Computer Algebra
Independent Integration Tests

Summer 2024

1-Algebraic-functions/1.2-Trinomial/1.2.1-Quadratic-trinomial/1.2.1.2/91-1.2.1.2-c

Nasser M. Abbasi

May 18, 2024 Compiled on May 18, 2024 at 2:23am

Contents

CHAPTER 1

INTRODUCTION

This report gives the result of running the computer algebra independent integration test.The download section in on the main webpage contains links to download the problems in plain text format used for all CAS systems. The number of integrals in this report is [**404**]. This is test number [91].

1.1 Listing of CAS systems tested

The following are the CAS systems tested:

1. Mathematica 14 (January 9, 2024) on windows 10 pro.
2. Rubi 4.17.3 (Sept 25, 2023) on Mathematica 14 on windows 10m pro.
3. Maple 2024 (March 1, 2024) on windows 10 pro.
4. Maxima 5.47 (June 1, 2023) using Lisp SBCL 2.4.0 on Linux Manjaro 23.1.2 KDE via sagemath 10.3.
5. FriCAS 1.3.10 built with sbcl 2.3.11 (January 10, 2024) on Linux Manjaro 23.1.2 KDE via sagemath 10.3.
6. Giac/Xcas 1.9.0-99 on Linux via sagemath 10.3.
7. Sympy 1.12 using Python 3.11.6 (Nov 14 2023, 09:36:21) [GCC 13.2.1 20230801] on Linux Manjaro 23.1.2 KDE.
8. Mupad using Matlab 2021a with Symbolic Math Toolbox Version 8.7 on windows 10.
9. Reduce CSL rev 6687 (January 9, 2024) on Linux Manjaro 23.1.2 KDE.

Maxima and Fricas and Giac are called using Sagemath. This was done using Sagemath `integrate` command by changing the name of the algorithm to use the different CAS systems.

Sympy was run directly in Python not via sagemath.

Reduce was called directly.

1.2 Results

Important note: A number of problems in this test suite have no antiderivative in closed form. This means the antiderivative of these integrals can not be expressed in terms of elementary, special functions or `Hypergeometric2F1` functions. `RootSum` and `RootOf` are not allowed. If a CAS returns the above integral unevaluated within the time limit, then the result is counted as passed and assigned an A grade.

However, if CAS times out, then it is assigned an F grade even if the integral is not integrable, as this implies CAS could not determine that the integral is not integrable in the time limit.

If a CAS returns an antiderivative to such an integral, it is assigned an A grade automatically and this special result is listed in the introduction section of each individual test report to make it easy to identify as this can be important result to investigate.

The results given in in the table below reflects the above.

System	% solved	% Failed
Rubi	100.00 (404)	0.00 (0)
Mathematica	100.00 (404)	0.00 (0)
Maple	83.66 (338)	16.34 (66)
Fricas	83.66 (338)	16.34 (66)
Reduce	82.18 (332)	17.82 (72)
Giac	73.27 (296)	26.73 (108)
Mupad	68.07 (275)	31.93 (129)
Maxima	50.50 (204)	49.50 (200)
Sympy	50.50 (204)	49.50 (200)

Table 1.1: Percentage solved for each CAS

The table below gives additional break down of the grading of quality of the antiderivatives generated by each CAS. The grading is given using the letters A,B,C and F with A being the best quality. The grading is accomplished by comparing the antiderivative generated with the optimal antiderivatives included in the test suite. The following table describes the meaning of these grades.

grade	description
A	Integral was solved and antiderivative is optimal in quality and leaf size.
B	Integral was solved and antiderivative is optimal in quality but leaf size is larger than twice the optimal antiderivatives leaf size.
C	Integral was solved and antiderivative is non-optimal in quality. This can be due to one or more of the following reasons 1. antiderivative contains a hypergeometric function and the optimal antiderivative does not. 2. antiderivative contains a special function and the optimal antiderivative does not. 3. antiderivative contains the imaginary unit and the optimal antiderivative does not.
F	Integral was not solved. Either the integral was returned unevaluated within the time limit, or it timed out, or CAS hanged or crashed or an exception was raised.

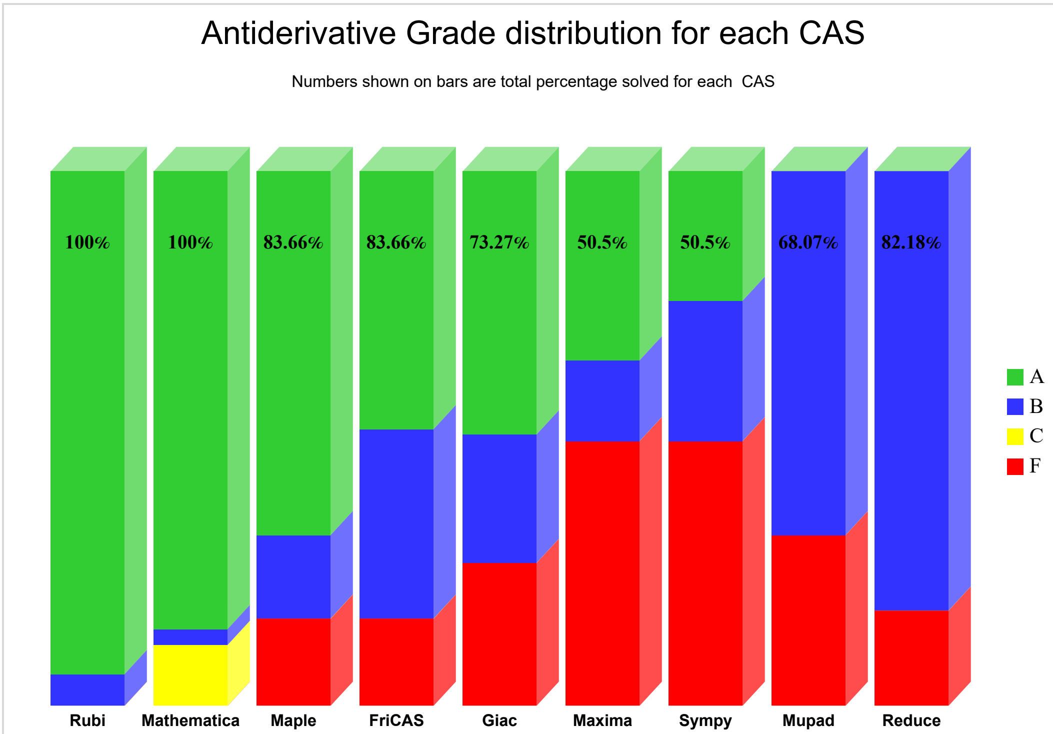
Table 1.2: Description of grading applied to integration result

Grading is implemented for all CAS systems. Based on the above, the following table summarizes the grading for this test suite.

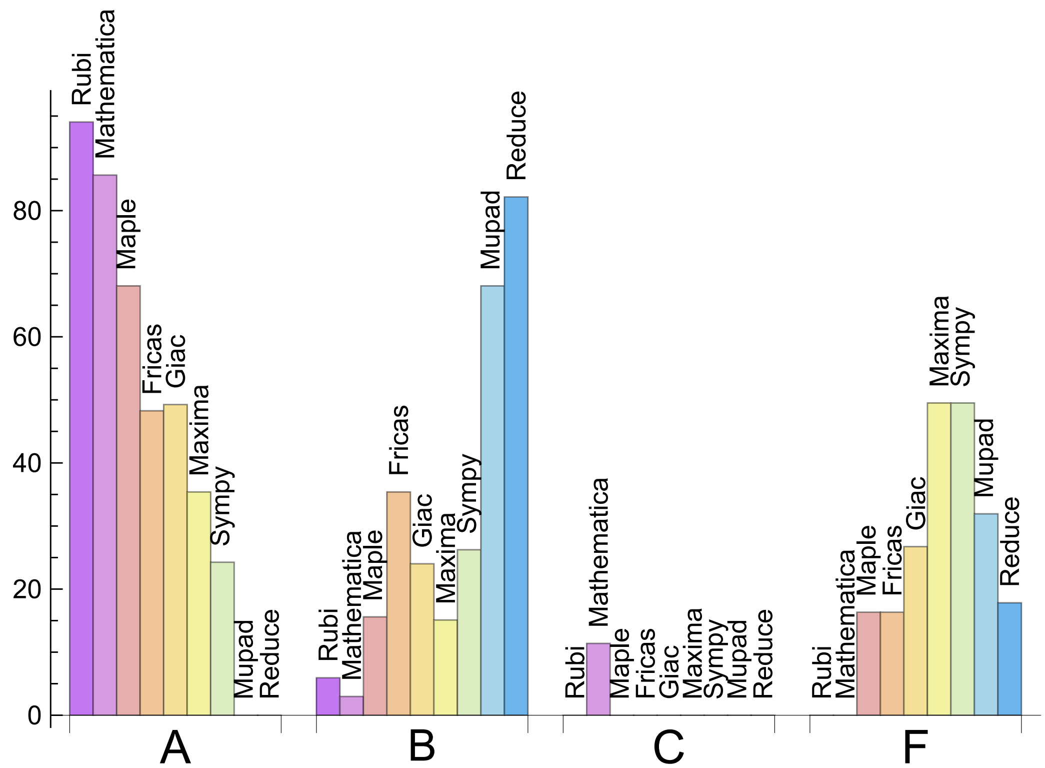
System	% A grade	% B grade	% C grade	% F grade
Rubi	94.059	5.941	0.000	0.000
Mathematica	85.644	2.970	11.386	0.000
Maple	68.069	15.594	0.000	16.337
Giac	49.257	24.010	0.000	26.733
Fricas	48.267	35.396	0.000	16.337
Maxima	35.396	15.099	0.000	49.505
Sympy	24.257	26.238	0.000	49.505
Mupad	0.000	68.069	0.000	31.931
Reduce	0.000	82.178	0.000	17.822

Table 1.3: Antiderivative Grade distribution of each CAS

The following is a Bar chart illustration of the data in the above table.



The figure below compares the grades of the CAS systems.



The following table shows the distribution of the different types of failures for each CAS. There are 3 types failures. The first is when CAS returns the input within the time limit, which means it could not solve it. This is the typical failure and given as **F**.

The second failure is due to time out. CAS could not solve the integral within the 3 minutes time limit which is assigned. This is assigned **F(-1)**.

The third is due to an exception generated, indicated as **F(-2)**. This most likely indicates an interface problem between sagemath and the CAS (applicable only to FriCAS, Maxima and Giac) or it could be an indication of an internal error in the CAS itself. This type of error requires more investigation to determine the cause.

System	Number failed	Percentage normal failure	Percentage timeout failure	Percentage exception failure
Rubi	0	0.00	0.00	0.00
Mathematica	0	0.00	0.00	0.00
Fricas	66	100.00	0.00	0.00
Maple	66	100.00	0.00	0.00
Reduce	72	100.00	0.00	0.00
Giac	108	70.37	0.00	29.63
Mupad	129	0.00	100.00	0.00
Maxima	200	48.50	0.00	51.50
Sympy	200	73.50	25.50	1.00

Table 1.4: Failure statistics for each CAS

1.3 Time and leaf size Performance

The table below summarizes the performance of each CAS system in terms of time used and leaf size of results.

Mean size is the average leaf size produced by the CAS (before any normalization). The Normalized mean is relative to the mean size of the optimal anti-derivative given in the input files.

For example, if CAS has **Normalized mean** of 3, then the mean size of its leaf size is 3 times as large as the mean size of the optimal leaf size.

Median size is value of leaf size where half the values are larger than this and half are smaller (before any normalization). i.e. The Middle value.

Similarly the **Normalized median** is relative to the median leaf size of the optimal.

For example, if a CAS has Normalized median of 1.2, then its median is 1.2 as large as the median leaf size of the optimal.

System	Mean time (sec)
Maxima	0.04
Giac	0.17
Mathematica	0.22
Reduce	0.27
Rubi	0.51
Fricas	1.35
Maple	2.03
Mupad	3.86
Sympy	4.56

Table 1.5: Time performance for each CAS

System	Mean size	Normalized mean	Median size	Normalized median
Mathematica	118.43	0.93	100.00	0.88
Maxima	174.57	1.81	115.50	1.47
Rubi	203.04	1.40	123.00	1.00
Maple	246.07	1.55	119.00	1.06
Giac	292.87	2.07	157.00	1.43
Reduce	366.31	2.42	197.00	1.98
Fricas	378.20	2.45	226.00	2.28
Mupad	409.65	3.14	130.00	1.42
Sympy	663.72	3.60	154.00	1.85

Table 1.6: Leaf size performance for each CAS

1.4 Performance based on number of rules Rubi used

This section shows how each CAS performed based on the number of rules Rubi needed to solve the same integral. One diagram is given for each CAS.

On the y axis is the percentage solved which Rubi itself needed the number of rules given the x axis. These plots show that as more rules are needed then most CAS system percentage of solving decreases which indicates the integral is becoming more complicated to solve.

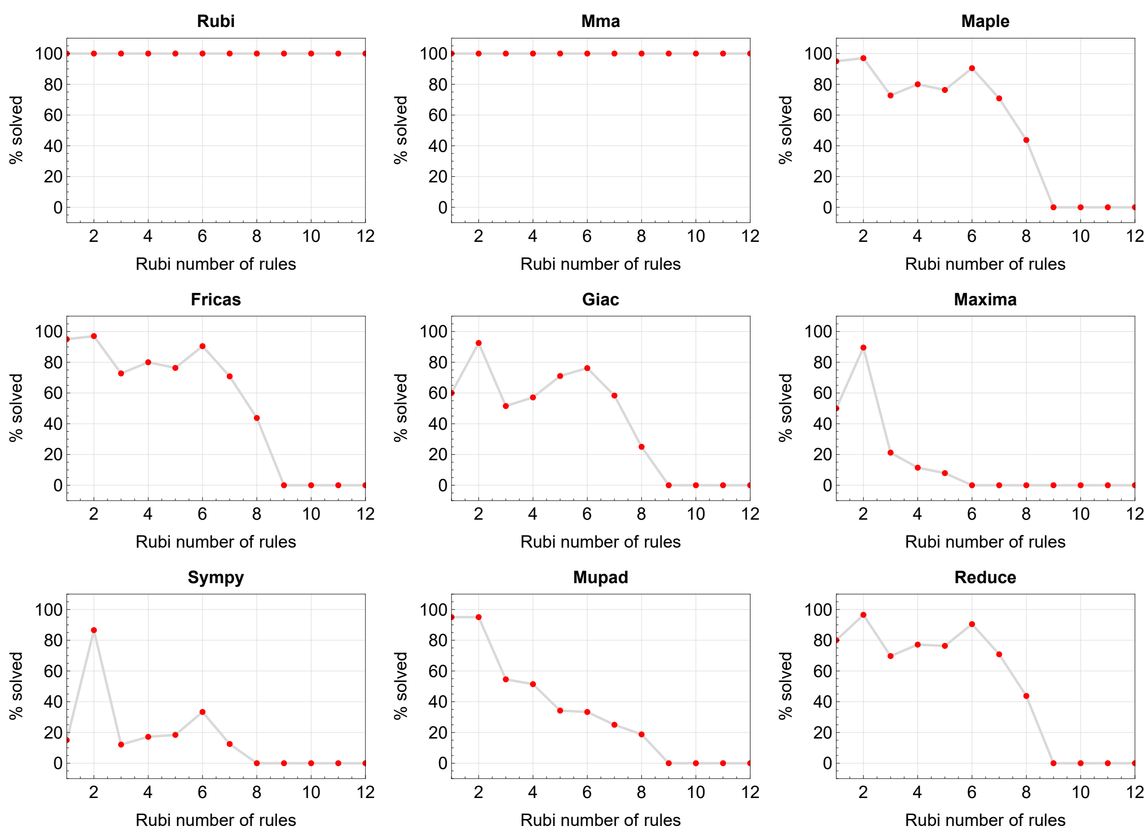


Figure 1.1: Solving statistics per number of Rubi rules used

1.5 Performance based on number of steps Rubi used

This section shows how each CAS performed based on the number of steps Rubi needed to solve the same integral. Note that the number of steps Rubi needed can be much higher than the number of rules, as the same rule could be used more than once.

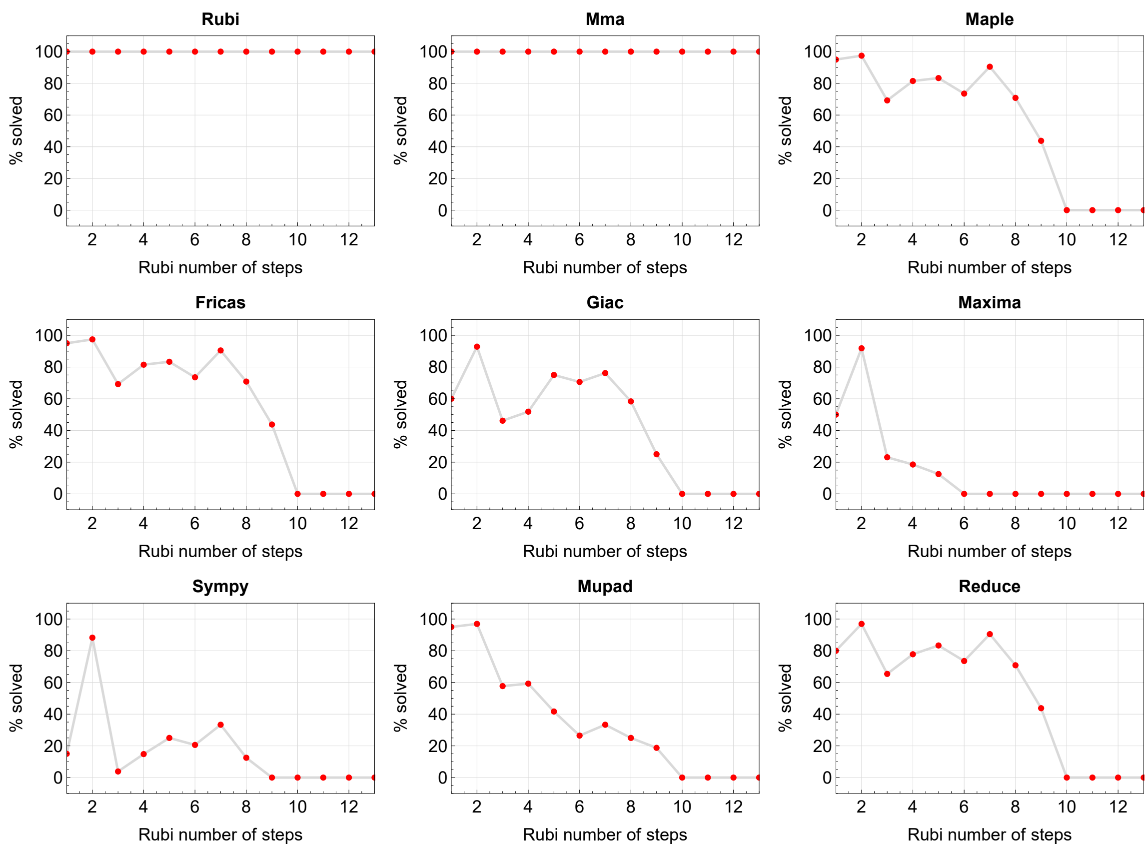


Figure 1.2: Solving statistics per number of Rubi steps used

The above diagram show that the precentage of solved intergals decreases for most CAS systems as the number of steps increases. As expected, for integrals that required less steps by Rubi, CAS systems had more success which indicates the integral was not as hard to solve. As Rubi needed more steps to solve the integral, the solved percentage decreased for most CAS systems which indicates the integral is becoming harder to solve.

1.6 Solved integrals histogram based on leaf size of result

The following shows the distribution of solved integrals for each CAS system based on leaf size of the antiderivatives produced by each CAS. It shows that most integrals solved produced leaf size less than about 100 to 150. The bin size used is 40.

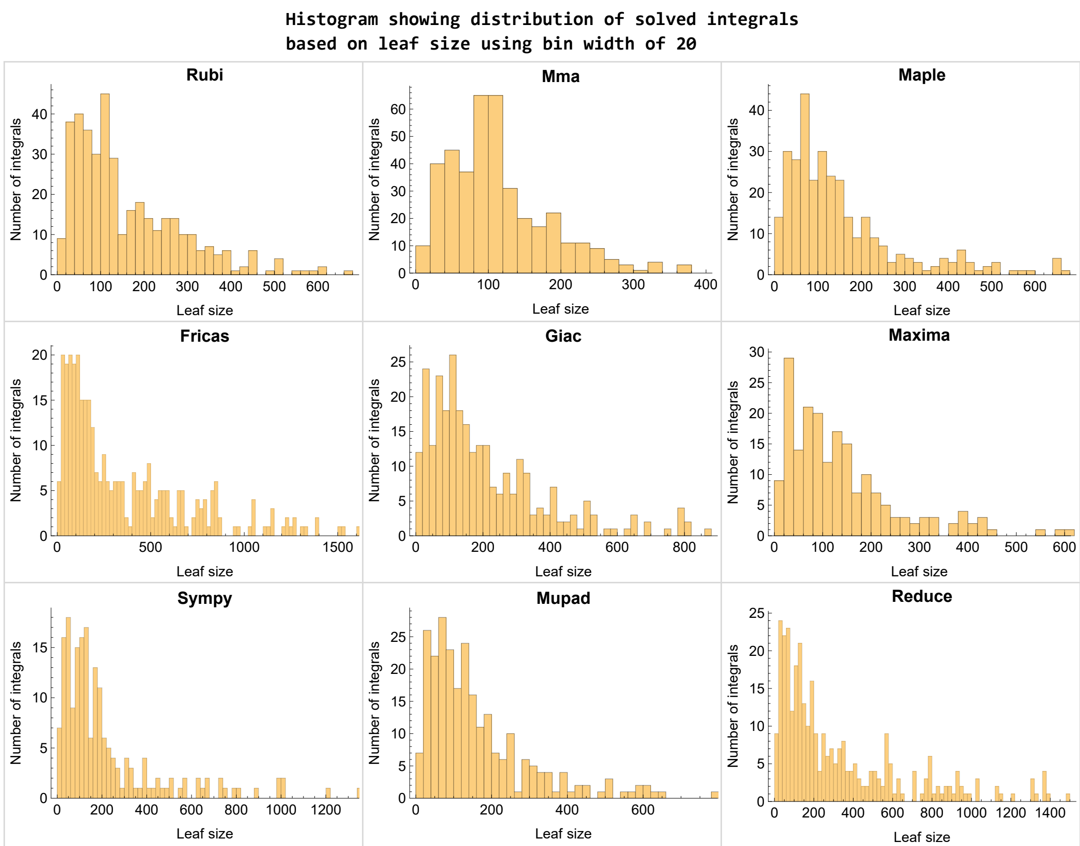


Figure 1.3: Solved integrals based on leaf size distribution

1.7 Solved integrals histogram based on CPU time used

The following shows the distribution of solved integrals for each CAS system based on CPU time used in seconds. The bin size used is 0.1 second.

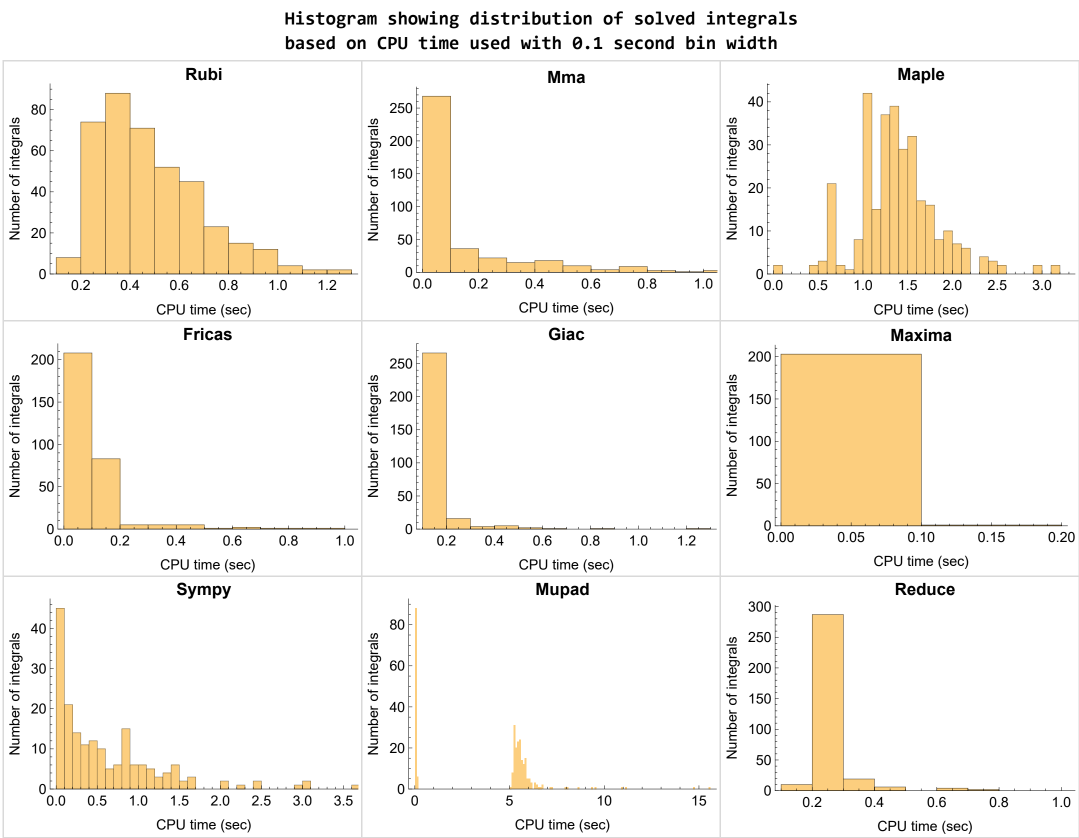


Figure 1.4: Solved integrals histogram based on CPU time used

1.8 Leaf size vs. CPU time used

The following shows the relation between the CPU time used to solve an integral and the leaf size of the antiderivative.

The result for Fricas, Maxima and Giac is shifted more to the right than the other CAS system due to the use of sagemath to call them, which causes an initial slight delay in the timing to start the integration due to overhead of starting a new process each time. This should also be taken into account when looking at the timing of these three CAS systems. Direct calls not using sagemath would result in faster timings, but current implementation uses sagemath as this makes testing much easier to do.

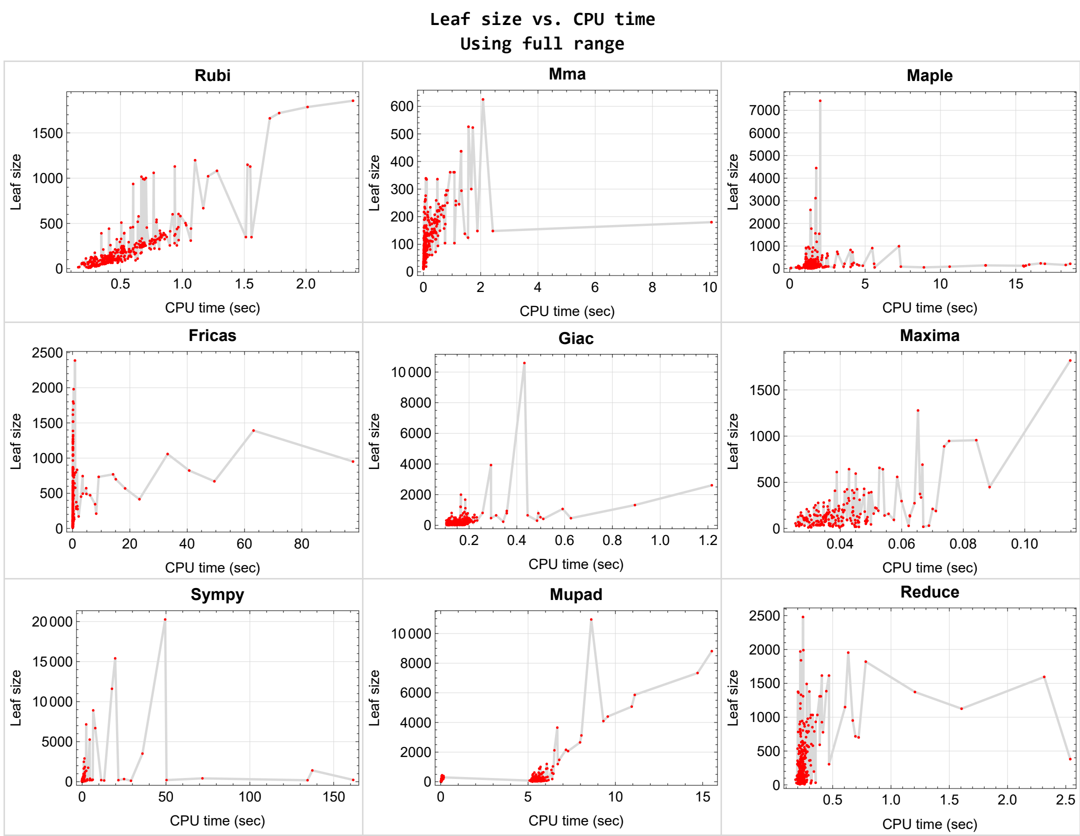


Figure 1.5: Leaf size vs. CPU time. Full range

1.9 list of integrals with no known antiderivative

{}

1.10 List of integrals solved by CAS but has no known antiderivative

Rubi {}

Mathematica {}

Maple {}

Maxima {}

Fricas {}

Sympy {}

Giac {}

Reduce {}

Mupad {}

1.11 list of integrals solved by CAS but failed verification

The following are integrals solved by CAS but the verification phase failed to verify the anti-derivative produced is correct. This does not necessarily mean that the anti-derivative is wrong as additional methods of verification might be needed, or more time is needed (3 minutes time limit was used). These integrals are listed here to make it possible to do further investigation to determine why the result could not be verified.

Rubi {330, 331, 332, 333, 334, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381}

Mathematica {}

Maple {}

Maxima Verification phase not currently implemented.

Fricas Verification phase not currently implemented.

Sympy Verification phase not currently implemented.

Giac Verification phase not currently implemented.

Reduce Verification phase not currently implemented.

Mupad Verification phase not currently implemented.

1.12 Timing

The command `AbsoluteTiming[]` was used in Mathematica to obtain the elapsed time for each integrate call. In Maple, the command `Usage` was used as in the following example

```
cpu_time := Usage(assign ('result_of_int',int(expr,x)),output='realtime'
```

For all other CAS systems, the elapsed time to complete each integral was found by taking the difference between the time after the call completed from the time before the call was made. This was done using Python's `time.time()` call.

All elapsed times shown are in seconds. A time limit of 3 CPU minutes was used for each integral. If the integrate command did not complete within this time limit, the integral was aborted and considered to have failed and assigned an F grade. The time used by failed integrals due to time out was not counted in the final statistics.

1.13 Verification

A verification phase was applied on the result of integration for `Rubi` and `Mathematica`.

Future version of this report will implement verification for the other CAS systems. For the integrals whose result was not run through a verification phase, it is assumed that the antiderivative was correct.

Verification phase also had 3 minutes time out. An integral whose result was not verified could still be correct, but further investigation is needed on those integrals. These integrals were marked in the summary table below and also in each integral separate section so they are easy to identify and locate.

1.14 Important notes about some of the results

Important note about Maxima results

Since tests were run in a batch mode, and using an automated script, then any integral where Maxima needed an interactive response from the user to answer a question during the evaluation of the integral will fail.

The exception raised is `ValueError`. Therefore Maxima results is lower than what would result if Maxima was run directly and each question was answered correctly.

The percentage of such failures were not counted for each test file, but for an example, for the `Timofeev` test file, there were about 14 such integrals out of total 705, or about 2 percent. This percentage can be higher or lower depending on the specific input test file.

Such integrals can be identified by looking at the output of the integration in each section for Maxima. The exception message will indicate the cause of error.

Maxima integrate was run using SageMath with the following settings set by default

```
'besselexpand : true'
'display2d : false'
'domain : complex'
'keepfloat : true'
'load(to_poly_solve)'
'load(simplify_sum)'
'load(abs_integrate)' 'load(diag)'
```

SageMath automatic loading of Maxima `abs_integrate` was found to cause some problems. So the following code was added to disable this effect.

```
from sage.interfaces.maxima_lib import maxima_lib
maxima_lib.set('extra_definite_integration_methods', '[]')
maxima_lib.set('extra_integration_methods', '[]')
```

See `https://ask.sagemath.org/question/43088/integrate-results-that-are-different-from-using-maxima/` for reference.

Important note about FriCAS result

There were few integrals which failed due to SageMath interface and not because FriCAS system could not do the integration.

These will fail With error `Exception raised: NotImplementedError`.

The number of such cases seems to be very small. About 1 or 2 percent of all integrals. These can be identified by looking at the exception message given in the result.

Important note about finding leaf size of antiderivative

For Mathematica, Rubi, and Maple, the builtin system function `LeafSize` was used to find the leaf size of each antiderivative.

The other CAS systems (SageMath and Sympy) do not have special builtin function for this purpose at this time. Therefore the leaf size for Fricas and Sympy antiderivative was determined using the following function, thanks to user `slelievre` at `https://ask.sagemath.org/question/57123/could-we-have-a-leaf_count-function-in-base-sagemath/`

```
def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)
```

For Sympy, which was called directly from Python, the following code was used to obtain the leafsize of its result

```
try:
  # 1.7 is a fudge factor since it is low side from actual leaf count
  leafCount = round(1.7*count_ops(anti))

  except Exception as ee:
        leafCount =1
```

Important note about Mupad results

Matlab's symbolic toolbox does not have a leaf count function to measure the size of the antiderivative. Maple was used to determine the leaf size of Mupad output by post processing Mupad result.

Currently no grading of the antiderivative for Mupad is implemented. If it can integrate the problem, it was assigned a B grade automatically as a placeholder. In the future, when grading function is implemented for Mupad, the tests will be rerun again.

The following is an example of using Matlab's symbolic toolbox (Mupad) to solve an integral

```
integrand = evalin(symengine,'cos(x)*sin(x)')
the_variable = evalin(symengine,'x')
anti = int(integrand,the_variable)
```

Which gives `sin(x)^2/2`

1.15 Current tree layout of integration tests

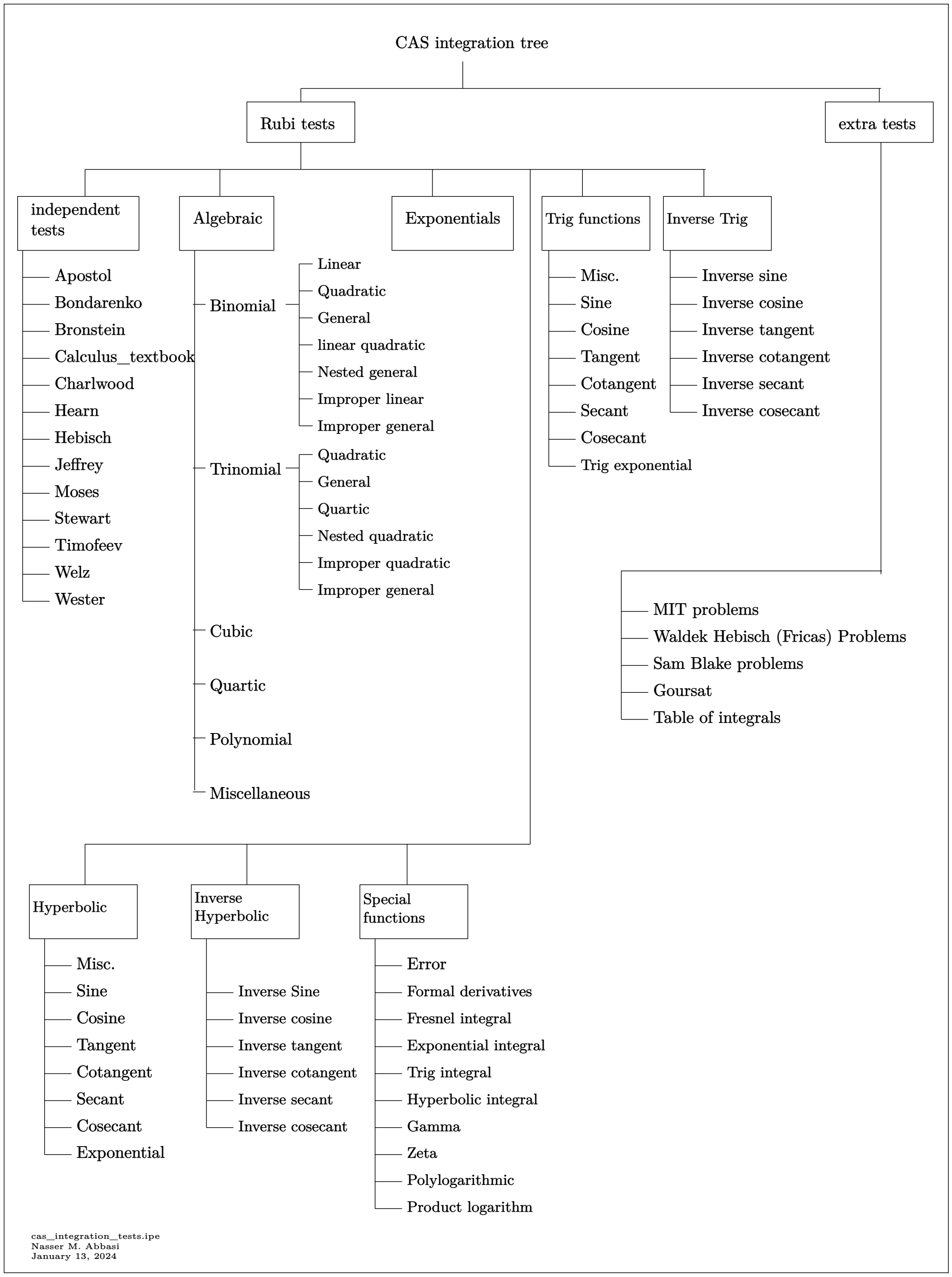
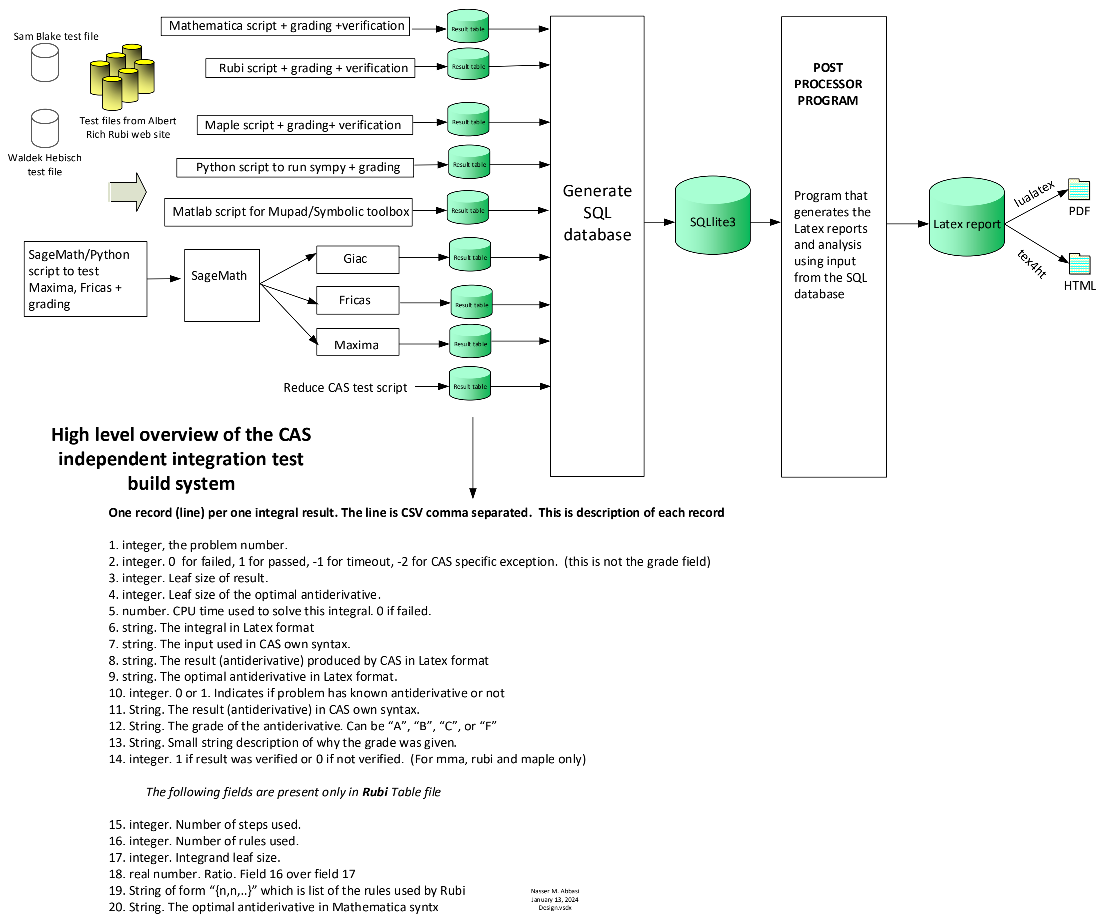


Figure 1.6: CAS integration tests tree

1.16 Design of the test system

The following diagram gives a high level view of the current test build system.



CHAPTER 2

DETAILED SUMMARY TABLES OF RESULTS

2.1 List of integrals sorted by grade for each CAS

Rubi

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 335, 336, 337, 338, 340, 341, 342, 343, 347, 348, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 367, 368, 372, 373, 374, 380, 381, 382, 383, 384, 385, 386, 387, 388, 389, 390, 392, 393, 394, 395, 396, 397, 398, 399, 400, 401, 402, 403, 404 }

B grade { 331, 332, 333, 334, 339, 344, 345, 346, 349, 361, 362, 363, 364, 365, 366, 369, 370, 371, 375, 376, 377, 378, 379, 391 }

C grade { }

F normal fail { }

F(-1) timedout fail { }

F(-2) exception fail { }

Mma

A grade { 2, 3, 4, 5, 6, 7, 8, 9, 10, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 28, 29, 30, 31, 32, 33, 34, 35, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 72, 73, 74, 75, 76, 77, 78, 79, 80, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 94, 95, 96, 97, 98, 99, 100, 101, 102, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 331, 332, 333, 334, 335, 336, 382, 383, 384, 385, 386, 387, 388, 389, 390, 391, 392, 393, 394, 395, 396, 397, 398, 399, 400, 401, 402, 403, 404 }

B grade { 1, 11, 12, 25, 26, 27, 36, 70, 71, 81, 93, 103 }

C grade { 330, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381 }

F normal fail { }

F(-1) timedout fail { }

F(-2) exception fail { }

Maple

A grade { 3, 4, 5, 6, 7, 8, 9, 10, 14, 15, 16, 17, 18, 19, 21, 22, 23, 24, 31, 32, 33, 34, 35, 38, 39, 40, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 73, 74, 75, 76, 77, 78, 79, 80, 82, 83, 84, 85, 86, 87, 88, 90, 91, 92, 95, 96, 97, 98, 99, 100, 101, 102, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 202, 203, 204, 206, 207, 208, 209, 211, 212, 213, 217, 218, 219, 220, 222, 223, 224, 225, 230, 231, 232, 233, 234, 235, 236, 237, 238, 244, 245, 246, 247, 250, 251, 252, 253, 254, 255, 260, 261, 262, 263, 264, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 297, 298, 299, 300, 301, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 328, 329, 384, 398, 399, 400, 403, 404 }

B grade { 1, 2, 11, 12, 13, 20, 25, 26, 27, 28, 29, 30, 36, 37, 41, 70, 71, 72, 81, 89, 93, 94, 103, 142, 200, 201, 205, 210, 214, 215, 216, 221, 226, 227, 228, 229, 239, 240, 241, 242, 243, 248, 249, 256, 257, 258, 259, 265, 266, 267, 295, 296, 302, 303, 304, 305, 306, 307, 326, 327, 382, 383, 397 }

C grade { }

F normal fail { 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 385, 386, 387, 388, 389, 390, 391, 392, 393, 394, 395, 396, 401, 402 }

F(-1) timedout fail { }

F(-2) exception fail { }

Fricas

A grade { 3, 4, 5, 6, 7, 8, 9, 10, 14, 15, 16, 17, 18, 19, 21, 22, 28, 32, 42, 43, 44, 45, 46, 47, 48, 54, 55, 56, 57, 63, 64, 65, 73, 74, 75, 76, 77, 78, 79, 80, 82, 83, 84, 85, 86, 87, 88, 90, 91, 92, 95, 96, 97, 99, 102, 105, 106, 107, 108, 109, 110, 111, 112, 113, 119, 120, 121, 122, 123, 131, 132, 133, 141, 143, 151, 152, 153, 154, 155, 159, 160, 161, 162, 163, 164, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 183, 184, 185, 186, 187, 188, 193, 200, 201, 202, 203, 204, 205, 206, 207, 210, 211, 212, 213, 214, 215, 216, 221, 222, 223, 224, 225, 226, 227, 228, 229, 234, 236, 237, 238, 239, 240, 241, 242, 243, 248, 249, 250, 252, 253, 254, 256, 257, 258, 259, 260, 262, 265, 266, 267, 269, 275, 276, 277, 278, 279, 280, 281, 282, 283, 287, 288, 289, 290, 291, 292, 293, 294, 297, 298, 299, 300, 302, 303, 304, 305, 308, 309, 310, 311, 312, 315, 316, 317, 318, 321, 322, 328, 329, 399, 400, 403, 404 }

B grade { 1, 2, 11, 12, 13, 20, 23, 24, 25, 26, 27, 29, 30, 31, 33, 34, 35, 36, 37, 38, 39, 40, 41, 49, 50, 51, 52, 53, 58, 59, 60, 61, 62, 66, 67, 68, 69, 70, 71, 72, 81, 89, 93, 94, 98, 100, 101, 103, 104, 114, 115, 116, 117, 118, 124, 125, 126, 127, 128, 129, 130, 134, 135, 136, 137, 138, 139, 140, 142, 144, 145, 146, 147, 148, 149, 150, 156, 157, 158, 165, 166, 167, 180, 181, 182, 189, 190, 191, 192, 194, 195, 196, 197, 198, 199, 208, 209, 217, 218, 219, 220, 230, 231, 232, 233, 235, 244, 245, 246, 247, 251, 255, 261, 263, 264, 268, 270, 271, 272, 273, 274, 284, 285, 286, 295, 296, 301, 306, 307, 313, 314, 319, 320, 323, 324, 325, 326, 327, 382, 383, 384, 397, 398 }

C grade { }

F normal fail { 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 385, 386, 387, 388, 389, 390, 391, 392, 393, 394, 395, 396, 401, 402 }

F(-1) timedout fail { }

F(-2) exception fail { }

Maxima

A grade { 3, 4, 5, 6, 7, 8, 9, 10, 14, 15, 16, 17, 18, 19, 21, 22, 28, 32, 33, 34, 35, 42, 43, 44, 45, 46, 47, 48, 52, 53, 54, 55, 56, 57, 61, 62, 63, 64, 65, 73, 74, 75, 76, 77, 78, 79, 80, 82, 83, 84, 85, 86, 87, 88, 90, 91, 92, 95, 96, 97, 99, 100, 101, 102, 105, 106, 107, 108, 109, 110, 111, 112, 113, 116, 117, 118, 119, 120, 121, 122, 123, 127, 128, 129, 130, 131, 132, 133, 138, 139, 140, 141, 143, 149, 150, 152, 153, 154, 155, 156, 157, 159, 160, 161, 162, 163, 164, 165, 167, 168, 169, 170, 171, 172, 275, 277, 278, 279, 280, 281, 282, 287, 288, 289, 290, 291, 297, 298, 299, 300, 301, 308, 309, 310, 311, 315, 316, 317, 321, 322, 323, 403, 404 }

B grade { 1, 2, 11, 12, 13, 20, 23, 24, 25, 26, 27, 29, 30, 31, 36, 37, 38, 39, 40, 41, 49, 50, 51, 58, 59, 60, 66, 67, 68, 69, 70, 71, 72, 81, 89, 93, 94, 98, 103, 104, 114, 115, 124, 125, 126, 134, 135, 136, 137, 142, 144, 145, 146, 147, 148, 151, 158, 166, 382, 383, 384 }

C grade { }

F normal fail { 283, 284, 285, 286, 292, 293, 294, 295, 296, 302, 303, 304, 305, 306, 307, 312, 313, 314, 318, 319, 320, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 385, 386, 387, 388, 389, 390, 391, 392, 393, 394, 395, 396, 397, 398, 399, 400, 401, 402 }

F(-1) timedout fail { }

F(-2) exception fail { 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 276 }

Giac

A grade { 3, 4, 5, 7, 8, 9, 10, 14, 15, 17, 18, 19, 21, 22, 23, 24, 32, 34, 35, 38, 39, 40, 42, 43, 44, 45, 46, 47, 48, 49, 52, 53, 54, 55, 56, 57, 61, 62, 63, 64, 65, 73, 74, 75, 77, 78, 79, 80, 83, 84, 86, 87, 88, 90, 91, 92, 96, 99, 100, 101, 102, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 116, 117, 118, 119, 120, 121, 122, 123, 124, 127, 128, 129, 130, 131, 132, 133, 134, 138, 139, 140, 141, 143, 144, 152, 153, 154, 155, 156, 160, 161, 162, 163, 164, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 184, 185, 186, 187, 188, 189, 192, 193, 194, 195, 196, 197, 198, 200, 201, 202, 203, 204, 205, 210, 211, 212, 213, 215, 221, 222, 223, 224, 226, 227, 234, 236, 238, 239, 248, 249, 250, 253, 261, 275, 277, 280, 281, 282, 283, 284, 285, 286, 292, 293, 294, 295, 296, 302, 303, 304, 305, 306, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 328, 329, 403, 404 }

B grade { 1, 2, 6, 11, 12, 13, 16, 20, 25, 26, 27, 28, 29, 30, 31, 33, 36, 37, 41, 50, 51, 58, 59, 60, 66, 67, 68, 69, 70, 71, 72, 76, 81, 82, 85, 89, 93, 94, 95, 97, 98, 103, 115, 125, 126, 135, 136, 137, 142, 145, 146, 147, 148, 149, 150, 151, 157, 158, 159, 165, 166, 167, 168, 182, 183, 190, 191, 199, 208, 214, 217, 225, 228, 235, 237, 240, 251, 263, 271, 273, 278, 279, 287, 288, 289, 290, 291, 297, 298, 299, 300, 301, 307, 327, 382, 383, 384 }

C grade { }

F normal fail { 262, 264, 270, 272, 274, 276, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 385, 386, 387, 388, 389, 390, 391, 392, 393, 394, 395, 396, 397, 398, 399, 400, 401, 402 }

F(-1) timedout fail { }

F(-2) exception fail { 206, 207, 209, 216, 218, 219, 220, 229, 230, 231, 232, 233, 241, 242, 243, 244, 245, 246, 247, 252, 254, 255, 256, 257, 258, 259, 260, 265, 266, 267, 268, 269 }

Mupad

A grade { }

B grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 202, 203, 206, 207, 208, 209, 211, 212, 217, 218, 219, 220, 223, 230, 231, 232, 233, 235, 244, 245, 246, 247, 250, 251, 252, 253, 254, 255, 260, 261, 262, 263, 264, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 287, 288, 289, 290, 291, 297, 298, 299, 300, 301, 308, 309, 310, 311, 315, 316, 317, 321, 322, 323, 382, 383, 384, 397, 398, 399, 400, 403, 404 }

C grade { }

F normal fail { }

F(-1) timedout fail { 148, 200, 201, 204, 205, 210, 213, 214, 215, 216, 221, 222, 224, 225, 226, 227, 228, 229, 234, 236, 237, 238, 239, 240, 241, 242, 243, 248, 249, 256, 257, 258, 259, 265, 266, 267, 283, 284, 285, 286, 292, 293, 294, 295, 296, 302, 303, 304, 305, 306, 307, 312, 313, 314, 318, 319, 320, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 385, 386, 387, 388, 389, 390, 391, 392, 393, 394, 395, 396, 401, 402 }

F(-2) exception fail { }

Sympy

A grade { 3, 4, 5, 6, 7, 8, 9, 14, 15, 17, 18, 19, 21, 32, 33, 34, 35, 42, 43, 44, 45, 46, 52, 53, 54, 55, 56, 61, 62, 63, 64, 73, 74, 75, 76, 77, 78, 79, 83, 84, 86, 87, 88, 90, 91, 96, 99, 100, 101, 102, 105, 107, 108, 109, 110, 111, 116, 117, 118, 119, 120, 121, 122, 127, 128, 129, 130, 131, 139, 140, 141, 143, 149, 150, 152, 153, 157, 158, 159, 160, 161, 165, 166, 167, 168, 169, 173, 175, 176, 177, 178, 179, 180, 213, 224, 225, 236, 277 }

B grade { 1, 2, 10, 11, 12, 13, 16, 20, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 36, 37, 38, 39, 40, 41, 47, 48, 49, 50, 51, 57, 58, 59, 60, 65, 66, 67, 68, 69, 70, 71, 72, 80, 81, 82, 85, 89, 92, 93, 94, 95, 97, 98, 103, 104, 106, 112, 113, 114, 115, 123, 124, 125, 126, 132, 133, 134, 135, 136, 137, 138, 142, 144, 145, 146, 147, 148, 151, 154, 155, 156, 162, 163, 164, 170, 171, 172, 200, 201, 202, 203, 210, 211, 212, 221, 222, 223, 234, 235, 248, 249, 250, 251, 382, 383, 384, 404 }

C grade { }

F normal fail { 186, 187, 188, 189, 190, 198, 199, 204, 205, 206, 207, 208, 209, 214, 215, 216, 217, 218, 219, 220, 226, 227, 228, 229, 230, 231, 232, 241, 242, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 280, 281, 282, 283, 284, 288, 289, 290, 291, 292, 299, 300, 301, 309, 310, 311, 312, 313, 314, 316, 317, 318, 319, 320, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 385, 386, 387, 391, 392, 393, 397, 398, 399, 400, 401 }

F(-1) timedout fail { 174, 181, 182, 183, 184, 185, 191, 192, 193, 194, 195, 196, 197, 233, 237, 238, 239, 240, 243, 244, 245, 246, 247, 278, 279, 285, 286, 287, 293, 294, 295, 296, 297, 298, 302, 303, 304, 305, 306, 307, 308, 315, 321, 322, 323, 389, 390, 394, 395, 396, 402 }

F(-2) exception fail { 388, 403 }

Reduce

A grade { }

B grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 382, 383, 384 }

C grade { }

F normal fail { 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 385, 386, 387, 388, 389, 390, 391, 392, 393, 394, 395, 396, 397, 398, 399, 400, 401, 402, 403, 404 }

F(-1) timedout fail { }

F(-2) exception fail { }

2.2 Detailed conclusion table per each integral for all CAS systems

Detailed conclusion table per each integral is given by the table below. The elapsed time is in seconds. For failed result it is given as **F(-1)** if the failure was due to timeout. It is given as **F(-2)** if the failure was due to an exception being raised, which could indicate a bug in the system. If the failure was due to integral not being evaluated within the time limit, then it is given as **F**.

In this table, the column `N.S.` means **normalized size** and is defined as $\frac{\text{antiderivative leaf size}}{\text{optimal antiderivative leaf size}}$. To make the table fit the page, the name `Mathematica` was abbreviated to `MMA`.

Problem 1	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	B	B	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	38	38	84	94	96	96	100	97	97	88
N.S.	1	1.00	2.21	2.47	2.53	2.53	2.63	2.55	2.55	2.32
time (sec)	N/A	0.298	0.018	0.635	0.036	0.066	0.027	0.151	0.235	5.470

Problem 2	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	B	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	38	38	67	70	69	69	73	72	73	65
N.S.	1	1.00	1.76	1.84	1.82	1.82	1.92	1.89	1.92	1.71
time (sec)	N/A	0.301	0.008	0.612	0.026	0.068	0.024	0.116	0.225	0.021

Problem 3	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	38	38	46	48	48	48	49	49	49	47
N.S.	1	1.00	1.21	1.26	1.26	1.26	1.29	1.29	1.29	1.24
time (sec)	N/A	0.320	0.007	0.452	0.034	0.066	0.020	0.135	0.207	0.028

Problem 4	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	28	28	32	25	24	24	26	24	25	25
N.S.	1	1.00	1.14	0.89	0.86	0.86	0.93	0.86	0.89	0.89
time (sec)	N/A	0.267	0.000	0.060	0.027	0.060	0.017	0.144	0.261	0.021

Problem 5	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	14	14	12	11	10	10	8	10	10	10
N.S.	1	1.00	0.86	0.79	0.71	0.71	0.57	0.71	0.71	0.71
time (sec)	N/A	0.239	0.001	0.671	0.040	0.062	0.024	0.149	0.220	0.010

Problem 6	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	25	26	25	24	20	117	28	26
N.S.	1	1.00	1.00	1.04	1.00	0.96	0.80	4.68	1.12	1.04
time (sec)	N/A	0.293	0.008	0.669	0.038	0.070	0.079	0.154	0.228	5.510

Problem 7	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	32	32	31	33	35	39	27	33	49	31
N.S.	1	1.00	0.97	1.03	1.09	1.22	0.84	1.03	1.53	0.97
time (sec)	N/A	0.307	0.010	0.664	0.035	0.069	0.094	0.166	0.221	0.033

Problem 8	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	28	28	26	25	38	38	39	24	37	39
N.S.	1	1.00	0.93	0.89	1.36	1.36	1.39	0.86	1.32	1.39
time (sec)	N/A	0.256	0.008	0.695	0.033	0.066	0.132	0.170	0.231	0.022

Problem 9	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	38	38	27	26	50	50	53	41	48	52
N.S.	1	1.00	0.71	0.68	1.32	1.32	1.39	1.08	1.26	1.37
time (sec)	N/A	0.311	0.008	0.658	0.037	0.069	0.190	0.169	0.225	5.535

Problem 10	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	38	38	27	26	61	61	65	25	59	63
N.S.	1	1.00	0.71	0.68	1.61	1.61	1.71	0.66	1.55	1.66
time (sec)	N/A	0.306	0.008	0.658	0.027	0.075	0.219	0.168	0.230	0.028

Problem 11	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	B	B	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	65	65	189	195	197	197	218	212	213	181
N.S.	1	1.00	2.91	3.00	3.03	3.03	3.35	3.26	3.28	2.78
time (sec)	N/A	0.477	0.026	1.211	0.032	0.067	0.041	0.171	0.243	5.583

Problem 12	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	B	B	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	65	65	148	157	156	156	168	170	172	144
N.S.	1	1.00	2.28	2.42	2.40	2.40	2.58	2.62	2.65	2.22
time (sec)	N/A	0.435	0.017	1.238	0.047	0.065	0.035	0.177	0.227	0.037

Problem 13	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	B	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	65	65	122	121	124	124	133	130	131	115
N.S.	1	1.00	1.88	1.86	1.91	1.91	2.05	2.00	2.02	1.77
time (sec)	N/A	0.401	0.012	1.082	0.026	0.064	0.033	0.160	0.229	5.536

Problem 14	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	65	77	79	69	72	81	87	89	90	74
N.S.	1	1.18	1.22	1.06	1.11	1.25	1.34	1.37	1.38	1.14
time (sec)	N/A	0.427	0.007	0.957	0.035	0.063	0.026	0.164	0.250	5.588

Problem 15	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	38	38	47	48	48	48	49	49	49	47
N.S.	1	1.00	1.24	1.26	1.26	1.26	1.29	1.29	1.29	1.24
time (sec)	N/A	0.322	0.008	1.386	0.034	0.066	0.038	0.154	0.218	0.027

Problem 16	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	14	14	14	13	20	20	19	84	21	20
N.S.	1	1.00	1.00	0.93	1.43	1.43	1.36	6.00	1.50	1.43
time (sec)	N/A	0.240	0.001	1.200	0.038	0.066	0.037	0.154	0.227	0.019

Problem 17	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	49	49	43	56	61	63	44	60	72	62
N.S.	1	1.00	0.88	1.14	1.24	1.29	0.90	1.22	1.47	1.27
time (sec)	N/A	0.334	0.013	1.244	0.031	0.069	0.130	0.132	0.228	0.042

Problem 18	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	51	51	47	63	67	92	60	65	112	71
N.S.	1	1.00	0.92	1.24	1.31	1.80	1.18	1.27	2.20	1.39
time (sec)	N/A	0.356	0.028	1.271	0.049	0.070	0.199	0.163	0.210	5.402

Problem 19	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	59	59	49	67	79	99	80	110	113	77
N.S.	1	1.00	0.83	1.14	1.34	1.68	1.36	1.86	1.92	1.31
time (sec)	N/A	0.375	0.017	1.238	0.035	0.069	0.240	0.162	0.221	0.041

Problem 20	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	B	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	28	28	53	60	84	84	88	59	69	80
N.S.	1	1.00	1.89	2.14	3.00	3.00	3.14	2.11	2.46	2.86
time (sec)	N/A	0.270	0.017	1.266	0.046	0.072	0.312	0.181	0.250	0.027

Problem 21	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	65	65	56	62	98	98	104	61	95	96
N.S.	1	1.00	0.86	0.95	1.51	1.51	1.60	0.94	1.46	1.48
time (sec)	N/A	0.369	0.015	1.214	0.035	0.072	0.424	0.143	0.223	5.408

Problem 22	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	65	65	57	62	109	109	116	61	106	107
N.S.	1	1.00	0.88	0.95	1.68	1.68	1.78	0.94	1.63	1.65
time (sec)	N/A	0.363	0.018	1.221	0.043	0.072	0.500	0.133	0.227	0.041

Problem 23	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	B	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	65	65	58	62	120	120	128	61	117	118
N.S.	1	1.00	0.89	0.95	1.85	1.85	1.97	0.94	1.80	1.82
time (sec)	N/A	0.361	0.018	1.223	0.035	0.072	0.637	0.140	0.244	5.948

Problem 24	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	B	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	65	65	57	62	131	131	139	61	128	129
N.S.	1	1.00	0.88	0.95	2.02	2.02	2.14	0.94	1.97	1.98
time (sec)	N/A	0.363	0.023	1.220	0.037	0.070	0.875	0.135	0.240	5.829

Problem 25	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	B	B	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	92	92	276	325	327	327	364	362	363	308
N.S.	1	1.00	3.00	3.53	3.55	3.55	3.96	3.93	3.95	3.35
time (sec)	N/A	0.622	0.076	1.188	0.043	0.076	0.056	0.140	0.236	5.866

Problem 26	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	B	B	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	92	92	235	273	277	277	308	303	305	261
N.S.	1	1.00	2.55	2.97	3.01	3.01	3.35	3.29	3.32	2.84
time (sec)	N/A	0.537	0.040	1.174	0.033	0.079	0.067	0.140	0.263	0.061

Problem 27	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	B	B	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	92	92	217	222	225	225	243	245	247	208
N.S.	1	1.00	2.36	2.41	2.45	2.45	2.64	2.66	2.68	2.26
time (sec)	N/A	0.502	0.019	1.105	0.031	0.072	0.062	0.138	0.219	5.722

Problem 28	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	A	A	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	92	104	161	172	140	167	190	188	189	152
N.S.	1	1.13	1.75	1.87	1.52	1.82	2.07	2.04	2.05	1.65
time (sec)	N/A	0.536	0.019	0.955	0.027	0.066	0.045	0.152	0.216	5.611

Problem 29	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	B	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	65	65	122	121	124	124	133	130	131	115
N.S.	1	1.00	1.88	1.86	1.91	1.91	2.05	2.00	2.02	1.77
time (sec)	N/A	0.422	0.010	1.364	0.034	0.072	0.084	0.132	0.249	5.580

Problem 30	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	B	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	38	38	67	73	69	69	73	155	73	65
N.S.	1	1.00	1.76	1.92	1.82	1.82	1.92	4.08	1.92	1.71
time (sec)	N/A	0.329	0.007	1.354	0.033	0.071	0.080	0.135	0.279	5.934

Problem 31	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	14	14	14	13	31	31	32	31	32	31
N.S.	1	1.00	1.00	0.93	2.21	2.21	2.29	2.21	2.29	2.21
time (sec)	N/A	0.252	0.001	1.212	0.030	0.065	0.077	0.162	0.353	0.025

Problem 32	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	73	73	74	109	114	116	83	115	132	118
N.S.	1	1.00	1.01	1.49	1.56	1.59	1.14	1.58	1.81	1.62
time (sec)	N/A	0.378	0.022	1.227	0.028	0.091	0.270	0.157	0.242	0.034

Problem 33	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	75	75	72	109	118	173	102	167	197	123
N.S.	1	1.00	0.96	1.45	1.57	2.31	1.36	2.23	2.63	1.64
time (sec)	N/A	0.424	0.042	1.250	0.035	0.072	0.433	0.141	0.209	0.041

Problem 34	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	78	78	114	114	125	188	128	112	209	130
N.S.	1	1.00	1.46	1.46	1.60	2.41	1.64	1.44	2.68	1.67
time (sec)	N/A	0.412	0.028	1.322	0.035	0.078	0.459	0.157	0.250	0.060

Problem 35	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	86	86	80	115	142	176	148	118	180	138
N.S.	1	1.00	0.93	1.34	1.65	2.05	1.72	1.37	2.09	1.60
time (sec)	N/A	0.435	0.029	1.264	0.044	0.074	0.602	0.159	0.221	5.295

Problem 36	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	B	B	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	28	28	91	104	143	143	155	111	122	135
N.S.	1	1.00	3.25	3.71	5.11	5.11	5.54	3.96	4.36	4.82
time (sec)	N/A	0.264	0.022	1.262	0.037	0.074	0.940	0.147	0.208	0.037

Problem 37	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	B	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	58	90	97	110	160	160	172	114	159	154
N.S.	1	1.55	1.67	1.90	2.76	2.76	2.97	1.97	2.74	2.66
time (sec)	N/A	0.422	0.024	1.292	0.051	0.075	1.554	0.152	0.235	5.381

Problem 38	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	B	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	92	92	97	110	171	171	184	114	170	165
N.S.	1	1.00	1.05	1.20	1.86	1.86	2.00	1.24	1.85	1.79
time (sec)	N/A	0.460	0.025	1.272	0.036	0.073	3.025	0.142	0.247	5.398

Problem 39	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	B	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	92	92	97	110	182	182	196	114	181	176
N.S.	1	1.00	1.05	1.20	1.98	1.98	2.13	1.24	1.97	1.91
time (sec)	N/A	0.409	0.022	1.263	0.048	0.079	5.788	0.158	0.230	5.439

Problem 40	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	B	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	92	92	97	110	193	193	207	114	192	187
N.S.	1	1.00	1.05	1.20	2.10	2.10	2.25	1.24	2.09	2.03
time (sec)	N/A	0.421	0.029	1.269	0.038	0.073	11.366	0.150	0.221	0.061

Problem 41	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	B	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	122	122	167	244	258	259	209	273	301	280
N.S.	1	1.00	1.37	2.00	2.11	2.12	1.71	2.24	2.47	2.30
time (sec)	N/A	0.477	0.049	1.534	0.041	0.089	0.271	0.133	0.210	5.386

Problem 42	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	98	98	115	168	177	179	136	184	208	189
N.S.	1	1.00	1.17	1.71	1.81	1.83	1.39	1.88	2.12	1.93
time (sec)	N/A	0.432	0.041	1.627	0.041	0.089	0.211	0.116	0.249	0.029

Problem 43	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	74	74	74	107	114	115	83	116	132	118
N.S.	1	1.00	1.00	1.45	1.54	1.55	1.12	1.57	1.78	1.59
time (sec)	N/A	0.390	0.034	1.464	0.041	0.075	0.166	0.148	0.215	0.032

Problem 44	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	49	50	43	56	60	62	44	60	72	62
N.S.	1	1.02	0.88	1.14	1.22	1.27	0.90	1.22	1.47	1.27
time (sec)	N/A	0.352	0.022	1.376	0.038	0.074	0.129	0.126	0.221	0.039

Problem 45	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	26	26	25	26	26	25	20	27	28	25
N.S.	1	1.00	0.96	1.00	1.00	0.96	0.77	1.04	1.08	0.96
time (sec)	N/A	0.302	0.011	1.454	0.037	0.080	0.078	0.126	0.217	0.026

Problem 46	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	10	10	10	11	10	10	7	11	10	10
N.S.	1	1.00	1.00	1.10	1.00	1.00	0.70	1.10	1.00	1.00
time (sec)	N/A	0.240	0.001	1.322	0.030	0.068	0.024	0.139	0.258	0.013

Problem 47	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	36	51	26	28	36	26	128	46	26	40
N.S.	1	1.42	0.72	0.78	1.00	0.72	3.56	1.28	0.72	1.11
time (sec)	N/A	0.349	0.015	1.309	0.032	0.076	0.158	0.132	0.214	0.042

Problem 48	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	57	57	53	57	92	93	233	94	115	76
N.S.	1	1.00	0.93	1.00	1.61	1.63	4.09	1.65	2.02	1.33
time (sec)	N/A	0.367	0.043	1.691	0.029	0.072	0.357	0.129	0.213	5.393

Problem 49	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	B	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	82	82	67	81	202	242	381	145	288	182
N.S.	1	1.00	0.82	0.99	2.46	2.95	4.65	1.77	3.51	2.22
time (sec)	N/A	0.416	0.041	1.725	0.036	0.078	0.557	0.114	0.204	5.657

Problem 50	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	107	107	107	103	361	425	570	243	499	312
N.S.	1	1.00	1.00	0.96	3.37	3.97	5.33	2.27	4.66	2.92
time (sec)	N/A	0.460	0.053	1.725	0.048	0.082	0.769	0.159	0.256	5.671

Problem 51	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	130	130	130	125	558	657	802	338	769	505
N.S.	1	1.00	1.00	0.96	4.29	5.05	6.17	2.60	5.92	3.88
time (sec)	N/A	0.529	0.032	1.736	0.059	0.089	1.038	0.141	0.225	5.703

Problem 52	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	104	104	165	175	183	267	155	188	297	203
N.S.	1	1.00	1.59	1.68	1.76	2.57	1.49	1.81	2.86	1.95
time (sec)	N/A	0.504	0.042	1.457	0.033	0.076	0.536	0.148	0.225	0.042

Problem 53	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	75	75	114	108	117	172	102	118	197	123
N.S.	1	1.00	1.52	1.44	1.56	2.29	1.36	1.57	2.63	1.64
time (sec)	N/A	0.414	0.024	1.398	0.029	0.079	0.379	0.121	0.226	5.828

Problem 54	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	51	51	47	63	67	92	60	65	112	71
N.S.	1	1.00	0.92	1.24	1.31	1.80	1.18	1.27	2.20	1.39
time (sec)	N/A	0.370	0.025	1.457	0.046	0.074	0.327	0.109	0.226	0.049

Problem 55	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	31	31	31	33	34	37	27	32	49	32
N.S.	1	1.00	1.00	1.06	1.10	1.19	0.87	1.03	1.58	1.03
time (sec)	N/A	0.324	0.008	1.414	0.031	0.080	0.123	0.131	0.221	0.031

Problem 56	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	12	12	12	13	13	13	10	12	12	12
N.S.	1	1.00	1.00	1.08	1.08	1.08	0.83	1.00	1.00	1.00
time (sec)	N/A	0.257	0.002	1.427	0.031	0.077	0.080	0.120	0.215	0.022

Problem 57	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	56	56	53	58	90	92	233	93	115	77
N.S.	1	1.00	0.95	1.04	1.61	1.64	4.16	1.66	2.05	1.38
time (sec)	N/A	0.362	0.021	1.290	0.045	0.085	0.343	0.136	0.223	5.839

Problem 58	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	81	106	66	82	208	241	406	166	459	182
N.S.	1	1.31	0.81	1.01	2.57	2.98	5.01	2.05	5.67	2.25
time (sec)	N/A	0.488	0.044	1.396	0.029	0.085	0.559	0.124	0.246	0.145

Problem 59	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	109	109	98	109	386	494	634	253	821	330
N.S.	1	1.00	0.90	1.00	3.54	4.53	5.82	2.32	7.53	3.03
time (sec)	N/A	0.474	0.062	1.683	0.045	0.087	0.836	0.134	0.219	5.850

Problem 60	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	133	133	230	254	271	416	258	264	470	291
N.S.	1	1.00	1.73	1.91	2.04	3.13	1.94	1.98	3.53	2.19
time (sec)	N/A	0.566	0.053	1.507	0.042	0.073	1.045	0.124	0.222	6.014

Problem 61	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	103	103	167	172	191	291	185	183	330	196
N.S.	1	1.00	1.62	1.67	1.85	2.83	1.80	1.78	3.20	1.90
time (sec)	N/A	0.487	0.036	1.421	0.039	0.080	0.653	0.110	0.205	0.055

Problem 62	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	78	78	114	114	125	188	128	112	209	130
N.S.	1	1.00	1.46	1.46	1.60	2.41	1.64	1.44	2.68	1.67
time (sec)	N/A	0.417	0.031	1.410	0.040	0.075	0.449	0.113	0.233	5.779

Problem 63	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	59	59	48	66	80	100	80	69	113	77
N.S.	1	1.00	0.81	1.12	1.36	1.69	1.36	1.17	1.92	1.31
time (sec)	N/A	0.374	0.018	1.398	0.040	0.074	0.248	0.114	0.242	6.032

Problem 64	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	28	28	26	25	38	38	39	24	37	39
N.S.	1	1.00	0.93	0.89	1.36	1.36	1.39	0.86	1.32	1.39
time (sec)	N/A	0.263	0.007	1.411	0.027	0.069	0.145	0.141	0.219	0.021

Problem 65	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	14	14	14	13	24	24	26	12	23	26
N.S.	1	1.00	1.00	0.93	1.71	1.71	1.86	0.86	1.64	1.86
time (sec)	N/A	0.248	0.002	1.388	0.040	0.065	0.096	0.152	0.226	0.020

Problem 66	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	82	82	67	81	202	242	381	165	289	183
N.S.	1	1.00	0.82	0.99	2.46	2.95	4.65	2.01	3.52	2.23
time (sec)	N/A	0.418	0.033	1.400	0.041	0.080	0.538	0.121	0.244	6.385

Problem 67	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	110	110	97	108	386	495	632	254	821	329
N.S.	1	1.00	0.88	0.98	3.51	4.50	5.75	2.31	7.46	2.99
time (sec)	N/A	0.476	0.064	1.390	0.049	0.101	0.839	0.114	0.222	6.116

Problem 68	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	143	160	128	140	594	760	881	345	1202	542
N.S.	1	1.12	0.90	0.98	4.15	5.31	6.16	2.41	8.41	3.79
time (sec)	N/A	0.615	0.070	1.245	0.045	0.101	1.171	0.110	0.223	6.455

Problem 69	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	170	170	154	165	889	1151	1217	458	1840	797
N.S.	1	1.00	0.91	0.97	5.23	6.77	7.16	2.69	10.82	4.69
time (sec)	N/A	0.663	0.149	1.602	0.074	0.101	1.667	0.116	0.228	6.337

Problem 70	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	B	B	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	39	39	117	123	121	121	136	127	127	122
N.S.	1	1.00	3.00	3.15	3.10	3.10	3.49	3.26	3.26	3.13
time (sec)	N/A	0.326	0.026	0.566	0.038	0.073	0.038	0.129	0.248	0.036

Problem 71	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	B	B	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	39	39	95	100	102	102	107	103	103	99
N.S.	1	1.00	2.44	2.56	2.62	2.62	2.74	2.64	2.64	2.54
time (sec)	N/A	0.323	0.014	0.612	0.029	0.072	0.027	0.146	0.218	5.746

Problem 72	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	B	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	39	39	73	76	75	75	80	78	79	75
N.S.	1	1.00	1.87	1.95	1.92	1.92	2.05	2.00	2.03	1.92
time (sec)	N/A	0.316	0.015	0.553	0.033	0.076	0.023	0.143	0.207	5.521

Problem 73	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	39	39	51	54	54	54	56	55	55	53
N.S.	1	1.00	1.31	1.38	1.38	1.38	1.44	1.41	1.41	1.36
time (sec)	N/A	0.332	0.013	0.415	0.025	0.079	0.020	0.125	0.208	0.028

Problem 74	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	34	34	38	31	30	30	32	30	31	31
N.S.	1	1.00	1.12	0.91	0.88	0.88	0.94	0.88	0.91	0.91
time (sec)	N/A	0.276	0.000	0.082	0.028	0.081	0.017	0.136	0.215	0.022

Problem 75	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	20	20	14	13	12	12	12	12	12	12
N.S.	1	1.00	0.70	0.65	0.60	0.60	0.60	0.60	0.60	0.60
time (sec)	N/A	0.262	0.001	0.612	0.031	0.084	0.023	0.116	0.256	0.012

Problem 76	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	26	26	30	31	31	30	26	119	33	30
N.S.	1	1.00	1.15	1.19	1.19	1.15	1.00	4.58	1.27	1.15
time (sec)	N/A	0.307	0.011	0.629	0.039	0.086	0.077	0.120	0.222	0.031

Problem 77	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	33	33	36	38	39	44	32	37	53	37
N.S.	1	1.00	1.09	1.15	1.18	1.33	0.97	1.12	1.61	1.12
time (sec)	N/A	0.324	0.010	0.613	0.041	0.084	0.103	0.105	0.206	0.034

Problem 78	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	35	35	29	30	43	43	44	29	31	30
N.S.	1	1.00	0.83	0.86	1.23	1.23	1.26	0.83	0.89	0.86
time (sec)	N/A	0.273	0.010	0.615	0.032	0.071	0.148	0.108	0.235	5.756

Problem 79	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	39	39	30	31	55	55	58	41	53	57
N.S.	1	1.00	0.77	0.79	1.41	1.41	1.49	1.05	1.36	1.46
time (sec)	N/A	0.319	0.009	0.621	0.036	0.085	0.186	0.143	0.217	0.025

Problem 80	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	39	39	30	31	66	66	70	30	64	68
N.S.	1	1.00	0.77	0.79	1.69	1.69	1.79	0.77	1.64	1.74
time (sec)	N/A	0.325	0.010	0.625	0.030	0.073	0.243	0.124	0.267	0.031

Problem 81	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	B	B	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	77	77	160	173	172	172	185	186	188	168
N.S.	1	1.00	2.08	2.25	2.23	2.23	2.40	2.42	2.44	2.18
time (sec)	N/A	0.513	0.024	1.246	0.027	0.080	0.038	0.159	0.270	0.042

Problem 82	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	77	77	120	137	140	140	150	146	147	135
N.S.	1	1.00	1.56	1.78	1.82	1.82	1.95	1.90	1.91	1.75
time (sec)	N/A	0.483	0.026	1.271	0.032	0.073	0.033	0.105	0.270	5.569

Problem 83	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	77	97	87	93	93	97	104	105	106	99
N.S.	1	1.26	1.13	1.21	1.21	1.26	1.35	1.36	1.38	1.29
time (sec)	N/A	0.475	0.022	1.075	0.031	0.085	0.028	0.135	0.250	0.026

Problem 84	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	54	54	54	64	64	64	66	65	65	63
N.S.	1	1.00	1.00	1.19	1.19	1.19	1.22	1.20	1.20	1.17
time (sec)	N/A	0.380	0.010	1.444	0.034	0.083	0.043	0.142	0.201	5.713

Problem 85	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	20	20	20	19	28	28	29	97	29	28
N.S.	1	1.00	1.00	0.95	1.40	1.40	1.45	4.85	1.45	1.40
time (sec)	N/A	0.263	0.002	1.347	0.026	0.089	0.043	0.119	0.210	0.024

Problem 86	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	61	62	52	66	72	72	53	72	82	69
N.S.	1	1.02	0.85	1.08	1.18	1.18	0.87	1.18	1.34	1.13
time (sec)	N/A	0.362	0.017	1.388	0.035	0.096	0.131	0.152	0.209	6.073

Problem 87	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	63	63	59	75	79	108	71	77	120	83
N.S.	1	1.00	0.94	1.19	1.25	1.71	1.13	1.22	1.90	1.32
time (sec)	N/A	0.389	0.030	1.368	0.029	0.076	0.203	0.137	0.244	0.054

Problem 88	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	71	71	59	78	90	112	90	120	113	89
N.S.	1	1.00	0.83	1.10	1.27	1.58	1.27	1.69	1.59	1.25
time (sec)	N/A	0.402	0.024	1.350	0.042	0.072	0.272	0.133	0.215	5.597

Problem 89	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	B	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	35	35	59	70	94	94	99	69	66	65
N.S.	1	1.00	1.69	2.00	2.69	2.69	2.83	1.97	1.89	1.86
time (sec)	N/A	0.266	0.025	1.401	0.042	0.071	0.380	0.141	0.220	5.526

Problem 90	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	77	77	61	72	108	108	114	71	105	85
N.S.	1	1.00	0.79	0.94	1.40	1.40	1.48	0.92	1.36	1.10
time (sec)	N/A	0.379	0.025	1.553	0.039	0.083	0.542	0.121	0.201	0.036

Problem 91	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	77	77	61	72	119	119	126	71	116	119
N.S.	1	1.00	0.79	0.94	1.55	1.55	1.64	0.92	1.51	1.55
time (sec)	N/A	0.389	0.022	1.310	0.046	0.080	0.800	0.133	0.235	5.557

Problem 92	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	77	77	61	72	130	130	138	71	127	130
N.S.	1	1.00	0.79	0.94	1.69	1.69	1.79	0.92	1.65	1.69
time (sec)	N/A	0.386	0.022	1.345	0.046	0.082	1.423	0.109	0.213	0.050

Problem 93	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	B	B	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	111	111	255	299	303	303	335	329	331	295
N.S.	1	1.00	2.30	2.69	2.73	2.73	3.02	2.96	2.98	2.66
time (sec)	N/A	0.639	0.050	1.325	0.037	0.070	0.048	0.137	0.238	5.453

Problem 94	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	B	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	111	111	211	248	251	251	270	271	273	242
N.S.	1	1.00	1.90	2.23	2.26	2.26	2.43	2.44	2.46	2.18
time (sec)	N/A	0.559	0.060	1.338	0.032	0.070	0.044	0.137	0.253	5.603

Problem 95	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	111	131	167	198	183	193	218	214	215	186
N.S.	1	1.18	1.50	1.78	1.65	1.74	1.96	1.93	1.94	1.68
time (sec)	N/A	0.586	0.044	1.161	0.030	0.071	0.039	0.140	0.237	0.044

Problem 96	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	91	91	123	147	150	150	160	156	157	145
N.S.	1	1.00	1.35	1.62	1.65	1.65	1.76	1.71	1.73	1.59
time (sec)	N/A	0.472	0.026	1.560	0.032	0.072	0.061	0.146	0.232	0.033

Problem 97	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	54	54	79	99	95	95	100	174	99	91
N.S.	1	1.00	1.46	1.83	1.76	1.76	1.85	3.22	1.83	1.69
time (sec)	N/A	0.343	0.019	1.533	0.028	0.074	0.064	0.132	0.215	5.485

Problem 98	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	20	20	20	19	45	45	49	45	46	45
N.S.	1	1.00	1.00	0.95	2.25	2.25	2.45	2.25	2.30	2.25
time (sec)	N/A	0.253	0.002	1.414	0.028	0.079	0.053	0.110	0.240	0.031

Problem 99	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	89	89	85	124	131	130	95	133	147	128
N.S.	1	1.00	0.96	1.39	1.47	1.46	1.07	1.49	1.65	1.44
time (sec)	N/A	0.394	0.025	1.369	0.035	0.083	0.197	0.139	0.233	0.037

Problem 100	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	94	94	129	125	136	193	117	182	207	141
N.S.	1	1.00	1.37	1.33	1.45	2.05	1.24	1.94	2.20	1.50
time (sec)	N/A	0.444	0.030	1.382	0.030	0.086	0.315	0.117	0.225	5.421

Problem 101	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	97	97	129	133	142	209	144	129	215	149
N.S.	1	1.00	1.33	1.37	1.46	2.15	1.48	1.33	2.22	1.54
time (sec)	N/A	0.446	0.038	1.364	0.034	0.077	0.520	0.149	0.219	5.399

Problem 102	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	105	105	92	133	158	194	163	136	181	157
N.S.	1	1.00	0.88	1.27	1.50	1.85	1.55	1.30	1.72	1.50
time (sec)	N/A	0.452	0.033	1.325	0.036	0.093	1.150	0.135	0.239	5.364

Problem 103	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	B	B	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	35	35	100	123	158	158	170	126	125	102
N.S.	1	1.00	2.86	3.51	4.51	4.51	4.86	3.60	3.57	2.91
time (sec)	N/A	0.272	0.027	1.310	0.056	0.081	2.969	0.145	0.254	0.049

Problem 104	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	B	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	73	108	103	129	175	175	187	129	174	135
N.S.	1	1.48	1.41	1.77	2.40	2.40	2.56	1.77	2.38	1.85
time (sec)	N/A	0.305	0.036	1.337	0.036	0.079	4.613	0.109	0.217	0.049

Problem 105	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	111	111	103	129	186	186	199	129	185	184
N.S.	1	1.00	0.93	1.16	1.68	1.68	1.79	1.16	1.67	1.66
time (sec)	N/A	0.284	0.027	1.397	0.044	0.087	21.631	0.125	0.216	5.697

Problem 106	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	111	111	103	129	197	197	211	129	196	195
N.S.	1	1.00	0.93	1.16	1.77	1.77	1.90	1.16	1.77	1.76
time (sec)	N/A	0.283	0.028	1.410	0.041	0.087	50.320	0.146	0.282	0.053

Problem 107	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	131	131	134	201	205	207	153	210	246	217
N.S.	1	1.00	1.02	1.53	1.56	1.58	1.17	1.60	1.88	1.66
time (sec)	N/A	0.296	0.043	1.734	0.038	0.076	0.254	0.130	0.217	0.038

Problem 108	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	100	100	91	132	135	137	99	136	162	138
N.S.	1	1.00	0.91	1.32	1.35	1.37	0.99	1.36	1.62	1.38
time (sec)	N/A	0.258	0.026	1.576	0.028	0.088	0.174	0.110	0.209	5.364

Problem 109	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	69	69	58	74	77	79	58	76	94	77
N.S.	1	1.00	0.84	1.07	1.12	1.14	0.84	1.10	1.36	1.12
time (sec)	N/A	0.235	0.019	1.578	0.035	0.081	0.131	0.107	0.210	5.483

Problem 110	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	38	38	35	39	38	35	32	39	42	39
N.S.	1	1.00	0.92	1.03	1.00	0.92	0.84	1.03	1.11	1.03
time (sec)	N/A	0.210	0.008	1.580	0.026	0.075	0.084	0.146	0.203	0.033

Problem 111	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	16	17	16	16	12	17	16	16
N.S.	1	1.00	1.00	1.06	1.00	1.00	0.75	1.06	1.00	1.00
time (sec)	N/A	0.159	0.001	1.478	0.046	0.074	0.024	0.113	0.226	5.244

Problem 112	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	47	69	33	34	47	33	172	57	33	51
N.S.	1	1.47	0.70	0.72	1.00	0.70	3.66	1.21	0.70	1.09
time (sec)	N/A	0.245	0.012	1.499	0.031	0.074	0.163	0.111	0.234	5.370

Problem 113	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	73	73	66	75	107	109	301	111	131	95
N.S.	1	1.00	0.90	1.03	1.47	1.49	4.12	1.52	1.79	1.30
time (sec)	N/A	0.268	0.022	1.763	0.039	0.081	0.374	0.126	0.220	5.262

Problem 114	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	B	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	108	108	102	107	228	266	471	158	307	220
N.S.	1	1.00	0.94	0.99	2.11	2.46	4.36	1.46	2.84	2.04
time (sec)	N/A	0.310	0.040	1.773	0.044	0.080	0.560	0.145	0.196	0.102

Problem 115	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	139	139	135	137	393	454	672	275	523	359
N.S.	1	1.00	0.97	0.99	2.83	3.27	4.83	1.98	3.76	2.58
time (sec)	N/A	0.347	0.050	1.829	0.050	0.086	0.803	0.117	0.234	5.277

Problem 116	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	219	219	339	402	398	545	345	418	631	625
N.S.	1	1.00	1.55	1.84	1.82	2.49	1.58	1.91	2.88	2.85
time (sec)	N/A	0.521	0.084	1.556	0.041	0.077	0.922	0.173	0.230	5.361

Problem 117	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	184	184	263	298	300	417	264	312	486	387
N.S.	1	1.00	1.43	1.62	1.63	2.27	1.43	1.70	2.64	2.10
time (sec)	N/A	0.441	0.066	1.575	0.043	0.078	0.597	0.118	0.214	0.050

Problem 118	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	145	145	196	208	214	305	185	222	357	242
N.S.	1	1.00	1.35	1.43	1.48	2.10	1.28	1.53	2.46	1.67
time (sec)	N/A	0.374	0.049	1.625	0.040	0.075	0.443	0.139	0.225	0.047

Problem 119	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	105	105	142	137	143	205	131	146	246	152
N.S.	1	1.00	1.35	1.30	1.36	1.95	1.25	1.39	2.34	1.45
time (sec)	N/A	0.308	0.037	1.664	0.030	0.082	0.319	0.150	0.244	5.413

Problem 120	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	74	74	65	86	89	117	85	86	150	96
N.S.	1	1.00	0.88	1.16	1.20	1.58	1.15	1.16	2.03	1.30
time (sec)	N/A	0.260	0.033	1.586	0.039	0.074	0.209	0.134	0.222	5.407

Problem 121	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	48	48	47	49	52	56	46	49	74	47
N.S.	1	1.00	0.98	1.02	1.08	1.17	0.96	1.02	1.54	0.98
time (sec)	N/A	0.237	0.012	1.556	0.044	0.088	0.108	0.131	0.208	5.263

Problem 122	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	18	18	18	19	18	18	17	18	18	18
N.S.	1	1.00	1.00	1.06	1.00	1.00	0.94	1.00	1.00	1.00
time (sec)	N/A	0.167	0.004	1.569	0.030	0.068	0.070	0.128	0.200	5.258

Problem 123	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	75	75	74	75	113	116	287	112	149	96
N.S.	1	1.00	0.99	1.00	1.51	1.55	3.83	1.49	1.99	1.28
time (sec)	N/A	0.244	0.030	1.595	0.028	0.079	0.386	0.157	0.248	0.072

Problem 124	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	B	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	106	143	86	107	236	277	486	195	523	223
N.S.	1	1.35	0.81	1.01	2.23	2.61	4.58	1.84	4.93	2.10
time (sec)	N/A	0.354	0.070	1.644	0.035	0.078	0.591	0.134	0.266	0.153

Problem 125	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	146	146	130	144	423	544	734	289	897	381
N.S.	1	1.00	0.89	0.99	2.90	3.73	5.03	1.98	6.14	2.61
time (sec)	N/A	0.345	0.065	1.952	0.042	0.089	0.910	0.122	0.216	5.414

Problem 126	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	176	176	160	174	641	807	996	480	1331	595
N.S.	1	1.00	0.91	0.99	3.64	4.59	5.66	2.73	7.56	3.38
time (sec)	N/A	0.417	0.093	2.014	0.054	0.088	1.301	0.121	0.228	5.815

Problem 127	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	221	221	337	399	408	606	386	403	716	516
N.S.	1	1.00	1.52	1.81	1.85	2.74	1.75	1.82	3.24	2.33
time (sec)	N/A	0.528	0.098	1.609	0.038	0.088	4.716	0.146	0.226	5.282

Problem 128	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	185	185	262	297	310	469	303	301	547	341
N.S.	1	1.00	1.42	1.61	1.68	2.54	1.64	1.63	2.96	1.84
time (sec)	N/A	0.608	0.066	1.658	0.046	0.084	3.049	0.119	0.221	0.064

Problem 129	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	142	142	191	211	224	338	226	214	393	232
N.S.	1	1.00	1.35	1.49	1.58	2.38	1.59	1.51	2.77	1.63
time (sec)	N/A	0.570	0.053	1.589	0.051	0.081	1.205	0.125	0.233	5.272

Problem 130	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	111	111	139	147	156	227	163	136	258	163
N.S.	1	1.00	1.25	1.32	1.41	2.05	1.47	1.23	2.32	1.47
time (sec)	N/A	0.484	0.063	1.729	0.036	0.086	0.509	0.139	0.216	0.077

Problem 131	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	85	85	65	90	105	126	109	89	142	106
N.S.	1	1.00	0.76	1.06	1.24	1.48	1.28	1.05	1.67	1.25
time (sec)	N/A	0.421	0.053	1.598	0.035	0.078	0.286	0.161	0.291	0.056

Problem 132	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	35	35	35	36	56	56	60	35	44	43
N.S.	1	1.00	1.00	1.03	1.60	1.60	1.71	1.00	1.26	1.23
time (sec)	N/A	0.276	0.012	1.666	0.027	0.071	0.168	0.121	0.275	5.164

Problem 133	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	20	20	20	19	35	35	39	18	35	37
N.S.	1	1.00	1.00	0.95	1.75	1.75	1.95	0.90	1.75	1.85
time (sec)	N/A	0.247	0.003	1.582	0.036	0.070	0.122	0.109	0.314	0.018

Problem 134	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	B	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	107	107	83	106	238	280	457	190	333	225
N.S.	1	1.00	0.78	0.99	2.22	2.62	4.27	1.78	3.11	2.10
time (sec)	N/A	0.458	0.061	1.674	0.036	0.081	0.812	0.134	0.227	5.212

Problem 135	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	142	142	127	142	429	555	736	286	911	392
N.S.	1	1.00	0.89	1.00	3.02	3.91	5.18	2.01	6.42	2.76
time (sec)	N/A	0.534	0.078	1.527	0.048	0.084	1.205	0.156	0.250	0.160

Problem 136	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	189	214	168	186	642	828	1001	386	1309	616
N.S.	1	1.13	0.89	0.98	3.40	4.38	5.30	2.04	6.93	3.26
time (sec)	N/A	0.745	0.097	1.572	0.043	0.094	1.401	0.114	0.246	5.827

Problem 137	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	223	223	201	218	947	1222	1357	503	1970	878
N.S.	1	1.00	0.90	0.98	4.25	5.48	6.09	2.26	8.83	3.94
time (sec)	N/A	0.774	0.143	1.806	0.075	0.106	2.282	0.143	0.222	5.680

Problem 138	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	217	217	335	399	424	644	425	398	749	452
N.S.	1	1.00	1.54	1.84	1.95	2.97	1.96	1.83	3.45	2.08
time (sec)	N/A	0.813	0.107	1.602	0.045	0.084	71.723	0.114	0.224	5.259

Problem 139	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	179	179	259	300	326	488	337	299	556	331
N.S.	1	1.00	1.45	1.68	1.82	2.73	1.88	1.67	3.11	1.85
time (sec)	N/A	0.670	0.066	1.587	0.044	0.089	25.011	0.141	0.257	0.082

Problem 140	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	146	146	194	216	243	347	257	206	379	246
N.S.	1	1.00	1.33	1.48	1.66	2.38	1.76	1.41	2.60	1.68
time (sec)	N/A	0.579	0.056	1.530	0.040	0.079	6.345	0.119	0.221	5.275

Problem 141	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	122	122	99	145	179	217	189	142	223	178
N.S.	1	1.00	0.81	1.19	1.47	1.78	1.55	1.16	1.83	1.46
time (sec)	N/A	0.490	0.039	1.491	0.038	0.084	1.405	0.143	0.244	5.283

Problem 142	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	B	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	35	35	65	76	113	113	121	75	87	81
N.S.	1	1.00	1.86	2.17	3.23	3.23	3.46	2.14	2.49	2.31
time (sec)	N/A	0.266	0.025	1.511	0.037	0.080	0.411	0.138	0.214	0.038

Problem 143	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	54	54	37	37	74	74	80	36	71	77
N.S.	1	1.00	0.69	0.69	1.37	1.37	1.48	0.67	1.31	1.43
time (sec)	N/A	0.341	0.015	1.506	0.035	0.079	0.235	0.140	0.237	5.198

Problem 144	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	B	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	20	20	20	19	52	52	58	18	52	54
N.S.	1	1.00	1.00	0.95	2.60	2.60	2.90	0.90	2.60	2.70
time (sec)	N/A	0.247	0.003	1.511	0.027	0.079	0.159	0.147	0.228	5.367

Problem 145	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	139	139	117	135	412	484	668	273	565	372
N.S.	1	1.00	0.84	0.97	2.96	3.48	4.81	1.96	4.06	2.68
time (sec)	N/A	0.505	0.077	1.629	0.044	0.096	0.880	0.122	0.219	0.135

Problem 146	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	173	173	157	173	656	837	1006	386	1362	617
N.S.	1	1.00	0.91	1.00	3.79	4.84	5.82	2.23	7.87	3.57
time (sec)	N/A	0.624	0.111	1.691	0.053	0.101	1.450	0.141	0.206	5.606

Problem 147	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	226	226	206	221	956	1242	1363	503	1990	891
N.S.	1	1.00	0.91	0.98	4.23	5.50	6.03	2.23	8.81	3.94
time (sec)	N/A	0.734	0.138	1.671	0.084	0.111	2.494	0.137	0.248	5.637

Problem 148	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	B	B	B	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	256	281	234	253	1278	1618	1748	635	2480	0
N.S.	1	1.10	0.91	0.99	4.99	6.32	6.83	2.48	9.69	0.00
time (sec)	N/A	0.937	0.239	1.639	0.065	0.134	3.606	0.165	0.245	0.000

Problem 149	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	43	43	34	32	38	98	53	313	97	34
N.S.	1	1.00	0.79	0.74	0.88	2.28	1.23	7.28	2.26	0.79
time (sec)	N/A	0.307	0.046	1.591	0.027	0.074	0.855	0.113	0.217	0.029

Problem 150	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	43	43	34	32	38	74	53	198	73	34
N.S.	1	1.00	0.79	0.74	0.88	1.72	1.23	4.60	1.70	0.79
time (sec)	N/A	0.301	0.038	1.571	0.034	0.074	0.845	0.158	0.223	5.187

Problem 151	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	43	43	34	32	90	51	104	105	49	34
N.S.	1	1.00	0.79	0.74	2.09	1.19	2.42	2.44	1.14	0.79
time (sec)	N/A	0.288	0.038	1.448	0.028	0.076	0.626	0.118	0.259	0.026

Problem 152	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	41	41	33	31	37	30	51	41	29	33
N.S.	1	1.00	0.80	0.76	0.90	0.73	1.24	1.00	0.71	0.80
time (sec)	N/A	0.289	0.037	0.711	0.034	0.078	0.770	0.145	0.228	0.024

Problem 153	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	39	39	31	31	42	40	58	39	31	30
N.S.	1	1.00	0.79	0.79	1.08	1.03	1.49	1.00	0.79	0.77
time (sec)	N/A	0.292	0.034	0.701	0.038	0.079	0.255	0.113	0.228	5.230

Problem 154	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	41	41	33	31	33	51	126	33	39	34
N.S.	1	1.00	0.80	0.76	0.80	1.24	3.07	0.80	0.95	0.83
time (sec)	N/A	0.285	0.041	0.678	0.033	0.077	0.429	0.175	0.237	0.026

Problem 155	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	43	43	34	32	34	63	187	34	50	34
N.S.	1	1.00	0.79	0.74	0.79	1.47	4.35	0.79	1.16	0.79
time (sec)	N/A	0.298	0.041	0.811	0.033	0.079	0.705	0.135	0.237	0.025

Problem 156	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	43	43	34	32	34	74	248	34	61	31
N.S.	1	1.00	0.79	0.74	0.79	1.72	5.77	0.79	1.42	0.72
time (sec)	N/A	0.299	0.044	0.678	0.033	0.076	1.356	0.117	0.238	5.351

Problem 157	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	83	83	67	65	80	184	110	566	190	80
N.S.	1	1.00	0.81	0.78	0.96	2.22	1.33	6.82	2.29	0.96
time (sec)	N/A	0.389	0.059	2.312	0.029	0.077	0.879	0.181	0.214	5.347

Problem 158	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	B	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	83	83	67	65	280	147	110	368	149	80
N.S.	1	1.00	0.81	0.78	3.37	1.77	1.33	4.43	1.80	0.96
time (sec)	N/A	0.371	0.057	2.179	0.035	0.079	0.895	0.153	0.202	5.178

Problem 159	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	83	83	67	65	80	109	110	208	108	80
N.S.	1	1.00	0.81	0.78	0.96	1.31	1.33	2.51	1.30	0.96
time (sec)	N/A	0.360	0.055	2.198	0.033	0.077	0.875	0.174	0.227	0.034

Problem 160	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	81	81	66	65	80	74	109	86	71	80
N.S.	1	1.00	0.81	0.80	0.99	0.91	1.35	1.06	0.88	0.99
time (sec)	N/A	0.356	0.042	1.386	0.041	0.081	0.879	0.137	0.220	5.086

Problem 161	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	79	79	65	65	87	83	133	94	72	80
N.S.	1	1.00	0.82	0.82	1.10	1.05	1.68	1.19	0.91	1.01
time (sec)	N/A	0.352	0.054	1.396	0.034	0.080	0.445	0.144	0.230	0.037

Problem 162	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	79	79	68	62	84	95	264	84	80	80
N.S.	1	1.00	0.86	0.78	1.06	1.20	3.34	1.06	1.01	1.01
time (sec)	N/A	0.358	0.054	1.458	0.044	0.085	0.711	0.194	0.236	0.036

Problem 163	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	81	81	67	65	77	105	388	80	91	78
N.S.	1	1.00	0.83	0.80	0.95	1.30	4.79	0.99	1.12	0.96
time (sec)	N/A	0.358	0.058	1.392	0.034	0.081	0.965	0.123	0.211	0.045

Problem 164	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	83	83	67	65	77	117	510	80	102	78
N.S.	1	1.00	0.81	0.78	0.93	1.41	6.14	0.96	1.23	0.94
time (sec)	N/A	0.364	0.053	1.369	0.032	0.082	1.387	0.148	0.240	5.148

Problem 165	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	119	119	111	97	137	335	178	1214	357	106
N.S.	1	1.00	0.93	0.82	1.15	2.82	1.50	10.20	3.00	0.89
time (sec)	N/A	0.439	0.101	2.556	0.027	0.079	0.984	0.164	0.198	5.162

Problem 166	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	B	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	119	119	111	97	611	283	178	874	299	106
N.S.	1	1.00	0.93	0.82	5.13	2.38	1.50	7.34	2.51	0.89
time (sec)	N/A	0.419	0.100	2.385	0.039	0.077	1.007	0.181	0.215	0.037

Problem 167	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	119	119	111	97	137	231	178	582	241	106
N.S.	1	1.00	0.93	0.82	1.15	1.94	1.50	4.89	2.03	0.89
time (sec)	N/A	0.424	0.083	2.397	0.039	0.087	1.129	0.128	0.240	0.037

Problem 168	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	119	119	111	97	137	179	178	337	183	106
N.S.	1	1.00	0.93	0.82	1.15	1.50	1.50	2.83	1.54	0.89
time (sec)	N/A	0.412	0.076	2.329	0.033	0.083	1.081	0.153	0.216	0.038

Problem 169	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	115	115	110	95	137	130	177	145	129	106
N.S.	1	1.00	0.96	0.83	1.19	1.13	1.54	1.26	1.12	0.92
time (sec)	N/A	0.422	0.078	1.473	0.036	0.081	1.105	0.146	0.224	0.037

Problem 170	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	113	113	109	105	144	139	230	167	130	133
N.S.	1	1.00	0.96	0.93	1.27	1.23	2.04	1.48	1.15	1.18
time (sec)	N/A	0.401	0.070	1.530	0.032	0.092	0.776	0.184	0.197	5.163

Problem 171	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	115	115	110	83	141	151	450	158	137	147
N.S.	1	1.00	0.96	0.72	1.23	1.31	3.91	1.37	1.19	1.28
time (sec)	N/A	0.400	0.099	1.536	0.035	0.081	1.078	0.191	0.200	5.407

Problem 172	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	113	113	109	106	140	163	654	150	149	129
N.S.	1	1.00	0.96	0.94	1.24	1.44	5.79	1.33	1.32	1.14
time (sec)	N/A	0.406	0.090	1.464	0.054	0.087	1.454	0.138	0.206	5.265

Problem 173	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	180	207	179	185	0	510	230	298	434	207
N.S.	1	1.15	0.99	1.03	0.00	2.83	1.28	1.66	2.41	1.15
time (sec)	N/A	0.533	0.206	4.471	0.000	0.103	161.604	0.150	0.232	0.040

Problem 174	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	A	F(-1)	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	147	170	135	146	0	366	0	209	288	165
N.S.	1	1.16	0.92	0.99	0.00	2.49	0.00	1.42	1.96	1.12
time (sec)	N/A	0.458	0.144	2.013	0.000	0.096	0.000	0.160	0.251	5.237

Problem 175	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	114	133	102	112	0	254	122	134	164	121
N.S.	1	1.17	0.89	0.98	0.00	2.23	1.07	1.18	1.44	1.06
time (sec)	N/A	0.406	0.193	1.991	0.000	0.284	29.125	0.157	0.220	0.053

Problem 176	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	83	96	83	76	0	191	90	83	70	67
N.S.	1	1.16	1.00	0.92	0.00	2.30	1.08	1.00	0.84	0.81
time (sec)	N/A	0.352	0.126	1.959	0.000	0.098	1.536	0.121	0.201	0.041

Problem 177	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	65	65	65	48	0	155	58	49	72	49
N.S.	1	1.00	1.00	0.74	0.00	2.38	0.89	0.75	1.11	0.75
time (sec)	N/A	0.309	0.059	1.780	0.000	0.079	1.303	0.150	0.218	5.255

Problem 178	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	91	91	91	76	0	233	92	89	107	75
N.S.	1	1.00	1.00	0.84	0.00	2.56	1.01	0.98	1.18	0.82
time (sec)	N/A	0.366	0.135	1.803	0.000	0.097	2.019	0.162	0.211	0.045

Problem 179	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	120	136	111	117	0	434	124	140	267	127
N.S.	1	1.13	0.92	0.98	0.00	3.62	1.03	1.17	2.22	1.06
time (sec)	N/A	0.432	0.265	1.937	0.000	0.112	2.003	0.153	0.234	5.243

Problem 180	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	B	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	153	181	145	146	0	756	160	217	503	171
N.S.	1	1.18	0.95	0.95	0.00	4.94	1.05	1.42	3.29	1.12
time (sec)	N/A	0.476	0.284	1.924	0.000	0.105	13.142	0.143	0.233	5.326

Problem 181	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	B	F(-1)	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	186	226	189	175	0	1130	0	310	788	213
N.S.	1	1.22	1.02	0.94	0.00	6.08	0.00	1.67	4.24	1.15
time (sec)	N/A	0.507	0.302	1.923	0.000	0.121	0.000	0.105	0.221	5.338

Problem 182	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	B	F(-1)	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	210	246	246	257	0	788	0	422	804	443
N.S.	1	1.17	1.17	1.22	0.00	3.75	0.00	2.01	3.83	2.11
time (sec)	N/A	0.514	0.528	4.240	0.000	0.105	0.000	0.177	0.244	0.076

Problem 183	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	A	F(-1)	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	178	209	191	204	0	586	0	312	570	290
N.S.	1	1.17	1.07	1.15	0.00	3.29	0.00	1.75	3.20	1.63
time (sec)	N/A	0.483	0.400	4.120	0.000	0.099	0.000	0.157	0.287	5.445

Problem 184	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	A	F(-1)	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	144	172	146	151	0	420	0	218	359	200
N.S.	1	1.19	1.01	1.05	0.00	2.92	0.00	1.51	2.49	1.39
time (sec)	N/A	0.424	0.355	4.161	0.000	0.098	0.000	0.160	0.299	5.298

Problem 185	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	A	F(-1)	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	112	135	110	129	0	282	0	142	174	140
N.S.	1	1.21	0.98	1.15	0.00	2.52	0.00	1.27	1.55	1.25
time (sec)	N/A	0.386	0.315	4.848	0.000	0.106	0.000	0.119	0.277	0.079

Problem 186	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	94	94	93	75	0	309	0	94	180	81
N.S.	1	1.00	0.99	0.80	0.00	3.29	0.00	1.00	1.91	0.86
time (sec)	N/A	0.336	0.211	4.037	0.000	0.093	0.000	0.131	0.222	5.193

Problem 187	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	101	101	100	82	0	353	0	111	207	97
N.S.	1	1.00	0.99	0.81	0.00	3.50	0.00	1.10	2.05	0.96
time (sec)	N/A	0.364	0.196	1.865	0.000	0.106	0.000	0.132	0.233	5.238

Problem 188	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	128	146	118	100	0	476	0	171	270	155
N.S.	1	1.14	0.92	0.78	0.00	3.72	0.00	1.34	2.11	1.21
time (sec)	N/A	0.416	0.417	1.951	0.000	0.101	0.000	0.111	0.250	5.243

Problem 189	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	B	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	158	191	156	153	0	854	0	253	585	200
N.S.	1	1.21	0.99	0.97	0.00	5.41	0.00	1.60	3.70	1.27
time (sec)	N/A	0.469	0.378	2.073	0.000	0.110	0.000	0.165	0.211	5.385

Problem 190	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	B	F	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	192	236	200	183	0	1304	0	340	933	244
N.S.	1	1.23	1.04	0.95	0.00	6.79	0.00	1.77	4.86	1.27
time (sec)	N/A	0.507	0.502	2.016	0.000	0.166	0.000	0.152	0.222	5.292

Problem 191	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	B	F(-1)	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	222	250	250	235	0	858	0	418	895	430
N.S.	1	1.13	1.13	1.06	0.00	3.86	0.00	1.88	4.03	1.94
time (sec)	N/A	0.521	0.835	16.651	0.000	0.154	0.000	0.185	0.256	5.455

Problem 192	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	B	F(-1)	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	186	213	207	180	0	638	0	303	594	319
N.S.	1	1.15	1.11	0.97	0.00	3.43	0.00	1.63	3.19	1.72
time (sec)	N/A	0.456	0.621	15.887	0.000	0.123	0.000	0.178	0.241	0.092

Problem 193	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	A	F(-1)	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	152	176	149	164	0	440	0	203	320	240
N.S.	1	1.16	0.98	1.08	0.00	2.89	0.00	1.34	2.11	1.58
time (sec)	N/A	0.423	0.575	18.312	0.000	0.096	0.000	0.165	0.220	5.338

Problem 194	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	B	F(-1)	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	130	135	118	101	0	492	0	133	341	171
N.S.	1	1.04	0.91	0.78	0.00	3.78	0.00	1.02	2.62	1.32
time (sec)	N/A	0.380	0.436	15.540	0.000	0.105	0.000	0.159	0.212	0.078

Problem 195	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	B	F(-1)	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	144	142	131	136	0	565	0	155	383	166
N.S.	1	0.99	0.91	0.94	0.00	3.92	0.00	1.08	2.66	1.15
time (sec)	N/A	0.401	0.505	15.659	0.000	0.105	0.000	0.138	0.249	5.266

Problem 196	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	B	F(-1)	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	146	158	125	129	0	654	0	175	434	177
N.S.	1	1.08	0.86	0.88	0.00	4.48	0.00	1.20	2.97	1.21
time (sec)	N/A	0.429	0.334	15.496	0.000	0.110	0.000	0.180	0.233	5.276

Problem 197	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	B	F(-1)	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	176	203	157	152	0	870	0	269	518	251
N.S.	1	1.15	0.89	0.86	0.00	4.94	0.00	1.53	2.94	1.43
time (sec)	N/A	0.471	0.721	2.000	0.000	0.131	0.000	0.113	0.255	5.292

Problem 198	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	B	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	208	248	202	180	0	1326	0	340	944	296
N.S.	1	1.19	0.97	0.87	0.00	6.38	0.00	1.63	4.54	1.42
time (sec)	N/A	0.510	0.709	1.980	0.000	0.197	0.000	0.165	0.238	0.171

Problem 199	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	B	F	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	244	293	257	209	0	1980	0	433	1491	344
N.S.	1	1.20	1.05	0.86	0.00	8.11	0.00	1.77	6.11	1.41
time (sec)	N/A	0.566	1.130	2.023	0.000	0.375	0.000	0.194	0.276	5.405

Problem 200	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F(-2)	A	B	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	358	351	278	1543	0	844	1646	400	790	0
N.S.	1	0.98	0.78	4.31	0.00	2.36	4.60	1.12	2.21	0.00
time (sec)	N/A	0.874	0.767	1.954	0.000	0.118	0.947	0.161	0.274	0.000

Problem 201	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F(-2)	A	B	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	298	284	223	816	0	672	989	305	579	0
N.S.	1	0.95	0.75	2.74	0.00	2.26	3.32	1.02	1.94	0.00
time (sec)	N/A	0.683	0.526	1.691	0.000	0.118	0.833	0.210	0.235	0.000

Problem 202	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	240	219	179	373	0	532	571	222	400	307
N.S.	1	0.91	0.75	1.55	0.00	2.22	2.38	0.92	1.67	1.28
time (sec)	N/A	0.525	0.371	1.481	0.000	0.110	1.188	0.182	0.268	5.841

Problem 203	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	144	159	144	154	0	412	306	156	251	131
N.S.	1	1.10	1.00	1.07	0.00	2.86	2.12	1.08	1.74	0.91
time (sec)	N/A	0.417	0.431	1.389	0.000	0.101	0.401	0.141	0.236	5.430

Problem 204	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	116	131	112	131	0	337	0	114	135	0
N.S.	1	1.13	0.97	1.13	0.00	2.91	0.00	0.98	1.16	0.00
time (sec)	N/A	0.413	0.216	1.769	0.000	0.107	0.000	0.141	0.233	0.000

Problem 205	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F(-2)	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	112	124	110	212	0	326	0	185	149	0
N.S.	1	1.11	0.98	1.89	0.00	2.91	0.00	1.65	1.33	0.00
time (sec)	N/A	0.403	0.164	1.872	0.000	0.127	0.000	0.201	0.241	0.000

Problem 206	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	A	F	F(-2)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	54	54	43	58	0	90	0	0	145	58
N.S.	1	1.00	0.80	1.07	0.00	1.67	0.00	0.00	2.69	1.07
time (sec)	N/A	0.285	0.035	2.198	0.000	0.187	0.000	0.000	0.260	5.542

Problem 207	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	A	F	F(-2)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	111	111	81	90	0	205	0	0	342	562
N.S.	1	1.00	0.73	0.81	0.00	1.85	0.00	0.00	3.08	5.06
time (sec)	N/A	0.387	0.235	2.437	0.000	0.426	0.000	0.000	0.245	5.655

Problem 208	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	B	F	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	171	186	124	146	0	373	0	698	603	877
N.S.	1	1.09	0.73	0.85	0.00	2.18	0.00	4.08	3.53	5.13
time (sec)	N/A	0.532	1.556	2.974	0.000	1.290	0.000	0.190	0.316	6.039

Problem 209	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	B	F	F(-2)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	231	261	180	217	0	572	0	0	926	1192
N.S.	1	1.13	0.78	0.94	0.00	2.48	0.00	0.00	4.01	5.16
time (sec)	N/A	0.682	10.063	3.633	0.000	4.762	0.000	0.000	0.407	6.700

Problem 210	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F(-2)	A	B	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	421	371	361	1188	0	1042	2909	519	1033	0
N.S.	1	0.88	0.86	2.82	0.00	2.48	6.91	1.23	2.45	0.00
time (sec)	N/A	0.829	0.932	1.786	0.000	0.137	1.420	0.232	0.328	0.000

Problem 211	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	363	306	295	559	0	844	1681	411	790	646
N.S.	1	0.84	0.81	1.54	0.00	2.33	4.63	1.13	2.18	1.78
time (sec)	N/A	0.654	0.804	1.497	0.000	0.273	1.270	0.203	0.278	5.881

Problem 212	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	217	246	239	247	0	666	1853	310	579	225
N.S.	1	1.13	1.10	1.14	0.00	3.07	8.54	1.43	2.67	1.04
time (sec)	N/A	0.526	0.669	1.463	0.000	0.130	1.622	0.187	0.280	5.502

Problem 213	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	A	A	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	245	215	180	230	0	532	751	229	400	0
N.S.	1	0.88	0.73	0.94	0.00	2.17	3.07	0.93	1.63	0.00
time (sec)	N/A	0.513	0.409	1.780	0.000	0.111	1.652	0.195	0.247	0.000

Problem 214	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F(-2)	A	F	B	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	174	194	150	311	0	418	0	471	253	0
N.S.	1	1.11	0.86	1.79	0.00	2.40	0.00	2.71	1.45	0.00
time (sec)	N/A	0.514	0.391	1.902	0.000	0.105	0.000	0.292	0.223	0.000

Problem 215	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F(-2)	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	160	186	148	392	0	414	0	224	347	0
N.S.	1	1.16	0.92	2.45	0.00	2.59	0.00	1.40	2.17	0.00
time (sec)	N/A	0.569	0.382	2.103	0.000	0.145	0.000	0.204	0.229	0.000

Problem 216	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F(-2)	A	F	F(-2)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	157	174	130	473	0	426	0	0	255	0
N.S.	1	1.11	0.83	3.01	0.00	2.71	0.00	0.00	1.62	0.00
time (sec)	N/A	0.512	0.306	2.429	0.000	0.264	0.000	0.000	0.265	0.000

Problem 217	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	B	F	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	54	54	43	58	0	130	0	806	229	901
N.S.	1	1.00	0.80	1.07	0.00	2.41	0.00	14.93	4.24	16.69
time (sec)	N/A	0.280	0.045	2.940	0.000	0.589	0.000	0.257	0.253	6.101

Problem 218	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	B	F	F(-2)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	111	111	61	90	0	275	0	0	467	1477
N.S.	1	1.00	0.55	0.81	0.00	2.48	0.00	0.00	4.21	13.31
time (sec)	N/A	0.379	0.208	3.572	0.000	1.767	0.000	0.000	0.269	6.785

Problem 219	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	B	F	F(-2)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	171	186	94	146	0	472	0	0	777	2067
N.S.	1	1.09	0.55	0.85	0.00	2.76	0.00	0.00	4.54	12.09
time (sec)	N/A	0.541	0.505	4.597	0.000	6.146	0.000	0.000	0.416	7.290

Problem 220	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	B	F	F(-2)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	231	261	138	217	0	699	0	0	1149	2657
N.S.	1	1.13	0.60	0.94	0.00	3.03	0.00	0.00	4.97	11.50
time (sec)	N/A	0.659	1.445	5.596	0.000	15.093	0.000	0.000	0.606	7.978

Problem 221	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F(-2)	A	B	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	544	458	526	1560	0	1520	8918	786	1615	0
N.S.	1	0.84	0.97	2.87	0.00	2.79	16.39	1.44	2.97	0.00
time (sec)	N/A	0.967	1.572	1.700	0.000	0.179	6.605	0.358	0.468	0.000

Problem 222	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	A	B	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	486	393	437	745	0	1270	5268	650	1308	0
N.S.	1	0.81	0.90	1.53	0.00	2.61	10.84	1.34	2.69	0.00
time (sec)	N/A	0.849	1.314	1.496	0.000	0.153	4.511	0.313	0.398	0.000

Problem 223	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	290	333	361	340	0	1034	6698	524	1033	319
N.S.	1	1.15	1.24	1.17	0.00	3.57	23.10	1.81	3.56	1.10
time (sec)	N/A	0.673	1.082	1.386	0.000	0.138	7.840	0.499	0.368	5.470

Problem 224	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	A	A	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	373	302	295	327	0	844	3516	415	790	0
N.S.	1	0.81	0.79	0.88	0.00	2.26	9.43	1.11	2.12	0.00
time (sec)	N/A	0.663	0.850	1.721	0.000	0.132	35.968	0.511	0.342	0.000

Problem 225	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	A	A	B	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	308	279	222	408	0	672	1416	1319	579	0
N.S.	1	0.91	0.72	1.32	0.00	2.18	4.60	4.28	1.88	0.00
time (sec)	N/A	0.678	0.548	1.856	0.000	0.115	137.235	0.894	0.330	0.000

Problem 226	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F(-2)	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	231	264	183	489	0	534	0	235	400	0
N.S.	1	1.14	0.79	2.12	0.00	2.31	0.00	1.02	1.73	0.00
time (sec)	N/A	0.616	0.548	2.136	0.000	0.120	0.000	0.343	0.258	0.000

Problem 227	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F(-2)	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	222	257	186	570	0	554	0	302	597	0
N.S.	1	1.16	0.84	2.57	0.00	2.50	0.00	1.36	2.69	0.00
time (sec)	N/A	0.956	0.492	2.517	0.000	0.243	0.000	0.484	0.246	0.000

Problem 228	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F(-2)	A	F	B	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	211	239	185	651	0	594	0	462	622	0
N.S.	1	1.13	0.88	3.09	0.00	2.82	0.00	2.19	2.95	0.00
time (sec)	N/A	0.676	0.465	3.193	0.000	0.353	0.000	0.626	0.262	0.000

Problem 229	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F(-2)	A	F	F(-2)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	206	224	173	732	0	578	0	0	491	0
N.S.	1	1.09	0.84	3.55	0.00	2.81	0.00	0.00	2.38	0.00
time (sec)	N/A	0.603	0.478	4.165	0.000	0.958	0.000	0.000	0.283	0.000

Problem 230	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	B	F	F(-2)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	54	54	43	58	0	170	0	0	305	2156
N.S.	1	1.00	0.80	1.07	0.00	3.15	0.00	0.00	5.65	39.93
time (sec)	N/A	0.284	0.050	5.631	0.000	2.136	0.000	0.000	0.468	7.171

Problem 231	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	B	F	F(-2)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	111	111	61	90	0	344	0	0	592	3125
N.S.	1	1.00	0.55	0.81	0.00	3.10	0.00	0.00	5.33	28.15
time (sec)	N/A	0.392	0.298	7.371	0.000	7.879	0.000	0.000	0.389	8.056

Problem 232	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	B	F	F(-2)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	171	186	104	146	0	569	0	0	951	4096
N.S.	1	1.09	0.61	0.85	0.00	3.33	0.00	0.00	5.56	23.95
time (sec)	N/A	0.523	0.751	12.989	0.000	18.334	0.000	0.000	0.672	9.316

Problem 233	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	B	F(-1)	F(-2)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	231	261	148	217	0	823	0	0	1372	5069
N.S.	1	1.13	0.64	0.94	0.00	3.56	0.00	0.00	5.94	21.94
time (sec)	N/A	0.665	1.884	16.928	0.000	40.705	0.000	0.000	1.205	10.937

Problem 234	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	A	B	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	609	480	625	931	0	1804	15410	936	1954	0
N.S.	1	0.79	1.03	1.53	0.00	2.96	25.30	1.54	3.21	0.00
time (sec)	N/A	1.031	2.079	1.543	0.000	0.186	19.707	0.358	0.633	0.000

Problem 235	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	363	420	523	433	0	1518	20264	786	1615	413
N.S.	1	1.16	1.44	1.19	0.00	4.18	55.82	2.17	4.45	1.14
time (sec)	N/A	0.818	1.722	1.398	0.000	0.173	49.501	0.490	0.407	5.430

Problem 236	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	A	A	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	496	389	437	424	0	1270	11608	651	1308	0
N.S.	1	0.78	0.88	0.85	0.00	2.56	23.40	1.31	2.64	0.00
time (sec)	N/A	0.822	1.315	1.724	0.000	0.157	17.790	0.445	0.387	0.000

Problem 237	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	A	F(-1)	B	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	436	366	361	505	0	1042	0	2611	1033	0
N.S.	1	0.84	0.83	1.16	0.00	2.39	0.00	5.99	2.37	0.00
time (sec)	N/A	0.827	1.059	1.848	0.000	0.125	0.000	1.216	0.314	0.000

Problem 238	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	A	F(-1)	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	371	345	279	586	0	844	0	412	790	0
N.S.	1	0.93	0.75	1.58	0.00	2.27	0.00	1.11	2.13	0.00
time (sec)	N/A	0.832	0.793	2.155	0.000	0.110	0.000	0.196	0.294	0.000

Problem 239	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F(-2)	A	F(-1)	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	288	328	225	667	0	674	0	315	579	0
N.S.	1	1.14	0.78	2.32	0.00	2.34	0.00	1.09	2.01	0.00
time (sec)	N/A	0.800	0.670	2.488	0.000	0.188	0.000	0.234	0.282	0.000

Problem 240	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F(-2)	A	F(-1)	B	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	281	348	221	748	0	726	0	1066	879	0
N.S.	1	1.24	0.79	2.66	0.00	2.58	0.00	3.79	3.13	0.00
time (sec)	N/A	1.559	0.549	3.147	0.000	0.427	0.000	0.592	0.291	0.000

Problem 241	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F(-2)	A	F	F(-2)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	277	310	222	829	0	786	0	0	991	0
N.S.	1	1.12	0.80	2.99	0.00	2.84	0.00	0.00	3.58	0.00
time (sec)	N/A	1.068	0.481	4.035	0.000	0.772	0.000	0.000	0.308	0.000

Problem 242	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F(-2)	A	F	F(-2)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	260	292	219	910	0	798	0	0	931	0
N.S.	1	1.12	0.84	3.50	0.00	3.07	0.00	0.00	3.58	0.00
time (sec)	N/A	0.782	0.486	5.473	0.000	1.161	0.000	0.000	0.354	0.000

Problem 243	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F(-2)	A	F(-1)	F(-2)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	255	274	207	991	0	744	0	0	701	0
N.S.	1	1.07	0.81	3.89	0.00	2.92	0.00	0.00	2.75	0.00
time (sec)	N/A	0.764	0.393	7.244	0.000	3.534	0.000	0.000	0.723	0.000

Problem 244	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	B	F(-1)	F(-2)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	54	54	53	58	0	210	0	0	381	4399
N.S.	1	1.00	0.98	1.07	0.00	3.89	0.00	0.00	7.06	81.46
time (sec)	N/A	0.279	0.040	8.901	0.000	8.259	0.000	0.000	2.538	9.565

Problem 245	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	B	F(-1)	F(-2)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	111	111	72	90	0	415	0	0	717	5860
N.S.	1	1.00	0.65	0.81	0.00	3.74	0.00	0.00	6.46	52.79
time (sec)	N/A	0.380	0.423	10.610	0.000	23.339	0.000	0.000	0.694	11.109

Problem 246	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	B	F(-1)	F(-2)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	171	186	104	146	0	670	0	0	1125	7337
N.S.	1	1.09	0.61	0.85	0.00	3.92	0.00	0.00	6.58	42.91
time (sec)	N/A	0.526	1.082	13.002	0.000	49.488	0.000	0.000	1.606	14.707

Problem 247	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	B	F(-1)	F(-2)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	231	261	148	217	0	951	0	0	1595	8811
N.S.	1	1.13	0.64	0.94	0.00	4.12	0.00	0.00	6.90	38.14
time (sec)	N/A	0.695	2.423	18.613	0.000	97.838	0.000	0.000	2.315	15.524

Problem 248	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F(-2)	A	B	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	240	267	187	888	0	534	782	230	400	0
N.S.	1	1.11	0.78	3.70	0.00	2.22	3.26	0.96	1.67	0.00
time (sec)	N/A	0.684	0.359	1.309	0.000	0.154	0.889	0.200	0.254	0.000

Problem 249	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F(-2)	A	B	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	180	200	155	442	0	418	502	163	253	0
N.S.	1	1.11	0.86	2.46	0.00	2.32	2.79	0.91	1.41	0.00
time (sec)	N/A	0.525	0.298	1.134	0.000	0.121	0.766	0.193	0.228	0.000

Problem 250	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	116	135	136	186	0	338	335	120	135	144
N.S.	1	1.16	1.17	1.60	0.00	2.91	2.89	1.03	1.16	1.24
time (sec)	N/A	0.386	0.278	0.990	0.000	0.106	0.681	0.181	0.236	5.900

Problem 251	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	68	82	94	62	0	244	218	156	58	49
N.S.	1	1.21	1.38	0.91	0.00	3.59	3.21	2.29	0.85	0.72
time (sec)	N/A	0.293	0.106	0.911	0.000	0.105	0.499	0.157	0.239	5.855

Problem 252	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	A	F	F(-2)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	52	52	42	50	0	59	0	0	71	50
N.S.	1	1.00	0.81	0.96	0.00	1.13	0.00	0.00	1.37	0.96
time (sec)	N/A	0.272	0.034	1.125	0.000	0.149	0.000	0.000	0.255	5.614

Problem 253	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	111	111	61	81	0	140	0	176	211	69
N.S.	1	1.00	0.55	0.73	0.00	1.26	0.00	1.59	1.90	0.62
time (sec)	N/A	0.375	0.095	1.185	0.000	0.296	0.000	0.181	0.230	5.729

Problem 254	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	A	F	F(-2)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	171	186	94	138	0	279	0	0	431	110
N.S.	1	1.09	0.55	0.81	0.00	1.63	0.00	0.00	2.52	0.64
time (sec)	N/A	0.527	0.143	1.327	0.000	1.286	0.000	0.000	0.261	6.172

Problem 255	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	B	F	F(-2)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	231	261	138	209	0	452	0	0	705	252
N.S.	1	1.13	0.60	0.90	0.00	1.96	0.00	0.00	3.05	1.09
time (sec)	N/A	0.666	0.236	1.460	0.000	2.968	0.000	0.000	0.271	5.883

Problem 256	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F(-2)	A	F	F(-2)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	287	349	234	3117	0	759	0	0	569	0
N.S.	1	1.22	0.82	10.86	0.00	2.64	0.00	0.00	1.98	0.00
time (sec)	N/A	1.514	0.581	1.709	0.000	0.642	0.000	0.000	0.285	0.000

Problem 257	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F(-2)	A	F	F(-2)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	226	257	180	1770	0	585	0	0	388	0
N.S.	1	1.14	0.80	7.83	0.00	2.59	0.00	0.00	1.72	0.00
time (sec)	N/A	0.899	0.479	1.431	0.000	0.307	0.000	0.000	0.259	0.000

Problem 258	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F(-2)	A	F	F(-2)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	163	190	143	960	0	447	0	0	230	0
N.S.	1	1.17	0.88	5.89	0.00	2.74	0.00	0.00	1.41	0.00
time (sec)	N/A	0.567	0.258	1.232	0.000	0.169	0.000	0.000	0.242	0.000

Problem 259	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F(-2)	A	F	F(-2)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	111	125	113	485	0	349	0	0	104	0
N.S.	1	1.13	1.02	4.37	0.00	3.14	0.00	0.00	0.94	0.00
time (sec)	N/A	0.407	0.153	1.090	0.000	0.142	0.000	0.000	0.257	0.000

Problem 260	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	A	F	F(-2)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	50	50	39	53	0	65	0	0	59	53
N.S.	1	1.00	0.78	1.06	0.00	1.30	0.00	0.00	1.18	1.06
time (sec)	N/A	0.272	0.034	0.978	0.000	0.178	0.000	0.000	0.232	5.295

Problem 261	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	B	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	62	62	49	60	0	153	0	101	143	75
N.S.	1	1.00	0.79	0.97	0.00	2.47	0.00	1.63	2.31	1.21
time (sec)	N/A	0.279	0.086	0.915	0.000	0.452	0.000	0.158	0.231	0.033

Problem 262	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	A	F	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	165	121	95	138	0	306	0	0	323	120
N.S.	1	0.73	0.58	0.84	0.00	1.85	0.00	0.00	1.96	0.73
time (sec)	N/A	0.372	0.136	1.174	0.000	1.257	0.000	0.000	0.225	5.594

Problem 263	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	B	F	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	226	196	136	216	0	494	0	3930	561	1005
N.S.	1	0.87	0.60	0.96	0.00	2.19	0.00	17.39	2.48	4.45
time (sec)	N/A	0.509	0.179	1.238	0.000	3.580	0.000	0.292	0.295	5.714

Problem 264	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	B	F	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	287	271	193	307	0	733	0	0	852	2121
N.S.	1	0.94	0.67	1.07	0.00	2.55	0.00	0.00	2.97	7.39
time (sec)	N/A	0.647	0.281	1.420	0.000	9.125	0.000	0.000	0.325	6.509

Problem 265	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F(-2)	A	F	F(-2)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	279	315	230	7420	0	833	0	0	773	0
N.S.	1	1.13	0.82	26.59	0.00	2.99	0.00	0.00	2.77	0.00
time (sec)	N/A	0.970	0.609	2.019	0.000	1.582	0.000	0.000	0.283	0.000

Problem 266	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F(-2)	A	F	F(-2)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	214	248	189	4447	0	641	0	0	491	0
N.S.	1	1.16	0.88	20.78	0.00	3.00	0.00	0.00	2.29	0.00
time (sec)	N/A	0.673	0.460	1.754	0.000	0.685	0.000	0.000	0.238	0.000

Problem 267	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F(-2)	A	F	F(-2)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	158	180	137	2599	0	469	0	0	198	0
N.S.	1	1.14	0.87	16.45	0.00	2.97	0.00	0.00	1.25	0.00
time (sec)	N/A	0.500	0.264	1.375	0.000	0.497	0.000	0.000	0.234	0.000

Problem 268	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	B	F	F(-2)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	54	54	46	58	0	103	0	0	123	58
N.S.	1	1.00	0.85	1.07	0.00	1.91	0.00	0.00	2.28	1.07
time (sec)	N/A	0.280	0.042	1.254	0.000	0.359	0.000	0.000	0.214	5.706

Problem 269	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	A	F	F(-2)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	108	116	59	85	0	156	0	0	168	72
N.S.	1	1.07	0.55	0.79	0.00	1.44	0.00	0.00	1.56	0.67
time (sec)	N/A	0.357	0.086	1.122	0.000	0.477	0.000	0.000	0.230	5.572

Problem 270	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	B	F	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	162	118	90	146	0	316	0	0	346	120
N.S.	1	0.73	0.56	0.90	0.00	1.95	0.00	0.00	2.14	0.74
time (sec)	N/A	0.388	0.148	1.008	0.000	1.693	0.000	0.000	0.224	5.570

Problem 271	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	B	F	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	132	132	132	153	0	491	0	366	647	131
N.S.	1	1.00	1.00	1.16	0.00	3.72	0.00	2.77	4.90	0.99
time (sec)	N/A	0.378	0.194	0.944	0.000	4.837	0.000	0.191	0.240	5.356

Problem 272	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	B	F	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	289	207	193	242	0	769	0	0	977	253
N.S.	1	0.72	0.67	0.84	0.00	2.66	0.00	0.00	3.38	0.88
time (sec)	N/A	0.481	0.277	1.201	0.000	14.142	0.000	0.000	0.286	5.852

Problem 273	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	B	F	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	352	282	259	323	0	1058	0	10592	1384	3654
N.S.	1	0.80	0.74	0.92	0.00	3.01	0.00	30.09	3.93	10.38
time (sec)	N/A	0.628	0.444	1.263	0.000	33.166	0.000	0.432	0.445	6.675

Problem 274	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	B	F	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	413	357	336	404	0	1392	0	0	1820	10949
N.S.	1	0.86	0.81	0.98	0.00	3.37	0.00	0.00	4.41	26.51
time (sec)	N/A	0.783	0.487	1.441	0.000	63.142	0.000	0.000	0.784	8.619

Problem 275	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	17	17	19	16	26	20	0	19	19	17
N.S.	1	1.00	1.12	0.94	1.53	1.18	0.00	1.12	1.12	1.00
time (sec)	N/A	0.225	0.091	0.665	0.039	0.086	0.000	0.179	0.239	5.258

Problem 276	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	A	F	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	48	48	45	54	0	62	0	0	59	47
N.S.	1	1.00	0.94	1.12	0.00	1.29	0.00	0.00	1.23	0.98
time (sec)	N/A	0.292	0.060	1.280	0.000	0.132	0.000	0.000	0.256	5.268

Problem 277	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	34	34	34	26	29	29	31	30	23	25
N.S.	1	1.00	1.00	0.76	0.85	0.85	0.91	0.88	0.68	0.74
time (sec)	N/A	0.272	0.093	0.671	0.030	0.092	0.351	0.181	0.312	5.450

Problem 278	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F(-1)	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	305	317	187	233	296	315	0	646	289	346
N.S.	1	1.04	0.61	0.76	0.97	1.03	0.00	2.12	0.95	1.13
time (sec)	N/A	0.819	0.156	1.187	0.060	0.096	0.000	0.170	0.312	5.593

Problem 279	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F(-1)	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	240	248	131	158	211	230	0	444	197	256
N.S.	1	1.03	0.55	0.66	0.88	0.96	0.00	1.85	0.82	1.07
time (sec)	N/A	0.690	0.107	1.177	0.070	0.094	0.000	0.186	0.339	5.586

Problem 280	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	175	179	88	100	140	159	0	276	122	180
N.S.	1	1.02	0.50	0.57	0.80	0.91	0.00	1.58	0.70	1.03
time (sec)	N/A	0.544	0.071	1.248	0.063	0.093	0.000	0.177	0.276	5.333

Problem 281	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	110	110	55	59	83	102	0	146	63	121
N.S.	1	1.00	0.50	0.54	0.75	0.93	0.00	1.33	0.57	1.10
time (sec)	N/A	0.386	0.045	1.116	0.049	0.080	0.000	0.186	0.319	5.294

Problem 282	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	48	48	37	40	18	57	0	18	25	49
N.S.	1	1.00	0.77	0.83	0.38	1.19	0.00	0.38	0.52	1.02
time (sec)	N/A	0.276	0.015	1.046	0.045	0.088	0.000	0.173	0.258	5.296

Problem 283	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	128	128	118	153	0	341	0	106	65	0
N.S.	1	1.00	0.92	1.20	0.00	2.66	0.00	0.83	0.51	0.00
time (sec)	N/A	0.480	0.106	1.085	0.000	0.097	0.000	0.151	0.256	0.000

Problem 284	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	B	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	129	129	112	153	0	482	0	111	168	0
N.S.	1	1.00	0.87	1.19	0.00	3.74	0.00	0.86	1.30	0.00
time (sec)	N/A	0.434	0.250	1.080	0.000	0.104	0.000	0.162	0.242	0.000

Problem 285	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	B	F(-1)	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	199	194	165	282	0	748	0	228	363	0
N.S.	1	0.97	0.83	1.42	0.00	3.76	0.00	1.15	1.82	0.00
time (sec)	N/A	0.583	0.452	1.088	0.000	0.128	0.000	0.187	0.308	0.000

Problem 286	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	B	F(-1)	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	264	271	200	447	0	1114	0	337	626	0
N.S.	1	1.03	0.76	1.69	0.00	4.22	0.00	1.28	2.37	0.00
time (sec)	N/A	0.696	0.708	1.085	0.000	0.134	0.000	0.190	0.247	0.000

Problem 287	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F(-1)	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	305	317	187	235	373	392	0	1115	374	424
N.S.	1	1.04	0.61	0.77	1.22	1.29	0.00	3.66	1.23	1.39
time (sec)	N/A	0.785	0.182	1.196	0.066	0.094	0.000	0.185	0.244	5.627

Problem 288	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	240	248	132	160	273	292	0	784	265	320
N.S.	1	1.03	0.55	0.67	1.14	1.22	0.00	3.27	1.10	1.33
time (sec)	N/A	0.680	0.133	1.235	0.064	0.096	0.000	0.194	0.261	5.558

Problem 289	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	175	179	88	102	189	208	0	500	173	230
N.S.	1	1.02	0.50	0.58	1.08	1.19	0.00	2.86	0.99	1.31
time (sec)	N/A	0.512	0.094	1.236	0.071	0.098	0.000	0.162	0.210	5.431

Problem 290	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	110	110	55	61	98	135	0	278	97	128
N.S.	1	1.00	0.50	0.55	0.89	1.23	0.00	2.53	0.88	1.16
time (sec)	N/A	0.391	0.058	1.217	0.049	0.104	0.000	0.159	0.207	5.399

Problem 291	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	48	48	37	42	43	74	0	114	42	62
N.S.	1	1.00	0.77	0.88	0.90	1.54	0.00	2.38	0.88	1.29
time (sec)	N/A	0.279	0.022	1.049	0.050	0.091	0.000	0.191	0.218	5.224

Problem 292	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	181	191	137	265	0	431	0	199	157	0
N.S.	1	1.06	0.76	1.46	0.00	2.38	0.00	1.10	0.87	0.00
time (sec)	N/A	0.585	0.185	1.088	0.000	0.108	0.000	0.170	0.242	0.000

Problem 293	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F(-1)	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	175	181	142	304	0	443	0	198	164	0
N.S.	1	1.03	0.81	1.74	0.00	2.53	0.00	1.13	0.94	0.00
time (sec)	N/A	0.578	0.296	1.066	0.000	0.110	0.000	0.181	0.224	0.000

Problem 294	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F(-1)	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	185	184	140	271	0	648	0	208	322	0
N.S.	1	0.99	0.76	1.46	0.00	3.50	0.00	1.12	1.74	0.00
time (sec)	N/A	0.520	0.446	1.061	0.000	0.119	0.000	0.220	0.204	0.000

Problem 295	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	B	F(-1)	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	250	249	198	447	0	966	0	322	560	0
N.S.	1	1.00	0.79	1.79	0.00	3.86	0.00	1.29	2.24	0.00
time (sec)	N/A	0.671	0.710	1.070	0.000	0.119	0.000	0.181	0.218	0.000

Problem 296	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	B	F(-1)	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	315	326	242	652	0	1386	0	532	864	0
N.S.	1	1.03	0.77	2.07	0.00	4.40	0.00	1.69	2.74	0.00
time (sec)	N/A	0.857	1.128	1.085	0.000	0.149	0.000	0.191	0.259	0.000

Problem 297	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F(-1)	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	305	317	197	235	448	467	0	1674	459	501
N.S.	1	1.04	0.65	0.77	1.47	1.53	0.00	5.49	1.50	1.64
time (sec)	N/A	0.771	0.161	1.191	0.089	0.097	0.000	0.184	0.208	5.769

Problem 298	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F(-1)	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	240	248	142	160	335	354	0	1195	333	383
N.S.	1	1.03	0.59	0.67	1.40	1.48	0.00	4.98	1.39	1.60
time (sec)	N/A	0.663	0.126	1.202	0.066	0.093	0.000	0.165	0.214	5.644

Problem 299	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	175	179	98	102	217	254	0	780	224	241
N.S.	1	1.02	0.56	0.58	1.24	1.45	0.00	4.46	1.28	1.38
time (sec)	N/A	0.522	0.076	1.214	0.052	0.089	0.000	0.175	0.244	5.516

Problem 300	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	110	110	65	61	132	169	0	520	131	156
N.S.	1	1.00	0.59	0.55	1.20	1.54	0.00	4.73	1.19	1.42
time (sec)	N/A	0.393	0.060	1.185	0.063	0.087	0.000	0.216	0.266	5.409

Problem 301	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	F	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	48	48	37	42	60	91	0	218	59	79
N.S.	1	1.00	0.77	0.88	1.25	1.90	0.00	4.54	1.23	1.65
time (sec)	N/A	0.276	0.032	1.039	0.045	0.088	0.000	0.190	0.225	5.291

Problem 302	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	A	F(-1)	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	240	254	170	427	0	570	0	323	286	0
N.S.	1	1.06	0.71	1.78	0.00	2.38	0.00	1.35	1.19	0.00
time (sec)	N/A	0.704	0.226	1.050	0.000	0.110	0.000	0.199	0.211	0.000

Problem 303	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	A	F(-1)	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	233	244	178	511	0	611	0	325	350	0
N.S.	1	1.05	0.76	2.19	0.00	2.62	0.00	1.39	1.50	0.00
time (sec)	N/A	0.684	0.389	1.066	0.000	0.113	0.000	0.208	0.229	0.000

Problem 304	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	A	F(-1)	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	236	236	184	517	0	594	0	335	310	0
N.S.	1	1.00	0.78	2.19	0.00	2.52	0.00	1.42	1.31	0.00
time (sec)	N/A	0.634	0.475	1.079	0.000	0.121	0.000	0.198	0.248	0.000

Problem 305	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	A	F(-1)	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	236	239	173	433	0	836	0	294	500	0
N.S.	1	1.01	0.73	1.83	0.00	3.54	0.00	1.25	2.12	0.00
time (sec)	N/A	0.619	0.763	1.081	0.000	0.125	0.000	0.225	0.212	0.000

Problem 306	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	B	F(-1)	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	301	304	245	652	0	1204	0	506	782	0
N.S.	1	1.01	0.81	2.17	0.00	4.00	0.00	1.68	2.60	0.00
time (sec)	N/A	0.780	1.243	1.070	0.000	0.134	0.000	0.222	0.203	0.000

Problem 307	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	B	F(-1)	B	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	366	381	300	900	0	1686	0	683	1128	0
N.S.	1	1.04	0.82	2.46	0.00	4.61	0.00	1.87	3.08	0.00
time (sec)	N/A	0.943	1.670	1.091	0.000	0.170	0.000	0.223	0.222	0.000

Problem 308	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	236	248	131	150	192	173	0	282	135	194
N.S.	1	1.05	0.56	0.64	0.81	0.73	0.00	1.19	0.57	0.82
time (sec)	N/A	0.658	0.134	1.056	0.052	0.086	0.000	0.152	0.246	5.746

Problem 309	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	173	179	87	92	122	117	0	166	77	131
N.S.	1	1.03	0.50	0.53	0.71	0.68	0.00	0.96	0.45	0.76
time (sec)	N/A	0.510	0.131	1.066	0.049	0.087	0.000	0.160	0.181	5.950

Problem 310	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	108	110	54	51	65	73	0	89	35	85
N.S.	1	1.02	0.50	0.47	0.60	0.68	0.00	0.82	0.32	0.79
time (sec)	N/A	0.378	0.045	1.065	0.046	0.088	0.000	0.157	0.208	5.776

Problem 311	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	46	46	35	32	18	49	0	36	17	54
N.S.	1	1.00	0.76	0.70	0.39	1.07	0.00	0.78	0.37	1.17
time (sec)	N/A	0.271	0.009	1.030	0.045	0.085	0.000	0.156	0.210	5.562

Problem 312	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	84	84	97	81	0	234	0	63	65	0
N.S.	1	1.00	1.15	0.96	0.00	2.79	0.00	0.75	0.77	0.00
time (sec)	N/A	0.322	0.083	1.042	0.000	0.098	0.000	0.155	0.258	0.000

Problem 313	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	B	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	140	139	135	162	0	557	0	124	193	0
N.S.	1	0.99	0.96	1.16	0.00	3.98	0.00	0.89	1.38	0.00
time (sec)	N/A	0.448	0.202	1.060	0.000	0.100	0.000	0.153	0.221	0.000

Problem 314	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	B	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	207	216	174	282	0	869	0	254	414	0
N.S.	1	1.04	0.84	1.36	0.00	4.20	0.00	1.23	2.00	0.00
time (sec)	N/A	0.570	0.293	1.089	0.000	0.108	0.000	0.155	0.225	0.000

Problem 315	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	171	175	87	102	79	141	0	114	78	174
N.S.	1	1.02	0.51	0.60	0.46	0.82	0.00	0.67	0.46	1.02
time (sec)	N/A	0.506	0.072	1.043	0.048	0.086	0.000	0.155	0.224	5.847

Problem 316	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	106	106	51	60	36	98	0	50	37	116
N.S.	1	1.00	0.48	0.57	0.34	0.92	0.00	0.47	0.35	1.09
time (sec)	N/A	0.397	0.052	1.043	0.045	0.085	0.000	0.154	0.235	5.857

Problem 317	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	46	46	35	42	18	74	0	18	19	82
N.S.	1	1.00	0.76	0.91	0.39	1.61	0.00	0.39	0.41	1.78
time (sec)	N/A	0.272	0.011	1.041	0.067	0.090	0.000	0.161	0.215	5.907

Problem 318	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	139	139	115	126	0	500	0	124	109	0
N.S.	1	1.00	0.83	0.91	0.00	3.60	0.00	0.89	0.78	0.00
time (sec)	N/A	0.482	0.095	1.066	0.000	0.094	0.000	0.174	0.232	0.000

Problem 319	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	B	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	196	213	141	225	0	764	0	266	267	0
N.S.	1	1.09	0.72	1.15	0.00	3.90	0.00	1.36	1.36	0.00
time (sec)	N/A	0.600	0.355	1.097	0.000	0.117	0.000	0.144	0.214	0.000

Problem 320	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	B	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	269	290	183	374	0	1140	0	374	503	0
N.S.	1	1.08	0.68	1.39	0.00	4.24	0.00	1.39	1.87	0.00
time (sec)	N/A	0.783	0.496	1.273	0.000	0.149	0.000	0.173	0.267	0.000

Problem 321	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	171	173	84	102	93	174	0	171	89	195
N.S.	1	1.01	0.49	0.60	0.54	1.02	0.00	1.00	0.52	1.14
time (sec)	N/A	0.519	0.080	1.082	0.057	0.100	0.000	0.178	0.194	5.538

Problem 322	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	108	108	53	60	50	131	0	79	48	147
N.S.	1	1.00	0.49	0.56	0.46	1.21	0.00	0.73	0.44	1.36
time (sec)	N/A	0.402	0.055	1.069	0.048	0.087	0.000	0.186	0.228	5.519

Problem 323	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	F(-1)	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	48	48	37	42	28	107	0	39	29	110
N.S.	1	1.00	0.77	0.88	0.58	2.23	0.00	0.81	0.60	2.29
time (sec)	N/A	0.274	0.018	1.069	0.062	0.082	0.000	0.184	0.227	5.456

Problem 324	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	B	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	196	273	135	224	0	788	0	197	273	0
N.S.	1	1.39	0.69	1.14	0.00	4.02	0.00	1.01	1.39	0.00
time (sec)	N/A	0.736	0.206	1.066	0.000	0.136	0.000	0.215	0.231	0.000

Problem 325	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	B	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	255	289	177	423	0	1236	0	322	584	0
N.S.	1	1.13	0.69	1.66	0.00	4.85	0.00	1.26	2.29	0.00
time (sec)	N/A	0.773	0.390	1.065	0.000	0.196	0.000	0.196	0.277	0.000

Problem 326	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	B	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	332	366	240	658	0	1778	0	469	930	0
N.S.	1	1.10	0.72	1.98	0.00	5.36	0.00	1.41	2.80	0.00
time (sec)	N/A	0.936	0.743	1.080	0.000	0.369	0.000	0.200	0.275	0.000

Problem 327	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	B	F	B	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	394	443	294	920	0	2388	0	742	1379	0
N.S.	1	1.12	0.75	2.34	0.00	6.06	0.00	1.88	3.50	0.00
time (sec)	N/A	1.070	1.327	1.076	0.000	0.843	0.000	0.187	0.300	0.000

Problem 328	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	52	52	52	58	0	144	0	31	44	0
N.S.	1	1.00	1.00	1.12	0.00	2.77	0.00	0.60	0.85	0.00
time (sec)	N/A	0.279	0.015	0.650	0.000	0.093	0.000	0.153	0.239	0.000

Problem 329	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	53	53	54	63	0	147	0	29	28	0
N.S.	1	1.00	1.02	1.19	0.00	2.77	0.00	0.55	0.53	0.00
time (sec)	N/A	0.278	0.139	0.645	0.000	0.089	0.000	0.150	0.189	0.000

Problem 330	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	F	F	F	F(-1)
verified	N/A	No	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	502	604	95	0	0	0	0	0	128	0
N.S.	1	1.20	0.19	0.00	0.00	0.00	0.00	0.00	0.25	0.00
time (sec)	N/A	0.969	0.070	0.000	0.000	0.000	0.000	0.000	2.165	0.000

Problem 331	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	B	A	F	F	F	F	F	F	F(-1)
verified	N/A	No	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	84	1855	102	0	0	0	0	0	143	0
N.S.	1	22.08	1.21	0.00	0.00	0.00	0.00	0.00	1.70	0.00
time (sec)	N/A	2.379	0.079	0.000	0.000	0.000	0.000	0.000	0.272	0.000

Problem 332	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	B	A	F	F	F	F	F	F	F(-1)
verified	N/A	No	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	84	1786	93	0	0	0	0	0	103	0
N.S.	1	21.26	1.11	0.00	0.00	0.00	0.00	0.00	1.23	0.00
time (sec)	N/A	2.012	0.051	0.000	0.000	0.000	0.000	0.000	0.268	0.000

Problem 333	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	B	A	F	F	F	F	F	F	F(-1)
verified	N/A	No	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	82	1719	88	0	0	0	0	0	63	0
N.S.	1	20.96	1.07	0.00	0.00	0.00	0.00	0.00	0.77	0.00
time (sec)	N/A	1.783	0.033	0.000	0.000	0.000	0.000	0.000	0.256	0.000

Problem 334	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	B	A	F	F	F	F	F	F	F(-1)
verified	N/A	No	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	108	1661	95	0	0	0	0	0	28	0
N.S.	1	15.38	0.88	0.00	0.00	0.00	0.00	0.00	0.26	0.00
time (sec)	N/A	1.707	0.036	0.000	0.000	0.000	0.000	0.000	0.211	0.000

Problem 335	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	82	116	98	0	0	0	0	0	62	0
N.S.	1	1.41	1.20	0.00	0.00	0.00	0.00	0.00	0.76	0.00
time (sec)	N/A	0.398	0.049	0.000	0.000	0.000	0.000	0.000	0.229	0.000

Problem 336	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	84	118	102	0	0	0	0	0	99	0
N.S.	1	1.40	1.21	0.00	0.00	0.00	0.00	0.00	1.18	0.00
time (sec)	N/A	0.377	0.054	0.000	0.000	0.000	0.000	0.000	0.223	0.000

Problem 337	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	F	F	F	F(-1)
verified	N/A	No	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	350	581	110	0	0	0	0	0	1122	0
N.S.	1	1.66	0.31	0.00	0.00	0.00	0.00	0.00	3.21	0.00
time (sec)	N/A	0.984	0.048	0.000	0.000	0.000	0.000	0.000	1.290	0.000

Problem 338	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	F	F	F	F(-1)
verified	N/A	No	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	292	514	88	0	0	0	0	0	697	0
N.S.	1	1.76	0.30	0.00	0.00	0.00	0.00	0.00	2.39	0.00
time (sec)	N/A	0.791	0.066	0.000	0.000	0.000	0.000	0.000	0.924	0.000

Problem 339	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	B	C	F	F	F	F	F	F	F(-1)
verified	N/A	No	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	206	454	96	0	0	0	0	0	536	0
N.S.	1	2.20	0.47	0.00	0.00	0.00	0.00	0.00	2.60	0.00
time (sec)	N/A	0.713	0.032	0.000	0.000	0.000	0.000	0.000	0.600	0.000

Problem 340	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	F	F	F	F(-1)
verified	N/A	No	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	171	236	98	0	0	0	0	0	254	0
N.S.	1	1.38	0.57	0.00	0.00	0.00	0.00	0.00	1.49	0.00
time (sec)	N/A	0.669	0.045	0.000	0.000	0.000	0.000	0.000	0.544	0.000

Problem 341	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	F	F	F	F(-1)
verified	N/A	No	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	176	235	106	0	0	0	0	0	979	0
N.S.	1	1.34	0.60	0.00	0.00	0.00	0.00	0.00	5.56	0.00
time (sec)	N/A	0.657	0.055	0.000	0.000	0.000	0.000	0.000	0.852	0.000

Problem 342	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	F	F	F	F(-1)
verified	N/A	No	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	237	304	110	0	0	0	0	0	0	0
N.S.	1	1.28	0.46	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.752	0.048	0.000	0.000	0.000	0.000	0.000	1.071	0.000

Problem 343	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	F	F	F	F(-1)
verified	N/A	No	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	304	369	110	0	0	0	0	0	0	0
N.S.	1	1.21	0.36	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.842	0.045	0.000	0.000	0.000	0.000	0.000	1.419	0.000

Problem 344	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	B	C	F	F	F	F	F	F	F(-1)
verified	N/A	No	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	346	1148	110	0	0	0	0	0	1122	0
N.S.	1	3.32	0.32	0.00	0.00	0.00	0.00	0.00	3.24	0.00
time (sec)	N/A	1.527	0.051	0.000	0.000	0.000	0.000	0.000	1.896	0.000

Problem 345	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	B	C	F	F	F	F	F	F	F(-1)
verified	N/A	No	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	288	1081	88	0	0	0	0	0	697	0
N.S.	1	3.75	0.31	0.00	0.00	0.00	0.00	0.00	2.42	0.00
time (sec)	N/A	1.280	0.064	0.000	0.000	0.000	0.000	0.000	1.170	0.000

Problem 346	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	B	C	F	F	F	F	F	F	F(-1)
verified	N/A	No	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	206	1021	96	0	0	0	0	0	538	0
N.S.	1	4.96	0.47	0.00	0.00	0.00	0.00	0.00	2.61	0.00
time (sec)	N/A	1.208	0.034	0.000	0.000	0.000	0.000	0.000	0.756	0.000

Problem 347	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	F	F	F	F(-1)
verified	N/A	No	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	169	326	98	0	0	0	0	0	257	0
N.S.	1	1.93	0.58	0.00	0.00	0.00	0.00	0.00	1.52	0.00
time (sec)	N/A	0.755	0.041	0.000	0.000	0.000	0.000	0.000	0.611	0.000

Problem 348	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	F	F	F	F(-1)
verified	N/A	No	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	166	307	106	0	0	0	0	0	967	0
N.S.	1	1.85	0.64	0.00	0.00	0.00	0.00	0.00	5.83	0.00
time (sec)	N/A	0.738	0.046	0.000	0.000	0.000	0.000	0.000	0.841	0.000

Problem 349	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	B	C	F	F	F	F	F	F	F(-1)
verified	N/A	No	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	178	373	110	0	0	0	0	0	0	0
N.S.	1	2.10	0.62	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.862	0.046	0.000	0.000	0.000	0.000	0.000	1.213	0.000

Problem 350	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	F	F	F	F(-1)
verified	N/A	No	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	241	438	110	0	0	0	0	0	0	0
N.S.	1	1.82	0.46	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.957	0.085	0.000	0.000	0.000	0.000	0.000	1.718	0.000

Problem 351	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	F	F	F	F(-1)
verified	N/A	No	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	304	503	110	0	0	0	0	0	0	0
N.S.	1	1.65	0.36	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.025	0.051	0.000	0.000	0.000	0.000	0.000	2.166	0.000

Problem 352	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	F	F	F	F(-1)
verified	N/A	No	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	473	668	112	0	0	0	0	0	0	0
N.S.	1	1.41	0.24	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.169	0.074	0.000	0.000	0.000	0.000	0.000	2.629	0.000

Problem 353	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	F	F	F	F(-1)
verified	N/A	No	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	415	601	112	0	0	0	0	0	1373	0
N.S.	1	1.45	0.27	0.00	0.00	0.00	0.00	0.00	3.31	0.00
time (sec)	N/A	0.921	0.053	0.000	0.000	0.000	0.000	0.000	2.128	0.000

Problem 354	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	F	F	F	F(-1)
verified	N/A	No	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	279	541	96	0	0	0	0	0	1125	0
N.S.	1	1.94	0.34	0.00	0.00	0.00	0.00	0.00	4.03	0.00
time (sec)	N/A	0.792	0.056	0.000	0.000	0.000	0.000	0.000	1.218	0.000

Problem 355	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	F	F	F	F(-1)
verified	N/A	No	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	297	341	98	0	0	0	0	0	694	0
N.S.	1	1.15	0.33	0.00	0.00	0.00	0.00	0.00	2.34	0.00
time (sec)	N/A	0.815	0.043	0.000	0.000	0.000	0.000	0.000	0.884	0.000

Problem 356	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	F	F	F	F(-1)
verified	N/A	No	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	226	288	98	0	0	0	0	0	585	0
N.S.	1	1.27	0.43	0.00	0.00	0.00	0.00	0.00	2.59	0.00
time (sec)	N/A	0.692	0.037	0.000	0.000	0.000	0.000	0.000	0.809	0.000

Problem 357	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	F	F	F	F(-1)
verified	N/A	No	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	214	280	108	0	0	0	0	0	1406	0
N.S.	1	1.31	0.50	0.00	0.00	0.00	0.00	0.00	6.57	0.00
time (sec)	N/A	0.686	0.050	0.000	0.000	0.000	0.000	0.000	1.504	0.000

Problem 358	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	F	F	F	F(-1)
verified	N/A	No	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	223	279	112	0	0	0	0	0	0	0
N.S.	1	1.25	0.50	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.743	0.057	0.000	0.000	0.000	0.000	0.000	1.833	0.000

Problem 359	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	F	F	F	F(-1)
verified	N/A	No	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	290	348	112	0	0	0	0	0	0	0
N.S.	1	1.20	0.39	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.824	0.059	0.000	0.000	0.000	0.000	0.000	2.780	0.000

Problem 360	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	F	F	F	F(-1)
verified	N/A	No	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	353	413	112	0	0	0	0	0	0	0
N.S.	1	1.17	0.32	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.901	0.069	0.000	0.000	0.000	0.000	0.000	3.478	0.000

Problem 361	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	B	C	F	F	F	F	F	F	F(-1)
verified	N/A	No	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	290	1128	102	0	0	0	0	0	143	0
N.S.	1	3.89	0.35	0.00	0.00	0.00	0.00	0.00	0.49	0.00
time (sec)	N/A	1.549	0.065	0.000	0.000	0.000	0.000	0.000	0.726	0.000

Problem 362	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	B	C	F	F	F	F	F	F	F(-1)
verified	N/A	No	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	230	1059	93	0	0	0	0	0	103	0
N.S.	1	4.60	0.40	0.00	0.00	0.00	0.00	0.00	0.45	0.00
time (sec)	N/A	0.768	0.052	0.000	0.000	0.000	0.000	0.000	0.473	0.000

Problem 363	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	B	C	F	F	F	F	F	F	F(-1)
verified	N/A	No	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	171	992	88	0	0	0	0	0	63	0
N.S.	1	5.80	0.51	0.00	0.00	0.00	0.00	0.00	0.37	0.00
time (sec)	N/A	0.683	0.034	0.000	0.000	0.000	0.000	0.000	0.311	0.000

Problem 364	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	B	C	F	F	F	F	F	F	F(-1)
verified	N/A	No	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	141	936	95	0	0	0	0	0	28	0
N.S.	1	6.64	0.67	0.00	0.00	0.00	0.00	0.00	0.20	0.00
time (sec)	N/A	0.602	0.047	0.000	0.000	0.000	0.000	0.000	0.230	0.000

Problem 365	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	B	C	F	F	F	F	F	F	F(-1)
verified	N/A	No	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	130	329	98	0	0	0	0	0	62	0
N.S.	1	2.53	0.75	0.00	0.00	0.00	0.00	0.00	0.48	0.00
time (sec)	N/A	0.505	0.047	0.000	0.000	0.000	0.000	0.000	0.217	0.000

Problem 366	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	B	C	F	F	F	F	F	F	F(-1)
verified	N/A	No	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	189	394	102	0	0	0	0	0	99	0
N.S.	1	2.08	0.54	0.00	0.00	0.00	0.00	0.00	0.52	0.00
time (sec)	N/A	0.531	0.048	0.000	0.000	0.000	0.000	0.000	0.253	0.000

Problem 367	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	F	F	F	F(-1)
verified	N/A	No	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	249	459	102	0	0	0	0	0	136	0
N.S.	1	1.84	0.41	0.00	0.00	0.00	0.00	0.00	0.55	0.00
time (sec)	N/A	0.601	0.051	0.000	0.000	0.000	0.000	0.000	0.323	0.000

Problem 368	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	F	F	F	F(-1)
verified	N/A	No	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	294	578	100	0	0	0	0	0	143	0
N.S.	1	1.97	0.34	0.00	0.00	0.00	0.00	0.00	0.49	0.00
time (sec)	N/A	0.646	0.061	0.000	0.000	0.000	0.000	0.000	0.655	0.000

Problem 369	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	B	C	F	F	F	F	F	F	F(-1)
verified	N/A	No	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	234	509	91	0	0	0	0	0	103	0
N.S.	1	2.18	0.39	0.00	0.00	0.00	0.00	0.00	0.44	0.00
time (sec)	N/A	0.507	0.046	0.000	0.000	0.000	0.000	0.000	0.406	0.000

Problem 370	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	B	C	F	F	F	F	F	F	F(-1)
verified	N/A	No	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	174	442	86	0	0	0	0	0	63	0
N.S.	1	2.54	0.49	0.00	0.00	0.00	0.00	0.00	0.36	0.00
time (sec)	N/A	0.407	0.034	0.000	0.000	0.000	0.000	0.000	0.287	0.000

Problem 371	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	B	C	F	F	F	F	F	F	F(-1)
verified	N/A	No	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	142	391	93	0	0	0	0	0	28	0
N.S.	1	2.75	0.65	0.00	0.00	0.00	0.00	0.00	0.20	0.00
time (sec)	N/A	0.346	0.032	0.000	0.000	0.000	0.000	0.000	0.204	0.000

Problem 372	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	F	F	F	F(-1)
verified	N/A	No	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	187	260	96	0	0	0	0	0	62	0
N.S.	1	1.39	0.51	0.00	0.00	0.00	0.00	0.00	0.33	0.00
time (sec)	N/A	0.419	0.050	0.000	0.000	0.000	0.000	0.000	0.218	0.000

Problem 373	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	F	F	F	F(-1)
verified	N/A	No	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	245	325	100	0	0	0	0	0	99	0
N.S.	1	1.33	0.41	0.00	0.00	0.00	0.00	0.00	0.40	0.00
time (sec)	N/A	0.470	0.049	0.000	0.000	0.000	0.000	0.000	0.256	0.000

Problem 374	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	F	F	F	F(-1)
verified	N/A	No	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	309	390	100	0	0	0	0	0	136	0
N.S.	1	1.26	0.32	0.00	0.00	0.00	0.00	0.00	0.44	0.00
time (sec)	N/A	0.530	0.065	0.000	0.000	0.000	0.000	0.000	0.261	0.000

Problem 375	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	B	C	F	F	F	F	F	F	F(-1)
verified	N/A	No	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	285	1198	100	0	0	0	0	0	287	0
N.S.	1	4.20	0.35	0.00	0.00	0.00	0.00	0.00	1.01	0.00
time (sec)	N/A	1.103	0.051	0.000	0.000	0.000	0.000	0.000	0.810	0.000

Problem 376	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	B	C	F	F	F	F	F	F	F(-1)
verified	N/A	No	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	224	1129	93	0	0	0	0	0	211	0
N.S.	1	5.04	0.42	0.00	0.00	0.00	0.00	0.00	0.94	0.00
time (sec)	N/A	0.939	0.038	0.000	0.000	0.000	0.000	0.000	0.544	0.000

Problem 377	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	B	C	F	F	F	F	F	F	F(-1)
verified	N/A	No	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	170	986	91	0	0	0	0	0	135	0
N.S.	1	5.80	0.54	0.00	0.00	0.00	0.00	0.00	0.79	0.00
time (sec)	N/A	0.694	0.031	0.000	0.000	0.000	0.000	0.000	0.354	0.000

Problem 378	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	B	C	F	F	F	F	F	F	F(-1)
verified	N/A	No	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	136	1002	86	0	0	0	0	0	64	0
N.S.	1	7.37	0.63	0.00	0.00	0.00	0.00	0.00	0.47	0.00
time (sec)	N/A	0.705	0.030	0.000	0.000	0.000	0.000	0.000	0.246	0.000

Problem 379	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	B	C	F	F	F	F	F	F	F(-1)
verified	N/A	No	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	198	1016	94	0	0	0	0	0	132	0
N.S.	1	5.13	0.47	0.00	0.00	0.00	0.00	0.00	0.67	0.00
time (sec)	N/A	0.669	0.029	0.000	0.000	0.000	0.000	0.000	0.248	0.000

Problem 380	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	F	F	F	F(-1)
verified	N/A	No	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	242	453	96	0	0	0	0	0	208	0
N.S.	1	1.87	0.40	0.00	0.00	0.00	0.00	0.00	0.86	0.00
time (sec)	N/A	0.581	0.051	0.000	0.000	0.000	0.000	0.000	0.311	0.000

Problem 381	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	F	F	F	F(-1)
verified	N/A	No	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	303	518	100	0	0	0	0	0	284	0
N.S.	1	1.71	0.33	0.00	0.00	0.00	0.00	0.00	0.94	0.00
time (sec)	N/A	0.639	0.047	0.000	0.000	0.000	0.000	0.000	0.358	0.000

Problem 382	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	B	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	130	130	114	436	1819	1156	7164	1999	1376	1202
N.S.	1	1.00	0.88	3.35	13.99	8.89	55.11	15.38	10.58	9.25
time (sec)	N/A	0.350	0.137	1.082	0.115	0.121	2.427	0.166	0.202	6.070

Problem 383	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	B	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	90	90	79	183	691	479	2494	805	535	486
N.S.	1	1.00	0.88	2.03	7.68	5.32	27.71	8.94	5.94	5.40
time (sec)	N/A	0.283	0.124	0.954	0.067	0.102	1.022	0.127	0.198	5.460

Problem 384	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	52	52	45	55	174	136	556	219	134	141
N.S.	1	1.00	0.87	1.06	3.35	2.62	10.69	4.21	2.58	2.71
time (sec)	N/A	0.214	0.070	0.511	0.044	0.096	0.422	0.115	0.194	5.383

Problem 385	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	54	54	54	0	0	0	0	0	36	0
N.S.	1	1.00	1.00	0.00	0.00	0.00	0.00	0.00	0.67	0.00
time (sec)	N/A	0.198	0.080	0.000	0.000	0.000	0.000	0.000	0.218	0.000

Problem 386	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	61	61	59	0	0	0	0	0	111	0
N.S.	1	1.00	0.97	0.00	0.00	0.00	0.00	0.00	1.82	0.00
time (sec)	N/A	0.198	0.094	0.000	0.000	0.000	0.000	0.000	0.217	0.000

Problem 387	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	64	64	63	0	0	0	0	0	220	0
N.S.	1	1.00	0.98	0.00	0.00	0.00	0.00	0.00	3.44	0.00
time (sec)	N/A	0.199	0.100	0.000	0.000	0.000	0.000	0.000	0.197	0.000

Problem 388	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F(-2)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	65	65	63	0	0	0	0	0	363	0
N.S.	1	1.00	0.97	0.00	0.00	0.00	0.00	0.00	5.58	0.00
time (sec)	N/A	0.204	0.127	0.000	0.000	0.000	0.000	0.000	0.264	0.000

Problem 389	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	89	124	112	0	0	0	0	0	0	0
N.S.	1	1.39	1.26	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.269	0.097	0.000	0.000	0.000	0.000	0.000	0.385	0.000

Problem 390	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	89	124	112	0	0	0	0	0	0	0
N.S.	1	1.39	1.26	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.260	0.081	0.000	0.000	0.000	0.000	0.000	0.360	0.000

Problem 391	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	B	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	87	176	94	0	0	0	0	0	0	0
N.S.	1	2.02	1.08	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.292	0.084	0.000	0.000	0.000	0.000	0.000	0.222	0.000

Problem 392	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	113	113	96	0	0	0	0	0	1594	0
N.S.	1	1.00	0.85	0.00	0.00	0.00	0.00	0.00	14.11	0.00
time (sec)	N/A	0.221	0.066	0.000	0.000	0.000	0.000	0.000	0.206	0.000

Problem 393	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	87	93	81	0	0	0	0	0	258	0
N.S.	1	1.07	0.93	0.00	0.00	0.00	0.00	0.00	2.97	0.00
time (sec)	N/A	0.255	0.066	0.000	0.000	0.000	0.000	0.000	0.206	0.000

Problem 394	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	86	120	108	0	0	0	0	0	0	0
N.S.	1	1.40	1.26	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.272	0.083	0.000	0.000	0.000	0.000	0.000	0.231	0.000

Problem 395	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	90	124	112	0	0	0	0	0	0	0
N.S.	1	1.38	1.24	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.265	0.083	0.000	0.000	0.000	0.000	0.000	0.240	0.000

Problem 396	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	92	126	107	0	0	0	0	0	0	0
N.S.	1	1.37	1.16	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.302	0.082	0.000	0.000	0.000	0.000	0.000	0.246	0.000

Problem 397	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	B	F	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	288	296	217	745	0	1051	0	0	135	1036
N.S.	1	1.03	0.75	2.59	0.00	3.65	0.00	0.00	0.47	3.60
time (sec)	N/A	0.495	0.153	1.829	0.000	0.169	0.000	0.000	0.186	6.425

Problem 398	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	B	F	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	206	212	131	381	0	580	0	0	115	595
N.S.	1	1.03	0.64	1.85	0.00	2.82	0.00	0.00	0.56	2.89
time (sec)	N/A	0.361	0.108	1.546	0.000	0.121	0.000	0.000	0.227	6.385

Problem 399	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	128	128	76	170	0	255	0	0	95	293
N.S.	1	1.00	0.59	1.33	0.00	1.99	0.00	0.00	0.74	2.29
time (sec)	N/A	0.263	0.089	1.537	0.000	0.107	0.000	0.000	0.182	5.802

Problem 400	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	60	60	49	75	0	92	0	0	75	150
N.S.	1	1.00	0.82	1.25	0.00	1.53	0.00	0.00	1.25	2.50
time (sec)	N/A	0.188	0.064	1.517	0.000	0.097	0.000	0.000	0.192	5.666

Problem 401	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	88	107	95	0	0	0	0	0	55	0
N.S.	1	1.22	1.08	0.00	0.00	0.00	0.00	0.00	0.62	0.00
time (sec)	N/A	0.293	0.061	0.000	0.000	0.000	0.000	0.000	0.206	0.000

Problem 402	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	91	116	97	0	0	0	0	0	40	0
N.S.	1	1.27	1.07	0.00	0.00	0.00	0.00	0.00	0.44	0.00
time (sec)	N/A	0.295	0.079	0.000	0.000	0.000	0.000	0.000	0.225	0.000

Problem 403	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F(-2)	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	54	54	42	57	33	57	0	81	38	57
N.S.	1	1.00	0.78	1.06	0.61	1.06	0.00	1.50	0.70	1.06
time (sec)	N/A	0.189	0.019	1.368	0.041	0.100	0.000	0.157	0.202	5.664

Problem 404	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	52	52	41	56	30	55	190	79	38	56
N.S.	1	1.00	0.79	1.08	0.58	1.06	3.65	1.52	0.73	1.08
time (sec)	N/A	0.188	0.011	1.360	0.069	0.088	134.398	0.130	0.205	5.547

2.3 Detailed conclusion table specific for Rubi results

The following table is specific to Rubi only. It gives additional statistics for each integral. the column **steps** is the number of steps used by Rubi to obtain the antiderivative. The **rules** column is the number of unique rules used. The **integrand size** column is the leaf size of the integrand. Finally the ratio $\frac{\text{number of rules}}{\text{integrand size}}$ is also given. The larger this ratio is, the harder the integral is to solve. In this test file, problem number [375] had the largest ratio of [.3243240000000000001]

Table 2.1: Rubi specific breakdown of results for each integral

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1	A	2	2	1.00	27	0.074
2	A	2	2	1.00	27	0.074
3	A	2	2	1.00	25	0.080
4	A	1	1	1.00	19	0.053
5	A	2	2	1.00	27	0.074
6	A	2	2	1.00	27	0.074
7	A	2	2	1.00	27	0.074
8	A	2	2	1.00	27	0.074
9	A	2	2	1.00	27	0.074
10	A	2	2	1.00	27	0.074
11	A	2	2	1.00	29	0.069
12	A	2	2	1.00	29	0.069
13	A	2	2	1.00	27	0.074
14	A	2	2	1.18	21	0.095
15	A	2	2	1.00	29	0.069
16	A	2	2	1.00	29	0.069
17	A	2	2	1.00	29	0.069
18	A	2	2	1.00	29	0.069
19	A	2	2	1.00	29	0.069
20	A	2	2	1.00	29	0.069
21	A	2	2	1.00	29	0.069

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
22	A	2	2	1.00	29	0.069
23	A	2	2	1.00	29	0.069
24	A	2	2	1.00	29	0.069
25	A	2	2	1.00	29	0.069
26	A	2	2	1.00	29	0.069
27	A	2	2	1.00	27	0.074
28	A	2	2	1.13	21	0.095
29	A	2	2	1.00	29	0.069
30	A	2	2	1.00	29	0.069
31	A	2	2	1.00	29	0.069
32	A	2	2	1.00	29	0.069
33	A	2	2	1.00	29	0.069
34	A	2	2	1.00	29	0.069
35	A	2	2	1.00	29	0.069
36	A	2	2	1.00	29	0.069
37	A	2	2	1.55	29	0.069
38	A	2	2	1.00	29	0.069
39	A	2	2	1.00	29	0.069
40	A	2	2	1.00	29	0.069
41	A	2	2	1.00	29	0.069
42	A	2	2	1.00	29	0.069
43	A	2	2	1.00	29	0.069
44	A	2	2	1.02	29	0.069
45	A	2	2	1.00	29	0.069
46	A	2	2	1.00	27	0.074
47	A	2	2	1.42	21	0.095
48	A	2	2	1.00	29	0.069
49	A	2	2	1.00	29	0.069
50	A	2	2	1.00	29	0.069
51	A	2	2	1.00	29	0.069
52	A	2	2	1.00	29	0.069
53	A	2	2	1.00	29	0.069

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
54	A	2	2	1.00	29	0.069
55	A	2	2	1.00	29	0.069
56	A	2	2	1.00	29	0.069
57	A	2	2	1.00	27	0.074
58	A	2	2	1.31	21	0.095
59	A	2	2	1.00	29	0.069
60	A	2	2	1.00	29	0.069
61	A	2	2	1.00	29	0.069
62	A	2	2	1.00	29	0.069
63	A	2	2	1.00	29	0.069
64	A	2	2	1.00	29	0.069
65	A	2	2	1.00	29	0.069
66	A	2	2	1.00	29	0.069
67	A	2	2	1.00	27	0.074
68	A	2	2	1.12	21	0.095
69	A	2	2	1.00	29	0.069
70	A	2	2	1.00	33	0.061
71	A	2	2	1.00	33	0.061
72	A	2	2	1.00	33	0.061
73	A	2	2	1.00	31	0.065
74	A	1	1	1.00	25	0.040
75	A	2	2	1.00	33	0.061
76	A	2	2	1.00	33	0.061
77	A	2	2	1.00	33	0.061
78	A	2	2	1.00	33	0.061
79	A	2	2	1.00	33	0.061
80	A	2	2	1.00	33	0.061
81	A	2	2	1.00	35	0.057
82	A	2	2	1.00	33	0.061
83	A	2	2	1.26	27	0.074
84	A	2	2	1.00	35	0.057
85	A	2	2	1.00	35	0.057

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
86	A	2	2	1.02	35	0.057
87	A	2	2	1.00	35	0.057
88	A	2	2	1.00	35	0.057
89	A	2	2	1.00	35	0.057
90	A	2	2	1.00	35	0.057
91	A	2	2	1.00	35	0.057
92	A	2	2	1.00	35	0.057
93	A	2	2	1.00	35	0.057
94	A	2	2	1.00	33	0.061
95	A	2	2	1.18	27	0.074
96	A	2	2	1.00	35	0.057
97	A	2	2	1.00	35	0.057
98	A	2	2	1.00	35	0.057
99	A	2	2	1.00	35	0.057
100	A	2	2	1.00	35	0.057
101	A	2	2	1.00	35	0.057
102	A	2	2	1.00	35	0.057
103	A	2	2	1.00	35	0.057
104	A	2	2	1.48	35	0.057
105	A	2	2	1.00	35	0.057
106	A	2	2	1.00	35	0.057
107	A	2	2	1.00	35	0.057
108	A	2	2	1.00	35	0.057
109	A	2	2	1.00	35	0.057
110	A	2	2	1.00	35	0.057
111	A	2	2	1.00	33	0.061
112	A	2	2	1.47	27	0.074
113	A	2	2	1.00	35	0.057
114	A	2	2	1.00	35	0.057
115	A	2	2	1.00	35	0.057
116	A	2	2	1.00	35	0.057
117	A	2	2	1.00	35	0.057

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
118	A	2	2	1.00	35	0.057
119	A	2	2	1.00	35	0.057
120	A	2	2	1.00	35	0.057
121	A	2	2	1.00	35	0.057
122	A	2	2	1.00	35	0.057
123	A	2	2	1.00	33	0.061
124	A	2	2	1.35	27	0.074
125	A	2	2	1.00	35	0.057
126	A	2	2	1.00	35	0.057
127	A	2	2	1.00	35	0.057
128	A	2	2	1.00	35	0.057
129	A	2	2	1.00	35	0.057
130	A	2	2	1.00	35	0.057
131	A	2	2	1.00	35	0.057
132	A	2	2	1.00	35	0.057
133	A	2	2	1.00	35	0.057
134	A	2	2	1.00	35	0.057
135	A	2	2	1.00	33	0.061
136	A	2	2	1.13	27	0.074
137	A	2	2	1.00	35	0.057
138	A	2	2	1.00	35	0.057
139	A	2	2	1.00	35	0.057
140	A	2	2	1.00	35	0.057
141	A	2	2	1.00	35	0.057
142	A	2	2	1.00	35	0.057
143	A	2	2	1.00	35	0.057
144	A	2	2	1.00	35	0.057
145	A	2	2	1.00	35	0.057
146	A	2	2	1.00	35	0.057
147	A	2	2	1.00	33	0.061
148	A	2	2	1.10	27	0.074
149	A	2	2	1.00	35	0.057

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
150	A	2	2	1.00	35	0.057
151	A	2	2	1.00	35	0.057
152	A	2	2	1.00	35	0.057
153	A	2	2	1.00	35	0.057
154	A	2	2	1.00	35	0.057
155	A	2	2	1.00	35	0.057
156	A	2	2	1.00	35	0.057
157	A	2	2	1.00	37	0.054
158	A	2	2	1.00	37	0.054
159	A	2	2	1.00	37	0.054
160	A	2	2	1.00	37	0.054
161	A	2	2	1.00	37	0.054
162	A	2	2	1.00	37	0.054
163	A	2	2	1.00	37	0.054
164	A	2	2	1.00	37	0.054
165	A	2	2	1.00	37	0.054
166	A	2	2	1.00	37	0.054
167	A	2	2	1.00	37	0.054
168	A	2	2	1.00	37	0.054
169	A	2	2	1.00	37	0.054
170	A	2	2	1.00	37	0.054
171	A	2	2	1.00	37	0.054
172	A	2	2	1.00	37	0.054
173	A	8	7	1.15	37	0.189
174	A	7	6	1.16	37	0.162
175	A	6	5	1.17	37	0.135
176	A	5	4	1.16	37	0.108
177	A	4	3	1.00	37	0.081
178	A	5	4	1.00	37	0.108
179	A	6	5	1.13	37	0.135
180	A	7	6	1.18	37	0.162
181	A	8	7	1.22	37	0.189

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
182	A	9	8	1.17	37	0.216
183	A	8	7	1.17	37	0.189
184	A	7	6	1.19	37	0.162
185	A	6	5	1.21	37	0.135
186	A	5	4	1.00	37	0.108
187	A	5	4	1.00	37	0.108
188	A	6	5	1.14	37	0.135
189	A	7	6	1.21	37	0.162
190	A	8	7	1.23	37	0.189
191	A	9	8	1.13	37	0.216
192	A	8	7	1.15	37	0.189
193	A	7	6	1.16	37	0.162
194	A	6	5	1.04	37	0.135
195	A	6	5	0.99	37	0.135
196	A	6	5	1.08	37	0.135
197	A	7	6	1.15	37	0.162
198	A	8	7	1.19	37	0.189
199	A	9	8	1.20	37	0.216
200	A	7	6	0.98	37	0.162
201	A	6	5	0.95	37	0.135
202	A	5	4	0.91	35	0.114
203	A	4	3	1.10	29	0.103
204	A	4	3	1.13	37	0.081
205	A	6	5	1.11	37	0.135
206	A	1	1	1.00	37	0.027
207	A	2	2	1.00	37	0.054
208	A	3	3	1.09	37	0.081
209	A	4	4	1.13	37	0.108
210	A	7	6	0.88	37	0.162
211	A	6	5	0.84	35	0.143
212	A	5	4	1.13	29	0.138
213	A	5	4	0.88	37	0.108

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
214	A	5	4	1.11	37	0.108
215	A	6	5	1.16	37	0.135
216	A	7	6	1.11	37	0.162
217	A	1	1	1.00	37	0.027
218	A	2	2	1.00	37	0.054
219	A	3	3	1.09	37	0.081
220	A	4	4	1.13	37	0.108
221	A	8	7	0.84	37	0.189
222	A	7	6	0.81	35	0.171
223	A	6	5	1.15	29	0.172
224	A	6	5	0.81	37	0.135
225	A	7	6	0.91	37	0.162
226	A	6	5	1.14	37	0.135
227	A	8	7	1.16	37	0.189
228	A	7	6	1.13	37	0.162
229	A	8	7	1.09	37	0.189
230	A	1	1	1.00	37	0.027
231	A	2	2	1.00	37	0.054
232	A	3	3	1.09	37	0.081
233	A	4	4	1.13	37	0.108
234	A	8	7	0.79	35	0.200
235	A	7	6	1.16	29	0.207
236	A	7	6	0.78	37	0.162
237	A	8	7	0.84	37	0.189
238	A	8	7	0.93	37	0.189
239	A	8	7	1.14	37	0.189
240	A	9	8	1.24	37	0.216
241	A	9	8	1.12	37	0.216
242	A	8	7	1.12	37	0.189
243	A	9	8	1.07	37	0.216
244	A	1	1	1.00	37	0.027
245	A	2	2	1.00	37	0.054

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
246	A	3	3	1.09	37	0.081
247	A	4	4	1.13	37	0.108
248	A	6	5	1.11	37	0.135
249	A	5	4	1.11	37	0.108
250	A	4	3	1.16	35	0.086
251	A	3	2	1.21	29	0.069
252	A	1	1	1.00	37	0.027
253	A	2	2	1.00	37	0.054
254	A	3	3	1.09	37	0.081
255	A	4	4	1.13	37	0.108
256	A	9	8	1.22	37	0.216
257	A	7	6	1.14	37	0.162
258	A	6	5	1.17	37	0.135
259	A	4	3	1.13	37	0.081
260	A	2	2	1.00	35	0.057
261	A	1	1	1.00	29	0.034
262	A	2	2	0.73	37	0.054
263	A	3	3	0.87	37	0.081
264	A	4	4	0.94	37	0.108
265	A	8	7	1.13	37	0.189
266	A	7	6	1.16	37	0.162
267	A	5	4	1.14	37	0.108
268	A	1	1	1.00	37	0.027
269	A	2	2	1.07	37	0.054
270	A	2	2	0.73	35	0.057
271	A	2	2	1.00	29	0.069
272	A	3	3	0.72	37	0.081
273	A	4	4	0.80	37	0.108
274	A	5	5	0.86	37	0.135
275	A	2	2	1.00	16	0.125
276	A	1	1	1.00	36	0.028
277	A	4	3	1.00	16	0.188

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
278	A	5	5	1.04	39	0.128
279	A	4	4	1.03	39	0.103
280	A	3	3	1.02	39	0.077
281	A	2	2	1.00	39	0.051
282	A	1	1	1.00	39	0.026
283	A	4	3	1.00	39	0.077
284	A	4	3	1.00	39	0.077
285	A	5	4	0.97	39	0.103
286	A	6	5	1.03	39	0.128
287	A	5	5	1.04	39	0.128
288	A	4	4	1.03	39	0.103
289	A	3	3	1.02	39	0.077
290	A	2	2	1.00	39	0.051
291	A	1	1	1.00	39	0.026
292	A	5	4	1.06	39	0.103
293	A	5	4	1.03	39	0.103
294	A	5	4	0.99	39	0.103
295	A	6	5	1.00	39	0.128
296	A	7	6	1.03	39	0.154
297	A	5	5	1.04	39	0.128
298	A	4	4	1.03	39	0.103
299	A	3	3	1.02	39	0.077
300	A	2	2	1.00	39	0.051
301	A	1	1	1.00	39	0.026
302	A	6	5	1.06	39	0.128
303	A	6	5	1.05	39	0.128
304	A	6	5	1.00	39	0.128
305	A	6	5	1.01	39	0.128
306	A	7	6	1.01	39	0.154
307	A	8	7	1.04	39	0.179
308	A	4	4	1.05	39	0.103
309	A	3	3	1.03	39	0.077

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
310	A	2	2	1.02	39	0.051
311	A	1	1	1.00	39	0.026
312	A	3	2	1.00	39	0.051
313	A	4	3	0.99	39	0.077
314	A	5	4	1.04	39	0.103
315	A	3	3	1.02	39	0.077
316	A	2	2	1.00	39	0.051
317	A	1	1	1.00	39	0.026
318	A	4	3	1.00	39	0.077
319	A	5	4	1.09	39	0.103
320	A	6	5	1.08	39	0.128
321	A	3	3	1.01	39	0.077
322	A	2	2	1.00	39	0.051
323	A	1	1	1.00	39	0.026
324	A	6	5	1.39	39	0.128
325	A	6	5	1.13	39	0.128
326	A	7	6	1.10	39	0.154
327	A	8	7	1.12	39	0.179
328	A	3	2	1.00	26	0.077
329	A	3	2	1.00	28	0.071
330	A	6	5	1.20	39	0.128
331	B	10	9	22.08	37	0.243
332	B	8	7	21.26	37	0.189
333	B	6	5	20.96	35	0.143
334	B	5	4	15.38	29	0.138
335	A	3	3	1.41	37	0.081
336	A	3	3	1.40	37	0.081
337	A	7	6	1.66	37	0.162
338	A	5	4	1.76	35	0.114
339	B	4	3	2.20	29	0.103
340	A	8	7	1.38	37	0.189
341	A	8	7	1.34	37	0.189

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
342	A	9	8	1.28	37	0.216
343	A	10	9	1.21	37	0.243
344	B	9	8	3.32	37	0.216
345	B	7	6	3.75	35	0.171
346	B	6	5	4.96	29	0.172
347	A	9	8	1.93	37	0.216
348	A	9	8	1.85	37	0.216
349	B	10	9	2.10	37	0.243
350	A	11	10	1.82	37	0.270
351	A	12	11	1.65	37	0.297
352	A	8	7	1.41	37	0.189
353	A	6	5	1.45	35	0.143
354	A	5	4	1.94	29	0.138
355	A	10	9	1.15	37	0.243
356	A	9	8	1.27	37	0.216
357	A	9	8	1.31	37	0.216
358	A	9	8	1.25	37	0.216
359	A	10	9	1.20	37	0.243
360	A	11	10	1.17	37	0.270
361	B	10	9	3.89	37	0.243
362	B	8	7	4.60	37	0.189
363	B	6	5	5.80	35	0.143
364	B	5	4	6.64	29	0.138
365	B	9	8	2.53	37	0.216
366	B	10	9	2.08	37	0.243
367	A	11	10	1.84	37	0.270
368	A	8	7	1.97	37	0.189
369	B	6	5	2.18	37	0.135
370	B	4	3	2.54	35	0.086
371	B	3	2	2.75	29	0.069
372	A	8	7	1.39	37	0.189
373	A	9	8	1.33	37	0.216

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
374	A	10	9	1.26	37	0.243
375	B	13	12	4.20	37	0.324
376	B	11	10	5.04	37	0.270
377	B	6	5	5.80	37	0.135
378	B	6	5	7.37	35	0.143
379	B	6	5	5.13	29	0.172
380	A	11	10	1.87	37	0.270
381	A	12	11	1.71	37	0.297
382	A	2	2	1.00	35	0.057
383	A	2	2	1.00	35	0.057
384	A	2	2	1.00	33	0.061
385	A	2	2	1.00	35	0.057
386	A	2	2	1.00	35	0.057
387	A	2	2	1.00	35	0.057
388	A	2	2	1.00	35	0.057
389	A	3	3	1.39	35	0.086
390	A	3	3	1.39	35	0.086
391	B	2	2	2.02	33	0.061
392	A	1	1	1.00	27	0.037
393	A	3	3	1.07	35	0.086
394	A	3	3	1.40	35	0.086
395	A	3	3	1.38	35	0.086
396	A	4	4	1.37	35	0.114
397	A	4	4	1.03	39	0.103
398	A	3	3	1.03	39	0.077
399	A	2	2	1.00	39	0.051
400	A	1	1	1.00	39	0.026
401	A	4	4	1.22	39	0.103
402	A	4	4	1.27	37	0.108
403	A	1	1	1.00	37	0.027
404	A	1	1	1.00	37	0.027

CHAPTER 3

LISTING OF INTEGRALS

3.1 $\int (a+bx)^3 \left(ac+(bc+ad)x+bdx^2\right) \, dx$

Optimal result

Integrand size = 27, antiderivative size = 38

$$\int (a+bx)^3 \left(ac+(bc+ad)x+bdx^2\right) \, dx = \frac{(bc-ad)(a+bx)^5}{5b^2}+\frac{d(a+bx)^6}{6b^2}$$

output
```
1/5*(-a*d+b*c)*(b*x+a)^5/b^2+1/6*d*(b*x+a)^6/b^2
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 84 vs. 2(38) = 76.

Time = 0.02 (sec) , antiderivative size = 84, normalized size of antiderivative = 2.21

$$\int (a+bx)^3 \left(ac+(bc+ad)x+bdx^2\right) \, dx = \frac{1}{30}x\big(15a^4(2c+dx)+20a^3bx(3c+2dx) + 15a^2b^2x^2(4c+3dx)+6ab^3x^3(5c+4dx) + b^4x^4(6c+5dx)\big)$$

input
```
Integrate[(a + b*x)^3*(a*c + (b*c + a*d)*x + b*d*x^2),x]
```

output
```
(x*(15*a^4*(2*c + d*x) + 20*a^3*b*x*(3*c + 2*d*x) + 15*a^2*b^2*x^2*(4*c +
3*d*x) + 6*a*b^3*x^3*(5*c + 4*d*x) + b^4*x^4*(6*c + 5*d*x)))/30
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.074, Rules used = {1121, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+bx)^3 \left(x(ad+bc)+ac+bdx^2\right) \, dx$$

$$\Big\downarrow \text{1121}$$

$$\int \left(\frac{(a+bx)^4(bc-ad)}{b}+\frac{d(a+bx)^5}{b}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{(a+bx)^5(bc-ad)}{5b^2}+\frac{d(a+bx)^6}{6b^2}$$

input
```
Int[(a + b*x)^3*(a*c + (b*c + a*d)*x + b*d*x^2),x]
```

output
```
((b*c - a*d)*(a + b*x)^5)/(5*b^2) + (d*(a + b*x)^6)/(6*b^2)
```

Defintions of rubi rules used

rule 1121
```
Int[((d_) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Int[ExpandIntegrand[(d + e*x)^(m + p)*(a/d + (c/e)*x)^p, x], x]
/; FreeQ[{a, b, c, d, e, m, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && (Int
egerQ[p] || (GtQ[a, 0] && GtQ[d, 0] && LtQ[c, 0]))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 93 vs. $2(34) = 68$.

Time = 0.64 (sec) , antiderivative size = 94, normalized size of antiderivative = 2.47

method	result
norman	$\frac{b^4 d\,x^6}{6}+\left(\frac{4}{5}a\,b^3 d+\frac{1}{5}b^4 c\right)x^5+\left(\frac{3}{2}a^2 d\,b^2+a\,b^3 c\right)x^4+\left(\frac{4}{3}a^3 bd+2a^2 b^2 c\right)x^3+\left(\frac{1}{2}a^4 d+2a^3 bc\right)$
gosper	$\frac{x\left(5b^4 d\,x^5+24x^4 a\,b^3 d+6x^4 b^4 c+45a^2 b^2 d\,x^3+30a\,b^3 c\,x^3+40a^3 bd\,x^2+60a^2 b^2 c\,x^2+15a^4 dx+60a^3 bcx+30c\,a^4\right)}{30}$
risch	$\frac{1}{6}b^4 d\,x^6+\frac{4}{5}x^5 a\,b^3 d+\frac{1}{5}b^4 c\,x^5+\frac{3}{2}a^2 b^2 d\,x^4+x^4 a\,b^3 c+\frac{4}{3}a^3 bd\,x^3+2a^2 b^2 c\,x^3+\frac{1}{2}a^4 d\,x^2+2a$
parallelrisch	$\frac{1}{6}b^4 d\,x^6+\frac{4}{5}x^5 a\,b^3 d+\frac{1}{5}b^4 c\,x^5+\frac{3}{2}a^2 b^2 d\,x^4+x^4 a\,b^3 c+\frac{4}{3}a^3 bd\,x^3+2a^2 b^2 c\,x^3+\frac{1}{2}a^4 d\,x^2+2a$
orering	$\frac{x\left(5b^4 d\,x^5+24x^4 a\,b^3 d+6x^4 b^4 c+45a^2 b^2 d\,x^3+30a\,b^3 c\,x^3+40a^3 bd\,x^2+60a^2 b^2 c\,x^2+15a^4 dx+60a^3 bcx+30c\,a^4\right)\left(ac+(ad+bc)x+\right.}{30(dx+c)(bx+a)}$
default	$\frac{b^4 d\,x^6}{6}+\frac{\left(3a\,b^3 d+b^3(ad+bc)\right)x^5}{5}+\frac{\left(3a^2 d\,b^2+3a\,b^2(ad+bc)+a\,b^3 c\right)x^4}{4}+\frac{\left(a^3 bd+3a^2 b(ad+bc)+3a^2 b^2 c\right)x^3}{3}+\frac{\left(a^3(ad+b\right.}{}$

input
```
int((b*x+a)^3*(a*c+(a*d+b*c)*x+b*d*x^2),x,method=_RETURNVERBOSE)
```

output
```
1/6*b^4*d*x^6+(4/5*a*b^3*d+1/5*b^4*c)*x^5+(3/2*a^2*d*b^2+a*b^3*c)*x^4+(4/3
*a^3*b*d+2*a^2*b^2*c)*x^3+(1/2*a^4*d+2*a^3*b*c)*x^2+a^4*c*x
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 96 vs. $2(34) = 68$.

Time = 0.07 (sec) , antiderivative size = 96, normalized size of antiderivative = 2.53

$$\int (a+bx)^3\left(ac+(bc+ad)x+bdx^2\right)\,dx=\frac{1}{6}\,b^4dx^6+a^4cx+\frac{1}{5}\left(b^4c+4\,ab^3d\right)x^5+\frac{1}{2}\left(2\,ab^3c+3\,a^2b^2d\right)x^4+\frac{2}{3}\left(3\,a^2b^2c+2\,a^3bd\right)x^3+\frac{1}{2}\left(4\,a^3bc+a^4d\right)x^2$$

input
```
integrate((b*x+a)^3*(a*c+(a*d+b*c)*x+b*d*x^2),x, algorithm="fricas")
```

output
```
1/6*b^4*d*x^6 + a^4*c*x + 1/5*(b^4*c + 4*a*b^3*d)*x^5 + 1/2*(2*a*b^3*c + 3
*a^2*b^2*d)*x^4 + 2/3*(3*a^2*b^2*c + 2*a^3*b*d)*x^3 + 1/2*(4*a^3*b*c + a^4
*d)*x^2
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 100 vs. $2(32) = 64$.

Time = 0.03 (sec) , antiderivative size = 100, normalized size of antiderivative = 2.63

$$\int (a+bx)^3 \left(ac+(bc+ad)x+bdx^2\right) dx = a^4cx + \frac{b^4dx^6}{6} + x^5 \cdot \left(\frac{4ab^3d}{5} + \frac{b^4c}{5}\right) + x^4 \cdot \left(\frac{3a^2b^2d}{2} + ab^3c\right) + x^3 \cdot \left(\frac{4a^3bd}{3} + 2a^2b^2c\right) + x^2\left(\frac{a^4d}{2} + 2a^3bc\right)$$

input
```
integrate((b*x+a)**3*(a*c+(a*d+b*c)*x+b*d*x**2),x)
```

output
```
a**4*c*x + b**4*d*x**6/6 + x**5*(4*a*b**3*d/5 + b**4*c/5) + x**4*(3*a**2*b
**2*d/2 + a*b**3*c) + x**3*(4*a**3*b*d/3 + 2*a**2*b**2*c) + x**2*(a**4*d/2
 + 2*a**3*b*c)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 96 vs. $2(34) = 68$.

Time = 0.04 (sec) , antiderivative size = 96, normalized size of antiderivative = 2.53

$$\int (a+bx)^3 \left(ac+(bc+ad)x+bdx^2\right) dx = \frac{1}{6}b^4dx^6 + a^4cx + \frac{1}{5}\left(b^4c + 4\,ab^3d\right)x^5 + \frac{1}{2}\left(2\,ab^3c + 3\,a^2b^2d\right)x^4 + \frac{2}{3}\left(3\,a^2b^2c + 2\,a^3bd\right)x^3 + \frac{1}{2}\left(4\,a^3bc + a^4d\right)x^2$$

input
```
integrate((b*x+a)^3*(a*c+(a*d+b*c)*x+b*d*x^2),x, algorithm="maxima")
```

output
```
1/6*b^4*d*x^6 + a^4*c*x + 1/5*(b^4*c + 4*a*b^3*d)*x^5 + 1/2*(2*a*b^3*c + 3
*a^2*b^2*d)*x^4 + 2/3*(3*a^2*b^2*c + 2*a^3*b*d)*x^3 + 1/2*(4*a^3*b*c + a^4
*d)*x^2
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 97 vs. $2(34) = 68$.

Time = 0.15 (sec) , antiderivative size = 97, normalized size of antiderivative = 2.55

$$\int (a+bx)^3 \left(ac+(bc+ad)x+bdx^2\right) dx = \frac{1}{6}b^4dx^6 + \frac{1}{5}b^4cx^5 + \frac{4}{5}ab^3dx^5 + ab^3cx^4 + \frac{3}{2}a^2b^2dx^4 + 2a^2b^2cx^3 + \frac{4}{3}a^3bdx^3 + 2a^3bcx^2 + \frac{1}{2}a^4dx^2 + a^4cx$$

input
```
integrate((b*x+a)^3*(a*c+(a*d+b*c)*x+b*d*x^2),x, algorithm="giac")
```

output
```
1/6*b^4*d*x^6 + 1/5*b^4*c*x^5 + 4/5*a*b^3*d*x^5 + a*b^3*c*x^4 + 3/2*a^2*b^
2*d*x^4 + 2*a^2*b^2*c*x^3 + 4/3*a^3*b*d*x^3 + 2*a^3*b*c*x^2 + 1/2*a^4*d*x^
2 + a^4*c*x
```

Mupad [B] (verification not implemented)

Time = 5.47 (sec) , antiderivative size = 88, normalized size of antiderivative = 2.32

$$\int (a+bx)^3 \left(ac+(bc+ad)x+bdx^2\right) dx = x^5 \left(\frac{c\,b^4}{5} + \frac{4\,a\,d\,b^3}{5}\right) + x^2 \left(\frac{d\,a^4}{2} + 2\,b\,c\,a^3\right) + \frac{b^4\,d\,x^6}{6} + a^4\,c\,x + \frac{2\,a^2\,b\,x^3\,(2\,a\,d + 3\,b\,c)}{3} + \frac{a\,b^2\,x^4\,(3\,a\,d + 2\,b\,c)}{2}$$

input
```
int((a + b*x)^3*(a*c + x*(a*d + b*c) + b*d*x^2),x)
```

output
```
x^5*((b^4*c)/5 + (4*a*b^3*d)/5) + x^2*((a^4*d)/2 + 2*a^3*b*c) + (b^4*d*x^6
)/6 + a^4*c*x + (2*a^2*b*x^3*(2*a*d + 3*b*c))/3 + (a*b^2*x^4*(3*a*d + 2*b*
c))/2
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 97, normalized size of antiderivative = 2.55

$$\int (a+bx)^3 \left(ac+(bc+ad)x+bdx^2\right) \, dx$$
$$= \frac{x(5b^4d\,x^5+24a\,b^3d\,x^4+6b^4c\,x^4+45a^2b^2d\,x^3+30a\,b^3c\,x^3+40a^3bd\,x^2+60a^2b^2c\,x^2+15a^4dx+60a^3bcx}{30}$$

input
```
int((b*x+a)^3*(a*c+(a*d+b*c)*x+b*d*x^2),x)
```

output
```
(x*(30*a**4*c + 15*a**4*d*x + 60*a**3*b*c*x + 40*a**3*b*d*x**2 + 60*a**2*b
**2*c*x**2 + 45*a**2*b**2*d*x**3 + 30*a*b**3*c*x**3 + 24*a*b**3*d*x**4 + 6
*b**4*c*x**4 + 5*b**4*d*x**5))/30
```

3.2 $\int (a+bx)^2 \left(ac+(bc+ad)x+bdx^2\right) \, dx$

Optimal result

Integrand size = 27, antiderivative size = 38

$$\int (a+bx)^2 \left(ac+(bc+ad)x+bdx^2\right) \, dx = \frac{(bc-ad)(a+bx)^4}{4b^2}+\frac{d(a+bx)^5}{5b^2}$$

output
```
1/4*(-a*d+b*c)*(b*x+a)^4/b^2+1/5*d*(b*x+a)^5/b^2
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.76

$$\int (a+bx)^2 \left(ac+(bc+ad)x+bdx^2\right) \, dx = a^3cx+\frac{1}{2}a^2(3bc+ad)x^2+ab(bc+ad)x^3 +\frac{1}{4}b^2(bc+3ad)x^4+\frac{1}{5}b^3dx^5$$

input
```
Integrate[(a + b*x)^2*(a*c + (b*c + a*d)*x + b*d*x^2),x]
```

output
```
a^3*c*x + (a^2*(3*b*c + a*d)*x^2)/2 + a*b*(b*c + a*d)*x^3 + (b^2*(b*c + 3*
a*d)*x^4)/4 + (b^3*d*x^5)/5
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.074$, Rules used = {1121, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+bx)^2 \left(x(ad+bc)+ac+bdx^2\right) \, dx$$

$$\Big\downarrow \text{1121}$$

$$\int \left(\frac{(a+bx)^3(bc-ad)}{b}+\frac{d(a+bx)^4}{b}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{(a+bx)^4(bc-ad)}{4b^2}+\frac{d(a+bx)^5}{5b^2}$$

input
```
Int[(a + b*x)^2*(a*c + (b*c + a*d)*x + b*d*x^2),x]
```

output
```
((b*c - a*d)*(a + b*x)^4)/(4*b^2) + (d*(a + b*x)^5)/(5*b^2)
```

Defintions of rubi rules used

rule 1121
```
Int[((d_) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Int[ExpandIntegrand[(d + e*x)^(m + p)*(a/d + (c/e)*x)^p, x], x]
/; FreeQ[{a, b, c, d, e, m, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && (Int
egerQ[p] || (GtQ[a, 0] && GtQ[d, 0] && LtQ[c, 0]))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 69 vs. $2(34) = 68$.

Time = 0.61 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.84

method	result	size
norman	$\frac{b^3 d\,x^5}{5}+\left(\frac{3}{4}da\,b^2+\frac{1}{4}b^3c\right)x^4+\left(a^2bd+ca\,b^2\right)x^3+\left(\frac{1}{2}a^3d+\frac{3}{2}c\,a^2b\right)x^2+a^3cx$	70
risch	$\frac{1}{5}b^3d\,x^5+\frac{3}{4}a\,b^2d\,x^4+\frac{1}{4}b^3c\,x^4+a^2bd\,x^3+a\,b^2c\,x^3+\frac{1}{2}a^3d\,x^2+\frac{3}{2}a^2bc\,x^2+a^3cx$	73
parallelrisch	$\frac{1}{5}b^3d\,x^5+\frac{3}{4}a\,b^2d\,x^4+\frac{1}{4}b^3c\,x^4+a^2bd\,x^3+a\,b^2c\,x^3+\frac{1}{2}a^3d\,x^2+\frac{3}{2}a^2bc\,x^2+a^3cx$	73
gosper	$\frac{x\left(4b^3d\,x^4+15a\,b^2d\,x^3+5b^3c\,x^3+20a^2bd\,x^2+20a\,b^2c\,x^2+10a^3dx+30a^2bcx+20c\,a^3\right)}{20}$	74
default	$\frac{b^3d\,x^5}{5}+\frac{\left(2da\,b^2+b^2(ad+bc)\right)x^4}{4}+\frac{\left(a^2bd+2ab(ad+bc)+ca\,b^2\right)x^3}{3}+\frac{\left(a^2(ad+bc)+2c\,a^2b\right)x^2}{2}+a^3cx$	94
orering	$\frac{x\left(4b^3d\,x^4+15a\,b^2d\,x^3+5b^3c\,x^3+20a^2bd\,x^2+20a\,b^2c\,x^2+10a^3dx+30a^2bcx+20c\,a^3\right)\left(ac+(ad+bc)x+bd\,x^2\right)}{20(dx+c)(bx+a)}$	107

input
```
int((b*x+a)^2*(a*c+(a*d+b*c)*x+b*d*x^2),x,method=_RETURNVERBOSE)
```

output
```
1/5*b^3*d*x^5+(3/4*d*a*b^2+1/4*b^3*c)*x^4+(a^2*b*d+a*b^2*c)*x^3+(1/2*a^3*d
+3/2*c*a^2*b)*x^2+a^3*c*x
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 69 vs. $2(34) = 68$.

Time = 0.07 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.82

$$\int (a+bx)^2\left(ac+(bc+ad)x+bdx^2\right)\,dx=\frac{1}{5}\,b^3dx^5+a^3cx+\frac{1}{4}\left(b^3c+3\,ab^2d\right)x^4+\left(ab^2c+a^2bd\right)x^3+\frac{1}{2}\left(3\,a^2bc+a^3d\right)x^2$$

input
```
integrate((b*x+a)^2*(a*c+(a*d+b*c)*x+b*d*x^2),x, algorithm="fricas")
```

output
```
1/5*b^3*d*x^5 + a^3*c*x + 1/4*(b^3*c + 3*a*b^2*d)*x^4 + (a*b^2*c + a^2*b*d
)*x^3 + 1/2*(3*a^2*b*c + a^3*d)*x^2
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 73 vs. $2(32) = 64$.

Time = 0.02 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.92

$$\int (a+bx)^2 \left(ac+(bc+ad)x+bdx^2\right) dx = a^3cx + \frac{b^3dx^5}{5} + x^4 \cdot \left(\frac{3ab^2d}{4} + \frac{b^3c}{4}\right) + x^3\left(a^2bd+ab^2c\right) + x^2\left(\frac{a^3d}{2} + \frac{3a^2bc}{2}\right)$$

input
```
integrate((b*x+a)**2*(a*c+(a*d+b*c)*x+b*d*x**2),x)
```

output
```
a**3*c*x + b**3*d*x**5/5 + x**4*(3*a*b**2*d/4 + b**3*c/4) + x**3*(a**2*b*d
 + a*b**2*c) + x**2*(a**3*d/2 + 3*a**2*b*c/2)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 69 vs. $2(34) = 68$.

Time = 0.03 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.82

$$\int (a+bx)^2 \left(ac+(bc+ad)x+bdx^2\right) dx = \frac{1}{5}b^3dx^5 + a^3cx + \frac{1}{4}\left(b^3c+3ab^2d\right)x^4 + \left(ab^2c+a^2bd\right)x^3 + \frac{1}{2}\left(3a^2bc+a^3d\right)x^2$$

input
```
integrate((b*x+a)^2*(a*c+(a*d+b*c)*x+b*d*x^2),x, algorithm="maxima")
```

output
```
1/5*b^3*d*x^5 + a^3*c*x + 1/4*(b^3*c + 3*a*b^2*d)*x^4 + (a*b^2*c + a^2*b*d
)*x^3 + 1/2*(3*a^2*b*c + a^3*d)*x^2
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 72 vs. $2(34) = 68$.

Time = 0.12 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.89

$$\int (a+bx)^2 \left(ac+(bc+ad)x+bdx^2\right) dx = \frac{1}{5} b^3dx^5 + \frac{1}{4} b^3cx^4 + \frac{3}{4} ab^2dx^4 + ab^2cx^3 + a^2bdx^3 + \frac{3}{2} a^2bcx^2 + \frac{1}{2} a^3dx^2 + a^3cx$$

input
```
integrate((b*x+a)^2*(a*c+(a*d+b*c)*x+b*d*x^2),x, algorithm="giac")
```

output
```
1/5*b^3*d*x^5 + 1/4*b^3*c*x^4 + 3/4*a*b^2*d*x^4 + a*b^2*c*x^3 + a^2*b*d*x^
3 + 3/2*a^2*b*c*x^2 + 1/2*a^3*d*x^2 + a^3*c*x
```

Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.71

$$\int (a+bx)^2 \left(ac+(bc+ad)x+bdx^2\right) dx = x^4 \left(\frac{c\,b^3}{4} + \frac{3\,a\,d\,b^2}{4}\right) + x^2 \left(\frac{d\,a^3}{2} + \frac{3\,b\,c\,a^2}{2}\right) + \frac{b^3\,d\,x^5}{5} + a^3\,c\,x + a\,b\,x^3\,(a\,d + b\,c)$$

input
```
int((a + b*x)^2*(a*c + x*(a*d + b*c) + b*d*x^2),x)
```

output
```
x^4*((b^3*c)/4 + (3*a*b^2*d)/4) + x^2*((a^3*d)/2 + (3*a^2*b*c)/2) + (b^3*d
*x^5)/5 + a^3*c*x + a*b*x^3*(a*d + b*c)
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.92

$$\int (a+bx)^2 \left(ac+(bc+ad)x+bdx^2\right)\,dx$$
$$=\frac{x(4b^3d\,x^4+15a\,b^2d\,x^3+5b^3c\,x^3+20a^2bd\,x^2+20a\,b^2c\,x^2+10a^3dx+30a^2bcx+20a^3c)}{20}$$

input

```
int((b*x+a)^2*(a*c+(a*d+b*c)*x+b*d*x^2),x)
```

output

```
(x*(20*a**3*c + 10*a**3*d*x + 30*a**2*b*c*x + 20*a**2*b*d*x**2 + 20*a*b**2
*c*x**2 + 15*a*b**2*d*x**3 + 5*b**3*c*x**3 + 4*b**3*d*x**4))/20
```

3.3 $\int (a+bx)\left(ac+(bc+ad)x+bdx^2\right)\,dx$

Optimal result

Integrand size = 25, antiderivative size = 38

$$\int (a+bx)\left(ac+(bc+ad)x+bdx^2\right)\,dx=\frac{(bc-ad)(a+bx)^3}{3b^2}+\frac{d(a+bx)^4}{4b^2}$$

output
```
1/3*(-a*d+b*c)*(b*x+a)^3/b^2+1/4*d*(b*x+a)^4/b^2
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.21

$$\int (a+bx)\left(ac+(bc+ad)x+bdx^2\right)\,dx=\frac{1}{12}x\left(6a^2(2c+dx)+4abx(3c+2dx)+b^2x^2(4c+3dx)\right)$$

input
```
Integrate[(a + b*x)*(a*c + (b*c + a*d)*x + b*d*x^2),x]
```

output
```
(x*(6*a^2*(2*c + d*x) + 4*a*b*x*(3*c + 2*d*x) + b^2*x^2*(4*c + 3*d*x)))/12
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.080, Rules used = {1121, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+bx)\left(x(ad+bc)+ac+bdx^2\right)dx$$

$$\Big\downarrow \text{1121}$$

$$\int \left(\frac{(a+bx)^2(bc-ad)}{b}+\frac{d(a+bx)^3}{b}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{(a+bx)^3(bc-ad)}{3b^2}+\frac{d(a+bx)^4}{4b^2}$$

input
```
Int[(a + b*x)*(a*c + (b*c + a*d)*x + b*d*x^2),x]
```

output
```
((b*c - a*d)*(a + b*x)^3)/(3*b^2) + (d*(a + b*x)^4)/(4*b^2)
```

Defintions of rubi rules used

rule 1121
```
Int[((d_) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Int[ExpandIntegrand[(d + e*x)^(m + p)*(a/d + (c/e)*x)^p, x], x]
/; FreeQ[{a, b, c, d, e, m, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && (Int
egerQ[p] || (GtQ[a, 0] && GtQ[d, 0] && LtQ[c, 0]))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.45 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.26

method	result	size
norman	$\frac{b^2 d\,x^4}{4}+\left(\frac{2}{3}dab+\frac{1}{3}b^2 c\right)x^3+\left(\frac{1}{2}a^2 d+abc\right)x^2+a^2 cx$	48
gosper	$\frac{x\left(3b^2 d\,x^3+8abd\,x^2+4b^2 c\,x^2+6a^2 dx+12abcx+12a^2 c\right)}{12}$	50
risch	$\frac{1}{4}b^2 d\,x^4+\frac{2}{3}abd\,x^3+\frac{1}{3}b^2 c\,x^3+\frac{1}{2}a^2 d\,x^2+abc\,x^2+a^2 cx$	50
parallelrisch	$\frac{1}{4}b^2 d\,x^4+\frac{2}{3}abd\,x^3+\frac{1}{3}b^2 c\,x^3+\frac{1}{2}a^2 d\,x^2+abc\,x^2+a^2 cx$	50
default	$\frac{b^2 d\,x^4}{4}+\frac{(dab+b(ad+bc))x^3}{3}+\frac{(a(ad+bc)+abc)x^2}{2}+a^2 cx$	55
orering	$\frac{x\left(3b^2 d\,x^3+8abd\,x^2+4b^2 c\,x^2+6a^2 dx+12abcx+12a^2 c\right)\left(ac+(ad+bc)x+bd\,x^2\right)}{12(dx+c)(bx+a)}$	83

input
```
int((b*x+a)*(a*c+(a*d+b*c)*x+b*d*x^2),x,method=_RETURNVERBOSE)
```

output
```
1/4*b^2*d*x^4+(2/3*d*a*b+1/3*b^2*c)*x^3+(1/2*a^2*d+a*b*c)*x^2+a^2*c*x
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.26

$$\int (a+bx)\left(ac+(bc+ad)x+bdx^2\right)\,dx=\frac{1}{4}b^2dx^4+a^2cx+\frac{1}{3}\left(b^2c+2\,abd\right)x^3+\frac{1}{2}\left(2\,abc+a^2d\right)x^2$$

input
```
integrate((b*x+a)*(a*c+(a*d+b*c)*x+b*d*x^2),x, algorithm="fricas")
```

output
```
1/4*b^2*d*x^4 + a^2*c*x + 1/3*(b^2*c + 2*a*b*d)*x^3 + 1/2*(2*a*b*c + a^2*d
)*x^2
```

Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.29

$$\int (a+bx)\left(ac+(bc+ad)x+bdx^2\right)\,dx = a^2cx + \frac{b^2dx^4}{4} + x^3\cdot\left(\frac{2abd}{3} + \frac{b^2c}{3}\right) + x^2\left(\frac{a^2d}{2} + abc\right)$$

input
```
integrate((b*x+a)*(a*c+(a*d+b*c)*x+b*d*x**2),x)
```

output
```
a**2*c*x + b**2*d*x**4/4 + x**3*(2*a*b*d/3 + b**2*c/3) + x**2*(a**2*d/2 +
a*b*c)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.26

$$\int (a+bx)\left(ac+(bc+ad)x+bdx^2\right)\,dx = \frac{1}{4}\,b^2dx^4 + a^2cx + \frac{1}{3}\,\left(b^2c + 2\,abd\right)x^3 + \frac{1}{2}\,\left(2\,abc + a^2d\right)x^2$$

input
```
integrate((b*x+a)*(a*c+(a*d+b*c)*x+b*d*x^2),x, algorithm="maxima")
```

output
```
1/4*b^2*d*x^4 + a^2*c*x + 1/3*(b^2*c + 2*a*b*d)*x^3 + 1/2*(2*a*b*c + a^2*d
)*x^2
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.29

$$\int (a+bx)\left(ac+(bc+ad)x+bdx^2\right)\,dx=\frac{1}{4}\,b^2dx^4+\frac{1}{3}\,b^2cx^3+\frac{2}{3}\,abdx^3+abcx^2+\frac{1}{2}\,a^2dx^2+a^2cx$$

input
```
integrate((b*x+a)*(a*c+(a*d+b*c)*x+b*d*x^2),x, algorithm="giac")
```

output
```
1/4*b^2*d*x^4 + 1/3*b^2*c*x^3 + 2/3*a*b*d*x^3 + a*b*c*x^2 + 1/2*a^2*d*x^2
+ a^2*c*x
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.24

$$\int (a+bx)\left(ac+(bc+ad)x+bdx^2\right)\,dx=x^2\left(\frac{d\,a^2}{2}+b\,c\,a\right)+x^3\left(\frac{c\,b^2}{3}+\frac{2\,a\,d\,b}{3}\right)+\frac{b^2\,d\,x^4}{4}+a^2\,c\,x$$

input
```
int((a + b*x)*(a*c + x*(a*d + b*c) + b*d*x^2),x)
```

output
```
x^2*((a^2*d)/2 + a*b*c) + x^3*((b^2*c)/3 + (2*a*b*d)/3) + (b^2*d*x^4)/4 +
a^2*c*x
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.29

$$\int (a+bx)\left(ac+(bc+ad)x+bdx^2\right)\,dx = \frac{x(3b^2d\,x^3+8abd\,x^2+4b^2c\,x^2+6a^2dx+12abcx+12a^2c)}{12}$$

input
```
int((b*x+a)*(a*c+(a*d+b*c)*x+b*d*x^2),x)
```

output
```
(x*(12*a**2*c + 6*a**2*d*x + 12*a*b*c*x + 8*a*b*d*x**2 + 4*b**2*c*x**2 + 3
*b**2*d*x**3))/12
```

3.4 $\int (ac + (bc + ad)x + bdx^2) \, dx$

Optimal result

Integrand size = 19, antiderivative size = 28

$$\int (ac + (bc + ad)x + bdx^2) \, dx = acx + \frac{1}{2}(bc + ad)x^2 + \frac{1}{3}bdx^3$$

output
```
a*c*x+1/2*(a*d+b*c)*x^2+1/3*b*d*x^3
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.14

$$\int (ac + (bc + ad)x + bdx^2) \, dx = acx + \frac{1}{2}bcx^2 + \frac{1}{2}adx^2 + \frac{1}{3}bdx^3$$

input
```
Integrate[a*c + (b*c + a*d)*x + b*d*x^2,x]
```

output
```
a*c*x + (b*c*x^2)/2 + (a*d*x^2)/2 + (b*d*x^3)/3
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.053$, Rules used = {2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(x(ad+bc)+ac+bdx^2\right) \, dx$$

$$\Big\downarrow \; 2009$$

$$\frac{1}{2}x^2(ad+bc)+acx+\frac{1}{3}bdx^3$$

input
```
Int[a*c + (b*c + a*d)*x + b*d*x^2,x]
```

output
```
a*c*x + ((b*c + a*d)*x^2)/2 + (b*d*x^3)/3
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.89

method	result	size
default	$acx+\frac{(ad+bc)x^2}{2}+\frac{bd\,x^3}{3}$	25
gosper	$\frac{x(2bd\,x^2+3adx+3cbx+6ac)}{6}$	26
norman	$\frac{bd\,x^3}{3}+\left(\frac{ad}{2}+\frac{bc}{2}\right)x^2+acx$	26
risch	$acx+\frac{1}{2}ad\,x^2+\frac{1}{2}bc\,x^2+\frac{1}{3}bd\,x^3$	27
parallelrisch	$acx+\frac{1}{2}ad\,x^2+\frac{1}{2}bc\,x^2+\frac{1}{3}bd\,x^3$	27
parts	$acx+\frac{1}{2}ad\,x^2+\frac{1}{2}bc\,x^2+\frac{1}{3}bd\,x^3$	27
orering	$\frac{x(2bd\,x^2+3adx+3cbx+6ac)(ac+(ad+bc)x+bd\,x^2)}{6(bx+a)(dx+c)}$	59

input
```
int(a*c+(a*d+b*c)*x+b*d*x^2,x,method=_RETURNVERBOSE)
```

output
```
a*c*x+1/2*(a*d+b*c)*x^2+1/3*b*d*x^3
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.86

$$\int \left(ac+(bc+ad)x+bdx^2\right)\,dx=\frac{1}{3}\,bdx^3+acx+\frac{1}{2}\,(bc+ad)x^2$$

input
```
integrate(a*c+(a*d+b*c)*x+b*d*x^2,x, algorithm="fricas")
```

output
```
1/3*b*d*x^3 + a*c*x + 1/2*(b*c + a*d)*x^2
```

Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.93

$$\int (ac + (bc + ad)x + bdx^2) \, dx = acx + \frac{bdx^3}{3} + x^2 \left(\frac{ad}{2} + \frac{bc}{2} \right)$$

input
```
integrate(a*c+(a*d+b*c)*x+b*d*x**2,x)
```

output
```
a*c*x + b*d*x**3/3 + x**2*(a*d/2 + b*c/2)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.86

$$\int (ac + (bc + ad)x + bdx^2) \, dx = \frac{1}{3} bdx^3 + acx + \frac{1}{2} (bc + ad)x^2$$

input
```
integrate(a*c+(a*d+b*c)*x+b*d*x^2,x, algorithm="maxima")
```

output
```
1/3*b*d*x^3 + a*c*x + 1/2*(b*c + a*d)*x^2
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.86

$$\int (ac + (bc + ad)x + bdx^2) \, dx = \frac{1}{3} bdx^3 + acx + \frac{1}{2} (bc + ad)x^2$$

input
```
integrate(a*c+(a*d+b*c)*x+b*d*x^2,x, algorithm="giac")
```

output
```
1/3*b*d*x^3 + a*c*x + 1/2*(b*c + a*d)*x^2
```

Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.89

$$\int (ac + (bc + ad)x + bdx^2) \, dx = \frac{b\,d\,x^3}{3} + \left(\frac{a\,d}{2} + \frac{b\,c}{2}\right)\,x^2 + a\,c\,x$$

input
```
int(a*c + x*(a*d + b*c) + b*d*x^2,x)
```

output
```
x^2*((a*d)/2 + (b*c)/2) + a*c*x + (b*d*x^3)/3
```

Reduce [B] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.89

$$\int (ac + (bc + ad)x + bdx^2) \, dx = \frac{x(2bd\,x^2 + 3adx + 3bcx + 6ac)}{6}$$

input
```
int(a*c+(a*d+b*c)*x+b*d*x^2,x)
```

output
```
(x*(6*a*c + 3*a*d*x + 3*b*c*x + 2*b*d*x**2))/6
```

3.5 $\int \frac{ac+(bc+ad)x+bdx^2}{a+bx} dx$

Optimal result

Integrand size = 27, antiderivative size = 14

$$\int \frac{ac + (bc + ad)x + bdx^2}{a + bx} dx = \frac{(c + dx)^2}{2d}$$

output
```
1/2*(d*x+c)^2/d
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.86

$$\int \frac{ac + (bc + ad)x + bdx^2}{a + bx} dx = cx + \frac{dx^2}{2}$$

input
```
Integrate[(a*c + (b*c + a*d)*x + b*d*x^2)/(a + b*x),x]
```

output
```
c*x + (d*x^2)/2
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.074$, Rules used = $\{1120, 17\}$

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x(ad+bc)+ac+bdx^2}{a+bx} \, dx$$

$$\Big\downarrow \text{1120}$$

$$\int (c+dx)dx$$

$$\Big\downarrow \text{17}$$

$$\frac{(c+dx)^2}{2d}$$

input
```
Int[(a*c + (b*c + a*d)*x + b*d*x^2)/(a + b*x),x]
```

output
```
(c + d*x)^2/(2*d)
```

Defintions of rubi rules used

rule 17
```
Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1
)/(b*(m + 1))), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]
```

rule 1120
```
Int[((d_) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Int[(d + e*x)^(m + p)*(a/d + (c/e)*x)^p, x] /; FreeQ[{a, b, c, d
, e, m}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[p] && (EqQ[m + p,
0] || EqQ[m + 2*p + 2, 0])
```

Maple [A] (verified)

Time = 0.67 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.79

method	result	size
gosper	$\frac{x(dx+2c)}{2}$	11
default	$\frac{1}{2}d\,x^2+cx$	11
norman	$\frac{1}{2}d\,x^2+cx$	11
risch	$\frac{1}{2}d\,x^2+cx$	11
parallelrisch	$\frac{1}{2}d\,x^2+cx$	11
parts	$\frac{1}{2}d\,x^2+cx$	11
orering	$\frac{x(dx+2c)\left(ac+(ad+bc)x+bd\,x^2\right)}{2(dx+c)(bx+a)}$	44

input
```
int((a*c+(a*d+b*c)*x+b*d*x^2)/(b*x+a),x,method=_RETURNVERBOSE)
```

output
```
1/2*x*(d*x+2*c)
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.71

$$\int \frac{ac+(bc+ad)x+bdx^2}{a+bx}\,dx=\frac{1}{2}\,dx^2+cx$$

input
```
integrate((a*c+(a*d+b*c)*x+b*d*x^2)/(b*x+a),x, algorithm="fricas")
```

output
```
1/2*d*x^2 + c*x
```

Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.57

$$\int \frac{ac+(bc+ad)x+bdx^2}{a+bx}\,dx = cx+\frac{dx^2}{2}$$

input
```
integrate((a*c+(a*d+b*c)*x+b*d*x**2)/(b*x+a),x)
```

output
```
c*x + d*x**2/2
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.71

$$\int \frac{ac+(bc+ad)x+bdx^2}{a+bx}\,dx = \frac{1}{2}\,dx^2+cx$$

input
```
integrate((a*c+(a*d+b*c)*x+b*d*x^2)/(b*x+a),x, algorithm="maxima")
```

output
```
1/2*d*x^2 + c*x
```

Giac [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.71

$$\int \frac{ac+(bc+ad)x+bdx^2}{a+bx}\,dx = \frac{1}{2}\,dx^2+cx$$

input
```
integrate((a*c+(a*d+b*c)*x+b*d*x^2)/(b*x+a),x, algorithm="giac")
```

output
```
1/2*d*x^2 + c*x
```

Mupad [B] (verification not implemented)

Time = 0.01 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.71

$$\int \frac{ac + (bc + ad)x + bdx^2}{a + bx} \, dx = \frac{d\,x^2}{2} + c\,x$$

input
```
int((a*c + x*(a*d + b*c) + b*d*x^2)/(a + b*x),x)
```

output
```
c*x + (d*x^2)/2
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.71

$$\int \frac{ac + (bc + ad)x + bdx^2}{a + bx} \, dx = \frac{x(dx + 2c)}{2}$$

input
```
int((a*c+(a*d+b*c)*x+b*d*x^2)/(b*x+a),x)
```

output
```
(x*(2*c + d*x))/2
```

3.6 $\int \frac{ac+(bc+ad)x+bdx^2}{(a+bx)^2} dx$

Optimal result

Integrand size = 27, antiderivative size = 25

$$\int \frac{ac+(bc+ad)x+bdx^2}{(a+bx)^2} dx = \frac{dx}{b} + \frac{(bc-ad)\log(a+bx)}{b^2}$$

output
```
d*x/b+(-a*d+b*c)*ln(b*x+a)/b^2
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

$$\int \frac{ac+(bc+ad)x+bdx^2}{(a+bx)^2} dx = \frac{dx}{b} + \frac{(bc-ad)\log(a+bx)}{b^2}$$

input
```
Integrate[(a*c + (b*c + a*d)*x + b*d*x^2)/(a + b*x)^2,x]
```

output
```
(d*x)/b + ((b*c - a*d)*Log[a + b*x])/b^2
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.074$, Rules used = {1121, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x(ad+bc)+ac+bdx^2}{(a+bx)^2}dx$$
$$\Big\downarrow \text{1121}$$
$$\int \left(\frac{bc-ad}{b(a+bx)}+\frac{d}{b}\right)dx$$
$$\Big\downarrow \text{2009}$$
$$\frac{(bc-ad)\log(a+bx)}{b^2}+\frac{dx}{b}$$

input
```
Int[(a*c + (b*c + a*d)*x + b*d*x^2)/(a + b*x)^2,x]
```

output
```
(d*x)/b + ((b*c - a*d)*Log[a + b*x])/b^2
```

Defintions of rubi rules used

rule 1121
```
Int[((d_) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Int[ExpandIntegrand[(d + e*x)^(m + p)*(a/d + (c/e)*x)^p, x], x]
/; FreeQ[{a, b, c, d, e, m, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && (Int
egerQ[p] || (GtQ[a, 0] && GtQ[d, 0] && LtQ[c, 0]))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.67 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.04

method	result	size
default	$\frac{dx}{b}+\frac{(-ad+bc)\ln(bx+a)}{b^2}$	26
parallelrisch	$-\frac{\ln(bx+a)ad-\ln(bx+a)bc-bdx}{b^2}$	31
risch	$\frac{dx}{b}-\frac{\ln(bx+a)ad}{b^2}+\frac{\ln(bx+a)c}{b}$	32
norman	$\frac{d\,x^2-\frac{a^2d}{b^2}}{bx+a}-\frac{(ad-bc)\ln(bx+a)}{b^2}$	44

input
```
int((a*c+(a*d+b*c)*x+b*d*x^2)/(b*x+a)^2,x,method=_RETURNVERBOSE)
```

output
```
d*x/b+(-a*d+b*c)*ln(b*x+a)/b^2
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.96

$$\int \frac{ac+(bc+ad)x+bdx^2}{(a+bx)^2}\,dx=\frac{bdx+(bc-ad)\log(bx+a)}{b^2}$$

input
```
integrate((a*c+(a*d+b*c)*x+b*d*x^2)/(b*x+a)^2,x, algorithm="fricas")
```

output
```
(b*d*x + (b*c - a*d)*log(b*x + a))/b^2
```

Sympy [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.80

$$\int \frac{ac+(bc+ad)x+bdx^2}{(a+bx)^2}\,dx=\frac{dx}{b}-\frac{(ad-bc)\log(a+bx)}{b^2}$$

input
```
integrate((a*c+(a*d+b*c)*x+b*d*x**2)/(b*x+a)**2,x)
```

output
```
d*x/b - (a*d - b*c)*log(a + b*x)/b**2
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

$$\int \frac{ac + (bc + ad)x + bdx^2}{(a + bx)^2}\,dx = \frac{dx}{b} + \frac{(bc - ad)\log(bx + a)}{b^2}$$

input
```
integrate((a*c+(a*d+b*c)*x+b*d*x^2)/(b*x+a)^2,x, algorithm="maxima")
```

output
```
d*x/b + (b*c - a*d)*log(b*x + a)/b^2
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 117 vs. 2(25) = 50.

Time = 0.15 (sec) , antiderivative size = 117, normalized size of antiderivative = 4.68

$$\int \frac{ac + (bc + ad)x + bdx^2}{(a + bx)^2}\,dx = bd\left(\frac{2\,a\log\left(\frac{|bx+a|}{(bx+a)^2|b|}\right)}{b^3} + \frac{bx + a}{b^3} - \frac{a^2}{(bx + a)b^3}\right) - \frac{(bc + ad)\left(\frac{\log\left(\frac{|bx+a|}{(bx+a)^2|b|}\right)}{b} - \frac{a}{(bx+a)b}\right)}{b} - \frac{ac}{(bx + a)b}$$

input
```
integrate((a*c+(a*d+b*c)*x+b*d*x^2)/(b*x+a)^2,x, algorithm="giac")
```

output
```
b*d*(2*a*log(abs(b*x + a)/((b*x + a)^2*abs(b)))/b^3 + (b*x + a)/b^3 - a^2/
((b*x + a)*b^3)) - (b*c + a*d)*(log(abs(b*x + a)/((b*x + a)^2*abs(b)))/b -
 a/((b*x + a)*b))/b - a*c/((b*x + a)*b)
```

Mupad [B] (verification not implemented)

Time = 5.51 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.04

$$\int \frac{ac+(bc+ad)x+bdx^2}{(a+bx)^2}\,dx=\frac{d\,x}{b}-\frac{\ln\left(a+b\,x\right)\,\left(a\,d-b\,c\right)}{b^2}$$

input
```
int((a*c + x*(a*d + b*c) + b*d*x^2)/(a + b*x)^2,x)
```

output
```
(d*x)/b - (log(a + b*x)*(a*d - b*c))/b^2
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.12

$$\int \frac{ac+(bc+ad)x+bdx^2}{(a+bx)^2}\,dx=\frac{-\log(bx+a)\,ad+\log(bx+a)\,bc+bdx}{b^2}$$

input
```
int((a*c+(a*d+b*c)*x+b*d*x^2)/(b*x+a)^2,x)
```

output
```
( - log(a + b*x)*a*d + log(a + b*x)*b*c + b*d*x)/b**2
```

3.7 $\int \frac{ac+(bc+ad)x+bdx^2}{(a+bx)^3} dx$

Optimal result

Integrand size = 27, antiderivative size = 32

$$\int \frac{ac+(bc+ad)x+bdx^2}{(a+bx)^3} dx = -\frac{bc-ad}{b^2(a+bx)} + \frac{d\log(a+bx)}{b^2}$$

output
```
-(-a*d+b*c)/b^2/(b*x+a)+d*ln(b*x+a)/b^2
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.97

$$\int \frac{ac+(bc+ad)x+bdx^2}{(a+bx)^3} dx = \frac{-bc+ad}{b^2(a+bx)} + \frac{d\log(a+bx)}{b^2}$$

input
```
Integrate[(a*c + (b*c + a*d)*x + b*d*x^2)/(a + b*x)^3,x]
```

output
```
(-(b*c) + a*d)/(b^2*(a + b*x)) + (d*Log[a + b*x])/b^2
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.074$, Rules used = {1121, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x(ad+bc)+ac+bdx^2}{(a+bx)^3}\,dx$$

$$\Big\downarrow \text{ 1121}$$

$$\int \left(\frac{bc-ad}{b(a+bx)^2}+\frac{d}{b(a+bx)}\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{d\log(a+bx)}{b^2}-\frac{bc-ad}{b^2(a+bx)}$$

input
```
Int[(a*c + (b*c + a*d)*x + b*d*x^2)/(a + b*x)^3,x]
```

output
```
-((b*c - a*d)/(b^2*(a + b*x))) + (d*Log[a + b*x])/b^2
```

Defintions of rubi rules used

rule 1121
```
Int[((d_) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Int[ExpandIntegrand[(d + e*x)^(m + p)*(a/d + (c/e)*x)^p, x], x]
/; FreeQ[{a, b, c, d, e, m, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && (Int
egerQ[p] || (GtQ[a, 0] && GtQ[d, 0] && LtQ[c, 0]))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.66 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.03

method	result	size
default	$-\frac{-ad+bc}{b^{2}(bx+a)}+\frac{d\ln(bx+a)}{b^{2}}$	33
risch	$\frac{ad}{b^{2}(bx+a)}-\frac{c}{b(bx+a)}+\frac{d\ln(bx+a)}{b^{2}}$	39
parallelrisch	$\frac{\ln(bx+a)xbd+\ln(bx+a)ad+ad-bc}{b^{2}(bx+a)}$	39
norman	$\frac{\frac{a(ad-bc)}{b^{2}}+\frac{(ad-bc)x}{b}}{(bx+a)^{2}}+\frac{d\ln(bx+a)}{b^{2}}$	48

input
```
int((a*c+(a*d+b*c)*x+b*d*x^2)/(b*x+a)^3,x,method=_RETURNVERBOSE)
```

output
```
-(-a*d+b*c)/b^2/(b*x+a)+d*ln(b*x+a)/b^2
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.22

$$\int \frac{ac+(bc+ad)x+bdx^2}{(a+bx)^3}\,dx=-\frac{bc-ad-(bdx+ad)\log(bx+a)}{b^3x+ab^2}$$

input
```
integrate((a*c+(a*d+b*c)*x+b*d*x^2)/(b*x+a)^3,x, algorithm="fricas")
```

output
```
-(b*c - a*d - (b*d*x + a*d)*log(b*x + a))/(b^3*x + a*b^2)
```

Sympy [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.84

$$\int \frac{ac + (bc + ad)x + bdx^2}{(a + bx)^3} \, dx = \frac{ad - bc}{ab^2 + b^3x} + \frac{d \log (a + bx)}{b^2}$$

input
```
integrate((a*c+(a*d+b*c)*x+b*d*x**2)/(b*x+a)**3,x)
```

output
```
(a*d - b*c)/(a*b**2 + b**3*x) + d*log(a + b*x)/b**2
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.09

$$\int \frac{ac + (bc + ad)x + bdx^2}{(a + bx)^3} \, dx = -\frac{bc - ad}{b^3x + ab^2} + \frac{d \log (bx + a)}{b^2}$$

input
```
integrate((a*c+(a*d+b*c)*x+b*d*x^2)/(b*x+a)^3,x, algorithm="maxima")
```

output
```
-(b*c - a*d)/(b^3*x + a*b^2) + d*log(b*x + a)/b^2
```

Giac [A] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.03

$$\int \frac{ac + (bc + ad)x + bdx^2}{(a + bx)^3} \, dx = \frac{d \log (|bx + a|)}{b^2} - \frac{bc - ad}{(bx + a)b^2}$$

input
```
integrate((a*c+(a*d+b*c)*x+b*d*x^2)/(b*x+a)^3,x, algorithm="giac")
```

output
```
d*log(abs(b*x + a))/b^2 - (b*c - a*d)/((b*x + a)*b^2)
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.97

$$\int \frac{ac+(bc+ad)x+bdx^2}{(a+bx)^3}\,dx=\frac{a\,d-b\,c}{b^2\,\left(a+b\,x\right)}+\frac{d\,\ln\left(a+b\,x\right)}{b^2}$$

input
```
int((a*c + x*(a*d + b*c) + b*d*x^2)/(a + b*x)^3,x)
```

output
```
(a*d - b*c)/(b^2*(a + b*x)) + (d*log(a + b*x))/b^2
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.53

$$\int \frac{ac+(bc+ad)x+bdx^2}{(a+bx)^3}\,dx=\frac{\log(bx+a)\,a^2d+\log(bx+a)\,abdx-abdx+b^2cx}{a\,b^2\,(bx+a)}$$

input
```
int((a*c+(a*d+b*c)*x+b*d*x^2)/(b*x+a)^3,x)
```

output
```
(log(a + b*x)*a**2*d + log(a + b*x)*a*b*d*x - a*b*d*x + b**2*c*x)/(a*b**2*
(a + b*x))
```

3.8 $\int \frac{ac+(bc+ad)x+bdx^2}{(a+bx)^4} \, dx$

Optimal result

Integrand size = 27, antiderivative size = 28

$$\int \frac{ac+(bc+ad)x+bdx^2}{(a+bx)^4} \, dx = -\frac{(c+dx)^2}{2(bc-ad)(a+bx)^2}$$

output
```
-1/2*(d*x+c)^2/(-a*d+b*c)/(b*x+a)^2
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.93

$$\int \frac{ac+(bc+ad)x+bdx^2}{(a+bx)^4} \, dx = -\frac{ad+b(c+2dx)}{2b^2(a+bx)^2}$$

input
```
Integrate[(a*c + (b*c + a*d)*x + b*d*x^2)/(a + b*x)^4,x]
```

output
```
-1/2*(a*d + b*(c + 2*d*x))/(b^2*(a + b*x)^2)
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.074$, Rules used = {1120, 48}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x(ad+bc)+ac+bdx^2}{(a+bx)^4}dx$$
$$\Big\downarrow \ 1120$$
$$\int \frac{c+dx}{(a+bx)^3}dx$$
$$\Big\downarrow \ 48$$
$$-\frac{(c+dx)^2}{2(a+bx)^2(bc-ad)}$$

input
```
Int[(a*c + (b*c + a*d)*x + b*d*x^2)/(a + b*x)^4,x]
```

output
```
-1/2*(c + d*x)^2/((b*c - a*d)*(a + b*x)^2)
```

Defintions of rubi rules used

rule 48
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && EqQ[m + n + 2, 0] && NeQ[m, -1]
```

rule 1120
```
Int[((d_) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Int[(d + e*x)^(m + p)*(a/d + (c/e)*x)^p, x] /; FreeQ[{a, b, c, d
, e, m}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[p] && (EqQ[m + p,
0] || EqQ[m + 2*p + 2, 0])
```

Maple [A] (verified)

Time = 0.70 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.89

method	result	size
gosper	$-\frac{2bdx+ad+bc}{2b^{2}(bx+a)^{2}}$	25
parallelrisch	$\frac{-2bdx-ad-bc}{2b^{2}(bx+a)^{2}}$	27
risch	$\frac{-\frac{dx}{b}-\frac{ad+bc}{2b^{2}}}{(bx+a)^{2}}$	29
default	$-\frac{d}{b^{2}(bx+a)}-\frac{-ad+bc}{2b^{2}(bx+a)^{2}}$	35
orering	$-\frac{(2bdx+ad+bc)\left(ac+(ad+bc)x+bd\,x^{2}\right)}{2b^{2}(bx+a)^{3}(dx+c)}$	51
norman	$\frac{-d\,x^{2}+\frac{a\left(-dab-b^{2}c\right)}{2b^{3}}+\frac{\left(-3dab-b^{2}c\right)x}{2b^{2}}}{(bx+a)^{3}}$	52

input
```
int((a*c+(a*d+b*c)*x+b*d*x^2)/(b*x+a)^4,x,method=_RETURNVERBOSE)
```

output
```
-1/2*(2*b*d*x+a*d+b*c)/b^2/(b*x+a)^2
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.36

$$\int \frac{ac+(bc+ad)x+bdx^2}{(a+bx)^4}\,dx=-\frac{2\,bdx+bc+ad}{2\,(b^4x^2+2\,ab^3x+a^2b^2)}$$

input
```
integrate((a*c+(a*d+b*c)*x+b*d*x^2)/(b*x+a)^4,x, algorithm="fricas")
```

output
```
-1/2*(2*b*d*x + b*c + a*d)/(b^4*x^2 + 2*a*b^3*x + a^2*b^2)
```

Sympy [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.39

$$\int \frac{ac+(bc+ad)x+bdx^2}{(a+bx)^4}\,dx = \frac{-ad-bc-2bdx}{2a^2b^2+4ab^3x+2b^4x^2}$$

input
```
integrate((a*c+(a*d+b*c)*x+b*d*x**2)/(b*x+a)**4,x)
```

output
```
(-a*d - b*c - 2*b*d*x)/(2*a**2*b**2 + 4*a*b**3*x + 2*b**4*x**2)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.36

$$\int \frac{ac+(bc+ad)x+bdx^2}{(a+bx)^4}\,dx = -\frac{2\,bdx+bc+ad}{2\,(b^4x^2+2\,ab^3x+a^2b^2)}$$

input
```
integrate((a*c+(a*d+b*c)*x+b*d*x^2)/(b*x+a)^4,x, algorithm="maxima")
```

output
```
-1/2*(2*b*d*x + b*c + a*d)/(b^4*x^2 + 2*a*b^3*x + a^2*b^2)
```

Giac [A] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.86

$$\int \frac{ac+(bc+ad)x+bdx^2}{(a+bx)^4}\,dx = -\frac{2\,bdx+bc+ad}{2\,(bx+a)^2b^2}$$

input
```
integrate((a*c+(a*d+b*c)*x+b*d*x^2)/(b*x+a)^4,x, algorithm="giac")
```

output
```
-1/2*(2*b*d*x + b*c + a*d)/((b*x + a)^2*b^2)
```

Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.39

$$\int \frac{ac+(bc+ad)x+bdx^2}{(a+bx)^4}\,dx = -\frac{\frac{a\,d+b\,c}{2\,b^2}+\frac{d\,x}{b}}{a^2+2\,a\,b\,x+b^2\,x^2}$$

input
```
int((a*c + x*(a*d + b*c) + b*d*x^2)/(a + b*x)^4,x)
```

output
```
-((a*d + b*c)/(2*b^2) + (d*x)/b)/(a^2 + b^2*x^2 + 2*a*b*x)
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.32

$$\int \frac{ac+(bc+ad)x+bdx^2}{(a+bx)^4}\,dx = \frac{bd\,x^2-ac}{2ab\,(b^2x^2+2abx+a^2)}$$

input
```
int((a*c+(a*d+b*c)*x+b*d*x^2)/(b*x+a)^4,x)
```

output
```
( - a*c + b*d*x**2)/(2*a*b*(a**2 + 2*a*b*x + b**2*x**2))
```

3.9 $\int \frac{ac+(bc+ad)x+bdx^2}{(a+bx)^5}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 38

$$\int \frac{ac+(bc+ad)x+bdx^2}{(a+bx)^5}\,dx = -\frac{bc-ad}{3b^2(a+bx)^3} - \frac{d}{2b^2(a+bx)^2}$$

output
```
-1/3*(-a*d+b*c)/b^2/(b*x+a)^3-1/2*d/b^2/(b*x+a)^2
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.71

$$\int \frac{ac+(bc+ad)x+bdx^2}{(a+bx)^5}\,dx = -\frac{2bc+ad+3bdx}{6b^2(a+bx)^3}$$

input
```
Integrate[(a*c + (b*c + a*d)*x + b*d*x^2)/(a + b*x)^5,x]
```

output
```
-1/6*(2*b*c + a*d + 3*b*d*x)/(b^2*(a + b*x)^3)
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.074, Rules used = {1121, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x(ad+bc)+ac+bdx^2}{(a+bx)^5}dx$$

$$\Big\downarrow \text{1121}$$

$$\int \left(\frac{bc-ad}{b(a+bx)^4}+\frac{d}{b(a+bx)^3}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{bc-ad}{3b^2(a+bx)^3}-\frac{d}{2b^2(a+bx)^2}$$

input
```
Int[(a*c + (b*c + a*d)*x + b*d*x^2)/(a + b*x)^5,x]
```

output
```
-1/3*(b*c - a*d)/(b^2*(a + b*x)^3) - d/(2*b^2*(a + b*x)^2)
```

Defintions of rubi rules used

rule 1121
```
Int[((d_) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Int[ExpandIntegrand[(d + e*x)^(m + p)*(a/d + (c/e)*x)^p, x], x]
/; FreeQ[{a, b, c, d, e, m, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && (Int
egerQ[p] || (GtQ[a, 0] && GtQ[d, 0] && LtQ[c, 0]))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.66 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.68

method	result	size
gosper	$-\frac{3bdx+ad+2bc}{6b^2(bx+a)^3}$	26
risch	$\frac{-\frac{dx}{2b}-\frac{ad+2bc}{6b^2}}{(bx+a)^3}$	30
parallelrisch	$\frac{-3b^2dx-dab-2b^2c}{6b^3(bx+a)^3}$	32
default	$-\frac{-ad+bc}{3b^2(bx+a)^3}-\frac{d}{2b^2(bx+a)^2}$	35
orering	$-\frac{(3bdx+ad+2bc)(ac+(ad+bc)x+bd\,x^2)}{6b^2(bx+a)^4(dx+c)}$	52
norman	$\frac{\frac{a\left(-da\,b^2-2b^3c\right)}{6b^4}-\frac{d\,x^2}{2}+\frac{\left(-2da\,b^2-b^3c\right)x}{3b^3}}{(bx+a)^4}$	56

input
```
int((a*c+(a*d+b*c)*x+b*d*x^2)/(b*x+a)^5,x,method=_RETURNVERBOSE)
```

output
```
-1/6/b^2*(3*b*d*x+a*d+2*b*c)/(b*x+a)^3
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.32

$$\int \frac{ac+(bc+ad)x+bdx^2}{(a+bx)^5}\,dx = -\frac{3\,bdx+2\,bc+ad}{6\,(b^5x^3+3\,ab^4x^2+3\,a^2b^3x+a^3b^2)}$$

input
```
integrate((a*c+(a*d+b*c)*x+b*d*x^2)/(b*x+a)^5,x, algorithm="fricas")
```

output
```
-1/6*(3*b*d*x + 2*b*c + a*d)/(b^5*x^3 + 3*a*b^4*x^2 + 3*a^2*b^3*x + a^3*b^
2)
```

Sympy [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.39

$$\int \frac{ac+(bc+ad)x+bdx^2}{(a+bx)^5}\,dx = \frac{-ad-2bc-3bdx}{6a^3b^2+18a^2b^3x+18ab^4x^2+6b^5x^3}$$

input
```
integrate((a*c+(a*d+b*c)*x+b*d*x**2)/(b*x+a)**5,x)
```

output
```
(-a*d - 2*b*c - 3*b*d*x)/(6*a**3*b**2 + 18*a**2*b**3*x + 18*a*b**4*x**2 +
6*b**5*x**3)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.32

$$\int \frac{ac+(bc+ad)x+bdx^2}{(a+bx)^5}\,dx = -\frac{3\,bdx+2\,bc+ad}{6\,(b^5x^3+3\,ab^4x^2+3\,a^2b^3x+a^3b^2)}$$

input
```
integrate((a*c+(a*d+b*c)*x+b*d*x^2)/(b*x+a)^5,x, algorithm="maxima")
```

output
```
-1/6*(3*b*d*x + 2*b*c + a*d)/(b^5*x^3 + 3*a*b^4*x^2 + 3*a^2*b^3*x + a^3*b^
2)
```

Giac [A] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.08

$$\int \frac{ac+(bc+ad)x+bdx^2}{(a+bx)^5}\,dx = -\frac{c}{3\,(bx+a)^3b}-\frac{d}{2\,(bx+a)^2b^2}+\frac{ad}{3\,(bx+a)^3b^2}$$

input
```
integrate((a*c+(a*d+b*c)*x+b*d*x^2)/(b*x+a)^5,x, algorithm="giac")
```

output
```
-1/3*c/((b*x + a)^3*b) - 1/2*d/((b*x + a)^2*b^2) + 1/3*a*d/((b*x + a)^3*b^
2)
```

Mupad [B] (verification not implemented)

Time = 5.53 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.37

$$\int \frac{ac + (bc + ad)x + bdx^2}{(a + bx)^5} \, dx = -\frac{\frac{a\,d+2\,b\,c}{6\,b^2} + \frac{d\,x}{2\,b}}{a^3 + 3\,a^2\,b\,x + 3\,a\,b^2\,x^2 + b^3\,x^3}$$

input
```
int((a*c + x*(a*d + b*c) + b*d*x^2)/(a + b*x)^5,x)
```

output
```
-((a*d + 2*b*c)/(6*b^2) + (d*x)/(2*b))/(a^3 + b^3*x^3 + 3*a*b^2*x^2 + 3*a^
2*b*x)
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.26

$$\int \frac{ac + (bc + ad)x + bdx^2}{(a + bx)^5} \, dx = \frac{-3bdx - ad - 2bc}{6b^2 \left(b^3x^3 + 3a\,b^2x^2 + 3a^2bx + a^3\right)}$$

input
```
int((a*c+(a*d+b*c)*x+b*d*x^2)/(b*x+a)^5,x)
```

output
```
( - a*d - 2*b*c - 3*b*d*x)/(6*b**2*(a**3 + 3*a**2*b*x + 3*a*b**2*x**2 + b*
*3*x**3))
```

3.10 $\int \frac{ac+(bc+ad)x+bdx^2}{(a+bx)^6} dx$

Optimal result

Integrand size = 27, antiderivative size = 38

$$\int \frac{ac+(bc+ad)x+bdx^2}{(a+bx)^6} dx = -\frac{bc-ad}{4b^2(a+bx)^4} - \frac{d}{3b^2(a+bx)^3}$$

output
```
-1/4*(-a*d+b*c)/b^2/(b*x+a)^4-1/3*d/b^2/(b*x+a)^3
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.71

$$\int \frac{ac+(bc+ad)x+bdx^2}{(a+bx)^6} dx = -\frac{3bc+ad+4bdx}{12b^2(a+bx)^4}$$

input
```
Integrate[(a*c + (b*c + a*d)*x + b*d*x^2)/(a + b*x)^6,x]
```

output
```
-1/12*(3*b*c + a*d + 4*b*d*x)/(b^2*(a + b*x)^4)
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.074$, Rules used = {1121, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x(ad+bc)+ac+bdx^2}{(a+bx)^6} \, dx$$

$$\Big\downarrow \ 1121$$

$$\int \left(\frac{bc-ad}{b(a+bx)^5}+\frac{d}{b(a+bx)^4}\right)dx$$

$$\Big\downarrow \ 2009$$

$$-\frac{bc-ad}{4b^2(a+bx)^4}-\frac{d}{3b^2(a+bx)^3}$$

input
```
Int[(a*c + (b*c + a*d)*x + b*d*x^2)/(a + b*x)^6,x]
```

output
```
-1/4*(b*c - a*d)/(b^2*(a + b*x)^4) - d/(3*b^2*(a + b*x)^3)
```

Defintions of rubi rules used

rule 1121
```
Int[((d_) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Int[ExpandIntegrand[(d + e*x)^(m + p)*(a/d + (c/e)*x)^p, x], x]
/; FreeQ[{a, b, c, d, e, m, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && (Int
egerQ[p] || (GtQ[a, 0] && GtQ[d, 0] && LtQ[c, 0]))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.66 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.68

method	result	size
gosper	$-\frac{4bdx+ad+3bc}{12b^{2}(bx+a)^{4}}$	26
risch	$\frac{-\frac{dx}{3b}-\frac{ad+3bc}{12b^{2}}}{(bx+a)^{4}}$	30
parallelrisch	$\frac{-4dx\,b^{3}-da\,b^{2}-3b^{3}c}{12b^{4}(bx+a)^{4}}$	34
default	$-\frac{-ad+bc}{4b^{2}(bx+a)^{4}}-\frac{d}{3b^{2}(bx+a)^{3}}$	35
orering	$-\frac{(4bdx+ad+3bc)(ac+(ad+bc)x+bd\,x^{2})}{12b^{2}(bx+a)^{5}(dx+c)}$	52
norman	$\frac{\frac{a(-a\,b^{3}d-3b^{4}c)}{12b^{5}}-\frac{d\,x^{2}}{3}+\frac{(-5a\,b^{3}d-3b^{4}c)x}{12b^{4}}}{(bx+a)^{5}}$	56

input
```
int((a*c+(a*d+b*c)*x+b*d*x^2)/(b*x+a)^6,x,method=_RETURNVERBOSE)
```

output
```
-1/12/b^2*(4*b*d*x+a*d+3*b*c)/(b*x+a)^4
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.61

$$\int \frac{ac+(bc+ad)x+bdx^2}{(a+bx)^6}\,dx = -\frac{4\,bdx+3\,bc+ad}{12\,(b^6x^4+4\,ab^5x^3+6\,a^2b^4x^2+4\,a^3b^3x+a^4b^2)}$$

input
```
integrate((a*c+(a*d+b*c)*x+b*d*x^2)/(b*x+a)^6,x, algorithm="fricas")
```

output
```
-1/12*(4*b*d*x + 3*b*c + a*d)/(b^6*x^4 + 4*a*b^5*x^3 + 6*a^2*b^4*x^2 + 4*a
^3*b^3*x + a^4*b^2)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 65 vs. $2(32) = 64$.

