\sqrt{a^2+2abx+b^2x^2}} - \frac{35e^2(bd-ae)^{3/2}(a+bx)\operatorname{arctanh}\left(\frac{\sqrt{b}\sqrt{d+ex}}{\sqrt{bd-ae}}\right)}{4b^{9/2}\sqrt{a^2+2abx+b^2x^2}}$$

output

```
35/4*e^2*(-a*e+b*d)*(b*x+a)*(e*x+d)^(1/2)/b^4/((b*x+a)^2)^(1/2)+35/12*e^2*(b*x+a)*(e*x+d)^(3/2)/b^3/((b*x+a)^2)^(1/2)-7/4*e*(e*x+d)^(5/2)/b^2/((b*x+a)^2)^(1/2)-1/2*(e*x+d)^(7/2)/b/(b*x+a)/((b*x+a)^2)^(1/2)-35/4*e^2*(-a*e+b*d)^(3/2)*(b*x+a)*arctanh(b^(1/2)*(e*x+d)^(1/2)/(-a*e+b*d)^(1/2))/b^(9/2)/((b*x+a)^2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.70 (sec) , antiderivative size = 185, normalized size of antiderivative = 0.73

$$\int \frac{(d+ex)^{7/2}}{(a^2+2abx+b^2x^2)^{3/2}} dx = \frac{e^2(a+bx)^3 \left(-\frac{\sqrt{b}\sqrt{d+ex}(105a^3e^3+35a^2be^2(-4d+5ex)+7ab^2e(3d^2-34dex+8e^2x^2))+b^3(6d^3+39d^2e^2x-80d^2e^2x^2-8e^3x^3))}{e^2(a+bx)^2} + b^3(6d^3+39d^2e^2x-80d^2e^2x^2-8e^3x^3) \right)}{12b^{9/2}((a+bx)^2)^3}$$

input

```
Integrate[(d + e*x)^(7/2)/(a^2 + 2*a*b*x + b^2*x^2)^(3/2), x]
```

output

```
(e^2*(a + b*x)^3*(-((Sqrt[b]*Sqrt[d + e*x]*(105*a^3*e^3 + 35*a^2*b*e^2*(-4*d + 5*e*x) + 7*a*b^2*e*(3*d^2 - 34*d*e*x + 8*e^2*x^2) + b^3*(6*d^3 + 39*d^2*e*x - 80*d^2*e^2*x^2 - 8*e^3*x^3)))/(e^2*(a + b*x)^2)) + 105*(-(b*d) + a*e)^(3/2)*ArcTan[(Sqrt[b]*Sqrt[d + e*x])/Sqrt[-(b*d) + a*e]])/(12*b^(9/2)*((a + b*x)^2)^(3/2))
```

Rubi [A] (verified)

Time = 0.50 (sec) , antiderivative size = 179, normalized size of antiderivative = 0.70, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$, Rules used = {1102, 27, 51, 51, 60, 60, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\begin{aligned} & \int \frac{(d+ex)^{7/2}}{(a^2+2abx+b^2x^2)^{3/2}} dx \\ & \quad \downarrow 1102 \\ & \frac{b^3(a+bx) \int \frac{(d+ex)^{7/2}}{b^3(a+bx)^3} dx}{\sqrt{a^2+2abx+b^2x^2}} \\ & \quad \downarrow 27 \\ & \frac{(a+bx) \int \frac{(d+ex)^{7/2}}{(a+bx)^3} dx}{\sqrt{a^2+2abx+b^2x^2}} \\ & \quad \downarrow 51 \end{aligned}$$

$$\begin{aligned}
 & \frac{(a+bx) \left(\frac{7e \int \frac{(d+ex)^{5/2}}{(a+bx)^2} dx}{4b} - \frac{(d+ex)^{7/2}}{2b(a+bx)^2} \right)}{\sqrt{a^2 + 2abx + b^2x^2}} \\
 & \quad \downarrow 51 \\
 & \frac{(a+bx) \left(\frac{7e \left(\frac{5e \int \frac{(d+ex)^{3/2}}{a+bx} dx}{2b} - \frac{(d+ex)^{5/2}}{b(a+bx)} \right)}{4b} - \frac{(d+ex)^{7/2}}{2b(a+bx)^2} \right)}{\sqrt{a^2 + 2abx + b^2x^2}} \\
 & \quad \downarrow 60 \\
 & \frac{(a+bx) \left(\frac{7e \left(\frac{5e \left(\frac{(bd-ae) \int \frac{\sqrt{d+ex}}{b(a+bx)} dx}{b} + \frac{2(d+ex)^{3/2}}{3b} \right)}{2b} - \frac{(d+ex)^{5/2}}{b(a+bx)} \right)}{4b} - \frac{(d+ex)^{7/2}}{2b(a+bx)^2} \right)}{\sqrt{a^2 + 2abx + b^2x^2}} \\
 & \quad \downarrow 60 \\
 & \frac{(a+bx) \left(\frac{7e \left(\frac{5e \left(\frac{(bd-ae) \int \frac{1}{(a+bx)\sqrt{d+ex}} dx}{b} + \frac{2\sqrt{d+ex}}{b} \right)}{2b} + \frac{2(d+ex)^{3/2}}{3b} \right)}{4b} - \frac{(d+ex)^{5/2}}{b(a+bx)} \right)}{\sqrt{a^2 + 2abx + b^2x^2}} - \frac{(d+ex)^{7/2}}{2b(a+bx)^2} \\
 & \quad \downarrow 73
 \end{aligned}$$

$$\frac{(a+bx) \left(\frac{5e \left(\frac{(bd-ae) \int \frac{1}{a + \frac{b(d+ex)}{e} - \frac{bd}{e}} + \frac{2\sqrt{d+ex}}{b} \right)}{b} + \frac{2(d+ex)^{3/2}}{3b} \right)}{7e \cdot 2b} - \frac{(d+ex)^{5/2}}{b(a+bx)} \right)}{4b} - \frac{(d+ex)^{7/2}}{2b(a+bx)^2}$$

$$\sqrt{a^2 + 2abx + b^2x^2}$$

↓ 221

$$\frac{(a+bx) \left(\frac{5e \left(\frac{(bd-ae) \left(\frac{2\sqrt{d+ex}}{b} - \frac{2\sqrt{bd-ae} \operatorname{arctanh} \left(\frac{\sqrt{b}\sqrt{d+ex}}{\sqrt{bd-ae}} \right)}{b^{3/2}} \right)}{b} + \frac{2(d+ex)^{3/2}}{3b} \right)}{7e \cdot 2b} - \frac{(d+ex)^{5/2}}{b(a+bx)} \right)}{4b} - \frac{(d+ex)^{7/2}}{2b(a+bx)^2}$$

$$\sqrt{a^2 + 2abx + b^2x^2}$$

input $\text{Int}[(d + e*x)^{(7/2)} / (a^2 + 2*a*b*x + b^2*x^2)^{(3/2)}, x]$

output $((a + b*x)*(-1/2*(d + e*x)^{(7/2)} / (b*(a + b*x)^2) + (7*e*(-(d + e*x)^{(5/2)} / (b*(a + b*x)))) + (5*e*((2*(d + e*x)^{(3/2)}) / (3*b) + ((b*d - a*e)*((2*\text{Sqrt}[d + e*x]) / b - (2*\text{Sqrt}[b*d - a*e]*\text{ArcTanh}[(\text{Sqrt}[b]*\text{Sqrt}[d + e*x]) / \text{Sqrt}[b*d - a*e]]) / b^{(3/2)})) / (2*b))) / (4*b)) / \text{Sqrt}[a^2 + 2*a*b*x + b^2*x^2]$

Defintions of rubi rules used

rule 27 $\text{Int}[(a_)*(F_x_), x_Symbol] \rightarrow \text{Simp}[a \text{ Int}[F_x, x], x] /; \text{FreeQ}[a, x] \ \&\& \ !\text{MatchQ}[F_x, (b_)*(G_x_)] /; \text{FreeQ}[b, x]$

rule 51 $\text{Int}[(a_.) + (b_.)*(x_)^{(m_)}*((c_.) + (d_.)*(x_)^{(n_)}, x_Symbol] \rightarrow \text{Simp}[(a + b*x)^{(m + 1)}*((c + d*x)^n / (b*(m + 1))), x] - \text{Simp}[d*(n / (b*(m + 1)) \text{Int}[(a + b*x)^{(m + 1)}*(c + d*x)^{(n - 1)}, x], x] /; \text{FreeQ}\{a, b, c, d, n\}, x] \ \&\& \ \text{ILtQ}[m, -1] \ \&\& \ \text{FractionQ}[n] \ \&\& \ \text{GtQ}[n, 0]$

rule 60 $\text{Int}[(a_.) + (b_.)*(x_)^{(m_)}*((c_.) + (d_.)*(x_)^{(n_)}, x_Symbol] \rightarrow \text{Simp}[(a + b*x)^{(m + 1)}*((c + d*x)^n / (b*(m + n + 1))), x] + \text{Simp}[n*(b*c - a*d) / (b*(m + n + 1)) \text{Int}[(a + b*x)^m*(c + d*x)^{(n - 1)}, x], x] /; \text{FreeQ}\{a, b, c, d\}, x] \ \&\& \ \text{GtQ}[n, 0] \ \&\& \ \text{NeQ}[m + n + 1, 0] \ \&\& \ !(\text{IGtQ}[m, 0] \ \&\& \ (!\text{IntegerQ}[n] \ || \ (\text{GtQ}[m, 0] \ \&\& \ \text{LtQ}[m - n, 0])) \ \&\& \ !\text{ILtQ}[m + n + 2, 0] \ \&\& \ \text{IntLinearQ}[a, b, c, d, m, n, x]$

rule 73 $\text{Int}[(a_.) + (b_.)*(x_)^{(m_)}*((c_.) + (d_.)*(x_)^{(n_)}, x_Symbol] \rightarrow \text{With}\{p = \text{Denominator}[m]\}, \text{Simp}[p/b \ \text{Subst}[\text{Int}[x^{(p*(m + 1) - 1)}*(c - a*(d/b) + d*(x^p/b)^n], x], x, (a + b*x)^{(1/p)}, x]] /; \text{FreeQ}\{a, b, c, d\}, x] \ \&\& \ \text{LtQ}[-1, m, 0] \ \&\& \ \text{LeQ}[-1, n, 0] \ \&\& \ \text{LeQ}[\text{Denominator}[n], \text{Denominator}[m]] \ \&\& \ \text{IntLinearQ}[a, b, c, d, m, n, x]$

rule 221 $\text{Int}[(a_.) + (b_.)*(x_)^2)^{-1}, x_Symbol] \rightarrow \text{Simp}[(\text{Rt}[-a/b, 2] / a) * \text{ArcTanh}[x / \text{Rt}[-a/b, 2]], x] /; \text{FreeQ}\{a, b\}, x] \ \&\& \ \text{NegQ}[a/b]$

rule 1102

```
Int[((d_.) + (e_.)*(x_.))^(m_.)*((a_.) + (b_.)*(x_.) + (c_.)*(x_.)^2)^(p_), x_Symbol]
:> Simp[(a + b*x + c*x^2)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x)^(2*FracPart[p]))
Int[(d + e*x)^m*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && EqQ[b^2 - 4*a*c, 0]
```

Maple [A] (verified)

Time = 1.55 (sec) , antiderivative size = 176, normalized size of antiderivative = 0.69

| method | result |
|---------|--|
| risch | $-\frac{2e^2(-bex+9ae-10bd)\sqrt{ex+d}\sqrt{(bx+a)^2}}{3b^4(bx+a)} + \frac{(2e^2a^2-4abde+2b^2d^2)e^2\left(-\frac{13(ex+d)^{\frac{3}{2}}b}{8} + \left(-\frac{11ae}{8} + \frac{11bd}{8}\right)\sqrt{ex+d} + \frac{35\arctan\left(\frac{b\sqrt{ex+d}}{\sqrt{b(ae-bd)}}\right)}{8\sqrt{b(ae-bd)}}\right)}{b^4(bx+a)}$ |
| default | $-\frac{\left(-8(ex+d)^{\frac{3}{2}}\sqrt{b(ae-bd)}b^3e^2x^2-105\arctan\left(\frac{b\sqrt{ex+d}}{\sqrt{b(ae-bd)}}\right)\right)a^2b^2e^4x^2+210\arctan\left(\frac{b\sqrt{ex+d}}{\sqrt{b(ae-bd)}}\right)ab^3de^3x^2-105\arctan\left(\frac{b\sqrt{ex+d}}{\sqrt{b(ae-bd)}}\right)}{b^4(bx+a)}$ |

input

```
int((e*x+d)^(7/2)/(b^2*x^2+2*a*b*x+a^2)^(3/2),x,method=_RETURNVERBOSE)
```

output

```
-2/3*e^2*(-b*e*x+9*a*e-10*b*d)*(e*x+d)^(1/2)/b^4*((b*x+a)^2)^(1/2)/(b*x+a)
+1/b^4*(2*a^2*e^2-4*a*b*d*e+2*b^2*d^2)*e^2*((-13/8*(e*x+d)^(3/2)*b+(-11/8*
a*e+11/8*b*d)*(e*x+d)^(1/2))/(b*(e*x+d)+a*e-b*d)^2+35/8/(b*(a*e-b*d))^(1/2)
)*arctan(b*(e*x+d)^(1/2)/(b*(a*e-b*d))^(1/2))*((b*x+a)^2)^(1/2)/(b*x+a)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 520, normalized size of antiderivative = 2.05

$$\int \frac{(d + ex)^{7/2}}{(a^2 + 2abx + b^2x^2)^{3/2}} dx = \left[\frac{105(a^2bde^2 - a^3e^3 + (b^3de^2 - ab^2e^3)x^2 + 2(ab^2de^2 - a^2be^3)x)\sqrt{\frac{bd-ae}{b}}}{(a^2 + 2abx + b^2x^2)^{3/2}} \right]$$

input

```
integrate((e*x+d)^(7/2)/(b^2*x^2+2*a*b*x+a^2)^(3/2),x, algorithm="fricas")
```

output

```
[-1/24*(105*(a^2*b*d*e^2 - a^3*e^3 + (b^3*d*e^2 - a*b^2*e^3)*x^2 + 2*(a*b^2*d*e^2 - a^2*b*e^3)*x)*sqrt((b*d - a*e)/b)*log((b*e*x + 2*b*d - a*e + 2*sqrt(e*x + d)*b*sqrt((b*d - a*e)/b))/(b*x + a)) - 2*(8*b^3*e^3*x^3 - 6*b^3*d^3 - 21*a*b^2*d^2*e + 140*a^2*b*d*e^2 - 105*a^3*e^3 + 8*(10*b^3*d*e^2 - 7*a*b^2*e^3)*x^2 - (39*b^3*d^2*e - 238*a*b^2*d*e^2 + 175*a^2*b*e^3)*x)*sqrt(e*x + d))/(b^6*x^2 + 2*a*b^5*x + a^2*b^4), -1/12*(105*(a^2*b*d*e^2 - a^3*e^3 + (b^3*d*e^2 - a*b^2*e^3)*x^2 + 2*(a*b^2*d*e^2 - a^2*b*e^3)*x)*sqrt(-(b*d - a*e)/b)*arctan(-sqrt(e*x + d)*b*sqrt(-(b*d - a*e)/b)/(b*d - a*e)) - (8*b^3*e^3*x^3 - 6*b^3*d^3 - 21*a*b^2*d^2*e + 140*a^2*b*d*e^2 - 105*a^3*e^3 + 8*(10*b^3*d*e^2 - 7*a*b^2*e^3)*x^2 - (39*b^3*d^2*e - 238*a*b^2*d*e^2 + 175*a^2*b*e^3)*x)*sqrt(e*x + d))/(b^6*x^2 + 2*a*b^5*x + a^2*b^4)]
```

Sympy [F]

$$\int \frac{(d + ex)^{7/2}}{(a^2 + 2abx + b^2x^2)^{3/2}} dx = \int \frac{(d + ex)^{7/2}}{((a + bx)^2)^{3/2}} dx$$

input

```
integrate((e*x+d)**(7/2)/(b**2*x**2+2*a*b*x+a**2)**(3/2), x)
```

output

```
Integral((d + e*x)**(7/2)/((a + b*x)**2)**(3/2), x)
```

Maxima [F]

$$\int \frac{(d + ex)^{7/2}}{(a^2 + 2abx + b^2x^2)^{3/2}} dx = \int \frac{(ex + d)^{7/2}}{(b^2x^2 + 2abx + a^2)^{3/2}} dx$$

input

```
integrate((e*x+d)^(7/2)/(b^2*x^2+2*a*b*x+a^2)^(3/2), x, algorithm="maxima")
```

output

```
integrate((e*x + d)^(7/2)/(b^2*x^2 + 2*a*b*x + a^2)^(3/2), x)
```

Giac [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 287, normalized size of antiderivative = 1.13

$$\int \frac{(d+ex)^{7/2}}{(a^2+2abx+b^2x^2)^{3/2}} dx = \frac{35(b^2d^2e^2 - 2abde^3 + a^2e^4) \arctan\left(\frac{\sqrt{ex+db}}{\sqrt{-b^2d+abe}}\right)}{4\sqrt{-b^2d+abe}b^4\operatorname{sgn}(bx+a)} - \frac{13(ex+d)^{\frac{3}{2}}b^3d^2e^2 - 11\sqrt{ex+db}b^3d^3e^2 - 26(ex+d)^{\frac{3}{2}}ab^2de^3 + 33\sqrt{ex+db}ab^2d^2e^3 + 13(ex+d)^{\frac{3}{2}}a^2be^4}{4((ex+d)b - bd + ae)^2b^4\operatorname{sgn}(bx+a)} + \frac{2\left((ex+d)^{\frac{3}{2}}b^6e^2 + 9\sqrt{ex+db}b^6de^2 - 9\sqrt{ex+db}ab^5e^3\right)}{3b^9\operatorname{sgn}(bx+a)}$$

input `integrate((e*x+d)^(7/2)/(b^2*x^2+2*a*b*x+a^2)^(3/2),x, algorithm="giac")`output `35/4*(b^2*d^2*e^2 - 2*a*b*d*e^3 + a^2*e^4)*arctan(sqrt(e*x + d)*b/sqrt(-b^2*d + a*b*e))/(sqrt(-b^2*d + a*b*e)*b^4*sgn(b*x + a)) - 1/4*(13*(e*x + d)^(3/2)*b^3*d^2*e^2 - 11*sqrt(e*x + d)*b^3*d^3*e^2 - 26*(e*x + d)^(3/2)*a*b^2*d*e^3 + 33*sqrt(e*x + d)*a*b^2*d^2*e^3 + 13*(e*x + d)^(3/2)*a^2*b*e^4 - 33*sqrt(e*x + d)*a^2*b*d*e^4 + 11*sqrt(e*x + d)*a^3*e^5)/(((e*x + d)*b - b*d + a*e)^2*b^4*sgn(b*x + a)) + 2/3*((e*x + d)^(3/2)*b^6*e^2 + 9*sqrt(e*x + d)*b^6*d*e^2 - 9*sqrt(e*x + d)*a*b^5*e^3)/(b^9*sgn(b*x + a))`**Mupad [F(-1)]**

Timed out.

$$\int \frac{(d+ex)^{7/2}}{(a^2+2abx+b^2x^2)^{3/2}} dx = \int \frac{(d+ex)^{7/2}}{(a^2+2abx+b^2x^2)^{3/2}} dx$$

input `int((d + e*x)^(7/2)/(a^2 + b^2*x^2 + 2*a*b*x)^(3/2),x)`output `int((d + e*x)^(7/2)/(a^2 + b^2*x^2 + 2*a*b*x)^(3/2), x)`

Reduce [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 464, normalized size of antiderivative = 1.83

$$\int \frac{(d+ex)^{7/2}}{(a^2+2abx+b^2x^2)^{3/2}} dx = \frac{105\sqrt{b}\sqrt{ae-bd} \operatorname{atan}\left(\frac{\sqrt{ex+db}}{\sqrt{b}\sqrt{ae-bd}}\right) a^3 e^3 - 105\sqrt{b}\sqrt{ae-bd} \operatorname{atan}\left(\frac{\sqrt{ex+db}}{\sqrt{b}\sqrt{ae-bd}}\right) c}{1}$$

input

```
int((e*x+d)^(7/2)/(b^2*x^2+2*a*b*x+a^2)^(3/2),x)
```

output

```
(105*sqrt(b)*sqrt(a*e - b*d)*atan((sqrt(d + e*x)*b)/(sqrt(b)*sqrt(a*e - b*d)))
*a**3*e**3 - 105*sqrt(b)*sqrt(a*e - b*d)*atan((sqrt(d + e*x)*b)/(sqrt(b)*sqrt(a*e - b*d)))
*a**2*b*d*e**2 + 210*sqrt(b)*sqrt(a*e - b*d)*atan((sqrt(d + e*x)*b)/(sqrt(b)*sqrt(a*e - b*d)))
*a**2*b*e**3*x - 210*sqrt(b)*sqrt(a*e - b*d)*atan((sqrt(d + e*x)*b)/(sqrt(b)*sqrt(a*e - b*d)))
*a*b**2*d*e**2*x + 105*sqrt(b)*sqrt(a*e - b*d)*atan((sqrt(d + e*x)*b)/(sqrt(b)*sqrt(a*e - b*d)))
*a*b**2*e**3*x**2 - 105*sqrt(b)*sqrt(a*e - b*d)*atan((sqrt(d + e*x)*b)/(sqrt(b)*sqrt(a*e - b*d)))
*b**3*d*e**2*x**2 - 105*sqrt(d + e*x)*a**3*b*e**3 + 140*sqrt(d + e*x)*a**2*b**2*d*e**2
- 175*sqrt(d + e*x)*a**2*b**2*e**3*x - 21*sqrt(d + e*x)*a*b**3*d**2*e + 238*sqrt(d + e*x)*a*b**3*d*e**2*x
- 56*sqrt(d + e*x)*a*b**3*e**3*x**2 - 6*sqrt(d + e*x)*b**4*d**3 - 39*sqrt(d + e*x)*b**4*d**2*e*x
+ 80*sqrt(d + e*x)*b**4*d*e**2*x**2 + 8*sqrt(d + e*x)*b**4*e**3*x**3)/(12*b**5*(a**2 + 2*a*b*x + b**2*x**2))
```

3.381 $\int \frac{(d+ex)^{5/2}}{(a^2+2abx+b^2x^2)^{3/2}} dx$

| | |
|---|------|
| Optimal result | 2816 |
| Mathematica [A] (verified) | 2816 |
| Rubi [A] (verified) | 2817 |
| Maple [A] (verified) | 2820 |
| Fricas [A] (verification not implemented) | 2820 |
| Sympy [F] | 2821 |
| Maxima [F] | 2821 |
| Giac [A] (verification not implemented) | 2822 |
| Mupad [F(-1)] | 2822 |
| Reduce [B] (verification not implemented) | 2823 |

Optimal result

Integrand size = 30, antiderivative size = 202

$$\int \frac{(d+ex)^{5/2}}{(a^2+2abx+b^2x^2)^{3/2}} dx = \frac{15e^2(a+bx)\sqrt{d+ex}}{4b^3\sqrt{a^2+2abx+b^2x^2}} - \frac{5e(d+ex)^{3/2}}{4b^2\sqrt{a^2+2abx+b^2x^2}} - \frac{(d+ex)^{5/2}}{2b(a+bx)\sqrt{a^2+2abx+b^2x^2}} - \frac{15e^2\sqrt{bd-ae}(a+bx)\operatorname{arctanh}\left(\frac{\sqrt{b}\sqrt{d+ex}}{\sqrt{bd-ae}}\right)}{4b^{7/2}\sqrt{a^2+2abx+b^2x^2}}$$

output

```
15/4*e^2*(b*x+a)*(e*x+d)^(1/2)/b^3/((b*x+a)^2)^(1/2)-5/4*e*(e*x+d)^(3/2)/b
^(2/((b*x+a)^2)^(1/2)-1/2*(e*x+d)^(5/2)/b/(b*x+a)/((b*x+a)^2)^(1/2)-15/4*e^
2*(-a*e+b*d)^(1/2)*(b*x+a)*arctanh(b^(1/2)*(e*x+d)^(1/2)/(-a*e+b*d)^(1/2))
/b^(7/2)/((b*x+a)^2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.45 (sec) , antiderivative size = 138, normalized size of antiderivative = 0.68

$$\int \frac{(d+ex)^{5/2}}{(a^2+2abx+b^2x^2)^{3/2}} dx = \frac{\sqrt{b}\sqrt{d+ex}(15a^2e^2-5abe(d-5ex)+b^2(-2d^2-9dex+8e^2x^2))-15e^2\sqrt{d+ex}}{4b^{7/2}(a+bx)\sqrt{(a+bx)^2}}$$

input

```
Integrate[(d + e*x)^(5/2)/(a^2 + 2*a*b*x + b^2*x^2)^(3/2), x]
```

output

```
(Sqrt[b]*Sqrt[d + e*x]*(15*a^2*e^2 - 5*a*b*e*(d - 5*e*x) + b^2*(-2*d^2 - 9
*d*e*x + 8*e^2*x^2)) - 15*e^2*Sqrt[-(b*d) + a*e]*(a + b*x)^2*ArcTan[(Sqrt[
b]*Sqrt[d + e*x])/Sqrt[-(b*d) + a*e]])/(4*b^(7/2)*(a + b*x)*Sqrt[(a + b*x)
^2])
```

Rubi [A] (verified)

Time = 0.46 (sec) , antiderivative size = 150, normalized size of antiderivative = 0.74, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.233$, Rules used = {1102, 27, 51, 51, 60, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\begin{aligned}
 & \int \frac{(d+ex)^{5/2}}{(a^2+2abx+b^2x^2)^{3/2}} dx \\
 & \quad \downarrow \text{1102} \\
 & \frac{b^3(a+bx) \int \frac{(d+ex)^{5/2}}{b^3(a+bx)^3} dx}{\sqrt{a^2+2abx+b^2x^2}} \\
 & \quad \downarrow \text{27} \\
 & \frac{(a+bx) \int \frac{(d+ex)^{5/2}}{(a+bx)^3} dx}{\sqrt{a^2+2abx+b^2x^2}} \\
 & \quad \downarrow \text{51} \\
 & \frac{(a+bx) \left(\frac{5e \int \frac{(d+ex)^{3/2}}{(a+bx)^2} dx}{4b} - \frac{(d+ex)^{5/2}}{2b(a+bx)^2} \right)}{\sqrt{a^2+2abx+b^2x^2}} \\
 & \quad \downarrow \text{51} \\
 & \frac{(a+bx) \left(\frac{5e \left(\frac{3e \int \frac{\sqrt{d+ex}}{a+bx} dx}{2b} - \frac{(d+ex)^{3/2}}{b(a+bx)} \right)}{4b} - \frac{(d+ex)^{5/2}}{2b(a+bx)^2} \right)}{\sqrt{a^2+2abx+b^2x^2}} \\
 & \quad \downarrow \text{60}
 \end{aligned}$$

$$\begin{aligned}
 & \frac{(a+bx) \left(\frac{5e \left(\frac{3e \left(\frac{(bd-ae) \int \frac{1}{(a+bx)\sqrt{d+ex}} dx + \frac{2\sqrt{d+ex}}{b} \right)}{2b} - \frac{(d+ex)^{3/2}}{b(a+bx)} \right)}{4b} - \frac{(d+ex)^{5/2}}{2b(a+bx)^2} \right)}{\sqrt{a^2 + 2abx + b^2x^2}} \\
 & \quad \downarrow \text{73} \\
 & \frac{(a+bx) \left(\frac{5e \left(\frac{3e \left(\frac{2(bd-ae) \int \frac{1}{a + \frac{b(d+ex)}{be} - \frac{bd}{e}} d\sqrt{d+ex}}{2b} + \frac{2\sqrt{d+ex}}{b} \right)}{2b} - \frac{(d+ex)^{3/2}}{b(a+bx)} \right)}{4b} - \frac{(d+ex)^{5/2}}{2b(a+bx)^2} \right)}{\sqrt{a^2 + 2abx + b^2x^2}} \\
 & \quad \downarrow \text{221} \\
 & \frac{(a+bx) \left(\frac{5e \left(\frac{3e \left(\frac{2\sqrt{d+ex}}{b} - \frac{2\sqrt{bd-ae} \operatorname{arctanh}\left(\frac{\sqrt{b}\sqrt{d+ex}}{\sqrt{bd-ae}}\right)}{b^{3/2}} \right)}{2b} - \frac{(d+ex)^{3/2}}{b(a+bx)} \right)}{4b} - \frac{(d+ex)^{5/2}}{2b(a+bx)^2} \right)}{\sqrt{a^2 + 2abx + b^2x^2}}
 \end{aligned}$$

input `Int[(d + e*x)^(5/2)/(a^2 + 2*a*b*x + b^2*x^2)^(3/2),x]`

output `((a + b*x)*(-1/2*(d + e*x)^(5/2)/(b*(a + b*x)^2) + (5*e*(-((d + e*x)^(3/2)/(b*(a + b*x)))) + (3*e*((2*sqrt[d + e*x])/b - (2*sqrt[b*d - a*e]*ArcTanh[(sqrt[b]*sqrt[d + e*x])/sqrt[b*d - a*e]])/b^(3/2)))/(2*b)))/(4*b))/sqrt[a^2 + 2*a*b*x + b^2*x^2]`

Definitions of rubi rules used

rule 27 `Int[(a_)*(F x_), x_Symbol] := Simp[a Int[F x, x], x] /; FreeQ[a, x] && !MatchQ[F x, (b_)*(G x_) /; FreeQ[b, x]]`

rule 51 `Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] := Simp[(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Simp[d*(n/(b*(m + 1))) Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d, n}, x] && ILtQ[m, -1] && FractionQ[n] && GtQ[n, 0]`

rule 60 `Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] := Simp[(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(b*(m + n + 1))) Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] && !(IGtQ[m, 0] && (!IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) && !ILtQ[m + n + 2, 0] && IntLinearQ[a, b, c, d, m, n, x]`

rule 73 `Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] := With[{p = Denominator[m]}, Simp[p/b Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) + d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntLinearQ[a, b, c, d, m, n, x]`

rule 221 `Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[-a/b, 2]/a)*ArcTanh[x/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]`

rule 1102 `Int[((d_.) + (e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] := Simp[(a + b*x + c*x^2)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x)^(2*FracPart[p])) Int[(d + e*x)^m*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && EqQ[b^2 - 4*a*c, 0]`

Maple [A] (verified)

Time = 1.57 (sec) , antiderivative size = 149, normalized size of antiderivative = 0.74

| method | result |
|---------|--|
| risch | $\frac{2e^2\sqrt{ex+d}\sqrt{(bx+a)^2}}{b^3(bx+a)} - \frac{(2ae-2bd)e^2\left(\frac{-\frac{9}{8}(ex+d)^{\frac{3}{2}}b + (-\frac{7ae}{8} + \frac{7bd}{8})\sqrt{ex+d}}{(b(ex+d)+ae-bd)^2} + \frac{15\arctan\left(\frac{b\sqrt{ex+d}}{\sqrt{b(ae-bd)}}\right)}{8\sqrt{b(ae-bd)}}\right)\sqrt{(bx+a)^2}}{b^3(bx+a)}$ |
| default | $\frac{\left(-15\arctan\left(\frac{b\sqrt{ex+d}}{\sqrt{b(ae-bd)}}\right)a b^2 e^3 x^2 + 15\arctan\left(\frac{b\sqrt{ex+d}}{\sqrt{b(ae-bd)}}\right)b^3 d e^2 x^2 + 8b^2 e^2 x^2 \sqrt{ex+d} \sqrt{b(ae-bd)} - 30\arctan\left(\frac{b\sqrt{ex+d}}{\sqrt{b(ae-bd)}}\right)a^2 b e^3\right)}{b^3(bx+a)}$ |

```
input int((e*x+d)^(5/2)/(b^2*x^2+2*a*b*x+a^2)^(3/2),x,method=_RETURNVERBOSE)
```

```
output 2/b^3*e^2*(e*x+d)^(1/2)*((b*x+a)^2)^(1/2)/(b*x+a)-1/b^3*(2*a*e-2*b*d)*e^2*
((-9/8*(e*x+d)^(3/2)*b+(-7/8*a*e+7/8*b*d)*(e*x+d)^(1/2))/(b*(e*x+d)+a*e-b*
d)^2+15/8/(b*(a*e-b*d))^(1/2)*arctan(b*(e*x+d)^(1/2)/(b*(a*e-b*d))^(1/2))
*((b*x+a)^2)^(1/2)/(b*x+a)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 344, normalized size of antiderivative = 1.70

$$\int \frac{(d+ex)^{5/2}}{(a^2+2abx+b^2x^2)^{3/2}} dx = \left[\frac{15(b^2e^2x^2+2abe^2x+a^2e^2)\sqrt{\frac{bd-ae}{b}} \log\left(\frac{beax+2bd-ae-2\sqrt{ex+db}\sqrt{\frac{bd-ae}{b}}}{bx+a}\right) + 2(\dots)}{8(b^5x^2+2ab^4x+\dots)} \right]$$

```
input integrate((e*x+d)^(5/2)/(b^2*x^2+2*a*b*x+a^2)^(3/2),x, algorithm="fricas")
```

output

```
[1/8*(15*(b^2*e^2*x^2 + 2*a*b*e^2*x + a^2*e^2)*sqrt((b*d - a*e)/b)*log((b*
e*x + 2*b*d - a*e - 2*sqrt(e*x + d)*b*sqrt((b*d - a*e)/b))/(b*x + a)) + 2*
(8*b^2*e^2*x^2 - 2*b^2*d^2 - 5*a*b*d*e + 15*a^2*e^2 - (9*b^2*d*e - 25*a*b*
e^2)*x)*sqrt(e*x + d))/(b^5*x^2 + 2*a*b^4*x + a^2*b^3), -1/4*(15*(b^2*e^2*
x^2 + 2*a*b*e^2*x + a^2*e^2)*sqrt(-(b*d - a*e)/b)*arctan(-sqrt(e*x + d)*b*
sqrt(-(b*d - a*e)/b)/(b*d - a*e)) - (8*b^2*e^2*x^2 - 2*b^2*d^2 - 5*a*b*d*e
+ 15*a^2*e^2 - (9*b^2*d*e - 25*a*b*e^2)*x)*sqrt(e*x + d))/(b^5*x^2 + 2*a*
b^4*x + a^2*b^3)]
```

Sympy [F]

$$\int \frac{(d + ex)^{5/2}}{(a^2 + 2abx + b^2x^2)^{3/2}} dx = \int \frac{(d + ex)^{5/2}}{((a + bx)^2)^{3/2}} dx$$

input

```
integrate((e*x+d)**(5/2)/(b**2*x**2+2*a*b*x+a**2)**(3/2),x)
```

output

```
Integral((d + e*x)**(5/2)/((a + b*x)**2)**(3/2), x)
```

Maxima [F]

$$\int \frac{(d + ex)^{5/2}}{(a^2 + 2abx + b^2x^2)^{3/2}} dx = \int \frac{(ex + d)^{5/2}}{(b^2x^2 + 2abx + a^2)^{3/2}} dx$$

input

```
integrate((e*x+d)^(5/2)/(b^2*x^2+2*a*b*x+a^2)^(3/2),x, algorithm="maxima")
```

output

```
integrate((e*x + d)^(5/2)/(b^2*x^2 + 2*a*b*x + a^2)^(3/2), x)
```

Giac [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 195, normalized size of antiderivative = 0.97

$$\int \frac{(d+ex)^{5/2}}{(a^2+2abx+b^2x^2)^{3/2}} dx = \frac{2\sqrt{ex+de^2}}{b^3\operatorname{sgn}(bx+a)} + \frac{15(bde^2-ae^3)\arctan\left(\frac{\sqrt{ex+db}}{\sqrt{-b^2d+abeb^3\operatorname{sgn}(bx+a)}}\right)}{4\sqrt{-b^2d+abeb^3\operatorname{sgn}(bx+a)}} \\ - \frac{9(ex+d)^{3/2}b^2de^2 - 7\sqrt{ex+db^2d^2e^2} - 9(ex+d)^{3/2}abe^3 + 14\sqrt{ex+db}de^3 - 7\sqrt{ex+da^2e^4}}{4((ex+d)b-bd+ae)^2b^3\operatorname{sgn}(bx+a)}$$

input `integrate((e*x+d)^(5/2)/(b^2*x^2+2*a*b*x+a^2)^(3/2),x, algorithm="giac")`

output `2*sqrt(e*x + d)*e^2/(b^3*sgn(b*x + a)) + 15/4*(b*d*e^2 - a*e^3)*arctan(sqrt(e*x + d)*b/sqrt(-b^2*d + a*b*e))/(sqrt(-b^2*d + a*b*e)*b^3*sgn(b*x + a)) - 1/4*(9*(e*x + d)^(3/2)*b^2*d*e^2 - 7*sqrt(e*x + d)*b^2*d^2*e^2 - 9*(e*x + d)^(3/2)*a*b*e^3 + 14*sqrt(e*x + d)*a*b*d*e^3 - 7*sqrt(e*x + d)*a^2*e^4)/(((e*x + d)*b - b*d + a*e)^2*b^3*sgn(b*x + a))`

Mupad [F(-1)]

Timed out.

$$\int \frac{(d+ex)^{5/2}}{(a^2+2abx+b^2x^2)^{3/2}} dx = \int \frac{(d+ex)^{5/2}}{(a^2+2abx+b^2x^2)^{3/2}} dx$$

input `int((d + e*x)^(5/2)/(a^2 + b^2*x^2 + 2*a*b*x)^(3/2),x)`

output `int((d + e*x)^(5/2)/(a^2 + b^2*x^2 + 2*a*b*x)^(3/2), x)`

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 246, normalized size of antiderivative = 1.22

$$\int \frac{(d+ex)^{5/2}}{(a^2+2abx+b^2x^2)^{3/2}} dx = \frac{-15\sqrt{b}\sqrt{ae-bd} \operatorname{atan}\left(\frac{\sqrt{ex+db}}{\sqrt{b}\sqrt{ae-bd}}\right) a^2 e^2 - 30\sqrt{b}\sqrt{ae-bd} \operatorname{atan}\left(\frac{\sqrt{ex+db}}{\sqrt{b}\sqrt{ae-bd}}\right) a}{(a^2+2abx+b^2x^2)^{3/2}}$$

input

```
int((e*x+d)^(5/2)/(b^2*x^2+2*a*b*x+a^2)^(3/2),x)
```

output

```
( - 15*sqrt(b)*sqrt(a*e - b*d)*atan((sqrt(d + e*x)*b)/(sqrt(b)*sqrt(a*e -
b*d)))**2*e**2 - 30*sqrt(b)*sqrt(a*e - b*d)*atan((sqrt(d + e*x)*b)/(sqrt
(b)*sqrt(a*e - b*d)))**2*x - 15*sqrt(b)*sqrt(a*e - b*d)*atan((sqrt(d
+ e*x)*b)/(sqrt(b)*sqrt(a*e - b*d)))**2*e**2*x**2 + 15*sqrt(d + e*x)*a*
*2*b*e**2 - 5*sqrt(d + e*x)*a*b**2*d*e + 25*sqrt(d + e*x)*a*b**2*e**2*x -
2*sqrt(d + e*x)*b**3*d**2 - 9*sqrt(d + e*x)*b**3*d*e*x + 8*sqrt(d + e*x)*b
**3*e**2*x**2)/(4*b**4*(a**2 + 2*a*b*x + b**2*x**2))
```

3.382
$$\int \frac{(d+ex)^{3/2}}{(a^2+2abx+b^2x^2)^{3/2}} dx$$

| | |
|---|------|
| Optimal result | 2824 |
| Mathematica [A] (verified) | 2824 |
| Rubi [A] (verified) | 2825 |
| Maple [A] (verified) | 2827 |
| Fricas [A] (verification not implemented) | 2827 |
| Sympy [F] | 2828 |
| Maxima [F] | 2828 |
| Giac [A] (verification not implemented) | 2829 |
| Mupad [F(-1)] | 2829 |
| Reduce [B] (verification not implemented) | 2829 |

Optimal result

Integrand size = 30, antiderivative size = 158

$$\int \frac{(d+ex)^{3/2}}{(a^2+2abx+b^2x^2)^{3/2}} dx = -\frac{3e\sqrt{d+ex}}{4b^2\sqrt{a^2+2abx+b^2x^2}} - \frac{(d+ex)^{3/2}}{2b(a+bx)\sqrt{a^2+2abx+b^2x^2}} - \frac{3e^2(a+bx)\operatorname{arctanh}\left(\frac{\sqrt{b}\sqrt{d+ex}}{\sqrt{bd-ae}}\right)}{4b^{5/2}\sqrt{bd-ae}\sqrt{a^2+2abx+b^2x^2}}$$

output

$$-3/4*e*(e*x+d)^{(1/2)}/b^2/((b*x+a)^2)^{(1/2)}-1/2*(e*x+d)^{(3/2)}/b/(b*x+a)/((b*x+a)^2)^{(1/2)}-3/4*e^2*(b*x+a)*\operatorname{arctanh}(b^{(1/2)}*(e*x+d)^{(1/2)}/(-a*e+b*d)^{(1/2)})/b^{(5/2)}/(-a*e+b*d)^{(1/2)}/((b*x+a)^2)^{(1/2)}$$

Mathematica [A] (verified)

Time = 0.36 (sec) , antiderivative size = 110, normalized size of antiderivative = 0.70

$$\int \frac{(d+ex)^{3/2}}{(a^2+2abx+b^2x^2)^{3/2}} dx = \frac{-\sqrt{b}\sqrt{d+ex}(2bd+3ae+5bex) + \frac{3e^2(a+bx)^2 \operatorname{arctan}\left(\frac{\sqrt{b}\sqrt{d+ex}}{\sqrt{-bd+ae}}\right)}{\sqrt{-bd+ae}}}{4b^{5/2}(a+bx)\sqrt{(a+bx)^2}}$$

input

`Integrate[(d + e*x)^(3/2)/(a^2 + 2*a*b*x + b^2*x^2)^(3/2), x]`

output

```
(-(Sqrt[b]*Sqrt[d + e*x]*(2*b*d + 3*a*e + 5*b*e*x)) + (3*e^2*(a + b*x)^2*ArcTan[(Sqrt[b]*Sqrt[d + e*x])/Sqrt[-(b*d) + a*e]])/Sqrt[-(b*d) + a*e])/(4*b^(5/2)*(a + b*x)*Sqrt[(a + b*x)^2])
```

Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 128, normalized size of antiderivative = 0.81, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {1102, 27, 51, 51, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\begin{aligned}
 & \int \frac{(d+ex)^{3/2}}{(a^2+2abx+b^2x^2)^{3/2}} dx \\
 & \quad \downarrow \text{1102} \\
 & \frac{b^3(a+bx) \int \frac{(d+ex)^{3/2}}{b^3(a+bx)^3} dx}{\sqrt{a^2+2abx+b^2x^2}} \\
 & \quad \downarrow \text{27} \\
 & \frac{(a+bx) \int \frac{(d+ex)^{3/2}}{(a+bx)^3} dx}{\sqrt{a^2+2abx+b^2x^2}} \\
 & \quad \downarrow \text{51} \\
 & \frac{(a+bx) \left(\frac{3e \int \frac{\sqrt{d+ex}}{(a+bx)^2} dx}{4b} - \frac{(d+ex)^{3/2}}{2b(a+bx)^2} \right)}{\sqrt{a^2+2abx+b^2x^2}} \\
 & \quad \downarrow \text{51} \\
 & \frac{(a+bx) \left(\frac{3e \left(\frac{e \int \frac{1}{(a+bx)\sqrt{d+ex}} dx}{2b} - \frac{\sqrt{d+ex}}{b(a+bx)} \right)}{4b} - \frac{(d+ex)^{3/2}}{2b(a+bx)^2} \right)}{\sqrt{a^2+2abx+b^2x^2}} \\
 & \quad \downarrow \text{73}
 \end{aligned}$$

$$\frac{(a+bx) \left(\frac{3e \left(\frac{\int \frac{1}{a + \frac{b(d+ex)}{e} - \frac{bd}{e}} d\sqrt{d+ex}}{4b} - \frac{\sqrt{d+ex}}{b(a+bx)} \right)}{\sqrt{a^2 + 2abx + b^2x^2}} - \frac{(d+ex)^{3/2}}{2b(a+bx)^2} \right)}{\sqrt{a^2 + 2abx + b^2x^2}}$$

↓ 221

$$\frac{(a+bx) \left(\frac{3e \left(-\frac{e \operatorname{arctanh} \left(\frac{\sqrt{b}\sqrt{d+ex}}{\sqrt{bd-ae}} \right)}{b^{3/2}\sqrt{bd-ae}} - \frac{\sqrt{d+ex}}{b(a+bx)} \right)}{4b} - \frac{(d+ex)^{3/2}}{2b(a+bx)^2} \right)}{\sqrt{a^2 + 2abx + b^2x^2}}$$

input `Int[(d + e*x)^(3/2)/(a^2 + 2*a*b*x + b^2*x^2)^(3/2),x]`

output `((a + b*x)*(-1/2*(d + e*x)^(3/2)/(b*(a + b*x)^2) + (3*e*(-(Sqrt[d + e*x]/(b*(a + b*x)))) - (e*ArcTanh[(Sqrt[b]*Sqrt[d + e*x])/Sqrt[b*d - a*e]])/(b^(3/2)*Sqrt[b*d - a*e]))/(4*b))/Sqrt[a^2 + 2*a*b*x + b^2*x^2]`

Defintions of rubi rules used

rule 27 `Int[(a_)*(F_x_), x_Symbol] := Simp[a Int[F_x, x], x] /; FreeQ[a, x] && !MatchQ[F_x, (b_)*(G_x_) /; FreeQ[b, x]]`

rule 51 `Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] := Simp[(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Simp[d*(n/(b*(m + 1)))] Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d, n}, x] && ILtQ[m, -1] && FractionQ[n] && GtQ[n, 0]`

rule 73 `Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] := With[{p = Denominator[m]}, Simp[p/b Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) + d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntLinearQ[a, b, c, d, m, n, x]`

rule 221 `Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[-a/b, 2]/a)*ArcTanh[x/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]`

rule 1102 `Int[((d_) + (e_)*(x_)^(m_))*((a_) + (b_)*(x_) + (c_)*(x_)^2)^(p_), x_Symbol] := Simp[(a + b*x + c*x^2)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x)^(2*FracPart[p])) Int[(d + e*x)^m*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && EqQ[b^2 - 4*a*c, 0]`

Maple [A] (verified)

Time = 1.54 (sec) , antiderivative size = 194, normalized size of antiderivative = 1.23

| method | result |
|---------|---|
| default | $\frac{\left(-3 \arctan\left(\frac{b\sqrt{ex+d}}{\sqrt{b(ae-bd)}}\right) b^2 e^2 x^2 - 6 \arctan\left(\frac{b\sqrt{ex+d}}{\sqrt{b(ae-bd)}}\right) ab e^2 x + 5(ex+d)^{\frac{3}{2}} \sqrt{b(ae-bd)} b - 3 \arctan\left(\frac{b\sqrt{ex+d}}{\sqrt{b(ae-bd)}}\right) e^2 a^2 + 3\sqrt{ex+d}}{4\sqrt{b(ae-bd)} b^2 (bx+a)^2^{\frac{3}{2}}}$ |

input `int((e*x+d)^(3/2)/(b^2*x^2+2*a*b*x+a^2)^(3/2), x, method=_RETURNVERBOSE)`

output `-1/4*(-3*arctan(b*(e*x+d)^(1/2)/(b*(a*e-b*d))^(1/2))*b^2*e^2*x^2-6*arctan(b*(e*x+d)^(1/2)/(b*(a*e-b*d))^(1/2))*a*b*e^2*x+5*(e*x+d)^(3/2)*(b*(a*e-b*d))^(1/2)*b-3*arctan(b*(e*x+d)^(1/2)/(b*(a*e-b*d))^(1/2))*e^2*a^2+3*(e*x+d)^(1/2)*a*e*(b*(a*e-b*d))^(1/2)-3*(e*x+d)^(1/2)*d*b*(b*(a*e-b*d))^(1/2)*(b*x+a)/(b*(a*e-b*d))^(1/2)/b^2/((b*x+a)^2)^(3/2)`

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 383, normalized size of antiderivative = 2.42

$$\int \frac{(d+ex)^{3/2}}{(a^2+2abx+b^2x^2)^{3/2}} dx = \frac{3(b^2e^2x^2+2abe^2x+a^2e^2)\sqrt{b^2d-abe} \log\left(\frac{be x+2bd-ae-2\sqrt{b^2d-abe}\sqrt{ex+d}}{bx+a}\right) -}{8(a^2b^4d-a^3b^3e+(b^6d-ab^5e)x^2+}$$

input `integrate((e*x+d)^(3/2)/(b^2*x^2+2*a*b*x+a^2)^(3/2), x, algorithm="fricas")`

output

```
[1/8*(3*(b^2*e^2*x^2 + 2*a*b*e^2*x + a^2*e^2)*sqrt(b^2*d - a*b*e)*log((b*e*x + 2*b*d - a*e - 2*sqrt(b^2*d - a*b*e)*sqrt(e*x + d))/(b*x + a)) - 2*(2*b^3*d^2 + a*b^2*d*e - 3*a^2*b*e^2 + 5*(b^3*d*e - a*b^2*e^2)*x)*sqrt(e*x + d))/(a^2*b^4*d - a^3*b^3*e + (b^6*d - a*b^5*e)*x^2 + 2*(a*b^5*d - a^2*b^4*e)*x), 1/4*(3*(b^2*e^2*x^2 + 2*a*b*e^2*x + a^2*e^2)*sqrt(-b^2*d + a*b*e)*arctan(sqrt(-b^2*d + a*b*e)*sqrt(e*x + d)/(b*e*x + b*d)) - (2*b^3*d^2 + a*b^2*d*e - 3*a^2*b*e^2 + 5*(b^3*d*e - a*b^2*e^2)*x)*sqrt(e*x + d))/(a^2*b^4*d - a^3*b^3*e + (b^6*d - a*b^5*e)*x^2 + 2*(a*b^5*d - a^2*b^4*e)*x)]
```

Sympy [F]

$$\int \frac{(d + ex)^{3/2}}{(a^2 + 2abx + b^2x^2)^{3/2}} dx = \int \frac{(d + ex)^{\frac{3}{2}}}{((a + bx)^2)^{\frac{3}{2}}} dx$$

input

```
integrate((e*x+d)**(3/2)/(b**2*x**2+2*a*b*x+a**2)**(3/2),x)
```

output

```
Integral((d + e*x)**(3/2)/((a + b*x)**2)**(3/2), x)
```

Maxima [F]

$$\int \frac{(d + ex)^{3/2}}{(a^2 + 2abx + b^2x^2)^{3/2}} dx = \int \frac{(ex + d)^{\frac{3}{2}}}{(b^2x^2 + 2abx + a^2)^{\frac{3}{2}}} dx$$

input

```
integrate((e*x+d)^(3/2)/(b^2*x^2+2*a*b*x+a^2)^(3/2),x, algorithm="maxima")
```

output

```
integrate((e*x + d)^(3/2)/(b^2*x^2 + 2*a*b*x + a^2)^(3/2), x)
```

Giac [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 124, normalized size of antiderivative = 0.78

$$\int \frac{(d+ex)^{3/2}}{(a^2+2abx+b^2x^2)^{3/2}} dx = \frac{3e^2 \arctan\left(\frac{\sqrt{ex+db}}{\sqrt{-b^2d+abe}}\right)}{4\sqrt{-b^2d+abe}b^2 \operatorname{sgn}(bx+a)} - \frac{5(ex+d)^{3/2}be^2 - 3\sqrt{ex+db}de^2 + 3\sqrt{ex+d}ae^3}{4((ex+d)b-bd+ae)^2b^2 \operatorname{sgn}(bx+a)}$$

input `integrate((e*x+d)^(3/2)/(b^2*x^2+2*a*b*x+a^2)^(3/2),x, algorithm="giac")`

output `3/4*e^2*arctan(sqrt(e*x + d)*b/sqrt(-b^2*d + a*b*e))/(sqrt(-b^2*d + a*b*e)*b^2*sgn(b*x + a)) - 1/4*(5*(e*x + d)^(3/2)*b*e^2 - 3*sqrt(e*x + d)*b*d*e^2 + 3*sqrt(e*x + d)*a*e^3)/(((e*x + d)*b - b*d + a*e)^2*b^2*sgn(b*x + a))`

Mupad [F(-1)]

Timed out.

$$\int \frac{(d+ex)^{3/2}}{(a^2+2abx+b^2x^2)^{3/2}} dx = \int \frac{(d+ex)^{3/2}}{(a^2+2abx+b^2x^2)^{3/2}} dx$$

input `int((d + e*x)^(3/2)/(a^2 + b^2*x^2 + 2*a*b*x)^(3/2),x)`

output `int((d + e*x)^(3/2)/(a^2 + b^2*x^2 + 2*a*b*x)^(3/2), x)`

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 259, normalized size of antiderivative = 1.64

$$\int \frac{(d+ex)^{3/2}}{(a^2+2abx+b^2x^2)^{3/2}} dx = \frac{3\sqrt{b}\sqrt{ae-bd} \operatorname{atan}\left(\frac{\sqrt{ex+db}}{\sqrt{b}\sqrt{ae-bd}}\right) a^2 e^2 + 6\sqrt{b}\sqrt{ae-bd} \operatorname{atan}\left(\frac{\sqrt{ex+db}}{\sqrt{b}\sqrt{ae-bd}}\right) ab e^2}{\dots}$$

input `int((e*x+d)^(3/2)/(b^2*x^2+2*a*b*x+a^2)^(3/2),x)`

output `(3*sqrt(b)*sqrt(a*e - b*d)*atan((sqrt(d + e*x)*b)/(sqrt(b)*sqrt(a*e - b*d)))*a**2*e**2 + 6*sqrt(b)*sqrt(a*e - b*d)*atan((sqrt(d + e*x)*b)/(sqrt(b)*sqrt(a*e - b*d)))*a*b*e**2*x + 3*sqrt(b)*sqrt(a*e - b*d)*atan((sqrt(d + e*x)*b)/(sqrt(b)*sqrt(a*e - b*d)))*b**2*e**2*x**2 - 3*sqrt(d + e*x)*a**2*b*e**2 + sqrt(d + e*x)*a*b**2*d*e - 5*sqrt(d + e*x)*a*b**2*e**2*x + 2*sqrt(d + e*x)*b**3*d**2 + 5*sqrt(d + e*x)*b**3*d*e*x)/(4*b**3*(a**3*e - a**2*b*d + 2*a**2*b*e*x - 2*a*b**2*d*x + a*b**2*e*x**2 - b**3*d*x**2))`

3.383 $\int \frac{\sqrt{d+ex}}{(a^2+2abx+b^2x^2)^{3/2}} dx$

| | |
|---|------|
| Optimal result | 2831 |
| Mathematica [A] (verified) | 2831 |
| Rubi [A] (verified) | 2832 |
| Maple [A] (verified) | 2834 |
| Fricas [A] (verification not implemented) | 2835 |
| Sympy [F] | 2835 |
| Maxima [F] | 2836 |
| Giac [A] (verification not implemented) | 2836 |
| Mupad [F(-1)] | 2837 |
| Reduce [B] (verification not implemented) | 2837 |

Optimal result

Integrand size = 30, antiderivative size = 168

$$\int \frac{\sqrt{d+ex}}{(a^2+2abx+b^2x^2)^{3/2}} dx = -\frac{e\sqrt{d+ex}}{4b(bd-ae)\sqrt{a^2+2abx+b^2x^2}} - \frac{\sqrt{d+ex}}{2b(a+bx)\sqrt{a^2+2abx+b^2x^2}} + \frac{e^2(a+bx)\operatorname{arctanh}\left(\frac{\sqrt{b}\sqrt{d+ex}}{\sqrt{bd-ae}}\right)}{4b^{3/2}(bd-ae)^{3/2}\sqrt{a^2+2abx+b^2x^2}}$$

output `-1/4*e*(e*x+d)^(1/2)/b/(-a*e+b*d)/((b*x+a)^2)^(1/2)-1/2*(e*x+d)^(1/2)/b/(b*x+a)/((b*x+a)^2)^(1/2)+1/4*e^2*(b*x+a)*arctanh(b^(1/2)*(e*x+d)^(1/2)/(-a*e+b*d)^(1/2))/b^(3/2)/(-a*e+b*d)^(3/2)/((b*x+a)^2)^(1/2)`

Mathematica [A] (verified)

Time = 0.34 (sec) , antiderivative size = 121, normalized size of antiderivative = 0.72

$$\int \frac{\sqrt{d+ex}}{(a^2+2abx+b^2x^2)^{3/2}} dx = \frac{\sqrt{b}\sqrt{-bd+ae}\sqrt{d+ex}(ae-b(2d+ex))-e^2(a+bx)^2\arctan\left(\frac{\sqrt{b}\sqrt{d+ex}}{\sqrt{-bd+ae}}\right)}{4b^{3/2}(-bd+ae)^{3/2}(a+bx)\sqrt{(a+bx)^2}}$$

input `Integrate[Sqrt[d + e*x]/(a^2 + 2*a*b*x + b^2*x^2)^(3/2), x]`

output `-1/4*(Sqrt[b]*Sqrt[-(b*d) + a*e]*Sqrt[d + e*x]*(a*e - b*(2*d + e*x)) - e^2*(a + b*x)^2*ArcTan[(Sqrt[b]*Sqrt[d + e*x])/Sqrt[-(b*d) + a*e]]/(b^(3/2)*(-(b*d) + a*e)^(3/2)*(a + b*x)*Sqrt[(a + b*x)^2])`

Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 134, normalized size of antiderivative = 0.80, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {1102, 27, 51, 52, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\begin{aligned}
 & \int \frac{\sqrt{d+ex}}{(a^2+2abx+b^2x^2)^{3/2}} dx \\
 & \quad \downarrow 1102 \\
 & \frac{b^3(a+bx) \int \frac{\sqrt{d+ex}}{b^3(a+bx)^3} dx}{\sqrt{a^2+2abx+b^2x^2}} \\
 & \quad \downarrow 27 \\
 & \frac{(a+bx) \int \frac{\sqrt{d+ex}}{(a+bx)^3} dx}{\sqrt{a^2+2abx+b^2x^2}} \\
 & \quad \downarrow 51 \\
 & \frac{(a+bx) \left(\frac{e \int \frac{1}{(a+bx)^2 \sqrt{d+ex}} dx}{4b} - \frac{\sqrt{d+ex}}{2b(a+bx)^2} \right)}{\sqrt{a^2+2abx+b^2x^2}} \\
 & \quad \downarrow 52 \\
 & \frac{(a+bx) \left(e \left(-\frac{e \int \frac{1}{(a+bx) \sqrt{d+ex}} dx}{2(bd-ae)} - \frac{\sqrt{d+ex}}{(a+bx)(bd-ae)} \right) - \frac{\sqrt{d+ex}}{2b(a+bx)^2} \right)}{\sqrt{a^2+2abx+b^2x^2}}
 \end{aligned}$$

$$\begin{array}{c}
 \downarrow 73 \\
 (a + bx) \left(\frac{e \left(\frac{\int \frac{1}{a + \frac{b(d+ex)}{e} - \frac{bd}{e}}{bd-ae} d\sqrt{d+ex}}{\frac{\sqrt{d+ex}}{(a+bx)(bd-ae)}} \right)}{4b} - \frac{\sqrt{d+ex}}{2b(a+bx)^2} \right) \\
 \hline
 \sqrt{a^2 + 2abx + b^2x^2} \\
 \downarrow 221 \\
 (a + bx) \left(\frac{e \left(\frac{e \operatorname{arctanh} \left(\frac{\sqrt{b}\sqrt{d+ex}}{\sqrt{bd-ae}} \right)}{\sqrt{b}(bd-ae)^{3/2}} - \frac{\sqrt{d+ex}}{(a+bx)(bd-ae)} \right)}{4b} - \frac{\sqrt{d+ex}}{2b(a+bx)^2} \right) \\
 \hline
 \sqrt{a^2 + 2abx + b^2x^2}
 \end{array}$$

input `Int[Sqrt[d + e*x]/(a^2 + 2*a*b*x + b^2*x^2)^(3/2),x]`

output `((a + b*x)*(-1/2*Sqrt[d + e*x]/(b*(a + b*x)^2) + (e*(-(Sqrt[d + e*x]/((b*d - a*e)*(a + b*x)))) + (e*ArcTanh[(Sqrt[b]*Sqrt[d + e*x])/Sqrt[b*d - a*e]])/(Sqrt[b]*(b*d - a*e)^(3/2))))/(4*b))/Sqrt[a^2 + 2*a*b*x + b^2*x^2]`

Defintions of rubi rules used

rule 27 `Int[(a_)*(Fx_), x_Symbol] := Simp[a Int[Fx, x], x] /; FreeQ[a, x] && !MatchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]`

rule 51 `Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] := Simp[(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Simp[d*(n/(b*(m + 1)))] Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x] /; FreeQ[{a, b, c, d, n}, x] && ILtQ[m, -1] && FractionQ[n] && GtQ[n, 0]`

rule 52 `Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] := Simp[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((m + n + 2)/((b*c - a*d)*(m + 1)))] Int[(a + b*x)^(m + 1)*(c + d*x)^n, x] /; FreeQ[{a, b, c, d, n}, x] && ILtQ[m, -1] && FractionQ[n] && LtQ[n, 0]`

rule 73 `Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] := With[
 {p = Denominator[m]}, Simp[p/b Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x] /; FreeQ[{a, b, c, d}, x] && Lt
 Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
 inearQ[a, b, c, d, m, n, x]`

rule 221 `Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
 /Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]`

rule 1102 `Int[((d_.) + (e_.)*(x_))^(m_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
 ymbol] := Simp[(a + b*x + c*x^2)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x)^(2*F
 racPart[p])) Int[(d + e*x)^m*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c,
 d, e, m, p}, x] && EqQ[b^2 - 4*a*c, 0]`

Maple [A] (verified)

Time = 1.47 (sec) , antiderivative size = 200, normalized size of antiderivative = 1.19

| method | result |
|---------|---|
| default | $\frac{\left(\arctan\left(\frac{b\sqrt{ex+d}}{\sqrt{b(ae-bd)}}\right)b^2e^2x^2+2\arctan\left(\frac{b\sqrt{ex+d}}{\sqrt{b(ae-bd)}}\right)abe^2x+(ex+d)^{\frac{3}{2}}\sqrt{b(ae-bd)}b+\arctan\left(\frac{b\sqrt{ex+d}}{\sqrt{b(ae-bd)}}\right)e^2a^2-\sqrt{ex+d}ae\sqrt{b(ae-bd)}}{4\sqrt{b(ae-bd)}(ae-bd)b((bx+a)^2)^{\frac{3}{2}}}$ |

input `int((e*x+d)^(1/2)/(b^2*x^2+2*a*b*x+a^2)^(3/2),x,method=_RETURNVERBOSE)`

output `1/4*(arctan(b*(e*x+d)^(1/2)/(b*(a*e-b*d))^(1/2))*b^2*e^2*x^2+2*arctan(b*(e
 x+d)^(1/2)/(b(a*e-b*d))^(1/2))*a*b*e^2*x+(e*x+d)^(3/2)*(b*(a*e-b*d))^(1/
 2)*b+arctan(b*(e*x+d)^(1/2)/(b*(a*e-b*d))^(1/2))*e^2*a^2-(e*x+d)^(1/2)*a*e
 (b(a*e-b*d))^(1/2)+(e*x+d)^(1/2)*d*b*(b*(a*e-b*d))^(1/2)*(b*x+a)/(b*(a
 e-b*d))^(1/2)/(a*e-b*d)/b/((b*x+a)^2)^(3/2)`

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 456, normalized size of antiderivative = 2.71

$$\int \frac{\sqrt{d+ex}}{(a^2+2abx+b^2x^2)^{3/2}} dx = \left[-\frac{(b^2e^2x^2+2abe^2x+a^2e^2)\sqrt{b^2d-abe} \log\left(\frac{bex+2bd-ae-2\sqrt{b^2d-abe}\sqrt{ex+d}}{bx+a}\right) + (b^2e^2x^2+2abe^2x+a^2e^2)\sqrt{-b^2d+abe} \arctan\left(\frac{\sqrt{-b^2d+abe}\sqrt{ex+d}}{bex+bd}\right) + (2b^3d^2-3ab^2de+a^2be^2+(b^3de-a^2b^2e^2)x)}{4(a^2b^4d^2-2a^3b^3de+a^4b^2e^2+(b^6d^2-2ab^5de+a^2b^4e^2)x^2+2(ab^5d^2-2a^2b^4de+a^3b^3e^2)x)} \right]$$

input `integrate((e*x+d)^(1/2)/(b^2*x^2+2*a*b*x+a^2)^(3/2),x, algorithm="fricas")`

output `[-1/8*((b^2*e^2*x^2 + 2*a*b*e^2*x + a^2*e^2)*sqrt(b^2*d - a*b*e)*log((b*e*x + 2*b*d - a*e - 2*sqrt(b^2*d - a*b*e)*sqrt(e*x + d))/(b*x + a)) + 2*(2*b^3*d^2 - 3*a*b^2*d*e + a^2*b*e^2 + (b^3*d*e - a*b^2*e^2)*x)*sqrt(e*x + d)] / (a^2*b^4*d^2 - 2*a^3*b^3*d*e + a^4*b^2*e^2 + (b^6*d^2 - 2*a*b^5*d*e + a^2*b^4*e^2)*x^2 + 2*(a*b^5*d^2 - 2*a^2*b^4*d*e + a^3*b^3*e^2)*x), -1/4*((b^2*e^2*x^2 + 2*a*b*e^2*x + a^2*e^2)*sqrt(-b^2*d + a*b*e)*arctan(sqrt(-b^2*d + a*b*e)*sqrt(e*x + d)/(b*e*x + b*d)) + (2*b^3*d^2 - 3*a*b^2*d*e + a^2*b*e^2 + (b^3*d*e - a*b^2*e^2)*x)*sqrt(e*x + d)] / (a^2*b^4*d^2 - 2*a^3*b^3*d*e + a^4*b^2*e^2 + (b^6*d^2 - 2*a*b^5*d*e + a^2*b^4*e^2)*x^2 + 2*(a*b^5*d^2 - 2*a^2*b^4*d*e + a^3*b^3*e^2)*x)]`

Sympy [F]

$$\int \frac{\sqrt{d+ex}}{(a^2+2abx+b^2x^2)^{3/2}} dx = \int \frac{\sqrt{d+ex}}{((a+bx)^2)^{3/2}} dx$$

input `integrate((e*x+d)**(1/2)/(b**2*x**2+2*a*b*x+a**2)**(3/2),x)`

output `Integral(sqrt(d + e*x)/((a + b*x)**2)**(3/2), x)`

Maxima [F]

$$\int \frac{\sqrt{d+ex}}{(a^2+2abx+b^2x^2)^{3/2}} dx = \int \frac{\sqrt{ex+d}}{(b^2x^2+2abx+a^2)^{3/2}} dx$$

input `integrate((e*x+d)^(1/2)/(b^2*x^2+2*a*b*x+a^2)^(3/2),x, algorithm="maxima")`

output `integrate(sqrt(e*x + d)/(b^2*x^2 + 2*a*b*x + a^2)^(3/2), x)`

Giac [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 150, normalized size of antiderivative = 0.89

$$\int \frac{\sqrt{d+ex}}{(a^2+2abx+b^2x^2)^{3/2}} dx = -\frac{e^2 \arctan\left(\frac{\sqrt{ex+db}}{\sqrt{-b^2d+abe}}\right)}{4(b^2d\operatorname{sgn}(bx+a) - ab\operatorname{sgn}(bx+a))\sqrt{-b^2d+abe}} - \frac{(ex+d)^{\frac{3}{2}}be^2 + \sqrt{ex+db}de^2 - \sqrt{ex+da}e^3}{4(b^2d\operatorname{sgn}(bx+a) - ab\operatorname{sgn}(bx+a))((ex+d)b - bd + ae)^2}$$

input `integrate((e*x+d)^(1/2)/(b^2*x^2+2*a*b*x+a^2)^(3/2),x, algorithm="giac")`

output `-1/4*e^2*arctan(sqrt(e*x + d)*b/sqrt(-b^2*d + a*b*e))/((b^2*d*sgn(b*x + a) - a*b*e*sgn(b*x + a))*sqrt(-b^2*d + a*b*e)) - 1/4*((e*x + d)^(3/2)*b*e^2 + sqrt(e*x + d)*b*d*e^2 - sqrt(e*x + d)*a*e^3)/((b^2*d*sgn(b*x + a) - a*b*e*sgn(b*x + a))*((e*x + d)*b - b*d + a*e)^2)`

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{d+ex}}{(a^2+2abx+b^2x^2)^{3/2}} dx = \int \frac{\sqrt{d+ex}}{(a^2+2abx+b^2x^2)^{3/2}} dx$$

input `int((d + e*x)^(1/2)/(a^2 + b^2*x^2 + 2*a*b*x)^(3/2), x)`

output `int((d + e*x)^(1/2)/(a^2 + b^2*x^2 + 2*a*b*x)^(3/2), x)`

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 301, normalized size of antiderivative = 1.79

$$\int \frac{\sqrt{d+ex}}{(a^2+2abx+b^2x^2)^{3/2}} dx = \frac{\sqrt{b}\sqrt{ae-bd}\operatorname{atan}\left(\frac{\sqrt{ex+db}}{\sqrt{b}\sqrt{ae-bd}}\right)a^2e^2 + 2\sqrt{b}\sqrt{ae-bd}\operatorname{atan}\left(\frac{\sqrt{ex+db}}{\sqrt{b}\sqrt{ae-bd}}\right)abe^2x}{4b^2(a^2b^2e^2x^2 - 2ab^3e^2x + b^4d^2x^2)}$$

input `int((e*x+d)^(1/2)/(b^2*x^2+2*a*b*x+a^2)^(3/2), x)`

output `(sqrt(b)*sqrt(a*e - b*d)*atan((sqrt(d + e*x)*b)/(sqrt(b)*sqrt(a*e - b*d)))
*a**2*e**2 + 2*sqrt(b)*sqrt(a*e - b*d)*atan((sqrt(d + e*x)*b)/(sqrt(b)*sqrt(a*e - b*d)))
*a*b*e**2*x + sqrt(b)*sqrt(a*e - b*d)*atan((sqrt(d + e*x)*b)/(sqrt(b)*sqrt(a*e - b*d)))
*b**2*e**2*x**2 - sqrt(d + e*x)*a**2*b*e**2 + 3*sqrt(d + e*x)*a*b**2*d*e
+ sqrt(d + e*x)*a*b**2*e**2*x - 2*sqrt(d + e*x)*b**3*d**2 - sqrt(d + e*x)*b**3*d*e*x
/(4*b**2*(a**4*e**2 - 2*a**3*b*d*e + 2*a**3*b*e**2*x + a**2*b**2*d**2 - 4*a**2*b**2*d*e*x + a**2*b**2*e**2*x**2
+ 2*a*b**3*d**2*x - 2*a*b**3*d*e*x**2 + b**4*d**2*x**2))`

3.384 $\int \frac{1}{\sqrt{d+ex}(a^2+2abx+b^2x^2)^{3/2}} dx$

| | |
|---|------|
| Optimal result | 2838 |
| Mathematica [A] (verified) | 2838 |
| Rubi [A] (verified) | 2839 |
| Maple [A] (verified) | 2841 |
| Fricas [B] (verification not implemented) | 2842 |
| Sympy [F] | 2842 |
| Maxima [F] | 2843 |
| Giac [A] (verification not implemented) | 2843 |
| Mupad [F(-1)] | 2844 |
| Reduce [B] (verification not implemented) | 2844 |

Optimal result

Integrand size = 30, antiderivative size = 172

$$\int \frac{1}{\sqrt{d+ex}(a^2+2abx+b^2x^2)^{3/2}} dx = \frac{3e\sqrt{d+ex}}{4(bd-ae)^2\sqrt{a^2+2abx+b^2x^2}} - \frac{\sqrt{d+ex}}{2(bd-ae)(a+bx)\sqrt{a^2+2abx+b^2x^2}} - \frac{3e^2(a+bx)\operatorname{arctanh}\left(\frac{\sqrt{b}\sqrt{d+ex}}{\sqrt{bd-ae}}\right)}{4\sqrt{b}(bd-ae)^{5/2}\sqrt{a^2+2abx+b^2x^2}}$$

output `3/4*e*(e*x+d)^(1/2)/(-a*e+b*d)^2/((b*x+a)^2)^(1/2)-1/2*(e*x+d)^(1/2)/(-a*e+b*d)/(b*x+a)/((b*x+a)^2)^(1/2)-3/4*e^2*(b*x+a)*arctanh(b^(1/2)*(e*x+d)^(1/2)/(-a*e+b*d)^(1/2))/b^(1/2)/(-a*e+b*d)^(5/2)/((b*x+a)^2)^(1/2)`

Mathematica [A] (verified)

Time = 0.24 (sec) , antiderivative size = 121, normalized size of antiderivative = 0.70

$$\int \frac{1}{\sqrt{d+ex}(a^2+2abx+b^2x^2)^{3/2}} dx = \frac{\sqrt{b}\sqrt{-bd+ae}\sqrt{d+ex}(-2bd+5ae+3bex)+3e^2(a+bx)^2\operatorname{arctan}\left(\frac{\sqrt{b}\sqrt{d+ex}}{\sqrt{-bd+ae}}\right)}{4\sqrt{b}(-bd+ae)^{5/2}(a+bx)\sqrt{(a+bx)^2}}$$

input `Integrate[1/(Sqrt[d + e*x]*(a^2 + 2*a*b*x + b^2*x^2)^(3/2)), x]`

output

```
(Sqrt[b]*Sqrt[-(b*d) + a*e]*Sqrt[d + e*x]*(-2*b*d + 5*a*e + 3*b*e*x) + 3*e
^2*(a + b*x)^2*ArcTan[(Sqrt[b]*Sqrt[d + e*x])/Sqrt[-(b*d) + a*e]]/(4*Sqrt
[b]*(-(b*d) + a*e)^(5/2)*(a + b*x)*Sqrt[(a + b*x)^2])
```

Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 148, normalized size of antiderivative = 0.86, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {1102, 27, 52, 52, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\begin{aligned}
 & \int \frac{1}{(a^2 + 2abx + b^2x^2)^{3/2} \sqrt{d+ex}} dx \\
 & \quad \downarrow \text{1102} \\
 & \frac{b^3(a+bx) \int \frac{1}{b^3(a+bx)^3 \sqrt{d+ex}} dx}{\sqrt{a^2 + 2abx + b^2x^2}} \\
 & \quad \downarrow \text{27} \\
 & \frac{(a+bx) \int \frac{1}{(a+bx)^3 \sqrt{d+ex}} dx}{\sqrt{a^2 + 2abx + b^2x^2}} \\
 & \quad \downarrow \text{52} \\
 & \frac{(a+bx) \left(-\frac{3e \int \frac{1}{(a+bx)^2 \sqrt{d+ex}} dx}{4(bd-ae)} - \frac{\sqrt{d+ex}}{2(a+bx)^2(bd-ae)} \right)}{\sqrt{a^2 + 2abx + b^2x^2}} \\
 & \quad \downarrow \text{52} \\
 & \frac{(a+bx) \left(-\frac{3e \left(-\frac{e \int \frac{1}{(a+bx) \sqrt{d+ex}} dx}{2(bd-ae)} - \frac{\sqrt{d+ex}}{(a+bx)(bd-ae)} \right)}{4(bd-ae)} - \frac{\sqrt{d+ex}}{2(a+bx)^2(bd-ae)} \right)}{\sqrt{a^2 + 2abx + b^2x^2}} \\
 & \quad \downarrow \text{73}
 \end{aligned}$$

$$\frac{(a+bx) \left(-\frac{3e \left(-\frac{\int \frac{1}{a+\frac{b(d+ex)}{e}-\frac{bd}{bd-ae}} d\sqrt{d+ex}}{bd-ae} - \frac{\sqrt{d+ex}}{(a+bx)(bd-ae)} \right)}{4(bd-ae)} - \frac{\sqrt{d+ex}}{2(a+bx)^2(bd-ae)} \right)}{\sqrt{a^2+2abx+b^2x^2}}$$

↓ 221

$$\frac{(a+bx) \left(-\frac{3e \left(\frac{e \operatorname{arctanh}\left(\frac{\sqrt{b}\sqrt{d+ex}}{\sqrt{bd-ae}}\right)}{\sqrt{b}(bd-ae)^{3/2}} - \frac{\sqrt{d+ex}}{(a+bx)(bd-ae)} \right)}{4(bd-ae)} - \frac{\sqrt{d+ex}}{2(a+bx)^2(bd-ae)} \right)}{\sqrt{a^2+2abx+b^2x^2}}$$

input `Int[1/(Sqrt[d + e*x]*(a^2 + 2*a*b*x + b^2*x^2)^(3/2)),x]`

output `((a + b*x)*(-1/2*Sqrt[d + e*x]/((b*d - a*e)*(a + b*x)^2) - (3*e*(-(Sqrt[d + e*x]/((b*d - a*e)*(a + b*x)))) + (e*ArcTanh[(Sqrt[b]*Sqrt[d + e*x])/Sqrt[b*d - a*e]])/(Sqrt[b]*(b*d - a*e)^(3/2))))/(4*(b*d - a*e)))/Sqrt[a^2 + 2*a*b*x + b^2*x^2]`

Defintions of rubi rules used

rule 27 `Int[(a_)*(Fx_), x_Symbol] := Simp[a Int[Fx, x], x] /; FreeQ[a, x] && !MatchQ[Fx, (b_)*(Gx_)] /; FreeQ[b, x]`

rule 52 `Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] := Simp[(a + b*x)^(m + 1)*(c + d*x)^(n + 1)/((b*c - a*d)*(m + 1)), x] - Simp[d*(c + d*x)^(n + 2)/((b*c - a*d)*(m + 1)) Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && ILtQ[m, -1] && FractionQ[n] && LtQ[n, 0]`

rule 73 `Int[((a_.) + (b_.)*(x_)^(m_))*((c_.) + (d_.)*(x_)^(n_)), x_Symbol] := With[
 {p = Denominator[m]}, Simp[p/b Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x] /; FreeQ[{a, b, c, d}, x] && Lt
 Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
 inearQ[a, b, c, d, m, n, x]`

rule 221 `Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
 /Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]`

rule 1102 `Int[((d_.) + (e_.)*(x_)^(m_.))*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
 ymbol] := Simp[(a + b*x + c*x^2)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x)^(2*F
 racPart[p])) Int[(d + e*x)^m*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c,
 d, e, m, p}, x] && EqQ[b^2 - 4*a*c, 0]`

Maple [A] (verified)

Time = 1.50 (sec) , antiderivative size = 203, normalized size of antiderivative = 1.18

| method | result |
|---------|---|
| default | $\frac{\left(3 \arctan\left(\frac{b\sqrt{ex+d}}{\sqrt{b(ae-bd)}}\right) b^2 e^2 x^2 + 6 \arctan\left(\frac{b\sqrt{ex+d}}{\sqrt{b(ae-bd)}}\right) a b e^2 x + 3\sqrt{ex+d} \sqrt{b(ae-bd)} b e x + 3 \arctan\left(\frac{b\sqrt{ex+d}}{\sqrt{b(ae-bd)}}\right) e^2 a^2 + 5\sqrt{ex+d} a e\right)}{4\sqrt{b(ae-bd)}(ae-bd)^2((bx+a)^2)^{\frac{3}{2}}}$ |

input `int(1/(e*x+d)^(1/2)/(b^2*x^2+2*a*b*x+a^2)^(3/2),x,method=_RETURNVERBOSE)`

output `1/4*(3*arctan(b*(e*x+d)^(1/2)/(b*(a*e-b*d))^(1/2))*b^2*e^2*x^2+6*arctan(b*
 (e*x+d)^(1/2)/(b*(a*e-b*d))^(1/2))*a*b*e^2*x+3*(e*x+d)^(1/2)*(b*(a*e-b*d))
 ^1/2)*b*e*x+3*arctan(b*(e*x+d)^(1/2)/(b*(a*e-b*d))^(1/2))*e^2*a^2+5*(e*x+
 d)^(1/2)*a*e*(b*(a*e-b*d))^(1/2)-2*(e*x+d)^(1/2)*d*b*(b*(a*e-b*d))^(1/2))*
 (b*x+a)/(b*(a*e-b*d))^(1/2)/(a*e-b*d)^2/((b*x+a)^2)^(3/2)`

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 268 vs. $2(119) = 238$.

Time = 0.10 (sec) , antiderivative size = 549, normalized size of antiderivative = 3.19

$$\int \frac{1}{\sqrt{d+ex}(a^2+2abx+b^2x^2)^{3/2}} dx = \left[\frac{3(b^2e^2x^2+2abe^2x+a^2e^2)\sqrt{b^2d-abe} \log\left(\frac{bex+2bd-ae-2\sqrt{b^2d-abe}\sqrt{d+ex}}{bx+a}\right)}{8(a^2b^4d^3-3a^3b^3d^2e+3a^4b^2de^2-a^5be^3+(b^6d^3-3ab^5d^2e+3a^4b^4d^2e^2-3a^3b^3d^2e^2+a^4b^2de^2-a^5be^3)x^2+2(a^2b^4d^3-3a^3b^3d^2e+3a^4b^2de^2-a^5be^3)x)} \right]$$

input `integrate(1/(e*x+d)^(1/2)/(b^2*x^2+2*a*b*x+a^2)^(3/2),x, algorithm="fricas")`

output `[1/8*(3*(b^2*e^2*x^2 + 2*a*b*e^2*x + a^2*e^2)*sqrt(b^2*d - a*b*e)*log((b*e*x + 2*b*d - a*e - 2*sqrt(b^2*d - a*b*e)*sqrt(e*x + d))/(b*x + a)) - 2*(2*b^3*d^2 - 7*a*b^2*d*e + 5*a^2*b*e^2 - 3*(b^3*d*e - a*b^2*e^2)*x)*sqrt(e*x + d))/(a^2*b^4*d^3 - 3*a^3*b^3*d^2*e + 3*a^4*b^2*d*e^2 - a^5*b*e^3 + (b^6*d^3 - 3*a*b^5*d^2*e + 3*a^2*b^4*d*e^2 - a^3*b^3*e^3)*x^2 + 2*(a*b^5*d^3 - 3*a^2*b^4*d^2*e + 3*a^3*b^3*d*e^2 - a^4*b^2*e^3)*x), 1/4*(3*(b^2*e^2*x^2 + 2*a*b*e^2*x + a^2*e^2)*sqrt(-b^2*d + a*b*e)*arctan(sqrt(-b^2*d + a*b*e)*sqrt(e*x + d)/(b*e*x + b*d)) - (2*b^3*d^2 - 7*a*b^2*d*e + 5*a^2*b*e^2 - 3*(b^3*d*e - a*b^2*e^2)*x)*sqrt(e*x + d))/(a^2*b^4*d^3 - 3*a^3*b^3*d^2*e + 3*a^4*b^2*d*e^2 - a^5*b*e^3 + (b^6*d^3 - 3*a*b^5*d^2*e + 3*a^2*b^4*d*e^2 - a^3*b^3*e^3)*x^2 + 2*(a*b^5*d^3 - 3*a^2*b^4*d^2*e + 3*a^3*b^3*d*e^2 - a^4*b^2*e^3)*x)]`

Sympy [F]

$$\int \frac{1}{\sqrt{d+ex}(a^2+2abx+b^2x^2)^{3/2}} dx = \int \frac{1}{\sqrt{d+ex}((a+bx)^2)^{3/2}} dx$$

input `integrate(1/(e*x+d)**(1/2)/(b**2*x**2+2*a*b*x+a**2)**(3/2),x)`

output `Integral(1/(sqrt(d + e*x)*((a + b*x)**2)**(3/2)), x)`

Maxima [F]

$$\int \frac{1}{\sqrt{d+ex} (a^2 + 2abx + b^2x^2)^{3/2}} dx = \int \frac{1}{(b^2x^2 + 2abx + a^2)^{\frac{3}{2}} \sqrt{ex+d}} dx$$

input `integrate(1/(e*x+d)^(1/2)/(b^2*x^2+2*a*b*x+a^2)^(3/2),x, algorithm="maxima")`

output `integrate(1/((b^2*x^2 + 2*a*b*x + a^2)^(3/2)*sqrt(e*x + d)), x)`

Giac [A] (verification not implemented)

Time = 1.36 (sec) , antiderivative size = 184, normalized size of antiderivative = 1.07

$$\int \frac{1}{\sqrt{d+ex} (a^2 + 2abx + b^2x^2)^{3/2}} dx = \frac{3e^2 \arctan\left(\frac{\sqrt{ex+db}}{\sqrt{-b^2d+abe}}\right)}{4(b^2d^2\operatorname{sgn}(bx+a) - 2abde\operatorname{sgn}(bx+a) + a^2e^2\operatorname{sgn}(bx+a))\sqrt{-b^2d+abe}} + \frac{3(ex+d)^{\frac{3}{2}}be^2 - 5\sqrt{ex+db}de^2 + 5\sqrt{ex+d}ae^3}{4(b^2d^2\operatorname{sgn}(bx+a) - 2abde\operatorname{sgn}(bx+a) + a^2e^2\operatorname{sgn}(bx+a))((ex+d)b - bd + ae)^2}$$

input `integrate(1/(e*x+d)^(1/2)/(b^2*x^2+2*a*b*x+a^2)^(3/2),x, algorithm="giac")`

output `3/4*e^2*arctan(sqrt(e*x + d)*b/sqrt(-b^2*d + a*b*e))/((b^2*d^2*sgn(b*x + a) - 2*a*b*d*e*sgn(b*x + a) + a^2*e^2*sgn(b*x + a))*sqrt(-b^2*d + a*b*e)) + 1/4*(3*(e*x + d)^(3/2)*b*e^2 - 5*sqrt(e*x + d)*b*d*e^2 + 5*sqrt(e*x + d)*a*e^3)/((b^2*d^2*sgn(b*x + a) - 2*a*b*d*e*sgn(b*x + a) + a^2*e^2*sgn(b*x + a))*((e*x + d)*b - b*d + a*e)^2)`

3.385 $\int \frac{1}{(d+ex)^{3/2}(a^2+2abx+b^2x^2)^{3/2}} dx$

| | |
|---|------|
| Optimal result | 2845 |
| Mathematica [A] (verified) | 2846 |
| Rubi [A] (verified) | 2846 |
| Maple [A] (verified) | 2849 |
| Fricas [B] (verification not implemented) | 2850 |
| Sympy [F] | 2851 |
| Maxima [F] | 2851 |
| Giac [A] (verification not implemented) | 2851 |
| Mupad [F(-1)] | 2852 |
| Reduce [B] (verification not implemented) | 2852 |

Optimal result

Integrand size = 30, antiderivative size = 223

$$\int \frac{1}{(d+ex)^{3/2}(a^2+2abx+b^2x^2)^{3/2}} dx = \frac{5e}{4(bd-ae)^2\sqrt{d+ex}\sqrt{a^2+2abx+b^2x^2}} - \frac{1}{2(bd-ae)(a+bx)\sqrt{d+ex}\sqrt{a^2+2abx+b^2x^2}} + \frac{15e^2(a+bx)}{4(bd-ae)^3\sqrt{d+ex}\sqrt{a^2+2abx+b^2x^2}} - \frac{15\sqrt{b}e^2(a+bx)\operatorname{arctanh}\left(\frac{\sqrt{b}\sqrt{d+ex}}{\sqrt{bd-ae}}\right)}{4(bd-ae)^{7/2}\sqrt{a^2+2abx+b^2x^2}}$$

output

```
5/4*e/(-a*e+b*d)^2/(e*x+d)^(1/2)/((b*x+a)^2)^(1/2)-1/2/(-a*e+b*d)/(b*x+a)/(e*x+d)^(1/2)/((b*x+a)^2)^(1/2)+15/4*e^2*(b*x+a)/(-a*e+b*d)^3/(e*x+d)^(1/2)/((b*x+a)^2)^(1/2)-15/4*b^(1/2)*e^2*(b*x+a)*arctanh(b^(1/2)*(e*x+d)^(1/2)/(-a*e+b*d)^2)/(-a*e+b*d)^(7/2)/((b*x+a)^2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.43 (sec) , antiderivative size = 155, normalized size of antiderivative = 0.70

$$\int \frac{1}{(d+ex)^{3/2}(a^2+2abx+b^2x^2)^{3/2}} dx = \frac{\sqrt{-bd+ae}(8a^2e^2+abe(9d+25ex)+b^2(-2d^2+5dex+15e^2x^2))+15\sqrt{be^2}(a+bx)^2\sqrt{d+ex}\arctan\left(\frac{\sqrt{b}\sqrt{d+ex}}{\sqrt{-bd+ae}}\right)}{4(-bd+ae)^{7/2}(a+bx)\sqrt{(a+bx)^2\sqrt{d+ex}}}$$

input

```
Integrate[1/((d + e*x)^(3/2)*(a^2 + 2*a*b*x + b^2*x^2)^(3/2)),x]
```

output

```
-1/4*(Sqrt[-(b*d) + a*e]*(8*a^2*e^2 + a*b*e*(9*d + 25*e*x) + b^2*(-2*d^2 + 5*d*e*x + 15*e^2*x^2)) + 15*Sqrt[b]*e^2*(a + b*x)^2*Sqrt[d + e*x]*ArcTan[(Sqrt[b]*Sqrt[d + e*x])/Sqrt[-(b*d) + a*e]]/((-b*d) + a*e)^(7/2)*(a + b*x)*Sqrt[(a + b*x)^2]*Sqrt[d + e*x])
```

Rubi [A] (verified)

Time = 0.50 (sec) , antiderivative size = 185, normalized size of antiderivative = 0.83, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.233$, Rules used = {1102, 27, 52, 52, 61, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\begin{aligned} & \int \frac{1}{(a^2 + 2abx + b^2x^2)^{3/2} (d + ex)^{3/2}} dx \\ & \quad \downarrow \text{1102} \\ & \frac{b^3(a + bx) \int \frac{1}{b^3(a+bx)^3(d+ex)^{3/2}} dx}{\sqrt{a^2 + 2abx + b^2x^2}} \\ & \quad \downarrow \text{27} \\ & \frac{(a + bx) \int \frac{1}{(a+bx)^3(d+ex)^{3/2}} dx}{\sqrt{a^2 + 2abx + b^2x^2}} \\ & \quad \downarrow \text{52} \end{aligned}$$

$$\begin{aligned}
 & \frac{(a + bx) \left(-\frac{5e \int \frac{1}{(a+bx)^2(d+ex)^{3/2}} dx}{4(bd-ae)} - \frac{1}{2(a+bx)^2\sqrt{d+ex}(bd-ae)} \right)}{\sqrt{a^2 + 2abx + b^2x^2}} \\
 & \quad \downarrow \text{52} \\
 & \frac{(a + bx) \left(-\frac{5e \left(-\frac{3e \int \frac{1}{(a+bx)(d+ex)^{3/2}} dx}{2(bd-ae)} - \frac{1}{(a+bx)\sqrt{d+ex}(bd-ae)} \right)}{4(bd-ae)} - \frac{1}{2(a+bx)^2\sqrt{d+ex}(bd-ae)} \right)}{\sqrt{a^2 + 2abx + b^2x^2}} \\
 & \quad \downarrow \text{61} \\
 & \frac{(a + bx) \left(-\frac{5e \left(\frac{3e \left(\frac{b \int \frac{1}{(a+bx)\sqrt{d+ex}} dx}{bd-ae} + \frac{2}{\sqrt{d+ex}(bd-ae)} \right)}{2(bd-ae)} - \frac{1}{(a+bx)\sqrt{d+ex}(bd-ae)} \right)}{4(bd-ae)} - \frac{1}{2(a+bx)^2\sqrt{d+ex}(bd-ae)} \right)}{\sqrt{a^2 + 2abx + b^2x^2}} \\
 & \quad \downarrow \text{73} \\
 & \frac{(a + bx) \left(-\frac{5e \left(\frac{3e \left(\frac{2b \int \frac{1}{a + \frac{b(d+ex) - bd}{e}} d\sqrt{d+ex}}{e(bd-ae)} + \frac{2}{\sqrt{d+ex}(bd-ae)} \right)}{2(bd-ae)} - \frac{1}{(a+bx)\sqrt{d+ex}(bd-ae)} \right)}{4(bd-ae)} - \frac{1}{2(a+bx)^2\sqrt{d+ex}(bd-ae)} \right)}{\sqrt{a^2 + 2abx + b^2x^2}} \\
 & \quad \downarrow \text{221}
 \end{aligned}$$

$$(a + bx) \left(\frac{5e \left(\frac{3e \left(\frac{2}{\sqrt{d+ex}(bd-ae)} - \frac{2\sqrt{b}\operatorname{arctanh}\left(\frac{\sqrt{b}\sqrt{d+ex}}{\sqrt{bd-ae}}\right)}{(bd-ae)^{3/2}} \right)}{2(bd-ae)} - \frac{1}{(a+bx)\sqrt{d+ex}(bd-ae)} \right)}{4(bd-ae)} - \frac{1}{2(a+bx)^2\sqrt{d+ex}(bd-ae)} \right)}{\sqrt{a^2 + 2abx + b^2x^2}}$$

input `Int[1/((d + e*x)^(3/2)*(a^2 + 2*a*b*x + b^2*x^2)^(3/2)),x]`

output `((a + b*x)*(-1/2*1/((b*d - a*e)*(a + b*x)^2*sqrt[d + e*x]) - (5*e*(-1/((b*d - a*e)*(a + b*x)*sqrt[d + e*x])) - (3*e*(2/((b*d - a*e)*sqrt[d + e*x]) - (2*sqrt[b]*ArcTanh[(sqrt[b]*sqrt[d + e*x])/sqrt[b*d - a*e]])/(b*d - a*e)^(3/2)))/(2*(b*d - a*e))))/(4*(b*d - a*e)))/sqrt[a^2 + 2*a*b*x + b^2*x^2]`

Defintions of rubi rules used

rule 27 `Int[(a_)*(Fx_), x_Symbol] := Simp[a Int[Fx, x], x] /; FreeQ[a, x] && !MatchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]`

rule 52 `Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] := Simp[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((m + n + 2)/((b*c - a*d)*(m + 1)) Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && ILtQ[m, -1] && FractionQ[n] && LtQ[n, 0]`

rule 61 `Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] := Simp[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((m + n + 2)/((b*c - a*d)*(m + 1)) Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && LtQ[m, -1] && !(LtQ[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && IntLinearQ[a, b, c, d, m, n, x]`

rule 73 `Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] := With[
 {p = Denominator[m]}, Simp[p/b Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x] /; FreeQ[{a, b, c, d}, x] && Lt
 Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
 inearQ[a, b, c, d, m, n, x]`

rule 221 `Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
 /Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]`

rule 1102 `Int[((d_.) + (e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
 ymbol] := Simp[(a + b*x + c*x^2)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x)^(2*F
 racPart[p])) Int[(d + e*x)^m*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c,
 d, e, m, p}, x] && EqQ[b^2 - 4*a*c, 0]`

Maple [A] (verified)

Time = 1.49 (sec) , antiderivative size = 285, normalized size of antiderivative = 1.28

| method | result |
|---------|--|
| default | $-\frac{\left(15 \arctan\left(\frac{b\sqrt{ex+d}}{\sqrt{b(ae-bd)}}\right)\sqrt{ex+d} b^3 e^2 x^2 + 30 \arctan\left(\frac{b\sqrt{ex+d}}{\sqrt{b(ae-bd)}}\right)\sqrt{ex+d} a b^2 e^2 x + 15 \sqrt{b(ae-bd)} b^2 e^2 x^2 + 15 \arctan\left(\frac{b\sqrt{ex+d}}{\sqrt{b(ae-bd)}}\right) 4\sqrt{ex+d} \sqrt{b(ae-bd)}\right)}{4\sqrt{ex+d} \sqrt{b(ae-bd)}}$ |

input `int(1/(e*x+d)^(3/2)/(b^2*x^2+2*a*b*x+a^2)^(3/2),x,method=_RETURNVERBOSE)`

output `-1/4*(15*arctan(b*(e*x+d)^(1/2)/(b*(a*e-b*d))^(1/2))*(e*x+d)^(1/2)*b^3*e^2
 *x^2+30*arctan(b*(e*x+d)^(1/2)/(b*(a*e-b*d))^(1/2))*(e*x+d)^(1/2)*a*b^2*e^2
 x+15(b*(a*e-b*d))^(1/2)*b^2*e^2*x^2+15*arctan(b*(e*x+d)^(1/2)/(b*(a*e-b
 d))^(1/2))(e*x+d)^(1/2)*a^2*b*e^2+25*(b*(a*e-b*d))^(1/2)*a*b*e^2*x+5*(b
 *(a*e-b*d))^(1/2)*b^2*d*e*x+8*(b*(a*e-b*d))^(1/2)*a^2*e^2+9*(b*(a*e-b*d))^(
 1/2)*a*b*d*e-2*(b*(a*e-b*d))^(1/2)*b^2*d^2)*(b*x+a)/(e*x+d)^(1/2)/(b*(a*e-
 b*d))^(1/2)/(a*e-b*d)^3/((b*x+a)^2)^(3/2)`

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 366 vs. $2(155) = 310$.

Time = 0.11 (sec) , antiderivative size = 762, normalized size of antiderivative = 3.42

$$\int \frac{1}{(d+ex)^{3/2}(a^2+2abx+b^2x^2)^{3/2}} dx = \left[-\frac{15(b^2e^3x^3 + a^2de^2 + (b^2de^2 + 2abe^3)x^2 + (2abde^2 + a^2de^2 + b^2d^2e)x + a^2d^2e)}{8(a^2b^3d^4 - 3a^3b^2d^3e + 3a^4bd^2e^2 - a^5de^3 + (b^5d^3e - 3ab^4d^2e^2 + 3a^2b^3d^2e^2 - 5a^3b^2d^2e^2 + a^4bd^2e^3 - a^5e^4)x^2 + (2ab^4d^4 - 5a^2b^3d^3e + 3a^3b^2d^2e^2 + a^4bd^2e^3 - a^5e^4)x) + \frac{1}{4}(15(b^2e^3x^3 + a^2d^2e^2 + (b^2d^2e^2 + 2a^2b^2e^3)x^2 + (2a^2b^2d^2e^2 + a^2d^2e^3)x)\sqrt{b/(b^2d^2 - a^2e^2)}\log((b^2e^3x^3 + a^2d^2e^2 + (b^2d^2e^2 + 2a^2b^2e^3)x^2 + (2a^2b^2d^2e^2 + a^2d^2e^3)x)\sqrt{b/(b^2d^2 - a^2e^2)})/(b^2x^2 + a^2) - 2(15b^2e^2x^2 - 2b^2d^2 + 9a^2b^2d^2e + 8a^2d^2e^2 + 5(b^2d^2e^2 + 5a^2b^2e^2)x)\sqrt{e^2x^2 + d^2})/(a^2b^3d^4 - 3a^3b^2d^3e + 3a^4bd^2e^2 - a^5d^2e^3 + (b^5d^3e - 3a^2b^4d^2e^2 + 3a^3b^2d^2e^3 - a^3b^2e^4)x^3 + (b^5d^4 - a^2b^4d^3e - 3a^2b^3d^2e^2 + 5a^3b^2d^2e^3 - 2a^4b^2e^4)x^2 + (2a^2b^4d^4 - 5a^2b^3d^3e + 3a^3b^2d^2e^2 + a^4bd^2e^3 - a^5e^4)x) + 1/4(15(b^2e^3x^3 + a^2d^2e^2 + (b^2d^2e^2 + 2a^2b^2e^3)x^2 + (2a^2b^2d^2e^2 + a^2d^2e^3)x)\sqrt{-b/(b^2d^2 - a^2e^2)}\arctan(\sqrt{e^2x^2 + d^2}\sqrt{-b/(b^2d^2 - a^2e^2)}) + (15b^2e^2x^2 - 2b^2d^2 + 9a^2b^2d^2e + 8a^2d^2e^2 + 5(b^2d^2e^2 + 5a^2b^2e^2)x)\sqrt{e^2x^2 + d^2})/(a^2b^3d^4 - 3a^3b^2d^3e + 3a^4bd^2e^2 - a^5d^2e^3 + (b^5d^3e - 3a^2b^4d^2e^2 + 3a^3b^2d^2e^3 - a^3b^2e^4)x^3 + (b^5d^4 - a^2b^4d^3e - 3a^2b^3d^2e^2 + 5a^3b^2d^2e^3 - 2a^4b^2e^4)x^2 + (2a^2b^4d^4 - 5a^2b^3d^3e + 3a^3b^2d^2e^2 + a^4bd^2e^3 - a^5e^4)x) \right]$$

input

```
integrate(1/(e*x+d)^(3/2)/(b^2*x^2+2*a*b*x+a^2)^(3/2),x, algorithm="fricas")
```

output

```
[-1/8*(15*(b^2*e^3*x^3 + a^2*d*e^2 + (b^2*d*e^2 + 2*a*b*e^3)*x^2 + (2*a*b*d*e^2 + a^2*e^3)*x)*sqrt(b/(b*d - a*e))*log((b*e*x + 2*b*d - a*e + 2*(b*d - a*e)*sqrt(e*x + d)*sqrt(b/(b*d - a*e)))/(b*x + a)) - 2*(15*b^2*e^2*x^2 - 2*b^2*d^2 + 9*a*b*d*e + 8*a^2*e^2 + 5*(b^2*d*e + 5*a*b*e^2)*x)*sqrt(e*x + d))/(a^2*b^3*d^4 - 3*a^3*b^2*d^3*e + 3*a^4*b*d^2*e^2 - a^5*d*e^3 + (b^5*d^3*e - 3*a*b^4*d^2*e^2 + 3*a^2*b^3*d^2*e^3 - a^3*b^2*e^4)*x^3 + (b^5*d^4 - a*b^4*d^3*e - 3*a^2*b^3*d^2*e^2 + 5*a^3*b^2*d^2*e^3 - 2*a^4*b*e^4)*x^2 + (2*a*b^4*d^4 - 5*a^2*b^3*d^3*e + 3*a^3*b^2*d^2*e^2 + a^4*b*d^2*e^3 - a^5*e^4)*x), 1/4*(15*(b^2*e^3*x^3 + a^2*d*e^2 + (b^2*d*e^2 + 2*a*b*e^3)*x^2 + (2*a*b*d*e^2 + a^2*e^3)*x)*sqrt(-b/(b*d - a*e))*arctan(sqrt(e*x + d)*sqrt(-b/(b*d - a*e))) + (15*b^2*e^2*x^2 - 2*b^2*d^2 + 9*a*b*d*e + 8*a^2*e^2 + 5*(b^2*d*e + 5*a*b*e^2)*x)*sqrt(e*x + d))/(a^2*b^3*d^4 - 3*a^3*b^2*d^3*e + 3*a^4*b*d^2*e^2 - a^5*d*e^3 + (b^5*d^3*e - 3*a*b^4*d^2*e^2 + 3*a^2*b^3*d^2*e^3 - a^3*b^2*e^4)*x^3 + (b^5*d^4 - a*b^4*d^3*e - 3*a^2*b^3*d^2*e^2 + 5*a^3*b^2*d^2*e^3 - 2*a^4*b*e^4)*x^2 + (2*a*b^4*d^4 - 5*a^2*b^3*d^3*e + 3*a^3*b^2*d^2*e^2 + a^4*b*d^2*e^3 - a^5*e^4)*x)]
```

Sympy [F]

$$\int \frac{1}{(d+ex)^{3/2} (a^2+2abx+b^2x^2)^{3/2}} dx = \int \frac{1}{(d+ex)^{\frac{3}{2}} ((a+bx)^2)^{\frac{3}{2}}} dx$$

input `integrate(1/(e*x+d)**(3/2)/(b**2*x**2+2*a*b*x+a**2)**(3/2),x)`

output `Integral(1/((d + e*x)**(3/2)*((a + b*x)**2)**(3/2)), x)`

Maxima [F]

$$\int \frac{1}{(d+ex)^{3/2} (a^2+2abx+b^2x^2)^{3/2}} dx = \int \frac{1}{(b^2x^2+2abx+a^2)^{\frac{3}{2}} (ex+d)^{\frac{3}{2}}} dx$$

input `integrate(1/(e*x+d)^(3/2)/(b^2*x^2+2*a*b*x+a^2)^(3/2),x, algorithm="maxima")`

output `integrate(1/((b^2*x^2 + 2*a*b*x + a^2)^(3/2)*(e*x + d)^(3/2)), x)`

Giac [A] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 306, normalized size of antiderivative = 1.37

$$\int \frac{1}{(d+ex)^{3/2} (a^2+2abx+b^2x^2)^{3/2}} dx = \frac{15 be^2 \arctan\left(\frac{\sqrt{ex+db}}{\sqrt{-b^2d+abe}}\right)}{4(b^3d^3\operatorname{sgn}(bx+a) - 3ab^2d^2\operatorname{sgn}(bx+a) + 3a^2bde^2\operatorname{sgn}(bx+a))} + \frac{2e^2}{(b^3d^3\operatorname{sgn}(bx+a) - 3ab^2d^2\operatorname{sgn}(bx+a) + 3a^2bde^2\operatorname{sgn}(bx+a) - a^3e^3\operatorname{sgn}(bx+a))\sqrt{ex+d}} + \frac{7(ex+d)^{\frac{3}{2}}b^2e^2 - 9\sqrt{ex+db}de^2 + 9\sqrt{ex+db}abe^3}{4(b^3d^3\operatorname{sgn}(bx+a) - 3ab^2d^2\operatorname{sgn}(bx+a) + 3a^2bde^2\operatorname{sgn}(bx+a) - a^3e^3\operatorname{sgn}(bx+a))((ex+d)b - bd + a)}$$

input `integrate(1/(e*x+d)^(3/2)/(b^2*x^2+2*a*b*x+a^2)^(3/2),x, algorithm="giac")`

output

```
15/4*b*e^2*arctan(sqrt(e*x + d)*b/sqrt(-b^2*d + a*b*e))/((b^3*d^3*sgn(b*x
+ a) - 3*a*b^2*d^2*e*sgn(b*x + a) + 3*a^2*b*d*e^2*sgn(b*x + a) - a^3*e^3*sgn(b*x + a))*sqrt(-b^2*d + a*b*e)) + 2*e^2/((b^3*d^3*sgn(b*x + a) - 3*a*b^2*d^2*e*sgn(b*x + a) + 3*a^2*b*d*e^2*sgn(b*x + a) - a^3*e^3*sgn(b*x + a))*sqrt(e*x + d)) + 1/4*(7*(e*x + d)^(3/2)*b^2*e^2 - 9*sqrt(e*x + d)*b^2*d*e^2 + 9*sqrt(e*x + d)*a*b*e^3)/((b^3*d^3*sgn(b*x + a) - 3*a*b^2*d^2*e*sgn(b*x + a) + 3*a^2*b*d*e^2*sgn(b*x + a) - a^3*e^3*sgn(b*x + a))*((e*x + d)*b - b*d + a*e)^2)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(d+ex)^{3/2} (a^2+2abx+b^2x^2)^{3/2}} dx = \int \frac{1}{(d+ex)^{3/2} (a^2+2abx+b^2x^2)^{3/2}} dx$$

input

```
int(1/((d + e*x)^(3/2)*(a^2 + b^2*x^2 + 2*a*b*x)^(3/2)), x)
```

output

```
int(1/((d + e*x)^(3/2)*(a^2 + b^2*x^2 + 2*a*b*x)^(3/2)), x)
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 437, normalized size of antiderivative = 1.96

$$\int \frac{1}{(d+ex)^{3/2} (a^2+2abx+b^2x^2)^{3/2}} dx = \frac{-15\sqrt{b}\sqrt{ex+d}\sqrt{ae-bd}\operatorname{atan}\left(\frac{\sqrt{ex+db}}{\sqrt{b}\sqrt{ae-bd}}\right)a^2e^2 - 30\sqrt{b}\sqrt{ex+d}}{4\sqrt{ex+d}(a^4b^2e^4x^2 - 4a^3b^3de^3)}$$

input

```
int(1/(e*x+d)^(3/2)/(b^2*x^2+2*a*b*x+a^2)^(3/2), x)
```

output

```
( - 15*sqrt(b)*sqrt(d + e*x)*sqrt(a*e - b*d)*atan((sqrt(d + e*x)*b)/(sqrt(b)*sqrt(a*e - b*d)))*a**2*e**2 - 30*sqrt(b)*sqrt(d + e*x)*sqrt(a*e - b*d)*atan((sqrt(d + e*x)*b)/(sqrt(b)*sqrt(a*e - b*d)))*a*b*e**2*x - 15*sqrt(b)*sqrt(d + e*x)*sqrt(a*e - b*d)*atan((sqrt(d + e*x)*b)/(sqrt(b)*sqrt(a*e - b*d)))*b**2*e**2*x**2 - 8*a**3*e**3 - a**2*b*d*e**2 - 25*a**2*b*e**3*x + 11*a*b**2*d**2*e + 20*a*b**2*d*e**2*x - 15*a*b**2*e**3*x**2 - 2*b**3*d**3 + 5*b**3*d**2*e*x + 15*b**3*d*e**2*x**2)/(4*sqrt(d + e*x)*(a**6*e**4 - 4*a**5*b*d*e**3 + 2*a**5*b*e**4*x + 6*a**4*b**2*d**2*e**2 - 8*a**4*b**2*d*e**3*x + a**4*b**2*e**4*x**2 - 4*a**3*b**3*d**3*e + 12*a**3*b**3*d**2*e**2*x - 4*a**3*b**3*d*e**3*x**2 + a**2*b**4*d**4 - 8*a**2*b**4*d**3*e*x + 6*a**2*b**4*d**2*e**2*x**2 + 2*a*b**5*d**4*x - 4*a*b**5*d**3*e*x**2 + b**6*d**4*x**2))
```

3.386 $\int \frac{1}{(d+ex)^{5/2}(a^2+2abx+b^2x^2)^{3/2}} dx$

| | |
|---|------|
| Optimal result | 2854 |
| Mathematica [A] (verified) | 2855 |
| Rubi [A] (verified) | 2855 |
| Maple [B] (verified) | 2859 |
| Fricas [B] (verification not implemented) | 2859 |
| Sympy [F] | 2860 |
| Maxima [F] | 2861 |
| Giac [B] (verification not implemented) | 2861 |
| Mupad [F(-1)] | 2862 |
| Reduce [B] (verification not implemented) | 2862 |

Optimal result

Integrand size = 30, antiderivative size = 275

$$\int \frac{1}{(d+ex)^{5/2}(a^2+2abx+b^2x^2)^{3/2}} dx = \frac{7e}{4(bd-ae)^2(d+ex)^{3/2}\sqrt{a^2+2abx+b^2x^2}} - \frac{1}{2(bd-ae)(a+bx)(d+ex)^{3/2}\sqrt{a^2+2abx+b^2x^2}} + \frac{35e^2(a+bx)}{12(bd-ae)^3(d+ex)^{3/2}\sqrt{a^2+2abx+b^2x^2}} + \frac{35be^2(a+bx)}{4(bd-ae)^4\sqrt{d+ex}\sqrt{a^2+2abx+b^2x^2}} - \frac{35b^{3/2}e^2(a+bx)\operatorname{arctanh}\left(\frac{\sqrt{b}\sqrt{d+ex}}{\sqrt{bd-ae}}\right)}{4(bd-ae)^{9/2}\sqrt{a^2+2abx+b^2x^2}}$$

output

```
7/4*e/(-a*e+b*d)^2/(e*x+d)^(3/2)/((b*x+a)^2)^(1/2)-1/2/(-a*e+b*d)/(b*x+a)/
(e*x+d)^(3/2)/((b*x+a)^2)^(1/2)+35/12*e^2*(b*x+a)/(-a*e+b*d)^3/(e*x+d)^(3/
2)/((b*x+a)^2)^(1/2)+35/4*b*e^2*(b*x+a)/(-a*e+b*d)^4/(e*x+d)^(1/2)/((b*x+a
)^2)^(1/2)-35/4*b^(3/2)*e^2*(b*x+a)*arctanh(b^(1/2)*(e*x+d)^(1/2)/(-a*e+b*
d)^(1/2))/(-a*e+b*d)^(9/2)/((b*x+a)^2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.76 (sec) , antiderivative size = 188, normalized size of antiderivative = 0.68

$$\int \frac{1}{(d+ex)^{5/2}(a^2+2abx+b^2x^2)^{3/2}} dx = \frac{e^2(a+bx)^3 \left(\frac{-8a^3e^3+8a^2be^2(10d+7ex)+ab^2e(39d^2+238dex+175e^2x^2)+b^3(-6d^3+21d^2ex+140dex^2+105e^3x^3)}{e^2(bd-ae)^4(a+bx)^2(d+ex)^{3/2}} \right)}{12((a+bx)^2)^{3/2}}$$

input `Integrate[1/((d + e*x)^(5/2)*(a^2 + 2*a*b*x + b^2*x^2)^(3/2)), x]`

output `(e^2*(a + b*x)^3*((-8*a^3*e^3 + 8*a^2*b*e^2*(10*d + 7*e*x) + a*b^2*e*(39*d^2 + 238*d*e*x + 175*e^2*x^2) + b^3*(-6*d^3 + 21*d^2*e*x + 140*d*e^2*x^2 + 105*e^3*x^3))/(e^2*(b*d - a*e)^4*(a + b*x)^2*(d + e*x)^(3/2)) + (105*b^(3/2)*ArcTan[(Sqrt[b]*Sqrt[d + e*x])/Sqrt[-(b*d) + a*e]])/(-(b*d) + a*e)^(9/2))/(12*((a + b*x)^2)^(3/2))`

Rubi [A] (verified)

Time = 0.55 (sec) , antiderivative size = 221, normalized size of antiderivative = 0.80, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$, Rules used = {1102, 27, 52, 52, 61, 61, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\begin{aligned} & \int \frac{1}{(a^2 + 2abx + b^2x^2)^{3/2} (d + ex)^{5/2}} dx \\ & \quad \downarrow \text{1102} \\ & \frac{b^3(a + bx) \int \frac{1}{b^3(a+bx)^3(d+ex)^{5/2}} dx}{\sqrt{a^2 + 2abx + b^2x^2}} \\ & \quad \downarrow \text{27} \\ & \frac{(a + bx) \int \frac{1}{(a+bx)^3(d+ex)^{5/2}} dx}{\sqrt{a^2 + 2abx + b^2x^2}} \\ & \quad \downarrow \text{52} \end{aligned}$$

$$(a + bx) \left(-\frac{7e \int \frac{1}{(a+bx)^2(d+ex)^{5/2}} dx}{4(bd-ae)} - \frac{1}{2(a+bx)^2(d+ex)^{3/2}(bd-ae)} \right)$$

$$\sqrt{a^2 + 2abx + b^2x^2}$$

52

$$(a + bx) \left(-\frac{7e \left(-\frac{5e \int \frac{1}{(a+bx)(d+ex)^{5/2}} dx}{2(bd-ae)} - \frac{1}{(a+bx)(d+ex)^{3/2}(bd-ae)} \right)}{4(bd-ae)} - \frac{1}{2(a+bx)^2(d+ex)^{3/2}(bd-ae)} \right)$$

$$\sqrt{a^2 + 2abx + b^2x^2}$$

61

$$(a + bx) \left(-\frac{7e \left(-\frac{5e \left(\frac{b \int \frac{1}{(a+bx)(d+ex)^{3/2}} dx}{bd-ae} + \frac{2}{3(d+ex)^{3/2}(bd-ae)} \right)}{2(bd-ae)} - \frac{1}{(a+bx)(d+ex)^{3/2}(bd-ae)} \right)}{4(bd-ae)} - \frac{1}{2(a+bx)^2(d+ex)^{3/2}(bd-ae)} \right)$$

$$\sqrt{a^2 + 2abx + b^2x^2}$$

61

$$(a + bx) \left(-\frac{7e \left(-\frac{5e \left(\frac{b \left(\frac{b \int \frac{1}{(a+bx)\sqrt{d+ex}} dx}{bd-ae} + \frac{2}{\sqrt{d+ex}(bd-ae)} \right)}{2(bd-ae)} + \frac{2}{3(d+ex)^{3/2}(bd-ae)} \right)}{4(bd-ae)} - \frac{1}{(a+bx)(d+ex)^{3/2}(bd-ae)} \right)}{4(bd-ae)} - \frac{1}{2(a+bx)^2(d+ex)^{3/2}(bd-ae)} \right)$$

$$\sqrt{a^2 + 2abx + b^2x^2}$$

73

$$\begin{aligned}
 & \left(\frac{(a+bx) \left(\frac{5e \left(b \left(\frac{2b \int \frac{1}{a + \frac{b(d+ex) - bd}{e(bd-ae)}} d\sqrt{d+ex} \right) + \frac{2}{\sqrt{d+ex}(bd-ae)} \right)}{bd-ae} + \frac{2}{3(d+ex)^{3/2}(bd-ae)} \right)}{2(bd-ae)} - \frac{1}{(a+bx)(d+ex)^{3/2}(bd-ae)} \right)}{4(bd-ae)} - \frac{1}{2(a+bx)^2(d+ex)} \right)
 \end{aligned}$$

$$\sqrt{a^2 + 2abx + b^2x^2}$$

↓ 221

$$\begin{aligned}
 & \left(\frac{(a+bx) \left(\frac{5e \left(b \left(\frac{2}{\sqrt{d+ex}(bd-ae)} - \frac{2\sqrt{b} \operatorname{arctanh} \left(\frac{\sqrt{b}\sqrt{d+ex}}{\sqrt{bd-ae}} \right)}{(bd-ae)^{3/2}} \right) + \frac{2}{3(d+ex)^{3/2}(bd-ae)} \right)}{2(bd-ae)} - \frac{1}{(a+bx)(d+ex)^{3/2}(bd-ae)} \right)}{4(bd-ae)} - \frac{1}{2(a+bx)^2(d+ex)} \right)
 \end{aligned}$$

$$\sqrt{a^2 + 2abx + b^2x^2}$$

input `Int[1/((d + e*x)^(5/2)*(a^2 + 2*a*b*x + b^2*x^2)^(3/2)),x]`

output `((a + b*x)*(-1/2*1/((b*d - a*e)*(a + b*x)^2*(d + e*x)^(3/2)) - (7*e*(-1/((b*d - a*e)*(a + b*x)*(d + e*x)^(3/2))) - (5*e*(2/(3*(b*d - a*e)*(d + e*x)^(3/2)) + (b*(2/((b*d - a*e)*Sqrt[d + e*x]) - (2*Sqrt[b]*ArcTanh[(Sqrt[b]*Sqrt[d + e*x])/Sqrt[b*d - a*e]])/(b*d - a*e)^(3/2)))/(b*d - a*e)))/(2*(b*d - a*e))))/(4*(b*d - a*e)))/Sqrt[a^2 + 2*a*b*x + b^2*x^2]`

Defintions of rubi rules used

rule 27 `Int[(a_)*(F_x_), x_Symbol] := Simp[a Int[F_x, x], x] /; FreeQ[a, x] && !MatchQ[F_x, (b_)*(G_x_) /; FreeQ[b, x]]`

rule 52 `Int[((a_.) + (b_.)*(x_)^(m_))*((c_.) + (d_.)*(x_)^(n_)), x_Symbol] := Simp[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((m + n + 2)/((b*c - a*d)*(m + 1))) Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && ILtQ[m, -1] && FractionQ[n] && LtQ[n, 0]`

rule 61 `Int[((a_.) + (b_.)*(x_)^(m_))*((c_.) + (d_.)*(x_)^(n_)), x_Symbol] := Simp[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((m + n + 2)/((b*c - a*d)*(m + 1))) Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && LtQ[m, -1] && !(LtQ[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && IntLinearQ[a, b, c, d, m, n, x]`

rule 73 `Int[((a_.) + (b_.)*(x_)^(m_))*((c_.) + (d_.)*(x_)^(n_)), x_Symbol] := With[{p = Denominator[m]}, Simp[p/b Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) + d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntLinearQ[a, b, c, d, m, n, x]`

rule 221 `Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[-a/b, 2]/a)*ArcTanh[x/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]`

rule 1102

```
Int[((d_.) + (e_.)*(x_.))^(m_.)*((a_.) + (b_.)*(x_.) + (c_.)*(x_.)^2)^(p_), x_Symbol]
:> Simp[(a + b*x + c*x^2)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x)^(2*FracPart[p]))
Int[(d + e*x)^m*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d, e, m, p}, x]
&& EqQ[b^2 - 4*a*c, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 387 vs. $2(192) = 384$.

Time = 1.51 (sec) , antiderivative size = 388, normalized size of antiderivative = 1.41

| method | result |
|---------|--|
| default | $\frac{\left(105(ex+d)^{\frac{3}{2}} \arctan\left(\frac{b\sqrt{ex+d}}{\sqrt{b(ae-bd)}}\right) b^4 e^2 x^2 + 210(ex+d)^{\frac{3}{2}} \arctan\left(\frac{b\sqrt{ex+d}}{\sqrt{b(ae-bd)}}\right) a b^3 e^2 x + 105(ex+d)^{\frac{3}{2}} \arctan\left(\frac{b\sqrt{ex+d}}{\sqrt{b(ae-bd)}}\right) a^2 b^2 e^2 + \dots}{\dots}$ |

input

```
int(1/(e*x+d)^(5/2)/(b^2*x^2+2*a*b*x+a^2)^(3/2),x,method=_RETURNVERBOSE)
```

output

```
1/12*(105*(e*x+d)^(3/2)*arctan(b*(e*x+d)^(1/2)/(b*(a*e-b*d))^(1/2))*b^4*e^2*x^2+210*(e*x+d)^(3/2)*arctan(b*(e*x+d)^(1/2)/(b*(a*e-b*d))^(1/2))*a*b^3*e^2*x+105*(e*x+d)^(3/2)*arctan(b*(e*x+d)^(1/2)/(b*(a*e-b*d))^(1/2))*a^2*b^2*e^2+105*(b*(a*e-b*d))^(1/2)*b^3*e^3*x^3+175*(b*(a*e-b*d))^(1/2)*a*b^2*e^3*x^2+140*(b*(a*e-b*d))^(1/2)*b^3*d*e^2*x^2+56*(b*(a*e-b*d))^(1/2)*a^2*b*e^3*x+238*(b*(a*e-b*d))^(1/2)*a*b^2*d*e^2*x+21*(b*(a*e-b*d))^(1/2)*b^3*d^2*e*x-8*(b*(a*e-b*d))^(1/2)*a^3*e^3+80*(b*(a*e-b*d))^(1/2)*a^2*b*d*e^2+39*(b*(a*e-b*d))^(1/2)*a*b^2*d^2*e-6*(b*(a*e-b*d))^(1/2)*b^3*d^3*(b*x+a)/(b*(a*e-b*d))^(1/2)/(e*x+d)^(3/2)/(a*e-b*d)^4/((b*x+a)^2)^(3/2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 588 vs. $2(192) = 384$.

Time = 0.13 (sec) , antiderivative size = 1206, normalized size of antiderivative = 4.39

$$\int \frac{1}{(d+ex)^{5/2} (a^2+2abx+b^2x^2)^{3/2}} dx = \text{Too large to display}$$

input `integrate(1/(e*x+d)^(5/2)/(b^2*x^2+2*a*b*x+a^2)^(3/2),x, algorithm="fricas")`

output `[1/24*(105*(b^3*e^4*x^4 + a^2*b*d^2*e^2 + 2*(b^3*d*e^3 + a*b^2*e^4)*x^3 + (b^3*d^2*e^2 + 4*a*b^2*d*e^3 + a^2*b*e^4)*x^2 + 2*(a*b^2*d^2*e^2 + a^2*b*d*e^3)*x)*sqrt(b/(b*d - a*e))*log((b*e*x + 2*b*d - a*e - 2*(b*d - a*e)*sqrt(e*x + d)*sqrt(b/(b*d - a*e)))/(b*x + a)) + 2*(105*b^3*e^3*x^3 - 6*b^3*d^3 + 39*a*b^2*d^2*e + 80*a^2*b*d*e^2 - 8*a^3*e^3 + 35*(4*b^3*d*e^2 + 5*a*b^2*e^3)*x^2 + 7*(3*b^3*d^2*e + 34*a*b^2*d*e^2 + 8*a^2*b*e^3)*x)*sqrt(e*x + d)]/(a^2*b^4*d^6 - 4*a^3*b^3*d^5*e + 6*a^4*b^2*d^4*e^2 - 4*a^5*b*d^3*e^3 + a^6*d^2*e^4 + (b^6*d^4*e^2 - 4*a*b^5*d^3*e^3 + 6*a^2*b^4*d^2*e^4 - 4*a^3*b^3*d*e^5 + a^4*b^2*e^6)*x^4 + 2*(b^6*d^5*e - 3*a*b^5*d^4*e^2 + 2*a^2*b^4*d^3*e^3 + 2*a^3*b^3*d^2*e^4 - 3*a^4*b^2*d*e^5 + a^5*b*e^6)*x^3 + (b^6*d^6 - 9*a^2*b^4*d^4*e^2 + 16*a^3*b^3*d^3*e^3 - 9*a^4*b^2*d^2*e^4 + a^6*e^6)*x^2 + 2*(a*b^5*d^6 - 3*a^2*b^4*d^5*e + 2*a^3*b^3*d^4*e^2 + 2*a^4*b^2*d^3*e^3 - 3*a^5*b*d^2*e^4 + a^6*d*e^5)*x), 1/12*(105*(b^3*e^4*x^4 + a^2*b*d^2*e^2 + 2*(b^3*d*e^3 + a*b^2*e^4)*x^3 + (b^3*d^2*e^2 + 4*a*b^2*d*e^3 + a^2*b*e^4)*x^2 + 2*(a*b^2*d^2*e^2 + a^2*b*d*e^3)*x)*sqrt(-b/(b*d - a*e))*arctan(sqrt(e*x + d)*sqrt(-b/(b*d - a*e))) + (105*b^3*e^3*x^3 - 6*b^3*d^3 + 39*a*b^2*d^2*e + 80*a^2*b*d*e^2 - 8*a^3*e^3 + 35*(4*b^3*d*e^2 + 5*a*b^2*e^3)*x^2 + 7*(3*b^3*d^2*e + 34*a*b^2*d*e^2 + 8*a^2*b*e^3)*x)*sqrt(e*x + d)]/(a^2*b^4*d^6 - 4*a^3*b^3*d^5*e + 6*a^4*b^2*d^4*e^2 - 4*a^5*b*d^3*e^3 + a^6*d^2*e^4 + (b^6*d^4*e^2 - 4*a*b^5*d^3*e^3 + 6*a^2*b^4*d^2*e^4 - 4*a^3*b^3*d*e^5...`

Sympy [F]

$$\int \frac{1}{(d+ex)^{5/2} (a^2+2abx+b^2x^2)^{3/2}} dx = \int \frac{1}{(d+ex)^{5/2} ((a+bx)^2)^{3/2}} dx$$

input `integrate(1/(e*x+d)**(5/2)/(b**2*x**2+2*a*b*x+a**2)**(3/2),x)`

output `Integral(1/((d + e*x)**(5/2)*((a + b*x)**2)**(3/2)), x)`

Maxima [F]

$$\int \frac{1}{(d+ex)^{5/2} (a^2+2abx+b^2x^2)^{3/2}} dx = \int \frac{1}{(b^2x^2+2abx+a^2)^{3/2} (ex+d)^{5/2}} dx$$

input `integrate(1/(e*x+d)^(5/2)/(b^2*x^2+2*a*b*x+a^2)^(3/2),x, algorithm="maxima")`

output `integrate(1/((b^2*x^2 + 2*a*b*x + a^2)^(3/2)*(e*x + d)^(5/2)), x)`

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 388 vs. $2(192) = 384$.

Time = 0.20 (sec) , antiderivative size = 388, normalized size of antiderivative = 1.41

$$\int \frac{1}{(d+ex)^{5/2} (a^2+2abx+b^2x^2)^{3/2}} dx = \frac{35b^2e^2 \arctan\left(\frac{b\sqrt{ex+d}}{\sqrt{-b^2d+abe}}\right)}{4(b^4d^4\operatorname{sgn}(bx+a) - 4ab^3d^3\operatorname{esgn}(bx+a) + 6a^2b^2d^2e^2\operatorname{sgn}(bx+a) - 4a^3bde^3\operatorname{sgn}(bx+a) + a^4e^4\operatorname{sgn}(bx+a))} + \frac{2(9(ex+d)be^2 + bde^2 - ae^3)}{3(b^4d^4\operatorname{sgn}(bx+a) - 4ab^3d^3\operatorname{esgn}(bx+a) + 6a^2b^2d^2e^2\operatorname{sgn}(bx+a) - 4a^3bde^3\operatorname{sgn}(bx+a) + a^4e^4\operatorname{sgn}(bx+a))} + \frac{11(ex+d)^{3/2}b^3e^2 - 13\sqrt{ex+d}b^3de^2 + 13\sqrt{ex+d}ab^2e^3}{4(b^4d^4\operatorname{sgn}(bx+a) - 4ab^3d^3\operatorname{esgn}(bx+a) + 6a^2b^2d^2e^2\operatorname{sgn}(bx+a) - 4a^3bde^3\operatorname{sgn}(bx+a) + a^4e^4\operatorname{sgn}(bx+a))}$$

input `integrate(1/(e*x+d)^(5/2)/(b^2*x^2+2*a*b*x+a^2)^(3/2),x, algorithm="giac")`

output `35/4*b^2*e^2*arctan(sqrt(e*x + d)*b/sqrt(-b^2*d + a*b*e))/((b^4*d^4*sgn(b*x + a) - 4*a*b^3*d^3*e*sgn(b*x + a) + 6*a^2*b^2*d^2*e^2*sgn(b*x + a) - 4*a^3*b*d*e^3*sgn(b*x + a) + a^4*e^4*sgn(b*x + a))*sqrt(-b^2*d + a*b*e)) + 2/3*(9*(e*x + d)*b*e^2 + b*d*e^2 - a*e^3)/((b^4*d^4*sgn(b*x + a) - 4*a*b^3*d^3*e*sgn(b*x + a) + 6*a^2*b^2*d^2*e^2*sgn(b*x + a) - 4*a^3*b*d*e^3*sgn(b*x + a) + a^4*e^4*sgn(b*x + a))*(e*x + d)^(3/2)) + 1/4*(11*(e*x + d)^(3/2)*b^3*e^2 - 13*sqrt(e*x + d)*b^3*d*e^2 + 13*sqrt(e*x + d)*a*b^2*e^3)/((b^4*d^4*sgn(b*x + a) - 4*a*b^3*d^3*e*sgn(b*x + a) + 6*a^2*b^2*d^2*e^2*sgn(b*x + a) - 4*a^3*b*d*e^3*sgn(b*x + a) + a^4*e^4*sgn(b*x + a))*((e*x + d)*b - b*d + a*e)^2)`

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(d+ex)^{5/2} (a^2+2abx+b^2x^2)^{3/2}} dx = \int \frac{1}{(d+ex)^{5/2} (a^2+2abx+b^2x^2)^{3/2}} dx$$

input `int(1/((d + e*x)^(5/2)*(a^2 + b^2*x^2 + 2*a*b*x)^(3/2)), x)`

output `int(1/((d + e*x)^(5/2)*(a^2 + b^2*x^2 + 2*a*b*x)^(3/2)), x)`

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 804, normalized size of antiderivative = 2.92

$$\int \frac{1}{(d+ex)^{5/2} (a^2+2abx+b^2x^2)^{3/2}} dx = \frac{105\sqrt{b}\sqrt{ex+d}\sqrt{ae-bd}\operatorname{atan}\left(\frac{\sqrt{ex+d}b}{\sqrt{b}\sqrt{ae-bd}}\right) + a^2bde^2 + 105\sqrt{b}\sqrt{ex+d}}{(d+ex)^{5/2} (a^2+2abx+b^2x^2)^{3/2}}$$

input `int(1/(e*x+d)^(5/2)/(b^2*x^2+2*a*b*x+a^2)^(3/2), x)`

output

```
(105*sqrt(b)*sqrt(d + e*x)*sqrt(a*e - b*d)*atan((sqrt(d + e*x)*b)/(sqrt(b)
*sqrt(a*e - b*d)))*a**2*b*d*e**2 + 105*sqrt(b)*sqrt(d + e*x)*sqrt(a*e - b*
d)*atan((sqrt(d + e*x)*b)/(sqrt(b)*sqrt(a*e - b*d)))*a**2*b*e**3*x + 210*sq
rt(b)*sqrt(d + e*x)*sqrt(a*e - b*d)*atan((sqrt(d + e*x)*b)/(sqrt(b)*sqrt(
a*e - b*d)))*a*b**2*d*e**2*x + 210*sqrt(b)*sqrt(d + e*x)*sqrt(a*e - b*d)*a
tan((sqrt(d + e*x)*b)/(sqrt(b)*sqrt(a*e - b*d)))*a*b**2*e**3*x**2 + 105*sq
rt(b)*sqrt(d + e*x)*sqrt(a*e - b*d)*atan((sqrt(d + e*x)*b)/(sqrt(b)*sqrt(a
*e - b*d)))*b**3*d*e**2*x**2 + 105*sqrt(b)*sqrt(d + e*x)*sqrt(a*e - b*d)*a
tan((sqrt(d + e*x)*b)/(sqrt(b)*sqrt(a*e - b*d)))*b**3*e**3*x**3 - 8*a**4*e
**4 + 88*a**3*b*d*e**3 + 56*a**3*b*e**4*x - 41*a**2*b**2*d**2*e**2 + 182*a
**2*b**2*d*e**3*x + 175*a**2*b**2*e**4*x**2 - 45*a*b**3*d**3*e - 217*a*b**
3*d**2*e**2*x - 35*a*b**3*d*e**3*x**2 + 105*a*b**3*e**4*x**3 + 6*b**4*d**4
- 21*b**4*d**3*e*x - 140*b**4*d**2*e**2*x**2 - 105*b**4*d*e**3*x**3)/(12*
sqrt(d + e*x)*(a**7*d*e**5 + a**7*e**6*x - 5*a**6*b*d**2*e**4 - 3*a**6*b*d
**5*x + 2*a**6*b*e**6*x**2 + 10*a**5*b**2*d**3*e**3 - 9*a**5*b**2*d*e**5
*x**2 + a**5*b**2*e**6*x**3 - 10*a**4*b**3*d**4*e**2 + 10*a**4*b**3*d**3*e
**3*x + 15*a**4*b**3*d**2*e**4*x**2 - 5*a**4*b**3*d*e**5*x**3 + 5*a**3*b**
4*d**5*e - 15*a**3*b**4*d**4*e**2*x - 10*a**3*b**4*d**3*e**3*x**2 + 10*a**
3*b**4*d**2*e**4*x**3 - a**2*b**5*d**6 + 9*a**2*b**5*d**5*e*x - 10*a**2*b*
**5*d**3*e**3*x**3 - 2*a*b**6*d**6*x + 3*a*b**6*d**5*e*x**2 + 5*a*b**6*d...
```

3.387 $\int \frac{1}{(d+ex)^{7/2}(a^2+2abx+b^2x^2)^{3/2}} dx$

| | |
|---|------|
| Optimal result | 2864 |
| Mathematica [A] (verified) | 2865 |
| Rubi [A] (verified) | 2865 |
| Maple [B] (verified) | 2871 |
| Fricas [B] (verification not implemented) | 2872 |
| Sympy [F] | 2873 |
| Maxima [F] | 2873 |
| Giac [B] (verification not implemented) | 2873 |
| Mupad [F(-1)] | 2874 |
| Reduce [B] (verification not implemented) | 2874 |

Optimal result

Integrand size = 30, antiderivative size = 329

$$\int \frac{1}{(d+ex)^{7/2}(a^2+2abx+b^2x^2)^{3/2}} dx = \frac{9e}{4(bd-ae)^2(d+ex)^{5/2}\sqrt{a^2+2abx+b^2x^2}} - \frac{1}{2(bd-ae)(a+bx)(d+ex)^{5/2}\sqrt{a^2+2abx+b^2x^2}} + \frac{63e^2(a+bx)}{20(bd-ae)^3(d+ex)^{5/2}\sqrt{a^2+2abx+b^2x^2}} + \frac{21be^2(a+bx)}{4(bd-ae)^4(d+ex)^{3/2}\sqrt{a^2+2abx+b^2x^2}} + \frac{63b^2e^2(a+bx)}{4(bd-ae)^5\sqrt{d+ex}\sqrt{a^2+2abx+b^2x^2}} - \frac{63b^{5/2}e^2(a+bx)\operatorname{arctanh}\left(\frac{\sqrt{b}\sqrt{d+ex}}{\sqrt{bd-ae}}\right)}{4(bd-ae)^{11/2}\sqrt{a^2+2abx+b^2x^2}}$$

output

```
9/4*e/(-a*e+b*d)^2/(e*x+d)^(5/2)/((b*x+a)^2)^(1/2)-1/2/(-a*e+b*d)/(b*x+a)/
(e*x+d)^(5/2)/((b*x+a)^2)^(1/2)+63/20*e^2*(b*x+a)/(-a*e+b*d)^3/(e*x+d)^(5/
2)/((b*x+a)^2)^(1/2)+21/4*b*e^2*(b*x+a)/(-a*e+b*d)^4/(e*x+d)^(3/2)/((b*x+a
)^2)^(1/2)+63/4*b^2*e^2*(b*x+a)/(-a*e+b*d)^5/(e*x+d)^(1/2)/((b*x+a)^2)^(1/
2)-63/4*b^(5/2)*e^2*(b*x+a)*arctanh(b^(1/2)*(e*x+d)^(1/2)/(-a*e+b*d)^(1/2)
)/(-a*e+b*d)^(11/2)/((b*x+a)^2)^(1/2)
```


$$\begin{aligned}
 & \downarrow 52 \\
 & \frac{(a+bx) \left(-\frac{9e \int \frac{1}{(a+bx)^2(d+ex)^{7/2}} dx}{4(bd-ae)} - \frac{1}{2(a+bx)^2(d+ex)^{5/2}(bd-ae)} \right)}{\sqrt{a^2+2abx+b^2x^2}} \\
 & \downarrow 52 \\
 & \frac{(a+bx) \left(-\frac{9e \left(-\frac{7e \int \frac{1}{(a+bx)(d+ex)^{7/2}} dx}{2(bd-ae)} - \frac{1}{(a+bx)(d+ex)^{5/2}(bd-ae)} \right)}{4(bd-ae)} - \frac{1}{2(a+bx)^2(d+ex)^{5/2}(bd-ae)} \right)}{\sqrt{a^2+2abx+b^2x^2}} \\
 & \downarrow 61 \\
 & \frac{(a+bx) \left(-\frac{9e \left(\frac{7e \left(\frac{b \int \frac{1}{(a+bx)(d+ex)^{5/2}} dx}{bd-ae} + \frac{2}{5(d+ex)^{5/2}(bd-ae)} \right)}{2(bd-ae)} - \frac{1}{(a+bx)(d+ex)^{5/2}(bd-ae)} \right)}{4(bd-ae)} - \frac{1}{2(a+bx)^2(d+ex)^{5/2}(bd-ae)} \right)}{\sqrt{a^2+2abx+b^2x^2}} \\
 & \downarrow 61 \\
 & \frac{(a+bx) \left(-\frac{9e \left(\frac{7e \left(\frac{b \left(\frac{b \int \frac{1}{(a+bx)(d+ex)^{3/2}} dx}{bd-ae} + \frac{2}{3(d+ex)^{3/2}(bd-ae)} \right)}{bd-ae} + \frac{2}{5(d+ex)^{5/2}(bd-ae)} \right)}{2(bd-ae)} - \frac{1}{(a+bx)(d+ex)^{5/2}(bd-ae)} \right)}{4(bd-ae)} - \frac{1}{2(a+bx)^2(d+ex)^{5/2}(bd-ae)} \right)}{\sqrt{a^2+2abx+b^2x^2}}
 \end{aligned}$$

↓ 61

$$\begin{aligned}
 & \left(\left(\left(\left(\left(\frac{b \int \frac{1}{(a+bx)\sqrt{d+ex}} dx}{bd-ae} + \frac{2}{\sqrt{d+ex}(bd-ae)} \right) + \frac{2}{3(d+ex)^{3/2}(bd-ae)} \right) + \frac{2}{5(d+ex)^{5/2}(bd-ae)} \right) + \frac{2}{7e} \right) + \frac{2}{9e} \right) - \frac{1}{(a+bx)(d+ex)^{5/2}(bd-ae)} \\
 & \frac{(a+bx)}{4(bd-ae)}
 \end{aligned}$$

$$\sqrt{a^2 + 2abx + b^2x^2}$$

↓ 73

$$\begin{aligned}
 & \left(\left(\left(\left(\frac{2b \int \frac{1}{a + \frac{b(d+ex) - bd}{e}} d\sqrt{d+ex}}{e(bd-ae)} + \frac{2}{\sqrt{d+ex}(bd-ae)} \right) + \frac{2}{3(d+ex)^{3/2}(bd-ae)} \right) + \frac{2}{5(d+ex)^{5/2}(bd-ae)} \right) - \frac{1}{(a+bx)(d+ex)^{5/2}(bd-ae)} \right) \\
 & \left(\frac{7e}{bd-ae} \right) \\
 & \left(\frac{9e}{2(bd-ae)} \right) \\
 & \left(\frac{(a+bx)}{4(bd-ae)} \right)
 \end{aligned}$$

$$\sqrt{a^2 + 2abx + b^2x^2}$$

$$\frac{(a + bx) \left(\frac{7e \left(\frac{b \left(\frac{2}{\sqrt{d+ex}(bd-ae)} - \frac{2\sqrt{b} \operatorname{arctanh} \left(\frac{\sqrt{b}\sqrt{d+ex}}{\sqrt{bd-ae}} \right)}{(bd-ae)^{3/2}} \right)}{bd-ae} + \frac{2}{3(d+ex)^{3/2}(bd-ae)} \right)}{bd-ae} + \frac{2}{5(d+ex)^{5/2}(bd-ae)} \right) - \frac{9e}{2(bd-ae)} - \frac{1}{(a+bx)(d+ex)^{5/2}(bd-ae)}}{4(bd-ae)}$$

$$\sqrt{a^2 + 2abx + b^2x^2}$$

input `Int[1/((d + e*x)^(7/2)*(a^2 + 2*a*b*x + b^2*x^2)^(3/2)),x]`

output

$$\frac{((a + bx)^{-1/2} * 1 / ((b*d - a*e) * (a + b*x)^2 * (d + e*x)^{5/2}) - (9*e * (-1 / ((b*d - a*e) * (a + b*x) * (d + e*x)^{5/2}))) - (7*e * (2 / (5 * (b*d - a*e) * (d + e*x)^{5/2})) + (b * (2 / (3 * (b*d - a*e) * (d + e*x)^{3/2})) + (b * (2 / ((b*d - a*e) * \sqrt{d + e*x})) - (2 * \sqrt{b} * \text{ArcTanh}[(\sqrt{b} * \sqrt{d + e*x}) / \sqrt{b*d - a*e}])) / (b*d - a*e)^{3/2})) / (b*d - a*e)) / (b*d - a*e)) / (2 * (b*d - a*e))) / (4 * (b*d - a*e)) / \sqrt{a^2 + 2*a*b*x + b^2*x^2}}$$

Defintions of rubi rules used

rule 27

$$\text{Int}[(a_*)(Fx_), x_Symbol] \rightarrow \text{Simp}[a \quad \text{Int}[Fx, x], x] \text{ ; FreeQ}[a, x] \&\& \text{!MatchQ}[Fx, (b_*)(Gx_) \text{ ; FreeQ}[b, x]$$

rule 52

$$\text{Int}[(a_.) + (b_.)(x_)^{(m_.)} * ((c_.) + (d_.)(x_)^{(n_.)}), x_Symbol] \rightarrow \text{Simp}[(a + b*x)^{(m + 1)} * ((c + d*x)^{(n + 1)} / ((b*c - a*d) * (m + 1))), x] - \text{Simp}[d * ((m + n + 2) / ((b*c - a*d) * (m + 1))) \quad \text{Int}[(a + b*x)^{(m + 1)} * (c + d*x)^n, x], x] \text{ ; FreeQ}[\{a, b, c, d, n\}, x] \&\& \text{ILtQ}[m, -1] \&\& \text{FractionQ}[n] \&\& \text{LtQ}[n, 0]$$

rule 61

$$\text{Int}[(a_.) + (b_.)(x_)^{(m_.)} * ((c_.) + (d_.)(x_)^{(n_.)}), x_Symbol] \rightarrow \text{Simp}[(a + b*x)^{(m + 1)} * ((c + d*x)^{(n + 1)} / ((b*c - a*d) * (m + 1))), x] - \text{Simp}[d * ((m + n + 2) / ((b*c - a*d) * (m + 1))) \quad \text{Int}[(a + b*x)^{(m + 1)} * (c + d*x)^n, x], x] \text{ ; FreeQ}[\{a, b, c, d, n\}, x] \&\& \text{LtQ}[m, -1] \&\& \text{!(LtQ}[n, -1] \&\& (\text{EqQ}[a, 0] \text{ || } (\text{NeQ}[c, 0] \&\& \text{LtQ}[m - n, 0] \&\& \text{IntegerQ}[n]))) \&\& \text{IntLinearQ}[a, b, c, d, m, n, x]$$

rule 73

$$\text{Int}[(a_.) + (b_.)(x_)^{(m_.)} * ((c_.) + (d_.)(x_)^{(n_.)}), x_Symbol] \rightarrow \text{With}[\{p = \text{Denominator}[m]\}, \text{Simp}[p/b \quad \text{Subst}[\text{Int}[x^{(p*(m + 1) - 1)} * (c - a*(d/b) + d*(x^p/b)^n, x], x, (a + b*x)^{(1/p)}, x]] \text{ ; FreeQ}[\{a, b, c, d\}, x] \&\& \text{LtQ}[-1, m, 0] \&\& \text{LeQ}[-1, n, 0] \&\& \text{LeQ}[\text{Denominator}[n], \text{Denominator}[m]] \&\& \text{IntLinearQ}[a, b, c, d, m, n, x]$$

rule 221

$$\text{Int}[(a_.) + (b_.)(x_)^2)^{-1}, x_Symbol] \rightarrow \text{Simp}[(\text{Rt}[-a/b, 2]/a) * \text{ArcTanh}[x / \text{Rt}[-a/b, 2]], x] \text{ ; FreeQ}[\{a, b\}, x] \&\& \text{NegQ}[a/b]$$

rule 1102

```
Int[((d_.) + (e_.)*(x_.))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol]
:> Simp[(a + b*x + c*x^2)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x)^(2*FracPart[p]))
Int[(d + e*x)^m*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d, e, m, p}, x]
&& EqQ[b^2 - 4*a*c, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 517 vs. $2(231) = 462$.

Time = 1.51 (sec) , antiderivative size = 518, normalized size of antiderivative = 1.57

| method | result |
|---------|--|
| default | $-\frac{\left(315(ex+d)^{\frac{5}{2}} \arctan\left(\frac{b\sqrt{ex+d}}{\sqrt{b(ae-bd)}}\right) b^5 e^2 x^2 + 630(ex+d)^{\frac{5}{2}} \arctan\left(\frac{b\sqrt{ex+d}}{\sqrt{b(ae-bd)}}\right) a b^4 e^2 x + 315(ex+d)^{\frac{5}{2}} \arctan\left(\frac{b\sqrt{ex+d}}{\sqrt{b(ae-bd)}}\right) a^2 b^3 e^2\right)}{\dots}$ |

input

```
int(1/(e*x+d)^(7/2)/(b^2*x^2+2*a*b*x+a^2)^(3/2),x,method=_RETURNVERBOSE)
```

output

```
-1/20*(315*(e*x+d)^(5/2)*arctan(b*(e*x+d)^(1/2)/(b*(a*e-b*d))^(1/2))*b^5*e
^2*x^2+630*(e*x+d)^(5/2)*arctan(b*(e*x+d)^(1/2)/(b*(a*e-b*d))^(1/2))*a*b^4
*e^2*x+315*(e*x+d)^(5/2)*arctan(b*(e*x+d)^(1/2)/(b*(a*e-b*d))^(1/2))*a^2*b
^3*e^2+315*(b*(a*e-b*d))^(1/2)*b^4*e^4*x^4+525*(b*(a*e-b*d))^(1/2)*a*b^3*e
^4*x^3+735*(b*(a*e-b*d))^(1/2)*b^4*d*e^3*x^3+168*(b*(a*e-b*d))^(1/2)*a^2*b
^2*e^4*x^2+1239*(b*(a*e-b*d))^(1/2)*a*b^3*d*e^3*x^2+483*(b*(a*e-b*d))^(1/2)
)*b^4*d^2*e^2*x^2-24*(b*(a*e-b*d))^(1/2)*a^3*b*e^4*x+408*(b*(a*e-b*d))^(1/2)
)*a^2*b^2*d*e^3*x+831*(b*(a*e-b*d))^(1/2)*a*b^3*d^2*e^2*x+45*(b*(a*e-b*d)
)^(1/2)*b^4*d^3*e*x+8*(b*(a*e-b*d))^(1/2)*a^4*e^4-56*(b*(a*e-b*d))^(1/2)*a
^3*b*d*e^3+288*(b*(a*e-b*d))^(1/2)*a^2*b^2*d^2*e^2+85*(b*(a*e-b*d))^(1/2)*
a*b^3*d^3*e-10*(b*(a*e-b*d))^(1/2)*b^4*d^4)*(b*x+a)/(e*x+d)^(5/2)/(b*(a*e-
b*d))^(1/2)/(a*e-b*d)^5/((b*x+a)^2)^(3/2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 904 vs. $2(231) = 462$.