Time = 0.22 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.71

$$\int \frac{ac + (bc + ad)x + bdx^2}{(a + bx)^6}\,dx = \frac{-ad - 3bc - 4bdx}{12a^4b^2 + 48a^3b^3x + 72a^2b^4x^2 + 48ab^5x^3 + 12b^6x^4}$$

input
```
integrate((a*c+(a*d+b*c)*x+b*d*x**2)/(b*x+a)**6,x)
```

output
```
(-a*d - 3*b*c - 4*b*d*x)/(12*a**4*b**2 + 48*a**3*b**3*x + 72*a**2*b**4*x**
2 + 48*a*b**5*x**3 + 12*b**6*x**4)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.61

$$\int \frac{ac + (bc + ad)x + bdx^2}{(a + bx)^6}\,dx = -\frac{4\,bdx + 3\,bc + ad}{12\,(b^6x^4 + 4\,ab^5x^3 + 6\,a^2b^4x^2 + 4\,a^3b^3x + a^4b^2)}$$

input
```
integrate((a*c+(a*d+b*c)*x+b*d*x^2)/(b*x+a)^6,x, algorithm="maxima")
```

output
```
-1/12*(4*b*d*x + 3*b*c + a*d)/(b^6*x^4 + 4*a*b^5*x^3 + 6*a^2*b^4*x^2 + 4*a
^3*b^3*x + a^4*b^2)
```

Giac [A] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.66

$$\int \frac{ac + (bc + ad)x + bdx^2}{(a + bx)^6}\,dx = -\frac{4\,bdx + 3\,bc + ad}{12\,(bx + a)^4b^2}$$

input
```
integrate((a*c+(a*d+b*c)*x+b*d*x^2)/(b*x+a)^6,x, algorithm="giac")
```

output
```
-1/12*(4*b*d*x + 3*b*c + a*d)/((b*x + a)^4*b^2)
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.66

$$\int \frac{ac+(bc+ad)x+bdx^2}{(a+bx)^6}\,dx = -\frac{\frac{a\,d+3\,b\,c}{12\,b^2}+\frac{d\,x}{3\,b}}{a^4+4\,a^3\,b\,x+6\,a^2\,b^2\,x^2+4\,a\,b^3\,x^3+b^4\,x^4}$$

input
```
int((a*c + x*(a*d + b*c) + b*d*x^2)/(a + b*x)^6,x)
```

output
```
-((a*d + 3*b*c)/(12*b^2) + (d*x)/(3*b))/(a^4 + b^4*x^4 + 4*a*b^3*x^3 + 6*a
^2*b^2*x^2 + 4*a^3*b*x)
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.55

$$\int \frac{ac+(bc+ad)x+bdx^2}{(a+bx)^6}\,dx = \frac{-4bdx-ad-3bc}{12b^2\,(b^4x^4+4a\,b^3x^3+6a^2b^2x^2+4a^3bx+a^4)}$$

input
```
int((a*c+(a*d+b*c)*x+b*d*x^2)/(b*x+a)^6,x)
```

output
```
( - a*d - 3*b*c - 4*b*d*x)/(12*b**2*(a**4 + 4*a**3*b*x + 6*a**2*b**2*x**2
+ 4*a*b**3*x**3 + b**4*x**4))
```

3.11 $\int (a+bx)^3 \left(ac+(bc+ad)x+bdx^2\right)^2 \, dx$

Optimal result

Integrand size = 29, antiderivative size = 65

$$\int (a+bx)^3 \left(ac+(bc+ad)x+bdx^2\right)^2 \, dx = \frac{(bc-ad)^2(a+bx)^6}{6b^3} + \frac{2d(bc-ad)(a+bx)^7}{7b^3} + \frac{d^2(a+bx)^8}{8b^3}$$

output
```
1/6*(-a*d+b*c)^2*(b*x+a)^6/b^3+2/7*d*(-a*d+b*c)*(b*x+a)^7/b^3+1/8*d^2*(b*x
+a)^8/b^3
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 189 vs. 2(65) = 130.

Time = 0.03 (sec) , antiderivative size = 189, normalized size of antiderivative = 2.91

$$\int (a+bx)^3\left(ac+(bc+ad)x+bdx^2\right)^2\,dx = a^5c^2x+\frac{1}{2}a^4c(5bc+2ad)x^2 + \frac{1}{3}a^3\left(10b^2c^2+10abcd+a^2d^2\right)x^3 + \frac{5}{4}a^2b\left(2b^2c^2+4abcd+a^2d^2\right)x^4 + ab^2\left(b^2c^2+4abcd+2a^2d^2\right)x^5 + \frac{1}{6}b^3\left(b^2c^2+10abcd+10a^2d^2\right)x^6 + \frac{1}{7}b^4d(2bc+5ad)x^7+\frac{1}{8}b^5d^2x^8$$

input
```
Integrate[(a + b*x)^3*(a*c + (b*c + a*d)*x + b*d*x^2)^2,x]
```

output
```
a^5*c^2*x + (a^4*c*(5*b*c + 2*a*d)*x^2)/2 + (a^3*(10*b^2*c^2 + 10*a*b*c*d
+ a^2*d^2)*x^3)/3 + (5*a^2*b*(2*b^2*c^2 + 4*a*b*c*d + a^2*d^2)*x^4)/4 + a*
b^2*(b^2*c^2 + 4*a*b*c*d + 2*a^2*d^2)*x^5 + (b^3*(b^2*c^2 + 10*a*b*c*d + 1
0*a^2*d^2)*x^6)/6 + (b^4*d*(2*b*c + 5*a*d)*x^7)/7 + (b^5*d^2*x^8)/8
```

Rubi [A] (verified)

Time = 0.48 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.069$, Rules used = {1121, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+bx)^3\left(x(ad+bc)+ac+bdx^2\right)^2\,dx$$

$$\Big\downarrow \; 1121$$

$$\int \left(\frac{2d(a+bx)^6(bc-ad)}{b^2}+\frac{(a+bx)^5(bc-ad)^2}{b^2}+\frac{d^2(a+bx)^7}{b^2}\right)dx$$
$$\Big\downarrow \text{2009}$$
$$\frac{2d(a+bx)^7(bc-ad)}{7b^3}+\frac{(a+bx)^6(bc-ad)^2}{6b^3}+\frac{d^2(a+bx)^8}{8b^3}$$

input
```
Int[(a + b*x)^3*(a*c + (b*c + a*d)*x + b*d*x^2)^2,x]
```

output
```
((b*c - a*d)^2*(a + b*x)^6)/(6*b^3) + (2*d*(b*c - a*d)*(a + b*x)^7)/(7*b^3
) + (d^2*(a + b*x)^8)/(8*b^3)
```

Defintions of rubi rules used

rule 1121
```
Int[((d_) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Int[ExpandIntegrand[(d + e*x)^(m + p)*(a/d + (c/e)*x)^p, x], x]
/; FreeQ[{a, b, c, d, e, m, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && (Int
egerQ[p] || (GtQ[a, 0] && GtQ[d, 0] && LtQ[c, 0]))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 194 vs. $2(59) = 118$.

Time = 1.21 (sec) , antiderivative size = 195, normalized size of antiderivative = 3.00

method	result
norman	$\frac{b^5d^2x^8}{8}+\left(\frac{5}{7}a\,b^4d^2+\frac{2}{7}b^5cd\right)x^7+\left(\frac{5}{3}d^2a^2b^3+\frac{5}{3}cda\,b^4+\frac{1}{6}b^5c^2\right)x^6+\left(2d^2a^3b^2+4a^2b^3cd+a\,b\right.$
risch	$\frac{1}{8}b^5d^2x^8+\frac{5}{7}x^7a\,b^4d^2+\frac{2}{7}x^7b^5cd+\frac{5}{3}x^6d^2a^2b^3+\frac{5}{3}x^6cda\,b^4+\frac{1}{6}x^6b^5c^2+2a^3b^2d^2x^5+4a^2b^3cd$
parallelrisch	$\frac{1}{8}b^5d^2x^8+\frac{5}{7}x^7a\,b^4d^2+\frac{2}{7}x^7b^5cd+\frac{5}{3}x^6d^2a^2b^3+\frac{5}{3}x^6cda\,b^4+\frac{1}{6}x^6b^5c^2+2a^3b^2d^2x^5+4a^2b^3cd$
gosper	$\frac{x(21b^5d^2x^7+120x^6a\,b^4d^2+48x^6b^5cd+280x^5d^2a^2b^3+280x^5cda\,b^4+28b^5c^2x^5+336a^3b^2d^2x^4+672a^2b^3cd\,x^4+168a\,b^4c^2x^4+2}{168}$
orering	$\frac{x(21b^5d^2x^7+120x^6a\,b^4d^2+48x^6b^5cd+280x^5d^2a^2b^3+280x^5cda\,b^4+28b^5c^2x^5+336a^3b^2d^2x^4+672a^2b^3cd\,x^4+168a\,b^4c^2x^4+2}{16}$
default	$\frac{b^5d^2x^8}{8}+\frac{(3a\,b^4d^2+2b^4d(ad+bc))x^7}{7}+\frac{\left(3d^2a^2b^3+6a\,b^3d(ad+bc)+b^3\left(2abcd+(ad+bc)^2\right)\right)x^6}{6}+\frac{\left(d^2a^3b^2+6a^2b^2d(ad+b\right.}{}$

input
```
int((b*x+a)^3*(a*c+(a*d+b*c)*x+b*d*x^2)^2,x,method=_RETURNVERBOSE)
```

output
```
1/8*b^5*d^2*x^8+(5/7*a*b^4*d^2+2/7*b^5*c*d)*x^7+(5/3*d^2*a^2*b^3+5/3*c*d*a
*b^4+1/6*b^5*c^2)*x^6+(2*a^3*b^2*d^2+4*a^2*b^3*c*d+a*b^4*c^2)*x^5+(5/4*a^4
*d^2*b+5*c*d*a^3*b^2+5/2*a^2*b^3*c^2)*x^4+(1/3*d^2*a^5+10/3*c*d*a^4*b+10/3
*a^3*b^2*c^2)*x^3+(c*d*a^5+5/2*c^2*a^4*b)*x^2+a^5*c^2*x
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 197 vs. $2(59) = 118$.

Time = 0.07 (sec) , antiderivative size = 197, normalized size of antiderivative = 3.03

$$\int (a+bx)^3\left(ac+(bc+ad)x+bdx^2\right)^2\,dx=\frac{1}{8}\,b^5d^2x^8+a^5c^2x+\frac{1}{7}\left(2\,b^5cd+5\,ab^4d^2\right)x^7+\frac{1}{6}\left(b^5c^2+10\,ab^4cd+10\,a^2b^3d^2\right)x^6+\left(ab^4c^2+4\,a^2b^3cd+2\,a^3b^2d^2\right)x^5+\frac{5}{4}\left(2\,a^2b^3c^2+4\,a^3b^2cd+a^4bd^2\right)x^4+\frac{1}{3}\left(10\,a^3b^2c^2+10\,a^4bcd+a^5d^2\right)x^3+\frac{1}{2}\left(5\,a^4bc^2+2\,a^5cd\right)x^2$$

input
```
integrate((b*x+a)^3*(a*c+(a*d+b*c)*x+b*d*x^2)^2,x, algorithm="fricas")
```

output
```
1/8*b^5*d^2*x^8 + a^5*c^2*x + 1/7*(2*b^5*c*d + 5*a*b^4*d^2)*x^7 + 1/6*(b^5
*c^2 + 10*a*b^4*c*d + 10*a^2*b^3*d^2)*x^6 + (a*b^4*c^2 + 4*a^2*b^3*c*d + 2
*a^3*b^2*d^2)*x^5 + 5/4*(2*a^2*b^3*c^2 + 4*a^3*b^2*c*d + a^4*b*d^2)*x^4 +
1/3*(10*a^3*b^2*c^2 + 10*a^4*b*c*d + a^5*d^2)*x^3 + 1/2*(5*a^4*b*c^2 + 2*a
^5*c*d)*x^2
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 218 vs. $2(56) = 112$.

Time = 0.04 (sec) , antiderivative size = 218, normalized size of antiderivative = 3.35

$$\int (a+bx)^3 \left(ac+(bc+ad)x+bdx^2\right)^2 \,dx = a^5c^2x + \frac{b^5d^2x^8}{8} + x^7 \cdot \left(\frac{5ab^4d^2}{7} + \frac{2b^5cd}{7}\right) + x^6 \cdot \left(\frac{5a^2b^3d^2}{3} + \frac{5ab^4cd}{3} + \frac{b^5c^2}{6}\right) + x^5 \cdot \left(2a^3b^2d^2 + 4a^2b^3cd + ab^4c^2\right) + x^4 \cdot \left(\frac{5a^4bd^2}{4} + 5a^3b^2cd + \frac{5a^2b^3c^2}{2}\right) + x^3\left(\frac{a^5d^2}{3} + \frac{10a^4bcd}{3} + \frac{10a^3b^2c^2}{3}\right) + x^2\left(a^5cd + \frac{5a^4bc^2}{2}\right)$$

input
```
integrate((b*x+a)**3*(a*c+(a*d+b*c)*x+b*d*x**2)**2,x)
```

output
```
a**5*c**2*x + b**5*d**2*x**8/8 + x**7*(5*a*b**4*d**2/7 + 2*b**5*c*d/7) + x
**6*(5*a**2*b**3*d**2/3 + 5*a*b**4*c*d/3 + b**5*c**2/6) + x**5*(2*a**3*b**
2*d**2 + 4*a**2*b**3*c*d + a*b**4*c**2) + x**4*(5*a**4*b*d**2/4 + 5*a**3*b
**2*c*d + 5*a**2*b**3*c**2/2) + x**3*(a**5*d**2/3 + 10*a**4*b*c*d/3 + 10*a
**3*b**2*c**2/3) + x**2*(a**5*c*d + 5*a**4*b*c**2/2)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 197 vs. $2(59) = 118$.

Time = 0.03 (sec) , antiderivative size = 197, normalized size of antiderivative = 3.03

$$\int (a+bx)^3 \left(ac+(bc+ad)x+bdx^2\right)^2 dx = \frac{1}{8}b^5d^2x^8 + a^5c^2x + \frac{1}{7}\left(2b^5cd + 5ab^4d^2\right)x^7 + \frac{1}{6}\left(b^5c^2 + 10ab^4cd + 10a^2b^3d^2\right)x^6 + \left(ab^4c^2 + 4a^2b^3cd + 2a^3b^2d^2\right)x^5 + \frac{5}{4}\left(2a^2b^3c^2 + 4a^3b^2cd + a^4bd^2\right)x^4 + \frac{1}{3}\left(10a^3b^2c^2 + 10a^4bcd + a^5d^2\right)x^3 + \frac{1}{2}\left(5a^4bc^2 + 2a^5cd\right)x^2$$

input
```
integrate((b*x+a)^3*(a*c+(a*d+b*c)*x+b*d*x^2)^2,x, algorithm="maxima")
```

output
```
1/8*b^5*d^2*x^8 + a^5*c^2*x + 1/7*(2*b^5*c*d + 5*a*b^4*d^2)*x^7 + 1/6*(b^5
*c^2 + 10*a*b^4*c*d + 10*a^2*b^3*d^2)*x^6 + (a*b^4*c^2 + 4*a^2*b^3*c*d + 2
*a^3*b^2*d^2)*x^5 + 5/4*(2*a^2*b^3*c^2 + 4*a^3*b^2*c*d + a^4*b*d^2)*x^4 +
1/3*(10*a^3*b^2*c^2 + 10*a^4*b*c*d + a^5*d^2)*x^3 + 1/2*(5*a^4*b*c^2 + 2*a
^5*c*d)*x^2
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 212 vs. $2(59) = 118$.

Time = 0.17 (sec) , antiderivative size = 212, normalized size of antiderivative = 3.26

$$\int (a+bx)^3 \left(ac+(bc+ad)x+bdx^2\right)^2 dx = \frac{1}{8}b^5d^2x^8 + \frac{2}{7}b^5cdx^7 + \frac{5}{7}ab^4d^2x^7 + \frac{1}{6}b^5c^2x^6 + \frac{5}{3}ab^4cdx^6 + \frac{5}{3}a^2b^3d^2x^6 + ab^4c^2x^5 + 4a^2b^3cdx^5 + 2a^3b^2d^2x^5 + \frac{5}{2}a^2b^3c^2x^4 + 5a^3b^2cdx^4 + \frac{5}{4}a^4bd^2x^4 + \frac{10}{3}a^3b^2c^2x^3 + \frac{10}{3}a^4bcdx^3 + \frac{1}{3}a^5d^2x^3 + \frac{5}{2}a^4bc^2x^2 + a^5cdx^2 + a^5c^2x$$

input
```
integrate((b*x+a)^3*(a*c+(a*d+b*c)*x+b*d*x^2)^2,x, algorithm="giac")
```

output
```
1/8*b^5*d^2*x^8 + 2/7*b^5*c*d*x^7 + 5/7*a*b^4*d^2*x^7 + 1/6*b^5*c^2*x^6 +
5/3*a*b^4*c*d*x^6 + 5/3*a^2*b^3*d^2*x^6 + a*b^4*c^2*x^5 + 4*a^2*b^3*c*d*x^
5 + 2*a^3*b^2*d^2*x^5 + 5/2*a^2*b^3*c^2*x^4 + 5*a^3*b^2*c*d*x^4 + 5/4*a^4*
b*d^2*x^4 + 10/3*a^3*b^2*c^2*x^3 + 10/3*a^4*b*c*d*x^3 + 1/3*a^5*d^2*x^3 +
5/2*a^4*b*c^2*x^2 + a^5*c*d*x^2 + a^5*c^2*x
```

Mupad [B] (verification not implemented)

Time = 5.58 (sec) , antiderivative size = 181, normalized size of antiderivative = 2.78

$$\int (a+bx)^3\left(ac+(bc+ad)x+bdx^2\right)^2\,dx = x^3\left(\frac{a^5\,d^2}{3}+\frac{10\,a^4\,b\,c\,d}{3}+\frac{10\,a^3\,b^2\,c^2}{3}\right) + x^6\left(\frac{5\,a^2\,b^3\,d^2}{3}+\frac{5\,a\,b^4\,c\,d}{3}+\frac{b^5\,c^2}{6}\right) + a^5\,c^2\,x+\frac{b^5\,d^2\,x^8}{8}+\frac{a^4\,c\,x^2\,(2\,a\,d+5\,b\,c)}{2} + \frac{b^4\,d\,x^7\,(5\,a\,d+2\,b\,c)}{7} + \frac{5\,a^2\,b\,x^4\,(a^2\,d^2+4\,a\,b\,c\,d+2\,b^2\,c^2)}{4} + a\,b^2\,x^5\,\left(2\,a^2\,d^2+4\,a\,b\,c\,d+b^2\,c^2\right)$$

input
```
int((a + b*x)^3*(a*c + x*(a*d + b*c) + b*d*x^2)^2,x)
```

output
```
x^3*((a^5*d^2)/3 + (10*a^3*b^2*c^2)/3 + (10*a^4*b*c*d)/3) + x^6*((b^5*c^2)
/6 + (5*a^2*b^3*d^2)/3 + (5*a*b^4*c*d)/3) + a^5*c^2*x + (b^5*d^2*x^8)/8 +
(a^4*c*x^2*(2*a*d + 5*b*c))/2 + (b^4*d*x^7*(5*a*d + 2*b*c))/7 + (5*a^2*b*x
^4*(a^2*d^2 + 2*b^2*c^2 + 4*a*b*c*d))/4 + a*b^2*x^5*(2*a^2*d^2 + b^2*c^2 +
 4*a*b*c*d)
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 213, normalized size of antiderivative = 3.28

$$\int (a+bx)^3 \left(ac+(bc+ad)x+bdx^2\right)^2 dx$$
$$= \frac{x(21b^5d^2x^7+120a\,b^4d^2x^6+48b^5cd\,x^6+280a^2b^3d^2x^5+280a\,b^4cd\,x^5+28b^5c^2x^5+336a^3b^2d^2x^4+672a^2b}{}$$

input
```
int((b*x+a)^3*(a*c+(a*d+b*c)*x+b*d*x^2)^2,x)
```

output
```
(x*(168*a**5*c**2 + 168*a**5*c*d*x + 56*a**5*d**2*x**2 + 420*a**4*b*c**2*x
 + 560*a**4*b*c*d*x**2 + 210*a**4*b*d**2*x**3 + 560*a**3*b**2*c**2*x**2 +
840*a**3*b**2*c*d*x**3 + 336*a**3*b**2*d**2*x**4 + 420*a**2*b**3*c**2*x**3
 + 672*a**2*b**3*c*d*x**4 + 280*a**2*b**3*d**2*x**5 + 168*a*b**4*c**2*x**4
 + 280*a*b**4*c*d*x**5 + 120*a*b**4*d**2*x**6 + 28*b**5*c**2*x**5 + 48*b**
5*c*d*x**6 + 21*b**5*d**2*x**7))/168
```

3.12 $\int (a+bx)^2 \left(ac+(bc+ad)x+bdx^2\right)^2 \, dx$

Optimal result

Integrand size = 29, antiderivative size = 65

$$\int (a+bx)^2 \left(ac+(bc+ad)x+bdx^2\right)^2 \, dx = \frac{(bc-ad)^2(a+bx)^5}{5b^3} + \frac{d(bc-ad)(a+bx)^6}{3b^3} + \frac{d^2(a+bx)^7}{7b^3}$$

output
```
1/5*(-a*d+b*c)^2*(b*x+a)^5/b^3+1/3*d*(-a*d+b*c)*(b*x+a)^6/b^3+1/7*d^2*(b*x
+a)^7/b^3
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 148 vs. 2(65) = 130.

Time = 0.02 (sec) , antiderivative size = 148, normalized size of antiderivative = 2.28

$$\int (a+bx)^2 \left(ac+(bc+ad)x+bdx^2\right)^2 \, dx = a^4c^2x + a^3c(2bc+ad)x^2 + \frac{1}{3}a^2\left(6b^2c^2+8abcd+a^2d^2\right)x^3 + ab\left(b^2c^2+3abcd+a^2d^2\right)x^4 + \frac{1}{5}b^2\left(b^2c^2+8abcd+6a^2d^2\right)x^5 + \frac{1}{3}b^3d(bc+2ad)x^6 + \frac{1}{7}b^4d^2x^7$$

input
```
Integrate[(a + b*x)^2*(a*c + (b*c + a*d)*x + b*d*x^2)^2,x]
```

output
```
a^4*c^2*x + a^3*c*(2*b*c + a*d)*x^2 + (a^2*(6*b^2*c^2 + 8*a*b*c*d + a^2*d^
2)*x^3)/3 + a*b*(b^2*c^2 + 3*a*b*c*d + a^2*d^2)*x^4 + (b^2*(b^2*c^2 + 8*a*
b*c*d + 6*a^2*d^2)*x^5)/5 + (b^3*d*(b*c + 2*a*d)*x^6)/3 + (b^4*d^2*x^7)/7
```

Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.069$, Rules used = {1121, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+bx)^2 \left(x(ad+bc)+ac+bdx^2\right)^2 \, dx$$

$$\Big\downarrow \text{1121}$$

$$\int \left(\frac{2d(a+bx)^5(bc-ad)}{b^2}+\frac{(a+bx)^4(bc-ad)^2}{b^2}+\frac{d^2(a+bx)^6}{b^2}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{d(a+bx)^6(bc-ad)}{3b^3}+\frac{(a+bx)^5(bc-ad)^2}{5b^3}+\frac{d^2(a+bx)^7}{7b^3}$$

input
```
Int[(a + b*x)^2*(a*c + (b*c + a*d)*x + b*d*x^2)^2,x]
```

output
```
((b*c - a*d)^2*(a + b*x)^5)/(5*b^3) + (d*(b*c - a*d)*(a + b*x)^6)/(3*b^3)
+ (d^2*(a + b*x)^7)/(7*b^3)
```

Defintions of rubi rules used

rule 1121
```
Int[((d_) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Int[ExpandIntegrand[(d + e*x)^(m + p)*(a/d + (c/e)*x)^p, x], x]
/; FreeQ[{a, b, c, d, e, m, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && (Int
egerQ[p] || (GtQ[a, 0] && GtQ[d, 0] && LtQ[c, 0]))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 156 vs. $2(59) = 118$.

Time = 1.24 (sec) , antiderivative size = 157, normalized size of antiderivative = 2.42

method	result
norman	$\frac{b^4 d^2 x^7}{7} + \left(\frac{2}{3} a\, b^3 d^2 + \frac{1}{3} b^4 c d\right) x^6 + \left(\frac{6}{5} d^2 a^2 b^2 + \frac{8}{5} c d a\, b^3 + \frac{1}{5} b^4 c^2\right) x^5 + (a^3 d^2 b + 3 c d\, a^2 b^2 + c^2 a\, b$
risch	$\frac{1}{7} b^4 d^2 x^7 + \frac{2}{3} a\, b^3 d^2 x^6 + \frac{1}{3} b^4 c d\, x^6 + \frac{6}{5} a^2 b^2 d^2 x^5 + \frac{8}{5} a\, b^3 c d\, x^5 + \frac{1}{5} b^4 c^2 x^5 + a^3 b\, d^2 x^4 + 3 a^2 b^2 c d\, x$
parallelrisch	$\frac{1}{7} b^4 d^2 x^7 + \frac{2}{3} a\, b^3 d^2 x^6 + \frac{1}{3} b^4 c d\, x^6 + \frac{6}{5} a^2 b^2 d^2 x^5 + \frac{8}{5} a\, b^3 c d\, x^5 + \frac{1}{5} b^4 c^2 x^5 + a^3 b\, d^2 x^4 + 3 a^2 b^2 c d\, x$
gosper	$x(15 b^4 d^2 x^6+70 a\, b^3 d^2 x^5+35 x^5 b^4 c d+126 a^2 b^2 d^2 x^4+168 a\, b^3 c d\, x^4+21 b^4 c^2 x^4+105 a^3 b\, d^2 x^3+315 a^2 b^2 c d\, x^3+105 a\, b^3 c^2 x^3+35 x$ / 105
orering	$x(15 b^4 d^2 x^6+70 a\, b^3 d^2 x^5+35 x^5 b^4 c d+126 a^2 b^2 d^2 x^4+168 a\, b^3 c d\, x^4+21 b^4 c^2 x^4+105 a^3 b\, d^2 x^3+315 a^2 b^2 c d\, x^3+105 a\, b^3 c^2 x^3+35 x$ / $105 (d x+c)^2 (b x+a)^2$
default	$\frac{b^4 d^2 x^7}{7} + \frac{(2 a\, b^3 d^2+2 b^3 d (a d+b c)) x^6}{6} + \frac{\left(d^2 a^2 b^2+4 a\, b^2 d (a d+b c)+b^2 \left(2 a b c d+(a d+b c)^2\right)\right) x^5}{5} + (2 a^2 b d (a d+b c)+2 a b (2 a$

input
```
int((b*x+a)^2*(a*c+(a*d+b*c)*x+b*d*x^2)^2,x,method=_RETURNVERBOSE)
```

output
```
1/7*b^4*d^2*x^7+(2/3*a*b^3*d^2+1/3*b^4*c*d)*x^6+(6/5*d^2*a^2*b^2+8/5*c*d*a
*b^3+1/5*b^4*c^2)*x^5+(a^3*b*d^2+3*a^2*b^2*c*d+a*b^3*c^2)*x^4+(1/3*a^4*d^2
+8/3*a^3*d*c*b+2*a^2*b^2*c^2)*x^3+(a^4*c*d+2*a^3*b*c^2)*x^2+a^4*c^2*x
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 156 vs. 2(59) = 118.

Time = 0.07 (sec) , antiderivative size = 156, normalized size of antiderivative = 2.40

$$\int (a+bx)^2 \left(ac+(bc+ad)x+bdx^2\right)^2 \, dx = \frac{1}{7} b^4 d^2 x^7 + a^4 c^2 x + \frac{1}{3} \left(b^4 c d + 2 a b^3 d^2\right) x^6 + \frac{1}{5} \left(b^4 c^2 + 8 a b^3 c d + 6 a^2 b^2 d^2\right) x^5 + \left(a b^3 c^2 + 3 a^2 b^2 c d + a^3 b d^2\right) x^4 + \frac{1}{3} \left(6 a^2 b^2 c^2 + 8 a^3 b c d + a^4 d^2\right) x^3 + \left(2 a^3 b c^2 + a^4 c d\right) x^2$$

input
```
integrate((b*x+a)^2*(a*c+(a*d+b*c)*x+b*d*x^2)^2,x, algorithm="fricas")
```

output
```
1/7*b^4*d^2*x^7 + a^4*c^2*x + 1/3*(b^4*c*d + 2*a*b^3*d^2)*x^6 + 1/5*(b^4*c
^2 + 8*a*b^3*c*d + 6*a^2*b^2*d^2)*x^5 + (a*b^3*c^2 + 3*a^2*b^2*c*d + a^3*b
*d^2)*x^4 + 1/3*(6*a^2*b^2*c^2 + 8*a^3*b*c*d + a^4*d^2)*x^3 + (2*a^3*b*c^2
 + a^4*c*d)*x^2
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 168 vs. 2(54) = 108.

Time = 0.03 (sec) , antiderivative size = 168, normalized size of antiderivative = 2.58

$$\int (a+bx)^2 \left(ac+(bc+ad)x+bdx^2\right)^2 \, dx = a^4 c^2 x + \frac{b^4 d^2 x^7}{7} + x^6 \cdot \left(\frac{2ab^3d^2}{3} + \frac{b^4cd}{3}\right) + x^5 \cdot \left(\frac{6a^2b^2d^2}{5} + \frac{8ab^3cd}{5} + \frac{b^4c^2}{5}\right) + x^4 \left(a^3bd^2 + 3a^2b^2cd + ab^3c^2\right) + x^3 \left(\frac{a^4d^2}{3} + \frac{8a^3bcd}{3} + 2a^2b^2c^2\right) + x^2 \left(a^4cd + 2a^3bc^2\right)$$

input
```
integrate((b*x+a)**2*(a*c+(a*d+b*c)*x+b*d*x**2)**2,x)
```

output
```
a**4*c**2*x + b**4*d**2*x**7/7 + x**6*(2*a*b**3*d**2/3 + b**4*c*d/3) + x**
5*(6*a**2*b**2*d**2/5 + 8*a*b**3*c*d/5 + b**4*c**2/5) + x**4*(a**3*b*d**2
+ 3*a**2*b**2*c*d + a*b**3*c**2) + x**3*(a**4*d**2/3 + 8*a**3*b*c*d/3 + 2*
a**2*b**2*c**2) + x**2*(a**4*c*d + 2*a**3*b*c**2)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 156 vs. $2(59) = 118$.

Time = 0.05 (sec) , antiderivative size = 156, normalized size of antiderivative = 2.40

$$\int (a+bx)^2 \left(ac+(bc+ad)x+bdx^2\right)^2 dx = \frac{1}{7} b^4 d^2 x^7 + a^4 c^2 x + \frac{1}{3} \left(b^4 cd + 2 ab^3 d^2\right) x^6 + \frac{1}{5} \left(b^4 c^2 + 8 ab^3 cd + 6 a^2 b^2 d^2\right) x^5 + \left(ab^3 c^2 + 3 a^2 b^2 cd + a^3 bd^2\right) x^4 + \frac{1}{3} \left(6 a^2 b^2 c^2 + 8 a^3 bcd + a^4 d^2\right) x^3 + \left(2 a^3 bc^2 + a^4 cd\right) x^2$$

input
```
integrate((b*x+a)^2*(a*c+(a*d+b*c)*x+b*d*x^2)^2,x, algorithm="maxima")
```

output
```
1/7*b^4*d^2*x^7 + a^4*c^2*x + 1/3*(b^4*c*d + 2*a*b^3*d^2)*x^6 + 1/5*(b^4*c
^2 + 8*a*b^3*c*d + 6*a^2*b^2*d^2)*x^5 + (a*b^3*c^2 + 3*a^2*b^2*c*d + a^3*b
*d^2)*x^4 + 1/3*(6*a^2*b^2*c^2 + 8*a^3*b*c*d + a^4*d^2)*x^3 + (2*a^3*b*c^2
 + a^4*c*d)*x^2
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 170 vs. $2(59) = 118$.

Time = 0.18 (sec) , antiderivative size = 170, normalized size of antiderivative = 2.62

$$\int (a+bx)^2 \left(ac+(bc+ad)x+bdx^2\right)^2 \, dx = \frac{1}{7}b^4d^2x^7+\frac{1}{3}b^4cdx^6+\frac{2}{3}ab^3d^2x^6 +\frac{1}{5}b^4c^2x^5+\frac{8}{5}ab^3cdx^5+\frac{6}{5}a^2b^2d^2x^5 +ab^3c^2x^4+3a^2b^2cdx^4+a^3bd^2x^4 +2a^2b^2c^2x^3+\frac{8}{3}a^3bcdx^3+\frac{1}{3}a^4d^2x^3 +2a^3bc^2x^2+a^4cdx^2+a^4c^2x$$

input
```
integrate((b*x+a)^2*(a*c+(a*d+b*c)*x+b*d*x^2)^2,x, algorithm="giac")
```

output
```
1/7*b^4*d^2*x^7 + 1/3*b^4*c*d*x^6 + 2/3*a*b^3*d^2*x^6 + 1/5*b^4*c^2*x^5 +
8/5*a*b^3*c*d*x^5 + 6/5*a^2*b^2*d^2*x^5 + a*b^3*c^2*x^4 + 3*a^2*b^2*c*d*x^
4 + a^3*b*d^2*x^4 + 2*a^2*b^2*c^2*x^3 + 8/3*a^3*b*c*d*x^3 + 1/3*a^4*d^2*x^
3 + 2*a^3*b*c^2*x^2 + a^4*c*d*x^2 + a^4*c^2*x
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 144, normalized size of antiderivative = 2.22

$$\int (a+bx)^2 \left(ac+(bc+ad)x+bdx^2\right)^2 \, dx = x^3\left(\frac{a^4d^2}{3}+\frac{8a^3bcd}{3}+2a^2b^2c^2\right) +x^5\left(\frac{6a^2b^2d^2}{5}+\frac{8ab^3cd}{5}+\frac{b^4c^2}{5}\right) +a^4c^2x+\frac{b^4d^2x^7}{7}+a^3cx^2(ad+2bc) +\frac{b^3dx^6(2ad+bc)}{3} +abx^4\left(a^2d^2+3abcd+b^2c^2\right)$$

input
```
int((a + b*x)^2*(a*c + x*(a*d + b*c) + b*d*x^2)^2,x)
```

output
```
x^3*((a^4*d^2)/3 + 2*a^2*b^2*c^2 + (8*a^3*b*c*d)/3) + x^5*((b^4*c^2)/5 + (
6*a^2*b^2*d^2)/5 + (8*a*b^3*c*d)/5) + a^4*c^2*x + (b^4*d^2*x^7)/7 + a^3*c*
x^2*(a*d + 2*b*c) + (b^3*d*x^6*(2*a*d + b*c))/3 + a*b*x^4*(a^2*d^2 + b^2*c
^2 + 3*a*b*c*d)
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 172, normalized size of antiderivative = 2.65

$$\int (a+bx)^2 \left(ac+(bc+ad)x+bdx^2\right)^2 \, dx$$
$$= \frac{x(15b^4d^2x^6+70a\,b^3d^2x^5+35b^4cd\,x^5+126a^2b^2d^2x^4+168a\,b^3cd\,x^4+21b^4c^2x^4+105a^3b\,d^2x^3+315a^2b^2c}{105}$$

input
```
int((b*x+a)^2*(a*c+(a*d+b*c)*x+b*d*x^2)^2,x)
```

output
```
(x*(105*a**4*c**2 + 105*a**4*c*d*x + 35*a**4*d**2*x**2 + 210*a**3*b*c**2*x
 + 280*a**3*b*c*d*x**2 + 105*a**3*b*d**2*x**3 + 210*a**2*b**2*c**2*x**2 +
315*a**2*b**2*c*d*x**3 + 126*a**2*b**2*d**2*x**4 + 105*a*b**3*c**2*x**3 +
168*a*b**3*c*d*x**4 + 70*a*b**3*d**2*x**5 + 21*b**4*c**2*x**4 + 35*b**4*c*
d*x**5 + 15*b**4*d**2*x**6))/105
```

3.13 $\int (a+bx) \left(ac+(bc+ad)x+bdx^2\right)^2 \, dx$

Optimal result

Integrand size = 27, antiderivative size = 65

$$\int (a+bx) \left(ac+(bc+ad)x+bdx^2\right)^2 \, dx = \frac{(bc-ad)^2(a+bx)^4}{4b^3} + \frac{2d(bc-ad)(a+bx)^5}{5b^3} + \frac{d^2(a+bx)^6}{6b^3}$$

output
```
1/4*(-a*d+b*c)^2*(b*x+a)^4/b^3+2/5*d*(-a*d+b*c)*(b*x+a)^5/b^3+1/6*d^2*(b*x
+a)^6/b^3
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 122, normalized size of antiderivative = 1.88

$$\int (a+bx) \left(ac+(bc+ad)x+bdx^2\right)^2 \, dx = a^3c^2x + \frac{1}{2}a^2c(3bc+2ad)x^2 + \frac{1}{3}a\left(3b^2c^2+6abcd+a^2d^2\right)x^3 + \frac{1}{4}b\left(b^2c^2+6abcd+3a^2d^2\right)x^4 + \frac{1}{5}b^2d(2bc+3ad)x^5 + \frac{1}{6}b^3d^2x^6$$

input
```
Integrate[(a + b*x)*(a*c + (b*c + a*d)*x + b*d*x^2)^2,x]
```

output
```
a^3*c^2*x + (a^2*c*(3*b*c + 2*a*d)*x^2)/2 + (a*(3*b^2*c^2 + 6*a*b*c*d + a^
2*d^2)*x^3)/3 + (b*(b^2*c^2 + 6*a*b*c*d + 3*a^2*d^2)*x^4)/4 + (b^2*d*(2*b*
c + 3*a*d)*x^5)/5 + (b^3*d^2*x^6)/6
```

Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.074$, Rules used = {1121, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+bx) \left(x(ad+bc)+ac+bdx^2\right)^2 \, dx$$

$$\Big\downarrow \text{1121}$$

$$\int \left(\frac{2d(a+bx)^4(bc-ad)}{b^2}+\frac{(a+bx)^3(bc-ad)^2}{b^2}+\frac{d^2(a+bx)^5}{b^2}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{2d(a+bx)^5(bc-ad)}{5b^3}+\frac{(a+bx)^4(bc-ad)^2}{4b^3}+\frac{d^2(a+bx)^6}{6b^3}$$

input
```
Int[(a + b*x)*(a*c + (b*c + a*d)*x + b*d*x^2)^2,x]
```

output
```
((b*c - a*d)^2*(a + b*x)^4)/(4*b^3) + (2*d*(b*c - a*d)*(a + b*x)^5)/(5*b^3
) + (d^2*(a + b*x)^6)/(6*b^3)
```

Defintions of rubi rules used

rule 1121
```
Int[((d_) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Int[ExpandIntegrand[(d + e*x)^(m + p)*(a/d + (c/e)*x)^p, x], x]
/; FreeQ[{a, b, c, d, e, m, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && (Int
egerQ[p] || (GtQ[a, 0] && GtQ[d, 0] && LtQ[c, 0]))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 120 vs. $2(59) = 118$.

Time = 1.08 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.86

method	result
norman	$\frac{b^3 d^2 x^6}{6}+\left(\frac{3}{5}a\,b^2 d^2+\frac{2}{5}b^3 cd\right)x^5+\left(\frac{3}{4}a^2 b\,d^2+\frac{3}{2}cda\,b^2+\frac{1}{4}b^3 c^2\right)x^4+\left(\frac{1}{3}a^3 d^2+2a^2 bcd+a\,c^2 b^2\right.$
risch	$\frac{1}{6}b^3 d^2 x^6+\frac{3}{5}a\,b^2 d^2 x^5+\frac{2}{5}b^3 cd\,x^5+\frac{3}{4}a^2 b\,d^2 x^4+\frac{3}{2}cda\,b^2 x^4+\frac{1}{4}b^3 c^2 x^4+\frac{1}{3}x^3 a^3 d^2+2a^2 bcd\,x^3$
parallelrisch	$\frac{1}{6}b^3 d^2 x^6+\frac{3}{5}a\,b^2 d^2 x^5+\frac{2}{5}b^3 cd\,x^5+\frac{3}{4}a^2 b\,d^2 x^4+\frac{3}{2}cda\,b^2 x^4+\frac{1}{4}b^3 c^2 x^4+\frac{1}{3}x^3 a^3 d^2+2a^2 bcd\,x^3$
gosper	$\frac{x\left(10b^3 d^2 x^5+36a\,b^2 d^2 x^4+24b^3 cd\,x^4+45d^2 x^3 a^2 b+90a\,b^2 cd\,x^3+15b^3 c^2 x^3+20a^3 d^2 x^2+120x^2 a^2 bcd+60a\,b^2 c^2 x^2+60a^3 cdx+9\right.}{60}$
default	$\frac{b^3 d^2 x^6}{6}+\frac{\left(a\,b^2 d^2+2b^2 d(ad+bc)\right)x^5}{5}+\frac{\left(2abd(ad+bc)+b\left(2abcd+(ad+bc)^2\right)\right)x^4}{4}+\frac{\left(a\left(2abcd+(ad+bc)^2\right)+2bac(ad+bc)\right.}{3}$
orering	$\frac{x\left(10b^3 d^2 x^5+36a\,b^2 d^2 x^4+24b^3 cd\,x^4+45d^2 x^3 a^2 b+90a\,b^2 cd\,x^3+15b^3 c^2 x^3+20a^3 d^2 x^2+120x^2 a^2 bcd+60a\,b^2 c^2 x^2+60a^3 cdx+9\right.}{60(dx+c)^2(bx+a)^2}$

input
```
int((b*x+a)*(a*c+(a*d+b*c)*x+b*d*x^2)^2,x,method=_RETURNVERBOSE)
```

output
```
1/6*b^3*d^2*x^6+(3/5*a*b^2*d^2+2/5*b^3*c*d)*x^5+(3/4*a^2*b*d^2+3/2*c*d*a*b
^2+1/4*b^3*c^2)*x^4+(1/3*a^3*d^2+2*a^2*b*c*d+a*c^2*b^2)*x^3+(a^3*d*c+3/2*a
^2*b*c^2)*x^2+a^3*c^2*x
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 124 vs. $2(59) = 118$.

Time = 0.06 (sec) , antiderivative size = 124, normalized size of antiderivative = 1.91

$$\int (a+bx)\left(ac+(bc+ad)x+bdx^2\right)^2\,dx = \frac{1}{6}\,b^3d^2x^6+a^3c^2x+\frac{1}{5}\left(2\,b^3cd+3\,ab^2d^2\right)x^5 + \frac{1}{4}\left(b^3c^2+6\,ab^2cd+3\,a^2bd^2\right)x^4 + \frac{1}{3}\left(3\,ab^2c^2+6\,a^2bcd+a^3d^2\right)x^3 + \frac{1}{2}\left(3\,a^2bc^2+2\,a^3cd\right)x^2$$

input
```
integrate((b*x+a)*(a*c+(a*d+b*c)*x+b*d*x^2)^2,x, algorithm="fricas")
```

output
```
1/6*b^3*d^2*x^6 + a^3*c^2*x + 1/5*(2*b^3*c*d + 3*a*b^2*d^2)*x^5 + 1/4*(b^3
*c^2 + 6*a*b^2*c*d + 3*a^2*b*d^2)*x^4 + 1/3*(3*a*b^2*c^2 + 6*a^2*b*c*d + a
^3*d^2)*x^3 + 1/2*(3*a^2*b*c^2 + 2*a^3*c*d)*x^2
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 133 vs. $2(56) = 112$.

Time = 0.03 (sec) , antiderivative size = 133, normalized size of antiderivative = 2.05

$$\int (a+bx)\left(ac+(bc+ad)x+bdx^2\right)^2\,dx = a^3c^2x+\frac{b^3d^2x^6}{6}+x^5\cdot\left(\frac{3ab^2d^2}{5}+\frac{2b^3cd}{5}\right) + x^4\cdot\left(\frac{3a^2bd^2}{4}+\frac{3ab^2cd}{2}+\frac{b^3c^2}{4}\right) + x^3\left(\frac{a^3d^2}{3}+2a^2bcd+ab^2c^2\right) + x^2\left(a^3cd+\frac{3a^2bc^2}{2}\right)$$

input
```
integrate((b*x+a)*(a*c+(a*d+b*c)*x+b*d*x**2)**2,x)
```

output
```
a**3*c**2*x + b**3*d**2*x**6/6 + x**5*(3*a*b**2*d**2/5 + 2*b**3*c*d/5) + x
**4*(3*a**2*b*d**2/4 + 3*a*b**2*c*d/2 + b**3*c**2/4) + x**3*(a**3*d**2/3 +
 2*a**2*b*c*d + a*b**2*c**2) + x**2*(a**3*c*d + 3*a**2*b*c**2/2)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 124 vs. 2(59) = 118.

Time = 0.03 (sec) , antiderivative size = 124, normalized size of antiderivative = 1.91

$$\int (a+bx)\left(ac+(bc+ad)x+bdx^2\right)^2\,dx = \frac{1}{6}\,b^3d^2x^6 + a^3c^2x + \frac{1}{5}\left(2\,b^3cd + 3\,ab^2d^2\right)x^5 + \frac{1}{4}\left(b^3c^2 + 6\,ab^2cd + 3\,a^2bd^2\right)x^4 + \frac{1}{3}\left(3\,ab^2c^2 + 6\,a^2bcd + a^3d^2\right)x^3 + \frac{1}{2}\left(3\,a^2bc^2 + 2\,a^3cd\right)x^2$$

input
```
integrate((b*x+a)*(a*c+(a*d+b*c)*x+b*d*x^2)^2,x, algorithm="maxima")
```

output
```
1/6*b^3*d^2*x^6 + a^3*c^2*x + 1/5*(2*b^3*c*d + 3*a*b^2*d^2)*x^5 + 1/4*(b^3
*c^2 + 6*a*b^2*c*d + 3*a^2*b*d^2)*x^4 + 1/3*(3*a*b^2*c^2 + 6*a^2*b*c*d + a
^3*d^2)*x^3 + 1/2*(3*a^2*b*c^2 + 2*a^3*c*d)*x^2
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 130 vs. 2(59) = 118.

Time = 0.16 (sec) , antiderivative size = 130, normalized size of antiderivative = 2.00

$$\int (a+bx)\left(ac+(bc+ad)x+bdx^2\right)^2\,dx = \frac{1}{6}\,b^3d^2x^6 + \frac{2}{5}\,b^3cdx^5 + \frac{3}{5}\,ab^2d^2x^5 + \frac{1}{4}\,b^3c^2x^4 + \frac{3}{2}\,ab^2cdx^4 + \frac{3}{4}\,a^2bd^2x^4 + ab^2c^2x^3 + 2\,a^2bcdx^3 + \frac{1}{3}\,a^3d^2x^3 + \frac{3}{2}\,a^2bc^2x^2 + a^3cdx^2 + a^3c^2x$$

input
```
integrate((b*x+a)*(a*c+(a*d+b*c)*x+b*d*x^2)^2,x, algorithm="giac")
```

output
```
1/6*b^3*d^2*x^6 + 2/5*b^3*c*d*x^5 + 3/5*a*b^2*d^2*x^5 + 1/4*b^3*c^2*x^4 +
3/2*a*b^2*c*d*x^4 + 3/4*a^2*b*d^2*x^4 + a*b^2*c^2*x^3 + 2*a^2*b*c*d*x^3 +
1/3*a^3*d^2*x^3 + 3/2*a^2*b*c^2*x^2 + a^3*c*d*x^2 + a^3*c^2*x
```

Mupad [B] (verification not implemented)

Time = 5.54 (sec) , antiderivative size = 115, normalized size of antiderivative = 1.77

$$\int (a+bx)\left(ac+(bc+ad)x+bdx^2\right)^2\,dx = x^3\left(\frac{a^3\,d^2}{3}+2\,a^2\,b\,c\,d+a\,b^2\,c^2\right)+x^4\left(\frac{3\,a^2\,b\,d^2}{4}+\frac{3\,a\,b^2\,c\,d}{2}+\frac{b^3\,c^2}{4}\right)+a^3\,c^2\,x+\frac{b^3\,d^2\,x^6}{6}+\frac{a^2\,c\,x^2\,(2\,a\,d+3\,b\,c)}{2}+\frac{b^2\,d\,x^5\,(3\,a\,d+2\,b\,c)}{5}$$

input
```
int((a + b*x)*(a*c + x*(a*d + b*c) + b*d*x^2)^2,x)
```

output
```
x^3*((a^3*d^2)/3 + a*b^2*c^2 + 2*a^2*b*c*d) + x^4*((b^3*c^2)/4 + (3*a^2*b*
d^2)/4 + (3*a*b^2*c*d)/2) + a^3*c^2*x + (b^3*d^2*x^6)/6 + (a^2*c*x^2*(2*a*
d + 3*b*c))/2 + (b^2*d*x^5*(3*a*d + 2*b*c))/5
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 131, normalized size of antiderivative = 2.02

$$\int (a+bx)\left(ac+(bc+ad)x+bdx^2\right)^2\,dx = \frac{x(10b^3d^2x^5+36a\,b^2d^2x^4+24b^3cd\,x^4+45a^2b\,d^2x^3+90a\,b^2cd\,x^3+15b^3c^2x^3+20a^3d^2x^2+120a^2bcd\,x^2+}{60}$$

input
```
int((b*x+a)*(a*c+(a*d+b*c)*x+b*d*x^2)^2,x)
```

output
```
(x*(60*a**3*c**2 + 60*a**3*c*d*x + 20*a**3*d**2*x**2 + 90*a**2*b*c**2*x +
120*a**2*b*c*d*x**2 + 45*a**2*b*d**2*x**3 + 60*a*b**2*c**2*x**2 + 90*a*b**
2*c*d*x**3 + 36*a*b**2*d**2*x**4 + 15*b**3*c**2*x**3 + 24*b**3*c*d*x**4 +
10*b**3*d**2*x**5))/60
```

3.14 $\int (ac+(bc+ad)x+bdx^2)^2\,dx$

Optimal result

Integrand size = 21, antiderivative size = 65

$$\int (ac+(bc+ad)x+bdx^2)^2\,dx=\frac{(bc-ad)^2(c+dx)^3}{3d^3}-\frac{b(bc-ad)(c+dx)^4}{2d^3}+\frac{b^2(c+dx)^5}{5d^3}$$

output
```
1/3*(-a*d+b*c)^2*(d*x+c)^3/d^3-1/2*b*(-a*d+b*c)*(d*x+c)^4/d^3+1/5*b^2*(d*x
+c)^5/d^3
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.22

$$\int (ac+(bc+ad)x+bdx^2)^2\,dx=a^2c^2x+ac(bc+ad)x^2+\frac{1}{3}\left(b^2c^2+4abcd+a^2d^2\right)x^3+\frac{1}{2}bd(bc+ad)x^4+\frac{1}{5}b^2d^2x^5$$

input
```
Integrate[(a*c + (b*c + a*d)*x + b*d*x^2)^2,x]
```

output
```
a^2*c^2*x + a*c*(b*c + a*d)*x^2 + ((b^2*c^2 + 4*a*b*c*d + a^2*d^2)*x^3)/3
+ (b*d*(b*c + a*d)*x^4)/2 + (b^2*d^2*x^5)/5
```

Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.18, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {1084, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(x(ad+bc)+ac+bdx^2\right)^2 dx$$

$$\Big\downarrow \; 1084$$

$$\frac{\int \left((bc+bdx)^4-2b^3(bc-ad)(c+dx)^3+b^2(bc-ad)^2(c+dx)^2\right)dx}{b^2d^2}$$

$$\Big\downarrow \; 2009$$

$$\frac{-\frac{b^3(c+dx)^4(bc-ad)}{2d}+\frac{b^2(c+dx)^3(bc-ad)^2}{3d}+\frac{b^4(c+dx)^5}{5d}}{b^2d^2}$$

input
```
Int[(a*c + (b*c + a*d)*x + b*d*x^2)^2,x]
```

output
```
((b^2*(b*c - a*d)^2*(c + d*x)^3)/(3*d) - (b^3*(b*c - a*d)*(c + d*x)^4)/(2*
d) + (b^4*(c + d*x)^5)/(5*d))/(b^2*d^2)
```

Defintions of rubi rules used

rule 1084
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{q = Rt[b^2
- 4*a*c, 2]}, Simp[1/c^p   Int[ExpandIntegrand[(b/2 - q/2 + c*x)^p*(b/2 + q
/2 + c*x)^p, x], x], x] /;  !FractionalPowerFactorQ[q]] /; FreeQ[{a, b, c},
 x] && IntegerQ[p] && NiceSqrtQ[b^2 - 4*a*c]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.96 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.06

method	result
default	$\frac{b^2d^2x^5}{5}+\frac{(ad+bc)bd\,x^4}{2}+\frac{\left(2abcd+(ad+bc)^2\right)x^3}{3}+ac(ad+bc)\,x^2+a^2c^2x$
norman	$\frac{b^2d^2x^5}{5}+\left(\frac{1}{2}ab\,d^2+\frac{1}{2}cd\,b^2\right)x^4+\left(\frac{1}{3}a^2d^2+\frac{4}{3}abcd+\frac{1}{3}b^2c^2\right)x^3+\left(a^2cd+ab\,c^2\right)x^2+a^2c^2x$
risch	$\frac{1}{5}b^2d^2x^5+\frac{1}{2}ab\,d^2x^4+\frac{1}{2}b^2cd\,x^4+\frac{1}{3}a^2d^2x^3+\frac{4}{3}abcd\,x^3+\frac{1}{3}b^2c^2x^3+a^2cd\,x^2+ab\,c^2x^2+a^2c^2$
parallelrisch	$\frac{1}{5}b^2d^2x^5+\frac{1}{2}ab\,d^2x^4+\frac{1}{2}b^2cd\,x^4+\frac{1}{3}a^2d^2x^3+\frac{4}{3}abcd\,x^3+\frac{1}{3}b^2c^2x^3+a^2cd\,x^2+ab\,c^2x^2+a^2c^2$
gosper	$\frac{x\left(6b^2d^2x^4+15d^2x^3ab+15b^2cd\,x^3+10a^2d^2x^2+40abcd\,x^2+10b^2c^2x^2+30a^2cdx+30ab\,c^2x+30a^2c^2\right)}{30}$
orering	$\frac{x\left(6b^2d^2x^4+15d^2x^3ab+15b^2cd\,x^3+10a^2d^2x^2+40abcd\,x^2+10b^2c^2x^2+30a^2cdx+30ab\,c^2x+30a^2c^2\right)\left(ac+(ad+bc)x+bd\,x^2\right)^2}{30(dx+c)^2(bx+a)^2}$

input
```
int((a*c+(a*d+b*c)*x+b*d*x^2)^2,x,method=_RETURNVERBOSE)
```

output
```
1/5*b^2*d^2*x^5+1/2*(a*d+b*c)*b*d*x^4+1/3*(2*a*b*c*d+(a*d+b*c)^2)*x^3+a*c*
(a*d+b*c)*x^2+a^2*c^2*x
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.25

$$\int \left(ac+(bc+ad)x+bdx^2\right)^2\,dx=\frac{1}{5}\,b^2d^2x^5+a^2c^2x+\frac{1}{2}\left(b^2cd+abd^2\right)x^4+\frac{1}{3}\left(b^2c^2+4\,abcd+a^2d^2\right)x^3+\left(abc^2+a^2cd\right)x^2$$

input
```
integrate((a*c+(a*d+b*c)*x+b*d*x^2)^2,x, algorithm="fricas")
```

output
```
1/5*b^2*d^2*x^5 + a^2*c^2*x + 1/2*(b^2*c*d + a*b*d^2)*x^4 + 1/3*(b^2*c^2 +
 4*a*b*c*d + a^2*d^2)*x^3 + (a*b*c^2 + a^2*c*d)*x^2
```

Sympy [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.34

$$\int \left(ac+(bc+ad)x+bdx^2\right)^2\,dx = a^2c^2x+\frac{b^2d^2x^5}{5}+x^4\left(\frac{abd^2}{2}+\frac{b^2cd}{2}\right)+x^3\left(\frac{a^2d^2}{3}+\frac{4abcd}{3}+\frac{b^2c^2}{3}\right)+x^2\left(a^2cd+abc^2\right)$$

input
```
integrate((a*c+(a*d+b*c)*x+b*d*x**2)**2,x)
```

output
```
a**2*c**2*x + b**2*d**2*x**5/5 + x**4*(a*b*d**2/2 + b**2*c*d/2) + x**3*(a*
*2*d**2/3 + 4*a*b*c*d/3 + b**2*c**2/3) + x**2*(a**2*c*d + a*b*c**2)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.11

$$\int \left(ac+(bc+ad)x+bdx^2\right)^2\,dx = \frac{1}{5}\,b^2d^2x^5+\frac{1}{2}\,(bc+ad)bdx^4+a^2c^2x+\frac{1}{3}\,(bc+ad)^2x^3+\frac{1}{3}\,\left(2\,bdx^3+3\,(bc+ad)x^2\right)ac$$

input
```
integrate((a*c+(a*d+b*c)*x+b*d*x^2)^2,x, algorithm="maxima")
```

output
```
1/5*b^2*d^2*x^5 + 1/2*(b*c + a*d)*b*d*x^4 + a^2*c^2*x + 1/3*(b*c + a*d)^2*
x^3 + 1/3*(2*b*d*x^3 + 3*(b*c + a*d)*x^2)*a*c
```

Giac [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.37

$$\int \left(ac+(bc+ad)x+bdx^2\right)^2 \, dx = \frac{1}{5}b^2d^2x^5+\frac{1}{2}b^2cdx^4+\frac{1}{2}abd^2x^4+\frac{1}{3}b^2c^2x^3 + \frac{4}{3}abcdx^3+\frac{1}{3}a^2d^2x^3+abc^2x^2+a^2cdx^2+a^2c^2x$$

input
```
integrate((a*c+(a*d+b*c)*x+b*d*x^2)^2,x, algorithm="giac")
```

output
```
1/5*b^2*d^2*x^5 + 1/2*b^2*c*d*x^4 + 1/2*a*b*d^2*x^4 + 1/3*b^2*c^2*x^3 + 4/
3*a*b*c*d*x^3 + 1/3*a^2*d^2*x^3 + a*b*c^2*x^2 + a^2*c*d*x^2 + a^2*c^2*x
```

Mupad [B] (verification not implemented)

Time = 5.59 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.14

$$\int \left(ac+(bc+ad)x+bdx^2\right)^2 \, dx = x^3\left(\frac{a^2\,d^2}{3}+\frac{4\,a\,b\,c\,d}{3}+\frac{b^2\,c^2}{3}\right)+a^2\,c^2\,x + \frac{b^2\,d^2\,x^5}{5}+a\,c\,x^2\,(a\,d+b\,c)+\frac{b\,d\,x^4\,(a\,d+b\,c)}{2}$$

input
```
int((a*c + x*(a*d + b*c) + b*d*x^2)^2,x)
```

output
```
x^3*((a^2*d^2)/3 + (b^2*c^2)/3 + (4*a*b*c*d)/3) + a^2*c^2*x + (b^2*d^2*x^5
)/5 + a*c*x^2*(a*d + b*c) + (b*d*x^4*(a*d + b*c))/2
```

Reduce [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.38

$$\int \left(ac+(bc+ad)x+bdx^2\right)^2\,dx$$
$$=\frac{x(6b^2d^2x^4+15ab\,d^2x^3+15b^2cd\,x^3+10a^2d^2x^2+40abcd\,x^2+10b^2c^2x^2+30a^2cdx+30ab\,c^2x+30a^2c^2)}{30}$$

input
```
int((a*c+(a*d+b*c)*x+b*d*x^2)^2,x)
```

output
```
(x*(30*a**2*c**2 + 30*a**2*c*d*x + 10*a**2*d**2*x**2 + 30*a*b*c**2*x + 40*
a*b*c*d*x**2 + 15*a*b*d**2*x**3 + 10*b**2*c**2*x**2 + 15*b**2*c*d*x**3 + 6
*b**2*d**2*x**4))/30
```

3.15 $\int \frac{(ac+(bc+ad)x+bdx^2)^2}{a+bx}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 38

$$\int \frac{(ac+(bc+ad)x+bdx^2)^2}{a+bx}\,dx=-\frac{(bc-ad)(c+dx)^3}{3d^2}+\frac{b(c+dx)^4}{4d^2}$$

output
```
-1/3*(-a*d+b*c)*(d*x+c)^3/d^2+1/4*b*(d*x+c)^4/d^2
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.24

$$\int \frac{(ac+(bc+ad)x+bdx^2)^2}{a+bx}\,dx=\frac{1}{12}x\left(12ac^2+6c(bc+2ad)x+4d(2bc+ad)x^2+3bd^2x^3\right)$$

input
```
Integrate[(a*c + (b*c + a*d)*x + b*d*x^2)^2/(a + b*x),x]
```

output
```
(x*(12*a*c^2 + 6*c*(b*c + 2*a*d)*x + 4*d*(2*b*c + a*d)*x^2 + 3*b*d^2*x^3))
/12
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.069$, Rules used = {1121, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(x(ad+bc)+ac+bdx^2)^2}{a+bx} \, dx$$

$$\Big\downarrow \text{1121}$$

$$\int \left(\frac{(c+dx)^2(ad-bc)}{d}+\frac{b(c+dx)^3}{d}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{b(c+dx)^4}{4d^2}-\frac{(c+dx)^3(bc-ad)}{3d^2}$$

input
```
Int[(a*c + (b*c + a*d)*x + b*d*x^2)^2/(a + b*x),x]
```

output
```
-1/3*((b*c - a*d)*(c + d*x)^3)/d^2 + (b*(c + d*x)^4)/(4*d^2)
```

Defintions of rubi rules used

rule 1121
```
Int[((d_) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Int[ExpandIntegrand[(d + e*x)^(m + p)*(a/d + (c/e)*x)^p, x], x]
/; FreeQ[{a, b, c, d, e, m, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && (Int
egerQ[p] || (GtQ[a, 0] && GtQ[d, 0] && LtQ[c, 0]))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 1.39 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.26

method	result	size
norman	$\frac{b\,d^{2} x^{4}}{4}+\left(\frac{1}{3} a\,d^{2}+\frac{2}{3} d b c \right) x^{3}+\left(a c d +\frac{1}{2} b\,c^{2}\right) x^{2}+a\,c^{2} x$	48
gosper	$\frac{x \left(3 b\,d^{2} x^{3}+4 a\,d^{2} x^{2}+8 b c d\,x^{2}+12 a d x c +6 c^{2} b x +12 a\,c^{2}\right)}{12}$	50
risch	$\frac{1}{4} b\,d^{2} x^{4}+\frac{1}{3} a\,d^{2} x^{3}+\frac{2}{3} b c d\,x^{3}+a d\,x^{2} c +\frac{1}{2} b\,c^{2} x^{2}+a\,c^{2} x$	50
parallelrisch	$\frac{1}{4} b\,d^{2} x^{4}+\frac{1}{3} a\,d^{2} x^{3}+\frac{2}{3} b c d\,x^{3}+a d\,x^{2} c +\frac{1}{2} b\,c^{2} x^{2}+a\,c^{2} x$	50
default	$\frac{b\,d^{2} x^{4}}{4}+\frac{\left(d b c +d \left(a d +b c \right)\right) x^{3}}{3}+\frac{\left(c \left(a d +b c \right)+a c d \right) x^{2}}{2}+a\,c^{2} x$	55
orering	$\frac{x \left(3 b\,d^{2} x^{3}+4 a\,d^{2} x^{2}+8 b c d\,x^{2}+12 a d x c +6 c^{2} b x +12 a\,c^{2}\right) \left(a c +\left(a d +b c \right) x +b d\,x^{2}\right)^{2}}{12 \left(b x +a \right)^{2} \left(d x +c \right)^{2}}$	85

input
```
int((a*c+(a*d+b*c)*x+b*d*x^2)^2/(b*x+a),x,method=_RETURNVERBOSE)
```

output
```
1/4*b*d^2*x^4+(1/3*a*d^2+2/3*d*b*c)*x^3+(a*c*d+1/2*b*c^2)*x^2+a*c^2*x
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.26

$$\int \frac{(ac+(bc+ad)x+bdx^2)^2}{a+bx}\,dx=\frac{1}{4}\,bd^2x^4+ac^2x+\frac{1}{3}\,(2\,bcd+ad^2)x^3+\frac{1}{2}\,(bc^2+2\,acd)x^2$$

input
```
integrate((a*c+(a*d+b*c)*x+b*d*x^2)^2/(b*x+a),x, algorithm="fricas")
```

output
```
1/4*b*d^2*x^4 + a*c^2*x + 1/3*(2*b*c*d + a*d^2)*x^3 + 1/2*(b*c^2 + 2*a*c*d
)*x^2
```

Sympy [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.29

$$\int \frac{(ac+(bc+ad)x+bdx^2)^2}{a+bx}\,dx = ac^2x+\frac{bd^2x^4}{4}+x^3\left(\frac{ad^2}{3}+\frac{2bcd}{3}\right)+x^2\left(acd+\frac{bc^2}{2}\right)$$

input
```
integrate((a*c+(a*d+b*c)*x+b*d*x**2)**2/(b*x+a),x)
```

output
```
a*c**2*x + b*d**2*x**4/4 + x**3*(a*d**2/3 + 2*b*c*d/3) + x**2*(a*c*d + b*c
**2/2)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.26

$$\int \frac{(ac+(bc+ad)x+bdx^2)^2}{a+bx}\,dx = \frac{1}{4}\,bd^2x^4+ac^2x+\frac{1}{3}\,(2\,bcd+ad^2)x^3+\frac{1}{2}\,(bc^2+2\,acd)x^2$$

input
```
integrate((a*c+(a*d+b*c)*x+b*d*x^2)^2/(b*x+a),x, algorithm="maxima")
```

output
```
1/4*b*d^2*x^4 + a*c^2*x + 1/3*(2*b*c*d + a*d^2)*x^3 + 1/2*(b*c^2 + 2*a*c*d
)*x^2
```

Giac [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.29

$$\int \frac{(ac+(bc+ad)x+bdx^2)^2}{a+bx}\,dx = \frac{1}{4}\,bd^2x^4+\frac{2}{3}\,bcdx^3+\frac{1}{3}\,ad^2x^3+\frac{1}{2}\,bc^2x^2+acdx^2+ac^2x$$

input
```
integrate((a*c+(a*d+b*c)*x+b*d*x^2)^2/(b*x+a),x, algorithm="giac")
```

output
```
1/4*b*d^2*x^4 + 2/3*b*c*d*x^3 + 1/3*a*d^2*x^3 + 1/2*b*c^2*x^2 + a*c*d*x^2
+ a*c^2*x
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.24

$$\int \frac{(ac+(bc+ad)x+bdx^2)^2}{a+bx}\,dx = x^2\left(\frac{b\,c^2}{2}+a\,d\,c\right)+x^3\left(\frac{a\,d^2}{3}+\frac{2\,b\,c\,d}{3}\right)+\frac{b\,d^2\,x^4}{4}+a\,c^2\,x$$

input
```
int((a*c + x*(a*d + b*c) + b*d*x^2)^2/(a + b*x),x)
```

output
```
x^2*((b*c^2)/2 + a*c*d) + x^3*((a*d^2)/3 + (2*b*c*d)/3) + (b*d^2*x^4)/4 +
a*c^2*x
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.29

$$\int \frac{(ac+(bc+ad)x+bdx^2)^2}{a+bx}\,dx = \frac{x(3b\,d^2x^3+4a\,d^2x^2+8bcd\,x^2+12acdx+6b\,c^2x+12a\,c^2)}{12}$$

input
```
int((a*c+(a*d+b*c)*x+b*d*x^2)^2/(b*x+a),x)
```

output
```
(x*(12*a*c**2 + 12*a*c*d*x + 4*a*d**2*x**2 + 6*b*c**2*x + 8*b*c*d*x**2 + 3
*b*d**2*x**3))/12
```

3.16 $\int \frac{(ac+(bc+ad)x+bdx^2)^2}{(a+bx)^2}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 14

$$\int \frac{(ac+(bc+ad)x+bdx^2)^2}{(a+bx)^2}\,dx=\frac{(c+dx)^3}{3d}$$

output
```
1/3*(d*x+c)^3/d
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00

$$\int \frac{(ac+(bc+ad)x+bdx^2)^2}{(a+bx)^2}\,dx=\frac{(c+dx)^3}{3d}$$

input
```
Integrate[(a*c + (b*c + a*d)*x + b*d*x^2)^2/(a + b*x)^2,x]
```

output
```
(c + d*x)^3/(3*d)
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.069, Rules used = {1120, 17}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(x(ad+bc)+ac+bdx^2)^2}{(a+bx)^2} dx$$

$$\Big\downarrow \; 1120$$

$$\int (c+dx)^2 dx$$

$$\Big\downarrow \; 17$$

$$\frac{(c+dx)^3}{3d}$$

input
```
Int[(a*c + (b*c + a*d)*x + b*d*x^2)^2/(a + b*x)^2,x]
```

output
```
(c + d*x)^3/(3*d)
```

Defintions of rubi rules used

rule 17
```
Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1
)/(b*(m + 1))), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]
```

rule 1120
```
Int[((d_) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Int[(d + e*x)^(m + p)*(a/d + (c/e)*x)^p, x] /; FreeQ[{a, b, c, d
, e, m}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[p] && (EqQ[m + p,
0] || EqQ[m + 2*p + 2, 0])
```

Maple [A] (verified)

Time = 1.20 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.93

method	result	size
default	$\frac{(dx+c)^3}{3d}$	13
parallelrisch	$\frac{1}{3}d^2x^3 + cd\,x^2 + c^2x$	21
gosper	$\frac{x(d^2x^2+3cdx+3c^2)}{3}$	22
risch	$\frac{d^2x^3}{3} + cd\,x^2 + c^2x + \frac{c^3}{3d}$	29
norman	$\frac{(\frac{1}{3}a\,d^2+dbc)x^3+(acd+b\,c^2)x^2+a\,c^2x+\frac{b\,d^2x^4}{3}}{bx+a}$	54
orering	$\frac{x(d^2x^2+3cdx+3c^2)(ac+(ad+bc)x+bd\,x^2)^2}{3(dx+c)^2(bx+a)^2}$	57

input
```
int((a*c+(a*d+b*c)*x+b*d*x^2)^2/(b*x+a)^2,x,method=_RETURNVERBOSE)
```

output
```
1/3*(d*x+c)^3/d
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.43

$$\int \frac{(ac+(bc+ad)x+bdx^2)^2}{(a+bx)^2}\,dx = \frac{1}{3}\,d^2x^3+cdx^2+c^2x$$

input
```
integrate((a*c+(a*d+b*c)*x+b*d*x^2)^2/(b*x+a)^2,x, algorithm="fricas")
```

output
```
1/3*d^2*x^3 + c*d*x^2 + c^2*x
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 19 vs. $2(8) = 16$.

Time = 0.04 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.36

$$\int \frac{(ac + (bc + ad)x + bdx^2)^2}{(a + bx)^2} \, dx = c^2x + cdx^2 + \frac{d^2x^3}{3}$$

input
```
integrate((a*c+(a*d+b*c)*x+b*d*x**2)**2/(b*x+a)**2,x)
```

output
```
c**2*x + c*d*x**2 + d**2*x**3/3
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.43

$$\int \frac{(ac + (bc + ad)x + bdx^2)^2}{(a + bx)^2} \, dx = \frac{1}{3} d^2x^3 + cdx^2 + c^2x$$

input
```
integrate((a*c+(a*d+b*c)*x+b*d*x^2)^2/(b*x+a)^2,x, algorithm="maxima")
```

output
```
1/3*d^2*x^3 + c*d*x^2 + c^2*x
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 84 vs. $2(12) = 24$.

Time = 0.15 (sec) , antiderivative size = 84, normalized size of antiderivative = 6.00

$$\int \frac{(ac + (bc + ad)x + bdx^2)^2}{(a + bx)^2} \, dx = \frac{\left(\frac{3\,b^2c^2}{(bx+a)^2} + \frac{3\,bcd}{bx+a} - \frac{6\,abcd}{(bx+a)^2} - \frac{3\,ad^2}{bx+a} + \frac{3\,a^2d^2}{(bx+a)^2} + d^2\right)(bx + a)^3}{3\,b^3}$$

input
```
integrate((a*c+(a*d+b*c)*x+b*d*x^2)^2/(b*x+a)^2,x, algorithm="giac")
```

output
```
1/3*(3*b^2*c^2/(b*x + a)^2 + 3*b*c*d/(b*x + a) - 6*a*b*c*d/(b*x + a)^2 - 3
*a*d^2/(b*x + a) + 3*a^2*d^2/(b*x + a)^2 + d^2)*(b*x + a)^3/b^3
```

Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.43

$$\int \frac{(ac + (bc + ad)x + bdx^2)^2}{(a + bx)^2} \, dx = c^2 \, x + c \, d \, x^2 + \frac{d^2 \, x^3}{3}$$

input
```
int((a*c + x*(a*d + b*c) + b*d*x^2)^2/(a + b*x)^2,x)
```

output
```
c^2*x + (d^2*x^3)/3 + c*d*x^2
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.50

$$\int \frac{(ac + (bc + ad)x + bdx^2)^2}{(a + bx)^2} \, dx = \frac{x(d^2x^2 + 3cdx + 3c^2)}{3}$$

input
```
int((a*c+(a*d+b*c)*x+b*d*x^2)^2/(b*x+a)^2,x)
```

output
```
(x*(3*c**2 + 3*c*d*x + d**2*x**2))/3
```

3.17 $\int \frac{(ac+(bc+ad)x+bdx^2)^2}{(a+bx)^3}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 49

$$\int \frac{(ac+(bc+ad)x+bdx^2)^2}{(a+bx)^3}\,dx = \frac{d(2bc-ad)x}{b^2}+\frac{d^2x^2}{2b}+\frac{(bc-ad)^2\log(a+bx)}{b^3}$$

output
```
d*(-a*d+2*b*c)*x/b^2+1/2*d^2*x^2/b+(-a*d+b*c)^2*ln(b*x+a)/b^3
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.88

$$\int \frac{(ac+(bc+ad)x+bdx^2)^2}{(a+bx)^3}\,dx = \frac{bdx(4bc-2ad+bdx)+2(bc-ad)^2\log(a+bx)}{2b^3}$$

input
```
Integrate[(a*c + (b*c + a*d)*x + b*d*x^2)^2/(a + b*x)^3,x]
```

output
```
(b*d*x*(4*b*c - 2*a*d + b*d*x) + 2*(b*c - a*d)^2*Log[a + b*x])/(2*b^3)
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.069, Rules used = {1121, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(x(ad+bc)+ac+bdx^2)^2}{(a+bx)^3}dx$$
$$\Big\downarrow \text{1121}$$
$$\int \left(\frac{(bc-ad)^2}{b^2(a+bx)}+\frac{d(bc-ad)}{b^2}+\frac{d(c+dx)}{b}\right)dx$$
$$\Big\downarrow \text{2009}$$
$$\frac{(bc-ad)^2\log(a+bx)}{b^3}+\frac{dx(bc-ad)}{b^2}+\frac{(c+dx)^2}{2b}$$

input
```
Int[(a*c + (b*c + a*d)*x + b*d*x^2)^2/(a + b*x)^3,x]
```

output
```
(d*(b*c - a*d)*x)/b^2 + (c + d*x)^2/(2*b) + ((b*c - a*d)^2*Log[a + b*x])/b
^3
```

Defintions of rubi rules used

rule 1121
```
Int[((d_) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Int[ExpandIntegrand[(d + e*x)^(m + p)*(a/d + (c/e)*x)^p, x], x]
/; FreeQ[{a, b, c, d, e, m, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && (Int
egerQ[p] || (GtQ[a, 0] && GtQ[d, 0] && LtQ[c, 0]))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 1.24 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.14

method	result	size
default	$-\frac{d\left(-\frac{1}{2}bd\,x^{2}+adx-2cbx\right)}{b^{2}}+\frac{\left(a^{2}d^{2}-2abcd+b^{2}c^{2}\right)\ln\left(bx+a\right)}{b^{3}}$	56
parallelrisch	$\frac{b^{2}d^{2}x^{2}+2\ln\left(bx+a\right)a^{2}d^{2}-4\ln\left(bx+a\right)abcd+2\ln\left(bx+a\right)b^{2}c^{2}-2ab\,d^{2}x+4b^{2}cxd}{2b^{3}}$	73
risch	$\frac{d^{2}x^{2}}{2b}-\frac{d^{2}ax}{b^{2}}+\frac{2cdx}{b}+\frac{\ln\left(bx+a\right)a^{2}d^{2}}{b^{3}}-\frac{2\ln\left(bx+a\right)acd}{b^{2}}+\frac{\ln\left(bx+a\right)c^{2}}{b}$	74
norman	$\frac{\frac{a^{2}\left(3a^{2}d^{2}-8abcd\right)}{2b^{3}}+\frac{b\,d^{2}x^{4}}{2}+\frac{2a\left(a^{2}d^{2}-3abcd\right)x}{b^{2}}+2bcd\,x^{3}}{\left(bx+a\right)^{2}}+\frac{\left(a^{2}d^{2}-2abcd+b^{2}c^{2}\right)\ln\left(bx+a\right)}{b^{3}}$	103

input
```
int((a*c+(a*d+b*c)*x+b*d*x^2)^2/(b*x+a)^3,x,method=_RETURNVERBOSE)
```

output
```
-d/b^2*(-1/2*b*d*x^2+a*d*x-2*c*b*x)+(a^2*d^2-2*a*b*c*d+b^2*c^2)/b^3*ln(b*x
+a)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.29

$$\int \frac{(ac+(bc+ad)x+bdx^2)^2}{(a+bx)^3}\,dx = \frac{b^2d^2x^2+2\,(2\,b^2cd-abd^2)x+2\,(b^2c^2-2\,abcd+a^2d^2)\log{(bx+a)}}{2\,b^3}$$

input
```
integrate((a*c+(a*d+b*c)*x+b*d*x^2)^2/(b*x+a)^3,x, algorithm="fricas")
```

output
```
1/2*(b^2*d^2*x^2 + 2*(2*b^2*c*d - a*b*d^2)*x + 2*(b^2*c^2 - 2*a*b*c*d + a^
2*d^2)*log(b*x + a))/b^3
```

Sympy [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.90

$$\int \frac{(ac+(bc+ad)x+bdx^2)^2}{(a+bx)^3}\,dx = x\left(-\frac{ad^2}{b^2}+\frac{2cd}{b}\right)+\frac{d^2x^2}{2b}+\frac{(ad-bc)^2\log{(a+bx)}}{b^3}$$

input
```
integrate((a*c+(a*d+b*c)*x+b*d*x**2)**2/(b*x+a)**3,x)
```

output
```
x*(-a*d**2/b**2 + 2*c*d/b) + d**2*x**2/(2*b) + (a*d - b*c)**2*log(a + b*x)
/b**3
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.24

$$\int \frac{(ac+(bc+ad)x+bdx^2)^2}{(a+bx)^3}\,dx = \frac{bd^2x^2+2\,(2\,bcd-ad^2)x}{2\,b^2} + \frac{(b^2c^2-2\,abcd+a^2d^2)\log{(bx+a)}}{b^3}$$

input
```
integrate((a*c+(a*d+b*c)*x+b*d*x^2)^2/(b*x+a)^3,x, algorithm="maxima")
```

output
```
1/2*(b*d^2*x^2 + 2*(2*b*c*d - a*d^2)*x)/b^2 + (b^2*c^2 - 2*a*b*c*d + a^2*d
^2)*log(b*x + a)/b^3
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.22

$$\int \frac{(ac+(bc+ad)x+bdx^2)^2}{(a+bx)^3}\,dx = \frac{bd^2x^2+4\,bcdx-2\,ad^2x}{2\,b^2} + \frac{(b^2c^2-2\,abcd+a^2d^2)\log{(|bx+a|)}}{b^3}$$

input
```
integrate((a*c+(a*d+b*c)*x+b*d*x^2)^2/(b*x+a)^3,x, algorithm="giac")
```

output
```
1/2*(b*d^2*x^2 + 4*b*c*d*x - 2*a*d^2*x)/b^2 + (b^2*c^2 - 2*a*b*c*d + a^2*d
^2)*log(abs(b*x + a))/b^3
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.27

$$\int \frac{(ac+(bc+ad)x+bdx^2)^2}{(a+bx)^3}\,dx = \frac{\ln(a+b\,x)\,\left(a^2\,d^2-2\,a\,b\,c\,d+b^2\,c^2\right)}{b^3} - x\left(\frac{a\,d^2}{b^2}-\frac{2\,c\,d}{b}\right)+\frac{d^2\,x^2}{2\,b}$$

input
```
int((a*c + x*(a*d + b*c) + b*d*x^2)^2/(a + b*x)^3,x)
```

output
```
(log(a + b*x)*(a^2*d^2 + b^2*c^2 - 2*a*b*c*d))/b^3 - x*((a*d^2)/b^2 - (2*c
*d)/b) + (d^2*x^2)/(2*b)
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.47

$$\int \frac{(ac+(bc+ad)x+bdx^2)^2}{(a+bx)^3}\,dx = \frac{2\log(bx+a)\,a^2d^2-4\log(bx+a)\,abcd+2\log(bx+a)\,b^2c^2-2ab\,d^2x+4b^2cdx+b^2d^2x^2}{2b^3}$$

input
```
int((a*c+(a*d+b*c)*x+b*d*x^2)^2/(b*x+a)^3,x)
```

output
```
(2*log(a + b*x)*a**2*d**2 - 4*log(a + b*x)*a*b*c*d + 2*log(a + b*x)*b**2*c
**2 - 2*a*b*d**2*x + 4*b**2*c*d*x + b**2*d**2*x**2)/(2*b**3)
```

3.18 $\int \frac{(ac+(bc+ad)x+bdx^2)^2}{(a+bx)^4} dx$

Optimal result

Integrand size = 29, antiderivative size = 51

$$\int \frac{(ac+(bc+ad)x+bdx^2)^2}{(a+bx)^4} dx = \frac{d^2x}{b^2} - \frac{(bc-ad)^2}{b^3(a+bx)} + \frac{2d(bc-ad)\log(a+bx)}{b^3}$$

output
```
d^2*x/b^2-(-a*d+b*c)^2/b^3/(b*x+a)+2*d*(-a*d+b*c)*ln(b*x+a)/b^3
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.92

$$\int \frac{(ac+(bc+ad)x+bdx^2)^2}{(a+bx)^4} dx = \frac{bd^2x - \frac{(bc-ad)^2}{a+bx} + 2d(bc-ad)\log(a+bx)}{b^3}$$

input
```
Integrate[(a*c + (b*c + a*d)*x + b*d*x^2)^2/(a + b*x)^4,x]
```

output
```
(b*d^2*x - (b*c - a*d)^2/(a + b*x) + 2*d*(b*c - a*d)*Log[a + b*x])/b^3
```

Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.069$, Rules used = {1121, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(x(ad+bc)+ac+bdx^2)^2}{(a+bx)^4} \, dx$$
$$\Big\downarrow \text{ 1121}$$
$$\int \left(\frac{2d(bc-ad)}{b^2(a+bx)}+\frac{(bc-ad)^2}{b^2(a+bx)^2}+\frac{d^2}{b^2}\right)dx$$
$$\Big\downarrow \text{ 2009}$$
$$-\frac{(bc-ad)^2}{b^3(a+bx)}+\frac{2d(bc-ad)\log(a+bx)}{b^3}+\frac{d^2x}{b^2}$$

input
```
Int[(a*c + (b*c + a*d)*x + b*d*x^2)^2/(a + b*x)^4,x]
```

output
```
(d^2*x)/b^2 - (b*c - a*d)^2/(b^3*(a + b*x)) + (2*d*(b*c - a*d)*Log[a + b*x
])/b^3
```

Defintions of rubi rules used

rule 1121
```
Int[((d_) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Int[ExpandIntegrand[(d + e*x)^(m + p)*(a/d + (c/e)*x)^p, x], x]
/; FreeQ[{a, b, c, d, e, m, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && (Int
egerQ[p] || (GtQ[a, 0] && GtQ[d, 0] && LtQ[c, 0]))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 1.27 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.24

method	result	size
default	$\frac{d^2x}{b^2}-\frac{a^2d^2-2abcd+b^2c^2}{b^3(bx+a)}-\frac{2d(ad-bc)\ln(bx+a)}{b^3}$	63
risch	$\frac{d^2x}{b^2}-\frac{a^2d^2}{b^3(bx+a)}+\frac{2acd}{b^2(bx+a)}-\frac{c^2}{b(bx+a)}-\frac{2d^2\ln(bx+a)a}{b^3}+\frac{2d\ln(bx+a)c}{b^2}$	86
parallelrisch	$-\frac{2\ln(bx+a)xab\,d^2-2\ln(bx+a)x\,b^2cd-b^2d^2x^2+2\ln(bx+a)a^2d^2-2\ln(bx+a)abcd+2a^2d^2-2abcd+b^2c^2}{b^3(bx+a)}$	100
norman	$\frac{b\,d^2x^4-\frac{a^2\left(4a^2d^2-2abcd+b^2c^2\right)}{b^3}-\frac{\left(7a^2d^2-2abcd+b^2c^2\right)x^2}{b}-\frac{a\left(10a^2d^2-4abcd+2b^2c^2\right)x}{b^2}}{(bx+a)^3}-\frac{2d(ad-bc)\ln(bx+a)}{b^3}$	129

input
```
int((a*c+(a*d+b*c)*x+b*d*x^2)^2/(b*x+a)^4,x,method=_RETURNVERBOSE)
```

output
```
d^2*x/b^2-(a^2*d^2-2*a*b*c*d+b^2*c^2)/b^3/(b*x+a)-2/b^3*d*(a*d-b*c)*ln(b*x
+a)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.80

$$\int\frac{(ac+(bc+ad)x+bdx^2)^2}{(a+bx)^4}\,dx$$
$$=\frac{b^2d^2x^2+abd^2x-b^2c^2+2\,abcd-a^2d^2+2\left(abcd-a^2d^2+(b^2cd-abd^2)x\right)\log\left(bx+a\right)}{b^4x+ab^3}$$

input
```
integrate((a*c+(a*d+b*c)*x+b*d*x^2)^2/(b*x+a)^4,x, algorithm="fricas")
```

output
```
(b^2*d^2*x^2 + a*b*d^2*x - b^2*c^2 + 2*a*b*c*d - a^2*d^2 + 2*(a*b*c*d - a^
2*d^2 + (b^2*c*d - a*b*d^2)*x)*log(b*x + a))/(b^4*x + a*b^3)
```

Sympy [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.18

$$\int \frac{(ac+(bc+ad)x+bdx^2)^2}{(a+bx)^4}\,dx = \frac{-a^2d^2+2abcd-b^2c^2}{ab^3+b^4x}+\frac{d^2x}{b^2}-\frac{2d(ad-bc)\log(a+bx)}{b^3}$$

input
```
integrate((a*c+(a*d+b*c)*x+b*d*x**2)**2/(b*x+a)**4,x)
```

output
```
(-a**2*d**2 + 2*a*b*c*d - b**2*c**2)/(a*b**3 + b**4*x) + d**2*x/b**2 - 2*d
*(a*d - b*c)*log(a + b*x)/b**3
```

Maxima [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.31

$$\int \frac{(ac+(bc+ad)x+bdx^2)^2}{(a+bx)^4}\,dx = \frac{d^2x}{b^2}-\frac{b^2c^2-2\,abcd+a^2d^2}{b^4x+ab^3}+\frac{2\,(bcd-ad^2)\log(bx+a)}{b^3}$$

input
```
integrate((a*c+(a*d+b*c)*x+b*d*x^2)^2/(b*x+a)^4,x, algorithm="maxima")
```

output
```
d^2*x/b^2 - (b^2*c^2 - 2*a*b*c*d + a^2*d^2)/(b^4*x + a*b^3) + 2*(b*c*d - a
*d^2)*log(b*x + a)/b^3
```

Giac [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.27

$$\int \frac{(ac+(bc+ad)x+bdx^2)^2}{(a+bx)^4}\,dx = \frac{d^2x}{b^2}+\frac{2\,(bcd-ad^2)\log(|bx+a|)}{b^3}-\frac{b^2c^2-2\,abcd+a^2d^2}{(bx+a)b^3}$$

input
```
integrate((a*c+(a*d+b*c)*x+b*d*x^2)^2/(b*x+a)^4,x, algorithm="giac")
```

output
```
d^2*x/b^2 + 2*(b*c*d - a*d^2)*log(abs(b*x + a))/b^3 - (b^2*c^2 - 2*a*b*c*d
 + a^2*d^2)/((b*x + a)*b^3)
```

Mupad [B] (verification not implemented)

Time = 5.40 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.39

$$\int \frac{(ac+(bc+ad)x+bdx^2)^2}{(a+bx)^4}\,dx = \frac{d^2\,x}{b^2} - \frac{a^2\,d^2-2\,a\,b\,c\,d+b^2\,c^2}{b\,\left(x\,b^3+a\,b^2\right)} - \frac{\ln\left(a+b\,x\right)\,\left(2\,a\,d^2-2\,b\,c\,d\right)}{b^3}$$

input
```
int((a*c + x*(a*d + b*c) + b*d*x^2)^2/(a + b*x)^4,x)
```

output
```
(d^2*x)/b^2 - (a^2*d^2 + b^2*c^2 - 2*a*b*c*d)/(b*(a*b^2 + b^3*x)) - (log(a
 + b*x)*(2*a*d^2 - 2*b*c*d))/b^3
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 112, normalized size of antiderivative = 2.20

$$\int \frac{(ac+(bc+ad)x+bdx^2)^2}{(a+bx)^4}\,dx = \frac{-2\log(bx+a)\,a^3d^2+2\log(bx+a)\,a^2bcd-2\log(bx+a)\,a^2b\,d^2x+2\log(bx+a)\,a\,b^2cdx+2a^2b\,d^2x-2}{a\,b^3\,(bx+a)}$$

input
```
int((a*c+(a*d+b*c)*x+b*d*x^2)^2/(b*x+a)^4,x)
```

output
```
( - 2*log(a + b*x)*a**3*d**2 + 2*log(a + b*x)*a**2*b*c*d - 2*log(a + b*x)*
a**2*b*d**2*x + 2*log(a + b*x)*a*b**2*c*d*x + 2*a**2*b*d**2*x - 2*a*b**2*c
*d*x + a*b**2*d**2*x**2 + b**3*c**2*x)/(a*b**3*(a + b*x))
```

3.19 $\int \frac{(ac+(bc+ad)x+bdx^2)^2}{(a+bx)^5} dx$

Optimal result

Integrand size = 29, antiderivative size = 59

$$\int \frac{(ac+(bc+ad)x+bdx^2)^2}{(a+bx)^5} dx = -\frac{(bc-ad)^2}{2b^3(a+bx)^2} - \frac{2d(bc-ad)}{b^3(a+bx)} + \frac{d^2\log(a+bx)}{b^3}$$

output
```
-1/2*(-a*d+b*c)^2/b^3/(b*x+a)^2-2*d*(-a*d+b*c)/b^3/(b*x+a)+d^2*ln(b*x+a)/b
^3
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.83

$$\int \frac{(ac+(bc+ad)x+bdx^2)^2}{(a+bx)^5} dx = \frac{-\frac{(bc-ad)(3ad+b(c+4dx))}{(a+bx)^2} + 2d^2\log(a+bx)}{2b^3}$$

input
```
Integrate[(a*c + (b*c + a*d)*x + b*d*x^2)^2/(a + b*x)^5,x]
```

output
```
(-(((b*c - a*d)*(3*a*d + b*(c + 4*d*x)))/(a + b*x)^2) + 2*d^2*Log[a + b*x]
)/(2*b^3)
```

Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.069$, Rules used = {1121, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(x(ad+bc)+ac+bdx^2\right)^2}{(a+bx)^5}\,dx$$

$$\Big\downarrow \ 1121$$

$$\int \left(\frac{2d(bc-ad)}{b^2(a+bx)^2}+\frac{(bc-ad)^2}{b^2(a+bx)^3}+\frac{d^2}{b^2(a+bx)}\right)dx$$

$$\Big\downarrow \ 2009$$

$$-\frac{2d(bc-ad)}{b^3(a+bx)}-\frac{(bc-ad)^2}{2b^3(a+bx)^2}+\frac{d^2\log(a+bx)}{b^3}$$

input
```
Int[(a*c + (b*c + a*d)*x + b*d*x^2)^2/(a + b*x)^5,x]
```

output
```
-1/2*(b*c - a*d)^2/(b^3*(a + b*x)^2) - (2*d*(b*c - a*d))/(b^3*(a + b*x)) +
 (d^2*Log[a + b*x])/b^3
```

Defintions of rubi rules used

rule 1121
```
Int[((d_) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Int[ExpandIntegrand[(d + e*x)^(m + p)*(a/d + (c/e)*x)^p, x], x]
/; FreeQ[{a, b, c, d, e, m, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && (Int
egerQ[p] || (GtQ[a, 0] && GtQ[d, 0] && LtQ[c, 0]))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 1.24 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.14

method	result	s
risch	$\frac{\frac{2d(ad-bc)x}{b^2}+\frac{3a^2d^2-2abcd-b^2c^2}{2b^3}}{(bx+a)^2}+\frac{d^2\ln(bx+a)}{b^3}$	6
default	$\frac{2d(ad-bc)}{b^3(bx+a)}+\frac{d^2\ln(bx+a)}{b^3}-\frac{a^2d^2-2abcd+b^2c^2}{2b^3(bx+a)^2}$	6
parallelrisch	$\frac{2\ln(bx+a)x^2b^2d^2+4\ln(bx+a)xab\,d^2+2\ln(bx+a)a^2d^2+4ab\,d^2x-4b^2cxd+3a^2d^2-2abcd-b^2c^2}{2b^3(bx+a)^2}$	9
norman	$\frac{\frac{a\left(5a^2b\,d^2-4cda\,b^2-b^3c^2\right)x}{b^3}+\frac{a^2\left(3a^2b\,d^2-2cda\,b^2-b^3c^2\right)}{2b^4}+\frac{2\left(ab\,d^2-cd\,b^2\right)x^3}{b}+\frac{\left(11a^2b\,d^2-10cda\,b^2-b^3c^2\right)x^2}{2b^2}}{(bx+a)^4}+\frac{d^2\ln(bx+a)}{b^3}$	1

input
```
int((a*c+(a*d+b*c)*x+b*d*x^2)^2/(b*x+a)^5,x,method=_RETURNVERBOSE)
```

output
```
(2/b^2*d*(a*d-b*c)*x+1/2*(3*a^2*d^2-2*a*b*c*d-b^2*c^2)/b^3)/(b*x+a)^2+d^2*
ln(b*x+a)/b^3
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.68

$$\int \frac{(ac+(bc+ad)x+bdx^2)^2}{(a+bx)^5}\,dx$$
$$= -\frac{b^2c^2+2\,abcd-3\,a^2d^2+4\,(b^2cd-abd^2)x-2\,(b^2d^2x^2+2\,abd^2x+a^2d^2)\log(bx+a)}{2\,(b^5x^2+2\,ab^4x+a^2b^3)}$$

input
```
integrate((a*c+(a*d+b*c)*x+b*d*x^2)^2/(b*x+a)^5,x, algorithm="fricas")
```

output
```
-1/2*(b^2*c^2 + 2*a*b*c*d - 3*a^2*d^2 + 4*(b^2*c*d - a*b*d^2)*x - 2*(b^2*d
^2*x^2 + 2*a*b*d^2*x + a^2*d^2)*log(b*x + a))/(b^5*x^2 + 2*a*b^4*x + a^2*b
^3)
```

Sympy [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.36

$$\int \frac{(ac+(bc+ad)x+bdx^2)^2}{(a+bx)^5}\,dx = \frac{3a^2d^2-2abcd-b^2c^2+x(4abd^2-4b^2cd)}{2a^2b^3+4ab^4x+2b^5x^2} + \frac{d^2\log{(a+bx)}}{b^3}$$

input
```
integrate((a*c+(a*d+b*c)*x+b*d*x**2)**2/(b*x+a)**5,x)
```

output
```
(3*a**2*d**2 - 2*a*b*c*d - b**2*c**2 + x*(4*a*b*d**2 - 4*b**2*c*d))/(2*a**
2*b**3 + 4*a*b**4*x + 2*b**5*x**2) + d**2*log(a + b*x)/b**3
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.34

$$\int \frac{(ac+(bc+ad)x+bdx^2)^2}{(a+bx)^5}\,dx = -\frac{b^2c^2+2\,abcd-3\,a^2d^2+4\,(b^2cd-abd^2)x}{2\,(b^5x^2+2\,ab^4x+a^2b^3)} + \frac{d^2\log{(bx+a)}}{b^3}$$

input
```
integrate((a*c+(a*d+b*c)*x+b*d*x^2)^2/(b*x+a)^5,x, algorithm="maxima")
```

output
```
-1/2*(b^2*c^2 + 2*a*b*c*d - 3*a^2*d^2 + 4*(b^2*c*d - a*b*d^2)*x)/(b^5*x^2
+ 2*a*b^4*x + a^2*b^3) + d^2*log(b*x + a)/b^3
```

Giac [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 110, normalized size of antiderivative = 1.86

$$\int \frac{(ac+(bc+ad)x+bdx^2)^2}{(a+bx)^5}\,dx = -\frac{d^2\log\left(\frac{|bx+a|}{(bx+a)^2|b|}\right)}{b^3} - \frac{\frac{b^5c^2}{(bx+a)^2}+\frac{4\,b^4cd}{bx+a}-\frac{2\,ab^4cd}{(bx+a)^2}-\frac{4\,ab^3d^2}{bx+a}+\frac{a^2b^3d^2}{(bx+a)^2}}{2\,b^6}$$

input
```
integrate((a*c+(a*d+b*c)*x+b*d*x^2)^2/(b*x+a)^5,x, algorithm="giac")
```

output
```
-d^2*log(abs(b*x + a)/((b*x + a)^2*abs(b)))/b^3 - 1/2*(b^5*c^2/(b*x + a)^2
 + 4*b^4*c*d/(b*x + a) - 2*a*b^4*c*d/(b*x + a)^2 - 4*a*b^3*d^2/(b*x + a) +
 a^2*b^3*d^2/(b*x + a)^2)/b^6
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.31

$$\int \frac{(ac+(bc+ad)x+bdx^2)^2}{(a+bx)^5}\,dx = \frac{d^2\,\ln(a+b\,x)}{b^3} - \frac{\frac{-3\,a^2\,d^2+2\,a\,b\,c\,d+b^2\,c^2}{2\,b^3}-\frac{2\,d\,x\,(a\,d-b\,c)}{b^2}}{a^2+2\,a\,b\,x+b^2\,x^2}$$

input
```
int((a*c + x*(a*d + b*c) + b*d*x^2)^2/(a + b*x)^5,x)
```

output
```
(d^2*log(a + b*x))/b^3 - ((b^2*c^2 - 3*a^2*d^2 + 2*a*b*c*d)/(2*b^3) - (2*d
*x*(a*d - b*c))/b^2)/(a^2 + b^2*x^2 + 2*a*b*x)
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.92

$$\int \frac{(ac+(bc+ad)x+bdx^2)^2}{(a+bx)^5}\,dx$$
$$= \frac{2\log(bx+a)\,a^3d^2+4\log(bx+a)\,a^2b\,d^2x+2\log(bx+a)\,a\,b^2d^2x^2+a^3d^2-a\,b^2c^2-2a\,b^2d^2x^2+2b^3cd\,x}{2a\,b^3\,(b^2x^2+2abx+a^2)}$$

input
```
int((a*c+(a*d+b*c)*x+b*d*x^2)^2/(b*x+a)^5,x)
```

output
```
(2*log(a + b*x)*a**3*d**2 + 4*log(a + b*x)*a**2*b*d**2*x + 2*log(a + b*x)*
a*b**2*d**2*x**2 + a**3*d**2 - a*b**2*c**2 - 2*a*b**2*d**2*x**2 + 2*b**3*c
*d*x**2)/(2*a*b**3*(a**2 + 2*a*b*x + b**2*x**2))
```

3.20 $\int \frac{(ac+(bc+ad)x+bdx^2)^2}{(a+bx)^6} dx$

Optimal result

Integrand size = 29, antiderivative size = 28

$$\int \frac{(ac+(bc+ad)x+bdx^2)^2}{(a+bx)^6} dx = -\frac{(c+dx)^3}{3(bc-ad)(a+bx)^3}$$

output
```
-1/3*(d*x+c)^3/(-a*d+b*c)/(b*x+a)^3
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.89

$$\int \frac{(ac+(bc+ad)x+bdx^2)^2}{(a+bx)^6} dx = -\frac{a^2d^2+abd(c+3dx)+b^2(c^2+3cdx+3d^2x^2)}{3b^3(a+bx)^3}$$

input
```
Integrate[(a*c + (b*c + a*d)*x + b*d*x^2)^2/(a + b*x)^6,x]
```

output
```
-1/3*(a^2*d^2 + a*b*d*(c + 3*d*x) + b^2*(c^2 + 3*c*d*x + 3*d^2*x^2))/(b^3*
(a + b*x)^3)
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.069$, Rules used = {1120, 48}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(x(ad+bc)+ac+bdx^2)^2}{(a+bx)^6}dx$$

$$\Big\downarrow \text{ 1120}$$

$$\int \frac{(c+dx)^2}{(a+bx)^4}dx$$

$$\Big\downarrow \text{ 48}$$

$$-\frac{(c+dx)^3}{3(a+bx)^3(bc-ad)}$$

input
```
Int[(a*c + (b*c + a*d)*x + b*d*x^2)^2/(a + b*x)^6,x]
```

output
```
-1/3*(c + d*x)^3/((b*c - a*d)*(a + b*x)^3)
```

Defintions of rubi rules used

rule 48
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && EqQ[m + n + 2, 0] && NeQ[m, -1]
```

rule 1120
```
Int[((d_) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Int[(d + e*x)^(m + p)*(a/d + (c/e)*x)^p, x] /; FreeQ[{a, b, c, d
, e, m}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[p] && (EqQ[m + p,
0] || EqQ[m + 2*p + 2, 0])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 59 vs. $2(26) = 52$.

Time = 1.27 (sec) , antiderivative size = 60, normalized size of antiderivative = 2.14

method	result
gosper	$-\frac{3b^2d^2x^2+3ab\,d^2x+3b^2cxd+a^2d^2+abcd+b^2c^2}{3b^3(bx+a)^3}$
risch	$\frac{-\frac{d^2x^2}{b}-\frac{d(ad+bc)x}{b^2}-\frac{a^2d^2+abcd+b^2c^2}{3b^3}}{(bx+a)^3}$
parallelrisch	$\frac{-3b^2d^2x^2-3ab\,d^2x-3b^2cxd-a^2d^2-abcd-b^2c^2}{3b^3(bx+a)^3}$
default	$-\frac{a^2d^2-2abcd+b^2c^2}{3b^3(bx+a)^3}-\frac{d^2}{b^3(bx+a)}+\frac{d(ad-bc)}{b^3(bx+a)^2}$
orering	$-\frac{(3b^2d^2x^2+3ab\,d^2x+3b^2cxd+a^2d^2+abcd+b^2c^2)(ac+(ad+bc)x+bd\,x^2)^2}{3b^3(bx+a)^5(dx+c)^2}$
norman	$\frac{-b\,d^2x^4+\frac{(-3a\,b^2d^2-b^3cd)x^3}{b^2}+\frac{a^2(-d^2a^2b^2-cda\,b^3-b^4c^2)}{3b^5}+\frac{(-10d^2a^2b^2-7cda\,b^3-b^4c^2)x^2}{3b^3}+\frac{a(-5d^2a^2b^2-5cda\,b^3-2b^4c^2)x}{3b^4}}{(bx+a)^5}$

input
```
int((a*c+(a*d+b*c)*x+b*d*x^2)^2/(b*x+a)^6,x,method=_RETURNVERBOSE)
```

output
```
-1/3*(3*b^2*d^2*x^2+3*a*b*d^2*x+3*b^2*c*d*x+a^2*d^2+a*b*c*d+b^2*c^2)/b^3/(
b*x+a)^3
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 84 vs. $2(26) = 52$.

Time = 0.07 (sec) , antiderivative size = 84, normalized size of antiderivative = 3.00

$$\int \frac{(ac+(bc+ad)x+bdx^2)^2}{(a+bx)^6}\,dx = -\frac{3\,b^2d^2x^2+b^2c^2+abcd+a^2d^2+3\,(b^2cd+abd^2)x}{3\,(b^6x^3+3\,ab^5x^2+3\,a^2b^4x+a^3b^3)}$$

input
```
integrate((a*c+(a*d+b*c)*x+b*d*x^2)^2/(b*x+a)^6,x, algorithm="fricas")
```

output
```
-1/3*(3*b^2*d^2*x^2 + b^2*c^2 + a*b*c*d + a^2*d^2 + 3*(b^2*c*d + a*b*d^2)*
x)/(b^6*x^3 + 3*a*b^5*x^2 + 3*a^2*b^4*x + a^3*b^3)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 88 vs. $2(22) = 44$.

Time = 0.31 (sec) , antiderivative size = 88, normalized size of antiderivative = 3.14

$$\int \frac{(ac + (bc + ad)x + bdx^2)^2}{(a + bx)^6} \, dx = \frac{-a^2d^2 - abcd - b^2c^2 - 3b^2d^2x^2 + x(-3abd^2 - 3b^2cd)}{3a^3b^3 + 9a^2b^4x + 9ab^5x^2 + 3b^6x^3}$$

input
```
integrate((a*c+(a*d+b*c)*x+b*d*x**2)**2/(b*x+a)**6,x)
```

output
```
(-a**2*d**2 - a*b*c*d - b**2*c**2 - 3*b**2*d**2*x**2 + x*(-3*a*b*d**2 - 3*
b**2*c*d))/(3*a**3*b**3 + 9*a**2*b**4*x + 9*a*b**5*x**2 + 3*b**6*x**3)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 84 vs. $2(26) = 52$.

Time = 0.05 (sec) , antiderivative size = 84, normalized size of antiderivative = 3.00

$$\int \frac{(ac + (bc + ad)x + bdx^2)^2}{(a + bx)^6} \, dx = -\frac{3\,b^2d^2x^2 + b^2c^2 + abcd + a^2d^2 + 3\,(b^2cd + abd^2)x}{3\,(b^6x^3 + 3\,ab^5x^2 + 3\,a^2b^4x + a^3b^3)}$$

input
```
integrate((a*c+(a*d+b*c)*x+b*d*x^2)^2/(b*x+a)^6,x, algorithm="maxima")
```

output
```
-1/3*(3*b^2*d^2*x^2 + b^2*c^2 + a*b*c*d + a^2*d^2 + 3*(b^2*c*d + a*b*d^2)*
x)/(b^6*x^3 + 3*a*b^5*x^2 + 3*a^2*b^4*x + a^3*b^3)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 59 vs. 2(26) = 52.

Time = 0.18 (sec) , antiderivative size = 59, normalized size of antiderivative = 2.11

$$\int \frac{(ac+(bc+ad)x+bdx^2)^2}{(a+bx)^6}\,dx = -\frac{3\,b^2d^2x^2+3\,b^2cdx+3\,abd^2x+b^2c^2+abcd+a^2d^2}{3\,(bx+a)^3b^3}$$

input
```
integrate((a*c+(a*d+b*c)*x+b*d*x^2)^2/(b*x+a)^6,x, algorithm="giac")
```

output
```
-1/3*(3*b^2*d^2*x^2 + 3*b^2*c*d*x + 3*a*b*d^2*x + b^2*c^2 + a*b*c*d + a^2*
d^2)/((b*x + a)^3*b^3)
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 80, normalized size of antiderivative = 2.86

$$\int \frac{(ac+(bc+ad)x+bdx^2)^2}{(a+bx)^6}\,dx = -\frac{\frac{a^2\,d^2+a\,b\,c\,d+b^2\,c^2}{3\,b^3}+\frac{d^2\,x^2}{b}+\frac{d\,x\,(a\,d+b\,c)}{b^2}}{a^3+3\,a^2\,b\,x+3\,a\,b^2\,x^2+b^3\,x^3}$$

input
```
int((a*c + x*(a*d + b*c) + b*d*x^2)^2/(a + b*x)^6,x)
```

output
```
-((a^2*d^2 + b^2*c^2 + a*b*c*d)/(3*b^3) + (d^2*x^2)/b + (d*x*(a*d + b*c))/
b^2)/(a^3 + b^3*x^3 + 3*a*b^2*x^2 + 3*a^2*b*x)
```

Reduce [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 69, normalized size of antiderivative = 2.46

$$\int \frac{(ac+(bc+ad)x+bdx^2)^2}{(a+bx)^6}\,dx = \frac{b^2d^2x^3-3abcdx-a^2cd-ab\,c^2}{3a\,b^2\,(b^3x^3+3a\,b^2x^2+3a^2bx+a^3)}$$

input
```
int((a*c+(a*d+b*c)*x+b*d*x^2)^2/(b*x+a)^6,x)
```

output
```
( - a**2*c*d - a*b*c**2 - 3*a*b*c*d*x + b**2*d**2*x**3)/(3*a*b**2*(a**3 +
3*a**2*b*x + 3*a*b**2*x**2 + b**3*x**3))
```

3.21 $\int \frac{(ac+(bc+ad)x+bdx^2)^2}{(a+bx)^7} dx$

Optimal result

Integrand size = 29, antiderivative size = 65

$$\int \frac{(ac+(bc+ad)x+bdx^2)^2}{(a+bx)^7} dx = -\frac{(bc-ad)^2}{4b^3(a+bx)^4} - \frac{2d(bc-ad)}{3b^3(a+bx)^3} - \frac{d^2}{2b^3(a+bx)^2}$$

output
```
-1/4*(-a*d+b*c)^2/b^3/(b*x+a)^4-2/3*d*(-a*d+b*c)/b^3/(b*x+a)^3-1/2*d^2/b^3
/(b*x+a)^2
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.86

$$\int \frac{(ac+(bc+ad)x+bdx^2)^2}{(a+bx)^7} dx = -\frac{a^2d^2+2abd(c+2dx)+b^2(3c^2+8cdx+6d^2x^2)}{12b^3(a+bx)^4}$$

input
```
Integrate[(a*c + (b*c + a*d)*x + b*d*x^2)^2/(a + b*x)^7,x]
```

output
```
-1/12*(a^2*d^2 + 2*a*b*d*(c + 2*d*x) + b^2*(3*c^2 + 8*c*d*x + 6*d^2*x^2))/
(b^3*(a + b*x)^4)
```

Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.069, Rules used = {1121, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(x(ad+bc)+ac+bdx^2\right)^2}{(a+bx)^7}dx$$

$$\Big\downarrow \ 1121$$

$$\int \left(\frac{2d(bc-ad)}{b^2(a+bx)^4}+\frac{(bc-ad)^2}{b^2(a+bx)^5}+\frac{d^2}{b^2(a+bx)^3}\right)dx$$

$$\Big\downarrow \ 2009$$

$$-\frac{2d(bc-ad)}{3b^3(a+bx)^3}-\frac{(bc-ad)^2}{4b^3(a+bx)^4}-\frac{d^2}{2b^3(a+bx)^2}$$

input
```
Int[(a*c + (b*c + a*d)*x + b*d*x^2)^2/(a + b*x)^7,x]
```

output
```
-1/4*(b*c - a*d)^2/(b^3*(a + b*x)^4) - (2*d*(b*c - a*d))/(3*b^3*(a + b*x)^
3) - d^2/(2*b^3*(a + b*x)^2)
```

Defintions of rubi rules used

rule 1121
```
Int[((d_) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Int[ExpandIntegrand[(d + e*x)^(m + p)*(a/d + (c/e)*x)^p, x], x]
/; FreeQ[{a, b, c, d, e, m, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && (Int
egerQ[p] || (GtQ[a, 0] && GtQ[d, 0] && LtQ[c, 0]))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 1.21 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.95

method	result	si
gosper	$-\frac{6b^2d^2x^2+4ab\,d^2x+8b^2cxd+a^2d^2+2abcd+3b^2c^2}{12b^3(bx+a)^4}$	6
risch	$\frac{-\frac{d^2x^2}{2b}-\frac{d(ad+2bc)x}{3b^2}-\frac{a^2d^2+2abcd+3b^2c^2}{12b^3}}{(bx+a)^4}$	6
parallelrisch	$\frac{-6b^3d^2x^2-4a\,b^2d^2x-8b^3cdx-a^2b\,d^2-2cda\,b^2-3b^3c^2}{12b^4(bx+a)^4}$	6
default	$\frac{2d(ad-bc)}{3b^3(bx+a)^3}-\frac{d^2}{2b^3(bx+a)^2}-\frac{a^2d^2-2abcd+b^2c^2}{4b^3(bx+a)^4}$	7
orering	$-\frac{(6b^2d^2x^2+4ab\,d^2x+8b^2cxd+a^2d^2+2abcd+3b^2c^2)(ac+(ad+bc)x+bd\,x^2)^2}{12b^3(bx+a)^6(dx+c)^2}$	9
norman	$\frac{\frac{a^2\left(-d^2a^2b^3-2cda\,b^4-3b^5c^2\right)}{12b^6}-\frac{b\,d^2x^4}{2}+\frac{2\left(-2a\,b^3d^2-b^4cd\right)x^3}{3b^3}+\frac{\left(-5d^2a^2b^3-6cda\,b^4-b^5c^2\right)x^2}{4b^4}+\frac{a\left(-d^2a^2b^3-2cda\,b^4-b^5c^2\right)x}{2b^5}}{(bx+a)^6}$	1

input
```
int((a*c+(a*d+b*c)*x+b*d*x^2)^2/(b*x+a)^7,x,method=_RETURNVERBOSE)
```

output
```
-1/12/b^3*(6*b^2*d^2*x^2+4*a*b*d^2*x+8*b^2*c*d*x+a^2*d^2+2*a*b*c*d+3*b^2*c
^2)/(b*x+a)^4
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 98, normalized size of antiderivative = 1.51

$$\int \frac{(ac+(bc+ad)x+bdx^2)^2}{(a+bx)^7}\,dx = -\frac{6\,b^2d^2x^2+3\,b^2c^2+2\,abcd+a^2d^2+4\,(2\,b^2cd+abd^2)x}{12\,(b^7x^4+4\,ab^6x^3+6\,a^2b^5x^2+4\,a^3b^4x+a^4b^3)}$$

input
```
integrate((a*c+(a*d+b*c)*x+b*d*x^2)^2/(b*x+a)^7,x, algorithm="fricas")
```

output
```
-1/12*(6*b^2*d^2*x^2 + 3*b^2*c^2 + 2*a*b*c*d + a^2*d^2 + 4*(2*b^2*c*d + a*
b*d^2)*x)/(b^7*x^4 + 4*a*b^6*x^3 + 6*a^2*b^5*x^2 + 4*a^3*b^4*x + a^4*b^3)
```

Sympy [A] (verification not implemented)

Time = 0.42 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.60

$$\int \frac{(ac+(bc+ad)x+bdx^2)^2}{(a+bx)^7}\,dx = \frac{-a^2d^2-2abcd-3b^2c^2-6b^2d^2x^2+x(-4abd^2-8b^2cd)}{12a^4b^3+48a^3b^4x+72a^2b^5x^2+48ab^6x^3+12b^7x^4}$$

input
```
integrate((a*c+(a*d+b*c)*x+b*d*x**2)**2/(b*x+a)**7,x)
```

output
```
(-a**2*d**2 - 2*a*b*c*d - 3*b**2*c**2 - 6*b**2*d**2*x**2 + x*(-4*a*b*d**2
- 8*b**2*c*d))/(12*a**4*b**3 + 48*a**3*b**4*x + 72*a**2*b**5*x**2 + 48*a*b
**6*x**3 + 12*b**7*x**4)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 98, normalized size of antiderivative = 1.51

$$\int \frac{(ac+(bc+ad)x+bdx^2)^2}{(a+bx)^7}\,dx = -\frac{6\,b^2d^2x^2+3\,b^2c^2+2\,abcd+a^2d^2+4\,(2\,b^2cd+abd^2)x}{12\,(b^7x^4+4\,ab^6x^3+6\,a^2b^5x^2+4\,a^3b^4x+a^4b^3)}$$

input
```
integrate((a*c+(a*d+b*c)*x+b*d*x^2)^2/(b*x+a)^7,x, algorithm="maxima")
```

output
```
-1/12*(6*b^2*d^2*x^2 + 3*b^2*c^2 + 2*a*b*c*d + a^2*d^2 + 4*(2*b^2*c*d + a*
b*d^2)*x)/(b^7*x^4 + 4*a*b^6*x^3 + 6*a^2*b^5*x^2 + 4*a^3*b^4*x + a^4*b^3)
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.94

$$\int \frac{(ac+(bc+ad)x+bdx^2)^2}{(a+bx)^7}\,dx = -\frac{6\,b^2d^2x^2+8\,b^2cdx+4\,abd^2x+3\,b^2c^2+2\,abcd+a^2d^2}{12\,(bx+a)^4b^3}$$

input
```
integrate((a*c+(a*d+b*c)*x+b*d*x^2)^2/(b*x+a)^7,x, algorithm="giac")
```

output
```
-1/12*(6*b^2*d^2*x^2 + 8*b^2*c*d*x + 4*a*b*d^2*x + 3*b^2*c^2 + 2*a*b*c*d +
 a^2*d^2)/((b*x + a)^4*b^3)
```

Mupad [B] (verification not implemented)

Time = 5.41 (sec) , antiderivative size = 96, normalized size of antiderivative = 1.48

$$\int \frac{(ac+(bc+ad)x+bdx^2)^2}{(a+bx)^7}\,dx = -\frac{\frac{a^2\,d^2+2\,a\,b\,c\,d+3\,b^2\,c^2}{12\,b^3}+\frac{d^2\,x^2}{2\,b}+\frac{d\,x\,(a\,d+2\,b\,c)}{3\,b^2}}{a^4+4\,a^3\,b\,x+6\,a^2\,b^2\,x^2+4\,a\,b^3\,x^3+b^4\,x^4}$$

input
```
int((a*c + x*(a*d + b*c) + b*d*x^2)^2/(a + b*x)^7,x)
```

output
```
-((a^2*d^2 + 3*b^2*c^2 + 2*a*b*c*d)/(12*b^3) + (d^2*x^2)/(2*b) + (d*x*(a*d
 + 2*b*c))/(3*b^2))/(a^4 + b^4*x^4 + 4*a*b^3*x^3 + 6*a^2*b^2*x^2 + 4*a^3*b
*x)
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.46

$$\int \frac{(ac+(bc+ad)x+bdx^2)^2}{(a+bx)^7}\,dx = \frac{-6b^2d^2x^2-4ab\,d^2x-8b^2cdx-a^2d^2-2abcd-3b^2c^2}{12b^3\,(b^4x^4+4a\,b^3x^3+6a^2b^2x^2+4a^3bx+a^4)}$$

input
```
int((a*c+(a*d+b*c)*x+b*d*x^2)^2/(b*x+a)^7,x)
```

output
```
( - a**2*d**2 - 2*a*b*c*d - 4*a*b*d**2*x - 3*b**2*c**2 - 8*b**2*c*d*x - 6*
b**2*d**2*x**2)/(12*b**3*(a**4 + 4*a**3*b*x + 6*a**2*b**2*x**2 + 4*a*b**3*
x**3 + b**4*x**4))
```

3.22 $\int \frac{(ac+(bc+ad)x+bdx^2)^2}{(a+bx)^8} dx$

Optimal result

Integrand size = 29, antiderivative size = 65

$$\int \frac{(ac+(bc+ad)x+bdx^2)^2}{(a+bx)^8} dx = -\frac{(bc-ad)^2}{5b^3(a+bx)^5} - \frac{d(bc-ad)}{2b^3(a+bx)^4} - \frac{d^2}{3b^3(a+bx)^3}$$

output
```
-1/5*(-a*d+b*c)^2/b^3/(b*x+a)^5-1/2*d*(-a*d+b*c)/b^3/(b*x+a)^4-1/3*d^2/b^3
/(b*x+a)^3
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.88

$$\int \frac{(ac+(bc+ad)x+bdx^2)^2}{(a+bx)^8} dx = -\frac{a^2d^2+abd(3c+5dx)+b^2(6c^2+15cdx+10d^2x^2)}{30b^3(a+bx)^5}$$

input
```
Integrate[(a*c + (b*c + a*d)*x + b*d*x^2)^2/(a + b*x)^8,x]
```

output
```
-1/30*(a^2*d^2 + a*b*d*(3*c + 5*d*x) + b^2*(6*c^2 + 15*c*d*x + 10*d^2*x^2)
)/(b^3*(a + b*x)^5)
```

Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.069, Rules used = {1121, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(x(ad+bc)+ac+bdx^2\right)^2}{(a+bx)^8}\,dx$$

$$\Big\downarrow \text{1121}$$

$$\int \left(\frac{2d(bc-ad)}{b^2(a+bx)^5}+\frac{(bc-ad)^2}{b^2(a+bx)^6}+\frac{d^2}{b^2(a+bx)^4}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{d(bc-ad)}{2b^3(a+bx)^4}-\frac{(bc-ad)^2}{5b^3(a+bx)^5}-\frac{d^2}{3b^3(a+bx)^3}$$

input
```
Int[(a*c + (b*c + a*d)*x + b*d*x^2)^2/(a + b*x)^8,x]
```

output
```
-1/5*(b*c - a*d)^2/(b^3*(a + b*x)^5) - (d*(b*c - a*d))/(2*b^3*(a + b*x)^4)
 - d^2/(3*b^3*(a + b*x)^3)
```

Defintions of rubi rules used

rule 1121
```
Int[((d_) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Int[ExpandIntegrand[(d + e*x)^(m + p)*(a/d + (c/e)*x)^p, x], x]
/; FreeQ[{a, b, c, d, e, m, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && (Int
egerQ[p] || (GtQ[a, 0] && GtQ[d, 0] && LtQ[c, 0]))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 1.22 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.95

method	result
gosper	$-\frac{10b^2d^2x^2+5ab\,d^2x+15b^2cxd+a^2d^2+3abcd+6b^2c^2}{30b^3(bx+a)^5}$
risch	$\frac{-\frac{d^2x^2}{3b}-\frac{d(ad+3bc)x}{6b^2}-\frac{a^2d^2+3abcd+6b^2c^2}{30b^3}}{(bx+a)^5}$
parallelrisch	$\frac{-10b^4d^2x^2-5a\,b^3d^2x-15b^4cdx-d^2a^2b^2-3cda\,b^3-6b^4c^2}{30b^5(bx+a)^5}$
default	$-\frac{a^2d^2-2abcd+b^2c^2}{5b^3(bx+a)^5}-\frac{d^2}{3b^3(bx+a)^3}+\frac{d(ad-bc)}{2b^3(bx+a)^4}$
orering	$-\frac{(10b^2d^2x^2+5ab\,d^2x+15b^2cxd+a^2d^2+3abcd+6b^2c^2)(ac+(ad+bc)x+bd\,x^2)^2}{30b^3(bx+a)^7(dx+c)^2}$
norman	$\frac{\frac{a^2(-a^2b^4d^2-3a\,b^5cd-6b^6c^2)}{30b^7}-\frac{b\,d^2x^4}{3}+\frac{(-5a\,b^4d^2-3b^5cd)x^3}{6b^4}+\frac{(-7a^2b^4d^2-11a\,b^5cd-2b^6c^2)x^2}{10b^5}+\frac{a(-7a^2b^4d^2-21a\,b^5cd-12b^6c^2)x}{30b^6}}{(bx+a)^7}$

input
```
int((a*c+(a*d+b*c)*x+b*d*x^2)^2/(b*x+a)^8,x,method=_RETURNVERBOSE)
```

output
```
-1/30/b^3*(10*b^2*d^2*x^2+5*a*b*d^2*x+15*b^2*c*d*x+a^2*d^2+3*a*b*c*d+6*b^2
*c^2)/(b*x+a)^5
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 109, normalized size of antiderivative = 1.68

$$\int \frac{(ac+(bc+ad)x+bdx^2)^2}{(a+bx)^8}\,dx = -\frac{10\,b^2d^2x^2+6\,b^2c^2+3\,abcd+a^2d^2+5\,(3\,b^2cd+abd^2)x}{30\,(b^8x^5+5\,ab^7x^4+10\,a^2b^6x^3+10\,a^3b^5x^2+5\,a^4b^4x+a^5b^3)}$$

input
```
integrate((a*c+(a*d+b*c)*x+b*d*x^2)^2/(b*x+a)^8,x, algorithm="fricas")
```

output
```
-1/30*(10*b^2*d^2*x^2 + 6*b^2*c^2 + 3*a*b*c*d + a^2*d^2 + 5*(3*b^2*c*d + a
*b*d^2)*x)/(b^8*x^5 + 5*a*b^7*x^4 + 10*a^2*b^6*x^3 + 10*a^3*b^5*x^2 + 5*a^
4*b^4*x + a^5*b^3)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 116 vs. 2(56) = 112.

Time = 0.50 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.78

$$\int \frac{(ac+(bc+ad)x+bdx^2)^2}{(a+bx)^8}\,dx = \frac{-a^2d^2-3abcd-6b^2c^2-10b^2d^2x^2+x(-5abd^2-15b^2cd)}{30a^5b^3+150a^4b^4x+300a^3b^5x^2+300a^2b^6x^3+150ab^7x^4+30b^8x^5}$$

input
```
integrate((a*c+(a*d+b*c)*x+b*d*x**2)**2/(b*x+a)**8,x)
```

output
```
(-a**2*d**2 - 3*a*b*c*d - 6*b**2*c**2 - 10*b**2*d**2*x**2 + x*(-5*a*b*d**2
 - 15*b**2*c*d))/(30*a**5*b**3 + 150*a**4*b**4*x + 300*a**3*b**5*x**2 + 30
0*a**2*b**6*x**3 + 150*a*b**7*x**4 + 30*b**8*x**5)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 109, normalized size of antiderivative = 1.68

$$\int \frac{(ac+(bc+ad)x+bdx^2)^2}{(a+bx)^8}\,dx = -\frac{10\,b^2d^2x^2+6\,b^2c^2+3\,abcd+a^2d^2+5\,(3\,b^2cd+abd^2)x}{30\,(b^8x^5+5\,ab^7x^4+10\,a^2b^6x^3+10\,a^3b^5x^2+5\,a^4b^4x+a^5b^3)}$$

input
```
integrate((a*c+(a*d+b*c)*x+b*d*x^2)^2/(b*x+a)^8,x, algorithm="maxima")
```

output
```
-1/30*(10*b^2*d^2*x^2 + 6*b^2*c^2 + 3*a*b*c*d + a^2*d^2 + 5*(3*b^2*c*d + a
*b*d^2)*x)/(b^8*x^5 + 5*a*b^7*x^4 + 10*a^2*b^6*x^3 + 10*a^3*b^5*x^2 + 5*a^
4*b^4*x + a^5*b^3)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.94

$$\int \frac{(ac+(bc+ad)x+bdx^2)^2}{(a+bx)^8}\,dx$$
$$= -\frac{10\,b^2d^2x^2+15\,b^2cdx+5\,abd^2x+6\,b^2c^2+3\,abcd+a^2d^2}{30\,(bx+a)^5b^3}$$

input
```
integrate((a*c+(a*d+b*c)*x+b*d*x^2)^2/(b*x+a)^8,x, algorithm="giac")
```

output
```
-1/30*(10*b^2*d^2*x^2 + 15*b^2*c*d*x + 5*a*b*d^2*x + 6*b^2*c^2 + 3*a*b*c*d
 + a^2*d^2)/((b*x + a)^5*b^3)
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.65

$$\int \frac{(ac+(bc+ad)x+bdx^2)^2}{(a+bx)^8}\,dx$$
$$= -\frac{\frac{a^2\,d^2+3\,a\,b\,c\,d+6\,b^2\,c^2}{30\,b^3}+\frac{d^2\,x^2}{3\,b}+\frac{d\,x\,(a\,d+3\,b\,c)}{6\,b^2}}{a^5+5\,a^4\,b\,x+10\,a^3\,b^2\,x^2+10\,a^2\,b^3\,x^3+5\,a\,b^4\,x^4+b^5\,x^5}$$

input
```
int((a*c + x*(a*d + b*c) + b*d*x^2)^2/(a + b*x)^8,x)
```

output
```
-((a^2*d^2 + 6*b^2*c^2 + 3*a*b*c*d)/(30*b^3) + (d^2*x^2)/(3*b) + (d*x*(a*d
 + 3*b*c))/(6*b^2))/(a^5 + b^5*x^5 + 5*a*b^4*x^4 + 10*a^3*b^2*x^2 + 10*a^2
*b^3*x^3 + 5*a^4*b*x)
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 106, normalized size of antiderivative = 1.63

$$\int \frac{(ac+(bc+ad)x+bdx^2)^2}{(a+bx)^8}\,dx = \frac{-10b^2d^2x^2-5ab\,d^2x-15b^2cdx-a^2d^2-3abcd-6b^2c^2}{30b^3\left(b^5x^5+5a\,b^4x^4+10a^2b^3x^3+10a^3b^2x^2+5a^4bx+a^5\right)}$$

input
```
int((a*c+(a*d+b*c)*x+b*d*x^2)^2/(b*x+a)^8,x)
```

output
```
( - a**2*d**2 - 3*a*b*c*d - 5*a*b*d**2*x - 6*b**2*c**2 - 15*b**2*c*d*x - 1
0*b**2*d**2*x**2)/(30*b**3*(a**5 + 5*a**4*b*x + 10*a**3*b**2*x**2 + 10*a**
2*b**3*x**3 + 5*a*b**4*x**4 + b**5*x**5))
```

3.23 $\int \frac{(ac+(bc+ad)x+bdx^2)^2}{(a+bx)^9}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 65

$$\int \frac{(ac+(bc+ad)x+bdx^2)^2}{(a+bx)^9}\,dx = -\frac{(bc-ad)^2}{6b^3(a+bx)^6} - \frac{2d(bc-ad)}{5b^3(a+bx)^5} - \frac{d^2}{4b^3(a+bx)^4}$$

output
```
-1/6*(-a*d+b*c)^2/b^3/(b*x+a)^6-2/5*d*(-a*d+b*c)/b^3/(b*x+a)^5-1/4*d^2/b^3
/(b*x+a)^4
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.89

$$\int \frac{(ac+(bc+ad)x+bdx^2)^2}{(a+bx)^9}\,dx = -\frac{a^2d^2+2abd(2c+3dx)+b^2(10c^2+24cdx+15d^2x^2)}{60b^3(a+bx)^6}$$

input
```
Integrate[(a*c + (b*c + a*d)*x + b*d*x^2)^2/(a + b*x)^9,x]
```

output
```
-1/60*(a^2*d^2 + 2*a*b*d*(2*c + 3*d*x) + b^2*(10*c^2 + 24*c*d*x + 15*d^2*x
^2))/(b^3*(a + b*x)^6)
```

Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.069, Rules used = {1121, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(x(ad+bc)+ac+bdx^2)^2}{(a+bx)^9}\,dx$$

$$\Big\downarrow \text{1121}$$

$$\int \left(\frac{2d(bc-ad)}{b^2(a+bx)^6}+\frac{(bc-ad)^2}{b^2(a+bx)^7}+\frac{d^2}{b^2(a+bx)^5}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{2d(bc-ad)}{5b^3(a+bx)^5}-\frac{(bc-ad)^2}{6b^3(a+bx)^6}-\frac{d^2}{4b^3(a+bx)^4}$$

input
```
Int[(a*c + (b*c + a*d)*x + b*d*x^2)^2/(a + b*x)^9,x]
```

output
```
-1/6*(b*c - a*d)^2/(b^3*(a + b*x)^6) - (2*d*(b*c - a*d))/(5*b^3*(a + b*x)^
5) - d^2/(4*b^3*(a + b*x)^4)
```

Defintions of rubi rules used

rule 1121
```
Int[((d_) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Int[ExpandIntegrand[(d + e*x)^(m + p)*(a/d + (c/e)*x)^p, x], x]
/; FreeQ[{a, b, c, d, e, m, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && (Int
egerQ[p] || (GtQ[a, 0] && GtQ[d, 0] && LtQ[c, 0]))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 1.22 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.95

method	result
gosper	$-\frac{15b^2d^2x^2+6ab\,d^2x+24b^2cxd+a^2d^2+4abcd+10b^2c^2}{60b^3(bx+a)^6}$
risch	$\frac{-\frac{d^2x^2}{4b}-\frac{d(ad+4bc)x}{10b^2}-\frac{a^2d^2+4abcd+10b^2c^2}{60b^3}}{(bx+a)^6}$
parallelrisch	$\frac{-15d^2x^2b^5-6a\,b^4d^2x-24b^5cdx-d^2a^2b^3-4cda\,b^4-10b^5c^2}{60b^6(bx+a)^6}$
default	$\frac{2d(ad-bc)}{5b^3(bx+a)^5}-\frac{a^2d^2-2abcd+b^2c^2}{6b^3(bx+a)^6}-\frac{d^2}{4b^3(bx+a)^4}$
orering	$-\frac{(15b^2d^2x^2+6ab\,d^2x+24b^2cxd+a^2d^2+4abcd+10b^2c^2)(ac+(ad+bc)x+bd\,x^2)^2}{60b^3(bx+a)^8(dx+c)^2}$
norman	$\frac{\frac{a^2(-a^2b^5d^2-4a\,b^6cd-10b^7c^2)}{60b^8}-\frac{b\,d^2x^4}{4}+\frac{(-3a\,b^5d^2-2cd\,b^6)x^3}{5b^5}+\frac{(-14a^2b^5d^2-26a\,b^6cd-5b^7c^2)x^2}{30b^6}+\frac{a(-2a^2b^5d^2-8a\,b^6cd-5b^7c^2)}{15b^7}}{(bx+a)^8}$

input
```
int((a*c+(a*d+b*c)*x+b*d*x^2)^2/(b*x+a)^9,x,method=_RETURNVERBOSE)
```

output
```
-1/60/b^3*(15*b^2*d^2*x^2+6*a*b*d^2*x+24*b^2*c*d*x+a^2*d^2+4*a*b*c*d+10*b^
2*c^2)/(b*x+a)^6
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 120 vs. 2(59) = 118.

Time = 0.07 (sec) , antiderivative size = 120, normalized size of antiderivative = 1.85

$$\int \frac{(ac+(bc+ad)x+bdx^2)^2}{(a+bx)^9}\,dx = -\frac{15\,b^2d^2x^2+10\,b^2c^2+4\,abcd+a^2d^2+6\,(4\,b^2cd+abd^2)x}{60\,(b^9x^6+6\,ab^8x^5+15\,a^2b^7x^4+20\,a^3b^6x^3+15\,a^4b^5x^2+6\,a^5b^4x+a^6b^3)}$$

input
```
integrate((a*c+(a*d+b*c)*x+b*d*x^2)^2/(b*x+a)^9,x, algorithm="fricas")
```

output
```
-1/60*(15*b^2*d^2*x^2 + 10*b^2*c^2 + 4*a*b*c*d + a^2*d^2 + 6*(4*b^2*c*d +
a*b*d^2)*x)/(b^9*x^6 + 6*a*b^8*x^5 + 15*a^2*b^7*x^4 + 20*a^3*b^6*x^3 + 15*
a^4*b^5*x^2 + 6*a^5*b^4*x + a^6*b^3)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 128 vs. $2(58) = 116$.

Time = 0.64 (sec) , antiderivative size = 128, normalized size of antiderivative = 1.97

$$\int \frac{(ac+(bc+ad)x+bdx^2)^2}{(a+bx)^9}\,dx = \frac{-a^2d^2-4abcd-10b^2c^2-15b^2d^2x^2+x(-6abd^2-24b^2cd)}{60a^6b^3+360a^5b^4x+900a^4b^5x^2+1200a^3b^6x^3+900a^2b^7x^4+360ab^8x^5+60b^9x^6}$$

input
```
integrate((a*c+(a*d+b*c)*x+b*d*x**2)**2/(b*x+a)**9,x)
```

output
```
(-a**2*d**2 - 4*a*b*c*d - 10*b**2*c**2 - 15*b**2*d**2*x**2 + x*(-6*a*b*d**
2 - 24*b**2*c*d))/(60*a**6*b**3 + 360*a**5*b**4*x + 900*a**4*b**5*x**2 + 1
200*a**3*b**6*x**3 + 900*a**2*b**7*x**4 + 360*a*b**8*x**5 + 60*b**9*x**6)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 120 vs. $2(59) = 118$.

Time = 0.03 (sec) , antiderivative size = 120, normalized size of antiderivative = 1.85

$$\int \frac{(ac+(bc+ad)x+bdx^2)^2}{(a+bx)^9}\,dx = -\frac{15\,b^2d^2x^2+10\,b^2c^2+4\,abcd+a^2d^2+6\,(4\,b^2cd+abd^2)x}{60\,(b^9x^6+6\,ab^8x^5+15\,a^2b^7x^4+20\,a^3b^6x^3+15\,a^4b^5x^2+6\,a^5b^4x+a^6b^3)}$$

input
```
integrate((a*c+(a*d+b*c)*x+b*d*x^2)^2/(b*x+a)^9,x, algorithm="maxima")
```

output
```
-1/60*(15*b^2*d^2*x^2 + 10*b^2*c^2 + 4*a*b*c*d + a^2*d^2 + 6*(4*b^2*c*d +
a*b*d^2)*x)/(b^9*x^6 + 6*a*b^8*x^5 + 15*a^2*b^7*x^4 + 20*a^3*b^6*x^3 + 15*
a^4*b^5*x^2 + 6*a^5*b^4*x + a^6*b^3)
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.94

$$\int \frac{(ac+(bc+ad)x+bdx^2)^2}{(a+bx)^9}\,dx = -\frac{15\,b^2d^2x^2+24\,b^2cdx+6\,abd^2x+10\,b^2c^2+4\,abcd+a^2d^2}{60\,(bx+a)^6b^3}$$

input
```
integrate((a*c+(a*d+b*c)*x+b*d*x^2)^2/(b*x+a)^9,x, algorithm="giac")
```

output
```
-1/60*(15*b^2*d^2*x^2 + 24*b^2*c*d*x + 6*a*b*d^2*x + 10*b^2*c^2 + 4*a*b*c*
d + a^2*d^2)/((b*x + a)^6*b^3)
```

Mupad [B] (verification not implemented)

Time = 5.95 (sec) , antiderivative size = 118, normalized size of antiderivative = 1.82

$$\int \frac{(ac+(bc+ad)x+bdx^2)^2}{(a+bx)^9}\,dx = -\frac{\frac{a^2\,d^2+4\,a\,b\,c\,d+10\,b^2\,c^2}{60\,b^3}+\frac{d^2\,x^2}{4\,b}+\frac{d\,x\,(a\,d+4\,b\,c)}{10\,b^2}}{a^6+6\,a^5\,b\,x+15\,a^4\,b^2\,x^2+20\,a^3\,b^3\,x^3+15\,a^2\,b^4\,x^4+6\,a\,b^5\,x^5+b^6\,x^6}$$

input
```
int((a*c + x*(a*d + b*c) + b*d*x^2)^2/(a + b*x)^9,x)
```

output
```
-((a^2*d^2 + 10*b^2*c^2 + 4*a*b*c*d)/(60*b^3) + (d^2*x^2)/(4*b) + (d*x*(a*
d + 4*b*c))/(10*b^2))/(a^6 + b^6*x^6 + 6*a*b^5*x^5 + 15*a^4*b^2*x^2 + 20*a
^3*b^3*x^3 + 15*a^2*b^4*x^4 + 6*a^5*b*x)
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.80

$$\int \frac{(ac+(bc+ad)x+bdx^2)^2}{(a+bx)^9}\,dx = \frac{-15b^2d^2x^2-6ab\,d^2x-24b^2cdx-a^2d^2-4abcd-10b^2c^2}{60b^3\left(b^6x^6+6a\,b^5x^5+15a^2b^4x^4+20a^3b^3x^3+15a^4b^2x^2+6a^5bx+a^6\right)}$$

input
```
int((a*c+(a*d+b*c)*x+b*d*x^2)^2/(b*x+a)^9,x)
```

output
```
( - a**2*d**2 - 4*a*b*c*d - 6*a*b*d**2*x - 10*b**2*c**2 - 24*b**2*c*d*x -
15*b**2*d**2*x**2)/(60*b**3*(a**6 + 6*a**5*b*x + 15*a**4*b**2*x**2 + 20*a*
*3*b**3*x**3 + 15*a**2*b**4*x**4 + 6*a*b**5*x**5 + b**6*x**6))
```

3.24 $\int \frac{(ac+(bc+ad)x+bdx^2)^2}{(a+bx)^{10}}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 65

$$\int \frac{(ac+(bc+ad)x+bdx^2)^2}{(a+bx)^{10}}\,dx=-\frac{(bc-ad)^2}{7b^3(a+bx)^7}-\frac{d(bc-ad)}{3b^3(a+bx)^6}-\frac{d^2}{5b^3(a+bx)^5}$$

output
```
-1/7*(-a*d+b*c)^2/b^3/(b*x+a)^7-1/3*d*(-a*d+b*c)/b^3/(b*x+a)^6-1/5*d^2/b^3
/(b*x+a)^5
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.88

$$\int \frac{(ac+(bc+ad)x+bdx^2)^2}{(a+bx)^{10}}\,dx=-\frac{a^2d^2+abd(5c+7dx)+b^2(15c^2+35cdx+21d^2x^2)}{105b^3(a+bx)^7}$$

input
```
Integrate[(a*c + (b*c + a*d)*x + b*d*x^2)^2/(a + b*x)^10,x]
```

output
```
-1/105*(a^2*d^2 + a*b*d*(5*c + 7*d*x) + b^2*(15*c^2 + 35*c*d*x + 21*d^2*x^
2))/(b^3*(a + b*x)^7)
```

Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.069$, Rules used = {1121, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(x(ad+bc)+ac+bdx^2\right)^2}{(a+bx)^{10}}\,dx$$

$$\Big\downarrow \text{1121}$$

$$\int \left(\frac{2d(bc-ad)}{b^2(a+bx)^7}+\frac{(bc-ad)^2}{b^2(a+bx)^8}+\frac{d^2}{b^2(a+bx)^6}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{d(bc-ad)}{3b^3(a+bx)^6}-\frac{(bc-ad)^2}{7b^3(a+bx)^7}-\frac{d^2}{5b^3(a+bx)^5}$$

input
```
Int[(a*c + (b*c + a*d)*x + b*d*x^2)^2/(a + b*x)^10,x]
```

output
```
-1/7*(b*c - a*d)^2/(b^3*(a + b*x)^7) - (d*(b*c - a*d))/(3*b^3*(a + b*x)^6)
 - d^2/(5*b^3*(a + b*x)^5)
```

Defintions of rubi rules used

rule 1121
```
Int[((d_) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Int[ExpandIntegrand[(d + e*x)^(m + p)*(a/d + (c/e)*x)^p, x], x]
/; FreeQ[{a, b, c, d, e, m, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && (Int
egerQ[p] || (GtQ[a, 0] && GtQ[d, 0] && LtQ[c, 0]))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 1.22 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.95

method	result
gosper	$-\frac{21b^2d^2x^2+7ab\,d^2x+35b^2cxd+a^2d^2+5abcd+15b^2c^2}{105b^3(bx+a)^7}$
risch	$\frac{-\frac{d^2x^2}{5b}-\frac{d(ad+5bc)x}{15b^2}-\frac{a^2d^2+5abcd+15b^2c^2}{105b^3}}{(bx+a)^7}$
parallelrisch	$\frac{-21d^2x^2b^6-7a\,b^5d^2x-35b^6cdx-a^2b^4d^2-5a\,b^5cd-15b^6c^2}{105b^7(bx+a)^7}$
default	$-\frac{d^2}{5b^3(bx+a)^5}-\frac{a^2d^2-2abcd+b^2c^2}{7b^3(bx+a)^7}+\frac{d(ad-bc)}{3b^3(bx+a)^6}$
orering	$-\frac{(21b^2d^2x^2+7ab\,d^2x+35b^2cxd+a^2d^2+5abcd+15b^2c^2)(ac+(ad+bc)x+bd\,x^2)^2}{105b^3(bx+a)^9(dx+c)^2}$
norman	$\frac{\frac{a^2(-a^2b^6d^2-5a\,b^7cd-15b^8c^2)}{105b^9}-\frac{b\,d^2x^4}{5}+\frac{(-7a\,b^6d^2-5cd\,b^7)x^3}{15b^6}+\frac{(-12a^2b^6d^2-25a\,b^7cd-5b^8c^2)x^2}{35b^7}+\frac{a(-3a^2b^6d^2-15a\,b^7cd-10b^8c^2}{35b^8}}{(bx+a)^9}$

input
```
int((a*c+(a*d+b*c)*x+b*d*x^2)^2/(b*x+a)^10,x,method=_RETURNVERBOSE)
```

output
```
-1/105/b^3*(21*b^2*d^2*x^2+7*a*b*d^2*x+35*b^2*c*d*x+a^2*d^2+5*a*b*c*d+15*b
^2*c^2)/(b*x+a)^7
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 131 vs. 2(59) = 118.

Time = 0.07 (sec) , antiderivative size = 131, normalized size of antiderivative = 2.02

$$\int \frac{(ac+(bc+ad)x+bdx^2)^2}{(a+bx)^{10}}\,dx = -\frac{21\,b^2d^2x^2+15\,b^2c^2+5\,abcd+a^2d^2+7\,(5\,b^2cd+abd^2)x}{105\,(b^{10}x^7+7\,ab^9x^6+21\,a^2b^8x^5+35\,a^3b^7x^4+35\,a^4b^6x^3+21\,a^5b^5x^2+7\,a^6b^4x+a^7b^3)}$$

input
```
integrate((a*c+(a*d+b*c)*x+b*d*x^2)^2/(b*x+a)^10,x, algorithm="fricas")
```

output
```
-1/105*(21*b^2*d^2*x^2 + 15*b^2*c^2 + 5*a*b*c*d + a^2*d^2 + 7*(5*b^2*c*d +
 a*b*d^2)*x)/(b^10*x^7 + 7*a*b^9*x^6 + 21*a^2*b^8*x^5 + 35*a^3*b^7*x^4 + 3
5*a^4*b^6*x^3 + 21*a^5*b^5*x^2 + 7*a^6*b^4*x + a^7*b^3)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 139 vs. $2(56) = 112$.

Time = 0.87 (sec) , antiderivative size = 139, normalized size of antiderivative = 2.14

$$\int \frac{(ac+(bc+ad)x+bdx^2)^2}{(a+bx)^{10}}\,dx = \frac{-a^2d^2-5abcd-15b^2c^2-21b^2d^2x^2+x(-7abd^2-35b^2cd)}{105a^7b^3+735a^6b^4x+2205a^5b^5x^2+3675a^4b^6x^3+3675a^3b^7x^4+2205a^2b^8x^5+735ab^9x^6+105b^{10}x^7}$$

input
```
integrate((a*c+(a*d+b*c)*x+b*d*x**2)**2/(b*x+a)**10,x)
```

output
```
(-a**2*d**2 - 5*a*b*c*d - 15*b**2*c**2 - 21*b**2*d**2*x**2 + x*(-7*a*b*d**
2 - 35*b**2*c*d))/(105*a**7*b**3 + 735*a**6*b**4*x + 2205*a**5*b**5*x**2 +
 3675*a**4*b**6*x**3 + 3675*a**3*b**7*x**4 + 2205*a**2*b**8*x**5 + 735*a*b
**9*x**6 + 105*b**10*x**7)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 131 vs. $2(59) = 118$.

Time = 0.04 (sec) , antiderivative size = 131, normalized size of antiderivative = 2.02

$$\int \frac{(ac+(bc+ad)x+bdx^2)^2}{(a+bx)^{10}}\,dx = -\frac{21\,b^2d^2x^2+15\,b^2c^2+5\,abcd+a^2d^2+7\,(5\,b^2cd+abd^2)x}{105\,(b^{10}x^7+7\,ab^9x^6+21\,a^2b^8x^5+35\,a^3b^7x^4+35\,a^4b^6x^3+21\,a^5b^5x^2+7\,a^6b^4x+a^7b^3)}$$

input
```
integrate((a*c+(a*d+b*c)*x+b*d*x^2)^2/(b*x+a)^10,x, algorithm="maxima")
```

output
```
-1/105*(21*b^2*d^2*x^2 + 15*b^2*c^2 + 5*a*b*c*d + a^2*d^2 + 7*(5*b^2*c*d +
 a*b*d^2)*x)/(b^10*x^7 + 7*a*b^9*x^6 + 21*a^2*b^8*x^5 + 35*a^3*b^7*x^4 + 3
5*a^4*b^6*x^3 + 21*a^5*b^5*x^2 + 7*a^6*b^4*x + a^7*b^3)
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.94

$$\int \frac{(ac + (bc + ad)x + bdx^2)^2}{(a + bx)^{10}}\,dx = -\frac{21\,b^2d^2x^2 + 35\,b^2cdx + 7\,abd^2x + 15\,b^2c^2 + 5\,abcd + a^2d^2}{105\,(bx + a)^7b^3}$$

input
```
integrate((a*c+(a*d+b*c)*x+b*d*x^2)^2/(b*x+a)^10,x, algorithm="giac")
```

output
```
-1/105*(21*b^2*d^2*x^2 + 35*b^2*c*d*x + 7*a*b*d^2*x + 15*b^2*c^2 + 5*a*b*c
*d + a^2*d^2)/((b*x + a)^7*b^3)
```

Mupad [B] (verification not implemented)

Time = 5.83 (sec) , antiderivative size = 129, normalized size of antiderivative = 1.98

$$\int \frac{(ac + (bc + ad)x + bdx^2)^2}{(a + bx)^{10}}\,dx = -\frac{\frac{a^2\,d^2+5\,a\,b\,c\,d+15\,b^2\,c^2}{105\,b^3} + \frac{d^2\,x^2}{5\,b} + \frac{d\,x\,(a\,d+5\,b\,c)}{15\,b^2}}{a^7 + 7\,a^6\,b\,x + 21\,a^5\,b^2\,x^2 + 35\,a^4\,b^3\,x^3 + 35\,a^3\,b^4\,x^4 + 21\,a^2\,b^5\,x^5 + 7\,a\,b^6\,x^6 + b^7\,x^7}$$

input
```
int((a*c + x*(a*d + b*c) + b*d*x^2)^2/(a + b*x)^10,x)
```

output
```
-((a^2*d^2 + 15*b^2*c^2 + 5*a*b*c*d)/(105*b^3) + (d^2*x^2)/(5*b) + (d*x*(a
*d + 5*b*c))/(15*b^2))/(a^7 + b^7*x^7 + 7*a*b^6*x^6 + 21*a^5*b^2*x^2 + 35*
a^4*b^3*x^3 + 35*a^3*b^4*x^4 + 21*a^2*b^5*x^5 + 7*a^6*b*x)
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 128, normalized size of antiderivative = 1.97

$$\int \frac{(ac+(bc+ad)x+bdx^2)^2}{(a+bx)^{10}}\,dx = \frac{-21b^2d^2x^2-7ab\,d^2x-35b^2cdx-a^2d^2-5abcd-15b^2c^2}{105b^3\left(b^7x^7+7a\,b^6x^6+21a^2b^5x^5+35a^3b^4x^4+35a^4b^3x^3+21a^5b^2x^2+7a^6bx+a^7\right)}$$

input
```
int((a*c+(a*d+b*c)*x+b*d*x^2)^2/(b*x+a)^10,x)
```

output
```
( - a**2*d**2 - 5*a*b*c*d - 7*a*b*d**2*x - 15*b**2*c**2 - 35*b**2*c*d*x -
21*b**2*d**2*x**2)/(105*b**3*(a**7 + 7*a**6*b*x + 21*a**5*b**2*x**2 + 35*a
**4*b**3*x**3 + 35*a**3*b**4*x**4 + 21*a**2*b**5*x**5 + 7*a*b**6*x**6 + b*
*7*x**7))
```

3.25 $\int (a+bx)^3 \left(ac+(bc+ad)x+bdx^2\right)^3 \, dx$

Optimal result

Integrand size = 29, antiderivative size = 92

$$\int (a+bx)^3 \left(ac+(bc+ad)x+bdx^2\right)^3 \, dx = \frac{(bc-ad)^3(a+bx)^7}{7b^4}+\frac{3d(bc-ad)^2(a+bx)^8}{8b^4} + \frac{d^2(bc-ad)(a+bx)^9}{3b^4}+\frac{d^3(a+bx)^{10}}{10b^4}$$

output
```
1/7*(-a*d+b*c)^3*(b*x+a)^7/b^4+3/8*d*(-a*d+b*c)^2*(b*x+a)^8/b^4+1/3*d^2*(-
a*d+b*c)*(b*x+a)^9/b^4+1/10*d^3*(b*x+a)^10/b^4
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 276 vs. $2(92) = 184$.

Time = 0.08 (sec) , antiderivative size = 276, normalized size of antiderivative = 3.00

$$\int (a+bx)^3 \left(ac+(bc+ad)x+bdx^2\right)^3 \, dx = \frac{1}{840}x\big(210a^6\left(4c^3+6c^2dx+4cd^2x^2+d^3x^3\right) + 252a^5bx\left(10c^3+20c^2dx+15cd^2x^2 + 4d^3x^3\right) + 210a^4b^2x^2\left(20c^3+45c^2dx + 36cd^2x^2+10d^3x^3\right) + 120a^3b^3x^3\left(35c^3 + 84c^2dx+70cd^2x^2+20d^3x^3\right) + 45a^2b^4x^4\left(56c^3+140c^2dx+120cd^2x^2 + 35d^3x^3\right) + 10ab^5x^5\left(84c^3+216c^2dx + 189cd^2x^2+56d^3x^3\right) + b^6x^6\left(120c^3 + 315c^2dx+280cd^2x^2+84d^3x^3\right)\big)$$

input
```
Integrate[(a + b*x)^3*(a*c + (b*c + a*d)*x + b*d*x^2)^3,x]
```

output
```
(x*(210*a^6*(4*c^3 + 6*c^2*d*x + 4*c*d^2*x^2 + d^3*x^3) + 252*a^5*b*x*(10*
c^3 + 20*c^2*d*x + 15*c*d^2*x^2 + 4*d^3*x^3) + 210*a^4*b^2*x^2*(20*c^3 + 4
5*c^2*d*x + 36*c*d^2*x^2 + 10*d^3*x^3) + 120*a^3*b^3*x^3*(35*c^3 + 84*c^2*
d*x + 70*c*d^2*x^2 + 20*d^3*x^3) + 45*a^2*b^4*x^4*(56*c^3 + 140*c^2*d*x +
120*c*d^2*x^2 + 35*d^3*x^3) + 10*a*b^5*x^5*(84*c^3 + 216*c^2*d*x + 189*c*d
^2*x^2 + 56*d^3*x^3) + b^6*x^6*(120*c^3 + 315*c^2*d*x + 280*c*d^2*x^2 + 84
*d^3*x^3)))/840
```

Rubi [A] (verified)

Time = 0.62 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.069$, Rules used = {1121, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+bx)^3 \left(x(ad+bc)+ac+bdx^2\right)^3 \, dx$$

$$\Big\downarrow \text{ 1121}$$

$$\int \left(\frac{3d^2(a+bx)^8(bc-ad)}{b^3}+\frac{3d(a+bx)^7(bc-ad)^2}{b^3}+\frac{(a+bx)^6(bc-ad)^3}{b^3}+\frac{d^3(a+bx)^9}{b^3}\right) dx$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{d^2(a+bx)^9(bc-ad)}{3b^4}+\frac{3d(a+bx)^8(bc-ad)^2}{8b^4}+\frac{(a+bx)^7(bc-ad)^3}{7b^4}+\frac{d^3(a+bx)^{10}}{10b^4}$$

input
```
Int[(a + b*x)^3*(a*c + (b*c + a*d)*x + b*d*x^2)^3,x]
```

output
```
((b*c - a*d)^3*(a + b*x)^7)/(7*b^4) + (3*d*(b*c - a*d)^2*(a + b*x)^8)/(8*b
^4) + (d^2*(b*c - a*d)*(a + b*x)^9)/(3*b^4) + (d^3*(a + b*x)^10)/(10*b^4)
```

Defintions of rubi rules used

rule 1121
```
Int[((d_) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Int[ExpandIntegrand[(d + e*x)^(m + p)*(a/d + (c/e)*x)^p, x], x]
/; FreeQ[{a, b, c, d, e, m, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && (Int
egerQ[p] || (GtQ[a, 0] && GtQ[d, 0] && LtQ[c, 0]))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 324 vs. $2(84) = 168$.

Time = 1.19 (sec) , antiderivative size = 325, normalized size of antiderivative = 3.53

method	result
norman	$\frac{b^6 d^3 x^{10}}{10} + \left(\frac{2}{3} d^3 a\, b^5 + \frac{1}{3} c\, d^2 b^6\right) x^9 + \left(\frac{15}{8} d^3 a^2 b^4 + \frac{9}{4} c\, d^2 a\, b^5 + \frac{3}{8} c^2 d\, b^6\right) x^8 + \left(\frac{20}{7} d^3 a^3 b^3 + \frac{45}{7} a^2 b^4$
risch	$\frac{5}{2} a^4 b^2 d^3 x^6 + \frac{15}{8} a^2 b^4 d^3 x^8 + \frac{1}{3} b^6 c\, d^2 x^9 + \frac{3}{2} a^6 c^2 d\, x^2 + 6 a^5 b\, c^2 d\, x^3 + \frac{9}{4} x^8 c\, d^2 a\, b^5 + \frac{45}{7} x^7 a^2 b^4 c\, d^2$
parallelrisch	$\frac{5}{2} a^4 b^2 d^3 x^6 + \frac{15}{8} a^2 b^4 d^3 x^8 + \frac{1}{3} b^6 c\, d^2 x^9 + \frac{3}{2} a^6 c^2 d\, x^2 + 6 a^5 b\, c^2 d\, x^3 + \frac{9}{4} x^8 c\, d^2 a\, b^5 + \frac{45}{7} x^7 a^2 b^4 c\, d^2$
gosper	$\frac{x(84 b^6 d^3 x^9 + 560 x^8 d^3 a\, b^5 + 280 x^8 c\, d^2 b^6 + 1575 x^7 d^3 a^2 b^4 + 1890 x^7 c\, d^2 a\, b^5 + 315 x^7 c^2 d\, b^6 + 2400 x^6 d^3 a^3 b^3 + 5400 x^6 a^2 b^4 c\, d^2 + 21}{}$
orering	$\frac{x(84 b^6 d^3 x^9 + 560 x^8 d^3 a\, b^5 + 280 x^8 c\, d^2 b^6 + 1575 x^7 d^3 a^2 b^4 + 1890 x^7 c\, d^2 a\, b^5 + 315 x^7 c^2 d\, b^6 + 2400 x^6 d^3 a^3 b^3 + 5400 x^6 a^2 b^4 c\, d^2 + 21}{}$
default	$\frac{b^6 d^3 x^{10}}{10} + \frac{(3 d^3 a\, b^5 + 3 b^5 (ad+bc) d^2) x^9}{9} + \frac{\left(3 d^3 a^2 b^4 + 9 a\, b^4 (ad+bc) d^2 + b^3 \left(a\, b^2 c\, d^2 + 2(ad+bc)^2 bd + bd\left(2abcd + (ad+bc)^2\right)\right.\right.}{8}$

input
```
int((b*x+a)^3*(a*c+(a*d+b*c)*x+b*d*x^2)^3,x,method=_RETURNVERBOSE)
```

output
```
1/10*b^6*d^3*x^10+(2/3*d^3*a*b^5+1/3*c*d^2*b^6)*x^9+(15/8*d^3*a^2*b^4+9/4*
c*d^2*a*b^5+3/8*c^2*d*b^6)*x^8+(20/7*d^3*a^3*b^3+45/7*a^2*b^4*c*d^2+18/7*a
*b^5*c^2*d+1/7*b^6*c^3)*x^7+(5/2*d^3*a^4*b^2+10*c*d^2*a^3*b^3+15/2*c^2*d*a
^2*b^4+c^3*a*b^5)*x^6+(6/5*a^5*b*d^3+9*c*d^2*a^4*b^2+12*c^2*d*a^3*b^3+3*c^
3*a^2*b^4)*x^5+(1/4*d^3*a^6+9/2*c*d^2*a^5*b+45/4*c^2*d*a^4*b^2+5*a^3*b^3*c
^3)*x^4+(a^6*c*d^2+6*a^5*b*c^2*d+5*a^4*b^2*c^3)*x^3+(3/2*c^2*d*a^6+3*c^3*a
^5*b)*x^2+a^6*c^3*x
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 327 vs. $2(84) = 168$.

Time = 0.08 (sec) , antiderivative size = 327, normalized size of antiderivative = 3.55

$$\int (a+bx)^3 \left(ac+(bc+ad)x+bdx^2\right)^3 \, dx$$
$$= \frac{1}{10}b^6d^3x^{10} + a^6c^3x + \frac{1}{3}\left(b^6cd^2 + 2\,ab^5d^3\right)x^9 + \frac{3}{8}\left(b^6c^2d + 6\,ab^5cd^2 + 5\,a^2b^4d^3\right)x^8$$
$$+ \frac{1}{7}\left(b^6c^3 + 18\,ab^5c^2d + 45\,a^2b^4cd^2 + 20\,a^3b^3d^3\right)x^7$$
$$+ \frac{1}{2}\left(2\,ab^5c^3 + 15\,a^2b^4c^2d + 20\,a^3b^3cd^2 + 5\,a^4b^2d^3\right)x^6$$
$$+ \frac{3}{5}\left(5\,a^2b^4c^3 + 20\,a^3b^3c^2d + 15\,a^4b^2cd^2 + 2\,a^5bd^3\right)x^5$$
$$+ \frac{1}{4}\left(20\,a^3b^3c^3 + 45\,a^4b^2c^2d + 18\,a^5bcd^2 + a^6d^3\right)x^4$$
$$+ \left(5\,a^4b^2c^3 + 6\,a^5bc^2d + a^6cd^2\right)x^3 + \frac{3}{2}\left(2\,a^5bc^3 + a^6c^2d\right)x^2$$

input
```
integrate((b*x+a)^3*(a*c+(a*d+b*c)*x+b*d*x^2)^3,x, algorithm="fricas")
```

output
```
1/10*b^6*d^3*x^10 + a^6*c^3*x + 1/3*(b^6*c*d^2 + 2*a*b^5*d^3)*x^9 + 3/8*(b
^6*c^2*d + 6*a*b^5*c*d^2 + 5*a^2*b^4*d^3)*x^8 + 1/7*(b^6*c^3 + 18*a*b^5*c^
2*d + 45*a^2*b^4*c*d^2 + 20*a^3*b^3*d^3)*x^7 + 1/2*(2*a*b^5*c^3 + 15*a^2*b
^4*c^2*d + 20*a^3*b^3*c*d^2 + 5*a^4*b^2*d^3)*x^6 + 3/5*(5*a^2*b^4*c^3 + 20
*a^3*b^3*c^2*d + 15*a^4*b^2*c*d^2 + 2*a^5*b*d^3)*x^5 + 1/4*(20*a^3*b^3*c^3
 + 45*a^4*b^2*c^2*d + 18*a^5*b*c*d^2 + a^6*d^3)*x^4 + (5*a^4*b^2*c^3 + 6*a
^5*b*c^2*d + a^6*c*d^2)*x^3 + 3/2*(2*a^5*b*c^3 + a^6*c^2*d)*x^2
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 364 vs. $2(80) = 160$.

Time = 0.06 (sec) , antiderivative size = 364, normalized size of antiderivative = 3.96

$$\int (a+bx)^3 \left(ac+(bc+ad)x+bdx^2\right)^3 \, dx$$
$$= a^6c^3x + \frac{b^6d^3x^{10}}{10} + x^9\cdot\left(\frac{2ab^5d^3}{3}+\frac{b^6cd^2}{3}\right) + x^8\cdot\left(\frac{15a^2b^4d^3}{8}+\frac{9ab^5cd^2}{4}+\frac{3b^6c^2d}{8}\right)$$
$$+ x^7\cdot\left(\frac{20a^3b^3d^3}{7}+\frac{45a^2b^4cd^2}{7}+\frac{18ab^5c^2d}{7}+\frac{b^6c^3}{7}\right)$$
$$+ x^6\cdot\left(\frac{5a^4b^2d^3}{2}+10a^3b^3cd^2+\frac{15a^2b^4c^2d}{2}+ab^5c^3\right)$$
$$+ x^5\cdot\left(\frac{6a^5bd^3}{5}+9a^4b^2cd^2+12a^3b^3c^2d+3a^2b^4c^3\right)$$
$$+ x^4\left(\frac{a^6d^3}{4}+\frac{9a^5bcd^2}{2}+\frac{45a^4b^2c^2d}{4}+5a^3b^3c^3\right)$$
$$+ x^3\left(a^6cd^2+6a^5bc^2d+5a^4b^2c^3\right) + x^2\cdot\left(\frac{3a^6c^2d}{2}+3a^5bc^3\right)$$

input
```
integrate((b*x+a)**3*(a*c+(a*d+b*c)*x+b*d*x**2)**3,x)
```

output
```
a**6*c**3*x + b**6*d**3*x**10/10 + x**9*(2*a*b**5*d**3/3 + b**6*c*d**2/3)
+ x**8*(15*a**2*b**4*d**3/8 + 9*a*b**5*c*d**2/4 + 3*b**6*c**2*d/8) + x**7*
(20*a**3*b**3*d**3/7 + 45*a**2*b**4*c*d**2/7 + 18*a*b**5*c**2*d/7 + b**6*c
**3/7) + x**6*(5*a**4*b**2*d**3/2 + 10*a**3*b**3*c*d**2 + 15*a**2*b**4*c**
2*d/2 + a*b**5*c**3) + x**5*(6*a**5*b*d**3/5 + 9*a**4*b**2*c*d**2 + 12*a**
3*b**3*c**2*d + 3*a**2*b**4*c**3) + x**4*(a**6*d**3/4 + 9*a**5*b*c*d**2/2
+ 45*a**4*b**2*c**2*d/4 + 5*a**3*b**3*c**3) + x**3*(a**6*c*d**2 + 6*a**5*b
*c**2*d + 5*a**4*b**2*c**3) + x**2*(3*a**6*c**2*d/2 + 3*a**5*b*c**3)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 327 vs. $2(84) = 168$.

Time = 0.04 (sec) , antiderivative size = 327, normalized size of antiderivative = 3.55

$$\int (a+bx)^3 \left(ac+(bc+ad)x+bdx^2\right)^3 \, dx$$
$$= \frac{1}{10} b^6 d^3 x^{10} + a^6 c^3 x + \frac{1}{3} \left(b^6 c d^2 + 2\, a b^5 d^3\right) x^9 + \frac{3}{8} \left(b^6 c^2 d + 6\, a b^5 c d^2 + 5\, a^2 b^4 d^3\right) x^8$$
$$+ \frac{1}{7} \left(b^6 c^3 + 18\, a b^5 c^2 d + 45\, a^2 b^4 c d^2 + 20\, a^3 b^3 d^3\right) x^7$$
$$+ \frac{1}{2} \left(2\, a b^5 c^3 + 15\, a^2 b^4 c^2 d + 20\, a^3 b^3 c d^2 + 5\, a^4 b^2 d^3\right) x^6$$
$$+ \frac{3}{5} \left(5\, a^2 b^4 c^3 + 20\, a^3 b^3 c^2 d + 15\, a^4 b^2 c d^2 + 2\, a^5 b d^3\right) x^5$$
$$+ \frac{1}{4} \left(20\, a^3 b^3 c^3 + 45\, a^4 b^2 c^2 d + 18\, a^5 b c d^2 + a^6 d^3\right) x^4$$
$$+ \left(5\, a^4 b^2 c^3 + 6\, a^5 b c^2 d + a^6 c d^2\right) x^3 + \frac{3}{2} \left(2\, a^5 b c^3 + a^6 c^2 d\right) x^2$$

input
```
integrate((b*x+a)^3*(a*c+(a*d+b*c)*x+b*d*x^2)^3,x, algorithm="maxima")
```

output
```
1/10*b^6*d^3*x^10 + a^6*c^3*x + 1/3*(b^6*c*d^2 + 2*a*b^5*d^3)*x^9 + 3/8*(b
^6*c^2*d + 6*a*b^5*c*d^2 + 5*a^2*b^4*d^3)*x^8 + 1/7*(b^6*c^3 + 18*a*b^5*c^
2*d + 45*a^2*b^4*c*d^2 + 20*a^3*b^3*d^3)*x^7 + 1/2*(2*a*b^5*c^3 + 15*a^2*b
^4*c^2*d + 20*a^3*b^3*c*d^2 + 5*a^4*b^2*d^3)*x^6 + 3/5*(5*a^2*b^4*c^3 + 20
*a^3*b^3*c^2*d + 15*a^4*b^2*c*d^2 + 2*a^5*b*d^3)*x^5 + 1/4*(20*a^3*b^3*c^3
 + 45*a^4*b^2*c^2*d + 18*a^5*b*c*d^2 + a^6*d^3)*x^4 + (5*a^4*b^2*c^3 + 6*a
^5*b*c^2*d + a^6*c*d^2)*x^3 + 3/2*(2*a^5*b*c^3 + a^6*c^2*d)*x^2
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 362 vs. $2(84) = 168$.

Time = 0.14 (sec) , antiderivative size = 362, normalized size of antiderivative = 3.93

$$\int (a+bx)^3 \left(ac+(bc+ad)x+bdx^2\right)^3 \, dx = \frac{1}{10}b^6d^3x^{10}+\frac{1}{3}b^6cd^2x^9+\frac{2}{3}ab^5d^3x^9+\frac{3}{8}b^6c^2dx^8+\frac{9}{4}ab^5cd^2x^8+\frac{15}{8}a^2b^4d^3x^8+\frac{1}{7}b^6c^3x^7+\frac{18}{7}ab^5c^2dx^7+\frac{45}{7}a^2b^4cd^2x^7+\frac{20}{7}a^3b^3d^3x^7+ab^5c^3x^6+\frac{15}{2}a^2b^4c^2dx^6+10a^3b^3cd^2x^6+\frac{5}{2}a^4b^2d^3x^6+3a^2b^4c^3x^5+12a^3b^3c^2dx^5+9a^4b^2cd^2x^5+\frac{6}{5}a^5bd^3x^5+5a^3b^3c^3x^4+\frac{45}{4}a^4b^2c^2dx^4+\frac{9}{2}a^5bcd^2x^4+\frac{1}{4}a^6d^3x^4+5a^4b^2c^3x^3+6a^5bc^2dx^3+a^6cd^2x^3+3a^5bc^3x^2+\frac{3}{2}a^6c^2dx^2+a^6c^3x$$

input

```
integrate((b*x+a)^3*(a*c+(a*d+b*c)*x+b*d*x^2)^3,x, algorithm="giac")
```

output

```
1/10*b^6*d^3*x^10 + 1/3*b^6*c*d^2*x^9 + 2/3*a*b^5*d^3*x^9 + 3/8*b^6*c^2*d*
x^8 + 9/4*a*b^5*c*d^2*x^8 + 15/8*a^2*b^4*d^3*x^8 + 1/7*b^6*c^3*x^7 + 18/7*
a*b^5*c^2*d*x^7 + 45/7*a^2*b^4*c*d^2*x^7 + 20/7*a^3*b^3*d^3*x^7 + a*b^5*c^
3*x^6 + 15/2*a^2*b^4*c^2*d*x^6 + 10*a^3*b^3*c*d^2*x^6 + 5/2*a^4*b^2*d^3*x^
6 + 3*a^2*b^4*c^3*x^5 + 12*a^3*b^3*c^2*d*x^5 + 9*a^4*b^2*c*d^2*x^5 + 6/5*a
^5*b*d^3*x^5 + 5*a^3*b^3*c^3*x^4 + 45/4*a^4*b^2*c^2*d*x^4 + 9/2*a^5*b*c*d^
2*x^4 + 1/4*a^6*d^3*x^4 + 5*a^4*b^2*c^3*x^3 + 6*a^5*b*c^2*d*x^3 + a^6*c*d^
2*x^3 + 3*a^5*b*c^3*x^2 + 3/2*a^6*c^2*d*x^2 + a^6*c^3*x
```

Mupad [B] (verification not implemented)

Time = 5.87 (sec) , antiderivative size = 308, normalized size of antiderivative = 3.35

$$\int (a+bx)^3 \left(ac+(bc+ad)x+bdx^2\right)^3 \, dx = x^5 \left(\frac{6 a^5 b d^3}{5} + 9 a^4 b^2 c d^2 + 12 a^3 b^3 c^2 d + 3 a^2 b^4 c^3\right) + x^6 \left(\frac{5 a^4 b^2 d^3}{2} + 10 a^3 b^3 c d^2 + \frac{15 a^2 b^4 c^2 d}{2} + a b^5 c^3\right) + x^4 \left(\frac{a^6 d^3}{4} + \frac{9 a^5 b c d^2}{2} + \frac{45 a^4 b^2 c^2 d}{4} + 5 a^3 b^3 c^3\right) + x^7 \left(\frac{20 a^3 b^3 d^3}{7} + \frac{45 a^2 b^4 c d^2}{7} + \frac{18 a b^5 c^2 d}{7} + \frac{b^6 c^3}{7}\right) + a^6 c^3 x + \frac{b^6 d^3 x^{10}}{10} + \frac{3 a^5 c^2 x^2 (a d + 2 b c)}{2} + \frac{b^5 d^2 x^9 (2 a d + b c)}{3} + a^4 c x^3 \left(a^2 d^2 + 6 a b c d + 5 b^2 c^2\right) + \frac{3 b^4 d x^8 (5 a^2 d^2 + 6 a b c d + b^2 c^2)}{8}$$

input
```
int((a + b*x)^3*(a*c + x*(a*d + b*c) + b*d*x^2)^3,x)
```

output
```
x^5*((6*a^5*b*d^3)/5 + 3*a^2*b^4*c^3 + 12*a^3*b^3*c^2*d + 9*a^4*b^2*c*d^2)
 + x^6*(a*b^5*c^3 + (5*a^4*b^2*d^3)/2 + (15*a^2*b^4*c^2*d)/2 + 10*a^3*b^3*
c*d^2) + x^4*((a^6*d^3)/4 + 5*a^3*b^3*c^3 + (45*a^4*b^2*c^2*d)/4 + (9*a^5*
b*c*d^2)/2) + x^7*((b^6*c^3)/7 + (20*a^3*b^3*d^3)/7 + (45*a^2*b^4*c*d^2)/7
 + (18*a*b^5*c^2*d)/7) + a^6*c^3*x + (b^6*d^3*x^10)/10 + (3*a^5*c^2*x^2*(a
*d + 2*b*c))/2 + (b^5*d^2*x^9*(2*a*d + b*c))/3 + a^4*c*x^3*(a^2*d^2 + 5*b^
2*c^2 + 6*a*b*c*d) + (3*b^4*d*x^8*(5*a^2*d^2 + b^2*c^2 + 6*a*b*c*d))/8
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 363, normalized size of antiderivative = 3.95

$$\int (a+bx)^3 \left(ac+(bc+ad)x+bdx^2\right)^3 \, dx$$
$$= \frac{x(84b^6d^3x^9 + 560a\,b^5d^3x^8 + 280b^6c\,d^2x^8 + 1575a^2b^4d^3x^7 + 1890a\,b^5c\,d^2x^7 + 315b^6c^2d\,x^7 + 2400a^3b^3d^3x^6}{}$$

input
```
int((b*x+a)^3*(a*c+(a*d+b*c)*x+b*d*x^2)^3,x)
```

output
```
(x*(840*a**6*c**3 + 1260*a**6*c**2*d*x + 840*a**6*c*d**2*x**2 + 210*a**6*d
**3*x**3 + 2520*a**5*b*c**3*x + 5040*a**5*b*c**2*d*x**2 + 3780*a**5*b*c*d*
*2*x**3 + 1008*a**5*b*d**3*x**4 + 4200*a**4*b**2*c**3*x**2 + 9450*a**4*b**
2*c**2*d*x**3 + 7560*a**4*b**2*c*d**2*x**4 + 2100*a**4*b**2*d**3*x**5 + 42
00*a**3*b**3*c**3*x**3 + 10080*a**3*b**3*c**2*d*x**4 + 8400*a**3*b**3*c*d*
*2*x**5 + 2400*a**3*b**3*d**3*x**6 + 2520*a**2*b**4*c**3*x**4 + 6300*a**2*
b**4*c**2*d*x**5 + 5400*a**2*b**4*c*d**2*x**6 + 1575*a**2*b**4*d**3*x**7 +
 840*a*b**5*c**3*x**5 + 2160*a*b**5*c**2*d*x**6 + 1890*a*b**5*c*d**2*x**7
+ 560*a*b**5*d**3*x**8 + 120*b**6*c**3*x**6 + 315*b**6*c**2*d*x**7 + 280*b
**6*c*d**2*x**8 + 84*b**6*d**3*x**9))/840
```

3.26 $\int (a+bx)^2 \left(ac+(bc+ad)x+bdx^2\right)^3 \, dx$

Optimal result

Integrand size = 29, antiderivative size = 92

$$\int (a+bx)^2 \left(ac+(bc+ad)x+bdx^2\right)^3 \, dx = \frac{(bc-ad)^3(a+bx)^6}{6b^4} + \frac{3d(bc-ad)^2(a+bx)^7}{7b^4} + \frac{3d^2(bc-ad)(a+bx)^8}{8b^4} + \frac{d^3(a+bx)^9}{9b^4}$$

output
```
1/6*(-a*d+b*c)^3*(b*x+a)^6/b^4+3/7*d*(-a*d+b*c)^2*(b*x+a)^7/b^4+3/8*d^2*(-
a*d+b*c)*(b*x+a)^8/b^4+1/9*d^3*(b*x+a)^9/b^4
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 235 vs. $2(92) = 184$.

Time = 0.04 (sec) , antiderivative size = 235, normalized size of antiderivative = 2.55

$$\int (a+bx)^2 \left(ac+(bc+ad)x+bdx^2\right)^3 \, dx = \frac{1}{504}x\left(126a^5\left(4c^3+6c^2dx+4cd^2x^2+d^3x^3\right)+126a^4bx\left(10c^3+20c^2dx+15cd^2x^2+4d^3x^3\right)+84a^3b^2x^2\left(20c^3+45c^2dx+36cd^2x^2+10d^3x^3\right)+36a^2b^3x^3\left(35c^3+84c^2dx+70cd^2x^2+20d^3x^3\right)+9ab^4x^4\left(56c^3+140c^2dx+120cd^2x^2+35d^3x^3\right)+b^5x^5\left(84c^3+216c^2dx+189cd^2x^2+56d^3x^3\right)\right)$$

input
```
Integrate[(a + b*x)^2*(a*c + (b*c + a*d)*x + b*d*x^2)^3,x]
```

output
```
(x*(126*a^5*(4*c^3 + 6*c^2*d*x + 4*c*d^2*x^2 + d^3*x^3) + 126*a^4*b*x*(10*
c^3 + 20*c^2*d*x + 15*c*d^2*x^2 + 4*d^3*x^3) + 84*a^3*b^2*x^2*(20*c^3 + 45
*c^2*d*x + 36*c*d^2*x^2 + 10*d^3*x^3) + 36*a^2*b^3*x^3*(35*c^3 + 84*c^2*d*
x + 70*c*d^2*x^2 + 20*d^3*x^3) + 9*a*b^4*x^4*(56*c^3 + 140*c^2*d*x + 120*c
*d^2*x^2 + 35*d^3*x^3) + b^5*x^5*(84*c^3 + 216*c^2*d*x + 189*c*d^2*x^2 + 5
6*d^3*x^3)))/504
```

Rubi [A] (verified)

Time = 0.54 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.069$, Rules used = {1121, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+bx)^2 \left(x(ad+bc)+ac+bdx^2\right)^3 \, dx$$

$$\Big\downarrow \text{1121}$$

$$\int \left(\frac{3d^2(a+bx)^7(bc-ad)}{b^3}+\frac{3d(a+bx)^6(bc-ad)^2}{b^3}+\frac{(a+bx)^5(bc-ad)^3}{b^3}+\frac{d^3(a+bx)^8}{b^3}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{3d^2(a+bx)^8(bc-ad)}{8b^4}+\frac{3d(a+bx)^7(bc-ad)^2}{7b^4}+\frac{(a+bx)^6(bc-ad)^3}{6b^4}+\frac{d^3(a+bx)^9}{9b^4}$$

input
```
Int[(a + b*x)^2*(a*c + (b*c + a*d)*x + b*d*x^2)^3,x]
```

output
```
((b*c - a*d)^3*(a + b*x)^6)/(6*b^4) + (3*d*(b*c - a*d)^2*(a + b*x)^7)/(7*b
^4) + (3*d^2*(b*c - a*d)*(a + b*x)^8)/(8*b^4) + (d^3*(a + b*x)^9)/(9*b^4)
```

Defintions of rubi rules used

rule 1121
```
Int[((d_) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Int[ExpandIntegrand[(d + e*x)^(m + p)*(a/d + (c/e)*x)^p, x], x]
/; FreeQ[{a, b, c, d, e, m, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && (Int
egerQ[p] || (GtQ[a, 0] && GtQ[d, 0] && LtQ[c, 0]))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 272 vs. $2(84) = 168$.

Time = 1.17 (sec) , antiderivative size = 273, normalized size of antiderivative = 2.97

method	result
norman	$\frac{b^5d^3x^9}{9}+\left(\frac{5}{8}a\,b^4d^3+\frac{3}{8}b^5c\,d^2\right)x^8+\left(\frac{10}{7}d^3a^2b^3+\frac{15}{7}a\,b^4c\,d^2+\frac{3}{7}b^5c^2d\right)x^7+\left(\frac{5}{3}d^3a^3b^2+5c\,d^2a^2\right.$
risch	$\frac{1}{9}b^5d^3x^9+\frac{5}{8}a\,b^4d^3x^8+\frac{3}{8}b^5c\,d^2x^8+\frac{10}{7}x^7d^3a^2b^3+\frac{15}{7}a\,b^4c\,d^2x^7+\frac{3}{7}b^5c^2d\,x^7+\frac{5}{3}x^6d^3a^3b^2+5$
parallelrisch	$\frac{1}{9}b^5d^3x^9+\frac{5}{8}a\,b^4d^3x^8+\frac{3}{8}b^5c\,d^2x^8+\frac{10}{7}x^7d^3a^2b^3+\frac{15}{7}a\,b^4c\,d^2x^7+\frac{3}{7}b^5c^2d\,x^7+\frac{5}{3}x^6d^3a^3b^2+5$
gosper	$x\left(56b^5d^3x^8+315a\,b^4d^3x^7+189x^7b^5c\,d^2+720a^2b^3d^3x^6+1080a\,b^4c\,d^2x^6+216x^6b^5c^2d+840x^5d^3a^3b^2+2520a^2b^3c\,d^2x^5+1260\right.$
orering	$x\left(56b^5d^3x^8+315a\,b^4d^3x^7+189x^7b^5c\,d^2+720a^2b^3d^3x^6+1080a\,b^4c\,d^2x^6+216x^6b^5c^2d+840x^5d^3a^3b^2+2520a^2b^3c\,d^2x^5+1260\right.$
default	$\frac{b^5d^3x^9}{9}+\frac{(2a\,b^4d^3+3b^4(ad+bc)d^2)x^8}{8}+\frac{\left(d^3a^2b^3+6a\,b^3(ad+bc)d^2+b^2\left(a\,b^2c\,d^2+2(ad+bc)^2bd+bd\left(2abcd+(ad+bc)^2\right)\right)\right)}{7}$

input
```
int((b*x+a)^2*(a*c+(a*d+b*c)*x+b*d*x^2)^3,x,method=_RETURNVERBOSE)
```

output
```
1/9*b^5*d^3*x^9+(5/8*a*b^4*d^3+3/8*b^5*c*d^2)*x^8+(10/7*d^3*a^2*b^3+15/7*a
*b^4*c*d^2+3/7*b^5*c^2*d)*x^7+(5/3*d^3*a^3*b^2+5*c*d^2*a^2*b^3+5/2*c^2*d*a
*b^4+1/6*b^5*c^3)*x^6+(a^4*b*d^3+6*a^3*b^2*c*d^2+6*a^2*b^3*c^2*d+a*b^4*c^3
)*x^5+(1/4*a^5*d^3+15/4*a^4*b*c*d^2+15/2*a^3*b^2*c^2*d+5/2*a^2*b^3*c^3)*x^
4+(c*d^2*a^5+5*a^4*b*c^2*d+10/3*a^3*b^2*c^3)*x^3+(3/2*c^2*d*a^5+5/2*a^4*b*
c^3)*x^2+a^5*c^3*x
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 277 vs. $2(84) = 168$.

Time = 0.08 (sec) , antiderivative size = 277, normalized size of antiderivative = 3.01

$$\int (a+bx)^2\left(ac+(bc+ad)x+bdx^2\right)^3\,dx$$
$$=\frac{1}{9}b^5d^3x^9+a^5c^3x+\frac{1}{8}\left(3\,b^5cd^2+5\,ab^4d^3\right)x^8+\frac{1}{7}\left(3\,b^5c^2d+15\,ab^4cd^2+10\,a^2b^3d^3\right)x^7$$
$$+\frac{1}{6}\left(b^5c^3+15\,ab^4c^2d+30\,a^2b^3cd^2+10\,a^3b^2d^3\right)x^6$$
$$+\left(ab^4c^3+6\,a^2b^3c^2d+6\,a^3b^2cd^2+a^4bd^3\right)x^5$$
$$+\frac{1}{4}\left(10\,a^2b^3c^3+30\,a^3b^2c^2d+15\,a^4bcd^2+a^5d^3\right)x^4$$
$$+\frac{1}{3}\left(10\,a^3b^2c^3+15\,a^4bc^2d+3\,a^5cd^2\right)x^3+\frac{1}{2}\left(5\,a^4bc^3+3\,a^5c^2d\right)x^2$$

input
```
integrate((b*x+a)^2*(a*c+(a*d+b*c)*x+b*d*x^2)^3,x, algorithm="fricas")
```

output
```
1/9*b^5*d^3*x^9 + a^5*c^3*x + 1/8*(3*b^5*c*d^2 + 5*a*b^4*d^3)*x^8 + 1/7*(3
*b^5*c^2*d + 15*a*b^4*c*d^2 + 10*a^2*b^3*d^3)*x^7 + 1/6*(b^5*c^3 + 15*a*b^
4*c^2*d + 30*a^2*b^3*c*d^2 + 10*a^3*b^2*d^3)*x^6 + (a*b^4*c^3 + 6*a^2*b^3*
c^2*d + 6*a^3*b^2*c*d^2 + a^4*b*d^3)*x^5 + 1/4*(10*a^2*b^3*c^3 + 30*a^3*b^
2*c^2*d + 15*a^4*b*c*d^2 + a^5*d^3)*x^4 + 1/3*(10*a^3*b^2*c^3 + 15*a^4*b*c
^2*d + 3*a^5*c*d^2)*x^3 + 1/2*(5*a^4*b*c^3 + 3*a^5*c^2*d)*x^2
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 308 vs. $2(82) = 164$.

Time = 0.07 (sec) , antiderivative size = 308, normalized size of antiderivative = 3.35

$$\int (a+bx)^2 \left(ac+(bc+ad)x+bdx^2\right)^3 \, dx = a^5c^3x + \frac{b^5d^3x^9}{9} + x^8 \cdot \left(\frac{5ab^4d^3}{8} + \frac{3b^5cd^2}{8}\right) + x^7 \cdot \left(\frac{10a^2b^3d^3}{7} + \frac{15ab^4cd^2}{7} + \frac{3b^5c^2d}{7}\right) + x^6 \cdot \left(\frac{5a^3b^2d^3}{3} + 5a^2b^3cd^2 + \frac{5ab^4c^2d}{2} + \frac{b^5c^3}{6}\right) + x^5\left(a^4bd^3 + 6a^3b^2cd^2 + 6a^2b^3c^2d + ab^4c^3\right) + x^4\left(\frac{a^5d^3}{4} + \frac{15a^4bcd^2}{4} + \frac{15a^3b^2c^2d}{2} + \frac{5a^2b^3c^3}{2}\right) + x^3\left(a^5cd^2 + 5a^4bc^2d + \frac{10a^3b^2c^3}{3}\right) + x^2 \cdot \left(\frac{3a^5c^2d}{2} + \frac{5a^4bc^3}{2}\right)$$

input
```
integrate((b*x+a)**2*(a*c+(a*d+b*c)*x+b*d*x**2)**3,x)
```

output
```
a**5*c**3*x + b**5*d**3*x**9/9 + x**8*(5*a*b**4*d**3/8 + 3*b**5*c*d**2/8)
+ x**7*(10*a**2*b**3*d**3/7 + 15*a*b**4*c*d**2/7 + 3*b**5*c**2*d/7) + x**6
*(5*a**3*b**2*d**3/3 + 5*a**2*b**3*c*d**2 + 5*a*b**4*c**2*d/2 + b**5*c**3/
6) + x**5*(a**4*b*d**3 + 6*a**3*b**2*c*d**2 + 6*a**2*b**3*c**2*d + a*b**4*
c**3) + x**4*(a**5*d**3/4 + 15*a**4*b*c*d**2/4 + 15*a**3*b**2*c**2*d/2 + 5
*a**2*b**3*c**3/2) + x**3*(a**5*c*d**2 + 5*a**4*b*c**2*d + 10*a**3*b**2*c*
*3/3) + x**2*(3*a**5*c**2*d/2 + 5*a**4*b*c**3/2)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 277 vs. $2(84) = 168$.

Time = 0.03 (sec) , antiderivative size = 277, normalized size of antiderivative = 3.01

$$\int (a+bx)^2 \left(ac+(bc+ad)x+bdx^2\right)^3 \, dx$$
$$= \frac{1}{9}b^5d^3x^9 + a^5c^3x + \frac{1}{8}\left(3\,b^5cd^2 + 5\,ab^4d^3\right)x^8 + \frac{1}{7}\left(3\,b^5c^2d + 15\,ab^4cd^2 + 10\,a^2b^3d^3\right)x^7 + \frac{1}{6}\left(b^5c^3 + 15\,ab^4c^2d + 30\,a^2b^3cd^2 + 10\,a^3b^2d^3\right)x^6 + \left(ab^4c^3 + 6\,a^2b^3c^2d + 6\,a^3b^2cd^2 + a^4bd^3\right)x^5 + \frac{1}{4}\left(10\,a^2b^3c^3 + 30\,a^3b^2c^2d + 15\,a^4bcd^2 + a^5d^3\right)x^4 + \frac{1}{3}\left(10\,a^3b^2c^3 + 15\,a^4bc^2d + 3\,a^5cd^2\right)x^3 + \frac{1}{2}\left(5\,a^4bc^3 + 3\,a^5c^2d\right)x^2$$

input
```
integrate((b*x+a)^2*(a*c+(a*d+b*c)*x+b*d*x^2)^3,x, algorithm="maxima")
```

output
```
1/9*b^5*d^3*x^9 + a^5*c^3*x + 1/8*(3*b^5*c*d^2 + 5*a*b^4*d^3)*x^8 + 1/7*(3
*b^5*c^2*d + 15*a*b^4*c*d^2 + 10*a^2*b^3*d^3)*x^7 + 1/6*(b^5*c^3 + 15*a*b^
4*c^2*d + 30*a^2*b^3*c*d^2 + 10*a^3*b^2*d^3)*x^6 + (a*b^4*c^3 + 6*a^2*b^3*
c^2*d + 6*a^3*b^2*c*d^2 + a^4*b*d^3)*x^5 + 1/4*(10*a^2*b^3*c^3 + 30*a^3*b^
2*c^2*d + 15*a^4*b*c*d^2 + a^5*d^3)*x^4 + 1/3*(10*a^3*b^2*c^3 + 15*a^4*b*c
^2*d + 3*a^5*c*d^2)*x^3 + 1/2*(5*a^4*b*c^3 + 3*a^5*c^2*d)*x^2
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 303 vs. $2(84) = 168$.

Time = 0.14 (sec) , antiderivative size = 303, normalized size of antiderivative = 3.29

$$\int (a+bx)^2\left(ac+(bc+ad)x+bdx^2\right)^3\,dx = \frac{1}{9}\,b^5d^3x^9+\frac{3}{8}\,b^5cd^2x^8+\frac{5}{8}\,ab^4d^3x^8+\frac{3}{7}\,b^5c^2dx^7+\frac{15}{7}\,ab^4cd^2x^7+\frac{10}{7}\,a^2b^3d^3x^7+\frac{1}{6}\,b^5c^3x^6+\frac{5}{2}\,ab^4c^2dx^6+5\,a^2b^3cd^2x^6+\frac{5}{3}\,a^3b^2d^3x^6+ab^4c^3x^5+6\,a^2b^3c^2dx^5+6\,a^3b^2cd^2x^5+a^4bd^3x^5+\frac{5}{2}\,a^2b^3c^3x^4+\frac{15}{2}\,a^3b^2c^2dx^4+\frac{15}{4}\,a^4bcd^2x^4+\frac{1}{4}\,a^5d^3x^4+\frac{10}{3}\,a^3b^2c^3x^3+5\,a^4bc^2dx^3+a^5cd^2x^3+\frac{5}{2}\,a^4bc^3x^2+\frac{3}{2}\,a^5c^2dx^2+a^5c^3x$$

input
```
integrate((b*x+a)^2*(a*c+(a*d+b*c)*x+b*d*x^2)^3,x, algorithm="giac")
```

output
```
1/9*b^5*d^3*x^9 + 3/8*b^5*c*d^2*x^8 + 5/8*a*b^4*d^3*x^8 + 3/7*b^5*c^2*d*x^
7 + 15/7*a*b^4*c*d^2*x^7 + 10/7*a^2*b^3*d^3*x^7 + 1/6*b^5*c^3*x^6 + 5/2*a*
b^4*c^2*d*x^6 + 5*a^2*b^3*c*d^2*x^6 + 5/3*a^3*b^2*d^3*x^6 + a*b^4*c^3*x^5
+ 6*a^2*b^3*c^2*d*x^5 + 6*a^3*b^2*c*d^2*x^5 + a^4*b*d^3*x^5 + 5/2*a^2*b^3*
c^3*x^4 + 15/2*a^3*b^2*c^2*d*x^4 + 15/4*a^4*b*c*d^2*x^4 + 1/4*a^5*d^3*x^4
+ 10/3*a^3*b^2*c^3*x^3 + 5*a^4*b*c^2*d*x^3 + a^5*c*d^2*x^3 + 5/2*a^4*b*c^3
*x^2 + 3/2*a^5*c^2*d*x^2 + a^5*c^3*x
```

Mupad [B] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 261, normalized size of antiderivative = 2.84

$$\int (a+bx)^2 \left(ac+(bc+ad)x+bdx^2\right)^3 \, dx = x^5 \left(a^4 b d^3 + 6 a^3 b^2 c d^2 + 6 a^2 b^3 c^2 d + a b^4 c^3\right) + x^4 \left(\frac{a^5 d^3}{4} + \frac{15 a^4 b c d^2}{4} + \frac{15 a^3 b^2 c^2 d}{2} + \frac{5 a^2 b^3 c^3}{2}\right) + x^6 \left(\frac{5 a^3 b^2 d^3}{3} + 5 a^2 b^3 c d^2 + \frac{5 a b^4 c^2 d}{2} + \frac{b^5 c^3}{6}\right) + a^5 c^3 x + \frac{b^5 d^3 x^9}{9} + \frac{a^4 c^2 x^2 (3 a d + 5 b c)}{2} + \frac{b^4 d^2 x^8 (5 a d + 3 b c)}{8} + \frac{a^3 c x^3 (3 a^2 d^2 + 15 a b c d + 10 b^2 c^2)}{3} + \frac{b^3 d x^7 (10 a^2 d^2 + 15 a b c d + 3 b^2 c^2)}{7}$$

input

```
int((a + b*x)^2*(a*c + x*(a*d + b*c) + b*d*x^2)^3,x)
```

output

```
x^5*(a*b^4*c^3 + a^4*b*d^3 + 6*a^2*b^3*c^2*d + 6*a^3*b^2*c*d^2) + x^4*((a^
5*d^3)/4 + (5*a^2*b^3*c^3)/2 + (15*a^3*b^2*c^2*d)/2 + (15*a^4*b*c*d^2)/4)
+ x^6*((b^5*c^3)/6 + (5*a^3*b^2*d^3)/3 + 5*a^2*b^3*c*d^2 + (5*a*b^4*c^2*d)
/2) + a^5*c^3*x + (b^5*d^3*x^9)/9 + (a^4*c^2*x^2*(3*a*d + 5*b*c))/2 + (b^4
*d^2*x^8*(5*a*d + 3*b*c))/8 + (a^3*c*x^3*(3*a^2*d^2 + 10*b^2*c^2 + 15*a*b*
c*d))/3 + (b^3*d*x^7*(10*a^2*d^2 + 3*b^2*c^2 + 15*a*b*c*d))/7
```

Reduce [B] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 305, normalized size of antiderivative = 3.32

$$\int (a+bx)^2 \left(ac+(bc+ad)x+bdx^2\right)^3 \, dx$$
$$= \frac{x(56b^5d^3x^8 + 315a\,b^4d^3x^7 + 189b^5c\,d^2x^7 + 720a^2b^3d^3x^6 + 1080a\,b^4c\,d^2x^6 + 216b^5c^2d\,x^6 + 840a^3b^2d^3x^5 + \cdots}{\cdots}$$

input
```
int((b*x+a)^2*(a*c+(a*d+b*c)*x+b*d*x^2)^3,x)
```

output
```
(x*(504*a**5*c**3 + 756*a**5*c**2*d*x + 504*a**5*c*d**2*x**2 + 126*a**5*d*
*3*x**3 + 1260*a**4*b*c**3*x + 2520*a**4*b*c**2*d*x**2 + 1890*a**4*b*c*d**
2*x**3 + 504*a**4*b*d**3*x**4 + 1680*a**3*b**2*c**3*x**2 + 3780*a**3*b**2*
c**2*d*x**3 + 3024*a**3*b**2*c*d**2*x**4 + 840*a**3*b**2*d**3*x**5 + 1260*
a**2*b**3*c**3*x**3 + 3024*a**2*b**3*c**2*d*x**4 + 2520*a**2*b**3*c*d**2*x
**5 + 720*a**2*b**3*d**3*x**6 + 504*a*b**4*c**3*x**4 + 1260*a*b**4*c**2*d*
x**5 + 1080*a*b**4*c*d**2*x**6 + 315*a*b**4*d**3*x**7 + 84*b**5*c**3*x**5
+ 216*b**5*c**2*d*x**6 + 189*b**5*c*d**2*x**7 + 56*b**5*d**3*x**8))/504
```

3.27 $\int (a+bx) \left(ac+(bc+ad)x+bdx^2\right)^3 \, dx$

Optimal result

Integrand size = 27, antiderivative size = 92

$$\int (a+bx) \left(ac+(bc+ad)x+bdx^2\right)^3 \, dx = \frac{(bc-ad)^3(a+bx)^5}{5b^4} + \frac{d(bc-ad)^2(a+bx)^6}{2b^4} + \frac{3d^2(bc-ad)(a+bx)^7}{7b^4} + \frac{d^3(a+bx)^8}{8b^4}$$

output
```
1/5*(-a*d+b*c)^3*(b*x+a)^5/b^4+1/2*d*(-a*d+b*c)^2*(b*x+a)^6/b^4+3/7*d^2*(-
a*d+b*c)*(b*x+a)^7/b^4+1/8*d^3*(b*x+a)^8/b^4
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 217 vs. $2(92) = 184$.

Time = 0.02 (sec) , antiderivative size = 217, normalized size of antiderivative = 2.36

$$\int (a+bx)\left(ac+(bc+ad)x+bdx^2\right)^3\,dx = a^4c^3x+\frac{1}{2}a^3c^2(4bc+3ad)x^2 + a^2c\left(2b^2c^2+4abcd+a^2d^2\right)x^3 + \frac{1}{4}a\left(4b^3c^3+18ab^2c^2d+12a^2bcd^2+a^3d^3\right)x^4 + \frac{1}{5}b\left(b^3c^3+12ab^2c^2d+18a^2bcd^2+4a^3d^3\right)x^5 + \frac{1}{2}b^2d\left(b^2c^2+4abcd+2a^2d^2\right)x^6 + \frac{1}{7}b^3d^2(3bc+4ad)x^7+\frac{1}{8}b^4d^3x^8$$

input
```
Integrate[(a + b*x)*(a*c + (b*c + a*d)*x + b*d*x^2)^3,x]
```

output
```
a^4*c^3*x + (a^3*c^2*(4*b*c + 3*a*d)*x^2)/2 + a^2*c*(2*b^2*c^2 + 4*a*b*c*d
 + a^2*d^2)*x^3 + (a*(4*b^3*c^3 + 18*a*b^2*c^2*d + 12*a^2*b*c*d^2 + a^3*d^
3)*x^4)/4 + (b*(b^3*c^3 + 12*a*b^2*c^2*d + 18*a^2*b*c*d^2 + 4*a^3*d^3)*x^5
)/5 + (b^2*d*(b^2*c^2 + 4*a*b*c*d + 2*a^2*d^2)*x^6)/2 + (b^3*d^2*(3*b*c +
4*a*d)*x^7)/7 + (b^4*d^3*x^8)/8
```

Rubi [A] (verified)

Time = 0.50 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.074$, Rules used = {1121, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+bx)\left(x(ad+bc)+ac+bdx^2\right)^3\,dx$$

$$\Big\downarrow \text{1121}$$

$$\int \left(\frac{3d^2(a+bx)^6(bc-ad)}{b^3}+\frac{3d(a+bx)^5(bc-ad)^2}{b^3}+\frac{(a+bx)^4(bc-ad)^3}{b^3}+\frac{d^3(a+bx)^7}{b^3}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{3d^2(a+bx)^7(bc-ad)}{7b^4}+\frac{d(a+bx)^6(bc-ad)^2}{2b^4}+\frac{(a+bx)^5(bc-ad)^3}{5b^4}+\frac{d^3(a+bx)^8}{8b^4}$$

input
```
Int[(a + b*x)*(a*c + (b*c + a*d)*x + b*d*x^2)^3,x]
```

output
```
((b*c - a*d)^3*(a + b*x)^5)/(5*b^4) + (d*(b*c - a*d)^2*(a + b*x)^6)/(2*b^4
) + (3*d^2*(b*c - a*d)*(a + b*x)^7)/(7*b^4) + (d^3*(a + b*x)^8)/(8*b^4)
```

Defintions of rubi rules used

rule 1121
```
Int[((d_) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Int[ExpandIntegrand[(d + e*x)^(m + p)*(a/d + (c/e)*x)^p, x], x]
/; FreeQ[{a, b, c, d, e, m, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && (Int
egerQ[p] || (GtQ[a, 0] && GtQ[d, 0] && LtQ[c, 0]))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 221 vs. $2(84) = 168$.

Time = 1.10 (sec) , antiderivative size = 222, normalized size of antiderivative = 2.41

method	result
norman	$\frac{b^4 d^3 x^8}{8}+\left(\frac{4}{7} a\, b^3 d^3+\frac{3}{7} b^4 c\, d^2\right) x^7+\left(a^2 b^2 d^3+2 c\, d^2 a\, b^3+\frac{1}{2} b^4 c^2 d\right) x^6+\left(\frac{4}{5} a^3 b\, d^3+\frac{18}{5} a^2 b^2 c\, d^2\right.$
risch	$\frac{1}{8} b^4 d^3 x^8+\frac{4}{7} a\, b^3 d^3 x^7+\frac{3}{7} b^4 c\, d^2 x^7+a^2 b^2 d^3 x^6+2 a\, b^3 c\, d^2 x^6+\frac{1}{2} b^4 c^2 d\, x^6+\frac{4}{5} a^3 b\, d^3 x^5+\frac{18}{5} a^2 b$
parallelrisch	$\frac{1}{8} b^4 d^3 x^8+\frac{4}{7} a\, b^3 d^3 x^7+\frac{3}{7} b^4 c\, d^2 x^7+a^2 b^2 d^3 x^6+2 a\, b^3 c\, d^2 x^6+\frac{1}{2} b^4 c^2 d\, x^6+\frac{4}{5} a^3 b\, d^3 x^5+\frac{18}{5} a^2 b$
gosper	$\frac{x\left(35 b^4 d^3 x^7+160 a\, b^3 d^3 x^6+120 b^4 c\, d^2 x^6+280 a^2 b^2 d^3 x^5+560 a\, b^3 c\, d^2 x^5+140 b^4 c^2 d\, x^5+224 a^3 b\, d^3 x^4+1008 a^2 b^2 c\, d^2 x^4+672 a\, b^3\right.}{}$
orering	$\frac{x\left(35 b^4 d^3 x^7+160 a\, b^3 d^3 x^6+120 b^4 c\, d^2 x^6+280 a^2 b^2 d^3 x^5+560 a\, b^3 c\, d^2 x^5+140 b^4 c^2 d\, x^5+224 a^3 b\, d^3 x^4+1008 a^2 b^2 c\, d^2 x^4+672 a\, b^3\right.}{}$
default	$\frac{b^4 d^3 x^8}{8}+\frac{\left(a\, b^3 d^3+3 b^3 (a d+b c) d^2\right) x^7}{7}+\frac{\left(3 a (a d+b c) b^2 d^2+b \left(a\, b^2 c\, d^2+2 (a d+b c)^2 b d+b d \left(2 a b c d+(a d+b c)^2\right)\right)\right) x^6}{6}+\frac{\left(a\left(\right.\right.}{}$

input
```
int((b*x+a)*(a*c+(a*d+b*c)*x+b*d*x^2)^3,x,method=_RETURNVERBOSE)
```

output
```
1/8*b^4*d^3*x^8+(4/7*a*b^3*d^3+3/7*b^4*c*d^2)*x^7+(a^2*b^2*d^3+2*c*d^2*a*b
^3+1/2*b^4*c^2*d)*x^6+(4/5*a^3*b*d^3+18/5*a^2*b^2*c*d^2+12/5*a*b^3*c^2*d+1
/5*b^4*c^3)*x^5+(1/4*a^4*d^3+3*a^3*b*c*d^2+9/2*a^2*b^2*c^2*d+a*b^3*c^3)*x^
4+(a^4*c*d^2+4*a^3*b*c^2*d+2*a^2*b^2*c^3)*x^3+(3/2*a^4*c^2*d+2*a^3*b*c^3)*
x^2+a^4*c^3*x
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 225 vs. $2(84) = 168$.

Time = 0.07 (sec) , antiderivative size = 225, normalized size of antiderivative = 2.45

$$\begin{aligned}
&\int (a+bx)\left(ac+(bc+ad)x+bdx^2\right)^3 dx \\
&= \frac{1}{8}\, b^4 d^3 x^8 + a^4 c^3 x + \frac{1}{7}\left(3\, b^4 c d^2 + 4\, a b^3 d^3\right) x^7 + \frac{1}{2}\left(b^4 c^2 d + 4\, a b^3 c d^2 + 2\, a^2 b^2 d^3\right) x^6 \\
&\quad + \frac{1}{5}\left(b^4 c^3 + 12\, a b^3 c^2 d + 18\, a^2 b^2 c d^2 + 4\, a^3 b d^3\right) x^5 \\
&\quad + \frac{1}{4}\left(4\, a b^3 c^3 + 18\, a^2 b^2 c^2 d + 12\, a^3 b c d^2 + a^4 d^3\right) x^4 \\
&\quad + \left(2\, a^2 b^2 c^3 + 4\, a^3 b c^2 d + a^4 c d^2\right) x^3 + \frac{1}{2}\left(4\, a^3 b c^3 + 3\, a^4 c^2 d\right) x^2
\end{aligned}$$

input
```
integrate((b*x+a)*(a*c+(a*d+b*c)*x+b*d*x^2)^3,x, algorithm="fricas")
```

output
```
1/8*b^4*d^3*x^8 + a^4*c^3*x + 1/7*(3*b^4*c*d^2 + 4*a*b^3*d^3)*x^7 + 1/2*(b
^4*c^2*d + 4*a*b^3*c*d^2 + 2*a^2*b^2*d^3)*x^6 + 1/5*(b^4*c^3 + 12*a*b^3*c^
2*d + 18*a^2*b^2*c*d^2 + 4*a^3*b*d^3)*x^5 + 1/4*(4*a*b^3*c^3 + 18*a^2*b^2*
c^2*d + 12*a^3*b*c*d^2 + a^4*d^3)*x^4 + (2*a^2*b^2*c^3 + 4*a^3*b*c^2*d + a
^4*c*d^2)*x^3 + 1/2*(4*a^3*b*c^3 + 3*a^4*c^2*d)*x^2
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 243 vs. $2(80) = 160$.

Time = 0.06 (sec) , antiderivative size = 243, normalized size of antiderivative = 2.64

$$\int (a+bx)\left(ac+(bc+ad)x+bdx^2\right)^3\,dx = a^4c^3x + \frac{b^4d^3x^8}{8} + x^7\cdot\left(\frac{4ab^3d^3}{7} + \frac{3b^4cd^2}{7}\right) + x^6\left(a^2b^2d^3 + 2ab^3cd^2 + \frac{b^4c^2d}{2}\right) + x^5\cdot\left(\frac{4a^3bd^3}{5} + \frac{18a^2b^2cd^2}{5} + \frac{12ab^3c^2d}{5} + \frac{b^4c^3}{5}\right) + x^4\left(\frac{a^4d^3}{4} + 3a^3bcd^2 + \frac{9a^2b^2c^2d}{2} + ab^3c^3\right) + x^3\left(a^4cd^2 + 4a^3bc^2d + 2a^2b^2c^3\right) + x^2\cdot\left(\frac{3a^4c^2d}{2} + 2a^3bc^3\right)$$

input
```
integrate((b*x+a)*(a*c+(a*d+b*c)*x+b*d*x**2)**3,x)
```

output
```
a**4*c**3*x + b**4*d**3*x**8/8 + x**7*(4*a*b**3*d**3/7 + 3*b**4*c*d**2/7)
+ x**6*(a**2*b**2*d**3 + 2*a*b**3*c*d**2 + b**4*c**2*d/2) + x**5*(4*a**3*b
*d**3/5 + 18*a**2*b**2*c*d**2/5 + 12*a*b**3*c**2*d/5 + b**4*c**3/5) + x**4
*(a**4*d**3/4 + 3*a**3*b*c*d**2 + 9*a**2*b**2*c**2*d/2 + a*b**3*c**3) + x*
*3*(a**4*c*d**2 + 4*a**3*b*c**2*d + 2*a**2*b**2*c**3) + x**2*(3*a**4*c**2*
d/2 + 2*a**3*b*c**3)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 225 vs. $2(84) = 168$.

Time = 0.03 (sec) , antiderivative size = 225, normalized size of antiderivative = 2.45

$$\int (a+bx)\left(ac+(bc+ad)x+bdx^2\right)^3\,dx$$
$$= \frac{1}{8}\,b^4d^3x^8 + a^4c^3x + \frac{1}{7}\left(3\,b^4cd^2 + 4\,ab^3d^3\right)x^7 + \frac{1}{2}\left(b^4c^2d + 4\,ab^3cd^2 + 2\,a^2b^2d^3\right)x^6$$
$$+ \frac{1}{5}\left(b^4c^3 + 12\,ab^3c^2d + 18\,a^2b^2cd^2 + 4\,a^3bd^3\right)x^5$$
$$+ \frac{1}{4}\left(4\,ab^3c^3 + 18\,a^2b^2c^2d + 12\,a^3bcd^2 + a^4d^3\right)x^4$$
$$+ \left(2\,a^2b^2c^3 + 4\,a^3bc^2d + a^4cd^2\right)x^3 + \frac{1}{2}\left(4\,a^3bc^3 + 3\,a^4c^2d\right)x^2$$

input
```
integrate((b*x+a)*(a*c+(a*d+b*c)*x+b*d*x^2)^3,x, algorithm="maxima")
```

output
```
1/8*b^4*d^3*x^8 + a^4*c^3*x + 1/7*(3*b^4*c*d^2 + 4*a*b^3*d^3)*x^7 + 1/2*(b
^4*c^2*d + 4*a*b^3*c*d^2 + 2*a^2*b^2*d^3)*x^6 + 1/5*(b^4*c^3 + 12*a*b^3*c^
2*d + 18*a^2*b^2*c*d^2 + 4*a^3*b*d^3)*x^5 + 1/4*(4*a*b^3*c^3 + 18*a^2*b^2*
c^2*d + 12*a^3*b*c*d^2 + a^4*d^3)*x^4 + (2*a^2*b^2*c^3 + 4*a^3*b*c^2*d + a
^4*c*d^2)*x^3 + 1/2*(4*a^3*b*c^3 + 3*a^4*c^2*d)*x^2
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 245 vs. $2(84) = 168$.

Time = 0.14 (sec) , antiderivative size = 245, normalized size of antiderivative = 2.66

$$\int (a+bx)\left(ac+(bc+ad)x+bdx^2\right)^3\,dx = \frac{1}{8}\,b^4d^3x^8 + \frac{3}{7}\,b^4cd^2x^7 + \frac{4}{7}\,ab^3d^3x^7 + \frac{1}{2}\,b^4c^2dx^6$$
$$+ 2\,ab^3cd^2x^6 + a^2b^2d^3x^6 + \frac{1}{5}\,b^4c^3x^5$$
$$+ \frac{12}{5}\,ab^3c^2dx^5 + \frac{18}{5}\,a^2b^2cd^2x^5 + \frac{4}{5}\,a^3bd^3x^5$$
$$+ ab^3c^3x^4 + \frac{9}{2}\,a^2b^2c^2dx^4 + 3\,a^3bcd^2x^4$$
$$+ \frac{1}{4}\,a^4d^3x^4 + 2\,a^2b^2c^3x^3 + 4\,a^3bc^2dx^3$$
$$+ a^4cd^2x^3 + 2\,a^3bc^3x^2 + \frac{3}{2}\,a^4c^2dx^2 + a^4c^3x$$

input
```
integrate((b*x+a)*(a*c+(a*d+b*c)*x+b*d*x^2)^3,x, algorithm="giac")
```

output
```
1/8*b^4*d^3*x^8 + 3/7*b^4*c*d^2*x^7 + 4/7*a*b^3*d^3*x^7 + 1/2*b^4*c^2*d*x^
6 + 2*a*b^3*c*d^2*x^6 + a^2*b^2*d^3*x^6 + 1/5*b^4*c^3*x^5 + 12/5*a*b^3*c^2
*d*x^5 + 18/5*a^2*b^2*c*d^2*x^5 + 4/5*a^3*b*d^3*x^5 + a*b^3*c^3*x^4 + 9/2*
a^2*b^2*c^2*d*x^4 + 3*a^3*b*c*d^2*x^4 + 1/4*a^4*d^3*x^4 + 2*a^2*b^2*c^3*x^
3 + 4*a^3*b*c^2*d*x^3 + a^4*c*d^2*x^3 + 2*a^3*b*c^3*x^2 + 3/2*a^4*c^2*d*x^
2 + a^4*c^3*x
```

Mupad [B] (verification not implemented)

Time = 5.72 (sec) , antiderivative size = 208, normalized size of antiderivative = 2.26

$$\int (a+bx)\left(ac+(bc+ad)x+bdx^2\right)^3\,dx = x^4\left(\frac{a^4\,d^3}{4}+3\,a^3\,b\,c\,d^2+\frac{9\,a^2\,b^2\,c^2\,d}{2}+a\,b^3\,c^3\right) + x^5\left(\frac{4\,a^3\,b\,d^3}{5}+\frac{18\,a^2\,b^2\,c\,d^2}{5}+\frac{12\,a\,b^3\,c^2\,d}{5}+\frac{b^4\,c^3}{5}\right)+a^4\,c^3\,x + \frac{b^4\,d^3\,x^8}{8}+\frac{a^3\,c^2\,x^2\,(3\,a\,d+4\,b\,c)}{2}+\frac{b^3\,d^2\,x^7\,(4\,a\,d+3\,b\,c)}{7} + a^2\,c\,x^3\left(a^2\,d^2+4\,a\,b\,c\,d+2\,b^2\,c^2\right)+\frac{b^2\,d\,x^6\,(2\,a^2\,d^2+4\,a\,b\,c\,d+b^2\,c^2)}{2}$$

input
```
int((a + b*x)*(a*c + x*(a*d + b*c) + b*d*x^2)^3,x)
```

output
```
x^4*((a^4*d^3)/4 + a*b^3*c^3 + (9*a^2*b^2*c^2*d)/2 + 3*a^3*b*c*d^2) + x^5*
((b^4*c^3)/5 + (4*a^3*b*d^3)/5 + (18*a^2*b^2*c*d^2)/5 + (12*a*b^3*c^2*d)/5
) + a^4*c^3*x + (b^4*d^3*x^8)/8 + (a^3*c^2*x^2*(3*a*d + 4*b*c))/2 + (b^3*d
^2*x^7*(4*a*d + 3*b*c))/7 + a^2*c*x^3*(a^2*d^2 + 2*b^2*c^2 + 4*a*b*c*d) +
(b^2*d*x^6*(2*a^2*d^2 + b^2*c^2 + 4*a*b*c*d))/2
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 247, normalized size of antiderivative = 2.68

$$\int (a+bx)\left(ac+(bc+ad)x+bdx^2\right)^3\,dx$$
$$= \frac{x(35b^4d^3x^7+160a\,b^3d^3x^6+120b^4c\,d^2x^6+280a^2b^2d^3x^5+560a\,b^3c\,d^2x^5+140b^4c^2d\,x^5+224a^3b\,d^3x^4+1}{}$$

input
```
int((b*x+a)*(a*c+(a*d+b*c)*x+b*d*x^2)^3,x)
```

output
```
(x*(280*a**4*c**3 + 420*a**4*c**2*d*x + 280*a**4*c*d**2*x**2 + 70*a**4*d**
3*x**3 + 560*a**3*b*c**3*x + 1120*a**3*b*c**2*d*x**2 + 840*a**3*b*c*d**2*x
**3 + 224*a**3*b*d**3*x**4 + 560*a**2*b**2*c**3*x**2 + 1260*a**2*b**2*c**2
*d*x**3 + 1008*a**2*b**2*c*d**2*x**4 + 280*a**2*b**2*d**3*x**5 + 280*a*b**
3*c**3*x**3 + 672*a*b**3*c**2*d*x**4 + 560*a*b**3*c*d**2*x**5 + 160*a*b**3
*d**3*x**6 + 56*b**4*c**3*x**4 + 140*b**4*c**2*d*x**5 + 120*b**4*c*d**2*x*
*6 + 35*b**4*d**3*x**7))/280
```

3.28 $\int \left(ac+(bc+ad)x+bdx^2\right)^3\,dx$

Optimal result

Integrand size = 21, antiderivative size = 92

$$\int \left(ac+(bc+ad)x+bdx^2\right)^3\,dx=-\frac{(bc-ad)^3(c+dx)^4}{4d^4}+\frac{3b(bc-ad)^2(c+dx)^5}{5d^4}-\frac{b^2(bc-ad)(c+dx)^6}{2d^4}+\frac{b^3(c+dx)^7}{7d^4}$$

output
```
-1/4*(-a*d+b*c)^3*(d*x+c)^4/d^4+3/5*b*(-a*d+b*c)^2*(d*x+c)^5/d^4-1/2*b^2*(
-a*d+b*c)*(d*x+c)^6/d^4+1/7*b^3*(d*x+c)^7/d^4
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 161, normalized size of antiderivative = 1.75

$$\int \left(ac+(bc+ad)x+bdx^2\right)^3\,dx=a^3c^3x+\frac{3}{2}a^2c^2(bc+ad)x^2+ac\left(b^2c^2+3abcd+a^2d^2\right)x^3+\frac{1}{4}\left(b^3c^3+9ab^2c^2d+9a^2bcd^2+a^3d^3\right)x^4+\frac{3}{5}bd\left(b^2c^2+3abcd+a^2d^2\right)x^5+\frac{1}{2}b^2d^2(bc+ad)x^6+\frac{1}{7}b^3d^3x^7$$

input
```
Integrate[(a*c + (b*c + a*d)*x + b*d*x^2)^3,x]
```

output
```
a^3*c^3*x + (3*a^2*c^2*(b*c + a*d)*x^2)/2 + a*c*(b^2*c^2 + 3*a*b*c*d + a^2
*d^2)*x^3 + ((b^3*c^3 + 9*a*b^2*c^2*d + 9*a^2*b*c*d^2 + a^3*d^3)*x^4)/4 +
(3*b*d*(b^2*c^2 + 3*a*b*c*d + a^2*d^2)*x^5)/5 + (b^2*d^2*(b*c + a*d)*x^6)/
2 + (b^3*d^3*x^7)/7
```

Rubi [A] (verified)

Time = 0.54 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.13, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {1084, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(x(ad+bc)+ac+bdx^2\right)^3 dx$$

$$\Big\downarrow \text{1084}$$

$$\frac{\int \left((bc+bdx)^6-3b^5(bc-ad)(c+dx)^5+3b^4(bc-ad)^2(c+dx)^4-b^3(bc-ad)^3(c+dx)^3\right)dx}{b^3d^3}$$

$$\Big\downarrow \text{2009}$$

$$\frac{-\frac{b^5(c+dx)^6(bc-ad)}{2d}+\frac{3b^4(c+dx)^5(bc-ad)^2}{5d}-\frac{b^3(c+dx)^4(bc-ad)^3}{4d}+\frac{b^6(c+dx)^7}{7d}}{b^3d^3}$$

input
```
Int[(a*c + (b*c + a*d)*x + b*d*x^2)^3,x]
```

output
```
(-1/4*(b^3*(b*c - a*d)^3*(c + d*x)^4)/d + (3*b^4*(b*c - a*d)^2*(c + d*x)^5
)/(5*d) - (b^5*(b*c - a*d)*(c + d*x)^6)/(2*d) + (b^6*(c + d*x)^7)/(7*d))/(
b^3*d^3)
```

Defintions of rubi rules used

rule 1084
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{q = Rt[b^2
- 4*a*c, 2]}, Simp[1/c^p   Int[ExpandIntegrand[(b/2 - q/2 + c*x)^p*(b/2 + q
/2 + c*x)^p, x], x], x] /;  !FractionalPowerFactorQ[q]] /; FreeQ[{a, b, c},
 x] && IntegerQ[p] && NiceSqrtQ[b^2 - 4*a*c]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 171 vs. $2(84) = 168$.

Time = 0.96 (sec) , antiderivative size = 172, normalized size of antiderivative = 1.87

method	result
norman	$\frac{b^3d^3x^7}{7}+\left(\frac{1}{2}a\,b^2d^3+\frac{1}{2}b^3c\,d^2\right)x^6+\left(\frac{3}{5}d^3a^2b+\frac{9}{5}a\,b^2c\,d^2+\frac{3}{5}b^3c^2d\right)x^5+\left(\frac{1}{4}a^3d^3+\frac{9}{4}a^2bc\,d^2+\right.$
risch	$\frac{1}{7}b^3d^3x^7+\frac{1}{2}a\,b^2d^3x^6+\frac{1}{2}b^3c\,d^2x^6+\frac{3}{5}a^2b\,d^3x^5+\frac{9}{5}a\,b^2c\,d^2x^5+\frac{3}{5}b^3c^2d\,x^5+\frac{1}{4}x^4a^3d^3+\frac{9}{4}a^2bc$
parallelrisch	$\frac{1}{7}b^3d^3x^7+\frac{1}{2}a\,b^2d^3x^6+\frac{1}{2}b^3c\,d^2x^6+\frac{3}{5}a^2b\,d^3x^5+\frac{9}{5}a\,b^2c\,d^2x^5+\frac{3}{5}b^3c^2d\,x^5+\frac{1}{4}x^4a^3d^3+\frac{9}{4}a^2bc$
gosper	$\frac{x(20b^3d^3x^6+70a\,b^2d^3x^5+70b^3c\,d^2x^5+84a^2b\,d^3x^4+252a\,b^2c\,d^2x^4+84b^3c^2d\,x^4+35a^3d^3x^3+315a^2bc\,d^2x^3+315a\,b^2c^2d\,x^3+35}{140}$
default	$\frac{b^3d^3x^7}{7}+\frac{(ad+bc)b^2d^2x^6}{2}+\frac{\left(a\,b^2c\,d^2+2(ad+bc)^2bd+bd\left(2abcd+(ad+bc)^2\right)\right)x^5}{5}+\frac{\left(4ac(ad+bc)bd+(ad+bc)\left(2abcd+(a\right.\right.}{4}$
orering	$\frac{x(20b^3d^3x^6+70a\,b^2d^3x^5+70b^3c\,d^2x^5+84a^2b\,d^3x^4+252a\,b^2c\,d^2x^4+84b^3c^2d\,x^4+35a^3d^3x^3+315a^2bc\,d^2x^3+315a\,b^2c^2d\,x^3+35}{140(bx+a)^3(dx+}$

input
```
int((a*c+(a*d+b*c)*x+b*d*x^2)^3,x,method=_RETURNVERBOSE)
```

output
```
1/7*b^3*d^3*x^7+(1/2*a*b^2*d^3+1/2*b^3*c*d^2)*x^6+(3/5*d^3*a^2*b+9/5*a*b^2
*c*d^2+3/5*b^3*c^2*d)*x^5+(1/4*a^3*d^3+9/4*a^2*b*c*d^2+9/4*a*b^2*c^2*d+1/4
*b^3*c^3)*x^4+(a^3*c*d^2+3*a^2*b*c^2*d+a*b^2*c^3)*x^3+(3/2*a^3*c^2*d+3/2*a
^2*b*c^3)*x^2+a^3*c^3*x
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 167, normalized size of antiderivative = 1.82

$$\int \left(ac+(bc+ad)x+bdx^2\right)^3\,dx=\frac{1}{7}\,b^3d^3x^7+a^3c^3x+\frac{1}{2}\,\left(b^3cd^2+ab^2d^3\right)x^6+\frac{3}{5}\,\left(b^3c^2d+3\,ab^2cd^2+a^2bd^3\right)x^5+\frac{1}{4}\,\left(b^3c^3+9\,ab^2c^2d+9\,a^2bcd^2+a^3d^3\right)x^4+\left(ab^2c^3+3\,a^2bc^2d+a^3cd^2\right)x^3+\frac{3}{2}\,\left(a^2bc^3+a^3c^2d\right)x^2$$

input
```
integrate((a*c+(a*d+b*c)*x+b*d*x^2)^3,x, algorithm="fricas")
```

output
```
1/7*b^3*d^3*x^7 + a^3*c^3*x + 1/2*(b^3*c*d^2 + a*b^2*d^3)*x^6 + 3/5*(b^3*c
^2*d + 3*a*b^2*c*d^2 + a^2*b*d^3)*x^5 + 1/4*(b^3*c^3 + 9*a*b^2*c^2*d + 9*a
^2*b*c*d^2 + a^3*d^3)*x^4 + (a*b^2*c^3 + 3*a^2*b*c^2*d + a^3*c*d^2)*x^3 +
3/2*(a^2*b*c^3 + a^3*c^2*d)*x^2
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 190 vs. $2(80) = 160$.

Time = 0.05 (sec) , antiderivative size = 190, normalized size of antiderivative = 2.07

$$\int \left(ac+(bc+ad)x+bdx^2\right)^3\,dx=a^3c^3x+\frac{b^3d^3x^7}{7}+x^6\left(\frac{ab^2d^3}{2}+\frac{b^3cd^2}{2}\right)+x^5\cdot\left(\frac{3a^2bd^3}{5}+\frac{9ab^2cd^2}{5}+\frac{3b^3c^2d}{5}\right)+x^4\left(\frac{a^3d^3}{4}+\frac{9a^2bcd^2}{4}+\frac{9ab^2c^2d}{4}+\frac{b^3c^3}{4}\right)+x^3\left(a^3cd^2+3a^2bc^2d+ab^2c^3\right)+x^2\cdot\left(\frac{3a^3c^2d}{2}+\frac{3a^2bc^3}{2}\right)$$

input
```
integrate((a*c+(a*d+b*c)*x+b*d*x**2)**3,x)
```

output
```
a**3*c**3*x + b**3*d**3*x**7/7 + x**6*(a*b**2*d**3/2 + b**3*c*d**2/2) + x*
*5*(3*a**2*b*d**3/5 + 9*a*b**2*c*d**2/5 + 3*b**3*c**2*d/5) + x**4*(a**3*d*
*3/4 + 9*a**2*b*c*d**2/4 + 9*a*b**2*c**2*d/4 + b**3*c**3/4) + x**3*(a**3*c
*d**2 + 3*a**2*b*c**2*d + a*b**2*c**3) + x**2*(3*a**3*c**2*d/2 + 3*a**2*b*
c**3/2)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 140, normalized size of antiderivative = 1.52

$$\begin{aligned}\int \left(ac+(bc+ad)x+bdx^2\right)^3\,dx &= \frac{1}{7}\,b^3d^3x^7+\frac{1}{2}\,(bc+ad)b^2d^2x^6+\frac{3}{5}\,(bc+ad)^2bdx^5+a^3c^3x \\ &\quad +\frac{1}{4}\,(bc+ad)^3x^4+\frac{1}{2}\,\left(2\,bdx^3+3\,(bc+ad)x^2\right)a^2c^2 \\ &\quad +\frac{1}{10}\,\left(6\,b^2d^2x^5+15\,(bc+ad)bdx^4+10\,(bc+ad)^2x^3\right)ac\end{aligned}$$

input
```
integrate((a*c+(a*d+b*c)*x+b*d*x^2)^3,x, algorithm="maxima")
```

output
```
1/7*b^3*d^3*x^7 + 1/2*(b*c + a*d)*b^2*d^2*x^6 + 3/5*(b*c + a*d)^2*b*d*x^5
+ a^3*c^3*x + 1/4*(b*c + a*d)^3*x^4 + 1/2*(2*b*d*x^3 + 3*(b*c + a*d)*x^2)*
a^2*c^2 + 1/10*(6*b^2*d^2*x^5 + 15*(b*c + a*d)*b*d*x^4 + 10*(b*c + a*d)^2*
x^3)*a*c
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 188 vs. 2(84) = 168.

Time = 0.15 (sec) , antiderivative size = 188, normalized size of antiderivative = 2.04

$$\begin{aligned}\int \left(ac+(bc+ad)x+bdx^2\right)^3\,dx &= \frac{1}{7}\,b^3d^3x^7+\frac{1}{2}\,b^3cd^2x^6+\frac{1}{2}\,ab^2d^3x^6+\frac{3}{5}\,b^3c^2dx^5 \\ &\quad +\frac{9}{5}\,ab^2cd^2x^5+\frac{3}{5}\,a^2bd^3x^5+\frac{1}{4}\,b^3c^3x^4+\frac{9}{4}\,ab^2c^2dx^4 \\ &\quad +\frac{9}{4}\,a^2bcd^2x^4+\frac{1}{4}\,a^3d^3x^4+ab^2c^3x^3+3\,a^2bc^2dx^3 \\ &\quad +a^3cd^2x^3+\frac{3}{2}\,a^2bc^3x^2+\frac{3}{2}\,a^3c^2dx^2+a^3c^3x\end{aligned}$$

input
```
integrate((a*c+(a*d+b*c)*x+b*d*x^2)^3,x, algorithm="giac")
```

output
```
1/7*b^3*d^3*x^7 + 1/2*b^3*c*d^2*x^6 + 1/2*a*b^2*d^3*x^6 + 3/5*b^3*c^2*d*x^
5 + 9/5*a*b^2*c*d^2*x^5 + 3/5*a^2*b*d^3*x^5 + 1/4*b^3*c^3*x^4 + 9/4*a*b^2*
c^2*d*x^4 + 9/4*a^2*b*c*d^2*x^4 + 1/4*a^3*d^3*x^4 + a*b^2*c^3*x^3 + 3*a^2*
b*c^2*d*x^3 + a^3*c*d^2*x^3 + 3/2*a^2*b*c^3*x^2 + 3/2*a^3*c^2*d*x^2 + a^3*
c^3*x
```

Mupad [B] (verification not implemented)

Time = 5.61 (sec) , antiderivative size = 152, normalized size of antiderivative = 1.65

$$\int \left(ac+(bc+ad)x+bdx^2\right)^3\,dx=x^4\left(\frac{a^3\,d^3}{4}+\frac{9\,a^2\,b\,c\,d^2}{4}+\frac{9\,a\,b^2\,c^2\,d}{4}+\frac{b^3\,c^3}{4}\right)+a^3\,c^3\,x+\frac{b^3\,d^3\,x^7}{7}+a\,c\,x^3\,\left(a^2\,d^2+3\,a\,b\,c\,d+b^2\,c^2\right)+\frac{3\,b\,d\,x^5\,\left(a^2\,d^2+3\,a\,b\,c\,d+b^2\,c^2\right)}{5}+\frac{3\,a^2\,c^2\,x^2\,\left(a\,d+b\,c\right)}{2}+\frac{b^2\,d^2\,x^6\,\left(a\,d+b\,c\right)}{2}$$

input
```
int((a*c + x*(a*d + b*c) + b*d*x^2)^3,x)
```

output
```
x^4*((a^3*d^3)/4 + (b^3*c^3)/4 + (9*a*b^2*c^2*d)/4 + (9*a^2*b*c*d^2)/4) +
a^3*c^3*x + (b^3*d^3*x^7)/7 + a*c*x^3*(a^2*d^2 + b^2*c^2 + 3*a*b*c*d) + (3
*b*d*x^5*(a^2*d^2 + b^2*c^2 + 3*a*b*c*d))/5 + (3*a^2*c^2*x^2*(a*d + b*c))/
2 + (b^2*d^2*x^6*(a*d + b*c))/2
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 189, normalized size of antiderivative = 2.05

$$\int (ac + (bc + ad)x + bdx^2)^3 \, dx$$
$$= \frac{x(20b^3d^3x^6 + 70a\,b^2d^3x^5 + 70b^3c\,d^2x^5 + 84a^2b\,d^3x^4 + 252a\,b^2c\,d^2x^4 + 84b^3c^2d\,x^4 + 35a^3d^3x^3 + 315a^2bc\,d}{}$$

input
```
int((a*c+(a*d+b*c)*x+b*d*x^2)^3,x)
```

output
```
(x*(140*a**3*c**3 + 210*a**3*c**2*d*x + 140*a**3*c*d**2*x**2 + 35*a**3*d**
3*x**3 + 210*a**2*b*c**3*x + 420*a**2*b*c**2*d*x**2 + 315*a**2*b*c*d**2*x*
*3 + 84*a**2*b*d**3*x**4 + 140*a*b**2*c**3*x**2 + 315*a*b**2*c**2*d*x**3 +
 252*a*b**2*c*d**2*x**4 + 70*a*b**2*d**3*x**5 + 35*b**3*c**3*x**3 + 84*b**
3*c**2*d*x**4 + 70*b**3*c*d**2*x**5 + 20*b**3*d**3*x**6))/140
```

3.29 $\int \frac{(ac+(bc+ad)x+bdx^2)^3}{a+bx}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 65

$$\int \frac{(ac+(bc+ad)x+bdx^2)^3}{a+bx}\,dx = \frac{(bc-ad)^2(c+dx)^4}{4d^3} - \frac{2b(bc-ad)(c+dx)^5}{5d^3} + \frac{b^2(c+dx)^6}{6d^3}$$

output
```
1/4*(-a*d+b*c)^2*(d*x+c)^4/d^3-2/5*b*(-a*d+b*c)*(d*x+c)^5/d^3+1/6*b^2*(d*x
+c)^6/d^3
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 122, normalized size of antiderivative = 1.88

$$\int \frac{(ac+(bc+ad)x+bdx^2)^3}{a+bx}\,dx = a^2c^3x + \frac{1}{2}ac^2(2bc+3ad)x^2 + \frac{1}{3}c\left(b^2c^2+6abcd+3a^2d^2\right)x^3 + \frac{1}{4}d\left(3b^2c^2+6abcd+a^2d^2\right)x^4 + \frac{1}{5}bd^2(3bc+2ad)x^5 + \frac{1}{6}b^2d^3x^6$$

input
```
Integrate[(a*c + (b*c + a*d)*x + b*d*x^2)^3/(a + b*x),x]
```

output
```
a^2*c^3*x + (a*c^2*(2*b*c + 3*a*d)*x^2)/2 + (c*(b^2*c^2 + 6*a*b*c*d + 3*a^
2*d^2)*x^3)/3 + (d*(3*b^2*c^2 + 6*a*b*c*d + a^2*d^2)*x^4)/4 + (b*d^2*(3*b*
c + 2*a*d)*x^5)/5 + (b^2*d^3*x^6)/6
```

Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.069$, Rules used = {1121, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(x(ad+bc)+ac+bdx^2\right)^3}{a+bx}\,dx$$

$$\Big\downarrow \text{ 1121}$$

$$\int \left(-\frac{2b(c+dx)^4(bc-ad)}{d^2}+\frac{(c+dx)^3(ad-bc)^2}{d^2}+\frac{b^2(c+dx)^5}{d^2}\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$-\frac{2b(c+dx)^5(bc-ad)}{5d^3}+\frac{(c+dx)^4(bc-ad)^2}{4d^3}+\frac{b^2(c+dx)^6}{6d^3}$$

input
```
Int[(a*c + (b*c + a*d)*x + b*d*x^2)^3/(a + b*x),x]
```

output
```
((b*c - a*d)^2*(c + d*x)^4)/(4*d^3) - (2*b*(b*c - a*d)*(c + d*x)^5)/(5*d^3
) + (b^2*(c + d*x)^6)/(6*d^3)
```

Defintions of rubi rules used

rule 1121
```
Int[((d_) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Int[ExpandIntegrand[(d + e*x)^(m + p)*(a/d + (c/e)*x)^p, x], x]
/; FreeQ[{a, b, c, d, e, m, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && (Int
egerQ[p] || (GtQ[a, 0] && GtQ[d, 0] && LtQ[c, 0]))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 120 vs. $2(59) = 118$.

Time = 1.36 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.86

method	result
norman	$\frac{b^2 d^3 x^6}{6} + (\frac{2}{5}ab\,d^3 + \frac{3}{5}b^2 c\,d^2)\,x^5 + (\frac{1}{4}a^2 d^3 + \frac{3}{2}abc\,d^2 + \frac{3}{4}b^2 c^2 d)\,x^4 + (a^2 c\,d^2 + 2ab\,c^2 d + \frac{1}{3}c^3 b^2$
risch	$\frac{1}{6}b^2 d^3 x^6 + \frac{2}{5}ab\,d^3 x^5 + \frac{3}{5}b^2 c\,d^2 x^5 + \frac{1}{4}x^4 a^2 d^3 + \frac{3}{2}abc\,d^2 x^4 + \frac{3}{4}b^2 c^2 d\,x^4 + x^3 a^2 c\,d^2 + 2abd\,x^3 c^2$
parallelrisch	$\frac{1}{6}b^2 d^3 x^6 + \frac{2}{5}ab\,d^3 x^5 + \frac{3}{5}b^2 c\,d^2 x^5 + \frac{1}{4}x^4 a^2 d^3 + \frac{3}{2}abc\,d^2 x^4 + \frac{3}{4}b^2 c^2 d\,x^4 + x^3 a^2 c\,d^2 + 2abd\,x^3 c^2$
gosper	$\frac{x(10b^2 d^3 x^5+24ab\,d^3 x^4+36b^2 c\,d^2 x^4+15a^2 d^3 x^3+90abc\,d^2 x^3+45b^2 c^2 d\,x^3+60a^2 c\,d^2 x^2+120ab\,c^2 x^2 d+20b^2 c^3 x^2+90a^2 c^2 dx+}{60}$
default	$\frac{b^2 d^3 x^6}{6} + \frac{(b^2 c\,d^2+2d^2 b(ad+bc))x^5}{5} + \frac{(2cbd(ad+bc)+d(2abcd+(ad+bc)^2))x^4}{4} + \frac{(c(2abcd+(ad+bc)^2)+2dac(ad+bc)}{3}$
orering	$\frac{x(10b^2 d^3 x^5+24ab\,d^3 x^4+36b^2 c\,d^2 x^4+15a^2 d^3 x^3+90abc\,d^2 x^3+45b^2 c^2 d\,x^3+60a^2 c\,d^2 x^2+120ab\,c^2 x^2 d+20b^2 c^3 x^2+90a^2 c^2 dx+}{60(bx+a)^3(dx+c)^3}$

input
```
int((a*c+(a*d+b*c)*x+b*d*x^2)^3/(b*x+a),x,method=_RETURNVERBOSE)
```

output
```
1/6*b^2*d^3*x^6+(2/5*a*b*d^3+3/5*b^2*c*d^2)*x^5+(1/4*a^2*d^3+3/2*a*b*c*d^2
+3/4*b^2*c^2*d)*x^4+(a^2*c*d^2+2*a*b*c^2*d+1/3*c^3*b^2)*x^3+(3/2*a^2*c^2*d
+a*b*c^3)*x^2+a^2*c^3*x
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 124 vs. $2(59) = 118$.

Time = 0.07 (sec) , antiderivative size = 124, normalized size of antiderivative = 1.91

$$\int \frac{(ac + (bc + ad)x + bdx^2)^3}{a + bx} \, dx = \frac{1}{6} b^2 d^3 x^6 + a^2 c^3 x + \frac{1}{5} \left(3 b^2 c d^2 + 2 a b d^3\right) x^5 + \frac{1}{4} \left(3 b^2 c^2 d + 6 a b c d^2 + a^2 d^3\right) x^4 + \frac{1}{3} \left(b^2 c^3 + 6 a b c^2 d + 3 a^2 c d^2\right) x^3 + \frac{1}{2} \left(2 a b c^3 + 3 a^2 c^2 d\right) x^2$$

input
```
integrate((a*c+(a*d+b*c)*x+b*d*x^2)^3/(b*x+a),x, algorithm="fricas")
```

output
```
1/6*b^2*d^3*x^6 + a^2*c^3*x + 1/5*(3*b^2*c*d^2 + 2*a*b*d^3)*x^5 + 1/4*(3*b
^2*c^2*d + 6*a*b*c*d^2 + a^2*d^3)*x^4 + 1/3*(b^2*c^3 + 6*a*b*c^2*d + 3*a^2
*c*d^2)*x^3 + 1/2*(2*a*b*c^3 + 3*a^2*c^2*d)*x^2
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 133 vs. $2(56) = 112$.

Time = 0.08 (sec) , antiderivative size = 133, normalized size of antiderivative = 2.05

$$\int \frac{(ac + (bc + ad)x + bdx^2)^3}{a + bx} \, dx = a^2 c^3 x + \frac{b^2 d^3 x^6}{6} + x^5 \cdot \left(\frac{2abd^3}{5} + \frac{3b^2 c d^2}{5}\right) + x^4 \left(\frac{a^2 d^3}{4} + \frac{3abcd^2}{2} + \frac{3b^2 c^2 d}{4}\right) + x^3 \left(a^2 c d^2 + 2abc^2 d + \frac{b^2 c^3}{3}\right) + x^2 \cdot \left(\frac{3a^2 c^2 d}{2} + abc^3\right)$$

input
```
integrate((a*c+(a*d+b*c)*x+b*d*x**2)**3/(b*x+a),x)
```

output
```
a**2*c**3*x + b**2*d**3*x**6/6 + x**5*(2*a*b*d**3/5 + 3*b**2*c*d**2/5) + x
**4*(a**2*d**3/4 + 3*a*b*c*d**2/2 + 3*b**2*c**2*d/4) + x**3*(a**2*c*d**2 +
 2*a*b*c**2*d + b**2*c**3/3) + x**2*(3*a**2*c**2*d/2 + a*b*c**3)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 124 vs. $2(59) = 118$.

Time = 0.03 (sec) , antiderivative size = 124, normalized size of antiderivative = 1.91

$$\int \frac{(ac+(bc+ad)x+bdx^2)^3}{a+bx}\,dx = \frac{1}{6}\,b^2d^3x^6 + a^2c^3x + \frac{1}{5}\left(3\,b^2cd^2 + 2\,abd^3\right)x^5 + \frac{1}{4}\left(3\,b^2c^2d + 6\,abcd^2 + a^2d^3\right)x^4 + \frac{1}{3}\left(b^2c^3 + 6\,abc^2d + 3\,a^2cd^2\right)x^3 + \frac{1}{2}\left(2\,abc^3 + 3\,a^2c^2d\right)x^2$$

input
```
integrate((a*c+(a*d+b*c)*x+b*d*x^2)^3/(b*x+a),x, algorithm="maxima")
```

output
```
1/6*b^2*d^3*x^6 + a^2*c^3*x + 1/5*(3*b^2*c*d^2 + 2*a*b*d^3)*x^5 + 1/4*(3*b
^2*c^2*d + 6*a*b*c*d^2 + a^2*d^3)*x^4 + 1/3*(b^2*c^3 + 6*a*b*c^2*d + 3*a^2
*c*d^2)*x^3 + 1/2*(2*a*b*c^3 + 3*a^2*c^2*d)*x^2
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 130 vs. $2(59) = 118$.

Time = 0.13 (sec) , antiderivative size = 130, normalized size of antiderivative = 2.00

$$\int \frac{(ac+(bc+ad)x+bdx^2)^3}{a+bx}\,dx = \frac{1}{6}\,b^2d^3x^6 + \frac{3}{5}\,b^2cd^2x^5 + \frac{2}{5}\,abd^3x^5 + \frac{3}{4}\,b^2c^2dx^4 + \frac{3}{2}\,abcd^2x^4 + \frac{1}{4}\,a^2d^3x^4 + \frac{1}{3}\,b^2c^3x^3 + 2\,abc^2dx^3 + a^2cd^2x^3 + abc^3x^2 + \frac{3}{2}\,a^2c^2dx^2 + a^2c^3x$$

input
```
integrate((a*c+(a*d+b*c)*x+b*d*x^2)^3/(b*x+a),x, algorithm="giac")
```

output
```
1/6*b^2*d^3*x^6 + 3/5*b^2*c*d^2*x^5 + 2/5*a*b*d^3*x^5 + 3/4*b^2*c^2*d*x^4
+ 3/2*a*b*c*d^2*x^4 + 1/4*a^2*d^3*x^4 + 1/3*b^2*c^3*x^3 + 2*a*b*c^2*d*x^3
+ a^2*c*d^2*x^3 + a*b*c^3*x^2 + 3/2*a^2*c^2*d*x^2 + a^2*c^3*x
```

Mupad [B] (verification not implemented)

Time = 5.58 (sec) , antiderivative size = 115, normalized size of antiderivative = 1.77

$$\int \frac{(ac+(bc+ad)x+bdx^2)^3}{a+bx}\,dx = x^3\left(a^2\,c\,d^2+2\,a\,b\,c^2\,d+\frac{b^2\,c^3}{3}\right) + x^4\left(\frac{a^2\,d^3}{4}+\frac{3\,a\,b\,c\,d^2}{2}+\frac{3\,b^2\,c^2\,d}{4}\right)+a^2\,c^3\,x+\frac{b^2\,d^3\,x^6}{6} + \frac{a\,c^2\,x^2\,(3\,a\,d+2\,b\,c)}{2}+\frac{b\,d^2\,x^5\,(2\,a\,d+3\,b\,c)}{5}$$

input
```
int((a*c + x*(a*d + b*c) + b*d*x^2)^3/(a + b*x),x)
```

output
```
x^3*((b^2*c^3)/3 + a^2*c*d^2 + 2*a*b*c^2*d) + x^4*((a^2*d^3)/4 + (3*b^2*c^
2*d)/4 + (3*a*b*c*d^2)/2) + a^2*c^3*x + (b^2*d^3*x^6)/6 + (a*c^2*x^2*(3*a*
d + 2*b*c))/2 + (b*d^2*x^5*(2*a*d + 3*b*c))/5
```

Reduce [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 131, normalized size of antiderivative = 2.02

$$\int \frac{(ac+(bc+ad)x+bdx^2)^3}{a+bx}\,dx = \frac{x(10b^2d^3x^5+24ab\,d^3x^4+36b^2c\,d^2x^4+15a^2d^3x^3+90abc\,d^2x^3+45b^2c^2d\,x^3+60a^2c\,d^2x^2+120ab\,c^2d\,x^2}{60}$$

input
```
int((a*c+(a*d+b*c)*x+b*d*x^2)^3/(b*x+a),x)
```

output
```
(x*(60*a**2*c**3 + 90*a**2*c**2*d*x + 60*a**2*c*d**2*x**2 + 15*a**2*d**3*x
**3 + 60*a*b*c**3*x + 120*a*b*c**2*d*x**2 + 90*a*b*c*d**2*x**3 + 24*a*b*d*
*3*x**4 + 20*b**2*c**3*x**2 + 45*b**2*c**2*d*x**3 + 36*b**2*c*d**2*x**4 +
10*b**2*d**3*x**5))/60
```

3.30 $\int \frac{(ac+(bc+ad)x+bdx^2)^3}{(a+bx)^2} dx$

Optimal result

Integrand size = 29, antiderivative size = 38

$$\int \frac{(ac+(bc+ad)x+bdx^2)^3}{(a+bx)^2} dx = -\frac{(bc-ad)(c+dx)^4}{4d^2} + \frac{b(c+dx)^5}{5d^2}$$

output
```
-1/4*(-a*d+b*c)*(d*x+c)^4/d^2+1/5*b*(d*x+c)^5/d^2
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.76

$$\int \frac{(ac+(bc+ad)x+bdx^2)^3}{(a+bx)^2} dx = ac^3x + \frac{1}{2}c^2(bc+3ad)x^2 + cd(bc+ad)x^3 + \frac{1}{4}d^2(3bc+ad)x^4 + \frac{1}{5}bd^3x^5$$

input
```
Integrate[(a*c + (b*c + a*d)*x + b*d*x^2)^3/(a + b*x)^2,x]
```

output
```
a*c^3*x + (c^2*(b*c + 3*a*d)*x^2)/2 + c*d*(b*c + a*d)*x^3 + (d^2*(3*b*c +
a*d)*x^4)/4 + (b*d^3*x^5)/5
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.069, Rules used = {1121, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(x(ad+bc)+ac+bdx^2)^3}{(a+bx)^2}dx$$

$$\Big\downarrow \text{1121}$$

$$\int \left(\frac{(c+dx)^3(ad-bc)}{d}+\frac{b(c+dx)^4}{d}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{b(c+dx)^5}{5d^2}-\frac{(c+dx)^4(bc-ad)}{4d^2}$$

input
```
Int[(a*c + (b*c + a*d)*x + b*d*x^2)^3/(a + b*x)^2,x]
```

output
```
-1/4*((b*c - a*d)*(c + d*x)^4)/d^2 + (b*(c + d*x)^5)/(5*d^2)
```

Defintions of rubi rules used

rule 1121
```
Int[((d_) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Int[ExpandIntegrand[(d + e*x)^(m + p)*(a/d + (c/e)*x)^p, x], x]
/; FreeQ[{a, b, c, d, e, m, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && (Int
egerQ[p] || (GtQ[a, 0] && GtQ[d, 0] && LtQ[c, 0]))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 72 vs. $2(34) = 68$.

Time = 1.35 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.92

method	result
risch	$\frac{1}{5}b\,d^3x^5+\frac{1}{4}a\,d^3x^4+\frac{3}{4}bc\,d^2x^4+ac\,d^2x^3+b\,c^2d\,x^3+\frac{3}{2}a\,c^2d\,x^2+\frac{1}{2}b\,c^3x^2+a\,c^3x$
parallelrisch	$\frac{1}{5}b\,d^3x^5+\frac{1}{4}a\,d^3x^4+\frac{3}{4}bc\,d^2x^4+ac\,d^2x^3+b\,c^2d\,x^3+\frac{3}{2}a\,c^2d\,x^2+\frac{1}{2}b\,c^3x^2+a\,c^3x$
gosper	$\frac{x(4b\,d^3x^4+5a\,x^3d^3+15x^3bc\,d^2+20a\,d^2x^2c+20b\,c^2d\,x^2+30adx\,c^2+10b\,c^3x+20c^3a)}{20}$
default	$\frac{b\,d^3x^5}{5}+\frac{(2bc\,d^2+d^2(ad+bc))x^4}{4}+\frac{(b\,c^2d+2cd(ad+bc)+a\,d^2c)x^3}{3}+\frac{(c^2(ad+bc)+2a\,c^2d)x^2}{2}+a\,c^3x$
orering	$\frac{x(4b\,d^3x^4+5a\,x^3d^3+15x^3bc\,d^2+20a\,d^2x^2c+20b\,c^2d\,x^2+30adx\,c^2+10b\,c^3x+20c^3a)(ac+(ad+bc)x+bd\,x^2)^3}{20(bx+a)^3(dx+c)^3}$
norman	$\frac{(\frac{9}{20}ab\,d^3+\frac{3}{4}b^2c\,d^2)x^5+(\frac{3}{2}a^2c^2d+\frac{3}{2}ab\,c^3)x^2+(\frac{1}{4}a^2d^3+\frac{7}{4}abc\,d^2+b^2c^2d)x^4+(a^2c\,d^2+\frac{5}{2}ab\,c^2d+\frac{1}{2}c^3b^2)x^3+a^2c^3x+\frac{b^2d^3x^6}{5}}{bx+a}$

input
```
int((a*c+(a*d+b*c)*x+b*d*x^2)^3/(b*x+a)^2,x,method=_RETURNVERBOSE)
```

output
```
1/5*b*d^3*x^5+1/4*a*d^3*x^4+3/4*b*c*d^2*x^4+a*c*d^2*x^3+b*c^2*d*x^3+3/2*a*
c^2*d*x^2+1/2*b*c^3*x^2+a*c^3*x
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 69 vs. $2(34) = 68$.

Time = 0.07 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.82

$$\int \frac{(ac+(bc+ad)x+bdx^2)^3}{(a+bx)^2}\,dx = \frac{1}{5}\,bd^3x^5+ac^3x+\frac{1}{4}\,(3\,bcd^2+ad^3)x^4 + (bc^2d+acd^2)x^3+\frac{1}{2}\,(bc^3+3\,ac^2d)x^2$$

input
```
integrate((a*c+(a*d+b*c)*x+b*d*x^2)^3/(b*x+a)^2,x, algorithm="fricas")
```

output
```
1/5*b*d^3*x^5 + a*c^3*x + 1/4*(3*b*c*d^2 + a*d^3)*x^4 + (b*c^2*d + a*c*d^2
)*x^3 + 1/2*(b*c^3 + 3*a*c^2*d)*x^2
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 73 vs. $2(32) = 64$.

Time = 0.08 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.92

$$\int \frac{(ac+(bc+ad)x+bdx^2)^3}{(a+bx)^2}\,dx = ac^3x + \frac{bd^3x^5}{5} + x^4\left(\frac{ad^3}{4} + \frac{3bcd^2}{4}\right) + x^3\left(acd^2 + bc^2d\right) + x^2\cdot\left(\frac{3ac^2d}{2} + \frac{bc^3}{2}\right)$$

input
```
integrate((a*c+(a*d+b*c)*x+b*d*x**2)**3/(b*x+a)**2,x)
```

output
```
a*c**3*x + b*d**3*x**5/5 + x**4*(a*d**3/4 + 3*b*c*d**2/4) + x**3*(a*c*d**2
 + b*c**2*d) + x**2*(3*a*c**2*d/2 + b*c**3/2)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 69 vs. $2(34) = 68$.

Time = 0.03 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.82

$$\int \frac{(ac+(bc+ad)x+bdx^2)^3}{(a+bx)^2}\,dx = \frac{1}{5}\,bd^3x^5 + ac^3x + \frac{1}{4}\left(3\,bcd^2 + ad^3\right)x^4 + \left(bc^2d + acd^2\right)x^3 + \frac{1}{2}\left(bc^3 + 3\,ac^2d\right)x^2$$

input
```
integrate((a*c+(a*d+b*c)*x+b*d*x^2)^3/(b*x+a)^2,x, algorithm="maxima")
```

output
```
1/5*b*d^3*x^5 + a*c^3*x + 1/4*(3*b*c*d^2 + a*d^3)*x^4 + (b*c^2*d + a*c*d^2
)*x^3 + 1/2*(b*c^3 + 3*a*c^2*d)*x^2
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 155 vs. $2(34) = 68$.

Time = 0.14 (sec) , antiderivative size = 155, normalized size of antiderivative = 4.08

$$\int \frac{(ac + (bc + ad)x + bdx^2)^3}{(a + bx)^2}\, dx = \frac{\left(\frac{10\,b^3c^3}{(bx+a)^3} + \frac{20\,b^2c^2d}{(bx+a)^2} - \frac{30\,ab^2c^2d}{(bx+a)^3} + \frac{15\,bcd^2}{bx+a} - \frac{40\,abcd^2}{(bx+a)^2} + \frac{30\,a^2bcd^2}{(bx+a)^3} - \frac{15\,ad^3}{bx+a} + \frac{20\,a^2d^3}{(bx+a)^2} - \frac{10\,a^3d^3}{(bx+a)^3} + 4\,d^3\right)(bx + a)^5}{20\,b^4}$$

input
```
integrate((a*c+(a*d+b*c)*x+b*d*x^2)^3/(b*x+a)^2,x, algorithm="giac")
```

output
```
1/20*(10*b^3*c^3/(b*x + a)^3 + 20*b^2*c^2*d/(b*x + a)^2 - 30*a*b^2*c^2*d/(
b*x + a)^3 + 15*b*c*d^2/(b*x + a) - 40*a*b*c*d^2/(b*x + a)^2 + 30*a^2*b*c*
d^2/(b*x + a)^3 - 15*a*d^3/(b*x + a) + 20*a^2*d^3/(b*x + a)^2 - 10*a^3*d^3
/(b*x + a)^3 + 4*d^3)*(b*x + a)^5/b^4
```

Mupad [B] (verification not implemented)

Time = 5.93 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.71

$$\int \frac{(ac + (bc + ad)x + bdx^2)^3}{(a + bx)^2}\, dx = x^2\left(\frac{b\,c^3}{2} + \frac{3\,a\,d\,c^2}{2}\right) + x^4\left(\frac{a\,d^3}{4} + \frac{3\,b\,c\,d^2}{4}\right) + \frac{b\,d^3\,x^5}{5} + a\,c^3\,x + c\,d\,x^3\,(a\,d + b\,c)$$

input
```
int((a*c + x*(a*d + b*c) + b*d*x^2)^3/(a + b*x)^2,x)
```

output
```
x^2*((b*c^3)/2 + (3*a*c^2*d)/2) + x^4*((a*d^3)/4 + (3*b*c*d^2)/4) + (b*d^3
*x^5)/5 + a*c^3*x + c*d*x^3*(a*d + b*c)
```

Reduce [B] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.92

$$\int \frac{(ac+(bc+ad)x+bdx^2)^3}{(a+bx)^2}\,dx$$
$$= \frac{x(4b\,d^3x^4+5a\,d^3x^3+15bc\,d^2x^3+20ac\,d^2x^2+20b\,c^2d\,x^2+30a\,c^2dx+10b\,c^3x+20a\,c^3)}{20}$$

input
```
int((a*c+(a*d+b*c)*x+b*d*x^2)^3/(b*x+a)^2,x)
```

output
```
(x*(20*a*c**3 + 30*a*c**2*d*x + 20*a*c*d**2*x**2 + 5*a*d**3*x**3 + 10*b*c*
*3*x + 20*b*c**2*d*x**2 + 15*b*c*d**2*x**3 + 4*b*d**3*x**4))/20
```

3.31 $\int \frac{(ac+(bc+ad)x+bdx^2)^3}{(a+bx)^3} dx$

Optimal result

Integrand size = 29, antiderivative size = 14

$$\int \frac{(ac+(bc+ad)x+bdx^2)^3}{(a+bx)^3} dx = \frac{(c+dx)^4}{4d}$$

output
```
1/4*(d*x+c)^4/d
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00

$$\int \frac{(ac+(bc+ad)x+bdx^2)^3}{(a+bx)^3} dx = \frac{(c+dx)^4}{4d}$$

input
```
Integrate[(a*c + (b*c + a*d)*x + b*d*x^2)^3/(a + b*x)^3,x]
```

output
```
(c + d*x)^4/(4*d)
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.069$, Rules used = {1120, 17}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(x(ad+bc)+ac+bdx^2)^3}{(a+bx)^3}dx$$

$$\Big\downarrow \ 1120$$

$$\int (c+dx)^3dx$$

$$\Big\downarrow \ 17$$

$$\frac{(c+dx)^4}{4d}$$

input
```
Int[(a*c + (b*c + a*d)*x + b*d*x^2)^3/(a + b*x)^3,x]
```

output
```
(c + d*x)^4/(4*d)
```

Defintions of rubi rules used

rule 17
```
Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1
)/(b*(m + 1))), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]
```

rule 1120
```
Int[((d_) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Int[(d + e*x)^(m + p)*(a/d + (c/e)*x)^p, x] /; FreeQ[{a, b, c, d
, e, m}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[p] && (EqQ[m + p,
0] || EqQ[m + 2*p + 2, 0])
```

Maple [A] (verified)

Time = 1.21 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.93

method	result
default	$\frac{(dx+c)^4}{4d}$
parallelrisch	$\frac{1}{4}d^3x^4+c\,d^2x^3+\frac{3}{2}c^2d\,x^2+c^3x$
gosper	$\frac{x\left(d^3x^3+4c\,d^2x^2+6c^2dx+4c^3\right)}{4}$
risch	$\frac{d^3x^4}{4}+c\,d^2x^3+\frac{3c^2d\,x^2}{2}+c^3x+\frac{c^4}{4d}$
orering	$\frac{x\left(d^3x^3+4c\,d^2x^2+6c^2dx+4c^3\right)\left(ac+(ad+bc)x+bd\,x^2\right)^3}{4(dx+c)^3(bx+a)^3}$
norman	$\frac{\left(\frac{1}{2}ab\,d^3+b^2c\,d^2\right)x^5+\left(\frac{1}{4}a^2d^3+2abc\,d^2+\frac{3}{2}b^2c^2d\right)x^4+\left(a^2c\,d^2+3ab\,c^2d+c^3b^2\right)x^3-\frac{a^2\left(3a^2c^2d+4ab\,c^3\right)}{2b^2}+\frac{b^2d^3x^6}{4}-\frac{a\left(3a^2c^2d+3ab\right.}{b}}{(bx+a)^2}$

input
```
int((a*c+(a*d+b*c)*x+b*d*x^2)^3/(b*x+a)^3,x,method=_RETURNVERBOSE)
```

output
```
1/4*(d*x+c)^4/d
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 31 vs. 2(12) = 24.

Time = 0.06 (sec) , antiderivative size = 31, normalized size of antiderivative = 2.21

$$\int \frac{(ac+(bc+ad)x+bdx^2)^3}{(a+bx)^3}\,dx = \frac{1}{4}\,d^3x^4+cd^2x^3+\frac{3}{2}\,c^2dx^2+c^3x$$

input
```
integrate((a*c+(a*d+b*c)*x+b*d*x^2)^3/(b*x+a)^3,x, algorithm="fricas")
```

output
```
1/4*d^3*x^4 + c*d^2*x^3 + 3/2*c^2*d*x^2 + c^3*x
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 32 vs. $2(8) = 16$.

Time = 0.08 (sec) , antiderivative size = 32, normalized size of antiderivative = 2.29

$$\int \frac{(ac + (bc + ad)x + bdx^2)^3}{(a + bx)^3}\, dx = c^3x + \frac{3c^2dx^2}{2} + cd^2x^3 + \frac{d^3x^4}{4}$$

input
```
integrate((a*c+(a*d+b*c)*x+b*d*x**2)**3/(b*x+a)**3,x)
```

output
```
c**3*x + 3*c**2*d*x**2/2 + c*d**2*x**3 + d**3*x**4/4
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 31 vs. $2(12) = 24$.

Time = 0.03 (sec) , antiderivative size = 31, normalized size of antiderivative = 2.21

$$\int \frac{(ac + (bc + ad)x + bdx^2)^3}{(a + bx)^3}\, dx = \frac{1}{4}\, d^3x^4 + cd^2x^3 + \frac{3}{2}\, c^2dx^2 + c^3x$$

input
```
integrate((a*c+(a*d+b*c)*x+b*d*x^2)^3/(b*x+a)^3,x, algorithm="maxima")
```

output
```
1/4*d^3*x^4 + c*d^2*x^3 + 3/2*c^2*d*x^2 + c^3*x
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 31 vs. $2(12) = 24$.

Time = 0.16 (sec) , antiderivative size = 31, normalized size of antiderivative = 2.21

$$\int \frac{(ac + (bc + ad)x + bdx^2)^3}{(a + bx)^3}\, dx = \frac{1}{4}\, d^3x^4 + cd^2x^3 + \frac{3}{2}\, c^2dx^2 + c^3x$$

input
```
integrate((a*c+(a*d+b*c)*x+b*d*x^2)^3/(b*x+a)^3,x, algorithm="giac")
```

output
```
1/4*d^3*x^4 + c*d^2*x^3 + 3/2*c^2*d*x^2 + c^3*x
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 31, normalized size of antiderivative = 2.21

$$\int \frac{(ac+(bc+ad)x+bdx^2)^3}{(a+bx)^3}\,dx = c^3\,x+\frac{3\,c^2\,d\,x^2}{2}+c\,d^2\,x^3+\frac{d^3\,x^4}{4}$$

input
```
int((a*c + x*(a*d + b*c) + b*d*x^2)^3/(a + b*x)^3,x)
```

output
```
c^3*x + (d^3*x^4)/4 + (3*c^2*d*x^2)/2 + c*d^2*x^3
```

Reduce [B] (verification not implemented)

Time = 0.35 (sec) , antiderivative size = 32, normalized size of antiderivative = 2.29

$$\int \frac{(ac+(bc+ad)x+bdx^2)^3}{(a+bx)^3}\,dx = \frac{x(d^3x^3+4c\,d^2x^2+6c^2dx+4c^3)}{4}$$

input
```
int((a*c+(a*d+b*c)*x+b*d*x^2)^3/(b*x+a)^3,x)
```

output
```
(x*(4*c**3 + 6*c**2*d*x + 4*c*d**2*x**2 + d**3*x**3))/4
```

3.32 $\int \frac{(ac+(bc+ad)x+bdx^2)^3}{(a+bx)^4} dx$

Optimal result

Integrand size = 29, antiderivative size = 73

$$\int \frac{(ac+(bc+ad)x+bdx^2)^3}{(a+bx)^4} dx = \frac{d(bc-ad)^2x}{b^3} + \frac{(bc-ad)(c+dx)^2}{2b^2} + \frac{(c+dx)^3}{3b} + \frac{(bc-ad)^3\log(a+bx)}{b^4}$$

output
```
d*(-a*d+b*c)^2*x/b^3+1/2*(-a*d+b*c)*(d*x+c)^2/b^2+1/3*(d*x+c)^3/b+(-a*d+b*
c)^3*ln(b*x+a)/b^4
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.01

$$\int \frac{(ac+(bc+ad)x+bdx^2)^3}{(a+bx)^4} dx = \frac{bdx(6a^2d^2-3abd(6c+dx)+b^2(18c^2+9cdx+2d^2x^2))+6(bc-ad)^3\log(a+bx)}{6b^4}$$

input
```
Integrate[(a*c + (b*c + a*d)*x + b*d*x^2)^3/(a + b*x)^4,x]
```

output
```
(b*d*x*(6*a^2*d^2 - 3*a*b*d*(6*c + d*x) + b^2*(18*c^2 + 9*c*d*x + 2*d^2*x^
2)) + 6*(b*c - a*d)^3*Log[a + b*x])/(6*b^4)
```

Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.069$, Rules used = $\{1121, 2009\}$

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(x(ad+bc)+ac+bdx^2\right)^3}{(a+bx)^4}\,dx$$

$$\Big\downarrow \text{1121}$$

$$\int \left(\frac{(bc-ad)^3}{b^3(a+bx)}+\frac{d(bc-ad)^2}{b^3}+\frac{d(c+dx)(bc-ad)}{b^2}+\frac{d(c+dx)^2}{b}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{(bc-ad)^3\log(a+bx)}{b^4}+\frac{dx(bc-ad)^2}{b^3}+\frac{(c+dx)^2(bc-ad)}{2b^2}+\frac{(c+dx)^3}{3b}$$

input
```
Int[(a*c + (b*c + a*d)*x + b*d*x^2)^3/(a + b*x)^4,x]
```

output
```
(d*(b*c - a*d)^2*x)/b^3 + ((b*c - a*d)*(c + d*x)^2)/(2*b^2) + (c + d*x)^3/
(3*b) + ((b*c - a*d)^3*Log[a + b*x])/b^4
```

Defintions of rubi rules used

rule 1121
```
Int[((d_) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Int[ExpandIntegrand[(d + e*x)^(m + p)*(a/d + (c/e)*x)^p, x], x]
/; FreeQ[{a, b, c, d, e, m, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && (Int
egerQ[p] || (GtQ[a, 0] && GtQ[d, 0] && LtQ[c, 0]))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 1.23 (sec) , antiderivative size = 109, normalized size of antiderivative = 1.49

method	result
default	$\frac{d\left(\frac{1}{3}x^3b^2d^2-\frac{1}{2}ab\,d^2x^2+\frac{3}{2}b^2c\,x^2d+a^2d^2x-3abcdx+3x\,b^2c^2\right)}{b^3}+\frac{\left(-a^3d^3+3a^2bc\,d^2-3a\,b^2c^2d+b^3c^3\right)\ln(bx+a)}{b^4}$
risch	$\frac{x^3d^3}{3b}-\frac{d^3a\,x^2}{2b^2}+\frac{3d^2c\,x^2}{2b}+\frac{d^3a^2x}{b^3}-\frac{3d^2acx}{b^2}+\frac{3dx\,c^2}{b}-\frac{\ln(bx+a)a^3d^3}{b^4}+\frac{3\ln(bx+a)a^2c\,d^2}{b^3}-\frac{3\ln(bx+a)a\,c^2d}{b^2}$
parallelrisch	$-\frac{-2x^3b^3d^3+3a\,b^2d^3x^2-9x^2b^3c\,d^2+6\ln(bx+a)a^3d^3-18\ln(bx+a)a^2bc\,d^2+18\ln(bx+a)a\,b^2c^2d-6\ln(bx+a)b^3c^3-6a^2b\,d^3x+}{6b^4}$
norman	$\frac{\left(\frac{1}{2}ab\,d^3+\frac{3}{2}b^2c\,d^2\right)x^5+\left(\frac{1}{2}a^2d^3+\frac{3}{2}abc\,d^2+3b^2c^2d\right)x^4-\frac{a^3\left(11a^3d^3-27a^2bc\,d^2+54a\,b^2c^2d\right)}{6b^4}+\frac{b^2d^3x^6}{3}-\frac{3a\left(a^3d^3-2a^2bc\,d^2+6a\,b^2c^2d\right.}{b^2}}{(bx+a)^3}$

input
```
int((a*c+(a*d+b*c)*x+b*d*x^2)^3/(b*x+a)^4,x,method=_RETURNVERBOSE)
```

output
```
d/b^3*(1/3*x^3*b^2*d^2-1/2*a*b*d^2*x^2+3/2*b^2*c*x^2*d+a^2*d^2*x-3*a*b*c*d
*x+3*x*b^2*c^2)+(-a^3*d^3+3*a^2*b*c*d^2-3*a*b^2*c^2*d+b^3*c^3)/b^4*ln(b*x+
a)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.59

$$\int \frac{(ac + (bc + ad)x + bdx^2)^3}{(a + bx)^4} \, dx$$
$$= \frac{2 \, b^3 d^3 x^3 + 3 \, (3 \, b^3 c d^2 - a b^2 d^3) x^2 + 6 \, (3 \, b^3 c^2 d - 3 \, a b^2 c d^2 + a^2 b d^3) x + 6 \, (b^3 c^3 - 3 \, a b^2 c^2 d + 3 \, a^2 b c d^2 - a^3 d^3}{6 \, b^4}$$

input
```
integrate((a*c+(a*d+b*c)*x+b*d*x^2)^3/(b*x+a)^4,x, algorithm="fricas")
```

output
```
1/6*(2*b^3*d^3*x^3 + 3*(3*b^3*c*d^2 - a*b^2*d^3)*x^2 + 6*(3*b^3*c^2*d - 3*
a*b^2*c*d^2 + a^2*b*d^3)*x + 6*(b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 -
a^3*d^3)*log(b*x + a))/b^4
```

Sympy [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.14

$$\int \frac{(ac + (bc + ad)x + bdx^2)^3}{(a + bx)^4} \, dx = x^2 \left(-\frac{ad^3}{2b^2} + \frac{3cd^2}{2b} \right) + x \left(\frac{a^2 d^3}{b^3} - \frac{3acd^2}{b^2} + \frac{3c^2 d}{b} \right) + \frac{d^3 x^3}{3b} - \frac{(ad - bc)^3 \log (a + bx)}{b^4}$$

input
```
integrate((a*c+(a*d+b*c)*x+b*d*x**2)**3/(b*x+a)**4,x)
```

output
```
x**2*(-a*d**3/(2*b**2) + 3*c*d**2/(2*b)) + x*(a**2*d**3/b**3 - 3*a*c*d**2/
b**2 + 3*c**2*d/b) + d**3*x**3/(3*b) - (a*d - b*c)**3*log(a + b*x)/b**4
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.56

$$\int \frac{(ac+(bc+ad)x+bdx^2)^3}{(a+bx)^4}\,dx = \frac{2\,b^2d^3x^3+3\,(3\,b^2cd^2-abd^3)x^2+6\,(3\,b^2c^2d-3\,abcd^2+a^2d^3)x}{6\,b^3} + \frac{(b^3c^3-3\,ab^2c^2d+3\,a^2bcd^2-a^3d^3)\log(bx+a)}{b^4}$$

input
```
integrate((a*c+(a*d+b*c)*x+b*d*x^2)^3/(b*x+a)^4,x, algorithm="maxima")
```

output
```
1/6*(2*b^2*d^3*x^3 + 3*(3*b^2*c*d^2 - a*b*d^3)*x^2 + 6*(3*b^2*c^2*d - 3*a*
b*c*d^2 + a^2*d^3)*x)/b^3 + (b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3
*d^3)*log(b*x + a)/b^4
```

Giac [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 115, normalized size of antiderivative = 1.58

$$\int \frac{(ac+(bc+ad)x+bdx^2)^3}{(a+bx)^4}\,dx = \frac{2\,b^2d^3x^3+9\,b^2cd^2x^2-3\,abd^3x^2+18\,b^2c^2dx-18\,abcd^2x+6\,a^2d^3x}{6\,b^3} + \frac{(b^3c^3-3\,ab^2c^2d+3\,a^2bcd^2-a^3d^3)\log(|bx+a|)}{b^4}$$

input
```
integrate((a*c+(a*d+b*c)*x+b*d*x^2)^3/(b*x+a)^4,x, algorithm="giac")
```

output
```
1/6*(2*b^2*d^3*x^3 + 9*b^2*c*d^2*x^2 - 3*a*b*d^3*x^2 + 18*b^2*c^2*d*x - 18
*a*b*c*d^2*x + 6*a^2*d^3*x)/b^3 + (b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2
 - a^3*d^3)*log(abs(b*x + a))/b^4
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 118, normalized size of antiderivative = 1.62

$$\int \frac{(ac+(bc+ad)x+bdx^2)^3}{(a+bx)^4}\,dx = x\left(\frac{3\,c^2\,d}{b}+\frac{a\left(\frac{a\,d^3}{b^2}-\frac{3\,c\,d^2}{b}\right)}{b}\right) - x^2\left(\frac{a\,d^3}{2\,b^2}-\frac{3\,c\,d^2}{2\,b}\right)+\frac{d^3\,x^3}{3\,b} - \frac{\ln\left(a+b\,x\right)\,\left(a^3\,d^3-3\,a^2\,b\,c\,d^2+3\,a\,b^2\,c^2\,d-b^3\,c^3\right)}{b^4}$$

input
```
int((a*c + x*(a*d + b*c) + b*d*x^2)^3/(a + b*x)^4,x)
```

output
```
x*((3*c^2*d)/b + (a*((a*d^3)/b^2 - (3*c*d^2)/b))/b) - x^2*((a*d^3)/(2*b^2)
 - (3*c*d^2)/(2*b)) + (d^3*x^3)/(3*b) - (log(a + b*x)*(a^3*d^3 - b^3*c^3 +
 3*a*b^2*c^2*d - 3*a^2*b*c*d^2))/b^4
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 132, normalized size of antiderivative = 1.81

$$\int \frac{(ac+(bc+ad)x+bdx^2)^3}{(a+bx)^4}\,dx = \frac{-6\log(bx+a)\,a^3d^3+18\log(bx+a)\,a^2bc\,d^2-18\log(bx+a)\,a\,b^2c^2d+6\log(bx+a)\,b^3c^3+6a^2b\,d^3x-1}{6b^4}$$

input
```
int((a*c+(a*d+b*c)*x+b*d*x^2)^3/(b*x+a)^4,x)
```

output
```
( - 6*log(a + b*x)*a**3*d**3 + 18*log(a + b*x)*a**2*b*c*d**2 - 18*log(a +
b*x)*a*b**2*c**2*d + 6*log(a + b*x)*b**3*c**3 + 6*a**2*b*d**3*x - 18*a*b**
2*c*d**2*x - 3*a*b**2*d**3*x**2 + 18*b**3*c**2*d*x + 9*b**3*c*d**2*x**2 +
2*b**3*d**3*x**3)/(6*b**4)
```

3.33 $\int \frac{(ac+(bc+ad)x+bdx^2)^3}{(a+bx)^5}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 75

$$\int \frac{(ac+(bc+ad)x+bdx^2)^3}{(a+bx)^5}\,dx = \frac{d^2(3bc-2ad)x}{b^3}+\frac{d^3x^2}{2b^2}-\frac{(bc-ad)^3}{b^4(a+bx)}+\frac{3d(bc-ad)^2\log(a+bx)}{b^4}$$

output
```
d^2*(-2*a*d+3*b*c)*x/b^3+1/2*d^3*x^2/b^2-(-a*d+b*c)^3/b^4/(b*x+a)+3*d*(-a*
d+b*c)^2*ln(b*x+a)/b^4
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.96

$$\int \frac{(ac+(bc+ad)x+bdx^2)^3}{(a+bx)^5}\,dx = \frac{2bd^2(3bc-2ad)x+b^2d^3x^2-\frac{2(bc-ad)^3}{a+bx}+6d(bc-ad)^2\log(a+bx)}{2b^4}$$

input
```
Integrate[(a*c + (b*c + a*d)*x + b*d*x^2)^3/(a + b*x)^5,x]
```

output
```
(2*b*d^2*(3*b*c - 2*a*d)*x + b^2*d^3*x^2 - (2*(b*c - a*d)^3)/(a + b*x) + 6
*d*(b*c - a*d)^2*Log[a + b*x])/(2*b^4)
```

Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.069$, Rules used = {1121, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(x(ad+bc)+ac+bdx^2\right)^3}{(a+bx)^5}\,dx$$

$$\Big\downarrow \text{1121}$$

$$\int \left(\frac{d^2(3bc-2ad)}{b^3}+\frac{3d(bc-ad)^2}{b^3(a+bx)}+\frac{(bc-ad)^3}{b^3(a+bx)^2}+\frac{d^3x}{b^2}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{(bc-ad)^3}{b^4(a+bx)}+\frac{3d(bc-ad)^2\log(a+bx)}{b^4}+\frac{d^2x(3bc-2ad)}{b^3}+\frac{d^3x^2}{2b^2}$$

input
```
Int[(a*c + (b*c + a*d)*x + b*d*x^2)^3/(a + b*x)^5,x]
```

output
```
(d^2*(3*b*c - 2*a*d)*x)/b^3 + (d^3*x^2)/(2*b^2) - (b*c - a*d)^3/(b^4*(a +
b*x)) + (3*d*(b*c - a*d)^2*Log[a + b*x])/b^4
```

Defintions of rubi rules used

rule 1121
```
Int[((d_) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Int[ExpandIntegrand[(d + e*x)^(m + p)*(a/d + (c/e)*x)^p, x], x]
/; FreeQ[{a, b, c, d, e, m, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && (Int
egerQ[p] || (GtQ[a, 0] && GtQ[d, 0] && LtQ[c, 0]))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 1.25 (sec) , antiderivative size = 109, normalized size of antiderivative = 1.45

method	result
default	$-\frac{d^2\left(-\frac{1}{2}bd\,x^2+2adx-3cbx\right)}{b^3}-\frac{-a^3d^3+3a^2bc\,d^2-3a\,b^2c^2d+b^3c^3}{b^4(bx+a)}+\frac{3d\left(a^2d^2-2abcd+b^2c^2\right)\ln(bx+a)}{b^4}$
risch	$\frac{d^3x^2}{2b^2}-\frac{2d^3ax}{b^3}+\frac{3c\,d^2x}{b^2}+\frac{a^3d^3}{b^4(bx+a)}-\frac{3a^2c\,d^2}{b^3(bx+a)}+\frac{3a\,c^2d}{b^2(bx+a)}-\frac{c^3}{b(bx+a)}+\frac{3d^3\ln(bx+a)a^2}{b^4}-\frac{6d^2\ln(bx+a)ac}{b^3}$
parallelrisch	$\frac{x^3b^3d^3+6\ln(bx+a)x\,a^2b\,d^3-12\ln(bx+a)xa\,b^2c\,d^2+6\ln(bx+a)x\,b^3c^2d-3a\,b^2d^3x^2+6x^2b^3c\,d^2+6\ln(bx+a)a^3d^3-12\ln(bx+a}{2b^4(bx+a)}$
norman	$\frac{\frac{a^3\left(6a^3d^3-15a^2bc\,d^2+3a\,b^2c^2d-b^3c^3\right)}{b^4}+\frac{\left(11a^3d^3-33a^2bc\,d^2+3a\,b^2c^2d-b^3c^3\right)x^3}{b}+\frac{a^2\left(21a^3d^3-54a^2bc\,d^2+9a\,b^2c^2d-3b^3c^3\right)x}{b^3}+\frac{b^2d^3x}{2}}{(bx+a)^4}$

input
```
int((a*c+(a*d+b*c)*x+b*d*x^2)^3/(b*x+a)^5,x,method=_RETURNVERBOSE)
```

output
```
-d^2/b^3*(-1/2*b*d*x^2+2*a*d*x-3*c*b*x)-1/b^4*(-a^3*d^3+3*a^2*b*c*d^2-3*a*
b^2*c^2*d+b^3*c^3)/(b*x+a)+3/b^4*d*(a^2*d^2-2*a*b*c*d+b^2*c^2)*ln(b*x+a)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 173 vs. 2(73) = 146.

Time = 0.07 (sec) , antiderivative size = 173, normalized size of antiderivative = 2.31

$$\int \frac{(ac+(bc+ad)x+bdx^2)^3}{(a+bx)^5}\,dx$$
$$=\frac{b^3d^3x^3-2\,b^3c^3+6\,ab^2c^2d-6\,a^2bcd^2+2\,a^3d^3+3\,(2\,b^3cd^2-ab^2d^3)x^2+2\,(3\,ab^2cd^2-2\,a^2bd^3)x+6\,(ab^2}{2\,(b^5x+ab^4)}$$

input
```
integrate((a*c+(a*d+b*c)*x+b*d*x^2)^3/(b*x+a)^5,x, algorithm="fricas")
```

output
```
1/2*(b^3*d^3*x^3 - 2*b^3*c^3 + 6*a*b^2*c^2*d - 6*a^2*b*c*d^2 + 2*a^3*d^3 +
 3*(2*b^3*c*d^2 - a*b^2*d^3)*x^2 + 2*(3*a*b^2*c*d^2 - 2*a^2*b*d^3)*x + 6*(
a*b^2*c^2*d - 2*a^2*b*c*d^2 + a^3*d^3 + (b^3*c^2*d - 2*a*b^2*c*d^2 + a^2*b
*d^3)*x)*log(b*x + a))/(b^5*x + a*b^4)
```

Sympy [A] (verification not implemented)

Time = 0.43 (sec) , antiderivative size = 102, normalized size of antiderivative = 1.36

$$\int \frac{(ac+(bc+ad)x+bdx^2)^3}{(a+bx)^5}\,dx = x\left(-\frac{2ad^3}{b^3}+\frac{3cd^2}{b^2}\right)+\frac{a^3d^3-3a^2bcd^2+3ab^2c^2d-b^3c^3}{ab^4+b^5x} + \frac{d^3x^2}{2b^2}+\frac{3d(ad-bc)^2\log(a+bx)}{b^4}$$

input
```
integrate((a*c+(a*d+b*c)*x+b*d*x**2)**3/(b*x+a)**5,x)
```

output
```
x*(-2*a*d**3/b**3 + 3*c*d**2/b**2) + (a**3*d**3 - 3*a**2*b*c*d**2 + 3*a*b*
*2*c**2*d - b**3*c**3)/(a*b**4 + b**5*x) + d**3*x**2/(2*b**2) + 3*d*(a*d -
 b*c)**2*log(a + b*x)/b**4
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 118, normalized size of antiderivative = 1.57

$$\int \frac{(ac+(bc+ad)x+bdx^2)^3}{(a+bx)^5}\,dx = -\frac{b^3c^3-3\,ab^2c^2d+3\,a^2bcd^2-a^3d^3}{b^5x+ab^4} + \frac{bd^3x^2+2\,(3\,bcd^2-2\,ad^3)x}{2\,b^3} + \frac{3\,(b^2c^2d-2\,abcd^2+a^2d^3)\log(bx+a)}{b^4}$$

input
```
integrate((a*c+(a*d+b*c)*x+b*d*x^2)^3/(b*x+a)^5,x, algorithm="maxima")
```

output
```
-(b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)/(b^5*x + a*b^4) + 1/2
*(b*d^3*x^2 + 2*(3*b*c*d^2 - 2*a*d^3)*x)/b^3 + 3*(b^2*c^2*d - 2*a*b*c*d^2
+ a^2*d^3)*log(b*x + a)/b^4
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 167 vs. 2(73) = 146.

Time = 0.14 (sec) , antiderivative size = 167, normalized size of antiderivative = 2.23

$$\int \frac{(ac+(bc+ad)x+bdx^2)^3}{(a+bx)^5}\,dx = \frac{\left(d^3+\frac{6\,(b^2cd^2-abd^3)}{(bx+a)b}\right)(bx+a)^2}{2\,b^4} - \frac{3\,(b^2c^2d-2\,abcd^2+a^2d^3)\log\left(\frac{|bx+a|}{(bx+a)^2|b|}\right)}{b^4} - \frac{\frac{b^5c^3}{bx+a}-\frac{3\,ab^4c^2d}{bx+a}+\frac{3\,a^2b^3cd^2}{bx+a}-\frac{a^3b^2d^3}{bx+a}}{b^6}$$

input
```
integrate((a*c+(a*d+b*c)*x+b*d*x^2)^3/(b*x+a)^5,x, algorithm="giac")
```

output
```
1/2*(d^3 + 6*(b^2*c*d^2 - a*b*d^3)/((b*x + a)*b))*(b*x + a)^2/b^4 - 3*(b^2
*c^2*d - 2*a*b*c*d^2 + a^2*d^3)*log(abs(b*x + a)/((b*x + a)^2*abs(b)))/b^4
 - (b^5*c^3/(b*x + a) - 3*a*b^4*c^2*d/(b*x + a) + 3*a^2*b^3*c*d^2/(b*x + a
) - a^3*b^2*d^3/(b*x + a))/b^6
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 123, normalized size of antiderivative = 1.64

$$\int \frac{(ac+(bc+ad)x+bdx^2)^3}{(a+bx)^5}\,dx = \frac{\ln(a+b\,x)\,\left(3\,a^2\,d^3-6\,a\,b\,c\,d^2+3\,b^2\,c^2\,d\right)}{b^4} - x\left(\frac{2\,a\,d^3}{b^3}-\frac{3\,c\,d^2}{b^2}\right) + \frac{a^3\,d^3-3\,a^2\,b\,c\,d^2+3\,a\,b^2\,c^2\,d-b^3\,c^3}{b\,\left(x\,b^4+a\,b^3\right)} + \frac{d^3\,x^2}{2\,b^2}$$

input
```
int((a*c + x*(a*d + b*c) + b*d*x^2)^3/(a + b*x)^5,x)
```

output
```
(log(a + b*x)*(3*a^2*d^3 + 3*b^2*c^2*d - 6*a*b*c*d^2))/b^4 - x*((2*a*d^3)/
b^3 - (3*c*d^2)/b^2) + (a^3*d^3 - b^3*c^3 + 3*a*b^2*c^2*d - 3*a^2*b*c*d^2)
/(b*(a*b^3 + b^4*x)) + (d^3*x^2)/(2*b^2)
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 197, normalized size of antiderivative = 2.63

$$\int \frac{(ac + (bc + ad)x + bdx^2)^3}{(a + bx)^5} dx$$
$$= \frac{6\log(bx + a)\, a^4d^3 - 12\log(bx + a)\, a^3bc\, d^2 + 6\log(bx + a)\, a^3b\, d^3x + 6\log(bx + a)\, a^2b^2c^2d - 12\log(bx +}{}$$

input
```
int((a*c+(a*d+b*c)*x+b*d*x^2)^3/(b*x+a)^5,x)
```

output
```
(6*log(a + b*x)*a**4*d**3 - 12*log(a + b*x)*a**3*b*c*d**2 + 6*log(a + b*x)
*a**3*b*d**3*x + 6*log(a + b*x)*a**2*b**2*c**2*d - 12*log(a + b*x)*a**2*b*
*2*c*d**2*x + 6*log(a + b*x)*a*b**3*c**2*d*x - 6*a**3*b*d**3*x + 12*a**2*b
**2*c*d**2*x - 3*a**2*b**2*d**3*x**2 - 6*a*b**3*c**2*d*x + 6*a*b**3*c*d**2
*x**2 + a*b**3*d**3*x**3 + 2*b**4*c**3*x)/(2*a*b**4*(a + b*x))
```

3.34 $\int \frac{(ac+(bc+ad)x+bdx^2)^3}{(a+bx)^6} dx$

Optimal result

Integrand size = 29, antiderivative size = 78

$$\int \frac{(ac+(bc+ad)x+bdx^2)^3}{(a+bx)^6} dx = \frac{d^3x}{b^3} - \frac{(bc-ad)^3}{2b^4(a+bx)^2} - \frac{3d(bc-ad)^2}{b^4(a+bx)} + \frac{3d^2(bc-ad)\log(a+bx)}{b^4}$$

output
```
d^3*x/b^3-1/2*(-a*d+b*c)^3/b^4/(b*x+a)^2-3*d*(-a*d+b*c)^2/b^4/(b*x+a)+3*d^
2*(-a*d+b*c)*ln(b*x+a)/b^4
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.46

$$\int \frac{(ac+(bc+ad)x+bdx^2)^3}{(a+bx)^6} dx = \frac{-5a^3d^3 + a^2bd^2(9c-4dx) + ab^2d(-3c^2+12cdx+4d^2x^2) - b^3(c^3+6c^2dx-2d^3x^3) - 6d^2(-bc+ad)(a}{2b^4(a+bx)^2}$$

input
```
Integrate[(a*c + (b*c + a*d)*x + b*d*x^2)^3/(a + b*x)^6,x]
```

output
```
(-5*a^3*d^3 + a^2*b*d^2*(9*c - 4*d*x) + a*b^2*d*(-3*c^2 + 12*c*d*x + 4*d^2
*x^2) - b^3*(c^3 + 6*c^2*d*x - 2*d^3*x^3) - 6*d^2*(-(b*c) + a*d)*(a + b*x)
^2*Log[a + b*x])/(2*b^4*(a + b*x)^2)
```

Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.069, Rules used = {1121, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(x(ad+bc)+ac+bdx^2\right)^3}{(a+bx)^6} \, dx$$

$$\Big\downarrow \text{ 1121}$$

$$\int \left(\frac{3d^2(bc-ad)}{b^3(a+bx)}+\frac{3d(bc-ad)^2}{b^3(a+bx)^2}+\frac{(bc-ad)^3}{b^3(a+bx)^3}+\frac{d^3}{b^3}\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{3d^2(bc-ad)\log(a+bx)}{b^4}-\frac{3d(bc-ad)^2}{b^4(a+bx)}-\frac{(bc-ad)^3}{2b^4(a+bx)^2}+\frac{d^3x}{b^3}$$

input
```
Int[(a*c + (b*c + a*d)*x + b*d*x^2)^3/(a + b*x)^6,x]
```

output
```
(d^3*x)/b^3 - (b*c - a*d)^3/(2*b^4*(a + b*x)^2) - (3*d*(b*c - a*d)^2)/(b^4
*(a + b*x)) + (3*d^2*(b*c - a*d)*Log[a + b*x])/b^4
```

Defintions of rubi rules used

rule 1121
```
Int[((d_) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Int[ExpandIntegrand[(d + e*x)^(m + p)*(a/d + (c/e)*x)^p, x], x]
/; FreeQ[{a, b, c, d, e, m, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && (Int
egerQ[p] || (GtQ[a, 0] && GtQ[d, 0] && LtQ[c, 0]))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 1.32 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.46

method	result
default	$\frac{d^3 x}{b^3}-\frac{3d\left(a^2d^2-2abcd+b^2c^2\right)}{b^4(bx+a)}-\frac{3d^2(ad-bc)\ln(bx+a)}{b^4}-\frac{-a^3d^3+3a^2bc\,d^2-3a\,b^2c^2d+b^3c^3}{2b^4(bx+a)^2}$
risch	$\frac{d^3x}{b^3}+\frac{\left(-3a^2d^3+6abc\,d^2-3b^2c^2d\right)x-\frac{5a^3d^3-9a^2bc\,d^2+3a\,b^2c^2d+b^3c^3}{2b}}{b^3(bx+a)^2}-\frac{3d^3\ln(bx+a)a}{b^4}+\frac{3d^2\ln(bx+a)c}{b^3}$
parallelrisch	$-\frac{6\ln(bx+a)x^2a\,b^2d^3-6\ln(bx+a)x^2b^3c\,d^2-2x^3b^3d^3+12\ln(bx+a)x\,a^2b\,d^3-12\ln(bx+a)xa\,b^2c\,d^2+6\ln(bx+a)a^3d^3-6\ln(bx}{2b^4(bx+a)^2}$
norman	$\frac{b^2d^3x^6-\frac{a^3\left(15a^3b\,d^3-9a^2b^2c\,d^2+3a\,b^3c^2d+b^4c^3\right)}{2b^5}-\frac{\left(18d^3a^2b-6a\,b^2c\,d^2+3b^3c^2d\right)x^4}{b}-\frac{\left(103a^3b\,d^3-45a^2b^2c\,d^2+21a\,b^3c^2d+b^4c^3\right)x^3}{2b^2}-}{(bx+a)^5}$

input
```
int((a*c+(a*d+b*c)*x+b*d*x^2)^3/(b*x+a)^6,x,method=_RETURNVERBOSE)
```

output
```
d^3*x/b^3-3/b^4*d*(a^2*d^2-2*a*b*c*d+b^2*c^2)/(b*x+a)-3/b^4*d^2*(a*d-b*c)*
ln(b*x+a)-1/2/b^4*(-a^3*d^3+3*a^2*b*c*d^2-3*a*b^2*c^2*d+b^3*c^3)/(b*x+a)^2
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 188 vs. $2(76) = 152$.

Time = 0.08 (sec) , antiderivative size = 188, normalized size of antiderivative = 2.41

$$\int \frac{(ac + (bc + ad)x + bdx^2)^3}{(a + bx)^6}\,dx = \frac{2\,b^3d^3x^3 + 4\,ab^2d^3x^2 - b^3c^3 - 3\,ab^2c^2d + 9\,a^2bcd^2 - 5\,a^3d^3 - 2\,(3\,b^3c^2d - 6\,ab^2cd^2 + 2\,a^2bd^3)x + 6\,(a^2bcd}{2\,(b^6x^2 + 2\,ab^5x + a^2b^4)}$$

input
```
integrate((a*c+(a*d+b*c)*x+b*d*x^2)^3/(b*x+a)^6,x, algorithm="fricas")
```

output
```
1/2*(2*b^3*d^3*x^3 + 4*a*b^2*d^3*x^2 - b^3*c^3 - 3*a*b^2*c^2*d + 9*a^2*b*c
*d^2 - 5*a^3*d^3 - 2*(3*b^3*c^2*d - 6*a*b^2*c*d^2 + 2*a^2*b*d^3)*x + 6*(a^
2*b*c*d^2 - a^3*d^3 + (b^3*c*d^2 - a*b^2*d^3)*x^2 + 2*(a*b^2*c*d^2 - a^2*b
*d^3)*x)*log(b*x + a))/(b^6*x^2 + 2*a*b^5*x + a^2*b^4)
```

Sympy [A] (verification not implemented)

Time = 0.46 (sec) , antiderivative size = 128, normalized size of antiderivative = 1.64

$$\int \frac{(ac + (bc + ad)x + bdx^2)^3}{(a + bx)^6}\,dx = \frac{-5a^3d^3 + 9a^2bcd^2 - 3ab^2c^2d - b^3c^3 + x(-6a^2bd^3 + 12ab^2cd^2 - 6b^3c^2d)}{2a^2b^4 + 4ab^5x + 2b^6x^2} + \frac{d^3x}{b^3} - \frac{3d^2(ad - bc)\log(a + bx)}{b^4}$$

input
```
integrate((a*c+(a*d+b*c)*x+b*d*x**2)**3/(b*x+a)**6,x)
```

output
```
(-5*a**3*d**3 + 9*a**2*b*c*d**2 - 3*a*b**2*c**2*d - b**3*c**3 + x*(-6*a**2
*b*d**3 + 12*a*b**2*c*d**2 - 6*b**3*c**2*d))/(2*a**2*b**4 + 4*a*b**5*x + 2
*b**6*x**2) + d**3*x/b**3 - 3*d**2*(a*d - b*c)*log(a + b*x)/b**4
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 125, normalized size of antiderivative = 1.60

$$\int \frac{(ac+(bc+ad)x+bdx^2)^3}{(a+bx)^6}\,dx = \frac{d^3x}{b^3} - \frac{b^3c^3+3\,ab^2c^2d-9\,a^2bcd^2+5\,a^3d^3+6\,(b^3c^2d-2\,ab^2cd^2+a^2bd^3)x}{2\,(b^6x^2+2\,ab^5x+a^2b^4)} + \frac{3\,(bcd^2-ad^3)\log(bx+a)}{b^4}$$

input
```
integrate((a*c+(a*d+b*c)*x+b*d*x^2)^3/(b*x+a)^6,x, algorithm="maxima")
```

output
```
d^3*x/b^3 - 1/2*(b^3*c^3 + 3*a*b^2*c^2*d - 9*a^2*b*c*d^2 + 5*a^3*d^3 + 6*(
b^3*c^2*d - 2*a*b^2*c*d^2 + a^2*b*d^3)*x)/(b^6*x^2 + 2*a*b^5*x + a^2*b^4)
+ 3*(b*c*d^2 - a*d^3)*log(b*x + a)/b^4
```

Giac [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.44

$$\int \frac{(ac+(bc+ad)x+bdx^2)^3}{(a+bx)^6}\,dx = \frac{d^3x}{b^3} + \frac{3\,(bcd^2-ad^3)\log(|bx+a|)}{b^4} - \frac{b^3c^3+3\,ab^2c^2d-9\,a^2bcd^2+5\,a^3d^3+6\,(b^3c^2d-2\,ab^2cd^2+a^2bd^3)x}{2\,(bx+a)^2b^4}$$

input
```
integrate((a*c+(a*d+b*c)*x+b*d*x^2)^3/(b*x+a)^6,x, algorithm="giac")
```

output
```
d^3*x/b^3 + 3*(b*c*d^2 - a*d^3)*log(abs(b*x + a))/b^4 - 1/2*(b^3*c^3 + 3*a
*b^2*c^2*d - 9*a^2*b*c*d^2 + 5*a^3*d^3 + 6*(b^3*c^2*d - 2*a*b^2*c*d^2 + a^
2*b*d^3)*x)/((b*x + a)^2*b^4)
```

Mupad [B] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 130, normalized size of antiderivative = 1.67

$$\int \frac{(ac+(bc+ad)x+bdx^2)^3}{(a+bx)^6}\,dx$$
$$= \frac{d^3\,x}{b^3} - \frac{\ln(a+b\,x)\,\left(3\,a\,d^3-3\,b\,c\,d^2\right)}{b^4} - \frac{\frac{5\,a^3\,d^3-9\,a^2\,b\,c\,d^2+3\,a\,b^2\,c^2\,d+b^3\,c^3}{2\,b}+x\,\left(3\,a^2\,d^3-6\,a\,b\,c\,d^2+3\,b^2\,c^2\,d\right)}{a^2\,b^3+2\,a\,b^4\,x+b^5\,x^2}$$

input
```
int((a*c + x*(a*d + b*c) + b*d*x^2)^3/(a + b*x)^6,x)
```

output
```
(d^3*x)/b^3 - (log(a + b*x)*(3*a*d^3 - 3*b*c*d^2))/b^4 - ((5*a^3*d^3 + b^3
*c^3 + 3*a*b^2*c^2*d - 9*a^2*b*c*d^2)/(2*b) + x*(3*a^2*d^3 + 3*b^2*c^2*d -
 6*a*b*c*d^2))/(a^2*b^3 + b^5*x^2 + 2*a*b^4*x)
```

Reduce [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 209, normalized size of antiderivative = 2.68

$$\int \frac{(ac+(bc+ad)x+bdx^2)^3}{(a+bx)^6}\,dx$$
$$= \frac{-6\log(bx+a)\,a^4d^3+6\log(bx+a)\,a^3bc\,d^2-12\log(bx+a)\,a^3b\,d^3x+12\log(bx+a)\,a^2b^2c\,d^2x-6\log(b}{2a\,b}$$

input
```
int((a*c+(a*d+b*c)*x+b*d*x^2)^3/(b*x+a)^6,x)
```

output
```
( - 6*log(a + b*x)*a**4*d**3 + 6*log(a + b*x)*a**3*b*c*d**2 - 12*log(a + b
*x)*a**3*b*d**3*x + 12*log(a + b*x)*a**2*b**2*c*d**2*x - 6*log(a + b*x)*a*
*2*b**2*d**3*x**2 + 6*log(a + b*x)*a*b**3*c*d**2*x**2 - 3*a**4*d**3 + 3*a*
*3*b*c*d**2 + 6*a**2*b**2*d**3*x**2 - a*b**3*c**3 - 6*a*b**3*c*d**2*x**2 +
 2*a*b**3*d**3*x**3 + 3*b**4*c**2*d*x**2)/(2*a*b**4*(a**2 + 2*a*b*x + b**2
*x**2))
```

3.35 $\int \frac{(ac+(bc+ad)x+bdx^2)^3}{(a+bx)^7}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 86

$$\int \frac{(ac+(bc+ad)x+bdx^2)^3}{(a+bx)^7}\,dx = -\frac{(bc-ad)^3}{3b^4(a+bx)^3} - \frac{3d(bc-ad)^2}{2b^4(a+bx)^2} - \frac{3d^2(bc-ad)}{b^4(a+bx)} + \frac{d^3\log(a+bx)}{b^4}$$

output
```
-1/3*(-a*d+b*c)^3/b^4/(b*x+a)^3-3/2*d*(-a*d+b*c)^2/b^4/(b*x+a)^2-3*d^2*(-a
*d+b*c)/b^4/(b*x+a)+d^3*ln(b*x+a)/b^4
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.93

$$\int \frac{(ac+(bc+ad)x+bdx^2)^3}{(a+bx)^7}\,dx = \frac{-\frac{(bc-ad)(11a^2d^2+abd(5c+27dx)+b^2(2c^2+9cdx+18d^2x^2))}{(a+bx)^3}+6d^3\log(a+bx)}{6b^4}$$

input
```
Integrate[(a*c + (b*c + a*d)*x + b*d*x^2)^3/(a + b*x)^7,x]
```

output
```
(-(((b*c - a*d)*(11*a^2*d^2 + a*b*d*(5*c + 27*d*x) + b^2*(2*c^2 + 9*c*d*x
+ 18*d^2*x^2)))/(a + b*x)^3) + 6*d^3*Log[a + b*x])/(6*b^4)
```

Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.069$, Rules used = {1121, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(x(ad+bc)+ac+bdx^2)^3}{(a+bx)^7}\,dx$$

$$\Big\downarrow \text{1121}$$

$$\int \left(\frac{3d^2(bc-ad)}{b^3(a+bx)^2}+\frac{3d(bc-ad)^2}{b^3(a+bx)^3}+\frac{(bc-ad)^3}{b^3(a+bx)^4}+\frac{d^3}{b^3(a+bx)}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{3d^2(bc-ad)}{b^4(a+bx)}-\frac{3d(bc-ad)^2}{2b^4(a+bx)^2}-\frac{(bc-ad)^3}{3b^4(a+bx)^3}+\frac{d^3\log(a+bx)}{b^4}$$

input
```
Int[(a*c + (b*c + a*d)*x + b*d*x^2)^3/(a + b*x)^7,x]
```

output
```
-1/3*(b*c - a*d)^3/(b^4*(a + b*x)^3) - (3*d*(b*c - a*d)^2)/(2*b^4*(a + b*x
)^2) - (3*d^2*(b*c - a*d))/(b^4*(a + b*x)) + (d^3*Log[a + b*x])/b^4
```

Defintions of rubi rules used

rule 1121
```
Int[((d_) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Int[ExpandIntegrand[(d + e*x)^(m + p)*(a/d + (c/e)*x)^p, x], x]
/; FreeQ[{a, b, c, d, e, m, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && (Int
egerQ[p] || (GtQ[a, 0] && GtQ[d, 0] && LtQ[c, 0]))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 1.26 (sec) , antiderivative size = 115, normalized size of antiderivative = 1.34

method	result
risch	$\frac{\frac{3d^2(ad-bc)x^2}{b^2}+\frac{3d\left(3a^2d^2-2abcd-b^2c^2\right)x}{2b^3}+\frac{11a^3d^3-6a^2bc\,d^2-3a\,b^2c^2d-2b^3c^3}{6b^4}}{(bx+a)^3}+\frac{d^3\ln(bx+a)}{b^4}$
default	$-\frac{-a^3d^3+3a^2bc\,d^2-3a\,b^2c^2d+b^3c^3}{3b^4(bx+a)^3}+\frac{3d^2(ad-bc)}{b^4(bx+a)}+\frac{d^3\ln(bx+a)}{b^4}-\frac{3d\left(a^2d^2-2abcd+b^2c^2\right)}{2b^4(bx+a)^2}$
parallelrisch	$\frac{6\ln(bx+a)x^3b^3d^3+18\ln(bx+a)x^2a\,b^2d^3+18\ln(bx+a)x\,a^2b\,d^3+18a\,b^2d^3x^2-18x^2b^3c\,d^2+6\ln(bx+a)a^3d^3+27a^2b\,d^3x-18xa}{6b^4(bx+a)^3}$
norman	$\frac{\frac{a\left(22d^3a^3b^2-15c\,d^2a^2b^3-6c^2da\,b^4-b^5c^3\right)x^2}{b^4}+\frac{a^2\left(10d^3a^3b^2-6c\,d^2a^2b^3-3c^2da\,b^4-b^5c^3\right)x}{b^5}+\frac{a^3\left(11d^3a^3b^2-6c\,d^2a^2b^3-3c^2da\,b^4-2b^5c\right.}{6b^6}}{(bx+a)^6}$

input
```
int((a*c+(a*d+b*c)*x+b*d*x^2)^3/(b*x+a)^7,x,method=_RETURNVERBOSE)
```

output
```
(3*d^2*(a*d-b*c)/b^2*x^2+3/2*d*(3*a^2*d^2-2*a*b*c*d-b^2*c^2)/b^3*x+1/6*(11
*a^3*d^3-6*a^2*b*c*d^2-3*a*b^2*c^2*d-2*b^3*c^3)/b^4)/(b*x+a)^3+d^3*ln(b*x+
a)/b^4
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 176 vs. $2(82) = 164$.

Time = 0.07 (sec) , antiderivative size = 176, normalized size of antiderivative = 2.05

$$\int \frac{\left(ac + (bc + ad)x + bdx^2\right)^3}{(a + bx)^7}\,dx = -\frac{2\,b^3c^3 + 3\,ab^2c^2d + 6\,a^2bcd^2 - 11\,a^3d^3 + 18\,(b^3cd^2 - ab^2d^3)x^2 + 9\,(b^3c^2d + 2\,ab^2cd^2 - 3\,a^2bd^3)x - 6\,(b^3}{6\,(b^7x^3 + 3\,ab^6x^2 + 3\,a^2b^5x + a^3b^4)}$$

input
```
integrate((a*c+(a*d+b*c)*x+b*d*x^2)^3/(b*x+a)^7,x, algorithm="fricas")
```

output
```
-1/6*(2*b^3*c^3 + 3*a*b^2*c^2*d + 6*a^2*b*c*d^2 - 11*a^3*d^3 + 18*(b^3*c*d
^2 - a*b^2*d^3)*x^2 + 9*(b^3*c^2*d + 2*a*b^2*c*d^2 - 3*a^2*b*d^3)*x - 6*(b
^3*d^3*x^3 + 3*a*b^2*d^3*x^2 + 3*a^2*b*d^3*x + a^3*d^3)*log(b*x + a))/(b^7
*x^3 + 3*a*b^6*x^2 + 3*a^2*b^5*x + a^3*b^4)
```

Sympy [A] (verification not implemented)

Time = 0.60 (sec) , antiderivative size = 148, normalized size of antiderivative = 1.72

$$\int \frac{\left(ac + (bc + ad)x + bdx^2\right)^3}{(a + bx)^7}\,dx = \frac{11a^3d^3 - 6a^2bcd^2 - 3ab^2c^2d - 2b^3c^3 + x^2 \cdot (18ab^2d^3 - 18b^3cd^2) + x(27a^2bd^3 - 18ab^2cd^2 - 9b^3c^2d)}{6a^3b^4 + 18a^2b^5x + 18ab^6x^2 + 6b^7x^3} + \frac{d^3\log{(a + bx)}}{b^4}$$

input
```
integrate((a*c+(a*d+b*c)*x+b*d*x**2)**3/(b*x+a)**7,x)
```

output
```
(11*a**3*d**3 - 6*a**2*b*c*d**2 - 3*a*b**2*c**2*d - 2*b**3*c**3 + x**2*(18
*a*b**2*d**3 - 18*b**3*c*d**2) + x*(27*a**2*b*d**3 - 18*a*b**2*c*d**2 - 9*
b**3*c**2*d))/(6*a**3*b**4 + 18*a**2*b**5*x + 18*a*b**6*x**2 + 6*b**7*x**3
) + d**3*log(a + b*x)/b**4
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 142, normalized size of antiderivative = 1.65

$$\int \frac{(ac+(bc+ad)x+bdx^2)^3}{(a+bx)^7}\,dx = -\frac{2\,b^3c^3+3\,ab^2c^2d+6\,a^2bcd^2-11\,a^3d^3+18\,(b^3cd^2-ab^2d^3)x^2+9\,(b^3c^2d+2\,ab^2cd^2-3\,a^2bd^3)x}{6\,(b^7x^3+3\,ab^6x^2+3\,a^2b^5x+a^3b^4)} + \frac{d^3\log(bx+a)}{b^4}$$

input
```
integrate((a*c+(a*d+b*c)*x+b*d*x^2)^3/(b*x+a)^7,x, algorithm="maxima")
```

output
```
-1/6*(2*b^3*c^3 + 3*a*b^2*c^2*d + 6*a^2*b*c*d^2 - 11*a^3*d^3 + 18*(b^3*c*d
^2 - a*b^2*d^3)*x^2 + 9*(b^3*c^2*d + 2*a*b^2*c*d^2 - 3*a^2*b*d^3)*x)/(b^7*
x^3 + 3*a*b^6*x^2 + 3*a^2*b^5*x + a^3*b^4) + d^3*log(b*x + a)/b^4
```

Giac [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 118, normalized size of antiderivative = 1.37

$$\int \frac{(ac+(bc+ad)x+bdx^2)^3}{(a+bx)^7}\,dx = \frac{d^3\log(|bx+a|)}{b^4} - \frac{18\,(b^2cd^2-abd^3)x^2+9\,(b^2c^2d+2\,abcd^2-3\,a^2d^3)x+\frac{2\,b^3c^3+3\,ab^2c^2d+6\,a^2bcd^2-11\,a^3d^3}{b}}{6\,(bx+a)^3b^3}$$

input
```
integrate((a*c+(a*d+b*c)*x+b*d*x^2)^3/(b*x+a)^7,x, algorithm="giac")
```

output
```
d^3*log(abs(b*x + a))/b^4 - 1/6*(18*(b^2*c*d^2 - a*b*d^3)*x^2 + 9*(b^2*c^2
*d + 2*a*b*c*d^2 - 3*a^2*d^3)*x + (2*b^3*c^3 + 3*a*b^2*c^2*d + 6*a^2*b*c*d
^2 - 11*a^3*d^3)/b)/((b*x + a)^3*b^3)
```

Mupad [B] (verification not implemented)

Time = 5.29 (sec) , antiderivative size = 138, normalized size of antiderivative = 1.60

$$\int \frac{(ac+(bc+ad)x+bdx^2)^3}{(a+bx)^7}\,dx$$
$$= \frac{d^3\ln(a+b\,x)}{b^4} - \frac{\frac{-11\,a^3\,d^3+6\,a^2\,b\,c\,d^2+3\,a\,b^2\,c^2\,d+2\,b^3\,c^3}{6\,b^4}+\frac{3\,x\,(-3\,a^2\,d^3+2\,a\,b\,c\,d^2+b^2\,c^2\,d)}{2\,b^3}-\frac{3\,d^2\,x^2\,(a\,d-b\,c)}{b^2}}{a^3+3\,a^2\,b\,x+3\,a\,b^2\,x^2+b^3\,x^3}$$

input
```
int((a*c + x*(a*d + b*c) + b*d*x^2)^3/(a + b*x)^7,x)
```

output
```
(d^3*log(a + b*x))/b^4 - ((2*b^3*c^3 - 11*a^3*d^3 + 3*a*b^2*c^2*d + 6*a^2*
b*c*d^2)/(6*b^4) + (3*x*(b^2*c^2*d - 3*a^2*d^3 + 2*a*b*c*d^2))/(2*b^3) - (
3*d^2*x^2*(a*d - b*c))/b^2)/(a^3 + b^3*x^3 + 3*a*b^2*x^2 + 3*a^2*b*x)
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 180, normalized size of antiderivative = 2.09

$$\int \frac{(ac+(bc+ad)x+bdx^2)^3}{(a+bx)^7}\,dx$$
$$= \frac{6\log(bx+a)\,a^4d^3+18\log(bx+a)\,a^3b\,d^3x+18\log(bx+a)\,a^2b^2d^3x^2+6\log(bx+a)\,a\,b^3d^3x^3+5a^4d^3+}{6a\,b^4\,(b^3x^3+3a\,b^2x^2+3a^2bx+a^3)}$$

input
```
int((a*c+(a*d+b*c)*x+b*d*x^2)^3/(b*x+a)^7,x)
```

output
```
(6*log(a + b*x)*a**4*d**3 + 18*log(a + b*x)*a**3*b*d**3*x + 18*log(a + b*x
)*a**2*b**2*d**3*x**2 + 6*log(a + b*x)*a*b**3*d**3*x**3 + 5*a**4*d**3 + 9*
a**3*b*d**3*x - 3*a**2*b**2*c**2*d - 2*a*b**3*c**3 - 9*a*b**3*c**2*d*x - 6
*a*b**3*d**3*x**3 + 6*b**4*c*d**2*x**3)/(6*a*b**4*(a**3 + 3*a**2*b*x + 3*a
*b**2*x**2 + b**3*x**3))
```

3.36 $\int \frac{(ac+(bc+ad)x+bdx^2)^3}{(a+bx)^8} dx$

Optimal result

Integrand size = 29, antiderivative size = 28

$$\int \frac{(ac+(bc+ad)x+bdx^2)^3}{(a+bx)^8} dx = -\frac{(c+dx)^4}{4(bc-ad)(a+bx)^4}$$

output
```
-1/4*(d*x+c)^4/(-a*d+b*c)/(b*x+a)^4
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 91 vs. 2(28) = 56.

Time = 0.02 (sec) , antiderivative size = 91, normalized size of antiderivative = 3.25

$$\int \frac{(ac+(bc+ad)x+bdx^2)^3}{(a+bx)^8} dx = -\frac{a^3d^3+a^2bd^2(c+4dx)+ab^2d(c^2+4cdx+6d^2x^2)+b^3(c^3+4c^2dx+6cd^2x^2+4d^3x^3)}{4b^4(a+bx)^4}$$

input
```
Integrate[(a*c + (b*c + a*d)*x + b*d*x^2)^3/(a + b*x)^8,x]
```

output
```
-1/4*(a^3*d^3 + a^2*b*d^2*(c + 4*d*x) + a*b^2*d*(c^2 + 4*c*d*x + 6*d^2*x^2
) + b^3*(c^3 + 4*c^2*d*x + 6*c*d^2*x^2 + 4*d^3*x^3))/(b^4*(a + b*x)^4)
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.069$, Rules used = {1120, 48}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(x(ad+bc)+ac+bdx^2)^3}{(a+bx)^8}dx$$

$$\Big\downarrow \text{1120}$$

$$\int \frac{(c+dx)^3}{(a+bx)^5}dx$$

$$\Big\downarrow \text{48}$$

$$-\frac{(c+dx)^4}{4(a+bx)^4(bc-ad)}$$

input
```
Int[(a*c + (b*c + a*d)*x + b*d*x^2)^3/(a + b*x)^8,x]
```

output
```
-1/4*(c + d*x)^4/((b*c - a*d)*(a + b*x)^4)
```

Defintions of rubi rules used

rule 48
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && EqQ[m + n + 2, 0] && NeQ[m, -1]
```

rule 1120
```
Int[((d_) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Int[(d + e*x)^(m + p)*(a/d + (c/e)*x)^p, x] /; FreeQ[{a, b, c, d
, e, m}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[p] && (EqQ[m + p,
0] || EqQ[m + 2*p + 2, 0])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 103 vs. $2(26) = 52$.

Time = 1.26 (sec) , antiderivative size = 104, normalized size of antiderivative = 3.71

method	result
risch	$\frac{-\frac{x^3d^3}{b}-\frac{3d^2(ad+bc)x^2}{2b^2}-\frac{d\left(a^2d^2+abcd+b^2c^2\right)x}{b^3}-\frac{a^3d^3+a^2bc\,d^2+a\,b^2c^2d+b^3c^3}{4b^4}}{(bx+a)^4}$
gosper	$-\frac{4x^3b^3d^3+6a\,b^2d^3x^2+6x^2b^3c\,d^2+4a^2b\,d^3x+4xa\,b^2c\,d^2+4x\,b^3c^2d+a^3d^3+a^2bc\,d^2+a\,b^2c^2d+b^3c^3}{4b^4(bx+a)^4}$
parallelrisch	$\frac{-4x^3b^3d^3-6a\,b^2d^3x^2-6x^2b^3c\,d^2-4a^2b\,d^3x-4xa\,b^2c\,d^2-4x\,b^3c^2d-a^3d^3-a^2bc\,d^2-a\,b^2c^2d-b^3c^3}{4b^4(bx+a)^4}$
default	$-\frac{d(a^2d^2-2abcd+b^2c^2)}{b^4(bx+a)^3}-\frac{d^3}{b^4(bx+a)}+\frac{3d^2(ad-bc)}{2b^4(bx+a)^2}-\frac{-a^3d^3+3a^2bc\,d^2-3a\,b^2c^2d+b^3c^3}{4b^4(bx+a)^4}$
orering	$-\frac{\left(4x^3b^3d^3+6a\,b^2d^3x^2+6x^2b^3c\,d^2+4a^2b\,d^3x+4xa\,b^2c\,d^2+4x\,b^3c^2d+a^3d^3+a^2bc\,d^2+a\,b^2c^2d+b^3c^3\right)\left(ac+(ad+bc)x+bd\,x^2\right)^3}{4b^4(bx+a)^7(dx+c)^3}$
norman	$\frac{-b^2d^3x^6+\frac{a^3\left(-d^3a^3b^3-a^2b^4c\,d^2-a\,b^5c^2d-b^6c^3\right)}{4b^7}+\frac{3\left(-3a\,b^3d^3-b^4c\,d^2\right)x^5}{2b^2}+\frac{\left(-17d^3a^2b^3-11a\,b^4c\,d^2-2b^5c^2d\right)x^4}{2b^3}+\frac{\left(-35d^3a^3b^3-3\right.}{}}{(bx+a)^7}$

input
```
int((a*c+(a*d+b*c)*x+b*d*x^2)^3/(b*x+a)^8,x,method=_RETURNVERBOSE)
```

output
```
(-1/b*x^3*d^3-3/2*d^2*(a*d+b*c)/b^2*x^2-d*(a^2*d^2+a*b*c*d+b^2*c^2)/b^3*x-
1/4*(a^3*d^3+a^2*b*c*d^2+a*b^2*c^2*d+b^3*c^3)/b^4)/(b*x+a)^4
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 143 vs. 2(26) = 52.

Time = 0.07 (sec) , antiderivative size = 143, normalized size of antiderivative = 5.11

$$\int \frac{(ac+(bc+ad)x+bdx^2)^3}{(a+bx)^8}\,dx = -\frac{4\,b^3d^3x^3+b^3c^3+ab^2c^2d+a^2bcd^2+a^3d^3+6\,(b^3cd^2+ab^2d^3)x^2+4\,(b^3c^2d+ab^2cd^2+a^2bd^3)x}{4\,(b^8x^4+4\,ab^7x^3+6\,a^2b^6x^2+4\,a^3b^5x+a^4b^4)}$$

input
```
integrate((a*c+(a*d+b*c)*x+b*d*x^2)^3/(b*x+a)^8,x, algorithm="fricas")
```

output
```
-1/4*(4*b^3*d^3*x^3 + b^3*c^3 + a*b^2*c^2*d + a^2*b*c*d^2 + a^3*d^3 + 6*(b
^3*c*d^2 + a*b^2*d^3)*x^2 + 4*(b^3*c^2*d + a*b^2*c*d^2 + a^2*b*d^3)*x)/(b^
8*x^4 + 4*a*b^7*x^3 + 6*a^2*b^6*x^2 + 4*a^3*b^5*x + a^4*b^4)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 155 vs. 2(22) = 44.

Time = 0.94 (sec) , antiderivative size = 155, normalized size of antiderivative = 5.54

$$\int \frac{(ac+(bc+ad)x+bdx^2)^3}{(a+bx)^8}\,dx = \frac{-a^3d^3-a^2bcd^2-ab^2c^2d-b^3c^3-4b^3d^3x^3+x^2(-6ab^2d^3-6b^3cd^2)+x(-4a^2bd^3-4ab^2cd^2-4b^3c^2d)}{4a^4b^4+16a^3b^5x+24a^2b^6x^2+16ab^7x^3+4b^8x^4}$$

input
```
integrate((a*c+(a*d+b*c)*x+b*d*x**2)**3/(b*x+a)**8,x)
```

output
```
(-a**3*d**3 - a**2*b*c*d**2 - a*b**2*c**2*d - b**3*c**3 - 4*b**3*d**3*x**3
 + x**2*(-6*a*b**2*d**3 - 6*b**3*c*d**2) + x*(-4*a**2*b*d**3 - 4*a*b**2*c*
d**2 - 4*b**3*c**2*d))/(4*a**4*b**4 + 16*a**3*b**5*x + 24*a**2*b**6*x**2 +
 16*a*b**7*x**3 + 4*b**8*x**4)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 143 vs. $2(26) = 52$.

Time = 0.04 (sec) , antiderivative size = 143, normalized size of antiderivative = 5.11

$$\int \frac{(ac+(bc+ad)x+bdx^2)^3}{(a+bx)^8}\,dx = -\frac{4\,b^3d^3x^3+b^3c^3+ab^2c^2d+a^2bcd^2+a^3d^3+6\,(b^3cd^2+ab^2d^3)x^2+4\,(b^3c^2d+ab^2cd^2+a^2bd^3)x}{4\,(b^8x^4+4\,ab^7x^3+6\,a^2b^6x^2+4\,a^3b^5x+a^4b^4)}$$

input
```
integrate((a*c+(a*d+b*c)*x+b*d*x^2)^3/(b*x+a)^8,x, algorithm="maxima")
```

output
```
-1/4*(4*b^3*d^3*x^3 + b^3*c^3 + a*b^2*c^2*d + a^2*b*c*d^2 + a^3*d^3 + 6*(b
^3*c*d^2 + a*b^2*d^3)*x^2 + 4*(b^3*c^2*d + a*b^2*c*d^2 + a^2*b*d^3)*x)/(b^
8*x^4 + 4*a*b^7*x^3 + 6*a^2*b^6*x^2 + 4*a^3*b^5*x + a^4*b^4)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 111 vs. $2(26) = 52$.

Time = 0.15 (sec) , antiderivative size = 111, normalized size of antiderivative = 3.96

$$\int \frac{(ac+(bc+ad)x+bdx^2)^3}{(a+bx)^8}\,dx = -\frac{4\,b^3d^3x^3+6\,b^3cd^2x^2+6\,ab^2d^3x^2+4\,b^3c^2dx+4\,ab^2cd^2x+4\,a^2bd^3x+b^3c^3+ab^2c^2d+a^2bcd^2+a^3d^3}{4\,(bx+a)^4b^4}$$

input
```
integrate((a*c+(a*d+b*c)*x+b*d*x^2)^3/(b*x+a)^8,x, algorithm="giac")
```

output
```
-1/4*(4*b^3*d^3*x^3 + 6*b^3*c*d^2*x^2 + 6*a*b^2*d^3*x^2 + 4*b^3*c^2*d*x +
4*a*b^2*c*d^2*x + 4*a^2*b*d^3*x + b^3*c^3 + a*b^2*c^2*d + a^2*b*c*d^2 + a^
3*d^3)/((b*x + a)^4*b^4)
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 135, normalized size of antiderivative = 4.82

$$\int \frac{(ac+(bc+ad)x+bdx^2)^3}{(a+bx)^8}\,dx = -\frac{\frac{a^3\,d^3+a^2\,b\,c\,d^2+a\,b^2\,c^2\,d+b^3\,c^3}{4\,b^4}+\frac{d^3\,x^3}{b}+\frac{d\,x\,\left(a^2\,d^2+a\,b\,c\,d+b^2\,c^2\right)}{b^3}+\frac{3\,d^2\,x^2\,\left(a\,d+b\,c\right)}{2\,b^2}}{a^4+4\,a^3\,b\,x+6\,a^2\,b^2\,x^2+4\,a\,b^3\,x^3+b^4\,x^4}$$

input
```
int((a*c + x*(a*d + b*c) + b*d*x^2)^3/(a + b*x)^8,x)
```

output
```
-((a^3*d^3 + b^3*c^3 + a*b^2*c^2*d + a^2*b*c*d^2)/(4*b^4) + (d^3*x^3)/b +
(d*x*(a^2*d^2 + b^2*c^2 + a*b*c*d))/b^3 + (3*d^2*x^2*(a*d + b*c))/(2*b^2))
/(a^4 + b^4*x^4 + 4*a*b^3*x^3 + 6*a^2*b^2*x^2 + 4*a^3*b*x)
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 122, normalized size of antiderivative = 4.36

$$\int \frac{(ac+(bc+ad)x+bdx^2)^3}{(a+bx)^8}\,dx = \frac{b^3d^3x^4-6a\,b^2c\,d^2x^2-4a^2bc\,d^2x-4a\,b^2c^2dx-a^3c\,d^2-a^2b\,c^2d-a\,b^2c^3}{4a\,b^3\left(b^4x^4+4a\,b^3x^3+6a^2b^2x^2+4a^3bx+a^4\right)}$$

input
```
int((a*c+(a*d+b*c)*x+b*d*x^2)^3/(b*x+a)^8,x)
```

output
```
( - a**3*c*d**2 - a**2*b*c**2*d - 4*a**2*b*c*d**2*x - a*b**2*c**3 - 4*a*b*
*2*c**2*d*x - 6*a*b**2*c*d**2*x**2 + b**3*d**3*x**4)/(4*a*b**3*(a**4 + 4*a
**3*b*x + 6*a**2*b**2*x**2 + 4*a*b**3*x**3 + b**4*x**4))
```

3.37 $\int \frac{\left(ac+(bc+ad)x+bdx^2\right)^3}{(a+bx)^9} dx$

Optimal result

Integrand size = 29, antiderivative size = 58

$$\int \frac{\left(ac+(bc+ad)x+bdx^2\right)^3}{(a+bx)^9} dx = -\frac{(c+dx)^4}{5(bc-ad)(a+bx)^5} + \frac{d(c+dx)^4}{20(bc-ad)^2(a+bx)^4}$$

output
```
-1/5*(d*x+c)^4/(-a*d+b*c)/(b*x+a)^5+1/20*d*(d*x+c)^4/(-a*d+b*c)^2/(b*x+a)^
4
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.67

$$\int \frac{\left(ac+(bc+ad)x+bdx^2\right)^3}{(a+bx)^9} dx = -\frac{a^3d^3+a^2bd^2(2c+5dx)+ab^2d(3c^2+10cdx+10d^2x^2)+b^3(4c^3+15c^2dx+20cd^2x^2+10d^3x^3)}{20b^4(a+bx)^5}$$

input
```
Integrate[(a*c + (b*c + a*d)*x + b*d*x^2)^3/(a + b*x)^9,x]
```

output
```
-1/20*(a^3*d^3 + a^2*b*d^2*(2*c + 5*d*x) + a*b^2*d*(3*c^2 + 10*c*d*x + 10*
d^2*x^2) + b^3*(4*c^3 + 15*c^2*d*x + 20*c*d^2*x^2 + 10*d^3*x^3))/(b^4*(a +
 b*x)^5)
```

Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.55, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.069, Rules used = {1121, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(x(ad+bc)+ac+bdx^2\right)^3}{(a+bx)^9}dx$$

$$\Big\downarrow \text{1121}$$

$$\int \left(\frac{3d^2(bc-ad)}{b^3(a+bx)^4}+\frac{3d(bc-ad)^2}{b^3(a+bx)^5}+\frac{(bc-ad)^3}{b^3(a+bx)^6}+\frac{d^3}{b^3(a+bx)^3}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{d^2(bc-ad)}{b^4(a+bx)^3}-\frac{3d(bc-ad)^2}{4b^4(a+bx)^4}-\frac{(bc-ad)^3}{5b^4(a+bx)^5}-\frac{d^3}{2b^4(a+bx)^2}$$

input
```
Int[(a*c + (b*c + a*d)*x + b*d*x^2)^3/(a + b*x)^9,x]
```

output
```
-1/5*(b*c - a*d)^3/(b^4*(a + b*x)^5) - (3*d*(b*c - a*d)^2)/(4*b^4*(a + b*x
)^4) - (d^2*(b*c - a*d))/(b^4*(a + b*x)^3) - d^3/(2*b^4*(a + b*x)^2)
```

Defintions of rubi rules used

rule 1121
```
Int[((d_) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Int[ExpandIntegrand[(d + e*x)^(m + p)*(a/d + (c/e)*x)^p, x], x]
/; FreeQ[{a, b, c, d, e, m, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && (Int
egerQ[p] || (GtQ[a, 0] && GtQ[d, 0] && LtQ[c, 0]))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 109 vs. $2(54) = 108$.

Time = 1.29 (sec) , antiderivative size = 110, normalized size of antiderivative = 1.90

method	result
risch	$\frac{-\frac{x^3d^3}{2b}-\frac{d^2(ad+2bc)x^2}{2b^2}-\frac{d\left(a^2d^2+2abcd+3b^2c^2\right)x}{4b^3}-\frac{a^3d^3+2a^2bc\,d^2+3a\,b^2c^2d+4b^3c^3}{20b^4}}{(bx+a)^5}$
gosper	$-\frac{10x^3b^3d^3+10a\,b^2d^3x^2+20x^2b^3c\,d^2+5a^2b\,d^3x+10xa\,b^2c\,d^2+15x\,b^3c^2d+a^3d^3+2a^2bc\,d^2+3a\,b^2c^2d+4b^3c^3}{20b^4(bx+a)^5}$
default	$-\frac{-a^3d^3+3a^2bc\,d^2-3a\,b^2c^2d+b^3c^3}{5b^4(bx+a)^5}+\frac{d^2(ad-bc)}{b^4(bx+a)^3}-\frac{d^3}{2b^4(bx+a)^2}-\frac{3d\left(a^2d^2-2abcd+b^2c^2\right)}{4b^4(bx+a)^4}$
parallelrisch	$\frac{-10d^3x^3b^4-10a\,b^3d^3x^2-20b^4c\,d^2x^2-5a^2b^2d^3x-10a\,b^3c\,d^2x-15b^4c^2dx-a^3b\,d^3-2a^2b^2c\,d^2-3a\,b^3c^2d-4b^4c^3}{20b^5(bx+a)^5}$
orering	$-\frac{\left(10x^3b^3d^3+10a\,b^2d^3x^2+20x^2b^3c\,d^2+5a^2b\,d^3x+10xa\,b^2c\,d^2+15x\,b^3c^2d+a^3d^3+2a^2bc\,d^2+3a\,b^2c^2d+4b^3c^3\right)(ac+(ad+bc)x}{20b^4(bx+a)^8(dx+c)^3}$
norman	$\frac{\frac{\left(-2a\,b^4d^3-b^5c\,d^2\right)x^5}{b^3}+\frac{a^3\left(-a^3b^4d^3-2a^2b^5c\,d^2-3a\,c^2d\,b^6-4c^3b^7\right)}{20b^8}-\frac{b^2d^3x^6}{2}+\frac{\left(-13d^3a^2b^4-14c\,d^2a\,b^5-3c^2d\,b^6\right)x^4}{4b^4}+\frac{\left(-14a^3b^4d^3-\right.}{}}{(bx+}$

input
```
int((a*c+(a*d+b*c)*x+b*d*x^2)^3/(b*x+a)^9,x,method=_RETURNVERBOSE)
```

output
```
(-1/2/b*x^3*d^3-1/2/b^2*d^2*(a*d+2*b*c)*x^2-1/4/b^3*d*(a^2*d^2+2*a*b*c*d+3
*b^2*c^2)*x-1/20/b^4*(a^3*d^3+2*a^2*b*c*d^2+3*a*b^2*c^2*d+4*b^3*c^3))/(b*x
+a)^5
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 160 vs. $2(54) = 108$.

Time = 0.07 (sec) , antiderivative size = 160, normalized size of antiderivative = 2.76

$$\int \frac{(ac + (bc + ad)x + bdx^2)^3}{(a + bx)^9}\, dx = -\frac{10\, b^3 d^3 x^3 + 4\, b^3 c^3 + 3\, ab^2 c^2 d + 2\, a^2 bcd^2 + a^3 d^3 + 10\, (2\, b^3 cd^2 + ab^2 d^3) x^2 + 5\, (3\, b^3 c^2 d + 2\, ab^2 cd^2 + a^2 bc}{20\, (b^9 x^5 + 5\, ab^8 x^4 + 10\, a^2 b^7 x^3 + 10\, a^3 b^6 x^2 + 5\, a^4 b^5 x + a^5 b^4)}$$

input
```
integrate((a*c+(a*d+b*c)*x+b*d*x^2)^3/(b*x+a)^9,x, algorithm="fricas")
```

output
```
-1/20*(10*b^3*d^3*x^3 + 4*b^3*c^3 + 3*a*b^2*c^2*d + 2*a^2*b*c*d^2 + a^3*d^
3 + 10*(2*b^3*c*d^2 + a*b^2*d^3)*x^2 + 5*(3*b^3*c^2*d + 2*a*b^2*c*d^2 + a^
2*b*d^3)*x)/(b^9*x^5 + 5*a*b^8*x^4 + 10*a^2*b^7*x^3 + 10*a^3*b^6*x^2 + 5*a
^4*b^5*x + a^5*b^4)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 172 vs. $2(46) = 92$.

Time = 1.55 (sec) , antiderivative size = 172, normalized size of antiderivative = 2.97

$$\int \frac{(ac + (bc + ad)x + bdx^2)^3}{(a + bx)^9}\, dx = \frac{-a^3 d^3 - 2a^2 bcd^2 - 3ab^2 c^2 d - 4b^3 c^3 - 10b^3 d^3 x^3 + x^2(-10ab^2 d^3 - 20b^3 cd^2) + x(-5a^2 bd^3 - 10ab^2 cd^2 - 1}{20a^5 b^4 + 100a^4 b^5 x + 200a^3 b^6 x^2 + 200a^2 b^7 x^3 + 100ab^8 x^4 + 20b^9 x^5}$$

input
```
integrate((a*c+(a*d+b*c)*x+b*d*x**2)**3/(b*x+a)**9,x)
```

output
```
(-a**3*d**3 - 2*a**2*b*c*d**2 - 3*a*b**2*c**2*d - 4*b**3*c**3 - 10*b**3*d*
*3*x**3 + x**2*(-10*a*b**2*d**3 - 20*b**3*c*d**2) + x*(-5*a**2*b*d**3 - 10
*a*b**2*c*d**2 - 15*b**3*c**2*d))/(20*a**5*b**4 + 100*a**4*b**5*x + 200*a*
*3*b**6*x**2 + 200*a**2*b**7*x**3 + 100*a*b**8*x**4 + 20*b**9*x**5)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 160 vs. $2(54) = 108$.

Time = 0.05 (sec) , antiderivative size = 160, normalized size of antiderivative = 2.76

$$\int \frac{(ac + (bc + ad)x + bdx^2)^3}{(a + bx)^9} \, dx = -\frac{10\, b^3 d^3 x^3 + 4\, b^3 c^3 + 3\, ab^2 c^2 d + 2\, a^2 bcd^2 + a^3 d^3 + 10\,(2\, b^3 cd^2 + ab^2 d^3)x^2 + 5\,(3\, b^3 c^2 d + 2\, ab^2 cd^2 + a^2 bc}{20\,(b^9 x^5 + 5\, ab^8 x^4 + 10\, a^2 b^7 x^3 + 10\, a^3 b^6 x^2 + 5\, a^4 b^5 x + a^5 b^4)}$$

input
```
integrate((a*c+(a*d+b*c)*x+b*d*x^2)^3/(b*x+a)^9,x, algorithm="maxima")
```

output
```
-1/20*(10*b^3*d^3*x^3 + 4*b^3*c^3 + 3*a*b^2*c^2*d + 2*a^2*b*c*d^2 + a^3*d^
3 + 10*(2*b^3*c*d^2 + a*b^2*d^3)*x^2 + 5*(3*b^3*c^2*d + 2*a*b^2*c*d^2 + a^
2*b*d^3)*x)/(b^9*x^5 + 5*a*b^8*x^4 + 10*a^2*b^7*x^3 + 10*a^3*b^6*x^2 + 5*a
^4*b^5*x + a^5*b^4)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 114 vs. $2(54) = 108$.

Time = 0.15 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.97

$$\int \frac{(ac + (bc + ad)x + bdx^2)^3}{(a + bx)^9} \, dx = -\frac{10\, b^3 d^3 x^3 + 20\, b^3 cd^2 x^2 + 10\, ab^2 d^3 x^2 + 15\, b^3 c^2 dx + 10\, ab^2 cd^2 x + 5\, a^2 bd^3 x + 4\, b^3 c^3 + 3\, ab^2 c^2 d + 2\, a^2 bcd}{20\,(bx + a)^5 b^4}$$

input
```
integrate((a*c+(a*d+b*c)*x+b*d*x^2)^3/(b*x+a)^9,x, algorithm="giac")
```

output
```
-1/20*(10*b^3*d^3*x^3 + 20*b^3*c*d^2*x^2 + 10*a*b^2*d^3*x^2 + 15*b^3*c^2*d
*x + 10*a*b^2*c*d^2*x + 5*a^2*b*d^3*x + 4*b^3*c^3 + 3*a*b^2*c^2*d + 2*a^2*
b*c*d^2 + a^3*d^3)/((b*x + a)^5*b^4)
```

Mupad [B] (verification not implemented)

Time = 5.38 (sec) , antiderivative size = 154, normalized size of antiderivative = 2.66

$$\int \frac{(ac+(bc+ad)x+bdx^2)^3}{(a+bx)^9}\,dx = -\frac{\frac{a^3\,d^3+2\,a^2\,b\,c\,d^2+3\,a\,b^2\,c^2\,d+4\,b^3\,c^3}{20\,b^4}+\frac{d^3\,x^3}{2\,b}+\frac{d\,x\,\left(a^2\,d^2+2\,a\,b\,c\,d+3\,b^2\,c^2\right)}{4\,b^3}+\frac{d^2\,x^2\,\left(a\,d+2\,b\,c\right)}{2\,b^2}}{a^5+5\,a^4\,b\,x+10\,a^3\,b^2\,x^2+10\,a^2\,b^3\,x^3+5\,a\,b^4\,x^4+b^5\,x^5}$$

input
```
int((a*c + x*(a*d + b*c) + b*d*x^2)^3/(a + b*x)^9,x)
```

output
```
-((a^3*d^3 + 4*b^3*c^3 + 3*a*b^2*c^2*d + 2*a^2*b*c*d^2)/(20*b^4) + (d^3*x^
3)/(2*b) + (d*x*(a^2*d^2 + 3*b^2*c^2 + 2*a*b*c*d))/(4*b^3) + (d^2*x^2*(a*d
 + 2*b*c))/(2*b^2))/(a^5 + b^5*x^5 + 5*a*b^4*x^4 + 10*a^3*b^2*x^2 + 10*a^2
*b^3*x^3 + 5*a^4*b*x)
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 159, normalized size of antiderivative = 2.74

$$\int \frac{(ac+(bc+ad)x+bdx^2)^3}{(a+bx)^9}\,dx = \frac{-10b^3d^3x^3-10a\,b^2d^3x^2-20b^3c\,d^2x^2-5a^2b\,d^3x-10a\,b^2c\,d^2x-15b^3c^2dx-a^3d^3-2a^2bc\,d^2-3a\,b^2c^2d}{20b^4\left(b^5x^5+5a\,b^4x^4+10a^2b^3x^3+10a^3b^2x^2+5a^4bx+a^5\right)}$$

input
```
int((a*c+(a*d+b*c)*x+b*d*x^2)^3/(b*x+a)^9,x)
```

output
```
( - a**3*d**3 - 2*a**2*b*c*d**2 - 5*a**2*b*d**3*x - 3*a*b**2*c**2*d - 10*a
*b**2*c*d**2*x - 10*a*b**2*d**3*x**2 - 4*b**3*c**3 - 15*b**3*c**2*d*x - 20
*b**3*c*d**2*x**2 - 10*b**3*d**3*x**3)/(20*b**4*(a**5 + 5*a**4*b*x + 10*a*
*3*b**2*x**2 + 10*a**2*b**3*x**3 + 5*a*b**4*x**4 + b**5*x**5))
```

3.38 $\int \frac{(ac+(bc+ad)x+bdx^2)^3}{(a+bx)^{10}}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 92

$$\int \frac{(ac+(bc+ad)x+bdx^2)^3}{(a+bx)^{10}}\,dx = -\frac{(bc-ad)^3}{6b^4(a+bx)^6} - \frac{3d(bc-ad)^2}{5b^4(a+bx)^5} - \frac{3d^2(bc-ad)}{4b^4(a+bx)^4} - \frac{d^3}{3b^4(a+bx)^3}$$

output
```
-1/6*(-a*d+b*c)^3/b^4/(b*x+a)^6-3/5*d*(-a*d+b*c)^2/b^4/(b*x+a)^5-3/4*d^2*(
-a*d+b*c)/b^4/(b*x+a)^4-1/3*d^3/b^4/(b*x+a)^3
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.05

$$\int \frac{(ac+(bc+ad)x+bdx^2)^3}{(a+bx)^{10}}\,dx = -\frac{a^3d^3+3a^2bd^2(c+2dx)+3ab^2d(2c^2+6cdx+5d^2x^2)+b^3(10c^3+36c^2dx+45cd^2x^2+20d^3x^3)}{60b^4(a+bx)^6}$$

input
```
Integrate[(a*c + (b*c + a*d)*x + b*d*x^2)^3/(a + b*x)^10,x]
```

output
```
-1/60*(a^3*d^3 + 3*a^2*b*d^2*(c + 2*d*x) + 3*a*b^2*d*(2*c^2 + 6*c*d*x + 5*
d^2*x^2) + b^3*(10*c^3 + 36*c^2*d*x + 45*c*d^2*x^2 + 20*d^3*x^3))/(b^4*(a
+ b*x)^6)
```

Rubi [A] (verified)

Time = 0.46 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.069, Rules used = {1121, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(x(ad+bc)+ac+bdx^2\right)^3}{(a+bx)^{10}}dx$$

$$\Big\downarrow \text{1121}$$

$$\int \left(\frac{3d^2(bc-ad)}{b^3(a+bx)^5}+\frac{3d(bc-ad)^2}{b^3(a+bx)^6}+\frac{(bc-ad)^3}{b^3(a+bx)^7}+\frac{d^3}{b^3(a+bx)^4}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{3d^2(bc-ad)}{4b^4(a+bx)^4}-\frac{3d(bc-ad)^2}{5b^4(a+bx)^5}-\frac{(bc-ad)^3}{6b^4(a+bx)^6}-\frac{d^3}{3b^4(a+bx)^3}$$

input
```
Int[(a*c + (b*c + a*d)*x + b*d*x^2)^3/(a + b*x)^10,x]
```

output
```
-1/6*(b*c - a*d)^3/(b^4*(a + b*x)^6) - (3*d*(b*c - a*d)^2)/(5*b^4*(a + b*x
)^5) - (3*d^2*(b*c - a*d))/(4*b^4*(a + b*x)^4) - d^3/(3*b^4*(a + b*x)^3)
```

Defintions of rubi rules used

rule 1121
```
Int[((d_) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Int[ExpandIntegrand[(d + e*x)^(m + p)*(a/d + (c/e)*x)^p, x], x]
/; FreeQ[{a, b, c, d, e, m, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && (Int
egerQ[p] || (GtQ[a, 0] && GtQ[d, 0] && LtQ[c, 0]))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 1.27 (sec) , antiderivative size = 110, normalized size of antiderivative = 1.20

method	result
risch	$\frac{-\frac{x^3 d^3}{3b}-\frac{d^2(ad+3bc)x^2}{4b^2}-\frac{d\left(a^2d^2+3abcd+6b^2c^2\right)x}{10b^3}-\frac{a^3d^3+3a^2bc\,d^2+6a\,b^2c^2d+10b^3c^3}{60b^4}}{(bx+a)^6}$
gosper	$-\frac{20x^3b^3d^3+15a\,b^2d^3x^2+45x^2b^3c\,d^2+6a^2b\,d^3x+18xa\,b^2c\,d^2+36x\,b^3c^2d+a^3d^3+3a^2bc\,d^2+6a\,b^2c^2d+10b^3c^3}{60b^4(bx+a)^6}$
default	$-\frac{3d\left(a^2d^2-2abcd+b^2c^2\right)}{5b^4(bx+a)^5}-\frac{d^3}{3b^4(bx+a)^3}-\frac{-a^3d^3+3a^2bc\,d^2-3a\,b^2c^2d+b^3c^3}{6b^4(bx+a)^6}+\frac{3d^2(ad-bc)}{4b^4(bx+a)^4}$
parallelrisch	$\frac{-20d^3x^3b^5-15a\,b^4d^3x^2-45b^5c\,d^2x^2-6a^2b^3d^3x-18a\,b^4c\,d^2x-36b^5c^2dx-d^3a^3b^2-3c\,d^2a^2b^3-6c^2da\,b^4-10b^5c^3}{60b^6(bx+a)^6}$
orering	$-\frac{\left(20x^3b^3d^3+15a\,b^2d^3x^2+45x^2b^3c\,d^2+6a^2b\,d^3x+18xa\,b^2c\,d^2+36x\,b^3c^2d+a^3d^3+3a^2bc\,d^2+6a\,b^2c^2d+10b^3c^3\right)(ac+(ad+bc)}{60b^4(bx+a)^9(dx+c)^3}$
norman	$\frac{\frac{a^3\left(-a^3b^5d^3-3a^2b^6c\,d^2-6a\,b^7c^2d-10c^3b^8\right)}{60b^9}-\frac{b^2d^3x^6}{3}+\frac{\left(-5d^3a\,b^5-3c\,d^2b^6\right)x^5}{4b^4}+\frac{\left(-37a^2b^5d^3-51a\,d^2c\,b^6-12c^2d\,b^7\right)x^4}{20b^5}+\frac{(-42a^3b^5d}{}}{(bx}$

input
```
int((a*c+(a*d+b*c)*x+b*d*x^2)^3/(b*x+a)^10,x,method=_RETURNVERBOSE)
```

output
```
(-1/3/b*x^3*d^3-1/4/b^2*d^2*(a*d+3*b*c)*x^2-1/10/b^3*d*(a^2*d^2+3*a*b*c*d+
6*b^2*c^2)*x-1/60/b^4*(a^3*d^3+3*a^2*b*c*d^2+6*a*b^2*c^2*d+10*b^3*c^3))/(b
*x+a)^6
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 171 vs. $2(84) = 168$.

Time = 0.07 (sec) , antiderivative size = 171, normalized size of antiderivative = 1.86

$$\int \frac{(ac + (bc + ad)x + bdx^2)^3}{(a + bx)^{10}}\,dx = -\frac{20\,b^3d^3x^3 + 10\,b^3c^3 + 6\,ab^2c^2d + 3\,a^2bcd^2 + a^3d^3 + 15\,(3\,b^3cd^2 + ab^2d^3)x^2 + 6\,(6\,b^3c^2d + 3\,ab^2cd^2 + a^2}{60\,(b^{10}x^6 + 6\,ab^9x^5 + 15\,a^2b^8x^4 + 20\,a^3b^7x^3 + 15\,a^4b^6x^2 + 6\,a^5b^5x + a^6b^4)}$$

input
```
integrate((a*c+(a*d+b*c)*x+b*d*x^2)^3/(b*x+a)^10,x, algorithm="fricas")
```

output
```
-1/60*(20*b^3*d^3*x^3 + 10*b^3*c^3 + 6*a*b^2*c^2*d + 3*a^2*b*c*d^2 + a^3*d
^3 + 15*(3*b^3*c*d^2 + a*b^2*d^3)*x^2 + 6*(6*b^3*c^2*d + 3*a*b^2*c*d^2 + a
^2*b*d^3)*x)/(b^10*x^6 + 6*a*b^9*x^5 + 15*a^2*b^8*x^4 + 20*a^3*b^7*x^3 + 1
5*a^4*b^6*x^2 + 6*a^5*b^5*x + a^6*b^4)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 184 vs. $2(83) = 166$.

Time = 3.03 (sec) , antiderivative size = 184, normalized size of antiderivative = 2.00

$$\int \frac{(ac + (bc + ad)x + bdx^2)^3}{(a + bx)^{10}}\,dx = \frac{-a^3d^3 - 3a^2bcd^2 - 6ab^2c^2d - 10b^3c^3 - 20b^3d^3x^3 + x^2(-15ab^2d^3 - 45b^3cd^2) + x(-6a^2bd^3 - 18ab^2cd^2 -}{60a^6b^4 + 360a^5b^5x + 900a^4b^6x^2 + 1200a^3b^7x^3 + 900a^2b^8x^4 + 360ab^9x^5 + 60b^{10}x^6}$$

input
```
integrate((a*c+(a*d+b*c)*x+b*d*x**2)**3/(b*x+a)**10,x)
```

output
```
(-a**3*d**3 - 3*a**2*b*c*d**2 - 6*a*b**2*c**2*d - 10*b**3*c**3 - 20*b**3*d
**3*x**3 + x**2*(-15*a*b**2*d**3 - 45*b**3*c*d**2) + x*(-6*a**2*b*d**3 - 1
8*a*b**2*c*d**2 - 36*b**3*c**2*d))/(60*a**6*b**4 + 360*a**5*b**5*x + 900*a
**4*b**6*x**2 + 1200*a**3*b**7*x**3 + 900*a**2*b**8*x**4 + 360*a*b**9*x**5
 + 60*b**10*x**6)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 171 vs. $2(84) = 168$.

Time = 0.04 (sec) , antiderivative size = 171, normalized size of antiderivative = 1.86

$$\int \frac{(ac + (bc + ad)x + bdx^2)^3}{(a + bx)^{10}}\, dx = -\frac{20\, b^3d^3x^3 + 10\, b^3c^3 + 6\, ab^2c^2d + 3\, a^2bcd^2 + a^3d^3 + 15\, (3\, b^3cd^2 + ab^2d^3)x^2 + 6\, (6\, b^3c^2d + 3\, ab^2cd^2 + a^2}{60\, (b^{10}x^6 + 6\, ab^9x^5 + 15\, a^2b^8x^4 + 20\, a^3b^7x^3 + 15\, a^4b^6x^2 + 6\, a^5b^5x + a^6b^4)}$$

input
```
integrate((a*c+(a*d+b*c)*x+b*d*x^2)^3/(b*x+a)^10,x, algorithm="maxima")
```

output
```
-1/60*(20*b^3*d^3*x^3 + 10*b^3*c^3 + 6*a*b^2*c^2*d + 3*a^2*b*c*d^2 + a^3*d
^3 + 15*(3*b^3*c*d^2 + a*b^2*d^3)*x^2 + 6*(6*b^3*c^2*d + 3*a*b^2*c*d^2 + a
^2*b*d^3)*x)/(b^10*x^6 + 6*a*b^9*x^5 + 15*a^2*b^8*x^4 + 20*a^3*b^7*x^3 + 1
5*a^4*b^6*x^2 + 6*a^5*b^5*x + a^6*b^4)
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.24

$$\int \frac{(ac + (bc + ad)x + bdx^2)^3}{(a + bx)^{10}}\, dx = -\frac{20\, b^3d^3x^3 + 45\, b^3cd^2x^2 + 15\, ab^2d^3x^2 + 36\, b^3c^2dx + 18\, ab^2cd^2x + 6\, a^2bd^3x + 10\, b^3c^3 + 6\, ab^2c^2d + 3\, a^2b}{60\, (bx + a)^6b^4}$$

input
```
integrate((a*c+(a*d+b*c)*x+b*d*x^2)^3/(b*x+a)^10,x, algorithm="giac")
```

output
```
-1/60*(20*b^3*d^3*x^3 + 45*b^3*c*d^2*x^2 + 15*a*b^2*d^3*x^2 + 36*b^3*c^2*d
*x + 18*a*b^2*c*d^2*x + 6*a^2*b*d^3*x + 10*b^3*c^3 + 6*a*b^2*c^2*d + 3*a^2
*b*c*d^2 + a^3*d^3)/((b*x + a)^6*b^4)
```

Mupad [B] (verification not implemented)

Time = 5.40 (sec) , antiderivative size = 165, normalized size of antiderivative = 1.79

$$\int \frac{(ac+(bc+ad)x+bdx^2)^3}{(a+bx)^{10}}\,dx = -\frac{\frac{a^3\,d^3+3\,a^2\,b\,c\,d^2+6\,a\,b^2\,c^2\,d+10\,b^3\,c^3}{60\,b^4}+\frac{d^3\,x^3}{3\,b}+\frac{d\,x\,\left(a^2\,d^2+3\,a\,b\,c\,d+6\,b^2\,c^2\right)}{10\,b^3}+\frac{d^2\,x^2\,\left(a\,d+3\,b\,c\right)}{4\,b^2}}{a^6+6\,a^5\,b\,x+15\,a^4\,b^2\,x^2+20\,a^3\,b^3\,x^3+15\,a^2\,b^4\,x^4+6\,a\,b^5\,x^5+b^6\,x^6}$$

input
```
int((a*c + x*(a*d + b*c) + b*d*x^2)^3/(a + b*x)^10,x)
```

output
```
-((a^3*d^3 + 10*b^3*c^3 + 6*a*b^2*c^2*d + 3*a^2*b*c*d^2)/(60*b^4) + (d^3*x
^3)/(3*b) + (d*x*(a^2*d^2 + 6*b^2*c^2 + 3*a*b*c*d))/(10*b^3) + (d^2*x^2*(a
*d + 3*b*c))/(4*b^2))/(a^6 + b^6*x^6 + 6*a*b^5*x^5 + 15*a^4*b^2*x^2 + 20*a
^3*b^3*x^3 + 15*a^2*b^4*x^4 + 6*a^5*b*x)
```

Reduce [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 170, normalized size of antiderivative = 1.85

$$\int \frac{(ac+(bc+ad)x+bdx^2)^3}{(a+bx)^{10}}\,dx = \frac{-20b^3d^3x^3-15a\,b^2d^3x^2-45b^3c\,d^2x^2-6a^2b\,d^3x-18a\,b^2c\,d^2x-36b^3c^2dx-a^3d^3-3a^2bc\,d^2-6a\,b^2c^2d}{60b^4\left(b^6x^6+6a\,b^5x^5+15a^2b^4x^4+20a^3b^3x^3+15a^4b^2x^2+6a^5bx+a^6\right)}$$

input
```
int((a*c+(a*d+b*c)*x+b*d*x^2)^3/(b*x+a)^10,x)
```

output
```
( - a**3*d**3 - 3*a**2*b*c*d**2 - 6*a**2*b*d**3*x - 6*a*b**2*c**2*d - 18*a
*b**2*c*d**2*x - 15*a*b**2*d**3*x**2 - 10*b**3*c**3 - 36*b**3*c**2*d*x - 4
5*b**3*c*d**2*x**2 - 20*b**3*d**3*x**3)/(60*b**4*(a**6 + 6*a**5*b*x + 15*a
**4*b**2*x**2 + 20*a**3*b**3*x**3 + 15*a**2*b**4*x**4 + 6*a*b**5*x**5 + b*
*6*x**6))
```

3.39 $\int \frac{\left(ac+(bc+ad)x+bdx^2\right)^3}{(a+bx)^{11}}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 92

$$\int \frac{\left(ac+(bc+ad)x+bdx^2\right)^3}{(a+bx)^{11}}\,dx = -\frac{(bc-ad)^3}{7b^4(a+bx)^7}-\frac{d(bc-ad)^2}{2b^4(a+bx)^6}-\frac{3d^2(bc-ad)}{5b^4(a+bx)^5}-\frac{d^3}{4b^4(a+bx)^4}$$

output
```
-1/7*(-a*d+b*c)^3/b^4/(b*x+a)^7-1/2*d*(-a*d+b*c)^2/b^4/(b*x+a)^6-3/5*d^2*(
-a*d+b*c)/b^4/(b*x+a)^5-1/4*d^3/b^4/(b*x+a)^4
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.05

$$\int \frac{\left(ac+(bc+ad)x+bdx^2\right)^3}{(a+bx)^{11}}\,dx = -\frac{a^3d^3+a^2bd^2(4c+7dx)+ab^2d(10c^2+28cdx+21d^2x^2)+b^3(20c^3+70c^2dx+84cd^2x^2+35d^3x^3)}{140b^4(a+bx)^7}$$

input
```
Integrate[(a*c + (b*c + a*d)*x + b*d*x^2)^3/(a + b*x)^11,x]
```

output
```
-1/140*(a^3*d^3 + a^2*b*d^2*(4*c + 7*d*x) + a*b^2*d*(10*c^2 + 28*c*d*x + 2
1*d^2*x^2) + b^3*(20*c^3 + 70*c^2*d*x + 84*c*d^2*x^2 + 35*d^3*x^3))/(b^4*(
a + b*x)^7)
```

Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.069$, Rules used = {1121, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(x(ad+bc)+ac+bdx^2\right)^3}{(a+bx)^{11}}\,dx$$

$$\Big\downarrow \text{1121}$$

$$\int \left(\frac{3d^2(bc-ad)}{b^3(a+bx)^6}+\frac{3d(bc-ad)^2}{b^3(a+bx)^7}+\frac{(bc-ad)^3}{b^3(a+bx)^8}+\frac{d^3}{b^3(a+bx)^5}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{3d^2(bc-ad)}{5b^4(a+bx)^5}-\frac{d(bc-ad)^2}{2b^4(a+bx)^6}-\frac{(bc-ad)^3}{7b^4(a+bx)^7}-\frac{d^3}{4b^4(a+bx)^4}$$

input
```
Int[(a*c + (b*c + a*d)*x + b*d*x^2)^3/(a + b*x)^11,x]
```

output
```
-1/7*(b*c - a*d)^3/(b^4*(a + b*x)^7) - (d*(b*c - a*d)^2)/(2*b^4*(a + b*x)^
6) - (3*d^2*(b*c - a*d))/(5*b^4*(a + b*x)^5) - d^3/(4*b^4*(a + b*x)^4)
```

Defintions of rubi rules used

rule 1121
```
Int[((d_) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Int[ExpandIntegrand[(d + e*x)^(m + p)*(a/d + (c/e)*x)^p, x], x]
/; FreeQ[{a, b, c, d, e, m, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && (Int
egerQ[p] || (GtQ[a, 0] && GtQ[d, 0] && LtQ[c, 0]))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 1.26 (sec) , antiderivative size = 110, normalized size of antiderivative = 1.20

method	result
risch	$\frac{-\frac{x^3 d^3}{4b}-\frac{3d^2(ad+4bc)x^2}{20b^2}-\frac{d\left(a^2d^2+4abcd+10b^2c^2\right)x}{20b^3}-\frac{a^3d^3+4a^2bc\,d^2+10a\,b^2c^2d+20b^3c^3}{140b^4}}{(bx+a)^7}$
gosper	$-\frac{35x^3b^3d^3+21a\,b^2d^3x^2+84x^2b^3c\,d^2+7a^2b\,d^3x+28xa\,b^2c\,d^2+70x\,b^3c^2d+a^3d^3+4a^2bc\,d^2+10a\,b^2c^2d+20b^3c^3}{140b^4(bx+a)^7}$
default	$\frac{3d^2(ad-bc)}{5b^4(bx+a)^5}-\frac{-a^3d^3+3a^2bc\,d^2-3a\,b^2c^2d+b^3c^3}{7b^4(bx+a)^7}-\frac{d\left(a^2d^2-2abcd+b^2c^2\right)}{2b^4(bx+a)^6}-\frac{d^3}{4b^4(bx+a)^4}$
parallelrisch	$\frac{-35d^3x^3b^6-21a\,b^5d^3x^2-84b^6c\,d^2x^2-7a^2b^4d^3x-28a\,b^5c\,d^2x-70b^6c^2dx-d^3a^3b^3-4a^2b^4c\,d^2-10a\,b^5c^2d-20b^6c^3}{140b^7(bx+a)^7}$
orering	$-\frac{\left(35x^3b^3d^3+21a\,b^2d^3x^2+84x^2b^3c\,d^2+7a^2b\,d^3x+28xa\,b^2c\,d^2+70x\,b^3c^2d+a^3d^3+4a^2bc\,d^2+10a\,b^2c^2d+20b^3c^3\right)\left(ac+(ad+bc\right.}{140b^4(bx+a)^{10}(dx+c)^3}$
norman	$\frac{\frac{a^3\left(-a^3b^6d^3-4a^2b^7c\,d^2-10a\,b^8c^2d-20b^9c^3\right)}{140b^{10}}-\frac{b^2d^3x^6}{4}+\frac{3\left(-3a\,d^3b^6-2c\,d^2b^7\right)x^5}{10b^5}+\frac{\left(-5a^2b^6d^3-8a\,d^2c\,b^7-2c^2d\,b^8\right)x^4}{4b^6}+\frac{\left(-6a^3b^6d^3-\right.}{}}{(bx+a}$

input
```
int((a*c+(a*d+b*c)*x+b*d*x^2)^3/(b*x+a)^11,x,method=_RETURNVERBOSE)
```

output
```
(-1/4/b*x^3*d^3-3/20/b^2*d^2*(a*d+4*b*c)*x^2-1/20/b^3*d*(a^2*d^2+4*a*b*c*d
+10*b^2*c^2)*x-1/140/b^4*(a^3*d^3+4*a^2*b*c*d^2+10*a*b^2*c^2*d+20*b^3*c^3)
)/(b*x+a)^7
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 182 vs. $2(84) = 168$.

Time = 0.08 (sec) , antiderivative size = 182, normalized size of antiderivative = 1.98

$$\int \frac{(ac+(bc+ad)x+bdx^2)^3}{(a+bx)^{11}}\,dx = -\frac{35\,b^3d^3x^3+20\,b^3c^3+10\,ab^2c^2d+4\,a^2bcd^2+a^3d^3+21\,(4\,b^3cd^2+ab^2d^3)x^2+7\,(10\,b^3c^2d+4\,ab^2cd^2+}{140\,(b^{11}x^7+7\,ab^{10}x^6+21\,a^2b^9x^5+35\,a^3b^8x^4+35\,a^4b^7x^3+21\,a^5b^6x^2+7\,a^6b^5x+a^7b^4)}$$

input
```
integrate((a*c+(a*d+b*c)*x+b*d*x^2)^3/(b*x+a)^11,x, algorithm="fricas")
```

output
```
-1/140*(35*b^3*d^3*x^3 + 20*b^3*c^3 + 10*a*b^2*c^2*d + 4*a^2*b*c*d^2 + a^3
*d^3 + 21*(4*b^3*c*d^2 + a*b^2*d^3)*x^2 + 7*(10*b^3*c^2*d + 4*a*b^2*c*d^2
+ a^2*b*d^3)*x)/(b^11*x^7 + 7*a*b^10*x^6 + 21*a^2*b^9*x^5 + 35*a^3*b^8*x^4
 + 35*a^4*b^7*x^3 + 21*a^5*b^6*x^2 + 7*a^6*b^5*x + a^7*b^4)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 196 vs. $2(82) = 164$.

Time = 5.79 (sec) , antiderivative size = 196, normalized size of antiderivative = 2.13

$$\int \frac{(ac+(bc+ad)x+bdx^2)^3}{(a+bx)^{11}}\,dx = \frac{-a^3d^3-4a^2bcd^2-10ab^2c^2d-20b^3c^3-35b^3d^3x^3+x^2(-21ab^2d^3-84b^3cd^2)+x(-7a^2bd^3-28ab^2cd^2-}{140a^7b^4+980a^6b^5x+2940a^5b^6x^2+4900a^4b^7x^3+4900a^3b^8x^4+2940a^2b^9x^5+980ab^{10}x^6+140b^1}$$

input
```
integrate((a*c+(a*d+b*c)*x+b*d*x**2)**3/(b*x+a)**11,x)
```

output
```
(-a**3*d**3 - 4*a**2*b*c*d**2 - 10*a*b**2*c**2*d - 20*b**3*c**3 - 35*b**3*
d**3*x**3 + x**2*(-21*a*b**2*d**3 - 84*b**3*c*d**2) + x*(-7*a**2*b*d**3 -
28*a*b**2*c*d**2 - 70*b**3*c**2*d))/(140*a**7*b**4 + 980*a**6*b**5*x + 294
0*a**5*b**6*x**2 + 4900*a**4*b**7*x**3 + 4900*a**3*b**8*x**4 + 2940*a**2*b
**9*x**5 + 980*a*b**10*x**6 + 140*b**11*x**7)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 182 vs. $2(84) = 168$.

Time = 0.05 (sec) , antiderivative size = 182, normalized size of antiderivative = 1.98

$$\int \frac{(ac+(bc+ad)x+bdx^2)^3}{(a+bx)^{11}}\,dx = -\frac{35\,b^3d^3x^3+20\,b^3c^3+10\,ab^2c^2d+4\,a^2bcd^2+a^3d^3+21\,(4\,b^3cd^2+ab^2d^3)x^2+7\,(10\,b^3c^2d+4\,ab^2cd^2+\cdots}{140\,(b^{11}x^7+7\,ab^{10}x^6+21\,a^2b^9x^5+35\,a^3b^8x^4+35\,a^4b^7x^3+21\,a^5b^6x^2+7\,a^6b^5x+a^7b^4)}$$

input
```
integrate((a*c+(a*d+b*c)*x+b*d*x^2)^3/(b*x+a)^11,x, algorithm="maxima")
```

output
```
-1/140*(35*b^3*d^3*x^3 + 20*b^3*c^3 + 10*a*b^2*c^2*d + 4*a^2*b*c*d^2 + a^3
*d^3 + 21*(4*b^3*c*d^2 + a*b^2*d^3)*x^2 + 7*(10*b^3*c^2*d + 4*a*b^2*c*d^2
+ a^2*b*d^3)*x)/(b^11*x^7 + 7*a*b^10*x^6 + 21*a^2*b^9*x^5 + 35*a^3*b^8*x^4
 + 35*a^4*b^7*x^3 + 21*a^5*b^6*x^2 + 7*a^6*b^5*x + a^7*b^4)
```

Giac [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.24

$$\int \frac{(ac+(bc+ad)x+bdx^2)^3}{(a+bx)^{11}}\,dx = -\frac{35\,b^3d^3x^3+84\,b^3cd^2x^2+21\,ab^2d^3x^2+70\,b^3c^2dx+28\,ab^2cd^2x+7\,a^2bd^3x+20\,b^3c^3+10\,ab^2c^2d+4\,a^2\cdots}{140\,(bx+a)^7b^4}$$

input
```
integrate((a*c+(a*d+b*c)*x+b*d*x^2)^3/(b*x+a)^11,x, algorithm="giac")
```

output
```
-1/140*(35*b^3*d^3*x^3 + 84*b^3*c*d^2*x^2 + 21*a*b^2*d^3*x^2 + 70*b^3*c^2*
d*x + 28*a*b^2*c*d^2*x + 7*a^2*b*d^3*x + 20*b^3*c^3 + 10*a*b^2*c^2*d + 4*a
^2*b*c*d^2 + a^3*d^3)/((b*x + a)^7*b^4)
```

Mupad [B] (verification not implemented)

Time = 5.44 (sec) , antiderivative size = 176, normalized size of antiderivative = 1.91

$$\int \frac{(ac+(bc+ad)x+bdx^2)^3}{(a+bx)^{11}}\,dx = -\frac{\frac{a^3\,d^3+4\,a^2\,b\,c\,d^2+10\,a\,b^2\,c^2\,d+20\,b^3\,c^3}{140\,b^4}+\frac{d^3\,x^3}{4\,b}+\frac{d\,x\,\left(a^2\,d^2+4\,a\,b\,c\,d+10\,b^2\,c^2\right)}{20\,b^3}+\frac{3\,d^2\,x^2\,\left(a\,d+4\,b\,c\right)}{20\,b^2}}{a^7+7\,a^6\,b\,x+21\,a^5\,b^2\,x^2+35\,a^4\,b^3\,x^3+35\,a^3\,b^4\,x^4+21\,a^2\,b^5\,x^5+7\,a\,b^6\,x^6+b^7\,x^7}$$

input
```
int((a*c + x*(a*d + b*c) + b*d*x^2)^3/(a + b*x)^11,x)
```

output
```
-((a^3*d^3 + 20*b^3*c^3 + 10*a*b^2*c^2*d + 4*a^2*b*c*d^2)/(140*b^4) + (d^3
*x^3)/(4*b) + (d*x*(a^2*d^2 + 10*b^2*c^2 + 4*a*b*c*d))/(20*b^3) + (3*d^2*x
^2*(a*d + 4*b*c))/(20*b^2))/(a^7 + b^7*x^7 + 7*a*b^6*x^6 + 21*a^5*b^2*x^2
+ 35*a^4*b^3*x^3 + 35*a^3*b^4*x^4 + 21*a^2*b^5*x^5 + 7*a^6*b*x)
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 181, normalized size of antiderivative = 1.97

$$\int \frac{(ac+(bc+ad)x+bdx^2)^3}{(a+bx)^{11}}\,dx = \frac{-35b^3d^3x^3-21a\,b^2d^3x^2-84b^3c\,d^2x^2-7a^2b\,d^3x-28a\,b^2c\,d^2x-70b^3c^2dx-a^3d^3-4a^2bc\,d^2-10a\,b^2c^2}{140b^4\left(b^7x^7+7a\,b^6x^6+21a^2b^5x^5+35a^3b^4x^4+35a^4b^3x^3+21a^5b^2x^2+7a^6bx+a^7\right)}$$

input
```
int((a*c+(a*d+b*c)*x+b*d*x^2)^3/(b*x+a)^11,x)
```

output
```
( - a**3*d**3 - 4*a**2*b*c*d**2 - 7*a**2*b*d**3*x - 10*a*b**2*c**2*d - 28*
a*b**2*c*d**2*x - 21*a*b**2*d**3*x**2 - 20*b**3*c**3 - 70*b**3*c**2*d*x -
84*b**3*c*d**2*x**2 - 35*b**3*d**3*x**3)/(140*b**4*(a**7 + 7*a**6*b*x + 21
*a**5*b**2*x**2 + 35*a**4*b**3*x**3 + 35*a**3*b**4*x**4 + 21*a**2*b**5*x**
5 + 7*a*b**6*x**6 + b**7*x**7))
```

3.40 $\int \frac{(ac+(bc+ad)x+bdx^2)^3}{(a+bx)^{12}} \, dx$

Optimal result

Integrand size = 29, antiderivative size = 92

$$\int \frac{(ac+(bc+ad)x+bdx^2)^3}{(a+bx)^{12}} \, dx = -\frac{(bc-ad)^3}{8b^4(a+bx)^8} - \frac{3d(bc-ad)^2}{7b^4(a+bx)^7} - \frac{d^2(bc-ad)}{2b^4(a+bx)^6} - \frac{d^3}{5b^4(a+bx)^5}$$

output
```
-1/8*(-a*d+b*c)^3/b^4/(b*x+a)^8-3/7*d*(-a*d+b*c)^2/b^4/(b*x+a)^7-1/2*d^2*(
-a*d+b*c)/b^4/(b*x+a)^6-1/5*d^3/b^4/(b*x+a)^5
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.05

$$\int \frac{(ac+(bc+ad)x+bdx^2)^3}{(a+bx)^{12}} \, dx = -\frac{a^3d^3+a^2bd^2(5c+8dx)+ab^2d(15c^2+40cdx+28d^2x^2)+b^3(35c^3+120c^2dx+140cd^2x^2+56d^3x^3)}{280b^4(a+bx)^8}$$

input
```
Integrate[(a*c + (b*c + a*d)*x + b*d*x^2)^3/(a + b*x)^12,x]
```

output
```
-1/280*(a^3*d^3 + a^2*b*d^2*(5*c + 8*d*x) + a*b^2*d*(15*c^2 + 40*c*d*x + 2
8*d^2*x^2) + b^3*(35*c^3 + 120*c^2*d*x + 140*c*d^2*x^2 + 56*d^3*x^3))/(b^4
*(a + b*x)^8)
```

Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.069$, Rules used = {1121, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(x(ad+bc)+ac+bdx^2\right)^3}{(a+bx)^{12}}\,dx$$

$$\Big\downarrow \text{1121}$$

$$\int \left(\frac{3d^2(bc-ad)}{b^3(a+bx)^7}+\frac{3d(bc-ad)^2}{b^3(a+bx)^8}+\frac{(bc-ad)^3}{b^3(a+bx)^9}+\frac{d^3}{b^3(a+bx)^6}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{d^2(bc-ad)}{2b^4(a+bx)^6}-\frac{3d(bc-ad)^2}{7b^4(a+bx)^7}-\frac{(bc-ad)^3}{8b^4(a+bx)^8}-\frac{d^3}{5b^4(a+bx)^5}$$

input
```
Int[(a*c + (b*c + a*d)*x + b*d*x^2)^3/(a + b*x)^12,x]
```

output
```
-1/8*(b*c - a*d)^3/(b^4*(a + b*x)^8) - (3*d*(b*c - a*d)^2)/(7*b^4*(a + b*x
)^7) - (d^2*(b*c - a*d))/(2*b^4*(a + b*x)^6) - d^3/(5*b^4*(a + b*x)^5)
```

Defintions of rubi rules used

rule 1121
```
Int[((d_) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Int[ExpandIntegrand[(d + e*x)^(m + p)*(a/d + (c/e)*x)^p, x], x]
/; FreeQ[{a, b, c, d, e, m, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && (Int
egerQ[p] || (GtQ[a, 0] && GtQ[d, 0] && LtQ[c, 0]))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 1.27 (sec) , antiderivative size = 110, normalized size of antiderivative = 1.20

method	result
risch	$\frac{-\frac{x^3 d^3}{5b}-\frac{d^2(ad+5bc)x^2}{10b^2}-\frac{d\left(a^2d^2+5abcd+15b^2c^2\right)x}{35b^3}-\frac{a^3d^3+5a^2bc\,d^2+15a\,b^2c^2d+35b^3c^3}{280b^4}}{(bx+a)^8}$
gosper	$-\frac{56x^3b^3d^3+28a\,b^2d^3x^2+140x^2b^3c\,d^2+8a^2b\,d^3x+40xa\,b^2c\,d^2+120x\,b^3c^2d+a^3d^3+5a^2bc\,d^2+15a\,b^2c^2d+35b^3c^3}{280b^4(bx+a)^8}$
default	$-\frac{-a^3d^3+3a^2bc\,d^2-3a\,b^2c^2d+b^3c^3}{8b^4(bx+a)^8}-\frac{d^3}{5b^4(bx+a)^5}-\frac{3d\left(a^2d^2-2abcd+b^2c^2\right)}{7b^4(bx+a)^7}+\frac{d^2(ad-bc)}{2b^4(bx+a)^6}$
parallelrisch	$\frac{-56d^3x^3b^7-28a\,b^6d^3x^2-140b^7c\,d^2x^2-8a^2b^5d^3x-40a\,b^6c\,d^2x-120b^7c^2dx-a^3b^4d^3-5a^2b^5c\,d^2-15a\,c^2d\,b^6-35c^3b^7}{280b^8(bx+a)^8}$
orering	$-\frac{\left(56x^3b^3d^3+28a\,b^2d^3x^2+140x^2b^3c\,d^2+8a^2b\,d^3x+40xa\,b^2c\,d^2+120x\,b^3c^2d+a^3d^3+5a^2bc\,d^2+15a\,b^2c^2d+35b^3c^3\right)(ac+(ad+}{280b^4(bx+a)^{11}(dx+c)^3}$
norman	$\frac{\frac{a^3\left(-a^3b^7d^3-5a^2b^8c\,d^2-15a\,b^9c^2d-35b^{10}c^3\right)}{280b^{11}}-\frac{b^2d^3x^6}{5}+\frac{\left(-7a\,d^3b^7-5c\,d^2b^8\right)x^5}{10b^6}+\frac{\left(-13a^2b^7d^3-23a\,d^2c\,b^8-6c^2d\,b^9\right)x^4}{14b^7}+\frac{\left(-33a^3b^7\right.}{}}{}$

input
```
int((a*c+(a*d+b*c)*x+b*d*x^2)^3/(b*x+a)^12,x,method=_RETURNVERBOSE)
```

output
```
(-1/5/b*x^3*d^3-1/10/b^2*d^2*(a*d+5*b*c)*x^2-1/35/b^3*d*(a^2*d^2+5*a*b*c*d
+15*b^2*c^2)*x-1/280/b^4*(a^3*d^3+5*a^2*b*c*d^2+15*a*b^2*c^2*d+35*b^3*c^3)
)/(b*x+a)^8
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 193 vs. $2(84) = 168$.

Time = 0.07 (sec) , antiderivative size = 193, normalized size of antiderivative = 2.10

$$\int \frac{(ac+(bc+ad)x+bdx^2)^3}{(a+bx)^{12}}\,dx = -\frac{56\,b^3d^3x^3+35\,b^3c^3+15\,ab^2c^2d+5\,a^2bcd^2+a^3d^3+28\,(5\,b^3cd^2+ab^2d^3)x^2+8\,(15\,b^3c^2d+5\,ab^2cd^2+}{280\,(b^{12}x^8+8\,ab^{11}x^7+28\,a^2b^{10}x^6+56\,a^3b^9x^5+70\,a^4b^8x^4+56\,a^5b^7x^3+28\,a^6b^6x^2+8\,a^7b^5x+a^8}$$

input
```
integrate((a*c+(a*d+b*c)*x+b*d*x^2)^3/(b*x+a)^12,x, algorithm="fricas")
```

output
```
-1/280*(56*b^3*d^3*x^3 + 35*b^3*c^3 + 15*a*b^2*c^2*d + 5*a^2*b*c*d^2 + a^3
*d^3 + 28*(5*b^3*c*d^2 + a*b^2*d^3)*x^2 + 8*(15*b^3*c^2*d + 5*a*b^2*c*d^2
+ a^2*b*d^3)*x)/(b^12*x^8 + 8*a*b^11*x^7 + 28*a^2*b^10*x^6 + 56*a^3*b^9*x^
5 + 70*a^4*b^8*x^4 + 56*a^5*b^7*x^3 + 28*a^6*b^6*x^2 + 8*a^7*b^5*x + a^8*b
^4)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 207 vs. $2(82) = 164$.

Time = 11.37 (sec) , antiderivative size = 207, normalized size of antiderivative = 2.25

$$\int \frac{(ac+(bc+ad)x+bdx^2)^3}{(a+bx)^{12}}\,dx = \frac{-a^3d^3-5a^2bcd^2-15ab^2c^2d-35b^3c^3-56b^3d^3x^3+x^2(-28ab^2d^3-140b^3cd^2)+x(-8a^2bd^3-40ab^2c}{280a^8b^4+2240a^7b^5x+7840a^6b^6x^2+15680a^5b^7x^3+19600a^4b^8x^4+15680a^3b^9x^5+7840a^2b^{10}x^6+2240a}$$

input
```
integrate((a*c+(a*d+b*c)*x+b*d*x**2)**3/(b*x+a)**12,x)
```

output
```
(-a**3*d**3 - 5*a**2*b*c*d**2 - 15*a*b**2*c**2*d - 35*b**3*c**3 - 56*b**3*
d**3*x**3 + x**2*(-28*a*b**2*d**3 - 140*b**3*c*d**2) + x*(-8*a**2*b*d**3 -
 40*a*b**2*c*d**2 - 120*b**3*c**2*d))/(280*a**8*b**4 + 2240*a**7*b**5*x +
7840*a**6*b**6*x**2 + 15680*a**5*b**7*x**3 + 19600*a**4*b**8*x**4 + 15680*
a**3*b**9*x**5 + 7840*a**2*b**10*x**6 + 2240*a*b**11*x**7 + 280*b**12*x**8
)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 193 vs. $2(84) = 168$.

Time = 0.04 (sec) , antiderivative size = 193, normalized size of antiderivative = 2.10

$$\int \frac{(ac + (bc + ad)x + bdx^2)^3}{(a + bx)^{12}}\,dx = -\frac{56\,b^3d^3x^3 + 35\,b^3c^3 + 15\,ab^2c^2d + 5\,a^2bcd^2 + a^3d^3 + 28\,(5\,b^3cd^2 + ab^2d^3)x^2 + 8\,(15\,b^3c^2d + 5\,ab^2cd^2 + \ldots}{280\,(b^{12}x^8 + 8\,ab^{11}x^7 + 28\,a^2b^{10}x^6 + 56\,a^3b^9x^5 + 70\,a^4b^8x^4 + 56\,a^5b^7x^3 + 28\,a^6b^6x^2 + 8\,a^7b^5x + a^8\ldots}$$

input
```
integrate((a*c+(a*d+b*c)*x+b*d*x^2)^3/(b*x+a)^12,x, algorithm="maxima")
```

output
```
-1/280*(56*b^3*d^3*x^3 + 35*b^3*c^3 + 15*a*b^2*c^2*d + 5*a^2*b*c*d^2 + a^3
*d^3 + 28*(5*b^3*c*d^2 + a*b^2*d^3)*x^2 + 8*(15*b^3*c^2*d + 5*a*b^2*c*d^2
+ a^2*b*d^3)*x)/(b^12*x^8 + 8*a*b^11*x^7 + 28*a^2*b^10*x^6 + 56*a^3*b^9*x^
5 + 70*a^4*b^8*x^4 + 56*a^5*b^7*x^3 + 28*a^6*b^6*x^2 + 8*a^7*b^5*x + a^8*b
^4)
```

Giac [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.24

$$\int \frac{(ac + (bc + ad)x + bdx^2)^3}{(a + bx)^{12}}\,dx = -\frac{56\,b^3d^3x^3 + 140\,b^3cd^2x^2 + 28\,ab^2d^3x^2 + 120\,b^3c^2dx + 40\,ab^2cd^2x + 8\,a^2bd^3x + 35\,b^3c^3 + 15\,ab^2c^2d + 5\ldots}{280\,(bx + a)^8b^4}$$

input
```
integrate((a*c+(a*d+b*c)*x+b*d*x^2)^3/(b*x+a)^12,x, algorithm="giac")
```

output
```
-1/280*(56*b^3*d^3*x^3 + 140*b^3*c*d^2*x^2 + 28*a*b^2*d^3*x^2 + 120*b^3*c^
2*d*x + 40*a*b^2*c*d^2*x + 8*a^2*b*d^3*x + 35*b^3*c^3 + 15*a*b^2*c^2*d + 5
*a^2*b*c*d^2 + a^3*d^3)/((b*x + a)^8*b^4)
```

Mupad [B] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 187, normalized size of antiderivative = 2.03

$$\int \frac{(ac+(bc+ad)x+bdx^2)^3}{(a+bx)^{12}}\,dx = -\frac{\frac{a^3\,d^3+5\,a^2\,b\,c\,d^2+15\,a\,b^2\,c^2\,d+35\,b^3\,c^3}{280\,b^4}+\frac{d^3\,x^3}{5\,b}+\frac{d\,x\,\left(a^2\,d^2+5\,a\,b\,c\,d+15\,b^2\,c^2\right)}{35\,b^3}+\frac{d^2\,x^2\,\left(a\,d+5\,b\,c\right)}{10\,b^2}}{a^8+8\,a^7\,b\,x+28\,a^6\,b^2\,x^2+56\,a^5\,b^3\,x^3+70\,a^4\,b^4\,x^4+56\,a^3\,b^5\,x^5+28\,a^2\,b^6\,x^6+8\,a\,b^7\,x^7+b^8\,x^8}$$

input
```
int((a*c + x*(a*d + b*c) + b*d*x^2)^3/(a + b*x)^12,x)
```

output
```
-((a^3*d^3 + 35*b^3*c^3 + 15*a*b^2*c^2*d + 5*a^2*b*c*d^2)/(280*b^4) + (d^3
*x^3)/(5*b) + (d*x*(a^2*d^2 + 15*b^2*c^2 + 5*a*b*c*d))/(35*b^3) + (d^2*x^2
*(a*d + 5*b*c))/(10*b^2))/(a^8 + b^8*x^8 + 8*a*b^7*x^7 + 28*a^6*b^2*x^2 +
56*a^5*b^3*x^3 + 70*a^4*b^4*x^4 + 56*a^3*b^5*x^5 + 28*a^2*b^6*x^6 + 8*a^7*
b*x)
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 192, normalized size of antiderivative = 2.09

$$\int \frac{(ac+(bc+ad)x+bdx^2)^3}{(a+bx)^{12}}\,dx = \frac{-56b^3d^3x^3-28a\,b^2d^3x^2-140b^3c\,d^2x^2-8a^2b\,d^3x-40a\,b^2c\,d^2x-120b^3c^2dx-a^3d^3-5a^2bc\,d^2-15a\,b^2}{280b^4\left(b^8x^8+8a\,b^7x^7+28a^2b^6x^6+56a^3b^5x^5+70a^4b^4x^4+56a^5b^3x^3+28a^6b^2x^2+8a^7bx+a^8\right)}$$

input
```
int((a*c+(a*d+b*c)*x+b*d*x^2)^3/(b*x+a)^12,x)
```

output
```
( - a**3*d**3 - 5*a**2*b*c*d**2 - 8*a**2*b*d**3*x - 15*a*b**2*c**2*d - 40*
a*b**2*c*d**2*x - 28*a*b**2*d**3*x**2 - 35*b**3*c**3 - 120*b**3*c**2*d*x -
 140*b**3*c*d**2*x**2 - 56*b**3*d**3*x**3)/(280*b**4*(a**8 + 8*a**7*b*x +
28*a**6*b**2*x**2 + 56*a**5*b**3*x**3 + 70*a**4*b**4*x**4 + 56*a**3*b**5*x
**5 + 28*a**2*b**6*x**6 + 8*a*b**7*x**7 + b**8*x**8))
```

3.41 $\int \frac{(a+bx)^6}{ac+(bc+ad)x+bdx^2}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 122

$$\int \frac{(a+bx)^6}{ac+(bc+ad)x+bdx^2}\,dx = \frac{b(bc-ad)^4x}{d^5} - \frac{(bc-ad)^3(a+bx)^2}{2d^4} + \frac{(bc-ad)^2(a+bx)^3}{3d^3} - \frac{(bc-ad)(a+bx)^4}{4d^2} + \frac{(a+bx)^5}{5d} - \frac{(bc-ad)^5\log(c+dx)}{d^6}$$

output
```
b*(-a*d+b*c)^4*x/d^5-1/2*(-a*d+b*c)^3*(b*x+a)^2/d^4+1/3*(-a*d+b*c)^2*(b*x+
a)^3/d^3-1/4*(-a*d+b*c)*(b*x+a)^4/d^2+1/5*(b*x+a)^5/d-(-a*d+b*c)^5*ln(d*x+
c)/d^6
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 167, normalized size of antiderivative = 1.37

$$\int \frac{(a+bx)^6}{ac+(bc+ad)x+bdx^2}\,dx$$
$$= \frac{bdx(300a^4d^4+300a^3bd^3(-2c+dx)+100a^2b^2d^2(6c^2-3cdx+2d^2x^2)+25ab^3d(-12c^3+6c^2dx-4cd^2x}{60d^6}$$

input
```
Integrate[(a + b*x)^6/(a*c + (b*c + a*d)*x + b*d*x^2),x]
```

output
```
(b*d*x*(300*a^4*d^4 + 300*a^3*b*d^3*(-2*c + d*x) + 100*a^2*b^2*d^2*(6*c^2
- 3*c*d*x + 2*d^2*x^2) + 25*a*b^3*d*(-12*c^3 + 6*c^2*d*x - 4*c*d^2*x^2 + 3
*d^3*x^3) + b^4*(60*c^4 - 30*c^3*d*x + 20*c^2*d^2*x^2 - 15*c*d^3*x^3 + 12*
d^4*x^4)) - 60*(b*c - a*d)^5*Log[c + d*x])/(60*d^6)
```

Rubi [A] (verified)

Time = 0.48 (sec) , antiderivative size = 122, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.069, Rules used = {1121, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx)^6}{x(ad+bc)+ac+bdx^2}\,dx$$

$$\Big\downarrow \text{1121}$$

$$\int \left(\frac{(ad-bc)^5}{d^5(c+dx)}+\frac{b(bc-ad)^4}{d^5}-\frac{b(a+bx)(bc-ad)^3}{d^4}+\frac{b(a+bx)^2(bc-ad)^2}{d^3}-\frac{b(a+bx)^3(bc-ad)}{d^2}+\frac{b(a+bx)}{d}\right.$$

$$\Big\downarrow \text{2009}$$

$$-\frac{(bc-ad)^5\log(c+dx)}{d^6}+\frac{bx(bc-ad)^4}{d^5}-\frac{(a+bx)^2(bc-ad)^3}{2d^4}+\frac{(a+bx)^3(bc-ad)^2}{3d^3}-\frac{(a+bx)^4(bc-ad)}{4d^2}+\frac{(a+bx)^5}{5d}$$

input
```
Int[(a + b*x)^6/(a*c + (b*c + a*d)*x + b*d*x^2),x]
```

output
```
(b*(b*c - a*d)^4*x)/d^5 - ((b*c - a*d)^3*(a + b*x)^2)/(2*d^4) + ((b*c - a*
d)^2*(a + b*x)^3)/(3*d^3) - ((b*c - a*d)*(a + b*x)^4)/(4*d^2) + (a + b*x)^
5/(5*d) - ((b*c - a*d)^5*Log[c + d*x])/d^6
```

Defintions of rubi rules used

rule 1121
```
Int[((d_) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Int[ExpandIntegrand[(d + e*x)^(m + p)*(a/d + (c/e)*x)^p, x], x]
/; FreeQ[{a, b, c, d, e, m, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && (Int
egerQ[p] || (GtQ[a, 0] && GtQ[d, 0] && LtQ[c, 0]))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 243 vs. $2(114) = 228$.

Time = 1.53 (sec) , antiderivative size = 244, normalized size of antiderivative = 2.00

method	result
norman	$\frac{b\left(5a^4d^4-10a^3bc\,d^3+10a^2b^2c^2d^2-5a\,b^3c^3d+c^4b^4\right)x}{d^5}+\frac{b^5x^5}{5d}+\frac{b^2\left(10a^3d^3-10a^2bc\,d^2+5a\,b^2c^2d-b^3c^3\right)x^2}{2d^4}+\frac{b^3\left(10a^2d^2-\right.}{}$
default	$\frac{b\left(\frac{1}{5}b^4x^5d^4+\frac{5}{4}a\,b^3d^4x^4-\frac{1}{4}b^4c\,x^4d^3+\frac{10}{3}x^3a^2b^2d^4-\frac{5}{3}a\,b^3c\,d^3x^3+\frac{1}{3}x^3b^4c^2d^2+5a^3b\,d^4x^2-5x^2a^2b^2c\,d^3+\frac{5}{2}a\,b^3c^2d^2x^2-\frac{1}{2}x^2b^4c\right.}{d^5}$
risch	$\frac{b^5x^5}{5d}+\frac{5b^4a\,x^4}{4d}-\frac{b^5c\,x^4}{4d^2}+\frac{10b^3x^3a^2}{3d}-\frac{5b^4ac\,x^3}{3d^2}+\frac{b^5x^3c^2}{3d^3}+\frac{5b^2a^3x^2}{d}-\frac{5b^3x^2a^2c}{d^2}+\frac{5b^4a\,c^2x^2}{2d^3}-\frac{b^5x^2c^3}{2d^4}+$
parallelrisch	$\frac{12b^5d^5x^5+75a\,b^4d^5x^4-15b^5c\,d^4x^4+200a^2b^3d^5x^3-100a\,b^4c\,d^4x^3+20b^5c^2d^3x^3+300a^3b^2d^5x^2-300a^2b^3c\,d^4x^2+150a\,b^4c^2d^3}{}$

input
```
int((b*x+a)^6/(a*c+(a*d+b*c)*x+b*d*x^2),x,method=_RETURNVERBOSE)
```

output
```
b*(5*a^4*d^4-10*a^3*b*c*d^3+10*a^2*b^2*c^2*d^2-5*a*b^3*c^3*d+b^4*c^4)/d^5*
x+1/5*b^5/d*x^5+1/2*b^2/d^4*(10*a^3*d^3-10*a^2*b*c*d^2+5*a*b^2*c^2*d-b^3*c
^3)*x^2+1/3*b^3/d^3*(10*a^2*d^2-5*a*b*c*d+b^2*c^2)*x^3+1/4*b^4/d^2*(5*a*d-
b*c)*x^4+(a^5*d^5-5*a^4*b*c*d^4+10*a^3*b^2*c^2*d^3-10*a^2*b^3*c^3*d^2+5*a*
b^4*c^4*d-b^5*c^5)/d^6*ln(d*x+c)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 259 vs. $2(114) = 228$.

Time = 0.09 (sec) , antiderivative size = 259, normalized size of antiderivative = 2.12

$$\int \frac{(a+bx)^6}{ac+(bc+ad)x+bdx^2}\,dx$$
$$= \frac{12\,b^5d^5x^5 - 15\,(b^5cd^4 - 5\,ab^4d^5)x^4 + 20\,(b^5c^2d^3 - 5\,ab^4cd^4 + 10\,a^2b^3d^5)x^3 - 30\,(b^5c^3d^2 - 5\,ab^4c^2d^3 + 10\,\ldots}{}$$

input
```
integrate((b*x+a)^6/(a*c+(a*d+b*c)*x+b*d*x^2),x, algorithm="fricas")
```

output
```
1/60*(12*b^5*d^5*x^5 - 15*(b^5*c*d^4 - 5*a*b^4*d^5)*x^4 + 20*(b^5*c^2*d^3
- 5*a*b^4*c*d^4 + 10*a^2*b^3*d^5)*x^3 - 30*(b^5*c^3*d^2 - 5*a*b^4*c^2*d^3
+ 10*a^2*b^3*c*d^4 - 10*a^3*b^2*d^5)*x^2 + 60*(b^5*c^4*d - 5*a*b^4*c^3*d^2
 + 10*a^2*b^3*c^2*d^3 - 10*a^3*b^2*c*d^4 + 5*a^4*b*d^5)*x - 60*(b^5*c^5 -
5*a*b^4*c^4*d + 10*a^2*b^3*c^3*d^2 - 10*a^3*b^2*c^2*d^3 + 5*a^4*b*c*d^4 -
a^5*d^5)*log(d*x + c))/d^6
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 209 vs. $2(104) = 208$.

Time = 0.27 (sec) , antiderivative size = 209, normalized size of antiderivative = 1.71

$$\int \frac{(a+bx)^6}{ac+(bc+ad)x+bdx^2}\,dx = \frac{b^5x^5}{5d} + x^4\cdot\left(\frac{5ab^4}{4d} - \frac{b^5c}{4d^2}\right) + x^3\cdot\left(\frac{10a^2b^3}{3d} - \frac{5ab^4c}{3d^2} + \frac{b^5c^2}{3d^3}\right) + x^2\cdot\left(\frac{5a^3b^2}{d} - \frac{5a^2b^3c}{d^2} + \frac{5ab^4c^2}{2d^3} - \frac{b^5c^3}{2d^4}\right) + x\left(\frac{5a^4b}{d} - \frac{10a^3b^2c}{d^2} + \frac{10a^2b^3c^2}{d^3} - \frac{5ab^4c^3}{d^4} + \frac{b^5c^4}{d^5}\right) + \frac{(ad-bc)^5\log(c+dx)}{d^6}$$

input
```
integrate((b*x+a)**6/(a*c+(a*d+b*c)*x+b*d*x**2),x)
```

output
```
b**5*x**5/(5*d) + x**4*(5*a*b**4/(4*d) - b**5*c/(4*d**2)) + x**3*(10*a**2*
b**3/(3*d) - 5*a*b**4*c/(3*d**2) + b**5*c**2/(3*d**3)) + x**2*(5*a**3*b**2
/d - 5*a**2*b**3*c/d**2 + 5*a*b**4*c**2/(2*d**3) - b**5*c**3/(2*d**4)) + x
*(5*a**4*b/d - 10*a**3*b**2*c/d**2 + 10*a**2*b**3*c**2/d**3 - 5*a*b**4*c**
3/d**4 + b**5*c**4/d**5) + (a*d - b*c)**5*log(c + d*x)/d**6
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 258 vs. $2(114) = 228$.

Time = 0.04 (sec) , antiderivative size = 258, normalized size of antiderivative = 2.11

$$\int \frac{(a+bx)^6}{ac+(bc+ad)x+bdx^2}\,dx$$
$$= \frac{12\,b^5d^4x^5 - 15\,(b^5cd^3 - 5\,ab^4d^4)x^4 + 20\,(b^5c^2d^2 - 5\,ab^4cd^3 + 10\,a^2b^3d^4)x^3 - 30\,(b^5c^3d - 5\,ab^4c^2d^2 + 10\,a}{60\,d^5}$$
$$- \frac{(b^5c^5 - 5\,ab^4c^4d + 10\,a^2b^3c^3d^2 - 10\,a^3b^2c^2d^3 + 5\,a^4bcd^4 - a^5d^5)\log(dx+c)}{d^6}$$

input
```
integrate((b*x+a)^6/(a*c+(a*d+b*c)*x+b*d*x^2),x, algorithm="maxima")
```

output
```
1/60*(12*b^5*d^4*x^5 - 15*(b^5*c*d^3 - 5*a*b^4*d^4)*x^4 + 20*(b^5*c^2*d^2
- 5*a*b^4*c*d^3 + 10*a^2*b^3*d^4)*x^3 - 30*(b^5*c^3*d - 5*a*b^4*c^2*d^2 +
10*a^2*b^3*c*d^3 - 10*a^3*b^2*d^4)*x^2 + 60*(b^5*c^4 - 5*a*b^4*c^3*d + 10*
a^2*b^3*c^2*d^2 - 10*a^3*b^2*c*d^3 + 5*a^4*b*d^4)*x)/d^5 - (b^5*c^5 - 5*a*
b^4*c^4*d + 10*a^2*b^3*c^3*d^2 - 10*a^3*b^2*c^2*d^3 + 5*a^4*b*c*d^4 - a^5*
d^5)*log(d*x + c)/d^6
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 273 vs. $2(114) = 228$.

Time = 0.13 (sec) , antiderivative size = 273, normalized size of antiderivative = 2.24

$$\int \frac{(a+bx)^6}{ac+(bc+ad)x+bdx^2}\,dx$$
$$= \frac{12\,b^5d^4x^5 - 15\,b^5cd^3x^4 + 75\,ab^4d^4x^4 + 20\,b^5c^2d^2x^3 - 100\,ab^4cd^3x^3 + 200\,a^2b^3d^4x^3 - 30\,b^5c^3dx^2 + 150\,ab^4}{}$$
$$- \frac{(b^5c^5 - 5\,ab^4c^4d + 10\,a^2b^3c^3d^2 - 10\,a^3b^2c^2d^3 + 5\,a^4bcd^4 - a^5d^5)\log(|dx+c|)}{d^6}$$

input
```
integrate((b*x+a)^6/(a*c+(a*d+b*c)*x+b*d*x^2),x, algorithm="giac")
```

output
```
1/60*(12*b^5*d^4*x^5 - 15*b^5*c*d^3*x^4 + 75*a*b^4*d^4*x^4 + 20*b^5*c^2*d^
2*x^3 - 100*a*b^4*c*d^3*x^3 + 200*a^2*b^3*d^4*x^3 - 30*b^5*c^3*d*x^2 + 150
*a*b^4*c^2*d^2*x^2 - 300*a^2*b^3*c*d^3*x^2 + 300*a^3*b^2*d^4*x^2 + 60*b^5*
c^4*x - 300*a*b^4*c^3*d*x + 600*a^2*b^3*c^2*d^2*x - 600*a^3*b^2*c*d^3*x +
300*a^4*b*d^4*x)/d^5 - (b^5*c^5 - 5*a*b^4*c^4*d + 10*a^2*b^3*c^3*d^2 - 10*
a^3*b^2*c^2*d^3 + 5*a^4*b*c*d^4 - a^5*d^5)*log(abs(d*x + c))/d^6
```

Mupad [B] (verification not implemented)

Time = 5.39 (sec) , antiderivative size = 280, normalized size of antiderivative = 2.30

$$\int \frac{(a+bx)^6}{ac+(bc+ad)x+bdx^2}\,dx$$
$$= x\left(\frac{5\,a^4\,b}{d}-\frac{c\left(\frac{10\,a^3\,b^2}{d}+\frac{c\left(\frac{c\left(\frac{5\,a\,b^4}{d}-\frac{b^5\,c}{d^2}\right)}{d}-\frac{10\,a^2\,b^3}{d}\right)}{d}\right)}{d}\right)+x^4\left(\frac{5\,a\,b^4}{4\,d}-\frac{b^5\,c}{4\,d^2}\right)+x^2\left(\frac{5\,a^3\,b^2}{d}+\frac{c\left(\frac{c\left(\frac{5\,a\,b^4}{d}-\frac{b^5\,c}{d^2}\right)}{d}-\frac{10\,a^2\,b^3}{d}\right)}{2\,d}\right)-x^3\left(\frac{c\left(\frac{5\,a\,b^4}{d}-\frac{b^5\,c}{d^2}\right)}{3\,d}-\frac{10\,a^2\,b^3}{3\,d}\right)+\frac{b^5\,x^5}{5\,d}+\frac{\ln\left(c+d\,x\right)\,\left(a^5\,d^5-5\,a^4\,b\,c\,d^4+10\,a^3\,b^2\,c^2\,d^3-10\,a^2\,b^3\,c^3\,d^2+5\,a\,b^4\,c^4\,d-b^5\,c^5\right)}{d^6}$$

input
```
int((a + b*x)^6/(a*c + x*(a*d + b*c) + b*d*x^2),x)
```

output
```
x*((5*a^4*b)/d - (c*((10*a^3*b^2)/d + (c*((c*((5*a*b^4)/d - (b^5*c)/d^2))/
d - (10*a^2*b^3)/d))/d))/d) + x^4*((5*a*b^4)/(4*d) - (b^5*c)/(4*d^2)) + x^
2*((5*a^3*b^2)/d + (c*((c*((5*a*b^4)/d - (b^5*c)/d^2))/d - (10*a^2*b^3)/d)
)/(2*d)) - x^3*((c*((5*a*b^4)/d - (b^5*c)/d^2))/(3*d) - (10*a^2*b^3)/(3*d)
) + (b^5*x^5)/(5*d) + (log(c + d*x)*(a^5*d^5 - b^5*c^5 - 10*a^2*b^3*c^3*d^
2 + 10*a^3*b^2*c^2*d^3 + 5*a*b^4*c^4*d - 5*a^4*b*c*d^4))/d^6
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 301, normalized size of antiderivative = 2.47

$$\int \frac{(a+bx)^6}{ac+(bc+ad)x+bdx^2}\,dx$$
$$= \frac{60\log(dx+c)\,a^5d^5-300\log(dx+c)\,a^4bc\,d^4+600\log(dx+c)\,a^3b^2c^2d^3-600\log(dx+c)\,a^2b^3c^3d^2+30}{}$$

input
```
int((b*x+a)^6/(a*c+(a*d+b*c)*x+b*d*x^2),x)
```

output
```
(60*log(c + d*x)*a**5*d**5 - 300*log(c + d*x)*a**4*b*c*d**4 + 600*log(c +
d*x)*a**3*b**2*c**2*d**3 - 600*log(c + d*x)*a**2*b**3*c**3*d**2 + 300*log(
c + d*x)*a*b**4*c**4*d - 60*log(c + d*x)*b**5*c**5 + 300*a**4*b*d**5*x - 6
00*a**3*b**2*c*d**4*x + 300*a**3*b**2*d**5*x**2 + 600*a**2*b**3*c**2*d**3*
x - 300*a**2*b**3*c*d**4*x**2 + 200*a**2*b**3*d**5*x**3 - 300*a*b**4*c**3*
d**2*x + 150*a*b**4*c**2*d**3*x**2 - 100*a*b**4*c*d**4*x**3 + 75*a*b**4*d*
*5*x**4 + 60*b**5*c**4*d*x - 30*b**5*c**3*d**2*x**2 + 20*b**5*c**2*d**3*x*
*3 - 15*b**5*c*d**4*x**4 + 12*b**5*d**5*x**5)/(60*d**6)
```

3.42 $\int \frac{(a+bx)^5}{ac+(bc+ad)x+bdx^2} \, dx$

Optimal result

Integrand size = 29, antiderivative size = 98

$$\int \frac{(a+bx)^5}{ac+(bc+ad)x+bdx^2} \, dx = -\frac{b(bc-ad)^3x}{d^4} + \frac{(bc-ad)^2(a+bx)^2}{2d^3} - \frac{(bc-ad)(a+bx)^3}{3d^2} + \frac{(a+bx)^4}{4d} + \frac{(bc-ad)^4\log(c+dx)}{d^5}$$

output
```
-b*(-a*d+b*c)^3*x/d^4+1/2*(-a*d+b*c)^2*(b*x+a)^2/d^3-1/3*(-a*d+b*c)*(b*x+a
)^3/d^2+1/4*(b*x+a)^4/d+(-a*d+b*c)^4*ln(d*x+c)/d^5
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 115, normalized size of antiderivative = 1.17

$$\int \frac{(a+bx)^5}{ac+(bc+ad)x+bdx^2} \, dx = \frac{bdx(48a^3d^3 + 36a^2bd^2(-2c+dx) + 8ab^2d(6c^2-3cdx+2d^2x^2) + b^3(-12c^3+6c^2dx-4cd^2x^2+3d^3x^3))}{12d^5}$$

input
```
Integrate[(a + b*x)^5/(a*c + (b*c + a*d)*x + b*d*x^2),x]
```

output
```
(b*d*x*(48*a^3*d^3 + 36*a^2*b*d^2*(-2*c + d*x) + 8*a*b^2*d*(6*c^2 - 3*c*d*
x + 2*d^2*x^2) + b^3*(-12*c^3 + 6*c^2*d*x - 4*c*d^2*x^2 + 3*d^3*x^3)) + 12
*(b*c - a*d)^4*Log[c + d*x])/(12*d^5)
```

Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 98, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.069, Rules used = {1121, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx)^5}{x(ad+bc)+ac+bdx^2}\,dx$$

$$\Big\downarrow \; 1121$$

$$\int \left(\frac{(ad-bc)^4}{d^4(c+dx)}-\frac{b(bc-ad)^3}{d^4}+\frac{b(a+bx)(bc-ad)^2}{d^3}-\frac{b(a+bx)^2(bc-ad)}{d^2}+\frac{b(a+bx)^3}{d}\right)dx$$

$$\Big\downarrow \; 2009$$

$$\frac{(bc-ad)^4\log(c+dx)}{d^5}-\frac{bx(bc-ad)^3}{d^4}+\frac{(a+bx)^2(bc-ad)^2}{2d^3}-\frac{(a+bx)^3(bc-ad)}{3d^2}+\frac{(a+bx)^4}{4d}$$

input
```
Int[(a + b*x)^5/(a*c + (b*c + a*d)*x + b*d*x^2),x]
```

output
```
-((b*(b*c - a*d)^3*x)/d^4) + ((b*c - a*d)^2*(a + b*x)^2)/(2*d^3) - ((b*c -
 a*d)*(a + b*x)^3)/(3*d^2) + (a + b*x)^4/(4*d) + ((b*c - a*d)^4*Log[c + d*
x])/d^5
```

Defintions of rubi rules used

rule 1121
```
Int[((d_) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Int[ExpandIntegrand[(d + e*x)^(m + p)*(a/d + (c/e)*x)^p, x], x]
/; FreeQ[{a, b, c, d, e, m, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && (Int
egerQ[p] || (GtQ[a, 0] && GtQ[d, 0] && LtQ[c, 0]))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 1.63 (sec) , antiderivative size = 168, normalized size of antiderivative = 1.71

method	result
norman	$\frac{b\left(4a^3d^3-6a^2bc\,d^2+4a\,b^2c^2d-b^3c^3\right)x}{d^4}+\frac{b^4x^4}{4d}+\frac{b^2\left(6a^2d^2-4abcd+b^2c^2\right)x^2}{2d^3}+\frac{b^3(4ad-bc)x^3}{3d^2}+\frac{(a^4d^4-4a^3bc\,d^3+6a^2b^2}{}$
default	$\frac{b\left(\frac{b^3d^3x^4}{4}+\frac{\left((2ad-bc)b^2d^2+2a\,b^2d^3\right)x^3}{3}+\frac{\left(2(2ad-bc)ab\,d^2+bd\left(2a^2d^2-2abcd+b^2c^2\right)\right)x^2}{2}+(2ad-bc)\left(2a^2d^2-2abcd+b^2c^2\right)x\right)}{d^4}+$
risch	$\frac{b^4x^4}{4d}+\frac{4b^3a\,x^3}{3d}-\frac{b^4c\,x^3}{3d^2}+\frac{3b^2a^2x^2}{d}-\frac{2b^3ac\,x^2}{d^2}+\frac{b^4c^2x^2}{2d^3}+\frac{4b\,a^3x}{d}-\frac{6b^2a^2cx}{d^2}+\frac{4b^3a\,c^2x}{d^3}-\frac{b^4c^3x}{d^4}+\frac{\ln(dx+}{d}$
parallelrisch	$\frac{3b^4d^4x^4+16a\,b^3d^4x^3-4b^4c\,d^3x^3+36a^2b^2d^4x^2-24a\,b^3c\,d^3x^2+6b^4c^2d^2x^2+12\ln(dx+c)a^4d^4-48\ln(dx+c)a^3bc\,d^3+72\ln(dx+}{12d^5}$

input
```
int((b*x+a)^5/(a*c+(a*d+b*c)*x+b*d*x^2),x,method=_RETURNVERBOSE)
```

output
```
b*(4*a^3*d^3-6*a^2*b*c*d^2+4*a*b^2*c^2*d-b^3*c^3)/d^4*x+1/4*b^4*x^4/d+1/2*
b^2/d^3*(6*a^2*d^2-4*a*b*c*d+b^2*c^2)*x^2+1/3*b^3/d^2*(4*a*d-b*c)*x^3+(a^4
*d^4-4*a^3*b*c*d^3+6*a^2*b^2*c^2*d^2-4*a*b^3*c^3*d+b^4*c^4)/d^5*ln(d*x+c)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 179, normalized size of antiderivative = 1.83

$$\int \frac{(a+bx)^5}{ac+(bc+ad)x+bdx^2}\,dx$$
$$= \frac{3\,b^4d^4x^4 - 4\,(b^4cd^3 - 4\,ab^3d^4)x^3 + 6\,(b^4c^2d^2 - 4\,ab^3cd^3 + 6\,a^2b^2d^4)x^2 - 12\,(b^4c^3d - 4\,ab^3c^2d^2 + 6\,a^2b^2cd^3}{12\,d^5}$$

input
```
integrate((b*x+a)^5/(a*c+(a*d+b*c)*x+b*d*x^2),x, algorithm="fricas")
```

output
```
1/12*(3*b^4*d^4*x^4 - 4*(b^4*c*d^3 - 4*a*b^3*d^4)*x^3 + 6*(b^4*c^2*d^2 - 4
*a*b^3*c*d^3 + 6*a^2*b^2*d^4)*x^2 - 12*(b^4*c^3*d - 4*a*b^3*c^2*d^2 + 6*a^
2*b^2*c*d^3 - 4*a^3*b*d^4)*x + 12*(b^4*c^4 - 4*a*b^3*c^3*d + 6*a^2*b^2*c^2
*d^2 - 4*a^3*b*c*d^3 + a^4*d^4)*log(d*x + c))/d^5
```

Sympy [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 136, normalized size of antiderivative = 1.39

$$\int \frac{(a+bx)^5}{ac+(bc+ad)x+bdx^2}\,dx = \frac{b^4x^4}{4d} + x^3\cdot\left(\frac{4ab^3}{3d} - \frac{b^4c}{3d^2}\right) + x^2\cdot\left(\frac{3a^2b^2}{d} - \frac{2ab^3c}{d^2} + \frac{b^4c^2}{2d^3}\right) + x\left(\frac{4a^3b}{d} - \frac{6a^2b^2c}{d^2} + \frac{4ab^3c^2}{d^3} - \frac{b^4c^3}{d^4}\right) + \frac{(ad-bc)^4\log(c+dx)}{d^5}$$

input
```
integrate((b*x+a)**5/(a*c+(a*d+b*c)*x+b*d*x**2),x)
```

output
```
b**4*x**4/(4*d) + x**3*(4*a*b**3/(3*d) - b**4*c/(3*d**2)) + x**2*(3*a**2*b
**2/d - 2*a*b**3*c/d**2 + b**4*c**2/(2*d**3)) + x*(4*a**3*b/d - 6*a**2*b**
2*c/d**2 + 4*a*b**3*c**2/d**3 - b**4*c**3/d**4) + (a*d - b*c)**4*log(c + d
*x)/d**5
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 177, normalized size of antiderivative = 1.81

$$\int \frac{(a+bx)^5}{ac+(bc+ad)x+bdx^2}\,dx$$
$$= \frac{3\,b^4d^3x^4 - 4\,(b^4cd^2 - 4\,ab^3d^3)x^3 + 6\,(b^4c^2d - 4\,ab^3cd^2 + 6\,a^2b^2d^3)x^2 - 12\,(b^4c^3 - 4\,ab^3c^2d + 6\,a^2b^2cd^2 -}{12\,d^4}$$
$$+ \frac{(b^4c^4 - 4\,ab^3c^3d + 6\,a^2b^2c^2d^2 - 4\,a^3bcd^3 + a^4d^4)\log(dx+c)}{d^5}$$

input
```
integrate((b*x+a)^5/(a*c+(a*d+b*c)*x+b*d*x^2),x, algorithm="maxima")
```

output
```
1/12*(3*b^4*d^3*x^4 - 4*(b^4*c*d^2 - 4*a*b^3*d^3)*x^3 + 6*(b^4*c^2*d - 4*a
*b^3*c*d^2 + 6*a^2*b^2*d^3)*x^2 - 12*(b^4*c^3 - 4*a*b^3*c^2*d + 6*a^2*b^2*
c*d^2 - 4*a^3*b*d^3)*x)/d^4 + (b^4*c^4 - 4*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2
 - 4*a^3*b*c*d^3 + a^4*d^4)*log(d*x + c)/d^5
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 184, normalized size of antiderivative = 1.88

$$\int \frac{(a+bx)^5}{ac+(bc+ad)x+bdx^2}\,dx$$
$$= \frac{3\,b^4d^3x^4 - 4\,b^4cd^2x^3 + 16\,ab^3d^3x^3 + 6\,b^4c^2dx^2 - 24\,ab^3cd^2x^2 + 36\,a^2b^2d^3x^2 - 12\,b^4c^3x + 48\,ab^3c^2dx - 72}{12\,d^4}$$
$$+ \frac{(b^4c^4 - 4\,ab^3c^3d + 6\,a^2b^2c^2d^2 - 4\,a^3bcd^3 + a^4d^4)\log(|dx+c|)}{d^5}$$

input
```
integrate((b*x+a)^5/(a*c+(a*d+b*c)*x+b*d*x^2),x, algorithm="giac")
```

output
```
1/12*(3*b^4*d^3*x^4 - 4*b^4*c*d^2*x^3 + 16*a*b^3*d^3*x^3 + 6*b^4*c^2*d*x^2
 - 24*a*b^3*c*d^2*x^2 + 36*a^2*b^2*d^3*x^2 - 12*b^4*c^3*x + 48*a*b^3*c^2*d
*x - 72*a^2*b^2*c*d^2*x + 48*a^3*b*d^3*x)/d^4 + (b^4*c^4 - 4*a*b^3*c^3*d +
 6*a^2*b^2*c^2*d^2 - 4*a^3*b*c*d^3 + a^4*d^4)*log(abs(d*x + c))/d^5
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 189, normalized size of antiderivative = 1.93

$$\int \frac{(a+bx)^5}{ac+(bc+ad)x+bdx^2}\,dx$$
$$= x^3\left(\frac{4\,a\,b^3}{3\,d}-\frac{b^4\,c}{3\,d^2}\right)+x\left(\frac{4\,a^3\,b}{d}+\frac{c\left(\frac{c\left(\frac{4\,a\,b^3}{d}-\frac{b^4\,c}{d^2}\right)}{d}-\frac{6\,a^2\,b^2}{d}\right)}{d}\right)$$
$$-x^2\left(\frac{c\left(\frac{4\,a\,b^3}{d}-\frac{b^4\,c}{d^2}\right)}{2\,d}-\frac{3\,a^2\,b^2}{d}\right)$$
$$+\frac{\ln\left(c+d\,x\right)\,\left(a^4\,d^4-4\,a^3\,b\,c\,d^3+6\,a^2\,b^2\,c^2\,d^2-4\,a\,b^3\,c^3\,d+b^4\,c^4\right)}{d^5}+\frac{b^4\,x^4}{4\,d}$$

input
```
int((a + b*x)^5/(a*c + x*(a*d + b*c) + b*d*x^2),x)
```

output
```
x^3*((4*a*b^3)/(3*d) - (b^4*c)/(3*d^2)) + x*((4*a^3*b)/d + (c*((c*((4*a*b^
3)/d - (b^4*c)/d^2))/d - (6*a^2*b^2)/d))/d) - x^2*((c*((4*a*b^3)/d - (b^4*
c)/d^2))/(2*d) - (3*a^2*b^2)/d) + (log(c + d*x)*(a^4*d^4 + b^4*c^4 + 6*a^2
*b^2*c^2*d^2 - 4*a*b^3*c^3*d - 4*a^3*b*c*d^3))/d^5 + (b^4*x^4)/(4*d)
```

Reduce [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 208, normalized size of antiderivative = 2.12

$$\int \frac{(a+bx)^5}{ac+(bc+ad)x+bdx^2}\,dx$$
$$= \frac{12\log(dx+c)\,a^4d^4-48\log(dx+c)\,a^3bc\,d^3+72\log(dx+c)\,a^2b^2c^2d^2-48\log(dx+c)\,a\,b^3c^3d+12\log(c}{}$$

input
```
int((b*x+a)^5/(a*c+(a*d+b*c)*x+b*d*x^2),x)
```

output

```
(12*log(c + d*x)*a**4*d**4 - 48*log(c + d*x)*a**3*b*c*d**3 + 72*log(c + d*
x)*a**2*b**2*c**2*d**2 - 48*log(c + d*x)*a*b**3*c**3*d + 12*log(c + d*x)*b
**4*c**4 + 48*a**3*b*d**4*x - 72*a**2*b**2*c*d**3*x + 36*a**2*b**2*d**4*x*
*2 + 48*a*b**3*c**2*d**2*x - 24*a*b**3*c*d**3*x**2 + 16*a*b**3*d**4*x**3 -
 12*b**4*c**3*d*x + 6*b**4*c**2*d**2*x**2 - 4*b**4*c*d**3*x**3 + 3*b**4*d*
*4*x**4)/(12*d**5)
```

3.43 $\int \frac{(a+bx)^4}{ac+(bc+ad)x+bdx^2} dx$

Optimal result

Integrand size = 29, antiderivative size = 74

$$\int \frac{(a+bx)^4}{ac+(bc+ad)x+bdx^2} dx = \frac{b(bc-ad)^2x}{d^3} - \frac{(bc-ad)(a+bx)^2}{2d^2} + \frac{(a+bx)^3}{3d} - \frac{(bc-ad)^3\log(c+dx)}{d^4}$$

output
```
b*(-a*d+b*c)^2*x/d^3-1/2*(-a*d+b*c)*(b*x+a)^2/d^2+1/3*(b*x+a)^3/d-(-a*d+b*
c)^3*ln(d*x+c)/d^4
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.00

$$\int \frac{(a+bx)^4}{ac+(bc+ad)x+bdx^2} dx = \frac{bdx(18a^2d^2+9abd(-2c+dx)+b^2(6c^2-3cdx+2d^2x^2))-6(bc-ad)^3\log(c+dx)}{6d^4}$$

input
```
Integrate[(a + b*x)^4/(a*c + (b*c + a*d)*x + b*d*x^2),x]
```

output
```
(b*d*x*(18*a^2*d^2 + 9*a*b*d*(-2*c + d*x) + b^2*(6*c^2 - 3*c*d*x + 2*d^2*x
^2)) - 6*(b*c - a*d)^3*Log[c + d*x])/(6*d^4)
```

Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.069$, Rules used = {1121, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx)^4}{x(ad+bc)+ac+bdx^2}\,dx$$

$$\Big\downarrow \text{1121}$$

$$\int \left(\frac{(ad-bc)^3}{d^3(c+dx)}+\frac{b(bc-ad)^2}{d^3}-\frac{b(a+bx)(bc-ad)}{d^2}+\frac{b(a+bx)^2}{d}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{(bc-ad)^3\log(c+dx)}{d^4}+\frac{bx(bc-ad)^2}{d^3}-\frac{(a+bx)^2(bc-ad)}{2d^2}+\frac{(a+bx)^3}{3d}$$

input
```
Int[(a + b*x)^4/(a*c + (b*c + a*d)*x + b*d*x^2),x]
```

output
```
(b*(b*c - a*d)^2*x)/d^3 - ((b*c - a*d)*(a + b*x)^2)/(2*d^2) + (a + b*x)^3/
(3*d) - ((b*c - a*d)^3*Log[c + d*x])/d^4
```

Defintions of rubi rules used

rule 1121
```
Int[((d_) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Int[ExpandIntegrand[(d + e*x)^(m + p)*(a/d + (c/e)*x)^p, x], x]
/; FreeQ[{a, b, c, d, e, m, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && (Int
egerQ[p] || (GtQ[a, 0] && GtQ[d, 0] && LtQ[c, 0]))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 1.46 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.45

method	result
norman	$\frac{b(3a^2d^2-3abcd+b^2c^2)x}{d^3}+\frac{b^3x^3}{3d}+\frac{b^2(3ad-bc)x^2}{2d^2}+\frac{(a^3d^3-3a^2bc\,d^2+3a\,b^2c^2d-b^3c^3)\ln(dx+c)}{d^4}$
default	$\frac{b(\frac{1}{3}x^3b^2d^2+\frac{3}{2}ab\,d^2x^2-\frac{1}{2}b^2c\,x^2d+3a^2d^2x-3abcdx+x\,b^2c^2)}{d^3}+\frac{(a^3d^3-3a^2bc\,d^2+3a\,b^2c^2d-b^3c^3)\ln(dx+c)}{d^4}$
risch	$\frac{b^3x^3}{3d}+\frac{3b^2a\,x^2}{2d}-\frac{b^3c\,x^2}{2d^2}+\frac{3b\,a^2x}{d}-\frac{3b^2acx}{d^2}+\frac{b^3x\,c^2}{d^3}+\frac{\ln(dx+c)a^3}{d}-\frac{3\ln(dx+c)a^2bc}{d^2}+\frac{3\ln(dx+c)a\,b^2c^2}{d^3}-$
parallelrisch	$\frac{2x^3b^3d^3+9a\,b^2d^3x^2-3x^2b^3c\,d^2+6\ln(dx+c)a^3d^3-18\ln(dx+c)a^2bc\,d^2+18\ln(dx+c)a\,b^2c^2d-6\ln(dx+c)b^3c^3+18a^2b\,d^3x-18}{6d^4}$

input
```
int((b*x+a)^4/(a*c+(a*d+b*c)*x+b*d*x^2),x,method=_RETURNVERBOSE)
```

output
```
b*(3*a^2*d^2-3*a*b*c*d+b^2*c^2)/d^3*x+1/3*b^3/d*x^3+1/2*b^2/d^2*(3*a*d-b*c
)*x^2+(a^3*d^3-3*a^2*b*c*d^2+3*a*b^2*c^2*d-b^3*c^3)/d^4*ln(d*x+c)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 115, normalized size of antiderivative = 1.55

$$\int \frac{(a+bx)^4}{ac+(bc+ad)x+bdx^2}\,dx = \frac{2\,b^3d^3x^3-3\,(b^3cd^2-3\,ab^2d^3)x^2+6\,(b^3c^2d-3\,ab^2cd^2+3\,a^2bd^3)x-6\,(b^3c^3-3\,ab^2c^2d+3\,a^2bcd^2-a^3d^3}{6\,d^4}$$

input
```
integrate((b*x+a)^4/(a*c+(a*d+b*c)*x+b*d*x^2),x, algorithm="fricas")
```

output
```
1/6*(2*b^3*d^3*x^3 - 3*(b^3*c*d^2 - 3*a*b^2*d^3)*x^2 + 6*(b^3*c^2*d - 3*a*
b^2*c*d^2 + 3*a^2*b*d^3)*x - 6*(b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 -
a^3*d^3)*log(d*x + c))/d^4
```

Sympy [A] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.12

$$\int \frac{(a+bx)^4}{ac+(bc+ad)x+bdx^2}\,dx = \frac{b^3x^3}{3d}+x^2\cdot\left(\frac{3ab^2}{2d}-\frac{b^3c}{2d^2}\right) + x\left(\frac{3a^2b}{d}-\frac{3ab^2c}{d^2}+\frac{b^3c^2}{d^3}\right)+\frac{(ad-bc)^3\log(c+dx)}{d^4}$$

input
```
integrate((b*x+a)**4/(a*c+(a*d+b*c)*x+b*d*x**2),x)
```

output
```
b**3*x**3/(3*d) + x**2*(3*a*b**2/(2*d) - b**3*c/(2*d**2)) + x*(3*a**2*b/d
- 3*a*b**2*c/d**2 + b**3*c**2/d**3) + (a*d - b*c)**3*log(c + d*x)/d**4
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.54

$$\int \frac{(a+bx)^4}{ac+(bc+ad)x+bdx^2}\,dx = \frac{2\,b^3d^2x^3-3\,(b^3cd-3\,ab^2d^2)x^2+6\,(b^3c^2-3\,ab^2cd+3\,a^2bd^2)x}{6\,d^3} - \frac{(b^3c^3-3\,ab^2c^2d+3\,a^2bcd^2-a^3d^3)\log(dx+c)}{d^4}$$

input
```
integrate((b*x+a)^4/(a*c+(a*d+b*c)*x+b*d*x^2),x, algorithm="maxima")
```

output
```
1/6*(2*b^3*d^2*x^3 - 3*(b^3*c*d - 3*a*b^2*d^2)*x^2 + 6*(b^3*c^2 - 3*a*b^2*
c*d + 3*a^2*b*d^2)*x)/d^3 - (b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3
*d^3)*log(d*x + c)/d^4
```

Giac [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.57

$$\int \frac{(a+bx)^4}{ac+(bc+ad)x+bdx^2}\,dx = \frac{2\,b^3d^2x^3-3\,b^3cdx^2+9\,ab^2d^2x^2+6\,b^3c^2x-18\,ab^2cdx+18\,a^2bd^2x}{6\,d^3} - \frac{(b^3c^3-3\,ab^2c^2d+3\,a^2bcd^2-a^3d^3)\log(|dx+c|)}{d^4}$$

input
```
integrate((b*x+a)^4/(a*c+(a*d+b*c)*x+b*d*x^2),x, algorithm="giac")
```

output
```
1/6*(2*b^3*d^2*x^3 - 3*b^3*c*d*x^2 + 9*a*b^2*d^2*x^2 + 6*b^3*c^2*x - 18*a*
b^2*c*d*x + 18*a^2*b*d^2*x)/d^3 - (b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2
 - a^3*d^3)*log(abs(d*x + c))/d^4
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 118, normalized size of antiderivative = 1.59

$$\int \frac{(a+bx)^4}{ac+(bc+ad)x+bdx^2}\,dx = x^2\left(\frac{3\,a\,b^2}{2\,d}-\frac{b^3\,c}{2\,d^2}\right)+x\left(\frac{3\,a^2\,b}{d}-\frac{c\left(\frac{3\,a\,b^2}{d}-\frac{b^3\,c}{d^2}\right)}{d}\right) + \frac{\ln(c+d\,x)\,\left(a^3\,d^3-3\,a^2\,b\,c\,d^2+3\,a\,b^2\,c^2\,d-b^3\,c^3\right)}{d^4} + \frac{b^3\,x^3}{3\,d}$$

input
```
int((a + b*x)^4/(a*c + x*(a*d + b*c) + b*d*x^2),x)
```

output
```
x^2*((3*a*b^2)/(2*d) - (b^3*c)/(2*d^2)) + x*((3*a^2*b)/d - (c*((3*a*b^2)/d
 - (b^3*c)/d^2))/d) + (log(c + d*x)*(a^3*d^3 - b^3*c^3 + 3*a*b^2*c^2*d - 3
*a^2*b*c*d^2))/d^4 + (b^3*x^3)/(3*d)
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 132, normalized size of antiderivative = 1.78

$$\int \frac{(a+bx)^4}{ac+(bc+ad)x+bdx^2}\,dx$$
$$= \frac{6\log(dx+c)\,a^3d^3-18\log(dx+c)\,a^2bc\,d^2+18\log(dx+c)\,a\,b^2c^2d-6\log(dx+c)\,b^3c^3+18a^2b\,d^3x-18}{6d^4}$$

input
```
int((b*x+a)^4/(a*c+(a*d+b*c)*x+b*d*x^2),x)
```

output
```
(6*log(c + d*x)*a**3*d**3 - 18*log(c + d*x)*a**2*b*c*d**2 + 18*log(c + d*x
)*a*b**2*c**2*d - 6*log(c + d*x)*b**3*c**3 + 18*a**2*b*d**3*x - 18*a*b**2*
c*d**2*x + 9*a*b**2*d**3*x**2 + 6*b**3*c**2*d*x - 3*b**3*c*d**2*x**2 + 2*b
**3*d**3*x**3)/(6*d**4)
```

3.44 $\int \frac{(a+bx)^3}{ac+(bc+ad)x+bdx^2} \, dx$

Optimal result

Integrand size = 29, antiderivative size = 49

$$\int \frac{(a+bx)^3}{ac+(bc+ad)x+bdx^2} \, dx = -\frac{b(bc-2ad)x}{d^2} + \frac{b^2x^2}{2d} + \frac{(bc-ad)^2 \log(c+dx)}{d^3}$$

output
```
-b*(-2*a*d+b*c)*x/d^2+1/2*b^2*x^2/d+(-a*d+b*c)^2*ln(d*x+c)/d^3
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.88

$$\int \frac{(a+bx)^3}{ac+(bc+ad)x+bdx^2} \, dx = \frac{bdx(-2bc+4ad+bdx)+2(bc-ad)^2 \log(c+dx)}{2d^3}$$

input
```
Integrate[(a + b*x)^3/(a*c + (b*c + a*d)*x + b*d*x^2),x]
```

output
```
(b*d*x*(-2*b*c + 4*a*d + b*d*x) + 2*(b*c - a*d)^2*Log[c + d*x])/(2*d^3)
```

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.02, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.069, Rules used = {1121, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx)^3}{x(ad+bc)+ac+bdx^2}\,dx$$

$$\Big\downarrow \text{1121}$$

$$\int \left(\frac{(ad-bc)^2}{d^2(c+dx)}-\frac{b(bc-ad)}{d^2}+\frac{b(a+bx)}{d}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{(bc-ad)^2\log(c+dx)}{d^3}-\frac{bx(bc-ad)}{d^2}+\frac{(a+bx)^2}{2d}$$

input
```
Int[(a + b*x)^3/(a*c + (b*c + a*d)*x + b*d*x^2),x]
```

output
```
-((b*(b*c - a*d)*x)/d^2) + (a + b*x)^2/(2*d) + ((b*c - a*d)^2*Log[c + d*x]
)/d^3
```

Defintions of rubi rules used

rule 1121
```
Int[((d_) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Int[ExpandIntegrand[(d + e*x)^(m + p)*(a/d + (c/e)*x)^p, x], x]
/; FreeQ[{a, b, c, d, e, m, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && (Int
egerQ[p] || (GtQ[a, 0] && GtQ[d, 0] && LtQ[c, 0]))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 1.38 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.14

method	result	size
default	$\frac{b\left(\frac{1}{2}bd\,x^2+2adx-cbx\right)}{d^2}+\frac{\left(a^2d^2-2abcd+b^2c^2\right)\ln\left(dx+c\right)}{d^3}$	56
norman	$\frac{b(2ad-bc)x}{d^2}+\frac{b^2x^2}{2d}+\frac{\left(a^2d^2-2abcd+b^2c^2\right)\ln\left(dx+c\right)}{d^3}$	59
parallelrisch	$\frac{b^2d^2x^2+2\ln\left(dx+c\right)a^2d^2-4\ln\left(dx+c\right)abcd+2\ln\left(dx+c\right)b^2c^2+4ab\,d^2x-2b^2cxd}{2d^3}$	73
risch	$\frac{b^2x^2}{2d}+\frac{2bax}{d}-\frac{b^2cx}{d^2}+\frac{\ln\left(dx+c\right)a^2}{d}-\frac{2\ln\left(dx+c\right)abc}{d^2}+\frac{\ln\left(dx+c\right)b^2c^2}{d^3}$	74

input
```
int((b*x+a)^3/(a*c+(a*d+b*c)*x+b*d*x^2),x,method=_RETURNVERBOSE)
```

output
```
b/d^2*(1/2*b*d*x^2+2*a*d*x-c*b*x)+(a^2*d^2-2*a*b*c*d+b^2*c^2)/d^3*ln(d*x+c
)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.27

$$\int \frac{(a+bx)^3}{ac+(bc+ad)x+bdx^2}\,dx = \frac{b^2d^2x^2-2\left(b^2cd-2\,abd^2\right)x+2\left(b^2c^2-2\,abcd+a^2d^2\right)\log\left(dx+c\right)}{2\,d^3}$$

input
```
integrate((b*x+a)^3/(a*c+(a*d+b*c)*x+b*d*x^2),x, algorithm="fricas")
```

output
```
1/2*(b^2*d^2*x^2 - 2*(b^2*c*d - 2*a*b*d^2)*x + 2*(b^2*c^2 - 2*a*b*c*d + a^
2*d^2)*log(d*x + c))/d^3
```

Sympy [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.90

$$\int \frac{(a+bx)^3}{ac+(bc+ad)x+bdx^2}\,dx = \frac{b^2x^2}{2d} + x\left(\frac{2ab}{d} - \frac{b^2c}{d^2}\right) + \frac{(ad-bc)^2\log(c+dx)}{d^3}$$

input
```
integrate((b*x+a)**3/(a*c+(a*d+b*c)*x+b*d*x**2),x)
```

output
```
b**2*x**2/(2*d) + x*(2*a*b/d - b**2*c/d**2) + (a*d - b*c)**2*log(c + d*x)/
d**3
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.22

$$\int \frac{(a+bx)^3}{ac+(bc+ad)x+bdx^2}\,dx = \frac{b^2dx^2 - 2\,(b^2c - 2\,abd)x}{2\,d^2} + \frac{(b^2c^2 - 2\,abcd + a^2d^2)\log(dx+c)}{d^3}$$

input
```
integrate((b*x+a)^3/(a*c+(a*d+b*c)*x+b*d*x^2),x, algorithm="maxima")
```

output
```
1/2*(b^2*d*x^2 - 2*(b^2*c - 2*a*b*d)*x)/d^2 + (b^2*c^2 - 2*a*b*c*d + a^2*d
^2)*log(d*x + c)/d^3
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.22

$$\int \frac{(a+bx)^3}{ac+(bc+ad)x+bdx^2}\,dx = \frac{b^2dx^2 - 2\,b^2cx + 4\,abdx}{2\,d^2} + \frac{(b^2c^2 - 2\,abcd + a^2d^2)\log(|dx+c|)}{d^3}$$

input
```
integrate((b*x+a)^3/(a*c+(a*d+b*c)*x+b*d*x^2),x, algorithm="giac")
```

output
```
1/2*(b^2*d*x^2 - 2*b^2*c*x + 4*a*b*d*x)/d^2 + (b^2*c^2 - 2*a*b*c*d + a^2*d
^2)*log(abs(d*x + c))/d^3
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.27

$$\int \frac{(a+bx)^3}{ac+(bc+ad)x+bdx^2}\,dx = \frac{\ln(c+d\,x)\,\left(a^2\,d^2-2\,a\,b\,c\,d+b^2\,c^2\right)}{d^3} - x\left(\frac{b^2\,c}{d^2}-\frac{2\,a\,b}{d}\right)+\frac{b^2\,x^2}{2\,d}$$

input
```
int((a + b*x)^3/(a*c + x*(a*d + b*c) + b*d*x^2),x)
```

output
```
(log(c + d*x)*(a^2*d^2 + b^2*c^2 - 2*a*b*c*d))/d^3 - x*((b^2*c)/d^2 - (2*a
*b)/d) + (b^2*x^2)/(2*d)
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.47

$$\int \frac{(a+bx)^3}{ac+(bc+ad)x+bdx^2}\,dx = \frac{2\log(dx+c)\,a^2d^2-4\log(dx+c)\,abcd+2\log(dx+c)\,b^2c^2+4ab\,d^2x-2b^2cdx+b^2d^2x^2}{2d^3}$$

input
```
int((b*x+a)^3/(a*c+(a*d+b*c)*x+b*d*x^2),x)
```

output
```
(2*log(c + d*x)*a**2*d**2 - 4*log(c + d*x)*a*b*c*d + 2*log(c + d*x)*b**2*c
**2 + 4*a*b*d**2*x - 2*b**2*c*d*x + b**2*d**2*x**2)/(2*d**3)
```

3.45 $\int \frac{(a+bx)^2}{ac+(bc+ad)x+bdx^2}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 26

$$\int \frac{(a+bx)^2}{ac+(bc+ad)x+bdx^2}\,dx=\frac{bx}{d}-\frac{(bc-ad)\log(c+dx)}{d^2}$$

output
```
b*x/d-(-a*d+b*c)*ln(d*x+c)/d^2
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.96

$$\int \frac{(a+bx)^2}{ac+(bc+ad)x+bdx^2}\,dx=\frac{bx}{d}+\frac{(-bc+ad)\log(c+dx)}{d^2}$$

input
```
Integrate[(a + b*x)^2/(a*c + (b*c + a*d)*x + b*d*x^2),x]
```

output
```
(b*x)/d + ((-(b*c) + a*d)*Log[c + d*x])/d^2
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.069$, Rules used = {1121, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx)^2}{x(ad+bc)+ac+bdx^2} \, dx$$

$$\Big\downarrow \text{1121}$$

$$\int \left(\frac{ad-bc}{d(c+dx)}+\frac{b}{d}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{bx}{d}-\frac{(bc-ad)\log(c+dx)}{d^2}$$

input
```
Int[(a + b*x)^2/(a*c + (b*c + a*d)*x + b*d*x^2),x]
```

output
```
(b*x)/d - ((b*c - a*d)*Log[c + d*x])/d^2
```

Defintions of rubi rules used

rule 1121
```
Int[((d_) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Int[ExpandIntegrand[(d + e*x)^(m + p)*(a/d + (c/e)*x)^p, x], x]
/; FreeQ[{a, b, c, d, e, m, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && (Int
egerQ[p] || (GtQ[a, 0] && GtQ[d, 0] && LtQ[c, 0]))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 1.45 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00

method	result	size
default	$\frac{bx}{d}+\frac{(ad-bc)\ln(dx+c)}{d^2}$	26
norman	$\frac{bx}{d}+\frac{(ad-bc)\ln(dx+c)}{d^2}$	26
parallelrisch	$\frac{\ln(dx+c)ad-\ln(dx+c)bc+bdx}{d^2}$	29
risch	$\frac{bx}{d}+\frac{\ln(dx+c)a}{d}-\frac{\ln(dx+c)bc}{d^2}$	32

input
```
int((b*x+a)^2/(a*c+(a*d+b*c)*x+b*d*x^2),x,method=_RETURNVERBOSE)
```

output
```
b*x/d+(a*d-b*c)/d^2*ln(d*x+c)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.96

$$\int \frac{(a+bx)^2}{ac+(bc+ad)x+bdx^2}\,dx=\frac{bdx-(bc-ad)\log(dx+c)}{d^2}$$

input
```
integrate((b*x+a)^2/(a*c+(a*d+b*c)*x+b*d*x^2),x, algorithm="fricas")
```

output
```
(b*d*x - (b*c - a*d)*log(d*x + c))/d^2
```

Sympy [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.77

$$\int \frac{(a+bx)^2}{ac+(bc+ad)x+bdx^2}\,dx=\frac{bx}{d}+\frac{(ad-bc)\log(c+dx)}{d^2}$$

input
```
integrate((b*x+a)**2/(a*c+(a*d+b*c)*x+b*d*x**2),x)
```

output
```
b*x/d + (a*d - b*c)*log(c + d*x)/d**2
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00

$$\int \frac{(a+bx)^2}{ac+(bc+ad)x+bdx^2}\,dx=\frac{bx}{d}-\frac{(bc-ad)\log(dx+c)}{d^2}$$

input
```
integrate((b*x+a)^2/(a*c+(a*d+b*c)*x+b*d*x^2),x, algorithm="maxima")
```

output
```
b*x/d - (b*c - a*d)*log(d*x + c)/d^2
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.04

$$\int \frac{(a+bx)^2}{ac+(bc+ad)x+bdx^2}\,dx=\frac{bx}{d}-\frac{(bc-ad)\log(|dx+c|)}{d^2}$$

input
```
integrate((b*x+a)^2/(a*c+(a*d+b*c)*x+b*d*x^2),x, algorithm="giac")
```

output
```
b*x/d - (b*c - a*d)*log(abs(d*x + c))/d^2
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.96

$$\int \frac{(a+bx)^2}{ac+(bc+ad)x+bdx^2}\,dx=\frac{\ln(c+d\,x)\ (a\,d-b\,c)}{d^2}+\frac{b\,x}{d}$$

input
```
int((a + b*x)^2/(a*c + x*(a*d + b*c) + b*d*x^2),x)
```

output
```
(log(c + d*x)*(a*d - b*c))/d^2 + (b*x)/d
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.08

$$\int \frac{(a+bx)^2}{ac+(bc+ad)x+bdx^2}\,dx=\frac{\log(dx+c)\,ad-\log(dx+c)\,bc+bdx}{d^2}$$

input
```
int((b*x+a)^2/(a*c+(a*d+b*c)*x+b*d*x^2),x)
```

output
```
(log(c + d*x)*a*d - log(c + d*x)*b*c + b*d*x)/d**2
```

3.46 $\int \frac{a+bx}{ac+(bc+ad)x+bdx^2}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 10

$$\int \frac{a+bx}{ac+(bc+ad)x+bdx^2}\,dx=\frac{\log(c+dx)}{d}$$

output
```
ln(d*x+c)/d
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00

$$\int \frac{a+bx}{ac+(bc+ad)x+bdx^2}\,dx=\frac{\log(c+dx)}{d}$$

input
```
Integrate[(a + b*x)/(a*c + (b*c + a*d)*x + b*d*x^2),x]
```

output
```
Log[c + d*x]/d
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.074, Rules used = {1120, 16}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+bx}{x(ad+bc)+ac+bdx^2}dx$$
$$\Big\downarrow \ 1120$$
$$\int \frac{1}{c+dx}dx$$
$$\Big\downarrow \ 16$$
$$\frac{\log(c+dx)}{d}$$

input
```
Int[(a + b*x)/(a*c + (b*c + a*d)*x + b*d*x^2),x]
```

output
```
Log[c + d*x]/d
```

Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 1120
```
Int[((d_) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Int[(d + e*x)^(m + p)*(a/d + (c/e)*x)^p, x] /; FreeQ[{a, b, c, d
, e, m}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[p] && (EqQ[m + p,
0] || EqQ[m + 2*p + 2, 0])
```

Maple [A] (verified)

Time = 1.32 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.10

method	result	size
default	$\frac{\ln(dx+c)}{d}$	11
norman	$\frac{\ln(dx+c)}{d}$	11
risch	$\frac{\ln(dx+c)}{d}$	11
parallelrisch	$\frac{\ln(dx+c)}{d}$	11

input
```
int((b*x+a)/(a*c+(a*d+b*c)*x+b*d*x^2),x,method=_RETURNVERBOSE)
```

output
```
ln(d*x+c)/d
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00

$$\int \frac{a+bx}{ac+(bc+ad)x+bdx^2}\,dx=\frac{\log(dx+c)}{d}$$

input
```
integrate((b*x+a)/(a*c+(a*d+b*c)*x+b*d*x^2),x, algorithm="fricas")
```

output
```
log(d*x + c)/d
```

Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.70

$$\int \frac{a+bx}{ac+(bc+ad)x+bdx^2}\,dx=\frac{\log(c+dx)}{d}$$

input
```
integrate((b*x+a)/(a*c+(a*d+b*c)*x+b*d*x**2),x)
```

output
```
log(c + d*x)/d
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00

$$\int \frac{a+bx}{ac+(bc+ad)x+bdx^2}\,dx=\frac{\log(dx+c)}{d}$$

input
```
integrate((b*x+a)/(a*c+(a*d+b*c)*x+b*d*x^2),x, algorithm="maxima")
```

output
```
log(d*x + c)/d
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.10

$$\int \frac{a+bx}{ac+(bc+ad)x+bdx^2}\,dx=\frac{\log(|dx+c|)}{d}$$

input
```
integrate((b*x+a)/(a*c+(a*d+b*c)*x+b*d*x^2),x, algorithm="giac")
```

output
```
log(abs(d*x + c))/d
```

Mupad [B] (verification not implemented)

Time = 0.01 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00

$$\int \frac{a+bx}{ac+(bc+ad)x+bdx^2}\,dx=\frac{\ln(c+d\,x)}{d}$$

input
```
int((a + b*x)/(a*c + x*(a*d + b*c) + b*d*x^2),x)
```

output
```
log(c + d*x)/d
```

Reduce [B] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00

$$\int \frac{a+bx}{ac+(bc+ad)x+bdx^2}\,dx=\frac{\log(dx+c)}{d}$$

input
```
int((b*x+a)/(a*c+(a*d+b*c)*x+b*d*x^2),x)
```

output
```
log(c + d*x)/d
```

3.47 $\int \frac{1}{ac+(bc+ad)x+bdx^2}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 36

$$\int \frac{1}{ac+(bc+ad)x+bdx^2}\,dx=\frac{\log(a+bx)}{bc-ad}-\frac{\log(c+dx)}{bc-ad}$$

output
```
ln(b*x+a)/(-a*d+b*c)-ln(d*x+c)/(-a*d+b*c)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.72

$$\int \frac{1}{ac+(bc+ad)x+bdx^2}\,dx=\frac{\log(a+bx)-\log(c+dx)}{bc-ad}$$

input
```
Integrate[(a*c + (b*c + a*d)*x + b*d*x^2)^(-1),x]
```

output
```
(Log[a + b*x] - Log[c + d*x])/(b*c - a*d)
```

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.42, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {1081, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x(ad+bc)+ac+bdx^2}dx$$

$$\Big\downarrow \text{1081}$$

$$bd\int \left(\frac{1}{d(bc-ad)(a+bx)}-\frac{1}{b(bc-ad)(c+dx)}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$bd\left(\frac{\log(a+bx)}{bd(bc-ad)}-\frac{\log(c+dx)}{bd(bc-ad)}\right)$$

input
```
Int[(a*c + (b*c + a*d)*x + b*d*x^2)^(-1),x]
```

output
```
b*d*(Log[a + b*x]/(b*d*(b*c - a*d)) - Log[c + d*x]/(b*d*(b*c - a*d)))
```

Defintions of rubi rules used

rule 1081
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = Rt[b^2
- 4*a*c, 2]}, Simp[c   Int[ExpandIntegrand[1/((b/2 - q/2 + c*x)*(b/2 + q/2
+ c*x)), x], x], x]] /; FreeQ[{a, b, c}, x] && NiceSqrtQ[b^2 - 4*a*c]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 1.31 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.78

method	result	size
parallelrisch	$-\frac{\ln(bx+a)-\ln(dx+c)}{ad-bc}$	28
default	$-\frac{\ln(bx+a)}{ad-bc}+\frac{\ln(dx+c)}{ad-bc}$	37
norman	$-\frac{\ln(bx+a)}{ad-bc}+\frac{\ln(dx+c)}{ad-bc}$	37
risch	$-\frac{\ln(bx+a)}{ad-bc}+\frac{\ln(-dx-c)}{ad-bc}$	40

input
```
int(1/(a*c+(a*d+b*c)*x+b*d*x^2),x,method=_RETURNVERBOSE)
```

output
```
-(ln(b*x+a)-ln(d*x+c))/(a*d-b*c)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.72

$$\int \frac{1}{ac+(bc+ad)x+bdx^2}\,dx=\frac{\log(bx+a)-\log(dx+c)}{bc-ad}$$

input
```
integrate(1/(a*c+(a*d+b*c)*x+b*d*x^2),x, algorithm="fricas")
```

output
```
(log(b*x + a) - log(d*x + c))/(b*c - a*d)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 128 vs. $2(26) = 52$.