Time = 0.21 (sec) , antiderivative size = 1838, normalized size of antiderivative = 5.59

$$\int \frac{1}{(d+ex)^{7/2} (a^2+2abx+b^2x^2)^{3/2}} dx = \text{Too large to display}$$

input `integrate(1/(e*x+d)^(7/2)/(b^2*x^2+2*a*b*x+a^2)^(3/2),x, algorithm="fricas")`

output

```
[-1/40*(315*(b^4*e^5*x^5 + a^2*b^2*d^3*e^2 + (3*b^4*d*e^4 + 2*a*b^3*e^5)*x^4 + (3*b^4*d^2*e^3 + 6*a*b^3*d*e^4 + a^2*b^2*e^5)*x^3 + (b^4*d^3*e^2 + 6*a*b^3*d^2*e^3 + 3*a^2*b^2*d*e^4)*x^2 + (2*a*b^3*d^3*e^2 + 3*a^2*b^2*d^2*e^3)*x)*sqrt(b/(b*d - a*e))*log((b*e*x + 2*b*d - a*e + 2*(b*d - a*e)*sqrt(e*x + d))*sqrt(b/(b*d - a*e)))/(b*x + a) - 2*(315*b^4*e^4*x^4 - 10*b^4*d^4 + 85*a*b^3*d^3*e + 288*a^2*b^2*d^2*e^2 - 56*a^3*b*d*e^3 + 8*a^4*e^4 + 105*(7*b^4*d*e^3 + 5*a*b^3*e^4)*x^3 + 21*(23*b^4*d^2*e^2 + 59*a*b^3*d*e^3 + 8*a^2*b^2*e^4)*x^2 + 3*(15*b^4*d^3*e + 277*a*b^3*d^2*e^2 + 136*a^2*b^2*d*e^3 - 8*a^3*b*e^4)*x)*sqrt(e*x + d)/(a^2*b^5*d^8 - 5*a^3*b^4*d^7*e + 10*a^4*b^3*d^6*e^2 - 10*a^5*b^2*d^5*e^3 + 5*a^6*b*d^4*e^4 - a^7*d^3*e^5 + (b^7*d^5*e^3 - 5*a*b^6*d^4*e^4 + 10*a^2*b^5*d^3*e^5 - 10*a^3*b^4*d^2*e^6 + 5*a^4*b^3*d*e^7 - a^5*b^2*e^8)*x^5 + (3*b^7*d^6*e^2 - 13*a*b^6*d^5*e^3 + 20*a^2*b^5*d^4*e^4 - 10*a^3*b^4*d^3*e^5 - 5*a^4*b^3*d^2*e^6 + 7*a^5*b^2*d*e^7 - 2*a^6*b*e^8)*x^4 + (3*b^7*d^7*e - 9*a*b^6*d^6*e^2 + a^2*b^5*d^5*e^3 + 25*a^3*b^4*d^4*e^4 - 35*a^4*b^3*d^3*e^5 + 17*a^5*b^2*d^2*e^6 - a^6*b*d*e^7 - a^7*e^8)*x^3 + (b^7*d^8 + a*b^6*d^7*e - 17*a^2*b^5*d^6*e^2 + 35*a^3*b^4*d^5*e^3 - 25*a^4*b^3*d^4*e^4 - a^5*b^2*d^3*e^5 + 9*a^6*b*d^2*e^6 - 3*a^7*d*e^7)*x^2 + (2*a*b^6*d^8 - 7*a^2*b^5*d^7*e + 5*a^3*b^4*d^6*e^2 + 10*a^4*b^3*d^5*e^3 - 20*a^5*b^2*d^4*e^4 + 13*a^6*b*d^3*e^5 - 3*a^7*d^2*e^6)*x), 1/20*(315*(b^4*e^5*x^5 + a^2*b^2*d^3*e^2 + (3*b^4*d*e^4 + 2*a*b^3*e^5)*x^4 + (3...
```

Sympy [F]

$$\int \frac{1}{(d+ex)^{7/2} (a^2+2abx+b^2x^2)^{3/2}} dx = \int \frac{1}{(d+ex)^{7/2} ((a+bx)^2)^{3/2}} dx$$

input `integrate(1/(e*x+d)**(7/2)/(b**2*x**2+2*a*b*x+a**2)**(3/2), x)`

output `Integral(1/((d + e*x)**(7/2)*((a + b*x)**2)**(3/2)), x)`

Maxima [F]

$$\int \frac{1}{(d+ex)^{7/2} (a^2+2abx+b^2x^2)^{3/2}} dx = \int \frac{1}{(b^2x^2+2abx+a^2)^{3/2} (ex+d)^{7/2}} dx$$

input `integrate(1/(e*x+d)^(7/2)/(b^2*x^2+2*a*b*x+a^2)^(3/2), x, algorithm="maxima")`

output `integrate(1/((b^2*x^2 + 2*a*b*x + a^2)^(3/2)*(e*x + d)^(7/2)), x)`

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 494 vs. $2(231) = 462$.

Time = 1.35 (sec) , antiderivative size = 494, normalized size of antiderivative = 1.50

$$\int \frac{1}{(d+ex)^{7/2} (a^2+2abx+b^2x^2)^{3/2}} dx = \frac{1}{4(b^5d^5\operatorname{sgn}(bx+a) - 5ab^4d^4\operatorname{esgn}(bx+a) + 10a^2b^3d^3e^2\operatorname{sgn}(bx+a) - 10a^3b^2d^2e^3\operatorname{sgn}(bx+a) + 5a^4bde^4\operatorname{sgn}(bx+a) - 5a^5d^5\operatorname{sgn}(bx+a))} + \frac{15(ex+d)^{3/2}b^4e^2 - 17\sqrt{ex+d}b^4de^2 + 17\sqrt{ex+d}b^4de^2}{4(b^5d^5\operatorname{sgn}(bx+a) - 5ab^4d^4\operatorname{esgn}(bx+a) + 10a^2b^3d^3e^2\operatorname{sgn}(bx+a) - 10a^3b^2d^2e^3\operatorname{sgn}(bx+a) + 5a^4bde^4\operatorname{sgn}(bx+a) - 5a^5d^5\operatorname{sgn}(bx+a))} + \frac{2(30(ex+d)^2b^2e^2 + 5(ex+d)b^2de^2 + b^2d^2e^2 - 5(ex+d)abe^3 - 2abd^2e^3)}{5(b^5d^5\operatorname{sgn}(bx+a) - 5ab^4d^4\operatorname{esgn}(bx+a) + 10a^2b^3d^3e^2\operatorname{sgn}(bx+a) - 10a^3b^2d^2e^3\operatorname{sgn}(bx+a) + 5a^4bde^4\operatorname{sgn}(bx+a) - 5a^5d^5\operatorname{sgn}(bx+a))}$$

input `integrate(1/(e*x+d)^(7/2)/(b^2*x^2+2*a*b*x+a^2)^(3/2),x, algorithm="giac")`

output
$$\frac{63}{4}b^3e^2\arctan\left(\frac{\sqrt{ex+d}b}{\sqrt{-b^2d+abe}}\right)/((b^5d^5\operatorname{sgn}(bx+a) - 5ab^4d^4e\operatorname{sgn}(bx+a) + 10a^2b^3d^3e^2\operatorname{sgn}(bx+a) - 10a^3b^2d^2e^3\operatorname{sgn}(bx+a) + 5a^4bde^4\operatorname{sgn}(bx+a) - a^5e^5\operatorname{sgn}(bx+a))\sqrt{-b^2d+abe}) + \frac{1}{4}(15(ex+d)^{3/2}b^4e^2 - 17\sqrt{ex+d}b^4de^2 + 17\sqrt{ex+d}ab^3e^3)/((b^5d^5\operatorname{sgn}(bx+a) - 5ab^4d^4e\operatorname{sgn}(bx+a) + 10a^2b^3d^3e^2\operatorname{sgn}(bx+a) - 10a^3b^2d^2e^3\operatorname{sgn}(bx+a) + 5a^4bde^4\operatorname{sgn}(bx+a) - a^5e^5\operatorname{sgn}(bx+a))((ex+d)b - bd + ae)^2) + \frac{2}{5}(30(ex+d)^2b^2e^2 + 5(ex+d)b^2de^2 + b^2d^2e^2 - 5(ex+d)abe^3 - 2abde^3 + a^2e^4)/((b^5d^5\operatorname{sgn}(bx+a) - 5ab^4d^4e\operatorname{sgn}(bx+a) + 10a^2b^3d^3e^2\operatorname{sgn}(bx+a) - 10a^3b^2d^2e^3\operatorname{sgn}(bx+a) + 5a^4bde^4\operatorname{sgn}(bx+a) - a^5e^5\operatorname{sgn}(bx+a))(ex+d)^{5/2})$$

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(d+ex)^{7/2}(a^2+2abx+b^2x^2)^{3/2}} dx = \int \frac{1}{(d+ex)^{7/2}(a^2+2abx+b^2x^2)^{3/2}} dx$$

input `int(1/((d + e*x)^(7/2)*(a^2 + b^2*x^2 + 2*a*b*x)^(3/2)),x)`

output `int(1/((d + e*x)^(7/2)*(a^2 + b^2*x^2 + 2*a*b*x)^(3/2)), x)`

Reduce [B] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 1299, normalized size of antiderivative = 3.95

$$\int \frac{1}{(d+ex)^{7/2}(a^2+2abx+b^2x^2)^{3/2}} dx = \text{Too large to display}$$

input `int(1/(e*x+d)^(7/2)/(b^2*x^2+2*a*b*x+a^2)^(3/2),x)`

output

```
( - 315*sqrt(b)*sqrt(d + e*x)*sqrt(a*e - b*d)*atan((sqrt(d + e*x)*b)/(sqrt
(b)*sqrt(a*e - b*d)))*a**2*b**2*d**2*e**2 - 630*sqrt(b)*sqrt(d + e*x)*sqrt
(a*e - b*d)*atan((sqrt(d + e*x)*b)/(sqrt(b)*sqrt(a*e - b*d)))*a**2*b**2*d*
e**3*x - 315*sqrt(b)*sqrt(d + e*x)*sqrt(a*e - b*d)*atan((sqrt(d + e*x)*b)/
(sqrt(b)*sqrt(a*e - b*d)))*a**2*b**2*e**4*x**2 - 630*sqrt(b)*sqrt(d + e*x)
*sqrt(a*e - b*d)*atan((sqrt(d + e*x)*b)/(sqrt(b)*sqrt(a*e - b*d)))*a*b**3*
d**2*e**2*x - 1260*sqrt(b)*sqrt(d + e*x)*sqrt(a*e - b*d)*atan((sqrt(d + e*
x)*b)/(sqrt(b)*sqrt(a*e - b*d)))*a*b**3*d*e**3*x**2 - 630*sqrt(b)*sqrt(d +
e*x)*sqrt(a*e - b*d)*atan((sqrt(d + e*x)*b)/(sqrt(b)*sqrt(a*e - b*d)))*a*
b**3*e**4*x**3 - 315*sqrt(b)*sqrt(d + e*x)*sqrt(a*e - b*d)*atan((sqrt(d +
e*x)*b)/(sqrt(b)*sqrt(a*e - b*d)))*b**4*d**2*e**2*x**2 - 630*sqrt(b)*sqrt(
d + e*x)*sqrt(a*e - b*d)*atan((sqrt(d + e*x)*b)/(sqrt(b)*sqrt(a*e - b*d))
)*b**4*d*e**3*x**3 - 315*sqrt(b)*sqrt(d + e*x)*sqrt(a*e - b*d)*atan((sqrt(d
+ e*x)*b)/(sqrt(b)*sqrt(a*e - b*d)))*b**4*e**4*x**4 - 8*a**5*e**5 + 64*a*
*4*b*d*e**4 + 24*a**4*b*e**5*x - 344*a**3*b**2*d**2*e**3 - 432*a**3*b**2*d
*e**4*x - 168*a**3*b**2*e**5*x**2 + 203*a**2*b**3*d**3*e**2 - 423*a**2*b**
3*d**2*e**3*x - 1071*a**2*b**3*d*e**4*x**2 - 525*a**2*b**3*e**5*x**3 + 95*
a*b**4*d**4*e + 786*a*b**4*d**3*e**2*x + 756*a*b**4*d**2*e**3*x**2 - 210*a
*b**4*d*e**4*x**3 - 315*a*b**4*e**5*x**4 - 10*b**5*d**5 + 45*b**5*d**4*e*x
+ 483*b**5*d**3*e**2*x**2 + 735*b**5*d**2*e**3*x**3 + 315*b**5*d*e**4*...
```


3.388 $\int \frac{(d+ex)^{13/2}}{(a^2+2abx+b^2x^2)^{5/2}} dx$

| | |
|---|------|
| Optimal result | 2876 |
| Mathematica [A] (verified) | 2877 |
| Rubi [A] (verified) | 2877 |
| Maple [A] (verified) | 2889 |
| Fricas [B] (verification not implemented) | 2890 |
| Sympy [F(-1)] | 2891 |
| Maxima [F] | 2891 |
| Giac [B] (verification not implemented) | 2891 |
| Mupad [F(-1)] | 2892 |
| Reduce [B] (verification not implemented) | 2893 |

Optimal result

Integrand size = 30, antiderivative size = 400

$$\int \frac{(d+ex)^{13/2}}{(a^2+2abx+b^2x^2)^{5/2}} dx = \frac{3003e^4(bd-ae)^2(a+bx)\sqrt{d+ex}}{64b^7\sqrt{a^2+2abx+b^2x^2}} + \frac{1001e^4(bd-ae)(a+bx)(d+ex)^{3/2}}{64b^6\sqrt{a^2+2abx+b^2x^2}} + \frac{3003e^4(a+bx)(d+ex)^{5/2}}{320b^5\sqrt{a^2+2abx+b^2x^2}} - \frac{429e^3(d+ex)^{7/2}}{64b^4\sqrt{a^2+2abx+b^2x^2}} - \frac{143e^2(d+ex)^{9/2}}{96b^3(a+bx)\sqrt{a^2+2abx+b^2x^2}} - \frac{13e(d+ex)^{11/2}}{24b^2(a+bx)^2\sqrt{a^2+2abx+b^2x^2}} - \frac{(d+ex)^{13/2}}{4b(a+bx)^3\sqrt{a^2+2abx+b^2x^2}} - \frac{3003e^4(bd-ae)^{5/2}(a+bx)\operatorname{arctanh}\left(\frac{\sqrt{b}\sqrt{d+ex}}{\sqrt{bd-ae}}\right)}{64b^{15/2}\sqrt{a^2+2abx+b^2x^2}}$$

output

```
3003/64*e^4*(-a*e+b*d)^2*(b*x+a)*(e*x+d)^(1/2)/b^7/((b*x+a)^2)^(1/2)+1001/
64*e^4*(-a*e+b*d)*(b*x+a)*(e*x+d)^(3/2)/b^6/((b*x+a)^2)^(1/2)+3003/320*e^4
*(b*x+a)*(e*x+d)^(5/2)/b^5/((b*x+a)^2)^(1/2)-429/64*e^3*(e*x+d)^(7/2)/b^4/
((b*x+a)^2)^(1/2)-143/96*e^2*(e*x+d)^(9/2)/b^3/(b*x+a)/((b*x+a)^2)^(1/2)-1
3/24*e*(e*x+d)^(11/2)/b^2/(b*x+a)^2/((b*x+a)^2)^(1/2)-1/4*(e*x+d)^(13/2)/b
/(b*x+a)^3/((b*x+a)^2)^(1/2)-3003/64*e^4*(-a*e+b*d)^(5/2)*(b*x+a)*arctanh(
b^(1/2)*(e*x+d)^(1/2)/(-a*e+b*d)^(1/2))/b^(15/2)/((b*x+a)^2)^(1/2)
```

Mathematica [A] (verified)

Time = 1.64 (sec) , antiderivative size = 374, normalized size of antiderivative = 0.94

$$\int \frac{(d+ex)^{13/2}}{(a^2+2abx+b^2x^2)^{5/2}} dx = \frac{e^4(a+bx)^5 \left(-\sqrt{b\sqrt{d+ex}}(-45045a^6e^6+15015a^5be^5(7d-11ex)-3003a^4b^2e^4(23d^2-129dex+73e^2) \right)}{\dots}$$

input

```
Integrate[(d + e*x)^(13/2)/(a^2 + 2*a*b*x + b^2*x^2)^(5/2), x]
```

output

```
(e^4*(a + b*x)^5*(-((Sqrt[b]*Sqrt[d + e*x]*(-45045*a^6*e^6 + 15015*a^5*b*e^5*(7*d - 11*e*x) - 3003*a^4*b^2*e^4*(23*d^2 - 129*d*e*x + 73*e^2*x^2) + 429*a^3*b^3*e^3*(15*d^3 - 599*d^2*e*x + 1207*d*e^2*x^2 - 279*e^3*x^3) + 143*a^2*b^4*e^2*(10*d^4 + 175*d^3*e*x - 2433*d^2*e^2*x^2 + 1999*d*e^3*x^3 - 128*e^4*x^4) + 13*a*b^5*e*(40*d^5 + 420*d^4*e*x + 2765*d^3*e^2*x^2 - 15077*d^2*e^3*x^3 + 3456*d*e^4*x^4 + 128*e^5*x^5) + b^6*(240*d^6 + 1960*d^5*e*x + 7630*d^4*e^2*x^2 + 22155*d^3*e^3*x^3 - 32384*d^2*e^4*x^4 - 3968*d*e^5*x^5 - 384*e^6*x^6)))/(e^4*(a + b*x)^4)) - 45045*(-(b*d) + a*e)^(5/2)*ArcTan[(Sqrt[b]*Sqrt[d + e*x])/Sqrt[-(b*d) + a*e]])/(960*b^(15/2)*((a + b*x)^2)^(5/2))
```

Rubi [A] (verified)

Time = 0.62 (sec) , antiderivative size = 272, normalized size of antiderivative = 0.68, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.367$, Rules used = {1102, 27, 51, 51, 51, 51, 60, 60, 60, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex)^{13/2}}{(a^2+2abx+b^2x^2)^{5/2}} dx$$

$$\downarrow 1102$$

$$\frac{b^5(a+bx) \int \frac{(d+ex)^{13/2}}{b^5(a+bx)^5} dx}{\sqrt{a^2+2abx+b^2x^2}}$$

$$\downarrow 27$$

$$\begin{aligned}
& \frac{(a+bx) \int \frac{(d+ex)^{13/2}}{(a+bx)^5} dx}{\sqrt{a^2+2abx+b^2x^2}} \\
& \quad \downarrow 51 \\
& \frac{(a+bx) \left(\frac{13e \int \frac{(d+ex)^{11/2}}{(a+bx)^4} dx}{8b} - \frac{(d+ex)^{13/2}}{4b(a+bx)^4} \right)}{\sqrt{a^2+2abx+b^2x^2}} \\
& \quad \downarrow 51 \\
& \frac{(a+bx) \left(\frac{13e \left(\frac{11e \int \frac{(d+ex)^{9/2}}{(a+bx)^3} dx}{6b} - \frac{(d+ex)^{11/2}}{3b(a+bx)^3} \right)}{8b} - \frac{(d+ex)^{13/2}}{4b(a+bx)^4} \right)}{\sqrt{a^2+2abx+b^2x^2}} \\
& \quad \downarrow 51 \\
& \frac{(a+bx) \left(\frac{13e \left(\frac{11e \left(\frac{9e \int \frac{(d+ex)^{7/2}}{(a+bx)^2} dx}{4b} - \frac{(d+ex)^{9/2}}{2b(a+bx)^2} \right)}{6b} - \frac{(d+ex)^{11/2}}{3b(a+bx)^3} \right)}{8b} - \frac{(d+ex)^{13/2}}{4b(a+bx)^4} \right)}{\sqrt{a^2+2abx+b^2x^2}} \\
& \quad \downarrow 51
\end{aligned}$$

$$\left((a + bx) \left[\frac{13e \left(\frac{11e \left(\frac{9e \left(\frac{7e \int \frac{(d+ex)^{5/2}}{a+bx} dx - \frac{(d+ex)^{7/2}}{b(a+bx)} \right)}{4b} - \frac{(d+ex)^{9/2}}{2b(a+bx)^2} \right)}{6b} - \frac{(d+ex)^{11/2}}{3b(a+bx)^3} \right)}{8b} - \frac{(d+ex)^{13/2}}{4b(a+bx)^4} \right] \right)$$

$$\sqrt{a^2 + 2abx + b^2x^2}$$

↓ 60

$$\left((a + bx) \left[\frac{7e \left(\frac{(bd-ae) \int \frac{(d+ex)^{3/2}}{a+bx} dx + 2(d+ex)^{5/2}}{5b} \right)}{2b} - \frac{(d+ex)^{7/2}}{b(a+bx)} \right] \right.$$

$$\left. \frac{11e}{4b} \left[\frac{(d+ex)^{9/2}}{2b(a+bx)^2} \right] \right.$$

$$\left. \frac{13e}{6b} \left[\frac{(d+ex)^{11/2}}{3b(a+bx)^3} \right] \right.$$

$$\left. \frac{(d+ex)^{13/2}}{4b(a+bx)^4} \right)$$

$$\sqrt{a^2 + 2abx + b^2x^2}$$

$$\begin{aligned}
 & \left(\frac{7e}{9e} \left(\frac{(bd-ae) \int \frac{\sqrt{d+ex}}{a+bx} dx + \frac{2(d+ex)^{3/2}}{3b}}{b} + \frac{2(d+ex)^{5/2}}{5b} \right) - \frac{(d+ex)^{7/2}}{b(a+bx)} \right) \\
 & \frac{11e}{11e} \left(\frac{\left(\frac{7e}{9e} \left(\frac{(bd-ae) \int \frac{\sqrt{d+ex}}{a+bx} dx + \frac{2(d+ex)^{3/2}}{3b}}{b} + \frac{2(d+ex)^{5/2}}{5b} \right) - \frac{(d+ex)^{7/2}}{b(a+bx)} \right)}{2b} - \frac{(d+ex)^{9/2}}{2b(a+bx)^2} \right) \\
 & \frac{13e}{13e} \left(\frac{\left(\frac{11e}{11e} \left(\frac{\left(\frac{7e}{9e} \left(\frac{(bd-ae) \int \frac{\sqrt{d+ex}}{a+bx} dx + \frac{2(d+ex)^{3/2}}{3b}}{b} + \frac{2(d+ex)^{5/2}}{5b} \right) - \frac{(d+ex)^{7/2}}{b(a+bx)} \right)}{2b} - \frac{(d+ex)^{9/2}}{2b(a+bx)^2} \right)}{4b} - \frac{(d+ex)^{11/2}}{3b(a+bx)^3} \right) \right) \\
 & \frac{(a+bx)}{(a+bx)} \left(\frac{\left(\frac{13e}{13e} \left(\frac{\left(\frac{11e}{11e} \left(\frac{\left(\frac{7e}{9e} \left(\frac{(bd-ae) \int \frac{\sqrt{d+ex}}{a+bx} dx + \frac{2(d+ex)^{3/2}}{3b}}{b} + \frac{2(d+ex)^{5/2}}{5b} \right) - \frac{(d+ex)^{7/2}}{b(a+bx)} \right)}{2b} - \frac{(d+ex)^{9/2}}{2b(a+bx)^2} \right)}{4b} - \frac{(d+ex)^{11/2}}{3b(a+bx)^3} \right) \right)}{6b} - \frac{(d+ex)^{13/2}}{4b(a+bx)^4} \right) \right)
 \end{aligned}$$

↓ 60

$$\left(\frac{7e}{9e} \left(\frac{(bd-ae) \left(\frac{(bd-ae) \int \frac{1}{(a+bx)\sqrt{d+ex}} dx + 2\sqrt{d+ex}}{b} \right) + \frac{2(d+ex)^{3/2}}{3b}}{b} \right) + \frac{2(d+ex)^{5/2}}{5b} \right) - \frac{(d+ex)^{7/2}}{b(a+bx)} \right) - \frac{(d+ex)^{9/2}}{2b(a+bx)^2}$$

$$\frac{11e}{4b} - \frac{(d+ex)^{9/2}}{2b(a+bx)^2}$$

$$\frac{13e}{6b}$$

↓ 73

$$\left(\frac{(bd-ae) \left(\frac{2(bd-ae) \int \frac{1}{a + \frac{b(d+ex)}{e} - \frac{bd}{e}} d\sqrt{d+ex} + \frac{2\sqrt{d+ex}}{b} \right)}{b} + \frac{2(d+ex)^{3/2}}{3b} \right) + \frac{2(d+ex)^{5/2}}{5b} - \frac{(d+ex)^{7/2}}{b(a+bx)}$$

11e

13e

6b

↓ 221

$$\begin{aligned}
 & \left((bd-ae) \left(\frac{2\sqrt{d+ex}}{b} - \frac{2\sqrt{bd-ae} \operatorname{arctanh}\left(\frac{\sqrt{b}\sqrt{d+ex}}{\sqrt{bd-ae}}\right)}{b^{3/2}} \right) + \frac{2(d+ex)^{3/2}}{3b} \right) \\
 7e & \frac{\hspace{10em}}{b} + \frac{2(d+ex)^{5/2}}{5b} \\
 9e & \frac{\hspace{10em}}{2b} - \frac{(d+ex)^{7/2}}{b(a+bx)} \\
 11e & \frac{\hspace{10em}}{4b} - \frac{(d+ex)^{9/2}}{2b(a+bx)^2} \\
 13e & \frac{\hspace{10em}}{6b} - \frac{(d+ex)^{11/2}}{2b(a+bx)^3}
 \end{aligned}$$

input `Int[(d + e*x)^(13/2)/(a^2 + 2*a*b*x + b^2*x^2)^(5/2),x]`

output `((a + b*x)*(-1/4*(d + e*x)^(13/2)/(b*(a + b*x)^4) + (13*e*(-1/3*(d + e*x)^(11/2)/(b*(a + b*x)^3) + (11*e*(-1/2*(d + e*x)^(9/2)/(b*(a + b*x)^2) + (9*e*(-((d + e*x)^(7/2)/(b*(a + b*x)))) + (7*e*((2*(d + e*x)^(5/2))/(5*b) + ((b*d - a*e)*((2*(d + e*x)^(3/2))/(3*b) + ((b*d - a*e)*((2*Sqrt[d + e*x])/b - (2*Sqrt[b*d - a*e]*ArcTanh[(Sqrt[b]*Sqrt[d + e*x])/Sqrt[b*d - a*e]])/b^(3/2))))/b))/b)/(2*b)))/(4*b)))/(6*b)))/(8*b))/Sqrt[a^2 + 2*a*b*x + b^2*x^2]`

Defintions of rubi rules used

rule 27 `Int[(a_)*(F_x_), x_Symbol] := Simp[a Int[F_x, x], x] /; FreeQ[a, x] && !MatchQ[F_x, (b_)*(G_x_)] /; FreeQ[b, x]`

rule 51 `Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] := Simp[(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Simp[d*(n/(b*(m + 1))) Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d, n}, x] && ILtQ[m, -1] && FractionQ[n] && GtQ[n, 0]`

rule 60 `Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] := Simp[(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*(b*c - a*d)/(b*(m + n + 1)) Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] && !(IGtQ[m, 0] && (!IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) && !ILtQ[m + n + 2, 0] && IntLinearQ[a, b, c, d, m, n, x]`

rule 73 `Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] := With[{p = Denominator[m]}, Simp[p/b Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) + d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x] /; FreeQ[{a, b, c, d}, x] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntLinearQ[a, b, c, d, m, n, x]`

rule 221 `Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[-a/b, 2]/a)*ArcTanh[x/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]`

rule 1102 `Int[((d_.) + (e_.)*(x_)^(m_.))*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] := Simp[(a + b*x + c*x^2)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x)^(2*FracPart[p])) Int[(d + e*x)^m*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && EqQ[b^2 - 4*a*c, 0]`

Maple [A] (verified)

Time = 1.61 (sec) , antiderivative size = 311, normalized size of antiderivative = 0.78

| method | result |
|---------|--|
| risch | $\frac{2e^4(3b^2e^2x^2 - 25xab e^2 + 31b^2dex + 225e^2a^2 - 475abde + 253b^2d^2)\sqrt{ex+d}\sqrt{(bx+a)^2}}{15b^7(bx+a)} - \frac{(2e^3a^3 - 6a^2bd e^2 + 6ab^2d^2e - 2b^3d^3)e^4}{(bx+a)}$ |
| default | Expression too large to display |

input `int((e*x+d)^(13/2)/(b^2*x^2+2*a*b*x+a^2)^(5/2),x,method=_RETURNVERBOSE)`

output `2/15*e^4*(3*b^2*e^2*x^2-25*a*b*e^2*x+31*b^2*d*e*x+225*a^2*e^2-475*a*b*d*e+253*b^2*d^2)*(e*x+d)^(1/2)/b^7*((b*x+a)^2)^(1/2)/(b*x+a)-1/b^7*(2*a^3*e^3-6*a^2*b*d*e^2+6*a*b^2*d^2*e-2*b^3*d^3)*e^4*((-1477/128*(e*x+d)^(7/2)*b^3-1767/384*(a*e-b*d)*b^2*(e*x+d)^(5/2)+(-10633/384*a^2*b*e^2+10633/192*a*b^2*d*e-10633/384*b^3*d^2)*(e*x+d)^(3/2)+(-1083/128*e^3*a^3+3249/128*a^2*b*d*e^2-3249/128*a*b^2*d^2*e+1083/128*b^3*d^3)*(e*x+d)^(1/2))/(b*(e*x+d)+a*e-b*d)^4+3003/128/(b*(a*e-b*d))^(1/2)*arctan(b*(e*x+d)^(1/2)/(b*(a*e-b*d))^(1/2)))*((b*x+a)^2)^(1/2)/(b*x+a)`

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 638 vs. $2(272) = 544$.

Time = 0.12 (sec) , antiderivative size = 1286, normalized size of antiderivative = 3.22

$$\int \frac{(d + ex)^{13/2}}{(a^2 + 2abx + b^2x^2)^{5/2}} dx = \text{Too large to display}$$

input `integrate((e*x+d)^(13/2)/(b^2*x^2+2*a*b*x+a^2)^(5/2),x, algorithm="fricas")`

output `[1/1920*(45045*(a^4*b^2*d^2*e^4 - 2*a^5*b*d*e^5 + a^6*e^6 + (b^6*d^2*e^4 - 2*a*b^5*d*e^5 + a^2*b^4*e^6)*x^4 + 4*(a*b^5*d^2*e^4 - 2*a^2*b^4*d*e^5 + a^3*b^3*e^6)*x^3 + 6*(a^2*b^4*d^2*e^4 - 2*a^3*b^3*d*e^5 + a^4*b^2*e^6)*x^2 + 4*(a^3*b^3*d^2*e^4 - 2*a^4*b^2*d*e^5 + a^5*b*e^6)*x)*sqrt((b*d - a*e)/b) *log((b*e*x + 2*b*d - a*e - 2*sqrt(e*x + d)*b*sqrt((b*d - a*e)/b))/(b*x + a)) + 2*(384*b^6*e^6*x^6 - 240*b^6*d^6 - 520*a*b^5*d^5*e - 1430*a^2*b^4*d^4*e^2 - 6435*a^3*b^3*d^3*e^3 + 69069*a^4*b^2*d^2*e^4 - 105105*a^5*b*d*e^5 + 45045*a^6*e^6 + 128*(31*b^6*d*e^5 - 13*a*b^5*e^6)*x^5 + 128*(253*b^6*d^2*e^4 - 351*a*b^5*d*e^5 + 143*a^2*b^4*e^6)*x^4 - (22155*b^6*d^3*e^3 - 19600*1*a*b^5*d^2*e^4 + 285857*a^2*b^4*d*e^5 - 119691*a^3*b^3*e^6)*x^3 - (7630*b^6*d^4*e^2 + 35945*a*b^5*d^3*e^3 - 347919*a^2*b^4*d^2*e^4 + 517803*a^3*b^3*d*e^5 - 219219*a^4*b^2*d*e^6)*x^2 - (1960*b^6*d^5*e + 5460*a*b^5*d^4*e^2 + 25025*a^2*b^4*d^3*e^3 - 256971*a^3*b^3*d^2*e^4 + 387387*a^4*b^2*d*e^5 - 165165*a^5*b*e^6)*x)*sqrt(e*x + d))/(b^11*x^4 + 4*a*b^10*x^3 + 6*a^2*b^9*x^2 + 4*a^3*b^8*x + a^4*b^7), -1/960*(45045*(a^4*b^2*d^2*e^4 - 2*a^5*b*d*e^5 + a^6*e^6 + (b^6*d^2*e^4 - 2*a*b^5*d*e^5 + a^2*b^4*e^6)*x^4 + 4*(a*b^5*d^2*e^4 - 2*a^2*b^4*d*e^5 + a^3*b^3*e^6)*x^3 + 6*(a^2*b^4*d^2*e^4 - 2*a^3*b^3*d*e^5 + a^4*b^2*e^6)*x^2 + 4*(a^3*b^3*d^2*e^4 - 2*a^4*b^2*d*e^5 + a^5*b*e^6)*x)*sqrt(-(b*d - a*e)/b)*arctan(-sqrt(e*x + d)*b*sqrt(-(b*d - a*e)/b)/(b*d - a*e)) - (384*b^6*e^6*x^6 - 240*b^6*d^6 - 520*a*b^5*d^5*e - 1430*a...`

Sympy [F(-1)]

Timed out.

$$\int \frac{(d + ex)^{13/2}}{(a^2 + 2abx + b^2x^2)^{5/2}} dx = \text{Timed out}$$

input `integrate((e*x+d)**(13/2)/(b**2*x**2+2*a*b*x+a**2)**(5/2),x)`

output `Timed out`

Maxima [F]

$$\int \frac{(d + ex)^{13/2}}{(a^2 + 2abx + b^2x^2)^{5/2}} dx = \int \frac{(ex + d)^{\frac{13}{2}}}{(b^2x^2 + 2abx + a^2)^{\frac{5}{2}}} dx$$

input `integrate((e*x+d)^(13/2)/(b^2*x^2+2*a*b*x+a^2)^(5/2),x, algorithm="maxima")`

output `integrate((e*x + d)^(13/2)/(b^2*x^2 + 2*a*b*x + a^2)^(5/2), x)`

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 651 vs. $2(272) = 544$.

Time = 0.19 (sec) , antiderivative size = 651, normalized size of antiderivative = 1.63

$$\int \frac{(d + ex)^{13/2}}{(a^2 + 2abx + b^2x^2)^{5/2}} dx = \text{Too large to display}$$

input `integrate((e*x+d)^(13/2)/(b^2*x^2+2*a*b*x+a^2)^(5/2),x, algorithm="giac")`

output

```

3003/64*(b^3*d^3*e^4 - 3*a*b^2*d^2*e^5 + 3*a^2*b*d*e^6 - a^3*e^7)*arctan(
sqrt(e*x + d)*b/sqrt(-b^2*d + a*b*e))/(sqrt(-b^2*d + a*b*e)*b^7*sgn(b*x +
a)) - 1/192*(4431*(e*x + d)^(7/2)*b^6*d^3*e^4 - 11767*(e*x + d)^(5/2)*b^6*d
^4*e^4 + 10633*(e*x + d)^(3/2)*b^6*d^5*e^4 - 3249*sqrt(e*x + d)*b^6*d^6*e
^4 - 13293*(e*x + d)^(7/2)*a*b^5*d^2*e^5 + 47068*(e*x + d)^(5/2)*a*b^5*d^3*
e^5 - 53165*(e*x + d)^(3/2)*a*b^5*d^4*e^5 + 19494*sqrt(e*x + d)*a*b^5*d^5*
e^5 + 13293*(e*x + d)^(7/2)*a^2*b^4*d*e^6 - 70602*(e*x + d)^(5/2)*a^2*b^4*
d^2*e^6 + 106330*(e*x + d)^(3/2)*a^2*b^4*d^3*e^6 - 48735*sqrt(e*x + d)*a^2
*b^4*d^4*e^6 - 4431*(e*x + d)^(7/2)*a^3*b^3*e^7 + 47068*(e*x + d)^(5/2)*a^
3*b^3*d*e^7 - 106330*(e*x + d)^(3/2)*a^3*b^3*d^2*e^7 + 64980*sqrt(e*x + d)
*a^3*b^3*d^3*e^7 - 11767*(e*x + d)^(5/2)*a^4*b^2*e^8 + 53165*(e*x + d)^(3/
2)*a^4*b^2*d*e^8 - 48735*sqrt(e*x + d)*a^4*b^2*d^2*e^8 - 10633*(e*x + d)^(
3/2)*a^5*b*e^9 + 19494*sqrt(e*x + d)*a^5*b*d*e^9 - 3249*sqrt(e*x + d)*a^6*
e^10)/(((e*x + d)*b - b*d + a*e)^4*b^7*sgn(b*x + a)) + 2/15*(3*(e*x + d)^(
5/2)*b^20*e^4 + 25*(e*x + d)^(3/2)*b^20*d*e^4 + 225*sqrt(e*x + d)*b^20*d^2
*e^4 - 25*(e*x + d)^(3/2)*a*b^19*e^5 - 450*sqrt(e*x + d)*a*b^19*d*e^5 + 22
5*sqrt(e*x + d)*a^2*b^18*e^6)/(b^25*sgn(b*x + a))

```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d + ex)^{13/2}}{(a^2 + 2abx + b^2x^2)^{5/2}} dx = \int \frac{(d + ex)^{13/2}}{(a^2 + 2abx + b^2x^2)^{5/2}} dx$$

input

```
int((d + e*x)^(13/2)/(a^2 + b^2*x^2 + 2*a*b*x)^(5/2), x)
```

output

```
int((d + e*x)^(13/2)/(a^2 + b^2*x^2 + 2*a*b*x)^(5/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 1308, normalized size of antiderivative = 3.27

$$\int \frac{(d + ex)^{13/2}}{(a^2 + 2abx + b^2x^2)^{5/2}} dx = \text{Too large to display}$$

input `int((e*x+d)^(13/2)/(b^2*x^2+2*a*b*x+a^2)^(5/2),x)`

output `(- 45045*sqrt(b)*sqrt(a*e - b*d)*atan((sqrt(d + e*x)*b)/(sqrt(b)*sqrt(a*e - b*d)))*a**6*e**6 + 90090*sqrt(b)*sqrt(a*e - b*d)*atan((sqrt(d + e*x)*b)/(sqrt(b)*sqrt(a*e - b*d)))*a**5*b*d*e**5 - 180180*sqrt(b)*sqrt(a*e - b*d)*atan((sqrt(d + e*x)*b)/(sqrt(b)*sqrt(a*e - b*d)))*a**5*b*e**6*x - 45045*sqrt(b)*sqrt(a*e - b*d)*atan((sqrt(d + e*x)*b)/(sqrt(b)*sqrt(a*e - b*d)))*a**4*b**2*d**2*e**4 + 360360*sqrt(b)*sqrt(a*e - b*d)*atan((sqrt(d + e*x)*b)/(sqrt(b)*sqrt(a*e - b*d)))*a**4*b**2*d*e**5*x - 270270*sqrt(b)*sqrt(a*e - b*d)*atan((sqrt(d + e*x)*b)/(sqrt(b)*sqrt(a*e - b*d)))*a**4*b**2*e**6*x**2 - 180180*sqrt(b)*sqrt(a*e - b*d)*atan((sqrt(d + e*x)*b)/(sqrt(b)*sqrt(a*e - b*d)))*a**3*b**3*d**2*e**4*x + 540540*sqrt(b)*sqrt(a*e - b*d)*atan((sqrt(d + e*x)*b)/(sqrt(b)*sqrt(a*e - b*d)))*a**3*b**3*d*e**5*x**2 - 180180*sqrt(b)*sqrt(a*e - b*d)*atan((sqrt(d + e*x)*b)/(sqrt(b)*sqrt(a*e - b*d)))*a**3*b**3*e**6*x**3 - 270270*sqrt(b)*sqrt(a*e - b*d)*atan((sqrt(d + e*x)*b)/(sqrt(b)*sqrt(a*e - b*d)))*a**2*b**4*d**2*e**4*x**2 + 360360*sqrt(b)*sqrt(a*e - b*d)*atan((sqrt(d + e*x)*b)/(sqrt(b)*sqrt(a*e - b*d)))*a**2*b**4*d*e**5*x**3 - 45045*sqrt(b)*sqrt(a*e - b*d)*atan((sqrt(d + e*x)*b)/(sqrt(b)*sqrt(a*e - b*d)))*a**2*b**4*e**6*x**4 - 180180*sqrt(b)*sqrt(a*e - b*d)*atan((sqrt(d + e*x)*b)/(sqrt(b)*sqrt(a*e - b*d)))*a*b**5*d**2*e**4*x**3 + 90090*sqrt(b)*sqrt(a*e - b*d)*atan((sqrt(d + e*x)*b)/(sqrt(b)*sqrt(a*e - b*d)))*a*b**5*d*e**5*x**4 - 45045*sqrt(b)*sqrt(a*e - b*d)*atan((sqrt(d + e...`

3.389
$$\int \frac{(d+ex)^{11/2}}{(a^2+2abx+b^2x^2)^{5/2}} dx$$

| | |
|---|------|
| Optimal result | 2894 |
| Mathematica [A] (verified) | 2895 |
| Rubi [A] (verified) | 2895 |
| Maple [A] (verified) | 2905 |
| Fricas [B] (verification not implemented) | 2906 |
| Sympy [F(-1)] | 2907 |
| Maxima [F] | 2907 |
| Giac [B] (verification not implemented) | 2907 |
| Mupad [F(-1)] | 2908 |
| Reduce [B] (verification not implemented) | 2908 |

Optimal result

Integrand size = 30, antiderivative size = 346

$$\int \frac{(d+ex)^{11/2}}{(a^2+2abx+b^2x^2)^{5/2}} dx = \frac{1155e^4(bd-ae)(a+bx)\sqrt{d+ex}}{64b^6\sqrt{a^2+2abx+b^2x^2}} + \frac{385e^4(a+bx)(d+ex)^{3/2}}{64b^5\sqrt{a^2+2abx+b^2x^2}} - \frac{231e^3(d+ex)^{5/2}}{64b^4\sqrt{a^2+2abx+b^2x^2}} - \frac{33e^2(d+ex)^{7/2}}{32b^3(a+bx)\sqrt{a^2+2abx+b^2x^2}} - \frac{11e(d+ex)^{9/2}}{24b^2(a+bx)^2\sqrt{a^2+2abx+b^2x^2}} - \frac{(d+ex)^{11/2}}{4b(a+bx)^3\sqrt{a^2+2abx+b^2x^2}} - \frac{1155e^4(bd-ae)^{3/2}(a+bx)\operatorname{arctanh}\left(\frac{\sqrt{b}\sqrt{d+ex}}{\sqrt{bd-ae}}\right)}{64b^{13/2}\sqrt{a^2+2abx+b^2x^2}}$$

output

```
1155/64*e^4*(-a*e+b*d)*(b*x+a)*(e*x+d)^(1/2)/b^6/((b*x+a)^2)^(1/2)+385/64*
e^4*(b*x+a)*(e*x+d)^(3/2)/b^5/((b*x+a)^2)^(1/2)-231/64*e^3*(e*x+d)^(5/2)/b
^4/((b*x+a)^2)^(1/2)-33/32*e^2*(e*x+d)^(7/2)/b^3/(b*x+a)/((b*x+a)^2)^(1/2)
-11/24*e*(e*x+d)^(9/2)/b^2/(b*x+a)^2/((b*x+a)^2)^(1/2)-1/4*(e*x+d)^(11/2)/
b/(b*x+a)^3/((b*x+a)^2)^(1/2)-1155/64*e^4*(-a*e+b*d)^(3/2)*(b*x+a)*arctanh
(b^(1/2)*(e*x+d)^(1/2)/(-a*e+b*d)^(1/2))/b^(13/2)/((b*x+a)^2)^(1/2)
```

Mathematica [A] (verified)

Time = 1.54 (sec) , antiderivative size = 300, normalized size of antiderivative = 0.87

$$\int \frac{(d+ex)^{11/2}}{(a^2+2abx+b^2x^2)^{5/2}} dx = \frac{e^4(a+bx)^5 \left(-\frac{\sqrt{b}\sqrt{d+ex}(3465a^5e^5+1155a^4be^4(-4d+11ex)+231a^3b^2e^3(3d^2-74dex+73e^2x^2)+99a^2b^3e^2(2d^3+27d^2ex-232de^2x^2+93e^3x^3)+11ab^4e(8d^4+68d^3ex+345d^2e^2x^2-1162de^3x^3+128e^4x^4)+b^5(48d^5+328d^4ex+1030d^3e^2x^2+2295d^2e^3x^3-2048de^4x^4-128e^5x^5))}{e^4(a+bx)^4} + 3465(-bd+ae)^{3/2} \operatorname{ArcTan}\left[\frac{\sqrt{b}\sqrt{d+ex}}{\sqrt{-(bd+ae)}}\right]}{(192b^{13/2})(a+bx)^2)^{5/2}} \right)}{e^4(a+bx)^5}$$

input

```
Integrate[(d + e*x)^(11/2)/(a^2 + 2*a*b*x + b^2*x^2)^(5/2), x]
```

output

```
(e^4*(a + b*x)^5*(-((Sqrt[b]*Sqrt[d + e*x]*(3465*a^5*e^5 + 1155*a^4*b*e^4*(-4*d + 11*e*x) + 231*a^3*b^2*e^3*(3*d^2 - 74*d*e*x + 73*e^2*x^2) + 99*a^2*b^3*e^2*(2*d^3 + 27*d^2*e*x - 232*d*e^2*x^2 + 93*e^3*x^3) + 11*a*b^4*e*(8*d^4 + 68*d^3*e*x + 345*d^2*e^2*x^2 - 1162*d*e^3*x^3 + 128*e^4*x^4) + b^5*(48*d^5 + 328*d^4*e*x + 1030*d^3*e^2*x^2 + 2295*d^2*e^3*x^3 - 2048*d*e^4*x^4 - 128*e^5*x^5)))/(e^4*(a + b*x)^4)) + 3465*(-(b*d) + a*e)^(3/2)*ArcTan[(Sqrt[b]*Sqrt[d + e*x])/Sqrt[-(b*d) + a*e]])/(192*b^(13/2)*(a + b*x)^2)^(5/2))
```

Rubi [A] (verified)

Time = 0.56 (sec) , antiderivative size = 243, normalized size of antiderivative = 0.70, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {1102, 27, 51, 51, 51, 51, 60, 60, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex)^{11/2}}{(a^2+2abx+b^2x^2)^{5/2}} dx$$

$$\downarrow 1102$$

$$\frac{b^5(a+bx) \int \frac{(d+ex)^{11/2}}{b^5(a+bx)^5} dx}{\sqrt{a^2+2abx+b^2x^2}}$$

$$\downarrow 27$$

$$\begin{aligned}
 & \frac{(a+bx) \int \frac{(d+ex)^{11/2}}{(a+bx)^5} dx}{\sqrt{a^2+2abx+b^2x^2}} \\
 & \quad \downarrow 51 \\
 & \frac{(a+bx) \left(\frac{11e \int \frac{(d+ex)^{9/2}}{(a+bx)^4} dx}{8b} - \frac{(d+ex)^{11/2}}{4b(a+bx)^4} \right)}{\sqrt{a^2+2abx+b^2x^2}} \\
 & \quad \downarrow 51 \\
 & \frac{(a+bx) \left(\frac{11e \left(\frac{3e \int \frac{(d+ex)^{7/2}}{(a+bx)^3} dx}{2b} - \frac{(d+ex)^{9/2}}{3b(a+bx)^3} \right)}{8b} - \frac{(d+ex)^{11/2}}{4b(a+bx)^4} \right)}{\sqrt{a^2+2abx+b^2x^2}} \\
 & \quad \downarrow 51 \\
 & \frac{(a+bx) \left(\frac{11e \left(\frac{3e \left(\frac{7e \int \frac{(d+ex)^{5/2}}{(a+bx)^2} dx}{4b} - \frac{(d+ex)^{7/2}}{2b(a+bx)^2} \right)}{2b} - \frac{(d+ex)^{9/2}}{3b(a+bx)^3} \right)}{8b} - \frac{(d+ex)^{11/2}}{4b(a+bx)^4} \right)}{\sqrt{a^2+2abx+b^2x^2}} \\
 & \quad \downarrow 51
 \end{aligned}$$

$$\frac{(a+bx) \left(\frac{11e \left(\frac{3e \left(\frac{7e \left(\frac{5e \int \frac{(d+ex)^{3/2}}{a+bx} dx - \frac{(d+ex)^{5/2}}{b(a+bx)} \right)}{4b} - \frac{(d+ex)^{7/2}}{2b(a+bx)^2} \right)}{2b} - \frac{(d+ex)^{9/2}}{3b(a+bx)^3} \right)}{8b} - \frac{(d+ex)^{11/2}}{4b(a+bx)^4} \right)}{\sqrt{a^2 + 2abx + b^2x^2}}$$

\downarrow 60

$$\begin{aligned}
 & \left(\frac{(bd-ae) \int \frac{1}{(a+bx)\sqrt{d+ex}} dx + 2\sqrt{d+ex}}{b} + \frac{2(d+ex)^{3/2}}{3b} \right) \\
 & \frac{5e}{7e} \left(\frac{\left(\frac{(bd-ae) \int \frac{1}{(a+bx)\sqrt{d+ex}} dx + 2\sqrt{d+ex}}{b} + \frac{2(d+ex)^{3/2}}{3b} \right)}{2b} - \frac{(d+ex)^{5/2}}{b(a+bx)} \right) \\
 & \frac{3e}{3e} \left(\frac{\left(\frac{(bd-ae) \int \frac{1}{(a+bx)\sqrt{d+ex}} dx + 2\sqrt{d+ex}}{b} + \frac{2(d+ex)^{3/2}}{3b} \right)}{4b} - \frac{(d+ex)^{7/2}}{2b(a+bx)^2} \right) \\
 & \frac{11e}{11e} \left(\frac{\left(\frac{(bd-ae) \int \frac{1}{(a+bx)\sqrt{d+ex}} dx + 2\sqrt{d+ex}}{b} + \frac{2(d+ex)^{3/2}}{3b} \right)}{2b} - \frac{(d+ex)^{9/2}}{3b(a+bx)^3} \right) \\
 & \frac{(a+bx)}{(a+bx)} \left(\frac{\left(\frac{(bd-ae) \int \frac{1}{(a+bx)\sqrt{d+ex}} dx + 2\sqrt{d+ex}}{b} + \frac{2(d+ex)^{3/2}}{3b} \right)}{8b} - \frac{(d+ex)^{11/2}}{4b(a+bx)^4} \right)
 \end{aligned}$$

↓ 73

$$\begin{aligned}
 & \left(\frac{(bd-ae) \int \frac{1}{a + \frac{b(d+ex)}{e} - \frac{bd}{e}} d\sqrt{d+ex} + \frac{2\sqrt{d+ex}}{b}}{b} + \frac{2(d+ex)^{3/2}}{3b} \right) \\
 & \frac{5e}{7e} \left(\frac{\dots}{2b} - \frac{(d+ex)^{5/2}}{b(a+bx)} \right) \\
 & \frac{3e}{4b} \left(\dots - \frac{(d+ex)^{7/2}}{2b(a+bx)^2} \right) \\
 & \frac{11e}{2b} \left(\dots - \frac{(d+ex)^{9/2}}{3b(a+bx)^3} \right) \\
 & \frac{(a+bx)}{8b}
 \end{aligned}$$

↓ 221

$$\begin{aligned}
 & \left(\frac{5e}{b} \left((bd-ae) \left(\frac{2\sqrt{d+ex}}{b} - \frac{2\sqrt{bd-ae} \operatorname{arctanh} \left(\frac{\sqrt{b}\sqrt{d+ex}}{\sqrt{bd-ae}} \right)}{b^{3/2}} \right) + \frac{2(d+ex)^{3/2}}{3b} \right) \right. \\
 & \left. - \frac{(d+ex)^{5/2}}{b(a+bx)} \right) \\
 & \left(\frac{3e}{4b} \right) - \frac{(d+ex)^{7/2}}{2b(a+bx)^2} \\
 & \left(\frac{11e}{2b} \right) - \frac{(d+ex)^{9/2}}{3b(a+bx)^3} \\
 & \frac{(a+bx)}{8b}
 \end{aligned}$$

input `Int[(d + e*x)^(11/2)/(a^2 + 2*a*b*x + b^2*x^2)^(5/2),x]`

output `((a + b*x)*(-1/4*(d + e*x)^(11/2)/(b*(a + b*x)^4) + (11*e*(-1/3*(d + e*x)^(9/2)/(b*(a + b*x)^3) + (3*e*(-1/2*(d + e*x)^(7/2)/(b*(a + b*x)^2) + (7*e*(-((d + e*x)^(5/2)/(b*(a + b*x)))) + (5*e*((2*(d + e*x)^(3/2))/(3*b) + ((b*d - a*e)*((2*Sqrt[d + e*x])/b - (2*Sqrt[b*d - a*e]*ArcTanh[(Sqrt[b]*Sqrt[d + e*x])/Sqrt[b*d - a*e]])/b^(3/2)))/b))/(2*b)))/(4*b)))/(2*b)))/(8*b))/Sqrt[a^2 + 2*a*b*x + b^2*x^2]`

Defintions of rubi rules used

rule 27 `Int[(a_)*(F_x_), x_Symbol] := Simp[a Int[F_x, x], x] /; FreeQ[a, x] && !MatchQ[F_x, (b_)*(G_x_) /; FreeQ[b, x]]`

rule 51 `Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] := Simp[(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Simp[d*(n/(b*(m + 1)))] Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d, n}, x] && ILtQ[m, -1] && FractionQ[n] && GtQ[n, 0]`

rule 60 `Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] := Simp[(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(b*(m + n + 1)))] Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] && !(IGtQ[m, 0] && (!IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) && !ILtQ[m + n + 2, 0] && IntLinearQ[a, b, c, d, m, n, x]`

rule 73 `Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] := With[{p = Denominator[m]}, Simp[p/b Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) + d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntLinearQ[a, b, c, d, m, n, x]`

rule 221 `Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[-a/b, 2]/a)*ArcTanh[x/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]`

rule 1102 `Int[((d_.) + (e_.)*(x_)^(m_.))*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] := Simp[(a + b*x + c*x^2)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x)^(2*FracPart[p])) Int[(d + e*x)^m*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && EqQ[b^2 - 4*a*c, 0]`

Maple [A] (verified)

Time = 1.55 (sec) , antiderivative size = 260, normalized size of antiderivative = 0.75

| method | result |
|---------|--|
| risch | $-\frac{2e^{4(-bx+15ae-16bd)}\sqrt{ex+d}\sqrt{(bx+a)^2}}{3b^6(bx+a)} + \frac{(2e^2a^2-4abde+2b^2d^2)e^4}{\left(\frac{-765(ex+d)^{\frac{7}{2}}b^3}{128} - \frac{5855(ae-bd)b^2(ex+d)^{\frac{5}{2}}}{384} + \left(-\frac{5153}{384}a^2be\right)\right)}$ |
| default | Expression too large to display |

input `int((e*x+d)^(11/2)/(b^2*x^2+2*a*b*x+a^2)^(5/2),x,method=_RETURNVERBOSE)`

output `-2/3*e^4*(-b*e*x+15*a*e-16*b*d)*(e*x+d)^(1/2)/b^6*((b*x+a)^2)^(1/2)/(b*x+a)+1/b^6*(2*a^2*e^2-4*a*b*d*e+2*b^2*d^2)*e^4*((-765/128*(e*x+d)^(7/2)*b^3-5855/384*(a*e-b*d)*b^2*(e*x+d)^(5/2)+(-5153/384*a^2*b*e^2+5153/192*a*b^2*d*e-5153/384*b^3*d^2)*(e*x+d)^(3/2)+(-515/128*e^3*a^3+1545/128*a^2*b*d*e^2-1545/128*a*b^2*d^2*e+515/128*b^3*d^3)*(e*x+d)^(1/2))/(b*(e*x+d)+a*e-b*d)^4+1155/128/(b*(a*e-b*d))^(1/2)*arctan(b*(e*x+d)^(1/2)/(b*(a*e-b*d))^(1/2))*((b*x+a)^2)^(1/2)/(b*x+a)`

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 479 vs. $2(233) = 466$.

Time = 0.12 (sec) , antiderivative size = 968, normalized size of antiderivative = 2.80

$$\int \frac{(d + ex)^{11/2}}{(a^2 + 2abx + b^2x^2)^{5/2}} dx = \text{Too large to display}$$

input `integrate((e*x+d)^(11/2)/(b^2*x^2+2*a*b*x+a^2)^(5/2),x, algorithm="fricas")`

output `[-1/384*(3465*(a^4*b*d*e^4 - a^5*e^5 + (b^5*d*e^4 - a*b^4*e^5)*x^4 + 4*(a*b^4*d*e^4 - a^2*b^3*e^5)*x^3 + 6*(a^2*b^3*d*e^4 - a^3*b^2*e^5)*x^2 + 4*(a^3*b^2*d*e^4 - a^4*b*e^5)*x)*sqrt((b*d - a*e)/b)*log((b*e*x + 2*b*d - a*e + 2*sqrt(e*x + d)*b*sqrt((b*d - a*e)/b))/(b*x + a)) - 2*(128*b^5*e^5*x^5 - 48*b^5*d^5 - 88*a*b^4*d^4*e - 198*a^2*b^3*d^3*e^2 - 693*a^3*b^2*d^2*e^3 + 4620*a^4*b*d*e^4 - 3465*a^5*e^5 + 128*(16*b^5*d*e^4 - 11*a*b^4*e^5)*x^4 - (2295*b^5*d^2*e^3 - 12782*a*b^4*d*e^4 + 9207*a^2*b^3*e^5)*x^3 - (1030*b^5*d^3*e^2 + 3795*a*b^4*d^2*e^3 - 22968*a^2*b^3*d*e^4 + 16863*a^3*b^2*e^5)*x^2 - (328*b^5*d^4*e + 748*a*b^4*d^3*e^2 + 2673*a^2*b^3*d^2*e^3 - 17094*a^3*b^2*d*e^4 + 12705*a^4*b*e^5)*x)*sqrt(e*x + d))/(b^10*x^4 + 4*a*b^9*x^3 + 6*a^2*b^8*x^2 + 4*a^3*b^7*x + a^4*b^6), -1/192*(3465*(a^4*b*d*e^4 - a^5*e^5 + (b^5*d*e^4 - a*b^4*e^5)*x^4 + 4*(a*b^4*d*e^4 - a^2*b^3*e^5)*x^3 + 6*(a^2*b^3*d*e^4 - a^3*b^2*e^5)*x^2 + 4*(a^3*b^2*d*e^4 - a^4*b*e^5)*x)*sqrt(-(b*d - a*e)/b)*arctan(-sqrt(e*x + d)*b*sqrt(-(b*d - a*e)/b)/(b*d - a*e)) - (128*b^5*e^5*x^5 - 48*b^5*d^5 - 88*a*b^4*d^4*e - 198*a^2*b^3*d^3*e^2 - 693*a^3*b^2*d^2*e^3 + 4620*a^4*b*d*e^4 - 3465*a^5*e^5 + 128*(16*b^5*d*e^4 - 11*a*b^4*e^5)*x^4 - (2295*b^5*d^2*e^3 - 12782*a*b^4*d*e^4 + 9207*a^2*b^3*e^5)*x^3 - (1030*b^5*d^3*e^2 + 3795*a*b^4*d^2*e^3 - 22968*a^2*b^3*d*e^4 + 16863*a^3*b^2*e^5)*x^2 - (328*b^5*d^4*e + 748*a*b^4*d^3*e^2 + 2673*a^2*b^3*d^2*e^3 - 17094*a^3*b^2*d*e^4 + 12705*a^4*b*e^5)*x)*sqrt(e*x + d))/(b^10*...`

Sympy [F(-1)]

Timed out.

$$\int \frac{(d + ex)^{11/2}}{(a^2 + 2abx + b^2x^2)^{5/2}} dx = \text{Timed out}$$

input `integrate((e*x+d)**(11/2)/(b**2*x**2+2*a*b*x+a**2)**(5/2),x)`

output `Timed out`

Maxima [F]

$$\int \frac{(d + ex)^{11/2}}{(a^2 + 2abx + b^2x^2)^{5/2}} dx = \int \frac{(ex + d)^{\frac{11}{2}}}{(b^2x^2 + 2abx + a^2)^{\frac{5}{2}}} dx$$

input `integrate((e*x+d)^(11/2)/(b^2*x^2+2*a*b*x+a^2)^(5/2),x, algorithm="maxima")`

output `integrate((e*x + d)^(11/2)/(b^2*x^2 + 2*a*b*x + a^2)^(5/2), x)`

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 498 vs. 2(233) = 466.

Time = 0.18 (sec) , antiderivative size = 498, normalized size of antiderivative = 1.44

$$\int \frac{(d + ex)^{11/2}}{(a^2 + 2abx + b^2x^2)^{5/2}} dx = \frac{1155 (b^2d^2e^4 - 2abde^5 + a^2e^6) \arctan\left(\frac{\sqrt{ex+db}}{\sqrt{-b^2d+abe}}\right)}{64 \sqrt{-b^2d+abe} b^6 \operatorname{sgn}(bx+a)} - \frac{2295 (ex+d)^{\frac{7}{2}} b^5 d^2 e^4 - 5855 (ex+d)^{\frac{5}{2}} b^5 d^3 e^4 + 5153 (ex+d)^{\frac{3}{2}} b^5 d^4 e^4 - 1545 \sqrt{ex+db} d^5 e^4 - 4590 (ex+d)^{\frac{1}{2}} b^5 d^5 e^4}{3 b^{15} \operatorname{sgn}(bx+a)} + \frac{2 \left((ex+d)^{\frac{3}{2}} b^{10} e^4 + 15 \sqrt{ex+db} b^{10} d e^4 - 15 \sqrt{ex+db} b^9 e^5 \right)}{3 b^{15} \operatorname{sgn}(bx+a)}$$

input `integrate((e*x+d)^(11/2)/(b^2*x^2+2*a*b*x+a^2)^(5/2),x, algorithm="giac")`

output `1155/64*(b^2*d^2*e^4 - 2*a*b*d*e^5 + a^2*e^6)*arctan(sqrt(e*x + d)*b/sqrt(-b^2*d + a*b*e))/(sqrt(-b^2*d + a*b*e)*b^6*sgn(b*x + a)) - 1/192*(2295*(e*x + d)^(7/2)*b^5*d^2*e^4 - 5855*(e*x + d)^(5/2)*b^5*d^3*e^4 + 5153*(e*x + d)^(3/2)*b^5*d^4*e^4 - 1545*sqrt(e*x + d)*b^5*d^5*e^4 - 4590*(e*x + d)^(7/2)*a*b^4*d*e^5 + 17565*(e*x + d)^(5/2)*a*b^4*d^2*e^5 - 20612*(e*x + d)^(3/2)*a*b^4*d^3*e^5 + 7725*sqrt(e*x + d)*a*b^4*d^4*e^5 + 2295*(e*x + d)^(7/2)*a^2*b^3*e^6 - 17565*(e*x + d)^(5/2)*a^2*b^3*d*e^6 + 30918*(e*x + d)^(3/2)*a^2*b^3*d^2*e^6 - 15450*sqrt(e*x + d)*a^2*b^3*d^3*e^6 + 5855*(e*x + d)^(5/2)*a^3*b^2*e^7 - 20612*(e*x + d)^(3/2)*a^3*b^2*d*e^7 + 15450*sqrt(e*x + d)*a^3*b^2*d^2*e^7 + 5153*(e*x + d)^(3/2)*a^4*b*e^8 - 7725*sqrt(e*x + d)*a^4*b*d*e^8 + 1545*sqrt(e*x + d)*a^5*e^9)/(((e*x + d)*b - b*d + a*e)^4*b^6*sgn(b*x + a)) + 2/3*((e*x + d)^(3/2)*b^10*e^4 + 15*sqrt(e*x + d)*b^10*d*e^4 - 15*sqrt(e*x + d)*a*b^9*e^5)/(b^15*sgn(b*x + a))`

Mupad [F(-1)]

Timed out.

$$\int \frac{(d + ex)^{11/2}}{(a^2 + 2abx + b^2x^2)^{5/2}} dx = \int \frac{(d + ex)^{11/2}}{(a^2 + 2abx + b^2x^2)^{5/2}} dx$$

input `int((d + e*x)^(11/2)/(a^2 + b^2*x^2 + 2*a*b*x)^(5/2),x)`

output `int((d + e*x)^(11/2)/(a^2 + b^2*x^2 + 2*a*b*x)^(5/2), x)`

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 907, normalized size of antiderivative = 2.62

$$\int \frac{(d + ex)^{11/2}}{(a^2 + 2abx + b^2x^2)^{5/2}} dx = \text{Too large to display}$$

input `int((e*x+d)^(11/2)/(b^2*x^2+2*a*b*x+a^2)^(5/2),x)`

output

```
(3465*sqrt(b)*sqrt(a*e - b*d)*atan((sqrt(d + e*x)*b)/(sqrt(b)*sqrt(a*e - b
*d)))*a**5*e**5 - 3465*sqrt(b)*sqrt(a*e - b*d)*atan((sqrt(d + e*x)*b)/(sqr
t(b)*sqrt(a*e - b*d)))*a**4*b*d*e**4 + 13860*sqrt(b)*sqrt(a*e - b*d)*atan(
(sqrt(d + e*x)*b)/(sqrt(b)*sqrt(a*e - b*d)))*a**4*b*e**5*x - 13860*sqrt(b)
*sqrt(a*e - b*d)*atan((sqrt(d + e*x)*b)/(sqrt(b)*sqrt(a*e - b*d)))*a**3*b*
*2*d*e**4*x + 20790*sqrt(b)*sqrt(a*e - b*d)*atan((sqrt(d + e*x)*b)/(sqrt(b)
)*sqrt(a*e - b*d)))*a**3*b**2*e**5*x**2 - 20790*sqrt(b)*sqrt(a*e - b*d)*at
an((sqrt(d + e*x)*b)/(sqrt(b)*sqrt(a*e - b*d)))*a**2*b**3*d*e**4*x**2 + 13
860*sqrt(b)*sqrt(a*e - b*d)*atan((sqrt(d + e*x)*b)/(sqrt(b)*sqrt(a*e - b*d
)))*a**2*b**3*e**5*x**3 - 13860*sqrt(b)*sqrt(a*e - b*d)*atan((sqrt(d + e*x
)*b)/(sqrt(b)*sqrt(a*e - b*d)))*a*b**4*d*e**4*x**3 + 3465*sqrt(b)*sqrt(a*e
- b*d)*atan((sqrt(d + e*x)*b)/(sqrt(b)*sqrt(a*e - b*d)))*a*b**4*e**5*x**4
- 3465*sqrt(b)*sqrt(a*e - b*d)*atan((sqrt(d + e*x)*b)/(sqrt(b)*sqrt(a*e -
b*d)))*b**5*d*e**4*x**4 - 3465*sqrt(d + e*x)*a**5*b*e**5 + 4620*sqrt(d +
e*x)*a**4*b**2*d*e**4 - 12705*sqrt(d + e*x)*a**4*b**2*e**5*x - 693*sqrt(d
+ e*x)*a**3*b**3*d**2*e**3 + 17094*sqrt(d + e*x)*a**3*b**3*d*e**4*x - 1686
3*sqrt(d + e*x)*a**3*b**3*e**5*x**2 - 198*sqrt(d + e*x)*a**2*b**4*d**3*e**
2 - 2673*sqrt(d + e*x)*a**2*b**4*d**2*e**3*x + 22968*sqrt(d + e*x)*a**2*b
**4*d*e**4*x**2 - 9207*sqrt(d + e*x)*a**2*b**4*e**5*x**3 - 88*sqrt(d + e*x)
*a*b**5*d**4*e - 748*sqrt(d + e*x)*a*b**5*d**3*e**2*x - 3795*sqrt(d + e...
```

3.390
$$\int \frac{(d+ex)^{9/2}}{(a^2+2abx+b^2x^2)^{5/2}} dx$$

| | |
|---|------|
| Optimal result | 2910 |
| Mathematica [A] (verified) | 2911 |
| Rubi [A] (verified) | 2911 |
| Maple [A] (verified) | 2917 |
| Fricas [A] (verification not implemented) | 2917 |
| Sympy [F(-1)] | 2918 |
| Maxima [F] | 2918 |
| Giac [A] (verification not implemented) | 2919 |
| Mupad [F(-1)] | 2919 |
| Reduce [B] (verification not implemented) | 2920 |

Optimal result

Integrand size = 30, antiderivative size = 294

$$\int \frac{(d+ex)^{9/2}}{(a^2+2abx+b^2x^2)^{5/2}} dx = \frac{315e^4(a+bx)\sqrt{d+ex}}{64b^5\sqrt{a^2+2abx+b^2x^2}} - \frac{105e^3(d+ex)^{3/2}}{64b^4\sqrt{a^2+2abx+b^2x^2}}$$

$$- \frac{21e^2(d+ex)^{5/2}}{32b^3(a+bx)\sqrt{a^2+2abx+b^2x^2}} - \frac{3e(d+ex)^{7/2}}{8b^2(a+bx)^2\sqrt{a^2+2abx+b^2x^2}}$$

$$- \frac{(d+ex)^{9/2}}{4b(a+bx)^3\sqrt{a^2+2abx+b^2x^2}} - \frac{315e^4\sqrt{bd-ae}(a+bx)\operatorname{arctanh}\left(\frac{\sqrt{b}\sqrt{d+ex}}{\sqrt{bd-ae}}\right)}{64b^{11/2}\sqrt{a^2+2abx+b^2x^2}}$$

output

```
315/64*e^4*(b*x+a)*(e*x+d)^(1/2)/b^5/((b*x+a)^2)^(1/2)-105/64*e^3*(e*x+d)^(3/2)/b^4/((b*x+a)^2)^(1/2)-21/32*e^2*(e*x+d)^(5/2)/b^3/(b*x+a)/((b*x+a)^2)^(1/2)-3/8*e*(e*x+d)^(7/2)/b^2/(b*x+a)^2/((b*x+a)^2)^(1/2)-1/4*(e*x+d)^(9/2)/b/(b*x+a)^3/((b*x+a)^2)^(1/2)-315/64*e^4*(-a*e+b*d)^(1/2)*(b*x+a)*arctanh(b^(1/2)*(e*x+d)^(1/2)/(-a*e+b*d)^(1/2))/b^(11/2)/((b*x+a)^2)^(1/2)
```

Mathematica [A] (verified)

Time = 1.08 (sec) , antiderivative size = 232, normalized size of antiderivative = 0.79

$$\int \frac{(d+ex)^{9/2}}{(a^2+2abx+b^2x^2)^{5/2}} dx = \frac{-\sqrt{b}\sqrt{d+ex}(-315a^4e^4+105a^3be^3(d-11ex)+21a^2b^2e^2(2d^2+19dex-7$$

input

```
Integrate[(d + e*x)^(9/2)/(a^2 + 2*a*b*x + b^2*x^2)^(5/2), x]
```

output

```
(-(Sqrt[b]*Sqrt[d + e*x]*(-315*a^4*e^4 + 105*a^3*b*e^3*(d - 11*e*x) + 21*a^2*b^2*e^2*(2*d^2 + 19*d*e*x - 73*e^2*x^2) + 3*a*b^3*e*(8*d^3 + 52*d^2*e*x + 185*d*e^2*x^2 - 279*e^3*x^3) + b^4*(16*d^4 + 88*d^3*e*x + 210*d^2*e^2*x^2 + 325*d*e^3*x^3 - 128*e^4*x^4))) - 315*e^4*Sqrt[-(b*d) + a*e]*(a + b*x)^4*ArcTan[(Sqrt[b]*Sqrt[d + e*x])/Sqrt[-(b*d) + a*e]])/(64*b^(11/2)*(a + b*x)^3*Sqrt[(a + b*x)^2])
```

Rubi [A] (verified)

Time = 0.53 (sec) , antiderivative size = 214, normalized size of antiderivative = 0.73, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.300$, Rules used = {1102, 27, 51, 51, 51, 51, 60, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex)^{9/2}}{(a^2+2abx+b^2x^2)^{5/2}} dx$$

$$\downarrow 1102$$

$$\frac{b^5(a+bx) \int \frac{(d+ex)^{9/2}}{b^5(a+bx)^5} dx}{\sqrt{a^2+2abx+b^2x^2}}$$

$$\downarrow 27$$

$$\frac{(a+bx) \int \frac{(d+ex)^{9/2}}{(a+bx)^5} dx}{\sqrt{a^2+2abx+b^2x^2}}$$

$$\begin{array}{c}
 \downarrow 51 \\
 (a + bx) \left(\frac{9e \int \frac{(d+ex)^{7/2} dx}{(a+bx)^4} - \frac{(d+ex)^{9/2}}{4b(a+bx)^4}}{\sqrt{a^2 + 2abx + b^2x^2}} \right) \\
 \downarrow 51 \\
 (a + bx) \left(\frac{9e \left(\frac{7e \int \frac{(d+ex)^{5/2} dx}{(a+bx)^3} - \frac{(d+ex)^{7/2}}{3b(a+bx)^3}}{8b} - \frac{(d+ex)^{9/2}}{4b(a+bx)^4} \right)}{\sqrt{a^2 + 2abx + b^2x^2}} \right) \\
 \downarrow 51 \\
 (a + bx) \left(\frac{9e \left(\frac{7e \left(\frac{5e \int \frac{(d+ex)^{3/2} dx}{(a+bx)^2} - \frac{(d+ex)^{5/2}}{2b(a+bx)^2} \right)}{6b} - \frac{(d+ex)^{7/2}}{3b(a+bx)^3} \right)}{8b} - \frac{(d+ex)^{9/2}}{4b(a+bx)^4} \right)}{\sqrt{a^2 + 2abx + b^2x^2}} \\
 \downarrow 51 \\
 (a + bx) \left(\frac{9e \left(\frac{7e \left(\frac{5e \left(\frac{3e \int \frac{\sqrt{d+ex}}{a+bx} dx}{2b} - \frac{(d+ex)^{3/2}}{b(a+bx)} \right)}{4b} - \frac{(d+ex)^{5/2}}{2b(a+bx)^2} \right)}{6b} - \frac{(d+ex)^{7/2}}{3b(a+bx)^3} \right)}{8b} - \frac{(d+ex)^{9/2}}{4b(a+bx)^4} \right)}{\sqrt{a^2 + 2abx + b^2x^2}}
 \end{array}$$

↓ 60

$$\begin{aligned}
 & \left(\left(\left(\left(\left(\frac{3e \left(\frac{(bd-ae) \int \frac{1}{(a+bx)\sqrt{d+ex}} dx + 2\sqrt{d+ex}}{b} \right)}{2b} - \frac{(d+ex)^{3/2}}{b(a+bx)} \right)}{4b} - \frac{(d+ex)^{5/2}}{2b(a+bx)^2} \right)}{6b} - \frac{(d+ex)^{7/2}}{3b(a+bx)^3} \right)}{8b} - \frac{(d+ex)^{9/2}}{4b(a+bx)^4} \right) \\
 & (a+bx)
 \end{aligned}$$

$$\sqrt{a^2 + 2abx + b^2x^2}$$

↓ 73

$$\begin{aligned}
 & \left(\frac{3e \left(\frac{2(bd-ae) \int \frac{1}{a + \frac{b(d+ex)}{e} - \frac{bd}{e}} + \frac{2\sqrt{d+ex}}{b}} d\sqrt{d+ex} \right)}{2b} - \frac{(d+ex)^{3/2}}{b(a+bx)} \right) \\
 & \frac{7e}{4b} \left(\frac{(d+ex)^{5/2}}{2b(a+bx)^2} \right) \\
 & \frac{9e}{6b} \left(\frac{(d+ex)^{7/2}}{3b(a+bx)^3} \right) \\
 & \frac{(a+bx)}{8b} \left(\frac{(d+ex)^{9/2}}{4b(a+bx)^4} \right)
 \end{aligned}$$

$$\sqrt{a^2 + 2abx + b^2x^2}$$

$$\frac{(a + bx) \left(\frac{5e \left(\frac{3e \left(\frac{2\sqrt{d+ex}}{b} - \frac{2\sqrt{bd-ae} \operatorname{arctanh} \left(\frac{\sqrt{b}\sqrt{d+ex}}{\sqrt{bd-ae}} \right)}{b^{3/2}} \right)}{2b} - \frac{(d+ex)^{3/2}}{b(a+bx)} \right)}{4b} - \frac{(d+ex)^{5/2}}{2b(a+bx)^2} \right)}{6b} - \frac{(d+ex)^{7/2}}{3b(a+bx)^3} \right)}{8b} - \frac{(d+ex)^{9/2}}{4b(a+bx)^4}$$

$$\sqrt{a^2 + 2abx + b^2x^2}$$

input `Int[(d + e*x)^(9/2)/(a^2 + 2*a*b*x + b^2*x^2)^(5/2),x]`

output

$$\begin{aligned} & ((a + b*x)*(-1/4*(d + e*x)^{(9/2)}/(b*(a + b*x)^4) + (9*e*(-1/3*(d + e*x)^{(7/2)}/(b*(a + b*x)^3) + (7*e*(-1/2*(d + e*x)^{(5/2)}/(b*(a + b*x)^2) + (5*e*(-(d + e*x)^{(3/2)}/(b*(a + b*x))) + (3*e*((2*\text{Sqrt}[d + e*x])/b - (2*\text{Sqrt}[b*d - a*e]*\text{ArcTanh}[(\text{Sqrt}[b]*\text{Sqrt}[d + e*x])/(\text{Sqrt}[b*d - a*e])]/b^{(3/2)})/(2*b)))/(4*b)))/(6*b)))/(8*b))/\text{Sqrt}[a^2 + 2*a*b*x + b^2*x^2] \end{aligned}$$

Defintions of rubi rules used

rule 27

$$\text{Int}[(a_*)(Fx_), x_Symbol] \rightarrow \text{Simp}[a \text{ Int}[Fx, x], x] /; \text{FreeQ}[a, x] \ \&\& \ !\text{MatchQ}[Fx, (b_*)(Gx_) /; \text{FreeQ}[b, x]]$$

rule 51

$$\begin{aligned} & \text{Int}[((a_.) + (b_.)*(x_))^{(m_)}*((c_.) + (d_.)*(x_))^{(n_)}, x_Symbol] \rightarrow \text{Simp}[\\ & (a + b*x)^{(m + 1)}*((c + d*x)^n/(b*(m + 1))), x] - \text{Simp}[d*(n/(b*(m + 1))) \\ & \text{Int}[(a + b*x)^{(m + 1)}*(c + d*x)^{(n - 1)}, x], x] /; \text{FreeQ}[\{a, b, c, d, n\}, x] \\ & \ \&\& \ \text{ILtQ}[m, -1] \ \&\& \ \text{FractionQ}[n] \ \&\& \ \text{GtQ}[n, 0] \end{aligned}$$

rule 60

$$\begin{aligned} & \text{Int}[((a_.) + (b_.)*(x_))^{(m_)}*((c_.) + (d_.)*(x_))^{(n_)}, x_Symbol] \rightarrow \text{Simp}[\\ & (a + b*x)^{(m + 1)}*((c + d*x)^n/(b*(m + n + 1))), x] + \text{Simp}[n*((b*c - a*d)/(\\ & b*(m + n + 1))) \text{ Int}[(a + b*x)^m*(c + d*x)^{(n - 1)}, x], x] /; \text{FreeQ}[\{a, b, \\ & c, d\}, x] \ \&\& \ \text{GtQ}[n, 0] \ \&\& \ \text{NeQ}[m + n + 1, 0] \ \&\& \ !(\text{IGtQ}[m, 0] \ \&\& \ (\ !\text{Integer} \\ & \text{Q}[n] \ || \ (\text{GtQ}[m, 0] \ \&\& \ \text{LtQ}[m - n, 0]))) \ \&\& \ !\text{ILtQ}[m + n + 2, 0] \ \&\& \ \text{IntLinear} \\ & \text{Q}[a, b, c, d, m, n, x] \end{aligned}$$

rule 73

$$\begin{aligned} & \text{Int}[((a_.) + (b_.)*(x_))^{(m_)}*((c_.) + (d_.)*(x_))^{(n_)}, x_Symbol] \rightarrow \text{With}[\\ & \{p = \text{Denominator}[m]\}, \text{Simp}[p/b \text{ Subst}[\text{Int}[x^{(p*(m + 1) - 1)}*(c - a*(d/b) + \\ & d*(x^p/b)^n], x], x, (a + b*x)^{(1/p)}, x]] /; \text{FreeQ}[\{a, b, c, d\}, x] \ \&\& \ \text{Lt} \\ & \text{Q}[-1, m, 0] \ \&\& \ \text{LeQ}[-1, n, 0] \ \&\& \ \text{LeQ}[\text{Denominator}[n], \text{Denominator}[m]] \ \&\& \ \text{IntL} \\ & \text{inearQ}[a, b, c, d, m, n, x] \end{aligned}$$

rule 221

$$\text{Int}[(a + b*x^2)^{-1}, x_Symbol] \rightarrow \text{Simp}[(\text{Rt}[-a/b, 2]/a)*\text{ArcTanh}[x/\text{Rt}[-a/b, 2]], x] /; \text{FreeQ}[\{a, b\}, x] \ \&\& \ \text{NegQ}[a/b]$$

rule 1102

```
Int[((d_.) + (e_.)*(x_.))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] := Simp[(a + b*x + c*x^2)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x)^(2*F
racPart[p])) Int[(d + e*x)^m*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c,
d, e, m, p}, x] && EqQ[b^2 - 4*a*c, 0]
```

Maple [A] (verified)

Time = 1.68 (sec) , antiderivative size = 233, normalized size of antiderivative = 0.79

| method | result |
|---------|---|
| risch | $\frac{2e^4\sqrt{ex+d}\sqrt{bx+a}^2}{b^5(bx+a)} - \frac{(2ae-2bd)e^4\left(-\frac{325(ex+d)^{\frac{7}{2}}b^3}{128} - \frac{765(ae-bd)b^2(ex+d)^{\frac{5}{2}}}{128} + \frac{(-\frac{643}{128}a^2be^2 + \frac{643}{64}ab^2de - \frac{643}{128}b^3d^2)(ex+d)^{\frac{3}{2}} + (-\frac{187}{128}e^3a^3 + 561/128a^2bde - 561/128ab^2d^2e + 187/128b^3d^3)(ex+d)^{\frac{1}{2}}\right)}{b^5(bx+a)}$ |
| default | $\frac{\left(512ab^3e^4x^3\sqrt{b(ae-bd)}\sqrt{ex+d} + 768a^2b^2e^4x^2\sqrt{b(ae-bd)}\sqrt{ex+d} + 315\arctan\left(\frac{b\sqrt{ex+d}}{\sqrt{b(ae-bd)}}\right)b^5de^4x^4 + 325(ex+d)^{\frac{7}{2}}\sqrt{b(ae-bd)}\right)}{b^5(bx+a)}$ |

input

```
int((e*x+d)^(9/2)/(b^2*x^2+2*a*b*x+a^2)^(5/2),x,method=_RETURNVERBOSE)
```

output

```
2*e^4/b^5*(e*x+d)^(1/2)*((b*x+a)^2)^(1/2)/(b*x+a)-1/b^5*(2*a*e-2*b*d)*e^4*
((-325/128*(e*x+d)^(7/2)*b^3-765/128*(a*e-b*d)*b^2*(e*x+d)^(5/2)+(-643/128
*a^2*b*e^2+643/64*a*b^2*d*e-643/128*b^3*d^2)*(e*x+d)^(3/2)+(-187/128*e^3*a
^3+561/128*a^2*b*d*e^2-561/128*a*b^2*d^2*e+187/128*b^3*d^3)*(e*x+d)^(1/2))
/(b*(e*x+d)+a*e-b*d)^4+315/128/(b*(a*e-b*d))^(1/2)*arctan(b*(e*x+d)^(1/2)/
(b*(a*e-b*d))^(1/2))*((b*x+a)^2)^(1/2)/(b*x+a)
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 680, normalized size of antiderivative = 2.31

$$\int \frac{(d+ex)^{9/2}}{(a^2+2abx+b^2x^2)^{5/2}} dx = \frac{315(b^4e^4x^4 + 4ab^3e^4x^3 + 6a^2b^2e^4x^2 + 4a^3be^4x + a^4e^4)\sqrt{\frac{bd-ae}{b}} \log\left(\frac{bex+2}{b}\right)}{b^5(bx+a)}$$

input

```
integrate((e*x+d)^(9/2)/(b^2*x^2+2*a*b*x+a^2)^(5/2),x, algorithm="fricas")
```

output

```
[1/128*(315*(b^4*e^4*x^4 + 4*a*b^3*e^4*x^3 + 6*a^2*b^2*e^4*x^2 + 4*a^3*b*e^4*x + a^4*e^4)*sqrt((b*d - a*e)/b)*log((b*e*x + 2*b*d - a*e - 2*sqrt(e*x + d))*b*sqrt((b*d - a*e)/b))/(b*x + a) + 2*(128*b^4*e^4*x^4 - 16*b^4*d^4 - 24*a*b^3*d^3*e - 42*a^2*b^2*d^2*e^2 - 105*a^3*b*d*e^3 + 315*a^4*e^4 - (325*b^4*d*e^3 - 837*a*b^3*e^4)*x^3 - 3*(70*b^4*d^2*e^2 + 185*a*b^3*d*e^3 - 511*a^2*b^2*e^4)*x^2 - (88*b^4*d^3*e + 156*a*b^3*d^2*e^2 + 399*a^2*b^2*d*e^3 - 1155*a^3*b*e^4)*x)*sqrt(e*x + d))/(b^9*x^4 + 4*a*b^8*x^3 + 6*a^2*b^7*x^2 + 4*a^3*b^6*x + a^4*b^5), -1/64*(315*(b^4*e^4*x^4 + 4*a*b^3*e^4*x^3 + 6*a^2*b^2*e^4*x^2 + 4*a^3*b*e^4*x + a^4*e^4)*sqrt(-(b*d - a*e)/b)*arctan(-sqrt(e*x + d)*b*sqrt(-(b*d - a*e)/b)/(b*d - a*e)) - (128*b^4*e^4*x^4 - 16*b^4*d^4 - 24*a*b^3*d^3*e - 42*a^2*b^2*d^2*e^2 - 105*a^3*b*d*e^3 + 315*a^4*e^4 - (325*b^4*d*e^3 - 837*a*b^3*e^4)*x^3 - 3*(70*b^4*d^2*e^2 + 185*a*b^3*d*e^3 - 511*a^2*b^2*e^4)*x^2 - (88*b^4*d^3*e + 156*a*b^3*d^2*e^2 + 399*a^2*b^2*d*e^3 - 1155*a^3*b*e^4)*x)*sqrt(e*x + d))/(b^9*x^4 + 4*a*b^8*x^3 + 6*a^2*b^7*x^2 + 4*a^3*b^6*x + a^4*b^5)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(d + ex)^{9/2}}{(a^2 + 2abx + b^2x^2)^{5/2}} dx = \text{Timed out}$$

input

```
integrate((e*x+d)**(9/2)/(b**2*x**2+2*a*b*x+a**2)**(5/2),x)
```

output

Timed out

Maxima [F]

$$\int \frac{(d + ex)^{9/2}}{(a^2 + 2abx + b^2x^2)^{5/2}} dx = \int \frac{(ex + d)^{\frac{9}{2}}}{(b^2x^2 + 2abx + a^2)^{\frac{5}{2}}} dx$$

input

```
integrate((e*x+d)^(9/2)/(b^2*x^2+2*a*b*x+a^2)^(5/2),x, algorithm="maxima")
```

output

```
integrate((e*x + d)^(9/2)/(b^2*x^2 + 2*a*b*x + a^2)^(5/2), x)
```

Giac [A] (verification not implemented)

Time = 1.36 (sec) , antiderivative size = 364, normalized size of antiderivative = 1.24

$$\int \frac{(d+ex)^{9/2}}{(a^2+2abx+b^2x^2)^{5/2}} dx = \frac{2\sqrt{ex+d}e^4}{b^5\operatorname{sgn}(bx+a)} + \frac{315(bde^4 - ae^5)\arctan\left(\frac{\sqrt{ex+db}}{\sqrt{-b^2d+abe}}\right)}{64\sqrt{-b^2d+abe}b^5\operatorname{sgn}(bx+a)}$$

$$\frac{325(ex+d)^{7/2}b^4de^4 - 765(ex+d)^{5/2}b^4d^2e^4 + 643(ex+d)^{3/2}b^4d^3e^4 - 187\sqrt{ex+db}b^4d^4e^4 - 325(ex+d)^{7/2}a}{b^5\operatorname{sgn}(bx+a)}$$

input `integrate((e*x+d)^(9/2)/(b^2*x^2+2*a*b*x+a^2)^(5/2),x, algorithm="giac")`

output `2*sqrt(e*x + d)*e^4/(b^5*sgn(b*x + a)) + 315/64*(b*d*e^4 - a*e^5)*arctan(sqrt(e*x + d)*b/sqrt(-b^2*d + a*b*e))/(sqrt(-b^2*d + a*b*e)*b^5*sgn(b*x + a)) - 1/64*(325*(e*x + d)^(7/2)*b^4*d*e^4 - 765*(e*x + d)^(5/2)*b^4*d^2*e^4 + 643*(e*x + d)^(3/2)*b^4*d^3*e^4 - 187*sqrt(e*x + d)*b^4*d^4*e^4 - 325*(e*x + d)^(7/2)*a*b^3*e^5 + 1530*(e*x + d)^(5/2)*a*b^3*d*e^5 - 1929*(e*x + d)^(3/2)*a*b^3*d^2*e^5 + 748*sqrt(e*x + d)*a*b^3*d^3*e^5 - 765*(e*x + d)^(5/2)*a^2*b^2*e^6 + 1929*(e*x + d)^(3/2)*a^2*b^2*d*e^6 - 1122*sqrt(e*x + d)*a^2*b^2*d^2*e^6 - 643*(e*x + d)^(3/2)*a^3*b*e^7 + 748*sqrt(e*x + d)*a^3*b*d*e^7 - 187*sqrt(e*x + d)*a^4*e^8)/(((e*x + d)*b - b*d + a*e)^4*b^5*sgn(b*x + a))`

Mupad [F(-1)]

Timed out.

$$\int \frac{(d+ex)^{9/2}}{(a^2+2abx+b^2x^2)^{5/2}} dx = \int \frac{(d+ex)^{9/2}}{(a^2+2abx+b^2x^2)^{5/2}} dx$$

input `int((d + e*x)^(9/2)/(a^2 + b^2*x^2 + 2*a*b*x)^(5/2), x)`

output `int((d + e*x)^(9/2)/(a^2 + b^2*x^2 + 2*a*b*x)^(5/2), x)`

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 543, normalized size of antiderivative = 1.85

$$\int \frac{(d+ex)^{9/2}}{(a^2+2abx+b^2x^2)^{5/2}} dx = \frac{-315\sqrt{b}\sqrt{ae-bd} \operatorname{atan}\left(\frac{\sqrt{ex+db}}{\sqrt{b}\sqrt{ae-bd}}\right) a^4 e^4 - 1260\sqrt{b}\sqrt{ae-bd} \operatorname{atan}\left(\frac{\sqrt{ex+db}}{\sqrt{b}\sqrt{ae-bd}}\right)}{(a^2+2abx+b^2x^2)^{5/2}}$$

input

```
int((e*x+d)^(9/2)/(b^2*x^2+2*a*b*x+a^2)^(5/2),x)
```

output

```
( - 315*sqrt(b)*sqrt(a*e - b*d)*atan((sqrt(d + e*x)*b)/(sqrt(b)*sqrt(a*e -
b*d)))*a**4*e**4 - 1260*sqrt(b)*sqrt(a*e - b*d)*atan((sqrt(d + e*x)*b)/(s
qrt(b)*sqrt(a*e - b*d)))*a**3*b*e**4*x - 1890*sqrt(b)*sqrt(a*e - b*d)*atan
((sqrt(d + e*x)*b)/(sqrt(b)*sqrt(a*e - b*d)))*a**2*b**2*e**4*x**2 - 1260*s
qrt(b)*sqrt(a*e - b*d)*atan((sqrt(d + e*x)*b)/(sqrt(b)*sqrt(a*e - b*d)))*a
*b**3*e**4*x**3 - 315*sqrt(b)*sqrt(a*e - b*d)*atan((sqrt(d + e*x)*b)/(sqrt
(b)*sqrt(a*e - b*d)))*b**4*e**4*x**4 + 315*sqrt(d + e*x)*a**4*b*e**4 - 105
*sqrt(d + e*x)*a**3*b**2*d*e**3 + 1155*sqrt(d + e*x)*a**3*b**2*e**4*x - 42
*sqrt(d + e*x)*a**2*b**3*d**2*e**2 - 399*sqrt(d + e*x)*a**2*b**3*d*e**3*x
+ 1533*sqrt(d + e*x)*a**2*b**3*e**4*x**2 - 24*sqrt(d + e*x)*a*b**4*d**3*e
- 156*sqrt(d + e*x)*a*b**4*d**2*e**2*x - 555*sqrt(d + e*x)*a*b**4*d*e**3*x
**2 + 837*sqrt(d + e*x)*a*b**4*e**4*x**3 - 16*sqrt(d + e*x)*b**5*d**4 - 88
*sqrt(d + e*x)*b**5*d**3*e*x - 210*sqrt(d + e*x)*b**5*d**2*e**2*x**2 - 325
*sqrt(d + e*x)*b**5*d*e**3*x**3 + 128*sqrt(d + e*x)*b**5*e**4*x**4)/(64*b*
*6*(a**4 + 4*a**3*b*x + 6*a**2*b**2*x**2 + 4*a*b**3*x**3 + b**4*x**4))
```

3.391
$$\int \frac{(d+ex)^{7/2}}{(a^2+2abx+b^2x^2)^{5/2}} dx$$

| | |
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Optimal result

Integrand size = 30, antiderivative size = 250

$$\int \frac{(d+ex)^{7/2}}{(a^2+2abx+b^2x^2)^{5/2}} dx = -\frac{35e^3\sqrt{d+ex}}{64b^4\sqrt{a^2+2abx+b^2x^2}} - \frac{35e^2(d+ex)^{3/2}}{96b^3(a+bx)\sqrt{a^2+2abx+b^2x^2}} - \frac{7e(d+ex)^{5/2}}{24b^2(a+bx)^2\sqrt{a^2+2abx+b^2x^2}} - \frac{(d+ex)^{7/2}}{4b(a+bx)^3\sqrt{a^2+2abx+b^2x^2}} - \frac{35e^4(a+bx)\operatorname{arctanh}\left(\frac{\sqrt{b}\sqrt{d+ex}}{\sqrt{bd-ae}}\right)}{64b^{9/2}\sqrt{bd-ae}\sqrt{a^2+2abx+b^2x^2}}$$

output

```
-35/64*e^3*(e*x+d)^(1/2)/b^4/((b*x+a)^2)^(1/2)-35/96*e^2*(e*x+d)^(3/2)/b^3
/(b*x+a)/((b*x+a)^2)^(1/2)-7/24*e*(e*x+d)^(5/2)/b^2/(b*x+a)^2/((b*x+a)^2)^(
1/2)-1/4*(e*x+d)^(7/2)/b/(b*x+a)^3/((b*x+a)^2)^(1/2)-35/64*e^4*(b*x+a)*ar
ctanh(b^(1/2)*(e*x+d)^(1/2)/(-a*e+b*d)^(1/2))/b^(9/2)/(-a*e+b*d)^(1/2)/(b
*x+a)^2)^(1/2)
```

Mathematica [A] (verified)

Time = 1.02 (sec) , antiderivative size = 185, normalized size of antiderivative = 0.74

$$\int \frac{(d+ex)^{7/2}}{(a^2+2abx+b^2x^2)^{5/2}} dx = \frac{e^4(a+bx)^5 \left(-\frac{\sqrt{b}\sqrt{d+ex}(105a^3e^3+35a^2be^2(2d+11ex)+7ab^2e(8d^2+36dex+73e^2x^2)+b^3(48d^3+20d^2ex+326de^2x^2+279e^3x^3))}{e^4(a+bx)^4} + (105\text{ArcTan}[\frac{\sqrt{b}\sqrt{d+ex}}{\sqrt{-(b*d)+a*e}}])/\sqrt{-(b*d)+a*e}) \right)}{192b^{9/2}((a+bx)^2)^{5/2}}$$

input `Integrate[(d + e*x)^(7/2)/(a^2 + 2*a*b*x + b^2*x^2)^(5/2), x]`

output `(e^4*(a + b*x)^5*(-((Sqrt[b]*Sqrt[d + e*x]*(105*a^3*e^3 + 35*a^2*b*e^2*(2*d + 11*e*x) + 7*a*b^2*e*(8*d^2 + 36*d*e*x + 73*e^2*x^2) + b^3*(48*d^3 + 20*d^2*e*x + 326*d*e^2*x^2 + 279*e^3*x^3)))/(e^4*(a + b*x)^4)) + (105*ArcTan[(Sqrt[b]*Sqrt[d + e*x])/Sqrt[-(b*d) + a*e]])/Sqrt[-(b*d) + a*e]))/(192*b^(9/2)*((a + b*x)^2)^(5/2))`

Rubi [A] (verified)

Time = 0.50 (sec) , antiderivative size = 192, normalized size of antiderivative = 0.77, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$, Rules used = {1102, 27, 51, 51, 51, 51, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\begin{aligned} & \int \frac{(d+ex)^{7/2}}{(a^2+2abx+b^2x^2)^{5/2}} dx \\ & \quad \downarrow \text{1102} \\ & \frac{b^5(a+bx) \int \frac{(d+ex)^{7/2}}{b^5(a+bx)^5} dx}{\sqrt{a^2+2abx+b^2x^2}} \\ & \quad \downarrow \text{27} \\ & \frac{(a+bx) \int \frac{(d+ex)^{7/2}}{(a+bx)^5} dx}{\sqrt{a^2+2abx+b^2x^2}} \\ & \quad \downarrow \text{51} \end{aligned}$$

$$\begin{aligned}
 & \frac{(a+bx) \left(\frac{7e \int \frac{(d+ex)^{5/2}}{(a+bx)^4} dx}{8b} - \frac{(d+ex)^{7/2}}{4b(a+bx)^4} \right)}{\sqrt{a^2 + 2abx + b^2x^2}} \\
 & \quad \downarrow 51 \\
 & \frac{(a+bx) \left(\frac{7e \left(\frac{5e \int \frac{(d+ex)^{3/2}}{(a+bx)^3} dx}{6b} - \frac{(d+ex)^{5/2}}{3b(a+bx)^3} \right)}{8b} - \frac{(d+ex)^{7/2}}{4b(a+bx)^4} \right)}{\sqrt{a^2 + 2abx + b^2x^2}} \\
 & \quad \downarrow 51 \\
 & \frac{(a+bx) \left(\frac{7e \left(\frac{5e \left(\frac{3e \int \frac{\sqrt{d+ex}}{(a+bx)^2} dx}{4b} - \frac{(d+ex)^{3/2}}{2b(a+bx)^2} \right)}{6b} - \frac{(d+ex)^{5/2}}{3b(a+bx)^3} \right)}{8b} - \frac{(d+ex)^{7/2}}{4b(a+bx)^4} \right)}{\sqrt{a^2 + 2abx + b^2x^2}} \\
 & \quad \downarrow 51 \\
 & \frac{(a+bx) \left(\frac{7e \left(\frac{5e \left(\frac{3e \left(\frac{e \int \frac{1}{(a+bx)\sqrt{d+ex}} dx}{2b} - \frac{\sqrt{d+ex}}{b(a+bx)} \right)}{4b} - \frac{(d+ex)^{3/2}}{2b(a+bx)^2} \right)}{6b} - \frac{(d+ex)^{5/2}}{3b(a+bx)^3} \right)}{8b} - \frac{(d+ex)^{7/2}}{4b(a+bx)^4} \right)}{\sqrt{a^2 + 2abx + b^2x^2}} \\
 & \quad \downarrow 73
 \end{aligned}$$

$$\left(\frac{(a+bx) \left(\frac{5e \left(\frac{3e \left(\frac{\int \frac{1}{a + \frac{b(d+ex) - bd}{e} - \frac{\sqrt{d+ex}}{b(a+bx)}}{4b} - \frac{(d+ex)^{3/2}}{2b(a+bx)^2} \right)}{6b} - \frac{(d+ex)^{5/2}}{3b(a+bx)^3} \right)}{8b} - \frac{(d+ex)^{7/2}}{4b(a+bx)^4} \right)}{\sqrt{a^2 + 2abx + b^2x^2}} \right)$$

$$\sqrt{a^2 + 2abx + b^2x^2}$$

221

$$\left(\frac{(a+bx) \left(\frac{5e \left(\frac{3e \left(-\frac{e \operatorname{arctanh} \left(\frac{\sqrt{b}\sqrt{d+ex}}{\sqrt{bd-ae}} \right)}{b^{3/2}\sqrt{bd-ae}} - \frac{\sqrt{d+ex}}{b(a+bx)} \right)}{4b} - \frac{(d+ex)^{3/2}}{2b(a+bx)^2} \right)}{6b} - \frac{(d+ex)^{5/2}}{3b(a+bx)^3} \right)}{8b} - \frac{(d+ex)^{7/2}}{4b(a+bx)^4} \right)}{\sqrt{a^2 + 2abx + b^2x^2}}$$

$$\sqrt{a^2 + 2abx + b^2x^2}$$

input

```
Int[(d + e*x)^(7/2)/(a^2 + 2*a*b*x + b^2*x^2)^(5/2),x]
```

output

```
((a + b*x)*(-1/4*(d + e*x)^(7/2)/(b*(a + b*x)^4) + (7*e*(-1/3*(d + e*x)^(5/2)/(b*(a + b*x)^3) + (5*e*(-1/2*(d + e*x)^(3/2)/(b*(a + b*x)^2) + (3*e*(-(Sqrt[d + e*x]/(b*(a + b*x))) - (e*ArcTanh[(Sqrt[b]*Sqrt[d + e*x])/Sqrt[b*d - a*e]])/(b^(3/2)*Sqrt[b*d - a*e])))/(4*b)))/(6*b)))/(8*b))/Sqrt[a^2 + 2*a*b*x + b^2*x^2]
```

Defintions of rubi rules used

rule 27

```
Int[(a_)*(Fx_), x_Symbol] := Simp[a Int[Fx, x], x] /; FreeQ[a, x] && !MatchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] := Simp[(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Simp[d*(n/(b*(m + 1))) Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d, n}, x] && ILtQ[m, -1] && FractionQ[n] && GtQ[n, 0]
```

rule 73

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] := With[{p = Denominator[m]}, Simp[p/b Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) + d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

rule 221

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[-a/b, 2]/a)*ArcTanh[x/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 1102

```
Int[((d_.) + (e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] := Simp[(a + b*x + c*x^2)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x)^(2*FracPart[p])) Int[(d + e*x)^m*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && EqQ[b^2 - 4*a*c, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 466 vs. $2(167) = 334$.

Time = 1.61 (sec) , antiderivative size = 467, normalized size of antiderivative = 1.87

| method | result |
|---------|--|
| default | $-\left(-105 \arctan\left(\frac{b\sqrt{ex+d}}{\sqrt{b(ae-bd)}}\right) b^4 e^4 x^4 - 420 \arctan\left(\frac{b\sqrt{ex+d}}{\sqrt{b(ae-bd)}}\right) a b^3 e^4 x^3 + 279 (ex+d)^{\frac{7}{2}} \sqrt{b(ae-bd)} b^3 - 630 \arctan\left(\frac{b\sqrt{ex+d}}{\sqrt{b(ae-bd)}}\right) a^2\right)$ |

input `int((e*x+d)^(7/2)/(b^2*x^2+2*a*b*x+a^2)^(5/2),x,method=_RETURNVERBOSE)`

output
$$\begin{aligned} & -1/192*(-105*\arctan(b*(e*x+d)^(1/2)/(b*(a*e-b*d))^(1/2))*b^4*e^4*x^4-420* \\ & \arctan(b*(e*x+d)^(1/2)/(b*(a*e-b*d))^(1/2))*a*b^3*e^4*x^3+279*(e*x+d)^(7/2) \\ & *(b*(a*e-b*d))^(1/2)*b^3-630*\arctan(b*(e*x+d)^(1/2)/(b*(a*e-b*d))^(1/2))* \\ & a^2*b^2*e^4*x^2+511*(e*x+d)^(5/2)*(b*(a*e-b*d))^(1/2)*a*b^2*e-511*(e*x+d)^(\\ & 5/2)*(b*(a*e-b*d))^(1/2)*b^3*d-420*\arctan(b*(e*x+d)^(1/2)/(b*(a*e-b*d))^(1 \\ & /2))*a^3*b*e^4*x+385*(e*x+d)^(3/2)*(b*(a*e-b*d))^(1/2)*a^2*b*e^2-770*(e*x+ \\ & d)^(3/2)*(b*(a*e-b*d))^(1/2)*a*b^2*d*e+385*(e*x+d)^(3/2)*(b*(a*e-b*d))^(1/ \\ & 2)*b^3*d^2-105*\arctan(b*(e*x+d)^(1/2)/(b*(a*e-b*d))^(1/2))*a^4*e^4+105*(e* \\ & x+d)^(1/2)*(b*(a*e-b*d))^(1/2)*a^3*e^3-315*(e*x+d)^(1/2)*a^2*d*b*e^2*(b*(a \\ & *e-b*d))^(1/2)+315*(e*x+d)^(1/2)*(b*(a*e-b*d))^(1/2)*a*b^2*d^2*e-105*(e*x+ \\ & d)^(1/2)*(b*(a*e-b*d))^(1/2)*b^3*d^3*(b*x+a)/(b*(a*e-b*d))^(1/2)/b^4/((b* \\ & x+a)^2)^(5/2) \end{aligned}$$

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 376 vs. $2(167) = 334$.

Time = 0.11 (sec) , antiderivative size = 765, normalized size of antiderivative = 3.06

$$\int \frac{(d+ex)^{7/2}}{(a^2+2abx+b^2x^2)^{5/2}} dx = \left[\frac{105(b^4e^4x^4+4ab^3e^4x^3+6a^2b^2e^4x^2+4a^3be^4x+a^4e^4)\sqrt{b^2d-abe} \log\left(\frac{be}{b^2d-abe}\right)}{\dots} \right]$$

input `integrate((e*x+d)^(7/2)/(b^2*x^2+2*a*b*x+a^2)^(5/2),x, algorithm="fricas")`

output

```
[1/384*(105*(b^4*e^4*x^4 + 4*a*b^3*e^4*x^3 + 6*a^2*b^2*e^4*x^2 + 4*a^3*b*e^4*x + a^4*e^4)*sqrt(b^2*d - a*b*e)*log((b*e*x + 2*b*d - a*e - 2*sqrt(b^2*d - a*b*e)*sqrt(e*x + d))/(b*x + a)) - 2*(48*b^5*d^4 + 8*a*b^4*d^3*e + 14*a^2*b^3*d^2*e^2 + 35*a^3*b^2*d*e^3 - 105*a^4*b*e^4 + 279*(b^5*d*e^3 - a*b^4*e^4)*x^3 + (326*b^5*d^2*e^2 + 185*a*b^4*d*e^3 - 511*a^2*b^3*e^4)*x^2 + (200*b^5*d^3*e + 52*a*b^4*d^2*e^2 + 133*a^2*b^3*d*e^3 - 385*a^3*b^2*e^4)*x)*sqrt(e*x + d)/(a^4*b^6*d - a^5*b^5*e + (b^10*d - a*b^9*e)*x^4 + 4*(a*b^9*d - a^2*b^8*e)*x^3 + 6*(a^2*b^8*d - a^3*b^7*e)*x^2 + 4*(a^3*b^7*d - a^4*b^6*e)*x), 1/192*(105*(b^4*e^4*x^4 + 4*a*b^3*e^4*x^3 + 6*a^2*b^2*e^4*x^2 + 4*a^3*b*e^4*x + a^4*e^4)*sqrt(-b^2*d + a*b*e)*arctan(sqrt(-b^2*d + a*b*e)*sqrt(e*x + d)/(b*e*x + b*d)) - (48*b^5*d^4 + 8*a*b^4*d^3*e + 14*a^2*b^3*d^2*e^2 + 35*a^3*b^2*d*e^3 - 105*a^4*b*e^4 + 279*(b^5*d*e^3 - a*b^4*e^4)*x^3 + (326*b^5*d^2*e^2 + 185*a*b^4*d*e^3 - 511*a^2*b^3*e^4)*x^2 + (200*b^5*d^3*e + 52*a*b^4*d^2*e^2 + 133*a^2*b^3*d*e^3 - 385*a^3*b^2*e^4)*x)*sqrt(e*x + d)/(a^4*b^6*d - a^5*b^5*e + (b^10*d - a*b^9*e)*x^4 + 4*(a*b^9*d - a^2*b^8*e)*x^3 + 6*(a^2*b^8*d - a^3*b^7*e)*x^2 + 4*(a^3*b^7*d - a^4*b^6*e)*x)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(d + ex)^{7/2}}{(a^2 + 2abx + b^2x^2)^{5/2}} dx = \text{Timed out}$$

input

```
integrate((e*x+d)**(7/2)/(b**2*x**2+2*a*b*x+a**2)**(5/2),x)
```

output

Timed out

Maxima [F]

$$\int \frac{(d + ex)^{7/2}}{(a^2 + 2abx + b^2x^2)^{5/2}} dx = \int \frac{(ex + d)^{7/2}}{(b^2x^2 + 2abx + a^2)^{5/2}} dx$$

input

```
integrate((e*x+d)^(7/2)/(b^2*x^2+2*a*b*x+a^2)^(5/2),x, algorithm="maxima")
```

output `integrate((e*x + d)^(7/2)/(b^2*x^2 + 2*a*b*x + a^2)^(5/2), x)`

Giac [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 251, normalized size of antiderivative = 1.00

$$\int \frac{(d + ex)^{7/2}}{(a^2 + 2abx + b^2x^2)^{5/2}} dx = \frac{35 e^4 \arctan\left(\frac{\sqrt{ex+db}}{\sqrt{-b^2d+abe}}\right)}{64 \sqrt{-b^2d + abe} b^4 \operatorname{sgn}(bx + a)} - \frac{279 (ex + d)^{7/2} b^3 e^4 - 511 (ex + d)^{5/2} b^3 d e^4 + 385 (ex + d)^{3/2} b^3 d^2 e^4 - 105 \sqrt{ex + db} b^3 d^3 e^4 + 511 (ex + d)^{5/2} a b^2}{192 ((ex + d)b -$$

input `integrate((e*x+d)^(7/2)/(b^2*x^2+2*a*b*x+a^2)^(5/2),x, algorithm="giac")`

output `35/64*e^4*arctan(sqrt(e*x + d)*b/sqrt(-b^2*d + a*b*e))/(sqrt(-b^2*d + a*b*e)*b^4*sgn(b*x + a)) - 1/192*(279*(e*x + d)^(7/2)*b^3*e^4 - 511*(e*x + d)^(5/2)*b^3*d*e^4 + 385*(e*x + d)^(3/2)*b^3*d^2*e^4 - 105*sqrt(e*x + d)*b^3*d^3*e^4 + 511*(e*x + d)^(5/2)*a*b^2*e^5 - 770*(e*x + d)^(3/2)*a*b^2*d*e^5 + 315*sqrt(e*x + d)*a*b^2*d^2*e^5 + 385*(e*x + d)^(3/2)*a^2*b*e^6 - 315*sqrt(e*x + d)*a^2*b*d*e^6 + 105*sqrt(e*x + d)*a^3*e^7)/(((e*x + d)*b - b*d + a*e)^4*b^4*sgn(b*x + a))`

Mupad [F(-1)]

Timed out.

$$\int \frac{(d + ex)^{7/2}}{(a^2 + 2abx + b^2x^2)^{5/2}} dx = \int \frac{(d + ex)^{7/2}}{(a^2 + 2abx + b^2x^2)^{5/2}} dx$$

input `int((d + e*x)^(7/2)/(a^2 + b^2*x^2 + 2*a*b*x)^(5/2), x)`

output `int((d + e*x)^(7/2)/(a^2 + b^2*x^2 + 2*a*b*x)^(5/2), x)`

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 583, normalized size of antiderivative = 2.33

$$\int \frac{(d+ex)^{7/2}}{(a^2+2abx+b^2x^2)^{5/2}} dx = \frac{105\sqrt{b}\sqrt{ae-bd}\operatorname{atan}\left(\frac{\sqrt{ex+db}}{\sqrt{b}\sqrt{ae-bd}}\right)a^4e^4 + 420\sqrt{b}\sqrt{ae-bd}\operatorname{atan}\left(\frac{\sqrt{ex+db}}{\sqrt{b}\sqrt{ae-bd}}\right)}{c}$$

input

```
int((e*x+d)^(7/2)/(b^2*x^2+2*a*b*x+a^2)^(5/2),x)
```

output

```
(105*sqrt(b)*sqrt(a*e - b*d)*atan((sqrt(d + e*x)*b)/(sqrt(b)*sqrt(a*e - b*d)))*a**4*e**4 + 420*sqrt(b)*sqrt(a*e - b*d)*atan((sqrt(d + e*x)*b)/(sqrt(b)*sqrt(a*e - b*d)))*a**3*b*e**4*x + 630*sqrt(b)*sqrt(a*e - b*d)*atan((sqrt(d + e*x)*b)/(sqrt(b)*sqrt(a*e - b*d)))*a**2*b**2*e**4*x**2 + 420*sqrt(b)*sqrt(a*e - b*d)*atan((sqrt(d + e*x)*b)/(sqrt(b)*sqrt(a*e - b*d)))*a*b**3*e**4*x**3 + 105*sqrt(b)*sqrt(a*e - b*d)*atan((sqrt(d + e*x)*b)/(sqrt(b)*sqrt(a*e - b*d)))*b**4*e**4*x**4 - 105*sqrt(d + e*x)*a**4*b*e**4 + 35*sqrt(d + e*x)*a**3*b**2*d*e**3 - 385*sqrt(d + e*x)*a**3*b**2*e**4*x + 14*sqrt(d + e*x)*a**2*b**3*d**2*e**2 + 133*sqrt(d + e*x)*a**2*b**3*d*e**3*x - 511*sqrt(d + e*x)*a**2*b**3*e**4*x**2 + 8*sqrt(d + e*x)*a*b**4*d**3*e + 52*sqrt(d + e*x)*a*b**4*d**2*e**2*x + 185*sqrt(d + e*x)*a*b**4*d*e**3*x**2 - 279*sqrt(d + e*x)*a*b**4*e**4*x**3 + 48*sqrt(d + e*x)*b**5*d**4 + 200*sqrt(d + e*x)*b**5*d**3*e*x + 326*sqrt(d + e*x)*b**5*d**2*e**2*x**2 + 279*sqrt(d + e*x)*b**5*d*e**3*x**3)/(192*b**5*(a**5*e - a**4*b*d + 4*a**4*b*e*x - 4*a**3*b**2*d*x + 6*a**3*b**2*e*x**2 - 6*a**2*b**3*d*x**2 + 4*a**2*b**3*e*x**3 - 4*a*b**4*d*x**3 + a*b**4*e*x**4 - b**5*d*x**4))
```

3.392 $\int \frac{(d+ex)^{5/2}}{(a^2+2abx+b^2x^2)^{5/2}} dx$

| | |
|---|------|
| Optimal result | 2930 |
| Mathematica [A] (verified) | 2931 |
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Optimal result

Integrand size = 30, antiderivative size = 260

$$\int \frac{(d+ex)^{5/2}}{(a^2+2abx+b^2x^2)^{5/2}} dx = -\frac{5e^3\sqrt{d+ex}}{64b^3(bd-ae)\sqrt{a^2+2abx+b^2x^2}} - \frac{5e^2\sqrt{d+ex}}{32b^3(a+bx)\sqrt{a^2+2abx+b^2x^2}} - \frac{5e(d+ex)^{3/2}}{24b^2(a+bx)^2\sqrt{a^2+2abx+b^2x^2}} - \frac{(d+ex)^{5/2}}{4b(a+bx)^3\sqrt{a^2+2abx+b^2x^2}} + \frac{5e^4(a+bx)\operatorname{arctanh}\left(\frac{\sqrt{b}\sqrt{d+ex}}{\sqrt{bd-ae}}\right)}{64b^{7/2}(bd-ae)^{3/2}\sqrt{a^2+2abx+b^2x^2}}$$

output

```
-5/64*e^3*(e*x+d)^(1/2)/b^3/(-a*e+b*d)/((b*x+a)^2)^(1/2)-5/32*e^2*(e*x+d)^(1/2)/b^3/(b*x+a)/((b*x+a)^2)^(1/2)-5/24*e*(e*x+d)^(3/2)/b^2/(b*x+a)^2/((b*x+a)^2)^(1/2)-1/4*(e*x+d)^(5/2)/b/(b*x+a)^3/((b*x+a)^2)^(1/2)+5/64*e^4*(b*x+a)*arctanh(b^(1/2)*(e*x+d)^(1/2)/(-a*e+b*d)^(1/2))/b^(7/2)/(-a*e+b*d)^(3/2)/((b*x+a)^2)^(1/2)
```

Mathematica [A] (verified)

Time = 1.01 (sec) , antiderivative size = 194, normalized size of antiderivative = 0.75

$$\int \frac{(d+ex)^{5/2}}{(a^2+2abx+b^2x^2)^{5/2}} dx = \frac{e^4(a+bx)^5 \left(\frac{\sqrt{b}\sqrt{d+ex}(-15a^3e^3-5a^2be^2(2d+11ex)-ab^2e(8d^2+36dex+73e^2x^2)+b^3(48d^3+136d^2ex+118de^2x^2+15e^3x^3))}{e^4(-bd+ae)(a+bx)^4} + (15\text{ArcTan}[\frac{\sqrt{b}\sqrt{d+ex}}{\sqrt{-(b*d)+a*e}}]) / (-(b*d)+a*e)^{(3/2)} \right)}{192b^{7/2}((a+bx)^2)^{5/2}}$$

input `Integrate[(d + e*x)^(5/2)/(a^2 + 2*a*b*x + b^2*x^2)^(5/2), x]`

output `(e^4*(a + b*x)^5*((Sqrt[b]*Sqrt[d + e*x]*(-15*a^3*e^3 - 5*a^2*b*e^2*(2*d + 11*e*x) - a*b^2*e*(8*d^2 + 36*d*e*x + 73*e^2*x^2) + b^3*(48*d^3 + 136*d^2*e*x + 118*d*e^2*x^2 + 15*e^3*x^3)))/(e^4*(-(b*d) + a*e)*(a + b*x)^4) + (15*ArcTan[(Sqrt[b]*Sqrt[d + e*x])/Sqrt[-(b*d) + a*e]])/(-(b*d) + a*e)^(3/2)))/(192*b^(7/2)*((a + b*x)^2)^(5/2))`

Rubi [A] (verified)

Time = 0.52 (sec) , antiderivative size = 198, normalized size of antiderivative = 0.76, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$, Rules used = {1102, 27, 51, 51, 52, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\begin{aligned} & \int \frac{(d+ex)^{5/2}}{(a^2+2abx+b^2x^2)^{5/2}} dx \\ & \quad \downarrow \text{1102} \\ & \frac{b^5(a+bx) \int \frac{(d+ex)^{5/2}}{b^5(a+bx)^5} dx}{\sqrt{a^2+2abx+b^2x^2}} \\ & \quad \downarrow \text{27} \\ & \frac{(a+bx) \int \frac{(d+ex)^{5/2}}{(a+bx)^5} dx}{\sqrt{a^2+2abx+b^2x^2}} \\ & \quad \downarrow \text{51} \end{aligned}$$

$$\begin{aligned}
 & \frac{(a+bx) \left(\frac{5e \int \frac{(d+ex)^{3/2}}{(a+bx)^4} dx}{8b} - \frac{(d+ex)^{5/2}}{4b(a+bx)^4} \right)}{\sqrt{a^2 + 2abx + b^2x^2}} \\
 & \quad \downarrow 51 \\
 & \frac{(a+bx) \left(\frac{5e \left(\frac{e \int \frac{\sqrt{d+ex}}{(a+bx)^3} dx}{2b} - \frac{(d+ex)^{3/2}}{3b(a+bx)^3} \right)}{8b} - \frac{(d+ex)^{5/2}}{4b(a+bx)^4} \right)}{\sqrt{a^2 + 2abx + b^2x^2}} \\
 & \quad \downarrow 51 \\
 & \frac{(a+bx) \left(\frac{5e \left(\frac{e \left(\frac{e \int \frac{1}{(a+bx)^2 \sqrt{d+ex}} dx}{4b} - \frac{\sqrt{d+ex}}{2b(a+bx)^2} \right)}{2b} - \frac{(d+ex)^{3/2}}{3b(a+bx)^3} \right)}{8b} - \frac{(d+ex)^{5/2}}{4b(a+bx)^4} \right)}{\sqrt{a^2 + 2abx + b^2x^2}} \\
 & \quad \downarrow 52 \\
 & \frac{(a+bx) \left(\frac{5e \left(\frac{e \left(\frac{e \int \frac{1}{(a+bx)\sqrt{d+ex}} dx}{2(bd-ae)} - \frac{\sqrt{d+ex}}{(a+bx)(bd-ae)} \right)}{4b} - \frac{\sqrt{d+ex}}{2b(a+bx)^2} \right)}{2b} - \frac{(d+ex)^{3/2}}{3b(a+bx)^3} \right)}{8b} - \frac{(d+ex)^{5/2}}{4b(a+bx)^4} \right)}{\sqrt{a^2 + 2abx + b^2x^2}} \\
 & \quad \downarrow 73
 \end{aligned}$$

$$\left(\frac{(a+bx) \left(\frac{5e \left(\frac{e \left(\frac{\int \frac{1}{a + \frac{b(d+ex) - bd}{e} d\sqrt{d+ex}}{bd-ae} - \frac{\sqrt{d+ex}}{(a+bx)(bd-ae)} \right)}{4b} - \frac{\sqrt{d+ex}}{2b(a+bx)^2} \right)}{2b} - \frac{(d+ex)^{3/2}}{3b(a+bx)^3} \right)}{8b} - \frac{(d+ex)^{5/2}}{4b(a+bx)^4} \right) \right)$$

$$\sqrt{a^2 + 2abx + b^2x^2}$$

221

$$\left(\frac{(a+bx) \left(\frac{5e \left(\frac{e \left(\frac{e \operatorname{arctanh} \left(\frac{\sqrt{b}\sqrt{d+ex}}{\sqrt{bd-ae}} \right) - \frac{\sqrt{d+ex}}{(a+bx)(bd-ae)} \right)}{4b} - \frac{\sqrt{d+ex}}{2b(a+bx)^2} \right)}{2b} - \frac{(d+ex)^{3/2}}{3b(a+bx)^3} \right)}{8b} - \frac{(d+ex)^{5/2}}{4b(a+bx)^4} \right) \right)$$

$$\sqrt{a^2 + 2abx + b^2x^2}$$

input

```
Int[(d + e*x)^(5/2)/(a^2 + 2*a*b*x + b^2*x^2)^(5/2),x]
```

output

$$\begin{aligned} & ((a + b*x)*(-1/4*(d + e*x)^{(5/2)}/(b*(a + b*x)^4) + (5*e*(-1/3*(d + e*x)^{(3/2)}/(b*(a + b*x)^3) + (e*(-1/2*\sqrt{d + e*x})/(b*(a + b*x)^2) + (e*(-\sqrt{d + e*x})/((b*d - a*e)*(a + b*x))) + (e*\text{ArcTanh}[(\sqrt{b}*\sqrt{d + e*x})/\sqrt{b*d - a*e}])/(\sqrt{b}*(b*d - a*e)^{(3/2)})))/(4*b)))/(2*b)))/(8*b))/\sqrt{a^2 + 2*a*b*x + b^2*x^2} \end{aligned}$$

Defintions of rubi rules used

rule 27

$$\text{Int}[(a_*)(Fx_), x_Symbol] \rightarrow \text{Simp}[a \text{ Int}[Fx, x], x] \text{ ; FreeQ}[a, x] \ \&\& \ !\text{MatchQ}[Fx, (b_*)(Gx_) \text{ ; FreeQ}[b, x]$$

rule 51

$$\begin{aligned} & \text{Int}[((a_.) + (b_.)*(x_))^{(m_)}*((c_.) + (d_.)*(x_))^{(n_)}, x_Symbol] \rightarrow \text{Simp}[\\ & (a + b*x)^{(m + 1)}*((c + d*x)^n/(b*(m + 1))), x] - \text{Simp}[d*(n/(b*(m + 1))) \\ & \text{Int}[(a + b*x)^{(m + 1)}*(c + d*x)^{(n - 1)}, x], x] \text{ ; FreeQ}[\{a, b, c, d, n\}, x] \\ & \ \&\& \ \text{ILtQ}[m, -1] \ \&\& \ \text{FractionQ}[n] \ \&\& \ \text{GtQ}[n, 0] \end{aligned}$$

rule 52

$$\begin{aligned} & \text{Int}[((a_.) + (b_.)*(x_))^{(m_)}*((c_.) + (d_.)*(x_))^{(n_)}, x_Symbol] \rightarrow \text{Simp}[\\ & (a + b*x)^{(m + 1)}*((c + d*x)^{(n + 1)}/((b*c - a*d)*(m + 1))), x] - \text{Simp}[d*((m + n + 2)/((b*c - a*d)*(m + 1))) \\ & \text{Int}[(a + b*x)^{(m + 1)}*(c + d*x)^n, x], x] \text{ ; FreeQ}[\{a, b, c, d, n\}, x] \ \&\& \ \text{ILtQ}[m, -1] \ \&\& \ \text{FractionQ}[n] \ \&\& \ \text{LtQ}[n, 0] \end{aligned}$$

rule 73

$$\begin{aligned} & \text{Int}[((a_.) + (b_.)*(x_))^{(m_)}*((c_.) + (d_.)*(x_))^{(n_)}, x_Symbol] \rightarrow \text{With}[\\ & \{p = \text{Denominator}[m]\}, \text{Simp}[p/b \text{ Subst}[\text{Int}[x^{(p*(m + 1) - 1)}*(c - a*(d/b) + \\ & d*(x^p/b))^{(n)}, x], x, (a + b*x)^{(1/p)}, x]] \text{ ; FreeQ}[\{a, b, c, d\}, x] \ \&\& \ \text{LtQ}[-1, m, 0] \\ & \ \&\& \ \text{LeQ}[-1, n, 0] \ \&\& \ \text{LeQ}[\text{Denominator}[n], \text{Denominator}[m]] \ \&\& \ \text{IntLinearQ}[a, b, c, d, m, n, x] \end{aligned}$$

rule 221

$$\text{Int}[((a_) + (b_.)*(x_)^2)^{-1}, x_Symbol] \rightarrow \text{Simp}[(\text{Rt}[-a/b, 2]/a)*\text{ArcTanh}[x/\text{Rt}[-a/b, 2]], x] \text{ ; FreeQ}[\{a, b\}, x] \ \&\& \ \text{NegQ}[a/b]$$

rule 1102

```
Int[((d_.) + (e_.)*(x_.))^(m_.)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol]
:> Simp[(a + b*x + c*x^2)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x)^(2*FracPart[p]))
Int[(d + e*x)^m*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d, e, m, p}, x]
&& EqQ[b^2 - 4*a*c, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 476 vs. $2(177) = 354$.

Time = 1.57 (sec) , antiderivative size = 477, normalized size of antiderivative = 1.83

| method | result |
|---------|---|
| default | $\frac{\left(15 \arctan\left(\frac{b\sqrt{ex+d}}{\sqrt{b(ae-bd)}}\right) b^4 e^4 x^4 + 60 \arctan\left(\frac{b\sqrt{ex+d}}{\sqrt{b(ae-bd)}}\right) a b^3 e^4 x^3 + 15 (ex+d)^{\frac{7}{2}} \sqrt{b(ae-bd)} b^3 + 90 \arctan\left(\frac{b\sqrt{ex+d}}{\sqrt{b(ae-bd)}}\right) a^2 b^2 e^4 x^2 - \dots}{\dots}$ |

input

```
int((e*x+d)^(5/2)/(b^2*x^2+2*a*b*x+a^2)^(5/2),x,method=_RETURNVERBOSE)
```

output

```
1/192*(15*arctan(b*(e*x+d)^(1/2)/(b*(a*e-b*d))^(1/2))*b^4*e^4*x^4+60*arctan(b*(e*x+d)^(1/2)/(b*(a*e-b*d))^(1/2))*a*b^3*e^4*x^3+15*(e*x+d)^(7/2)*(b*(a*e-b*d))^(1/2)*b^3+90*arctan(b*(e*x+d)^(1/2)/(b*(a*e-b*d))^(1/2))*a^2*b^2*e^4*x^2-73*(e*x+d)^(5/2)*(b*(a*e-b*d))^(1/2)*a*b^2*e+73*(e*x+d)^(5/2)*(b*(a*e-b*d))^(1/2)*b^3*d+60*arctan(b*(e*x+d)^(1/2)/(b*(a*e-b*d))^(1/2))*a^3*b*e^4*x-55*(e*x+d)^(3/2)*(b*(a*e-b*d))^(1/2)*a^2*b*e^2+110*(e*x+d)^(3/2)*(b*(a*e-b*d))^(1/2)*a*b^2*d*e-55*(e*x+d)^(3/2)*(b*(a*e-b*d))^(1/2)*b^3*d^2+15*arctan(b*(e*x+d)^(1/2)/(b*(a*e-b*d))^(1/2))*a^4*e^4-15*(e*x+d)^(1/2)*(b*(a*e-b*d))^(1/2)*a^3*e^3+45*(e*x+d)^(1/2)*a^2*d*b*e^2*(b*(a*e-b*d))^(1/2)-45*(e*x+d)^(1/2)*(b*(a*e-b*d))^(1/2)*a*b^2*d^2*e+15*(e*x+d)^(1/2)*(b*(a*e-b*d))^(1/2)*b^3*d^3*(b*x+a)/(b*(a*e-b*d))^(1/2)/(a*e-b*d)/b^3/((b*x+a)^2)^(5/2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 440 vs. $2(177) = 354$.

Time = 0.11 (sec) , antiderivative size = 894, normalized size of antiderivative = 3.44

$$\int \frac{(d + ex)^{5/2}}{(a^2 + 2abx + b^2x^2)^{5/2}} dx = \text{Too large to display}$$

input `integrate((e*x+d)^(5/2)/(b^2*x^2+2*a*b*x+a^2)^(5/2),x, algorithm="fricas")`

output `[-1/384*(15*(b^4*e^4*x^4 + 4*a*b^3*e^4*x^3 + 6*a^2*b^2*e^4*x^2 + 4*a^3*b*e^4*x + a^4*e^4)*sqrt(b^2*d - a*b*e)*log((b*e*x + 2*b*d - a*e - 2*sqrt(b^2*d - a*b*e)*sqrt(e*x + d))/(b*x + a)) + 2*(48*b^5*d^4 - 56*a*b^4*d^3*e - 2*a^2*b^3*d^2*e^2 - 5*a^3*b^2*d*e^3 + 15*a^4*b*e^4 + 15*(b^5*d*e^3 - a*b^4*e^4)*x^3 + (118*b^5*d^2*e^2 - 191*a*b^4*d*e^3 + 73*a^2*b^3*e^4)*x^2 + (136*b^5*d^3*e - 172*a*b^4*d^2*e^2 - 19*a^2*b^3*d*e^3 + 55*a^3*b^2*e^4)*x)*sqrt(e*x + d)/(a^4*b^6*d^2 - 2*a^5*b^5*d*e + a^6*b^4*e^2 + (b^10*d^2 - 2*a*b^9*d*e + a^2*b^8*e^2)*x^4 + 4*(a*b^9*d^2 - 2*a^2*b^8*d*e + a^3*b^7*e^2)*x^3 + 6*(a^2*b^8*d^2 - 2*a^3*b^7*d*e + a^4*b^6*e^2)*x^2 + 4*(a^3*b^7*d^2 - 2*a^4*b^6*d*e + a^5*b^5*e^2)*x), -1/192*(15*(b^4*e^4*x^4 + 4*a*b^3*e^4*x^3 + 6*a^2*b^2*e^4*x^2 + 4*a^3*b*e^4*x + a^4*e^4)*sqrt(-b^2*d + a*b*e)*arctan(sqrt(-b^2*d + a*b*e)*sqrt(e*x + d)/(b*e*x + b*d)) + (48*b^5*d^4 - 56*a*b^4*d^3*e - 2*a^2*b^3*d^2*e^2 - 5*a^3*b^2*d*e^3 + 15*a^4*b*e^4 + 15*(b^5*d*e^3 - a*b^4*e^4)*x^3 + (118*b^5*d^2*e^2 - 191*a*b^4*d*e^3 + 73*a^2*b^3*e^4)*x^2 + (136*b^5*d^3*e - 172*a*b^4*d^2*e^2 - 19*a^2*b^3*d*e^3 + 55*a^3*b^2*e^4)*x)*sqrt(e*x + d)/(a^4*b^6*d^2 - 2*a^5*b^5*d*e + a^6*b^4*e^2 + (b^10*d^2 - 2*a*b^9*d*e + a^2*b^8*e^2)*x^4 + 4*(a*b^9*d^2 - 2*a^2*b^8*d*e + a^3*b^7*e^2)*x^3 + 6*(a^2*b^8*d^2 - 2*a^3*b^7*d*e + a^4*b^6*e^2)*x^2 + 4*(a^3*b^7*d^2 - 2*a^4*b^6*d*e + a^5*b^5*e^2)*x)]`

Sympy [F(-1)]

Timed out.

$$\int \frac{(d + ex)^{5/2}}{(a^2 + 2abx + b^2x^2)^{5/2}} dx = \text{Timed out}$$

input `integrate((e*x+d)**(5/2)/(b**2*x**2+2*a*b*x+a**2)**(5/2),x)`

output `Timed out`

Maxima [F]

$$\int \frac{(d + ex)^{5/2}}{(a^2 + 2abx + b^2x^2)^{5/2}} dx = \int \frac{(ex + d)^{5/2}}{(b^2x^2 + 2abx + a^2)^{5/2}} dx$$

input `integrate((e*x+d)^(5/2)/(b^2*x^2+2*a*b*x+a^2)^(5/2),x, algorithm="maxima")`

output `integrate((e*x + d)^(5/2)/(b^2*x^2 + 2*a*b*x + a^2)^(5/2), x)`

Giac [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 283, normalized size of antiderivative = 1.09

$$\int \frac{(d + ex)^{5/2}}{(a^2 + 2abx + b^2x^2)^{5/2}} dx = -\frac{5e^4 \arctan\left(\frac{\sqrt{ex+db}}{\sqrt{-b^2d+abe}}\right)}{64(b^4 \operatorname{dsgn}(bx+a) - ab^3 \operatorname{esgn}(bx+a))\sqrt{-b^2d+abe}} - \frac{15(ex+d)^{7/2}b^3e^4 + 73(ex+d)^{5/2}b^3de^4 - 55(ex+d)^{3/2}b^3d^2e^4 + 15\sqrt{ex+db}b^3d^3e^4 - 73(ex+d)^{5/2}ab^2e^5 + 15d^{5/2}b^3e^4}{192(b^4 \operatorname{dsgn}(bx+a) - ab^3 \operatorname{esgn}(bx+a))\sqrt{-b^2d+abe}}$$

input `integrate((e*x+d)^(5/2)/(b^2*x^2+2*a*b*x+a^2)^(5/2),x, algorithm="giac")`

output

```
-5/64*e^4*arctan(sqrt(e*x + d)*b/sqrt(-b^2*d + a*b*e))/((b^4*d*sgn(b*x + a)
) - a*b^3*e*sgn(b*x + a))*sqrt(-b^2*d + a*b*e)) - 1/192*(15*(e*x + d)^(7/2)
)*b^3*e^4 + 73*(e*x + d)^(5/2)*b^3*d*e^4 - 55*(e*x + d)^(3/2)*b^3*d^2*e^4
+ 15*sqrt(e*x + d)*b^3*d^3*e^4 - 73*(e*x + d)^(5/2)*a*b^2*e^5 + 110*(e*x +
d)^(3/2)*a*b^2*d*e^5 - 45*sqrt(e*x + d)*a*b^2*d^2*e^5 - 55*(e*x + d)^(3/2)
)*a^2*b*e^6 + 45*sqrt(e*x + d)*a^2*b*d*e^6 - 15*sqrt(e*x + d)*a^3*e^7)/((b
^4*d*sgn(b*x + a) - a*b^3*e*sgn(b*x + a))*((e*x + d)*b - b*d + a*e)^4)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d + ex)^{5/2}}{(a^2 + 2abx + b^2x^2)^{5/2}} dx = \int \frac{(d + ex)^{5/2}}{(a^2 + 2abx + b^2x^2)^{5/2}} dx$$

input

```
int((d + e*x)^(5/2)/(a^2 + b^2*x^2 + 2*a*b*x)^(5/2), x)
```

output

```
int((d + e*x)^(5/2)/(a^2 + b^2*x^2 + 2*a*b*x)^(5/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 661, normalized size of antiderivative = 2.54

$$\int \frac{(d + ex)^{5/2}}{(a^2 + 2abx + b^2x^2)^{5/2}} dx = \frac{15\sqrt{b}\sqrt{ae - bd} \operatorname{atan}\left(\frac{\sqrt{ex+db}}{\sqrt{b}\sqrt{ae-bd}}\right) a^4 e^4 + 60\sqrt{b}\sqrt{ae - bd} \operatorname{atan}\left(\frac{\sqrt{ex+db}}{\sqrt{b}\sqrt{ae-bd}}\right) a^3 b}{(a^2 + 2abx + b^2x^2)^{5/2}}$$

input

```
int((e*x+d)^(5/2)/(b^2*x^2+2*a*b*x+a^2)^(5/2), x)
```

output

```
(15*sqrt(b)*sqrt(a*e - b*d)*atan((sqrt(d + e*x)*b)/(sqrt(b)*sqrt(a*e - b*d
)))*a**4*e**4 + 60*sqrt(b)*sqrt(a*e - b*d)*atan((sqrt(d + e*x)*b)/(sqrt(b)
*sqrt(a*e - b*d)))*a**3*b*e**4*x + 90*sqrt(b)*sqrt(a*e - b*d)*atan((sqrt(d
+ e*x)*b)/(sqrt(b)*sqrt(a*e - b*d)))*a**2*b**2*e**4*x**2 + 60*sqrt(b)*sqr
t(a*e - b*d)*atan((sqrt(d + e*x)*b)/(sqrt(b)*sqrt(a*e - b*d)))*a*b**3*e**4
*x**3 + 15*sqrt(b)*sqrt(a*e - b*d)*atan((sqrt(d + e*x)*b)/(sqrt(b)*sqrt(a*
e - b*d)))*b**4*e**4*x**4 - 15*sqrt(d + e*x)*a**4*b*e**4 + 5*sqrt(d + e*x)
*a**3*b**2*d*e**3 - 55*sqrt(d + e*x)*a**3*b**2*e**4*x + 2*sqrt(d + e*x)*a*
**2*b**3*d**2*e**2 + 19*sqrt(d + e*x)*a**2*b**3*d*e**3*x - 73*sqrt(d + e*x)
*a**2*b**3*e**4*x**2 + 56*sqrt(d + e*x)*a*b**4*d**3*e + 172*sqrt(d + e*x)*
a*b**4*d**2*e**2*x + 191*sqrt(d + e*x)*a*b**4*d*e**3*x**2 + 15*sqrt(d + e*
x)*a*b**4*e**4*x**3 - 48*sqrt(d + e*x)*b**5*d**4 - 136*sqrt(d + e*x)*b**5*
d**3*e*x - 118*sqrt(d + e*x)*b**5*d**2*e**2*x**2 - 15*sqrt(d + e*x)*b**5*d
**3*x**3)/(192*b**4*(a**6*e**2 - 2*a**5*b*d*e + 4*a**5*b*e**2*x + a**4*b
**2*d**2 - 8*a**4*b**2*d*e*x + 6*a**4*b**2*e**2*x**2 + 4*a**3*b**3*d**2*x
- 12*a**3*b**3*d*e*x**2 + 4*a**3*b**3*e**2*x**3 + 6*a**2*b**4*d**2*x**2 -
8*a**2*b**4*d*e*x**3 + a**2*b**4*e**2*x**4 + 4*a*b**5*d**2*x**3 - 2*a*b**5
*d*e*x**4 + b**6*d**2*x**4))
```


3.393 $\int \frac{(d+ex)^{3/2}}{(a^2+2abx+b^2x^2)^{5/2}} dx$

| | |
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| Optimal result | 2940 |
| Mathematica [A] (verified) | 2941 |
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Optimal result

Integrand size = 30, antiderivative size = 270

$$\int \frac{(d+ex)^{3/2}}{(a^2+2abx+b^2x^2)^{5/2}} dx = \frac{3e^3\sqrt{d+ex}}{64b^2(bd-ae)^2\sqrt{a^2+2abx+b^2x^2}} - \frac{e^2\sqrt{d+ex}}{8b^2(a+bx)^2\sqrt{a^2+2abx+b^2x^2}} - \frac{32b^2(bd-ae)(a+bx)\sqrt{a^2+2abx+b^2x^2}}{4b(a+bx)^3\sqrt{a^2+2abx+b^2x^2}} - \frac{3e^4(a+bx)\operatorname{arctanh}\left(\frac{\sqrt{b}\sqrt{d+ex}}{\sqrt{bd-ae}}\right)}{64b^{5/2}(bd-ae)^{5/2}\sqrt{a^2+2abx+b^2x^2}}$$

output

```
3/64*e^3*(e*x+d)^(1/2)/b^2/(-a*e+b*d)^2/((b*x+a)^2)^(1/2)-1/8*e*(e*x+d)^(1/2)/b^2/(b*x+a)^2/((b*x+a)^2)^(1/2)-1/32*e^2*(e*x+d)^(1/2)/b^2/(-a*e+b*d)/(b*x+a)/((b*x+a)^2)^(1/2)-1/4*(e*x+d)^(3/2)/b/(b*x+a)^3/((b*x+a)^2)^(1/2)-3/64*e^4*(b*x+a)*arctanh(b^(1/2)*(e*x+d)^(1/2)/(-a*e+b*d)^(1/2))/b^(5/2)/(-a*e+b*d)^(5/2)/((b*x+a)^2)^(1/2)
```

Mathematica [A] (verified)

Time = 1.07 (sec) , antiderivative size = 194, normalized size of antiderivative = 0.72

$$\int \frac{(d+ex)^{3/2}}{(a^2+2abx+b^2x^2)^{5/2}} dx = \frac{e^4(a+bx)^5 \left(-\frac{\sqrt{b}\sqrt{d+ex}(3a^3e^3+a^2be^2(2d+11ex)-ab^2e(24d^2+44dex+11e^2x^2))+b^3(16d^3+24d^2e)}{e^4(bd-ae)^2(a+bx)^4} \right)}{64b^{5/2}((a+bx)^2)^{5/2}}$$

input `Integrate[(d + e*x)^(3/2)/(a^2 + 2*a*b*x + b^2*x^2)^(5/2), x]`

output `(e^4*(a + b*x)^5*(-((Sqrt[b]*Sqrt[d + e*x]*(3*a^3*e^3 + a^2*b*e^2*(2*d + 11*e*x) - a*b^2*e*(24*d^2 + 44*d*e*x + 11*e^2*x^2) + b^3*(16*d^3 + 24*d^2*e*x + 2*d*e^2*x^2 - 3*e^3*x^3)))/(e^4*(b*d - a*e)^2*(a + b*x)^4)) + (3*ArcTan[(Sqrt[b]*Sqrt[d + e*x])/Sqrt[-(b*d) + a*e]]/(-(b*d) + a*e)^(5/2)))/(64*b^(5/2)*((a + b*x)^2)^(5/2))`

Rubi [A] (verified)

Time = 0.53 (sec) , antiderivative size = 212, normalized size of antiderivative = 0.79, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$, Rules used = {1102, 27, 51, 51, 52, 52, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\begin{aligned} & \int \frac{(d+ex)^{3/2}}{(a^2+2abx+b^2x^2)^{5/2}} dx \\ & \quad \downarrow \text{1102} \\ & \frac{b^5(a+bx) \int \frac{(d+ex)^{3/2}}{b^5(a+bx)^5} dx}{\sqrt{a^2+2abx+b^2x^2}} \\ & \quad \downarrow \text{27} \\ & \frac{(a+bx) \int \frac{(d+ex)^{3/2}}{(a+bx)^5} dx}{\sqrt{a^2+2abx+b^2x^2}} \\ & \quad \downarrow \text{51} \end{aligned}$$

$$\frac{(a + bx) \left(\frac{3e \int \frac{\sqrt{d+ex}}{(a+bx)^4} dx}{8b} - \frac{(d+ex)^{3/2}}{4b(a+bx)^4} \right)}{\sqrt{a^2 + 2abx + b^2x^2}}$$

↓ 51

$$\frac{(a + bx) \left(\frac{3e \left(\frac{e \int \frac{1}{(a+bx)^3 \sqrt{d+ex}} dx}{6b} - \frac{\sqrt{d+ex}}{3b(a+bx)^3} \right)}{8b} - \frac{(d+ex)^{3/2}}{4b(a+bx)^4} \right)}{\sqrt{a^2 + 2abx + b^2x^2}}$$

↓ 52

$$\frac{(a + bx) \left(\frac{3e \left(\frac{e \left(-\frac{3e \int \frac{1}{(a+bx)^2 \sqrt{d+ex}} dx}{4(bd-ae)} - \frac{\sqrt{d+ex}}{2(a+bx)^2(bd-ae)} \right)}{6b} - \frac{\sqrt{d+ex}}{3b(a+bx)^3} \right)}{8b} - \frac{(d+ex)^{3/2}}{4b(a+bx)^4} \right)}{\sqrt{a^2 + 2abx + b^2x^2}}$$

↓ 52

$$\frac{(a + bx) \left(\frac{3e \left(\frac{e \left(-\frac{3e \left(-\frac{e \int \frac{1}{(a+bx) \sqrt{d+ex}} dx}{2(bd-ae)} - \frac{\sqrt{d+ex}}{(a+bx)(bd-ae)} \right)}{4(bd-ae)} - \frac{\sqrt{d+ex}}{2(a+bx)^2(bd-ae)} \right)}{6b} - \frac{\sqrt{d+ex}}{3b(a+bx)^3} \right)}{8b} - \frac{(d+ex)^{3/2}}{4b(a+bx)^4} \right)}{\sqrt{a^2 + 2abx + b^2x^2}}$$

↓ 73

$$\left((a + bx) \left[\frac{3e \left(\frac{\int \frac{1}{a + \frac{b(d+ex) - bd}{e} bd - ae} d\sqrt{d+ex}}{(a+bx)(bd-ae)} - \frac{\sqrt{d+ex}}{2(a+bx)^2(bd-ae)} \right)}{4(bd-ae)} - \frac{\sqrt{d+ex}}{2(a+bx)^2(bd-ae)} \right] - \frac{\sqrt{d+ex}}{3b(a+bx)^3} \right) - \frac{(d+ex)^{3/2}}{4b(a+bx)^4}$$

$$\sqrt{a^2 + 2abx + b^2x^2}$$

221

$$\left((a + bx) \left[\frac{3e \left(\frac{e \operatorname{arctanh} \left(\frac{\sqrt{b}\sqrt{d+ex}}{\sqrt{bd-ae}} \right)}{\sqrt{b}(bd-ae)^{3/2}} - \frac{\sqrt{d+ex}}{(a+bx)(bd-ae)} \right)}{4(bd-ae)} - \frac{\sqrt{d+ex}}{2(a+bx)^2(bd-ae)} \right] - \frac{\sqrt{d+ex}}{3b(a+bx)^3} \right) - \frac{(d+ex)^{3/2}}{4b(a+bx)^4}$$

$$\sqrt{a^2 + 2abx + b^2x^2}$$

input `Int[(d + e*x)^(3/2)/(a^2 + 2*a*b*x + b^2*x^2)^(5/2),x]`

output

$$\frac{\left((a + bx)^{-1/4} (d + ex)^{3/2} / (b(a + bx)^4) + (3e(-1/3\sqrt{d + ex}) / (b(a + bx)^3) + (e(-1/2\sqrt{d + ex}) / ((b*d - a*e)(a + bx)^2) - (3e * (-\sqrt{d + ex}) / ((b*d - a*e)(a + bx))) + (e * \text{ArcTanh}[\sqrt{b} * \sqrt{d + ex}] / \sqrt{b*d - a*e}) / (\sqrt{b} * (b*d - a*e)^{3/2})) / (4 * (b*d - a*e))) / (6 * b) \right) / (8 * b) / \sqrt{a^2 + 2 * a * b * x + b^2 * x^2}$$

Defintions of rubi rules used

rule 27

$$\text{Int}[(a_*)(Fx_), x_Symbol] \rightarrow \text{Simp}[a \text{ Int}[Fx, x], x] /; \text{FreeQ}[a, x] \ \&\& \ !\text{MatchQ}[Fx, (b_*)(Gx_)] /; \text{FreeQ}[b, x]$$

rule 51

$$\text{Int}[(a_.) + (b_.) * (x_)^m * ((c_.) + (d_.) * (x_))^{n_}, x_Symbol] \rightarrow \text{Simp}[(a + b*x)^{m+1} * ((c + d*x)^n / (b*(m+1))), x] - \text{Simp}[d*(n/(b*(m+1))) \text{Int}[(a + b*x)^{m+1} * (c + d*x)^{n-1}, x], x] /; \text{FreeQ}\{a, b, c, d, n\}, x] \ \&\& \ \text{ILtQ}[m, -1] \ \&\& \ \text{FractionQ}[n] \ \&\& \ \text{GtQ}[n, 0]$$

rule 52

$$\text{Int}[(a_.) + (b_.) * (x_)^m * ((c_.) + (d_.) * (x_))^{n_}, x_Symbol] \rightarrow \text{Simp}[(a + b*x)^{m+1} * ((c + d*x)^{n+1} / ((b*c - a*d)*(m+1))), x] - \text{Simp}[d * ((m + n + 2) / ((b*c - a*d)*(m+1))) \text{Int}[(a + b*x)^{m+1} * (c + d*x)^n, x], x] /; \text{FreeQ}\{a, b, c, d, n\}, x] \ \&\& \ \text{ILtQ}[m, -1] \ \&\& \ \text{FractionQ}[n] \ \&\& \ \text{LtQ}[n, 0]$$

rule 73

$$\text{Int}[(a_.) + (b_.) * (x_)^m * ((c_.) + (d_.) * (x_))^{n_}, x_Symbol] \rightarrow \text{With}\{p = \text{Denominator}[m]\}, \text{Simp}[p/b \text{ Subst}[\text{Int}[x^{p*(m+1)-1} * (c - a*(d/b) + d*(x^p/b))^{n_}, x], x, (a + b*x)^{1/p}], x]] /; \text{FreeQ}\{a, b, c, d\}, x] \ \&\& \ \text{LtQ}[-1, m, 0] \ \&\& \ \text{LeQ}[-1, n, 0] \ \&\& \ \text{LeQ}[\text{Denominator}[n], \text{Denominator}[m]] \ \&\& \ \text{IntLinearQ}[a, b, c, d, m, n, x]$$

rule 221

$$\text{Int}[(a_.) + (b_.) * (x_)^2)^{-1}, x_Symbol] \rightarrow \text{Simp}[(\text{Rt}[-a/b, 2]/a) * \text{ArcTanh}[x / \text{Rt}[-a/b, 2]], x] /; \text{FreeQ}\{a, b\}, x] \ \&\& \ \text{NegQ}[a/b]$$

rule 1102

```
Int[((d_.) + (e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol]
:> Simp[(a + b*x + c*x^2)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x)^(2*FracPart[p]))
Int[(d + e*x)^m*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d, e, m, p}, x]
&& EqQ[b^2 - 4*a*c, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 476 vs. $2(187) = 374$.

Time = 1.53 (sec) , antiderivative size = 477, normalized size of antiderivative = 1.77

| method | result |
|---------|--|
| default | $\frac{(bx+a) \left(3 \arctan\left(\frac{b\sqrt{ex+d}}{\sqrt{b(ae-bd)}}\right) b^4 e^4 x^4 + 12 \arctan\left(\frac{b\sqrt{ex+d}}{\sqrt{b(ae-bd)}}\right) a b^3 e^4 x^3 + 3(ex+d)^{\frac{7}{2}} \sqrt{b(ae-bd)} b^3 + 18 \arctan\left(\frac{b\sqrt{ex+d}}{\sqrt{b(ae-bd)}}\right) a^2 b^2 e^4 \right)}{\dots}$ |

input

```
int((e*x+d)^(3/2)/(b^2*x^2+2*a*b*x+a^2)^(5/2),x,method=_RETURNVERBOSE)
```

output

```
1/64*(b*x+a)*(3*arctan(b*(e*x+d)^(1/2)/(b*(a*e-b*d))^(1/2))*b^4*e^4*x^4+12
*arctan(b*(e*x+d)^(1/2)/(b*(a*e-b*d))^(1/2))*a*b^3*e^4*x^3+3*(e*x+d)^(7/2)
*(b*(a*e-b*d))^(1/2)*b^3+18*arctan(b*(e*x+d)^(1/2)/(b*(a*e-b*d))^(1/2))*a^
2*b^2*e^4*x^2+11*(e*x+d)^(5/2)*(b*(a*e-b*d))^(1/2)*a*b^2*e-11*(e*x+d)^(5/2)
*(b*(a*e-b*d))^(1/2)*b^3*d+12*arctan(b*(e*x+d)^(1/2)/(b*(a*e-b*d))^(1/2))
*a^3*b*e^4*x-11*(e*x+d)^(3/2)*(b*(a*e-b*d))^(1/2)*a^2*b*e^2+22*(e*x+d)^(3/
2)*(b*(a*e-b*d))^(1/2)*a*b^2*d*e-11*(e*x+d)^(3/2)*(b*(a*e-b*d))^(1/2)*b^3*
d^2+3*arctan(b*(e*x+d)^(1/2)/(b*(a*e-b*d))^(1/2))*a^4*e^4-3*(e*x+d)^(1/2)*
(b*(a*e-b*d))^(1/2)*a^3*e^3+9*(e*x+d)^(1/2)*a^2*d*b*e^2*(b*(a*e-b*d))^(1/2)
)-9*(e*x+d)^(1/2)*(b*(a*e-b*d))^(1/2)*a*b^2*d^2*e+3*(e*x+d)^(1/2)*(b*(a*e-
b*d))^(1/2)*b^3*d^3)/(b*(a*e-b*d))^(1/2)/b^2/(a*e-b*d)^2/((b*x+a)^2)^(5/2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 515 vs. $2(187) = 374$.

Time = 0.11 (sec) , antiderivative size = 1043, normalized size of antiderivative = 3.86

$$\int \frac{(d+ex)^{3/2}}{(a^2+2abx+b^2x^2)^{5/2}} dx = \text{Too large to display}$$

input `integrate((e*x+d)^(3/2)/(b^2*x^2+2*a*b*x+a^2)^(5/2),x, algorithm="fricas")`

output `[1/128*(3*(b^4*e^4*x^4 + 4*a*b^3*e^4*x^3 + 6*a^2*b^2*e^4*x^2 + 4*a^3*b*e^4*x + a^4*e^4)*sqrt(b^2*d - a*b*e)*log((b*e*x + 2*b*d - a*e - 2*sqrt(b^2*d - a*b*e)*sqrt(e*x + d))/(b*x + a)) - 2*(16*b^5*d^4 - 40*a*b^4*d^3*e + 26*a^2*b^3*d^2*e^2 + a^3*b^2*d*e^3 - 3*a^4*b*e^4 - 3*(b^5*d*e^3 - a*b^4*e^4)*x^3 + (2*b^5*d^2*e^2 - 13*a*b^4*d*e^3 + 11*a^2*b^3*e^4)*x^2 + (24*b^5*d^3*e - 68*a*b^4*d^2*e^2 + 55*a^2*b^3*d*e^3 - 11*a^3*b^2*e^4)*x)*sqrt(e*x + d))/(a^4*b^6*d^3 - 3*a^5*b^5*d^2*e + 3*a^6*b^4*d*e^2 - a^7*b^3*e^3 + (b^10*d^3 - 3*a*b^9*d^2*e + 3*a^2*b^8*d*e^2 - a^3*b^7*e^3)*x^4 + 4*(a*b^9*d^3 - 3*a^2*b^8*d^2*e + 3*a^3*b^7*d*e^2 - a^4*b^6*e^3)*x^3 + 6*(a^2*b^8*d^3 - 3*a^3*b^7*d^2*e + 3*a^4*b^6*d*e^2 - a^5*b^5*e^3)*x^2 + 4*(a^3*b^7*d^3 - 3*a^4*b^6*d^2*e + 3*a^5*b^5*d*e^2 - a^6*b^4*e^3)*x), 1/64*(3*(b^4*e^4*x^4 + 4*a*b^3*e^4*x^3 + 6*a^2*b^2*e^4*x^2 + 4*a^3*b*e^4*x + a^4*e^4)*sqrt(-b^2*d + a*b*e)*arctan(sqrt(-b^2*d + a*b*e)*sqrt(e*x + d)/(b*e*x + b*d)) - (16*b^5*d^4 - 40*a*b^4*d^3*e + 26*a^2*b^3*d^2*e^2 + a^3*b^2*d*e^3 - 3*a^4*b*e^4 - 3*(b^5*d*e^3 - a*b^4*e^4)*x^3 + (2*b^5*d^2*e^2 - 13*a*b^4*d*e^3 + 11*a^2*b^3*e^4)*x^2 + (24*b^5*d^3*e - 68*a*b^4*d^2*e^2 + 55*a^2*b^3*d*e^3 - 11*a^3*b^2*e^4)*x)*sqrt(e*x + d))/(a^4*b^6*d^3 - 3*a^5*b^5*d^2*e + 3*a^6*b^4*d*e^2 - a^7*b^3*e^3 + (b^10*d^3 - 3*a*b^9*d^2*e + 3*a^2*b^8*d*e^2 - a^3*b^7*e^3)*x^4 + 4*(a*b^9*d^3 - 3*a^2*b^8*d^2*e + 3*a^3*b^7*d*e^2 - a^4*b^6*e^3)*x^3 + 6*(a^2*b^8*d^3 - 3*a^3*b^7*d^2*e + 3*a^4*b^6*d*e^2 - a^5*b^5*e^3)*...`

Sympy [F]

$$\int \frac{(d+ex)^{3/2}}{(a^2+2abx+b^2x^2)^{5/2}} dx = \int \frac{(d+ex)^{3/2}}{((a+bx)^2)^{5/2}} dx$$

input `integrate((e*x+d)**(3/2)/(b**2*x**2+2*a*b*x+a**2)**(5/2),x)`

output `Integral((d + e*x)**(3/2)/((a + b*x)**2)**(5/2), x)`

Maxima [F]

$$\int \frac{(d+ex)^{3/2}}{(a^2+2abx+b^2x^2)^{5/2}} dx = \int \frac{(ex+d)^{3/2}}{(b^2x^2+2abx+a^2)^{5/2}} dx$$

input `integrate((e*x+d)^(3/2)/(b^2*x^2+2*a*b*x+a^2)^(5/2),x, algorithm="maxima")`

output `integrate((e*x + d)^(3/2)/(b^2*x^2 + 2*a*b*x + a^2)^(5/2), x)`

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 321, normalized size of antiderivative = 1.19

$$\int \frac{(d+ex)^{3/2}}{(a^2+2abx+b^2x^2)^{5/2}} dx = \frac{3e^4 \arctan\left(\frac{\sqrt{ex+db}}{\sqrt{-b^2d+abe}}\right)}{64(b^4d^2\operatorname{sgn}(bx+a) - 2ab^3d\operatorname{sgn}(bx+a) + a^2b^2e^2\operatorname{sgn}(bx+a))\sqrt{-b^2d+abe}} + \frac{3(ex+d)^{7/2}b^3e^4 - 11(ex+d)^{5/2}b^3de^4 - 11(ex+d)^{3/2}b^3d^2e^4 + 3\sqrt{ex+db}b^3d^3e^4 + 11(ex+d)^{5/2}ab^2e^5 + 22(ex+d)^{3/2}a^2b^2e^5}{64(b^4d^2\operatorname{sgn}(bx+a) - 2ab^3d\operatorname{sgn}(bx+a) + a^2b^2e^2\operatorname{sgn}(bx+a))\sqrt{-b^2d+abe}}$$

input `integrate((e*x+d)^(3/2)/(b^2*x^2+2*a*b*x+a^2)^(5/2),x, algorithm="giac")`

output

```
3/64*e^4*arctan(sqrt(e*x + d)*b/sqrt(-b^2*d + a*b*e))/((b^4*d^2*sgn(b*x +
a) - 2*a*b^3*d*e*sgn(b*x + a) + a^2*b^2*e^2*sgn(b*x + a))*sqrt(-b^2*d + a*
b*e)) + 1/64*(3*(e*x + d)^(7/2)*b^3*e^4 - 11*(e*x + d)^(5/2)*b^3*d*e^4 - 1
1*(e*x + d)^(3/2)*b^3*d^2*e^4 + 3*sqrt(e*x + d)*b^3*d^3*e^4 + 11*(e*x + d)
^(5/2)*a*b^2*e^5 + 22*(e*x + d)^(3/2)*a*b^2*d*e^5 - 9*sqrt(e*x + d)*a*b^2*
d^2*e^5 - 11*(e*x + d)^(3/2)*a^2*b*e^6 + 9*sqrt(e*x + d)*a^2*b*d*e^6 - 3*s
qrt(e*x + d)*a^3*e^7)/((b^4*d^2*sgn(b*x + a) - 2*a*b^3*d*e*sgn(b*x + a) +
a^2*b^2*e^2*sgn(b*x + a))*((e*x + d)*b - b*d + a*e)^4)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d + ex)^{3/2}}{(a^2 + 2abx + b^2x^2)^{5/2}} dx = \int \frac{(d + ex)^{3/2}}{(a^2 + 2abx + b^2x^2)^{5/2}} dx$$

input

```
int((d + e*x)^(3/2)/(a^2 + b^2*x^2 + 2*a*b*x)^(5/2), x)
```

output

```
int((d + e*x)^(3/2)/(a^2 + b^2*x^2 + 2*a*b*x)^(5/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 742, normalized size of antiderivative = 2.75

$$\int \frac{(d + ex)^{3/2}}{(a^2 + 2abx + b^2x^2)^{5/2}} dx = \frac{3\sqrt{b}\sqrt{ae - bd} \operatorname{atan}\left(\frac{\sqrt{ex+db}}{\sqrt{b}\sqrt{ae-bd}}\right) a^4 e^4 + 12\sqrt{b}\sqrt{ae - bd} \operatorname{atan}\left(\frac{\sqrt{ex+db}}{\sqrt{b}\sqrt{ae-bd}}\right) a^3 b e}{(a^2 + 2abx + b^2x^2)^{5/2}}$$

input

```
int((e*x+d)^(3/2)/(b^2*x^2+2*a*b*x+a^2)^(5/2), x)
```

output

```
(3*sqrt(b)*sqrt(a*e - b*d)*atan((sqrt(d + e*x)*b)/(sqrt(b)*sqrt(a*e - b*d)))*a**4*e**4 + 12*sqrt(b)*sqrt(a*e - b*d)*atan((sqrt(d + e*x)*b)/(sqrt(b)*sqrt(a*e - b*d)))*a**3*b*e**4*x + 18*sqrt(b)*sqrt(a*e - b*d)*atan((sqrt(d + e*x)*b)/(sqrt(b)*sqrt(a*e - b*d)))*a**2*b**2*e**4*x**2 + 12*sqrt(b)*sqrt(a*e - b*d)*atan((sqrt(d + e*x)*b)/(sqrt(b)*sqrt(a*e - b*d)))*a*b**3*e**4*x**3 + 3*sqrt(b)*sqrt(a*e - b*d)*atan((sqrt(d + e*x)*b)/(sqrt(b)*sqrt(a*e - b*d)))*b**4*e**4*x**4 - 3*sqrt(d + e*x)*a**4*b*e**4 + sqrt(d + e*x)*a**3*b**2*d*e**3 - 11*sqrt(d + e*x)*a**3*b**2*e**4*x + 26*sqrt(d + e*x)*a**2*b**3*d**2*e**2 + 55*sqrt(d + e*x)*a**2*b**3*d*e**3*x + 11*sqrt(d + e*x)*a**2*b**3*e**4*x**2 - 40*sqrt(d + e*x)*a*b**4*d**3*e - 68*sqrt(d + e*x)*a*b**4*d**2*e**2*x - 13*sqrt(d + e*x)*a*b**4*d*e**3*x**2 + 3*sqrt(d + e*x)*a*b**4*e**4*x**3 + 16*sqrt(d + e*x)*b**5*d**4 + 24*sqrt(d + e*x)*b**5*d**3*e*x + 2*sqrt(d + e*x)*b**5*d**2*e**2*x**2 - 3*sqrt(d + e*x)*b**5*d*e**3*x**3)/(64*b**3*(a**7*e**3 - 3*a**6*b*d*e**2 + 4*a**6*b*e**3*x + 3*a**5*b**2*d**2*e - 12*a**5*b**2*d*e**2*x + 6*a**5*b**2*e**3*x**2 - a**4*b**3*d**3 + 12*a**4*b**3*d**2*e*x - 18*a**4*b**3*d*e**2*x**2 + 4*a**4*b**3*e**3*x**3 - 4*a**3*b**4*d**3*x + 18*a**3*b**4*d**2*e*x**2 - 12*a**3*b**4*d*e**2*x**3 + a**3*b**4*e**3*x**4 - 6*a**2*b**5*d**3*x**2 + 12*a**2*b**5*d**2*e*x**3 - 3*a**2*b**5*d*e**2*x**4 - 4*a*b**6*d**3*x**3 + 3*a*b**6*d**2*e*x**4 - b**7*d**3*x**4))
```

3.394 $\int \frac{\sqrt{d+ex}}{(a^2+2abx+b^2x^2)^{5/2}} dx$

| | |
|---|------|
| Optimal result | 2950 |
| Mathematica [A] (verified) | 2951 |
| Rubi [A] (verified) | 2951 |
| Maple [B] (verified) | 2955 |
| Fricas [B] (verification not implemented) | 2956 |
| Sympy [F] | 2957 |
| Maxima [F] | 2957 |
| Giac [A] (verification not implemented) | 2957 |
| Mupad [F(-1)] | 2958 |
| Reduce [B] (verification not implemented) | 2958 |

Optimal result

Integrand size = 30, antiderivative size = 280

$$\int \frac{\sqrt{d+ex}}{(a^2+2abx+b^2x^2)^{5/2}} dx = -\frac{5e^3\sqrt{d+ex}}{64b(bd-ae)^3\sqrt{a^2+2abx+b^2x^2}} - \frac{\sqrt{d+ex}}{4b(a+bx)^3\sqrt{a^2+2abx+b^2x^2}} - \frac{e\sqrt{d+ex}}{24b(bd-ae)(a+bx)^2\sqrt{a^2+2abx+b^2x^2}} + \frac{5e^2\sqrt{d+ex}}{96b(bd-ae)^2(a+bx)\sqrt{a^2+2abx+b^2x^2}} + \frac{5e^4(a+bx)\operatorname{arctanh}\left(\frac{\sqrt{b}\sqrt{d+ex}}{\sqrt{bd-ae}}\right)}{64b^{3/2}(bd-ae)^{7/2}\sqrt{a^2+2abx+b^2x^2}}$$

output

```
-5/64*e^3*(e*x+d)^(1/2)/b/(-a*e+b*d)^3/((b*x+a)^2)^(1/2)-1/4*(e*x+d)^(1/2)
/b/(b*x+a)^3/((b*x+a)^2)^(1/2)-1/24*e*(e*x+d)^(1/2)/b/(-a*e+b*d)/(b*x+a)^2
/((b*x+a)^2)^(1/2)+5/96*e^2*(e*x+d)^(1/2)/b/(-a*e+b*d)^2/(b*x+a)/((b*x+a)
2)^(1/2)+5/64*e^4*(b*x+a)*arctanh(b^(1/2)*(e*x+d)^(1/2)/(-a*e+b*d)^(1/2))/
b^(3/2)/(-a*e+b*d)^(7/2)/((b*x+a)^2)^(1/2)
```

Mathematica [A] (verified)

Time = 1.05 (sec) , antiderivative size = 192, normalized size of antiderivative = 0.69

$$\int \frac{\sqrt{d+ex}}{(a^2+2abx+b^2x^2)^{5/2}} dx = \frac{e^4(a+bx)^5 \left(\frac{\sqrt{b}\sqrt{d+ex}(-15a^3e^3+a^2be^2(118d+73ex)+ab^2e(-136d^2-36dex+55e^2x^2)+b^3(48d^3+8d^2ex-10d^2e^2x^2+15e^3x^3))}{e^4(-bd+ae)^3(a+bx)^4} + (15\text{ArcTan}[\frac{\sqrt{b}\sqrt{d+ex}}{\sqrt{-(b*d)+a*e}}]) / (-(b*d)+a*e)^{(7/2)} \right)}{192b^{3/2}((a+bx)^2)^{5/2}}$$

input `Integrate[Sqrt[d + e*x]/(a^2 + 2*a*b*x + b^2*x^2)^(5/2), x]`

output `(e^4*(a + b*x)^5*((Sqrt[b]*Sqrt[d + e*x]*(-15*a^3*e^3 + a^2*b*e^2*(118*d + 73*e*x) + a*b^2*e*(-136*d^2 - 36*d*e*x + 55*e^2*x^2) + b^3*(48*d^3 + 8*d^2*e*x - 10*d^2*e^2*x^2 + 15*e^3*x^3)))/(e^4*(-(b*d) + a*e)^3*(a + b*x)^4) + (15*ArcTan[(Sqrt[b]*Sqrt[d + e*x])/Sqrt[-(b*d) + a*e]])/(-(b*d) + a*e)^(7/2)))/(192*b^(3/2)*((a + b*x)^2)^(5/2))`

Rubi [A] (verified)

Time = 0.54 (sec) , antiderivative size = 226, normalized size of antiderivative = 0.81, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$, Rules used = {1102, 27, 51, 52, 52, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\begin{aligned} & \int \frac{\sqrt{d+ex}}{(a^2+2abx+b^2x^2)^{5/2}} dx \\ & \quad \downarrow \text{1102} \\ & \frac{b^5(a+bx) \int \frac{\sqrt{d+ex}}{b^5(a+bx)^5} dx}{\sqrt{a^2+2abx+b^2x^2}} \\ & \quad \downarrow \text{27} \\ & \frac{(a+bx) \int \frac{\sqrt{d+ex}}{(a+bx)^5} dx}{\sqrt{a^2+2abx+b^2x^2}} \\ & \quad \downarrow \text{51} \end{aligned}$$

$$\begin{aligned}
 & \frac{(a+bx) \left(\frac{e \int \frac{1}{(a+bx)^4 \sqrt{d+ex}} dx}{8b} - \frac{\sqrt{d+ex}}{4b(a+bx)^4} \right)}{\sqrt{a^2 + 2abx + b^2x^2}} \\
 & \quad \downarrow 52 \\
 & \frac{(a+bx) \left(\frac{e \left(-\frac{5e \int \frac{1}{(a+bx)^3 \sqrt{d+ex}} dx}{6(bd-ae)} - \frac{\sqrt{d+ex}}{3(a+bx)^3(bd-ae)} \right)}{8b} - \frac{\sqrt{d+ex}}{4b(a+bx)^4} \right)}{\sqrt{a^2 + 2abx + b^2x^2}} \\
 & \quad \downarrow 52 \\
 & \frac{(a+bx) \left(\frac{e \left(-\frac{5e \left(-\frac{3e \int \frac{1}{(a+bx)^2 \sqrt{d+ex}} dx}{4(bd-ae)} - \frac{\sqrt{d+ex}}{2(a+bx)^2(bd-ae)} \right)}{6(bd-ae)} - \frac{\sqrt{d+ex}}{3(a+bx)^3(bd-ae)} \right)}{8b} - \frac{\sqrt{d+ex}}{4b(a+bx)^4} \right)}{\sqrt{a^2 + 2abx + b^2x^2}} \\
 & \quad \downarrow 52 \\
 & \frac{(a+bx) \left(\frac{e \left(\frac{5e \left(\frac{3e \left(-\frac{e \int \frac{1}{(a+bx) \sqrt{d+ex}} dx}{2(bd-ae)} - \frac{\sqrt{d+ex}}{(a+bx)(bd-ae)} \right)}{4(bd-ae)} - \frac{\sqrt{d+ex}}{2(a+bx)^2(bd-ae)} \right)}{6(bd-ae)} - \frac{\sqrt{d+ex}}{3(a+bx)^3(bd-ae)} \right)}{8b} - \frac{\sqrt{d+ex}}{4b(a+bx)^4} \right)}{\sqrt{a^2 + 2abx + b^2x^2}} \\
 & \quad \downarrow 73
 \end{aligned}$$

$$\left(\begin{array}{l} (a+bx) \\ e \\ 5e \end{array} \left(\begin{array}{l} \int \frac{1}{a + \frac{b(d+ex) - bd}{e}} d\sqrt{d+ex} \\ - \frac{bd - ae}{(a+bx)(bd-ae)} \\ - \frac{\sqrt{d+ex}}{2(a+bx)^2(bd-ae)} \end{array} \right) \right) - \frac{\sqrt{d+ex}}{3(a+bx)^3(bd-ae)} - \frac{\sqrt{d+ex}}{4b(a+bx)^4}$$

$$\sqrt{a^2 + 2abx + b^2x^2}$$

↓ 221

$$\left(\begin{array}{l} (a+bx) \\ e \\ 5e \end{array} \left(\begin{array}{l} e \operatorname{arctanh} \left(\frac{\sqrt{b}\sqrt{d+ex}}{\sqrt{bd-ae}} \right) \\ - \frac{\sqrt{bd-ae}}{(a+bx)(bd-ae)} \\ - \frac{\sqrt{d+ex}}{2(a+bx)^2(bd-ae)} \end{array} \right) \right) - \frac{\sqrt{d+ex}}{3(a+bx)^3(bd-ae)} - \frac{\sqrt{d+ex}}{4b(a+bx)^4}$$

$$\sqrt{a^2 + 2abx + b^2x^2}$$

input `Int[Sqrt[d + e*x]/(a^2 + 2*a*b*x + b^2*x^2)^(5/2),x]`

output `((a + b*x)*(-1/4*Sqrt[d + e*x]/(b*(a + b*x)^4) + (e*(-1/3*Sqrt[d + e*x]/((b*d - a*e)*(a + b*x)^3) - (5*e*(-1/2*Sqrt[d + e*x]/((b*d - a*e)*(a + b*x)^2) - (3*e*(-(Sqrt[d + e*x]/((b*d - a*e)*(a + b*x)))) + (e*ArcTanh[(Sqrt[b]*Sqrt[d + e*x])/Sqrt[b*d - a*e]])/(Sqrt[b]*(b*d - a*e)^(3/2)))))/(4*(b*d - a*e))))/(6*(b*d - a*e)))/(8*b))/Sqrt[a^2 + 2*a*b*x + b^2*x^2]`

Defintions of rubi rules used

rule 27 `Int[(a_)*(F_x_), x_Symbol] := Simp[a Int[F_x, x], x] /; FreeQ[a, x] && !MatchQ[F_x, (b_)*(G_x_) /; FreeQ[b, x]]`

rule 51 `Int[((a_.) + (b_.)*(x_)^(m_))*((c_.) + (d_.)*(x_)^(n_)), x_Symbol] := Simp[(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Simp[d*(n/(b*(m + 1))) Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d, n}, x] && ILtQ[m, -1] && FractionQ[n] && GtQ[n, 0]`

rule 52 `Int[((a_.) + (b_.)*(x_)^(m_))*((c_.) + (d_.)*(x_)^(n_)), x_Symbol] := Simp[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((m + n + 2)/((b*c - a*d)*(m + 1))) Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && ILtQ[m, -1] && FractionQ[n] && LtQ[n, 0]`

rule 73 `Int[((a_.) + (b_.)*(x_)^(m_))*((c_.) + (d_.)*(x_)^(n_)), x_Symbol] := With[{p = Denominator[m]}, Simp[p/b Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) + d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x] /; FreeQ[{a, b, c, d}, x] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntegerQ[a, b, c, d, m, n, x]`

rule 221 `Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[-a/b, 2]/a)*ArcTanh[x/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]`

rule 1102

```
Int[((d_.) + (e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] := Simp[(a + b*x + c*x^2)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x)^(2*F
racPart[p])) Int[(d + e*x)^m*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c,
d, e, m, p}, x] && EqQ[b^2 - 4*a*c, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 499 vs. $2(197) = 394$.

Time = 1.58 (sec) , antiderivative size = 500, normalized size of antiderivative = 1.79

| method | result |
|---------|---|
| default | $\frac{(bx+a) \left(15 \arctan\left(\frac{b\sqrt{ex+d}}{\sqrt{b(ae-bd)}}\right) b^4 e^4 x^4 + 60 \arctan\left(\frac{b\sqrt{ex+d}}{\sqrt{b(ae-bd)}}\right) a b^3 e^4 x^3 + 15 (ex+d)^{\frac{7}{2}} \sqrt{b(ae-bd)} b^3 + 90 \arctan\left(\frac{b\sqrt{ex+d}}{\sqrt{b(ae-bd)}}\right) a^2 b^2 \right)}{\dots}$ |

input

```
int((e*x+d)^(1/2)/(b^2*x^2+2*a*b*x+a^2)^(5/2),x,method=_RETURNVERBOSE)
```

output

```
1/192*(b*x+a)*(15*arctan(b*(e*x+d)^(1/2)/(b*(a*e-b*d))^(1/2))*b^4*e^4*x^4+
60*arctan(b*(e*x+d)^(1/2)/(b*(a*e-b*d))^(1/2))*a*b^3*e^4*x^3+15*(e*x+d)^(7
/2)*(b*(a*e-b*d))^(1/2)*b^3+90*arctan(b*(e*x+d)^(1/2)/(b*(a*e-b*d))^(1/2))
*a^2*b^2*e^4*x^2+55*(e*x+d)^(5/2)*(b*(a*e-b*d))^(1/2)*a*b^2*e-55*(e*x+d)^(
5/2)*(b*(a*e-b*d))^(1/2)*b^3*d+60*arctan(b*(e*x+d)^(1/2)/(b*(a*e-b*d))^(1/
2))*a^3*b*e^4*x+73*(e*x+d)^(3/2)*(b*(a*e-b*d))^(1/2)*a^2*b*e^2-146*(e*x+d)
^(3/2)*(b*(a*e-b*d))^(1/2)*a*b^2*d*e+73*(e*x+d)^(3/2)*(b*(a*e-b*d))^(1/2)*
b^3*d^2+15*arctan(b*(e*x+d)^(1/2)/(b*(a*e-b*d))^(1/2))*a^4*e^4-15*(e*x+d)^(
1/2)*(b*(a*e-b*d))^(1/2)*a^3*e^3+45*(e*x+d)^(1/2)*a^2*d*b*e^2*(b*(a*e-b*d
))^(1/2)-45*(e*x+d)^(1/2)*(b*(a*e-b*d))^(1/2)*a*b^2*d^2*e+15*(e*x+d)^(1/2)
*(b*(a*e-b*d))^(1/2)*b^3*d^3)/(b*(a*e-b*d))^(1/2)/b/(a*e-b*d)/(a^2*e^2-2*a
*b*d*e+b^2*d^2)/((b*x+a)^2)^(5/2)
```


Sympy [F]

$$\int \frac{\sqrt{d+ex}}{(a^2+2abx+b^2x^2)^{5/2}} dx = \int \frac{\sqrt{d+ex}}{((a+bx)^2)^{5/2}} dx$$

input `integrate((e*x+d)**(1/2)/(b**2*x**2+2*a*b*x+a**2)**(5/2),x)`

output `Integral(sqrt(d + e*x)/((a + b*x)**2)**(5/2), x)`

Maxima [F]

$$\int \frac{\sqrt{d+ex}}{(a^2+2abx+b^2x^2)^{5/2}} dx = \int \frac{\sqrt{ex+d}}{(b^2x^2+2abx+a^2)^{5/2}} dx$$

input `integrate((e*x+d)^(1/2)/(b^2*x^2+2*a*b*x+a^2)^(5/2),x, algorithm="maxima")`

output `integrate(sqrt(e*x + d)/(b^2*x^2 + 2*a*b*x + a^2)^(5/2), x)`

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 359, normalized size of antiderivative = 1.28

$$\int \frac{\sqrt{d+ex}}{(a^2+2abx+b^2x^2)^{5/2}} dx =$$

$$5e^4 \arctan\left(\frac{\sqrt{ex+db}}{\sqrt{-b^2d+abe}}\right)$$

$$\frac{64(b^4d^3\operatorname{sgn}(bx+a) - 3ab^3d^2\operatorname{esgn}(bx+a) + 3a^2b^2de^2\operatorname{sgn}(bx+a) - a^3be^3\operatorname{sgn}(bx+a))\sqrt{-b^2d+abe} - 15(ex+d)^{7/2}b^3e^4 - 55(ex+d)^{5/2}b^3de^4 + 73(ex+d)^{3/2}b^3d^2e^4 + 15\sqrt{ex+db}d^3e^4 + 55(ex+d)^{5/2}ab^2e^5 - 1}{192(b^4d^3\operatorname{sgn}(bx+a) - 3ab^3d^2\operatorname{esgn}(bx+a) + 3a^2b^2de^2\operatorname{sgn}(bx+a))\sqrt{-b^2d+abe}}$$

input `integrate((e*x+d)^(1/2)/(b^2*x^2+2*a*b*x+a^2)^(5/2),x, algorithm="giac")`

output

```
-5/64*e^4*arctan(sqrt(e*x + d)*b/sqrt(-b^2*d + a*b*e))/((b^4*d^3*sgn(b*x +
a) - 3*a*b^3*d^2*e*sgn(b*x + a) + 3*a^2*b^2*d*e^2*sgn(b*x + a) - a^3*b*e^
3*sgn(b*x + a))*sqrt(-b^2*d + a*b*e)) - 1/192*(15*(e*x + d)^(7/2)*b^3*e^4
- 55*(e*x + d)^(5/2)*b^3*d*e^4 + 73*(e*x + d)^(3/2)*b^3*d^2*e^4 + 15*sqrt(
e*x + d)*b^3*d^3*e^4 + 55*(e*x + d)^(5/2)*a*b^2*e^5 - 146*(e*x + d)^(3/2)*
a*b^2*d*e^5 - 45*sqrt(e*x + d)*a*b^2*d^2*e^5 + 73*(e*x + d)^(3/2)*a^2*b*e^
6 + 45*sqrt(e*x + d)*a^2*b*d*e^6 - 15*sqrt(e*x + d)*a^3*e^7)/((b^4*d^3*sgn
(b*x + a) - 3*a*b^3*d^2*e*sgn(b*x + a) + 3*a^2*b^2*d*e^2*sgn(b*x + a) - a^
3*b*e^3*sgn(b*x + a))*((e*x + d)*b - b*d + a*e)^4)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{d+ex}}{(a^2+2abx+b^2x^2)^{5/2}} dx = \int \frac{\sqrt{d+ex}}{(a^2+2abx+b^2x^2)^{5/2}} dx$$

input

```
int((d + e*x)^(1/2)/(a^2 + b^2*x^2 + 2*a*b*x)^(5/2), x)
```

output

```
int((d + e*x)^(1/2)/(a^2 + b^2*x^2 + 2*a*b*x)^(5/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 821, normalized size of antiderivative = 2.93

$$\int \frac{\sqrt{d+ex}}{(a^2+2abx+b^2x^2)^{5/2}} dx = \frac{15\sqrt{b}\sqrt{ae-bd} \operatorname{atan}\left(\frac{\sqrt{ex+db}}{\sqrt{b}\sqrt{ae-bd}}\right) a^4 e^4 + 60\sqrt{b}\sqrt{ae-bd} \operatorname{atan}\left(\frac{\sqrt{ex+db}}{\sqrt{b}\sqrt{ae-bd}}\right) a^3 b}{a^4 e^4 + 60\sqrt{b}\sqrt{ae-bd} \operatorname{atan}\left(\frac{\sqrt{ex+db}}{\sqrt{b}\sqrt{ae-bd}}\right) a^3 b}$$

input

```
int((e*x+d)^(1/2)/(b^2*x^2+2*a*b*x+a^2)^(5/2), x)
```

output

```
(15*sqrt(b)*sqrt(a*e - b*d)*atan((sqrt(d + e*x)*b)/(sqrt(b)*sqrt(a*e - b*d
)))**4**e**4 + 60*sqrt(b)*sqrt(a*e - b*d)*atan((sqrt(d + e*x)*b)/(sqrt(b)
*sqrt(a*e - b*d)))*a**3*b**e**4*x + 90*sqrt(b)*sqrt(a*e - b*d)*atan((sqrt(d
+ e*x)*b)/(sqrt(b)*sqrt(a*e - b*d)))*a**2*b**2**e**4*x**2 + 60*sqrt(b)*sqr
t(a*e - b*d)*atan((sqrt(d + e*x)*b)/(sqrt(b)*sqrt(a*e - b*d)))*a*b**3**e**4
*x**3 + 15*sqrt(b)*sqrt(a*e - b*d)*atan((sqrt(d + e*x)*b)/(sqrt(b)*sqrt(a*
e - b*d)))*b**4**e**4*x**4 - 15*sqrt(d + e*x)*a**4*b**e**4 + 133*sqrt(d + e*
x)*a**3*b**2*d**e**3 + 73*sqrt(d + e*x)*a**3*b**2**e**4*x - 254*sqrt(d + e*x
)*a**2*b**3*d**2**e**2 - 109*sqrt(d + e*x)*a**2*b**3*d**e**3*x + 55*sqrt(d +
e*x)*a**2*b**3**e**4*x**2 + 184*sqrt(d + e*x)*a*b**4*d**3*e + 44*sqrt(d +
e*x)*a*b**4*d**2**e**2*x - 65*sqrt(d + e*x)*a*b**4*d**e**3*x**2 + 15*sqrt(d
+ e*x)*a*b**4**e**4*x**3 - 48*sqrt(d + e*x)*b**5*d**4 - 8*sqrt(d + e*x)*b**
5*d**3**e*x + 10*sqrt(d + e*x)*b**5*d**2**e**2*x**2 - 15*sqrt(d + e*x)*b**5*
d**e**3*x**3)/(192*b**2*(a**8**e**4 - 4*a**7*b*d**e**3 + 4*a**7*b**e**4*x + 6*
a**6*b**2*d**2**e**2 - 16*a**6*b**2*d**e**3*x + 6*a**6*b**2**e**4*x**2 - 4*a*
**5*b**3*d**3*e + 24*a**5*b**3*d**2**e**2*x - 24*a**5*b**3*d**e**3*x**2 + 4*a
**5*b**3**e**4*x**3 + a**4*b**4*d**4 - 16*a**4*b**4*d**3**e*x + 36*a**4*b**4
*d**2**e**2*x**2 - 16*a**4*b**4*d**e**3*x**3 + a**4*b**4**e**4*x**4 + 4*a**3*
b**5*d**4*x - 24*a**3*b**5*d**3**e*x**2 + 24*a**3*b**5*d**2**e**2*x**3 - 4*a
**3*b**5*d**e**3*x**4 + 6*a**2*b**6*d**4*x**2 - 16*a**2*b**6*d**3**e*x**3...
```

3.395 $\int \frac{1}{\sqrt{d+ex}(a^2+2abx+b^2x^2)^{5/2}} dx$

| | |
|---|------|
| Optimal result | 2960 |
| Mathematica [A] (verified) | 2961 |
| Rubi [A] (verified) | 2961 |
| Maple [B] (verified) | 2965 |
| Fricas [B] (verification not implemented) | 2965 |
| Sympy [F] | 2966 |
| Maxima [F] | 2967 |
| Giac [B] (verification not implemented) | 2967 |
| Mupad [F(-1)] | 2968 |
| Reduce [B] (verification not implemented) | 2968 |

Optimal result

Integrand size = 30, antiderivative size = 278

$$\int \frac{1}{\sqrt{d+ex}(a^2+2abx+b^2x^2)^{5/2}} dx = \frac{35e^3\sqrt{d+ex}}{64(bd-ae)^4\sqrt{a^2+2abx+b^2x^2}} - \frac{\sqrt{d+ex}}{4(bd-ae)(a+bx)^3\sqrt{a^2+2abx+b^2x^2}} + \frac{7e\sqrt{d+ex}}{24(bd-ae)^2(a+bx)^2\sqrt{a^2+2abx+b^2x^2}} - \frac{35e^2\sqrt{d+ex}}{96(bd-ae)^3(a+bx)\sqrt{a^2+2abx+b^2x^2}} - \frac{35e^4(a+bx)\operatorname{arctanh}\left(\frac{\sqrt{b}\sqrt{d+ex}}{\sqrt{bd-ae}}\right)}{64\sqrt{b}(bd-ae)^{9/2}\sqrt{a^2+2abx+b^2x^2}}$$

output

```
35/64*e^3*(e*x+d)^(1/2)/(-a*e+b*d)^4/((b*x+a)^2)^(1/2)-1/4*(e*x+d)^(1/2)/(-a*e+b*d)/(b*x+a)^3/((b*x+a)^2)^(1/2)+7/24*e*(e*x+d)^(1/2)/(-a*e+b*d)^2/(b*x+a)^2/((b*x+a)^2)^(1/2)-35/96*e^2*(e*x+d)^(1/2)/(-a*e+b*d)^3/(b*x+a)/((b*x+a)^2)^(1/2)-35/64*e^4*(b*x+a)*arctanh(b^(1/2)*(e*x+d)^(1/2)/(-a*e+b*d)^(1/2))/b^(1/2)/(-a*e+b*d)^(9/2)/((b*x+a)^2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.57 (sec) , antiderivative size = 187, normalized size of antiderivative = 0.67

$$\int \frac{1}{\sqrt{d+ex}(a^2+2abx+b^2x^2)^{5/2}} dx = \frac{e^4(a+bx)^5 \left(\frac{\sqrt{d+ex}(279a^3e^3+a^2be^2(-326d+511ex)+ab^2e(200d^2-252dex+385e^2x^2)}{e^4(bd-ae)^4(a+bx)^4} \right)}{192((a+bx)^2)}$$

input `Integrate[1/(Sqrt[d + e*x]*(a^2 + 2*a*b*x + b^2*x^2)^(5/2)), x]`

output `(e^4*(a + b*x)^5*((Sqrt[d + e*x]*(279*a^3*e^3 + a^2*b*e^2*(-326*d + 511*e*x) + a*b^2*e*(200*d^2 - 252*d*e*x + 385*e^2*x^2) + b^3*(-48*d^3 + 56*d^2*e*x - 70*d*e^2*x^2 + 105*e^3*x^3)))/(e^4*(b*d - a*e)^4*(a + b*x)^4) + (105*ArcTan[(Sqrt[b]*Sqrt[d + e*x])/Sqrt[-(b*d) + a*e]])/(Sqrt[b]*(-(b*d) + a*e)^(9/2)))/(192*((a + b*x)^2)^(5/2))`

Rubi [A] (verified)

Time = 0.57 (sec) , antiderivative size = 240, normalized size of antiderivative = 0.86, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$, Rules used = {1102, 27, 52, 52, 52, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\begin{aligned} & \int \frac{1}{(a^2 + 2abx + b^2x^2)^{5/2} \sqrt{d+ex}} dx \\ & \quad \downarrow \text{1102} \\ & \frac{b^5(a+bx) \int \frac{1}{b^5(a+bx)^5 \sqrt{d+ex}} dx}{\sqrt{a^2 + 2abx + b^2x^2}} \\ & \quad \downarrow \text{27} \\ & \frac{(a+bx) \int \frac{1}{(a+bx)^5 \sqrt{d+ex}} dx}{\sqrt{a^2 + 2abx + b^2x^2}} \\ & \quad \downarrow \text{52} \end{aligned}$$

$$\begin{aligned}
 & \frac{(a+bx) \left(-\frac{7e \int \frac{1}{(a+bx)^4 \sqrt{d+ex}} dx}{8(bd-ae)} - \frac{\sqrt{d+ex}}{4(a+bx)^4(bd-ae)} \right)}{\sqrt{a^2 + 2abx + b^2x^2}} \\
 & \quad \downarrow 52 \\
 & \frac{(a+bx) \left(-\frac{7e \left(-\frac{5e \int \frac{1}{(a+bx)^3 \sqrt{d+ex}} dx}{6(bd-ae)} - \frac{\sqrt{d+ex}}{3(a+bx)^3(bd-ae)} \right)}{8(bd-ae)} - \frac{\sqrt{d+ex}}{4(a+bx)^4(bd-ae)} \right)}{\sqrt{a^2 + 2abx + b^2x^2}} \\
 & \quad \downarrow 52 \\
 & \frac{(a+bx) \left(-\frac{7e \left(\frac{5e \left(-\frac{3e \int \frac{1}{(a+bx)^2 \sqrt{d+ex}} dx}{4(bd-ae)} - \frac{\sqrt{d+ex}}{2(a+bx)^2(bd-ae)} \right)}{6(bd-ae)} - \frac{\sqrt{d+ex}}{3(a+bx)^3(bd-ae)} \right)}{8(bd-ae)} - \frac{\sqrt{d+ex}}{4(a+bx)^4(bd-ae)} \right)}{\sqrt{a^2 + 2abx + b^2x^2}} \\
 & \quad \downarrow 52 \\
 & \frac{(a+bx) \left(-\frac{7e \left(\frac{5e \left(\frac{3e \left(-\frac{e \int \frac{1}{(a+bx) \sqrt{d+ex}} dx}{2(bd-ae)} - \frac{\sqrt{d+ex}}{(a+bx)(bd-ae)} \right)}{4(bd-ae)} - \frac{\sqrt{d+ex}}{2(a+bx)^2(bd-ae)} \right)}{6(bd-ae)} - \frac{\sqrt{d+ex}}{3(a+bx)^3(bd-ae)} \right)}{8(bd-ae)} - \frac{\sqrt{d+ex}}{4(a+bx)^4(bd-ae)} \right)}{\sqrt{a^2 + 2abx + b^2x^2}} \\
 & \quad \downarrow 73
 \end{aligned}$$

$$\left((a+bx) \left[\frac{5e \left(\frac{3e \left(\frac{\int \frac{1}{a + \frac{b(d+ex) - bd}{e} d\sqrt{d+ex}}{bd-ae} - \frac{\sqrt{d+ex}}{(a+bx)(bd-ae)} \right)}{4(bd-ae)} - \frac{\sqrt{d+ex}}{2(a+bx)^2(bd-ae)} \right)}{6(bd-ae)} - \frac{\sqrt{d+ex}}{3(a+bx)^3(bd-ae)} \right] - \frac{\sqrt{d+ex}}{4(a+bx)^4(bd-ae)} \right)$$

$$\sqrt{a^2 + 2abx + b^2x^2}$$

↓ 221

$$\left((a+bx) \left[\frac{5e \left(\frac{3e \left(\frac{e \operatorname{arctanh} \left(\frac{\sqrt{b}\sqrt{d+ex}}{\sqrt{bd-ae}} \right)}{\sqrt{b}(bd-ae)^{3/2}} - \frac{\sqrt{d+ex}}{(a+bx)(bd-ae)} \right)}{4(bd-ae)} - \frac{\sqrt{d+ex}}{2(a+bx)^2(bd-ae)} \right)}{6(bd-ae)} - \frac{\sqrt{d+ex}}{3(a+bx)^3(bd-ae)} \right] - \frac{\sqrt{d+ex}}{4(a+bx)^4(bd-ae)} \right)$$

$$\sqrt{a^2 + 2abx + b^2x^2}$$

input `Int[1/(Sqrt[d + e*x]*(a^2 + 2*a*b*x + b^2*x^2)^(5/2)),x]`

output `((a + b*x)*(-1/4*Sqrt[d + e*x]/((b*d - a*e)*(a + b*x)^4) - (7*e*(-1/3*Sqrt[d + e*x]/((b*d - a*e)*(a + b*x)^3) - (5*e*(-1/2*Sqrt[d + e*x]/((b*d - a*e)*(a + b*x)^2) - (3*e*(-(Sqrt[d + e*x]/((b*d - a*e)*(a + b*x)))) + (e*ArcTanh[(Sqrt[b]*Sqrt[d + e*x])/Sqrt[b*d - a*e]])/(Sqrt[b]*(b*d - a*e)^(3/2)))))/(4*(b*d - a*e))))/(6*(b*d - a*e)))/(8*(b*d - a*e)))/Sqrt[a^2 + 2*a*b*x + b^2*x^2]`

Defintions of rubi rules used

rule 27 `Int[(a_)*(F_x_), x_Symbol] := Simp[a Int[F_x, x], x] /; FreeQ[a, x] && !MatchQ[F_x, (b_)*(G_x_) /; FreeQ[b, x]]`

rule 52 `Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] := Simp[(a + b*x)^(m + 1)*(c + d*x)^(n + 1)/((b*c - a*d)*(m + 1)), x] - Simp[d*((m + n + 2)/((b*c - a*d)*(m + 1)) Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && ILtQ[m, -1] && FractionQ[n] && LtQ[n, 0]`

rule 73 `Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] := With[{p = Denominator[m]}, Simp[p/b Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) + d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntLinearQ[a, b, c, d, m, n, x]`

rule 221 `Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[-a/b, 2]/a)*ArcTanh[x/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]`

rule 1102 `Int[((d_.) + (e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] := Simp[(a + b*x + c*x^2)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x)^(2*FracPart[p])) Int[(d + e*x)^m*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && EqQ[b^2 - 4*a*c, 0]`

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 496 vs. $2(195) = 390$.

Time = 1.52 (sec) , antiderivative size = 497, normalized size of antiderivative = 1.79

| method | result |
|---------|---|
| default | $\left(105 \arctan\left(\frac{b\sqrt{ex+d}}{\sqrt{b(ae-bd)}}\right) b^4 e^4 x^4 + 420 \arctan\left(\frac{b\sqrt{ex+d}}{\sqrt{b(ae-bd)}}\right) a b^3 e^4 x^3 + 105 b^3 e^3 x^3 \sqrt{b(ae-bd)} \sqrt{ex+d} + 630 \arctan\left(\frac{b\sqrt{ex+d}}{\sqrt{b(ae-bd)}}\right) a^2 b\right)$ |

input `int(1/(e*x+d)^(1/2)/(b^2*x^2+2*a*b*x+a^2)^(5/2),x,method=_RETURNVERBOSE)`

output
$$\frac{1}{192} \left(105 \arctan\left(\frac{b\sqrt{ex+d}}{\sqrt{b(ae-bd)}}\right) b^4 e^4 x^4 + 420 \arctan\left(\frac{b\sqrt{ex+d}}{\sqrt{b(ae-bd)}}\right) a b^3 e^4 x^3 + 105 b^3 e^3 x^3 \sqrt{b(ae-bd)} \sqrt{ex+d} + 630 \arctan\left(\frac{b\sqrt{ex+d}}{\sqrt{b(ae-bd)}}\right) a^2 b \right)$$

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 656 vs. $2(195) = 390$.

Time = 0.14 (sec) , antiderivative size = 1325, normalized size of antiderivative = 4.77

$$\int \frac{1}{\sqrt{d+ex} (a^2+2abx+b^2x^2)^{5/2}} dx = \text{Too large to display}$$

input `integrate(1/(e*x+d)^(1/2)/(b^2*x^2+2*a*b*x+a^2)^(5/2),x, algorithm="fricas")`

output

```
[1/384*(105*(b^4*e^4*x^4 + 4*a*b^3*e^4*x^3 + 6*a^2*b^2*e^4*x^2 + 4*a^3*b*e^4*x + a^4*e^4)*sqrt(b^2*d - a*b*e)*log((b*e*x + 2*b*d - a*e - 2*sqrt(b^2*d - a*b*e)*sqrt(e*x + d))/(b*x + a)) - 2*(48*b^5*d^4 - 248*a*b^4*d^3*e + 526*a^2*b^3*d^2*e^2 - 605*a^3*b^2*d*e^3 + 279*a^4*b*e^4 - 105*(b^5*d*e^3 - a*b^4*e^4)*x^3 + 35*(2*b^5*d^2*e^2 - 13*a*b^4*d*e^3 + 11*a^2*b^3*e^4)*x^2 - 7*(8*b^5*d^3*e - 44*a*b^4*d^2*e^2 + 109*a^2*b^3*d*e^3 - 73*a^3*b^2*e^4)*x)*sqrt(e*x + d))/(a^4*b^6*d^5 - 5*a^5*b^5*d^4*e + 10*a^6*b^4*d^3*e^2 - 10*a^7*b^3*d^2*e^3 + 5*a^8*b^2*d*e^4 - a^9*b*e^5 + (b^10*d^5 - 5*a*b^9*d^4*e + 10*a^2*b^8*d^3*e^2 - 10*a^3*b^7*d^2*e^3 + 5*a^4*b^6*d*e^4 - a^5*b^5*e^5)*x^4 + 4*(a*b^9*d^5 - 5*a^2*b^8*d^4*e + 10*a^3*b^7*d^3*e^2 - 10*a^4*b^6*d^2*e^3 + 5*a^5*b^5*d*e^4 - a^6*b^4*e^5)*x^3 + 6*(a^2*b^8*d^5 - 5*a^3*b^7*d^4*e + 10*a^4*b^6*d^3*e^2 - 10*a^5*b^5*d^2*e^3 + 5*a^6*b^4*d*e^4 - a^7*b^3*e^5)*x^2 + 4*(a^3*b^7*d^5 - 5*a^4*b^6*d^4*e + 10*a^5*b^5*d^3*e^2 - 10*a^6*b^4*d^2*e^3 + 5*a^7*b^3*d*e^4 - a^8*b^2*e^5)*x), 1/192*(105*(b^4*e^4*x^4 + 4*a*b^3*e^4*x^3 + 6*a^2*b^2*e^4*x^2 + 4*a^3*b*e^4*x + a^4*e^4)*sqrt(-b^2*d + a*b*e)*arctan(sqrt(-b^2*d + a*b*e)*sqrt(e*x + d)/(b*e*x + b*d)) - (48*b^5*d^4 - 248*a*b^4*d^3*e + 526*a^2*b^3*d^2*e^2 - 605*a^3*b^2*d*e^3 + 279*a^4*b*e^4 - 105*(b^5*d*e^3 - a*b^4*e^4)*x^3 + 35*(2*b^5*d^2*e^2 - 13*a*b^4*d*e^3 + 11*a^2*b^3*e^4)*x^2 - 7*(8*b^5*d^3*e - 44*a*b^4*d^2*e^2 + 109*a^2*b^3*d*e^3 - 73*a^3*b^2*e^4)*x)*sqrt(e*x + d))/(a^4*b^6*d^5 - 5*a^5*b^...
```

SymPy [F]

$$\int \frac{1}{\sqrt{d+ex} (a^2 + 2abx + b^2x^2)^{5/2}} dx = \int \frac{1}{\sqrt{d+ex} ((a+bx)^2)^{5/2}} dx$$

input

```
integrate(1/(e*x+d)**(1/2)/(b**2*x**2+2*a*b*x+a**2)**(5/2), x)
```

output

```
Integral(1/(sqrt(d + e*x)*((a + b*x)**2)**(5/2)), x)
```

Maxima [F]

$$\int \frac{1}{\sqrt{d+ex} (a^2 + 2abx + b^2x^2)^{5/2}} dx = \int \frac{1}{(b^2x^2 + 2abx + a^2)^{5/2} \sqrt{ex+d}} dx$$

input `integrate(1/(e*x+d)^(1/2)/(b^2*x^2+2*a*b*x+a^2)^(5/2),x, algorithm="maxima")`

output `integrate(1/((b^2*x^2 + 2*a*b*x + a^2)^(5/2)*sqrt(e*x + d)), x)`

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 391 vs. 2(195) = 390.

Time = 0.14 (sec) , antiderivative size = 391, normalized size of antiderivative = 1.41

$$\int \frac{1}{\sqrt{d+ex} (a^2 + 2abx + b^2x^2)^{5/2}} dx = \frac{35 e^4 \arctan\left(\frac{\sqrt{ex+d}}{\sqrt{-b^2d+abe}}\right) + 105 (ex+d)^{7/2} b^3 e^4 - 385 (ex+d)^{5/2} b^3 d e^4 + 511 (ex+d)^{3/2} b^3 d^2 e^4 - 279 \sqrt{ex+d} b^3 d^3 e^4 + 385 (ex+d)^{5/2} a b^2 e^4}{192 (b^4 d^4 \operatorname{sgn}(bx+a) - 4 a b^3 d^3 \operatorname{sgn}(bx+a) + 6 a^2 b^2 d^2 e^2 \operatorname{sgn}(bx+a) - 4 a^3 b d e^3 \operatorname{sgn}(bx+a) + a^4 e^4 \operatorname{sgn}(bx+a)) \sqrt{-b^2 d + a b e}} + \frac{1}{192} \frac{105 (ex+d)^{7/2} b^3 e^4 - 385 (ex+d)^{5/2} b^3 d e^4 + 511 (ex+d)^{3/2} b^3 d^2 e^4 - 279 \sqrt{ex+d} b^3 d^3 e^4 + 385 (ex+d)^{5/2} a b^2 e^4}{192 (b^4 d^4 \operatorname{sgn}(bx+a) - 4 a b^3 d^3 \operatorname{sgn}(bx+a) + 6 a^2 b^2 d^2 e^2 \operatorname{sgn}(bx+a) - 4 a^3 b d e^3 \operatorname{sgn}(bx+a) + a^4 e^4 \operatorname{sgn}(bx+a)) \sqrt{-b^2 d + a b e}}$$

input `integrate(1/(e*x+d)^(1/2)/(b^2*x^2+2*a*b*x+a^2)^(5/2),x, algorithm="giac")`

output `35/64*e^4*arctan(sqrt(e*x + d)*b/sqrt(-b^2*d + a*b*e))/((b^4*d^4*sgn(b*x + a) - 4*a*b^3*d^3*e*sgn(b*x + a) + 6*a^2*b^2*d^2*e^2*sgn(b*x + a) - 4*a^3*b*d*e^3*sgn(b*x + a) + a^4*e^4*sgn(b*x + a))*sqrt(-b^2*d + a*b*e)) + 1/192*(105*(e*x + d)^(7/2)*b^3*e^4 - 385*(e*x + d)^(5/2)*b^3*d*e^4 + 511*(e*x + d)^(3/2)*b^3*d^2*e^4 - 279*sqrt(e*x + d)*b^3*d^3*e^4 + 385*(e*x + d)^(5/2)*a*b^2*e^4 - 1022*(e*x + d)^(3/2)*a*b^2*d*e^4 + 837*sqrt(e*x + d)*a*b^2*d^2*e^4 + 511*(e*x + d)^(3/2)*a^2*b*e^4 - 837*sqrt(e*x + d)*a^2*b*d*e^4 + 279*sqrt(e*x + d)*a^3*e^4)/((b^4*d^4*sgn(b*x + a) - 4*a*b^3*d^3*e*sgn(b*x + a) + 6*a^2*b^2*d^2*e^2*sgn(b*x + a) - 4*a^3*b*d*e^3*sgn(b*x + a) + a^4*e^4*sgn(b*x + a))*((e*x + d)*b - b*d + a*e)^4)`

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{d+ex} (a^2 + 2abx + b^2x^2)^{5/2}} dx = \int \frac{1}{\sqrt{d+ex} (a^2 + 2abx + b^2x^2)^{5/2}} dx$$

input `int(1/((d + e*x)^(1/2)*(a^2 + b^2*x^2 + 2*a*b*x)^(5/2)),x)`

output `int(1/((d + e*x)^(1/2)*(a^2 + b^2*x^2 + 2*a*b*x)^(5/2)), x)`

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 903, normalized size of antiderivative = 3.25

$$\int \frac{1}{\sqrt{d+ex} (a^2 + 2abx + b^2x^2)^{5/2}} dx = \text{Too large to display}$$

input `int(1/(e*x+d)^(1/2)/(b^2*x^2+2*a*b*x+a^2)^(5/2),x)`

output

```
(105*sqrt(b)*sqrt(a*e - b*d)*atan((sqrt(d + e*x)*b)/(sqrt(b)*sqrt(a*e - b*d)))
*a**4*e**4 + 420*sqrt(b)*sqrt(a*e - b*d)*atan((sqrt(d + e*x)*b)/(sqrt(b)*sqrt(a*e - b*d)))
*a**3*b*e**4*x + 630*sqrt(b)*sqrt(a*e - b*d)*atan((sqrt(d + e*x)*b)/(sqrt(b)*sqrt(a*e - b*d)))
*a**2*b**2*e**4*x**2 + 420*sqrt(b)*sqrt(a*e - b*d)*atan((sqrt(d + e*x)*b)/(sqrt(b)*sqrt(a*e - b*d)))
*a*b**3*e**4*x**3 + 105*sqrt(b)*sqrt(a*e - b*d)*atan((sqrt(d + e*x)*b)/(sqrt(b)*sqrt(a*e - b*d)))
*b**4*e**4*x**4 + 279*sqrt(d + e*x)*a**4*b*e**4 - 605*sqrt(d + e*x)*a**3*b**2*d*e**3 + 511*sqrt(d + e*x)*a**3*b**2*e**4*x + 526*sqrt(d + e*x)*a**2*b**3*d**2*e**2 - 763*sqrt(d + e*x)*a**2*b**3*d*e**3*x + 385*sqrt(d + e*x)*a**2*b**3*e**4*x**2 - 248*sqrt(d + e*x)*a*b**4*d**3*e + 308*sqrt(d + e*x)*a*b**4*d**2*e**2*x - 455*sqrt(d + e*x)*a*b**4*d*e**3*x**2 + 105*sqrt(d + e*x)*a*b**4*e**4*x**3 + 48*sqrt(d + e*x)*b**5*d**4 - 56*sqrt(d + e*x)*b**5*d**3*e*x + 70*sqrt(d + e*x)*b**5*d**2*e**2*x**2 - 105*sqrt(d + e*x)*b**5*d*e**3*x**3)/(192*b*(a**9*e**5 - 5*a**8*b*d*e**4 + 4*a**8*b*e**5*x + 10*a**7*b**2*d**2*e**3 - 20*a**7*b**2*d*e**4*x + 6*a**7*b**2*e**5*x**2 - 10*a**6*b**3*d**3*e**2 + 40*a**6*b**3*d**2*e**3*x - 30*a**6*b**3*d*e**4*x**2 + 4*a**6*b**3*e**5*x**3 + 5*a**5*b**4*d**4*e - 40*a**5*b**4*d**3*e**2*x + 60*a**5*b**4*d**2*e**3*x**2 - 20*a**5*b**4*d*e**4*x**3 + a**5*b**4*e**5*x**4 - a**4*b**5*d**5 + 20*a**4*b**5*d**4*e*x - 60*a**4*b**5*d**3*e**2*x**2 + 40*a**4*b**5*d**2*e**3*x**3 - 5*a**4*b**5*d*e**4*x**4 - 4*a...
```

3.396 $\int \frac{1}{(d+ex)^{3/2}(a^2+2abx+b^2x^2)^{5/2}} dx$

| | |
|---|------|
| Optimal result | 2970 |
| Mathematica [A] (verified) | 2971 |
| Rubi [A] (verified) | 2971 |
| Maple [B] (verified) | 2977 |
| Fricas [B] (verification not implemented) | 2978 |
| Sympy [F] | 2979 |
| Maxima [F] | 2979 |
| Giac [B] (verification not implemented) | 2979 |
| Mupad [F(-1)] | 2980 |
| Reduce [B] (verification not implemented) | 2981 |

Optimal result

Integrand size = 30, antiderivative size = 329

$$\int \frac{1}{(d+ex)^{3/2}(a^2+2abx+b^2x^2)^{5/2}} dx = \frac{105e^3}{64(bd-ae)^4\sqrt{d+ex}\sqrt{a^2+2abx+b^2x^2}} - \frac{1}{4(bd-ae)(a+bx)^3\sqrt{d+ex}\sqrt{a^2+2abx+b^2x^2}} + \frac{3e}{8(bd-ae)^2(a+bx)^2\sqrt{d+ex}\sqrt{a^2+2abx+b^2x^2}} - \frac{21e^2}{32(bd-ae)^3(a+bx)\sqrt{d+ex}\sqrt{a^2+2abx+b^2x^2}} + \frac{315e^4(a+bx)}{64(bd-ae)^5\sqrt{d+ex}\sqrt{a^2+2abx+b^2x^2}} - \frac{315\sqrt{b}e^4(a+bx)\operatorname{arctanh}\left(\frac{\sqrt{b}\sqrt{d+ex}}{\sqrt{bd-ae}}\right)}{64(bd-ae)^{11/2}\sqrt{a^2+2abx+b^2x^2}}$$

output

```
105/64*e^3/(-a*e+b*d)^4/(e*x+d)^(1/2)/((b*x+a)^2)^(1/2)-1/4/(-a*e+b*d)/(b*x+a)^3/(e*x+d)^(1/2)/((b*x+a)^2)^(1/2)+3/8*e/(-a*e+b*d)^2/(b*x+a)^2/(e*x+d)^(1/2)/((b*x+a)^2)^(1/2)-21/32*e^2/(-a*e+b*d)^3/(b*x+a)/(e*x+d)^(1/2)/((b*x+a)^2)^(1/2)+315/64*e^4*(b*x+a)/(-a*e+b*d)^5/(e*x+d)^(1/2)/((b*x+a)^2)^(1/2)-315/64*b^(1/2)*e^4*(b*x+a)*arctanh(b^(1/2)*(e*x+d)^(1/2)/(-a*e+b*d)^(1/2))/(-a*e+b*d)^(11/2)/((b*x+a)^2)^(1/2)
```

Mathematica [A] (verified)

Time = 1.26 (sec) , antiderivative size = 247, normalized size of antiderivative = 0.75

$$\int \frac{1}{(d+ex)^{3/2}(a^2+2abx+b^2x^2)^{5/2}} dx = \frac{\sqrt{-bd+ae}(128a^4e^4+a^3be^3(325d+837ex)+3a^2b^2e^2(-70d^2+185dex+511e^2x^2)+ab^3e(88d^3-156d^2ex+399d^2e^2x^2+1155e^3x^3)+b^4(-16d^4+24d^3ex-42d^2e^2x^2+105d^2e^3x^3+315e^4x^4))+315\sqrt{b}e^4(a+bx)^4\sqrt{d+ex}\operatorname{ArcTan}\left[\frac{\sqrt{b}\sqrt{d+ex}}{\sqrt{-bd+ae}}\right]}{64(-bd+ae)^{11/2}(a+bx)^3\sqrt{(a+bx)^2}\sqrt{d+ex}}$$

input

```
Integrate[1/((d + e*x)^(3/2)*(a^2 + 2*a*b*x + b^2*x^2)^(5/2)), x]
```

output

```
-1/64*(Sqrt[-(b*d) + a*e]*(128*a^4*e^4 + a^3*b*e^3*(325*d + 837*e*x) + 3*a^2*b^2*e^2*(-70*d^2 + 185*d*e*x + 511*e^2*x^2) + a*b^3*e*(88*d^3 - 156*d^2*e*x + 399*d^2*e^2*x^2 + 1155*e^3*x^3) + b^4*(-16*d^4 + 24*d^3*e*x - 42*d^2*e^2*x^2 + 105*d^2*e^3*x^3 + 315*e^4*x^4)) + 315*Sqrt[b]*e^4*(a + b*x)^4*Sqrt[d + e*x]*ArcTan[(Sqrt[b]*Sqrt[d + e*x])/Sqrt[-(b*d) + a*e]]/((-b*d) + a*e)^(11/2)*(a + b*x)^3*Sqrt[(a + b*x)^2]*Sqrt[d + e*x])
```

Rubi [A] (verified)

Time = 0.61 (sec) , antiderivative size = 277, normalized size of antiderivative = 0.84, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.300$, Rules used = {1102, 27, 52, 52, 52, 52, 61, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\begin{aligned} & \int \frac{1}{(a^2+2abx+b^2x^2)^{5/2}(d+ex)^{3/2}} dx \\ & \quad \downarrow \text{1102} \\ & \frac{b^5(a+bx) \int \frac{1}{b^5(a+bx)^5(d+ex)^{3/2}} dx}{\sqrt{a^2+2abx+b^2x^2}} \\ & \quad \downarrow \text{27} \\ & \frac{(a+bx) \int \frac{1}{(a+bx)^5(d+ex)^{3/2}} dx}{\sqrt{a^2+2abx+b^2x^2}} \end{aligned}$$

$$\begin{aligned}
 & \downarrow 52 \\
 & \frac{(a+bx) \left(-\frac{9e \int \frac{1}{(a+bx)^4(d+ex)^{3/2}} dx}{8(bd-ae)} - \frac{1}{4(a+bx)^4 \sqrt{d+ex}(bd-ae)} \right)}{\sqrt{a^2 + 2abx + b^2x^2}} \\
 & \downarrow 52 \\
 & \frac{(a+bx) \left(-\frac{9e \left(-\frac{7e \int \frac{1}{(a+bx)^3(d+ex)^{3/2}} dx}{6(bd-ae)} - \frac{1}{3(a+bx)^3 \sqrt{d+ex}(bd-ae)} \right)}{8(bd-ae)} - \frac{1}{4(a+bx)^4 \sqrt{d+ex}(bd-ae)} \right)}{\sqrt{a^2 + 2abx + b^2x^2}} \\
 & \downarrow 52 \\
 & \frac{(a+bx) \left(-\frac{9e \left(-\frac{7e \left(-\frac{5e \int \frac{1}{(a+bx)^2(d+ex)^{3/2}} dx}{4(bd-ae)} - \frac{1}{2(a+bx)^2 \sqrt{d+ex}(bd-ae)} \right)}{6(bd-ae)} - \frac{1}{3(a+bx)^3 \sqrt{d+ex}(bd-ae)} \right)}{8(bd-ae)} - \frac{1}{4(a+bx)^4 \sqrt{d+ex}(bd-ae)} \right)}{\sqrt{a^2 + 2abx + b^2x^2}} \\
 & \downarrow 52 \\
 & \frac{(a+bx) \left(-\frac{9e \left(-\frac{7e \left(-\frac{5e \left(-\frac{3e \int \frac{1}{(a+bx)(d+ex)^{3/2}} dx}{2(bd-ae)} - \frac{1}{(a+bx) \sqrt{d+ex}(bd-ae)} \right)}{4(bd-ae)} - \frac{1}{2(a+bx)^2 \sqrt{d+ex}(bd-ae)} \right)}{6(bd-ae)} - \frac{1}{3(a+bx)^3 \sqrt{d+ex}(bd-ae)} \right)}{8(bd-ae)} - \frac{1}{4(a+bx)^4 \sqrt{d+ex}(bd-ae)} \right)}{\sqrt{a^2 + 2abx + b^2x^2}}
 \end{aligned}$$

↓ 61

$$\begin{aligned}
 & \left(\left(\left(\left(\left(\frac{3e \left(\frac{b \int \frac{1}{(a+bx)\sqrt{d+ex}} dx}{bd-ae} + \frac{2}{\sqrt{d+ex}(bd-ae)} \right)}{2(bd-ae)} - \frac{1}{(a+bx)\sqrt{d+ex}(bd-ae)} \right)}{4(bd-ae)} - \frac{1}{2(a+bx)^2\sqrt{d+ex}(bd-ae)} \right)}{6(bd-ae)} - \frac{1}{3(a+bx)^3\sqrt{d+ex}(bd-ae)} \right)}{8(bd-ae)} \right) \\
 & \frac{(a+bx)}{\sqrt{a^2 + 2abx + b^2x^2}}
 \end{aligned}$$

↓ 73

$$\left((a + bx) \left[\frac{9e}{8(bd - ae)} - \frac{7e}{4(bd - ae)} \left(\frac{3e}{2(bd - ae)} \left(\frac{2b \int \frac{1}{a + \frac{b(d+ex) - bd}{e(bd - ae)}} + \frac{2}{\sqrt{d+ex}(bd - ae)} \right) - \frac{1}{(a + bx)\sqrt{d+ex}(bd - ae)} \right) - \frac{1}{2(a + bx)^2 \sqrt{d+ex}(bd - ae)} \right] \right)$$

$$\sqrt{a^2 + 2abx + b^2x^2}$$

$$\frac{(a+bx) \left(\frac{3e \left(\frac{2}{\sqrt{d+ex}(bd-ae)} - \frac{2\sqrt{b} \operatorname{arctanh}\left(\frac{\sqrt{b}\sqrt{d+ex}}{\sqrt{bd-ae}}\right)}{(bd-ae)^{3/2}} \right)}{2(bd-ae)} - \frac{1}{(a+bx)\sqrt{d+ex}(bd-ae)} \right)}{4(bd-ae)} - \frac{1}{2(a+bx)^2\sqrt{d+ex}(bd-ae)}$$

$$\frac{7e \left(\frac{5e \left(\frac{3e \left(\frac{2}{\sqrt{d+ex}(bd-ae)} - \frac{2\sqrt{b} \operatorname{arctanh}\left(\frac{\sqrt{b}\sqrt{d+ex}}{\sqrt{bd-ae}}\right)}{(bd-ae)^{3/2}} \right)}{2(bd-ae)} - \frac{1}{(a+bx)\sqrt{d+ex}(bd-ae)} \right)}{4(bd-ae)} - \frac{1}{2(a+bx)^2\sqrt{d+ex}(bd-ae)} \right)}{6(bd-ae)} - \frac{1}{3(a+bx)^3}$$

$$\frac{9e \left(\frac{7e \left(\frac{5e \left(\frac{3e \left(\frac{2}{\sqrt{d+ex}(bd-ae)} - \frac{2\sqrt{b} \operatorname{arctanh}\left(\frac{\sqrt{b}\sqrt{d+ex}}{\sqrt{bd-ae}}\right)}{(bd-ae)^{3/2}} \right)}{2(bd-ae)} - \frac{1}{(a+bx)\sqrt{d+ex}(bd-ae)} \right)}{4(bd-ae)} - \frac{1}{2(a+bx)^2\sqrt{d+ex}(bd-ae)} \right)}{6(bd-ae)} - \frac{1}{3(a+bx)^3} \right)}{8(bd-ae)}$$

$$\frac{(a+bx)}{\sqrt{a^2 + 2abx + b^2x^2}}$$

```
input Int[1/((d + e*x)^(3/2)*(a^2 + 2*a*b*x + b^2*x^2)^(5/2)),x]
```

output

$$\begin{aligned} & ((a + b*x)*(-1/4*1/((b*d - a*e)*(a + b*x)^4*\text{Sqrt}[d + e*x]) - (9*e*(-1/3*1/ \\ & ((b*d - a*e)*(a + b*x)^3*\text{Sqrt}[d + e*x]) - (7*e*(-1/2*1/((b*d - a*e)*(a + b \\ & *x)^2*\text{Sqrt}[d + e*x]) - (5*e*(-1/((b*d - a*e)*(a + b*x)*\text{Sqrt}[d + e*x])) - \\ & (3*e*(2/((b*d - a*e)*\text{Sqrt}[d + e*x]) - (2*\text{Sqrt}[b]*\text{ArcTanh}[(\text{Sqrt}[b]*\text{Sqrt}[d + \\ & e*x])/ \text{Sqrt}[b*d - a*e]])/(b*d - a*e)^{(3/2)})/(2*(b*d - a*e))))/(4*(b*d - a \\ & *e))))/(6*(b*d - a*e)))/(8*(b*d - a*e)))/\text{Sqrt}[a^2 + 2*a*b*x + b^2*x^2] \end{aligned}$$

Defintions of rubi rules used

rule 27

$$\text{Int}[(a_*)(F_x_), x_Symbol] \rightarrow \text{Simp}[a \text{ Int}[F_x, x], x] \text{ /; FreeQ}[a, x] \ \&\& \ !\text{MatchQ}[F_x, (b_*)(G_x_) \text{ /; FreeQ}[b, x]$$

rule 52

$$\text{Int}[(a_.) + (b_.)*(x_)^{(m_)}*((c_.) + (d_.)*(x_)^{(n_)}), x_Symbol] \rightarrow \text{Simp}[(a + b*x)^{(m + 1)}*((c + d*x)^{(n + 1)}/((b*c - a*d)*(m + 1))), x] - \text{Simp}[d*((m + n + 2)/((b*c - a*d)*(m + 1))) \text{ Int}[(a + b*x)^{(m + 1)}*(c + d*x)^n, x], x] \text{ /; FreeQ}[\{a, b, c, d, n\}, x] \ \&\& \ \text{ILtQ}[m, -1] \ \&\& \ \text{FractionQ}[n] \ \&\& \ \text{LtQ}[n, 0]$$

rule 61

$$\text{Int}[(a_.) + (b_.)*(x_)^{(m_)}*((c_.) + (d_.)*(x_)^{(n_)}), x_Symbol] \rightarrow \text{Simp}[(a + b*x)^{(m + 1)}*((c + d*x)^{(n + 1)}/((b*c - a*d)*(m + 1))), x] - \text{Simp}[d*((m + n + 2)/((b*c - a*d)*(m + 1))) \text{ Int}[(a + b*x)^{(m + 1)}*(c + d*x)^n, x], x] \text{ /; FreeQ}[\{a, b, c, d, n\}, x] \ \&\& \ \text{LtQ}[m, -1] \ \&\& \ !(\text{LtQ}[n, -1] \ \&\& \ (\text{EqQ}[a, 0] \ || \ (\text{NeQ}[c, 0] \ \&\& \ \text{LtQ}[m - n, 0] \ \&\& \ \text{IntegerQ}[n]))) \ \&\& \ \text{IntLinearQ}[a, b, c, d, m, n, x]$$

rule 73

$$\text{Int}[(a_.) + (b_.)*(x_)^{(m_)}*((c_.) + (d_.)*(x_)^{(n_)}), x_Symbol] \rightarrow \text{With}[\{p = \text{Denominator}[m]\}, \text{Simp}[p/b \text{ Subst}[\text{Int}[x^{(p*(m + 1) - 1)}*(c - a*(d/b) + d*(x^{p/b})^n, x], x, (a + b*x)^{(1/p)}, x]] \text{ /; FreeQ}[\{a, b, c, d\}, x] \ \&\& \ \text{LtQ}[-1, m, 0] \ \&\& \ \text{LeQ}[-1, n, 0] \ \&\& \ \text{LeQ}[\text{Denominator}[n], \text{Denominator}[m]] \ \&\& \ \text{IntLinearQ}[a, b, c, d, m, n, x]$$

rule 221

$$\text{Int}[(a_.) + (b_.)*(x_)^2)^{-1}, x_Symbol] \rightarrow \text{Simp}[(\text{Rt}[-a/b, 2]/a)*\text{ArcTanh}[x/\text{Rt}[-a/b, 2]], x] \text{ /; FreeQ}[\{a, b\}, x] \ \&\& \ \text{NegQ}[a/b]$$

rule 1102

```
Int[((d_.) + (e_.)*(x_.))^(m_.)*((a_.) + (b_.)*(x_.) + (c_.)*(x_.)^2)^(p_), x_Symbol]
:> Simp[(a + b*x + c*x^2)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x)^(2*FracPart[p]))
Int[(d + e*x)^m*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d, e, m, p}, x]
&& EqQ[b^2 - 4*a*c, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 601 vs. $2(231) = 462$.

Time = 1.55 (sec) , antiderivative size = 602, normalized size of antiderivative = 1.83

| method | result |
|---------|--|
| default | $-\frac{\left(315\sqrt{ex+d} \arctan\left(\frac{b\sqrt{ex+d}}{\sqrt{b(ae-bd)}}\right) b^5 e^4 x^4 + 1260\sqrt{ex+d} \arctan\left(\frac{b\sqrt{ex+d}}{\sqrt{b(ae-bd)}}\right) a b^4 e^4 x^3 + 1890\sqrt{ex+d} \arctan\left(\frac{b\sqrt{ex+d}}{\sqrt{b(ae-bd)}}\right) a^2 b^3 e^4}{\dots}$ |

input

```
int(1/(e*x+d)^(3/2)/(b^2*x^2+2*a*b*x+a^2)^(5/2),x,method=_RETURNVERBOSE)
```

output

```
-1/64*(315*(e*x+d)^(1/2)*arctan(b*(e*x+d)^(1/2)/(b*(a*e-b*d))^(1/2))*b^5*e^4*x^4+1260*(e*x+d)^(1/2)*arctan(b*(e*x+d)^(1/2)/(b*(a*e-b*d))^(1/2))*a*b^4*e^4*x^3+1890*(e*x+d)^(1/2)*arctan(b*(e*x+d)^(1/2)/(b*(a*e-b*d))^(1/2))*a^2*b^3*e^4*x^2+315*(b*(a*e-b*d))^(1/2)*b^4*e^4*x^4+1260*arctan(b*(e*x+d)^(1/2)/(b*(a*e-b*d))^(1/2))*a^3*b^2*e^4*x*(e*x+d)^(1/2)+1155*(b*(a*e-b*d))^(1/2)*a*b^3*e^4*x^3+105*(b*(a*e-b*d))^(1/2)*b^4*d*e^3*x^3+315*(e*x+d)^(1/2)*arctan(b*(e*x+d)^(1/2)/(b*(a*e-b*d))^(1/2))*a^4*b*e^4+1533*(b*(a*e-b*d))^(1/2)*a^2*b^2*e^4*x^2+399*(b*(a*e-b*d))^(1/2)*a*b^3*d*e^3*x^2-42*(b*(a*e-b*d))^(1/2)*b^4*d^2*e^2*x^2+837*(b*(a*e-b*d))^(1/2)*a^3*b*e^4*x+555*(b*(a*e-b*d))^(1/2)*a^2*b^2*d*e^3*x-156*(b*(a*e-b*d))^(1/2)*a*b^3*d^2*e^2*x+24*(b*(a*e-b*d))^(1/2)*b^4*d^3*e*x+128*(b*(a*e-b*d))^(1/2)*a^4*e^4+325*(b*(a*e-b*d))^(1/2)*a^3*b*d*e^3-210*(b*(a*e-b*d))^(1/2)*a^2*b^2*d^2*e^2+88*(b*(a*e-b*d))^(1/2)*a*b^3*d^3*e-16*(b*(a*e-b*d))^(1/2)*b^4*d^4*(b*x+a)/(b*(a*e-b*d))^(1/2)/(e*x+d)^(1/2)/(a*e-b*d)^5/((b*x+a)^2)^(5/2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 842 vs. $2(231) = 462$.

Time = 0.20 (sec) , antiderivative size = 1714, normalized size of antiderivative = 5.21

$$\int \frac{1}{(d+ex)^{3/2} (a^2+2abx+b^2x^2)^{5/2}} dx = \text{Too large to display}$$

input `integrate(1/(e*x+d)^(3/2)/(b^2*x^2+2*a*b*x+a^2)^(5/2),x, algorithm="fricas")`

output `[-1/128*(315*(b^4*e^5*x^5 + a^4*d*e^4 + (b^4*d*e^4 + 4*a*b^3*e^5)*x^4 + 2*(2*a*b^3*d*e^4 + 3*a^2*b^2*e^5)*x^3 + 2*(3*a^2*b^2*d*e^4 + 2*a^3*b*e^5)*x^2 + (4*a^3*b*d*e^4 + a^4*e^5)*x)*sqrt(b/(b*d - a*e))*log((b*e*x + 2*b*d - a*e + 2*(b*d - a*e)*sqrt(e*x + d)*sqrt(b/(b*d - a*e)))/(b*x + a)) - 2*(315*b^4*e^4*x^4 - 16*b^4*d^4 + 88*a*b^3*d^3*e - 210*a^2*b^2*d^2*e^2 + 325*a^3*b*d*e^3 + 128*a^4*e^4 + 105*(b^4*d*e^3 + 11*a*b^3*e^4)*x^3 - 21*(2*b^4*d^2*e^2 - 19*a*b^3*d*e^3 - 73*a^2*b^2*e^4)*x^2 + 3*(8*b^4*d^3*e - 52*a*b^3*d^2*e^2 + 185*a^2*b^2*d*e^3 + 279*a^3*b*e^4)*x)*sqrt(e*x + d)]/(a^4*b^5*d^6 - 5*a^5*b^4*d^5*e + 10*a^6*b^3*d^4*e^2 - 10*a^7*b^2*d^3*e^3 + 5*a^8*b*d^2*e^4 - a^9*d*e^5 + (b^9*d^5*e - 5*a*b^8*d^4*e^2 + 10*a^2*b^7*d^3*e^3 - 10*a^3*b^6*d^2*e^4 + 5*a^4*b^5*d*e^5 - a^5*b^4*e^6)*x^5 + (b^9*d^6 - a*b^8*d^5*e - 10*a^2*b^7*d^4*e^2 + 30*a^3*b^6*d^3*e^3 - 35*a^4*b^5*d^2*e^4 + 19*a^5*b^4*d*e^5 - 4*a^6*b^3*e^6)*x^4 + 2*(2*a*b^8*d^6 - 7*a^2*b^7*d^5*e + 5*a^3*b^6*d^4*e^2 + 10*a^4*b^5*d^3*e^3 - 20*a^5*b^4*d^2*e^4 + 13*a^6*b^3*d*e^5 - 3*a^7*b^2*e^6)*x^3 + 2*(3*a^2*b^7*d^6 - 13*a^3*b^6*d^5*e + 20*a^4*b^5*d^4*e^2 - 10*a^5*b^4*d^3*e^3 - 5*a^6*b^3*d^2*e^4 + 7*a^7*b^2*d*e^5 - 2*a^8*b*e^6)*x^2 + (4*a^3*b^6*d^6 - 19*a^4*b^5*d^5*e + 35*a^5*b^4*d^4*e^2 - 30*a^6*b^3*d^3*e^3 + 10*a^7*b^2*d^2*e^4 + a^8*b*d*e^5 - a^9*e^6)*x), 1/64*(315*(b^4*e^5*x^5 + a^4*d*e^4 + (b^4*d*e^4 + 4*a*b^3*e^5)*x^4 + 2*(2*a*b^3*d*e^4 + 3*a^2*b^2*e^5)*x^3 + 2*(3*a^2*b^2*d*e^4 + 2*a^3*b*e^5)*x^2 + (4*a^3*b*d*e^4 + a^4*e^5)*x)`

Sympy [F]

$$\int \frac{1}{(d+ex)^{3/2} (a^2+2abx+b^2x^2)^{5/2}} dx = \int \frac{1}{(d+ex)^{\frac{3}{2}} ((a+bx)^2)^{\frac{5}{2}}} dx$$

input `integrate(1/(e*x+d)**(3/2)/(b**2*x**2+2*a*b*x+a**2)**(5/2), x)`

output `Integral(1/((d + e*x)**(3/2)*((a + b*x)**2)**(5/2)), x)`

Maxima [F]

$$\int \frac{1}{(d+ex)^{3/2} (a^2+2abx+b^2x^2)^{5/2}} dx = \int \frac{1}{(b^2x^2+2abx+a^2)^{\frac{5}{2}} (ex+d)^{\frac{3}{2}}} dx$$

input `integrate(1/(e*x+d)^(3/2)/(b^2*x^2+2*a*b*x+a^2)^(5/2), x, algorithm="maxima")`

output `integrate(1/((b^2*x^2 + 2*a*b*x + a^2)^(5/2)*(e*x + d)^(3/2)), x)`

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 553 vs. $2(231) = 462$.

Time = 0.13 (sec) , antiderivative size = 553, normalized size of antiderivative = 1.68

$$\int \frac{1}{(d+ex)^{3/2} (a^2+2abx+b^2x^2)^{5/2}} dx = \frac{1}{64 (b^5 d^5 \operatorname{sgn}(bx+a) - 5 ab^4 d^4 \operatorname{esgn}(bx+a) + 10 a^2 b^3 d^3 e^2 \operatorname{sgn}(bx+a) - 5 a^3 b^2 d^2 e^3 \operatorname{sgn}(bx+a) + 5 a^4 b d e^4 \operatorname{sgn}(bx+a) - 187 (ex+d)^{\frac{7}{2}} b^4 e^4 - 643 (ex+d)^{\frac{5}{2}} b^4 d e^4 + 765 (ex+d)^{\frac{3}{2}} b^4 d^2 e^4 - 325 \sqrt{ex+db^4 d^3 e^4} + 643 (ex+d)^{\frac{5}{2}} ab^3 e^4 - 643 (ex+d)^{\frac{3}{2}} ab^3 d e^4 + 187 (ex+d)^{\frac{1}{2}} ab^3 d^2 e^4 - 187 (ex+d)^{\frac{1}{2}} ab^3 d^3 e^4 + 187 (ex+d)^{\frac{1}{2}} ab^3 d^4 e^4 - 187 (ex+d)^{\frac{1}{2}} ab^3 d^5 e^4)}{64 (b^5 d^5 \operatorname{sgn}(bx+a) - 5 ab^4 d^4 \operatorname{esgn}(bx+a) + 10 a^2 b^3 d^3 e^2 \operatorname{sgn}(bx+a) - 5 a^3 b^2 d^2 e^3 \operatorname{sgn}(bx+a) + 5 a^4 b d e^4 \operatorname{sgn}(bx+a) - 187 (ex+d)^{\frac{7}{2}} b^4 e^4 - 643 (ex+d)^{\frac{5}{2}} b^4 d e^4 + 765 (ex+d)^{\frac{3}{2}} b^4 d^2 e^4 - 325 \sqrt{ex+db^4 d^3 e^4} + 643 (ex+d)^{\frac{5}{2}} ab^3 e^4 - 643 (ex+d)^{\frac{3}{2}} ab^3 d e^4 + 187 (ex+d)^{\frac{1}{2}} ab^3 d^2 e^4 - 187 (ex+d)^{\frac{1}{2}} ab^3 d^3 e^4 + 187 (ex+d)^{\frac{1}{2}} ab^3 d^4 e^4 - 187 (ex+d)^{\frac{1}{2}} ab^3 d^5 e^4)}$$

input `integrate(1/(e*x+d)^(3/2)/(b^2*x^2+2*a*b*x+a^2)^(5/2),x, algorithm="giac")`

output `315/64*b*e^4*arctan(sqrt(e*x + d)*b/sqrt(-b^2*d + a*b*e))/((b^5*d^5*sgn(b*x + a) - 5*a*b^4*d^4*e*sgn(b*x + a) + 10*a^2*b^3*d^3*e^2*sgn(b*x + a) - 10*a^3*b^2*d^2*e^3*sgn(b*x + a) + 5*a^4*b*d*e^4*sgn(b*x + a) - a^5*e^5*sgn(b*x + a))*sqrt(-b^2*d + a*b*e)) + 2*e^4/((b^5*d^5*sgn(b*x + a) - 5*a*b^4*d^4*e*sgn(b*x + a) + 10*a^2*b^3*d^3*e^2*sgn(b*x + a) - 10*a^3*b^2*d^2*e^3*sgn(b*x + a) + 5*a^4*b*d*e^4*sgn(b*x + a) - a^5*e^5*sgn(b*x + a))*sqrt(e*x + d)) + 1/64*(187*(e*x + d)^(7/2)*b^4*e^4 - 643*(e*x + d)^(5/2)*b^4*d*e^4 + 765*(e*x + d)^(3/2)*b^4*d^2*e^4 - 325*sqrt(e*x + d)*b^4*d^3*e^4 + 643*(e*x + d)^(5/2)*a*b^3*e^5 - 1530*(e*x + d)^(3/2)*a*b^3*d*e^5 + 975*sqrt(e*x + d)*a*b^3*d^2*e^5 + 765*(e*x + d)^(3/2)*a^2*b^2*e^6 - 975*sqrt(e*x + d)*a^2*b^2*d*e^6 + 325*sqrt(e*x + d)*a^3*b*e^7)/((b^5*d^5*sgn(b*x + a) - 5*a*b^4*d^4*e*sgn(b*x + a) + 10*a^2*b^3*d^3*e^2*sgn(b*x + a) - 10*a^3*b^2*d^2*e^3*sgn(b*x + a) + 5*a^4*b*d*e^4*sgn(b*x + a) - a^5*e^5*sgn(b*x + a))*((e*x + d)*b - b*d + a*e)^4)`

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(d + ex)^{3/2} (a^2 + 2abx + b^2x^2)^{5/2}} dx = \int \frac{1}{(d + ex)^{3/2} (a^2 + 2abx + b^2x^2)^{5/2}} dx$$

input `int(1/((d + e*x)^(3/2)*(a^2 + b^2*x^2 + 2*a*b*x)^(5/2)),x)`

output `int(1/((d + e*x)^(3/2)*(a^2 + b^2*x^2 + 2*a*b*x)^(5/2)), x)`

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 1014, normalized size of antiderivative = 3.08

$$\int \frac{1}{(d+ex)^{3/2}(a^2+2abx+b^2x^2)^{5/2}} dx = \text{Too large to display}$$

input

```
int(1/(e*x+d)^(3/2)/(b^2*x^2+2*a*b*x+a^2)^(5/2),x)
```

output

```
( - 315*sqrt(b)*sqrt(d + e*x)*sqrt(a*e - b*d)*atan((sqrt(d + e*x)*b)/(sqrt
(b)*sqrt(a*e - b*d)))*a**4*e**4 - 1260*sqrt(b)*sqrt(d + e*x)*sqrt(a*e - b*
d)*atan((sqrt(d + e*x)*b)/(sqrt(b)*sqrt(a*e - b*d)))*a**3*b*e**4*x - 1890*
sqrt(b)*sqrt(d + e*x)*sqrt(a*e - b*d)*atan((sqrt(d + e*x)*b)/(sqrt(b)*sqrt
(a*e - b*d)))*a**2*b**2*e**4*x**2 - 1260*sqrt(b)*sqrt(d + e*x)*sqrt(a*e -
b*d)*atan((sqrt(d + e*x)*b)/(sqrt(b)*sqrt(a*e - b*d)))*a*b**3*e**4*x**3 -
315*sqrt(b)*sqrt(d + e*x)*sqrt(a*e - b*d)*atan((sqrt(d + e*x)*b)/(sqrt(b)*
sqrt(a*e - b*d)))*b**4*e**4*x**4 - 128*a**5*e**5 - 197*a**4*b*d*e**4 - 837
*a**4*b*e**5*x + 535*a**3*b**2*d**2*e**3 + 282*a**3*b**2*d*e**4*x - 1533*a
**3*b**2*e**5*x**2 - 298*a**2*b**3*d**3*e**2 + 711*a**2*b**3*d**2*e**3*x +
1134*a**2*b**3*d*e**4*x**2 - 1155*a**2*b**3*e**5*x**3 + 104*a*b**4*d**4*e
- 180*a*b**4*d**3*e**2*x + 441*a*b**4*d**2*e**3*x**2 + 1050*a*b**4*d*e**4
*x**3 - 315*a*b**4*e**5*x**4 - 16*b**5*d**5 + 24*b**5*d**4*e*x - 42*b**5*d
**3*e**2*x**2 + 105*b**5*d**2*e**3*x**3 + 315*b**5*d*e**4*x**4)/(64*sqrt(d
+ e*x)*(a**10*e**6 - 6*a**9*b*d*e**5 + 4*a**9*b*e**6*x + 15*a**8*b**2*d**
2*e**4 - 24*a**8*b**2*d*e**5*x + 6*a**8*b**2*e**6*x**2 - 20*a**7*b**3*d**3
*e**3 + 60*a**7*b**3*d**2*e**4*x - 36*a**7*b**3*d*e**5*x**2 + 4*a**7*b**3*
e**6*x**3 + 15*a**6*b**4*d**4*e**2 - 80*a**6*b**4*d**3*e**3*x + 90*a**6*b*
**4*d**2*e**4*x**2 - 24*a**6*b**4*d*e**5*x**3 + a**6*b**4*e**6*x**4 - 6*a**
5*b**5*d**5*e + 60*a**5*b**5*d**4*e**2*x - 120*a**5*b**5*d**3*e**3*x**2...
```

3.397 $\int \frac{1}{(d+ex)^{5/2}(a^2+2abx+b^2x^2)^{5/2}} dx$

| | |
|---|------|
| Optimal result | 2982 |
| Mathematica [A] (verified) | 2983 |
| Rubi [A] (verified) | 2983 |
| Maple [B] (verified) | 2993 |
| Fricas [B] (verification not implemented) | 2994 |
| Sympy [F] | 2995 |
| Maxima [F] | 2996 |
| Giac [B] (verification not implemented) | 2996 |
| Mupad [F(-1)] | 2997 |
| Reduce [B] (verification not implemented) | 2997 |

Optimal result

Integrand size = 30, antiderivative size = 381

$$\int \frac{1}{(d+ex)^{5/2}(a^2+2abx+b^2x^2)^{5/2}} dx = \frac{231e^3}{64(bd-ae)^4(d+ex)^{3/2}\sqrt{a^2+2abx+b^2x^2}}$$

$$- \frac{1}{4(bd-ae)(a+bx)^3(d+ex)^{3/2}\sqrt{a^2+2abx+b^2x^2}}$$

$$+ \frac{11e}{24(bd-ae)^2(a+bx)^2(d+ex)^{3/2}\sqrt{a^2+2abx+b^2x^2}}$$

$$- \frac{33e^2}{32(bd-ae)^3(a+bx)(d+ex)^{3/2}\sqrt{a^2+2abx+b^2x^2}}$$

$$+ \frac{385e^4(a+bx)}{64(bd-ae)^5(d+ex)^{3/2}\sqrt{a^2+2abx+b^2x^2}}$$

$$+ \frac{1155be^4(a+bx)}{64(bd-ae)^6\sqrt{d+ex}\sqrt{a^2+2abx+b^2x^2}} - \frac{1155b^{3/2}e^4(a+bx)\operatorname{arctanh}\left(\frac{\sqrt{b}\sqrt{d+ex}}{\sqrt{bd-ae}}\right)}{64(bd-ae)^{13/2}\sqrt{a^2+2abx+b^2x^2}}$$

output

$$\frac{231}{64}e^3/(-a*e+b*d)^4/(e*x+d)^{(3/2)}/((b*x+a)^2)^{(1/2)}-1/4/(-a*e+b*d)/(b*x+a)^3/(e*x+d)^{(3/2)}/((b*x+a)^2)^{(1/2)}+11/24*e/(-a*e+b*d)^2/(b*x+a)^2/(e*x+d)^{(3/2)}/((b*x+a)^2)^{(1/2)}-33/32*e^2/(-a*e+b*d)^3/(b*x+a)/(e*x+d)^{(3/2)}/((b*x+a)^2)^{(1/2)}+385/64*e^4*(b*x+a)/(-a*e+b*d)^5/(e*x+d)^{(3/2)}/((b*x+a)^2)^{(1/2)}+1155/64*b*e^4*(b*x+a)/(-a*e+b*d)^6/(e*x+d)^{(1/2)}/((b*x+a)^2)^{(1/2)}-1155/64*b^(3/2)*e^4*(b*x+a)*\operatorname{arctanh}(b^(1/2)*(e*x+d)^{(1/2)}/(-a*e+b*d)^{(1/2)})/(-a*e+b*d)^{(13/2)}/((b*x+a)^2)^{(1/2)}$$
Mathematica [A] (verified)

Time = 1.52 (sec) , antiderivative size = 301, normalized size of antiderivative = 0.79

$$\int \frac{1}{(d+ex)^{5/2}(a^2+2abx+b^2x^2)^{5/2}} dx = \frac{e^4(a+bx)^5}{-128a^5e^5+128a^4be^4(16d+11ex)+a^3b^2e^3(2295d^2+12782dex+9207e^2)}$$

input

`Integrate[1/((d + e*x)^(5/2)*(a^2 + 2*a*b*x + b^2*x^2)^(5/2)), x]`

output

$$\frac{(e^4*(a + b*x)^5*((-128*a^5*e^5 + 128*a^4*b*e^4*(16*d + 11*e*x) + a^3*b^2*e^3*(2295*d^2 + 12782*d*e*x + 9207*e^2*x^2) + a^2*b^3*e^2*(-1030*d^3 + 3795*d^2*e*x + 22968*d*e^2*x^2 + 16863*e^3*x^3) + a*b^4*e*(328*d^4 - 748*d^3*e*x + 2673*d^2*e^2*x^2 + 17094*d*e^3*x^3 + 12705*e^4*x^4) + b^5*(-48*d^5 + 88*d^4*e*x - 198*d^3*e^2*x^2 + 693*d^2*e^3*x^3 + 4620*d*e^4*x^4 + 3465*e^5*x^5)))/(e^4*(b*d - a*e)^6*(a + b*x)^4*(d + e*x)^{(3/2)}) + (3465*b^(3/2)*\operatorname{ArcTan}[(\operatorname{Sqrt}[b]*\operatorname{Sqrt}[d + e*x])/(\operatorname{Sqrt}[-(b*d) + a*e])]/(-(b*d) + a*e)^{(13/2)})}{(192*((a + b*x)^2)^{(5/2)}}$$
Rubi [A] (verified)

Time = 0.63 (sec) , antiderivative size = 313, normalized size of antiderivative = 0.82, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {1102, 27, 52, 52, 52, 52, 61, 61, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\begin{aligned}
 & \int \frac{1}{(a^2 + 2abx + b^2x^2)^{5/2} (d + ex)^{5/2}} dx \\
 & \quad \downarrow \text{1102} \\
 & \frac{b^5(a + bx) \int \frac{1}{b^5(a+bx)^5(d+ex)^{5/2}} dx}{\sqrt{a^2 + 2abx + b^2x^2}} \\
 & \quad \downarrow \text{27} \\
 & \frac{(a + bx) \int \frac{1}{(a+bx)^5(d+ex)^{5/2}} dx}{\sqrt{a^2 + 2abx + b^2x^2}} \\
 & \quad \downarrow \text{52} \\
 & \frac{(a + bx) \left(-\frac{11e \int \frac{1}{(a+bx)^4(d+ex)^{5/2}} dx}{8(bd-ae)} - \frac{1}{4(a+bx)^4(d+ex)^{3/2}(bd-ae)} \right)}{\sqrt{a^2 + 2abx + b^2x^2}} \\
 & \quad \downarrow \text{52} \\
 & \frac{(a + bx) \left(-\frac{11e \left(-\frac{3e \int \frac{1}{(a+bx)^3(d+ex)^{5/2}} dx}{2(bd-ae)} - \frac{1}{3(a+bx)^3(d+ex)^{3/2}(bd-ae)} \right)}{8(bd-ae)} - \frac{1}{4(a+bx)^4(d+ex)^{3/2}(bd-ae)} \right)}{\sqrt{a^2 + 2abx + b^2x^2}} \\
 & \quad \downarrow \text{52} \\
 & \frac{(a + bx) \left(-\frac{11e \left(-\frac{3e \left(-\frac{7e \int \frac{1}{(a+bx)^2(d+ex)^{5/2}} dx}{4(bd-ae)} - \frac{1}{2(a+bx)^2(d+ex)^{3/2}(bd-ae)} \right)}{2(bd-ae)} - \frac{1}{3(a+bx)^3(d+ex)^{3/2}(bd-ae)} \right)}{8(bd-ae)} - \frac{1}{4(a+bx)^4(d+ex)^{3/2}(bd-ae)} \right)}{\sqrt{a^2 + 2abx + b^2x^2}} \\
 & \quad \downarrow \text{52} \\
 & \frac{(a + bx) \left(-\frac{11e \left(-\frac{3e \left(-\frac{7e \int \frac{1}{(a+bx)^2(d+ex)^{5/2}} dx}{4(bd-ae)} - \frac{1}{2(a+bx)^2(d+ex)^{3/2}(bd-ae)} \right)}{2(bd-ae)} - \frac{1}{3(a+bx)^3(d+ex)^{3/2}(bd-ae)} \right)}{8(bd-ae)} - \frac{1}{4(a+bx)^4(d+ex)^{3/2}(bd-ae)} \right)}{\sqrt{a^2 + 2abx + b^2x^2}}
 \end{aligned}$$

$$\begin{aligned}
 & \left((a + bx) \left[\frac{11e}{8(bd - ae)} - \frac{3e \left(\frac{7e \left(-\frac{5e \int \frac{1}{(a+bx)(d+ex)^{5/2}} dx}{2(bd - ae)} - \frac{1}{(a+bx)(d+ex)^{3/2}(bd - ae)} \right)}{4(bd - ae)} - \frac{1}{2(a+bx)^2(d+ex)^{3/2}(bd - ae)} \right)}{2(bd - ae)} - \frac{1}{3(a+bx)^3(d+ex)^{3/2}(bd - ae)} \right] \right)
 \end{aligned}$$

$$\sqrt{a^2 + 2abx + b^2x^2}$$

↓ 61

$$\left((a+bx) \left[\frac{11e}{2(bd-ae)} - \frac{3e}{4(bd-ae)} \left(\frac{7e}{2(bd-ae)} \left(\frac{5e}{3(d+ex)^{3/2}(bd-ae)} + \frac{b \int \frac{1}{(a+bx)(d+ex)^{3/2}} dx}{bd-ae} \right) - \frac{1}{(a+bx)(d+ex)^{3/2}(bd-ae)} \right) - \frac{1}{2(a+bx)^2(d+ex)^{3/2}(bd-ae)} \right] \right)$$

$$\sqrt{a^2 + 2abx + b^2x^2}$$

$$\begin{aligned}
 & \left(\frac{5e}{bd-ae} \left(b \left(\frac{\int \frac{1}{(a+bx)\sqrt{d+ex}} dx}{bd-ae} + \frac{2}{\sqrt{d+ex}(bd-ae)} \right) + \frac{2}{3(d+ex)^{3/2}(bd-ae)} \right) - \frac{1}{(a+bx)(d+ex)^{3/2}(bd-ae)} \right) \\
 & \frac{3e}{4(bd-ae)} - \frac{1}{2(a+bx)^2(d+ex)} \\
 & \frac{11e}{2(bd-ae)} \\
 & \frac{(a+bx)}{8(bd-ae)}
 \end{aligned}$$

↓ 73

$$\begin{aligned}
 & \left(\frac{2b \int \frac{1}{a + \frac{b(d+ex) - bd}{e}} d\sqrt{d+ex}}{e(bd-ae)} + \frac{2}{\sqrt{d+ex}(bd-ae)} \right) \\
 & \frac{5e}{bd-ae} + \frac{2}{3(d+ex)^{3/2}(bd-ae)} \\
 & \frac{7e}{2(bd-ae)} - \frac{1}{(a+bx)(d+ex)^{3/2}(bd-ae)} \\
 & \frac{3e}{4(bd-ae)} - \frac{1}{2(a+bx)(d+ex)^{3/2}(bd-ae)} \\
 & \frac{11e}{2(bd-ae)} \\
 & \frac{(a+bx)}{8(bd-ae)}
 \end{aligned}$$

↓ 221

$$\begin{aligned}
 & \left(\frac{5e}{7e} \left(\frac{b \left(\frac{2}{\sqrt{d+ex}(bd-ae)} - \frac{2\sqrt{b} \operatorname{arctanh} \left(\frac{\sqrt{b}\sqrt{d+ex}}{\sqrt{bd-ae}} \right)}{(bd-ae)^{3/2}} \right)}{bd-ae} + \frac{2}{3(d+ex)^{3/2}(bd-ae)} \right) - \frac{1}{(a+bx)(d+ex)^{3/2}(bd-ae)} \right) \\
 & \frac{3e}{4(bd-ae)} \qquad \qquad \qquad \frac{2(a+bx)}{2(bd-ae)} \\
 & \frac{11e}{2(bd-ae)} \\
 & \frac{(a+bx)}{8(bd-ae)}
 \end{aligned}$$

input `Int[1/((d + e*x)^(5/2)*(a^2 + 2*a*b*x + b^2*x^2)^(5/2)),x]`

output `((a + b*x)*(-1/4*1/((b*d - a*e)*(a + b*x)^4*(d + e*x)^(3/2)) - (11*e*(-1/3*1/((b*d - a*e)*(a + b*x)^3*(d + e*x)^(3/2)) - (3*e*(-1/2*1/((b*d - a*e)*(a + b*x)^2*(d + e*x)^(3/2)) - (7*e*(-1/((b*d - a*e)*(a + b*x)*(d + e*x)^(3/2))) - (5*e*(2/(3*(b*d - a*e)*(d + e*x)^(3/2)) + (b*(2/((b*d - a*e)*Sqrt[d + e*x]) - (2*Sqrt[b]*ArcTanh[(Sqrt[b]*Sqrt[d + e*x])/Sqrt[b*d - a*e]])/(b*d - a*e)^(3/2)))/(b*d - a*e)))/(2*(b*d - a*e)))/(4*(b*d - a*e)))/(2*(b*d - a*e)))/(8*(b*d - a*e)))/Sqrt[a^2 + 2*a*b*x + b^2*x^2]`

Defintions of rubi rules used

rule 27 `Int[(a_)*(F_x_), x_Symbol] := Simp[a Int[F_x, x], x] /; FreeQ[a, x] && !MatchQ[F_x, (b_)*(G_x_)] /; FreeQ[b, x]`

rule 52 `Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] := Simp[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((m + n + 2)/((b*c - a*d)*(m + 1)) Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && ILtQ[m, -1] && FractionQ[n] && LtQ[n, 0]`

rule 61 `Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] := Simp[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((m + n + 2)/((b*c - a*d)*(m + 1)) Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && LtQ[m, -1] && !(LtQ[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && IntLinearQ[a, b, c, d, m, n, x]`

rule 73 `Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] := With[{p = Denominator[m]}, Simp[p/b Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) + d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x] /; FreeQ[{a, b, c, d}, x] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntLinearQ[a, b, c, d, m, n, x]`

rule 221 `Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[-a/b, 2]/a)*ArcTanh[x/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]`

rule 1102 `Int[((d_) + (e_)*(x_)^(m_))*((a_) + (b_)*(x_) + (c_)*(x_)^2)^(p_), x_Symbol] := Simp[(a + b*x + c*x^2)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x)^(2*FracPart[p])) Int[(d + e*x)^m*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && EqQ[b^2 - 4*a*c, 0]`

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 762 vs. $2(268) = 536$.

Time = 1.52 (sec) , antiderivative size = 763, normalized size of antiderivative = 2.00

| method | result |
|---------|---|
| default | $\frac{\left(12705\sqrt{b(ae-bd)} a b^4 e^5 x^4 + 4620\sqrt{b(ae-bd)} b^5 d e^4 x^4 + 88\sqrt{b(ae-bd)} b^5 d^4 e x + 3465(e x + d)^{\frac{3}{2}} \arctan\left(\frac{b\sqrt{e x + d}}{\sqrt{b(ae-bd)}}\right) a^4 b^2 e^4 + 12782\right)}{...}$ |

input `int(1/(e*x+d)^(5/2)/(b^2*x^2+2*a*b*x+a^2)^(5/2),x,method=_RETURNVERBOSE)`

output

```

1/192*(12705*(b*(a*e-b*d))^(1/2)*a*b^4*e^5*x^4+4620*(b*(a*e-b*d))^(1/2)*b^
5*d*e^4*x^4+88*(b*(a*e-b*d))^(1/2)*b^5*d^4*e*x+3465*(e*x+d)^(3/2)*arctan(b
*(e*x+d)^(1/2)/(b*(a*e-b*d))^(1/2))*a^4*b^2*e^4+12782*(b*(a*e-b*d))^(1/2)*
a^3*b^2*d*e^4*x+2048*(b*(a*e-b*d))^(1/2)*a^4*b*d*e^4+9207*(b*(a*e-b*d))^(1
/2)*a^3*b^2*e^5*x^2+3465*(b*(a*e-b*d))^(1/2)*b^5*e^5*x^5+16863*(b*(a*e-b*d
))^(1/2)*a^2*b^3*e^5*x^3+2673*(b*(a*e-b*d))^(1/2)*a*b^4*d^2*e^3*x^2+1408*(
b*(a*e-b*d))^(1/2)*a^4*b*e^5*x+13860*(e*x+d)^(3/2)*arctan(b*(e*x+d)^(1/2)/
(b*(a*e-b*d))^(1/2))*a^3*b^3*e^4*x+22968*(b*(a*e-b*d))^(1/2)*a^2*b^3*d*e^4
*x^2-748*(b*(a*e-b*d))^(1/2)*a*b^4*d^3*e^2*x+3465*(e*x+d)^(3/2)*arctan(b*(
e*x+d)^(1/2)/(b*(a*e-b*d))^(1/2))*b^6*e^4*x^4+17094*(b*(a*e-b*d))^(1/2)*a*
b^4*d*e^4*x^3+693*(b*(a*e-b*d))^(1/2)*b^5*d^2*e^3*x^3-198*(b*(a*e-b*d))^(1
/2)*b^5*d^3*e^2*x^2+20790*(e*x+d)^(3/2)*arctan(b*(e*x+d)^(1/2)/(b*(a*e-b*d
))^(1/2))*a^2*b^4*e^4*x^2+13860*(e*x+d)^(3/2)*arctan(b*(e*x+d)^(1/2)/(b*(a
*e-b*d))^(1/2))*a*b^5*e^4*x^3+3795*(b*(a*e-b*d))^(1/2)*a^2*b^3*d^2*e^3*x+2
295*(b*(a*e-b*d))^(1/2)*a^3*b^2*d^2*e^3-1030*(b*(a*e-b*d))^(1/2)*a^2*b^3*d
^3*e^2+328*(b*(a*e-b*d))^(1/2)*a*b^4*d^4*e-128*(b*(a*e-b*d))^(1/2)*a^5*e^5
-48*(b*(a*e-b*d))^(1/2)*b^5*d^5)*(b*x+a)/(e*x+d)^(3/2)/(b*(a*e-b*d))^(1/2)
/(a*e-b*d)^6/((b*x+a)^2)^(5/2)

```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1222 vs. $2(268) = 536$.

Time = 0.36 (sec) , antiderivative size = 2474, normalized size of antiderivative = 6.49

$$\int \frac{1}{(d+ex)^{5/2}(a^2+2abx+b^2x^2)^{5/2}} dx = \text{Too large to display}$$

input

```

integrate(1/(e*x+d)^(5/2)/(b^2*x^2+2*a*b*x+a^2)^(5/2),x, algorithm="fricas
")

```

output

```
[1/384*(3465*(b^5*e^6*x^6 + a^4*b*d^2*e^4 + 2*(b^5*d*e^5 + 2*a*b^4*e^6)*x^5 + (b^5*d^2*e^4 + 8*a*b^4*d*e^5 + 6*a^2*b^3*e^6)*x^4 + 4*(a*b^4*d^2*e^4 + 3*a^2*b^3*d*e^5 + a^3*b^2*e^6)*x^3 + (6*a^2*b^3*d^2*e^4 + 8*a^3*b^2*d*e^5 + a^4*b*e^6)*x^2 + 2*(2*a^3*b^2*d^2*e^4 + a^4*b*d*e^5)*x)*sqrt(b/(b*d - a*e))*log((b*e*x + 2*b*d - a*e - 2*(b*d - a*e)*sqrt(e*x + d)*sqrt(b/(b*d - a*e)))/(b*x + a)) + 2*(3465*b^5*e^5*x^5 - 48*b^5*d^5 + 328*a*b^4*d^4*e - 1030*a^2*b^3*d^3*e^2 + 2295*a^3*b^2*d^2*e^3 + 2048*a^4*b*d*e^4 - 128*a^5*e^5 + 1155*(4*b^5*d*e^4 + 11*a*b^4*e^5)*x^4 + 231*(3*b^5*d^2*e^3 + 74*a*b^4*d*e^4 + 73*a^2*b^3*e^5)*x^3 - 99*(2*b^5*d^3*e^2 - 27*a*b^4*d^2*e^3 - 232*a^2*b^3*d*e^4 - 93*a^3*b^2*e^5)*x^2 + 11*(8*b^5*d^4*e - 68*a*b^4*d^3*e^2 + 345*a^2*b^3*d^2*e^3 + 1162*a^3*b^2*d*e^4 + 128*a^4*b*e^5)*x)*sqrt(e*x + d))/(a^4*b^6*d^8 - 6*a^5*b^5*d^7*e + 15*a^6*b^4*d^6*e^2 - 20*a^7*b^3*d^5*e^3 + 15*a^8*b^2*d^4*e^4 - 6*a^9*b*d^3*e^5 + a^10*d^2*e^6 + (b^10*d^6*e^2 - 6*a*b^9*d^5*e^3 + 15*a^2*b^8*d^4*e^4 - 20*a^3*b^7*d^3*e^5 + 15*a^4*b^6*d^2*e^6 - 6*a^5*b^5*d*e^7 + a^6*b^4*e^8)*x^6 + 2*(b^10*d^7*e - 4*a*b^9*d^6*e^2 + 3*a^2*b^8*d^5*e^3 + 10*a^3*b^7*d^4*e^4 - 25*a^4*b^6*d^3*e^5 + 24*a^5*b^5*d^2*e^6 - 11*a^6*b^4*d*e^7 + 2*a^7*b^3*e^8)*x^5 + (b^10*d^8 + 2*a*b^9*d^7*e - 27*a^2*b^8*d^6*e^2 + 64*a^3*b^7*d^5*e^3 - 55*a^4*b^6*d^4*e^4 - 6*a^5*b^5*d^3*e^5 + 43*a^6*b^4*d^2*e^6 - 28*a^7*b^3*d*e^7 + 6*a^8*b^2*e^8)*x^4 + 4*(a*b^9*d^8 - 3*a^2*b^8*d^7*e - 2*a^3*b^7*d^6*e^2 + 19*a^4*b^6*d^5*e...
```

SymPy [F]

$$\int \frac{1}{(d+ex)^{5/2} (a^2+2abx+b^2x^2)^{5/2}} dx = \int \frac{1}{(d+ex)^{5/2} ((a+bx)^2)^{5/2}} dx$$

input

```
integrate(1/(e*x+d)**(5/2)/(b**2*x**2+2*a*b*x+a**2)**(5/2), x)
```

output

```
Integral(1/((d + e*x)**(5/2)*((a + b*x)**2)**(5/2)), x)
```


Maxima [F]

$$\int \frac{1}{(d+ex)^{5/2}(a^2+2abx+b^2x^2)^{5/2}} dx = \int \frac{1}{(b^2x^2+2abx+a^2)^{5/2}(ex+d)^{5/2}} dx$$

input `integrate(1/(e*x+d)^(5/2)/(b^2*x^2+2*a*b*x+a^2)^(5/2),x, algorithm="maxima")`

output `integrate(1/((b^2*x^2 + 2*a*b*x + a^2)^(5/2)*(e*x + d)^(5/2)), x)`

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 635 vs. $2(268) = 536$.

Time = 0.17 (sec) , antiderivative size = 635, normalized size of antiderivative = 1.67

$$\int \frac{1}{(d+ex)^{5/2}(a^2+2abx+b^2x^2)^{5/2}} dx = \text{Too large to display}$$

input `integrate(1/(e*x+d)^(5/2)/(b^2*x^2+2*a*b*x+a^2)^(5/2),x, algorithm="giac")`

output `1155/64*b^2*e^4*arctan(sqrt(e*x + d)*b/sqrt(-b^2*d + a*b*e))/((b^6*d^6*sgn(b*x + a) - 6*a*b^5*d^5*e*sgn(b*x + a) + 15*a^2*b^4*d^4*e^2*sgn(b*x + a) - 20*a^3*b^3*d^3*e^3*sgn(b*x + a) + 15*a^4*b^2*d^2*e^4*sgn(b*x + a) - 6*a^5*b*d*e^5*sgn(b*x + a) + a^6*e^6*sgn(b*x + a))*sqrt(-b^2*d + a*b*e)) + 2/3*(15*(e*x + d)*b*e^4 + b*d*e^4 - a*e^5)/((b^6*d^6*sgn(b*x + a) - 6*a*b^5*d^5*e*sgn(b*x + a) + 15*a^2*b^4*d^4*e^2*sgn(b*x + a) - 20*a^3*b^3*d^3*e^3*sgn(b*x + a) + 15*a^4*b^2*d^2*e^4*sgn(b*x + a) - 6*a^5*b*d*e^5*sgn(b*x + a) + a^6*e^6*sgn(b*x + a))*(e*x + d)^(3/2)) + 1/192*(1545*(e*x + d)^(7/2)*b^5*e^4 - 5153*(e*x + d)^(5/2)*b^5*d*e^4 + 5855*(e*x + d)^(3/2)*b^5*d^2*e^4 - 2295*sqrt(e*x + d)*b^5*d^3*e^4 + 5153*(e*x + d)^(5/2)*a*b^4*e^5 - 11710*(e*x + d)^(3/2)*a*b^4*d*e^5 + 6885*sqrt(e*x + d)*a*b^4*d^2*e^5 + 5855*(e*x + d)^(3/2)*a^2*b^3*e^6 - 6885*sqrt(e*x + d)*a^2*b^3*d*e^6 + 2295*sqrt(e*x + d)*a^3*b^2*e^7)/((b^6*d^6*sgn(b*x + a) - 6*a*b^5*d^5*e*sgn(b*x + a) + 15*a^2*b^4*d^4*e^2*sgn(b*x + a) - 20*a^3*b^3*d^3*e^3*sgn(b*x + a) + 15*a^4*b^2*d^2*e^4*sgn(b*x + a) - 6*a^5*b*d*e^5*sgn(b*x + a) + a^6*e^6*sgn(b*x + a)))*((e*x + d)*b - b*d + a*e)^4)`

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(d+ex)^{5/2} (a^2+2abx+b^2x^2)^{5/2}} dx = \int \frac{1}{(d+ex)^{5/2} (a^2+2abx+b^2x^2)^{5/2}} dx$$

input `int(1/((d + e*x)^(5/2)*(a^2 + b^2*x^2 + 2*a*b*x)^(5/2)),x)`

output `int(1/((d + e*x)^(5/2)*(a^2 + b^2*x^2 + 2*a*b*x)^(5/2)), x)`

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 1661, normalized size of antiderivative = 4.36

$$\int \frac{1}{(d+ex)^{5/2} (a^2+2abx+b^2x^2)^{5/2}} dx = \text{Too large to display}$$

input `int(1/(e*x+d)^(5/2)/(b^2*x^2+2*a*b*x+a^2)^(5/2),x)`

output