Time = 0.16 (sec) , antiderivative size = 128, normalized size of antiderivative = 3.56

$$\int \frac{1}{ac+(bc+ad)x+bdx^2}\,dx = \frac{\log\left(x+\frac{-\frac{a^2d^2}{ad-bc}+\frac{2abcd}{ad-bc}+ad-\frac{b^2c^2}{ad-bc}+bc}{2bd}\right)}{ad-bc} - \frac{\log\left(x+\frac{\frac{a^2d^2}{ad-bc}-\frac{2abcd}{ad-bc}+ad+\frac{b^2c^2}{ad-bc}+bc}{2bd}\right)}{ad-bc}$$

input
```
integrate(1/(a*c+(a*d+b*c)*x+b*d*x**2),x)
```

output
```
log(x + (-a**2*d**2/(a*d - b*c) + 2*a*b*c*d/(a*d - b*c) + a*d - b**2*c**2/
(a*d - b*c) + b*c)/(2*b*d))/(a*d - b*c) - log(x + (a**2*d**2/(a*d - b*c) -
 2*a*b*c*d/(a*d - b*c) + a*d + b**2*c**2/(a*d - b*c) + b*c)/(2*b*d))/(a*d
- b*c)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.00

$$\int \frac{1}{ac+(bc+ad)x+bdx^2}\,dx = \frac{\log(bx+a)}{bc-ad} - \frac{\log(dx+c)}{bc-ad}$$

input
```
integrate(1/(a*c+(a*d+b*c)*x+b*d*x^2),x, algorithm="maxima")
```

output
```
log(b*x + a)/(b*c - a*d) - log(d*x + c)/(b*c - a*d)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.28

$$\int \frac{1}{ac+(bc+ad)x+bdx^2}\,dx = \frac{b\log(|bx+a|)}{b^2c-abd} - \frac{d\log(|dx+c|)}{bcd-ad^2}$$

input
```
integrate(1/(a*c+(a*d+b*c)*x+b*d*x^2),x, algorithm="giac")
```

output
```
b*log(abs(b*x + a))/(b^2*c - a*b*d) - d*log(abs(d*x + c))/(b*c*d - a*d^2)
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.11

$$\int \frac{1}{ac + (bc + ad)x + bdx^2} \, dx = \frac{\operatorname{atan}\left(\frac{b\,c\,2\mathrm{i}+b\,d\,x\,2\mathrm{i}}{a\,d-b\,c} + 1\mathrm{i}\right)\,2\mathrm{i}}{a\,d - b\,c}$$

input
```
int(1/(a*c + x*(a*d + b*c) + b*d*x^2),x)
```

output
```
(atan((b*c*2i + b*d*x*2i)/(a*d - b*c) + 1i)*2i)/(a*d - b*c)
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.72

$$\int \frac{1}{ac + (bc + ad)x + bdx^2} \, dx = \frac{-\log(bx + a) + \log(dx + c)}{ad - bc}$$

input
```
int(1/(a*c+(a*d+b*c)*x+b*d*x^2),x)
```

output
```
( - log(a + b*x) + log(c + d*x))/(a*d - b*c)
```

3.48 $\int \frac{1}{(a+bx)(ac+(bc+ad)x+bdx^2)}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 57

$$\int \frac{1}{(a+bx)\,(ac+(bc+ad)x+bdx^2)}\,dx = -\frac{1}{(bc-ad)(a+bx)} - \frac{d\log(a+bx)}{(bc-ad)^2} + \frac{d\log(c+dx)}{(bc-ad)^2}$$

output
```
-1/(-a*d+b*c)/(b*x+a)-d*ln(b*x+a)/(-a*d+b*c)^2+d*ln(d*x+c)/(-a*d+b*c)^2
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.93

$$\int \frac{1}{(a+bx)\,(ac+(bc+ad)x+bdx^2)}\,dx = \frac{-bc+ad-d(a+bx)\log(a+bx)+d(a+bx)\log(c+dx)}{(bc-ad)^2(a+bx)}$$

input
```
Integrate[1/((a + b*x)*(a*c + (b*c + a*d)*x + b*d*x^2)),x]
```

output
```
(-(b*c) + a*d - d*(a + b*x)*Log[a + b*x] + d*(a + b*x)*Log[c + d*x])/((b*c
 - a*d)^2*(a + b*x))
```

Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.069, Rules used = {1121, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a+bx)\left(x(ad+bc)+ac+bdx^2\right)}dx$$

$$\Big\downarrow \text{1121}$$

$$\int \left(\frac{d^2}{(c+dx)(bc-ad)^2}-\frac{bd}{(a+bx)(bc-ad)^2}+\frac{b}{(a+bx)^2(bc-ad)}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{1}{(a+bx)(bc-ad)}-\frac{d\log(a+bx)}{(bc-ad)^2}+\frac{d\log(c+dx)}{(bc-ad)^2}$$

input
```
Int[1/((a + b*x)*(a*c + (b*c + a*d)*x + b*d*x^2)),x]
```

output
```
-(1/((b*c - a*d)*(a + b*x))) - (d*Log[a + b*x])/(b*c - a*d)^2 + (d*Log[c +
 d*x])/(b*c - a*d)^2
```

Defintions of rubi rules used

rule 1121
```
Int[((d_) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Int[ExpandIntegrand[(d + e*x)^(m + p)*(a/d + (c/e)*x)^p, x], x]
/; FreeQ[{a, b, c, d, e, m, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && (Int
egerQ[p] || (GtQ[a, 0] && GtQ[d, 0] && LtQ[c, 0]))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 1.69 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.00

method	result	size
default	$\frac{1}{(ad-bc)(bx+a)}-\frac{d\ln(bx+a)}{(ad-bc)^2}+\frac{d\ln(dx+c)}{(ad-bc)^2}$	57
risch	$\frac{1}{(ad-bc)(bx+a)}+\frac{d\ln(-dx-c)}{a^2d^2-2abcd+b^2c^2}-\frac{d\ln(bx+a)}{a^2d^2-2abcd+b^2c^2}$	86
norman	$-\frac{bx}{a(ad-bc)(bx+a)}+\frac{d\ln(dx+c)}{a^2d^2-2abcd+b^2c^2}-\frac{d\ln(bx+a)}{a^2d^2-2abcd+b^2c^2}$	89
parallelrisch	$-\frac{\ln(bx+a)xabd-\ln(dx+c)xabd+\ln(bx+a)a^2d-\ln(dx+c)a^2d+abdx-b^2cx}{(a^2d^2-2abcd+b^2c^2)(bx+a)a}$	95

input
```
int(1/(b*x+a)/(a*c+(a*d+b*c)*x+b*d*x^2),x,method=_RETURNVERBOSE)
```

output
```
1/(a*d-b*c)/(b*x+a)-d/(a*d-b*c)^2*ln(b*x+a)+d/(a*d-b*c)^2*ln(d*x+c)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 93, normalized size of antiderivative = 1.63

$$\int \frac{1}{(a+bx)\,(ac+(bc+ad)x+bdx^2)}\,dx = -\frac{bc-ad+(bdx+ad)\log(bx+a)-(bdx+ad)\log(dx+c)}{ab^2c^2-2\,a^2bcd+a^3d^2+(b^3c^2-2\,ab^2cd+a^2bd^2)x}$$

input
```
integrate(1/(b*x+a)/(a*c+(a*d+b*c)*x+b*d*x^2),x, algorithm="fricas")
```

output
```
-(b*c - a*d + (b*d*x + a*d)*log(b*x + a) - (b*d*x + a*d)*log(d*x + c))/(a*
b^2*c^2 - 2*a^2*b*c*d + a^3*d^2 + (b^3*c^2 - 2*a*b^2*c*d + a^2*b*d^2)*x)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 233 vs. $2(46) = 92$.

Time = 0.36 (sec) , antiderivative size = 233, normalized size of antiderivative = 4.09

$$\int \frac{1}{(a+bx)\left(ac+(bc+ad)x+bdx^2\right)}\,dx = \frac{d\log\left(x+\frac{-\frac{a^3d^4}{(ad-bc)^2}+\frac{3a^2bcd^3}{(ad-bc)^2}-\frac{3ab^2c^2d^2}{(ad-bc)^2}+ad^2+\frac{b^3c^3d}{(ad-bc)^2}+bcd}{2bd^2}\right)}{(ad-bc)^2} - \frac{d\log\left(x+\frac{\frac{a^3d^4}{(ad-bc)^2}-\frac{3a^2bcd^3}{(ad-bc)^2}+\frac{3ab^2c^2d^2}{(ad-bc)^2}+ad^2-\frac{b^3c^3d}{(ad-bc)^2}+bcd}{2bd^2}\right)}{(ad-bc)^2} + \frac{1}{a^2d-abc+x\left(abd-b^2c\right)}$$

input
```
integrate(1/(b*x+a)/(a*c+(a*d+b*c)*x+b*d*x**2),x)
```

output
```
d*log(x + (-a**3*d**4/(a*d - b*c)**2 + 3*a**2*b*c*d**3/(a*d - b*c)**2 - 3*
a*b**2*c**2*d**2/(a*d - b*c)**2 + a*d**2 + b**3*c**3*d/(a*d - b*c)**2 + b*
c*d)/(2*b*d**2))/(a*d - b*c)**2 - d*log(x + (a**3*d**4/(a*d - b*c)**2 - 3*
a**2*b*c*d**3/(a*d - b*c)**2 + 3*a*b**2*c**2*d**2/(a*d - b*c)**2 + a*d**2
- b**3*c**3*d/(a*d - b*c)**2 + b*c*d)/(2*b*d**2))/(a*d - b*c)**2 + 1/(a**2
*d - a*b*c + x*(a*b*d - b**2*c))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.61

$$\int \frac{1}{(a+bx)\left(ac+(bc+ad)x+bdx^2\right)}\,dx = -\frac{d\log\left(bx+a\right)}{b^2c^2-2\,abcd+a^2d^2}+\frac{d\log\left(dx+c\right)}{b^2c^2-2\,abcd+a^2d^2} - \frac{1}{abc-a^2d+\left(b^2c-abd\right)x}$$

input
```
integrate(1/(b*x+a)/(a*c+(a*d+b*c)*x+b*d*x^2),x, algorithm="maxima")
```

output
```
-d*log(b*x + a)/(b^2*c^2 - 2*a*b*c*d + a^2*d^2) + d*log(d*x + c)/(b^2*c^2
- 2*a*b*c*d + a^2*d^2) - 1/(a*b*c - a^2*d + (b^2*c - a*b*d)*x)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.65

$$\int \frac{1}{(a+bx)\left(ac+(bc+ad)x+bdx^2\right)}\,dx = -\frac{bd\log\left(|bx+a|\right)}{b^3c^2-2\,ab^2cd+a^2bd^2} + \frac{d^2\log\left(|dx+c|\right)}{b^2c^2d-2\,abcd^2+a^2d^3} - \frac{1}{(bc-ad)(bx+a)}$$

input
```
integrate(1/(b*x+a)/(a*c+(a*d+b*c)*x+b*d*x^2),x, algorithm="giac")
```

output
```
-b*d*log(abs(b*x + a))/(b^3*c^2 - 2*a*b^2*c*d + a^2*b*d^2) + d^2*log(abs(d
*x + c))/(b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3) - 1/((b*c - a*d)*(b*x + a))
```

Mupad [B] (verification not implemented)

Time = 5.39 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.33

$$\int \frac{1}{(a+bx)\left(ac+(bc+ad)x+bdx^2\right)}\,dx = \frac{1}{(a\,d-b\,c)\,(a+b\,x)} - \frac{2\,d\,\mathrm{atanh}\left(\frac{a^2\,d^2-b^2\,c^2}{{(a\,d-b\,c)}^2}+\frac{2\,b\,d\,x}{a\,d-b\,c}\right)}{{(a\,d-b\,c)}^2}$$

input
```
int(1/((a + b*x)*(a*c + x*(a*d + b*c) + b*d*x^2)),x)
```

output
```
1/((a*d - b*c)*(a + b*x)) - (2*d*atanh((a^2*d^2 - b^2*c^2)/(a*d - b*c)^2 +
 (2*b*d*x)/(a*d - b*c)))/(a*d - b*c)^2
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 115, normalized size of antiderivative = 2.02

$$\int \frac{1}{(a+bx)\left(ac+(bc+ad)x+bdx^2\right)}\,dx$$
$$=\frac{-\log(bx+a)\,a^2d-\log(bx+a)\,abdx+\log(dx+c)\,a^2d+\log(dx+c)\,abdx-abdx+b^2cx}{a\,(a^2b\,d^2x-2a\,b^2cdx+b^3c^2x+a^3d^2-2a^2bcd+a\,b^2c^2)}$$

input
```
int(1/(b*x+a)/(a*c+(a*d+b*c)*x+b*d*x^2),x)
```

output
```
( - log(a + b*x)*a**2*d - log(a + b*x)*a*b*d*x + log(c + d*x)*a**2*d + log
(c + d*x)*a*b*d*x - a*b*d*x + b**2*c*x)/(a*(a**3*d**2 - 2*a**2*b*c*d + a**
2*b*d**2*x + a*b**2*c**2 - 2*a*b**2*c*d*x + b**3*c**2*x))
```

3.49 $\int \frac{1}{(a+bx)^2(ac+(bc+ad)x+bdx^2)}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 82

$$\int \frac{1}{(a+bx)^2\,(ac+(bc+ad)x+bdx^2)}\,dx = -\frac{1}{2(bc-ad)(a+bx)^2}+\frac{d}{(bc-ad)^2(a+bx)} + \frac{d^2\log(a+bx)}{(bc-ad)^3}-\frac{d^2\log(c+dx)}{(bc-ad)^3}$$

output
```
-1/2/(-a*d+b*c)/(b*x+a)^2+d/(-a*d+b*c)^2/(b*x+a)+d^2*ln(b*x+a)/(-a*d+b*c)^
3-d^2*ln(d*x+c)/(-a*d+b*c)^3
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.82

$$\int \frac{1}{(a+bx)^2\,(ac+(bc+ad)x+bdx^2)}\,dx = \frac{\frac{(bc-ad)(-bc+3ad+2bdx)}{(a+bx)^2}+2d^2\log(a+bx)-2d^2\log(c+dx)}{2(bc-ad)^3}$$

input
```
Integrate[1/((a + b*x)^2*(a*c + (b*c + a*d)*x + b*d*x^2)),x]
```

output
```
(((b*c - a*d)*(-(b*c) + 3*a*d + 2*b*d*x))/(a + b*x)^2 + 2*d^2*Log[a + b*x]
 - 2*d^2*Log[c + d*x])/(2*(b*c - a*d)^3)
```

Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.069$, Rules used = $\{1121, 2009\}$

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a+bx)^2\left(x(ad+bc)+ac+bdx^2\right)}dx$$

$$\Big\downarrow \text{1121}$$

$$\int \left(-\frac{d^3}{(c+dx)(bc-ad)^3}+\frac{bd^2}{(a+bx)(bc-ad)^3}-\frac{bd}{(a+bx)^2(bc-ad)^2}+\frac{b}{(a+bx)^3(bc-ad)}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{d^2\log(a+bx)}{(bc-ad)^3}-\frac{d^2\log(c+dx)}{(bc-ad)^3}+\frac{d}{(a+bx)(bc-ad)^2}-\frac{1}{2(a+bx)^2(bc-ad)}$$

input
```
Int[1/((a + b*x)^2*(a*c + (b*c + a*d)*x + b*d*x^2)),x]
```

output
```
-1/2*1/((b*c - a*d)*(a + b*x)^2) + d/((b*c - a*d)^2*(a + b*x)) + (d^2*Log[
a + b*x])/(b*c - a*d)^3 - (d^2*Log[c + d*x])/(b*c - a*d)^3
```

Defintions of rubi rules used

rule 1121
```
Int[((d_) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Int[ExpandIntegrand[(d + e*x)^(m + p)*(a/d + (c/e)*x)^p, x], x]
/; FreeQ[{a, b, c, d, e, m, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && (Int
egerQ[p] || (GtQ[a, 0] && GtQ[d, 0] && LtQ[c, 0]))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 1.72 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.99

method	result
default	$\frac{1}{2(ad-bc)(bx+a)^2}+\frac{d}{(ad-bc)^2(bx+a)}-\frac{d^2\ln(bx+a)}{(ad-bc)^3}+\frac{d^2\ln(dx+c)}{(ad-bc)^3}$
risch	$\frac{\frac{bdx}{a^2d^2-2abcd+b^2c^2}+\frac{3ad-bc}{2a^2d^2-4abcd+2b^2c^2}}{(bx+a)^2}+\frac{d^2\ln(-dx-c)}{a^3d^3-3a^2bc\,d^2+3a\,b^2c^2d-b^3c^3}-\frac{d^2\ln(bx+a)}{a^3d^3-3a^2bc\,d^2+3a\,b^2c^2d-b^3c^3}$
norman	$\frac{\frac{bdx}{a^2d^2-2abcd+b^2c^2}+\frac{3da\,b^2-b^3c}{2b^2\left(a^2d^2-2abcd+b^2c^2\right)}}{(bx+a)^2}+\frac{d^2\ln(dx+c)}{a^3d^3-3a^2bc\,d^2+3a\,b^2c^2d-b^3c^3}-\frac{d^2\ln(bx+a)}{a^3d^3-3a^2bc\,d^2+3a\,b^2c^2d-b^3c^3}$
parallelrisch	$-\frac{2\ln(bx+a)x^2b^4d^2-2\ln(dx+c)x^2b^4d^2+4\ln(bx+a)xa\,b^3d^2-4\ln(dx+c)xa\,b^3d^2+2\ln(bx+a)a^2b^2d^2-2\ln(dx+c)a^2b^2d^2-2x}{2(a^3d^3-3a^2bc\,d^2+3a\,b^2c^2d-b^3c^3)(bx+a)^2b^2}$

input
```
int(1/(b*x+a)^2/(a*c+(a*d+b*c)*x+b*d*x^2),x,method=_RETURNVERBOSE)
```

output
```
1/2/(a*d-b*c)/(b*x+a)^2+d/(a*d-b*c)^2/(b*x+a)-d^2/(a*d-b*c)^3*ln(b*x+a)+d^
2/(a*d-b*c)^3*ln(d*x+c)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 242 vs. $2(80) = 160$.

Time = 0.08 (sec) , antiderivative size = 242, normalized size of antiderivative = 2.95

$$\int \frac{1}{(a+bx)^2\,(ac+(bc+ad)x+bdx^2)}\,dx =$$
$$-\frac{b^2c^2-4\,abcd+3\,a^2d^2-2\,(b^2cd-abd^2)x-2\,(b^2d^2x^2+2\,abd^2x+a^2d^2)\log\left(bx+a\right)+2\,(b^2d^2x^2+2\,a}{2\,(a^2b^3c^3-3\,a^3b^2c^2d+3\,a^4bcd^2-a^5d^3+(b^5c^3-3\,ab^4c^2d+3\,a^2b^3cd^2-a^3b^2d^3)x^2+2\,(ab^4c^3-3\,a^2b^3}$$

input
```
integrate(1/(b*x+a)^2/(a*c+(a*d+b*c)*x+b*d*x^2),x, algorithm="fricas")
```

output
```
-1/2*(b^2*c^2 - 4*a*b*c*d + 3*a^2*d^2 - 2*(b^2*c*d - a*b*d^2)*x - 2*(b^2*d
^2*x^2 + 2*a*b*d^2*x + a^2*d^2)*log(b*x + a) + 2*(b^2*d^2*x^2 + 2*a*b*d^2*
x + a^2*d^2)*log(d*x + c))/(a^2*b^3*c^3 - 3*a^3*b^2*c^2*d + 3*a^4*b*c*d^2
- a^5*d^3 + (b^5*c^3 - 3*a*b^4*c^2*d + 3*a^2*b^3*c*d^2 - a^3*b^2*d^3)*x^2
+ 2*(a*b^4*c^3 - 3*a^2*b^3*c^2*d + 3*a^3*b^2*c*d^2 - a^4*b*d^3)*x)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 381 vs. $2(68) = 136$.

Time = 0.56 (sec) , antiderivative size = 381, normalized size of antiderivative = 4.65

$$\int \frac{1}{(a+bx)^2\,(ac+(bc+ad)x+bdx^2)}\,dx = \frac{d^2\log\left(x+\frac{-\frac{a^4d^6}{(ad-bc)^3}+\frac{4a^3bcd^5}{(ad-bc)^3}-\frac{6a^2b^2c^2d^4}{(ad-bc)^3}+\frac{4ab^3c^3d^3}{(ad-bc)^3}+ad^3-\frac{b^4c^4d^2}{(ad-bc)^3}+bcd^2}{2bd^3}\right)}{(ad-bc)^3} - \frac{d^2\log\left(x+\frac{\frac{a^4d^6}{(ad-bc)^3}-\frac{4a^3bcd^5}{(ad-bc)^3}+\frac{6a^2b^2c^2d^4}{(ad-bc)^3}-\frac{4ab^3c^3d^3}{(ad-bc)^3}+ad^3+\frac{b^4c^4d^2}{(ad-bc)^3}+bcd^2}{2bd^3}\right)}{(ad-bc)^3} + \frac{3ad-bc+2bdx}{2a^4d^2-4a^3bcd+2a^2b^2c^2+x^2\cdot(2a^2b^2d^2-4ab^3cd+2b^4c^2)+x\,(4a^3bd^2-8a^2b^2cd+4ab^3c^2)}$$

input
```
integrate(1/(b*x+a)**2/(a*c+(a*d+b*c)*x+b*d*x**2),x)
```

output
```
d**2*log(x + (-a**4*d**6/(a*d - b*c)**3 + 4*a**3*b*c*d**5/(a*d - b*c)**3 -
 6*a**2*b**2*c**2*d**4/(a*d - b*c)**3 + 4*a*b**3*c**3*d**3/(a*d - b*c)**3
+ a*d**3 - b**4*c**4*d**2/(a*d - b*c)**3 + b*c*d**2)/(2*b*d**3))/(a*d - b*
c)**3 - d**2*log(x + (a**4*d**6/(a*d - b*c)**3 - 4*a**3*b*c*d**5/(a*d - b*
c)**3 + 6*a**2*b**2*c**2*d**4/(a*d - b*c)**3 - 4*a*b**3*c**3*d**3/(a*d - b
*c)**3 + a*d**3 + b**4*c**4*d**2/(a*d - b*c)**3 + b*c*d**2)/(2*b*d**3))/(a
*d - b*c)**3 + (3*a*d - b*c + 2*b*d*x)/(2*a**4*d**2 - 4*a**3*b*c*d + 2*a**
2*b**2*c**2 + x**2*(2*a**2*b**2*d**2 - 4*a*b**3*c*d + 2*b**4*c**2) + x*(4*
a**3*b*d**2 - 8*a**2*b**2*c*d + 4*a*b**3*c**2))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 202 vs. $2(80) = 160$.

Time = 0.04 (sec) , antiderivative size = 202, normalized size of antiderivative = 2.46

$$\int \frac{1}{(a+bx)^2 (ac+(bc+ad)x+bdx^2)}\,dx = \frac{d^2\log(bx+a)}{b^3c^3-3\,ab^2c^2d+3\,a^2bcd^2-a^3d^3} - \frac{d^2\log(dx+c)}{b^3c^3-3\,ab^2c^2d+3\,a^2bcd^2-a^3d^3} + \frac{2\,bdx-bc+3\,ad}{2\,(a^2b^2c^2-2\,a^3bcd+a^4d^2+(b^4c^2-2\,ab^3cd+a^2b^2d^2)x^2+2\,(ab^3c^2-2\,a^2b^2cd+a^3bd^2)x)}$$

input
```
integrate(1/(b*x+a)^2/(a*c+(a*d+b*c)*x+b*d*x^2),x, algorithm="maxima")
```

output
```
d^2*log(b*x + a)/(b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3) - d^2
*log(d*x + c)/(b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3) + 1/2*(2
*b*d*x - b*c + 3*a*d)/(a^2*b^2*c^2 - 2*a^3*b*c*d + a^4*d^2 + (b^4*c^2 - 2*
a*b^3*c*d + a^2*b^2*d^2)*x^2 + 2*(a*b^3*c^2 - 2*a^2*b^2*c*d + a^3*b*d^2)*x
)
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 145, normalized size of antiderivative = 1.77

$$\int \frac{1}{(a+bx)^2 (ac+(bc+ad)x+bdx^2)}\,dx = -\frac{bd^2\log\left(\left|-\frac{bc}{bx+a}+\frac{ad}{bx+a}-d\right|\right)}{b^4c^3-3\,ab^3c^2d+3\,a^2b^2cd^2-a^3bd^3} - \frac{\frac{b^3c}{(bx+a)^2}-\frac{2\,b^2d}{bx+a}-\frac{ab^2d}{(bx+a)^2}}{2\,(b^4c^2-2\,ab^3cd+a^2b^2d^2)}$$

input
```
integrate(1/(b*x+a)^2/(a*c+(a*d+b*c)*x+b*d*x^2),x, algorithm="giac")
```

output
```
-b*d^2*log(abs(-b*c/(b*x + a) + a*d/(b*x + a) - d))/(b^4*c^3 - 3*a*b^3*c^2
*d + 3*a^2*b^2*c*d^2 - a^3*b*d^3) - 1/2*(b^3*c/(b*x + a)^2 - 2*b^2*d/(b*x
+ a) - a*b^2*d/(b*x + a)^2)/(b^4*c^2 - 2*a*b^3*c*d + a^2*b^2*d^2)
```

Mupad [B] (verification not implemented)

Time = 5.66 (sec) , antiderivative size = 182, normalized size of antiderivative = 2.22

$$\int \frac{1}{(a+bx)^2\,(ac+(bc+ad)x+bdx^2)}\,dx = \frac{\frac{3\,a\,d-b\,c}{2\,(a^2\,d^2-2\,a\,b\,c\,d+b^2\,c^2)}+\frac{b\,d\,x}{a^2\,d^2-2\,a\,b\,c\,d+b^2\,c^2}}{a^2+2\,a\,b\,x+b^2\,x^2} - \frac{2\,d^2\,\mathrm{atanh}\left(\frac{a^3\,d^3-a^2\,b\,c\,d^2-a\,b^2\,c^2\,d+b^3\,c^3}{(a\,d-b\,c)^3}+\frac{2\,b\,d\,x\,(a^2\,d^2-2\,a\,b\,c\,d+b^2\,c^2)}{(a\,d-b\,c)^3}\right)}{(a\,d-b\,c)^3}$$

input
```
int(1/((a + b*x)^2*(a*c + x*(a*d + b*c) + b*d*x^2)),x)
```

output
```
((3*a*d - b*c)/(2*(a^2*d^2 + b^2*c^2 - 2*a*b*c*d)) + (b*d*x)/(a^2*d^2 + b^
2*c^2 - 2*a*b*c*d))/(a^2 + b^2*x^2 + 2*a*b*x) - (2*d^2*atanh((a^3*d^3 + b^
3*c^3 - a*b^2*c^2*d - a^2*b*c*d^2)/(a*d - b*c)^3 + (2*b*d*x*(a^2*d^2 + b^2
*c^2 - 2*a*b*c*d))/(a*d - b*c)^3))/(a*d - b*c)^3
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 288, normalized size of antiderivative = 3.51

$$\int \frac{1}{(a+bx)^2\,(ac+(bc+ad)x+bdx^2)}\,dx = \frac{-2\log(bx+a)\,a^3d^2-4\log(bx+a)\,a^2b\,d^2x-2\log(bx+a)\,a\,b^2d^2x^2+2\log(dx+c)\,a^3d^2+4\log(dx+c)}{2a\,(a^3b^2d^3x^2-3a^2b^3c\,d^2x^2+3a\,b^4c^2d\,x^2-b^5c^3x^2+2a^4b\,d^3x-6a^3b^2c\,d^2x+6a^2b}$$

input
```
int(1/(b*x+a)^2/(a*c+(a*d+b*c)*x+b*d*x^2),x)
```

output
```
( - 2*log(a + b*x)*a**3*d**2 - 4*log(a + b*x)*a**2*b*d**2*x - 2*log(a + b*
x)*a*b**2*d**2*x**2 + 2*log(c + d*x)*a**3*d**2 + 4*log(c + d*x)*a**2*b*d**
2*x + 2*log(c + d*x)*a*b**2*d**2*x**2 + 2*a**3*d**2 - 3*a**2*b*c*d + a*b**
2*c**2 - a*b**2*d**2*x**2 + b**3*c*d*x**2)/(2*a*(a**5*d**3 - 3*a**4*b*c*d*
*2 + 2*a**4*b*d**3*x + 3*a**3*b**2*c**2*d - 6*a**3*b**2*c*d**2*x + a**3*b*
*2*d**3*x**2 - a**2*b**3*c**3 + 6*a**2*b**3*c**2*d*x - 3*a**2*b**3*c*d**2*
x**2 - 2*a*b**4*c**3*x + 3*a*b**4*c**2*d*x**2 - b**5*c**3*x**2))
```

3.50 $\int \frac{1}{(a+bx)^3(ac+(bc+ad)x+bdx^2)}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 107

$$\int \frac{1}{(a+bx)^3\left(ac+(bc+ad)x+bdx^2\right)}\,dx = -\frac{1}{3(bc-ad)(a+bx)^3} + \frac{d}{2(bc-ad)^2(a+bx)^2} - \frac{d^2}{(bc-ad)^3(a+bx)} - \frac{d^3\log(a+bx)}{(bc-ad)^4} + \frac{d^3\log(c+dx)}{(bc-ad)^4}$$

output
```
-1/3/(-a*d+b*c)/(b*x+a)^3+1/2*d/(-a*d+b*c)^2/(b*x+a)^2-d^2/(-a*d+b*c)^3/(b
*x+a)-d^3*ln(b*x+a)/(-a*d+b*c)^4+d^3*ln(d*x+c)/(-a*d+b*c)^4
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.00

$$\int \frac{1}{(a+bx)^3\left(ac+(bc+ad)x+bdx^2\right)}\,dx = \frac{1}{3(-bc+ad)(a+bx)^3} + \frac{d}{2(bc-ad)^2(a+bx)^2} - \frac{d^2}{(bc-ad)^3(a+bx)} - \frac{d^3\log(a+bx)}{(bc-ad)^4} + \frac{d^3\log(c+dx)}{(bc-ad)^4}$$

input
```
Integrate[1/((a + b*x)^3*(a*c + (b*c + a*d)*x + b*d*x^2)),x]
```

output
```
1/(3*(-(b*c) + a*d)*(a + b*x)^3) + d/(2*(b*c - a*d)^2*(a + b*x)^2) - d^2/(
(b*c - a*d)^3*(a + b*x)) - (d^3*Log[a + b*x])/(b*c - a*d)^4 + (d^3*Log[c +
 d*x])/(b*c - a*d)^4
```

Rubi [A] (verified)

Time = 0.46 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.069, Rules used = {1121, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a+bx)^3 (x(ad+bc)+ac+bdx^2)} \, dx$$

$$\Big\downarrow \text{1121}$$

$$\int \left(\frac{d^4}{(c+dx)(bc-ad)^4}-\frac{bd^3}{(a+bx)(bc-ad)^4}+\frac{bd^2}{(a+bx)^2(bc-ad)^3}-\frac{bd}{(a+bx)^3(bc-ad)^2}+\frac{b}{(a+bx)^4(bc-ad}\right.$$

$$\Big\downarrow \text{2009}$$

$$-\frac{d^3\log(a+bx)}{(bc-ad)^4}+\frac{d^3\log(c+dx)}{(bc-ad)^4}-\frac{d^2}{(a+bx)(bc-ad)^3}+\frac{d}{2(a+bx)^2(bc-ad)^2}-\frac{1}{3(a+bx)^3(bc-ad)}$$

input
```
Int[1/((a + b*x)^3*(a*c + (b*c + a*d)*x + b*d*x^2)),x]
```

output
```
-1/3*1/((b*c - a*d)*(a + b*x)^3) + d/(2*(b*c - a*d)^2*(a + b*x)^2) - d^2/(
(b*c - a*d)^3*(a + b*x)) - (d^3*Log[a + b*x])/(b*c - a*d)^4 + (d^3*Log[c +
 d*x])/(b*c - a*d)^4
```

Defintions of rubi rules used

rule 1121
```
Int[((d_) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Int[ExpandIntegrand[(d + e*x)^(m + p)*(a/d + (c/e)*x)^p, x], x]
/; FreeQ[{a, b, c, d, e, m, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && (Int
egerQ[p] || (GtQ[a, 0] && GtQ[d, 0] && LtQ[c, 0]))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 1.72 (sec) , antiderivative size = 103, normalized size of antiderivative = 0.96

method	result
default	$\frac{1}{3(ad-bc)(bx+a)^{3}}+\frac{d}{2(ad-bc)^{2}(bx+a)^{2}}+\frac{d^{2}}{(ad-bc)^{3}(bx+a)}-\frac{d^{3}\ln(bx+a)}{(ad-bc)^{4}}+\frac{d^{3}\ln(dx+c)}{(ad-bc)^{4}}$
risch	$\frac{\frac{b^{2}d^{2}x^{2}}{a^{3}d^{3}-3a^{2}bc\,d^{2}+3a\,b^{2}c^{2}d-b^{3}c^{3}}+\frac{(5ad-bc)bdx}{2a^{3}d^{3}-6a^{2}bc\,d^{2}+6a\,b^{2}c^{2}d-2b^{3}c^{3}}+\frac{11a^{2}d^{2}-7abcd+2b^{2}c^{2}}{6a^{3}d^{3}-18a^{2}bc\,d^{2}+18a\,b^{2}c^{2}d-6b^{3}c^{3}}}{(bx+a)^{3}}+\frac{d^{3}\ln(\cdots}{a^{4}d^{4}-4a^{3}bc\,d^{3}+6a^{2}\cdots}$
parallelrisch	$-\frac{18a^{2}b^{4}c\,d^{2}+2b^{6}c^{3}+18\ln(bx+a)x\,a^{2}b^{4}d^{3}-18\ln(dx+c)x\,a^{2}b^{4}d^{3}+18xa\,b^{5}c\,d^{2}+18\ln(bx+a)x^{2}a\,b^{5}d^{3}-18\ln(dx+c)x^{2}a\,b^{5}d^{3}\cdots}{6(a^{4}d^{4}-4a^{3}bc\,d^{3}\cdots}$
norman	$\frac{\frac{b^{2}d^{2}x^{2}}{a^{3}d^{3}-3a^{2}bc\,d^{2}+3a\,b^{2}c^{2}d-b^{3}c^{3}}+\frac{11d^{2}a^{2}b^{3}-7cda\,b^{4}+2b^{5}c^{2}}{6b^{3}\left(a^{3}d^{3}-3a^{2}bc\,d^{2}+3a\,b^{2}c^{2}d-b^{3}c^{3}\right)}+\frac{\left(5a\,b^{3}d^{2}-b^{4}cd\right)x}{2b^{2}\left(a^{3}d^{3}-3a^{2}bc\,d^{2}+3a\,b^{2}c^{2}d-b^{3}c^{3}\right)}}{(bx+a)^{3}}+\frac{\cdots}{a^{4}d^{4}-4a^{3}bc\,d^{3}+\cdots}$

input
```
int(1/(b*x+a)^3/(a*c+(a*d+b*c)*x+b*d*x^2),x,method=_RETURNVERBOSE)
```

output
```
1/3/(a*d-b*c)/(b*x+a)^3+1/2*d/(a*d-b*c)^2/(b*x+a)^2+d^2/(a*d-b*c)^3/(b*x+a
)-d^3/(a*d-b*c)^4*ln(b*x+a)+d^3/(a*d-b*c)^4*ln(d*x+c)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 425 vs. $2(103) = 206$.

Time = 0.08 (sec) , antiderivative size = 425, normalized size of antiderivative = 3.97

$$\int \frac{1}{(a+bx)^3\,(ac+(bc+ad)x+bdx^2)}\,dx = -\frac{2\,b^3c^3 - 9\,ab^2c^2d + 18\,a^2bcd^2 - 11\,a^3d^3 + 6\,(b^3cd^2 - ab^2d^3)x^2 - 3\,(b^3c^2d - 6\,ab^2cd^2 + 5\,a^2bc}{6\,(a^3b^4c^4 - 4\,a^4b^3c^3d + 6\,a^5b^2c^2d^2 - 4\,a^6bcd^3 + a^7d^4 + (b^7c^4 - 4\,ab^6c^3d + 6\,a^2b^5c^2d^2 - 4\,a^3b^4cd^3 + a^4b}$$

input
```
integrate(1/(b*x+a)^3/(a*c+(a*d+b*c)*x+b*d*x^2),x, algorithm="fricas")
```

output
```
-1/6*(2*b^3*c^3 - 9*a*b^2*c^2*d + 18*a^2*b*c*d^2 - 11*a^3*d^3 + 6*(b^3*c*d
^2 - a*b^2*d^3)*x^2 - 3*(b^3*c^2*d - 6*a*b^2*c*d^2 + 5*a^2*b*d^3)*x + 6*(b
^3*d^3*x^3 + 3*a*b^2*d^3*x^2 + 3*a^2*b*d^3*x + a^3*d^3)*log(b*x + a) - 6*(
b^3*d^3*x^3 + 3*a*b^2*d^3*x^2 + 3*a^2*b*d^3*x + a^3*d^3)*log(d*x + c))/(a^
3*b^4*c^4 - 4*a^4*b^3*c^3*d + 6*a^5*b^2*c^2*d^2 - 4*a^6*b*c*d^3 + a^7*d^4
+ (b^7*c^4 - 4*a*b^6*c^3*d + 6*a^2*b^5*c^2*d^2 - 4*a^3*b^4*c*d^3 + a^4*b^3
*d^4)*x^3 + 3*(a*b^6*c^4 - 4*a^2*b^5*c^3*d + 6*a^3*b^4*c^2*d^2 - 4*a^4*b^3
*c*d^3 + a^5*b^2*d^4)*x^2 + 3*(a^2*b^5*c^4 - 4*a^3*b^4*c^3*d + 6*a^4*b^3*c
^2*d^2 - 4*a^5*b^2*c*d^3 + a^6*b*d^4)*x)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 570 vs. $2(88) = 176$.

Time = 0.77 (sec) , antiderivative size = 570, normalized size of antiderivative = 5.33

$$\int \frac{1}{(a+bx)^3\,(ac+(bc+ad)x+bdx^2)}\,dx = \frac{d^3\log\left(x+\frac{-\frac{a^5d^8}{(ad-bc)^4}+\frac{5a^4bcd^7}{(ad-bc)^4}-\frac{10a^3b^2c^2d^6}{(ad-bc)^4}+\frac{10a^2b^3c^3d^5}{(ad-bc)^4}-\frac{5ab^4c^4d^4}{(ad-bc)^4}+ad^4+\frac{b^5c^5d^3}{(ad-bc)^4}+bcd^3}{2bd^4}\right)}{(ad-bc)^4} - \frac{d^3\log\left(x+\frac{\frac{a^5d^8}{(ad-bc)^4}-\frac{5a^4bcd^7}{(ad-bc)^4}+\frac{10a^3b^2c^2d^6}{(ad-bc)^4}-\frac{10a^2b^3c^3d^5}{(ad-bc)^4}+\frac{5ab^4c^4d^4}{(ad-bc)^4}+ad^4-\frac{b^5c^5d^3}{(ad-bc)^4}+bcd^3}{2bd^4}\right)}{(ad-bc)^4} + \frac{11a^2d^2 - 7abcd + 2b^2c^2 + 6b^2d^2x}{6a^6d^3 - 18a^5bcd^2 + 18a^4b^2c^2d - 6a^3b^3c^3 + x^3\cdot(6a^3b^3d^3 - 18a^2b^4cd^2 + 18ab^5c^2d - 6b^6c^3) + x^2\cdot(18a^4}$$

input
```
integrate(1/(b*x+a)**3/(a*c+(a*d+b*c)*x+b*d*x**2),x)
```

output
```
d**3*log(x + (-a**5*d**8/(a*d - b*c)**4 + 5*a**4*b*c*d**7/(a*d - b*c)**4 -
 10*a**3*b**2*c**2*d**6/(a*d - b*c)**4 + 10*a**2*b**3*c**3*d**5/(a*d - b*c
)**4 - 5*a*b**4*c**4*d**4/(a*d - b*c)**4 + a*d**4 + b**5*c**5*d**3/(a*d -
b*c)**4 + b*c*d**3)/(2*b*d**4))/(a*d - b*c)**4 - d**3*log(x + (a**5*d**8/(
a*d - b*c)**4 - 5*a**4*b*c*d**7/(a*d - b*c)**4 + 10*a**3*b**2*c**2*d**6/(a
*d - b*c)**4 - 10*a**2*b**3*c**3*d**5/(a*d - b*c)**4 + 5*a*b**4*c**4*d**4/
(a*d - b*c)**4 + a*d**4 - b**5*c**5*d**3/(a*d - b*c)**4 + b*c*d**3)/(2*b*d
**4))/(a*d - b*c)**4 + (11*a**2*d**2 - 7*a*b*c*d + 2*b**2*c**2 + 6*b**2*d*
*2*x**2 + x*(15*a*b*d**2 - 3*b**2*c*d))/(6*a**6*d**3 - 18*a**5*b*c*d**2 +
18*a**4*b**2*c**2*d - 6*a**3*b**3*c**3 + x**3*(6*a**3*b**3*d**3 - 18*a**2*
b**4*c*d**2 + 18*a*b**5*c**2*d - 6*b**6*c**3) + x**2*(18*a**4*b**2*d**3 -
54*a**3*b**3*c*d**2 + 54*a**2*b**4*c**2*d - 18*a*b**5*c**3) + x*(18*a**5*b
*d**3 - 54*a**4*b**2*c*d**2 + 54*a**3*b**3*c**2*d - 18*a**2*b**4*c**3))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 361 vs. $2(103) = 206$.

Time = 0.05 (sec) , antiderivative size = 361, normalized size of antiderivative = 3.37

$$\int \frac{1}{(a+bx)^3\,(ac+(bc+ad)x+bdx^2)}\,dx$$
$$= -\frac{d^3\log(bx+a)}{b^4c^4-4\,ab^3c^3d+6\,a^2b^2c^2d^2-4\,a^3bcd^3+a^4d^4}$$
$$+\frac{d^3\log(dx+c)}{b^4c^4-4\,ab^3c^3d+6\,a^2b^2c^2d^2-4\,a^3bcd^3+a^4d^4}$$
$$-\frac{6\,b^2d^2x^2+2\,b^2c^2-7\,abcd+11\,a^2d^2-3\,(b}{6\,(a^3b^3c^3-3\,a^4b^2c^2d+3\,a^5bcd^2-a^6d^3+(b^6c^3-3\,ab^5c^2d+3\,a^2b^4cd^2-a^3b^3d^3)x^3+3\,(ab^5c^3-3\,a^2b^4}$$

input
```
integrate(1/(b*x+a)^3/(a*c+(a*d+b*c)*x+b*d*x^2),x, algorithm="maxima")
```

output
```
-d^3*log(b*x + a)/(b^4*c^4 - 4*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 - 4*a^3*b*c
*d^3 + a^4*d^4) + d^3*log(d*x + c)/(b^4*c^4 - 4*a*b^3*c^3*d + 6*a^2*b^2*c^
2*d^2 - 4*a^3*b*c*d^3 + a^4*d^4) - 1/6*(6*b^2*d^2*x^2 + 2*b^2*c^2 - 7*a*b*
c*d + 11*a^2*d^2 - 3*(b^2*c*d - 5*a*b*d^2)*x)/(a^3*b^3*c^3 - 3*a^4*b^2*c^2
*d + 3*a^5*b*c*d^2 - a^6*d^3 + (b^6*c^3 - 3*a*b^5*c^2*d + 3*a^2*b^4*c*d^2
- a^3*b^3*d^3)*x^3 + 3*(a*b^5*c^3 - 3*a^2*b^4*c^2*d + 3*a^3*b^3*c*d^2 - a^
4*b^2*d^3)*x^2 + 3*(a^2*b^4*c^3 - 3*a^3*b^3*c^2*d + 3*a^4*b^2*c*d^2 - a^5*
b*d^3)*x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 243 vs. $2(103) = 206$.

Time = 0.16 (sec) , antiderivative size = 243, normalized size of antiderivative = 2.27

$$\int \frac{1}{(a+bx)^3\left(ac+(bc+ad)x+bdx^2\right)}\,dx$$
$$= -\frac{bd^3\log(|bx+a|)}{b^5c^4-4\,ab^4c^3d+6\,a^2b^3c^2d^2-4\,a^3b^2cd^3+a^4bd^4}$$
$$+\frac{d^4\log(|dx+c|)}{b^4c^4d-4\,ab^3c^3d^2+6\,a^2b^2c^2d^3-4\,a^3bcd^4+a^4d^5}$$
$$-\frac{2\,b^3c^3-9\,ab^2c^2d+18\,a^2bcd^2-11\,a^3d^3+6\,(b^3cd^2-ab^2d^3)x^2-3\,(b^3c^2d-6\,ab^2cd^2+5\,a^2bd^3)x}{6\,(bc-ad)^4(bx+a)^3}$$

input
```
integrate(1/(b*x+a)^3/(a*c+(a*d+b*c)*x+b*d*x^2),x, algorithm="giac")
```

output
```
-b*d^3*log(abs(b*x + a))/(b^5*c^4 - 4*a*b^4*c^3*d + 6*a^2*b^3*c^2*d^2 - 4*
a^3*b^2*c*d^3 + a^4*b*d^4) + d^4*log(abs(d*x + c))/(b^4*c^4*d - 4*a*b^3*c^
3*d^2 + 6*a^2*b^2*c^2*d^3 - 4*a^3*b*c*d^4 + a^4*d^5) - 1/6*(2*b^3*c^3 - 9*
a*b^2*c^2*d + 18*a^2*b*c*d^2 - 11*a^3*d^3 + 6*(b^3*c*d^2 - a*b^2*d^3)*x^2
- 3*(b^3*c^2*d - 6*a*b^2*c*d^2 + 5*a^2*b*d^3)*x)/((b*c - a*d)^4*(b*x + a)^
3)
```

Mupad [B] (verification not implemented)

Time = 5.67 (sec) , antiderivative size = 312, normalized size of antiderivative = 2.92

$$\int \frac{1}{(a+bx)^3 (ac+(bc+ad)x+bdx^2)} dx$$
$$= \frac{\frac{11\,a^2\,d^2-7\,a\,b\,c\,d+2\,b^2\,c^2}{6\,(a^3\,d^3-3\,a^2\,b\,c\,d^2+3\,a\,b^2\,c^2\,d-b^3\,c^3)} - \frac{d\,x\,(b^2\,c-5\,a\,b\,d)}{2\,(a^3\,d^3-3\,a^2\,b\,c\,d^2+3\,a\,b^2\,c^2\,d-b^3\,c^3)} + \frac{b^2\,d^2\,x^2}{a^3\,d^3-3\,a^2\,b\,c\,d^2+3\,a\,b^2\,c^2\,d-b^3\,c^3}}{a^3+3\,a^2\,b\,x+3\,a\,b^2\,x^2+b^3\,x^3} - \frac{2\,d^3\,\mathrm{atanh}\left(\frac{a^4\,d^4-2\,a^3\,b\,c\,d^3+2\,a\,b^3\,c^3\,d-b^4\,c^4}{(a\,d-b\,c)^4} + \frac{2\,b\,d\,x\,(a^3\,d^3-3\,a^2\,b\,c\,d^2+3\,a\,b^2\,c^2\,d-b^3\,c^3)}{(a\,d-b\,c)^4}\right)}{(a\,d-b\,c)^4}$$

input
```
int(1/((a + b*x)^3*(a*c + x*(a*d + b*c) + b*d*x^2)),x)
```

output
```
((11*a^2*d^2 + 2*b^2*c^2 - 7*a*b*c*d)/(6*(a^3*d^3 - b^3*c^3 + 3*a*b^2*c^2*
d - 3*a^2*b*c*d^2)) - (d*x*(b^2*c - 5*a*b*d))/(2*(a^3*d^3 - b^3*c^3 + 3*a*
b^2*c^2*d - 3*a^2*b*c*d^2)) + (b^2*d^2*x^2)/(a^3*d^3 - b^3*c^3 + 3*a*b^2*c
^2*d - 3*a^2*b*c*d^2))/(a^3 + b^3*x^3 + 3*a*b^2*x^2 + 3*a^2*b*x) - (2*d^3*
atanh((a^4*d^4 - b^4*c^4 + 2*a*b^3*c^3*d - 2*a^3*b*c*d^3)/(a*d - b*c)^4 +
(2*b*d*x*(a^3*d^3 - b^3*c^3 + 3*a*b^2*c^2*d - 3*a^2*b*c*d^2))/(a*d - b*c)^
4))/(a*d - b*c)^4
```

Reduce [B] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 499, normalized size of antiderivative = 4.66

$$\int \frac{1}{(a+bx)^3 (ac+(bc+ad)x+bdx^2)} dx$$
$$= \frac{-6\log(bx+a)\,a^4d^3 - 18\log(bx+a)\,a^3b\,d^3x - 18\log(bx+a)\,a^2b^2d^3x^2 - 6\log(bx+a)\,a\,b^3d^3x^3 + 6\log(}{6a\,(a^4b^3d^4x^3 - 4a^3b^4c\,d^3x^3 + 6a^2b^5c^2d^2x^3 - 4a\,b^6c^3d\,x^3 + b^7c^4x^3 + 3a^5b^2d^4x^2 - 12a}$$

input
```
int(1/(b*x+a)^3/(a*c+(a*d+b*c)*x+b*d*x^2),x)
```

output
```
( - 6*log(a + b*x)*a**4*d**3 - 18*log(a + b*x)*a**3*b*d**3*x - 18*log(a +
b*x)*a**2*b**2*d**3*x**2 - 6*log(a + b*x)*a*b**3*d**3*x**3 + 6*log(c + d*x
)*a**4*d**3 + 18*log(c + d*x)*a**3*b*d**3*x + 18*log(c + d*x)*a**2*b**2*d*
*3*x**2 + 6*log(c + d*x)*a*b**3*d**3*x**3 + 9*a**4*d**3 - 16*a**3*b*c*d**2
 + 9*a**3*b*d**3*x + 9*a**2*b**2*c**2*d - 12*a**2*b**2*c*d**2*x - 2*a*b**3
*c**3 + 3*a*b**3*c**2*d*x - 2*a*b**3*d**3*x**3 + 2*b**4*c*d**2*x**3)/(6*a*
(a**7*d**4 - 4*a**6*b*c*d**3 + 3*a**6*b*d**4*x + 6*a**5*b**2*c**2*d**2 - 1
2*a**5*b**2*c*d**3*x + 3*a**5*b**2*d**4*x**2 - 4*a**4*b**3*c**3*d + 18*a**
4*b**3*c**2*d**2*x - 12*a**4*b**3*c*d**3*x**2 + a**4*b**3*d**4*x**3 + a**3
*b**4*c**4 - 12*a**3*b**4*c**3*d*x + 18*a**3*b**4*c**2*d**2*x**2 - 4*a**3*
b**4*c*d**3*x**3 + 3*a**2*b**5*c**4*x - 12*a**2*b**5*c**3*d*x**2 + 6*a**2*
b**5*c**2*d**2*x**3 + 3*a*b**6*c**4*x**2 - 4*a*b**6*c**3*d*x**3 + b**7*c**
4*x**3))
```

3.51 $\int \frac{1}{(a+bx)^4(ac+(bc+ad)x+bdx^2)}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 130

$$\int \frac{1}{(a+bx)^4\,(ac+(bc+ad)x+bdx^2)}\,dx = -\frac{1}{4(bc-ad)(a+bx)^4} + \frac{d}{3(bc-ad)^2(a+bx)^3} - \frac{d^2}{2(bc-ad)^3(a+bx)^2} + \frac{d^3}{(bc-ad)^4(a+bx)} + \frac{d^4\log(a+bx)}{(bc-ad)^5} - \frac{d^4\log(c+dx)}{(bc-ad)^5}$$

output
```
-1/4/(-a*d+b*c)/(b*x+a)^4+1/3*d/(-a*d+b*c)^2/(b*x+a)^3-1/2*d^2/(-a*d+b*c)^
3/(b*x+a)^2+d^3/(-a*d+b*c)^4/(b*x+a)+d^4*ln(b*x+a)/(-a*d+b*c)^5-d^4*ln(d*x
+c)/(-a*d+b*c)^5
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 130, normalized size of antiderivative = 1.00

$$\int \frac{1}{(a+bx)^4 (ac+(bc+ad)x+bdx^2)}\,dx = \frac{1}{4(-bc+ad)(a+bx)^4} + \frac{d}{3(bc-ad)^2(a+bx)^3} - \frac{d^2}{2(bc-ad)^3(a+bx)^2} + \frac{d^3}{(bc-ad)^4(a+bx)} + \frac{d^4\log(a+bx)}{(bc-ad)^5} - \frac{d^4\log(c+dx)}{(bc-ad)^5}$$

input
```
Integrate[1/((a + b*x)^4*(a*c + (b*c + a*d)*x + b*d*x^2)),x]
```

output
```
1/(4*(-(b*c) + a*d)*(a + b*x)^4) + d/(3*(b*c - a*d)^2*(a + b*x)^3) - d^2/(
2*(b*c - a*d)^3*(a + b*x)^2) + d^3/((b*c - a*d)^4*(a + b*x)) + (d^4*Log[a
+ b*x])/(b*c - a*d)^5 - (d^4*Log[c + d*x])/(b*c - a*d)^5
```

Rubi [A] (verified)

Time = 0.53 (sec) , antiderivative size = 130, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.069$, Rules used = {1121, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a+bx)^4 (x(ad+bc)+ac+bdx^2)}\,dx$$

$$\Big\downarrow \text{1121}$$

$$\int \left(-\frac{d^5}{(c+dx)(bc-ad)^5} + \frac{bd^4}{(a+bx)(bc-ad)^5} - \frac{bd^3}{(a+bx)^2(bc-ad)^4} + \frac{bd^2}{(a+bx)^3(bc-ad)^3} - \frac{bd}{(a+bx)^4(bc-a$$

$$\Big\downarrow \text{2009}$$

$$\frac{d^4\log(a+bx)}{(bc-ad)^5}-\frac{d^4\log(c+dx)}{(bc-ad)^5}+\frac{d^3}{(a+bx)(bc-ad)^4}-\frac{d^2}{2(a+bx)^2(bc-ad)^3}+\frac{d}{3(a+bx)^3(bc-ad)^2}-\frac{1}{4(a+bx)^4(bc-ad)}$$

input
```
Int[1/((a + b*x)^4*(a*c + (b*c + a*d)*x + b*d*x^2)),x]
```

output
```
-1/4*1/((b*c - a*d)*(a + b*x)^4) + d/(3*(b*c - a*d)^2*(a + b*x)^3) - d^2/(
2*(b*c - a*d)^3*(a + b*x)^2) + d^3/((b*c - a*d)^4*(a + b*x)) + (d^4*Log[a
+ b*x])/(b*c - a*d)^5 - (d^4*Log[c + d*x])/(b*c - a*d)^5
```

Defintions of rubi rules used

rule 1121
```
Int[((d_) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Int[ExpandIntegrand[(d + e*x)^(m + p)*(a/d + (c/e)*x)^p, x], x]
/; FreeQ[{a, b, c, d, e, m, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && (Int
egerQ[p] || (GtQ[a, 0] && GtQ[d, 0] && LtQ[c, 0]))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 1.74 (sec) , antiderivative size = 125, normalized size of antiderivative = 0.96

method	result
default	$\frac{1}{4(ad-bc)(bx+a)^4}+\frac{d}{3(ad-bc)^2(bx+a)^3}+\frac{d^2}{2(ad-bc)^3(bx+a)^2}+\frac{d^3}{(ad-bc)^4(bx+a)}-\frac{d^4\ln(bx+a)}{(ad-bc)^5}+\frac{d^4\ln(dx+c)}{(ad-bc)^5}$
parallelrisch	$-\frac{-6x^2b^8c^2d^2-52x\,a^3b^5d^4+4x\,b^8c^3d+12\ln(bx+a)x^4b^8d^4-12\ln(dx+c)x^4b^8d^4+12\ln(bx+a)a^4b^4d^4-12\ln(dx+c)a^4b^4d^4-1\cdots}{\cdots}$
risch	$\frac{\frac{b^3d^3x^3}{a^4d^4-4a^3bc\,d^3+6a^2b^2c^2d^2-4a\,b^3c^3d+c^4b^4}+\frac{(7ad-bc)d^2b^2x^2}{2a^4d^4-8a^3bc\,d^3+12a^2b^2c^2d^2-8a\,b^3c^3d+2c^4b^4}+\frac{bd\left(13a^2d^2-5abcd+b^2c^2\right)x}{3a^4d^4-12a^3bc\,d^3+18a^2b^2c^2d^2-12a\,b^3\cdots}}{(bx+a)^4}$
norman	$\frac{\frac{b^3d^3x^3}{a^4d^4-4a^3bc\,d^3+6a^2b^2c^2d^2-4a\,b^3c^3d+c^4b^4}+\frac{25a^3b^4d^3-23a^2b^5c\,d^2+13a\,c^2d\,b^6-3c^3b^7}{12b^4\left(a^4d^4-4a^3bc\,d^3+6a^2b^2c^2d^2-4a\,b^3c^3d+c^4b^4\right)}+\frac{\left(7a\,b^4d^3-b^5c\,d^2\right)x^2}{2b^2\left(a^4d^4-4a^3bc\,d^3+6a^2b^2c^2d^2-4\cdots\right)}}{(bx+a)^4}$

input
```
int(1/(b*x+a)^4/(a*c+(a*d+b*c)*x+b*d*x^2),x,method=_RETURNVERBOSE)
```

output
```
1/4/(a*d-b*c)/(b*x+a)^4+1/3*d/(a*d-b*c)^2/(b*x+a)^3+1/2*d^2/(a*d-b*c)^3/(b
*x+a)^2+d^3/(a*d-b*c)^4/(b*x+a)-d^4/(a*d-b*c)^5*ln(b*x+a)+d^4/(a*d-b*c)^5*
ln(d*x+c)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 657 vs. $2(124) = 248$.

Time = 0.09 (sec) , antiderivative size = 657, normalized size of antiderivative = 5.05

$$\int \frac{1}{(a+bx)^4\,(ac+(bc+ad)x+bdx^2)}\,dx = -\frac{3\,b^4c^4-16\,ab^3c^3d+36\,a^2b^2c^2d^2-48\,a^3bcd^3+25\,a^4d^4-12\,(b^4cd^3-ab}{12\,(a^4b^5c^5-5\,a^5b^4c^4d+10\,a^6b^3c^3d^2-10\,a^7b^2c^2d^3+5\,a^8bcd^4-a^9d^5+(b^9c^5-5\,ab^8c^4d+10\,a^2b^7c^3d^2}$$

input
```
integrate(1/(b*x+a)^4/(a*c+(a*d+b*c)*x+b*d*x^2),x, algorithm="fricas")
```

output
```
-1/12*(3*b^4*c^4 - 16*a*b^3*c^3*d + 36*a^2*b^2*c^2*d^2 - 48*a^3*b*c*d^3 +
25*a^4*d^4 - 12*(b^4*c*d^3 - a*b^3*d^4)*x^3 + 6*(b^4*c^2*d^2 - 8*a*b^3*c*d
^3 + 7*a^2*b^2*d^4)*x^2 - 4*(b^4*c^3*d - 6*a*b^3*c^2*d^2 + 18*a^2*b^2*c*d^
3 - 13*a^3*b*d^4)*x - 12*(b^4*d^4*x^4 + 4*a*b^3*d^4*x^3 + 6*a^2*b^2*d^4*x^
2 + 4*a^3*b*d^4*x + a^4*d^4)*log(b*x + a) + 12*(b^4*d^4*x^4 + 4*a*b^3*d^4*
x^3 + 6*a^2*b^2*d^4*x^2 + 4*a^3*b*d^4*x + a^4*d^4)*log(d*x + c))/(a^4*b^5*
c^5 - 5*a^5*b^4*c^4*d + 10*a^6*b^3*c^3*d^2 - 10*a^7*b^2*c^2*d^3 + 5*a^8*b*
c*d^4 - a^9*d^5 + (b^9*c^5 - 5*a*b^8*c^4*d + 10*a^2*b^7*c^3*d^2 - 10*a^3*b
^6*c^2*d^3 + 5*a^4*b^5*c*d^4 - a^5*b^4*d^5)*x^4 + 4*(a*b^8*c^5 - 5*a^2*b^7
*c^4*d + 10*a^3*b^6*c^3*d^2 - 10*a^4*b^5*c^2*d^3 + 5*a^5*b^4*c*d^4 - a^6*b
^3*d^5)*x^3 + 6*(a^2*b^7*c^5 - 5*a^3*b^6*c^4*d + 10*a^4*b^5*c^3*d^2 - 10*a
^5*b^4*c^2*d^3 + 5*a^6*b^3*c*d^4 - a^7*b^2*d^5)*x^2 + 4*(a^3*b^6*c^5 - 5*a
^4*b^5*c^4*d + 10*a^5*b^4*c^3*d^2 - 10*a^6*b^3*c^2*d^3 + 5*a^7*b^2*c*d^4 -
 a^8*b*d^5)*x)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 802 vs. $2(109) = 218$.

Time = 1.04 (sec) , antiderivative size = 802, normalized size of antiderivative = 6.17

$$\int \frac{1}{(a+bx)^4\,(ac+(bc+ad)x+bdx^2)}\,dx$$
$$= \frac{d^4\log\left(x+\frac{-\frac{a^6d^{10}}{(ad-bc)^5}+\frac{6a^5bcd^9}{(ad-bc)^5}-\frac{15a^4b^2c^2d^8}{(ad-bc)^5}+\frac{20a^3b^3c^3d^7}{(ad-bc)^5}-\frac{15a^2b^4c^4d^6}{(ad-bc)^5}+\frac{6ab^5c^5d^5}{(ad-bc)^5}+ad^5-\frac{b^6c^6d^4}{(ad-bc)^5}+bcd^4}{2bd^5}\right)}{(ad-bc)^5}$$
$$-\frac{d^4\log\left(x+\frac{\frac{a^6d^{10}}{(ad-bc)^5}-\frac{6a^5bcd^9}{(ad-bc)^5}+\frac{15a^4b^2c^2d^8}{(ad-bc)^5}-\frac{20a^3b^3c^3d^7}{(ad-bc)^5}+\frac{15a^2b^4c^4d^6}{(ad-bc)^5}-\frac{6ab^5c^5d^5}{(ad-bc)^5}+ad^5+\frac{b^6c^6d^4}{(ad-bc)^5}+bcd^4}{2bd^5}\right)}{(ad-bc)^5}$$
$$+\frac{}{12a^8d^4-48a^7bcd^3+72a^6b^2c^2d^2-48a^5b^3c^3d+12a^4b^4c^4+x^4\cdot(12a^4b^4d^4-48a^3b^5cd^3+72a^2b^6c^2d^2-}$$

input
```
integrate(1/(b*x+a)**4/(a*c+(a*d+b*c)*x+b*d*x**2),x)
```

output
```
d**4*log(x + (-a**6*d**10/(a*d - b*c)**5 + 6*a**5*b*c*d**9/(a*d - b*c)**5
- 15*a**4*b**2*c**2*d**8/(a*d - b*c)**5 + 20*a**3*b**3*c**3*d**7/(a*d - b*
c)**5 - 15*a**2*b**4*c**4*d**6/(a*d - b*c)**5 + 6*a*b**5*c**5*d**5/(a*d -
b*c)**5 + a*d**5 - b**6*c**6*d**4/(a*d - b*c)**5 + b*c*d**4)/(2*b*d**5))/(
a*d - b*c)**5 - d**4*log(x + (a**6*d**10/(a*d - b*c)**5 - 6*a**5*b*c*d**9/
(a*d - b*c)**5 + 15*a**4*b**2*c**2*d**8/(a*d - b*c)**5 - 20*a**3*b**3*c**3
*d**7/(a*d - b*c)**5 + 15*a**2*b**4*c**4*d**6/(a*d - b*c)**5 - 6*a*b**5*c*
*5*d**5/(a*d - b*c)**5 + a*d**5 + b**6*c**6*d**4/(a*d - b*c)**5 + b*c*d**4
)/(2*b*d**5))/(a*d - b*c)**5 + (25*a**3*d**3 - 23*a**2*b*c*d**2 + 13*a*b**
2*c**2*d - 3*b**3*c**3 + 12*b**3*d**3*x**3 + x**2*(42*a*b**2*d**3 - 6*b**3
*c*d**2) + x*(52*a**2*b*d**3 - 20*a*b**2*c*d**2 + 4*b**3*c**2*d))/(12*a**8
*d**4 - 48*a**7*b*c*d**3 + 72*a**6*b**2*c**2*d**2 - 48*a**5*b**3*c**3*d +
12*a**4*b**4*c**4 + x**4*(12*a**4*b**4*d**4 - 48*a**3*b**5*c*d**3 + 72*a**
2*b**6*c**2*d**2 - 48*a*b**7*c**3*d + 12*b**8*c**4) + x**3*(48*a**5*b**3*d
**4 - 192*a**4*b**4*c*d**3 + 288*a**3*b**5*c**2*d**2 - 192*a**2*b**6*c**3*
d + 48*a*b**7*c**4) + x**2*(72*a**6*b**2*d**4 - 288*a**5*b**3*c*d**3 + 432
*a**4*b**4*c**2*d**2 - 288*a**3*b**5*c**3*d + 72*a**2*b**6*c**4) + x*(48*a
**7*b*d**4 - 192*a**6*b**2*c*d**3 + 288*a**5*b**3*c**2*d**2 - 192*a**4*b**
4*c**3*d + 48*a**3*b**5*c**4))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 558 vs. $2(124) = 248$.

Time = 0.06 (sec) , antiderivative size = 558, normalized size of antiderivative = 4.29

$$\int \frac{1}{(a+bx)^4 (ac+(bc+ad)x+bdx^2)}\,dx$$
$$= \frac{d^4 \log(bx+a)}{b^5c^5 - 5\,ab^4c^4d + 10\,a^2b^3c^3d^2 - 10\,a^3b^2c^2d^3 + 5\,a^4bcd^4 - a^5d^5}$$
$$- \frac{d^4 \log(dx+c)}{b^5c^5 - 5\,ab^4c^4d + 10\,a^2b^3c^3d^2 - 10\,a^3b^2c^2d^3 + 5\,a^4bcd^4 - a^5d^5}$$
$$+ \frac{12\,b^3d^3x^3 - 3}{12\,(a^4b^4c^4 - 4\,a^5b^3c^3d + 6\,a^6b^2c^2d^2 - 4\,a^7bcd^3 + a^8d^4 + (b^8c^4 - 4\,ab^7c^3d + 6\,a^2b^6c^2d^2 - 4\,a^3b^5cd^3 + a^4}$$

input
```
integrate(1/(b*x+a)^4/(a*c+(a*d+b*c)*x+b*d*x^2),x, algorithm="maxima")
```

output
```
d^4*log(b*x + a)/(b^5*c^5 - 5*a*b^4*c^4*d + 10*a^2*b^3*c^3*d^2 - 10*a^3*b^
2*c^2*d^3 + 5*a^4*b*c*d^4 - a^5*d^5) - d^4*log(d*x + c)/(b^5*c^5 - 5*a*b^4
*c^4*d + 10*a^2*b^3*c^3*d^2 - 10*a^3*b^2*c^2*d^3 + 5*a^4*b*c*d^4 - a^5*d^5
) + 1/12*(12*b^3*d^3*x^3 - 3*b^3*c^3 + 13*a*b^2*c^2*d - 23*a^2*b*c*d^2 + 2
5*a^3*d^3 - 6*(b^3*c*d^2 - 7*a*b^2*d^3)*x^2 + 4*(b^3*c^2*d - 5*a*b^2*c*d^2
 + 13*a^2*b*d^3)*x)/(a^4*b^4*c^4 - 4*a^5*b^3*c^3*d + 6*a^6*b^2*c^2*d^2 - 4
*a^7*b*c*d^3 + a^8*d^4 + (b^8*c^4 - 4*a*b^7*c^3*d + 6*a^2*b^6*c^2*d^2 - 4*
a^3*b^5*c*d^3 + a^4*b^4*d^4)*x^4 + 4*(a*b^7*c^4 - 4*a^2*b^6*c^3*d + 6*a^3*
b^5*c^2*d^2 - 4*a^4*b^4*c*d^3 + a^5*b^3*d^4)*x^3 + 6*(a^2*b^6*c^4 - 4*a^3*
b^5*c^3*d + 6*a^4*b^4*c^2*d^2 - 4*a^5*b^3*c*d^3 + a^6*b^2*d^4)*x^2 + 4*(a^
3*b^5*c^4 - 4*a^4*b^4*c^3*d + 6*a^5*b^3*c^2*d^2 - 4*a^6*b^2*c*d^3 + a^7*b*
d^4)*x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 338 vs. $2(124) = 248$.

Time = 0.14 (sec) , antiderivative size = 338, normalized size of antiderivative = 2.60

$$\int \frac{1}{(a+bx)^4 (ac+(bc+ad)x+bdx^2)}\,dx$$
$$= \frac{bd^4 \log(|bx+a|)}{b^6c^5 - 5\,ab^5c^4d + 10\,a^2b^4c^3d^2 - 10\,a^3b^3c^2d^3 + 5\,a^4b^2cd^4 - a^5bd^5}$$
$$- \frac{d^5 \log(|dx+c|)}{b^5c^5d - 5\,ab^4c^4d^2 + 10\,a^2b^3c^3d^3 - 10\,a^3b^2c^2d^4 + 5\,a^4bcd^5 - a^5d^6}$$
$$- \frac{3\,b^4c^4 - 16\,ab^3c^3d + 36\,a^2b^2c^2d^2 - 48\,a^3bcd^3 + 25\,a^4d^4 - 12\,(b^4cd^3 - ab^3d^4)x^3 + 6\,(b^4c^2d^2 - 8\,ab^3cd^3 +}{12\,(bc-ad)^5(bx+a)^4}$$

input
```
integrate(1/(b*x+a)^4/(a*c+(a*d+b*c)*x+b*d*x^2),x, algorithm="giac")
```

output
```
b*d^4*log(abs(b*x + a))/(b^6*c^5 - 5*a*b^5*c^4*d + 10*a^2*b^4*c^3*d^2 - 10
*a^3*b^3*c^2*d^3 + 5*a^4*b^2*c*d^4 - a^5*b*d^5) - d^5*log(abs(d*x + c))/(b
^5*c^5*d - 5*a*b^4*c^4*d^2 + 10*a^2*b^3*c^3*d^3 - 10*a^3*b^2*c^2*d^4 + 5*a
^4*b*c*d^5 - a^5*d^6) - 1/12*(3*b^4*c^4 - 16*a*b^3*c^3*d + 36*a^2*b^2*c^2*
d^2 - 48*a^3*b*c*d^3 + 25*a^4*d^4 - 12*(b^4*c*d^3 - a*b^3*d^4)*x^3 + 6*(b^
4*c^2*d^2 - 8*a*b^3*c*d^3 + 7*a^2*b^2*d^4)*x^2 - 4*(b^4*c^3*d - 6*a*b^3*c^
2*d^2 + 18*a^2*b^2*c*d^3 - 13*a^3*b*d^4)*x)/((b*c - a*d)^5*(b*x + a)^4)
```

Mupad [B] (verification not implemented)

Time = 5.70 (sec) , antiderivative size = 505, normalized size of antiderivative = 3.88

$$\int \frac{1}{(a+bx)^4 (ac+(bc+ad)x+bdx^2)}\,dx$$
$$= \frac{\frac{25\,a^3\,d^3-23\,a^2\,b\,c\,d^2+13\,a\,b^2\,c^2\,d-3\,b^3\,c^3}{12\,(a^4\,d^4-4\,a^3\,b\,c\,d^3+6\,a^2\,b^2\,c^2\,d^2-4\,a\,b^3\,c^3\,d+b^4\,c^4)} - \frac{d^2\,x^2\,(b^3\,c-7\,a\,b^2\,d)}{2\,(a^4\,d^4-4\,a^3\,b\,c\,d^3+6\,a^2\,b^2\,c^2\,d^2-4\,a\,b^3\,c^3\,d+b^4\,c^4)} + \frac{d\,x\,(13\,a^2\,b\,d^2-5\,a}{3\,(a^4\,d^4-4\,a^3\,b\,c\,d^3+6\,a^2\,b^2\,c}}{a^4+4\,a^3\,b\,x+6\,a^2\,b^2\,x^2+4\,a\,b^3\,x^3+b^4\,x^4}$$
$$- \frac{2\,d^4\,\mathrm{atanh}\left(\frac{a^5\,d^5-3\,a^4\,b\,c\,d^4+2\,a^3\,b^2\,c^2\,d^3+2\,a^2\,b^3\,c^3\,d^2-3\,a\,b^4\,c^4\,d+b^5\,c^5}{(a\,d-b\,c)^5} + \frac{2\,b\,d\,x\,(a^4\,d^4-4\,a^3\,b\,c\,d^3+6\,a^2\,b^2\,c^2\,d^2-4\,a\,b^3\,c^3\,d+b^4\,c^4)}{(a\,d-b\,c)^5}\right.}{(a\,d-b\,c)^5}$$

input
```
int(1/((a + b*x)^4*(a*c + x*(a*d + b*c) + b*d*x^2)),x)
```

output
```
((25*a^3*d^3 - 3*b^3*c^3 + 13*a*b^2*c^2*d - 23*a^2*b*c*d^2)/(12*(a^4*d^4 +
 b^4*c^4 + 6*a^2*b^2*c^2*d^2 - 4*a*b^3*c^3*d - 4*a^3*b*c*d^3)) - (d^2*x^2*
(b^3*c - 7*a*b^2*d))/(2*(a^4*d^4 + b^4*c^4 + 6*a^2*b^2*c^2*d^2 - 4*a*b^3*c
^3*d - 4*a^3*b*c*d^3)) + (d*x*(b^3*c^2 + 13*a^2*b*d^2 - 5*a*b^2*c*d))/(3*(
a^4*d^4 + b^4*c^4 + 6*a^2*b^2*c^2*d^2 - 4*a*b^3*c^3*d - 4*a^3*b*c*d^3)) +
(b^3*d^3*x^3)/(a^4*d^4 + b^4*c^4 + 6*a^2*b^2*c^2*d^2 - 4*a*b^3*c^3*d - 4*a
^3*b*c*d^3))/(a^4 + b^4*x^4 + 4*a*b^3*x^3 + 6*a^2*b^2*x^2 + 4*a^3*b*x) - (
2*d^4*atanh((a^5*d^5 + b^5*c^5 + 2*a^2*b^3*c^3*d^2 + 2*a^3*b^2*c^2*d^3 - 3
*a*b^4*c^4*d - 3*a^4*b*c*d^4)/(a*d - b*c)^5 + (2*b*d*x*(a^4*d^4 + b^4*c^4
+ 6*a^2*b^2*c^2*d^2 - 4*a*b^3*c^3*d - 4*a^3*b*c*d^3))/(a*d - b*c)^5))/(a*d
 - b*c)^5
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 769, normalized size of antiderivative = 5.92

$$\int \frac{1}{(a+bx)^4\,(ac+(bc+ad)x+bdx^2)}\,dx$$
$$= \frac{-12\log(bx+a)\,a^5d^4 - 48\log(bx+a)\,a^4b\,d^4x - 72\log(bx+a)\,a^3b^2d^4x^2 - 48\log(bx+a)\,a^2b^3d^4x^3 - 12}{12a\,(a^5b^4d^5x^4 - 5a^4b^5c\,d^4x^4 + 10a^3b^6c^2d^3x^4 - 10a^2b^7c^3d^2x^4 + 5a\,b^8c^4d\,x^4 - b^9c^5x^4 + 4a^6b^3d^5}$$

input
```
int(1/(b*x+a)^4/(a*c+(a*d+b*c)*x+b*d*x^2),x)
```

output
```
( - 12*log(a + b*x)*a**5*d**4 - 48*log(a + b*x)*a**4*b*d**4*x - 72*log(a +
 b*x)*a**3*b**2*d**4*x**2 - 48*log(a + b*x)*a**2*b**3*d**4*x**3 - 12*log(a
 + b*x)*a*b**4*d**4*x**4 + 12*log(c + d*x)*a**5*d**4 + 48*log(c + d*x)*a**
4*b*d**4*x + 72*log(c + d*x)*a**3*b**2*d**4*x**2 + 48*log(c + d*x)*a**2*b*
*3*d**4*x**3 + 12*log(c + d*x)*a*b**4*d**4*x**4 + 22*a**5*d**4 - 45*a**4*b
*c*d**3 + 40*a**4*b*d**4*x + 36*a**3*b**2*c**2*d**2 - 60*a**3*b**2*c*d**3*
x + 24*a**3*b**2*d**4*x**2 - 16*a**2*b**3*c**3*d + 24*a**2*b**3*c**2*d**2*
x - 30*a**2*b**3*c*d**3*x**2 + 3*a*b**4*c**4 - 4*a*b**4*c**3*d*x + 6*a*b**
4*c**2*d**2*x**2 - 3*a*b**4*d**4*x**4 + 3*b**5*c*d**3*x**4)/(12*a*(a**9*d*
*5 - 5*a**8*b*c*d**4 + 4*a**8*b*d**5*x + 10*a**7*b**2*c**2*d**3 - 20*a**7*
b**2*c*d**4*x + 6*a**7*b**2*d**5*x**2 - 10*a**6*b**3*c**3*d**2 + 40*a**6*b
**3*c**2*d**3*x - 30*a**6*b**3*c*d**4*x**2 + 4*a**6*b**3*d**5*x**3 + 5*a**
5*b**4*c**4*d - 40*a**5*b**4*c**3*d**2*x + 60*a**5*b**4*c**2*d**3*x**2 - 2
0*a**5*b**4*c*d**4*x**3 + a**5*b**4*d**5*x**4 - a**4*b**5*c**5 + 20*a**4*b
**5*c**4*d*x - 60*a**4*b**5*c**3*d**2*x**2 + 40*a**4*b**5*c**2*d**3*x**3 -
 5*a**4*b**5*c*d**4*x**4 - 4*a**3*b**6*c**5*x + 30*a**3*b**6*c**4*d*x**2 -
 40*a**3*b**6*c**3*d**2*x**3 + 10*a**3*b**6*c**2*d**3*x**4 - 6*a**2*b**7*c
**5*x**2 + 20*a**2*b**7*c**4*d*x**3 - 10*a**2*b**7*c**3*d**2*x**4 - 4*a*b*
*8*c**5*x**3 + 5*a*b**8*c**4*d*x**4 - b**9*c**5*x**4))
```

3.52 $\int \frac{(a+bx)^6}{\left(ac+(bc+ad)x+bdx^2\right)^2}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 104

$$\int \frac{(a+bx)^6}{\left(ac+(bc+ad)x+bdx^2\right)^2}\,dx = \frac{6b^2(bc-ad)^2x}{d^4}-\frac{(bc-ad)^4}{d^5(c+dx)}-\frac{2b^3(bc-ad)(c+dx)^2}{d^5}+\frac{b^4(c+dx)^3}{3d^5}-\frac{4b(bc-ad)^3\log(c+dx)}{d^5}$$

output
```
6*b^2*(-a*d+b*c)^2*x/d^4-(-a*d+b*c)^4/d^5/(d*x+c)-2*b^3*(-a*d+b*c)*(d*x+c)
^2/d^5+1/3*b^4*(d*x+c)^3/d^5-4*b*(-a*d+b*c)^3*ln(d*x+c)/d^5
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 165, normalized size of antiderivative = 1.59

$$\int \frac{(a+bx)^6}{\left(ac+(bc+ad)x+bdx^2\right)^2}\,dx = \frac{12a^3bcd^3-3a^4d^4+18a^2b^2d^2(-c^2+cdx+d^2x^2)+6ab^3d(2c^3-4c^2dx-3cd^2x^2+d^3x^3)+b^4(-3c^4+9c}{3d^5(c+dx)}$$

input
```
Integrate[(a + b*x)^6/(a*c + (b*c + a*d)*x + b*d*x^2)^2,x]
```

output
```
(12*a^3*b*c*d^3 - 3*a^4*d^4 + 18*a^2*b^2*d^2*(-c^2 + c*d*x + d^2*x^2) + 6*
a*b^3*d*(2*c^3 - 4*c^2*d*x - 3*c*d^2*x^2 + d^3*x^3) + b^4*(-3*c^4 + 9*c^3*
d*x + 6*c^2*d^2*x^2 - 2*c*d^3*x^3 + d^4*x^4) - 12*b*(b*c - a*d)^3*(c + d*x
)*Log[c + d*x])/(3*d^5*(c + d*x))
```

Rubi [A] (verified)

Time = 0.50 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.069$, Rules used = {1121, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx)^6}{(x(ad+bc)+ac+bdx^2)^2}\,dx$$

$$\Big\downarrow \text{ 1121}$$

$$\int \left(-\frac{4b^3(c+dx)(bc-ad)}{d^4}+\frac{6b^2(bc-ad)^2}{d^4}-\frac{4b(bc-ad)^3}{d^4(c+dx)}+\frac{(ad-bc)^4}{d^4(c+dx)^2}+\frac{b^4(c+dx)^2}{d^4}\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$-\frac{2b^3(c+dx)^2(bc-ad)}{d^5}+\frac{6b^2x(bc-ad)^2}{d^4}-\frac{(bc-ad)^4}{d^5(c+dx)}-\frac{4b(bc-ad)^3\log(c+dx)}{d^5}+\frac{b^4(c+dx)^3}{3d^5}$$

input
```
Int[(a + b*x)^6/(a*c + (b*c + a*d)*x + b*d*x^2)^2,x]
```

output
```
(6*b^2*(b*c - a*d)^2*x)/d^4 - (b*c - a*d)^4/(d^5*(c + d*x)) - (2*b^3*(b*c
- a*d)*(c + d*x)^2)/d^5 + (b^4*(c + d*x)^3)/(3*d^5) - (4*b*(b*c - a*d)^3*L
og[c + d*x])/d^5
```

Defintions of rubi rules used

rule 1121
```
Int[((d_) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Int[ExpandIntegrand[(d + e*x)^(m + p)*(a/d + (c/e)*x)^p, x], x]
/; FreeQ[{a, b, c, d, e, m, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && (Int
egerQ[p] || (GtQ[a, 0] && GtQ[d, 0] && LtQ[c, 0]))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 1.46 (sec) , antiderivative size = 175, normalized size of antiderivative = 1.68

method	result
default	$\frac{b^2\left(\frac{1}{3}x^3b^2d^2+2ab\,d^2x^2-b^2c\,x^2d+6a^2d^2x-8abcdx+3x\,b^2c^2\right)}{d^4}-\frac{a^4d^4-4a^3bc\,d^3+6a^2b^2c^2d^2-4a\,b^3c^3d+c^4b^4}{d^5(dx+c)}+\frac{4b\left(a^3d^3-3\right.}{}$
risch	$\frac{b^4x^3}{3d^2}+\frac{2b^3a\,x^2}{d^2}-\frac{b^4c\,x^2}{d^3}+\frac{6b^2a^2x}{d^2}-\frac{8b^3acx}{d^3}+\frac{3b^4x\,c^2}{d^4}-\frac{a^4}{d(dx+c)}+\frac{4a^3bc}{d^2(dx+c)}-\frac{6a^2b^2c^2}{d^3(dx+c)}+\frac{4a\,b^3c^3}{d^4(dx+c)}-$
norman	$\frac{\frac{b^5x^5}{3d}+\frac{2b^3\left(12a^2d^2-10abcd+3b^2c^2\right)x^3}{3d^3}+\frac{b^4(7ad-2bc)x^4}{3d^2}-\frac{a\left(a^4b\,d^4+2a^3b^2c\,d^3+6a^2b^3c^2d^2-10a\,b^4c^3d+4b^5c^4\right)}{d^5b}-\frac{\left(7a^4b^2d^4-4d^3c\,a^3b^3\right.}{}}{(bx+a)(dx+c)}$
parallelrisch	$\frac{b^4d^4x^4+6a\,b^3d^4x^3-2b^4c\,d^3x^3+12\ln(dx+c)x\,a^3b\,d^4-36\ln(dx+c)x\,a^2b^2c\,d^3+36\ln(dx+c)xa\,b^3c^2d^2-12\ln(dx+c)x\,b^4c^3d+18}{}$

input
```
int((b*x+a)^6/(a*c+(a*d+b*c)*x+b*d*x^2)^2,x,method=_RETURNVERBOSE)
```

output
```
b^2/d^4*(1/3*x^3*b^2*d^2+2*a*b*d^2*x^2-b^2*c*x^2*d+6*a^2*d^2*x-8*a*b*c*d*x
+3*x*b^2*c^2)-(a^4*d^4-4*a^3*b*c*d^3+6*a^2*b^2*c^2*d^2-4*a*b^3*c^3*d+b^4*c
^4)/d^5/(d*x+c)+4*b/d^5*(a^3*d^3-3*a^2*b*c*d^2+3*a*b^2*c^2*d-b^3*c^3)*ln(d
*x+c)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 267 vs. $2(102) = 204$.

Time = 0.08 (sec) , antiderivative size = 267, normalized size of antiderivative = 2.57

$$\int \frac{(a+bx)^6}{(ac+(bc+ad)x+bdx^2)^2}\,dx$$
$$= \frac{b^4d^4x^4 - 3\,b^4c^4 + 12\,ab^3c^3d - 18\,a^2b^2c^2d^2 + 12\,a^3bcd^3 - 3\,a^4d^4 - 2\,(b^4cd^3 - 3\,ab^3d^4)x^3 + 6\,(b^4c^2d^2 - 3\,ab}{}$$

input
```
integrate((b*x+a)^6/(a*c+(a*d+b*c)*x+b*d*x^2)^2,x, algorithm="fricas")
```

output
```
1/3*(b^4*d^4*x^4 - 3*b^4*c^4 + 12*a*b^3*c^3*d - 18*a^2*b^2*c^2*d^2 + 12*a^
3*b*c*d^3 - 3*a^4*d^4 - 2*(b^4*c*d^3 - 3*a*b^3*d^4)*x^3 + 6*(b^4*c^2*d^2 -
 3*a*b^3*c*d^3 + 3*a^2*b^2*d^4)*x^2 + 3*(3*b^4*c^3*d - 8*a*b^3*c^2*d^2 + 6
*a^2*b^2*c*d^3)*x - 12*(b^4*c^4 - 3*a*b^3*c^3*d + 3*a^2*b^2*c^2*d^2 - a^3*
b*c*d^3 + (b^4*c^3*d - 3*a*b^3*c^2*d^2 + 3*a^2*b^2*c*d^3 - a^3*b*d^4)*x)*l
og(d*x + c))/(d^6*x + c*d^5)
```

Sympy [A] (verification not implemented)

Time = 0.54 (sec) , antiderivative size = 155, normalized size of antiderivative = 1.49

$$\int \frac{(a+bx)^6}{(ac+(bc+ad)x+bdx^2)^2}\,dx = \frac{b^4x^3}{3d^2} + \frac{4b(ad-bc)^3\log{(c+dx)}}{d^5} + x^2 \cdot \left(\frac{2ab^3}{d^2} - \frac{b^4c}{d^3}\right) + x\left(\frac{6a^2b^2}{d^2} - \frac{8ab^3c}{d^3} + \frac{3b^4c^2}{d^4}\right) + \frac{-a^4d^4 + 4a^3bcd^3 - 6a^2b^2c^2d^2 + 4ab^3c^3d - b^4c^4}{cd^5 + d^6x}$$

input
```
integrate((b*x+a)**6/(a*c+(a*d+b*c)*x+b*d*x**2)**2,x)
```

output
```
b**4*x**3/(3*d**2) + 4*b*(a*d - b*c)**3*log(c + d*x)/d**5 + x**2*(2*a*b**3
/d**2 - b**4*c/d**3) + x*(6*a**2*b**2/d**2 - 8*a*b**3*c/d**3 + 3*b**4*c**2
/d**4) + (-a**4*d**4 + 4*a**3*b*c*d**3 - 6*a**2*b**2*c**2*d**2 + 4*a*b**3*
c**3*d - b**4*c**4)/(c*d**5 + d**6*x)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 183, normalized size of antiderivative = 1.76

$$\int \frac{(a+bx)^6}{(ac+(bc+ad)x+bdx^2)^2}\,dx = -\frac{b^4c^4-4\,ab^3c^3d+6\,a^2b^2c^2d^2-4\,a^3bcd^3+a^4d^4}{d^6x+cd^5} + \frac{b^4d^2x^3-3\,(b^4cd-2\,ab^3d^2)x^2+3\,(3\,b^4c^2-8\,ab^3cd+6\,a^2b^2d^2)x}{3\,d^4} - \frac{4\,(b^4c^3-3\,ab^3c^2d+3\,a^2b^2cd^2-a^3bd^3)\log(dx+c)}{d^5}$$

input
```
integrate((b*x+a)^6/(a*c+(a*d+b*c)*x+b*d*x^2)^2,x, algorithm="maxima")
```

output
```
-(b^4*c^4 - 4*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 - 4*a^3*b*c*d^3 + a^4*d^4)/(
d^6*x + c*d^5) + 1/3*(b^4*d^2*x^3 - 3*(b^4*c*d - 2*a*b^3*d^2)*x^2 + 3*(3*b
^4*c^2 - 8*a*b^3*c*d + 6*a^2*b^2*d^2)*x)/d^4 - 4*(b^4*c^3 - 3*a*b^3*c^2*d
+ 3*a^2*b^2*c*d^2 - a^3*b*d^3)*log(d*x + c)/d^5
```

Giac [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 188, normalized size of antiderivative = 1.81

$$\int \frac{(a+bx)^6}{(ac+(bc+ad)x+bdx^2)^2}\,dx = -\frac{4\,(b^4c^3-3\,ab^3c^2d+3\,a^2b^2cd^2-a^3bd^3)\log(|dx+c|)}{d^5} + \frac{b^4d^4x^3-3\,b^4cd^3x^2+6\,ab^3d^4x^2+9\,b^4c^2d^2x-24\,ab^3cd^3x+18\,a^2b^2d^4x}{3\,d^6} - \frac{b^4c^4-4\,ab^3c^3d+6\,a^2b^2c^2d^2-4\,a^3bcd^3+a^4d^4}{(dx+c)d^5}$$

input
```
integrate((b*x+a)^6/(a*c+(a*d+b*c)*x+b*d*x^2)^2,x, algorithm="giac")
```

output
```
-4*(b^4*c^3 - 3*a*b^3*c^2*d + 3*a^2*b^2*c*d^2 - a^3*b*d^3)*log(abs(d*x + c
))/d^5 + 1/3*(b^4*d^4*x^3 - 3*b^4*c*d^3*x^2 + 6*a*b^3*d^4*x^2 + 9*b^4*c^2*
d^2*x - 24*a*b^3*c*d^3*x + 18*a^2*b^2*d^4*x)/d^6 - (b^4*c^4 - 4*a*b^3*c^3*
d + 6*a^2*b^2*c^2*d^2 - 4*a^3*b*c*d^3 + a^4*d^4)/((d*x + c)*d^5)
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 203, normalized size of antiderivative = 1.95

$$\int \frac{(a+bx)^6}{(ac+(bc+ad)x+bdx^2)^2}\,dx = x^2\left(\frac{2\,a\,b^3}{d^2}-\frac{b^4\,c}{d^3}\right)-x\left(\frac{2\,c\,\left(\frac{4\,a\,b^3}{d^2}-\frac{2\,b^4\,c}{d^3}\right)}{d}-\frac{6\,a^2\,b^2}{d^2}+\frac{b^4\,c^2}{d^4}\right)+\frac{b^4\,x^3}{3\,d^2}-\frac{\ln\left(c+d\,x\right)\,\left(-4\,a^3\,b\,d^3+12\,a^2\,b^2\,c\,d^2-12\,a\,b^3\,c^2\,d+4\,b^4\,c^3\right)}{d^5}-\frac{a^4\,d^4-4\,a^3\,b\,c\,d^3+6\,a^2\,b^2\,c^2\,d^2-4\,a\,b^3\,c^3\,d+b^4\,c^4}{d\,\left(x\,d^5+c\,d^4\right)}$$

input
```
int((a + b*x)^6/(a*c + x*(a*d + b*c) + b*d*x^2)^2,x)
```

output
```
x^2*((2*a*b^3)/d^2 - (b^4*c)/d^3) - x*((2*c*((4*a*b^3)/d^2 - (2*b^4*c)/d^3
))/d - (6*a^2*b^2)/d^2 + (b^4*c^2)/d^4) + (b^4*x^3)/(3*d^2) - (log(c + d*x
)*(4*b^4*c^3 - 4*a^3*b*d^3 + 12*a^2*b^2*c*d^2 - 12*a*b^3*c^2*d))/d^5 - (a^
4*d^4 + b^4*c^4 + 6*a^2*b^2*c^2*d^2 - 4*a*b^3*c^3*d - 4*a^3*b*c*d^3)/(d*(c
*d^4 + d^5*x))
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 297, normalized size of antiderivative = 2.86

$$\int \frac{(a+bx)^6}{(ac+(bc+ad)x+bdx^2)^2}\,dx = \frac{12\log(dx+c)\,a^3b\,c^2d^3+12\log(dx+c)\,a^3bc\,d^4x-36\log(dx+c)\,a^2b^2c^3d^2-36\log(dx+c)\,a^2b^2c^2d^3x+}{}$$

input
```
int((b*x+a)^6/(a*c+(a*d+b*c)*x+b*d*x^2)^2,x)
```

output
```
(12*log(c + d*x)*a**3*b*c**2*d**3 + 12*log(c + d*x)*a**3*b*c*d**4*x - 36*l
og(c + d*x)*a**2*b**2*c**3*d**2 - 36*log(c + d*x)*a**2*b**2*c**2*d**3*x +
36*log(c + d*x)*a*b**3*c**4*d + 36*log(c + d*x)*a*b**3*c**3*d**2*x - 12*lo
g(c + d*x)*b**4*c**5 - 12*log(c + d*x)*b**4*c**4*d*x + 3*a**4*d**5*x - 12*
a**3*b*c*d**4*x + 36*a**2*b**2*c**2*d**3*x + 18*a**2*b**2*c*d**4*x**2 - 36
*a*b**3*c**3*d**2*x - 18*a*b**3*c**2*d**3*x**2 + 6*a*b**3*c*d**4*x**3 + 12
*b**4*c**4*d*x + 6*b**4*c**3*d**2*x**2 - 2*b**4*c**2*d**3*x**3 + b**4*c*d*
*4*x**4)/(3*c*d**5*(c + d*x))
```

3.53 $\int \frac{(a+bx)^5}{(ac+(bc+ad)x+bdx^2)^2} \, dx$

Optimal result

Integrand size = 29, antiderivative size = 75

$$\int \frac{(a+bx)^5}{(ac+(bc+ad)x+bdx^2)^2} \, dx = -\frac{b^2(2bc-3ad)x}{d^3} + \frac{b^3x^2}{2d^2} + \frac{(bc-ad)^3}{d^4(c+dx)} + \frac{3b(bc-ad)^2\log(c+dx)}{d^4}$$

output
```
-b^2*(-3*a*d+2*b*c)*x/d^3+1/2*b^3*x^2/d^2+(-a*d+b*c)^3/d^4/(d*x+c)+3*b*(-a
*d+b*c)^2*ln(d*x+c)/d^4
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.52

$$\int \frac{(a+bx)^5}{(ac+(bc+ad)x+bdx^2)^2} \, dx = -\frac{b^2(2bc-3ad)x}{d^3} + \frac{b^3x^2}{2d^2} + \frac{b^3c^3-3ab^2c^2d+3a^2bcd^2-a^3d^3}{d^4(c+dx)} + \frac{3(b^3c^2-2ab^2cd+a^2bd^2)\log(c+dx)}{d^4}$$

input
```
Integrate[(a + b*x)^5/(a*c + (b*c + a*d)*x + b*d*x^2)^2,x]
```

output
```
-((b^2*(2*b*c - 3*a*d)*x)/d^3) + (b^3*x^2)/(2*d^2) + (b^3*c^3 - 3*a*b^2*c^
2*d + 3*a^2*b*c*d^2 - a^3*d^3)/(d^4*(c + d*x)) + (3*(b^3*c^2 - 2*a*b^2*c*d
 + a^2*b*d^2)*Log[c + d*x])/d^4
```

Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.069$, Rules used = {1121, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx)^5}{(x(ad+bc)+ac+bdx^2)^2}\,dx$$

$$\Big\downarrow \text{1121}$$

$$\int \left(-\frac{b^2(2bc-3ad)}{d^3}+\frac{3b(bc-ad)^2}{d^3(c+dx)}+\frac{(ad-bc)^3}{d^3(c+dx)^2}+\frac{b^3x}{d^2}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{b^2x(2bc-3ad)}{d^3}+\frac{(bc-ad)^3}{d^4(c+dx)}+\frac{3b(bc-ad)^2\log(c+dx)}{d^4}+\frac{b^3x^2}{2d^2}$$

input
```
Int[(a + b*x)^5/(a*c + (b*c + a*d)*x + b*d*x^2)^2,x]
```

output
```
-((b^2*(2*b*c - 3*a*d)*x)/d^3) + (b^3*x^2)/(2*d^2) + (b*c - a*d)^3/(d^4*(c
 + d*x)) + (3*b*(b*c - a*d)^2*Log[c + d*x])/d^4
```

Defintions of rubi rules used

rule 1121
```
Int[((d_) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Int[ExpandIntegrand[(d + e*x)^(m + p)*(a/d + (c/e)*x)^p, x], x]
/; FreeQ[{a, b, c, d, e, m, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && (Int
egerQ[p] || (GtQ[a, 0] && GtQ[d, 0] && LtQ[c, 0]))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 1.40 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.44

method	result
default	$\frac{b^2\left(\frac{1}{2}bd\,x^2+3adx-2cbx\right)}{d^3}-\frac{a^3d^3-3a^2bc\,d^2+3a\,b^2c^2d-b^3c^3}{d^4(dx+c)}+\frac{3b\left(a^2d^2-2abcd+b^2c^2\right)\ln(dx+c)}{d^4}$
risch	$\frac{b^3x^2}{2d^2}+\frac{3b^2ax}{d^2}-\frac{2b^3cx}{d^3}-\frac{a^3}{d(dx+c)}+\frac{3a^2bc}{d^2(dx+c)}-\frac{3a\,b^2c^2}{d^3(dx+c)}+\frac{b^3c^3}{d^4(dx+c)}+\frac{3b\ln(dx+c)a^2}{d^2}-\frac{6b^2\ln(dx+c)ac}{d^3}+$
norman	$\frac{\frac{b^4x^4}{2d}+\frac{b^3(7ad-3bc)x^3}{2d^2}-\frac{a\left(2a^3b\,d^3+9a\,b^3c^2d-6b^4c^3\right)}{2d^4b}-\frac{\left(8d^3a^3b^2-3c\,d^2a^2b^3+9c^2da\,b^4-6b^5c^3\right)x}{2d^4b}}{(bx+a)(dx+c)}+\frac{3b\left(a^2d^2-2abcd+b^2c^2\right)\ln(dx}{d^4}$
parallelrisch	$\frac{x^3b^3d^3+6\ln(dx+c)x\,a^2b\,d^3-12\ln(dx+c)xa\,b^2c\,d^2+6\ln(dx+c)x\,b^3c^2d+6a\,b^2d^3x^2-3x^2b^3c\,d^2+6\ln(dx+c)a^2bc\,d^2-12\ln(dx}{2d^4(dx+c)}$

input
```
int((b*x+a)^5/(a*c+(a*d+b*c)*x+b*d*x^2)^2,x,method=_RETURNVERBOSE)
```

output
```
b^2/d^3*(1/2*b*d*x^2+3*a*d*x-2*c*b*x)-(a^3*d^3-3*a^2*b*c*d^2+3*a*b^2*c^2*d
-b^3*c^3)/d^4/(d*x+c)+3*b/d^4*(a^2*d^2-2*a*b*c*d+b^2*c^2)*ln(d*x+c)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 172 vs. 2(73) = 146.

Time = 0.08 (sec) , antiderivative size = 172, normalized size of antiderivative = 2.29

$$\int \frac{(a+bx)^5}{(ac+(bc+ad)x+bdx^2)^2}\,dx$$
$$=\frac{b^3d^3x^3+2\,b^3c^3-6\,ab^2c^2d+6\,a^2bcd^2-2\,a^3d^3-3\,(b^3cd^2-2\,ab^2d^3)x^2-2\,(2\,b^3c^2d-3\,ab^2cd^2)x+6\,(b^3c}{2\,(d^5x+cd^4)}$$

input
```
integrate((b*x+a)^5/(a*c+(a*d+b*c)*x+b*d*x^2)^2,x, algorithm="fricas")
```

output
```
1/2*(b^3*d^3*x^3 + 2*b^3*c^3 - 6*a*b^2*c^2*d + 6*a^2*b*c*d^2 - 2*a^3*d^3 -
 3*(b^3*c*d^2 - 2*a*b^2*d^3)*x^2 - 2*(2*b^3*c^2*d - 3*a*b^2*c*d^2)*x + 6*(
b^3*c^3 - 2*a*b^2*c^2*d + a^2*b*c*d^2 + (b^3*c^2*d - 2*a*b^2*c*d^2 + a^2*b
*d^3)*x)*log(d*x + c))/(d^5*x + c*d^4)
```

Sympy [A] (verification not implemented)

Time = 0.38 (sec) , antiderivative size = 102, normalized size of antiderivative = 1.36

$$\int \frac{(a+bx)^5}{(ac+(bc+ad)x+bdx^2)^2}\,dx = \frac{b^3x^2}{2d^2} + \frac{3b(ad-bc)^2\log(c+dx)}{d^4} + x\left(\frac{3ab^2}{d^2} - \frac{2b^3c}{d^3}\right) + \frac{-a^3d^3+3a^2bcd^2-3ab^2c^2d+b^3c^3}{cd^4+d^5x}$$

input
```
integrate((b*x+a)**5/(a*c+(a*d+b*c)*x+b*d*x**2)**2,x)
```

output
```
b**3*x**2/(2*d**2) + 3*b*(a*d - b*c)**2*log(c + d*x)/d**4 + x*(3*a*b**2/d*
*2 - 2*b**3*c/d**3) + (-a**3*d**3 + 3*a**2*b*c*d**2 - 3*a*b**2*c**2*d + b*
*3*c**3)/(c*d**4 + d**5*x)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.56

$$\int \frac{(a+bx)^5}{(ac+(bc+ad)x+bdx^2)^2}\,dx = \frac{b^3c^3-3\,ab^2c^2d+3\,a^2bcd^2-a^3d^3}{d^5x+cd^4} + \frac{b^3dx^2-2\,(2\,b^3c-3\,ab^2d)x}{2\,d^3} + \frac{3\,(b^3c^2-2\,ab^2cd+a^2bd^2)\log(dx+c)}{d^4}$$

input
```
integrate((b*x+a)^5/(a*c+(a*d+b*c)*x+b*d*x^2)^2,x, algorithm="maxima")
```

output
```
(b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)/(d^5*x + c*d^4) + 1/2*
(b^3*d*x^2 - 2*(2*b^3*c - 3*a*b^2*d)*x)/d^3 + 3*(b^3*c^2 - 2*a*b^2*c*d + a
^2*b*d^2)*log(d*x + c)/d^4
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 118, normalized size of antiderivative = 1.57

$$\int \frac{(a+bx)^5}{(ac+(bc+ad)x+bdx^2)^2}\,dx = \frac{3\left(b^3c^2-2\,ab^2cd+a^2bd^2\right)\log\left(|dx+c|\right)}{d^4} + \frac{b^3d^2x^2-4\,b^3cdx+6\,ab^2d^2x}{2\,d^4} + \frac{b^3c^3-3\,ab^2c^2d+3\,a^2bcd^2-a^3d^3}{(dx+c)d^4}$$

input
```
integrate((b*x+a)^5/(a*c+(a*d+b*c)*x+b*d*x^2)^2,x, algorithm="giac")
```

output
```
3*(b^3*c^2 - 2*a*b^2*c*d + a^2*b*d^2)*log(abs(d*x + c))/d^4 + 1/2*(b^3*d^2
*x^2 - 4*b^3*c*d*x + 6*a*b^2*d^2*x)/d^4 + (b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2
*b*c*d^2 - a^3*d^3)/((d*x + c)*d^4)
```

Mupad [B] (verification not implemented)

Time = 5.83 (sec) , antiderivative size = 123, normalized size of antiderivative = 1.64

$$\int \frac{(a+bx)^5}{(ac+(bc+ad)x+bdx^2)^2}\,dx = x\left(\frac{3\,a\,b^2}{d^2}-\frac{2\,b^3\,c}{d^3}\right) + \frac{\ln\left(c+d\,x\right)\,\left(3\,a^2\,b\,d^2-6\,a\,b^2\,c\,d+3\,b^3\,c^2\right)}{d^4} - \frac{a^3\,d^3-3\,a^2\,b\,c\,d^2+3\,a\,b^2\,c^2\,d-b^3\,c^3}{d\,\left(x\,d^4+c\,d^3\right)} + \frac{b^3\,x^2}{2\,d^2}$$

input
```
int((a + b*x)^5/(a*c + x*(a*d + b*c) + b*d*x^2)^2,x)
```

output
```
x*((3*a*b^2)/d^2 - (2*b^3*c)/d^3) + (log(c + d*x)*(3*b^3*c^2 + 3*a^2*b*d^2
 - 6*a*b^2*c*d))/d^4 - (a^3*d^3 - b^3*c^3 + 3*a*b^2*c^2*d - 3*a^2*b*c*d^2)
/(d*(c*d^3 + d^4*x)) + (b^3*x^2)/(2*d^2)
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 197, normalized size of antiderivative = 2.63

$$\int \frac{(a+bx)^5}{(ac+(bc+ad)x+bdx^2)^2}\,dx$$
$$= \frac{6\log(dx+c)\,a^2b\,c^2d^2 + 6\log(dx+c)\,a^2bc\,d^3x - 12\log(dx+c)\,a\,b^2c^3d - 12\log(dx+c)\,a\,b^2c^2d^2x + 6\log}{}$$

input
```
int((b*x+a)^5/(a*c+(a*d+b*c)*x+b*d*x^2)^2,x)
```

output
```
(6*log(c + d*x)*a**2*b*c**2*d**2 + 6*log(c + d*x)*a**2*b*c*d**3*x - 12*log
(c + d*x)*a*b**2*c**3*d - 12*log(c + d*x)*a*b**2*c**2*d**2*x + 6*log(c + d
*x)*b**3*c**4 + 6*log(c + d*x)*b**3*c**3*d*x + 2*a**3*d**4*x - 6*a**2*b*c*
d**3*x + 12*a*b**2*c**2*d**2*x + 6*a*b**2*c*d**3*x**2 - 6*b**3*c**3*d*x -
3*b**3*c**2*d**2*x**2 + b**3*c*d**3*x**3)/(2*c*d**4*(c + d*x))
```

3.54 $\int \frac{(a+bx)^4}{(ac+(bc+ad)x+bdx^2)^2} dx$

Optimal result

Integrand size = 29, antiderivative size = 51

$$\int \frac{(a+bx)^4}{(ac+(bc+ad)x+bdx^2)^2} dx = \frac{b^2x}{d^2} - \frac{(bc-ad)^2}{d^3(c+dx)} - \frac{2b(bc-ad)\log(c+dx)}{d^3}$$

output
```
b^2*x/d^2-(-a*d+b*c)^2/d^3/(d*x+c)-2*b*(-a*d+b*c)*ln(d*x+c)/d^3
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.92

$$\int \frac{(a+bx)^4}{(ac+(bc+ad)x+bdx^2)^2} dx = \frac{b^2dx - \frac{(bc-ad)^2}{c+dx} + 2b(-bc+ad)\log(c+dx)}{d^3}$$

input
```
Integrate[(a + b*x)^4/(a*c + (b*c + a*d)*x + b*d*x^2)^2,x]
```

output
```
(b^2*d*x - (b*c - a*d)^2/(c + d*x) + 2*b*(-(b*c) + a*d)*Log[c + d*x])/d^3
```

Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.069, Rules used = {1121, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx)^4}{(x(ad+bc)+ac+bdx^2)^2}\,dx$$

$$\Big\downarrow \text{1121}$$

$$\int \left(-\frac{2b(bc-ad)}{d^2(c+dx)}+\frac{(ad-bc)^2}{d^2(c+dx)^2}+\frac{b^2}{d^2}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{(bc-ad)^2}{d^3(c+dx)}-\frac{2b(bc-ad)\log(c+dx)}{d^3}+\frac{b^2x}{d^2}$$

input
```
Int[(a + b*x)^4/(a*c + (b*c + a*d)*x + b*d*x^2)^2,x]
```

output
```
(b^2*x)/d^2 - (b*c - a*d)^2/(d^3*(c + d*x)) - (2*b*(b*c - a*d)*Log[c + d*x
])/d^3
```

Defintions of rubi rules used

rule 1121
```
Int[((d_) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Int[ExpandIntegrand[(d + e*x)^(m + p)*(a/d + (c/e)*x)^p, x], x]
/; FreeQ[{a, b, c, d, e, m, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && (Int
egerQ[p] || (GtQ[a, 0] && GtQ[d, 0] && LtQ[c, 0]))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 1.46 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.24

method	result	size
default	$\frac{b^{2}x}{d^{2}}-\frac{a^{2}d^{2}-2abcd+b^{2}c^{2}}{d^{3}(dx+c)}+\frac{2b(ad-bc)\ln(dx+c)}{d^{3}}$	63
risch	$\frac{b^{2}x}{d^{2}}-\frac{a^{2}}{d(dx+c)}+\frac{2abc}{d^{2}(dx+c)}-\frac{b^{2}c^{2}}{d^{3}(dx+c)}+\frac{2b\ln(dx+c)a}{d^{2}}-\frac{2b^{2}c\ln(dx+c)}{d^{3}}$	86
parallelrisch	$\frac{2\ln(dx+c)xab\,d^{2}-2\ln(dx+c)x\,b^{2}cd+b^{2}d^{2}x^{2}+2\ln(dx+c)abcd-2\ln(dx+c)b^{2}c^{2}-a^{2}d^{2}+2abcd-2b^{2}c^{2}}{d^{3}(dx+c)}$	99
norman	$\frac{\frac{b^{3}x^{3}}{d}-\frac{a\left(a^{2}b\,d^{2}-cda\,b^{2}+2b^{3}c^{2}\right)}{d^{3}b}-\frac{\left(2d^{2}a^{2}b^{2}-cda\,b^{3}+2b^{4}c^{2}\right)x}{d^{3}b}}{(bx+a)(dx+c)}+\frac{2b(ad-bc)\ln(dx+c)}{d^{3}}$	119

input
```
int((b*x+a)^4/(a*c+(a*d+b*c)*x+b*d*x^2)^2,x,method=_RETURNVERBOSE)
```

output
```
b^2*x/d^2-(a^2*d^2-2*a*b*c*d+b^2*c^2)/d^3/(d*x+c)+2*b/d^3*(a*d-b*c)*ln(d*x
+c)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.80

$$\int \frac{(a+bx)^4}{(ac+(bc+ad)x+bdx^2)^2}\,dx$$
$$=\frac{b^2d^2x^2+b^2cdx-b^2c^2+2\,abcd-a^2d^2-2\,(b^2c^2-abcd+(b^2cd-abd^2)x)\log(dx+c)}{d^4x+cd^3}$$

input
```
integrate((b*x+a)^4/(a*c+(a*d+b*c)*x+b*d*x^2)^2,x, algorithm="fricas")
```

output
```
(b^2*d^2*x^2 + b^2*c*d*x - b^2*c^2 + 2*a*b*c*d - a^2*d^2 - 2*(b^2*c^2 - a*
b*c*d + (b^2*c*d - a*b*d^2)*x)*log(d*x + c))/(d^4*x + c*d^3)
```

Sympy [A] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.18

$$\int \frac{(a+bx)^4}{(ac+(bc+ad)x+bdx^2)^2}\,dx = \frac{b^2x}{d^2} + \frac{2b(ad-bc)\log(c+dx)}{d^3} + \frac{-a^2d^2+2abcd-b^2c^2}{cd^3+d^4x}$$

input
```
integrate((b*x+a)**4/(a*c+(a*d+b*c)*x+b*d*x**2)**2,x)
```

output
```
b**2*x/d**2 + 2*b*(a*d - b*c)*log(c + d*x)/d**3 + (-a**2*d**2 + 2*a*b*c*d
- b**2*c**2)/(c*d**3 + d**4*x)
```

Maxima [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.31

$$\int \frac{(a+bx)^4}{(ac+(bc+ad)x+bdx^2)^2}\,dx = \frac{b^2x}{d^2} - \frac{b^2c^2-2\,abcd+a^2d^2}{d^4x+cd^3} - \frac{2\,(b^2c-abd)\log(dx+c)}{d^3}$$

input
```
integrate((b*x+a)^4/(a*c+(a*d+b*c)*x+b*d*x^2)^2,x, algorithm="maxima")
```

output
```
b^2*x/d^2 - (b^2*c^2 - 2*a*b*c*d + a^2*d^2)/(d^4*x + c*d^3) - 2*(b^2*c - a
*b*d)*log(d*x + c)/d^3
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.27

$$\int \frac{(a+bx)^4}{(ac+(bc+ad)x+bdx^2)^2}\,dx = \frac{b^2x}{d^2} - \frac{2\,(b^2c-abd)\log(|dx+c|)}{d^3} - \frac{b^2c^2-2\,abcd+a^2d^2}{(dx+c)d^3}$$

input
```
integrate((b*x+a)^4/(a*c+(a*d+b*c)*x+b*d*x^2)^2,x, algorithm="giac")
```

output
```
b^2*x/d^2 - 2*(b^2*c - a*b*d)*log(abs(d*x + c))/d^3 - (b^2*c^2 - 2*a*b*c*d
 + a^2*d^2)/((d*x + c)*d^3)
```

Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.39

$$\int \frac{(a+bx)^4}{(ac+(bc+ad)x+bdx^2)^2}\,dx = \frac{b^2\,x}{d^2} - \frac{a^2\,d^2-2\,a\,b\,c\,d+b^2\,c^2}{d\,\left(x\,d^3+c\,d^2\right)} - \frac{\ln\left(c+d\,x\right)\,\left(2\,b^2\,c-2\,a\,b\,d\right)}{d^3}$$

input
```
int((a + b*x)^4/(a*c + x*(a*d + b*c) + b*d*x^2)^2,x)
```

output
```
(b^2*x)/d^2 - (a^2*d^2 + b^2*c^2 - 2*a*b*c*d)/(d*(c*d^2 + d^3*x)) - (log(c
 + d*x)*(2*b^2*c - 2*a*b*d))/d^3
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 112, normalized size of antiderivative = 2.20

$$\int \frac{(a+bx)^4}{(ac+(bc+ad)x+bdx^2)^2}\,dx = \frac{2\log(dx+c)\,ab\,c^2d+2\log(dx+c)\,abc\,d^2x-2\log(dx+c)\,b^2c^3-2\log(dx+c)\,b^2c^2dx+a^2d^3x-2abc\,d}{c\,d^3\,(dx+c)}$$

input
```
int((b*x+a)^4/(a*c+(a*d+b*c)*x+b*d*x^2)^2,x)
```

output
```
(2*log(c + d*x)*a*b*c**2*d + 2*log(c + d*x)*a*b*c*d**2*x - 2*log(c + d*x)*
b**2*c**3 - 2*log(c + d*x)*b**2*c**2*d*x + a**2*d**3*x - 2*a*b*c*d**2*x +
2*b**2*c**2*d*x + b**2*c*d**2*x**2)/(c*d**3*(c + d*x))
```

3.55 $\int \frac{(a+bx)^3}{(ac+(bc+ad)x+bdx^2)^2} dx$

Optimal result

Integrand size = 29, antiderivative size = 31

$$\int \frac{(a+bx)^3}{(ac+(bc+ad)x+bdx^2)^2} dx = \frac{bc-ad}{d^2(c+dx)} + \frac{b\log(c+dx)}{d^2}$$

output
```
(-a*d+b*c)/d^2/(d*x+c)+b*ln(d*x+c)/d^2
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.00

$$\int \frac{(a+bx)^3}{(ac+(bc+ad)x+bdx^2)^2} dx = \frac{bc-ad}{d^2(c+dx)} + \frac{b\log(c+dx)}{d^2}$$

input
```
Integrate[(a + b*x)^3/(a*c + (b*c + a*d)*x + b*d*x^2)^2,x]
```

output
```
(b*c - a*d)/(d^2*(c + d*x)) + (b*Log[c + d*x])/d^2
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.069$, Rules used = $\{1121, 2009\}$

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx)^3}{(x(ad+bc)+ac+bdx^2)^2}dx$$
$$\Big\downarrow \text{1121}$$
$$\int \left(\frac{ad-bc}{d(c+dx)^2}+\frac{b}{d(c+dx)}\right)dx$$
$$\Big\downarrow \text{2009}$$
$$\frac{bc-ad}{d^2(c+dx)}+\frac{b\log(c+dx)}{d^2}$$

input
```
Int[(a + b*x)^3/(a*c + (b*c + a*d)*x + b*d*x^2)^2,x]
```

output
```
(b*c - a*d)/(d^2*(c + d*x)) + (b*Log[c + d*x])/d^2
```

Defintions of rubi rules used

rule 1121
```
Int[((d_) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Int[ExpandIntegrand[(d + e*x)^(m + p)*(a/d + (c/e)*x)^p, x], x]
/; FreeQ[{a, b, c, d, e, m, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && (Int
egerQ[p] || (GtQ[a, 0] && GtQ[d, 0] && LtQ[c, 0]))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 1.41 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.06

method	result	size
default	$-\frac{ad-bc}{d^2(dx+c)}+\frac{b\ln(dx+c)}{d^2}$	33
risch	$-\frac{a}{d(dx+c)}+\frac{bc}{d^2(dx+c)}+\frac{b\ln(dx+c)}{d^2}$	39
parallelrisch	$\frac{\ln(dx+c)xbd+\ln(dx+c)bc-ad+bc}{d^2(dx+c)}$	39
norman	$\frac{-\frac{a\left(dab-b^2c\right)}{d^2b}-\frac{\left(da\,b^2-b^3c\right)x}{d^2b}}{(bx+a)(dx+c)}+\frac{b\ln(dx+c)}{d^2}$	71

input
```
int((b*x+a)^3/(a*c+(a*d+b*c)*x+b*d*x^2)^2,x,method=_RETURNVERBOSE)
```

output
```
-(a*d-b*c)/d^2/(d*x+c)+b*ln(d*x+c)/d^2
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.19

$$\int \frac{(a+bx)^3}{(ac+(bc+ad)x+bdx^2)^2}\,dx=\frac{bc-ad+(bdx+bc)\log(dx+c)}{d^3x+cd^2}$$

input
```
integrate((b*x+a)^3/(a*c+(a*d+b*c)*x+b*d*x^2)^2,x, algorithm="fricas")
```

output
```
(b*c - a*d + (b*d*x + b*c)*log(d*x + c))/(d^3*x + c*d^2)
```

Sympy [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.87

$$\int \frac{(a+bx)^3}{\left(ac+(bc+ad)x+bdx^2\right)^2}\,dx = \frac{b\log\left(c+dx\right)}{d^2}+\frac{-ad+bc}{cd^2+d^3x}$$

input
```
integrate((b*x+a)**3/(a*c+(a*d+b*c)*x+b*d*x**2)**2,x)
```

output
```
b*log(c + d*x)/d**2 + (-a*d + b*c)/(c*d**2 + d**3*x)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.10

$$\int \frac{(a+bx)^3}{\left(ac+(bc+ad)x+bdx^2\right)^2}\,dx = \frac{bc-ad}{d^3x+cd^2}+\frac{b\log\left(dx+c\right)}{d^2}$$

input
```
integrate((b*x+a)^3/(a*c+(a*d+b*c)*x+b*d*x^2)^2,x, algorithm="maxima")
```

output
```
(b*c - a*d)/(d^3*x + c*d^2) + b*log(d*x + c)/d^2
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.03

$$\int \frac{(a+bx)^3}{\left(ac+(bc+ad)x+bdx^2\right)^2}\,dx = \frac{b\log\left(|dx+c|\right)}{d^2}+\frac{bc-ad}{(dx+c)d^2}$$

input
```
integrate((b*x+a)^3/(a*c+(a*d+b*c)*x+b*d*x^2)^2,x, algorithm="giac")
```

output
```
b*log(abs(d*x + c))/d^2 + (b*c - a*d)/((d*x + c)*d^2)
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.03

$$\int \frac{(a+bx)^3}{(ac+(bc+ad)x+bdx^2)^2}\,dx = \frac{b\,\ln\left(c+d\,x\right)}{d^2} - \frac{a\,d-b\,c}{d^2\,\left(c+d\,x\right)}$$

input
```
int((a + b*x)^3/(a*c + x*(a*d + b*c) + b*d*x^2)^2,x)
```

output
```
(b*log(c + d*x))/d^2 - (a*d - b*c)/(d^2*(c + d*x))
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.58

$$\int \frac{(a+bx)^3}{(ac+(bc+ad)x+bdx^2)^2}\,dx = \frac{\log(dx+c)\,b\,c^2+\log(dx+c)\,bcdx+a\,d^2x-bcdx}{c\,d^2\,(dx+c)}$$

input
```
int((b*x+a)^3/(a*c+(a*d+b*c)*x+b*d*x^2)^2,x)
```

output
```
(log(c + d*x)*b*c**2 + log(c + d*x)*b*c*d*x + a*d**2*x - b*c*d*x)/(c*d**2*
(c + d*x))
```

3.56 $\int \frac{(a+bx)^2}{(ac+(bc+ad)x+bdx^2)^2}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 12

$$\int \frac{(a+bx)^2}{(ac+(bc+ad)x+bdx^2)^2}\,dx = -\frac{1}{d(c+dx)}$$

output
```
-1/d/(d*x+c)
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00

$$\int \frac{(a+bx)^2}{(ac+(bc+ad)x+bdx^2)^2}\,dx = -\frac{1}{d(c+dx)}$$

input
```
Integrate[(a + b*x)^2/(a*c + (b*c + a*d)*x + b*d*x^2)^2,x]
```

output
```
-(1/(d*(c + d*x)))
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.069, Rules used = {1120, 17}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx)^2}{(x(ad+bc)+ac+bdx^2)^2}dx$$

$$\Big\downarrow \text{ 1120}$$

$$\int \frac{1}{(c+dx)^2}dx$$

$$\Big\downarrow \text{ 17}$$

$$-\frac{1}{d(c+dx)}$$

input
```
Int[(a + b*x)^2/(a*c + (b*c + a*d)*x + b*d*x^2)^2,x]
```

output
```
-(1/(d*(c + d*x)))
```

Defintions of rubi rules used

rule 17
```
Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1
)/(b*(m + 1))), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]
```

rule 1120
```
Int[((d_) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Int[(d + e*x)^(m + p)*(a/d + (c/e)*x)^p, x] /; FreeQ[{a, b, c, d
, e, m}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[p] && (EqQ[m + p,
0] || EqQ[m + 2*p + 2, 0])
```

Maple [A] (verified)

Time = 1.43 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.08

method	result	size
gosper	$-\frac{1}{d(dx+c)}$	13
default	$-\frac{1}{d(dx+c)}$	13
risch	$-\frac{1}{d(dx+c)}$	13
parallelrisch	$-\frac{1}{d(dx+c)}$	13
norman	$\frac{-\frac{bx}{d}-\frac{a}{d}}{(bx+a)(dx+c)}$	30
orering	$-\frac{(dx+c)(bx+a)^2}{d(ac+(ad+bc)x+bd\,x^2)^2}$	39

input
```
int((b*x+a)^2/(a*c+(a*d+b*c)*x+b*d*x^2)^2,x,method=_RETURNVERBOSE)
```

output
```
-1/d/(d*x+c)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.08

$$\int \frac{(a+bx)^2}{(ac+(bc+ad)x+bdx^2)^2}\,dx=-\frac{1}{d^2x+cd}$$

input
```
integrate((b*x+a)^2/(a*c+(a*d+b*c)*x+b*d*x^2)^2,x, algorithm="fricas")
```

output
```
-1/(d^2*x + c*d)
```

Sympy [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.83

$$\int \frac{(a+bx)^2}{(ac+(bc+ad)x+bdx^2)^2}\,dx = -\frac{1}{cd+d^2x}$$

input
```
integrate((b*x+a)**2/(a*c+(a*d+b*c)*x+b*d*x**2)**2,x)
```

output
```
-1/(c*d + d**2*x)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.08

$$\int \frac{(a+bx)^2}{(ac+(bc+ad)x+bdx^2)^2}\,dx = -\frac{1}{d^2x+cd}$$

input
```
integrate((b*x+a)^2/(a*c+(a*d+b*c)*x+b*d*x^2)^2,x, algorithm="maxima")
```

output
```
-1/(d^2*x + c*d)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00

$$\int \frac{(a+bx)^2}{(ac+(bc+ad)x+bdx^2)^2}\,dx = -\frac{1}{(dx+c)d}$$

input
```
integrate((b*x+a)^2/(a*c+(a*d+b*c)*x+b*d*x^2)^2,x, algorithm="giac")
```

output
```
-1/((d*x + c)*d)
```

Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00

$$\int \frac{(a+bx)^2}{(ac+(bc+ad)x+bdx^2)^2}\,dx = -\frac{1}{d\,(c+d\,x)}$$

input
```
int((a + b*x)^2/(a*c + x*(a*d + b*c) + b*d*x^2)^2,x)
```

output
```
-1/(d*(c + d*x))
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00

$$\int \frac{(a+bx)^2}{(ac+(bc+ad)x+bdx^2)^2}\,dx = \frac{x}{c\,(dx+c)}$$

input
```
int((b*x+a)^2/(a*c+(a*d+b*c)*x+b*d*x^2)^2,x)
```

output
```
x/(c*(c + d*x))
```

3.57 $\int \frac{a+bx}{(ac+(bc+ad)x+bdx^2)^2}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 56

$$\int \frac{a+bx}{(ac+(bc+ad)x+bdx^2)^2}\,dx=\frac{1}{(bc-ad)(c+dx)}+\frac{b\log(a+bx)}{(bc-ad)^2}-\frac{b\log(c+dx)}{(bc-ad)^2}$$

output
```
1/(-a*d+b*c)/(d*x+c)+b*ln(b*x+a)/(-a*d+b*c)^2-b*ln(d*x+c)/(-a*d+b*c)^2
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.95

$$\int \frac{a+bx}{(ac+(bc+ad)x+bdx^2)^2}\,dx =\frac{bc-ad+b(c+dx)\log(a+bx)-b(c+dx)\log(c+dx)}{(bc-ad)^2(c+dx)}$$

input
```
Integrate[(a + b*x)/(a*c + (b*c + a*d)*x + b*d*x^2)^2,x]
```

output
```
(b*c - a*d + b*(c + d*x)*Log[a + b*x] - b*(c + d*x)*Log[c + d*x])/((b*c -
a*d)^2*(c + d*x))
```

Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.074$, Rules used = {1121, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+bx}{(x(ad+bc)+ac+bdx^2)^2}dx$$

$$\Big\downarrow \text{1121}$$

$$\int \left(\frac{b^2}{(a+bx)(bc-ad)^2}-\frac{bd}{(c+dx)(bc-ad)^2}-\frac{d}{(c+dx)^2(bc-ad)}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{(c+dx)(bc-ad)}+\frac{b\log(a+bx)}{(bc-ad)^2}-\frac{b\log(c+dx)}{(bc-ad)^2}$$

input
```
Int[(a + b*x)/(a*c + (b*c + a*d)*x + b*d*x^2)^2,x]
```

output
```
1/((b*c - a*d)*(c + d*x)) + (b*Log[a + b*x])/(b*c - a*d)^2 - (b*Log[c + d*
x])/(b*c - a*d)^2
```

Defintions of rubi rules used

rule 1121
```
Int[((d_) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Int[ExpandIntegrand[(d + e*x)^(m + p)*(a/d + (c/e)*x)^p, x], x]
/; FreeQ[{a, b, c, d, e, m, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && (Int
egerQ[p] || (GtQ[a, 0] && GtQ[d, 0] && LtQ[c, 0]))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 1.29 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.04

method	result	size
default	$\frac{b\ln(bx+a)}{(ad-bc)^2}-\frac{1}{(ad-bc)(dx+c)}-\frac{b\ln(dx+c)}{(ad-bc)^2}$	58
risch	$-\frac{1}{(ad-bc)(dx+c)}+\frac{b\ln(-bx-a)}{a^2d^2-2abcd+b^2c^2}-\frac{b\ln(dx+c)}{a^2d^2-2abcd+b^2c^2}$	87
parallelrisch	$\frac{\ln(bx+a)xbcd-\ln(dx+c)xbcd+\ln(bx+a)b\,c^2-\ln(dx+c)b\,c^2+a\,d^2x-bcdx}{(a^2d^2-2abcd+b^2c^2)(dx+c)c}$	94
norman	$\frac{-\frac{a}{ad-bc}-\frac{bx}{ad-bc}}{(bx+a)(dx+c)}+\frac{b\ln(bx+a)}{a^2d^2-2abcd+b^2c^2}-\frac{b\ln(dx+c)}{a^2d^2-2abcd+b^2c^2}$	108

input
```
int((b*x+a)/(a*c+(a*d+b*c)*x+b*d*x^2)^2,x,method=_RETURNVERBOSE)
```

output
```
b/(a*d-b*c)^2*ln(b*x+a)-1/(a*d-b*c)/(d*x+c)-b/(a*d-b*c)^2*ln(d*x+c)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.64

$$\int \frac{a+bx}{(ac+(bc+ad)x+bdx^2)^2}\,dx = \frac{bc-ad+(bdx+bc)\log(bx+a)-(bdx+bc)\log(dx+c)}{b^2c^3-2\,abc^2d+a^2cd^2+(b^2c^2d-2\,abcd^2+a^2d^3)x}$$

input
```
integrate((b*x+a)/(a*c+(a*d+b*c)*x+b*d*x^2)^2,x, algorithm="fricas")
```

output
```
(b*c - a*d + (b*d*x + b*c)*log(b*x + a) - (b*d*x + b*c)*log(d*x + c))/(b^2
*c^3 - 2*a*b*c^2*d + a^2*c*d^2 + (b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)*x)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 233 vs. $2(46) = 92$.

Time = 0.34 (sec) , antiderivative size = 233, normalized size of antiderivative = 4.16

$$\int \frac{a+bx}{(ac+(bc+ad)x+bdx^2)^2}\,dx = -\frac{b\log\left(x+\frac{-\frac{a^3bd^3}{(ad-bc)^2}+\frac{3a^2b^2cd^2}{(ad-bc)^2}-\frac{3ab^3c^2d}{(ad-bc)^2}+abd+\frac{b^4c^3}{(ad-bc)^2}+b^2c}{2b^2d}\right)}{(ad-bc)^2} + \frac{b\log\left(x+\frac{\frac{a^3bd^3}{(ad-bc)^2}-\frac{3a^2b^2cd^2}{(ad-bc)^2}+\frac{3ab^3c^2d}{(ad-bc)^2}+abd-\frac{b^4c^3}{(ad-bc)^2}+b^2c}{2b^2d}\right)}{(ad-bc)^2} - \frac{1}{acd-bc^2+x(ad^2-bcd)}$$

input
```
integrate((b*x+a)/(a*c+(a*d+b*c)*x+b*d*x**2)**2,x)
```

output
```
-b*log(x + (-a**3*b*d**3/(a*d - b*c)**2 + 3*a**2*b**2*c*d**2/(a*d - b*c)**
2 - 3*a*b**3*c**2*d/(a*d - b*c)**2 + a*b*d + b**4*c**3/(a*d - b*c)**2 + b*
*2*c)/(2*b**2*d))/(a*d - b*c)**2 + b*log(x + (a**3*b*d**3/(a*d - b*c)**2 -
 3*a**2*b**2*c*d**2/(a*d - b*c)**2 + 3*a*b**3*c**2*d/(a*d - b*c)**2 + a*b*
d - b**4*c**3/(a*d - b*c)**2 + b**2*c)/(2*b**2*d))/(a*d - b*c)**2 - 1/(a*c
*d - b*c**2 + x*(a*d**2 - b*c*d))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.61

$$\int \frac{a+bx}{(ac+(bc+ad)x+bdx^2)^2}\,dx = \frac{b\log(bx+a)}{b^2c^2-2\,abcd+a^2d^2} - \frac{b\log(dx+c)}{b^2c^2-2\,abcd+a^2d^2} + \frac{1}{bc^2-acd+(bcd-ad^2)x}$$

input
```
integrate((b*x+a)/(a*c+(a*d+b*c)*x+b*d*x^2)^2,x, algorithm="maxima")
```

output
```
b*log(b*x + a)/(b^2*c^2 - 2*a*b*c*d + a^2*d^2) - b*log(d*x + c)/(b^2*c^2 -
 2*a*b*c*d + a^2*d^2) + 1/(b*c^2 - a*c*d + (b*c*d - a*d^2)*x)
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 93, normalized size of antiderivative = 1.66

$$\int \frac{a+bx}{(ac+(bc+ad)x+bdx^2)^2}\,dx = \frac{b^2\log(|bx+a|)}{b^3c^2-2\,ab^2cd+a^2bd^2} - \frac{bd\log(|dx+c|)}{b^2c^2d-2\,abcd^2+a^2d^3} + \frac{1}{(bc-ad)(dx+c)}$$

input
```
integrate((b*x+a)/(a*c+(a*d+b*c)*x+b*d*x^2)^2,x, algorithm="giac")
```

output
```
b^2*log(abs(b*x + a))/(b^3*c^2 - 2*a*b^2*c*d + a^2*b*d^2) - b*d*log(abs(d*
x + c))/(b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3) + 1/((b*c - a*d)*(d*x + c))
```

Mupad [B] (verification not implemented)

Time = 5.84 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.38

$$\int \frac{a+bx}{(ac+(bc+ad)x+bdx^2)^2}\,dx = \frac{2\,b\,\mathrm{atanh}\left(\frac{a^2\,d^2-b^2\,c^2}{(a\,d-b\,c)^2}+\frac{2\,b\,d\,x}{a\,d-b\,c}\right)}{(a\,d-b\,c)^2} - \frac{1}{(a\,d-b\,c)\,(c+d\,x)}$$

input
```
int((a + b*x)/(a*c + x*(a*d + b*c) + b*d*x^2)^2,x)
```

output
```
(2*b*atanh((a^2*d^2 - b^2*c^2)/(a*d - b*c)^2 + (2*b*d*x)/(a*d - b*c)))/(a*
d - b*c)^2 - 1/((a*d - b*c)*(c + d*x))
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 115, normalized size of antiderivative = 2.05

$$\int \frac{a+bx}{(ac+(bc+ad)x+bdx^2)^2}\,dx$$
$$=\frac{\log(bx+a)\,b\,c^2+\log(bx+a)\,bcdx-\log(dx+c)\,b\,c^2-\log(dx+c)\,bcdx+a\,d^2x-bcdx}{c\,(a^2d^3x-2abc\,d^2x+b^2c^2dx+a^2c\,d^2-2ab\,c^2d+b^2c^3)}$$

input
```
int((b*x+a)/(a*c+(a*d+b*c)*x+b*d*x^2)^2,x)
```

output
```
(log(a + b*x)*b*c**2 + log(a + b*x)*b*c*d*x - log(c + d*x)*b*c**2 - log(c
+ d*x)*b*c*d*x + a*d**2*x - b*c*d*x)/(c*(a**2*c*d**2 + a**2*d**3*x - 2*a*b
*c**2*d - 2*a*b*c*d**2*x + b**2*c**3 + b**2*c**2*d*x))
```

3.58 $\int \frac{1}{(ac+(bc+ad)x+bdx^2)^2}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 81

$$\int \frac{1}{(ac+(bc+ad)x+bdx^2)^2}\,dx = -\frac{b}{(bc-ad)^2(a+bx)} - \frac{d}{(bc-ad)^2(c+dx)} - \frac{2bd\log(a+bx)}{(bc-ad)^3} + \frac{2bd\log(c+dx)}{(bc-ad)^3}$$

output
```
-b/(-a*d+b*c)^2/(b*x+a)-d/(-a*d+b*c)^2/(d*x+c)-2*b*d*ln(b*x+a)/(-a*d+b*c)^
3+2*b*d*ln(d*x+c)/(-a*d+b*c)^3
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.81

$$\int \frac{1}{(ac+(bc+ad)x+bdx^2)^2}\,dx = \frac{\frac{b(-bc+ad)}{a+bx} + \frac{d(-bc+ad)}{c+dx} - 2bd\log(a+bx) + 2bd\log(c+dx)}{(bc-ad)^3}$$

input
```
Integrate[(a*c + (b*c + a*d)*x + b*d*x^2)^(-2),x]
```

output
```
((b*(-(b*c) + a*d))/(a + b*x) + (d*(-(b*c) + a*d))/(c + d*x) - 2*b*d*Log[a
 + b*x] + 2*b*d*Log[c + d*x])/(b*c - a*d)^3
```

Rubi [A] (verified)

Time = 0.49 (sec) , antiderivative size = 106, normalized size of antiderivative = 1.31, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {1084, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(x(ad+bc)+ac+bdx^2)^2}dx$$

$$\Big\downarrow \text{1084}$$

$$b^2d^2\int \left(\frac{2}{b(bc-ad)^3(c+dx)}+\frac{1}{b^2(bc-ad)^2(c+dx)^2}-\frac{2}{d(bc-ad)^3(a+bx)}+\frac{1}{d^2(bc-ad)^2(a+bx)^2}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$b^2d^2\left(-\frac{1}{b^2d(c+dx)(bc-ad)^2}-\frac{1}{bd^2(a+bx)(bc-ad)^2}-\frac{2\log(a+bx)}{bd(bc-ad)^3}+\frac{2\log(c+dx)}{bd(bc-ad)^3}\right)$$

input
```
Int[(a*c + (b*c + a*d)*x + b*d*x^2)^(-2),x]
```

output
```
b^2*d^2*(-(1/(b*d^2*(b*c - a*d)^2*(a + b*x))) - 1/(b^2*d*(b*c - a*d)^2*(c
+ d*x)) - (2*Log[a + b*x])/(b*d*(b*c - a*d)^3) + (2*Log[c + d*x])/(b*d*(b*
c - a*d)^3))
```

Defintions of rubi rules used

rule 1084
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{q = Rt[b^2
- 4*a*c, 2]}, Simp[1/c^p   Int[ExpandIntegrand[(b/2 - q/2 + c*x)^p*(b/2 + q
/2 + c*x)^p, x], x], x] /;  !FractionalPowerFactorQ[q]] /; FreeQ[{a, b, c},
 x] && IntegerQ[p] && NiceSqrtQ[b^2 - 4*a*c]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 1.40 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.01

method	result
default	$-\frac{b}{(ad-bc)^2(bx+a)}+\frac{2bd\ln(bx+a)}{(ad-bc)^3}-\frac{d}{(ad-bc)^2(dx+c)}-\frac{2bd\ln(dx+c)}{(ad-bc)^3}$
risch	$\frac{-\frac{2bdx}{a^2d^2-2abcd+b^2c^2}-\frac{ad+bc}{a^2d^2-2abcd+b^2c^2}}{bd\,x^2+adx+cbx+ac}-\frac{2bd\ln(dx+c)}{a^3d^3-3a^2bc\,d^2+3a\,b^2c^2d-b^3c^3}+\frac{2bd\ln(-bx-a)}{a^3d^3-3a^2bc\,d^2+3a\,b^2c^2d-b^3c^3}$
norman	$\frac{\frac{-ab\,d^2-cd\,b^2}{db\left(a^2d^2-2abcd+b^2c^2\right)}-\frac{2bdx}{a^2d^2-2abcd+b^2c^2}}{(bx+a)(dx+c)}+\frac{2bd\ln(bx+a)}{a^3d^3-3a^2bc\,d^2+3a\,b^2c^2d-b^3c^3}-\frac{2bd\ln(dx+c)}{a^3d^3-3a^2bc\,d^2+3a\,b^2c^2d-b^3c^3}$
parallelrisch	$\frac{2\ln(bx+a)x^2b^3d^3-2\ln(dx+c)x^2b^3d^3+2\ln(bx+a)xa\,b^2d^3+2\ln(bx+a)x\,b^3c\,d^2-2\ln(dx+c)xa\,b^2d^3-2\ln(dx+c)x\,b^3c\,d^2+2\ln}{(a^3d^3-3a^2bc\,d^2+3a\,b^2c^2d-b^3c^3)(bd\,x^2+adx+cbx+ac)}$

input
```
int(1/(a*c+(a*d+b*c)*x+b*d*x^2)^2,x,method=_RETURNVERBOSE)
```

output
```
-b/(a*d-b*c)^2/(b*x+a)+2*b/(a*d-b*c)^3*d*ln(b*x+a)-d/(a*d-b*c)^2/(d*x+c)-2
*b/(a*d-b*c)^3*d*ln(d*x+c)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 241 vs. 2(81) = 162.

Time = 0.09 (sec) , antiderivative size = 241, normalized size of antiderivative = 2.98

$$\int \frac{1}{(ac+(bc+ad)x+bdx^2)^2}\,dx = -\frac{b^2c^2-a^2d^2+2\,(b^2cd-abd^2)x+2\,(b^2d^2x^2+abcd+(b^2cd+abd^2)x)\log(bx+a)-2\,(b^2d^2x^2+abcd}{ab^3c^4-3\,a^2b^2c^3d+3\,a^3bc^2d^2-a^4cd^3+(b^4c^3d-3\,ab^3c^2d^2+3\,a^2b^2cd^3-a^3bd^4)x^2+(b^4c^4-2\,ab}$$

input
```
integrate(1/(a*c+(a*d+b*c)*x+b*d*x^2)^2,x, algorithm="fricas")
```

output
```
-(b^2*c^2 - a^2*d^2 + 2*(b^2*c*d - a*b*d^2)*x + 2*(b^2*d^2*x^2 + a*b*c*d +
 (b^2*c*d + a*b*d^2)*x)*log(b*x + a) - 2*(b^2*d^2*x^2 + a*b*c*d + (b^2*c*d
 + a*b*d^2)*x)*log(d*x + c))/(a*b^3*c^4 - 3*a^2*b^2*c^3*d + 3*a^3*b*c^2*d^
2 - a^4*c*d^3 + (b^4*c^3*d - 3*a*b^3*c^2*d^2 + 3*a^2*b^2*c*d^3 - a^3*b*d^4
)*x^2 + (b^4*c^4 - 2*a*b^3*c^3*d + 2*a^3*b*c*d^3 - a^4*d^4)*x)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 406 vs. $2(70) = 140$.

Time = 0.56 (sec) , antiderivative size = 406, normalized size of antiderivative = 5.01

$$\begin{aligned}\int \frac{1}{(ac+(bc+ad)x+bdx^2)^2}\,dx &= -\frac{2bd\log\left(x+\frac{-\frac{2a^4bd^5}{(ad-bc)^3}+\frac{8a^3b^2cd^4}{(ad-bc)^3}-\frac{12a^2b^3c^2d^3}{(ad-bc)^3}+\frac{8ab^4c^3d^2}{(ad-bc)^3}+2abd^2-\frac{2b^5c^4d}{(ad-bc)^3}+2b^2cd}{4b^2d^2}\right)}{(ad-bc)^3} \\ &+\frac{2bd\log\left(x+\frac{\frac{2a^4bd^5}{(ad-bc)^3}-\frac{8a^3b^2cd^4}{(ad-bc)^3}+\frac{12a^2b^3c^2d^3}{(ad-bc)^3}-\frac{8ab^4c^3d^2}{(ad-bc)^3}+2abd^2+\frac{2b^5c^4d}{(ad-bc)^3}+2b^2cd}{4b^2d^2}\right)}{(ad-bc)^3} \\ &+\frac{-ad-bc-2bdx}{a^3cd^2-2a^2bc^2d+ab^2c^3+x^2\left(a^2bd^3-2ab^2cd^2+b^3c^2d\right)+x\left(a^3d^3-a^2bcd^2-ab^2c^2d+b^3c^3\right)}\end{aligned}$$

input
```
integrate(1/(a*c+(a*d+b*c)*x+b*d*x**2)**2,x)
```

output
```
-2*b*d*log(x + (-2*a**4*b*d**5/(a*d - b*c)**3 + 8*a**3*b**2*c*d**4/(a*d -
b*c)**3 - 12*a**2*b**3*c**2*d**3/(a*d - b*c)**3 + 8*a*b**4*c**3*d**2/(a*d
- b*c)**3 + 2*a*b*d**2 - 2*b**5*c**4*d/(a*d - b*c)**3 + 2*b**2*c*d)/(4*b**
2*d**2))/(a*d - b*c)**3 + 2*b*d*log(x + (2*a**4*b*d**5/(a*d - b*c)**3 - 8*
a**3*b**2*c*d**4/(a*d - b*c)**3 + 12*a**2*b**3*c**2*d**3/(a*d - b*c)**3 -
8*a*b**4*c**3*d**2/(a*d - b*c)**3 + 2*a*b*d**2 + 2*b**5*c**4*d/(a*d - b*c)
**3 + 2*b**2*c*d)/(4*b**2*d**2))/(a*d - b*c)**3 + (-a*d - b*c - 2*b*d*x)/(
a**3*c*d**2 - 2*a**2*b*c**2*d + a*b**2*c**3 + x**2*(a**2*b*d**3 - 2*a*b**2
*c*d**2 + b**3*c**2*d) + x*(a**3*d**3 - a**2*b*c*d**2 - a*b**2*c**2*d + b*
*3*c**3))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 208 vs. $2(81) = 162$.

Time = 0.03 (sec) , antiderivative size = 208, normalized size of antiderivative = 2.57

$$\int \frac{1}{(ac+(bc+ad)x+bdx^2)^2}\,dx = -\frac{2\,bd\log(bx+a)}{b^3c^3-3\,ab^2c^2d+3\,a^2bcd^2-a^3d^3}+\frac{2\,bd\log(dx+c)}{b^3c^3-3\,ab^2c^2d+3\,a^2bcd^2-a^3d^3} - \frac{2\,bdx+bc+ad}{ab^2c^3-2\,a^2bc^2d+a^3cd^2+(b^3c^2d-2\,ab^2cd^2+a^2bd^3)x^2+(b^3c^3-ab^2c^2d-a^2bcd^2+a^3d^3)x}$$

input
```
integrate(1/(a*c+(a*d+b*c)*x+b*d*x^2)^2,x, algorithm="maxima")
```

output
```
-2*b*d*log(b*x + a)/(b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3) +
2*b*d*log(d*x + c)/(b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3) - (
2*b*d*x + b*c + a*d)/(a*b^2*c^3 - 2*a^2*b*c^2*d + a^3*c*d^2 + (b^3*c^2*d -
 2*a*b^2*c*d^2 + a^2*b*d^3)*x^2 + (b^3*c^3 - a*b^2*c^2*d - a^2*b*c*d^2 + a
^3*d^3)*x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 166 vs. $2(81) = 162$.

Time = 0.12 (sec) , antiderivative size = 166, normalized size of antiderivative = 2.05

$$\int \frac{1}{(ac+(bc+ad)x+bdx^2)^2}\,dx = -\frac{2\,b^2d\log\left(|bx+a|\right)}{b^4c^3-3\,ab^3c^2d+3\,a^2b^2cd^2-a^3bd^3} + \frac{2\,bd^2\log\left(|dx+c|\right)}{b^3c^3d-3\,ab^2c^2d^2+3\,a^2bcd^3-a^3d^4} - \frac{2\,bdx+bc+ad}{(b^2c^2-2\,abcd+a^2d^2)(bdx^2+bcx+adx+ac)}$$

input
```
integrate(1/(a*c+(a*d+b*c)*x+b*d*x^2)^2,x, algorithm="giac")
```

output
```
-2*b^2*d*log(abs(b*x + a))/(b^4*c^3 - 3*a*b^3*c^2*d + 3*a^2*b^2*c*d^2 - a^
3*b*d^3) + 2*b*d^2*log(abs(d*x + c))/(b^3*c^3*d - 3*a*b^2*c^2*d^2 + 3*a^2*
b*c*d^3 - a^3*d^4) - (2*b*d*x + b*c + a*d)/((b^2*c^2 - 2*a*b*c*d + a^2*d^2
)*(b*d*x^2 + b*c*x + a*d*x + a*c))
```

Mupad [B] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 182, normalized size of antiderivative = 2.25

$$\int \frac{1}{(ac+(bc+ad)x+bdx^2)^2}\,dx = \frac{4\,b\,d\,\mathrm{atanh}\left(\frac{a^3\,d^3-a^2\,b\,c\,d^2-a\,b^2\,c^2\,d+b^3\,c^3}{(a\,d-b\,c)^3}+\frac{2\,b\,d\,x\,(a^2\,d^2-2\,a\,b\,c\,d+b^2\,c^2)}{(a\,d-b\,c)^3}\right)}{(a\,d-b\,c)^3} - \frac{\frac{a\,d+b\,c}{a^2\,d^2-2\,a\,b\,c\,d+b^2\,c^2}+\frac{2\,b\,d\,x}{a^2\,d^2-2\,a\,b\,c\,d+b^2\,c^2}}{b\,d\,x^2+(a\,d+b\,c)\,x+a\,c}$$

input
```
int(1/(a*c + x*(a*d + b*c) + b*d*x^2)^2,x)
```

output
```
(4*b*d*atanh((a^3*d^3 + b^3*c^3 - a*b^2*c^2*d - a^2*b*c*d^2)/(a*d - b*c)^3
 + (2*b*d*x*(a^2*d^2 + b^2*c^2 - 2*a*b*c*d))/(a*d - b*c)^3))/(a*d - b*c)^3
 - ((a*d + b*c)/(a^2*d^2 + b^2*c^2 - 2*a*b*c*d) + (2*b*d*x)/(a^2*d^2 + b^2
*c^2 - 2*a*b*c*d))/(a*c + x*(a*d + b*c) + b*d*x^2)
```

Reduce [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 459, normalized size of antiderivative = 5.67

$$\int \frac{1}{(ac+(bc+ad)x+bdx^2)^2}\,dx$$
$$= \frac{2\log(bx+a)\,a^2bc\,d^2+2\log(bx+a)\,a^2b\,d^3x+2\log(bx+a)\,a\,b^2c^2d+4\log(bx+a)\,a\,b^2c\,d^2x+2\log(bx-}{}$$

input
```
int(1/(a*c+(a*d+b*c)*x+b*d*x^2)^2,x)
```

output
```
(2*log(a + b*x)*a**2*b*c*d**2 + 2*log(a + b*x)*a**2*b*d**3*x + 2*log(a + b
*x)*a*b**2*c**2*d + 4*log(a + b*x)*a*b**2*c*d**2*x + 2*log(a + b*x)*a*b**2
*d**3*x**2 + 2*log(a + b*x)*b**3*c**2*d*x + 2*log(a + b*x)*b**3*c*d**2*x**
2 - 2*log(c + d*x)*a**2*b*c*d**2 - 2*log(c + d*x)*a**2*b*d**3*x - 2*log(c
+ d*x)*a*b**2*c**2*d - 4*log(c + d*x)*a*b**2*c*d**2*x - 2*log(c + d*x)*a*b
**2*d**3*x**2 - 2*log(c + d*x)*b**3*c**2*d*x - 2*log(c + d*x)*b**3*c*d**2*
x**2 - a**3*d**3 + a**2*b*c*d**2 - a*b**2*c**2*d + 2*a*b**2*d**3*x**2 + b*
*3*c**3 - 2*b**3*c*d**2*x**2)/(a**5*c*d**4 + a**5*d**5*x - 2*a**4*b*c**2*d
**3 - a**4*b*c*d**4*x + a**4*b*d**5*x**2 - 2*a**3*b**2*c**2*d**3*x - 2*a**
3*b**2*c*d**4*x**2 + 2*a**2*b**3*c**4*d + 2*a**2*b**3*c**3*d**2*x - a*b**4
*c**5 + a*b**4*c**4*d*x + 2*a*b**4*c**3*d**2*x**2 - b**5*c**5*x - b**5*c**
4*d*x**2)
```

3.59 $\int \frac{1}{(a+bx)\left(ac+(bc+ad)x+bdx^2\right)^2}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 109

$$\int \frac{1}{(a+bx)\left(ac+(bc+ad)x+bdx^2\right)^2}\,dx = -\frac{b}{2(bc-ad)^2(a+bx)^2} + \frac{2bd}{(bc-ad)^3(a+bx)} + \frac{d^2}{(bc-ad)^3(c+dx)} + \frac{3bd^2\log(a+bx)}{(bc-ad)^4} - \frac{3bd^2\log(c+dx)}{(bc-ad)^4}$$

output

```
-1/2*b/(-a*d+b*c)^2/(b*x+a)^2+2*b*d/(-a*d+b*c)^3/(b*x+a)+d^2/(-a*d+b*c)^3/
(d*x+c)+3*b*d^2*ln(b*x+a)/(-a*d+b*c)^4-3*b*d^2*ln(d*x+c)/(-a*d+b*c)^4
```

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 98, normalized size of antiderivative = 0.90

$$\int \frac{1}{(a+bx)\left(ac+(bc+ad)x+bdx^2\right)^2}\,dx = \frac{-\frac{b(bc-ad)^2}{(a+bx)^2} + \frac{4bd(bc-ad)}{a+bx} + \frac{2d^2(bc-ad)}{c+dx} + 6bd^2\log(a+bx) - 6bd^2\log(c+dx)}{2(bc-ad)^4}$$

input
```
Integrate[1/((a + b*x)*(a*c + (b*c + a*d)*x + b*d*x^2)^2),x]
```

output
```
(-((b*(b*c - a*d)^2)/(a + b*x)^2) + (4*b*d*(b*c - a*d))/(a + b*x) + (2*d^2
*(b*c - a*d))/(c + d*x) + 6*b*d^2*Log[a + b*x] - 6*b*d^2*Log[c + d*x])/(2*
(b*c - a*d)^4)
```

Rubi [A] (verified)

Time = 0.47 (sec) , antiderivative size = 109, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.069, Rules used = {1121, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a+bx)\left(x(ad+bc)+ac+bdx^2\right)^2}dx$$

$$\Big\downarrow \text{1121}$$

$$\int \left(\frac{3b^2d^2}{(a+bx)(bc-ad)^4}-\frac{2b^2d}{(a+bx)^2(bc-ad)^3}+\frac{b^2}{(a+bx)^3(bc-ad)^2}-\frac{3bd^3}{(c+dx)(bc-ad)^4}-\frac{d^3}{(c+dx)^2(bc-ad}\right.$$

$$\Big\downarrow \text{2009}$$

$$\frac{d^2}{(c+dx)(bc-ad)^3}+\frac{3bd^2\log(a+bx)}{(bc-ad)^4}-\frac{3bd^2\log(c+dx)}{(bc-ad)^4}+\frac{2bd}{(a+bx)(bc-ad)^3}-\frac{b}{2(a+bx)^2(bc-ad)^2}$$

input
```
Int[1/((a + b*x)*(a*c + (b*c + a*d)*x + b*d*x^2)^2),x]
```

output
```
-1/2*b/((b*c - a*d)^2*(a + b*x)^2) + (2*b*d)/((b*c - a*d)^3*(a + b*x)) + d
^2/((b*c - a*d)^3*(c + d*x)) + (3*b*d^2*Log[a + b*x])/(b*c - a*d)^4 - (3*b
*d^2*Log[c + d*x])/(b*c - a*d)^4
```

Defintions of rubi rules used

rule 1121
```
Int[((d_) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Int[ExpandIntegrand[(d + e*x)^(m + p)*(a/d + (c/e)*x)^p, x], x]
/; FreeQ[{a, b, c, d, e, m, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && (Int
egerQ[p] || (GtQ[a, 0] && GtQ[d, 0] && LtQ[c, 0]))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 1.68 (sec) , antiderivative size = 109, normalized size of antiderivative = 1.00

method	result
default	$-\frac{b}{2(ad-bc)^{2}(bx+a)^{2}}+\frac{3b\,d^{2}\ln(bx+a)}{(ad-bc)^{4}}-\frac{2bd}{(ad-bc)^{3}(bx+a)}-\frac{d^{2}}{(ad-bc)^{3}(dx+c)}-\frac{3b\,d^{2}\ln(dx+c)}{(ad-bc)^{4}}$
risch	$\frac{-\frac{3b^{2}d^{2}x^{2}}{a^{3}d^{3}-3a^{2}bc\,d^{2}+3a\,b^{2}c^{2}d-b^{3}c^{3}}-\frac{3(3ad+bc)bdx}{2\left(a^{3}d^{3}-3a^{2}bc\,d^{2}+3a\,b^{2}c^{2}d-b^{3}c^{3}\right)}-\frac{2a^{2}d^{2}+5abcd-b^{2}c^{2}}{2\left(a^{3}d^{3}-3a^{2}bc\,d^{2}+3a\,b^{2}c^{2}d-b^{3}c^{3}\right)}}{(bx+a)(bd\,x^{2}+adx+cbx+ac)}-\frac{3b\,d}{a^{4}d^{4}-4a^{3}bc\,d^{3}+6}$
norman	$\frac{-\frac{3b^{2}d^{2}x^{2}}{a^{3}d^{3}-3a^{2}bc\,d^{2}+3a\,b^{2}c^{2}d-b^{3}c^{3}}+\frac{-2a^{2}b^{2}d^{3}-5c\,d^{2}a\,b^{3}+b^{4}c^{2}d}{2d\,b^{2}\left(a^{3}d^{3}-3a^{2}bc\,d^{2}+3a\,b^{2}c^{2}d-b^{3}c^{3}\right)}+\frac{\left(-9a\,b^{3}d^{3}-3b^{4}c\,d^{2}\right)x}{2d\,b^{2}\left(a^{3}d^{3}-3a^{2}bc\,d^{2}+3a\,b^{2}c^{2}d-b^{3}c^{3}\right)}}{(dx+c)(bx+a)^{2}}+\frac{}{a^{4}d^{4}-4a^{3}b}$
parallelrisch	$6a\,b^{4}c^{2}d^{2}-3a^{2}b^{3}c\,d^{3}+6\ln(bx+a)x^{3}b^{5}d^{4}-6\ln(dx+c)x^{3}b^{5}d^{4}+12\ln(bx+a)xa\,b^{4}c\,d^{3}-12\ln(dx+c)xa\,b^{4}c\,d^{3}-12\ln(dx+c)x^{2}a$

input
```
int(1/(b*x+a)/(a*c+(a*d+b*c)*x+b*d*x^2)^2,x,method=_RETURNVERBOSE)
```

output
```
-1/2*b/(a*d-b*c)^2/(b*x+a)^2+3*b/(a*d-b*c)^4*d^2*ln(b*x+a)-2*b/(a*d-b*c)^3
*d/(b*x+a)-d^2/(a*d-b*c)^3/(d*x+c)-3*b/(a*d-b*c)^4*d^2*ln(d*x+c)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 494 vs. $2(107) = 214$.

Time = 0.09 (sec) , antiderivative size = 494, normalized size of antiderivative = 4.53

$$\int \frac{1}{(a+bx)\left(ac+(bc+ad)x+bdx^2\right)^2}\,dx = -\frac{b^3c^3 - 6\,ab^2c^2d + 3\,a^2bcd^2 + 2\,a^3d^3 - 6\,(b^3cd^2 - ab^2d^3)x^2 - 3\,(b^3c^2d + 2\,ab^2cd^2 - 3\,a^2bd^3)x - 6\,(b^3d^3}{2\,(a^2b^4c^5 - 4\,a^3b^3c^4d + 6\,a^4b^2c^3d^2 - 4\,a^5bc^2d^3 + a^6cd^4 + (b^6c^4d - 4\,ab^5c^3d^2 + 6\,a^2b^4c^2d^3 - 4\,a^3b^3cd^4 +}$$

input
```
integrate(1/(b*x+a)/(a*c+(a*d+b*c)*x+b*d*x^2)^2,x, algorithm="fricas")
```

output
```
-1/2*(b^3*c^3 - 6*a*b^2*c^2*d + 3*a^2*b*c*d^2 + 2*a^3*d^3 - 6*(b^3*c*d^2 -
 a*b^2*d^3)*x^2 - 3*(b^3*c^2*d + 2*a*b^2*c*d^2 - 3*a^2*b*d^3)*x - 6*(b^3*d
^3*x^3 + a^2*b*c*d^2 + (b^3*c*d^2 + 2*a*b^2*d^3)*x^2 + (2*a*b^2*c*d^2 + a^
2*b*d^3)*x)*log(b*x + a) + 6*(b^3*d^3*x^3 + a^2*b*c*d^2 + (b^3*c*d^2 + 2*a
*b^2*d^3)*x^2 + (2*a*b^2*c*d^2 + a^2*b*d^3)*x)*log(d*x + c))/(a^2*b^4*c^5
- 4*a^3*b^3*c^4*d + 6*a^4*b^2*c^3*d^2 - 4*a^5*b*c^2*d^3 + a^6*c*d^4 + (b^6
*c^4*d - 4*a*b^5*c^3*d^2 + 6*a^2*b^4*c^2*d^3 - 4*a^3*b^3*c*d^4 + a^4*b^2*d
^5)*x^3 + (b^6*c^5 - 2*a*b^5*c^4*d - 2*a^2*b^4*c^3*d^2 + 8*a^3*b^3*c^2*d^3
 - 7*a^4*b^2*c*d^4 + 2*a^5*b*d^5)*x^2 + (2*a*b^5*c^5 - 7*a^2*b^4*c^4*d + 8
*a^3*b^3*c^3*d^2 - 2*a^4*b^2*c^2*d^3 - 2*a^5*b*c*d^4 + a^6*d^5)*x)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 634 vs. $2(97) = 194$.

Time = 0.84 (sec) , antiderivative size = 634, normalized size of antiderivative = 5.82

$$\int \frac{1}{(a+bx)\left(ac+(bc+ad)x+bdx^2\right)^2}\,dx$$
$$= -\frac{3bd^2\log\left(x+\frac{-\frac{3a^5bd^7}{(ad-bc)^4}+\frac{15a^4b^2cd^6}{(ad-bc)^4}-\frac{30a^3b^3c^2d^5}{(ad-bc)^4}+\frac{30a^2b^4c^3d^4}{(ad-bc)^4}-\frac{15ab^5c^4d^3}{(ad-bc)^4}+3abd^3+\frac{3b^6c^5d^2}{(ad-bc)^4}+3b^2cd^2}{6b^2d^3}\right)}{(ad-bc)^4}$$
$$+\frac{3bd^2\log\left(x+\frac{\frac{3a^5bd^7}{(ad-bc)^4}-\frac{15a^4b^2cd^6}{(ad-bc)^4}+\frac{30a^3b^3c^2d^5}{(ad-bc)^4}-\frac{30a^2b^4c^3d^4}{(ad-bc)^4}+\frac{15ab^5c^4d^3}{(ad-bc)^4}+3abd^3-\frac{3b^6c^5d^2}{(ad-bc)^4}+3b^2cd^2}{6b^2d^3}\right)}{(ad-bc)^4}$$
$$+\frac{-2a^2d^2-5abcd+b^2c^2-6}{2a^5cd^3-6a^4bc^2d^2+6a^3b^2c^3d-2a^2b^3c^4+x^3\cdot\left(2a^3b^2d^4-6a^2b^3cd^3+6ab^4c^2d^2-2b^5c^3d\right)+x^2\cdot\left(4a^4ba\right.}$$

input
```
integrate(1/(b*x+a)/(a*c+(a*d+b*c)*x+b*d*x**2)**2,x)
```

output
```
-3*b*d**2*log(x + (-3*a**5*b*d**7/(a*d - b*c)**4 + 15*a**4*b**2*c*d**6/(a*
d - b*c)**4 - 30*a**3*b**3*c**2*d**5/(a*d - b*c)**4 + 30*a**2*b**4*c**3*d*
*4/(a*d - b*c)**4 - 15*a*b**5*c**4*d**3/(a*d - b*c)**4 + 3*a*b*d**3 + 3*b*
*6*c**5*d**2/(a*d - b*c)**4 + 3*b**2*c*d**2)/(6*b**2*d**3))/(a*d - b*c)**4
 + 3*b*d**2*log(x + (3*a**5*b*d**7/(a*d - b*c)**4 - 15*a**4*b**2*c*d**6/(a
*d - b*c)**4 + 30*a**3*b**3*c**2*d**5/(a*d - b*c)**4 - 30*a**2*b**4*c**3*d
**4/(a*d - b*c)**4 + 15*a*b**5*c**4*d**3/(a*d - b*c)**4 + 3*a*b*d**3 - 3*b
**6*c**5*d**2/(a*d - b*c)**4 + 3*b**2*c*d**2)/(6*b**2*d**3))/(a*d - b*c)**
4 + (-2*a**2*d**2 - 5*a*b*c*d + b**2*c**2 - 6*b**2*d**2*x**2 + x*(-9*a*b*d
**2 - 3*b**2*c*d))/(2*a**5*c*d**3 - 6*a**4*b*c**2*d**2 + 6*a**3*b**2*c**3*
d - 2*a**2*b**3*c**4 + x**3*(2*a**3*b**2*d**4 - 6*a**2*b**3*c*d**3 + 6*a*b
**4*c**2*d**2 - 2*b**5*c**3*d) + x**2*(4*a**4*b*d**4 - 10*a**3*b**2*c*d**3
 + 6*a**2*b**3*c**2*d**2 + 2*a*b**4*c**3*d - 2*b**5*c**4) + x*(2*a**5*d**4
 - 2*a**4*b*c*d**3 - 6*a**3*b**2*c**2*d**2 + 10*a**2*b**3*c**3*d - 4*a*b**
4*c**4))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 386 vs. $2(107) = 214$.

Time = 0.04 (sec) , antiderivative size = 386, normalized size of antiderivative = 3.54

$$\int \frac{1}{(a+bx)\left(ac+(bc+ad)x+bdx^2\right)^2}\,dx$$
$$= \frac{3\,bd^2\log\left(bx+a\right)}{b^4c^4-4\,ab^3c^3d+6\,a^2b^2c^2d^2-4\,a^3bcd^3+a^4d^4} - \frac{3\,bd^2\log\left(dx+c\right)}{b^4c^4-4\,ab^3c^3d+6\,a^2b^2c^2d^2-4\,a^3bcd^3+a^4d^4} + \frac{6\,b^2d^2x^2-b^2c^2+5\,abcd+2\,a^2 \cdots}{2\left(a^2b^3c^4-3\,a^3b^2c^3d+3\,a^4bc^2d^2-a^5cd^3+(b^5c^3d-3\,ab^4c^2d^2+3\,a^2b^3cd^3-a^3b^2d^4)x^3+(b^5c^4-ab^4c^3 \cdots\right.}$$

input
```
integrate(1/(b*x+a)/(a*c+(a*d+b*c)*x+b*d*x^2)^2,x, algorithm="maxima")
```

output
```
3*b*d^2*log(b*x + a)/(b^4*c^4 - 4*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 - 4*a^3*
b*c*d^3 + a^4*d^4) - 3*b*d^2*log(d*x + c)/(b^4*c^4 - 4*a*b^3*c^3*d + 6*a^2
*b^2*c^2*d^2 - 4*a^3*b*c*d^3 + a^4*d^4) + 1/2*(6*b^2*d^2*x^2 - b^2*c^2 + 5
*a*b*c*d + 2*a^2*d^2 + 3*(b^2*c*d + 3*a*b*d^2)*x)/(a^2*b^3*c^4 - 3*a^3*b^2
*c^3*d + 3*a^4*b*c^2*d^2 - a^5*c*d^3 + (b^5*c^3*d - 3*a*b^4*c^2*d^2 + 3*a^
2*b^3*c*d^3 - a^3*b^2*d^4)*x^3 + (b^5*c^4 - a*b^4*c^3*d - 3*a^2*b^3*c^2*d^
2 + 5*a^3*b^2*c*d^3 - 2*a^4*b*d^4)*x^2 + (2*a*b^4*c^4 - 5*a^2*b^3*c^3*d +
3*a^3*b^2*c^2*d^2 + a^4*b*c*d^3 - a^5*d^4)*x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 253 vs. $2(107) = 214$.

Time = 0.13 (sec) , antiderivative size = 253, normalized size of antiderivative = 2.32

$$\int \frac{1}{(a+bx)\left(ac+(bc+ad)x+bdx^2\right)^2}\,dx$$
$$= \frac{3\,b^2d^2\log\left(|bx+a|\right)}{b^5c^4-4\,ab^4c^3d+6\,a^2b^3c^2d^2-4\,a^3b^2cd^3+a^4bd^4} - \frac{3\,bd^3\log\left(|dx+c|\right)}{b^4c^4d-4\,ab^3c^3d^2+6\,a^2b^2c^2d^3-4\,a^3bcd^4+a^4d^5} - \frac{b^3c^3-6\,ab^2c^2d+3\,a^2bcd^2+2\,a^3d^3-6\,(b^3cd^2-ab^2d^3)x^2-3\,(b^3c^2d+2\,ab^2cd^2-3\,a^2bd^3)x}{2\,(bc-ad)^4(bx+a)^2(dx+c)}$$

input
```
integrate(1/(b*x+a)/(a*c+(a*d+b*c)*x+b*d*x^2)^2,x, algorithm="giac")
```

output
```
3*b^2*d^2*log(abs(b*x + a))/(b^5*c^4 - 4*a*b^4*c^3*d + 6*a^2*b^3*c^2*d^2 -
 4*a^3*b^2*c*d^3 + a^4*b*d^4) - 3*b*d^3*log(abs(d*x + c))/(b^4*c^4*d - 4*a
*b^3*c^3*d^2 + 6*a^2*b^2*c^2*d^3 - 4*a^3*b*c*d^4 + a^4*d^5) - 1/2*(b^3*c^3
 - 6*a*b^2*c^2*d + 3*a^2*b*c*d^2 + 2*a^3*d^3 - 6*(b^3*c*d^2 - a*b^2*d^3)*x
^2 - 3*(b^3*c^2*d + 2*a*b^2*c*d^2 - 3*a^2*b*d^3)*x)/((b*c - a*d)^4*(b*x +
a)^2*(d*x + c))
```

Mupad [B] (verification not implemented)

Time = 5.85 (sec) , antiderivative size = 330, normalized size of antiderivative = 3.03

$$\int \frac{1}{(a+bx)\left(ac+(bc+ad)x+bdx^2\right)^2}\,dx$$
$$=\frac{6\,b\,d^2\,\mathrm{atanh}\left(\frac{a^4\,d^4-2\,a^3\,b\,c\,d^3+2\,a\,b^3\,c^3\,d-b^4\,c^4}{{\left(a\,d-b\,c\right)}^4}+\frac{2\,b\,d\,x\,\left(a^3\,d^3-3\,a^2\,b\,c\,d^2+3\,a\,b^2\,c^2\,d-b^3\,c^3\right)}{{\left(a\,d-b\,c\right)}^4}\right)}{{\left(a\,d-b\,c\right)}^4}$$
$$-\frac{\frac{2\,a^2\,d^2+5\,a\,b\,c\,d-b^2\,c^2}{2\,\left(a^3\,d^3-3\,a^2\,b\,c\,d^2+3\,a\,b^2\,c^2\,d-b^3\,c^3\right)}+\frac{3\,d\,x\,\left(c\,b^2+3\,a\,d\,b\right)}{2\,\left(a^3\,d^3-3\,a^2\,b\,c\,d^2+3\,a\,b^2\,c^2\,d-b^3\,c^3\right)}+\frac{3\,b^2\,d^2\,x^2}{a^3\,d^3-3\,a^2\,b\,c\,d^2+3\,a\,b^2\,c^2\,d-b^3\,c^3}}{x\,\left(d\,a^2+2\,b\,c\,a\right)+a^2\,c+x^2\,\left(c\,b^2+2\,a\,d\,b\right)+b^2\,d\,x^3}$$

input
```
int(1/((a + b*x)*(a*c + x*(a*d + b*c) + b*d*x^2)^2),x)
```

output
```
(6*b*d^2*atanh((a^4*d^4 - b^4*c^4 + 2*a*b^3*c^3*d - 2*a^3*b*c*d^3)/(a*d -
b*c)^4 + (2*b*d*x*(a^3*d^3 - b^3*c^3 + 3*a*b^2*c^2*d - 3*a^2*b*c*d^2))/(a*
d - b*c)^4))/(a*d - b*c)^4 - ((2*a^2*d^2 - b^2*c^2 + 5*a*b*c*d)/(2*(a^3*d^
3 - b^3*c^3 + 3*a*b^2*c^2*d - 3*a^2*b*c*d^2)) + (3*d*x*(b^2*c + 3*a*b*d))/
(2*(a^3*d^3 - b^3*c^3 + 3*a*b^2*c^2*d - 3*a^2*b*c*d^2)) + (3*b^2*d^2*x^2)/
(a^3*d^3 - b^3*c^3 + 3*a*b^2*c^2*d - 3*a^2*b*c*d^2))/(x*(a^2*d + 2*a*b*c)
+ a^2*c + x^2*(b^2*c + 2*a*b*d) + b^2*d*x^3)
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 821, normalized size of antiderivative = 7.53

$$\int \frac{1}{(a+bx)\left(ac+(bc+ad)x+bdx^2\right)^2}\,dx = \text{Too large to display}$$

input
```
int(1/(b*x+a)/(a*c+(a*d+b*c)*x+b*d*x^2)^2,x)
```

output
```
(12*log(a + b*x)*a**3*b*c*d**3 + 12*log(a + b*x)*a**3*b*d**4*x + 6*log(a +
 b*x)*a**2*b**2*c**2*d**2 + 30*log(a + b*x)*a**2*b**2*c*d**3*x + 24*log(a
+ b*x)*a**2*b**2*d**4*x**2 + 12*log(a + b*x)*a*b**3*c**2*d**2*x + 24*log(a
 + b*x)*a*b**3*c*d**3*x**2 + 12*log(a + b*x)*a*b**3*d**4*x**3 + 6*log(a +
b*x)*b**4*c**2*d**2*x**2 + 6*log(a + b*x)*b**4*c*d**3*x**3 - 12*log(c + d*
x)*a**3*b*c*d**3 - 12*log(c + d*x)*a**3*b*d**4*x - 6*log(c + d*x)*a**2*b**
2*c**2*d**2 - 30*log(c + d*x)*a**2*b**2*c*d**3*x - 24*log(c + d*x)*a**2*b*
*2*d**4*x**2 - 12*log(c + d*x)*a*b**3*c**2*d**2*x - 24*log(c + d*x)*a*b**3
*c*d**3*x**2 - 12*log(c + d*x)*a*b**3*d**4*x**3 - 6*log(c + d*x)*b**4*c**2
*d**2*x**2 - 6*log(c + d*x)*b**4*c*d**3*x**3 - 4*a**4*d**4 - 2*a**3*b*c*d*
*3 - 12*a**3*b*d**4*x + 3*a**2*b**2*c**2*d**2 + 9*a**2*b**2*c*d**3*x + 4*a
*b**3*c**3*d + 6*a*b**3*d**4*x**3 - b**4*c**4 + 3*b**4*c**3*d*x - 6*b**4*c
*d**3*x**3)/(2*(2*a**7*c*d**5 + 2*a**7*d**6*x - 7*a**6*b*c**2*d**4 - 3*a**
6*b*c*d**5*x + 4*a**6*b*d**6*x**2 + 8*a**5*b**2*c**3*d**3 - 6*a**5*b**2*c*
*2*d**4*x - 12*a**5*b**2*c*d**5*x**2 + 2*a**5*b**2*d**6*x**3 - 2*a**4*b**3
*c**4*d**2 + 14*a**4*b**3*c**3*d**3*x + 9*a**4*b**3*c**2*d**4*x**2 - 7*a**
4*b**3*c*d**5*x**3 - 2*a**3*b**4*c**5*d - 6*a**3*b**4*c**4*d**2*x + 4*a**3
*b**4*c**3*d**3*x**2 + 8*a**3*b**4*c**2*d**4*x**3 + a**2*b**5*c**6 - 3*a**
2*b**5*c**5*d*x - 6*a**2*b**5*c**4*d**2*x**2 - 2*a**2*b**5*c**3*d**3*x**3
+ 2*a*b**6*c**6*x - 2*a*b**6*c**4*d**2*x**3 + b**7*c**6*x**2 + b**7*c**...
```

3.60 $\int \frac{(a+bx)^8}{\left(ac+(bc+ad)x+bdx^2\right)^3}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 133

$$\int \frac{(a+bx)^8}{\left(ac+(bc+ad)x+bdx^2\right)^3}\,dx = \frac{10b^3(bc-ad)^2x}{d^5}+\frac{(bc-ad)^5}{2d^6(c+dx)^2} - \frac{5b(bc-ad)^4}{d^6(c+dx)}-\frac{5b^4(bc-ad)(c+dx)^2}{2d^6} + \frac{b^5(c+dx)^3}{3d^6}-\frac{10b^2(bc-ad)^3\log(c+dx)}{d^6}$$

output

```
10*b^3*(-a*d+b*c)^2*x/d^5+1/2*(-a*d+b*c)^5/d^6/(d*x+c)^2-5*b*(-a*d+b*c)^4/
d^6/(d*x+c)-5/2*b^4*(-a*d+b*c)*(d*x+c)^2/d^6+1/3*b^5*(d*x+c)^3/d^6-10*b^2*
(-a*d+b*c)^3*ln(d*x+c)/d^6
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 230, normalized size of antiderivative = 1.73

$$\int \frac{(a+bx)^8}{\left(ac+(bc+ad)x+bdx^2\right)^3}\,dx = \frac{-3a^5d^5-15a^4bd^4(c+2dx)+30a^3b^2cd^3(3c+4dx)+30a^2b^3d^2(-5c^3-4c^2dx+4cd^2x^2+2d^3x^3)+15ab}{}$$

input
```
Integrate[(a + b*x)^8/(a*c + (b*c + a*d)*x + b*d*x^2)^3,x]
```

output
```
(-3*a^5*d^5 - 15*a^4*b*d^4*(c + 2*d*x) + 30*a^3*b^2*c*d^3*(3*c + 4*d*x) +
30*a^2*b^3*d^2*(-5*c^3 - 4*c^2*d*x + 4*c*d^2*x^2 + 2*d^3*x^3) + 15*a*b^4*d
*(7*c^4 + 2*c^3*d*x - 11*c^2*d^2*x^2 - 4*c*d^3*x^3 + d^4*x^4) + b^5*(-27*c
^5 + 6*c^4*d*x + 63*c^3*d^2*x^2 + 20*c^2*d^3*x^3 - 5*c*d^4*x^4 + 2*d^5*x^5
) - 60*b^2*(b*c - a*d)^3*(c + d*x)^2*Log[c + d*x])/(6*d^6*(c + d*x)^2)
```

Rubi [A] (verified)

Time = 0.57 (sec) , antiderivative size = 133, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.069$, Rules used = {1121, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx)^8}{\left(x(ad+bc)+ac+bdx^2\right)^3}dx$$

$$\Big\downarrow \text{1121}$$

$$\int \left(-\frac{5b^4(c+dx)(bc-ad)}{d^5}+\frac{10b^3(bc-ad)^2}{d^5}-\frac{10b^2(bc-ad)^3}{d^5(c+dx)}+\frac{5b(bc-ad)^4}{d^5(c+dx)^2}+\frac{(ad-bc)^5}{d^5(c+dx)^3}+\frac{b^5(c+dx)^2}{d^5}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{5b^4(c+dx)^2(bc-ad)}{2d^6}+\frac{10b^3x(bc-ad)^2}{d^5}-\frac{10b^2(bc-ad)^3\log(c+dx)}{d^6}-\frac{5b(bc-ad)^4}{d^6(c+dx)}+\frac{(bc-ad)^5}{2d^6(c+dx)^2}+\frac{b^5(c+dx)^3}{3d^6}$$

input
```
Int[(a + b*x)^8/(a*c + (b*c + a*d)*x + b*d*x^2)^3,x]
```

output
```
(10*b^3*(b*c - a*d)^2*x)/d^5 + (b*c - a*d)^5/(2*d^6*(c + d*x)^2) - (5*b*(b
*c - a*d)^4)/(d^6*(c + d*x)) - (5*b^4*(b*c - a*d)*(c + d*x)^2)/(2*d^6) + (
b^5*(c + d*x)^3)/(3*d^6) - (10*b^2*(b*c - a*d)^3*Log[c + d*x])/d^6
```

Defintions of rubi rules used

rule 1121
```
Int[((d_) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Int[ExpandIntegrand[(d + e*x)^(m + p)*(a/d + (c/e)*x)^p, x], x]
/; FreeQ[{a, b, c, d, e, m, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && (Int
egerQ[p] || (GtQ[a, 0] && GtQ[d, 0] && LtQ[c, 0]))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 1.51 (sec) , antiderivative size = 254, normalized size of antiderivative = 1.91

method	result
default	$\frac{b^3\left(\frac{1}{3}x^3b^2d^2+\frac{5}{2}ab\,d^2x^2-\frac{3}{2}b^2c\,x^2d+10a^2d^2x-15abcdx+6x\,b^2c^2\right)}{d^5}-\frac{5b\left(a^4d^4-4a^3bc\,d^3+6a^2b^2c^2d^2-4a\,b^3c^3d+c^4b^4\right)}{d^6(dx+c)}-\frac{a^5}{}$
risch	$\frac{b^5x^3}{3d^3}+\frac{5b^4a\,x^2}{2d^3}-\frac{3b^5c\,x^2}{2d^4}+\frac{10b^3a^2x}{d^3}-\frac{15b^4acx}{d^4}+\frac{6b^5x\,c^2}{d^5}+\frac{\left(-5a^4b\,d^4+20b^2c\,d^3a^3-30a^2b^3c^2d^2+20a\,b^4c^3d-5b^5\right.}{}$
norman	$\frac{\frac{b^7x^7}{3d}+\frac{b^5\left(46a^2d^2-35abcd+10b^2c^2\right)x^5}{3d^3}+\frac{b^6(19ad-5bc)x^6}{6d^2}-\frac{\left(198a^5b^4d^5+190a^4c\,d^4b^5+305a^3c^2d^3b^6-415c^3d^2a^2b^7+10a\,c^4d\,b^8+90c^5b^9\right.}{6d^6b^2}}{}$
parallelrisch	$\frac{60a^2b^3d^5x^3+90a^3b^2c^2d^3-270a^2b^3c^3d^2+270a\,b^4c^4d-15a^4bc\,d^4-120b^5c^4dx-3a^5d^5+120a^3b^2c\,d^4x-360a^2b^3c^2d^3x+360a\,b^4}{}$

input
```
int((b*x+a)^8/(a*c+(a*d+b*c)*x+b*d*x^2)^3,x,method=_RETURNVERBOSE)
```

output
```
b^3/d^5*(1/3*x^3*b^2*d^2+5/2*a*b*d^2*x^2-3/2*b^2*c*x^2*d+10*a^2*d^2*x-15*a
*b*c*d*x+6*x*b^2*c^2)-5*b/d^6*(a^4*d^4-4*a^3*b*c*d^3+6*a^2*b^2*c^2*d^2-4*a
*b^3*c^3*d+b^4*c^4)/(d*x+c)-1/2*(a^5*d^5-5*a^4*b*c*d^4+10*a^3*b^2*c^2*d^3-
10*a^2*b^3*c^3*d^2+5*a*b^4*c^4*d-b^5*c^5)/d^6/(d*x+c)^2+10*b^2/d^6*(a^3*d^
3-3*a^2*b*c*d^2+3*a*b^2*c^2*d-b^3*c^3)*ln(d*x+c)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 416 vs. $2(127) = 254$.

Time = 0.07 (sec) , antiderivative size = 416, normalized size of antiderivative = 3.13

$$\int \frac{(a+bx)^8}{(ac+(bc+ad)x+bdx^2)^3}\,dx$$
$$= \frac{2\,b^5d^5x^5 - 27\,b^5c^5 + 105\,ab^4c^4d - 150\,a^2b^3c^3d^2 + 90\,a^3b^2c^2d^3 - 15\,a^4bcd^4 - 3\,a^5d^5 - 5\,(b^5cd^4 - 3\,ab^4d^5)x}{}$$

input
```
integrate((b*x+a)^8/(a*c+(a*d+b*c)*x+b*d*x^2)^3,x, algorithm="fricas")
```

output
```
1/6*(2*b^5*d^5*x^5 - 27*b^5*c^5 + 105*a*b^4*c^4*d - 150*a^2*b^3*c^3*d^2 +
90*a^3*b^2*c^2*d^3 - 15*a^4*b*c*d^4 - 3*a^5*d^5 - 5*(b^5*c*d^4 - 3*a*b^4*d
^5)*x^4 + 20*(b^5*c^2*d^3 - 3*a*b^4*c*d^4 + 3*a^2*b^3*d^5)*x^3 + 3*(21*b^5
*c^3*d^2 - 55*a*b^4*c^2*d^3 + 40*a^2*b^3*c*d^4)*x^2 + 6*(b^5*c^4*d + 5*a*b
^4*c^3*d^2 - 20*a^2*b^3*c^2*d^3 + 20*a^3*b^2*c*d^4 - 5*a^4*b*d^5)*x - 60*(
b^5*c^5 - 3*a*b^4*c^4*d + 3*a^2*b^3*c^3*d^2 - a^3*b^2*c^2*d^3 + (b^5*c^3*d
^2 - 3*a*b^4*c^2*d^3 + 3*a^2*b^3*c*d^4 - a^3*b^2*d^5)*x^2 + 2*(b^5*c^4*d -
 3*a*b^4*c^3*d^2 + 3*a^2*b^3*c^2*d^3 - a^3*b^2*c*d^4)*x)*log(d*x + c))/(d^
8*x^2 + 2*c*d^7*x + c^2*d^6)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 258 vs. $2(121) = 242$.

Time = 1.04 (sec) , antiderivative size = 258, normalized size of antiderivative = 1.94

$$\int \frac{(a+bx)^8}{(ac+(bc+ad)x+bdx^2)^3}\,dx$$
$$= \frac{b^5x^3}{3d^3} + \frac{10b^2(ad-bc)^3\log{(c+dx)}}{d^6} + x^2\cdot\left(\frac{5ab^4}{2d^3} - \frac{3b^5c}{2d^4}\right) + x\left(\frac{10a^2b^3}{d^3} - \frac{15ab^4c}{d^4} + \frac{6b^5c^2}{d^5}\right)$$
$$+ \frac{-a^5d^5 - 5a^4bcd^4 + 30a^3b^2c^2d^3 - 50a^2b^3c^3d^2 + 35ab^4c^4d - 9b^5c^5 + x(-10a^4bd^5 + 40a^3b^2cd^4 - 60a^2b^3c}{2c^2d^6 + 4cd^7x + 2d^8x^2}$$

input
```
integrate((b*x+a)**8/(a*c+(a*d+b*c)*x+b*d*x**2)**3,x)
```

output
```
b**5*x**3/(3*d**3) + 10*b**2*(a*d - b*c)**3*log(c + d*x)/d**6 + x**2*(5*a*
b**4/(2*d**3) - 3*b**5*c/(2*d**4)) + x*(10*a**2*b**3/d**3 - 15*a*b**4*c/d*
*4 + 6*b**5*c**2/d**5) + (-a**5*d**5 - 5*a**4*b*c*d**4 + 30*a**3*b**2*c**2
*d**3 - 50*a**2*b**3*c**3*d**2 + 35*a*b**4*c**4*d - 9*b**5*c**5 + x*(-10*a
**4*b*d**5 + 40*a**3*b**2*c*d**4 - 60*a**2*b**3*c**2*d**3 + 40*a*b**4*c**3
*d**2 - 10*b**5*c**4*d))/(2*c**2*d**6 + 4*c*d**7*x + 2*d**8*x**2)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 271 vs. $2(127) = 254$.

Time = 0.04 (sec) , antiderivative size = 271, normalized size of antiderivative = 2.04

$$\int \frac{(a+bx)^8}{(ac+(bc+ad)x+bdx^2)^3}\,dx = -\frac{9\,b^5c^5 - 35\,ab^4c^4d + 50\,a^2b^3c^3d^2 - 30\,a^3b^2c^2d^3 + 5\,a^4bcd^4 + a^5d^5 + 10\,(b^5c^4d - 4\,ab^4c^3d^2 + 6\,a^2b^3c^2d^3}{2\,(d^8x^2 + 2\,cd^7x + c^2d^6)} + \frac{2\,b^5d^2x^3 - 3\,(3\,b^5cd - 5\,ab^4d^2)x^2 + 6\,(6\,b^5c^2 - 15\,ab^4cd + 10\,a^2b^3d^2)x}{6\,d^5} - \frac{10\,(b^5c^3 - 3\,ab^4c^2d + 3\,a^2b^3cd^2 - a^3b^2d^3)\log(dx+c)}{d^6}$$

input
```
integrate((b*x+a)^8/(a*c+(a*d+b*c)*x+b*d*x^2)^3,x, algorithm="maxima")
```

output
```
-1/2*(9*b^5*c^5 - 35*a*b^4*c^4*d + 50*a^2*b^3*c^3*d^2 - 30*a^3*b^2*c^2*d^3
 + 5*a^4*b*c*d^4 + a^5*d^5 + 10*(b^5*c^4*d - 4*a*b^4*c^3*d^2 + 6*a^2*b^3*c
^2*d^3 - 4*a^3*b^2*c*d^4 + a^4*b*d^5)*x)/(d^8*x^2 + 2*c*d^7*x + c^2*d^6) +
 1/6*(2*b^5*d^2*x^3 - 3*(3*b^5*c*d - 5*a*b^4*d^2)*x^2 + 6*(6*b^5*c^2 - 15*
a*b^4*c*d + 10*a^2*b^3*d^2)*x)/d^5 - 10*(b^5*c^3 - 3*a*b^4*c^2*d + 3*a^2*b
^3*c*d^2 - a^3*b^2*d^3)*log(d*x + c)/d^6
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 264 vs. $2(127) = 254$.

Time = 0.12 (sec) , antiderivative size = 264, normalized size of antiderivative = 1.98

$$\int \frac{(a+bx)^8}{(ac+(bc+ad)x+bdx^2)^3}\,dx$$
$$= -\frac{10\,(b^5c^3 - 3\,ab^4c^2d + 3\,a^2b^3cd^2 - a^3b^2d^3)\log(|dx+c|)}{d^6} - \frac{9\,b^5c^5 - 35\,ab^4c^4d + 50\,a^2b^3c^3d^2 - 30\,a^3b^2c^2d^3 + 5\,a^4bcd^4 + a^5d^5 + 10\,(b^5c^4d - 4\,ab^4c^3d^2 + 6\,a^2b^3c^2d^3 -}{2\,(dx+c)^2d^6} + \frac{2\,b^5d^6x^3 - 9\,b^5cd^5x^2 + 15\,ab^4d^6x^2 + 36\,b^5c^2d^4x - 90\,ab^4cd^5x + 60\,a^2b^3d^6x}{6\,d^9}$$

input
```
integrate((b*x+a)^8/(a*c+(a*d+b*c)*x+b*d*x^2)^3,x, algorithm="giac")
```

output
```
-10*(b^5*c^3 - 3*a*b^4*c^2*d + 3*a^2*b^3*c*d^2 - a^3*b^2*d^3)*log(abs(d*x
+ c))/d^6 - 1/2*(9*b^5*c^5 - 35*a*b^4*c^4*d + 50*a^2*b^3*c^3*d^2 - 30*a^3*
b^2*c^2*d^3 + 5*a^4*b*c*d^4 + a^5*d^5 + 10*(b^5*c^4*d - 4*a*b^4*c^3*d^2 +
6*a^2*b^3*c^2*d^3 - 4*a^3*b^2*c*d^4 + a^4*b*d^5)*x)/((d*x + c)^2*d^6) + 1/
6*(2*b^5*d^6*x^3 - 9*b^5*c*d^5*x^2 + 15*a*b^4*d^6*x^2 + 36*b^5*c^2*d^4*x -
 90*a*b^4*c*d^5*x + 60*a^2*b^3*d^6*x)/d^9
```

Mupad [B] (verification not implemented)

Time = 6.01 (sec) , antiderivative size = 291, normalized size of antiderivative = 2.19

$$\int \frac{(a+bx)^8}{(ac+(bc+ad)x+bdx^2)^3}\,dx = x^2\left(\frac{5\,a\,b^4}{2\,d^3} - \frac{3\,b^5\,c}{2\,d^4}\right) - \frac{\frac{a^5\,d^5+5\,a^4\,b\,c\,d^4-30\,a^3\,b^2\,c^2\,d^3+50\,a^2\,b^3\,c^3\,d^2-35\,a\,b^4\,c^4\,d+9\,b^5\,c^5}{2\,d} + x\,(5\,a^4\,b\,d^4 - 20\,a^3\,b^2\,c\,d^3 + 30\,a^2\,b^3\,c^2\,d^2 - 20\,a\,b^4}{c^2\,d^5 + 2\,c\,d^6\,x + d^7\,x^2} - x\left(\frac{3\,c\left(\frac{5\,a\,b^4}{d^3} - \frac{3\,b^5\,c}{d^4}\right)}{d} - \frac{10\,a^2\,b^3}{d^3} + \frac{3\,b^5\,c^2}{d^5}\right) - \frac{\ln(c+d\,x)\,(-10\,a^3\,b^2\,d^3 + 30\,a^2\,b^3\,c\,d^2 - 30\,a\,b^4\,c^2\,d + 10\,b^5\,c^3)}{d^6} + \frac{b^5\,x^3}{3\,d^3}$$

input
```
int((a + b*x)^8/(a*c + x*(a*d + b*c) + b*d*x^2)^3,x)
```

output
```
x^2*((5*a*b^4)/(2*d^3) - (3*b^5*c)/(2*d^4)) - ((a^5*d^5 + 9*b^5*c^5 + 50*a
^2*b^3*c^3*d^2 - 30*a^3*b^2*c^2*d^3 - 35*a*b^4*c^4*d + 5*a^4*b*c*d^4)/(2*d
) + x*(5*b^5*c^4 + 5*a^4*b*d^4 - 20*a^3*b^2*c*d^3 + 30*a^2*b^3*c^2*d^2 - 2
0*a*b^4*c^3*d))/(c^2*d^5 + d^7*x^2 + 2*c*d^6*x) - x*((3*c*((5*a*b^4)/d^3 -
 (3*b^5*c)/d^4))/d - (10*a^2*b^3)/d^3 + (3*b^5*c^2)/d^5) - (log(c + d*x)*(
10*b^5*c^3 - 10*a^3*b^2*d^3 + 30*a^2*b^3*c*d^2 - 30*a*b^4*c^2*d))/d^6 + (b
^5*x^3)/(3*d^3)
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 470, normalized size of antiderivative = 3.53

$$\int \frac{(a+bx)^8}{(ac+(bc+ad)x+bdx^2)^3}\,dx = \frac{60\log(dx+c)\,a^3b^2c^3d^3 - 180\log(dx+c)\,a^2b^3c^4d^2 + 180\log(dx+c)\,a\,b^4c^5d - 120\log(dx+c)\,b^5c^5dx - \cdots}{}$$

input
```
int((b*x+a)^8/(a*c+(a*d+b*c)*x+b*d*x^2)^3,x)
```

output
```
(60*log(c + d*x)*a**3*b**2*c**3*d**3 + 120*log(c + d*x)*a**3*b**2*c**2*d**
4*x + 60*log(c + d*x)*a**3*b**2*c*d**5*x**2 - 180*log(c + d*x)*a**2*b**3*c
**4*d**2 - 360*log(c + d*x)*a**2*b**3*c**3*d**3*x - 180*log(c + d*x)*a**2*
b**3*c**2*d**4*x**2 + 180*log(c + d*x)*a*b**4*c**5*d + 360*log(c + d*x)*a*
b**4*c**4*d**2*x + 180*log(c + d*x)*a*b**4*c**3*d**3*x**2 - 60*log(c + d*x
)*b**5*c**6 - 120*log(c + d*x)*b**5*c**5*d*x - 60*log(c + d*x)*b**5*c**4*d
**2*x**2 - 3*a**5*c*d**5 + 15*a**4*b*d**6*x**2 + 30*a**3*b**2*c**3*d**3 -
60*a**3*b**2*c*d**5*x**2 - 90*a**2*b**3*c**4*d**2 + 180*a**2*b**3*c**2*d**
4*x**2 + 60*a**2*b**3*c*d**5*x**3 + 90*a*b**4*c**5*d - 180*a*b**4*c**3*d**
3*x**2 - 60*a*b**4*c**2*d**4*x**3 + 15*a*b**4*c*d**5*x**4 - 30*b**5*c**6 +
 60*b**5*c**4*d**2*x**2 + 20*b**5*c**3*d**3*x**3 - 5*b**5*c**2*d**4*x**4 +
 2*b**5*c*d**5*x**5)/(6*c*d**6*(c**2 + 2*c*d*x + d**2*x**2))
```

3.61 $\int \frac{(a+bx)^7}{(ac+(bc+ad)x+bdx^2)^3}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 103

$$\int \frac{(a+bx)^7}{(ac+(bc+ad)x+bdx^2)^3}\,dx = -\frac{b^3(3bc-4ad)x}{d^4}+\frac{b^4x^2}{2d^3}-\frac{(bc-ad)^4}{2d^5(c+dx)^2}+\frac{4b(bc-ad)^3}{d^5(c+dx)}+\frac{6b^2(bc-ad)^2\log(c+dx)}{d^5}$$

output
```
-b^3*(-4*a*d+3*b*c)*x/d^4+1/2*b^4*x^2/d^3-1/2*(-a*d+b*c)^4/d^5/(d*x+c)^2+4
*b*(-a*d+b*c)^3/d^5/(d*x+c)+6*b^2*(-a*d+b*c)^2*ln(d*x+c)/d^5
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 167, normalized size of antiderivative = 1.62

$$\int \frac{(a+bx)^7}{(ac+(bc+ad)x+bdx^2)^3}\,dx = \frac{-a^4d^4-4a^3bd^3(c+2dx)+6a^2b^2cd^2(3c+4dx)+4ab^3d(-5c^3-4c^2dx+4cd^2x^2+2d^3x^3)+b^4(7c^4+2c}{2d^5(c+dx)^2}$$

input
```
Integrate[(a + b*x)^7/(a*c + (b*c + a*d)*x + b*d*x^2)^3,x]
```

output
```
(-(a^4*d^4) - 4*a^3*b*d^3*(c + 2*d*x) + 6*a^2*b^2*c*d^2*(3*c + 4*d*x) + 4*
a*b^3*d*(-5*c^3 - 4*c^2*d*x + 4*c*d^2*x^2 + 2*d^3*x^3) + b^4*(7*c^4 + 2*c^
3*d*x - 11*c^2*d^2*x^2 - 4*c*d^3*x^3 + d^4*x^4) + 12*b^2*(b*c - a*d)^2*(c
+ d*x)^2*Log[c + d*x])/(2*d^5*(c + d*x)^2)
```

Rubi [A] (verified)

Time = 0.49 (sec) , antiderivative size = 103, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.069$, Rules used = {1121, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx)^7}{(x(ad+bc)+ac+bdx^2)^3}\,dx$$

$$\Big\downarrow \text{1121}$$

$$\int \left(-\frac{b^3(3bc-4ad)}{d^4}+\frac{6b^2(bc-ad)^2}{d^4(c+dx)}-\frac{4b(bc-ad)^3}{d^4(c+dx)^2}+\frac{(ad-bc)^4}{d^4(c+dx)^3}+\frac{b^4x}{d^3}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{b^3x(3bc-4ad)}{d^4}+\frac{6b^2(bc-ad)^2\log(c+dx)}{d^5}+\frac{4b(bc-ad)^3}{d^5(c+dx)}-\frac{(bc-ad)^4}{2d^5(c+dx)^2}+\frac{b^4x^2}{2d^3}$$

input
```
Int[(a + b*x)^7/(a*c + (b*c + a*d)*x + b*d*x^2)^3,x]
```

output
```
-((b^3*(3*b*c - 4*a*d)*x)/d^4) + (b^4*x^2)/(2*d^3) - (b*c - a*d)^4/(2*d^5*
(c + d*x)^2) + (4*b*(b*c - a*d)^3)/(d^5*(c + d*x)) + (6*b^2*(b*c - a*d)^2*
Log[c + d*x])/d^5
```

Defintions of rubi rules used

rule 1121
```
Int[((d_) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Int[ExpandIntegrand[(d + e*x)^(m + p)*(a/d + (c/e)*x)^p, x], x]
/; FreeQ[{a, b, c, d, e, m, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && (Int
egerQ[p] || (GtQ[a, 0] && GtQ[d, 0] && LtQ[c, 0]))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 1.42 (sec) , antiderivative size = 172, normalized size of antiderivative = 1.67

method	result
default	$\frac{b^3\left(\frac{1}{2}bd\,x^2+4adx-3cbx\right)}{d^4}-\frac{4b\left(a^3d^3-3a^2bc\,d^2+3a\,b^2c^2d-b^3c^3\right)}{d^5(dx+c)}-\frac{a^4d^4-4a^3bc\,d^3+6a^2b^2c^2d^2-4a\,b^3c^3d+c^4b^4}{2d^5(dx+c)^2}+\frac{6b^2(a^{\cdots}}{\cdots}$ ⋯
risch	$\frac{b^4x^2}{2d^3}+\frac{4b^3ax}{d^3}-\frac{3b^4cx}{d^4}+\frac{\left(-4a^3b\,d^3+12a^2b^2c\,d^2-12a\,b^3c^2d+4b^4c^3\right)x-\frac{a^4d^4+4a^3bc\,d^3-18a^2b^2c^2d^2+20a\,b^3c^3d-7c^4b^4}{2d}}{d^4(dx+c)^2}+$ ⋯
parallelrisch	$\frac{b^4d^4x^4+12\ln(dx+c)x^2a^2b^2d^4-24\ln(dx+c)x^2a\,b^3c\,d^3+12\ln(dx+c)x^2b^4c^2d^2+8a\,b^3d^4x^3-4b^4c\,d^3x^3+24\ln(dx+c)x\,a^2b^2c\,d^3\cdots}{\cdots}$ ⋯
norman	$\frac{\frac{b^5(5ad-2bc)x^5}{d^2}+\frac{b^6x^6}{2d}-\frac{a^2\left(a^4b^2d^4+4d^3c\,a^3b^3-a^2c^2d^2b^4+28a\,c^3d\,b^5-18b^6c^4\right)}{2b^2d^5}-\frac{\left(34a^4b^4d^4+16a^3b^5c\,d^3+63a^2b^6c^2d^2-20a\,b^7c^3d-18\cdots\right.}{2d^5b^2}\cdots}{(dx+c)^2(bx+a)^2\cdots}$ ⋯

input
```
int((b*x+a)^7/(a*c+(a*d+b*c)*x+b*d*x^2)^3,x,method=_RETURNVERBOSE)
```

output
```
b^3/d^4*(1/2*b*d*x^2+4*a*d*x-3*c*b*x)-4*b/d^5*(a^3*d^3-3*a^2*b*c*d^2+3*a*b
^2*c^2*d-b^3*c^3)/(d*x+c)-1/2*(a^4*d^4-4*a^3*b*c*d^3+6*a^2*b^2*c^2*d^2-4*a
*b^3*c^3*d+b^4*c^4)/d^5/(d*x+c)^2+6*b^2/d^5*(a^2*d^2-2*a*b*c*d+b^2*c^2)*ln
(d*x+c)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 291 vs. $2(99) = 198$.

Time = 0.08 (sec) , antiderivative size = 291, normalized size of antiderivative = 2.83

$$\int \frac{(a+bx)^7}{(ac+(bc+ad)x+bdx^2)^3}\,dx = \frac{b^4d^4x^4 + 7\,b^4c^4 - 20\,ab^3c^3d + 18\,a^2b^2c^2d^2 - 4\,a^3bcd^3 - a^4d^4 - 4\,(b^4cd^3 - 2\,ab^3d^4)x^3 - (11\,b^4c^2d^2 - 16\,ab}{}$$

input
```
integrate((b*x+a)^7/(a*c+(a*d+b*c)*x+b*d*x^2)^3,x, algorithm="fricas")
```

output
```
1/2*(b^4*d^4*x^4 + 7*b^4*c^4 - 20*a*b^3*c^3*d + 18*a^2*b^2*c^2*d^2 - 4*a^3
*b*c*d^3 - a^4*d^4 - 4*(b^4*c*d^3 - 2*a*b^3*d^4)*x^3 - (11*b^4*c^2*d^2 - 1
6*a*b^3*c*d^3)*x^2 + 2*(b^4*c^3*d - 8*a*b^3*c^2*d^2 + 12*a^2*b^2*c*d^3 - 4
*a^3*b*d^4)*x + 12*(b^4*c^4 - 2*a*b^3*c^3*d + a^2*b^2*c^2*d^2 + (b^4*c^2*d
^2 - 2*a*b^3*c*d^3 + a^2*b^2*d^4)*x^2 + 2*(b^4*c^3*d - 2*a*b^3*c^2*d^2 + a
^2*b^2*c*d^3)*x)*log(d*x + c))/(d^7*x^2 + 2*c*d^6*x + c^2*d^5)
```

Sympy [A] (verification not implemented)

Time = 0.65 (sec) , antiderivative size = 185, normalized size of antiderivative = 1.80

$$\int \frac{(a+bx)^7}{(ac+(bc+ad)x+bdx^2)^3}\,dx = \frac{b^4x^2}{2d^3} + \frac{6b^2(ad-bc)^2\log(c+dx)}{d^5} + x\left(\frac{4ab^3}{d^3} - \frac{3b^4c}{d^4}\right) + \frac{-a^4d^4 - 4a^3bcd^3 + 18a^2b^2c^2d^2 - 20ab^3c^3d + 7b^4c^4 + x(-8a^3bd^4 + 24a^2b^2cd^3 - 24ab^3c^2d^2 + 8b^4c^3d)}{2c^2d^5 + 4cd^6x + 2d^7x^2}$$

input
```
integrate((b*x+a)**7/(a*c+(a*d+b*c)*x+b*d*x**2)**3,x)
```

output
```
b**4*x**2/(2*d**3) + 6*b**2*(a*d - b*c)**2*log(c + d*x)/d**5 + x*(4*a*b**3
/d**3 - 3*b**4*c/d**4) + (-a**4*d**4 - 4*a**3*b*c*d**3 + 18*a**2*b**2*c**2
*d**2 - 20*a*b**3*c**3*d + 7*b**4*c**4 + x*(-8*a**3*b*d**4 + 24*a**2*b**2*
c*d**3 - 24*a*b**3*c**2*d**2 + 8*b**4*c**3*d))/(2*c**2*d**5 + 4*c*d**6*x +
 2*d**7*x**2)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 191, normalized size of antiderivative = 1.85

$$\int \frac{(a+bx)^7}{(ac+(bc+ad)x+bdx^2)^3}\,dx = \frac{7\,b^4c^4-20\,ab^3c^3d+18\,a^2b^2c^2d^2-4\,a^3bcd^3-a^4d^4+8\,(b^4c^3d-3\,ab^3c^2d^2+3\,a^2b^2cd^3-a^3bd^4)x}{2\,(d^7x^2+2\,cd^6x+c^2d^5)} + \frac{b^4dx^2-2\,(3\,b^4c-4\,ab^3d)x}{2\,d^4} + \frac{6\,(b^4c^2-2\,ab^3cd+a^2b^2d^2)\log\left(dx+c\right)}{d^5}$$

input
```
integrate((b*x+a)^7/(a*c+(a*d+b*c)*x+b*d*x^2)^3,x, algorithm="maxima")
```

output
```
1/2*(7*b^4*c^4 - 20*a*b^3*c^3*d + 18*a^2*b^2*c^2*d^2 - 4*a^3*b*c*d^3 - a^4
*d^4 + 8*(b^4*c^3*d - 3*a*b^3*c^2*d^2 + 3*a^2*b^2*c*d^3 - a^3*b*d^4)*x)/(d
^7*x^2 + 2*c*d^6*x + c^2*d^5) + 1/2*(b^4*d*x^2 - 2*(3*b^4*c - 4*a*b^3*d)*x
)/d^4 + 6*(b^4*c^2 - 2*a*b^3*c*d + a^2*b^2*d^2)*log(d*x + c)/d^5
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 183, normalized size of antiderivative = 1.78

$$\int \frac{(a+bx)^7}{(ac+(bc+ad)x+bdx^2)^3}\,dx = \frac{6\,(b^4c^2-2\,ab^3cd+a^2b^2d^2)\log\left(|dx+c|\right)}{d^5} + \frac{b^4d^3x^2-6\,b^4cd^2x+8\,ab^3d^3x}{2\,d^6} + \frac{7\,b^4c^4-20\,ab^3c^3d+18\,a^2b^2c^2d^2-4\,a^3bcd^3-a^4d^4+8\,(b^4c^3d-3\,ab^3c^2d^2+3\,a^2b^2cd^3-a^3bd^4)x}{2\,(dx+c)^2d^5}$$

input
```
integrate((b*x+a)^7/(a*c+(a*d+b*c)*x+b*d*x^2)^3,x, algorithm="giac")
```

output
```
6*(b^4*c^2 - 2*a*b^3*c*d + a^2*b^2*d^2)*log(abs(d*x + c))/d^5 + 1/2*(b^4*d
^3*x^2 - 6*b^4*c*d^2*x + 8*a*b^3*d^3*x)/d^6 + 1/2*(7*b^4*c^4 - 20*a*b^3*c^
3*d + 18*a^2*b^2*c^2*d^2 - 4*a^3*b*c*d^3 - a^4*d^4 + 8*(b^4*c^3*d - 3*a*b^
3*c^2*d^2 + 3*a^2*b^2*c*d^3 - a^3*b*d^4)*x)/((d*x + c)^2*d^5)
```

Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 196, normalized size of antiderivative = 1.90

$$\int \frac{(a+bx)^7}{(ac+(bc+ad)x+bdx^2)^3}\,dx = x\left(\frac{4\,a\,b^3}{d^3}-\frac{3\,b^4\,c}{d^4}\right) - \frac{\frac{a^4\,d^4+4\,a^3\,b\,c\,d^3-18\,a^2\,b^2\,c^2\,d^2+20\,a\,b^3\,c^3\,d-7\,b^4\,c^4}{2\,d}-x\,\left(-4\,a^3\,b\,d^3+12\,a^2\,b^2\,c\,d^2-12\,a\,b^3\,c^2\,d+4\,b^4\,c^3\right)}{c^2\,d^4+2\,c\,d^5\,x+d^6\,x^2} + \frac{b^4\,x^2}{2\,d^3} + \frac{\ln\left(c+d\,x\right)\,\left(6\,a^2\,b^2\,d^2-12\,a\,b^3\,c\,d+6\,b^4\,c^2\right)}{d^5}$$

input
```
int((a + b*x)^7/(a*c + x*(a*d + b*c) + b*d*x^2)^3,x)
```

output
```
x*((4*a*b^3)/d^3 - (3*b^4*c)/d^4) - ((a^4*d^4 - 7*b^4*c^4 - 18*a^2*b^2*c^2
*d^2 + 20*a*b^3*c^3*d + 4*a^3*b*c*d^3)/(2*d) - x*(4*b^4*c^3 - 4*a^3*b*d^3
+ 12*a^2*b^2*c*d^2 - 12*a*b^3*c^2*d))/(c^2*d^4 + d^6*x^2 + 2*c*d^5*x) + (b
^4*x^2)/(2*d^3) + (log(c + d*x)*(6*b^4*c^2 + 6*a^2*b^2*d^2 - 12*a*b^3*c*d)
)/d^5
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 330, normalized size of antiderivative = 3.20

$$\int \frac{(a+bx)^7}{(ac+(bc+ad)x+bdx^2)^3}\,dx = \frac{12\log(dx+c)\,a^2b^2c^3d^2+24\log(dx+c)\,a^2b^2c^2d^3x+12\log(dx+c)\,a^2b^2c\,d^4x^2-24\log(dx+c)\,a\,b^3c^4d-}{}$$

input
```
int((b*x+a)^7/(a*c+(a*d+b*c)*x+b*d*x^2)^3,x)
```

output

```
(12*log(c + d*x)*a**2*b**2*c**3*d**2 + 24*log(c + d*x)*a**2*b**2*c**2*d**3
*x + 12*log(c + d*x)*a**2*b**2*c*d**4*x**2 - 24*log(c + d*x)*a*b**3*c**4*d
 - 48*log(c + d*x)*a*b**3*c**3*d**2*x - 24*log(c + d*x)*a*b**3*c**2*d**3*x
**2 + 12*log(c + d*x)*b**4*c**5 + 24*log(c + d*x)*b**4*c**4*d*x + 12*log(c
 + d*x)*b**4*c**3*d**2*x**2 - a**4*c*d**4 + 4*a**3*b*d**5*x**2 + 6*a**2*b*
*2*c**3*d**2 - 12*a**2*b**2*c*d**4*x**2 - 12*a*b**3*c**4*d + 24*a*b**3*c**
2*d**3*x**2 + 8*a*b**3*c*d**4*x**3 + 6*b**4*c**5 - 12*b**4*c**3*d**2*x**2
- 4*b**4*c**2*d**3*x**3 + b**4*c*d**4*x**4)/(2*c*d**5*(c**2 + 2*c*d*x + d*
*2*x**2))
```

3.62 $\int \frac{(a+bx)^6}{(ac+(bc+ad)x+bdx^2)^3}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 78

$$\int \frac{(a+bx)^6}{(ac+(bc+ad)x+bdx^2)^3}\,dx = \frac{b^3x}{d^3}+\frac{(bc-ad)^3}{2d^4(c+dx)^2}-\frac{3b(bc-ad)^2}{d^4(c+dx)} - \frac{3b^2(bc-ad)\log(c+dx)}{d^4}$$

output
```
b^3*x/d^3+1/2*(-a*d+b*c)^3/d^4/(d*x+c)^2-3*b*(-a*d+b*c)^2/d^4/(d*x+c)-3*b^
2*(-a*d+b*c)*ln(d*x+c)/d^4
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.46

$$\int \frac{(a+bx)^6}{(ac+(bc+ad)x+bdx^2)^3}\,dx = \frac{-a^3d^3-3a^2bd^2(c+2dx)+3ab^2cd(3c+4dx)+b^3(-5c^3-4c^2dx+4cd^2x^2+2d^3x^3)-6b^2(bc-ad)(c+}{2d^4(c+dx)^2}$$

input
```
Integrate[(a + b*x)^6/(a*c + (b*c + a*d)*x + b*d*x^2)^3,x]
```

output
```
(-(a^3*d^3) - 3*a^2*b*d^2*(c + 2*d*x) + 3*a*b^2*c*d*(3*c + 4*d*x) + b^3*(-
5*c^3 - 4*c^2*d*x + 4*c*d^2*x^2 + 2*d^3*x^3) - 6*b^2*(b*c - a*d)*(c + d*x)
^2*Log[c + d*x])/(2*d^4*(c + d*x)^2)
```

Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.069, Rules used = {1121, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx)^6}{(x(ad+bc)+ac+bdx^2)^3}\,dx$$

$$\Big\downarrow \text{1121}$$

$$\int \left(-\frac{3b^2(bc-ad)}{d^3(c+dx)}+\frac{3b(bc-ad)^2}{d^3(c+dx)^2}+\frac{(ad-bc)^3}{d^3(c+dx)^3}+\frac{b^3}{d^3}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{3b^2(bc-ad)\log(c+dx)}{d^4}-\frac{3b(bc-ad)^2}{d^4(c+dx)}+\frac{(bc-ad)^3}{2d^4(c+dx)^2}+\frac{b^3x}{d^3}$$

input
```
Int[(a + b*x)^6/(a*c + (b*c + a*d)*x + b*d*x^2)^3,x]
```

output
```
(b^3*x)/d^3 + (b*c - a*d)^3/(2*d^4*(c + d*x)^2) - (3*b*(b*c - a*d)^2)/(d^4
*(c + d*x)) - (3*b^2*(b*c - a*d)*Log[c + d*x])/d^4
```

Defintions of rubi rules used

rule 1121
```
Int[((d_) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Int[ExpandIntegrand[(d + e*x)^(m + p)*(a/d + (c/e)*x)^p, x], x]
/; FreeQ[{a, b, c, d, e, m, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && (Int
egerQ[p] || (GtQ[a, 0] && GtQ[d, 0] && LtQ[c, 0]))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 1.41 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.46

method	result
default	$\frac{b^3 x}{d^3}-\frac{3b(a^2d^2-2abcd+b^2c^2)}{d^4(dx+c)}-\frac{a^3d^3-3a^2bc\,d^2+3a\,b^2c^2d-b^3c^3}{2d^4(dx+c)^2}+\frac{3b^2(ad-bc)\ln(dx+c)}{d^4}$
risch	$\frac{b^3 x}{d^3}+\frac{(-3a^2b\,d^2+6cda\,b^2-3b^3c^2)x-\frac{a^3d^3+3a^2bc\,d^2-9a\,b^2c^2d+5b^3c^3}{2d}}{d^3(dx+c)^2}+\frac{3b^2\ln(dx+c)a}{d^3}-\frac{3b^3\ln(dx+c)c}{d^4}$
parallelrisch	$\frac{6\ln(dx+c)x^2a\,b^2d^3-6\ln(dx+c)x^2b^3c\,d^2+2x^3b^3d^3+12\ln(dx+c)xa\,b^2c\,d^2-12\ln(dx+c)x\,b^3c^2d+6\ln(dx+c)a\,b^2c^2d-6\ln(dx-}{2d^4(dx+c)^2}$
norman	$\frac{\frac{b^5x^5}{d}-\frac{(17a^3b^4d^3-5a^2b^5c\,d^2+19a\,c^2d\,b^6+9c^3b^7)x^2}{2d^4b^2}-\frac{2(3a^2d^2b^4-acd\,b^5+3b^6c^2)x^3}{d^3b}-\frac{(d^3a^3b^2+3c\,d^2a^2b^3-5c^2da\,b^4+9b^5c^3)a^2}{2d^4b^2}-\frac{a(4}{}}{(dx+c)^2(bx+a)^2}$

input
```
int((b*x+a)^6/(a*c+(a*d+b*c)*x+b*d*x^2)^3,x,method=_RETURNVERBOSE)
```

output
```
b^3*x/d^3-3*b/d^4*(a^2*d^2-2*a*b*c*d+b^2*c^2)/(d*x+c)-1/2*(a^3*d^3-3*a^2*b
*c*d^2+3*a*b^2*c^2*d-b^3*c^3)/d^4/(d*x+c)^2+3*b^2/d^4*(a*d-b*c)*ln(d*x+c)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 188 vs. $2(76) = 152$.

Time = 0.07 (sec) , antiderivative size = 188, normalized size of antiderivative = 2.41

$$\int \frac{(a+bx)^6}{(ac+(bc+ad)x+bdx^2)^3}\,dx$$
$$=\frac{2\,b^3d^3x^3+4\,b^3cd^2x^2-5\,b^3c^3+9\,ab^2c^2d-3\,a^2bcd^2-a^3d^3-2\,(2\,b^3c^2d-6\,ab^2cd^2+3\,a^2bd^3)x-6\,(b^3c^3-}{2\,(d^6x^2+2\,cd^5x+c^2d^4)}$$

input
```
integrate((b*x+a)^6/(a*c+(a*d+b*c)*x+b*d*x^2)^3,x, algorithm="fricas")
```

output
```
1/2*(2*b^3*d^3*x^3 + 4*b^3*c*d^2*x^2 - 5*b^3*c^3 + 9*a*b^2*c^2*d - 3*a^2*b
*c*d^2 - a^3*d^3 - 2*(2*b^3*c^2*d - 6*a*b^2*c*d^2 + 3*a^2*b*d^3)*x - 6*(b^
3*c^3 - a*b^2*c^2*d + (b^3*c*d^2 - a*b^2*d^3)*x^2 + 2*(b^3*c^2*d - a*b^2*c
*d^2)*x)*log(d*x + c))/(d^6*x^2 + 2*c*d^5*x + c^2*d^4)
```

Sympy [A] (verification not implemented)

Time = 0.45 (sec) , antiderivative size = 128, normalized size of antiderivative = 1.64

$$\int \frac{(a+bx)^6}{(ac+(bc+ad)x+bdx^2)^3}\,dx = \frac{b^3x}{d^3}+\frac{3b^2(ad-bc)\log{(c+dx)}}{d^4} + \frac{-a^3d^3-3a^2bcd^2+9ab^2c^2d-5b^3c^3+x(-6a^2bd^3+12ab^2cd^2-6b^3c^2d)}{2c^2d^4+4cd^5x+2d^6x^2}$$

input
```
integrate((b*x+a)**6/(a*c+(a*d+b*c)*x+b*d*x**2)**3,x)
```

output
```
b**3*x/d**3 + 3*b**2*(a*d - b*c)*log(c + d*x)/d**4 + (-a**3*d**3 - 3*a**2*
b*c*d**2 + 9*a*b**2*c**2*d - 5*b**3*c**3 + x*(-6*a**2*b*d**3 + 12*a*b**2*c
*d**2 - 6*b**3*c**2*d))/(2*c**2*d**4 + 4*c*d**5*x + 2*d**6*x**2)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 125, normalized size of antiderivative = 1.60

$$\int \frac{(a+bx)^6}{(ac+(bc+ad)x+bdx^2)^3}\,dx = \frac{b^3x}{d^3}-\frac{5\,b^3c^3-9\,ab^2c^2d+3\,a^2bcd^2+a^3d^3+6\,(b^3c^2d-2\,ab^2cd^2+a^2bd^3)x}{2\,(d^6x^2+2\,cd^5x+c^2d^4)} - \frac{3\,(b^3c-ab^2d)\log{(dx+c)}}{d^4}$$

input
```
integrate((b*x+a)^6/(a*c+(a*d+b*c)*x+b*d*x^2)^3,x, algorithm="maxima")
```

output
```
b^3*x/d^3 - 1/2*(5*b^3*c^3 - 9*a*b^2*c^2*d + 3*a^2*b*c*d^2 + a^3*d^3 + 6*(
b^3*c^2*d - 2*a*b^2*c*d^2 + a^2*b*d^3)*x)/(d^6*x^2 + 2*c*d^5*x + c^2*d^4)
- 3*(b^3*c - a*b^2*d)*log(d*x + c)/d^4
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.44

$$\int \frac{(a+bx)^6}{(ac+(bc+ad)x+bdx^2)^3}\,dx = \frac{b^3x}{d^3} - \frac{3\,(b^3c-ab^2d)\log\left(|dx+c|\right)}{d^4} - \frac{5\,b^3c^3-9\,ab^2c^2d+3\,a^2bcd^2+a^3d^3+6\,(b^3c^2d-2\,ab^2cd^2+a^2bd^3)x}{2\,(dx+c)^2d^4}$$

input
```
integrate((b*x+a)^6/(a*c+(a*d+b*c)*x+b*d*x^2)^3,x, algorithm="giac")
```

output
```
b^3*x/d^3 - 3*(b^3*c - a*b^2*d)*log(abs(d*x + c))/d^4 - 1/2*(5*b^3*c^3 - 9
*a*b^2*c^2*d + 3*a^2*b*c*d^2 + a^3*d^3 + 6*(b^3*c^2*d - 2*a*b^2*c*d^2 + a^
2*b*d^3)*x)/((d*x + c)^2*d^4)
```

Mupad [B] (verification not implemented)

Time = 5.78 (sec) , antiderivative size = 130, normalized size of antiderivative = 1.67

$$\int \frac{(a+bx)^6}{(ac+(bc+ad)x+bdx^2)^3}\,dx = \frac{b^3\,x}{d^3} - \frac{\ln\left(c+d\,x\right)\,\left(3\,b^3\,c-3\,a\,b^2\,d\right)}{d^4} - \frac{\frac{a^3\,d^3+3\,a^2\,b\,c\,d^2-9\,a\,b^2\,c^2\,d+5\,b^3\,c^3}{2\,d}+x\,\left(3\,a^2\,b\,d^2-6\,a\,b^2\,c\,d+3\,b^3\,c^2\right)}{c^2\,d^3+2\,c\,d^4\,x+d^5\,x^2}$$

input
```
int((a + b*x)^6/(a*c + x*(a*d + b*c) + b*d*x^2)^3,x)
```

output
```
(b^3*x)/d^3 - (log(c + d*x)*(3*b^3*c - 3*a*b^2*d))/d^4 - ((a^3*d^3 + 5*b^3
*c^3 - 9*a*b^2*c^2*d + 3*a^2*b*c*d^2)/(2*d) + x*(3*b^3*c^2 + 3*a^2*b*d^2 -
 6*a*b^2*c*d))/(c^2*d^3 + d^5*x^2 + 2*c*d^4*x)
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 209, normalized size of antiderivative = 2.68

$$\int \frac{(a+bx)^6}{(ac+(bc+ad)x+bdx^2)^3}\,dx$$
$$= \frac{6\log(dx+c)\,a\,b^2c^3d+12\log(dx+c)\,a\,b^2c^2d^2x+6\log(dx+c)\,a\,b^2c\,d^3x^2-6\log(dx+c)\,b^3c^4-12\log(d}{2c\,d^4\,(}$$

input
```
int((b*x+a)^6/(a*c+(a*d+b*c)*x+b*d*x^2)^3,x)
```

output
```
(6*log(c + d*x)*a*b**2*c**3*d + 12*log(c + d*x)*a*b**2*c**2*d**2*x + 6*log
(c + d*x)*a*b**2*c*d**3*x**2 - 6*log(c + d*x)*b**3*c**4 - 12*log(c + d*x)*
b**3*c**3*d*x - 6*log(c + d*x)*b**3*c**2*d**2*x**2 - a**3*c*d**3 + 3*a**2*
b*d**4*x**2 + 3*a*b**2*c**3*d - 6*a*b**2*c*d**3*x**2 - 3*b**3*c**4 + 6*b**
3*c**2*d**2*x**2 + 2*b**3*c*d**3*x**3)/(2*c*d**4*(c**2 + 2*c*d*x + d**2*x*
*2))
```

3.63 $\int \frac{(a+bx)^5}{\left(ac+(bc+ad)x+bdx^2\right)^3} \, dx$

Optimal result

Integrand size = 29, antiderivative size = 59

$$\int \frac{(a+bx)^5}{\left(ac+(bc+ad)x+bdx^2\right)^3} \, dx = -\frac{(bc-ad)^2}{2d^3(c+dx)^2}+\frac{2b(bc-ad)}{d^3(c+dx)}+\frac{b^2\log(c+dx)}{d^3}$$

output
```
-1/2*(-a*d+b*c)^2/d^3/(d*x+c)^2+2*b*(-a*d+b*c)/d^3/(d*x+c)+b^2*ln(d*x+c)/d
^3
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.81

$$\int \frac{(a+bx)^5}{\left(ac+(bc+ad)x+bdx^2\right)^3} \, dx = \frac{\frac{(bc-ad)(3bc+ad+4bdx)}{(c+dx)^2}+2b^2\log(c+dx)}{2d^3}$$

input
```
Integrate[(a + b*x)^5/(a*c + (b*c + a*d)*x + b*d*x^2)^3,x]
```

output
```
(((b*c - a*d)*(3*b*c + a*d + 4*b*d*x))/(c + d*x)^2 + 2*b^2*Log[c + d*x])/(
2*d^3)
```

Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.069$, Rules used = {1121, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx)^5}{(x(ad+bc)+ac+bdx^2)^3}\,dx$$

$$\Big\downarrow \text{1121}$$

$$\int \left(-\frac{2b(bc-ad)}{d^2(c+dx)^2}+\frac{(ad-bc)^2}{d^2(c+dx)^3}+\frac{b^2}{d^2(c+dx)}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{2b(bc-ad)}{d^3(c+dx)}-\frac{(bc-ad)^2}{2d^3(c+dx)^2}+\frac{b^2\log(c+dx)}{d^3}$$

input
```
Int[(a + b*x)^5/(a*c + (b*c + a*d)*x + b*d*x^2)^3,x]
```

output
```
-1/2*(b*c - a*d)^2/(d^3*(c + d*x)^2) + (2*b*(b*c - a*d))/(d^3*(c + d*x)) +
 (b^2*Log[c + d*x])/d^3
```

Defintions of rubi rules used

rule 1121
```
Int[((d_) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Int[ExpandIntegrand[(d + e*x)^(m + p)*(a/d + (c/e)*x)^p, x], x]
/; FreeQ[{a, b, c, d, e, m, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && (Int
egerQ[p] || (GtQ[a, 0] && GtQ[d, 0] && LtQ[c, 0]))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 1.40 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.12

method	result	size
risch	$\frac{-\frac{2b(ad-bc)x}{d^2}-\frac{a^2d^2+2abcd-3b^2c^2}{2d^3}}{(dx+c)^2}+\frac{b^2\ln(dx+c)}{d^3}$	66
default	$-\frac{2b(ad-bc)}{d^3(dx+c)}-\frac{a^2d^2-2abcd+b^2c^2}{2d^3(dx+c)^2}+\frac{b^2\ln(dx+c)}{d^3}$	69
parallelrisch	$\frac{2\ln(dx+c)x^2b^2d^2+4\ln(dx+c)x\,b^2cd+2\ln(dx+c)b^2c^2-4ab\,d^2x+4b^2cxd-a^2d^2-2abcd+3b^2c^2}{2d^3(dx+c)^2}$	97
norman	$\frac{-\frac{\left(9a^2d^2b^4-6acd\,b^5-3b^6c^2\right)x^2}{2d^3b^2}-\frac{2\left(ad\,b^4-b^5c\right)x^3}{d^2b}-\frac{\left(d^2a^2b^2+2cda\,b^3-3b^4c^2\right)a^2}{2d^3b^2}-\frac{a\left(3d^2a^2b^3-3b^5c^2\right)x}{d^3b^2}}{(dx+c)^2(bx+a)^2}+\frac{b^2\ln(dx+c)}{d^3}$	162

input
```
int((b*x+a)^5/(a*c+(a*d+b*c)*x+b*d*x^2)^3,x,method=_RETURNVERBOSE)
```

output
```
(-2*b/d^2*(a*d-b*c)*x-1/2*(a^2*d^2+2*a*b*c*d-3*b^2*c^2)/d^3)/(d*x+c)^2+b^2
*ln(d*x+c)/d^3
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.69

$$\int \frac{(a+bx)^5}{(ac+(bc+ad)x+bdx^2)^3}\,dx = \frac{3\,b^2c^2-2\,abcd-a^2d^2+4\,(b^2cd-abd^2)x+2\,(b^2d^2x^2+2\,b^2cdx+b^2c^2)\log{(dx+c)}}{2\,(d^5x^2+2\,cd^4x+c^2d^3)}$$

input
```
integrate((b*x+a)^5/(a*c+(a*d+b*c)*x+b*d*x^2)^3,x, algorithm="fricas")
```

output
```
1/2*(3*b^2*c^2 - 2*a*b*c*d - a^2*d^2 + 4*(b^2*c*d - a*b*d^2)*x + 2*(b^2*d^
2*x^2 + 2*b^2*c*d*x + b^2*c^2)*log(d*x + c))/(d^5*x^2 + 2*c*d^4*x + c^2*d^
3)
```

Sympy [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.36

$$\int \frac{(a+bx)^5}{(ac+(bc+ad)x+bdx^2)^3}\,dx = \frac{b^2\log(c+dx)}{d^3} + \frac{-a^2d^2-2abcd+3b^2c^2+x(-4abd^2+4b^2cd)}{2c^2d^3+4cd^4x+2d^5x^2}$$

input
```
integrate((b*x+a)**5/(a*c+(a*d+b*c)*x+b*d*x**2)**3,x)
```

output
```
b**2*log(c + d*x)/d**3 + (-a**2*d**2 - 2*a*b*c*d + 3*b**2*c**2 + x*(-4*a*b
*d**2 + 4*b**2*c*d))/(2*c**2*d**3 + 4*c*d**4*x + 2*d**5*x**2)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.36

$$\int \frac{(a+bx)^5}{(ac+(bc+ad)x+bdx^2)^3}\,dx = \frac{3\,b^2c^2-2\,abcd-a^2d^2+4\,(b^2cd-abd^2)x}{2\,(d^5x^2+2\,cd^4x+c^2d^3)} + \frac{b^2\log(dx+c)}{d^3}$$

input
```
integrate((b*x+a)^5/(a*c+(a*d+b*c)*x+b*d*x^2)^3,x, algorithm="maxima")
```

output
```
1/2*(3*b^2*c^2 - 2*a*b*c*d - a^2*d^2 + 4*(b^2*c*d - a*b*d^2)*x)/(d^5*x^2 +
 2*c*d^4*x + c^2*d^3) + b^2*log(d*x + c)/d^3
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.17

$$\int \frac{(a+bx)^5}{(ac+(bc+ad)x+bdx^2)^3}\,dx = \frac{b^2\log(|dx+c|)}{d^3} + \frac{4\,(b^2c-abd)x+\frac{3\,b^2c^2-2\,abcd-a^2d^2}{d}}{2\,(dx+c)^2d^2}$$

input
```
integrate((b*x+a)^5/(a*c+(a*d+b*c)*x+b*d*x^2)^3,x, algorithm="giac")
```

output
```
b^2*log(abs(d*x + c))/d^3 + 1/2*(4*(b^2*c - a*b*d)*x + (3*b^2*c^2 - 2*a*b*
c*d - a^2*d^2)/d)/((d*x + c)^2*d^2)
```

Mupad [B] (verification not implemented)

Time = 6.03 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.31

$$\int \frac{(a+bx)^5}{(ac+(bc+ad)x+bdx^2)^3}\,dx = \frac{b^2\,\ln(c+d\,x)}{d^3} - \frac{\frac{a^2\,d^2+2\,a\,b\,c\,d-3\,b^2\,c^2}{2\,d^3}+\frac{2\,b\,x\,(a\,d-b\,c)}{d^2}}{c^2+2\,c\,d\,x+d^2\,x^2}$$

input
```
int((a + b*x)^5/(a*c + x*(a*d + b*c) + b*d*x^2)^3,x)
```

output
```
(b^2*log(c + d*x))/d^3 - ((a^2*d^2 - 3*b^2*c^2 + 2*a*b*c*d)/(2*d^3) + (2*b
*x*(a*d - b*c))/d^2)/(c^2 + d^2*x^2 + 2*c*d*x)
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.92

$$\int \frac{(a+bx)^5}{(ac+(bc+ad)x+bdx^2)^3}\,dx$$
$$= \frac{2\log(dx+c)\,b^2c^3+4\log(dx+c)\,b^2c^2dx+2\log(dx+c)\,b^2c\,d^2x^2-a^2c\,d^2+2ab\,d^3x^2+b^2c^3-2b^2c\,d^2x^2}{2c\,d^3\,(d^2x^2+2cdx+c^2)}$$

input
```
int((b*x+a)^5/(a*c+(a*d+b*c)*x+b*d*x^2)^3,x)
```

output
```
(2*log(c + d*x)*b**2*c**3 + 4*log(c + d*x)*b**2*c**2*d*x + 2*log(c + d*x)*
b**2*c*d**2*x**2 - a**2*c*d**2 + 2*a*b*d**3*x**2 + b**2*c**3 - 2*b**2*c*d*
*2*x**2)/(2*c*d**3*(c**2 + 2*c*d*x + d**2*x**2))
```

3.64 $\int \frac{(a+bx)^4}{(ac+(bc+ad)x+bdx^2)^3}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 28

$$\int \frac{(a+bx)^4}{(ac+(bc+ad)x+bdx^2)^3}\,dx = \frac{(a+bx)^2}{2(bc-ad)(c+dx)^2}$$

output
```
1/2*(b*x+a)^2/(-a*d+b*c)/(d*x+c)^2
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.93

$$\int \frac{(a+bx)^4}{(ac+(bc+ad)x+bdx^2)^3}\,dx = -\frac{ad+b(c+2dx)}{2d^2(c+dx)^2}$$

input
```
Integrate[(a + b*x)^4/(a*c + (b*c + a*d)*x + b*d*x^2)^3,x]
```

output
```
-1/2*(a*d + b*(c + 2*d*x))/(d^2*(c + d*x)^2)
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.069$, Rules used = {1120, 48}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx)^4}{(x(ad+bc)+ac+bdx^2)^3}dx$$

$$\Big\downarrow \text{ 1120}$$

$$\int \frac{a+bx}{(c+dx)^3}dx$$

$$\Big\downarrow \text{ 48}$$

$$\frac{(a+bx)^2}{2(c+dx)^2(bc-ad)}$$

input
```
Int[(a + b*x)^4/(a*c + (b*c + a*d)*x + b*d*x^2)^3,x]
```

output
```
(a + b*x)^2/(2*(b*c - a*d)*(c + d*x)^2)
```

Defintions of rubi rules used

rule 48
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && EqQ[m + n + 2, 0] && NeQ[m, -1]
```

rule 1120
```
Int[((d_) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Int[(d + e*x)^(m + p)*(a/d + (c/e)*x)^p, x] /; FreeQ[{a, b, c, d
, e, m}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[p] && (EqQ[m + p,
0] || EqQ[m + 2*p + 2, 0])
```

Maple [A] (verified)

Time = 1.41 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.89

method	result	size
gosper	$-\frac{2bdx+ad+bc}{2(dx+c)^{2} d^{2}}$	25
parallelrisch	$\frac{-2bdx-ad-bc}{2d^{2} (dx+c)^{2}}$	27
risch	$\frac{-\frac{bx}{d}-\frac{ad+bc}{2d^{2}}}{(dx+c)^{2}}$	29
default	$-\frac{b}{d^{2}(dx+c)}-\frac{ad-bc}{2d^{2}(dx+c)^{2}}$	35
orering	$-\frac{(2bdx+ad+bc)(bx+a)^{3}(dx+c)}{2d^{2}(ac+(ad+bc)x+bd\,x^{2})^{3}}$	51
norman	$\frac{-\frac{b^{3}x^{3}}{d}-\frac{\left(da\,b^{2}+b^{3}c\right)a^{2}}{2b^{2}d^{2}}+\frac{\left(-5ad\,b^{4}-b^{5}c\right)x^{2}}{2b^{2}d^{2}}-\frac{a\left(2a\,b^{3}d+b^{4}c\right)x}{b^{2}d^{2}}}{(dx+c)^{2}(bx+a)^{2}}$	99

input
```
int((b*x+a)^4/(a*c+(a*d+b*c)*x+b*d*x^2)^3,x,method=_RETURNVERBOSE)
```

output
```
-1/2*(2*b*d*x+a*d+b*c)/(d*x+c)^2/d^2
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.36

$$\int \frac{(a+bx)^4}{(ac+(bc+ad)x+bdx^2)^3}\,dx = -\frac{2\,bdx+bc+ad}{2\,(d^4x^2+2\,cd^3x+c^2d^2)}$$

input
```
integrate((b*x+a)^4/(a*c+(a*d+b*c)*x+b*d*x^2)^3,x, algorithm="fricas")
```

output
```
-1/2*(2*b*d*x + b*c + a*d)/(d^4*x^2 + 2*c*d^3*x + c^2*d^2)
```

Sympy [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.39

$$\int \frac{(a+bx)^4}{(ac+(bc+ad)x+bdx^2)^3}\,dx = \frac{-ad-bc-2bdx}{2c^2d^2+4cd^3x+2d^4x^2}$$

input
```
integrate((b*x+a)**4/(a*c+(a*d+b*c)*x+b*d*x**2)**3,x)
```

output
```
(-a*d - b*c - 2*b*d*x)/(2*c**2*d**2 + 4*c*d**3*x + 2*d**4*x**2)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.36

$$\int \frac{(a+bx)^4}{(ac+(bc+ad)x+bdx^2)^3}\,dx = -\frac{2\,bdx+bc+ad}{2\,(d^4x^2+2\,cd^3x+c^2d^2)}$$

input
```
integrate((b*x+a)^4/(a*c+(a*d+b*c)*x+b*d*x^2)^3,x, algorithm="maxima")
```

output
```
-1/2*(2*b*d*x + b*c + a*d)/(d^4*x^2 + 2*c*d^3*x + c^2*d^2)
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.86

$$\int \frac{(a+bx)^4}{(ac+(bc+ad)x+bdx^2)^3}\,dx = -\frac{2\,bdx+bc+ad}{2\,(dx+c)^2d^2}$$

input
```
integrate((b*x+a)^4/(a*c+(a*d+b*c)*x+b*d*x^2)^3,x, algorithm="giac")
```

output
```
-1/2*(2*b*d*x + b*c + a*d)/((d*x + c)^2*d^2)
```

Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.39

$$\int \frac{(a+bx)^4}{(ac+(bc+ad)x+bdx^2)^3}\,dx = -\frac{\frac{a\,d+b\,c}{2\,d^2}+\frac{b\,x}{d}}{c^2+2\,c\,d\,x+d^2\,x^2}$$

input
```
int((a + b*x)^4/(a*c + x*(a*d + b*c) + b*d*x^2)^3,x)
```

output
```
-((a*d + b*c)/(2*d^2) + (b*x)/d)/(c^2 + d^2*x^2 + 2*c*d*x)
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.32

$$\int \frac{(a+bx)^4}{(ac+(bc+ad)x+bdx^2)^3}\,dx = \frac{bd\,x^2-ac}{2cd\,(d^2x^2+2cdx+c^2)}$$

input
```
int((b*x+a)^4/(a*c+(a*d+b*c)*x+b*d*x^2)^3,x)
```

output
```
( - a*c + b*d*x**2)/(2*c*d*(c**2 + 2*c*d*x + d**2*x**2))
```

3.65 $\int \frac{(a+bx)^3}{\left(ac+(bc+ad)x+bdx^2\right)^3} \, dx$

Optimal result

Integrand size = 29, antiderivative size = 14

$$\int \frac{(a+bx)^3}{\left(ac+(bc+ad)x+bdx^2\right)^3} \, dx=-\frac{1}{2d(c+dx)^2}$$

output
```
-1/2/d/(d*x+c)^2
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00

$$\int \frac{(a+bx)^3}{\left(ac+(bc+ad)x+bdx^2\right)^3} \, dx=-\frac{1}{2d(c+dx)^2}$$

input
```
Integrate[(a + b*x)^3/(a*c + (b*c + a*d)*x + b*d*x^2)^3,x]
```

output
```
-1/2*1/(d*(c + d*x)^2)
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.069, Rules used = {1120, 17}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx)^3}{(x(ad+bc)+ac+bdx^2)^3} \, dx$$
$$\Big\downarrow \text{ 1120}$$
$$\int \frac{1}{(c+dx)^3}dx$$
$$\Big\downarrow \text{ 17}$$
$$-\frac{1}{2d(c+dx)^2}$$

input
```
Int[(a + b*x)^3/(a*c + (b*c + a*d)*x + b*d*x^2)^3,x]
```

output
```
-1/2*1/(d*(c + d*x)^2)
```

Defintions of rubi rules used

rule 17
```
Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1
)/(b*(m + 1))), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]
```

rule 1120
```
Int[((d_) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Int[(d + e*x)^(m + p)*(a/d + (c/e)*x)^p, x] /; FreeQ[{a, b, c, d
, e, m}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[p] && (EqQ[m + p,
0] || EqQ[m + 2*p + 2, 0])
```

Maple [A] (verified)

Time = 1.39 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.93

method	result	size
gosper	$-\frac{1}{2d(dx+c)^{2}}$	13
default	$-\frac{1}{2d(dx+c)^{2}}$	13
risch	$-\frac{1}{2d(dx+c)^{2}}$	13
parallelrisch	$-\frac{1}{2d(dx+c)^{2}}$	13
orering	$-\frac{(dx+c)(bx+a)^{3}}{2d(ac+(ad+bc)x+bd\,x^{2})^{3}}$	39
norman	$\frac{-\frac{abx}{d}-\frac{a^{2}}{2d}-\frac{b^{2}x^{2}}{2d}}{(dx+c)^{2}(bx+a)^{2}}$	44

input
```
int((b*x+a)^3/(a*c+(a*d+b*c)*x+b*d*x^2)^3,x,method=_RETURNVERBOSE)
```

output
```
-1/2/d/(d*x+c)^2
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.71

$$\int \frac{(a+bx)^3}{(ac+(bc+ad)x+bdx^2)^3}\,dx = -\frac{1}{2\,(d^3x^2+2\,cd^2x+c^2d)}$$

input
```
integrate((b*x+a)^3/(a*c+(a*d+b*c)*x+b*d*x^2)^3,x, algorithm="fricas")
```

output
```
-1/2/(d^3*x^2 + 2*c*d^2*x + c^2*d)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 26 vs. $2(12) = 24$.