```
(3465*sqrt(b)*sqrt(d + e*x)*sqrt(a*e - b*d)*atan((sqrt(d + e*x)*b)/(sqrt(b)
)*sqrt(a*e - b*d)))*a**4*b*d**e**4 + 3465*sqrt(b)*sqrt(d + e*x)*sqrt(a*e -
b*d)*atan((sqrt(d + e*x)*b)/(sqrt(b)*sqrt(a*e - b*d)))*a**4*b*e**5*x + 138
60*sqrt(b)*sqrt(d + e*x)*sqrt(a*e - b*d)*atan((sqrt(d + e*x)*b)/(sqrt(b)*s
qrt(a*e - b*d)))*a**3*b**2*d**e**4*x + 13860*sqrt(b)*sqrt(d + e*x)*sqrt(a*e
- b*d)*atan((sqrt(d + e*x)*b)/(sqrt(b)*sqrt(a*e - b*d)))*a**3*b**2*e**5*x
**2 + 20790*sqrt(b)*sqrt(d + e*x)*sqrt(a*e - b*d)*atan((sqrt(d + e*x)*b)/(
sqrt(b)*sqrt(a*e - b*d)))*a**2*b**3*d**e**4*x**2 + 20790*sqrt(b)*sqrt(d + e
*x)*sqrt(a*e - b*d)*atan((sqrt(d + e*x)*b)/(sqrt(b)*sqrt(a*e - b*d)))*a**2
*b**3*e**5*x**3 + 13860*sqrt(b)*sqrt(d + e*x)*sqrt(a*e - b*d)*atan((sqrt(d
+ e*x)*b)/(sqrt(b)*sqrt(a*e - b*d)))*a*b**4*d**e**4*x**3 + 13860*sqrt(b)*s
qrt(d + e*x)*sqrt(a*e - b*d)*atan((sqrt(d + e*x)*b)/(sqrt(b)*sqrt(a*e - b*
d)))*a*b**4*e**5*x**4 + 3465*sqrt(b)*sqrt(d + e*x)*sqrt(a*e - b*d)*atan((s
qrt(d + e*x)*b)/(sqrt(b)*sqrt(a*e - b*d)))*b**5*d**e**4*x**4 + 3465*sqrt(b)
*sqrt(d + e*x)*sqrt(a*e - b*d)*atan((sqrt(d + e*x)*b)/(sqrt(b)*sqrt(a*e -
b*d)))*b**5*e**5*x**5 - 128*a**6*e**6 + 2176*a**5*b*d**e**5 + 1408*a**5*b*e
**6*x + 247*a**4*b**2*d**2*e**4 + 11374*a**4*b**2*d**e**5*x + 9207*a**4*b**
2*e**6*x**2 - 3325*a**3*b**3*d**3*e**3 - 8987*a**3*b**3*d**2*e**4*x + 1376
1*a**3*b**3*d**e**5*x**2 + 16863*a**3*b**3*e**6*x**3 + 1358*a**2*b**4*d**4*
e**2 - 4543*a**2*b**4*d**3*e**3*x - 20295*a**2*b**4*d**2*e**4*x**2 + 23...
```

3.398 $\int \frac{1}{(d+ex)^{7/2}(a^2+2abx+b^2x^2)^{5/2}} dx$

| | |
|---|------|
| Optimal result | 2999 |
| Mathematica [A] (verified) | 3000 |
| Rubi [A] (verified) | 3001 |
| Maple [B] (verified) | 3012 |
| Fricas [B] (verification not implemented) | 3013 |
| Sympy [F] | 3014 |
| Maxima [F] | 3014 |
| Giac [B] (verification not implemented) | 3014 |
| Mupad [F(-1)] | 3015 |
| Reduce [B] (verification not implemented) | 3016 |

Optimal result

Integrand size = 30, antiderivative size = 435

$$\int \frac{1}{(d+ex)^{7/2}(a^2+2abx+b^2x^2)^{5/2}} dx = \frac{429e^3}{64(bd-ae)^4(d+ex)^{5/2}\sqrt{a^2+2abx+b^2x^2}}$$

$$- \frac{1}{4(bd-ae)(a+bx)^3(d+ex)^{5/2}\sqrt{a^2+2abx+b^2x^2}}$$

$$+ \frac{13e}{24(bd-ae)^2(a+bx)^2(d+ex)^{5/2}\sqrt{a^2+2abx+b^2x^2}}$$

$$- \frac{143e^2}{96(bd-ae)^3(a+bx)(d+ex)^{5/2}\sqrt{a^2+2abx+b^2x^2}}$$

$$+ \frac{3003e^4(a+bx)}{320(bd-ae)^5(d+ex)^{5/2}\sqrt{a^2+2abx+b^2x^2}}$$

$$+ \frac{1001be^4(a+bx)}{64(bd-ae)^6(d+ex)^{3/2}\sqrt{a^2+2abx+b^2x^2}}$$

$$+ \frac{3003b^2e^4(a+bx)}{64(bd-ae)^7\sqrt{d+ex}\sqrt{a^2+2abx+b^2x^2}} - \frac{3003b^{5/2}e^4(a+bx)\operatorname{arctanh}\left(\frac{\sqrt{b}\sqrt{d+ex}}{\sqrt{bd-ae}}\right)}{64(bd-ae)^{15/2}\sqrt{a^2+2abx+b^2x^2}}$$

output

$$\begin{aligned} & 429/64*e^3/(-a*e+b*d)^4/(e*x+d)^(5/2)/((b*x+a)^2)^(1/2)-1/4/(-a*e+b*d)/(b* \\ & x+a)^3/(e*x+d)^(5/2)/((b*x+a)^2)^(1/2)+13/24*e/(-a*e+b*d)^2/(b*x+a)^2/(e*x \\ & +d)^(5/2)/((b*x+a)^2)^(1/2)-143/96*e^2/(-a*e+b*d)^3/(b*x+a)/(e*x+d)^(5/2)/ \\ & ((b*x+a)^2)^(1/2)+3003/320*e^4*(b*x+a)/(-a*e+b*d)^5/(e*x+d)^(5/2)/((b*x+a) \\ & ^2)^(1/2)+1001/64*b*e^4*(b*x+a)/(-a*e+b*d)^6/(e*x+d)^(3/2)/((b*x+a)^2)^(1/2) \\ & +3003/64*b^2*e^4*(b*x+a)/(-a*e+b*d)^7/(e*x+d)^(1/2)/((b*x+a)^2)^(1/2)-30 \\ & 03/64*b^(5/2)*e^4*(b*x+a)*arctanh(b^(1/2)*(e*x+d)^(1/2)/(-a*e+b*d)^(1/2))/ \\ & (-a*e+b*d)^(15/2)/((b*x+a)^2)^(1/2) \end{aligned}$$
Mathematica [A] (verified)

Time = 1.87 (sec) , antiderivative size = 378, normalized size of antiderivative = 0.87

$$\int \frac{1}{(d+ex)^{7/2}(a^2+2abx+b^2x^2)^{5/2}} dx = \frac{e^4(a+bx)^5 \left(\frac{-384a^6e^6+128a^5be^5(31d+13ex)-128a^4b^2e^4(253d^2+351dex+143e^2}{(d+ex)^{7/2}(a^2+2abx+b^2x^2)^{5/2}} \right)}{e^4(a+bx)^5}$$

input

`Integrate[1/((d + e*x)^(7/2)*(a^2 + 2*a*b*x + b^2*x^2)^(5/2)),x]`

output

$$\begin{aligned} & (e^4*(a + b*x)^5*((-384*a^6*e^6 + 128*a^5*b*e^5*(31*d + 13*e*x) - 128*a^4*b^2*e^4*(253*d^2 + 351*d*e*x + 143*e^2*x^2) - a^3*b^3*e^3*(22155*d^3 + 196 \\ & 001*d^2*e*x + 285857*d*e^2*x^2 + 119691*e^3*x^3) - a^2*b^4*e^2*(-7630*d^4 \\ & + 35945*d^3*e*x + 347919*d^2*e^2*x^2 + 517803*d*e^3*x^3 + 219219*e^4*x^4) \\ & - a*b^5*e*(1960*d^5 - 5460*d^4*e*x + 25025*d^3*e^2*x^2 + 256971*d^2*e^3*x^3 + 387387*d*e^4*x^4 + 165165*e^5*x^5) + b^6*(240*d^6 - 520*d^5*e*x + 1430 \\ & *d^4*e^2*x^2 - 6435*d^3*e^3*x^3 - 69069*d^2*e^4*x^4 - 105105*d*e^5*x^5 - 4 \\ & 5045*e^6*x^6))/(e^4*(-(b*d) + a*e)^7*(a + b*x)^4*(d + e*x)^(5/2)) - (45045 \\ & *b^(5/2)*ArcTan[(Sqrt[b]*Sqrt[d + e*x])/Sqrt[-(b*d) + a*e]]/(-(b*d) + a*e) \\ & ^{(15/2)))/(960*((a + b*x)^2)^(5/2)) \end{aligned}$$

Rubi [A] (verified)

Time = 0.68 (sec) , antiderivative size = 349, normalized size of antiderivative = 0.80, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.367$, Rules used = {1102, 27, 52, 52, 52, 61, 61, 61, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\begin{aligned}
 & \int \frac{1}{(a^2 + 2abx + b^2x^2)^{5/2} (d + ex)^{7/2}} dx \\
 & \quad \downarrow \text{1102} \\
 & \frac{b^5(a + bx) \int \frac{1}{b^5(a+bx)^5(d+ex)^{7/2}} dx}{\sqrt{a^2 + 2abx + b^2x^2}} \\
 & \quad \downarrow \text{27} \\
 & \frac{(a + bx) \int \frac{1}{(a+bx)^5(d+ex)^{7/2}} dx}{\sqrt{a^2 + 2abx + b^2x^2}} \\
 & \quad \downarrow \text{52} \\
 & \frac{(a + bx) \left(-\frac{13e \int \frac{1}{(a+bx)^4(d+ex)^{7/2}} dx}{8(bd-ae)} - \frac{1}{4(a+bx)^4(d+ex)^{5/2}(bd-ae)} \right)}{\sqrt{a^2 + 2abx + b^2x^2}} \\
 & \quad \downarrow \text{52} \\
 & \frac{(a + bx) \left(-\frac{13e \left(-\frac{11e \int \frac{1}{(a+bx)^3(d+ex)^{7/2}} dx}{6(bd-ae)} - \frac{1}{3(a+bx)^3(d+ex)^{5/2}(bd-ae)} \right)}{8(bd-ae)} - \frac{1}{4(a+bx)^4(d+ex)^{5/2}(bd-ae)} \right)}{\sqrt{a^2 + 2abx + b^2x^2}} \\
 & \quad \downarrow \text{52}
 \end{aligned}$$

$$(a + bx) \left(- \frac{13e \left(- \frac{11e \left(- \frac{9e \int \frac{1}{(a+bx)^2(d+ex)^{7/2}} dx}{4(bd-ae)} - \frac{1}{2(a+bx)^2(d+ex)^{5/2}(bd-ae)} \right)}{6(bd-ae)} - \frac{1}{3(a+bx)^3(d+ex)^{5/2}(bd-ae)} \right)}{8(bd-ae)} - \frac{1}{4(a+bx)^4(d+ex)^{5/2}(bd-ae)} \right)$$

$$\sqrt{a^2 + 2abx + b^2x^2}$$

↓ 52

$$(a + bx) \left(- \frac{13e \left(- \frac{11e \left(- \frac{9e \int \frac{1}{(a+bx)(d+ex)^{7/2}} dx}{2(bd-ae)} - \frac{1}{(a+bx)(d+ex)^{5/2}(bd-ae)} \right)}{4(bd-ae)} - \frac{1}{2(a+bx)^2(d+ex)^{5/2}(bd-ae)} \right)}{6(bd-ae)} - \frac{1}{3(a+bx)^3(d+ex)^{5/2}(bd-ae)} \right)$$

$$\sqrt{a^2 + 2abx + b^2x^2}$$

↓ 61

$$\begin{aligned}
 & \left(\begin{aligned} & \left(\begin{aligned} & 9e \left(\frac{7e \left(\frac{b \int \frac{1}{(a+bx)(d+ex)^{5/2}} dx}{bd-ae} + \frac{2}{5(d+ex)^{5/2}(bd-ae)} \right)}{2(bd-ae)} - \frac{1}{(a+bx)(d+ex)^{5/2}(bd-ae)} \right) \\ & 11e - \frac{\hspace{15em}}{4(bd-ae)} - \frac{1}{2(a+bx)^2(d+ex)^{5/2}(bd-ae)} \end{aligned} \right) \\ & 13e - \frac{\hspace{15em}}{6(bd-ae)} \end{aligned} \right) \\
 & (a+bx) - \frac{\hspace{15em}}{8(bd-ae)} \\
 & \hspace{15em} \sqrt{a^2 + 2abx + b^2x^2}
 \end{aligned}$$

$$\begin{aligned}
 & \left(\left(\left(\left(\left(\frac{b \int \frac{1}{(a+bx)(d+ex)^{3/2}} dx}{bd-ae} + \frac{2}{3(d+ex)^{3/2}(bd-ae)} \right) + \frac{2}{5(d+ex)^{5/2}(bd-ae)} \right) - \frac{1}{(a+bx)(d+ex)^{5/2}(bd-ae)} \right) \right. \\
 & \left. - \frac{7e}{9e} \frac{\left(\frac{b \int \frac{1}{(a+bx)(d+ex)^{3/2}} dx}{bd-ae} + \frac{2}{3(d+ex)^{3/2}(bd-ae)} + \frac{2}{5(d+ex)^{5/2}(bd-ae)} \right)}{2(bd-ae)} \right) \\
 & \left. - \frac{11e}{11e} \frac{\left(\frac{b \int \frac{1}{(a+bx)(d+ex)^{3/2}} dx}{bd-ae} + \frac{2}{3(d+ex)^{3/2}(bd-ae)} + \frac{2}{5(d+ex)^{5/2}(bd-ae)} \right)}{4(bd-ae)} \right) \\
 & \left. - \frac{13e}{13e} \frac{\left(\frac{b \int \frac{1}{(a+bx)(d+ex)^{3/2}} dx}{bd-ae} + \frac{2}{3(d+ex)^{3/2}(bd-ae)} + \frac{2}{5(d+ex)^{5/2}(bd-ae)} \right)}{6(bd-ae)} \right) \\
 & \left. - \frac{(a+bx)}{(a+bx)} \frac{\left(\frac{b \int \frac{1}{(a+bx)(d+ex)^{3/2}} dx}{bd-ae} + \frac{2}{3(d+ex)^{3/2}(bd-ae)} + \frac{2}{5(d+ex)^{5/2}(bd-ae)} \right)}{8(bd-ae)} \right)
 \end{aligned}$$

↓ 61

$$\begin{aligned}
 & \left(\left(\left(\left(\frac{b \int \frac{1}{(a+bx)\sqrt{d+ex}} dx}{bd-ae} + \frac{2}{\sqrt{d+ex}(bd-ae)} \right) + \frac{2}{3(d+ex)^{3/2}(bd-ae)} \right) + \frac{2}{5(d+ex)^{5/2}(bd-ae)} \right) \right. \\
 7e & \left. \frac{b}{bd-ae} \right) \\
 9e & \frac{\phantom{\left(\left(\left(\left(\frac{b \int \frac{1}{(a+bx)\sqrt{d+ex}} dx}{bd-ae} + \frac{2}{\sqrt{d+ex}(bd-ae)} \right) + \frac{2}{3(d+ex)^{3/2}(bd-ae)} \right) + \frac{2}{5(d+ex)^{5/2}(bd-ae)} \right) \frac{b}{bd-ae}}}{2(bd-ae)} - \frac{1}{(a+bx)(d+ex)^{5/2}} \\
 11e & \frac{\phantom{\left(\left(\left(\left(\frac{b \int \frac{1}{(a+bx)\sqrt{d+ex}} dx}{bd-ae} + \frac{2}{\sqrt{d+ex}(bd-ae)} \right) + \frac{2}{3(d+ex)^{3/2}(bd-ae)} \right) + \frac{2}{5(d+ex)^{5/2}(bd-ae)} \right) \frac{b}{bd-ae}}}{4(bd-ae)} \\
 13e & \frac{\phantom{\left(\left(\left(\left(\frac{b \int \frac{1}{(a+bx)\sqrt{d+ex}} dx}{bd-ae} + \frac{2}{\sqrt{d+ex}(bd-ae)} \right) + \frac{2}{3(d+ex)^{3/2}(bd-ae)} \right) + \frac{2}{5(d+ex)^{5/2}(bd-ae)} \right) \frac{b}{bd-ae}}}{6(bd-ae)}
 \end{aligned}$$

↓ 73

| | | |
|--|-----|--|
| | | $\left(\frac{b \left(\frac{2b \int \frac{1}{a + \frac{b(d+ex) - bd}{e(bd-ae)}} d\sqrt{d+ex} + \frac{2}{\sqrt{d+ex}(bd-ae)} \right)}{bd-ae} + \frac{2}{3(d+ex)^{3/2}(bd-ae)} \right)$ |
| | 7e | $\frac{bd-ae}{bd-ae} + \frac{2}{5(d+ex)^{5/2}(bd-ae)}$ |
| | 9e | $\frac{2(bd-ae)}{(a+bx)}$ |
| | 11e | $4(bd-ae)$ |
| | 13e | $6(bd-ae)$ |

↓ 221

| | | | |
|--|-----|---|--|
| | | $7e \left(\frac{b \left(\frac{2}{\sqrt{d+ex}(bd-ae)} - \frac{2\sqrt{b}\operatorname{arctanh}\left(\frac{\sqrt{b}\sqrt{d+ex}}{\sqrt{bd-ae}}\right)}{(bd-ae)^{3/2}} \right)}{bd-ae} + \frac{2}{3(d+ex)^{3/2}(bd-ae)} \right)$ | |
| | 9e | $\frac{2(bd-ae)}{(a+bx)(bd-ae)}$ | |
| | 11e | $4(bd-ae)$ | |
| | 13e | $6(bd-ae)$ | |

input `Int[1/((d + e*x)^(7/2)*(a^2 + 2*a*b*x + b^2*x^2)^(5/2)),x]`

output `((a + b*x)*(-1/4*1/((b*d - a*e)*(a + b*x)^4*(d + e*x)^(5/2)) - (13*e*(-1/3*1/((b*d - a*e)*(a + b*x)^3*(d + e*x)^(5/2)) - (11*e*(-1/2*1/((b*d - a*e)*(a + b*x)^2*(d + e*x)^(5/2)) - (9*e*(-1/((b*d - a*e)*(a + b*x)*(d + e*x)^(5/2))) - (7*e*(2/(5*(b*d - a*e)*(d + e*x)^(5/2)) + (b*(2/(3*(b*d - a*e)*(d + e*x)^(3/2)) + (b*(2/((b*d - a*e)*Sqrt[d + e*x])) - (2*Sqrt[b]*ArcTanh[(Sqrt[b]*Sqrt[d + e*x])/Sqrt[b*d - a*e]])/(b*d - a*e)^(3/2)))/(b*d - a*e)))/(b*d - a*e)))/(2*(b*d - a*e)))/(4*(b*d - a*e)))/(6*(b*d - a*e)))/(8*(b*d - a*e)))/Sqrt[a^2 + 2*a*b*x + b^2*x^2]`

Defintions of rubi rules used

rule 27 `Int[(a_)*(F_x_), x_Symbol] := Simp[a Int[F_x, x], x] /; FreeQ[a, x] && !MatchQ[F_x, (b_)*(G_x_)] /; FreeQ[b, x]`

rule 52 `Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] := Simp[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((m + n + 2)/((b*c - a*d)*(m + 1)) Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && ILtQ[m, -1] && FractionQ[n] && LtQ[n, 0]`

rule 61 `Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] := Simp[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((m + n + 2)/((b*c - a*d)*(m + 1)) Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && LtQ[m, -1] && !(LtQ[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && IntLinearQ[a, b, c, d, m, n, x]`

rule 73 `Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] := With[{p = Denominator[m]}, Simp[p/b Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) + d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntLinearQ[a, b, c, d, m, n, x]`

output

```

-1/960*(45045*(e*x+d)^(5/2)*arctan(b*(e*x+d)^(1/2)/(b*(a*e-b*d))^(1/2))*a^
4*b^3*e^4+256971*(b*(a*e-b*d))^(1/2)*a*b^5*d^2*e^4*x^3+347919*(b*(a*e-b*d)
)^(1/2)*a^2*b^4*d^2*e^4*x^2+25025*(b*(a*e-b*d))^(1/2)*a*b^5*d^3*e^3*x^2+19
6001*(b*(a*e-b*d))^(1/2)*a^3*b^3*d^2*e^4*x+35945*(b*(a*e-b*d))^(1/2)*a^2*b
^4*d^3*e^3*x-5460*(b*(a*e-b*d))^(1/2)*a*b^5*d^4*e^2*x+45045*(b*(a*e-b*d))^(
1/2)*b^6*e^6*x^6+270270*(e*x+d)^(5/2)*arctan(b*(e*x+d)^(1/2)/(b*(a*e-b*d)
)^(1/2))*a^2*b^5*e^4*x^2+180180*(e*x+d)^(5/2)*arctan(b*(e*x+d)^(1/2)/(b*(a
*e-b*d))^(1/2))*a^3*b^4*e^4*x+45045*(e*x+d)^(5/2)*arctan(b*(e*x+d)^(1/2)/(
b*(a*e-b*d))^(1/2))*b^7*e^4*x^4+180180*(e*x+d)^(5/2)*arctan(b*(e*x+d)^(1/2)
)/(b*(a*e-b*d))^(1/2))*a*b^6*e^4*x^3+520*(b*(a*e-b*d))^(1/2)*b^6*d^5*e*x+3
2384*(b*(a*e-b*d))^(1/2)*a^4*b^2*d^2*e^4+22155*(b*(a*e-b*d))^(1/2)*a^3*b^3
*d^3*e^3-7630*(b*(a*e-b*d))^(1/2)*a^2*b^4*d^4*e^2+1960*(b*(a*e-b*d))^(1/2)
*a*b^5*d^5*e+285857*(b*(a*e-b*d))^(1/2)*a^3*b^3*d*e^5*x^2+44928*(b*(a*e-b*
d))^(1/2)*a^4*b^2*d*e^5*x+387387*(b*(a*e-b*d))^(1/2)*a*b^5*d*e^5*x^4+51780
3*(b*(a*e-b*d))^(1/2)*a^2*b^4*d*e^5*x^3+69069*(b*(a*e-b*d))^(1/2)*b^6*d^2*
e^4*x^4+6435*(b*(a*e-b*d))^(1/2)*b^6*d^3*e^3*x^3-1430*(b*(a*e-b*d))^(1/2)*
b^6*d^4*e^2*x^2+165165*(b*(a*e-b*d))^(1/2)*a*b^5*e^6*x^5+105105*(b*(a*e-b*
d))^(1/2)*b^6*d*e^5*x^5+219219*(b*(a*e-b*d))^(1/2)*a^2*b^4*e^6*x^4+119691*
(b*(a*e-b*d))^(1/2)*a^3*b^3*e^6*x^3+18304*(b*(a*e-b*d))^(1/2)*a^4*b^2*e^6*
x^2-1664*(b*(a*e-b*d))^(1/2)*a^5*b*e^6*x-3968*(b*(a*e-b*d))^(1/2)*a^5*b...

```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1659 vs. $2(307) = 614$.

Time = 0.69 (sec) , antiderivative size = 3348, normalized size of antiderivative = 7.70

$$\int \frac{1}{(d+ex)^{7/2} (a^2+2abx+b^2x^2)^{5/2}} dx = \text{Too large to display}$$

input

```

integrate(1/(e*x+d)^(7/2)/(b^2*x^2+2*a*b*x+a^2)^(5/2),x, algorithm="fricas
")

```

output

Too large to include

Sympy [F]

$$\int \frac{1}{(d+ex)^{7/2} (a^2+2abx+b^2x^2)^{5/2}} dx = \int \frac{1}{(d+ex)^{7/2} ((a+bx)^2)^{5/2}} dx$$

input `integrate(1/(e*x+d)**(7/2)/(b**2*x**2+2*a*b*x+a**2)**(5/2),x)`

output `Integral(1/((d + e*x)**(7/2)*((a + b*x)**2)**(5/2)), x)`

Maxima [F]

$$\int \frac{1}{(d+ex)^{7/2} (a^2+2abx+b^2x^2)^{5/2}} dx = \int \frac{1}{(b^2x^2+2abx+a^2)^{5/2} (ex+d)^{7/2}} dx$$

input `integrate(1/(e*x+d)^(7/2)/(b^2*x^2+2*a*b*x+a^2)^(5/2),x, algorithm="maxima")`

output `integrate(1/((b^2*x^2 + 2*a*b*x + a^2)^(5/2)*(e*x + d)^(7/2)), x)`

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 743 vs. 2(307) = 614.

Time = 0.17 (sec) , antiderivative size = 743, normalized size of antiderivative = 1.71

$$\int \frac{1}{(d+ex)^{7/2} (a^2+2abx+b^2x^2)^{5/2}} dx = \text{Too large to display}$$

input `integrate(1/(e*x+d)^(7/2)/(b^2*x^2+2*a*b*x+a^2)^(5/2),x, algorithm="giac")`

output

```

3003/64*b^3*e^4*arctan(sqrt(e*x + d)*b/sqrt(-b^2*d + a*b*e))/((b^7*d^7*sgn
(b*x + a) - 7*a*b^6*d^6*e*sgn(b*x + a) + 21*a^2*b^5*d^5*e^2*sgn(b*x + a) -
35*a^3*b^4*d^4*e^3*sgn(b*x + a) + 35*a^4*b^3*d^3*e^4*sgn(b*x + a) - 21*a^
5*b^2*d^2*e^5*sgn(b*x + a) + 7*a^6*b*d*e^6*sgn(b*x + a) - a^7*e^7*sgn(b*x
+ a))*sqrt(-b^2*d + a*b*e)) + 2/15*(225*(e*x + d)^2*b^2*e^4 + 25*(e*x + d)
*b^2*d*e^4 + 3*b^2*d^2*e^4 - 25*(e*x + d)*a*b*e^5 - 6*a*b*d*e^5 + 3*a^2*e^
6)/((b^7*d^7*sgn(b*x + a) - 7*a*b^6*d^6*e*sgn(b*x + a) + 21*a^2*b^5*d^5*e^
2*sgn(b*x + a) - 35*a^3*b^4*d^4*e^3*sgn(b*x + a) + 35*a^4*b^3*d^3*e^4*sgn(
b*x + a) - 21*a^5*b^2*d^2*e^5*sgn(b*x + a) + 7*a^6*b*d*e^6*sgn(b*x + a) -
a^7*e^7*sgn(b*x + a))*(e*x + d)^(5/2)) + 1/192*(3249*(e*x + d)^(7/2)*b^6*e
^4 - 10633*(e*x + d)^(5/2)*b^6*d*e^4 + 11767*(e*x + d)^(3/2)*b^6*d^2*e^4 -
4431*sqrt(e*x + d)*b^6*d^3*e^4 + 10633*(e*x + d)^(5/2)*a*b^5*e^5 - 23534*
(e*x + d)^(3/2)*a*b^5*d*e^5 + 13293*sqrt(e*x + d)*a*b^5*d^2*e^5 + 11767*(e
*x + d)^(3/2)*a^2*b^4*e^6 - 13293*sqrt(e*x + d)*a^2*b^4*d*e^6 + 4431*sqrt(
e*x + d)*a^3*b^3*e^7)/((b^7*d^7*sgn(b*x + a) - 7*a*b^6*d^6*e*sgn(b*x + a)
+ 21*a^2*b^5*d^5*e^2*sgn(b*x + a) - 35*a^3*b^4*d^4*e^3*sgn(b*x + a) + 35*a
^4*b^3*d^3*e^4*sgn(b*x + a) - 21*a^5*b^2*d^2*e^5*sgn(b*x + a) + 7*a^6*b*d*
e^6*sgn(b*x + a) - a^7*e^7*sgn(b*x + a))*((e*x + d)*b - b*d + a*e)^4)

```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(d + ex)^{7/2} (a^2 + 2abx + b^2x^2)^{5/2}} dx = \int \frac{1}{(d + ex)^{7/2} (a^2 + 2abx + b^2x^2)^{5/2}} dx$$

input

```
int(1/((d + e*x)^(7/2)*(a^2 + b^2*x^2 + 2*a*b*x)^(5/2)),x)
```

output

```
int(1/((d + e*x)^(7/2)*(a^2 + b^2*x^2 + 2*a*b*x)^(5/2)), x)
```

Reduce [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 2284, normalized size of antiderivative = 5.25

$$\int \frac{1}{(d+ex)^{7/2}(a^2+2abx+b^2x^2)^{5/2}} dx = \text{Too large to display}$$

input `int(1/(e*x+d)^(7/2)/(b^2*x^2+2*a*b*x+a^2)^(5/2),x)`

output

```
( - 45045*sqrt(b)*sqrt(d + e*x)*sqrt(a*e - b*d)*atan((sqrt(d + e*x)*b)/(sqrt(b)*sqrt(a*e - b*d)))*a**4*b**2*d**2*e**4 - 90090*sqrt(b)*sqrt(d + e*x)*sqrt(a*e - b*d)*atan((sqrt(d + e*x)*b)/(sqrt(b)*sqrt(a*e - b*d)))*a**4*b**2*d**5*x - 45045*sqrt(b)*sqrt(d + e*x)*sqrt(a*e - b*d)*atan((sqrt(d + e*x)*b)/(sqrt(b)*sqrt(a*e - b*d)))*a**4*b**2*e**6*x**2 - 180180*sqrt(b)*sqrt(d + e*x)*sqrt(a*e - b*d)*atan((sqrt(d + e*x)*b)/(sqrt(b)*sqrt(a*e - b*d)))*a**3*b**3*d**2*e**4*x - 360360*sqrt(b)*sqrt(d + e*x)*sqrt(a*e - b*d)*atan((sqrt(d + e*x)*b)/(sqrt(b)*sqrt(a*e - b*d)))*a**3*b**3*d*e**5*x**2 - 180180*sqrt(b)*sqrt(d + e*x)*sqrt(a*e - b*d)*atan((sqrt(d + e*x)*b)/(sqrt(b)*sqrt(a*e - b*d)))*a**3*b**3*e**6*x**3 - 270270*sqrt(b)*sqrt(d + e*x)*sqrt(a*e - b*d)*atan((sqrt(d + e*x)*b)/(sqrt(b)*sqrt(a*e - b*d)))*a**2*b**4*d**2*e**4*x**2 - 540540*sqrt(b)*sqrt(d + e*x)*sqrt(a*e - b*d)*atan((sqrt(d + e*x)*b)/(sqrt(b)*sqrt(a*e - b*d)))*a**2*b**4*d*e**5*x**3 - 270270*sqrt(b)*sqrt(d + e*x)*sqrt(a*e - b*d)*atan((sqrt(d + e*x)*b)/(sqrt(b)*sqrt(a*e - b*d)))*a**2*b**4*e**6*x**4 - 180180*sqrt(b)*sqrt(d + e*x)*sqrt(a*e - b*d)*atan((sqrt(d + e*x)*b)/(sqrt(b)*sqrt(a*e - b*d)))*a*b**5*d**2*e**4*x**3 - 360360*sqrt(b)*sqrt(d + e*x)*sqrt(a*e - b*d)*atan((sqrt(d + e*x)*b)/(sqrt(b)*sqrt(a*e - b*d)))*a*b**5*d*e**5*x**4 - 180180*sqrt(b)*sqrt(d + e*x)*sqrt(a*e - b*d)*atan((sqrt(d + e*x)*b)/(sqrt(b)*sqrt(a*e - b*d)))*a*b**5*e**6*x**5 - 45045*sqrt(b)*sqrt(d + e*x)*sqrt(a*e - b*d)*atan((sqrt(d + e*x)*...
```

3.399 $\int (ad + bdx)^m (a^2c + 2abcx + b^2cx^2)^3 dx$

| | |
|---|------|
| Optimal result | 3017 |
| Mathematica [A] (verified) | 3017 |
| Rubi [A] (verified) | 3018 |
| Maple [A] (verified) | 3019 |
| Fricas [B] (verification not implemented) | 3020 |
| Sympy [B] (verification not implemented) | 3020 |
| Maxima [B] (verification not implemented) | 3021 |
| Giac [B] (verification not implemented) | 3022 |
| Mupad [B] (verification not implemented) | 3023 |
| Reduce [B] (verification not implemented) | 3023 |

Optimal result

Integrand size = 33, antiderivative size = 27

$$\int (ad + bdx)^m (a^2c + 2abcx + b^2cx^2)^3 dx = \frac{c^3(ad + bdx)^{7+m}}{bd^7(7+m)}$$

output

```
c^3*(b*d*x+a*d)^(7+m)/b/d^7/(7+m)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.04

$$\int (ad + bdx)^m (a^2c + 2abcx + b^2cx^2)^3 dx = \frac{c^3(a + bx)^7(d(a + bx))^m}{b(7+m)}$$

input

```
Integrate[(a*d + b*d*x)^m*(a^2*c + 2*a*b*c*x + b^2*c*x^2)^3,x]
```

output

```
(c^3*(a + b*x)^7*(d*(a + b*x))^m)/(b*(7 + m))
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.121$, Rules used = {1098, 27, 35, 17}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a^2c + 2abcx + b^2cx^2)^3 (ad + bdx)^m dx$$

$$\downarrow 1098$$

$$\frac{\int b^6c^6(a + bx)^6(ad + bxd)^m dx}{b^6c^3}$$

$$\downarrow 27$$

$$c^3 \int (a + bx)^6(ad + bxd)^m dx$$

$$\downarrow 35$$

$$\frac{c^3 \int (ad + bxd)^{m+6} dx}{d^6}$$

$$\downarrow 17$$

$$\frac{c^3(ad + bdx)^{m+7}}{bd^7(m + 7)}$$

input

$$\text{Int}[(a*d + b*d*x)^m*(a^2*c + 2*a*b*c*x + b^2*c*x^2)^3,x]$$

output

$$(c^3*(a*d + b*d*x)^(7 + m))/(b*d^7*(7 + m))$$

Definitions of rubi rules used

rule 17 $\text{Int}[(c_.)*((a_.) + (b_.)*(x_))^{(m_.)}, x_Symbol] \rightarrow \text{Simp}[c*((a + b*x)^{(m + 1})/(b*(m + 1))), x] /;$ $\text{FreeQ}\{a, b, c, m\}, x] \ \&\& \ \text{NeQ}[m, -1]$

rule 27 $\text{Int}[(a_)*(F x_), x_Symbol] \rightarrow \text{Simp}[a \ \text{Int}[F x, x], x] /;$ $\text{FreeQ}[a, x] \ \&\& \ !\text{MatchQ}[F x, (b_)*(G x_)] /;$ $\text{FreeQ}[b, x]$

rule 35 $\text{Int}[(u_.)*((a_.) + (b_.)*(x_))^{(m_.)}*((c_.) + (d_.)*(x_))^{(n_.)}, x_Symbol] \rightarrow \text{Simp}[(b/d)^m \ \text{Int}[u*(c + d*x)^{(m + n)}, x], x] /;$ $\text{FreeQ}\{a, b, c, d, m, n\}, x] \ \&\& \ \text{EqQ}[b*c - a*d, 0] \ \&\& \ \text{IntegerQ}[m] \ \&\& \ !(\text{IntegerQ}[n] \ \&\& \ \text{SimplerQ}[a + b*x, c + d*x])$

rule 1098 $\text{Int}[(d_.) + (e_.)*(x_))^{(m_.)}*((a_.) + (b_.)*(x_.) + (c_.)*(x_.)^2)^{(p_.)}, x_Symbol] \rightarrow \text{Simp}[1/c^p \ \text{Int}[(d + e*x)^m*(b/2 + c*x)^{(2*p)}, x], x] /;$ $\text{FreeQ}\{a, b, c, d, e, m\}, x] \ \&\& \ \text{EqQ}[b^2 - 4*a*c, 0] \ \&\& \ \text{IntegerQ}[p]$

Maple [A] (verified)

Time = 1.55 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.70

| method | result |
|--------------|--|
| gospers | $\frac{(bx+a)(bdx+ad)^m c^3 (b^2 x^2 + 2abx + a^2)^3}{b(7+m)}$ |
| orering | $\frac{(bx+a)(bdx+ad)^m (b^2 c x^2 + 2abcx + a^2 c)^3}{b(7+m)}$ |
| risch | $\frac{c^3 (x^7 b^7 + 7b^6 a x^6 + 21a^2 x^5 b^5 + 35b^4 x^4 a^3 + 35b^3 x^3 a^4 + 21x^2 b^2 a^5 + 7x b a^6 + a^7) (d(bx+a))^m}{b(7+m)}$ |
| parallelrisc | $\frac{x^7 (d(bx+a))^m b^7 c^3 + 7x^6 (d(bx+a))^m a b^6 c^3 + 21x^5 (d(bx+a))^m a^2 b^5 c^3 + 35x^4 (d(bx+a))^m a^3 b^4 c^3 + 35x^3 (d(bx+a))^m a^4 b^3 c^3 + 21x^2 (d(bx+a))^m a^5 b^2 c^3 + 7x (d(bx+a))^m a^6 b c^3 + a^7 (d(bx+a))^m c^3}{b(7+m)}$ |
| norman | $\frac{a^7 c^3 e^{m \ln(bdx+ad)}}{b(7+m)} + \frac{b^6 c^3 x^7 e^{m \ln(bdx+ad)}}{7+m} + \frac{7a^6 c^3 x e^{m \ln(bdx+ad)}}{7+m} + \frac{7a b^5 c^3 x^6 e^{m \ln(bdx+ad)}}{7+m} + \frac{21a^2 b^4 c^3 x^5 e^{m \ln(bdx+ad)}}{7+m}$ |

input $\text{int}((b*d*x+a*d)^m*(b^2*c*x^2+2*a*b*x+a^2*c)^3,x,\text{method}=_RETURNVERBOSE)$

output $(b*x+a)*(b*d*x+a*d)^m*c^3*(b^2*x^2+2*a*b*x+a^2)^3/b/(7+m)$

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 116 vs. $2(27) = 54$.

Time = 0.08 (sec) , antiderivative size = 116, normalized size of antiderivative = 4.30

$$\int (ad + bdx)^m (a^2c + 2abcx + b^2cx^2)^3 dx$$

$$= \frac{(b^7c^3x^7 + 7ab^6c^3x^6 + 21a^2b^5c^3x^5 + 35a^3b^4c^3x^4 + 35a^4b^3c^3x^3 + 21a^5b^2c^3x^2 + 7a^6bc^3x + a^7c^3)(bdx + a^2c)}{bm + 7b}$$

input `integrate((b*d*x+a*d)^m*(b^2*c*x^2+2*a*b*c*x+a^2*c)^3,x, algorithm="fricas")`

output `(b^7*c^3*x^7 + 7*a*b^6*c^3*x^6 + 21*a^2*b^5*c^3*x^5 + 35*a^3*b^4*c^3*x^4 + 35*a^4*b^3*c^3*x^3 + 21*a^5*b^2*c^3*x^2 + 7*a^6*b*c^3*x + a^7*c^3)*(b*d*x + a*d)^m/(b*m + 7*b)`

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 279 vs. $2(22) = 44$.

Time = 0.97 (sec) , antiderivative size = 279, normalized size of antiderivative = 10.33

$$\int (ad + bdx)^m (a^2c + 2abcx + b^2cx^2)^3 dx$$

$$= \begin{cases} \frac{c^3x}{ad^7} \\ a^6c^3x(ad)^m \\ \frac{c^3 \log(\frac{a}{b} + x)}{bd^7} \\ \frac{a^7c^3(ad+bdx)^m}{bm+7b} + \frac{7a^6bc^3x(ad+bdx)^m}{bm+7b} + \frac{21a^5b^2c^3x^2(ad+bdx)^m}{bm+7b} + \frac{35a^4b^3c^3x^3(ad+bdx)^m}{bm+7b} + \frac{35a^3b^4c^3x^4(ad+bdx)^m}{bm+7b} + \frac{21a^2b^5c^3x^5(ad+bdx)^m}{bm+7b} + \frac{7a^6bc^3x(ad+bdx)^m}{bm+7b} + \frac{a^7c^3(ad+bdx)^m}{bm+7b} \end{cases}$$

input `integrate((b*d*x+a*d)**m*(b**2*c*x**2+2*a*b*c*x+a**2*c)**3,x)`

output

```
Piecewise((c**3*x/(a*d**7), Eq(b, 0) & Eq(m, -7)), (a**6*c**3*x*(a*d)**m,
Eq(b, 0)), (c**3*log(a/b + x)/(b*d**7), Eq(m, -7)), (a**7*c**3*(a*d + b*d*
x)**m/(b*m + 7*b) + 7*a**6*b*c**3*x*(a*d + b*d*x)**m/(b*m + 7*b) + 21*a**5
*b**2*c**3*x**2*(a*d + b*d*x)**m/(b*m + 7*b) + 35*a**4*b**3*c**3*x**3*(a*d
+ b*d*x)**m/(b*m + 7*b) + 35*a**3*b**4*c**3*x**4*(a*d + b*d*x)**m/(b*m +
7*b) + 21*a**2*b**5*c**3*x**5*(a*d + b*d*x)**m/(b*m + 7*b) + 7*a*b**6*c**3
*x**6*(a*d + b*d*x)**m/(b*m + 7*b) + b**7*c**3*x**7*(a*d + b*d*x)**m/(b*m
+ 7*b), True))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 928 vs. $2(27) = 54$.

Time = 0.09 (sec) , antiderivative size = 928, normalized size of antiderivative = 34.37

$$\int (ad + bdx)^m (a^2c + 2abcx + b^2cx^2)^3 dx = \text{Too large to display}$$

input

```
integrate((b*d*x+a*d)^m*(b^2*c*x^2+2*a*b*c*x+a^2*c)^3,x, algorithm="maxima
")
```

output

```

6*(b^2*d^m*(m + 1)*x^2 + a*b*d^m*m*x - a^2*d^m)*(b*x + a)^m*a^5*c^3/((m^2
+ 3*m + 2)*b) + 15*((m^2 + 3*m + 2)*b^3*d^m*x^3 + (m^2 + m)*a*b^2*d^m*x^2
- 2*a^2*b*d^m*m*x + 2*a^3*d^m)*(b*x + a)^m*a^4*c^3/((m^3 + 6*m^2 + 11*m +
6)*b) + (b*d*x + a*d)^(m + 1)*a^6*c^3/(b*d*(m + 1)) + 20*((m^3 + 6*m^2 + 1
1*m + 6)*b^4*d^m*x^4 + (m^3 + 3*m^2 + 2*m)*a*b^3*d^m*x^3 - 3*(m^2 + m)*a^2
*b^2*d^m*x^2 + 6*a^3*b*d^m*m*x - 6*a^4*d^m)*(b*x + a)^m*a^3*c^3/((m^4 + 10
*m^3 + 35*m^2 + 50*m + 24)*b) + 15*((m^4 + 10*m^3 + 35*m^2 + 50*m + 24)*b^
5*d^m*x^5 + (m^4 + 6*m^3 + 11*m^2 + 6*m)*a*b^4*d^m*x^4 - 4*(m^3 + 3*m^2 +
2*m)*a^2*b^3*d^m*x^3 + 12*(m^2 + m)*a^3*b^2*d^m*x^2 - 24*a^4*b*d^m*m*x + 2
4*a^5*d^m)*(b*x + a)^m*a^2*c^3/((m^5 + 15*m^4 + 85*m^3 + 225*m^2 + 274*m +
120)*b) + 6*((m^5 + 15*m^4 + 85*m^3 + 225*m^2 + 274*m + 120)*b^6*d^m*x^6
+ (m^5 + 10*m^4 + 35*m^3 + 50*m^2 + 24*m)*a*b^5*d^m*x^5 - 5*(m^4 + 6*m^3 +
11*m^2 + 6*m)*a^2*b^4*d^m*x^4 + 20*(m^3 + 3*m^2 + 2*m)*a^3*b^3*d^m*x^3 -
60*(m^2 + m)*a^4*b^2*d^m*x^2 + 120*a^5*b*d^m*m*x - 120*a^6*d^m)*(b*x + a)^
m*a*c^3/((m^6 + 21*m^5 + 175*m^4 + 735*m^3 + 1624*m^2 + 1764*m + 720)*b) +
((m^6 + 21*m^5 + 175*m^4 + 735*m^3 + 1624*m^2 + 1764*m + 720)*b^7*d^m*x^7
+ (m^6 + 15*m^5 + 85*m^4 + 225*m^3 + 274*m^2 + 120*m)*a*b^6*d^m*x^6 - 6*(
m^5 + 10*m^4 + 35*m^3 + 50*m^2 + 24*m)*a^2*b^5*d^m*x^5 + 30*(m^4 + 6*m^3 +
11*m^2 + 6*m)*a^3*b^4*d^m*x^4 - 120*(m^3 + 3*m^2 + 2*m)*a^4*b^3*d^m*x^3 +
360*(m^2 + m)*a^5*b^2*d^m*x^2 - 720*a^6*b*d^m*m*x + 720*a^7*d^m)*(b*x ...

```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 186 vs. $2(27) = 54$.

Time = 0.12 (sec) , antiderivative size = 186, normalized size of antiderivative = 6.89

$$\int (ad + bdx)^m (a^2c + 2abcx + b^2cx^2)^3 dx$$

$$= \frac{(bdx + ad)^m b^7 c^3 x^7 + 7(bdx + ad)^m ab^6 c^3 x^6 + 21(bdx + ad)^m a^2 b^5 c^3 x^5 + 35(bdx + ad)^m a^3 b^4 c^3 x^4 + 35(bdx + ad)^m a^4 b^3 c^3 x^3 + 21(bdx + ad)^m a^5 b^2 c^3 x^2 + 7(bdx + ad)^m a^6 b c^3 x + (bdx + ad)^m a^7 c^3}{bm + 7b}$$

input

```
integrate((b*d*x+a*d)^m*(b^2*c*x^2+2*a*b*c*x+a^2*c)^3,x, algorithm="giac")
```

output

```

((b*d*x + a*d)^m*b^7*c^3*x^7 + 7*(b*d*x + a*d)^m*a*b^6*c^3*x^6 + 21*(b*d*x
+ a*d)^m*a^2*b^5*c^3*x^5 + 35*(b*d*x + a*d)^m*a^3*b^4*c^3*x^4 + 35*(b*d*x
+ a*d)^m*a^4*b^3*c^3*x^3 + 21*(b*d*x + a*d)^m*a^5*b^2*c^3*x^2 + 7*(b*d*x
+ a*d)^m*a^6*b*c^3*x + (b*d*x + a*d)^m*a^7*c^3)/(b*m + 7*b)

```

Mupad [B] (verification not implemented)

Time = 5.66 (sec) , antiderivative size = 147, normalized size of antiderivative = 5.44

$$\int (ad+bdx)^m (a^2c+2abcx+b^2cx^2)^3 dx = (ad+bdx)^m \left(\frac{7a^6c^3x}{m+7} + \frac{a^7c^3}{b(m+7)} + \frac{b^6c^3x^7}{m+7} \right. \\ \left. + \frac{21a^5bc^3x^2}{m+7} + \frac{7ab^5c^3x^6}{m+7} + \frac{35a^4b^2c^3x^3}{m+7} \right. \\ \left. + \frac{35a^3b^3c^3x^4}{m+7} + \frac{21a^2b^4c^3x^5}{m+7} \right)$$

input `int((a*d + b*d*x)^m*(a^2*c + b^2*c*x^2 + 2*a*b*c*x)^3,x)`output `(a*d + b*d*x)^m*((7*a^6*c^3*x)/(m + 7) + (a^7*c^3)/(b*(m + 7)) + (b^6*c^3*x^7)/(m + 7) + (21*a^5*b*c^3*x^2)/(m + 7) + (7*a*b^5*c^3*x^6)/(m + 7) + (35*a^4*b^2*c^3*x^3)/(m + 7) + (35*a^3*b^3*c^3*x^4)/(m + 7) + (21*a^2*b^4*c^3*x^5)/(m + 7))`**Reduce [B] (verification not implemented)**

Time = 0.26 (sec) , antiderivative size = 93, normalized size of antiderivative = 3.44

$$\int (ad+bdx)^m (a^2c+2abcx+b^2cx^2)^3 dx \\ = \frac{(bdx+ad)^m c^3 (b^7x^7 + 7ab^6x^6 + 21a^2b^5x^5 + 35a^3b^4x^4 + 35a^4b^3x^3 + 21a^5b^2x^2 + 7a^6bx + a^7)}{b(m+7)}$$

input `int((b*d*x+a*d)^m*(b^2*c*x^2+2*a*b*c*x+a^2*c)^3,x)`output `((a*d + b*d*x)**m*c**3*(a**7 + 7*a**6*b*x + 21*a**5*b**2*x**2 + 35*a**4*b**3*x**3 + 35*a**3*b**4*x**4 + 21*a**2*b**5*x**5 + 7*a*b**6*x**6 + b**7*x**7))/(b*(m + 7))`

3.400 $\int (ad + bdx)^m (a^2c + 2abcx + b^2cx^2)^2 dx$

| | |
|---|------|
| Optimal result | 3024 |
| Mathematica [A] (verified) | 3024 |
| Rubi [A] (verified) | 3025 |
| Maple [A] (verified) | 3026 |
| Fricas [B] (verification not implemented) | 3027 |
| Sympy [B] (verification not implemented) | 3027 |
| Maxima [B] (verification not implemented) | 3028 |
| Giac [B] (verification not implemented) | 3029 |
| Mupad [B] (verification not implemented) | 3029 |
| Reduce [B] (verification not implemented) | 3030 |

Optimal result

Integrand size = 33, antiderivative size = 27

$$\int (ad + bdx)^m (a^2c + 2abcx + b^2cx^2)^2 dx = \frac{c^2(ad + bdx)^{5+m}}{bd^5(5+m)}$$

output

```
c^2*(b*d*x+a*d)^(5+m)/b/d^5/(5+m)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.04

$$\int (ad + bdx)^m (a^2c + 2abcx + b^2cx^2)^2 dx = \frac{c^2(a + bx)^5(d(a + bx))^m}{b(5 + m)}$$

input

```
Integrate[(a*d + b*d*x)^m*(a^2*c + 2*a*b*c*x + b^2*c*x^2)^2,x]
```

output

```
(c^2*(a + b*x)^5*(d*(a + b*x))^m)/(b*(5 + m))
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.121$, Rules used = {1098, 27, 35, 17}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a^2c + 2abcx + b^2cx^2)^2 (ad + bdx)^m dx$$

$$\downarrow 1098$$

$$\frac{\int b^4c^4(a + bx)^4(ad + bxd)^m dx}{b^4c^2}$$

$$\downarrow 27$$

$$c^2 \int (a + bx)^4(ad + bxd)^m dx$$

$$\downarrow 35$$

$$\frac{c^2 \int (ad + bxd)^{m+4} dx}{d^4}$$

$$\downarrow 17$$

$$\frac{c^2(ad + bdx)^{m+5}}{bd^5(m + 5)}$$

input `Int[(a*d + b*d*x)^m*(a^2*c + 2*a*b*c*x + b^2*c*x^2)^2,x]`

output `(c^2*(a*d + b*d*x)^(5 + m))/(b*d^5*(5 + m))`

Definitions of rubi rules used

rule 17 $\text{Int}[(c_.)*((a_.) + (b_.)*(x_.))^{(m_.)}, x_Symbol] \rightarrow \text{Simp}[c*((a + b*x)^{(m + 1})/(b*(m + 1))), x] /;$ $\text{FreeQ}[\{a, b, c, m\}, x] \ \&\& \ \text{NeQ}[m, -1]$

rule 27 $\text{Int}[(a_)*(F_x_), x_Symbol] \rightarrow \text{Simp}[a \ \text{Int}[F_x, x], x] /;$ $\text{FreeQ}[a, x] \ \&\& \ \text{!MatchQ}[F_x, (b_)*(G_x_)] /;$ $\text{FreeQ}[b, x]$

rule 35 $\text{Int}[(u_.)*((a_) + (b_.)*(x_.))^{(m_.)*((c_) + (d_.)*(x_.))^{(n_.)}, x_Symbol] \rightarrow \text{Simp}[(b/d)^m \ \text{Int}[u*(c + d*x)^{(m + n)}, x], x] /;$ $\text{FreeQ}[\{a, b, c, d, m, n\}, x] \ \&\& \ \text{EqQ}[b*c - a*d, 0] \ \&\& \ \text{IntegerQ}[m] \ \&\& \ \text{!(IntegerQ}[n] \ \&\& \ \text{SimplerQ}[a + b*x, c + d*x])$

rule 1098 $\text{Int}(((d_.) + (e_.)*(x_.))^{(m_.)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^{(p_.)}, x_Symbol] \rightarrow \text{Simp}[1/c^p \ \text{Int}[(d + e*x)^m*(b/2 + c*x)^{(2*p)}, x], x] /;$ $\text{FreeQ}[\{a, b, c, d, e, m\}, x] \ \&\& \ \text{EqQ}[b^2 - 4*a*c, 0] \ \&\& \ \text{IntegerQ}[p]$

Maple [A] (verified)

Time = 1.34 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.70

| method | result |
|--------------|--|
| gospers | $\frac{(bx+a)(bdx+ad)^m c^2 (b^2 x^2 + 2abx + a^2)^2}{b(5+m)}$ |
| orering | $\frac{(bx+a)(bdx+ad)^m (b^2 c x^2 + 2abcx + a^2 c)^2}{b(5+m)}$ |
| risch | $\frac{c^2 (b^5 x^5 + 5a b^4 x^4 + 10a^2 b^3 x^3 + 10a^3 b^2 x^2 + 5a^4 b x + a^5) (d(bx+a))^m}{b(5+m)}$ |
| parallelrisc | $\frac{x^5 (d(bx+a))^m a b^5 c^2 + 5x^4 (d(bx+a))^m a^2 b^4 c^2 + 10x^3 (d(bx+a))^m a^3 b^3 c^2 + 10x^2 (d(bx+a))^m a^4 b^2 c^2 + 5x (d(bx+a))^m a^5 b c^2 + (d(bx+a))^m a^6 c^2}{a(5+m)b}$ |
| norman | $\frac{a^5 c^2 e^{m \ln(bdx+ad)}}{b(5+m)} + \frac{b^4 c^2 x^5 e^{m \ln(bdx+ad)}}{5+m} + \frac{5a^4 c^2 x e^{m \ln(bdx+ad)}}{5+m} + \frac{5a b^3 c^2 x^4 e^{m \ln(bdx+ad)}}{5+m} + \frac{10a^2 b^2 c^2 x^3 e^{m \ln(bdx+ad)}}{5+m}$ |

input $\text{int}((b*d*x+a*d)^m*(b^2*c*x^2+2*a*b*x+a^2*c)^2,x,\text{method}=_RETURNVERBOSE)$

output $(b*x+a)*(b*d*x+a*d)^m*c^2*(b^2*x^2+2*a*b*x+a^2)^2/b/(5+m)$

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 88 vs. $2(27) = 54$.

Time = 0.09 (sec) , antiderivative size = 88, normalized size of antiderivative = 3.26

$$\int (ad + bdx)^m (a^2c + 2abcx + b^2cx^2)^2 dx$$

$$= \frac{(b^5c^2x^5 + 5ab^4c^2x^4 + 10a^2b^3c^2x^3 + 10a^3b^2c^2x^2 + 5a^4bc^2x + a^5c^2)(bdx + ad)^m}{bm + 5b}$$

input

```
integrate((b*d*x+a*d)^m*(b^2*c*x^2+2*a*b*c*x+a^2*c)^2,x, algorithm="fricas")
```

output

```
(b^5*c^2*x^5 + 5*a*b^4*c^2*x^4 + 10*a^2*b^3*c^2*x^3 + 10*a^3*b^2*c^2*x^2 + 5*a^4*b*c^2*x + a^5*c^2)*(b*d*x + a*d)^m/(b*m + 5*b)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 214 vs. $2(22) = 44$.

Time = 0.55 (sec) , antiderivative size = 214, normalized size of antiderivative = 7.93

$$\int (ad + bdx)^m (a^2c + 2abcx + b^2cx^2)^2 dx$$

$$= \begin{cases} \frac{c^2x}{ad^5} \\ a^4c^2x(ad)^m \\ \frac{c^2 \log\left(\frac{a}{b} + x\right)}{bd^5} \\ \frac{a^5c^2(ad+bdx)^m}{bm+5b} + \frac{5a^4bc^2x(ad+bdx)^m}{bm+5b} + \frac{10a^3b^2c^2x^2(ad+bdx)^m}{bm+5b} + \frac{10a^2b^3c^2x^3(ad+bdx)^m}{bm+5b} + \frac{5ab^4c^2x^4(ad+bdx)^m}{bm+5b} + \frac{b^5c^2x^5(ad+bdx)^m}{bm+5b} \end{cases}$$

input

```
integrate((b*d*x+a*d)**m*(b**2*c*x**2+2*a*b*c*x+a**2*c)**2,x)
```


output

```
Piecewise((c**2*x/(a*d**5), Eq(b, 0) & Eq(m, -5)), (a**4*c**2*x*(a*d)**m,
Eq(b, 0)), (c**2*log(a/b + x)/(b*d**5), Eq(m, -5)), (a**5*c**2*(a*d + b*d*
x)**m/(b*m + 5*b) + 5*a**4*b*c**2*x*(a*d + b*d*x)**m/(b*m + 5*b) + 10*a**3
*b**2*c**2*x**2*(a*d + b*d*x)**m/(b*m + 5*b) + 10*a**2*b**3*c**2*x**3*(a*d
+ b*d*x)**m/(b*m + 5*b) + 5*a*b**4*c**2*x**4*(a*d + b*d*x)**m/(b*m + 5*b)
+ b**5*c**2*x**5*(a*d + b*d*x)**m/(b*m + 5*b), True))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 457 vs. $2(27) = 54$.

Time = 0.07 (sec) , antiderivative size = 457, normalized size of antiderivative = 16.93

$$\int (ad + bdx)^m (a^2c + 2abcx + b^2cx^2)^2 dx$$

$$= \frac{4(b^2d^m(m+1)x^2 + abd^m mx - a^2d^m)(bx+a)^m a^3c^2}{(m^2 + 3m + 2)b}$$

$$+ \frac{6((m^2 + 3m + 2)b^3d^m x^3 + (m^2 + m)ab^2d^m x^2 - 2a^2bd^m mx + 2a^3d^m)(bx+a)^m a^2c^2}{(m^3 + 6m^2 + 11m + 6)b}$$

$$+ \frac{(bdx + ad)^{m+1} a^4 c^2}{bd(m+1)}$$

$$+ \frac{4((m^3 + 6m^2 + 11m + 6)b^4d^m x^4 + (m^3 + 3m^2 + 2m)ab^3d^m x^3 - 3(m^2 + m)a^2b^2d^m x^2 + 6a^3bd^m mx - 4a^4d^m x)(bx+a)^m a^2c^2}{(m^4 + 10m^3 + 35m^2 + 50m + 24)b}$$

$$+ \frac{((m^4 + 10m^3 + 35m^2 + 50m + 24)b^5d^m x^5 + (m^4 + 6m^3 + 11m^2 + 6m)ab^4d^m x^4 - 4(m^3 + 3m^2 + 2m)a^2b^3d^m x^3 + 6a^3bd^m mx - 4a^4d^m x)(bx+a)^m a^2c^2}{(m^5 + 15m^4 + 85m^3 + 225m^2 + 270m + 108)b}$$

input

```
integrate((b*d*x+a*d)^m*(b^2*c*x^2+2*a*b*c*x+a^2*c)^2,x, algorithm="maxima
")
```

output

$$4*(b^2*d^m*(m+1)*x^2 + a*b*d^m*m*x - a^2*d^m)*(b*x+a)^m*a^3*c^2/((m^2+3*m+2)*b) + 6*((m^2+3*m+2)*b^3*d^m*x^3 + (m^2+m)*a*b^2*d^m*x^2 - 2*a^2*b*d^m*m*x + 2*a^3*d^m)*(b*x+a)^m*a^2*c^2/((m^3+6*m^2+11*m+6)*b) + (b*d*x+a*d)^(m+1)*a^4*c^2/(b*d*(m+1)) + 4*((m^3+6*m^2+11*m+6)*b^4*d^m*x^4 + (m^3+3*m^2+2*m)*a*b^3*d^m*x^3 - 3*(m^2+m)*a^2*b^2*d^m*x^2 + 6*a^3*b*d^m*m*x - 6*a^4*d^m)*(b*x+a)^m*a*c^2/((m^4+10*m^3+35*m^2+50*m+24)*b) + ((m^4+10*m^3+35*m^2+50*m+24)*b^5*d^m*x^5 + (m^4+6*m^3+11*m^2+6*m)*a*b^4*d^m*x^4 - 4*(m^3+3*m^2+2*m)*a^2*b^3*d^m*x^3 + 12*(m^2+m)*a^3*b^2*d^m*x^2 - 24*a^4*b*d^m*m*x + 24*a^5*d^m)*(b*x+a)^m*c^2/((m^5+15*m^4+85*m^3+225*m^2+274*m+120)*b)$$
Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 138 vs. $2(27) = 54$.

Time = 0.13 (sec) , antiderivative size = 138, normalized size of antiderivative = 5.11

$$\int (ad + bdx)^m (a^2c + 2abcx + b^2cx^2)^2 dx = \frac{(bdx + ad)^m b^5 c^2 x^5 + 5(bdx + ad)^m ab^4 c^2 x^4 + 10(bdx + ad)^m a^2 b^3 c^2 x^3 + 10(bdx + ad)^m a^3 b^2 c^2 x^2 + 5(bdx + ad)^m a^4 b c^2 x + (bdx + ad)^m a^5 c^2}{bm + 5b}$$

input

```
integrate((b*d*x+a*d)^m*(b^2*c*x^2+2*a*b*c*x+a^2*c)^2,x, algorithm="giac")
```

output

$$((b*d*x + a*d)^m*b^5*c^2*x^5 + 5*(b*d*x + a*d)^m*a*b^4*c^2*x^4 + 10*(b*d*x + a*d)^m*a^2*b^3*c^2*x^3 + 10*(b*d*x + a*d)^m*a^3*b^2*c^2*x^2 + 5*(b*d*x + a*d)^m*a^4*b*c^2*x + (b*d*x + a*d)^m*a^5*c^2)/(b*m + 5*b)$$
Mupad [B] (verification not implemented)

Time = 5.60 (sec) , antiderivative size = 109, normalized size of antiderivative = 4.04

$$\int (ad + bdx)^m (a^2c + 2abcx + b^2cx^2)^2 dx = (ad + bdx)^m \left(\frac{5a^4c^2x}{m+5} + \frac{a^5c^2}{b(m+5)} + \frac{b^4c^2x^5}{m+5} + \frac{10a^3b^2c^2x^2}{m+5} + \frac{5ab^3c^2x^4}{m+5} + \frac{10a^2b^2c^2x^3}{m+5} \right)$$

input `int((a*d + b*d*x)^m*(a^2*c + b^2*c*x^2 + 2*a*b*c*x)^2,x)`

output `(a*d + b*d*x)^m*((5*a^4*c^2*x)/(m + 5) + (a^5*c^2)/(b*(m + 5)) + (b^4*c^2*x^5)/(m + 5) + (10*a^3*b*c^2*x^2)/(m + 5) + (5*a*b^3*c^2*x^4)/(m + 5) + (10*a^2*b^2*c^2*x^3)/(m + 5))`

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 71, normalized size of antiderivative = 2.63

$$\int (ad + bdx)^m (a^2c + 2abcx + b^2cx^2)^2 dx$$

$$= \frac{(bdx + ad)^m c^2 (b^5x^5 + 5ab^4x^4 + 10a^2b^3x^3 + 10a^3b^2x^2 + 5a^4bx + a^5)}{b(m + 5)}$$

input `int((b*d*x+a*d)^m*(b^2*c*x^2+2*a*b*c*x+a^2*c)^2,x)`

output `((a*d + b*d*x)**m*c**2*(a**5 + 5*a**4*b*x + 10*a**3*b**2*x**2 + 10*a**2*b**3*x**3 + 5*a*b**4*x**4 + b**5*x**5))/(b*(m + 5))`

3.401 $\int (ad + bdx)^m (a^2c + 2abcx + b^2cx^2) dx$

| | |
|---|------|
| Optimal result | 3031 |
| Mathematica [A] (verified) | 3031 |
| Rubi [A] (verified) | 3032 |
| Maple [A] (verified) | 3033 |
| Fricas [B] (verification not implemented) | 3034 |
| Sympy [B] (verification not implemented) | 3034 |
| Maxima [B] (verification not implemented) | 3035 |
| Giac [B] (verification not implemented) | 3035 |
| Mupad [B] (verification not implemented) | 3036 |
| Reduce [B] (verification not implemented) | 3036 |

Optimal result

Integrand size = 31, antiderivative size = 25

$$\int (ad + bdx)^m (a^2c + 2abcx + b^2cx^2) dx = \frac{c(ad + bdx)^{3+m}}{bd^3(3+m)}$$

output

```
c*(b*d*x+a*d)^(3+m)/b/d^3/(3+m)
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.04

$$\int (ad + bdx)^m (a^2c + 2abcx + b^2cx^2) dx = \frac{c(a + bx)^3(d(a + bx))^m}{b(3+m)}$$

input

```
Integrate[(a*d + b*d*x)^m*(a^2*c + 2*a*b*c*x + b^2*c*x^2),x]
```

output

```
(c*(a + b*x)^3*(d*(a + b*x))^m)/(b*(3 + m))
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.129$, Rules used = {1098, 27, 35, 17}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a^2c + 2abcx + b^2cx^2) (ad + bdx)^m dx$$

$$\downarrow 1098$$

$$\frac{\int b^2c^2(a + bx)^2(ad + bxd)^m dx}{b^2c}$$

$$\downarrow 27$$

$$c \int (a + bx)^2(ad + bxd)^m dx$$

$$\downarrow 35$$

$$\frac{c \int (ad + bxd)^{m+2} dx}{d^2}$$

$$\downarrow 17$$

$$\frac{c(ad + bdx)^{m+3}}{bd^3(m + 3)}$$

input `Int[(a*d + b*d*x)^m*(a^2*c + 2*a*b*c*x + b^2*c*x^2),x]`

output `(c*(a*d + b*d*x)^(3 + m))/(b*d^3*(3 + m))`

Definitions of rubi rules used

rule 17 $\text{Int}[(c_)*(a_)+(b_)*(x_)]^{(m_)}, x_Symbol] \rightarrow \text{Simp}[c*((a + b*x)^{(m + 1)})/(b*(m + 1)), x] /;$ FreeQ[{a, b, c, m}, x] && NeQ[m, -1]

rule 27 $\text{Int}[(a_)*(F_x), x_Symbol] \rightarrow \text{Simp}[a \text{ Int}[F_x, x], x] /;$ FreeQ[a, x] && !MatchQ[F_x, (b_)*(G_x)] /; FreeQ[b, x]

rule 35 $\text{Int}[(u_)*((a_)+(b_)*(x_)]^{(m_)*((c_)+(d_)*(x_)]^{(n_)}, x_Symbol] \rightarrow \text{Simp}[(b/d)^m \text{ Int}[u*(c + d*x)^{(m + n)}, x], x] /;$ FreeQ[{a, b, c, d, m, n}, x] && EqQ[b*c - a*d, 0] && IntegerQ[m] && !(IntegerQ[n] && SimplerQ[a + b*x, c + d*x])

rule 1098 $\text{Int}(((d_)+(e_)*(x_)]^{(m_)*((a_)+(b_)*(x_)+(c_)*(x_)^2)^{(p_)}, x_Symbol] \rightarrow \text{Simp}[1/c^p \text{ Int}[(d + e*x)^m*(b/2 + c*x)^{(2*p)}, x], x] /;$ FreeQ[{a, b, c, d, e, m}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]

Maple [A] (verified)

Time = 0.76 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.68

| method | result | size |
|---------------|--|------|
| gospers | $\frac{(bx+a)(bdx+ad)^m c(b^2x^2+2abx+a^2)}{b(3+m)}$ | 42 |
| orering | $\frac{(bx+a)(bdx+ad)^m (b^2cx^2+2abcx+a^2c)}{b(3+m)}$ | 45 |
| risch | $\frac{c(b^3x^3+3ab^2x^2+3a^2bx+a^3)(d(bx+a))^m}{b(3+m)}$ | 47 |
| parallelrisch | $\frac{x^3(d(bx+a))^m b^3c+3x^2(d(bx+a))^m a b^2c+3x(d(bx+a))^m a^2bc+(d(bx+a))^m a^3c}{b(3+m)}$ | 78 |
| norman | $\frac{b^2cx^3e^{m \ln(bdx+ad)}}{3+m} + \frac{ca^3e^{m \ln(bdx+ad)}}{b(3+m)} + \frac{3ca^2xe^{m \ln(bdx+ad)}}{3+m} + \frac{3cabx^2e^{m \ln(bdx+ad)}}{3+m}$ | 101 |

input $\text{int}((b*d*x+a*d)^m*(b^2*c*x^2+2*a*b*c*x+a^2*c), x, \text{method}=_RETURNVERBOSE)$

output $(b*x+a)*(b*d*x+a*d)^m*c*(b^2*x^2+2*a*b*x+a^2)/b/(3+m)$

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 52 vs. $2(25) = 50$.

Time = 0.10 (sec) , antiderivative size = 52, normalized size of antiderivative = 2.08

$$\int (ad+bdx)^m (a^2c+2abcx+b^2cx^2) dx = \frac{(b^3cx^3 + 3ab^2cx^2 + 3a^2bcx + a^3c)(bdx + ad)^m}{bm + 3b}$$

input `integrate((b*d*x+a*d)^m*(b^2*c*x^2+2*a*b*c*x+a^2*c),x, algorithm="fricas")`

output `(b^3*c*x^3 + 3*a*b^2*c*x^2 + 3*a^2*b*c*x + a^3*c)*(b*d*x + a*d)^m/(b*m + 3*b)`

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 138 vs. $2(20) = 40$.

Time = 0.33 (sec) , antiderivative size = 138, normalized size of antiderivative = 5.52

$$\int (ad + bdx)^m (a^2c + 2abcx + b^2cx^2) dx = \begin{cases} \frac{cx}{ad^3} & \text{for } b = 0 \wedge m = -3 \\ a^2cx(ad)^m & \text{for } b = 0 \\ \frac{c \log\left(\frac{a}{b} + x\right)}{bd^3} & \text{for } m = -3 \\ \frac{a^3c(ad+bdx)^m}{bm+3b} + \frac{3a^2bcx(ad+bdx)^m}{bm+3b} + \frac{3ab^2cx^2(ad+bdx)^m}{bm+3b} + \frac{b^3cx^3(ad+bdx)^m}{bm+3b} & \text{otherwise} \end{cases}$$

input `integrate((b*d*x+a*d)**m*(b**2*c*x**2+2*a*b*c*x+a**2*c),x)`

output `Piecewise((c*x/(a*d**3), Eq(b, 0) & Eq(m, -3)), (a**2*c*x*(a*d)**m, Eq(b, 0)), (c*log(a/b + x)/(b*d**3), Eq(m, -3)), (a**3*c*(a*d + b*d*x)**m/(b*m + 3*b) + 3*a**2*b*c*x*(a*d + b*d*x)**m/(b*m + 3*b) + 3*a*b**2*c*x**2*(a*d + b*d*x)**m/(b*m + 3*b) + b**3*c*x**3*(a*d + b*d*x)**m/(b*m + 3*b), True))`

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 164 vs. $2(25) = 50$.

Time = 0.05 (sec) , antiderivative size = 164, normalized size of antiderivative = 6.56

$$\int (ad + bdx)^m (a^2c + 2abcx + b^2cx^2) dx$$

$$= \frac{2(b^2d^m(m+1)x^2 + abd^m mx - a^2d^m)(bx+a)^m ac}{(m^2 + 3m + 2)b}$$

$$+ \frac{((m^2 + 3m + 2)b^3d^m x^3 + (m^2 + m)ab^2d^m x^2 - 2a^2bd^m mx + 2a^3d^m)(bx+a)^m c}{(m^3 + 6m^2 + 11m + 6)b}$$

$$+ \frac{(bdx + ad)^{m+1} a^2c}{bd(m+1)}$$

input `integrate((b*d*x+a*d)^m*(b^2*c*x^2+2*a*b*c*x+a^2*c),x, algorithm="maxima")`

output `2*(b^2*d^m*(m+1)*x^2 + a*b*d^m*m*x - a^2*d^m)*(b*x+a)^m*a*c/((m^2+3*m+2)*b) + ((m^2+3*m+2)*b^3*d^m*x^3 + (m^2+m)*a*b^2*d^m*x^2 - 2*a^2*b*d^m*m*x + 2*a^3*d^m)*(b*x+a)^m*c/((m^3+6*m^2+11*m+6)*b) + (b*d*x+a*d)^(m+1)*a^2*c/(b*d*(m+1))`

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 82 vs. $2(25) = 50$.

Time = 0.16 (sec) , antiderivative size = 82, normalized size of antiderivative = 3.28

$$\int (ad + bdx)^m (a^2c + 2abcx + b^2cx^2) dx$$

$$= \frac{(bdx + ad)^m b^3cx^3 + 3(bdx + ad)^m ab^2cx^2 + 3(bdx + ad)^m a^2bcx + (bdx + ad)^m a^3c}{bm + 3b}$$

input `integrate((b*d*x+a*d)^m*(b^2*c*x^2+2*a*b*c*x+a^2*c),x, algorithm="giac")`

output `((b*d*x+a*d)^m*b^3*c*x^3 + 3*(b*d*x+a*d)^m*a*b^2*c*x^2 + 3*(b*d*x+a*d)^m*a^2*b*c*x + (b*d*x+a*d)^m*a^3*c)/(b*m+3*b)`

Mupad [B] (verification not implemented)

Time = 5.68 (sec) , antiderivative size = 63, normalized size of antiderivative = 2.52

$$\int (ad + bdx)^m (a^2c + 2abcx + b^2cx^2) dx = (ad + bdx)^m \left(\frac{3a^2cx}{m+3} + \frac{a^3c}{b(m+3)} + \frac{b^2cx^3}{m+3} + \frac{3abcx^2}{m+3} \right)$$

input `int((a*d + b*d*x)^m*(a^2*c + b^2*c*x^2 + 2*a*b*c*x),x)`output `(a*d + b*d*x)^m*((3*a^2*c*x)/(m + 3) + (a^3*c)/(b*(m + 3)) + (b^2*c*x^3)/(m + 3) + (3*a*b*c*x^2)/(m + 3))`**Reduce [B] (verification not implemented)**

Time = 0.21 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.88

$$\int (ad + bdx)^m (a^2c + 2abcx + b^2cx^2) dx = \frac{(bdx + ad)^m c(b^3x^3 + 3ab^2x^2 + 3a^2bx + a^3)}{b(m+3)}$$

input `int((b*d*x+a*d)^m*(b^2*c*x^2+2*a*b*c*x+a^2*c),x)`output `((a*d + b*d*x)**m*c*(a**3 + 3*a**2*b*x + 3*a*b**2*x**2 + b**3*x**3))/(b*(m + 3))`

3.402 $\int (ad + bdx)^m dx$

| | |
|---|------|
| Optimal result | 3037 |
| Mathematica [A] (verified) | 3037 |
| Rubi [A] (verified) | 3038 |
| Maple [A] (verified) | 3039 |
| Fricas [A] (verification not implemented) | 3039 |
| Sympy [A] (verification not implemented) | 3040 |
| Maxima [A] (verification not implemented) | 3040 |
| Giac [A] (verification not implemented) | 3040 |
| Mupad [B] (verification not implemented) | 3041 |
| Reduce [B] (verification not implemented) | 3041 |

Optimal result

Integrand size = 10, antiderivative size = 24

$$\int (ad + bdx)^m dx = \frac{(ad + bdx)^{1+m}}{bd(1+m)}$$

output

```
(b*d*x+a*d)^(1+m)/b/d/(1+m)
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.96

$$\int (ad + bdx)^m dx = \frac{(a + bx)(d(a + bx))^m}{b(1+m)}$$

input

```
Integrate[(a*d + b*d*x)^m,x]
```

output

```
((a + b*x)*(d*(a + b*x))^m)/(b*(1 + m))
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {17}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (ad + bdx)^m dx$$

$$\downarrow 17$$

$$\frac{(ad + bdx)^{m+1}}{bd(m + 1)}$$

input `Int[(a*d + b*d*x)^m,x]`

output `(a*d + b*d*x)^(1 + m)/(b*d*(1 + m))`

Defintions of rubi rules used

rule 17 `Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1))/(b*(m + 1)), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]`

Maple [A] (verified)

Time = 0.66 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

| method | result | size |
|---------------|--|------|
| risch | $\frac{(bx+a)(d(bx+a))^m}{b(1+m)}$ | 24 |
| gospers | $\frac{(bx+a)(bdx+ad)^m}{b(1+m)}$ | 25 |
| default | $\frac{(bdx+ad)^{1+m}}{bd(1+m)}$ | 25 |
| orering | $\frac{(bx+a)(bdx+ad)^m}{b(1+m)}$ | 25 |
| parallelrisch | $\frac{x(d(bx+a))^m ab + (d(bx+a))^m a^2}{(1+m)ab}$ | 40 |
| norman | $\frac{x e^{m \ln(bdx+ad)}}{1+m} + \frac{a e^{m \ln(bdx+ad)}}{b(1+m)}$ | 43 |

input `int((b*d*x+a*d)^m,x,method=_RETURNVERBOSE)`output `(b*x+a)/b/(1+m)*(d*(b*x+a))^m`**Fricas [A] (verification not implemented)**

Time = 0.08 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.96

$$\int (ad + bdx)^m dx = \frac{(bx + a)(bdx + ad)^m}{bm + b}$$

input `integrate((b*d*x+a*d)^m,x, algorithm="fricas")`output `(b*x + a)*(b*d*x + a*d)^m/(b*m + b)`

Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.21

$$\int (ad + bdx)^m dx = \frac{\begin{cases} \frac{(ad+bdx)^{m+1}}{m+1} & \text{for } m \neq -1 \\ \log(ad + bdx) & \text{otherwise} \end{cases}}{bd}$$

input `integrate((b*d*x+a*d)**m,x)`output `Piecewise(((a*d + b*d*x)**(m + 1)/(m + 1), Ne(m, -1)), (log(a*d + b*d*x), True))/(b*d)`**Maxima [A] (verification not implemented)**

Time = 0.03 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int (ad + bdx)^m dx = \frac{(bdx + ad)^{m+1}}{bd(m+1)}$$

input `integrate((b*d*x+a*d)^m,x, algorithm="maxima")`output `(b*d*x + a*d)^(m + 1)/(b*d*(m + 1))`**Giac [A] (verification not implemented)**

Time = 0.11 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int (ad + bdx)^m dx = \frac{(bdx + ad)^{m+1}}{bd(m+1)}$$

input `integrate((b*d*x+a*d)^m,x, algorithm="giac")`output `(b*d*x + a*d)^(m + 1)/(b*d*(m + 1))`

Mupad [B] (verification not implemented)

Time = 5.58 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int (ad + bdx)^m dx = \frac{(ad + bdx)^m (a + bx)}{b(m + 1)}$$

input `int((a*d + b*d*x)^m,x)`

output `((a*d + b*d*x)^m*(a + b*x))/(b*(m + 1))`

Reduce [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int (ad + bdx)^m dx = \frac{(bdx + ad)^m (bx + a)}{b(m + 1)}$$

input `int((b*d*x+a*d)^m,x)`

output `((a*d + b*d*x)**m*(a + b*x))/(b*(m + 1))`

3.403 $\int \frac{(ad+bdx)^m}{a^2c+2abcx+b^2cx^2} dx$

| | |
|---|------|
| Optimal result | 3042 |
| Mathematica [A] (verified) | 3042 |
| Rubi [A] (verified) | 3043 |
| Maple [A] (verified) | 3044 |
| Fricas [A] (verification not implemented) | 3045 |
| Sympy [B] (verification not implemented) | 3045 |
| Maxima [A] (verification not implemented) | 3046 |
| Giac [F] | 3046 |
| Mupad [B] (verification not implemented) | 3046 |
| Reduce [B] (verification not implemented) | 3047 |

Optimal result

Integrand size = 33, antiderivative size = 28

$$\int \frac{(ad + bdx)^m}{a^2c + 2abcx + b^2cx^2} dx = -\frac{d(ad + bdx)^{-1+m}}{bc(1 - m)}$$

output

```
-d*(b*d*x+a*d)^(-1+m)/b/c/(1-m)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.86

$$\int \frac{(ad + bdx)^m}{a^2c + 2abcx + b^2cx^2} dx = \frac{d(d(a + bx))^{-1+m}}{bc(-1 + m)}$$

input

```
Integrate[(a*d + b*d*x)^m/(a^2*c + 2*a*b*c*x + b^2*c*x^2), x]
```

output

```
(d*(d*(a + b*x))^( -1 + m))/(b*c*(-1 + m))
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.121$, Rules used = {1098, 27, 35, 17}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\begin{aligned}
 & \int \frac{(ad + bdx)^m}{a^2c + 2abcx + b^2cx^2} dx \\
 & \quad \downarrow \text{1098} \\
 & b^2c \int \frac{(ad + bxd)^m}{b^2c^2(a + bx)^2} dx \\
 & \quad \downarrow \text{27} \\
 & \frac{\int \frac{(ad + bxd)^m}{(a + bx)^2} dx}{c} \\
 & \quad \downarrow \text{35} \\
 & \frac{d^2 \int (ad + bxd)^{m-2} dx}{c} \\
 & \quad \downarrow \text{17} \\
 & -\frac{d(ad + bdx)^{m-1}}{bc(1 - m)}
 \end{aligned}$$

input `Int[(a*d + b*d*x)^m/(a^2*c + 2*a*b*c*x + b^2*c*x^2),x]`

output `-((d*(a*d + b*d*x)^(-1 + m))/(b*c*(1 - m)))`

Definitions of rubi rules used

rule 17 $\text{Int}[(c_)*(a_)+(b_)*(x_)]^{(m_)}, x_Symbol] \rightarrow \text{Simp}[c*((a + b*x)^{(m + 1)})/(b*(m + 1))], x] /;$ FreeQ[{a, b, c, m}, x] && NeQ[m, -1]

rule 27 $\text{Int}[(a_)*(F_x), x_Symbol] \rightarrow \text{Simp}[a \text{ Int}[F_x, x], x] /;$ FreeQ[a, x] && !MatchQ[F_x, (b_)*(G_x)] /; FreeQ[b, x]

rule 35 $\text{Int}[(u_)*((a_) + (b_)*(x_)]^{(m_)}*((c_) + (d_)*(x_)]^{(n_)}, x_Symbol] \rightarrow \text{Simp}[(b/d)^m \text{ Int}[u*(c + d*x)^{(m + n)}, x], x] /;$ FreeQ[{a, b, c, d, m, n}, x] && EqQ[b*c - a*d, 0] && IntegerQ[m] && !(IntegerQ[n] && SimplerQ[a + b*x, c + d*x])

rule 1098 $\text{Int}(((d_) + (e_)*(x_)]^{(m_)}*((a_) + (b_)*(x_) + (c_)*(x_)^2)^{(p_)}, x_Symbol] \rightarrow \text{Simp}[1/c^p \text{ Int}[(d + e*x)^m*(b/2 + c*x)^{(2*p)}, x], x] /;$ FreeQ[{a, b, c, d, e, m}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]

Maple [A] (verified)

Time = 1.45 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.04

| method | result | size |
|---------------|---|------|
| risch | $\frac{(d(bx+a))^m}{cb(m-1)(bx+a)}$ | 29 |
| parallelrisch | $\frac{(d(bx+a))^m}{cb(m-1)(bx+a)}$ | 29 |
| gosper | $\frac{(bdx+ad)^m}{(bx+a)cb(m-1)}$ | 30 |
| norman | $\frac{e^{m \ln(bdx+ad)}}{cb(m-1)(bx+a)}$ | 32 |
| orering | $\frac{(bx+a)(bdx+ad)^m}{b(m-1)(b^2cx^2+2abcx+a^2c)}$ | 47 |

input $\text{int}((b*d*x+a*d)^m/(b^2*c*x^2+2*a*b*c*x+a^2*c), x, \text{method}=_RETURNVERBOSE)$

output $1/c/b/(m-1)/(b*x+a)*(d*(b*x+a))^m$

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.39

$$\int \frac{(ad + bdx)^m}{a^2c + 2abcx + b^2cx^2} dx = \frac{(bdx + ad)^m}{abcm - abc + (b^2cm - b^2c)x}$$

input `integrate((b*d*x+a*d)^m/(b^2*c*x^2+2*a*b*c*x+a^2*c),x, algorithm="fricas")`

output `(b*d*x + a*d)^m/(a*b*c*m - a*b*c + (b^2*c*m - b^2*c)*x)`

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 65 vs. 2(20) = 40.

Time = 0.36 (sec) , antiderivative size = 65, normalized size of antiderivative = 2.32

$$\int \frac{(ad + bdx)^m}{a^2c + 2abcx + b^2cx^2} dx = \begin{cases} \frac{dx}{ac} & \text{for } b = 0 \wedge m = 1 \\ \frac{x(ad)^m}{a^2c} & \text{for } b = 0 \\ \frac{d \log(\frac{a}{b} + x)}{bc} & \text{for } m = 1 \\ \frac{(ad + bdx)^m}{abcm - abc + b^2cmx - b^2cx} & \text{otherwise} \end{cases}$$

input `integrate((b*d*x+a*d)**m/(b**2*c*x**2+2*a*b*c*x+a**2*c),x)`

output `Piecewise((d*x/(a*c), Eq(b, 0) & Eq(m, 1)), (x*(a*d)**m/(a**2*c), Eq(b, 0)), (d*log(a/b + x)/(b*c), Eq(m, 1)), ((a*d + b*d*x)**m/(a*b*c*m - a*b*c + b**2*c*m*x - b**2*c*x), True))`

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.07

$$\int \frac{(ad + bdx)^m}{a^2c + 2abcx + b^2cx^2} dx = \frac{(bx + a)^m d^m}{b^2c(m-1)x + abc(m-1)}$$

input `integrate((b*d*x+a*d)^m/(b^2*c*x^2+2*a*b*c*x+a^2*c),x, algorithm="maxima")`output `(b*x + a)^m*d^m/(b^2*c*(m - 1)*x + a*b*c*(m - 1))`**Giac [F]**

$$\int \frac{(ad + bdx)^m}{a^2c + 2abcx + b^2cx^2} dx = \int \frac{(bdx + ad)^m}{b^2cx^2 + 2abcx + a^2c} dx$$

input `integrate((b*d*x+a*d)^m/(b^2*c*x^2+2*a*b*c*x+a^2*c),x, algorithm="giac")`output `integrate((b*d*x + a*d)^m/(b^2*c*x^2 + 2*a*b*c*x + a^2*c), x)`**Mupad [B] (verification not implemented)**

Time = 5.76 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.11

$$\int \frac{(ad + bdx)^m}{a^2c + 2abcx + b^2cx^2} dx = \frac{(ad + bdx)^m}{b^2c \left(x + \frac{a}{b}\right) (m-1)}$$

input `int((a*d + b*d*x)^m/(a^2*c + b^2*c*x^2 + 2*a*b*c*x),x)`output `(a*d + b*d*x)^m/(b^2*c*(x + a/b)*(m - 1))`

Reduce [B] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.21

$$\int \frac{(ad + bdx)^m}{a^2c + 2abcx + b^2cx^2} dx = \frac{(bdx + ad)^m}{bc(bmx + am - bx - a)}$$

input `int((b*d*x+a*d)^m/(b^2*c*x^2+2*a*b*c*x+a^2*c),x)`

output `(a*d + b*d*x)**m/(b*c*(a*m - a + b*m*x - b*x))`

$$3.404 \quad \int \frac{(ad+bdx)^m}{(a^2c+2abcx+b^2cx^2)^2} dx$$

| | |
|---|------|
| Optimal result | 3048 |
| Mathematica [A] (verified) | 3048 |
| Rubi [A] (verified) | 3049 |
| Maple [A] (verified) | 3050 |
| Fricas [B] (verification not implemented) | 3051 |
| Sympy [B] (verification not implemented) | 3051 |
| Maxima [B] (verification not implemented) | 3052 |
| Giac [F] | 3052 |
| Mupad [B] (verification not implemented) | 3053 |
| Reduce [B] (verification not implemented) | 3053 |

Optimal result

Integrand size = 33, antiderivative size = 30

$$\int \frac{(ad + bdx)^m}{(a^2c + 2abcx + b^2cx^2)^2} dx = -\frac{d^3(ad + bdx)^{-3+m}}{bc^2(3 - m)}$$

output `-d^3*(b*d*x+a*d)^(-3+m)/b/c^2/(3-m)`

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.93

$$\int \frac{(ad + bdx)^m}{(a^2c + 2abcx + b^2cx^2)^2} dx = \frac{(d(a + bx))^m}{bc^2(-3 + m)(a + bx)^3}$$

input `Integrate[(a*d + b*d*x)^m/(a^2*c + 2*a*b*c*x + b^2*c*x^2)^2,x]`

output `(d*(a + b*x))^m/(b*c^2*(-3 + m)*(a + b*x)^3)`

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.121$, Rules used = {1098, 27, 35, 17}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(ad + bdx)^m}{(a^2c + 2abcx + b^2cx^2)^2} dx$$

$$\downarrow 1098$$

$$b^4c^2 \int \frac{(ad + bxd)^m}{b^4c^4(a + bx)^4} dx$$

$$\downarrow 27$$

$$\frac{\int \frac{(ad+bx)^m}{(a+bx)^4} dx}{c^2}$$

$$\downarrow 35$$

$$\frac{d^4 \int (ad + bxd)^{m-4} dx}{c^2}$$

$$\downarrow 17$$

$$-\frac{d^3(ad + bdx)^{m-3}}{bc^2(3 - m)}$$

input `Int[(a*d + b*d*x)^m/(a^2*c + 2*a*b*c*x + b^2*c*x^2)^2,x]`

output `-((d^3*(a*d + b*d*x)^(-3 + m))/(b*c^2*(3 - m)))`

Definitions of rubi rules used

rule 17 $\text{Int}[(c_)*(a_)+(b_)*(x_)]^{(m_)}, x_Symbol] \rightarrow \text{Simp}[c*((a + b*x)^{(m + 1)})/(b*(m + 1)), x] /; \text{FreeQ}\{a, b, c, m\}, x \ \&\& \ \text{NeQ}[m, -1]$

rule 27 $\text{Int}[(a_)*(F_x), x_Symbol] \rightarrow \text{Simp}[a \ \text{Int}[F_x, x], x] /; \text{FreeQ}[a, x] \ \&\& \ !\text{MatchQ}[F_x, (b_)*(G_x)] /; \text{FreeQ}[b, x]$

rule 35 $\text{Int}[(u_)*((a_)+(b_)*(x_)]^{(m_)*((c_)+(d_)*(x_)]^{(n_)}, x_Symbol] \rightarrow \text{Simp}[(b/d)^m \ \text{Int}[u*(c + d*x)^{(m + n)}, x], x] /; \text{FreeQ}\{a, b, c, d, m, n\}, x \ \&\& \ \text{EqQ}[b*c - a*d, 0] \ \&\& \ \text{IntegerQ}[m] \ \&\& \ !(\text{IntegerQ}[n] \ \&\& \ \text{SimplerQ}[a + b*x, c + d*x])$

rule 1098 $\text{Int}(((d_)+(e_)*(x_)]^{(m_)*((a_)+(b_)*(x_)+(c_)*(x_)^2)^{(p_)}, x_Symbol] \rightarrow \text{Simp}[1/c^p \ \text{Int}[(d + e*x)^m*(b/2 + c*x)^{(2*p)}, x], x] /; \text{FreeQ}\{a, b, c, d, e, m\}, x \ \&\& \ \text{EqQ}[b^2 - 4*a*c, 0] \ \&\& \ \text{IntegerQ}[p]$

Maple [A] (verified)

Time = 1.57 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.97

| method | result | size |
|---------------|---|------|
| risch | $\frac{(d(bx+a))^m}{b(-3+m)c^2(bx+a)^3}$ | 29 |
| parallelrisch | $\frac{(d(bx+a))^m}{b(-3+m)c^2(bx+a)^3}$ | 29 |
| norman | $\frac{e^{m \ln(bdx+ad)}}{b c^2 (-3+m)(bx+a)^3}$ | 32 |
| orering | $\frac{(bx+a)(bdx+ad)^m}{b(-3+m)(b^2c x^2+2abcx+a^2c)^2}$ | 47 |
| gosper | $\frac{(bdx+ad)^m}{(bx+a)(b^2x^2+2abx+a^2)c^2b(-3+m)}$ | 48 |

input $\text{int}((b*d*x+a*d)^m/(b^2*c*x^2+2*a*b*c*x+a^2*c)^2, x, \text{method}=_RETURNVERBOSE)$

output $1/b/(-3+m)/c^2/(b*x+a)^3*(d*(b*x+a))^m$

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 103 vs. $2(27) = 54$.

Time = 0.09 (sec) , antiderivative size = 103, normalized size of antiderivative = 3.43

$$\int \frac{(ad + bdx)^m}{(a^2c + 2abcx + b^2cx^2)^2} dx$$

$$= \frac{(bdx + ad)^m}{a^3bc^2m - 3a^3bc^2 + (b^4c^2m - 3b^4c^2)x^3 + 3(ab^3c^2m - 3ab^3c^2)x^2 + 3(a^2b^2c^2m - 3a^2b^2c^2)x}$$

input `integrate((b*d*x+a*d)^m/(b^2*c*x^2+2*a*b*c*x+a^2*c)^2,x, algorithm="fricas")`

output `(b*d*x + a*d)^m/(a^3*b*c^2*m - 3*a^3*b*c^2 + (b^4*c^2*m - 3*b^4*c^2)*x^3 + 3*(a*b^3*c^2*m - 3*a*b^3*c^2)*x^2 + 3*(a^2*b^2*c^2*m - 3*a^2*b^2*c^2)*x)`

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 148 vs. $2(24) = 48$.

Time = 0.76 (sec) , antiderivative size = 148, normalized size of antiderivative = 4.93

$$\int \frac{(ad + bdx)^m}{(a^2c + 2abcx + b^2cx^2)^2} dx$$

$$= \begin{cases} \frac{d^3x}{ac^2} & \text{for } b = 0 \wedge m = 3 \\ \frac{x(ad)^m}{a^4c^2} & \text{for } b = 0 \\ \frac{d^3 \log\left(\frac{a}{b} + x\right)}{bc^2} & \text{for } m = 3 \\ \frac{(ad+bdx)^m}{a^3bc^2m - 3a^3bc^2 + 3a^2b^2c^2mx - 9a^2b^2c^2x + 3ab^3c^2mx^2 - 9ab^3c^2x^2 + b^4c^2mx^3 - 3b^4c^2x^3} & \text{otherwise} \end{cases}$$

input `integrate((b*d*x+a*d)**m/(b**2*c*x**2+2*a*b*c*x+a**2*c)**2,x)`

output

```
Piecewise((d**3*x/(a*c**2), Eq(b, 0) & Eq(m, 3)), (x*(a*d)**m/(a**4*c**2),
Eq(b, 0)), (d**3*log(a/b + x)/(b*c**2), Eq(m, 3)), ((a*d + b*d*x)**m/(a**
3*b*c**2*m - 3*a**3*b*c**2 + 3*a**2*b**2*c**2*m*x - 9*a**2*b**2*c**2*x + 3
*a*b**3*c**2*m*x**2 - 9*a*b**3*c**2*x**2 + b**4*c**2*m*x**3 - 3*b**4*c**2*
x**3), True))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 68 vs. $2(27) = 54$.

Time = 0.05 (sec) , antiderivative size = 68, normalized size of antiderivative = 2.27

$$\int \frac{(ad + bdx)^m}{(a^2c + 2abcx + b^2cx^2)^2} dx$$

$$= \frac{(bx + a)^m d^m}{b^4c^2(m-3)x^3 + 3ab^3c^2(m-3)x^2 + 3a^2b^2c^2(m-3)x + a^3bc^2(m-3)}$$

input

```
integrate((b*d*x+a*d)^m/(b^2*c*x^2+2*a*b*c*x+a^2*c)^2,x, algorithm="maxima
")
```

output

```
(b*x + a)^m*d^m/(b^4*c^2*(m - 3)*x^3 + 3*a*b^3*c^2*(m - 3)*x^2 + 3*a^2*b^2
*c^2*(m - 3)*x + a^3*b*c^2*(m - 3))
```

Giac [F]

$$\int \frac{(ad + bdx)^m}{(a^2c + 2abcx + b^2cx^2)^2} dx = \int \frac{(bdx + ad)^m}{(b^2cx^2 + 2abcx + a^2c)^2} dx$$

input

```
integrate((b*d*x+a*d)^m/(b^2*c*x^2+2*a*b*c*x+a^2*c)^2,x, algorithm="giac")
```

output

```
integrate((b*d*x + a*d)^m/(b^2*c*x^2 + 2*a*b*c*x + a^2*c)^2, x)
```

Mupad [B] (verification not implemented)

Time = 5.73 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.77

$$\int \frac{(ad + bdx)^m}{(a^2c + 2abcx + b^2cx^2)^2} dx = \frac{(ad + bdx)^m}{b^4c^2(m-3)\left(x^3 + \frac{a^3}{b^3} + \frac{3ax^2}{b} + \frac{3a^2x}{b^2}\right)}$$

input `int((a*d + b*d*x)^m/(a^2*c + b^2*c*x^2 + 2*a*b*c*x)^2,x)`output `(a*d + b*d*x)^m/(b^4*c^2*(m - 3)*(x^3 + a^3/b^3 + (3*a*x^2)/b + (3*a^2*x)/b^2))`**Reduce [B] (verification not implemented)**

Time = 0.32 (sec) , antiderivative size = 80, normalized size of antiderivative = 2.67

$$\int \frac{(ad + bdx)^m}{(a^2c + 2abcx + b^2cx^2)^2} dx = \frac{(bdx + ad)^m}{bc^2(b^3mx^3 + 3ab^2mx^2 - 3b^3x^3 + 3a^2bmx - 9ab^2x^2 + a^3m - 9a^2bx - 3a^3)}$$

input `int((b*d*x+a*d)^m/(b^2*c*x^2+2*a*b*c*x+a^2*c)^2,x)`output `(a*d + b*d*x)**m/(b*c**2*(a**3*m - 3*a**3 + 3*a**2*b*m*x - 9*a**2*b*x + 3*a*b**2*m*x**2 - 9*a*b**2*x**2 + b**3*m*x**3 - 3*b**3*x**3))`

$$3.405 \quad \int \frac{(ad+bdx)^m}{(a^2c+2abcx+b^2cx^2)^3} dx$$

| | |
|---|------|
| Optimal result | 3054 |
| Mathematica [A] (verified) | 3054 |
| Rubi [A] (verified) | 3055 |
| Maple [A] (verified) | 3056 |
| Fricas [B] (verification not implemented) | 3057 |
| Sympy [B] (verification not implemented) | 3057 |
| Maxima [B] (verification not implemented) | 3058 |
| Giac [F] | 3058 |
| Mupad [B] (verification not implemented) | 3059 |
| Reduce [B] (verification not implemented) | 3059 |

Optimal result

Integrand size = 33, antiderivative size = 30

$$\int \frac{(ad + bdx)^m}{(a^2c + 2abcx + b^2cx^2)^3} dx = -\frac{d^5(ad + bdx)^{-5+m}}{bc^3(5 - m)}$$

output `-d^5*(b*d*x+a*d)^(-5+m)/b/c^3/(5-m)`

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.93

$$\int \frac{(ad + bdx)^m}{(a^2c + 2abcx + b^2cx^2)^3} dx = \frac{(d(a + bx))^m}{bc^3(-5 + m)(a + bx)^5}$$

input `Integrate[(a*d + b*d*x)^m/(a^2*c + 2*a*b*c*x + b^2*c*x^2)^3,x]`

output `(d*(a + b*x))^m/(b*c^3*(-5 + m)*(a + b*x)^5)`

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.121$, Rules used = {1098, 27, 35, 17}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\begin{aligned}
 & \int \frac{(ad + bdx)^m}{(a^2c + 2abcx + b^2cx^2)^3} dx \\
 & \quad \downarrow 1098 \\
 & b^6c^3 \int \frac{(ad + bxd)^m}{b^6c^6(a + bx)^6} dx \\
 & \quad \downarrow 27 \\
 & \frac{\int \frac{(ad+bx)^m}{(a+bx)^6} dx}{c^3} \\
 & \quad \downarrow 35 \\
 & \frac{d^6 \int (ad + bxd)^{m-6} dx}{c^3} \\
 & \quad \downarrow 17 \\
 & -\frac{d^5(ad + bdx)^{m-5}}{bc^3(5 - m)}
 \end{aligned}$$

input `Int[(a*d + b*d*x)^m/(a^2*c + 2*a*b*c*x + b^2*c*x^2)^3,x]`

output `-((d^5*(a*d + b*d*x)^(-5 + m))/(b*c^3*(5 - m)))`

Definitions of rubi rules used

rule 17 $\text{Int}[(c_)*(a_)+(b_)*(x_)]^{(m_)}, x_Symbol] \rightarrow \text{Simp}[c*((a + b*x)^{(m + 1)})/(b*(m + 1)), x] /; \text{FreeQ}\{a, b, c, m\}, x] \ \&\& \ \text{NeQ}[m, -1]$

rule 27 $\text{Int}[(a_)*(F_x), x_Symbol] \rightarrow \text{Simp}[a \ \text{Int}[F_x, x], x] /; \text{FreeQ}[a, x] \ \&\& \ !\text{MatchQ}[F_x, (b_)*(G_x)] /; \text{FreeQ}[b, x]$

rule 35 $\text{Int}[(u_)*((a_)+(b_)*(x_)]^{(m_)*((c_)+(d_)*(x_)]^{(n_)}, x_Symbol] \rightarrow \text{Simp}[(b/d)^m \ \text{Int}[u*(c + d*x)^{(m + n)}, x], x] /; \text{FreeQ}\{a, b, c, d, m, n\}, x] \ \&\& \ \text{EqQ}[b*c - a*d, 0] \ \&\& \ \text{IntegerQ}[m] \ \&\& \ !(\text{IntegerQ}[n] \ \&\& \ \text{SimplerQ}[a + b*x, c + d*x])$

rule 1098 $\text{Int}[(d_)+(e_)*(x_)]^{(m_)*((a_)+(b_)*(x_)+(c_)*(x_)^2)^{(p_)}, x_Symbol] \rightarrow \text{Simp}[1/c^p \ \text{Int}[(d + e*x)^m*(b/2 + c*x)^{(2*p)}, x], x] /; \text{FreeQ}\{a, b, c, d, e, m\}, x] \ \&\& \ \text{EqQ}[b^2 - 4*a*c, 0] \ \&\& \ \text{IntegerQ}[p]$

Maple [A] (verified)

Time = 1.88 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.97

| method | result | size |
|---------------|--|------|
| risch | $\frac{(d(bx+a))^m}{b(-5+m)c^3(bx+a)^5}$ | 29 |
| parallelrisch | $\frac{(d(bx+a))^m}{b(-5+m)c^3(bx+a)^5}$ | 29 |
| norman | $\frac{e^{m \ln(bdx+ad)}}{b c^3(-5+m)(bx+a)^5}$ | 32 |
| orering | $\frac{(bx+a)(bdx+ad)^m}{b(-5+m)(b^2c x^2+2abcx+a^2c)^3}$ | 47 |
| gosper | $\frac{(bdx+ad)^m}{(bx+a)(b^2x^2+2abx+a^2)^2 c^3 b(-5+m)}$ | 48 |

input $\text{int}((b*d*x+a*d)^m/(b^2*c*x^2+2*a*b*c*x+a^2*c)^3, x, \text{method}=_RETURNVERBOSE)$

output $1/b/(-5+m)/c^3/(b*x+a)^5*(d*(b*x+a))^m$

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 159 vs. $2(27) = 54$.

Time = 0.08 (sec) , antiderivative size = 159, normalized size of antiderivative = 5.30

$$\int \frac{(ad + bdx)^m}{(a^2c + 2abcx + b^2cx^2)^3} dx$$

$$= \frac{(bdx + ad)^m}{a^5bc^3m - 5a^5bc^3 + (b^6c^3m - 5b^6c^3)x^5 + 5(ab^5c^3m - 5ab^5c^3)x^4 + 10(a^2b^4c^3m - 5a^2b^4c^3)x^3 + 10(a^3b^3c^3m - 5a^3b^3c^3)x^2 + 5(a^4b^2c^3m - 5a^4b^2c^3)x + 5a^5bc^3}$$

input `integrate((b*d*x+a*d)^m/(b^2*c*x^2+2*a*b*c*x+a^2*c)^3,x, algorithm="fricas")`

output `(b*d*x + a*d)^m/(a^5*b*c^3*m - 5*a^5*b*c^3 + (b^6*c^3*m - 5*b^6*c^3)*x^5 + 5*(a*b^5*c^3*m - 5*a*b^5*c^3)*x^4 + 10*(a^2*b^4*c^3*m - 5*a^2*b^4*c^3)*x^3 + 10*(a^3*b^3*c^3*m - 5*a^3*b^3*c^3)*x^2 + 5*(a^4*b^2*c^3*m - 5*a^4*b^2*c^3)*x)`

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 212 vs. $2(24) = 48$.

Time = 1.74 (sec) , antiderivative size = 212, normalized size of antiderivative = 7.07

$$\int \frac{(ad + bdx)^m}{(a^2c + 2abcx + b^2cx^2)^3} dx$$

$$= \begin{cases} \frac{d^5 x}{ac^3} \\ \frac{x(ad)^m}{a^6 c^3} \\ \frac{d^5 \log\left(\frac{a}{b} + x\right)}{bc^3} \end{cases} \frac{(ad+bdx)^m}{a^5bc^3m - 5a^5bc^3 + 5a^4b^2c^3mx - 25a^4b^2c^3x + 10a^3b^3c^3mx^2 - 50a^3b^3c^3x^2 + 10a^2b^4c^3mx^3 - 50a^2b^4c^3x^3 + 5ab^5c^3mx^4 - 25ab^5c^3x^4 + b^6c^3mx^5 - 5b^6c^3x^5}$$

input `integrate((b*d*x+a*d)**m/(b**2*c*x**2+2*a*b*c*x+a**2*c)**3,x)`

output

```
Piecewise((d**5*x/(a*c**3), Eq(b, 0) & Eq(m, 5)), (x*(a*d)**m/(a**6*c**3),
Eq(b, 0)), (d**5*log(a/b + x)/(b*c**3), Eq(m, 5)), ((a*d + b*d*x)**m/(a**
5*b*c**3*m - 5*a**5*b*c**3 + 5*a**4*b**2*c**3*m*x - 25*a**4*b**2*c**3*x +
10*a**3*b**3*c**3*m*x**2 - 50*a**3*b**3*c**3*x**2 + 10*a**2*b**4*c**3*m*x*
*3 - 50*a**2*b**4*c**3*x**3 + 5*a*b**5*c**3*m*x**4 - 25*a*b**5*c**3*x**4 +
b**6*c**3*m*x**5 - 5*b**6*c**3*x**5), True))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 102 vs. $2(27) = 54$.

Time = 0.05 (sec) , antiderivative size = 102, normalized size of antiderivative = 3.40

$$\int \frac{(ad + bdx)^m}{(a^2c + 2abcx + b^2cx^2)^3} dx$$

$$= \frac{(bx + a)^m d^m}{b^6 c^3 (m - 5)x^5 + 5 ab^5 c^3 (m - 5)x^4 + 10 a^2 b^4 c^3 (m - 5)x^3 + 10 a^3 b^3 c^3 (m - 5)x^2 + 5 a^4 b^2 c^3 (m - 5)x + a^5}$$

input

```
integrate((b*d*x+a*d)^m/(b^2*c*x^2+2*a*b*c*x+a^2*c)^3,x, algorithm="maxima
")
```

output

```
(b*x + a)^m*d^m/(b^6*c^3*(m - 5)*x^5 + 5*a*b^5*c^3*(m - 5)*x^4 + 10*a^2*b^
4*c^3*(m - 5)*x^3 + 10*a^3*b^3*c^3*(m - 5)*x^2 + 5*a^4*b^2*c^3*(m - 5)*x +
a^5*b*c^3*(m - 5))
```

Giac [F]

$$\int \frac{(ad + bdx)^m}{(a^2c + 2abcx + b^2cx^2)^3} dx = \int \frac{(bdx + ad)^m}{(b^2cx^2 + 2abcx + a^2c)^3} dx$$

input

```
integrate((b*d*x+a*d)^m/(b^2*c*x^2+2*a*b*c*x+a^2*c)^3,x, algorithm="giac")
```

output

```
integrate((b*d*x + a*d)^m/(b^2*c*x^2 + 2*a*b*c*x + a^2*c)^3, x)
```

Mupad [B] (verification not implemented)

Time = 5.31 (sec) , antiderivative size = 75, normalized size of antiderivative = 2.50

$$\int \frac{(ad + bdx)^m}{(a^2c + 2abcx + b^2cx^2)^3} dx = \frac{(ad + bdx)^m}{b^6 c^3 (m - 5) \left(x^5 + \frac{a^5}{b^5} + \frac{5ax^4}{b} + \frac{5a^4x}{b^4} + \frac{10a^2x^3}{b^2} + \frac{10a^3x^2}{b^3}\right)}$$

input `int((a*d + b*d*x)^m/(a^2*c + b^2*c*x^2 + 2*a*b*c*x)^3,x)`output `(a*d + b*d*x)^m/(b^6*c^3*(m - 5)*(x^5 + a^5/b^5 + (5*a*x^4)/b + (5*a^4*x)/b^4 + (10*a^2*x^3)/b^2 + (10*a^3*x^2)/b^3))`**Reduce [B] (verification not implemented)**

Time = 0.26 (sec) , antiderivative size = 126, normalized size of antiderivative = 4.20

$$\int \frac{(ad + bdx)^m}{(a^2c + 2abcx + b^2cx^2)^3} dx = \frac{(bdx + ad)^m}{b^6 c^3 (b^5 m x^5 + 5 a b^4 m x^4 - 5 b^5 x^5 + 10 a^2 b^3 m x^3 - 25 a b^4 x^4 + 10 a^3 b^2 m x^2 - 50 a^2 b^3 x^3 + 5 a^4 b m x - 50 a^3 b^2)}$$

input `int((b*d*x+a*d)^m/(b^2*c*x^2+2*a*b*c*x+a^2*c)^3,x)`output `(a*d + b*d*x)**m/(b*c**3*(a**5*m - 5*a**5 + 5*a**4*b*m*x - 25*a**4*b*x + 10*a**3*b**2*m*x**2 - 50*a**3*b**2*x**2 + 10*a**2*b**3*m*x**3 - 50*a**2*b**3*x**3 + 5*a*b**4*m*x**4 - 25*a*b**4*x**4 + b**5*m*x**5 - 5*b**5*x**5))`

3.406 $\int (ad+bdx)^m (a^2c + 2abcx + b^2cx^2)^{3/2} dx$

| | |
|---|------|
| Optimal result | 3060 |
| Mathematica [A] (verified) | 3060 |
| Rubi [A] (verified) | 3061 |
| Maple [A] (verified) | 3062 |
| Fricas [A] (verification not implemented) | 3063 |
| Sympy [F] | 3063 |
| Maxima [A] (verification not implemented) | 3063 |
| Giac [B] (verification not implemented) | 3064 |
| Mupad [F(-1)] | 3064 |
| Reduce [B] (verification not implemented) | 3065 |

Optimal result

Integrand size = 35, antiderivative size = 49

$$\int (ad + bdx)^m (a^2c + 2abcx + b^2cx^2)^{3/2} dx = \frac{c(ad + bdx)^{3+m} \sqrt{a^2c + 2abcx + b^2cx^2}}{bd^3(4 + m)}$$

output

```
c*(b*d*x+a*d)^(3+m)*(b^2*c*x^2+2*a*b*c*x+a^2*c)^(1/2)/b/d^3/(4+m)
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.73

$$\int (ad + bdx)^m (a^2c + 2abcx + b^2cx^2)^{3/2} dx = \frac{(a + bx)(d(a + bx))^m (c(a + bx)^2)^{3/2}}{b(4 + m)}$$

input

```
Integrate[(a*d + b*d*x)^m*(a^2*c + 2*a*b*c*x + b^2*c*x^2)^(3/2),x]
```

output

```
((a + b*x)*(d*(a + b*x))^m*(c*(a + b*x)^2)^(3/2))/(b*(4 + m))
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.14, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.114$, Rules used = {1102, 27, 35, 17}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\begin{aligned}
 & \int (a^2c + 2abcx + b^2cx^2)^{3/2} (ad + bdx)^m dx \\
 & \quad \downarrow \text{1102} \\
 & \frac{\sqrt{a^2c + 2abcx + b^2cx^2} \int b^3c^3(a + bx)^3(ad + bdx)^m dx}{b^3c^2(a + bx)} \\
 & \quad \downarrow \text{27} \\
 & \frac{c\sqrt{a^2c + 2abcx + b^2cx^2} \int (a + bx)^3(ad + bdx)^m dx}{a + bx} \\
 & \quad \downarrow \text{35} \\
 & \frac{c\sqrt{a^2c + 2abcx + b^2cx^2} \int (ad + bdx)^{m+3} dx}{d^3(a + bx)} \\
 & \quad \downarrow \text{17} \\
 & \frac{c\sqrt{a^2c + 2abcx + b^2cx^2}(ad + bdx)^{m+4}}{bd^4(m + 4)(a + bx)}
 \end{aligned}$$

input `Int[(a*d + b*d*x)^m*(a^2*c + 2*a*b*c*x + b^2*c*x^2)^(3/2),x]`

output `(c*(a*d + b*d*x)^(4 + m)*Sqrt[a^2*c + 2*a*b*c*x + b^2*c*x^2])/(b*d^4*(4 + m)*(a + b*x))`

Definitions of rubi rules used

rule 17 `Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] := Simp[c*((a + b*x)^(m + 1))/(b*(m + 1))], x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]`

rule 27 `Int[(a_)*(Fx_), x_Symbol] := Simp[a Int[Fx, x], x] /; FreeQ[a, x] && !MatchQ[Fx, (b_)*(Gx_)] /; FreeQ[b, x]`

rule 35 `Int[(u_.)*((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(n_.), x_Symbol] := Simp[(b/d)^m Int[u*(c + d*x)^(m + n), x], x] /; FreeQ[{a, b, c, d, m, n}, x] && EqQ[b*c - a*d, 0] && IntegerQ[m] && !(IntegerQ[n] && SimplerQ[a + b*x, c + d*x])`

rule 1102 `Int[((d_.) + (e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] := Simp[(a + b*x + c*x^2)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x)^(2*FracPart[p])) Int[(d + e*x)^m*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && EqQ[b^2 - 4*a*c, 0]`

Maple [A] (verified)

Time = 1.28 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.96

| method | result | size |
|---------|---|------|
| gospers | $\frac{(bx+a)(bdx+ad)^m (b^2cx^2+2abcx+a^2c)^{\frac{3}{2}}}{b(4+m)}$ | 47 |
| orering | $\frac{(bx+a)(bdx+ad)^m (b^2cx^2+2abcx+a^2c)^{\frac{3}{2}}}{b(4+m)}$ | 47 |
| risch | $\frac{c\sqrt{c(bx+a)^2 (b^4x^4+4ab^3x^3+6a^2b^2x^2+4a^3bx+a^4)}(d(bx+a))^m}{(bx+a)b(4+m)}$ | 76 |

input `int((b*d*x+a*d)^m*(b^2*c*x^2+2*a*b*c*x+a^2*c)^(3/2),x,method=_RETURNVERBOSE)`

output `(b*x+a)/b/(4+m)*(b*d*x+a*d)^m*(b^2*c*x^2+2*a*b*c*x+a^2*c)^(3/2)`

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.51

$$\int (ad + bdx)^m (a^2c + 2abcx + b^2cx^2)^{3/2} dx = \frac{(b^3cx^3 + 3ab^2cx^2 + 3a^2bcx + a^3c)\sqrt{b^2cx^2 + 2abcx + a^2c}(bdx + ad)^m}{bm + 4b}$$

input `integrate((b*d*x+a*d)^m*(b^2*c*x^2+2*a*b*c*x+a^2*c)^(3/2),x, algorithm="fricas")`

output `(b^3*c*x^3 + 3*a*b^2*c*x^2 + 3*a^2*b*c*x + a^3*c)*sqrt(b^2*c*x^2 + 2*a*b*c*x + a^2*c)*(b*d*x + a*d)^m/(b*m + 4*b)`

Sympy [F]

$$\int (ad + bdx)^m (a^2c + 2abcx + b^2cx^2)^{3/2} dx = \int (c(a + bx)^2)^{\frac{3}{2}} (d(a + bx))^m dx$$

input `integrate((b*d*x+a*d)**m*(b**2*c*x**2+2*a*b*c*x+a**2*c)**(3/2),x)`

output `Integral((c*(a + b*x)**2)**(3/2)*(d*(a + b*x))**m, x)`

Maxima [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.73

$$\int (ad + bdx)^m (a^2c + 2abcx + b^2cx^2)^{3/2} dx = \frac{(b^4c^{\frac{3}{2}}d^m x^4 + 4ab^3c^{\frac{3}{2}}d^m x^3 + 6a^2b^2c^{\frac{3}{2}}d^m x^2 + 4a^3bc^{\frac{3}{2}}d^m x + a^4c^{\frac{3}{2}}d^m)(bx + a)^m}{b(m + 4)}$$

input `integrate((b*d*x+a*d)^m*(b^2*c*x^2+2*a*b*c*x+a^2*c)^(3/2),x, algorithm="maxima")`

output $(b^4c^{3/2}d^m x^4 + 4a^3b^3c^{3/2}d^m x^3 + 6a^2b^2c^{3/2}d^m x^2 + 4a^3b^3c^{3/2}d^m x + a^4c^{3/2}d^m)(bx + a)^m/(b(m + 4))$

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 137 vs. $2(47) = 94$.

Time = 0.14 (sec) , antiderivative size = 137, normalized size of antiderivative = 2.80

$$\int (ad + bdx)^m (a^2c + 2abcx + b^2cx^2)^{3/2} dx = \frac{((bdx + ad)^m b^4 cx^4 \operatorname{sgn}(bx + a) + 4(bdx + ad)^m ab^3 cx^3 \operatorname{sgn}(bx + a) + 6(bdx + ad)^m a^2 b^2 cx^2 \operatorname{sgn}(bx + a) + (bdx + ad)^m a^3 \operatorname{sgn}(bx + a)) \sqrt{c}}{bm + 4b}$$

input `integrate((b*d*x+a*d)^m*(b^2*c*x^2+2*a*b*c*x+a^2*c)^(3/2),x, algorithm="giac")`

output $((b*d*x + a*d)^m*b^4*c*x^4*\operatorname{sgn}(b*x + a) + 4*(b*d*x + a*d)^m*a*b^3*c*x^3*\operatorname{sgn}(b*x + a) + 6*(b*d*x + a*d)^m*a^2*b^2*c*x^2*\operatorname{sgn}(b*x + a) + 4*(b*d*x + a*d)^m*a^3*b*c*x*\operatorname{sgn}(b*x + a) + (b*d*x + a*d)^m*a^4*c*\operatorname{sgn}(b*x + a))*\operatorname{sqrt}(c)/(b*m + 4*b)$

Mupad [F(-1)]

Timed out.

$$\int (ad + bdx)^m (a^2c + 2abcx + b^2cx^2)^{3/2} dx = \int (ad + bdx)^m (ca^2 + 2cabx + cb^2x^2)^{3/2} dx$$

input `int((a*d + b*d*x)^m*(a^2*c + b^2*c*x^2 + 2*a*b*c*x)^(3/2),x)`

output `int((a*d + b*d*x)^m*(a^2*c + b^2*c*x^2 + 2*a*b*c*x)^(3/2), x)`

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.22

$$\int (ad + bdx)^m (a^2c + 2abcx + b^2cx^2)^{3/2} dx = \frac{\sqrt{c}(bdx + ad)^m c(b^4x^4 + 4ab^3x^3 + 6a^2b^2x^2 + 4a^3bx + a^4)}{b(m + 4)}$$

input `int((b*d*x+a*d)^m*(b^2*c*x^2+2*a*b*c*x+a^2*c)^(3/2),x)`

output `(sqrt(c)*(a*d + b*d*x)**m*c*(a**4 + 4*a**3*b*x + 6*a**2*b**2*x**2 + 4*a*b*
*3*x**3 + b**4*x**4))/(b*(m + 4))`

3.407 $\int (ad + bdx)^m \sqrt{a^2c + 2abcx + b^2cx^2} dx$

| | |
|---|------|
| Optimal result | 3066 |
| Mathematica [A] (verified) | 3066 |
| Rubi [A] (verified) | 3067 |
| Maple [A] (verified) | 3068 |
| Fricas [A] (verification not implemented) | 3069 |
| Sympy [F] | 3069 |
| Maxima [A] (verification not implemented) | 3069 |
| Giac [A] (verification not implemented) | 3070 |
| Mupad [F(-1)] | 3070 |
| Reduce [B] (verification not implemented) | 3070 |

Optimal result

Integrand size = 35, antiderivative size = 48

$$\int (ad + bdx)^m \sqrt{a^2c + 2abcx + b^2cx^2} dx = \frac{(ad + bdx)^{1+m} \sqrt{a^2c + 2abcx + b^2cx^2}}{bd(2 + m)}$$

output

```
(b*d*x+a*d)^(1+m)*(b^2*c*x^2+2*a*b*c*x+a^2*c)^(1/2)/b/d/(2+m)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.75

$$\int (ad + bdx)^m \sqrt{a^2c + 2abcx + b^2cx^2} dx = \frac{(a + bx)(d(a + bx))^m \sqrt{c(a + bx)^2}}{b(2 + m)}$$

input

```
Integrate[(a*d + b*d*x)^m*Sqrt[a^2*c + 2*a*b*c*x + b^2*c*x^2],x]
```

output

```
((a + b*x)*(d*(a + b*x))^m*Sqrt[c*(a + b*x)^2])/(b*(2 + m))
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.15, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.114$, Rules used = {1102, 27, 35, 17}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\begin{aligned}
 & \int \sqrt{a^2c + 2abcx + b^2cx^2}(ad + bdx)^m dx \\
 & \quad \downarrow \text{1102} \\
 & \frac{\sqrt{a^2c + 2abcx + b^2cx^2} \int bc(a + bx)(ad + bdx)^m dx}{bc(a + bx)} \\
 & \quad \downarrow \text{27} \\
 & \frac{\sqrt{a^2c + 2abcx + b^2cx^2} \int (a + bx)(ad + bdx)^m dx}{a + bx} \\
 & \quad \downarrow \text{35} \\
 & \frac{\sqrt{a^2c + 2abcx + b^2cx^2} \int (ad + bdx)^{m+1} dx}{d(a + bx)} \\
 & \quad \downarrow \text{17} \\
 & \frac{\sqrt{a^2c + 2abcx + b^2cx^2}(ad + bdx)^{m+2}}{bd^2(m + 2)(a + bx)}
 \end{aligned}$$

input `Int[(a*d + b*d*x)^m*sqrt[a^2*c + 2*a*b*c*x + b^2*c*x^2],x]`

output `((a*d + b*d*x)^(2 + m)*sqrt[a^2*c + 2*a*b*c*x + b^2*c*x^2])/(b*d^2*(2 + m)*(a + b*x))`

Definitions of rubi rules used

rule 17 $\text{Int}[(c_)*((a_)+(b_)*(x_))^{(m_)}, x_Symbol] \rightarrow \text{Simp}[c*((a+b*x)^{(m+1)})/(b*(m+1)), x] /; \text{FreeQ}\{a, b, c, m\}, x] \ \&\& \ \text{NeQ}[m, -1]$

rule 27 $\text{Int}[(a_)*(F_x), x_Symbol] \rightarrow \text{Simp}[a \ \text{Int}[F_x, x], x] /; \text{FreeQ}[a, x] \ \&\& \ !\text{MatchQ}[F_x, (b_)*(G_x)] /; \text{FreeQ}[b, x]$

rule 35 $\text{Int}[(u_)*((a_)+(b_)*(x_))^{(m_)}*((c_)+(d_)*(x_))^{(n_)}, x_Symbol] \rightarrow \text{Simp}[(b/d)^m \ \text{Int}[u*(c+d*x)^{(m+n)}, x], x] /; \text{FreeQ}\{a, b, c, d, m, n\}, x] \ \&\& \ \text{EqQ}[b*c - a*d, 0] \ \&\& \ \text{IntegerQ}[m] \ \&\& \ !(\text{IntegerQ}[n] \ \&\& \ \text{SimplerQ}[a+b*x, c+d*x])$

rule 1102 $\text{Int}[(d_)+(e_)*(x_))^{(m_)}*((a_)+(b_)*(x_)+(c_)*(x_)^2)^{(p_)}, x_Symbol] \rightarrow \text{Simp}[(a+b*x+c*x^2)^{\text{FracPart}[p]}/(c^{\text{IntPart}[p]}*(b/2+c*x)^{(2*\text{FracPart}[p])}) \ \text{Int}[(d+e*x)^m*(b/2+c*x)^{(2*p)}, x], x] /; \text{FreeQ}\{a, b, c, d, e, m, p\}, x] \ \&\& \ \text{EqQ}[b^2 - 4*a*c, 0]$

Maple [A] (verified)

Time = 1.08 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.98

| method | result | size |
|---------|--|------|
| gospers | $\frac{(bx+a)(bdx+ad)^m \sqrt{b^2cx^2+2abcx+a^2c}}{b(2+m)}$ | 47 |
| orering | $\frac{(bx+a)(bdx+ad)^m \sqrt{b^2cx^2+2abcx+a^2c}}{b(2+m)}$ | 47 |
| risch | $\frac{\sqrt{c(bx+a)^2} (b^2x^2+2abx+a^2)(d(bx+a))^m}{(bx+a)b(2+m)}$ | 53 |

input $\text{int}((b*d*x+a*d)^m*(b^2*c*x^2+2*a*b*c*x+a^2*c)^{(1/2)}, x, \text{method}=_RETURNVERBOSE)$

output $(b*x+a)/b/(2+m)*(b*d*x+a*d)^m*(b^2*c*x^2+2*a*b*c*x+a^2*c)^{(1/2)}$

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.98

$$\int (ad + bdx)^m \sqrt{a^2c + 2abcx + b^2cx^2} dx = \frac{\sqrt{b^2cx^2 + 2abcx + a^2c}(bx + a)(bdx + ad)^m}{bm + 2b}$$

input `integrate((b*d*x+a*d)^m*(b^2*c*x^2+2*a*b*c*x+a^2*c)^(1/2),x, algorithm="fricas")`

output `sqrt(b^2*c*x^2 + 2*a*b*c*x + a^2*c)*(b*x + a)*(b*d*x + a*d)^m/(b*m + 2*b)`

Sympy [F]

$$\int (ad + bdx)^m \sqrt{a^2c + 2abcx + b^2cx^2} dx = \int \sqrt{c(a + bx)^2} (d(a + bx))^m dx$$

input `integrate((b*d*x+a*d)**m*(b**2*c*x**2+2*a*b*c*x+a**2*c)**(1/2),x)`

output `Integral(sqrt(c*(a + b*x)**2)*(d*(a + b*x))**m, x)`

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.06

$$\int (ad + bdx)^m \sqrt{a^2c + 2abcx + b^2cx^2} dx = \frac{(b^2\sqrt{cd^m}x^2 + 2ab\sqrt{cd^m}x + a^2\sqrt{cd^m})(bx + a)^m}{b(m + 2)}$$

input `integrate((b*d*x+a*d)^m*(b^2*c*x^2+2*a*b*c*x+a^2*c)^(1/2),x, algorithm="maxima")`

output `(b^2*sqrt(c)*d^m*x^2 + 2*a*b*sqrt(c)*d^m*x + a^2*sqrt(c)*d^m)*(b*x + a)^m/(b*(m + 2))`

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.62

$$\int (ad + bdx)^m \sqrt{a^2c + 2abcx + b^2cx^2} dx$$

$$= \frac{((bdx + ad)^m b^2 x^2 \operatorname{sgn}(bx + a) + 2(bdx + ad)^m abx \operatorname{sgn}(bx + a) + (bdx + ad)^m a^2 \operatorname{sgn}(bx + a)) \sqrt{c}}{bm + 2b}$$

input `integrate((b*d*x+a*d)^m*(b^2*c*x^2+2*a*b*c*x+a^2*c)^(1/2),x, algorithm="giac")`

output `((b*d*x + a*d)^m*b^2*x^2*sgn(b*x + a) + 2*(b*d*x + a*d)^m*a*b*x*sgn(b*x + a) + (b*d*x + a*d)^m*a^2*sgn(b*x + a))*sqrt(c)/(b*m + 2*b)`

Mupad [F(-1)]

Timed out.

$$\int (ad + bdx)^m \sqrt{a^2c + 2abcx + b^2cx^2} dx = \int (ad + bdx)^m \sqrt{ca^2 + 2cabx + cb^2x^2} dx$$

input `int((a*d + b*d*x)^m*(a^2*c + b^2*c*x^2 + 2*a*b*c*x)^(1/2),x)`

output `int((a*d + b*d*x)^m*(a^2*c + b^2*c*x^2 + 2*a*b*c*x)^(1/2), x)`

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.77

$$\int (ad + bdx)^m \sqrt{a^2c + 2abcx + b^2cx^2} dx = \frac{\sqrt{c}(bdx + ad)^m (b^2x^2 + 2abx + a^2)}{b(m + 2)}$$

input `int((b*d*x+a*d)^m*(b^2*c*x^2+2*a*b*c*x+a^2*c)^(1/2),x)`

output `(sqrt(c)*(a*d + b*d*x)**m*(a**2 + 2*a*b*x + b**2*x**2))/(b*(m + 2))`

3.408 $\int \frac{(ad+bdx)^m}{\sqrt{a^2c+2abcx+b^2cx^2}} dx$

| | |
|---|------|
| Optimal result | 3071 |
| Mathematica [A] (verified) | 3071 |
| Rubi [A] (verified) | 3072 |
| Maple [A] (verified) | 3073 |
| Fricas [A] (verification not implemented) | 3074 |
| Sympy [F] | 3074 |
| Maxima [A] (verification not implemented) | 3074 |
| Giac [F] | 3075 |
| Mupad [B] (verification not implemented) | 3075 |
| Reduce [B] (verification not implemented) | 3075 |

Optimal result

Integrand size = 35, antiderivative size = 46

$$\int \frac{(ad + bdx)^m}{\sqrt{a^2c + 2abcx + b^2cx^2}} dx = \frac{(ad + bdx)^{1+m}}{b d m \sqrt{a^2c + 2abcx + b^2cx^2}}$$

output

$$(b*d*x+a*d)^{(1+m)}/b/d/m/(b^2*c*x^2+2*a*b*c*x+a^2*c)^{(1/2)}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.74

$$\int \frac{(ad + bdx)^m}{\sqrt{a^2c + 2abcx + b^2cx^2}} dx = \frac{(a + bx)(d(a + bx))^m}{b m \sqrt{c(a + bx)^2}}$$

input

$$\text{Integrate}[(a*d + b*d*x)^m/\text{Sqrt}[a^2*c + 2*a*b*c*x + b^2*c*x^2], x]$$

output

$$((a + b*x)*(d*(a + b*x))^m)/(b*m*\text{Sqrt}[c*(a + b*x)^2])$$

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.114$, Rules used = {1102, 27, 35, 17}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\begin{aligned}
 & \int \frac{(ad + bdx)^m}{\sqrt{a^2c + 2abcx + b^2cx^2}} dx \\
 & \quad \downarrow \text{1102} \\
 & \frac{bc(a + bx) \int \frac{(ad+bx)^m}{bc(a+bx)} dx}{\sqrt{a^2c + 2abcx + b^2cx^2}} \\
 & \quad \downarrow \text{27} \\
 & \frac{(a + bx) \int \frac{(ad+bx)^m}{a+bx} dx}{\sqrt{a^2c + 2abcx + b^2cx^2}} \\
 & \quad \downarrow \text{35} \\
 & \frac{d(a + bx) \int (ad + bdx)^{m-1} dx}{\sqrt{a^2c + 2abcx + b^2cx^2}} \\
 & \quad \downarrow \text{17} \\
 & \frac{(a + bx)(ad + bdx)^m}{bm\sqrt{a^2c + 2abcx + b^2cx^2}}
 \end{aligned}$$

input `Int[(a*d + b*d*x)^m/Sqrt[a^2*c + 2*a*b*c*x + b^2*c*x^2],x]`

output `((a + b*x)*(a*d + b*d*x)^m)/(b*m*Sqrt[a^2*c + 2*a*b*c*x + b^2*c*x^2])`

Definitions of rubi rules used

rule 17 $\text{Int}[(c_)*((a_)+(b_)*(x_))^{(m_)}, x_Symbol] \rightarrow \text{Simp}[c*((a+b*x)^{(m+1)})/(b*(m+1)), x] /; \text{FreeQ}\{a, b, c, m\}, x] \ \&\& \ \text{NeQ}[m, -1]$

rule 27 $\text{Int}[(a_)*(F_x), x_Symbol] \rightarrow \text{Simp}[a \ \text{Int}[F_x, x], x] /; \text{FreeQ}[a, x] \ \&\& \ !\text{MatchQ}[F_x, (b_)*(G_x)] /; \text{FreeQ}[b, x]$

rule 35 $\text{Int}[(u_)*((a_)+(b_)*(x_))^{(m_)*}((c_)+(d_)*(x_))^{(n_)}, x_Symbol] \rightarrow \text{Simp}[(b/d)^m \ \text{Int}[u*(c+d*x)^{(m+n)}, x], x] /; \text{FreeQ}\{a, b, c, d, m, n\}, x] \ \&\& \ \text{EqQ}[b*c - a*d, 0] \ \&\& \ \text{IntegerQ}[m] \ \&\& \ !(\text{IntegerQ}[n] \ \&\& \ \text{SimplerQ}[a+b*x, c+d*x])$

rule 1102 $\text{Int}(((d_)+(e_)*(x_))^{(m_)*}((a_)+(b_)*(x_)+(c_)*(x_)^2)^{(p_)}, x_Symbol] \rightarrow \text{Simp}[(a+b*x+c*x^2)^{\text{FracPart}[p]}/(c^{\text{IntPart}[p]}*(b/2+c*x)^{(2*\text{FracPart}[p])}) \ \text{Int}[(d+e*x)^m*(b/2+c*x)^{(2*p)}, x], x] /; \text{FreeQ}\{a, b, c, d, e, m, p\}, x] \ \&\& \ \text{EqQ}[b^2 - 4*a*c, 0]$

Maple [A] (verified)

Time = 1.35 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.72

| method | result | size |
|---------|--|------|
| risch | $\frac{(bx+a)(d(bx+a))^m}{\sqrt{c(bx+a)^2}bm}$ | 33 |
| gospers | $\frac{(bx+a)(bdx+ad)^m}{bm\sqrt{b^2cx^2+2abcx+a^2c}}$ | 45 |
| orering | $\frac{(bx+a)(bdx+ad)^m}{bm\sqrt{b^2cx^2+2abcx+a^2c}}$ | 45 |

input $\text{int}((b*d*x+a*d)^m/(b^2*c*x^2+2*a*b*c*x+a^2*c)^{(1/2)}, x, \text{method}=_RETURNVERBOSE)$

output $1/(c*(b*x+a)^2)^{(1/2)}*(b*x+a)/b/m*(d*(b*x+a))^m$

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.04

$$\int \frac{(ad + bdx)^m}{\sqrt{a^2c + 2abcx + b^2cx^2}} dx = \frac{\sqrt{b^2cx^2 + 2abcx + a^2c}(bdx + ad)^m}{b^2cmx + abcm}$$

input `integrate((b*d*x+a*d)^m/(b^2*c*x^2+2*a*b*c*x+a^2*c)^(1/2),x, algorithm="fricas")`

output `sqrt(b^2*c*x^2 + 2*a*b*c*x + a^2*c)*(b*d*x + a*d)^m/(b^2*c*m*x + a*b*c*m)`

Sympy [F]

$$\int \frac{(ad + bdx)^m}{\sqrt{a^2c + 2abcx + b^2cx^2}} dx = \int \frac{(d(a + bx))^m}{\sqrt{c(a + bx)^2}} dx$$

input `integrate((b*d*x+a*d)**m/(b**2*c*x**2+2*a*b*c*x+a**2*c)**(1/2),x)`

output `Integral((d*(a + b*x))**m/sqrt(c*(a + b*x)**2), x)`

Maxima [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.43

$$\int \frac{(ad + bdx)^m}{\sqrt{a^2c + 2abcx + b^2cx^2}} dx = \frac{(bx + a)^m d^m}{b\sqrt{cm}}$$

input `integrate((b*d*x+a*d)^m/(b^2*c*x^2+2*a*b*c*x+a^2*c)^(1/2),x, algorithm="maxima")`

output `(b*x + a)^m*d^m/(b*sqrt(c)*m)`

Giac [F]

$$\int \frac{(ad + bdx)^m}{\sqrt{a^2c + 2abcx + b^2cx^2}} dx = \int \frac{(bdx + ad)^m}{\sqrt{b^2cx^2 + 2abcx + a^2c}} dx$$

input `integrate((b*d*x+a*d)^m/(b^2*c*x^2+2*a*b*c*x+a^2*c)^(1/2),x, algorithm="giac")`

output `integrate((b*d*x + a*d)^m/sqrt(b^2*c*x^2 + 2*a*b*c*x + a^2*c), x)`

Mupad [B] (verification not implemented)

Time = 5.43 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.11

$$\int \frac{(ad + bdx)^m}{\sqrt{a^2c + 2abcx + b^2cx^2}} dx = \frac{(ad + bdx)^m \sqrt{ca^2 + 2cabbx + cb^2x^2}}{b^2cm \left(x + \frac{a}{b}\right)}$$

input `int((a*d + b*d*x)^m/(a^2*c + b^2*c*x^2 + 2*a*b*c*x)^(1/2),x)`

output `((a*d + b*d*x)^m*(a^2*c + b^2*c*x^2 + 2*a*b*c*x)^(1/2))/(b^2*c*m*(x + a/b))`

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.48

$$\int \frac{(ad + bdx)^m}{\sqrt{a^2c + 2abcx + b^2cx^2}} dx = \frac{\sqrt{c}(bdx + ad)^m}{bcm}$$

input `int((b*d*x+a*d)^m/(b^2*c*x^2+2*a*b*c*x+a^2*c)^(1/2),x)`

output `(sqrt(c)*(a*d + b*d*x)**m)/(b*c*m)`

$$3.409 \quad \int \frac{(ad+bdx)^m}{(a^2c+2abcx+b^2cx^2)^{3/2}} dx$$

| | |
|---|------|
| Optimal result | 3076 |
| Mathematica [A] (verified) | 3076 |
| Rubi [A] (verified) | 3077 |
| Maple [A] (verified) | 3078 |
| Fricas [B] (verification not implemented) | 3079 |
| Sympy [F] | 3079 |
| Maxima [A] (verification not implemented) | 3079 |
| Giac [F] | 3080 |
| Mupad [F(-1)] | 3080 |
| Reduce [B] (verification not implemented) | 3080 |

Optimal result

Integrand size = 35, antiderivative size = 52

$$\int \frac{(ad+bdx)^m}{(a^2c+2abcx+b^2cx^2)^{3/2}} dx = -\frac{d(ad+bdx)^{-1+m}}{bc(2-m)\sqrt{a^2c+2abcx+b^2cx^2}}$$

output

```
-d*(b*d*x+a*d)^(-1+m)/b/c/(2-m)/(b^2*c*x^2+2*a*b*c*x+a^2*c)^(1/2)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.69

$$\int \frac{(ad+bdx)^m}{(a^2c+2abcx+b^2cx^2)^{3/2}} dx = \frac{(a+bx)(d(a+bx))^m}{b(-2+m)(c(a+bx)^2)^{3/2}}$$

input

```
Integrate[(a*d + b*d*x)^m/(a^2*c + 2*a*b*c*x + b^2*c*x^2)^(3/2), x]
```

output

```
((a + b*x)*(d*(a + b*x))^m)/(b*(-2 + m)*(c*(a + b*x)^2)^(3/2))
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.13, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.114$, Rules used = {1102, 27, 35, 17}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\begin{aligned}
 & \int \frac{(ad + bdx)^m}{(a^2c + 2abcx + b^2cx^2)^{3/2}} dx \\
 & \quad \downarrow \text{1102} \\
 & \frac{b^3c^2(a + bx) \int \frac{(ad + bdx)^m}{b^3c^3(a + bx)^3} dx}{\sqrt{a^2c + 2abcx + b^2cx^2}} \\
 & \quad \downarrow \text{27} \\
 & \frac{(a + bx) \int \frac{(ad + bdx)^m}{(a + bx)^3} dx}{c\sqrt{a^2c + 2abcx + b^2cx^2}} \\
 & \quad \downarrow \text{35} \\
 & \frac{d^3(a + bx) \int (ad + bdx)^{m-3} dx}{c\sqrt{a^2c + 2abcx + b^2cx^2}} \\
 & \quad \downarrow \text{17} \\
 & -\frac{d^2(a + bx)(ad + bdx)^{m-2}}{bc(2 - m)\sqrt{a^2c + 2abcx + b^2cx^2}}
 \end{aligned}$$

input `Int[(a*d + b*d*x)^m/(a^2*c + 2*a*b*c*x + b^2*c*x^2)^(3/2),x]`

output `-((d^2*(a + b*x)*(a*d + b*d*x)^(-2 + m))/(b*c*(2 - m)*Sqrt[a^2*c + 2*a*b*c*x + b^2*c*x^2]))`

Definitions of rubi rules used

rule 17 $\text{Int}[(c_)*(a_)+(b_)*(x_)]^{(m_)}, x_Symbol] \rightarrow \text{Simp}[c*((a + b*x)^{(m + 1)})/(b*(m + 1)), x] /; \text{FreeQ}[\{a, b, c, m\}, x] \ \&\& \ \text{NeQ}[m, -1]$

rule 27 $\text{Int}[(a_)*(F_x_), x_Symbol] \rightarrow \text{Simp}[a \ \text{Int}[F_x, x], x] /; \text{FreeQ}[a, x] \ \&\& \ !\text{MatchQ}[F_x, (b_)*(G_x_)] /; \text{FreeQ}[b, x]$

rule 35 $\text{Int}[(u_)*((a_) + (b_)*(x_))^{(m_)}*((c_) + (d_)*(x_))^{(n_)}, x_Symbol] \rightarrow \text{Simp}[(b/d)^m \ \text{Int}[u*(c + d*x)^{(m + n)}, x], x] /; \text{FreeQ}[\{a, b, c, d, m, n\}, x] \ \&\& \ \text{EqQ}[b*c - a*d, 0] \ \&\& \ \text{IntegerQ}[m] \ \&\& \ !(\text{IntegerQ}[n] \ \&\& \ \text{SimplerQ}[a + b*x, c + d*x])$

rule 1102 $\text{Int}(((d_) + (e_)*(x_))^{(m_)}*((a_) + (b_)*(x_) + (c_)*(x_)^2)^{(p_)}, x_Symbol] \rightarrow \text{Simp}[(a + b*x + c*x^2)^{\text{FracPart}[p]}/(c^{\text{IntPart}[p]}*(b/2 + c*x)^{(2*\text{FracPart}[p])}) \ \text{Int}[(d + e*x)^m*(b/2 + c*x)^{(2*p)}, x], x] /; \text{FreeQ}[\{a, b, c, d, e, m, p\}, x] \ \&\& \ \text{EqQ}[b^2 - 4*a*c, 0]$

Maple [A] (verified)

Time = 1.38 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.77

| method | result | size |
|---------|---|------|
| risch | $\frac{(d(bx+a))^m}{c(bx+a)\sqrt{c(bx+a)^2 b(-2+m)}}$ | 40 |
| gospers | $\frac{(bx+a)(bdx+ad)^m}{b(-2+m)(b^2c x^2+2abcx+a^2c)^{\frac{3}{2}}}$ | 47 |
| orering | $\frac{(bx+a)(bdx+ad)^m}{b(-2+m)(b^2c x^2+2abcx+a^2c)^{\frac{3}{2}}}$ | 47 |

input $\text{int}((b*d*x+a*d)^m/(b^2*c*x^2+2*a*b*c*x+a^2*c)^{(3/2)}, x, \text{method}=_RETURNVERBOSE)$

output $1/c/(b*x+a)/(c*(b*x+a)^2)^{(1/2)}/b/(-2+m)*(d*(b*x+a))^m$

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 125 vs. $2(47) = 94$.

Time = 0.09 (sec) , antiderivative size = 125, normalized size of antiderivative = 2.40

$$\int \frac{(ad + bdx)^m}{(a^2c + 2abcx + b^2cx^2)^{3/2}} dx = \frac{\sqrt{b^2cx^2 + 2abcx + a^2c}(bdx + ad)^m}{a^3bc^2m - 2a^3bc^2 + (b^4c^2m - 2b^4c^2)x^3 + 3(ab^3c^2m - 2ab^3c^2)x^2 + 3(a^2b^3c^2m - 2a^2b^3c^2)x}$$

input `integrate((b*d*x+a*d)^m/(b^2*c*x^2+2*a*b*c*x+a^2*c)^(3/2),x, algorithm="fricas")`

output `sqrt(b^2*c*x^2 + 2*a*b*c*x + a^2*c)*(b*d*x + a*d)^m/(a^3*b*c^2*m - 2*a^3*b*c^2 + (b^4*c^2*m - 2*b^4*c^2)*x^3 + 3*(a*b^3*c^2*m - 2*a*b^3*c^2)*x^2 + 3*(a^2*b^3*c^2*m - 2*a^2*b^3*c^2)*x)`

Sympy [F]

$$\int \frac{(ad + bdx)^m}{(a^2c + 2abcx + b^2cx^2)^{3/2}} dx = \int \frac{(d(a + bx))^m}{(c(a + bx)^2)^{\frac{3}{2}}} dx$$

input `integrate((b*d*x+a*d)**m/(b**2*c*x**2+2*a*b*c*x+a**2*c)**(3/2),x)`

output `Integral((d*(a + b*x))**m/(c*(a + b*x)**2)**(3/2), x)`

Maxima [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.04

$$\int \frac{(ad + bdx)^m}{(a^2c + 2abcx + b^2cx^2)^{3/2}} dx = \frac{(bx + a)^m \sqrt{cd}^m}{b^3c^2(m - 2)x^2 + 2ab^2c^2(m - 2)x + a^2bc^2(m - 2)}$$

input `integrate((b*d*x+a*d)^m/(b^2*c*x^2+2*a*b*c*x+a^2*c)^(3/2),x, algorithm="maxima")`

output $(b*x + a)^m * \text{sqrt}(c) * d^m / (b^3 * c^2 * (m - 2) * x^2 + 2 * a * b^2 * c^2 * (m - 2) * x + a^2 * b * c^2 * (m - 2))$

Giac [F]

$$\int \frac{(ad + bdx)^m}{(a^2c + 2abcx + b^2cx^2)^{3/2}} dx = \int \frac{(bdx + ad)^m}{(b^2cx^2 + 2abcx + a^2c)^{3/2}} dx$$

input `integrate((b*d*x+a*d)^m/(b^2*c*x^2+2*a*b*c*x+a^2*c)^(3/2),x, algorithm="giac")`

output `integrate((b*d*x + a*d)^m/(b^2*c*x^2 + 2*a*b*c*x + a^2*c)^(3/2), x)`

Mupad [F(-1)]

Timed out.

$$\int \frac{(ad + bdx)^m}{(a^2c + 2abcx + b^2cx^2)^{3/2}} dx = \int \frac{(ad + bdx)^m}{(ca^2 + 2cabx + cb^2x^2)^{3/2}} dx$$

input `int((a*d + b*d*x)^m/(a^2*c + b^2*c*x^2 + 2*a*b*c*x)^(3/2),x)`

output `int((a*d + b*d*x)^m/(a^2*c + b^2*c*x^2 + 2*a*b*c*x)^(3/2), x)`

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.13

$$\int \frac{(ad + bdx)^m}{(a^2c + 2abcx + b^2cx^2)^{3/2}} dx = \frac{\sqrt{c} (bdx + ad)^m}{b c^2 (b^2 m x^2 + 2 ab m x - 2 b^2 x^2 + a^2 m - 4 ab x - 2 a^2)}$$

input `int((b*d*x+a*d)^m/(b^2*c*x^2+2*a*b*c*x+a^2*c)^(3/2),x)`

output $(\sqrt{c}(a*d + b*d*x)**m)/(b*c**2*(a**2*m - 2*a**2 + 2*a*b*m*x - 4*a*b*x + b**2*m*x**2 - 2*b**2*x**2))$

3.410 $\int \frac{(ad+bdx)^m}{(a^2c+2abcx+b^2cx^2)^{5/2}} dx$

| | |
|---|------|
| Optimal result | 3082 |
| Mathematica [A] (verified) | 3082 |
| Rubi [A] (verified) | 3083 |
| Maple [A] (verified) | 3084 |
| Fricas [B] (verification not implemented) | 3085 |
| Sympy [F] | 3085 |
| Maxima [A] (verification not implemented) | 3085 |
| Giac [F] | 3086 |
| Mupad [F(-1)] | 3086 |
| Reduce [B] (verification not implemented) | 3087 |

Optimal result

Integrand size = 35, antiderivative size = 54

$$\int \frac{(ad + bdx)^m}{(a^2c + 2abcx + b^2cx^2)^{5/2}} dx = -\frac{d^3(ad + bdx)^{-3+m}}{bc^2(4 - m)\sqrt{a^2c + 2abcx + b^2cx^2}}$$

output `-d^3*(b*d*x+a*d)^(-3+m)/b/c^2/(4-m)/(b^2*c*x^2+2*a*b*c*x+a^2*c)^(1/2)`

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.67

$$\int \frac{(ad + bdx)^m}{(a^2c + 2abcx + b^2cx^2)^{5/2}} dx = \frac{(a + bx)(d(a + bx))^m}{b(-4 + m)(c(a + bx)^2)^{5/2}}$$

input `Integrate[(a*d + b*d*x)^m/(a^2*c + 2*a*b*c*x + b^2*c*x^2)^(5/2), x]`

output `((a + b*x)*(d*(a + b*x))^m)/(b*(-4 + m)*(c*(a + b*x)^2)^(5/2))`

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.09, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.114$, Rules used = {1102, 27, 35, 17}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\begin{aligned}
 & \int \frac{(ad + bdx)^m}{(a^2c + 2abcx + b^2cx^2)^{5/2}} dx \\
 & \quad \downarrow \text{1102} \\
 & \frac{b^5c^3(a + bx) \int \frac{(ad+bx)^m}{b^5c^5(a+bx)^5} dx}{\sqrt{a^2c + 2abcx + b^2cx^2}} \\
 & \quad \downarrow \text{27} \\
 & \frac{(a + bx) \int \frac{(ad+bx)^m}{(a+bx)^5} dx}{c^2\sqrt{a^2c + 2abcx + b^2cx^2}} \\
 & \quad \downarrow \text{35} \\
 & \frac{d^5(a + bx) \int (ad + bdx)^{m-5} dx}{c^2\sqrt{a^2c + 2abcx + b^2cx^2}} \\
 & \quad \downarrow \text{17} \\
 & -\frac{d^4(a + bx)(ad + bdx)^{m-4}}{bc^2(4 - m)\sqrt{a^2c + 2abcx + b^2cx^2}}
 \end{aligned}$$

input `Int[(a*d + b*d*x)^m/(a^2*c + 2*a*b*c*x + b^2*c*x^2)^(5/2),x]`

output `-((d^4*(a + b*x)*(a*d + b*d*x)^(-4 + m))/(b*c^2*(4 - m)*Sqrt[a^2*c + 2*a*b*c*x + b^2*c*x^2]))`

Definitions of rubi rules used

rule 17 $\text{Int}[(c_)*(a_)+(b_)*(x_)]^{(m_)}, x_Symbol] \rightarrow \text{Simp}[c*((a + b*x)^{(m + 1)})/(b*(m + 1)), x] /; \text{FreeQ}[\{a, b, c, m\}, x] \ \&\& \ \text{NeQ}[m, -1]$

rule 27 $\text{Int}[(a_)*(F_x_), x_Symbol] \rightarrow \text{Simp}[a \ \text{Int}[F_x, x], x] /; \text{FreeQ}[a, x] \ \&\& \ !\text{MatchQ}[F_x, (b_)*(G_x_)] /; \text{FreeQ}[b, x]$

rule 35 $\text{Int}[(u_)*((a_) + (b_)*(x_))^{(m_)}*((c_) + (d_)*(x_))^{(n_)}, x_Symbol] \rightarrow \text{Simp}[(b/d)^m \ \text{Int}[u*(c + d*x)^{(m + n)}, x], x] /; \text{FreeQ}[\{a, b, c, d, m, n\}, x] \ \&\& \ \text{EqQ}[b*c - a*d, 0] \ \&\& \ \text{IntegerQ}[m] \ \&\& \ !(\text{IntegerQ}[n] \ \&\& \ \text{SimplerQ}[a + b*x, c + d*x])$

rule 1102 $\text{Int}(((d_) + (e_)*(x_))^{(m_)}*((a_) + (b_)*(x_) + (c_)*(x_)^2)^{(p_)}, x_Symbol] \rightarrow \text{Simp}[(a + b*x + c*x^2)^{\text{FracPart}[p]}/(c^{\text{IntPart}[p]}*(b/2 + c*x)^{(2*\text{FracPart}[p])}) \ \text{Int}[(d + e*x)^m*(b/2 + c*x)^{(2*p)}, x], x] /; \text{FreeQ}[\{a, b, c, d, e, m, p\}, x] \ \&\& \ \text{EqQ}[b^2 - 4*a*c, 0]$

Maple [A] (verified)

Time = 1.33 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.74

| method | result | size |
|---------|--|------|
| risch | $\frac{(d(bx+a))^m}{c^2(bx+a)^3 \sqrt{c(bx+a)^2} b(-4+m)}$ | 40 |
| gospers | $\frac{(bx+a)(bdx+ad)^m}{b(-4+m)(b^2cx^2+2abcx+a^2c)^{\frac{5}{2}}}$ | 47 |
| orering | $\frac{(bx+a)(bdx+ad)^m}{b(-4+m)(b^2cx^2+2abcx+a^2c)^{\frac{5}{2}}}$ | 47 |

input $\text{int}((b*d*x+a*d)^m/(b^2*c*x^2+2*a*b*c*x+a^2*c)^{(5/2)}, x, \text{method}=_RETURNVERBOSE)$

output $1/c^2/(b*x+a)^3/(c*(b*x+a)^2)^{(1/2)}/b/(-4+m)*(d*(b*x+a))^m$

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 181 vs. $2(49) = 98$.

Time = 0.09 (sec) , antiderivative size = 181, normalized size of antiderivative = 3.35

$$\int \frac{(ad + bdx)^m}{(a^2c + 2abcx + b^2cx^2)^{5/2}} dx = \frac{\sqrt{b^2cx^2 + 2abca} (a^5bc^3m - 4a^5bc^3 + (b^6c^3m - 4b^6c^3)x^5 + 5(ab^5c^3m - 4ab^5c^3)x^4 + 10(a^2b^4c^3m - 4a^2b^4c^3)x^3 + 10(a^3b^3c^3m - 4a^3b^3c^3)x^2 + 5(a^4b^2c^3m - 4a^4b^2c^3)x)}{(a^5bc^3m - 4a^5bc^3 + (b^6c^3m - 4b^6c^3)x^5 + 5(ab^5c^3m - 4ab^5c^3)x^4 + 10(a^2b^4c^3m - 4a^2b^4c^3)x^3 + 10(a^3b^3c^3m - 4a^3b^3c^3)x^2 + 5(a^4b^2c^3m - 4a^4b^2c^3)x)}$$

input `integrate((b*d*x+a*d)^m/(b^2*c*x^2+2*a*b*c*x+a^2*c)^(5/2),x, algorithm="fricas")`

output `sqrt(b^2*c*x^2 + 2*a*b*c*x + a^2*c)*(b*d*x + a*d)^m/(a^5*b*c^3*m - 4*a^5*b*c^3 + (b^6*c^3*m - 4*b^6*c^3)*x^5 + 5*(a*b^5*c^3*m - 4*a*b^5*c^3)*x^4 + 10*(a^2*b^4*c^3*m - 4*a^2*b^4*c^3)*x^3 + 10*(a^3*b^3*c^3*m - 4*a^3*b^3*c^3)*x^2 + 5*(a^4*b^2*c^3*m - 4*a^4*b^2*c^3)*x)`

Sympy [F]

$$\int \frac{(ad + bdx)^m}{(a^2c + 2abcx + b^2cx^2)^{5/2}} dx = \int \frac{(d(a + bx))^m}{(c(a + bx)^2)^{\frac{5}{2}}} dx$$

input `integrate((b*d*x+a*d)**m/(b**2*c*x**2+2*a*b*c*x+a**2*c)**(5/2),x)`

output `Integral((d*(a + b*x))**m/(c*(a + b*x)**2)**(5/2), x)`

Maxima [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.63

$$\int \frac{(ad + bdx)^m}{(a^2c + 2abcx + b^2cx^2)^{5/2}} dx = \frac{(bx + a)^m \sqrt{cd^m}}{b^5c^3(m - 4)x^4 + 4ab^4c^3(m - 4)x^3 + 6a^2b^3c^3(m - 4)x^2 + 4a^3b^2c^3(m - 4)x + a^4c^3(m - 4)}$$

input `integrate((b*d*x+a*d)^m/(b^2*c*x^2+2*a*b*c*x+a^2*c)^(5/2),x, algorithm="maxima")`

output `(b*x + a)^m*sqrt(c)*d^m/(b^5*c^3*(m - 4)*x^4 + 4*a*b^4*c^3*(m - 4)*x^3 + 6*a^2*b^3*c^3*(m - 4)*x^2 + 4*a^3*b^2*c^3*(m - 4)*x + a^4*b*c^3*(m - 4))`

Giac [F]

$$\int \frac{(ad + bdx)^m}{(a^2c + 2abcx + b^2cx^2)^{5/2}} dx = \int \frac{(bdx + ad)^m}{(b^2cx^2 + 2abcx + a^2c)^{5/2}} dx$$

input `integrate((b*d*x+a*d)^m/(b^2*c*x^2+2*a*b*c*x+a^2*c)^(5/2),x, algorithm="giac")`

output `integrate((b*d*x + a*d)^m/(b^2*c*x^2 + 2*a*b*c*x + a^2*c)^(5/2), x)`

Mupad [F(-1)]

Timed out.

$$\int \frac{(ad + bdx)^m}{(a^2c + 2abcx + b^2cx^2)^{5/2}} dx = \int \frac{(ad + bdx)^m}{(ca^2 + 2cabx + cb^2x^2)^{5/2}} dx$$

input `int((a*d + b*d*x)^m/(a^2*c + b^2*c*x^2 + 2*a*b*c*x)^(5/2),x)`

output `int((a*d + b*d*x)^m/(a^2*c + b^2*c*x^2 + 2*a*b*c*x)^(5/2), x)`

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.94

$$\int \frac{(ad + bdx)^m}{(a^2c + 2abcx + b^2cx^2)^{5/2}} dx = \frac{\sqrt{c}(bdx + ad)^m}{bc^3(b^4mx^4 + 4ab^3mx^3 - 4b^4x^4 + 6a^2b^2mx^2 - 16ab^3x^3 + 4a^3bmx - 24a^4x^4)}$$

input `int((b*d*x+a*d)^m/(b^2*c*x^2+2*a*b*c*x+a^2*c)^(5/2),x)`

output `(sqrt(c)*(a*d + b*d*x)**m)/(b*c**3*(a**4*m - 4*a**4 + 4*a**3*b*m*x - 16*a**3*b*x + 6*a**2*b**2*m*x**2 - 24*a**2*b**2*x**2 + 4*a*b**3*m*x**3 - 16*a*b**3*x**3 + b**4*m*x**4 - 4*b**4*x**4))`

3.411 $\int (d + ex)^m (a^2 + 2abx + b^2x^2)^3 dx$

| | |
|---|------|
| Optimal result | 3088 |
| Mathematica [A] (verified) | 3089 |
| Rubi [A] (verified) | 3089 |
| Maple [B] (verified) | 3091 |
| Fricas [B] (verification not implemented) | 3092 |
| Sympy [B] (verification not implemented) | 3093 |
| Maxima [B] (verification not implemented) | 3094 |
| Giac [B] (verification not implemented) | 3095 |
| Mupad [B] (verification not implemented) | 3096 |
| Reduce [B] (verification not implemented) | 3096 |

Optimal result

Integrand size = 26, antiderivative size = 206

$$\int (d + ex)^m (a^2 + 2abx + b^2x^2)^3 dx = \frac{(bd - ae)^6 (d + ex)^{1+m}}{e^7 (1 + m)} - \frac{6b(bd - ae)^5 (d + ex)^{2+m}}{e^7 (2 + m)} + \frac{15b^2 (bd - ae)^4 (d + ex)^{3+m}}{e^7 (3 + m)} - \frac{20b^3 (bd - ae)^3 (d + ex)^{4+m}}{e^7 (4 + m)} + \frac{15b^4 (bd - ae)^2 (d + ex)^{5+m}}{e^7 (5 + m)} - \frac{6b^5 (bd - ae) (d + ex)^{6+m}}{e^7 (6 + m)} + \frac{b^6 (d + ex)^{7+m}}{e^7 (7 + m)}$$

output

```
(-a*e+b*d)^6*(e*x+d)^(1+m)/e^7/(1+m)-6*b*(-a*e+b*d)^5*(e*x+d)^(2+m)/e^7/(2+m)+15*b^2*(-a*e+b*d)^4*(e*x+d)^(3+m)/e^7/(3+m)-20*b^3*(-a*e+b*d)^3*(e*x+d)^(4+m)/e^7/(4+m)+15*b^4*(-a*e+b*d)^2*(e*x+d)^(5+m)/e^7/(5+m)-6*b^5*(-a*e+b*d)*(e*x+d)^(6+m)/e^7/(6+m)+b^6*(e*x+d)^(7+m)/e^7/(7+m)
```

Mathematica [A] (verified)

Time = 0.21 (sec) , antiderivative size = 175, normalized size of antiderivative = 0.85

$$\int (d + ex)^m (a^2 + 2abx + b^2x^2)^3 dx$$

$$= \frac{(d + ex)^{1+m} \left(\frac{(bd-ae)^6}{1+m} - \frac{6b(bd-ae)^5(d+ex)}{2+m} + \frac{15b^2(bd-ae)^4(d+ex)^2}{3+m} - \frac{20b^3(bd-ae)^3(d+ex)^3}{4+m} + \frac{15b^4(bd-ae)^2(d+ex)^4}{5+m} - \frac{6b^5(bd-ae)(d+ex)^5}{6+m} + \frac{b^6(d+ex)^6}{7+m} \right)}{e^7}$$

input

```
Integrate[(d + e*x)^m*(a^2 + 2*a*b*x + b^2*x^2)^3,x]
```

output

```
((d + e*x)^(1 + m)*((b*d - a*e)^6/(1 + m) - (6*b*(b*d - a*e)^5*(d + e*x))/(2 + m) + (15*b^2*(b*d - a*e)^4*(d + e*x)^2)/(3 + m) - (20*b^3*(b*d - a*e)^3*(d + e*x)^3)/(4 + m) + (15*b^4*(b*d - a*e)^2*(d + e*x)^4)/(5 + m) - (6*b^5*(b*d - a*e)*(d + e*x)^5)/(6 + m) + (b^6*(d + e*x)^6)/(7 + m))/e^7
```

Rubi [A] (verified)

Time = 0.66 (sec) , antiderivative size = 206, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {1098, 27, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a^2 + 2abx + b^2x^2)^3 (d + ex)^m dx$$

$$\downarrow 1098$$

$$\frac{\int b^6(a + bx)^6(d + ex)^m dx}{b^6}$$

$$\downarrow 27$$

$$\int (a + bx)^6(d + ex)^m dx$$

$$\downarrow 53$$

$$\int \left(-\frac{6b^5(bd - ae)(d + ex)^{m+5}}{e^6} + \frac{15b^4(bd - ae)^2(d + ex)^{m+4}}{e^6} - \frac{20b^3(bd - ae)^3(d + ex)^{m+3}}{e^6} + \frac{15b^2(bd - ae)^4(d + ex)^{m+2}}{e^6} - \frac{6b(bd - ae)^5(d + ex)^{m+1}}{e^6} + \frac{b^6(bd - ae)^6(d + ex)^m}{e^6} \right) dx$$

↓ 2009

$$\frac{6b^5(bd - ae)(d + ex)^{m+6}}{e^7(m + 6)} + \frac{15b^4(bd - ae)^2(d + ex)^{m+5}}{e^7(m + 5)} - \frac{20b^3(bd - ae)^3(d + ex)^{m+4}}{e^7(m + 4)} + \frac{15b^2(bd - ae)^4(d + ex)^{m+3}}{e^7(m + 3)} + \frac{(bd - ae)^6(d + ex)^{m+1}}{e^7(m + 1)} - \frac{6b(bd - ae)^5(d + ex)^{m+2}}{e^7(m + 2)} + \frac{b^6(d + ex)^{m+7}}{e^7(m + 7)}$$

input `Int[(d + e*x)^m*(a^2 + 2*a*b*x + b^2*x^2)^3,x]`

output `((b*d - a*e)^6*(d + e*x)^(1 + m))/(e^7*(1 + m)) - (6*b*(b*d - a*e)^5*(d + e*x)^(2 + m))/(e^7*(2 + m)) + (15*b^2*(b*d - a*e)^4*(d + e*x)^(3 + m))/(e^7*(3 + m)) - (20*b^3*(b*d - a*e)^3*(d + e*x)^(4 + m))/(e^7*(4 + m)) + (15*b^4*(b*d - a*e)^2*(d + e*x)^(5 + m))/(e^7*(5 + m)) - (6*b^5*(b*d - a*e)*(d + e*x)^(6 + m))/(e^7*(6 + m)) + (b^6*(d + e*x)^(7 + m))/(e^7*(7 + m))`

Defintions of rubi rules used

rule 27 `Int[(a_)*(F_x_), x_Symbol] := Simp[a Int[F_x, x], x] /; FreeQ[a, x] && !MatchQ[F_x, (b_)*(G_x_)] /; FreeQ[b, x]`

rule 53 `Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] := Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && IGtQ[m, 0] && (!IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])`

rule 1098 `Int[((d_.) + (e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] := Simp[1/c^p Int[(d + e*x)^m*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]`

rule 2009 `Int[u_, x_Symbol] := Simp[IntSum[u, x], x] /; SumQ[u]`

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1693 vs. $2(206) = 412$.

Time = 1.39 (sec) , antiderivative size = 1694, normalized size of antiderivative = 8.22

| method | result | size |
|-------------|---------------------------------|------|
| norman | Expression too large to display | 1694 |
| gosper | Expression too large to display | 2157 |
| oring | Expression too large to display | 2185 |
| risch | Expression too large to display | 2687 |
| parallelsch | Expression too large to display | 3994 |

input

```
int((e*x+d)^m*(b^2*x^2+2*a*b*x+a^2)^3,x,method=_RETURNVERBOSE)
```

output

```
b^6/(7+m)*x^7*exp(m*ln(e*x+d))+d*(a^6*e^6*m^6+27*a^6*e^6*m^5-6*a^5*b*d*e^5
*m^5+295*a^6*e^6*m^4-150*a^5*b*d*e^5*m^4+30*a^4*b^2*d^2*e^4*m^4+1665*a^6*e
^6*m^3-1470*a^5*b*d*e^5*m^3+660*a^4*b^2*d^2*e^4*m^3-120*a^3*b^3*d^3*e^3*m^
3+5104*a^6*e^6*m^2-7050*a^5*b*d*e^5*m^2+5370*a^4*b^2*d^2*e^4*m^2-2160*a^3*
b^3*d^3*e^3*m^2+360*a^2*b^4*d^4*e^2*m^2+8028*a^6*e^6*m-16524*a^5*b*d*e^5*m
+19140*a^4*b^2*d^2*e^4*m-12840*a^3*b^3*d^3*e^3*m+4680*a^2*b^4*d^4*e^2*m-72
0*a*b^5*d^5*e*m+5040*a^6*e^6-15120*a^5*b*d*e^5+25200*a^4*b^2*d^2*e^4-25200
*a^3*b^3*d^3*e^3+15120*a^2*b^4*d^4*e^2-5040*a*b^5*d^5*e+720*b^6*d^6)/e^7/(
m^7+28*m^6+322*m^5+1960*m^4+6769*m^3+13132*m^2+13068*m+5040)*exp(m*ln(e*x+
d))+(a^6*e^6*m^6+6*a^5*b*d*e^5*m^6+27*a^6*e^6*m^5+150*a^5*b*d*e^5*m^5-30*a
^4*b^2*d^2*e^4*m^5+295*a^6*e^6*m^4+1470*a^5*b*d*e^5*m^4-660*a^4*b^2*d^2*e^
4*m^4+120*a^3*b^3*d^3*e^3*m^4+1665*a^6*e^6*m^3+7050*a^5*b*d*e^5*m^3-5370*a
^4*b^2*d^2*e^4*m^3+2160*a^3*b^3*d^3*e^3*m^3-360*a^2*b^4*d^4*e^2*m^3+5104*a
^6*e^6*m^2+16524*a^5*b*d*e^5*m^2-19140*a^4*b^2*d^2*e^4*m^2+12840*a^3*b^3*d
^3*e^3*m^2-4680*a^2*b^4*d^4*e^2*m^2+720*a*b^5*d^5*e*m^2+8028*a^6*e^6*m+151
20*a^5*b*d*e^5*m-25200*a^4*b^2*d^2*e^4*m+25200*a^3*b^3*d^3*e^3*m-15120*a^2
*b^4*d^4*e^2*m+5040*a*b^5*d^5*e*m-720*b^6*d^6*m+5040*a^6*e^6)/e^6/(m^7+28*
m^6+322*m^5+1960*m^4+6769*m^3+13132*m^2+13068*m+5040)*x*exp(m*ln(e*x+d))+
(6*a*e*m+b*d*m+42*a*e)*b^5/e/(m^2+13*m+42)*x^6*exp(m*ln(e*x+d))+3*(5*a^2*e^
2*m^2+2*a*b*d*e*m^2+65*a^2*e^2*m+14*a*b*d*e*m-2*b^2*d^2*m+210*a^2*e^2)*...
```


Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2230 vs. $2(206) = 412$.

Time = 0.12 (sec) , antiderivative size = 2230, normalized size of antiderivative = 10.83

$$\int (d + ex)^m (a^2 + 2abx + b^2x^2)^3 dx = \text{Too large to display}$$

input `integrate((e*x+d)^m*(b^2*x^2+2*a*b*x+a^2)^3,x, algorithm="fricas")`

output

```
(a^6*d*e^6*m^6 + 720*b^6*d^7 - 5040*a*b^5*d^6*e + 15120*a^2*b^4*d^5*e^2 -
25200*a^3*b^3*d^4*e^3 + 25200*a^4*b^2*d^3*e^4 - 15120*a^5*b*d^2*e^5 + 5040
*a^6*d*e^6 + (b^6*e^7*m^6 + 21*b^6*e^7*m^5 + 175*b^6*e^7*m^4 + 735*b^6*e^7
*m^3 + 1624*b^6*e^7*m^2 + 1764*b^6*e^7*m + 720*b^6*e^7)*x^7 + (5040*a*b^5*
e^7 + (b^6*d*e^6 + 6*a*b^5*e^7)*m^6 + 3*(5*b^6*d*e^6 + 44*a*b^5*e^7)*m^5 +
5*(17*b^6*d*e^6 + 228*a*b^5*e^7)*m^4 + 15*(15*b^6*d*e^6 + 328*a*b^5*e^7)*
m^3 + 2*(137*b^6*d*e^6 + 5547*a*b^5*e^7)*m^2 + 12*(10*b^6*d*e^6 + 1019*a*b
^5*e^7)*m)*x^6 - 3*(2*a^5*b*d^2*e^5 - 9*a^6*d*e^6)*m^5 + 3*(5040*a^2*b^4*e
^7 + (2*a*b^5*d*e^6 + 5*a^2*b^4*e^7)*m^6 - (2*b^6*d^2*e^5 - 34*a*b^5*d*e^6
- 115*a^2*b^4*e^7)*m^5 - 5*(4*b^6*d^2*e^5 - 42*a*b^5*d*e^6 - 207*a^2*b^4*
e^7)*m^4 - 5*(14*b^6*d^2*e^5 - 118*a*b^5*d*e^6 - 925*a^2*b^4*e^7)*m^3 - 4*
(25*b^6*d^2*e^5 - 187*a*b^5*d*e^6 - 2680*a^2*b^4*e^7)*m^2 - 12*(4*b^6*d^2*
e^5 - 28*a*b^5*d*e^6 - 1005*a^2*b^4*e^7)*m)*x^5 + 5*(6*a^4*b^2*d^3*e^4 - 3
0*a^5*b*d^2*e^5 + 59*a^6*d*e^6)*m^4 + 5*(5040*a^3*b^3*e^7 + (3*a^2*b^4*d*e
^6 + 4*a^3*b^3*e^7)*m^6 - 3*(2*a*b^5*d^2*e^5 - 19*a^2*b^4*d*e^6 - 32*a^3*b
^3*e^7)*m^5 + (6*b^6*d^3*e^4 - 78*a*b^5*d^2*e^5 + 393*a^2*b^4*d*e^6 + 904*
a^3*b^3*e^7)*m^4 + 3*(12*b^6*d^3*e^4 - 106*a*b^5*d^2*e^5 + 401*a^2*b^4*d*e
^6 + 1408*a^3*b^3*e^7)*m^3 + 2*(33*b^6*d^3*e^4 - 249*a*b^5*d^2*e^5 + 810*a
^2*b^4*d*e^6 + 5090*a^3*b^3*e^7)*m^2 + 36*(b^6*d^3*e^4 - 7*a*b^5*d^2*e^5 +
21*a^2*b^4*d*e^6 + 328*a^3*b^3*e^7)*m)*x^4 - 15*(8*a^3*b^3*d^4*e^3 - 4...
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 28410 vs. $2(182) = 364$.

Time = 6.04 (sec) , antiderivative size = 28410, normalized size of antiderivative = 137.91

$$\int (d + ex)^m (a^2 + 2abx + b^2x^2)^3 dx = \text{Too large to display}$$

input `integrate((e*x+d)**m*(b**2*x**2+2*a*b*x+a**2)**3,x)`

output

```
Piecewise(((d**m*(a**6*x + 3*a**5*b*x**2 + 5*a**4*b**2*x**3 + 5*a**3*b**3*x**4 + 3*a**2*b**4*x**5 + a*b**5*x**6 + b**6*x**7/7), Eq(e, 0)), (-10*a**6*e**6/(60*d**6*e**7 + 360*d**5*e**8*x + 900*d**4*e**9*x**2 + 1200*d**3*e**10*x**3 + 900*d**2*e**11*x**4 + 360*d*e**12*x**5 + 60*e**13*x**6) - 12*a**5*b*d*e**5/(60*d**6*e**7 + 360*d**5*e**8*x + 900*d**4*e**9*x**2 + 1200*d**3*e**10*x**3 + 900*d**2*e**11*x**4 + 360*d*e**12*x**5 + 60*e**13*x**6) - 72*a**5*b**6*x/(60*d**6*e**7 + 360*d**5*e**8*x + 900*d**4*e**9*x**2 + 1200*d**3*e**10*x**3 + 900*d**2*e**11*x**4 + 360*d*e**12*x**5 + 60*e**13*x**6) - 15*a**4*b**2*d**2*e**4/(60*d**6*e**7 + 360*d**5*e**8*x + 900*d**4*e**9*x**2 + 1200*d**3*e**10*x**3 + 900*d**2*e**11*x**4 + 360*d*e**12*x**5 + 60*e**13*x**6) - 90*a**4*b**2*d**5*x/(60*d**6*e**7 + 360*d**5*e**8*x + 900*d**4*e**9*x**2 + 1200*d**3*e**10*x**3 + 900*d**2*e**11*x**4 + 360*d*e**12*x**5 + 60*e**13*x**6) - 225*a**4*b**2*e**6*x**2/(60*d**6*e**7 + 360*d**5*e**8*x + 900*d**4*e**9*x**2 + 1200*d**3*e**10*x**3 + 900*d**2*e**11*x**4 + 360*d*e**12*x**5 + 60*e**13*x**6) - 20*a**3*b**3*d**3*e**3/(60*d**6*e**7 + 360*d**5*e**8*x + 900*d**4*e**9*x**2 + 1200*d**3*e**10*x**3 + 900*d**2*e**11*x**4 + 360*d*e**12*x**5 + 60*e**13*x**6) - 120*a**3*b**3*d**2*e**4*x/(60*d**6*e**7 + 360*d**5*e**8*x + 900*d**4*e**9*x**2 + 1200*d**3*e**10*x**3 + 900*d**2*e**11*x**4 + 360*d*e**12*x**5 + 60*e**13*x**6) - 300*a**3*b**3*d*e**5*x**2/(60*d**6*e**7 + 360*d**5*e**8*x + 900*d**4*e**9*x**2 + 120...
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 818 vs. $2(206) = 412$.

Time = 0.07 (sec) , antiderivative size = 818, normalized size of antiderivative = 3.97

$$\int (d + ex)^m (a^2 + 2abx + b^2x^2)^3 dx = \text{Too large to display}$$

input `integrate((e*x+d)^m*(b^2*x^2+2*a*b*x+a^2)^3,x, algorithm="maxima")`

output

```
6*(e^2*(m + 1)*x^2 + d*e*m*x - d^2)*(e*x + d)^m*a^5*b/((m^2 + 3*m + 2)*e^2)
+ (e*x + d)^(m + 1)*a^6/(e*(m + 1)) + 15*((m^2 + 3*m + 2)*e^3*x^3 + (m^2
+ m)*d*e^2*x^2 - 2*d^2*e*m*x + 2*d^3)*(e*x + d)^m*a^4*b^2/((m^3 + 6*m^2 +
11*m + 6)*e^3) + 20*((m^3 + 6*m^2 + 11*m + 6)*e^4*x^4 + (m^3 + 3*m^2 + 2*
m)*d*e^3*x^3 - 3*(m^2 + m)*d^2*e^2*x^2 + 6*d^3*e*m*x - 6*d^4)*(e*x + d)^m*
a^3*b^3/((m^4 + 10*m^3 + 35*m^2 + 50*m + 24)*e^4) + 15*((m^4 + 10*m^3 + 35
*m^2 + 50*m + 24)*e^5*x^5 + (m^4 + 6*m^3 + 11*m^2 + 6*m)*d*e^4*x^4 - 4*(m^
3 + 3*m^2 + 2*m)*d^2*e^3*x^3 + 12*(m^2 + m)*d^3*e^2*x^2 - 24*d^4*e*m*x + 2
4*d^5)*(e*x + d)^m*a^2*b^4/((m^5 + 15*m^4 + 85*m^3 + 225*m^2 + 274*m + 120
)*e^5) + 6*((m^5 + 15*m^4 + 85*m^3 + 225*m^2 + 274*m + 120)*e^6*x^6 + (m^5
+ 10*m^4 + 35*m^3 + 50*m^2 + 24*m)*d*e^5*x^5 - 5*(m^4 + 6*m^3 + 11*m^2 +
6*m)*d^2*e^4*x^4 + 20*(m^3 + 3*m^2 + 2*m)*d^3*e^3*x^3 - 60*(m^2 + m)*d^4*e
^2*x^2 + 120*d^5*e*m*x - 120*d^6)*(e*x + d)^m*a*b^5/((m^6 + 21*m^5 + 175*m
^4 + 735*m^3 + 1624*m^2 + 1764*m + 720)*e^6) + ((m^6 + 21*m^5 + 175*m^4 +
735*m^3 + 1624*m^2 + 1764*m + 720)*e^7*x^7 + (m^6 + 15*m^5 + 85*m^4 + 225*
m^3 + 274*m^2 + 120*m)*d*e^6*x^6 - 6*(m^5 + 10*m^4 + 35*m^3 + 50*m^2 + 24*
m)*d^2*e^5*x^5 + 30*(m^4 + 6*m^3 + 11*m^2 + 6*m)*d^3*e^4*x^4 - 120*(m^3 +
3*m^2 + 2*m)*d^4*e^3*x^3 + 360*(m^2 + m)*d^5*e^2*x^2 - 720*d^6*e*m*x + 720
*d^7)*(e*x + d)^m*b^6/((m^7 + 28*m^6 + 322*m^5 + 1960*m^4 + 6769*m^3 + 131
32*m^2 + 13068*m + 5040)*e^7)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 3878 vs. $2(206) = 412$.

Time = 0.19 (sec) , antiderivative size = 3878, normalized size of antiderivative = 18.83

$$\int (d + ex)^m (a^2 + 2abx + b^2x^2)^3 dx = \text{Too large to display}$$

input `integrate((e*x+d)^m*(b^2*x^2+2*a*b*x+a^2)^3,x, algorithm="giac")`

output

```
((e*x + d)^m*b^6*e^7*m^6*x^7 + (e*x + d)^m*b^6*d*e^6*m^6*x^6 + 6*(e*x + d)^m*a*b^5*e^7*m^6*x^6 + 21*(e*x + d)^m*b^6*e^7*m^5*x^7 + 6*(e*x + d)^m*a*b^5*d*e^6*m^6*x^5 + 15*(e*x + d)^m*a^2*b^4*e^7*m^6*x^5 + 15*(e*x + d)^m*b^6*d*e^6*m^5*x^6 + 132*(e*x + d)^m*a*b^5*e^7*m^5*x^6 + 175*(e*x + d)^m*b^6*e^7*m^4*x^7 + 15*(e*x + d)^m*a^2*b^4*d*e^6*m^6*x^4 + 20*(e*x + d)^m*a^3*b^3*e^7*m^6*x^4 - 6*(e*x + d)^m*b^6*d^2*e^5*m^5*x^5 + 102*(e*x + d)^m*a*b^5*d*e^6*m^5*x^5 + 345*(e*x + d)^m*a^2*b^4*e^7*m^5*x^5 + 85*(e*x + d)^m*b^6*d*e^6*m^4*x^6 + 1140*(e*x + d)^m*a*b^5*e^7*m^4*x^6 + 735*(e*x + d)^m*b^6*e^7*m^3*x^7 + 20*(e*x + d)^m*a^3*b^3*d*e^6*m^6*x^3 + 15*(e*x + d)^m*a^4*b^2*e^7*m^6*x^3 - 30*(e*x + d)^m*a*b^5*d^2*e^5*m^5*x^4 + 285*(e*x + d)^m*a^2*b^4*d*e^6*m^5*x^4 + 480*(e*x + d)^m*a^3*b^3*e^7*m^5*x^4 - 60*(e*x + d)^m*b^6*d^2*e^5*m^4*x^5 + 630*(e*x + d)^m*a*b^5*d*e^6*m^4*x^5 + 3105*(e*x + d)^m*a^2*b^4*e^7*m^4*x^5 + 225*(e*x + d)^m*b^6*d*e^6*m^3*x^6 + 4920*(e*x + d)^m*a*b^5*e^7*m^3*x^6 + 1624*(e*x + d)^m*b^6*e^7*m^2*x^7 + 15*(e*x + d)^m*a^4*b^2*d*e^6*m^6*x^2 + 6*(e*x + d)^m*a^5*b*e^7*m^6*x^2 - 60*(e*x + d)^m*a^2*b^4*d^2*e^5*m^5*x^3 + 420*(e*x + d)^m*a^3*b^3*d*e^6*m^5*x^3 + 375*(e*x + d)^m*a^4*b^2*e^7*m^5*x^3 + 30*(e*x + d)^m*b^6*d^3*e^4*m^4*x^4 - 390*(e*x + d)^m*a*b^5*d^2*e^5*m^4*x^4 + 1965*(e*x + d)^m*a^2*b^4*d*e^6*m^4*x^4 + 4520*(e*x + d)^m*a^3*b^3*e^7*m^4*x^4 - 210*(e*x + d)^m*b^6*d^2*e^5*m^3*x^5 + 1770*(e*x + d)^m*a*b^5*d*e^6*m^3*x^5 + 13875*(e*x + d)^m*a^2*b^4*e^7*m^3*...
```

Mupad [B] (verification not implemented)

Time = 6.18 (sec) , antiderivative size = 1898, normalized size of antiderivative = 9.21

$$\int (d + ex)^m (a^2 + 2abx + b^2x^2)^3 dx = \text{Too large to display}$$

input `int((d + e*x)^m*(a^2 + b^2*x^2 + 2*a*b*x)^3,x)`

output `((d + e*x)^m*(720*b^6*d^7 + 5040*a^6*d*e^6 - 15120*a^5*b*d^2*e^5 + 5104*a^6*d*e^6*m^2 + 1665*a^6*d*e^6*m^3 + 295*a^6*d*e^6*m^4 + 27*a^6*d*e^6*m^5 + a^6*d*e^6*m^6 + 15120*a^2*b^4*d^5*e^2 - 25200*a^3*b^3*d^4*e^3 + 25200*a^4*b^2*d^3*e^4 - 5040*a*b^5*d^6*e + 8028*a^6*d*e^6*m - 720*a*b^5*d^6*e*m + 360*a^2*b^4*d^5*e^2*m^2 - 2160*a^3*b^3*d^4*e^3*m^2 + 5370*a^4*b^2*d^3*e^4*m^2 - 120*a^3*b^3*d^4*e^3*m^3 + 660*a^4*b^2*d^3*e^4*m^3 + 30*a^4*b^2*d^3*e^4*m^4 - 16524*a^5*b*d^2*e^5*m + 4680*a^2*b^4*d^5*e^2*m - 12840*a^3*b^3*d^4*e^3*m + 19140*a^4*b^2*d^3*e^4*m - 7050*a^5*b*d^2*e^5*m^2 - 1470*a^5*b*d^2*e^5*m^3 - 150*a^5*b*d^2*e^5*m^4 - 6*a^5*b*d^2*e^5*m^5))/(e^7*(13068*m + 13132*m^2 + 6769*m^3 + 1960*m^4 + 322*m^5 + 28*m^6 + m^7 + 5040)) + (x*(d + e*x)^m*(5040*a^6*e^7 + 8028*a^6*e^7*m + 5104*a^6*e^7*m^2 + 1665*a^6*e^7*m^3 + 295*a^6*e^7*m^4 + 27*a^6*e^7*m^5 + a^6*e^7*m^6 - 720*b^6*d^6*e*m + 15120*a^5*b*d^6*e^6*m - 4680*a^2*b^4*d^4*e^3*m^2 + 12840*a^3*b^3*d^3*e^4*m^2 - 19140*a^4*b^2*d^2*e^5*m^2 - 360*a^2*b^4*d^4*e^3*m^3 + 2160*a^3*b^3*d^3*e^4*m^3 - 5370*a^4*b^2*d^2*e^5*m^3 + 120*a^3*b^3*d^3*e^4*m^4 - 660*a^4*b^2*d^2*e^5*m^4 - 30*a^4*b^2*d^2*e^5*m^5 + 5040*a*b^5*d^5*e^2*m + 16524*a^5*b*d^5*e^2*m^2 + 7050*a^5*b*d^5*e^2*m^3 + 1470*a^5*b*d^5*e^2*m^4 + 150*a^5*b*d^5*e^2*m^5 + 6*a^5*b*d^5*e^2*m^6 - 15120*a^2*b^4*d^4*e^3*m + 25200*a^3*b^3*d^3*e^4*m - 25200*a^4*b^2*d^2*e^5*m + 720*a*b^5*d^5*e^2*m^2))/(e^7*(13068*m + 13132*m^2 + 6769*m^3 + 1960*m^4 + 322*m^5 + 28*m^6 + m^7 + 5040)) + (b^6*x^7*...`

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 2686, normalized size of antiderivative = 13.04

$$\int (d + ex)^m (a^2 + 2abx + b^2x^2)^3 dx = \text{Too large to display}$$

input `int((e*x+d)^m*(b^2*x^2+2*a*b*x+a^2)^3,x)`

output

```

((d + e*x)**m*(a**6*d*e**6*m**6 + 27*a**6*d*e**6*m**5 + 295*a**6*d*e**6*m**
*4 + 1665*a**6*d*e**6*m**3 + 5104*a**6*d*e**6*m**2 + 8028*a**6*d*e**6*m +
5040*a**6*d*e**6 + a**6*e**7*m**6*x + 27*a**6*e**7*m**5*x + 295*a**6*e**7*
m**4*x + 1665*a**6*e**7*m**3*x + 5104*a**6*e**7*m**2*x + 8028*a**6*e**7*m*
x + 5040*a**6*e**7*x - 6*a**5*b*d**2*e**5*m**5 - 150*a**5*b*d**2*e**5*m**4
- 1470*a**5*b*d**2*e**5*m**3 - 7050*a**5*b*d**2*e**5*m**2 - 16524*a**5*b*
d**2*e**5*m - 15120*a**5*b*d**2*e**5 + 6*a**5*b*d*e**6*m**6*x + 150*a**5*b
*d*e**6*m**5*x + 1470*a**5*b*d*e**6*m**4*x + 7050*a**5*b*d*e**6*m**3*x + 1
6524*a**5*b*d*e**6*m**2*x + 15120*a**5*b*d*e**6*m*x + 6*a**5*b*e**7*m**6*x
**2 + 156*a**5*b*e**7*m**5*x**2 + 1620*a**5*b*e**7*m**4*x**2 + 8520*a**5*b
*e**7*m**3*x**2 + 23574*a**5*b*e**7*m**2*x**2 + 31644*a**5*b*e**7*m*x**2 +
15120*a**5*b*e**7*x**2 + 30*a**4*b**2*d**3*e**4*m**4 + 660*a**4*b**2*d**3
*e**4*m**3 + 5370*a**4*b**2*d**3*e**4*m**2 + 19140*a**4*b**2*d**3*e**4*m +
25200*a**4*b**2*d**3*e**4 - 30*a**4*b**2*d**2*e**5*m**5*x - 660*a**4*b**2
*d**2*e**5*m**4*x - 5370*a**4*b**2*d**2*e**5*m**3*x - 19140*a**4*b**2*d**2
*e**5*m**2*x - 25200*a**4*b**2*d**2*e**5*m*x + 15*a**4*b**2*d*e**6*m**6*x*
*2 + 345*a**4*b**2*d*e**6*m**5*x**2 + 3015*a**4*b**2*d*e**6*m**4*x**2 + 12
255*a**4*b**2*d*e**6*m**3*x**2 + 22170*a**4*b**2*d*e**6*m**2*x**2 + 12600*
a**4*b**2*d*e**6*m*x**2 + 15*a**4*b**2*e**7*m**6*x**3 + 375*a**4*b**2*e**7
*m**5*x**3 + 3705*a**4*b**2*e**7*m**4*x**3 + 18285*a**4*b**2*e**7*m**3*...

```

3.412 $\int (d + ex)^m (a^2 + 2abx + b^2x^2)^2 dx$

| | |
|---|------|
| Optimal result | 3098 |
| Mathematica [A] (verified) | 3098 |
| Rubi [A] (verified) | 3099 |
| Maple [B] (verified) | 3100 |
| Fricas [B] (verification not implemented) | 3101 |
| Sympy [B] (verification not implemented) | 3102 |
| Maxima [B] (verification not implemented) | 3103 |
| Giac [B] (verification not implemented) | 3104 |
| Mupad [B] (verification not implemented) | 3105 |
| Reduce [B] (verification not implemented) | 3106 |

Optimal result

Integrand size = 26, antiderivative size = 142

$$\int (d + ex)^m (a^2 + 2abx + b^2x^2)^2 dx = \frac{(bd - ae)^4 (d + ex)^{1+m}}{e^5 (1 + m)} - \frac{4b(bd - ae)^3 (d + ex)^{2+m}}{e^5 (2 + m)} + \frac{6b^2 (bd - ae)^2 (d + ex)^{3+m}}{e^5 (3 + m)} - \frac{4b^3 (bd - ae) (d + ex)^{4+m}}{e^5 (4 + m)} + \frac{b^4 (d + ex)^{5+m}}{e^5 (5 + m)}$$

output

```
(-a*e+b*d)^4*(e*x+d)^(1+m)/e^5/(1+m)-4*b*(-a*e+b*d)^3*(e*x+d)^(2+m)/e^5/(2+m)+6*b^2*(-a*e+b*d)^2*(e*x+d)^(3+m)/e^5/(3+m)-4*b^3*(-a*e+b*d)*(e*x+d)^(4+m)/e^5/(4+m)+b^4*(e*x+d)^(5+m)/e^5/(5+m)
```

Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 121, normalized size of antiderivative = 0.85

$$\int (d + ex)^m (a^2 + 2abx + b^2x^2)^2 dx = \frac{(d + ex)^{1+m} \left(\frac{(bd-ae)^4}{1+m} - \frac{4b(bd-ae)^3(d+ex)}{2+m} + \frac{6b^2(bd-ae)^2(d+ex)^2}{3+m} - \frac{4b^3(bd-ae)(d+ex)^3}{4+m} + \frac{b^4(d+ex)^4}{5+m} \right)}{e^5}$$

input `Integrate[(d + e*x)^m*(a^2 + 2*a*b*x + b^2*x^2)^2,x]`

output
$$\frac{((d + e*x)^{(1 + m)}*((b*d - a*e)^4/(1 + m) - (4*b*(b*d - a*e)^3*(d + e*x))/(2 + m) + (6*b^2*(b*d - a*e)^2*(d + e*x)^2)/(3 + m) - (4*b^3*(b*d - a*e)*(d + e*x)^3)/(4 + m) + (b^4*(d + e*x)^4)/(5 + m))}{e^5}$$

Rubi [A] (verified)

Time = 0.53 (sec) , antiderivative size = 142, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {1098, 27, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a^2 + 2abx + b^2x^2)^2 (d + ex)^m dx$$

$$\downarrow 1098$$

$$\frac{\int b^4(a + bx)^4(d + ex)^m dx}{b^4}$$

$$\downarrow 27$$

$$\int (a + bx)^4(d + ex)^m dx$$

$$\downarrow 53$$

$$\int \left(-\frac{4b^3(bd - ae)(d + ex)^{m+3}}{e^4} + \frac{6b^2(bd - ae)^2(d + ex)^{m+2}}{e^4} + \frac{(ae - bd)^4(d + ex)^m}{e^4} - \frac{4b(bd - ae)^3(d + ex)^{m+1}}{e^4} \right) dx$$

$$\downarrow 2009$$

$$-\frac{4b^3(bd - ae)(d + ex)^{m+4}}{e^5(m + 4)} + \frac{6b^2(bd - ae)^2(d + ex)^{m+3}}{e^5(m + 3)} + \frac{(bd - ae)^4(d + ex)^{m+1}}{e^5(m + 1)} - \frac{4b(bd - ae)^3(d + ex)^{m+2}}{e^5(m + 2)} + \frac{b^4(d + ex)^{m+5}}{e^5(m + 5)}$$

input `Int[(d + e*x)^m*(a^2 + 2*a*b*x + b^2*x^2)^2,x]`

output
$$\frac{(b*d - a*e)^4*(d + e*x)^{(1 + m)}}{(e^5*(1 + m))} - \frac{(4*b*(b*d - a*e)^3*(d + e*x)^{(2 + m))}}{(e^5*(2 + m))} + \frac{(6*b^2*(b*d - a*e)^2*(d + e*x)^{(3 + m))}}{(e^5*(3 + m))} - \frac{(4*b^3*(b*d - a*e)*(d + e*x)^{(4 + m))}}{(e^5*(4 + m))} + \frac{(b^4*(d + e*x)^{(5 + m))}}{(e^5*(5 + m))}$$

Defintions of rubi rules used

rule 27
$$\text{Int}[(a_)*(F x_), x_Symbol] \rightarrow \text{Simp}[a \text{ Int}[F x, x], x] /; \text{FreeQ}[a, x] \ \&\& \ !\text{MatchQ}[F x, (b_)*(G x_)] /; \text{FreeQ}[b, x]$$

rule 53
$$\text{Int}[(a_.) + (b_.)*(x_))^{(m_.)}*((c_.) + (d_.)*(x_))^{(n_.)}, x_Symbol] \rightarrow \text{Int}[\text{ExpandIntegrand}[(a + b*x)^m*(c + d*x)^n, x], x] /; \text{FreeQ}[\{a, b, c, d, n\}, x] \ \&\& \ \text{IGtQ}[m, 0] \ \&\& \ (\ !\text{IntegerQ}[n] \ || \ (\text{EqQ}[c, 0] \ \&\& \ \text{LeQ}[7*m + 4*n + 4, 0]) \ || \ \text{LtQ}[9*m + 5*(n + 1), 0] \ || \ \text{GtQ}[m + n + 2, 0])$$

rule 1098
$$\text{Int}[(d_.) + (e_.)*(x_))^{(m_.)}*((a_.) + (b_.)*(x_.) + (c_.)*(x_.)^2)^{(p_.)}, x_Symbol] \rightarrow \text{Simp}[1/c^p \text{ Int}[(d + e*x)^m*(b/2 + c*x)^(2*p), x], x] /; \text{FreeQ}[\{a, b, c, d, e, m\}, x] \ \&\& \ \text{EqQ}[b^2 - 4*a*c, 0] \ \&\& \ \text{IntegerQ}[p]$$

rule 2009
$$\text{Int}[u_, x_Symbol] \rightarrow \text{Simp}[\text{IntSum}[u, x], x] /; \text{SumQ}[u]$$

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 734 vs. 2(142) = 284.

Time = 1.27 (sec) , antiderivative size = 735, normalized size of antiderivative = 5.18

| method | result |
|--------------|--|
| norman | $\frac{b^4 x^5 e^{m \ln(ex+d)}}{5+m} + \frac{d(a^4 e^4 m^4 + 14a^4 e^4 m^3 - 4a^3 b d e^3 m^3 + 71a^4 e^4 m^2 - 48a^3 b d e^3 m^2 + 12a^2 b^2 d^2 e^2 m^2 + 154a^4 e^4 m - 188a^3 b d e^3 m)}{e^5(m^5 + 15m^4 + 85m^3)}$ |
| gospers | $\frac{(ex+d)^{1+m}(b^4 e^4 m^4 x^4 + 4a b^3 e^4 m^4 x^3 + 10b^4 e^4 m^3 x^4 + 6a^2 b^2 e^4 m^4 x^2 + 44a b^3 e^4 m^3 x^3 - 4b^4 d e^3 m^3 x^3 + 35b^4 e^4 m^2 x^4 + 4a^3 b e^4 m^4 x^4)}{e^5(m^5 + 15m^4 + 85m^3)}$ |
| oring | $(b^4 e^4 m^4 x^4 + 4a b^3 e^4 m^4 x^3 + 10b^4 e^4 m^3 x^4 + 6a^2 b^2 e^4 m^4 x^2 + 44a b^3 e^4 m^3 x^3 - 4b^4 d e^3 m^3 x^3 + 35b^4 e^4 m^2 x^4 + 4a^3 b e^4 m^4 x^4 + 72a^2 b^2 e^4 m^4 x^4)$ |
| risch | Expression too large to display |
| parallelrisc | Expression too large to display |

input `int((e*x+d)^m*(b^2*x^2+2*a*b*x+a^2)^2,x,method=_RETURNVERBOSE)`

output

```

b^4/(5+m)*x^5*exp(m*ln(e*x+d))+d*(a^4*e^4*m^4+14*a^4*e^4*m^3-4*a^3*b*d*e^3
*m^3+71*a^4*e^4*m^2-48*a^3*b*d*e^3*m^2+12*a^2*b^2*d^2*e^2*m^2+154*a^4*e^4*
m-188*a^3*b*d*e^3*m+108*a^2*b^2*d^2*e^2*m-24*a*b^3*d^3*e*m+120*a^4*e^4-240
*a^3*b*d*e^3+240*a^2*b^2*d^2*e^2-120*a*b^3*d^3*e+24*b^4*d^4)/e^5/(m^5+15*m
^4+85*m^3+225*m^2+274*m+120)*exp(m*ln(e*x+d))+(a^4*e^4*m^4+4*a^3*b*d*e^3*m
^4+14*a^4*e^4*m^3+48*a^3*b*d*e^3*m^3-12*a^2*b^2*d^2*e^2*m^3+71*a^4*e^4*m^2
+188*a^3*b*d*e^3*m^2-108*a^2*b^2*d^2*e^2*m^2+24*a*b^3*d^3*e*m^2+154*a^4*e^
4*m+240*a^3*b*d*e^3*m-240*a^2*b^2*d^2*e^2*m+120*a*b^3*d^3*e*m-24*b^4*d^4*m
+120*a^4*e^4)/e^4/(m^5+15*m^4+85*m^3+225*m^2+274*m+120)*x*exp(m*ln(e*x+d))
+b^3*(4*a*e*m+b*d*m+20*a*e)/e/(m^2+9*m+20)*x^4*exp(m*ln(e*x+d))+2*(3*a^2*e
^2*m^2+2*a*b*d*e*m^2+27*a^2*e^2*m+10*a*b*d*e*m-2*b^2*d^2*m+60*a^2*e^2)/e^2
*b^2/(m^3+12*m^2+47*m+60)*x^3*exp(m*ln(e*x+d))+2*(2*a^3*e^3*m^3+3*a^2*b*d*
e^2*m^3+24*a^3*e^3*m^2+27*a^2*b*d*e^2*m^2-6*a*b^2*d^2*e*m^2+94*a^3*e^3*m+6
0*a^2*b*d*e^2*m-30*a*b^2*d^2*e*m+6*b^3*d^3*m+120*a^3*e^3)*b/e^3/(m^4+14*m^
3+71*m^2+154*m+120)*x^2*exp(m*ln(e*x+d))

```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 901 vs. $2(142) = 284$.

Time = 0.10 (sec) , antiderivative size = 901, normalized size of antiderivative = 6.35

$$\int (d + ex)^m (a^2 + 2abx + b^2x^2)^2 dx = \text{Too large to display}$$

input `integrate((e*x+d)^m*(b^2*x^2+2*a*b*x+a^2)^2,x, algorithm="fricas")`

output

```
(a^4*d*e^4*m^4 + 24*b^4*d^5 - 120*a*b^3*d^4*e + 240*a^2*b^2*d^3*e^2 - 240*
a^3*b*d^2*e^3 + 120*a^4*d*e^4 + (b^4*e^5*m^4 + 10*b^4*e^5*m^3 + 35*b^4*e^5
*m^2 + 50*b^4*e^5*m + 24*b^4*e^5)*x^5 + (120*a*b^3*e^5 + (b^4*d*e^4 + 4*a*
b^3*e^5)*m^4 + 2*(3*b^4*d*e^4 + 22*a*b^3*e^5)*m^3 + (11*b^4*d*e^4 + 164*a*
b^3*e^5)*m^2 + 2*(3*b^4*d*e^4 + 122*a*b^3*e^5)*m)*x^4 - 2*(2*a^3*b*d^2*e^3
- 7*a^4*d*e^4)*m^3 + 2*(120*a^2*b^2*e^5 + (2*a*b^3*d*e^4 + 3*a^2*b^2*e^5)
*m^4 - 2*(b^4*d^2*e^3 - 8*a*b^3*d*e^4 - 18*a^2*b^2*e^5)*m^3 - (6*b^4*d^2*e
^3 - 34*a*b^3*d*e^4 - 147*a^2*b^2*e^5)*m^2 - 2*(2*b^4*d^2*e^3 - 10*a*b^3*d
*e^4 - 117*a^2*b^2*e^5)*m)*x^3 + (12*a^2*b^2*d^3*e^2 - 48*a^3*b*d^2*e^3 +
71*a^4*d*e^4)*m^2 + 2*(120*a^3*b*e^5 + (3*a^2*b^2*d*e^4 + 2*a^3*b*e^5)*m^4
- 2*(3*a*b^3*d^2*e^3 - 15*a^2*b^2*d*e^4 - 13*a^3*b*e^5)*m^3 + (6*b^4*d^3*
e^2 - 36*a*b^3*d^2*e^3 + 87*a^2*b^2*d*e^4 + 118*a^3*b*e^5)*m^2 + 2*(3*b^4*
d^3*e^2 - 15*a*b^3*d^2*e^3 + 30*a^2*b^2*d*e^4 + 107*a^3*b*e^5)*m)*x^2 - 2*
(12*a*b^3*d^4*e - 54*a^2*b^2*d^3*e^2 + 94*a^3*b*d^2*e^3 - 77*a^4*d*e^4)*m
+ (120*a^4*e^5 + (4*a^3*b*d*e^4 + a^4*e^5)*m^4 - 2*(6*a^2*b^2*d^2*e^3 - 24
*a^3*b*d*e^4 - 7*a^4*e^5)*m^3 + (24*a*b^3*d^3*e^2 - 108*a^2*b^2*d^2*e^3 +
188*a^3*b*d*e^4 + 71*a^4*e^5)*m^2 - 2*(12*b^4*d^4*e - 60*a*b^3*d^3*e^2 + 1
20*a^2*b^2*d^2*e^3 - 120*a^3*b*d*e^4 - 77*a^4*e^5)*m)*x)*(e*x + d)^m/(e^5*
m^5 + 15*e^5*m^4 + 85*e^5*m^3 + 225*e^5*m^2 + 274*e^5*m + 120*e^5)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 8719 vs. $2(124) = 248$.

Time = 1.83 (sec) , antiderivative size = 8719, normalized size of antiderivative = 61.40

$$\int (d + ex)^m (a^2 + 2abx + b^2x^2)^2 dx = \text{Too large to display}$$

input

```
integrate((e*x+d)**m*(b**2*x**2+2*a*b*x+a**2)**2,x)
```

output

```
Piecewise((d**m*(a**4*x + 2*a**3*b*x**2 + 2*a**2*b**2*x**3 + a*b**3*x**4 +
b**4*x**5/5), Eq(e, 0)), (-3*a**4*e**4/(12*d**4*e**5 + 48*d**3*e**6*x + 7
2*d**2*e**7*x**2 + 48*d*e**8*x**3 + 12*e**9*x**4) - 4*a**3*b*d*e**3/(12*d*
*4*e**5 + 48*d**3*e**6*x + 72*d**2*e**7*x**2 + 48*d*e**8*x**3 + 12*e**9*x*
*4) - 16*a**3*b*e**4*x/(12*d**4*e**5 + 48*d**3*e**6*x + 72*d**2*e**7*x**2
+ 48*d*e**8*x**3 + 12*e**9*x**4) - 6*a**2*b**2*d**2*e**2/(12*d**4*e**5 + 4
8*d**3*e**6*x + 72*d**2*e**7*x**2 + 48*d*e**8*x**3 + 12*e**9*x**4) - 24*a*
*2*b**2*d*e**3*x/(12*d**4*e**5 + 48*d**3*e**6*x + 72*d**2*e**7*x**2 + 48*d
*e**8*x**3 + 12*e**9*x**4) - 36*a**2*b**2*e**4*x**2/(12*d**4*e**5 + 48*d**
3*e**6*x + 72*d**2*e**7*x**2 + 48*d*e**8*x**3 + 12*e**9*x**4) - 12*a*b**3*
d**3*e/(12*d**4*e**5 + 48*d**3*e**6*x + 72*d**2*e**7*x**2 + 48*d*e**8*x**3
+ 12*e**9*x**4) - 48*a*b**3*d**2*e**2*x/(12*d**4*e**5 + 48*d**3*e**6*x +
72*d**2*e**7*x**2 + 48*d*e**8*x**3 + 12*e**9*x**4) - 72*a*b**3*d*e**3*x**2
/(12*d**4*e**5 + 48*d**3*e**6*x + 72*d**2*e**7*x**2 + 48*d*e**8*x**3 + 12*
e**9*x**4) - 48*a*b**3*e**4*x**3/(12*d**4*e**5 + 48*d**3*e**6*x + 72*d**2*
e**7*x**2 + 48*d*e**8*x**3 + 12*e**9*x**4) + 12*b**4*d**4*log(d/e + x)/(12
*d**4*e**5 + 48*d**3*e**6*x + 72*d**2*e**7*x**2 + 48*d*e**8*x**3 + 12*e**9
*x**4) + 25*b**4*d**4/(12*d**4*e**5 + 48*d**3*e**6*x + 72*d**2*e**7*x**2 +
48*d*e**8*x**3 + 12*e**9*x**4) + 48*b**4*d**3*e*x*log(d/e + x)/(12*d**4*e
**5 + 48*d**3*e**6*x + 72*d**2*e**7*x**2 + 48*d*e**8*x**3 + 12*e**9*x**...
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 392 vs. $2(142) = 284$.

Time = 0.06 (sec) , antiderivative size = 392, normalized size of antiderivative = 2.76

$$\int (d + ex)^m (a^2 + 2abx + b^2x^2)^2 dx$$

$$= \frac{4(e^2(m+1)x^2 + demx - d^2)(ex + d)^m a^3 b}{(m^2 + 3m + 2)e^2} + \frac{(ex + d)^{m+1} a^4}{e(m+1)}$$

$$+ \frac{6((m^2 + 3m + 2)e^3 x^3 + (m^2 + m)de^2 x^2 - 2d^2 emx + 2d^3)(ex + d)^m a^2 b^2}{(m^3 + 6m^2 + 11m + 6)e^3}$$

$$+ \frac{4((m^3 + 6m^2 + 11m + 6)e^4 x^4 + (m^3 + 3m^2 + 2m)de^3 x^3 - 3(m^2 + m)d^2 e^2 x^2 + 6d^3 emx - 6d^4)(ex + d)^m a b^3}{(m^4 + 10m^3 + 35m^2 + 50m + 24)e^4}$$

$$+ \frac{((m^4 + 10m^3 + 35m^2 + 50m + 24)e^5 x^5 + (m^4 + 6m^3 + 11m^2 + 6m)de^4 x^4 - 4(m^3 + 3m^2 + 2m)d^3 e^3 x^3 + 4m^2 d^2 e^2 x^2 - 4m d^3 e x + 4d^4)(ex + d)^m a^4}{(m^5 + 15m^4 + 85m^3 + 225m^2 + 274m + 124)e^5}$$

input

```
integrate((e*x+d)^m*(b^2*x^2+2*a*b*x+a^2)^2,x, algorithm="maxima")
```

output

```

4*(e^2*(m + 1)*x^2 + d*e*m*x - d^2)*(e*x + d)^m*a^3*b/((m^2 + 3*m + 2)*e^2
) + (e*x + d)^(m + 1)*a^4/(e*(m + 1)) + 6*((m^2 + 3*m + 2)*e^3*x^3 + (m^2
+ m)*d*e^2*x^2 - 2*d^2*e*m*x + 2*d^3)*(e*x + d)^m*a^2*b^2/((m^3 + 6*m^2 +
11*m + 6)*e^3) + 4*((m^3 + 6*m^2 + 11*m + 6)*e^4*x^4 + (m^3 + 3*m^2 + 2*m)
*d*e^3*x^3 - 3*(m^2 + m)*d^2*e^2*x^2 + 6*d^3*e*m*x - 6*d^4)*(e*x + d)^m*a*
b^3/((m^4 + 10*m^3 + 35*m^2 + 50*m + 24)*e^4) + ((m^4 + 10*m^3 + 35*m^2 +
50*m + 24)*e^5*x^5 + (m^4 + 6*m^3 + 11*m^2 + 6*m)*d*e^4*x^4 - 4*(m^3 + 3*m
^2 + 2*m)*d^2*e^3*x^3 + 12*(m^2 + m)*d^3*e^2*x^2 - 24*d^4*e*m*x + 24*d^5)*
(e*x + d)^m*b^4/((m^5 + 15*m^4 + 85*m^3 + 225*m^2 + 274*m + 120)*e^5)

```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1528 vs. $2(142) = 284$.

Time = 0.15 (sec) , antiderivative size = 1528, normalized size of antiderivative = 10.76

$$\int (d + ex)^m (a^2 + 2abx + b^2x^2)^2 dx = \text{Too large to display}$$

input

```

integrate((e*x+d)^m*(b^2*x^2+2*a*b*x+a^2)^2,x, algorithm="giac")

```


input `int((d + e*x)^m*(a^2 + b^2*x^2 + 2*a*b*x)^2,x)`

output

```
((d + e*x)^m*(24*b^4*d^5 + 120*a^4*d*e^4 - 240*a^3*b*d^2*e^3 + 71*a^4*d*e^4*m^2 + 14*a^4*d*e^4*m^3 + a^4*d*e^4*m^4 + 240*a^2*b^2*d^3*e^2 - 120*a*b^3*d^4*e + 154*a^4*d*e^4*m - 24*a*b^3*d^4*e*m + 12*a^2*b^2*d^3*e^2*m^2 - 188*a^3*b*d^2*e^3*m + 108*a^2*b^2*d^3*e^2*m - 48*a^3*b*d^2*e^3*m^2 - 4*a^3*b*d^2*e^3*m^3))/(e^5*(274*m + 225*m^2 + 85*m^3 + 15*m^4 + m^5 + 120)) + (x*(d + e*x)^m*(120*a^4*e^5 + 154*a^4*e^5*m + 71*a^4*e^5*m^2 + 14*a^4*e^5*m^3 + a^4*e^5*m^4 - 24*b^4*d^4*e*m + 240*a^3*b*d*e^4*m - 108*a^2*b^2*d^2*e^3*m^2 - 12*a^2*b^2*d^2*e^3*m^3 + 120*a*b^3*d^3*e^2*m + 188*a^3*b*d*e^4*m^2 + 48*a^3*b*d*e^4*m^3 + 4*a^3*b*d*e^4*m^4 - 240*a^2*b^2*d^2*e^3*m + 24*a*b^3*d^3*e^2*m^2))/(e^5*(274*m + 225*m^2 + 85*m^3 + 15*m^4 + m^5 + 120)) + (b^4*x^5*(d + e*x)^m*(50*m + 35*m^2 + 10*m^3 + m^4 + 24))/(274*m + 225*m^2 + 85*m^3 + 15*m^4 + m^5 + 120) + (2*b^2*x^3*(d + e*x)^m*(3*m + m^2 + 2)*(60*a^2*e^2 + 27*a^2*e^2*m - 2*b^2*d^2*m + 3*a^2*e^2*m^2 + 10*a*b*d*e*m + 2*a*b*d*e*m^2))/(e^2*(274*m + 225*m^2 + 85*m^3 + 15*m^4 + m^5 + 120)) + (b^3*x^4*(d + e*x)^m*(20*a*e + 4*a*e*m + b*d*m)*(11*m + 6*m^2 + m^3 + 6))/(e*(274*m + 225*m^2 + 85*m^3 + 15*m^4 + m^5 + 120)) + (2*b*x^2*(m + 1)*(d + e*x)^m*(120*a^3*e^3 + 94*a^3*e^3*m + 6*b^3*d^3*m + 24*a^3*e^3*m^2 + 2*a^3*e^3*m^3 - 30*a*b^2*d^2*e*m + 60*a^2*b*d*e^2*m - 6*a*b^2*d^2*e*m^2 + 27*a^2*b*d*e^2*m^2 + 3*a^2*b*d*e^2*m^3))/(e^3*(274*m + 225*m^2 + 85*m^3 + 15*m^4 + m^5 + 120))
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 1028, normalized size of antiderivative = 7.24

$$\int (d + ex)^m (a^2 + 2abx + b^2x^2)^2 dx = \text{Too large to display}$$

input `int((e*x+d)^m*(b^2*x^2+2*a*b*x+a^2)^2,x)`

output

```

((d + e*x)**m*(a**4*d***4*m**4 + 14*a**4*d***4*m**3 + 71*a**4*d***4*m**
2 + 154*a**4*d***4*m + 120*a**4*d***4 + a**4*e**5*m**4*x + 14*a**4*e**5*
m**3*x + 71*a**4*e**5*m**2*x + 154*a**4*e**5*m*x + 120*a**4*e**5*x - 4*a**
3*b*d**2*e**3*m**3 - 48*a**3*b*d**2*e**3*m**2 - 188*a**3*b*d**2*e**3*m - 2
40*a**3*b*d**2*e**3 + 4*a**3*b*d***4*m**4*x + 48*a**3*b*d***4*m**3*x + 1
88*a**3*b*d***4*m**2*x + 240*a**3*b*d***4*m*x + 4*a**3*b*e**5*m**4*x**2
+ 52*a**3*b*e**5*m**3*x**2 + 236*a**3*b*e**5*m**2*x**2 + 428*a**3*b*e**5*m
*x**2 + 240*a**3*b*e**5*x**2 + 12*a**2*b**2*d**3*e**2*m**2 + 108*a**2*b**2
*d**3*e**2*m + 240*a**2*b**2*d**3*e**2 - 12*a**2*b**2*d**2*e**3*m**3*x - 1
08*a**2*b**2*d**2*e**3*m**2*x - 240*a**2*b**2*d**2*e**3*m*x + 6*a**2*b**2*
d***4*m**4*x**2 + 60*a**2*b**2*d***4*m**3*x**2 + 174*a**2*b**2*d***4*m**
*2*x**2 + 120*a**2*b**2*d***4*m*x**2 + 6*a**2*b**2*e**5*m**4*x**3 + 72*a*
*2*b**2*e**5*m**3*x**3 + 294*a**2*b**2*e**5*m**2*x**3 + 468*a**2*b**2*e**5
*m*x**3 + 240*a**2*b**2*e**5*x**3 - 24*a*b**3*d**4*e*m - 120*a*b**3*d**4*
+ 24*a*b**3*d**3*e**2*m**2*x + 120*a*b**3*d**3*e**2*m*x - 12*a*b**3*d**2*
e**3*m**3*x**2 - 72*a*b**3*d**2*e**3*m**2*x**2 - 60*a*b**3*d**2*e**3*m*x**
2 + 4*a*b**3*d***4*m**4*x**3 + 32*a*b**3*d***4*m**3*x**3 + 68*a*b**3*d*
**4*m**2*x**3 + 40*a*b**3*d***4*m*x**3 + 4*a*b**3*e**5*m**4*x**4 + 44*a*b
**3*e**5*m**3*x**4 + 164*a*b**3*e**5*m**2*x**4 + 244*a*b**3*e**5*m*x**4 +
120*a*b**3*e**5*x**4 + 24*b**4*d**5 - 24*b**4*d**4*e*m*x + 12*b**4*d**3...

```


3.413 $\int (d + ex)^m (a^2 + 2abx + b^2x^2) dx$

| | |
|---|------|
| Optimal result | 3108 |
| Mathematica [A] (verified) | 3108 |
| Rubi [A] (verified) | 3109 |
| Maple [B] (verified) | 3110 |
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Optimal result

Integrand size = 24, antiderivative size = 78

$$\int (d + ex)^m (a^2 + 2abx + b^2x^2) dx = \frac{(bd - ae)^2(d + ex)^{1+m}}{e^3(1 + m)} - \frac{2b(bd - ae)(d + ex)^{2+m}}{e^3(2 + m)} + \frac{b^2(d + ex)^{3+m}}{e^3(3 + m)}$$

output

$$(-a*e+b*d)^2*(e*x+d)^(1+m)/e^3/(1+m)-2*b*(-a*e+b*d)*(e*x+d)^(2+m)/e^3/(2+m)+b^2*(e*x+d)^(3+m)/e^3/(3+m)$$

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.86

$$\int (d + ex)^m (a^2 + 2abx + b^2x^2) dx = \frac{(d + ex)^{1+m} \left(\frac{(bd - ae)^2}{1+m} - \frac{2b(bd - ae)(d + ex)}{2+m} + \frac{b^2(d + ex)^2}{3+m} \right)}{e^3}$$

input

$$\text{Integrate}[(d + e*x)^m*(a^2 + 2*a*b*x + b^2*x^2), x]$$

output

$$((d + e*x)^(1 + m)*((b*d - a*e)^2/(1 + m) - (2*b*(b*d - a*e)*(d + e*x))/(2 + m) + (b^2*(d + e*x)^2)/(3 + m))/e^3$$

Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {1098, 27, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\begin{aligned}
 & \int (a^2 + 2abx + b^2x^2)(d + ex)^m dx \\
 & \quad \downarrow \text{1098} \\
 & \quad \int \frac{b^2(a + bx)^2(d + ex)^m dx}{b^2} \\
 & \quad \downarrow \text{27} \\
 & \quad \int (a + bx)^2(d + ex)^m dx \\
 & \quad \downarrow \text{53} \\
 & \quad \int \left(\frac{(ae - bd)^2(d + ex)^m}{e^2} - \frac{2b(bd - ae)(d + ex)^{m+1}}{e^2} + \frac{b^2(d + ex)^{m+2}}{e^2} \right) dx \\
 & \quad \downarrow \text{2009} \\
 & \quad \frac{(bd - ae)^2(d + ex)^{m+1}}{e^3(m + 1)} - \frac{2b(bd - ae)(d + ex)^{m+2}}{e^3(m + 2)} + \frac{b^2(d + ex)^{m+3}}{e^3(m + 3)}
 \end{aligned}$$

input `Int[(d + e*x)^m*(a^2 + 2*a*b*x + b^2*x^2), x]`

output `((b*d - a*e)^2*(d + e*x)^(1 + m))/(e^3*(1 + m)) - (2*b*(b*d - a*e)*(d + e*x)^(2 + m))/(e^3*(2 + m)) + (b^2*(d + e*x)^(3 + m))/(e^3*(3 + m))`

output

```
1/e^3*(e*x+d)^(1+m)/(m^3+6*m^2+11*m+6)*(b^2*e^2*m^2*x^2+2*a*b*e^2*m^2*x+3*
b^2*e^2*m*x^2+a^2*e^2*m^2+8*a*b*e^2*m*x-2*b^2*d*e*m*x+2*b^2*e^2*x^2+5*a^2*
e^2*m-2*a*b*d*e*m+6*a*b*e^2*x-2*b^2*d*e*x+6*a^2*e^2-6*a*b*d*e+2*b^2*d^2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 237 vs. $2(78) = 156$.

Time = 0.09 (sec) , antiderivative size = 237, normalized size of antiderivative = 3.04

$$\int (d + ex)^m (a^2 + 2abx + b^2x^2) dx$$

$$= \frac{(a^2de^2m^2 + 2b^2d^3 - 6abd^2e + 6a^2de^2 + (b^2e^3m^2 + 3b^2e^3m + 2b^2e^3)x^3 + (6abe^3 + (b^2de^2 + 2abe^3)m^2 - e^3m^3))}{e^3m^3}$$

input

```
integrate((e*x+d)^m*(b^2*x^2+2*a*b*x+a^2),x, algorithm="fricas")
```

output

```
(a^2*d*e^2*m^2 + 2*b^2*d^3 - 6*a*b*d^2*e + 6*a^2*d*e^2 + (b^2*e^3*m^2 + 3*
b^2*e^3*m + 2*b^2*e^3)*x^3 + (6*a*b*e^3 + (b^2*d*e^2 + 2*a*b*e^3)*m^2 + (b
^2*d*e^2 + 8*a*b*e^3)*m)*x^2 - (2*a*b*d^2*e - 5*a^2*d*e^2)*m + (6*a^2*e^3
+ (2*a*b*d*e^2 + a^2*e^3)*m^2 - (2*b^2*d^2*e - 6*a*b*d*e^2 - 5*a^2*e^3)*m)
*x)*(e*x + d)^m/(e^3*m^3 + 6*e^3*m^2 + 11*e^3*m + 6*e^3)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1506 vs. $2(66) = 132$.

Time = 0.56 (sec) , antiderivative size = 1506, normalized size of antiderivative = 19.31

$$\int (d + ex)^m (a^2 + 2abx + b^2x^2) dx = \text{Too large to display}$$

input

```
integrate((e*x+d)**m*(b**2*x**2+2*a*b*x+a**2),x)
```

output

```
Piecewise((d**m*(a**2*x + a*b*x**2 + b**2*x**3/3), Eq(e, 0)), (-a**2*e**2/
(2*d**2*e**3 + 4*d*e**4*x + 2*e**5*x**2) - 2*a*b*d*e/(2*d**2*e**3 + 4*d*e
**4*x + 2*e**5*x**2) - 4*a*b*e**2*x/(2*d**2*e**3 + 4*d*e**4*x + 2*e**5*x**2
) + 2*b**2*d**2*log(d/e + x)/(2*d**2*e**3 + 4*d*e**4*x + 2*e**5*x**2) + 3*
b**2*d**2/(2*d**2*e**3 + 4*d*e**4*x + 2*e**5*x**2) + 4*b**2*d*e*x*log(d/e
+ x)/(2*d**2*e**3 + 4*d*e**4*x + 2*e**5*x**2) + 4*b**2*d*e*x/(2*d**2*e**3
+ 4*d*e**4*x + 2*e**5*x**2) + 2*b**2*e**2*x**2*log(d/e + x)/(2*d**2*e**3 +
4*d*e**4*x + 2*e**5*x**2), Eq(m, -3)), (-a**2*e**2/(d*e**3 + e**4*x) + 2*
a*b*d*e*log(d/e + x)/(d*e**3 + e**4*x) + 2*a*b*d*e/(d*e**3 + e**4*x) + 2*a
*b*e**2*x*log(d/e + x)/(d*e**3 + e**4*x) - 2*b**2*d**2*log(d/e + x)/(d*e**
3 + e**4*x) - 2*b**2*d**2/(d*e**3 + e**4*x) - 2*b**2*d*e*x*log(d/e + x)/(d
*e**3 + e**4*x) + b**2*e**2*x**2/(d*e**3 + e**4*x), Eq(m, -2)), (a**2*log(
d/e + x)/e - 2*a*b*d*log(d/e + x)/e**2 + 2*a*b*x/e + b**2*d**2*log(d/e + x
)/e**3 - b**2*d*x/e**2 + b**2*x**2/(2*e), Eq(m, -1)), (a**2*d*e**2*m**2*(d
+ e*x)**m/(e**3*m**3 + 6*e**3*m**2 + 11*e**3*m + 6*e**3) + 5*a**2*d*e**2*
m*(d + e*x)**m/(e**3*m**3 + 6*e**3*m**2 + 11*e**3*m + 6*e**3) + 6*a**2*d*e
**2*(d + e*x)**m/(e**3*m**3 + 6*e**3*m**2 + 11*e**3*m + 6*e**3) + a**2*e**
3*m**2*x*(d + e*x)**m/(e**3*m**3 + 6*e**3*m**2 + 11*e**3*m + 6*e**3) + 5*a
**2*e**3*m*x*(d + e*x)**m/(e**3*m**3 + 6*e**3*m**2 + 11*e**3*m + 6*e**3) +
6*a**2*e**3*x*(d + e*x)**m/(e**3*m**3 + 6*e**3*m**2 + 11*e**3*m + 6*e...
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 138, normalized size of antiderivative = 1.77

$$\int (d + ex)^m (a^2 + 2abx + b^2x^2) dx$$

$$= \frac{2(e^2(m+1)x^2 + demx - d^2)(ex + d)^m ab}{(m^2 + 3m + 2)e^2} + \frac{(ex + d)^{m+1} a^2}{e(m+1)}$$

$$+ \frac{((m^2 + 3m + 2)e^3x^3 + (m^2 + m)de^2x^2 - 2d^2emx + 2d^3)(ex + d)^m b^2}{(m^3 + 6m^2 + 11m + 6)e^3}$$

input

```
integrate((e*x+d)^m*(b^2*x^2+2*a*b*x+a^2),x, algorithm="maxima")
```

output

```
2*(e^2*(m + 1)*x^2 + d*e*m*x - d^2)*(e*x + d)^m*a*b/((m^2 + 3*m + 2)*e^2)
+ (e*x + d)^(m + 1)*a^2/(e*(m + 1)) + ((m^2 + 3*m + 2)*e^3*x^3 + (m^2 + m)
*d*e^2*x^2 - 2*d^2*e*m*x + 2*d^3)*(e*x + d)^m*b^2/((m^3 + 6*m^2 + 11*m + 6
)*e^3)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 385 vs. $2(78) = 156$.

Time = 0.14 (sec) , antiderivative size = 385, normalized size of antiderivative = 4.94

$$\int (d + ex)^m (a^2 + 2abx + b^2x^2) dx$$

$$= \frac{(ex + d)^m b^2 e^3 m^2 x^3 + (ex + d)^m b^2 d e^2 m^2 x^2 + 2(ex + d)^m a b e^3 m^2 x^2 + 3(ex + d)^m b^2 e^3 m x^3 + 2(ex + d)^m (a^2 + 2abx + b^2 x^2) dx}{1}$$

input `integrate((e*x+d)^m*(b^2*x^2+2*a*b*x+a^2),x, algorithm="giac")`

output

```
((e*x + d)^m*b^2*e^3*m^2*x^3 + (e*x + d)^m*b^2*d*e^2*m^2*x^2 + 2*(e*x + d)
^m*a*b*e^3*m^2*x^2 + 3*(e*x + d)^m*b^2*e^3*m*x^3 + 2*(e*x + d)^m*a*b*d*e^2
*m^2*x + (e*x + d)^m*a^2*e^3*m^2*x + (e*x + d)^m*b^2*d*e^2*m*x^2 + 8*(e*x
+ d)^m*a*b*e^3*m*x^2 + 2*(e*x + d)^m*b^2*e^3*x^3 + (e*x + d)^m*a^2*d*e^2*m
^2 - 2*(e*x + d)^m*b^2*d^2*e*m*x + 6*(e*x + d)^m*a*b*d*e^2*m*x + 5*(e*x +
d)^m*a^2*e^3*m*x + 6*(e*x + d)^m*a*b*e^3*x^2 - 2*(e*x + d)^m*a*b*d^2*e*m +
5*(e*x + d)^m*a^2*d*e^2*m + 6*(e*x + d)^m*a^2*e^3*x + 2*(e*x + d)^m*b^2*d
^3 - 6*(e*x + d)^m*a*b*d^2*e + 6*(e*x + d)^m*a^2*d*e^2)/(e^3*m^3 + 6*e^3*m
^2 + 11*e^3*m + 6*e^3)
```

Mupad [B] (verification not implemented)

Time = 5.35 (sec) , antiderivative size = 226, normalized size of antiderivative = 2.90

$$\int (d + ex)^m (a^2 + 2abx + b^2x^2) dx$$

$$= (d + ex)^m \left(\frac{b^2 x^3 (m^2 + 3m + 2)}{m^3 + 6m^2 + 11m + 6} + \frac{d(a^2 e^2 m^2 + 5a^2 e^2 m + 6a^2 e^2 - 2abdem - 6abde + 2b^2 d^2)}{e^3 (m^3 + 6m^2 + 11m + 6)} + \frac{x(a^2 e^3 m^2 + 5a^2 e^3 m + 6a^2 e^3 + 2abde^2 m^2 + 6abde^2 m - 2b^2 d^2 em)}{e^3 (m^3 + 6m^2 + 11m + 6)} + \frac{bx^2 (m + 1) (6ae + 2aem + bdm)}{e (m^3 + 6m^2 + 11m + 6)} \right)$$

input `int((d + e*x)^m*(a^2 + b^2*x^2 + 2*a*b*x),x)`

output

```
(d + e*x)^m*((b^2*x^3*(3*m + m^2 + 2))/(11*m + 6*m^2 + m^3 + 6) + (d*(6*a^2*e^2 + 2*b^2*d^2 + 5*a^2*e^2*m + a^2*e^2*m^2 - 6*a*b*d*e - 2*a*b*d*e*m))/(e^3*(11*m + 6*m^2 + m^3 + 6)) + (x*(6*a^2*e^3 + 5*a^2*e^3*m + a^2*e^3*m^2 - 2*b^2*d^2*e*m + 2*a*b*d*e^2*m^2 + 6*a*b*d*e^2*m))/(e^3*(11*m + 6*m^2 + m^3 + 6)) + (b*x^2*(m + 1)*(6*a*e + 2*a*e*m + b*d*m))/(e*(11*m + 6*m^2 + m^3 + 6)))
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 241, normalized size of antiderivative = 3.09

$$\int (d + ex)^m (a^2 + 2abx + b^2x^2) dx$$

$$= \frac{(ex + d)^m (b^2e^3m^2x^3 + 2abe^3m^2x^2 + b^2de^2m^2x^2 + 3b^2e^3mx^3 + a^2e^3m^2x + 2abde^2m^2x + 8abe^3mx^2 + \dots)}{\dots}$$

input

```
int((e*x+d)^m*(b^2*x^2+2*a*b*x+a^2),x)
```

output

```
((d + e*x)**m*(a**2*d*e**2*m**2 + 5*a**2*d*e**2*m + 6*a**2*d*e**2 + a**2*e**3*m**2*x + 5*a**2*e**3*m*x + 6*a**2*e**3*x - 2*a*b*d**2*e*m - 6*a*b*d**2*e + 2*a*b*d*e**2*m**2*x + 6*a*b*d*e**2*m*x + 2*a*b*e**3*m**2*x**2 + 8*a*b*e**3*m*x**2 + 6*a*b*e**3*x**2 + 2*b**2*d**3 - 2*b**2*d**2*e*m*x + b**2*d*e**2*m**2*x**2 + b**2*d*e**2*m*x**2 + b**2*e**3*m**2*x**3 + 3*b**2*e**3*m*x**3 + 2*b**2*e**3*x**3))/(e**3*(m**3 + 6*m**2 + 11*m + 6))
```

3.414 $\int \frac{(d+ex)^m}{a^2+2abx+b^2x^2} dx$

| | |
|----------------------------|------|
| Optimal result | 3115 |
| Mathematica [A] (verified) | 3115 |
| Rubi [A] (verified) | 3116 |
| Maple [F] | 3117 |
| Fricas [F] | 3117 |
| Sympy [F] | 3118 |
| Maxima [F] | 3118 |
| Giac [F] | 3118 |
| Mupad [F(-1)] | 3119 |
| Reduce [F] | 3119 |

Optimal result

Integrand size = 26, antiderivative size = 51

$$\int \frac{(d+ex)^m}{a^2+2abx+b^2x^2} dx = \frac{e(d+ex)^{1+m} \operatorname{Hypergeometric2F1}\left(2, 1+m, 2+m, \frac{b(d+ex)}{bd-ae}\right)}{(bd-ae)^2(1+m)}$$

output

```
e*(e*x+d)^(1+m)*hypergeom([2, 1+m],[2+m],b*(e*x+d)/(-a*e+b*d))/(-a*e+b*d)^2/(1+m)
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.02

$$\int \frac{(d+ex)^m}{a^2+2abx+b^2x^2} dx = \frac{e(d+ex)^{1+m} \operatorname{Hypergeometric2F1}\left(2, 1+m, 2+m, -\frac{b(d+ex)}{-bd+ae}\right)}{(-bd+ae)^2(1+m)}$$

input

```
Integrate[(d + e*x)^m/(a^2 + 2*a*b*x + b^2*x^2),x]
```

output

```
(e*(d + e*x)^(1 + m)*Hypergeometric2F1[2, 1 + m, 2 + m, -((b*(d + e*x))/(-b*d) + a*e)])/((-b*d) + a*e)^2*(1 + m)
```


Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.115$, Rules used = {1098, 27, 78}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex)^m}{a^2+2abx+b^2x^2} dx$$

$$\downarrow 1098$$

$$b^2 \int \frac{(d+ex)^m}{b^2(a+bx)^2} dx$$

$$\downarrow 27$$

$$\int \frac{(d+ex)^m}{(a+bx)^2} dx$$

$$\downarrow 78$$

$$\frac{e(d+ex)^{m+1} \text{Hypergeometric2F1}\left(2, m+1, m+2, \frac{b(d+ex)}{bd-ae}\right)}{(m+1)(bd-ae)^2}$$

input `Int[(d + e*x)^m/(a^2 + 2*a*b*x + b^2*x^2), x]`

output `(e*(d + e*x)^(1 + m)*Hypergeometric2F1[2, 1 + m, 2 + m, (b*(d + e*x))/(b*d - a*e)])/((b*d - a*e)^2*(1 + m))`

Defintions of rubi rules used

rule 27 `Int[(a_)*(F_x_), x_Symbol] := Simp[a Int[F_x, x], x] /; FreeQ[a, x] && !MatchQ[F_x, (b_)*(G_x_) /; FreeQ[b, x]]`

rule 78 `Int[((a_) + (b_)*(x_))^(m_)*((c_) + (d_)*(x_))^(n_), x_Symbol] := Simp[(b*c - a*d)^n*((a + b*x)^(m + 1)/(b^(n + 1)*(m + 1)))*Hypergeometric2F1[-n, m + 1, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m}, x] && !IntegerQ[m] && IntegerQ[n]`

rule 1098 `Int[((d_) + (e_)*(x_))^(m_)*((a_) + (b_)*(x_) + (c_)*(x_)^2)^(p_), x_Symbol] := Simp[1/c^p Int[(d + e*x)^m*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]`

Maple [F]

$$\int \frac{(ex + d)^m}{b^2x^2 + 2abx + a^2} dx$$

input `int((e*x+d)^m/(b^2*x^2+2*a*b*x+a^2),x)`

output `int((e*x+d)^m/(b^2*x^2+2*a*b*x+a^2),x)`

Fricas [F]

$$\int \frac{(d + ex)^m}{a^2 + 2abx + b^2x^2} dx = \int \frac{(ex + d)^m}{b^2x^2 + 2abx + a^2} dx$$

input `integrate((e*x+d)^m/(b^2*x^2+2*a*b*x+a^2),x, algorithm="fricas")`

output `integral((e*x + d)^m/(b^2*x^2 + 2*a*b*x + a^2), x)`

Sympy [F]

$$\int \frac{(d + ex)^m}{a^2 + 2abx + b^2x^2} dx = \int \frac{(d + ex)^m}{(a + bx)^2} dx$$

input `integrate((e*x+d)**m/(b**2*x**2+2*a*b*x+a**2),x)`

output `Integral((d + e*x)**m/(a + b*x)**2, x)`

Maxima [F]

$$\int \frac{(d + ex)^m}{a^2 + 2abx + b^2x^2} dx = \int \frac{(ex + d)^m}{b^2x^2 + 2abx + a^2} dx$$

input `integrate((e*x+d)^m/(b^2*x^2+2*a*b*x+a^2),x, algorithm="maxima")`

output `integrate((e*x + d)^m/(b^2*x^2 + 2*a*b*x + a^2), x)`

Giac [F]

$$\int \frac{(d + ex)^m}{a^2 + 2abx + b^2x^2} dx = \int \frac{(ex + d)^m}{b^2x^2 + 2abx + a^2} dx$$

input `integrate((e*x+d)^m/(b^2*x^2+2*a*b*x+a^2),x, algorithm="giac")`

output `integrate((e*x + d)^m/(b^2*x^2 + 2*a*b*x + a^2), x)`

Mupad [F(-1)]

Timed out.

$$\int \frac{(d+ex)^m}{a^2+2abx+b^2x^2} dx = \int \frac{(d+ex)^m}{a^2+2abx+b^2x^2} dx$$

input `int((d + e*x)^m/(a^2 + b^2*x^2 + 2*a*b*x), x)`output `int((d + e*x)^m/(a^2 + b^2*x^2 + 2*a*b*x), x)`**Reduce [F]**

$$\int \frac{(d+ex)^m}{a^2+2abx+b^2x^2} dx = \text{Too large to display}$$

input `int((e*x+d)^m/(b^2*x^2+2*a*b*x+a^2), x)`

output

```

((d + e*x)**m*d + int(((d + e*x)**m*x)/(a**3*d*e*m + a**3*e**2*m*x - a**2*
b*d**2 + 2*a**2*b*d*e*m*x - a**2*b*d*e*x + 2*a**2*b*e**2*m*x**2 - 2*a*b**2
*d**2*x + a*b**2*d*e*m*x**2 - 2*a*b**2*d*e*x**2 + a*b**2*e**2*m*x**3 - b**
3*d**2*x**2 - b**3*d*e*x**3),x)*a**3*e**3*m**2 - int(((d + e*x)**m*x)/(a**
3*d*e*m + a**3*e**2*m*x - a**2*b*d**2 + 2*a**2*b*d*e*m*x - a**2*b*d*e*x +
2*a**2*b*e**2*m*x**2 - 2*a*b**2*d**2*x + a*b**2*d*e*m*x**2 - 2*a*b**2*d*e*
x**2 + a*b**2*e**2*m*x**3 - b**3*d**2*x**2 - b**3*d*e*x**3),x)*a**2*b*d*e*
**2*m**2 - int(((d + e*x)**m*x)/(a**3*d*e*m + a**3*e**2*m*x - a**2*b*d**2 +
2*a**2*b*d*e*m*x - a**2*b*d*e*x + 2*a**2*b*e**2*m*x**2 - 2*a*b**2*d**2*x
+ a*b**2*d*e*m*x**2 - 2*a*b**2*d*e*x**2 + a*b**2*e**2*m*x**3 - b**3*d**2*x
**2 - b**3*d*e*x**3),x)*a**2*b*d*e**2*m + int(((d + e*x)**m*x)/(a**3*d*e*m
+ a**3*e**2*m*x - a**2*b*d**2 + 2*a**2*b*d*e*m*x - a**2*b*d*e*x + 2*a**2*
b*e**2*m*x**2 - 2*a*b**2*d**2*x + a*b**2*d*e*m*x**2 - 2*a*b**2*d*e*x**2 +
a*b**2*e**2*m*x**3 - b**3*d**2*x**2 - b**3*d*e*x**3),x)*a**2*b*e**3*m**2*x
+ int(((d + e*x)**m*x)/(a**3*d*e*m + a**3*e**2*m*x - a**2*b*d**2 + 2*a**2
*b*d*e*m*x - a**2*b*d*e*x + 2*a**2*b*e**2*m*x**2 - 2*a*b**2*d**2*x + a*b**
2*d*e*m*x**2 - 2*a*b**2*d*e*x**2 + a*b**2*e**2*m*x**3 - b**3*d**2*x**2 - b
**3*d*e*x**3),x)*a*b**2*d**2*e*m - int(((d + e*x)**m*x)/(a**3*d*e*m + a**3
*e**2*m*x - a**2*b*d**2 + 2*a**2*b*d*e*m*x - a**2*b*d*e*x + 2*a**2*b*e**2*
m*x**2 - 2*a*b**2*d**2*x + a*b**2*d*e*m*x**2 - 2*a*b**2*d*e*x**2 + a*b*...

```

3.415 $\int \frac{(d+ex)^m}{(a^2+2abx+b^2x^2)^2} dx$

| | |
|----------------------------|------|
| Optimal result | 3121 |
| Mathematica [A] (verified) | 3121 |
| Rubi [A] (verified) | 3122 |
| Maple [F] | 3123 |
| Fricas [F] | 3123 |
| Sympy [F] | 3124 |
| Maxima [F] | 3124 |
| Giac [F] | 3124 |
| Mupad [F(-1)] | 3125 |
| Reduce [F] | 3125 |

Optimal result

Integrand size = 26, antiderivative size = 53

$$\int \frac{(d+ex)^m}{(a^2+2abx+b^2x^2)^2} dx = \frac{e^3(d+ex)^{1+m} \operatorname{Hypergeometric2F1}\left(4, 1+m, 2+m, \frac{b(d+ex)}{bd-ae}\right)}{(bd-ae)^4(1+m)}$$

output

```
e^3*(e*x+d)^(1+m)*hypergeom([4, 1+m], [2+m], b*(e*x+d)/(-a*e+b*d))/(-a*e+b*d)^4/(1+m)
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.02

$$\int \frac{(d+ex)^m}{(a^2+2abx+b^2x^2)^2} dx = \frac{e^3(d+ex)^{1+m} \operatorname{Hypergeometric2F1}\left(4, 1+m, 2+m, -\frac{b(d+ex)}{-bd+ae}\right)}{(-bd+ae)^4(1+m)}$$

input

```
Integrate[(d + e*x)^m/(a^2 + 2*a*b*x + b^2*x^2)^2,x]
```

output

```
(e^3*(d + e*x)^(1 + m)*Hypergeometric2F1[4, 1 + m, 2 + m, -((b*(d + e*x))/
(-b*d) + a*e))]/((-b*d) + a*e)^4*(1 + m))
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.115$, Rules used = {1098, 27, 78}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d + ex)^m}{(a^2 + 2abx + b^2x^2)^2} dx$$

$$\downarrow 1098$$

$$b^4 \int \frac{(d + ex)^m}{b^4(a + bx)^4} dx$$

$$\downarrow 27$$

$$\int \frac{(d + ex)^m}{(a + bx)^4} dx$$

$$\downarrow 78$$

$$\frac{e^3(d + ex)^{m+1} \text{Hypergeometric2F1}\left(4, m + 1, m + 2, \frac{b(d+ex)}{bd-ae}\right)}{(m + 1)(bd - ae)^4}$$

input

```
Int[(d + e*x)^m/(a^2 + 2*a*b*x + b^2*x^2)^2,x]
```

output

```
(e^3*(d + e*x)^(1 + m)*Hypergeometric2F1[4, 1 + m, 2 + m, (b*(d + e*x))/(b
*d - a*e)]/((b*d - a*e)^4*(1 + m))
```

Defintions of rubi rules used

rule 27 `Int[(a_)*(F_x_), x_Symbol] := Simp[a Int[F_x, x], x] /; FreeQ[a, x] && !MatchQ[F_x, (b_)*(G_x_) /; FreeQ[b, x]]`

rule 78 `Int[((a_) + (b_)*(x_))^(m_)*((c_) + (d_)*(x_))^(n_), x_Symbol] := Simp[(b*c - a*d)^n*(a + b*x)^(m + 1)/(b^(n + 1)*(m + 1))*Hypergeometric2F1[-n, m + 1, m + 2, (-d)*(a + b*x)/(b*c - a*d)], x] /; FreeQ[{a, b, c, d, m}, x] && !IntegerQ[m] && IntegerQ[n]`

rule 1098 `Int[((d_) + (e_)*(x_))^(m_)*((a_) + (b_)*(x_) + (c_)*(x_)^2)^(p_), x_Symbol] := Simp[1/c^p Int[(d + e*x)^m*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]`

Maple [F]

$$\int \frac{(ex + d)^m}{(b^2x^2 + 2abx + a^2)^2} dx$$

input `int((e*x+d)^m/(b^2*x^2+2*a*b*x+a^2)^2,x)`

output `int((e*x+d)^m/(b^2*x^2+2*a*b*x+a^2)^2,x)`

Fricas [F]

$$\int \frac{(d + ex)^m}{(a^2 + 2abx + b^2x^2)^2} dx = \int \frac{(ex + d)^m}{(b^2x^2 + 2abx + a^2)^2} dx$$

input `integrate((e*x+d)^m/(b^2*x^2+2*a*b*x+a^2)^2,x, algorithm="fricas")`

output `integral((e*x + d)^m/(b^4*x^4 + 4*a*b^3*x^3 + 6*a^2*b^2*x^2 + 4*a^3*b*x + a^4), x)`

Sympy [F]

$$\int \frac{(d + ex)^m}{(a^2 + 2abx + b^2x^2)^2} dx = \int \frac{(d + ex)^m}{(a + bx)^4} dx$$

input `integrate((e*x+d)**m/(b**2*x**2+2*a*b*x+a**2)**2,x)`

output `Integral((d + e*x)**m/(a + b*x)**4, x)`

Maxima [F]

$$\int \frac{(d + ex)^m}{(a^2 + 2abx + b^2x^2)^2} dx = \int \frac{(ex + d)^m}{(b^2x^2 + 2abx + a^2)^2} dx$$

input `integrate((e*x+d)^m/(b^2*x^2+2*a*b*x+a^2)^2,x, algorithm="maxima")`

output `integrate((e*x + d)^m/(b^2*x^2 + 2*a*b*x + a^2)^2, x)`

Giac [F]

$$\int \frac{(d + ex)^m}{(a^2 + 2abx + b^2x^2)^2} dx = \int \frac{(ex + d)^m}{(b^2x^2 + 2abx + a^2)^2} dx$$

input `integrate((e*x+d)^m/(b^2*x^2+2*a*b*x+a^2)^2,x, algorithm="giac")`

output `integrate((e*x + d)^m/(b^2*x^2 + 2*a*b*x + a^2)^2, x)`

Mupad [F(-1)]

Timed out.

$$\int \frac{(d + ex)^m}{(a^2 + 2abx + b^2x^2)^2} dx = \int \frac{(d + ex)^m}{(a^2 + 2abx + b^2x^2)^2} dx$$

input `int((d + e*x)^m/(a^2 + b^2*x^2 + 2*a*b*x)^2,x)`output `int((d + e*x)^m/(a^2 + b^2*x^2 + 2*a*b*x)^2, x)`**Reduce [F]**

$$\int \frac{(d + ex)^m}{(a^2 + 2abx + b^2x^2)^2} dx = \text{too large to display}$$

input `int((e*x+d)^m/(b^2*x^2+2*a*b*x+a^2)^2,x)`

output

```

((d + e*x)**m*d + int(((d + e*x)**m*x)/(a**5*d*e*m + a**5*e**2*m*x - 3*a**
4*b*d**2 + 4*a**4*b*d*e*m*x - 3*a**4*b*d*e*x + 4*a**4*b*e**2*m*x**2 - 12*a
**3*b**2*d**2*x + 6*a**3*b**2*d*e*m*x**2 - 12*a**3*b**2*d*e*x**2 + 6*a**3*
b**2*e**2*m*x**3 - 18*a**2*b**3*d**2*x**2 + 4*a**2*b**3*d*e*m*x**3 - 18*a*
**2*b**3*d*e*x**3 + 4*a**2*b**3*e**2*m*x**4 - 12*a*b**4*d**2*x**3 + a*b**4*
d*e*m*x**4 - 12*a*b**4*d*e*x**4 + a*b**4*e**2*m*x**5 - 3*b**5*d**2*x**4 -
3*b**5*d*e*x**5),x)*a**5*e**3*m**2 - int(((d + e*x)**m*x)/(a**5*d*e*m + a
**5*e**2*m*x - 3*a**4*b*d**2 + 4*a**4*b*d*e*m*x - 3*a**4*b*d*e*x + 4*a**4*b
*e**2*m*x**2 - 12*a**3*b**2*d**2*x + 6*a**3*b**2*d*e*m*x**2 - 12*a**3*b**2
*d*e*x**2 + 6*a**3*b**2*e**2*m*x**3 - 18*a**2*b**3*d**2*x**2 + 4*a**2*b**3
*d*e*m*x**3 - 18*a**2*b**3*d*e*x**3 + 4*a**2*b**3*e**2*m*x**4 - 12*a*b**4*
d**2*x**3 + a*b**4*d*e*m*x**4 - 12*a*b**4*d*e*x**4 + a*b**4*e**2*m*x**5 -
3*b**5*d**2*x**4 - 3*b**5*d*e*x**5),x)*a**4*b*d*e**2*m**2 - 3*int(((d + e*
x)**m*x)/(a**5*d*e*m + a**5*e**2*m*x - 3*a**4*b*d**2 + 4*a**4*b*d*e*m*x -
3*a**4*b*d*e*x + 4*a**4*b*e**2*m*x**2 - 12*a**3*b**2*d**2*x + 6*a**3*b**2*
d*e*m*x**2 - 12*a**3*b**2*d*e*x**2 + 6*a**3*b**2*e**2*m*x**3 - 18*a**2*b**
3*d**2*x**2 + 4*a**2*b**3*d*e*m*x**3 - 18*a**2*b**3*d*e*x**3 + 4*a**2*b**3
*e**2*m*x**4 - 12*a*b**4*d**2*x**3 + a*b**4*d*e*m*x**4 - 12*a*b**4*d*e*x**
4 + a*b**4*e**2*m*x**5 - 3*b**5*d**2*x**4 - 3*b**5*d*e*x**5),x)*a**4*b*d*e
**2*m + 3*int(((d + e*x)**m*x)/(a**5*d*e*m + a**5*e**2*m*x - 3*a**4*b*d...

```

3.416 $\int \frac{(d+ex)^m}{(a^2+2abx+b^2x^2)^3} dx$

| | |
|----------------------------|------|
| Optimal result | 3127 |
| Mathematica [A] (verified) | 3127 |
| Rubi [A] (verified) | 3128 |
| Maple [F] | 3129 |
| Fricas [F] | 3129 |
| Sympy [F] | 3130 |
| Maxima [F] | 3130 |
| Giac [F] | 3130 |
| Mupad [F(-1)] | 3131 |
| Reduce [F] | 3131 |

Optimal result

Integrand size = 26, antiderivative size = 53

$$\int \frac{(d+ex)^m}{(a^2+2abx+b^2x^2)^3} dx = \frac{e^5(d+ex)^{1+m} \operatorname{Hypergeometric2F1}\left(6, 1+m, 2+m, \frac{b(d+ex)}{bd-ae}\right)}{(bd-ae)^6(1+m)}$$

output

```
e^5*(e*x+d)^(1+m)*hypergeom([6, 1+m], [2+m], b*(e*x+d)/(-a*e+b*d))/(-a*e+b*d)^6/(1+m)
```

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.02

$$\int \frac{(d+ex)^m}{(a^2+2abx+b^2x^2)^3} dx = \frac{e^5(d+ex)^{1+m} \operatorname{Hypergeometric2F1}\left(6, 1+m, 2+m, -\frac{b(d+ex)}{-bd+ae}\right)}{(-bd+ae)^6(1+m)}$$

input

```
Integrate[(d + e*x)^m/(a^2 + 2*a*b*x + b^2*x^2)^3,x]
```

output

```
(e^5*(d + e*x)^(1 + m)*Hypergeometric2F1[6, 1 + m, 2 + m, -((b*(d + e*x))/
(-(b*d) + a*e))]/((-(b*d) + a*e)^6*(1 + m))
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.115$, Rules used = {1098, 27, 78}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d + ex)^m}{(a^2 + 2abx + b^2x^2)^3} dx$$

$$\downarrow 1098$$

$$b^6 \int \frac{(d + ex)^m}{b^6(a + bx)^6} dx$$

$$\downarrow 27$$

$$\int \frac{(d + ex)^m}{(a + bx)^6} dx$$

$$\downarrow 78$$

$$\frac{e^5(d + ex)^{m+1} \text{Hypergeometric2F1}\left(6, m + 1, m + 2, \frac{b(d+ex)}{bd-ae}\right)}{(m + 1)(bd - ae)^6}$$

input

```
Int[(d + e*x)^m/(a^2 + 2*a*b*x + b^2*x^2)^3,x]
```

output

```
(e^5*(d + e*x)^(1 + m)*Hypergeometric2F1[6, 1 + m, 2 + m, (b*(d + e*x))/(b
*d - a*e)]/((b*d - a*e)^6*(1 + m))
```

Definitions of rubi rules used

rule 27 `Int[(a_)*(F_x_), x_Symbol] := Simp[a Int[F_x, x], x] /; FreeQ[a, x] && !MatchQ[F_x, (b_)*(G_x_) /; FreeQ[b, x]]`

rule 78 `Int[((a_) + (b_)*(x_))^(m_)*((c_) + (d_)*(x_))^(n_), x_Symbol] := Simp[(b*c - a*d)^n*(a + b*x)^(m + 1)/(b^(n + 1)*(m + 1))*Hypergeometric2F1[-n, m + 1, m + 2, (-d)*(a + b*x)/(b*c - a*d)], x] /; FreeQ[{a, b, c, d, m}, x] && !IntegerQ[m] && IntegerQ[n]`

rule 1098 `Int[((d_) + (e_)*(x_))^(m_)*((a_) + (b_)*(x_) + (c_)*(x_)^2)^(p_), x_Symbol] := Simp[1/c^p Int[(d + e*x)^m*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]`

Maple [F]

$$\int \frac{(ex + d)^m}{(b^2x^2 + 2abx + a^2)^3} dx$$

input `int((e*x+d)^m/(b^2*x^2+2*a*b*x+a^2)^3,x)`

output `int((e*x+d)^m/(b^2*x^2+2*a*b*x+a^2)^3,x)`

Fricas [F]

$$\int \frac{(d + ex)^m}{(a^2 + 2abx + b^2x^2)^3} dx = \int \frac{(ex + d)^m}{(b^2x^2 + 2abx + a^2)^3} dx$$

input `integrate((e*x+d)^m/(b^2*x^2+2*a*b*x+a^2)^3,x, algorithm="fricas")`

output `integral((e*x + d)^m/(b^6*x^6 + 6*a*b^5*x^5 + 15*a^2*b^4*x^4 + 20*a^3*b^3*x^3 + 15*a^4*b^2*x^2 + 6*a^5*b*x + a^6), x)`

Sympy [F]

$$\int \frac{(d + ex)^m}{(a^2 + 2abx + b^2x^2)^3} dx = \int \frac{(d + ex)^m}{(a + bx)^6} dx$$

input `integrate((e*x+d)**m/(b**2*x**2+2*a*b*x+a**2)**3,x)`

output `Integral((d + e*x)**m/(a + b*x)**6, x)`

Maxima [F]

$$\int \frac{(d + ex)^m}{(a^2 + 2abx + b^2x^2)^3} dx = \int \frac{(ex + d)^m}{(b^2x^2 + 2abx + a^2)^3} dx$$

input `integrate((e*x+d)^m/(b^2*x^2+2*a*b*x+a^2)^3,x, algorithm="maxima")`

output `integrate((e*x + d)^m/(b^2*x^2 + 2*a*b*x + a^2)^3, x)`

Giac [F]

$$\int \frac{(d + ex)^m}{(a^2 + 2abx + b^2x^2)^3} dx = \int \frac{(ex + d)^m}{(b^2x^2 + 2abx + a^2)^3} dx$$

input `integrate((e*x+d)^m/(b^2*x^2+2*a*b*x+a^2)^3,x, algorithm="giac")`

output `integrate((e*x + d)^m/(b^2*x^2 + 2*a*b*x + a^2)^3, x)`

Mupad [F(-1)]

Timed out.

$$\int \frac{(d + ex)^m}{(a^2 + 2abx + b^2x^2)^3} dx = \int \frac{(d + ex)^m}{(a^2 + 2abx + b^2x^2)^3} dx$$

input `int((d + e*x)^m/(a^2 + b^2*x^2 + 2*a*b*x)^3,x)`output `int((d + e*x)^m/(a^2 + b^2*x^2 + 2*a*b*x)^3, x)`**Reduce [F]**

$$\int \frac{(d + ex)^m}{(a^2 + 2abx + b^2x^2)^3} dx = \text{too large to display}$$

input `int((e*x+d)^m/(b^2*x^2+2*a*b*x+a^2)^3,x)`

output

```

((d + e*x)**m*d + int(((d + e*x)**m*x)/(a**7*d*e*m + a**7*e**2*m*x - 5*a**
6*b*d**2 + 6*a**6*b*d*e*m*x - 5*a**6*b*d*e*x + 6*a**6*b*e**2*m*x**2 - 30*a
**5*b**2*d**2*x + 15*a**5*b**2*d*e*m*x**2 - 30*a**5*b**2*d*e*x**2 + 15*a**
5*b**2*e**2*m*x**3 - 75*a**4*b**3*d**2*x**2 + 20*a**4*b**3*d*e*m*x**3 - 75
*a**4*b**3*d*e*x**3 + 20*a**4*b**3*e**2*m*x**4 - 100*a**3*b**4*d**2*x**3 +
15*a**3*b**4*d*e*m*x**4 - 100*a**3*b**4*d*e*x**4 + 15*a**3*b**4*e**2*m*x*
*5 - 75*a**2*b**5*d**2*x**4 + 6*a**2*b**5*d*e*m*x**5 - 75*a**2*b**5*d*e*x*
*5 + 6*a**2*b**5*e**2*m*x**6 - 30*a*b**6*d**2*x**5 + a*b**6*d*e*m*x**6 - 3
0*a*b**6*d*e*x**6 + a*b**6*e**2*m*x**7 - 5*b**7*d**2*x**6 - 5*b**7*d*e*x**
7),x)*a**7*e**3*m**2 - int(((d + e*x)**m*x)/(a**7*d*e*m + a**7*e**2*m*x -
5*a**6*b*d**2 + 6*a**6*b*d*e*m*x - 5*a**6*b*d*e*x + 6*a**6*b*e**2*m*x**2 -
30*a**5*b**2*d**2*x + 15*a**5*b**2*d*e*m*x**2 - 30*a**5*b**2*d*e*x**2 + 1
5*a**5*b**2*e**2*m*x**3 - 75*a**4*b**3*d**2*x**2 + 20*a**4*b**3*d*e*m*x**3
- 75*a**4*b**3*d*e*x**3 + 20*a**4*b**3*e**2*m*x**4 - 100*a**3*b**4*d**2*x
**3 + 15*a**3*b**4*d*e*m*x**4 - 100*a**3*b**4*d*e*x**4 + 15*a**3*b**4*e**2
*m*x**5 - 75*a**2*b**5*d**2*x**4 + 6*a**2*b**5*d*e*m*x**5 - 75*a**2*b**5*d
*e*x**5 + 6*a**2*b**5*e**2*m*x**6 - 30*a*b**6*d**2*x**5 + a*b**6*d*e*m*x**
6 - 30*a*b**6*d*e*x**6 + a*b**6*e**2*m*x**7 - 5*b**7*d**2*x**6 - 5*b**7*d*
e*x**7),x)*a**6*b*d*e**2*m**2 - 5*int(((d + e*x)**m*x)/(a**7*d*e*m + a**7*
e**2*m*x - 5*a**6*b*d**2 + 6*a**6*b*d*e*m*x - 5*a**6*b*d*e*x + 6*a**6*b...

```

3.417 $\int (d + ex)^m (a^2 + 2abx + b^2x^2)^{5/2} dx$

| | |
|---|------|
| Optimal result | 3133 |
| Mathematica [A] (verified) | 3134 |
| Rubi [A] (verified) | 3134 |
| Maple [B] (verified) | 3136 |
| Fricas [B] (verification not implemented) | 3137 |
| Sympy [F(-2)] | 3138 |
| Maxima [B] (verification not implemented) | 3139 |
| Giac [B] (verification not implemented) | 3139 |
| Mupad [F(-1)] | 3140 |
| Reduce [B] (verification not implemented) | 3141 |

Optimal result

Integrand size = 28, antiderivative size = 337

$$\begin{aligned}
 & \int (d + ex)^m (a^2 + 2abx + b^2x^2)^{5/2} dx = \\
 & - \frac{(bd - ae)^5 (d + ex)^{1+m} \sqrt{a^2 + 2abx + b^2x^2}}{e^6 (1 + m)(a + bx)} \\
 & + \frac{5b(bd - ae)^4 (d + ex)^{2+m} \sqrt{a^2 + 2abx + b^2x^2}}{e^6 (2 + m)(a + bx)} \\
 & - \frac{10b^2(bd - ae)^3 (d + ex)^{3+m} \sqrt{a^2 + 2abx + b^2x^2}}{e^6 (3 + m)(a + bx)} \\
 & + \frac{10b^3(bd - ae)^2 (d + ex)^{4+m} \sqrt{a^2 + 2abx + b^2x^2}}{e^6 (4 + m)(a + bx)} \\
 & - \frac{5b^4(bd - ae)(d + ex)^{5+m} \sqrt{a^2 + 2abx + b^2x^2}}{e^6 (5 + m)(a + bx)} \\
 & + \frac{b^5 (d + ex)^{6+m} \sqrt{a^2 + 2abx + b^2x^2}}{e^6 (6 + m)(a + bx)}
 \end{aligned}$$

output

$$\begin{aligned}
& -(-a*e+b*d)^5*(e*x+d)^{(1+m)}*((b*x+a)^2)^{(1/2)}/e^{6/(1+m)}/(b*x+a)+5*b*(-a*e+ \\
& b*d)^4*(e*x+d)^{(2+m)}*((b*x+a)^2)^{(1/2)}/e^{6/(2+m)}/(b*x+a)-10*b^2*(-a*e+b*d) \\
& ^3*(e*x+d)^{(3+m)}*((b*x+a)^2)^{(1/2)}/e^{6/(3+m)}/(b*x+a)+10*b^3*(-a*e+b*d)^2*(\\
& e*x+d)^{(4+m)}*((b*x+a)^2)^{(1/2)}/e^{6/(4+m)}/(b*x+a)-5*b^4*(-a*e+b*d)*(e*x+d)^{(\\
& 5+m)}*((b*x+a)^2)^{(1/2)}/e^{6/(5+m)}/(b*x+a)+b^5*(e*x+d)^{(6+m)}*((b*x+a)^2)^{(1 \\
& /2)}/e^{6/(6+m)}/(b*x+a)
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.23 (sec) , antiderivative size = 167, normalized size of antiderivative = 0.50

$$\int (d+ex)^m (a^2+2abx+b^2x^2)^{5/2} dx = \frac{((a+bx)^2)^{5/2} (d+ex)^{1+m} \left(-\frac{(bd-ae)^5}{1+m} + \frac{5b(bd-ae)^4(d+ex)}{2+m} - \frac{10b^2(bd-ae)^3(d+ex)^2}{3+m} + \frac{10b^3(bd-ae)^2(d+ex)^3}{4+m} - \frac{5b^4(bd-ae)(d+ex)^4}{5+m} + \frac{b^5(d+ex)^5}{6+m} \right)}{e^6(a+bx)^5}$$

input

Integrate[(d + e*x)^m*(a^2 + 2*a*b*x + b^2*x^2)^(5/2),x]

output

$$\begin{aligned}
& (((a+b*x)^2)^{(5/2)}*(d+e*x)^{(1+m)}*(-((b*d-a*e)^5/(1+m)) + (5*b*(b \\
& *d-a*e)^4*(d+e*x))/(2+m) - (10*b^2*(b*d-a*e)^3*(d+e*x)^2)/(3+m) \\
&) + (10*b^3*(b*d-a*e)^2*(d+e*x)^3)/(4+m) - (5*b^4*(b*d-a*e)*(d+e \\
& *x)^4)/(5+m) + (b^5*(d+e*x)^5)/(6+m))/e^{6*(a+b*x)^5}
\end{aligned}$$

Rubi [A] (verified)Time = 0.63 (sec) , antiderivative size = 203, normalized size of antiderivative = 0.60, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {1102, 27, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a^2+2abx+b^2x^2)^{5/2} (d+ex)^m dx$$

↓ 1102

$$\frac{\sqrt{a^2 + 2abx + b^2x^2} \int b^5(a + bx)^5(d + ex)^m dx}{b^5(a + bx)}$$

↓ 27

$$\frac{\sqrt{a^2 + 2abx + b^2x^2} \int (a + bx)^5(d + ex)^m dx}{a + bx}$$

↓ 53

$$\frac{\sqrt{a^2 + 2abx + b^2x^2} \int \left(\frac{(ae-bd)^5(d+ex)^m}{e^5} + \frac{5b(bd-ae)^4(d+ex)^{m+1}}{e^5} - \frac{10b^2(bd-ae)^3(d+ex)^{m+2}}{e^5} + \frac{10b^3(bd-ae)^2(d+ex)^{m+3}}{e^5} - \frac{5b^4(bd-ae)(d+ex)^{m+4}}{e^5} + \frac{b^5(d+ex)^{m+5}}{e^5} \right) dx}{a + bx}$$

↓ 2009

$$\frac{\sqrt{a^2 + 2abx + b^2x^2} \left(-\frac{5b^4(bd-ae)(d+ex)^{m+5}}{e^6(m+5)} + \frac{10b^3(bd-ae)^2(d+ex)^{m+4}}{e^6(m+4)} - \frac{10b^2(bd-ae)^3(d+ex)^{m+3}}{e^6(m+3)} - \frac{(bd-ae)^5(d+ex)^{m+1}}{e^6(m+1)} + \frac{b^5(d+ex)^{m+5}}{e^6(m+5)} \right)}{a + bx}$$

input `Int[(d + e*x)^m*(a^2 + 2*a*b*x + b^2*x^2)^(5/2),x]`

output `(Sqrt[a^2 + 2*a*b*x + b^2*x^2]*(-(((b*d - a*e)^5*(d + e*x)^(1 + m))/(e^6*(1 + m))) + (5*b*(b*d - a*e)^4*(d + e*x)^(2 + m))/(e^6*(2 + m)) - (10*b^2*(b*d - a*e)^3*(d + e*x)^(3 + m))/(e^6*(3 + m)) + (10*b^3*(b*d - a*e)^2*(d + e*x)^(4 + m))/(e^6*(4 + m)) - (5*b^4*(b*d - a*e)*(d + e*x)^(5 + m))/(e^6*(5 + m)) + (b^5*(d + e*x)^(6 + m))/(e^6*(6 + m))))/(a + b*x)`

Defintions of rubi rules used

rule 27 `Int[(a_)*(Fx_), x_Symbol] := Simp[a Int[Fx, x], x] /; FreeQ[a, x] && !MatchQ[Fx, (b_)*(Gx_)] /; FreeQ[b, x]`

rule 53 `Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] := Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && IGtQ[m, 0] && (!IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])`

rule 1102

```
Int[((d_.) + (e_.)*(x_)^(m_.))*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol]
:> Simp[(a + b*x + c*x^2)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x)^(2*FracPart[p]))
Int[(d + e*x)^m*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d, e, m, p}, x]
&& EqQ[b^2 - 4*a*c, 0]
```

rule 2009

```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1360 vs. $2(271) = 542$.

Time = 1.24 (sec) , antiderivative size = 1361, normalized size of antiderivative = 4.04

| method | result | size |
|---------|---------------------------------|------|
| gospers | Expression too large to display | 1361 |
| orering | Expression too large to display | 1373 |
| risch | Expression too large to display | 1745 |

input

```
int((e*x+d)^m*(b^2*x^2+2*a*b*x+a^2)^(5/2), x, method=_RETURNVERBOSE)
```

output

```

1/e^6*(e*x+d)^(1+m)/(b*x+a)^5*((b*x+a)^2)^(5/2)/(m^6+21*m^5+175*m^4+735*m^
3+1624*m^2+1764*m+720)*(b^5*e^5*m^5*x^5+5*a*b^4*e^5*m^5*x^4+15*b^5*e^5*m^4
*x^5+10*a^2*b^3*e^5*m^5*x^3+80*a*b^4*e^5*m^4*x^4-5*b^5*d*e^4*m^4*x^4+85*b^
5*e^5*m^3*x^5+10*a^3*b^2*e^5*m^5*x^2+170*a^2*b^3*e^5*m^4*x^3-20*a*b^4*d*e^
4*m^4*x^3+475*a*b^4*e^5*m^3*x^4-50*b^5*d*e^4*m^3*x^4+225*b^5*e^5*m^2*x^5+5
*a^4*b*e^5*m^5*x+180*a^3*b^2*e^5*m^4*x^2-30*a^2*b^3*d*e^4*m^4*x^2+1070*a^2
*b^3*e^5*m^3*x^3-240*a*b^4*d*e^4*m^3*x^3+1300*a*b^4*e^5*m^2*x^4+20*b^5*d^2
*e^3*m^3*x^3-175*b^5*d*e^4*m^2*x^4+274*b^5*e^5*m*x^5+a^5*e^5*m^5+95*a^4*b*
e^5*m^4*x-20*a^3*b^2*d*e^4*m^4*x+1210*a^3*b^2*e^5*m^3*x^2-420*a^2*b^3*d*e^
4*m^3*x^2+3070*a^2*b^3*e^5*m^2*x^3+60*a*b^4*d^2*e^3*m^3*x^2-940*a*b^4*d*e^
4*m^2*x^3+1620*a*b^4*e^5*m*x^4+120*b^5*d^2*e^3*m^2*x^3-250*b^5*d*e^4*m*x^4
+120*b^5*e^5*x^5+20*a^5*e^5*m^4-5*a^4*b*d*e^4*m^4+685*a^4*b*e^5*m^3*x-320*
a^3*b^2*d*e^4*m^3*x+3720*a^3*b^2*e^5*m^2*x^2+60*a^2*b^3*d^2*e^3*m^3*x-1950
*a^2*b^3*d*e^4*m^2*x^2+3960*a^2*b^3*e^5*m*x^3+540*a*b^4*d^2*e^3*m^2*x^2-14
40*a*b^4*d*e^4*m*x^3+720*a*b^4*e^5*x^4-60*b^5*d^3*e^2*m^2*x^2+220*b^5*d^2*
e^3*m*x^3-120*b^5*d*e^4*x^4+155*a^5*e^5*m^3-90*a^4*b*d*e^4*m^3+2305*a^4*b*
e^5*m^2*x+20*a^3*b^2*d^2*e^3*m^3-1780*a^3*b^2*d*e^4*m^2*x+5080*a^3*b^2*e^5
*m*x^2+720*a^2*b^3*d^2*e^3*m^2*x-3360*a^2*b^3*d*e^4*m*x^2+1800*a^2*b^3*e^5
*x^3-120*a*b^4*d^3*e^2*m^2*x+1200*a*b^4*d^2*e^3*m*x^2-720*a*b^4*d*e^4*x^3-
180*b^5*d^3*e^2*m*x^2+120*b^5*d^2*e^3*x^3+580*a^5*e^5*m^2-595*a^4*b*d*e...

```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1460 vs. $2(271) = 542$.

Time = 0.09 (sec) , antiderivative size = 1460, normalized size of antiderivative = 4.33

$$\int (d + ex)^m (a^2 + 2abx + b^2x^2)^{5/2} dx = \text{Too large to display}$$

input

```
integrate((e*x+d)^m*(b^2*x^2+2*a*b*x+a^2)^(5/2),x, algorithm="fricas")
```

output

```
(a^5*d*e^5*m^5 - 120*b^5*d^6 + 720*a*b^4*d^5*e - 1800*a^2*b^3*d^4*e^2 + 24
00*a^3*b^2*d^3*e^3 - 1800*a^4*b*d^2*e^4 + 720*a^5*d*e^5 + (b^5*e^6*m^5 + 1
5*b^5*e^6*m^4 + 85*b^5*e^6*m^3 + 225*b^5*e^6*m^2 + 274*b^5*e^6*m + 120*b^5
*e^6)*x^6 + (720*a*b^4*e^6 + (b^5*d*e^5 + 5*a*b^4*e^6)*m^5 + 10*(b^5*d*e^5
+ 8*a*b^4*e^6)*m^4 + 5*(7*b^5*d*e^5 + 95*a*b^4*e^6)*m^3 + 50*(b^5*d*e^5 +
26*a*b^4*e^6)*m^2 + 12*(2*b^5*d*e^5 + 135*a*b^4*e^6)*m)*x^5 - 5*(a^4*b*d^
2*e^4 - 4*a^5*d*e^5)*m^4 + 5*(360*a^2*b^3*e^6 + (a*b^4*d*e^5 + 2*a^2*b^3*e
^6)*m^5 - (b^5*d^2*e^4 - 12*a*b^4*d*e^5 - 34*a^2*b^3*e^6)*m^4 - (6*b^5*d^2
*e^4 - 47*a*b^4*d*e^5 - 214*a^2*b^3*e^6)*m^3 - (11*b^5*d^2*e^4 - 72*a*b^4*
d*e^5 - 614*a^2*b^3*e^6)*m^2 - 6*(b^5*d^2*e^4 - 6*a*b^4*d*e^5 - 132*a^2*b^
3*e^6)*m)*x^4 + 5*(4*a^3*b^2*d^3*e^3 - 18*a^4*b*d^2*e^4 + 31*a^5*d*e^5)*m^
3 + 10*(240*a^3*b^2*e^6 + (a^2*b^3*d*e^5 + a^3*b^2*e^6)*m^5 - 2*(a*b^4*d^2
*e^4 - 7*a^2*b^3*d*e^5 - 9*a^3*b^2*e^6)*m^4 + (2*b^5*d^3*e^3 - 18*a*b^4*d^
2*e^4 + 65*a^2*b^3*d*e^5 + 121*a^3*b^2*e^6)*m^3 + 2*(3*b^5*d^3*e^3 - 20*a*
b^4*d^2*e^4 + 56*a^2*b^3*d*e^5 + 186*a^3*b^2*e^6)*m^2 + 4*(b^5*d^3*e^3 - 6
*a*b^4*d^2*e^4 + 15*a^2*b^3*d*e^5 + 127*a^3*b^2*e^6)*m)*x^3 - 5*(12*a^2*b^
3*d^4*e^2 - 60*a^3*b^2*d^3*e^3 + 119*a^4*b*d^2*e^4 - 116*a^5*d*e^5)*m^2 +
5*(360*a^4*b*e^6 + (2*a^3*b^2*d*e^5 + a^4*b*e^6)*m^5 - (6*a^2*b^3*d^2*e^4
- 32*a^3*b^2*d*e^5 - 19*a^4*b*e^6)*m^4 + (12*a*b^4*d^3*e^3 - 72*a^2*b^3*d^
2*e^4 + 178*a^3*b^2*d*e^5 + 137*a^4*b*e^6)*m^3 - (12*b^5*d^4*e^2 - 84*a...
```

Sympy [F(-2)]

Exception generated.

$$\int (d + ex)^m (a^2 + 2abx + b^2x^2)^{5/2} dx = \text{Exception raised: HeuristicGCDFailed}$$

input

```
integrate((e*x+d)**m*(b**2*x**2+2*a*b*x+a**2)**(5/2),x)
```

output

```
Exception raised: HeuristicGCDFailed >> no luck
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 792 vs. $2(271) = 542$.

Time = 0.07 (sec) , antiderivative size = 792, normalized size of antiderivative = 2.35

$$\int (d + ex)^m (a^2 + 2abx + b^2x^2)^{5/2} dx = \text{Too large to display}$$

input `integrate((e*x+d)^m*(b^2*x^2+2*a*b*x+a^2)^(5/2),x, algorithm="maxima")`

output

```
((m^5 + 15*m^4 + 85*m^3 + 225*m^2 + 274*m + 120)*b^5*e^6*x^6 - 60*(m^2 + 1
1*m + 30)*a^2*b^3*d^4*e^2 + 20*(m^3 + 15*m^2 + 74*m + 120)*a^3*b^2*d^3*e^3
- 5*(m^4 + 18*m^3 + 119*m^2 + 342*m + 360)*a^4*b*d^2*e^4 + (m^5 + 20*m^4
+ 155*m^3 + 580*m^2 + 1044*m + 720)*a^5*d*e^5 + 120*a*b^4*d^5*e*(m + 6) -
120*b^5*d^6 + ((m^5 + 10*m^4 + 35*m^3 + 50*m^2 + 24*m)*b^5*d*e^5 + 5*(m^5
+ 16*m^4 + 95*m^3 + 260*m^2 + 324*m + 144)*a*b^4*e^6)*x^5 - 5*((m^4 + 6*m^
3 + 11*m^2 + 6*m)*b^5*d^2*e^4 - (m^5 + 12*m^4 + 47*m^3 + 72*m^2 + 36*m)*a*
b^4*d*e^5 - 2*(m^5 + 17*m^4 + 107*m^3 + 307*m^2 + 396*m + 180)*a^2*b^3*e^6
)*x^4 + 10*(2*(m^3 + 3*m^2 + 2*m)*b^5*d^3*e^3 - 2*(m^4 + 9*m^3 + 20*m^2 +
12*m)*a*b^4*d^2*e^4 + (m^5 + 14*m^4 + 65*m^3 + 112*m^2 + 60*m)*a^2*b^3*d*e
^5 + (m^5 + 18*m^4 + 121*m^3 + 372*m^2 + 508*m + 240)*a^3*b^2*e^6)*x^3 - 5
*(12*(m^2 + m)*b^5*d^4*e^2 - 12*(m^3 + 7*m^2 + 6*m)*a*b^4*d^3*e^3 + 6*(m^4
+ 12*m^3 + 41*m^2 + 30*m)*a^2*b^3*d^2*e^4 - 2*(m^5 + 16*m^4 + 89*m^3 + 19
4*m^2 + 120*m)*a^3*b^2*d*e^5 - (m^5 + 19*m^4 + 137*m^3 + 461*m^2 + 702*m +
360)*a^4*b*e^6)*x^2 - (120*(m^2 + 6*m)*a*b^4*d^4*e^2 - 60*(m^3 + 11*m^2 +
30*m)*a^2*b^3*d^3*e^3 + 20*(m^4 + 15*m^3 + 74*m^2 + 120*m)*a^3*b^2*d^2*e^
4 - 5*(m^5 + 18*m^4 + 119*m^3 + 342*m^2 + 360*m)*a^4*b*d*e^5 - (m^5 + 20*m
^4 + 155*m^3 + 580*m^2 + 1044*m + 720)*a^5*e^6 - 120*b^5*d^5*e*m)*x)*(e*x
+ d)^m/((m^6 + 21*m^5 + 175*m^4 + 735*m^3 + 1624*m^2 + 1764*m + 720)*e^6)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 3197 vs. $2(271) = 542$.