Time = 0.10 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.86

$$\int \frac{(a+bx)^3}{(ac+(bc+ad)x+bdx^2)^3}\,dx = -\frac{1}{2c^2d+4cd^2x+2d^3x^2}$$

input
```
integrate((b*x+a)**3/(a*c+(a*d+b*c)*x+b*d*x**2)**3,x)
```

output
```
-1/(2*c**2*d + 4*c*d**2*x + 2*d**3*x**2)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.71

$$\int \frac{(a+bx)^3}{(ac+(bc+ad)x+bdx^2)^3}\,dx = -\frac{1}{2\,(d^3x^2+2\,cd^2x+c^2d)}$$

input
```
integrate((b*x+a)^3/(a*c+(a*d+b*c)*x+b*d*x^2)^3,x, algorithm="maxima")
```

output
```
-1/2/(d^3*x^2 + 2*c*d^2*x + c^2*d)
```

Giac [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.86

$$\int \frac{(a+bx)^3}{(ac+(bc+ad)x+bdx^2)^3}\,dx = -\frac{1}{2\,(dx+c)^2d}$$

input
```
integrate((b*x+a)^3/(a*c+(a*d+b*c)*x+b*d*x^2)^3,x, algorithm="giac")
```

output
```
-1/2/((d*x + c)^2*d)
```

Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.86

$$\int \frac{(a+bx)^3}{(ac+(bc+ad)x+bdx^2)^3}\,dx = -\frac{1}{2\,c^2\,d+4\,c\,d^2\,x+2\,d^3\,x^2}$$

input
```
int((a + b*x)^3/(a*c + x*(a*d + b*c) + b*d*x^2)^3,x)
```

output
```
-1/(2*c^2*d + 2*d^3*x^2 + 4*c*d^2*x)
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.64

$$\int \frac{(a+bx)^3}{(ac+(bc+ad)x+bdx^2)^3}\,dx = -\frac{1}{2d\,(d^2x^2+2cdx+c^2)}$$

input
```
int((b*x+a)^3/(a*c+(a*d+b*c)*x+b*d*x^2)^3,x)
```

output
```
( - 1)/(2*d*(c**2 + 2*c*d*x + d**2*x**2))
```

3.66 $\int \frac{(a+bx)^2}{\left(ac+(bc+ad)x+bdx^2\right)^3} \, dx$

Optimal result

Integrand size = 29, antiderivative size = 82

$$\int \frac{(a+bx)^2}{\left(ac+(bc+ad)x+bdx^2\right)^3} \, dx = \frac{1}{2(bc-ad)(c+dx)^2}+\frac{b}{(bc-ad)^2(c+dx)} +\frac{b^2\log(a+bx)}{(bc-ad)^3}-\frac{b^2\log(c+dx)}{(bc-ad)^3}$$

output
```
1/2/(-a*d+b*c)/(d*x+c)^2+b/(-a*d+b*c)^2/(d*x+c)+b^2*ln(b*x+a)/(-a*d+b*c)^3
-b^2*ln(d*x+c)/(-a*d+b*c)^3
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.82

$$\int \frac{(a+bx)^2}{\left(ac+(bc+ad)x+bdx^2\right)^3} \, dx = \frac{\frac{(bc-ad)(3bc-ad+2bdx)}{(c+dx)^2}+2b^2\log(a+bx)-2b^2\log(c+dx)}{2(bc-ad)^3}$$

input
```
Integrate[(a + b*x)^2/(a*c + (b*c + a*d)*x + b*d*x^2)^3,x]
```

output
```
(((b*c - a*d)*(3*b*c - a*d + 2*b*d*x))/(c + d*x)^2 + 2*b^2*Log[a + b*x] -
2*b^2*Log[c + d*x])/(2*(b*c - a*d)^3)
```

Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.069, Rules used = {1121, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx)^2}{(x(ad+bc)+ac+bdx^2)^3}\,dx$$

$$\Big\downarrow \text{1121}$$

$$\int \left(\frac{b^3}{(a+bx)(bc-ad)^3}-\frac{b^2d}{(c+dx)(bc-ad)^3}-\frac{bd}{(c+dx)^2(bc-ad)^2}-\frac{d}{(c+dx)^3(bc-ad)}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{b^2\log(a+bx)}{(bc-ad)^3}-\frac{b^2\log(c+dx)}{(bc-ad)^3}+\frac{b}{(c+dx)(bc-ad)^2}+\frac{1}{2(c+dx)^2(bc-ad)}$$

input
```
Int[(a + b*x)^2/(a*c + (b*c + a*d)*x + b*d*x^2)^3,x]
```

output
```
1/(2*(b*c - a*d)*(c + d*x)^2) + b/((b*c - a*d)^2*(c + d*x)) + (b^2*Log[a +
 b*x])/(b*c - a*d)^3 - (b^2*Log[c + d*x])/(b*c - a*d)^3
```

Defintions of rubi rules used

rule 1121
```
Int[((d_) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Int[ExpandIntegrand[(d + e*x)^(m + p)*(a/d + (c/e)*x)^p, x], x]
/; FreeQ[{a, b, c, d, e, m, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && (Int
egerQ[p] || (GtQ[a, 0] && GtQ[d, 0] && LtQ[c, 0]))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 1.40 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.99

method	result
default	$-\frac{b^{2} \ln \left(b x+a \right)}{\left(a d-b c \right)^{3}}-\frac{1}{2 \left(a d-b c \right) \left(d x+c \right)^{2}}+\frac{b^{2} \ln \left(d x+c \right)}{\left(a d-b c \right)^{3}}+\frac{b}{\left(a d-b c \right)^{2} \left(d x+c \right)}$
risch	$\frac{\frac{b d x}{a^{2} d^{2}-2 a b c d+b^{2} c^{2}}-\frac{a d-3 b c}{2 \left(a^{2} d^{2}-2 a b c d+b^{2} c^{2}\right)}}{\left(d x+c \right)^{2}}+\frac{b^{2} \ln \left(-d x-c \right)}{a^{3} d^{3}-3 a^{2} b c \,d^{2}+3 a \,b^{2} c^{2} d-b^{3} c^{3}}-\frac{b^{2} \ln \left(b x+a \right)}{a^{3} d^{3}-3 a^{2} b c \,d^{2}+3 a \,b^{2} c^{2} d-b^{3} c^{3}}$
parallelrisch	$-\frac{2 \ln \left(b x+a \right) x^{2} b^{2} d^{4}-2 \ln \left(d x+c \right) x^{2} b^{2} d^{4}+4 \ln \left(b x+a \right) x \,b^{2} c \,d^{3}-4 \ln \left(d x+c \right) x \,b^{2} c \,d^{3}+2 \ln \left(b x+a \right) b^{2} c^{2} d^{2}-2 \ln \left(d x+c \right) b^{2} c^{2} d^{2}-2 a}{2 \left(a^{3} d^{3}-3 a^{2} b c \,d^{2}+3 a \,b^{2} c^{2} d-b^{3} c^{3}\right) \left(d x+c \right)^{2} d^{2}}$
norman	$\frac{\frac{b^{3} d \,x^{3}}{a^{2} d^{2}-2 a b c d+b^{2} c^{2}}+\frac{3 c a \,b^{2} x}{a^{2} d^{2}-2 a b c d+b^{2} c^{2}}+\frac{a^{2} \left(-a \,b^{2} d^{3}+3 b^{3} c \,d^{2}\right)}{2 d^{2} b^{2} \left(a^{2} d^{2}-2 a b c d+b^{2} c^{2}\right)}+\frac{\left(3 a \,b^{4} d^{3}+3 b^{5} c \,d^{2}\right) x^{2}}{2 d^{2} b^{2} \left(a^{2} d^{2}-2 a b c d+b^{2} c^{2}\right)}}{\left(d x+c \right)^{2} \left(b x+a \right)^{2}}+\frac{b^{2} \ln \left(d x+c \right)}{a^{3} d^{3}-3 a^{2} b c \,d^{2}+3 a \,b^{2} c^{2} d-b}$

input
```
int((b*x+a)^2/(a*c+(a*d+b*c)*x+b*d*x^2)^3,x,method=_RETURNVERBOSE)
```

output
```
-b^2/(a*d-b*c)^3*ln(b*x+a)-1/2/(a*d-b*c)/(d*x+c)^2+b^2/(a*d-b*c)^3*ln(d*x+
c)+b/(a*d-b*c)^2/(d*x+c)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 242 vs. $2(80) = 160$.

Time = 0.08 (sec) , antiderivative size = 242, normalized size of antiderivative = 2.95

$$\int \frac{(a+bx)^2}{(ac+(bc+ad)x+bdx^2)^3}\,dx$$
$$= \frac{3\,b^2c^2 - 4\,abcd + a^2d^2 + 2\,(b^2cd - abd^2)x + 2\,(b^2d^2x^2 + 2\,b^2cdx + b^2c^2)\log(bx+a) - 2\,(b^2d^2x^2 + 2\,b^2c}{2\,(b^3c^5 - 3\,ab^2c^4d + 3\,a^2bc^3d^2 - a^3c^2d^3 + (b^3c^3d^2 - 3\,ab^2c^2d^3 + 3\,a^2bcd^4 - a^3d^5)x^2 + 2\,(b^3c^4d - 3\,ab^2c^3d^2}$$

input
```
integrate((b*x+a)^2/(a*c+(a*d+b*c)*x+b*d*x^2)^3,x, algorithm="fricas")
```

output
```
1/2*(3*b^2*c^2 - 4*a*b*c*d + a^2*d^2 + 2*(b^2*c*d - a*b*d^2)*x + 2*(b^2*d^
2*x^2 + 2*b^2*c*d*x + b^2*c^2)*log(b*x + a) - 2*(b^2*d^2*x^2 + 2*b^2*c*d*x
 + b^2*c^2)*log(d*x + c))/(b^3*c^5 - 3*a*b^2*c^4*d + 3*a^2*b*c^3*d^2 - a^3
*c^2*d^3 + (b^3*c^3*d^2 - 3*a*b^2*c^2*d^3 + 3*a^2*b*c*d^4 - a^3*d^5)*x^2 +
 2*(b^3*c^4*d - 3*a*b^2*c^3*d^2 + 3*a^2*b*c^2*d^3 - a^3*c*d^4)*x)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 381 vs. $2(68) = 136$.

Time = 0.54 (sec) , antiderivative size = 381, normalized size of antiderivative = 4.65

$$\int \frac{(a+bx)^2}{(ac+(bc+ad)x+bdx^2)^3}\,dx$$
$$= \frac{b^2\log\left(x + \frac{-\frac{a^4b^2d^4}{(ad-bc)^3} + \frac{4a^3b^3cd^3}{(ad-bc)^3} - \frac{6a^2b^4c^2d^2}{(ad-bc)^3} + \frac{4ab^5c^3d}{(ad-bc)^3} + ab^2d - \frac{b^6c^4}{(ad-bc)^3} + b^3c}{2b^3d}\right)}{(ad-bc)^3}$$
$$- \frac{b^2\log\left(x + \frac{\frac{a^4b^2d^4}{(ad-bc)^3} - \frac{4a^3b^3cd^3}{(ad-bc)^3} + \frac{6a^2b^4c^2d^2}{(ad-bc)^3} - \frac{4ab^5c^3d}{(ad-bc)^3} + ab^2d + \frac{b^6c^4}{(ad-bc)^3} + b^3c}{2b^3d}\right)}{(ad-bc)^3}$$
$$+ \frac{-ad + 3bc + 2bdx}{2a^2c^2d^2 - 4abc^3d + 2b^2c^4 + x^2\cdot(2a^2d^4 - 4abcd^3 + 2b^2c^2d^2) + x\,(4a^2cd^3 - 8abc^2d^2 + 4b^2c^3d)}$$

input
```
integrate((b*x+a)**2/(a*c+(a*d+b*c)*x+b*d*x**2)**3,x)
```

output
```
b**2*log(x + (-a**4*b**2*d**4/(a*d - b*c)**3 + 4*a**3*b**3*c*d**3/(a*d - b
*c)**3 - 6*a**2*b**4*c**2*d**2/(a*d - b*c)**3 + 4*a*b**5*c**3*d/(a*d - b*c
)**3 + a*b**2*d - b**6*c**4/(a*d - b*c)**3 + b**3*c)/(2*b**3*d))/(a*d - b*
c)**3 - b**2*log(x + (a**4*b**2*d**4/(a*d - b*c)**3 - 4*a**3*b**3*c*d**3/(
a*d - b*c)**3 + 6*a**2*b**4*c**2*d**2/(a*d - b*c)**3 - 4*a*b**5*c**3*d/(a*
d - b*c)**3 + a*b**2*d + b**6*c**4/(a*d - b*c)**3 + b**3*c)/(2*b**3*d))/(a
*d - b*c)**3 + (-a*d + 3*b*c + 2*b*d*x)/(2*a**2*c**2*d**2 - 4*a*b*c**3*d +
 2*b**2*c**4 + x**2*(2*a**2*d**4 - 4*a*b*c*d**3 + 2*b**2*c**2*d**2) + x*(4
*a**2*c*d**3 - 8*a*b*c**2*d**2 + 4*b**2*c**3*d))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 202 vs. $2(80) = 160$.

Time = 0.04 (sec) , antiderivative size = 202, normalized size of antiderivative = 2.46

$$\int \frac{(a+bx)^2}{(ac+(bc+ad)x+bdx^2)^3}\,dx$$
$$= \frac{b^2\log(bx+a)}{b^3c^3-3\,ab^2c^2d+3\,a^2bcd^2-a^3d^3} - \frac{b^2\log(dx+c)}{b^3c^3-3\,ab^2c^2d+3\,a^2bcd^2-a^3d^3} + \frac{2\,bdx+3\,bc-ad}{2\,(b^2c^4-2\,abc^3d+a^2c^2d^2+(b^2c^2d^2-2\,abcd^3+a^2d^4)x^2+2\,(b^2c^3d-2\,abc^2d^2+a^2cd^3)x)}$$

input
```
integrate((b*x+a)^2/(a*c+(a*d+b*c)*x+b*d*x^2)^3,x, algorithm="maxima")
```

output
```
b^2*log(b*x + a)/(b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3) - b^2
*log(d*x + c)/(b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3) + 1/2*(2
*b*d*x + 3*b*c - a*d)/(b^2*c^4 - 2*a*b*c^3*d + a^2*c^2*d^2 + (b^2*c^2*d^2
- 2*a*b*c*d^3 + a^2*d^4)*x^2 + 2*(b^2*c^3*d - 2*a*b*c^2*d^2 + a^2*c*d^3)*x
)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 165 vs. $2(80) = 160$.

Time = 0.12 (sec) , antiderivative size = 165, normalized size of antiderivative = 2.01

$$\int \frac{(a+bx)^2}{(ac+(bc+ad)x+bdx^2)^3}\,dx = \frac{b^3\log(|bx+a|)}{b^4c^3-3\,ab^3c^2d+3\,a^2b^2cd^2-a^3bd^3} - \frac{b^2d\log(|dx+c|)}{b^3c^3d-3\,ab^2c^2d^2+3\,a^2bcd^3-a^3d^4} + \frac{3\,b^2c^2-4\,abcd+a^2d^2+2\,(b^2cd-abd^2)x}{2\,(bc-ad)^3(dx+c)^2}$$

input
```
integrate((b*x+a)^2/(a*c+(a*d+b*c)*x+b*d*x^2)^3,x, algorithm="giac")
```

output
```
b^3*log(abs(b*x + a))/(b^4*c^3 - 3*a*b^3*c^2*d + 3*a^2*b^2*c*d^2 - a^3*b*d
^3) - b^2*d*log(abs(d*x + c))/(b^3*c^3*d - 3*a*b^2*c^2*d^2 + 3*a^2*b*c*d^3
 - a^3*d^4) + 1/2*(3*b^2*c^2 - 4*a*b*c*d + a^2*d^2 + 2*(b^2*c*d - a*b*d^2)
*x)/((b*c - a*d)^3*(d*x + c)^2)
```

Mupad [B] (verification not implemented)

Time = 6.39 (sec) , antiderivative size = 183, normalized size of antiderivative = 2.23

$$\int \frac{(a+bx)^2}{(ac+(bc+ad)x+bdx^2)^3}\,dx = -\frac{\frac{a\,d-3\,b\,c}{2\,(a^2\,d^2-2\,a\,b\,c\,d+b^2\,c^2)}-\frac{b\,d\,x}{a^2\,d^2-2\,a\,b\,c\,d+b^2\,c^2}}{c^2+2\,c\,d\,x+d^2\,x^2} - \frac{2\,b^2\,\mathrm{atanh}\left(\frac{a^3\,d^3-a^2\,b\,c\,d^2-a\,b^2\,c^2\,d+b^3\,c^3}{(a\,d-b\,c)^3}+\frac{2\,b\,d\,x\,(a^2\,d^2-2\,a\,b\,c\,d+b^2\,c^2)}{(a\,d-b\,c)^3}\right)}{(a\,d-b\,c)^3}$$

input
```
int((a + b*x)^2/(a*c + x*(a*d + b*c) + b*d*x^2)^3,x)
```

output
```
- ((a*d - 3*b*c)/(2*(a^2*d^2 + b^2*c^2 - 2*a*b*c*d)) - (b*d*x)/(a^2*d^2 +
b^2*c^2 - 2*a*b*c*d))/(c^2 + d^2*x^2 + 2*c*d*x) - (2*b^2*atanh((a^3*d^3 +
b^3*c^3 - a*b^2*c^2*d - a^2*b*c*d^2)/(a*d - b*c)^3 + (2*b*d*x*(a^2*d^2 + b
^2*c^2 - 2*a*b*c*d))/(a*d - b*c)^3))/(a*d - b*c)^3
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 289, normalized size of antiderivative = 3.52

$$\int \frac{(a+bx)^2}{(ac+(bc+ad)x+bdx^2)^3}\,dx = \frac{-2\log(bx+a)\,b^2c^3-4\log(bx+a)\,b^2c^2dx-2\log(bx+a)\,b^2c\,d^2x^2+2\log(dx+c)\,b^2c^3+4\log(dx+c)\,b}{2c\,(a^3d^5x^2-3a^2bc\,d^4x^2+3a\,b^2c^2d^3x^2-b^3c^3d^2x^2+2a^3c\,d^4x-6a^2b\,c^2d^3x+6a\,b^2c}$$

input
```
int((b*x+a)^2/(a*c+(a*d+b*c)*x+b*d*x^2)^3,x)
```

output
```
( - 2*log(a + b*x)*b**2*c**3 - 4*log(a + b*x)*b**2*c**2*d*x - 2*log(a + b*
x)*b**2*c*d**2*x**2 + 2*log(c + d*x)*b**2*c**3 + 4*log(c + d*x)*b**2*c**2*
d*x + 2*log(c + d*x)*b**2*c*d**2*x**2 - a**2*c*d**2 + 3*a*b*c**2*d - a*b*d
**3*x**2 - 2*b**2*c**3 + b**2*c*d**2*x**2)/(2*c*(a**3*c**2*d**3 + 2*a**3*c
*d**4*x + a**3*d**5*x**2 - 3*a**2*b*c**3*d**2 - 6*a**2*b*c**2*d**3*x - 3*a
**2*b*c*d**4*x**2 + 3*a*b**2*c**4*d + 6*a*b**2*c**3*d**2*x + 3*a*b**2*c**2
*d**3*x**2 - b**3*c**5 - 2*b**3*c**4*d*x - b**3*c**3*d**2*x**2))
```

3.67 $\int \frac{a+bx}{\left(ac+(bc+ad)x+bdx^2\right)^3}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 110

$$\int \frac{a+bx}{\left(ac+(bc+ad)x+bdx^2\right)^3}\,dx = -\frac{b^2}{(bc-ad)^3(a+bx)} - \frac{d}{2(bc-ad)^2(c+dx)^2} - \frac{2bd}{(bc-ad)^3(c+dx)} - \frac{3b^2d\log(a+bx)}{(bc-ad)^4} + \frac{3b^2d\log(c+dx)}{(bc-ad)^4}$$

output
```
-b^2/(-a*d+b*c)^3/(b*x+a)-1/2*d/(-a*d+b*c)^2/(d*x+c)^2-2*b*d/(-a*d+b*c)^3/
(d*x+c)-3*b^2*d*ln(b*x+a)/(-a*d+b*c)^4+3*b^2*d*ln(d*x+c)/(-a*d+b*c)^4
```

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 97, normalized size of antiderivative = 0.88

$$\int \frac{a+bx}{\left(ac+(bc+ad)x+bdx^2\right)^3}\,dx = -\frac{\frac{2b^2(bc-ad)}{a+bx} + \frac{d(bc-ad)^2}{(c+dx)^2} + \frac{4bd(bc-ad)}{c+dx} + 6b^2d\log(a+bx) - 6b^2d\log(c+dx)}{2(bc-ad)^4}$$

input
```
Integrate[(a + b*x)/(a*c + (b*c + a*d)*x + b*d*x^2)^3,x]
```

output
```
-1/2*((2*b^2*(b*c - a*d))/(a + b*x) + (d*(b*c - a*d)^2)/(c + d*x)^2 + (4*b
*d*(b*c - a*d))/(c + d*x) + 6*b^2*d*Log[a + b*x] - 6*b^2*d*Log[c + d*x])/(
b*c - a*d)^4
```

Rubi [A] (verified)

Time = 0.48 (sec) , antiderivative size = 110, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.074$, Rules used = {1121, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+bx}{(x(ad+bc)+ac+bdx^2)^3}\,dx$$

$$\Big\downarrow \text{1121}$$

$$\int \left(-\frac{3b^3d}{(a+bx)(bc-ad)^4}+\frac{b^3}{(a+bx)^2(bc-ad)^3}+\frac{3b^2d^2}{(c+dx)(bc-ad)^4}+\frac{2bd^2}{(c+dx)^2(bc-ad)^3}+\frac{d^2}{(c+dx)^3(bc-a$$

$$\Big\downarrow \text{2009}$$

$$-\frac{b^2}{(a+bx)(bc-ad)^3}-\frac{3b^2d\log(a+bx)}{(bc-ad)^4}+\frac{3b^2d\log(c+dx)}{(bc-ad)^4}-\frac{2bd}{(c+dx)(bc-ad)^3}-\frac{d}{2(c+dx)^2(bc-ad)^2}$$

input
```
Int[(a + b*x)/(a*c + (b*c + a*d)*x + b*d*x^2)^3,x]
```

output
```
-(b^2/((b*c - a*d)^3*(a + b*x))) - d/(2*(b*c - a*d)^2*(c + d*x)^2) - (2*b*
d)/((b*c - a*d)^3*(c + d*x)) - (3*b^2*d*Log[a + b*x])/(b*c - a*d)^4 + (3*b
^2*d*Log[c + d*x])/(b*c - a*d)^4
```

Defintions of rubi rules used

rule 1121
```
Int[((d_) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Int[ExpandIntegrand[(d + e*x)^(m + p)*(a/d + (c/e)*x)^p, x], x]
/; FreeQ[{a, b, c, d, e, m, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && (Int
egerQ[p] || (GtQ[a, 0] && GtQ[d, 0] && LtQ[c, 0]))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 1.39 (sec) , antiderivative size = 108, normalized size of antiderivative = 0.98

method	result
default	$\frac{b^{2}}{(ad-bc)^{3}(bx+a)}-\frac{3b^{2}d\ln(bx+a)}{(ad-bc)^{4}}-\frac{d}{2(ad-bc)^{2}(dx+c)^{2}}+\frac{3b^{2}d\ln(dx+c)}{(ad-bc)^{4}}+\frac{2db}{(ad-bc)^{3}(dx+c)}$
risch	$\frac{\frac{3b^{2}d^{2}x^{2}}{a^{3}d^{3}-3a^{2}bc\,d^{2}+3a\,b^{2}c^{2}d-b^{3}c^{3}}+\frac{3(ad+3bc)bdx}{2\left(a^{3}d^{3}-3a^{2}bc\,d^{2}+3a\,b^{2}c^{2}d-b^{3}c^{3}\right)}-\frac{a^{2}d^{2}-5abcd-2b^{2}c^{2}}{2\left(a^{3}d^{3}-3a^{2}bc\,d^{2}+3a\,b^{2}c^{2}d-b^{3}c^{3}\right)}}{(dx+c)(bd\,x^{2}+adx+cbx+ac)}-\frac{3b^{2}d\,\mathrm{l}}{a^{4}d^{4}-4a^{3}bc\,d^{3}+6a^{2}}$
parallelrisch	$-\frac{-6a^{2}c\,d^{4}b^{2}+3a\,b^{3}c^{2}d^{3}+a^{3}b\,d^{5}+12\ln(bx+a)xa\,b^{3}c\,d^{4}-12\ln(dx+c)xa\,b^{3}c\,d^{4}-6xa\,b^{3}c\,d^{4}+6\ln(bx+a)x^{3}b^{4}d^{5}-6\ln(dx+c)x}{}$
norman	$\frac{\frac{3b^{3}d^{2}x^{3}}{a^{3}d^{3}-3a^{2}bc\,d^{2}+3a\,b^{2}c^{2}d-b^{3}c^{3}}+\frac{\left(a^{2}b^{3}d^{4}+7a\,b^{4}c\,d^{3}+b^{5}c^{2}d^{2}\right)x}{d^{2}b^{2}\left(a^{3}d^{3}-3a^{2}bc\,d^{2}+3a\,b^{2}c^{2}d-b^{3}c^{3}\right)}+\frac{a\left(-a^{2}b^{2}d^{4}+5a\,b^{3}c\,d^{3}+2b^{4}c^{2}d^{2}\right)}{2d^{2}b^{2}\left(a^{3}d^{3}-3a^{2}bc\,d^{2}+3a\,b^{2}c^{2}d-b^{3}c^{3}\right)}+\frac{(9a\,b^{4}d}{2d^{2}b^{2}\left(a^{3}d^{3}-3a^{2}\right.}}{(dx+c)^{2}(bx+a)^{2}}$

input
```
int((b*x+a)/(a*c+(a*d+b*c)*x+b*d*x^2)^3,x,method=_RETURNVERBOSE)
```

output
```
b^2/(a*d-b*c)^3/(b*x+a)-3*b^2/(a*d-b*c)^4*d*ln(b*x+a)-1/2*d/(a*d-b*c)^2/(d
*x+c)^2+3*b^2/(a*d-b*c)^4*d*ln(d*x+c)+2*d/(a*d-b*c)^3*b/(d*x+c)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 495 vs. $2(108) = 216$.

Time = 0.10 (sec) , antiderivative size = 495, normalized size of antiderivative = 4.50

$$\int \frac{a+bx}{\left(ac+(bc+ad)x+bdx^2\right)^3}\,dx =$$
$$-\frac{2\,b^3c^3+3\,ab^2c^2d-6\,a^2bcd^2+a^3d^3+6\,(b^3cd^2-ab^2d^3)x^2+3\,(3\,b^3c^2d-2\,ab^2cd^2-a^2bd^3)x+6\,(b^3d^3}{2\,(ab^4c^6-4\,a^2b^3c^5d+6\,a^3b^2c^4d^2-4\,a^4bc^3d^3+a^5c^2d^4+(b^5c^4d^2-4\,ab^4c^3d^3+6\,a^2b^3c^2d^4-4\,a^3b^2cd^5}$$

input
```
integrate((b*x+a)/(a*c+(a*d+b*c)*x+b*d*x^2)^3,x, algorithm="fricas")
```

output
```
-1/2*(2*b^3*c^3 + 3*a*b^2*c^2*d - 6*a^2*b*c*d^2 + a^3*d^3 + 6*(b^3*c*d^2 -
 a*b^2*d^3)*x^2 + 3*(3*b^3*c^2*d - 2*a*b^2*c*d^2 - a^2*b*d^3)*x + 6*(b^3*d
^3*x^3 + a*b^2*c^2*d + (2*b^3*c*d^2 + a*b^2*d^3)*x^2 + (b^3*c^2*d + 2*a*b^
2*c*d^2)*x)*log(b*x + a) - 6*(b^3*d^3*x^3 + a*b^2*c^2*d + (2*b^3*c*d^2 + a
*b^2*d^3)*x^2 + (b^3*c^2*d + 2*a*b^2*c*d^2)*x)*log(d*x + c))/(a*b^4*c^6 -
4*a^2*b^3*c^5*d + 6*a^3*b^2*c^4*d^2 - 4*a^4*b*c^3*d^3 + a^5*c^2*d^4 + (b^5
*c^4*d^2 - 4*a*b^4*c^3*d^3 + 6*a^2*b^3*c^2*d^4 - 4*a^3*b^2*c*d^5 + a^4*b*d
^6)*x^3 + (2*b^5*c^5*d - 7*a*b^4*c^4*d^2 + 8*a^2*b^3*c^3*d^3 - 2*a^3*b^2*c
^2*d^4 - 2*a^4*b*c*d^5 + a^5*d^6)*x^2 + (b^5*c^6 - 2*a*b^4*c^5*d - 2*a^2*b
^3*c^4*d^2 + 8*a^3*b^2*c^3*d^3 - 7*a^4*b*c^2*d^4 + 2*a^5*c*d^5)*x)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 632 vs. $2(97) = 194$.

Time = 0.84 (sec) , antiderivative size = 632, normalized size of antiderivative = 5.75

$$\int \frac{a+bx}{(ac+(bc+ad)x+bdx^2)^3}\,dx$$
$$= \frac{3b^2d\log\left(x+\frac{-\frac{3a^5b^2d^6}{(ad-bc)^4}+\frac{15a^4b^3cd^5}{(ad-bc)^4}-\frac{30a^3b^4c^2d^4}{(ad-bc)^4}+\frac{30a^2b^5c^3d^3}{(ad-bc)^4}-\frac{15ab^6c^4d^2}{(ad-bc)^4}+3ab^2d^2+\frac{3b^7c^5d}{(ad-bc)^4}+3b^3cd}{6b^3d^2}\right)}{(ad-bc)^4}$$
$$-\frac{3b^2d\log\left(x+\frac{\frac{3a^5b^2d^6}{(ad-bc)^4}-\frac{15a^4b^3cd^5}{(ad-bc)^4}+\frac{30a^3b^4c^2d^4}{(ad-bc)^4}-\frac{30a^2b^5c^3d^3}{(ad-bc)^4}+\frac{15ab^6c^4d^2}{(ad-bc)^4}+3ab^2d^2-\frac{3b^7c^5d}{(ad-bc)^4}+3b^3cd}{6b^3d^2}\right)}{(ad-bc)^4}$$
$$+\frac{-a^2d^2+5abcd+2b^2c^2+}{2a^4c^2d^3-6a^3bc^3d^2+6a^2b^2c^4d-2ab^3c^5+x^3\cdot(2a^3bd^5-6a^2b^2cd^4+6ab^3c^2d^3-2b^4c^3d^2)+x^2\cdot(2a^4d^5}$$

input
```
integrate((b*x+a)/(a*c+(a*d+b*c)*x+b*d*x**2)**3,x)
```

output
```
3*b**2*d*log(x + (-3*a**5*b**2*d**6/(a*d - b*c)**4 + 15*a**4*b**3*c*d**5/(
a*d - b*c)**4 - 30*a**3*b**4*c**2*d**4/(a*d - b*c)**4 + 30*a**2*b**5*c**3*
d**3/(a*d - b*c)**4 - 15*a*b**6*c**4*d**2/(a*d - b*c)**4 + 3*a*b**2*d**2 +
 3*b**7*c**5*d/(a*d - b*c)**4 + 3*b**3*c*d)/(6*b**3*d**2))/(a*d - b*c)**4
- 3*b**2*d*log(x + (3*a**5*b**2*d**6/(a*d - b*c)**4 - 15*a**4*b**3*c*d**5/
(a*d - b*c)**4 + 30*a**3*b**4*c**2*d**4/(a*d - b*c)**4 - 30*a**2*b**5*c**3
*d**3/(a*d - b*c)**4 + 15*a*b**6*c**4*d**2/(a*d - b*c)**4 + 3*a*b**2*d**2
- 3*b**7*c**5*d/(a*d - b*c)**4 + 3*b**3*c*d)/(6*b**3*d**2))/(a*d - b*c)**4
 + (-a**2*d**2 + 5*a*b*c*d + 2*b**2*c**2 + 6*b**2*d**2*x**2 + x*(3*a*b*d**
2 + 9*b**2*c*d))/(2*a**4*c**2*d**3 - 6*a**3*b*c**3*d**2 + 6*a**2*b**2*c**4
*d - 2*a*b**3*c**5 + x**3*(2*a**3*b*d**5 - 6*a**2*b**2*c*d**4 + 6*a*b**3*c
**2*d**3 - 2*b**4*c**3*d**2) + x**2*(2*a**4*d**5 - 2*a**3*b*c*d**4 - 6*a**
2*b**2*c**2*d**3 + 10*a*b**3*c**3*d**2 - 4*b**4*c**4*d) + x*(4*a**4*c*d**4
 - 10*a**3*b*c**2*d**3 + 6*a**2*b**2*c**3*d**2 + 2*a*b**3*c**4*d - 2*b**4*
c**5))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 386 vs. $2(108) = 216$.

Time = 0.05 (sec) , antiderivative size = 386, normalized size of antiderivative = 3.51

$$\int \frac{a+bx}{(ac+(bc+ad)x+bdx^2)^3}\,dx = -\frac{3\,b^2d\log(bx+a)}{b^4c^4-4\,ab^3c^3d+6\,a^2b^2c^2d^2-4\,a^3bcd^3+a^4d^4} + \frac{3\,b^2d\log(dx+c)}{b^4c^4-4\,ab^3c^3d+6\,a^2b^2c^2d^2-4\,a^3bcd^3+a^4d^4} - \frac{6\,b^2d^2x^2+2\,b^2c^2+5\,abcd-a^2}{2\,(ab^3c^5-3\,a^2b^2c^4d+3\,a^3bc^3d^2-a^4c^2d^3+(b^4c^3d^2-3\,ab^3c^2d^3+3\,a^2b^2cd^4-a^3bd^5)x^3+(2\,b^4c^4d-5}$$

input
```
integrate((b*x+a)/(a*c+(a*d+b*c)*x+b*d*x^2)^3,x, algorithm="maxima")
```

output
```
-3*b^2*d*log(b*x + a)/(b^4*c^4 - 4*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 - 4*a^3
*b*c*d^3 + a^4*d^4) + 3*b^2*d*log(d*x + c)/(b^4*c^4 - 4*a*b^3*c^3*d + 6*a^
2*b^2*c^2*d^2 - 4*a^3*b*c*d^3 + a^4*d^4) - 1/2*(6*b^2*d^2*x^2 + 2*b^2*c^2
+ 5*a*b*c*d - a^2*d^2 + 3*(3*b^2*c*d + a*b*d^2)*x)/(a*b^3*c^5 - 3*a^2*b^2*
c^4*d + 3*a^3*b*c^3*d^2 - a^4*c^2*d^3 + (b^4*c^3*d^2 - 3*a*b^3*c^2*d^3 + 3
*a^2*b^2*c*d^4 - a^3*b*d^5)*x^3 + (2*b^4*c^4*d - 5*a*b^3*c^3*d^2 + 3*a^2*b
^2*c^2*d^3 + a^3*b*c*d^4 - a^4*d^5)*x^2 + (b^4*c^5 - a*b^3*c^4*d - 3*a^2*b
^2*c^3*d^2 + 5*a^3*b*c^2*d^3 - 2*a^4*c*d^4)*x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 254 vs. $2(108) = 216$.

Time = 0.11 (sec) , antiderivative size = 254, normalized size of antiderivative = 2.31

$$\int \frac{a+bx}{(ac+(bc+ad)x+bdx^2)^3}\,dx = -\frac{3\,b^3d\log(|bx+a|)}{b^5c^4-4\,ab^4c^3d+6\,a^2b^3c^2d^2-4\,a^3b^2cd^3+a^4bd^4} + \frac{3\,b^2d^2\log(|dx+c|)}{b^4c^4d-4\,ab^3c^3d^2+6\,a^2b^2c^2d^3-4\,a^3bcd^4+a^4d^5} - \frac{2\,b^3c^3+3\,ab^2c^2d-6\,a^2bcd^2+a^3d^3+6\,(b^3cd^2-ab^2d^3)x^2+3\,(3\,b^3c^2d-2\,ab^2cd^2-a^2bd^3)x}{2\,(bc-ad)^4(bx+a)(dx+c)^2}$$

input
```
integrate((b*x+a)/(a*c+(a*d+b*c)*x+b*d*x^2)^3,x, algorithm="giac")
```

output
```
-3*b^3*d*log(abs(b*x + a))/(b^5*c^4 - 4*a*b^4*c^3*d + 6*a^2*b^3*c^2*d^2 -
4*a^3*b^2*c*d^3 + a^4*b*d^4) + 3*b^2*d^2*log(abs(d*x + c))/(b^4*c^4*d - 4*
a*b^3*c^3*d^2 + 6*a^2*b^2*c^2*d^3 - 4*a^3*b*c*d^4 + a^4*d^5) - 1/2*(2*b^3*
c^3 + 3*a*b^2*c^2*d - 6*a^2*b*c*d^2 + a^3*d^3 + 6*(b^3*c*d^2 - a*b^2*d^3)*
x^2 + 3*(3*b^3*c^2*d - 2*a*b^2*c*d^2 - a^2*b*d^3)*x)/((b*c - a*d)^4*(b*x +
 a)*(d*x + c)^2)
```

Mupad [B] (verification not implemented)

Time = 6.12 (sec) , antiderivative size = 329, normalized size of antiderivative = 2.99

$$\int \frac{a+bx}{(ac+(bc+ad)x+bdx^2)^3}\,dx$$
$$= \frac{\frac{-a^2\,d^2+5\,a\,b\,c\,d+2\,b^2\,c^2}{2\,(a^3\,d^3-3\,a^2\,b\,c\,d^2+3\,a\,b^2\,c^2\,d-b^3\,c^3)}+\frac{3\,b\,x\,(a\,d^2+3\,b\,c\,d)}{2\,(a^3\,d^3-3\,a^2\,b\,c\,d^2+3\,a\,b^2\,c^2\,d-b^3\,c^3)}+\frac{3\,b^2\,d^2\,x^2}{a^3\,d^3-3\,a^2\,b\,c\,d^2+3\,a\,b^2\,c^2\,d-b^3\,c^3}}{x\,(b\,c^2+2\,a\,d\,c)+a\,c^2+x^2\,(a\,d^2+2\,b\,c\,d)+b\,d^2\,x^3}$$
$$-\frac{6\,b^2\,d\,\mathrm{atanh}\left(\frac{a^4\,d^4-2\,a^3\,b\,c\,d^3+2\,a\,b^3\,c^3\,d-b^4\,c^4}{(a\,d-b\,c)^4}+\frac{2\,b\,d\,x\,(a^3\,d^3-3\,a^2\,b\,c\,d^2+3\,a\,b^2\,c^2\,d-b^3\,c^3)}{(a\,d-b\,c)^4}\right)}{(a\,d-b\,c)^4}$$

input
```
int((a + b*x)/(a*c + x*(a*d + b*c) + b*d*x^2)^3,x)
```

output
```
((2*b^2*c^2 - a^2*d^2 + 5*a*b*c*d)/(2*(a^3*d^3 - b^3*c^3 + 3*a*b^2*c^2*d -
 3*a^2*b*c*d^2)) + (3*b*x*(a*d^2 + 3*b*c*d))/(2*(a^3*d^3 - b^3*c^3 + 3*a*b
^2*c^2*d - 3*a^2*b*c*d^2)) + (3*b^2*d^2*x^2)/(a^3*d^3 - b^3*c^3 + 3*a*b^2*
c^2*d - 3*a^2*b*c*d^2))/(x*(b*c^2 + 2*a*c*d) + a*c^2 + x^2*(a*d^2 + 2*b*c*
d) + b*d^2*x^3) - (6*b^2*d*atanh((a^4*d^4 - b^4*c^4 + 2*a*b^3*c^3*d - 2*a^
3*b*c*d^3)/(a*d - b*c)^4 + (2*b*d*x*(a^3*d^3 - b^3*c^3 + 3*a*b^2*c^2*d - 3
*a^2*b*c*d^2))/(a*d - b*c)^4))/(a*d - b*c)^4
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 821, normalized size of antiderivative = 7.46

$$\int \frac{a+bx}{(ac+(bc+ad)x+bdx^2)^3}\,dx = \text{Too large to display}$$

input
```
int((b*x+a)/(a*c+(a*d+b*c)*x+b*d*x^2)^3,x)
```

output
```
( - 6*log(a + b*x)*a**2*b**2*c**2*d**2 - 12*log(a + b*x)*a**2*b**2*c*d**3*
x - 6*log(a + b*x)*a**2*b**2*d**4*x**2 - 12*log(a + b*x)*a*b**3*c**3*d - 3
0*log(a + b*x)*a*b**3*c**2*d**2*x - 24*log(a + b*x)*a*b**3*c*d**3*x**2 - 6
*log(a + b*x)*a*b**3*d**4*x**3 - 12*log(a + b*x)*b**4*c**3*d*x - 24*log(a
+ b*x)*b**4*c**2*d**2*x**2 - 12*log(a + b*x)*b**4*c*d**3*x**3 + 6*log(c +
d*x)*a**2*b**2*c**2*d**2 + 12*log(c + d*x)*a**2*b**2*c*d**3*x + 6*log(c +
d*x)*a**2*b**2*d**4*x**2 + 12*log(c + d*x)*a*b**3*c**3*d + 30*log(c + d*x)
*a*b**3*c**2*d**2*x + 24*log(c + d*x)*a*b**3*c*d**3*x**2 + 6*log(c + d*x)*
a*b**3*d**4*x**3 + 12*log(c + d*x)*b**4*c**3*d*x + 24*log(c + d*x)*b**4*c*
*2*d**2*x**2 + 12*log(c + d*x)*b**4*c*d**3*x**3 - a**4*d**4 + 4*a**3*b*c*d
**3 + 3*a**3*b*d**4*x + 3*a**2*b**2*c**2*d**2 - 2*a*b**3*c**3*d + 9*a*b**3
*c**2*d**2*x - 6*a*b**3*d**4*x**3 - 4*b**4*c**4 - 12*b**4*c**3*d*x + 6*b**
4*c*d**3*x**3)/(2*(a**6*c**2*d**5 + 2*a**6*c*d**6*x + a**6*d**7*x**2 - 2*a
**5*b*c**3*d**4 - 3*a**5*b*c**2*d**5*x + a**5*b*d**7*x**3 - 2*a**4*b**2*c*
*4*d**3 - 6*a**4*b**2*c**3*d**4*x - 6*a**4*b**2*c**2*d**5*x**2 - 2*a**4*b*
*2*c*d**6*x**3 + 8*a**3*b**3*c**5*d**2 + 14*a**3*b**3*c**4*d**3*x + 4*a**3
*b**3*c**3*d**4*x**2 - 2*a**3*b**3*c**2*d**5*x**3 - 7*a**2*b**4*c**6*d - 6
*a**2*b**4*c**5*d**2*x + 9*a**2*b**4*c**4*d**3*x**2 + 8*a**2*b**4*c**3*d**
4*x**3 + 2*a*b**5*c**7 - 3*a*b**5*c**6*d*x - 12*a*b**5*c**5*d**2*x**2 - 7*
a*b**5*c**4*d**3*x**3 + 2*b**6*c**7*x + 4*b**6*c**6*d*x**2 + 2*b**6*c**...
```

3.68 $\int \frac{1}{(ac+(bc+ad)x+bdx^2)^3}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 143

$$\int \frac{1}{(ac+(bc+ad)x+bdx^2)^3}\,dx = -\frac{b^2}{2(bc-ad)^3(a+bx)^2}+\frac{3b^2d}{(bc-ad)^4(a+bx)} + \frac{d^2}{2(bc-ad)^3(c+dx)^2}+\frac{3bd^2}{(bc-ad)^4(c+dx)} + \frac{6b^2d^2\log(a+bx)}{(bc-ad)^5}-\frac{6b^2d^2\log(c+dx)}{(bc-ad)^5}$$

output
```
-1/2*b^2/(-a*d+b*c)^3/(b*x+a)^2+3*b^2*d/(-a*d+b*c)^4/(b*x+a)+1/2*d^2/(-a*d
+b*c)^3/(d*x+c)^2+3*b*d^2/(-a*d+b*c)^4/(d*x+c)+6*b^2*d^2*ln(b*x+a)/(-a*d+b
*c)^5-6*b^2*d^2*ln(d*x+c)/(-a*d+b*c)^5
```

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 128, normalized size of antiderivative = 0.90

$$\int \frac{1}{(ac+(bc+ad)x+bdx^2)^3}\,dx = \frac{-\frac{b^2(bc-ad)^2}{(a+bx)^2}+\frac{6b^2d(bc-ad)}{a+bx}+\frac{d^2(bc-ad)^2}{(c+dx)^2}+\frac{6bd^2(bc-ad)}{c+dx}+12b^2d^2\log(a+bx)-12b^2d^2\log(c+dx)}{2(bc-ad)^5}$$

input
```
Integrate[(a*c + (b*c + a*d)*x + b*d*x^2)^(-3),x]
```

output
```
(-((b^2*(b*c - a*d)^2)/(a + b*x)^2) + (6*b^2*d*(b*c - a*d))/(a + b*x) + (d
^2*(b*c - a*d)^2)/(c + d*x)^2 + (6*b*d^2*(b*c - a*d))/(c + d*x) + 12*b^2*d
^2*Log[a + b*x] - 12*b^2*d^2*Log[c + d*x])/(2*(b*c - a*d)^5)
```

Rubi [A] (verified)

Time = 0.62 (sec) , antiderivative size = 160, normalized size of antiderivative = 1.12, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {1084, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\left(x(ad+bc)+ac+bdx^2\right)^3}\,dx$$

$$\Big\downarrow \text{1084}$$

$$b^3d^3\int \left(-\frac{6}{b(bc-ad)^5(c+dx)}-\frac{3}{b^2(bc-ad)^4(c+dx)^2}-\frac{1}{b^3(bc-ad)^3(c+dx)^3}+\frac{6}{d(bc-ad)^5(a+bx)}-\frac{}{d^2(bc}\right.$$

$$\Big\downarrow \text{2009}$$

$$b^3d^3\left(\frac{1}{2b^3d(c+dx)^2(bc-ad)^3}+\frac{3}{b^2d(c+dx)(bc-ad)^4}-\frac{1}{2bd^3(a+bx)^2(bc-ad)^3}+\frac{3}{bd^2(a+bx)(bc-ad)^4}+\frac{6}{b}\right.$$

input
```
Int[(a*c + (b*c + a*d)*x + b*d*x^2)^(-3),x]
```

output
```
b^3*d^3*(-1/2*1/(b*d^3*(b*c - a*d)^3*(a + b*x)^2) + 3/(b*d^2*(b*c - a*d)^4
*(a + b*x)) + 1/(2*b^3*d*(b*c - a*d)^3*(c + d*x)^2) + 3/(b^2*d*(b*c - a*d)
^4*(c + d*x)) + (6*Log[a + b*x])/(b*d*(b*c - a*d)^5) - (6*Log[c + d*x])/(b
*d*(b*c - a*d)^5))
```

Defintions of rubi rules used

rule 1084
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{q = Rt[b^2
 - 4*a*c, 2]}, Simp[1/c^p   Int[ExpandIntegrand[(b/2 - q/2 + c*x)^p*(b/2 + q
/2 + c*x)^p, x], x], x] /;  !FractionalPowerFactorQ[q]] /; FreeQ[{a, b, c},
 x] && IntegerQ[p] && NiceSqrtQ[b^2 - 4*a*c]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 1.24 (sec) , antiderivative size = 140, normalized size of antiderivative = 0.98

method	result
default	$\frac{b^2}{2(ad-bc)^3(bx+a)^2}-\frac{6b^2d^2\ln(bx+a)}{(ad-bc)^5}+\frac{3b^2d}{(ad-bc)^4(bx+a)}-\frac{d^2}{2(ad-bc)^3(dx+c)^2}+\frac{6b^2d^2\ln(dx+c)}{(ad-bc)^5}+\frac{3d^2b}{(ad-bc)^4(dx+c}$
risch	$\frac{\frac{6b^3d^3x^3}{a^4d^4-4a^3bc\,d^3+6a^2b^2c^2d^2-4a\,b^3c^3d+c^4b^4}+\frac{9b^2d^2(ad+bc)x^2}{a^4d^4-4a^3bc\,d^3+6a^2b^2c^2d^2-4a\,b^3c^3d+c^4b^4}+\frac{2\left(a^2d^2+7abcd+b^2c^2\right)bdx}{a^4d^4-4a^3bc\,d^3+6a^2b^2c^2d^2-4a\,b^3c^3d+c^4}}{(bd\,x^2+adx+cbx+ac)^2}$
norman	$\frac{\frac{\left(9a\,b^4d^5+9b^5c\,d^4\right)x^2}{d^2b^2\left(a^4d^4-4a^3bc\,d^3+6a^2b^2c^2d^2-4a\,b^3c^3d+c^4b^4\right)}+\frac{-a^3d^5b^2+7a^2b^3c\,d^4+7d^3a\,c^2b^4-c^3d^2b^5}{2d^2b^2\left(a^4d^4-4a^3bc\,d^3+6a^2b^2c^2d^2-4a\,b^3c^3d+c^4b^4\right)}+\frac{6b^3d^3x^3}{a^4d^4-4a^3bc\,d^3+6a^2b^2c^2d}}{(dx+c)^2(bx+a)^2}$
parallelrisch	$-\frac{-24x\,a^2b^4c\,d^5+24xa\,b^5c^2d^4+24\ln(bx+a)x^3a\,b^5d^6+24\ln(bx+a)x^3b^6c\,d^5-24\ln(dx+c)x^3a\,b^5d^6-24\ln(dx+c)x^3b^6c\,d^5+12}{}$

input
```
int(1/(a*c+(a*d+b*c)*x+b*d*x^2)^3,x,method=_RETURNVERBOSE)
```

output
```
1/2*b^2/(a*d-b*c)^3/(b*x+a)^2-6*b^2/(a*d-b*c)^5*d^2*ln(b*x+a)+3*b^2/(a*d-b
*c)^4*d/(b*x+a)-1/2*d^2/(a*d-b*c)^3/(d*x+c)^2+6*b^2/(a*d-b*c)^5*d^2*ln(d*x
+c)+3*d^2/(a*d-b*c)^4*b/(d*x+c)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 760 vs. $2(139) = 278$.

Time = 0.10 (sec) , antiderivative size = 760, normalized size of antiderivative = 5.31

$$\int \frac{1}{(ac+(bc+ad)x+bdx^2)^3}\,dx = -\frac{b^4c^4 - 8\,ab^3c^3d + 8\,a^3bcd^3 - a^4d^4 - 12\,(b^4cd^3 - ab^3d^4)x^3 - 18\,(b^4c^2d^2 - a^2b^2d^4)x^2 - 4\,(b^4c^3d +}{2\,(a^2b^5c^7 - 5\,a^3b^4c^6d + 10\,a^4b^3c^5d^2 - 10\,a^5b^2c^4d^3 + 5\,a^6bc^3d^4 - a^7c^2d^5 + (b^7c^5d^2 - 5\,ab^6c^4d^3 + 10\,a^2b^5}$$

input
```
integrate(1/(a*c+(a*d+b*c)*x+b*d*x^2)^3,x, algorithm="fricas")
```

output
```
-1/2*(b^4*c^4 - 8*a*b^3*c^3*d + 8*a^3*b*c*d^3 - a^4*d^4 - 12*(b^4*c*d^3 -
a*b^3*d^4)*x^3 - 18*(b^4*c^2*d^2 - a^2*b^2*d^4)*x^2 - 4*(b^4*c^3*d + 6*a*b
^3*c^2*d^2 - 6*a^2*b^2*c*d^3 - a^3*b*d^4)*x - 12*(b^4*d^4*x^4 + a^2*b^2*c^
2*d^2 + 2*(b^4*c*d^3 + a*b^3*d^4)*x^3 + (b^4*c^2*d^2 + 4*a*b^3*c*d^3 + a^2
*b^2*d^4)*x^2 + 2*(a*b^3*c^2*d^2 + a^2*b^2*c*d^3)*x)*log(b*x + a) + 12*(b^
4*d^4*x^4 + a^2*b^2*c^2*d^2 + 2*(b^4*c*d^3 + a*b^3*d^4)*x^3 + (b^4*c^2*d^2
 + 4*a*b^3*c*d^3 + a^2*b^2*d^4)*x^2 + 2*(a*b^3*c^2*d^2 + a^2*b^2*c*d^3)*x)
*log(d*x + c))/(a^2*b^5*c^7 - 5*a^3*b^4*c^6*d + 10*a^4*b^3*c^5*d^2 - 10*a^
5*b^2*c^4*d^3 + 5*a^6*b*c^3*d^4 - a^7*c^2*d^5 + (b^7*c^5*d^2 - 5*a*b^6*c^4
*d^3 + 10*a^2*b^5*c^3*d^4 - 10*a^3*b^4*c^2*d^5 + 5*a^4*b^3*c*d^6 - a^5*b^2
*d^7)*x^4 + 2*(b^7*c^6*d - 4*a*b^6*c^5*d^2 + 5*a^2*b^5*c^4*d^3 - 5*a^4*b^3
*c^2*d^5 + 4*a^5*b^2*c*d^6 - a^6*b*d^7)*x^3 + (b^7*c^7 - a*b^6*c^6*d - 9*a
^2*b^5*c^5*d^2 + 25*a^3*b^4*c^4*d^3 - 25*a^4*b^3*c^3*d^4 + 9*a^5*b^2*c^2*d
^5 + a^6*b*c*d^6 - a^7*d^7)*x^2 + 2*(a*b^6*c^7 - 4*a^2*b^5*c^6*d + 5*a^3*b
^4*c^5*d^2 - 5*a^5*b^2*c^3*d^4 + 4*a^6*b*c^2*d^5 - a^7*c*d^6)*x)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 881 vs. $2(128) = 256$.

Time = 1.17 (sec) , antiderivative size = 881, normalized size of antiderivative = 6.16

$$\int \frac{1}{(ac+(bc+ad)x+bdx^2)^3}\,dx = \text{Too large to display}$$

input
```
integrate(1/(a*c+(a*d+b*c)*x+b*d*x**2)**3,x)
```

output
```
6*b**2*d**2*log(x + (-6*a**6*b**2*d**8/(a*d - b*c)**5 + 36*a**5*b**3*c*d**
7/(a*d - b*c)**5 - 90*a**4*b**4*c**2*d**6/(a*d - b*c)**5 + 120*a**3*b**5*c
**3*d**5/(a*d - b*c)**5 - 90*a**2*b**6*c**4*d**4/(a*d - b*c)**5 + 36*a*b**
7*c**5*d**3/(a*d - b*c)**5 + 6*a*b**2*d**3 - 6*b**8*c**6*d**2/(a*d - b*c)*
*5 + 6*b**3*c*d**2)/(12*b**3*d**3))/(a*d - b*c)**5 - 6*b**2*d**2*log(x + (
6*a**6*b**2*d**8/(a*d - b*c)**5 - 36*a**5*b**3*c*d**7/(a*d - b*c)**5 + 90*
a**4*b**4*c**2*d**6/(a*d - b*c)**5 - 120*a**3*b**5*c**3*d**5/(a*d - b*c)**
5 + 90*a**2*b**6*c**4*d**4/(a*d - b*c)**5 - 36*a*b**7*c**5*d**3/(a*d - b*c
)**5 + 6*a*b**2*d**3 + 6*b**8*c**6*d**2/(a*d - b*c)**5 + 6*b**3*c*d**2)/(1
2*b**3*d**3))/(a*d - b*c)**5 + (-a**3*d**3 + 7*a**2*b*c*d**2 + 7*a*b**2*c*
*2*d - b**3*c**3 + 12*b**3*d**3*x**3 + x**2*(18*a*b**2*d**3 + 18*b**3*c*d*
*2) + x*(4*a**2*b*d**3 + 28*a*b**2*c*d**2 + 4*b**3*c**2*d))/(2*a**6*c**2*d
**4 - 8*a**5*b*c**3*d**3 + 12*a**4*b**2*c**4*d**2 - 8*a**3*b**3*c**5*d + 2
*a**2*b**4*c**6 + x**4*(2*a**4*b**2*d**6 - 8*a**3*b**3*c*d**5 + 12*a**2*b*
*4*c**2*d**4 - 8*a*b**5*c**3*d**3 + 2*b**6*c**4*d**2) + x**3*(4*a**5*b*d**
6 - 12*a**4*b**2*c*d**5 + 8*a**3*b**3*c**2*d**4 + 8*a**2*b**4*c**3*d**3 -
12*a*b**5*c**4*d**2 + 4*b**6*c**5*d) + x**2*(2*a**6*d**6 - 18*a**4*b**2*c*
*2*d**4 + 32*a**3*b**3*c**3*d**3 - 18*a**2*b**4*c**4*d**2 + 2*b**6*c**6) +
 x*(4*a**6*c*d**5 - 12*a**5*b*c**2*d**4 + 8*a**4*b**2*c**3*d**3 + 8*a**3*b
**3*c**4*d**2 - 12*a**2*b**4*c**5*d + 4*a*b**5*c**6))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 594 vs. $2(139) = 278$.

Time = 0.05 (sec) , antiderivative size = 594, normalized size of antiderivative = 4.15

$$\int \frac{1}{(ac+(bc+ad)x+bdx^2)^3}\,dx = \frac{6\,b^2d^2\log(bx+a)}{b^5c^5-5\,ab^4c^4d+10\,a^2b^3c^3d^2-10\,a^3b^2c^2d^3+5\,a^4bcd^4-a^5d^5} - \frac{6\,b^2d^2\log(dx+c)}{b^5c^5-5\,ab^4c^4d+10\,a^2b^3c^3d^2-10\,a^3b^2c^2d^3+5\,a^4bcd^4-a^5d^5} + \frac{}{2\,(a^2b^4c^6-4\,a^3b^3c^5d+6\,a^4b^2c^4d^2-4\,a^5bc^3d^3+a^6c^2d^4+(b^6c^4d^2-4\,ab^5c^3d^3+6\,a^2b^4c^2d^4-4\,a^3b^3cd^5}$$

input
```
integrate(1/(a*c+(a*d+b*c)*x+b*d*x^2)^3,x, algorithm="maxima")
```

output
```
6*b^2*d^2*log(b*x + a)/(b^5*c^5 - 5*a*b^4*c^4*d + 10*a^2*b^3*c^3*d^2 - 10*
a^3*b^2*c^2*d^3 + 5*a^4*b*c*d^4 - a^5*d^5) - 6*b^2*d^2*log(d*x + c)/(b^5*c
^5 - 5*a*b^4*c^4*d + 10*a^2*b^3*c^3*d^2 - 10*a^3*b^2*c^2*d^3 + 5*a^4*b*c*d
^4 - a^5*d^5) + 1/2*(12*b^3*d^3*x^3 - b^3*c^3 + 7*a*b^2*c^2*d + 7*a^2*b*c*
d^2 - a^3*d^3 + 18*(b^3*c*d^2 + a*b^2*d^3)*x^2 + 4*(b^3*c^2*d + 7*a*b^2*c*
d^2 + a^2*b*d^3)*x)/(a^2*b^4*c^6 - 4*a^3*b^3*c^5*d + 6*a^4*b^2*c^4*d^2 - 4
*a^5*b*c^3*d^3 + a^6*c^2*d^4 + (b^6*c^4*d^2 - 4*a*b^5*c^3*d^3 + 6*a^2*b^4*
c^2*d^4 - 4*a^3*b^3*c*d^5 + a^4*b^2*d^6)*x^4 + 2*(b^6*c^5*d - 3*a*b^5*c^4*
d^2 + 2*a^2*b^4*c^3*d^3 + 2*a^3*b^3*c^2*d^4 - 3*a^4*b^2*c*d^5 + a^5*b*d^6)
*x^3 + (b^6*c^6 - 9*a^2*b^4*c^4*d^2 + 16*a^3*b^3*c^3*d^3 - 9*a^4*b^2*c^2*d
^4 + a^6*d^6)*x^2 + 2*(a*b^5*c^6 - 3*a^2*b^4*c^5*d + 2*a^3*b^3*c^4*d^2 + 2
*a^4*b^2*c^3*d^3 - 3*a^5*b*c^2*d^4 + a^6*c*d^5)*x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 345 vs. $2(139) = 278$.

Time = 0.11 (sec) , antiderivative size = 345, normalized size of antiderivative = 2.41

$$\int \frac{1}{(ac+(bc+ad)x+bdx^2)^3}\,dx$$
$$= \frac{6\,b^3d^2\log(|bx+a|)}{b^6c^5-5\,ab^5c^4d+10\,a^2b^4c^3d^2-10\,a^3b^3c^2d^3+5\,a^4b^2cd^4-a^5bd^5}$$
$$-\frac{6\,b^2d^3\log(|dx+c|)}{b^5c^5d-5\,ab^4c^4d^2+10\,a^2b^3c^3d^3-10\,a^3b^2c^2d^4+5\,a^4bcd^5-a^5d^6}$$
$$+\frac{12\,b^3d^3x^3+18\,b^3cd^2x^2+18\,ab^2d^3x^2+4\,b^3c^2dx+28\,ab^2cd^2x+4\,a^2bd^3x-b^3c^3+7\,ab^2c^2d+7\,a^2bcd^2 -}{2\,(b^4c^4-4\,ab^3c^3d+6\,a^2b^2c^2d^2-4\,a^3bcd^3+a^4d^4)(bdx^2+bcx+adx+ac)^2}$$

input
```
integrate(1/(a*c+(a*d+b*c)*x+b*d*x^2)^3,x, algorithm="giac")
```

output
```
6*b^3*d^2*log(abs(b*x + a))/(b^6*c^5 - 5*a*b^5*c^4*d + 10*a^2*b^4*c^3*d^2
- 10*a^3*b^3*c^2*d^3 + 5*a^4*b^2*c*d^4 - a^5*b*d^5) - 6*b^2*d^3*log(abs(d*
x + c))/(b^5*c^5*d - 5*a*b^4*c^4*d^2 + 10*a^2*b^3*c^3*d^3 - 10*a^3*b^2*c^2
*d^4 + 5*a^4*b*c*d^5 - a^5*d^6) + 1/2*(12*b^3*d^3*x^3 + 18*b^3*c*d^2*x^2 +
 18*a*b^2*d^3*x^2 + 4*b^3*c^2*d*x + 28*a*b^2*c*d^2*x + 4*a^2*b*d^3*x - b^3
*c^3 + 7*a*b^2*c^2*d + 7*a^2*b*c*d^2 - a^3*d^3)/((b^4*c^4 - 4*a*b^3*c^3*d
+ 6*a^2*b^2*c^2*d^2 - 4*a^3*b*c*d^3 + a^4*d^4)*(b*d*x^2 + b*c*x + a*d*x +
a*c)^2)
```

Mupad [B] (verification not implemented)

Time = 6.45 (sec) , antiderivative size = 542, normalized size of antiderivative = 3.79

$$\int \frac{1}{(ac+(bc+ad)x+bdx^2)^3}\,dx$$
$$= \frac{\frac{6\,b^3\,d^3\,x^3}{a^4\,d^4-4\,a^3\,b\,c\,d^3+6\,a^2\,b^2\,c^2\,d^2-4\,a\,b^3\,c^3\,d+b^4\,c^4}-\frac{a^3\,d^3-7\,a^2\,b\,c\,d^2-7\,a\,b^2\,c^2\,d+b^3\,c^3}{2\,(a^4\,d^4-4\,a^3\,b\,c\,d^3+6\,a^2\,b^2\,c^2\,d^2-4\,a\,b^3\,c^3\,d+b^4\,c^4)}+\frac{9\,b\,d\,x^2\,(c\,b^2\,d+a\,b\,d^{\cdots}}{a^4\,d^4-4\,a^3\,b\,c\,d^3+6\,a^2\,b^2\,c^2\,d^2-4\cdots}}{x\,(2\,d\,a^2\,c+2\,b\,a\,c^2)+x^2\,(a^2\,d^2+4\,a\,b\,c\,d+b^2\,c^2)+x^3\,(2\,c\,b^2\,d+2\,a\,b\,d^{\cdots}}$$
$$-\frac{12\,b^2\,d^2\,\mathrm{atanh}\left(\frac{a^5\,d^5-3\,a^4\,b\,c\,d^4+2\,a^3\,b^2\,c^2\,d^3+2\,a^2\,b^3\,c^3\,d^2-3\,a\,b^4\,c^4\,d+b^5\,c^5}{(a\,d-b\,c)^5}+\frac{2\,b\,d\,x\,(a^4\,d^4-4\,a^3\,b\,c\,d^3+6\,a^2\,b^2\,c^2\,d^2-4\,a\,b^3\,c^3\,d+b^{\cdots}}{(a\,d-b\,c)^5}\right.}{(a\,d-b\,c)^5}$$

input
```
int(1/(a*c + x*(a*d + b*c) + b*d*x^2)^3,x)
```

output
```
((6*b^3*d^3*x^3)/(a^4*d^4 + b^4*c^4 + 6*a^2*b^2*c^2*d^2 - 4*a*b^3*c^3*d -
4*a^3*b*c*d^3) - (a^3*d^3 + b^3*c^3 - 7*a*b^2*c^2*d - 7*a^2*b*c*d^2)/(2*(a
^4*d^4 + b^4*c^4 + 6*a^2*b^2*c^2*d^2 - 4*a*b^3*c^3*d - 4*a^3*b*c*d^3)) + (
9*b*d*x^2*(a*b*d^2 + b^2*c*d))/(a^4*d^4 + b^4*c^4 + 6*a^2*b^2*c^2*d^2 - 4*
a*b^3*c^3*d - 4*a^3*b*c*d^3) + (2*b*d*x*(a^2*d^2 + b^2*c^2 + 7*a*b*c*d))/(
a^4*d^4 + b^4*c^4 + 6*a^2*b^2*c^2*d^2 - 4*a*b^3*c^3*d - 4*a^3*b*c*d^3))/(x
*(2*a*b*c^2 + 2*a^2*c*d) + x^2*(a^2*d^2 + b^2*c^2 + 4*a*b*c*d) + x^3*(2*a*
b*d^2 + 2*b^2*c*d) + a^2*c^2 + b^2*d^2*x^4) - (12*b^2*d^2*atanh((a^5*d^5 +
 b^5*c^5 + 2*a^2*b^3*c^3*d^2 + 2*a^3*b^2*c^2*d^3 - 3*a*b^4*c^4*d - 3*a^4*b
*c*d^4)/(a*d - b*c)^5 + (2*b*d*x*(a^4*d^4 + b^4*c^4 + 6*a^2*b^2*c^2*d^2 -
4*a*b^3*c^3*d - 4*a^3*b*c*d^3))/(a*d - b*c)^5))/(a*d - b*c)^5
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 1202, normalized size of antiderivative = 8.41

$$\int \frac{1}{(ac+(bc+ad)x+bdx^2)^3}\,dx = \text{Too large to display}$$

input
```
int(1/(a*c+(a*d+b*c)*x+b*d*x^2)^3,x)
```

output
```
( - 12*log(a + b*x)*a**3*b**2*c**2*d**3 - 24*log(a + b*x)*a**3*b**2*c*d**4
*x - 12*log(a + b*x)*a**3*b**2*d**5*x**2 - 12*log(a + b*x)*a**2*b**3*c**3*
d**2 - 48*log(a + b*x)*a**2*b**3*c**2*d**3*x - 60*log(a + b*x)*a**2*b**3*c
*d**4*x**2 - 24*log(a + b*x)*a**2*b**3*d**5*x**3 - 24*log(a + b*x)*a*b**4*
c**3*d**2*x - 60*log(a + b*x)*a*b**4*c**2*d**3*x**2 - 48*log(a + b*x)*a*b*
*4*c*d**4*x**3 - 12*log(a + b*x)*a*b**4*d**5*x**4 - 12*log(a + b*x)*b**5*c
**3*d**2*x**2 - 24*log(a + b*x)*b**5*c**2*d**3*x**3 - 12*log(a + b*x)*b**5
*c*d**4*x**4 + 12*log(c + d*x)*a**3*b**2*c**2*d**3 + 24*log(c + d*x)*a**3*
b**2*c*d**4*x + 12*log(c + d*x)*a**3*b**2*d**5*x**2 + 12*log(c + d*x)*a**2
*b**3*c**3*d**2 + 48*log(c + d*x)*a**2*b**3*c**2*d**3*x + 60*log(c + d*x)*
a**2*b**3*c*d**4*x**2 + 24*log(c + d*x)*a**2*b**3*d**5*x**3 + 24*log(c + d
*x)*a*b**4*c**3*d**2*x + 60*log(c + d*x)*a*b**4*c**2*d**3*x**2 + 48*log(c
+ d*x)*a*b**4*c*d**4*x**3 + 12*log(c + d*x)*a*b**4*d**5*x**4 + 12*log(c +
d*x)*b**5*c**3*d**2*x**2 + 24*log(c + d*x)*b**5*c**2*d**3*x**3 + 12*log(c
+ d*x)*b**5*c*d**4*x**4 - a**5*d**5 + 7*a**4*b*c*d**4 + 4*a**4*b*d**5*x +
2*a**3*b**2*c**2*d**3 + 16*a**3*b**2*c*d**4*x + 12*a**3*b**2*d**5*x**2 - 2
*a**2*b**3*c**3*d**2 - 7*a*b**4*c**4*d - 16*a*b**4*c**3*d**2*x - 6*a*b**4*
d**5*x**4 + b**5*c**5 - 4*b**5*c**4*d*x - 12*b**5*c**3*d**2*x**2 + 6*b**5*
c*d**4*x**4)/(2*(a**8*c**2*d**6 + 2*a**8*c*d**7*x + a**8*d**8*x**2 - 4*a**
7*b*c**3*d**5 - 6*a**7*b*c**2*d**6*x + 2*a**7*b*d**8*x**3 + 5*a**6*b**2...
```

3.69 $\int \frac{1}{(a+bx)\left(ac+(bc+ad)x+bdx^2\right)^3}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 170

$$\int \frac{1}{(a+bx)\left(ac+(bc+ad)x+bdx^2\right)^3}\,dx = -\frac{b^2}{3(bc-ad)^3(a+bx)^3} + \frac{3b^2d}{2(bc-ad)^4(a+bx)^2} - \frac{6b^2d^2}{(bc-ad)^5(a+bx)} - \frac{d^3}{2(bc-ad)^4(c+dx)^2} - \frac{4bd^3}{(bc-ad)^5(c+dx)} - \frac{10b^2d^3\log(a+bx)}{(bc-ad)^6} + \frac{10b^2d^3\log(c+dx)}{(bc-ad)^6}$$

output
```
-1/3*b^2/(-a*d+b*c)^3/(b*x+a)^3+3/2*b^2*d/(-a*d+b*c)^4/(b*x+a)^2-6*b^2*d^2
/(-a*d+b*c)^5/(b*x+a)-1/2*d^3/(-a*d+b*c)^4/(d*x+c)^2-4*b*d^3/(-a*d+b*c)^5/
(d*x+c)-10*b^2*d^3*ln(b*x+a)/(-a*d+b*c)^6+10*b^2*d^3*ln(d*x+c)/(-a*d+b*c)^
6
```

Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 154, normalized size of antiderivative = 0.91

$$\int \frac{1}{(a+bx)\left(ac+(bc+ad)x+bdx^2\right)^3}\,dx = -\frac{\frac{2b^2(bc-ad)^3}{(a+bx)^3}-\frac{9b^2d(bc-ad)^2}{(a+bx)^2}+\frac{36b^2d^2(bc-ad)}{a+bx}+\frac{3d^3(bc-ad)^2}{(c+dx)^2}+\frac{24bd^3(bc-ad)}{c+dx}+60b^2d^3\log(a+bx)-60b^2d^3\log(c+}{6(bc-ad)^6}$$

input
```
Integrate[1/((a + b*x)*(a*c + (b*c + a*d)*x + b*d*x^2)^3),x]
```

output
```
-1/6*((2*b^2*(b*c - a*d)^3)/(a + b*x)^3 - (9*b^2*d*(b*c - a*d)^2)/(a + b*x
)^2 + (36*b^2*d^2*(b*c - a*d))/(a + b*x) + (3*d^3*(b*c - a*d)^2)/(c + d*x)
^2 + (24*b*d^3*(b*c - a*d))/(c + d*x) + 60*b^2*d^3*Log[a + b*x] - 60*b^2*d
^3*Log[c + d*x])/(b*c - a*d)^6
```

Rubi [A] (verified)

Time = 0.66 (sec) , antiderivative size = 170, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.069$, Rules used = {1121, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a+bx)\left(x(ad+bc)+ac+bdx^2\right)^3}\,dx$$

$$\Big\downarrow \text{1121}$$

$$\int \left(-\frac{10b^3d^3}{(a+bx)(bc-ad)^6}+\frac{6b^3d^2}{(a+bx)^2(bc-ad)^5}-\frac{3b^3d}{(a+bx)^3(bc-ad)^4}+\frac{b^3}{(a+bx)^4(bc-ad)^3}+\frac{10b^2d^4}{(c+dx)(bc-a}\right.$$

$$\Big\downarrow \text{2009}$$

$$-\frac{10b^2d^3\log(a+bx)}{(bc-ad)^6}+\frac{10b^2d^3\log(c+dx)}{(bc-ad)^6}-\frac{6b^2d^2}{(a+bx)(bc-ad)^5}+\frac{3b^2d}{2(a+bx)^2(bc-ad)^4}-\frac{b^2}{3(a+bx)^3(bc-ad)^3}-\frac{4bd^3}{(c+dx)(bc-ad)^5}-\frac{d^3}{2(c+dx)^2(bc-ad)^4}$$

input

```
Int[1/((a + b*x)*(a*c + (b*c + a*d)*x + b*d*x^2)^3),x]
```

output

```
-1/3*b^2/((b*c - a*d)^3*(a + b*x)^3) + (3*b^2*d)/(2*(b*c - a*d)^4*(a + b*x
)^2) - (6*b^2*d^2)/((b*c - a*d)^5*(a + b*x)) - d^3/(2*(b*c - a*d)^4*(c + d
*x)^2) - (4*b*d^3)/((b*c - a*d)^5*(c + d*x)) - (10*b^2*d^3*Log[a + b*x])/(
b*c - a*d)^6 + (10*b^2*d^3*Log[c + d*x])/(b*c - a*d)^6
```

Defintions of rubi rules used

rule 1121

```
Int[((d_) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Int[ExpandIntegrand[(d + e*x)^(m + p)*(a/d + (c/e)*x)^p, x], x]
/; FreeQ[{a, b, c, d, e, m, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && (Int
egerQ[p] || (GtQ[a, 0] && GtQ[d, 0] && LtQ[c, 0]))
```

rule 2009

```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 1.60 (sec) , antiderivative size = 165, normalized size of antiderivative = 0.97

method	result
default	$\frac{b^2}{3(ad-bc)^3(bx+a)^3}-\frac{10b^2d^3\ln(bx+a)}{(ad-bc)^6}+\frac{6b^2d^2}{(ad-bc)^5(bx+a)}+\frac{3b^2d}{2(ad-bc)^4(bx+a)^2}-\frac{d^3}{2(ad-bc)^4(dx+c)^2}+\frac{10b^2d^3\ln(\cdots}{(ad-bc\cdots}$
risch	$\frac{\frac{10b^4d^4x^4}{a^5d^5-5a^4bc\,d^4+10a^3b^2c^2d^3-10a^2b^3c^3d^2+5a\,b^4c^4d-b^5c^5}+\frac{5b^3d^3(5ad+3bc)x^3}{a^5d^5-5a^4bc\,d^4+10a^3b^2c^2d^3-10a^2b^3c^3d^2+5a\,b^4c^4d-b^5c^5}+\frac{\cdots}{3\left(a^5d^5-5a^4\cdots\right.}}{\cdots}$
norman	$\frac{\frac{10b^4d^4x^4}{a^5d^5-5a^4bc\,d^4+10a^3b^2c^2d^3-10a^2b^3c^3d^2+5a\,b^4c^4d-b^5c^5}+\frac{\left(25a\,b^5d^6+15b^6c\,d^5\right)x^3}{d^2b^2\left(a^5d^5-5a^4bc\,d^4+10a^3b^2c^2d^3-10a^2b^3c^3d^2+5a\,b^4c^4d-b^5c^5\right)}+\frac{-\cdots}{6d^2b^3\cdots}}{\cdots}$
parallelrisch	$-\frac{-60x^4a\,b^7d^7+60x^4b^8c\,d^6-150x^3a^2b^6d^7+90x^3b^8c^2d^5-110x^2a^3b^5d^7+20x^2b^8c^3d^4-15x\,a^4b^4d^7+60\ln(bx+a)x^5b^8d^7-60\ln\cdots}{\cdots}$

input

```
int(1/(b*x+a)/(a*c+(a*d+b*c)*x+b*d*x^2)^3,x,method=_RETURNVERBOSE)
```

output
```
1/3*b^2/(a*d-b*c)^3/(b*x+a)^3-10*b^2/(a*d-b*c)^6*d^3*ln(b*x+a)+6*b^2/(a*d-
b*c)^5*d^2/(b*x+a)+3/2*b^2/(a*d-b*c)^4*d/(b*x+a)^2-1/2*d^3/(a*d-b*c)^4/(d*
x+c)^2+10*b^2/(a*d-b*c)^6*d^3*ln(d*x+c)+4*d^3/(a*d-b*c)^5*b/(d*x+c)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1151 vs. $2(164) = 328$.

Time = 0.10 (sec) , antiderivative size = 1151, normalized size of antiderivative = 6.77

$$\int \frac{1}{(a+bx)\,(ac+(bc+ad)x+bdx^2)^3}\,dx = \text{Too large to display}$$

input
```
integrate(1/(b*x+a)/(a*c+(a*d+b*c)*x+b*d*x^2)^3,x, algorithm="fricas")
```

output
```
-1/6*(2*b^5*c^5 - 15*a*b^4*c^4*d + 60*a^2*b^3*c^3*d^2 - 20*a^3*b^2*c^2*d^3
 - 30*a^4*b*c*d^4 + 3*a^5*d^5 + 60*(b^5*c*d^4 - a*b^4*d^5)*x^4 + 30*(3*b^5
*c^2*d^3 + 2*a*b^4*c*d^4 - 5*a^2*b^3*d^5)*x^3 + 10*(2*b^5*c^3*d^2 + 21*a*b
^4*c^2*d^3 - 12*a^2*b^3*c*d^4 - 11*a^3*b^2*d^5)*x^2 - 5*(b^5*c^4*d - 12*a*
b^4*c^3*d^2 - 24*a^2*b^3*c^2*d^3 + 32*a^3*b^2*c*d^4 + 3*a^4*b*d^5)*x + 60*
(b^5*d^5*x^5 + a^3*b^2*c^2*d^3 + (2*b^5*c*d^4 + 3*a*b^4*d^5)*x^4 + (b^5*c^
2*d^3 + 6*a*b^4*c*d^4 + 3*a^2*b^3*d^5)*x^3 + (3*a*b^4*c^2*d^3 + 6*a^2*b^3*
c*d^4 + a^3*b^2*d^5)*x^2 + (3*a^2*b^3*c^2*d^3 + 2*a^3*b^2*c*d^4)*x)*log(b*
x + a) - 60*(b^5*d^5*x^5 + a^3*b^2*c^2*d^3 + (2*b^5*c*d^4 + 3*a*b^4*d^5)*x
^4 + (b^5*c^2*d^3 + 6*a*b^4*c*d^4 + 3*a^2*b^3*d^5)*x^3 + (3*a*b^4*c^2*d^3
+ 6*a^2*b^3*c*d^4 + a^3*b^2*d^5)*x^2 + (3*a^2*b^3*c^2*d^3 + 2*a^3*b^2*c*d^
4)*x)*log(d*x + c))/(a^3*b^6*c^8 - 6*a^4*b^5*c^7*d + 15*a^5*b^4*c^6*d^2 -
20*a^6*b^3*c^5*d^3 + 15*a^7*b^2*c^4*d^4 - 6*a^8*b*c^3*d^5 + a^9*c^2*d^6 +
(b^9*c^6*d^2 - 6*a*b^8*c^5*d^3 + 15*a^2*b^7*c^4*d^4 - 20*a^3*b^6*c^3*d^5 +
 15*a^4*b^5*c^2*d^6 - 6*a^5*b^4*c*d^7 + a^6*b^3*d^8)*x^5 + (2*b^9*c^7*d -
9*a*b^8*c^6*d^2 + 12*a^2*b^7*c^5*d^3 + 5*a^3*b^6*c^4*d^4 - 30*a^4*b^5*c^3*
d^5 + 33*a^5*b^4*c^2*d^6 - 16*a^6*b^3*c*d^7 + 3*a^7*b^2*d^8)*x^4 + (b^9*c^
8 - 18*a^2*b^7*c^6*d^2 + 52*a^3*b^6*c^5*d^3 - 60*a^4*b^5*c^4*d^4 + 24*a^5*
b^4*c^3*d^5 + 10*a^6*b^3*c^2*d^6 - 12*a^7*b^2*c*d^7 + 3*a^8*b*d^8)*x^3 + (
3*a*b^8*c^8 - 12*a^2*b^7*c^7*d + 10*a^3*b^6*c^6*d^2 + 24*a^4*b^5*c^5*d^...
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1217 vs. $2(153) = 306$.

Time = 1.67 (sec) , antiderivative size = 1217, normalized size of antiderivative = 7.16

$$\int \frac{1}{(a+bx)\left(ac+(bc+ad)x+bdx^2\right)^3}\,dx = \text{Too large to display}$$

input
```
integrate(1/(b*x+a)/(a*c+(a*d+b*c)*x+b*d*x**2)**3,x)
```

output
```
10*b**2*d**3*log(x + (-10*a**7*b**2*d**10/(a*d - b*c)**6 + 70*a**6*b**3*c*
d**9/(a*d - b*c)**6 - 210*a**5*b**4*c**2*d**8/(a*d - b*c)**6 + 350*a**4*b*
*5*c**3*d**7/(a*d - b*c)**6 - 350*a**3*b**6*c**4*d**6/(a*d - b*c)**6 + 210
*a**2*b**7*c**5*d**5/(a*d - b*c)**6 - 70*a*b**8*c**6*d**4/(a*d - b*c)**6 +
 10*a*b**2*d**4 + 10*b**9*c**7*d**3/(a*d - b*c)**6 + 10*b**3*c*d**3)/(20*b
**3*d**4))/(a*d - b*c)**6 - 10*b**2*d**3*log(x + (10*a**7*b**2*d**10/(a*d
- b*c)**6 - 70*a**6*b**3*c*d**9/(a*d - b*c)**6 + 210*a**5*b**4*c**2*d**8/(
a*d - b*c)**6 - 350*a**4*b**5*c**3*d**7/(a*d - b*c)**6 + 350*a**3*b**6*c**
4*d**6/(a*d - b*c)**6 - 210*a**2*b**7*c**5*d**5/(a*d - b*c)**6 + 70*a*b**8
*c**6*d**4/(a*d - b*c)**6 + 10*a*b**2*d**4 - 10*b**9*c**7*d**3/(a*d - b*c)
**6 + 10*b**3*c*d**3)/(20*b**3*d**4))/(a*d - b*c)**6 + (-3*a**4*d**4 + 27*
a**3*b*c*d**3 + 47*a**2*b**2*c**2*d**2 - 13*a*b**3*c**3*d + 2*b**4*c**4 +
60*b**4*d**4*x**4 + x**3*(150*a*b**3*d**4 + 90*b**4*c*d**3) + x**2*(110*a*
*2*b**2*d**4 + 230*a*b**3*c*d**3 + 20*b**4*c**2*d**2) + x*(15*a**3*b*d**4
+ 175*a**2*b**2*c*d**3 + 55*a*b**3*c**2*d**2 - 5*b**4*c**3*d))/(6*a**8*c**
2*d**5 - 30*a**7*b*c**3*d**4 + 60*a**6*b**2*c**4*d**3 - 60*a**5*b**3*c**5*
d**2 + 30*a**4*b**4*c**6*d - 6*a**3*b**5*c**7 + x**5*(6*a**5*b**3*d**7 - 3
0*a**4*b**4*c*d**6 + 60*a**3*b**5*c**2*d**5 - 60*a**2*b**6*c**3*d**4 + 30*
a*b**7*c**4*d**3 - 6*b**8*c**5*d**2) + x**4*(18*a**6*b**2*d**7 - 78*a**5*b
**3*c*d**6 + 120*a**4*b**4*c**2*d**5 - 60*a**3*b**5*c**3*d**4 - 30*a**2...
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 889 vs. $2(164) = 328$.

Time = 0.07 (sec) , antiderivative size = 889, normalized size of antiderivative = 5.23

$$\int \frac{1}{(a+bx)\left(ac+(bc+ad)x+bdx^2\right)^3}\,dx = \text{Too large to display}$$

input
```
integrate(1/(b*x+a)/(a*c+(a*d+b*c)*x+b*d*x^2)^3,x, algorithm="maxima")
```

output
```
-10*b^2*d^3*log(b*x + a)/(b^6*c^6 - 6*a*b^5*c^5*d + 15*a^2*b^4*c^4*d^2 - 2
0*a^3*b^3*c^3*d^3 + 15*a^4*b^2*c^2*d^4 - 6*a^5*b*c*d^5 + a^6*d^6) + 10*b^2
*d^3*log(d*x + c)/(b^6*c^6 - 6*a*b^5*c^5*d + 15*a^2*b^4*c^4*d^2 - 20*a^3*b
^3*c^3*d^3 + 15*a^4*b^2*c^2*d^4 - 6*a^5*b*c*d^5 + a^6*d^6) - 1/6*(60*b^4*d
^4*x^4 + 2*b^4*c^4 - 13*a*b^3*c^3*d + 47*a^2*b^2*c^2*d^2 + 27*a^3*b*c*d^3
- 3*a^4*d^4 + 30*(3*b^4*c*d^3 + 5*a*b^3*d^4)*x^3 + 10*(2*b^4*c^2*d^2 + 23*
a*b^3*c*d^3 + 11*a^2*b^2*d^4)*x^2 - 5*(b^4*c^3*d - 11*a*b^3*c^2*d^2 - 35*a
^2*b^2*c*d^3 - 3*a^3*b*d^4)*x)/(a^3*b^5*c^7 - 5*a^4*b^4*c^6*d + 10*a^5*b^3
*c^5*d^2 - 10*a^6*b^2*c^4*d^3 + 5*a^7*b*c^3*d^4 - a^8*c^2*d^5 + (b^8*c^5*d
^2 - 5*a*b^7*c^4*d^3 + 10*a^2*b^6*c^3*d^4 - 10*a^3*b^5*c^2*d^5 + 5*a^4*b^4
*c*d^6 - a^5*b^3*d^7)*x^5 + (2*b^8*c^6*d - 7*a*b^7*c^5*d^2 + 5*a^2*b^6*c^4
*d^3 + 10*a^3*b^5*c^3*d^4 - 20*a^4*b^4*c^2*d^5 + 13*a^5*b^3*c*d^6 - 3*a^6*
b^2*d^7)*x^4 + (b^8*c^7 + a*b^7*c^6*d - 17*a^2*b^6*c^5*d^2 + 35*a^3*b^5*c^
4*d^3 - 25*a^4*b^4*c^3*d^4 - a^5*b^3*c^2*d^5 + 9*a^6*b^2*c*d^6 - 3*a^7*b*d
^7)*x^3 + (3*a*b^7*c^7 - 9*a^2*b^6*c^6*d + a^3*b^5*c^5*d^2 + 25*a^4*b^4*c^
4*d^3 - 35*a^5*b^3*c^3*d^4 + 17*a^6*b^2*c^2*d^5 - a^7*b*c*d^6 - a^8*d^7)*x
^2 + (3*a^2*b^6*c^7 - 13*a^3*b^5*c^6*d + 20*a^4*b^4*c^5*d^2 - 10*a^5*b^3*c
^4*d^3 - 5*a^6*b^2*c^3*d^4 + 7*a^7*b*c^2*d^5 - 2*a^8*c*d^6)*x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 458 vs. $2(164) = 328$.

Time = 0.12 (sec) , antiderivative size = 458, normalized size of antiderivative = 2.69

$$\int \frac{1}{(a+bx)\left(ac+(bc+ad)x+bdx^2\right)^3}\,dx$$
$$= -\frac{10\,b^3d^3\log\left(|bx+a|\right)}{b^7c^6-6\,ab^6c^5d+15\,a^2b^5c^4d^2-20\,a^3b^4c^3d^3+15\,a^4b^3c^2d^4-6\,a^5b^2cd^5+a^6bd^6}$$
$$+\frac{10\,b^2d^4\log\left(|dx+c|\right)}{b^6c^6d-6\,ab^5c^5d^2+15\,a^2b^4c^4d^3-20\,a^3b^3c^3d^4+15\,a^4b^2c^2d^5-6\,a^5bcd^6+a^6d^7}$$
$$-\frac{2\,b^5c^5-15\,ab^4c^4d+60\,a^2b^3c^3d^2-20\,a^3b^2c^2d^3-30\,a^4bcd^4+3\,a^5d^5+60\,(b^5cd^4-ab^4d^5)x^4+30\,(3\,b^5c}{}$$

input
```
integrate(1/(b*x+a)/(a*c+(a*d+b*c)*x+b*d*x^2)^3,x, algorithm="giac")
```

output
```
-10*b^3*d^3*log(abs(b*x + a))/(b^7*c^6 - 6*a*b^6*c^5*d + 15*a^2*b^5*c^4*d^
2 - 20*a^3*b^4*c^3*d^3 + 15*a^4*b^3*c^2*d^4 - 6*a^5*b^2*c*d^5 + a^6*b*d^6)
 + 10*b^2*d^4*log(abs(d*x + c))/(b^6*c^6*d - 6*a*b^5*c^5*d^2 + 15*a^2*b^4*
c^4*d^3 - 20*a^3*b^3*c^3*d^4 + 15*a^4*b^2*c^2*d^5 - 6*a^5*b*c*d^6 + a^6*d^
7) - 1/6*(2*b^5*c^5 - 15*a*b^4*c^4*d + 60*a^2*b^3*c^3*d^2 - 20*a^3*b^2*c^2
*d^3 - 30*a^4*b*c*d^4 + 3*a^5*d^5 + 60*(b^5*c*d^4 - a*b^4*d^5)*x^4 + 30*(3
*b^5*c^2*d^3 + 2*a*b^4*c*d^4 - 5*a^2*b^3*d^5)*x^3 + 10*(2*b^5*c^3*d^2 + 21
*a*b^4*c^2*d^3 - 12*a^2*b^3*c*d^4 - 11*a^3*b^2*d^5)*x^2 - 5*(b^5*c^4*d - 1
2*a*b^4*c^3*d^2 - 24*a^2*b^3*c^2*d^3 + 32*a^3*b^2*c*d^4 + 3*a^4*b*d^5)*x)/
((b*c - a*d)^6*(b*x + a)^3*(d*x + c)^2)
```

Mupad [B] (verification not implemented)

Time = 6.34 (sec) , antiderivative size = 797, normalized size of antiderivative = 4.69

$$\int \frac{1}{(a+bx)\left(ac+(bc+ad)x+bdx^2\right)^3}\,dx$$
$$= \frac{\frac{-3\,a^4\,d^4+27\,a^3\,b\,c\,d^3+47\,a^2\,b^2\,c^2\,d^2-13\,a\,b^3\,c^3\,d+2\,b^4\,c^4}{6\,(a^5\,d^5-5\,a^4\,b\,c\,d^4+10\,a^3\,b^2\,c^2\,d^3-10\,a^2\,b^3\,c^3\,d^2+5\,a\,b^4\,c^4\,d-b^5\,c^5)}+\frac{5\,d\,x\,(3\,a^3\,b\,d^3+35\,a^2\,b^2\,c\,d^2+11\,a\,b^3\,c^2\,d-b^4\,c^3)}{6\,(a^5\,d^5-5\,a^4\,b\,c\,d^4+10\,a^3\,b^2\,c^2\,d^3-10\,a^2\,b^3\,c^3\,d^2+5\,a\,b^4\,c^4\,d-b^5\,c^5)}+}{x^2\,\left(a^3\,d^2+6\,a^2\,b\,c\,d+3\,a\,b^2\,c^2\right)+x^3\,\left(3\,a^2\right.}$$
$$-\frac{20\,b^2\,d^3\,\mathrm{atanh}\left(\frac{a^6\,d^6-4\,a^5\,b\,c\,d^5+5\,a^4\,b^2\,c^2\,d^4-5\,a^2\,b^4\,c^4\,d^2+4\,a\,b^5\,c^5\,d-b^6\,c^6}{(a\,d-b\,c)^6}+\frac{2\,b\,d\,x\,(a^5\,d^5-5\,a^4\,b\,c\,d^4+10\,a^3\,b^2\,c^2\,d^3-10\,a^2\,b^3\,c}{(a\,d-b\,c)^6}\right.}{(a\,d-b\,c)^6}$$

input
```
int(1/((a + b*x)*(a*c + x*(a*d + b*c) + b*d*x^2)^3),x)
```

output
```
((2*b^4*c^4 - 3*a^4*d^4 + 47*a^2*b^2*c^2*d^2 - 13*a*b^3*c^3*d + 27*a^3*b*c
*d^3)/(6*(a^5*d^5 - b^5*c^5 - 10*a^2*b^3*c^3*d^2 + 10*a^3*b^2*c^2*d^3 + 5*
a*b^4*c^4*d - 5*a^4*b*c*d^4)) + (5*d*x*(3*a^3*b*d^3 - b^4*c^3 + 35*a^2*b^2
*c*d^2 + 11*a*b^3*c^2*d))/(6*(a^5*d^5 - b^5*c^5 - 10*a^2*b^3*c^3*d^2 + 10*
a^3*b^2*c^2*d^3 + 5*a*b^4*c^4*d - 5*a^4*b*c*d^4)) + (10*b^4*d^4*x^4)/(a^5*
d^5 - b^5*c^5 - 10*a^2*b^3*c^3*d^2 + 10*a^3*b^2*c^2*d^3 + 5*a*b^4*c^4*d -
5*a^4*b*c*d^4) + (5*d^2*x^2*(2*b^4*c^2 + 11*a^2*b^2*d^2 + 23*a*b^3*c*d))/(
3*(a^5*d^5 - b^5*c^5 - 10*a^2*b^3*c^3*d^2 + 10*a^3*b^2*c^2*d^3 + 5*a*b^4*c
^4*d - 5*a^4*b*c*d^4)) + (5*b*d^2*x^3*(5*a*b^2*d^2 + 3*b^3*c*d))/(a^5*d^5
- b^5*c^5 - 10*a^2*b^3*c^3*d^2 + 10*a^3*b^2*c^2*d^3 + 5*a*b^4*c^4*d - 5*a^
4*b*c*d^4))/(x^2*(a^3*d^2 + 3*a*b^2*c^2 + 6*a^2*b*c*d) + x^3*(b^3*c^2 + 3*
a^2*b*d^2 + 6*a*b^2*c*d) + x*(3*a^2*b*c^2 + 2*a^3*c*d) + x^4*(3*a*b^2*d^2
+ 2*b^3*c*d) + a^3*c^2 + b^3*d^2*x^5) - (20*b^2*d^3*atanh((a^6*d^6 - b^6*c
^6 - 5*a^2*b^4*c^4*d^2 + 5*a^4*b^2*c^2*d^4 + 4*a*b^5*c^5*d - 4*a^5*b*c*d^5
)/(a*d - b*c)^6 + (2*b*d*x*(a^5*d^5 - b^5*c^5 - 10*a^2*b^3*c^3*d^2 + 10*a^
3*b^2*c^2*d^3 + 5*a*b^4*c^4*d - 5*a^4*b*c*d^4))/(a*d - b*c)^6))/(a*d - b*c
)^6
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 1840, normalized size of antiderivative = 10.82

$$\int \frac{1}{(a+bx)\,(ac+(bc+ad)x+bdx^2)^3}\,dx = \text{Too large to display}$$

input
```
int(1/(b*x+a)/(a*c+(a*d+b*c)*x+b*d*x^2)^3,x)
```

output

```
( - 180*log(a + b*x)*a**4*b**2*c**2*d**4 - 360*log(a + b*x)*a**4*b**2*c*d*
*5*x - 180*log(a + b*x)*a**4*b**2*d**6*x**2 - 120*log(a + b*x)*a**3*b**3*c
**3*d**3 - 780*log(a + b*x)*a**3*b**3*c**2*d**4*x - 1200*log(a + b*x)*a**3
*b**3*c*d**5*x**2 - 540*log(a + b*x)*a**3*b**3*d**6*x**3 - 360*log(a + b*x
)*a**2*b**4*c**3*d**3*x - 1260*log(a + b*x)*a**2*b**4*c**2*d**4*x**2 - 144
0*log(a + b*x)*a**2*b**4*c*d**5*x**3 - 540*log(a + b*x)*a**2*b**4*d**6*x**
4 - 360*log(a + b*x)*a*b**5*c**3*d**3*x**2 - 900*log(a + b*x)*a*b**5*c**2*
d**4*x**3 - 720*log(a + b*x)*a*b**5*c*d**5*x**4 - 180*log(a + b*x)*a*b**5*
d**6*x**5 - 120*log(a + b*x)*b**6*c**3*d**3*x**3 - 240*log(a + b*x)*b**6*c
**2*d**4*x**4 - 120*log(a + b*x)*b**6*c*d**5*x**5 + 180*log(c + d*x)*a**4*
b**2*c**2*d**4 + 360*log(c + d*x)*a**4*b**2*c*d**5*x + 180*log(c + d*x)*a*
*4*b**2*d**6*x**2 + 120*log(c + d*x)*a**3*b**3*c**3*d**3 + 780*log(c + d*x
)*a**3*b**3*c**2*d**4*x + 1200*log(c + d*x)*a**3*b**3*c*d**5*x**2 + 540*lo
g(c + d*x)*a**3*b**3*d**6*x**3 + 360*log(c + d*x)*a**2*b**4*c**3*d**3*x +
1260*log(c + d*x)*a**2*b**4*c**2*d**4*x**2 + 1440*log(c + d*x)*a**2*b**4*c
*d**5*x**3 + 540*log(c + d*x)*a**2*b**4*d**6*x**4 + 360*log(c + d*x)*a*b**
5*c**3*d**3*x**2 + 900*log(c + d*x)*a*b**5*c**2*d**4*x**3 + 720*log(c + d*
x)*a*b**5*c*d**5*x**4 + 180*log(c + d*x)*a*b**5*d**6*x**5 + 120*log(c + d*
x)*b**6*c**3*d**3*x**3 + 240*log(c + d*x)*b**6*c**2*d**4*x**4 + 120*log(c
+ d*x)*b**6*c*d**5*x**5 - 9*a**6*d**6 + 84*a**5*b*c*d**5 + 45*a**5*b*d*...
```

3.70 $\int (d+ex)^4 \left(ade+\left(cd^2+ae^2\right)x+cdex^2\right) dx$

Optimal result

Integrand size = 33, antiderivative size = 39

$$\int (d+ex)^4 \left(ade+\left(cd^2+ae^2\right)x+cdex^2\right) dx=\frac{1}{6}\left(a-\frac{cd^2}{e^2}\right)(d+ex)^6+\frac{cd(d+ex)^7}{7e^2}$$

output
```
1/6*(a-c*d^2/e^2)*(e*x+d)^6+1/7*c*d*(e*x+d)^7/e^2
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 117 vs. 2(39) = 78.

Time = 0.03 (sec) , antiderivative size = 117, normalized size of antiderivative = 3.00

$$\begin{aligned}&\int (d+ex)^4 \left(ade+\left(cd^2+ae^2\right)x+cdex^2\right) dx\\&=\frac{1}{42}x\big(7ae\big(6d^5+15d^4ex+20d^3e^2x^2+15d^2e^3x^3+6de^4x^4+e^5x^5\big)\\&\quad+cdx\big(21d^5+70d^4ex+105d^3e^2x^2+84d^2e^3x^3+35de^4x^4+6e^5x^5\big)\big)\end{aligned}$$

input
```
Integrate[(d + e*x)^4*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2),x]
```

output
```
(x*(7*a*e*(6*d^5 + 15*d^4*e*x + 20*d^3*e^2*x^2 + 15*d^2*e^3*x^3 + 6*d*e^4*
x^4 + e^5*x^5) + c*d*x*(21*d^5 + 70*d^4*e*x + 105*d^3*e^2*x^2 + 84*d^2*e^3
*x^3 + 35*d*e^4*x^4 + 6*e^5*x^5)))/42
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.061$, Rules used = {1121, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (d+ex)^4 \left(x\left(ae^2+cd^2\right)+ade+cdex^2\right) dx$$

$$\Big\downarrow \text{1121}$$

$$\int \left(\frac{(d+ex)^5 \left(ae^2-cd^2\right)}{e}+\frac{cd(d+ex)^6}{e}\right) dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{6}(d+ex)^6 \left(a-\frac{cd^2}{e^2}\right)+\frac{cd(d+ex)^7}{7e^2}$$

input
```
Int[(d + e*x)^4*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2),x]
```

output
```
((a - (c*d^2)/e^2)*(d + e*x)^6)/6 + (c*d*(d + e*x)^7)/(7*e^2)
```

Defintions of rubi rules used

rule 1121
```
Int[((d_) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Int[ExpandIntegrand[(d + e*x)^(m + p)*(a/d + (c/e)*x)^p, x], x]
/; FreeQ[{a, b, c, d, e, m, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && (Int
egerQ[p] || (GtQ[a, 0] && GtQ[d, 0] && LtQ[c, 0]))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 122 vs. $2(35) = 70$.

Time = 0.57 (sec) , antiderivative size = 123, normalized size of antiderivative = 3.15

method	result
norman	$\frac{cd\,e^5x^7}{7}+\left(\frac{1}{6}e^6a+\frac{5}{6}c\,d^2e^4\right)x^6+\left(a\,e^5d+2c\,d^3e^3\right)x^5+\left(\frac{5}{2}a\,e^4d^2+\frac{5}{2}c\,d^4e^2\right)x^4+(\frac{10}{3}a\,e^3d^3+$
gosper	$\frac{x(6cd\,e^5x^6+7x^5e^6a+35x^5c\,d^2e^4+42ad\,e^5x^4+84c\,d^3e^3x^4+105x^3a\,e^4d^2+105x^3c\,d^4e^2+140x^2a\,e^3d^3+70x^2c\,d^5e+105xa\,e^2d^4}{42}$
risch	$\frac{1}{7}cd\,e^5x^7+\frac{1}{6}x^6e^6a+\frac{5}{6}x^6c\,d^2e^4+ad\,e^5x^5+2c\,d^3e^3x^5+\frac{5}{2}x^4a\,e^4d^2+\frac{5}{2}x^4c\,d^4e^2+\frac{10}{3}x^3a\,e^3d^3$
parallelrisch	$\frac{1}{7}cd\,e^5x^7+\frac{1}{6}x^6e^6a+\frac{5}{6}x^6c\,d^2e^4+ad\,e^5x^5+2c\,d^3e^3x^5+\frac{5}{2}x^4a\,e^4d^2+\frac{5}{2}x^4c\,d^4e^2+\frac{10}{3}x^3a\,e^3d^3$
orering	$\frac{x(6cd\,e^5x^6+7x^5e^6a+35x^5c\,d^2e^4+42ad\,e^5x^4+84c\,d^3e^3x^4+105x^3a\,e^4d^2+105x^3c\,d^4e^2+140x^2a\,e^3d^3+70x^2c\,d^5e+105xa\,e^2d^4}{42(cdx+ae)(ex+d)}$
default	$\frac{cd\,e^5x^7}{7}+\frac{(4c\,d^2e^4+e^4(a\,e^2+c\,d^2))x^6}{6}+\frac{(6c\,d^3e^3+4d\,e^3(a\,e^2+c\,d^2)+a\,e^5d)x^5}{5}+\frac{(4c\,d^4e^2+6d^2e^2(a\,e^2+c\,d^2)+4a\,e^4d^2)}{4}$

input
```
int((e*x+d)^4*(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e),x,method=_RETURNVERBOSE)
```

output
```
1/7*c*d*e^5*x^7+(1/6*e^6*a+5/6*c*d^2*e^4)*x^6+(a*d*e^5+2*c*d^3*e^3)*x^5+(5
/2*a*e^4*d^2+5/2*c*d^4*e^2)*x^4+(10/3*a*e^3*d^3+5/3*c*d^5*e)*x^3+(5/2*a*e^
2*d^4+1/2*c*d^6)*x^2+a*e*d^5*x
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 121 vs. $2(35) = 70$.

Time = 0.07 (sec) , antiderivative size = 121, normalized size of antiderivative = 3.10

$$\int (d+ex)^4 \left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)\,dx = \frac{1}{7}\,cde^5x^7+ad^5ex+\frac{1}{6}\,\left(5\,cd^2e^4+ae^6\right)x^6+\left(2\,cd^3e^3+ade^5\right)x^5 + \frac{5}{2}\,\left(cd^4e^2+ad^2e^4\right)x^4+\frac{5}{3}\,\left(cd^5e+2\,ad^3e^3\right)x^3+\frac{1}{2}\,\left(cd^6+5\,ad^4e^2\right)x^2$$

input
```
integrate((e*x+d)^4*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2),x, algorithm="fricas
")
```

output
```
1/7*c*d*e^5*x^7 + a*d^5*e*x + 1/6*(5*c*d^2*e^4 + a*e^6)*x^6 + (2*c*d^3*e^3
 + a*d*e^5)*x^5 + 5/2*(c*d^4*e^2 + a*d^2*e^4)*x^4 + 5/3*(c*d^5*e + 2*a*d^3
*e^3)*x^3 + 1/2*(c*d^6 + 5*a*d^4*e^2)*x^2
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 136 vs. $2(34) = 68$.

Time = 0.04 (sec) , antiderivative size = 136, normalized size of antiderivative = 3.49

$$\int (d+ex)^4 \left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)\,dx = ad^5ex+\frac{cde^5x^7}{7}+x^6\left(\frac{ae^6}{6}+\frac{5cd^2e^4}{6}\right)+x^5\left(ade^5+2cd^3e^3\right)+x^4 \cdot\left(\frac{5ad^2e^4}{2}+\frac{5cd^4e^2}{2}\right)+x^3\cdot\left(\frac{10ad^3e^3}{3}+\frac{5cd^5e}{3}\right)+x^2\cdot\left(\frac{5ad^4e^2}{2}+\frac{cd^6}{2}\right)$$

input
```
integrate((e*x+d)**4*(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2),x)
```

output
```
a*d**5*e*x + c*d*e**5*x**7/7 + x**6*(a*e**6/6 + 5*c*d**2*e**4/6) + x**5*(a
*d*e**5 + 2*c*d**3*e**3) + x**4*(5*a*d**2*e**4/2 + 5*c*d**4*e**2/2) + x**3
*(10*a*d**3*e**3/3 + 5*c*d**5*e/3) + x**2*(5*a*d**4*e**2/2 + c*d**6/2)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 121 vs. $2(35) = 70$.

Time = 0.04 (sec) , antiderivative size = 121, normalized size of antiderivative = 3.10

$$\int (d+ex)^4 \left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)\,dx = \frac{1}{7}cde^5x^7+ad^5ex+\frac{1}{6}\left(5\,cd^2e^4+ae^6\right)x^6+\left(2\,cd^3e^3+ade^5\right)x^5 + \frac{5}{2}\left(cd^4e^2+ad^2e^4\right)x^4+\frac{5}{3}\left(cd^5e+2\,ad^3e^3\right)x^3+\frac{1}{2}\left(cd^6+5\,ad^4e^2\right)x^2$$

input
```
integrate((e*x+d)^4*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2),x, algorithm="maxima
")
```

output
```
1/7*c*d*e^5*x^7 + a*d^5*e*x + 1/6*(5*c*d^2*e^4 + a*e^6)*x^6 + (2*c*d^3*e^3
 + a*d*e^5)*x^5 + 5/2*(c*d^4*e^2 + a*d^2*e^4)*x^4 + 5/3*(c*d^5*e + 2*a*d^3
*e^3)*x^3 + 1/2*(c*d^6 + 5*a*d^4*e^2)*x^2
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 127 vs. $2(35) = 70$.

Time = 0.13 (sec) , antiderivative size = 127, normalized size of antiderivative = 3.26

$$\int (d+ex)^4 \left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)\,dx = \frac{1}{7}cde^5x^7+\frac{5}{6}cd^2e^4x^6+\frac{1}{6}ae^6x^6+2\,cd^3e^3x^5+ade^5x^5+\frac{5}{2}cd^4e^2x^4 + \frac{5}{2}ad^2e^4x^4+\frac{5}{3}cd^5ex^3+\frac{10}{3}ad^3e^3x^3+\frac{1}{2}cd^6x^2+\frac{5}{2}ad^4e^2x^2+ad^5ex$$

input
```
integrate((e*x+d)^4*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2),x, algorithm="giac")
```

output
```
1/7*c*d*e^5*x^7 + 5/6*c*d^2*e^4*x^6 + 1/6*a*e^6*x^6 + 2*c*d^3*e^3*x^5 + a*
d*e^5*x^5 + 5/2*c*d^4*e^2*x^4 + 5/2*a*d^2*e^4*x^4 + 5/3*c*d^5*e*x^3 + 10/3
*a*d^3*e^3*x^3 + 1/2*c*d^6*x^2 + 5/2*a*d^4*e^2*x^2 + a*d^5*e*x
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 122, normalized size of antiderivative = 3.13

$$\int (d+ex)^4 \left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)\,dx = x^4\left(\frac{5\,c\,d^4\,e^2}{2}+\frac{5\,a\,d^2\,e^4}{2}\right)+x^2\left(\frac{c\,d^6}{2}+\frac{5\,a\,d^4\,e^2}{2}\right)+x^6\left(\frac{5\,c\,d^2\,e^4}{6}+\frac{a\,e^6}{6}\right) + x^5\left(2\,c\,d^3\,e^3+a\,d\,e^5\right)+x^3\left(\frac{5\,c\,d^5\,e}{3}+\frac{10\,a\,d^3\,e^3}{3}\right)+a\,d^5\,e\,x+\frac{c\,d\,e^5\,x^7}{7}$$

input
```
int((d + e*x)^4*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2),x)
```

output
```
x^4*((5*a*d^2*e^4)/2 + (5*c*d^4*e^2)/2) + x^2*((c*d^6)/2 + (5*a*d^4*e^2)/2
) + x^6*((a*e^6)/6 + (5*c*d^2*e^4)/6) + x^5*(2*c*d^3*e^3 + a*d*e^5) + x^3*
((10*a*d^3*e^3)/3 + (5*c*d^5*e)/3) + a*d^5*e*x + (c*d*e^5*x^7)/7
```

Reduce [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 127, normalized size of antiderivative = 3.26

$$\int (d+ex)^4 \left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)\,dx = \frac{x(6cd\,e^5x^6+7a\,e^6x^5+35c\,d^2e^4x^5+42ad\,e^5x^4+84c\,d^3e^3x^4+105a\,d^2e^4x^3+105c\,d^4e^2x^3+140a\,d^3e^3x^2}{42}$$

input
```
int((e*x+d)^4*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2),x)
```

output
```
(x*(42*a*d**5*e + 105*a*d**4*e**2*x + 140*a*d**3*e**3*x**2 + 105*a*d**2*e*
*4*x**3 + 42*a*d*e**5*x**4 + 7*a*e**6*x**5 + 21*c*d**6*x + 70*c*d**5*e*x**
2 + 105*c*d**4*e**2*x**3 + 84*c*d**3*e**3*x**4 + 35*c*d**2*e**4*x**5 + 6*c
*d*e**5*x**6))/42
```

3.71 $\int (d+ex)^3 \left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)\,dx$

Optimal result

Integrand size = 33, antiderivative size = 39

$$\int (d+ex)^3 \left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)\,dx=\frac{1}{5}\left(a-\frac{cd^2}{e^2}\right)(d+ex)^5+\frac{cd(d+ex)^6}{6e^2}$$

output
```
1/5*(a-c*d^2/e^2)*(e*x+d)^5+1/6*c*d*(e*x+d)^6/e^2
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 95 vs. 2(39) = 78.

Time = 0.01 (sec) , antiderivative size = 95, normalized size of antiderivative = 2.44

$$\int (d+ex)^3 \left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)\,dx$$
$$=\frac{1}{30}x\left(6ae\left(5d^4+10d^3ex+10d^2e^2x^2+5de^3x^3+e^4x^4\right)+cdx\left(15d^4+40d^3ex+45d^2e^2x^2+24de^3x^3+5e^4x^4\right)\right)$$

input
```
Integrate[(d + e*x)^3*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2),x]
```

output
```
(x*(6*a*e*(5*d^4 + 10*d^3*e*x + 10*d^2*e^2*x^2 + 5*d*e^3*x^3 + e^4*x^4) +
c*d*x*(15*d^4 + 40*d^3*e*x + 45*d^2*e^2*x^2 + 24*d*e^3*x^3 + 5*e^4*x^4)))/
30
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.061$, Rules used = {1121, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (d+ex)^3 \left(x\left(ae^2+cd^2\right)+ade+cdex^2\right) dx$$

$$\Big\downarrow \text{1121}$$

$$\int \left(\frac{(d+ex)^4 \left(ae^2-cd^2\right)}{e}+\frac{cd(d+ex)^5}{e}\right) dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{5}(d+ex)^5 \left(a-\frac{cd^2}{e^2}\right)+\frac{cd(d+ex)^6}{6e^2}$$

input
```
Int[(d + e*x)^3*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2),x]
```

output
```
((a - (c*d^2)/e^2)*(d + e*x)^5)/5 + (c*d*(d + e*x)^6)/(6*e^2)
```

Defintions of rubi rules used

rule 1121
```
Int[((d_) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Int[ExpandIntegrand[(d + e*x)^(m + p)*(a/d + (c/e)*x)^p, x], x]
/; FreeQ[{a, b, c, d, e, m, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && (Int
egerQ[p] || (GtQ[a, 0] && GtQ[d, 0] && LtQ[c, 0]))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 99 vs. $2(35) = 70$.

Time = 0.61 (sec) , antiderivative size = 100, normalized size of antiderivative = 2.56

method	result
norman	$\frac{cd\,e^4x^6}{6} + \left(\frac{1}{5}a\,e^5 + \frac{4}{5}c\,d^2e^3\right)x^5 + \left(ad\,e^4 + \frac{3}{2}d^3e^2c\right)x^4 + \left(2a\,e^3d^2 + \frac{4}{3}c\,d^4e\right)x^3 + \left(2a\,e^2d^3 + \frac{1}{2}\right.$
gosper	$\frac{x(5cd\,e^4x^5+6x^4a\,e^5+24x^4c\,d^2e^3+30x^3ad\,e^4+45x^3d^3e^2c+60x^2a\,e^3d^2+40x^2c\,d^4e+60xa\,e^2d^3+15xc\,d^5+30ae\,d^4)}{30}$
risch	$\frac{1}{6}cd\,e^4x^6 + \frac{1}{5}x^5a\,e^5 + \frac{4}{5}x^5c\,d^2e^3 + x^4ad\,e^4 + \frac{3}{2}x^4d^3e^2c + 2x^3a\,e^3d^2 + \frac{4}{3}x^3c\,d^4e + 2x^2a\,e^2d^3 +$
parallelrisch	$\frac{1}{6}cd\,e^4x^6 + \frac{1}{5}x^5a\,e^5 + \frac{4}{5}x^5c\,d^2e^3 + x^4ad\,e^4 + \frac{3}{2}x^4d^3e^2c + 2x^3a\,e^3d^2 + \frac{4}{3}x^3c\,d^4e + 2x^2a\,e^2d^3 +$
orering	$\frac{x(5cd\,e^4x^5+6x^4a\,e^5+24x^4c\,d^2e^3+30x^3ad\,e^4+45x^3d^3e^2c+60x^2a\,e^3d^2+40x^2c\,d^4e+60xa\,e^2d^3+15xc\,d^5+30ae\,d^4)\left(ade+(a\,e\right.}{30(cdx+ae)(ex+d)}$
default	$\frac{cd\,e^4x^6}{6} + \frac{(3c\,d^2e^3+e^3(a\,e^2+c\,d^2))x^5}{5} + \frac{(3d^3e^2c+3d\,e^2(a\,e^2+c\,d^2)+ad\,e^4)x^4}{4} + \frac{(c\,d^4e+3d^2e(a\,e^2+c\,d^2)+3a\,e^3d^2)x^3}{3}$

input
```
int((e*x+d)^3*(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e),x,method=_RETURNVERBOSE)
```

output
```
1/6*c*d*e^4*x^6+(1/5*a*e^5+4/5*c*d^2*e^3)*x^5+(a*d*e^4+3/2*d^3*e^2*c)*x^4+
(2*a*e^3*d^2+4/3*c*d^4*e)*x^3+(2*a*e^2*d^3+1/2*c*d^5)*x^2+a*e*d^4*x
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 102 vs. $2(35) = 70$.

Time = 0.07 (sec) , antiderivative size = 102, normalized size of antiderivative = 2.62

$$\int (d+ex)^3 \left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)\,dx = \frac{1}{6}\,cde^4x^6+ad^4ex+\frac{1}{5}\left(4\,cd^2e^3+ae^5\right)x^5+\frac{1}{2}\left(3\,cd^3e^2+2\,ade^4\right)x^4 + \frac{2}{3}\left(2\,cd^4e+3\,ad^2e^3\right)x^3+\frac{1}{2}\left(cd^5+4\,ad^3e^2\right)x^2$$

input
```
integrate((e*x+d)^3*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2),x, algorithm="fricas
")
```

output
```
1/6*c*d*e^4*x^6 + a*d^4*e*x + 1/5*(4*c*d^2*e^3 + a*e^5)*x^5 + 1/2*(3*c*d^3
*e^2 + 2*a*d*e^4)*x^4 + 2/3*(2*c*d^4*e + 3*a*d^2*e^3)*x^3 + 1/2*(c*d^5 + 4
*a*d^3*e^2)*x^2
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 107 vs. $2(34) = 68$.

Time = 0.03 (sec) , antiderivative size = 107, normalized size of antiderivative = 2.74

$$\int (d+ex)^3 \left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)\,dx = ad^4ex+\frac{cde^4x^6}{6}+x^5\left(\frac{ae^5}{5}+\frac{4cd^2e^3}{5}\right)+x^4\left(ade^4+\frac{3cd^3e^2}{2}\right) + x^3\cdot\left(2ad^2e^3+\frac{4cd^4e}{3}\right)+x^2\cdot\left(2ad^3e^2+\frac{cd^5}{2}\right)$$

input
```
integrate((e*x+d)**3*(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2),x)
```

output
```
a*d**4*e*x + c*d*e**4*x**6/6 + x**5*(a*e**5/5 + 4*c*d**2*e**3/5) + x**4*(a
*d*e**4 + 3*c*d**3*e**2/2) + x**3*(2*a*d**2*e**3 + 4*c*d**4*e/3) + x**2*(2
*a*d**3*e**2 + c*d**5/2)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 102 vs. $2(35) = 70$.

Time = 0.03 (sec) , antiderivative size = 102, normalized size of antiderivative = 2.62

$$\int (d+ex)^3 \left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)\,dx = \frac{1}{6}\,cde^4x^6+ad^4ex+\frac{1}{5}\,\left(4\,cd^2e^3+ae^5\right)x^5+\frac{1}{2}\,\left(3\,cd^3e^2+2\,ade^4\right)x^4 + \frac{2}{3}\,\left(2\,cd^4e+3\,ad^2e^3\right)x^3+\frac{1}{2}\,\left(cd^5+4\,ad^3e^2\right)x^2$$

input

```
integrate((e*x+d)^3*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2),x, algorithm="maxima
")
```

output

```
1/6*c*d*e^4*x^6 + a*d^4*e*x + 1/5*(4*c*d^2*e^3 + a*e^5)*x^5 + 1/2*(3*c*d^3
*e^2 + 2*a*d*e^4)*x^4 + 2/3*(2*c*d^4*e + 3*a*d^2*e^3)*x^3 + 1/2*(c*d^5 + 4
*a*d^3*e^2)*x^2
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 103 vs. $2(35) = 70$.

Time = 0.15 (sec) , antiderivative size = 103, normalized size of antiderivative = 2.64

$$\int (d+ex)^3 \left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)\,dx = \frac{1}{6}\,cde^4x^6+\frac{4}{5}\,cd^2e^3x^5+\frac{1}{5}\,ae^5x^5+\frac{3}{2}\,cd^3e^2x^4+ade^4x^4 + \frac{4}{3}\,cd^4ex^3+2\,ad^2e^3x^3+\frac{1}{2}\,cd^5x^2+2\,ad^3e^2x^2+ad^4ex$$

input

```
integrate((e*x+d)^3*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2),x, algorithm="giac")
```

output

```
1/6*c*d*e^4*x^6 + 4/5*c*d^2*e^3*x^5 + 1/5*a*e^5*x^5 + 3/2*c*d^3*e^2*x^4 +
a*d*e^4*x^4 + 4/3*c*d^4*e*x^3 + 2*a*d^2*e^3*x^3 + 1/2*c*d^5*x^2 + 2*a*d^3*
e^2*x^2 + a*d^4*e*x
```

Mupad [B] (verification not implemented)

Time = 5.75 (sec) , antiderivative size = 99, normalized size of antiderivative = 2.54

$$\int (d+ex)^3 \left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)\,dx = x^2\left(\frac{c\,d^5}{2}+2\,a\,d^3\,e^2\right)+x^5\left(\frac{4\,c\,d^2\,e^3}{5}+\frac{a\,e^5}{5}\right)+x^4\left(\frac{3\,c\,d^3\,e^2}{2}+a\,d\,e^4\right)+x^3\left(\frac{4\,c\,d^4\,e}{3}+2\,a\,d^2\,e^3\right)+a\,d^4\,e\,x+\frac{c\,d\,e^4\,x^6}{6}$$

input
```
int((d + e*x)^3*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2),x)
```

output
```
x^2*((c*d^5)/2 + 2*a*d^3*e^2) + x^5*((a*e^5)/5 + (4*c*d^2*e^3)/5) + x^4*((
3*c*d^3*e^2)/2 + a*d*e^4) + x^3*(2*a*d^2*e^3 + (4*c*d^4*e)/3) + a*d^4*e*x
+ (c*d*e^4*x^6)/6
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 103, normalized size of antiderivative = 2.64

$$\int (d+ex)^3 \left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)\,dx = \frac{x(5cd\,e^4x^5+6a\,e^5x^4+24c\,d^2e^3x^4+30ad\,e^4x^3+45c\,d^3e^2x^3+60a\,d^2e^3x^2+40c\,d^4e\,x^2+60a\,d^3e^2x+15}{30}$$

input
```
int((e*x+d)^3*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2),x)
```

output
```
(x*(30*a*d**4*e + 60*a*d**3*e**2*x + 60*a*d**2*e**3*x**2 + 30*a*d*e**4*x**
3 + 6*a*e**5*x**4 + 15*c*d**5*x + 40*c*d**4*e*x**2 + 45*c*d**3*e**2*x**3 +
 24*c*d**2*e**3*x**4 + 5*c*d*e**4*x**5))/30
```

3.72 $\int (d+ex)^2 \left(ade+\left(cd^2+ae^2\right) x+cdex^2\right) \, dx$

Optimal result

Integrand size = 33, antiderivative size = 39

$$\int (d+ex)^2 \left(ade+\left(cd^2+ae^2\right) x+cdex^2\right) \, dx=\frac{1}{4}\left(a-\frac{cd^2}{e^2}\right) (d+ex)^4+\frac{cd(d+ex)^5}{5e^2}$$

output
```
1/4*(a-c*d^2/e^2)*(e*x+d)^4+1/5*c*d*(e*x+d)^5/e^2
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.87

$$\int (d+ex)^2 \left(ade+\left(cd^2+ae^2\right) x+cdex^2\right) \, dx$$
$$=\frac{1}{20}x\left(5ae\left(4d^3+6d^2ex+4de^2x^2+e^3x^3\right)+cdx\left(10d^3+20d^2ex+15de^2x^2+4e^3x^3\right)\right)$$

input
```
Integrate[(d + e*x)^2*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2),x]
```

output
```
(x*(5*a*e*(4*d^3 + 6*d^2*e*x + 4*d*e^2*x^2 + e^3*x^3) + c*d*x*(10*d^3 + 20
*d^2*e*x + 15*d*e^2*x^2 + 4*e^3*x^3)))/20
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.061, Rules used = {1121, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (d+ex)^2 \left(x\left(ae^2+cd^2\right)+ade+cdex^2\right) \, dx$$

$$\Big\downarrow \ 1121$$

$$\int \left(\frac{(d+ex)^3\left(ae^2-cd^2\right)}{e}+\frac{cd(d+ex)^4}{e}\right)dx$$

$$\Big\downarrow \ 2009$$

$$\frac{1}{4}(d+ex)^4\left(a-\frac{cd^2}{e^2}\right)+\frac{cd(d+ex)^5}{5e^2}$$

input
```
Int[(d + e*x)^2*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2),x]
```

output
```
((a - (c*d^2)/e^2)*(d + e*x)^4)/4 + (c*d*(d + e*x)^5)/(5*e^2)
```

Defintions of rubi rules used

rule 1121
```
Int[((d_) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Int[ExpandIntegrand[(d + e*x)^(m + p)*(a/d + (c/e)*x)^p, x], x]
/; FreeQ[{a, b, c, d, e, m, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && (Int
egerQ[p] || (GtQ[a, 0] && GtQ[d, 0] && LtQ[c, 0]))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 75 vs. $2(35) = 70$.

Time = 0.55 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.95

method	result
norman	$\frac{cd\,e^3x^5}{5}+\left(\frac{1}{4}e^4a+\frac{3}{4}d^2e^2c\right)x^4+\left(ad\,e^3+c\,d^3e\right)x^3+\left(\frac{3}{2}a\,d^2e^2+\frac{1}{2}c\,d^4\right)x^2+x\,d^3ea$
risch	$\frac{1}{5}cd\,e^3x^5+\frac{1}{4}x^4e^4a+\frac{3}{4}x^4d^2e^2c+x^3ad\,e^3+c\,d^3e\,x^3+\frac{3}{2}x^2a\,d^2e^2+\frac{1}{2}x^2c\,d^4+x\,d^3ea$
parallelrisch	$\frac{1}{5}cd\,e^3x^5+\frac{1}{4}x^4e^4a+\frac{3}{4}x^4d^2e^2c+x^3ad\,e^3+c\,d^3e\,x^3+\frac{3}{2}x^2a\,d^2e^2+\frac{1}{2}x^2c\,d^4+x\,d^3ea$
gosper	$\frac{x\left(4cd\,e^3x^4+5x^3e^4a+15x^3d^2e^2c+20ad\,e^3x^2+20c\,d^3e\,x^2+30xa\,d^2e^2+10xc\,d^4+20d^3ea\right)}{20}$
default	$\frac{cd\,e^3x^5}{5}+\frac{\left(2d^2e^2c+e^2\left(a\,e^2+c\,d^2\right)\right)x^4}{4}+\frac{\left(c\,d^3e+2de\left(a\,e^2+c\,d^2\right)+ad\,e^3\right)x^3}{3}+\frac{\left(d^2\left(a\,e^2+c\,d^2\right)+2a\,d^2e^2\right)x^2}{2}+x\,d^3ea$
orering	$\frac{x\left(4cd\,e^3x^4+5x^3e^4a+15x^3d^2e^2c+20ad\,e^3x^2+20c\,d^3e\,x^2+30xa\,d^2e^2+10xc\,d^4+20d^3ea\right)\left(ade+\left(a\,e^2+c\,d^2\right)x+cd\,x^2e\right)}{20(cdx+ae)(ex+d)}$

input
```
int((e*x+d)^2*(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e),x,method=_RETURNVERBOSE)
```

output
```
1/5*c*d*e^3*x^5+(1/4*e^4*a+3/4*d^2*e^2*c)*x^4+(a*d*e^3+c*d^3*e)*x^3+(3/2*a
*d^2*e^2+1/2*c*d^4)*x^2+x*d^3*e*a
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 75 vs. $2(35) = 70$.

Time = 0.08 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.92

$$\int (d+ex)^2\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)\,dx$$
$$=\frac{1}{5}cde^3x^5+ad^3ex+\frac{1}{4}\left(3\,cd^2e^2+ae^4\right)x^4+\left(cd^3e+ade^3\right)x^3+\frac{1}{2}\left(cd^4+3\,ad^2e^2\right)x^2$$

input
```
integrate((e*x+d)^2*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2),x, algorithm="fricas
")
```

output
```
1/5*c*d*e^3*x^5 + a*d^3*e*x + 1/4*(3*c*d^2*e^2 + a*e^4)*x^4 + (c*d^3*e + a
*d*e^3)*x^3 + 1/2*(c*d^4 + 3*a*d^2*e^2)*x^2
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 80 vs. 2(34) = 68.

Time = 0.02 (sec) , antiderivative size = 80, normalized size of antiderivative = 2.05

$$\int (d+ex)^2 \left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)\,dx$$
$$= ad^3ex + \frac{cde^3x^5}{5} + x^4\left(\frac{ae^4}{4} + \frac{3cd^2e^2}{4}\right) + x^3\left(ade^3 + cd^3e\right) + x^2\cdot\left(\frac{3ad^2e^2}{2} + \frac{cd^4}{2}\right)$$

input
```
integrate((e*x+d)**2*(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2),x)
```

output
```
a*d**3*e*x + c*d*e**3*x**5/5 + x**4*(a*e**4/4 + 3*c*d**2*e**2/4) + x**3*(a
*d*e**3 + c*d**3*e) + x**2*(3*a*d**2*e**2/2 + c*d**4/2)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 75 vs. 2(35) = 70.

Time = 0.03 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.92

$$\int (d+ex)^2 \left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)\,dx$$
$$= \frac{1}{5}\,cde^3x^5 + ad^3ex + \frac{1}{4}\left(3\,cd^2e^2 + ae^4\right)x^4 + \left(cd^3e + ade^3\right)x^3 + \frac{1}{2}\left(cd^4 + 3\,ad^2e^2\right)x^2$$

input
```
integrate((e*x+d)^2*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2),x, algorithm="maxima
")
```

output
```
1/5*c*d*e^3*x^5 + a*d^3*e*x + 1/4*(3*c*d^2*e^2 + a*e^4)*x^4 + (c*d^3*e + a
*d*e^3)*x^3 + 1/2*(c*d^4 + 3*a*d^2*e^2)*x^2
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 78 vs. $2(35) = 70$.

Time = 0.14 (sec) , antiderivative size = 78, normalized size of antiderivative = 2.00

$$\int (d+ex)^2 \left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)\,dx = \frac{1}{5}\,cde^3x^5+\frac{3}{4}\,cd^2e^2x^4+\frac{1}{4}\,ae^4x^4+cd^3ex^3+ade^3x^3+\frac{1}{2}\,cd^4x^2+\frac{3}{2}\,ad^2e^2x^2+ad^3ex$$

input
```
integrate((e*x+d)^2*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2),x, algorithm="giac")
```

output
```
1/5*c*d*e^3*x^5 + 3/4*c*d^2*e^2*x^4 + 1/4*a*e^4*x^4 + c*d^3*e*x^3 + a*d*e^
3*x^3 + 1/2*c*d^4*x^2 + 3/2*a*d^2*e^2*x^2 + a*d^3*e*x
```

Mupad [B] (verification not implemented)

Time = 5.52 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.92

$$\int (d+ex)^2 \left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)\,dx = x^2\left(\frac{c\,d^4}{2}+\frac{3\,a\,d^2\,e^2}{2}\right)+x^4\left(\frac{3\,c\,d^2\,e^2}{4}+\frac{a\,e^4}{4}\right)+x^3\left(c\,d^3\,e+a\,d\,e^3\right)+a\,d^3\,e\,x+\frac{c\,d\,e^3\,x^5}{5}$$

input
```
int((d + e*x)^2*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2),x)
```

output
```
x^2*((c*d^4)/2 + (3*a*d^2*e^2)/2) + x^4*((a*e^4)/4 + (3*c*d^2*e^2)/4) + x^
3*(a*d*e^3 + c*d^3*e) + a*d^3*e*x + (c*d*e^3*x^5)/5
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 79, normalized size of antiderivative = 2.03

$$\int (d+ex)^2 \left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)\,dx$$
$$=\frac{x(4cd\,e^3x^4+5a\,e^4x^3+15c\,d^2e^2x^3+20ad\,e^3x^2+20c\,d^3e\,x^2+30a\,d^2e^2x+10c\,d^4x+20a\,d^3e)}{20}$$

input
```
int((e*x+d)^2*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2),x)
```

output
```
(x*(20*a*d**3*e + 30*a*d**2*e**2*x + 20*a*d*e**3*x**2 + 5*a*e**4*x**3 + 10
*c*d**4*x + 20*c*d**3*e*x**2 + 15*c*d**2*e**2*x**3 + 4*c*d*e**3*x**4))/20
```

3.73 $\int (d+ex) \left(ade+\left(cd^2+ae^2\right) x+cdex^2\right) \, dx$

Optimal result

Integrand size = 31, antiderivative size = 39

$$\int (d+ex) \left(ade+\left(cd^2+ae^2\right) x+cdex^2\right) \, dx=\frac{1}{3} \left(a-\frac{cd^2}{e^2}\right) (d+ex)^3+\frac{cd(d+ex)^4}{4e^2}$$

output
```
1/3*(a-c*d^2/e^2)*(e*x+d)^3+1/4*c*d*(e*x+d)^4/e^2
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.31

$$\int (d+ex) \left(ade+\left(cd^2+ae^2\right) x+cdex^2\right) \, dx$$
$$=\frac{1}{12} x \left(4ae\left(3d^2+3dex+e^2x^2\right)+cdx\left(6d^2+8dex+3e^2x^2\right)\right)$$

input
```
Integrate[(d + e*x)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2),x]
```

output
```
(x*(4*a*e*(3*d^2 + 3*d*e*x + e^2*x^2) + c*d*x*(6*d^2 + 8*d*e*x + 3*e^2*x^2
)))/12
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.065, Rules used = {1121, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (d+ex)\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)dx$$

$$\Big\downarrow \text{1121}$$

$$\int \left(\frac{(d+ex)^2\left(ae^2-cd^2\right)}{e}+\frac{cd(d+ex)^3}{e}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{3}(d+ex)^3\left(a-\frac{cd^2}{e^2}\right)+\frac{cd(d+ex)^4}{4e^2}$$

input
```
Int[(d + e*x)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2),x]
```

output
```
((a - (c*d^2)/e^2)*(d + e*x)^3)/3 + (c*d*(d + e*x)^4)/(4*e^2)
```

Defintions of rubi rules used

rule 1121
```
Int[((d_) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Int[ExpandIntegrand[(d + e*x)^(m + p)*(a/d + (c/e)*x)^p, x], x]
/; FreeQ[{a, b, c, d, e, m, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && (Int
egerQ[p] || (GtQ[a, 0] && GtQ[d, 0] && LtQ[c, 0]))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.42 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.38

method	result	size
norman	$\frac{cd\,e^2x^4}{4}+\left(\frac{1}{3}a\,e^3+\frac{2}{3}c\,d^2e\right)x^3+\left(ad\,e^2+\frac{1}{2}c\,d^3\right)x^2+xa\,d^2e$	54
gosper	$\frac{x\left(3cd\,x^3e^2+4a\,e^3x^2+8c\,d^2e\,x^2+12d\,e^2ax+6d^3cx+12a\,d^2e\right)}{12}$	56
risch	$\frac{1}{4}cd\,e^2x^4+\frac{1}{3}x^3a\,e^3+\frac{2}{3}x^3c\,d^2e+ad\,e^2x^2+\frac{1}{2}d^3c\,x^2+xa\,d^2e$	56
parallelrisch	$\frac{1}{4}cd\,e^2x^4+\frac{1}{3}x^3a\,e^3+\frac{2}{3}x^3c\,d^2e+ad\,e^2x^2+\frac{1}{2}d^3c\,x^2+xa\,d^2e$	56
default	$\frac{cd\,e^2x^4}{4}+\frac{\left(c\,d^2e+e\left(a\,e^2+c\,d^2\right)\right)x^3}{3}+\frac{\left(d\left(a\,e^2+c\,d^2\right)+ad\,e^2\right)x^2}{2}+xa\,d^2e$	69
orering	$\frac{x\left(3cd\,x^3e^2+4a\,e^3x^2+8c\,d^2e\,x^2+12d\,e^2ax+6d^3cx+12a\,d^2e\right)\left(ade+\left(a\,e^2+c\,d^2\right)x+cd\,x^2e\right)}{12(cdx+ae)(ex+d)}$	98

input
```
int((e*x+d)*(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e),x,method=_RETURNVERBOSE)
```

output
```
1/4*c*d*e^2*x^4+(1/3*a*e^3+2/3*c*d^2*e)*x^3+(a*d*e^2+1/2*c*d^3)*x^2+x*a*d^
2*e
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.38

$$\int (d+ex)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)\,dx$$
$$=\frac{1}{4}\,cde^2x^4+ad^2ex+\frac{1}{3}\,(2\,cd^2e+ae^3)x^3+\frac{1}{2}\,(cd^3+2\,ade^2)x^2$$

input
```
integrate((e*x+d)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2),x, algorithm="fricas")
```

output
```
1/4*c*d*e^2*x^4 + a*d^2*e*x + 1/3*(2*c*d^2*e + a*e^3)*x^3 + 1/2*(c*d^3 + 2
*a*d*e^2)*x^2
```

Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.44

$$\int (d+ex)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)\,dx = ad^2ex+\frac{cde^2x^4}{4}+x^3\left(\frac{ae^3}{3}+\frac{2cd^2e}{3}\right)+x^2\left(ade^2+\frac{cd^3}{2}\right)$$

input
```
integrate((e*x+d)*(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2),x)
```

output
```
a*d**2*e*x + c*d*e**2*x**4/4 + x**3*(a*e**3/3 + 2*c*d**2*e/3) + x**2*(a*d*
e**2 + c*d**3/2)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.38

$$\int (d+ex)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)\,dx = \frac{1}{4}\,cde^2x^4+ad^2ex+\frac{1}{3}\,(2\,cd^2e+ae^3)x^3+\frac{1}{2}\,(cd^3+2\,ade^2)x^2$$

input
```
integrate((e*x+d)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2),x, algorithm="maxima")
```

output
```
1/4*c*d*e^2*x^4 + a*d^2*e*x + 1/3*(2*c*d^2*e + a*e^3)*x^3 + 1/2*(c*d^3 + 2
*a*d*e^2)*x^2
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.41

$$\int (d+ex)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)\,dx = \frac{1}{4}\,cde^2x^4+\frac{2}{3}\,cd^2ex^3+\frac{1}{3}\,ae^3x^3+\frac{1}{2}\,cd^3x^2+ade^2x^2+ad^2ex$$

input
```
integrate((e*x+d)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2),x, algorithm="giac")
```

output
```
1/4*c*d*e^2*x^4 + 2/3*c*d^2*e*x^3 + 1/3*a*e^3*x^3 + 1/2*c*d^3*x^2 + a*d*e^
2*x^2 + a*d^2*e*x
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.36

$$\int (d+ex)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)\,dx = x^2\left(\frac{c\,d^3}{2}+a\,d\,e^2\right)+x^3\left(\frac{2\,c\,d^2\,e}{3}+\frac{a\,e^3}{3}\right)+a\,d^2\,e\,x+\frac{c\,d\,e^2\,x^4}{4}$$

input
```
int((d + e*x)*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2),x)
```

output
```
x^2*((c*d^3)/2 + a*d*e^2) + x^3*((a*e^3)/3 + (2*c*d^2*e)/3) + a*d^2*e*x +
(c*d*e^2*x^4)/4
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.41

$$\int (d+ex)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)\,dx$$
$$=\frac{x(3cd\,e^2x^3+4a\,e^3x^2+8c\,d^2e\,x^2+12ad\,e^2x+6c\,d^3x+12a\,d^2e)}{12}$$

input
```
int((e*x+d)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2),x)
```

output
```
(x*(12*a*d**2*e + 12*a*d*e**2*x + 4*a*e**3*x**2 + 6*c*d**3*x + 8*c*d**2*e*
x**2 + 3*c*d*e**2*x**3))/12
```

3.74 $\int (ade + (cd^2 + ae^2) x + cdex^2) \, dx$

Optimal result

Integrand size = 25, antiderivative size = 34

$$\int (ade + (cd^2 + ae^2) x + cdex^2) \, dx = adex + \frac{1}{2}(cd^2 + ae^2) x^2 + \frac{1}{3}cdex^3$$

output
```
a*d*e*x+1/2*(a*e^2+c*d^2)*x^2+1/3*c*d*e*x^3
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.12

$$\int (ade + (cd^2 + ae^2) x + cdex^2) \, dx = adex + \frac{1}{2}cd^2x^2 + \frac{1}{2}ae^2x^2 + \frac{1}{3}cdex^3$$

input
```
Integrate[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2,x]
```

output
```
a*d*e*x + (c*d^2*x^2)/2 + (a*e^2*x^2)/2 + (c*d*e*x^3)/3
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.040, Rules used = {2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)\,dx$$

$$\Big\downarrow \; 2009$$

$$\frac{1}{2}x^2\left(ae^2+cd^2\right)+adex+\frac{1}{3}cdex^3$$

input
```
Int[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2,x]
```

output
```
a*d*e*x + ((c*d^2 + a*e^2)*x^2)/2 + (c*d*e*x^3)/3
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.91

method	result	size
default	$adex+\frac{(a\,e^{2}+c\,d^{2})\,x^{2}}{2}+\frac{cde\,x^{3}}{3}$	31
gosper	$\frac{x\,(2cd\,x^{2}e+3a\,e^{2}x+3c\,d^{2}x+6ade)}{6}$	32
norman	$\frac{cde\,x^{3}}{3}+\left(\frac{c\,d^{2}}{2}+\frac{a\,e^{2}}{2}\right)x^{2}+adex$	32
risch	$adex+\frac{1}{2}a\,e^{2}x^{2}+\frac{1}{2}c\,d^{2}x^{2}+\frac{1}{3}cde\,x^{3}$	33
parallelrisch	$adex+\frac{1}{2}a\,e^{2}x^{2}+\frac{1}{2}c\,d^{2}x^{2}+\frac{1}{3}cde\,x^{3}$	33
parts	$adex+\frac{1}{2}a\,e^{2}x^{2}+\frac{1}{2}c\,d^{2}x^{2}+\frac{1}{3}cde\,x^{3}$	33
orering	$\frac{x\,(2cd\,x^{2}e+3a\,e^{2}x+3c\,d^{2}x+6ade)\,(ade+(a\,e^{2}+c\,d^{2})\,x+cd\,x^{2}e)}{6(ex+d)\,(cdx+ae)}$	74

input
```
int(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e,x,method=_RETURNVERBOSE)
```

output
```
a*d*e*x+1/2*(a*e^2+c*d^2)*x^2+1/3*c*d*e*x^3
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.88

$$\int \left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)\,dx=\frac{1}{3}\,cdex^3+adex+\frac{1}{2}\left(cd^2+ae^2\right)x^2$$

input
```
integrate(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2,x, algorithm="fricas")
```

output
```
1/3*c*d*e*x^3 + a*d*e*x + 1/2*(c*d^2 + a*e^2)*x^2
```

Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.94

$$\int \left(ade + \left(cd^2 + ae^2\right)x + cdex^2\right) dx = adex + \frac{cdex^3}{3} + x^2\left(\frac{ae^2}{2} + \frac{cd^2}{2}\right)$$

input
```
integrate(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2,x)
```

output
```
a*d*e*x + c*d*e*x**3/3 + x**2*(a*e**2/2 + c*d**2/2)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.88

$$\int \left(ade + \left(cd^2 + ae^2\right)x + cdex^2\right) dx = \frac{1}{3}cdex^3 + adex + \frac{1}{2}\left(cd^2 + ae^2\right)x^2$$

input
```
integrate(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2,x, algorithm="maxima")
```

output
```
1/3*c*d*e*x^3 + a*d*e*x + 1/2*(c*d^2 + a*e^2)*x^2
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.88

$$\int \left(ade + \left(cd^2 + ae^2\right)x + cdex^2\right) dx = \frac{1}{3}cdex^3 + adex + \frac{1}{2}\left(cd^2 + ae^2\right)x^2$$

input
```
integrate(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2,x, algorithm="giac")
```

output
```
1/3*c*d*e*x^3 + a*d*e*x + 1/2*(c*d^2 + a*e^2)*x^2
```

Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.91

$$\int \left(ade + \left(cd^2 + ae^2\right)x + cdex^2\right) \, dx = \frac{c\,d\,e\,x^3}{3} + \left(\frac{c\,d^2}{2} + \frac{a\,e^2}{2}\right)x^2 + a\,d\,e\,x$$

input
```
int(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2,x)
```

output
```
x^2*((a*e^2)/2 + (c*d^2)/2) + a*d*e*x + (c*d*e*x^3)/3
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.91

$$\int \left(ade + \left(cd^2 + ae^2\right)x + cdex^2\right) \, dx = \frac{x(2cde\,x^2 + 3a\,e^2x + 3c\,d^2x + 6ade)}{6}$$

input
```
int(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2,x)
```

output
```
(x*(6*a*d*e + 3*a*e**2*x + 3*c*d**2*x + 2*c*d*e*x**2))/6
```

3.75 $\int \frac{ade+(cd^2+ae^2)x+cdex^2}{d+ex}\,dx$

Optimal result

Integrand size = 33, antiderivative size = 20

$$\int \frac{ade+(cd^2+ae^2)\,x+cdex^2}{d+ex}\,dx=\frac{(ae+cdx)^2}{2cd}$$

output
```
1/2*(c*d*x+a*e)^2/c/d
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.70

$$\int \frac{ade+(cd^2+ae^2)\,x+cdex^2}{d+ex}\,dx=aex+\frac{1}{2}cdx^2$$

input
```
Integrate[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)/(d + e*x),x]
```

output
```
a*e*x + (c*d*x^2)/2
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.061$, Rules used = {1120, 17}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x(ae^2+cd^2)+ade+cdex^2}{d+ex}dx$$
$$\Big\downarrow \text{ 1120}$$
$$\int (ae+cdx)dx$$
$$\Big\downarrow \text{ 17}$$
$$\frac{(ae+cdx)^2}{2cd}$$

input
```
Int[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)/(d + e*x),x]
```

output
```
(a*e + c*d*x)^2/(2*c*d)
```

Defintions of rubi rules used

rule 17
```
Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1
)/(b*(m + 1))), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]
```

rule 1120
```
Int[((d_) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Int[(d + e*x)^(m + p)*(a/d + (c/e)*x)^p, x] /; FreeQ[{a, b, c, d
, e, m}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[p] && (EqQ[m + p,
0] || EqQ[m + 2*p + 2, 0])
```

Maple [A] (verified)

Time = 0.61 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.65

method	result	size
gosper	$\frac{x(cdx+2ae)}{2}$	13
default	$\frac{1}{2}cd\,x^2+aex$	13
norman	$\frac{1}{2}cd\,x^2+aex$	13
risch	$\frac{1}{2}cd\,x^2+aex$	13
parallelrisch	$\frac{1}{2}cd\,x^2+aex$	13
parts	$\frac{1}{2}cd\,x^2+aex$	13
orering	$\frac{x(cdx+2ae)\left(ade+\left(a\,e^2+c\,d^2\right)x+cd\,x^2e\right)}{2(cdx+ae)(ex+d)}$	55

input
```
int((a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)/(e*x+d),x,method=_RETURNVERBOSE)
```

output
```
1/2*x*(c*d*x+2*a*e)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.60

$$\int \frac{ade+(cd^2+ae^2)\,x+cdex^2}{d+ex}\,dx=\frac{1}{2}\,cdx^2+aex$$

input
```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)/(e*x+d),x, algorithm="fricas")
```

output
```
1/2*c*d*x^2 + a*e*x
```

Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.60

$$\int \frac{ade + (cd^2 + ae^2)\,x + cdex^2}{d + ex}\,dx = aex + \frac{cdx^2}{2}$$

input
```
integrate((a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)/(e*x+d),x)
```

output
```
a*e*x + c*d*x**2/2
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.60

$$\int \frac{ade + (cd^2 + ae^2)\,x + cdex^2}{d + ex}\,dx = \frac{1}{2}\,cdx^2 + aex$$

input
```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)/(e*x+d),x, algorithm="maxima")
```

output
```
1/2*c*d*x^2 + a*e*x
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.60

$$\int \frac{ade + (cd^2 + ae^2)\,x + cdex^2}{d + ex}\,dx = \frac{1}{2}\,cdx^2 + aex$$

input
```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)/(e*x+d),x, algorithm="giac")
```

output
```
1/2*c*d*x^2 + a*e*x
```

Mupad [B] (verification not implemented)

Time = 0.01 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.60

$$\int \frac{ade+(cd^2+ae^2)\,x+cdex^2}{d+ex}\,dx=\frac{c\,d\,x^2}{2}+a\,e\,x$$

input
```
int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)/(d + e*x),x)
```

output
```
a*e*x + (c*d*x^2)/2
```

Reduce [B] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.60

$$\int \frac{ade+(cd^2+ae^2)\,x+cdex^2}{d+ex}\,dx=\frac{x(cdx+2ae)}{2}$$

input
```
int((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)/(e*x+d),x)
```

output
```
(x*(2*a*e + c*d*x))/2
```

3.76 $\int \frac{ade+(cd^2+ae^2)x+cdex^2}{(d+ex)^2}\,dx$

Optimal result

Integrand size = 33, antiderivative size = 26

$$\int \frac{ade+(cd^2+ae^2)\,x+cdex^2}{(d+ex)^2}\,dx=\frac{cdx}{e}+\left(a-\frac{cd^2}{e^2}\right)\log(d+ex)$$

output
```
c*d*x/e+(a-c*d^2/e^2)*ln(e*x+d)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.15

$$\int \frac{ade+(cd^2+ae^2)\,x+cdex^2}{(d+ex)^2}\,dx=\frac{cdx}{e}+\frac{(-cd^2+ae^2)\log(d+ex)}{e^2}$$

input
```
Integrate[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)/(d + e*x)^2,x]
```

output
```
(c*d*x)/e + ((-(c*d^2) + a*e^2)*Log[d + e*x])/e^2
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.061$, Rules used = {1121, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x(ae^2+cd^2)+ade+cdex^2}{(d+ex)^2} \, dx$$

$$\Big\downarrow \text{1121}$$

$$\int \left(\frac{ae^2-cd^2}{e(d+ex)}+\frac{cd}{e}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\left(a-\frac{cd^2}{e^2}\right)\log(d+ex)+\frac{cdx}{e}$$

input
```
Int[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)/(d + e*x)^2,x]
```

output
```
(c*d*x)/e + (a - (c*d^2)/e^2)*Log[d + e*x]
```

Defintions of rubi rules used

rule 1121
```
Int[((d_) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Int[ExpandIntegrand[(d + e*x)^(m + p)*(a/d + (c/e)*x)^p, x], x]
/; FreeQ[{a, b, c, d, e, m, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && (Int
egerQ[p] || (GtQ[a, 0] && GtQ[d, 0] && LtQ[c, 0]))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.63 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.19

method	result	size
default	$\frac{cdx}{e}+\frac{\left(a\,e^{2}-c\,d^{2}\right)\ln\left(ex+d\right)}{e^{2}}$	31
risch	$\frac{cdx}{e}+\ln\left(ex+d\right)a-\frac{\ln\left(ex+d\right)c\,d^{2}}{e^{2}}$	32
parallelrisch	$\frac{\ln\left(ex+d\right)a\,e^{2}-\ln\left(ex+d\right)c\,d^{2}+cdxe}{e^{2}}$	34
norman	$\frac{cd\,x^{2}-\frac{d^{3}c}{e^{2}}}{ex+d}+\frac{\left(a\,e^{2}-c\,d^{2}\right)\ln\left(ex+d\right)}{e^{2}}$	48

input
```
int((a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)/(e*x+d)^2,x,method=_RETURNVERBOSE)
```

output
```
c*d*x/e+(a*e^2-c*d^2)/e^2*ln(e*x+d)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.15

$$\int \frac{ade+(cd^2+ae^2)\,x+cdex^2}{(d+ex)^2}\,dx=\frac{cdex-(cd^2-ae^2)\log(ex+d)}{e^2}$$

input
```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)/(e*x+d)^2,x, algorithm="fricas
")
```

output
```
(c*d*e*x - (c*d^2 - a*e^2)*log(e*x + d))/e^2
```

Sympy [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00

$$\int \frac{ade+(cd^2+ae^2)\,x+cdex^2}{(d+ex)^2}\,dx=\frac{cdx}{e}+\frac{(ae^2-cd^2)\log{(d+ex)}}{e^2}$$

input
```
integrate((a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)/(e*x+d)**2,x)
```

output
```
c*d*x/e + (a*e**2 - c*d**2)*log(d + e*x)/e**2
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.19

$$\int \frac{ade+(cd^2+ae^2)\,x+cdex^2}{(d+ex)^2}\,dx=\frac{cdx}{e}-\frac{(cd^2-ae^2)\log{(ex+d)}}{e^2}$$

input
```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)/(e*x+d)^2,x, algorithm="maxima
")
```

output
```
c*d*x/e - (c*d^2 - a*e^2)*log(e*x + d)/e^2
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 119 vs. $2(26) = 52$.

Time = 0.12 (sec) , antiderivative size = 119, normalized size of antiderivative = 4.58

$$\int \frac{ade+(cd^2+ae^2)\,x+cdex^2}{(d+ex)^2}\,dx=cde\left(\frac{2\,d\log\left(\frac{|ex+d|}{(ex+d)^2|e|}\right)}{e^3}+\frac{ex+d}{e^3}-\frac{d^2}{(ex+d)e^3}\right)-\frac{ad}{ex+d}-\frac{(cd^2+ae^2)\left(\frac{\log\left(\frac{|ex+d|}{(ex+d)^2|e|}\right)}{e}-\frac{d}{(ex+d)e}\right)}{e}$$

input
```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)/(e*x+d)^2,x, algorithm="giac")
```

output
```
c*d*e*(2*d*log(abs(e*x + d)/((e*x + d)^2*abs(e)))/e^3 + (e*x + d)/e^3 - d^
2/((e*x + d)*e^3)) - a*d/(e*x + d) - (c*d^2 + a*e^2)*(log(abs(e*x + d)/((e
*x + d)^2*abs(e)))/e - d/((e*x + d)*e))/e
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.15

$$\int \frac{ade + (cd^2 + ae^2)\,x + cdex^2}{(d + ex)^2}\,dx = \frac{\ln\left(d + e\,x\right)\,\left(a\,e^2 - c\,d^2\right)}{e^2} + \frac{c\,d\,x}{e}$$

input
```
int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)/(d + e*x)^2,x)
```

output
```
(log(d + e*x)*(a*e^2 - c*d^2))/e^2 + (c*d*x)/e
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.27

$$\int \frac{ade + (cd^2 + ae^2)\,x + cdex^2}{(d + ex)^2}\,dx = \frac{\log(ex + d)\,a\,e^2 - \log(ex + d)\,c\,d^2 + cdex}{e^2}$$

input
```
int((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)/(e*x+d)^2,x)
```

output
```
(log(d + e*x)*a*e**2 - log(d + e*x)*c*d**2 + c*d*e*x)/e**2
```

3.77 $\int \frac{ade+(cd^2+ae^2)x+cdex^2}{(d+ex)^3}\,dx$

Optimal result

Integrand size = 33, antiderivative size = 33

$$\int \frac{ade+(cd^2+ae^2)\,x+cdex^2}{(d+ex)^3}\,dx=-\frac{a-\frac{cd^2}{e^2}}{d+ex}+\frac{cd\log(d+ex)}{e^2}$$

output
```
-(a-c*d^2/e^2)/(e*x+d)+c*d*ln(e*x+d)/e^2
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.09

$$\int \frac{ade+(cd^2+ae^2)\,x+cdex^2}{(d+ex)^3}\,dx=\frac{cd^2-ae^2}{e^2(d+ex)}+\frac{cd\log(d+ex)}{e^2}$$

input
```
Integrate[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)/(d + e*x)^3,x]
```

output
```
(c*d^2 - a*e^2)/(e^2*(d + e*x)) + (c*d*Log[d + e*x])/e^2
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.061, Rules used = {1121, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x(ae^2+cd^2)+ade+cdex^2}{(d+ex)^3}dx$$

$$\Big\downarrow \text{1121}$$

$$\int \left(\frac{ae^2-cd^2}{e(d+ex)^2}+\frac{cd}{e(d+ex)}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{cd\log(d+ex)}{e^2}-\frac{a-\frac{cd^2}{e^2}}{d+ex}$$

input
```
Int[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)/(d + e*x)^3,x]
```

output
```
-((a - (c*d^2)/e^2)/(d + e*x)) + (c*d*Log[d + e*x])/e^2
```

Defintions of rubi rules used

rule 1121
```
Int[((d_) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Int[ExpandIntegrand[(d + e*x)^(m + p)*(a/d + (c/e)*x)^p, x], x]
/; FreeQ[{a, b, c, d, e, m, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && (Int
egerQ[p] || (GtQ[a, 0] && GtQ[d, 0] && LtQ[c, 0]))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.61 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.15

method	result	size
default	$\frac{cd\ln(ex+d)}{e^2}-\frac{a\,e^2-c\,d^2}{e^2(ex+d)}$	38
risch	$\frac{cd\ln(ex+d)}{e^2}-\frac{a}{ex+d}+\frac{c\,d^2}{e^2(ex+d)}$	39
parallelrisch	$\frac{\ln(ex+d)xcde+\ln(ex+d)c\,d^2-a\,e^2+c\,d^2}{e^2(ex+d)}$	46
norman	$\frac{\frac{(a\,d\,e^2-c\,d^3)x^2}{d^2}+\frac{(a\,d\,e^2-c\,d^3)x}{de}}{(ex+d)^2}+\frac{cd\ln(ex+d)}{e^2}$	64

input
```
int((a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)/(e*x+d)^3,x,method=_RETURNVERBOSE)
```

output
```
c*d*ln(e*x+d)/e^2-(a*e^2-c*d^2)/e^2/(e*x+d)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.33

$$\int \frac{ade+(cd^2+ae^2)\,x+cdex^2}{(d+ex)^3}\,dx=\frac{cd^2-ae^2+(cdex+cd^2)\log{(ex+d)}}{e^3x+de^2}$$

input
```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)/(e*x+d)^3,x, algorithm="fricas
")
```

output
```
(c*d^2 - a*e^2 + (c*d*e*x + c*d^2)*log(e*x + d))/(e^3*x + d*e^2)
```

Sympy [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.97

$$\int \frac{ade + (cd^2 + ae^2)\,x + cdex^2}{(d+ex)^3}\,dx = \frac{cd\log{(d+ex)}}{e^2} + \frac{-ae^2 + cd^2}{de^2 + e^3x}$$

input
```
integrate((a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)/(e*x+d)**3,x)
```

output
```
c*d*log(d + e*x)/e**2 + (-a*e**2 + c*d**2)/(d*e**2 + e**3*x)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.18

$$\int \frac{ade + (cd^2 + ae^2)\,x + cdex^2}{(d+ex)^3}\,dx = \frac{cd\log{(ex+d)}}{e^2} + \frac{cd^2 - ae^2}{e^3x + de^2}$$

input
```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)/(e*x+d)^3,x, algorithm="maxima
")
```

output
```
c*d*log(e*x + d)/e^2 + (c*d^2 - a*e^2)/(e^3*x + d*e^2)
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.12

$$\int \frac{ade + (cd^2 + ae^2)\,x + cdex^2}{(d+ex)^3}\,dx = \frac{cd\log{(|ex+d|)}}{e^2} + \frac{cd^2 - ae^2}{(ex+d)e^2}$$

input
```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)/(e*x+d)^3,x, algorithm="giac")
```

output
```
c*d*log(abs(e*x + d))/e^2 + (c*d^2 - a*e^2)/((e*x + d)*e^2)
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.12

$$\int \frac{ade + (cd^2 + ae^2)\,x + cdex^2}{(d+ex)^3}\,dx = \frac{c\,d\,\ln\left(d+e\,x\right)}{e^2} - \frac{a\,e^2 - c\,d^2}{e^2\,\left(d+e\,x\right)}$$

input
```
int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)/(d + e*x)^3,x)
```

output
```
(c*d*log(d + e*x))/e^2 - (a*e^2 - c*d^2)/(e^2*(d + e*x))
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.61

$$\int \frac{ade + (cd^2 + ae^2)\,x + cdex^2}{(d+ex)^3}\,dx = \frac{\log(ex+d)\,c\,d^3 + \log(ex+d)\,c\,d^2ex + a\,e^3x - c\,d^2ex}{d\,e^2\,(ex+d)}$$

input
```
int((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)/(e*x+d)^3,x)
```

output
```
(log(d + e*x)*c*d**3 + log(d + e*x)*c*d**2*e*x + a*e**3*x - c*d**2*e*x)/(d
*e**2*(d + e*x))
```

3.78 $\int \frac{ade+(cd^2+ae^2)x+cdex^2}{(d+ex)^4}\,dx$

Optimal result

Integrand size = 33, antiderivative size = 35

$$\int \frac{ade+(cd^2+ae^2)\,x+cdex^2}{(d+ex)^4}\,dx=\frac{(ae+cdx)^2}{2\,(cd^2-ae^2)\,(d+ex)^2}$$

output
```
1/2*(c*d*x+a*e)^2/(-a*e^2+c*d^2)/(e*x+d)^2
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.83

$$\int \frac{ade+(cd^2+ae^2)\,x+cdex^2}{(d+ex)^4}\,dx=-\frac{ae^2+cd(d+2ex)}{2e^2(d+ex)^2}$$

input
```
Integrate[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)/(d + e*x)^4,x]
```

output
```
-1/2*(a*e^2 + c*d*(d + 2*e*x))/(e^2*(d + e*x)^2)
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.061, Rules used = {1120, 48}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x(ae^2+cd^2)+ade+cdex^2}{(d+ex)^4}dx$$

$$\Big\downarrow \text{ 1120}$$

$$\int \frac{ae+cdx}{(d+ex)^3}dx$$

$$\Big\downarrow \text{ 48}$$

$$\frac{(ae+cdx)^2}{2(d+ex)^2(cd^2-ae^2)}$$

input
```
Int[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)/(d + e*x)^4,x]
```

output
```
(a*e + c*d*x)^2/(2*(c*d^2 - a*e^2)*(d + e*x)^2)
```

Defintions of rubi rules used

rule 48
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && EqQ[m + n + 2, 0] && NeQ[m, -1]
```

rule 1120
```
Int[((d_) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Int[(d + e*x)^(m + p)*(a/d + (c/e)*x)^p, x] /; FreeQ[{a, b, c, d
, e, m}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[p] && (EqQ[m + p,
0] || EqQ[m + 2*p + 2, 0])
```

Maple [A] (verified)

Time = 0.62 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.86

method	result	size
gosper	$-\frac{2cdxe+a\,e^{2}+c\,d^{2}}{2(ex+d)^{2}e^{2}}$	30
parallelrisch	$\frac{-2cdxe-a\,e^{2}-c\,d^{2}}{2e^{2}(ex+d)^{2}}$	32
risch	$\frac{-\frac{cdx}{e}-\frac{a\,e^{2}+c\,d^{2}}{2e^{2}}}{(ex+d)^{2}}$	34
default	$-\frac{cd}{e^{2}(ex+d)}-\frac{a\,e^{2}-c\,d^{2}}{2e^{2}(ex+d)^{2}}$	40
norman	$\frac{-\frac{d\left(a\,e^{3}+c\,d^{2}e\right)}{2e^{3}}-\frac{\left(a\,e^{3}+3c\,d^{2}e\right)x}{2e^{2}}-cd\,x^{2}}{(ex+d)^{3}}$	54
orering	$-\frac{(2cdxe+a\,e^{2}+c\,d^{2})\left(ade+(a\,e^{2}+c\,d^{2})x+cd\,x^{2}e\right)}{2e^{2}(cdx+ae)(ex+d)^{3}}$	65

input
```
int((a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)/(e*x+d)^4,x,method=_RETURNVERBOSE)
```

output
```
-1/2*(2*c*d*e*x+a*e^2+c*d^2)/(e*x+d)^2/e^2
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.23

$$\int \frac{ade+(cd^2+ae^2)\,x+cdex^2}{(d+ex)^4}\,dx=-\frac{2\,cdex+cd^2+ae^2}{2\,(e^4x^2+2\,de^3x+d^2e^2)}$$

input
```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)/(e*x+d)^4,x, algorithm="fricas
")
```

output
```
-1/2*(2*c*d*e*x + c*d^2 + a*e^2)/(e^4*x^2 + 2*d*e^3*x + d^2*e^2)
```

Sympy [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.26

$$\int \frac{ade + (cd^2 + ae^2)x + cdex^2}{(d + ex)^4} dx = \frac{-ae^2 - cd^2 - 2cdex}{2d^2e^2 + 4de^3x + 2e^4x^2}$$

input
```
integrate((a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)/(e*x+d)**4,x)
```

output
```
(-a*e**2 - c*d**2 - 2*c*d*e*x)/(2*d**2*e**2 + 4*d*e**3*x + 2*e**4*x**2)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.23

$$\int \frac{ade + (cd^2 + ae^2)x + cdex^2}{(d + ex)^4} dx = -\frac{2\,cdex + cd^2 + ae^2}{2\,(e^4x^2 + 2\,de^3x + d^2e^2)}$$

input
```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)/(e*x+d)^4,x, algorithm="maxima
")
```

output
```
-1/2*(2*c*d*e*x + c*d^2 + a*e^2)/(e^4*x^2 + 2*d*e^3*x + d^2*e^2)
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.83

$$\int \frac{ade + (cd^2 + ae^2)x + cdex^2}{(d + ex)^4} dx = -\frac{2\,cdex + cd^2 + ae^2}{2\,(ex + d)^2e^2}$$

input
```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)/(e*x+d)^4,x, algorithm="giac")
```

output
```
-1/2*(2*c*d*e*x + c*d^2 + a*e^2)/((e*x + d)^2*e^2)
```

Mupad [B] (verification not implemented)

Time = 5.76 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.86

$$\int \frac{ade + (cd^2 + ae^2)\,x + cdex^2}{(d+ex)^4}\,dx = -\frac{\frac{a}{2} - \frac{c\,x^2}{2}}{d^2 + 2\,d\,e\,x + e^2\,x^2}$$

input
```
int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)/(d + e*x)^4,x)
```

output
```
-(a/2 - (c*x^2)/2)/(d^2 + e^2*x^2 + 2*d*e*x)
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.89

$$\int \frac{ade + (cd^2 + ae^2)\,x + cdex^2}{(d+ex)^4}\,dx = \frac{c\,x^2 - a}{2e^2x^2 + 4dex + 2d^2}$$

input
```
int((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)/(e*x+d)^4,x)
```

output
```
( - a + c*x**2)/(2*(d**2 + 2*d*e*x + e**2*x**2))
```

3.79 $\int \frac{ade+(cd^2+ae^2)x+cdex^2}{(d+ex)^5}\,dx$

Optimal result

Integrand size = 33, antiderivative size = 39

$$\int \frac{ade+(cd^2+ae^2)\,x+cdex^2}{(d+ex)^5}\,dx=-\frac{a-\frac{cd^2}{e^2}}{3(d+ex)^3}-\frac{cd}{2e^2(d+ex)^2}$$

output
```
-1/3*(a-c*d^2/e^2)/(e*x+d)^3-1/2*c*d/e^2/(e*x+d)^2
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.77

$$\int \frac{ade+(cd^2+ae^2)\,x+cdex^2}{(d+ex)^5}\,dx=-\frac{2ae^2+cd(d+3ex)}{6e^2(d+ex)^3}$$

input
```
Integrate[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)/(d + e*x)^5,x]
```

output
```
-1/6*(2*a*e^2 + c*d*(d + 3*e*x))/(e^2*(d + e*x)^3)
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.061$, Rules used = {1121, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x(ae^2+cd^2)+ade+cdex^2}{(d+ex)^5}\,dx$$

$$\Big\downarrow \text{1121}$$

$$\int \left(\frac{ae^2-cd^2}{e(d+ex)^4}+\frac{cd}{e(d+ex)^3}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{a-\frac{cd^2}{e^2}}{3(d+ex)^3}-\frac{cd}{2e^2(d+ex)^2}$$

input
```
Int[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)/(d + e*x)^5,x]
```

output
```
-1/3*(a - (c*d^2)/e^2)/(d + e*x)^3 - (c*d)/(2*e^2*(d + e*x)^2)
```

Defintions of rubi rules used

rule 1121
```
Int[((d_) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Int[ExpandIntegrand[(d + e*x)^(m + p)*(a/d + (c/e)*x)^p, x], x]
/; FreeQ[{a, b, c, d, e, m, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && (Int
egerQ[p] || (GtQ[a, 0] && GtQ[d, 0] && LtQ[c, 0]))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.62 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.79

method	result	size
gosper	$-\frac{3cdxe+2a\,e^{2}+c\,d^{2}}{6e^{2}\left(ex+d\right)^{3}}$	31
risch	$\frac{-\frac{cdx}{2e}-\frac{2a\,e^{2}+c\,d^{2}}{6e^{2}}}{\left(ex+d\right)^{3}}$	35
parallelrisch	$\frac{-3cdx\,e^{2}-2a\,e^{3}-c\,d^{2}e}{6e^{3}\left(ex+d\right)^{3}}$	35
default	$-\frac{a\,e^{2}-c\,d^{2}}{3e^{2}\left(ex+d\right)^{3}}-\frac{cd}{2e^{2}\left(ex+d\right)^{2}}$	40
norman	$\frac{-\frac{d\left(2e^{4}a+d^{2}e^{2}c\right)}{6e^{4}}-\frac{\left(e^{4}a+2d^{2}e^{2}c\right)x}{3e^{3}}-\frac{cd\,x^{2}}{2}}{\left(ex+d\right)^{4}}$	59
orering	$-\frac{\left(3cdxe+2a\,e^{2}+c\,d^{2}\right)\left(ade+\left(a\,e^{2}+c\,d^{2}\right)x+cd\,x^{2}e\right)}{6e^{2}\left(cdx+ae\right)\left(ex+d\right)^{4}}$	66

input
```
int((a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)/(e*x+d)^5,x,method=_RETURNVERBOSE)
```

output
```
-1/6/e^2*(3*c*d*e*x+2*a*e^2+c*d^2)/(e*x+d)^3
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.41

$$\int \frac{ade+(cd^2+ae^2)\,x+cdex^2}{(d+ex)^5}\,dx=-\frac{3\,cdex+cd^2+2\,ae^2}{6\,(e^5x^3+3\,de^4x^2+3\,d^2e^3x+d^3e^2)}$$

input
```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)/(e*x+d)^5,x, algorithm="fricas
")
```

output
```
-1/6*(3*c*d*e*x + c*d^2 + 2*a*e^2)/(e^5*x^3 + 3*d*e^4*x^2 + 3*d^2*e^3*x +
d^3*e^2)
```

Sympy [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.49

$$\int \frac{ade + (cd^2 + ae^2)\,x + cdex^2}{(d+ex)^5}\,dx = \frac{-2ae^2 - cd^2 - 3cdex}{6d^3e^2 + 18d^2e^3x + 18de^4x^2 + 6e^5x^3}$$

input
```
integrate((a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)/(e*x+d)**5,x)
```

output
```
(-2*a*e**2 - c*d**2 - 3*c*d*e*x)/(6*d**3*e**2 + 18*d**2*e**3*x + 18*d*e**4
*x**2 + 6*e**5*x**3)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.41

$$\int \frac{ade + (cd^2 + ae^2)\,x + cdex^2}{(d+ex)^5}\,dx = -\frac{3\,cdex + cd^2 + 2\,ae^2}{6\,(e^5x^3 + 3\,de^4x^2 + 3\,d^2e^3x + d^3e^2)}$$

input
```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)/(e*x+d)^5,x, algorithm="maxima
")
```

output
```
-1/6*(3*c*d*e*x + c*d^2 + 2*a*e^2)/(e^5*x^3 + 3*d*e^4*x^2 + 3*d^2*e^3*x +
d^3*e^2)
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.05

$$\int \frac{ade + (cd^2 + ae^2)\,x + cdex^2}{(d+ex)^5}\,dx = -\frac{a}{3\,(ex+d)^3} - \frac{cd}{2\,(ex+d)^2e^2} + \frac{cd^2}{3\,(ex+d)^3e^2}$$

input
```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)/(e*x+d)^5,x, algorithm="giac")
```

output
```
-1/3*a/(e*x + d)^3 - 1/2*c*d/((e*x + d)^2*e^2) + 1/3*c*d^2/((e*x + d)^3*e^
2)
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.46

$$\int \frac{ade+(cd^2+ae^2)\,x+cdex^2}{(d+ex)^5}\,dx=-\frac{\frac{c\,d^2+2\,a\,e^2}{6\,e^2}+\frac{c\,d\,x}{2\,e}}{d^3+3\,d^2\,e\,x+3\,d\,e^2\,x^2+e^3\,x^3}$$

input
```
int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)/(d + e*x)^5,x)
```

output
```
-((2*a*e^2 + c*d^2)/(6*e^2) + (c*d*x)/(2*e))/(d^3 + e^3*x^3 + 3*d*e^2*x^2
+ 3*d^2*e*x)
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.36

$$\int \frac{ade+(cd^2+ae^2)\,x+cdex^2}{(d+ex)^5}\,dx=\frac{-3cdex-2a\,e^2-c\,d^2}{6e^2\,(e^3x^3+3d\,e^2x^2+3d^2ex+d^3)}$$

input
```
int((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)/(e*x+d)^5,x)
```

output
```
( - 2*a*e**2 - c*d**2 - 3*c*d*e*x)/(6*e**2*(d**3 + 3*d**2*e*x + 3*d*e**2*x
**2 + e**3*x**3))
```

3.80 $\int \frac{ade+(cd^2+ae^2)x+cdex^2}{(d+ex)^6}\,dx$

Optimal result

Integrand size = 33, antiderivative size = 39

$$\int \frac{ade+(cd^2+ae^2)\,x+cdex^2}{(d+ex)^6}\,dx=-\frac{a-\frac{cd^2}{e^2}}{4(d+ex)^4}-\frac{cd}{3e^2(d+ex)^3}$$

output
```
-1/4*(a-c*d^2/e^2)/(e*x+d)^4-1/3*c*d/e^2/(e*x+d)^3
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.77

$$\int \frac{ade+(cd^2+ae^2)\,x+cdex^2}{(d+ex)^6}\,dx=-\frac{3ae^2+cd(d+4ex)}{12e^2(d+ex)^4}$$

input
```
Integrate[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)/(d + e*x)^6,x]
```

output
```
-1/12*(3*a*e^2 + c*d*(d + 4*e*x))/(e^2*(d + e*x)^4)
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.061$, Rules used = {1121, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x(ae^2+cd^2)+ade+cdex^2}{(d+ex)^6}\,dx$$

$$\Big\downarrow \text{1121}$$

$$\int \left(\frac{ae^2-cd^2}{e(d+ex)^5}+\frac{cd}{e(d+ex)^4}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{a-\frac{cd^2}{e^2}}{4(d+ex)^4}-\frac{cd}{3e^2(d+ex)^3}$$

input
```
Int[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)/(d + e*x)^6,x]
```

output
```
-1/4*(a - (c*d^2)/e^2)/(d + e*x)^4 - (c*d)/(3*e^2*(d + e*x)^3)
```

Defintions of rubi rules used

rule 1121
```
Int[((d_) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Int[ExpandIntegrand[(d + e*x)^(m + p)*(a/d + (c/e)*x)^p, x], x]
/; FreeQ[{a, b, c, d, e, m, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && (Int
egerQ[p] || (GtQ[a, 0] && GtQ[d, 0] && LtQ[c, 0]))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.62 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.79

method	result	size
gosper	$-\frac{4cdxe+3a\,e^{2}+c\,d^{2}}{12e^{2}\left(ex+d\right)^{4}}$	31
risch	$\frac{-\frac{cdx}{3e}-\frac{3a\,e^{2}+c\,d^{2}}{12e^{2}}}{\left(ex+d\right)^{4}}$	35
parallelrisch	$\frac{-4d\,e^{3}cx-3e^{4}a-d^{2}e^{2}c}{12e^{4}\left(ex+d\right)^{4}}$	37
default	$-\frac{cd}{3e^{2}\left(ex+d\right)^{3}}-\frac{a\,e^{2}-c\,d^{2}}{4e^{2}\left(ex+d\right)^{4}}$	40
norman	$\frac{-\frac{d\left(3a\,e^{5}+c\,d^{2}e^{3}\right)}{12e^{5}}-\frac{\left(3a\,e^{5}+5c\,d^{2}e^{3}\right)x}{12e^{4}}-\frac{cd\,x^{2}}{3}}{\left(ex+d\right)^{5}}$	60
orering	$-\frac{\left(4cdxe+3a\,e^{2}+c\,d^{2}\right)\left(ade+\left(a\,e^{2}+c\,d^{2}\right)x+cd\,x^{2}e\right)}{12e^{2}\left(cdx+ae\right)\left(ex+d\right)^{5}}$	66

input
```
int((a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)/(e*x+d)^6,x,method=_RETURNVERBOSE)
```

output
```
-1/12/e^2*(4*c*d*e*x+3*a*e^2+c*d^2)/(e*x+d)^4
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.69

$$\int \frac{ade+(cd^2+ae^2)\,x+cdex^2}{(d+ex)^6}\,dx = -\frac{4\,cdex+cd^2+3\,ae^2}{12\,(e^6x^4+4\,de^5x^3+6\,d^2e^4x^2+4\,d^3e^3x+d^4e^2)}$$

input
```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)/(e*x+d)^6,x, algorithm="fricas
")
```

output
```
-1/12*(4*c*d*e*x + c*d^2 + 3*a*e^2)/(e^6*x^4 + 4*d*e^5*x^3 + 6*d^2*e^4*x^2
 + 4*d^3*e^3*x + d^4*e^2)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 70 vs. $2(34) = 68$.

Time = 0.24 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.79

$$\int \frac{ade + (cd^2 + ae^2)\,x + cdex^2}{(d + ex)^6}\,dx = \frac{-3ae^2 - cd^2 - 4cdex}{12d^4e^2 + 48d^3e^3x + 72d^2e^4x^2 + 48de^5x^3 + 12e^6x^4}$$

input
```
integrate((a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)/(e*x+d)**6,x)
```

output
```
(-3*a*e**2 - c*d**2 - 4*c*d*e*x)/(12*d**4*e**2 + 48*d**3*e**3*x + 72*d**2*
e**4*x**2 + 48*d*e**5*x**3 + 12*e**6*x**4)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.69

$$\int \frac{ade + (cd^2 + ae^2)\,x + cdex^2}{(d + ex)^6}\,dx = -\frac{4\,cdex + cd^2 + 3\,ae^2}{12\,(e^6x^4 + 4\,de^5x^3 + 6\,d^2e^4x^2 + 4\,d^3e^3x + d^4e^2)}$$

input
```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)/(e*x+d)^6,x, algorithm="maxima
")
```

output
```
-1/12*(4*c*d*e*x + c*d^2 + 3*a*e^2)/(e^6*x^4 + 4*d*e^5*x^3 + 6*d^2*e^4*x^2
 + 4*d^3*e^3*x + d^4*e^2)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.77

$$\int \frac{ade + (cd^2 + ae^2)\,x + cdex^2}{(d + ex)^6}\,dx = -\frac{4\,cdex + cd^2 + 3\,ae^2}{12\,(ex + d)^4e^2}$$

input
```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)/(e*x+d)^6,x, algorithm="giac")
```

output
```
-1/12*(4*c*d*e*x + c*d^2 + 3*a*e^2)/((e*x + d)^4*e^2)
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.74

$$\int \frac{ade + (cd^2 + ae^2)x + cdex^2}{(d+ex)^6}\,dx = -\frac{\frac{c\,d^2+3\,a\,e^2}{12\,e^2} + \frac{c\,d\,x}{3\,e}}{d^4 + 4\,d^3\,e\,x + 6\,d^2\,e^2\,x^2 + 4\,d\,e^3\,x^3 + e^4\,x^4}$$

input
```
int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)/(d + e*x)^6,x)
```

output
```
-((3*a*e^2 + c*d^2)/(12*e^2) + (c*d*x)/(3*e))/(d^4 + e^4*x^4 + 4*d*e^3*x^3
 + 6*d^2*e^2*x^2 + 4*d^3*e*x)
```

Reduce [B] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.64

$$\int \frac{ade + (cd^2 + ae^2)x + cdex^2}{(d+ex)^6}\,dx = \frac{-4cdex - 3a\,e^2 - c\,d^2}{12e^2\left(e^4x^4 + 4d\,e^3x^3 + 6d^2e^2x^2 + 4d^3ex + d^4\right)}$$

input
```
int((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)/(e*x+d)^6,x)
```

output
```
( - 3*a*e**2 - c*d**2 - 4*c*d*e*x)/(12*e**2*(d**4 + 4*d**3*e*x + 6*d**2*e*
*2*x**2 + 4*d*e**3*x**3 + e**4*x**4))
```

3.81 $\int (d+ex)^2 \left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^2 dx$

Optimal result

Integrand size = 35, antiderivative size = 77

$$\int (d+ex)^2 \left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^2 dx = \frac{\left(cd^2-ae^2\right)^2 (d+ex)^5}{5e^3}-\frac{cd\left(cd^2-ae^2\right)(d+ex)^6}{3e^3}+\frac{c^2d^2(d+ex)^7}{7e^3}$$

output
```
1/5*(-a*e^2+c*d^2)^2*(e*x+d)^5/e^3-1/3*c*d*(-a*e^2+c*d^2)*(e*x+d)^6/e^3+1/
7*c^2*d^2*(e*x+d)^7/e^3
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 160 vs. 2(77) = 154.

Time = 0.02 (sec) , antiderivative size = 160, normalized size of antiderivative = 2.08

$$\begin{aligned}\int (d+ex)^2 \left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^2 dx &= \frac{1}{15}acdex^2\left(15d^4+40d^3ex+45d^2e^2x^2+24de^3x^3+5e^4x^4\right)\\&+\frac{1}{105}c^2d^2x^3\left(35d^4+105d^3ex+126d^2e^2x^2+70de^3x^3+15e^4x^4\right)\\&+a^2\left(d^4e^2x+2d^3e^3x^2+2d^2e^4x^3+de^5x^4+\frac{e^6x^5}{5}\right)\end{aligned}$$

input
```
Integrate[(d + e*x)^2*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^2,x]
```

output
```
(a*c*d*e*x^2*(15*d^4 + 40*d^3*e*x + 45*d^2*e^2*x^2 + 24*d*e^3*x^3 + 5*e^4*
x^4))/15 + (c^2*d^2*x^3*(35*d^4 + 105*d^3*e*x + 126*d^2*e^2*x^2 + 70*d*e^3
*x^3 + 15*e^4*x^4))/105 + a^2*(d^4*e^2*x + 2*d^3*e^3*x^2 + 2*d^2*e^4*x^3 +
 d*e^5*x^4 + (e^6*x^5)/5)
```

Rubi [A] (verified)

Time = 0.51 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.057$, Rules used = {1121, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (d+ex)^2 \left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^2 dx$$

$$\Big\downarrow \text{ 1121}$$

$$\int \left(-\frac{2cd(d+ex)^5\left(cd^2-ae^2\right)}{e^2}+\frac{(d+ex)^4\left(ae^2-cd^2\right)^2}{e^2}+\frac{c^2d^2(d+ex)^6}{e^2}\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$-\frac{cd(d+ex)^6\left(cd^2-ae^2\right)}{3e^3}+\frac{(d+ex)^5\left(cd^2-ae^2\right)^2}{5e^3}+\frac{c^2d^2(d+ex)^7}{7e^3}$$

input
```
Int[(d + e*x)^2*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^2,x]
```

output
```
((c*d^2 - a*e^2)^2*(d + e*x)^5)/(5*e^3) - (c*d*(c*d^2 - a*e^2)*(d + e*x)^6
)/(3*e^3) + (c^2*d^2*(d + e*x)^7)/(7*e^3)
```

Defintions of rubi rules used

rule 1121
```
Int[((d_) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Int[ExpandIntegrand[(d + e*x)^(m + p)*(a/d + (c/e)*x)^p, x], x]
/; FreeQ[{a, b, c, d, e, m, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && (Int
egerQ[p] || (GtQ[a, 0] && GtQ[d, 0] && LtQ[c, 0]))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 172 vs. $2(71) = 142$.

Time = 1.25 (sec) , antiderivative size = 173, normalized size of antiderivative = 2.25

method	result
norman	$\frac{c^2d^2e^4x^7}{7} + (\frac{1}{3}acd\,e^5 + \frac{2}{3}c^2d^3e^3)\,x^6 + (\frac{1}{5}a^2e^6 + \frac{8}{5}ac\,d^2e^4 + \frac{6}{5}c^2d^4e^2)\,x^5 + (e^5a^2d + 3ac\,d^3e^3 +$
risch	$\frac{1}{7}c^2d^2e^4x^7 + \frac{1}{3}x^6acd\,e^5 + \frac{2}{3}x^6c^2d^3e^3 + \frac{1}{5}x^5a^2e^6 + \frac{8}{5}x^5ac\,d^2e^4 + \frac{6}{5}x^5c^2d^4e^2 + a^2d\,e^5x^4 + 3ac$
parallelrisch	$\frac{1}{7}c^2d^2e^4x^7 + \frac{1}{3}x^6acd\,e^5 + \frac{2}{3}x^6c^2d^3e^3 + \frac{1}{5}x^5a^2e^6 + \frac{8}{5}x^5ac\,d^2e^4 + \frac{6}{5}x^5c^2d^4e^2 + a^2d\,e^5x^4 + 3ac$
gosper	$x(15c^2d^2e^4x^6+35x^5acd\,e^5+70x^5c^2d^3e^3+21x^4a^2e^6+168x^4ac\,d^2e^4+126x^4c^2d^4e^2+105a^2d\,e^5x^3+315ac\,d^3e^3x^3+105c^2d^5e\,x$ / 105
orering	$x(15c^2d^2e^4x^6+35x^5acd\,e^5+70x^5c^2d^3e^3+21x^4a^2e^6+168x^4ac\,d^2e^4+126x^4c^2d^4e^2+105a^2d\,e^5x^3+315ac\,d^3e^3x^3+105c^2d^5e\,x$ / $105(cdx+ae)^2(ex$
default	$\frac{c^2d^2e^4x^7}{7} + \frac{(2c^2d^3e^3+2e^3dc(a\,e^2+c\,d^2))x^6}{6} + \frac{(c^2d^4e^2+4d^2e^2c(a\,e^2+c\,d^2)+e^2(2ac\,d^2e^2+(a\,e^2+c\,d^2)^2))x^5}{5} + (2d^3e$

input
```
int((e*x+d)^2*(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^2,x,method=_RETURNVERBOSE)
```

output
```
1/7*c^2*d^2*e^4*x^7+(1/3*a*c*d*e^5+2/3*c^2*d^3*e^3)*x^6+(1/5*a^2*e^6+8/5*a
*c*d^2*e^4+6/5*c^2*d^4*e^2)*x^5+(a^2*d*e^5+3*a*c*d^3*e^3+c^2*d^5*e)*x^4+(2
*a^2*d^2*e^4+8/3*a*d^4*e^2*c+1/3*d^6*c^2)*x^3+(2*a^2*d^3*e^3+a*c*d^5*e)*x^
2+e^2*a^2*d^4*x
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 172 vs. 2(71) = 142.

Time = 0.08 (sec) , antiderivative size = 172, normalized size of antiderivative = 2.23

$$\int (d+ex)^2 \left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^2\,dx = \frac{1}{7}c^2d^2e^4x^7+a^2d^4e^2x+\frac{1}{3}\left(2c^2d^3e^3+acde^5\right)x^6 + \frac{1}{5}\left(6c^2d^4e^2+8acd^2e^4+a^2e^6\right)x^5+\left(c^2d^5e+3acd^3e^3+a^2de^5\right)x^4 + \frac{1}{3}\left(c^2d^6+8acd^4e^2+6a^2d^2e^4\right)x^3+\left(acd^5e+2a^2d^3e^3\right)x^2$$

input
```
integrate((e*x+d)^2*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^2,x, algorithm="fric
as")
```

output
```
1/7*c^2*d^2*e^4*x^7 + a^2*d^4*e^2*x + 1/3*(2*c^2*d^3*e^3 + a*c*d*e^5)*x^6
+ 1/5*(6*c^2*d^4*e^2 + 8*a*c*d^2*e^4 + a^2*e^6)*x^5 + (c^2*d^5*e + 3*a*c*d
^3*e^3 + a^2*d*e^5)*x^4 + 1/3*(c^2*d^6 + 8*a*c*d^4*e^2 + 6*a^2*d^2*e^4)*x^
3 + (a*c*d^5*e + 2*a^2*d^3*e^3)*x^2
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 185 vs. 2(66) = 132.

Time = 0.04 (sec) , antiderivative size = 185, normalized size of antiderivative = 2.40

$$\int (d+ex)^2 \left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^2\,dx = a^2d^4e^2x+\frac{c^2d^2e^4x^7}{7}+x^6\left(\frac{acde^5}{3}+\frac{2c^2d^3e^3}{3}\right) + x^5\left(\frac{a^2e^6}{5}+\frac{8acd^2e^4}{5}+\frac{6c^2d^4e^2}{5}\right)+x^4\left(a^2de^5+3acd^3e^3+c^2d^5e\right) + x^3\cdot\left(2a^2d^2e^4+\frac{8acd^4e^2}{3}+\frac{c^2d^6}{3}\right)+x^2\cdot\left(2a^2d^3e^3+acd^5e\right)$$

input
```
integrate((e*x+d)**2*(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**2,x)
```

output
```
a**2*d**4*e**2*x + c**2*d**2*e**4*x**7/7 + x**6*(a*c*d*e**5/3 + 2*c**2*d**
3*e**3/3) + x**5*(a**2*e**6/5 + 8*a*c*d**2*e**4/5 + 6*c**2*d**4*e**2/5) +
x**4*(a**2*d*e**5 + 3*a*c*d**3*e**3 + c**2*d**5*e) + x**3*(2*a**2*d**2*e**
4 + 8*a*c*d**4*e**2/3 + c**2*d**6/3) + x**2*(2*a**2*d**3*e**3 + a*c*d**5*e
)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 172 vs. $2(71) = 142$.

Time = 0.03 (sec) , antiderivative size = 172, normalized size of antiderivative = 2.23

$$\begin{aligned}\int (d+ex)^2 \left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^2 \, dx &= \frac{1}{7}\,c^2d^2e^4x^7+a^2d^4e^2x+\frac{1}{3}\left(2\,c^2d^3e^3+acde^5\right)x^6 \\ &\quad+\frac{1}{5}\left(6\,c^2d^4e^2+8\,acd^2e^4+a^2e^6\right)x^5+\left(c^2d^5e+3\,acd^3e^3+a^2de^5\right)x^4 \\ &\quad+\frac{1}{3}\left(c^2d^6+8\,acd^4e^2+6\,a^2d^2e^4\right)x^3+\left(acd^5e+2\,a^2d^3e^3\right)x^2\end{aligned}$$

input
```
integrate((e*x+d)^2*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^2,x, algorithm="maxi
ma")
```

output
```
1/7*c^2*d^2*e^4*x^7 + a^2*d^4*e^2*x + 1/3*(2*c^2*d^3*e^3 + a*c*d*e^5)*x^6
+ 1/5*(6*c^2*d^4*e^2 + 8*a*c*d^2*e^4 + a^2*e^6)*x^5 + (c^2*d^5*e + 3*a*c*d
^3*e^3 + a^2*d*e^5)*x^4 + 1/3*(c^2*d^6 + 8*a*c*d^4*e^2 + 6*a^2*d^2*e^4)*x^
3 + (a*c*d^5*e + 2*a^2*d^3*e^3)*x^2
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 186 vs. $2(71) = 142$.

Time = 0.16 (sec) , antiderivative size = 186, normalized size of antiderivative = 2.42

$$\int (d+ex)^2 \left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^2 dx$$
$$= \frac{1}{7}c^2d^2e^4x^7 + \frac{2}{3}c^2d^3e^3x^6 + \frac{1}{3}acde^5x^6 + \frac{6}{5}c^2d^4e^2x^5 + \frac{8}{5}acd^2e^4x^5 + \frac{1}{5}a^2e^6x^5 + c^2d^5ex^4 + 3\,acd^3e^3x^4 + a^2de^5x^4 + \frac{1}{3}c^2d^6x^3 + \frac{8}{3}acd^4e^2x^3 + 2\,a^2d^2e^4x^3 + acd^5ex^2 + 2\,a^2d^3e^3x^2 + a^2d^4e^2x$$

input
```
integrate((e*x+d)^2*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^2,x, algorithm="giac
")
```

output
```
1/7*c^2*d^2*e^4*x^7 + 2/3*c^2*d^3*e^3*x^6 + 1/3*a*c*d*e^5*x^6 + 6/5*c^2*d^
4*e^2*x^5 + 8/5*a*c*d^2*e^4*x^5 + 1/5*a^2*e^6*x^5 + c^2*d^5*e*x^4 + 3*a*c*
d^3*e^3*x^4 + a^2*d*e^5*x^4 + 1/3*c^2*d^6*x^3 + 8/3*a*c*d^4*e^2*x^3 + 2*a^
2*d^2*e^4*x^3 + a*c*d^5*e*x^2 + 2*a^2*d^3*e^3*x^2 + a^2*d^4*e^2*x
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 168, normalized size of antiderivative = 2.18

$$\int (d+ex)^2 \left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^2 dx$$
$$= x^3\left(2\,a^2\,d^2\,e^4 + \frac{8\,a\,c\,d^4\,e^2}{3} + \frac{c^2\,d^6}{3}\right) + x^5\left(\frac{a^2\,e^6}{5} + \frac{8\,a\,c\,d^2\,e^4}{5} + \frac{6\,c^2\,d^4\,e^2}{5}\right) + x^4\left(a^2\,d\,e^5 + 3\,a\,c\,d^3\,e^3 + c^2\,d^5\,e\right) + a^2\,d^4\,e^2\,x + \frac{c^2\,d^2\,e^4\,x^7}{7} + a\,d^3\,e\,x^2\left(c\,d^2 + 2\,a\,e^2\right) + \frac{c\,d\,e^3\,x^6\left(2\,c\,d^2 + a\,e^2\right)}{3}$$

input
```
int((d + e*x)^2*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^2,x)
```

output
```
x^3*((c^2*d^6)/3 + 2*a^2*d^2*e^4 + (8*a*c*d^4*e^2)/3) + x^5*((a^2*e^6)/5 +
 (6*c^2*d^4*e^2)/5 + (8*a*c*d^2*e^4)/5) + x^4*(a^2*d*e^5 + c^2*d^5*e + 3*a
*c*d^3*e^3) + a^2*d^4*e^2*x + (c^2*d^2*e^4*x^7)/7 + a*d^3*e*x^2*(2*a*e^2 +
 c*d^2) + (c*d*e^3*x^6*(a*e^2 + 2*c*d^2))/3
```

Reduce [B] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 188, normalized size of antiderivative = 2.44

$$\int (d+ex)^2 \left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^2 \, dx$$
$$= \frac{x(15c^2d^2e^4x^6+35acd\,e^5x^5+70c^2d^3e^3x^5+21a^2e^6x^4+168ac\,d^2e^4x^4+126c^2d^4e^2x^4+105a^2d\,e^5x^3+315}{}$$

input
```
int((e*x+d)^2*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^2,x)
```

output
```
(x*(105*a**2*d**4*e**2 + 210*a**2*d**3*e**3*x + 210*a**2*d**2*e**4*x**2 +
105*a**2*d*e**5*x**3 + 21*a**2*e**6*x**4 + 105*a*c*d**5*e*x + 280*a*c*d**4
*e**2*x**2 + 315*a*c*d**3*e**3*x**3 + 168*a*c*d**2*e**4*x**4 + 35*a*c*d*e*
*5*x**5 + 35*c**2*d**6*x**2 + 105*c**2*d**5*e*x**3 + 126*c**2*d**4*e**2*x*
*4 + 70*c**2*d**3*e**3*x**5 + 15*c**2*d**2*e**4*x**6))/105
```

3.82 $\int (d+ex) \left(ade+\left(cd^2+ae^2\right) x+cdex^2\right)^2 \, dx$

Optimal result

Integrand size = 33, antiderivative size = 77

$$\int (d+ex) \left(ade+\left(cd^2+ae^2\right) x+cdex^2\right)^2 \, dx = \frac{\left(cd^2-ae^2\right)^2 (d+ex)^4}{4e^3}-\frac{2cd(cd^2-ae^2)\,(d+ex)^5}{5e^3}+\frac{c^2d^2(d+ex)^6}{6e^3}$$

output
```
1/4*(-a*e^2+c*d^2)^2*(e*x+d)^4/e^3-2/5*c*d*(-a*e^2+c*d^2)*(e*x+d)^5/e^3+1/
6*c^2*d^2*(e*x+d)^6/e^3
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 120, normalized size of antiderivative = 1.56

$$\int (d+ex) \left(ade+\left(cd^2+ae^2\right) x+cdex^2\right)^2 \, dx = \frac{1}{60}x\left(15a^2e^2\left(4d^3+6d^2ex+4de^2x^2+e^3x^3\right)+6acdex\left(10d^3+20d^2ex+15de^2x^2+4e^3x^3\right)+c^2d^2x^2\left(20d^3+45d^2ex+36de^2x^2+10e^3x^3\right)\right)$$

input
```
Integrate[(d + e*x)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^2,x]
```

output
```
(x*(15*a^2*e^2*(4*d^3 + 6*d^2*e*x + 4*d*e^2*x^2 + e^3*x^3) + 6*a*c*d*e*x*(
10*d^3 + 20*d^2*e*x + 15*d*e^2*x^2 + 4*e^3*x^3) + c^2*d^2*x^2*(20*d^3 + 45
*d^2*e*x + 36*d*e^2*x^2 + 10*e^3*x^3)))/60
```

Rubi [A] (verified)

Time = 0.48 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.061$, Rules used = {1121, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (d+ex)\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^2 dx$$

$$\Big\downarrow \text{1121}$$

$$\int \left(-\frac{2cd(d+ex)^4\left(cd^2-ae^2\right)}{e^2}+\frac{(d+ex)^3\left(ae^2-cd^2\right)^2}{e^2}+\frac{c^2d^2(d+ex)^5}{e^2}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{2cd(d+ex)^5\left(cd^2-ae^2\right)}{5e^3}+\frac{(d+ex)^4\left(cd^2-ae^2\right)^2}{4e^3}+\frac{c^2d^2(d+ex)^6}{6e^3}$$

input
```
Int[(d + e*x)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^2,x]
```

output
```
((c*d^2 - a*e^2)^2*(d + e*x)^4)/(4*e^3) - (2*c*d*(c*d^2 - a*e^2)*(d + e*x)
^5)/(5*e^3) + (c^2*d^2*(d + e*x)^6)/(6*e^3)
```

Defintions of rubi rules used

rule 1121
```
Int[((d_) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Int[ExpandIntegrand[(d + e*x)^(m + p)*(a/d + (c/e)*x)^p, x], x]
/; FreeQ[{a, b, c, d, e, m, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && (Int
egerQ[p] || (GtQ[a, 0] && GtQ[d, 0] && LtQ[c, 0]))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 1.27 (sec) , antiderivative size = 137, normalized size of antiderivative = 1.78

method	result
norman	$\frac{c^2d^2e^3x^6}{6}+\left(\frac{2}{5}acd\,e^4+\frac{3}{5}c^2d^3e^2\right)x^5+\left(\frac{1}{4}a^2e^5+\frac{3}{2}ac\,d^2e^3+\frac{3}{4}c^2d^4e\right)x^4+\left(a^2d\,e^4+2ac\,d^3e^2+\right.$
risch	$\frac{1}{6}c^2d^2e^3x^6+\frac{2}{5}x^5acd\,e^4+\frac{3}{5}x^5c^2d^3e^2+\frac{1}{4}x^4a^2e^5+\frac{3}{2}x^4ac\,d^2e^3+\frac{3}{4}x^4c^2d^4e+x^3a^2d\,e^4+2x^3a$
parallelrisch	$\frac{1}{6}c^2d^2e^3x^6+\frac{2}{5}x^5acd\,e^4+\frac{3}{5}x^5c^2d^3e^2+\frac{1}{4}x^4a^2e^5+\frac{3}{2}x^4ac\,d^2e^3+\frac{3}{4}x^4c^2d^4e+x^3a^2d\,e^4+2x^3a$
gosper	$\frac{x(10c^2d^2e^3x^5+24x^4acd\,e^4+36x^4c^2d^3e^2+15x^3a^2e^5+90x^3ac\,d^2e^3+45x^3c^2d^4e+60x^2a^2d\,e^4+120x^2ac\,d^3e^2+20x^2c^2d^5+90x\,e}{60}$
orering	$\frac{x(10c^2d^2e^3x^5+24x^4acd\,e^4+36x^4c^2d^3e^2+15x^3a^2e^5+90x^3ac\,d^2e^3+45x^3c^2d^4e+60x^2a^2d\,e^4+120x^2ac\,d^3e^2+20x^2c^2d^5+90x\,e}{60(cdx+ae)^2(ex+d)^2}$
default	$\frac{c^2d^2e^3x^6}{6}+\frac{(c^2d^3e^2+2e^2dc(a\,e^2+c\,d^2))x^5}{5}+\frac{\left(2d^2ec(a\,e^2+c\,d^2)+e\left(2ac\,d^2e^2+(a\,e^2+c\,d^2)^2\right)\right)x^4}{4}+\frac{\left(d\left(2ac\,d^2e^2+(a\,e\right.\right.}{}$

input
```
int((e*x+d)*(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^2,x,method=_RETURNVERBOSE)
```

output
```
1/6*c^2*d^2*e^3*x^6+(2/5*a*c*d*e^4+3/5*c^2*d^3*e^2)*x^5+(1/4*a^2*e^5+3/2*a
*c*d^2*e^3+3/4*c^2*d^4*e)*x^4+(a^2*d*e^4+2*a*c*d^3*e^2+1/3*c^2*d^5)*x^3+(3
/2*e^3*a^2*d^2+a*c*d^4*e)*x^2+e^2*a^2*d^3*x
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 140, normalized size of antiderivative = 1.82

$$\int (d+ex)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^2\,dx = \frac{1}{6}\,c^2d^2e^3x^6+a^2d^3e^2x+\frac{1}{5}\left(3\,c^2d^3e^2+2\,acde^4\right)x^5+\frac{1}{4}\left(3\,c^2d^4e+6\,acd^2e^3+a^2e^5\right)x^4 + \frac{1}{3}\left(c^2d^5+6\,acd^3e^2+3\,a^2de^4\right)x^3+\frac{1}{2}\left(2\,acd^4e+3\,a^2d^2e^3\right)x^2$$

input
```
integrate((e*x+d)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^2,x, algorithm="fricas
")
```

output
```
1/6*c^2*d^2*e^3*x^6 + a^2*d^3*e^2*x + 1/5*(3*c^2*d^3*e^2 + 2*a*c*d*e^4)*x^
5 + 1/4*(3*c^2*d^4*e + 6*a*c*d^2*e^3 + a^2*e^5)*x^4 + 1/3*(c^2*d^5 + 6*a*c
*d^3*e^2 + 3*a^2*d*e^4)*x^3 + 1/2*(2*a*c*d^4*e + 3*a^2*d^2*e^3)*x^2
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 150 vs. 2(68) = 136.

Time = 0.03 (sec) , antiderivative size = 150, normalized size of antiderivative = 1.95

$$\int (d+ex)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^2\,dx = a^2d^3e^2x+\frac{c^2d^2e^3x^6}{6}+x^5\cdot\left(\frac{2acde^4}{5}+\frac{3c^2d^3e^2}{5}\right)+x^4\left(\frac{a^2e^5}{4}+\frac{3acd^2e^3}{2}+\frac{3c^2d^4e}{4}\right) + x^3\left(a^2de^4+2acd^3e^2+\frac{c^2d^5}{3}\right)+x^2\cdot\left(\frac{3a^2d^2e^3}{2}+acd^4e\right)$$

input
```
integrate((e*x+d)*(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**2,x)
```

output
```
a**2*d**3*e**2*x + c**2*d**2*e**3*x**6/6 + x**5*(2*a*c*d*e**4/5 + 3*c**2*d
**3*e**2/5) + x**4*(a**2*e**5/4 + 3*a*c*d**2*e**3/2 + 3*c**2*d**4*e/4) + x
**3*(a**2*d*e**4 + 2*a*c*d**3*e**2 + c**2*d**5/3) + x**2*(3*a**2*d**2*e**3
/2 + a*c*d**4*e)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 140, normalized size of antiderivative = 1.82

$$\int (d+ex)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^2\,dx$$
$$=\frac{1}{6}\,c^2d^2e^3x^6+a^2d^3e^2x+\frac{1}{5}\left(3\,c^2d^3e^2+2\,acde^4\right)x^5+\frac{1}{4}\left(3\,c^2d^4e+6\,acd^2e^3+a^2e^5\right)x^4+\frac{1}{3}\left(c^2d^5+6\,acd^3e^2+3\,a^2de^4\right)x^3+\frac{1}{2}\left(2\,acd^4e+3\,a^2d^2e^3\right)x^2$$

input
```
integrate((e*x+d)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^2,x, algorithm="maxima
")
```

output
```
1/6*c^2*d^2*e^3*x^6 + a^2*d^3*e^2*x + 1/5*(3*c^2*d^3*e^2 + 2*a*c*d*e^4)*x^
5 + 1/4*(3*c^2*d^4*e + 6*a*c*d^2*e^3 + a^2*e^5)*x^4 + 1/3*(c^2*d^5 + 6*a*c
*d^3*e^2 + 3*a^2*d*e^4)*x^3 + 1/2*(2*a*c*d^4*e + 3*a^2*d^2*e^3)*x^2
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 146 vs. 2(71) = 142.