Time = 0.20 (sec) , antiderivative size = 3197, normalized size of antiderivative = 9.49

$$\int (d + ex)^m (a^2 + 2abx + b^2x^2)^{5/2} dx = \text{Too large to display}$$

input `integrate((e*x+d)^m*(b^2*x^2+2*a*b*x+a^2)^(5/2),x, algorithm="giac")`

output `((e*x + d)^m*b^5*e^6*m^5*x^6*sgn(b*x + a) + (e*x + d)^m*b^5*d*e^5*m^5*x^5*sgn(b*x + a) + 5*(e*x + d)^m*a*b^4*e^6*m^5*x^5*sgn(b*x + a) + 15*(e*x + d)^m*b^5*e^6*m^4*x^6*sgn(b*x + a) + 5*(e*x + d)^m*a*b^4*d*e^5*m^5*x^4*sgn(b*x + a) + 10*(e*x + d)^m*a^2*b^3*e^6*m^5*x^4*sgn(b*x + a) + 10*(e*x + d)^m*b^5*d*e^5*m^4*x^5*sgn(b*x + a) + 80*(e*x + d)^m*a*b^4*e^6*m^4*x^5*sgn(b*x + a) + 85*(e*x + d)^m*b^5*e^6*m^3*x^6*sgn(b*x + a) + 10*(e*x + d)^m*a^2*b^3*d*e^5*m^5*x^3*sgn(b*x + a) + 10*(e*x + d)^m*a^3*b^2*e^6*m^5*x^3*sgn(b*x + a) - 5*(e*x + d)^m*b^5*d^2*e^4*m^4*x^4*sgn(b*x + a) + 60*(e*x + d)^m*a*b^4*d*e^5*m^4*x^4*sgn(b*x + a) + 170*(e*x + d)^m*a^2*b^3*e^6*m^4*x^4*sgn(b*x + a) + 35*(e*x + d)^m*b^5*d*e^5*m^3*x^5*sgn(b*x + a) + 475*(e*x + d)^m*a*b^4*e^6*m^3*x^5*sgn(b*x + a) + 225*(e*x + d)^m*b^5*e^6*m^2*x^6*sgn(b*x + a) + 10*(e*x + d)^m*a^3*b^2*d*e^5*m^5*x^2*sgn(b*x + a) + 5*(e*x + d)^m*a^4*b*e^6*m^5*x^2*sgn(b*x + a) - 20*(e*x + d)^m*a*b^4*d^2*e^4*m^4*x^3*sgn(b*x + a) + 140*(e*x + d)^m*a^2*b^3*d*e^5*m^4*x^3*sgn(b*x + a) + 180*(e*x + d)^m*a^3*b^2*e^6*m^4*x^3*sgn(b*x + a) - 30*(e*x + d)^m*b^5*d^2*e^4*m^3*x^4*sgn(b*x + a) + 235*(e*x + d)^m*a*b^4*d*e^5*m^3*x^4*sgn(b*x + a) + 1070*(e*x + d)^m*a^2*b^3*e^6*m^3*x^4*sgn(b*x + a) + 50*(e*x + d)^m*b^5*d*e^5*m^2*x^5*sgn(b*x + a) + 1300*(e*x + d)^m*a*b^4*e^6*m^2*x^5*sgn(b*x + a) + 274*(e*x + d)^m*b^5*e^6*m*x^6*sgn(b*x + a) + 5*(e*x + d)^m*a^4*b*d*e^5*m^5*x*sgn(b*x + a) + (e*x + d)^m*a^5*e^6*m^5*x*sgn(b*x + a) - 30*(e*x + d)^m*a^2*...`

Mupad [F(-1)]

Timed out.

$$\int (d + ex)^m (a^2 + 2abx + b^2x^2)^{5/2} dx = \int (d + ex)^m (a^2 + 2abx + b^2x^2)^{5/2} dx$$

input `int((d + e*x)^m*(a^2 + b^2*x^2 + 2*a*b*x)^(5/2),x)`

output `int((d + e*x)^m*(a^2 + b^2*x^2 + 2*a*b*x)^(5/2), x)`

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 1728, normalized size of antiderivative = 5.13

$$\int (d + ex)^m (a^2 + 2abx + b^2x^2)^{5/2} dx = \text{Too large to display}$$

input `int((e*x+d)^m*(b^2*x^2+2*a*b*x+a^2)^(5/2),x)`

output

```
((d + e*x)**m*(a**5*d*e**5*m**5 + 20*a**5*d*e**5*m**4 + 155*a**5*d*e**5*m**3 + 580*a**5*d*e**5*m**2 + 1044*a**5*d*e**5*m + 720*a**5*d*e**5 + a**5*e**6*m**5*x + 20*a**5*e**6*m**4*x + 155*a**5*e**6*m**3*x + 580*a**5*e**6*m**2*x + 1044*a**5*e**6*m*x + 720*a**5*e**6 - 5*a**4*b*d**2*e**4*m**4 - 90*a**4*b*d**2*e**4*m**3 - 595*a**4*b*d**2*e**4*m**2 - 1710*a**4*b*d**2*e**4*m - 1800*a**4*b*d**2*e**4 + 5*a**4*b*d*e**5*m**5*x + 90*a**4*b*d*e**5*m**4*x + 595*a**4*b*d*e**5*m**3*x + 1710*a**4*b*d*e**5*m**2*x + 1800*a**4*b*d*e**5*m*x + 5*a**4*b*e**6*m**5*x**2 + 95*a**4*b*e**6*m**4*x**2 + 685*a**4*b*e**6*m**3*x**2 + 2305*a**4*b*e**6*m**2*x**2 + 3510*a**4*b*e**6*m*x**2 + 1800*a**4*b*e**6*x**2 + 20*a**3*b**2*d**3*e**3*m**3 + 300*a**3*b**2*d**3*e**3*m**2 + 1480*a**3*b**2*d**3*e**3*m + 2400*a**3*b**2*d**3*e**3 - 20*a**3*b**2*d**2*e**4*m**4*x - 300*a**3*b**2*d**2*e**4*m**3*x - 1480*a**3*b**2*d**2*e**4*m**2*x - 2400*a**3*b**2*d**2*e**4*m*x + 10*a**3*b**2*d*e**5*m**5*x**2 + 160*a**3*b**2*d*e**5*m**4*x**2 + 890*a**3*b**2*d*e**5*m**3*x**2 + 1940*a**3*b**2*d*e**5*m**2*x**2 + 1200*a**3*b**2*d*e**5*m*x**2 + 10*a**3*b**2*e**6*m**5*x**3 + 180*a**3*b**2*e**6*m**4*x**3 + 1210*a**3*b**2*e**6*m**3*x**3 + 3720*a**3*b**2*e**6*m**2*x**3 + 5080*a**3*b**2*e**6*m*x**3 + 2400*a**3*b**2*e**6*x**3 - 60*a**2*b**3*d**4*e**2*m**2 - 660*a**2*b**3*d**4*e**2*m - 1800*a**2*b**3*d**4*e**2 + 60*a**2*b**3*d**3*e**3*m**3*x + 660*a**2*b**3*d**3*e**3*m**2*x + 1800*a**2*b**3*d**3*e**3*m*x - 30*a**2*b**3*d**...
```

3.418 $\int (d + ex)^m (a^2 + 2abx + b^2x^2)^{3/2} dx$

| | |
|---|------|
| Optimal result | 3142 |
| Mathematica [A] (verified) | 3143 |
| Rubi [A] (verified) | 3143 |
| Maple [B] (verified) | 3145 |
| Fricas [B] (verification not implemented) | 3145 |
| Sympy [F] | 3146 |
| Maxima [A] (verification not implemented) | 3146 |
| Giac [B] (verification not implemented) | 3147 |
| Mupad [F(-1)] | 3148 |
| Reduce [B] (verification not implemented) | 3149 |

Optimal result

Integrand size = 28, antiderivative size = 219

$$\int (d + ex)^m (a^2 + 2abx + b^2x^2)^{3/2} dx =$$

$$\frac{(bd - ae)^3 (d + ex)^{1+m} \sqrt{a^2 + 2abx + b^2x^2}}{e^4 (1 + m) (a + bx)}$$

$$+ \frac{3b (bd - ae)^2 (d + ex)^{2+m} \sqrt{a^2 + 2abx + b^2x^2}}{e^4 (2 + m) (a + bx)}$$

$$- \frac{3b^2 (bd - ae) (d + ex)^{3+m} \sqrt{a^2 + 2abx + b^2x^2}}{e^4 (3 + m) (a + bx)} + \frac{b^3 (d + ex)^{4+m} \sqrt{a^2 + 2abx + b^2x^2}}{e^4 (4 + m) (a + bx)}$$

output

```

-(-a*e+b*d)^3*(e*x+d)^(1+m)*((b*x+a)^2)^(1/2)/e^4/(1+m)/(b*x+a)+3*b*(-a*e+
b*d)^2*(e*x+d)^(2+m)*((b*x+a)^2)^(1/2)/e^4/(2+m)/(b*x+a)-3*b^2*(-a*e+b*d)*
(e*x+d)^(3+m)*((b*x+a)^2)^(1/2)/e^4/(3+m)/(b*x+a)+b^3*(e*x+d)^(4+m)*((b*x+
a)^2)^(1/2)/e^4/(4+m)/(b*x+a)
    
```

Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 113, normalized size of antiderivative = 0.52

$$\int (d + ex)^m (a^2 + 2abx + b^2x^2)^{3/2} dx = \frac{((a + bx)^2)^{3/2} (d + ex)^{1+m} \left(-\frac{(bd-ae)^3}{1+m} + \frac{3b(bd-ae)^2(d+ex)}{2+m} - \frac{3b^2(bd-ae)(d+ex)^2}{3+m} + \frac{b^3(d+ex)^3}{4+m} \right)}{e^4(a + bx)^3}$$

input

```
Integrate[(d + e*x)^m*(a^2 + 2*a*b*x + b^2*x^2)^(3/2),x]
```

output

```
((a + b*x)^2)^(3/2)*(d + e*x)^(1 + m)*(-((b*d - a*e)^3/(1 + m)) + (3*b*(b*d - a*e)^2*(d + e*x))/(2 + m) - (3*b^2*(b*d - a*e)*(d + e*x)^2)/(3 + m) + (b^3*(d + e*x)^3)/(4 + m))/(e^4*(a + b*x)^3)
```

Rubi [A] (verified)

Time = 0.49 (sec) , antiderivative size = 139, normalized size of antiderivative = 0.63, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {1102, 27, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\begin{aligned} & \int (a^2 + 2abx + b^2x^2)^{3/2} (d + ex)^m dx \\ & \quad \downarrow \text{1102} \\ & \frac{\sqrt{a^2 + 2abx + b^2x^2} \int b^3(a + bx)^3(d + ex)^m dx}{b^3(a + bx)} \\ & \quad \downarrow \text{27} \\ & \frac{\sqrt{a^2 + 2abx + b^2x^2} \int (a + bx)^3(d + ex)^m dx}{a + bx} \\ & \quad \downarrow \text{53} \end{aligned}$$

$$\frac{\sqrt{a^2 + 2abx + b^2x^2} \int \left(\frac{(ae-bd)^3(d+ex)^m}{e^3} + \frac{3b(bd-ae)^2(d+ex)^{m+1}}{e^3} - \frac{3b^2(bd-ae)(d+ex)^{m+2}}{e^3} + \frac{b^3(d+ex)^{m+3}}{e^3} \right) dx}{a + bx}$$

↓ 2009

$$\frac{\sqrt{a^2 + 2abx + b^2x^2} \left(-\frac{3b^2(bd-ae)(d+ex)^{m+3}}{e^4(m+3)} - \frac{(bd-ae)^3(d+ex)^{m+1}}{e^4(m+1)} + \frac{3b(bd-ae)^2(d+ex)^{m+2}}{e^4(m+2)} + \frac{b^3(d+ex)^{m+4}}{e^4(m+4)} \right)}{a + bx}$$

input `Int[(d + e*x)^m*(a^2 + 2*a*b*x + b^2*x^2)^(3/2),x]`

output `(Sqrt[a^2 + 2*a*b*x + b^2*x^2]*(-(((b*d - a*e)^3*(d + e*x)^(1 + m))/(e^4*(1 + m))) + (3*b*(b*d - a*e)^2*(d + e*x)^(2 + m))/(e^4*(2 + m)) - (3*b^2*(b*d - a*e)*(d + e*x)^(3 + m))/(e^4*(3 + m)) + (b^3*(d + e*x)^(4 + m))/(e^4*(4 + m))))/(a + b*x)`

Defintions of rubi rules used

rule 27 `Int[(a_)*(F_x_), x_Symbol] := Simp[a Int[F_x, x], x] /; FreeQ[a, x] && !MatchQ[F_x, (b_)*(G_x_) /; FreeQ[b, x]]`

rule 53 `Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] := Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && IGtQ[m, 0] && (!IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])`

rule 1102 `Int[((d_.) + (e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] := Simp[(a + b*x + c*x^2)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x)^(2*FracPart[p])) Int[(d + e*x)^m*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && EqQ[b^2 - 4*a*c, 0]`

rule 2009 `Int[u_, x_Symbol] := Simp[IntSum[u, x], x] /; SumQ[u]`

output

```
(a^3*d*e^3*m^3 - 6*b^3*d^4 + 24*a*b^2*d^3*e - 36*a^2*b*d^2*e^2 + 24*a^3*d*
e^3 + (b^3*e^4*m^3 + 6*b^3*e^4*m^2 + 11*b^3*e^4*m + 6*b^3*e^4)*x^4 + (24*a
*b^2*e^4 + (b^3*d*e^3 + 3*a*b^2*e^4)*m^3 + 3*(b^3*d*e^3 + 7*a*b^2*e^4)*m^2
+ 2*(b^3*d*e^3 + 21*a*b^2*e^4)*m)*x^3 - 3*(a^2*b*d^2*e^2 - 3*a^3*d*e^3)*m
^2 + 3*(12*a^2*b*e^4 + (a*b^2*d*e^3 + a^2*b*e^4)*m^3 - (b^3*d^2*e^2 - 5*a*
b^2*d*e^3 - 8*a^2*b*e^4)*m^2 - (b^3*d^2*e^2 - 4*a*b^2*d*e^3 - 19*a^2*b*e^4
)*m)*x^2 + (6*a*b^2*d^3*e - 21*a^2*b*d^2*e^2 + 26*a^3*d*e^3)*m + (24*a^3*e
^4 + (3*a^2*b*d*e^3 + a^3*e^4)*m^3 - 3*(2*a*b^2*d^2*e^2 - 7*a^2*b*d*e^3 -
3*a^3*e^4)*m^2 + 2*(3*b^3*d^3*e - 12*a*b^2*d^2*e^2 + 18*a^2*b*d*e^3 + 13*a
^3*e^4)*m)*x)*(e*x + d)^m/(e^4*m^4 + 10*e^4*m^3 + 35*e^4*m^2 + 50*e^4*m +
24*e^4)
```

Sympy [F]

$$\int (d + ex)^m (a^2 + 2abx + b^2x^2)^{3/2} dx = \int (d + ex)^m ((a + bx)^2)^{3/2} dx$$

input

```
integrate((e*x+d)**m*(b**2*x**2+2*a*b*x+a**2)**(3/2),x)
```

output

```
Integral((d + e*x)**m*((a + b*x)**2)**(3/2), x)
```

Maxima [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 305, normalized size of antiderivative = 1.39

$$\int (d + ex)^m (a^2 + 2abx + b^2x^2)^{3/2} dx = \frac{((m^3 + 6m^2 + 11m + 6)b^3e^4x^4 - 3(m^2 + 7m + 12)a^2bd^2e^2 + (m^3 + 9m^2 + 26m + 24)a^3d^2e^2)}{(d + ex)^{m+1}}$$

input

```
integrate((e*x+d)^m*(b^2*x^2+2*a*b*x+a^2)^(3/2),x, algorithm="maxima")
```

output

```
((m^3 + 6*m^2 + 11*m + 6)*b^3*e^4*x^4 - 3*(m^2 + 7*m + 12)*a^2*b*d^2*e^2 +
(m^3 + 9*m^2 + 26*m + 24)*a^3*d*e^3 + 6*a*b^2*d^3*e*(m + 4) - 6*b^3*d^4 +
((m^3 + 3*m^2 + 2*m)*b^3*d*e^3 + 3*(m^3 + 7*m^2 + 14*m + 8)*a*b^2*e^4)*x^
3 - 3*((m^2 + m)*b^3*d^2*e^2 - (m^3 + 5*m^2 + 4*m)*a*b^2*d*e^3 - (m^3 + 8*
m^2 + 19*m + 12)*a^2*b*e^4)*x^2 - (6*(m^2 + 4*m)*a*b^2*d^2*e^2 - 3*(m^3 +
7*m^2 + 12*m)*a^2*b*d*e^3 - (m^3 + 9*m^2 + 26*m + 24)*a^3*e^4 - 6*b^3*d^3*
e*m)*x)*(e*x + d)^m/((m^4 + 10*m^3 + 35*m^2 + 50*m + 24)*e^4)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1073 vs. $2(175) = 350$.

Time = 0.16 (sec) , antiderivative size = 1073, normalized size of antiderivative = 4.90

$$\int (d + ex)^m (a^2 + 2abx + b^2x^2)^{3/2} dx = \text{Too large to display}$$

input

```
integrate((e*x+d)^m*(b^2*x^2+2*a*b*x+a^2)^(3/2),x, algorithm="giac")
```


output

```

((e*x + d)^m*b^3*e^4*m^3*x^4*sgn(b*x + a) + (e*x + d)^m*b^3*d*e^3*m^3*x^3*
sgn(b*x + a) + 3*(e*x + d)^m*a*b^2*e^4*m^3*x^3*sgn(b*x + a) + 6*(e*x + d)^
m*b^3*e^4*m^2*x^4*sgn(b*x + a) + 3*(e*x + d)^m*a*b^2*d*e^3*m^3*x^2*sgn(b*x
+ a) + 3*(e*x + d)^m*a^2*b*e^4*m^3*x^2*sgn(b*x + a) + 3*(e*x + d)^m*b^3*d
*e^3*m^2*x^3*sgn(b*x + a) + 21*(e*x + d)^m*a*b^2*e^4*m^2*x^3*sgn(b*x + a)
+ 11*(e*x + d)^m*b^3*e^4*m*x^4*sgn(b*x + a) + 3*(e*x + d)^m*a^2*b*d*e^3*m^
3*x*sgn(b*x + a) + (e*x + d)^m*a^3*e^4*m^3*x*sgn(b*x + a) - 3*(e*x + d)^m*
b^3*d^2*e^2*m^2*x^2*sgn(b*x + a) + 15*(e*x + d)^m*a*b^2*d*e^3*m^2*x^2*sgn(
b*x + a) + 24*(e*x + d)^m*a^2*b*e^4*m^2*x^2*sgn(b*x + a) + 2*(e*x + d)^m*b
^3*d*e^3*m*x^3*sgn(b*x + a) + 42*(e*x + d)^m*a*b^2*e^4*m*x^3*sgn(b*x + a)
+ 6*(e*x + d)^m*b^3*e^4*x^4*sgn(b*x + a) + (e*x + d)^m*a^3*d*e^3*m^3*sgn(b
*x + a) - 6*(e*x + d)^m*a*b^2*d^2*e^2*m^2*x*sgn(b*x + a) + 21*(e*x + d)^m*
a^2*b*d*e^3*m^2*x*sgn(b*x + a) + 9*(e*x + d)^m*a^3*e^4*m^2*x*sgn(b*x + a)
- 3*(e*x + d)^m*b^3*d^2*e^2*m*x^2*sgn(b*x + a) + 12*(e*x + d)^m*a*b^2*d*e^
3*m*x^2*sgn(b*x + a) + 57*(e*x + d)^m*a^2*b*e^4*m*x^2*sgn(b*x + a) + 24*(e
*x + d)^m*a*b^2*e^4*x^3*sgn(b*x + a) - 3*(e*x + d)^m*a^2*b*d^2*e^2*m^2*sgn
(b*x + a) + 9*(e*x + d)^m*a^3*d*e^3*m^2*sgn(b*x + a) + 6*(e*x + d)^m*b^3*d
^3*e*m*x*sgn(b*x + a) - 24*(e*x + d)^m*a*b^2*d^2*e^2*m*x*sgn(b*x + a) + 36
*(e*x + d)^m*a^2*b*d*e^3*m*x*sgn(b*x + a) + 26*(e*x + d)^m*a^3*e^4*m*x*sgn
(b*x + a) + 36*(e*x + d)^m*a^2*b*e^4*x^2*sgn(b*x + a) + 6*(e*x + d)^m*a...

```

Mupad [F(-1)]

Timed out.

$$\int (d + ex)^m (a^2 + 2abx + b^2x^2)^{3/2} dx = \int (d + ex)^m (a^2 + 2abx + b^2x^2)^{3/2} dx$$

input

```
int((d + e*x)^m*(a^2 + b^2*x^2 + 2*a*b*x)^(3/2),x)
```

output

```
int((d + e*x)^m*(a^2 + b^2*x^2 + 2*a*b*x)^(3/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 546, normalized size of antiderivative = 2.49

$$\int (d + ex)^m (a^2 + 2abx + b^2x^2)^{3/2} dx = \frac{(ex + d)^m (b^3e^4m^3x^4 + 3ab^2e^4m^3x^3 + b^3de^3m^3x^3 + 6b^3e^4m^2x^4 + 3a^2be^4m^3x^2 + 3ab^2de^3m^3x^2 + 3a^2be^4m^2x^4 + 3ab^2de^3m^3x^2 + 3a^2be^4m^2x^4 + 3ab^2de^3m^3x^2)}{(ex + d)^m (b^3e^4m^3x^4 + 3ab^2e^4m^3x^3 + b^3de^3m^3x^3 + 6b^3e^4m^2x^4 + 3a^2be^4m^3x^2 + 3ab^2de^3m^3x^2 + 3a^2be^4m^2x^4 + 3ab^2de^3m^3x^2 + 3a^2be^4m^2x^4 + 3ab^2de^3m^3x^2)}$$

input

```
int((e*x+d)^m*(b^2*x^2+2*a*b*x+a^2)^(3/2),x)
```

output

```
((d + e*x)**m*(a**3*d*e**3*m**3 + 9*a**3*d*e**3*m**2 + 26*a**3*d*e**3*m + 24*a**3*d*e**3 + a**3*e**4*m**3*x + 9*a**3*e**4*m**2*x + 26*a**3*e**4*m*x + 24*a**3*e**4*x - 3*a**2*b*d**2*e**2*m**2 - 21*a**2*b*d**2*e**2*m - 36*a**2*b*d**2*e**2 + 3*a**2*b*d*e**3*m**3*x + 21*a**2*b*d*e**3*m**2*x + 36*a**2*b*d*e**3*m*x + 3*a**2*b*e**4*m**3*x**2 + 24*a**2*b*e**4*m**2*x**2 + 57*a**2*b*e**4*m*x**2 + 36*a**2*b*e**4*x**2 + 6*a*b**2*d**3*e*m + 24*a*b**2*d**3*e - 6*a*b**2*d**2*e**2*m**2*x - 24*a*b**2*d**2*e**2*m*x + 3*a*b**2*d*e**3*m**3*x**2 + 15*a*b**2*d*e**3*m**2*x**2 + 12*a*b**2*d*e**3*m*x**2 + 3*a*b**2*e**4*m**3*x**3 + 21*a*b**2*e**4*m**2*x**3 + 42*a*b**2*e**4*m*x**3 + 24*a*b**2*e**4*x**3 - 6*b**3*d**4 + 6*b**3*d**3*e*m*x - 3*b**3*d**2*e**2*m**2*x**2 - 3*b**3*d**2*e**2*m*x**2 + b**3*d*e**3*m**3*x**3 + 3*b**3*d*e**3*m**2*x**3 + 2*b**3*d*e**3*m*x**3 + b**3*e**4*m**3*x**4 + 6*b**3*e**4*m**2*x**4 + 11*b**3*e**4*m*x**4 + 6*b**3*e**4*x**4))/(e**4*(m**4 + 10*m**3 + 35*m**2 + 50*m + 24))
```

3.419 $\int (d + ex)^m \sqrt{a^2 + 2abx + b^2x^2} dx$

| | |
|---|------|
| Optimal result | 3150 |
| Mathematica [A] (verified) | 3150 |
| Rubi [A] (verified) | 3151 |
| Maple [A] (verified) | 3152 |
| Fricas [A] (verification not implemented) | 3153 |
| Sympy [F(-2)] | 3153 |
| Maxima [A] (verification not implemented) | 3153 |
| Giac [B] (verification not implemented) | 3154 |
| Mupad [B] (verification not implemented) | 3154 |
| Reduce [B] (verification not implemented) | 3155 |

Optimal result

Integrand size = 28, antiderivative size = 101

$$\int (d + ex)^m \sqrt{a^2 + 2abx + b^2x^2} dx = -\frac{(bd - ae)(d + ex)^{1+m} \sqrt{a^2 + 2abx + b^2x^2}}{e^2(1 + m)(a + bx)} + \frac{b(d + ex)^{2+m} \sqrt{a^2 + 2abx + b^2x^2}}{e^2(2 + m)(a + bx)}$$

output

$$-(-a*e+b*d)*(e*x+d)^{(1+m)*((b*x+a)^2)^{(1/2)}/e^2/(1+m)/(b*x+a)+b*(e*x+d)^{(2+m)*((b*x+a)^2)^{(1/2)}/e^2/(2+m)/(b*x+a)}$$

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.58

$$\int (d + ex)^m \sqrt{a^2 + 2abx + b^2x^2} dx = \frac{\sqrt{(a + bx)^2}(d + ex)^{1+m}(-bd + ae(2 + m) + be(1 + m)x)}{e^2(1 + m)(2 + m)(a + bx)}$$

input

`Integrate[(d + e*x)^m*Sqrt[a^2 + 2*a*b*x + b^2*x^2],x]`

output

```
(Sqrt[(a + b*x)^2]*(d + e*x)^(1 + m)*(-(b*d) + a*e*(2 + m) + b*e*(1 + m)*x
))/ (e^2*(1 + m)*(2 + m)*(a + b*x))
```

Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.74, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {1102, 27, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{a^2 + 2abx + b^2x^2}(d + ex)^m dx$$

$$\downarrow \text{1102}$$

$$\frac{\sqrt{a^2 + 2abx + b^2x^2} \int b(a + bx)(d + ex)^m dx}{b(a + bx)}$$

$$\downarrow \text{27}$$

$$\frac{\sqrt{a^2 + 2abx + b^2x^2} \int (a + bx)(d + ex)^m dx}{a + bx}$$

$$\downarrow \text{53}$$

$$\frac{\sqrt{a^2 + 2abx + b^2x^2} \int \left(\frac{(ae - bd)(d + ex)^m}{e} + \frac{b(d + ex)^{m+1}}{e} \right) dx}{a + bx}$$

$$\downarrow \text{2009}$$

$$\frac{\sqrt{a^2 + 2abx + b^2x^2} \left(\frac{b(d + ex)^{m+2}}{e^2(m+2)} - \frac{(bd - ae)(d + ex)^{m+1}}{e^2(m+1)} \right)}{a + bx}$$

input

```
Int[(d + e*x)^m*Sqrt[a^2 + 2*a*b*x + b^2*x^2], x]
```

output

```
(Sqrt[a^2 + 2*a*b*x + b^2*x^2]*(-(((b*d - a*e)*(d + e*x)^(1 + m))/(e^2*(1
+ m))) + (b*(d + e*x)^(2 + m))/(e^2*(2 + m))))/(a + b*x)
```

Definitions of rubi rules used

rule 27 $\text{Int}[(a_*)(Fx_), x_Symbol] \rightarrow \text{Simp}[a \text{ Int}[Fx, x], x] /; \text{FreeQ}[a, x] \ \&\& \ !\text{MatchQ}[Fx, (b_*)(Gx_)] /; \text{FreeQ}[b, x]$

rule 53 $\text{Int}[((a_.) + (b_.)(x_))^{(m_.)}*((c_.) + (d_.)(x_))^{(n_.)}, x_Symbol] \rightarrow \text{Int}[\text{ExpandIntegrand}[(a + b*x)^m*(c + d*x)^n, x], x] /; \text{FreeQ}[\{a, b, c, d, n\}, x] \ \&\& \ \text{IGtQ}[m, 0] \ \&\& \ (\ !\text{IntegerQ}[n] \ || \ (\text{EqQ}[c, 0] \ \&\& \ \text{LeQ}[7*m + 4*n + 4, 0]) \ || \ \text{LtQ}[9*m + 5*(n + 1), 0] \ || \ \text{GtQ}[m + n + 2, 0])$

rule 1102 $\text{Int}[((d_.) + (e_.)(x_))^{(m_.)}*((a_) + (b_.)(x_) + (c_.)(x_)^2)^{(p_)}, x_Symbol] \rightarrow \text{Simp}[(a + b*x + c*x^2)^{\text{FracPart}[p]} / (c^{\text{IntPart}[p]}*(b/2 + c*x)^{(2*\text{FracPart}[p])}) \ \text{Int}[(d + e*x)^m*(b/2 + c*x)^{(2*p)}, x], x] /; \text{FreeQ}[\{a, b, c, d, e, m, p\}, x] \ \&\& \ \text{EqQ}[b^2 - 4*a*c, 0]$

rule 2009 $\text{Int}[u_, x_Symbol] \rightarrow \text{Simp}[\text{IntSum}[u, x], x] /; \text{SumQ}[u]$

Maple [A] (verified)

Time = 0.68 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.61

| method | result | size |
|---------|---|------|
| gospers | $\frac{(ex+d)^{1+m} \sqrt{(bx+a)^2 (bemx+aem+bx+2ae-bd)}}{e^2(bx+a)(m^2+3m+2)}$ | 62 |
| orering | $\frac{\sqrt{(bx+a)^2} (ex+d)^m (bemx+aem+bx+2ae-bd)(ex+d)}{(bx+a)e^2(m^2+3m+2)}$ | 65 |
| risch | $\frac{\sqrt{(bx+a)^2} (be^2x^2m+ae^2mx+bdemx+be^2x^2+adem+2ae^2x+2ade-bd^2)(ex+d)^m}{(bx+a)e^2(2+m)(1+m)}$ | 92 |

input $\text{int}((e*x+d)^m*((b*x+a)^2)^{(1/2)}, x, \text{method}=_RETURNVERBOSE)$

output $1/e^2*(e*x+d)^{(1+m)}/(b*x+a)/(m^2+3*m+2)*((b*x+a)^2)^{(1/2)}*(b*e*m*x+a*e*m+b*e*x+2*a*e-b*d)$

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.82

$$\int (d + ex)^m \sqrt{a^2 + 2abx + b^2x^2} dx$$

$$= \frac{(adem - bd^2 + 2ade + (be^2m + be^2)x^2 + (2ae^2 + (bde + ae^2)m)x)(ex + d)^m}{e^2m^2 + 3e^2m + 2e^2}$$

input `integrate((e*x+d)^m*((b*x+a)^2)^(1/2),x, algorithm="fricas")`

output `(a*d*e*m - b*d^2 + 2*a*d*e + (b*e^2*m + b*e^2)*x^2 + (2*a*e^2 + (b*d*e + a*e^2)*m)*x)*(e*x + d)^m/(e^2*m^2 + 3*e^2*m + 2*e^2)`

Sympy [F(-2)]

Exception generated.

$$\int (d + ex)^m \sqrt{a^2 + 2abx + b^2x^2} dx = \text{Exception raised: HeuristicGCDFailed}$$

input `integrate((e*x+d)**m*((b*x+a)**2)**(1/2),x)`

output `Exception raised: HeuristicGCDFailed >> no luck`

Maxima [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.61

$$\int (d + ex)^m \sqrt{a^2 + 2abx + b^2x^2} dx$$

$$= \frac{(be^2(m + 1)x^2 + ade(m + 2) - bd^2 + (ae^2(m + 2) + bdem)x)(ex + d)^m}{(m^2 + 3m + 2)e^2}$$

input `integrate((e*x+d)^m*((b*x+a)^2)^(1/2),x, algorithm="maxima")`

output

$$(b \cdot e^{2(m+1)} x^2 + a \cdot d \cdot e^{(m+2)} - b \cdot d^2 + (a \cdot e^{2(m+2)} + b \cdot d \cdot e^m) x) \cdot (e x + d)^m / ((m^2 + 3m + 2) e^2)$$

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 180 vs. $2(79) = 158$.

Time = 0.16 (sec) , antiderivative size = 180, normalized size of antiderivative = 1.78

$$\int (d + ex)^m \sqrt{a^2 + 2abx + b^2 x^2} dx$$

$$= \frac{(ex + d)^m b e^2 m x^2 \operatorname{sgn}(bx + a) + (ex + d)^m b d e m x \operatorname{sgn}(bx + a) + (ex + d)^m a e^2 m x \operatorname{sgn}(bx + a) + (ex + d)^m a^2 \operatorname{sgn}(bx + a)}{e^2 (m^2 + 3m + 2)}$$

input

```
integrate((e*x+d)^m*((b*x+a)^2)^(1/2),x, algorithm="giac")
```

output

$$\frac{((e x + d)^m b e^{2m} x^2 \operatorname{sgn}(b x + a) + (e x + d)^m b d e^m x \operatorname{sgn}(b x + a) + (e x + d)^m a e^{2m} x \operatorname{sgn}(b x + a) + (e x + d)^m b e^{2x} x^2 \operatorname{sgn}(b x + a) + (e x + d)^m a d e^m \operatorname{sgn}(b x + a) + 2(e x + d)^m a e^{2x} x \operatorname{sgn}(b x + a) - (e x + d)^m b d^2 \operatorname{sgn}(b x + a) + 2(e x + d)^m a d e \operatorname{sgn}(b x + a))}{(e^{2m} (m^2 + 3e^{2m} + 2e^2))}$$

Mupad [B] (verification not implemented)

Time = 5.70 (sec) , antiderivative size = 129, normalized size of antiderivative = 1.28

$$\int (d + ex)^m \sqrt{a^2 + 2abx + b^2 x^2} dx$$

$$= \frac{(d + ex)^m \left(\frac{x^2 \sqrt{(a+bx)^2} (m+1)}{m^2 + 3m + 2} + \frac{x \sqrt{(a+bx)^2} (2ae^2 + ae^2 m + b d e m)}{b e^2 (m^2 + 3m + 2)} + \frac{d \sqrt{(a+bx)^2} (2ae - bd + a e m)}{b e^2 (m^2 + 3m + 2)} \right)}{x + \frac{a}{b}}$$

input

```
int(((a + b*x)^2)^(1/2)*(d + e*x)^m,x)
```

output

$$\frac{((d + ex)^m ((x^2 (a + bx)^2)^{(1/2)} (m + 1)) / (3m + m^2 + 2) + (x ((a + bx)^2)^{(1/2)} (2ae^2 + ae^{2m} + bde^m)) / (be^{2(3m + m^2 + 2)}) + (d * ((a + bx)^2)^{(1/2)} (2ae - bd + ae^m)) / (be^{2(3m + m^2 + 2)})) / (x + a/b)}$$
Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.74

$$\int (d + ex)^m \sqrt{a^2 + 2abx + b^2x^2} dx$$

$$= \frac{(ex + d)^m (be^2mx^2 + ae^2mx + bdemx + be^2x^2 + adem + 2ae^2x + 2ade - bd^2)}{e^2(m^2 + 3m + 2)}$$

input

`int((e*x+d)^m*((b*x+a)^2)^(1/2),x)`

output

$$\frac{((d + ex)**m*(a*d*e*m + 2*a*d*e + a*e**2*m*x + 2*a*e**2*x - b*d**2 + b*d*e*m*x + b*e**2*m*x**2 + b*e**2*x**2)) / (e**2*(m**2 + 3*m + 2))}$$

3.420 $\int \frac{(d+ex)^m}{\sqrt{a^2+2abx+b^2x^2}} dx$

| | |
|----------------------------|------|
| Optimal result | 3156 |
| Mathematica [A] (verified) | 3156 |
| Rubi [A] (verified) | 3157 |
| Maple [F] | 3158 |
| Fricas [F] | 3158 |
| Sympy [F] | 3159 |
| Maxima [F] | 3159 |
| Giac [F] | 3159 |
| Mupad [F(-1)] | 3160 |
| Reduce [F] | 3160 |

Optimal result

Integrand size = 28, antiderivative size = 76

$$\int \frac{(d+ex)^m}{\sqrt{a^2+2abx+b^2x^2}} dx$$

$$= -\frac{(a+bx)(d+ex)^{1+m} \operatorname{Hypergeometric2F1}\left(1, 1+m, 2+m, \frac{b(d+ex)}{bd-ae}\right)}{(bd-ae)(1+m)\sqrt{a^2+2abx+b^2x^2}}$$

output

```
- (b*x+a)*(e*x+d)^(1+m)*hypergeom([1, 1+m], [2+m], b*(e*x+d)/(-a*e+b*d))/(-a*e+b*d)/(1+m)/((b*x+a)^2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.88

$$\int \frac{(d+ex)^m}{\sqrt{a^2+2abx+b^2x^2}} dx$$

$$= -\frac{(a+bx)(d+ex)^{1+m} \operatorname{Hypergeometric2F1}\left(1, 1+m, 2+m, \frac{b(d+ex)}{bd-ae}\right)}{(bd-ae)(1+m)\sqrt{(a+bx)^2}}$$

input

```
Integrate[(d + e*x)^m/Sqrt[a^2 + 2*a*b*x + b^2*x^2], x]
```

output

$$-\left(\frac{(a + bx)(d + ex)^{(1 + m)} \text{Hypergeometric2F1}[1, 1 + m, 2 + m, (b(d + ex))/(bd - ae)]}{(bd - ae)(1 + m) \sqrt{(a + bx)^2}}\right)$$
Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.107$, Rules used = {1102, 27, 78}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\begin{aligned} & \int \frac{(d + ex)^m}{\sqrt{a^2 + 2abx + b^2x^2}} dx \\ & \quad \downarrow \text{1102} \\ & \frac{b(a + bx) \int \frac{(d+ex)^m}{b(a+bx)} dx}{\sqrt{a^2 + 2abx + b^2x^2}} \\ & \quad \downarrow \text{27} \\ & \frac{(a + bx) \int \frac{(d+ex)^m}{a+bx} dx}{\sqrt{a^2 + 2abx + b^2x^2}} \\ & \quad \downarrow \text{78} \\ & \frac{(a + bx)(d + ex)^{m+1} \text{Hypergeometric2F1}\left(1, m + 1, m + 2, \frac{b(d+ex)}{bd-ae}\right)}{(m + 1)\sqrt{a^2 + 2abx + b^2x^2}(bd - ae)} \end{aligned}$$

input

$$\text{Int}[(d + ex)^m/\text{Sqrt}[a^2 + 2*a*b*x + b^2*x^2], x]$$

output

$$-\left(\frac{(a + bx)(d + ex)^{(1 + m)} \text{Hypergeometric2F1}[1, 1 + m, 2 + m, (b(d + ex))/(bd - ae)]}{(bd - ae)(1 + m) \sqrt{a^2 + 2*a*b*x + b^2*x^2}}\right)$$

Defintions of rubi rules used

rule 27 `Int[(a_)*(F_x_), x_Symbol] := Simp[a Int[F_x, x], x] /; FreeQ[a, x] && !MatchQ[F_x, (b_)*(G_x_) /; FreeQ[b, x]]`

rule 78 `Int[((a_) + (b_)*(x_))^(m_)*((c_) + (d_)*(x_))^(n_), x_Symbol] := Simp[(b*c - a*d)^(n+1)*(a + b*x)^(m+1)/(b^(n+1)*(m+1))*Hypergeometric2F1[-n, m+1, m+2, (-d)*(a + b*x)/(b*c - a*d)], x] /; FreeQ[{a, b, c, d, m}, x] && !IntegerQ[m] && IntegerQ[n]`

rule 1102 `Int[((d_) + (e_)*(x_))^(m_)*((a_) + (b_)*(x_) + (c_)*(x_)^2)^(p_), x_Symbol] := Simp[(a + b*x + c*x^2)^(FracPart[p])/(c^(IntPart[p])*(b/2 + c*x)^(2*FracPart[p])) Int[(d + e*x)^(m+1)*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && EqQ[b^2 - 4*a*c, 0]`

Maple [F]

$$\int \frac{(ex + d)^m}{\sqrt{(bx + a)^2}} dx$$

input `int((e*x+d)^m/((b*x+a)^2)^(1/2),x)`

output `int((e*x+d)^m/((b*x+a)^2)^(1/2),x)`

Fricas [F]

$$\int \frac{(d + ex)^m}{\sqrt{a^2 + 2abx + b^2x^2}} dx = \int \frac{(ex + d)^m}{\sqrt{(bx + a)^2}} dx$$

input `integrate((e*x+d)^m/((b*x+a)^2)^(1/2),x, algorithm="fricas")`

output `integral((e*x + d)^m/sqrt(b^2*x^2 + 2*a*b*x + a^2), x)`

Sympy [F]

$$\int \frac{(d + ex)^m}{\sqrt{a^2 + 2abx + b^2x^2}} dx = \int \frac{(d + ex)^m}{\sqrt{(a + bx)^2}} dx$$

input `integrate((e*x+d)**m/((b*x+a)**2)**(1/2), x)`

output `Integral((d + e*x)**m/sqrt((a + b*x)**2), x)`

Maxima [F]

$$\int \frac{(d + ex)^m}{\sqrt{a^2 + 2abx + b^2x^2}} dx = \int \frac{(ex + d)^m}{\sqrt{(bx + a)^2}} dx$$

input `integrate((e*x+d)^m/((b*x+a)^2)^(1/2), x, algorithm="maxima")`

output `integrate((e*x + d)^m/sqrt((b*x + a)^2), x)`

Giac [F]

$$\int \frac{(d + ex)^m}{\sqrt{a^2 + 2abx + b^2x^2}} dx = \int \frac{(ex + d)^m}{\sqrt{(bx + a)^2}} dx$$

input `integrate((e*x+d)^m/((b*x+a)^2)^(1/2), x, algorithm="giac")`

output `integrate((e*x + d)^m/sqrt((b*x + a)^2), x)`

Mupad [F(-1)]

Timed out.

$$\int \frac{(d + ex)^m}{\sqrt{a^2 + 2abx + b^2x^2}} dx = \int \frac{(d + ex)^m}{\sqrt{(a + bx)^2}} dx$$

input `int((d + e*x)^m/((a + b*x)^2)^(1/2),x)`output `int((d + e*x)^m/((a + b*x)^2)^(1/2), x)`**Reduce [F]**

$$\int \frac{(d + ex)^m}{\sqrt{a^2 + 2abx + b^2x^2}} dx$$

$$= \frac{(ex + d)^m d + \left(\int \frac{(ex+d)^m x}{be x^2 + aex + bdx + ad} dx \right) a e^2 m - \left(\int \frac{(ex+d)^m x}{be x^2 + aex + bdx + ad} dx \right) b d e m}{a e m}$$

input `int((e*x+d)^m/((b*x+a)^2)^(1/2),x)`output `((d + e*x)**m*d + int(((d + e*x)**m*x)/(a*d + a*e*x + b*d*x + b*e*x**2),x) *a*e**2*m - int(((d + e*x)**m*x)/(a*d + a*e*x + b*d*x + b*e*x**2),x)*b*d*e *m)/(a*e*m)`

3.421
$$\int \frac{(d+ex)^m}{(a^2+2abx+b^2x^2)^{3/2}} dx$$

| | |
|----------------------------|------|
| Optimal result | 3161 |
| Mathematica [A] (verified) | 3161 |
| Rubi [A] (verified) | 3162 |
| Maple [F] | 3163 |
| Fricas [F] | 3163 |
| Sympy [F(-2)] | 3164 |
| Maxima [F] | 3164 |
| Giac [F] | 3164 |
| Mupad [F(-1)] | 3165 |
| Reduce [F] | 3165 |

Optimal result

Integrand size = 28, antiderivative size = 79

$$\int \frac{(d+ex)^m}{(a^2+2abx+b^2x^2)^{3/2}} dx = \frac{e^2(a+bx)(d+ex)^{1+m} \operatorname{Hypergeometric2F1}\left(3, 1+m, 2+m, \frac{b(d+ex)}{bd-ae}\right)}{(bd-ae)^3(1+m)\sqrt{a^2+2abx+b^2x^2}}$$

output

```
-e^2*(b*x+a)*(e*x+d)^(1+m)*hypergeom([3, 1+m], [2+m], b*(e*x+d)/(-a*e+b*d))/(-a*e+b*d)^3/(1+m)/((b*x+a)^2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.91

$$\int \frac{(d+ex)^m}{(a^2+2abx+b^2x^2)^{3/2}} dx = \frac{e^2(a+bx)^3(d+ex)^{1+m} \operatorname{Hypergeometric2F1}\left(3, 1+m, 2+m, -\frac{b(d+ex)}{-bd+ae}\right)}{(-bd+ae)^3(1+m)((a+bx)^2)^{3/2}}$$

input

```
Integrate[(d + e*x)^m/(a^2 + 2*a*b*x + b^2*x^2)^(3/2), x]
```

output

```
(e^2*(a + b*x)^3*(d + e*x)^(1 + m)*Hypergeometric2F1[3, 1 + m, 2 + m, -(b
*(d + e*x))/(-(b*d) + a*e)]/((-b*d) + a*e)^3*(1 + m)*((a + b*x)^2)^(3/2
))
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.107$, Rules used = {1102, 27, 78}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d + ex)^m}{(a^2 + 2abx + b^2x^2)^{3/2}} dx$$

$$\downarrow 1102$$

$$\frac{b^3(a + bx) \int \frac{(d+ex)^m}{b^3(a+bx)^3} dx}{\sqrt{a^2 + 2abx + b^2x^2}}$$

$$\downarrow 27$$

$$\frac{(a + bx) \int \frac{(d+ex)^m}{(a+bx)^3} dx}{\sqrt{a^2 + 2abx + b^2x^2}}$$

$$\downarrow 78$$

$$\frac{e^2(a + bx)(d + ex)^{m+1} \text{Hypergeometric2F1}\left(3, m + 1, m + 2, \frac{b(d+ex)}{bd-ae}\right)}{(m + 1)\sqrt{a^2 + 2abx + b^2x^2}(bd - ae)^3}$$

input

```
Int[(d + e*x)^m/(a^2 + 2*a*b*x + b^2*x^2)^(3/2),x]
```

output

```
-((e^2*(a + b*x)*(d + e*x)^(1 + m)*Hypergeometric2F1[3, 1 + m, 2 + m, (b*(
d + e*x))/(b*d - a*e)]/((b*d - a*e)^3*(1 + m)*Sqrt[a^2 + 2*a*b*x + b^2*x^
2]))
```

Definitions of rubi rules used

rule 27 `Int[(a_)*(F_x_), x_Symbol] := Simp[a Int[F_x, x], x] /; FreeQ[a, x] && !MatchQ[F_x, (b_)*(G_x_)] /; FreeQ[b, x]`

rule 78 `Int[((a_) + (b_)*(x_))^(m_)*((c_) + (d_)*(x_))^(n_), x_Symbol] := Simp[(b*c - a*d)^n*(a + b*x)^(m + 1)/(b^(n + 1)*(m + 1))*Hypergeometric2F1[-n, m + 1, m + 2, (-d)*(a + b*x)/(b*c - a*d)], x] /; FreeQ[{a, b, c, d, m}, x] && !IntegerQ[m] && IntegerQ[n]`

rule 1102 `Int[((d_) + (e_)*(x_))^(m_)*((a_) + (b_)*(x_) + (c_)*(x_)^2)^(p_), x_Symbol] := Simp[(a + b*x + c*x^2)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x)^(2*FracPart[p])) Int[(d + e*x)^m*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && EqQ[b^2 - 4*a*c, 0]`

Maple [F]

$$\int \frac{(ex + d)^m}{(b^2x^2 + 2abx + a^2)^{\frac{3}{2}}} dx$$

input `int((e*x+d)^m/(b^2*x^2+2*a*b*x+a^2)^(3/2), x)`

output `int((e*x+d)^m/(b^2*x^2+2*a*b*x+a^2)^(3/2), x)`

Fricas [F]

$$\int \frac{(d + ex)^m}{(a^2 + 2abx + b^2x^2)^{3/2}} dx = \int \frac{(ex + d)^m}{(b^2x^2 + 2abx + a^2)^{\frac{3}{2}}} dx$$

input `integrate((e*x+d)^m/(b^2*x^2+2*a*b*x+a^2)^(3/2), x, algorithm="fricas")`

output `integral(sqrt(b^2*x^2 + 2*a*b*x + a^2)*(e*x + d)^m/(b^4*x^4 + 4*a*b^3*x^3 + 6*a^2*b^2*x^2 + 4*a^3*b*x + a^4), x)`

Sympy [F(-2)]

Exception generated.

$$\int \frac{(d + ex)^m}{(a^2 + 2abx + b^2x^2)^{3/2}} dx = \text{Exception raised: HeuristicGCDFailed}$$

input `integrate((e*x+d)**m/(b**2*x**2+2*a*b*x+a**2)**(3/2),x)`

output Exception raised: HeuristicGCDFailed >> no luck

Maxima [F]

$$\int \frac{(d + ex)^m}{(a^2 + 2abx + b^2x^2)^{3/2}} dx = \int \frac{(ex + d)^m}{(b^2x^2 + 2abx + a^2)^{\frac{3}{2}}} dx$$

input `integrate((e*x+d)^m/(b^2*x^2+2*a*b*x+a^2)^(3/2),x, algorithm="maxima")`

output `integrate((e*x + d)^m/(b^2*x^2 + 2*a*b*x + a^2)^(3/2), x)`

Giac [F]

$$\int \frac{(d + ex)^m}{(a^2 + 2abx + b^2x^2)^{3/2}} dx = \int \frac{(ex + d)^m}{(b^2x^2 + 2abx + a^2)^{\frac{3}{2}}} dx$$

input `integrate((e*x+d)^m/(b^2*x^2+2*a*b*x+a^2)^(3/2),x, algorithm="giac")`

output `integrate((e*x + d)^m/(b^2*x^2 + 2*a*b*x + a^2)^(3/2), x)`

Mupad [F(-1)]

Timed out.

$$\int \frac{(d + ex)^m}{(a^2 + 2abx + b^2x^2)^{3/2}} dx = \int \frac{(d + ex)^m}{(a^2 + 2abx + b^2x^2)^{3/2}} dx$$

input `int((d + e*x)^m/(a^2 + b^2*x^2 + 2*a*b*x)^(3/2),x)`output `int((d + e*x)^m/(a^2 + b^2*x^2 + 2*a*b*x)^(3/2), x)`**Reduce [F]**

$$\int \frac{(d + ex)^m}{(a^2 + 2abx + b^2x^2)^{3/2}} dx = \text{too large to display}$$

input `int((e*x+d)^m/(b^2*x^2+2*a*b*x+a^2)^(3/2),x)`

output

```

((d + e*x)**m*d + int(((d + e*x)**m*x)/(a**4*d*e**m + a**4*e**2*m*x - 2*a**
3*b*d**2 + 3*a**3*b*d*e**m*x - 2*a**3*b*d*e*x + 3*a**3*b*e**2*m*x**2 - 6*a**
2*b**2*d**2*x + 3*a**2*b**2*d*e**m*x**2 - 6*a**2*b**2*d*e*x**2 + 3*a**2*b**
2*e**2*m*x**3 - 6*a*b**3*d**2*x**2 + a*b**3*d*e**m*x**3 - 6*a*b**3*d*e*x**
3 + a*b**3*e**2*m*x**4 - 2*b**4*d**2*x**3 - 2*b**4*d*e*x**4),x)*a**4*e**3*
m**2 - int(((d + e*x)**m*x)/(a**4*d*e**m + a**4*e**2*m*x - 2*a**3*b*d**2 +
3*a**3*b*d*e**m*x - 2*a**3*b*d*e*x + 3*a**3*b*e**2*m*x**2 - 6*a**2*b**2*d**
2*x + 3*a**2*b**2*d*e**m*x**2 - 6*a**2*b**2*d*e*x**2 + 3*a**2*b**2*e**2*m*x
**3 - 6*a*b**3*d**2*x**2 + a*b**3*d*e**m*x**3 - 6*a*b**3*d*e*x**3 + a*b**3*
e**2*m*x**4 - 2*b**4*d**2*x**3 - 2*b**4*d*e*x**4),x)*a**3*b*d*e**2*m**2 -
2*int(((d + e*x)**m*x)/(a**4*d*e**m + a**4*e**2*m*x - 2*a**3*b*d**2 + 3*a**
3*b*d*e**m*x - 2*a**3*b*d*e*x + 3*a**3*b*e**2*m*x**2 - 6*a**2*b**2*d**2*x +
3*a**2*b**2*d*e**m*x**2 - 6*a**2*b**2*d*e*x**2 + 3*a**2*b**2*e**2*m*x**3 -
6*a*b**3*d**2*x**2 + a*b**3*d*e**m*x**3 - 6*a*b**3*d*e*x**3 + a*b**3*e**2*
m*x**4 - 2*b**4*d**2*x**3 - 2*b**4*d*e*x**4),x)*a**2*b*d*e**2*m + 2*int(((
d + e*x)**m*x)/(a**4*d*e**m + a**4*e**2*m*x - 2*a**3*b*d**2 + 3*a**3*b*d*e**
m*x - 2*a**3*b*d*e*x + 3*a**3*b*e**2*m*x**2 - 6*a**2*b**2*d**2*x + 3*a**2*
b**2*d*e**m*x**2 - 6*a**2*b**2*d*e*x**2 + 3*a**2*b**2*e**2*m*x**3 - 6*a*b**
3*d**2*x**2 + a*b**3*d*e**m*x**3 - 6*a*b**3*d*e*x**3 + a*b**3*e**2*m*x**4 -
2*b**4*d**2*x**3 - 2*b**4*d*e*x**4),x)*a**3*b*e**3*m**2*x + 2*int(((d ...

```

$$3.422 \quad \int \frac{(d+ex)^m}{(a^2+2abx+b^2x^2)^{5/2}} dx$$

| | |
|----------------------------|------|
| Optimal result | 3167 |
| Mathematica [A] (verified) | 3167 |
| Rubi [A] (verified) | 3168 |
| Maple [F] | 3169 |
| Fricas [F] | 3169 |
| Sympy [F(-2)] | 3170 |
| Maxima [F] | 3170 |
| Giac [F] | 3170 |
| Mupad [F(-1)] | 3171 |
| Reduce [F] | 3171 |

Optimal result

Integrand size = 28, antiderivative size = 79

$$\int \frac{(d+ex)^m}{(a^2+2abx+b^2x^2)^{5/2}} dx = \frac{e^4(a+bx)(d+ex)^{1+m} \operatorname{Hypergeometric2F1}\left(5, 1+m, 2+m, \frac{b(d+ex)}{bd-ae}\right)}{(bd-ae)^5(1+m)\sqrt{a^2+2abx+b^2x^2}}$$

output

```
-e^4*(b*x+a)*(e*x+d)^(1+m)*hypergeom([5, 1+m], [2+m], b*(e*x+d)/(-a*e+b*d))/
(-a*e+b*d)^5/(1+m)/((b*x+a)^2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.91

$$\int \frac{(d+ex)^m}{(a^2+2abx+b^2x^2)^{5/2}} dx = \frac{e^4(a+bx)^5(d+ex)^{1+m} \operatorname{Hypergeometric2F1}\left(5, 1+m, 2+m, -\frac{b(d+ex)}{-bd+ae}\right)}{(-bd+ae)^5(1+m)((a+bx)^2)^{5/2}}$$

input

```
Integrate[(d + e*x)^m/(a^2 + 2*a*b*x + b^2*x^2)^(5/2), x]
```

output

```
(e^4*(a + b*x)^5*(d + e*x)^(1 + m)*Hypergeometric2F1[5, 1 + m, 2 + m, -(b
*(d + e*x))/(-(b*d) + a*e)]/((-b*d) + a*e)^5*(1 + m)*((a + b*x)^2)^(5/2
))
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.107$, Rules used = {1102, 27, 78}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d + ex)^m}{(a^2 + 2abx + b^2x^2)^{5/2}} dx$$

$$\downarrow 1102$$

$$\frac{b^5(a + bx) \int \frac{(d+ex)^m}{b^5(a+bx)^5} dx}{\sqrt{a^2 + 2abx + b^2x^2}}$$

$$\downarrow 27$$

$$\frac{(a + bx) \int \frac{(d+ex)^m}{(a+bx)^5} dx}{\sqrt{a^2 + 2abx + b^2x^2}}$$

$$\downarrow 78$$

$$\frac{e^4(a + bx)(d + ex)^{m+1} \text{Hypergeometric2F1}\left(5, m + 1, m + 2, \frac{b(d+ex)}{bd-ae}\right)}{(m + 1)\sqrt{a^2 + 2abx + b^2x^2}(bd - ae)^5}$$

input

```
Int[(d + e*x)^m/(a^2 + 2*a*b*x + b^2*x^2)^(5/2),x]
```

output

```
-((e^4*(a + b*x)*(d + e*x)^(1 + m)*Hypergeometric2F1[5, 1 + m, 2 + m, (b*(
d + e*x))/(b*d - a*e)]/((b*d - a*e)^5*sqrt[a^2 + 2*a*b*x + b^2*x^
2]))
```

Definitions of rubi rules used

rule 27 `Int[(a_)*(F_x_), x_Symbol] := Simp[a Int[F_x, x], x] /; FreeQ[a, x] && !MatchQ[F_x, (b_)*(G_x_)] /; FreeQ[b, x]`

rule 78 `Int[((a_) + (b_)*(x_))^(m_)*((c_) + (d_)*(x_))^(n_), x_Symbol] := Simp[(b*c - a*d)^n*(a + b*x)^(m + 1)/(b^(n + 1)*(m + 1))*Hypergeometric2F1[-n, m + 1, m + 2, (-d)*(a + b*x)/(b*c - a*d)], x] /; FreeQ[{a, b, c, d, m}, x] && !IntegerQ[m] && IntegerQ[n]`

rule 1102 `Int[((d_) + (e_)*(x_))^(m_)*((a_) + (b_)*(x_) + (c_)*(x_)^2)^(p_), x_Symbol] := Simp[(a + b*x + c*x^2)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x)^(2*FracPart[p])) Int[(d + e*x)^m*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && EqQ[b^2 - 4*a*c, 0]`

Maple [F]

$$\int \frac{(ex + d)^m}{(b^2x^2 + 2abx + a^2)^{\frac{5}{2}}} dx$$

input `int((e*x+d)^m/(b^2*x^2+2*a*b*x+a^2)^(5/2), x)`

output `int((e*x+d)^m/(b^2*x^2+2*a*b*x+a^2)^(5/2), x)`

Fricas [F]

$$\int \frac{(d + ex)^m}{(a^2 + 2abx + b^2x^2)^{5/2}} dx = \int \frac{(ex + d)^m}{(b^2x^2 + 2abx + a^2)^{\frac{5}{2}}} dx$$

input `integrate((e*x+d)^m/(b^2*x^2+2*a*b*x+a^2)^(5/2), x, algorithm="fricas")`

output `integral(sqrt(b^2*x^2 + 2*a*b*x + a^2)*(e*x + d)^m/(b^6*x^6 + 6*a*b^5*x^5 + 15*a^2*b^4*x^4 + 20*a^3*b^3*x^3 + 15*a^4*b^2*x^2 + 6*a^5*b*x + a^6), x)`

Sympy [F(-2)]

Exception generated.

$$\int \frac{(d + ex)^m}{(a^2 + 2abx + b^2x^2)^{5/2}} dx = \text{Exception raised: HeuristicGCDFailed}$$

input `integrate((e*x+d)**m/(b**2*x**2+2*a*b*x+a**2)**(5/2),x)`

output `Exception raised: HeuristicGCDFailed >> no luck`

Maxima [F]

$$\int \frac{(d + ex)^m}{(a^2 + 2abx + b^2x^2)^{5/2}} dx = \int \frac{(ex + d)^m}{(b^2x^2 + 2abx + a^2)^{5/2}} dx$$

input `integrate((e*x+d)^m/(b^2*x^2+2*a*b*x+a^2)^(5/2),x, algorithm="maxima")`

output `integrate((e*x + d)^m/(b^2*x^2 + 2*a*b*x + a^2)^(5/2), x)`

Giac [F]

$$\int \frac{(d + ex)^m}{(a^2 + 2abx + b^2x^2)^{5/2}} dx = \int \frac{(ex + d)^m}{(b^2x^2 + 2abx + a^2)^{5/2}} dx$$

input `integrate((e*x+d)^m/(b^2*x^2+2*a*b*x+a^2)^(5/2),x, algorithm="giac")`

output `integrate((e*x + d)^m/(b^2*x^2 + 2*a*b*x + a^2)^(5/2), x)`

Mupad [F(-1)]

Timed out.

$$\int \frac{(d + ex)^m}{(a^2 + 2abx + b^2x^2)^{5/2}} dx = \int \frac{(d + ex)^m}{(a^2 + 2abx + b^2x^2)^{5/2}} dx$$

input `int((d + e*x)^m/(a^2 + b^2*x^2 + 2*a*b*x)^(5/2), x)`output `int((d + e*x)^m/(a^2 + b^2*x^2 + 2*a*b*x)^(5/2), x)`**Reduce [F]**

$$\int \frac{(d + ex)^m}{(a^2 + 2abx + b^2x^2)^{5/2}} dx = \text{too large to display}$$

input `int((e*x+d)^m/(b^2*x^2+2*a*b*x+a^2)^(5/2), x)`

output

```

((d + e*x)**m*d + int(((d + e*x)**m*x)/(a**6*d*e*m + a**6*e**2*m*x - 4*a**
5*b*d**2 + 5*a**5*b*d*e*m*x - 4*a**5*b*d*e*x + 5*a**5*b*e**2*m*x**2 - 20*a
**4*b**2*d**2*x + 10*a**4*b**2*d*e*m*x**2 - 20*a**4*b**2*d*e*x**2 + 10*a**
4*b**2*e**2*m*x**3 - 40*a**3*b**3*d**2*x**2 + 10*a**3*b**3*d*e*m*x**3 - 40
*a**3*b**3*d*e*x**3 + 10*a**3*b**3*e**2*m*x**4 - 40*a**2*b**4*d**2*x**3 +
5*a**2*b**4*d*e*m*x**4 - 40*a**2*b**4*d*e*x**4 + 5*a**2*b**4*e**2*m*x**5 -
20*a*b**5*d**2*x**4 + a*b**5*d*e*m*x**5 - 20*a*b**5*d*e*x**5 + a*b**5*e**
2*m*x**6 - 4*b**6*d**2*x**5 - 4*b**6*d*e*x**6),x)*a**6*e**3*m**2 - int(((d
+ e*x)**m*x)/(a**6*d*e*m + a**6*e**2*m*x - 4*a**5*b*d**2 + 5*a**5*b*d*e*m
*x - 4*a**5*b*d*e*x + 5*a**5*b*e**2*m*x**2 - 20*a**4*b**2*d**2*x + 10*a**4
*b**2*d*e*m*x**2 - 20*a**4*b**2*d*e*x**2 + 10*a**4*b**2*e**2*m*x**3 - 40*a
**3*b**3*d**2*x**2 + 10*a**3*b**3*d*e*m*x**3 - 40*a**3*b**3*d*e*x**3 + 10*
a**3*b**3*e**2*m*x**4 - 40*a**2*b**4*d**2*x**3 + 5*a**2*b**4*d*e*m*x**4 -
40*a**2*b**4*d*e*x**4 + 5*a**2*b**4*e**2*m*x**5 - 20*a*b**5*d**2*x**4 + a*
b**5*d*e*m*x**5 - 20*a*b**5*d*e*x**5 + a*b**5*e**2*m*x**6 - 4*b**6*d**2*x*
*5 - 4*b**6*d*e*x**6),x)*a**5*b*d*e**2*m**2 - 4*int(((d + e*x)**m*x)/(a**6
*d*e*m + a**6*e**2*m*x - 4*a**5*b*d**2 + 5*a**5*b*d*e*m*x - 4*a**5*b*d*e*x
+ 5*a**5*b*e**2*m*x**2 - 20*a**4*b**2*d**2*x + 10*a**4*b**2*d*e*m*x**2 -
20*a**4*b**2*d*e*x**2 + 10*a**4*b**2*e**2*m*x**3 - 40*a**3*b**3*d**2*x**2
+ 10*a**3*b**3*d*e*m*x**3 - 40*a**3*b**3*d*e*x**3 + 10*a**3*b**3*e**2*m...

```

3.423 $\int (d + ex)^3 (cd^2 + 2cdex + ce^2x^2)^p dx$

| | |
|---|------|
| Optimal result | 3173 |
| Mathematica [A] (verified) | 3173 |
| Rubi [A] (verified) | 3174 |
| Maple [A] (verified) | 3174 |
| Fricas [A] (verification not implemented) | 3175 |
| Sympy [B] (verification not implemented) | 3176 |
| Maxima [B] (verification not implemented) | 3176 |
| Giac [A] (verification not implemented) | 3177 |
| Mupad [B] (verification not implemented) | 3177 |
| Reduce [B] (verification not implemented) | 3178 |

Optimal result

Integrand size = 30, antiderivative size = 39

$$\int (d + ex)^3 (cd^2 + 2cdex + ce^2x^2)^p dx = \frac{(cd^2 + 2cdex + ce^2x^2)^{2+p}}{2c^2e(2+p)}$$

output $1/2*(c*e^2*x^2+2*c*d*e*x+c*d^2)^(2+p)/c^2/e/(2+p)$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.74

$$\int (d + ex)^3 (cd^2 + 2cdex + ce^2x^2)^p dx = \frac{(d + ex)^4 (c(d + ex)^2)^p}{e(4 + 2p)}$$

input $\text{Integrate}[(d + e*x)^3*(c*d^2 + 2*c*d*e*x + c*e^2*x^2)^p,x]$

output $((d + e*x)^4*(c*(d + e*x)^2)^p)/(e*(4 + 2*p))$

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.033$, Rules used = {1099}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (d + ex)^3 (cd^2 + 2cdex + ce^2x^2)^p dx$$

$$\downarrow 1099$$

$$\frac{(cd^2 + 2cdex + ce^2x^2)^{p+2}}{2c^2e(p+2)}$$

input `Int[(d + e*x)^3*(c*d^2 + 2*c*d*e*x + c*e^2*x^2)^p,x]`

output `(c*d^2 + 2*c*d*e*x + c*e^2*x^2)^(2 + p)/(2*c^2*e*(2 + p))`

Defintions of rubi rules used

rule 1099 `Int[((d_) + (e_)*(x_))^(m_)*((a_) + (b_)*(x_) + (c_)*(x_)^2)^(p_), x_Symbol] := Simp[e^m*((a + b*x + c*x^2)^(p + (m + 1)/2)/(c^((m + 1)/2)*(m + 2*p + 1))), x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[b^2 - 4*a*c, 0] && EqQ[2*c*d - b*e, 0] && IntegerQ[(m - 1)/2]`

Maple [A] (verified)

Time = 1.50 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.03

| method | result |
|-------------|--|
| gospers | $\frac{(ex+d)^4 (x^2 c e^2 + 2cdxe + c d^2)^p}{2e(2+p)}$ |
| orering | $\frac{(ex+d)^4 (x^2 c e^2 + 2cdxe + c d^2)^p}{2e(2+p)}$ |
| risch | $\frac{(e^4 x^4 + 4d e^3 x^3 + 6d^2 e^2 x^2 + 4d^3 ex + d^4) (c(ex+d)^2)^p}{2e(2+p)}$ |
| parallelsch | $\frac{x^4 (c(e^2 x^2 + 2dex + d^2))^p d e^4 + 4x^3 (c(e^2 x^2 + 2dex + d^2))^p d^2 e^3 + 6x^2 (c(e^2 x^2 + 2dex + d^2))^p d^3 e^2 + 4x (c(e^2 x^2 + 2dex + d^2))^p d^4 e}{2(2+p)ed}$ |
| norman | $\frac{2d^3 x e^{p \ln(x^2 c e^2 + 2cdxe + c d^2)}}{2+p} + \frac{d^4 e^{p \ln(x^2 c e^2 + 2cdxe + c d^2)}}{2e(2+p)} + \frac{e^3 x^4 e^{p \ln(x^2 c e^2 + 2cdxe + c d^2)}}{4+2p} + \frac{2d e^2 x^3 e^{p \ln(x^2 c e^2 + 2cdxe + c d^2)}}{2+p}$ |

```
input int((e*x+d)^3*(c*e^2*x^2+2*c*d*e*x+c*d^2)^p,x,method=_RETURNVERBOSE)
```

```
output 1/2*(e*x+d)^4/e/(2+p)*(c*e^2*x^2+2*c*d*e*x+c*d^2)^p
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.82

$$\int (d + ex)^3 (cd^2 + 2cdex + ce^2x^2)^p dx = \frac{(e^4 x^4 + 4de^3 x^3 + 6d^2 e^2 x^2 + 4d^3 ex + d^4)(ce^2 x^2 + 2cdex + cd^2)^p}{2(ep + 2e)}$$

```
input integrate((e*x+d)^3*(c*e^2*x^2+2*c*d*e*x+c*d^2)^p,x, algorithm="fricas")
```

```
output 1/2*(e^4*x^4 + 4*d*e^3*x^3 + 6*d^2*e^2*x^2 + 4*d^3*e*x + d^4)*(c*e^2*x^2 + 2*c*d*e*x + c*d^2)^p/(e*p + 2*e)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 233 vs. $2(34) = 68$.

Time = 0.36 (sec) , antiderivative size = 233, normalized size of antiderivative = 5.97

$$\int (d + ex)^3 (cd^2 + 2cdex + ce^2x^2)^p dx$$

$$= \begin{cases} \frac{x}{c^2d} \\ d^3x(cd^2)^p \\ \frac{\log\left(\frac{d}{e} + x\right)}{c^2e} \\ \frac{d^4(cd^2 + 2cdex + ce^2x^2)^p}{2ep + 4e} + \frac{4d^3ex(cd^2 + 2cdex + ce^2x^2)^p}{2ep + 4e} + \frac{6d^2e^2x^2(cd^2 + 2cdex + ce^2x^2)^p}{2ep + 4e} + \frac{4de^3x^3(cd^2 + 2cdex + ce^2x^2)^p}{2ep + 4e} + \frac{e^4x^4(cd^2 + 2cdex + ce^2x^2)^p}{2ep + 4e} \end{cases}$$

input `integrate((e*x+d)**3*(c*e**2*x**2+2*c*d*e*x+c*d**2)**p,x)`

output `Piecewise((x/(c**2*d), Eq(e, 0) & Eq(p, -2)), (d**3*x*(c*d**2)**p, Eq(e, 0)), (log(d/e + x)/(c**2*e), Eq(p, -2)), (d**4*(c*d**2 + 2*c*d*e*x + c*e**2*x**2)**p/(2*e*p + 4*e) + 4*d**3*e*x*(c*d**2 + 2*c*d*e*x + c*e**2*x**2)**p/(2*e*p + 4*e) + 6*d**2*e**2*x**2*(c*d**2 + 2*c*d*e*x + c*e**2*x**2)**p/(2*e*p + 4*e) + 4*d*e**3*x**3*(c*d**2 + 2*c*d*e*x + c*e**2*x**2)**p/(2*e*p + 4*e) + e**4*x**4*(c*d**2 + 2*c*d*e*x + c*e**2*x**2)**p/(2*e*p + 4*e), True))`

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 313 vs. $2(37) = 74$.

Time = 0.06 (sec) , antiderivative size = 313, normalized size of antiderivative = 8.03

$$\int (d + ex)^3 (cd^2 + 2cdex + ce^2x^2)^p dx$$

$$= \frac{(c^p ex + c^p d)(ex + d)^{2p} d^3}{e(2p + 1)} + \frac{3(c^p e^2(2p + 1)x^2 + 2c^p d e p x - c^p d^2)(ex + d)^{2p} d^2}{2(2p^2 + 3p + 1)e}$$

$$+ \frac{3((2p^2 + 3p + 1)c^p e^3 x^3 + (2p^2 + p)c^p d e^2 x^2 - 2c^p d^2 e p x + c^p d^3)(ex + d)^{2p} d}{(4p^3 + 12p^2 + 11p + 3)e}$$

$$+ \frac{((4p^3 + 12p^2 + 11p + 3)c^p e^4 x^4 + 2(2p^3 + 3p^2 + p)c^p d e^3 x^3 - 3(2p^2 + p)c^p d^2 e^2 x^2 + 6c^p d^3 e p x - 3c^p d^4)(ex + d)^{2p}}{2(4p^4 + 20p^3 + 35p^2 + 25p + 6)e}$$

input `integrate((e*x+d)^3*(c*e^2*x^2+2*c*d*e*x+c*d^2)^p,x, algorithm="maxima")`

output $(c^p e^x + c^p d)(e^x + d)^{(2p)} d^3 / (e^{(2p+1)}) + 3/2 (c^p e^{2(2p+1)} x^2 + 2c^p d e^p x - c^p d^2)(e^x + d)^{(2p)} d^2 / ((2p^2 + 3p + 1)e) + 3((2p^2 + 3p + 1)c^p e^3 x^3 + (2p^2 + p)c^p d e^2 x^2 - 2c^p d^2 e^p x + c^p d^3)(e^x + d)^{(2p)} d / ((4p^3 + 12p^2 + 11p + 3)e) + 1/2((4p^3 + 12p^2 + 11p + 3)c^p e^4 x^4 + 2(2p^3 + 3p^2 + p)c^p d e^3 x^3 - 3(2p^2 + p)c^p d^2 e^2 x^2 + 6c^p d^3 e^p x - 3c^p d^4)(e^x + d)^{(2p)} / ((4p^4 + 20p^3 + 35p^2 + 25p + 6)e)$

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.46

$$\int (d+ex)^3 (cd^2+2cdex+ce^2x^2)^p dx = \frac{(ce^2x^2+2cdex+cd^2)^2 (ce^2x^2+2cdex+cd^2)^p}{2c^2e(p+2)}$$

input `integrate((e*x+d)^3*(c*e^2*x^2+2*c*d*e*x+c*d^2)^p,x, algorithm="giac")`

output $1/2*(c*e^2*x^2+2*c*d*e*x+c*d^2)^2*(c*e^2*x^2+2*c*d*e*x+c*d^2)^p/(c^2*e*(p+2))$

Mupad [B] (verification not implemented)

Time = 5.64 (sec) , antiderivative size = 90, normalized size of antiderivative = 2.31

$$\int (d+ex)^3 (cd^2+2cdex+ce^2x^2)^p dx = (cd^2+2cdex+ce^2x^2)^p \left(\frac{d^4}{2e(p+2)} + \frac{e^3x^4}{2(p+2)} + \frac{2d^3x}{p+2} + \frac{3d^2ex^2}{p+2} + \frac{2de^2x^3}{p+2} \right)$$

input `int((d+e*x)^3*(c*d^2+c*e^2*x^2+2*c*d*e*x)^p,x)`

output $(c*d^2+c*e^2*x^2+2*c*d*e*x)^p*(d^4/(2*e*(p+2))+(e^3*x^4)/(2*(p+2)))+(2*d^3*x)/(p+2)+(3*d^2*e*x^2)/(p+2)+(2*d*e^2*x^3)/(p+2)$

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.79

$$\int (d + ex)^3 (cd^2 + 2cdex + ce^2x^2)^p dx$$

$$= \frac{(ce^2x^2 + 2cdex + cd^2)^p (e^4x^4 + 4de^3x^3 + 6d^2e^2x^2 + 4d^3ex + d^4)}{2e(p + 2)}$$

input `int((e*x+d)^3*(c*e^2*x^2+2*c*d*e*x+c*d^2)^p,x)`output `((c*d**2 + 2*c*d*e*x + c*e**2*x**2)**p*(d**4 + 4*d**3*e*x + 6*d**2*e**2*x**2 + 4*d*e**3*x**3 + e**4*x**4))/(2*e*(p + 2))`

3.424 $\int (d + ex) (cd^2 + 2cdex + ce^2x^2)^p dx$

| | |
|---|------|
| Optimal result | 3179 |
| Mathematica [A] (verified) | 3179 |
| Rubi [A] (verified) | 3180 |
| Maple [A] (verified) | 3180 |
| Fricas [A] (verification not implemented) | 3181 |
| Sympy [B] (verification not implemented) | 3181 |
| Maxima [A] (verification not implemented) | 3182 |
| Giac [A] (verification not implemented) | 3183 |
| Mupad [B] (verification not implemented) | 3183 |
| Reduce [B] (verification not implemented) | 3183 |

Optimal result

Integrand size = 28, antiderivative size = 39

$$\int (d + ex) (cd^2 + 2cdex + ce^2x^2)^p dx = \frac{(cd^2 + 2cdex + ce^2x^2)^{1+p}}{2ce(1+p)}$$

output $1/2*(c*e^2*x^2+2*c*d*e*x+c*d^2)^(p+1)/c/e/(p+1)$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.74

$$\int (d + ex) (cd^2 + 2cdex + ce^2x^2)^p dx = \frac{(d + ex)^2 (c(d + ex)^2)^p}{e(2 + 2p)}$$

input $\text{Integrate}[(d + e*x)*(c*d^2 + 2*c*d*e*x + c*e^2*x^2)^p, x]$

output $((d + e*x)^2*(c*(d + e*x)^2)^p)/(e*(2 + 2*p))$

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.036$, Rules used = {1099}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (d + ex) (cd^2 + 2cdex + ce^2x^2)^p dx$$

$$\downarrow 1099$$

$$\frac{(cd^2 + 2cdex + ce^2x^2)^{p+1}}{2ce(p+1)}$$

input `Int[(d + e*x)*(c*d^2 + 2*c*d*e*x + c*e^2*x^2)^p,x]`

output `(c*d^2 + 2*c*d*e*x + c*e^2*x^2)^(1 + p)/(2*c*e*(1 + p))`

Defintions of rubi rules used

rule 1099

```
Int[((d_) + (e_)*(x_))^(m_)*((a_) + (b_)*(x_) + (c_)*(x_)^2)^(p_), x_Sy
mbol] := Simp[e^m*((a + b*x + c*x^2)^(p + (m + 1)/2)/(c^((m + 1)/2)*(m + 2*
p + 1))), x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[b^2 - 4*a*c, 0] && EqQ[
2*c*d - b*e, 0] && IntegerQ[(m - 1)/2]
```

Maple [A] (verified)

Time = 1.35 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.97

| method | result | size |
|---------------|---|------|
| risch | $\frac{(e^2x^2+2dex+d^2)(c(ex+d)^2)^p}{2e(p+1)}$ | 38 |
| gospers | $\frac{(ex+d)^2(x^2ce^2+2cdxe+cd^2)^p}{2e(p+1)}$ | 40 |
| orering | $\frac{(ex+d)^2(x^2ce^2+2cdxe+cd^2)^p}{2e(p+1)}$ | 40 |
| parallelrisch | $\frac{x^2(c(e^2x^2+2dex+d^2))^p d e^2 + 2x(c(e^2x^2+2dex+d^2))^p d^2 e + (c(e^2x^2+2dex+d^2))^p d^3}{2de(p+1)}$ | 94 |
| norman | $\frac{dx e^{p \ln(x^2 c e^2 + 2cdxe + cd^2)}}{p+1} + \frac{d^2 e^{p \ln(x^2 c e^2 + 2cdxe + cd^2)}}{2e(p+1)} + \frac{e x^2 e^{p \ln(x^2 c e^2 + 2cdxe + cd^2)}}{2+2p}$ | 106 |

input `int((e*x+d)*(c*e^2*x^2+2*c*d*e*x+c*d^2)^p,x,method=_RETURNVERBOSE)`

output `1/2*(e^2*x^2+2*d*e*x+d^2)/e/(p+1)*(c*(e*x+d)^2)^p`

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.21

$$\int (d+ex)(cd^2+2cdex+ce^2x^2)^p dx = \frac{(e^2x^2+2dex+d^2)(ce^2x^2+2cdex+cd^2)^p}{2(ep+e)}$$

input `integrate((e*x+d)*(c*e^2*x^2+2*c*d*e*x+c*d^2)^p,x, algorithm="fricas")`

output `1/2*(e^2*x^2 + 2*d*e*x + d^2)*(c*e^2*x^2 + 2*c*d*e*x + c*d^2)^p/(e*p + e)`

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 139 vs. $2(32) = 64$.

Time = 0.26 (sec) , antiderivative size = 139, normalized size of antiderivative = 3.56

$$\int (d + ex) (cd^2 + 2cdex + ce^2x^2)^p dx$$

$$= \begin{cases} \frac{x}{cd} & \text{for } e = 0 \wedge p = -1 \\ dx(cd^2)^p & \text{for } e = 0 \\ \frac{\log\left(\frac{d}{e} + x\right)}{ce} & \text{for } p = -1 \\ \frac{d^2(cd^2 + 2cdex + ce^2x^2)^p}{2ep + 2e} + \frac{2dex(cd^2 + 2cdex + ce^2x^2)^p}{2ep + 2e} + \frac{e^2x^2(cd^2 + 2cdex + ce^2x^2)^p}{2ep + 2e} & \text{otherwise} \end{cases}$$

input `integrate((e*x+d)*(c*e**2*x**2+2*c*d*e*x+c*d**2)**p,x)`

output `Piecewise((x/(c*d), Eq(e, 0) & Eq(p, -1)), (d*x*(c*d**2)**p, Eq(e, 0)), (log(d/e + x)/(c*e), Eq(p, -1)), (d**2*(c*d**2 + 2*c*d*e*x + c*e**2*x**2)**p/(2*e*p + 2*e) + 2*d*e*x*(c*d**2 + 2*c*d*e*x + c*e**2*x**2)**p/(2*e*p + 2*e) + e**2*x**2*(c*d**2 + 2*c*d*e*x + c*e**2*x**2)**p/(2*e*p + 2*e), True))`

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.95

$$\int (d + ex) (cd^2 + 2cdex + ce^2x^2)^p dx = \frac{(ce^2x^2 + 2cdex + cd^2)^{p+1}}{2ce(p+1)}$$

input `integrate((e*x+d)*(c*e^2*x^2+2*c*d*e*x+c*d^2)^p,x, algorithm="maxima")`

output `1/2*(c*e^2*x^2 + 2*c*d*e*x + c*d^2)^(p + 1)/(c*e*(p + 1))`

Giac [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.95

$$\int (d + ex) (cd^2 + 2cdex + ce^2x^2)^p dx = \frac{(ce^2x^2 + 2cdex + cd^2)^{p+1}}{2ce(p+1)}$$

input `integrate((e*x+d)*(c*e^2*x^2+2*c*d*e*x+c*d^2)^p,x, algorithm="giac")`

output `1/2*(c*e^2*x^2 + 2*c*d*e*x + c*d^2)^(p + 1)/(c*e*(p + 1))`

Mupad [B] (verification not implemented)

Time = 5.75 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.46

$$\int (d + ex) (cd^2 + 2cdex + ce^2x^2)^p dx = (cd^2 + 2cdex + ce^2x^2)^p \left(\frac{d^2}{2e(p+1)} + \frac{dx}{p+1} + \frac{ex^2}{2(p+1)} \right)$$

input `int((d + e*x)*(c*d^2 + c*e^2*x^2 + 2*c*d*e*x)^p,x)`

output `(c*d^2 + c*e^2*x^2 + 2*c*d*e*x)^p*(d^2/(2*e*(p + 1)) + (d*x)/(p + 1) + (e*x^2)/(2*(p + 1)))`

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.23

$$\int (d + ex) (cd^2 + 2cdex + ce^2x^2)^p dx = \frac{(ce^2x^2 + 2cdex + cd^2)^p (e^2x^2 + 2dex + d^2)}{2e(p+1)}$$

input `int((e*x+d)*(c*e^2*x^2+2*c*d*e*x+c*d^2)^p,x)`

output $((c*d**2 + 2*c*d*e*x + c*e**2*x**2)**p*(d**2 + 2*d*e*x + e**2*x**2))/(2*e*(p + 1))$

3.425 $\int \frac{(cd^2 + 2cdex + ce^2x^2)^p}{d + ex} dx$

| | |
|---|------|
| Optimal result | 3185 |
| Mathematica [A] (verified) | 3185 |
| Rubi [A] (verified) | 3186 |
| Maple [A] (verified) | 3186 |
| Fricas [A] (verification not implemented) | 3187 |
| Sympy [A] (verification not implemented) | 3188 |
| Maxima [A] (verification not implemented) | 3188 |
| Giac [F] | 3189 |
| Mupad [B] (verification not implemented) | 3189 |
| Reduce [B] (verification not implemented) | 3189 |

Optimal result

Integrand size = 30, antiderivative size = 32

$$\int \frac{(cd^2 + 2cdex + ce^2x^2)^p}{d + ex} dx = \frac{(cd^2 + 2cdex + ce^2x^2)^p}{2ep}$$

output `1/2*(c*e^2*x^2+2*c*d*e*x+c*d^2)^p/e/p`

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.66

$$\int \frac{(cd^2 + 2cdex + ce^2x^2)^p}{d + ex} dx = \frac{(c(d + ex)^2)^p}{2ep}$$

input `Integrate[(c*d^2 + 2*c*d*e*x + c*e^2*x^2)^p/(d + e*x),x]`

output `(c*(d + e*x)^2)^p/(2*e*p)`

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.033$, Rules used = {1099}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(cd^2 + 2cdex + ce^2x^2)^p}{d + ex} dx$$

↓ 1099

$$\frac{(cd^2 + 2cdex + ce^2x^2)^p}{2ep}$$

input `Int[(c*d^2 + 2*c*d*e*x + c*e^2*x^2)^p/(d + e*x),x]`

output `(c*d^2 + 2*c*d*e*x + c*e^2*x^2)^p/(2*e*p)`

Defintions of rubi rules used

rule 1099

```
Int[((d_) + (e_)*(x_))^(m_)*((a_) + (b_)*(x_) + (c_)*(x_)^2)^(p_), x_Sy
mbol] := Simp[e^m*((a + b*x + c*x^2)^(p + (m + 1)/2)/(c^((m + 1)/2)*(m + 2*
p + 1))), x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[b^2 - 4*a*c, 0] && EqQ[
2*c*d - b*e, 0] && IntegerQ[(m - 1)/2]
```

Maple [A] (verified)

Time = 1.57 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.62

| method | result | size |
|-------------------|---|------|
| risch | $\frac{(c(ex+d)^2)^p}{2ep}$ | 20 |
| parallelrisc | $\frac{(c(e^2x^2+2dex+d^2))^p}{2ep}$ | 29 |
| gosper | $\frac{(x^2ce^2+2cdxe+cd^2)^p}{2ep}$ | 31 |
| derivativedivides | $\frac{(x^2ce^2+2cdxe+cd^2)^p}{2ep}$ | 31 |
| default | $\frac{(x^2ce^2+2cdxe+cd^2)^p}{2ep}$ | 31 |
| orering | $\frac{(x^2ce^2+2cdxe+cd^2)^p}{2ep}$ | 31 |
| norman | $\frac{e^{p \ln(x^2ce^2+2cdxe+cd^2)}}{2ep}$ | 33 |

input `int((c*e^2*x^2+2*c*d*e*x+c*d^2)^p/(e*x+d),x,method=_RETURNVERBOSE)`

output `1/2/e/p*(c*(e*x+d)^2)^p`

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.94

$$\int \frac{(cd^2 + 2cdex + ce^2x^2)^p}{d + ex} dx = \frac{(ce^2x^2 + 2cdex + cd^2)^p}{2ep}$$

input `integrate((c*e^2*x^2+2*c*d*e*x+c*d^2)^p/(e*x+d),x, algorithm="fricas")`

output `1/2*(c*e^2*x^2 + 2*c*d*e*x + c*d^2)^p/(e*p)`

Sympy [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.50

$$\int \frac{(cd^2 + 2cdex + ce^2x^2)^p}{d + ex} dx = \begin{cases} \frac{x}{d} & \text{for } e = 0 \wedge p = 0 \\ \frac{x(cd^2)^p}{d} & \text{for } e = 0 \\ \frac{\log\left(\frac{d}{e} + x\right)}{e} & \text{for } p = 0 \\ \frac{(cd^2 + 2cdex + ce^2x^2)^p}{2ep} & \text{otherwise} \end{cases}$$

input `integrate((c**2*x**2+2*c*d*e*x+c*d**2)**p/(e*x+d),x)`output `Piecewise((x/d, Eq(e, 0) & Eq(p, 0)), (x*(c*d**2)**p/d, Eq(e, 0)), (log(d/e + x)/e, Eq(p, 0)), ((c*d**2 + 2*c*d*e*x + c**2*x**2)**p/(2*e*p), True))`**Maxima [A] (verification not implemented)**

Time = 0.04 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.62

$$\int \frac{(cd^2 + 2cdex + ce^2x^2)^p}{d + ex} dx = \frac{(ex + d)^{2p} c^p}{2ep}$$

input `integrate((c*e^2*x^2+2*c*d*e*x+c*d^2)^p/(e*x+d),x, algorithm="maxima")`output `1/2*(e*x + d)^(2*p)*c^p/(e*p)`

Giac [F]

$$\int \frac{(cd^2 + 2cdex + ce^2x^2)^p}{d + ex} dx = \int \frac{(ce^2x^2 + 2cdex + cd^2)^p}{ex + d} dx$$

input `integrate((c*e^2*x^2+2*c*d*e*x+c*d^2)^p/(e*x+d),x, algorithm="giac")`

output `integrate((c*e^2*x^2 + 2*c*d*e*x + c*d^2)^p/(e*x + d), x)`

Mupad [B] (verification not implemented)

Time = 5.37 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.94

$$\int \frac{(cd^2 + 2cdex + ce^2x^2)^p}{d + ex} dx = \frac{(cd^2 + 2cdex + ce^2x^2)^p}{2ep}$$

input `int((c*d^2 + c*e^2*x^2 + 2*c*d*e*x)^p/(d + e*x),x)`

output `(c*d^2 + c*e^2*x^2 + 2*c*d*e*x)^p/(2*e*p)`

Reduce [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.94

$$\int \frac{(cd^2 + 2cdex + ce^2x^2)^p}{d + ex} dx = \frac{(ce^2x^2 + 2cdex + cd^2)^p}{2ep}$$

input `int((c*e^2*x^2+2*c*d*e*x+c*d^2)^p/(e*x+d),x)`

output `(c*d**2 + 2*c*d*e*x + c*e**2*x**2)**p/(2*e*p)`

$$3.426 \quad \int \frac{(cd^2 + 2cdex + ce^2x^2)^p}{(d+ex)^3} dx$$

| | |
|---|------|
| Optimal result | 3190 |
| Mathematica [A] (verified) | 3190 |
| Rubi [A] (verified) | 3191 |
| Maple [A] (verified) | 3191 |
| Fricas [A] (verification not implemented) | 3192 |
| Sympy [B] (verification not implemented) | 3193 |
| Maxima [A] (verification not implemented) | 3193 |
| Giac [F] | 3194 |
| Mupad [B] (verification not implemented) | 3194 |
| Reduce [B] (verification not implemented) | 3194 |

Optimal result

Integrand size = 30, antiderivative size = 39

$$\int \frac{(cd^2 + 2cdex + ce^2x^2)^p}{(d+ex)^3} dx = -\frac{c(cd^2 + 2cdex + ce^2x^2)^{-1+p}}{2e(1-p)}$$

output `-1/2*c*(c*e^2*x^2+2*c*d*e*x+c*d^2)^(-1+p)/e/(1-p)`

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.64

$$\int \frac{(cd^2 + 2cdex + ce^2x^2)^p}{(d+ex)^3} dx = \frac{c(c(d+ex)^2)^{-1+p}}{e(-2+2p)}$$

input `Integrate[(c*d^2 + 2*c*d*e*x + c*e^2*x^2)^p/(d + e*x)^3,x]`

output `(c*(c*(d + e*x)^2)^(-1 + p))/(e*(-2 + 2*p))`

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.033$, Rules used = {1099}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(cd^2 + 2cdex + ce^2x^2)^p}{(d + ex)^3} dx$$

↓ 1099

$$-\frac{c(cd^2 + 2cdex + ce^2x^2)^{p-1}}{2e(1-p)}$$

input `Int[(c*d^2 + 2*c*d*e*x + c*e^2*x^2)^p/(d + e*x)^3,x]`

output `-1/2*(c*(c*d^2 + 2*c*d*e*x + c*e^2*x^2)^(-1 + p))/(e*(1 - p))`

Defintions of rubi rules used

rule 1099

```
Int[((d_) + (e_)*(x_))^(m_)*((a_) + (b_)*(x_) + (c_)*(x_)^2)^(p_), x_Sy
mbol] :> Simp[e^m*((a + b*x + c*x^2)^(p + (m + 1)/2)/(c^((m + 1)/2)*(m + 2*
p + 1))), x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[b^2 - 4*a*c, 0] && EqQ[
2*c*d - b*e, 0] && IntegerQ[(m - 1)/2]
```

Maple [A] (verified)

Time = 1.65 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.74

| method | result | size |
|--------------|---|------|
| risch | $\frac{(c(ex+d)^2)^p}{2e(p-1)(ex+d)^2}$ | 29 |
| parallelrisc | $\frac{(c(e^2x^2+2dex+d^2))^p}{2(ex+d)^2(p-1)e}$ | 38 |
| gosper | $\frac{(x^2ce^2+2cdxe+cd^2)^p}{2(ex+d)^2e(p-1)}$ | 40 |
| orering | $\frac{(x^2ce^2+2cdxe+cd^2)^p}{2(ex+d)^2e(p-1)}$ | 40 |
| norman | $\frac{e^{p \ln(x^2ce^2+2cdxe+cd^2)}}{2e(p-1)(ex+d)^2}$ | 42 |

input `int((c*e^2*x^2+2*c*d*e*x+c*d^2)^p/(e*x+d)^3,x,method=_RETURNVERBOSE)`

output `1/2/e/(p-1)/(e*x+d)^2*(c*(e*x+d)^2)^p`

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.79

$$\int \frac{(cd^2 + 2cdex + ce^2x^2)^p}{(d + ex)^3} dx = \frac{(ce^2x^2 + 2cdex + cd^2)^p}{2(d^2ep - d^2e + (e^3p - e^3)x^2 + 2(de^2p - de^2)x)}$$

input `integrate((c*e^2*x^2+2*c*d*e*x+c*d^2)^p/(e*x+d)^3,x, algorithm="fricas")`

output `1/2*(c*e^2*x^2 + 2*c*d*e*x + c*d^2)^p/(d^2*e*p - d^2*e + (e^3*p - e^3)*x^2 + 2*(d*e^2*p - d*e^2)*x)`

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 100 vs. $2(34) = 68$.

Time = 0.44 (sec) , antiderivative size = 100, normalized size of antiderivative = 2.56

$$\int \frac{(cd^2 + 2cdex + ce^2x^2)^p}{(d + ex)^3} dx = \begin{cases} \frac{cx}{d} & \text{for } e = 0 \wedge p = 1 \\ \frac{x(cd^2)^p}{d^3} & \text{for } e = 0 \\ \frac{c \log\left(\frac{d}{e} + x\right)}{e} & \text{for } p = 1 \\ \frac{(cd^2 + 2cdex + ce^2x^2)^p}{2d^2ep - 2d^2e + 4de^2px - 4de^2x + 2e^3px^2 - 2e^3x^2} & \text{otherwise} \end{cases}$$

input `integrate((c*e**2*x**2+2*c*d*e*x+c*d**2)**p/(e*x+d)**3,x)`

output `Piecewise((c*x/d, Eq(e, 0) & Eq(p, 1)), (x*(c*d**2)**p/d**3, Eq(e, 0)), (c*log(d/e + x)/e, Eq(p, 1)), ((c*d**2 + 2*c*d*e*x + c*e**2*x**2)**p/(2*d**2*e*p - 2*d**2*e + 4*d*e**2*p*x - 4*d*e**2*x + 2*e**3*p*x**2 - 2*e**3*x**2), True))`

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.15

$$\int \frac{(cd^2 + 2cdex + ce^2x^2)^p}{(d + ex)^3} dx = \frac{(ex + d)^{2p}c^p}{2(e^3(p - 1)x^2 + 2de^2(p - 1)x + d^2e(p - 1))}$$

input `integrate((c*e^2*x^2+2*c*d*e*x+c*d^2)^p/(e*x+d)^3,x, algorithm="maxima")`

output `1/2*(e*x + d)^(2*p)*c^p/(e^3*(p - 1)*x^2 + 2*d*e^2*(p - 1)*x + d^2*e*(p - 1))`

Giac [F]

$$\int \frac{(cd^2 + 2cdex + ce^2x^2)^p}{(d + ex)^3} dx = \int \frac{(ce^2x^2 + 2cdex + cd^2)^p}{(ex + d)^3} dx$$

input `integrate((c*e^2*x^2+2*c*d*e*x+c*d^2)^p/(e*x+d)^3,x, algorithm="giac")`

output `integrate((c*e^2*x^2 + 2*c*d*e*x + c*d^2)^p/(e*x + d)^3, x)`

Mupad [B] (verification not implemented)

Time = 5.22 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.33

$$\int \frac{(cd^2 + 2cdex + ce^2x^2)^p}{(d + ex)^3} dx = \frac{(cd^2 + 2cdex + ce^2x^2)^p}{2e^3(p-1)\left(x^2 + \frac{d^2}{e^2} + \frac{2dx}{e}\right)}$$

input `int((c*d^2 + c*e^2*x^2 + 2*c*d*e*x)^p/(d + e*x)^3,x)`

output `(c*d^2 + c*e^2*x^2 + 2*c*d*e*x)^p/(2*e^3*(p - 1)*(x^2 + d^2/e^2 + (2*d*x)/e))`

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.72

$$\int \frac{(cd^2 + 2cdex + ce^2x^2)^p}{(d + ex)^3} dx = \frac{(ce^2x^2 + 2cdex + cd^2)^p}{2e(e^2px^2 + 2dep - e^2x^2 + d^2p - 2dex - d^2)}$$

input `int((c*e^2*x^2+2*c*d*e*x+c*d^2)^p/(e*x+d)^3,x)`

output `(c*d**2 + 2*c*d*e*x + c*e**2*x**2)**p/(2*e*(d**2*p - d**2 + 2*d*e*p*x - 2*d*e*x + e**2*p*x**2 - e**2*x**2))`

$$3.427 \quad \int \frac{(cd^2 + 2cdex + ce^2x^2)^p}{(d+ex)^5} dx$$

| | |
|---|------|
| Optimal result | 3195 |
| Mathematica [A] (verified) | 3195 |
| Rubi [A] (verified) | 3196 |
| Maple [A] (verified) | 3196 |
| Fricas [B] (verification not implemented) | 3197 |
| Sympy [B] (verification not implemented) | 3198 |
| Maxima [A] (verification not implemented) | 3198 |
| Giac [F] | 3199 |
| Mupad [B] (verification not implemented) | 3199 |
| Reduce [B] (verification not implemented) | 3199 |

Optimal result

Integrand size = 30, antiderivative size = 41

$$\int \frac{(cd^2 + 2cdex + ce^2x^2)^p}{(d+ex)^5} dx = -\frac{c^2(cd^2 + 2cdex + ce^2x^2)^{-2+p}}{2e(2-p)}$$

output $-1/2*c^2*(c*e^2*x^2+2*c*d*e*x+c*d^2)^{-2+p}/e/(2-p)$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.71

$$\int \frac{(cd^2 + 2cdex + ce^2x^2)^p}{(d+ex)^5} dx = \frac{(c(d+ex)^2)^p}{e(-4+2p)(d+ex)^4}$$

input $\text{Integrate}[(c*d^2 + 2*c*d*e*x + c*e^2*x^2)^p/(d + e*x)^5, x]$

output $(c*(d + e*x)^2)^p/(e*(-4 + 2*p)*(d + e*x)^4)$

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.033$, Rules used = {1099}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(cd^2 + 2cdex + ce^2x^2)^p}{(d + ex)^5} dx$$

↓ 1099

$$-\frac{c^2(cd^2 + 2cdex + ce^2x^2)^{p-2}}{2e(2-p)}$$

input `Int[(c*d^2 + 2*c*d*e*x + c*e^2*x^2)^p/(d + e*x)^5,x]`

output `-1/2*(c^2*(c*d^2 + 2*c*d*e*x + c*e^2*x^2)^(-2 + p))/(e*(2 - p))`

Defintions of rubi rules used

rule 1099 `Int[((d_) + (e_)*(x_))^(m_)*((a_) + (b_)*(x_) + (c_)*(x_)^2)^(p_), x_Symbol] := Simp[e^m*((a + b*x + c*x^2)^(p + (m + 1)/2)/(c^((m + 1)/2)*(m + 2*p + 1))), x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[b^2 - 4*a*c, 0] && EqQ[2*c*d - b*e, 0] && IntegerQ[(m - 1)/2]`

Maple [A] (verified)

Time = 1.70 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.71

| method | result | size |
|---------------|--|------|
| risch | $\frac{(c(ex+d)^2)^p}{2e(-2+p)(ex+d)^4}$ | 29 |
| parallelrisch | $\frac{(c(e^2x^2+2dex+d^2))^p}{2e(ex+d)^4(-2+p)}$ | 38 |
| gospers | $\frac{(x^2ce^2+2cdxe+cd^2)^p}{2(ex+d)^4e(-2+p)}$ | 40 |
| orering | $\frac{(x^2ce^2+2cdxe+cd^2)^p}{2(ex+d)^4e(-2+p)}$ | 40 |
| norman | $\frac{e^{p \ln(x^2ce^2+2cdxe+cd^2)}}{2e(-2+p)(ex+d)^4}$ | 42 |

input `int((c*e^2*x^2+2*c*d*e*x+c*d^2)^p/(e*x+d)^5,x,method=_RETURNVERBOSE)`

output `1/2/e/(-2+p)/(e*x+d)^4*(c*(e*x+d)^2)^p`

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 114 vs. $2(37) = 74$.

Time = 0.08 (sec) , antiderivative size = 114, normalized size of antiderivative = 2.78

$$\int \frac{(cd^2 + 2cdex + ce^2x^2)^p}{(d + ex)^5} dx$$

$$= \frac{(ce^2x^2 + 2cdex + cd^2)^p}{2(d^4ep - 2d^4e + (e^5p - 2e^5)x^4 + 4(de^4p - 2de^4)x^3 + 6(d^2e^3p - 2d^2e^3)x^2 + 4(d^3e^2p - 2d^3e^2)x)}$$

input `integrate((c*e^2*x^2+2*c*d*e*x+c*d^2)^p/(e*x+d)^5,x, algorithm="fricas")`

output `1/2*(c*e^2*x^2 + 2*c*d*e*x + c*d^2)^p/(d^4*e*p - 2*d^4*e + (e^5*p - 2*e^5)*x^4 + 4*(d*e^4*p - 2*d*e^4)*x^3 + 6*(d^2*e^3*p - 2*d^2*e^3)*x^2 + 4*(d^3*e^2*p - 2*d^3*e^2)*x)`

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 155 vs. $2(36) = 72$.

Time = 0.93 (sec) , antiderivative size = 155, normalized size of antiderivative = 3.78

$$\int \frac{(cd^2 + 2cdex + ce^2x^2)^p}{(d + ex)^5} dx$$

$$= \begin{cases} \frac{c^2x}{d} & \text{for } e = 0 \wedge p = 2 \\ \frac{x(cd^2)^p}{d^5} & \text{for } e = 0 \\ \frac{c^2 \log\left(\frac{d}{e} + x\right)}{e} & \text{for } p = 2 \\ \frac{(cd^2 + 2cdex + ce^2x^2)^p}{2d^4ep - 4d^4e + 8d^3e^2px - 16d^3e^2x + 12d^2e^3px^2 - 24d^2e^3x^2 + 8de^4px^3 - 16de^4x^3 + 2e^5px^4 - 4e^5x^4} & \text{otherwise} \end{cases}$$

input `integrate((c**2*x**2+2*c*d*e*x+c*d**2)**p/(e*x+d)**5,x)`

output `Piecewise((c**2*x/d, Eq(e, 0) & Eq(p, 2)), (x*(c*d**2)**p/d**5, Eq(e, 0)), (c**2*log(d/e + x)/e, Eq(p, 2)), ((c*d**2 + 2*c*d*e*x + c*e**2*x**2)**p/(2*d**4*e*p - 4*d**4*e + 8*d**3*e**2*p*x - 16*d**3*e**2*x + 12*d**2*e**3*p*x**2 - 24*d**2*e**3*x**2 + 8*d*e**4*p*x**3 - 16*d*e**4*x**3 + 2*e**5*p*x**4 - 4*e**5*x**4), True))`

Maxima [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.78

$$\int \frac{(cd^2 + 2cdex + ce^2x^2)^p}{(d + ex)^5} dx$$

$$= \frac{(ex + d)^{2p}c^p}{2(e^5(p-2)x^4 + 4de^4(p-2)x^3 + 6d^2e^3(p-2)x^2 + 4d^3e^2(p-2)x + d^4e(p-2))}$$

input `integrate((c*e^2*x^2+2*c*d*e*x+c*d^2)^p/(e*x+d)^5,x, algorithm="maxima")`

output `1/2*(e*x + d)^(2*p)*c^p/(e^5*(p - 2)*x^4 + 4*d*e^4*(p - 2)*x^3 + 6*d^2*e^3*(p - 2)*x^2 + 4*d^3*e^2*(p - 2)*x + d^4*e*(p - 2))`

Giac [F]

$$\int \frac{(cd^2 + 2cdex + ce^2x^2)^p}{(d + ex)^5} dx = \int \frac{(ce^2x^2 + 2cdex + cd^2)^p}{(ex + d)^5} dx$$

input `integrate((c*e^2*x^2+2*c*d*e*x+c*d^2)^p/(e*x+d)^5,x, algorithm="giac")`

output `integrate((c*e^2*x^2 + 2*c*d*e*x + c*d^2)^p/(e*x + d)^5, x)`

Mupad [B] (verification not implemented)

Time = 5.21 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.80

$$\int \frac{(cd^2 + 2cdex + ce^2x^2)^p}{(d + ex)^5} dx = \frac{(cd^2 + 2cdex + ce^2x^2)^p}{2e^5(p-2)\left(x^4 + \frac{d^4}{e^4} + \frac{4dx^3}{e} + \frac{4d^3x}{e^3} + \frac{6d^2x^2}{e^2}\right)}$$

input `int((c*d^2 + c*e^2*x^2 + 2*c*d*e*x)^p/(d + e*x)^5,x)`

output `(c*d^2 + c*e^2*x^2 + 2*c*d*e*x)^p/(2*e^5*(p - 2)*(x^4 + d^4/e^4 + (4*d*x^3)/e + (4*d^3*x)/e^3 + (6*d^2*x^2)/e^2))`

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 113, normalized size of antiderivative = 2.76

$$\int \frac{(cd^2 + 2cdex + ce^2x^2)^p}{(d + ex)^5} dx = \frac{(ce^2x^2 + 2cdex + cd^2)^p}{2e(e^4px^4 + 4de^3px^3 - 2e^4x^4 + 6d^2e^2px^2 - 8de^3x^3 + 4d^3epx - 12d^2e^2x^2 + d^4p - 8d^3ex - 2d^4)}$$

input `int((c*e^2*x^2+2*c*d*e*x+c*d^2)^p/(e*x+d)^5,x)`

output

```
(c*d**2 + 2*c*d*e*x + c*e**2*x**2)**p/(2*e*(d**4*p - 2*d**4 + 4*d**3*e*p*x  
- 8*d**3*e*x + 6*d**2*e**2*p*x**2 - 12*d**2*e**2*x**2 + 4*d*e**3*p*x**3 -  
8*d*e**3*x**3 + e**4*p*x**4 - 2*e**4*x**4))
```

3.428 $\int (d + ex)^4 (cd^2 + 2cdex + ce^2x^2)^p dx$

| | |
|---|------|
| Optimal result | 3201 |
| Mathematica [A] (verified) | 3201 |
| Rubi [A] (verified) | 3202 |
| Maple [A] (verified) | 3203 |
| Fricas [B] (verification not implemented) | 3204 |
| Sympy [B] (verification not implemented) | 3204 |
| Maxima [B] (verification not implemented) | 3205 |
| Giac [B] (verification not implemented) | 3206 |
| Mupad [B] (verification not implemented) | 3207 |
| Reduce [B] (verification not implemented) | 3207 |

Optimal result

Integrand size = 30, antiderivative size = 40

$$\int (d + ex)^4 (cd^2 + 2cdex + ce^2x^2)^p dx = \frac{(d + ex)^5 (cd^2 + 2cdex + ce^2x^2)^p}{e(5 + 2p)}$$

output

```
(e*x+d)^5*(c*e^2*x^2+2*c*d*e*x+c*d^2)^p/e/(5+2*p)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.72

$$\int (d + ex)^4 (cd^2 + 2cdex + ce^2x^2)^p dx = \frac{(d + ex)^5 (c(d + ex)^2)^p}{e(5 + 2p)}$$

input

```
Integrate[(d + e*x)^4*(c*d^2 + 2*c*d*e*x + c*e^2*x^2)^p,x]
```

output

```
((d + e*x)^5*(c*(d + e*x)^2)^p)/(e*(5 + 2*p))
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.22, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {1102, 35, 17}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (d + ex)^4 (cd^2 + 2cdex + ce^2x^2)^p dx$$

$$\downarrow 1102$$

$$(cde + ce^2x)^{-2p} (cd^2 + 2cdex + ce^2x^2)^p \int (d + ex)^4 (cxe^2 + cde)^{2p} dx$$

$$\downarrow 35$$

$$\frac{(cde + ce^2x)^{-2p} (cd^2 + 2cdex + ce^2x^2)^p \int (cxe^2 + cde)^{2(p+2)} dx}{c^4e^4}$$

$$\downarrow 17$$

$$\frac{(cde + ce^2x)^5 (cd^2 + 2cdex + ce^2x^2)^p}{c^5e^6(2p + 5)}$$

input `Int[(d + e*x)^4*(c*d^2 + 2*c*d*e*x + c*e^2*x^2)^p,x]`

output `((c*d*e + c*e^2*x)^5*(c*d^2 + 2*c*d*e*x + c*e^2*x^2)^p)/(c^5*e^6*(5 + 2*p))`

Defintions of rubi rules used

rule 17 `Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] := Simp[c*((a + b*x)^(m + 1))/(b*(m + 1)), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]`

```
rule 35 Int[(u_.)*((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(n_.), x_Symbol] :=
  Simp[(b/d)^m Int[u*(c + d*x)^(m + n), x], x] /; FreeQ[{a, b, c, d, m, n}, x]
  && EqQ[b*c - a*d, 0] && IntegerQ[m] && !(IntegerQ[n] && SimplerQ[a + b*x, c + d*x])
```

```
rule 1102 Int[((d_.) + (e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :=
  Simp[(a + b*x + c*x^2)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x)^(2*FracPart[p]))
  Int[(d + e*x)^m*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d, e, m, p}, x]
  && EqQ[b^2 - 4*a*c, 0]
```

Maple [A] (verified)

Time = 1.51 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.02

| method | result |
|---------------|---|
| gospers | $\frac{(ex+d)^5 (x^2ce^2+2cdxe+cd^2)^p}{e^{(5+2p)}}$ |
| orering | $\frac{(ex+d)^5 (x^2ce^2+2cdxe+cd^2)^p}{e^{(5+2p)}}$ |
| risch | $\frac{(e^5x^5+5de^4x^4+10d^2e^3x^3+10d^3e^2x^2+5d^4ex+d^5)(c(ex+d)^2)^p}{e^{(5+2p)}}$ |
| parallelrisch | $\frac{x^5(c(e^2x^2+2dex+d^2))^p d e^5 + 5x^4(c(e^2x^2+2dex+d^2))^p d^2 e^4 + 10x^3(c(e^2x^2+2dex+d^2))^p d^3 e^3 + 10x^2(c(e^2x^2+2dex+d^2))^p d^4 e^2 + 5x(c(e^2x^2+2dex+d^2))^p d^5 e}{(5+2p)de}$ |
| norman | $\frac{d^5 e^{p \ln(x^2ce^2+2cdxe+cd^2)}}{e^{(5+2p)}} + \frac{e^4 x^5 e^{p \ln(x^2ce^2+2cdxe+cd^2)}}{5+2p} + \frac{5d^4 x e^{p \ln(x^2ce^2+2cdxe+cd^2)}}{5+2p} + \frac{5d e^3 x^4 e^{p \ln(x^2ce^2+2cdxe+cd^2)}}{5+2p}$ |

```
input int((e*x+d)^4*(c*e^2*x^2+2*c*d*e*x+c*d^2)^p,x,method=_RETURNVERBOSE)
```

```
output (e*x+d)^5*(c*e^2*x^2+2*c*d*e*x+c*d^2)^p/e/(5+2*p)
```


Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 82 vs. $2(40) = 80$.

Time = 0.09 (sec) , antiderivative size = 82, normalized size of antiderivative = 2.05

$$\int (d + ex)^4 (cd^2 + 2cdex + ce^2x^2)^p dx$$

$$= \frac{(e^5x^5 + 5de^4x^4 + 10d^2e^3x^3 + 10d^3e^2x^2 + 5d^4ex + d^5)(ce^2x^2 + 2cdex + cd^2)^p}{2ep + 5e}$$

input `integrate((e*x+d)^4*(c*e^2*x^2+2*c*d*e*x+c*d^2)^p,x, algorithm="fricas")`

output `(e^5*x^5 + 5*d*e^4*x^4 + 10*d^2*e^3*x^3 + 10*d^3*e^2*x^2 + 5*d^4*e*x + d^5)*(c*e^2*x^2 + 2*c*d*e*x + c*d^2)^p/(2*e*p + 5*e)`

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 306 vs. $2(36) = 72$.

Time = 3.73 (sec) , antiderivative size = 306, normalized size of antiderivative = 7.65

$$\int (d + ex)^4 (cd^2 + 2cdex + ce^2x^2)^p dx$$

$$= \begin{cases} \frac{d^4 x}{(cd^2)^{\frac{5}{2}}} \\ d^4 x (cd^2)^p \\ \frac{\left(\frac{d}{e} + x\right) \log\left(\frac{d}{e} + x\right)}{c^2 \sqrt{ce^2 \left(\frac{d}{e} + x\right)^2}} \\ \frac{d^5 (cd^2 + 2cdex + ce^2x^2)^p}{2ep + 5e} + \frac{5d^4 ex (cd^2 + 2cdex + ce^2x^2)^p}{2ep + 5e} + \frac{10d^3 e^2 x^2 (cd^2 + 2cdex + ce^2x^2)^p}{2ep + 5e} + \frac{10d^2 e^3 x^3 (cd^2 + 2cdex + ce^2x^2)^p}{2ep + 5e} + \frac{5de^4 x^4 (cd^2 + 2cdex + ce^2x^2)^p}{2ep + 5e} \end{cases}$$

input `integrate((e*x+d)**4*(c*e**2*x**2+2*c*d*e*x+c*d**2)**p,x)`

output

```
Piecewise((d**4*x/(c*d**2)**(5/2), Eq(e, 0) & Eq(p, -5/2)), (d**4*x*(c*d**2)**p, Eq(e, 0)), ((d/e + x)*log(d/e + x)/(c**2*sqrt(c*e**2*(d/e + x)**2)), Eq(p, -5/2)), (d**5*(c*d**2 + 2*c*d*e*x + c*e**2*x**2)**p/(2*e*p + 5*e) + 5*d**4*e*x*(c*d**2 + 2*c*d*e*x + c*e**2*x**2)**p/(2*e*p + 5*e) + 10*d**3*e**2*x**2*(c*d**2 + 2*c*d*e*x + c*e**2*x**2)**p/(2*e*p + 5*e) + 10*d**2*e**3*x**3*(c*d**2 + 2*c*d*e*x + c*e**2*x**2)**p/(2*e*p + 5*e) + 5*d*e**4*x**4*(c*d**2 + 2*c*d*e*x + c*e**2*x**2)**p/(2*e*p + 5*e) + e**5*x**5*(c*d**2 + 2*c*d*e*x + c*e**2*x**2)**p/(2*e*p + 5*e), True))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 483 vs. $2(40) = 80$.

Time = 0.07 (sec) , antiderivative size = 483, normalized size of antiderivative = 12.08

$$\int (d + ex)^4 (cd^2 + 2cdex + ce^2x^2)^p dx$$

$$= \frac{(c^p ex + c^p d)(ex + d)^{2p} d^4}{e(2p + 1)} + \frac{2(c^p e^2(2p + 1)x^2 + 2c^p depx - c^p d^2)(ex + d)^{2p} d^3}{(2p^2 + 3p + 1)e}$$

$$+ \frac{6((2p^2 + 3p + 1)c^p e^3 x^3 + (2p^2 + p)c^p de^2 x^2 - 2c^p d^2 epx + c^p d^3)(ex + d)^{2p} d^2}{(4p^3 + 12p^2 + 11p + 3)e}$$

$$+ \frac{2((4p^3 + 12p^2 + 11p + 3)c^p e^4 x^4 + 2(2p^3 + 3p^2 + p)c^p de^3 x^3 - 3(2p^2 + p)c^p d^2 e^2 x^2 + 6c^p d^3 epx - 3c^p d^4)(ex + d)^{2p} d}{(4p^4 + 20p^3 + 35p^2 + 25p + 6)e}$$

$$+ \frac{((4p^4 + 20p^3 + 35p^2 + 25p + 6)c^p e^5 x^5 + (4p^4 + 12p^3 + 11p^2 + 3p)c^p de^4 x^4 - 4(2p^3 + 3p^2 + p)c^p d^3 e^3 x^3 + 6c^p d^4 epx - 3c^p d^5)(ex + d)^{2p}}{(8p^5 + 60p^4 + 170p^3 + 225p^2 + 137p + 30)e}$$

input

```
integrate((e*x+d)^4*(c*e^2*x^2+2*c*d*e*x+c*d^2)^p,x, algorithm="maxima")
```

output

```
(c^p*e*x + c^p*d)*(e*x + d)^(2*p)*d^4/(e*(2*p + 1)) + 2*(c^p*e^2*(2*p + 1)
*x^2 + 2*c^p*d*e*p*x - c^p*d^2)*(e*x + d)^(2*p)*d^3/((2*p^2 + 3*p + 1)*e)
+ 6*((2*p^2 + 3*p + 1)*c^p*e^3*x^3 + (2*p^2 + p)*c^p*d*e^2*x^2 - 2*c^p*d^2
*e*p*x + c^p*d^3)*(e*x + d)^(2*p)*d^2/((4*p^3 + 12*p^2 + 11*p + 3)*e) + 2*
((4*p^3 + 12*p^2 + 11*p + 3)*c^p*e^4*x^4 + 2*(2*p^3 + 3*p^2 + p)*c^p*d*e^3
*x^3 - 3*(2*p^2 + p)*c^p*d^2*e^2*x^2 + 6*c^p*d^3*e*p*x - 3*c^p*d^4)*(e*x +
d)^(2*p)*d/((4*p^4 + 20*p^3 + 35*p^2 + 25*p + 6)*e) + ((4*p^4 + 20*p^3 +
35*p^2 + 25*p + 6)*c^p*e^5*x^5 + (4*p^4 + 12*p^3 + 11*p^2 + 3*p)*c^p*d*e^4
*x^4 - 4*(2*p^3 + 3*p^2 + p)*c^p*d^2*e^3*x^3 + 6*(2*p^2 + p)*c^p*d^3*e^2*x
^2 - 12*c^p*d^4*e*p*x + 6*c^p*d^5)*(e*x + d)^(2*p)/((8*p^5 + 60*p^4 + 170*
p^3 + 225*p^2 + 137*p + 30)*e)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 193 vs. $2(40) = 80$.

Time = 0.16 (sec) , antiderivative size = 193, normalized size of antiderivative = 4.82

$$\int (d + ex)^4 (cd^2 + 2cdex + ce^2x^2)^p dx$$

$$= \frac{(ce^2x^2 + 2cdex + cd^2)^p e^5 x^5 + 5(ce^2x^2 + 2cdex + cd^2)^p de^4 x^4 + 10(ce^2x^2 + 2cdex + cd^2)^p d^2 e^3 x^3 + 10(ce^2x^2 + 2cdex + cd^2)^p d e^2 x^2 + 5(ce^2x^2 + 2cdex + cd^2)^p d^2 e x + 5(ce^2x^2 + 2cdex + cd^2)^p d^3}{2ep + 5e}$$

input

```
integrate((e*x+d)^4*(c*e^2*x^2+2*c*d*e*x+c*d^2)^p,x, algorithm="giac")
```

output

```
((c*e^2*x^2 + 2*c*d*e*x + c*d^2)^p*e^5*x^5 + 5*(c*e^2*x^2 + 2*c*d*e*x + c*
d^2)^p*d*e^4*x^4 + 10*(c*e^2*x^2 + 2*c*d*e*x + c*d^2)^p*d^2*e^3*x^3 + 10*(
c*e^2*x^2 + 2*c*d*e*x + c*d^2)^p*d^3*e^2*x^2 + 5*(c*e^2*x^2 + 2*c*d*e*x +
c*d^2)^p*d^4*e*x + (c*e^2*x^2 + 2*c*d*e*x + c*d^2)^p*d^5)/(2*e*p + 5*e)
```

Mupad [B] (verification not implemented)

Time = 5.31 (sec) , antiderivative size = 115, normalized size of antiderivative = 2.88

$$\int (d+ex)^4 (cd^2+2cdex+ce^2x^2)^p dx = (cd^2+2cdex+ce^2x^2)^p \left(\frac{5d^4x}{2p+5} + \frac{d^5}{e(2p+5)} + \frac{e^4x^5}{2p+5} + \frac{10d^2e^2x^3}{2p+5} + \frac{10d^3ex^2}{2p+5} + \frac{5de^3x^4}{2p+5} \right)$$

input `int((d + e*x)^4*(c*d^2 + c*e^2*x^2 + 2*c*d*e*x)^p,x)`output `(c*d^2 + c*e^2*x^2 + 2*c*d*e*x)^p*((5*d^4*x)/(2*p + 5) + d^5/(e*(2*p + 5)) + (e^4*x^5)/(2*p + 5) + (10*d^2*e^2*x^3)/(2*p + 5) + (10*d^3*e*x^2)/(2*p + 5) + (5*d*e^3*x^4)/(2*p + 5))`**Reduce [B] (verification not implemented)**

Time = 0.27 (sec) , antiderivative size = 82, normalized size of antiderivative = 2.05

$$\int (d+ex)^4 (cd^2+2cdex+ce^2x^2)^p dx = \frac{(ce^2x^2+2cdex+cd^2)^p (e^5x^5+5de^4x^4+10d^2e^3x^3+10d^3e^2x^2+5d^4ex+d^5)}{e(2p+5)}$$

input `int((e*x+d)^4*(c*e^2*x^2+2*c*d*e*x+c*d^2)^p,x)`output `((c*d**2 + 2*c*d*e*x + c*e**2*x**2)**p*(d**5 + 5*d**4*e*x + 10*d**3*e**2*x**2 + 10*d**2*e**3*x**3 + 5*d*e**4*x**4 + e**5*x**5))/(e*(2*p + 5))`

3.429 $\int (d + ex)^2 (cd^2 + 2cdex + ce^2x^2)^p dx$

| | |
|---|------|
| Optimal result | 3208 |
| Mathematica [A] (verified) | 3208 |
| Rubi [A] (verified) | 3209 |
| Maple [A] (verified) | 3210 |
| Fricas [A] (verification not implemented) | 3211 |
| Sympy [B] (verification not implemented) | 3211 |
| Maxima [B] (verification not implemented) | 3212 |
| Giac [B] (verification not implemented) | 3212 |
| Mupad [B] (verification not implemented) | 3213 |
| Reduce [B] (verification not implemented) | 3213 |

Optimal result

Integrand size = 30, antiderivative size = 40

$$\int (d + ex)^2 (cd^2 + 2cdex + ce^2x^2)^p dx = \frac{(d + ex)^3 (cd^2 + 2cdex + ce^2x^2)^p}{e(3 + 2p)}$$

output

```
(e*x+d)^3*(c*e^2*x^2+2*c*d*e*x+c*d^2)^p/e/(3+2*p)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.85

$$\int (d + ex)^2 (cd^2 + 2cdex + ce^2x^2)^p dx = \frac{(d + ex) (c(d + ex)^2)^{1+p}}{ce(1 + 2(1 + p))}$$

input

```
Integrate[(d + e*x)^2*(c*d^2 + 2*c*d*e*x + c*e^2*x^2)^p,x]
```

output

```
((d + e*x)*(c*(d + e*x)^2)^(1 + p))/(c*e*(1 + 2*(1 + p)))
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.22, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {1102, 35, 17}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (d + ex)^2 (cd^2 + 2cdex + ce^2x^2)^p dx$$

$$\downarrow 1102$$

$$(cde + ce^2x)^{-2p} (cd^2 + 2cdex + ce^2x^2)^p \int (d + ex)^2 (cxe^2 + cde)^{2p} dx$$

$$\downarrow 35$$

$$\frac{(cde + ce^2x)^{-2p} (cd^2 + 2cdex + ce^2x^2)^p \int (cxe^2 + cde)^{2(p+1)} dx}{c^2e^2}$$

$$\downarrow 17$$

$$\frac{(cde + ce^2x)^3 (cd^2 + 2cdex + ce^2x^2)^p}{c^3e^4(2p + 3)}$$

input `Int[(d + e*x)^2*(c*d^2 + 2*c*d*e*x + c*e^2*x^2)^p,x]`

output `((c*d*e + c*e^2*x)^3*(c*d^2 + 2*c*d*e*x + c*e^2*x^2)^p)/(c^3*e^4*(3 + 2*p))`

Defintions of rubi rules used

rule 17 `Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] := Simp[c*((a + b*x)^(m + 1))/(b*(m + 1)), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]`

```
rule 35 Int[(u_)*((a_) + (b_)*(x_))^(m_)*((c_) + (d_)*(x_))^(n_), x_Symbol] :=
Simp[(b/d)^m Int[u*(c + d*x)^(m + n), x], x] /; FreeQ[{a, b, c, d, m, n}, x]
&& EqQ[b*c - a*d, 0] && IntegerQ[m] && !(IntegerQ[n] && SimplerQ[a + b*x, c + d*x])
```

```
rule 1102 Int[((d_) + (e_)*(x_))^(m_)*((a_) + (b_)*(x_) + (c_)*(x_)^2)^(p_), x_Symbol] :=
Simp[(a + b*x + c*x^2)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x)^(2*FracPart[p]))
Int[(d + e*x)^m*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d, e, m, p}, x]
&& EqQ[b^2 - 4*a*c, 0]
```

Maple [A] (verified)

Time = 1.46 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.02

| method | result |
|---------------|---|
| gospers | $\frac{(ex+d)^3(x^2ce^2+2cdxe+cd^2)^p}{e^{(3+2p)}}$ |
| orering | $\frac{(ex+d)^3(x^2ce^2+2cdxe+cd^2)^p}{e^{(3+2p)}}$ |
| risch | $\frac{(e^3x^3+3de^2x^2+3d^2ex+d^3)(c(ex+d)^2)^p}{e^{(3+2p)}}$ |
| parallelrisch | $\frac{x^3(c(e^2x^2+2dex+d^2))^p d e^3 + 3x^2(c(e^2x^2+2dex+d^2))^p d^2 e^2 + 3x(c(e^2x^2+2dex+d^2))^p d^3 e + (c(e^2x^2+2dex+d^2))^p d^4}{d^{(3+2p)}e}$ |
| norman | $\frac{d^3 e^{p \ln(x^2 c e^2 + 2 c d x e + c d^2)}}{e^{(3+2p)}} + \frac{e^2 x^3 e^{p \ln(x^2 c e^2 + 2 c d x e + c d^2)}}{3+2p} + \frac{3 d^2 x e^{p \ln(x^2 c e^2 + 2 c d x e + c d^2)}}{3+2p} + \frac{3 d e x^2 e^{p \ln(x^2 c e^2 + 2 c d x e + c d^2)}}{3+2p}$ |

```
input int((e*x+d)^2*(c*e^2*x^2+2*c*d*e*x+c*d^2)^p,x,method=_RETURNVERBOSE)
```

```
output (e*x+d)^3*(c*e^2*x^2+2*c*d*e*x+c*d^2)^p/e/(3+2*p)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.50

$$\int (d + ex)^2 (cd^2 + 2cdex + ce^2x^2)^p dx = \frac{(e^3x^3 + 3de^2x^2 + 3d^2ex + d^3)(ce^2x^2 + 2cdex + cd^2)^p}{2ep + 3e}$$

```
input integrate((e*x+d)^2*(c*e^2*x^2+2*c*d*e*x+c*d^2)^p,x, algorithm="fricas")
```

```
output (e^3*x^3 + 3*d*e^2*x^2 + 3*d^2*e*x + d^3)*(c*e^2*x^2 + 2*c*d*e*x + c*d^2)^p/(2*e*p + 3*e)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 216 vs. 2(36) = 72.

Time = 2.49 (sec) , antiderivative size = 216, normalized size of antiderivative = 5.40

$$\int (d + ex)^2 (cd^2 + 2cdex + ce^2x^2)^p dx = \begin{cases} \frac{d^2x}{(cd^2)^{\frac{3}{2}}} & \text{for } e = 0 \\ d^2x(cd^2)^p & \text{for } e = 0 \\ \frac{(\frac{d}{e}+x) \log(\frac{d}{e}+x)}{c\sqrt{ce^2(\frac{d}{e}+x)^2}} & \text{for } p = -\frac{1}{2} \\ \frac{d^3(cd^2+2cdex+ce^2x^2)^p}{2ep+3e} + \frac{3d^2ex(cd^2+2cdex+ce^2x^2)^p}{2ep+3e} + \frac{3de^2x^2(cd^2+2cdex+ce^2x^2)^p}{2ep+3e} + \frac{e^3x^3(cd^2+2cdex+ce^2x^2)^p}{2ep+3e} & \text{otherwise} \end{cases}$$

```
input integrate((e*x+d)**2*(c*e**2*x**2+2*c*d*e*x+c*d**2)**p,x)
```


output

```
Piecewise((d**2*x/(c*d**2)**(3/2), Eq(e, 0) & Eq(p, -3/2)), (d**2*x*(c*d**2)**p, Eq(e, 0)), ((d/e + x)*log(d/e + x)/(c*sqrt(c*e**2*(d/e + x)**2)), Eq(p, -3/2)), (d**3*(c*d**2 + 2*c*d*e*x + c*e**2*x**2)**p/(2*e*p + 3*e) + 3*d**2*e*x*(c*d**2 + 2*c*d*e*x + c*e**2*x**2)**p/(2*e*p + 3*e) + 3*d*e**2*x**2*(c*d**2 + 2*c*d*e*x + c*e**2*x**2)**p/(2*e*p + 3*e) + e**3*x**3*(c*d**2 + 2*c*d*e*x + c*e**2*x**2)**p/(2*e*p + 3*e), True))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 182 vs. $2(40) = 80$.

Time = 0.05 (sec) , antiderivative size = 182, normalized size of antiderivative = 4.55

$$\int (d + ex)^2 (cd^2 + 2cdex + ce^2x^2)^p dx$$

$$= \frac{(c^p ex + c^p d)(ex + d)^{2p} d^2}{e(2p + 1)} + \frac{(c^p e^2(2p + 1)x^2 + 2c^p dex - c^p d^2)(ex + d)^{2p} d}{(2p^2 + 3p + 1)e}$$

$$+ \frac{((2p^2 + 3p + 1)c^p e^3 x^3 + (2p^2 + p)c^p de^2 x^2 - 2c^p d^2 ep x + c^p d^3)(ex + d)^{2p}}{(4p^3 + 12p^2 + 11p + 3)e}$$

input

```
integrate((e*x+d)^2*(c*e^2*x^2+2*c*d*e*x+c*d^2)^p,x, algorithm="maxima")
```

output

```
(c^p*e*x + c^p*d)*(e*x + d)^(2*p)*d^2/(e*(2*p + 1)) + (c^p*e^2*(2*p + 1)*x^2 + 2*c^p*d*e*p*x - c^p*d^2)*(e*x + d)^(2*p)*d/((2*p^2 + 3*p + 1)*e) + ((2*p^2 + 3*p + 1)*c^p*e^3*x^3 + (2*p^2 + p)*c^p*d*e^2*x^2 - 2*c^p*d^2*e*p*x + c^p*d^3)*(e*x + d)^(2*p)/((4*p^3 + 12*p^2 + 11*p + 3)*e)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 127 vs. $2(40) = 80$.

Time = 0.14 (sec) , antiderivative size = 127, normalized size of antiderivative = 3.18

$$\int (d + ex)^2 (cd^2 + 2cdex + ce^2x^2)^p dx$$

$$= \frac{(ce^2x^2 + 2cdex + cd^2)^p e^3 x^3 + 3(ce^2x^2 + 2cdex + cd^2)^p de^2 x^2 + 3(ce^2x^2 + 2cdex + cd^2)^p d^2 ex + (ce^2x^2 + 2cdex + cd^2)^p d^2}{2ep + 3e}$$

input `integrate((e*x+d)^2*(c*e^2*x^2+2*c*d*e*x+c*d^2)^p,x, algorithm="giac")`

output `((c*e^2*x^2 + 2*c*d*e*x + c*d^2)^p*e^3*x^3 + 3*(c*e^2*x^2 + 2*c*d*e*x + c*d^2)^p*d*e^2*x^2 + 3*(c*e^2*x^2 + 2*c*d*e*x + c*d^2)^p*d^2*e*x + (c*e^2*x^2 + 2*c*d*e*x + c*d^2)^p*d^3)/(2*e*p + 3*e)`

Mupad [B] (verification not implemented)

Time = 5.18 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.98

$$\int (d + ex)^2 (cd^2 + 2cdex + ce^2x^2)^p dx = (cd^2 + 2cdex + ce^2x^2)^p \left(\frac{3d^2x}{2p+3} + \frac{d^3}{e(2p+3)} + \frac{e^2x^3}{2p+3} + \frac{3dex^2}{2p+3} \right)$$

input `int((d + e*x)^2*(c*d^2 + c*e^2*x^2 + 2*c*d*e*x)^p,x)`

output `(c*d^2 + c*e^2*x^2 + 2*c*d*e*x)^p*((3*d^2*x)/(2*p + 3) + d^3/(e*(2*p + 3)) + (e^2*x^3)/(2*p + 3) + (3*d*e*x^2)/(2*p + 3))`

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.50

$$\int (d + ex)^2 (cd^2 + 2cdex + ce^2x^2)^p dx = \frac{(ce^2x^2 + 2cdex + cd^2)^p (e^3x^3 + 3de^2x^2 + 3d^2ex + d^3)}{e(2p+3)}$$

input `int((e*x+d)^2*(c*e^2*x^2+2*c*d*e*x+c*d^2)^p,x)`

output `((c*d**2 + 2*c*d*e*x + c*e**2*x**2)**p*(d**3 + 3*d**2*e*x + 3*d*e**2*x**2 + e**3*x**3))/(e*(2*p + 3))`

3.430 $\int (cd^2 + 2cdex + ce^2x^2)^p dx$

| | |
|---|------|
| Optimal result | 3214 |
| Mathematica [A] (verified) | 3214 |
| Rubi [A] (verified) | 3215 |
| Maple [A] (verified) | 3216 |
| Fricas [A] (verification not implemented) | 3216 |
| Sympy [B] (verification not implemented) | 3217 |
| Maxima [A] (verification not implemented) | 3217 |
| Giac [A] (verification not implemented) | 3218 |
| Mupad [B] (verification not implemented) | 3218 |
| Reduce [B] (verification not implemented) | 3218 |

Optimal result

Integrand size = 22, antiderivative size = 38

$$\int (cd^2 + 2cdex + ce^2x^2)^p dx = \frac{(d + ex)(cd^2 + 2cdex + ce^2x^2)^p}{e(1 + 2p)}$$

output

```
(e*x+d)*(c*e^2*x^2+2*c*d*e*x+c*d^2)^p/e/(1+2*p)
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.71

$$\int (cd^2 + 2cdex + ce^2x^2)^p dx = \frac{(d + ex)(c(d + ex)^2)^p}{e(1 + 2p)}$$

input

```
Integrate[(c*d^2 + 2*c*d*e*x + c*e^2*x^2)^p,x]
```

output

```
((d + e*x)*(c*(d + e*x)^2)^p)/(e*(1 + 2*p))
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.24, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {1079, 17}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (cd^2 + 2cdex + ce^2x^2)^p dx$$

$$\downarrow 1079$$

$$(cde + ce^2x)^{-2p} (cd^2 + 2cdex + ce^2x^2)^p \int (cxe^2 + cde)^{2p} dx$$

$$\downarrow 17$$

$$\frac{(cde + ce^2x) (cd^2 + 2cdex + ce^2x^2)^p}{ce^2(2p + 1)}$$

input `Int[(c*d^2 + 2*c*d*e*x + c*e^2*x^2)^p,x]`

output `((c*d*e + c*e^2*x)*(c*d^2 + 2*c*d*e*x + c*e^2*x^2)^p)/(c*e^2*(1 + 2*p))`

Defintions of rubi rules used

rule 17 `Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] := Simp[c*((a + b*x)^(m + 1))/(b*(m + 1)), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]`

rule 1079 `Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] := Simp[(a + b*x + c*x^2)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x)^(2*FracPart[p])) Int[(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c, p}, x] && EqQ[b^2 - 4*a*c, 0]`

Maple [A] (verified)

Time = 1.32 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.74

| method | result | size |
|---------------|---|------|
| risch | $\frac{(ex+d)(c(ex+d)^2)^p}{e^{(2p+1)}}$ | 28 |
| gosper | $\frac{(ex+d)(x^2ce^2+2cdxe+cd^2)^p}{e^{(2p+1)}}$ | 39 |
| orering | $\frac{(ex+d)(x^2ce^2+2cdxe+cd^2)^p}{e^{(2p+1)}}$ | 39 |
| parallelrisch | $\frac{x(c(e^2x^2+2dex+d^2))^p de + (c(e^2x^2+2dex+d^2))^p d^2}{(2p+1)de}$ | 64 |
| norman | $\frac{x e^{p \ln(x^2 c e^2 + 2 c d x e + c d^2)}}{2p+1} + \frac{d e^{p \ln(x^2 c e^2 + 2 c d x e + c d^2)}}{e^{(2p+1)}}$ | 71 |

input `int((c*e^2*x^2+2*c*d*e*x+c*d^2)^p,x,method=_RETURNVERBOSE)`output `(e*x+d)/e/(2*p+1)*(c*(e*x+d)^2)^p`**Fricas [A] (verification not implemented)**

Time = 0.09 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.95

$$\int (cd^2 + 2cdex + ce^2x^2)^p dx = \frac{(ex+d)(ce^2x^2 + 2cdex + cd^2)^p}{2ep + e}$$

input `integrate((c*e^2*x^2+2*c*d*e*x+c*d^2)^p,x, algorithm="fricas")`output `(e*x + d)*(c*e^2*x^2 + 2*c*d*e*x + c*d^2)^p/(2*e*p + e)`

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 116 vs. $2(34) = 68$.

Time = 0.51 (sec) , antiderivative size = 116, normalized size of antiderivative = 3.05

$$\int (cd^2 + 2cdex + ce^2x^2)^p dx = \begin{cases} \frac{x}{\sqrt{cd^2}} & \text{for } e = 0 \wedge p = -\frac{1}{2} \\ x(cd^2)^p & \text{for } e = 0 \\ \frac{\left(\frac{d}{e} + x\right) \log\left(\frac{d}{e} + x\right)}{\sqrt{ce^2\left(\frac{d}{e} + x\right)^2}} & \text{for } p = -\frac{1}{2} \\ \frac{d(cd^2 + 2cdex + ce^2x^2)^p}{2ep + e} + \frac{ex(cd^2 + 2cdex + ce^2x^2)^p}{2ep + e} & \text{otherwise} \end{cases}$$

input `integrate((c*e**2*x**2+2*c*d*e*x+c*d**2)**p,x)`

output `Piecewise((x/sqrt(c*d**2), Eq(e, 0) & Eq(p, -1/2)), (x*(c*d**2)**p, Eq(e, 0)), ((d/e + x)*log(d/e + x)/sqrt(c*e**2*(d/e + x)**2), Eq(p, -1/2)), (d*(c*d**2 + 2*c*d*e*x + c*e**2*x**2)**p/(2*e*p + e) + e*x*(c*d**2 + 2*c*d*e*x + c*e**2*x**2)**p/(2*e*p + e), True))`

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.84

$$\int (cd^2 + 2cdex + ce^2x^2)^p dx = \frac{(c^p ex + c^p d)(ex + d)^{2p}}{e(2p + 1)}$$

input `integrate((c*e^2*x^2+2*c*d*e*x+c*d^2)^p,x, algorithm="maxima")`

output `(c^p*e*x + c^p*d)*(e*x + d)^(2*p)/(e*(2*p + 1))`

Giac [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.55

$$\int (cd^2 + 2cdex + ce^2x^2)^p dx = \frac{(ce^2x^2 + 2cdex + cd^2)^p ex + (ce^2x^2 + 2cdex + cd^2)^p d}{2ep + e}$$

input `integrate((c*e^2*x^2+2*c*d*e*x+c*d^2)^p,x, algorithm="giac")`

output `((c*e^2*x^2 + 2*c*d*e*x + c*d^2)^p*e*x + (c*e^2*x^2 + 2*c*d*e*x + c*d^2)^p*d)/(2*e*p + e)`

Mupad [B] (verification not implemented)

Time = 5.42 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.18

$$\int (cd^2 + 2cdex + ce^2x^2)^p dx = \left(\frac{x}{2p+1} + \frac{d}{e(2p+1)} \right) (cd^2 + 2cdex + ce^2x^2)^p$$

input `int((c*d^2 + c*e^2*x^2 + 2*c*d*e*x)^p,x)`

output `(x/(2*p + 1) + d/(e*(2*p + 1)))*(c*d^2 + c*e^2*x^2 + 2*c*d*e*x)^p`

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.00

$$\int (cd^2 + 2cdex + ce^2x^2)^p dx = \frac{(ce^2x^2 + 2cdex + cd^2)^p (ex + d)}{e(2p+1)}$$

input `int((c*e^2*x^2+2*c*d*e*x+c*d^2)^p,x)`

output `((c*d**2 + 2*c*d*e*x + c*e**2*x**2)**p*(d + e*x))/(e*(2*p + 1))`

$$3.431 \quad \int \frac{(cd^2 + 2cdex + ce^2x^2)^p}{(d+ex)^2} dx$$

| | |
|---|------|
| Optimal result | 3219 |
| Mathematica [A] (verified) | 3219 |
| Rubi [A] (verified) | 3220 |
| Maple [A] (verified) | 3221 |
| Fricas [A] (verification not implemented) | 3222 |
| Sympy [F] | 3222 |
| Maxima [A] (verification not implemented) | 3223 |
| Giac [F] | 3223 |
| Mupad [B] (verification not implemented) | 3223 |
| Reduce [B] (verification not implemented) | 3224 |

Optimal result

Integrand size = 30, antiderivative size = 41

$$\int \frac{(cd^2 + 2cdex + ce^2x^2)^p}{(d+ex)^2} dx = -\frac{(cd^2 + 2cdex + ce^2x^2)^p}{e(1-2p)(d+ex)}$$

output $-(c*e^{2*x^2}+2*c*d*e*x+c*d^2)^p/e/(1-2*p)/(e*x+d)$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.78

$$\int \frac{(cd^2 + 2cdex + ce^2x^2)^p}{(d+ex)^2} dx = \frac{c(d+ex)(c(d+ex)^2)^{-1+p}}{e(1+2(-1+p))}$$

input $\text{Integrate}[(c*d^2 + 2*c*d*e*x + c*e^{2*x^2})^p/(d + e*x)^2,x]$

output $(c*(d + e*x)*(c*(d + e*x)^2)^{-1 + p})/(e*(1 + 2*(-1 + p)))$

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.10, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {1102, 35, 17}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(cd^2 + 2cdex + ce^2x^2)^p}{(d + ex)^2} dx$$

$$\downarrow \text{1102}$$

$$(cde + ce^2x)^{-2p} (cd^2 + 2cdex + ce^2x^2)^p \int \frac{(cxe^2 + cde)^{2p}}{(d + ex)^2} dx$$

$$\downarrow \text{35}$$

$$c^2e^2(cde + ce^2x)^{-2p} (cd^2 + 2cdex + ce^2x^2)^p \int (cxe^2 + cde)^{-2(1-p)} dx$$

$$\downarrow \text{17}$$

$$-\frac{c(cd^2 + 2cdex + ce^2x^2)^p}{(1 - 2p)(cde + ce^2x)}$$

input `Int[(c*d^2 + 2*c*d*e*x + c*e^2*x^2)^p/(d + e*x)^2,x]`

output `-((c*(c*d^2 + 2*c*d*e*x + c*e^2*x^2)^p)/((1 - 2*p)*(c*d*e + c*e^2*x)))`

Defintions of rubi rules used

rule 17 `Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] := Simp[c*((a + b*x)^(m + 1))/(b*(m + 1)), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]`

rule 35 `Int[(u_.)*((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(n_.), x_Symbol] :=
Simp[(b/d)^m Int[u*(c + d*x)^(m + n), x], x] /; FreeQ[{a, b, c, d, m, n},
x] && EqQ[b*c - a*d, 0] && IntegerQ[m] && !(IntegerQ[n] && SimplerQ[a +
b*x, c + d*x])`

rule 1102 `Int[((d_.) + (e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] := Simp[(a + b*x + c*x^2)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x)^(2*F
racPart[p])) Int[(d + e*x)^m*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c,
d, e, m, p}, x] && EqQ[b^2 - 4*a*c, 0]`

Maple [A] (verified)

Time = 1.60 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.73

| method | result | size |
|--------------|--|------|
| risch | $\frac{(c(ex+d)^2)^p}{e(-1+2p)(ex+d)}$ | 30 |
| parallelrisc | $\frac{(c(e^2x^2+2dex+d^2))^p}{e(ex+d)(-1+2p)}$ | 39 |
| gosper | $\frac{(x^2ce^2+2cdxe+cd^2)^p}{(ex+d)e(-1+2p)}$ | 41 |
| orering | $\frac{(x^2ce^2+2cdxe+cd^2)^p}{(ex+d)e(-1+2p)}$ | 41 |
| norman | $\frac{e^{p \ln(x^2ce^2+2cdxe+cd^2)}}{e(-1+2p)(ex+d)}$ | 43 |

input `int((c*e^2*x^2+2*c*d*e*x+c*d^2)^p/(e*x+d)^2,x,method=_RETURNVERBOSE)`

output `1/e/(-1+2*p)/(e*x+d)*(c*(e*x+d)^2)^p`

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.20

$$\int \frac{(cd^2 + 2cdex + ce^2x^2)^p}{(d + ex)^2} dx = \frac{(ce^2x^2 + 2cdex + cd^2)^p}{2dep - de + (2e^2p - e^2)x}$$

input `integrate((c*e^2*x^2+2*c*d*e*x+c*d^2)^p/(e*x+d)^2,x, algorithm="fricas")`

output `(c*e^2*x^2 + 2*c*d*e*x + c*d^2)^p/(2*d*e*p - d*e + (2*e^2*p - e^2)*x)`

Sympy [F]

$$\int \frac{(cd^2 + 2cdex + ce^2x^2)^p}{(d + ex)^2} dx = \begin{cases} \frac{x\sqrt{cd^2}}{d^2} & \text{for } e = 0 \wedge p = \frac{1}{2} \\ \frac{x(cd^2)^p}{d^2} & \text{for } e = 0 \\ \int \frac{\sqrt{c(d+ex)^2}}{(d+ex)^2} dx & \text{for } p = \frac{1}{2} \\ \frac{(cd^2+2cdex+ce^2x^2)^p}{2dep-de+2e^2px-e^2x} & \text{otherwise} \end{cases}$$

input `integrate((c*e**2*x**2+2*c*d*e*x+c*d**2)**p/(e*x+d)**2,x)`

output `Piecewise((x*sqrt(c*d**2)/d**2, Eq(e, 0) & Eq(p, 1/2)), (x*(c*d**2)**p/d**2, Eq(e, 0)), (Integral(sqrt(c*(d + e*x)**2)/(d + e*x)**2, x), Eq(p, 1/2)), ((c*d**2 + 2*c*d*e*x + c*e**2*x**2)**p/(2*d*e*p - d*e + 2*e**2*p*x - e**2*x), True))`

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.83

$$\int \frac{(cd^2 + 2cdex + ce^2x^2)^p}{(d + ex)^2} dx = \frac{(ex + d)^{2p} c^p}{e^2(2p - 1)x + de(2p - 1)}$$

input `integrate((c*e^2*x^2+2*c*d*e*x+c*d^2)^p/(e*x+d)^2,x, algorithm="maxima")`output `(e*x + d)^(2*p)*c^p/(e^2*(2*p - 1)*x + d*e*(2*p - 1))`**Giac [F]**

$$\int \frac{(cd^2 + 2cdex + ce^2x^2)^p}{(d + ex)^2} dx = \int \frac{(ce^2x^2 + 2cdex + cd^2)^p}{(ex + d)^2} dx$$

input `integrate((c*e^2*x^2+2*c*d*e*x+c*d^2)^p/(e*x+d)^2,x, algorithm="giac")`output `integrate((c*e^2*x^2 + 2*c*d*e*x + c*d^2)^p/(e*x + d)^2, x)`**Mupad [B] (verification not implemented)**

Time = 5.14 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.02

$$\int \frac{(cd^2 + 2cdex + ce^2x^2)^p}{(d + ex)^2} dx = \frac{(cd^2 + 2cdex + ce^2x^2)^p}{e^2(2p - 1)(x + \frac{d}{e})}$$

input `int((c*d^2 + c*e^2*x^2 + 2*c*d*e*x)^p/(d + e*x)^2,x)`output `(c*d^2 + c*e^2*x^2 + 2*c*d*e*x)^p/(e^2*(2*p - 1)*(x + d/e))`

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.10

$$\int \frac{(cd^2 + 2cdex + ce^2x^2)^p}{(d + ex)^2} dx = \frac{(ce^2x^2 + 2cdex + cd^2)^p}{e(2epx + 2dp - ex - d)}$$

input `int((c*e^2*x^2+2*c*d*e*x+c*d^2)^p/(e*x+d)^2,x)`

output `(c*d**2 + 2*c*d*e*x + c*e**2*x**2)**p/(e*(2*d*p - d + 2*e*p*x - e*x))`

3.432 $\int \frac{(cd^2 + 2cdex + ce^2x^2)^p}{(d+ex)^4} dx$

| | |
|---|------|
| Optimal result | 3225 |
| Mathematica [A] (verified) | 3225 |
| Rubi [A] (verified) | 3226 |
| Maple [A] (verified) | 3227 |
| Fricas [B] (verification not implemented) | 3228 |
| Sympy [F] | 3228 |
| Maxima [A] (verification not implemented) | 3229 |
| Giac [F] | 3229 |
| Mupad [B] (verification not implemented) | 3229 |
| Reduce [B] (verification not implemented) | 3230 |

Optimal result

Integrand size = 30, antiderivative size = 41

$$\int \frac{(cd^2 + 2cdex + ce^2x^2)^p}{(d + ex)^4} dx = -\frac{(cd^2 + 2cdex + ce^2x^2)^p}{e(3 - 2p)(d + ex)^3}$$

output `-(c*e^2*x^2+2*c*d*e*x+c*d^2)^p/e/(3-2*p)/(e*x+d)^3`

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.71

$$\int \frac{(cd^2 + 2cdex + ce^2x^2)^p}{(d + ex)^4} dx = \frac{(c(d + ex)^2)^p}{e(-3 + 2p)(d + ex)^3}$$

input `Integrate[(c*d^2 + 2*c*d*e*x + c*e^2*x^2)^p/(d + e*x)^4,x]`

output `(c*(d + e*x)^2)^p/(e*(-3 + 2*p)*(d + e*x)^3)`

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.22, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {1102, 35, 17}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(cd^2 + 2cdex + ce^2x^2)^p}{(d + ex)^4} dx$$

↓ 1102

$$(cde + ce^2x)^{-2p} (cd^2 + 2cdex + ce^2x^2)^p \int \frac{(cxe^2 + cde)^{2p}}{(d + ex)^4} dx$$

↓ 35

$$c^4e^4(cde + ce^2x)^{-2p} (cd^2 + 2cdex + ce^2x^2)^p \int (cxe^2 + cde)^{-2(2-p)} dx$$

↓ 17

$$\frac{c^3e^2(cd^2 + 2cdex + ce^2x^2)^p}{(3 - 2p)(cde + ce^2x)^3}$$

input `Int[(c*d^2 + 2*c*d*e*x + c*e^2*x^2)^p/(d + e*x)^4,x]`

output `-((c^3*e^2*(c*d^2 + 2*c*d*e*x + c*e^2*x^2)^p)/((3 - 2*p)*(c*d*e + c*e^2*x)^3))`

Defintions of rubi rules used

rule 17 `Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] := Simp[c*((a + b*x)^(m + 1))/(b*(m + 1)), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]`

rule 35

```
Int[(u_.)*((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(n_.), x_Symbol] :=
Simp[(b/d)^m Int[u*(c + d*x)^(m + n), x], x] /; FreeQ[{a, b, c, d, m, n}, x]
&& EqQ[b*c - a*d, 0] && IntegerQ[m] && !(IntegerQ[n] && SimplerQ[a + b*x, c + d*x])
```

rule 1102

```
Int[((d_.) + (e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :=
Simp[(a + b*x + c*x^2)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x)^(2*FracPart[p]))
Int[(d + e*x)^m*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d, e, m, p}, x]
&& EqQ[b^2 - 4*a*c, 0]
```

Maple [A] (verified)

Time = 1.62 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.73

| method | result | size |
|---------------|--|------|
| risch | $\frac{(c(ex+d)^2)^p}{e^{(2p-3)(ex+d)^3}}$ | 30 |
| parallelrisch | $\frac{(c(e^2x^2+2dex+d^2))^p}{e^{(ex+d)^3(2p-3)}}$ | 39 |
| gosper | $\frac{(x^2ce^2+2cdxe+cd^2)^p}{(ex+d)^3e^{(2p-3)}}$ | 41 |
| orering | $\frac{(x^2ce^2+2cdxe+cd^2)^p}{(ex+d)^3e^{(2p-3)}}$ | 41 |
| norman | $\frac{e^{p \ln(x^2ce^2+2cdxe+cd^2)}}{e^{(2p-3)(ex+d)^3}}$ | 43 |

input

```
int((c*e^2*x^2+2*c*d*e*x+c*d^2)^p/(e*x+d)^4,x,method=_RETURNVERBOSE)
```

output

```
1/e/(2*p-3)/(e*x+d)^3*(c*(e*x+d)^2)^p
```


Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 95 vs. $2(40) = 80$.

Time = 0.08 (sec) , antiderivative size = 95, normalized size of antiderivative = 2.32

$$\int \frac{(cd^2 + 2cdex + ce^2x^2)^p}{(d + ex)^4} dx$$

$$= \frac{(ce^2x^2 + 2cdex + cd^2)^p}{2d^3ep - 3d^3e + (2e^4p - 3e^4)x^3 + 3(2de^3p - 3de^3)x^2 + 3(2d^2e^2p - 3d^2e^2)x}$$

input `integrate((c*e^2*x^2+2*c*d*e*x+c*d^2)^p/(e*x+d)^4,x, algorithm="fricas")`

output `(c*e^2*x^2 + 2*c*d*e*x + c*d^2)^p/(2*d^3*e*p - 3*d^3*e + (2*e^4*p - 3*e^4)*x^3 + 3*(2*d*e^3*p - 3*d*e^3)*x^2 + 3*(2*d^2*e^2*p - 3*d^2*e^2)*x)`

Sympy [F]

$$\int \frac{(cd^2 + 2cdex + ce^2x^2)^p}{(d + ex)^4} dx$$

$$= \begin{cases} \frac{x(cd^2)^{\frac{3}{2}}}{d^4} & \text{for } e = 0 \wedge p = \frac{3}{2} \\ \frac{x(cd^2)^p}{d^4} & \text{for } e = 0 \\ \int \frac{(c(d+ex)^2)^{\frac{3}{2}}}{(d+ex)^4} dx & \text{for } p = \frac{3}{2} \\ \frac{(cd^2+2cdex+ce^2x^2)^p}{2d^3ep-3d^3e+6d^2e^2px-9d^2e^2x+6de^3px^2-9de^3x^2+2e^4px^3-3e^4x^3} & \text{otherwise} \end{cases}$$

input `integrate((c*e**2*x**2+2*c*d*e*x+c*d**2)**p/(e*x+d)**4,x)`

output `Piecewise((x*(c*d**2)**(3/2)/d**4, Eq(e, 0) & Eq(p, 3/2)), (x*(c*d**2)**p/d**4, Eq(e, 0)), (Integral((c*(d + e*x)**2)**(3/2)/(d + e*x)**4, x), Eq(p, 3/2)), ((c*d**2 + 2*c*d*e*x + c*e**2*x**2)**p/(2*d**3*e*p - 3*d**3*e + 6*d**2*e**2*p*x - 9*d**2*e**2*x + 6*d*e**3*p*x**2 - 9*d*e**3*x**2 + 2*e**4*p*x**3 - 3*e**4*x**3), True))`

Maxima [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.61

$$\int \frac{(cd^2 + 2cdex + ce^2x^2)^p}{(d + ex)^4} dx$$

$$= \frac{(ex + d)^{2p} c^p}{e^4(2p - 3)x^3 + 3de^3(2p - 3)x^2 + 3d^2e^2(2p - 3)x + d^3e(2p - 3)}$$

input `integrate((c*e^2*x^2+2*c*d*e*x+c*d^2)^p/(e*x+d)^4,x, algorithm="maxima")`output `(e*x + d)^(2*p)*c^p/(e^4*(2*p - 3)*x^3 + 3*d*e^3*(2*p - 3)*x^2 + 3*d^2*e^2*(2*p - 3)*x + d^3*e*(2*p - 3))`**Giac [F]**

$$\int \frac{(cd^2 + 2cdex + ce^2x^2)^p}{(d + ex)^4} dx = \int \frac{(ce^2x^2 + 2cdex + cd^2)^p}{(ex + d)^4} dx$$

input `integrate((c*e^2*x^2+2*c*d*e*x+c*d^2)^p/(e*x+d)^4,x, algorithm="giac")`output `integrate((c*e^2*x^2 + 2*c*d*e*x + c*d^2)^p/(e*x + d)^4, x)`**Mupad [B] (verification not implemented)**

Time = 5.22 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.56

$$\int \frac{(cd^2 + 2cdex + ce^2x^2)^p}{(d + ex)^4} dx = \frac{(cd^2 + 2cdex + ce^2x^2)^p}{e^4(2p - 3) \left(x^3 + \frac{d^3}{e^3} + \frac{3dx^2}{e} + \frac{3d^2x}{e^2} \right)}$$

input `int((c*d^2 + c*e^2*x^2 + 2*c*d*e*x)^p/(d + e*x)^4,x)`output `(c*d^2 + c*e^2*x^2 + 2*c*d*e*x)^p/(e^4*(2*p - 3)*(x^3 + d^3/e^3 + (3*d*x^2)/e + (3*d^2*x)/e^2))`

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 91, normalized size of antiderivative = 2.22

$$\int \frac{(cd^2 + 2cdex + ce^2x^2)^p}{(d + ex)^4} dx$$

$$= \frac{(ce^2x^2 + 2cdex + cd^2)^p}{e(2e^3px^3 + 6de^2px^2 - 3e^3x^3 + 6d^2epx - 9de^2x^2 + 2d^3p - 9d^2ex - 3d^3)}$$

input `int((c*e^2*x^2+2*c*d*e*x+c*d^2)^p/(e*x+d)^4,x)`output `(c*d**2 + 2*c*d*e*x + c*e**2*x**2)**p/(e*(2*d**3*p - 3*d**3 + 6*d**2*e*p*x - 9*d**2*e*x + 6*d*e**2*p*x**2 - 9*d*e**2*x**2 + 2*e**3*p*x**3 - 3*e**3*x**3))`

3.433 $\int (d + ex)^3 (a^2 + 2abx + b^2x^2)^p dx$

| | |
|---|------|
| Optimal result | 3231 |
| Mathematica [A] (verified) | 3232 |
| Rubi [A] (verified) | 3232 |
| Maple [B] (verified) | 3233 |
| Fricas [B] (verification not implemented) | 3234 |
| Sympy [F] | 3235 |
| Maxima [A] (verification not implemented) | 3236 |
| Giac [B] (verification not implemented) | 3237 |
| Mupad [B] (verification not implemented) | 3238 |
| Reduce [B] (verification not implemented) | 3239 |

Optimal result

Integrand size = 26, antiderivative size = 181

$$\int (d + ex)^3 (a^2 + 2abx + b^2x^2)^p dx = \frac{(bd - ae)^3(a + bx) (a^2 + 2abx + b^2x^2)^p}{b^4(1 + 2p)} + \frac{3e(bd - ae)^2(a + bx)^2 (a^2 + 2abx + b^2x^2)^p}{2b^4(1 + p)} + \frac{3e^2(bd - ae)(a + bx)^3 (a^2 + 2abx + b^2x^2)^p}{b^4(3 + 2p)} + \frac{e^3(a + bx)^4 (a^2 + 2abx + b^2x^2)^p}{2b^4(2 + p)}$$

output

```
(-a*e+b*d)^3*(b*x+a)*(b^2*x^2+2*a*b*x+a^2)^p/b^4/(1+2*p)+3/2*e*(-a*e+b*d)^2*(b*x+a)^2*(b^2*x^2+2*a*b*x+a^2)^p/b^4/(p+1)+3*e^2*(-a*e+b*d)*(b*x+a)^3*(b^2*x^2+2*a*b*x+a^2)^p/b^4/(3+2*p)+1/2*e^3*(b*x+a)^4*(b^2*x^2+2*a*b*x+a^2)^p/b^4/(2+p)
```

Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 107, normalized size of antiderivative = 0.59

$$\int (d + ex)^3 (a^2 + 2abx + b^2x^2)^p dx$$

$$= \frac{(a + bx) ((a + bx)^2)^p \left(\frac{2(bd - ae)^3}{1 + 2p} + \frac{3e(bd - ae)^2(a + bx)}{1 + p} + \frac{6e^2(bd - ae)(a + bx)^2}{3 + 2p} + \frac{e^3(a + bx)^3}{2 + p} \right)}{2b^4}$$

input

```
Integrate[(d + e*x)^3*(a^2 + 2*a*b*x + b^2*x^2)^p,x]
```

output

```
((a + b*x)*((a + b*x)^2)^p*((2*(b*d - a*e)^3)/(1 + 2*p) + (3*e*(b*d - a*e)^2*(a + b*x))/(1 + p) + (6*e^2*(b*d - a*e)*(a + b*x)^2)/(3 + 2*p) + (e^3*(a + b*x)^3)/(2 + p))/((2*b^4))
```

Rubi [A] (verified)Time = 0.56 (sec) , antiderivative size = 175, normalized size of antiderivative = 0.97, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.115$, Rules used = {1102, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (d + ex)^3 (a^2 + 2abx + b^2x^2)^p dx$$

$$\downarrow \text{1102}$$

$$(ab + b^2x)^{-2p} (a^2 + 2abx + b^2x^2)^p \int (xb^2 + ab)^{2p} (d + ex)^3 dx$$

$$\downarrow \text{53}$$

$$(ab + b^2x)^{-2p} (a^2 + 2abx + b^2x^2)^p \int \left(\frac{(bd - ae)^3 (xb^2 + ab)^{2p}}{b^3} + \frac{3e(bd - ae)^2 (xb^2 + ab)^{2p+1}}{b^4} + \frac{3e^2(bd - ae) (xb^2 + ab)^{2p+2}}{b^5} \right) dx$$

$$\downarrow \text{2009}$$

$$(ab + b^2x)^{-2p} (a^2 + 2abx + b^2x^2)^p \left(\frac{e^3(ab + b^2x)^{2(p+2)}}{2b^8(p+2)} + \frac{3e^2(bd - ae)(ab + b^2x)^{2p+3}}{b^7(2p+3)} + \frac{3e(bd - ae)^2(ab + b^2x)^{2p+4}}{2b^6(p+1)} \right)$$

input `Int[(d + e*x)^3*(a^2 + 2*a*b*x + b^2*x^2)^p,x]`

output `((a^2 + 2*a*b*x + b^2*x^2)^p*((3*e*(b*d - a*e)^2*(a*b + b^2*x)^(2*(1 + p)))/(2*b^6*(1 + p)) + (e^3*(a*b + b^2*x)^(2*(2 + p)))/(2*b^8*(2 + p)) + ((b*d - a*e)^3*(a*b + b^2*x)^(1 + 2*p))/(b^5*(1 + 2*p)) + (3*e^2*(b*d - a*e)*(a*b + b^2*x)^(3 + 2*p))/(b^7*(3 + 2*p))))/(a*b + b^2*x)^(2*p)`

Definitions of rubi rules used

rule 53 `Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] := Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && IGtQ[m, 0] && (!IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])`

rule 1102 `Int[((d_.) + (e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] := Simp[(a + b*x + c*x^2)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x)^(2*FracPart[p])) Int[(d + e*x)^m*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && EqQ[b^2 - 4*a*c, 0]`

rule 2009 `Int[u_, x_Symbol] := Simp[IntSum[u, x], x] /; SumQ[u]`

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 404 vs. $2(177) = 354$.

Time = 1.49 (sec) , antiderivative size = 405, normalized size of antiderivative = 2.24

| method | result |
|--------------|---|
| gospers | $-\frac{(b^2x^2+2abx+a^2)^p(-4b^3e^3p^3x^3-12b^3de^2p^3x^2-12b^3e^3p^2x^3+6ab^2e^3p^2x^2-12b^3d^2ep^3x-42b^3de^2p^2x^2-11b^3e^3px^3+12a^3e^3p^2x^2)}{(b^2x^2+2abx+a^2)^p(-4b^3e^3p^3x^3-12b^3de^2p^3x^2-12b^3e^3p^2x^3+6ab^2e^3p^2x^2-12b^3d^2ep^3x-42b^3de^2p^2x^2-11b^3e^3px^3+12a^3e^3p^2x^2)}$ |
| orering | $-\frac{(b^2x^2+2abx+a^2)^p(-4b^3e^3p^3x^3-12b^3de^2p^3x^2-12b^3e^3p^2x^3+6ab^2e^3p^2x^2-12b^3d^2ep^3x-42b^3de^2p^2x^2-11b^3e^3px^3+12a^3e^3p^2x^2)}{(b^2x^2+2abx+a^2)^p(-4b^3e^3p^3x^3-12b^3de^2p^3x^2-12b^3e^3p^2x^3+6ab^2e^3p^2x^2-12b^3d^2ep^3x-42b^3de^2p^2x^2-11b^3e^3px^3+12a^3e^3p^2x^2)}$ |
| norman | $\frac{(6ab^2d^2ep^3+2b^3d^3p^3-6a^2bd^2e^2p^2+21ab^2d^2ep^2+9b^3d^3p^2+3a^3e^3p-12a^2bd^2ep+18ab^2d^2ep+13b^3d^3p+6b^3d^3)x e^{p \ln(b^2x^2+2abx+a^2)}}{b^3(4p^4+20p^3+35p^2+25p+6)}$ |
| risch | $-\frac{(-4b^4e^3p^3x^4-4ab^3e^3p^3x^3-12b^4de^2p^3x^3-12b^4e^3p^2x^4-12ab^3de^2p^3x^2-6ab^3e^3p^2x^3-12b^4d^2ep^3x^2-42b^4de^2p^2x^3-11b^4e^3px^4+12a^3e^3p^2x^2)}{(b^2x^2+2abx+a^2)^p(-4b^3e^3p^3x^3-12b^3de^2p^3x^2-12b^3e^3p^2x^3+6ab^2e^3p^2x^2-12b^3d^2ep^3x-42b^3de^2p^2x^2-11b^3e^3px^3+12a^3e^3p^2x^2)}$ |
| parallelrisc | Expression too large to display |

```
input int((e*x+d)^3*(b^2*x^2+2*a*b*x+a^2)^p,x,method=_RETURNVERBOSE)
```

```
output -1/2*(b^2*x^2+2*a*b*x+a^2)^p*(-4*b^3*e^3*p^3*x^3-12*b^3*d*e^2*p^3*x^2-12*b^3*e^3*p^2*x^3+6*a*b^2*e^3*p^2*x^2-12*b^3*d^2*e*p^3*x-42*b^3*d*e^2*p^2*x^2-11*b^3*e^3*p*x^3+12*a*b^2*d*e^2*p^2*x+9*a*b^2*e^3*p*x^2-4*b^3*d^3*p^3-48*b^3*d^2*e*p^2*x-42*b^3*d*e^2*p*x^2-3*b^3*e^3*x^3-6*a^2*b*e^3*p*x+6*a*b^2*d^2*e*p^2+30*a*b^2*d*e^2*p*x+3*a*b^2*e^3*x^2-18*b^3*d^3*p^2-57*b^3*d^2*e*p*x-12*b^3*d*e^2*x^2-6*a^2*b*d*e^2*p-3*a^2*b*e^3*x+21*a*b^2*d^2*e*p+12*a*b^2*d*e^2*x-26*b^3*d^3*p-18*b^3*d^2*e*x+3*a^3*e^3-12*a^2*b*d*e^2+18*a*b^2*d^2*e-12*b^3*d^3)*(b*x+a)/b^4/(4*p^4+20*p^3+35*p^2+25*p+6)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 516 vs. 2(177) = 354.
 Time = 0.09 (sec) , antiderivative size = 516, normalized size of antiderivative = 2.85

$$\int (d + ex)^3 (a^2 + 2abx + b^2x^2)^p dx$$

$$= \frac{(4ab^3d^3p^3 + 12ab^3d^3 - 18a^2b^2d^2e + 12a^3bde^2 - 3a^4e^3 + (4b^4e^3p^3 + 12b^4e^3p^2 + 11b^4e^3p + 3b^4e^3)x^4 + \dots}{(b^2x^2 + 2abx + a^2)^p(-4b^3e^3p^3x^3 - \dots)}$$

```
input integrate((e*x+d)^3*(b^2*x^2+2*a*b*x+a^2)^p,x, algorithm="fricas")
```

output

```

1/2*(4*a*b^3*d^3*p^3 + 12*a*b^3*d^3 - 18*a^2*b^2*d^2*e + 12*a^3*b*d*e^2 -
3*a^4*e^3 + (4*b^4*e^3*p^3 + 12*b^4*e^3*p^2 + 11*b^4*e^3*p + 3*b^4*e^3)*x^
4 + 2*(6*b^4*d*e^2 + 2*(3*b^4*d*e^2 + a*b^3*e^3)*p^3 + 3*(7*b^4*d*e^2 + a*
b^3*e^3)*p^2 + (21*b^4*d*e^2 + a*b^3*e^3)*p)*x^3 + 6*(3*a*b^3*d^3 - a^2*b^
2*d^2*e)*p^2 + 3*(6*b^4*d^2*e + 4*(b^4*d^2*e + a*b^3*d*e^2)*p^3 + 2*(8*b^4
*d^2*e + 5*a*b^3*d*e^2 - a^2*b^2*e^3)*p^2 + (19*b^4*d^2*e + 4*a*b^3*d*e^2
- a^2*b^2*e^3)*p)*x^2 + (26*a*b^3*d^3 - 21*a^2*b^2*d^2*e + 6*a^3*b*d*e^2)*
p + 2*(6*b^4*d^3 + 2*(b^4*d^3 + 3*a*b^3*d^2*e)*p^3 + 3*(3*b^4*d^3 + 7*a*b^
3*d^2*e - 2*a^2*b^2*d*e^2)*p^2 + (13*b^4*d^3 + 18*a*b^3*d^2*e - 12*a^2*b^2
*d*e^2 + 3*a^3*b*e^3)*p)*x)*(b^2*x^2 + 2*a*b*x + a^2)^p/(4*b^4*p^4 + 20*b^
4*p^3 + 35*b^4*p^2 + 25*b^4*p + 6*b^4)

```

SymPy [F]

$$\int (d + ex)^3 (a^2 + 2abx + b^2x^2)^p dx = \text{Too large to display}$$

input

```
integrate((e*x+d)**3*(b**2*x**2+2*a*b*x+a**2)**p,x)
```


output

```
Piecewise(((d**3*x + 3*d**2*e*x**2/2 + d*e**2*x**3 + e**3*x**4/4)*(a**2)**
p, Eq(b, 0)), (6*a**3*e**3*log(a/b + x)/(6*a**3*b**4 + 18*a**2*b**5*x + 18
*a*b**6*x**2 + 6*b**7*x**3) + 11*a**3*e**3/(6*a**3*b**4 + 18*a**2*b**5*x +
18*a*b**6*x**2 + 6*b**7*x**3) - 6*a**2*b*d*e**2/(6*a**3*b**4 + 18*a**2*b*
*5*x + 18*a*b**6*x**2 + 6*b**7*x**3) + 18*a**2*b*e**3*x*log(a/b + x)/(6*a*
*3*b**4 + 18*a**2*b**5*x + 18*a*b**6*x**2 + 6*b**7*x**3) + 27*a**2*b*e**3*
x/(6*a**3*b**4 + 18*a**2*b**5*x + 18*a*b**6*x**2 + 6*b**7*x**3) - 3*a*b**2
*d**2*e/(6*a**3*b**4 + 18*a**2*b**5*x + 18*a*b**6*x**2 + 6*b**7*x**3) - 18
*a*b**2*d*e**2*x/(6*a**3*b**4 + 18*a**2*b**5*x + 18*a*b**6*x**2 + 6*b**7*x
**3) + 18*a*b**2*e**3*x**2*log(a/b + x)/(6*a**3*b**4 + 18*a**2*b**5*x + 18
*a*b**6*x**2 + 6*b**7*x**3) + 18*a*b**2*e**3*x**2/(6*a**3*b**4 + 18*a**2*b
**5*x + 18*a*b**6*x**2 + 6*b**7*x**3) - 2*b**3*d**3/(6*a**3*b**4 + 18*a**2
*b**5*x + 18*a*b**6*x**2 + 6*b**7*x**3) - 9*b**3*d**2*e*x/(6*a**3*b**4 + 1
8*a**2*b**5*x + 18*a*b**6*x**2 + 6*b**7*x**3) - 18*b**3*d*e**2*x**2/(6*a**
3*b**4 + 18*a**2*b**5*x + 18*a*b**6*x**2 + 6*b**7*x**3) + 6*b**3*e**3*x**3
*log(a/b + x)/(6*a**3*b**4 + 18*a**2*b**5*x + 18*a*b**6*x**2 + 6*b**7*x**3
), Eq(p, -2)), (Integral((d + e*x)**3/((a + b*x)**2)**(3/2), x), Eq(p, -3/
2)), (6*a**3*e**3*log(a/b + x)/(2*a*b**4 + 2*b**5*x) + 6*a**3*e**3/(2*a*b*
*4 + 2*b**5*x) - 12*a**2*b*d*e**2*log(a/b + x)/(2*a*b**4 + 2*b**5*x) - 12*
a**2*b*d*e**2/(2*a*b**4 + 2*b**5*x) + 6*a**2*b*e**3*x*log(a/b + x)/(2*a...
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 276, normalized size of antiderivative = 1.52

$$\int (d + ex)^3 (a^2 + 2abx + b^2x^2)^p dx$$

$$= \frac{(bx + a)(bx + a)^{2p}d^3}{b(2p + 1)} + \frac{3(b^2(2p + 1)x^2 + 2abpx - a^2)(bx + a)^{2p}d^2e}{2(2p^2 + 3p + 1)b^2}$$

$$+ \frac{3((2p^2 + 3p + 1)b^3x^3 + (2p^2 + p)ab^2x^2 - 2a^2bpx + a^3)(bx + a)^{2p}de^2}{(4p^3 + 12p^2 + 11p + 3)b^3}$$

$$+ \frac{((4p^3 + 12p^2 + 11p + 3)b^4x^4 + 2(2p^3 + 3p^2 + p)ab^3x^3 - 3(2p^2 + p)a^2b^2x^2 + 6a^3bpx - 3a^4)(bx + a)^{2p}de^3}{2(4p^4 + 20p^3 + 35p^2 + 25p + 6)b^4}$$

input

```
integrate((e*x+d)^3*(b^2*x^2+2*a*b*x+a^2)^p,x, algorithm="maxima")
```

output

```
(b*x + a)*(b*x + a)^(2*p)*d^3/(b*(2*p + 1)) + 3/2*(b^2*(2*p + 1)*x^2 + 2*a
*b*p*x - a^2)*(b*x + a)^(2*p)*d^2*e/((2*p^2 + 3*p + 1)*b^2) + 3*((2*p^2 +
3*p + 1)*b^3*x^3 + (2*p^2 + p)*a*b^2*x^2 - 2*a^2*b*p*x + a^3)*(b*x + a)^(2
*p)*d*e^2/((4*p^3 + 12*p^2 + 11*p + 3)*b^3) + 1/2*((4*p^3 + 12*p^2 + 11*p
+ 3)*b^4*x^4 + 2*(2*p^3 + 3*p^2 + p)*a*b^3*x^3 - 3*(2*p^2 + p)*a^2*b^2*x^2
+ 6*a^3*b*p*x - 3*a^4)*(b*x + a)^(2*p)*e^3/((4*p^4 + 20*p^3 + 35*p^2 + 25
*p + 6)*b^4)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1279 vs. $2(177) = 354$.

Time = 0.16 (sec) , antiderivative size = 1279, normalized size of antiderivative = 7.07

$$\int (d + ex)^3 (a^2 + 2abx + b^2x^2)^p dx = \text{Too large to display}$$

input

```
integrate((e*x+d)^3*(b^2*x^2+2*a*b*x+a^2)^p,x, algorithm="giac")
```

output

```

1/2*(4*(b^2*x^2 + 2*a*b*x + a^2)^p*b^4*e^3*p^3*x^4 + 12*(b^2*x^2 + 2*a*b*x
+ a^2)^p*b^4*d*e^2*p^3*x^3 + 4*(b^2*x^2 + 2*a*b*x + a^2)^p*a*b^3*e^3*p^3*
x^3 + 12*(b^2*x^2 + 2*a*b*x + a^2)^p*b^4*e^3*p^2*x^4 + 12*(b^2*x^2 + 2*a*b
*x + a^2)^p*b^4*d^2*e*p^3*x^2 + 12*(b^2*x^2 + 2*a*b*x + a^2)^p*a*b^3*d*e^2
*p^3*x^2 + 42*(b^2*x^2 + 2*a*b*x + a^2)^p*b^4*d*e^2*p^2*x^3 + 6*(b^2*x^2 +
2*a*b*x + a^2)^p*a*b^3*e^3*p^2*x^3 + 11*(b^2*x^2 + 2*a*b*x + a^2)^p*b^4*e
^3*p*x^4 + 4*(b^2*x^2 + 2*a*b*x + a^2)^p*b^4*d^3*p^3*x + 12*(b^2*x^2 + 2*a
*b*x + a^2)^p*a*b^3*d^2*e*p^3*x + 48*(b^2*x^2 + 2*a*b*x + a^2)^p*b^4*d^2*e
*p^2*x^2 + 30*(b^2*x^2 + 2*a*b*x + a^2)^p*a*b^3*d*e^2*p^2*x^2 - 6*(b^2*x^2
+ 2*a*b*x + a^2)^p*a^2*b^2*e^3*p^2*x^2 + 42*(b^2*x^2 + 2*a*b*x + a^2)^p*b
^4*d*e^2*p*x^3 + 2*(b^2*x^2 + 2*a*b*x + a^2)^p*a*b^3*e^3*p*x^3 + 3*(b^2*x^
2 + 2*a*b*x + a^2)^p*b^4*e^3*x^4 + 4*(b^2*x^2 + 2*a*b*x + a^2)^p*a*b^3*d^3
*p^3 + 18*(b^2*x^2 + 2*a*b*x + a^2)^p*b^4*d^3*p^2*x + 42*(b^2*x^2 + 2*a*b*
x + a^2)^p*a*b^3*d^2*e*p^2*x - 12*(b^2*x^2 + 2*a*b*x + a^2)^p*a^2*b^2*d*e^
2*p^2*x + 57*(b^2*x^2 + 2*a*b*x + a^2)^p*b^4*d^2*e*p*x^2 + 12*(b^2*x^2 + 2
*a*b*x + a^2)^p*a*b^3*d*e^2*p*x^2 - 3*(b^2*x^2 + 2*a*b*x + a^2)^p*a^2*b^2*
e^3*p*x^2 + 12*(b^2*x^2 + 2*a*b*x + a^2)^p*b^4*d*e^2*x^3 + 18*(b^2*x^2 + 2
*a*b*x + a^2)^p*a*b^3*d^3*p^2 - 6*(b^2*x^2 + 2*a*b*x + a^2)^p*a^2*b^2*d^2*
e*p^2 + 26*(b^2*x^2 + 2*a*b*x + a^2)^p*b^4*d^3*p*x + 36*(b^2*x^2 + 2*a*b*x
+ a^2)^p*a*b^3*d^2*e*p*x - 24*(b^2*x^2 + 2*a*b*x + a^2)^p*a^2*b^2*d*e^...

```

Mupad [B] (verification not implemented)

Time = 5.35 (sec) , antiderivative size = 484, normalized size of antiderivative = 2.67

$$\begin{aligned}
& \int (d + ex)^3 (a^2 + 2abx + b^2x^2)^p dx \\
&= (a^2 + 2abx + b^2x^2)^p \left(\frac{a(-3a^3e^3 + 6a^2bde^2p + 12a^2bde^2 - 6ab^2d^2ep^2 - 21ab^2d^2ep - 18ab^2d^2}{2b^4(4p^4 + 20p^3 + 35p^2 + 25p + 6)} \right. \\
&\quad + \frac{e^3x^4(4p^3 + 12p^2 + 11p + 3)}{2(4p^4 + 20p^3 + 35p^2 + 25p + 6)} \\
&\quad + \frac{x(6a^3be^3p - 12a^2b^2de^2p^2 - 24a^2b^2de^2p + 12ab^3d^2ep^3 + 42ab^3d^2ep^2 + 36ab^3d^2ep + 4b^4d^3}{2b^4(4p^4 + 20p^3 + 35p^2 + 25p + 6)} \\
&\quad + \frac{3ex^2(2p + 1)(-a^2e^2p + 2abdep^2 + 4abdep + 2b^2d^2p^2 + 7b^2d^2p + 6b^2d^2)}{2b^2(4p^4 + 20p^3 + 35p^2 + 25p + 6)} \\
&\quad \left. + \frac{e^2x^3(2p^2 + 3p + 1)(6bd + aep + 3bdp)}{b(4p^4 + 20p^3 + 35p^2 + 25p + 6)} \right)
\end{aligned}$$

input `int((d + e*x)^3*(a^2 + b^2*x^2 + 2*a*b*x)^p,x)`

output
$$(a^2 + b^2x^2 + 2abx)^p \left(\frac{a(12b^3d^3 - 3a^3e^3 + 26b^3d^3p + 18b^3d^3p^2 + 4b^3d^3p^3 - 18ab^2d^2e + 12a^2bde^2 - 21ab^2d^2ep + 6a^2bde^2p - 6ab^2d^2ep^2)}{(2b^4(25p + 35p^2 + 20p^3 + 4p^4 + 6))} + \frac{e^3x^4(11p + 12p^2 + 4p^3 + 3)}{(2(25p + 35p^2 + 20p^3 + 4p^4 + 6))} + \frac{x(12b^4d^3 + 26b^4d^3p + 18b^4d^3p^2 + 4b^4d^3p^3 + 6a^3b^3e^3p + 36ab^3d^2ep - 24a^2b^2de^2p + 42ab^3d^2ep^2 + 12ab^3d^2ep^3 - 12a^2b^2de^2p^2)}{(2b^4(25p + 35p^2 + 20p^3 + 4p^4 + 6))} + \frac{(3ex^2(2p + 1)(6b^2d^2 - a^2e^2p + 7b^2d^2p + 2b^2d^2p^2 + 4abd^2ep + 2abd^2ep^2)}{(2b^2(25p + 35p^2 + 20p^3 + 4p^4 + 6))} + \frac{e^2x^3(3p + 2p^2 + 1)(6bd + aep + 3bd^2p)}{(b(25p + 35p^2 + 20p^3 + 4p^4 + 6))} \right)$$

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 564, normalized size of antiderivative = 3.12

$$\int (d + ex)^3 (a^2 + 2abx + b^2x^2)^p dx$$

$$= \frac{(b^2x^2 + 2abx + a^2)^p (4b^4e^3p^3x^4 + 4ab^3e^3p^3x^3 + 12b^4de^2p^3x^3 + 12b^4e^3p^2x^4 + 12ab^3de^2p^3x^2 + 6ab^3e^3p^2x^2 + 6ab^3e^3p^2x^2 + 6ab^3e^3p^2x^2)}{(b^2x^2 + 2abx + a^2)^p (4b^4e^3p^3x^4 + 4ab^3e^3p^3x^3 + 12b^4de^2p^3x^3 + 12b^4e^3p^2x^4 + 12ab^3de^2p^3x^2 + 6ab^3e^3p^2x^2 + 6ab^3e^3p^2x^2)}$$

input `int((e*x+d)^3*(b^2*x^2+2*a*b*x+a^2)^p,x)`

output

$$\frac{((a^{**2} + 2*a*b*x + b^{**2}*x^{**2})*p*(- 3*a^{**4}*e^{**3} + 6*a^{**3}*b*d*e^{**2}*p + 12*a^{**3}*b*d*e^{**2} + 6*a^{**3}*b*e^{**3}*p*x - 6*a^{**2}*b^{**2}*d^{**2}*e*p^{**2} - 21*a^{**2}*b^{**2}*d^{**2}*e*p - 18*a^{**2}*b^{**2}*d^{**2}*e - 12*a^{**2}*b^{**2}*d*e^{**2}*p^{**2}*x - 24*a^{**2}*b^{**2}*d*e^{**2}*p*x - 6*a^{**2}*b^{**2}*e^{**3}*p^{**2}*x^{**2} - 3*a^{**2}*b^{**2}*e^{**3}*p*x^{**2} + 4*a*b^{**3}*d^{**3}*p^{**3} + 18*a*b^{**3}*d^{**3}*p^{**2} + 26*a*b^{**3}*d^{**3}*p + 12*a*b^{**3}*d^{**3} + 12*a*b^{**3}*d^{**2}*e*p^{**3}*x + 42*a*b^{**3}*d^{**2}*e*p^{**2}*x + 36*a*b^{**3}*d^{**2}*e*p*x + 12*a*b^{**3}*d*e^{**2}*p^{**3}*x^{**2} + 30*a*b^{**3}*d*e^{**2}*p^{**2}*x^{**2} + 12*a*b^{**3}*d*e^{**2}*p*x^{**2} + 4*a*b^{**3}*e^{**3}*p^{**3}*x^{**3} + 6*a*b^{**3}*e^{**3}*p^{**2}*x^{**3} + 2*a*b^{**3}*e^{**3}*p*x^{**3} + 4*b^{**4}*d^{**3}*p^{**3}*x + 18*b^{**4}*d^{**3}*p^{**2}*x + 26*b^{**4}*d^{**3}*p*x + 12*b^{**4}*d^{**3}*x + 12*b^{**4}*d^{**2}*e*p^{**3}*x^{**2} + 48*b^{**4}*d^{**2}*e*p^{**2}*x^{**2} + 57*b^{**4}*d^{**2}*e*p*x^{**2} + 18*b^{**4}*d^{**2}*e*x^{**2} + 12*b^{**4}*d*e^{**2}*p^{**3}*x^{**3} + 42*b^{**4}*d*e^{**2}*p^{**2}*x^{**3} + 42*b^{**4}*d*e^{**2}*p*x^{**3} + 12*b^{**4}*d*e^{**2}*x^{**3} + 4*b^{**4}*e^{**3}*p^{**3}*x^{**4} + 12*b^{**4}*e^{**3}*p^{**2}*x^{**4} + 11*b^{**4}*e^{**3}*p*x^{**4} + 3*b^{**4}*e^{**3}*x^{**4})/(2*b^{**4}*(4*p^{**4} + 20*p^{**3} + 35*p^{**2} + 25*p + 6))$$

3.434 $\int (d + ex)^2 (a^2 + 2abx + b^2x^2)^p dx$

| | |
|---|------|
| Optimal result | 3241 |
| Mathematica [A] (verified) | 3241 |
| Rubi [A] (verified) | 3242 |
| Maple [A] (verified) | 3243 |
| Fricas [A] (verification not implemented) | 3244 |
| Sympy [F] | 3244 |
| Maxima [A] (verification not implemented) | 3245 |
| Giac [B] (verification not implemented) | 3246 |
| Mupad [B] (verification not implemented) | 3247 |
| Reduce [B] (verification not implemented) | 3247 |

Optimal result

Integrand size = 26, antiderivative size = 127

$$\int (d + ex)^2 (a^2 + 2abx + b^2x^2)^p dx = \frac{(bd - ae)^2(a + bx)(a^2 + 2abx + b^2x^2)^p}{b^3(1 + 2p)} + \frac{e(bd - ae)(a + bx)^2(a^2 + 2abx + b^2x^2)^p}{b^3(1 + p)} + \frac{e^2(a + bx)^3(a^2 + 2abx + b^2x^2)^p}{b^3(3 + 2p)}$$

output

```
(-a*e+b*d)^2*(b*x+a)*(b^2*x^2+2*a*b*x+a^2)^p/b^3/(1+2*p)+e*(-a*e+b*d)*(b*x+a)^2*(b^2*x^2+2*a*b*x+a^2)^p/b^3/(p+1)+e^2*(b*x+a)^3*(b^2*x^2+2*a*b*x+a^2)^p/b^3/(3+2*p)
```

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.59

$$\int (d + ex)^2 (a^2 + 2abx + b^2x^2)^p dx = \frac{(a + bx) ((a + bx)^2)^p \left(\frac{(bd - ae)^2}{1 + 2p} + \frac{e(bd - ae)(a + bx)}{1 + p} + \frac{e^2(a + bx)^2}{3 + 2p} \right)}{b^3}$$

input `Integrate[(d + e*x)^2*(a^2 + 2*a*b*x + b^2*x^2)^p,x]`

output $((a + b*x)*((a + b*x)^2)^p*((b*d - a*e)^2/(1 + 2*p) + (e*(b*d - a*e)*(a + b*x))/(1 + p) + (e^2*(a + b*x)^2)/(3 + 2*p))/b^3$

Rubi [A] (verified)

Time = 0.47 (sec) , antiderivative size = 131, normalized size of antiderivative = 1.03, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.115$, Rules used = {1102, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (d + ex)^2 (a^2 + 2abx + b^2x^2)^p dx$$

$$\downarrow 1102$$

$$(ab + b^2x)^{-2p} (a^2 + 2abx + b^2x^2)^p \int (xb^2 + ab)^{2p} (d + ex)^2 dx$$

$$\downarrow 53$$

$$(ab + b^2x)^{-2p} (a^2 + 2abx + b^2x^2)^p \int \left(\frac{(bd - ae)^2 (xb^2 + ab)^{2p}}{b^2} + \frac{2e(bd - ae) (xb^2 + ab)^{2p+1}}{b^3} + \frac{e^2 (xb^2 + ab)^{2p+2}}{b^4} \right) dx$$

$$\downarrow 2009$$

$$(ab + b^2x)^{-2p} (a^2 + 2abx + b^2x^2)^p \left(\frac{e^2 (ab + b^2x)^{2p+3}}{b^6(2p+3)} + \frac{e(bd - ae) (ab + b^2x)^{2(p+1)}}{b^5(p+1)} + \frac{(bd - ae)^2 (ab + b^2x)^{2p}}{b^4(2p+1)} \right)$$

input `Int[(d + e*x)^2*(a^2 + 2*a*b*x + b^2*x^2)^p,x]`

output $((a^2 + 2*a*b*x + b^2*x^2)^p*((e*(b*d - a*e)*(a*b + b^2*x)^(2*(1 + p)))/(b^5*(1 + p)) + ((b*d - a*e)^2*(a*b + b^2*x)^(1 + 2*p))/(b^4*(1 + 2*p)) + (e^2*(a*b + b^2*x)^(3 + 2*p))/(b^6*(3 + 2*p)))/(a*b + b^2*x)^(2*p)$

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 249, normalized size of antiderivative = 1.96

$$\int (d + ex)^2 (a^2 + 2abx + b^2x^2)^p dx$$

$$= \frac{(2ab^2d^2p^2 + 3ab^2d^2 - 3a^2bde + a^3e^2 + (2b^3e^2p^2 + 3b^3e^2p + b^3e^2)x^3 + (3b^3de + 2(2b^3de + ab^2e^2)p^2 + 4b^3de)p)x^2 + (2ab^2d^2p^2 + 3ab^2d^2 - 3a^2bde + a^3e^2 + (2b^3e^2p^2 + 3b^3e^2p + b^3e^2)x^3 + (3b^3de + 2(2b^3de + ab^2e^2)p^2 + 4b^3de)p)x + (2ab^2d^2p^2 + 3ab^2d^2 - 3a^2bde + a^3e^2 + (2b^3e^2p^2 + 3b^3e^2p + b^3e^2)x^3 + (3b^3de + 2(2b^3de + ab^2e^2)p^2 + 4b^3de)p)}{4b^3p^3 + 12b^3p^2 + 11b^3p + 3b^3}$$

input `integrate((e*x+d)^2*(b^2*x^2+2*a*b*x+a^2)^p,x, algorithm="fricas")`

output `(2*a*b^2*d^2*p^2 + 3*a*b^2*d^2 - 3*a^2*b*d*e + a^3*e^2 + (2*b^3*e^2*p^2 + 3*b^3*e^2*p + b^3*e^2)*x^3 + (3*b^3*d*e + 2*(2*b^3*d*e + a*b^2*e^2)*p^2 + (8*b^3*d*e + a*b^2*e^2)*p)*x^2 + (5*a*b^2*d^2 - 2*a^2*b*d*e)*p + (3*b^3*d^2 + 2*(b^3*d^2 + 2*a*b^2*d*e)*p^2 + (5*b^3*d^2 + 6*a*b^2*d*e - 2*a^2*b*e^2)*p)*x*(b^2*x^2 + 2*a*b*x + a^2)^p/(4*b^3*p^3 + 12*b^3*p^2 + 11*b^3*p + 3*b^3)`

Sympy [F]

$$\int (d + ex)^2 (a^2 + 2abx + b^2x^2)^p dx = \text{Too large to display}$$

input `integrate((e*x+d)**2*(b**2*x**2+2*a*b*x+a**2)**p,x)`

output

```
Piecewise(((d**2*x + d*e*x**2 + e**2*x**3/3)*(a**2)**p, Eq(b, 0)), (Integral((d + e*x)**2/((a + b*x)**2)**(3/2), x), Eq(p, -3/2)), (-2*a**2*e**2*log(a/b + x)/(a*b**3 + b**4*x) - 2*a**2*e**2/(a*b**3 + b**4*x) + 2*a*b*d*e*log(a/b + x)/(a*b**3 + b**4*x) + 2*a*b*d*e/(a*b**3 + b**4*x) - 2*a*b*e**2*x*log(a/b + x)/(a*b**3 + b**4*x) - b**2*d**2/(a*b**3 + b**4*x) + 2*b**2*d*e*x*log(a/b + x)/(a*b**3 + b**4*x) + b**2*e**2*x**2/(a*b**3 + b**4*x), Eq(p, -1)), (Piecewise(((e**2*x/(2*b**2) + (-3*a*e**2/(2*b) + 2*d*e)/b**2)*sqrt(a**2 + 2*a*b*x + b**2*x**2) + (a/b + x)*(-a**2*e**2/(2*b**2) - a*(-3*a*e**2/(2*b) + 2*d*e)/b + d**2)*log(a/b + x)/sqrt(b**2*(a/b + x)**2), Ne(b**2, 0)), ((sqrt(a**2 + 2*a*b*x)*(a**2*e**2 - 4*a*b*d*e + 4*b**2*d**2)/(4*b**2) + (a**2 + 2*a*b*x)**(3/2)*(-a*e**2 + 2*b*d*e)/(6*a*b**2) + e**2*(a**2 + 2*a*b*x)**(5/2)/(20*a**2*b**2)))/(a*b), Ne(a*b, 0)), (Piecewise((d**2*x, Eq(e, 0)), ((d + e*x)**3/(3*e), True))/sqrt(a**2), True)), Eq(p, -1/2)), (a**3*e**2*(a**2 + 2*a*b*x + b**2*x**2)**p/(4*b**3*p**3 + 12*b**3*p**2 + 11*b**3*p + 3*b**3) - 2*a**2*b*d*e*p*(a**2 + 2*a*b*x + b**2*x**2)**p/(4*b**3*p**3 + 12*b**3*p**2 + 11*b**3*p + 3*b**3) - 3*a**2*b*d*e*(a**2 + 2*a*b*x + b**2*x**2)**p/(4*b**3*p**3 + 12*b**3*p**2 + 11*b**3*p + 3*b**3) - 2*a**2*b*e**2*p*x*(a**2 + 2*a*b*x + b**2*x**2)**p/(4*b**3*p**3 + 12*b**3*p**2 + 11*b**3*p + 3*b**3) + 2*a*b**2*d**2*p**2*(a**2 + 2*a*b*x + b**2*x**2)**p/(4*b**3*p**3 + 12*b**3*p**2 + 11*b**3*p + 3*b**3) + 5*a*b**2*d**2*p*(a**2 ...
```

Maxima [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 157, normalized size of antiderivative = 1.24

$$\int (d + ex)^2 (a^2 + 2abx + b^2x^2)^p dx$$

$$= \frac{(bx + a)(bx + a)^{2p}d^2}{b(2p + 1)} + \frac{(b^2(2p + 1)x^2 + 2abpx - a^2)(bx + a)^{2p}de}{(2p^2 + 3p + 1)b^2}$$

$$+ \frac{((2p^2 + 3p + 1)b^3x^3 + (2p^2 + p)ab^2x^2 - 2a^2bpx + a^3)(bx + a)^{2p}e^2}{(4p^3 + 12p^2 + 11p + 3)b^3}$$

input

```
integrate((e*x+d)^2*(b^2*x^2+2*a*b*x+a^2)^p,x, algorithm="maxima")
```

output

```
(b*x + a)*(b*x + a)^(2*p)*d^2/(b*(2*p + 1)) + (b^2*(2*p + 1)*x^2 + 2*a*b*x*x - a^2)*(b*x + a)^(2*p)*d*e/((2*p^2 + 3*p + 1)*b^2) + ((2*p^2 + 3*p + 1)*b^3*x^3 + (2*p^2 + p)*a*b^2*x^2 - 2*a^2*b*p*x + a^3)*(b*x + a)^(2*p)*e^2/((4*p^3 + 12*p^2 + 11*p + 3)*b^3)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 608 vs. $2(127) = 254$.

Time = 0.15 (sec) , antiderivative size = 608, normalized size of antiderivative = 4.79

$$\int (d + ex)^2 (a^2 + 2abx + b^2x^2)^p dx$$

$$= \frac{2(b^2x^2 + 2abx + a^2)^p b^3 e^2 p^2 x^3 + 4(b^2x^2 + 2abx + a^2)^p b^3 d e p^2 x^2 + 2(b^2x^2 + 2abx + a^2)^p a b^2 e^2 p^2 x^2 + 3(b^2x^2 + 2abx + a^2)^p a^2 b e^2 p^2 x + 3(b^2x^2 + 2abx + a^2)^p a^3 e^2 p^2}{(4b^3 p^3 + 12b^3 p^2 + 11b^3 p + 3b^3)}$$

input

```
integrate((e*x+d)^2*(b^2*x^2+2*a*b*x+a^2)^p,x, algorithm="giac")
```

output

```
(2*(b^2*x^2 + 2*a*b*x + a^2)^p*b^3*e^2*p^2*x^3 + 4*(b^2*x^2 + 2*a*b*x + a^2)^p*b^3*d*e*p^2*x^2 + 2*(b^2*x^2 + 2*a*b*x + a^2)^p*a*b^2*e^2*p^2*x^2 + 3*(b^2*x^2 + 2*a*b*x + a^2)^p*b^3*e^2*p*x^3 + 2*(b^2*x^2 + 2*a*b*x + a^2)^p*b^3*d^2*p^2*x + 4*(b^2*x^2 + 2*a*b*x + a^2)^p*a*b^2*d*e*p^2*x + 8*(b^2*x^2 + 2*a*b*x + a^2)^p*b^3*d*e*p*x^2 + (b^2*x^2 + 2*a*b*x + a^2)^p*a*b^2*e^2*p*x^2 + (b^2*x^2 + 2*a*b*x + a^2)^p*b^3*e^2*x^3 + 2*(b^2*x^2 + 2*a*b*x + a^2)^p*a*b^2*d^2*p^2 + 5*(b^2*x^2 + 2*a*b*x + a^2)^p*b^3*d^2*p*x + 6*(b^2*x^2 + 2*a*b*x + a^2)^p*a*b^2*d*e*p*x - 2*(b^2*x^2 + 2*a*b*x + a^2)^p*a^2*b*e^2*p*x + 3*(b^2*x^2 + 2*a*b*x + a^2)^p*b^3*d*e*x^2 + 5*(b^2*x^2 + 2*a*b*x + a^2)^p*a*b^2*d^2*p - 2*(b^2*x^2 + 2*a*b*x + a^2)^p*a^2*b*d*e*p + 3*(b^2*x^2 + 2*a*b*x + a^2)^p*b^3*d^2*x + 3*(b^2*x^2 + 2*a*b*x + a^2)^p*a*b^2*d^2 - 3*(b^2*x^2 + 2*a*b*x + a^2)^p*a^2*b*d*e + (b^2*x^2 + 2*a*b*x + a^2)^p*a^3*e^2)/(4*b^3*p^3 + 12*b^3*p^2 + 11*b^3*p + 3*b^3)
```

Mupad [B] (verification not implemented)

Time = 5.16 (sec) , antiderivative size = 250, normalized size of antiderivative = 1.97

$$\int (d + ex)^2 (a^2 + 2abx + b^2x^2)^p dx$$

$$= (a^2 + 2abx + b^2x^2)^p \left(\frac{e^2 x^3 (2p^2 + 3p + 1)}{4p^3 + 12p^2 + 11p + 3} \right.$$

$$+ \frac{x(-2a^2 b e^2 p + 4a b^2 d e p^2 + 6a b^2 d e p + 2b^3 d^2 p^2 + 5b^3 d^2 p + 3b^3 d^2)}{b^3 (4p^3 + 12p^2 + 11p + 3)}$$

$$+ \frac{a(a^2 e^2 - 2ab d e p - 3ab d e + 2b^2 d^2 p^2 + 5b^2 d^2 p + 3b^2 d^2)}{b^3 (4p^3 + 12p^2 + 11p + 3)}$$

$$\left. + \frac{e x^2 (2p + 1) (3bd + a e p + 2bd p)}{b (4p^3 + 12p^2 + 11p + 3)} \right)$$

input `int((d + e*x)^2*(a^2 + b^2*x^2 + 2*a*b*x)^p,x)`output `(a^2 + b^2*x^2 + 2*a*b*x)^p*((e^2*x^3*(3*p + 2*p^2 + 1))/(11*p + 12*p^2 + 4*p^3 + 3) + (x*(3*b^3*d^2 + 5*b^3*d^2*p + 2*b^3*d^2*p^2 - 2*a^2*b*e^2*p + 4*a*b^2*d*e*p^2 + 6*a*b^2*d*e*p))/(b^3*(11*p + 12*p^2 + 4*p^3 + 3)) + (a*(a^2*e^2 + 3*b^2*d^2 + 5*b^2*d^2*p + 2*b^2*d^2*p^2 - 3*a*b*d*e - 2*a*b*d*e*p))/(b^3*(11*p + 12*p^2 + 4*p^3 + 3)) + (e*x^2*(2*p + 1)*(3*b*d + a*e*p + 2*b*d*p))/(b*(11*p + 12*p^2 + 4*p^3 + 3)))`**Reduce [B] (verification not implemented)**

Time = 0.21 (sec) , antiderivative size = 256, normalized size of antiderivative = 2.02

$$\int (d + ex)^2 (a^2 + 2abx + b^2x^2)^p dx$$

$$= \frac{(b^2x^2 + 2abx + a^2)^p (2b^3e^2p^2x^3 + 2ab^2e^2p^2x^2 + 4b^3de p^2x^2 + 3b^3e^2p x^3 + 4ab^2de p^2x + ab^2e^2p x^2 + 2b^3d^2x^2 + 2ab^2de p x + ab^2e^2p x^2 + 2b^3d^2x^2 + 2ab^2de p x + ab^2e^2p x^2 + 2b^3d^2x^2)}{b^3(4p^3 + 12p^2 + 11p + 3)}$$

input `int((e*x+d)^2*(b^2*x^2+2*a*b*x+a^2)^p,x)`

output

```
((a**2 + 2*a*b*x + b**2*x**2)**p*(a**3*e**2 - 2*a**2*b*d*e*p - 3*a**2*b*d*  
e - 2*a**2*b*e**2*p*x + 2*a*b**2*d**2*p**2 + 5*a*b**2*d**2*p + 3*a*b**2*d*  
*2 + 4*a*b**2*d*e*p**2*x + 6*a*b**2*d*e*p*x + 2*a*b**2*e**2*p**2*x**2 + a*  
b**2*e**2*p*x**2 + 2*b**3*d**2*p**2*x + 5*b**3*d**2*p*x + 3*b**3*d**2*x +  
4*b**3*d*e*p**2*x**2 + 8*b**3*d*e*p*x**2 + 3*b**3*d*e*x**2 + 2*b**3*e**2*p  
**2*x**3 + 3*b**3*e**2*p*x**3 + b**3*e**2*x**3))/(b**3*(4*p**3 + 12*p**2 +  
11*p + 3))
```

3.435 $\int (d + ex) (a^2 + 2abx + b^2x^2)^p dx$

| | |
|---|------|
| Optimal result | 3249 |
| Mathematica [A] (verified) | 3249 |
| Rubi [A] (verified) | 3250 |
| Maple [A] (verified) | 3251 |
| Fricas [A] (verification not implemented) | 3252 |
| Sympy [B] (verification not implemented) | 3252 |
| Maxima [A] (verification not implemented) | 3253 |
| Giac [B] (verification not implemented) | 3254 |
| Mupad [B] (verification not implemented) | 3254 |
| Reduce [B] (verification not implemented) | 3255 |

Optimal result

Integrand size = 24, antiderivative size = 76

$$\int (d + ex) (a^2 + 2abx + b^2x^2)^p dx = \frac{(bd - ae)(a + bx) (a^2 + 2abx + b^2x^2)^p}{b^2(1 + 2p)} + \frac{e(a^2 + 2abx + b^2x^2)^{1+p}}{2b^2(1 + p)}$$

output

$(-a*e+b*d)*(b*x+a)*(b^2*x^2+2*a*b*x+a^2)^p/b^2/(1+2*p)+1/2*e*(b^2*x^2+2*a*b*x+a^2)^(p+1)/b^2/(p+1)$

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.71

$$\int (d + ex) (a^2 + 2abx + b^2x^2)^p dx = \frac{(a + bx) ((a + bx)^2)^p (-ae + 2bd(1 + p) + be(1 + 2p)x)}{2b^2(1 + p)(1 + 2p)}$$

input

`Integrate[(d + e*x)*(a^2 + 2*a*b*x + b^2*x^2)^p,x]`

output

$$\frac{((a + b*x)*((a + b*x)^2)^p*(-(a*e) + 2*b*d*(1 + p) + b*e*(1 + 2*p)*x))/(2*b^2*(1 + p)*(1 + 2*p))$$

Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.05, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {1100, 1079, 17}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (d + ex) (a^2 + 2abx + b^2x^2)^p dx$$

$$\downarrow 1100$$

$$\frac{(bd - ae) \int (a^2 + 2bxa + b^2x^2)^p dx}{b} + \frac{e(a^2 + 2abx + b^2x^2)^{p+1}}{2b^2(p + 1)}$$

$$\downarrow 1079$$

$$\frac{(bd - ae) (a^2 + 2abx + b^2x^2)^p (ab + b^2x)^{-2p} \int (xb^2 + ab)^{2p} dx}{b} + \frac{e(a^2 + 2abx + b^2x^2)^{p+1}}{2b^2(p + 1)}$$

$$\downarrow 17$$

$$\frac{e(a^2 + 2abx + b^2x^2)^{p+1}}{2b^2(p + 1)} + \frac{(ab + b^2x) (bd - ae) (a^2 + 2abx + b^2x^2)^p}{b^3(2p + 1)}$$

input

$$\text{Int}[(d + e*x)*(a^2 + 2*a*b*x + b^2*x^2)^p, x]$$

output

$$\frac{((b*d - a*e)*(a*b + b^2*x)*(a^2 + 2*a*b*x + b^2*x^2)^p)/(b^3*(1 + 2*p)) + (e*(a^2 + 2*a*b*x + b^2*x^2)^(1 + p))/(2*b^2*(1 + p))$$

Defintions of rubi rules used

rule 17 $\text{Int}[(c_.)*((a_.) + (b_.)*(x_))^m, x_Symbol] \rightarrow \text{Simp}[c*((a + b*x)^{m + 1})/(b*(m + 1)), x] /; \text{FreeQ}[\{a, b, c, m\}, x] \ \&\& \ \text{NeQ}[m, -1]$

rule 1079 $\text{Int}[(a + (b_.)*(x) + (c_.)*(x)^2)^p, x_Symbol] \rightarrow \text{Simp}[(a + b*x + c*x^2)^{\text{FracPart}[p]} / (c^{\text{IntPart}[p]} * (b/2 + c*x)^{(2*\text{FracPart}[p])}) \ \text{Int}[(b/2 + c*x)^{(2*p)}, x], x] /; \text{FreeQ}[\{a, b, c, p\}, x] \ \&\& \ \text{EqQ}[b^2 - 4*a*c, 0]$

rule 1100 $\text{Int}[(d_.) + (e_.)*(x))*((a + (b_.)*(x) + (c_.)*(x)^2)^p, x_Symbol] \rightarrow \text{Simp}[e*((a + b*x + c*x^2)^{p + 1}) / (2*c*(p + 1)), x] + \text{Simp}[(2*c*d - b*e) / (2*c) \ \text{Int}[(a + b*x + c*x^2)^p, x], x] /; \text{FreeQ}[\{a, b, c, d, e, p\}, x] \ \&\& \ \text{EqQ}[b^2 - 4*a*c, 0]$

Maple [A] (verified)

Time = 1.34 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.86

| method | result |
|---------------|--|
| gospers | $-\frac{(b^2x^2+2abx+a^2)^p(-2bepx-2bdp-bex+ae-2bd)(bx+a)}{2b^2(2p^2+3p+1)}$ |
| orering | $-\frac{(b^2x^2+2abx+a^2)^p(-2bepx-2bdp-bex+ae-2bd)(bx+a)}{2b^2(2p^2+3p+1)}$ |
| risch | $-\frac{(-2b^2epx^2-2abepx-2b^2dp-x^2b^2e-2abdpx-2b^2dx+a^2e-2dab)(bx+a)^p}{2b^2(p+1)(2p+1)}$ |
| norman | $\frac{(aep+bdp+bd)x e^{p \ln(b^2x^2+2abx+a^2)}}{b(2p^2+3p+1)} + \frac{e x^2 e^{p \ln(b^2x^2+2abx+a^2)}}{2+2p} - \frac{a(-2bdp+ae-2bd)e^{p \ln(b^2x^2+2abx+a^2)}}{2b^2(2p^2+3p+1)}$ |
| parallelrisch | $\frac{2x^2(b^2x^2+2abx+a^2)^p a b^2 ep + x^2(b^2x^2+2abx+a^2)^p a b^2 e + 2x(b^2x^2+2abx+a^2)^p a^2 bep + 2x(b^2x^2+2abx+a^2)^p a b^2 dp + 2x(b^2x^2+2abx+a^2)^p a b^2 e}{2(p+1)(2p+1)a b^2}$ |

input $\text{int}((e*x+d)*(b^2*x^2+2*a*b*x+a^2)^p, x, \text{method}=_RETURNVERBOSE)$

output $-1/2*(b^2*x^2+2*a*b*x+a^2)^p*(-2*b*e*p*x-2*b*d*p-b*e*x+a*e-2*b*d)*(b*x+a)/b^2/(2*p^2+3*p+1)$

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 96, normalized size of antiderivative = 1.26

$$\int (d + ex) (a^2 + 2abx + b^2x^2)^p dx$$

$$= \frac{(2abd p + 2abd - a^2e + (2b^2ep + b^2e)x^2 + 2(b^2d + (b^2d + abe)p)x)(b^2x^2 + 2abx + a^2)^p}{2(2b^2p^2 + 3b^2p + b^2)}$$

input `integrate((e*x+d)*(b^2*x^2+2*a*b*x+a^2)^p,x, algorithm="fricas")`

output `1/2*(2*a*b*d*p + 2*a*b*d - a^2*e + (2*b^2*e*p + b^2*e)*x^2 + 2*(b^2*d + (b^2*d + a*b*e)*p)*x)*(b^2*x^2 + 2*a*b*x + a^2)^p/(2*b^2*p^2 + 3*b^2*p + b^2)`

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 604 vs. 2(68) = 136.

Time = 0.85 (sec) , antiderivative size = 604, normalized size of antiderivative = 7.95

$$\int (d + ex) (a^2 + 2abx + b^2x^2)^p dx$$

$$= \left\{ \begin{array}{l} \left(dx + \frac{ex^2}{2} \right) (a^2)^p \\ \frac{ae \log\left(\frac{a}{b} + x\right)}{ab^2 + b^3x} + \frac{ae}{ab^2 + b^3x} - \frac{bd}{ab^2 + b^3x} + \frac{bex \log\left(\frac{a}{b} + x\right)}{ab^2 + b^3x} \\ \frac{\left(\frac{a}{b} + x\right) \left(-\frac{ae}{b} + d\right) \log\left(\frac{a}{b} + x\right)}{\sqrt{b^2\left(\frac{a}{b} + x\right)^2}} + \frac{e\sqrt{a^2 + 2abx + b^2x^2}}{b^2} \quad \text{for } b^2 \neq 0 \\ \frac{2d\sqrt{a^2 + 2abx} + \frac{e\left(-a^2\sqrt{a^2 + 2abx} + \frac{(a^2 + 2abx)^{\frac{3}{2}}}{3}\right)}{ab}}{2ab} \quad \text{for } ab \neq 0 \\ \frac{dx + \frac{ex^2}{2}}{\sqrt{a^2}} \quad \text{otherwise} \end{array} \right.$$

$$- \frac{a^2e(a^2 + 2abx + b^2x^2)^p}{4b^2p^2 + 6b^2p + 2b^2} + \frac{2abd p(a^2 + 2abx + b^2x^2)^p}{4b^2p^2 + 6b^2p + 2b^2} + \frac{2abd(a^2 + 2abx + b^2x^2)^p}{4b^2p^2 + 6b^2p + 2b^2} + \frac{2abepx(a^2 + 2abx + b^2x^2)^p}{4b^2p^2 + 6b^2p + 2b^2} + \frac{2b^2dpx(a^2 + 2abx + b^2x^2)^p}{4b^2p^2 + 6b^2p + 2b^2}$$

input `integrate((e*x+d)*(b**2*x**2+2*a*b*x+a**2)**p,x)`

output

```
Piecewise(((d*x + e*x**2/2)*(a**2)**p, Eq(b, 0)), (a*e*log(a/b + x)/(a*b**2 + b**3*x) + a*e/(a*b**2 + b**3*x) - b*d/(a*b**2 + b**3*x) + b*e*x*log(a/b + x)/(a*b**2 + b**3*x), Eq(p, -1)), (Piecewise(((a/b + x)*(-a*e/b + d)*log(a/b + x)/sqrt(b**2*(a/b + x)**2) + e*sqrt(a**2 + 2*a*b*x + b**2*x**2)/b**2, Ne(b**2, 0)), ((2*d*sqrt(a**2 + 2*a*b*x) + e*(-a**2*sqrt(a**2 + 2*a*b*x) + (a**2 + 2*a*b*x)**(3/2)/3)/(a*b))/(2*a*b), Ne(a*b, 0))), ((d*x + e*x**2/2)/sqrt(a**2), True)), Eq(p, -1/2)), (-a**2*e*(a**2 + 2*a*b*x + b**2*x**2)**p/(4*b**2*p**2 + 6*b**2*p + 2*b**2) + 2*a*b*d*p*(a**2 + 2*a*b*x + b**2*x**2)**p/(4*b**2*p**2 + 6*b**2*p + 2*b**2) + 2*a*b*d*(a**2 + 2*a*b*x + b**2*x**2)**p/(4*b**2*p**2 + 6*b**2*p + 2*b**2) + 2*b**2*d*p*x*(a**2 + 2*a*b*x + b**2*x**2)**p/(4*b**2*p**2 + 6*b**2*p + 2*b**2) + 2*b**2*d*x*(a**2 + 2*a*b*x + b**2*x**2)**p/(4*b**2*p**2 + 6*b**2*p + 2*b**2) + 2*b**2*e*p*x**2*(a**2 + 2*a*b*x + b**2*x**2)**p/(4*b**2*p**2 + 6*b**2*p + 2*b**2) + b**2*e*x**2*(a**2 + 2*a*b*x + b**2*x**2)**p/(4*b**2*p**2 + 6*b**2*p + 2*b**2), True))
```

Maxima [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.03

$$\int (d + ex) (a^2 + 2abx + b^2x^2)^p dx = \frac{(bx + a)(bx + a)^{2p}d}{b(2p + 1)} + \frac{(b^2(2p + 1)x^2 + 2abpx - a^2)(bx + a)^{2p}e}{2(2p^2 + 3p + 1)b^2}$$

input

```
integrate((e*x+d)*(b^2*x^2+2*a*b*x+a^2)^p,x, algorithm="maxima")
```

output

```
(b*x + a)*(b*x + a)^(2*p)*d/(b*(2*p + 1)) + 1/2*(b^2*(2*p + 1)*x^2 + 2*a*b*p*x - a^2)*(b*x + a)^(2*p)*e/((2*p^2 + 3*p + 1)*b^2)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 224 vs. $2(74) = 148$.

Time = 0.17 (sec) , antiderivative size = 224, normalized size of antiderivative = 2.95

$$\int (d + ex) (a^2 + 2abx + b^2x^2)^p dx$$

$$= \frac{2(b^2x^2 + 2abx + a^2)^p b^2 e p x^2 + 2(b^2x^2 + 2abx + a^2)^p b^2 d p x + 2(b^2x^2 + 2abx + a^2)^p a b e p x + (b^2x^2 + 2abx + a^2)^p a^2 e p x^2 + 2(b^2x^2 + 2abx + a^2)^p a^2 d p x + 2(b^2x^2 + 2abx + a^2)^p a^2 b e p x + (b^2x^2 + 2abx + a^2)^p a^2 b d p x}{2(b^2x^2 + 2abx + a^2)^p (2p^2 + 3p + 1)}$$

input `integrate((e*x+d)*(b^2*x^2+2*a*b*x+a^2)^p,x, algorithm="giac")`

output
$$\frac{1}{2} \cdot \frac{2 \cdot (b^2 x^2 + 2 a b x + a^2)^p b^2 e p x^2 + 2 \cdot (b^2 x^2 + 2 a b x + a^2)^p b^2 d p x + (b^2 x^2 + 2 a b x + a^2)^p a b e p x + 2 \cdot (b^2 x^2 + 2 a b x + a^2)^p a b d p x + 2 \cdot (b^2 x^2 + 2 a b x + a^2)^p a^2 e p x^2 + 2 \cdot (b^2 x^2 + 2 a b x + a^2)^p a^2 d p x + 2 \cdot (b^2 x^2 + 2 a b x + a^2)^p a^2 b e p x + (b^2 x^2 + 2 a b x + a^2)^p a^2 b d p x}{2 \cdot (b^2 x^2 + 2 a b x + a^2)^p (2 p^2 + 3 p + 1)}$$

Mupad [B] (verification not implemented)

Time = 5.45 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.47

$$\int (d + ex) (a^2 + 2abx + b^2x^2)^p dx = \left(\frac{x(2b^2d + 2b^2dp + 2abep)}{2b^2(2p^2 + 3p + 1)} + \frac{a(2bd - ae + 2bdp)}{2b^2(2p^2 + 3p + 1)} + \frac{ex^2(2p + 1)}{2(2p^2 + 3p + 1)} \right) (a^2 + 2abx + b^2x^2)^p$$

input `int((d + e*x)*(a^2 + b^2*x^2 + 2*a*b*x)^p,x)`

output
$$\left(\frac{x(2b^2d + 2b^2dp + 2abep)}{2b^2(3p + 2p^2 + 1)} + \frac{a(2bd - ae + 2bdp)}{2b^2(3p + 2p^2 + 1)} + \frac{ex^2(2p + 1)}{2(3p + 2p^2 + 1)} \right) (a^2 + b^2x^2 + 2abx)^p$$

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 93, normalized size of antiderivative = 1.22

$$\int (d + ex) (a^2 + 2abx + b^2x^2)^p dx$$

$$= \frac{(b^2x^2 + 2abx + a^2)^p (2b^2epx^2 + 2abepx + 2b^2dpx + b^2ex^2 + 2abdpx + 2b^2dx - a^2e + 2abd)}{2b^2(2p^2 + 3p + 1)}$$

input `int((e*x+d)*(b^2*x^2+2*a*b*x+a^2)^p,x)`output `((a**2 + 2*a*b*x + b**2*x**2)**p*(- a**2*e + 2*a*b*d*p + 2*a*b*d + 2*a*b*e*p*x + 2*b**2*d*p*x + 2*b**2*d*x + 2*b**2*e*p*x**2 + b**2*e*x**2))/(2*b**2*(2*p**2 + 3*p + 1))`

3.436 $\int (a^2 + 2abx + b^2x^2)^p dx$

| | |
|---|------|
| Optimal result | 3256 |
| Mathematica [A] (verified) | 3256 |
| Rubi [A] (verified) | 3257 |
| Maple [A] (verified) | 3258 |
| Fricas [A] (verification not implemented) | 3258 |
| Sympy [B] (verification not implemented) | 3259 |
| Maxima [A] (verification not implemented) | 3259 |
| Giac [A] (verification not implemented) | 3260 |
| Mupad [B] (verification not implemented) | 3260 |
| Reduce [B] (verification not implemented) | 3260 |

Optimal result

Integrand size = 18, antiderivative size = 34

$$\int (a^2 + 2abx + b^2x^2)^p dx = \frac{(a + bx)(a^2 + 2abx + b^2x^2)^p}{b(1 + 2p)}$$

output

```
(b*x+a)*(b^2*x^2+2*a*b*x+a^2)^p/b/(1+2*p)
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.74

$$\int (a^2 + 2abx + b^2x^2)^p dx = \frac{(a + bx)((a + bx)^2)^p}{b(1 + 2p)}$$

input

```
Integrate[(a^2 + 2*a*b*x + b^2*x^2)^p,x]
```

output

```
((a + b*x)*((a + b*x)^2)^p)/(b*(1 + 2*p))
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.12, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {1079, 17}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a^2 + 2abx + b^2x^2)^p dx$$

$$\downarrow 1079$$

$$(ab + b^2x)^{-2p} (a^2 + 2abx + b^2x^2)^p \int (xb^2 + ab)^{2p} dx$$

$$\downarrow 17$$

$$\frac{(ab + b^2x) (a^2 + 2abx + b^2x^2)^p}{b^2(2p + 1)}$$

input `Int[(a^2 + 2*a*b*x + b^2*x^2)^p,x]`

output `((a*b + b^2*x)*(a^2 + 2*a*b*x + b^2*x^2)^p)/(b^2*(1 + 2*p))`

Defintions of rubi rules used

rule 17 `Int[(c_.)*((a_.) + (b_.)*(x_)^(m_.), x_Symbol] := Simp[c*((a + b*x)^(m + 1))/(b*(m + 1)), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]`

rule 1079 `Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] := Simp[(a + b*x + c*x^2)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x)^(2*FracPart[p])) Int[(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c, p}, x] && EqQ[b^2 - 4*a*c, 0]`

Maple [A] (verified)

Time = 1.33 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.76

| method | result | size |
|--------------|--|------|
| risch | $\frac{(bx+a)((bx+a)^2)^p}{b(2p+1)}$ | 26 |
| gospers | $\frac{(bx+a)(b^2x^2+2abx+a^2)^p}{b(2p+1)}$ | 35 |
| orering | $\frac{(bx+a)(b^2x^2+2abx+a^2)^p}{b(2p+1)}$ | 35 |
| paralelrisch | $\frac{x(b^2x^2+2abx+a^2)^p ab + (b^2x^2+2abx+a^2)^p a^2}{(2p+1)ab}$ | 60 |
| norman | $\frac{x e^{p \ln(b^2x^2+2abx+a^2)}}{2p+1} + \frac{a e^{p \ln(b^2x^2+2abx+a^2)}}{b(2p+1)}$ | 63 |

input `int((b^2*x^2+2*a*b*x+a^2)^p,x,method=_RETURNVERBOSE)`

output `(b*x+a)/b/(2*p+1)*((b*x+a)^2)^p`

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.94

$$\int (a^2 + 2abx + b^2x^2)^p dx = \frac{(bx+a)(b^2x^2+2abx+a^2)^p}{2bp+b}$$

input `integrate((b^2*x^2+2*a*b*x+a^2)^p,x, algorithm="fricas")`

output `(b*x + a)*(b^2*x^2 + 2*a*b*x + a^2)^p/(2*b*p + b)`

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 100 vs. $2(29) = 58$.

Time = 0.48 (sec) , antiderivative size = 100, normalized size of antiderivative = 2.94

$$\int (a^2 + 2abx + b^2x^2)^p dx = \begin{cases} \frac{x}{\sqrt{a^2}} & \text{for } b = 0 \wedge p = -\frac{1}{2} \\ x(a^2)^p & \text{for } b = 0 \\ \frac{(\frac{a}{b}+x) \log(\frac{a}{b}+x)}{\sqrt{b^2(\frac{a}{b}+x)^2}} & \text{for } p = -\frac{1}{2} \\ \frac{a(a^2+2abx+b^2x^2)^p}{2bp+b} + \frac{bx(a^2+2abx+b^2x^2)^p}{2bp+b} & \text{otherwise} \end{cases}$$

input `integrate((b**2*x**2+2*a*b*x+a**2)**p,x)`

output `Piecewise((x/sqrt(a**2), Eq(b, 0) & Eq(p, -1/2)), (x*(a**2)**p, Eq(b, 0)), ((a/b + x)*log(a/b + x)/sqrt(b**2*(a/b + x)**2), Eq(p, -1/2)), (a*(a**2 + 2*a*b*x + b**2*x**2)**p/(2*b*p + b) + b*x*(a**2 + 2*a*b*x + b**2*x**2)**p/(2*b*p + b), True))`

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.74

$$\int (a^2 + 2abx + b^2x^2)^p dx = \frac{(bx + a)(bx + a)^{2p}}{b(2p + 1)}$$

input `integrate((b^2*x^2+2*a*b*x+a^2)^p,x, algorithm="maxima")`

output `(b*x + a)*(b*x + a)^(2*p)/(b*(2*p + 1))`

Giac [A] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.50

$$\int (a^2 + 2abx + b^2x^2)^p dx = \frac{(b^2x^2 + 2abx + a^2)^p bx + (b^2x^2 + 2abx + a^2)^p a}{2bp + b}$$

input `integrate((b^2*x^2+2*a*b*x+a^2)^p,x, algorithm="giac")`

output `((b^2*x^2 + 2*a*b*x + a^2)^p*b*x + (b^2*x^2 + 2*a*b*x + a^2)^p*a)/(2*b*p + b)`

Mupad [B] (verification not implemented)

Time = 5.60 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.21

$$\int (a^2 + 2abx + b^2x^2)^p dx = \left(\frac{x}{2p+1} + \frac{a}{b(2p+1)} \right) (a^2 + 2abx + b^2x^2)^p$$

input `int((a^2 + b^2*x^2 + 2*a*b*x)^p,x)`

output `(x/(2*p + 1) + a/(b*(2*p + 1)))*(a^2 + b^2*x^2 + 2*a*b*x)^p`

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.00

$$\int (a^2 + 2abx + b^2x^2)^p dx = \frac{(b^2x^2 + 2abx + a^2)^p (bx + a)}{b(2p+1)}$$

input `int((b^2*x^2+2*a*b*x+a^2)^p,x)`

output `((a**2 + 2*a*b*x + b**2*x**2)**p*(a + b*x))/(b*(2*p + 1))`

3.437 $\int \frac{(a^2+2abx+b^2x^2)^p}{d+ex} dx$

| | |
|----------------------------|------|
| Optimal result | 3261 |
| Mathematica [A] (verified) | 3261 |
| Rubi [A] (verified) | 3262 |
| Maple [F] | 3263 |
| Fricas [F] | 3263 |
| Sympy [F] | 3264 |
| Maxima [F] | 3264 |
| Giac [F] | 3264 |
| Mupad [F(-1)] | 3265 |
| Reduce [F] | 3265 |

Optimal result

Integrand size = 26, antiderivative size = 71

$$\int \frac{(a^2 + 2abx + b^2x^2)^p}{d + ex} dx = \frac{(a + bx)(a^2 + 2abx + b^2x^2)^p \operatorname{Hypergeometric2F1}\left(1, 1 + 2p, 2(1 + p), -\frac{e(a+bx)}{bd-ae}\right)}{(bd - ae)(1 + 2p)}$$

output

```
(b*x+a)*(b^2*x^2+2*a*b*x+a^2)^p*hypergeom([1, 1+2*p],[2*p+2],-e*(b*x+a)/(-a*e+b*d))/(-a*e+b*d)/(1+2*p)
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.87

$$\int \frac{(a^2 + 2abx + b^2x^2)^p}{d + ex} dx = \frac{(a + bx)((a + bx)^2)^p \operatorname{Hypergeometric2F1}\left(1, 1 + 2p, 2 + 2p, \frac{e(a+bx)}{-bd+ae}\right)}{(-bd + ae)(1 + 2p)}$$

input

```
Integrate[(a^2 + 2*a*b*x + b^2*x^2)^p/(d + e*x),x]
```

output

$$-\left(\frac{(a + bx)(a + bx)^2}{-(bd) + ae}\right)^p \text{Hypergeometric2F1}\left[1, 1 + 2p, 2 + 2p, \frac{e(a + bx)}{-(bd) + ae}\right] / \left(\frac{e(a + bx)}{-(bd) + ae}\right)^{1 + 2p}$$
Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.10, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {1102, 78}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a^2 + 2abx + b^2x^2)^p}{d + ex} dx$$

$$\downarrow 1102$$

$$(ab + b^2x)^{-2p} (a^2 + 2abx + b^2x^2)^p \int \frac{(xb^2 + ab)^{2p}}{d + ex} dx$$

$$\downarrow 78$$

$$\frac{(ab + b^2x) (a^2 + 2abx + b^2x^2)^p \text{Hypergeometric2F1}\left(1, 2p + 1, 2(p + 1), -\frac{e(a + bx)}{bd - ae}\right)}{b(2p + 1)(bd - ae)}$$

input

$$\text{Int}[(a^2 + 2*a*b*x + b^2*x^2)^p/(d + e*x), x]$$

output

$$\left(\frac{(a*b + b^2*x)(a^2 + 2*a*b*x + b^2*x^2)^p}{b*(bd - a*e)} \text{Hypergeometric2F1}\left[1, 1 + 2*p, 2*(1 + p), -\frac{e*(a + b*x)}{b*d - a*e}\right]\right) / (b*(b*d - a*e)*(1 + 2*p))$$

Definitions of rubi rules used

rule 78

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(b
*c - a*d)^n*((a + b*x)^(m + 1)/(b^(n + 1)*(m + 1)))*Hypergeometric2F1[-n, m
+ 1, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m}, x]
&& !IntegerQ[m] && IntegerQ[n]
```

rule 1102

```
Int[((d_.) + (e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(a + b*x + c*x^2)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x)^(2*F
racPart[p])) Int[(d + e*x)^m*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c,
d, e, m, p}, x] && EqQ[b^2 - 4*a*c, 0]
```

Maple [F]

$$\int \frac{(b^2x^2 + 2abx + a^2)^p}{ex + d} dx$$

input

```
int((b^2*x^2+2*a*b*x+a^2)^p/(e*x+d),x)
```

output

```
int((b^2*x^2+2*a*b*x+a^2)^p/(e*x+d),x)
```

Fricas [F]

$$\int \frac{(a^2 + 2abx + b^2x^2)^p}{d + ex} dx = \int \frac{(b^2x^2 + 2abx + a^2)^p}{ex + d} dx$$

input

```
integrate((b^2*x^2+2*a*b*x+a^2)^p/(e*x+d),x, algorithm="fricas")
```

output

```
integral((b^2*x^2 + 2*a*b*x + a^2)^p/(e*x + d), x)
```

Sympy [F]

$$\int \frac{(a^2 + 2abx + b^2x^2)^p}{d + ex} dx = \int \frac{((a + bx)^2)^p}{d + ex} dx$$

input `integrate((b**2*x**2+2*a*b*x+a**2)**p/(e*x+d),x)`

output `Integral(((a + b*x)**2)**p/(d + e*x), x)`

Maxima [F]

$$\int \frac{(a^2 + 2abx + b^2x^2)^p}{d + ex} dx = \int \frac{(b^2x^2 + 2abx + a^2)^p}{ex + d} dx$$

input `integrate((b^2*x^2+2*a*b*x+a^2)^p/(e*x+d),x, algorithm="maxima")`

output `integrate((b^2*x^2 + 2*a*b*x + a^2)^p/(e*x + d), x)`

Giac [F]

$$\int \frac{(a^2 + 2abx + b^2x^2)^p}{d + ex} dx = \int \frac{(b^2x^2 + 2abx + a^2)^p}{ex + d} dx$$

input `integrate((b^2*x^2+2*a*b*x+a^2)^p/(e*x+d),x, algorithm="giac")`

output `integrate((b^2*x^2 + 2*a*b*x + a^2)^p/(e*x + d), x)`

Mupad [F(-1)]

Timed out.

$$\int \frac{(a^2 + 2abx + b^2x^2)^p}{d + ex} dx = \int \frac{(a^2 + 2abx + b^2x^2)^p}{d + ex} dx$$

input `int((a^2 + b^2*x^2 + 2*a*b*x)^p/(d + e*x), x)`output `int((a^2 + b^2*x^2 + 2*a*b*x)^p/(d + e*x), x)`**Reduce [F]**

$$\int \frac{(a^2 + 2abx + b^2x^2)^p}{d + ex} dx$$

$$= \frac{(b^2x^2 + 2abx + a^2)^p a - 2 \left(\int \frac{(b^2x^2 + 2abx + a^2)^p x}{be x^2 + aex + bdx + ad} dx \right) abep + 2 \left(\int \frac{(b^2x^2 + 2abx + a^2)^p x}{be x^2 + aex + bdx + ad} dx \right) b^2 dp}{2bdp}$$

input `int((b^2*x^2+2*a*b*x+a^2)^p/(e*x+d), x)`output `((a**2 + 2*a*b*x + b**2*x**2)**p*a - 2*int(((a**2 + 2*a*b*x + b**2*x**2)**p*x)/(a*d + a*e*x + b*d*x + b*e*x**2), x)*a*b*e*p + 2*int(((a**2 + 2*a*b*x + b**2*x**2)**p*x)/(a*d + a*e*x + b*d*x + b*e*x**2), x)*b**2*d*p)/(2*b*d*p)`

3.438 $\int \frac{(a^2+2abx+b^2x^2)^p}{(d+ex)^2} dx$

| | |
|----------------------------|------|
| Optimal result | 3266 |
| Mathematica [A] (verified) | 3266 |
| Rubi [A] (verified) | 3267 |
| Maple [F] | 3268 |
| Fricas [F] | 3268 |
| Sympy [F] | 3269 |
| Maxima [F] | 3269 |
| Giac [F] | 3269 |
| Mupad [F(-1)] | 3270 |
| Reduce [F] | 3270 |

Optimal result

Integrand size = 26, antiderivative size = 72

$$\int \frac{(a^2 + 2abx + b^2x^2)^p}{(d + ex)^2} dx = \frac{b(a + bx)(a^2 + 2abx + b^2x^2)^p \operatorname{Hypergeometric2F1}\left(2, 1 + 2p, 2(1 + p), -\frac{e(a+bx)}{bd-ae}\right)}{(bd - ae)^2(1 + 2p)}$$

output

```
b*(b*x+a)*(b^2*x^2+2*a*b*x+a^2)^p*hypergeom([2, 1+2*p],[2*p+2],-e*(b*x+a)/(-a*e+b*d))/(-a*e+b*d)^2/(1+2*p)
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.88

$$\int \frac{(a^2 + 2abx + b^2x^2)^p}{(d + ex)^2} dx = \frac{b(a + bx)((a + bx)^2)^p \operatorname{Hypergeometric2F1}\left(2, 1 + 2p, 2 + 2p, -\frac{e(a+bx)}{bd-ae}\right)}{(bd - ae)^2(1 + 2p)}$$

input

```
Integrate[(a^2 + 2*a*b*x + b^2*x^2)^p/(d + e*x)^2,x]
```

output

```
(b*(a + b*x)*((a + b*x)^2)^p*Hypergeometric2F1[2, 1 + 2*p, 2 + 2*p, -((e*(a + b*x))/(b*d - a*e))]/((b*d - a*e)^2*(1 + 2*p))
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.04, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {1102, 78}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a^2 + 2abx + b^2x^2)^p}{(d + ex)^2} dx$$

$$\downarrow 1102$$

$$(ab + b^2x)^{-2p} (a^2 + 2abx + b^2x^2)^p \int \frac{(xb^2 + ab)^{2p}}{(d + ex)^2} dx$$

$$\downarrow 78$$

$$\frac{(ab + b^2x) (a^2 + 2abx + b^2x^2)^p \text{Hypergeometric2F1} \left(2, 2p + 1, 2(p + 1), -\frac{e(a+bx)}{bd-ae} \right)}{(2p + 1)(bd - ae)^2}$$

input

```
Int[(a^2 + 2*a*b*x + b^2*x^2)^p/(d + e*x)^2,x]
```

output

```
((a*b + b^2*x)*(a^2 + 2*a*b*x + b^2*x^2)^p*Hypergeometric2F1[2, 1 + 2*p, 2*(1 + p), -((e*(a + b*x))/(b*d - a*e))]/((b*d - a*e)^2*(1 + 2*p))
```


Definitions of rubi rules used

rule 78

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] := Simp[(b
*c - a*d)^n*((a + b*x)^(m + 1)/(b^(n + 1)*(m + 1)))*Hypergeometric2F1[-n, m
+ 1, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m}, x]
&& !IntegerQ[m] && IntegerQ[n]
```

rule 1102

```
Int[((d_.) + (e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] := Simp[(a + b*x + c*x^2)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x)^(2*F
racPart[p])) Int[(d + e*x)^m*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c,
d, e, m, p}, x] && EqQ[b^2 - 4*a*c, 0]
```

Maple [F]

$$\int \frac{(b^2x^2 + 2abx + a^2)^p}{(ex + d)^2} dx$$

input

```
int((b^2*x^2+2*a*b*x+a^2)^p/(e*x+d)^2,x)
```

output

```
int((b^2*x^2+2*a*b*x+a^2)^p/(e*x+d)^2,x)
```

Fricas [F]

$$\int \frac{(a^2 + 2abx + b^2x^2)^p}{(d + ex)^2} dx = \int \frac{(b^2x^2 + 2abx + a^2)^p}{(ex + d)^2} dx$$

input

```
integrate((b^2*x^2+2*a*b*x+a^2)^p/(e*x+d)^2,x, algorithm="fricas")
```

output

```
integral((b^2*x^2 + 2*a*b*x + a^2)^p/(e^2*x^2 + 2*d*e*x + d^2), x)
```

Sympy [F]

$$\int \frac{(a^2 + 2abx + b^2x^2)^p}{(d + ex)^2} dx = \int \frac{((a + bx)^2)^p}{(d + ex)^2} dx$$

input `integrate((b**2*x**2+2*a*b*x+a**2)**p/(e*x+d)**2,x)`

output `Integral(((a + b*x)**2)**p/(d + e*x)**2, x)`

Maxima [F]

$$\int \frac{(a^2 + 2abx + b^2x^2)^p}{(d + ex)^2} dx = \int \frac{(b^2x^2 + 2abx + a^2)^p}{(ex + d)^2} dx$$

input `integrate((b^2*x^2+2*a*b*x+a^2)^p/(e*x+d)^2,x, algorithm="maxima")`

output `integrate((b^2*x^2 + 2*a*b*x + a^2)^p/(e*x + d)^2, x)`

Giac [F]

$$\int \frac{(a^2 + 2abx + b^2x^2)^p}{(d + ex)^2} dx = \int \frac{(b^2x^2 + 2abx + a^2)^p}{(ex + d)^2} dx$$

input `integrate((b^2*x^2+2*a*b*x+a^2)^p/(e*x+d)^2,x, algorithm="giac")`

output `integrate((b^2*x^2 + 2*a*b*x + a^2)^p/(e*x + d)^2, x)`

Mupad [F(-1)]

Timed out.

$$\int \frac{(a^2 + 2abx + b^2x^2)^p}{(d + ex)^2} dx = \int \frac{(a^2 + 2abx + b^2x^2)^p}{(d + ex)^2} dx$$

input `int((a^2 + b^2*x^2 + 2*a*b*x)^p/(d + e*x)^2,x)`output `int((a^2 + b^2*x^2 + 2*a*b*x)^p/(d + e*x)^2, x)`**Reduce [F]**

$$\int \frac{(a^2 + 2abx + b^2x^2)^p}{(d + ex)^2} dx = \text{Too large to display}$$

input `int((b^2*x^2+2*a*b*x+a^2)^p/(e*x+d)^2,x)`

output

```
( - (a**2 + 2*a*b*x + b**2*x**2)**p*a + 2*int(((a**2 + 2*a*b*x + b**2*x**2)
)**p*x)/(a**2*d**2*e + 2*a**2*d*e**2*x + a**2*e**3*x**2 - 2*a*b*d**3*p - 4
*a*b*d**2*e*p*x + a*b*d**2*e*x - 2*a*b*d*e**2*p*x**2 + 2*a*b*d*e**2*x**2 +
a*b*e**3*x**3 - 2*b**2*d**3*p*x - 4*b**2*d**2*e*p*x**2 - 2*b**2*d*e**2*p*
x**3),x)*a**2*b*d*e**2*p + 2*int(((a**2 + 2*a*b*x + b**2*x**2)**p*x)/(a**2
*d**2*e + 2*a**2*d*e**2*x + a**2*e**3*x**2 - 2*a*b*d**3*p - 4*a*b*d**2*e*p
*x + a*b*d**2*e*x - 2*a*b*d*e**2*p*x**2 + 2*a*b*d*e**2*x**2 + a*b*e**3*x**
3 - 2*b**2*d**3*p*x - 4*b**2*d**2*e*p*x**2 - 2*b**2*d*e**2*p*x**3),x)*a**2
*b*e**3*p*x - 4*int(((a**2 + 2*a*b*x + b**2*x**2)**p*x)/(a**2*d**2*e + 2*a
**2*d*e**2*x + a**2*e**3*x**2 - 2*a*b*d**3*p - 4*a*b*d**2*e*p*x + a*b*d**2
*e*x - 2*a*b*d*e**2*p*x**2 + 2*a*b*d*e**2*x**2 + a*b*e**3*x**3 - 2*b**2*d*
**3*p*x - 4*b**2*d**2*e*p*x**2 - 2*b**2*d*e**2*p*x**3),x)*a*b**2*d**2*e*p**
2 - 2*int(((a**2 + 2*a*b*x + b**2*x**2)**p*x)/(a**2*d**2*e + 2*a**2*d*e**2
*x + a**2*e**3*x**2 - 2*a*b*d**3*p - 4*a*b*d**2*e*p*x + a*b*d**2*e*x - 2*a
*b*d*e**2*p*x**2 + 2*a*b*d*e**2*x**2 + a*b*e**3*x**3 - 2*b**2*d**3*p*x - 4
*b**2*d**2*e*p*x**2 - 2*b**2*d*e**2*p*x**3),x)*a*b**2*d**2*e*p - 4*int(((a
**2 + 2*a*b*x + b**2*x**2)**p*x)/(a**2*d**2*e + 2*a**2*d*e**2*x + a**2*e**
3*x**2 - 2*a*b*d**3*p - 4*a*b*d**2*e*p*x + a*b*d**2*e*x - 2*a*b*d*e**2*p*x
**2 + 2*a*b*d*e**2*x**2 + a*b*e**3*x**3 - 2*b**2*d**3*p*x - 4*b**2*d**2*e*
p*x**2 - 2*b**2*d*e**2*p*x**3),x)*a*b**2*d*e**2*p**2*x - 2*int(((a**2 +...
```

3.439 $\int \frac{(a^2+2abx+b^2x^2)^p}{(d+ex)^3} dx$

| | |
|----------------------------|------|
| Optimal result | 3272 |
| Mathematica [A] (verified) | 3272 |
| Rubi [A] (verified) | 3273 |
| Maple [F] | 3274 |
| Fricas [F] | 3274 |
| Sympy [F] | 3275 |
| Maxima [F] | 3275 |
| Giac [F] | 3275 |
| Mupad [F(-1)] | 3276 |
| Reduce [F] | 3276 |

Optimal result

Integrand size = 26, antiderivative size = 74

$$\int \frac{(a^2 + 2abx + b^2x^2)^p}{(d + ex)^3} dx = \frac{b^2(a + bx)(a^2 + 2abx + b^2x^2)^p \operatorname{Hypergeometric2F1}\left(3, 1 + 2p, 2(1 + p), -\frac{e(a+bx)}{bd-ae}\right)}{(bd - ae)^3(1 + 2p)}$$

output

```
b^2*(b*x+a)*(b^2*x^2+2*a*b*x+a^2)^p*hypergeom([3, 1+2*p],[2*p+2],-e*(b*x+a)/(-a*e+b*d))/(-a*e+b*d)^3/(1+2*p)
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.88

$$\int \frac{(a^2 + 2abx + b^2x^2)^p}{(d + ex)^3} dx = \frac{b^2(a + bx)((a + bx)^2)^p \operatorname{Hypergeometric2F1}\left(3, 1 + 2p, 2 + 2p, -\frac{e(a+bx)}{bd-ae}\right)}{(bd - ae)^3(1 + 2p)}$$

input

```
Integrate[(a^2 + 2*a*b*x + b^2*x^2)^p/(d + e*x)^3,x]
```

output

$$(b^2(a + bx)((a + bx)^2)^p \text{Hypergeometric2F1}[3, 1 + 2p, 2 + 2p, -((e(a + bx))/(bd - ae))]) / ((bd - ae)^{3(1 + 2p)})$$
Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.03, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {1102, 78}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a^2 + 2abx + b^2x^2)^p}{(d + ex)^3} dx$$

$$\downarrow 1102$$

$$(ab + b^2x)^{-2p} (a^2 + 2abx + b^2x^2)^p \int \frac{(xb^2 + ab)^{2p}}{(d + ex)^3} dx$$

$$\downarrow 78$$

$$\frac{b(ab + b^2x) (a^2 + 2abx + b^2x^2)^p \text{Hypergeometric2F1} \left(3, 2p + 1, 2(p + 1), -\frac{e(a+bx)}{bd-ae} \right)}{(2p + 1)(bd - ae)^3}$$

input

$$\text{Int}[(a^2 + 2*a*b*x + b^2*x^2)^p / (d + e*x)^3, x]$$

output

$$(b*(a*b + b^2*x)*(a^2 + 2*a*b*x + b^2*x^2)^p \text{Hypergeometric2F1}[3, 1 + 2p, 2*(1 + p), -((e*(a + b*x))/(b*d - a*e))]) / ((b*d - a*e)^{3*(1 + 2*p)})$$

Definitions of rubi rules used

rule 78

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(b
*c - a*d)^n*((a + b*x)^(m + 1)/(b^(n + 1)*(m + 1)))*Hypergeometric2F1[-n, m
+ 1, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m}, x]
&& !IntegerQ[m] && IntegerQ[n]
```

rule 1102

```
Int[((d_.) + (e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(a + b*x + c*x^2)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x)^(2*F
racPart[p])) Int[(d + e*x)^m*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c,
d, e, m, p}, x] && EqQ[b^2 - 4*a*c, 0]
```

Maple [F]

$$\int \frac{(b^2x^2 + 2abx + a^2)^p}{(ex + d)^3} dx$$

input

```
int((b^2*x^2+2*a*b*x+a^2)^p/(e*x+d)^3,x)
```

output

```
int((b^2*x^2+2*a*b*x+a^2)^p/(e*x+d)^3,x)
```

Fricas [F]

$$\int \frac{(a^2 + 2abx + b^2x^2)^p}{(d + ex)^3} dx = \int \frac{(b^2x^2 + 2abx + a^2)^p}{(ex + d)^3} dx$$

input

```
integrate((b^2*x^2+2*a*b*x+a^2)^p/(e*x+d)^3,x, algorithm="fricas")
```

output

```
integral((b^2*x^2 + 2*a*b*x + a^2)^p/(e^3*x^3 + 3*d*e^2*x^2 + 3*d^2*e*x +
d^3), x)
```

Sympy [F]

$$\int \frac{(a^2 + 2abx + b^2x^2)^p}{(d + ex)^3} dx = \int \frac{((a + bx)^2)^p}{(d + ex)^3} dx$$

input `integrate((b**2*x**2+2*a*b*x+a**2)**p/(e*x+d)**3,x)`

output `Integral(((a + b*x)**2)**p/(d + e*x)**3, x)`

Maxima [F]

$$\int \frac{(a^2 + 2abx + b^2x^2)^p}{(d + ex)^3} dx = \int \frac{(b^2x^2 + 2abx + a^2)^p}{(ex + d)^3} dx$$

input `integrate((b^2*x^2+2*a*b*x+a^2)^p/(e*x+d)^3,x, algorithm="maxima")`

output `integrate((b^2*x^2 + 2*a*b*x + a^2)^p/(e*x + d)^3, x)`

Giac [F]

$$\int \frac{(a^2 + 2abx + b^2x^2)^p}{(d + ex)^3} dx = \int \frac{(b^2x^2 + 2abx + a^2)^p}{(ex + d)^3} dx$$

input `integrate((b^2*x^2+2*a*b*x+a^2)^p/(e*x+d)^3,x, algorithm="giac")`

output `integrate((b^2*x^2 + 2*a*b*x + a^2)^p/(e*x + d)^3, x)`

Mupad [F(-1)]

Timed out.

$$\int \frac{(a^2 + 2abx + b^2x^2)^p}{(d + ex)^3} dx = \int \frac{(a^2 + 2abx + b^2x^2)^p}{(d + ex)^3} dx$$

input `int((a^2 + b^2*x^2 + 2*a*b*x)^p/(d + e*x)^3,x)`output `int((a^2 + b^2*x^2 + 2*a*b*x)^p/(d + e*x)^3, x)`**Reduce [F]**

$$\int \frac{(a^2 + 2abx + b^2x^2)^p}{(d + ex)^3} dx = \text{too large to display}$$

input `int((b^2*x^2+2*a*b*x+a^2)^p/(e*x+d)^3,x)`

output

```
( - (a**2 + 2*a*b*x + b**2*x**2)**p*a + 2*int(((a**2 + 2*a*b*x + b**2*x**2)
)**p*x)/(a**2*d**3*e + 3*a**2*d**2*e**2*x + 3*a**2*d*e**3*x**2 + a**2*e**4
*x**3 - a*b*d**4*p - 3*a*b*d**3*e*p*x + a*b*d**3*e*x - 3*a*b*d**2*e**2*p*x
**2 + 3*a*b*d**2*e**2*x**2 - a*b*d*e**3*p*x**3 + 3*a*b*d*e**3*x**3 + a*b*e
**4*x**4 - b**2*d**4*p*x - 3*b**2*d**3*e*p*x**2 - 3*b**2*d**2*e**2*p*x**3
- b**2*d*e**3*p*x**4),x)*a**2*b*d**2*e**2*p + 4*int(((a**2 + 2*a*b*x + b**
2*x**2)**p*x)/(a**2*d**3*e + 3*a**2*d**2*e**2*x + 3*a**2*d*e**3*x**2 + a**
2*e**4*x**3 - a*b*d**4*p - 3*a*b*d**3*e*p*x + a*b*d**3*e*x - 3*a*b*d**2*e*
**2*p*x**2 + 3*a*b*d**2*e**2*x**2 - a*b*d*e**3*p*x**3 + 3*a*b*d*e**3*x**3 +
a*b*e**4*x**4 - b**2*d**4*p*x - 3*b**2*d**3*e*p*x**2 - 3*b**2*d**2*e**2*p
*x**3 - b**2*d*e**3*p*x**4),x)*a**2*b*d*e**3*p*x + 2*int(((a**2 + 2*a*b*x
+ b**2*x**2)**p*x)/(a**2*d**3*e + 3*a**2*d**2*e**2*x + 3*a**2*d*e**3*x**2
+ a**2*e**4*x**3 - a*b*d**4*p - 3*a*b*d**3*e*p*x + a*b*d**3*e*x - 3*a*b*d*
**2*e**2*p*x**2 + 3*a*b*d**2*e**2*x**2 - a*b*d*e**3*p*x**3 + 3*a*b*d*e**3*x
**3 + a*b*e**4*x**4 - b**2*d**4*p*x - 3*b**2*d**3*e*p*x**2 - 3*b**2*d**2*e
**2*p*x**3 - b**2*d*e**3*p*x**4),x)*a**2*b*e**4*p*x**2 - 2*int(((a**2 + 2*
a*b*x + b**2*x**2)**p*x)/(a**2*d**3*e + 3*a**2*d**2*e**2*x + 3*a**2*d*e**3
*x**2 + a**2*e**4*x**3 - a*b*d**4*p - 3*a*b*d**3*e*p*x + a*b*d**3*e*x - 3*
a*b*d**2*e**2*p*x**2 + 3*a*b*d**2*e**2*x**2 - a*b*d*e**3*p*x**3 + 3*a*b*d*
e**3*x**3 + a*b*e**4*x**4 - b**2*d**4*p*x - 3*b**2*d**3*e*p*x**2 - 3*b...
```

3.440 $\int (d + ex)^{3/2} (a^2 + 2abx + b^2x^2)^p dx$

| | |
|----------------------------|------|
| Optimal result | 3278 |
| Mathematica [A] (verified) | 3278 |
| Rubi [A] (verified) | 3279 |
| Maple [F] | 3280 |
| Fricas [F] | 3281 |
| Sympy [F] | 3281 |
| Maxima [F] | 3281 |
| Giac [F] | 3282 |
| Mupad [F(-1)] | 3282 |
| Reduce [F] | 3282 |

Optimal result

Integrand size = 28, antiderivative size = 83

$$\int (d + ex)^{3/2} (a^2 + 2abx + b^2x^2)^p dx = \frac{2 \left(-\frac{e(a+bx)}{bd-ae} \right)^{-2p} (d + ex)^{5/2} (a^2 + 2abx + b^2x^2)^p \operatorname{Hypergeometric2F1} \left(\frac{5}{2}, -2p, \frac{7}{2}, \frac{b(d+ex)}{bd-ae} \right)}{5e}$$

output `2/5*(e*x+d)^(5/2)*(b^2*x^2+2*a*b*x+a^2)^p*hypergeom([5/2, -2*p], [7/2], b*(e*x+d)/(-a*e+b*d))/e/((-e*(b*x+a)/(-a*e+b*d))^(2*p))`

Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.88

$$\int (d + ex)^{3/2} (a^2 + 2abx + b^2x^2)^p dx = \frac{2 \left(\frac{e(a+bx)}{-bd+ae} \right)^{-2p} ((a + bx)^2)^p (d + ex)^{5/2} \operatorname{Hypergeometric2F1} \left(\frac{5}{2}, -2p, \frac{7}{2}, \frac{b(d+ex)}{bd-ae} \right)}{5e}$$

input `Integrate[(d + e*x)^(3/2)*(a^2 + 2*a*b*x + b^2*x^2)^p,x]`

output

$$(2*((a + b*x)^2)^p*(d + e*x)^(5/2)*Hypergeometric2F1[5/2, -2*p, 7/2, (b*(d + e*x))/(b*d - a*e)]/(5*e*((e*(a + b*x))/(-(b*d) + a*e))^(2*p))$$
Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.107$, Rules used = {1102, 80, 79}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (d + ex)^{3/2} (a^2 + 2abx + b^2x^2)^p dx$$

$$\downarrow 1102$$

$$(ab + b^2x)^{-2p} (a^2 + 2abx + b^2x^2)^p \int (xb^2 + ab)^{2p} (d + ex)^{3/2} dx$$

$$\downarrow 80$$

$$(a^2 + 2abx + b^2x^2)^p \left(-\frac{e(a + bx)}{bd - ae} \right)^{-2p} \int (d + ex)^{3/2} \left(-\frac{bx e}{bd - ae} - \frac{ae}{bd - ae} \right)^{2p} dx$$

$$\downarrow 79$$

$$\frac{2(d + ex)^{5/2} (a^2 + 2abx + b^2x^2)^p \left(-\frac{e(a + bx)}{bd - ae} \right)^{-2p} \text{Hypergeometric2F1} \left(\frac{5}{2}, -2p, \frac{7}{2}, \frac{b(d + ex)}{bd - ae} \right)}{5e}$$

input

$$\text{Int}[(d + e*x)^(3/2)*(a^2 + 2*a*b*x + b^2*x^2)^p,x]$$

output

$$(2*(d + e*x)^(5/2)*(a^2 + 2*a*b*x + b^2*x^2)^p*Hypergeometric2F1[5/2, -2*p, 7/2, (b*(d + e*x))/(b*d - a*e)]/(5*e*((e*(a + b*x))/(b*d - a*e))^(2*p))$$

Definitions of rubi rules used

rule 79

```
Int[((a_) + (b_)*(x_))^(m_)*((c_) + (d_)*(x_))^(n_), x_Symbol] := Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&& !IntegerQ[m] && !IntegerQ[n] && GtQ[b/(b*c - a*d), 0] && (RationalQ[m]
|| !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

rule 80

```
Int[((a_) + (b_)*(x_))^(m_)*((c_) + (d_)*(x_))^(n_), x_Symbol] := Simp[(c
+ d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*(b*((c + d*x)/(b*c - a*d))
^FracPart[n]) Int[(a + b*x)^m*Simp[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d)
), x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && !IntegerQ[m] && !Integ
erQ[n] && (RationalQ[m] || !SimplerQ[n + 1, m + 1])
```

rule 1102

```
Int[((d_) + (e_)*(x_))^(m_)*((a_) + (b_)*(x_) + (c_)*(x_)^2)^(p_), x_S
ymbol] := Simp[(a + b*x + c*x^2)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x)^(2*F
racPart[p])) Int[(d + e*x)^m*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c,
d, e, m, p}, x] && EqQ[b^2 - 4*a*c, 0]
```

Maple [F]

$$\int (ex + d)^{\frac{3}{2}} (b^2x^2 + 2abx + a^2)^p dx$$

input

```
int((e*x+d)^(3/2)*(b^2*x^2+2*a*b*x+a^2)^p,x)
```

output

```
int((e*x+d)^(3/2)*(b^2*x^2+2*a*b*x+a^2)^p,x)
```

Fricas [F]

$$\int (d + ex)^{3/2} (a^2 + 2abx + b^2x^2)^p dx = \int (ex + d)^{\frac{3}{2}} (b^2x^2 + 2abx + a^2)^p dx$$

input `integrate((e*x+d)^(3/2)*(b^2*x^2+2*a*b*x+a^2)^p,x, algorithm="fricas")`

output `integral((e*x + d)^(3/2)*(b^2*x^2 + 2*a*b*x + a^2)^p, x)`

Sympy [F]

$$\int (d + ex)^{3/2} (a^2 + 2abx + b^2x^2)^p dx = \int (d + ex)^{\frac{3}{2}} ((a + bx)^2)^p dx$$

input `integrate((e*x+d)**(3/2)*(b**2*x**2+2*a*b*x+a**2)**p,x)`

output `Integral((d + e*x)**(3/2)*((a + b*x)**2)**p, x)`

Maxima [F]

$$\int (d + ex)^{3/2} (a^2 + 2abx + b^2x^2)^p dx = \int (ex + d)^{\frac{3}{2}} (b^2x^2 + 2abx + a^2)^p dx$$

input `integrate((e*x+d)^(3/2)*(b^2*x^2+2*a*b*x+a^2)^p,x, algorithm="maxima")`

output `integrate((e*x + d)^(3/2)*(b^2*x^2 + 2*a*b*x + a^2)^p, x)`

Giac [F]

$$\int (d + ex)^{3/2} (a^2 + 2abx + b^2x^2)^p dx = \int (ex + d)^{\frac{3}{2}} (b^2x^2 + 2abx + a^2)^p dx$$

input `integrate((e*x+d)^(3/2)*(b^2*x^2+2*a*b*x+a^2)^p,x, algorithm="giac")`

output `integrate((e*x + d)^(3/2)*(b^2*x^2 + 2*a*b*x + a^2)^p, x)`

Mupad [F(-1)]

Timed out.

$$\int (d + ex)^{3/2} (a^2 + 2abx + b^2x^2)^p dx = \int (d + ex)^{3/2} (a^2 + 2abx + b^2x^2)^p dx$$

input `int((d + e*x)^(3/2)*(a^2 + b^2*x^2 + 2*a*b*x)^p,x)`

output `int((d + e*x)^(3/2)*(a^2 + b^2*x^2 + 2*a*b*x)^p, x)`

Reduce [F]

$$\int (d + ex)^{3/2} (a^2 + 2abx + b^2x^2)^p dx = \text{too large to display}$$

input `int((e*x+d)^(3/2)*(b^2*x^2+2*a*b*x+a^2)^p,x)`

output

```
(2*( - 8*sqrt(d + e*x)*(a**2 + 2*a*b*x + b**2*x**2)**p*a**2*d*e*p + 4*sqrt
(d + e*x)*(a**2 + 2*a*b*x + b**2*x**2)**p*a**2*e**2*p*x + 16*sqrt(d + e*x)
*(a**2 + 2*a*b*x + b**2*x**2)**p*a*b*d**2*p**2 + 24*sqrt(d + e*x)*(a**2 +
2*a*b*x + b**2*x**2)**p*a*b*d**2*p + 3*sqrt(d + e*x)*(a**2 + 2*a*b*x + b**
2*x**2)**p*a*b*d**2 + 16*sqrt(d + e*x)*(a**2 + 2*a*b*x + b**2*x**2)**p*a*b
*d*e*p**2*x + 4*sqrt(d + e*x)*(a**2 + 2*a*b*x + b**2*x**2)**p*a*b*d*e*p*x
+ 6*sqrt(d + e*x)*(a**2 + 2*a*b*x + b**2*x**2)**p*a*b*d*e*x + 4*sqrt(d + e
*x)*(a**2 + 2*a*b*x + b**2*x**2)**p*a*b*e**2*p*x**2 + 3*sqrt(d + e*x)*(a**
2 + 2*a*b*x + b**2*x**2)**p*a*b*e**2*x**2 + 16*sqrt(d + e*x)*(a**2 + 2*a*b
*x + b**2*x**2)**p*b**2*d**2*p**2*x + 24*sqrt(d + e*x)*(a**2 + 2*a*b*x + b
**2*x**2)**p*b**2*d**2*p*x + 16*sqrt(d + e*x)*(a**2 + 2*a*b*x + b**2*x**2)
**p*b**2*d*e*p**2*x**2 + 12*sqrt(d + e*x)*(a**2 + 2*a*b*x + b**2*x**2)**p*
b**2*d*e*p*x**2 - 96*int((sqrt(d + e*x)*(a**2 + 2*a*b*x + b**2*x**2)**p*x)
/(16*a**2*d*e*p**2 + 32*a**2*d*e*p + 15*a**2*d*e + 16*a**2*e**2*p**2*x + 3
2*a**2*e**2*p*x + 15*a**2*e**2*x + 64*a*b*d**2*p**3 + 128*a*b*d**2*p**2 +
60*a*b*d**2*p + 64*a*b*d*e*p**3*x + 144*a*b*d*e*p**2*x + 92*a*b*d*e*p*x +
15*a*b*d*e*x + 16*a*b*e**2*p**2*x**2 + 32*a*b*e**2*p*x**2 + 15*a*b*e**2*x*
*2 + 64*b**2*d**2*p**3*x + 128*b**2*d**2*p**2*x + 60*b**2*d**2*p*x + 64*b*
*2*d*e*p**3*x**2 + 128*b**2*d*e*p**2*x**2 + 60*b**2*d*e*p*x**2),x)*a**4*e
*4*p**3 - 192*int((sqrt(d + e*x)*(a**2 + 2*a*b*x + b**2*x**2)**p*x)/(16...
```


3.441 $\int \sqrt{d + ex}(a^2 + 2abx + b^2x^2)^p dx$

| | |
|----------------------------|------|
| Optimal result | 3284 |
| Mathematica [A] (verified) | 3284 |
| Rubi [A] (verified) | 3285 |
| Maple [F] | 3286 |
| Fricas [F] | 3287 |
| Sympy [F] | 3287 |
| Maxima [F] | 3287 |
| Giac [F] | 3288 |
| Mupad [F(-1)] | 3288 |
| Reduce [F] | 3288 |

Optimal result

Integrand size = 28, antiderivative size = 83

$$\int \sqrt{d + ex}(a^2 + 2abx + b^2x^2)^p dx$$

$$= \frac{2 \left(-\frac{e(a+bx)}{bd-ae}\right)^{-2p} (d + ex)^{3/2} (a^2 + 2abx + b^2x^2)^p \operatorname{Hypergeometric2F1}\left(\frac{3}{2}, -2p, \frac{5}{2}, \frac{b(d+ex)}{bd-ae}\right)}{3e}$$

output `2/3*(e*x+d)^(3/2)*(b^2*x^2+2*a*b*x+a^2)^p*hypergeom([3/2, -2*p], [5/2], b*(e*x+d)/(-a*e+b*d))/e/((-e*(b*x+a)/(-a*e+b*d))^(2*p))`

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.88

$$\int \sqrt{d + ex}(a^2 + 2abx + b^2x^2)^p dx$$

$$= \frac{2 \left(\frac{e(a+bx)}{-bd+ae}\right)^{-2p} ((a + bx)^2)^p (d + ex)^{3/2} \operatorname{Hypergeometric2F1}\left(\frac{3}{2}, -2p, \frac{5}{2}, \frac{b(d+ex)}{bd-ae}\right)}{3e}$$

input `Integrate[Sqrt[d + e*x]*(a^2 + 2*a*b*x + b^2*x^2)^p,x]`

output

$$(2*((a + b*x)^2)^p*(d + e*x)^{(3/2)}*Hypergeometric2F1[3/2, -2*p, 5/2, (b*(d + e*x))/(b*d - a*e]])/(3*e*((e*(a + b*x))/(-(b*d) + a*e))^{(2*p)})$$
Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.107$, Rules used = {1102, 80, 79}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{d+ex}(a^2+2abx+b^2x^2)^p dx$$

$$\downarrow 1102$$

$$(ab+b^2x)^{-2p}(a^2+2abx+b^2x^2)^p \int (xb^2+ab)^{2p} \sqrt{d+ex} dx$$

$$\downarrow 80$$

$$(a^2+2abx+b^2x^2)^p \left(-\frac{e(a+bx)}{bd-ae}\right)^{-2p} \int \sqrt{d+ex} \left(-\frac{bx e}{bd-ae} - \frac{ae}{bd-ae}\right)^{2p} dx$$

$$\downarrow 79$$

$$\frac{2(d+ex)^{3/2}(a^2+2abx+b^2x^2)^p \left(-\frac{e(a+bx)}{bd-ae}\right)^{-2p} \text{Hypergeometric2F1}\left(\frac{3}{2}, -2p, \frac{5}{2}, \frac{b(d+ex)}{bd-ae}\right)}{3e}$$

input

$$\text{Int}[\text{Sqrt}[d + e*x]*(a^2 + 2*a*b*x + b^2*x^2)^p, x]$$

output

$$(2*(d + e*x)^{(3/2)}*(a^2 + 2*a*b*x + b^2*x^2)^p*Hypergeometric2F1[3/2, -2*p, 5/2, (b*(d + e*x))/(b*d - a*e]])/(3*e*((e*(a + b*x))/(-(b*d) + a*e))^{(2*p)})$$

Definitions of rubi rules used

rule 79

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] := Simp[((a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x] && !IntegerQ[m] && !IntegerQ[n] && GtQ[b/(b*c - a*d), 0] && (RationalQ[m] || !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

rule 80

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] := Simp[(c + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*(b*((c + d*x)/(b*c - a*d)))^FracPart[n]) Int[(a + b*x)^m*Simp[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d))], x]^n, x] /; FreeQ[{a, b, c, d, m, n}, x] && !IntegerQ[m] && !IntegerQ[n] && (RationalQ[m] || !SimplerQ[n + 1, m + 1])
```

rule 1102

```
Int[((d_.) + (e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] := Simp[(a + b*x + c*x^2)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x)^(2*FracPart[p])) Int[(d + e*x)^m*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && EqQ[b^2 - 4*a*c, 0]
```

Maple [F]

$$\int \sqrt{ex + d} (b^2x^2 + 2abx + a^2)^p dx$$

input

```
int((e*x+d)^(1/2)*(b^2*x^2+2*a*b*x+a^2)^p,x)
```

output

```
int((e*x+d)^(1/2)*(b^2*x^2+2*a*b*x+a^2)^p,x)
```

Fricas [F]

$$\int \sqrt{d+ex}(a^2+2abx+b^2x^2)^p dx = \int \sqrt{ex+d}(b^2x^2+2abx+a^2)^p dx$$

input `integrate((e*x+d)^(1/2)*(b^2*x^2+2*a*b*x+a^2)^p,x, algorithm="fricas")`

output `integral(sqrt(e*x + d)*(b^2*x^2 + 2*a*b*x + a^2)^p, x)`

Sympy [F]

$$\int \sqrt{d+ex}(a^2+2abx+b^2x^2)^p dx = \int \sqrt{d+ex}((a+bx)^2)^p dx$$

input `integrate((e*x+d)**(1/2)*(b**2*x**2+2*a*b*x+a**2)**p,x)`

output `Integral(sqrt(d + e*x)*((a + b*x)**2)**p, x)`

Maxima [F]

$$\int \sqrt{d+ex}(a^2+2abx+b^2x^2)^p dx = \int \sqrt{ex+d}(b^2x^2+2abx+a^2)^p dx$$

input `integrate((e*x+d)^(1/2)*(b^2*x^2+2*a*b*x+a^2)^p,x, algorithm="maxima")`

output `integrate(sqrt(e*x + d)*(b^2*x^2 + 2*a*b*x + a^2)^p, x)`

Giac [F]

$$\int \sqrt{d+ex}(a^2+2abx+b^2x^2)^p dx = \int \sqrt{ex+d}(b^2x^2+2abx+a^2)^p dx$$

input `integrate((e*x+d)^(1/2)*(b^2*x^2+2*a*b*x+a^2)^p,x, algorithm="giac")`

output `integrate(sqrt(e*x + d)*(b^2*x^2 + 2*a*b*x + a^2)^p, x)`

Mupad [F(-1)]

Timed out.

$$\int \sqrt{d+ex}(a^2+2abx+b^2x^2)^p dx = \int \sqrt{d+ex}(a^2+2abx+b^2x^2)^p dx$$

input `int((d + e*x)^(1/2)*(a^2 + b^2*x^2 + 2*a*b*x)^p,x)`

output `int((d + e*x)^(1/2)*(a^2 + b^2*x^2 + 2*a*b*x)^p, x)`

Reduce [F]

$$\int \sqrt{d+ex}(a^2+2abx+b^2x^2)^p dx = \text{too large to display}$$

input `int((e*x+d)^(1/2)*(b^2*x^2+2*a*b*x+a^2)^p,x)`

output

```

(2*(4*sqrt(d + e*x)*(a**2 + 2*a*b*x + b**2*x**2)**p*a*d*p + sqrt(d + e*x)*
(a**2 + 2*a*b*x + b**2*x**2)**p*a*d + sqrt(d + e*x)*(a**2 + 2*a*b*x + b**2
*x**2)**p*a*e*x + 4*sqrt(d + e*x)*(a**2 + 2*a*b*x + b**2*x**2)**p*b*d*p*x
+ 8*int((sqrt(d + e*x)*(a**2 + 2*a*b*x + b**2*x**2)**p*x)/(4*a**2*d*e*p +
3*a**2*d*e + 4*a**2*e**2*p*x + 3*a**2*e**2*x + 16*a*b*d**2*p**2 + 12*a*b*d
**2*p + 16*a*b*d*e*p**2*x + 16*a*b*d*e*p*x + 3*a*b*d*e*x + 4*a*b*e**2*p*x*
*2 + 3*a*b*e**2*x**2 + 16*b**2*d**2*p**2*x + 12*b**2*d**2*p*x + 16*b**2*d*
e*p**2*x**2 + 12*b**2*d*e*p*x**2),x)*a**3*e**3*p**2 + 6*int((sqrt(d + e*x)
*(a**2 + 2*a*b*x + b**2*x**2)**p*x)/(4*a**2*d*e*p + 3*a**2*d*e + 4*a**2*e*
**2*p*x + 3*a**2*e**2*x + 16*a*b*d**2*p**2 + 12*a*b*d**2*p + 16*a*b*d*e*p**
2*x + 16*a*b*d*e*p*x + 3*a*b*d*e*x + 4*a*b*e**2*p*x**2 + 3*a*b*e**2*x**2 +
16*b**2*d**2*p**2*x + 12*b**2*d**2*p*x + 16*b**2*d*e*p**2*x**2 + 12*b**2*
d*e*p*x**2),x)*a**3*e**3*p + 32*int((sqrt(d + e*x)*(a**2 + 2*a*b*x + b**2*
x**2)**p*x)/(4*a**2*d*e*p + 3*a**2*d*e + 4*a**2*e**2*p*x + 3*a**2*e**2*x +
16*a*b*d**2*p**2 + 12*a*b*d**2*p + 16*a*b*d*e*p**2*x + 16*a*b*d*e*p*x + 3
*a*b*d*e*x + 4*a*b*e**2*p*x**2 + 3*a*b*e**2*x**2 + 16*b**2*d**2*p**2*x + 1
2*b**2*d**2*p*x + 16*b**2*d*e*p**2*x**2 + 12*b**2*d*e*p*x**2),x)*a**2*b*d*
e**2*p**3 + 8*int((sqrt(d + e*x)*(a**2 + 2*a*b*x + b**2*x**2)**p*x)/(4*a**
2*d*e*p + 3*a**2*d*e + 4*a**2*e**2*p*x + 3*a**2*e**2*x + 16*a*b*d**2*p**2
+ 12*a*b*d**2*p + 16*a*b*d*e*p**2*x + 16*a*b*d*e*p*x + 3*a*b*d*e*x + 4*...

```

3.442 $\int \frac{(a^2+2abx+b^2x^2)^p}{\sqrt{d+ex}} dx$

| | |
|----------------------------|------|
| Optimal result | 3290 |
| Mathematica [A] (verified) | 3290 |
| Rubi [A] (verified) | 3291 |
| Maple [F] | 3292 |
| Fricas [F] | 3293 |
| Sympy [F] | 3293 |
| Maxima [F] | 3293 |
| Giac [F] | 3294 |
| Mupad [F(-1)] | 3294 |
| Reduce [F] | 3294 |

Optimal result

Integrand size = 28, antiderivative size = 81

$$\int \frac{(a^2 + 2abx + b^2x^2)^p}{\sqrt{d + ex}} dx = \frac{2 \left(-\frac{e(a+bx)}{bd-ae} \right)^{-2p} \sqrt{d + ex} (a^2 + 2abx + b^2x^2)^p \operatorname{Hypergeometric2F1} \left(\frac{1}{2}, -2p, \frac{3}{2}, \frac{b(d+ex)}{bd-ae} \right)}{e}$$

output

```
2*(e*x+d)^(1/2)*(b^2*x^2+2*a*b*x+a^2)^p*hypergeom([1/2, -2*p], [3/2], b*(e*x+d)/(-a*e+b*d))/e/((-e*(b*x+a)/(-a*e+b*d))^(2*p))
```

Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.88

$$\int \frac{(a^2 + 2abx + b^2x^2)^p}{\sqrt{d + ex}} dx = \frac{2 \left(\frac{e(a+bx)}{-bd+ae} \right)^{-2p} ((a + bx)^2)^p \sqrt{d + ex} \operatorname{Hypergeometric2F1} \left(\frac{1}{2}, -2p, \frac{3}{2}, \frac{b(d+ex)}{bd-ae} \right)}{e}$$

input

```
Integrate[(a^2 + 2*a*b*x + b^2*x^2)^p/Sqrt[d + e*x],x]
```

output

$$(2*((a + b*x)^2)^p*\text{Sqrt}[d + e*x]*\text{Hypergeometric2F1}[1/2, -2*p, 3/2, (b*(d + e*x))/(b*d - a*e)])/(e*((e*(a + b*x))/(-(b*d) + a*e))^(2*p))$$
Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.107$, Rules used = {1102, 80, 79}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\begin{aligned} & \int \frac{(a^2 + 2abx + b^2x^2)^p}{\sqrt{d + ex}} dx \\ & \quad \downarrow \text{1102} \\ & (ab + b^2x)^{-2p} (a^2 + 2abx + b^2x^2)^p \int \frac{(xb^2 + ab)^{2p}}{\sqrt{d + ex}} dx \\ & \quad \downarrow \text{80} \\ & (a^2 + 2abx + b^2x^2)^p \left(-\frac{e(a + bx)}{bd - ae} \right)^{-2p} \int \frac{\left(-\frac{bx e}{bd - ae} - \frac{ae}{bd - ae} \right)^{2p}}{\sqrt{d + ex}} dx \\ & \quad \downarrow \text{79} \\ & \frac{2\sqrt{d + ex} (a^2 + 2abx + b^2x^2)^p \left(-\frac{e(a + bx)}{bd - ae} \right)^{-2p} \text{Hypergeometric2F1} \left(\frac{1}{2}, -2p, \frac{3}{2}, \frac{b(d + ex)}{bd - ae} \right)}{e} \end{aligned}$$

input

$$\text{Int}[(a^2 + 2*a*b*x + b^2*x^2)^p/\text{Sqrt}[d + e*x], x]$$

output

$$(2*\text{Sqrt}[d + e*x]*(a^2 + 2*a*b*x + b^2*x^2)^p*\text{Hypergeometric2F1}[1/2, -2*p, 3/2, (b*(d + e*x))/(b*d - a*e)])/(e*(-((e*(a + b*x))/(b*d - a*e)))^(2*p))$$

Definitions of rubi rules used

rule 79

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] := Simp[((a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x] && !IntegerQ[m] && !IntegerQ[n] && GtQ[b/(b*c - a*d), 0] && (RationalQ[m] || !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

rule 80

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] := Simp[(c + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*(b*((c + d*x)/(b*c - a*d)))^FracPart[n]) Int[(a + b*x)^m*Simp[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d)), x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && !IntegerQ[m] && !IntegerQ[n] && (RationalQ[m] || !SimplerQ[n + 1, m + 1])
```

rule 1102

```
Int[((d_.) + (e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] := Simp[(a + b*x + c*x^2)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x)^(2*FracPart[p])) Int[(d + e*x)^m*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && EqQ[b^2 - 4*a*c, 0]
```

Maple [F]

$$\int \frac{(b^2x^2 + 2abx + a^2)^p}{\sqrt{ex + d}} dx$$

input

```
int((b^2*x^2+2*a*b*x+a^2)^p/(e*x+d)^(1/2),x)
```

output

```
int((b^2*x^2+2*a*b*x+a^2)^p/(e*x+d)^(1/2),x)
```

Fricas [F]

$$\int \frac{(a^2 + 2abx + b^2x^2)^p}{\sqrt{d + ex}} dx = \int \frac{(b^2x^2 + 2abx + a^2)^p}{\sqrt{ex + d}} dx$$

input `integrate((b^2*x^2+2*a*b*x+a^2)^p/(e*x+d)^(1/2),x, algorithm="fricas")`

output `integral((b^2*x^2 + 2*a*b*x + a^2)^p/sqrt(e*x + d), x)`

Sympy [F]

$$\int \frac{(a^2 + 2abx + b^2x^2)^p}{\sqrt{d + ex}} dx = \int \frac{((a + bx)^2)^p}{\sqrt{d + ex}} dx$$

input `integrate((b**2*x**2+2*a*b*x+a**2)**p/(e*x+d)**(1/2),x)`

output `Integral(((a + b*x)**2)**p/sqrt(d + e*x), x)`

Maxima [F]

$$\int \frac{(a^2 + 2abx + b^2x^2)^p}{\sqrt{d + ex}} dx = \int \frac{(b^2x^2 + 2abx + a^2)^p}{\sqrt{ex + d}} dx$$

input `integrate((b^2*x^2+2*a*b*x+a^2)^p/(e*x+d)^(1/2),x, algorithm="maxima")`

output `integrate((b^2*x^2 + 2*a*b*x + a^2)^p/sqrt(e*x + d), x)`

Giac [F]

$$\int \frac{(a^2 + 2abx + b^2x^2)^p}{\sqrt{d + ex}} dx = \int \frac{(b^2x^2 + 2abx + a^2)^p}{\sqrt{ex + d}} dx$$

input `integrate((b^2*x^2+2*a*b*x+a^2)^p/(e*x+d)^(1/2),x, algorithm="giac")`

output `integrate((b^2*x^2 + 2*a*b*x + a^2)^p/sqrt(e*x + d), x)`

Mupad [F(-1)]

Timed out.

$$\int \frac{(a^2 + 2abx + b^2x^2)^p}{\sqrt{d + ex}} dx = \int \frac{(a^2 + 2abx + b^2x^2)^p}{\sqrt{d + ex}} dx$$

input `int((a^2 + b^2*x^2 + 2*a*b*x)^p/(d + e*x)^(1/2),x)`

output `int((a^2 + b^2*x^2 + 2*a*b*x)^p/(d + e*x)^(1/2), x)`

Reduce [F]

$$\int \frac{(a^2 + 2abx + b^2x^2)^p}{\sqrt{d + ex}} dx$$

$$= \frac{2\sqrt{ex + d} (b^2x^2 + 2abx + a^2)^p a - 4 \left(\int \frac{\sqrt{ex+d} (b^2x^2+2abx+a^2)^p x}{4b^2dep x^2+4abdep x+ab e^2x^2+4b^2d^2px+a^2e^2x+4ab d^2p+abdex+a^2de} dx \right) a^2 b e^2 p - \dots}{\dots}$$

input `int((b^2*x^2+2*a*b*x+a^2)^p/(e*x+d)^(1/2),x)`

output

```
(2*(sqrt(d + e*x)*(a**2 + 2*a*b*x + b**2*x**2)**p*a - 2*int((sqrt(d + e*x)
*(a**2 + 2*a*b*x + b**2*x**2)**p*x)/(a**2*d*e + a**2*e**2*x + 4*a*b*d**2*p
+ 4*a*b*d*e*p*x + a*b*d*e*x + a*b*e**2*x**2 + 4*b**2*d**2*p*x + 4*b**2*d*
e*p*x**2),x)*a**2*b*e**2*p - 8*int((sqrt(d + e*x)*(a**2 + 2*a*b*x + b**2*x
**2)**p*x)/(a**2*d*e + a**2*e**2*x + 4*a*b*d**2*p + 4*a*b*d*e*p*x + a*b*d*
e*x + a*b*e**2*x**2 + 4*b**2*d**2*p*x + 4*b**2*d*e*p*x**2),x)*a*b**2*d*e*p
**2 + 2*int((sqrt(d + e*x)*(a**2 + 2*a*b*x + b**2*x**2)**p*x)/(a**2*d*e +
a**2*e**2*x + 4*a*b*d**2*p + 4*a*b*d*e*p*x + a*b*d*e*x + a*b*e**2*x**2 + 4
*b**2*d**2*p*x + 4*b**2*d*e*p*x**2),x)*a*b**2*d*e*p + 8*int((sqrt(d + e*x)
*(a**2 + 2*a*b*x + b**2*x**2)**p*x)/(a**2*d*e + a**2*e**2*x + 4*a*b*d**2*p
+ 4*a*b*d*e*p*x + a*b*d*e*x + a*b*e**2*x**2 + 4*b**2*d**2*p*x + 4*b**2*d*
e*p*x**2),x)*b**3*d**2*p**2))/(a*e + 4*b*d*p)
```

3.443 $\int \frac{(a^2+2abx+b^2x^2)^p}{(d+ex)^{3/2}} dx$

| | |
|----------------------------|------|
| Optimal result | 3296 |
| Mathematica [A] (verified) | 3296 |
| Rubi [A] (verified) | 3297 |
| Maple [F] | 3298 |
| Fricas [F] | 3299 |
| Sympy [F] | 3299 |
| Maxima [F] | 3299 |
| Giac [F] | 3300 |
| Mupad [F(-1)] | 3300 |
| Reduce [F] | 3300 |

Optimal result

Integrand size = 28, antiderivative size = 81

$$\int \frac{(a^2 + 2abx + b^2x^2)^p}{(d + ex)^{3/2}} dx = \frac{2 \left(-\frac{e(a+bx)}{bd-ae} \right)^{-2p} (a^2 + 2abx + b^2x^2)^p \operatorname{Hypergeometric2F1} \left(-\frac{1}{2}, -2p, \frac{1}{2}, \frac{b(d+ex)}{bd-ae} \right)}{e\sqrt{d+ex}}$$

output `-2*(b^2*x^2+2*a*b*x+a^2)^p*hypergeom([-1/2, -2*p], [1/2], b*(e*x+d)/(-a*e+b*d))/e/((-e*(b*x+a)/(-a*e+b*d))^(2*p))/(e*x+d)^(1/2)`

Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.88

$$\int \frac{(a^2 + 2abx + b^2x^2)^p}{(d + ex)^{3/2}} dx = \frac{2 \left(\frac{e(a+bx)}{-bd+ae} \right)^{-2p} ((a + bx)^2)^p \operatorname{Hypergeometric2F1} \left(-\frac{1}{2}, -2p, \frac{1}{2}, \frac{b(d+ex)}{bd-ae} \right)}{e\sqrt{d+ex}}$$

input `Integrate[(a^2 + 2*a*b*x + b^2*x^2)^p/(d + e*x)^(3/2),x]`

output

```
(-2*((a + b*x)^2)^p*Hypergeometric2F1[-1/2, -2*p, 1/2, (b*(d + e*x))/(b*d - a*e)]/(e*((e*(a + b*x))/(-b*d) + a*e))^(2*p)*Sqrt[d + e*x])
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.107$, Rules used = {1102, 80, 79}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a^2 + 2abx + b^2x^2)^p}{(d + ex)^{3/2}} dx$$

↓ 1102

$$(ab + b^2x)^{-2p} (a^2 + 2abx + b^2x^2)^p \int \frac{(xb^2 + ab)^{2p}}{(d + ex)^{3/2}} dx$$

↓ 80

$$(a^2 + 2abx + b^2x^2)^p \left(-\frac{e(a + bx)}{bd - ae} \right)^{-2p} \int \frac{\left(-\frac{bx}{bd - ae} - \frac{ae}{bd - ae} \right)^{2p}}{(d + ex)^{3/2}} dx$$

↓ 79

$$\frac{2(a^2 + 2abx + b^2x^2)^p \left(-\frac{e(a + bx)}{bd - ae} \right)^{-2p} \text{Hypergeometric2F1} \left(-\frac{1}{2}, -2p, \frac{1}{2}, \frac{b(d + ex)}{bd - ae} \right)}{e\sqrt{d + ex}}$$

input

```
Int[(a^2 + 2*a*b*x + b^2*x^2)^p/(d + e*x)^(3/2),x]
```

output

```
(-2*(a^2 + 2*a*b*x + b^2*x^2)^p*Hypergeometric2F1[-1/2, -2*p, 1/2, (b*(d + e*x))/(b*d - a*e)]/(e*(-((e*(a + b*x))/(b*d - a*e))))^(2*p)*Sqrt[d + e*x])
```

Definitions of rubi rules used

rule 79

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] := Simp[((a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x] && !IntegerQ[m] && !IntegerQ[n] && GtQ[b/(b*c - a*d), 0] && (RationalQ[m] || !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

rule 80

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] := Simp[(c + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*(b*((c + d*x)/(b*c - a*d)))^FracPart[n]) Int[(a + b*x)^m*Simp[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d)), x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && !IntegerQ[m] && !IntegerQ[n] && (RationalQ[m] || !SimplerQ[n + 1, m + 1])
```

rule 1102

```
Int[((d_.) + (e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] := Simp[(a + b*x + c*x^2)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x)^(2*FracPart[p])) Int[(d + e*x)^m*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && EqQ[b^2 - 4*a*c, 0]
```

Maple [F]

$$\int \frac{(b^2x^2 + 2abx + a^2)^p}{(ex + d)^{\frac{3}{2}}} dx$$

input

```
int((b^2*x^2+2*a*b*x+a^2)^p/(e*x+d)^(3/2),x)
```

output

```
int((b^2*x^2+2*a*b*x+a^2)^p/(e*x+d)^(3/2),x)
```

Fricas [F]

$$\int \frac{(a^2 + 2abx + b^2x^2)^p}{(d + ex)^{3/2}} dx = \int \frac{(b^2x^2 + 2abx + a^2)^p}{(ex + d)^{\frac{3}{2}}} dx$$

input `integrate((b^2*x^2+2*a*b*x+a^2)^p/(e*x+d)^(3/2),x, algorithm="fricas")`

output `integral(sqrt(e*x + d)*(b^2*x^2 + 2*a*b*x + a^2)^p/(e^2*x^2 + 2*d*e*x + d^2), x)`

Sympy [F]

$$\int \frac{(a^2 + 2abx + b^2x^2)^p}{(d + ex)^{3/2}} dx = \int \frac{((a + bx)^2)^p}{(d + ex)^{\frac{3}{2}}} dx$$

input `integrate((b**2*x**2+2*a*b*x+a**2)**p/(e*x+d)**(3/2),x)`

output `Integral(((a + b*x)**2)**p/(d + e*x)**(3/2), x)`

Maxima [F]

$$\int \frac{(a^2 + 2abx + b^2x^2)^p}{(d + ex)^{3/2}} dx = \int \frac{(b^2x^2 + 2abx + a^2)^p}{(ex + d)^{\frac{3}{2}}} dx$$

input `integrate((b^2*x^2+2*a*b*x+a^2)^p/(e*x+d)^(3/2),x, algorithm="maxima")`

output `integrate((b^2*x^2 + 2*a*b*x + a^2)^p/(e*x + d)^(3/2), x)`

Giac [F]

$$\int \frac{(a^2 + 2abx + b^2x^2)^p}{(d + ex)^{3/2}} dx = \int \frac{(b^2x^2 + 2abx + a^2)^p}{(ex + d)^{\frac{3}{2}}} dx$$

input `integrate((b^2*x^2+2*a*b*x+a^2)^p/(e*x+d)^(3/2),x, algorithm="giac")`

output `integrate((b^2*x^2 + 2*a*b*x + a^2)^p/(e*x + d)^(3/2), x)`

Mupad [F(-1)]

Timed out.

$$\int \frac{(a^2 + 2abx + b^2x^2)^p}{(d + ex)^{3/2}} dx = \int \frac{(a^2 + 2abx + b^2x^2)^p}{(d + ex)^{3/2}} dx$$

input `int((a^2 + b^2*x^2 + 2*a*b*x)^p/(d + e*x)^(3/2), x)`

output `int((a^2 + b^2*x^2 + 2*a*b*x)^p/(d + e*x)^(3/2), x)`

Reduce [F]

$$\int \frac{(a^2 + 2abx + b^2x^2)^p}{(d + ex)^{3/2}} dx = \text{Too large to display}$$

input `int((b^2*x^2+2*a*b*x+a^2)^p/(e*x+d)^(3/2), x)`

output

```

(2*( - sqrt(d + e*x)*(a**2 + 2*a*b*x + b**2*x**2)**p*a + 2*int((sqrt(d + e
*x)*(a**2 + 2*a*b*x + b**2*x**2)**p*x)/(a**2*d**2*e + 2*a**2*d*e**2*x + a
**2*e**3*x**2 - 4*a*b*d**3*p - 8*a*b*d**2*e*p*x + a*b*d**2*e*x - 4*a*b*d*e
**2*p*x**2 + 2*a*b*d*e**2*x**2 + a*b*e**3*x**3 - 4*b**2*d**3*p*x - 8*b**2*d
**2*e*p*x**2 - 4*b**2*d*e**2*p*x**3),x)*a**2*b*d*e**2*p + 2*int((sqrt(d +
e*x)*(a**2 + 2*a*b*x + b**2*x**2)**p*x)/(a**2*d**2*e + 2*a**2*d*e**2*x + a
**2*e**3*x**2 - 4*a*b*d**3*p - 8*a*b*d**2*e*p*x + a*b*d**2*e*x - 4*a*b*d*e
**2*p*x**2 + 2*a*b*d*e**2*x**2 + a*b*e**3*x**3 - 4*b**2*d**3*p*x - 8*b**2*
d**2*e*p*x**2 - 4*b**2*d*e**2*p*x**3),x)*a**2*b*e**3*p*x - 8*int((sqrt(d +
e*x)*(a**2 + 2*a*b*x + b**2*x**2)**p*x)/(a**2*d**2*e + 2*a**2*d*e**2*x +
a**2*e**3*x**2 - 4*a*b*d**3*p - 8*a*b*d**2*e*p*x + a*b*d**2*e*x - 4*a*b*d*
e**2*p*x**2 + 2*a*b*d*e**2*x**2 + a*b*e**3*x**3 - 4*b**2*d**3*p*x - 8*b**2
*d**2*e*p*x**2 - 4*b**2*d*e**2*p*x**3),x)*a*b**2*d**2*e*p**2 - 2*int((sqrt
(d + e*x)*(a**2 + 2*a*b*x + b**2*x**2)**p*x)/(a**2*d**2*e + 2*a**2*d*e**2*
x + a**2*e**3*x**2 - 4*a*b*d**3*p - 8*a*b*d**2*e*p*x + a*b*d**2*e*x - 4*a*
b*d*e**2*p*x**2 + 2*a*b*d*e**2*x**2 + a*b*e**3*x**3 - 4*b**2*d**3*p*x - 8*
b**2*d**2*e*p*x**2 - 4*b**2*d*e**2*p*x**3),x)*a*b**2*d**2*e*p - 8*int((sqr
t(d + e*x)*(a**2 + 2*a*b*x + b**2*x**2)**p*x)/(a**2*d**2*e + 2*a**2*d*e**2
*x + a**2*e**3*x**2 - 4*a*b*d**3*p - 8*a*b*d**2*e*p*x + a*b*d**2*e*x - 4*a
*b*d*e**2*p*x**2 + 2*a*b*d*e**2*x**2 + a*b*e**3*x**3 - 4*b**2*d**3*p*x ...