Time = 0.10 (sec) , antiderivative size = 146, normalized size of antiderivative = 1.90

$$\int (d+ex)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^2\,dx$$
$$=\frac{1}{6}\,c^2d^2e^3x^6+\frac{3}{5}\,c^2d^3e^2x^5+\frac{2}{5}\,acde^4x^5+\frac{3}{4}\,c^2d^4ex^4+\frac{3}{2}\,acd^2e^3x^4+\frac{1}{4}\,a^2e^5x^4+\frac{1}{3}\,c^2d^5x^3+2\,acd^3e^2x^3+a^2de^4x^3+acd^4ex^2+\frac{3}{2}\,a^2d^2e^3x^2+a^2d^3e^2x$$

input
```
integrate((e*x+d)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^2,x, algorithm="giac")
```

output
```
1/6*c^2*d^2*e^3*x^6 + 3/5*c^2*d^3*e^2*x^5 + 2/5*a*c*d*e^4*x^5 + 3/4*c^2*d^
4*e*x^4 + 3/2*a*c*d^2*e^3*x^4 + 1/4*a^2*e^5*x^4 + 1/3*c^2*d^5*x^3 + 2*a*c*
d^3*e^2*x^3 + a^2*d*e^4*x^3 + a*c*d^4*e*x^2 + 3/2*a^2*d^2*e^3*x^2 + a^2*d^
3*e^2*x
```

Mupad [B] (verification not implemented)

Time = 5.57 (sec) , antiderivative size = 135, normalized size of antiderivative = 1.75

$$\int (d+ex)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^2\,dx = x^3\left(a^2\,d\,e^4+2\,a\,c\,d^3\,e^2+\frac{c^2\,d^5}{3}\right)+x^4\left(\frac{a^2\,e^5}{4}+\frac{3\,a\,c\,d^2\,e^3}{2}+\frac{3\,c^2\,d^4\,e}{4}\right)+a^2\,d^3\,e^2\,x+\frac{c^2\,d^2\,e^3\,x^6}{6}+\frac{a\,d^2\,e\,x^2\,\left(2\,c\,d^2+3\,a\,e^2\right)}{2}+\frac{c\,d\,e^2\,x^5\,\left(3\,c\,d^2+2\,a\,e^2\right)}{5}$$

input
```
int((d + e*x)*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^2,x)
```

output
```
x^3*((c^2*d^5)/3 + a^2*d*e^4 + 2*a*c*d^3*e^2) + x^4*((a^2*e^5)/4 + (3*c^2*
d^4*e)/4 + (3*a*c*d^2*e^3)/2) + a^2*d^3*e^2*x + (c^2*d^2*e^3*x^6)/6 + (a*d
^2*e*x^2*(3*a*e^2 + 2*c*d^2))/2 + (c*d*e^2*x^5*(2*a*e^2 + 3*c*d^2))/5
```

Reduce [B] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 147, normalized size of antiderivative = 1.91

$$\int (d+ex)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^2\,dx = \frac{x(10c^2d^2e^3x^5+24acd\,e^4x^4+36c^2d^3e^2x^4+15a^2e^5x^3+90ac\,d^2e^3x^3+45c^2d^4e\,x^3+60a^2d\,e^4x^2+120ac\,d}{60}$$

input
```
int((e*x+d)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^2,x)
```

output
```
(x*(60*a**2*d**3*e**2 + 90*a**2*d**2*e**3*x + 60*a**2*d*e**4*x**2 + 15*a**
2*e**5*x**3 + 60*a*c*d**4*e*x + 120*a*c*d**3*e**2*x**2 + 90*a*c*d**2*e**3*
x**3 + 24*a*c*d*e**4*x**4 + 20*c**2*d**5*x**2 + 45*c**2*d**4*e*x**3 + 36*c
**2*d**3*e**2*x**4 + 10*c**2*d**2*e**3*x**5))/60
```

3.83 $\int \left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^2\,dx$

Optimal result

Integrand size = 27, antiderivative size = 77

$$\int \left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^2\,dx=\frac{\left(cd^2-ae^2\right)^2(d+ex)^3}{3e^3}-\frac{cd\left(cd^2-ae^2\right)(d+ex)^4}{2e^3}+\frac{c^2d^2(d+ex)^5}{5e^3}$$

output
```
1/3*(-a*e^2+c*d^2)^2*(e*x+d)^3/e^3-1/2*c*d*(-a*e^2+c*d^2)*(e*x+d)^4/e^3+1/
5*c^2*d^2*(e*x+d)^5/e^3
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.13

$$\int \left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^2\,dx=\frac{1}{30}x\left(10a^2e^2\left(3d^2+3dex+e^2x^2\right)+5acdex\left(6d^2+8dex+3e^2x^2\right)+c^2d^2x^2\left(10d^2+15dex+6e^2x^2\right)\right)$$

input
```
Integrate[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^2,x]
```

output
```
(x*(10*a^2*e^2*(3*d^2 + 3*d*e*x + e^2*x^2) + 5*a*c*d*e*x*(6*d^2 + 8*d*e*x
+ 3*e^2*x^2) + c^2*d^2*x^2*(10*d^2 + 15*d*e*x + 6*e^2*x^2)))/30
```

Rubi [A] (verified)

Time = 0.48 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.26, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.074$, Rules used = {1084, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^2 dx$$

$$\Big\downarrow \text{ 1084}$$

$$\frac{\int \left(\left(cd^2+cexd\right)^4-2c^3d^3\left(cd^2-ae^2\right)(d+ex)^3+c^2d^2\left(cd^2-ae^2\right)^2(d+ex)^2\right)dx}{c^2d^2e^2}$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{-\frac{c^3d^3(d+ex)^4\left(cd^2-ae^2\right)}{2e}+\frac{c^2d^2(d+ex)^3\left(cd^2-ae^2\right)^2}{3e}+\frac{c^4d^4(d+ex)^5}{5e}}{c^2d^2e^2}$$

input
```
Int[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^2,x]
```

output
```
((c^2*d^2*(c*d^2 - a*e^2)^2*(d + e*x)^3)/(3*e) - (c^3*d^3*(c*d^2 - a*e^2)*
(d + e*x)^4)/(2*e) + (c^4*d^4*(d + e*x)^5)/(5*e))/(c^2*d^2*e^2)
```

Defintions of rubi rules used

rule 1084
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{q = Rt[b^2
- 4*a*c, 2]}, Simp[1/c^p   Int[ExpandIntegrand[(b/2 - q/2 + c*x)^p*(b/2 + q
/2 + c*x)^p, x], x], x] /;  !FractionalPowerFactorQ[q]] /; FreeQ[{a, b, c},
 x] && IntegerQ[p] && NiceSqrtQ[b^2 - 4*a*c]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 1.08 (sec) , antiderivative size = 93, normalized size of antiderivative = 1.21

method	result
default	$\frac{e^2c^2d^2x^5}{5}+\frac{(a\,e^2+c\,d^2)dec\,x^4}{2}+\frac{\left(2ac\,d^2e^2+(a\,e^2+c\,d^2)^2\right)x^3}{3}+ade(a\,e^2+c\,d^2)\,x^2+d^2e^2a^2x$
norman	$\frac{e^2c^2d^2x^5}{5}+\left(\frac{1}{2}d\,e^3ac+\frac{1}{2}d^3e\,c^2\right)x^4+\left(\frac{1}{3}a^2e^4+\frac{4}{3}ac\,d^2e^2+\frac{1}{3}c^2d^4\right)x^3+(a^2d\,e^3+ac\,d^3e)\,x^2+$
risch	$\frac{1}{5}e^2c^2d^2x^5+\frac{1}{2}acd\,e^3x^4+\frac{1}{2}c^2d^3e\,x^4+\frac{1}{3}x^3a^2e^4+\frac{4}{3}x^3ac\,d^2e^2+\frac{1}{3}c^2d^4x^3+a^2d\,e^3x^2+ac\,d^3e$
parallelrisch	$\frac{1}{5}e^2c^2d^2x^5+\frac{1}{2}acd\,e^3x^4+\frac{1}{2}c^2d^3e\,x^4+\frac{1}{3}x^3a^2e^4+\frac{4}{3}x^3ac\,d^2e^2+\frac{1}{3}c^2d^4x^3+a^2d\,e^3x^2+ac\,d^3e$
gosper	$\frac{x(6e^2c^2d^2x^4+15x^3d\,e^3ac+15x^3d^3e\,c^2+10x^2a^2e^4+40x^2ac\,d^2e^2+10x^2c^2d^4+30a^2d\,e^3x+30ac\,d^3ex+30d^2e^2a^2)}{30}$
orering	$\frac{x(6e^2c^2d^2x^4+15x^3d\,e^3ac+15x^3d^3e\,c^2+10x^2a^2e^4+40x^2ac\,d^2e^2+10x^2c^2d^4+30a^2d\,e^3x+30ac\,d^3ex+30d^2e^2a^2)(ade+(a\,e^2+}{30(ex+d)^2(cdx+ae)^2}$

input
```
int((a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^2,x,method=_RETURNVERBOSE)
```

output
```
1/5*e^2*c^2*d^2*x^5+1/2*(a*e^2+c*d^2)*d*e*c*x^4+1/3*(2*a*c*d^2*e^2+(a*e^2+
c*d^2)^2)*x^3+a*d*e*(a*e^2+c*d^2)*x^2+d^2*e^2*a^2*x
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.26

$$\int \left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^2\,dx=\frac{1}{5}\,c^2d^2e^2x^5+a^2d^2e^2x+\frac{1}{2}\left(c^2d^3e+acde^3\right)x^4+\frac{1}{3}\left(c^2d^4+4\,acd^2e^2+a^2e^4\right)x^3+\left(acd^3e+a^2de^3\right)x^2$$

input
```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^2,x, algorithm="fricas")
```

output
```
1/5*c^2*d^2*e^2*x^5 + a^2*d^2*e^2*x + 1/2*(c^2*d^3*e + a*c*d*e^3)*x^4 + 1/
3*(c^2*d^4 + 4*a*c*d^2*e^2 + a^2*e^4)*x^3 + (a*c*d^3*e + a^2*d*e^3)*x^2
```

Sympy [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.35

$$\int \left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^2\,dx=a^2d^2e^2x+\frac{c^2d^2e^2x^5}{5}+x^4\left(\frac{acde^3}{2}+\frac{c^2d^3e}{2}\right)+x^3\left(\frac{a^2e^4}{3}+\frac{4acd^2e^2}{3}+\frac{c^2d^4}{3}\right)+x^2\left(a^2de^3+acd^3e\right)$$

input
```
integrate((a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**2,x)
```

output
```
a**2*d**2*e**2*x + c**2*d**2*e**2*x**5/5 + x**4*(a*c*d*e**3/2 + c**2*d**3*
e/2) + x**3*(a**2*e**4/3 + 4*a*c*d**2*e**2/3 + c**2*d**4/3) + x**2*(a**2*d
*e**3 + a*c*d**3*e)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 93, normalized size of antiderivative = 1.21

$$\int \left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^2\,dx=\frac{1}{5}\,c^2d^2e^2x^5+\frac{1}{2}\,\left(cd^2+ae^2\right)cdex^4+a^2d^2e^2x+\frac{1}{3}\,\left(cd^2+ae^2\right)^2x^3+\frac{1}{3}\,\left(2\,cdex^3+3\,\left(cd^2+ae^2\right)x^2\right)ade$$

input
```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^2,x, algorithm="maxima")
```

output
```
1/5*c^2*d^2*e^2*x^5 + 1/2*(c*d^2 + a*e^2)*c*d*e*x^4 + a^2*d^2*e^2*x + 1/3*
(c*d^2 + a*e^2)^2*x^3 + 1/3*(2*c*d*e*x^3 + 3*(c*d^2 + a*e^2)*x^2)*a*d*e
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.36

$$\int \left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^2\,dx=\frac{1}{5}\,c^2d^2e^2x^5+\frac{1}{2}\,c^2d^3ex^4+\frac{1}{2}\,acde^3x^4+\frac{1}{3}\,c^2d^4x^3+\frac{4}{3}\,acd^2e^2x^3+\frac{1}{3}\,a^2e^4x^3+acd^3ex^2+a^2de^3x^2+a^2d^2e^2x$$

input
```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^2,x, algorithm="giac")
```

output
```
1/5*c^2*d^2*e^2*x^5 + 1/2*c^2*d^3*e*x^4 + 1/2*a*c*d*e^3*x^4 + 1/3*c^2*d^4*
x^3 + 4/3*a*c*d^2*e^2*x^3 + 1/3*a^2*e^4*x^3 + a*c*d^3*e*x^2 + a^2*d*e^3*x^
2 + a^2*d^2*e^2*x
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.29

$$\int \left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^2\,dx=x^3\left(\frac{a^2\,e^4}{3}+\frac{4\,a\,c\,d^2\,e^2}{3}+\frac{c^2\,d^4}{3}\right)+x^2\left(a^2\,d\,e^3+c\,a\,d^3\,e\right)+x^4\left(\frac{c^2\,d^3\,e}{2}+\frac{a\,c\,d\,e^3}{2}\right)+a^2\,d^2\,e^2\,x+\frac{c^2\,d^2\,e^2\,x^5}{5}$$

input
```
int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^2,x)
```

output
```
x^3*((a^2*e^4)/3 + (c^2*d^4)/3 + (4*a*c*d^2*e^2)/3) + x^2*(a^2*d*e^3 + a*c
*d^3*e) + x^4*((c^2*d^3*e)/2 + (a*c*d*e^3)/2) + a^2*d^2*e^2*x + (c^2*d^2*e
^2*x^5)/5
```

Reduce [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 106, normalized size of antiderivative = 1.38

$$\int \left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^2\,dx$$
$$=\frac{x(6c^2d^2e^2x^4+15acd\,e^3x^3+15c^2d^3e\,x^3+10a^2e^4x^2+40ac\,d^2e^2x^2+10c^2d^4x^2+30a^2d\,e^3x+30ac\,d^3ex+}{30}$$

input
```
int((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^2,x)
```

output
```
(x*(30*a**2*d**2*e**2 + 30*a**2*d*e**3*x + 10*a**2*e**4*x**2 + 30*a*c*d**3
*e*x + 40*a*c*d**2*e**2*x**2 + 15*a*c*d*e**3*x**3 + 10*c**2*d**4*x**2 + 15
*c**2*d**3*e*x**3 + 6*c**2*d**2*e**2*x**4))/30
```

3.84 $\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^2}{d+ex}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 54

$$\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^2}{d+ex}\,dx=\frac{\left(cd^2-ae^2\right)(ae+cdx)^3}{3c^2d^2}+\frac{e(ae+cdx)^4}{4c^2d^2}$$

output
```
1/3*(-a*e^2+c*d^2)*(c*d*x+a*e)^3/c^2/d^2+1/4*e*(c*d*x+a*e)^4/c^2/d^2
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.00

$$\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^2}{d+ex}\,dx=\frac{1}{12}x\big(6a^2e^2(2d+ex)+4acdex(3d+2ex)+c^2d^2x^2(4d+3ex)\big)$$

input
```
Integrate[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^2/(d + e*x),x]
```

output
```
(x*(6*a^2*e^2*(2*d + e*x) + 4*a*c*d*e*x*(3*d + 2*e*x) + c^2*d^2*x^2*(4*d +
 3*e*x)))/12
```

Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.057$, Rules used = {1121, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(x(ae^2+cd^2)+ade+cdex^2)^2}{d+ex}\,dx$$

$$\Big\downarrow \text{1121}$$

$$\int \left(\frac{(cd^2-ae^2)(ae+cdx)^2}{cd}+\frac{e(ae+cdx)^3}{cd}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{(cd^2-ae^2)(ae+cdx)^3}{3c^2d^2}+\frac{e(ae+cdx)^4}{4c^2d^2}$$

input
```
Int[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^2/(d + e*x),x]
```

output
```
((c*d^2 - a*e^2)*(a*e + c*d*x)^3)/(3*c^2*d^2) + (e*(a*e + c*d*x)^4)/(4*c^2
*d^2)
```

Defintions of rubi rules used

rule 1121
```
Int[((d_) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Int[ExpandIntegrand[(d + e*x)^(m + p)*(a/d + (c/e)*x)^p, x], x]
/; FreeQ[{a, b, c, d, e, m, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && (Int
egerQ[p] || (GtQ[a, 0] && GtQ[d, 0] && LtQ[c, 0]))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 1.44 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.19

method	result	size
norman	$\frac{d^2 e\, c^2 x^4}{4}+\left(\frac{2}{3} a d\, e^2 c+\frac{1}{3} c^2 d^3\right) x^3+\left(\frac{1}{2} a^2 e^3+d^2 e a c\right) x^2+a^2 d\, e^2 x$	64
gosper	$\frac{x\left(3 d^2 e\, c^2 x^3+8 x^2 a d\, e^2 c+4 c^2 d^3 x^2+6 x\, a^2 e^3+12 x\, d^2 e a c+12 a^2 d\, e^2\right)}{12}$	66
risch	$\frac{1}{4} d^2 e\, c^2 x^4+\frac{2}{3} x^3 a d\, e^2 c+\frac{1}{3} c^2 d^3 x^3+\frac{1}{2} x^2 a^2 e^3+x^2 d^2 e a c+a^2 d\, e^2 x$	66
parallelrisch	$\frac{1}{4} d^2 e\, c^2 x^4+\frac{2}{3} x^3 a d\, e^2 c+\frac{1}{3} c^2 d^3 x^3+\frac{1}{2} x^2 a^2 e^3+x^2 d^2 e a c+a^2 d\, e^2 x$	66
default	$\frac{d^2 e\, c^2 x^4}{4}+\frac{\left(a d\, e^2 c+c d\left(a\, e^2+c\, d^2\right)\right) x^3}{3}+\frac{\left(a e\left(a\, e^2+c\, d^2\right)+d^2 e a c\right) x^2}{2}+a^2 d\, e^2 x$	77
orering	$\frac{x\left(3 d^2 e\, c^2 x^3+8 x^2 a d\, e^2 c+4 c^2 d^3 x^2+6 x\, a^2 e^3+12 x\, d^2 e a c+12 a^2 d\, e^2\right)\left(a d e+\left(a\, e^2+c\, d^2\right) x+c d\, x^2 e\right)^2}{12 (e x+d)^2 (c d x+a e)^2}$	110

input
```
int((a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^2/(e*x+d),x,method=_RETURNVERBOSE)
```

output
```
1/4*d^2*e*c^2*x^4+(2/3*a*d*e^2*c+1/3*c^2*d^3)*x^3+(1/2*a^2*e^3+d^2*e*a*c)*
x^2+a^2*d*e^2*x
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.19

$$\int \frac{(ade+(cd^2+ae^2)\,x+cdex^2)^2}{d+ex}\,dx = \frac{1}{4}\,c^2d^2ex^4+a^2de^2x+\frac{1}{3}\,(c^2d^3+2\,acde^2)x^3 + \frac{1}{2}\,(2\,acd^2e+a^2e^3)x^2$$

input
```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^2/(e*x+d),x, algorithm="fricas
")
```

output
```
1/4*c^2*d^2*e*x^4 + a^2*d*e^2*x + 1/3*(c^2*d^3 + 2*a*c*d*e^2)*x^3 + 1/2*(2
*a*c*d^2*e + a^2*e^3)*x^2
```

Sympy [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.22

$$\int \frac{(ade + (cd^2 + ae^2)\,x + cdex^2)^2}{d + ex}\,dx = a^2de^2x + \frac{c^2d^2ex^4}{4} + x^3 \cdot \left(\frac{2acde^2}{3} + \frac{c^2d^3}{3}\right) + x^2\left(\frac{a^2e^3}{2} + acd^2e\right)$$

input
```
integrate((a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**2/(e*x+d),x)
```

output
```
a**2*d*e**2*x + c**2*d**2*e*x**4/4 + x**3*(2*a*c*d*e**2/3 + c**2*d**3/3) +
 x**2*(a**2*e**3/2 + a*c*d**2*e)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.19

$$\int \frac{(ade + (cd^2 + ae^2)\,x + cdex^2)^2}{d + ex}\,dx = \frac{1}{4}\,c^2d^2ex^4 + a^2de^2x + \frac{1}{3}\,(c^2d^3 + 2\,acde^2)x^3 + \frac{1}{2}\,(2\,acd^2e + a^2e^3)x^2$$

input
```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^2/(e*x+d),x, algorithm="maxima
")
```

output
```
1/4*c^2*d^2*e*x^4 + a^2*d*e^2*x + 1/3*(c^2*d^3 + 2*a*c*d*e^2)*x^3 + 1/2*(2
*a*c*d^2*e + a^2*e^3)*x^2
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.20

$$\int \frac{(ade + (cd^2 + ae^2) x + cdex^2)^2}{d + ex} dx = \frac{1}{4} c^2 d^2 e x^4 + \frac{1}{3} c^2 d^3 x^3 + \frac{2}{3} acde^2 x^3 + acd^2 e x^2 + \frac{1}{2} a^2 e^3 x^2 + a^2 de^2 x$$

input
```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^2/(e*x+d),x, algorithm="giac")
```

output
```
1/4*c^2*d^2*e*x^4 + 1/3*c^2*d^3*x^3 + 2/3*a*c*d*e^2*x^3 + a*c*d^2*e*x^2 +
1/2*a^2*e^3*x^2 + a^2*d*e^2*x
```

Mupad [B] (verification not implemented)

Time = 5.71 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.17

$$\int \frac{(ade + (cd^2 + ae^2) x + cdex^2)^2}{d + ex} dx = x^2 \left(\frac{a^2 e^3}{2} + c\, a\, d^2\, e\right) + x^3 \left(\frac{c^2 d^3}{3} + \frac{2\, a\, c\, d\, e^2}{3}\right) + \frac{c^2 d^2\, e\, x^4}{4} + a^2\, d\, e^2\, x$$

input
```
int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^2/(d + e*x),x)
```

output
```
x^2*((a^2*e^3)/2 + a*c*d^2*e) + x^3*((c^2*d^3)/3 + (2*a*c*d*e^2)/3) + (c^2
*d^2*e*x^4)/4 + a^2*d*e^2*x
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.20

$$\int \frac{(ade + (cd^2 + ae^2)x + cdex^2)^2}{d + ex}\,dx$$
$$= \frac{x(3c^2d^2e\,x^3 + 8acd\,e^2x^2 + 4c^2d^3x^2 + 6a^2e^3x + 12ac\,d^2ex + 12a^2d\,e^2)}{12}$$

input
```
int((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^2/(e*x+d),x)
```

output
```
(x*(12*a**2*d*e**2 + 6*a**2*e**3*x + 12*a*c*d**2*e*x + 8*a*c*d*e**2*x**2 +
 4*c**2*d**3*x**2 + 3*c**2*d**2*e*x**3))/12
```

3.85 $\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^2}{(d+ex)^2}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 20

$$\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^2}{(d+ex)^2}\,dx=\frac{(ae+cdx)^3}{3cd}$$

output
```
1/3*(c*d*x+a*e)^3/c/d
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^2}{(d+ex)^2}\,dx=\frac{(ae+cdx)^3}{3cd}$$

input
```
Integrate[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^2/(d + e*x)^2,x]
```

output
```
(a*e + c*d*x)^3/(3*c*d)
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.057, Rules used = {1120, 17}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(x(ae^2+cd^2)+ade+cdex^2)^2}{(d+ex)^2}dx$$

$$\Big\downarrow \text{1120}$$

$$\int (ae+cdx)^2dx$$

$$\Big\downarrow \text{17}$$

$$\frac{(ae+cdx)^3}{3cd}$$

input
```
Int[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^2/(d + e*x)^2,x]
```

output
```
(a*e + c*d*x)^3/(3*c*d)
```

Defintions of rubi rules used

rule 17
```
Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1
)/(b*(m + 1))), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]
```

rule 1120
```
Int[((d_) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Int[(d + e*x)^(m + p)*(a/d + (c/e)*x)^p, x] /; FreeQ[{a, b, c, d
, e, m}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[p] && (EqQ[m + p,
0] || EqQ[m + 2*p + 2, 0])
```

Maple [A] (verified)

Time = 1.35 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.95

method	result	size
default	$\frac{(cdx+ae)^{3}}{3cd}$	19
parallelrisch	$\frac{1}{3}d^{2}c^{2}x^{3}+acde\,x^{2}+a^{2}e^{2}x$	29
gosper	$\frac{x\left(d^{2}c^{2}x^{2}+3acdex+3e^{2}a^{2}\right)}{3}$	30
risch	$\frac{d^{2}c^{2}x^{3}}{3}+acde\,x^{2}+a^{2}e^{2}x+\frac{e^{3}a^{3}}{3cd}$	43
norman	$\frac{\left(ad\,e^{2}c+\frac{1}{3}c^{2}d^{3}\right)x^{3}+\left(a^{2}e^{3}+d^{2}eac\right)x^{2}+a^{2}d\,e^{2}x+\frac{d^{2}e\,c^{2}x^{4}}{3}}{ex+d}$	70
orering	$\frac{x\left(d^{2}c^{2}x^{2}+3acdex+3e^{2}a^{2}\right)\left(ade+\left(a\,e^{2}+c\,d^{2}\right)x+cd\,x^{2}e\right)^{2}}{3(cdx+ae)^{2}(ex+d)^{2}}$	74

input
```
int((a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^2/(e*x+d)^2,x,method=_RETURNVERBOSE)
```

output
```
1/3*(c*d*x+a*e)^3/c/d
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.40

$$\int \frac{\left(ade+\left(cd^{2}+ae^{2}\right)x+cdex^{2}\right)^{2}}{(d+ex)^{2}}\,dx=\frac{1}{3}\,c^{2}d^{2}x^{3}+acdex^{2}+a^{2}e^{2}x$$

input
```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^2/(e*x+d)^2,x, algorithm="fric
as")
```

output
```
1/3*c^2*d^2*x^3 + a*c*d*e*x^2 + a^2*e^2*x
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 29 vs. $2(14) = 28$.

Time = 0.04 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.45

$$\int \frac{(ade + (cd^2 + ae^2)x + cdex^2)^2}{(d + ex)^2} dx = a^2e^2x + acdex^2 + \frac{c^2d^2x^3}{3}$$

input
```
integrate((a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**2/(e*x+d)**2,x)
```

output
```
a**2*e**2*x + a*c*d*e*x**2 + c**2*d**2*x**3/3
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.40

$$\int \frac{(ade + (cd^2 + ae^2)x + cdex^2)^2}{(d + ex)^2} dx = \frac{1}{3}c^2d^2x^3 + acdex^2 + a^2e^2x$$

input
```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^2/(e*x+d)^2,x, algorithm="maxi
ma")
```

output
```
1/3*c^2*d^2*x^3 + a*c*d*e*x^2 + a^2*e^2*x
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 97 vs. $2(18) = 36$.

Time = 0.12 (sec) , antiderivative size = 97, normalized size of antiderivative = 4.85

$$\int \frac{(ade + (cd^2 + ae^2)x + cdex^2)^2}{(d + ex)^2} dx = \frac{\left(c^2d^2 - \frac{3c^2d^3}{ex+d} + \frac{3c^2d^4}{(ex+d)^2} + \frac{3acde^2}{ex+d} - \frac{6acd^2e^2}{(ex+d)^2} + \frac{3a^2e^4}{(ex+d)^2}\right)(ex + d)^3}{3e^3}$$

input
```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^2/(e*x+d)^2,x, algorithm="giac
")
```

output
```
1/3*(c^2*d^2 - 3*c^2*d^3/(e*x + d) + 3*c^2*d^4/(e*x + d)^2 + 3*a*c*d*e^2/(
e*x + d) - 6*a*c*d^2*e^2/(e*x + d)^2 + 3*a^2*e^4/(e*x + d)^2)*(e*x + d)^3/
e^3
```

Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.40

$$\int \frac{(ade + (cd^2 + ae^2)x + cdex^2)^2}{(d + ex)^2}\,dx = a^2\,e^2\,x + a\,c\,d\,e\,x^2 + \frac{c^2\,d^2\,x^3}{3}$$

input
```
int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^2/(d + e*x)^2,x)
```

output
```
a^2*e^2*x + (c^2*d^2*x^3)/3 + a*c*d*e*x^2
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.45

$$\int \frac{(ade + (cd^2 + ae^2)x + cdex^2)^2}{(d + ex)^2}\,dx = \frac{x(c^2d^2x^2 + 3acdex + 3a^2e^2)}{3}$$

input
```
int((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^2/(e*x+d)^2,x)
```

output
```
(x*(3*a**2*e**2 + 3*a*c*d*e*x + c**2*d**2*x**2))/3
```

3.86 $\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^2}{(d+ex)^3} \, dx$

Optimal result

Integrand size = 35, antiderivative size = 61

$$\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^2}{(d+ex)^3} \, dx = -\frac{cd\left(cd^2-2ae^2\right)x}{e^2}+\frac{c^2d^2x^2}{2e}+\frac{\left(cd^2-ae^2\right)^2\log(d+ex)}{e^3}$$

output
```
-c*d*(-2*a*e^2+c*d^2)*x/e^2+1/2*c^2*d^2*x^2/e+(-a*e^2+c*d^2)^2*ln(e*x+d)/e
^3
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.85

$$\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^2}{(d+ex)^3} \, dx = \frac{cdex(4ae^2+cd(-2d+ex))+2\left(cd^2-ae^2\right)^2\log(d+ex)}{2e^3}$$

input
```
Integrate[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^2/(d + e*x)^3,x]
```

output
```
(c*d*e*x*(4*a*e^2 + c*d*(-2*d + e*x)) + 2*(c*d^2 - a*e^2)^2*Log[d + e*x])/
(2*e^3)
```

Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.02, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.057, Rules used = {1121, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^2}{(d+ex)^3}dx$$

$$\Big\downarrow \text{ 1121}$$

$$\int \left(\frac{\left(ae^2-cd^2\right)^2}{e^2(d+ex)}-\frac{cd\left(cd^2-ae^2\right)}{e^2}+\frac{cd(ae+cdx)}{e}\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$-\frac{cdx\left(cd^2-ae^2\right)}{e^2}+\frac{\left(cd^2-ae^2\right)^2\log(d+ex)}{e^3}+\frac{(ae+cdx)^2}{2e}$$

input
```
Int[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^2/(d + e*x)^3,x]
```

output
```
-((c*d*(c*d^2 - a*e^2)*x)/e^2) + (a*e + c*d*x)^2/(2*e) + ((c*d^2 - a*e^2)^
2*Log[d + e*x])/e^3
```

Defintions of rubi rules used

rule 1121
```
Int[((d_) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Int[ExpandIntegrand[(d + e*x)^(m + p)*(a/d + (c/e)*x)^p, x], x]
/; FreeQ[{a, b, c, d, e, m, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && (Int
egerQ[p] || (GtQ[a, 0] && GtQ[d, 0] && LtQ[c, 0]))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 1.39 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.08

method	result	size
default	$\frac{cd\left(\frac{1}{2}cd\,x^{2}e+2a\,e^{2}x-c\,d^{2}x\right)}{e^{2}}+\frac{\left(a^{2}e^{4}-2ac\,d^{2}e^{2}+c^{2}d^{4}\right)\ln\left(ex+d\right)}{e^{3}}$	66
risch	$\frac{c^{2}d^{2}x^{2}}{2e}+2adxc-\frac{c^{2}d^{3}x}{e^{2}}+\ln\left(ex+d\right)a^{2}e-\frac{2\ln\left(ex+d\right)ac\,d^{2}}{e}+\frac{\ln\left(ex+d\right)c^{2}d^{4}}{e^{3}}$	77
parallelrisch	$\frac{x^{2}c^{2}d^{2}e^{2}+2\ln\left(ex+d\right)a^{2}e^{4}-4\ln\left(ex+d\right)ac\,d^{2}e^{2}+2\ln\left(ex+d\right)c^{2}d^{4}+4xacd\,e^{3}-2x\,c^{2}d^{3}e}{2e^{3}}$	83
norman	$\frac{-\frac{d^{2}\left(8ac\,d^{2}e^{2}-3c^{2}d^{4}\right)}{2e^{3}}-\frac{2d\left(3ac\,d^{2}e^{2}-c^{2}d^{4}\right)x}{e^{2}}+\frac{d^{2}e\,c^{2}x^{4}}{2}+2x^{3}ad\,e^{2}c}{\left(ex+d\right)^{2}}+\frac{\left(a^{2}e^{4}-2ac\,d^{2}e^{2}+c^{2}d^{4}\right)\ln\left(ex+d\right)}{e^{3}}$	122

input
```
int((a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^2/(e*x+d)^3,x,method=_RETURNVERBOSE)
```

output
```
c*d/e^2*(1/2*c*d*x^2*e+2*a*e^2*x-c*d^2*x)+(a^2*e^4-2*a*c*d^2*e^2+c^2*d^4)/
e^3*ln(e*x+d)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.18

$$\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^2}{(d+ex)^3}\,dx$$
$$=\frac{c^2d^2e^2x^2-2\,(c^2d^3e-2\,acde^3)x+2\,(c^2d^4-2\,acd^2e^2+a^2e^4)\log\left(ex+d\right)}{2\,e^3}$$

input
```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^2/(e*x+d)^3,x, algorithm="fric
as")
```

output
```
1/2*(c^2*d^2*e^2*x^2 - 2*(c^2*d^3*e - 2*a*c*d*e^3)*x + 2*(c^2*d^4 - 2*a*c*
d^2*e^2 + a^2*e^4)*log(e*x + d))/e^3
```

Sympy [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.87

$$\int \frac{(ade + (cd^2 + ae^2)\,x + cdex^2)^2}{(d + ex)^3}\,dx = \frac{c^2d^2x^2}{2e} + x\left(2acd - \frac{c^2d^3}{e^2}\right) + \frac{(ae^2 - cd^2)^2\log(d + ex)}{e^3}$$

input
```
integrate((a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**2/(e*x+d)**3,x)
```

output
```
c**2*d**2*x**2/(2*e) + x*(2*a*c*d - c**2*d**3/e**2) + (a*e**2 - c*d**2)**2
*log(d + e*x)/e**3
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.18

$$\int \frac{(ade + (cd^2 + ae^2)\,x + cdex^2)^2}{(d + ex)^3}\,dx = \frac{c^2d^2ex^2 - 2\,(c^2d^3 - 2\,acde^2)x}{2\,e^2} + \frac{(c^2d^4 - 2\,acd^2e^2 + a^2e^4)\log(ex + d)}{e^3}$$

input
```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^2/(e*x+d)^3,x, algorithm="maxi
ma")
```

output
```
1/2*(c^2*d^2*e*x^2 - 2*(c^2*d^3 - 2*a*c*d*e^2)*x)/e^2 + (c^2*d^4 - 2*a*c*d
^2*e^2 + a^2*e^4)*log(e*x + d)/e^3
```

Giac [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.18

$$\int \frac{(ade + (cd^2 + ae^2)x + cdex^2)^2}{(d+ex)^3}\,dx = \frac{c^2d^2ex^2 - 2\,c^2d^3x + 4\,acde^2x}{2\,e^2} + \frac{(c^2d^4 - 2\,acd^2e^2 + a^2e^4)\log(|ex+d|)}{e^3}$$

input
```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^2/(e*x+d)^3,x, algorithm="giac
")
```

output
```
1/2*(c^2*d^2*e*x^2 - 2*c^2*d^3*x + 4*a*c*d*e^2*x)/e^2 + (c^2*d^4 - 2*a*c*d
^2*e^2 + a^2*e^4)*log(abs(e*x + d))/e^3
```

Mupad [B] (verification not implemented)

Time = 6.07 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.13

$$\int \frac{(ade + (cd^2 + ae^2)x + cdex^2)^2}{(d+ex)^3}\,dx = x\left(2\,a\,c\,d - \frac{c^2\,d^3}{e^2}\right) + \frac{\ln(d+e\,x)\,\left(a^2\,e^4 - 2\,a\,c\,d^2\,e^2 + c^2\,d^4\right)}{e^3} + \frac{c^2\,d^2\,x^2}{2\,e}$$

input
```
int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^2/(d + e*x)^3,x)
```

output
```
x*(2*a*c*d - (c^2*d^3)/e^2) + (log(d + e*x)*(a^2*e^4 + c^2*d^4 - 2*a*c*d^2
*e^2))/e^3 + (c^2*d^2*x^2)/(2*e)
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.34

$$\int \frac{(ade + (cd^2 + ae^2)\,x + cdex^2)^2}{(d+ex)^3}\,dx = \frac{2\log(ex+d)\,a^2e^4 - 4\log(ex+d)\,ac\,d^2e^2 + 2\log(ex+d)\,c^2d^4 + 4acd\,e^3x - 2c^2d^3ex + c^2d^2e^2x^2}{2e^3}$$

input
```
int((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^2/(e*x+d)^3,x)
```

output
```
(2*log(d + e*x)*a**2*e**4 - 4*log(d + e*x)*a*c*d**2*e**2 + 2*log(d + e*x)*
c**2*d**4 + 4*a*c*d*e**3*x - 2*c**2*d**3*e*x + c**2*d**2*e**2*x**2)/(2*e**
3)
```

3.87 $\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^2}{(d+ex)^4}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 63

$$\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^2}{(d+ex)^4}\,dx = \frac{c^2d^2x}{e^2}-\frac{\left(cd^2-ae^2\right)^2}{e^3(d+ex)}-\frac{2cd\left(cd^2-ae^2\right)\log(d+ex)}{e^3}$$

output
```
c^2*d^2*x/e^2-(-a*e^2+c*d^2)^2/e^3/(e*x+d)-2*c*d*(-a*e^2+c*d^2)*ln(e*x+d)/
e^3
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.94

$$\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^2}{(d+ex)^4}\,dx = \frac{c^2d^2ex-\frac{\left(cd^2-ae^2\right)^2}{d+ex}+2cd\left(-cd^2+ae^2\right)\log(d+ex)}{e^3}$$

input
```
Integrate[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^2/(d + e*x)^4,x]
```

output
```
(c^2*d^2*e*x - (c*d^2 - a*e^2)^2/(d + e*x) + 2*c*d*(-(c*d^2) + a*e^2)*Log[
d + e*x])/e^3
```

Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.057$, Rules used = $\{1121, 2009\}$

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^2}{(d+ex)^4}\,dx$$

$$\Big\downarrow \text{ 1121}$$

$$\int \left(-\frac{2cd\left(cd^2-ae^2\right)}{e^2(d+ex)}+\frac{\left(ae^2-cd^2\right)^2}{e^2(d+ex)^2}+\frac{c^2d^2}{e^2}\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$-\frac{\left(cd^2-ae^2\right)^2}{e^3(d+ex)}-\frac{2cd\left(cd^2-ae^2\right)\log(d+ex)}{e^3}+\frac{c^2d^2x}{e^2}$$

input
```
Int[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^2/(d + e*x)^4,x]
```

output
```
(c^2*d^2*x)/e^2 - (c*d^2 - a*e^2)^2/(e^3*(d + e*x)) - (2*c*d*(c*d^2 - a*e^
2)*Log[d + e*x])/e^3
```

Defintions of rubi rules used

rule 1121
```
Int[((d_) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Int[ExpandIntegrand[(d + e*x)^(m + p)*(a/d + (c/e)*x)^p, x], x]
/; FreeQ[{a, b, c, d, e, m, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && (Int
egerQ[p] || (GtQ[a, 0] && GtQ[d, 0] && LtQ[c, 0]))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 1.37 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.19

method	result
default	$\frac{c^2 d^2 x}{e^2}+\frac{2 d c \left(a\,e^2-c\,d^2\right) \ln(e x+d)}{e^3}-\frac{a^2 e^4-2 a c\,d^2 e^2+c^2 d^4}{e^3 (e x+d)}$
risch	$\frac{c^2 d^2 x}{e^2}+\frac{2 d c \ln(e x+d) a}{e}-\frac{2 d^3 c^2 \ln(e x+d)}{e^3}-\frac{e\,a^2}{e x+d}+\frac{2 a c\,d^2}{e (e x+d)}-\frac{c^2 d^4}{e^3 (e x+d)}$
parallelrisch	$\frac{2 \ln(e x+d) x a c d\,e^3-2 \ln(e x+d) x\,c^2 d^3 e+x^2 c^2 d^2 e^2+2 \ln(e x+d) a c\,d^2 e^2-2 \ln(e x+d) c^2 d^4-a^2 e^4+2 a c\,d^2 e^2-2 c^2 d^4}{e^3 (e x+d)}$
norman	$\frac{d^2 e\,c^2 x^4-\frac{d^2 \left(a^2 e^4-2 a c\,d^2 e^2+4 c^2 d^4\right)}{e^3}-\frac{\left(a^2 e^4-2 a c\,d^2 e^2+7 c^2 d^4\right) x^2}{e}-\frac{d \left(2 a^2 e^4-4 a c\,d^2 e^2+10 c^2 d^4\right) x}{e^2}}{(e x+d)^3}+\frac{2 d c \left(a\,e^2-c\,d^2\right) \ln(e x+d)}{e^3}$

input
```
int((a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^2/(e*x+d)^4,x,method=_RETURNVERBOSE)
```

output
```
c^2*d^2*x/e^2+2*d/e^3*c*(a*e^2-c*d^2)*ln(e*x+d)-(a^2*e^4-2*a*c*d^2*e^2+c^2
*d^4)/e^3/(e*x+d)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.71

$$\int \frac{\left(ade+(cd^2+ae^2)\,x+cdex^2\right)^2}{(d+ex)^4}\,dx$$
$$=\frac{c^2d^2e^2x^2+c^2d^3ex-c^2d^4+2\,acd^2e^2-a^2e^4-2\,(c^2d^4-acd^2e^2+(c^2d^3e-acde^3)x)\log(ex+d)}{e^4x+de^3}$$

input
```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^2/(e*x+d)^4,x, algorithm="fric
as")
```

output
```
(c^2*d^2*e^2*x^2 + c^2*d^3*e*x - c^2*d^4 + 2*a*c*d^2*e^2 - a^2*e^4 - 2*(c^
2*d^4 - a*c*d^2*e^2 + (c^2*d^3*e - a*c*d*e^3)*x)*log(e*x + d))/(e^4*x + d*
e^3)
```

Sympy [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.13

$$\int \frac{(ade + (cd^2 + ae^2)\,x + cdex^2)^2}{(d+ex)^4}\,dx = \frac{c^2d^2x}{e^2} + \frac{2cd(ae^2 - cd^2)\log{(d+ex)}}{e^3} + \frac{-a^2e^4 + 2acd^2e^2 - c^2d^4}{de^3 + e^4x}$$

input
```
integrate((a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**2/(e*x+d)**4,x)
```

output
```
c**2*d**2*x/e**2 + 2*c*d*(a*e**2 - c*d**2)*log(d + e*x)/e**3 + (-a**2*e**4
 + 2*a*c*d**2*e**2 - c**2*d**4)/(d*e**3 + e**4*x)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.25

$$\int \frac{(ade + (cd^2 + ae^2)\,x + cdex^2)^2}{(d+ex)^4}\,dx = \frac{c^2d^2x}{e^2} - \frac{c^2d^4 - 2\,acd^2e^2 + a^2e^4}{e^4x + de^3} - \frac{2\,(c^2d^3 - acde^2)\log{(ex+d)}}{e^3}$$

input
```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^2/(e*x+d)^4,x, algorithm="maxi
ma")
```

output
```
c^2*d^2*x/e^2 - (c^2*d^4 - 2*a*c*d^2*e^2 + a^2*e^4)/(e^4*x + d*e^3) - 2*(c
^2*d^3 - a*c*d*e^2)*log(e*x + d)/e^3
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.22

$$\int \frac{(ade + (cd^2 + ae^2)x + cdex^2)^2}{(d + ex)^4} dx = \frac{c^2d^2x}{e^2} - \frac{2(c^2d^3 - acde^2)\log(|ex + d|)}{e^3} - \frac{c^2d^4 - 2acd^2e^2 + a^2e^4}{(ex + d)e^3}$$

input
```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^2/(e*x+d)^4,x, algorithm="giac
")
```

output
```
c^2*d^2*x/e^2 - 2*(c^2*d^3 - a*c*d*e^2)*log(abs(e*x + d))/e^3 - (c^2*d^4 -
 2*a*c*d^2*e^2 + a^2*e^4)/((e*x + d)*e^3)
```

Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.32

$$\int \frac{(ade + (cd^2 + ae^2)x + cdex^2)^2}{(d + ex)^4} dx = \frac{c^2 d^2 x}{e^2} - \frac{a^2 e^4 - 2acd^2 e^2 + c^2 d^4}{e(xe^3 + de^2)} - \frac{\ln(d + ex)(2c^2 d^3 - 2acde^2)}{e^3}$$

input
```
int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^2/(d + e*x)^4,x)
```

output
```
(c^2*d^2*x)/e^2 - (a^2*e^4 + c^2*d^4 - 2*a*c*d^2*e^2)/(e*(d*e^2 + e^3*x))
- (log(d + e*x)*(2*c^2*d^3 - 2*a*c*d*e^2))/e^3
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 120, normalized size of antiderivative = 1.90

$$\int \frac{(ade + (cd^2 + ae^2) x + cdex^2)^2}{(d + ex)^4} dx$$
$$= \frac{2\log(ex+d)\,ac\,d^3e^2 + 2\log(ex+d)\,ac\,d^2e^3x - 2\log(ex+d)\,c^2d^5 - 2\log(ex+d)\,c^2d^4ex + a^2e^5x - 2ac}{d\,e^3\,(ex+d)}$$

input
```
int((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^2/(e*x+d)^4,x)
```

output
```
(2*log(d + e*x)*a*c*d**3*e**2 + 2*log(d + e*x)*a*c*d**2*e**3*x - 2*log(d +
 e*x)*c**2*d**5 - 2*log(d + e*x)*c**2*d**4*e*x + a**2*e**5*x - 2*a*c*d**2*
e**3*x + 2*c**2*d**4*e*x + c**2*d**3*e**2*x**2)/(d*e**3*(d + e*x))
```

3.88 $\int \frac{(ade+(cd^2+ae^2)x+cdex^2)^2}{(d+ex)^5}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 71

$$\int \frac{(ade+(cd^2+ae^2)\,x+cdex^2)^2}{(d+ex)^5}\,dx = -\frac{(cd^2-ae^2)^2}{2e^3(d+ex)^2}+\frac{2cd(cd^2-ae^2)}{e^3(d+ex)}+\frac{c^2d^2\log(d+ex)}{e^3}$$

output
```
-1/2*(-a*e^2+c*d^2)^2/e^3/(e*x+d)^2+2*c*d*(-a*e^2+c*d^2)/e^3/(e*x+d)+c^2*d
^2*ln(e*x+d)/e^3
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.83

$$\int \frac{(ade+(cd^2+ae^2)\,x+cdex^2)^2}{(d+ex)^5}\,dx = \frac{\frac{(cd^2-ae^2)(ae^2+cd(3d+4ex))}{(d+ex)^2}+2c^2d^2\log(d+ex)}{2e^3}$$

input
```
Integrate[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^2/(d + e*x)^5,x]
```

output
```
(((c*d^2 - a*e^2)*(a*e^2 + c*d*(3*d + 4*e*x)))/(d + e*x)^2 + 2*c^2*d^2*Log
[d + e*x])/(2*e^3)
```

Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.057$, Rules used = {1121, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^2}{(d+ex)^5}dx$$

$$\Big\downarrow \text{1121}$$

$$\int \left(-\frac{2cd\left(cd^2-ae^2\right)}{e^2(d+ex)^2}+\frac{\left(ae^2-cd^2\right)^2}{e^2(d+ex)^3}+\frac{c^2d^2}{e^2(d+ex)}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{2cd\left(cd^2-ae^2\right)}{e^3(d+ex)}-\frac{\left(cd^2-ae^2\right)^2}{2e^3(d+ex)^2}+\frac{c^2d^2\log(d+ex)}{e^3}$$

input
```
Int[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^2/(d + e*x)^5,x]
```

output
```
-1/2*(c*d^2 - a*e^2)^2/(e^3*(d + e*x)^2) + (2*c*d*(c*d^2 - a*e^2))/(e^3*(d
 + e*x)) + (c^2*d^2*Log[d + e*x])/e^3
```

Defintions of rubi rules used

rule 1121
```
Int[((d_) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Int[ExpandIntegrand[(d + e*x)^(m + p)*(a/d + (c/e)*x)^p, x], x]
/; FreeQ[{a, b, c, d, e, m, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && (Int
egerQ[p] || (GtQ[a, 0] && GtQ[d, 0] && LtQ[c, 0]))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 1.35 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.10

method	result
risch	$\frac{-\frac{2dc\left(a\,e^{2}-c\,d^{2}\right)x}{e^{2}}-\frac{a^{2}e^{4}+2ac\,d^{2}e^{2}-3c^{2}d^{4}}{2e^{3}}}{\left(ex+d\right)^{2}}+\frac{c^{2}d^{2}\ln\left(ex+d\right)}{e^{3}}$
default	$\frac{c^{2}d^{2}\ln\left(ex+d\right)}{e^{3}}-\frac{2dc\left(a\,e^{2}-c\,d^{2}\right)}{e^{3}\left(ex+d\right)}-\frac{a^{2}e^{4}-2ac\,d^{2}e^{2}+c^{2}d^{4}}{2e^{3}\left(ex+d\right)^{2}}$
parallelrisch	$\frac{2\ln\left(ex+d\right)x^{2}c^{2}d^{2}e^{2}+4\ln\left(ex+d\right)x\,c^{2}d^{3}e+2\ln\left(ex+d\right)c^{2}d^{4}-4xacd\,e^{3}+4x\,c^{2}d^{3}e-a^{2}e^{4}-2ac\,d^{2}e^{2}+3c^{2}d^{4}}{2e^{3}\left(ex+d\right)^{2}}$
norman	$\frac{-\frac{d^{2}\left(a^{2}e^{5}+2ac\,d^{2}e^{3}-3c^{2}d^{4}e\right)}{2e^{4}}-\frac{\left(a^{2}e^{5}+10ac\,d^{2}e^{3}-11c^{2}d^{4}e\right)x^{2}}{2e^{2}}-\frac{2d\left(ac\,e^{3}-d^{2}e\,c^{2}\right)x^{3}}{e}-\frac{d\left(a^{2}e^{5}+4ac\,d^{2}e^{3}-5c^{2}d^{4}e\right)x}{e^{3}}}{\left(ex+d\right)^{4}}+\frac{c^{2}d^{2}\ln\left(ex\right.}{e^{3}}$

input
```
int((a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^2/(e*x+d)^5,x,method=_RETURNVERBOSE)
```

output
```
(-2*d/e^2*c*(a*e^2-c*d^2)*x-1/2*(a^2*e^4+2*a*c*d^2*e^2-3*c^2*d^4)/e^3)/(e*
x+d)^2+c^2*d^2*ln(e*x+d)/e^3
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.58

$$\int \frac{(ade + (cd^2 + ae^2)\,x + cdex^2)^2}{(d + ex)^5}\,dx$$
$$= \frac{3\,c^2d^4 - 2\,acd^2e^2 - a^2e^4 + 4\,(c^2d^3e - acde^3)x + 2\,(c^2d^2e^2x^2 + 2\,c^2d^3ex + c^2d^4)\log(ex + d)}{2\,(e^5x^2 + 2\,de^4x + d^2e^3)}$$

input
```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^2/(e*x+d)^5,x, algorithm="fric
as")
```

output
```
1/2*(3*c^2*d^4 - 2*a*c*d^2*e^2 - a^2*e^4 + 4*(c^2*d^3*e - a*c*d*e^3)*x + 2
*(c^2*d^2*e^2*x^2 + 2*c^2*d^3*e*x + c^2*d^4)*log(e*x + d))/(e^5*x^2 + 2*d*
e^4*x + d^2*e^3)
```

Sympy [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.27

$$\int \frac{(ade + (cd^2 + ae^2)\,x + cdex^2)^2}{(d + ex)^5}\,dx$$
$$= \frac{c^2d^2\log(d + ex)}{e^3} + \frac{-a^2e^4 - 2acd^2e^2 + 3c^2d^4 + x(-4acde^3 + 4c^2d^3e)}{2d^2e^3 + 4de^4x + 2e^5x^2}$$

input
```
integrate((a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**2/(e*x+d)**5,x)
```

output
```
c**2*d**2*log(d + e*x)/e**3 + (-a**2*e**4 - 2*a*c*d**2*e**2 + 3*c**2*d**4
+ x*(-4*a*c*d*e**3 + 4*c**2*d**3*e))/(2*d**2*e**3 + 4*d*e**4*x + 2*e**5*x*
*2)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.27

$$\int \frac{(ade + (cd^2 + ae^2) x + cdex^2)^2}{(d + ex)^5} \, dx = \frac{c^2 d^2 \log(ex + d)}{e^3} + \frac{3 c^2 d^4 - 2 acd^2 e^2 - a^2 e^4 + 4 (c^2 d^3 e - acde^3) x}{2 (e^5 x^2 + 2 de^4 x + d^2 e^3)}$$

input
```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^2/(e*x+d)^5,x, algorithm="maxi
ma")
```

output
```
c^2*d^2*log(e*x + d)/e^3 + 1/2*(3*c^2*d^4 - 2*a*c*d^2*e^2 - a^2*e^4 + 4*(c
^2*d^3*e - a*c*d*e^3)*x)/(e^5*x^2 + 2*d*e^4*x + d^2*e^3)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 120, normalized size of antiderivative = 1.69

$$\int \frac{(ade + (cd^2 + ae^2) x + cdex^2)^2}{(d + ex)^5} \, dx = -\frac{c^2 d^2 \log\left(\frac{|ex+d|}{(ex+d)^2 |e|}\right)}{e^3} + \frac{\frac{4 c^2 d^3 e^3}{ex+d} - \frac{c^2 d^4 e^3}{(ex+d)^2} - \frac{4 acde^5}{ex+d} + \frac{2 acd^2 e^5}{(ex+d)^2} - \frac{a^2 e^7}{(ex+d)^2}}{2 e^6}$$

input
```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^2/(e*x+d)^5,x, algorithm="giac
")
```

output
```
-c^2*d^2*log(abs(e*x + d)/((e*x + d)^2*abs(e)))/e^3 + 1/2*(4*c^2*d^3*e^3/(
e*x + d) - c^2*d^4*e^3/(e*x + d)^2 - 4*a*c*d*e^5/(e*x + d) + 2*a*c*d^2*e^5
/(e*x + d)^2 - a^2*e^7/(e*x + d)^2)/e^6
```

Mupad [B] (verification not implemented)

Time = 5.60 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.25

$$\int \frac{\left(ade + (cd^2 + ae^2)\,x + cdex^2\right)^2}{(d+ex)^5}\,dx = \frac{c^2\,d^2\,\ln\left(d+e\,x\right)}{e^3} - \frac{\frac{a^2\,e^4+2\,a\,c\,d^2\,e^2-3\,c^2\,d^4}{2\,e^3}+\frac{2\,c\,d\,x\,\left(a\,e^2-c\,d^2\right)}{e^2}}{d^2+2\,d\,e\,x+e^2\,x^2}$$

input
```
int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^2/(d + e*x)^5,x)
```

output
```
(c^2*d^2*log(d + e*x))/e^3 - ((a^2*e^4 - 3*c^2*d^4 + 2*a*c*d^2*e^2)/(2*e^3
) + (2*c*d*x*(a*e^2 - c*d^2))/e^2)/(d^2 + e^2*x^2 + 2*d*e*x)
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.59

$$\int \frac{\left(ade + (cd^2 + ae^2)\,x + cdex^2\right)^2}{(d+ex)^5}\,dx = \frac{2\log(ex+d)\,c^2d^4 + 4\log(ex+d)\,c^2d^3ex + 2\log(ex+d)\,c^2d^2e^2x^2 - a^2e^4 + 2ac\,e^4x^2 + c^2d^4 - 2c^2d^2e^2x^2}{2e^3\left(e^2x^2 + 2dex + d^2\right)}$$

input
```
int((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^2/(e*x+d)^5,x)
```

output
```
(2*log(d + e*x)*c**2*d**4 + 4*log(d + e*x)*c**2*d**3*e*x + 2*log(d + e*x)*
c**2*d**2*e**2*x**2 - a**2*e**4 + 2*a*c*e**4*x**2 + c**2*d**4 - 2*c**2*d**
2*e**2*x**2)/(2*e**3*(d**2 + 2*d*e*x + e**2*x**2))
```

3.89 $\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^2}{(d+ex)^6}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 35

$$\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^2}{(d+ex)^6}\,dx=\frac{(ae+cdx)^3}{3\left(cd^2-ae^2\right)(d+ex)^3}$$

output
```
1/3*(c*d*x+a*e)^3/(-a*e^2+c*d^2)/(e*x+d)^3
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.69

$$\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^2}{(d+ex)^6}\,dx$$
$$=-\frac{a^2e^4+acde^2(d+3ex)+c^2d^2(d^2+3dex+3e^2x^2)}{3e^3(d+ex)^3}$$

input
```
Integrate[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^2/(d + e*x)^6,x]
```

output
```
-1/3*(a^2*e^4 + a*c*d*e^2*(d + 3*e*x) + c^2*d^2*(d^2 + 3*d*e*x + 3*e^2*x^2
))/(e^3*(d + e*x)^3)
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.057, Rules used = {1120, 48}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^2}{(d+ex)^6}dx$$

$$\Big\downarrow \text{1120}$$

$$\int \frac{(ae+cdx)^2}{(d+ex)^4}dx$$

$$\Big\downarrow \text{48}$$

$$\frac{(ae+cdx)^3}{3(d+ex)^3\left(cd^2-ae^2\right)}$$

input
```
Int[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^2/(d + e*x)^6,x]
```

output
```
(a*e + c*d*x)^3/(3*(c*d^2 - a*e^2)*(d + e*x)^3)
```

Defintions of rubi rules used

rule 48
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && EqQ[m + n + 2, 0] && NeQ[m, -1]
```

rule 1120
```
Int[((d_) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Int[(d + e*x)^(m + p)*(a/d + (c/e)*x)^p, x] /; FreeQ[{a, b, c, d
, e, m}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[p] && (EqQ[m + p,
0] || EqQ[m + 2*p + 2, 0])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 69 vs. $2(33) = 66$.

Time = 1.40 (sec) , antiderivative size = 70, normalized size of antiderivative = 2.00

method	result
gosper	$-\frac{3x^2c^2d^2e^2+3xacd\,e^3+3x\,c^2d^3e+a^2e^4+ac\,d^2e^2+c^2d^4}{3e^3(ex+d)^3}$
risch	$\frac{-\frac{c^2d^2x^2}{e}-\frac{cd\left(a\,e^2+c\,d^2\right)x}{e^2}-\frac{a^2e^4+ac\,d^2e^2+c^2d^4}{3e^3}}{(ex+d)^3}$
parallelrisch	$\frac{-3x^2c^2d^2e^2-3xacd\,e^3-3x\,c^2d^3e-a^2e^4-ac\,d^2e^2-c^2d^4}{3e^3(ex+d)^3}$
default	$-\frac{a^2e^4-2ac\,d^2e^2+c^2d^4}{3e^3(ex+d)^3}-\frac{c^2d^2}{e^3(ex+d)}-\frac{dc(a\,e^2-c\,d^2)}{e^3(ex+d)^2}$
orering	$-\frac{\left(3x^2c^2d^2e^2+3xacd\,e^3+3x\,c^2d^3e+a^2e^4+ac\,d^2e^2+c^2d^4\right)\left(ade+\left(a\,e^2+c\,d^2\right)x+cd\,x^2e\right)^2}{3e^3(cdx+ae)^2(ex+d)^5}$
norman	$\frac{-\frac{d^2\left(a^2e^6+ac\,d^2e^4+c^2d^4e^2\right)}{3e^5}-\frac{\left(a^2e^6+7ac\,d^2e^4+10c^2d^4e^2\right)x^2}{3e^3}-d^2e\,c^2x^4-\frac{d\left(ac\,e^4+3e^2c^2d^2\right)x^3}{e^2}-\frac{d\left(2a^2e^6+5ac\,d^2e^4+5c^2d^4e^2\right)x}{3e^4}}{(ex+d)^5}$

input
```
int((a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^2/(e*x+d)^6,x,method=_RETURNVERBOSE)
```

output
```
-1/3*(3*c^2*d^2*e^2*x^2+3*a*c*d*e^3*x+3*c^2*d^3*e*x+a^2*e^4+a*c*d^2*e^2+c^
2*d^4)/e^3/(e*x+d)^3
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 94 vs. $2(33) = 66$.

Time = 0.07 (sec) , antiderivative size = 94, normalized size of antiderivative = 2.69

$$\int \frac{(ade+(cd^2+ae^2)\,x+cdex^2)^2}{(d+ex)^6}\,dx = -\frac{3\,c^2d^2e^2x^2+c^2d^4+acd^2e^2+a^2e^4+3\,(c^2d^3e+acde^3)x}{3\,(e^6x^3+3\,de^5x^2+3\,d^2e^4x+d^3e^3)}$$

input
```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^2/(e*x+d)^6,x, algorithm="fric
as")
```

output
```
-1/3*(3*c^2*d^2*e^2*x^2 + c^2*d^4 + a*c*d^2*e^2 + a^2*e^4 + 3*(c^2*d^3*e +
 a*c*d*e^3)*x)/(e^6*x^3 + 3*d*e^5*x^2 + 3*d^2*e^4*x + d^3*e^3)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 99 vs. 2(27) = 54.

Time = 0.38 (sec) , antiderivative size = 99, normalized size of antiderivative = 2.83

$$\int \frac{(ade+(cd^2+ae^2)x+cdex^2)^2}{(d+ex)^6}\,dx = \frac{-a^2e^4-acd^2e^2-c^2d^4-3c^2d^2e^2x^2+x(-3acde^3-3c^2d^3e)}{3d^3e^3+9d^2e^4x+9de^5x^2+3e^6x^3}$$

input
```
integrate((a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**2/(e*x+d)**6,x)
```

output
```
(-a**2*e**4 - a*c*d**2*e**2 - c**2*d**4 - 3*c**2*d**2*e**2*x**2 + x*(-3*a*
c*d*e**3 - 3*c**2*d**3*e))/(3*d**3*e**3 + 9*d**2*e**4*x + 9*d*e**5*x**2 +
3*e**6*x**3)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 94 vs. 2(33) = 66.

Time = 0.04 (sec) , antiderivative size = 94, normalized size of antiderivative = 2.69

$$\int \frac{(ade+(cd^2+ae^2)x+cdex^2)^2}{(d+ex)^6}\,dx = -\frac{3\,c^2d^2e^2x^2+c^2d^4+acd^2e^2+a^2e^4+3\,(c^2d^3e+acde^3)x}{3\,(e^6x^3+3\,de^5x^2+3\,d^2e^4x+d^3e^3)}$$

input
```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^2/(e*x+d)^6,x, algorithm="maxi
ma")
```

output
```
-1/3*(3*c^2*d^2*e^2*x^2 + c^2*d^4 + a*c*d^2*e^2 + a^2*e^4 + 3*(c^2*d^3*e +
 a*c*d*e^3)*x)/(e^6*x^3 + 3*d*e^5*x^2 + 3*d^2*e^4*x + d^3*e^3)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 69 vs. $2(33) = 66$.

Time = 0.14 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.97

$$\int \frac{(ade + (cd^2 + ae^2)x + cdex^2)^2}{(d + ex)^6} dx$$
$$= -\frac{3c^2d^2e^2x^2 + 3c^2d^3ex + 3acde^3x + c^2d^4 + acd^2e^2 + a^2e^4}{3(ex + d)^3e^3}$$

input
```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^2/(e*x+d)^6,x, algorithm="giac
")
```

output
```
-1/3*(3*c^2*d^2*e^2*x^2 + 3*c^2*d^3*e*x + 3*a*c*d*e^3*x + c^2*d^4 + a*c*d^
2*e^2 + a^2*e^4)/((e*x + d)^3*e^3)
```

Mupad [B] (verification not implemented)

Time = 5.53 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.86

$$\int \frac{(ade + (cd^2 + ae^2)x + cdex^2)^2}{(d + ex)^6} dx = -\frac{\frac{a^2 e}{3} - d\left(\frac{c^2 x^3}{3} - a c x\right) + \frac{a c d^2}{3 e}}{d^3 + 3 d^2 e x + 3 d e^2 x^2 + e^3 x^3}$$

input
```
int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^2/(d + e*x)^6,x)
```

output
```
-((a^2*e)/3 - d*((c^2*x^3)/3 - a*c*x) + (a*c*d^2)/(3*e))/(d^3 + e^3*x^3 +
3*d*e^2*x^2 + 3*d^2*e*x)
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.89

$$\int \frac{(ade + (cd^2 + ae^2)x + cdex^2)^2}{(d + ex)^6} dx = \frac{c^2de\,x^3 - 3acdex - a^2e^2 - ac\,d^2}{3e\,(e^3x^3 + 3d\,e^2x^2 + 3d^2ex + d^3)}$$

input
```
int((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^2/(e*x+d)^6,x)
```

output
```
( - a**2*e**2 - a*c*d**2 - 3*a*c*d*e*x + c**2*d*e*x**3)/(3*e*(d**3 + 3*d**
2*e*x + 3*d*e**2*x**2 + e**3*x**3))
```

3.90 $\int \frac{(ade+(cd^2+ae^2)x+cdex^2)^2}{(d+ex)^7}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 77

$$\int \frac{(ade+(cd^2+ae^2)x+cdex^2)^2}{(d+ex)^7}\,dx = -\frac{(cd^2-ae^2)^2}{4e^3(d+ex)^4}+\frac{2cd(cd^2-ae^2)}{3e^3(d+ex)^3}-\frac{c^2d^2}{2e^3(d+ex)^2}$$

output
```
-1/4*(-a*e^2+c*d^2)^2/e^3/(e*x+d)^4+2/3*c*d*(-a*e^2+c*d^2)/e^3/(e*x+d)^3-1
/2*c^2*d^2/e^3/(e*x+d)^2
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.79

$$\int \frac{(ade+(cd^2+ae^2)x+cdex^2)^2}{(d+ex)^7}\,dx = -\frac{3a^2e^4+2acde^2(d+4ex)+c^2d^2(d^2+4dex+6e^2x^2)}{12e^3(d+ex)^4}$$

input
```
Integrate[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^2/(d + e*x)^7,x]
```

output
```
-1/12*(3*a^2*e^4 + 2*a*c*d*e^2*(d + 4*e*x) + c^2*d^2*(d^2 + 4*d*e*x + 6*e^
2*x^2))/(e^3*(d + e*x)^4)
```

Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.057$, Rules used = {1121, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^2}{(d+ex)^7}dx$$

$$\Big\downarrow \text{ 1121}$$

$$\int \left(-\frac{2cd\left(cd^2-ae^2\right)}{e^2(d+ex)^4}+\frac{\left(ae^2-cd^2\right)^2}{e^2(d+ex)^5}+\frac{c^2d^2}{e^2(d+ex)^3}\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{2cd\left(cd^2-ae^2\right)}{3e^3(d+ex)^3}-\frac{\left(cd^2-ae^2\right)^2}{4e^3(d+ex)^4}-\frac{c^2d^2}{2e^3(d+ex)^2}$$

input
```
Int[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^2/(d + e*x)^7,x]
```

output
```
-1/4*(c*d^2 - a*e^2)^2/(e^3*(d + e*x)^4) + (2*c*d*(c*d^2 - a*e^2))/(3*e^3*
(d + e*x)^3) - (c^2*d^2)/(2*e^3*(d + e*x)^2)
```

Defintions of rubi rules used

rule 1121
```
Int[((d_) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Int[ExpandIntegrand[(d + e*x)^(m + p)*(a/d + (c/e)*x)^p, x], x]
/; FreeQ[{a, b, c, d, e, m, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && (Int
egerQ[p] || (GtQ[a, 0] && GtQ[d, 0] && LtQ[c, 0]))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 1.55 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.94

method	result
gosper	$-\frac{6x^2c^2d^2e^2+8xacd\,e^3+4x\,c^2d^3e+3a^2e^4+2ac\,d^2e^2+c^2d^4}{12e^3(ex+d)^4}$
risch	$\frac{-\frac{c^2d^2x^2}{2e}-\frac{dc\left(2a\,e^2+c\,d^2\right)x}{3e^2}-\frac{3a^2e^4+2ac\,d^2e^2+c^2d^4}{12e^3}}{(ex+d)^4}$
parallelrisch	$\frac{-6d^2c^2x^2e^3-8acd\,e^4x-4c^2d^3e^2x-3a^2e^5-2ac\,d^2e^3-c^2d^4e}{12e^4(ex+d)^4}$
default	$-\frac{2dc(a\,e^2-c\,d^2)}{3e^3(ex+d)^3}-\frac{a^2e^4-2ac\,d^2e^2+c^2d^4}{4e^3(ex+d)^4}-\frac{c^2d^2}{2e^3(ex+d)^2}$
orering	$-\frac{(6x^2c^2d^2e^2+8xacd\,e^3+4x\,c^2d^3e+3a^2e^4+2ac\,d^2e^2+c^2d^4)\left(ade+\left(a\,e^2+c\,d^2\right)x+cd\,x^2e\right)^2}{12e^3(cdx+ae)^2(ex+d)^6}$
norman	$\frac{-\frac{d^2\left(3a^2e^7+2a\,d^2c\,e^5+c^2d^4e^3\right)}{12e^6}-\frac{\left(a^2e^7+6a\,d^2c\,e^5+5c^2d^4e^3\right)x^2}{4e^4}-\frac{2d\left(ac\,e^5+2c^2d^2e^3\right)x^3}{3e^3}-\frac{d\left(a^2e^7+2a\,d^2c\,e^5+c^2d^4e^3\right)x}{2e^5}-\frac{d^2e\,c^2x^4}{2}}{(ex+d)^6}$

input
```
int((a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^2/(e*x+d)^7,x,method=_RETURNVERBOSE)
```

output
```
-1/12/e^3*(6*c^2*d^2*e^2*x^2+8*a*c*d*e^3*x+4*c^2*d^3*e*x+3*a^2*e^4+2*a*c*d
^2*e^2+c^2*d^4)/(e*x+d)^4
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.40

$$\int \frac{(ade+(cd^2+ae^2)x+cdex^2)^2}{(d+ex)^7}\,dx = -\frac{6\,c^2d^2e^2x^2+c^2d^4+2\,acd^2e^2+3\,a^2e^4+4\,(c^2d^3e+2\,acde^3)x}{12\,(e^7x^4+4\,de^6x^3+6\,d^2e^5x^2+4\,d^3e^4x+d^4e^3)}$$

input
```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^2/(e*x+d)^7,x, algorithm="fric
as")
```

output
```
-1/12*(6*c^2*d^2*e^2*x^2 + c^2*d^4 + 2*a*c*d^2*e^2 + 3*a^2*e^4 + 4*(c^2*d^
3*e + 2*a*c*d*e^3)*x)/(e^7*x^4 + 4*d*e^6*x^3 + 6*d^2*e^5*x^2 + 4*d^3*e^4*x
 + d^4*e^3)
```

Sympy [A] (verification not implemented)

Time = 0.54 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.48

$$\int \frac{(ade+(cd^2+ae^2)x+cdex^2)^2}{(d+ex)^7}\,dx = \frac{-3a^2e^4-2acd^2e^2-c^2d^4-6c^2d^2e^2x^2+x(-8acde^3-4c^2d^3e)}{12d^4e^3+48d^3e^4x+72d^2e^5x^2+48de^6x^3+12e^7x^4}$$

input
```
integrate((a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**2/(e*x+d)**7,x)
```

output
```
(-3*a**2*e**4 - 2*a*c*d**2*e**2 - c**2*d**4 - 6*c**2*d**2*e**2*x**2 + x*(-
8*a*c*d*e**3 - 4*c**2*d**3*e))/(12*d**4*e**3 + 48*d**3*e**4*x + 72*d**2*e*
*5*x**2 + 48*d*e**6*x**3 + 12*e**7*x**4)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.40

$$\int \frac{(ade + (cd^2 + ae^2)x + cdex^2)^2}{(d+ex)^7}\,dx = -\frac{6\,c^2d^2e^2x^2 + c^2d^4 + 2\,acd^2e^2 + 3\,a^2e^4 + 4\,(c^2d^3e + 2\,acde^3)x}{12\,(e^7x^4 + 4\,de^6x^3 + 6\,d^2e^5x^2 + 4\,d^3e^4x + d^4e^3)}$$

input
```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^2/(e*x+d)^7,x, algorithm="maxi
ma")
```

output
```
-1/12*(6*c^2*d^2*e^2*x^2 + c^2*d^4 + 2*a*c*d^2*e^2 + 3*a^2*e^4 + 4*(c^2*d^
3*e + 2*a*c*d*e^3)*x)/(e^7*x^4 + 4*d*e^6*x^3 + 6*d^2*e^5*x^2 + 4*d^3*e^4*x
 + d^4*e^3)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.92

$$\int \frac{(ade + (cd^2 + ae^2)x + cdex^2)^2}{(d+ex)^7}\,dx = -\frac{6\,c^2d^2e^2x^2 + 4\,c^2d^3ex + 8\,acde^3x + c^2d^4 + 2\,acd^2e^2 + 3\,a^2e^4}{12\,(ex+d)^4e^3}$$

input
```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^2/(e*x+d)^7,x, algorithm="giac
")
```

output
```
-1/12*(6*c^2*d^2*e^2*x^2 + 4*c^2*d^3*e*x + 8*a*c*d*e^3*x + c^2*d^4 + 2*a*c
*d^2*e^2 + 3*a^2*e^4)/((e*x + d)^4*e^3)
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.10

$$\int \frac{(ade+(cd^2+ae^2)x+cdex^2)^2}{(d+ex)^7}\,dx = -\frac{\frac{a^2\,e}{4}-d\left(\frac{c^2\,x^3}{3}-\frac{2\,a\,c\,x}{3}\right)-\frac{c^2\,e\,x^4}{12}+\frac{a\,c\,d^2}{6\,e}}{d^4+4\,d^3\,e\,x+6\,d^2\,e^2\,x^2+4\,d\,e^3\,x^3+e^4\,x^4}$$

input
```
int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^2/(d + e*x)^7,x)
```

output
```
-((a^2*e)/4 - d*((c^2*x^3)/3 - (2*a*c*x)/3) - (c^2*e*x^4)/12 + (a*c*d^2)/(
6*e))/(d^4 + e^4*x^4 + 4*d*e^3*x^3 + 6*d^2*e^2*x^2 + 4*d^3*e*x)
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.36

$$\int \frac{(ade+(cd^2+ae^2)x+cdex^2)^2}{(d+ex)^7}\,dx = \frac{-6c^2d^2e^2x^2-8acd\,e^3x-4c^2d^3ex-3a^2e^4-2ac\,d^2e^2-c^2d^4}{12e^3\,(e^4x^4+4d\,e^3x^3+6d^2e^2x^2+4d^3ex+d^4)}$$

input
```
int((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^2/(e*x+d)^7,x)
```

output
```
( - 3*a**2*e**4 - 2*a*c*d**2*e**2 - 8*a*c*d*e**3*x - c**2*d**4 - 4*c**2*d*
*3*e*x - 6*c**2*d**2*e**2*x**2)/(12*e**3*(d**4 + 4*d**3*e*x + 6*d**2*e**2*
x**2 + 4*d*e**3*x**3 + e**4*x**4))
```

3.91 $\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^2}{(d+ex)^8} dx$

Optimal result

Integrand size = 35, antiderivative size = 77

$$\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^2}{(d+ex)^8} dx = -\frac{\left(cd^2-ae^2\right)^2}{5e^3(d+ex)^5}+\frac{cd\left(cd^2-ae^2\right)}{2e^3(d+ex)^4}-\frac{c^2d^2}{3e^3(d+ex)^3}$$

output
```
-1/5*(-a*e^2+c*d^2)^2/e^3/(e*x+d)^5+1/2*c*d*(-a*e^2+c*d^2)/e^3/(e*x+d)^4-1
/3*c^2*d^2/e^3/(e*x+d)^3
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.79

$$\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^2}{(d+ex)^8} dx = -\frac{6a^2e^4+3acde^2(d+5ex)+c^2d^2\left(d^2+5dex+10e^2x^2\right)}{30e^3(d+ex)^5}$$

input
```
Integrate[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^2/(d + e*x)^8,x]
```

output
```
-1/30*(6*a^2*e^4 + 3*a*c*d*e^2*(d + 5*e*x) + c^2*d^2*(d^2 + 5*d*e*x + 10*e
^2*x^2))/(e^3*(d + e*x)^5)
```

Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.057$, Rules used = $\{1121, 2009\}$

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^2}{(d+ex)^8}dx$$

$$\Big\downarrow \text{1121}$$

$$\int \left(-\frac{2cd\left(cd^2-ae^2\right)}{e^2(d+ex)^5}+\frac{\left(ae^2-cd^2\right)^2}{e^2(d+ex)^6}+\frac{c^2d^2}{e^2(d+ex)^4}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{cd\left(cd^2-ae^2\right)}{2e^3(d+ex)^4}-\frac{\left(cd^2-ae^2\right)^2}{5e^3(d+ex)^5}-\frac{c^2d^2}{3e^3(d+ex)^3}$$

input
```
Int[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^2/(d + e*x)^8,x]
```

output
```
-1/5*(c*d^2 - a*e^2)^2/(e^3*(d + e*x)^5) + (c*d*(c*d^2 - a*e^2))/(2*e^3*(d
 + e*x)^4) - (c^2*d^2)/(3*e^3*(d + e*x)^3)
```

Defintions of rubi rules used

rule 1121
```
Int[((d_) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Int[ExpandIntegrand[(d + e*x)^(m + p)*(a/d + (c/e)*x)^p, x], x]
/; FreeQ[{a, b, c, d, e, m, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && (Int
egerQ[p] || (GtQ[a, 0] && GtQ[d, 0] && LtQ[c, 0]))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 1.31 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.94

method	result
gosper	$-\frac{10x^2c^2d^2e^2+15xacd\,e^3+5x\,c^2d^3e+6a^2e^4+3ac\,d^2e^2+c^2d^4}{30e^3(ex+d)^5}$
risch	$\frac{-\frac{c^2d^2x^2}{3e}-\frac{dc\left(3a\,e^2+c\,d^2\right)x}{6e^2}-\frac{6a^2e^4+3ac\,d^2e^2+c^2d^4}{30e^3}}{(ex+d)^5}$
parallelrisch	$\frac{-10d^2c^2x^2e^4-15acd\,e^5x-5c^2d^3e^3x-6a^2e^6-3ac\,d^2e^4-c^2d^4e^2}{30e^5(ex+d)^5}$
default	$-\frac{c^2d^2}{3e^3(ex+d)^3}-\frac{dc(a\,e^2-c\,d^2)}{2e^3(ex+d)^4}-\frac{a^2e^4-2ac\,d^2e^2+c^2d^4}{5e^3(ex+d)^5}$
orering	$-\frac{(10x^2c^2d^2e^2+15xacd\,e^3+5x\,c^2d^3e+6a^2e^4+3ac\,d^2e^2+c^2d^4)\left(ade+(a\,e^2+c\,d^2)x+cd\,x^2e\right)^2}{30e^3(cdx+ae)^2(ex+d)^7}$
norman	$\frac{-\frac{d^2\left(6e^8a^2+3a\,d^2c\,e^6+c^2d^4e^4\right)}{30e^7}-\frac{\left(2e^8a^2+11a\,d^2c\,e^6+7c^2d^4e^4\right)x^2}{10e^5}-\frac{d\left(3ac\,e^6+5c^2d^2e^4\right)x^3}{6e^4}-\frac{d\left(12e^8a^2+21a\,d^2c\,e^6+7c^2d^4e^4\right)x}{30e^6}-\frac{d^2e}{}}{(ex+d)^7}$

input
```
int((a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^2/(e*x+d)^8,x,method=_RETURNVERBOSE)
```

output
```
-1/30/e^3*(10*c^2*d^2*e^2*x^2+15*a*c*d*e^3*x+5*c^2*d^3*e*x+6*a^2*e^4+3*a*c
*d^2*e^2+c^2*d^4)/(e*x+d)^5
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.55

$$\int \frac{(ade + (cd^2 + ae^2)x + cdex^2)^2}{(d + ex)^8} dx$$
$$= -\frac{10c^2d^2e^2x^2 + c^2d^4 + 3acd^2e^2 + 6a^2e^4 + 5(c^2d^3e + 3acde^3)x}{30(e^8x^5 + 5de^7x^4 + 10d^2e^6x^3 + 10d^3e^5x^2 + 5d^4e^4x + d^5e^3)}$$

input
```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^2/(e*x+d)^8,x, algorithm="fric
as")
```

output
```
-1/30*(10*c^2*d^2*e^2*x^2 + c^2*d^4 + 3*a*c*d^2*e^2 + 6*a^2*e^4 + 5*(c^2*d
^3*e + 3*a*c*d*e^3)*x)/(e^8*x^5 + 5*d*e^7*x^4 + 10*d^2*e^6*x^3 + 10*d^3*e^
5*x^2 + 5*d^4*e^4*x + d^5*e^3)
```

Sympy [A] (verification not implemented)

Time = 0.80 (sec) , antiderivative size = 126, normalized size of antiderivative = 1.64

$$\int \frac{(ade + (cd^2 + ae^2)x + cdex^2)^2}{(d + ex)^8} dx$$
$$= \frac{-6a^2e^4 - 3acd^2e^2 - c^2d^4 - 10c^2d^2e^2x^2 + x(-15acde^3 - 5c^2d^3e)}{30d^5e^3 + 150d^4e^4x + 300d^3e^5x^2 + 300d^2e^6x^3 + 150de^7x^4 + 30e^8x^5}$$

input
```
integrate((a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**2/(e*x+d)**8,x)
```

output
```
(-6*a**2*e**4 - 3*a*c*d**2*e**2 - c**2*d**4 - 10*c**2*d**2*e**2*x**2 + x*(
-15*a*c*d*e**3 - 5*c**2*d**3*e))/(30*d**5*e**3 + 150*d**4*e**4*x + 300*d**
3*e**5*x**2 + 300*d**2*e**6*x**3 + 150*d*e**7*x**4 + 30*e**8*x**5)
```

Maxima [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.55

$$\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^2}{(d+ex)^8}\,dx = -\frac{10\,c^2d^2e^2x^2+c^2d^4+3\,acd^2e^2+6\,a^2e^4+5\,(c^2d^3e+3\,acde^3)x}{30\,(e^8x^5+5\,de^7x^4+10\,d^2e^6x^3+10\,d^3e^5x^2+5\,d^4e^4x+d^5e^3)}$$

input
```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^2/(e*x+d)^8,x, algorithm="maxi
ma")
```

output
```
-1/30*(10*c^2*d^2*e^2*x^2 + c^2*d^4 + 3*a*c*d^2*e^2 + 6*a^2*e^4 + 5*(c^2*d
^3*e + 3*a*c*d*e^3)*x)/(e^8*x^5 + 5*d*e^7*x^4 + 10*d^2*e^6*x^3 + 10*d^3*e^
5*x^2 + 5*d^4*e^4*x + d^5*e^3)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.92

$$\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^2}{(d+ex)^8}\,dx = -\frac{10\,c^2d^2e^2x^2+5\,c^2d^3ex+15\,acde^3x+c^2d^4+3\,acd^2e^2+6\,a^2e^4}{30\,(ex+d)^5e^3}$$

input
```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^2/(e*x+d)^8,x, algorithm="giac
")
```

output
```
-1/30*(10*c^2*d^2*e^2*x^2 + 5*c^2*d^3*e*x + 15*a*c*d*e^3*x + c^2*d^4 + 3*a
*c*d^2*e^2 + 6*a^2*e^4)/((e*x + d)^5*e^3)
```

Mupad [B] (verification not implemented)

Time = 5.56 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.55

$$\int \frac{(ade + (cd^2 + ae^2)x + cdex^2)^2}{(d+ex)^8}\,dx$$
$$= -\frac{\frac{6\,a^2\,e^4+3\,a\,c\,d^2\,e^2+c^2\,d^4}{30\,e^3}+\frac{c^2\,d^2\,x^2}{3\,e}+\frac{c\,d\,x\,\left(c\,d^2+3\,a\,e^2\right)}{6\,e^2}}{d^5+5\,d^4\,e\,x+10\,d^3\,e^2\,x^2+10\,d^2\,e^3\,x^3+5\,d\,e^4\,x^4+e^5\,x^5}$$

input
```
int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^2/(d + e*x)^8,x)
```

output
```
-((6*a^2*e^4 + c^2*d^4 + 3*a*c*d^2*e^2)/(30*e^3) + (c^2*d^2*x^2)/(3*e) + (
c*d*x*(3*a*e^2 + c*d^2))/(6*e^2))/(d^5 + e^5*x^5 + 5*d*e^4*x^4 + 10*d^3*e^
2*x^2 + 10*d^2*e^3*x^3 + 5*d^4*e*x)
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.51

$$\int \frac{(ade + (cd^2 + ae^2)x + cdex^2)^2}{(d+ex)^8}\,dx$$
$$= \frac{-10c^2d^2e^2x^2 - 15acd\,e^3x - 5c^2d^3ex - 6a^2e^4 - 3ac\,d^2e^2 - c^2d^4}{30e^3\left(e^5x^5 + 5d\,e^4x^4 + 10d^2e^3x^3 + 10d^3e^2x^2 + 5d^4ex + d^5\right)}$$

input
```
int((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^2/(e*x+d)^8,x)
```

output
```
( - 6*a**2*e**4 - 3*a*c*d**2*e**2 - 15*a*c*d*e**3*x - c**2*d**4 - 5*c**2*d
**3*e*x - 10*c**2*d**2*e**2*x**2)/(30*e**3*(d**5 + 5*d**4*e*x + 10*d**3*e*
*2*x**2 + 10*d**2*e**3*x**3 + 5*d*e**4*x**4 + e**5*x**5))
```

3.92 $\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^2}{(d+ex)^9} \, dx$

Optimal result

Integrand size = 35, antiderivative size = 77

$$\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^2}{(d+ex)^9} \, dx=-\frac{\left(cd^2-ae^2\right)^2}{6e^3(d+ex)^6}+\frac{2cd\left(cd^2-ae^2\right)}{5e^3(d+ex)^5}-\frac{c^2d^2}{4e^3(d+ex)^4}$$

output
```
-1/6*(-a*e^2+c*d^2)^2/e^3/(e*x+d)^6+2/5*c*d*(-a*e^2+c*d^2)/e^3/(e*x+d)^5-1
/4*c^2*d^2/e^3/(e*x+d)^4
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.79

$$\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^2}{(d+ex)^9} \, dx = -\frac{10a^2e^4+4acde^2(d+6ex)+c^2d^2(d^2+6dex+15e^2x^2)}{60e^3(d+ex)^6}$$

input
```
Integrate[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^2/(d + e*x)^9,x]
```

output
```
-1/60*(10*a^2*e^4 + 4*a*c*d*e^2*(d + 6*e*x) + c^2*d^2*(d^2 + 6*d*e*x + 15*
e^2*x^2))/(e^3*(d + e*x)^6)
```

Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.057$, Rules used = {1121, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^2}{(d+ex)^9}dx$$

$$\Big\downarrow \text{1121}$$

$$\int \left(-\frac{2cd\left(cd^2-ae^2\right)}{e^2(d+ex)^6}+\frac{\left(ae^2-cd^2\right)^2}{e^2(d+ex)^7}+\frac{c^2d^2}{e^2(d+ex)^5}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{2cd\left(cd^2-ae^2\right)}{5e^3(d+ex)^5}-\frac{\left(cd^2-ae^2\right)^2}{6e^3(d+ex)^6}-\frac{c^2d^2}{4e^3(d+ex)^4}$$

input
```
Int[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^2/(d + e*x)^9,x]
```

output
```
-1/6*(c*d^2 - a*e^2)^2/(e^3*(d + e*x)^6) + (2*c*d*(c*d^2 - a*e^2))/(5*e^3*
(d + e*x)^5) - (c^2*d^2)/(4*e^3*(d + e*x)^4)
```

Defintions of rubi rules used

rule 1121
```
Int[((d_) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Int[ExpandIntegrand[(d + e*x)^(m + p)*(a/d + (c/e)*x)^p, x], x]
/; FreeQ[{a, b, c, d, e, m, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && (Int
egerQ[p] || (GtQ[a, 0] && GtQ[d, 0] && LtQ[c, 0]))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 1.34 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.94

method	result
gosper	$-\frac{15x^2c^2d^2e^2+24xacd\,e^3+6x\,c^2d^3e+10a^2e^4+4ac\,d^2e^2+c^2d^4}{60e^3(ex+d)^6}$
risch	$\frac{-\frac{c^2d^2x^2}{4e}-\frac{dc\left(4a\,e^2+c\,d^2\right)x}{10e^2}-\frac{10a^2e^4+4ac\,d^2e^2+c^2d^4}{60e^3}}{(ex+d)^6}$
parallelrisch	$\frac{-15d^2c^2x^2e^5-24acd\,e^6x-6c^2d^3e^4x-10a^2e^7-4a\,d^2c\,e^5-c^2d^4e^3}{60e^6(ex+d)^6}$
default	$-\frac{c^2d^2}{4e^3(ex+d)^4}-\frac{2dc\left(a\,e^2-c\,d^2\right)}{5e^3(ex+d)^5}-\frac{a^2e^4-2ac\,d^2e^2+c^2d^4}{6e^3(ex+d)^6}$
orering	$-\frac{\left(15x^2c^2d^2e^2+24xacd\,e^3+6x\,c^2d^3e+10a^2e^4+4ac\,d^2e^2+c^2d^4\right)\left(ade+\left(a\,e^2+c\,d^2\right)x+cd\,x^2e\right)^2}{60e^3(cdx+ae)^2(ex+d)^8}$
norman	$\frac{-\frac{d^2\left(10e^9a^2+4a\,d^2c\,e^7+c^2d^4e^5\right)}{60e^8}-\frac{\left(5e^9a^2+26a\,d^2c\,e^7+14c^2d^4e^5\right)x^2}{30e^6}-\frac{d\left(2ac\,e^7+3c^2d^2e^5\right)x^3}{5e^5}-\frac{d\left(5e^9a^2+8a\,d^2c\,e^7+2c^2d^4e^5\right)x}{15e^7}-\frac{d^2e}{}}{(ex+d)^8}$

input
```
int((a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^2/(e*x+d)^9,x,method=_RETURNVERBOSE)
```

output
```
-1/60/e^3*(15*c^2*d^2*e^2*x^2+24*a*c*d*e^3*x+6*c^2*d^3*e*x+10*a^2*e^4+4*a*
c*d^2*e^2+c^2*d^4)/(e*x+d)^6
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 130, normalized size of antiderivative = 1.69

$$\int \frac{\left(ade + (cd^2 + ae^2)\,x + cdex^2\right)^2}{(d+ex)^9}\,dx$$
$$= -\frac{15\,c^2d^2e^2x^2 + c^2d^4 + 4\,acd^2e^2 + 10\,a^2e^4 + 6\,(c^2d^3e + 4\,acde^3)x}{60\,(e^9x^6 + 6\,de^8x^5 + 15\,d^2e^7x^4 + 20\,d^3e^6x^3 + 15\,d^4e^5x^2 + 6\,d^5e^4x + d^6e^3)}$$

input
```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^2/(e*x+d)^9,x, algorithm="fric
as")
```

output
```
-1/60*(15*c^2*d^2*e^2*x^2 + c^2*d^4 + 4*a*c*d^2*e^2 + 10*a^2*e^4 + 6*(c^2*
d^3*e + 4*a*c*d*e^3)*x)/(e^9*x^6 + 6*d*e^8*x^5 + 15*d^2*e^7*x^4 + 20*d^3*e
^6*x^3 + 15*d^4*e^5*x^2 + 6*d^5*e^4*x + d^6*e^3)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 138 vs. 2(68) = 136.

Time = 1.42 (sec) , antiderivative size = 138, normalized size of antiderivative = 1.79

$$\int \frac{\left(ade + (cd^2 + ae^2)\,x + cdex^2\right)^2}{(d+ex)^9}\,dx$$
$$= \frac{-10a^2e^4 - 4acd^2e^2 - c^2d^4 - 15c^2d^2e^2x^2 + x(-24acde^3 - 6c^2d^3e)}{60d^6e^3 + 360d^5e^4x + 900d^4e^5x^2 + 1200d^3e^6x^3 + 900d^2e^7x^4 + 360de^8x^5 + 60e^9x^6}$$

input
```
integrate((a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**2/(e*x+d)**9,x)
```

output
```
(-10*a**2*e**4 - 4*a*c*d**2*e**2 - c**2*d**4 - 15*c**2*d**2*e**2*x**2 + x*
(-24*a*c*d*e**3 - 6*c**2*d**3*e))/(60*d**6*e**3 + 360*d**5*e**4*x + 900*d*
*4*e**5*x**2 + 1200*d**3*e**6*x**3 + 900*d**2*e**7*x**4 + 360*d*e**8*x**5
+ 60*e**9*x**6)
```

Maxima [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 130, normalized size of antiderivative = 1.69

$$\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^2}{(d+ex)^9}\,dx = -\frac{15\,c^2d^2e^2x^2+c^2d^4+4\,acd^2e^2+10\,a^2e^4+6\,(c^2d^3e+4\,acde^3)x}{60\,(e^9x^6+6\,de^8x^5+15\,d^2e^7x^4+20\,d^3e^6x^3+15\,d^4e^5x^2+6\,d^5e^4x+d^6e^3)}$$

input
```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^2/(e*x+d)^9,x, algorithm="maxi
ma")
```

output
```
-1/60*(15*c^2*d^2*e^2*x^2 + c^2*d^4 + 4*a*c*d^2*e^2 + 10*a^2*e^4 + 6*(c^2*
d^3*e + 4*a*c*d*e^3)*x)/(e^9*x^6 + 6*d*e^8*x^5 + 15*d^2*e^7*x^4 + 20*d^3*e
^6*x^3 + 15*d^4*e^5*x^2 + 6*d^5*e^4*x + d^6*e^3)
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.92

$$\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^2}{(d+ex)^9}\,dx = -\frac{15\,c^2d^2e^2x^2+6\,c^2d^3ex+24\,acde^3x+c^2d^4+4\,acd^2e^2+10\,a^2e^4}{60\,(ex+d)^6e^3}$$

input
```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^2/(e*x+d)^9,x, algorithm="giac
")
```

output
```
-1/60*(15*c^2*d^2*e^2*x^2 + 6*c^2*d^3*e*x + 24*a*c*d*e^3*x + c^2*d^4 + 4*a
*c*d^2*e^2 + 10*a^2*e^4)/((e*x + d)^6*e^3)
```

Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 130, normalized size of antiderivative = 1.69

$$\int \frac{(ade + (cd^2 + ae^2)x + cdex^2)^2}{(d+ex)^9} dx$$
$$= -\frac{\frac{10\,a^2\,e^4+4\,a\,c\,d^2\,e^2+c^2\,d^4}{60\,e^3} + \frac{c^2\,d^2\,x^2}{4\,e} + \frac{c\,d\,x\,(c\,d^2+4\,a\,e^2)}{10\,e^2}}{d^6 + 6\,d^5\,e\,x + 15\,d^4\,e^2\,x^2 + 20\,d^3\,e^3\,x^3 + 15\,d^2\,e^4\,x^4 + 6\,d\,e^5\,x^5 + e^6\,x^6}$$

input
```
int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^2/(d + e*x)^9,x)
```

output
```
-((10*a^2*e^4 + c^2*d^4 + 4*a*c*d^2*e^2)/(60*e^3) + (c^2*d^2*x^2)/(4*e) +
(c*d*x*(4*a*e^2 + c*d^2))/(10*e^2))/(d^6 + e^6*x^6 + 6*d*e^5*x^5 + 15*d^4*
e^2*x^2 + 20*d^3*e^3*x^3 + 15*d^2*e^4*x^4 + 6*d^5*e*x)
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 127, normalized size of antiderivative = 1.65

$$\int \frac{(ade + (cd^2 + ae^2)x + cdex^2)^2}{(d+ex)^9} dx$$
$$= \frac{-15c^2d^2e^2x^2 - 24acd\,e^3x - 6c^2d^3ex - 10a^2e^4 - 4ac\,d^2e^2 - c^2d^4}{60e^3\,(e^6x^6 + 6d\,e^5x^5 + 15d^2e^4x^4 + 20d^3e^3x^3 + 15d^4e^2x^2 + 6d^5ex + d^6)}$$

input
```
int((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^2/(e*x+d)^9,x)
```

output
```
( - 10*a**2*e**4 - 4*a*c*d**2*e**2 - 24*a*c*d*e**3*x - c**2*d**4 - 6*c**2*
d**3*e*x - 15*c**2*d**2*e**2*x**2)/(60*e**3*(d**6 + 6*d**5*e*x + 15*d**4*e
**2*x**2 + 20*d**3*e**3*x**3 + 15*d**2*e**4*x**4 + 6*d*e**5*x**5 + e**6*x*
*6))
```

3.93 $\int (d+ex)^2 \left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^3 dx$

Optimal result

Integrand size = 35, antiderivative size = 111

$$\int (d+ex)^2 \left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^3 dx = -\frac{\left(cd^2-ae^2\right)^3 (d+ex)^6}{6e^4}+\frac{3cd\left(cd^2-ae^2\right)^2 (d+ex)^7}{7e^4} - \frac{3c^2d^2\left(cd^2-ae^2\right) (d+ex)^8}{8e^4}+\frac{c^3d^3(d+ex)^9}{9e^4}$$

output
```
-1/6*(-a*e^2+c*d^2)^3*(e*x+d)^6/e^4+3/7*c*d*(-a*e^2+c*d^2)^2*(e*x+d)^7/e^4
-3/8*c^2*d^2*(-a*e^2+c*d^2)*(e*x+d)^8/e^4+1/9*c^3*d^3*(e*x+d)^9/e^4
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 255 vs. $2(111) = 222$.

Time = 0.05 (sec) , antiderivative size = 255, normalized size of antiderivative = 2.30

$$\int (d+ex)^2 \left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^3\,dx$$
$$=\frac{1}{504}x\left(84a^3e^3\left(6d^5+15d^4ex+20d^3e^2x^2+15d^2e^3x^3+6de^4x^4+e^5x^5\right)+36a^2cde^2x\left(21d^5+70d^4ex+105d^3e^2x^2+84d^2e^3x^3+35de^4x^4+6e^5x^5\right)+9ac^2d^2ex^2\left(56d^5+210d^4ex+336d^3e^2x^2+280d^2e^3x^3+120de^4x^4+21e^5x^5\right)+c^3d^3x^3\left(126d^5+504d^4ex+840d^3e^2x^2+720d^2e^3x^3+315de^4x^4+56e^5x^5\right)\right)$$

input
```
Integrate[(d + e*x)^2*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^3,x]
```

output
```
(x*(84*a^3*e^3*(6*d^5 + 15*d^4*e*x + 20*d^3*e^2*x^2 + 15*d^2*e^3*x^3 + 6*d
*e^4*x^4 + e^5*x^5) + 36*a^2*c*d*e^2*x*(21*d^5 + 70*d^4*e*x + 105*d^3*e^2*
x^2 + 84*d^2*e^3*x^3 + 35*d*e^4*x^4 + 6*e^5*x^5) + 9*a*c^2*d^2*e*x^2*(56*d
^5 + 210*d^4*e*x + 336*d^3*e^2*x^2 + 280*d^2*e^3*x^3 + 120*d*e^4*x^4 + 21*
e^5*x^5) + c^3*d^3*x^3*(126*d^5 + 504*d^4*e*x + 840*d^3*e^2*x^2 + 720*d^2*
e^3*x^3 + 315*d*e^4*x^4 + 56*e^5*x^5)))/504
```

Rubi [A] (verified)

Time = 0.64 (sec) , antiderivative size = 111, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}=0.057$, Rules used = {1121, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (d+ex)^2 \left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^3\,dx$$

$$\Big\downarrow \text{ 1121}$$

$$\int \left(-\frac{3c^2d^2(d+ex)^7\left(cd^2-ae^2\right)}{e^3}+\frac{3cd(d+ex)^6\left(cd^2-ae^2\right)^2}{e^3}+\frac{(d+ex)^5\left(ae^2-cd^2\right)^3}{e^3}+\frac{c^3d^3(d+ex)^8}{e^3}\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$-\frac{3c^2d^2(d+ex)^8\left(cd^2-ae^2\right)}{8e^4}+\frac{3cd(d+ex)^7\left(cd^2-ae^2\right)^2}{7e^4}-\frac{(d+ex)^6\left(cd^2-ae^2\right)^3}{6e^4}+\frac{c^3d^3(d+ex)^9}{9e^4}$$

input
```
Int[(d + e*x)^2*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^3,x]
```

output
```
-1/6*((c*d^2 - a*e^2)^3*(d + e*x)^6)/e^4 + (3*c*d*(c*d^2 - a*e^2)^2*(d + e
*x)^7)/(7*e^4) - (3*c^2*d^2*(c*d^2 - a*e^2)*(d + e*x)^8)/(8*e^4) + (c^3*d^
3*(d + e*x)^9)/(9*e^4)
```

Defintions of rubi rules used

rule 1121
```
Int[((d_) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Int[ExpandIntegrand[(d + e*x)^(m + p)*(a/d + (c/e)*x)^p, x], x]
/; FreeQ[{a, b, c, d, e, m, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && (Int
egerQ[p] || (GtQ[a, 0] && GtQ[d, 0] && LtQ[c, 0]))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 298 vs. $2(103) = 206$.

Time = 1.32 (sec) , antiderivative size = 299, normalized size of antiderivative = 2.69

method	result
norman	$\frac{d^3c^3e^5x^9}{9}+(\frac{3}{8}a\,e^6c^2d^2+\frac{5}{8}d^4c^3e^4)x^8+(\frac{3}{7}a^2e^7cd+\frac{15}{7}a\,e^5c^2d^3+\frac{10}{7}d^5c^3e^3)x^7+(\frac{1}{6}a^3e^8+\frac{5}{2}a$
risch	$\frac{1}{9}d^3c^3e^5x^9+\frac{3}{8}x^8a\,e^6c^2d^2+\frac{5}{8}x^8d^4c^3e^4+\frac{3}{7}x^7a^2e^7cd+\frac{15}{7}x^7a\,e^5c^2d^3+\frac{10}{7}x^7d^5c^3e^3+\frac{1}{6}x^6a^3e^8$
parallelrisch	$\frac{1}{9}d^3c^3e^5x^9+\frac{3}{8}x^8a\,e^6c^2d^2+\frac{5}{8}x^8d^4c^3e^4+\frac{3}{7}x^7a^2e^7cd+\frac{15}{7}x^7a\,e^5c^2d^3+\frac{10}{7}x^7d^5c^3e^3+\frac{1}{6}x^6a^3e^8$
gosper	$x(56d^3c^3e^5x^8+189x^7a\,e^6c^2d^2+315x^7d^4c^3e^4+216x^6a^2e^7cd+1080x^6a\,e^5c^2d^3+720x^6d^5c^3e^3+84x^5a^3e^8+1260x^5a^2c\,d^2e^6+2$
orering	$x(56d^3c^3e^5x^8+189x^7a\,e^6c^2d^2+315x^7d^4c^3e^4+216x^6a^2e^7cd+1080x^6a\,e^5c^2d^3+720x^6d^5c^3e^3+84x^5a^3e^8+1260x^5a^2c\,d^2e^6+2$
default	$\frac{d^3c^3e^5x^9}{9}+\frac{(2d^4c^3e^4+3e^4(a\,e^2+c\,d^2)c^2d^2)x^8}{8}+\frac{(d^5c^3e^3+6d^3e^3(a\,e^2+c\,d^2)c^2+e^2(a\,c^2d^3e^3+2(a\,e^2+c\,d^2)^2dec+dec(2}{7}$

input
```
int((e*x+d)^2*(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^3,x,method=_RETURNVERBOSE)
```

output
```
1/9*d^3*c^3*e^5*x^9+(3/8*a*e^6*c^2*d^2+5/8*d^4*c^3*e^4)*x^8+(3/7*a^2*e^7*c
*d+15/7*a*e^5*c^2*d^3+10/7*d^5*c^3*e^3)*x^7+(1/6*a^3*e^8+5/2*a^2*c*d^2*e^6
+5*a*c^2*d^4*e^4+5/3*c^3*d^6*e^2)*x^6+(a^3*d*e^7+6*a^2*c*d^3*e^5+6*a*c^2*d
^5*e^3+c^3*d^7*e)*x^5+(5/2*e^6*a^3*d^2+15/2*a^2*e^4*c*d^4+15/4*a*e^2*c^2*d
^6+1/4*d^8*c^3)*x^4+(10/3*e^5*a^3*d^3+5*a^2*e^3*c*d^5+a*e*c^2*d^7)*x^3+(5/
2*e^4*a^3*d^4+3/2*a^2*e^2*c*d^6)*x^2+e^3*a^3*d^5*x
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 303 vs. $2(103) = 206$.

Time = 0.07 (sec) , antiderivative size = 303, normalized size of antiderivative = 2.73

$$\begin{aligned}\int (d+ex)^2 \left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^3 dx &= \frac{1}{9}c^3d^3e^5x^9 + a^3d^5e^3x + \frac{1}{8}\left(5c^3d^4e^4+3ac^2d^2e^6\right)x^8 \\ &+ \frac{1}{7}\left(10c^3d^5e^3+15ac^2d^3e^5+3a^2cde^7\right)x^7 \\ &+ \frac{1}{6}\left(10c^3d^6e^2+30ac^2d^4e^4+15a^2cd^2e^6+a^3e^8\right)x^6 \\ &+ \left(c^3d^7e+6ac^2d^5e^3+6a^2cd^3e^5+a^3de^7\right)x^5 \\ &+ \frac{1}{4}\left(c^3d^8+15ac^2d^6e^2+30a^2cd^4e^4+10a^3d^2e^6\right)x^4 \\ &+ \frac{1}{3}\left(3ac^2d^7e+15a^2cd^5e^3+10a^3d^3e^5\right)x^3 + \frac{1}{2}\left(3a^2cd^6e^2+5a^3d^4e^4\right)x^2\end{aligned}$$

input
```
integrate((e*x+d)^2*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^3,x, algorithm="fric
as")
```

output
```
1/9*c^3*d^3*e^5*x^9 + a^3*d^5*e^3*x + 1/8*(5*c^3*d^4*e^4 + 3*a*c^2*d^2*e^6
)*x^8 + 1/7*(10*c^3*d^5*e^3 + 15*a*c^2*d^3*e^5 + 3*a^2*c*d*e^7)*x^7 + 1/6*
(10*c^3*d^6*e^2 + 30*a*c^2*d^4*e^4 + 15*a^2*c*d^2*e^6 + a^3*e^8)*x^6 + (c^
3*d^7*e + 6*a*c^2*d^5*e^3 + 6*a^2*c*d^3*e^5 + a^3*d*e^7)*x^5 + 1/4*(c^3*d^
8 + 15*a*c^2*d^6*e^2 + 30*a^2*c*d^4*e^4 + 10*a^3*d^2*e^6)*x^4 + 1/3*(3*a*c
^2*d^7*e + 15*a^2*c*d^5*e^3 + 10*a^3*d^3*e^5)*x^3 + 1/2*(3*a^2*c*d^6*e^2 +
 5*a^3*d^4*e^4)*x^2
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 335 vs. $2(100) = 200$.

Time = 0.05 (sec) , antiderivative size = 335, normalized size of antiderivative = 3.02

$$\int (d+ex)^2 \left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^3 dx$$
$$= a^3d^5e^3x + \frac{c^3d^3e^5x^9}{9} + x^8 \cdot \left(\frac{3ac^2d^2e^6}{8} + \frac{5c^3d^4e^4}{8}\right) + x^7 \cdot \left(\frac{3a^2cde^7}{7} + \frac{15ac^2d^3e^5}{7} + \frac{10c^3d^5e^3}{7}\right) + x^6\left(\frac{a^3e^8}{6} + \frac{5a^2cd^2e^6}{2} + 5ac^2d^4e^4 + \frac{5c^3d^6e^2}{3}\right) + x^5\left(a^3de^7 + 6a^2cd^3e^5 + 6ac^2d^5e^3 + c^3d^7e\right) + x^4 \cdot \left(\frac{5a^3d^2e^6}{2} + \frac{15a^2cd^4e^4}{2} + \frac{15ac^2d^6e^2}{4} + \frac{c^3d^8}{4}\right) + x^3 \cdot \left(\frac{10a^3d^3e^5}{3} + 5a^2cd^5e^3 + ac^2d^7e\right) + x^2 \cdot \left(\frac{5a^3d^4e^4}{2} + \frac{3a^2cd^6e^2}{2}\right)$$

input
```
integrate((e*x+d)**2*(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**3,x)
```

output
```
a**3*d**5*e**3*x + c**3*d**3*e**5*x**9/9 + x**8*(3*a*c**2*d**2*e**6/8 + 5*
c**3*d**4*e**4/8) + x**7*(3*a**2*c*d*e**7/7 + 15*a*c**2*d**3*e**5/7 + 10*c
**3*d**5*e**3/7) + x**6*(a**3*e**8/6 + 5*a**2*c*d**2*e**6/2 + 5*a*c**2*d**
4*e**4 + 5*c**3*d**6*e**2/3) + x**5*(a**3*d*e**7 + 6*a**2*c*d**3*e**5 + 6*
a*c**2*d**5*e**3 + c**3*d**7*e) + x**4*(5*a**3*d**2*e**6/2 + 15*a**2*c*d**
4*e**4/2 + 15*a*c**2*d**6*e**2/4 + c**3*d**8/4) + x**3*(10*a**3*d**3*e**5/
3 + 5*a**2*c*d**5*e**3 + a*c**2*d**7*e) + x**2*(5*a**3*d**4*e**4/2 + 3*a**
2*c*d**6*e**2/2)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 303 vs. $2(103) = 206$.

Time = 0.04 (sec) , antiderivative size = 303, normalized size of antiderivative = 2.73

$$
\begin{aligned}
&\int (d+ex)^2 \left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^3 \, dx \\
&= \frac{1}{9}\,c^3d^3e^5x^9 + a^3d^5e^3x + \frac{1}{8}\left(5\,c^3d^4e^4 + 3\,ac^2d^2e^6\right)x^8 \\
&\quad + \frac{1}{7}\left(10\,c^3d^5e^3 + 15\,ac^2d^3e^5 + 3\,a^2cde^7\right)x^7 \\
&\quad + \frac{1}{6}\left(10\,c^3d^6e^2 + 30\,ac^2d^4e^4 + 15\,a^2cd^2e^6 + a^3e^8\right)x^6 \\
&\quad + \left(c^3d^7e + 6\,ac^2d^5e^3 + 6\,a^2cd^3e^5 + a^3de^7\right)x^5 \\
&\quad + \frac{1}{4}\left(c^3d^8 + 15\,ac^2d^6e^2 + 30\,a^2cd^4e^4 + 10\,a^3d^2e^6\right)x^4 \\
&\quad + \frac{1}{3}\left(3\,ac^2d^7e + 15\,a^2cd^5e^3 + 10\,a^3d^3e^5\right)x^3 + \frac{1}{2}\left(3\,a^2cd^6e^2 + 5\,a^3d^4e^4\right)x^2
\end{aligned}
$$

input
```
integrate((e*x+d)^2*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^3,x, algorithm="maxi
ma")
```

output
```
1/9*c^3*d^3*e^5*x^9 + a^3*d^5*e^3*x + 1/8*(5*c^3*d^4*e^4 + 3*a*c^2*d^2*e^6
)*x^8 + 1/7*(10*c^3*d^5*e^3 + 15*a*c^2*d^3*e^5 + 3*a^2*c*d*e^7)*x^7 + 1/6*
(10*c^3*d^6*e^2 + 30*a*c^2*d^4*e^4 + 15*a^2*c*d^2*e^6 + a^3*e^8)*x^6 + (c^
3*d^7*e + 6*a*c^2*d^5*e^3 + 6*a^2*c*d^3*e^5 + a^3*d*e^7)*x^5 + 1/4*(c^3*d^
8 + 15*a*c^2*d^6*e^2 + 30*a^2*c*d^4*e^4 + 10*a^3*d^2*e^6)*x^4 + 1/3*(3*a*c
^2*d^7*e + 15*a^2*c*d^5*e^3 + 10*a^3*d^3*e^5)*x^3 + 1/2*(3*a^2*c*d^6*e^2 +
 5*a^3*d^4*e^4)*x^2
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 329 vs. $2(103) = 206$.

Time = 0.14 (sec) , antiderivative size = 329, normalized size of antiderivative = 2.96

$$\int (d+ex)^2 \left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^3 dx$$
$$= \frac{1}{9}c^3d^3e^5x^9 + \frac{5}{8}c^3d^4e^4x^8 + \frac{3}{8}ac^2d^2e^6x^8 + \frac{10}{7}c^3d^5e^3x^7 + \frac{15}{7}ac^2d^3e^5x^7 + \frac{3}{7}a^2cde^7x^7 + \frac{5}{3}c^3d^6e^2x^6 + 5\,ac^2d^4e^4x^6 + \frac{5}{2}a^2cd^2e^6x^6 + \frac{1}{6}a^3e^8x^6 + c^3d^7ex^5 + 6\,ac^2d^5e^3x^5 + 6\,a^2cd^3e^5x^5 + a^3de^7x^5 + \frac{1}{4}c^3d^8x^4 + \frac{15}{4}ac^2d^6e^2x^4 + \frac{15}{2}a^2cd^4e^4x^4 + \frac{5}{2}a^3d^2e^6x^4 + ac^2d^7ex^3 + 5\,a^2cd^5e^3x^3 + \frac{10}{3}a^3d^3e^5x^3 + \frac{3}{2}a^2cd^6e^2x^2 + \frac{5}{2}a^3d^4e^4x^2 + a^3d^5e^3x$$

input

```
integrate((e*x+d)^2*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^3,x, algorithm="giac
")
```

output

```
1/9*c^3*d^3*e^5*x^9 + 5/8*c^3*d^4*e^4*x^8 + 3/8*a*c^2*d^2*e^6*x^8 + 10/7*c
^3*d^5*e^3*x^7 + 15/7*a*c^2*d^3*e^5*x^7 + 3/7*a^2*c*d*e^7*x^7 + 5/3*c^3*d^
6*e^2*x^6 + 5*a*c^2*d^4*e^4*x^6 + 5/2*a^2*c*d^2*e^6*x^6 + 1/6*a^3*e^8*x^6
+ c^3*d^7*e*x^5 + 6*a*c^2*d^5*e^3*x^5 + 6*a^2*c*d^3*e^5*x^5 + a^3*d*e^7*x^
5 + 1/4*c^3*d^8*x^4 + 15/4*a*c^2*d^6*e^2*x^4 + 15/2*a^2*c*d^4*e^4*x^4 + 5/
2*a^3*d^2*e^6*x^4 + a*c^2*d^7*e*x^3 + 5*a^2*c*d^5*e^3*x^3 + 10/3*a^3*d^3*e
^5*x^3 + 3/2*a^2*c*d^6*e^2*x^2 + 5/2*a^3*d^4*e^4*x^2 + a^3*d^5*e^3*x
```

Mupad [B] (verification not implemented)

Time = 5.45 (sec) , antiderivative size = 295, normalized size of antiderivative = 2.66

$$\int (d+ex)^2 \left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^3 dx$$
$$= x^4 \left(\frac{5\,a^3\,d^2\,e^6}{2} + \frac{15\,a^2\,c\,d^4\,e^4}{2} + \frac{15\,a\,c^2\,d^6\,e^2}{4} + \frac{c^3\,d^8}{4}\right) + x^6 \left(\frac{a^3\,e^8}{6} + \frac{5\,a^2\,c\,d^2\,e^6}{2} + 5\,a\,c^2\,d^4\,e^4 + \frac{5\,c^3\,d^6\,e^2}{3}\right) + x^5 \left(a^3\,d\,e^7 + 6\,a^2\,c\,d^3\,e^5 + 6\,a\,c^2\,d^5\,e^3 + c^3\,d^7\,e\right) + a^3\,d^5\,e^3\,x + \frac{c^3\,d^3\,e^5\,x^9}{9} + \frac{a\,d^3\,e\,x^3\,(10\,a^2\,e^4 + 15\,a\,c\,d^2\,e^2 + 3\,c^2\,d^4)}{3} + \frac{c\,d\,e^3\,x^7\,(3\,a^2\,e^4 + 15\,a\,c\,d^2\,e^2 + 10\,c^2\,d^4)}{7} + \frac{a^2\,d^4\,e^2\,x^2\,(3\,c\,d^2 + 5\,a\,e^2)}{2} + \frac{c^2\,d^2\,e^4\,x^8\,(5\,c\,d^2 + 3\,a\,e^2)}{8}$$

input
```
int((d + e*x)^2*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^3,x)
```

output
```
x^4*((c^3*d^8)/4 + (5*a^3*d^2*e^6)/2 + (15*a*c^2*d^6*e^2)/4 + (15*a^2*c*d^
4*e^4)/2) + x^6*((a^3*e^8)/6 + (5*c^3*d^6*e^2)/3 + 5*a*c^2*d^4*e^4 + (5*a^
2*c*d^2*e^6)/2) + x^5*(a^3*d*e^7 + c^3*d^7*e + 6*a*c^2*d^5*e^3 + 6*a^2*c*d
^3*e^5) + a^3*d^5*e^3*x + (c^3*d^3*e^5*x^9)/9 + (a*d^3*e*x^3*(10*a^2*e^4 +
 3*c^2*d^4 + 15*a*c*d^2*e^2))/3 + (c*d*e^3*x^7*(3*a^2*e^4 + 10*c^2*d^4 + 1
5*a*c*d^2*e^2))/7 + (a^2*d^4*e^2*x^2*(5*a*e^2 + 3*c*d^2))/2 + (c^2*d^2*e^4
*x^8*(3*a*e^2 + 5*c*d^2))/8
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 331, normalized size of antiderivative = 2.98

$$\int (d+ex)^2 \left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^3 \, dx$$
$$= \frac{x(56c^3d^3e^5x^8 + 189a\,c^2d^2e^6x^7 + 315c^3d^4e^4x^7 + 216a^2cd\,e^7x^6 + 1080a\,c^2d^3e^5x^6 + 720c^3d^5e^3x^6 + 84a^3e^8x}{}$$

input
```
int((e*x+d)^2*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^3,x)
```

output
```
(x*(504*a**3*d**5*e**3 + 1260*a**3*d**4*e**4*x + 1680*a**3*d**3*e**5*x**2
+ 1260*a**3*d**2*e**6*x**3 + 504*a**3*d*e**7*x**4 + 84*a**3*e**8*x**5 + 75
6*a**2*c*d**6*e**2*x + 2520*a**2*c*d**5*e**3*x**2 + 3780*a**2*c*d**4*e**4*
x**3 + 3024*a**2*c*d**3*e**5*x**4 + 1260*a**2*c*d**2*e**6*x**5 + 216*a**2*
c*d*e**7*x**6 + 504*a*c**2*d**7*e*x**2 + 1890*a*c**2*d**6*e**2*x**3 + 3024
*a*c**2*d**5*e**3*x**4 + 2520*a*c**2*d**4*e**4*x**5 + 1080*a*c**2*d**3*e**
5*x**6 + 189*a*c**2*d**2*e**6*x**7 + 126*c**3*d**8*x**3 + 504*c**3*d**7*e*
x**4 + 840*c**3*d**6*e**2*x**5 + 720*c**3*d**5*e**3*x**6 + 315*c**3*d**4*e
**4*x**7 + 56*c**3*d**3*e**5*x**8))/504
```

3.94 $\int (d+ex) \left(ade+\left(cd^2+ae^2\right) x+cdex^2\right)^3 \, dx$

Optimal result

Integrand size = 33, antiderivative size = 111

$$\int (d+ex) \left(ade+\left(cd^2+ae^2\right) x+cdex^2\right)^3 \, dx = -\frac{\left(cd^2-ae^2\right)^3 (d+ex)^5}{5e^4}+\frac{cd\left(cd^2-ae^2\right)^2 (d+ex)^6}{2e^4} - \frac{3c^2d^2\left(cd^2-ae^2\right) (d+ex)^7}{7e^4}+\frac{c^3d^3(d+ex)^8}{8e^4}$$

output
```
-1/5*(-a*e^2+c*d^2)^3*(e*x+d)^5/e^4+1/2*c*d*(-a*e^2+c*d^2)^2*(e*x+d)^6/e^4
-3/7*c^2*d^2*(-a*e^2+c*d^2)*(e*x+d)^7/e^4+1/8*c^3*d^3*(e*x+d)^8/e^4
```

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 211, normalized size of antiderivative = 1.90

$$\int (d+ex) \left(ade+\left(cd^2+ae^2\right) x+cdex^2\right)^3 \, dx = \frac{1}{280}x\left(56a^3e^3\left(5d^4+10d^3ex+10d^2e^2x^2+5de^3x^3+e^4x^4\right) + 28a^2cde^2x\left(15d^4+40d^3ex+45d^2e^2x^2+24de^3x^3+5e^4x^4\right) + 8ac^2d^2ex^2\left(35d^4+105d^3ex+126d^2e^2x^2+70de^3x^3+15e^4x^4\right) + c^3d^3x^3\left(70d^4+224d^3ex+280d^2e^2x^2+160de^3x^3+35e^4x^4\right)\right)$$

input
```
Integrate[(d + e*x)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^3,x]
```

output
```
(x*(56*a^3*e^3*(5*d^4 + 10*d^3*e*x + 10*d^2*e^2*x^2 + 5*d*e^3*x^3 + e^4*x^
4) + 28*a^2*c*d*e^2*x*(15*d^4 + 40*d^3*e*x + 45*d^2*e^2*x^2 + 24*d*e^3*x^3
 + 5*e^4*x^4) + 8*a*c^2*d^2*e*x^2*(35*d^4 + 105*d^3*e*x + 126*d^2*e^2*x^2
+ 70*d*e^3*x^3 + 15*e^4*x^4) + c^3*d^3*x^3*(70*d^4 + 224*d^3*e*x + 280*d^2
*e^2*x^2 + 160*d*e^3*x^3 + 35*e^4*x^4)))/280
```

Rubi [A] (verified)

Time = 0.56 (sec) , antiderivative size = 111, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.061$, Rules used = {1121, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (d+ex) \left(x \left(ae^2+cd^2\right)+ade+cdex^2\right)^3 dx$$

$$\Big\downarrow \text{1121}$$

$$\int \left(-\frac{3c^2d^2(d+ex)^6 \left(cd^2-ae^2\right)}{e^3}+\frac{3cd(d+ex)^5 \left(cd^2-ae^2\right)^2}{e^3}+\frac{(d+ex)^4 \left(ae^2-cd^2\right)^3}{e^3}+\frac{c^3d^3(d+ex)^7}{e^3}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{3c^2d^2(d+ex)^7 \left(cd^2-ae^2\right)}{7e^4}+\frac{cd(d+ex)^6 \left(cd^2-ae^2\right)^2}{2e^4}-\frac{(d+ex)^5 \left(cd^2-ae^2\right)^3}{5e^4}+\frac{c^3d^3(d+ex)^8}{8e^4}$$

input
```
Int[(d + e*x)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^3,x]
```

output
```
-1/5*((c*d^2 - a*e^2)^3*(d + e*x)^5)/e^4 + (c*d*(c*d^2 - a*e^2)^2*(d + e*x
)^6)/(2*e^4) - (3*c^2*d^2*(c*d^2 - a*e^2)*(d + e*x)^7)/(7*e^4) + (c^3*d^3*
(d + e*x)^8)/(8*e^4)
```

Defintions of rubi rules used

rule 1121
```
Int[((d_) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Int[ExpandIntegrand[(d + e*x)^(m + p)*(a/d + (c/e)*x)^p, x], x]
/; FreeQ[{a, b, c, d, e, m, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && (Int
egerQ[p] || (GtQ[a, 0] && GtQ[d, 0] && LtQ[c, 0]))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 247 vs. $2(103) = 206$.

Time = 1.34 (sec) , antiderivative size = 248, normalized size of antiderivative = 2.23

method	result
norman	$\frac{d^3c^3e^4x^8}{8} + \left(\frac{3}{7}a\,e^5c^2d^2 + \frac{4}{7}d^4c^3e^3\right)x^7 + \left(\frac{1}{2}a^2e^6cd + 2a\,e^4c^2d^3 + d^5c^3e^2\right)x^6 + \left(\frac{1}{5}a^3e^7 + \frac{12}{5}a^2e^5$
risch	$\frac{1}{8}d^3c^3e^4x^8 + \frac{3}{7}x^7a\,e^5c^2d^2 + \frac{4}{7}x^7d^4c^3e^3 + \frac{1}{2}x^6a^2e^6cd + 2x^6a\,e^4c^2d^3 + x^6d^5c^3e^2 + \frac{1}{5}x^5a^3e^7 +$
parallelrisch	$\frac{1}{8}d^3c^3e^4x^8 + \frac{3}{7}x^7a\,e^5c^2d^2 + \frac{4}{7}x^7d^4c^3e^3 + \frac{1}{2}x^6a^2e^6cd + 2x^6a\,e^4c^2d^3 + x^6d^5c^3e^2 + \frac{1}{5}x^5a^3e^7 +$
gosper	$x(35d^3c^3e^4x^7+120x^6a\,e^5c^2d^2+160x^6d^4c^3e^3+140x^5a^2e^6cd+560x^5a\,e^4c^2d^3+280x^5d^5c^3e^2+56x^4a^3e^7+672x^4a^2e^5c\,d^2+100$
orering	$x(35d^3c^3e^4x^7+120x^6a\,e^5c^2d^2+160x^6d^4c^3e^3+140x^5a^2e^6cd+560x^5a\,e^4c^2d^3+280x^5d^5c^3e^2+56x^4a^3e^7+672x^4a^2e^5c\,d^2+100$
default	$\frac{d^3c^3e^4x^8}{8} + \frac{(d^4c^3e^3+3e^3(a\,e^2+c\,d^2)c^2d^2)x^7}{7} + \frac{(3d^3(a\,e^2+c\,d^2)e^2c^2+e(a\,c^2d^3e^3+2(a\,e^2+c\,d^2)^2dec+dec(2ac\,d^2e^2+(a}{6}$

input
```
int((e*x+d)*(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^3,x,method=_RETURNVERBOSE)
```

output
```
1/8*d^3*c^3*e^4*x^8+(3/7*a*e^5*c^2*d^2+4/7*d^4*c^3*e^3)*x^7+(1/2*a^2*e^6*c
*d+2*a*e^4*c^2*d^3+d^5*c^3*e^2)*x^6+(1/5*a^3*e^7+12/5*a^2*e^5*c*d^2+18/5*a
*e^3*c^2*d^4+4/5*d^6*c^3*e)*x^5+(e^6*a^3*d+9/2*a^2*c*d^3*e^4+3*a*c^2*d^5*e
^2+1/4*c^3*d^7)*x^4+(2*a^3*d^2*e^5+4*a^2*c*d^4*e^3+a*c^2*d^6*e)*x^3+(2*e^4
*a^3*d^3+3/2*a^2*e^2*c*d^5)*x^2+e^3*a^3*d^4*x
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 251 vs. $2(103) = 206$.

Time = 0.07 (sec) , antiderivative size = 251, normalized size of antiderivative = 2.26

$$\int (d+ex)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^3 dx$$
$$= \frac{1}{8}c^3d^3e^4x^8 + a^3d^4e^3x + \frac{1}{7}\left(4c^3d^4e^3 + 3ac^2d^2e^5\right)x^7 + \frac{1}{2}\left(2c^3d^5e^2 + 4ac^2d^3e^4 + a^2cde^6\right)x^6 + \frac{1}{5}\left(4c^3d^6e + 18ac^2d^4e^3 + 12a^2cd^2e^5 + a^3e^7\right)x^5 + \frac{1}{4}\left(c^3d^7 + 12ac^2d^5e^2 + 18a^2cd^3e^4 + 4a^3de^6\right)x^4 + \left(ac^2d^6e + 4a^2cd^4e^3 + 2a^3d^2e^5\right)x^3 + \frac{1}{2}\left(3a^2cd^5e^2 + 4a^3d^3e^4\right)x^2$$

input
```
integrate((e*x+d)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^3,x, algorithm="fricas
")
```

output
```
1/8*c^3*d^3*e^4*x^8 + a^3*d^4*e^3*x + 1/7*(4*c^3*d^4*e^3 + 3*a*c^2*d^2*e^5
)*x^7 + 1/2*(2*c^3*d^5*e^2 + 4*a*c^2*d^3*e^4 + a^2*c*d*e^6)*x^6 + 1/5*(4*c
^3*d^6*e + 18*a*c^2*d^4*e^3 + 12*a^2*c*d^2*e^5 + a^3*e^7)*x^5 + 1/4*(c^3*d
^7 + 12*a*c^2*d^5*e^2 + 18*a^2*c*d^3*e^4 + 4*a^3*d*e^6)*x^4 + (a*c^2*d^6*e
 + 4*a^2*c*d^4*e^3 + 2*a^3*d^2*e^5)*x^3 + 1/2*(3*a^2*c*d^5*e^2 + 4*a^3*d^3
*e^4)*x^2
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 270 vs. $2(99) = 198$.

Time = 0.04 (sec) , antiderivative size = 270, normalized size of antiderivative = 2.43

$$\int (d+ex)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^3\,dx$$
$$=a^3d^4e^3x+\frac{c^3d^3e^4x^8}{8}+x^7\cdot\left(\frac{3ac^2d^2e^5}{7}+\frac{4c^3d^4e^3}{7}\right)+x^6\left(\frac{a^2cde^6}{2}+2ac^2d^3e^4+c^3d^5e^2\right)+x^5\left(\frac{a^3e^7}{5}+\frac{12a^2cd^2e^5}{5}+\frac{18ac^2d^4e^3}{5}+\frac{4c^3d^6e}{5}\right)+x^4\left(a^3de^6+\frac{9a^2cd^3e^4}{2}+3ac^2d^5e^2+\frac{c^3d^7}{4}\right)+x^3\cdot\left(2a^3d^2e^5+4a^2cd^4e^3+ac^2d^6e\right)+x^2\cdot\left(2a^3d^3e^4+\frac{3a^2cd^5e^2}{2}\right)$$

input
```
integrate((e*x+d)*(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**3,x)
```

output
```
a**3*d**4*e**3*x + c**3*d**3*e**4*x**8/8 + x**7*(3*a*c**2*d**2*e**5/7 + 4*
c**3*d**4*e**3/7) + x**6*(a**2*c*d*e**6/2 + 2*a*c**2*d**3*e**4 + c**3*d**5
*e**2) + x**5*(a**3*e**7/5 + 12*a**2*c*d**2*e**5/5 + 18*a*c**2*d**4*e**3/5
 + 4*c**3*d**6*e/5) + x**4*(a**3*d*e**6 + 9*a**2*c*d**3*e**4/2 + 3*a*c**2*
d**5*e**2 + c**3*d**7/4) + x**3*(2*a**3*d**2*e**5 + 4*a**2*c*d**4*e**3 + a
*c**2*d**6*e) + x**2*(2*a**3*d**3*e**4 + 3*a**2*c*d**5*e**2/2)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 251 vs. 2(103) = 206.

Time = 0.03 (sec) , antiderivative size = 251, normalized size of antiderivative = 2.26

$$\int (d+ex)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^3\,dx$$
$$=\frac{1}{8}\,c^3d^3e^4x^8+a^3d^4e^3x+\frac{1}{7}\left(4\,c^3d^4e^3+3\,ac^2d^2e^5\right)x^7+\frac{1}{2}\left(2\,c^3d^5e^2+4\,ac^2d^3e^4+a^2cde^6\right)x^6+\frac{1}{5}\left(4\,c^3d^6e+18\,ac^2d^4e^3+12\,a^2cd^2e^5+a^3e^7\right)x^5+\frac{1}{4}\left(c^3d^7+12\,ac^2d^5e^2+18\,a^2cd^3e^4+4\,a^3de^6\right)x^4+\left(ac^2d^6e+4\,a^2cd^4e^3+2\,a^3d^2e^5\right)x^3+\frac{1}{2}\left(3\,a^2cd^5e^2+4\,a^3d^3e^4\right)x^2$$

input
```
integrate((e*x+d)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^3,x, algorithm="maxima
")
```

output
```
1/8*c^3*d^3*e^4*x^8 + a^3*d^4*e^3*x + 1/7*(4*c^3*d^4*e^3 + 3*a*c^2*d^2*e^5
)*x^7 + 1/2*(2*c^3*d^5*e^2 + 4*a*c^2*d^3*e^4 + a^2*c*d*e^6)*x^6 + 1/5*(4*c
^3*d^6*e + 18*a*c^2*d^4*e^3 + 12*a^2*c*d^2*e^5 + a^3*e^7)*x^5 + 1/4*(c^3*d
^7 + 12*a*c^2*d^5*e^2 + 18*a^2*c*d^3*e^4 + 4*a^3*d*e^6)*x^4 + (a*c^2*d^6*e
 + 4*a^2*c*d^4*e^3 + 2*a^3*d^2*e^5)*x^3 + 1/2*(3*a^2*c*d^5*e^2 + 4*a^3*d^3
*e^4)*x^2
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 271 vs. $2(103) = 206$.

Time = 0.14 (sec) , antiderivative size = 271, normalized size of antiderivative = 2.44

$$\begin{aligned}\int (d+ex)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^3\,dx &= \frac{1}{8}\,c^3d^3e^4x^8+\frac{4}{7}\,c^3d^4e^3x^7+\frac{3}{7}\,ac^2d^2e^5x^7+c^3d^5e^2x^6+2\,ac^2d^3e^4x^6\\ &\quad+\frac{1}{2}\,a^2cde^6x^6+\frac{4}{5}\,c^3d^6ex^5+\frac{18}{5}\,ac^2d^4e^3x^5+\frac{12}{5}\,a^2cd^2e^5x^5+\frac{1}{5}\,a^3e^7x^5\\ &\quad+\frac{1}{4}\,c^3d^7x^4+3\,ac^2d^5e^2x^4+\frac{9}{2}\,a^2cd^3e^4x^4+a^3de^6x^4+ac^2d^6ex^3\\ &\quad+4\,a^2cd^4e^3x^3+2\,a^3d^2e^5x^3+\frac{3}{2}\,a^2cd^5e^2x^2+2\,a^3d^3e^4x^2+a^3d^4e^3x\end{aligned}$$

input
```
integrate((e*x+d)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^3,x, algorithm="giac")
```

output
```
1/8*c^3*d^3*e^4*x^8 + 4/7*c^3*d^4*e^3*x^7 + 3/7*a*c^2*d^2*e^5*x^7 + c^3*d^
5*e^2*x^6 + 2*a*c^2*d^3*e^4*x^6 + 1/2*a^2*c*d*e^6*x^6 + 4/5*c^3*d^6*e*x^5
+ 18/5*a*c^2*d^4*e^3*x^5 + 12/5*a^2*c*d^2*e^5*x^5 + 1/5*a^3*e^7*x^5 + 1/4*
c^3*d^7*x^4 + 3*a*c^2*d^5*e^2*x^4 + 9/2*a^2*c*d^3*e^4*x^4 + a^3*d*e^6*x^4
+ a*c^2*d^6*e*x^3 + 4*a^2*c*d^4*e^3*x^3 + 2*a^3*d^2*e^5*x^3 + 3/2*a^2*c*d^
5*e^2*x^2 + 2*a^3*d^3*e^4*x^2 + a^3*d^4*e^3*x
```

Mupad [B] (verification not implemented)

Time = 5.60 (sec) , antiderivative size = 242, normalized size of antiderivative = 2.18

$$\int (d+ex)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^3\,dx$$
$$=x^4\left(a^3\,d\,e^6+\frac{9\,a^2\,c\,d^3\,e^4}{2}+3\,a\,c^2\,d^5\,e^2+\frac{c^3\,d^7}{4}\right)+x^5\left(\frac{a^3\,e^7}{5}+\frac{12\,a^2\,c\,d^2\,e^5}{5}+\frac{18\,a\,c^2\,d^4\,e^3}{5}+\frac{4\,c^3\,d^6\,e}{5}\right)+a^3\,d^4\,e^3\,x+\frac{c^3\,d^3\,e^4\,x^8}{8}+a\,d^2\,e\,x^3\left(2\,a^2\,e^4+4\,a\,c\,d^2\,e^2+c^2\,d^4\right)+\frac{c\,d\,e^2\,x^6\left(a^2\,e^4+4\,a\,c\,d^2\,e^2+2\,c^2\,d^4\right)}{2}+\frac{a^2\,d^3\,e^2\,x^2\left(3\,c\,d^2+4\,a\,e^2\right)}{2}+\frac{c^2\,d^2\,e^3\,x^7\left(4\,c\,d^2+3\,a\,e^2\right)}{7}$$

input
```
int((d + e*x)*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^3,x)
```

output
```
x^4*((c^3*d^7)/4 + a^3*d*e^6 + 3*a*c^2*d^5*e^2 + (9*a^2*c*d^3*e^4)/2) + x^
5*((a^3*e^7)/5 + (4*c^3*d^6*e)/5 + (18*a*c^2*d^4*e^3)/5 + (12*a^2*c*d^2*e^
5)/5) + a^3*d^4*e^3*x + (c^3*d^3*e^4*x^8)/8 + a*d^2*e*x^3*(2*a^2*e^4 + c^2
*d^4 + 4*a*c*d^2*e^2) + (c*d*e^2*x^6*(a^2*e^4 + 2*c^2*d^4 + 4*a*c*d^2*e^2)
)/2 + (a^2*d^3*e^2*x^2*(4*a*e^2 + 3*c*d^2))/2 + (c^2*d^2*e^3*x^7*(3*a*e^2
+ 4*c*d^2))/7
```

Reduce [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 273, normalized size of antiderivative = 2.46

$$\int (d+ex)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^3\,dx$$
$$=\frac{x(35c^3d^3e^4x^7+120a\,c^2d^2e^5x^6+160c^3d^4e^3x^6+140a^2cd\,e^6x^5+560a\,c^2d^3e^4x^5+280c^3d^5e^2x^5+56a^3e^7x^4}{}$$

input
```
int((e*x+d)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^3,x)
```

output

```
(x*(280*a**3*d**4*e**3 + 560*a**3*d**3*e**4*x + 560*a**3*d**2*e**5*x**2 +
280*a**3*d*e**6*x**3 + 56*a**3*e**7*x**4 + 420*a**2*c*d**5*e**2*x + 1120*a
**2*c*d**4*e**3*x**2 + 1260*a**2*c*d**3*e**4*x**3 + 672*a**2*c*d**2*e**5*x
**4 + 140*a**2*c*d*e**6*x**5 + 280*a*c**2*d**6*e*x**2 + 840*a*c**2*d**5*e*
*2*x**3 + 1008*a*c**2*d**4*e**3*x**4 + 560*a*c**2*d**3*e**4*x**5 + 120*a*c
**2*d**2*e**5*x**6 + 70*c**3*d**7*x**3 + 224*c**3*d**6*e*x**4 + 280*c**3*d
**5*e**2*x**5 + 160*c**3*d**4*e**3*x**6 + 35*c**3*d**3*e**4*x**7))/280
```

3.95 $\int \left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^3\,dx$

Optimal result

Integrand size = 27, antiderivative size = 111

$$\int \left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^3\,dx=-\frac{\left(cd^2-ae^2\right)^3(d+ex)^4}{4e^4}+\frac{3cd\left(cd^2-ae^2\right)^2(d+ex)^5}{5e^4}-\frac{c^2d^2\left(cd^2-ae^2\right)(d+ex)^6}{2e^4}+\frac{c^3d^3(d+ex)^7}{7e^4}$$

output
```
-1/4*(-a*e^2+c*d^2)^3*(e*x+d)^4/e^4+3/5*c*d*(-a*e^2+c*d^2)^2*(e*x+d)^5/e^4
-1/2*c^2*d^2*(-a*e^2+c*d^2)*(e*x+d)^6/e^4+1/7*c^3*d^3*(e*x+d)^7/e^4
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 167, normalized size of antiderivative = 1.50

$$\int \left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^3\,dx=\frac{1}{140}x\left(35a^3e^3\left(4d^3+6d^2ex+4de^2x^2+e^3x^3\right)+21a^2cde^2x\left(10d^3+20d^2ex+15de^2x^2+4e^3x^3\right)+7ac^2d^2ex^2\left(20d^3+45d^2ex+36de^2x^2+10e^3x^3\right)+c^3d^3x^3\left(35d^3+84d^2ex+70de^2x^2+20e^3x^3\right)\right)$$

input
```
Integrate[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^3,x]
```

output
```
(x*(35*a^3*e^3*(4*d^3 + 6*d^2*e*x + 4*d*e^2*x^2 + e^3*x^3) + 21*a^2*c*d*e^
2*x*(10*d^3 + 20*d^2*e*x + 15*d*e^2*x^2 + 4*e^3*x^3) + 7*a*c^2*d^2*e*x^2*(
20*d^3 + 45*d^2*e*x + 36*d*e^2*x^2 + 10*e^3*x^3) + c^3*d^3*x^3*(35*d^3 + 8
4*d^2*e*x + 70*d*e^2*x^2 + 20*e^3*x^3)))/140
```

Rubi [A] (verified)

Time = 0.59 (sec) , antiderivative size = 131, normalized size of antiderivative = 1.18, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.074$, Rules used = {1084, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^3 dx$$

$$\Big\downarrow \text{1084}$$

$$\frac{\int \left(\left(cd^2+cexd\right)^6-3c^5d^5\left(cd^2-ae^2\right)(d+ex)^5+3c^4d^4\left(cd^2-ae^2\right)^2(d+ex)^4-c^3d^3\left(cd^2-ae^2\right)^3(d+ex)^3\right)dx}{c^3d^3e^3}$$

$$\Big\downarrow \text{2009}$$

$$\frac{-\frac{c^5d^5(d+ex)^6\left(cd^2-ae^2\right)}{2e}+\frac{3c^4d^4(d+ex)^5\left(cd^2-ae^2\right)^2}{5e}-\frac{c^3d^3(d+ex)^4\left(cd^2-ae^2\right)^3}{4e}+\frac{c^6d^6(d+ex)^7}{7e}}{c^3d^3e^3}$$

input
```
Int[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^3,x]
```

output
```
(-1/4*(c^3*d^3*(c*d^2 - a*e^2)^3*(d + e*x)^4)/e + (3*c^4*d^4*(c*d^2 - a*e^
2)^2*(d + e*x)^5)/(5*e) - (c^5*d^5*(c*d^2 - a*e^2)*(d + e*x)^6)/(2*e) + (c
^6*d^6*(d + e*x)^7)/(7*e))/(c^3*d^3*e^3)
```

Defintions of rubi rules used

rule 1084
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{q = Rt[b^2
- 4*a*c, 2]}, Simp[1/c^p   Int[ExpandIntegrand[(b/2 - q/2 + c*x)^p*(b/2 + q
/2 + c*x)^p, x], x], x] /;  !FractionalPowerFactorQ[q]] /; FreeQ[{a, b, c},
 x] && IntegerQ[p] && NiceSqrtQ[b^2 - 4*a*c]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 1.16 (sec) , antiderivative size = 198, normalized size of antiderivative = 1.78

method	result
norman	$\frac{e^3d^3c^3x^7}{7}+\left(\frac{1}{2}a\,c^2d^2e^4+\frac{1}{2}c^3d^4e^2\right)x^6+\left(\frac{3}{5}d\,e^5a^2c+\frac{9}{5}a\,c^2d^3e^3+\frac{3}{5}c^3d^5e\right)x^5+\left(\frac{1}{4}e^6a^3+\frac{9}{4}d^2e^4\right.$
risch	$\frac{1}{7}e^3d^3c^3x^7+\frac{1}{2}a\,c^2d^2e^4x^6+\frac{1}{2}c^3d^4e^2x^6+\frac{3}{5}x^5d\,e^5a^2c+\frac{9}{5}x^5a\,c^2d^3e^3+\frac{3}{5}x^5c^3d^5e+\frac{1}{4}x^4e^6a^3+$
parallelrisch	$\frac{1}{7}e^3d^3c^3x^7+\frac{1}{2}a\,c^2d^2e^4x^6+\frac{1}{2}c^3d^4e^2x^6+\frac{3}{5}x^5d\,e^5a^2c+\frac{9}{5}x^5a\,c^2d^3e^3+\frac{3}{5}x^5c^3d^5e+\frac{1}{4}x^4e^6a^3+$
gosper	$\frac{x\left(20e^3d^3c^3x^6+70x^5a\,c^2d^2e^4+70x^5c^3d^4e^2+84x^4d\,e^5a^2c+252x^4a\,c^2d^3e^3+84x^4c^3d^5e+35x^3e^6a^3+315x^3d^2e^4a^2c+315x^3d^4e\right.}{140}$
orering	$\frac{x\left(20e^3d^3c^3x^6+70x^5a\,c^2d^2e^4+70x^5c^3d^4e^2+84x^4d\,e^5a^2c+252x^4a\,c^2d^3e^3+84x^4c^3d^5e+35x^3e^6a^3+315x^3d^2e^4a^2c+315x^3d^4e\right.}{140(e}$
default	$\frac{e^3d^3c^3x^7}{7}+\frac{(a\,e^2+c\,d^2)e^2c^2d^2x^6}{2}+\frac{\left(a\,c^2d^3e^3+2(a\,e^2+c\,d^2)^2dec+dec\left(2ac\,d^2e^2+(a\,e^2+c\,d^2)^2\right)\right)x^5}{5}+\frac{\left(4a\,d^2e^2c(a\,e^2\right.}{}$

input
```
int((a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^3,x,method=_RETURNVERBOSE)
```

output
```
1/7*e^3*d^3*c^3*x^7+(1/2*a*c^2*d^2*e^4+1/2*c^3*d^4*e^2)*x^6+(3/5*d*e^5*a^2
*c+9/5*a*c^2*d^3*e^3+3/5*c^3*d^5*e)*x^5+(1/4*e^6*a^3+9/4*d^2*e^4*a^2*c+9/4
*d^4*e^2*a*c^2+1/4*d^6*c^3)*x^4+(a^3*d*e^5+3*a^2*c*d^3*e^3+a*c^2*d^5*e)*x^
3+(3/2*d^2*e^4*a^3+3/2*d^4*a^2*e^2*c)*x^2+d^3*e^3*a^3*x
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 193, normalized size of antiderivative = 1.74

$$\int \left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^3\,dx=\frac{1}{7}c^3d^3e^3x^7+a^3d^3e^3x+\frac{1}{2}\left(c^3d^4e^2+ac^2d^2e^4\right)x^6+\frac{3}{5}\left(c^3d^5e+3\,ac^2d^3e^3+a^2cde^5\right)x^5+\frac{1}{4}\left(c^3d^6+9\,ac^2d^4e^2+9\,a^2cd^2e^4+a^3e^6\right)x^4+\left(ac^2d^5e+3\,a^2cd^3e^3+a^3de^5\right)x^3+\frac{3}{2}\left(a^2cd^4e^2+a^3d^2e^4\right)x^2$$

input
```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^3,x, algorithm="fricas")
```

output
```
1/7*c^3*d^3*e^3*x^7 + a^3*d^3*e^3*x + 1/2*(c^3*d^4*e^2 + a*c^2*d^2*e^4)*x^
6 + 3/5*(c^3*d^5*e + 3*a*c^2*d^3*e^3 + a^2*c*d*e^5)*x^5 + 1/4*(c^3*d^6 + 9
*a*c^2*d^4*e^2 + 9*a^2*c*d^2*e^4 + a^3*e^6)*x^4 + (a*c^2*d^5*e + 3*a^2*c*d
^3*e^3 + a^3*d*e^5)*x^3 + 3/2*(a^2*c*d^4*e^2 + a^3*d^2*e^4)*x^2
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 218 vs. $2(99) = 198$.

Time = 0.04 (sec) , antiderivative size = 218, normalized size of antiderivative = 1.96

$$\int \left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^3\,dx=a^3d^3e^3x+\frac{c^3d^3e^3x^7}{7}+x^6\left(\frac{ac^2d^2e^4}{2}+\frac{c^3d^4e^2}{2}\right)+x^5\cdot\left(\frac{3a^2cde^5}{5}+\frac{9ac^2d^3e^3}{5}+\frac{3c^3d^5e}{5}\right)+x^4\left(\frac{a^3e^6}{4}+\frac{9a^2cd^2e^4}{4}+\frac{9ac^2d^4e^2}{4}+\frac{c^3d^6}{4}\right)+x^3\left(a^3de^5+3a^2cd^3e^3+ac^2d^5e\right)+x^2\cdot\left(\frac{3a^3d^2e^4}{2}+\frac{3a^2cd^4e^2}{2}\right)$$

input
```
integrate((a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**3,x)
```

output
```
a**3*d**3*e**3*x + c**3*d**3*e**3*x**7/7 + x**6*(a*c**2*d**2*e**4/2 + c**3
*d**4*e**2/2) + x**5*(3*a**2*c*d*e**5/5 + 9*a*c**2*d**3*e**3/5 + 3*c**3*d*
*5*e/5) + x**4*(a**3*e**6/4 + 9*a**2*c*d**2*e**4/4 + 9*a*c**2*d**4*e**2/4
+ c**3*d**6/4) + x**3*(a**3*d*e**5 + 3*a**2*c*d**3*e**3 + a*c**2*d**5*e) +
 x**2*(3*a**3*d**2*e**4/2 + 3*a**2*c*d**4*e**2/2)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 183, normalized size of antiderivative = 1.65

$$\begin{aligned}
&\int \left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^3\,dx\\
&=\frac{1}{7}\,c^3d^3e^3x^7+\frac{1}{2}\left(cd^2+ae^2\right)c^2d^2e^2x^6+a^3d^3e^3x+\frac{3}{5}\left(cd^2+ae^2\right)^2cdex^5\\
&\quad+\frac{1}{2}\left(2\,cdex^3+3\left(cd^2+ae^2\right)x^2\right)a^2d^2e^2+\frac{1}{4}\left(cd^2+ae^2\right)^3x^4\\
&\quad+\frac{1}{10}\left(6\,c^2d^2e^2x^5+15\left(cd^2+ae^2\right)cdex^4+10\left(cd^2+ae^2\right)^2x^3\right)ade
\end{aligned}$$

input
```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^3,x, algorithm="maxima")
```

output
```
1/7*c^3*d^3*e^3*x^7 + 1/2*(c*d^2 + a*e^2)*c^2*d^2*e^2*x^6 + a^3*d^3*e^3*x
+ 3/5*(c*d^2 + a*e^2)^2*c*d*e*x^5 + 1/2*(2*c*d*e*x^3 + 3*(c*d^2 + a*e^2)*x
^2)*a^2*d^2*e^2 + 1/4*(c*d^2 + a*e^2)^3*x^4 + 1/10*(6*c^2*d^2*e^2*x^5 + 15
*(c*d^2 + a*e^2)*c*d*e*x^4 + 10*(c*d^2 + a*e^2)^2*x^3)*a*d*e
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 214 vs. $2(103) = 206$.

Time = 0.14 (sec) , antiderivative size = 214, normalized size of antiderivative = 1.93

$$\int \left(ade + \left(cd^2 + ae^2\right)x + cdex^2\right)^3 dx = \frac{1}{7}c^3d^3e^3x^7 + \frac{1}{2}c^3d^4e^2x^6 + \frac{1}{2}ac^2d^2e^4x^6 + \frac{3}{5}c^3d^5ex^5 + \frac{9}{5}ac^2d^3e^3x^5 + \frac{3}{5}a^2cde^5x^5 + \frac{1}{4}c^3d^6x^4 + \frac{9}{4}ac^2d^4e^2x^4 + \frac{9}{4}a^2cd^2e^4x^4 + \frac{1}{4}a^3e^6x^4 + ac^2d^5ex^3 + 3a^2cd^3e^3x^3 + a^3de^5x^3 + \frac{3}{2}a^2cd^4e^2x^2 + \frac{3}{2}a^3d^2e^4x^2 + a^3d^3e^3x$$

input
```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^3,x, algorithm="giac")
```

output
```
1/7*c^3*d^3*e^3*x^7 + 1/2*c^3*d^4*e^2*x^6 + 1/2*a*c^2*d^2*e^4*x^6 + 3/5*c^
3*d^5*e*x^5 + 9/5*a*c^2*d^3*e^3*x^5 + 3/5*a^2*c*d*e^5*x^5 + 1/4*c^3*d^6*x^
4 + 9/4*a*c^2*d^4*e^2*x^4 + 9/4*a^2*c*d^2*e^4*x^4 + 1/4*a^3*e^6*x^4 + a*c^
2*d^5*e*x^3 + 3*a^2*c*d^3*e^3*x^3 + a^3*d*e^5*x^3 + 3/2*a^2*c*d^4*e^2*x^2
+ 3/2*a^3*d^2*e^4*x^2 + a^3*d^3*e^3*x
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 186, normalized size of antiderivative = 1.68

$$\int \left(ade + \left(cd^2 + ae^2\right)x + cdex^2\right)^3 dx = x^4\left(\frac{a^3e^6}{4} + \frac{9a^2cd^2e^4}{4} + \frac{9ac^2d^4e^2}{4} + \frac{c^3d^6}{4}\right) + a^3d^3e^3x + \frac{c^3d^3e^3x^7}{7} + adex^3\left(a^2e^4 + 3acd^2e^2 + c^2d^4\right) + \frac{3cdex^5\left(a^2e^4 + 3acd^2e^2 + c^2d^4\right)}{5} + \frac{3a^2d^2e^2x^2\left(cd^2 + ae^2\right)}{2} + \frac{c^2d^2e^2x^6\left(cd^2 + ae^2\right)}{2}$$

input
```
int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^3,x)
```

output
```
x^4*((a^3*e^6)/4 + (c^3*d^6)/4 + (9*a*c^2*d^4*e^2)/4 + (9*a^2*c*d^2*e^4)/4
) + a^3*d^3*e^3*x + (c^3*d^3*e^3*x^7)/7 + a*d*e*x^3*(a^2*e^4 + c^2*d^4 + 3
*a*c*d^2*e^2) + (3*c*d*e*x^5*(a^2*e^4 + c^2*d^4 + 3*a*c*d^2*e^2))/5 + (3*a
^2*d^2*e^2*x^2*(a*e^2 + c*d^2))/2 + (c^2*d^2*e^2*x^6*(a*e^2 + c*d^2))/2
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 215, normalized size of antiderivative = 1.94

$$\int \left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^3\,dx$$
$$=\frac{x(20c^3d^3e^3x^6+70a\,c^2d^2e^4x^5+70c^3d^4e^2x^5+84a^2cd\,e^5x^4+252a\,c^2d^3e^3x^4+84c^3d^5e\,x^4+35a^3e^6x^3+31}{}$$

input
```
int((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^3,x)
```

output
```
(x*(140*a**3*d**3*e**3 + 210*a**3*d**2*e**4*x + 140*a**3*d*e**5*x**2 + 35*
a**3*e**6*x**3 + 210*a**2*c*d**4*e**2*x + 420*a**2*c*d**3*e**3*x**2 + 315*
a**2*c*d**2*e**4*x**3 + 84*a**2*c*d*e**5*x**4 + 140*a*c**2*d**5*e*x**2 + 3
15*a*c**2*d**4*e**2*x**3 + 252*a*c**2*d**3*e**3*x**4 + 70*a*c**2*d**2*e**4
*x**5 + 35*c**3*d**6*x**3 + 84*c**3*d**5*e*x**4 + 70*c**3*d**4*e**2*x**5 +
 20*c**3*d**3*e**3*x**6))/140
```

3.96 $\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^3}{d+ex} \, dx$

Optimal result

Integrand size = 35, antiderivative size = 91

$$\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^3}{d+ex} \, dx=\frac{\left(cd^2-ae^2\right)^2(ae+cdx)^4}{4c^3d^3}+\frac{2e\left(cd^2-ae^2\right)(ae+cdx)^5}{5c^3d^3}+\frac{e^2(ae+cdx)^6}{6c^3d^3}$$

output
```
1/4*(-a*e^2+c*d^2)^2*(c*d*x+a*e)^4/c^3/d^3+2/5*e*(-a*e^2+c*d^2)*(c*d*x+a*e
)^5/c^3/d^3+1/6*e^2*(c*d*x+a*e)^6/c^3/d^3
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 123, normalized size of antiderivative = 1.35

$$\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^3}{d+ex} \, dx=\frac{1}{60}x\left(20a^3e^3\left(3d^2+3dex+e^2x^2\right)+15a^2cde^2x\left(6d^2+8dex+3e^2x^2\right)+6ac^2d^2ex^2\left(10d^2+15dex+6e^2x^2\right)+c^3d^3x^3\left(15d^2+24dex+10e^2x^2\right)\right)$$

input
```
Integrate[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^3/(d + e*x),x]
```

output
```
(x*(20*a^3*e^3*(3*d^2 + 3*d*e*x + e^2*x^2) + 15*a^2*c*d*e^2*x*(6*d^2 + 8*d
*e*x + 3*e^2*x^2) + 6*a*c^2*d^2*e*x^2*(10*d^2 + 15*d*e*x + 6*e^2*x^2) + c^
3*d^3*x^3*(15*d^2 + 24*d*e*x + 10*e^2*x^2)))/60
```

Rubi [A] (verified)

Time = 0.47 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.057$, Rules used = {1121, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^3}{d+ex}\,dx$$

$$\Big\downarrow \text{1121}$$

$$\int \left(\frac{e^2(ae+cdx)^5}{c^2d^2}+\frac{2e\left(cd^2-ae^2\right)(ae+cdx)^4}{c^2d^2}+\frac{\left(cd^2-ae^2\right)^2(ae+cdx)^3}{c^2d^2}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{e^2(ae+cdx)^6}{6c^3d^3}+\frac{2e\left(cd^2-ae^2\right)(ae+cdx)^5}{5c^3d^3}+\frac{\left(cd^2-ae^2\right)^2(ae+cdx)^4}{4c^3d^3}$$

input
```
Int[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^3/(d + e*x),x]
```

output
```
((c*d^2 - a*e^2)^2*(a*e + c*d*x)^4)/(4*c^3*d^3) + (2*e*(c*d^2 - a*e^2)*(a*
e + c*d*x)^5)/(5*c^3*d^3) + (e^2*(a*e + c*d*x)^6)/(6*c^3*d^3)
```

Defintions of rubi rules used

rule 1121
```
Int[((d_) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Int[ExpandIntegrand[(d + e*x)^(m + p)*(a/d + (c/e)*x)^p, x], x]
/; FreeQ[{a, b, c, d, e, m, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && (Int
egerQ[p] || (GtQ[a, 0] && GtQ[d, 0] && LtQ[c, 0]))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 1.56 (sec) , antiderivative size = 147, normalized size of antiderivative = 1.62

method	result
norman	$\frac{e^2d^3c^3x^6}{6}+\left(\frac{3}{5}e^3a\,c^2d^2+\frac{2}{5}d^4e\,c^3\right)x^5+\left(\frac{3}{4}d\,e^4a^2c+\frac{3}{2}d^3e^2a\,c^2+\frac{1}{4}d^5c^3\right)x^4+\left(\frac{1}{3}a^3e^5+2d^2e^3a\right.$
risch	$\frac{1}{6}e^2d^3c^3x^6+\frac{3}{5}x^5e^3a\,c^2d^2+\frac{2}{5}x^5d^4e\,c^3+\frac{3}{4}x^4d\,e^4a^2c+\frac{3}{2}x^4d^3e^2a\,c^2+\frac{1}{4}d^5c^3x^4+\frac{1}{3}x^3a^3e^5+2$
parallelrisch	$\frac{1}{6}e^2d^3c^3x^6+\frac{3}{5}x^5e^3a\,c^2d^2+\frac{2}{5}x^5d^4e\,c^3+\frac{3}{4}x^4d\,e^4a^2c+\frac{3}{2}x^4d^3e^2a\,c^2+\frac{1}{4}d^5c^3x^4+\frac{1}{3}x^3a^3e^5+2$
gosper	$\frac{x\left(10e^2d^3c^3x^5+36x^4e^3a\,c^2d^2+24x^4d^4e\,c^3+45x^3d\,e^4a^2c+90x^3d^3e^2a\,c^2+15x^3d^5c^3+20x^2a^3e^5+120x^2d^2e^3a^2c+60x^2d^4ae\,c^2\right.}{60}$
orering	$\frac{x\left(10e^2d^3c^3x^5+36x^4e^3a\,c^2d^2+24x^4d^4e\,c^3+45x^3d\,e^4a^2c+90x^3d^3e^2a\,c^2+15x^3d^5c^3+20x^2a^3e^5+120x^2d^2e^3a^2c+60x^2d^4ae\,c^2\right.}{60(ex+d)^3(cdx+ae)^3}$
default	$\frac{e^2d^3c^3x^6}{6}+\frac{\left(e^3a\,c^2d^2+2c^2d^2e\left(a\,e^2+c\,d^2\right)\right)x^5}{5}+\frac{\left(2a\,e^2dc\left(a\,e^2+c\,d^2\right)+cd\left(2ac\,d^2e^2+\left(a\,e^2+c\,d^2\right)^2\right)\right)x^4}{4}+\frac{\left(ae\left(2ac\,d^2e\right.\right.}{}$

input
```
int((a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^3/(e*x+d),x,method=_RETURNVERBOSE)
```

output
```
1/6*e^2*d^3*c^3*x^6+(3/5*e^3*a*c^2*d^2+2/5*d^4*e*c^3)*x^5+(3/4*d*e^4*a^2*c
+3/2*d^3*e^2*a*c^2+1/4*d^5*c^3)*x^4+(1/3*a^3*e^5+2*d^2*e^3*a^2*c+d^4*a*e*c
^2)*x^3+(d*e^4*a^3+3/2*d^3*e^2*a^2*c)*x^2+d^2*e^3*a^3*x
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 150, normalized size of antiderivative = 1.65

$$\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^3}{d+ex}\,dx = \frac{1}{6}\,c^3d^3e^2x^6+a^3d^2e^3x+\frac{1}{5}\left(2\,c^3d^4e+3\,ac^2d^2e^3\right)x^5 + \frac{1}{4}\left(c^3d^5+6\,ac^2d^3e^2+3\,a^2cde^4\right)x^4 + \frac{1}{3}\left(3\,ac^2d^4e+6\,a^2cd^2e^3+a^3e^5\right)x^3 + \frac{1}{2}\left(3\,a^2cd^3e^2+2\,a^3de^4\right)x^2$$

input
```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^3/(e*x+d),x, algorithm="fricas
")
```

output
```
1/6*c^3*d^3*e^2*x^6 + a^3*d^2*e^3*x + 1/5*(2*c^3*d^4*e + 3*a*c^2*d^2*e^3)*
x^5 + 1/4*(c^3*d^5 + 6*a*c^2*d^3*e^2 + 3*a^2*c*d*e^4)*x^4 + 1/3*(3*a*c^2*d
^4*e + 6*a^2*c*d^2*e^3 + a^3*e^5)*x^3 + 1/2*(3*a^2*c*d^3*e^2 + 2*a^3*d*e^4
)*x^2
```

Sympy [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 160, normalized size of antiderivative = 1.76

$$\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^3}{d+ex}\,dx = a^3d^2e^3x+\frac{c^3d^3e^2x^6}{6}+x^5\cdot\left(\frac{3ac^2d^2e^3}{5}+\frac{2c^3d^4e}{5}\right) + x^4\cdot\left(\frac{3a^2cde^4}{4}+\frac{3ac^2d^3e^2}{2}+\frac{c^3d^5}{4}\right) + x^3\left(\frac{a^3e^5}{3}+2a^2cd^2e^3+ac^2d^4e\right) + x^2\left(a^3de^4+\frac{3a^2cd^3e^2}{2}\right)$$

input
```
integrate((a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**3/(e*x+d),x)
```

output
```
a**3*d**2*e**3*x + c**3*d**3*e**2*x**6/6 + x**5*(3*a*c**2*d**2*e**3/5 + 2*
c**3*d**4*e/5) + x**4*(3*a**2*c*d*e**4/4 + 3*a*c**2*d**3*e**2/2 + c**3*d**
5/4) + x**3*(a**3*e**5/3 + 2*a**2*c*d**2*e**3 + a*c**2*d**4*e) + x**2*(a**
3*d*e**4 + 3*a**2*c*d**3*e**2/2)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 150, normalized size of antiderivative = 1.65

$$\int \frac{(ade+(cd^2+ae^2)x+cdex^2)^3}{d+ex}\,dx = \frac{1}{6}\,c^3d^3e^2x^6+a^3d^2e^3x+\frac{1}{5}\,(2\,c^3d^4e+3\,ac^2d^2e^3)x^5+\frac{1}{4}\,(c^3d^5+6\,ac^2d^3e^2+3\,a^2cde^4)x^4+\frac{1}{3}\,(3\,ac^2d^4e+6\,a^2cd^2e^3+a^3e^5)x^3+\frac{1}{2}\,(3\,a^2cd^3e^2+2\,a^3de^4)x^2$$

input
```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^3/(e*x+d),x, algorithm="maxima
")
```

output
```
1/6*c^3*d^3*e^2*x^6 + a^3*d^2*e^3*x + 1/5*(2*c^3*d^4*e + 3*a*c^2*d^2*e^3)*
x^5 + 1/4*(c^3*d^5 + 6*a*c^2*d^3*e^2 + 3*a^2*c*d*e^4)*x^4 + 1/3*(3*a*c^2*d
^4*e + 6*a^2*c*d^2*e^3 + a^3*e^5)*x^3 + 1/2*(3*a^2*c*d^3*e^2 + 2*a^3*d*e^4
)*x^2
```

Giac [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 156, normalized size of antiderivative = 1.71

$$\int \frac{(ade+(cd^2+ae^2)x+cdex^2)^3}{d+ex}\,dx = \frac{1}{6}\,c^3d^3e^2x^6+\frac{2}{5}\,c^3d^4ex^5+\frac{3}{5}\,ac^2d^2e^3x^5+\frac{1}{4}\,c^3d^5x^4+\frac{3}{2}\,ac^2d^3e^2x^4+\frac{3}{4}\,a^2cde^4x^4+ac^2d^4ex^3+2\,a^2cd^2e^3x^3+\frac{1}{3}\,a^3e^5x^3+\frac{3}{2}\,a^2cd^3e^2x^2+a^3de^4x^2+a^3d^2e^3x$$

input
```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^3/(e*x+d),x, algorithm="giac")
```

output
```
1/6*c^3*d^3*e^2*x^6 + 2/5*c^3*d^4*e*x^5 + 3/5*a*c^2*d^2*e^3*x^5 + 1/4*c^3*
d^5*x^4 + 3/2*a*c^2*d^3*e^2*x^4 + 3/4*a^2*c*d*e^4*x^4 + a*c^2*d^4*e*x^3 +
2*a^2*c*d^2*e^3*x^3 + 1/3*a^3*e^5*x^3 + 3/2*a^2*c*d^3*e^2*x^2 + a^3*d*e^4*
x^2 + a^3*d^2*e^3*x
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 145, normalized size of antiderivative = 1.59

$$\int \frac{(ade + (cd^2 + ae^2)\,x + cdex^2)^3}{d + ex}\,dx = x^3\left(\frac{a^3\,e^5}{3} + 2\,a^2\,c\,d^2\,e^3 + a\,c^2\,d^4\,e\right) + x^4\left(\frac{3\,a^2\,c\,d\,e^4}{4} + \frac{3\,a\,c^2\,d^3\,e^2}{2} + \frac{c^3\,d^5}{4}\right) + a^3\,d^2\,e^3\,x + \frac{c^3\,d^3\,e^2\,x^6}{6} + \frac{a^2\,d\,e^2\,x^2\,(3\,c\,d^2 + 2\,a\,e^2)}{2} + \frac{c^2\,d^2\,e\,x^5\,(2\,c\,d^2 + 3\,a\,e^2)}{5}$$

input
```
int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^3/(d + e*x),x)
```

output
```
x^3*((a^3*e^5)/3 + 2*a^2*c*d^2*e^3 + a*c^2*d^4*e) + x^4*((c^3*d^5)/4 + (3*
a*c^2*d^3*e^2)/2 + (3*a^2*c*d*e^4)/4) + a^3*d^2*e^3*x + (c^3*d^3*e^2*x^6)/
6 + (a^2*d*e^2*x^2*(2*a*e^2 + 3*c*d^2))/2 + (c^2*d^2*e*x^5*(3*a*e^2 + 2*c*
d^2))/5
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 157, normalized size of antiderivative = 1.73

$$\int \frac{(ade + (cd^2 + ae^2)\,x + cdex^2)^3}{d + ex}\,dx$$
$$= \frac{x(10c^3d^3e^2x^5 + 36a\,c^2d^2e^3x^4 + 24c^3d^4e\,x^4 + 45a^2cd\,e^4x^3 + 90a\,c^2d^3e^2x^3 + 15c^3d^5x^3 + 20a^3e^5x^2 + 120a^2}{60}$$

input
```
int((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^3/(e*x+d),x)
```

output
```
(x*(60*a**3*d**2*e**3 + 60*a**3*d*e**4*x + 20*a**3*e**5*x**2 + 90*a**2*c*d
**3*e**2*x + 120*a**2*c*d**2*e**3*x**2 + 45*a**2*c*d*e**4*x**3 + 60*a*c**2
*d**4*e*x**2 + 90*a*c**2*d**3*e**2*x**3 + 36*a*c**2*d**2*e**3*x**4 + 15*c*
*3*d**5*x**3 + 24*c**3*d**4*e*x**4 + 10*c**3*d**3*e**2*x**5))/60
```

3.97 $\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^3}{(d+ex)^2}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 54

$$\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^3}{(d+ex)^2}\,dx=\frac{\left(cd^2-ae^2\right)(ae+cdx)^4}{4c^2d^2}+\frac{e(ae+cdx)^5}{5c^2d^2}$$

output
```
1/4*(-a*e^2+c*d^2)*(c*d*x+a*e)^4/c^2/d^2+1/5*e*(c*d*x+a*e)^5/c^2/d^2
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.46

$$\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^3}{(d+ex)^2}\,dx=\frac{1}{20}x\left(10a^3e^3(2d+ex)+10a^2cde^2x(3d+2ex)+5ac^2d^2ex^2(4d+3ex)+c^3d^3x^3(5d+4ex)\right)$$

input
```
Integrate[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^3/(d + e*x)^2,x]
```

output
```
(x*(10*a^3*e^3*(2*d + e*x) + 10*a^2*c*d*e^2*x*(3*d + 2*e*x) + 5*a*c^2*d^2*
e*x^2*(4*d + 3*e*x) + c^3*d^3*x^3*(5*d + 4*e*x)))/20
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.057$, Rules used = {1121, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(x(ae^2+cd^2)+ade+cdex^2)^3}{(d+ex)^2}\,dx$$

$$\Big\downarrow \text{1121}$$

$$\int \left(\frac{(cd^2-ae^2)(ae+cdx)^3}{cd}+\frac{e(ae+cdx)^4}{cd}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{(cd^2-ae^2)(ae+cdx)^4}{4c^2d^2}+\frac{e(ae+cdx)^5}{5c^2d^2}$$

input
```
Int[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^3/(d + e*x)^2,x]
```

output
```
((c*d^2 - a*e^2)*(a*e + c*d*x)^4)/(4*c^2*d^2) + (e*(a*e + c*d*x)^5)/(5*c^2
*d^2)
```

Defintions of rubi rules used

rule 1121
```
Int[((d_) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Int[ExpandIntegrand[(d + e*x)^(m + p)*(a/d + (c/e)*x)^p, x], x]
/; FreeQ[{a, b, c, d, e, m, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && (Int
egerQ[p] || (GtQ[a, 0] && GtQ[d, 0] && LtQ[c, 0]))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 1.53 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.83

method	result
risch	$\frac{1}{5}e\,d^3c^3x^5+\frac{3}{4}x^4d^2e^2a\,c^2+\frac{1}{4}d^4c^3x^4+a^2cd\,e^3x^3+a\,c^2d^3e\,x^3+\frac{1}{2}x^2e^4a^3+\frac{3}{2}x^2d^2e^2a^2c+a^3d$
parallelrisch	$\frac{1}{5}e\,d^3c^3x^5+\frac{3}{4}x^4d^2e^2a\,c^2+\frac{1}{4}d^4c^3x^4+a^2cd\,e^3x^3+a\,c^2d^3e\,x^3+\frac{1}{2}x^2e^4a^3+\frac{3}{2}x^2d^2e^2a^2c+a^3d$
gosper	$\frac{x\left(4e\,d^3c^3x^4+15d^2e^2a\,c^2x^3+5d^4c^3x^3+20a^2cd\,e^3x^2+20a\,c^2d^3e\,x^2+10e^4a^3x+30d^2e^2a^2cx+20a^3d\,e^3\right)}{20}$
default	$\frac{e\,d^3c^3x^5}{5}+\frac{\left(2d^2e^2a\,c^2+c^2d^2\left(a\,e^2+c\,d^2\right)\right)x^4}{4}+\frac{\left(a^2cd\,e^3+2acde\left(a\,e^2+c\,d^2\right)+a\,c^2d^3e\right)x^3}{3}+\frac{\left(e^2a^2\left(a\,e^2+c\,d^2\right)+2d^2e^2a^2c\right.}{2}$
orering	$\frac{x\left(4e\,d^3c^3x^4+15d^2e^2a\,c^2x^3+5d^4c^3x^3+20a^2cd\,e^3x^2+20a\,c^2d^3e\,x^2+10e^4a^3x+30d^2e^2a^2cx+20a^3d\,e^3\right)\left(ade+\left(a\,e^2+c\,d^2\right)x+cd\right.}{20(ex+d)^3(cdx+ae)^3}$
norman	$\frac{\left(\frac{3}{2}d\,e^4a^3+\frac{3}{2}d^3e^2a^2c\right)x^2+\left(\frac{3}{4}e^3a\,c^2d^2+\frac{9}{20}d^4e\,c^3\right)x^5+\left(\frac{1}{2}a^3e^5+\frac{5}{2}d^2e^3a^2c+d^4ae\,c^2\right)x^3+\left(d\,e^4a^2c+\frac{7}{4}d^3e^2a\,c^2+\frac{1}{4}d^5c^3\right)x^4+d^2e}{ex+d}$

input
```
int((a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^3/(e*x+d)^2,x,method=_RETURNVERBOSE)
```

output
```
1/5*e*d^3*c^3*x^5+3/4*x^4*d^2*e^2*a*c^2+1/4*d^4*c^3*x^4+a^2*c*d*e^3*x^3+a*
c^2*d^3*e*x^3+1/2*x^2*e^4*a^3+3/2*x^2*d^2*e^2*a^2*c+a^3*d*e^3*x
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.76

$$\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^3}{(d+ex)^2}\,dx=\frac{1}{5}\,c^3d^3ex^5+a^3de^3x+\frac{1}{4}\,\left(c^3d^4+3\,ac^2d^2e^2\right)x^4+\left(ac^2d^3e+a^2cde^3\right)x^3+\frac{1}{2}\,\left(3\,a^2cd^2e^2+a^3e^4\right)x^2$$

input
```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^3/(e*x+d)^2,x, algorithm="fric
as")
```

output
```
1/5*c^3*d^3*e*x^5 + a^3*d*e^3*x + 1/4*(c^3*d^4 + 3*a*c^2*d^2*e^2)*x^4 + (a
*c^2*d^3*e + a^2*c*d*e^3)*x^3 + 1/2*(3*a^2*c*d^2*e^2 + a^3*e^4)*x^2
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 100 vs. $2(49) = 98$.

Time = 0.06 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.85

$$\int \frac{\left(ade+(cd^2+ae^2)\,x+cdex^2\right)^3}{(d+ex)^2}\,dx = a^3de^3x+\frac{c^3d^3ex^5}{5}+x^4\cdot\left(\frac{3ac^2d^2e^2}{4}+\frac{c^3d^4}{4}\right)+x^3\left(a^2cde^3+ac^2d^3e\right)+x^2\left(\frac{a^3e^4}{2}+\frac{3a^2cd^2e^2}{2}\right)$$

input
```
integrate((a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**3/(e*x+d)**2,x)
```

output
```
a**3*d*e**3*x + c**3*d**3*e*x**5/5 + x**4*(3*a*c**2*d**2*e**2/4 + c**3*d**
4/4) + x**3*(a**2*c*d*e**3 + a*c**2*d**3*e) + x**2*(a**3*e**4/2 + 3*a**2*c
*d**2*e**2/2)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.76

$$\int \frac{\left(ade+(cd^2+ae^2)\,x+cdex^2\right)^3}{(d+ex)^2}\,dx = \frac{1}{5}\,c^3d^3ex^5+a^3de^3x+\frac{1}{4}\,(c^3d^4+3\,ac^2d^2e^2)x^4+(ac^2d^3e+a^2cde^3)x^3+\frac{1}{2}\,(3\,a^2cd^2e^2+a^3e^4)x^2$$

input
```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^3/(e*x+d)^2,x, algorithm="maxi
ma")
```

output
```
1/5*c^3*d^3*e*x^5 + a^3*d*e^3*x + 1/4*(c^3*d^4 + 3*a*c^2*d^2*e^2)*x^4 + (a
*c^2*d^3*e + a^2*c*d*e^3)*x^3 + 1/2*(3*a^2*c*d^2*e^2 + a^3*e^4)*x^2
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 174 vs. $2(50) = 100$.

Time = 0.13 (sec) , antiderivative size = 174, normalized size of antiderivative = 3.22

$$\int \frac{(ade + (cd^2 + ae^2)x + cdex^2)^3}{(d+ex)^2}\,dx$$
$$= \frac{\left(4\,c^3d^3 - \frac{15\,c^3d^4}{ex+d} + \frac{20\,c^3d^5}{(ex+d)^2} - \frac{10\,c^3d^6}{(ex+d)^3} + \frac{15\,ac^2d^2e^2}{ex+d} - \frac{40\,ac^2d^3e^2}{(ex+d)^2} + \frac{30\,ac^2d^4e^2}{(ex+d)^3} + \frac{20\,a^2cde^4}{(ex+d)^2} - \frac{30\,a^2cd^2e^4}{(ex+d)^3} + \frac{10\,a^3e^6}{(ex+d)^3}\right)(e:}{20\,e^4}$$

input
```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^3/(e*x+d)^2,x, algorithm="giac
")
```

output
```
1/20*(4*c^3*d^3 - 15*c^3*d^4/(e*x + d) + 20*c^3*d^5/(e*x + d)^2 - 10*c^3*d
^6/(e*x + d)^3 + 15*a*c^2*d^2*e^2/(e*x + d) - 40*a*c^2*d^3*e^2/(e*x + d)^2
 + 30*a*c^2*d^4*e^2/(e*x + d)^3 + 20*a^2*c*d*e^4/(e*x + d)^2 - 30*a^2*c*d^
2*e^4/(e*x + d)^3 + 10*a^3*e^6/(e*x + d)^3)*(e*x + d)^5/e^4
```

Mupad [B] (verification not implemented)

Time = 5.48 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.69

$$\int \frac{(ade + (cd^2 + ae^2)x + cdex^2)^3}{(d+ex)^2}\,dx = x^2\left(\frac{a^3\,e^4}{2} + \frac{3\,c\,a^2\,d^2\,e^2}{2}\right) + x^4\left(\frac{c^3\,d^4}{4} + \frac{3\,a\,c^2\,d^2\,e^2}{4}\right) + \frac{c^3\,d^3\,e\,x^5}{5} + a^3\,d\,e^3\,x + a\,c\,d\,e\,x^3\,\left(c\,d^2 + a\,e^2\right)$$

input
```
int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^3/(d + e*x)^2,x)
```

output
```
x^2*((a^3*e^4)/2 + (3*a^2*c*d^2*e^2)/2) + x^4*((c^3*d^4)/4 + (3*a*c^2*d^2*
e^2)/4) + (c^3*d^3*e*x^5)/5 + a^3*d*e^3*x + a*c*d*e*x^3*(a*e^2 + c*d^2)
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.83

$$\int \frac{(ade + (cd^2 + ae^2)x + cdex^2)^3}{(d + ex)^2}\,dx$$
$$= \frac{x(4c^3d^3e\,x^4 + 15a\,c^2d^2e^2x^3 + 5c^3d^4x^3 + 20a^2cd\,e^3x^2 + 20a\,c^2d^3e\,x^2 + 10a^3e^4x + 30a^2c\,d^2e^2x + 20a^3d\,e^3)}{20}$$

input
```
int((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^3/(e*x+d)^2,x)
```

output
```
(x*(20*a**3*d*e**3 + 10*a**3*e**4*x + 30*a**2*c*d**2*e**2*x + 20*a**2*c*d*
e**3*x**2 + 20*a*c**2*d**3*e*x**2 + 15*a*c**2*d**2*e**2*x**3 + 5*c**3*d**4
*x**3 + 4*c**3*d**3*e*x**4))/20
```

3.98 $\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^3}{(d+ex)^3} dx$

Optimal result

Integrand size = 35, antiderivative size = 20

$$\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^3}{(d+ex)^3} dx = \frac{(ae+cdx)^4}{4cd}$$

output
```
1/4*(c*d*x+a*e)^4/c/d
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^3}{(d+ex)^3} dx = \frac{(ae+cdx)^4}{4cd}$$

input
```
Integrate[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^3/(d + e*x)^3,x]
```

output
```
(a*e + c*d*x)^4/(4*c*d)
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.057$, Rules used = {1120, 17}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(x(ae^2+cd^2)+ade+cdex^2)^3}{(d+ex)^3}dx$$

$$\Big\downarrow \text{1120}$$

$$\int (ae+cdx)^3dx$$

$$\Big\downarrow \text{17}$$

$$\frac{(ae+cdx)^4}{4cd}$$

input
```
Int[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^3/(d + e*x)^3,x]
```

output
```
(a*e + c*d*x)^4/(4*c*d)
```

Defintions of rubi rules used

rule 17
```
Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1
)/(b*(m + 1))), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]
```

rule 1120
```
Int[((d_) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Int[(d + e*x)^(m + p)*(a/d + (c/e)*x)^p, x] /; FreeQ[{a, b, c, d
, e, m}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[p] && (EqQ[m + p,
0] || EqQ[m + 2*p + 2, 0])
```

Maple [A] (verified)

Time = 1.41 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.95

method	result
default	$\frac{(cdx+ae)^4}{4cd}$
parallelrisch	$\frac{1}{4}c^3d^3x^4+a\,c^2d^2e\,x^3+\frac{3}{2}a^2cd\,e^2x^2+e^3a^3x$
gosper	$\frac{x\left(d^3c^3x^3+4a\,c^2d^2e\,x^2+6a^2cd\,e^2x+4e^3a^3\right)}{4}$
risch	$\frac{c^3d^3x^4}{4}+a\,c^2d^2e\,x^3+\frac{3a^2cd\,e^2x^2}{2}+e^3a^3x+\frac{a^4e^4}{4cd}$
orering	$\frac{x\left(d^3c^3x^3+4a\,c^2d^2e\,x^2+6a^2cd\,e^2x+4e^3a^3\right)\left(ade+\left(a\,e^2+c\,d^2\right)x+cd\,x^2e\right)^3}{4(cdx+ae)^3(ex+d)^3}$
norman	$\frac{\left(e^3a\,c^2d^2+\frac{1}{2}d^4e\,c^3\right)x^5+\left(\frac{3}{2}d\,e^4a^2c+2d^3e^2a\,c^2+\frac{1}{4}d^5c^3\right)x^4+\left(a^3e^5+3d^2e^3a^2c+d^4ae\,c^2\right)x^3-\frac{d^2\left(4d\,e^4a^3+3d^3e^2a^2c\right)}{2e^2}-\frac{d\left(3d\,e^4a^3\right.}{}}{(ex+d)^2}$

input
```
int((a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^3/(e*x+d)^3,x,method=_RETURNVERBOSE)
```

output
```
1/4*(c*d*x+a*e)^4/c/d
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 45 vs. 2(18) = 36.

Time = 0.08 (sec) , antiderivative size = 45, normalized size of antiderivative = 2.25

$$\int \frac{(ade+(cd^2+ae^2)\,x+cdex^2)^3}{(d+ex)^3}\,dx = \frac{1}{4}\,c^3d^3x^4+ac^2d^2ex^3+\frac{3}{2}\,a^2cde^2x^2+a^3e^3x$$

input
```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^3/(e*x+d)^3,x, algorithm="fric
as")
```

output
```
1/4*c^3*d^3*x^4 + a*c^2*d^2*e*x^3 + 3/2*a^2*c*d*e^2*x^2 + a^3*e^3*x
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 49 vs. $2(14) = 28$.

Time = 0.05 (sec) , antiderivative size = 49, normalized size of antiderivative = 2.45

$$\int \frac{(ade + (cd^2 + ae^2) x + cdex^2)^3}{(d + ex)^3} dx = a^3e^3x + \frac{3a^2cde^2x^2}{2} + ac^2d^2ex^3 + \frac{c^3d^3x^4}{4}$$

input
```
integrate((a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**3/(e*x+d)**3,x)
```

output
```
a**3*e**3*x + 3*a**2*c*d*e**2*x**2/2 + a*c**2*d**2*e*x**3 + c**3*d**3*x**4
/4
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 45 vs. $2(18) = 36$.

Time = 0.03 (sec) , antiderivative size = 45, normalized size of antiderivative = 2.25

$$\int \frac{(ade + (cd^2 + ae^2) x + cdex^2)^3}{(d + ex)^3} dx = \frac{1}{4} c^3d^3x^4 + ac^2d^2ex^3 + \frac{3}{2} a^2cde^2x^2 + a^3e^3x$$

input
```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^3/(e*x+d)^3,x, algorithm="maxi
ma")
```

output
```
1/4*c^3*d^3*x^4 + a*c^2*d^2*e*x^3 + 3/2*a^2*c*d*e^2*x^2 + a^3*e^3*x
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 45 vs. $2(18) = 36$.

Time = 0.11 (sec) , antiderivative size = 45, normalized size of antiderivative = 2.25

$$\int \frac{(ade + (cd^2 + ae^2)x + cdex^2)^3}{(d + ex)^3}\,dx = \frac{1}{4}c^3d^3x^4 + ac^2d^2ex^3 + \frac{3}{2}a^2cde^2x^2 + a^3e^3x$$

input
```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^3/(e*x+d)^3,x, algorithm="giac
")
```

output
```
1/4*c^3*d^3*x^4 + a*c^2*d^2*e*x^3 + 3/2*a^2*c*d*e^2*x^2 + a^3*e^3*x
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 45, normalized size of antiderivative = 2.25

$$\int \frac{(ade + (cd^2 + ae^2)x + cdex^2)^3}{(d + ex)^3}\,dx = a^3\,e^3\,x + \frac{3\,a^2\,c\,d\,e^2\,x^2}{2} + a\,c^2\,d^2\,e\,x^3 + \frac{c^3\,d^3\,x^4}{4}$$

input
```
int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^3/(d + e*x)^3,x)
```

output
```
a^3*e^3*x + (c^3*d^3*x^4)/4 + (3*a^2*c*d*e^2*x^2)/2 + a*c^2*d^2*e*x^3
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 46, normalized size of antiderivative = 2.30

$$\int \frac{(ade + (cd^2 + ae^2)x + cdex^2)^3}{(d + ex)^3}\,dx = \frac{x(c^3d^3x^3 + 4a\,c^2d^2e\,x^2 + 6a^2cd\,e^2x + 4a^3e^3)}{4}$$

input
```
int((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^3/(e*x+d)^3,x)
```

output
```
(x*(4*a**3*e**3 + 6*a**2*c*d*e**2*x + 4*a*c**2*d**2*e*x**2 + c**3*d**3*x**
3))/4
```

3.99 $\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^3}{(d+ex)^4}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 89

$$\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^3}{(d+ex)^4}\,dx=\frac{cd\left(cd^2-ae^2\right)^2x}{e^3}+\frac{1}{2}\left(a-\frac{cd^2}{e^2}\right)(ae+cdx)^2+\frac{(ae+cdx)^3}{3e}-\frac{\left(cd^2-ae^2\right)^3\log(d+ex)}{e^4}$$

output
```
c*d*(-a*e^2+c*d^2)^2*x/e^3+1/2*(a-c*d^2/e^2)*(c*d*x+a*e)^2+1/3*(c*d*x+a*e)
^3/e-(-a*e^2+c*d^2)^3*ln(e*x+d)/e^4
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.96

$$\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^3}{(d+ex)^4}\,dx$$
$$=\frac{cdex\left(18a^2e^4+9acde^2(-2d+ex)+c^2d^2\left(6d^2-3dex+2e^2x^2\right)\right)-6\left(cd^2-ae^2\right)^3\log(d+ex)}{6e^4}$$

input
```
Integrate[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^3/(d + e*x)^4,x]
```

output
```
(c*d*e*x*(18*a^2*e^4 + 9*a*c*d*e^2*(-2*d + e*x) + c^2*d^2*(6*d^2 - 3*d*e*x
 + 2*e^2*x^2)) - 6*(c*d^2 - a*e^2)^3*Log[d + e*x])/(6*e^4)
```

Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.057, Rules used = {1121, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^3}{(d+ex)^4}dx$$

$$\Big\downarrow \text{1121}$$

$$\int \left(-\frac{cd\left(cd^2-ae^2\right)(ae+cdx)}{e^2}+\frac{\left(ae^2-cd^2\right)^3}{e^3(d+ex)}+\frac{cd\left(cd^2-ae^2\right)^2}{e^3}+\frac{cd(ae+cdx)^2}{e}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{2}\left(a-\frac{cd^2}{e^2}\right)(ae+cdx)^2-\frac{\left(cd^2-ae^2\right)^3\log(d+ex)}{e^4}+\frac{cdx\left(cd^2-ae^2\right)^2}{e^3}+\frac{(ae+cdx)^3}{3e}$$

input
```
Int[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^3/(d + e*x)^4,x]
```

output
```
(c*d*(c*d^2 - a*e^2)^2*x)/e^3 + ((a - (c*d^2)/e^2)*(a*e + c*d*x)^2)/2 + (a
*e + c*d*x)^3/(3*e) - ((c*d^2 - a*e^2)^3*Log[d + e*x])/e^4
```

Defintions of rubi rules used

rule 1121
```
Int[((d_) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Int[ExpandIntegrand[(d + e*x)^(m + p)*(a/d + (c/e)*x)^p, x], x]
/; FreeQ[{a, b, c, d, e, m, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && (Int
egerQ[p] || (GtQ[a, 0] && GtQ[d, 0] && LtQ[c, 0]))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 1.37 (sec) , antiderivative size = 124, normalized size of antiderivative = 1.39

method	result
default	$\frac{cd\left(\frac{1}{3}x^3c^2d^2e^2+\frac{3}{2}x^2acd\,e^3-\frac{1}{2}x^2c^2d^3e+3a^2e^4x-3ac\,d^2e^2x+c^2d^4x\right)}{e^3}+\frac{\left(e^6a^3-3d^2e^4a^2c+3d^4e^2a\,c^2-d^6c^3\right)\ln(ex+d)}{e^4}$
risch	$\frac{c^3d^3x^3}{3e}+\frac{3a\,c^2d^2x^2}{2}-\frac{c^3d^4x^2}{2e^2}+3cde\,a^2x-\frac{3c^2d^3ax}{e}+\frac{c^3d^5x}{e^3}+e^2\ln\left(ex+d\right)a^3-3\ln\left(ex+d\right)d^2$
parallelrisch	$\frac{2c^3d^3e^3x^3+9x^2a\,c^2d^2e^4-3c^3d^4e^2x^2+6\ln(ex+d)a^3e^6-18\ln(ex+d)a^2c\,d^2e^4+18\ln(ex+d)a\,c^2d^4e^2-6\ln(ex+d)c^3d^6+18x\,a^2}{6e^4}$
norman	$\frac{\left(\frac{3}{2}e^3a\,c^2d^2+\frac{1}{2}d^4e\,c^3\right)x^5+\left(3d\,e^4a^2c+\frac{3}{2}d^3e^2a\,c^2+\frac{1}{2}d^5c^3\right)x^4-\frac{d^3\left(54d^2e^4a^2c-27d^4e^2a\,c^2+11d^6c^3\right)}{6e^4}-\frac{3d\left(6d^2e^4a^2c-2d^4e^2a\,c^2+d^6\right.}{e^2}}{(ex+d)^3}$

input
```
int((a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^3/(e*x+d)^4,x,method=_RETURNVERBOSE)
```

output
```
c*d/e^3*(1/3*x^3*c^2*d^2*e^2+3/2*x^2*a*c*d*e^3-1/2*x^2*c^2*d^3*e+3*a^2*e^4
*x-3*a*c*d^2*e^2*x+c^2*d^4*x)+(a^3*e^6-3*a^2*c*d^2*e^4+3*a*c^2*d^4*e^2-c^3
*d^6)/e^4*ln(e*x+d)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 130, normalized size of antiderivative = 1.46

$$\int \frac{(ade+(cd^2+ae^2)\,x+cdex^2)^3}{(d+ex)^4}\,dx$$
$$= \frac{2\,c^3d^3e^3x^3-3\,(c^3d^4e^2-3\,ac^2d^2e^4)x^2+6\,(c^3d^5e-3\,ac^2d^3e^3+3\,a^2cde^5)x-6\,(c^3d^6-3\,ac^2d^4e^2+3\,a^2cd}{6\,e^4}$$

input
```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^3/(e*x+d)^4,x, algorithm="fric
as")
```

output
```
1/6*(2*c^3*d^3*e^3*x^3 - 3*(c^3*d^4*e^2 - 3*a*c^2*d^2*e^4)*x^2 + 6*(c^3*d^
5*e - 3*a*c^2*d^3*e^3 + 3*a^2*c*d*e^5)*x - 6*(c^3*d^6 - 3*a*c^2*d^4*e^2 +
3*a^2*c*d^2*e^4 - a^3*e^6)*log(e*x + d))/e^4
```

Sympy [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.07

$$\int \frac{(ade+(cd^2+ae^2)\,x+cdex^2)^3}{(d+ex)^4}\,dx = \frac{c^3d^3x^3}{3e}+x^2\cdot\left(\frac{3ac^2d^2}{2}-\frac{c^3d^4}{2e^2}\right) + x\left(3a^2cde-\frac{3ac^2d^3}{e}+\frac{c^3d^5}{e^3}\right) + \frac{(ae^2-cd^2)^3\log(d+ex)}{e^4}$$

input
```
integrate((a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**3/(e*x+d)**4,x)
```

output
```
c**3*d**3*x**3/(3*e) + x**2*(3*a*c**2*d**2/2 - c**3*d**4/(2*e**2)) + x*(3*
a**2*c*d*e - 3*a*c**2*d**3/e + c**3*d**5/e**3) + (a*e**2 - c*d**2)**3*log(
d + e*x)/e**4
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 131, normalized size of antiderivative = 1.47

$$\int \frac{\left(ade + (cd^2 + ae^2)\,x + cdex^2\right)^3}{(d+ex)^4}\,dx = \frac{2\,c^3d^3e^2x^3 - 3\,(c^3d^4e - 3\,ac^2d^2e^3)x^2 + 6\,(c^3d^5 - 3\,ac^2d^3e^2 + 3\,a^2cde^4)x}{6\,e^3} - \frac{(c^3d^6 - 3\,ac^2d^4e^2 + 3\,a^2cd^2e^4 - a^3e^6)\log(ex+d)}{e^4}$$

input
```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^3/(e*x+d)^4,x, algorithm="maxi
ma")
```

output
```
1/6*(2*c^3*d^3*e^2*x^3 - 3*(c^3*d^4*e - 3*a*c^2*d^2*e^3)*x^2 + 6*(c^3*d^5
- 3*a*c^2*d^3*e^2 + 3*a^2*c*d*e^4)*x)/e^3 - (c^3*d^6 - 3*a*c^2*d^4*e^2 + 3
*a^2*c*d^2*e^4 - a^3*e^6)*log(e*x + d)/e^4
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 133, normalized size of antiderivative = 1.49

$$\int \frac{\left(ade + (cd^2 + ae^2)\,x + cdex^2\right)^3}{(d+ex)^4}\,dx = \frac{2\,c^3d^3e^2x^3 - 3\,c^3d^4ex^2 + 9\,ac^2d^2e^3x^2 + 6\,c^3d^5x - 18\,ac^2d^3e^2x + 18\,a^2cde^4x}{6\,e^3} - \frac{(c^3d^6 - 3\,ac^2d^4e^2 + 3\,a^2cd^2e^4 - a^3e^6)\log(|ex+d|)}{e^4}$$

input
```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^3/(e*x+d)^4,x, algorithm="giac
")
```

output
```
1/6*(2*c^3*d^3*e^2*x^3 - 3*c^3*d^4*e*x^2 + 9*a*c^2*d^2*e^3*x^2 + 6*c^3*d^5
*x - 18*a*c^2*d^3*e^2*x + 18*a^2*c*d*e^4*x)/e^3 - (c^3*d^6 - 3*a*c^2*d^4*e
^2 + 3*a^2*c*d^2*e^4 - a^3*e^6)*log(abs(e*x + d))/e^4
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 128, normalized size of antiderivative = 1.44

$$\int \frac{(ade + (cd^2 + ae^2)x + cdex^2)^3}{(d + ex)^4} dx$$
$$= x^2 \left(\frac{3\,a\,c^2\,d^2}{2} - \frac{c^3\,d^4}{2\,e^2}\right) - x \left(\frac{d\left(3\,a\,c^2\,d^2 - \frac{c^3\,d^4}{e^2}\right)}{e} - 3\,a^2\,c\,d\,e\right)$$
$$+ \frac{\ln(d + e\,x)\,\left(a^3\,e^6 - 3\,a^2\,c\,d^2\,e^4 + 3\,a\,c^2\,d^4\,e^2 - c^3\,d^6\right)}{e^4} + \frac{c^3\,d^3\,x^3}{3\,e}$$

input
```
int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^3/(d + e*x)^4,x)
```

output
```
x^2*((3*a*c^2*d^2)/2 - (c^3*d^4)/(2*e^2)) - x*((d*(3*a*c^2*d^2 - (c^3*d^4)
/e^2))/e - 3*a^2*c*d*e) + (log(d + e*x)*(a^3*e^6 - c^3*d^6 + 3*a*c^2*d^4*e
^2 - 3*a^2*c*d^2*e^4))/e^4 + (c^3*d^3*x^3)/(3*e)
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 147, normalized size of antiderivative = 1.65

$$\int \frac{(ade + (cd^2 + ae^2)x + cdex^2)^3}{(d + ex)^4} dx$$
$$= \frac{6\log(ex + d)\,a^3e^6 - 18\log(ex + d)\,a^2c\,d^2e^4 + 18\log(ex + d)\,a\,c^2d^4e^2 - 6\log(ex + d)\,c^3d^6 + 18a^2cd\,e^5x}{6e^4}$$

input
```
int((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^3/(e*x+d)^4,x)
```

output
```
(6*log(d + e*x)*a**3*e**6 - 18*log(d + e*x)*a**2*c*d**2*e**4 + 18*log(d +
e*x)*a*c**2*d**4*e**2 - 6*log(d + e*x)*c**3*d**6 + 18*a**2*c*d*e**5*x - 18
*a*c**2*d**3*e**3*x + 9*a*c**2*d**2*e**4*x**2 + 6*c**3*d**5*e*x - 3*c**3*d
**4*e**2*x**2 + 2*c**3*d**3*e**3*x**3)/(6*e**4)
```

3.100 $\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^3}{(d+ex)^5} \, dx$

Optimal result

Integrand size = 35, antiderivative size = 94

$$\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^3}{(d+ex)^5} \, dx = -\frac{c^2d^2\left(2cd^2-3ae^2\right)x}{e^3}+\frac{c^3d^3x^2}{2e^2}+\frac{\left(cd^2-ae^2\right)^3}{e^4(d+ex)}+\frac{3cd\left(cd^2-ae^2\right)^2\log(d+ex)}{e^4}$$

output
```
-c^2*d^2*(-3*a*e^2+2*c*d^2)*x/e^3+1/2*c^3*d^3*x^2/e^2+(-a*e^2+c*d^2)^3/e^4
/(e*x+d)+3*c*d*(-a*e^2+c*d^2)^2*ln(e*x+d)/e^4
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 129, normalized size of antiderivative = 1.37

$$\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^3}{(d+ex)^5} \, dx = \frac{6a^2cd^2e^4-2a^3e^6+6ac^2d^2e^2\left(-d^2+dex+e^2x^2\right)+c^3d^3\left(2d^3-4d^2ex-3de^2x^2+e^3x^3\right)+6cd\left(cd^2-ae^2\right)}{2e^4(d+ex)}$$

input
```
Integrate[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^3/(d + e*x)^5,x]
```

output
```
(6*a^2*c*d^2*e^4 - 2*a^3*e^6 + 6*a*c^2*d^2*e^2*(-d^2 + d*e*x + e^2*x^2) +
c^3*d^3*(2*d^3 - 4*d^2*e*x - 3*d*e^2*x^2 + e^3*x^3) + 6*c*d*(c*d^2 - a*e^2
)^2*(d + e*x)*Log[d + e*x])/(2*e^4*(d + e*x))
```

Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.057, Rules used = {1121, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^3}{(d+ex)^5}\,dx$$

$$\Big\downarrow \text{1121}$$

$$\int \left(-\frac{c^2d^2\left(2cd^2-3ae^2\right)}{e^3}+\frac{3cd\left(cd^2-ae^2\right)^2}{e^3(d+ex)}+\frac{\left(ae^2-cd^2\right)^3}{e^3(d+ex)^2}+\frac{c^3d^3x}{e^2}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{c^2d^2x\left(2cd^2-3ae^2\right)}{e^3}+\frac{\left(cd^2-ae^2\right)^3}{e^4(d+ex)}+\frac{3cd\left(cd^2-ae^2\right)^2\log(d+ex)}{e^4}+\frac{c^3d^3x^2}{2e^2}$$

input
```
Int[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^3/(d + e*x)^5,x]
```

output
```
-((c^2*d^2*(2*c*d^2 - 3*a*e^2)*x)/e^3) + (c^3*d^3*x^2)/(2*e^2) + (c*d^2 -
a*e^2)^3/(e^4*(d + e*x)) + (3*c*d*(c*d^2 - a*e^2)^2*Log[d + e*x])/e^4
```

Defintions of rubi rules used

rule 1121
```
Int[((d_) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Int[ExpandIntegrand[(d + e*x)^(m + p)*(a/d + (c/e)*x)^p, x], x]
/; FreeQ[{a, b, c, d, e, m, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && (Int
egerQ[p] || (GtQ[a, 0] && GtQ[d, 0] && LtQ[c, 0]))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 1.38 (sec) , antiderivative size = 125, normalized size of antiderivative = 1.33

method	result
default	$\frac{c^{2} d^{2} \left(\frac{1}{2} c d \,x^{2} e+3 a \,e^{2} x-2 c \,d^{2} x\right)}{e^{3}}+\frac{3 d c \left(a^{2} e^{4}-2 a c \,d^{2} e^{2}+c^{2} d^{4}\right) \ln \left(e x+d\right)}{e^{4}}-\frac{e^{6} a^{3}-3 d^{2} e^{4} a^{2} c+3 d^{4} e^{2} a \,c^{2}-d^{6} c^{3}}{e^{4} \left(e x+d\right)}$
risch	$\frac{c^{3} d^{3} x^{2}}{2 e^{2}}+\frac{3 c^{2} d^{2} a x}{e}-\frac{2 c^{3} d^{4} x}{e^{3}}+3 d c \ln \left(e x+d\right) a^{2}-\frac{6 d^{3} c^{2} \ln \left(e x+d\right) a}{e^{2}}+\frac{3 d^{5} c^{3} \ln \left(e x+d\right)}{e^{4}}-\frac{e^{2} a^{3}}{e x+d}+\frac{3 d^{2} a^{2} c}{e x+d}$
parallelrisch	$\frac{c^{3} d^{3} e^{3} x^{3}+6 \ln \left(e x+d\right) x \,a^{2} c d \,e^{5}-12 \ln \left(e x+d\right) x a \,c^{2} d^{3} e^{3}+6 \ln \left(e x+d\right) x \,c^{3} d^{5} e+6 x^{2} a \,c^{2} d^{2} e^{4}-3 c^{3} d^{4} e^{2} x^{2}+6 \ln \left(e x+d\right) a^{2} c \,d^{2} e^{4}-}{2 e^{4} \left(e x+d\right)}$
norman	$\frac{-\frac{d^{3} \left(e^{6} a^{3}-3 d^{2} e^{4} a^{2} c+15 d^{4} e^{2} a \,c^{2}-6 d^{6} c^{3}\right)}{e^{4}}-\frac{\left(e^{6} a^{3}-3 d^{2} e^{4} a^{2} c+33 d^{4} e^{2} a \,c^{2}-11 d^{6} c^{3}\right) x^{3}}{e}-\frac{3 d \left(2 e^{6} a^{3}-6 d^{2} e^{4} a^{2} c+46 d^{4} e^{2} a \,c^{2}-17 d^{6} c^{3}\right) x}{2 e^{2}}}{\left(e x+d\right)^{4}}$

input
```
int((a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^3/(e*x+d)^5,x,method=_RETURNVERBOSE)
```

output
```
c^2*d^2/e^3*(1/2*c*d*x^2*e+3*a*e^2*x-2*c*d^2*x)+3*d/e^4*c*(a^2*e^4-2*a*c*d
^2*e^2+c^2*d^4)*ln(e*x+d)-(a^3*e^6-3*a^2*c*d^2*e^4+3*a*c^2*d^4*e^2-c^3*d^6
)/e^4/(e*x+d)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 193 vs. $2(92) = 184$.

Time = 0.09 (sec) , antiderivative size = 193, normalized size of antiderivative = 2.05

$$\int \frac{\left(ade + (cd^2 + ae^2)x + cdex^2\right)^3}{(d + ex)^5}\,dx$$
$$= \frac{c^3d^3e^3x^3 + 2\,c^3d^6 - 6\,ac^2d^4e^2 + 6\,a^2cd^2e^4 - 2\,a^3e^6 - 3\,(c^3d^4e^2 - 2\,ac^2d^2e^4)x^2 - 2\,(2\,c^3d^5e - 3\,ac^2d^3e^3)x}{2\,(e^5x + de^4)}$$

input
```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^3/(e*x+d)^5,x, algorithm="fric
as")
```

output
```
1/2*(c^3*d^3*e^3*x^3 + 2*c^3*d^6 - 6*a*c^2*d^4*e^2 + 6*a^2*c*d^2*e^4 - 2*a
^3*e^6 - 3*(c^3*d^4*e^2 - 2*a*c^2*d^2*e^4)*x^2 - 2*(2*c^3*d^5*e - 3*a*c^2*
d^3*e^3)*x + 6*(c^3*d^6 - 2*a*c^2*d^4*e^2 + a^2*c*d^2*e^4 + (c^3*d^5*e - 2
*a*c^2*d^3*e^3 + a^2*c*d*e^5)*x)*log(e*x + d))/(e^5*x + d*e^4)
```

Sympy [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.24

$$\int \frac{\left(ade + (cd^2 + ae^2)x + cdex^2\right)^3}{(d + ex)^5}\,dx = \frac{c^3d^3x^2}{2e^2} + \frac{3cd(ae^2 - cd^2)^2\log(d + ex)}{e^4} + x\left(\frac{3ac^2d^2}{e} - \frac{2c^3d^4}{e^3}\right) + \frac{-a^3e^6 + 3a^2cd^2e^4 - 3ac^2d^4e^2 + c^3d^6}{de^4 + e^5x}$$

input
```
integrate((a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**3/(e*x+d)**5,x)
```

output
```
c**3*d**3*x**2/(2*e**2) + 3*c*d*(a*e**2 - c*d**2)**2*log(d + e*x)/e**4 + x
*(3*a*c**2*d**2/e - 2*c**3*d**4/e**3) + (-a**3*e**6 + 3*a**2*c*d**2*e**4 -
 3*a*c**2*d**4*e**2 + c**3*d**6)/(d*e**4 + e**5*x)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 136, normalized size of antiderivative = 1.45

$$\int \frac{(ade + (cd^2 + ae^2)x + cdex^2)^3}{(d + ex)^5}\,dx = \frac{c^3d^6 - 3\,ac^2d^4e^2 + 3\,a^2cd^2e^4 - a^3e^6}{e^5x + de^4} + \frac{c^3d^3ex^2 - 2\,(2\,c^3d^4 - 3\,ac^2d^2e^2)x}{2\,e^3} + \frac{3\,(c^3d^5 - 2\,ac^2d^3e^2 + a^2cde^4)\log{(ex + d)}}{e^4}$$

input
```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^3/(e*x+d)^5,x, algorithm="maxi
ma")
```

output
```
(c^3*d^6 - 3*a*c^2*d^4*e^2 + 3*a^2*c*d^2*e^4 - a^3*e^6)/(e^5*x + d*e^4) +
1/2*(c^3*d^3*e*x^2 - 2*(2*c^3*d^4 - 3*a*c^2*d^2*e^2)*x)/e^3 + 3*(c^3*d^5 -
 2*a*c^2*d^3*e^2 + a^2*c*d*e^4)*log(e*x + d)/e^4
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 182, normalized size of antiderivative = 1.94

$$\int \frac{(ade + (cd^2 + ae^2)x + cdex^2)^3}{(d + ex)^5}\,dx = \frac{\left(c^3d^3 - \frac{6\,(c^3d^4e - ac^2d^2e^3)}{(ex+d)e}\right)(ex + d)^2}{2\,e^4} - \frac{3\,(c^3d^5 - 2\,ac^2d^3e^2 + a^2cde^4)\log\left(\frac{|ex+d|}{(ex+d)^2|e|}\right)}{e^4} + \frac{\frac{c^3d^6e^2}{ex+d} - \frac{3\,ac^2d^4e^4}{ex+d} + \frac{3\,a^2cd^2e^6}{ex+d} - \frac{a^3e^8}{ex+d}}{e^6}$$

input
```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^3/(e*x+d)^5,x, algorithm="giac
")
```

output
```
1/2*(c^3*d^3 - 6*(c^3*d^4*e - a*c^2*d^2*e^3)/((e*x + d)*e))*(e*x + d)^2/e^
4 - 3*(c^3*d^5 - 2*a*c^2*d^3*e^2 + a^2*c*d*e^4)*log(abs(e*x + d)/((e*x + d
)^2*abs(e)))/e^4 + (c^3*d^6*e^2/(e*x + d) - 3*a*c^2*d^4*e^4/(e*x + d) + 3*
a^2*c*d^2*e^6/(e*x + d) - a^3*e^8/(e*x + d))/e^6
```

Mupad [B] (verification not implemented)

Time = 5.42 (sec) , antiderivative size = 141, normalized size of antiderivative = 1.50

$$\int \frac{(ade + (cd^2 + ae^2)x + cdex^2)^3}{(d + ex)^5} dx = \frac{\ln(d + e\,x)\,(3\,a^2\,c\,d\,e^4 - 6\,a\,c^2\,d^3\,e^2 + 3\,c^3\,d^5)}{e^4} - \frac{a^3\,e^6 - 3\,a^2\,c\,d^2\,e^4 + 3\,a\,c^2\,d^4\,e^2 - c^3\,d^6}{e\,(x\,e^4 + d\,e^3)} - x\left(\frac{2\,c^3\,d^4}{e^3} - \frac{3\,a\,c^2\,d^2}{e}\right) + \frac{c^3\,d^3\,x^2}{2\,e^2}$$

input
```
int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^3/(d + e*x)^5,x)
```

output
```
(log(d + e*x)*(3*c^3*d^5 - 6*a*c^2*d^3*e^2 + 3*a^2*c*d*e^4))/e^4 - (a^3*e^
6 - c^3*d^6 + 3*a*c^2*d^4*e^2 - 3*a^2*c*d^2*e^4)/(e*(d*e^3 + e^4*x)) - x*(
(2*c^3*d^4)/e^3 - (3*a*c^2*d^2)/e) + (c^3*d^3*x^2)/(2*e^2)
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 207, normalized size of antiderivative = 2.20

$$\int \frac{(ade + (cd^2 + ae^2)x + cdex^2)^3}{(d + ex)^5} dx = \frac{6\log(ex + d)\,a^2 c\,d^3 e^4 + 6\log(ex + d)\,a^2 c\,d^2 e^5 x - 12\log(ex + d)\,a\,c^2 d^5 e^2 - 12\log(ex + d)\,a\,c^2 d^4 e^3 x + 6\,\mathrm{l}}{}$$

input
```
int((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^3/(e*x+d)^5,x)
```

output
```
(6*log(d + e*x)*a**2*c*d**3*e**4 + 6*log(d + e*x)*a**2*c*d**2*e**5*x - 12*
log(d + e*x)*a*c**2*d**5*e**2 - 12*log(d + e*x)*a*c**2*d**4*e**3*x + 6*log
(d + e*x)*c**3*d**7 + 6*log(d + e*x)*c**3*d**6*e*x + 2*a**3*e**7*x - 6*a**
2*c*d**2*e**5*x + 12*a*c**2*d**4*e**3*x + 6*a*c**2*d**3*e**4*x**2 - 6*c**3
*d**6*e*x - 3*c**3*d**5*e**2*x**2 + c**3*d**4*e**3*x**3)/(2*d*e**4*(d + e*
x))
```

3.101 $\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^3}{(d+ex)^6}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 97

$$\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^3}{(d+ex)^6}\,dx = \frac{c^3d^3x}{e^3}+\frac{\left(cd^2-ae^2\right)^3}{2e^4(d+ex)^2}-\frac{3cd\left(cd^2-ae^2\right)^2}{e^4(d+ex)} - \frac{3c^2d^2\left(cd^2-ae^2\right)\log(d+ex)}{e^4}$$

output
```
c^3*d^3*x/e^3+1/2*(-a*e^2+c*d^2)^3/e^4/(e*x+d)^2-3*c*d*(-a*e^2+c*d^2)^2/e^
4/(e*x+d)-3*c^2*d^2*(-a*e^2+c*d^2)*ln(e*x+d)/e^4
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 129, normalized size of antiderivative = 1.33

$$\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^3}{(d+ex)^6}\,dx$$
$$= \frac{-a^3e^6-3a^2cde^4(d+2ex)+3ac^2d^3e^2(3d+4ex)+c^3d^3\left(-5d^3-4d^2ex+4de^2x^2+2e^3x^3\right)-6c^2d^2\left(cd^2-\right.}{2e^4(d+ex)^2}$$

input
```
Integrate[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^3/(d + e*x)^6,x]
```

output
```
(-(a^3*e^6) - 3*a^2*c*d*e^4*(d + 2*e*x) + 3*a*c^2*d^3*e^2*(3*d + 4*e*x) +
c^3*d^3*(-5*d^3 - 4*d^2*e*x + 4*d*e^2*x^2 + 2*e^3*x^3) - 6*c^2*d^2*(c*d^2
- a*e^2)*(d + e*x)^2*Log[d + e*x])/(2*e^4*(d + e*x)^2)
```

Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.057$, Rules used = {1121, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^3}{(d+ex)^6}\,dx$$

$$\Big\downarrow \text{1121}$$

$$\int \left(-\frac{3c^2d^2\left(cd^2-ae^2\right)}{e^3(d+ex)}+\frac{3cd\left(cd^2-ae^2\right)^2}{e^3(d+ex)^2}+\frac{\left(ae^2-cd^2\right)^3}{e^3(d+ex)^3}+\frac{c^3d^3}{e^3}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{3c^2d^2\left(cd^2-ae^2\right)\log(d+ex)}{e^4}-\frac{3cd\left(cd^2-ae^2\right)^2}{e^4(d+ex)}+\frac{\left(cd^2-ae^2\right)^3}{2e^4(d+ex)^2}+\frac{c^3d^3x}{e^3}$$

input
```
Int[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^3/(d + e*x)^6,x]
```

output
```
(c^3*d^3*x)/e^3 + (c*d^2 - a*e^2)^3/(2*e^4*(d + e*x)^2) - (3*c*d*(c*d^2 -
a*e^2)^2)/(e^4*(d + e*x)) - (3*c^2*d^2*(c*d^2 - a*e^2)*Log[d + e*x])/e^4
```

Defintions of rubi rules used

rule 1121
```
Int[((d_) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Int[ExpandIntegrand[(d + e*x)^(m + p)*(a/d + (c/e)*x)^p, x], x]
/; FreeQ[{a, b, c, d, e, m, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && (Int
egerQ[p] || (GtQ[a, 0] && GtQ[d, 0] && LtQ[c, 0]))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 1.36 (sec) , antiderivative size = 133, normalized size of antiderivative = 1.37

method	result
default	$\frac{c^3d^3x}{e^3}+\frac{3c^2d^2\left(a\,e^2-c\,d^2\right)\ln\left(ex+d\right)}{e^4}-\frac{3dc\left(a^2e^4-2ac\,d^2e^2+c^2d^4\right)}{e^4\left(ex+d\right)}-\frac{e^6a^3-3d^2e^4a^2c+3d^4e^2a\,c^2-d^6c^3}{2e^4\left(ex+d\right)^2}$
risch	$\frac{c^3d^3x}{e^3}+\frac{\left(-3d\,e^4a^2c+6d^3e^2a\,c^2-3d^5c^3\right)x-\frac{e^6a^3+3d^2e^4a^2c-9d^4e^2a\,c^2+5d^6c^3}{2e}}{e^3\left(ex+d\right)^2}+\frac{3c^2d^2\ln\left(ex+d\right)a}{e^2}-\frac{3c^3d^4\ln\left(ex+d\right)}{e^4}$
parallelrisch	$\frac{6\ln\left(ex+d\right)x^2a\,c^2d^2e^4-6\ln\left(ex+d\right)x^2c^3d^4e^2+2c^3d^3e^3x^3+12\ln\left(ex+d\right)xa\,c^2d^3e^3-12\ln\left(ex+d\right)x\,c^3d^5e+6\ln\left(ex+d\right)a\,c^2d^4e^2-}{2e^4\left(ex+d\right)^2}$
norman	$\frac{e^2d^3c^3x^6-\frac{d^3\left(a^3e^7+3a^2e^5c\,d^2-9a\,e^3c^2d^4+15d^6c^3e\right)}{2e^5}-\frac{\left(a^3e^7+21a^2e^5c\,d^2-45a\,e^3c^2d^4+103d^6c^3e\right)x^3}{2e^2}-\frac{d\left(3e^5c\,a^2-6e^3a\,c^2d^2+18d^4e\right.}{e}}{\left(ex+d\right)^5}$

input
```
int((a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^3/(e*x+d)^6,x,method=_RETURNVERBOSE)
```

output
```
c^3*d^3*x/e^3+3/e^4*c^2*d^2*(a*e^2-c*d^2)*ln(e*x+d)-3*d/e^4*c*(a^2*e^4-2*a
*c*d^2*e^2+c^2*d^4)/(e*x+d)-1/2*(a^3*e^6-3*a^2*c*d^2*e^4+3*a*c^2*d^4*e^2-c
^3*d^6)/e^4/(e*x+d)^2
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 209 vs. $2(95) = 190$.

Time = 0.08 (sec) , antiderivative size = 209, normalized size of antiderivative = 2.15

$$\int \frac{(ade + (cd^2 + ae^2)x + cdex^2)^3}{(d + ex)^6}\,dx$$
$$= \frac{2\,c^3d^3e^3x^3 + 4\,c^3d^4e^2x^2 - 5\,c^3d^6 + 9\,ac^2d^4e^2 - 3\,a^2cd^2e^4 - a^3e^6 - 2\,(2\,c^3d^5e - 6\,ac^2d^3e^3 + 3\,a^2cde^5)x - \cdots}{2\,(e^6x^2 + 2\,de^5x + d^2e^4)}$$

input
```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^3/(e*x+d)^6,x, algorithm="fric
as")
```

output
```
1/2*(2*c^3*d^3*e^3*x^3 + 4*c^3*d^4*e^2*x^2 - 5*c^3*d^6 + 9*a*c^2*d^4*e^2 -
 3*a^2*c*d^2*e^4 - a^3*e^6 - 2*(2*c^3*d^5*e - 6*a*c^2*d^3*e^3 + 3*a^2*c*d*
e^5)*x - 6*(c^3*d^6 - a*c^2*d^4*e^2 + (c^3*d^4*e^2 - a*c^2*d^2*e^4)*x^2 +
2*(c^3*d^5*e - a*c^2*d^3*e^3)*x)*log(e*x + d))/(e^6*x^2 + 2*d*e^5*x + d^2*
e^4)
```

Sympy [A] (verification not implemented)

Time = 0.52 (sec) , antiderivative size = 144, normalized size of antiderivative = 1.48

$$\int \frac{(ade + (cd^2 + ae^2)x + cdex^2)^3}{(d + ex)^6}\,dx$$
$$= \frac{c^3d^3x}{e^3} + \frac{3c^2d^2(ae^2 - cd^2)\log(d + ex)}{e^4}$$
$$+ \frac{-a^3e^6 - 3a^2cd^2e^4 + 9ac^2d^4e^2 - 5c^3d^6 + x(-6a^2cde^5 + 12ac^2d^3e^3 - 6c^3d^5e)}{2d^2e^4 + 4de^5x + 2e^6x^2}$$

input
```
integrate((a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**3/(e*x+d)**6,x)
```

output
```
c**3*d**3*x/e**3 + 3*c**2*d**2*(a*e**2 - c*d**2)*log(d + e*x)/e**4 + (-a**
3*e**6 - 3*a**2*c*d**2*e**4 + 9*a*c**2*d**4*e**2 - 5*c**3*d**6 + x*(-6*a**
2*c*d*e**5 + 12*a*c**2*d**3*e**3 - 6*c**3*d**5*e))/(2*d**2*e**4 + 4*d*e**5
*x + 2*e**6*x**2)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 142, normalized size of antiderivative = 1.46

$$\int \frac{(ade+(cd^2+ae^2)x+cdex^2)^3}{(d+ex)^6}\,dx = \frac{c^3d^3x}{e^3} - \frac{5\,c^3d^6-9\,ac^2d^4e^2+3\,a^2cd^2e^4+a^3e^6+6\,(c^3d^5e-2\,ac^2d^3e^3+a^2cde^5)x}{2\,(e^6x^2+2\,de^5x+d^2e^4)} - \frac{3\,(c^3d^4-ac^2d^2e^2)\log{(ex+d)}}{e^4}$$

input
```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^3/(e*x+d)^6,x, algorithm="maxi
ma")
```

output
```
c^3*d^3*x/e^3 - 1/2*(5*c^3*d^6 - 9*a*c^2*d^4*e^2 + 3*a^2*c*d^2*e^4 + a^3*e
^6 + 6*(c^3*d^5*e - 2*a*c^2*d^3*e^3 + a^2*c*d*e^5)*x)/(e^6*x^2 + 2*d*e^5*x
 + d^2*e^4) - 3*(c^3*d^4 - a*c^2*d^2*e^2)*log(e*x + d)/e^4
```

Giac [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 129, normalized size of antiderivative = 1.33

$$\int \frac{(ade+(cd^2+ae^2)x+cdex^2)^3}{(d+ex)^6}\,dx = \frac{c^3d^3x}{e^3} - \frac{3\,(c^3d^4-ac^2d^2e^2)\log{(|ex+d|)}}{e^4} - \frac{5\,c^3d^6-9\,ac^2d^4e^2+3\,a^2cd^2e^4+a^3e^6+6\,(c^3d^5e-2\,ac^2d^3e^3+a^2cde^5)x}{2\,(ex+d)^2e^4}$$

input
```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^3/(e*x+d)^6,x, algorithm="giac
")
```

output
```
c^3*d^3*x/e^3 - 3*(c^3*d^4 - a*c^2*d^2*e^2)*log(abs(e*x + d))/e^4 - 1/2*(5
*c^3*d^6 - 9*a*c^2*d^4*e^2 + 3*a^2*c*d^2*e^4 + a^3*e^6 + 6*(c^3*d^5*e - 2*
a*c^2*d^3*e^3 + a^2*c*d*e^5)*x)/((e*x + d)^2*e^4)
```

Mupad [B] (verification not implemented)

Time = 5.40 (sec) , antiderivative size = 149, normalized size of antiderivative = 1.54

$$\int \frac{(ade + (cd^2 + ae^2)x + cdex^2)^3}{(d + ex)^6} dx$$
$$= \frac{c^3 d^3 x}{e^3} - \frac{\ln(d + e x)\ (3 c^3 d^4 - 3 a c^2 d^2 e^2)}{e^4}$$
$$- \frac{\frac{a^3 e^6 + 3 a^2 c d^2 e^4 - 9 a c^2 d^4 e^2 + 5 c^3 d^6}{2 e} + x (3 a^2 c d e^4 - 6 a c^2 d^3 e^2 + 3 c^3 d^5)}{d^2 e^3 + 2 d e^4 x + e^5 x^2}$$

input
```
int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^3/(d + e*x)^6,x)
```

output
```
(c^3*d^3*x)/e^3 - (log(d + e*x)*(3*c^3*d^4 - 3*a*c^2*d^2*e^2))/e^4 - ((a^3
*e^6 + 5*c^3*d^6 - 9*a*c^2*d^4*e^2 + 3*a^2*c*d^2*e^4)/(2*e) + x*(3*c^3*d^5
 - 6*a*c^2*d^3*e^2 + 3*a^2*c*d*e^4))/(d^2*e^3 + e^5*x^2 + 2*d*e^4*x)
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 215, normalized size of antiderivative = 2.22

$$\int \frac{(ade + (cd^2 + ae^2)x + cdex^2)^3}{(d + ex)^6} dx$$
$$= \frac{6 \log(ex + d)\, a c^2 d^4 e^2 + 12 \log(ex + d)\, a c^2 d^3 e^3 x + 6 \log(ex + d)\, a c^2 d^2 e^4 x^2 - 6 \log(ex + d)\, c^3 d^6 - 12 \log}{2e^4}$$

input
```
int((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^3/(e*x+d)^6,x)
```

output
```
(6*log(d + e*x)*a*c**2*d**4*e**2 + 12*log(d + e*x)*a*c**2*d**3*e**3*x + 6*
log(d + e*x)*a*c**2*d**2*e**4*x**2 - 6*log(d + e*x)*c**3*d**6 - 12*log(d +
 e*x)*c**3*d**5*e*x - 6*log(d + e*x)*c**3*d**4*e**2*x**2 - a**3*e**6 + 3*a
**2*c*e**6*x**2 + 3*a*c**2*d**4*e**2 - 6*a*c**2*d**2*e**4*x**2 - 3*c**3*d*
*6 + 6*c**3*d**4*e**2*x**2 + 2*c**3*d**3*e**3*x**3)/(2*e**4*(d**2 + 2*d*e*
x + e**2*x**2))
```

3.102 $\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^3}{(d+ex)^7}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 105

$$\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^3}{(d+ex)^7}\,dx=\frac{\left(cd^2-ae^2\right)^3}{3e^4(d+ex)^3}-\frac{3cd\left(cd^2-ae^2\right)^2}{2e^4(d+ex)^2}+\frac{3c^2d^2\left(cd^2-ae^2\right)}{e^4(d+ex)}+\frac{c^3d^3\log(d+ex)}{e^4}$$

output
```
1/3*(-a*e^2+c*d^2)^3/e^4/(e*x+d)^3-3/2*c*d*(-a*e^2+c*d^2)^2/e^4/(e*x+d)^2+
3*c^2*d^2*(-a*e^2+c*d^2)/e^4/(e*x+d)+c^3*d^3*ln(e*x+d)/e^4
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.88

$$\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^3}{(d+ex)^7}\,dx=\frac{\frac{\left(cd^2-ae^2\right)\left(2a^2e^4+acde^2(5d+9ex)+c^2d^2\left(11d^2+27dex+18e^2x^2\right)\right)}{(d+ex)^3}+6c^3d^3\log(d+ex)}{6e^4}$$

input
```
Integrate[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^3/(d + e*x)^7,x]
```

output
```
(((c*d^2 - a*e^2)*(2*a^2*e^4 + a*c*d*e^2*(5*d + 9*e*x) + c^2*d^2*(11*d^2 +
 27*d*e*x + 18*e^2*x^2)))/(d + e*x)^3 + 6*c^3*d^3*Log[d + e*x])/(6*e^4)
```

Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.057$, Rules used = {1121, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^3}{(d+ex)^7}dx$$

$$\Big\downarrow \text{ 1121}$$

$$\int \left(-\frac{3c^2d^2\left(cd^2-ae^2\right)}{e^3(d+ex)^2}+\frac{3cd\left(cd^2-ae^2\right)^2}{e^3(d+ex)^3}+\frac{\left(ae^2-cd^2\right)^3}{e^3(d+ex)^4}+\frac{c^3d^3}{e^3(d+ex)}\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{3c^2d^2\left(cd^2-ae^2\right)}{e^4(d+ex)}-\frac{3cd\left(cd^2-ae^2\right)^2}{2e^4(d+ex)^2}+\frac{\left(cd^2-ae^2\right)^3}{3e^4(d+ex)^3}+\frac{c^3d^3\log(d+ex)}{e^4}$$

input
```
Int[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^3/(d + e*x)^7,x]
```

output
```
(c*d^2 - a*e^2)^3/(3*e^4*(d + e*x)^3) - (3*c*d*(c*d^2 - a*e^2)^2)/(2*e^4*(
d + e*x)^2) + (3*c^2*d^2*(c*d^2 - a*e^2))/(e^4*(d + e*x)) + (c^3*d^3*Log[d
 + e*x])/e^4
```

Defintions of rubi rules used

rule 1121
```
Int[((d_) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Int[ExpandIntegrand[(d + e*x)^(m + p)*(a/d + (c/e)*x)^p, x], x]
/; FreeQ[{a, b, c, d, e, m, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && (Int
egerQ[p] || (GtQ[a, 0] && GtQ[d, 0] && LtQ[c, 0]))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 1.32 (sec) , antiderivative size = 133, normalized size of antiderivative = 1.27

method	result
risch	$\frac{-\frac{3c^2d^2\left(a e^2-c d^2\right)x^2}{e^2}-\frac{3cd\left(a^2e^4+2ac d^2e^2-3c^2d^4\right)x}{2e^3}-\frac{2e^6a^3+3d^2e^4a^2c+6d^4e^2a c^2-11d^6c^3}{6e^4}}{(ex+d)^3}+\frac{c^3d^3\ln(ex+d)}{e^4}$
default	$-\frac{e^6a^3-3d^2e^4a^2c+3d^4e^2a c^2-d^6c^3}{3e^4(ex+d)^3}+\frac{c^3d^3\ln(ex+d)}{e^4}-\frac{3c^2d^2(a e^2-c d^2)}{e^4(ex+d)}-\frac{3dc(a^2e^4-2ac d^2e^2+c^2d^4)}{2e^4(ex+d)^2}$
parallelrisch	$\frac{6\ln(ex+d)x^3c^3d^3e^3+18\ln(ex+d)x^2c^3d^4e^2+18\ln(ex+d)x c^3d^5e-18x^2a c^2d^2e^4+18c^3d^4e^2x^2+6\ln(ex+d)c^3d^6-9x a^2cd e^5-\cdots}{6e^4(ex+d)^3}$
norman	$\frac{-\frac{d^3\left(2a^3e^8+3a^2c d^2e^6+6a c^2d^4e^4-11c^3d^6e^2\right)}{6e^6}-\frac{\left(a^3e^8+15a^2c d^2e^6+57a c^2d^4e^4-73c^3d^6e^2\right)x^3}{3e^3}-\frac{3d\left(a c^2d e^4-e^2d^3c^3\right)x^5}{e}-\frac{3d\left(a^2c e^6\cdots\right.}{\cdots}}{(ex+d)^6}$

input
```
int((a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^3/(e*x+d)^7,x,method=_RETURNVERBOSE)
```

output
```
(-3*c^2*d^2*(a*e^2-c*d^2)/e^2*x^2-3/2*c*d*(a^2*e^4+2*a*c*d^2*e^2-3*c^2*d^4
)/e^3*x-1/6*(2*a^3*e^6+3*a^2*c*d^2*e^4+6*a*c^2*d^4*e^2-11*c^3*d^6)/e^4)/(e
*x+d)^3+c^3*d^3*ln(e*x+d)/e^4
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 194, normalized size of antiderivative = 1.85

$$\int \frac{(ade+(cd^2+ae^2)x+cdex^2)^3}{(d+ex)^7}\,dx$$
$$= \frac{11\,c^3d^6-6\,ac^2d^4e^2-3\,a^2cd^2e^4-2\,a^3e^6+18\,(c^3d^4e^2-ac^2d^2e^4)x^2+9\,(3\,c^3d^5e-2\,ac^2d^3e^3-a^2cde^5)x+}{6\,(e^7x^3+3\,de^6x^2+3\,d^2e^5x+d^3e^4)}$$

input
```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^3/(e*x+d)^7,x, algorithm="fric
as")
```

output
```
1/6*(11*c^3*d^6 - 6*a*c^2*d^4*e^2 - 3*a^2*c*d^2*e^4 - 2*a^3*e^6 + 18*(c^3*
d^4*e^2 - a*c^2*d^2*e^4)*x^2 + 9*(3*c^3*d^5*e - 2*a*c^2*d^3*e^3 - a^2*c*d*
e^5)*x + 6*(c^3*d^3*e^3*x^3 + 3*c^3*d^4*e^2*x^2 + 3*c^3*d^5*e*x + c^3*d^6)
*log(e*x + d))/(e^7*x^3 + 3*d*e^6*x^2 + 3*d^2*e^5*x + d^3*e^4)
```

Sympy [A] (verification not implemented)

Time = 1.15 (sec) , antiderivative size = 163, normalized size of antiderivative = 1.55

$$\int \frac{(ade+(cd^2+ae^2)x+cdex^2)^3}{(d+ex)^7}\,dx = \frac{c^3d^3\log(d+ex)}{e^4}$$
$$+\frac{-2a^3e^6-3a^2cd^2e^4-6ac^2d^4e^2+11c^3d^6+x^2(-18ac^2d^2e^4+18c^3d^4e^2)+x(-9a^2cde^5-18ac^2d^3e^3+2}{6d^3e^4+18d^2e^5x+18de^6x^2+6e^7x^3}$$

input
```
integrate((a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**3/(e*x+d)**7,x)
```

output
```
c**3*d**3*log(d + e*x)/e**4 + (-2*a**3*e**6 - 3*a**2*c*d**2*e**4 - 6*a*c**
2*d**4*e**2 + 11*c**3*d**6 + x**2*(-18*a*c**2*d**2*e**4 + 18*c**3*d**4*e**
2) + x*(-9*a**2*c*d*e**5 - 18*a*c**2*d**3*e**3 + 27*c**3*d**5*e))/(6*d**3*
e**4 + 18*d**2*e**5*x + 18*d*e**6*x**2 + 6*e**7*x**3)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 158, normalized size of antiderivative = 1.50

$$\int \frac{(ade + (cd^2 + ae^2)x + cdex^2)^3}{(d+ex)^7}\,dx = \frac{c^3d^3\log(ex+d)}{e^4} + \frac{11\,c^3d^6 - 6\,ac^2d^4e^2 - 3\,a^2cd^2e^4 - 2\,a^3e^6 + 18\,(c^3d^4e^2 - ac^2d^2e^4)x^2 + 9\,(3\,c^3d^5e - 2\,ac^2d^3e^3 - a^2cde^5)x}{6\,(e^7x^3 + 3\,de^6x^2 + 3\,d^2e^5x + d^3e^4)}$$

input
```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^3/(e*x+d)^7,x, algorithm="maxi
ma")
```

output
```
c^3*d^3*log(e*x + d)/e^4 + 1/6*(11*c^3*d^6 - 6*a*c^2*d^4*e^2 - 3*a^2*c*d^2
*e^4 - 2*a^3*e^6 + 18*(c^3*d^4*e^2 - a*c^2*d^2*e^4)*x^2 + 9*(3*c^3*d^5*e -
 2*a*c^2*d^3*e^3 - a^2*c*d*e^5)*x)/(e^7*x^3 + 3*d*e^6*x^2 + 3*d^2*e^5*x +
d^3*e^4)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 136, normalized size of antiderivative = 1.30

$$\int \frac{(ade + (cd^2 + ae^2)x + cdex^2)^3}{(d+ex)^7}\,dx = \frac{c^3d^3\log(|ex+d|)}{e^4} + \frac{18\,(c^3d^4e - ac^2d^2e^3)x^2 + 9\,(3\,c^3d^5 - 2\,ac^2d^3e^2 - a^2cde^4)x + \frac{11\,c^3d^6 - 6\,ac^2d^4e^2 - 3\,a^2cd^2e^4 - 2\,a^3e^6}{e}}{6\,(ex+d)^3e^3}$$

input
```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^3/(e*x+d)^7,x, algorithm="giac
")
```

output
```
c^3*d^3*log(abs(e*x + d))/e^4 + 1/6*(18*(c^3*d^4*e - a*c^2*d^2*e^3)*x^2 +
9*(3*c^3*d^5 - 2*a*c^2*d^3*e^2 - a^2*c*d*e^4)*x + (11*c^3*d^6 - 6*a*c^2*d^
4*e^2 - 3*a^2*c*d^2*e^4 - 2*a^3*e^6)/e)/((e*x + d)^3*e^3)
```

Mupad [B] (verification not implemented)

Time = 5.36 (sec) , antiderivative size = 157, normalized size of antiderivative = 1.50

$$\int \frac{(ade + (cd^2 + ae^2)x + cdex^2)^3}{(d + ex)^7} \, dx$$
$$= \frac{c^3 d^3 \ln(d + e x)}{e^4} - \frac{\frac{2 a^3 e^6 + 3 a^2 c d^2 e^4 + 6 a c^2 d^4 e^2 - 11 c^3 d^6}{6 e^4} + \frac{3 x (a^2 c d e^4 + 2 a c^2 d^3 e^2 - 3 c^3 d^5)}{2 e^3} + \frac{3 c^2 d^2 x^2 (a e^2 - c d^2)}{e^2}}{d^3 + 3 d^2 e x + 3 d e^2 x^2 + e^3 x^3}$$

input
```
int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^3/(d + e*x)^7,x)
```

output
```
(c^3*d^3*log(d + e*x))/e^4 - ((2*a^3*e^6 - 11*c^3*d^6 + 6*a*c^2*d^4*e^2 +
3*a^2*c*d^2*e^4)/(6*e^4) + (3*x*(2*a*c^2*d^3*e^2 - 3*c^3*d^5 + a^2*c*d*e^4
))/(2*e^3) + (3*c^2*d^2*x^2*(a*e^2 - c*d^2))/e^2)/(d^3 + e^3*x^3 + 3*d*e^2
*x^2 + 3*d^2*e*x)
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 181, normalized size of antiderivative = 1.72

$$\int \frac{(ade + (cd^2 + ae^2)x + cdex^2)^3}{(d + ex)^7} \, dx$$
$$= \frac{6 \log(ex + d) c^3 d^6 + 18 \log(ex + d) c^3 d^5 ex + 18 \log(ex + d) c^3 d^4 e^2 x^2 + 6 \log(ex + d) c^3 d^3 e^3 x^3 - 2a^3 e^6 -}{6e^4 (e^3 x^3 + 3d e^2 x^2 + 3d^2 ex + d^3)}$$

input
```
int((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^3/(e*x+d)^7,x)
```

output
```
(6*log(d + e*x)*c**3*d**6 + 18*log(d + e*x)*c**3*d**5*e*x + 18*log(d + e*x
)*c**3*d**4*e**2*x**2 + 6*log(d + e*x)*c**3*d**3*e**3*x**3 - 2*a**3*e**6 -
 3*a**2*c*d**2*e**4 - 9*a**2*c*d*e**5*x + 6*a*c**2*d*e**5*x**3 + 5*c**3*d*
*6 + 9*c**3*d**5*e*x - 6*c**3*d**3*e**3*x**3)/(6*e**4*(d**3 + 3*d**2*e*x +
 3*d*e**2*x**2 + e**3*x**3))
```

3.103 $\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^3}{(d+ex)^8}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 35

$$\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^3}{(d+ex)^8}\,dx=\frac{(ae+cdx)^4}{4\left(cd^2-ae^2\right)(d+ex)^4}$$

output
```
1/4*(c*d*x+a*e)^4/(-a*e^2+c*d^2)/(e*x+d)^4
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 100 vs. 2(35) = 70.

Time = 0.03 (sec) , antiderivative size = 100, normalized size of antiderivative = 2.86

$$\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^3}{(d+ex)^8}\,dx = -\frac{a^3e^6+a^2cde^4(d+4ex)+ac^2d^2e^2(d^2+4dex+6e^2x^2)+c^3d^3(d^3+4d^2ex+6de^2x^2+4e^3x^3)}{4e^4(d+ex)^4}$$

input
```
Integrate[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^3/(d + e*x)^8,x]
```

output
```
-1/4*(a^3*e^6 + a^2*c*d*e^4*(d + 4*e*x) + a*c^2*d^2*e^2*(d^2 + 4*d*e*x + 6
*e^2*x^2) + c^3*d^3*(d^3 + 4*d^2*e*x + 6*d*e^2*x^2 + 4*e^3*x^3))/(e^4*(d +
 e*x)^4)
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.057$, Rules used = {1120, 48}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^3}{(d+ex)^8}dx$$

$$\Big\downarrow \text{ 1120}$$

$$\int \frac{(ae+cdx)^3}{(d+ex)^5}dx$$

$$\Big\downarrow \text{ 48}$$

$$\frac{(ae+cdx)^4}{4(d+ex)^4\left(cd^2-ae^2\right)}$$

input
```
Int[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^3/(d + e*x)^8,x]
```

output
```
(a*e + c*d*x)^4/(4*(c*d^2 - a*e^2)*(d + e*x)^4)
```

Defintions of rubi rules used

rule 48
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && EqQ[m + n + 2, 0] && NeQ[m, -1]
```

rule 1120
```
Int[((d_) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Int[(d + e*x)^(m + p)*(a/d + (c/e)*x)^p, x] /; FreeQ[{a, b, c, d
, e, m}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[p] && (EqQ[m + p,
0] || EqQ[m + 2*p + 2, 0])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 122 vs. $2(33) = 66$.

Time = 1.31 (sec) , antiderivative size = 123, normalized size of antiderivative = 3.51

method	result
risch	$\frac{-\frac{c^3 d^3 x^3}{e}-\frac{3c^2 d^2 \left(a e^2+c d^2\right) x^2}{2e^2}-\frac{dc\left(a^2 e^4+ac d^2 e^2+c^2 d^4\right) x}{e^3}-\frac{e^6 a^3+d^2 e^4 a^2 c+d^4 e^2 a c^2+d^6 c^3}{4e^4}}{(ex+d)^4}$
gosper	$-\frac{4c^3 d^3 e^3 x^3+6x^2 a c^2 d^2 e^4+6c^3 d^4 e^2 x^2+4x a^2 cd e^5+4xa c^2 d^3 e^3+4c^3 d^5 ex+e^6 a^3+d^2 e^4 a^2 c+d^4 e^2 a c^2+d^6 c^3}{4e^4 (ex+d)^4}$
parallelrisch	$\frac{-4c^3 d^3 e^3 x^3-6x^2 a c^2 d^2 e^4-6c^3 d^4 e^2 x^2-4x a^2 cd e^5-4xa c^2 d^3 e^3-4c^3 d^5 ex-e^6 a^3-d^2 e^4 a^2 c-d^4 e^2 a c^2-d^6 c^3}{4e^4 (ex+d)^4}$
default	$-\frac{dc(a^2 e^4-2ac d^2 e^2+c^2 d^4)}{e^4 (ex+d)^3}-\frac{e^6 a^3-3d^2 e^4 a^2 c+3d^4 e^2 a c^2-d^6 c^3}{4e^4 (ex+d)^4}-\frac{d^3 c^3}{e^4 (ex+d)}-\frac{3c^2 d^2 (a e^2-c d^2)}{2e^4 (ex+d)^2}$
orering	$-\frac{(4c^3 d^3 e^3 x^3+6x^2 a c^2 d^2 e^4+6c^3 d^4 e^2 x^2+4x a^2 cd e^5+4xa c^2 d^3 e^3+4c^3 d^5 ex+e^6 a^3+d^2 e^4 a^2 c+d^4 e^2 a c^2+d^6 c^3)(ade+(a e^2+c}{4e^4 (cdx+ae)^3 (ex+d)^7}$
norman	$\frac{-\frac{d^3\left(a^3 e^9+a^2 c d^2 e^7+d^4 a c^2 e^5+c^3 d^6 e^3\right)}{4e^7}-\frac{\left(a^3 e^9+13a^2 c d^2 e^7+31d^4 a c^2 e^5+35c^3 d^6 e^3\right) x^3}{4e^4}-e^2 d^3 c^3 x^6-\frac{d(2a^2 c e^7+11a e^5 c^2 d^2+17d^4 c}{2e^3}}{(ex+d}$

input
```
int((a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^3/(e*x+d)^8,x,method=_RETURNVERBOSE)
```

output
```
(-c^3*d^3/e*x^3-3/2*c^2*d^2*(a*e^2+c*d^2)/e^2*x^2-d*c*(a^2*e^4+a*c*d^2*e^2
+c^2*d^4)/e^3*x-1/4*(a^3*e^6+a^2*c*d^2*e^4+a*c^2*d^4*e^2+c^3*d^6)/e^4)/(e*
x+d)^4
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 158 vs. $2(33) = 66$.

Time = 0.08 (sec) , antiderivative size = 158, normalized size of antiderivative = 4.51

$$\int \frac{\left(ade + \left(cd^2 + ae^2\right)x + cdex^2\right)^3}{(d + ex)^8}\,dx = -\frac{4\,c^3d^3e^3x^3 + c^3d^6 + ac^2d^4e^2 + a^2cd^2e^4 + a^3e^6 + 6\,(c^3d^4e^2 + ac^2d^2e^4)x^2 + 4\,(c^3d^5e + ac^2d^3e^3 + a^2cde^5)}{4\,(e^8x^4 + 4\,de^7x^3 + 6\,d^2e^6x^2 + 4\,d^3e^5x + d^4e^4)}$$

input
```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^3/(e*x+d)^8,x, algorithm="fric
as")
```

output
```
-1/4*(4*c^3*d^3*e^3*x^3 + c^3*d^6 + a*c^2*d^4*e^2 + a^2*c*d^2*e^4 + a^3*e^
6 + 6*(c^3*d^4*e^2 + a*c^2*d^2*e^4)*x^2 + 4*(c^3*d^5*e + a*c^2*d^3*e^3 + a
^2*c*d*e^5)*x)/(e^8*x^4 + 4*d*e^7*x^3 + 6*d^2*e^6*x^2 + 4*d^3*e^5*x + d^4*
e^4)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 170 vs. $2(27) = 54$.

Time = 2.97 (sec) , antiderivative size = 170, normalized size of antiderivative = 4.86

$$\int \frac{\left(ade + \left(cd^2 + ae^2\right)x + cdex^2\right)^3}{(d + ex)^8}\,dx = \frac{-a^3e^6 - a^2cd^2e^4 - ac^2d^4e^2 - c^3d^6 - 4c^3d^3e^3x^3 + x^2(-6ac^2d^2e^4 - 6c^3d^4e^2) + x(-4a^2cde^5 - 4ac^2d^3e^3 -}{4d^4e^4 + 16d^3e^5x + 24d^2e^6x^2 + 16de^7x^3 + 4e^8x^4}$$

input
```
integrate((a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**3/(e*x+d)**8,x)
```

output
```
(-a**3*e**6 - a**2*c*d**2*e**4 - a*c**2*d**4*e**2 - c**3*d**6 - 4*c**3*d**
3*e**3*x**3 + x**2*(-6*a*c**2*d**2*e**4 - 6*c**3*d**4*e**2) + x*(-4*a**2*c
*d*e**5 - 4*a*c**2*d**3*e**3 - 4*c**3*d**5*e))/(4*d**4*e**4 + 16*d**3*e**5
*x + 24*d**2*e**6*x**2 + 16*d*e**7*x**3 + 4*e**8*x**4)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 158 vs. $2(33) = 66$.

Time = 0.06 (sec) , antiderivative size = 158, normalized size of antiderivative = 4.51

$$\int \frac{(ade + (cd^2 + ae^2)x + cdex^2)^3}{(d + ex)^8}\,dx = -\frac{4\,c^3d^3e^3x^3 + c^3d^6 + ac^2d^4e^2 + a^2cd^2e^4 + a^3e^6 + 6\,(c^3d^4e^2 + ac^2d^2e^4)x^2 + 4\,(c^3d^5e + ac^2d^3e^3 + a^2cde^5)}{4\,(e^8x^4 + 4\,de^7x^3 + 6\,d^2e^6x^2 + 4\,d^3e^5x + d^4e^4)}$$

input
```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^3/(e*x+d)^8,x, algorithm="maxi
ma")
```

output
```
-1/4*(4*c^3*d^3*e^3*x^3 + c^3*d^6 + a*c^2*d^4*e^2 + a^2*c*d^2*e^4 + a^3*e^
6 + 6*(c^3*d^4*e^2 + a*c^2*d^2*e^4)*x^2 + 4*(c^3*d^5*e + a*c^2*d^3*e^3 + a
^2*c*d*e^5)*x)/(e^8*x^4 + 4*d*e^7*x^3 + 6*d^2*e^6*x^2 + 4*d^3*e^5*x + d^4*
e^4)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 126 vs. $2(33) = 66$.

Time = 0.15 (sec) , antiderivative size = 126, normalized size of antiderivative = 3.60

$$\int \frac{(ade + (cd^2 + ae^2)x + cdex^2)^3}{(d + ex)^8}\,dx = -\frac{4\,c^3d^3e^3x^3 + 6\,c^3d^4e^2x^2 + 6\,ac^2d^2e^4x^2 + 4\,c^3d^5ex + 4\,ac^2d^3e^3x + 4\,a^2cde^5x + c^3d^6 + ac^2d^4e^2 + a^2cd^2e}{4\,(ex + d)^4e^4}$$

input
```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^3/(e*x+d)^8,x, algorithm="giac
")
```

output
```
-1/4*(4*c^3*d^3*e^3*x^3 + 6*c^3*d^4*e^2*x^2 + 6*a*c^2*d^2*e^4*x^2 + 4*c^3*
d^5*e*x + 4*a*c^2*d^3*e^3*x + 4*a^2*c*d*e^5*x + c^3*d^6 + a*c^2*d^4*e^2 +
a^2*c*d^2*e^4 + a^3*e^6)/((e*x + d)^4*e^4)
```

Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 102, normalized size of antiderivative = 2.91

$$\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^3}{(d+ex)^8}\,dx = -\frac{d\left(a^2\,c\,e\,x-a\,c^2\,e\,x^3\right)+\frac{a^3\,e^2}{4}+d^2\left(\frac{a^2\,c}{4}-\frac{c^3\,x^4}{4}\right)-\frac{a\,c^2\,e^2\,x^4}{4}}{d^4+4\,d^3\,e\,x+6\,d^2\,e^2\,x^2+4\,d\,e^3\,x^3+e^4\,x^4}$$

input
```
int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^3/(d + e*x)^8,x)
```

output
```
-(d*(a^2*c*e*x - a*c^2*e*x^3) + (a^3*e^2)/4 + d^2*((a^2*c)/4 - (c^3*x^4)/4
) - (a*c^2*e^2*x^4)/4)/(d^4 + e^4*x^4 + 4*d*e^3*x^3 + 6*d^2*e^2*x^2 + 4*d^
3*e*x)
```

Reduce [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 125, normalized size of antiderivative = 3.57

$$\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^3}{(d+ex)^8}\,dx = \frac{c^3d^2e^2x^4-6a\,c^2d^2e^2x^2-4a^2cd\,e^3x-4a\,c^2d^3ex-a^3e^4-a^2c\,d^2e^2-a\,c^2d^4}{4e^2\left(e^4x^4+4d\,e^3x^3+6d^2e^2x^2+4d^3ex+d^4\right)}$$

input
```
int((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^3/(e*x+d)^8,x)
```

output
```
( - a**3*e**4 - a**2*c*d**2*e**2 - 4*a**2*c*d*e**3*x - a*c**2*d**4 - 4*a*c
**2*d**3*e*x - 6*a*c**2*d**2*e**2*x**2 + c**3*d**2*e**2*x**4)/(4*e**2*(d**
4 + 4*d**3*e*x + 6*d**2*e**2*x**2 + 4*d*e**3*x**3 + e**4*x**4))
```

3.104 $\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^3}{(d+ex)^9} \, dx$

Optimal result

Integrand size = 35, antiderivative size = 73

$$\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^3}{(d+ex)^9} \, dx = \frac{(ae+cdx)^4}{5\left(cd^2-ae^2\right)(d+ex)^5}+\frac{cd(ae+cdx)^4}{20\left(cd^2-ae^2\right)^2(d+ex)^4}$$

output
```
1/5*(c*d*x+a*e)^4/(-a*e^2+c*d^2)/(e*x+d)^5+1/20*c*d*(c*d*x+a*e)^4/(-a*e^2+
c*d^2)^2/(e*x+d)^4
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 103, normalized size of antiderivative = 1.41

$$\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^3}{(d+ex)^9} \, dx = -\frac{4a^3e^6+3a^2cde^4(d+5ex)+2ac^2d^2e^2(d^2+5dex+10e^2x^2)+c^3d^3(d^3+5d^2ex+10de^2x^2+10e^3x^3)}{20e^4(d+ex)^5}$$

input
```
Integrate[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^3/(d + e*x)^9,x]
```

output
```
-1/20*(4*a^3*e^6 + 3*a^2*c*d*e^4*(d + 5*e*x) + 2*a*c^2*d^2*e^2*(d^2 + 5*d*
e*x + 10*e^2*x^2) + c^3*d^3*(d^3 + 5*d^2*e*x + 10*d*e^2*x^2 + 10*e^3*x^3))
/(e^4*(d + e*x)^5)
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.48, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.057$, Rules used = {1121, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^3}{(d+ex)^9}dx$$

$$\Big\downarrow \text{1121}$$

$$\int \left(-\frac{3c^2d^2\left(cd^2-ae^2\right)}{e^3(d+ex)^4}+\frac{3cd\left(cd^2-ae^2\right)^2}{e^3(d+ex)^5}+\frac{\left(ae^2-cd^2\right)^3}{e^3(d+ex)^6}+\frac{c^3d^3}{e^3(d+ex)^3}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{c^2d^2\left(cd^2-ae^2\right)}{e^4(d+ex)^3}-\frac{3cd\left(cd^2-ae^2\right)^2}{4e^4(d+ex)^4}+\frac{\left(cd^2-ae^2\right)^3}{5e^4(d+ex)^5}-\frac{c^3d^3}{2e^4(d+ex)^2}$$

input
```
Int[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^3/(d + e*x)^9,x]
```

output
```
(c*d^2 - a*e^2)^3/(5*e^4*(d + e*x)^5) - (3*c*d*(c*d^2 - a*e^2)^2)/(4*e^4*(
d + e*x)^4) + (c^2*d^2*(c*d^2 - a*e^2))/(e^4*(d + e*x)^3) - (c^3*d^3)/(2*e
^4*(d + e*x)^2)
```

Defintions of rubi rules used

rule 1121
```
Int[((d_) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Int[ExpandIntegrand[(d + e*x)^(m + p)*(a/d + (c/e)*x)^p, x], x]
/; FreeQ[{a, b, c, d, e, m, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && (Int
egerQ[p] || (GtQ[a, 0] && GtQ[d, 0] && LtQ[c, 0]))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 1.34 (sec) , antiderivative size = 129, normalized size of antiderivative = 1.77

method	result
risch	$\frac{-\frac{c^3d^3x^3}{2e}-\frac{d^2c^2\left(2a\,e^2+c\,d^2\right)x^2}{2e^2}-\frac{dc\left(3a^2e^4+2ac\,d^2e^2+c^2d^4\right)x}{4e^3}-\frac{4e^6a^3+3d^2e^4a^2c+2d^4e^2a\,c^2+d^6c^3}{20e^4}}{(ex+d)^5}$
gosper	$-\frac{10c^3d^3e^3x^3+20x^2a\,c^2d^2e^4+10c^3d^4e^2x^2+15x\,a^2cd\,e^5+10xa\,c^2d^3e^3+5c^3d^5ex+4e^6a^3+3d^2e^4a^2c+2d^4e^2a\,c^2+d^6c^3}{20e^4(ex+d)^5}$
parallelrisch	$\frac{-10d^3c^3x^3e^4-20a\,c^2d^2e^5x^2-10c^3d^4e^3x^2-15a^2cd\,e^6x-10a\,c^2d^3e^4x-5c^3d^5e^2x-4a^3e^7-3a^2e^5c\,d^2-2a\,e^3c^2d^4-d^6c^3e}{20e^5(ex+d)^5}$
default	$-\frac{c^2d^2\left(a\,e^2-c\,d^2\right)}{e^4(ex+d)^3}-\frac{3dc\left(a^2e^4-2ac\,d^2e^2+c^2d^4\right)}{4e^4(ex+d)^4}-\frac{e^6a^3-3d^2e^4a^2c+3d^4e^2a\,c^2-d^6c^3}{5e^4(ex+d)^5}-\frac{d^3c^3}{2e^4(ex+d)^2}$
orering	$-\frac{\left(10c^3d^3e^3x^3+20x^2a\,c^2d^2e^4+10c^3d^4e^2x^2+15x\,a^2cd\,e^5+10xa\,c^2d^3e^3+5c^3d^5ex+4e^6a^3+3d^2e^4a^2c+2d^4e^2a\,c^2+d^6c^3\right)(ade}{20e^4(cdx+ae)^3(ex+d)^8}$
norman	$\frac{-\frac{d^3\left(4a^3e^{10}+3a^2c\,d^2e^8+2d^4a\,c^2e^6+c^3d^6e^4\right)}{20e^8}-\frac{\left(a^3e^{10}+12a^2c\,d^2e^8+23d^4a\,c^2e^6+14c^3d^6e^4\right)x^3}{5e^5}-\frac{d\left(3a^2c\,e^8+14a\,e^6c^2d^2+13d^4c^3e^4\right)x^4}{4e^4}}{(e}$

input
```
int((a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^3/(e*x+d)^9,x,method=_RETURNVERBOSE)
```

output
```
(-1/2*c^3*d^3/e*x^3-1/2*d^2/e^2*c^2*(2*a*e^2+c*d^2)*x^2-1/4*d*c/e^3*(3*a^2
*e^4+2*a*c*d^2*e^2+c^2*d^4)*x-1/20/e^4*(4*a^3*e^6+3*a^2*c*d^2*e^4+2*a*c^2*
d^4*e^2+c^3*d^6))/(e*x+d)^5
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 175 vs. $2(69) = 138$.

Time = 0.08 (sec) , antiderivative size = 175, normalized size of antiderivative = 2.40

$$\int \frac{(ade + (cd^2 + ae^2)x + cdex^2)^3}{(d + ex)^9} dx = -\frac{10 c^3 d^3 e^3 x^3 + c^3 d^6 + 2 ac^2 d^4 e^2 + 3 a^2 c d^2 e^4 + 4 a^3 e^6 + 10 (c^3 d^4 e^2 + 2 ac^2 d^2 e^4) x^2 + 5 (c^3 d^5 e + 2 ac^2 d^3 e^3}{20 (e^9 x^5 + 5 de^8 x^4 + 10 d^2 e^7 x^3 + 10 d^3 e^6 x^2 + 5 d^4 e^5 x + d^5 e^4)}$$

input
```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^3/(e*x+d)^9,x, algorithm="fric
as")
```

output
```
-1/20*(10*c^3*d^3*e^3*x^3 + c^3*d^6 + 2*a*c^2*d^4*e^2 + 3*a^2*c*d^2*e^4 +
4*a^3*e^6 + 10*(c^3*d^4*e^2 + 2*a*c^2*d^2*e^4)*x^2 + 5*(c^3*d^5*e + 2*a*c^
2*d^3*e^3 + 3*a^2*c*d*e^5)*x)/(e^9*x^5 + 5*d*e^8*x^4 + 10*d^2*e^7*x^3 + 10
*d^3*e^6*x^2 + 5*d^4*e^5*x + d^5*e^4)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 187 vs. $2(61) = 122$.

Time = 4.61 (sec) , antiderivative size = 187, normalized size of antiderivative = 2.56

$$\int \frac{(ade + (cd^2 + ae^2)x + cdex^2)^3}{(d + ex)^9} dx = \frac{-4a^3e^6 - 3a^2cd^2e^4 - 2ac^2d^4e^2 - c^3d^6 - 10c^3d^3e^3x^3 + x^2(-20ac^2d^2e^4 - 10c^3d^4e^2) + x(-15a^2cde^5 - 10a}{20d^5e^4 + 100d^4e^5x + 200d^3e^6x^2 + 200d^2e^7x^3 + 100de^8x^4 + 20e^9x^5}$$

input
```
integrate((a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**3/(e*x+d)**9,x)
```

output
```
(-4*a**3*e**6 - 3*a**2*c*d**2*e**4 - 2*a*c**2*d**4*e**2 - c**3*d**6 - 10*c
**3*d**3*e**3*x**3 + x**2*(-20*a*c**2*d**2*e**4 - 10*c**3*d**4*e**2) + x*(
-15*a**2*c*d*e**5 - 10*a*c**2*d**3*e**3 - 5*c**3*d**5*e))/(20*d**5*e**4 +
100*d**4*e**5*x + 200*d**3*e**6*x**2 + 200*d**2*e**7*x**3 + 100*d*e**8*x**
4 + 20*e**9*x**5)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 175 vs. 2(69) = 138.

Time = 0.04 (sec) , antiderivative size = 175, normalized size of antiderivative = 2.40

$$\int \frac{(ade + (cd^2 + ae^2)\,x + cdex^2)^3}{(d + ex)^9}\,dx = -\frac{10\,c^3d^3e^3x^3 + c^3d^6 + 2\,ac^2d^4e^2 + 3\,a^2cd^2e^4 + 4\,a^3e^6 + 10\,(c^3d^4e^2 + 2\,ac^2d^2e^4)x^2 + 5\,(c^3d^5e + 2\,ac^2d^3e^3 \cdots}{20\,(e^9x^5 + 5\,de^8x^4 + 10\,d^2e^7x^3 + 10\,d^3e^6x^2 + 5\,d^4e^5x + d^5e^4)}$$

input
```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^3/(e*x+d)^9,x, algorithm="maxi
ma")
```

output
```
-1/20*(10*c^3*d^3*e^3*x^3 + c^3*d^6 + 2*a*c^2*d^4*e^2 + 3*a^2*c*d^2*e^4 +
4*a^3*e^6 + 10*(c^3*d^4*e^2 + 2*a*c^2*d^2*e^4)*x^2 + 5*(c^3*d^5*e + 2*a*c^
2*d^3*e^3 + 3*a^2*c*d*e^5)*x)/(e^9*x^5 + 5*d*e^8*x^4 + 10*d^2*e^7*x^3 + 10
*d^3*e^6*x^2 + 5*d^4*e^5*x + d^5*e^4)
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 129, normalized size of antiderivative = 1.77

$$\int \frac{(ade + (cd^2 + ae^2)\,x + cdex^2)^3}{(d + ex)^9}\,dx = -\frac{10\,c^3d^3e^3x^3 + 10\,c^3d^4e^2x^2 + 20\,ac^2d^2e^4x^2 + 5\,c^3d^5ex + 10\,ac^2d^3e^3x + 15\,a^2cde^5x + c^3d^6 + 2\,ac^2d^4e^2 + \cdots}{20\,(ex + d)^5e^4}$$

input
```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^3/(e*x+d)^9,x, algorithm="giac
")
```

output
```
-1/20*(10*c^3*d^3*e^3*x^3 + 10*c^3*d^4*e^2*x^2 + 20*a*c^2*d^2*e^4*x^2 + 5*
c^3*d^5*e*x + 10*a*c^2*d^3*e^3*x + 15*a^2*c*d*e^5*x + c^3*d^6 + 2*a*c^2*d^
4*e^2 + 3*a^2*c*d^2*e^4 + 4*a^3*e^6)/((e*x + d)^5*e^4)
```

Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 135, normalized size of antiderivative = 1.85

$$\int \frac{(ade + (cd^2 + ae^2)x + cdex^2)^3}{(d + ex)^9} dx$$
$$= -\frac{d^2 \left(\frac{3a^2c}{20} + a\,c^2\,x^2 - \frac{c^3\,x^4}{4}\right) - d\left(\frac{c^3\,e\,x^5}{20} - \frac{3\,a^2\,c\,e\,x}{4}\right) + \frac{a^3\,e^2}{5} + \frac{a\,c^2\,d^4}{10\,e^2} + \frac{a\,c^2\,d^3\,x}{2\,e}}{d^5 + 5\,d^4\,e\,x + 10\,d^3\,e^2\,x^2 + 10\,d^2\,e^3\,x^3 + 5\,d\,e^4\,x^4 + e^5\,x^5}$$

input
```
int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^3/(d + e*x)^9,x)
```

output
```
-(d^2*((3*a^2*c)/20 - (c^3*x^4)/4 + a*c^2*x^2) - d*((c^3*e*x^5)/20 - (3*a^
2*c*e*x)/4) + (a^3*e^2)/5 + (a*c^2*d^4)/(10*e^2) + (a*c^2*d^3*x)/(2*e))/(d
^5 + e^5*x^5 + 5*d*e^4*x^4 + 10*d^3*e^2*x^2 + 10*d^2*e^3*x^3 + 5*d^4*e*x)
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 174, normalized size of antiderivative = 2.38

$$\int \frac{(ade + (cd^2 + ae^2)x + cdex^2)^3}{(d + ex)^9} dx$$
$$= \frac{-10c^3d^3e^3x^3 - 20a\,c^2d^2e^4x^2 - 10c^3d^4e^2x^2 - 15a^2cd\,e^5x - 10a\,c^2d^3e^3x - 5c^3d^5ex - 4a^3e^6 - 3a^2c\,d^2e^4 -}{20e^4\left(e^5x^5 + 5d\,e^4x^4 + 10d^2e^3x^3 + 10d^3e^2x^2 + 5d^4ex + d^5\right)}$$

input
```
int((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^3/(e*x+d)^9,x)
```

output
```
( - 4*a**3*e**6 - 3*a**2*c*d**2*e**4 - 15*a**2*c*d*e**5*x - 2*a*c**2*d**4*
e**2 - 10*a*c**2*d**3*e**3*x - 20*a*c**2*d**2*e**4*x**2 - c**3*d**6 - 5*c*
*3*d**5*e*x - 10*c**3*d**4*e**2*x**2 - 10*c**3*d**3*e**3*x**3)/(20*e**4*(d
**5 + 5*d**4*e*x + 10*d**3*e**2*x**2 + 10*d**2*e**3*x**3 + 5*d*e**4*x**4 +
 e**5*x**5))
```

3.105 $\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^3}{(d+ex)^{10}}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 111

$$\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^3}{(d+ex)^{10}}\,dx = \frac{\left(cd^2-ae^2\right)^3}{6e^4(d+ex)^6}-\frac{3cd\left(cd^2-ae^2\right)^2}{5e^4(d+ex)^5}+\frac{3c^2d^2\left(cd^2-ae^2\right)}{4e^4(d+ex)^4}-\frac{c^3d^3}{3e^4(d+ex)^3}$$

output
```
1/6*(-a*e^2+c*d^2)^3/e^4/(e*x+d)^6-3/5*c*d*(-a*e^2+c*d^2)^2/e^4/(e*x+d)^5+
3/4*c^2*d^2*(-a*e^2+c*d^2)/e^4/(e*x+d)^4-1/3*c^3*d^3/e^4/(e*x+d)^3
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 103, normalized size of antiderivative = 0.93

$$\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^3}{(d+ex)^{10}}\,dx = -\frac{10a^3e^6+6a^2cde^4(d+6ex)+3ac^2d^2e^2\left(d^2+6dex+15e^2x^2\right)+c^3d^3\left(d^3+6d^2ex+15de^2x^2+20e^3x^3\right)}{60e^4(d+ex)^6}$$

input
```
Integrate[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^3/(d + e*x)^10,x]
```

output
```
-1/60*(10*a^3*e^6 + 6*a^2*c*d*e^4*(d + 6*e*x) + 3*a*c^2*d^2*e^2*(d^2 + 6*d
*e*x + 15*e^2*x^2) + c^3*d^3*(d^3 + 6*d^2*e*x + 15*d*e^2*x^2 + 20*e^3*x^3)
)/(e^4*(d + e*x)^6)
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 111, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.057$, Rules used = {1121, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^3}{(d+ex)^{10}}dx$$

$$\Big\downarrow \textbf{1121}$$

$$\int \left(-\frac{3c^2d^2\left(cd^2-ae^2\right)}{e^3(d+ex)^5}+\frac{3cd\left(cd^2-ae^2\right)^2}{e^3(d+ex)^6}+\frac{\left(ae^2-cd^2\right)^3}{e^3(d+ex)^7}+\frac{c^3d^3}{e^3(d+ex)^4}\right)dx$$

$$\Big\downarrow \textbf{2009}$$

$$\frac{3c^2d^2\left(cd^2-ae^2\right)}{4e^4(d+ex)^4}-\frac{3cd\left(cd^2-ae^2\right)^2}{5e^4(d+ex)^5}+\frac{\left(cd^2-ae^2\right)^3}{6e^4(d+ex)^6}-\frac{c^3d^3}{3e^4(d+ex)^3}$$

input
```
Int[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^3/(d + e*x)^10,x]
```

output
```
(c*d^2 - a*e^2)^3/(6*e^4*(d + e*x)^6) - (3*c*d*(c*d^2 - a*e^2)^2)/(5*e^4*(
d + e*x)^5) + (3*c^2*d^2*(c*d^2 - a*e^2))/(4*e^4*(d + e*x)^4) - (c^3*d^3)/
(3*e^4*(d + e*x)^3)
```

Defintions of rubi rules used

rule 1121
```
Int[((d_) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Int[ExpandIntegrand[(d + e*x)^(m + p)*(a/d + (c/e)*x)^p, x], x]
/; FreeQ[{a, b, c, d, e, m, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && (Int
egerQ[p] || (GtQ[a, 0] && GtQ[d, 0] && LtQ[c, 0]))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 1.40 (sec) , antiderivative size = 129, normalized size of antiderivative = 1.16

method	result
risch	$\frac{-\frac{c^3 d^3 x^3}{3e}-\frac{d^2 c^2 \left(3a\,e^2+c\,d^2\right) x^2}{4e^2}-\frac{dc\left(6a^2 e^4+3ac\,d^2 e^2+c^2 d^4\right) x}{10e^3}-\frac{10e^6 a^3+6d^2 e^4 a^2 c+3d^4 e^2 a\,c^2+d^6 c^3}{60e^4}}{(ex+d)^6}$
gosper	$-\frac{20c^3 d^3 e^3 x^3+45x^2 a\,c^2 d^2 e^4+15c^3 d^4 e^2 x^2+36x\,a^2 cd\,e^5+18xa\,c^2 d^3 e^3+6c^3 d^5 ex+10e^6 a^3+6d^2 e^4 a^2 c+3d^4 e^2 a\,c^2+d^6 c^3}{60e^4 (ex+d)^6}$
parallelrisch	$\frac{-20d^3 c^3 x^3 e^5-45a\,c^2 d^2 e^6 x^2-15c^3 d^4 e^4 x^2-36a^2 cd\,e^7 x-18a\,c^2 d^3 e^5 x-6c^3 d^5 e^3 x-10a^3 e^8-6a^2 c\,d^2 e^6-3a\,c^2 d^4 e^4-c^3 d^6 e^2}{60e^6 (ex+d)^6}$
default	$-\frac{c^3 d^3}{3e^4 (ex+d)^3}-\frac{3c^2 d^2 \left(a\,e^2-c\,d^2\right)}{4e^4 (ex+d)^4}-\frac{3dc\left(a^2 e^4-2ac\,d^2 e^2+c^2 d^4\right)}{5e^4 (ex+d)^5}-\frac{e^6 a^3-3d^2 e^4 a^2 c+3d^4 e^2 a\,c^2-d^6 c^3}{6e^4 (ex+d)^6}$
orering	$-\frac{\left(20c^3 d^3 e^3 x^3+45x^2 a\,c^2 d^2 e^4+15c^3 d^4 e^2 x^2+36x\,a^2 cd\,e^5+18xa\,c^2 d^3 e^3+6c^3 d^5 ex+10e^6 a^3+6d^2 e^4 a^2 c+3d^4 e^2 a\,c^2+d^6 c^3\right)(ad}{60e^4 (cdx+ae)^3 (ex+d)^9}$
norman	$\frac{-\frac{d^3\left(10a^3 e^{11}+6a^2 c\,d^2 e^9+3d^4 a\,c^2 e^7+c^3 d^6 e^5\right)}{60e^9}-\frac{\left(5a^3 e^{11}+57a^2 c\,d^2 e^9+96d^4 a\,c^2 e^7+42c^3 d^6 e^5\right) x^3}{30e^6}-\frac{d\left(12a^2 c\,e^9+51a\,c^2 d^2 e^7+37d^4 c^3 e^5\right.}{20e^5}}{}$

input
```
int((a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^3/(e*x+d)^10,x,method=_RETURNVERBOSE
)
```

output
```
(-1/3*c^3*d^3/e*x^3-1/4*d^2*c^2/e^2*(3*a*e^2+c*d^2)*x^2-1/10*d*c/e^3*(6*a^
2*e^4+3*a*c*d^2*e^2+c^2*d^4)*x-1/60/e^4*(10*a^3*e^6+6*a^2*c*d^2*e^4+3*a*c^
2*d^4*e^2+c^3*d^6))/(e*x+d)^6
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 186, normalized size of antiderivative = 1.68

$$\int \frac{(ade + (cd^2 + ae^2)x + cdex^2)^3}{(d+ex)^{10}}\,dx = -\frac{20\,c^3d^3e^3x^3 + c^3d^6 + 3\,ac^2d^4e^2 + 6\,a^2cd^2e^4 + 10\,a^3e^6 + 15\,(c^3d^4e^2 + 3\,ac^2d^2e^4)x^2 + 6\,(c^3d^5e + 3\,ac^2d^3e}{60\,(e^{10}x^6 + 6\,de^9x^5 + 15\,d^2e^8x^4 + 20\,d^3e^7x^3 + 15\,d^4e^6x^2 + 6\,d^5e^5x + d^6e^4)}$$

input
```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^3/(e*x+d)^10,x, algorithm="fri
cas")
```

output
```
-1/60*(20*c^3*d^3*e^3*x^3 + c^3*d^6 + 3*a*c^2*d^4*e^2 + 6*a^2*c*d^2*e^4 +
10*a^3*e^6 + 15*(c^3*d^4*e^2 + 3*a*c^2*d^2*e^4)*x^2 + 6*(c^3*d^5*e + 3*a*c
^2*d^3*e^3 + 6*a^2*c*d*e^5)*x)/(e^10*x^6 + 6*d*e^9*x^5 + 15*d^2*e^8*x^4 +
20*d^3*e^7*x^3 + 15*d^4*e^6*x^2 + 6*d^5*e^5*x + d^6*e^4)
```

Sympy [A] (verification not implemented)

Time = 21.63 (sec) , antiderivative size = 199, normalized size of antiderivative = 1.79

$$\int \frac{(ade + (cd^2 + ae^2)x + cdex^2)^3}{(d+ex)^{10}}\,dx = \frac{-10a^3e^6 - 6a^2cd^2e^4 - 3ac^2d^4e^2 - c^3d^6 - 20c^3d^3e^3x^3 + x^2(-45ac^2d^2e^4 - 15c^3d^4e^2) + x(-36a^2cde^5 - 18}{60d^6e^4 + 360d^5e^5x + 900d^4e^6x^2 + 1200d^3e^7x^3 + 900d^2e^8x^4 + 360de^9x^5 + 60e^{10}x^6}$$

input
```
integrate((a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**3/(e*x+d)**10,x)
```

output
```
(-10*a**3*e**6 - 6*a**2*c*d**2*e**4 - 3*a*c**2*d**4*e**2 - c**3*d**6 - 20*
c**3*d**3*e**3*x**3 + x**2*(-45*a*c**2*d**2*e**4 - 15*c**3*d**4*e**2) + x*
(-36*a**2*c*d*e**5 - 18*a*c**2*d**3*e**3 - 6*c**3*d**5*e))/(60*d**6*e**4 +
 360*d**5*e**5*x + 900*d**4*e**6*x**2 + 1200*d**3*e**7*x**3 + 900*d**2*e**
8*x**4 + 360*d*e**9*x**5 + 60*e**10*x**6)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 186, normalized size of antiderivative = 1.68

$$\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^3}{(d+ex)^{10}}\,dx = -\frac{20\,c^3d^3e^3x^3+c^3d^6+3\,ac^2d^4e^2+6\,a^2cd^2e^4+10\,a^3e^6+15\,(c^3d^4e^2+3\,ac^2d^2e^4)x^2+6\,(c^3d^5e+3\,ac^2d^3e}{60\,(e^{10}x^6+6\,de^9x^5+15\,d^2e^8x^4+20\,d^3e^7x^3+15\,d^4e^6x^2+6\,d^5e^5x+d^6e^4)}$$

input
```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^3/(e*x+d)^10,x, algorithm="max
ima")
```

output
```
-1/60*(20*c^3*d^3*e^3*x^3 + c^3*d^6 + 3*a*c^2*d^4*e^2 + 6*a^2*c*d^2*e^4 +
10*a^3*e^6 + 15*(c^3*d^4*e^2 + 3*a*c^2*d^2*e^4)*x^2 + 6*(c^3*d^5*e + 3*a*c
^2*d^3*e^3 + 6*a^2*c*d*e^5)*x)/(e^10*x^6 + 6*d*e^9*x^5 + 15*d^2*e^8*x^4 +
20*d^3*e^7*x^3 + 15*d^4*e^6*x^2 + 6*d^5*e^5*x + d^6*e^4)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 129, normalized size of antiderivative = 1.16

$$\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^3}{(d+ex)^{10}}\,dx = -\frac{20\,c^3d^3e^3x^3+15\,c^3d^4e^2x^2+45\,ac^2d^2e^4x^2+6\,c^3d^5ex+18\,ac^2d^3e^3x+36\,a^2cde^5x+c^3d^6+3\,ac^2d^4e^2+}{60\,(ex+d)^6e^4}$$

input
```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^3/(e*x+d)^10,x, algorithm="gia
c")
```

output
```
-1/60*(20*c^3*d^3*e^3*x^3 + 15*c^3*d^4*e^2*x^2 + 45*a*c^2*d^2*e^4*x^2 + 6*
c^3*d^5*e*x + 18*a*c^2*d^3*e^3*x + 36*a^2*c*d*e^5*x + c^3*d^6 + 3*a*c^2*d^
4*e^2 + 6*a^2*c*d^2*e^4 + 10*a^3*e^6)/((e*x + d)^6*e^4)
```

Mupad [B] (verification not implemented)

Time = 5.70 (sec) , antiderivative size = 184, normalized size of antiderivative = 1.66

$$\int \frac{(ade+(cd^2+ae^2)x+cdex^2)^3}{(d+ex)^{10}}\,dx$$
$$= -\frac{\frac{10\,a^3\,e^6+6\,a^2\,c\,d^2\,e^4+3\,a\,c^2\,d^4\,e^2+c^3\,d^6}{60\,e^4}+\frac{c^3\,d^3\,x^3}{3\,e}+\frac{c\,d\,x\,\left(6\,a^2\,e^4+3\,a\,c\,d^2\,e^2+c^2\,d^4\right)}{10\,e^3}+\frac{c^2\,d^2\,x^2\,\left(c\,d^2+3\,a\,e^2\right)}{4\,e^2}}{d^6+6\,d^5\,e\,x+15\,d^4\,e^2\,x^2+20\,d^3\,e^3\,x^3+15\,d^2\,e^4\,x^4+6\,d\,e^5\,x^5+e^6\,x^6}$$

input
```
int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^3/(d + e*x)^10,x)
```

output
```
-((10*a^3*e^6 + c^3*d^6 + 3*a*c^2*d^4*e^2 + 6*a^2*c*d^2*e^4)/(60*e^4) + (c
^3*d^3*x^3)/(3*e) + (c*d*x*(6*a^2*e^4 + c^2*d^4 + 3*a*c*d^2*e^2))/(10*e^3)
 + (c^2*d^2*x^2*(3*a*e^2 + c*d^2))/(4*e^2))/(d^6 + e^6*x^6 + 6*d*e^5*x^5 +
 15*d^4*e^2*x^2 + 20*d^3*e^3*x^3 + 15*d^2*e^4*x^4 + 6*d^5*e*x)
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 185, normalized size of antiderivative = 1.67

$$\int \frac{(ade+(cd^2+ae^2)x+cdex^2)^3}{(d+ex)^{10}}\,dx$$
$$= \frac{-20c^3d^3e^3x^3-45a\,c^2d^2e^4x^2-15c^3d^4e^2x^2-36a^2cd\,e^5x-18a\,c^2d^3e^3x-6c^3d^5ex-10a^3e^6-6a^2c\,d^2e^4 -}{60e^4\left(e^6x^6+6d\,e^5x^5+15d^2e^4x^4+20d^3e^3x^3+15d^4e^2x^2+6d^5ex+d^6\right)}$$

input
```
int((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^3/(e*x+d)^10,x)
```

output
```
( - 10*a**3*e**6 - 6*a**2*c*d**2*e**4 - 36*a**2*c*d*e**5*x - 3*a*c**2*d**4
*e**2 - 18*a*c**2*d**3*e**3*x - 45*a*c**2*d**2*e**4*x**2 - c**3*d**6 - 6*c
**3*d**5*e*x - 15*c**3*d**4*e**2*x**2 - 20*c**3*d**3*e**3*x**3)/(60*e**4*(
d**6 + 6*d**5*e*x + 15*d**4*e**2*x**2 + 20*d**3*e**3*x**3 + 15*d**2*e**4*x
**4 + 6*d*e**5*x**5 + e**6*x**6))
```

3.106 $\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^3}{(d+ex)^{11}}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 111

$$\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^3}{(d+ex)^{11}}\,dx=\frac{\left(cd^2-ae^2\right)^3}{7e^4(d+ex)^7}-\frac{cd\left(cd^2-ae^2\right)^2}{2e^4(d+ex)^6}+\frac{3c^2d^2\left(cd^2-ae^2\right)}{5e^4(d+ex)^5}-\frac{c^3d^3}{4e^4(d+ex)^4}$$

output
```
1/7*(-a*e^2+c*d^2)^3/e^4/(e*x+d)^7-1/2*c*d*(-a*e^2+c*d^2)^2/e^4/(e*x+d)^6+
3/5*c^2*d^2*(-a*e^2+c*d^2)/e^4/(e*x+d)^5-1/4*c^3*d^3/e^4/(e*x+d)^4
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 103, normalized size of antiderivative = 0.93

$$\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^3}{(d+ex)^{11}}\,dx=-\frac{20a^3e^6+10a^2cde^4(d+7ex)+4ac^2d^2e^2(d^2+7dex+21e^2x^2)+c^3d^3(d^3+7d^2ex+21de^2x^2+35e^3x^3)}{140e^4(d+ex)^7}$$

input
```
Integrate[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^3/(d + e*x)^11,x]
```

output
```
-1/140*(20*a^3*e^6 + 10*a^2*c*d*e^4*(d + 7*e*x) + 4*a*c^2*d^2*e^2*(d^2 + 7
*d*e*x + 21*e^2*x^2) + c^3*d^3*(d^3 + 7*d^2*e*x + 21*d*e^2*x^2 + 35*e^3*x^
3))/(e^4*(d + e*x)^7)
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 111, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.057, Rules used = {1121, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^3}{(d+ex)^{11}}\,dx$$

$$\Big\downarrow \text{1121}$$

$$\int \left(-\frac{3c^2d^2\left(cd^2-ae^2\right)}{e^3(d+ex)^6}+\frac{3cd\left(cd^2-ae^2\right)^2}{e^3(d+ex)^7}+\frac{\left(ae^2-cd^2\right)^3}{e^3(d+ex)^8}+\frac{c^3d^3}{e^3(d+ex)^5}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{3c^2d^2\left(cd^2-ae^2\right)}{5e^4(d+ex)^5}-\frac{cd\left(cd^2-ae^2\right)^2}{2e^4(d+ex)^6}+\frac{\left(cd^2-ae^2\right)^3}{7e^4(d+ex)^7}-\frac{c^3d^3}{4e^4(d+ex)^4}$$

input
```
Int[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^3/(d + e*x)^11,x]
```

output
```
(c*d^2 - a*e^2)^3/(7*e^4*(d + e*x)^7) - (c*d*(c*d^2 - a*e^2)^2)/(2*e^4*(d
+ e*x)^6) + (3*c^2*d^2*(c*d^2 - a*e^2))/(5*e^4*(d + e*x)^5) - (c^3*d^3)/(4
*e^4*(d + e*x)^4)
```

Defintions of rubi rules used

rule 1121
```
Int[((d_) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Int[ExpandIntegrand[(d + e*x)^(m + p)*(a/d + (c/e)*x)^p, x], x]
/; FreeQ[{a, b, c, d, e, m, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && (Int
egerQ[p] || (GtQ[a, 0] && GtQ[d, 0] && LtQ[c, 0]))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 1.41 (sec) , antiderivative size = 129, normalized size of antiderivative = 1.16

method	result
risch	$\frac{-\frac{c^3 d^3 x^3}{4e}-\frac{3d^2c^2\left(4a\,e^2+c\,d^2\right)x^2}{20e^2}-\frac{dc\left(10a^2e^4+4ac\,d^2e^2+c^2d^4\right)x}{20e^3}-\frac{20e^6a^3+10d^2e^4a^2c+4d^4e^2a\,c^2+d^6c^3}{140e^4}}{(ex+d)^7}$
gosper	$-\frac{35c^3d^3e^3x^3+84x^2a\,c^2d^2e^4+21c^3d^4e^2x^2+70x\,a^2cd\,e^5+28xa\,c^2d^3e^3+7c^3d^5ex+20e^6a^3+10d^2e^4a^2c+4d^4e^2a\,c^2+d^6c^3}{140e^4(ex+d)^7}$
parallelrisch	$\frac{-35d^3c^3x^3e^6-84a\,c^2d^2e^7x^2-21c^3d^4e^5x^2-70a^2cd\,e^8x-28a\,c^2d^3e^6x-7c^3d^5e^4x-20a^3e^9-10a^2c\,d^2e^7-4d^4a\,c^2e^5-c^3d^6e^3}{140e^7(ex+d)^7}$
default	$-\frac{c^3d^3}{4e^4(ex+d)^4}-\frac{e^6a^3-3d^2e^4a^2c+3d^4e^2a\,c^2-d^6c^3}{7e^4(ex+d)^7}-\frac{3c^2d^2\left(a\,e^2-c\,d^2\right)}{5e^4(ex+d)^5}-\frac{dc\left(a^2e^4-2ac\,d^2e^2+c^2d^4\right)}{2e^4(ex+d)^6}$
orering	$-\frac{\left(35c^3d^3e^3x^3+84x^2a\,c^2d^2e^4+21c^3d^4e^2x^2+70x\,a^2cd\,e^5+28xa\,c^2d^3e^3+7c^3d^5ex+20e^6a^3+10d^2e^4a^2c+4d^4e^2a\,c^2+d^6c^3\right)(a}{140e^4(cdx+ae)^3(ex+d)^{10}}$
norman	$-\frac{d^3\left(20a^3e^{12}+10a^2c\,d^2e^{10}+4d^4a\,c^2e^8+c^3d^6e^6\right)}{140e^{10}}-\frac{\left(a^3e^{12}+11a^2c\,d^2e^{10}+17d^4a\,c^2e^8+6c^3d^6e^6\right)x^3}{7e^7}-\frac{d\left(2a^2c\,e^{10}+8a\,c^2d^2e^8+5d^4c^3e^6\right)}{4e^6}$

input
```
int((a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^3/(e*x+d)^11,x,method=_RETURNVERBOSE
)
```

output
```
(-1/4*c^3*d^3/e*x^3-3/20*d^2*c^2/e^2*(4*a*e^2+c*d^2)*x^2-1/20*d*c/e^3*(10*
a^2*e^4+4*a*c*d^2*e^2+c^2*d^4)*x-1/140/e^4*(20*a^3*e^6+10*a^2*c*d^2*e^4+4*
a*c^2*d^4*e^2+c^3*d^6))/(e*x+d)^7
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 197, normalized size of antiderivative = 1.77

$$\int \frac{(ade + (cd^2 + ae^2)x + cdex^2)^3}{(d+ex)^{11}}\,dx = -\frac{35\,c^3d^3e^3x^3 + c^3d^6 + 4\,ac^2d^4e^2 + 10\,a^2cd^2e^4 + 20\,a^3e^6 + 21\,(c^3d^4e^2 + 4\,ac^2d^2e^4)x^2 + 7\,(c^3d^5e + 4\,ac^2d^3e^3 + 10\,a^2cde^5)x}{140\,(e^{11}x^7 + 7\,de^{10}x^6 + 21\,d^2e^9x^5 + 35\,d^3e^8x^4 + 35\,d^4e^7x^3 + 21\,d^5e^6x^2 + 7\,d^6e^5x + d^7e^4)}$$

input
```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^3/(e*x+d)^11,x, algorithm="fri
cas")
```

output
```
-1/140*(35*c^3*d^3*e^3*x^3 + c^3*d^6 + 4*a*c^2*d^4*e^2 + 10*a^2*c*d^2*e^4
+ 20*a^3*e^6 + 21*(c^3*d^4*e^2 + 4*a*c^2*d^2*e^4)*x^2 + 7*(c^3*d^5*e + 4*a
*c^2*d^3*e^3 + 10*a^2*c*d*e^5)*x)/(e^11*x^7 + 7*d*e^10*x^6 + 21*d^2*e^9*x^
5 + 35*d^3*e^8*x^4 + 35*d^4*e^7*x^3 + 21*d^5*e^6*x^2 + 7*d^6*e^5*x + d^7*e
^4)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 211 vs. 2(99) = 198.

Time = 50.32 (sec) , antiderivative size = 211, normalized size of antiderivative = 1.90

$$\int \frac{(ade + (cd^2 + ae^2)x + cdex^2)^3}{(d+ex)^{11}}\,dx = \frac{-20a^3e^6 - 10a^2cd^2e^4 - 4ac^2d^4e^2 - c^3d^6 - 35c^3d^3e^3x^3 + x^2(-84ac^2d^2e^4 - 21c^3d^4e^2) + x(-70a^2cde^5 - 28ac^2d^3e^3 - 7c^3d^5e)}{140d^7e^4 + 980d^6e^5x + 2940d^5e^6x^2 + 4900d^4e^7x^3 + 4900d^3e^8x^4 + 2940d^2e^9x^5 + 980de^{10}x^6 + 140e^{11}x^7}$$

input
```
integrate((a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**3/(e*x+d)**11,x)
```

output
```
(-20*a**3*e**6 - 10*a**2*c*d**2*e**4 - 4*a*c**2*d**4*e**2 - c**3*d**6 - 35
*c**3*d**3*e**3*x**3 + x**2*(-84*a*c**2*d**2*e**4 - 21*c**3*d**4*e**2) + x
*(-70*a**2*c*d*e**5 - 28*a*c**2*d**3*e**3 - 7*c**3*d**5*e))/(140*d**7*e**4
 + 980*d**6*e**5*x + 2940*d**5*e**6*x**2 + 4900*d**4*e**7*x**3 + 4900*d**3
*e**8*x**4 + 2940*d**2*e**9*x**5 + 980*d*e**10*x**6 + 140*e**11*x**7)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 197, normalized size of antiderivative = 1.77

$$\int \frac{(ade + (cd^2 + ae^2)\,x + cdex^2)^3}{(d + ex)^{11}}\,dx = -\frac{35\,c^3d^3e^3x^3 + c^3d^6 + 4\,ac^2d^4e^2 + 10\,a^2cd^2e^4 + 20\,a^3e^6 + 21\,(c^3d^4e^2 + 4\,ac^2d^2e^4)x^2 + 7\,(c^3d^5e + 4\,ac^2d^3}{140\,(e^{11}x^7 + 7\,de^{10}x^6 + 21\,d^2e^9x^5 + 35\,d^3e^8x^4 + 35\,d^4e^7x^3 + 21\,d^5e^6x^2 + 7\,d^6e^5x + d^7e}$$

input
```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^3/(e*x+d)^11,x, algorithm="max
ima")
```

output
```
-1/140*(35*c^3*d^3*e^3*x^3 + c^3*d^6 + 4*a*c^2*d^4*e^2 + 10*a^2*c*d^2*e^4
+ 20*a^3*e^6 + 21*(c^3*d^4*e^2 + 4*a*c^2*d^2*e^4)*x^2 + 7*(c^3*d^5*e + 4*a
*c^2*d^3*e^3 + 10*a^2*c*d*e^5)*x)/(e^11*x^7 + 7*d*e^10*x^6 + 21*d^2*e^9*x^
5 + 35*d^3*e^8*x^4 + 35*d^4*e^7*x^3 + 21*d^5*e^6*x^2 + 7*d^6*e^5*x + d^7*e
^4)
```

Giac [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 129, normalized size of antiderivative = 1.16

$$\int \frac{(ade + (cd^2 + ae^2)\,x + cdex^2)^3}{(d + ex)^{11}}\,dx = -\frac{35\,c^3d^3e^3x^3 + 21\,c^3d^4e^2x^2 + 84\,ac^2d^2e^4x^2 + 7\,c^3d^5ex + 28\,ac^2d^3e^3x + 70\,a^2cde^5x + c^3d^6 + 4\,ac^2d^4e^2 +}{140\,(ex + d)^7e^4}$$

input
```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^3/(e*x+d)^11,x, algorithm="gia
c")
```

output
```
-1/140*(35*c^3*d^3*e^3*x^3 + 21*c^3*d^4*e^2*x^2 + 84*a*c^2*d^2*e^4*x^2 + 7
*c^3*d^5*e*x + 28*a*c^2*d^3*e^3*x + 70*a^2*c*d*e^5*x + c^3*d^6 + 4*a*c^2*d
^4*e^2 + 10*a^2*c*d^2*e^4 + 20*a^3*e^6)/((e*x + d)^7*e^4)
```

Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 195, normalized size of antiderivative = 1.76

$$\int \frac{(ade + (cd^2 + ae^2)x + cdex^2)^3}{(d + ex)^{11}}\,dx = -\frac{\frac{20\,a^3\,e^6+10\,a^2\,c\,d^2\,e^4+4\,a\,c^2\,d^4\,e^2+c^3\,d^6}{140\,e^4}+\frac{c^3\,d^3\,x^3}{4\,e}+\frac{c\,d\,x\,\left(10\,a^2\,e^4+4\,a\,c\,d^2\,e^2+c^2\,d^4\right)}{20\,e^3}+\frac{3\,c^2\,d^2\,x^2\,\left(c\,d^2+4\,a\,e^2\right)}{20\,e^2}}{d^7+7\,d^6\,e\,x+21\,d^5\,e^2\,x^2+35\,d^4\,e^3\,x^3+35\,d^3\,e^4\,x^4+21\,d^2\,e^5\,x^5+7\,d\,e^6\,x^6+e^7\,x^7}$$

input
```
int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^3/(d + e*x)^11,x)
```

output
```
-((20*a^3*e^6 + c^3*d^6 + 4*a*c^2*d^4*e^2 + 10*a^2*c*d^2*e^4)/(140*e^4) +
(c^3*d^3*x^3)/(4*e) + (c*d*x*(10*a^2*e^4 + c^2*d^4 + 4*a*c*d^2*e^2))/(20*e
^3) + (3*c^2*d^2*x^2*(4*a*e^2 + c*d^2))/(20*e^2))/(d^7 + e^7*x^7 + 7*d*e^6
*x^6 + 21*d^5*e^2*x^2 + 35*d^4*e^3*x^3 + 35*d^3*e^4*x^4 + 21*d^2*e^5*x^5 +
 7*d^6*e*x)
```

Reduce [B] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 196, normalized size of antiderivative = 1.77

$$\int \frac{(ade + (cd^2 + ae^2)x + cdex^2)^3}{(d + ex)^{11}}\,dx = \frac{-35c^3d^3e^3x^3 - 84a\,c^2d^2e^4x^2 - 21c^3d^4e^2x^2 - 70a^2cd\,e^5x - 28a\,c^2d^3e^3x - 7c^3d^5ex - 20a^3e^6 - 10a^2c\,d^2e^4}{140e^4\,(e^7x^7 + 7d\,e^6x^6 + 21d^2e^5x^5 + 35d^3e^4x^4 + 35d^4e^3x^3 + 21d^5e^2x^2 + 7d^6ex + d^7)}$$

input
```
int((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^3/(e*x+d)^11,x)
```

output
```
( - 20*a**3*e**6 - 10*a**2*c*d**2*e**4 - 70*a**2*c*d*e**5*x - 4*a*c**2*d**
4*e**2 - 28*a*c**2*d**3*e**3*x - 84*a*c**2*d**2*e**4*x**2 - c**3*d**6 - 7*
c**3*d**5*e*x - 21*c**3*d**4*e**2*x**2 - 35*c**3*d**3*e**3*x**3)/(140*e**4
*(d**7 + 7*d**6*e*x + 21*d**5*e**2*x**2 + 35*d**4*e**3*x**3 + 35*d**3*e**4
*x**4 + 21*d**2*e**5*x**5 + 7*d*e**6*x**6 + e**7*x**7))
```

3.107 $\int \frac{(d+ex)^5}{ade+(cd^2+ae^2)x+cdex^2}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 131

$$\int \frac{(d+ex)^5}{ade+(cd^2+ae^2)\,x+cdex^2}\,dx = \frac{e(cd^2-ae^2)^3\,x}{c^4d^4}+\frac{(cd^2-ae^2)^2\,(d+ex)^2}{2c^3d^3}+\frac{(cd^2-ae^2)\,(d+ex)^3}{3c^2d^2}+\frac{(d+ex)^4}{4cd}+\frac{(cd^2-ae^2)^4\log(ae+cdx)}{c^5d^5}$$

output
```
e*(-a*e^2+c*d^2)^3*x/c^4/d^4+1/2*(-a*e^2+c*d^2)^2*(e*x+d)^2/c^3/d^3+1/3*(-
a*e^2+c*d^2)*(e*x+d)^3/c^2/d^2+1/4*(e*x+d)^4/c/d+(-a*e^2+c*d^2)^4*ln(c*d*x
+a*e)/c^5/d^5
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 134, normalized size of antiderivative = 1.02

$$\int \frac{(d+ex)^5}{ade+(cd^2+ae^2)\,x+cdex^2}\,dx$$
$$= \frac{cdex(-12a^3e^6+6a^2cde^4(8d+ex)-4ac^2d^2e^2(18d^2+6dex+e^2x^2)+c^3d^3(48d^3+36d^2ex+16de^2x^2+3}{12c^5d^5}$$

input
```
Integrate[(d + e*x)^5/(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2),x]
```

output
```
(c*d*e*x*(-12*a^3*e^6 + 6*a^2*c*d*e^4*(8*d + e*x) - 4*a*c^2*d^2*e^2*(18*d^
2 + 6*d*e*x + e^2*x^2) + c^3*d^3*(48*d^3 + 36*d^2*e*x + 16*d*e^2*x^2 + 3*e
^3*x^3)) + 12*(c*d^2 - a*e^2)^4*Log[a*e + c*d*x])/(12*c^5*d^5)
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 131, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.057$, Rules used = {1121, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex)^5}{x(ae^2+cd^2)+ade+cdex^2}\,dx$$

$$\Big\downarrow \text{1121}$$

$$\int \left(\frac{(cd^2-ae^2)^4}{c^4d^4(ae+cdx)}+\frac{e(cd^2-ae^2)^3}{c^4d^4}+\frac{e(d+ex)(cd^2-ae^2)^2}{c^3d^3}+\frac{e(d+ex)^2(cd^2-ae^2)}{c^2d^2}+\frac{e(d+ex)^3}{cd}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{(cd^2-ae^2)^4\log(ae+cdx)}{c^5d^5}+\frac{ex(cd^2-ae^2)^3}{c^4d^4}+\frac{(d+ex)^2(cd^2-ae^2)^2}{2c^3d^3}+\frac{(d+ex)^3(cd^2-ae^2)}{3c^2d^2}+\frac{(d+ex)^4}{4cd}$$

input
```
Int[(d + e*x)^5/(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2),x]
```

output
```
(e*(c*d^2 - a*e^2)^3*x)/(c^4*d^4) + ((c*d^2 - a*e^2)^2*(d + e*x)^2)/(2*c^3
*d^3) + ((c*d^2 - a*e^2)*(d + e*x)^3)/(3*c^2*d^2) + (d + e*x)^4/(4*c*d) +
((c*d^2 - a*e^2)^4*Log[a*e + c*d*x])/(c^5*d^5)
```

Defintions of rubi rules used

rule 1121
```
Int[((d_) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Int[ExpandIntegrand[(d + e*x)^(m + p)*(a/d + (c/e)*x)^p, x], x]
/; FreeQ[{a, b, c, d, e, m, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && (Int
egerQ[p] || (GtQ[a, 0] && GtQ[d, 0] && LtQ[c, 0]))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 1.73 (sec) , antiderivative size = 201, normalized size of antiderivative = 1.53

method	result
norman	$\frac{e^4x^4}{4dc}+\frac{e^2(a^2e^4-4acd^2e^2+6c^2d^4)x^2}{2c^3d^3}-\frac{e^3(ae^2-4cd^2)x^3}{3c^2d^2}-\frac{e(e^6a^3-4d^2e^4a^2c+6d^4e^2ac^2-4d^6c^3)x}{d^4c^4}+\frac{(a^4e^8-4a^3cd$
default	$-\frac{e\left(-\frac{x^4c^3d^3e^3}{4}+\frac{((ae^2-2cd^2)e^2c^2d^2-2c^3d^4e^2)x^3}{3}+\frac{(2(ae^2-2cd^2)d^3ec^2-dec(a^2e^4-2acd^2e^2+2c^2d^4))x^2}{2}+(ae^2-2cd^2)(a^2e\right.}{d^4c^4}$
risch	$\frac{e^4x^4}{4dc}-\frac{e^5x^3a}{3d^2c^2}+\frac{4e^3x^3}{3c}-\frac{2e^4x^2a}{dc^2}+\frac{3e^2dx^2}{c}+\frac{e^6x^2a^2}{2d^3c^3}-\frac{e^7a^3x}{d^4c^4}+\frac{4e^5a^2x}{d^2c^3}-\frac{6e^3ax}{c^2}+\frac{4ed^2x}{c}+\frac{\ln(cdx+ae)a}{d^5c^5}$
parallelrisch	$\frac{3c^4d^4e^4x^4-4ac^3d^3e^5x^3+16c^4d^5e^3x^3+6a^2c^2d^2e^6x^2-24ac^3d^4e^4x^2+36c^4d^6e^2x^2+12\ln(cdx+ae)a^4e^8-48\ln(cdx+ae)a^3cd^2}{12}$

input
```
int((e*x+d)^5/(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e),x,method=_RETURNVERBOSE)
```

output
```
1/4/d/c*e^4*x^4+1/2/c^3/d^3*e^2*(a^2*e^4-4*a*c*d^2*e^2+6*c^2*d^4)*x^2-1/3/
c^2/d^2*e^3*(a*e^2-4*c*d^2)*x^3-e*(a^3*e^6-4*a^2*c*d^2*e^4+6*a*c^2*d^4*e^2
-4*c^3*d^6)/d^4/c^4*x+(a^4*e^8-4*a^3*c*d^2*e^6+6*a^2*c^2*d^4*e^4-4*a*c^3*d
^6*e^2+c^4*d^8)/d^5/c^5*ln(c*d*x+a*e)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 207, normalized size of antiderivative = 1.58

$$\int \frac{(d+ex)^5}{ade+(cd^2+ae^2)\,x+cdex^2}\,dx$$
$$= \frac{3\,c^4d^4e^4x^4+4\,(4\,c^4d^5e^3-ac^3d^3e^5)x^3+6\,(6\,c^4d^6e^2-4\,ac^3d^4e^4+a^2c^2d^2e^6)x^2+12\,(4\,c^4d^7e-6\,ac^3d^5e^3+}{12\,c^5d^5}$$

input
```
integrate((e*x+d)^5/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2),x, algorithm="fricas
")
```

output
```
1/12*(3*c^4*d^4*e^4*x^4 + 4*(4*c^4*d^5*e^3 - a*c^3*d^3*e^5)*x^3 + 6*(6*c^4
*d^6*e^2 - 4*a*c^3*d^4*e^4 + a^2*c^2*d^2*e^6)*x^2 + 12*(4*c^4*d^7*e - 6*a*
c^3*d^5*e^3 + 4*a^2*c^2*d^3*e^5 - a^3*c*d*e^7)*x + 12*(c^4*d^8 - 4*a*c^3*d
^6*e^2 + 6*a^2*c^2*d^4*e^4 - 4*a^3*c*d^2*e^6 + a^4*e^8)*log(c*d*x + a*e))/
(c^5*d^5)
```

Sympy [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 153, normalized size of antiderivative = 1.17

$$\int \frac{(d+ex)^5}{ade+(cd^2+ae^2)\,x+cdex^2}\,dx = x^3\left(-\frac{ae^5}{3c^2d^2}+\frac{4e^3}{3c}\right)+x^2\left(\frac{a^2e^6}{2c^3d^3}-\frac{2ae^4}{c^2d}+\frac{3de^2}{c}\right)$$
$$+x\left(-\frac{a^3e^7}{c^4d^4}+\frac{4a^2e^5}{c^3d^2}-\frac{6ae^3}{c^2}+\frac{4d^2e}{c}\right)$$
$$+\frac{e^4x^4}{4cd}+\frac{(ae^2-cd^2)^4\log\left(ae+cdx\right)}{c^5d^5}$$

input
```
integrate((e*x+d)**5/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2),x)
```

output
```
x**3*(-a*e**5/(3*c**2*d**2) + 4*e**3/(3*c)) + x**2*(a**2*e**6/(2*c**3*d**3
) - 2*a*e**4/(c**2*d) + 3*d*e**2/c) + x*(-a**3*e**7/(c**4*d**4) + 4*a**2*e
**5/(c**3*d**2) - 6*a*e**3/c**2 + 4*d**2*e/c) + e**4*x**4/(4*c*d) + (a*e**
2 - c*d**2)**4*log(a*e + c*d*x)/(c**5*d**5)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 205, normalized size of antiderivative = 1.56

$$\int \frac{(d+ex)^5}{ade+(cd^2+ae^2)x+cdex^2}\,dx$$
$$= \frac{3c^3d^3e^4x^4+4(4c^3d^4e^3-ac^2d^2e^5)x^3+6(6c^3d^5e^2-4ac^2d^3e^4+a^2cde^6)x^2+12(4c^3d^6e-6ac^2d^4e^3+4}{12c^4d^4}$$
$$+\frac{(c^4d^8-4ac^3d^6e^2+6a^2c^2d^4e^4-4a^3cd^2e^6+a^4e^8)\log(cdx+ae)}{c^5d^5}$$

input
```
integrate((e*x+d)^5/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2),x, algorithm="maxima
")
```

output
```
1/12*(3*c^3*d^3*e^4*x^4 + 4*(4*c^3*d^4*e^3 - a*c^2*d^2*e^5)*x^3 + 6*(6*c^3
*d^5*e^2 - 4*a*c^2*d^3*e^4 + a^2*c*d*e^6)*x^2 + 12*(4*c^3*d^6*e - 6*a*c^2*
d^4*e^3 + 4*a^2*c*d^2*e^5 - a^3*e^7)*x)/(c^4*d^4) + (c^4*d^8 - 4*a*c^3*d^6
*e^2 + 6*a^2*c^2*d^4*e^4 - 4*a^3*c*d^2*e^6 + a^4*e^8)*log(c*d*x + a*e)/(c^
5*d^5)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 210, normalized size of antiderivative = 1.60

$$\int \frac{(d+ex)^5}{ade+(cd^2+ae^2)x+cdex^2}\,dx$$
$$= \frac{3c^3d^3e^4x^4+16c^3d^4e^3x^3-4ac^2d^2e^5x^3+36c^3d^5e^2x^2-24ac^2d^3e^4x^2+6a^2cde^6x^2+48c^3d^6ex-72ac^2d}{12c^4d^4}$$
$$+\frac{(c^4d^8-4ac^3d^6e^2+6a^2c^2d^4e^4-4a^3cd^2e^6+a^4e^8)\log(|cdx+ae|)}{c^5d^5}$$

input
```
integrate((e*x+d)^5/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2),x, algorithm="giac")
```

output
```
1/12*(3*c^3*d^3*e^4*x^4 + 16*c^3*d^4*e^3*x^3 - 4*a*c^2*d^2*e^5*x^3 + 36*c^
3*d^5*e^2*x^2 - 24*a*c^2*d^3*e^4*x^2 + 6*a^2*c*d*e^6*x^2 + 48*c^3*d^6*e*x
- 72*a*c^2*d^4*e^3*x + 48*a^2*c*d^2*e^5*x - 12*a^3*e^7*x)/(c^4*d^4) + (c^4
*d^8 - 4*a*c^3*d^6*e^2 + 6*a^2*c^2*d^4*e^4 - 4*a^3*c*d^2*e^6 + a^4*e^8)*lo
g(abs(c*d*x + a*e))/(c^5*d^5)
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 217, normalized size of antiderivative = 1.66

$$\int \frac{(d+ex)^5}{ade+(cd^2+ae^2)x+cdex^2}\,dx$$
$$= x^3\left(\frac{4\,e^3}{3\,c}-\frac{a\,e^5}{3\,c^2\,d^2}\right)+x\left(\frac{4\,d^2\,e}{c}-\frac{a\,e\left(\frac{6\,d\,e^2}{c}-\frac{a\,e\left(\frac{4\,e^3}{c}-\frac{a\,e^5}{c^2\,d^2}\right)}{c\,d}\right)}{c\,d}\right)$$
$$+x^2\left(\frac{3\,d\,e^2}{c}-\frac{a\,e\left(\frac{4\,e^3}{c}-\frac{a\,e^5}{c^2\,d^2}\right)}{2\,c\,d}\right)$$
$$+\frac{\ln\left(a\,e+c\,d\,x\right)\,\left(a^4\,e^8-4\,a^3\,c\,d^2\,e^6+6\,a^2\,c^2\,d^4\,e^4-4\,a\,c^3\,d^6\,e^2+c^4\,d^8\right)}{c^5\,d^5}+\frac{e^4\,x^4}{4\,c\,d}$$

input
```
int((d + e*x)^5/(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2),x)
```

output
```
x^3*((4*e^3)/(3*c) - (a*e^5)/(3*c^2*d^2)) + x*((4*d^2*e)/c - (a*e*((6*d*e^
2)/c - (a*e*((4*e^3)/c - (a*e^5)/(c^2*d^2)))/(c*d)))/(c*d)) + x^2*((3*d*e^
2)/c - (a*e*((4*e^3)/c - (a*e^5)/(c^2*d^2)))/(2*c*d)) + (log(a*e + c*d*x)*
(a^4*e^8 + c^4*d^8 - 4*a*c^3*d^6*e^2 - 4*a^3*c*d^2*e^6 + 6*a^2*c^2*d^4*e^4
))/(c^5*d^5) + (e^4*x^4)/(4*c*d)
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 246, normalized size of antiderivative = 1.88

$$\int \frac{(d+ex)^5}{ade+(cd^2+ae^2)x+cdex^2}\,dx$$
$$= \frac{12\log(cdx+ae)\,a^4e^8-48\log(cdx+ae)\,a^3c\,d^2e^6+72\log(cdx+ae)\,a^2c^2d^4e^4-48\log(cdx+ae)\,a\,c^3d^6e}{}$$

input
```
int((e*x+d)^5/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2),x)
```

output
```
(12*log(a*e + c*d*x)*a**4*e**8 - 48*log(a*e + c*d*x)*a**3*c*d**2*e**6 + 72
*log(a*e + c*d*x)*a**2*c**2*d**4*e**4 - 48*log(a*e + c*d*x)*a*c**3*d**6*e*
*2 + 12*log(a*e + c*d*x)*c**4*d**8 - 12*a**3*c*d*e**7*x + 48*a**2*c**2*d**
3*e**5*x + 6*a**2*c**2*d**2*e**6*x**2 - 72*a*c**3*d**5*e**3*x - 24*a*c**3*
d**4*e**4*x**2 - 4*a*c**3*d**3*e**5*x**3 + 48*c**4*d**7*e*x + 36*c**4*d**6
*e**2*x**2 + 16*c**4*d**5*e**3*x**3 + 3*c**4*d**4*e**4*x**4)/(12*c**5*d**5
)
```

3.108 $\int \frac{(d+ex)^4}{ade+(cd^2+ae^2)x+cdex^2} dx$

Optimal result

Integrand size = 35, antiderivative size = 100

$$\int \frac{(d+ex)^4}{ade+(cd^2+ae^2)x+cdex^2} dx = \frac{e(cd^2-ae^2)^2 x}{c^3d^3} + \frac{(cd^2-ae^2)(d+ex)^2}{2c^2d^2} + \frac{(d+ex)^3}{3cd} + \frac{(cd^2-ae^2)^3 \log(ae+cdx)}{c^4d^4}$$

output
```
e*(-a*e^2+c*d^2)^2*x/c^3/d^3+1/2*(-a*e^2+c*d^2)*(e*x+d)^2/c^2/d^2+1/3*(e*x
+d)^3/c/d+(-a*e^2+c*d^2)^3*ln(c*d*x+a*e)/c^4/d^4
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.91

$$\int \frac{(d+ex)^4}{ade+(cd^2+ae^2)x+cdex^2} dx = \frac{cdex(6a^2e^4-3acde^2(6d+ex)+c^2d^2(18d^2+9dex+2e^2x^2))+6(cd^2-ae^2)^3 \log(ae+cdx)}{6c^4d^4}$$

input
```
Integrate[(d + e*x)^4/(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2),x]
```

output
```
(c*d*e*x*(6*a^2*e^4 - 3*a*c*d*e^2*(6*d + e*x) + c^2*d^2*(18*d^2 + 9*d*e*x
+ 2*e^2*x^2)) + 6*(c*d^2 - a*e^2)^3*Log[a*e + c*d*x])/(6*c^4*d^4)
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.057$, Rules used = {1121, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex)^4}{x\left(ae^2+cd^2\right)+ade+cdex^2}dx$$

$$\Big\downarrow \text{1121}$$

$$\int \left(\frac{\left(cd^2-ae^2\right)^3}{c^3d^3(ae+cdx)}+\frac{e\left(cd^2-ae^2\right)^2}{c^3d^3}+\frac{e(d+ex)\left(cd^2-ae^2\right)}{c^2d^2}+\frac{e(d+ex)^2}{cd}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{\left(cd^2-ae^2\right)^3\log(ae+cdx)}{c^4d^4}+\frac{ex\left(cd^2-ae^2\right)^2}{c^3d^3}+\frac{(d+ex)^2\left(cd^2-ae^2\right)}{2c^2d^2}+\frac{(d+ex)^3}{3cd}$$

input
```
Int[(d + e*x)^4/(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2),x]
```

output
```
(e*(c*d^2 - a*e^2)^2*x)/(c^3*d^3) + ((c*d^2 - a*e^2)*(d + e*x)^2)/(2*c^2*d
^2) + (d + e*x)^3/(3*c*d) + ((c*d^2 - a*e^2)^3*Log[a*e + c*d*x])/(c^4*d^4)
```

Defintions of rubi rules used

rule 1121
```
Int[((d_) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Int[ExpandIntegrand[(d + e*x)^(m + p)*(a/d + (c/e)*x)^p, x], x]
/; FreeQ[{a, b, c, d, e, m, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && (Int
egerQ[p] || (GtQ[a, 0] && GtQ[d, 0] && LtQ[c, 0]))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 1.58 (sec) , antiderivative size = 132, normalized size of antiderivative = 1.32

method	result
default	$\frac{e\left(\frac{1}{3}x^3c^2d^2e^2-\frac{1}{2}x^2acd\,e^3+\frac{3}{2}x^2c^2d^3e+a^2e^4x-3ac\,d^2e^2x+3c^2d^4x\right)}{d^3c^3}+\frac{\left(-e^6a^3+3d^2e^4a^2c-3d^4e^2a\,c^2+d^6c^3\right)\ln(cdx+ae)}{d^4c^4}$
norman	$\frac{e\left(a^2e^4-3ac\,d^2e^2+3c^2d^4\right)x}{d^3c^3}+\frac{e^3x^3}{3cd}-\frac{e^2\left(a\,e^2-3c\,d^2\right)x^2}{2c^2d^2}-\frac{\left(e^6a^3-3d^2e^4a^2c+3d^4e^2a\,c^2-d^6c^3\right)\ln(cdx+ae)}{d^4c^4}$
risch	$\frac{e^3x^3}{3cd}-\frac{e^4x^2a}{2d^2c^2}+\frac{3e^2x^2}{2c}+\frac{e^5a^2x}{d^3c^3}-\frac{3e^3ax}{d\,c^2}+\frac{3edx}{c}-\frac{\ln(cdx+ae)e^6a^3}{d^4c^4}+\frac{3\ln(cdx+ae)e^4a^2}{d^2c^3}-\frac{3\ln(cdx+ae)e^2a}{c^2}$
parallelrisch	$-\frac{-2c^3d^3e^3x^3+3x^2a\,c^2d^2e^4-9c^3d^4e^2x^2+6\ln(cdx+ae)a^3e^6-18\ln(cdx+ae)a^2c\,d^2e^4+18\ln(cdx+ae)a\,c^2d^4e^2-6\ln(cdx+ae}{6d^4c^4}$

input
```
int((e*x+d)^4/(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e),x,method=_RETURNVERBOSE)
```

output
```
e/d^3/c^3*(1/3*x^3*c^2*d^2*e^2-1/2*x^2*a*c*d*e^3+3/2*x^2*c^2*d^3*e+a^2*e^4
*x-3*a*c*d^2*e^2*x+3*c^2*d^4*x)+(-a^3*e^6+3*a^2*c*d^2*e^4-3*a*c^2*d^4*e^2+
c^3*d^6)/d^4/c^4*ln(c*d*x+a*e)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 137, normalized size of antiderivative = 1.37

$$\int \frac{(d+ex)^4}{ade+(cd^2+ae^2)\,x+cdex^2}\,dx$$
$$=\frac{2\,c^3d^3e^3x^3+3\,(3\,c^3d^4e^2-ac^2d^2e^4)x^2+6\,(3\,c^3d^5e-3\,ac^2d^3e^3+a^2cde^5)x+6\,(c^3d^6-3\,ac^2d^4e^2+3\,a^2cd^2}{6\,c^4d^4}$$

input
```
integrate((e*x+d)^4/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2),x, algorithm="fricas
")
```

output
```
1/6*(2*c^3*d^3*e^3*x^3 + 3*(3*c^3*d^4*e^2 - a*c^2*d^2*e^4)*x^2 + 6*(3*c^3*
d^5*e - 3*a*c^2*d^3*e^3 + a^2*c*d*e^5)*x + 6*(c^3*d^6 - 3*a*c^2*d^4*e^2 +
3*a^2*c*d^2*e^4 - a^3*e^6)*log(c*d*x + a*e))/(c^4*d^4)
```

Sympy [A] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 99, normalized size of antiderivative = 0.99

$$\int \frac{(d+ex)^4}{ade+(cd^2+ae^2)\,x+cdex^2}\,dx = x^2\left(-\frac{ae^4}{2c^2d^2}+\frac{3e^2}{2c}\right)+x\left(\frac{a^2e^5}{c^3d^3}-\frac{3ae^3}{c^2d}+\frac{3de}{c}\right) + \frac{e^3x^3}{3cd}-\frac{(ae^2-cd^2)^3\log{(ae+cdx)}}{c^4d^4}$$

input
```
integrate((e*x+d)**4/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2),x)
```

output
```
x**2*(-a*e**4/(2*c**2*d**2) + 3*e**2/(2*c)) + x*(a**2*e**5/(c**3*d**3) - 3
*a*e**3/(c**2*d) + 3*d*e/c) + e**3*x**3/(3*c*d) - (a*e**2 - c*d**2)**3*log
(a*e + c*d*x)/(c**4*d**4)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 135, normalized size of antiderivative = 1.35

$$\int \frac{(d+ex)^4}{ade+(cd^2+ae^2)\,x+cdex^2}\,dx = \frac{2\,c^2d^2e^3x^3+3\,(3\,c^2d^3e^2-acde^4)x^2+6\,(3\,c^2d^4e-3\,acd^2e^3+a^2e^5)x}{6\,c^3d^3} + \frac{(c^3d^6-3\,ac^2d^4e^2+3\,a^2cd^2e^4-a^3e^6)\log{(cdx+ae)}}{c^4d^4}$$

input
```
integrate((e*x+d)^4/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2),x, algorithm="maxima
")
```

output
```
1/6*(2*c^2*d^2*e^3*x^3 + 3*(3*c^2*d^3*e^2 - a*c*d*e^4)*x^2 + 6*(3*c^2*d^4*
e - 3*a*c*d^2*e^3 + a^2*e^5)*x)/(c^3*d^3) + (c^3*d^6 - 3*a*c^2*d^4*e^2 + 3
*a^2*c*d^2*e^4 - a^3*e^6)*log(c*d*x + a*e)/(c^4*d^4)
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 136, normalized size of antiderivative = 1.36

$$\int \frac{(d+ex)^4}{ade+(cd^2+ae^2)\,x+cdex^2}\,dx = \frac{2\,c^2d^2e^3x^3+9\,c^2d^3e^2x^2-3\,acde^4x^2+18\,c^2d^4ex-18\,acd^2e^3x+6\,a^2e^5x}{6\,c^3d^3} + \frac{(c^3d^6-3\,ac^2d^4e^2+3\,a^2cd^2e^4-a^3e^6)\log(|cdx+ae|)}{c^4d^4}$$

input
```
integrate((e*x+d)^4/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2),x, algorithm="giac")
```

output
```
1/6*(2*c^2*d^2*e^3*x^3 + 9*c^2*d^3*e^2*x^2 - 3*a*c*d*e^4*x^2 + 18*c^2*d^4*
e*x - 18*a*c*d^2*e^3*x + 6*a^2*e^5*x)/(c^3*d^3) + (c^3*d^6 - 3*a*c^2*d^4*e
^2 + 3*a^2*c*d^2*e^4 - a^3*e^6)*log(abs(c*d*x + a*e))/(c^4*d^4)
```

Mupad [B] (verification not implemented)

Time = 5.36 (sec) , antiderivative size = 138, normalized size of antiderivative = 1.38

$$\int \frac{(d+ex)^4}{ade+(cd^2+ae^2)\,x+cdex^2}\,dx = x\left(\frac{3\,d\,e}{c}-\frac{a\,e\left(\frac{3\,e^2}{c}-\frac{a\,e^4}{c^2\,d^2}\right)}{c\,d}\right)+x^2\left(\frac{3\,e^2}{2\,c}-\frac{a\,e^4}{2\,c^2\,d^2}\right)+\frac{e^3\,x^3}{3\,c\,d} - \frac{\ln(a\,e+c\,d\,x)\,\left(a^3\,e^6-3\,a^2\,c\,d^2\,e^4+3\,a\,c^2\,d^4\,e^2-c^3\,d^6\right)}{c^4\,d^4}$$

input
```
int((d + e*x)^4/(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2),x)
```

output
```
x*((3*d*e)/c - (a*e*((3*e^2)/c - (a*e^4)/(c^2*d^2)))/(c*d)) + x^2*((3*e^2)
/(2*c) - (a*e^4)/(2*c^2*d^2)) + (e^3*x^3)/(3*c*d) - (log(a*e + c*d*x)*(a^3
*e^6 - c^3*d^6 + 3*a*c^2*d^4*e^2 - 3*a^2*c*d^2*e^4))/(c^4*d^4)
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 162, normalized size of antiderivative = 1.62

$$\int \frac{(d+ex)^4}{ade+(cd^2+ae^2)\,x+cdex^2}\,dx$$
$$= \frac{-6\log(cdx+ae)\,a^3e^6+18\log(cdx+ae)\,a^2c\,d^2e^4-18\log(cdx+ae)\,a\,c^2d^4e^2+6\log(cdx+ae)\,c^3d^6+6}{6c^4d^4}$$

input
```
int((e*x+d)^4/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2),x)
```

output
```
( - 6*log(a*e + c*d*x)*a**3*e**6 + 18*log(a*e + c*d*x)*a**2*c*d**2*e**4 -
18*log(a*e + c*d*x)*a*c**2*d**4*e**2 + 6*log(a*e + c*d*x)*c**3*d**6 + 6*a*
*2*c*d*e**5*x - 18*a*c**2*d**3*e**3*x - 3*a*c**2*d**2*e**4*x**2 + 18*c**3*
d**5*e*x + 9*c**3*d**4*e**2*x**2 + 2*c**3*d**3*e**3*x**3)/(6*c**4*d**4)
```

3.109 $\int \frac{(d+ex)^3}{ade+(cd^2+ae^2)x+cdex^2} dx$

Optimal result

Integrand size = 35, antiderivative size = 69

$$\int \frac{(d+ex)^3}{ade+(cd^2+ae^2)\,x+cdex^2}\,dx=\frac{e(2cd^2-ae^2)\,x}{c^2d^2}+\frac{e^2x^2}{2cd}+\frac{(cd^2-ae^2)^2\log(ae+cdx)}{c^3d^3}$$

output
```
e*(-a*e^2+2*c*d^2)*x/c^2/d^2+1/2*e^2*x^2/c/d+(-a*e^2+c*d^2)^2*ln(c*d*x+a*e
)/c^3/d^3
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.84

$$\int \frac{(d+ex)^3}{ade+(cd^2+ae^2)\,x+cdex^2}\,dx =\frac{cdex(-2ae^2+cd(4d+ex))+2(cd^2-ae^2)^2\log(ae+cdx)}{2c^3d^3}$$

input
```
Integrate[(d + e*x)^3/(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2),x]
```

output
```
(c*d*e*x*(-2*a*e^2 + c*d*(4*d + e*x)) + 2*(c*d^2 - a*e^2)^2*Log[a*e + c*d*
x])/(2*c^3*d^3)
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.057, Rules used = {1121, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex)^3}{x(ae^2+cd^2)+ade+cdex^2}dx$$

$$\Big\downarrow \text{1121}$$

$$\int \left(\frac{(cd^2-ae^2)^2}{c^2d^2(ae+cdx)}+\frac{e(cd^2-ae^2)}{c^2d^2}+\frac{e(d+ex)}{cd}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{(cd^2-ae^2)^2\log(ae+cdx)}{c^3d^3}+\frac{ex(cd^2-ae^2)}{c^2d^2}+\frac{(d+ex)^2}{2cd}$$

input
```
Int[(d + e*x)^3/(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2),x]
```

output
```
(e*(c*d^2 - a*e^2)*x)/(c^2*d^2) + (d + e*x)^2/(2*c*d) + ((c*d^2 - a*e^2)^2
*Log[a*e + c*d*x])/(c^3*d^3)
```

Defintions of rubi rules used

rule 1121
```
Int[((d_) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Int[ExpandIntegrand[(d + e*x)^(m + p)*(a/d + (c/e)*x)^p, x], x]
/; FreeQ[{a, b, c, d, e, m, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && (Int
egerQ[p] || (GtQ[a, 0] && GtQ[d, 0] && LtQ[c, 0]))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 1.58 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.07

method	result	size
default	$-\frac{e\left(-\frac{1}{2}cd\,x^2e+a\,e^2x-2c\,d^2x\right)}{c^2d^2}+\frac{\left(a^2e^4-2ac\,d^2e^2+c^2d^4\right)\ln\left(cdx+ae\right)}{c^3d^3}$	74
norman	$\frac{e^2x^2}{2cd}-\frac{e\left(a\,e^2-2c\,d^2\right)x}{c^2d^2}+\frac{\left(a^2e^4-2ac\,d^2e^2+c^2d^4\right)\ln\left(cdx+ae\right)}{c^3d^3}$	79
risch	$\frac{e^2x^2}{2cd}-\frac{e^3ax}{c^2d^2}+\frac{2ex}{c}+\frac{\ln\left(cdx+ae\right)a^2e^4}{c^3d^3}-\frac{2\ln\left(cdx+ae\right)a\,e^2}{c^2d}+\frac{d\ln\left(cdx+ae\right)}{c}$	93
parallelrisch	$\frac{x^2c^2d^2e^2+2\ln\left(cdx+ae\right)a^2e^4-4\ln\left(cdx+ae\right)ac\,d^2e^2+2\ln\left(cdx+ae\right)c^2d^4-2xacd\,e^3+4x\,c^2d^3e}{2d^3c^3}$	95

input
```
int((e*x+d)^3/(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e),x,method=_RETURNVERBOSE)
```

output
```
-e/c^2/d^2*(-1/2*c*d*x^2*e+a*e^2*x-2*c*d^2*x)+(a^2*e^4-2*a*c*d^2*e^2+c^2*d
^4)/c^3/d^3*ln(c*d*x+a*e)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.14

$$\int \frac{(d+ex)^3}{ade+(cd^2+ae^2)\,x+cdex^2}\,dx = \frac{c^2d^2e^2x^2+2\,(2\,c^2d^3e-acde^3)x+2\,(c^2d^4-2\,acd^2e^2+a^2e^4)\log\left(cdx+ae\right)}{2\,c^3d^3}$$

input
```
integrate((e*x+d)^3/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2),x, algorithm="fricas
")
```

output
```
1/2*(c^2*d^2*e^2*x^2 + 2*(2*c^2*d^3*e - a*c*d*e^3)*x + 2*(c^2*d^4 - 2*a*c*
d^2*e^2 + a^2*e^4)*log(c*d*x + a*e))/(c^3*d^3)
```

Sympy [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.84

$$\int \frac{(d+ex)^3}{ade+(cd^2+ae^2)\,x+cdex^2}\,dx = x\left(-\frac{ae^3}{c^2d^2}+\frac{2e}{c}\right)+\frac{e^2x^2}{2cd}+\frac{(ae^2-cd^2)^2\log{(ae+cdx)}}{c^3d^3}$$

input
```
integrate((e*x+d)**3/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2),x)
```

output
```
x*(-a*e**3/(c**2*d**2) + 2*e/c) + e**2*x**2/(2*c*d) + (a*e**2 - c*d**2)**2
*log(a*e + c*d*x)/(c**3*d**3)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.12

$$\int \frac{(d+ex)^3}{ade+(cd^2+ae^2)\,x+cdex^2}\,dx = \frac{cde^2x^2+2\,(2\,cd^2e-ae^3)x}{2\,c^2d^2} + \frac{(c^2d^4-2\,acd^2e^2+a^2e^4)\log{(cdx+ae)}}{c^3d^3}$$

input
```
integrate((e*x+d)^3/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2),x, algorithm="maxima
")
```

output
```
1/2*(c*d*e^2*x^2 + 2*(2*c*d^2*e - a*e^3)*x)/(c^2*d^2) + (c^2*d^4 - 2*a*c*d
^2*e^2 + a^2*e^4)*log(c*d*x + a*e)/(c^3*d^3)
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.10

$$\int \frac{(d+ex)^3}{ade+(cd^2+ae^2)\,x+cdex^2}\,dx = \frac{cde^2x^2+4\,cd^2ex-2\,ae^3x}{2\,c^2d^2} + \frac{(c^2d^4-2\,acd^2e^2+a^2e^4)\log{(|cdx+ae|)}}{c^3d^3}$$

input
```
integrate((e*x+d)^3/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2),x, algorithm="giac")
```

output
```
1/2*(c*d*e^2*x^2 + 4*c*d^2*e*x - 2*a*e^3*x)/(c^2*d^2) + (c^2*d^4 - 2*a*c*d
^2*e^2 + a^2*e^4)*log(abs(c*d*x + a*e))/(c^3*d^3)
```

Mupad [B] (verification not implemented)

Time = 5.48 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.12

$$\int \frac{(d+ex)^3}{ade+(cd^2+ae^2)x+cdex^2}\,dx = x\left(\frac{2\,e}{c}-\frac{a\,e^3}{c^2\,d^2}\right)+\frac{\ln\left(a\,e+c\,d\,x\right)\,\left(a^2\,e^4-2\,a\,c\,d^2\,e^2+c^2\,d^4\right)}{c^3\,d^3}+\frac{e^2\,x^2}{2\,c\,d}$$

input
```
int((d + e*x)^3/(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2),x)
```

output
```
x*((2*e)/c - (a*e^3)/(c^2*d^2)) + (log(a*e + c*d*x)*(a^2*e^4 + c^2*d^4 - 2
*a*c*d^2*e^2))/(c^3*d^3) + (e^2*x^2)/(2*c*d)
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.36

$$\int \frac{(d+ex)^3}{ade+(cd^2+ae^2)x+cdex^2}\,dx = \frac{2\log(cdx+ae)\,a^2e^4-4\log(cdx+ae)\,ac\,d^2e^2+2\log(cdx+ae)\,c^2d^4-2acd\,e^3x+4c^2d^3ex+c^2d^2e^2x^2}{2c^3d^3}$$

input
```
int((e*x+d)^3/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2),x)
```

output
```
(2*log(a*e + c*d*x)*a**2*e**4 - 4*log(a*e + c*d*x)*a*c*d**2*e**2 + 2*log(a
*e + c*d*x)*c**2*d**4 - 2*a*c*d*e**3*x + 4*c**2*d**3*e*x + c**2*d**2*e**2*
x**2)/(2*c**3*d**3)
```

3.110 $\int \frac{(d+ex)^2}{ade+(cd^2+ae^2)x+cdex^2}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 38

$$\int \frac{(d+ex)^2}{ade+(cd^2+ae^2)\,x+cdex^2}\,dx=\frac{ex}{cd}+\frac{(cd^2-ae^2)\log(ae+cdx)}{c^2d^2}$$

output
```
e*x/c/d+(-a*e^2+c*d^2)*ln(c*d*x+a*e)/c^2/d^2
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.92

$$\int \frac{(d+ex)^2}{ade+(cd^2+ae^2)\,x+cdex^2}\,dx=\frac{cdex+(cd^2-ae^2)\log(ae+cdx)}{c^2d^2}$$

input
```
Integrate[(d + e*x)^2/(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2),x]
```

output
```
(c*d*e*x + (c*d^2 - a*e^2)*Log[a*e + c*d*x])/(c^2*d^2)
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.057$, Rules used = {1121, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex)^2}{x(ae^2+cd^2)+ade+cdex^2}\,dx$$

$$\Big\downarrow \text{1121}$$

$$\int \left(\frac{cd^2-ae^2}{cd(ae+cdx)}+\frac{e}{cd}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{(cd^2-ae^2)\log(ae+cdx)}{c^2d^2}+\frac{ex}{cd}$$

input
```
Int[(d + e*x)^2/(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2),x]
```

output
```
(e*x)/(c*d) + ((c*d^2 - a*e^2)*Log[a*e + c*d*x])/(c^2*d^2)
```

Defintions of rubi rules used

rule 1121
```
Int[((d_) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Int[ExpandIntegrand[(d + e*x)^(m + p)*(a/d + (c/e)*x)^p, x], x]
/; FreeQ[{a, b, c, d, e, m, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && (Int
egerQ[p] || (GtQ[a, 0] && GtQ[d, 0] && LtQ[c, 0]))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 1.58 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.03

method	result	size
default	$\frac{ex}{cd}+\frac{\left(-a\,e^{2}+c\,d^{2}\right)\ln\left(cdx+ae\right)}{c^{2}d^{2}}$	39
norman	$\frac{ex}{cd}-\frac{\left(a\,e^{2}-c\,d^{2}\right)\ln\left(cdx+ae\right)}{c^{2}d^{2}}$	40
risch	$\frac{ex}{cd}-\frac{\ln\left(cdx+ae\right)a\,e^{2}}{c^{2}d^{2}}+\frac{\ln\left(cdx+ae\right)}{c}$	45
parallelrisch	$-\frac{\ln\left(cdx+ae\right)a\,e^{2}-\ln\left(cdx+ae\right)c\,d^{2}-cdxe}{c^{2}d^{2}}$	45

input
```
int((e*x+d)^2/(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e),x,method=_RETURNVERBOSE)
```

output
```
e*x/c/d+(-a*e^2+c*d^2)*ln(c*d*x+a*e)/c^2/d^2
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.92

$$\int \frac{(d+ex)^2}{ade+(cd^2+ae^2)\,x+cdex^2}\,dx=\frac{cdex+(cd^2-ae^2)\log(cdx+ae)}{c^2d^2}$$

input
```
integrate((e*x+d)^2/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2),x, algorithm="fricas
")
```

output
```
(c*d*e*x + (c*d^2 - a*e^2)*log(c*d*x + a*e))/(c^2*d^2)
```

Sympy [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.84

$$\int \frac{(d+ex)^2}{ade+(cd^2+ae^2)\,x+cdex^2}\,dx=\frac{ex}{cd}-\frac{(ae^2-cd^2)\log{(ae+cdx)}}{c^2d^2}$$

input
```
integrate((e*x+d)**2/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2),x)
```

output
```
e*x/(c*d) - (a*e**2 - c*d**2)*log(a*e + c*d*x)/(c**2*d**2)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.00

$$\int \frac{(d+ex)^2}{ade+(cd^2+ae^2)\,x+cdex^2}\,dx=\frac{ex}{cd}+\frac{(cd^2-ae^2)\log{(cdx+ae)}}{c^2d^2}$$

input
```
integrate((e*x+d)^2/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2),x, algorithm="maxima
")
```

output
```
e*x/(c*d) + (c*d^2 - a*e^2)*log(c*d*x + a*e)/(c^2*d^2)
```

Giac [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.03

$$\int \frac{(d+ex)^2}{ade+(cd^2+ae^2)\,x+cdex^2}\,dx=\frac{ex}{cd}+\frac{(cd^2-ae^2)\log{(|cdx+ae|)}}{c^2d^2}$$

input
```
integrate((e*x+d)^2/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2),x, algorithm="giac")
```

output
```
e*x/(c*d) + (c*d^2 - a*e^2)*log(abs(c*d*x + a*e))/(c^2*d^2)
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.03

$$\int \frac{(d+ex)^2}{ade+(cd^2+ae^2)\,x+cdex^2}\,dx=\frac{e\,x}{c\,d}-\frac{\ln\left(a\,e+c\,d\,x\right)\,\left(a\,e^2-c\,d^2\right)}{c^2\,d^2}$$

input
```
int((d + e*x)^2/(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2),x)
```

output
```
(e*x)/(c*d) - (log(a*e + c*d*x)*(a*e^2 - c*d^2))/(c^2*d^2)
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.11

$$\int \frac{(d+ex)^2}{ade+(cd^2+ae^2)\,x+cdex^2}\,dx=\frac{-\log(cdx+ae)\,a\,e^2+\log(cdx+ae)\,c\,d^2+cdex}{c^2d^2}$$

input
```
int((e*x+d)^2/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2),x)
```

output
```
( - log(a*e + c*d*x)*a*e**2 + log(a*e + c*d*x)*c*d**2 + c*d*e*x)/(c**2*d**
2)
```

3.111 $\int \frac{d+ex}{ade+(cd^2+ae^2)x+cdex^2}\,dx$

Optimal result

Integrand size = 33, antiderivative size = 16

$$\int \frac{d+ex}{ade+(cd^2+ae^2)\,x+cdex^2}\,dx=\frac{\log(ae+cdx)}{cd}$$

output
```
ln(c*d*x+a*e)/c/d
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int \frac{d+ex}{ade+(cd^2+ae^2)\,x+cdex^2}\,dx=\frac{\log(ae+cdx)}{cd}$$

input
```
Integrate[(d + e*x)/(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2),x]
```

output
```
Log[a*e + c*d*x]/(c*d)
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.061, Rules used = {1120, 16}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{d+ex}{x\left(ae^2+cd^2\right)+ade+cdex^2}dx$$

$$\Big\downarrow \text{ 1120}$$

$$\int \frac{1}{ae+cdx}dx$$

$$\Big\downarrow \text{ 16}$$

$$\frac{\log(ae+cdx)}{cd}$$

input
```
Int[(d + e*x)/(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2),x]
```

output
```
Log[a*e + c*d*x]/(c*d)
```

Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 1120
```
Int[((d_) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Int[(d + e*x)^(m + p)*(a/d + (c/e)*x)^p, x] /; FreeQ[{a, b, c, d
, e, m}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[p] && (EqQ[m + p,
0] || EqQ[m + 2*p + 2, 0])
```

Maple [A] (verified)

Time = 1.48 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.06

method	result	size
default	$\frac{\ln(cdx+ae)}{cd}$	17
norman	$\frac{\ln(cdx+ae)}{cd}$	17
risch	$\frac{\ln(cdx+ae)}{cd}$	17
parallelrisch	$\frac{\ln(cdx+ae)}{cd}$	17

input
```
int((e*x+d)/(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e),x,method=_RETURNVERBOSE)
```

output
```
ln(c*d*x+a*e)/c/d
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int \frac{d+ex}{ade+(cd^2+ae^2)\,x+cdex^2}\,dx = \frac{\log(cdx+ae)}{cd}$$

input
```
integrate((e*x+d)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2),x, algorithm="fricas")
```

output
```
log(c*d*x + a*e)/(c*d)
```

Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.75

$$\int \frac{d+ex}{ade+(cd^2+ae^2)\,x+cdex^2}\,dx = \frac{\log(ae+cdx)}{cd}$$

input
```
integrate((e*x+d)/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2),x)
```

output
```
log(a*e + c*d*x)/(c*d)
```

Maxima [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int \frac{d+ex}{ade+(cd^2+ae^2)\,x+cdex^2}\,dx=\frac{\log\left(cdx+ae\right)}{cd}$$

input
```
integrate((e*x+d)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2),x, algorithm="maxima")
```

output
```
log(c*d*x + a*e)/(c*d)
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.06

$$\int \frac{d+ex}{ade+(cd^2+ae^2)\,x+cdex^2}\,dx=\frac{\log\left(|cdx+ae|\right)}{cd}$$

input
```
integrate((e*x+d)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2),x, algorithm="giac")
```

output
```
log(abs(c*d*x + a*e))/(c*d)
```

Mupad [B] (verification not implemented)

Time = 5.24 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int \frac{d+ex}{ade+(cd^2+ae^2)\,x+cdex^2}\,dx=\frac{\ln\left(a\,e+c\,d\,x\right)}{c\,d}$$

input
```
int((d + e*x)/(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2),x)
```

output
```
log(a*e + c*d*x)/(c*d)
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int \frac{d+ex}{ade+(cd^2+ae^2)\,x+cdex^2}\,dx=\frac{\log(cdx+ae)}{cd}$$

input
```
int((e*x+d)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2),x)
```

output
```
log(a*e + c*d*x)/(c*d)
```

3.112 $\int \frac{1}{ade+(cd^2+ae^2)x+cdex^2}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 47

$$\int \frac{1}{ade+(cd^2+ae^2)\,x+cdex^2}\,dx=\frac{\log(ae+cdx)}{cd^2-ae^2}-\frac{\log(d+ex)}{cd^2-ae^2}$$

output
```
ln(c*d*x+a*e)/(-a*e^2+c*d^2)-ln(e*x+d)/(-a*e^2+c*d^2)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.70

$$\int \frac{1}{ade+(cd^2+ae^2)\,x+cdex^2}\,dx=\frac{\log(ae+cdx)-\log(d+ex)}{cd^2-ae^2}$$

input
```
Integrate[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(-1),x]
```

output
```
(Log[a*e + c*d*x] - Log[d + e*x])/(c*d^2 - a*e^2)
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.47, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.074, Rules used = {1081, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x\left(ae^2+cd^2\right)+ade+cdex^2}dx$$

$$\Big\downarrow \text{1081}$$

$$cde\int \left(\frac{1}{e\left(cd^2-ae^2\right)\left(ae+cdx\right)}-\frac{1}{cd\left(cd^2-ae^2\right)\left(d+ex\right)}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$cde\left(\frac{\log(ae+cdx)}{cde\left(cd^2-ae^2\right)}-\frac{\log(d+ex)}{cde\left(cd^2-ae^2\right)}\right)$$

input
```
Int[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(-1),x]
```

output
```
c*d*e*(Log[a*e + c*d*x]/(c*d*e*(c*d^2 - a*e^2)) - Log[d + e*x]/(c*d*e*(c*d
^2 - a*e^2)))
```

Defintions of rubi rules used

rule 1081
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = Rt[b^2
- 4*a*c, 2]}, Simp[c   Int[ExpandIntegrand[1/((b/2 - q/2 + c*x)*(b/2 + q/2
+ c*x)), x], x], x]] /; FreeQ[{a, b, c}, x] && NiceSqrtQ[b^2 - 4*a*c]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 1.50 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.72

method	result	size
parallelrisch	$\frac{\ln(ex+d)-\ln(cdx+ae)}{a\,e^2-c\,d^2}$	34
default	$-\frac{\ln(cdx+ae)}{a\,e^2-c\,d^2}+\frac{\ln(ex+d)}{a\,e^2-c\,d^2}$	48
norman	$-\frac{\ln(cdx+ae)}{a\,e^2-c\,d^2}+\frac{\ln(ex+d)}{a\,e^2-c\,d^2}$	48
risch	$\frac{\ln(-ex-d)}{a\,e^2-c\,d^2}-\frac{\ln(cdx+ae)}{a\,e^2-c\,d^2}$	51

input
```
int(1/(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e),x,method=_RETURNVERBOSE)
```

output
```
(ln(e*x+d)-ln(c*d*x+a*e))/(a*e^2-c*d^2)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.70

$$\int \frac{1}{ade+(cd^2+ae^2)\,x+cdex^2}\,dx=\frac{\log{(cdx+ae)}-\log{(ex+d)}}{cd^2-ae^2}$$

input
```
integrate(1/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2),x, algorithm="fricas")
```

output
```
(log(c*d*x + a*e) - log(e*x + d))/(c*d^2 - a*e^2)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 172 vs. $2(36) = 72$.

Time = 0.16 (sec) , antiderivative size = 172, normalized size of antiderivative = 3.66

$$\int \frac{1}{ade+(cd^2+ae^2)\,x+cdex^2}\,dx = \frac{\log\left(x+\frac{-\frac{a^2e^4}{ae^2-cd^2}+\frac{2acd^2e^2}{ae^2-cd^2}+ae^2-\frac{c^2d^4}{ae^2-cd^2}+cd^2}{2cde}\right)}{ae^2-cd^2} - \frac{\log\left(x+\frac{\frac{a^2e^4}{ae^2-cd^2}-\frac{2acd^2e^2}{ae^2-cd^2}+ae^2+\frac{c^2d^4}{ae^2-cd^2}+cd^2}{2cde}\right)}{ae^2-cd^2}$$

input
```
integrate(1/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2),x)
```

output
```
log(x + (-a**2*e**4/(a*e**2 - c*d**2) + 2*a*c*d**2*e**2/(a*e**2 - c*d**2)
+ a*e**2 - c**2*d**4/(a*e**2 - c*d**2) + c*d**2)/(2*c*d*e))/(a*e**2 - c*d*
*2) - log(x + (a**2*e**4/(a*e**2 - c*d**2) - 2*a*c*d**2*e**2/(a*e**2 - c*d
**2) + a*e**2 + c**2*d**4/(a*e**2 - c*d**2) + c*d**2)/(2*c*d*e))/(a*e**2 -
 c*d**2)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.00

$$\int \frac{1}{ade+(cd^2+ae^2)\,x+cdex^2}\,dx = \frac{\log(cdx+ae)}{cd^2-ae^2} - \frac{\log(ex+d)}{cd^2-ae^2}$$

input
```
integrate(1/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2),x, algorithm="maxima")
```

output
```
log(c*d*x + a*e)/(c*d^2 - a*e^2) - log(e*x + d)/(c*d^2 - a*e^2)
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.21

$$\int \frac{1}{ade+(cd^2+ae^2)\,x+cdex^2}\,dx = \frac{cd\log\left(|cdx+ae|\right)}{c^2d^3-acde^2} - \frac{e\log\left(|ex+d|\right)}{cd^2e-ae^3}$$

input
```
integrate(1/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2),x, algorithm="giac")
```

output
```
c*d*log(abs(c*d*x + a*e))/(c^2*d^3 - a*c*d*e^2) - e*log(abs(e*x + d))/(c*d
^2*e - a*e^3)
```

Mupad [B] (verification not implemented)

Time = 5.37 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.09

$$\int \frac{1}{ade+(cd^2+ae^2)\,x+cdex^2}\,dx = \frac{\mathrm{atan}\left(\frac{2\mathrm{i}\,c\,d^2+2\mathrm{i}\,c\,e\,x\,d}{a\,e^2-c\,d^2}+1\mathrm{i}\right)\,2\mathrm{i}}{a\,e^2-c\,d^2}$$

input
```
int(1/(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2),x)
```

output
```
(atan((c*d^2*2i + c*d*e*x*2i)/(a*e^2 - c*d^2) + 1i)*2i)/(a*e^2 - c*d^2)
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.70

$$\int \frac{1}{ade+(cd^2+ae^2)\,x+cdex^2}\,dx = \frac{-\log(cdx+ae)+\log(ex+d)}{a\,e^2-c\,d^2}$$

input
```
int(1/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2),x)
```

output
```
( - log(a*e + c*d*x) + log(d + e*x))/(a*e**2 - c*d**2)
```

3.113 $\int \frac{1}{(d+ex)(ade+(cd^2+ae^2)x+cdex^2)}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 73

$$\int \frac{1}{(d+ex)\left(ade+(cd^2+ae^2)\,x+cdex^2\right)}\,dx = \frac{1}{(cd^2-ae^2)\,(d+ex)}+\frac{cd\log(ae+cdx)}{(cd^2-ae^2)^2}-\frac{cd\log(d+ex)}{(cd^2-ae^2)^2}$$

output
```
1/(-a*e^2+c*d^2)/(e*x+d)+c*d*ln(c*d*x+a*e)/(-a*e^2+c*d^2)^2-c*d*ln(e*x+d)/
(-a*e^2+c*d^2)^2
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.90

$$\int \frac{1}{(d+ex)\left(ade+(cd^2+ae^2)\,x+cdex^2\right)}\,dx = \frac{cd^2-ae^2+cd(d+ex)\log(ae+cdx)-cd(d+ex)\log(d+ex)}{(cd^2-ae^2)^2\,(d+ex)}$$

input
```
Integrate[1/((d + e*x)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)),x]
```

output
```
(c*d^2 - a*e^2 + c*d*(d + e*x)*Log[a*e + c*d*x] - c*d*(d + e*x)*Log[d + e*
x])/((c*d^2 - a*e^2)^2*(d + e*x))
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.057$, Rules used = {1121, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(d+ex)\,(x\,(ae^2+cd^2)+ade+cdex^2)}dx$$

$$\Big\downarrow \text{1121}$$

$$\int \left(\frac{c^2d^2}{(cd^2-ae^2)^2\,(ae+cdx)}-\frac{cde}{(d+ex)\,(cd^2-ae^2)^2}-\frac{e}{(d+ex)^2\,(cd^2-ae^2)}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{(d+ex)\,(cd^2-ae^2)}+\frac{cd\log(ae+cdx)}{(cd^2-ae^2)^2}-\frac{cd\log(d+ex)}{(cd^2-ae^2)^2}$$

input
```
Int[1/((d + e*x)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)),x]
```

output
```
1/((c*d^2 - a*e^2)*(d + e*x)) + (c*d*Log[a*e + c*d*x])/(c*d^2 - a*e^2)^2 -
 (c*d*Log[d + e*x])/(c*d^2 - a*e^2)^2
```

Defintions of rubi rules used

rule 1121
```
Int[((d_) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Int[ExpandIntegrand[(d + e*x)^(m + p)*(a/d + (c/e)*x)^p, x], x]
/; FreeQ[{a, b, c, d, e, m, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && (Int
egerQ[p] || (GtQ[a, 0] && GtQ[d, 0] && LtQ[c, 0]))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 1.76 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.03

method	result	size
default	$\frac{cd\ln(cdx+ae)}{(a\,e^2-c\,d^2)^2}-\frac{1}{(a\,e^2-c\,d^2)(ex+d)}-\frac{cd\ln(ex+d)}{(a\,e^2-c\,d^2)^2}$	75
risch	$-\frac{1}{(a\,e^2-c\,d^2)(ex+d)}-\frac{cd\ln(ex+d)}{a^2e^4-2ac\,d^2e^2+c^2d^4}+\frac{cd\ln(-cdx-ae)}{a^2e^4-2ac\,d^2e^2+c^2d^4}$	103
norman	$\frac{ex}{d(a\,e^2-c\,d^2)(ex+d)}+\frac{cd\ln(cdx+ae)}{a^2e^4-2ac\,d^2e^2+c^2d^4}-\frac{cd\ln(ex+d)}{a^2e^4-2ac\,d^2e^2+c^2d^4}$	105
parallelrisch	$-\frac{\ln(ex+d)xcd\,e^2-\ln(cdx+ae)xcd\,e^2+\ln(ex+d)c\,d^2e-\ln(cdx+ae)c\,d^2e+a\,e^3-c\,d^2e}{(a^2e^4-2ac\,d^2e^2+c^2d^4)(ex+d)e}$	111

input
```
int(1/(e*x+d)/(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e),x,method=_RETURNVERBOSE)
```

output
```
c*d/(a*e^2-c*d^2)^2*ln(c*d*x+a*e)-1/(a*e^2-c*d^2)/(e*x+d)-c*d/(a*e^2-c*d^2
)^2*ln(e*x+d)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 109, normalized size of antiderivative = 1.49

$$\int \frac{1}{(d+ex)\,(ade+(cd^2+ae^2)\,x+cdex^2)}\,dx = \frac{cd^2-ae^2+(cdex+cd^2)\log(cdx+ae)-(cdex+cd^2)\log(ex+d)}{c^2d^5-2\,acd^3e^2+a^2de^4+(c^2d^4e-2\,acd^2e^3+a^2e^5)x}$$

input
```
integrate(1/(e*x+d)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2),x, algorithm="fricas
")
```

output
```
(c*d^2 - a*e^2 + (c*d*e*x + c*d^2)*log(c*d*x + a*e) - (c*d*e*x + c*d^2)*lo
g(e*x + d))/(c^2*d^5 - 2*a*c*d^3*e^2 + a^2*d*e^4 + (c^2*d^4*e - 2*a*c*d^2*
e^3 + a^2*e^5)*x)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 301 vs. $2(63) = 126$.

Time = 0.37 (sec) , antiderivative size = 301, normalized size of antiderivative = 4.12

$$\int \frac{1}{(d+ex)\left(ade+(cd^2+ae^2)\,x+cdex^2\right)}\,dx = -\frac{cd\log\left(x+\frac{-\frac{a^3cde^6}{(ae^2-cd^2)^2}+\frac{3a^2c^2d^3e^4}{(ae^2-cd^2)^2}-\frac{3ac^3d^5e^2}{(ae^2-cd^2)^2}+acde^2+\frac{c^4d^7}{(ae^2-cd^2)^2}+c^2d^3}{2c^2d^2e}\right)}{(ae^2-cd^2)^2} + \frac{cd\log\left(x+\frac{\frac{a^3cde^6}{(ae^2-cd^2)^2}-\frac{3a^2c^2d^3e^4}{(ae^2-cd^2)^2}+\frac{3ac^3d^5e^2}{(ae^2-cd^2)^2}+acde^2-\frac{c^4d^7}{(ae^2-cd^2)^2}+c^2d^3}{2c^2d^2e}\right)}{(ae^2-cd^2)^2} - \frac{1}{ade^2-cd^3+x\,(ae^3-cd^2e)}$$

input
```
integrate(1/(e*x+d)/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2),x)
```

output
```
-c*d*log(x + (-a**3*c*d*e**6/(a*e**2 - c*d**2)**2 + 3*a**2*c**2*d**3*e**4/
(a*e**2 - c*d**2)**2 - 3*a*c**3*d**5*e**2/(a*e**2 - c*d**2)**2 + a*c*d*e**
2 + c**4*d**7/(a*e**2 - c*d**2)**2 + c**2*d**3)/(2*c**2*d**2*e))/(a*e**2 -
 c*d**2)**2 + c*d*log(x + (a**3*c*d*e**6/(a*e**2 - c*d**2)**2 - 3*a**2*c**
2*d**3*e**4/(a*e**2 - c*d**2)**2 + 3*a*c**3*d**5*e**2/(a*e**2 - c*d**2)**2
 + a*c*d*e**2 - c**4*d**7/(a*e**2 - c*d**2)**2 + c**2*d**3)/(2*c**2*d**2*e
))/(a*e**2 - c*d**2)**2 - 1/(a*d*e**2 - c*d**3 + x*(a*e**3 - c*d**2*e))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.47

$$\int \frac{1}{(d+ex)\left(ade+(cd^2+ae^2)\,x+cdex^2\right)}\,dx$$
$$=\frac{cd\log\left(cdx+ae\right)}{c^2d^4-2\,acd^2e^2+a^2e^4}-\frac{cd\log\left(ex+d\right)}{c^2d^4-2\,acd^2e^2+a^2e^4}+\frac{1}{cd^3-ade^2+(cd^2e-ae^3)x}$$

input
```
integrate(1/(e*x+d)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2),x, algorithm="maxima
")
```

output
```
c*d*log(c*d*x + a*e)/(c^2*d^4 - 2*a*c*d^2*e^2 + a^2*e^4) - c*d*log(e*x + d
)/(c^2*d^4 - 2*a*c*d^2*e^2 + a^2*e^4) + 1/(c*d^3 - a*d*e^2 + (c*d^2*e - a*
e^3)*x)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 111, normalized size of antiderivative = 1.52

$$\int \frac{1}{(d+ex)\left(ade+(cd^2+ae^2)\,x+cdex^2\right)}\,dx$$
$$=\frac{c^2d^2\log\left(|cdx+ae|\right)}{c^3d^5-2\,ac^2d^3e^2+a^2cde^4}-\frac{cde\log\left(|ex+d|\right)}{c^2d^4e-2\,acd^2e^3+a^2e^5}+\frac{1}{(cd^2-ae^2)(ex+d)}$$

input
```
integrate(1/(e*x+d)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2),x, algorithm="giac")
```

output
```
c^2*d^2*log(abs(c*d*x + a*e))/(c^3*d^5 - 2*a*c^2*d^3*e^2 + a^2*c*d*e^4) -
c*d*e*log(abs(e*x + d))/(c^2*d^4*e - 2*a*c*d^2*e^3 + a^2*e^5) + 1/((c*d^2
- a*e^2)*(e*x + d))
```

Mupad [B] (verification not implemented)

Time = 5.26 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.30

$$\int \frac{1}{(d+ex)\,(ade+(cd^2+ae^2)\,x+cdex^2)}\,dx$$
$$=\frac{2\,c\,d\,\mathrm{atanh}\left(\frac{a^2\,e^4-c^2\,d^4}{{\left(a\,e^2-c\,d^2\right)}^2}+\frac{2\,c\,d\,e\,x}{a\,e^2-c\,d^2}\right)}{{\left(a\,e^2-c\,d^2\right)}^2}-\frac{1}{\left(a\,e^2-c\,d^2\right)\,\left(d+e\,x\right)}$$

input
```
int(1/((d + e*x)*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)),x)
```

output
```
(2*c*d*atanh((a^2*e^4 - c^2*d^4)/(a*e^2 - c*d^2)^2 + (2*c*d*e*x)/(a*e^2 -
c*d^2)))/(a*e^2 - c*d^2)^2 - 1/((a*e^2 - c*d^2)*(d + e*x))
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 131, normalized size of antiderivative = 1.79

$$\int \frac{1}{(d+ex)\,(ade+(cd^2+ae^2)\,x+cdex^2)}\,dx$$
$$=\frac{\log(cdx+ae)\,c\,d^3+\log(cdx+ae)\,c\,d^2ex-\log(ex+d)\,c\,d^3-\log(ex+d)\,c\,d^2ex+a\,e^3x-c\,d^2ex}{d\,(a^2e^5x-2ac\,d^2e^3x+c^2d^4ex+a^2d\,e^4-2ac\,d^3e^2+c^2d^5)}$$

input
```
int(1/(e*x+d)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2),x)
```

output
```
(log(a*e + c*d*x)*c*d**3 + log(a*e + c*d*x)*c*d**2*e*x - log(d + e*x)*c*d*
*3 - log(d + e*x)*c*d**2*e*x + a*e**3*x - c*d**2*e*x)/(d*(a**2*d*e**4 + a*
*2*e**5*x - 2*a*c*d**3*e**2 - 2*a*c*d**2*e**3*x + c**2*d**5 + c**2*d**4*e*
x))
```

3.114 $\int \frac{1}{(d+ex)^2(ade+(cd^2+ae^2)x+cdex^2)}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 108

$$\begin{aligned}\int \frac{1}{(d+ex)^2\,(ade+(cd^2+ae^2)\,x+cdex^2)}\,dx &= \frac{1}{2\,(cd^2-ae^2)\,(d+ex)^2}+\frac{cd}{(cd^2-ae^2)^2\,(d+ex)}\\ &\quad+\frac{c^2d^2\log(ae+cdx)}{(cd^2-ae^2)^3}-\frac{c^2d^2\log(d+ex)}{(cd^2-ae^2)^3}\end{aligned}$$

output
```
1/2/(-a*e^2+c*d^2)/(e*x+d)^2+c*d/(-a*e^2+c*d^2)^2/(e*x+d)+c^2*d^2*ln(c*d*x
+a*e)/(-a*e^2+c*d^2)^3-c^2*d^2*ln(e*x+d)/(-a*e^2+c*d^2)^3
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 102, normalized size of antiderivative = 0.94

$$\begin{aligned}&\int \frac{1}{(d+ex)^2\,(ade+(cd^2+ae^2)\,x+cdex^2)}\,dx\\ &=\frac{(cd^2-ae^2)\,(-ae^2+cd(3d+2ex))+2c^2d^2(d+ex)^2\log(ae+cdx)-2c^2d^2(d+ex)^2\log(d+ex)}{2\,(cd^2-ae^2)^3\,(d+ex)^2}\end{aligned}$$

input
```
Integrate[1/((d + e*x)^2*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)),x]
```

output
```
((c*d^2 - a*e^2)*(-(a*e^2) + c*d*(3*d + 2*e*x)) + 2*c^2*d^2*(d + e*x)^2*Lo
g[a*e + c*d*x] - 2*c^2*d^2*(d + e*x)^2*Log[d + e*x])/(2*(c*d^2 - a*e^2)^3*
(d + e*x)^2)
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.057$, Rules used = {1121, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(d+ex)^2 (x (ae^2+cd^2)+ade+cdex^2)}dx$$

$$\Big\downarrow \text{1121}$$

$$\int \left(\frac{c^3d^3}{(cd^2-ae^2)^3 (ae+cdx)}-\frac{c^2d^2e}{(d+ex) (cd^2-ae^2)^3}-\frac{cde}{(d+ex)^2 (cd^2-ae^2)^2}-\frac{e}{(d+ex)^3 (cd^2-ae^2)}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{c^2d^2 \log(ae+cdx)}{(cd^2-ae^2)^3}-\frac{c^2d^2 \log(d+ex)}{(cd^2-ae^2)^3}+\frac{cd}{(d+ex) (cd^2-ae^2)^2}+\frac{1}{2(d+ex)^2 (cd^2-ae^2)}$$

input
```
Int[1/((d + e*x)^2*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)),x]
```

output
```
1/(2*(c*d^2 - a*e^2)*(d + e*x)^2) + (c*d)/((c*d^2 - a*e^2)^2*(d + e*x)) +
(c^2*d^2*Log[a*e + c*d*x])/(c*d^2 - a*e^2)^3 - (c^2*d^2*Log[d + e*x])/(c*d
^2 - a*e^2)^3
```

Defintions of rubi rules used

rule 1121
```
Int[((d_) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Int[ExpandIntegrand[(d + e*x)^(m + p)*(a/d + (c/e)*x)^p, x], x]
/; FreeQ[{a, b, c, d, e, m, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && (Int
egerQ[p] || (GtQ[a, 0] && GtQ[d, 0] && LtQ[c, 0]))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 1.77 (sec) , antiderivative size = 107, normalized size of antiderivative = 0.99

method	result
default	$-\frac{c^2 d^2 \ln(cdx+ae)}{(a\,e^2-c\,d^2)^3}-\frac{1}{2(a\,e^2-c\,d^2)(ex+d)^2}+\frac{c^2 d^2 \ln(ex+d)}{(a\,e^2-c\,d^2)^3}+\frac{cd}{(a\,e^2-c\,d^2)^2(ex+d)}$
risch	$\frac{\frac{decx}{a^2e^4-2ac\,d^2e^2+c^2d^4}-\frac{a\,e^2-3c\,d^2}{2\left(a^2e^4-2ac\,d^2e^2+c^2d^4\right)}}{(ex+d)^2}-\frac{c^2d^2\ln(cdx+ae)}{e^6a^3-3d^2e^4a^2c+3d^4e^2a\,c^2-d^6c^3}+\frac{c^2d^2\ln(-ex-d)}{e^6a^3-3d^2e^4a^2c+3d^4e^2a\,c^2-d^6c^3}$
norman	$\frac{\frac{-a\,e^3+2c\,d^2e}{2e\left(a^2e^4-2ac\,d^2e^2+c^2d^4\right)}-\frac{e^2c\,x^2}{2\left(a^2e^4-2ac\,d^2e^2+c^2d^4\right)}}{(ex+d)^2}+\frac{c^2d^2\ln(ex+d)}{e^6a^3-3d^2e^4a^2c+3d^4e^2a\,c^2-d^6c^3}-\frac{c^2d^2\ln(cdx+ae)}{e^6a^3-3d^2e^4a^2c+3d^4e^2a\,c^2-d^6}$
parallelrisch	$\frac{2\ln(ex+d)x^2c^2d^2e^4-2\ln(cdx+ae)x^2c^2d^2e^4+4\ln(ex+d)x\,c^2d^3e^3-4\ln(cdx+ae)x\,c^2d^3e^3+2\ln(ex+d)c^2d^4e^2-2\ln(cdx+ae)}{2(e^6a^3-3d^2e^4a^2c+3d^4e^2a\,c^2-d^6c^3)(ex+d)^2e^2}$

input
```
int(1/(e*x+d)^2/(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e),x,method=_RETURNVERBOSE)
```

output
```
-c^2*d^2/(a*e^2-c*d^2)^3*ln(c*d*x+a*e)-1/2/(a*e^2-c*d^2)/(e*x+d)^2+c^2*d^2
/(a*e^2-c*d^2)^3*ln(e*x+d)+c*d/(a*e^2-c*d^2)^2/(e*x+d)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 266 vs. $2(106) = 212$.

Time = 0.08 (sec) , antiderivative size = 266, normalized size of antiderivative = 2.46

$$\int \frac{1}{(d+ex)^2 (ade + (cd^2+ae^2)x + cdex^2)} dx = \frac{3c^2d^4 - 4acd^2e^2 + a^2e^4 + 2(c^2d^3e - acde^3)x + 2(c^2d^2e^2x^2 + 2c^2d^3ex + c^2d^4)\log(cdx+ae) - 2(c^2d^2e}{2(c^3d^8 - 3ac^2d^6e^2 + 3a^2cd^4e^4 - a^3d^2e^6 + (c^3d^6e^2 - 3ac^2d^4e^4 + 3a^2cd^2e^6 - a^3e^8)x^2 + 2(c^3d^7e - 3}$$

input
```
integrate(1/(e*x+d)^2/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2),x, algorithm="fric
as")
```

output
```
1/2*(3*c^2*d^4 - 4*a*c*d^2*e^2 + a^2*e^4 + 2*(c^2*d^3*e - a*c*d*e^3)*x + 2
*(c^2*d^2*e^2*x^2 + 2*c^2*d^3*e*x + c^2*d^4)*log(c*d*x + a*e) - 2*(c^2*d^2
*e^2*x^2 + 2*c^2*d^3*e*x + c^2*d^4)*log(e*x + d))/(c^3*d^8 - 3*a*c^2*d^6*e
^2 + 3*a^2*c*d^4*e^4 - a^3*d^2*e^6 + (c^3*d^6*e^2 - 3*a*c^2*d^4*e^4 + 3*a^
2*c*d^2*e^6 - a^3*e^8)*x^2 + 2*(c^3*d^7*e - 3*a*c^2*d^5*e^3 + 3*a^2*c*d^3*
e^5 - a^3*d*e^7)*x)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 471 vs. $2(94) = 188$.

Time = 0.56 (sec) , antiderivative size = 471, normalized size of antiderivative = 4.36

$$\int \frac{1}{(d+ex)^2 (ade + (cd^2+ae^2)x + cdex^2)} dx = \frac{c^2d^2 \log\left(x + \frac{-\frac{a^4c^2d^2e^8}{(ae^2-cd^2)^3} + \frac{4a^3c^3d^4e^6}{(ae^2-cd^2)^3} - \frac{6a^2c^4d^6e^4}{(ae^2-cd^2)^3} + \frac{4ac^5d^8e^2}{(ae^2-cd^2)^3} + ac^2d^2e^2 - \frac{c^6d^{10}}{(ae^2-cd^2)^3} + c^3d^4}{2c^3d^3e}\right)}{(ae^2-cd^2)^3} - \frac{c^2d^2 \log\left(x + \frac{\frac{a^4c^2d^2e^8}{(ae^2-cd^2)^3} - \frac{4a^3c^3d^4e^6}{(ae^2-cd^2)^3} + \frac{6a^2c^4d^6e^4}{(ae^2-cd^2)^3} - \frac{4ac^5d^8e^2}{(ae^2-cd^2)^3} + ac^2d^2e^2 + \frac{c^6d^{10}}{(ae^2-cd^2)^3} + c^3d^4}{2c^3d^3e}\right)}{(ae^2-cd^2)^3} + \frac{-ae^2 + 3cd^2 + 2cdex}{2a^2d^2e^4 - 4acd^4e^2 + 2c^2d^6 + x^2 \cdot (2a^2e^6 - 4acd^2e^4 + 2c^2d^4e^2) + x(4a^2de^5 - 8acd^3e^3 + 4c^2d^5e)}$$

input
```
integrate(1/(e*x+d)**2/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2),x)
```

output
```
c**2*d**2*log(x + (-a**4*c**2*d**2*e**8/(a*e**2 - c*d**2)**3 + 4*a**3*c**3
*d**4*e**6/(a*e**2 - c*d**2)**3 - 6*a**2*c**4*d**6*e**4/(a*e**2 - c*d**2)*
*3 + 4*a*c**5*d**8*e**2/(a*e**2 - c*d**2)**3 + a*c**2*d**2*e**2 - c**6*d**
10/(a*e**2 - c*d**2)**3 + c**3*d**4)/(2*c**3*d**3*e))/(a*e**2 - c*d**2)**3
 - c**2*d**2*log(x + (a**4*c**2*d**2*e**8/(a*e**2 - c*d**2)**3 - 4*a**3*c*
*3*d**4*e**6/(a*e**2 - c*d**2)**3 + 6*a**2*c**4*d**6*e**4/(a*e**2 - c*d**2
)**3 - 4*a*c**5*d**8*e**2/(a*e**2 - c*d**2)**3 + a*c**2*d**2*e**2 + c**6*d
**10/(a*e**2 - c*d**2)**3 + c**3*d**4)/(2*c**3*d**3*e))/(a*e**2 - c*d**2)*
*3 + (-a*e**2 + 3*c*d**2 + 2*c*d*e*x)/(2*a**2*d**2*e**4 - 4*a*c*d**4*e**2
+ 2*c**2*d**6 + x**2*(2*a**2*e**6 - 4*a*c*d**2*e**4 + 2*c**2*d**4*e**2) +
x*(4*a**2*d*e**5 - 8*a*c*d**3*e**3 + 4*c**2*d**5*e))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 228 vs. $2(106) = 212$.

Time = 0.04 (sec) , antiderivative size = 228, normalized size of antiderivative = 2.11

$$\int \frac{1}{(d+ex)^2 (ade+(cd^2+ae^2)x+cdex^2)}\,dx = \frac{c^2d^2\log(cdx+ae)}{c^3d^6-3\,ac^2d^4e^2+3\,a^2cd^2e^4-a^3e^6} - \frac{c^2d^2\log(ex+d)}{c^3d^6-3\,ac^2d^4e^2+3\,a^2cd^2e^4-a^3e^6} + \frac{2\,cdex+3\,cd^2-ae^2}{2\,(c^2d^6-2\,acd^4e^2+a^2d^2e^4+(c^2d^4e^2-2\,acd^2e^4+a^2e^6)x^2+2\,(c^2d^5e-2\,acd^3e^3+a^2de^5)x)}$$

input
```
integrate(1/(e*x+d)^2/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2),x, algorithm="maxi
ma")
```

output
```
c^2*d^2*log(c*d*x + a*e)/(c^3*d^6 - 3*a*c^2*d^4*e^2 + 3*a^2*c*d^2*e^4 - a^
3*e^6) - c^2*d^2*log(e*x + d)/(c^3*d^6 - 3*a*c^2*d^4*e^2 + 3*a^2*c*d^2*e^4
 - a^3*e^6) + 1/2*(2*c*d*e*x + 3*c*d^2 - a*e^2)/(c^2*d^6 - 2*a*c*d^4*e^2 +
 a^2*d^2*e^4 + (c^2*d^4*e^2 - 2*a*c*d^2*e^4 + a^2*e^6)*x^2 + 2*(c^2*d^5*e
- 2*a*c*d^3*e^3 + a^2*d*e^5)*x)
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 158, normalized size of antiderivative = 1.46

$$\int \frac{1}{(d+ex)^2(ade+(cd^2+ae^2)x+cdex^2)}\,dx = \frac{c^2d^2e\log\left(\left|cd-\frac{cd^2}{ex+d}+\frac{ae^2}{ex+d}\right|\right)}{c^3d^6e-3\,ac^2d^4e^3+3\,a^2cd^2e^5-a^3e^7}+\frac{\frac{2\,cde^2}{ex+d}+\frac{cd^2e^2}{(ex+d)^2}-\frac{ae^4}{(ex+d)^2}}{2\,(c^2d^4e^2-2\,acd^2e^4+a^2e^6)}$$

input
```
integrate(1/(e*x+d)^2/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2),x, algorithm="giac
")
```

output
```
c^2*d^2*e*log(abs(c*d - c*d^2/(e*x + d) + a*e^2/(e*x + d)))/(c^3*d^6*e - 3
*a*c^2*d^4*e^3 + 3*a^2*c*d^2*e^5 - a^3*e^7) + 1/2*(2*c*d*e^2/(e*x + d) + c
*d^2*e^2/(e*x + d)^2 - a*e^4/(e*x + d)^2)/(c^2*d^4*e^2 - 2*a*c*d^2*e^4 + a
^2*e^6)
```

Mupad [B] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 220, normalized size of antiderivative = 2.04

$$\int \frac{1}{(d+ex)^2(ade+(cd^2+ae^2)x+cdex^2)}\,dx = -\frac{\frac{a\,e^2-3\,c\,d^2}{2\,(a^2\,e^4-2\,a\,c\,d^2\,e^2+c^2\,d^4)}-\frac{c\,d\,e\,x}{a^2\,e^4-2\,a\,c\,d^2\,e^2+c^2\,d^4}}{d^2+2\,d\,e\,x+e^2\,x^2} - \frac{2\,c^2\,d^2\,\mathrm{atanh}\left(\frac{a^3\,e^6-a^2\,c\,d^2\,e^4-a\,c^2\,d^4\,e^2+c^3\,d^6}{(a\,e^2-c\,d^2)^3}+\frac{2\,c\,d\,e\,x\,(a^2\,e^4-2\,a\,c\,d^2\,e^2+c^2\,d^4)}{(a\,e^2-c\,d^2)^3}\right)}{(a\,e^2-c\,d^2)^3}$$

input
```
int(1/((d + e*x)^2*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)),x)
```

output
```
- ((a*e^2 - 3*c*d^2)/(2*(a^2*e^4 + c^2*d^4 - 2*a*c*d^2*e^2)) - (c*d*e*x)/(
a^2*e^4 + c^2*d^4 - 2*a*c*d^2*e^2))/(d^2 + e^2*x^2 + 2*d*e*x) - (2*c^2*d^2
*atanh((a^3*e^6 + c^3*d^6 - a*c^2*d^4*e^2 - a^2*c*d^2*e^4)/(a*e^2 - c*d^2)
^3 + (2*c*d*e*x*(a^2*e^4 + c^2*d^4 - 2*a*c*d^2*e^2))/(a*e^2 - c*d^2)^3))/(
a*e^2 - c*d^2)^3
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 307, normalized size of antiderivative = 2.84

$$\int \frac{1}{(d+ex)^2\,(ade+(cd^2+ae^2)\,x+cdex^2)}\,dx$$
$$= \frac{-2\log(cdx+ae)\,c^2d^4-4\log(cdx+ae)\,c^2d^3ex-2\log(cdx+ae)\,c^2d^2e^2x^2+2\log(ex+d)\,c^2d^4+4\log(e}{2a^3e^8x^2-6a^2c\,d^2e^6x^2+6a\,c^2d^4e^4x^2-2c^3d^6e^2x^2+4a^3d\,e^7x-12a^2c\,d^3e^5x+12}$$

input
```
int(1/(e*x+d)^2/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2),x)
```

output
```
( - 2*log(a*e + c*d*x)*c**2*d**4 - 4*log(a*e + c*d*x)*c**2*d**3*e*x - 2*lo
g(a*e + c*d*x)*c**2*d**2*e**2*x**2 + 2*log(d + e*x)*c**2*d**4 + 4*log(d +
e*x)*c**2*d**3*e*x + 2*log(d + e*x)*c**2*d**2*e**2*x**2 - a**2*e**4 + 3*a*
c*d**2*e**2 - a*c*e**4*x**2 - 2*c**2*d**4 + c**2*d**2*e**2*x**2)/(2*(a**3*
d**2*e**6 + 2*a**3*d*e**7*x + a**3*e**8*x**2 - 3*a**2*c*d**4*e**4 - 6*a**2
*c*d**3*e**5*x - 3*a**2*c*d**2*e**6*x**2 + 3*a*c**2*d**6*e**2 + 6*a*c**2*d
**5*e**3*x + 3*a*c**2*d**4*e**4*x**2 - c**3*d**8 - 2*c**3*d**7*e*x - c**3*
d**6*e**2*x**2))
```

3.115 $\int \frac{1}{(d+ex)^3\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 139

$$\int \frac{1}{(d+ex)^3\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)}\,dx = \frac{1}{3\left(cd^2-ae^2\right)(d+ex)^3}+\frac{cd}{2\left(cd^2-ae^2\right)^2(d+ex)^2} + \frac{c^2d^2}{\left(cd^2-ae^2\right)^3(d+ex)}+\frac{c^3d^3\log(ae+cdx)}{\left(cd^2-ae^2\right)^4}-\frac{c^3d^3\log(d+ex)}{\left(cd^2-ae^2\right)^4}$$

output
```
1/3/(-a*e^2+c*d^2)/(e*x+d)^3+1/2*c*d/(-a*e^2+c*d^2)^2/(e*x+d)^2+c^2*d^2/(-
a*e^2+c*d^2)^3/(e*x+d)+c^3*d^3*ln(c*d*x+a*e)/(-a*e^2+c*d^2)^4-c^3*d^3*ln(e
*x+d)/(-a*e^2+c*d^2)^4
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 135, normalized size of antiderivative = 0.97

$$\int \frac{1}{(d+ex)^3\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)}\,dx = \frac{\left(cd^2-ae^2\right)\left(2a^2e^4-acde^2(7d+3ex)+c^2d^2\left(11d^2+15dex+6e^2x^2\right)\right)+6c^3d^3(d+ex)^3\log(ae+cdx)-}{6\left(cd^2-ae^2\right)^4(d+ex)^3}$$

input
```
Integrate[1/((d + e*x)^3*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)),x]
```

output
```
((c*d^2 - a*e^2)*(2*a^2*e^4 - a*c*d*e^2*(7*d + 3*e*x) + c^2*d^2*(11*d^2 +
15*d*e*x + 6*e^2*x^2)) + 6*c^3*d^3*(d + e*x)^3*Log[a*e + c*d*x] - 6*c^3*d^
3*(d + e*x)^3*Log[d + e*x])/(6*(c*d^2 - a*e^2)^4*(d + e*x)^3)
```

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 139, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.057, Rules used = {1121, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(d+ex)^3 \left(x \left(ae^2+cd^2\right)+ade+cdex^2\right)} \, dx$$

$$\Big\downarrow \text{1121}$$

$$\int \left(\frac{c^4d^4}{\left(cd^2-ae^2\right)^4 (ae+cdx)}-\frac{c^3d^3e}{(d+ex)\left(cd^2-ae^2\right)^4}-\frac{c^2d^2e}{(d+ex)^2\left(cd^2-ae^2\right)^3}-\frac{cde}{(d+ex)^3\left(cd^2-ae^2\right)^2}-\frac{}{(d+\right.$$

$$\Big\downarrow \text{2009}$$

$$\frac{c^3d^3\log(ae+cdx)}{\left(cd^2-ae^2\right)^4}-\frac{c^3d^3\log(d+ex)}{\left(cd^2-ae^2\right)^4}+\frac{c^2d^2}{(d+ex)\left(cd^2-ae^2\right)^3}+\frac{cd}{2(d+ex)^2\left(cd^2-ae^2\right)^2}+\frac{1}{3(d+ex)^3\left(cd^2-ae^2\right)}$$

input
```
Int[1/((d + e*x)^3*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)),x]
```

output
```
1/(3*(c*d^2 - a*e^2)*(d + e*x)^3) + (c*d)/(2*(c*d^2 - a*e^2)^2*(d + e*x)^2
) + (c^2*d^2)/((c*d^2 - a*e^2)^3*(d + e*x)) + (c^3*d^3*Log[a*e + c*d*x])/(
c*d^2 - a*e^2)^4 - (c^3*d^3*Log[d + e*x])/(c*d^2 - a*e^2)^4
```

Defintions of rubi rules used

rule 1121
```
Int[((d_) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Int[ExpandIntegrand[(d + e*x)^(m + p)*(a/d + (c/e)*x)^p, x], x]
/; FreeQ[{a, b, c, d, e, m, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && (Int
egerQ[p] || (GtQ[a, 0] && GtQ[d, 0] && LtQ[c, 0]))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 1.83 (sec) , antiderivative size = 137, normalized size of antiderivative = 0.99

method	result
default	$\frac{c^{3} d^{3} \ln \left(c d x+a e\right)}{\left(a \,e^{2}-c \,d^{2}\right)^{4}}-\frac{1}{3 \left(a \,e^{2}-c \,d^{2}\right) \left(e x+d\right)^{3}}-\frac{c^{2} d^{2}}{\left(a \,e^{2}-c \,d^{2}\right)^{3} \left(e x+d\right)}+\frac{c d}{2 \left(a \,e^{2}-c \,d^{2}\right)^{2} \left(e x+d\right)^{2}}-\frac{c^{3} d^{3} \ln \left(e x+d\right)}{\left(a \,e^{2}-c \,d^{2}\right)^{4}}$
risch	$\frac{-\frac{e^{2} c^{2} d^{2} x^{2}}{e^{6} a^{3}-3 d^{2} e^{4} a^{2} c+3 d^{4} e^{2} a \,c^{2}-d^{6} c^{3}}+\frac{\left(a \,e^{2}-5 c \,d^{2}\right) e c d x}{2 e^{6} a^{3}-6 d^{2} e^{4} a^{2} c+6 d^{4} e^{2} a \,c^{2}-2 d^{6} c^{3}}-\frac{2 a^{2} e^{4}-7 a c \,d^{2} e^{2}+11 c^{2} d^{4}}{6 \left(e^{6} a^{3}-3 d^{2} e^{4} a^{2} c+3 d^{4} e^{2} a \,c^{2}-d^{6} c^{3}\right)}}{\left(e x+d\right)^{3}}-\frac{}{a^{4} e^{8}-4 a^{3} c \,d}$
parallelrisch	$-\frac{2 a^{3} d^{2} e^{7}-6 c^{3} d^{8} e+x^{3} a^{2} c d \,e^{8}-6 x^{3} a \,c^{2} d^{3} e^{6}+3 x^{2} a^{2} c \,d^{2} e^{7}-12 x^{2} a \,c^{2} d^{4} e^{5}+6 \ln \left(e x+d\right) x^{3} c^{3} d^{5} e^{4}-6 \ln \left(c d x+a e\right) x^{3} c^{3} d^{5} e^{4}+18}{6}$
norman	$\frac{\frac{-a^{2} e^{5}+3 a c \,d^{2} e^{3}-3 c^{2} d^{4} e}{3 e \left(e^{6} a^{3}-3 d^{2} e^{4} a^{2} c+3 d^{4} e^{2} a \,c^{2}-d^{6} c^{3}\right)}+\frac{e^{2} \left(-d \,e^{3} a c+5 d^{3} e \,c^{2}\right) x^{3}}{6 d^{2} \left(e^{6} a^{3}-3 d^{2} e^{4} a^{2} c+3 d^{4} e^{2} a \,c^{2}-d^{6} c^{3}\right)}+\frac{\left(-d \,e^{3} a c+3 d^{3} e \,c^{2}\right) e \,x^{2}}{2 d \left(e^{6} a^{3}-3 d^{2} e^{4} a^{2} c+3 d^{4} e^{2} a \,c^{2}-d^{6} c^{3}\right)}}{\left(e x+d\right)^{3}}+\frac{}{a^{4} e^{8}-}$

input
```
int(1/(e*x+d)^3/(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e),x,method=_RETURNVERBOSE)
```

output
```
c^3*d^3/(a*e^2-c*d^2)^4*ln(c*d*x+a*e)-1/3/(a*e^2-c*d^2)/(e*x+d)^3-c^2*d^2/
(a*e^2-c*d^2)^3/(e*x+d)+1/2*c*d/(a*e^2-c*d^2)^2/(e*x+d)^2-c^3*d^3/(a*e^2-c
*d^2)^4*ln(e*x+d)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 454 vs. $2(135) = 270$.

Time = 0.09 (sec) , antiderivative size = 454, normalized size of antiderivative = 3.27

$$\int \frac{1}{(d+ex)^3\,(ade+(cd^2+ae^2)\,x+cdex^2)}\,dx$$
$$= \frac{11\,c^3d^6 - 18\,ac^2d^4e^2 + 9\,a^2cd^2e^4 - 2\,a^3e^6 + 6\,(c^3d^4e^2 - ac^2d^2e^4)x^2 + 3\,(5\,c^3d^5e - 6\,ac^2d^3e^3 + a^2c}{6\,(c^4d^{11} - 4\,ac^3d^9e^2 + 6\,a^2c^2d^7e^4 - 4\,a^3cd^5e^6 + a^4d^3e^8 + (c^4d^8e^3 - 4\,ac^3d^6e^5 + 6\,a^2c^2d^4e^7 - 4\,a^3cd^2e^9 +}$$

input
```
integrate(1/(e*x+d)^3/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2),x, algorithm="fric
as")
```

output
```
1/6*(11*c^3*d^6 - 18*a*c^2*d^4*e^2 + 9*a^2*c*d^2*e^4 - 2*a^3*e^6 + 6*(c^3*
d^4*e^2 - a*c^2*d^2*e^4)*x^2 + 3*(5*c^3*d^5*e - 6*a*c^2*d^3*e^3 + a^2*c*d*
e^5)*x + 6*(c^3*d^3*e^3*x^3 + 3*c^3*d^4*e^2*x^2 + 3*c^3*d^5*e*x + c^3*d^6)
*log(c*d*x + a*e) - 6*(c^3*d^3*e^3*x^3 + 3*c^3*d^4*e^2*x^2 + 3*c^3*d^5*e*x
 + c^3*d^6)*log(e*x + d))/(c^4*d^11 - 4*a*c^3*d^9*e^2 + 6*a^2*c^2*d^7*e^4
- 4*a^3*c*d^5*e^6 + a^4*d^3*e^8 + (c^4*d^8*e^3 - 4*a*c^3*d^6*e^5 + 6*a^2*c
^2*d^4*e^7 - 4*a^3*c*d^2*e^9 + a^4*e^11)*x^3 + 3*(c^4*d^9*e^2 - 4*a*c^3*d^
7*e^4 + 6*a^2*c^2*d^5*e^6 - 4*a^3*c*d^3*e^8 + a^4*d*e^10)*x^2 + 3*(c^4*d^1
0*e - 4*a*c^3*d^8*e^3 + 6*a^2*c^2*d^6*e^5 - 4*a^3*c*d^4*e^7 + a^4*d^2*e^9)
*x)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 672 vs. $2(121) = 242$.

Time = 0.80 (sec) , antiderivative size = 672, normalized size of antiderivative = 4.83

$$\int \frac{1}{(d+ex)^3\,(ade+(cd^2+ae^2)\,x+cdex^2)}\,dx = -\frac{c^3d^3\log\left(x+\frac{-\frac{a^5c^3d^3e^{10}}{(ae^2-cd^2)^4}+\frac{5a^4c^4d^5e^8}{(ae^2-cd^2)^4}-\frac{10a^3c^5d^7e^6}{(ae^2-cd^2)^4}+\frac{10a^2c^6d^9e^4}{(ae^2-cd^2)^4}-\frac{5ac^7d^{11}e^2}{(ae^2-cd^2)^4}+ac^3d^3e^2+\frac{c^8d^{13}}{(ae^2-cd^2)^4}+c^4d^5}{2c^4d^4e}\right)}{(ae^2-cd^2)^4} + \frac{c^3d^3\log\left(x+\frac{\frac{a^5c^3d^3e^{10}}{(ae^2-cd^2)^4}-\frac{5a^4c^4d^5e^8}{(ae^2-cd^2)^4}+\frac{10a^3c^5d^7e^6}{(ae^2-cd^2)^4}-\frac{10a^2c^6d^9e^4}{(ae^2-cd^2)^4}+\frac{5ac^7d^{11}e^2}{(ae^2-cd^2)^4}+ac^3d^3e^2-\frac{c^8d^{13}}{(ae^2-cd^2)^4}+c^4d^5}{2c^4d^4e}\right)}{(ae^2-cd^2)^4} + \frac{-2a^2e^4+7acd^2e^2-11c^2d^4-6c^2d}{6a^3d^3e^6-18a^2cd^5e^4+18ac^2d^7e^2-6c^3d^9+x^3\cdot(6a^3e^9-18a^2cd^2e^7+18ac^2d^4e^5-6c^3d^6e^3)+x^2\cdot(18}$$

input
```
integrate(1/(e*x+d)**3/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2),x)
```

output
```
-c**3*d**3*log(x + (-a**5*c**3*d**3*e**10/(a*e**2 - c*d**2)**4 + 5*a**4*c*
*4*d**5*e**8/(a*e**2 - c*d**2)**4 - 10*a**3*c**5*d**7*e**6/(a*e**2 - c*d**
2)**4 + 10*a**2*c**6*d**9*e**4/(a*e**2 - c*d**2)**4 - 5*a*c**7*d**11*e**2/
(a*e**2 - c*d**2)**4 + a*c**3*d**3*e**2 + c**8*d**13/(a*e**2 - c*d**2)**4
+ c**4*d**5)/(2*c**4*d**4*e))/(a*e**2 - c*d**2)**4 + c**3*d**3*log(x + (a*
*5*c**3*d**3*e**10/(a*e**2 - c*d**2)**4 - 5*a**4*c**4*d**5*e**8/(a*e**2 -
c*d**2)**4 + 10*a**3*c**5*d**7*e**6/(a*e**2 - c*d**2)**4 - 10*a**2*c**6*d*
*9*e**4/(a*e**2 - c*d**2)**4 + 5*a*c**7*d**11*e**2/(a*e**2 - c*d**2)**4 +
a*c**3*d**3*e**2 - c**8*d**13/(a*e**2 - c*d**2)**4 + c**4*d**5)/(2*c**4*d*
*4*e))/(a*e**2 - c*d**2)**4 + (-2*a**2*e**4 + 7*a*c*d**2*e**2 - 11*c**2*d*
*4 - 6*c**2*d**2*e**2*x**2 + x*(3*a*c*d*e**3 - 15*c**2*d**3*e))/(6*a**3*d*
*3*e**6 - 18*a**2*c*d**5*e**4 + 18*a*c**2*d**7*e**2 - 6*c**3*d**9 + x**3*(
6*a**3*e**9 - 18*a**2*c*d**2*e**7 + 18*a*c**2*d**4*e**5 - 6*c**3*d**6*e**3
) + x**2*(18*a**3*d*e**8 - 54*a**2*c*d**3*e**6 + 54*a*c**2*d**5*e**4 - 18*
c**3*d**7*e**2) + x*(18*a**3*d**2*e**7 - 54*a**2*c*d**4*e**5 + 54*a*c**2*d
**6*e**3 - 18*c**3*d**8*e))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 393 vs. $2(135) = 270$.

Time = 0.05 (sec) , antiderivative size = 393, normalized size of antiderivative = 2.83

$$\int \frac{1}{(d+ex)^3\,(ade+(cd^2+ae^2)\,x+cdex^2)}\,dx$$
$$= \frac{c^3d^3\log\left(cdx+ae\right)}{c^4d^8-4\,ac^3d^6e^2+6\,a^2c^2d^4e^4-4\,a^3cd^2e^6+a^4e^8}$$
$$-\frac{c^3d^3\log\left(ex+d\right)}{c^4d^8-4\,ac^3d^6e^2+6\,a^2c^2d^4e^4-4\,a^3cd^2e^6+a^4e^8}$$
$$+\frac{6\,c^2d^2e^2x^2+11\,c^2d^4-7\,acd^2e^2+2\,a^2e^4+3}{6\,(c^3d^9-3\,ac^2d^7e^2+3\,a^2cd^5e^4-a^3d^3e^6+(c^3d^6e^3-3\,ac^2d^4e^5+3\,a^2cd^2e^7-a^3e^9)x^3+3\,(c^3d^7e^2-3\,a}$$

input
```
integrate(1/(e*x+d)^3/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2),x, algorithm="maxi
ma")
```

output
```
c^3*d^3*log(c*d*x + a*e)/(c^4*d^8 - 4*a*c^3*d^6*e^2 + 6*a^2*c^2*d^4*e^4 -
4*a^3*c*d^2*e^6 + a^4*e^8) - c^3*d^3*log(e*x + d)/(c^4*d^8 - 4*a*c^3*d^6*e
^2 + 6*a^2*c^2*d^4*e^4 - 4*a^3*c*d^2*e^6 + a^4*e^8) + 1/6*(6*c^2*d^2*e^2*x
^2 + 11*c^2*d^4 - 7*a*c*d^2*e^2 + 2*a^2*e^4 + 3*(5*c^2*d^3*e - a*c*d*e^3)*
x)/(c^3*d^9 - 3*a*c^2*d^7*e^2 + 3*a^2*c*d^5*e^4 - a^3*d^3*e^6 + (c^3*d^6*e
^3 - 3*a*c^2*d^4*e^5 + 3*a^2*c*d^2*e^7 - a^3*e^9)*x^3 + 3*(c^3*d^7*e^2 - 3
*a*c^2*d^5*e^4 + 3*a^2*c*d^3*e^6 - a^3*d*e^8)*x^2 + 3*(c^3*d^8*e - 3*a*c^2
*d^6*e^3 + 3*a^2*c*d^4*e^5 - a^3*d^2*e^7)*x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 275 vs. $2(135) = 270$.

Time = 0.12 (sec) , antiderivative size = 275, normalized size of antiderivative = 1.98

$$\int \frac{1}{(d+ex)^3\,(ade+(cd^2+ae^2)\,x+cdex^2)}\,dx$$
$$= \frac{c^4d^4\log\left(|cdx+ae|\right)}{c^5d^9-4\,ac^4d^7e^2+6\,a^2c^3d^5e^4-4\,a^3c^2d^3e^6+a^4cde^8}$$
$$-\frac{c^3d^3e\log\left(|ex+d|\right)}{c^4d^8e-4\,ac^3d^6e^3+6\,a^2c^2d^4e^5-4\,a^3cd^2e^7+a^4e^9}$$
$$+\frac{11\,c^3d^6-18\,ac^2d^4e^2+9\,a^2cd^2e^4-2\,a^3e^6+6\,(c^3d^4e^2-ac^2d^2e^4)x^2+3\,(5\,c^3d^5e-6\,ac^2d^3e^3+a^2cde^5)x}{6\,(cd^2-ae^2)^4(ex+d)^3}$$

input
```
integrate(1/(e*x+d)^3/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2),x, algorithm="giac
")
```

output
```
c^4*d^4*log(abs(c*d*x + a*e))/(c^5*d^9 - 4*a*c^4*d^7*e^2 + 6*a^2*c^3*d^5*e
^4 - 4*a^3*c^2*d^3*e^6 + a^4*c*d*e^8) - c^3*d^3*e*log(abs(e*x + d))/(c^4*d
^8*e - 4*a*c^3*d^6*e^3 + 6*a^2*c^2*d^4*e^5 - 4*a^3*c*d^2*e^7 + a^4*e^9) +
1/6*(11*c^3*d^6 - 18*a*c^2*d^4*e^2 + 9*a^2*c*d^2*e^4 - 2*a^3*e^6 + 6*(c^3*
d^4*e^2 - a*c^2*d^2*e^4)*x^2 + 3*(5*c^3*d^5*e - 6*a*c^2*d^3*e^3 + a^2*c*d*
e^5)*x)/((c*d^2 - a*e^2)^4*(e*x + d)^3)
```

Mupad [B] (verification not implemented)

Time = 5.28 (sec) , antiderivative size = 359, normalized size of antiderivative = 2.58

$$\int \frac{1}{(d+ex)^3\,(ade+(cd^2+ae^2)\,x+cdex^2)}\,dx = \frac{2\,c^3\,d^3\,\mathrm{atanh}\left(\frac{a^4\,e^8-2\,a^3\,c\,d^2\,e^6+2\,a\,c^3\,d^6\,e^2-c^4\,d^8}{{\left(a\,e^2-c\,d^2\right)}^4}+\frac{2\,c\,d\,e\,x\,\left(a^3\,e^6-3\,a^2\,c\,d^2\,e^4+3\,a\,c^2\,d^4\,e^2-c^3\,d^6\right)}{{\left(a\,e^2-c\,d^2\right)}^4}\right)}{{\left(a\,e^2-c\,d^2\right)}^4} - \frac{\frac{2\,a^2\,e^4-7\,a\,c\,d^2\,e^2+11\,c^2\,d^4}{6\,\left(a^3\,e^6-3\,a^2\,c\,d^2\,e^4+3\,a\,c^2\,d^4\,e^2-c^3\,d^6\right)}-\frac{c\,d\,x\,\left(a\,e^3-5\,c\,d^2\,e\right)}{2\,\left(a^3\,e^6-3\,a^2\,c\,d^2\,e^4+3\,a\,c^2\,d^4\,e^2-c^3\,d^6\right)}+\frac{c^2\,d^2\,e^2\,x^2}{a^3\,e^6-3\,a^2\,c\,d^2\,e^4+3\,a\,c^2\,d^4\,e^2-c^3\,d^6}}{d^3+3\,d^2\,e\,x+3\,d\,e^2\,x^2+e^3\,x^3}$$

input
```
int(1/((d + e*x)^3*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)),x)
```

output
```
(2*c^3*d^3*atanh((a^4*e^8 - c^4*d^8 + 2*a*c^3*d^6*e^2 - 2*a^3*c*d^2*e^6)/(
a*e^2 - c*d^2)^4 + (2*c*d*e*x*(a^3*e^6 - c^3*d^6 + 3*a*c^2*d^4*e^2 - 3*a^2
*c*d^2*e^4))/(a*e^2 - c*d^2)^4))/(a*e^2 - c*d^2)^4 - ((2*a^2*e^4 + 11*c^2*
d^4 - 7*a*c*d^2*e^2)/(6*(a^3*e^6 - c^3*d^6 + 3*a*c^2*d^4*e^2 - 3*a^2*c*d^2
*e^4)) - (c*d*x*(a*e^3 - 5*c*d^2*e))/(2*(a^3*e^6 - c^3*d^6 + 3*a*c^2*d^4*e
^2 - 3*a^2*c*d^2*e^4)) + (c^2*d^2*e^2*x^2)/(a^3*e^6 - c^3*d^6 + 3*a*c^2*d^
4*e^2 - 3*a^2*c*d^2*e^4))/(d^3 + e^3*x^3 + 3*d*e^2*x^2 + 3*d^2*e*x)
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 523, normalized size of antiderivative = 3.76

$$\int \frac{1}{(d+ex)^3 (ade+(cd^2+ae^2)x+cdex^2)} dx = \frac{6\log(cdx+ae)c^3d^6+18\log(cdx+ae)c^3d^5ex+18\log(cdx+ae)c^3d^4e^2x^2+6\log(cdx+ae)c^3d^3e^3x^3 -}{6a^4e^{11}x^3-24a^3c\,d^2e^9x^3+36a^2c^2d^4e^7x^3-24a\,c^3d^6e^5x^3+6c^4d^8e^3x^3+18a^4d\,e^{10}x^2-72a^3}$$

input
```
int(1/(e*x+d)^3/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2),x)
```

output
```
(6*log(a*e + c*d*x)*c**3*d**6 + 18*log(a*e + c*d*x)*c**3*d**5*e*x + 18*log
(a*e + c*d*x)*c**3*d**4*e**2*x**2 + 6*log(a*e + c*d*x)*c**3*d**3*e**3*x**3
 - 6*log(d + e*x)*c**3*d**6 - 18*log(d + e*x)*c**3*d**5*e*x - 18*log(d + e
*x)*c**3*d**4*e**2*x**2 - 6*log(d + e*x)*c**3*d**3*e**3*x**3 - 2*a**3*e**6
 + 9*a**2*c*d**2*e**4 + 3*a**2*c*d*e**5*x - 16*a*c**2*d**4*e**2 - 12*a*c**
2*d**3*e**3*x + 2*a*c**2*d*e**5*x**3 + 9*c**3*d**6 + 9*c**3*d**5*e*x - 2*c
**3*d**3*e**3*x**3)/(6*(a**4*d**3*e**8 + 3*a**4*d**2*e**9*x + 3*a**4*d*e**
10*x**2 + a**4*e**11*x**3 - 4*a**3*c*d**5*e**6 - 12*a**3*c*d**4*e**7*x - 1
2*a**3*c*d**3*e**8*x**2 - 4*a**3*c*d**2*e**9*x**3 + 6*a**2*c**2*d**7*e**4
+ 18*a**2*c**2*d**6*e**5*x + 18*a**2*c**2*d**5*e**6*x**2 + 6*a**2*c**2*d**
4*e**7*x**3 - 4*a*c**3*d**9*e**2 - 12*a*c**3*d**8*e**3*x - 12*a*c**3*d**7*
e**4*x**2 - 4*a*c**3*d**6*e**5*x**3 + c**4*d**11 + 3*c**4*d**10*e*x + 3*c*
*4*d**9*e**2*x**2 + c**4*d**8*e**3*x**3))
```

3.116 $\int \frac{(d+ex)^8}{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^2}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 219

$$\int \frac{(d+ex)^8}{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^2}\,dx = \frac{15e^2\left(cd^2-ae^2\right)^4x}{c^6d^6}-\frac{\left(cd^2-ae^2\right)^6}{c^7d^7(ae+cdx)}+\frac{10e^3\left(cd^2-ae^2\right)^3(ae+cdx)^2}{c^7d^7}+\frac{5e^4\left(cd^2-ae^2\right)^2(ae+cdx)^3}{c^7d^7}+\frac{3e^5\left(cd^2-ae^2\right)(ae+cdx)^4}{2c^7d^7}+\frac{e^6(ae+cdx)^5}{5c^7d^7}+\frac{6e\left(cd^2-ae^2\right)^5\log(ae+cdx)}{c^7d^7}$$

output
```
15*e^2*(-a*e^2+c*d^2)^4*x/c^6/d^6-(-a*e^2+c*d^2)^6/c^7/d^7/(c*d*x+a*e)+10*
e^3*(-a*e^2+c*d^2)^3*(c*d*x+a*e)^2/c^7/d^7+5*e^4*(-a*e^2+c*d^2)^2*(c*d*x+a
*e)^3/c^7/d^7+3/2*e^5*(-a*e^2+c*d^2)*(c*d*x+a*e)^4/c^7/d^7+1/5*e^6*(c*d*x+
a*e)^5/c^7/d^7+6*e*(-a*e^2+c*d^2)^5*ln(c*d*x+a*e)/c^7/d^7
```

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 339, normalized size of antiderivative = 1.55

$$\int \frac{(d+ex)^8}{(ade+(cd^2+ae^2)x+cdex^2)^2}\,dx$$
$$= \frac{-10a^6e^{12}+10a^5cde^{10}(6d+5ex)-30a^4c^2d^2e^8(5d^2+8dex-e^2x^2)+10a^3c^3d^3e^6(20d^3+45d^2ex-15de^2}{}$$

input
```
Integrate[(d + e*x)^8/(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^2,x]
```

output
```
(-10*a^6*e^12 + 10*a^5*c*d*e^10*(6*d + 5*e*x) - 30*a^4*c^2*d^2*e^8*(5*d^2
+ 8*d*e*x - e^2*x^2) + 10*a^3*c^3*d^3*e^6*(20*d^3 + 45*d^2*e*x - 15*d*e^2*
x^2 - e^3*x^3) - 5*a^2*c^4*d^4*e^4*(30*d^4 + 80*d^3*e*x - 60*d^2*e^2*x^2 -
 10*d*e^3*x^3 - e^4*x^4) + a*c^5*d^5*e^2*(60*d^5 + 150*d^4*e*x - 300*d^3*e
^2*x^2 - 100*d^2*e^3*x^3 - 25*d*e^4*x^4 - 3*e^5*x^5) + c^6*d^6*(-10*d^6 +
150*d^4*e^2*x^2 + 100*d^3*e^3*x^3 + 50*d^2*e^4*x^4 + 15*d*e^5*x^5 + 2*e^6*
x^6) - 60*e*(-(c*d^2) + a*e^2)^5*(a*e + c*d*x)*Log[a*e + c*d*x])/(10*c^7*d
^7*(a*e + c*d*x))
```

Rubi [A] (verified)

Time = 0.52 (sec) , antiderivative size = 219, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.057$, Rules used = {1121, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex)^8}{(x(ae^2+cd^2)+ade+cdex^2)^2}\,dx$$
$$\Big\downarrow \; 1121$$
$$\int \left(\frac{e^6(ae+cdx)^4}{c^6d^6}+\frac{20(cd^2e-ae^3)^3(ae+cdx)}{c^6d^6}+\frac{6e(cd^2-ae^2)^5}{c^6d^6(ae+cdx)}+\frac{(cd^2-ae^2)^6}{c^6d^6(ae+cdx)^2}+\frac{15e^2(cd^2-ae^2)^4}{c^6d^6}+\frac{6}{}\right.$$
$$\Big\downarrow \; 2009$$

$$\frac{e^6(ae+cdx)^5}{5c^7d^7}-\frac{(cd^2-ae^2)^6}{c^7d^7(ae+cdx)}+\frac{6e(cd^2-ae^2)^5\log(ae+cdx)}{c^7d^7}+\frac{3e^5(cd^2-ae^2)(ae+cdx)^4}{2c^7d^7}+\frac{5e^4(cd^2-ae^2)^2(ae+cdx)^3}{c^7d^7}+\frac{10e^3(cd^2-ae^2)^3(ae+cdx)^2}{c^7d^7}+\frac{15e^2x(cd^2-ae^2)^4}{c^6d^6}$$

input
```
Int[(d + e*x)^8/(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^2,x]
```

output
```
(15*e^2*(c*d^2 - a*e^2)^4*x)/(c^6*d^6) - (c*d^2 - a*e^2)^6/(c^7*d^7*(a*e +
 c*d*x)) + (10*e^3*(c*d^2 - a*e^2)^3*(a*e + c*d*x)^2)/(c^7*d^7) + (5*e^4*(
c*d^2 - a*e^2)^2*(a*e + c*d*x)^3)/(c^7*d^7) + (3*e^5*(c*d^2 - a*e^2)*(a*e
+ c*d*x)^4)/(2*c^7*d^7) + (e^6*(a*e + c*d*x)^5)/(5*c^7*d^7) + (6*e*(c*d^2
- a*e^2)^5*Log[a*e + c*d*x])/(c^7*d^7)
```

Defintions of rubi rules used

rule 1121
```
Int[((d_) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Int[ExpandIntegrand[(d + e*x)^(m + p)*(a/d + (c/e)*x)^p, x], x]
/; FreeQ[{a, b, c, d, e, m, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && (Int
egerQ[p] || (GtQ[a, 0] && GtQ[d, 0] && LtQ[c, 0]))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 1.56 (sec) , antiderivative size = 402, normalized size of antiderivative = 1.84

method	result
default	$\frac{e^2\left(\frac{1}{5}c^4d^4e^4x^5-\frac{1}{2}a\,c^3d^3e^5x^4+\frac{3}{2}c^4d^5e^3x^4+a^2c^2d^2e^6x^3-4a\,c^3d^4e^4x^3+5c^4d^6e^2x^3-2a^3cd\,e^7x^2+9a^2c^2d^3e^5x^2-15a\,c^3d^5e^3x^2}{d^6c^6}$
norman	$\frac{\frac{e^3\left(3a^4e^8-16a^3c\,d^2e^6+35a^2c^2d^4e^4-40a\,c^3d^6e^2+25c^4d^8\right)x^3}{d^5c^5}-\frac{6a^6e^{12}-27a^5d^2e^{10}c+45a^4d^4e^8c^2-30a^3d^6e^6c^3+9a\,d^{10}e^2c^5+d^{12}c^6}{c^7d^6}+}{}$
risch	$-\frac{4e^6a\,x^3}{d^2c^3}-\frac{2e^9a^3x^2}{d^5c^5}+\frac{9e^7a^2x^2}{d^3c^4}-\frac{15e^5a\,x^2}{d\,c^3}+\frac{5e^{10}a^4x}{d^6c^6}-\frac{24e^8a^3x}{d^4c^5}+\frac{45e^6a^2x}{d^2c^4}-\frac{40e^4ax}{c^3}+\frac{e^8a^2x^3}{d^4c^4}+\frac{e^6x^5}{5d^2c^2}$
parallelrisch	$-\frac{3x^5a\,c^5d^5e^7-5x^4a^2c^4d^4e^8+25x^4a\,c^5d^6e^6+10x^3a^3c^3d^3e^9-50x^3a^2c^4d^5e^7+100x^3a\,c^5d^7e^5+60a^6e^{12}+10d^{12}c^6+60\ln(cdx}{}$

input
```
int((e*x+d)^8/(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^2,x,method=_RETURNVERBOSE)
```

output
```
e^2/d^6/c^6*(1/5*c^4*d^4*e^4*x^5-1/2*a*c^3*d^3*e^5*x^4+3/2*c^4*d^5*e^3*x^4
+a^2*c^2*d^2*e^6*x^3-4*a*c^3*d^4*e^4*x^3+5*c^4*d^6*e^2*x^3-2*a^3*c*d*e^7*x
^2+9*a^2*c^2*d^3*e^5*x^2-15*a*c^3*d^5*e^3*x^2+10*c^4*d^7*e*x^2+5*a^4*e^8*x
-24*a^3*c*d^2*e^6*x+45*a^2*c^2*d^4*e^4*x-40*a*c^3*d^6*e^2*x+15*c^4*d^8*x)-
6/d^7*e/c^7*(a^5*e^10-5*a^4*c*d^2*e^8+10*a^3*c^2*d^4*e^6-10*a^2*c^3*d^6*e^
4+5*a*c^4*d^8*e^2-c^5*d^10)*ln(c*d*x+a*e)-1/c^7/d^7*(a^6*e^12-6*a^5*c*d^2*
e^10+15*a^4*c^2*d^4*e^8-20*a^3*c^3*d^6*e^6+15*a^2*c^4*d^8*e^4-6*a*c^5*d^10
*e^2+c^6*d^12)/(c*d*x+a*e)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 545 vs. $2(215) = 430$.

Time = 0.08 (sec) , antiderivative size = 545, normalized size of antiderivative = 2.49

$$\int \frac{(d+ex)^8}{(ade+(cd^2+ae^2)x+cdex^2)^2}\,dx$$
$$= \frac{2\,c^6d^6e^6x^6 - 10\,c^6d^{12} + 60\,ac^5d^{10}e^2 - 150\,a^2c^4d^8e^4 + 200\,a^3c^3d^6e^6 - 150\,a^4c^2d^4e^8 + 60\,a^5cd^2e^{10} - 10\,a^6e}{}$$

input
```
integrate((e*x+d)^8/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^2,x, algorithm="fric
as")
```

output
```
1/10*(2*c^6*d^6*e^6*x^6 - 10*c^6*d^12 + 60*a*c^5*d^10*e^2 - 150*a^2*c^4*d^
8*e^4 + 200*a^3*c^3*d^6*e^6 - 150*a^4*c^2*d^4*e^8 + 60*a^5*c*d^2*e^10 - 10
*a^6*e^12 + 3*(5*c^6*d^7*e^5 - a*c^5*d^5*e^7)*x^5 + 5*(10*c^6*d^8*e^4 - 5*
a*c^5*d^6*e^6 + a^2*c^4*d^4*e^8)*x^4 + 10*(10*c^6*d^9*e^3 - 10*a*c^5*d^7*e
^5 + 5*a^2*c^4*d^5*e^7 - a^3*c^3*d^3*e^9)*x^3 + 30*(5*c^6*d^10*e^2 - 10*a*
c^5*d^8*e^4 + 10*a^2*c^4*d^6*e^6 - 5*a^3*c^3*d^4*e^8 + a^4*c^2*d^2*e^10)*x
^2 + 10*(15*a*c^5*d^9*e^3 - 40*a^2*c^4*d^7*e^5 + 45*a^3*c^3*d^5*e^7 - 24*a
^4*c^2*d^3*e^9 + 5*a^5*c*d*e^11)*x + 60*(a*c^5*d^10*e^2 - 5*a^2*c^4*d^8*e^
4 + 10*a^3*c^3*d^6*e^6 - 10*a^4*c^2*d^4*e^8 + 5*a^5*c*d^2*e^10 - a^6*e^12
+ (c^6*d^11*e - 5*a*c^5*d^9*e^3 + 10*a^2*c^4*d^7*e^5 - 10*a^3*c^3*d^5*e^7
+ 5*a^4*c^2*d^3*e^9 - a^5*c*d*e^11)*x)*log(c*d*x + a*e))/(c^8*d^8*x + a*c^
7*d^7*e)
```

Sympy [A] (verification not implemented)

Time = 0.92 (sec) , antiderivative size = 345, normalized size of antiderivative = 1.58

$$\int \frac{(d+ex)^8}{(ade+(cd^2+ae^2)\,x+cdex^2)^2}\,dx$$
$$= x^4\left(-\frac{ae^7}{2c^3d^3}+\frac{3e^5}{2c^2d}\right)+x^3\left(\frac{a^2e^8}{c^4d^4}-\frac{4ae^6}{c^3d^2}+\frac{5e^4}{c^2}\right)$$
$$+x^2\left(-\frac{2a^3e^9}{c^5d^5}+\frac{9a^2e^7}{c^4d^3}-\frac{15ae^5}{c^3d}+\frac{10de^3}{c^2}\right)$$
$$+x\left(\frac{5a^4e^{10}}{c^6d^6}-\frac{24a^3e^8}{c^5d^4}+\frac{45a^2e^6}{c^4d^2}-\frac{40ae^4}{c^3}+\frac{15d^2e^2}{c^2}\right)$$
$$+\frac{-a^6e^{12}+6a^5cd^2e^{10}-15a^4c^2d^4e^8+20a^3c^3d^6e^6-15a^2c^4d^8e^4+6ac^5d^{10}e^2-c^6d^{12}}{ac^7d^7e+c^8d^8x}$$
$$+\frac{e^6x^5}{5c^2d^2}-\frac{6e(ae^2-cd^2)^5\log{(ae+cdx)}}{c^7d^7}$$

input
```
integrate((e*x+d)**8/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**2,x)
```

output
```
x**4*(-a*e**7/(2*c**3*d**3) + 3*e**5/(2*c**2*d)) + x**3*(a**2*e**8/(c**4*d
**4) - 4*a*e**6/(c**3*d**2) + 5*e**4/c**2) + x**2*(-2*a**3*e**9/(c**5*d**5
) + 9*a**2*e**7/(c**4*d**3) - 15*a*e**5/(c**3*d) + 10*d*e**3/c**2) + x*(5*
a**4*e**10/(c**6*d**6) - 24*a**3*e**8/(c**5*d**4) + 45*a**2*e**6/(c**4*d**
2) - 40*a*e**4/c**3 + 15*d**2*e**2/c**2) + (-a**6*e**12 + 6*a**5*c*d**2*e*
*10 - 15*a**4*c**2*d**4*e**8 + 20*a**3*c**3*d**6*e**6 - 15*a**2*c**4*d**8*
e**4 + 6*a*c**5*d**10*e**2 - c**6*d**12)/(a*c**7*d**7*e + c**8*d**8*x) + e
**6*x**5/(5*c**2*d**2) - 6*e*(a*e**2 - c*d**2)**5*log(a*e + c*d*x)/(c**7*d
**7)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 398, normalized size of antiderivative = 1.82

$$\int \frac{(d+ex)^8}{\left(ade+(cd^2+ae^2)x+cdex^2\right)^2}\,dx = -\frac{c^6d^{12}-6\,ac^5d^{10}e^2+15\,a^2c^4d^8e^4-20\,a^3c^3d^6e^6+15\,a^4c^2d^4e^8-6\,a^5cd^2e^{10}+a^6e^{12}}{c^8d^8x+ac^7d^7e} + \frac{2\,c^4d^4e^6x^5+5\,(3\,c^4d^5e^5-ac^3d^3e^7)x^4+10\,(5\,c^4d^6e^4-4\,ac^3d^4e^6+a^2c^2d^2e^8)x^3+10\,(10\,c^4d^7e^3-15\,ac}{10\,c^6} + \frac{6\,(c^5d^{10}e-5\,ac^4d^8e^3+10\,a^2c^3d^6e^5-10\,a^3c^2d^4e^7+5\,a^4cd^2e^9-a^5e^{11})\log(cdx+ae)}{c^7d^7}$$

input
```
integrate((e*x+d)^8/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^2,x, algorithm="maxi
ma")
```

output
```
-(c^6*d^12 - 6*a*c^5*d^10*e^2 + 15*a^2*c^4*d^8*e^4 - 20*a^3*c^3*d^6*e^6 +
15*a^4*c^2*d^4*e^8 - 6*a^5*c*d^2*e^10 + a^6*e^12)/(c^8*d^8*x + a*c^7*d^7*e
) + 1/10*(2*c^4*d^4*e^6*x^5 + 5*(3*c^4*d^5*e^5 - a*c^3*d^3*e^7)*x^4 + 10*(
5*c^4*d^6*e^4 - 4*a*c^3*d^4*e^6 + a^2*c^2*d^2*e^8)*x^3 + 10*(10*c^4*d^7*e^
3 - 15*a*c^3*d^5*e^5 + 9*a^2*c^2*d^3*e^7 - 2*a^3*c*d*e^9)*x^2 + 10*(15*c^4
*d^8*e^2 - 40*a*c^3*d^6*e^4 + 45*a^2*c^2*d^4*e^6 - 24*a^3*c*d^2*e^8 + 5*a^
4*e^10)*x)/(c^6*d^6) + 6*(c^5*d^10*e - 5*a*c^4*d^8*e^3 + 10*a^2*c^3*d^6*e^
5 - 10*a^3*c^2*d^4*e^7 + 5*a^4*c*d^2*e^9 - a^5*e^11)*log(c*d*x + a*e)/(c^7
*d^7)
```

Giac [A] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 418, normalized size of antiderivative = 1.91

$$\int \frac{(d+ex)^8}{\left(ade+(cd^2+ae^2)x+cdex^2\right)^2}\,dx = \frac{6\,(c^5d^{10}e-5\,ac^4d^8e^3+10\,a^2c^3d^6e^5-10\,a^3c^2d^4e^7+5\,a^4cd^2e^9-a^5e^{11})\log(|cdx+ae|)}{c^7d^7} - \frac{c^6d^{12}-6\,ac^5d^{10}e^2+15\,a^2c^4d^8e^4-20\,a^3c^3d^6e^6+15\,a^4c^2d^4e^8-6\,a^5cd^2e^{10}+a^6e^{12}}{(cdx+ae)c^7d^7} + \frac{2\,c^8d^8e^6x^5+15\,c^8d^9e^5x^4-5\,ac^7d^7e^7x^4+50\,c^8d^{10}e^4x^3-40\,ac^7d^8e^6x^3+10\,a^2c^6d^6e^8x^3+100\,c^8d^{11}e^3x^2}{}$$

input
```
integrate((e*x+d)^8/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^2,x, algorithm="giac
")
```

output
```
6*(c^5*d^10*e - 5*a*c^4*d^8*e^3 + 10*a^2*c^3*d^6*e^5 - 10*a^3*c^2*d^4*e^7
+ 5*a^4*c*d^2*e^9 - a^5*e^11)*log(abs(c*d*x + a*e))/(c^7*d^7) - (c^6*d^12
- 6*a*c^5*d^10*e^2 + 15*a^2*c^4*d^8*e^4 - 20*a^3*c^3*d^6*e^6 + 15*a^4*c^2*
d^4*e^8 - 6*a^5*c*d^2*e^10 + a^6*e^12)/((c*d*x + a*e)*c^7*d^7) + 1/10*(2*c
^8*d^8*e^6*x^5 + 15*c^8*d^9*e^5*x^4 - 5*a*c^7*d^7*e^7*x^4 + 50*c^8*d^10*e^
4*x^3 - 40*a*c^7*d^8*e^6*x^3 + 10*a^2*c^6*d^6*e^8*x^3 + 100*c^8*d^11*e^3*x
^2 - 150*a*c^7*d^9*e^5*x^2 + 90*a^2*c^6*d^7*e^7*x^2 - 20*a^3*c^5*d^5*e^9*x
^2 + 150*c^8*d^12*e^2*x - 400*a*c^7*d^10*e^4*x + 450*a^2*c^6*d^8*e^6*x - 2
40*a^3*c^5*d^6*e^8*x + 50*a^4*c^4*d^4*e^10*x)/(c^10*d^10)
```

Mupad [B] (verification not implemented)

Time = 5.36 (sec) , antiderivative size = 625, normalized size of antiderivative = 2.85

$$\int \frac{(d+ex)^8}{(ade+(cd^2+ae^2)x+cdex^2)^2}\,dx = x^4\left(\frac{3e^5}{2c^2d}-\frac{ae^7}{2c^3d^3}\right) + x^2\left(\frac{10de^3}{c^2}+\frac{ae\left(\frac{a^2e^8}{c^4d^4}-\frac{15e^4}{c^2}+\frac{2ae\left(\frac{6e^5}{c^2d}-\frac{2ae^7}{c^3d^3}\right)}{cd}\right)}{cd}-\frac{a^2e^2\left(\frac{6e^5}{c^2d}-\frac{2ae^7}{c^3d^3}\right)}{2c^2d^2}\right) - x^3\left(\frac{a^2e^8}{3c^4d^4}-\frac{5e^4}{c^2}+\frac{2ae\left(\frac{6e^5}{c^2d}-\frac{2ae^7}{c^3d^3}\right)}{3cd}\right) + x\left(\frac{15d^2e^2}{c^2}+\frac{a^2e^2\left(\frac{a^2e^8}{c^4d^4}-\frac{15e^4}{c^2}+\frac{2ae\left(\frac{6e^5}{c^2d}-\frac{2ae^7}{c^3d^3}\right)}{cd}\right)}{c^2d^2}-\frac{2ae\left(\frac{20de^3}{c^2}+\frac{2ae\left(\frac{a^2e^8}{c^4d^4}-\frac{15e^4}{c^2}+\frac{2ae\left(\frac{6e^5}{c^2d}-\frac{2ae^7}{c^3d^3}\right)}{cd}\right)}{cd}-\frac{a^2e^2\left(\frac{6e^5}{c^2d}-\frac{2ae^7}{c^3d^3}\right)}{c^2d^2}\right)}{cd}\right) - \frac{a^6e^{12}-6a^5cd^2e^{10}+15a^4c^2d^4e^8-20a^3c^3d^6e^6+15a^2c^4d^8e^4-6ac^5d^{10}e^2+c^6d^{12}}{cd\,(xc^7d^7+aec^6d^6)} + \frac{e^6x^5}{5c^2d^2} - \frac{\ln(ae+cdx)\,(6a^5e^{11}-30a^4cd^2e^9+60a^3c^2d^4e^7-60a^2c^3d^6e^5+30ac^4d^8e^3-6c^5d^{10}e)}{c^7d^7}$$

input

```
int((d + e*x)^8/(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^2,x)
```

output
```
x^4*((3*e^5)/(2*c^2*d) - (a*e^7)/(2*c^3*d^3)) + x^2*((10*d*e^3)/c^2 + (a*e
*((a^2*e^8)/(c^4*d^4) - (15*e^4)/c^2 + (2*a*e*((6*e^5)/(c^2*d) - (2*a*e^7)
/(c^3*d^3)))/(c*d)))/(c*d) - (a^2*e^2*((6*e^5)/(c^2*d) - (2*a*e^7)/(c^3*d^
3)))/(2*c^2*d^2)) - x^3*((a^2*e^8)/(3*c^4*d^4) - (5*e^4)/c^2 + (2*a*e*((6*
e^5)/(c^2*d) - (2*a*e^7)/(c^3*d^3)))/(3*c*d)) + x*((15*d^2*e^2)/c^2 + (a^2
*e^2*((a^2*e^8)/(c^4*d^4) - (15*e^4)/c^2 + (2*a*e*((6*e^5)/(c^2*d) - (2*a*
e^7)/(c^3*d^3)))/(c*d)))/(c^2*d^2) - (2*a*e*((20*d*e^3)/c^2 + (2*a*e*((a^2
*e^8)/(c^4*d^4) - (15*e^4)/c^2 + (2*a*e*((6*e^5)/(c^2*d) - (2*a*e^7)/(c^3*
d^3)))/(c*d)))/(c*d) - (a^2*e^2*((6*e^5)/(c^2*d) - (2*a*e^7)/(c^3*d^3)))/(
c^2*d^2)))/(c*d)) - (a^6*e^12 + c^6*d^12 - 6*a*c^5*d^10*e^2 - 6*a^5*c*d^2*
e^10 + 15*a^2*c^4*d^8*e^4 - 20*a^3*c^3*d^6*e^6 + 15*a^4*c^2*d^4*e^8)/(c*d*
(c^7*d^7*x + a*c^6*d^6*e)) + (e^6*x^5)/(5*c^2*d^2) - (log(a*e + c*d*x)*(6*
a^5*e^11 - 6*c^5*d^10*e + 30*a*c^4*d^8*e^3 - 30*a^4*c*d^2*e^9 - 60*a^2*c^3
*d^6*e^5 + 60*a^3*c^2*d^4*e^7))/(c^7*d^7)
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 631, normalized size of antiderivative = 2.88

$$\int \frac{(d+ex)^8}{(ade+(cd^2+ae^2)x+cdex^2)^2}\,dx$$
$$= \frac{-60\log(cdx+ae)\,a^6cd\,e^{12}x+300\log(cdx+ae)\,a^5c^2d^3e^{10}x-600\log(cdx+ae)\,a^4c^3d^5e^8x+600\log(cd\ldots}{}$$

input
```
int((e*x+d)^8/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^2,x)
```

output
```
( - 60*log(a*e + c*d*x)*a**7*e**13 + 300*log(a*e + c*d*x)*a**6*c*d**2*e**1
1 - 60*log(a*e + c*d*x)*a**6*c*d*e**12*x - 600*log(a*e + c*d*x)*a**5*c**2*
d**4*e**9 + 300*log(a*e + c*d*x)*a**5*c**2*d**3*e**10*x + 600*log(a*e + c*
d*x)*a**4*c**3*d**6*e**7 - 600*log(a*e + c*d*x)*a**4*c**3*d**5*e**8*x - 30
0*log(a*e + c*d*x)*a**3*c**4*d**8*e**5 + 600*log(a*e + c*d*x)*a**3*c**4*d*
*7*e**6*x + 60*log(a*e + c*d*x)*a**2*c**5*d**10*e**3 - 300*log(a*e + c*d*x
)*a**2*c**5*d**9*e**4*x + 60*log(a*e + c*d*x)*a*c**6*d**11*e**2*x + 60*a**
6*c*d*e**12*x - 300*a**5*c**2*d**3*e**10*x + 30*a**5*c**2*d**2*e**11*x**2
+ 600*a**4*c**3*d**5*e**8*x - 150*a**4*c**3*d**4*e**9*x**2 - 10*a**4*c**3*
d**3*e**10*x**3 - 600*a**3*c**4*d**7*e**6*x + 300*a**3*c**4*d**6*e**7*x**2
 + 50*a**3*c**4*d**5*e**8*x**3 + 5*a**3*c**4*d**4*e**9*x**4 + 300*a**2*c**
5*d**9*e**4*x - 300*a**2*c**5*d**8*e**5*x**2 - 100*a**2*c**5*d**7*e**6*x**
3 - 25*a**2*c**5*d**6*e**7*x**4 - 3*a**2*c**5*d**5*e**8*x**5 - 60*a*c**6*d
**11*e**2*x + 150*a*c**6*d**10*e**3*x**2 + 100*a*c**6*d**9*e**4*x**3 + 50*
a*c**6*d**8*e**5*x**4 + 15*a*c**6*d**7*e**6*x**5 + 2*a*c**6*d**6*e**7*x**6
 + 10*c**7*d**13*x)/(10*a*c**7*d**7*e*(a*e + c*d*x))
```

3.117 $\int \frac{(d+ex)^7}{(ade+(cd^2+ae^2)x+cdex^2)^2}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 184

$$\int \frac{(d+ex)^7}{(ade+(cd^2+ae^2)\,x+cdex^2)^2}\,dx = \frac{10e^2(cd^2-ae^2)^3\,x}{c^5d^5} - \frac{(cd^2-ae^2)^5}{c^6d^6(ae+cdx)} + \frac{5e^3(cd^2-ae^2)^2\,(ae+cdx)^2}{c^6d^6} + \frac{5e^4(cd^2-ae^2)\,(ae+cdx)^3}{3c^6d^6} + \frac{e^5(ae+cdx)^4}{4c^6d^6} + \frac{5e(cd^2-ae^2)^4\log(ae+cdx)}{c^6d^6}$$

output
```
10*e^2*(-a*e^2+c*d^2)^3*x/c^5/d^5-(-a*e^2+c*d^2)^5/c^6/d^6/(c*d*x+a*e)+5*e
^3*(-a*e^2+c*d^2)^2*(c*d*x+a*e)^2/c^6/d^6+5/3*e^4*(-a*e^2+c*d^2)*(c*d*x+a*
e)^3/c^6/d^6+1/4*e^5*(c*d*x+a*e)^4/c^6/d^6+5*e*(-a*e^2+c*d^2)^4*ln(c*d*x+a
*e)/c^6/d^6
```

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 263, normalized size of antiderivative = 1.43

$$\int \frac{(d+ex)^7}{(ade+(cd^2+ae^2)x+cdex^2)^2}dx$$
$$= \frac{12a^5e^{10}-12a^4cde^8(5d+4ex)+30a^3c^2d^2e^6(4d^2+6dex-e^2x^2)-10a^2c^3d^3e^4(12d^3+24d^2ex-12de^2x^2}{}$$

input
```
Integrate[(d + e*x)^7/(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^2,x]
```

output
```
(12*a^5*e^10 - 12*a^4*c*d*e^8*(5*d + 4*e*x) + 30*a^3*c^2*d^2*e^6*(4*d^2 +
6*d*e*x - e^2*x^2) - 10*a^2*c^3*d^3*e^4*(12*d^3 + 24*d^2*e*x - 12*d*e^2*x^
2 - e^3*x^3) + 5*a*c^4*d^4*e^2*(12*d^4 + 24*d^3*e*x - 36*d^2*e^2*x^2 - 8*d
*e^3*x^3 - e^4*x^4) + c^5*d^5*(-12*d^5 + 120*d^3*e^2*x^2 + 60*d^2*e^3*x^3
+ 20*d*e^4*x^4 + 3*e^5*x^5) + 60*e*(c*d^2 - a*e^2)^4*(a*e + c*d*x)*Log[a*e
 + c*d*x])/(12*c^6*d^6*(a*e + c*d*x))
```

Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 184, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.057, Rules used = {1121, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex)^7}{(x(ae^2+cd^2)+ade+cdex^2)^2}dx$$
$$\Big\downarrow \text{1121}$$
$$\int \left(\frac{e^5(ae+cdx)^3}{c^5d^5}+\frac{5e\left(cd^2-ae^2\right)^4}{c^5d^5(ae+cdx)}+\frac{\left(cd^2-ae^2\right)^5}{c^5d^5(ae+cdx)^2}+\frac{10e^2\left(cd^2-ae^2\right)^3}{c^5d^5}+\frac{5\left(cd^2e^4-ae^6\right)(ae+cdx)^2}{c^5d^5}+\right.$$
$$\Big\downarrow \text{2009}$$

$$\frac{e^5(ae+cdx)^4}{4c^6d^6}-\frac{\left(cd^2-ae^2\right)^5}{c^6d^6(ae+cdx)}+\frac{5e\left(cd^2-ae^2\right)^4\log(ae+cdx)}{c^6d^6}+\frac{5e^4\left(cd^2-ae^2\right)(ae+cdx)^3}{3c^6d^6}+\frac{5e^3\left(cd^2-ae^2\right)^2(ae+cdx)^2}{c^6d^6}+\frac{10e^2x\left(cd^2-ae^2\right)^3}{c^5d^5}$$

input
```
Int[(d + e*x)^7/(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^2,x]
```

output
```
(10*e^2*(c*d^2 - a*e^2)^3*x)/(c^5*d^5) - (c*d^2 - a*e^2)^5/(c^6*d^6*(a*e +
 c*d*x)) + (5*e^3*(c*d^2 - a*e^2)^2*(a*e + c*d*x)^2)/(c^6*d^6) + (5*e^4*(c
*d^2 - a*e^2)*(a*e + c*d*x)^3)/(3*c^6*d^6) + (e^5*(a*e + c*d*x)^4)/(4*c^6*
d^6) + (5*e*(c*d^2 - a*e^2)^4*Log[a*e + c*d*x])/(c^6*d^6)
```

Defintions of rubi rules used

rule 1121
```
Int[((d_) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Int[ExpandIntegrand[(d + e*x)^(m + p)*(a/d + (c/e)*x)^p, x], x]
/; FreeQ[{a, b, c, d, e, m, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && (Int
egerQ[p] || (GtQ[a, 0] && GtQ[d, 0] && LtQ[c, 0]))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 1.58 (sec) , antiderivative size = 298, normalized size of antiderivative = 1.62

method	result
default	$-\frac{e^2\left(-\frac{1}{4}x^4c^3d^3e^3+\frac{2}{3}x^3a\,c^2d^2e^4-\frac{5}{3}x^3c^3d^4e^2-\frac{3}{2}x^2a^2cd\,e^5+5x^2a\,c^2d^3e^3-5x^2c^3d^5e+4e^6a^3x-15d^2e^4a^2cx+20d^4e^2a\,c^2x-1\right.}{d^5c^5}$
risch	$\frac{e^5x^4}{4d^2c^2}-\frac{2e^6x^3a}{3d^3c^3}+\frac{5e^4x^3}{3d\,c^2}+\frac{3e^7x^2a^2}{2d^4c^4}-\frac{5e^5x^2a}{d^2c^3}+\frac{5e^3x^2}{c^2}-\frac{4e^8a^3x}{d^5c^5}+\frac{15e^6a^2x}{d^3c^4}-\frac{20e^4ax}{d\,c^3}+\frac{10e^2dx}{c^2}+\frac{a^5}{c^6d^6(c}$
norman	$\frac{\frac{10a^5e^{10}-35a^4c\,d^2e^8+40a^3c^2d^4e^6-10a^2c^3d^6e^4-10a\,c^4d^8e^2-2c^5d^{10}}{2c^6d^5}+\frac{e^6x^6}{4cd}+\frac{\left(10a^5e^{12}-35a^4c\,d^2e^{10}+45a^3c^2d^4e^8-30a^2c^3d^6e^6+20a\right.}{2d^6c^6e}}{(ex+d)(cd}$
parallelrisch	$\frac{60a^5e^{10}-12c^5d^{10}-240\ln(cdx+ae)x\,a^3c^2d^3e^7+360\ln(cdx+ae)x\,a^2c^3d^5e^5-240\ln(cdx+ae)xa\,c^4d^7e^3+120x^2c^5d^8e^2+3x^5e^5}{}$

input
```
int((e*x+d)^7/(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^2,x,method=_RETURNVERBOSE)
```

output
```
-e^2/d^5/c^5*(-1/4*x^4*c^3*d^3*e^3+2/3*x^3*a*c^2*d^2*e^4-5/3*x^3*c^3*d^4*e
^2-3/2*x^2*a^2*c*d*e^5+5*x^2*a*c^2*d^3*e^3-5*x^2*c^3*d^5*e+4*e^6*a^3*x-15*
d^2*e^4*a^2*c*x+20*d^4*e^2*a*c^2*x-10*d^6*c^3*x)+5/d^6*e/c^6*(a^4*e^8-4*a^
3*c*d^2*e^6+6*a^2*c^2*d^4*e^4-4*a*c^3*d^6*e^2+c^4*d^8)*ln(c*d*x+a*e)-1/c^6
/d^6*(-a^5*e^10+5*a^4*c*d^2*e^8-10*a^3*c^2*d^4*e^6+10*a^2*c^3*d^6*e^4-5*a*
c^4*d^8*e^2+c^5*d^10)/(c*d*x+a*e)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 417 vs. $2(180) = 360$.

Time = 0.08 (sec) , antiderivative size = 417, normalized size of antiderivative = 2.27

$$\int \frac{(d+ex)^7}{(ade+(cd^2+ae^2)x+cdex^2)^2}\,dx$$
$$= \frac{3\,c^5d^5e^5x^5 - 12\,c^5d^{10} + 60\,ac^4d^8e^2 - 120\,a^2c^3d^6e^4 + 120\,a^3c^2d^4e^6 - 60\,a^4cd^2e^8 + 12\,a^5e^{10} + 5\,(4\,c^5d^6e^4 -}{}$$

input
```
integrate((e*x+d)^7/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^2,x, algorithm="fric
as")
```

output
```
1/12*(3*c^5*d^5*e^5*x^5 - 12*c^5*d^10 + 60*a*c^4*d^8*e^2 - 120*a^2*c^3*d^6
*e^4 + 120*a^3*c^2*d^4*e^6 - 60*a^4*c*d^2*e^8 + 12*a^5*e^10 + 5*(4*c^5*d^6
*e^4 - a*c^4*d^4*e^6)*x^4 + 10*(6*c^5*d^7*e^3 - 4*a*c^4*d^5*e^5 + a^2*c^3*
d^3*e^7)*x^3 + 30*(4*c^5*d^8*e^2 - 6*a*c^4*d^6*e^4 + 4*a^2*c^3*d^4*e^6 - a
^3*c^2*d^2*e^8)*x^2 + 12*(10*a*c^4*d^7*e^3 - 20*a^2*c^3*d^5*e^5 + 15*a^3*c
^2*d^3*e^7 - 4*a^4*c*d*e^9)*x + 60*(a*c^4*d^8*e^2 - 4*a^2*c^3*d^6*e^4 + 6*
a^3*c^2*d^4*e^6 - 4*a^4*c*d^2*e^8 + a^5*e^10 + (c^5*d^9*e - 4*a*c^4*d^7*e^
3 + 6*a^2*c^3*d^5*e^5 - 4*a^3*c^2*d^3*e^7 + a^4*c*d*e^9)*x)*log(c*d*x + a*
e))/(c^7*d^7*x + a*c^6*d^6*e)
```

Sympy [A] (verification not implemented)

Time = 0.60 (sec) , antiderivative size = 264, normalized size of antiderivative = 1.43

$$\int \frac{(d+ex)^7}{(ade+(cd^2+ae^2)x+cdex^2)^2}\,dx = x^3\left(-\frac{2ae^6}{3c^3d^3}+\frac{5e^4}{3c^2d}\right)+x^2\cdot\left(\frac{3a^2e^7}{2c^4d^4}-\frac{5ae^5}{c^3d^2}+\frac{5e^3}{c^2}\right) + x\left(-\frac{4a^3e^8}{c^5d^5}+\frac{15a^2e^6}{c^4d^3}-\frac{20ae^4}{c^3d}+\frac{10de^2}{c^2}\right) + \frac{a^5e^{10}-5a^4cd^2e^8+10a^3c^2d^4e^6-10a^2c^3d^6e^4+5ac^4d^8e^2-c^5d^{10}}{ac^6d^6e+c^7d^7x} + \frac{e^5x^4}{4c^2d^2}+\frac{5e(ae^2-cd^2)^4\log(ae+cdx)}{c^6d^6}$$

input
```
integrate((e*x+d)**7/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**2,x)
```

output
```
x**3*(-2*a*e**6/(3*c**3*d**3) + 5*e**4/(3*c**2*d)) + x**2*(3*a**2*e**7/(2*
c**4*d**4) - 5*a*e**5/(c**3*d**2) + 5*e**3/c**2) + x*(-4*a**3*e**8/(c**5*d
**5) + 15*a**2*e**6/(c**4*d**3) - 20*a*e**4/(c**3*d) + 10*d*e**2/c**2) + (
a**5*e**10 - 5*a**4*c*d**2*e**8 + 10*a**3*c**2*d**4*e**6 - 10*a**2*c**3*d*
*6*e**4 + 5*a*c**4*d**8*e**2 - c**5*d**10)/(a*c**6*d**6*e + c**7*d**7*x) +
 e**5*x**4/(4*c**2*d**2) + 5*e*(a*e**2 - c*d**2)**4*log(a*e + c*d*x)/(c**6
*d**6)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 300, normalized size of antiderivative = 1.63

$$\int \frac{(d+ex)^7}{(ade+(cd^2+ae^2)x+cdex^2)^2}\,dx = -\frac{c^5d^{10}-5\,ac^4d^8e^2+10\,a^2c^3d^6e^4-10\,a^3c^2d^4e^6+5\,a^4cd^2e^8-a^5e^{10}}{c^7d^7x+ac^6d^6e} + \frac{3\,c^3d^3e^5x^4+4\,(5\,c^3d^4e^4-2\,ac^2d^2e^6)x^3+6\,(10\,c^3d^5e^3-10\,ac^2d^3e^5+3\,a^2cde^7)x^2+12\,(10\,c^3d^6e^2-20}{12\,c^5d^5} + \frac{5\,(c^4d^8e-4\,ac^3d^6e^3+6\,a^2c^2d^4e^5-4\,a^3cd^2e^7+a^4e^9)\log(cdx+ae)}{c^6d^6}$$

input
```
integrate((e*x+d)^7/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^2,x, algorithm="maxi
ma")
```

output
```
-(c^5*d^10 - 5*a*c^4*d^8*e^2 + 10*a^2*c^3*d^6*e^4 - 10*a^3*c^2*d^4*e^6 + 5
*a^4*c*d^2*e^8 - a^5*e^10)/(c^7*d^7*x + a*c^6*d^6*e) + 1/12*(3*c^3*d^3*e^5
*x^4 + 4*(5*c^3*d^4*e^4 - 2*a*c^2*d^2*e^6)*x^3 + 6*(10*c^3*d^5*e^3 - 10*a*
c^2*d^3*e^5 + 3*a^2*c*d*e^7)*x^2 + 12*(10*c^3*d^6*e^2 - 20*a*c^2*d^4*e^4 +
 15*a^2*c*d^2*e^6 - 4*a^3*e^8)*x)/(c^5*d^5) + 5*(c^4*d^8*e - 4*a*c^3*d^6*e
^3 + 6*a^2*c^2*d^4*e^5 - 4*a^3*c*d^2*e^7 + a^4*e^9)*log(c*d*x + a*e)/(c^6*
d^6)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 312, normalized size of antiderivative = 1.70

$$
\begin{aligned}
&\int \frac{(d+ex)^7}{(ade+(cd^2+ae^2)x+cdex^2)^2}\,dx \\
&= \frac{5\,(c^4d^8e-4\,ac^3d^6e^3+6\,a^2c^2d^4e^5-4\,a^3cd^2e^7+a^4e^9)\log\left(|cdx+ae|\right)}{c^6d^6} \\
&\quad - \frac{c^5d^{10}-5\,ac^4d^8e^2+10\,a^2c^3d^6e^4-10\,a^3c^2d^4e^6+5\,a^4cd^2e^8-a^5e^{10}}{(cdx+ae)c^6d^6} \\
&\quad + \frac{3\,c^6d^6e^5x^4+20\,c^6d^7e^4x^3-8\,ac^5d^5e^6x^3+60\,c^6d^8e^3x^2-60\,ac^5d^6e^5x^2+18\,a^2c^4d^4e^7x^2+120\,c^6d^9e^2x-}{12\,c^8d^8}
\end{aligned}
$$

input
```
integrate((e*x+d)^7/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^2,x, algorithm="giac
")
```

output
```
5*(c^4*d^8*e - 4*a*c^3*d^6*e^3 + 6*a^2*c^2*d^4*e^5 - 4*a^3*c*d^2*e^7 + a^4
*e^9)*log(abs(c*d*x + a*e))/(c^6*d^6) - (c^5*d^10 - 5*a*c^4*d^8*e^2 + 10*a
^2*c^3*d^6*e^4 - 10*a^3*c^2*d^4*e^6 + 5*a^4*c*d^2*e^8 - a^5*e^10)/((c*d*x
+ a*e)*c^6*d^6) + 1/12*(3*c^6*d^6*e^5*x^4 + 20*c^6*d^7*e^4*x^3 - 8*a*c^5*d
^5*e^6*x^3 + 60*c^6*d^8*e^3*x^2 - 60*a*c^5*d^6*e^5*x^2 + 18*a^2*c^4*d^4*e^
7*x^2 + 120*c^6*d^9*e^2*x - 240*a*c^5*d^7*e^4*x + 180*a^2*c^4*d^5*e^6*x -
48*a^3*c^3*d^3*e^8*x)/(c^8*d^8)
```

Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 387, normalized size of antiderivative = 2.10

$$\int \frac{(d+ex)^7}{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^2}\,dx = x\left(\frac{10\,d\,e^2}{c^2}+\frac{2\,a\,e\left(\frac{a^2\,e^7}{c^4\,d^4}-\frac{10\,e^3}{c^2}+\frac{2\,a\,e\left(\frac{5\,e^4}{c^2\,d}-\frac{2\,a\,e^6}{c^3\,d^3}\right)}{c\,d}\right)}{c\,d}-\frac{a^2\,e^2\left(\frac{5\,e^4}{c^2\,d}-\frac{2\,a\,e^6}{c^3\,d^3}\right)}{c^2\,d^2}\right) + x^3\left(\frac{5\,e^4}{3\,c^2\,d}-\frac{2\,a\,e^6}{3\,c^3\,d^3}\right)-x^2\left(\frac{a^2\,e^7}{2\,c^4\,d^4}-\frac{5\,e^3}{c^2}+\frac{a\,e\left(\frac{5\,e^4}{c^2\,d}-\frac{2\,a\,e^6}{c^3\,d^3}\right)}{c\,d}\right)+\frac{e^5\,x^4}{4\,c^2\,d^2} + \frac{\ln\left(a\,e+c\,d\,x\right)\left(5\,a^4\,e^9-20\,a^3\,c\,d^2\,e^7+30\,a^2\,c^2\,d^4\,e^5-20\,a\,c^3\,d^6\,e^3+5\,c^4\,d^8\,e\right)}{c^6\,d^6} + \frac{a^5\,e^{10}-5\,a^4\,c\,d^2\,e^8+10\,a^3\,c^2\,d^4\,e^6-10\,a^2\,c^3\,d^6\,e^4+5\,a\,c^4\,d^8\,e^2-c^5\,d^{10}}{c\,d\left(x\,c^6\,d^6+a\,e\,c^5\,d^5\right)}$$

input

```
int((d + e*x)^7/(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^2,x)
```

output

```
x*((10*d*e^2)/c^2 + (2*a*e*((a^2*e^7)/(c^4*d^4) - (10*e^3)/c^2 + (2*a*e*((
5*e^4)/(c^2*d) - (2*a*e^6)/(c^3*d^3)))/(c*d)))/(c*d) - (a^2*e^2*((5*e^4)/(
c^2*d) - (2*a*e^6)/(c^3*d^3)))/(c^2*d^2)) + x^3*((5*e^4)/(3*c^2*d) - (2*a*
e^6)/(3*c^3*d^3)) - x^2*((a^2*e^7)/(2*c^4*d^4) - (5*e^3)/c^2 + (a*e*((5*e^
4)/(c^2*d) - (2*a*e^6)/(c^3*d^3)))/(c*d)) + (e^5*x^4)/(4*c^2*d^2) + (log(a
*e + c*d*x)*(5*a^4*e^9 + 5*c^4*d^8*e - 20*a*c^3*d^6*e^3 - 20*a^3*c*d^2*e^7
 + 30*a^2*c^2*d^4*e^5))/(c^6*d^6) + (a^5*e^10 - c^5*d^10 + 5*a*c^4*d^8*e^2
 - 5*a^4*c*d^2*e^8 - 10*a^2*c^3*d^6*e^4 + 10*a^3*c^2*d^4*e^6)/(c*d*(c^6*d^
6*x + a*c^5*d^5*e))
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 486, normalized size of antiderivative = 2.64

$$\int \frac{(d+ex)^7}{(ade+(cd^2+ae^2)x+cdex^2)^2}\,dx$$
$$= \frac{10a^3c^3d^3e^8x^3+240a^2c^4d^7e^4x-180a^2c^4d^6e^5x^2-40a^2c^4d^5e^6x^3-5a^2c^4d^4e^7x^4-60a\,c^5d^9e^2x+120a\,c^5d^8}{}$$

input
```
int((e*x+d)^7/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^2,x)
```

output
```
(60*log(a*e + c*d*x)*a**6*e**11 - 240*log(a*e + c*d*x)*a**5*c*d**2*e**9 +
60*log(a*e + c*d*x)*a**5*c*d*e**10*x + 360*log(a*e + c*d*x)*a**4*c**2*d**4
*e**7 - 240*log(a*e + c*d*x)*a**4*c**2*d**3*e**8*x - 240*log(a*e + c*d*x)*
a**3*c**3*d**6*e**5 + 360*log(a*e + c*d*x)*a**3*c**3*d**5*e**6*x + 60*log(
a*e + c*d*x)*a**2*c**4*d**8*e**3 - 240*log(a*e + c*d*x)*a**2*c**4*d**7*e**
4*x + 60*log(a*e + c*d*x)*a*c**5*d**9*e**2*x - 60*a**5*c*d*e**10*x + 240*a
**4*c**2*d**3*e**8*x - 30*a**4*c**2*d**2*e**9*x**2 - 360*a**3*c**3*d**5*e*
*6*x + 120*a**3*c**3*d**4*e**7*x**2 + 10*a**3*c**3*d**3*e**8*x**3 + 240*a*
*2*c**4*d**7*e**4*x - 180*a**2*c**4*d**6*e**5*x**2 - 40*a**2*c**4*d**5*e**
6*x**3 - 5*a**2*c**4*d**4*e**7*x**4 - 60*a*c**5*d**9*e**2*x + 120*a*c**5*d
**8*e**3*x**2 + 60*a*c**5*d**7*e**4*x**3 + 20*a*c**5*d**6*e**5*x**4 + 3*a*
c**5*d**5*e**6*x**5 + 12*c**6*d**11*x)/(12*a*c**6*d**6*e*(a*e + c*d*x))
```

3.118 $\int \frac{(d+ex)^6}{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^2}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 145

$$\int \frac{(d+ex)^6}{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^2}\,dx = \frac{e^2\left(6c^2d^4-8acd^2e^2+3a^2e^4\right)x}{c^4d^4} + \frac{e^3\left(2cd^2-ae^2\right)x^2}{c^3d^3} + \frac{e^4x^3}{3c^2d^2} - \frac{\left(cd^2-ae^2\right)^4}{c^5d^5(ae+cdx)} + \frac{4e\left(cd^2-ae^2\right)^3\log(ae+cdx)}{c^5d^5}$$

output
```
e^2*(3*a^2*e^4-8*a*c*d^2*e^2+6*c^2*d^4)*x/c^4/d^4+e^3*(-a*e^2+2*c*d^2)*x^2
/c^3/d^3+1/3*e^4*x^3/c^2/d^2-(-a*e^2+c*d^2)^4/c^5/d^5/(c*d*x+a*e)+4*e*(-a*
e^2+c*d^2)^3*ln(c*d*x+a*e)/c^5/d^5
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 196, normalized size of antiderivative = 1.35

$$\int \frac{(d+ex)^6}{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^2}\,dx = \frac{-3a^4e^8+3a^3cde^6(4d+3ex)-6a^2c^2d^2e^4\left(3d^2+4dex-e^2x^2\right)+2ac^3d^3e^2\left(6d^3+9d^2ex-9de^2x^2-e^3x^3\right)}{3c^5d^5(ae+cdx}$$

input
```
Integrate[(d + e*x)^6/(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^2,x]
```

output
```
(-3*a^4*e^8 + 3*a^3*c*d*e^6*(4*d + 3*e*x) - 6*a^2*c^2*d^2*e^4*(3*d^2 + 4*d
*e*x - e^2*x^2) + 2*a*c^3*d^3*e^2*(6*d^3 + 9*d^2*e*x - 9*d*e^2*x^2 - e^3*x
^3) + c^4*d^4*(-3*d^4 + 18*d^2*e^2*x^2 + 6*d*e^3*x^3 + e^4*x^4) - 12*e*(-(
c*d^2) + a*e^2)^3*(a*e + c*d*x)*Log[a*e + c*d*x])/(3*c^5*d^5*(a*e + c*d*x)
)
```

Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 145, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.057, Rules used = {1121, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex)^6}{\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^2}dx$$

$$\Big\downarrow \text{ 1121}$$

$$\int \left(\frac{3a^2e^6-8acd^2e^4+6c^2d^4e^2}{c^4d^4}+\frac{4e\left(cd^2-ae^2\right)^3}{c^4d^4(ae+cdx)}+\frac{\left(cd^2-ae^2\right)^4}{c^4d^4(ae+cdx)^2}+\frac{2e^3x\left(2cd^2-ae^2\right)}{c^3d^3}+\frac{e^4x^2}{c^2d^2}\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{e^2x\left(3a^2e^4-8acd^2e^2+6c^2d^4\right)}{c^4d^4}-\frac{\left(cd^2-ae^2\right)^4}{c^5d^5(ae+cdx)}+\frac{4e\left(cd^2-ae^2\right)^3\log(ae+cdx)}{c^5d^5}+\frac{e^3x^2\left(2cd^2-ae^2\right)}{c^3d^3}+\frac{e^4x^3}{3c^2d^2}$$

input
```
Int[(d + e*x)^6/(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^2,x]
```

output
```
(e^2*(6*c^2*d^4 - 8*a*c*d^2*e^2 + 3*a^2*e^4)*x)/(c^4*d^4) + (e^3*(2*c*d^2
- a*e^2)*x^2)/(c^3*d^3) + (e^4*x^3)/(3*c^2*d^2) - (c*d^2 - a*e^2)^4/(c^5*d
^5*(a*e + c*d*x)) + (4*e*(c*d^2 - a*e^2)^3*Log[a*e + c*d*x])/(c^5*d^5)
```

Defintions of rubi rules used

rule 1121
```
Int[((d_) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Int[ExpandIntegrand[(d + e*x)^(m + p)*(a/d + (c/e)*x)^p, x], x]
/; FreeQ[{a, b, c, d, e, m, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && (Int
egerQ[p] || (GtQ[a, 0] && GtQ[d, 0] && LtQ[c, 0]))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 1.62 (sec) , antiderivative size = 208, normalized size of antiderivative = 1.43

method	result
default	$\frac{e^{2}\left(\frac{1}{3}x^{3}c^{2}d^{2}e^{2}-x^{2}acd\,e^{3}+2x^{2}c^{2}d^{3}e+3a^{2}e^{4}x-8ac\,d^{2}e^{2}x+6c^{2}d^{4}x\right)}{d^{4}c^{4}}-\frac{4e\left(e^{6}a^{3}-3d^{2}e^{4}a^{2}c+3d^{4}e^{2}a\,c^{2}-d^{6}c^{3}\right)\ln\left(cdx+ae\right)}{d^{5}c^{5}}-$
risch	$\frac{e^{4}x^{3}}{3c^{2}d^{2}}-\frac{e^{5}x^{2}a}{d^{3}c^{3}}+\frac{2e^{3}x^{2}}{d\,c^{2}}+\frac{3e^{6}a^{2}x}{d^{4}c^{4}}-\frac{8e^{4}ax}{d^{2}c^{3}}+\frac{6e^{2}x}{c^{2}}-\frac{4e^{7}\ln\left(cdx+ae\right)a^{3}}{d^{5}c^{5}}+\frac{12e^{5}\ln\left(cdx+ae\right)a^{2}}{d^{3}c^{4}}-\frac{12e^{3}\ln(cdx+}{d\,c^{3}}$
norman	$\frac{-\frac{4a^{4}e^{8}-10a^{3}c\,d^{2}e^{6}+6a^{2}c^{2}d^{4}e^{4}+2a\,c^{3}d^{6}e^{2}+c^{4}d^{8}}{c^{5}d^{4}}+\frac{e^{5}x^{5}}{3cd}-\frac{\left(4a^{4}e^{10}-10a^{3}c\,d^{2}e^{8}+8a^{2}c^{2}d^{4}e^{6}-4a\,c^{3}d^{6}e^{4}+7c^{4}d^{8}e^{2}\right)x}{d^{5}c^{5}e}+\frac{2e^{3}\left(3a^{2}e^{4}-1\right.}{}}{\left(ex+d\right)\left(cdx+ae\right)}$
parallelrisch	$-\frac{-c^{4}d^{4}e^{4}x^{4}+2a\,c^{3}d^{3}e^{5}x^{3}-6c^{4}d^{5}e^{3}x^{3}+12\ln\left(cdx+ae\right)x\,a^{3}cd\,e^{7}-36\ln\left(cdx+ae\right)x\,a^{2}c^{2}d^{3}e^{5}+36\ln\left(cdx+ae\right)xa\,c^{3}d^{5}e^{3}-12\ln}{}$

input
```
int((e*x+d)^6/(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^2,x,method=_RETURNVERBOSE)
```

output
```
e^2/d^4/c^4*(1/3*x^3*c^2*d^2*e^2-x^2*a*c*d*e^3+2*x^2*c^2*d^3*e+3*a^2*e^4*x
-8*a*c*d^2*e^2*x+6*c^2*d^4*x)-4/d^5*e/c^5*(a^3*e^6-3*a^2*c*d^2*e^4+3*a*c^2
*d^4*e^2-c^3*d^6)*ln(c*d*x+a*e)-(a^4*e^8-4*a^3*c*d^2*e^6+6*a^2*c^2*d^4*e^4
-4*a*c^3*d^6*e^2+c^4*d^8)/d^5/c^5/(c*d*x+a*e)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 305 vs. 2(143) = 286.

Time = 0.08 (sec) , antiderivative size = 305, normalized size of antiderivative = 2.10

$$\int \frac{(d+ex)^6}{\left(ade+(cd^2+ae^2)\,x+cdex^2\right)^2}\,dx = \frac{c^4d^4e^4x^4-3\,c^4d^8+12\,ac^3d^6e^2-18\,a^2c^2d^4e^4+12\,a^3cd^2e^6-3\,a^4e^8+2\,(3\,c^4d^5e^3-ac^3d^3e^5)x^3+6\,(3\,c^4d^6\cdots}{}$$

input
```
integrate((e*x+d)^6/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^2,x, algorithm="fric
as")
```

output
```
1/3*(c^4*d^4*e^4*x^4 - 3*c^4*d^8 + 12*a*c^3*d^6*e^2 - 18*a^2*c^2*d^4*e^4 +
 12*a^3*c*d^2*e^6 - 3*a^4*e^8 + 2*(3*c^4*d^5*e^3 - a*c^3*d^3*e^5)*x^3 + 6*
(3*c^4*d^6*e^2 - 3*a*c^3*d^4*e^4 + a^2*c^2*d^2*e^6)*x^2 + 3*(6*a*c^3*d^5*e
^3 - 8*a^2*c^2*d^3*e^5 + 3*a^3*c*d*e^7)*x + 12*(a*c^3*d^6*e^2 - 3*a^2*c^2*
d^4*e^4 + 3*a^3*c*d^2*e^6 - a^4*e^8 + (c^4*d^7*e - 3*a*c^3*d^5*e^3 + 3*a^2
*c^2*d^3*e^5 - a^3*c*d*e^7)*x)*log(c*d*x + a*e))/(c^6*d^6*x + a*c^5*d^5*e)
```

Sympy [A] (verification not implemented)

Time = 0.44 (sec) , antiderivative size = 185, normalized size of antiderivative = 1.28

$$\begin{aligned}\int \frac{(d+ex)^6}{\left(ade+(cd^2+ae^2)\,x+cdex^2\right)^2}\,dx &= x^2\left(-\frac{ae^5}{c^3d^3}+\frac{2e^3}{c^2d}\right)+x\left(\frac{3a^2e^6}{c^4d^4}-\frac{8ae^4}{c^3d^2}+\frac{6e^2}{c^2}\right)\\ &\quad+\frac{-a^4e^8+4a^3cd^2e^6-6a^2c^2d^4e^4+4ac^3d^6e^2-c^4d^8}{ac^5d^5e+c^6d^6x}\\ &\quad+\frac{e^4x^3}{3c^2d^2}-\frac{4e(ae^2-cd^2)^3\log{(ae+cdx)}}{c^5d^5}\end{aligned}$$

input
```
integrate((e*x+d)**6/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**2,x)
```

output
```
x**2*(-a*e**5/(c**3*d**3) + 2*e**3/(c**2*d)) + x*(3*a**2*e**6/(c**4*d**4)
- 8*a*e**4/(c**3*d**2) + 6*e**2/c**2) + (-a**4*e**8 + 4*a**3*c*d**2*e**6 -
 6*a**2*c**2*d**4*e**4 + 4*a*c**3*d**6*e**2 - c**4*d**8)/(a*c**5*d**5*e +
c**6*d**6*x) + e**4*x**3/(3*c**2*d**2) - 4*e*(a*e**2 - c*d**2)**3*log(a*e
+ c*d*x)/(c**5*d**5)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 214, normalized size of antiderivative = 1.48

$$\int \frac{(d+ex)^6}{(ade+(cd^2+ae^2)\,x+cdex^2)^2}\,dx = -\frac{c^4d^8-4\,ac^3d^6e^2+6\,a^2c^2d^4e^4-4\,a^3cd^2e^6+a^4e^8}{c^6d^6x+ac^5d^5e} + \frac{c^2d^2e^4x^3+3\,(2\,c^2d^3e^3-acde^5)x^2+3\,(6\,c^2d^4e^2-8\,acd^2e^4+3\,a^2e^6)x}{3\,c^4d^4} + \frac{4\,(c^3d^6e-3\,ac^2d^4e^3+3\,a^2cd^2e^5-a^3e^7)\log{(cdx+ae)}}{c^5d^5}$$

input
```
integrate((e*x+d)^6/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^2,x, algorithm="maxi
ma")
```

output
```
-(c^4*d^8 - 4*a*c^3*d^6*e^2 + 6*a^2*c^2*d^4*e^4 - 4*a^3*c*d^2*e^6 + a^4*e^
8)/(c^6*d^6*x + a*c^5*d^5*e) + 1/3*(c^2*d^2*e^4*x^3 + 3*(2*c^2*d^3*e^3 - a
*c*d*e^5)*x^2 + 3*(6*c^2*d^4*e^2 - 8*a*c*d^2*e^4 + 3*a^2*e^6)*x)/(c^4*d^4)
 + 4*(c^3*d^6*e - 3*a*c^2*d^4*e^3 + 3*a^2*c*d^2*e^5 - a^3*e^7)*log(c*d*x +
 a*e)/(c^5*d^5)
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 222, normalized size of antiderivative = 1.53

$$\int \frac{(d+ex)^6}{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^2}\,dx$$
$$= \frac{4\left(c^3d^6e-3\,ac^2d^4e^3+3\,a^2cd^2e^5-a^3e^7\right)\log\left(\left|cdx+ae\right|\right)}{c^5d^5} - \frac{c^4d^8-4\,ac^3d^6e^2+6\,a^2c^2d^4e^4-4\,a^3cd^2e^6+a^4e^8}{(cdx+ae)c^5d^5} + \frac{c^4d^4e^4x^3+6\,c^4d^5e^3x^2-3\,ac^3d^3e^5x^2+18\,c^4d^6e^2x-24\,ac^3d^4e^4x+9\,a^2c^2d^2e^6x}{3\,c^6d^6}$$

input

```
integrate((e*x+d)^6/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^2,x, algorithm="giac
")
```

output

```
4*(c^3*d^6*e - 3*a*c^2*d^4*e^3 + 3*a^2*c*d^2*e^5 - a^3*e^7)*log(abs(c*d*x
+ a*e))/(c^5*d^5) - (c^4*d^8 - 4*a*c^3*d^6*e^2 + 6*a^2*c^2*d^4*e^4 - 4*a^3
*c*d^2*e^6 + a^4*e^8)/((c*d*x + a*e)*c^5*d^5) + 1/3*(c^4*d^4*e^4*x^3 + 6*c
^4*d^5*e^3*x^2 - 3*a*c^3*d^3*e^5*x^2 + 18*c^4*d^6*e^2*x - 24*a*c^3*d^4*e^4
*x + 9*a^2*c^2*d^2*e^6*x)/(c^6*d^6)
```

Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 242, normalized size of antiderivative = 1.67

$$\int \frac{(d+ex)^6}{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^2}\,dx$$
$$= x^2\left(\frac{2\,e^3}{c^2\,d}-\frac{a\,e^5}{c^3\,d^3}\right) - x\left(\frac{a^2\,e^6}{c^4\,d^4}-\frac{6\,e^2}{c^2}+\frac{2\,a\,e\left(\frac{4\,e^3}{c^2\,d}-\frac{2\,a\,e^5}{c^3\,d^3}\right)}{c\,d}\right) - \frac{\ln\left(a\,e+c\,d\,x\right)\left(4\,a^3\,e^7-12\,a^2\,c\,d^2\,e^5+12\,a\,c^2\,d^4\,e^3-4\,c^3\,d^6\,e\right)}{c^5\,d^5} - \frac{a^4\,e^8-4\,a^3\,c\,d^2\,e^6+6\,a^2\,c^2\,d^4\,e^4-4\,a\,c^3\,d^6\,e^2+c^4\,d^8}{c\,d\,\left(x\,c^5\,d^5+a\,e\,c^4\,d^4\right)} + \frac{e^4\,x^3}{3\,c^2\,d^2}$$

input

```
int((d + e*x)^6/(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^2,x)
```

output
```
x^2*((2*e^3)/(c^2*d) - (a*e^5)/(c^3*d^3)) - x*((a^2*e^6)/(c^4*d^4) - (6*e^
2)/c^2 + (2*a*e*((4*e^3)/(c^2*d) - (2*a*e^5)/(c^3*d^3)))/(c*d)) - (log(a*e
 + c*d*x)*(4*a^3*e^7 - 4*c^3*d^6*e + 12*a*c^2*d^4*e^3 - 12*a^2*c*d^2*e^5))
/(c^5*d^5) - (a^4*e^8 + c^4*d^8 - 4*a*c^3*d^6*e^2 - 4*a^3*c*d^2*e^6 + 6*a^
2*c^2*d^4*e^4)/(c*d*(c^5*d^5*x + a*c^4*d^4*e)) + (e^4*x^3)/(3*c^2*d^2)
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 357, normalized size of antiderivative = 2.46

$$\int \frac{(d+ex)^6}{(ade+(cd^2+ae^2)x+cdex^2)^2}\,dx$$
$$= \frac{-12\log(cdx+ae)\,a^5e^9+36\log(cdx+ae)\,a^4c\,d^2e^7-12\log(cdx+ae)\,a^4cd\,e^8x-36\log(cdx+ae)\,a^3c^2d}{}$$

input
```
int((e*x+d)^6/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^2,x)
```

output
```
( - 12*log(a*e + c*d*x)*a**5*e**9 + 36*log(a*e + c*d*x)*a**4*c*d**2*e**7 -
 12*log(a*e + c*d*x)*a**4*c*d*e**8*x - 36*log(a*e + c*d*x)*a**3*c**2*d**4*
e**5 + 36*log(a*e + c*d*x)*a**3*c**2*d**3*e**6*x + 12*log(a*e + c*d*x)*a**
2*c**3*d**6*e**3 - 36*log(a*e + c*d*x)*a**2*c**3*d**5*e**4*x + 12*log(a*e
+ c*d*x)*a*c**4*d**7*e**2*x + 12*a**4*c*d*e**8*x - 36*a**3*c**2*d**3*e**6*
x + 6*a**3*c**2*d**2*e**7*x**2 + 36*a**2*c**3*d**5*e**4*x - 18*a**2*c**3*d
**4*e**5*x**2 - 2*a**2*c**3*d**3*e**6*x**3 - 12*a*c**4*d**7*e**2*x + 18*a*
c**4*d**6*e**3*x**2 + 6*a*c**4*d**5*e**4*x**3 + a*c**4*d**4*e**5*x**4 + 3*
c**5*d**9*x)/(3*a*c**5*d**5*e*(a*e + c*d*x))
```

3.119 $\int \frac{(d+ex)^5}{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^2} \, dx$

Optimal result

Integrand size = 35, antiderivative size = 105

$$\int \frac{(d+ex)^5}{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^2} \, dx = \frac{e^2\left(3cd^2-2ae^2\right)x}{c^3d^3}+\frac{e^3x^2}{2c^2d^2}-\frac{\left(cd^2-ae^2\right)^3}{c^4d^4(ae+cdx)} + \frac{3e\left(cd^2-ae^2\right)^2\log(ae+cdx)}{c^4d^4}$$

output
```
e^2*(-2*a*e^2+3*c*d^2)*x/c^3/d^3+1/2*e^3*x^2/c^2/d^2-(-a*e^2+c*d^2)^3/c^4/
d^4/(c*d*x+a*e)+3*e*(-a*e^2+c*d^2)^2*ln(c*d*x+a*e)/c^4/d^4
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 142, normalized size of antiderivative = 1.35

$$\int \frac{(d+ex)^5}{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^2} \, dx = -\frac{e^2\left(-3cd^2+2ae^2\right)x}{c^3d^3}+\frac{e^3x^2}{2c^2d^2} + \frac{-c^3d^6+3ac^2d^4e^2-3a^2cd^2e^4+a^3e^6}{c^4d^4(ae+cdx)} + \frac{3\left(c^2d^4e-2acd^2e^3+a^2e^5\right)\log(ae+cdx)}{c^4d^4}$$

input
```
Integrate[(d + e*x)^5/(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^2,x]
```

output
```
-((e^2*(-3*c*d^2 + 2*a*e^2)*x)/(c^3*d^3)) + (e^3*x^2)/(2*c^2*d^2) + (-(c^3
*d^6) + 3*a*c^2*d^4*e^2 - 3*a^2*c*d^2*e^4 + a^3*e^6)/(c^4*d^4*(a*e + c*d*x
)) + (3*(c^2*d^4*e - 2*a*c*d^2*e^3 + a^2*e^5)*Log[a*e + c*d*x])/(c^4*d^4)
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.057$, Rules used = {1121, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex)^5}{(x(ae^2+cd^2)+ade+cdex^2)^2}dx$$

$$\Big\downarrow \text{1121}$$

$$\int \left(\frac{3e(cd^2-ae^2)^2}{c^3d^3(ae+cdx)}+\frac{(cd^2-ae^2)^3}{c^3d^3(ae+cdx)^2}+\frac{3cd^2e^2-2ae^4}{c^3d^3}+\frac{e^3x}{c^2d^2}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{(cd^2-ae^2)^3}{c^4d^4(ae+cdx)}+\frac{3e(cd^2-ae^2)^2\log(ae+cdx)}{c^4d^4}+\frac{e^2x(3cd^2-2ae^2)}{c^3d^3}+\frac{e^3x^2}{2c^2d^2}$$

input
```
Int[(d + e*x)^5/(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^2,x]
```

output
```
(e^2*(3*c*d^2 - 2*a*e^2)*x)/(c^3*d^3) + (e^3*x^2)/(2*c^2*d^2) - (c*d^2 - a
*e^2)^3/(c^4*d^4*(a*e + c*d*x)) + (3*e*(c*d^2 - a*e^2)^2*Log[a*e + c*d*x])
/(c^4*d^4)
```

Defintions of rubi rules used

rule 1121
```
Int[((d_) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Int[ExpandIntegrand[(d + e*x)^(m + p)*(a/d + (c/e)*x)^p, x], x]
/; FreeQ[{a, b, c, d, e, m, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && (Int
egerQ[p] || (GtQ[a, 0] && GtQ[d, 0] && LtQ[c, 0]))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 1.66 (sec) , antiderivative size = 137, normalized size of antiderivative = 1.30

method	result
default	$-\frac{e^{2}\left(-\frac{1}{2}c d\,x^{2} e+2a\,e^{2} x-3c\,d^{2} x\right)}{d^{3} c^{3}}+\frac{3 e\left(a^{2} e^{4}-2 a c\,d^{2} e^{2}+c^{2} d^{4}\right) \ln\left(c d x+a e\right)}{d^{4} c^{4}}-\frac{-e^{6} a^{3}+3 d^{2} e^{4} a^{2} c-3 d^{4} e^{2} a\,c^{2}+d^{6} c^{3}}{d^{4} c^{4}\left(c d x+a e\right)}$
risch	$\frac{e^{3} x^{2}}{2 c^{2} d^{2}}-\frac{2 e^{4} a x}{d^{3} c^{3}}+\frac{3 e^{2} x}{d\,c^{2}}+\frac{e^{6} a^{3}}{d^{4} c^{4}\left(c d x+a e\right)}-\frac{3 e^{4} a^{2}}{d^{2} c^{3}\left(c d x+a e\right)}+\frac{3 e^{2} a}{c^{2}\left(c d x+a e\right)}-\frac{d^{2}}{c\left(c d x+a e\right)}+\frac{3 e^{5} \ln\left(c d x+a e\right) a^{2}}{d^{4} c^{4}}-6$
norman	$\frac{\frac{6 e^{6} a^{3}-9 d^{2} e^{4} a^{2} c-2 d^{6} c^{3}}{2 d^{3} c^{4}}+\frac{e^{4} x^{4}}{2 d c}+\frac{\left(6 a^{3} e^{8}-9 a^{2} c\,d^{2} e^{6}+3 a\,c^{2} d^{4} e^{4}-8 c^{3} d^{6} e^{2}\right) x}{2 d^{4} c^{4} e}-\frac{e^{3}\left(3 a\,e^{2}-7 c\,d^{2}\right) x^{3}}{2 c^{2} d^{2}}}{\left(e x+d\right)\left(c d x+a e\right)}+\frac{3 e\left(a^{2} e^{4}-2 a c\,d^{2} e^{2}+c^{2} d^{4}\right) \ln}{d^{4} c^{4}}$
parallelrisch	$\frac{c^{3} d^{3} e^{3} x^{3}+6 \ln\left(c d x+a e\right) x\,a^{2} c d\,e^{5}-12 \ln\left(c d x+a e\right) x a\,c^{2} d^{3} e^{3}+6 \ln\left(c d x+a e\right) x\,c^{3} d^{5} e-3 x^{2} a\,c^{2} d^{2} e^{4}+6 c^{3} d^{4} e^{2} x^{2}+6 \ln\left(c d x+a e\right) a}{2 d^{4} c^{4}\left(c d x+a e\right)}$

input
```
int((e*x+d)^5/(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^2,x,method=_RETURNVERBOSE)
```

output
```
-e^2/d^3/c^3*(-1/2*c*d*x^2*e+2*a*e^2*x-3*c*d^2*x)+3/d^4*e/c^4*(a^2*e^4-2*a
*c*d^2*e^2+c^2*d^4)*ln(c*d*x+a*e)-1/d^4/c^4*(-a^3*e^6+3*a^2*c*d^2*e^4-3*a*
c^2*d^4*e^2+c^3*d^6)/(c*d*x+a*e)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 205, normalized size of antiderivative = 1.95

$$\int \frac{(d+ex)^5}{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^2}\,dx = \frac{c^3d^3e^3x^3 - 2\,c^3d^6 + 6\,ac^2d^4e^2 - 6\,a^2cd^2e^4 + 2\,a^3e^6 + 3\,(2\,c^3d^4e^2 - ac^2d^2e^4)x^2 + 2\,(3\,ac^2d^3e^3 - 2\,a^2cde^5)x}{2\,(c^5d^5x + ac^4d^4e)}$$

input
```
integrate((e*x+d)^5/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^2,x, algorithm="fric
as")
```

output
```
1/2*(c^3*d^3*e^3*x^3 - 2*c^3*d^6 + 6*a*c^2*d^4*e^2 - 6*a^2*c*d^2*e^4 + 2*a
^3*e^6 + 3*(2*c^3*d^4*e^2 - a*c^2*d^2*e^4)*x^2 + 2*(3*a*c^2*d^3*e^3 - 2*a^
2*c*d*e^5)*x + 6*(a*c^2*d^4*e^2 - 2*a^2*c*d^2*e^4 + a^3*e^6 + (c^3*d^5*e -
 2*a*c^2*d^3*e^3 + a^2*c*d*e^5)*x)*log(c*d*x + a*e))/(c^5*d^5*x + a*c^4*d^
4*e)
```

Sympy [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 131, normalized size of antiderivative = 1.25

$$\int \frac{(d+ex)^5}{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^2}\,dx = x\left(-\frac{2ae^4}{c^3d^3}+\frac{3e^2}{c^2d}\right) + \frac{a^3e^6-3a^2cd^2e^4+3ac^2d^4e^2-c^3d^6}{ac^4d^4e+c^5d^5x} + \frac{e^3x^2}{2c^2d^2} + \frac{3e\left(ae^2-cd^2\right)^2\log\left(ae+cdx\right)}{c^4d^4}$$

input
```
integrate((e*x+d)**5/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**2,x)
```

output
```
x*(-2*a*e**4/(c**3*d**3) + 3*e**2/(c**2*d)) + (a**3*e**6 - 3*a**2*c*d**2*e
**4 + 3*a*c**2*d**4*e**2 - c**3*d**6)/(a*c**4*d**4*e + c**5*d**5*x) + e**3
*x**2/(2*c**2*d**2) + 3*e*(a*e**2 - c*d**2)**2*log(a*e + c*d*x)/(c**4*d**4
)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 143, normalized size of antiderivative = 1.36

$$\int \frac{(d+ex)^5}{\left(ade+(cd^2+ae^2)\,x+cdex^2\right)^2}\,dx = -\frac{c^3d^6-3\,ac^2d^4e^2+3\,a^2cd^2e^4-a^3e^6}{c^5d^5x+ac^4d^4e} + \frac{cde^3x^2+2\,(3\,cd^2e^2-2\,ae^4)x}{2\,c^3d^3} + \frac{3\,(c^2d^4e-2\,acd^2e^3+a^2e^5)\log\left(cdx+ae\right)}{c^4d^4}$$

input
```
integrate((e*x+d)^5/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^2,x, algorithm="maxi
ma")
```

output
```
-(c^3*d^6 - 3*a*c^2*d^4*e^2 + 3*a^2*c*d^2*e^4 - a^3*e^6)/(c^5*d^5*x + a*c^
4*d^4*e) + 1/2*(c*d*e^3*x^2 + 2*(3*c*d^2*e^2 - 2*a*e^4)*x)/(c^3*d^3) + 3*(
c^2*d^4*e - 2*a*c*d^2*e^3 + a^2*e^5)*log(c*d*x + a*e)/(c^4*d^4)
```

Giac [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 146, normalized size of antiderivative = 1.39

$$\int \frac{(d+ex)^5}{\left(ade+(cd^2+ae^2)\,x+cdex^2\right)^2}\,dx = \frac{3\,(c^2d^4e-2\,acd^2e^3+a^2e^5)\log\left(|cdx+ae|\right)}{c^4d^4} + \frac{c^2d^2e^3x^2+6\,c^2d^3e^2x-4\,acde^4x}{2\,c^4d^4} - \frac{c^3d^6-3\,ac^2d^4e^2+3\,a^2cd^2e^4-a^3e^6}{(cdx+ae)c^4d^4}$$

input
```
integrate((e*x+d)^5/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^2,x, algorithm="giac
")
```

output
```
3*(c^2*d^4*e - 2*a*c*d^2*e^3 + a^2*e^5)*log(abs(c*d*x + a*e))/(c^4*d^4) +
1/2*(c^2*d^2*e^3*x^2 + 6*c^2*d^3*e^2*x - 4*a*c*d*e^4*x)/(c^4*d^4) - (c^3*d
^6 - 3*a*c^2*d^4*e^2 + 3*a^2*c*d^2*e^4 - a^3*e^6)/((c*d*x + a*e)*c^4*d^4)
```

Mupad [B] (verification not implemented)

Time = 5.41 (sec) , antiderivative size = 152, normalized size of antiderivative = 1.45

$$\int \frac{(d+ex)^5}{(ade+(cd^2+ae^2)x+cdex^2)^2}\,dx = x\left(\frac{3\,e^2}{c^2\,d}-\frac{2\,a\,e^4}{c^3\,d^3}\right)+\frac{e^3\,x^2}{2\,c^2\,d^2} + \frac{\ln(a\,e+c\,d\,x)\,\left(3\,a^2\,e^5-6\,a\,c\,d^2\,e^3+3\,c^2\,d^4\,e\right)}{c^4\,d^4} + \frac{a^3\,e^6-3\,a^2\,c\,d^2\,e^4+3\,a\,c^2\,d^4\,e^2-c^3\,d^6}{c\,d\,\left(x\,c^4\,d^4+a\,e\,c^3\,d^3\right)}$$

input
```
int((d + e*x)^5/(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^2,x)
```

output
```
x*((3*e^2)/(c^2*d) - (2*a*e^4)/(c^3*d^3)) + (e^3*x^2)/(2*c^2*d^2) + (log(a
*e + c*d*x)*(3*a^2*e^5 + 3*c^2*d^4*e - 6*a*c*d^2*e^3))/(c^4*d^4) + (a^3*e^
6 - c^3*d^6 + 3*a*c^2*d^4*e^2 - 3*a^2*c*d^2*e^4)/(c*d*(c^4*d^4*x + a*c^3*d
^3*e))
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 246, normalized size of antiderivative = 2.34

$$\int \frac{(d+ex)^5}{(ade+(cd^2+ae^2)x+cdex^2)^2}\,dx$$
$$= \frac{6\log(cdx+ae)\,a^4e^7-12\log(cdx+ae)\,a^3c\,d^2e^5+6\log(cdx+ae)\,a^3cd\,e^6x+6\log(cdx+ae)\,a^2c^2d^4e^3-}{}$$

input
```
int((e*x+d)^5/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^2,x)
```

output
```
(6*log(a*e + c*d*x)*a**4*e**7 - 12*log(a*e + c*d*x)*a**3*c*d**2*e**5 + 6*l
og(a*e + c*d*x)*a**3*c*d*e**6*x + 6*log(a*e + c*d*x)*a**2*c**2*d**4*e**3 -
 12*log(a*e + c*d*x)*a**2*c**2*d**3*e**4*x + 6*log(a*e + c*d*x)*a*c**3*d**
5*e**2*x - 6*a**3*c*d*e**6*x + 12*a**2*c**2*d**3*e**4*x - 3*a**2*c**2*d**2
*e**5*x**2 - 6*a*c**3*d**5*e**2*x + 6*a*c**3*d**4*e**3*x**2 + a*c**3*d**3*
e**4*x**3 + 2*c**4*d**7*x)/(2*a*c**4*d**4*e*(a*e + c*d*x))
```

3.120 $\int \frac{(d+ex)^4}{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^2}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 74

$$\int \frac{(d+ex)^4}{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^2}\,dx = \frac{e^2x}{c^2d^2}-\frac{\left(cd^2-ae^2\right)^2}{c^3d^3(ae+cdx)}+\frac{2e\left(cd^2-ae^2\right)\log(ae+cdx)}{c^3d^3}$$

output
```
e^2*x/c^2/d^2-(-a*e^2+c*d^2)^2/c^3/d^3/(c*d*x+a*e)+2*e*(-a*e^2+c*d^2)*ln(c
*d*x+a*e)/c^3/d^3
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.88

$$\int \frac{(d+ex)^4}{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^2}\,dx = \frac{cde^2x-\frac{\left(cd^2-ae^2\right)^2}{ae+cdx}+2\left(cd^2e-ae^3\right)\log(ae+cdx)}{c^3d^3}$$

input
```
Integrate[(d + e*x)^4/(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^2,x]
```

output
```
(c*d*e^2*x - (c*d^2 - a*e^2)^2/(a*e + c*d*x) + 2*(c*d^2*e - a*e^3)*Log[a*e
 + c*d*x])/(c^3*d^3)
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.057$, Rules used = {1121, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex)^4}{(x(ae^2+cd^2)+ade+cdex^2)^2}dx$$

$$\Big\downarrow \text{1121}$$

$$\int \left(\frac{2(cd^2e-ae^3)}{c^2d^2(ae+cdx)}+\frac{(cd^2-ae^2)^2}{c^2d^2(ae+cdx)^2}+\frac{e^2}{c^2d^2}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{(cd^2-ae^2)^2}{c^3d^3(ae+cdx)}+\frac{2e(cd^2-ae^2)\log(ae+cdx)}{c^3d^3}+\frac{e^2x}{c^2d^2}$$

input
```
Int[(d + e*x)^4/(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^2,x]
```

output
```
(e^2*x)/(c^2*d^2) - (c*d^2 - a*e^2)^2/(c^3*d^3*(a*e + c*d*x)) + (2*e*(c*d^
2 - a*e^2)*Log[a*e + c*d*x])/(c^3*d^3)
```

Defintions of rubi rules used

rule 1121
```
Int[((d_) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Int[ExpandIntegrand[(d + e*x)^(m + p)*(a/d + (c/e)*x)^p, x], x]
/; FreeQ[{a, b, c, d, e, m, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && (Int
egerQ[p] || (GtQ[a, 0] && GtQ[d, 0] && LtQ[c, 0]))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 1.59 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.16

method	result
default	$\frac{e^{2} x}{c^{2} d^{2}}-\frac{2 e \left(a \,e^{2}-c \,d^{2}\right) \ln \left(c d x+a e\right)}{d^{3} c^{3}}-\frac{a^{2} e^{4}-2 a c \,d^{2} e^{2}+c^{2} d^{4}}{c^{3} d^{3} \left(c d x+a e\right)}$
risch	$\frac{e^{2} x}{c^{2} d^{2}}-\frac{2 e^{3} \ln \left(c d x+a e\right) a}{d^{3} c^{3}}+\frac{2 e \ln \left(c d x+a e\right)}{d \,c^{2}}-\frac{a^{2} e^{4}}{c^{3} d^{3} \left(c d x+a e\right)}+\frac{2 a \,e^{2}}{c^{2} d \left(c d x+a e\right)}-\frac{d}{c \left(c d x+a e\right)}$
parallelrisch	$-\frac{2 \ln \left(c d x+a e\right) x a c d \,e^{3}-2 \ln \left(c d x+a e\right) x \,c^{2} d^{3} e-x^{2} c^{2} d^{2} e^{2}+2 \ln \left(c d x+a e\right) a^{2} e^{4}-2 \ln \left(c d x+a e\right) a c \,d^{2} e^{2}+2 a^{2} e^{4}-2 a c \,d^{2} e^{2}+c^{2} d^{4}}{d^{3} c^{3} \left(c d x+a e\right)}$
norman	$\frac{\frac{e^{3} x^{3}}{c d}-\frac{2 a^{2} e^{4}-a c \,d^{2} e^{2}+c^{2} d^{4}}{d^{2} c^{3}}-\frac{\left(2 a^{2} e^{6}-a c \,d^{2} e^{4}+2 c^{2} d^{4} e^{2}\right) x}{d^{3} c^{3} e}}{\left(e x+d\right) \left(c d x+a e\right)}-\frac{2 e \left(a \,e^{2}-c \,d^{2}\right) \ln \left(c d x+a e\right)}{d^{3} c^{3}}$

input
```
int((e*x+d)^4/(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^2,x,method=_RETURNVERBOSE)
```

output
```
e^2*x/c^2/d^2-2/d^3*e/c^3*(a*e^2-c*d^2)*ln(c*d*x+a*e)-(a^2*e^4-2*a*c*d^2*e
^2+c^2*d^4)/c^3/d^3/(c*d*x+a*e)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.58

$$\int \frac{(d+ex)^4}{(ade+(cd^2+ae^2)x+cdex^2)^2}\,dx$$
$$=\frac{c^2d^2e^2x^2+acde^3x-c^2d^4+2\,acd^2e^2-a^2e^4+2\,(acd^2e^2-a^2e^4+(c^2d^3e-acde^3)x)\log(cdx+ae)}{c^4d^4x+ac^3d^3e}$$

input
```
integrate((e*x+d)^4/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^2,x, algorithm="fric
as")
```

output
```
(c^2*d^2*e^2*x^2 + a*c*d*e^3*x - c^2*d^4 + 2*a*c*d^2*e^2 - a^2*e^4 + 2*(a*
c*d^2*e^2 - a^2*e^4 + (c^2*d^3*e - a*c*d*e^3)*x)*log(c*d*x + a*e))/(c^4*d^
4*x + a*c^3*d^3*e)
```

Sympy [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.15

$$\int \frac{(d+ex)^4}{(ade+(cd^2+ae^2)\,x+cdex^2)^2}\,dx = \frac{-a^2e^4+2acd^2e^2-c^2d^4}{ac^3d^3e+c^4d^4x}+\frac{e^2x}{c^2d^2} - \frac{2e(ae^2-cd^2)\log{(ae+cdx)}}{c^3d^3}$$

input
```
integrate((e*x+d)**4/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**2,x)
```

output
```
(-a**2*e**4 + 2*a*c*d**2*e**2 - c**2*d**4)/(a*c**3*d**3*e + c**4*d**4*x) +
 e**2*x/(c**2*d**2) - 2*e*(a*e**2 - c*d**2)*log(a*e + c*d*x)/(c**3*d**3)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.20

$$\int \frac{(d+ex)^4}{(ade+(cd^2+ae^2)\,x+cdex^2)^2}\,dx = -\frac{c^2d^4-2\,acd^2e^2+a^2e^4}{c^4d^4x+ac^3d^3e}+\frac{e^2x}{c^2d^2} + \frac{2\,(cd^2e-ae^3)\log{(cdx+ae)}}{c^3d^3}$$

input
```
integrate((e*x+d)^4/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^2,x, algorithm="maxi
ma")
```

output
```
-(c^2*d^4 - 2*a*c*d^2*e^2 + a^2*e^4)/(c^4*d^4*x + a*c^3*d^3*e) + e^2*x/(c^
2*d^2) + 2*(c*d^2*e - a*e^3)*log(c*d*x + a*e)/(c^3*d^3)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.16

$$\int \frac{(d+ex)^4}{\left(ade+(cd^2+ae^2)\,x+cdex^2\right)^2}\,dx = \frac{e^2x}{c^2d^2}+\frac{2\,(cd^2e-ae^3)\log\left(|cdx+ae|\right)}{c^3d^3} - \frac{c^2d^4-2\,acd^2e^2+a^2e^4}{(cdx+ae)c^3d^3}$$

input
```
integrate((e*x+d)^4/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^2,x, algorithm="giac
")
```

output
```
e^2*x/(c^2*d^2) + 2*(c*d^2*e - a*e^3)*log(abs(c*d*x + a*e))/(c^3*d^3) - (c
^2*d^4 - 2*a*c*d^2*e^2 + a^2*e^4)/((c*d*x + a*e)*c^3*d^3)
```

Mupad [B] (verification not implemented)

Time = 5.41 (sec) , antiderivative size = 96, normalized size of antiderivative = 1.30

$$\int \frac{(d+ex)^4}{\left(ade+(cd^2+ae^2)\,x+cdex^2\right)^2}\,dx = \frac{e^2\,x}{c^2\,d^2}-\frac{\ln\left(a\,e+c\,d\,x\right)\,\left(2\,a\,e^3-2\,c\,d^2\,e\right)}{c^3\,d^3} - \frac{a^2\,e^4-2\,a\,c\,d^2\,e^2+c^2\,d^4}{c\,d\,\left(x\,c^3\,d^3+a\,e\,c^2\,d^2\right)}$$

input
```
int((d + e*x)^4/(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^2,x)
```

output
```
(e^2*x)/(c^2*d^2) - (log(a*e + c*d*x)*(2*a*e^3 - 2*c*d^2*e))/(c^3*d^3) - (
a^2*e^4 + c^2*d^4 - 2*a*c*d^2*e^2)/(c*d*(c^3*d^3*x + a*c^2*d^2*e))
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 150, normalized size of antiderivative = 2.03

$$\int \frac{(d+ex)^4}{\left(ade+(cd^2+ae^2)\,x+cdex^2\right)^2}\,dx$$
$$= \frac{-2\log(cdx+ae)\,a^3e^5+2\log(cdx+ae)\,a^2c\,d^2e^3-2\log(cdx+ae)\,a^2cd\,e^4x+2\log(cdx+ae)\,a\,c^2d^3e^2x}{a\,c^3d^3e\,(cdx+ae)}$$

input
```
int((e*x+d)^4/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^2,x)
```

output
```
( - 2*log(a*e + c*d*x)*a**3*e**5 + 2*log(a*e + c*d*x)*a**2*c*d**2*e**3 - 2
*log(a*e + c*d*x)*a**2*c*d*e**4*x + 2*log(a*e + c*d*x)*a*c**2*d**3*e**2*x
+ 2*a**2*c*d*e**4*x - 2*a*c**2*d**3*e**2*x + a*c**2*d**2*e**3*x**2 + c**3*
d**5*x)/(a*c**3*d**3*e*(a*e + c*d*x))
```

3.121 $\int \frac{(d+ex)^3}{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^2}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 48

$$\int \frac{(d+ex)^3}{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^2}\,dx=-\frac{cd^2-ae^2}{c^2d^2(ae+cdx)}+\frac{e\log(ae+cdx)}{c^2d^2}$$

output
```
-(-a*e^2+c*d^2)/c^2/d^2/(c*d*x+a*e)+e*ln(c*d*x+a*e)/c^2/d^2
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.98

$$\int \frac{(d+ex)^3}{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^2}\,dx=\frac{-cd^2+ae^2}{c^2d^2(ae+cdx)}+\frac{e\log(ae+cdx)}{c^2d^2}$$

input
```
Integrate[(d + e*x)^3/(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^2,x]
```

output
```
(-(c*d^2) + a*e^2)/(c^2*d^2*(a*e + c*d*x)) + (e*Log[a*e + c*d*x])/(c^2*d^2
)
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.057$, Rules used = {1121, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex)^3}{(x(ae^2+cd^2)+ade+cdex^2)^2}dx$$

$$\Big\downarrow \text{1121}$$

$$\int \left(\frac{cd^2-ae^2}{cd(ae+cdx)^2}+\frac{e}{cd(ae+cdx)}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{e\log(ae+cdx)}{c^2d^2}-\frac{cd^2-ae^2}{c^2d^2(ae+cdx)}$$

input
```
Int[(d + e*x)^3/(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^2,x]
```

output
```
-((c*d^2 - a*e^2)/(c^2*d^2*(a*e + c*d*x))) + (e*Log[a*e + c*d*x])/(c^2*d^2
)
```

Defintions of rubi rules used

rule 1121
```
Int[((d_) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Int[ExpandIntegrand[(d + e*x)^(m + p)*(a/d + (c/e)*x)^p, x], x]
/; FreeQ[{a, b, c, d, e, m, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && (Int
egerQ[p] || (GtQ[a, 0] && GtQ[d, 0] && LtQ[c, 0]))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 1.56 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.02

method	result	size
default	$-\frac{-a\,e^{2}+c\,d^{2}}{c^{2} d^{2} (c d x+a e)}+\frac{e \ln (c d x+a e)}{c^{2} d^{2}}$	49
risch	$\frac{a\,e^{2}}{c^{2} d^{2} (c d x+a e)}-\frac{1}{c (c d x+a e)}+\frac{e \ln (c d x+a e)}{c^{2} d^{2}}$	55
parallelrisch	$\frac{\ln (c d x+a e) x c d e+\ln (c d x+a e) a\,e^{2}+a\,e^{2}-c\,d^{2}}{c^{2} d^{2} (c d x+a e)}$	58
norman	$\frac{\frac{a\,e^{2}-c\,d^{2}}{d\,c^{2}}+\frac{\left(e^{4} a-d^{2} e^{2} c\right) x}{c^{2} d^{2} e}}{(e x+d) (c d x+a e)}+\frac{e \ln (c d x+a e)}{c^{2} d^{2}}$	83

input
```
int((e*x+d)^3/(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^2,x,method=_RETURNVERBOSE)
```

output
```
-(-a*e^2+c*d^2)/c^2/d^2/(c*d*x+a*e)+e*ln(c*d*x+a*e)/c^2/d^2
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.17

$$\int \frac{(d+ex)^3}{(ade+(cd^2+ae^2)\,x+cdex^2)^2}\,dx=-\frac{cd^2-ae^2-(cdex+ae^2)\log(cdx+ae)}{c^3d^3x+ac^2d^2e}$$

input
```
integrate((e*x+d)^3/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^2,x, algorithm="fric
as")
```

output
```
-(c*d^2 - a*e^2 - (c*d*e*x + a*e^2)*log(c*d*x + a*e))/(c^3*d^3*x + a*c^2*d
^2*e)
```

Sympy [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.96

$$\int \frac{(d+ex)^3}{\left(ade+(cd^2+ae^2)\,x+cdex^2\right)^2}\,dx=\frac{ae^2-cd^2}{ac^2d^2e+c^3d^3x}+\frac{e\log\left(ae+cdx\right)}{c^2d^2}$$

input
```
integrate((e*x+d)**3/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**2,x)
```

output
```
(a*e**2 - c*d**2)/(a*c**2*d**2*e + c**3*d**3*x) + e*log(a*e + c*d*x)/(c**2
*d**2)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.08

$$\int \frac{(d+ex)^3}{\left(ade+(cd^2+ae^2)\,x+cdex^2\right)^2}\,dx=-\frac{cd^2-ae^2}{c^3d^3x+ac^2d^2e}+\frac{e\log\left(cdx+ae\right)}{c^2d^2}$$

input
```
integrate((e*x+d)^3/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^2,x, algorithm="maxi
ma")
```

output
```
-(c*d^2 - a*e^2)/(c^3*d^3*x + a*c^2*d^2*e) + e*log(c*d*x + a*e)/(c^2*d^2)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.02

$$\int \frac{(d+ex)^3}{\left(ade+(cd^2+ae^2)\,x+cdex^2\right)^2}\,dx=\frac{e\log\left(|cdx+ae|\right)}{c^2d^2}-\frac{cd^2-ae^2}{(cdx+ae)c^2d^2}$$

input
```
integrate((e*x+d)^3/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^2,x, algorithm="giac
")
```

output
```
e*log(abs(c*d*x + a*e))/(c^2*d^2) - (c*d^2 - a*e^2)/((c*d*x + a*e)*c^2*d^2
)
```

Mupad [B] (verification not implemented)

Time = 5.26 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.98

$$\int \frac{(d+ex)^3}{\left(ade+(cd^2+ae^2)\,x+cdex^2\right)^2}\,dx = \frac{a\,e^2-c\,d^2}{c^2\,d^2\,\left(a\,e+c\,d\,x\right)}+\frac{e\,\ln\left(a\,e+c\,d\,x\right)}{c^2\,d^2}$$

input
```
int((d + e*x)^3/(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^2,x)
```

output
```
(a*e^2 - c*d^2)/(c^2*d^2*(a*e + c*d*x)) + (e*log(a*e + c*d*x))/(c^2*d^2)
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.54

$$\int \frac{(d+ex)^3}{\left(ade+(cd^2+ae^2)\,x+cdex^2\right)^2}\,dx = \frac{\log(cdx+ae)\,a^2e^3+\log(cdx+ae)\,acd\,e^2x-acd\,e^2x+c^2d^3x}{a\,c^2d^2e\,(cdx+ae)}$$

input
```
int((e*x+d)^3/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^2,x)
```

output
```
(log(a*e + c*d*x)*a**2*e**3 + log(a*e + c*d*x)*a*c*d*e**2*x - a*c*d*e**2*x
 + c**2*d**3*x)/(a*c**2*d**2*e*(a*e + c*d*x))
```

3.122 $\int \frac{(d+ex)^2}{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^2}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 18

$$\int \frac{(d+ex)^2}{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^2}\,dx=-\frac{1}{cd(ae+cdx)}$$

output
```
-1/c/d/(c*d*x+a*e)
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int \frac{(d+ex)^2}{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^2}\,dx=-\frac{1}{cd(ae+cdx)}$$

input
```
Integrate[(d + e*x)^2/(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^2,x]
```

output
```
-(1/(c*d*(a*e + c*d*x)))
```

Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.057, Rules used = {1120, 17}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex)^2}{\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^2}dx$$

$$\Big\downarrow \text{ 1120}$$

$$\int \frac{1}{(ae+cdx)^2}dx$$

$$\Big\downarrow \text{ 17}$$

$$-\frac{1}{cd(ae+cdx)}$$

input
```
Int[(d + e*x)^2/(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^2,x]
```

output
```
-(1/(c*d*(a*e + c*d*x)))
```

Defintions of rubi rules used

rule 17
```
Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1
)/(b*(m + 1))), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]
```

rule 1120
```
Int[((d_) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Int[(d + e*x)^(m + p)*(a/d + (c/e)*x)^p, x] /; FreeQ[{a, b, c, d
, e, m}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[p] && (EqQ[m + p,
0] || EqQ[m + 2*p + 2, 0])
```

Maple [A] (verified)

Time = 1.57 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.06

method	result	size
gosper	$-\frac{1}{cd(cdx+ae)}$	19
default	$-\frac{1}{cd(cdx+ae)}$	19
risch	$-\frac{1}{cd(cdx+ae)}$	19
parallelrisch	$-\frac{1}{cd(cdx+ae)}$	19
norman	$\frac{-\frac{1}{c}-\frac{ex}{cd}}{(ex+d)(cdx+ae)}$	35
orering	$-\frac{(cdx+ae)(ex+d)^2}{cd(ade+(a\,e^2+c\,d^2)x+cd\,x^2e)^2}$	51

input
```
int((e*x+d)^2/(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^2,x,method=_RETURNVERBOSE)
```

output
```
-1/c/d/(c*d*x+a*e)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int \frac{(d+ex)^2}{(ade+(cd^2+ae^2)\,x+cdex^2)^2}\,dx=-\frac{1}{c^2d^2x+acde}$$

input
```
integrate((e*x+d)^2/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^2,x, algorithm="fric
as")
```

output
```
-1/(c^2*d^2*x + a*c*d*e)
```

Sympy [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.94

$$\int \frac{(d+ex)^2}{\left(ade+(cd^2+ae^2)\,x+cdex^2\right)^2}\,dx=-\frac{1}{acde+c^2d^2x}$$

input
```
integrate((e*x+d)**2/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**2,x)
```

output
```
-1/(a*c*d*e + c**2*d**2*x)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int \frac{(d+ex)^2}{\left(ade+(cd^2+ae^2)\,x+cdex^2\right)^2}\,dx=-\frac{1}{c^2d^2x+acde}$$

input
```
integrate((e*x+d)^2/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^2,x, algorithm="maxi
ma")
```

output
```
-1/(c^2*d^2*x + a*c*d*e)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int \frac{(d+ex)^2}{\left(ade+(cd^2+ae^2)\,x+cdex^2\right)^2}\,dx=-\frac{1}{(cdx+ae)cd}$$

input
```
integrate((e*x+d)^2/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^2,x, algorithm="giac
")
```

output
```
-1/((c*d*x + a*e)*c*d)
```

Mupad [B] (verification not implemented)

Time = 5.26 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int \frac{(d+ex)^2}{(ade+(cd^2+ae^2)\,x+cdex^2)^2}\,dx = -\frac{1}{c\,d\,(a\,e+c\,d\,x)}$$

input
```
int((d + e*x)^2/(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^2,x)
```

output
```
-1/(c*d*(a*e + c*d*x))
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int \frac{(d+ex)^2}{(ade+(cd^2+ae^2)\,x+cdex^2)^2}\,dx = \frac{x}{ae\,(cdx+ae)}$$

input
```
int((e*x+d)^2/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^2,x)
```

output
```
x/(a*e*(a*e + c*d*x))
```

3.123 $\int \frac{d+ex}{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^2}\,dx$

Optimal result

Integrand size = 33, antiderivative size = 75

$$\int \frac{d+ex}{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^2}\,dx=-\frac{1}{\left(cd^2-ae^2\right)(ae+cdx)}-\frac{e\log(ae+cdx)}{\left(cd^2-ae^2\right)^2}+\frac{e\log(d+ex)}{\left(cd^2-ae^2\right)^2}$$

output
```
-1/(-a*e^2+c*d^2)/(c*d*x+a*e)-e*ln(c*d*x+a*e)/(-a*e^2+c*d^2)^2+e*ln(e*x+d)
/(-a*e^2+c*d^2)^2
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.99

$$\int \frac{d+ex}{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^2}\,dx=\frac{1}{\left(-cd^2+ae^2\right)(ae+cdx)}-\frac{e\log(ae+cdx)}{\left(-cd^2+ae^2\right)^2}+\frac{e\log(d+ex)}{\left(-cd^2+ae^2\right)^2}$$

input
```
Integrate[(d + e*x)/(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^2,x]
```

output
```
1/((-(c*d^2) + a*e^2)*(a*e + c*d*x)) - (e*Log[a*e + c*d*x])/(-(c*d^2) + a*
e^2)^2 + (e*Log[d + e*x])/(-(c*d^2) + a*e^2)^2
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.061, Rules used = {1121, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{d+ex}{\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^2}dx$$

$$\Big\downarrow \text{ 1121}$$

$$\int \left(\frac{e^2}{(d+ex)\left(cd^2-ae^2\right)^2}-\frac{cde}{\left(cd^2-ae^2\right)^2(ae+cdx)}+\frac{cd}{\left(cd^2-ae^2\right)(ae+cdx)^2}\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$-\frac{1}{\left(cd^2-ae^2\right)(ae+cdx)}-\frac{e\log(ae+cdx)}{\left(cd^2-ae^2\right)^2}+\frac{e\log(d+ex)}{\left(cd^2-ae^2\right)^2}$$

input
```
Int[(d + e*x)/(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^2,x]
```

output
```
-(1/((c*d^2 - a*e^2)*(a*e + c*d*x))) - (e*Log[a*e + c*d*x])/(c*d^2 - a*e^2
)^2 + (e*Log[d + e*x])/(c*d^2 - a*e^2)^2
```

Defintions of rubi rules used

rule 1121
```
Int[((d_) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Int[ExpandIntegrand[(d + e*x)^(m + p)*(a/d + (c/e)*x)^p, x], x]
/; FreeQ[{a, b, c, d, e, m, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && (Int
egerQ[p] || (GtQ[a, 0] && GtQ[d, 0] && LtQ[c, 0]))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 1.60 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.00

method	result	size
default	$\frac{1}{(a\,e^{2}-c\,d^{2})(cdx+ae)}-\frac{e\ln(cdx+ae)}{(a\,e^{2}-c\,d^{2})^{2}}+\frac{e\ln(ex+d)}{(a\,e^{2}-c\,d^{2})^{2}}$	75
risch	$\frac{1}{(a\,e^{2}-c\,d^{2})(cdx+ae)}-\frac{e\ln(cdx+ae)}{a^{2}e^{4}-2ac\,d^{2}e^{2}+c^{2}d^{4}}+\frac{e\ln(-ex-d)}{a^{2}e^{4}-2ac\,d^{2}e^{2}+c^{2}d^{4}}$	104
parallelrisch	$\frac{\ln(ex+d)x\,c^{2}d^{2}e-\ln(cdx+ae)x\,c^{2}d^{2}e+\ln(ex+d)acd\,e^{2}-\ln(cdx+ae)acd\,e^{2}+ad\,e^{2}c-c^{2}d^{3}}{(a^{2}e^{4}-2ac\,d^{2}e^{2}+c^{2}d^{4})(cdx+ae)cd}$	125
norman	$\frac{\frac{d}{a\,e^{2}-c\,d^{2}}+\frac{ex}{a\,e^{2}-c\,d^{2}}}{(ex+d)(cdx+ae)}+\frac{e\ln(ex+d)}{a^{2}e^{4}-2ac\,d^{2}e^{2}+c^{2}d^{4}}-\frac{e\ln(cdx+ae)}{a^{2}e^{4}-2ac\,d^{2}e^{2}+c^{2}d^{4}}$	128

input
```
int((e*x+d)/(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^2,x,method=_RETURNVERBOSE)
```

output
```
1/(a*e^2-c*d^2)/(c*d*x+a*e)-e/(a*e^2-c*d^2)^2*ln(c*d*x+a*e)+e/(a*e^2-c*d^2
)^2*ln(e*x+d)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.55

$$\int \frac{d+ex}{(ade+(cd^2+ae^2)\,x+cdex^2)^2}\,dx$$
$$=-\frac{cd^2-ae^2+(cdex+ae^2)\log(cdx+ae)-(cdex+ae^2)\log(ex+d)}{ac^2d^4e-2\,a^2cd^2e^3+a^3e^5+(c^3d^5-2\,ac^2d^3e^2+a^2cde^4)x}$$

input
```
integrate((e*x+d)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^2,x, algorithm="fricas
")
```

output
```
-(c*d^2 - a*e^2 + (c*d*e*x + a*e^2)*log(c*d*x + a*e) - (c*d*e*x + a*e^2)*l
og(e*x + d))/(a*c^2*d^4*e - 2*a^2*c*d^2*e^3 + a^3*e^5 + (c^3*d^5 - 2*a*c^2
*d^3*e^2 + a^2*c*d*e^4)*x)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 287 vs. $2(63) = 126$.

Time = 0.39 (sec) , antiderivative size = 287, normalized size of antiderivative = 3.83

$$\int \frac{d+ex}{(ade+(cd^2+ae^2)x+cdex^2)^2}\,dx = \frac{e\log\left(x+\frac{-\frac{a^3e^7}{(ae^2-cd^2)^2}+\frac{3a^2cd^2e^5}{(ae^2-cd^2)^2}-\frac{3ac^2d^4e^3}{(ae^2-cd^2)^2}+ae^3+\frac{c^3d^6e}{(ae^2-cd^2)^2}+cd^2e}{2cde^2}\right)}{(ae^2-cd^2)^2} - \frac{e\log\left(x+\frac{\frac{a^3e^7}{(ae^2-cd^2)^2}-\frac{3a^2cd^2e^5}{(ae^2-cd^2)^2}+\frac{3ac^2d^4e^3}{(ae^2-cd^2)^2}+ae^3-\frac{c^3d^6e}{(ae^2-cd^2)^2}+cd^2e}{2cde^2}\right)}{(ae^2-cd^2)^2} + \frac{1}{a^2e^3-acd^2e+x(acde^2-c^2d^3)}$$

input
```
integrate((e*x+d)/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**2,x)
```

output
```
e*log(x + (-a**3*e**7/(a*e**2 - c*d**2)**2 + 3*a**2*c*d**2*e**5/(a*e**2 -
c*d**2)**2 - 3*a*c**2*d**4*e**3/(a*e**2 - c*d**2)**2 + a*e**3 + c**3*d**6*
e/(a*e**2 - c*d**2)**2 + c*d**2*e)/(2*c*d*e**2))/(a*e**2 - c*d**2)**2 - e*
log(x + (a**3*e**7/(a*e**2 - c*d**2)**2 - 3*a**2*c*d**2*e**5/(a*e**2 - c*d
**2)**2 + 3*a*c**2*d**4*e**3/(a*e**2 - c*d**2)**2 + a*e**3 - c**3*d**6*e/(
a*e**2 - c*d**2)**2 + c*d**2*e)/(2*c*d*e**2))/(a*e**2 - c*d**2)**2 + 1/(a*
*2*e**3 - a*c*d**2*e + x*(a*c*d*e**2 - c**2*d**3))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.51

$$\int \frac{d+ex}{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^2}\,dx = -\frac{e\log\left(cdx+ae\right)}{c^2d^4-2\,acd^2e^2+a^2e^4} + \frac{e\log\left(ex+d\right)}{c^2d^4-2\,acd^2e^2+a^2e^4} - \frac{1}{acd^2e-a^2e^3+(c^2d^3-acde^2)x}$$

input
```
integrate((e*x+d)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^2,x, algorithm="maxima
")
```

output
```
-e*log(c*d*x + a*e)/(c^2*d^4 - 2*a*c*d^2*e^2 + a^2*e^4) + e*log(e*x + d)/(
c^2*d^4 - 2*a*c*d^2*e^2 + a^2*e^4) - 1/(a*c*d^2*e - a^2*e^3 + (c^2*d^3 - a
*c*d*e^2)*x)
```

Giac [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.49

$$\int \frac{d+ex}{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^2}\,dx = -\frac{cde\log\left(|cdx+ae|\right)}{c^3d^5-2\,ac^2d^3e^2+a^2cde^4} + \frac{e^2\log\left(|ex+d|\right)}{c^2d^4e-2\,acd^2e^3+a^2e^5} - \frac{1}{(cd^2-ae^2)(cdx+ae)}$$

input
```
integrate((e*x+d)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^2,x, algorithm="giac")
```

output
```
-c*d*e*log(abs(c*d*x + a*e))/(c^3*d^5 - 2*a*c^2*d^3*e^2 + a^2*c*d*e^4) + e
^2*log(abs(e*x + d))/(c^2*d^4*e - 2*a*c*d^2*e^3 + a^2*e^5) - 1/((c*d^2 - a
*e^2)*(c*d*x + a*e))
```

Mupad [B] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 96, normalized size of antiderivative = 1.28

$$\int \frac{d+ex}{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^2}\,dx=\frac{1}{\left(a\,e+c\,d\,x\right)\,\left(a\,e^2-c\,d^2\right)}-\frac{2\,e\,\mathrm{atanh}\left(\frac{a^2\,e^4-c^2\,d^4}{{\left(a\,e^2-c\,d^2\right)}^2}+\frac{2\,c\,d\,e\,x}{a\,e^2-c\,d^2}\right)}{{\left(a\,e^2-c\,d^2\right)}^2}$$

input
```
int((d + e*x)/(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^2,x)
```

output
```
1/((a*e + c*d*x)*(a*e^2 - c*d^2)) - (2*e*atanh((a^2*e^4 - c^2*d^4)/(a*e^2
- c*d^2)^2 + (2*c*d*e*x)/(a*e^2 - c*d^2)))/(a*e^2 - c*d^2)^2
```

Reduce [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 149, normalized size of antiderivative = 1.99

$$\int \frac{d+ex}{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^2}\,dx =\frac{-\log(cdx+ae)\,a^2e^3-\log(cdx+ae)\,acd\,e^2x+\log(ex+d)\,a^2e^3+\log(ex+d)\,acd\,e^2x-acd\,e^2x+c^2d^3x}{ae\left(a^2cd\,e^4x-2a\,c^2d^3e^2x+c^3d^5x+a^3e^5-2a^2c\,d^2e^3+a\,c^2d^4e\right)}$$

input
```
int((e*x+d)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^2,x)
```

output
```
( - log(a*e + c*d*x)*a**2*e**3 - log(a*e + c*d*x)*a*c*d*e**2*x + log(d + e
*x)*a**2*e**3 + log(d + e*x)*a*c*d*e**2*x - a*c*d*e**2*x + c**2*d**3*x)/(a
*e*(a**3*e**5 - 2*a**2*c*d**2*e**3 + a**2*c*d*e**4*x + a*c**2*d**4*e - 2*a
*c**2*d**3*e**2*x + c**3*d**5*x))
```

3.124 $\int \frac{1}{\left(ade+(cd^2+ae^2)x+cdex^2\right)^2}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 106

$$\int \frac{1}{(ade+(cd^2+ae^2)x+cdex^2)^2}\,dx = -\frac{cd}{(cd^2-ae^2)^2(ae+cdx)} - \frac{e}{(cd^2-ae^2)^2(d+ex)} - \frac{2cde\log(ae+cdx)}{(cd^2-ae^2)^3} + \frac{2cde\log(d+ex)}{(cd^2-ae^2)^3}$$

output
```
-c*d/(-a*e^2+c*d^2)^2/(c*d*x+a*e)-e/(-a*e^2+c*d^2)^2/(e*x+d)-2*c*d*e*ln(c*
d*x+a*e)/(-a*e^2+c*d^2)^3+2*c*d*e*ln(e*x+d)/(-a*e^2+c*d^2)^3
```

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.81

$$\int \frac{1}{(ade+(cd^2+ae^2)x+cdex^2)^2}\,dx = \frac{\frac{(cd^2-ae^2)(ae^2+cd(d+2ex))}{(ae+cdx)(d+ex)}+2cde\log(ae+cdx)-2cde\log(d+ex)}{(-cd^2+ae^2)^3}$$

input
```
Integrate[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(-2),x]
```

output
```
(((c*d^2 - a*e^2)*(a*e^2 + c*d*(d + 2*e*x)))/((a*e + c*d*x)*(d + e*x)) + 2
*c*d*e*Log[a*e + c*d*x] - 2*c*d*e*Log[d + e*x])/(-(c*d^2) + a*e^2)^3
```

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 143, normalized size of antiderivative = 1.35, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.074$, Rules used = {1084, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^2}dx$$

$$\Big\downarrow \text{1084}$$

$$c^2d^2e^2\int \left(\frac{2}{cd\left(cd^2-ae^2\right)^3(d+ex)}+\frac{1}{c^2d^2\left(cd^2-ae^2\right)^2(d+ex)^2}-\frac{2}{e\left(cd^2-ae^2\right)^3(ae+cdx)}+\frac{1}{e^2\left(cd^2-ae^2\right)^2}\right.$$

$$\Big\downarrow \text{2009}$$

$$c^2d^2e^2\left(-\frac{1}{c^2d^2e(d+ex)\left(cd^2-ae^2\right)^2}-\frac{1}{cde^2\left(cd^2-ae^2\right)^2(ae+cdx)}-\frac{2\log(ae+cdx)}{cde\left(cd^2-ae^2\right)^3}+\frac{2\log(d+ex)}{cde\left(cd^2-ae^2\right)^3}\right)$$

input
```
Int[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(-2),x]
```

output
```
c^2*d^2*e^2*(-(1/(c*d*e^2*(c*d^2 - a*e^2)^2*(a*e + c*d*x))) - 1/(c^2*d^2*e
*(c*d^2 - a*e^2)^2*(d + e*x)) - (2*Log[a*e + c*d*x])/(c*d*e*(c*d^2 - a*e^2
)^3) + (2*Log[d + e*x])/(c*d*e*(c*d^2 - a*e^2)^3))
```

Defintions of rubi rules used

rule 1084
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{q = Rt[b^2
- 4*a*c, 2]}, Simp[1/c^p   Int[ExpandIntegrand[(b/2 - q/2 + c*x)^p*(b/2 + q
/2 + c*x)^p, x], x], x] /;  !FractionalPowerFactorQ[q]] /; FreeQ[{a, b, c},
 x] && IntegerQ[p] && NiceSqrtQ[b^2 - 4*a*c]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 1.64 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.01

method	result
default	$-\frac{cd}{(ae^2-cd^2)^2(cdx+ae)}+\frac{2cde\ln(cdx+ae)}{(ae^2-cd^2)^3}-\frac{e}{(ae^2-cd^2)^2(ex+d)}-\frac{2cde\ln(ex+d)}{(ae^2-cd^2)^3}$
norman	$\frac{\frac{-ace^2-c^2d^2}{c\left(a^2e^4-2acd^2e^2+c^2d^4\right)}-\frac{2decx}{a^2e^4-2acd^2e^2+c^2d^4}}{(ex+d)(cdx+ae)}-\frac{2dec\ln(ex+d)}{e^6a^3-3d^2e^4a^2c+3d^4e^2ac^2-d^6c^3}+\frac{2dec\ln(cdx+ae)}{e^6a^3-3d^2e^4a^2c+3d^4e^2ac^2-d^6c^3}$
risch	$\frac{-\frac{2decx}{a^2e^4-2acd^2e^2+c^2d^4}-\frac{ae^2+cd^2}{a^2e^4-2acd^2e^2+c^2d^4}}{cdx^2e+ae^2x+cd^2x+ade}+\frac{2dec\ln(-cdx-ae)}{e^6a^3-3d^2e^4a^2c+3d^4e^2ac^2-d^6c^3}-\frac{2dec\ln(ex+d)}{e^6a^3-3d^2e^4a^2c+3d^4e^2ac^2-d^6c^3}$
parallelrisch	$-\frac{2\ln(ex+d)x^2c^3d^3e^3-2\ln(cdx+ae)x^2c^3d^3e^3+2\ln(ex+d)xac^2d^2e^4+2\ln(ex+d)xc^3d^4e^2-2\ln(cdx+ae)xac^2d^2e^4-2\ln(cd}{(e^6a^3-3d^2e^4a^2c+3d^4e^2ac^2-d^6c^3)(cdx^2e+}$

input
```
int(1/(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^2,x,method=_RETURNVERBOSE)
```

output
```
-c*d/(a*e^2-c*d^2)^2/(c*d*x+a*e)+2*c*d/(a*e^2-c*d^2)^3*e*ln(c*d*x+a*e)-e/(
a*e^2-c*d^2)^2/(e*x+d)-2*c*d/(a*e^2-c*d^2)^3*e*ln(e*x+d)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 277 vs. 2(106) = 212.

Time = 0.08 (sec) , antiderivative size = 277, normalized size of antiderivative = 2.61

$$\int \frac{1}{(ade+(cd^2+ae^2)x+cdex^2)^2}\,dx =$$

$$-\frac{c^2d^4-a^2e^4+2(c^2d^3e-acde^3)x+2(c^2d^2e^2x^2+acd^2e^2+(c^2d^3e+acde^3)x)\log(cdx+ae)-2(c^2d^2e}{ac^3d^7e-3a^2c^2d^5e^3+3a^3cd^3e^5-a^4de^7+(c^4d^7e-3ac^3d^5e^3+3a^2c^2d^3e^5-a^3cde^7)x^2+(c}$$

input
```
integrate(1/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^2,x, algorithm="fricas")
```

output
```
-(c^2*d^4 - a^2*e^4 + 2*(c^2*d^3*e - a*c*d*e^3)*x + 2*(c^2*d^2*e^2*x^2 + a
*c*d^2*e^2 + (c^2*d^3*e + a*c*d*e^3)*x)*log(c*d*x + a*e) - 2*(c^2*d^2*e^2*
x^2 + a*c*d^2*e^2 + (c^2*d^3*e + a*c*d*e^3)*x)*log(e*x + d))/(a*c^3*d^7*e
- 3*a^2*c^2*d^5*e^3 + 3*a^3*c*d^3*e^5 - a^4*d*e^7 + (c^4*d^7*e - 3*a*c^3*d
^5*e^3 + 3*a^2*c^2*d^3*e^5 - a^3*c*d*e^7)*x^2 + (c^4*d^8 - 2*a*c^3*d^6*e^2
 + 2*a^3*c*d^2*e^6 - a^4*e^8)*x)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 486 vs. $2(95) = 190$.