```

3.444 $\int \frac{(a^2+2abx+b^2x^2)^p}{(d+ex)^{5/2}} dx$

| | |
|----------------------------|------|
| Optimal result | 3302 |
| Mathematica [A] (verified) | 3302 |
| Rubi [A] (verified) | 3303 |
| Maple [F] | 3304 |
| Fricas [F] | 3305 |
| Sympy [F] | 3305 |
| Maxima [F] | 3305 |
| Giac [F] | 3306 |
| Mupad [F(-1)] | 3306 |
| Reduce [F] | 3306 |

Optimal result

Integrand size = 28, antiderivative size = 83

$$\int \frac{(a^2 + 2abx + b^2x^2)^p}{(d + ex)^{5/2}} dx = \frac{2\left(-\frac{e(a+bx)}{bd-ae}\right)^{-2p} (a^2 + 2abx + b^2x^2)^p \text{Hypergeometric2F1}\left(-\frac{3}{2}, -2p, -\frac{1}{2}, \frac{b(d+ex)}{bd-ae}\right)}{3e(d + ex)^{3/2}}$$

output

```
-2/3*(b^2*x^2+2*a*b*x+a^2)^p*hypergeom([-3/2, -2*p], [-1/2], b*(e*x+d)/(-a*e+b*d))/e/((-e*(b*x+a)/(-a*e+b*d))^(2*p))/(e*x+d)^(3/2)
```

Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.88

$$\int \frac{(a^2 + 2abx + b^2x^2)^p}{(d + ex)^{5/2}} dx = \frac{2\left(\frac{e(a+bx)}{-bd+ae}\right)^{-2p} ((a + bx)^2)^p \text{Hypergeometric2F1}\left(-\frac{3}{2}, -2p, -\frac{1}{2}, \frac{b(d+ex)}{bd-ae}\right)}{3e(d + ex)^{3/2}}$$

input

```
Integrate[(a^2 + 2*a*b*x + b^2*x^2)^p/(d + e*x)^(5/2),x]
```

output

$$\frac{(-2*((a + b*x)^2)^p * \text{Hypergeometric2F1}[-3/2, -2*p, -1/2, (b*(d + e*x))/(b*d - a*e)])}{(3*e*((e*(a + b*x))/(-b*d + a*e))^{(2*p)}*(d + e*x)^{(3/2)})}$$

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.107$, Rules used = {1102, 80, 79}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\begin{aligned} & \int \frac{(a^2 + 2abx + b^2x^2)^p}{(d + ex)^{5/2}} dx \\ & \quad \downarrow \text{1102} \\ & (ab + b^2x)^{-2p} (a^2 + 2abx + b^2x^2)^p \int \frac{(xb^2 + ab)^{2p}}{(d + ex)^{5/2}} dx \\ & \quad \downarrow \text{80} \\ & (a^2 + 2abx + b^2x^2)^p \left(-\frac{e(a + bx)}{bd - ae} \right)^{-2p} \int \frac{\left(-\frac{bx}{bd - ae} - \frac{ae}{bd - ae} \right)^{2p}}{(d + ex)^{5/2}} dx \\ & \quad \downarrow \text{79} \\ & \frac{2(a^2 + 2abx + b^2x^2)^p \left(-\frac{e(a + bx)}{bd - ae} \right)^{-2p} \text{Hypergeometric2F1} \left(-\frac{3}{2}, -2p, -\frac{1}{2}, \frac{b(d + ex)}{bd - ae} \right)}{3e(d + ex)^{3/2}} \end{aligned}$$

input

$$\text{Int}[(a^2 + 2*a*b*x + b^2*x^2)^p/(d + e*x)^(5/2),x]$$

output

$$\frac{(-2*(a^2 + 2*a*b*x + b^2*x^2)^p * \text{Hypergeometric2F1}[-3/2, -2*p, -1/2, (b*(d + e*x))/(b*d - a*e)])}{(3*e*((e*(a + b*x))/(b*d - a*e))^{(2*p)}*(d + e*x)^{(3/2)})}$$

Defintions of rubi rules used

rule 79

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] := Simp[((a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x] && !IntegerQ[m] && !IntegerQ[n] && GtQ[b/(b*c - a*d), 0] && (RationalQ[m] || !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

rule 80

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] := Simp[(c + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*(b*((c + d*x)/(b*c - a*d)))^FracPart[n]) Int[(a + b*x)^m*Simp[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d)), x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && !IntegerQ[m] && !IntegerQ[n] && (RationalQ[m] || !SimplerQ[n + 1, m + 1])
```

rule 1102

```
Int[((d_.) + (e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] := Simp[(a + b*x + c*x^2)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x)^(2*FracPart[p])) Int[(d + e*x)^m*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && EqQ[b^2 - 4*a*c, 0]
```

Maple [F]

$$\int \frac{(b^2x^2 + 2abx + a^2)^p}{(ex + d)^{\frac{5}{2}}} dx$$

input

```
int((b^2*x^2+2*a*b*x+a^2)^p/(e*x+d)^(5/2),x)
```

output

```
int((b^2*x^2+2*a*b*x+a^2)^p/(e*x+d)^(5/2),x)
```

Fricas [F]

$$\int \frac{(a^2 + 2abx + b^2x^2)^p}{(d + ex)^{5/2}} dx = \int \frac{(b^2x^2 + 2abx + a^2)^p}{(ex + d)^{\frac{5}{2}}} dx$$

input `integrate((b^2*x^2+2*a*b*x+a^2)^p/(e*x+d)^(5/2),x, algorithm="fricas")`

output `integral(sqrt(e*x + d)*(b^2*x^2 + 2*a*b*x + a^2)^p/(e^3*x^3 + 3*d*e^2*x^2 + 3*d^2*e*x + d^3), x)`

Sympy [F]

$$\int \frac{(a^2 + 2abx + b^2x^2)^p}{(d + ex)^{5/2}} dx = \int \frac{((a + bx)^2)^p}{(d + ex)^{\frac{5}{2}}} dx$$

input `integrate((b**2*x**2+2*a*b*x+a**2)**p/(e*x+d)**(5/2),x)`

output `Integral(((a + b*x)**2)**p/(d + e*x)**(5/2), x)`

Maxima [F]

$$\int \frac{(a^2 + 2abx + b^2x^2)^p}{(d + ex)^{5/2}} dx = \int \frac{(b^2x^2 + 2abx + a^2)^p}{(ex + d)^{\frac{5}{2}}} dx$$

input `integrate((b^2*x^2+2*a*b*x+a^2)^p/(e*x+d)^(5/2),x, algorithm="maxima")`

output `integrate((b^2*x^2 + 2*a*b*x + a^2)^p/(e*x + d)^(5/2), x)`

Giac [F]

$$\int \frac{(a^2 + 2abx + b^2x^2)^p}{(d + ex)^{5/2}} dx = \int \frac{(b^2x^2 + 2abx + a^2)^p}{(ex + d)^{\frac{5}{2}}} dx$$

input `integrate((b^2*x^2+2*a*b*x+a^2)^p/(e*x+d)^(5/2),x, algorithm="giac")`

output `integrate((b^2*x^2 + 2*a*b*x + a^2)^p/(e*x + d)^(5/2), x)`

Mupad [F(-1)]

Timed out.

$$\int \frac{(a^2 + 2abx + b^2x^2)^p}{(d + ex)^{5/2}} dx = \int \frac{(a^2 + 2abx + b^2x^2)^p}{(d + ex)^{5/2}} dx$$

input `int((a^2 + b^2*x^2 + 2*a*b*x)^p/(d + e*x)^(5/2), x)`

output `int((a^2 + b^2*x^2 + 2*a*b*x)^p/(d + e*x)^(5/2), x)`

Reduce [F]

$$\int \frac{(a^2 + 2abx + b^2x^2)^p}{(d + ex)^{5/2}} dx = \text{too large to display}$$

input `int((b^2*x^2+2*a*b*x+a^2)^p/(e*x+d)^(5/2), x)`

output

```
(2*( - sqrt(d + e*x)*(a**2 + 2*a*b*x + b**2*x**2)**p*a + 6*int((sqrt(d + e
*x)*(a**2 + 2*a*b*x + b**2*x**2)**p*x)/(3*a**2*d**3*e + 9*a**2*d**2*e**2*x
+ 9*a**2*d*e**3*x**2 + 3*a**2*e**4*x**3 - 4*a*b*d**4*p - 12*a*b*d**3*e*p*
x + 3*a*b*d**3*e*x - 12*a*b*d**2*e**2*p*x**2 + 9*a*b*d**2*e**2*x**2 - 4*a*
b*d*e**3*p*x**3 + 9*a*b*d*e**3*x**3 + 3*a*b*e**4*x**4 - 4*b**2*d**4*p*x -
12*b**2*d**3*e*p*x**2 - 12*b**2*d**2*e**2*p*x**3 - 4*b**2*d*e**3*p*x**4),x
)*a**2*b*d**2*e**2*p + 12*int((sqrt(d + e*x)*(a**2 + 2*a*b*x + b**2*x**2)*
*p*x)/(3*a**2*d**3*e + 9*a**2*d**2*e**2*x + 9*a**2*d*e**3*x**2 + 3*a**2*e*
**4*x**3 - 4*a*b*d**4*p - 12*a*b*d**3*e*p*x + 3*a*b*d**3*e*x - 12*a*b*d**2*
e**2*p*x**2 + 9*a*b*d**2*e**2*x**2 - 4*a*b*d*e**3*p*x**3 + 9*a*b*d*e**3*x*
**3 + 3*a*b*e**4*x**4 - 4*b**2*d**4*p*x - 12*b**2*d**3*e*p*x**2 - 12*b**2*d
**2*e**2*p*x**3 - 4*b**2*d*e**3*p*x**4),x)*a**2*b*d*e**3*p*x + 6*int((sqrt
(d + e*x)*(a**2 + 2*a*b*x + b**2*x**2)**p*x)/(3*a**2*d**3*e + 9*a**2*d**2*
e**2*x + 9*a**2*d*e**3*x**2 + 3*a**2*e**4*x**3 - 4*a*b*d**4*p - 12*a*b*d**
3*e*p*x + 3*a*b*d**3*e*x - 12*a*b*d**2*e**2*p*x**2 + 9*a*b*d**2*e**2*x**2
- 4*a*b*d*e**3*p*x**3 + 9*a*b*d*e**3*x**3 + 3*a*b*e**4*x**4 - 4*b**2*d**4*
p*x - 12*b**2*d**3*e*p*x**2 - 12*b**2*d**2*e**2*p*x**3 - 4*b**2*d*e**3*p*x
**4),x)*a**2*b*e**4*p*x**2 - 8*int((sqrt(d + e*x)*(a**2 + 2*a*b*x + b**2*x
**2)**p*x)/(3*a**2*d**3*e + 9*a**2*d**2*e**2*x + 9*a**2*d*e**3*x**2 + 3*a*
**2*e**4*x**3 - 4*a*b*d**4*p - 12*a*b*d**3*e*p*x + 3*a*b*d**3*e*x - 12*a...
```


3.445 $\int (d + ex)^m (cd^2 + 2cdex + ce^2x^2)^p dx$

| | |
|---|------|
| Optimal result | 3308 |
| Mathematica [A] (verified) | 3308 |
| Rubi [A] (verified) | 3309 |
| Maple [A] (verified) | 3310 |
| Fricas [A] (verification not implemented) | 3310 |
| Sympy [F] | 3311 |
| Maxima [A] (verification not implemented) | 3311 |
| Giac [A] (verification not implemented) | 3312 |
| Mupad [B] (verification not implemented) | 3312 |
| Reduce [B] (verification not implemented) | 3312 |

Optimal result

Integrand size = 30, antiderivative size = 43

$$\int (d + ex)^m (cd^2 + 2cdex + ce^2x^2)^p dx = \frac{(d + ex)^{1+m} (cd^2 + 2cdex + ce^2x^2)^p}{e(1 + m + 2p)}$$

output

```
(e*x+d)^(1+m)*(c*e^2*x^2+2*c*d*e*x+c*d^2)^p/e/(1+m+2*p)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.74

$$\int (d + ex)^m (cd^2 + 2cdex + ce^2x^2)^p dx = \frac{(d + ex)^{1+m} (c(d + ex)^2)^p}{e(1 + m + 2p)}$$

input

```
Integrate[(d + e*x)^m*(c*d^2 + 2*c*d*e*x + c*e^2*x^2)^p,x]
```

output

```
((d + e*x)^(1 + m)*(c*(d + e*x)^2)^p)/(e*(1 + m + 2*p))
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {1102, 37, 17}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (d + ex)^m (cd^2 + 2cdex + ce^2x^2)^p dx$$

$$\downarrow 1102$$

$$(cde + ce^2x)^{-2p} (cd^2 + 2cdex + ce^2x^2)^p \int (d + ex)^m (cxe^2 + cde)^{2p} dx$$

$$\downarrow 37$$

$$(d + ex)^{-2p} (cd^2 + 2cdex + ce^2x^2)^p \int (d + ex)^{m+2p} dx$$

$$\downarrow 17$$

$$\frac{(d + ex)^{m+1} (cd^2 + 2cdex + ce^2x^2)^p}{e(m + 2p + 1)}$$

input `Int[(d + e*x)^m*(c*d^2 + 2*c*d*e*x + c*e^2*x^2)^p,x]`

output `((d + e*x)^(1 + m)*(c*d^2 + 2*c*d*e*x + c*e^2*x^2)^p)/(e*(1 + m + 2*p))`

Defintions of rubi rules used

rule 17 `Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] := Simp[c*((a + b*x)^(m + 1))/(b*(m + 1))], x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]`

rule 37 `Int[(u_.)*((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] := Simp[(a + b*x)^m/(c + d*x)^m Int[u*(c + d*x)^(m + n), x], x] /; FreeQ[{a, b, c, d, m, n}, x] && EqQ[b*c - a*d, 0] && !SimplerQ[a + b*x, c + d*x]`

rule 1102

```
Int[((d_.) + (e_.)*(x_.))^(m_.)*((a_.) + (b_.)*(x_.) + (c_.)*(x_.)^2)^(p_), x_Symbol]
:> Simp[(a + b*x + c*x^2)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x)^(2*FracPart[p]))
Int[(d + e*x)^m*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && EqQ[b^2 - 4*a*c, 0]
```

Maple [A] (verified)

Time = 1.82 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.02

| method | result |
|---------------|--|
| gospers | $\frac{(ex+d)^{1+m}(x^2ce^2+2cdxe+cd^2)^p}{e^{(1+m+2p)}}$ |
| orering | $\frac{(ex+d)(ex+d)^m(x^2ce^2+2cdxe+cd^2)^p}{e^{(1+m+2p)}}$ |
| parallelrisch | $\frac{x(ex+d)^m(c(e^2x^2+2dex+d^2))^pe+(ex+d)^m(c(e^2x^2+2dex+d^2))^pd}{e^{(1+m+2p)}}$ |
| norman | $\frac{x e^{m \ln(ex+d)} e^{p \ln(x^2 c e^2 + 2 c d x e + c d^2)}}{1+m+2p} + \frac{d e^{m \ln(ex+d)} e^{p \ln(x^2 c e^2 + 2 c d x e + c d^2)}}{e^{(1+m+2p)}}$ |
| risch | $\frac{(ex+d)(ex+d)^m c^p (ex+d)^{2p} e^{-i\pi p \left(\operatorname{csgn}(i(ex+d)^2)^3 - 2 \operatorname{csgn}(i(ex+d)^2) \operatorname{csgn}(i(ex+d)) + \operatorname{csgn}(i(ex+d)^2) \operatorname{csgn}(i(ex+d))^2 - \operatorname{csgn}(i(ex+d)) \right)}}{e^{(1+m+2p)}}$ |

```
input int((e*x+d)^m*(c*e^2*x^2+2*c*d*e*x+c*d^2)^p,x,method=_RETURNVERBOSE)
```

```
output (e*x+d)^(1+m)*(c*e^2*x^2+2*c*d*e*x+c*d^2)^p/e/(1+m+2*p)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.91

$$\int (d + ex)^m (cd^2 + 2cdex + ce^2x^2)^p dx = \frac{(ex + d)(ex + d)^m e^{(2p \log(ex+d) + p \log(c))}}{em + 2ep + e}$$

```
input integrate((e*x+d)^m*(c*e^2*x^2+2*c*d*e*x+c*d^2)^p,x, algorithm="fricas")
```

```
output (e*x + d)*(e*x + d)^m*e^(2*p*log(e*x + d) + p*log(c))/(e*m + 2*e*p + e)
```

Sympy [F]

$$\int (d + ex)^m (cd^2 + 2cdex + ce^2x^2)^p dx$$

$$= \begin{cases} d^{-2p-1}x(cd^2)^p & \text{for } e = 0 \wedge m = -2p - 1 \\ d^m x(cd^2)^p & \text{for } e = 0 \\ \int (c(d + ex)^2)^p (d + ex)^{-2p-1} dx & \text{for } m = -2p - 1 \\ \frac{d(d+ex)^m (cd^2+2cdex+ce^2x^2)^p}{em+2ep+e} + \frac{ex(d+ex)^m (cd^2+2cdex+ce^2x^2)^p}{em+2ep+e} & \text{otherwise} \end{cases}$$

input `integrate((e*x+d)**m*(c*e**2*x**2+2*c*d*e*x+c*d**2)**p,x)`

output `Piecewise((d**(-2*p - 1)*x*(c*d**2)**p, Eq(e, 0) & Eq(m, -2*p - 1)), (d**m*x*(c*d**2)**p, Eq(e, 0)), (Integral((c*(d + e*x)**2)**p*(d + e*x)**(-2*p - 1), x), Eq(m, -2*p - 1)), (d*(d + e*x)**m*(c*d**2 + 2*c*d*e*x + c*e**2*x**2)**p/(e*m + 2*e*p + e) + e*x*(d + e*x)**m*(c*d**2 + 2*c*d*e*x + c*e**2*x**2)**p/(e*m + 2*e*p + e), True))`

Maxima [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.00

$$\int (d + ex)^m (cd^2 + 2cdex + ce^2x^2)^p dx = \frac{(c^p ex + c^p d)e^{(m \log(ex+d) + 2p \log(ex+d))}}{e(m + 2p + 1)}$$

input `integrate((e*x+d)^m*(c*e^2*x^2+2*c*d*e*x+c*d^2)^p,x, algorithm="maxima")`

output `(c^p*e*x + c^p*d)*e^(m*log(e*x + d) + 2*p*log(e*x + d))/(e*(m + 2*p + 1))`

Giac [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.44

$$\int (d + ex)^m (cd^2 + 2cdex + ce^2x^2)^p dx$$

$$= \frac{(ex + d)^m ex e^{(2p \log(ex+d) + p \log(c))} + (ex + d)^m de^{(2p \log(ex+d) + p \log(c))}}{em + 2ep + e}$$

input `integrate((e*x+d)^m*(c*e^2*x^2+2*c*d*e*x+c*d^2)^p,x, algorithm="giac")`

output `((e*x + d)^m*e*x*e^(2*p*log(e*x + d) + p*log(c)) + (e*x + d)^m*d*e^(2*p*log(e*x + d) + p*log(c)))/(e*m + 2*e*p + e)`

Mupad [B] (verification not implemented)

Time = 5.81 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.00

$$\int (d + ex)^m (cd^2 + 2cdex + ce^2x^2)^p dx = \frac{(d + ex)^{m+1} (cd^2 + 2cdex + ce^2x^2)^p}{e(m + 2p + 1)}$$

input `int((d + e*x)^m*(c*d^2 + c*e^2*x^2 + 2*c*d*e*x)^p,x)`

output `((d + e*x)^(m + 1)*(c*d^2 + c*e^2*x^2 + 2*c*d*e*x)^p)/(e*(m + 2*p + 1))`

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.07

$$\int (d + ex)^m (cd^2 + 2cdex + ce^2x^2)^p dx = \frac{(ex + d)^m (ce^2x^2 + 2cdex + cd^2)^p (ex + d)}{e(m + 2p + 1)}$$

input `int((e*x+d)^m*(c*e^2*x^2+2*c*d*e*x+c*d^2)^p,x)`

output $((d + e*x)**m*(c*d**2 + 2*c*d*e*x + c*e**2*x**2)**p*(d + e*x))/(e*(m + 2*p + 1))$

3.446 $\int (d + ex)^p (cd^2 + 2cdex + ce^2x^2)^{-p} dx$

| | |
|---|------|
| Optimal result | 3314 |
| Mathematica [A] (verified) | 3314 |
| Rubi [A] (verified) | 3315 |
| Maple [A] (verified) | 3316 |
| Fricas [A] (verification not implemented) | 3316 |
| Sympy [F(-2)] | 3317 |
| Maxima [A] (verification not implemented) | 3317 |
| Giac [A] (verification not implemented) | 3317 |
| Mupad [B] (verification not implemented) | 3318 |
| Reduce [F] | 3318 |

Optimal result

Integrand size = 32, antiderivative size = 44

$$\int (d + ex)^p (cd^2 + 2cdex + ce^2x^2)^{-p} dx = \frac{(d + ex)^{1+p} (cd^2 + 2cdex + ce^2x^2)^{-p}}{e(1 - p)}$$

output

```
(e*x+d)^(p+1)/e/(1-p)/((c*e^2*x^2+2*c*d*e*x+c*d^2)^p)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.75

$$\int (d + ex)^p (cd^2 + 2cdex + ce^2x^2)^{-p} dx = \frac{(d + ex)^{1+p} (c(d + ex)^2)^{-p}}{e(1 - p)}$$

input

```
Integrate[(d + e*x)^p/(c*d^2 + 2*c*d*e*x + c*e^2*x^2)^p,x]
```

output

```
(d + e*x)^(1 + p)/(e*(1 - p)*(c*(d + e*x)^2)^p)
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.094$, Rules used = {1102, 37, 17}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (d + ex)^p (cd^2 + 2cdex + ce^2x^2)^{-p} dx$$

$$\downarrow 1102$$

$$(cde + ce^2x)^{2p} (cd^2 + 2cdex + ce^2x^2)^{-p} \int (d + ex)^p (cxe^2 + cde)^{-2p} dx$$

$$\downarrow 37$$

$$(d + ex)^{2p} (cd^2 + 2cdex + ce^2x^2)^{-p} \int (d + ex)^{-p} dx$$

$$\downarrow 17$$

$$\frac{(d + ex)^{p+1} (cd^2 + 2cdex + ce^2x^2)^{-p}}{e(1 - p)}$$

input `Int[(d + e*x)^p/(c*d^2 + 2*c*d*e*x + c*e^2*x^2)^p,x]`

output `(d + e*x)^(1 + p)/(e*(1 - p)*(c*d^2 + 2*c*d*e*x + c*e^2*x^2)^p)`

Defintions of rubi rules used

rule 17 `Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] := Simp[c*((a + b*x)^(m + 1))/(b*(m + 1)), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]`

rule 37 `Int[(u_.)*((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] := Simp[(a + b*x)^m/(c + d*x)^m Int[u*(c + d*x)^(m + n), x], x] /; FreeQ[{a, b, c, d, m, n}, x] && EqQ[b*c - a*d, 0] && !SimplerQ[a + b*x, c + d*x]`

rule 1102

```
Int[((d_.) + (e_.)*(x_.))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol]
:> Simp[(a + b*x + c*x^2)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x)^(2*FracPart[p]))
Int[(d + e*x)^m*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d, e, m, p}, x]
&& EqQ[b^2 - 4*a*c, 0]
```

Maple [A] (verified)

Time = 1.71 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.00

| method | result |
|---------------|--|
| gospers | $-\frac{(ex+d)^{p+1}(x^2ce^2+2cdxe+cd^2)^{-p}}{e^{p-1}}$ |
| orering | $-\frac{(ex+d)(ex+d)^p(x^2ce^2+2cdxe+cd^2)^{-p}}{e^{p-1}}$ |
| parallemrisch | $\frac{(-x(ex+d)^pe-d(ex+d)^p)(c(e^2x^2+2dex+d^2))^{-p}}{(p-1)e}$ |
| norman | $\left(-\frac{x e^{\ln(ex+d)p}}{p-1} - \frac{d e^{\ln(ex+d)p}}{(p-1)e}\right) e^{-p \ln(x^2ce^2+2cdxe+cd^2)}$ |
| risch | $-\frac{(ex+d)(ex+d)^pc^{-p}(ex+d)^{-2p}e^{\frac{i\pi p}{2}(\operatorname{csgn}(i(ex+d)^2)^3 - 2\operatorname{csgn}(i(ex+d)^2)^2 \operatorname{csgn}(i(ex+d)) + \operatorname{csgn}(i(ex+d)^2) \operatorname{csgn}(i(ex+d))^2 - \operatorname{csgn}(i(ex+d)))}}{(p-1)e}$ |

input

```
int((e*x+d)^p/((c*e^2*x^2+2*c*d*e*x+c*d^2)^p),x,method=_RETURNVERBOSE)
```

output

```
-1/e/(p-1)*(e*x+d)^(p+1)/((c*e^2*x^2+2*c*d*e*x+c*d^2)^p)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.68

$$\int (d + ex)^p (cd^2 + 2cdex + ce^2x^2)^{-p} dx = -\frac{ex + d}{(ep - e)(ex + d)^p c^p}$$

input

```
integrate((e*x+d)^p/((c*e^2*x^2+2*c*d*e*x+c*d^2)^p),x, algorithm="fricas")
```

output

```
-(e*x + d)/((e*p - e)*(e*x + d)^p*c^p)
```

Sympy [F(-2)]

Exception generated.

$$\int (d + ex)^p (cd^2 + 2cdex + ce^2x^2)^{-p} dx = \text{Exception raised: TypeError}$$

input `integrate((e*x+d)**p/((c*e**2*x**2+2*c*d*e*x+c*d**2)**p),x)`

output `Exception raised: TypeError >> Invalid NaN comparison`

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.66

$$\int (d + ex)^p (cd^2 + 2cdex + ce^2x^2)^{-p} dx = -\frac{ex + d}{(ex + d)^p c^p e^{(p-1)}}$$

input `integrate((e*x+d)^p/((c*e^2*x^2+2*c*d*e*x+c*d^2)^p),x, algorithm="maxima")`

output `-(e*x + d)/((e*x + d)^p*c^p*e^(p - 1))`

Giac [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.43

$$\begin{aligned} & \int (d + ex)^p (cd^2 + 2cdex + ce^2x^2)^{-p} dx \\ &= -\frac{(ex + d)^p ex e^{(-2p \log(ex+d) - p \log(c))} + (ex + d)^p de^{(-2p \log(ex+d) - p \log(c))}}{ep - e} \end{aligned}$$

input `integrate((e*x+d)^p/((c*e^2*x^2+2*c*d*e*x+c*d^2)^p),x, algorithm="giac")`

output `-((e*x + d)^p*e*x*e^(-2*p*log(e*x + d) - p*log(c)) + (e*x + d)^p*d*e^(-2*p*log(e*x + d) - p*log(c)))/(e*p - e)`

Mupad [B] (verification not implemented)

Time = 6.50 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.98

$$\int (d + ex)^p (cd^2 + 2cdex + ce^2x^2)^{-p} dx = -\frac{(d + ex)^{p+1}}{e(p-1)(cd^2 + 2cdex + ce^2x^2)^p}$$

input `int((d + e*x)^p/(c*d^2 + c*e^2*x^2 + 2*c*d*e*x)^p,x)`output `-(d + e*x)^(p + 1)/(e*(p - 1)*(c*d^2 + c*e^2*x^2 + 2*c*d*e*x)^p)`**Reduce [F]**

$$\int (d + ex)^p (cd^2 + 2cdex + ce^2x^2)^{-p} dx = \int \frac{(ex + d)^p}{(ce^2x^2 + 2cdex + cd^2)^p} dx$$

input `int((e*x+d)^p/((c*e^2*x^2+2*c*d*e*x+c*d^2)^p),x)`output `int((d + e*x)**p/(c*d**2 + 2*c*d*e*x + c*e**2*x**2)**p,x)`

3.447 $\int (d+ex)^{-1-2p} (cd^2 + 2cdex + ce^2x^2)^p dx$

| | |
|---|------|
| Optimal result | 3319 |
| Mathematica [A] (verified) | 3319 |
| Rubi [A] (verified) | 3320 |
| Maple [B] (verified) | 3321 |
| Fricas [A] (verification not implemented) | 3321 |
| Sympy [F] | 3322 |
| Maxima [A] (verification not implemented) | 3322 |
| Giac [F] | 3322 |
| Mupad [F(-1)] | 3323 |
| Reduce [F] | 3323 |

Optimal result

Integrand size = 34, antiderivative size = 41

$$\int (d + ex)^{-1-2p} (cd^2 + 2cdex + ce^2x^2)^p dx = \frac{(d + ex)^{-2p} (cd^2 + 2cdex + ce^2x^2)^p \log(d + ex)}{e}$$

output

$$(c*e^{2*x^2+2*c*d*e*x+c*d^2})^p*\ln(e*x+d)/e/((e*x+d)^{(2*p)})$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.73

$$\int (d + ex)^{-1-2p} (cd^2 + 2cdex + ce^2x^2)^p dx = \frac{(d + ex)^{-2p} (c(d + ex)^2)^p \log(d + ex)}{e}$$

input

$$\text{Integrate}[(d + e*x)^{-1 - 2*p}*(c*d^2 + 2*c*d*e*x + c*e^2*x^2)^p,x]$$

output

$$((c*(d + e*x)^2)^p*\text{Log}[d + e*x])/e*(d + e*x)^{(2*p)}$$

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.088$, Rules used = {1102, 37, 16}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (d + ex)^{-2p-1} (cd^2 + 2cdex + ce^2x^2)^p dx$$

$$\downarrow 1102$$

$$(cde + ce^2x)^{-2p} (cd^2 + 2cdex + ce^2x^2)^p \int (d + ex)^{-2p-1} (cxe^2 + cde)^{2p} dx$$

$$\downarrow 37$$

$$(d + ex)^{-2p} (cd^2 + 2cdex + ce^2x^2)^p \int \frac{1}{d + ex} dx$$

$$\downarrow 16$$

$$\frac{(d + ex)^{-2p} \log(d + ex) (cd^2 + 2cdex + ce^2x^2)^p}{e}$$

input `Int[(d + e*x)^(-1 - 2*p)*(c*d^2 + 2*c*d*e*x + c*e^2*x^2)^p,x]`

output `((c*d^2 + 2*c*d*e*x + c*e^2*x^2)^p*Log[d + e*x])/(e*(d + e*x)^(2*p))`

Defintions of rubi rules used

rule 16 `Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] := Simp[c*(Log[RemoveContent[a + b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]`

rule 37 `Int[(u_.)*((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] := Simp[(a + b*x)^m/(c + d*x)^m Int[u*(c + d*x)^(m + n), x], x] /; FreeQ[{a, b, c, d, m, n}, x] && EqQ[b*c - a*d, 0] && !SimplerQ[a + b*x, c + d*x]`

rule 1102

```
Int[((d_.) + (e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol]
:> Simp[(a + b*x + c*x^2)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x)^(2*FracPart[p]))
Int[(d + e*x)^m*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d, e, m, p}, x]
&& EqQ[b^2 - 4*a*c, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 94 vs. $2(43) = 86$.

Time = 1.61 (sec) , antiderivative size = 95, normalized size of antiderivative = 2.32

| method | result | size |
|--------|---|------|
| norman | $x \ln(ex + d) e^{(-1-2p) \ln(ex+d)} e^{p \ln(x^2 c e^2 + 2cdxe + c d^2)} + \frac{d \ln(ex+d) e^{(-1-2p) \ln(ex+d)} e^{p \ln(x^2 c e^2 + 2cdxe + c d^2)}}{e}$ | 95 |

input

```
int((e*x+d)^(-1-2*p)*(c*e^2*x^2+2*c*d*e*x+c*d^2)^p,x,method=_RETURNVERBOSE)
```

output

```
x*ln(e*x+d)*exp((-1-2*p)*ln(e*x+d))*exp(p*ln(c*e^2*x^2+2*c*d*e*x+c*d^2))+
/e*ln(e*x+d)*exp((-1-2*p)*ln(e*x+d))*exp(p*ln(c*e^2*x^2+2*c*d*e*x+c*d^2))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.32

$$\int (d + ex)^{-1-2p} (cd^2 + 2cdex + ce^2x^2)^p dx = \frac{c^p \log(ex + d)}{e}$$

input

```
integrate((e*x+d)^(-1-2*p)*(c*e^2*x^2+2*c*d*e*x+c*d^2)^p,x, algorithm="fricas")
```

output

```
c^p*log(e*x + d)/e
```

Sympy [F]

$$\int (d + ex)^{-1-2p} (cd^2 + 2cdex + ce^2x^2)^p dx = \int (c(d + ex)^2)^p (d + ex)^{-2p-1} dx$$

input `integrate((e*x+d)**(-1-2*p))*(c*e**2*x**2+2*c*d*e*x+c*d**2)**p,x)`

output `Integral((c*(d + e*x)**2)**p*(d + e*x)**(-2*p - 1), x)`

Maxima [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.32

$$\int (d + ex)^{-1-2p} (cd^2 + 2cdex + ce^2x^2)^p dx = \frac{c^p \log(ex + d)}{e}$$

input `integrate((e*x+d)^(-1-2*p))*(c*e^2*x^2+2*c*d*e*x+c*d^2)^p,x, algorithm="maxima")`

output `c^p*log(e*x + d)/e`

Giac [F]

$$\int (d + ex)^{-1-2p} (cd^2 + 2cdex + ce^2x^2)^p dx = \int (ce^2x^2 + 2cdex + cd^2)^p (ex + d)^{-2p-1} dx$$

input `integrate((e*x+d)^(-1-2*p))*(c*e^2*x^2+2*c*d*e*x+c*d^2)^p,x, algorithm="giac")`

output `integrate((c*e^2*x^2 + 2*c*d*e*x + c*d^2)^p*(e*x + d)^(-2*p - 1), x)`

Mupad [F(-1)]

Timed out.

$$\int (d + ex)^{-1-2p} (cd^2 + 2cdex + ce^2x^2)^p dx = \int \frac{(cd^2 + 2cdex + ce^2x^2)^p}{(d + ex)^{2p+1}} dx$$

input `int((c*d^2 + c*e^2*x^2 + 2*c*d*e*x)^p/(d + e*x)^(2*p + 1), x)`

output `int((c*d^2 + c*e^2*x^2 + 2*c*d*e*x)^p/(d + e*x)^(2*p + 1), x)`

Reduce [F]

$$\int (d + ex)^{-1-2p} (cd^2 + 2cdex + ce^2x^2)^p dx = \int \frac{(ce^2x^2 + 2cdex + cd^2)^p}{(ex + d)^{2p} d + (ex + d)^{2p} ex} dx$$

input `int((e*x+d)^(-1-2*p)*(c*e^2*x^2+2*c*d*e*x+c*d^2)^p,x)`

output `int((c*d**2 + 2*c*d*e*x + c*e**2*x**2)**p/((d + e*x)**(2*p)*d + (d + e*x)*
*(2*p)*e*x), x)`

3.448 $\int (d+ex)^{-1+2p} (cd^2 + 2cdex + ce^2x^2)^{-p} dx$

| | |
|---|------|
| Optimal result | 3324 |
| Mathematica [A] (verified) | 3324 |
| Rubi [A] (verified) | 3325 |
| Maple [A] (verified) | 3326 |
| Fricas [A] (verification not implemented) | 3326 |
| Sympy [F] | 3327 |
| Maxima [A] (verification not implemented) | 3327 |
| Giac [F] | 3327 |
| Mupad [F(-1)] | 3328 |
| Reduce [F] | 3328 |

Optimal result

Integrand size = 36, antiderivative size = 43

$$\int (d + ex)^{-1+2p} (cd^2 + 2cdex + ce^2x^2)^{-p} dx = \frac{(d + ex)^{2p} (cd^2 + 2cdex + ce^2x^2)^{-p} \log(d + ex)}{e}$$

output

$$(e*x+d)^{(2*p)}*\ln(e*x+d)/e/((c*e^2*x^2+2*c*d*e*x+c*d^2)^p)$$

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.74

$$\int (d + ex)^{-1+2p} (cd^2 + 2cdex + ce^2x^2)^{-p} dx = \frac{(d + ex)^{2p} (c(d + ex)^2)^{-p} \log(d + ex)}{e}$$

input

$$\text{Integrate}[(d + e*x)^{-1 + 2*p}/(c*d^2 + 2*c*d*e*x + c*e^2*x^2)^p,x]$$

output

$$((d + e*x)^{(2*p)}*Log[d + e*x])/(e*(c*(d + e*x)^2)^p)$$

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {1102, 37, 16}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (d + ex)^{2p-1} (cd^2 + 2cdex + ce^2x^2)^{-p} dx$$

$$\downarrow 1102$$

$$(cde + ce^2x)^{2p} (cd^2 + 2cdex + ce^2x^2)^{-p} \int (d + ex)^{2p-1} (cxe^2 + cde)^{-2p} dx$$

$$\downarrow 37$$

$$(d + ex)^{2p} (cd^2 + 2cdex + ce^2x^2)^{-p} \int \frac{1}{d + ex} dx$$

$$\downarrow 16$$

$$\frac{(d + ex)^{2p} \log(d + ex) (cd^2 + 2cdex + ce^2x^2)^{-p}}{e}$$

input

```
Int[(d + e*x)^(-1 + 2*p)/(c*d^2 + 2*c*d*e*x + c*e^2*x^2)^p,x]
```

output

```
((d + e*x)^(2*p)*Log[d + e*x])/(e*(c*d^2 + 2*c*d*e*x + c*e^2*x^2)^p)
```

Defintions of rubi rules used

rule 16

```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a + b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 37

```
Int[(u_.)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Simp[(a + b*x)^m/(c + d*x)^m Int[u*(c + d*x)^(m + n), x], x] /; FreeQ[{a, b, c, d, m, n}, x] && EqQ[b*c - a*d, 0] && !SimplerQ[a + b*x, c + d*x]
```

rule 1102

```
Int[((d_.) + (e_.)*(x_.))^(m_.)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol]
:> Simp[(a + b*x + c*x^2)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x)^(2*FracPart[p]))
Int[(d + e*x)^m*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d, e, m, p}, x]
&& EqQ[b^2 - 4*a*c, 0]
```

Maple [A] (verified)

Time = 1.66 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.72

| method | result | size |
|--------|---|------|
| norman | $\left(x \ln(ex + d) e^{(-1+2p) \ln(ex+d)} + \frac{d \ln(ex+d) e^{(-1+2p) \ln(ex+d)}}{e}\right) e^{-p \ln(x^2 c e^2 + 2cdx + c d^2)}$ | 74 |

input

```
int((e*x+d)^(-1+2*p)/((c*e^2*x^2+2*c*d*e*x+c*d^2)^p),x,method=_RETURNVERBOSE)
```

output

```
(x*ln(e*x+d)*exp((-1+2*p)*ln(e*x+d))+d/e*ln(e*x+d)*exp((-1+2*p)*ln(e*x+d))
)/exp(p*ln(c*e^2*x^2+2*c*d*e*x+c*d^2))
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.35

$$\int (d + ex)^{-1+2p} (cd^2 + 2cdex + ce^2x^2)^{-p} dx = \frac{\log(ex + d)}{c^p e}$$

input

```
integrate((e*x+d)^(-1+2*p)/((c*e^2*x^2+2*c*d*e*x+c*d^2)^p),x, algorithm="fricas")
```

output

```
log(e*x + d)/(c^p*e)
```

Sympy [F]

$$\int (d + ex)^{-1+2p} (cd^2 + 2cdex + ce^2x^2)^{-p} dx = \int (c(d + ex)^2)^{-p} (d + ex)^{2p-1} dx$$

input `integrate((e*x+d)**(-1+2*p))/((c*e**2*x**2+2*c*d*e*x+c*d**2)**p), x)`

output `Integral((d + e*x)**(2*p - 1)/(c*(d + e*x)**2)**p, x)`

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.35

$$\int (d + ex)^{-1+2p} (cd^2 + 2cdex + ce^2x^2)^{-p} dx = \frac{\log(ex + d)}{c^p e}$$

input `integrate((e*x+d)^(-1+2*p))/((c*e^2*x^2+2*c*d*e*x+c*d^2)^p), x, algorithm="maxima")`

output `log(e*x + d)/(c^p*e)`

Giac [F]

$$\int (d + ex)^{-1+2p} (cd^2 + 2cdex + ce^2x^2)^{-p} dx = \int \frac{(ex + d)^{2p-1}}{(ce^2x^2 + 2cdex + cd^2)^p} dx$$

input `integrate((e*x+d)^(-1+2*p))/((c*e^2*x^2+2*c*d*e*x+c*d^2)^p), x, algorithm="giac")`

output `integrate((e*x + d)^(2*p - 1)/(c*e^2*x^2 + 2*c*d*e*x + c*d^2)^p, x)`

Mupad [F(-1)]

Timed out.

$$\int (d + ex)^{-1+2p} (cd^2 + 2cdex + ce^2x^2)^{-p} dx = \int \frac{(d + ex)^{2p-1}}{(cd^2 + 2cdex + ce^2x^2)^p} dx$$

input `int((d + e*x)^(2*p - 1)/(c*d^2 + c*e^2*x^2 + 2*c*d*e*x)^p, x)`

output `int((d + e*x)^(2*p - 1)/(c*d^2 + c*e^2*x^2 + 2*c*d*e*x)^p, x)`

Reduce [F]

$$\begin{aligned} & \int (d + ex)^{-1+2p} (cd^2 + 2cdex + ce^2x^2)^{-p} dx \\ &= \int \frac{(ex + d)^{2p}}{(ce^2x^2 + 2cdex + cd^2)^p d + (ce^2x^2 + 2cdex + cd^2)^p ex} dx \end{aligned}$$

input `int((e*x+d)^(-1+2*p)/((c*e^2*x^2+2*c*d*e*x+c*d^2)^p), x)`

output `int((d + e*x)**(2*p)/((c*d**2 + 2*c*d*e*x + c*e**2*x**2)**p*d + (c*d**2 + 2*c*d*e*x + c*e**2*x**2)**p*e*x), x)`

3.449 $\int (1+x)^m (1+2x+x^2)^p dx$

| | |
|---|------|
| Optimal result | 3329 |
| Mathematica [A] (verified) | 3329 |
| Rubi [A] (verified) | 3330 |
| Maple [C] (verified) | 3331 |
| Fricas [A] (verification not implemented) | 3331 |
| Sympy [F] | 3332 |
| Maxima [A] (verification not implemented) | 3332 |
| Giac [A] (verification not implemented) | 3332 |
| Mupad [B] (verification not implemented) | 3333 |
| Reduce [B] (verification not implemented) | 3333 |

Optimal result

Integrand size = 16, antiderivative size = 26

$$\int (1+x)^m (1+2x+x^2)^p dx = \frac{(1+x)^{1+m} (1+2x+x^2)^p}{1+m+2p}$$

output

```
(1+x)^(1+m)*(x^2+2*x+1)^p/(1+m+2*p)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.88

$$\int (1+x)^m (1+2x+x^2)^p dx = \frac{(1+x)^{1+m} ((1+x)^2)^p}{1+m+2p}$$

input

```
Integrate[(1+x)^m*(1+2*x+x^2)^p,x]
```

output

```
((1+x)^(1+m)*((1+x)^2)^p)/(1+m+2*p)
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {1102, 17}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (x+1)^m (x^2+2x+1)^p dx$$

$$\downarrow 1102$$

$$(x+1)^{-2p} (x^2+2x+1)^p \int (x+1)^{m+2p} dx$$

$$\downarrow 17$$

$$\frac{(x+1)^{m+1} (x^2+2x+1)^p}{m+2p+1}$$

input `Int[(1 + x)^m*(1 + 2*x + x^2)^p,x]`

output `((1 + x)^(1 + m)*(1 + 2*x + x^2)^p)/(1 + m + 2*p)`

Defintions of rubi rules used

rule 17 `Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] := Simp[c*((a + b*x)^(m + 1))/(b*(m + 1)), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]`

rule 1102 `Int[((d_.) + (e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] := Simp[(a + b*x + c*x^2)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x)^(2*FracPart[p])) Int[(d + e*x)^m*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && EqQ[b^2 - 4*a*c, 0]`

Maple [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3.

Time = 1.15 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.69

| method | result | size |
|-------------|--|------|
| meijerg | $x \operatorname{hypergeom}([1, -m - 2p], [2], -x)$ | 18 |
| gosper | $\frac{(x+1)^{1+m}(x^2+2x+1)^p}{1+m+2p}$ | 27 |
| orering | $\frac{(x+1)(x+1)^m(x^2+2x+1)^p}{1+m+2p}$ | 28 |
| parallelsch | $\frac{(x^2+2x+1)^p(x+1)^m x + (x+1)^m(x^2+2x+1)^p}{1+m+2p}$ | 44 |
| norman | $\frac{e^{m \ln(x+1)} e^{p \ln(x^2+2x+1)}}{1+m+2p} + \frac{x e^{m \ln(x+1)} e^{p \ln(x^2+2x+1)}}{1+m+2p}$ | 59 |
| risch | $\frac{(x+1)^m(x+1)(x+1)^{2p} e^{-\frac{i\pi \operatorname{csgn}(i(x+1)^2)p(-\operatorname{csgn}(i(x+1)^2)+\operatorname{csgn}(i(x+1)))^2}{2}}}{1+m+2p}$ | 61 |

input `int((x+1)^m*(x^2+2*x+1)^p,x,method=_RETURNVERBOSE)`

output `x*hypergeom([1,-m-2*p],[2],-x)`

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.92

$$\int (1+x)^m (1+2x+x^2)^p dx = \frac{(x+1)^m (x+1)^{2p} (x+1)}{m+2p+1}$$

input `integrate((1+x)^m*(x^2+2*x+1)^p,x, algorithm="fricas")`

output `(x + 1)^m*(x + 1)^(2*p)*(x + 1)/(m + 2*p + 1)`

Sympy [F]

$$\int (1+x)^m (1+2x+x^2)^p dx = \begin{cases} \frac{x(x+1)^m (x^2+2x+1)^p}{m+2p+1} + \frac{(x+1)^m (x^2+2x+1)^p}{m+2p+1} & \text{for } m \neq -2p-1 \\ \int (x+1)^{-2p-1} ((x+1)^2)^p dx & \text{otherwise} \end{cases}$$

input `integrate((1+x)**m*(x**2+2*x+1)**p,x)`

output `Piecewise((x*(x + 1)**m*(x**2 + 2*x + 1)**p/(m + 2*p + 1) + (x + 1)**m*(x**2 + 2*x + 1)**p/(m + 2*p + 1), Ne(m, -2*p - 1)), (Integral((x + 1)**(-2*p - 1)*((x + 1)**2)**p, x), True))`

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.04

$$\int (1+x)^m (1+2x+x^2)^p dx = \frac{(x+1)e^{(m \log(x+1)+2p \log(x+1))}}{m+2p+1}$$

input `integrate((1+x)^m*(x^2+2*x+1)^p,x, algorithm="maxima")`

output `(x + 1)*e^(m*log(x + 1) + 2*p*log(x + 1))/(m + 2*p + 1)`

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.42

$$\int (1+x)^m (1+2x+x^2)^p dx = \frac{(x+1)^m (x+1)^{2p} x + (x+1)^m (x+1)^{2p}}{m+2p+1}$$

input `integrate((1+x)^m*(x^2+2*x+1)^p,x, algorithm="giac")`

output `((x + 1)^m*(x + 1)^(2*p)*x + (x + 1)^m*(x + 1)^(2*p))/(m + 2*p + 1)`

Mupad [B] (verification not implemented)

Time = 6.06 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00

$$\int (1+x)^m (1+2x+x^2)^p dx = \frac{(x+1)^{m+1} (x^2+2x+1)^p}{m+2p+1}$$

input `int((x + 1)^m*(2*x + x^2 + 1)^p,x)`output `((x + 1)^(m + 1)*(2*x + x^2 + 1)^p)/(m + 2*p + 1)`**Reduce [B] (verification not implemented)**

Time = 0.22 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.04

$$\int (1+x)^m (1+2x+x^2)^p dx = \frac{(x+1)^m (x^2+2x+1)^p (x+1)}{m+2p+1}$$

input `int((1+x)^m*(x^2+2*x+1)^p,x)`output `((x + 1)**m*(x**2 + 2*x + 1)**p*(x + 1))/(m + 2*p + 1)`

3.450 $\int \left(\frac{be}{2c} + ex\right)^m \left(\frac{b^2}{4c} + bx + cx^2\right)^p dx$

| | |
|---|------|
| Optimal result | 3334 |
| Mathematica [A] (verified) | 3334 |
| Rubi [A] (verified) | 3335 |
| Maple [A] (verified) | 3336 |
| Fricas [A] (verification not implemented) | 3337 |
| Sympy [F] | 3337 |
| Maxima [A] (verification not implemented) | 3338 |
| Giac [B] (verification not implemented) | 3338 |
| Mupad [B] (verification not implemented) | 3339 |
| Reduce [B] (verification not implemented) | 3339 |

Optimal result

Integrand size = 37, antiderivative size = 50

$$\int \left(\frac{be}{2c} + ex\right)^m \left(\frac{b^2}{4c} + bx + cx^2\right)^p dx = \frac{\left(\frac{be}{2c} + ex\right)^{1+m} \left(\frac{b^2}{4c} + bx + cx^2\right)^p}{e(1+m+2p)}$$

output $(1/2*b*e/c+e*x)^{(1+m)}*(1/4*b^2/c+b*x+c*x^2)^p/e/(1+m+2*p)$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.08

$$\int \left(\frac{be}{2c} + ex\right)^m \left(\frac{b^2}{4c} + bx + cx^2\right)^p dx = \frac{2^{-1-2p}(b+2cx) \left(\frac{(b+2cx)^2}{c}\right)^p \left(\frac{be}{2c} + ex\right)^m}{c(1+m+2p)}$$

input `Integrate[((b*e)/(2*c) + e*x)^m*(b^2/(4*c) + b*x + c*x^2)^p,x]`

output $(2^{(-1-2*p)}*(b+2*c*x)*((b+2*c*x)^2/c)^p*((b*e)/(2*c) + e*x)^m)/(c*(1+m+2*p))$

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.14, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.081$, Rules used = {1102, 37, 17}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(\frac{b^2}{4c} + bx + cx^2 \right)^p \left(\frac{be}{2c} + ex \right)^m dx$$

$$\downarrow \text{1102}$$

$$\left(\frac{b}{2} + cx \right)^{-2p} \left(\frac{b^2}{4c} + bx + cx^2 \right)^p \int \left(\frac{b}{2} + cx \right)^{2p} \left(xe + \frac{be}{2c} \right)^m dx$$

$$\downarrow \text{37}$$

$$\left(\frac{b^2}{4c} + bx + cx^2 \right)^p \left(\frac{be}{2c} + ex \right)^m \left(\frac{b}{2} + cx \right)^{-m-2p} \int \left(\frac{b}{2} + cx \right)^{m+2p} dx$$

$$\downarrow \text{17}$$

$$\frac{\left(\frac{b}{2} + cx \right) \left(\frac{b^2}{4c} + bx + cx^2 \right)^p \left(\frac{be}{2c} + ex \right)^m}{c(m+2p+1)}$$

input `Int[((b*e)/(2*c) + e*x)^m*(b^2/(4*c) + b*x + c*x^2)^p,x]`

output `((b/2 + c*x)*((b*e)/(2*c) + e*x)^m*(b^2/(4*c) + b*x + c*x^2)^p)/(c*(1 + m + 2*p))`

Defintions of rubi rules used

rule 17 `Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] := Simp[c*((a + b*x)^(m + 1))/(b*(m + 1)), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]`

```
rule 37 Int[(u_)*((a_) + (b_)*(x_))^(m_)*((c_) + (d_)*(x_))^(n_), x_Symbol] := S
imp[(a + b*x)^m/(c + d*x)^m Int[u*(c + d*x)^(m + n), x], x] /; FreeQ[{a,
b, c, d, m, n}, x] && EqQ[b*c - a*d, 0] && !SimplerQ[a + b*x, c + d*x]
```

```
rule 1102 Int[((d_) + (e_)*(x_))^(m_)*((a_) + (b_)*(x_) + (c_)*(x_)^2)^(p_), x_S
ymbol] := Simp[(a + b*x + c*x^2)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x)^(2*F
racPart[p])) Int[(d + e*x)^m*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c,
d, e, m, p}, x] && EqQ[b^2 - 4*a*c, 0]
```

Maple [A] (verified)

Time = 2.49 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.04

| method | result |
|---------------|---|
| orering | $\frac{(2cx+b)\left(\frac{be}{2c}+ex\right)^m\left(\frac{b^2}{4c}+bx+cx^2\right)^p}{2c(1+m+2p)}$ |
| gospers | $\frac{(2cx+b)\left(\frac{e(2cx+b)}{2c}\right)^m\left(\frac{4c^2x^2+4cbx+b^2}{4c}\right)^p}{2c(1+m+2p)}$ |
| parallelrisch | $\frac{2x\left(\frac{e(2cx+b)}{2c}\right)^m\left(\frac{4c^2x^2+4cbx+b^2}{4c}\right)^p c + \left(\frac{e(2cx+b)}{2c}\right)^m\left(\frac{4c^2x^2+4cbx+b^2}{4c}\right)^p b}{2c(1+m+2p)}$ |
| norman | $\frac{x e^{m \ln\left(\frac{be}{2c}+ex\right)} e^{p \ln\left(\frac{b^2}{4c}+bx+cx^2\right)}}{1+m+2p} + \frac{b e^{m \ln\left(\frac{be}{2c}+ex\right)} e^{p \ln\left(\frac{b^2}{4c}+bx+cx^2\right)}}{2c(1+m+2p)}$ |
| risch | $\left(\frac{1}{2}\right)^m e^m (2cx+b)^m c^{-m} \left(\frac{1}{4}\right)^p (2cx+b)^{2p} c^{-p} (2cx+b) e^{-\frac{i\pi\left(\operatorname{csgn}(ie(2cx+b))^3 m - \operatorname{csgn}(ie(2cx+b))^2 \operatorname{csgn}(ie) m - \operatorname{csgn}(ie(2cx+b))^2 \operatorname{csgn}(ie)\right)}{2}}$ |

```
input int((1/2*b/c*e+e*x)^m*(1/4/c*b^2+b*x+c*x^2)^p,x,method=_RETURNVERBOSE)
```

```
output 1/2*(2*c*x+b)/c/(1+m+2*p)*(1/2*b/c*e+e*x)^m*(1/4/c*b^2+b*x+c*x^2)^p
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.26

$$\int \left(\frac{be}{2c} + ex\right)^m \left(\frac{b^2}{4c} + bx + cx^2\right)^p dx = \frac{(2cx + b) \left(\frac{2cex + be}{2c}\right)^m e^{\left(2p \log\left(\frac{2cex + be}{2c}\right) + p \log\left(\frac{c}{e^2}\right)\right)}}{2(cm + 2cp + c)}$$

input `integrate((1/2*b*e/c+e*x)^m*(1/4*b^2/c+b*x+c*x^2)^p,x, algorithm="fricas")`

output `1/2*(2*c*x + b)*(1/2*(2*c*e*x + b*e)/c)^m*e^(2*p*log(1/2*(2*c*e*x + b*e)/c) + p*log(c/e^2))/(c*m + 2*c*p + c)`

Sympy [F]

$$\int \left(\frac{be}{2c} + ex\right)^m \left(\frac{b^2}{4c} + bx + cx^2\right)^p dx = \begin{cases} \frac{b\left(\frac{be}{2c} + ex\right)^m \left(\frac{b^2}{4c} + bx + cx^2\right)^p}{2cm + 4cp + 2c} + \frac{2cx\left(\frac{be}{2c} + ex\right)^m \left(\frac{b^2}{4c} + bx + cx^2\right)^p}{2cm + 4cp + 2c} & \text{for } m \neq -2p - 1 \\ 2^{2p+1} \cdot 4^{-p} \int \left(\frac{be}{c} + 2ex\right)^{-2p-1} \left(\frac{b^2}{c} + 4bx + 4cx^2\right)^p dx & \text{otherwise} \end{cases}$$

input `integrate((1/2*b*e/c+e*x)**m*(1/4*b**2/c+b*x+c*x**2)**p,x)`

output `Piecewise((b*(b*e/(2*c) + e*x)**m*(b**2/(4*c) + b*x + c*x**2)**p/(2*c*m + 4*c*p + 2*c) + 2*c*x*(b*e/(2*c) + e*x)**m*(b**2/(4*c) + b*x + c*x**2)**p/(2*c*m + 4*c*p + 2*c), Ne(m, -2*p - 1)), (2**(2*p + 1)*Integral((b*e/c + 2*e*x)**(-2*p - 1)*(b**2/c + 4*b*x + 4*c*x**2)**p, x)/4**p, True))`

Maxima [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.58

$$\int \left(\frac{be}{2c} + ex\right)^m \left(\frac{b^2}{4c} + bx + cx^2\right)^p dx = \frac{(2ce^m x + be^m)c^{-m-p-1}e^{(m \log(2cx+b)+2p \log(2cx+b))}}{(2^{2p+2}p + 2^{2p+1})2^m + 2^{m+2p+1}m}$$

input `integrate((1/2*b*e/c+e*x)^m*(1/4*b^2/c+b*x+c*x^2)^p,x, algorithm="maxima")`

output `(2*c*e^m*x + b*e^m)*c^(-m - p - 1)*e^(m*log(2*c*x + b) + 2*p*log(2*c*x + b)) / ((2^(2*p + 2)*p + 2^(2*p + 1))*2^m + 2^(m + 2*p + 1)*m)`

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 110 vs. 2(46) = 92.

Time = 0.15 (sec) , antiderivative size = 110, normalized size of antiderivative = 2.20

$$\int \left(\frac{be}{2c} + ex\right)^m \left(\frac{b^2}{4c} + bx + cx^2\right)^p dx = \frac{2cx e^{(-m \log(2) - 2p \log(2) + m \log(2cx+b) + 2p \log(2cx+b) - m \log(c) - p \log(c) + m \log(e))} + b e^{(-m \log(2) - 2p \log(2) + m \log(2cx+b) + 2p \log(2cx+b) - m \log(c) - p \log(c) + m \log(e))}}{2(cm + 2cp + c)}$$

input `integrate((1/2*b*e/c+e*x)^m*(1/4*b^2/c+b*x+c*x^2)^p,x, algorithm="giac")`

output `1/2*(2*c*x*e^(-m*log(2) - 2*p*log(2) + m*log(2*c*x + b) + 2*p*log(2*c*x + b) - m*log(c) - p*log(c) + m*log(e)) + b*e^(-m*log(2) - 2*p*log(2) + m*log(2*c*x + b) + 2*p*log(2*c*x + b) - m*log(c) - p*log(c) + m*log(e)))/(c*m + 2*c*p + c)`

Mupad [B] (verification not implemented)

Time = 6.06 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.02

$$\int \left(\frac{be}{2c} + ex\right)^m \left(\frac{b^2}{4c} + bx + cx^2\right)^p dx = \frac{\left(ex + \frac{be}{2c}\right)^m (b + 2cx) \left(bx + cx^2 + \frac{b^2}{4c}\right)^p}{2c(m + 2p + 1)}$$

input `int((e*x + (b*e)/(2*c))^m*(b*x + c*x^2 + b^2/(4*c))^p,x)`output `((e*x + (b*e)/(2*c))^m*(b + 2*c*x)*(b*x + c*x^2 + b^2/(4*c))^p)/(2*c*(m + 2*p + 1))`**Reduce [B] (verification not implemented)**

Time = 0.22 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.32

$$\int \left(\frac{be}{2c} + ex\right)^m \left(\frac{b^2}{4c} + bx + cx^2\right)^p dx = \frac{(2cex + be)^m (4c^2x^2 + 4bcx + b^2)^p (2cx + b)}{2c^{m+p}2^m4^p c(m + 2p + 1)}$$

input `int((1/2*b*e/c+e*x)^m*(1/4*b^2/c+b*x+c*x^2)^p,x)`output `((b*e + 2*c*e*x)**m*(b**2 + 4*b*c*x + 4*c**2*x**2)**p*(b + 2*c*x))/(2*c**
(m + p)*2**m*4**p*c*(m + 2*p + 1))`

3.451 $\int (d + ex)^m (a^2 + 2abx + b^2x^2)^p dx$

| | |
|----------------------------|------|
| Optimal result | 3340 |
| Mathematica [A] (verified) | 3340 |
| Rubi [A] (verified) | 3341 |
| Maple [F] | 3342 |
| Fricas [F] | 3343 |
| Sympy [F(-2)] | 3343 |
| Maxima [F] | 3343 |
| Giac [F] | 3344 |
| Mupad [F(-1)] | 3344 |
| Reduce [F] | 3344 |

Optimal result

Integrand size = 26, antiderivative size = 77

$$\int (d + ex)^m (a^2 + 2abx + b^2x^2)^p dx = \frac{(a + bx)(d + ex)^{1+m} (a^2 + 2abx + b^2x^2)^p \operatorname{Hypergeometric2F1}\left(1, 2 + m + 2p, 2 + m, \frac{b(d+ex)}{bd-ae}\right)}{(bd - ae)(1 + m)}$$

output `-(b*x+a)*(e*x+d)^(1+m)*(b^2*x^2+2*a*b*x+a^2)^p*hypergeom([1, 2+m+2*p], [2+m], b*(e*x+d)/(-a*e+b*d))/(-a*e+b*d)/(1+m)`

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.97

$$\int (d + ex)^m (a^2 + 2abx + b^2x^2)^p dx = \frac{\left(\frac{e(a+bx)}{-bd+ae}\right)^{-2p} ((a + bx)^2)^p (d + ex)^{1+m} \operatorname{Hypergeometric2F1}\left(1 + m, -2p, 2 + m, \frac{b(d+ex)}{bd-ae}\right)}{e(1 + m)}$$

input `Integrate[(d + e*x)^m*(a^2 + 2*a*b*x + b^2*x^2)^p,x]`

output

```
((a + b*x)^2)^p*(d + e*x)^(1 + m)*Hypergeometric2F1[1 + m, -2*p, 2 + m, (
b*(d + e*x))/(b*d - a*e)]/(e*(1 + m)*((e*(a + b*x))/(-(b*d) + a*e))^(2*p)
)
```

Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.10, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.115$, Rules used = {1102, 80, 79}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a^2 + 2abx + b^2x^2)^p (d + ex)^m dx$$

$$\downarrow 1102$$

$$(ab + b^2x)^{-2p} (a^2 + 2abx + b^2x^2)^p \int (xb^2 + ab)^{2p} (d + ex)^m dx$$

$$\downarrow 80$$

$$(a^2 + 2abx + b^2x^2)^p \left(-\frac{e(a + bx)}{bd - ae}\right)^{-2p} \int (d + ex)^m \left(-\frac{bx e}{bd - ae} - \frac{ae}{bd - ae}\right)^{2p} dx$$

$$\downarrow 79$$

$$\frac{(a^2 + 2abx + b^2x^2)^p (d + ex)^{m+1} \left(-\frac{e(a+bx)}{bd-ae}\right)^{-2p} \text{Hypergeometric2F1}\left(m + 1, -2p, m + 2, \frac{b(d+ex)}{bd-ae}\right)}{e(m + 1)}$$

input

```
Int[(d + e*x)^m*(a^2 + 2*a*b*x + b^2*x^2)^p,x]
```

output

```
((d + e*x)^(1 + m)*(a^2 + 2*a*b*x + b^2*x^2)^p*Hypergeometric2F1[1 + m, -2
*p, 2 + m, (b*(d + e*x))/(b*d - a*e)]/(e*(1 + m)*(-(e*(a + b*x))/(b*d -
a*e)))^(2*p)
```

Definitions of rubi rules used

rule 79

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] := Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&& !IntegerQ[m] && !IntegerQ[n] && GtQ[b/(b*c - a*d), 0] && (RationalQ[m]
|| !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

rule 80

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] := Simp[(c
+ d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*(b*((c + d*x)/(b*c - a*d)))
^FracPart[n]) Int[(a + b*x)^m*Simp[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d)
), x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && !IntegerQ[m] && !Integ
erQ[n] && (RationalQ[m] || !SimplerQ[n + 1, m + 1])
```

rule 1102

```
Int[((d_.) + (e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] := Simp[(a + b*x + c*x^2)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x)^(2*F
racPart[p])) Int[(d + e*x)^m*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c,
d, e, m, p}, x] && EqQ[b^2 - 4*a*c, 0]
```

Maple [F]

$$\int (ex + d)^m (b^2x^2 + 2abx + a^2)^p dx$$

input

```
int((e*x+d)^m*(b^2*x^2+2*a*b*x+a^2)^p,x)
```

output

```
int((e*x+d)^m*(b^2*x^2+2*a*b*x+a^2)^p,x)
```

Fricas [F]

$$\int (d + ex)^m (a^2 + 2abx + b^2x^2)^p dx = \int (b^2x^2 + 2abx + a^2)^p (ex + d)^m dx$$

input `integrate((e*x+d)^m*(b^2*x^2+2*a*b*x+a^2)^p,x, algorithm="fricas")`

output `integral((b^2*x^2 + 2*a*b*x + a^2)^p*(e*x + d)^m, x)`

Sympy [F(-2)]

Exception generated.

$$\int (d + ex)^m (a^2 + 2abx + b^2x^2)^p dx = \text{Exception raised: HeuristicGCDFailed}$$

input `integrate((e*x+d)**m*(b**2*x**2+2*a*b*x+a**2)**p,x)`

output `Exception raised: HeuristicGCDFailed >> no luck`

Maxima [F]

$$\int (d + ex)^m (a^2 + 2abx + b^2x^2)^p dx = \int (b^2x^2 + 2abx + a^2)^p (ex + d)^m dx$$

input `integrate((e*x+d)^m*(b^2*x^2+2*a*b*x+a^2)^p,x, algorithm="maxima")`

output `integrate((b^2*x^2 + 2*a*b*x + a^2)^p*(e*x + d)^m, x)`

Giac [F]

$$\int (d + ex)^m (a^2 + 2abx + b^2x^2)^p dx = \int (b^2x^2 + 2abx + a^2)^p (ex + d)^m dx$$

input `integrate((e*x+d)^m*(b^2*x^2+2*a*b*x+a^2)^p,x, algorithm="giac")`

output `integrate((b^2*x^2 + 2*a*b*x + a^2)^p*(e*x + d)^m, x)`

Mupad [F(-1)]

Timed out.

$$\int (d + ex)^m (a^2 + 2abx + b^2x^2)^p dx = \int (d + ex)^m (a^2 + 2abx + b^2x^2)^p dx$$

input `int((d + e*x)^m*(a^2 + b^2*x^2 + 2*a*b*x)^p,x)`

output `int((d + e*x)^m*(a^2 + b^2*x^2 + 2*a*b*x)^p, x)`

Reduce [F]

$$\int (d + ex)^m (a^2 + 2abx + b^2x^2)^p dx = \text{too large to display}$$

input `int((e*x+d)^m*(b^2*x^2+2*a*b*x+a^2)^p,x)`

output

```

((d + e*x)**m*(a**2 + 2*a*b*x + b**2*x**2)**p*a*d*m + 2*(d + e*x)**m*(a**2
+ 2*a*b*x + b**2*x**2)**p*a*d*p + (d + e*x)**m*(a**2 + 2*a*b*x + b**2*x**
2)**p*a*e*m*x + 2*(d + e*x)**m*(a**2 + 2*a*b*x + b**2*x**2)**p*b*d*p*x + 2
*int(((d + e*x)**m*(a**2 + 2*a*b*x + b**2*x**2)**p*x)/(a**2*d*e*m**2 + 2*a
**2*d*e*m*p + a**2*d*e*m + a**2*e**2*m**2*x + 2*a**2*e**2*m*p*x + a**2*e**
2*m*x + 2*a*b*d**2*m*p + 4*a*b*d**2*p**2 + 2*a*b*d**2*p + a*b*d*e*m**2*x +
4*a*b*d*e*m*p*x + a*b*d*e*m*x + 4*a*b*d*e*p**2*x + 2*a*b*d*e*p*x + a*b*e*
**2*m**2*x**2 + 2*a*b*e**2*m*p*x**2 + a*b*e**2*m*x**2 + 2*b**2*d**2*m*p*x +
4*b**2*d**2*p**2*x + 2*b**2*d**2*p*x + 2*b**2*d*e*m*p*x**2 + 4*b**2*d*e*p
**2*x**2 + 2*b**2*d*e*p*x**2),x)*a**3*e**3*m**3*p + 4*int(((d + e*x)**m*(a
**2 + 2*a*b*x + b**2*x**2)**p*x)/(a**2*d*e*m**2 + 2*a**2*d*e*m*p + a**2*d*
e*m + a**2*e**2*m**2*x + 2*a**2*e**2*m*p*x + a**2*e**2*m*x + 2*a*b*d**2*m*
p + 4*a*b*d**2*p**2 + 2*a*b*d**2*p + a*b*d*e*m**2*x + 4*a*b*d*e*m*p*x + a*
b*d*e*m*x + 4*a*b*d*e*p**2*x + 2*a*b*d*e*p*x + a*b*e**2*m**2*x**2 + 2*a*b*
e**2*m*p*x**2 + a*b*e**2*m*x**2 + 2*b**2*d**2*m*p*x + 4*b**2*d**2*p**2*x +
2*b**2*d**2*p*x + 2*b**2*d*e*m*p*x**2 + 4*b**2*d*e*p**2*x**2 + 2*b**2*d*e
*p*x**2),x)*a**3*e**3*m**2*p**2 + 2*int(((d + e*x)**m*(a**2 + 2*a*b*x + b*
**2*x**2)**p*x)/(a**2*d*e*m**2 + 2*a**2*d*e*m*p + a**2*d*e*m + a**2*e**2*m*
**2*x + 2*a**2*e**2*m*p*x + a**2*e**2*m*x + 2*a*b*d**2*m*p + 4*a*b*d**2*p**
2 + 2*a*b*d**2*p + a*b*d*e*m**2*x + 4*a*b*d*e*m*p*x + a*b*d*e*m*x + 4*a...

```

3.452 $\int (d + ex)^m (a^2 + 2abx + b^2x^2)^{5+p} dx$

| | |
|----------------------------|------|
| Optimal result | 3346 |
| Mathematica [A] (verified) | 3346 |
| Rubi [A] (verified) | 3347 |
| Maple [F] | 3348 |
| Fricas [F] | 3349 |
| Sympy [F(-2)] | 3349 |
| Maxima [F] | 3349 |
| Giac [F] | 3350 |
| Mupad [F(-1)] | 3350 |
| Reduce [F] | 3350 |

Optimal result

Integrand size = 28, antiderivative size = 79

$$\int (d + ex)^m (a^2 + 2abx + b^2x^2)^{5+p} dx = \frac{(a + bx)^{11} (d + ex)^{1+m} (a^2 + 2abx + b^2x^2)^p \operatorname{Hypergeometric2F1}\left(1, 12 + m + 2p, 2 + m, \frac{b(d+ex)}{bd-ae}\right)}{(bd - ae)(1 + m)}$$

output

```
-(b*x+a)^11*(e*x+d)^(1+m)*(b^2*x^2+2*a*b*x+a^2)^p*hypergeom([1, 12+m+2*p], [2+m], b*(e*x+d)/(-a*e+b*d))/(-a*e+b*d)/(1+m)
```

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.10

$$\int (d + ex)^m (a^2 + 2abx + b^2x^2)^{5+p} dx = \frac{(bd - ae)^{10} \left(\frac{e(a+bx)}{-bd+ae}\right)^{-2p} ((a + bx)^2)^p (d + ex)^{1+m} \operatorname{Hypergeometric2F1}\left(1 + m, -2(5 + p), 2 + m, \frac{b(d+ex)}{bd-ae}\right)}{e^{11}(1 + m)}$$

input

```
Integrate[(d + e*x)^m*(a^2 + 2*a*b*x + b^2*x^2)^(5 + p),x]
```

output

$$\frac{((b*d - a*e)^{10} * ((a + b*x)^2)^p * (d + e*x)^{(1 + m)} * \text{Hypergeometric2F1}[1 + m, -2*(5 + p), 2 + m, (b*(d + e*x))/(b*d - a*e)] / (e^{11} * (1 + m) * ((e*(a + b*x)) / (- (b*d) + a*e))^{(2*p)})}$$
Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.23, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.107$, Rules used = {1102, 80, 79}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a^2 + 2abx + b^2x^2)^{p+5} (d + ex)^m dx$$

$$\downarrow 1102$$

$$\frac{(ab + b^2x)^{-2p} (a^2 + 2abx + b^2x^2)^p \int (xb^2 + ab)^{2(p+5)} (d + ex)^m dx}{b^{10}}$$

$$\downarrow 80$$

$$\frac{(bd - ae)^{10} (a^2 + 2abx + b^2x^2)^p \left(-\frac{e(a+bx)}{bd-ae}\right)^{-2p} \int (d + ex)^m \left(-\frac{bx}{bd-ae} - \frac{ae}{bd-ae}\right)^{2(p+5)} dx}{e^{10}}$$

$$\downarrow 79$$

$$\frac{(bd - ae)^{10} (a^2 + 2abx + b^2x^2)^p (d + ex)^{m+1} \left(-\frac{e(a+bx)}{bd-ae}\right)^{-2p} \text{Hypergeometric2F1}\left(m + 1, -2(p + 5), m + 2, \frac{b(d+bx)}{bd-ae}\right)}{e^{11}(m + 1)}$$

input

$$\text{Int}[(d + e*x)^m * (a^2 + 2*a*b*x + b^2*x^2)^{(5 + p)}, x]$$

output

$$\frac{((b*d - a*e)^{10} * (d + e*x)^{(1 + m)} * (a^2 + 2*a*b*x + b^2*x^2)^p * \text{Hypergeometric2F1}[1 + m, -2*(5 + p), 2 + m, (b*(d + e*x))/(b*d - a*e)] / (e^{11} * (1 + m) * (-((e*(a + b*x)) / (b*d - a*e)))^{(2*p)})}$$

Definitions of rubi rules used

rule 79

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] := Simp[((a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x] && !IntegerQ[m] && !IntegerQ[n] && GtQ[b/(b*c - a*d), 0] && (RationalQ[m] || !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

rule 80

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] := Simp[(c + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*(b*((c + d*x)/(b*c - a*d)))^FracPart[n]) Int[(a + b*x)^m*Simp[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d)), x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && !IntegerQ[m] && !IntegerQ[n] && (RationalQ[m] || !SimplerQ[n + 1, m + 1])
```

rule 1102

```
Int[((d_.) + (e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] := Simp[(a + b*x + c*x^2)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x)^(2*FracPart[p])) Int[(d + e*x)^m*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && EqQ[b^2 - 4*a*c, 0]
```

Maple [F]

$$\int (ex + d)^m (b^2x^2 + 2abx + a^2)^{5+p} dx$$

input

```
int((e*x+d)^m*(b^2*x^2+2*a*b*x+a^2)^(5+p),x)
```

output

```
int((e*x+d)^m*(b^2*x^2+2*a*b*x+a^2)^(5+p),x)
```

Fricas [F]

$$\int (d + ex)^m (a^2 + 2abx + b^2x^2)^{5+p} dx = \int (b^2x^2 + 2abx + a^2)^{p+5} (ex + d)^m dx$$

input `integrate((e*x+d)^m*(b^2*x^2+2*a*b*x+a^2)^(5+p),x, algorithm="fricas")`

output `integral((b^2*x^2 + 2*a*b*x + a^2)^(p + 5)*(e*x + d)^m, x)`

Sympy [F(-2)]

Exception generated.

$$\int (d + ex)^m (a^2 + 2abx + b^2x^2)^{5+p} dx = \text{Exception raised: HeuristicGCDFailed}$$

input `integrate((e*x+d)**m*(b**2*x**2+2*a*b*x+a**2)**(5+p),x)`

output `Exception raised: HeuristicGCDFailed >> no luck`

Maxima [F]

$$\int (d + ex)^m (a^2 + 2abx + b^2x^2)^{5+p} dx = \int (b^2x^2 + 2abx + a^2)^{p+5} (ex + d)^m dx$$

input `integrate((e*x+d)^m*(b^2*x^2+2*a*b*x+a^2)^(5+p),x, algorithm="maxima")`

output `integrate((b^2*x^2 + 2*a*b*x + a^2)^(p + 5)*(e*x + d)^m, x)`

Giac [F]

$$\int (d + ex)^m (a^2 + 2abx + b^2x^2)^{5+p} dx = \int (b^2x^2 + 2abx + a^2)^{p+5} (ex + d)^m dx$$

input `integrate((e*x+d)^m*(b^2*x^2+2*a*b*x+a^2)^(5+p),x, algorithm="giac")`

output `integrate((b^2*x^2 + 2*a*b*x + a^2)^(p + 5)*(e*x + d)^m, x)`

Mupad [F(-1)]

Timed out.

$$\int (d + ex)^m (a^2 + 2abx + b^2x^2)^{5+p} dx = \int (d + ex)^m (a^2 + 2abx + b^2x^2)^{p+5} dx$$

input `int((d + e*x)^m*(a^2 + b^2*x^2 + 2*a*b*x)^(p + 5),x)`

output `int((d + e*x)^m*(a^2 + b^2*x^2 + 2*a*b*x)^(p + 5), x)`

Reduce [F]

$$\int (d + ex)^m (a^2 + 2abx + b^2x^2)^{5+p} dx = \text{too large to display}$$

input `int((e*x+d)^m*(b^2*x^2+2*a*b*x+a^2)^(5+p),x)`

output `too large to display`

3.453 $\int (d + ex)^{-3-2p} (a^2 + 2abx + b^2x^2)^p dx$

| | |
|---|------|
| Optimal result | 3351 |
| Mathematica [A] (verified) | 3351 |
| Rubi [A] (verified) | 3352 |
| Maple [A] (verified) | 3353 |
| Fricas [A] (verification not implemented) | 3354 |
| Sympy [F] | 3354 |
| Maxima [F] | 3354 |
| Giac [F] | 3355 |
| Mupad [B] (verification not implemented) | 3355 |
| Reduce [F] | 3356 |

Optimal result

Integrand size = 30, antiderivative size = 115

$$\int (d + ex)^{-3-2p} (a^2 + 2abx + b^2x^2)^p dx = \frac{b(a + bx)(d + ex)^{-1-2p} (a^2 + 2abx + b^2x^2)^p}{2(bd - ae)^2(1 + p)(1 + 2p)} + \frac{(a + bx)(d + ex)^{-2(1+p)} (a^2 + 2abx + b^2x^2)^p}{2(bd - ae)(1 + p)}$$

output

```
1/2*b*(b*x+a)*(e*x+d)^(-1-2*p)*(b^2*x^2+2*a*b*x+a^2)^p/(-a*e+b*d)^(2/(p+1)/
(1+2*p))+1/2*(b*x+a)*(b^2*x^2+2*a*b*x+a^2)^p/(-a*e+b*d)/(p+1)/((e*x+d)^(2*p
+2))
```

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.63

$$\int (d + ex)^{-3-2p} (a^2 + 2abx + b^2x^2)^p dx = \frac{(a + bx) ((a + bx)^2)^p (d + ex)^{-2(1+p)} (2bd(1 + p) - ae(1 + 2p) + bex)}{2(bd - ae)^2(1 + p)(1 + 2p)}$$

input

```
Integrate[(d + e*x)^(-3 - 2*p)*(a^2 + 2*a*b*x + b^2*x^2)^p,x]
```

output

$$\left((a + b*x) * ((a + b*x)^2)^p * (2*b*d*(1 + p) - a*e*(1 + 2*p) + b*e*x) / (2*(b*d - a*e)^{2*(1 + p)} * (1 + 2*p) * (d + e*x)^{2*(1 + p)}) \right)$$

Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 133, normalized size of antiderivative = 1.16, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {1102, 55, 48}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a^2 + 2abx + b^2x^2)^p (d + ex)^{-2p-3} dx$$

$$\downarrow 1102$$

$$(ab + b^2x)^{-2p} (a^2 + 2abx + b^2x^2)^p \int (xb^2 + ab)^{2p} (d + ex)^{-2p-3} dx$$

$$\downarrow 55$$

$$(ab + b^2x)^{-2p} (a^2 + 2abx + b^2x^2)^p \left(\frac{b \int (xb^2 + ab)^{2p} (d + ex)^{-2(p+1)} dx}{2(p+1)(bd - ae)} + \frac{(ab + b^2x)^{2p+1} (d + ex)^{-2(p+1)}}{2b(p+1)(bd - ae)} \right)$$

$$\downarrow 48$$

$$(ab + b^2x)^{-2p} (a^2 + 2abx + b^2x^2)^p \left(\frac{(ab + b^2x)^{2p+1} (d + ex)^{-2p-1}}{2(p+1)(2p+1)(bd - ae)^2} + \frac{(ab + b^2x)^{2p+1} (d + ex)^{-2(p+1)}}{2b(p+1)(bd - ae)} \right)$$

input

$$\text{Int}[(d + e*x)^{-3 - 2*p} * (a^2 + 2*a*b*x + b^2*x^2)^p, x]$$

output

$$\left((a^2 + 2*a*b*x + b^2*x^2)^p * (((a*b + b^2*x)^{(1 + 2*p)} * (d + e*x)^{-1 - 2*p}) / (2*(b*d - a*e)^{2*(1 + p)} * (1 + 2*p)) + (a*b + b^2*x)^{(1 + 2*p)} / (2*b*(b*d - a*e) * (1 + p) * (d + e*x)^{2*(1 + p)})) / (a*b + b^2*x)^{(2*p)} \right)$$

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 219, normalized size of antiderivative = 1.90

$$\int (d + ex)^{-3-2p} (a^2 + 2abx + b^2x^2)^p dx$$

$$= \frac{(b^2e^2x^3 + 2abd^2 - a^2de + (3b^2de + 2(b^2de - abe^2)p)x^2 + 2(abd^2 - a^2de)p + (2b^2d^2 + 2abde - a^2e^2 + 2(b^2d^2 - 2abde + a^2e^2)p^2 + 3(b^2d^2 - 2abde + a^2e^2)p))}{2(b^2d^2 - 2abde + a^2e^2) + 2(b^2d^2 - 2abde + a^2e^2)p^2 + 3(b^2d^2 - 2abde + a^2e^2)p}$$

input `integrate((e*x+d)^(-3-2*p)*(b^2*x^2+2*a*b*x+a^2)^p,x, algorithm="fricas")`

output `1/2*(b^2*e^2*x^3 + 2*a*b*d^2 - a^2*d*e + (3*b^2*d*e + 2*(b^2*d*e - a*b*e^2)*p)*x^2 + 2*(a*b*d^2 - a^2*d*e)*p + (2*b^2*d^2 + 2*a*b*d*e - a^2*e^2 + 2*(b^2*d^2 - a^2*e^2)*p)*x)*(b^2*x^2 + 2*a*b*x + a^2)^p*(e*x + d)^(-2*p - 3) / (b^2*d^2 - 2*a*b*d*e + a^2*e^2 + 2*(b^2*d^2 - 2*a*b*d*e + a^2*e^2)*p^2 + 3*(b^2*d^2 - 2*a*b*d*e + a^2*e^2)*p)`

Sympy [F]

$$\int (d + ex)^{-3-2p} (a^2 + 2abx + b^2x^2)^p dx = \int (d + ex)^{-2p-3} ((a + bx)^2)^p dx$$

input `integrate((e*x+d)**(-3-2*p)*(b**2*x**2+2*a*b*x+a**2)**p,x)`

output `Integral((d + e*x)**(-2*p - 3)*((a + b*x)**2)**p, x)`

Maxima [F]

$$\int (d + ex)^{-3-2p} (a^2 + 2abx + b^2x^2)^p dx = \int (b^2x^2 + 2abx + a^2)^p (ex + d)^{-2p-3} dx$$

input `integrate((e*x+d)^(-3-2*p)*(b^2*x^2+2*a*b*x+a^2)^p,x, algorithm="maxima")`

output `integrate((b^2*x^2 + 2*a*b*x + a^2)^p*(e*x + d)^(-2*p - 3), x)`

Giac [F]

$$\int (d + ex)^{-3-2p} (a^2 + 2abx + b^2x^2)^p dx = \int (b^2x^2 + 2abx + a^2)^p (ex + d)^{-2p-3} dx$$

input `integrate((e*x+d)^(-3-2*p)*(b^2*x^2+2*a*b*x+a^2)^p,x, algorithm="giac")`

output `integrate((b^2*x^2 + 2*a*b*x + a^2)^p*(e*x + d)^(-2*p - 3), x)`

Mupad [B] (verification not implemented)

Time = 5.75 (sec) , antiderivative size = 259, normalized size of antiderivative = 2.25

$$\begin{aligned} & \int (d + ex)^{-3-2p} (a^2 + 2abx + b^2x^2)^p dx \\ &= (a^2 + 2abx + b^2x^2)^p \left(\frac{x(2b^2d^2 - a^2e^2 - 2a^2e^2p + 2b^2d^2p + 2abde)}{2(ae - bd)^2(d + ex)^{2p+3}(2p^2 + 3p + 1)} \right. \\ & \quad \left. + \frac{b^2e^2x^3}{2(ae - bd)^2(d + ex)^{2p+3}(2p^2 + 3p + 1)} \right. \\ & \quad \left. - \frac{ad(ae - 2bd + 2aep - 2bdp)}{2(ae - bd)^2(d + ex)^{2p+3}(2p^2 + 3p + 1)} \right. \\ & \quad \left. + \frac{bex^2(3bd - 2aep + 2bdp)}{2(ae - bd)^2(d + ex)^{2p+3}(2p^2 + 3p + 1)} \right) \end{aligned}$$

input `int((a^2 + b^2*x^2 + 2*a*b*x)^p/(d + e*x)^(2*p + 3),x)`

output `(a^2 + b^2*x^2 + 2*a*b*x)^p*((x*(2*b^2*d^2 - a^2*e^2 - 2*a^2*e^2*p + 2*b^2*d^2*p + 2*a*b*d*e))/(2*(a*e - b*d)^2*(d + e*x)^(2*p + 3)*(3*p + 2*p^2 + 1)) + (b^2*e^2*x^3)/(2*(a*e - b*d)^2*(d + e*x)^(2*p + 3)*(3*p + 2*p^2 + 1)) - (a*d*(a*e - 2*b*d + 2*a*e*p - 2*b*d*p))/(2*(a*e - b*d)^2*(d + e*x)^(2*p + 3)*(3*p + 2*p^2 + 1)) + (b*e*x^2*(3*b*d - 2*a*e*p + 2*b*d*p))/(2*(a*e - b*d)^2*(d + e*x)^(2*p + 3)*(3*p + 2*p^2 + 1)))`

Reduce [F]

$$\int (d + ex)^{-3-2p} (a^2 + 2abx + b^2x^2)^p dx$$

$$= \int \frac{(b^2x^2 + 2abx + a^2)^p}{(ex + d)^{2p} d^3 + 3(ex + d)^{2p} d^2ex + 3(ex + d)^{2p} d e^2x^2 + (ex + d)^{2p} e^3x^3} dx$$

input `int((e*x+d)^(-3-2*p)*(b^2*x^2+2*a*b*x+a^2)^p,x)`

output `int((a**2 + 2*a*b*x + b**2*x**2)**p/((d + e*x)**(2*p)*d**3 + 3*(d + e*x)**(2*p)*d**2*e*x + 3*(d + e*x)**(2*p)*d*e**2*x**2 + (d + e*x)**(2*p)*e**3*x**3),x)`

3.454 $\int (d + ex) (9 + 12x + 4x^2)^p dx$

| | |
|---|------|
| Optimal result | 3357 |
| Mathematica [A] (verified) | 3357 |
| Rubi [A] (verified) | 3358 |
| Maple [C] (verified) | 3359 |
| Fricas [A] (verification not implemented) | 3360 |
| Sympy [B] (verification not implemented) | 3360 |
| Maxima [A] (verification not implemented) | 3361 |
| Giac [B] (verification not implemented) | 3361 |
| Mupad [B] (verification not implemented) | 3362 |
| Reduce [B] (verification not implemented) | 3362 |

Optimal result

Integrand size = 18, antiderivative size = 60

$$\int (d + ex) (9 + 12x + 4x^2)^p dx = \frac{(2d - 3e)(3 + 2x) (9 + 12x + 4x^2)^p}{4(1 + 2p)} + \frac{e(9 + 12x + 4x^2)^{1+p}}{8(1 + p)}$$

output `(2*d-3*e)*(3+2*x)*(4*x^2+12*x+9)^p/(4+8*p)+e*(4*x^2+12*x+9)^(p+1)/(8*p+8)`

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.80

$$\int (d + ex) (9 + 12x + 4x^2)^p dx = \frac{(3 + 2x) ((3 + 2x)^2)^p (4d(1 + p) + e(-3 + (2 + 4p)x))}{8(1 + p)(1 + 2p)}$$

input `Integrate[(d + e*x)*(9 + 12*x + 4*x^2)^p,x]`

output `((3 + 2*x)*((3 + 2*x)^2)^p*(4*d*(1 + p) + e*(-3 + (2 + 4*p)*x)))/(8*(1 + p)*(1 + 2*p))`

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {1100, 1079, 17}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (4x^2 + 12x + 9)^p (d + ex) dx$$

$$\downarrow 1100$$

$$\frac{1}{2}(2d - 3e) \int (4x^2 + 12x + 9)^p dx + \frac{e(4x^2 + 12x + 9)^{p+1}}{8(p+1)}$$

$$\downarrow 1079$$

$$\frac{1}{2}(2d - 3e) (4x^2 + 12x + 9)^p (4x + 6)^{-2p} \int (4x + 6)^{2p} dx + \frac{e(4x^2 + 12x + 9)^{p+1}}{8(p+1)}$$

$$\downarrow 17$$

$$\frac{(4x + 6)(2d - 3e) (4x^2 + 12x + 9)^p}{8(2p + 1)} + \frac{e(4x^2 + 12x + 9)^{p+1}}{8(p + 1)}$$

input `Int[(d + e*x)*(9 + 12*x + 4*x^2)^p,x]`

output `((2*d - 3*e)*(6 + 4*x)*(9 + 12*x + 4*x^2)^p)/(8*(1 + 2*p)) + (e*(9 + 12*x + 4*x^2)^(1 + p))/(8*(1 + p))`

Defintions of rubi rules used

rule 17 `Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1))/(b*(m + 1)), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]`

rule 1079

```
Int[((a_) + (b_)*(x_) + (c_)*(x_)^2)^(p_), x_Symbol] := Simp[(a + b*x + c
*x^2)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x)^(2*FracPart[p])) Int[(b/2 + c
*x)^(2*p), x], x] /; FreeQ[{a, b, c, p}, x] && EqQ[b^2 - 4*a*c, 0]
```

rule 1100

```
Int[((d_) + (e_)*(x_))*((a_) + (b_)*(x_) + (c_)*(x_)^2)^(p_), x_Symbol]
:= Simp[e*((a + b*x + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Simp[(2*c*d - b*
e)/(2*c) Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& EqQ[b^2 - 4*a*c, 0]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3.

Time = 0.98 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.72

| method | result |
|---------------|--|
| meijerg | $\frac{e^{3^{2p}x^2} \operatorname{hypergeom}([2, -2p], [3], -\frac{2x}{3})}{2} + d3^{2p}x \operatorname{hypergeom}([1, -2p], [2], -\frac{2x}{3})$ |
| gospers | $\frac{(4x^2+12x+9)^p (4epx+4dp+2ex+4d-3e)(2x+3)}{16p^2+24p+8}$ |
| oring | $\frac{(4x^2+12x+9)^p (4epx+4dp+2ex+4d-3e)(2x+3)}{16p^2+24p+8}$ |
| risch | $\frac{(8epx^2+8dpx+12epx+4ex^2+12dp+8dx+12d-9e)((2x+3)^2)^p}{8(p+1)(2p+1)}$ |
| norman | $\frac{3(4dp+4d-3e)e^{p \ln(4x^2+12x+9)}}{8(2p^2+3p+1)} + \frac{ex^2e^{p \ln(4x^2+12x+9)}}{2+2p} + \frac{(2dp+3pe+2d)xe^{p \ln(4x^2+12x+9)}}{4p^2+6p+2}$ |
| parallelrisch | $\frac{8x^2(4x^2+12x+9)^p ep+4x^2(4x^2+12x+9)^p e+8x(4x^2+12x+9)^p dp+12x(4x^2+12x+9)^p ep+8x(4x^2+12x+9)^p d+12(4x^2+12x+9)^p}{16p^2+24p+8}$ |

input

```
int((e*x+d)*(4*x^2+12*x+9)^p,x,method=_RETURNVERBOSE)
```

output

```
1/2*e*3^(2*p)*x^2*hypergeom([2, -2*p], [3], -2/3*x)+d*3^(2*p)*x*hypergeom([1,
-2*p], [2], -2/3*x)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.07

$$\int (d + ex) (9 + 12x + 4x^2)^p dx$$

$$= \frac{(4(2ep + e)x^2 + 12dp + 4((2d + 3e)p + 2d)x + 12d - 9e)(4x^2 + 12x + 9)^p}{8(2p^2 + 3p + 1)}$$

input `integrate((e*x+d)*(4*x^2+12*x+9)^p,x, algorithm="fricas")`

output `1/8*(4*(2*e*p + e)*x^2 + 12*d*p + 4*((2*d + 3*e)*p + 2*d)*x + 12*d - 9*e)*(4*x^2 + 12*x + 9)^p/(2*p^2 + 3*p + 1)`

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 321 vs. 2(48) = 96.

Time = 0.81 (sec) , antiderivative size = 321, normalized size of antiderivative = 5.35

$$\int (d + ex) (9 + 12x + 4x^2)^p dx$$

$$= \begin{cases} -\frac{2d}{8x+12} + \frac{2ex \log(x+\frac{3}{2})}{8x+12} + \frac{3e \log(x+\frac{3}{2})}{8x+12} + \frac{3e}{8x+12} \\ \frac{e\sqrt{4x^2+12x+9}}{4} + \frac{(d-\frac{3e}{2})(x+\frac{3}{2}) \log(x+\frac{3}{2})}{2\sqrt{(x+\frac{3}{2})^2}} \\ \frac{8dp(4x^2+12x+9)^p}{16p^2+24p+8} + \frac{12dp(4x^2+12x+9)^p}{16p^2+24p+8} + \frac{8dx(4x^2+12x+9)^p}{16p^2+24p+8} + \frac{12d(4x^2+12x+9)^p}{16p^2+24p+8} + \frac{8epx^2(4x^2+12x+9)^p}{16p^2+24p+8} + \frac{12epx(4x^2+12x+9)^p}{16p^2+24p+8} \end{cases}$$

input `integrate((e*x+d)*(4*x**2+12*x+9)**p,x)`

output

```
Piecewise((-2*d/(8*x + 12) + 2*e*x*log(x + 3/2)/(8*x + 12) + 3*e*log(x + 3/2)/(8*x + 12) + 3*e/(8*x + 12), Eq(p, -1)), (e*sqrt(4*x**2 + 12*x + 9)/4 + (d - 3*e/2)*(x + 3/2)*log(x + 3/2)/(2*sqrt((x + 3/2)**2)), Eq(p, -1/2)), (8*d*p*x*(4*x**2 + 12*x + 9)**p/(16*p**2 + 24*p + 8) + 12*d*p*(4*x**2 + 12*x + 9)**p/(16*p**2 + 24*p + 8) + 8*d*x*(4*x**2 + 12*x + 9)**p/(16*p**2 + 24*p + 8) + 12*d*(4*x**2 + 12*x + 9)**p/(16*p**2 + 24*p + 8) + 8*e*p*x**2*(4*x**2 + 12*x + 9)**p/(16*p**2 + 24*p + 8) + 12*e*p*x*(4*x**2 + 12*x + 9)**p/(16*p**2 + 24*p + 8) + 4*e*x**2*(4*x**2 + 12*x + 9)**p/(16*p**2 + 24*p + 8) - 9*e*(4*x**2 + 12*x + 9)**p/(16*p**2 + 24*p + 8), True))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.08

$$\int (d + ex) (9 + 12x + 4x^2)^p dx = \frac{(4(2p + 1)x^2 + 12px - 9)e(2x + 3)^{2p}}{8(2p^2 + 3p + 1)} + \frac{d(2x + 3)^{2p}(2x + 3)}{2(2p + 1)}$$

input

```
integrate((e*x+d)*(4*x^2+12*x+9)^p,x, algorithm="maxima")
```

output

```
1/8*(4*(2*p + 1)*x^2 + 12*p*x - 9)*e*(2*x + 3)^(2*p)/(2*p^2 + 3*p + 1) + 1/2*d*(2*x + 3)^(2*p)*(2*x + 3)/(2*p + 1)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 148 vs. 2(56) = 112.

Time = 0.18 (sec) , antiderivative size = 148, normalized size of antiderivative = 2.47

$$\int (d + ex) (9 + 12x + 4x^2)^p dx = \frac{8(4x^2 + 12x + 9)^p epx^2 + 8(4x^2 + 12x + 9)^p dp x + 12(4x^2 + 12x + 9)^p ep x + 4(4x^2 + 12x + 9)^p ex^2}{8(2p^2 + 3p + 1)}$$

input

```
integrate((e*x+d)*(4*x^2+12*x+9)^p,x, algorithm="giac")
```

output

$$\frac{1}{8} \cdot (8 \cdot (4x^2 + 12x + 9)^p \cdot e \cdot p \cdot x^2 + 8 \cdot (4x^2 + 12x + 9)^p \cdot d \cdot p \cdot x + 12 \cdot (4x^2 + 12x + 9)^p \cdot e \cdot p \cdot x + 4 \cdot (4x^2 + 12x + 9)^p \cdot e \cdot x^2 + 12 \cdot (4x^2 + 12x + 9)^p \cdot d \cdot p + 8 \cdot (4x^2 + 12x + 9)^p \cdot d \cdot x + 12 \cdot (4x^2 + 12x + 9)^p \cdot d - 9 \cdot (4x^2 + 12x + 9)^p \cdot e) / (2 \cdot p^2 + 3 \cdot p + 1)$$

Mupad [B] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.45

$$\int (d + ex) (9 + 12x + 4x^2)^p dx = \left(\frac{12d - 9e + 12dp}{16p^2 + 24p + 8} + \frac{x(8d + 8dp + 12ep)}{16p^2 + 24p + 8} + \frac{4ex^2(2p + 1)}{16p^2 + 24p + 8} \right) (4x^2 + 12x + 9)^p$$

input

```
int((d + e*x)*(12*x + 4*x^2 + 9)^p,x)
```

output

$$\frac{((12 \cdot d - 9 \cdot e + 12 \cdot d \cdot p) / (24 \cdot p + 16 \cdot p^2 + 8) + (x \cdot (8 \cdot d + 8 \cdot d \cdot p + 12 \cdot e \cdot p)) / (24 \cdot p + 16 \cdot p^2 + 8) + (4 \cdot e \cdot x^2 \cdot (2 \cdot p + 1)) / (24 \cdot p + 16 \cdot p^2 + 8)) \cdot (12 \cdot x + 4 \cdot x^2 + 9)^p}$$

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.05

$$\int (d + ex) (9 + 12x + 4x^2)^p dx = \frac{(4x^2 + 12x + 9)^p (8epx^2 + 8dpx + 12epx + 4ex^2 + 12dp + 8dx + 12d - 9e)}{16p^2 + 24p + 8}$$

input

```
int((e*x+d)*(4*x^2+12*x+9)^p,x)
```

output

$$\frac{((4 \cdot x^{**2} + 12 \cdot x + 9) \cdot p \cdot (8 \cdot d \cdot p \cdot x + 12 \cdot d \cdot p + 8 \cdot d \cdot x + 12 \cdot d + 8 \cdot e \cdot p \cdot x^{**2} + 12 \cdot e \cdot p \cdot x + 4 \cdot e \cdot x^{**2} - 9 \cdot e)) / (8 \cdot (2 \cdot p^{**2} + 3 \cdot p + 1))}$$

CHAPTER 4

APPENDIX

| | | |
|-----|---|------|
| 4.1 | Listing of Grading functions | 3363 |
| 4.2 | Links to plain text integration problems used in this report for each CAS . | 3381 |

4.1 Listing of Grading functions

The following are the current version of the grading functions used for grading the quality of the antiderivative with reference to the optimal antiderivative included in the test suite.

There is a version for Maple and for Mathematica/Rubi. There is a version for grading Sympy and version for use with Sagemath.

The following are links to the current source code.

The following are the listings of source code of the grading functions.

Mathematica and Rubi grading function

```
(* Original version thanks to Albert Rich emailed on 03/21/2017 *)
(* ::Package:: *)

(* Nasser: April 7,2022. add second output which gives reason for the grade *)
(*                               Small rewrite of logic in main function to make it*)
(*                               match Maple's logic. No change in functionality otherwise*)

(* ::Subsection:: *)
(*GradeAntiderivative[result,optimal]*)
```



```

(* ::Text:: *)
(*If result and optimal are mathematical expressions, *)
(*      GradeAntiderivative[result,optimal] returns*)
(* "F" if the result fails to integrate an expression that*)
(*      is integrable*)
(* "C" if result involves higher level functions than necessary*)
(* "B" if result is more than twice the size of the optimal*)
(*      antiderivative*)
(* "A" if result can be considered optimal*)

GradeAntiderivative[result_,optimal_] := Module[{expnResult,expnOptimal,leafCountResult,leafCountOptimal},
  expnResult = ExpnType[result];
  expnOptimal = ExpnType[optimal];
  leafCountResult = LeafCount[result];
  leafCountOptimal = LeafCount[optimal];

  (*Print["expnResult=",expnResult," expnOptimal=",expnOptimal];*)
  If[expnResult<=expnOptimal,
    If[Not[FreeQ[result,Complex]], (*result contains complex*)
      If[Not[FreeQ[optimal,Complex]], (*optimal contains complex*)
        If[leafCountResult<=2*leafCountOptimal,
          finalresult={"A",""}
          ,(*ELSE*)
          finalresult={"B","Both result and optimal contain complex but leaf count
          ]
        ,(*ELSE*)
        finalresult={"C","Result contains complex when optimal does not."}
      ]
    ,(*ELSE*)(*result does not contains complex*)
    If[leafCountResult<=2*leafCountOptimal,
      finalresult={"A",""}
      ,(*ELSE*)
      finalresult={"B","Leaf count is larger than twice the leaf count of optimal.
      ]
    ]
  ,(*ELSE*)(*expnResult>expnOptimal*)
  If[FreeQ[result,Integrate] && FreeQ[result,Int],
    finalresult={"C","Result contains higher order function than in optimal. Order "
  ,

```

```

        finalresult={"F","Contains unresolved integral."}
    ]
];

finalresult
]

(* ::Text:: *)
(*The following summarizes the type number assigned an *)
(*expression based on the functions it involves*)
(*1 = rational function*)
(*2 = algebraic function*)
(*3 = elementary function*)
(*4 = special function*)
(*5 = hyperpergeometric function*)
(*6 = appell function*)
(*7 = rootsum function*)
(*8 = integrate function*)
(*9 = unknown function*)

ExpnType[expn_] :=
  If[AtomQ[expn],
    1,
  If[ListQ[expn],
    Max[Map[ExpnType,expn]],
  If[Head[expn]===Power,
    If[IntegerQ[expn[[2]]],
      ExpnType[expn[[1]]],
    If[Head[expn[[2]]]===Rational,
      If[IntegerQ[expn[[1]]] || Head[expn[[1]]]===Rational,
        1,
        Max[ExpnType[expn[[1]],2]],
      Max[ExpnType[expn[[1]],ExpnType[expn[[2]],3]],
    If[Head[expn]===Plus || Head[expn]===Times,
      Max[ExpnType[First[expn]],ExpnType[Rest[expn]]],
    If[ElementaryFunctionQ[Head[expn]],
      Max[3,ExpnType[expn[[1]]]],
    If[SpecialFunctionQ[Head[expn]],
      Apply[Max,Append[Map[ExpnType,Apply[List,expn]],4]],
    If[HypergeometricFunctionQ[Head[expn]],

```

```
Apply[Max, Append[Map[ExpnType, Apply[List, expn]], 5]],
If[AppellFunctionQ[Head[expn]],
Apply[Max, Append[Map[ExpnType, Apply[List, expn]], 6]],
If[Head[expn]===RootSum,
Apply[Max, Append[Map[ExpnType, Apply[List, expn]], 7]],
If[Head[expn]===Integrate || Head[expn]===Int,
Apply[Max, Append[Map[ExpnType, Apply[List, expn]], 8]],
9]]]]]]]]]]]]
```

```
ElementaryFunctionQ[func_] :=
MemberQ[{
Exp, Log,
Sin, Cos, Tan, Cot, Sec, Csc,
ArcSin, ArcCos, ArcTan, ArcCot, ArcSec, ArcCsc,
Sinh, Cosh, Tanh, Coth, Sech, Csch,
ArcSinh, ArcCosh, ArcTanh, ArcCoth, ArcSech, ArcCsch
}, func]
```

```
SpecialFunctionQ[func_] :=
MemberQ[{
Erf, Erfc, Erfi,
FresnelS, FresnelC,
ExpIntegralE, ExpIntegralEi, LogIntegral,
SinIntegral, CosIntegral, SinhIntegral, CoshIntegral,
Gamma, LogGamma, PolyGamma,
Zeta, PolyLog, ProductLog,
EllipticF, EllipticE, EllipticPi
}, func]
```

```
HypergeometricFunctionQ[func_] :=
MemberQ[{Hypergeometric1F1, Hypergeometric2F1, HypergeometricPFQ}, func]
```

```
AppellFunctionQ[func_] :=
MemberQ[{AppellF1}, func]
```

Maple grading function

```

# File: GradeAntiderivative.mpl
# Original version thanks to Albert Rich emailed on 03/21/2017

#Nasser 03/22/2017 Use Maple leaf count instead since buildin
#Nasser 03/23/2017 missing 'ln' for ElementaryFunctionQ added
#Nasser 03/24/2017 corrected the check for complex result
#Nasser 10/27/2017 check for leafsize and do not call ExpnType()
#                    if leaf size is "too large". Set at 500,000
#Nasser 12/22/2019 Added debug flag, added 'dilog' to special functions
#                    see problem 156, file Apostol_Problems
#Nasser 4/07/2022  add second output which gives reason for the grade

GradeAntiderivative := proc(result,optimal)
local leaf_count_result,
      leaf_count_optimal,
      ExpnType_result,
      ExpnType_optimal,
      debug:=false;

      leaf_count_result:=leafcount(result);
      #do NOT call ExpnType() if leaf size is too large. Recursion problem
      if leaf_count_result > 500000 then
          return "B","result has leaf size over 500,000. Avoiding possible recursion issue";
      fi;

      leaf_count_optimal := leafcount(optimal);
      ExpnType_result   := ExpnType(result);
      ExpnType_optimal  := ExpnType(optimal);

      if debug then
          print("ExpnType_result",ExpnType_result," ExpnType_optimal=",ExpnType_optimal);
      fi;

# If result and optimal are mathematical expressions,
# GradeAntiderivative[result,optimal] returns
# "F" if the result fails to integrate an expression that
#     is integrable
# "C" if result involves higher level functions than necessary
# "B" if result is more than twice the size of the optimal

```

```

#   antiderivative
#   "A" if result can be considered optimal

#This check below actually is not needed, since I only
#call this grading only for passed integrals. i.e. I check
#for "F" before calling this. But no harm of keeping it here.
#just in case.

if not type(result,freeof('int')) then
    return "F","Result contains unresolved integral";
fi;

if ExpnType_result<=ExpnType_optimal then
    if debug then
        print("ExpnType_result<=ExpnType_optimal");
    fi;
    if is_contains_complex(result) then
        if is_contains_complex(optimal) then
            if debug then
                print("both result and optimal complex");
            fi;
            if leaf_count_result<=2*leaf_count_optimal then
                return "A"," ";
            else
                return "B",cat("Both result and optimal contain complex but leaf count of
                                convert(leaf_count_result,string)," vs. $2 (" ,
                                convert(leaf_count_optimal,string)," ) = ",convert(2*leaf

        end if
    else #result contains complex but optimal is not
        if debug then
            print("result contains complex but optimal is not");
        fi;
        return "C","Result contains complex when optimal does not.";
    fi;
else # result do not contain complex
    # this assumes optimal do not as well. No check is needed here.
    if debug then
        print("result do not contain complex, this assumes optimal do not as well
    fi;

```

```

        if leaf_count_result<=2*leaf_count_optimal then
            if debug then
                print("leaf_count_result<=2*leaf_count_optimal");
            fi;
            return "A"," ";
        else
            if debug then
                print("leaf_count_result>2*leaf_count_optimal");
            fi;
            return "B",cat("Leaf count of result is larger than twice the leaf count of
                            convert(leaf_count_result,string)," $ vs. $2(",
                            convert(leaf_count_optimal,string),")=",convert(2*leaf_co
            fi;
        fi;
    else #ExpnType(result) > ExpnType(optimal)
        if debug then
            print("ExpnType(result) > ExpnType(optimal)");
        fi;
        return "C",cat("Result contains higher order function than in optimal. Order ",
                        convert(ExpnType_result,string)," vs. order ",
                        convert(ExpnType_optimal,string),".");
    fi;
end proc:

#
# is_contains_complex(result)
# takes expressions and returns true if it contains "I" else false
#
#Nasser 032417
is_contains_complex:= proc(expression)
    return (has(expression,I));
end proc:

# The following summarizes the type number assigned an expression
# based on the functions it involves
# 1 = rational function
# 2 = algebraic function
# 3 = elementary function
# 4 = special function
# 5 = hyperpergeometric function

```

```

# 6 = appell function
# 7 = rootsum function
# 8 = integrate function
# 9 = unknown function

ExpnType := proc(expn)
  if type(expn,'atomic') then
    1
  elif type(expn,'list') then
    apply(max,map(ExpnType,expn))
  elif type(expn,'sqrt') then
    if type(op(1,expn),'rational') then
      1
    else
      max(2,ExpnType(op(1,expn)))
    end if
  elif type(expn,'^^') then
    if type(op(2,expn),'integer') then
      ExpnType(op(1,expn))
    elif type(op(2,expn),'rational') then
      if type(op(1,expn),'rational') then
        1
      else
        max(2,ExpnType(op(1,expn)))
      end if
    else
      max(3,ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    end if
  elif type(expn,'+'') or type(expn,'*') then
    max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
  elif ElementaryFunctionQ(op(0,expn)) then
    max(3,ExpnType(op(1,expn)))
  elif SpecialFunctionQ(op(0,expn)) then
    max(4,apply(max,map(ExpnType,[op(expn)])))
  elif HypergeometricFunctionQ(op(0,expn)) then
    max(5,apply(max,map(ExpnType,[op(expn)])))
  elif AppellFunctionQ(op(0,expn)) then
    max(6,apply(max,map(ExpnType,[op(expn)])))
  elif op(0,expn)='int' then
    max(8,apply(max,map(ExpnType,[op(expn)]))) else
  9

```

```

    end if
end proc:

ElementaryFunctionQ := proc(func)
  member(func, [
    exp, log, ln,
    sin, cos, tan, cot, sec, csc,
    arcsin, arccos, arctan, arccot, arcsec, arccsc,
    sinh, cosh, tanh, coth, sech, csch,
    arcsinh, arccosh, arctanh, arccoth, arcsech, arccsch])
end proc:

SpecialFunctionQ := proc(func)
  member(func, [
    erf, erfc, erfi,
    FresnelS, FresnelC,
    Ei, Ei, Li, Si, Ci, Shi, Chi,
    GAMMA, lnGAMMA, Psi, Zeta, polylog, dilog, LambertW,
    EllipticF, EllipticE, EllipticPi])
end proc:

HypergeometricFunctionQ := proc(func)
  member(func, [Hypergeometric1F1, hypergeom, HypergeometricPFQ])
end proc:

AppellFunctionQ := proc(func)
  member(func, [AppellF1])
end proc:

# u is a sum or product. rest(u) returns all but the
# first term or factor of u.
rest := proc(u) local v;
  if nops(u)=2 then
    op(2,u)
  else
    apply(op(0,u), op(2..nops(u),u))
  end if
end proc:

#leafcount(u) returns the number of nodes in u.

```



```
#Nasser 3/23/17 Replaced by build-in leafCount from package in Maple
leafcount := proc(u)
  MmaTranslator[Mma][LeafCount](u);
end proc;
```

Sympy grading function

```
#Dec 24, 2019. Nasser M. Abbasi:
#      Port of original Maple grading function by
#      Albert Rich to use with Sympy/Python
#Dec 27, 2019 Nasser. Added `RootSum`. See problem 177, Timofeev file
#      added 'exp_polar'
from sympy import *

def leaf_count(expr):
  #sympy do not have leaf count function. This is approximation
  return round(1.7*count_ops(expr))

def is_sqrt(expr):
  if isinstance(expr,Pow):
    if expr.args[1] == Rational(1,2):
      return True
    else:
      return False
  else:
    return False

def is_elementary_function(func):
  return func in [exp,log,ln,sin,cos,tan,cot,sec,csc,
    asin,acos,atan,acot,asec,acsc,sinh,cosh,tanh,coth,sech,csch,
    asinh,acosh,atanh,acoth,asech,acsch
  ]

def is_special_function(func):
  return func in [ erf,erfc,erfi,
    fresnels,fresnelc,Ei,Ei,Li,Si,Ci,Shi,Chi,
    gamma,loggamma,digamma,zeta,polylog,LambertW,
    elliptic_f,elliptic_e,elliptic_pi,exp_polar
  ]
```

```

def is_hypergeometric_function(func):
    return func in [hyper]

def is_appell_function(func):
    return func in [appellf1]

def is_atom(expn):
    try:
        if expn.isAtom or isinstance(expn,int) or isinstance(expn,float):
            return True
        else:
            return False

    except AttributeError as error:
        return False

def expnType(expn):
    debug=False
    if debug:
        print("expn=",expn,"type(expn)=",type(expn))

    if is_atom(expn):
        return 1
    elif isinstance(expn,list):
        return max(map(expnType, expn)) #apply(max,map(ExpnType,expn))
    elif is_sqrt(expn):
        if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
            return 1
        else:
            return max(2,expnType(expn.args[0])) #max(2,ExpnType(op(1,expn)))
    elif isinstance(expn,Pow): #type(expn,'^')
        if isinstance(expn.args[1],Integer): #type(op(2,expn),'integer')
            return expnType(expn.args[0]) #ExpnType(op(1,expn))
        elif isinstance(expn.args[1],Rational): #type(op(2,expn),'rational')
            if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
                return 1
            else:
                return max(2,expnType(expn.args[0])) #max(2,ExpnType(op(1,expn)))
        else:
            return max(3,expnType(expn.args[0]),expnType(expn.args[1])) #max(3,ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    elif isinstance(expn,Add) or isinstance(expn,Mul): #type(expn,'+') or type(expn,'*')

```

```

    m1 = expnType(expn.args[0])
    m2 = expnType(list(expn.args[1:]))
    return max(m1,m2) #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
elif is_elementary_function(expn.func): #ElementaryFunctionQ(op(0,expn))
    return max(3,expnType(expn.args[0])) #max(3,ExpnType(op(1,expn)))
elif is_special_function(expn.func): #SpecialFunctionQ(op(0,expn))
    m1 = max(map(expnType, list(expn.args)))
    return max(4,m1) #max(4,apply(max,map(ExpnType,[op(expn)])))
elif is_hypergeometric_function(expn.func): #HypergeometricFunctionQ(op(0,expn))
    m1 = max(map(expnType, list(expn.args)))
    return max(5,m1) #max(5,apply(max,map(ExpnType,[op(expn)])))
elif is_appell_function(expn.func):
    m1 = max(map(expnType, list(expn.args)))
    return max(6,m1) #max(5,apply(max,map(ExpnType,[op(expn)])))
elif isinstance(expn,RootSum):
    m1 = max(map(expnType, list(expn.args))) #Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
    return max(7,m1)
elif str(expn).find("Integral") != -1:
    m1 = max(map(expnType, list(expn.args)))
    return max(8,m1) #max(5,apply(max,map(ExpnType,[op(expn)])))
else:
    return 9

#main function
def grade_antiderivative(result,optimal):

    #print("Enter grade_antiderivative for sagemath")
    #print("Enter grade_antiderivative, result=",result," optimal=",optimal)

    leaf_count_result = leaf_count(result)
    leaf_count_optimal = leaf_count(optimal)

    #print("leaf_count_result=",leaf_count_result)
    #print("leaf_count_optimal=",leaf_count_optimal)

    expnType_result = expnType(result)
    expnType_optimal = expnType(optimal)

    if str(result).find("Integral") != -1:
        grade = "F"
        grade_annotation = ""

```

```

else:
    if expnType_result <= expnType_optimal:
        if result.has(I):
            if optimal.has(I): #both result and optimal complex
                if leaf_count_result <= 2*leaf_count_optimal:
                    grade = "A"
                    grade_annotation = ""
                else:
                    grade = "B"
                    grade_annotation = "Both result and optimal contain complex but leaf count of result is lar
            else: #result contains complex but optimal is not
                grade = "C"
                grade_annotation = "Result contains complex when optimal does not."
        else: # result do not contain complex, this assumes optimal do not as well
            if leaf_count_result <= 2*leaf_count_optimal:
                grade = "A"
                grade_annotation = ""
            else:
                grade = "B"
                grade_annotation = "Leaf count of result is larger than twice the leaf count of optimal. "+str(
        else:
            grade = "C"
            grade_annotation = "Result contains higher order function than in optimal. Order "+str(ExpnType

#print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)

return grade, grade_annotation

```

SageMath grading function

```

#Dec 24, 2019. Nasser: Ported original Maple grading function by
#       Albert Rich to use with Sagemath. This is used to
#       grade Fricas, Giac and Maxima results.
#Dec 24, 2019. Nasser: Added 'exp_integral_e' and 'sng', 'sin_integral'
#       'arctan2', 'floor', 'abs', 'log_integral'
#June 4, 2022 Made default grade_annotation "none" instead of "" due
#       issue later when reading the file.
#July 14, 2022. Added ellipticF. This is until they fix sagemath, then remove it.

```

```

from sage.all import *
from sage.symbolic.operators import add_vararg, mul_vararg

debug=False;

def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    #print("Enter tree_size, expr is ",expr)

    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)

def is_sqrt(expr):
    if expr.operator() == operator.pow: #isinstance(expr,Pow):
        if expr.operands()[1]==1/2: #expr.args[1] == Rational(1,2):
            if debug: print ("expr is sqrt")
            return True
        else:
            return False
    else:
        return False

def is_elementary_function(func):
    #debug=False
    m = func.name() in ['exp','log','ln',
        'sin','cos','tan','cot','sec','csc',
        'arcsin','arccos','arctan','arccot','arcsec','arccsc',
        'sinh','cosh','tanh','coth','sech','csch',
        'arcsinh','arccosh','arctanh','arcoth','arcsech','arcsch','sgn',
        'arctan2','floor','abs'
    ]
    if debug:

```

```

    if m:
        print ("func ", func , " is elementary_function")
    else:
        print ("func ", func , " is NOT elementary_function")

    return m

def is_special_function(func):
    #debug=False
    if debug:
        print ("type(func)=", type(func))

    m= func.name() in ['erf','erfc','erfi','fresnel_sin','fresnel_cos','Ei',
        'Ei','Li','Si','sin_integral','Ci','cos_integral','Shi','sinh_integral',
        'Chi','cosh_integral','gamma','log_gamma','psi','zeta',
        'polylog','lambert_w','elliptic_f','elliptic_e','ellipticF',
        'elliptic_pi','exp_integral_e','log_integral',
        'weierstrassPInverse','weierstrass','weierstrassP','weierstrassZeta',
        'weierstrassPPrime','weierstrassSigma']

    if debug:
        print ("m=",m)
    if m:
        print ("func ", func , " is special_function")
    else:
        print ("func ", func , " is NOT special_function")

    return m

def is_hypergeometric_function(func):
    return func.name() in ['hypergeometric','hypergeometric_M','hypergeometric_U']

def is_appell_function(func):
    return func.name() in ['hypergeometric'] #[appellf1] can't find this in sagemath

def is_atom(expn):
    #debug=False

```

```

if debug:
    print ("Enter is_atom, expn=",expn)

if not hasattr(expn, 'parent'):
    return False

#thanks to answer at https://ask.sagemath.org/question/49179/what-is-sagemath-equivalent-to-atomic
try:
    if expn.parent() is SR:
        return expn.operator() is None
    if expn.parent() in (ZZ, QQ, AA, QQbar):
        return expn in expn.parent() # Should always return True
    if hasattr(expn.parent(),"base_ring") and hasattr(expn.parent(),"gens"):
        return expn in expn.parent().base_ring() or expn in expn.parent().gens()

    return False

except AttributeError as error:
    print("Exception,AttributeError in is_atom")
    print ("caught exception" , type(error).__name__ )
    return False

def expnType(expn):

    if debug:
        print (">>>>>Enter expnType, expn=", expn)
        print (">>>>>is_atom(expn)=", is_atom(expn))

    if is_atom(expn):
        return 1
    elif type(expn)==list: #instance(expn,list):
        return max(map(expnType, expn)) #apply(max,map(ExpnType,expn))
    elif is_sqrt(expn):
        if type(expn.operands()[0])==Rational: #type(instance(expn.args[0],Rational):
            return 1
        else:
            return max(2,expnType(expn.operands()[0])) #max(2,expnType(expn.args[0]))
    elif expn.operator() == operator.pow: #instance(expn,Pow)
        if type(expn.operands()[1])==Integer: #instance(expn.args[1],Integer)

```

```

    return expnType(expn.operands()[0]) #expnType(expn.args[0])
elif type(expn.operands()[1])==Rational: #isinstance(expn.args[1],Rational)
    if type(expn.operands()[0])==Rational: #isinstance(expn.args[0],Rational)
        return 1
    else:
        return max(2,expnType(expn.operands()[0])) #max(2,expnType(expn.args[0]))
else:
    return max(3,expnType(expn.operands()[0]),expnType(expn.operands()[1])) #max(3,expnType(expn
elif expn.operator() == add_vararg or expn.operator() == mul_vararg: #isinstance(expn,Add) or isins
    m1 = expnType(expn.operands()[0]) #expnType(expn.args[0])
    m2 = expnType(expn.operands()[1:]) #expnType(list(expn.args[1:]))
    return max(m1,m2) #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
elif is_elementary_function(expn.operator()): #is_elementary_function(expn.func)
    return max(3,expnType(expn.operands()[0]))
elif is_special_function(expn.operator()): #is_special_function(expn.func)
    m1 = max(map(expnType, expn.operands())) #max(map(expnType, list(expn.args)))
    return max(4,m1) #max(4,m1)
elif is_hypergeometric_function(expn.operator()): #is_hypergeometric_function(expn.func)
    m1 = max(map(expnType, expn.operands())) #max(map(expnType, list(expn.args)))
    return max(5,m1) #max(5,m1)
elif is_appell_function(expn.operator()):
    m1 = max(map(expnType, expn.operands())) #max(map(expnType, list(expn.args)))
    return max(6,m1) #max(6,m1)
elif str(expn).find("Integral") != -1: #this will never happen, since it
    #is checked before calling the grading function that is passed.
    #but kept it here.
    m1 = max(map(expnType, expn.operands())) #max(map(expnType, list(expn.args)))
    return max(8,m1) #max(5,apply(max,map(ExpnType,[op(expn)])))
else:
    return 9

#main function
def grade_antiderivative(result,optimal):

if debug:
    print ("Enter grade_antiderivative for sagemath")
    print ("Enter grade_antiderivative, result=",result)
    print ("Enter grade_antiderivative, optimal=",optimal)
    print ("type(anti)=", type(result))
    print ("type(optimal)=", type(optimal))

```



```

leaf_count_result = tree_size(result) #leaf_count(result)
leaf_count_optimal = tree_size(optimal) #leaf_count(optimal)

#if debug: print ("leaf_count_result=", leaf_count_result, "leaf_count_optimal=",leaf_count_optimal)

expnType_result = expnType(result)
expnType_optimal = expnType(optimal)

if debug: print ("expnType_result=", expnType_result, "expnType_optimal=",expnType_optimal)

if expnType_result <= expnType_optimal:
    if result.has(I):
        if optimal.has(I): #both result and optimal complex
            if leaf_count_result <= 2*leaf_count_optimal:
                grade = "A"
                grade_annotation = "none"
            else:
                grade = "B"
                grade_annotation = "Both result and optimal contain complex but leaf count of result is larger"
        else: #result contains complex but optimal is not
            grade = "C"
            grade_annotation = "Result contains complex when optimal does not."
    else: # result do not contain complex, this assumes optimal do not as well
        if leaf_count_result <= 2*leaf_count_optimal:
            grade = "A"
            grade_annotation = "none"
        else:
            grade = "B"
            grade_annotation = "Leaf count of result is larger than twice the leaf count of optimal. "+str(leaf_count_result - 2*leaf_count_optimal)
    else:
        grade = "C"
        grade_annotation = "Result contains higher order function than in optimal. Order "+str(expnType_result - expnType_optimal)

print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)

return grade, grade_annotation

```

4.2 Links to plain text integration problems used in this report for each CAS

1. Mathematica integration problems as .m file
2. Maple integration problems as .txt file
3. Sagemath integration problems as .sage file
4. Reduce integration problems as .txt file
5. Mupad integration problems as .txt file
6. Sympy integration problems as .py file