Time = 0.59 (sec) , antiderivative size = 486, normalized size of antiderivative = 4.58

$$\int \frac{1}{(ade + (cd^2 + ae^2)x + cdex^2)^2}\,dx = -\frac{2cde\log\left(x + \frac{-\frac{2a^4cde^9}{(ae^2-cd^2)^3} + \frac{8a^3c^2d^3e^7}{(ae^2-cd^2)^3} - \frac{12a^2c^3d^5e^5}{(ae^2-cd^2)^3} + \frac{8ac^4d^7e^3}{(ae^2-cd^2)^3} + 2acde^3 - \frac{2c^5d^9e}{(ae^2-cd^2)^3} + 2c^2d^3e}{4c^2d^2e^2}\right)}{(ae^2-cd^2)^3} + \frac{2cde\log\left(x + \frac{\frac{2a^4cde^9}{(ae^2-cd^2)^3} - \frac{8a^3c^2d^3e^7}{(ae^2-cd^2)^3} + \frac{12a^2c^3d^5e^5}{(ae^2-cd^2)^3} - \frac{8ac^4d^7e^3}{(ae^2-cd^2)^3} + 2acde^3 + \frac{2c^5d^9e}{(ae^2-cd^2)^3} + 2c^2d^3e}{4c^2d^2e^2}\right)}{(ae^2-cd^2)^3} + \frac{-ae^2 - cd^2 - 2cdex}{a^3de^5 - 2a^2cd^3e^3 + ac^2d^5e + x^2(a^2cde^5 - 2ac^2d^3e^3 + c^3d^5e) + x(a^3e^6 - a^2cd^2e^4 - ac^2d^4e^2 + c^3d^6)}$$

input
```
integrate(1/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**2,x)
```

output
```
-2*c*d*e*log(x + (-2*a**4*c*d*e**9/(a*e**2 - c*d**2)**3 + 8*a**3*c**2*d**3
*e**7/(a*e**2 - c*d**2)**3 - 12*a**2*c**3*d**5*e**5/(a*e**2 - c*d**2)**3 +
 8*a*c**4*d**7*e**3/(a*e**2 - c*d**2)**3 + 2*a*c*d*e**3 - 2*c**5*d**9*e/(a
*e**2 - c*d**2)**3 + 2*c**2*d**3*e)/(4*c**2*d**2*e**2))/(a*e**2 - c*d**2)*
*3 + 2*c*d*e*log(x + (2*a**4*c*d*e**9/(a*e**2 - c*d**2)**3 - 8*a**3*c**2*d
**3*e**7/(a*e**2 - c*d**2)**3 + 12*a**2*c**3*d**5*e**5/(a*e**2 - c*d**2)**
3 - 8*a*c**4*d**7*e**3/(a*e**2 - c*d**2)**3 + 2*a*c*d*e**3 + 2*c**5*d**9*e
/(a*e**2 - c*d**2)**3 + 2*c**2*d**3*e)/(4*c**2*d**2*e**2))/(a*e**2 - c*d**
2)**3 + (-a*e**2 - c*d**2 - 2*c*d*e*x)/(a**3*d*e**5 - 2*a**2*c*d**3*e**3 +
 a*c**2*d**5*e + x**2*(a**2*c*d*e**5 - 2*a*c**2*d**3*e**3 + c**3*d**5*e) +
 x*(a**3*e**6 - a**2*c*d**2*e**4 - a*c**2*d**4*e**2 + c**3*d**6))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 236 vs. $2(106) = 212$.

Time = 0.03 (sec) , antiderivative size = 236, normalized size of antiderivative = 2.23

$$\int \frac{1}{(ade + (cd^2 + ae^2)x + cdex^2)^2} dx = -\frac{2 cde \log(cdx + ae)}{c^3d^6 - 3 ac^2d^4e^2 + 3 a^2cd^2e^4 - a^3e^6} + \frac{2 cde \log(ex + d)}{c^3d^6 - 3 ac^2d^4e^2 + 3 a^2cd^2e^4 - a^3e^6} - \frac{2 cdex + cd^2 + ae^2}{ac^2d^5e - 2 a^2cd^3e^3 + a^3de^5 + (c^3d^5e - 2 ac^2d^3e^3 + a^2cde^5)x^2 + (c^3d^6 - ac^2d^4e^2 - a^2cd^2e^4 + a^3e^6)x}$$

input
```
integrate(1/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^2,x, algorithm="maxima")
```

output
```
-2*c*d*e*log(c*d*x + a*e)/(c^3*d^6 - 3*a*c^2*d^4*e^2 + 3*a^2*c*d^2*e^4 - a
^3*e^6) + 2*c*d*e*log(e*x + d)/(c^3*d^6 - 3*a*c^2*d^4*e^2 + 3*a^2*c*d^2*e^
4 - a^3*e^6) - (2*c*d*e*x + c*d^2 + a*e^2)/(a*c^2*d^5*e - 2*a^2*c*d^3*e^3
+ a^3*d*e^5 + (c^3*d^5*e - 2*a*c^2*d^3*e^3 + a^2*c*d*e^5)*x^2 + (c^3*d^6 -
 a*c^2*d^4*e^2 - a^2*c*d^2*e^4 + a^3*e^6)*x)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 195, normalized size of antiderivative = 1.84

$$\int \frac{1}{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^2}\,dx$$
$$= -\frac{2\,c^2d^2e\log\left(\left|cdx+ae\right|\right)}{c^4d^7-3\,ac^3d^5e^2+3\,a^2c^2d^3e^4-a^3cde^6}+\frac{2\,cde^2\log\left(\left|ex+d\right|\right)}{c^3d^6e-3\,ac^2d^4e^3+3\,a^2cd^2e^5-a^3e^7} - \frac{2\,cdex+cd^2+ae^2}{\left(c^2d^4-2\,acd^2e^2+a^2e^4\right)\left(cdex^2+cd^2x+ae^2x+ade\right)}$$

input
```
integrate(1/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^2,x, algorithm="giac")
```

output
```
-2*c^2*d^2*e*log(abs(c*d*x + a*e))/(c^4*d^7 - 3*a*c^3*d^5*e^2 + 3*a^2*c^2*
d^3*e^4 - a^3*c*d*e^6) + 2*c*d*e^2*log(abs(e*x + d))/(c^3*d^6*e - 3*a*c^2*
d^4*e^3 + 3*a^2*c*d^2*e^5 - a^3*e^7) - (2*c*d*e*x + c*d^2 + a*e^2)/((c^2*d
^4 - 2*a*c*d^2*e^2 + a^2*e^4)*(c*d*e*x^2 + c*d^2*x + a*e^2*x + a*d*e))
```

Mupad [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 223, normalized size of antiderivative = 2.10

$$\int \frac{1}{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^2}\,dx$$
$$= \frac{4\,c\,d\,e\,\mathrm{atanh}\left(\frac{a^3\,e^6-a^2\,c\,d^2\,e^4-a\,c^2\,d^4\,e^2+c^3\,d^6}{\left(a\,e^2-c\,d^2\right)^3}+\frac{2\,c\,d\,e\,x\,\left(a^2\,e^4-2\,a\,c\,d^2\,e^2+c^2\,d^4\right)}{\left(a\,e^2-c\,d^2\right)^3}\right)}{\left(a\,e^2-c\,d^2\right)^3} - \frac{\frac{c\,d^2+a\,e^2}{a^2\,e^4-2\,a\,c\,d^2\,e^2+c^2\,d^4}+\frac{2\,c\,d\,e\,x}{a^2\,e^4-2\,a\,c\,d^2\,e^2+c^2\,d^4}}{c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e}$$

input
```
int(1/(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^2,x)
```

output
```
(4*c*d*e*atanh((a^3*e^6 + c^3*d^6 - a*c^2*d^4*e^2 - a^2*c*d^2*e^4)/(a*e^2
- c*d^2)^3 + (2*c*d*e*x*(a^2*e^4 + c^2*d^4 - 2*a*c*d^2*e^2))/(a*e^2 - c*d^
2)^3))/(a*e^2 - c*d^2)^3 - ((a*e^2 + c*d^2)/(a^2*e^4 + c^2*d^4 - 2*a*c*d^2
*e^2) + (2*c*d*e*x)/(a^2*e^4 + c^2*d^4 - 2*a*c*d^2*e^2))/(x*(a*e^2 + c*d^2
) + a*d*e + c*d*e*x^2)
```

Reduce [B] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 523, normalized size of antiderivative = 4.93

$$\int \frac{1}{(ade + (cd^2 + ae^2)x + cdex^2)^2}\,dx$$
$$= \frac{2\log(cdx + ae)\,a^2c\,d^2e^4 + 2\log(cdx + ae)\,a^2cd\,e^5x + 2\log(cdx + ae)\,a\,c^2d^4e^2 + 4\log(cdx + ae)\,a\,c^2d^3e^3}{}$$

input
```
int(1/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^2,x)
```

output
```
(2*log(a*e + c*d*x)*a**2*c*d**2*e**4 + 2*log(a*e + c*d*x)*a**2*c*d*e**5*x
+ 2*log(a*e + c*d*x)*a*c**2*d**4*e**2 + 4*log(a*e + c*d*x)*a*c**2*d**3*e**
3*x + 2*log(a*e + c*d*x)*a*c**2*d**2*e**4*x**2 + 2*log(a*e + c*d*x)*c**3*d
**5*e*x + 2*log(a*e + c*d*x)*c**3*d**4*e**2*x**2 - 2*log(d + e*x)*a**2*c*d
**2*e**4 - 2*log(d + e*x)*a**2*c*d*e**5*x - 2*log(d + e*x)*a*c**2*d**4*e**
2 - 4*log(d + e*x)*a*c**2*d**3*e**3*x - 2*log(d + e*x)*a*c**2*d**2*e**4*x*
*2 - 2*log(d + e*x)*c**3*d**5*e*x - 2*log(d + e*x)*c**3*d**4*e**2*x**2 - a
**3*e**6 + a**2*c*d**2*e**4 - a*c**2*d**4*e**2 + 2*a*c**2*d**2*e**4*x**2 +
 c**3*d**6 - 2*c**3*d**4*e**2*x**2)/(a**5*d*e**9 + a**5*e**10*x - 2*a**4*c
*d**3*e**7 - a**4*c*d**2*e**8*x + a**4*c*d*e**9*x**2 - 2*a**3*c**2*d**4*e*
*6*x - 2*a**3*c**2*d**3*e**7*x**2 + 2*a**2*c**3*d**7*e**3 + 2*a**2*c**3*d*
*6*e**4*x - a*c**4*d**9*e + a*c**4*d**8*e**2*x + 2*a*c**4*d**7*e**3*x**2 -
 c**5*d**10*x - c**5*d**9*e*x**2)
```

3.125 $\int \frac{1}{(d+ex)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^2}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 146

$$\int \frac{1}{(d+ex)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^2}\,dx = -\frac{c^2d^2}{\left(cd^2-ae^2\right)^3(ae+cdx)}-\frac{e}{2\left(cd^2-ae^2\right)^2(d+ex)^2}-\frac{2cde}{\left(cd^2-ae^2\right)^3(d+ex)}-\frac{3c^2d^2e\log(ae+cdx)}{\left(cd^2-ae^2\right)^4}+\frac{3c^2d^2e\log(d+ex)}{\left(cd^2-ae^2\right)^4}$$

output
```
-c^2*d^2/(-a*e^2+c*d^2)^3/(c*d*x+a*e)-1/2*e/(-a*e^2+c*d^2)^2/(e*x+d)^2-2*c
*d*e/(-a*e^2+c*d^2)^3/(e*x+d)-3*c^2*d^2*e*ln(c*d*x+a*e)/(-a*e^2+c*d^2)^4+3
*c^2*d^2*e*ln(e*x+d)/(-a*e^2+c*d^2)^4
```

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 130, normalized size of antiderivative = 0.89

$$\int \frac{1}{(d+ex)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^2}\,dx = \frac{\frac{2c^2d^2\left(-cd^2+ae^2\right)}{ae+cdx}-\frac{e\left(cd^2-ae^2\right)^2}{(d+ex)^2}+\frac{4cde\left(-cd^2+ae^2\right)}{d+ex}-6c^2d^2e\log(ae+cdx)+6c^2d^2e\log(d+ex)}{2\left(cd^2-ae^2\right)^4}$$

input
```
Integrate[1/((d + e*x)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^2),x]
```

output
```
((2*c^2*d^2*(-(c*d^2) + a*e^2))/(a*e + c*d*x) - (e*(c*d^2 - a*e^2)^2)/(d +
 e*x)^2 + (4*c*d*e*(-(c*d^2) + a*e^2))/(d + e*x) - 6*c^2*d^2*e*Log[a*e + c
*d*x] + 6*c^2*d^2*e*Log[d + e*x])/(2*(c*d^2 - a*e^2)^4)
```

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 146, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.057$, Rules used = {1121, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(d+ex)\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^2}dx$$

$$\Big\downarrow \text{1121}$$

$$\int \left(-\frac{3c^3d^3e}{\left(cd^2-ae^2\right)^4(ae+cdx)}+\frac{c^3d^3}{\left(cd^2-ae^2\right)^3(ae+cdx)^2}+\frac{3c^2d^2e^2}{(d+ex)\left(cd^2-ae^2\right)^4}+\frac{2cde^2}{(d+ex)^2\left(cd^2-ae^2\right)^3}+\right.$$

$$\Big\downarrow \text{2009}$$

$$-\frac{c^2d^2}{\left(cd^2-ae^2\right)^3(ae+cdx)}-\frac{3c^2d^2e\log(ae+cdx)}{\left(cd^2-ae^2\right)^4}+\frac{3c^2d^2e\log(d+ex)}{\left(cd^2-ae^2\right)^4}-\frac{2cde}{(d+ex)\left(cd^2-ae^2\right)^3}-\frac{e}{2(d+ex)^2\left(cd^2-ae^2\right)^2}$$

input
```
Int[1/((d + e*x)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^2),x]
```

output
```
-((c^2*d^2)/((c*d^2 - a*e^2)^3*(a*e + c*d*x))) - e/(2*(c*d^2 - a*e^2)^2*(d
 + e*x)^2) - (2*c*d*e)/((c*d^2 - a*e^2)^3*(d + e*x)) - (3*c^2*d^2*e*Log[a*
e + c*d*x])/(c*d^2 - a*e^2)^4 + (3*c^2*d^2*e*Log[d + e*x])/(c*d^2 - a*e^2)
^4
```

Defintions of rubi rules used

rule 1121
```
Int[((d_) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Int[ExpandIntegrand[(d + e*x)^(m + p)*(a/d + (c/e)*x)^p, x], x]
/; FreeQ[{a, b, c, d, e, m, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && (Int
egerQ[p] || (GtQ[a, 0] && GtQ[d, 0] && LtQ[c, 0]))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 1.95 (sec) , antiderivative size = 144, normalized size of antiderivative = 0.99

method	result
default	$\frac{c^2d^2}{(a e^2-c d^2)^3(cdx+ae)}-\frac{3c^2d^2e\ln(cdx+ae)}{(a e^2-c d^2)^4}-\frac{e}{2(a e^2-c d^2)^2(ex+d)^2}+\frac{3c^2d^2e\ln(ex+d)}{(a e^2-c d^2)^4}+\frac{2ecd}{(a e^2-c d^2)^3(ex+d)}$
risch	$\frac{\frac{3e^2c^2d^2x^2}{e^6a^3-3d^2e^4a^2c+3d^4e^2a c^2-d^6c^3}+\frac{3\left(a e^2+3c d^2\right)ecdx}{2\left(e^6a^3-3d^2e^4a^2c+3d^4e^2a c^2-d^6c^3\right)}-\frac{a^2e^4-5ac d^2e^2-2c^2d^4}{2\left(e^6a^3-3d^2e^4a^2c+3d^4e^2a c^2-d^6c^3\right)}}{(ex+d)(cd x^2e+a e^2x+c d^2x+ade)}+\frac{}{a^4e^8-4a^3c d^2}$
norman	$\frac{\frac{3e^2c^2d^2x^2}{e^6a^3-3d^2e^4a^2c+3d^4e^2a c^2-d^6c^3}+\frac{-a^2c e^5+5e^3a c^2d^2+2d^4e c^3}{2ec\left(e^6a^3-3d^2e^4a^2c+3d^4e^2a c^2-d^6c^3\right)}+\frac{\left(3a c^2d e^5+9e^3d^3c^3\right)x}{2\left(e^6a^3-3d^2e^4a^2c+3d^4e^2a c^2-d^6c^3\right)c e^2}}{(cdx+ae)(ex+d)^2}+\frac{}{a^4e^8-4a}$
parallelrisch	$\frac{6a^2c^2e^6d^3-a^3c e^8d+3x a^2c^2d^2e^7+6xa c^3d^4e^5+6\ln(ex+d)x^3c^4d^4e^5-6\ln(cdx+ae)x^3c^4d^4e^5+12\ln(ex+d)x^2c^4d^5e^4-12\ln(c}{}$

input
```
int(1/(e*x+d)/(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^2,x,method=_RETURNVERBOSE)
```

output
```
c^2*d^2/(a*e^2-c*d^2)^3/(c*d*x+a*e)-3*c^2*d^2/(a*e^2-c*d^2)^4*e*ln(c*d*x+a
*e)-1/2*e/(a*e^2-c*d^2)^2/(e*x+d)^2+3*c^2*d^2/(a*e^2-c*d^2)^4*e*ln(e*x+d)+
2*e/(a*e^2-c*d^2)^3*c*d/(e*x+d)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 544 vs. $2(144) = 288$.

Time = 0.09 (sec) , antiderivative size = 544, normalized size of antiderivative = 3.73

$$\int \frac{1}{(d+ex)\,(ade+(cd^2+ae^2)\,x+cdex^2)^2}\,dx = -\frac{2\,c^3d^6+3\,ac^2d^4e^2-6\,a^2cd^2e^4+a^3e^6+6\,(c^3d^4e^2-ac^2d^2e^4)x^2+3\,(3\,c^3d^5e-2\,ac^2d^3e^3-a^2cde^5)x+6}{2\,(ac^4d^{10}e-4\,a^2c^3d^8e^3+6\,a^3c^2d^6e^5-4\,a^4cd^4e^7+a^5d^2e^9+(c^5d^9e^2-4\,ac^4d^7e^4+6\,a^2c^3d^5e^6-4}$$

input
```
integrate(1/(e*x+d)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^2,x, algorithm="fric
as")
```

output
```
-1/2*(2*c^3*d^6 + 3*a*c^2*d^4*e^2 - 6*a^2*c*d^2*e^4 + a^3*e^6 + 6*(c^3*d^4
*e^2 - a*c^2*d^2*e^4)*x^2 + 3*(3*c^3*d^5*e - 2*a*c^2*d^3*e^3 - a^2*c*d*e^5
)*x + 6*(c^3*d^3*e^3*x^3 + a*c^2*d^4*e^2 + (2*c^3*d^4*e^2 + a*c^2*d^2*e^4)
*x^2 + (c^3*d^5*e + 2*a*c^2*d^3*e^3)*x)*log(c*d*x + a*e) - 6*(c^3*d^3*e^3*
x^3 + a*c^2*d^4*e^2 + (2*c^3*d^4*e^2 + a*c^2*d^2*e^4)*x^2 + (c^3*d^5*e + 2
*a*c^2*d^3*e^3)*x)*log(e*x + d))/(a*c^4*d^10*e - 4*a^2*c^3*d^8*e^3 + 6*a^3
*c^2*d^6*e^5 - 4*a^4*c*d^4*e^7 + a^5*d^2*e^9 + (c^5*d^9*e^2 - 4*a*c^4*d^7*
e^4 + 6*a^2*c^3*d^5*e^6 - 4*a^3*c^2*d^3*e^8 + a^4*c*d*e^10)*x^3 + (2*c^5*d
^10*e - 7*a*c^4*d^8*e^3 + 8*a^2*c^3*d^6*e^5 - 2*a^3*c^2*d^4*e^7 - 2*a^4*c*
d^2*e^9 + a^5*e^11)*x^2 + (c^5*d^11 - 2*a*c^4*d^9*e^2 - 2*a^2*c^3*d^7*e^4
+ 8*a^3*c^2*d^5*e^6 - 7*a^4*c*d^3*e^8 + 2*a^5*d*e^10)*x)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 734 vs. $2(133) = 266$.

Time = 0.91 (sec) , antiderivative size = 734, normalized size of antiderivative = 5.03

$$\int \frac{1}{(d+ex)\left(ade+(cd^2+ae^2)x+cdex^2\right)^2}\,dx$$
$$= \frac{3c^2d^2e\log\left(x+\frac{-\frac{3a^5c^2d^2e^{11}}{(ae^2-cd^2)^4}+\frac{15a^4c^3d^4e^9}{(ae^2-cd^2)^4}-\frac{30a^3c^4d^6e^7}{(ae^2-cd^2)^4}+\frac{30a^2c^5d^8e^5}{(ae^2-cd^2)^4}-\frac{15ac^6d^{10}e^3}{(ae^2-cd^2)^4}+3ac^2d^2e^3+\frac{3c^7d^{12}e}{(ae^2-cd^2)^4}+3c^3d^4e}{6c^3d^3e^2}\right)}{(ae^2-cd^2)^4}$$
$$-\frac{3c^2d^2e\log\left(x+\frac{\frac{3a^5c^2d^2e^{11}}{(ae^2-cd^2)^4}-\frac{15a^4c^3d^4e^9}{(ae^2-cd^2)^4}+\frac{30a^3c^4d^6e^7}{(ae^2-cd^2)^4}-\frac{30a^2c^5d^8e^5}{(ae^2-cd^2)^4}+\frac{15ac^6d^{10}e^3}{(ae^2-cd^2)^4}+3ac^2d^2e^3-\frac{3c^7d^{12}e}{(ae^2-cd^2)^4}+3c^3d^4e}{6c^3d^3e^2}\right)}{(ae^2-cd^2)^4}$$
$$+\frac{-a^2e^4+5acd^2e^2+2c^2d^4}{2a^4d^2e^7-6a^3cd^4e^5+6a^2c^2d^6e^3-2ac^3d^8e+x^3\cdot(2a^3cde^8-6a^2c^2d^3e^6+6ac^3d^5e^4-2c^4d^7e^2)+x^2\cdot(}$$

input
```
integrate(1/(e*x+d)/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**2,x)
```

output
```
3*c**2*d**2*e*log(x + (-3*a**5*c**2*d**2*e**11/(a*e**2 - c*d**2)**4 + 15*a
**4*c**3*d**4*e**9/(a*e**2 - c*d**2)**4 - 30*a**3*c**4*d**6*e**7/(a*e**2 -
 c*d**2)**4 + 30*a**2*c**5*d**8*e**5/(a*e**2 - c*d**2)**4 - 15*a*c**6*d**1
0*e**3/(a*e**2 - c*d**2)**4 + 3*a*c**2*d**2*e**3 + 3*c**7*d**12*e/(a*e**2
- c*d**2)**4 + 3*c**3*d**4*e)/(6*c**3*d**3*e**2))/(a*e**2 - c*d**2)**4 - 3
*c**2*d**2*e*log(x + (3*a**5*c**2*d**2*e**11/(a*e**2 - c*d**2)**4 - 15*a**
4*c**3*d**4*e**9/(a*e**2 - c*d**2)**4 + 30*a**3*c**4*d**6*e**7/(a*e**2 - c
*d**2)**4 - 30*a**2*c**5*d**8*e**5/(a*e**2 - c*d**2)**4 + 15*a*c**6*d**10*
e**3/(a*e**2 - c*d**2)**4 + 3*a*c**2*d**2*e**3 - 3*c**7*d**12*e/(a*e**2 -
c*d**2)**4 + 3*c**3*d**4*e)/(6*c**3*d**3*e**2))/(a*e**2 - c*d**2)**4 + (-a
**2*e**4 + 5*a*c*d**2*e**2 + 2*c**2*d**4 + 6*c**2*d**2*e**2*x**2 + x*(3*a*
c*d*e**3 + 9*c**2*d**3*e))/(2*a**4*d**2*e**7 - 6*a**3*c*d**4*e**5 + 6*a**2
*c**2*d**6*e**3 - 2*a*c**3*d**8*e + x**3*(2*a**3*c*d*e**8 - 6*a**2*c**2*d*
*3*e**6 + 6*a*c**3*d**5*e**4 - 2*c**4*d**7*e**2) + x**2*(2*a**4*e**9 - 2*a
**3*c*d**2*e**7 - 6*a**2*c**2*d**4*e**5 + 10*a*c**3*d**6*e**3 - 4*c**4*d**
8*e) + x*(4*a**4*d*e**8 - 10*a**3*c*d**3*e**6 + 6*a**2*c**2*d**5*e**4 + 2*
a*c**3*d**7*e**2 - 2*c**4*d**9))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 423 vs. $2(144) = 288$.

Time = 0.04 (sec) , antiderivative size = 423, normalized size of antiderivative = 2.90

$$\int \frac{1}{(d+ex)\left(ade+(cd^2+ae^2)x+cdex^2\right)^2}\,dx$$
$$= -\frac{3\,c^2d^2e\log(cdx+ae)}{c^4d^8-4\,ac^3d^6e^2+6\,a^2c^2d^4e^4-4\,a^3cd^2e^6+a^4e^8} + \frac{3\,c^2d^2e\log(ex+d)}{c^4d^8-4\,ac^3d^6e^2+6\,a^2c^2d^4e^4-4\,a^3cd^2e^6+a^4e^8} - \frac{6\,c^2d^2e^2x^2+2\,c^2d^4+5\,acd^2e^2}{2\,(ac^3d^8e-3\,a^2c^2d^6e^3+3\,a^3cd^4e^5-a^4d^2e^7+(c^4d^7e^2-3\,ac^3d^5e^4+3\,a^2c^2d^3e^6-a^3cde^8)x^3+(2\,c^4d^8e}$$

input
```
integrate(1/(e*x+d)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^2,x, algorithm="maxi
ma")
```

output
```
-3*c^2*d^2*e*log(c*d*x + a*e)/(c^4*d^8 - 4*a*c^3*d^6*e^2 + 6*a^2*c^2*d^4*e
^4 - 4*a^3*c*d^2*e^6 + a^4*e^8) + 3*c^2*d^2*e*log(e*x + d)/(c^4*d^8 - 4*a*
c^3*d^6*e^2 + 6*a^2*c^2*d^4*e^4 - 4*a^3*c*d^2*e^6 + a^4*e^8) - 1/2*(6*c^2*
d^2*e^2*x^2 + 2*c^2*d^4 + 5*a*c*d^2*e^2 - a^2*e^4 + 3*(3*c^2*d^3*e + a*c*d
*e^3)*x)/(a*c^3*d^8*e - 3*a^2*c^2*d^6*e^3 + 3*a^3*c*d^4*e^5 - a^4*d^2*e^7
+ (c^4*d^7*e^2 - 3*a*c^3*d^5*e^4 + 3*a^2*c^2*d^3*e^6 - a^3*c*d*e^8)*x^3 +
(2*c^4*d^8*e - 5*a*c^3*d^6*e^3 + 3*a^2*c^2*d^4*e^5 + a^3*c*d^2*e^7 - a^4*e
^9)*x^2 + (c^4*d^9 - a*c^3*d^7*e^2 - 3*a^2*c^2*d^5*e^4 + 5*a^3*c*d^3*e^6 -
 2*a^4*d*e^8)*x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 289 vs. $2(144) = 288$.

Time = 0.12 (sec) , antiderivative size = 289, normalized size of antiderivative = 1.98

$$\int \frac{1}{(d+ex)\left(ade+(cd^2+ae^2)x+cdex^2\right)^2}\,dx = -\frac{3\,c^3d^3e\log\left(|cdx+ae|\right)}{c^5d^9-4\,ac^4d^7e^2+6\,a^2c^3d^5e^4-4\,a^3c^2d^3e^6+a^4cde^8} + \frac{3\,c^2d^2e^2\log\left(|ex+d|\right)}{c^4d^8e-4\,ac^3d^6e^3+6\,a^2c^2d^4e^5-4\,a^3cd^2e^7+a^4e^9} - \frac{2\,c^3d^6+3\,ac^2d^4e^2-6\,a^2cd^2e^4+a^3e^6+6\,(c^3d^4e^2-ac^2d^2e^4)x^2+3\,(3\,c^3d^5e-2\,ac^2d^3e^3-a^2cde^5)x}{2\,(cd^2-ae^2)^4(cdx+ae)(ex+d)^2}$$

input
```
integrate(1/(e*x+d)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^2,x, algorithm="giac
")
```

output
```
-3*c^3*d^3*e*log(abs(c*d*x + a*e))/(c^5*d^9 - 4*a*c^4*d^7*e^2 + 6*a^2*c^3*
d^5*e^4 - 4*a^3*c^2*d^3*e^6 + a^4*c*d*e^8) + 3*c^2*d^2*e^2*log(abs(e*x + d
))/(c^4*d^8*e - 4*a*c^3*d^6*e^3 + 6*a^2*c^2*d^4*e^5 - 4*a^3*c*d^2*e^7 + a^
4*e^9) - 1/2*(2*c^3*d^6 + 3*a*c^2*d^4*e^2 - 6*a^2*c*d^2*e^4 + a^3*e^6 + 6*
(c^3*d^4*e^2 - a*c^2*d^2*e^4)*x^2 + 3*(3*c^3*d^5*e - 2*a*c^2*d^3*e^3 - a^2
*c*d*e^5)*x)/((c*d^2 - a*e^2)^4*(c*d*x + a*e)*(e*x + d)^2)
```

Mupad [B] (verification not implemented)

Time = 5.41 (sec) , antiderivative size = 381, normalized size of antiderivative = 2.61

$$\int \frac{1}{(d+ex)\left(ade+(cd^2+ae^2)x+cdex^2\right)^2}\,dx = \frac{\frac{-a^2\,e^4+5\,a\,c\,d^2\,e^2+2\,c^2\,d^4}{2\,\left(a^3\,e^6-3\,a^2\,c\,d^2\,e^4+3\,a\,c^2\,d^4\,e^2-c^3\,d^6\right)}+\frac{3\,c\,d\,x\,\left(3\,c\,d^2\,e+a\,e^3\right)}{2\,\left(a^3\,e^6-3\,a^2\,c\,d^2\,e^4+3\,a\,c^2\,d^4\,e^2-c^3\,d^6\right)}+\frac{3\,c^2\,d^2\,e^2\,x^2}{a^3\,e^6-3\,a^2\,c\,d^2\,e^4+3\,a\,c^2\,d^4\,e^2-c^3\,d^6}}{x^2\,\left(2\,c\,d^2\,e+a\,e^3\right)+x\,\left(c\,d^3+2\,a\,d\,e^2\right)+a\,d^2\,e+c\,d\,e^2\,x^3} - \frac{6\,c^2\,d^2\,e\,\mathrm{atanh}\left(\frac{a^4\,e^8-2\,a^3\,c\,d^2\,e^6+2\,a\,c^3\,d^6\,e^2-c^4\,d^8}{{\left(a\,e^2-c\,d^2\right)}^4}+\frac{2\,c\,d\,e\,x\,\left(a^3\,e^6-3\,a^2\,c\,d^2\,e^4+3\,a\,c^2\,d^4\,e^2-c^3\,d^6\right)}{{\left(a\,e^2-c\,d^2\right)}^4}\right)}{{\left(a\,e^2-c\,d^2\right)}^4}$$

input
```
int(1/((d + e*x)*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^2),x)
```

output
```
((2*c^2*d^4 - a^2*e^4 + 5*a*c*d^2*e^2)/(2*(a^3*e^6 - c^3*d^6 + 3*a*c^2*d^4
*e^2 - 3*a^2*c*d^2*e^4)) + (3*c*d*x*(a*e^3 + 3*c*d^2*e))/(2*(a^3*e^6 - c^3
*d^6 + 3*a*c^2*d^4*e^2 - 3*a^2*c*d^2*e^4)) + (3*c^2*d^2*e^2*x^2)/(a^3*e^6
- c^3*d^6 + 3*a*c^2*d^4*e^2 - 3*a^2*c*d^2*e^4))/(x^2*(a*e^3 + 2*c*d^2*e) +
 x*(c*d^3 + 2*a*d*e^2) + a*d^2*e + c*d*e^2*x^3) - (6*c^2*d^2*e*atanh((a^4*
e^8 - c^4*d^8 + 2*a*c^3*d^6*e^2 - 2*a^3*c*d^2*e^6)/(a*e^2 - c*d^2)^4 + (2*
c*d*e*x*(a^3*e^6 - c^3*d^6 + 3*a*c^2*d^4*e^2 - 3*a^2*c*d^2*e^4))/(a*e^2 -
c*d^2)^4))/(a*e^2 - c*d^2)^4
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 897, normalized size of antiderivative = 6.14

$$\int \frac{1}{(d+ex)\left(ade+(cd^2+ae^2)x+cdex^2\right)^2}\,dx = \text{Too large to display}$$

input
```
int(1/(e*x+d)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^2,x)
```

output
```
( - 6*log(a*e + c*d*x)*a**2*c**2*d**4*e**4 - 12*log(a*e + c*d*x)*a**2*c**2
*d**3*e**5*x - 6*log(a*e + c*d*x)*a**2*c**2*d**2*e**6*x**2 - 12*log(a*e +
c*d*x)*a*c**3*d**6*e**2 - 30*log(a*e + c*d*x)*a*c**3*d**5*e**3*x - 24*log(
a*e + c*d*x)*a*c**3*d**4*e**4*x**2 - 6*log(a*e + c*d*x)*a*c**3*d**3*e**5*x
**3 - 12*log(a*e + c*d*x)*c**4*d**7*e*x - 24*log(a*e + c*d*x)*c**4*d**6*e*
*2*x**2 - 12*log(a*e + c*d*x)*c**4*d**5*e**3*x**3 + 6*log(d + e*x)*a**2*c*
*2*d**4*e**4 + 12*log(d + e*x)*a**2*c**2*d**3*e**5*x + 6*log(d + e*x)*a**2
*c**2*d**2*e**6*x**2 + 12*log(d + e*x)*a*c**3*d**6*e**2 + 30*log(d + e*x)*
a*c**3*d**5*e**3*x + 24*log(d + e*x)*a*c**3*d**4*e**4*x**2 + 6*log(d + e*x
)*a*c**3*d**3*e**5*x**3 + 12*log(d + e*x)*c**4*d**7*e*x + 24*log(d + e*x)*
c**4*d**6*e**2*x**2 + 12*log(d + e*x)*c**4*d**5*e**3*x**3 - a**4*e**8 + 4*
a**3*c*d**2*e**6 + 3*a**3*c*d*e**7*x + 3*a**2*c**2*d**4*e**4 - 2*a*c**3*d*
*6*e**2 + 9*a*c**3*d**5*e**3*x - 6*a*c**3*d**3*e**5*x**3 - 4*c**4*d**8 - 1
2*c**4*d**7*e*x + 6*c**4*d**5*e**3*x**3)/(2*(a**6*d**2*e**11 + 2*a**6*d*e*
*12*x + a**6*e**13*x**2 - 2*a**5*c*d**4*e**9 - 3*a**5*c*d**3*e**10*x + a**
5*c*d*e**12*x**3 - 2*a**4*c**2*d**6*e**7 - 6*a**4*c**2*d**5*e**8*x - 6*a**
4*c**2*d**4*e**9*x**2 - 2*a**4*c**2*d**3*e**10*x**3 + 8*a**3*c**3*d**8*e**
5 + 14*a**3*c**3*d**7*e**6*x + 4*a**3*c**3*d**6*e**7*x**2 - 2*a**3*c**3*d*
*5*e**8*x**3 - 7*a**2*c**4*d**10*e**3 - 6*a**2*c**4*d**9*e**4*x + 9*a**2*c
**4*d**8*e**5*x**2 + 8*a**2*c**4*d**7*e**6*x**3 + 2*a*c**5*d**12*e - 3*...
```

3.126 $\int \frac{1}{(d+ex)^2\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^2}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 176

$$\begin{aligned}\int \frac{1}{(d+ex)^2\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^2}\,dx &= -\frac{c^3d^3}{\left(cd^2-ae^2\right)^4(ae+cdx)}-\frac{e}{3\left(cd^2-ae^2\right)^2(d+ex)^3}-\frac{cde}{\left(cd^2-ae^2\right)^3(d+ex)^2}\\&\quad-\frac{3c^2d^2e}{\left(cd^2-ae^2\right)^4(d+ex)}-\frac{4c^3d^3e\log(ae+cdx)}{\left(cd^2-ae^2\right)^5}+\frac{4c^3d^3e\log(d+ex)}{\left(cd^2-ae^2\right)^5}\end{aligned}$$

output
```
-c^3*d^3/(-a*e^2+c*d^2)^4/(c*d*x+a*e)-1/3*e/(-a*e^2+c*d^2)^2/(e*x+d)^3-c*d
*e/(-a*e^2+c*d^2)^3/(e*x+d)^2-3*c^2*d^2*e/(-a*e^2+c*d^2)^4/(e*x+d)-4*c^3*d
^3*e*ln(c*d*x+a*e)/(-a*e^2+c*d^2)^5+4*c^3*d^3*e*ln(e*x+d)/(-a*e^2+c*d^2)^5
```

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 160, normalized size of antiderivative = 0.91

$$\int \frac{1}{(d+ex)^2\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^2}\,dx = \frac{\frac{3c^3d^3\left(cd^2-ae^2\right)}{ae+cdx}-\frac{e\left(-cd^2+ae^2\right)^3}{(d+ex)^3}+\frac{3cde\left(cd^2-ae^2\right)^2}{(d+ex)^2}+\frac{9c^2d^2e\left(cd^2-ae^2\right)}{d+ex}+12c^3d^3e\log(ae+cdx)-12c^3d^3e\log(d+e}{3\left(-cd^2+ae^2\right)^5}$$

input
```
Integrate[1/((d + e*x)^2*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^2),x]
```

output
```
((3*c^3*d^3*(c*d^2 - a*e^2))/(a*e + c*d*x) - (e*(-(c*d^2) + a*e^2)^3)/(d +
 e*x)^3 + (3*c*d*e*(c*d^2 - a*e^2)^2)/(d + e*x)^2 + (9*c^2*d^2*e*(c*d^2 -
a*e^2))/(d + e*x) + 12*c^3*d^3*e*Log[a*e + c*d*x] - 12*c^3*d^3*e*Log[d + e
*x])/(3*(-(c*d^2) + a*e^2)^5)
```

Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 176, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.057$, Rules used = {1121, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(d+ex)^2 \left(x \left(ae^2+cd^2\right)+ade+cdex^2\right)^2} \, dx$$

$$\Big\downarrow \text{1121}$$

$$\int \left(-\frac{4c^4d^4e}{\left(cd^2-ae^2\right)^5 (ae+cdx)}+\frac{c^4d^4}{\left(cd^2-ae^2\right)^4 (ae+cdx)^2}+\frac{4c^3d^3e^2}{(d+ex) \left(cd^2-ae^2\right)^5}+\frac{3c^2d^2e^2}{(d+ex)^2 \left(cd^2-ae^2\right)^4}+\right.$$

$$\Big\downarrow \text{2009}$$

$$-\frac{c^3d^3}{\left(cd^2-ae^2\right)^4 (ae+cdx)}-\frac{4c^3d^3e \log(ae+cdx)}{\left(cd^2-ae^2\right)^5}+\frac{4c^3d^3e \log(d+ex)}{\left(cd^2-ae^2\right)^5}-\frac{3c^2d^2e}{(d+ex) \left(cd^2-ae^2\right)^4}-\frac{cde}{(d+ex)^2 \left(cd^2-ae^2\right)^3}-\frac{e}{3(d+ex)^3 \left(cd^2-ae^2\right)^2}$$

input
```
Int[1/((d + e*x)^2*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^2),x]
```

output
```
-((c^3*d^3)/((c*d^2 - a*e^2)^4*(a*e + c*d*x))) - e/(3*(c*d^2 - a*e^2)^2*(d
 + e*x)^3) - (c*d*e)/((c*d^2 - a*e^2)^3*(d + e*x)^2) - (3*c^2*d^2*e)/((c*d
^2 - a*e^2)^4*(d + e*x)) - (4*c^3*d^3*e*Log[a*e + c*d*x])/(c*d^2 - a*e^2)^
5 + (4*c^3*d^3*e*Log[d + e*x])/(c*d^2 - a*e^2)^5
```

Defintions of rubi rules used

rule 1121
```
Int[((d_) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Int[ExpandIntegrand[(d + e*x)^(m + p)*(a/d + (c/e)*x)^p, x], x]
/; FreeQ[{a, b, c, d, e, m, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && (Int
egerQ[p] || (GtQ[a, 0] && GtQ[d, 0] && LtQ[c, 0]))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 2.01 (sec) , antiderivative size = 174, normalized size of antiderivative = 0.99

method	result
default	$-\frac{c^3 d^3}{(a\,e^2-c\,d^2)^4 (cdx+ae)}+\frac{4c^3 d^3 e \ln(cdx+ae)}{(a\,e^2-c\,d^2)^5}-\frac{e}{3(a\,e^2-c\,d^2)^2 (ex+d)^3}-\frac{4c^3 d^3 e \ln(ex+d)}{(a\,e^2-c\,d^2)^5}-\frac{3e\,c^2 d^2}{(a\,e^2-c\,d^2)^4 (ex+d)}+$
risch	$\frac{-\frac{4c^3 d^3 e^3 x^3}{a^4 e^8-4a^3 c\,d^2 e^6+6a^2 c^2 d^4 e^4-4a\,c^3 d^6 e^2+c^4 d^8}-\frac{2\left(a\,e^2+5c\,d^2\right) e^2 c^2 d^2 x^2}{a^4 e^8-4a^3 c\,d^2 e^6+6a^2 c^2 d^4 e^4-4a\,c^3 d^6 e^2+c^4 d^8}+\frac{2\left(a^2 e^4-8ac\,d^2 e^2-11c^2 d\right.}{3\left(a^4 e^8-4a^3 c\,d^2 e^6+6a^2 c^2 d^4 e^4-4\right.}}{(ex+d)^2 (cd\,x^2 e+a\,e^2 x+c\,d^2 x+ade)}$
norman	$\frac{\frac{\left(-2a\,c^3 d^2 e^6-10c^4 d^4 e^4\right) x^2}{e^2 c\left(a^4 e^8-4a^3 c\,d^2 e^6+6a^2 c^2 d^4 e^4-4a\,c^3 d^6 e^2+c^4 d^8\right)}-\frac{4c^3 d^3 e^3 x^3}{a^4 e^8-4a^3 c\,d^2 e^6+6a^2 c^2 d^4 e^4-4a\,c^3 d^6 e^2+c^4 d^8}+\frac{-a^3 c\,e^8+5a^2 c^2 d^2 e^6-13c}{3e^2 c\left(a^4 e^8-4a^3 c\,d^2 e^6+6a^2 c^2 d\right.}}{(cdx+ae)(ex+d)^3}$
parallelrisch	$-\frac{-2x\,a^3 c^2 d^2 e^{10}+18x\,a^2 c^3 d^4 e^8+6xa\,c^4 d^6 e^6+12\ln(ex+d) x^4 c^5 d^5 e^7-12\ln(cdx+ae) x^4 c^5 d^5 e^7+36\ln(ex+d) x^3 c^5 d^6 e^6-36\ln}{}$

input
```
int(1/(e*x+d)^2/(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^2,x,method=_RETURNVERBOS
E)
```

output
```
-c^3*d^3/(a*e^2-c*d^2)^4/(c*d*x+a*e)+4*c^3*d^3/(a*e^2-c*d^2)^5*e*ln(c*d*x+
a*e)-1/3*e/(a*e^2-c*d^2)^2/(e*x+d)^3-4*c^3*d^3/(a*e^2-c*d^2)^5*e*ln(e*x+d)
-3*e/(a*e^2-c*d^2)^4*c^2*d^2/(e*x+d)+e/(a*e^2-c*d^2)^3*c*d/(e*x+d)^2
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 807 vs. $2(174) = 348$.

Time = 0.09 (sec) , antiderivative size = 807, normalized size of antiderivative = 4.59

$$\int \frac{1}{(d+ex)^2\,(ade+(cd^2+ae^2)\,x+cdex^2)^2}\,dx = -\frac{3\,c^4d^8+10\,ac^3d^6e^2-18\,a^2c^2d^4e^4+6\,a^3cd^2e^6-a^4e^8+12\,(c^4d^5e^3-ac^3d^3e^5)x^3+6\,(5\,c^4d^6e^2}{3\,(ac^5d^{13}e-5\,a^2c^4d^{11}e^3+10\,a^3c^3d^9e^5-10\,a^4c^2d^7e^7+5\,a^5cd^5e^9-a^6d^3e^{11}+(c^6d^{11}e^3-5\,ac^5d^9e^5+1}$$

input
```
integrate(1/(e*x+d)^2/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^2,x, algorithm="fr
icas")
```

output
```
-1/3*(3*c^4*d^8 + 10*a*c^3*d^6*e^2 - 18*a^2*c^2*d^4*e^4 + 6*a^3*c*d^2*e^6
- a^4*e^8 + 12*(c^4*d^5*e^3 - a*c^3*d^3*e^5)*x^3 + 6*(5*c^4*d^6*e^2 - 4*a*
c^3*d^4*e^4 - a^2*c^2*d^2*e^6)*x^2 + 2*(11*c^4*d^7*e - 3*a*c^3*d^5*e^3 - 9
*a^2*c^2*d^3*e^5 + a^3*c*d*e^7)*x + 12*(c^4*d^4*e^4*x^4 + a*c^3*d^6*e^2 +
(3*c^4*d^5*e^3 + a*c^3*d^3*e^5)*x^3 + 3*(c^4*d^6*e^2 + a*c^3*d^4*e^4)*x^2
+ (c^4*d^7*e + 3*a*c^3*d^5*e^3)*x)*log(c*d*x + a*e) - 12*(c^4*d^4*e^4*x^4
+ a*c^3*d^6*e^2 + (3*c^4*d^5*e^3 + a*c^3*d^3*e^5)*x^3 + 3*(c^4*d^6*e^2 + a
*c^3*d^4*e^4)*x^2 + (c^4*d^7*e + 3*a*c^3*d^5*e^3)*x)*log(e*x + d))/(a*c^5*
d^13*e - 5*a^2*c^4*d^11*e^3 + 10*a^3*c^3*d^9*e^5 - 10*a^4*c^2*d^7*e^7 + 5*
a^5*c*d^5*e^9 - a^6*d^3*e^11 + (c^6*d^11*e^3 - 5*a*c^5*d^9*e^5 + 10*a^2*c^
4*d^7*e^7 - 10*a^3*c^3*d^5*e^9 + 5*a^4*c^2*d^3*e^11 - a^5*c*d*e^13)*x^4 +
(3*c^6*d^12*e^2 - 14*a*c^5*d^10*e^4 + 25*a^2*c^4*d^8*e^6 - 20*a^3*c^3*d^6*
e^8 + 5*a^4*c^2*d^4*e^10 + 2*a^5*c*d^2*e^12 - a^6*e^14)*x^3 + 3*(c^6*d^13*
e - 4*a*c^5*d^11*e^3 + 5*a^2*c^4*d^9*e^5 - 5*a^4*c^2*d^5*e^9 + 4*a^5*c*d^3
*e^11 - a^6*d*e^13)*x^2 + (c^6*d^14 - 2*a*c^5*d^12*e^2 - 5*a^2*c^4*d^10*e^
4 + 20*a^3*c^3*d^8*e^6 - 25*a^4*c^2*d^6*e^8 + 14*a^5*c*d^4*e^10 - 3*a^6*d^
2*e^12)*x)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 996 vs. $2(160) = 320$.

Time = 1.30 (sec) , antiderivative size = 996, normalized size of antiderivative = 5.66

$$\int \frac{1}{(d+ex)^2 \left(ade+(cd^2+ae^2)\,x+cdex^2\right)^2}\,dx = \text{Too large to display}$$

input
```
integrate(1/(e*x+d)**2/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**2,x)
```

output
```
-4*c**3*d**3*e*log(x + (-4*a**6*c**3*d**3*e**13/(a*e**2 - c*d**2)**5 + 24*
a**5*c**4*d**5*e**11/(a*e**2 - c*d**2)**5 - 60*a**4*c**5*d**7*e**9/(a*e**2
 - c*d**2)**5 + 80*a**3*c**6*d**9*e**7/(a*e**2 - c*d**2)**5 - 60*a**2*c**7
*d**11*e**5/(a*e**2 - c*d**2)**5 + 24*a*c**8*d**13*e**3/(a*e**2 - c*d**2)*
*5 + 4*a*c**3*d**3*e**3 - 4*c**9*d**15*e/(a*e**2 - c*d**2)**5 + 4*c**4*d**
5*e)/(8*c**4*d**4*e**2))/(a*e**2 - c*d**2)**5 + 4*c**3*d**3*e*log(x + (4*a
**6*c**3*d**3*e**13/(a*e**2 - c*d**2)**5 - 24*a**5*c**4*d**5*e**11/(a*e**2
 - c*d**2)**5 + 60*a**4*c**5*d**7*e**9/(a*e**2 - c*d**2)**5 - 80*a**3*c**6
*d**9*e**7/(a*e**2 - c*d**2)**5 + 60*a**2*c**7*d**11*e**5/(a*e**2 - c*d**2
)**5 - 24*a*c**8*d**13*e**3/(a*e**2 - c*d**2)**5 + 4*a*c**3*d**3*e**3 + 4*
c**9*d**15*e/(a*e**2 - c*d**2)**5 + 4*c**4*d**5*e)/(8*c**4*d**4*e**2))/(a*
e**2 - c*d**2)**5 + (-a**3*e**6 + 5*a**2*c*d**2*e**4 - 13*a*c**2*d**4*e**2
 - 3*c**3*d**6 - 12*c**3*d**3*e**3*x**3 + x**2*(-6*a*c**2*d**2*e**4 - 30*c
**3*d**4*e**2) + x*(2*a**2*c*d*e**5 - 16*a*c**2*d**3*e**3 - 22*c**3*d**5*e
))/(3*a**5*d**3*e**9 - 12*a**4*c*d**5*e**7 + 18*a**3*c**2*d**7*e**5 - 12*a
**2*c**3*d**9*e**3 + 3*a*c**4*d**11*e + x**4*(3*a**4*c*d*e**11 - 12*a**3*c
**2*d**3*e**9 + 18*a**2*c**3*d**5*e**7 - 12*a*c**4*d**7*e**5 + 3*c**5*d**9
*e**3) + x**3*(3*a**5*e**12 - 3*a**4*c*d**2*e**10 - 18*a**3*c**2*d**4*e**8
 + 42*a**2*c**3*d**6*e**6 - 33*a*c**4*d**8*e**4 + 9*c**5*d**10*e**2) + x**
2*(9*a**5*d*e**11 - 27*a**4*c*d**3*e**9 + 18*a**3*c**2*d**5*e**7 + 18*a...
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 641 vs. $2(174) = 348$.

Time = 0.05 (sec) , antiderivative size = 641, normalized size of antiderivative = 3.64

$$\int \frac{1}{(d+ex)^2 (ade+(cd^2+ae^2)x+cdex^2)^2}\,dx$$
$$= -\frac{4\,c^3d^3e\log(cdx+ae)}{c^5d^{10}-5\,ac^4d^8e^2+10\,a^2c^3d^6e^4-10\,a^3c^2d^4e^6+5\,a^4cd^2e^8-a^5e^{10}}$$
$$+\frac{4\,c^3d^3e\log(ex+d)}{c^5d^{10}-5\,ac^4d^8e^2+10\,a^2c^3d^6e^4-10\,a^3c^2d^4e^6+5\,a^4cd^2e^8-a^5e^{10}}$$
$$-\frac{}{3\,(ac^4d^{11}e-4\,a^2c^3d^9e^3+6\,a^3c^2d^7e^5-4\,a^4cd^5e^7+a^5d^3e^9+(c^5d^9e^3-4\,ac^4d^7e^5+6\,a^2c^3d^5e^7-4\,a^3c^2d}$$

input
```
integrate(1/(e*x+d)^2/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^2,x, algorithm="ma
xima")
```

output
```
-4*c^3*d^3*e*log(c*d*x + a*e)/(c^5*d^10 - 5*a*c^4*d^8*e^2 + 10*a^2*c^3*d^6
*e^4 - 10*a^3*c^2*d^4*e^6 + 5*a^4*c*d^2*e^8 - a^5*e^10) + 4*c^3*d^3*e*log(
e*x + d)/(c^5*d^10 - 5*a*c^4*d^8*e^2 + 10*a^2*c^3*d^6*e^4 - 10*a^3*c^2*d^4
*e^6 + 5*a^4*c*d^2*e^8 - a^5*e^10) - 1/3*(12*c^3*d^3*e^3*x^3 + 3*c^3*d^6 +
 13*a*c^2*d^4*e^2 - 5*a^2*c*d^2*e^4 + a^3*e^6 + 6*(5*c^3*d^4*e^2 + a*c^2*d
^2*e^4)*x^2 + 2*(11*c^3*d^5*e + 8*a*c^2*d^3*e^3 - a^2*c*d*e^5)*x)/(a*c^4*d
^11*e - 4*a^2*c^3*d^9*e^3 + 6*a^3*c^2*d^7*e^5 - 4*a^4*c*d^5*e^7 + a^5*d^3*
e^9 + (c^5*d^9*e^3 - 4*a*c^4*d^7*e^5 + 6*a^2*c^3*d^5*e^7 - 4*a^3*c^2*d^3*e
^9 + a^4*c*d*e^11)*x^4 + (3*c^5*d^10*e^2 - 11*a*c^4*d^8*e^4 + 14*a^2*c^3*d
^6*e^6 - 6*a^3*c^2*d^4*e^8 - a^4*c*d^2*e^10 + a^5*e^12)*x^3 + 3*(c^5*d^11*
e - 3*a*c^4*d^9*e^3 + 2*a^2*c^3*d^7*e^5 + 2*a^3*c^2*d^5*e^7 - 3*a^4*c*d^3*
e^9 + a^5*d*e^11)*x^2 + (c^5*d^12 - a*c^4*d^10*e^2 - 6*a^2*c^3*d^8*e^4 + 1
4*a^3*c^2*d^6*e^6 - 11*a^4*c*d^4*e^8 + 3*a^5*d^2*e^10)*x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 480 vs. $2(174) = 348$.

Time = 0.12 (sec) , antiderivative size = 480, normalized size of antiderivative = 2.73

$$\int \frac{1}{(d+ex)^2 \left(ade+(cd^2+ae^2)x+cdex^2\right)^2}\,dx$$
$$= -\frac{4\,c^3d^3e^2\log\left(\left|cd-\frac{cd^2}{ex+d}+\frac{ae^2}{ex+d}\right|\right)}{c^5d^{10}e-5\,ac^4d^8e^3+10\,a^2c^3d^6e^5-10\,a^3c^2d^4e^7+5\,a^4cd^2e^9-a^5e^{11}}$$
$$-\frac{c^4d^4e}{(cd^2-ae^2)^5\left(cd-\frac{cd^2}{ex+d}+\frac{ae^2}{ex+d}\right)}$$
$$-\frac{\frac{9\,c^4d^6e^7}{ex+d}+\frac{3\,c^4d^7e^7}{(ex+d)^2}+\frac{c^4d^8e^7}{(ex+d)^3}-\frac{18\,ac^3d^4e^9}{ex+d}-\frac{9\,ac^3d^5e^9}{(ex+d)^2}-\frac{4\,ac^3d^6e^9}{(ex+d)^3}+\frac{9\,a^2c^2d^2e^{11}}{ex+d}+\frac{9\,a^2c^2d^3e^{11}}{(ex+d)^2}+\frac{6\,a^2c^2d^4e^{11}}{(ex+d)^3}-\frac{3\,a^3cd}{(ex+c}}{3\,(c^6d^{12}e^6-6\,ac^5d^{10}e^8+15\,a^2c^4d^8e^{10}-20\,a^3c^3d^6e^{12}+15\,a^4c^2d^4e^{14}-6\,a^5cd^2e^{16}+a}$$

input
```
integrate(1/(e*x+d)^2/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^2,x, algorithm="gi
ac")
```

output
```
-4*c^3*d^3*e^2*log(abs(c*d - c*d^2/(e*x + d) + a*e^2/(e*x + d)))/(c^5*d^10
*e - 5*a*c^4*d^8*e^3 + 10*a^2*c^3*d^6*e^5 - 10*a^3*c^2*d^4*e^7 + 5*a^4*c*d
^2*e^9 - a^5*e^11) - c^4*d^4*e/((c*d^2 - a*e^2)^5*(c*d - c*d^2/(e*x + d) +
 a*e^2/(e*x + d))) - 1/3*(9*c^4*d^6*e^7/(e*x + d) + 3*c^4*d^7*e^7/(e*x + d
)^2 + c^4*d^8*e^7/(e*x + d)^3 - 18*a*c^3*d^4*e^9/(e*x + d) - 9*a*c^3*d^5*e
^9/(e*x + d)^2 - 4*a*c^3*d^6*e^9/(e*x + d)^3 + 9*a^2*c^2*d^2*e^11/(e*x + d
) + 9*a^2*c^2*d^3*e^11/(e*x + d)^2 + 6*a^2*c^2*d^4*e^11/(e*x + d)^3 - 3*a^
3*c*d*e^13/(e*x + d)^2 - 4*a^3*c*d^2*e^13/(e*x + d)^3 + a^4*e^15/(e*x + d)
^3)/(c^6*d^12*e^6 - 6*a*c^5*d^10*e^8 + 15*a^2*c^4*d^8*e^10 - 20*a^3*c^3*d^
6*e^12 + 15*a^4*c^2*d^4*e^14 - 6*a^5*c*d^2*e^16 + a^6*e^18)
```

Mupad [B] (verification not implemented)

Time = 5.82 (sec) , antiderivative size = 595, normalized size of antiderivative = 3.38

$$\int \frac{1}{(d+ex)^2 \left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^2}\,dx$$
$$= \frac{8\,c^3\,d^3\,e\,\mathrm{atanh}\left(\frac{a^5\,e^{10}-3\,a^4\,c\,d^2\,e^8+2\,a^3\,c^2\,d^4\,e^6+2\,a^2\,c^3\,d^6\,e^4-3\,a\,c^4\,d^8\,e^2+c^5\,d^{10}}{{\left(a\,e^2-c\,d^2\right)}^5}+\frac{2\,c\,d\,e\,x\,\left(a^4\,e^8-4\,a^3\,c\,d^2\,e^6+6\,a^2\,c^2\,d^4\,e^4-4\,a\,c^3\,d^6\right.}{{\left(a\,e^2-c\,d^2\right)}^5}\right.}{{\left(a\,e^2-c\,d^2\right)}^5}$$
$$-\frac{\frac{a^3\,e^6-5\,a^2\,c\,d^2\,e^4+13\,a\,c^2\,d^4\,e^2+3\,c^3\,d^6}{3\,\left(a^4\,e^8-4\,a^3\,c\,d^2\,e^6+6\,a^2\,c^2\,d^4\,e^4-4\,a\,c^3\,d^6\,e^2+c^4\,d^8\right)}+\frac{4\,c^3\,d^3\,e^3\,x^3}{a^4\,e^8-4\,a^3\,c\,d^2\,e^6+6\,a^2\,c^2\,d^4\,e^4-4\,a\,c^3\,d^6\,e^2+c^4\,d^8}+\frac{2\,c\,d\,x\,\left(-a^2\,e^5+8\right.}{3\,\left(a^4\,e^8-4\,a^3\,c\,d^2\,e^6+6\,a\right.}}{x\,\left(c\,d^4+3\,a\,d^2\,e^2\right)+x^3\,\left(3\,c\,d^2\,e^2+a\,e^4\right)+x^2\,\left(3\,c\,d^3\,e+3\,a\,d\right.}$$

input
```
int(1/((d + e*x)^2*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^2),x)
```

output
```
(8*c^3*d^3*e*atanh((a^5*e^10 + c^5*d^10 - 3*a*c^4*d^8*e^2 - 3*a^4*c*d^2*e^
8 + 2*a^2*c^3*d^6*e^4 + 2*a^3*c^2*d^4*e^6)/(a*e^2 - c*d^2)^5 + (2*c*d*e*x*
(a^4*e^8 + c^4*d^8 - 4*a*c^3*d^6*e^2 - 4*a^3*c*d^2*e^6 + 6*a^2*c^2*d^4*e^4
))/(a*e^2 - c*d^2)^5))/(a*e^2 - c*d^2)^5 - ((a^3*e^6 + 3*c^3*d^6 + 13*a*c^
2*d^4*e^2 - 5*a^2*c*d^2*e^4)/(3*(a^4*e^8 + c^4*d^8 - 4*a*c^3*d^6*e^2 - 4*a
^3*c*d^2*e^6 + 6*a^2*c^2*d^4*e^4)) + (4*c^3*d^3*e^3*x^3)/(a^4*e^8 + c^4*d^
8 - 4*a*c^3*d^6*e^2 - 4*a^3*c*d^2*e^6 + 6*a^2*c^2*d^4*e^4) + (2*c*d*x*(11*
c^2*d^4*e - a^2*e^5 + 8*a*c*d^2*e^3))/(3*(a^4*e^8 + c^4*d^8 - 4*a*c^3*d^6*
e^2 - 4*a^3*c*d^2*e^6 + 6*a^2*c^2*d^4*e^4)) + (2*c^2*d^2*x^2*(a*e^4 + 5*c*
d^2*e^2))/(a^4*e^8 + c^4*d^8 - 4*a*c^3*d^6*e^2 - 4*a^3*c*d^2*e^6 + 6*a^2*c
^2*d^4*e^4))/(x*(c*d^4 + 3*a*d^2*e^2) + x^3*(a*e^4 + 3*c*d^2*e^2) + x^2*(3
*a*d*e^3 + 3*c*d^3*e) + a*d^3*e + c*d*e^3*x^4)
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 1331, normalized size of antiderivative = 7.56

$$\int \frac{1}{(d+ex)^2 \left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^2}\,dx = \text{Too large to display}$$

input
```
int(1/(e*x+d)^2/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^2,x)
```

output
```
(12*log(a*e + c*d*x)*a**2*c**3*d**6*e**4 + 36*log(a*e + c*d*x)*a**2*c**3*d
**5*e**5*x + 36*log(a*e + c*d*x)*a**2*c**3*d**4*e**6*x**2 + 12*log(a*e + c
*d*x)*a**2*c**3*d**3*e**7*x**3 + 36*log(a*e + c*d*x)*a*c**4*d**8*e**2 + 12
0*log(a*e + c*d*x)*a*c**4*d**7*e**3*x + 144*log(a*e + c*d*x)*a*c**4*d**6*e
**4*x**2 + 72*log(a*e + c*d*x)*a*c**4*d**5*e**5*x**3 + 12*log(a*e + c*d*x)
*a*c**4*d**4*e**6*x**4 + 36*log(a*e + c*d*x)*c**5*d**9*e*x + 108*log(a*e +
 c*d*x)*c**5*d**8*e**2*x**2 + 108*log(a*e + c*d*x)*c**5*d**7*e**3*x**3 + 3
6*log(a*e + c*d*x)*c**5*d**6*e**4*x**4 - 12*log(d + e*x)*a**2*c**3*d**6*e*
*4 - 36*log(d + e*x)*a**2*c**3*d**5*e**5*x - 36*log(d + e*x)*a**2*c**3*d**
4*e**6*x**2 - 12*log(d + e*x)*a**2*c**3*d**3*e**7*x**3 - 36*log(d + e*x)*a
*c**4*d**8*e**2 - 120*log(d + e*x)*a*c**4*d**7*e**3*x - 144*log(d + e*x)*a
*c**4*d**6*e**4*x**2 - 72*log(d + e*x)*a*c**4*d**5*e**5*x**3 - 12*log(d +
e*x)*a*c**4*d**4*e**6*x**4 - 36*log(d + e*x)*c**5*d**9*e*x - 108*log(d + e
*x)*c**5*d**8*e**2*x**2 - 108*log(d + e*x)*c**5*d**7*e**3*x**3 - 36*log(d
+ e*x)*c**5*d**6*e**4*x**4 - a**5*e**10 + 3*a**4*c*d**2*e**8 + 2*a**4*c*d*
e**9*x - 12*a**3*c**2*d**3*e**7*x - 6*a**3*c**2*d**2*e**8*x**2 - 32*a**2*c
**3*d**6*e**4 - 24*a**2*c**3*d**5*e**5*x - 6*a**2*c**3*d**4*e**6*x**2 + 21
*a*c**4*d**8*e**2 - 20*a*c**4*d**7*e**3*x - 42*a*c**4*d**6*e**4*x**2 + 12*
a*c**4*d**4*e**6*x**4 + 9*c**5*d**10 + 54*c**5*d**9*e*x + 54*c**5*d**8*e**
2*x**2 - 12*c**5*d**6*e**4*x**4)/(3*(a**7*d**3*e**13 + 3*a**7*d**2*e**1...
```

3.127 $\int \frac{(d+ex)^9}{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^3}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 221

$$\int \frac{(d+ex)^9}{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^3}\,dx = \frac{20e^3\left(cd^2-ae^2\right)^3x}{c^6d^6} - \frac{\left(cd^2-ae^2\right)^6}{2c^7d^7(ae+cdx)^2} - \frac{6e\left(cd^2-ae^2\right)^5}{c^7d^7(ae+cdx)} + \frac{15e^4\left(cd^2-ae^2\right)^2(ae+cdx)^2}{2c^7d^7} + \frac{2e^5\left(cd^2-ae^2\right)(ae+cdx)^3}{c^7d^7} + \frac{e^6(ae+cdx)^4}{4c^7d^7} + \frac{15e^2\left(cd^2-ae^2\right)^4\log(ae+cdx)}{c^7d^7}$$

output
```
20*e^3*(-a*e^2+c*d^2)^3*x/c^6/d^6-1/2*(-a*e^2+c*d^2)^6/c^7/d^7/(c*d*x+a*e)
^2-6*e*(-a*e^2+c*d^2)^5/c^7/d^7/(c*d*x+a*e)+15/2*e^4*(-a*e^2+c*d^2)^2*(c*d
*x+a*e)^2/c^7/d^7+2*e^5*(-a*e^2+c*d^2)*(c*d*x+a*e)^3/c^7/d^7+1/4*e^6*(c*d*
x+a*e)^4/c^7/d^7+15*e^2*(-a*e^2+c*d^2)^4*ln(c*d*x+a*e)/c^7/d^7
```

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 337, normalized size of antiderivative = 1.52

$$\int \frac{(d+ex)^9}{(ade+(cd^2+ae^2)\,x+cdex^2)^3}\,dx$$
$$= \frac{22a^6e^{12}-4a^5cde^{10}(27d+4ex)+2a^4c^2d^2e^8(105d^2+12dex-34e^2x^2)-4a^3c^3d^3e^6(50d^3-15d^2ex-63a}{}$$

input
```
Integrate[(d + e*x)^9/(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^3,x]
```

output
```
(22*a^6*e^12 - 4*a^5*c*d*e^10*(27*d + 4*e*x) + 2*a^4*c^2*d^2*e^8*(105*d^2
+ 12*d*e*x - 34*e^2*x^2) - 4*a^3*c^3*d^3*e^6*(50*d^3 - 15*d^2*e*x - 63*d*e
^2*x^2 + 5*e^3*x^3) + 5*a^2*c^4*d^4*e^4*(18*d^4 - 32*d^3*e*x - 66*d^2*e^2*
x^2 + 16*d*e^3*x^3 + e^4*x^4) - 2*a*c^5*d^5*e^2*(6*d^5 - 60*d^4*e*x - 80*d
^3*e^2*x^2 + 60*d^2*e^3*x^3 + 10*d*e^4*x^4 + e^5*x^5) + c^6*d^6*(-2*d^6 -
24*d^5*e*x + 80*d^3*e^3*x^3 + 30*d^2*e^4*x^4 + 8*d*e^5*x^5 + e^6*x^6) + 60
*e^2*(c*d^2 - a*e^2)^4*(a*e + c*d*x)^2*Log[a*e + c*d*x])/(4*c^7*d^7*(a*e +
 c*d*x)^2)
```

Rubi [A] (verified)

Time = 0.53 (sec) , antiderivative size = 221, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.057$, Rules used = {1121, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex)^9}{(x(ae^2+cd^2)+ade+cdex^2)^3}\,dx$$
$$\Big\downarrow \text{1121}$$
$$\int \left(\frac{e^6(ae+cdx)^3}{c^6d^6}+\frac{20(cd^2e-ae^3)^3}{c^6d^6}+\frac{15e^2(cd^2-ae^2)^4}{c^6d^6(ae+cdx)}+\frac{6e(cd^2-ae^2)^5}{c^6d^6(ae+cdx)^2}+\frac{(cd^2-ae^2)^6}{c^6d^6(ae+cdx)^3}+\frac{6(cd^2e^5-a\right.$$
$$\Big\downarrow \text{2009}$$

$$\frac{e^6(ae+cdx)^4}{4c^7d^7}-\frac{6e\left(cd^2-ae^2\right)^5}{c^7d^7(ae+cdx)}-\frac{\left(cd^2-ae^2\right)^6}{2c^7d^7(ae+cdx)^2}+\frac{15e^2\left(cd^2-ae^2\right)^4\log(ae+cdx)}{c^7d^7}+\frac{2e^5\left(cd^2-ae^2\right)(ae+cdx)^3}{c^7d^7}+\frac{15e^4\left(cd^2-ae^2\right)^2(ae+cdx)^2}{2c^7d^7}+\frac{20e^3x\left(cd^2-ae^2\right)^3}{c^6d^6}$$

input
```
Int[(d + e*x)^9/(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^3,x]
```

output
```
(20*e^3*(c*d^2 - a*e^2)^3*x)/(c^6*d^6) - (c*d^2 - a*e^2)^6/(2*c^7*d^7*(a*e
 + c*d*x)^2) - (6*e*(c*d^2 - a*e^2)^5)/(c^7*d^7*(a*e + c*d*x)) + (15*e^4*(
c*d^2 - a*e^2)^2*(a*e + c*d*x)^2)/(2*c^7*d^7) + (2*e^5*(c*d^2 - a*e^2)*(a*
e + c*d*x)^3)/(c^7*d^7) + (e^6*(a*e + c*d*x)^4)/(4*c^7*d^7) + (15*e^2*(c*d
^2 - a*e^2)^4*Log[a*e + c*d*x])/(c^7*d^7)
```

Defintions of rubi rules used

rule 1121
```
Int[((d_) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Int[ExpandIntegrand[(d + e*x)^(m + p)*(a/d + (c/e)*x)^p, x], x]
/; FreeQ[{a, b, c, d, e, m, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && (Int
egerQ[p] || (GtQ[a, 0] && GtQ[d, 0] && LtQ[c, 0]))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 1.61 (sec) , antiderivative size = 399, normalized size of antiderivative = 1.81

method	result
default	$-\frac{e^3\left(-\frac{1}{4}x^4c^3d^3e^3+x^3ac^2d^2e^4-2x^3c^3d^4e^2-3x^2a^2cde^5+9x^2ac^2d^3e^3-\frac{15}{2}x^2c^3d^5e+10e^6a^3x-36d^2e^4a^2cx+45d^4e^2ac^2x-2\cdots\right.}{d^6c^6}$
risch	$\frac{e^6x^4}{4d^3c^3}-\frac{e^7x^3a}{d^4c^4}+\frac{2e^5x^3}{d^2c^3}+\frac{3e^8x^2a^2}{d^5c^5}-\frac{9e^6x^2a}{d^3c^4}+\frac{15e^4x^2}{2dc^3}-\frac{10e^9a^3x}{d^6c^6}+\frac{36e^7a^2x}{d^4c^5}-\frac{45e^5ax}{d^2c^4}+\frac{20e^3x}{c^3}+\frac{(6a^5e^1\cdots}{}$
norman	$\frac{\frac{90a^6e^{12}-320a^5d^2e^{10}c+375a^4d^4e^8c^2-100a^3d^6e^6c^3-100a^2d^8e^4c^4-12ad^{10}e^2c^5-2d^{12}c^6}{4c^7d^5}+\frac{e^8x^8}{4cd}+\frac{(90a^6e^{16}-80cd^2a^5e^{14}-425a^4c^2d^4\cdots}{}}{}$
parallelrisch	$\frac{120a^5cde^{11}x-480a^4c^2d^3e^9x+720a^3c^3d^5e^7x-480a^2c^4d^7e^5x-2x^5ac^5d^5e^7+5x^4a^2c^4d^4e^8-20x^4ac^5d^6e^6-20x^3a^3c^3d^3e^9+8\cdots}{}$

input
```
int((e*x+d)^9/(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^3,x,method=_RETURNVERBOSE)
```

output
```
-e^3/d^6/c^6*(-1/4*x^4*c^3*d^3*e^3+x^3*a*c^2*d^2*e^4-2*x^3*c^3*d^4*e^2-3*x
^2*a^2*c*d*e^5+9*x^2*a*c^2*d^3*e^3-15/2*x^2*c^3*d^5*e+10*e^6*a^3*x-36*d^2*
e^4*a^2*c*x+45*d^4*e^2*a*c^2*x-20*d^6*c^3*x)+15*e^2/c^7/d^7*(a^4*e^8-4*a^3
*c*d^2*e^6+6*a^2*c^2*d^4*e^4-4*a*c^3*d^6*e^2+c^4*d^8)*ln(c*d*x+a*e)-1/2/d^
7/c^7*(a^6*e^12-6*a^5*c*d^2*e^10+15*a^4*c^2*d^4*e^8-20*a^3*c^3*d^6*e^6+15*
a^2*c^4*d^8*e^4-6*a*c^5*d^10*e^2+c^6*d^12)/(c*d*x+a*e)^2+6/d^7*e/c^7*(a^5*
e^10-5*a^4*c*d^2*e^8+10*a^3*c^2*d^4*e^6-10*a^2*c^3*d^6*e^4+5*a*c^4*d^8*e^2
-c^5*d^10)/(c*d*x+a*e)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 606 vs. $2(215) = 430$.

Time = 0.09 (sec) , antiderivative size = 606, normalized size of antiderivative = 2.74

$$\int \frac{(d+ex)^9}{(ade+(cd^2+ae^2)\,x+cdex^2)^3}\,dx$$
$$= \frac{c^6d^6e^6x^6 - 2\,c^6d^{12} - 12\,ac^5d^{10}e^2 + 90\,a^2c^4d^8e^4 - 200\,a^3c^3d^6e^6 + 210\,a^4c^2d^4e^8 - 108\,a^5cd^2e^{10} + 22\,a^6e^{12} -}{}$$

input
```
integrate((e*x+d)^9/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^3,x, algorithm="fric
as")
```

output
```
1/4*(c^6*d^6*e^6*x^6 - 2*c^6*d^12 - 12*a*c^5*d^10*e^2 + 90*a^2*c^4*d^8*e^4
 - 200*a^3*c^3*d^6*e^6 + 210*a^4*c^2*d^4*e^8 - 108*a^5*c*d^2*e^10 + 22*a^6
*e^12 + 2*(4*c^6*d^7*e^5 - a*c^5*d^5*e^7)*x^5 + 5*(6*c^6*d^8*e^4 - 4*a*c^5
*d^6*e^6 + a^2*c^4*d^4*e^8)*x^4 + 20*(4*c^6*d^9*e^3 - 6*a*c^5*d^7*e^5 + 4*
a^2*c^4*d^5*e^7 - a^3*c^3*d^3*e^9)*x^3 + 2*(80*a*c^5*d^8*e^4 - 165*a^2*c^4
*d^6*e^6 + 126*a^3*c^3*d^4*e^8 - 34*a^4*c^2*d^2*e^10)*x^2 - 4*(6*c^6*d^11*
e - 30*a*c^5*d^9*e^3 + 40*a^2*c^4*d^7*e^5 - 15*a^3*c^3*d^5*e^7 - 6*a^4*c^2
*d^3*e^9 + 4*a^5*c*d*e^11)*x + 60*(a^2*c^4*d^8*e^4 - 4*a^3*c^3*d^6*e^6 + 6
*a^4*c^2*d^4*e^8 - 4*a^5*c*d^2*e^10 + a^6*e^12 + (c^6*d^10*e^2 - 4*a*c^5*d
^8*e^4 + 6*a^2*c^4*d^6*e^6 - 4*a^3*c^3*d^4*e^8 + a^4*c^2*d^2*e^10)*x^2 + 2
*(a*c^5*d^9*e^3 - 4*a^2*c^4*d^7*e^5 + 6*a^3*c^3*d^5*e^7 - 4*a^4*c^2*d^3*e^
9 + a^5*c*d*e^11)*x)*log(c*d*x + a*e))/(c^9*d^9*x^2 + 2*a*c^8*d^8*e*x + a^
2*c^7*d^7*e^2)
```

Sympy [A] (verification not implemented)

Time = 4.72 (sec) , antiderivative size = 386, normalized size of antiderivative = 1.75

$$\int \frac{(d+ex)^9}{(ade+(cd^2+ae^2)x+cdex^2)^3}\,dx = x^3\left(-\frac{ae^7}{c^4d^4}+\frac{2e^5}{c^3d^2}\right)+x^2 \cdot\left(\frac{3a^2e^8}{c^5d^5}-\frac{9ae^6}{c^4d^3}+\frac{15e^4}{2c^3d}\right)+x\left(-\frac{10a^3e^9}{c^6d^6}+\frac{36a^2e^7}{c^5d^4}-\frac{45ae^5}{c^4d^2}+\frac{20e^3}{c^3}\right) + \frac{11a^6e^{12}-54a^5cd^2e^{10}+105a^4c^2d^4e^8-100a^3c^3d^6e^6+45a^2c^4d^8e^4-6ac^5d^{10}e^2-c^6d^{12}+x(12a^5cde^{11}-}{2a^2c^7d^7e^2+4ac^8d^8ex+2c^9d^9x^2} + \frac{e^6x^4}{4c^3d^3}+\frac{15e^2(ae^2-cd^2)^4\log(ae+cdx)}{c^7d^7}$$

input
```
integrate((e*x+d)**9/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**3,x)
```

output
```
x**3*(-a*e**7/(c**4*d**4) + 2*e**5/(c**3*d**2)) + x**2*(3*a**2*e**8/(c**5*
d**5) - 9*a*e**6/(c**4*d**3) + 15*e**4/(2*c**3*d)) + x*(-10*a**3*e**9/(c**
6*d**6) + 36*a**2*e**7/(c**5*d**4) - 45*a*e**5/(c**4*d**2) + 20*e**3/c**3)
 + (11*a**6*e**12 - 54*a**5*c*d**2*e**10 + 105*a**4*c**2*d**4*e**8 - 100*a
**3*c**3*d**6*e**6 + 45*a**2*c**4*d**8*e**4 - 6*a*c**5*d**10*e**2 - c**6*d
**12 + x*(12*a**5*c*d*e**11 - 60*a**4*c**2*d**3*e**9 + 120*a**3*c**3*d**5*
e**7 - 120*a**2*c**4*d**7*e**5 + 60*a*c**5*d**9*e**3 - 12*c**6*d**11*e))/(
2*a**2*c**7*d**7*e**2 + 4*a*c**8*d**8*e*x + 2*c**9*d**9*x**2) + e**6*x**4/
(4*c**3*d**3) + 15*e**2*(a*e**2 - c*d**2)**4*log(a*e + c*d*x)/(c**7*d**7)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 408, normalized size of antiderivative = 1.85

$$\int \frac{(d+ex)^9}{(ade+(cd^2+ae^2)x+cdex^2)^3}\,dx = -\frac{c^6d^{12}+6\,ac^5d^{10}e^2-45\,a^2c^4d^8e^4+100\,a^3c^3d^6e^6-105\,a^4c^2d^4e^8+54\,a^5cd^2e^{10}-11\,a^6e^{12}+12\,(c^6d^{11}e-}{2\,(c^9d^9x^2+2\,ac^8d^8ex+a^2c^7d^7e^2)} + \frac{c^3d^3e^6x^4+4\,(2\,c^3d^4e^5-ac^2d^2e^7)x^3+6\,(5\,c^3d^5e^4-6\,ac^2d^3e^6+2\,a^2cde^8)x^2+4\,(20\,c^3d^6e^3-45\,ac^2d^4e}{4\,c^6d^6} + \frac{15\,(c^4d^8e^2-4\,ac^3d^6e^4+6\,a^2c^2d^4e^6-4\,a^3cd^2e^8+a^4e^{10})\log(cdx+ae)}{c^7d^7}$$

input
```
integrate((e*x+d)^9/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^3,x, algorithm="maxi
ma")
```

output
```
-1/2*(c^6*d^12 + 6*a*c^5*d^10*e^2 - 45*a^2*c^4*d^8*e^4 + 100*a^3*c^3*d^6*e
^6 - 105*a^4*c^2*d^4*e^8 + 54*a^5*c*d^2*e^10 - 11*a^6*e^12 + 12*(c^6*d^11*
e - 5*a*c^5*d^9*e^3 + 10*a^2*c^4*d^7*e^5 - 10*a^3*c^3*d^5*e^7 + 5*a^4*c^2*
d^3*e^9 - a^5*c*d*e^11)*x)/(c^9*d^9*x^2 + 2*a*c^8*d^8*e*x + a^2*c^7*d^7*e^
2) + 1/4*(c^3*d^3*e^6*x^4 + 4*(2*c^3*d^4*e^5 - a*c^2*d^2*e^7)*x^3 + 6*(5*c
^3*d^5*e^4 - 6*a*c^2*d^3*e^6 + 2*a^2*c*d*e^8)*x^2 + 4*(20*c^3*d^6*e^3 - 45
*a*c^2*d^4*e^5 + 36*a^2*c*d^2*e^7 - 10*a^3*e^9)*x)/(c^6*d^6) + 15*(c^4*d^8
*e^2 - 4*a*c^3*d^6*e^4 + 6*a^2*c^2*d^4*e^6 - 4*a^3*c*d^2*e^8 + a^4*e^10)*l
og(c*d*x + a*e)/(c^7*d^7)
```

Giac [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 403, normalized size of antiderivative = 1.82

$$\int \frac{(d+ex)^9}{(ade+(cd^2+ae^2)x+cdex^2)^3}\,dx = \frac{15\,(c^4d^8e^2-4\,ac^3d^6e^4+6\,a^2c^2d^4e^6-4\,a^3cd^2e^8+a^4e^{10})\log(|cdx+ae|)}{c^7d^7} - \frac{c^6d^{12}+6\,ac^5d^{10}e^2-45\,a^2c^4d^8e^4+100\,a^3c^3d^6e^6-105\,a^4c^2d^4e^8+54\,a^5cd^2e^{10}-11\,a^6e^{12}+12\,(c^6d^{11}e-}{2\,(cdx+ae)^2c^7d^7} + \frac{c^9d^9e^6x^4+8\,c^9d^{10}e^5x^3-4\,ac^8d^8e^7x^3+30\,c^9d^{11}e^4x^2-36\,ac^8d^9e^6x^2+12\,a^2c^7d^7e^8x^2+80\,c^9d^{12}e^3x-1}{4\,c^{12}d^{12}}$$

input
```
integrate((e*x+d)^9/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^3,x, algorithm="giac
")
```

output
```
15*(c^4*d^8*e^2 - 4*a*c^3*d^6*e^4 + 6*a^2*c^2*d^4*e^6 - 4*a^3*c*d^2*e^8 +
a^4*e^10)*log(abs(c*d*x + a*e))/(c^7*d^7) - 1/2*(c^6*d^12 + 6*a*c^5*d^10*e
^2 - 45*a^2*c^4*d^8*e^4 + 100*a^3*c^3*d^6*e^6 - 105*a^4*c^2*d^4*e^8 + 54*a
^5*c*d^2*e^10 - 11*a^6*e^12 + 12*(c^6*d^11*e - 5*a*c^5*d^9*e^3 + 10*a^2*c^
4*d^7*e^5 - 10*a^3*c^3*d^5*e^7 + 5*a^4*c^2*d^3*e^9 - a^5*c*d*e^11)*x)/((c*
d*x + a*e)^2*c^7*d^7) + 1/4*(c^9*d^9*e^6*x^4 + 8*c^9*d^10*e^5*x^3 - 4*a*c^
8*d^8*e^7*x^3 + 30*c^9*d^11*e^4*x^2 - 36*a*c^8*d^9*e^6*x^2 + 12*a^2*c^7*d^
7*e^8*x^2 + 80*c^9*d^12*e^3*x - 180*a*c^8*d^10*e^5*x + 144*a^2*c^7*d^8*e^7
*x - 40*a^3*c^6*d^6*e^9*x)/(c^12*d^12)
```

Mupad [B] (verification not implemented)

Time = 5.28 (sec) , antiderivative size = 516, normalized size of antiderivative = 2.33

$$\int \frac{(d+ex)^9}{(ade+(cd^2+ae^2)\,x+cdex^2)^3}\,dx$$

$$= x^3\left(\frac{2\,e^5}{c^3\,d^2}-\frac{a\,e^7}{c^4\,d^4}\right)-x^2\left(\frac{3\,a^2\,e^8}{2\,c^5\,d^5}-\frac{15\,e^4}{2\,c^3\,d}+\frac{3\,a\,e\left(\frac{6\,e^5}{c^3\,d^2}-\frac{3\,a\,e^7}{c^4\,d^4}\right)}{2\,c\,d}\right)$$

$$+\frac{x\,(6\,a^5\,e^{11}-30\,a^4\,c\,d^2\,e^9+60\,a^3\,c^2\,d^4\,e^7-60\,a^2\,c^3\,d^6\,e^5+30\,a\,c^4\,d^8\,e^3-6\,c^5\,d^{10}\,e)-\frac{-11\,a^6\,e^{12}+54\,a^5\,c\,d^2\,e}{}}{a^2\,c^6\,d^6\,e^2+2\,a\,c^7\,d^7\,e\,x+c^8\,d^8\,x^2}$$

$$+x\left(\frac{20\,e^3}{c^3}-\frac{a^3\,e^9}{c^6\,d^6}-\frac{3\,a^2\,e^2\left(\frac{6\,e^5}{c^3\,d^2}-\frac{3\,a\,e^7}{c^4\,d^4}\right)}{c^2\,d^2}+\frac{3\,a\,e\left(\frac{3\,a^2\,e^8}{c^5\,d^5}-\frac{15\,e^4}{c^3\,d}+\frac{3\,a\,e\left(\frac{6\,e^5}{c^3\,d^2}-\frac{3\,a\,e^7}{c^4\,d^4}\right)}{c\,d}\right)}{c\,d}\right)$$

$$+\frac{e^6\,x^4}{4\,c^3\,d^3}$$

$$+\frac{\ln(a\,e+c\,d\,x)\,(15\,a^4\,e^{10}-60\,a^3\,c\,d^2\,e^8+90\,a^2\,c^2\,d^4\,e^6-60\,a\,c^3\,d^6\,e^4+15\,c^4\,d^8\,e^2)}{c^7\,d^7}$$

input
```
int((d + e*x)^9/(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^3,x)
```

output
```
x^3*((2*e^5)/(c^3*d^2) - (a*e^7)/(c^4*d^4)) - x^2*((3*a^2*e^8)/(2*c^5*d^5)
 - (15*e^4)/(2*c^3*d) + (3*a*e*((6*e^5)/(c^3*d^2) - (3*a*e^7)/(c^4*d^4)))/
(2*c*d)) + (x*(6*a^5*e^11 - 6*c^5*d^10*e + 30*a*c^4*d^8*e^3 - 30*a^4*c*d^2
*e^9 - 60*a^2*c^3*d^6*e^5 + 60*a^3*c^2*d^4*e^7) - (c^6*d^12 - 11*a^6*e^12
+ 6*a*c^5*d^10*e^2 + 54*a^5*c*d^2*e^10 - 45*a^2*c^4*d^8*e^4 + 100*a^3*c^3*
d^6*e^6 - 105*a^4*c^2*d^4*e^8)/(2*c*d))/(c^8*d^8*x^2 + a^2*c^6*d^6*e^2 + 2
*a*c^7*d^7*e*x) + x*((20*e^3)/c^3 - (a^3*e^9)/(c^6*d^6) - (3*a^2*e^2*((6*e
^5)/(c^3*d^2) - (3*a*e^7)/(c^4*d^4)))/(c^2*d^2) + (3*a*e*((3*a^2*e^8)/(c^5
*d^5) - (15*e^4)/(c^3*d) + (3*a*e*((6*e^5)/(c^3*d^2) - (3*a*e^7)/(c^4*d^4)
))/(c*d)))/(c*d)) + (e^6*x^4)/(4*c^3*d^3) + (log(a*e + c*d*x)*(15*a^4*e^10
 + 15*c^4*d^8*e^2 - 60*a*c^3*d^6*e^4 - 60*a^3*c*d^2*e^8 + 90*a^2*c^2*d^4*e
^6))/(c^7*d^7)
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 716, normalized size of antiderivative = 3.24

$$\int \frac{(d+ex)^9}{(ade+(cd^2+ae^2)x+cdex^2)^3}\,dx$$
$$= \frac{-480\log(cdx+ae)\,a^3c^4d^7e^5x+360\log(cdx+ae)\,a^3c^4d^6e^6x^2+120\log(cdx+ae)\,a^2c^5d^9e^3x-240\log(c}{}$$

input
```
int((e*x+d)^9/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^3,x)
```

output
```
(60*log(a*e + c*d*x)*a**7*e**12 - 240*log(a*e + c*d*x)*a**6*c*d**2*e**10 +
 120*log(a*e + c*d*x)*a**6*c*d*e**11*x + 360*log(a*e + c*d*x)*a**5*c**2*d*
*4*e**8 - 480*log(a*e + c*d*x)*a**5*c**2*d**3*e**9*x + 60*log(a*e + c*d*x)
*a**5*c**2*d**2*e**10*x**2 - 240*log(a*e + c*d*x)*a**4*c**3*d**6*e**6 + 72
0*log(a*e + c*d*x)*a**4*c**3*d**5*e**7*x - 240*log(a*e + c*d*x)*a**4*c**3*
d**4*e**8*x**2 + 60*log(a*e + c*d*x)*a**3*c**4*d**8*e**4 - 480*log(a*e + c
*d*x)*a**3*c**4*d**7*e**5*x + 360*log(a*e + c*d*x)*a**3*c**4*d**6*e**6*x**
2 + 120*log(a*e + c*d*x)*a**2*c**5*d**9*e**3*x - 240*log(a*e + c*d*x)*a**2
*c**5*d**8*e**4*x**2 + 60*log(a*e + c*d*x)*a*c**6*d**10*e**2*x**2 + 30*a**
7*e**12 - 120*a**6*c*d**2*e**10 + 180*a**5*c**2*d**4*e**8 - 60*a**5*c**2*d
**2*e**10*x**2 - 120*a**4*c**3*d**6*e**6 + 240*a**4*c**3*d**4*e**8*x**2 -
20*a**4*c**3*d**3*e**9*x**3 + 30*a**3*c**4*d**8*e**4 - 360*a**3*c**4*d**6*
e**6*x**2 + 80*a**3*c**4*d**5*e**7*x**3 + 5*a**3*c**4*d**4*e**8*x**4 + 240
*a**2*c**5*d**8*e**4*x**2 - 120*a**2*c**5*d**7*e**5*x**3 - 20*a**2*c**5*d*
*6*e**6*x**4 - 2*a**2*c**5*d**5*e**7*x**5 - 2*a*c**6*d**12 - 60*a*c**6*d**
10*e**2*x**2 + 80*a*c**6*d**9*e**3*x**3 + 30*a*c**6*d**8*e**4*x**4 + 8*a*c
**6*d**7*e**5*x**5 + a*c**6*d**6*e**6*x**6 + 12*c**7*d**12*x**2)/(4*a*c**7
*d**7*(a**2*e**2 + 2*a*c*d*e*x + c**2*d**2*x**2))
```

3.128 $\int \frac{(d+ex)^8}{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^3}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 185

$$\begin{aligned}\int \frac{(d+ex)^8}{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^3}\,dx &= \frac{e^3(10c^2d^4-15acd^2e^2+6a^2e^4)\,x}{c^5d^5} \\ &\quad+\frac{e^4(5cd^2-3ae^2)\,x^2}{2c^4d^4}+\frac{e^5x^3}{3c^3d^3} \\ &\quad-\frac{\left(cd^2-ae^2\right)^5}{2c^6d^6(ae+cdx)^2}-\frac{5e\left(cd^2-ae^2\right)^4}{c^6d^6(ae+cdx)} \\ &\quad+\frac{10e^2\left(cd^2-ae^2\right)^3\log(ae+cdx)}{c^6d^6}\end{aligned}$$

output

```
e^3*(6*a^2*e^4-15*a*c*d^2*e^2+10*c^2*d^4)*x/c^5/d^5+1/2*e^4*(-3*a*e^2+5*c*
d^2)*x^2/c^4/d^4+1/3*e^5*x^3/c^3/d^3-1/2*(-a*e^2+c*d^2)^5/c^6/d^6/(c*d*x+a
*e)^2-5*e*(-a*e^2+c*d^2)^4/c^6/d^6/(c*d*x+a*e)+10*e^2*(-a*e^2+c*d^2)^3*ln(
c*d*x+a*e)/c^6/d^6
```

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 262, normalized size of antiderivative = 1.42

$$\int \frac{(d+ex)^8}{(ade+(cd^2+ae^2)x+cdex^2)^3}\,dx = \frac{-27a^5e^{10}+3a^4cde^8(35d+2ex)+3a^3c^2d^2e^6(-50d^2+10dex+21e^2x^2)+5a^2c^3d^3e^4(18d^3-24d^2ex-33}{}$$

input
```
Integrate[(d + e*x)^8/(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^3,x]
```

output
```
(-27*a^5*e^10 + 3*a^4*c*d*e^8*(35*d + 2*e*x) + 3*a^3*c^2*d^2*e^6*(-50*d^2
+ 10*d*e*x + 21*e^2*x^2) + 5*a^2*c^3*d^3*e^4*(18*d^3 - 24*d^2*e*x - 33*d*e
^2*x^2 + 4*e^3*x^3) - 5*a*c^4*d^4*e^2*(3*d^4 - 24*d^3*e*x - 24*d^2*e^2*x^2
 + 12*d*e^3*x^3 + e^4*x^4) + c^5*d^5*(-3*d^5 - 30*d^4*e*x + 60*d^2*e^3*x^3
 + 15*d*e^4*x^4 + 2*e^5*x^5) - 60*e^2*(-(c*d^2) + a*e^2)^3*(a*e + c*d*x)^2
*Log[a*e + c*d*x])/(6*c^6*d^6*(a*e + c*d*x)^2)
```

Rubi [A] (verified)

Time = 0.61 (sec) , antiderivative size = 185, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.057$, Rules used = {1121, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex)^8}{(x(ae^2+cd^2)+ade+cdex^2)^3}\,dx$$

$$\Big\downarrow \text{1121}$$

$$\int \left(\frac{6a^2e^7-15acd^2e^5+10c^2d^4e^3}{c^5d^5}+\frac{10e^2(cd^2-ae^2)^3}{c^5d^5(ae+cdx)}+\frac{5e(cd^2-ae^2)^4}{c^5d^5(ae+cdx)^2}+\frac{(cd^2-ae^2)^5}{c^5d^5(ae+cdx)^3}+\frac{e^4x(5cd^2-3ae^2}{c^4d^4}\right.$$

$$\Big\downarrow \text{2009}$$

$$\frac{e^3x\left(6a^2e^4-15acd^2e^2+10c^2d^4\right)}{c^5d^5}-\frac{5e\left(cd^2-ae^2\right)^4}{c^6d^6(ae+cdx)}-\frac{\left(cd^2-ae^2\right)^5}{2c^6d^6(ae+cdx)^2}+\frac{10e^2\left(cd^2-ae^2\right)^3\log(ae+cdx)}{c^6d^6}+\frac{e^4x^2\left(5cd^2-3ae^2\right)}{2c^4d^4}+\frac{e^5x^3}{3c^3d^3}$$

input
```
Int[(d + e*x)^8/(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^3,x]
```

output
```
(e^3*(10*c^2*d^4 - 15*a*c*d^2*e^2 + 6*a^2*e^4)*x)/(c^5*d^5) + (e^4*(5*c*d^
2 - 3*a*e^2)*x^2)/(2*c^4*d^4) + (e^5*x^3)/(3*c^3*d^3) - (c*d^2 - a*e^2)^5/
(2*c^6*d^6*(a*e + c*d*x)^2) - (5*e*(c*d^2 - a*e^2)^4)/(c^6*d^6*(a*e + c*d*
x)) + (10*e^2*(c*d^2 - a*e^2)^3*Log[a*e + c*d*x])/(c^6*d^6)
```

Defintions of rubi rules used

rule 1121
```
Int[((d_) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Int[ExpandIntegrand[(d + e*x)^(m + p)*(a/d + (c/e)*x)^p, x], x]
/; FreeQ[{a, b, c, d, e, m, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && (Int
egerQ[p] || (GtQ[a, 0] && GtQ[d, 0] && LtQ[c, 0]))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 1.66 (sec) , antiderivative size = 297, normalized size of antiderivative = 1.61

method	result
default	$\frac{e^3\left(\frac{1}{3}x^3c^2d^2e^2-\frac{3}{2}x^2acd\,e^3+\frac{5}{2}x^2c^2d^3e+6a^2e^4x-15ac\,d^2e^2x+10c^2d^4x\right)}{d^5c^5}-\frac{10e^2\left(e^6a^3-3d^2e^4a^2c+3d^4e^2a\,c^2-d^6c^3\right)\ln(cdx-}{c^6d^6}$
risch	$\frac{e^5x^3}{3c^3d^3}-\frac{3e^6x^2a}{2d^4c^4}+\frac{5e^4x^2}{2d^2c^3}+\frac{6e^7a^2x}{d^5c^5}-\frac{15e^5ax}{d^3c^4}+\frac{10e^3x}{d\,c^3}+\frac{\left(-5a^4e^9+20a^3d^2e^7c-30a^2e^5d^4c^2+20a\,e^3d^6c^3-5d^8e\,c^4\right.}{d^5c}$
norman	$\frac{-\frac{90a^5e^{10}-230a^4c\,d^2e^8+145a^3c^2d^4e^6+45a^2c^3d^6e^4+15a\,c^4d^8e^2+3c^5d^{10}}{6d^4c^6}+\frac{e^7x^7}{3cd}-\frac{\left(90a^5e^{14}+10a^4c\,d^2e^{12}-415a^3c^2d^4e^{10}+305a^2c^3d^6\right.}{6d^6c^6e^2}}{}$
parallelrisch	$-\frac{90a^5e^{10}+3c^5d^{10}+120a^4cd\,e^9x-360a^3c^2d^3e^7x+360a^2c^3d^5e^5x+30c^5d^9ex-360\ln(cdx+ae)x\,a^3c^2d^3e^7+360\ln(cdx+ae)x}{}$

input
```
int((e*x+d)^8/(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^3,x,method=_RETURNVERBOSE)
```

output
```
e^3/d^5/c^5*(1/3*x^3*c^2*d^2*e^2-3/2*x^2*a*c*d*e^3+5/2*x^2*c^2*d^3*e+6*a^2
*e^4*x-15*a*c*d^2*e^2*x+10*c^2*d^4*x)-10*e^2/c^6/d^6*(a^3*e^6-3*a^2*c*d^2*
e^4+3*a*c^2*d^4*e^2-c^3*d^6)*ln(c*d*x+a*e)-1/2/d^6/c^6*(-a^5*e^10+5*a^4*c*
d^2*e^8-10*a^3*c^2*d^4*e^6+10*a^2*c^3*d^6*e^4-5*a*c^4*d^8*e^2+c^5*d^10)/(c
*d*x+a*e)^2-5/d^6*e/c^6*(a^4*e^8-4*a^3*c*d^2*e^6+6*a^2*c^2*d^4*e^4-4*a*c^3
*d^6*e^2+c^4*d^8)/(c*d*x+a*e)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 469 vs. $2(179) = 358$.

Time = 0.08 (sec) , antiderivative size = 469, normalized size of antiderivative = 2.54

$$\int \frac{(d+ex)^8}{(ade+(cd^2+ae^2)x+cdex^2)^3}\,dx$$
$$= \frac{2c^5d^5e^5x^5 - 3c^5d^{10} - 15ac^4d^8e^2 + 90a^2c^3d^6e^4 - 150a^3c^2d^4e^6 + 105a^4cd^2e^8 - 27a^5e^{10} + 5(3c^5d^6e^4 - \cdots}{}$$

input
```
integrate((e*x+d)^8/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^3,x, algorithm="fric
as")
```

output
```
1/6*(2*c^5*d^5*e^5*x^5 - 3*c^5*d^10 - 15*a*c^4*d^8*e^2 + 90*a^2*c^3*d^6*e^
4 - 150*a^3*c^2*d^4*e^6 + 105*a^4*c*d^2*e^8 - 27*a^5*e^10 + 5*(3*c^5*d^6*e
^4 - a*c^4*d^4*e^6)*x^4 + 20*(3*c^5*d^7*e^3 - 3*a*c^4*d^5*e^5 + a^2*c^3*d^
3*e^7)*x^3 + 3*(40*a*c^4*d^6*e^4 - 55*a^2*c^3*d^4*e^6 + 21*a^3*c^2*d^2*e^8
)*x^2 - 6*(5*c^5*d^9*e - 20*a*c^4*d^7*e^3 + 20*a^2*c^3*d^5*e^5 - 5*a^3*c^2
*d^3*e^7 - a^4*c*d*e^9)*x + 60*(a^2*c^3*d^6*e^4 - 3*a^3*c^2*d^4*e^6 + 3*a^
4*c*d^2*e^8 - a^5*e^10 + (c^5*d^8*e^2 - 3*a*c^4*d^6*e^4 + 3*a^2*c^3*d^4*e^
6 - a^3*c^2*d^2*e^8)*x^2 + 2*(a*c^4*d^7*e^3 - 3*a^2*c^3*d^5*e^5 + 3*a^3*c^
2*d^3*e^7 - a^4*c*d*e^9)*x)*log(c*d*x + a*e))/(c^8*d^8*x^2 + 2*a*c^7*d^7*e
*x + a^2*c^6*d^6*e^2)
```

Sympy [A] (verification not implemented)

Time = 3.05 (sec) , antiderivative size = 303, normalized size of antiderivative = 1.64

$$\int \frac{(d+ex)^8}{(ade+(cd^2+ae^2)x+cdex^2)^3}\,dx = x^2\left(-\frac{3ae^6}{2c^4d^4}+\frac{5e^4}{2c^3d^2}\right)+x\left(\frac{6a^2e^7}{c^5d^5}-\frac{15ae^5}{c^4d^3}+\frac{10e^3}{c^3d}\right) + \frac{-9a^5e^{10}+35a^4cd^2e^8-50a^3c^2d^4e^6+30a^2c^3d^6e^4-5ac^4d^8e^2-c^5d^{10}+x(-10a^4cde^9+40a^3c^2d^3e^7-6}{2a^2c^6d^6e^2+4ac^7d^7ex+2c^8d^8x^2} + \frac{e^5x^3}{3c^3d^3}-\frac{10e^2(ae^2-cd^2)^3\log(ae+cdx)}{c^6d^6}$$

input
```
integrate((e*x+d)**8/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**3,x)
```

output
```
x**2*(-3*a*e**6/(2*c**4*d**4) + 5*e**4/(2*c**3*d**2)) + x*(6*a**2*e**7/(c*
*5*d**5) - 15*a*e**5/(c**4*d**3) + 10*e**3/(c**3*d)) + (-9*a**5*e**10 + 35
*a**4*c*d**2*e**8 - 50*a**3*c**2*d**4*e**6 + 30*a**2*c**3*d**6*e**4 - 5*a*
c**4*d**8*e**2 - c**5*d**10 + x*(-10*a**4*c*d*e**9 + 40*a**3*c**2*d**3*e**
7 - 60*a**2*c**3*d**5*e**5 + 40*a*c**4*d**7*e**3 - 10*c**5*d**9*e))/(2*a**
2*c**6*d**6*e**2 + 4*a*c**7*d**7*e*x + 2*c**8*d**8*x**2) + e**5*x**3/(3*c*
*3*d**3) - 10*e**2*(a*e**2 - c*d**2)**3*log(a*e + c*d*x)/(c**6*d**6)
```

Maxima [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 310, normalized size of antiderivative = 1.68

$$\int \frac{(d+ex)^8}{(ade+(cd^2+ae^2)x+cdex^2)^3}\,dx = -\frac{c^5d^{10}+5\,ac^4d^8e^2-30\,a^2c^3d^6e^4+50\,a^3c^2d^4e^6-35\,a^4cd^2e^8+9\,a^5e^{10}+10\,(c^5d^9e-4\,ac^4d^7e^3+6\,a^2c^3a}{2\,(c^8d^8x^2+2\,ac^7d^7ex+a^2c^6d^6e^2)} + \frac{2\,c^2d^2e^5x^3+3\,(5\,c^2d^3e^4-3\,acde^6)x^2+6\,(10\,c^2d^4e^3-15\,acd^2e^5+6\,a^2e^7)x}{6\,c^5d^5} + \frac{10\,(c^3d^6e^2-3\,ac^2d^4e^4+3\,a^2cd^2e^6-a^3e^8)\log(cdx+ae)}{c^6d^6}$$

input
```
integrate((e*x+d)^8/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^3,x, algorithm="maxi
ma")
```

output
```
-1/2*(c^5*d^10 + 5*a*c^4*d^8*e^2 - 30*a^2*c^3*d^6*e^4 + 50*a^3*c^2*d^4*e^6
 - 35*a^4*c*d^2*e^8 + 9*a^5*e^10 + 10*(c^5*d^9*e - 4*a*c^4*d^7*e^3 + 6*a^2
*c^3*d^5*e^5 - 4*a^3*c^2*d^3*e^7 + a^4*c*d*e^9)*x)/(c^8*d^8*x^2 + 2*a*c^7*
d^7*e*x + a^2*c^6*d^6*e^2) + 1/6*(2*c^2*d^2*e^5*x^3 + 3*(5*c^2*d^3*e^4 - 3
*a*c*d*e^6)*x^2 + 6*(10*c^2*d^4*e^3 - 15*a*c*d^2*e^5 + 6*a^2*e^7)*x)/(c^5*
d^5) + 10*(c^3*d^6*e^2 - 3*a*c^2*d^4*e^4 + 3*a^2*c*d^2*e^6 - a^3*e^8)*log(
c*d*x + a*e)/(c^6*d^6)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 301, normalized size of antiderivative = 1.63

$$\int \frac{(d+ex)^8}{\left(ade+(cd^2+ae^2)\,x+cdex^2\right)^3}\,dx$$
$$= \frac{10\,(c^3d^6e^2-3\,ac^2d^4e^4+3\,a^2cd^2e^6-a^3e^8)\log\left(|cdx+ae|\right)}{c^6d^6}$$
$$-\frac{c^5d^{10}+5\,ac^4d^8e^2-30\,a^2c^3d^6e^4+50\,a^3c^2d^4e^6-35\,a^4cd^2e^8+9\,a^5e^{10}+10\,(c^5d^9e-4\,ac^4d^7e^3+6\,a^2c^3d}{2\,(cdx+ae)^2c^6d^6}$$
$$+\frac{2\,c^6d^6e^5x^3+15\,c^6d^7e^4x^2-9\,ac^5d^5e^6x^2+60\,c^6d^8e^3x-90\,ac^5d^6e^5x+36\,a^2c^4d^4e^7x}{6\,c^9d^9}$$

input
```
integrate((e*x+d)^8/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^3,x, algorithm="giac
")
```

output
```
10*(c^3*d^6*e^2 - 3*a*c^2*d^4*e^4 + 3*a^2*c*d^2*e^6 - a^3*e^8)*log(abs(c*d
*x + a*e))/(c^6*d^6) - 1/2*(c^5*d^10 + 5*a*c^4*d^8*e^2 - 30*a^2*c^3*d^6*e^
4 + 50*a^3*c^2*d^4*e^6 - 35*a^4*c*d^2*e^8 + 9*a^5*e^10 + 10*(c^5*d^9*e - 4
*a*c^4*d^7*e^3 + 6*a^2*c^3*d^5*e^5 - 4*a^3*c^2*d^3*e^7 + a^4*c*d*e^9)*x)/(
(c*d*x + a*e)^2*c^6*d^6) + 1/6*(2*c^6*d^6*e^5*x^3 + 15*c^6*d^7*e^4*x^2 - 9
*a*c^5*d^5*e^6*x^2 + 60*c^6*d^8*e^3*x - 90*a*c^5*d^6*e^5*x + 36*a^2*c^4*d^
4*e^7*x)/(c^9*d^9)
```

Mupad [B] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 341, normalized size of antiderivative = 1.84

$$\int \frac{(d+ex)^8}{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^3}\,dx$$
$$= x^2\left(\frac{5\,e^4}{2\,c^3\,d^2}-\frac{3\,a\,e^6}{2\,c^4\,d^4}\right)-x\left(\frac{3\,a^2\,e^7}{c^5\,d^5}-\frac{10\,e^3}{c^3\,d}+\frac{3\,a\,e\left(\frac{5\,e^4}{c^3\,d^2}-\frac{3\,a\,e^6}{c^4\,d^4}\right)}{c\,d}\right)$$
$$-\frac{x\left(5\,a^4\,e^9-20\,a^3\,c\,d^2\,e^7+30\,a^2\,c^2\,d^4\,e^5-20\,a\,c^3\,d^6\,e^3+5\,c^4\,d^8\,e\right)+\frac{9\,a^5\,e^{10}-35\,a^4\,c\,d^2\,e^8+50\,a^3\,c^2\,d^4\,e^6-30\,a^2\,c^3}{2\,c\,d}}{a^2\,c^5\,d^5\,e^2+2\,a\,c^6\,d^6\,e\,x+c^7\,d^7\,x^2}$$
$$-\frac{\ln\left(a\,e+c\,d\,x\right)\left(10\,a^3\,e^8-30\,a^2\,c\,d^2\,e^6+30\,a\,c^2\,d^4\,e^4-10\,c^3\,d^6\,e^2\right)}{c^6\,d^6}+\frac{e^5\,x^3}{3\,c^3\,d^3}$$

input
```
int((d + e*x)^8/(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^3,x)
```

output
```
x^2*((5*e^4)/(2*c^3*d^2) - (3*a*e^6)/(2*c^4*d^4)) - x*((3*a^2*e^7)/(c^5*d^
5) - (10*e^3)/(c^3*d) + (3*a*e*((5*e^4)/(c^3*d^2) - (3*a*e^6)/(c^4*d^4)))/
(c*d)) - (x*(5*a^4*e^9 + 5*c^4*d^8*e - 20*a*c^3*d^6*e^3 - 20*a^3*c*d^2*e^7
 + 30*a^2*c^2*d^4*e^5) + (9*a^5*e^10 + c^5*d^10 + 5*a*c^4*d^8*e^2 - 35*a^4
*c*d^2*e^8 - 30*a^2*c^3*d^6*e^4 + 50*a^3*c^2*d^4*e^6)/(2*c*d))/(c^7*d^7*x^
2 + a^2*c^5*d^5*e^2 + 2*a*c^6*d^6*e*x) - (log(a*e + c*d*x)*(10*a^3*e^8 - 1
0*c^3*d^6*e^2 + 30*a*c^2*d^4*e^4 - 30*a^2*c*d^2*e^6))/(c^6*d^6) + (e^5*x^3
)/(3*c^3*d^3)
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 547, normalized size of antiderivative = 2.96

$$\int \frac{(d+ex)^8}{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^3}\,dx$$
$$= \frac{-120\log(cdx+ae)\,a^5cd\,e^9x+90a^5c\,d^2e^8-90a^4c^2d^4e^6+30a^3c^3d^6e^4+360\log(cdx+ae)\,a^4c^2d^3e^7x-6}{}$$

input
```
int((e*x+d)^8/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^3,x)
```

output
```
( - 60*log(a*e + c*d*x)*a**6*e**10 + 180*log(a*e + c*d*x)*a**5*c*d**2*e**8
 - 120*log(a*e + c*d*x)*a**5*c*d*e**9*x - 180*log(a*e + c*d*x)*a**4*c**2*d
**4*e**6 + 360*log(a*e + c*d*x)*a**4*c**2*d**3*e**7*x - 60*log(a*e + c*d*x
)*a**4*c**2*d**2*e**8*x**2 + 60*log(a*e + c*d*x)*a**3*c**3*d**6*e**4 - 360
*log(a*e + c*d*x)*a**3*c**3*d**5*e**5*x + 180*log(a*e + c*d*x)*a**3*c**3*d
**4*e**6*x**2 + 120*log(a*e + c*d*x)*a**2*c**4*d**7*e**3*x - 180*log(a*e +
 c*d*x)*a**2*c**4*d**6*e**4*x**2 + 60*log(a*e + c*d*x)*a*c**5*d**8*e**2*x*
*2 - 30*a**6*e**10 + 90*a**5*c*d**2*e**8 - 90*a**4*c**2*d**4*e**6 + 60*a**
4*c**2*d**2*e**8*x**2 + 30*a**3*c**3*d**6*e**4 - 180*a**3*c**3*d**4*e**6*x
**2 + 20*a**3*c**3*d**3*e**7*x**3 + 180*a**2*c**4*d**6*e**4*x**2 - 60*a**2
*c**4*d**5*e**5*x**3 - 5*a**2*c**4*d**4*e**6*x**4 - 3*a*c**5*d**10 - 60*a*
c**5*d**8*e**2*x**2 + 60*a*c**5*d**7*e**3*x**3 + 15*a*c**5*d**6*e**4*x**4
+ 2*a*c**5*d**5*e**5*x**5 + 15*c**6*d**10*x**2)/(6*a*c**6*d**6*(a**2*e**2
+ 2*a*c*d*e*x + c**2*d**2*x**2))
```

3.129 $\int \frac{(d+ex)^7}{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^3}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 142

$$\int \frac{(d+ex)^7}{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^3}\,dx = \frac{e^3(4cd^2-3ae^2)\,x}{c^4d^4}+\frac{e^4x^2}{2c^3d^3} - \frac{\left(cd^2-ae^2\right)^4}{2c^5d^5(ae+cdx)^2}-\frac{4e\left(cd^2-ae^2\right)^3}{c^5d^5(ae+cdx)} + \frac{6e^2\left(cd^2-ae^2\right)^2\log(ae+cdx)}{c^5d^5}$$

output
```
e^3*(-3*a*e^2+4*c*d^2)*x/c^4/d^4+1/2*e^4*x^2/c^3/d^3-1/2*(-a*e^2+c*d^2)^4/
c^5/d^5/(c*d*x+a*e)^2-4*e*(-a*e^2+c*d^2)^3/c^5/d^5/(c*d*x+a*e)+6*e^2*(-a*e
^2+c*d^2)^2*ln(c*d*x+a*e)/c^5/d^5
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 191, normalized size of antiderivative = 1.35

$$\int \frac{(d+ex)^7}{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^3}\,dx = \frac{7a^4e^8+2a^3cde^6(-10d+ex)+a^2c^2d^2e^4(18d^2-16dex-11e^2x^2)-4ac^3d^3e^2(d^3-6d^2ex-4de^2x^2+e^3x}{2c^5d^5(ae+cdx)^2}$$

input
```
Integrate[(d + e*x)^7/(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^3,x]
```

output
```
(7*a^4*e^8 + 2*a^3*c*d*e^6*(-10*d + e*x) + a^2*c^2*d^2*e^4*(18*d^2 - 16*d*
e*x - 11*e^2*x^2) - 4*a*c^3*d^3*e^2*(d^3 - 6*d^2*e*x - 4*d*e^2*x^2 + e^3*x
^3) + c^4*d^4*(-d^4 - 8*d^3*e*x + 8*d*e^3*x^3 + e^4*x^4) + 12*e^2*(c*d^2 -
 a*e^2)^2*(a*e + c*d*x)^2*Log[a*e + c*d*x])/(2*c^5*d^5*(a*e + c*d*x)^2)
```

Rubi [A] (verified)

Time = 0.57 (sec) , antiderivative size = 142, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.057$, Rules used = $\{1121, 2009\}$

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex)^7}{\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^3}dx$$

$$\Big\downarrow \text{1121}$$

$$\int \left(\frac{6\left(cd^2e-ae^3\right)^2}{c^4d^4(ae+cdx)}+\frac{4e\left(cd^2-ae^2\right)^3}{c^4d^4(ae+cdx)^2}+\frac{\left(cd^2-ae^2\right)^4}{c^4d^4(ae+cdx)^3}+\frac{4cd^2e^3-3ae^5}{c^4d^4}+\frac{e^4x}{c^3d^3}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{4e\left(cd^2-ae^2\right)^3}{c^5d^5(ae+cdx)}-\frac{\left(cd^2-ae^2\right)^4}{2c^5d^5(ae+cdx)^2}+\frac{6e^2\left(cd^2-ae^2\right)^2\log(ae+cdx)}{c^5d^5}+\frac{e^3x\left(4cd^2-3ae^2\right)}{c^4d^4}+\frac{e^4x^2}{2c^3d^3}$$

input
```
Int[(d + e*x)^7/(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^3,x]
```

output
```
(e^3*(4*c*d^2 - 3*a*e^2)*x)/(c^4*d^4) + (e^4*x^2)/(2*c^3*d^3) - (c*d^2 - a
*e^2)^4/(2*c^5*d^5*(a*e + c*d*x)^2) - (4*e*(c*d^2 - a*e^2)^3)/(c^5*d^5*(a*
e + c*d*x)) + (6*e^2*(c*d^2 - a*e^2)^2*Log[a*e + c*d*x])/(c^5*d^5)
```

Defintions of rubi rules used

rule 1121
```
Int[((d_) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Int[ExpandIntegrand[(d + e*x)^(m + p)*(a/d + (c/e)*x)^p, x], x]
/; FreeQ[{a, b, c, d, e, m, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && (Int
egerQ[p] || (GtQ[a, 0] && GtQ[d, 0] && LtQ[c, 0]))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 1.59 (sec) , antiderivative size = 211, normalized size of antiderivative = 1.49

method	result
default	$-\frac{e^{3}\left(-\frac{1}{2}cd\,x^{2}e+3a\,e^{2}x-4c\,d^{2}x\right)}{d^{4}c^{4}}+\frac{6e^{2}\left(a^{2}e^{4}-2ac\,d^{2}e^{2}+c^{2}d^{4}\right)\ln\left(cdx+ae\right)}{c^{5}d^{5}}-\frac{a^{4}e^{8}-4a^{3}c\,d^{2}e^{6}+6a^{2}c^{2}d^{4}e^{4}-4a\,c^{3}d^{6}e^{2}+c}{2d^{5}c^{5}\left(cdx+ae\right)^{2}}$
risch	$\frac{e^{4}x^{2}}{2c^{3}d^{3}}-\frac{3e^{5}ax}{d^{4}c^{4}}+\frac{4e^{3}x}{d^{2}c^{3}}+\frac{\left(4a^{3}e^{7}-12a^{2}e^{5}c\,d^{2}+12a\,e^{3}c^{2}d^{4}-4d^{6}c^{3}e\right)x+\frac{7a^{4}e^{8}-20a^{3}c\,d^{2}e^{6}+18a^{2}c^{2}d^{4}e^{4}-4a\,c^{3}d^{6}e^{2}-c^{4}d^{8}}{2cd}}{d^{4}c^{4}\left(cdx+ae\right)^{2}}$
parallelrisch	$\frac{c^{4}d^{4}e^{4}x^{4}+12\ln\left(cdx+ae\right)x^{2}a^{2}c^{2}d^{2}e^{6}-24\ln\left(cdx+ae\right)x^{2}a\,c^{3}d^{4}e^{4}+12\ln\left(cdx+ae\right)x^{2}c^{4}d^{6}e^{2}-4a\,c^{3}d^{3}e^{5}x^{3}+8c^{4}d^{5}e^{3}x^{3}+24\ln(}{}$
norman	$\frac{\frac{\left(18a^{4}e^{10}-16a^{3}c\,d^{2}e^{8}-15a^{2}d^{4}e^{6}c^{2}-9a\,d^{6}e^{4}c^{3}-5c^{4}d^{8}e^{2}\right)x}{d^{4}c^{5}e}+\frac{\left(12a^{3}e^{10}-16a^{2}c\,d^{2}e^{8}+d^{4}a\,c^{2}e^{6}-17c^{3}d^{6}e^{4}\right)x^{3}}{d^{4}c^{4}e}+\frac{18a^{4}e^{8}-28a^{3}c\,d^{2}e^{6}+a}{2}}{\left(ex+d\right)^{2}\left(cdx+\right.}$

input
```
int((e*x+d)^7/(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^3,x,method=_RETURNVERBOSE)
```

output
```
-e^3/d^4/c^4*(-1/2*c*d*x^2*e+3*a*e^2*x-4*c*d^2*x)+6*e^2/c^5/d^5*(a^2*e^4-2
*a*c*d^2*e^2+c^2*d^4)*ln(c*d*x+a*e)-1/2*(a^4*e^8-4*a^3*c*d^2*e^6+6*a^2*c^2
*d^4*e^4-4*a*c^3*d^6*e^2+c^4*d^8)/d^5/c^5/(c*d*x+a*e)^2+4/d^5*e/c^5*(a^3*e
^6-3*a^2*c*d^2*e^4+3*a*c^2*d^4*e^2-c^3*d^6)/(c*d*x+a*e)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 338 vs. $2(138) = 276$.

Time = 0.08 (sec) , antiderivative size = 338, normalized size of antiderivative = 2.38

$$\int \frac{(d+ex)^7}{(ade+(cd^2+ae^2)x+cdex^2)^3}\,dx = \frac{c^4d^4e^4x^4 - c^4d^8 - 4\,ac^3d^6e^2 + 18\,a^2c^2d^4e^4 - 20\,a^3cd^2e^6 + 7\,a^4e^8 + 4\,(2\,c^4d^5e^3 - ac^3d^3e^5)x^3 + (16\,ac^3d^4e}{}$$

input
```
integrate((e*x+d)^7/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^3,x, algorithm="fric
as")
```

output
```
1/2*(c^4*d^4*e^4*x^4 - c^4*d^8 - 4*a*c^3*d^6*e^2 + 18*a^2*c^2*d^4*e^4 - 20
*a^3*c*d^2*e^6 + 7*a^4*e^8 + 4*(2*c^4*d^5*e^3 - a*c^3*d^3*e^5)*x^3 + (16*a
*c^3*d^4*e^4 - 11*a^2*c^2*d^2*e^6)*x^2 - 2*(4*c^4*d^7*e - 12*a*c^3*d^5*e^3
 + 8*a^2*c^2*d^3*e^5 - a^3*c*d*e^7)*x + 12*(a^2*c^2*d^4*e^4 - 2*a^3*c*d^2*
e^6 + a^4*e^8 + (c^4*d^6*e^2 - 2*a*c^3*d^4*e^4 + a^2*c^2*d^2*e^6)*x^2 + 2*
(a*c^3*d^5*e^3 - 2*a^2*c^2*d^3*e^5 + a^3*c*d*e^7)*x)*log(c*d*x + a*e))/(c^
7*d^7*x^2 + 2*a*c^6*d^6*e*x + a^2*c^5*d^5*e^2)
```

Sympy [A] (verification not implemented)

Time = 1.21 (sec) , antiderivative size = 226, normalized size of antiderivative = 1.59

$$\int \frac{(d+ex)^7}{(ade+(cd^2+ae^2)x+cdex^2)^3}\,dx = x\left(-\frac{3ae^5}{c^4d^4} + \frac{4e^3}{c^3d^2}\right) + \frac{7a^4e^8 - 20a^3cd^2e^6 + 18a^2c^2d^4e^4 - 4ac^3d^6e^2 - c^4d^8 + x(8a^3cde^7 - 24a^2c^2d^3e^5 + 24ac^3d^5e^3 - 8c^4d^7e)}{2a^2c^5d^5e^2 + 4ac^6d^6ex + 2c^7d^7x^2} + \frac{e^4x^2}{2c^3d^3} + \frac{6e^2(ae^2-cd^2)^2\log(ae+cdx)}{c^5d^5}$$

input
```
integrate((e*x+d)**7/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**3,x)
```

output
```
x*(-3*a*e**5/(c**4*d**4) + 4*e**3/(c**3*d**2)) + (7*a**4*e**8 - 20*a**3*c*
d**2*e**6 + 18*a**2*c**2*d**4*e**4 - 4*a*c**3*d**6*e**2 - c**4*d**8 + x*(8
*a**3*c*d*e**7 - 24*a**2*c**2*d**3*e**5 + 24*a*c**3*d**5*e**3 - 8*c**4*d**
7*e))/(2*a**2*c**5*d**5*e**2 + 4*a*c**6*d**6*e*x + 2*c**7*d**7*x**2) + e**
4*x**2/(2*c**3*d**3) + 6*e**2*(a*e**2 - c*d**2)**2*log(a*e + c*d*x)/(c**5*
d**5)
```

Maxima [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 224, normalized size of antiderivative = 1.58

$$\int \frac{(d+ex)^7}{(ade+(cd^2+ae^2)x+cdex^2)^3}\,dx = -\frac{c^4d^8+4\,ac^3d^6e^2-18\,a^2c^2d^4e^4+20\,a^3cd^2e^6-7\,a^4e^8+8\,(c^4d^7e-3\,ac^3d^5e^3+3\,a^2c^2d^3e^5-a^3cde^7)x}{2\,(c^7d^7x^2+2\,ac^6d^6ex+a^2c^5d^5e^2)} + \frac{cde^4x^2+2\,(4\,cd^2e^3-3\,ae^5)x}{2\,c^4d^4} + \frac{6\,(c^2d^4e^2-2\,acd^2e^4+a^2e^6)\log{(cdx+ae)}}{c^5d^5}$$

input
```
integrate((e*x+d)^7/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^3,x, algorithm="maxi
ma")
```

output
```
-1/2*(c^4*d^8 + 4*a*c^3*d^6*e^2 - 18*a^2*c^2*d^4*e^4 + 20*a^3*c*d^2*e^6 -
7*a^4*e^8 + 8*(c^4*d^7*e - 3*a*c^3*d^5*e^3 + 3*a^2*c^2*d^3*e^5 - a^3*c*d*e
^7)*x)/(c^7*d^7*x^2 + 2*a*c^6*d^6*e*x + a^2*c^5*d^5*e^2) + 1/2*(c*d*e^4*x^
2 + 2*(4*c*d^2*e^3 - 3*a*e^5)*x)/(c^4*d^4) + 6*(c^2*d^4*e^2 - 2*a*c*d^2*e^
4 + a^2*e^6)*log(c*d*x + a*e)/(c^5*d^5)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 214, normalized size of antiderivative = 1.51

$$\int \frac{(d+ex)^7}{(ade+(cd^2+ae^2)x+cdex^2)^3}\,dx = \frac{6\,(c^2d^4e^2-2\,acd^2e^4+a^2e^6)\log{(|cdx+ae|)}}{c^5d^5} + \frac{c^3d^3e^4x^2+8\,c^3d^4e^3x-6\,ac^2d^2e^5x}{2\,c^6d^6} - \frac{c^4d^8+4\,ac^3d^6e^2-18\,a^2c^2d^4e^4+20\,a^3cd^2e^6-7\,a^4e^8+8\,(c^4d^7e-3\,ac^3d^5e^3+3\,a^2c^2d^3e^5-a^3cde^7)x}{2\,(cdx+ae)^2c^5d^5}$$

input
```
integrate((e*x+d)^7/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^3,x, algorithm="giac
")
```

output
```
6*(c^2*d^4*e^2 - 2*a*c*d^2*e^4 + a^2*e^6)*log(abs(c*d*x + a*e))/(c^5*d^5)
+ 1/2*(c^3*d^3*e^4*x^2 + 8*c^3*d^4*e^3*x - 6*a*c^2*d^2*e^5*x)/(c^6*d^6) -
1/2*(c^4*d^8 + 4*a*c^3*d^6*e^2 - 18*a^2*c^2*d^4*e^4 + 20*a^3*c*d^2*e^6 - 7
*a^4*e^8 + 8*(c^4*d^7*e - 3*a*c^3*d^5*e^3 + 3*a^2*c^2*d^3*e^5 - a^3*c*d*e^
7)*x)/((c*d*x + a*e)^2*c^5*d^5)
```

Mupad [B] (verification not implemented)

Time = 5.27 (sec) , antiderivative size = 232, normalized size of antiderivative = 1.63

$$\int \frac{(d+ex)^7}{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^3}\,dx = \frac{x\,\left(4\,a^3\,e^7-12\,a^2\,c\,d^2\,e^5+12\,a\,c^2\,d^4\,e^3-4\,c^3\,d^6\,e\right)-\frac{-7\,a^4\,e^8+20\,a^3\,c\,d^2\,e^6-18\,a^2\,c^2\,d^4\,e^4+4\,a\,c^3\,d^6\,e^2+c^4\,d^8}{2\,c\,d}}{a^2\,c^4\,d^4\,e^2+2\,a\,c^5\,d^5\,e\,x+c^6\,d^6\,x^2} + x\,\left(\frac{4\,e^3}{c^3\,d^2}-\frac{3\,a\,e^5}{c^4\,d^4}\right)+\frac{\ln\left(a\,e+c\,d\,x\right)\,\left(6\,a^2\,e^6-12\,a\,c\,d^2\,e^4+6\,c^2\,d^4\,e^2\right)}{c^5\,d^5}+\frac{e^4\,x^2}{2\,c^3\,d^3}$$

input
```
int((d + e*x)^7/(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^3,x)
```

output
```
(x*(4*a^3*e^7 - 4*c^3*d^6*e + 12*a*c^2*d^4*e^3 - 12*a^2*c*d^2*e^5) - (c^4*
d^8 - 7*a^4*e^8 + 4*a*c^3*d^6*e^2 + 20*a^3*c*d^2*e^6 - 18*a^2*c^2*d^4*e^4)
/(2*c*d))/(c^6*d^6*x^2 + a^2*c^4*d^4*e^2 + 2*a*c^5*d^5*e*x) + x*((4*e^3)/(
c^3*d^2) - (3*a*e^5)/(c^4*d^4)) + (log(a*e + c*d*x)*(6*a^2*e^6 + 6*c^2*d^4
*e^2 - 12*a*c*d^2*e^4))/(c^5*d^5) + (e^4*x^2)/(2*c^3*d^3)
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 393, normalized size of antiderivative = 2.77

$$\int \frac{(d+ex)^7}{(ade+(cd^2+ae^2)\,x+cdex^2)^3}\,dx$$
$$= \frac{12\log(cdx+ae)\,a^5e^8-24\log(cdx+ae)\,a^4c\,d^2e^6+24\log(cdx+ae)\,a^4cd\,e^7x+12\log(cdx+ae)\,a^3c^2d^4e}{}$$

input
```
int((e*x+d)^7/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^3,x)
```

output
```
(12*log(a*e + c*d*x)*a**5*e**8 - 24*log(a*e + c*d*x)*a**4*c*d**2*e**6 + 24
*log(a*e + c*d*x)*a**4*c*d*e**7*x + 12*log(a*e + c*d*x)*a**3*c**2*d**4*e**
4 - 48*log(a*e + c*d*x)*a**3*c**2*d**3*e**5*x + 12*log(a*e + c*d*x)*a**3*c
**2*d**2*e**6*x**2 + 24*log(a*e + c*d*x)*a**2*c**3*d**5*e**3*x - 24*log(a*
e + c*d*x)*a**2*c**3*d**4*e**4*x**2 + 12*log(a*e + c*d*x)*a*c**4*d**6*e**2
*x**2 + 6*a**5*e**8 - 12*a**4*c*d**2*e**6 + 6*a**3*c**2*d**4*e**4 - 12*a**
3*c**2*d**2*e**6*x**2 + 24*a**2*c**3*d**4*e**4*x**2 - 4*a**2*c**3*d**3*e**
5*x**3 - a*c**4*d**8 - 12*a*c**4*d**6*e**2*x**2 + 8*a*c**4*d**5*e**3*x**3
+ a*c**4*d**4*e**4*x**4 + 4*c**5*d**8*x**2)/(2*a*c**5*d**5*(a**2*e**2 + 2*
a*c*d*e*x + c**2*d**2*x**2))
```

3.130 $\int \frac{(d+ex)^6}{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^3}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 111

$$\int \frac{(d+ex)^6}{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^3}\,dx = \frac{e^3x}{c^3d^3}-\frac{\left(cd^2-ae^2\right)^3}{2c^4d^4(ae+cdx)^2}-\frac{3e\left(cd^2-ae^2\right)^2}{c^4d^4(ae+cdx)} + \frac{3e^2\left(cd^2-ae^2\right)\log(ae+cdx)}{c^4d^4}$$

output
```
e^3*x/c^3/d^3-1/2*(-a*e^2+c*d^2)^3/c^4/d^4/(c*d*x+a*e)^2-3*e*(-a*e^2+c*d^2
)^2/c^4/d^4/(c*d*x+a*e)+3*e^2*(-a*e^2+c*d^2)*ln(c*d*x+a*e)/c^4/d^4
```

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 139, normalized size of antiderivative = 1.25

$$\int \frac{(d+ex)^6}{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^3}\,dx = \frac{-5a^3e^6+a^2cde^4(9d-4ex)+ac^2d^2e^2\left(-3d^2+12dex+4e^2x^2\right)-c^3\left(d^6+6d^5ex-2d^3e^3x^3\right)-6e^2\left(-cd^2\right.}{2c^4d^4(ae+cdx)^2}$$

input
```
Integrate[(d + e*x)^6/(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^3,x]
```

output
```
(-5*a^3*e^6 + a^2*c*d*e^4*(9*d - 4*e*x) + a*c^2*d^2*e^2*(-3*d^2 + 12*d*e*x
 + 4*e^2*x^2) - c^3*(d^6 + 6*d^5*e*x - 2*d^3*e^3*x^3) - 6*e^2*(-(c*d^2) +
a*e^2)*(a*e + c*d*x)^2*Log[a*e + c*d*x])/(2*c^4*d^4*(a*e + c*d*x)^2)
```

Rubi [A] (verified)

Time = 0.48 (sec) , antiderivative size = 111, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.057$, Rules used = {1121, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex)^6}{(x(ae^2+cd^2)+ade+cdex^2)^3}\,dx$$

$$\Big\downarrow \text{ 1121}$$

$$\int \left(\frac{3e(cd^2-ae^2)^2}{c^3d^3(ae+cdx)^2}+\frac{(cd^2-ae^2)^3}{c^3d^3(ae+cdx)^3}+\frac{3(cd^2e^2-ae^4)}{c^3d^3(ae+cdx)}+\frac{e^3}{c^3d^3}\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$-\frac{3e(cd^2-ae^2)^2}{c^4d^4(ae+cdx)}-\frac{(cd^2-ae^2)^3}{2c^4d^4(ae+cdx)^2}+\frac{3e^2(cd^2-ae^2)\log(ae+cdx)}{c^4d^4}+\frac{e^3x}{c^3d^3}$$

input
```
Int[(d + e*x)^6/(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^3,x]
```

output
```
(e^3*x)/(c^3*d^3) - (c*d^2 - a*e^2)^3/(2*c^4*d^4*(a*e + c*d*x)^2) - (3*e*(
c*d^2 - a*e^2)^2)/(c^4*d^4*(a*e + c*d*x)) + (3*e^2*(c*d^2 - a*e^2)*Log[a*e
 + c*d*x])/(c^4*d^4)
```

Defintions of rubi rules used

rule 1121
```
Int[((d_) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Int[ExpandIntegrand[(d + e*x)^(m + p)*(a/d + (c/e)*x)^p, x], x]
/; FreeQ[{a, b, c, d, e, m, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && (Int
egerQ[p] || (GtQ[a, 0] && GtQ[d, 0] && LtQ[c, 0]))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 1.73 (sec) , antiderivative size = 147, normalized size of antiderivative = 1.32

method	result
default	$\frac{e^3 x}{c^3 d^3}-\frac{3e^2(a e^2-c d^2)\ln(cdx+ae)}{c^4 d^4}-\frac{-e^6 a^3+3d^2 e^4 a^2 c-3d^4 e^2 a c^2+d^6 c^3}{2c^4 d^4 (cdx+ae)^2}-\frac{3e(a^2 e^4-2ac d^2 e^2+c^2 d^4)}{d^4 c^4 (cdx+ae)}$
risch	$\frac{e^3 x}{c^3 d^3}+\frac{(-3a^2 e^5+6ac d^2 e^3-3c^2 d^4 e)x-\frac{5e^6 a^3-9d^2 e^4 a^2 c+3d^4 e^2 a c^2+d^6 c^3}{2cd}}{d^3 c^3 (cdx+ae)^2}-\frac{3e^4 \ln(cdx+ae)a}{c^4 d^4}+\frac{3e^2 \ln(cdx+ae)}{c^3 d^2}$
parallelrisch	$-\frac{6\ln(cdx+ae)x^2 a c^2 d^2 e^4-6\ln(cdx+ae)x^2 c^3 d^4 e^2-2c^3 d^3 e^3 x^3+12\ln(cdx+ae)x a^2 cd e^5-12\ln(cdx+ae)xa c^2 d^3 e^3+6\ln(cdx \cdots}{2d^4 c^4 (cdx+ae) \cdots}$
norman	$\frac{\frac{e^5 x^5}{cd}-\frac{9e^6 a^3-5d^2 e^4 a^2 c+3d^4 e^2 a c^2+d^6 c^3}{2c^4 d^2}-\frac{(9a^3 e^{10}+19a^2 c d^2 e^8-5d^4 a c^2 e^6+17c^3 d^6 e^4)x^2}{2d^4 c^4 e^2}-\frac{(9a^3 e^8+a^2 c d^2 e^6+a c^2 d^4 e^4+4c^3 d^6 e^2) \cdots}{d^3 c^4 e} \cdots}{(ex+d)^2 (cdx+ae)^2}$

input
```
int((e*x+d)^6/(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^3,x,method=_RETURNVERBOSE)
```

output
```
e^3*x/c^3/d^3-3*e^2/c^4/d^4*(a*e^2-c*d^2)*ln(c*d*x+a*e)-1/2/c^4/d^4*(-a^3*
e^6+3*a^2*c*d^2*e^4-3*a*c^2*d^4*e^2+c^3*d^6)/(c*d*x+a*e)^2-3/d^4*e/c^4*(a^
2*e^4-2*a*c*d^2*e^2+c^2*d^4)/(c*d*x+a*e)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 227 vs. $2(109) = 218$.

Time = 0.09 (sec) , antiderivative size = 227, normalized size of antiderivative = 2.05

$$\int \frac{(d+ex)^6}{\left(ade+(cd^2+ae^2)\,x+cdex^2\right)^3}\,dx = \frac{2\,c^3d^3e^3x^3+4\,ac^2d^2e^4x^2-c^3d^6-3\,ac^2d^4e^2+9\,a^2cd^2e^4-5\,a^3e^6-2\,(3\,c^3d^5e-6\,ac^2d^3e^3+2\,a^2cde^5)x+}{2\,(c^6d^6x^2+2\,ac^5d^5ex+a^2c}$$

input
```
integrate((e*x+d)^6/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^3,x, algorithm="fric
as")
```

output
```
1/2*(2*c^3*d^3*e^3*x^3 + 4*a*c^2*d^2*e^4*x^2 - c^3*d^6 - 3*a*c^2*d^4*e^2 +
 9*a^2*c*d^2*e^4 - 5*a^3*e^6 - 2*(3*c^3*d^5*e - 6*a*c^2*d^3*e^3 + 2*a^2*c*
d*e^5)*x + 6*(a^2*c*d^2*e^4 - a^3*e^6 + (c^3*d^4*e^2 - a*c^2*d^2*e^4)*x^2
+ 2*(a*c^2*d^3*e^3 - a^2*c*d*e^5)*x)*log(c*d*x + a*e))/(c^6*d^6*x^2 + 2*a*
c^5*d^5*e*x + a^2*c^4*d^4*e^2)
```

Sympy [A] (verification not implemented)

Time = 0.51 (sec) , antiderivative size = 163, normalized size of antiderivative = 1.47

$$\int \frac{(d+ex)^6}{\left(ade+(cd^2+ae^2)\,x+cdex^2\right)^3}\,dx = \frac{-5a^3e^6+9a^2cd^2e^4-3ac^2d^4e^2-c^3d^6+x(-6a^2cde^5+12ac^2d^3e^3-6c^3d^5e)}{2a^2c^4d^4e^2+4ac^5d^5ex+2c^6d^6x^2} + \frac{e^3x}{c^3d^3} - \frac{3e^2(ae^2-cd^2)\log{(ae+cdx)}}{c^4d^4}$$

input
```
integrate((e*x+d)**6/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**3,x)
```

output
```
(-5*a**3*e**6 + 9*a**2*c*d**2*e**4 - 3*a*c**2*d**4*e**2 - c**3*d**6 + x*(-
6*a**2*c*d*e**5 + 12*a*c**2*d**3*e**3 - 6*c**3*d**5*e))/(2*a**2*c**4*d**4*
e**2 + 4*a*c**5*d**5*e*x + 2*c**6*d**6*x**2) + e**3*x/(c**3*d**3) - 3*e**2
*(a*e**2 - c*d**2)*log(a*e + c*d*x)/(c**4*d**4)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 156, normalized size of antiderivative = 1.41

$$\int \frac{(d+ex)^6}{(ade+(cd^2+ae^2)x+cdex^2)^3}\,dx = -\frac{c^3d^6+3\,ac^2d^4e^2-9\,a^2cd^2e^4+5\,a^3e^6+6\,(c^3d^5e-2\,ac^2d^3e^3+a^2cde^5)x}{2\,(c^6d^6x^2+2\,ac^5d^5ex+a^2c^4d^4e^2)} + \frac{e^3x}{c^3d^3} + \frac{3\,(cd^2e^2-ae^4)\log(cdx+ae)}{c^4d^4}$$

input
```
integrate((e*x+d)^6/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^3,x, algorithm="maxi
ma")
```

output
```
-1/2*(c^3*d^6 + 3*a*c^2*d^4*e^2 - 9*a^2*c*d^2*e^4 + 5*a^3*e^6 + 6*(c^3*d^5
*e - 2*a*c^2*d^3*e^3 + a^2*c*d*e^5)*x)/(c^6*d^6*x^2 + 2*a*c^5*d^5*e*x + a^
2*c^4*d^4*e^2) + e^3*x/(c^3*d^3) + 3*(c*d^2*e^2 - a*e^4)*log(c*d*x + a*e)/
(c^4*d^4)
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 136, normalized size of antiderivative = 1.23

$$\int \frac{(d+ex)^6}{(ade+(cd^2+ae^2)x+cdex^2)^3}\,dx = \frac{e^3x}{c^3d^3} + \frac{3\,(cd^2e^2-ae^4)\log(|cdx+ae|)}{c^4d^4} - \frac{c^3d^6+3\,ac^2d^4e^2-9\,a^2cd^2e^4+5\,a^3e^6+6\,(c^3d^5e-2\,ac^2d^3e^3+a^2cde^5)x}{2\,(cdx+ae)^2c^4d^4}$$

input
```
integrate((e*x+d)^6/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^3,x, algorithm="giac
")
```

output
```
e^3*x/(c^3*d^3) + 3*(c*d^2*e^2 - a*e^4)*log(abs(c*d*x + a*e))/(c^4*d^4) -
1/2*(c^3*d^6 + 3*a*c^2*d^4*e^2 - 9*a^2*c*d^2*e^4 + 5*a^3*e^6 + 6*(c^3*d^5*
e - 2*a*c^2*d^3*e^3 + a^2*c*d*e^5)*x)/((c*d*x + a*e)^2*c^4*d^4)
```

Mupad [B] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 163, normalized size of antiderivative = 1.47

$$\int \frac{(d+ex)^6}{(ade+(cd^2+ae^2)x+cdex^2)^3}\,dx$$
$$= \frac{e^3\,x}{c^3\,d^3} - \frac{x\,(3\,a^2\,e^5-6\,a\,c\,d^2\,e^3+3\,c^2\,d^4\,e)+\frac{5\,a^3\,e^6-9\,a^2\,c\,d^2\,e^4+3\,a\,c^2\,d^4\,e^2+c^3\,d^6}{2\,c\,d}}{a^2\,c^3\,d^3\,e^2+2\,a\,c^4\,d^4\,e\,x+c^5\,d^5\,x^2}$$
$$- \frac{\ln(a\,e+c\,d\,x)\,(3\,a\,e^4-3\,c\,d^2\,e^2)}{c^4\,d^4}$$

input
```
int((d + e*x)^6/(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^3,x)
```

output
```
(e^3*x)/(c^3*d^3) - (x*(3*a^2*e^5 + 3*c^2*d^4*e - 6*a*c*d^2*e^3) + (5*a^3*
e^6 + c^3*d^6 + 3*a*c^2*d^4*e^2 - 9*a^2*c*d^2*e^4)/(2*c*d))/(c^5*d^5*x^2 +
 a^2*c^3*d^3*e^2 + 2*a*c^4*d^4*e*x) - (log(a*e + c*d*x)*(3*a*e^4 - 3*c*d^2
*e^2))/(c^4*d^4)
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 258, normalized size of antiderivative = 2.32

$$\int \frac{(d+ex)^6}{(ade+(cd^2+ae^2)x+cdex^2)^3}\,dx$$
$$= \frac{-6\log(cdx+ae)\,a^4e^6+6\log(cdx+ae)\,a^3c\,d^2e^4-12\log(cdx+ae)\,a^3cd\,e^5x+12\log(cdx+ae)\,a^2c^2d^3e}{}$$

input
```
int((e*x+d)^6/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^3,x)
```

output
```
( - 6*log(a*e + c*d*x)*a**4*e**6 + 6*log(a*e + c*d*x)*a**3*c*d**2*e**4 - 1
2*log(a*e + c*d*x)*a**3*c*d*e**5*x + 12*log(a*e + c*d*x)*a**2*c**2*d**3*e*
*3*x - 6*log(a*e + c*d*x)*a**2*c**2*d**2*e**4*x**2 + 6*log(a*e + c*d*x)*a*
c**3*d**4*e**2*x**2 - 3*a**4*e**6 + 3*a**3*c*d**2*e**4 + 6*a**2*c**2*d**2*
e**4*x**2 - a*c**3*d**6 - 6*a*c**3*d**4*e**2*x**2 + 2*a*c**3*d**3*e**3*x**
3 + 3*c**4*d**6*x**2)/(2*a*c**4*d**4*(a**2*e**2 + 2*a*c*d*e*x + c**2*d**2*
x**2))
```

3.131 $\int \frac{(d+ex)^5}{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^3}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 85

$$\int \frac{(d+ex)^5}{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^3}\,dx=-\frac{\left(cd^2-ae^2\right)^2}{2c^3d^3(ae+cdx)^2}-\frac{2e\left(cd^2-ae^2\right)}{c^3d^3(ae+cdx)}+\frac{e^2\log(ae+cdx)}{c^3d^3}$$

output
```
-1/2*(-a*e^2+c*d^2)^2/c^3/d^3/(c*d*x+a*e)^2-2*e*(-a*e^2+c*d^2)/c^3/d^3/(c*
d*x+a*e)+e^2*ln(c*d*x+a*e)/c^3/d^3
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.76

$$\int \frac{(d+ex)^5}{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^3}\,dx=\frac{-\frac{\left(cd^2-ae^2\right)\left(3ae^2+cd(d+4ex)\right)}{(ae+cdx)^2}+2e^2\log(ae+cdx)}{2c^3d^3}$$

input
```
Integrate[(d + e*x)^5/(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^3,x]
```

output
```
(-(((c*d^2 - a*e^2)*(3*a*e^2 + c*d*(d + 4*e*x)))/(a*e + c*d*x)^2) + 2*e^2*
Log[a*e + c*d*x])/(2*c^3*d^3)
```

Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.057$, Rules used = {1121, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex)^5}{\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^3}\,dx$$

$$\Big\downarrow \text{1121}$$

$$\int \left(\frac{2\left(cd^2e-ae^3\right)}{c^2d^2(ae+cdx)^2}+\frac{e^2}{c^2d^2(ae+cdx)}+\frac{\left(cd^2-ae^2\right)^2}{c^2d^2(ae+cdx)^3}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{e^2\log(ae+cdx)}{c^3d^3}-\frac{2e\left(cd^2-ae^2\right)}{c^3d^3(ae+cdx)}-\frac{\left(cd^2-ae^2\right)^2}{2c^3d^3(ae+cdx)^2}$$

input
```
Int[(d + e*x)^5/(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^3,x]
```

output
```
-1/2*(c*d^2 - a*e^2)^2/(c^3*d^3*(a*e + c*d*x)^2) - (2*e*(c*d^2 - a*e^2))/(
c^3*d^3*(a*e + c*d*x)) + (e^2*Log[a*e + c*d*x])/(c^3*d^3)
```

Defintions of rubi rules used

rule 1121
```
Int[((d_) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Int[ExpandIntegrand[(d + e*x)^(m + p)*(a/d + (c/e)*x)^p, x], x]
/; FreeQ[{a, b, c, d, e, m, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && (Int
egerQ[p] || (GtQ[a, 0] && GtQ[d, 0] && LtQ[c, 0]))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 1.60 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.06

method	result	size
risch	$\frac{\frac{2e\left(a\,e^{2}-c\,d^{2}\right)x}{d^{2}c^{2}}+\frac{3a^{2}e^{4}-2ac\,d^{2}e^{2}-c^{2}d^{4}}{2d^{3}c^{3}}}{\left(cdx+ae\right)^{2}}+\frac{e^{2}\ln\left(cdx+ae\right)}{c^{3}d^{3}}$	90
default	$\frac{e^{2}\ln\left(cdx+ae\right)}{c^{3}d^{3}}-\frac{a^{2}e^{4}-2ac\,d^{2}e^{2}+c^{2}d^{4}}{2c^{3}d^{3}\left(cdx+ae\right)^{2}}+\frac{2e\left(a\,e^{2}-c\,d^{2}\right)}{d^{3}c^{3}\left(cdx+ae\right)}$	95
parallelrisch	$\frac{2\ln\left(cdx+ae\right)x^{2}c^{2}d^{2}e^{2}+4\ln\left(cdx+ae\right)xacd\,e^{3}+2\ln\left(cdx+ae\right)a^{2}e^{4}+4xacd\,e^{3}-4x\,c^{2}d^{3}e+3a^{2}e^{4}-2ac\,d^{2}e^{2}-c^{2}d^{4}}{2d^{3}c^{3}\left(cdx+ae\right)^{2}}$	123
norman	$\frac{\frac{\left(3a^{2}e^{6}-3c^{2}d^{4}e^{2}\right)x}{d^{2}c^{3}e}+\frac{3a^{2}e^{4}-2ac\,d^{2}e^{2}-c^{2}d^{4}}{2c^{3}d}+\frac{\left(3e^{8}a^{2}+6a\,d^{2}c\,e^{6}-9d^{4}e^{4}c^{2}\right)x^{2}}{2d^{3}c^{3}e^{2}}+\frac{2\left(e^{6}a-c\,d^{2}e^{4}\right)x^{3}}{d^{2}c^{2}e}}{\left(ex+d\right)^{2}\left(cdx+ae\right)^{2}}+\frac{e^{2}\ln\left(cdx+ae\right)}{c^{3}d^{3}}$	179

input
```
int((e*x+d)^5/(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^3,x,method=_RETURNVERBOSE)
```

output
```
(2/d^2*e/c^2*(a*e^2-c*d^2)*x+1/2*(3*a^2*e^4-2*a*c*d^2*e^2-c^2*d^4)/d^3/c^3
)/(c*d*x+a*e)^2+e^2*ln(c*d*x+a*e)/c^3/d^3
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 126, normalized size of antiderivative = 1.48

$$\int \frac{(d+ex)^5}{(ade+(cd^2+ae^2)\,x+cdex^2)^3}\,dx = -\frac{c^2d^4+2\,acd^2e^2-3\,a^2e^4+4\,(c^2d^3e-acde^3)x-2\,(c^2d^2e^2x^2+2\,acde^3x+a^2e^4)\log{(cdx+ae)}}{2\,(c^5d^5x^2+2\,ac^4d^4ex+a^2c^3d^3e^2)}$$

input
```
integrate((e*x+d)^5/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^3,x, algorithm="fric
as")
```

output
```
-1/2*(c^2*d^4 + 2*a*c*d^2*e^2 - 3*a^2*e^4 + 4*(c^2*d^3*e - a*c*d*e^3)*x -
2*(c^2*d^2*e^2*x^2 + 2*a*c*d*e^3*x + a^2*e^4)*log(c*d*x + a*e))/(c^5*d^5*x
^2 + 2*a*c^4*d^4*e*x + a^2*c^3*d^3*e^2)
```

Sympy [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 109, normalized size of antiderivative = 1.28

$$\int \frac{(d+ex)^5}{(ade+(cd^2+ae^2)\,x+cdex^2)^3}\,dx = \frac{3a^2e^4-2acd^2e^2-c^2d^4+x(4acde^3-4c^2d^3e)}{2a^2c^3d^3e^2+4ac^4d^4ex+2c^5d^5x^2} + \frac{e^2\log{(ae+cdx)}}{c^3d^3}$$

input
```
integrate((e*x+d)**5/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**3,x)
```

output
```
(3*a**2*e**4 - 2*a*c*d**2*e**2 - c**2*d**4 + x*(4*a*c*d*e**3 - 4*c**2*d**3
*e))/(2*a**2*c**3*d**3*e**2 + 4*a*c**4*d**4*e*x + 2*c**5*d**5*x**2) + e**2
*log(a*e + c*d*x)/(c**3*d**3)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.24

$$\int \frac{(d+ex)^5}{(ade+(cd^2+ae^2)x+cdex^2)^3}\,dx = -\frac{c^2d^4+2\,acd^2e^2-3\,a^2e^4+4\,(c^2d^3e-acde^3)x}{2\,(c^5d^5x^2+2\,ac^4d^4ex+a^2c^3d^3e^2)} + \frac{e^2\log(cdx+ae)}{c^3d^3}$$

input
```
integrate((e*x+d)^5/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^3,x, algorithm="maxi
ma")
```

output
```
-1/2*(c^2*d^4 + 2*a*c*d^2*e^2 - 3*a^2*e^4 + 4*(c^2*d^3*e - a*c*d*e^3)*x)/(
c^5*d^5*x^2 + 2*a*c^4*d^4*e*x + a^2*c^3*d^3*e^2) + e^2*log(c*d*x + a*e)/(c
^3*d^3)
```

Giac [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.05

$$\int \frac{(d+ex)^5}{(ade+(cd^2+ae^2)x+cdex^2)^3}\,dx = \frac{e^2\log(|cdx+ae|)}{c^3d^3} - \frac{4\,(cd^2e-ae^3)x+\frac{c^2d^4+2\,acd^2e^2-3\,a^2e^4}{cd}}{2\,(cdx+ae)^2c^2d^2}$$

input
```
integrate((e*x+d)^5/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^3,x, algorithm="giac
")
```

output
```
e^2*log(abs(c*d*x + a*e))/(c^3*d^3) - 1/2*(4*(c*d^2*e - a*e^3)*x + (c^2*d^
4 + 2*a*c*d^2*e^2 - 3*a^2*e^4)/(c*d))/((c*d*x + a*e)^2*c^2*d^2)
```

Mupad [B] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 106, normalized size of antiderivative = 1.25

$$\int \frac{(d+ex)^5}{(ade+(cd^2+ae^2)\,x+cdex^2)^3}\,dx = \frac{e^2\,\ln\left(a\,e+c\,d\,x\right)}{c^3\,d^3} - \frac{\frac{-3\,a^2\,e^4+2\,a\,c\,d^2\,e^2+c^2\,d^4}{2\,c^3\,d^3}-\frac{2\,e\,x\,\left(a\,e^2-c\,d^2\right)}{c^2\,d^2}}{a^2\,e^2+2\,a\,c\,d\,e\,x+c^2\,d^2\,x^2}$$

input
```
int((d + e*x)^5/(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^3,x)
```

output
```
(e^2*log(a*e + c*d*x))/(c^3*d^3) - ((c^2*d^4 - 3*a^2*e^4 + 2*a*c*d^2*e^2)/
(2*c^3*d^3) - (2*e*x*(a*e^2 - c*d^2))/(c^2*d^2))/(a^2*e^2 + c^2*d^2*x^2 +
2*a*c*d*e*x)
```

Reduce [B] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 142, normalized size of antiderivative = 1.67

$$\int \frac{(d+ex)^5}{(ade+(cd^2+ae^2)\,x+cdex^2)^3}\,dx = \frac{2\log(cdx+ae)\,a^3e^4+4\log(cdx+ae)\,a^2cd\,e^3x+2\log(cdx+ae)\,a\,c^2d^2e^2x^2+a^3e^4-a\,c^2d^4-2a\,c^2d^2e^2}{2a\,c^3d^3\,(c^2d^2x^2+2acdex+a^2e^2)}$$

input
```
int((e*x+d)^5/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^3,x)
```

output
```
(2*log(a*e + c*d*x)*a**3*e**4 + 4*log(a*e + c*d*x)*a**2*c*d*e**3*x + 2*log
(a*e + c*d*x)*a*c**2*d**2*e**2*x**2 + a**3*e**4 - a*c**2*d**4 - 2*a*c**2*d
**2*e**2*x**2 + 2*c**3*d**4*x**2)/(2*a*c**3*d**3*(a**2*e**2 + 2*a*c*d*e*x
+ c**2*d**2*x**2))
```

3.132 $\int \frac{(d+ex)^4}{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^3}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 35

$$\int \frac{(d+ex)^4}{(ade+(cd^2+ae^2)\,x+cdex^2)^3}\,dx=-\frac{(d+ex)^2}{2\,(cd^2-ae^2)\,(ae+cdx)^2}$$

output
```
-1/2*(e*x+d)^2/(-a*e^2+c*d^2)/(c*d*x+a*e)^2
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.00

$$\int \frac{(d+ex)^4}{(ade+(cd^2+ae^2)\,x+cdex^2)^3}\,dx=-\frac{ae^2+cd(d+2ex)}{2c^2d^2(ae+cdx)^2}$$

input
```
Integrate[(d + e*x)^4/(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^3,x]
```

output
```
-1/2*(a*e^2 + c*d*(d + 2*e*x))/(c^2*d^2*(a*e + c*d*x)^2)
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.057$, Rules used = {1120, 48}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex)^4}{(x(ae^2+cd^2)+ade+cdex^2)^3}dx$$
$$\Big\downarrow \text{ 1120}$$
$$\int \frac{d+ex}{(ae+cdx)^3}dx$$
$$\Big\downarrow \text{ 48}$$
$$-\frac{(d+ex)^2}{2(cd^2-ae^2)(ae+cdx)^2}$$

input
```
Int[(d + e*x)^4/(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^3,x]
```

output
```
-1/2*(d + e*x)^2/((c*d^2 - a*e^2)*(a*e + c*d*x)^2)
```

Defintions of rubi rules used

rule 48
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && EqQ[m + n + 2, 0] && NeQ[m, -1]
```

rule 1120
```
Int[((d_) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Int[(d + e*x)^(m + p)*(a/d + (c/e)*x)^p, x] /; FreeQ[{a, b, c, d
, e, m}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[p] && (EqQ[m + p,
0] || EqQ[m + 2*p + 2, 0])
```

Maple [A] (verified)

Time = 1.67 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.03

method	result	size
gosper	$-\frac{2cdxe+a\,e^{2}+c\,d^{2}}{2c^{2}d^{2}\left(cdx+ae\right)^{2}}$	36
parallelrisch	$\frac{-2cdxe-a\,e^{2}-c\,d^{2}}{2c^{2}d^{2}\left(cdx+ae\right)^{2}}$	38
risch	$\frac{-\frac{ex}{cd}-\frac{a\,e^{2}+c\,d^{2}}{2c^{2}d^{2}}}{\left(cdx+ae\right)^{2}}$	42
default	$-\frac{-a\,e^{2}+c\,d^{2}}{2c^{2}d^{2}\left(cdx+ae\right)^{2}}-\frac{e}{c^{2}d^{2}\left(cdx+ae\right)}$	51
orering	$-\frac{\left(2cdxe+a\,e^{2}+c\,d^{2}\right)\left(cdx+ae\right)\left(ex+d\right)^{3}}{2c^{2}d^{2}\left(ade+\left(a\,e^{2}+c\,d^{2}\right)x+cd\,x^{2}e\right)^{3}}$	68
norman	$\frac{-\frac{e^{3}x^{3}}{cd}+\frac{\left(-e^{4}a-2d^{2}e^{2}c\right)x}{c^{2}de}+\frac{-a\,e^{2}-c\,d^{2}}{2c^{2}}+\frac{\left(-e^{6}a-5c\,d^{2}e^{4}\right)x^{2}}{2c^{2}d^{2}e^{2}}}{\left(ex+d\right)^{2}\left(cdx+ae\right)^{2}}$	109

input
```
int((e*x+d)^4/(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^3,x,method=_RETURNVERBOSE)
```

output
```
-1/2*(2*c*d*e*x+a*e^2+c*d^2)/c^2/d^2/(c*d*x+a*e)^2
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.60

$$\int \frac{(d+ex)^4}{(ade+(cd^2+ae^2)\,x+cdex^2)^3}\,dx=-\frac{2\,cdex+cd^2+ae^2}{2\,(c^4d^4x^2+2\,ac^3d^3ex+a^2c^2d^2e^2)}$$

input
```
integrate((e*x+d)^4/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^3,x, algorithm="fric
as")
```

output
```
-1/2*(2*c*d*e*x + c*d^2 + a*e^2)/(c^4*d^4*x^2 + 2*a*c^3*d^3*e*x + a^2*c^2*
d^2*e^2)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 60 vs. $2(29) = 58$.

Time = 0.17 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.71

$$\int \frac{(d+ex)^4}{(ade+(cd^2+ae^2)\,x+cdex^2)^3}\,dx = \frac{-ae^2-cd^2-2cdex}{2a^2c^2d^2e^2+4ac^3d^3ex+2c^4d^4x^2}$$

input
```
integrate((e*x+d)**4/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**3,x)
```

output
```
(-a*e**2 - c*d**2 - 2*c*d*e*x)/(2*a**2*c**2*d**2*e**2 + 4*a*c**3*d**3*e*x
+ 2*c**4*d**4*x**2)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.60

$$\int \frac{(d+ex)^4}{(ade+(cd^2+ae^2)\,x+cdex^2)^3}\,dx = -\frac{2\,cdex+cd^2+ae^2}{2\,(c^4d^4x^2+2\,ac^3d^3ex+a^2c^2d^2e^2)}$$

input
```
integrate((e*x+d)^4/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^3,x, algorithm="maxi
ma")
```

output
```
-1/2*(2*c*d*e*x + c*d^2 + a*e^2)/(c^4*d^4*x^2 + 2*a*c^3*d^3*e*x + a^2*c^2*
d^2*e^2)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.00

$$\int \frac{(d+ex)^4}{(ade+(cd^2+ae^2)\,x+cdex^2)^3}\,dx = -\frac{2\,cdex+cd^2+ae^2}{2\,(cdx+ae)^2c^2d^2}$$

input
```
integrate((e*x+d)^4/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^3,x, algorithm="giac
")
```

output
```
-1/2*(2*c*d*e*x + c*d^2 + a*e^2)/((c*d*x + a*e)^2*c^2*d^2)
```

Mupad [B] (verification not implemented)

Time = 5.16 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.23

$$\int \frac{(d+ex)^4}{(ade+(cd^2+ae^2)\,x+cdex^2)^3}\,dx = -\frac{\frac{1}{2\,c}-\frac{x^2}{2\,a}}{a^2\,e^2+2\,a\,c\,d\,e\,x+c^2\,d^2\,x^2}$$

input
```
int((d + e*x)^4/(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^3,x)
```

output
```
-(1/(2*c) - x^2/(2*a))/(a^2*e^2 + c^2*d^2*x^2 + 2*a*c*d*e*x)
```

Reduce [B] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.26

$$\int \frac{(d+ex)^4}{(ade+(cd^2+ae^2)\,x+cdex^2)^3}\,dx = \frac{c\,x^2-a}{2ac\,(c^2d^2x^2+2acdex+a^2e^2)}$$

input
```
int((e*x+d)^4/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^3,x)
```

output
```
( - a + c*x**2)/(2*a*c*(a**2*e**2 + 2*a*c*d*e*x + c**2*d**2*x**2))
```

3.133 $\int \frac{(d+ex)^3}{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^3}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 20

$$\int \frac{(d+ex)^3}{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^3}\,dx=-\frac{1}{2cd(ae+cdx)^2}$$

output
```
-1/2/c/d/(c*d*x+a*e)^2
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{(d+ex)^3}{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^3}\,dx=-\frac{1}{2cd(ae+cdx)^2}$$

input
```
Integrate[(d + e*x)^3/(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^3,x]
```

output
```
-1/2*1/(c*d*(a*e + c*d*x)^2)
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.057$, Rules used = {1120, 17}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex)^3}{(x(ae^2+cd^2)+ade+cdex^2)^3} \, dx$$

$$\Big\downarrow \text{ 1120}$$

$$\int \frac{1}{(ae+cdx)^3}dx$$

$$\Big\downarrow \text{ 17}$$

$$-\frac{1}{2cd(ae+cdx)^2}$$

input
```
Int[(d + e*x)^3/(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^3,x]
```

output
```
-1/2*1/(c*d*(a*e + c*d*x)^2)
```

Defintions of rubi rules used

rule 17
```
Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1
)/(b*(m + 1))), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]
```

rule 1120
```
Int[((d_) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Int[(d + e*x)^(m + p)*(a/d + (c/e)*x)^p, x] /; FreeQ[{a, b, c, d
, e, m}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[p] && (EqQ[m + p,
0] || EqQ[m + 2*p + 2, 0])
```

Maple [A] (verified)

Time = 1.58 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.95

method	result	size
gosper	$-\frac{1}{2cd(cdx+ae)^{2}}$	19
default	$-\frac{1}{2cd(cdx+ae)^{2}}$	19
risch	$-\frac{1}{2cd(cdx+ae)^{2}}$	19
parallelrisch	$-\frac{1}{2cd(cdx+ae)^{2}}$	19
norman	$\frac{-\frac{ex}{c}-\frac{d}{2c}-\frac{e^{2}x^{2}}{2cd}}{(ex+d)^{2}(cdx+ae)^{2}}$	47
orering	$-\frac{(cdx+ae)(ex+d)^{3}}{2cd(ade+(a\,e^{2}+c\,d^{2})x+cd\,x^{2}e)^{3}}$	51

input
```
int((e*x+d)^3/(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^3,x,method=_RETURNVERBOSE)
```

output
```
-1/2/c/d/(c*d*x+a*e)^2
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.75

$$\int \frac{(d+ex)^3}{(ade+(cd^2+ae^2)\,x+cdex^2)^3}\,dx=-\frac{1}{2\,(c^3d^3x^2+2\,ac^2d^2ex+a^2cde^2)}$$

input
```
integrate((e*x+d)^3/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^3,x, algorithm="fric
as")
```

output
```
-1/2/(c^3*d^3*x^2 + 2*a*c^2*d^2*e*x + a^2*c*d*e^2)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 39 vs. $2(17) = 34$.

Time = 0.12 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.95

$$\int \frac{(d+ex)^3}{(ade+(cd^2+ae^2)x+cdex^2)^3}\,dx = -\frac{1}{2a^2cde^2+4ac^2d^2ex+2c^3d^3x^2}$$

input
```
integrate((e*x+d)**3/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**3,x)
```

output
```
-1/(2*a**2*c*d*e**2 + 4*a*c**2*d**2*e*x + 2*c**3*d**3*x**2)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.75

$$\int \frac{(d+ex)^3}{(ade+(cd^2+ae^2)x+cdex^2)^3}\,dx = -\frac{1}{2\,(c^3d^3x^2+2\,ac^2d^2ex+a^2cde^2)}$$

input
```
integrate((e*x+d)^3/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^3,x, algorithm="maxi
ma")
```

output
```
-1/2/(c^3*d^3*x^2 + 2*a*c^2*d^2*e*x + a^2*c*d*e^2)
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.90

$$\int \frac{(d+ex)^3}{(ade+(cd^2+ae^2)x+cdex^2)^3}\,dx = -\frac{1}{2\,(cdx+ae)^2cd}$$

input
```
integrate((e*x+d)^3/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^3,x, algorithm="giac
")
```

output
```
-1/2/((c*d*x + a*e)^2*c*d)
```

Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.85

$$\int \frac{(d+ex)^3}{(ade+(cd^2+ae^2)\,x+cdex^2)^3}\,dx = -\frac{1}{2\,a^2\,c\,d\,e^2+4\,a\,c^2\,d^2\,e\,x+2\,c^3\,d^3\,x^2}$$

input
```
int((d + e*x)^3/(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^3,x)
```

output
```
-1/(2*c^3*d^3*x^2 + 2*a^2*c*d*e^2 + 4*a*c^2*d^2*e*x)
```

Reduce [B] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.75

$$\int \frac{(d+ex)^3}{(ade+(cd^2+ae^2)\,x+cdex^2)^3}\,dx = -\frac{1}{2cd\,(c^2d^2x^2+2acdex+a^2e^2)}$$

input
```
int((e*x+d)^3/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^3,x)
```

output
```
( - 1)/(2*c*d*(a**2*e**2 + 2*a*c*d*e*x + c**2*d**2*x**2))
```

3.134 $\int \frac{(d+ex)^2}{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^3}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 107

$$\int \frac{(d+ex)^2}{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^3}\,dx = -\frac{1}{2\left(cd^2-ae^2\right)(ae+cdx)^2} + \frac{e}{\left(cd^2-ae^2\right)^2(ae+cdx)} + \frac{e^2\log(ae+cdx)}{\left(cd^2-ae^2\right)^3} - \frac{e^2\log(d+ex)}{\left(cd^2-ae^2\right)^3}$$

output
```
-1/2/(-a*e^2+c*d^2)/(c*d*x+a*e)^2+e/(-a*e^2+c*d^2)^2/(c*d*x+a*e)+e^2*ln(c*
d*x+a*e)/(-a*e^2+c*d^2)^3-e^2*ln(e*x+d)/(-a*e^2+c*d^2)^3
```

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.78

$$\int \frac{(d+ex)^2}{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^3}\,dx = -\frac{\frac{\left(cd^2-ae^2\right)\left(-3ae^2+cd(d-2ex)\right)}{(ae+cdx)^2}-2e^2\log(ae+cdx)+2e^2\log(d+ex)}{2\left(cd^2-ae^2\right)^3}$$

input
```
Integrate[(d + e*x)^2/(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^3,x]
```

output
```
-1/2*(((c*d^2 - a*e^2)*(-3*a*e^2 + c*d*(d - 2*e*x)))/(a*e + c*d*x)^2 - 2*e
^2*Log[a*e + c*d*x] + 2*e^2*Log[d + e*x])/(c*d^2 - a*e^2)^3
```

Rubi [A] (verified)

Time = 0.46 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.057$, Rules used = {1121, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex)^2}{(x(ae^2+cd^2)+ade+cdex^2)^3} \, dx$$

$$\Big\downarrow \text{1121}$$

$$\int \left(\frac{cde^2}{(cd^2-ae^2)^3(ae+cdx)} - \frac{cde}{(cd^2-ae^2)^2(ae+cdx)^2} + \frac{cd}{(cd^2-ae^2)(ae+cdx)^3} - \frac{e^3}{(d+ex)(cd^2-ae^2)^3}\right) dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{e}{(cd^2-ae^2)^2(ae+cdx)} - \frac{1}{2(cd^2-ae^2)(ae+cdx)^2} + \frac{e^2\log(ae+cdx)}{(cd^2-ae^2)^3} - \frac{e^2\log(d+ex)}{(cd^2-ae^2)^3}$$

input
```
Int[(d + e*x)^2/(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^3,x]
```

output
```
-1/2*1/((c*d^2 - a*e^2)*(a*e + c*d*x)^2) + e/((c*d^2 - a*e^2)^2*(a*e + c*d
*x)) + (e^2*Log[a*e + c*d*x])/(c*d^2 - a*e^2)^3 - (e^2*Log[d + e*x])/(c*d^
2 - a*e^2)^3
```

Defintions of rubi rules used

rule 1121
```
Int[((d_) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Int[ExpandIntegrand[(d + e*x)^(m + p)*(a/d + (c/e)*x)^p, x], x]
/; FreeQ[{a, b, c, d, e, m, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && (Int
egerQ[p] || (GtQ[a, 0] && GtQ[d, 0] && LtQ[c, 0]))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 1.67 (sec) , antiderivative size = 106, normalized size of antiderivative = 0.99

method	result
default	$\frac{1}{2(a\,e^2-c\,d^2)(cdx+ae)^2}+\frac{e}{(a\,e^2-c\,d^2)^2(cdx+ae)}-\frac{e^2\ln(cdx+ae)}{(a\,e^2-c\,d^2)^3}+\frac{e^2\ln(ex+d)}{(a\,e^2-c\,d^2)^3}$
risch	$\frac{\frac{decx}{a^2e^4-2ac\,d^2e^2+c^2d^4}+\frac{3a\,e^2-c\,d^2}{2a^2e^4-4ac\,d^2e^2+2c^2d^4}}{(cdx+ae)^2}-\frac{e^2\ln(cdx+ae)}{e^6a^3-3d^2e^4a^2c+3d^4e^2a\,c^2-d^6c^3}+\frac{e^2\ln(-ex-d)}{e^6a^3-3d^2e^4a^2c+3d^4e^2a\,c^2-d^6c^3}$
parallelrisch	$\frac{2\ln(ex+d)x^2a\,c^3d^3e^3-2\ln(cdx+ae)x^2a\,c^3d^3e^3+4\ln(ex+d)x\,a^2c^2d^2e^4-4\ln(cdx+ae)x\,a^2c^2d^2e^4-x^2a\,c^3d^3e^3+x^2c^4d^5e+2}{2(e^6a^3-3d^2e^4a^2c+3d^4e^2a\,c^2-d^6c^3)(cdx+ae)^2acde}$
norman	$\frac{\frac{cd\,e^3x^3}{a^2e^4-2ac\,d^2e^2+c^2d^4}+\frac{3ad\,e^3x}{a^2e^4-2ac\,d^2e^2+c^2d^4}+\frac{3d^2e^2a\,c^2-d^4c^3}{2c^2\left(a^2e^4-2ac\,d^2e^2+c^2d^4\right)}+\frac{\left(3a\,e^6c^2d^2+3d^4c^3e^4\right)x^2}{2d^2c^2e^2\left(a^2e^4-2ac\,d^2e^2+c^2d^4\right)}}{(ex+d)^2(cdx+ae)^2}+\frac{e^2\ln(e}{e^6a^3-3d^2e^4a^2c+}$

input
```
int((e*x+d)^2/(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^3,x,method=_RETURNVERBOSE)
```

output
```
1/2/(a*e^2-c*d^2)/(c*d*x+a*e)^2+e/(a*e^2-c*d^2)^2/(c*d*x+a*e)-e^2/(a*e^2-c
*d^2)^3*ln(c*d*x+a*e)+e^2/(a*e^2-c*d^2)^3*ln(e*x+d)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 280 vs. $2(105) = 210$.

Time = 0.08 (sec) , antiderivative size = 280, normalized size of antiderivative = 2.62

$$\int \frac{(d+ex)^2}{(ade+(cd^2+ae^2)x+cdex^2)^3}\,dx = -\frac{c^2d^4-4\,acd^2e^2+3\,a^2e^4-2\,(c^2d^3e-acde^3)x-2\,(c^2d^2e^2x^2+2\,acde^3x+a^2e^4)\log{(cdx+ae)}+2\,(c^2}{2\,(a^2c^3d^6e^2-3\,a^3c^2d^4e^4+3\,a^4cd^2e^6-a^5e^8+(c^5d^8-3\,ac^4d^6e^2+3\,a^2c^3d^4e^4-a^3c^2d^2e^6)x^2+2\,(ac^4d^7}$$

input
```
integrate((e*x+d)^2/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^3,x, algorithm="fric
as")
```

output
```
-1/2*(c^2*d^4 - 4*a*c*d^2*e^2 + 3*a^2*e^4 - 2*(c^2*d^3*e - a*c*d*e^3)*x -
2*(c^2*d^2*e^2*x^2 + 2*a*c*d*e^3*x + a^2*e^4)*log(c*d*x + a*e) + 2*(c^2*d^
2*e^2*x^2 + 2*a*c*d*e^3*x + a^2*e^4)*log(e*x + d))/(a^2*c^3*d^6*e^2 - 3*a^
3*c^2*d^4*e^4 + 3*a^4*c*d^2*e^6 - a^5*e^8 + (c^5*d^8 - 3*a*c^4*d^6*e^2 + 3
*a^2*c^3*d^4*e^4 - a^3*c^2*d^2*e^6)*x^2 + 2*(a*c^4*d^7*e - 3*a^2*c^3*d^5*e
^3 + 3*a^3*c^2*d^3*e^5 - a^4*c*d*e^7)*x)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 457 vs. $2(92) = 184$.

Time = 0.81 (sec) , antiderivative size = 457, normalized size of antiderivative = 4.27

$$\int \frac{(d+ex)^2}{(ade+(cd^2+ae^2)x+cdex^2)^3}\,dx = \frac{e^2\log\left(x+\frac{-\frac{a^4e^{10}}{(ae^2-cd^2)^3}+\frac{4a^3cd^2e^8}{(ae^2-cd^2)^3}-\frac{6a^2c^2d^4e^6}{(ae^2-cd^2)^3}+\frac{4ac^3d^6e^4}{(ae^2-cd^2)^3}+ae^4-\frac{c^4d^8e^2}{(ae^2-cd^2)^3}+cd^2e^2}{2cde^3}\right)}{(ae^2-cd^2)^3} - \frac{e^2\log\left(x+\frac{\frac{a^4e^{10}}{(ae^2-cd^2)^3}-\frac{4a^3cd^2e^8}{(ae^2-cd^2)^3}+\frac{6a^2c^2d^4e^6}{(ae^2-cd^2)^3}-\frac{4ac^3d^6e^4}{(ae^2-cd^2)^3}+ae^4+\frac{c^4d^8e^2}{(ae^2-cd^2)^3}+cd^2e^2}{2cde^3}\right)}{(ae^2-cd^2)^3} + \frac{3ae^2-cd^2+2cdex}{2a^4e^6-4a^3cd^2e^4+2a^2c^2d^4e^2+x^2\cdot(2a^2c^2d^2e^4-4ac^3d^4e^2+2c^4d^6)+x\,(4a^3cde^5-8a^2c^2d^3e^3+4ac^3d}$$

input
```
integrate((e*x+d)**2/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**3,x)
```

output
```
e**2*log(x + (-a**4*e**10/(a*e**2 - c*d**2)**3 + 4*a**3*c*d**2*e**8/(a*e**
2 - c*d**2)**3 - 6*a**2*c**2*d**4*e**6/(a*e**2 - c*d**2)**3 + 4*a*c**3*d**
6*e**4/(a*e**2 - c*d**2)**3 + a*e**4 - c**4*d**8*e**2/(a*e**2 - c*d**2)**3
 + c*d**2*e**2)/(2*c*d*e**3))/(a*e**2 - c*d**2)**3 - e**2*log(x + (a**4*e*
*10/(a*e**2 - c*d**2)**3 - 4*a**3*c*d**2*e**8/(a*e**2 - c*d**2)**3 + 6*a**
2*c**2*d**4*e**6/(a*e**2 - c*d**2)**3 - 4*a*c**3*d**6*e**4/(a*e**2 - c*d**
2)**3 + a*e**4 + c**4*d**8*e**2/(a*e**2 - c*d**2)**3 + c*d**2*e**2)/(2*c*d
*e**3))/(a*e**2 - c*d**2)**3 + (3*a*e**2 - c*d**2 + 2*c*d*e*x)/(2*a**4*e**
6 - 4*a**3*c*d**2*e**4 + 2*a**2*c**2*d**4*e**2 + x**2*(2*a**2*c**2*d**2*e*
*4 - 4*a*c**3*d**4*e**2 + 2*c**4*d**6) + x*(4*a**3*c*d*e**5 - 8*a**2*c**2*
d**3*e**3 + 4*a*c**3*d**5*e))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 238 vs. $2(105) = 210$.

Time = 0.04 (sec) , antiderivative size = 238, normalized size of antiderivative = 2.22

$$\int \frac{(d+ex)^2}{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^3}\,dx = \frac{e^2\log\left(cdx+ae\right)}{c^3d^6-3\,ac^2d^4e^2+3\,a^2cd^2e^4-a^3e^6} - \frac{e^2\log\left(ex+d\right)}{c^3d^6-3\,ac^2d^4e^2+3\,a^2cd^2e^4-a^3e^6} + \frac{2\,cdex-cd^2+3\,ae^2}{2\left(a^2c^2d^4e^2-2\,a^3cd^2e^4+a^4e^6+\left(c^4d^6-2\,ac^3d^4e^2+a^2c^2d^2e^4\right)x^2+2\left(ac^3d^5e-2\,a^2c^2d^3e^3+a^3cde^5\right)x\right)}$$

input
```
integrate((e*x+d)^2/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^3,x, algorithm="maxi
ma")
```

output
```
e^2*log(c*d*x + a*e)/(c^3*d^6 - 3*a*c^2*d^4*e^2 + 3*a^2*c*d^2*e^4 - a^3*e^
6) - e^2*log(e*x + d)/(c^3*d^6 - 3*a*c^2*d^4*e^2 + 3*a^2*c*d^2*e^4 - a^3*e
^6) + 1/2*(2*c*d*e*x - c*d^2 + 3*a*e^2)/(a^2*c^2*d^4*e^2 - 2*a^3*c*d^2*e^4
 + a^4*e^6 + (c^4*d^6 - 2*a*c^3*d^4*e^2 + a^2*c^2*d^2*e^4)*x^2 + 2*(a*c^3*
d^5*e - 2*a^2*c^2*d^3*e^3 + a^3*c*d*e^5)*x)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 190, normalized size of antiderivative = 1.78

$$\int \frac{(d+ex)^2}{\left(ade+(cd^2+ae^2)\,x+cdex^2\right)^3}\,dx = \frac{cde^2\log\left(|cdx+ae|\right)}{c^4d^7-3\,ac^3d^5e^2+3\,a^2c^2d^3e^4-a^3cde^6} - \frac{e^3\log\left(|ex+d|\right)}{c^3d^6e-3\,ac^2d^4e^3+3\,a^2cd^2e^5-a^3e^7} - \frac{c^2d^4-4\,acd^2e^2+3\,a^2e^4-2\,(c^2d^3e-acde^3)x}{2\,(cd^2-ae^2)^3(cdx+ae)^2}$$

input
```
integrate((e*x+d)^2/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^3,x, algorithm="giac
")
```

output
```
c*d*e^2*log(abs(c*d*x + a*e))/(c^4*d^7 - 3*a*c^3*d^5*e^2 + 3*a^2*c^2*d^3*e
^4 - a^3*c*d*e^6) - e^3*log(abs(e*x + d))/(c^3*d^6*e - 3*a*c^2*d^4*e^3 + 3
*a^2*c*d^2*e^5 - a^3*e^7) - 1/2*(c^2*d^4 - 4*a*c*d^2*e^2 + 3*a^2*e^4 - 2*(
c^2*d^3*e - a*c*d*e^3)*x)/((c*d^2 - a*e^2)^3*(c*d*x + a*e)^2)
```

Mupad [B] (verification not implemented)

Time = 5.21 (sec) , antiderivative size = 225, normalized size of antiderivative = 2.10

$$\int \frac{(d+ex)^2}{\left(ade+(cd^2+ae^2)\,x+cdex^2\right)^3}\,dx = \frac{\frac{3\,a\,e^2-c\,d^2}{2\,(a^2\,e^4-2\,a\,c\,d^2\,e^2+c^2\,d^4)}+\frac{c\,d\,e\,x}{a^2\,e^4-2\,a\,c\,d^2\,e^2+c^2\,d^4}}{a^2\,e^2+2\,a\,c\,d\,e\,x+c^2\,d^2\,x^2} - \frac{2\,e^2\,\mathrm{atanh}\left(\frac{a^3\,e^6-a^2\,c\,d^2\,e^4-a\,c^2\,d^4\,e^2+c^3\,d^6}{(a\,e^2-c\,d^2)^3}+\frac{2\,c\,d\,e\,x\,(a^2\,e^4-2\,a\,c\,d^2\,e^2+c^2\,d^4)}{(a\,e^2-c\,d^2)^3}\right)}{(a\,e^2-c\,d^2)^3}$$

input
```
int((d + e*x)^2/(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^3,x)
```

output
```
((3*a*e^2 - c*d^2)/(2*(a^2*e^4 + c^2*d^4 - 2*a*c*d^2*e^2)) + (c*d*e*x)/(a^
2*e^4 + c^2*d^4 - 2*a*c*d^2*e^2))/(a^2*e^2 + c^2*d^2*x^2 + 2*a*c*d*e*x) -
(2*e^2*atanh((a^3*e^6 + c^3*d^6 - a*c^2*d^4*e^2 - a^2*c*d^2*e^4)/(a*e^2 -
c*d^2)^3 + (2*c*d*e*x*(a^2*e^4 + c^2*d^4 - 2*a*c*d^2*e^2))/(a*e^2 - c*d^2)
^3))/(a*e^2 - c*d^2)^3
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 333, normalized size of antiderivative = 3.11

$$\int \frac{(d+ex)^2}{(ade+(cd^2+ae^2)\,x+cdex^2)^3}\,dx$$
$$= \frac{-2\log(cdx+ae)\,a^3e^4-4\log(cdx+ae)\,a^2cd\,e^3x-2\log(cdx+ae)\,a\,c^2d^2e^2x^2+2\log(ex+d)\,a^3e^4+4\log}{2a\,(a^3c^2d^2e^6x^2-3a^2c^3d^4e^4x^2+3a\,c^4d^6e^2x^2-c^5d^8x^2+2a^4cd\,e^7x-6a^3c^2d^3e^5}$$

input
```
int((e*x+d)^2/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^3,x)
```

output
```
( - 2*log(a*e + c*d*x)*a**3*e**4 - 4*log(a*e + c*d*x)*a**2*c*d*e**3*x - 2*
log(a*e + c*d*x)*a*c**2*d**2*e**2*x**2 + 2*log(d + e*x)*a**3*e**4 + 4*log(
d + e*x)*a**2*c*d*e**3*x + 2*log(d + e*x)*a*c**2*d**2*e**2*x**2 + 2*a**3*e
**4 - 3*a**2*c*d**2*e**2 + a*c**2*d**4 - a*c**2*d**2*e**2*x**2 + c**3*d**4
*x**2)/(2*a*(a**5*e**8 - 3*a**4*c*d**2*e**6 + 2*a**4*c*d*e**7*x + 3*a**3*c
**2*d**4*e**4 - 6*a**3*c**2*d**3*e**5*x + a**3*c**2*d**2*e**6*x**2 - a**2*
c**3*d**6*e**2 + 6*a**2*c**3*d**5*e**3*x - 3*a**2*c**3*d**4*e**4*x**2 - 2*
a*c**4*d**7*e*x + 3*a*c**4*d**6*e**2*x**2 - c**5*d**8*x**2))
```

3.135 $\int \frac{d+ex}{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^3}\,dx$

Optimal result

Integrand size = 33, antiderivative size = 142

$$\int \frac{d+ex}{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^3}\,dx = -\frac{cd}{2\left(cd^2-ae^2\right)^2(ae+cdx)^2} + \frac{2cde}{\left(cd^2-ae^2\right)^3(ae+cdx)} + \frac{e^2}{\left(cd^2-ae^2\right)^3(d+ex)} + \frac{3cde^2\log(ae+cdx)}{\left(cd^2-ae^2\right)^4} - \frac{3cde^2\log(d+ex)}{\left(cd^2-ae^2\right)^4}$$

output

```
-1/2*c*d/(-a*e^2+c*d^2)^2/(c*d*x+a*e)^2+2*c*d*e/(-a*e^2+c*d^2)^3/(c*d*x+a*
e)+e^2/(-a*e^2+c*d^2)^3/(e*x+d)+3*c*d*e^2*ln(c*d*x+a*e)/(-a*e^2+c*d^2)^4-3
*c*d*e^2*ln(e*x+d)/(-a*e^2+c*d^2)^4
```

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 127, normalized size of antiderivative = 0.89

$$\int \frac{d+ex}{(ade+(cd^2+ae^2)x+cdex^2)^3}\,dx = \frac{-\frac{cd(cd^2-ae^2)^2}{(ae+cdx)^2}+\frac{4cde(cd^2-ae^2)}{ae+cdx}+\frac{2cd^2e^2-2ae^4}{d+ex}+6cde^2\log(ae+cdx)-6cde^2\log(d+ex)}{2(cd^2-ae^2)^4}$$

input
```
Integrate[(d + e*x)/(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^3,x]
```

output
```
(-((c*d*(c*d^2 - a*e^2)^2)/(a*e + c*d*x)^2) + (4*c*d*e*(c*d^2 - a*e^2))/(a
*e + c*d*x) + (2*c*d^2*e^2 - 2*a*e^4)/(d + e*x) + 6*c*d*e^2*Log[a*e + c*d*
x] - 6*c*d*e^2*Log[d + e*x])/(2*(c*d^2 - a*e^2)^4)
```

Rubi [A] (verified)

Time = 0.53 (sec) , antiderivative size = 142, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.061$, Rules used = {1121, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{d+ex}{(x(ae^2+cd^2)+ade+cdex^2)^3}\,dx$$

$$\Big\downarrow \text{1121}$$

$$\int \left(\frac{3c^2d^2e^2}{(cd^2-ae^2)^4(ae+cdx)}-\frac{2c^2d^2e}{(cd^2-ae^2)^3(ae+cdx)^2}+\frac{c^2d^2}{(cd^2-ae^2)^2(ae+cdx)^3}-\frac{3cde^3}{(d+ex)(cd^2-ae^2)^4}-\right.$$

$$\Big\downarrow \text{2009}$$

$$\frac{e^2}{(d+ex)(cd^2-ae^2)^3}+\frac{2cde}{(cd^2-ae^2)^3(ae+cdx)}-\frac{cd}{2(cd^2-ae^2)^2(ae+cdx)^2}+\frac{3cde^2\log(ae+cdx)}{(cd^2-ae^2)^4}-\frac{3cde^2\log(d+ex)}{(cd^2-ae^2)^4}$$

input
```
Int[(d + e*x)/(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^3,x]
```

output
```
-1/2*(c*d)/((c*d^2 - a*e^2)^2*(a*e + c*d*x)^2) + (2*c*d*e)/((c*d^2 - a*e^2
)^3*(a*e + c*d*x)) + e^2/((c*d^2 - a*e^2)^3*(d + e*x)) + (3*c*d*e^2*Log[a*
e + c*d*x])/(c*d^2 - a*e^2)^4 - (3*c*d*e^2*Log[d + e*x])/(c*d^2 - a*e^2)^4
```

Defintions of rubi rules used

rule 1121
```
Int[((d_) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Int[ExpandIntegrand[(d + e*x)^(m + p)*(a/d + (c/e)*x)^p, x], x]
/; FreeQ[{a, b, c, d, e, m, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && (Int
egerQ[p] || (GtQ[a, 0] && GtQ[d, 0] && LtQ[c, 0]))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 1.53 (sec) , antiderivative size = 142, normalized size of antiderivative = 1.00

method	result
default	$-\frac{cd}{2(a\,e^2-c\,d^2)^2(cdx+ae)^2}+\frac{3cd\,e^2\ln(cdx+ae)}{(a\,e^2-c\,d^2)^4}-\frac{2cde}{(a\,e^2-c\,d^2)^3(cdx+ae)}-\frac{e^2}{(a\,e^2-c\,d^2)^3(ex+d)}-\frac{3cd\,e^2\ln(ex+d)}{(a\,e^2-c\,d^2)^4}$
risch	$\frac{-\frac{3e^2c^2d^2x^2}{e^6a^3-3d^2e^4a^2c+3d^4e^2a\,c^2-d^6c^3}-\frac{3\left(3a\,e^2+c\,d^2\right)cdex}{2\left(e^6a^3-3d^2e^4a^2c+3d^4e^2a\,c^2-d^6c^3\right)}-\frac{2a^2e^4+5ac\,d^2e^2-c^2d^4}{2\left(e^6a^3-3d^2e^4a^2c+3d^4e^2a\,c^2-d^6c^3\right)}}{(cdx+ae)(cd\,x^2e+a\,e^2x+c\,d^2x+ade)}-\frac{}{a^4e^8-4a^3c}$
norman	$\frac{-\frac{3d^2c^2e^3x^3}{e^6a^3-3d^2e^4a^2c+3d^4e^2a\,c^2-d^6c^3}+\frac{\left(-a^2c^2d\,e^6-7a\,c^3d^3e^4-c^4d^5e^2\right)x}{ed\,c^2\left(e^6a^3-3d^2e^4a^2c+3d^4e^2a\,c^2-d^6c^3\right)}+\frac{-2a^2c^2d\,e^4-5a\,c^3d^3e^2+c^4d^5}{2c^2\left(e^6a^3-3d^2e^4a^2c+3d^4e^2a\,c^2-d^6c^3\right)}+\frac{}{2d^2c^2e^2(}}{(ex+d)^2(cdx+ae)^2}$
parallelrisch	$-\frac{12\ln(ex+d)x^2a\,c^4d^4e^5-12\ln(cdx+ae)x^2a\,c^4d^4e^5+6\ln(ex+d)x\,a^2c^3d^3e^6+12\ln(ex+d)xa\,c^4d^5e^4-6\ln(cdx+ae)x\,a^2c^3d^3}{}$

input
```
int((e*x+d)/(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^3,x,method=_RETURNVERBOSE)
```

output
```
-1/2*c*d/(a*e^2-c*d^2)^2/(c*d*x+a*e)^2+3*c*d/(a*e^2-c*d^2)^4*e^2*ln(c*d*x+
a*e)-2*c*d/(a*e^2-c*d^2)^3*e/(c*d*x+a*e)-e^2/(a*e^2-c*d^2)^3/(e*x+d)-3*c*d
/(a*e^2-c*d^2)^4*e^2*ln(e*x+d)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 555 vs. $2(140) = 280$.

Time = 0.08 (sec) , antiderivative size = 555, normalized size of antiderivative = 3.91

$$\int \frac{d+ex}{(ade+(cd^2+ae^2)\,x+cdex^2)^3}\,dx = -\frac{c^3d^6-6\,ac^2d^4e^2+3\,a^2cd^2e^4+2\,a^3e^6-6\,(c^3d^4e^2-ac^2d^2e^4)x^2-3\,(c^3d^5e+2\,ac^2d^3e^3-3\,a^2cde^5)x-\cdots}{2\,(a^2c^4d^9e^2-4\,a^3c^3d^7e^4+6\,a^4c^2d^5e^6-4\,a^5cd^3e^8+a^6de^{10}+(c^6d^{10}e-4\,ac^5d^8e^3+6\,a^2c^4d^6e^5-4\,a\cdots}$$

input
```
integrate((e*x+d)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^3,x, algorithm="fricas
")
```

output
```
-1/2*(c^3*d^6 - 6*a*c^2*d^4*e^2 + 3*a^2*c*d^2*e^4 + 2*a^3*e^6 - 6*(c^3*d^4
*e^2 - a*c^2*d^2*e^4)*x^2 - 3*(c^3*d^5*e + 2*a*c^2*d^3*e^3 - 3*a^2*c*d*e^5
)*x - 6*(c^3*d^3*e^3*x^3 + a^2*c*d^2*e^4 + (c^3*d^4*e^2 + 2*a*c^2*d^2*e^4)
*x^2 + (2*a*c^2*d^3*e^3 + a^2*c*d*e^5)*x)*log(c*d*x + a*e) + 6*(c^3*d^3*e^
3*x^3 + a^2*c*d^2*e^4 + (c^3*d^4*e^2 + 2*a*c^2*d^2*e^4)*x^2 + (2*a*c^2*d^3
*e^3 + a^2*c*d*e^5)*x)*log(e*x + d))/(a^2*c^4*d^9*e^2 - 4*a^3*c^3*d^7*e^4
+ 6*a^4*c^2*d^5*e^6 - 4*a^5*c*d^3*e^8 + a^6*d*e^10 + (c^6*d^10*e - 4*a*c^5
*d^8*e^3 + 6*a^2*c^4*d^6*e^5 - 4*a^3*c^3*d^4*e^7 + a^4*c^2*d^2*e^9)*x^3 +
(c^6*d^11 - 2*a*c^5*d^9*e^2 - 2*a^2*c^4*d^7*e^4 + 8*a^3*c^3*d^5*e^6 - 7*a^
4*c^2*d^3*e^8 + 2*a^5*c*d*e^10)*x^2 + (2*a*c^5*d^10*e - 7*a^2*c^4*d^8*e^3
+ 8*a^3*c^3*d^6*e^5 - 2*a^4*c^2*d^4*e^7 - 2*a^5*c*d^2*e^9 + a^6*e^11)*x)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 736 vs. $2(131) = 262$.

Time = 1.20 (sec) , antiderivative size = 736, normalized size of antiderivative = 5.18

$$\int \frac{d+ex}{(ade+(cd^2+ae^2)x+cdex^2)^3}\,dx = -\frac{3cde^2\log\left(x+\frac{-\frac{3a^5cde^{12}}{(ae^2-cd^2)^4}+\frac{15a^4c^2d^3e^{10}}{(ae^2-cd^2)^4}-\frac{30a^3c^3d^5e^8}{(ae^2-cd^2)^4}+\frac{30a^2c^4d^7e^6}{(ae^2-cd^2)^4}-\frac{15ac^5d^9e^4}{(ae^2-cd^2)^4}+3acde^4+\frac{3c^6d^{11}e^2}{(ae^2-cd^2)^4}+3c^2d^3e^2}{6c^2d^2e^3}\right)}{(ae^2-cd^2)^4} + \frac{3cde^2\log\left(x+\frac{\frac{3a^5cde^{12}}{(ae^2-cd^2)^4}-\frac{15a^4c^2d^3e^{10}}{(ae^2-cd^2)^4}+\frac{30a^3c^3d^5e^8}{(ae^2-cd^2)^4}-\frac{30a^2c^4d^7e^6}{(ae^2-cd^2)^4}+\frac{15ac^5d^9e^4}{(ae^2-cd^2)^4}+3acde^4-\frac{3c^6d^{11}e^2}{(ae^2-cd^2)^4}+3c^2d^3e^2}{6c^2d^2e^3}\right)}{(ae^2-cd^2)^4} + \frac{-2a^2e^4-5acd^2e^2+c^2d}{2a^5de^8-6a^4cd^3e^6+6a^3c^2d^5e^4-2a^2c^3d^7e^2+x^3\cdot(2a^3c^2d^2e^7-6a^2c^3d^4e^5+6ac^4d^6e^3-2c^5d^8e)+x^2\cdot}$$

input
```
integrate((e*x+d)/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**3,x)
```

output
```
-3*c*d*e**2*log(x + (-3*a**5*c*d*e**12/(a*e**2 - c*d**2)**4 + 15*a**4*c**2
*d**3*e**10/(a*e**2 - c*d**2)**4 - 30*a**3*c**3*d**5*e**8/(a*e**2 - c*d**2
)**4 + 30*a**2*c**4*d**7*e**6/(a*e**2 - c*d**2)**4 - 15*a*c**5*d**9*e**4/(
a*e**2 - c*d**2)**4 + 3*a*c*d*e**4 + 3*c**6*d**11*e**2/(a*e**2 - c*d**2)**
4 + 3*c**2*d**3*e**2)/(6*c**2*d**2*e**3))/(a*e**2 - c*d**2)**4 + 3*c*d*e**
2*log(x + (3*a**5*c*d*e**12/(a*e**2 - c*d**2)**4 - 15*a**4*c**2*d**3*e**10
/(a*e**2 - c*d**2)**4 + 30*a**3*c**3*d**5*e**8/(a*e**2 - c*d**2)**4 - 30*a
**2*c**4*d**7*e**6/(a*e**2 - c*d**2)**4 + 15*a*c**5*d**9*e**4/(a*e**2 - c*
d**2)**4 + 3*a*c*d*e**4 - 3*c**6*d**11*e**2/(a*e**2 - c*d**2)**4 + 3*c**2*
d**3*e**2)/(6*c**2*d**2*e**3))/(a*e**2 - c*d**2)**4 + (-2*a**2*e**4 - 5*a*
c*d**2*e**2 + c**2*d**4 - 6*c**2*d**2*e**2*x**2 + x*(-9*a*c*d*e**3 - 3*c**
2*d**3*e))/(2*a**5*d*e**8 - 6*a**4*c*d**3*e**6 + 6*a**3*c**2*d**5*e**4 - 2
*a**2*c**3*d**7*e**2 + x**3*(2*a**3*c**2*d**2*e**7 - 6*a**2*c**3*d**4*e**5
 + 6*a*c**4*d**6*e**3 - 2*c**5*d**8*e) + x**2*(4*a**4*c*d*e**8 - 10*a**3*c
**2*d**3*e**6 + 6*a**2*c**3*d**5*e**4 + 2*a*c**4*d**7*e**2 - 2*c**5*d**9)
+ x*(2*a**5*e**9 - 2*a**4*c*d**2*e**7 - 6*a**3*c**2*d**4*e**5 + 10*a**2*c*
*3*d**6*e**3 - 4*a*c**4*d**8*e))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 429 vs. $2(140) = 280$.

Time = 0.05 (sec) , antiderivative size = 429, normalized size of antiderivative = 3.02

$$\int \frac{d+ex}{(ade+(cd^2+ae^2)\,x+cdex^2)^3}\,dx = \frac{3\,cde^2\log\left(cdx+ae\right)}{c^4d^8-4\,ac^3d^6e^2+6\,a^2c^2d^4e^4-4\,a^3cd^2e^6+a^4e^8} - \frac{3\,cde^2\log\left(ex+d\right)}{c^4d^8-4\,ac^3d^6e^2+6\,a^2c^2d^4e^4-4\,a^3cd^2e^6+a^4e^8} + \frac{6\,c^2d^2e^2x^2-c^2d^4+5\,acd^2e}{2\,(a^2c^3d^7e^2-3\,a^3c^2d^5e^4+3\,a^4cd^3e^6-a^5de^8+(c^5d^8e-3\,ac^4d^6e^3+3\,a^2c^3d^4e^5-a^3c^2d^2e^7)x^3+(c^5d^9}$$

input
```
integrate((e*x+d)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^3,x, algorithm="maxima
")
```

output
```
3*c*d*e^2*log(c*d*x + a*e)/(c^4*d^8 - 4*a*c^3*d^6*e^2 + 6*a^2*c^2*d^4*e^4
- 4*a^3*c*d^2*e^6 + a^4*e^8) - 3*c*d*e^2*log(e*x + d)/(c^4*d^8 - 4*a*c^3*d
^6*e^2 + 6*a^2*c^2*d^4*e^4 - 4*a^3*c*d^2*e^6 + a^4*e^8) + 1/2*(6*c^2*d^2*e
^2*x^2 - c^2*d^4 + 5*a*c*d^2*e^2 + 2*a^2*e^4 + 3*(c^2*d^3*e + 3*a*c*d*e^3)
*x)/(a^2*c^3*d^7*e^2 - 3*a^3*c^2*d^5*e^4 + 3*a^4*c*d^3*e^6 - a^5*d*e^8 + (
c^5*d^8*e - 3*a*c^4*d^6*e^3 + 3*a^2*c^3*d^4*e^5 - a^3*c^2*d^2*e^7)*x^3 + (
c^5*d^9 - a*c^4*d^7*e^2 - 3*a^2*c^3*d^5*e^4 + 5*a^3*c^2*d^3*e^6 - 2*a^4*c*
d*e^8)*x^2 + (2*a*c^4*d^8*e - 5*a^2*c^3*d^6*e^3 + 3*a^3*c^2*d^4*e^5 + a^4*
c*d^2*e^7 - a^5*e^9)*x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 286 vs. $2(140) = 280$.

Time = 0.16 (sec) , antiderivative size = 286, normalized size of antiderivative = 2.01

$$\int \frac{d+ex}{(ade+(cd^2+ae^2)\,x+cdex^2)^3}\,dx = \frac{3\,c^2d^2e^2\log\left(|cdx+ae|\right)}{c^5d^9-4\,ac^4d^7e^2+6\,a^2c^3d^5e^4-4\,a^3c^2d^3e^6+a^4cde^8} - \frac{3\,cde^3\log\left(|ex+d|\right)}{c^4d^8e-4\,ac^3d^6e^3+6\,a^2c^2d^4e^5-4\,a^3cd^2e^7+a^4e^9} - \frac{c^3d^6-6\,ac^2d^4e^2+3\,a^2cd^2e^4+2\,a^3e^6-6\,(c^3d^4e^2-ac^2d^2e^4)x^2-3\,(c^3d^5e+2\,ac^2d^3e^3-3\,a^2cde^5)x}{2\,(cd^2-ae^2)^4(cdx+ae)^2(ex+d)}$$

input
```
integrate((e*x+d)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^3,x, algorithm="giac")
```

output
```
3*c^2*d^2*e^2*log(abs(c*d*x + a*e))/(c^5*d^9 - 4*a*c^4*d^7*e^2 + 6*a^2*c^3
*d^5*e^4 - 4*a^3*c^2*d^3*e^6 + a^4*c*d*e^8) - 3*c*d*e^3*log(abs(e*x + d))/
(c^4*d^8*e - 4*a*c^3*d^6*e^3 + 6*a^2*c^2*d^4*e^5 - 4*a^3*c*d^2*e^7 + a^4*e
^9) - 1/2*(c^3*d^6 - 6*a*c^2*d^4*e^2 + 3*a^2*c*d^2*e^4 + 2*a^3*e^6 - 6*(c^
3*d^4*e^2 - a*c^2*d^2*e^4)*x^2 - 3*(c^3*d^5*e + 2*a*c^2*d^3*e^3 - 3*a^2*c*
d*e^5)*x)/((c*d^2 - a*e^2)^4*(c*d*x + a*e)^2*(e*x + d))
```

Mupad [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 392, normalized size of antiderivative = 2.76

$$\int \frac{d+ex}{(ade+(cd^2+ae^2)x+cdex^2)^3}\,dx = \frac{6\,c\,d\,e^2\,\mathrm{atanh}\left(\frac{a^4\,e^8-2\,a^3\,c\,d^2\,e^6+2\,a\,c^3\,d^6\,e^2-c^4\,d^8}{{\left(a\,e^2-c\,d^2\right)}^4}+\frac{2\,c\,d\,e\,x\,\left(a^3\,e^6-3\,a^2\,c\,d^2\,e^4+3\,a\,c^2\,d^4\,e^2-c^3\,d^6\right)}{{\left(a\,e^2-c\,d^2\right)}^4}\right)}{{\left(a\,e^2-c\,d^2\right)}^4} - \frac{\frac{2\,a^2\,e^4+5\,a\,c\,d^2\,e^2-c^2\,d^4}{2\,\left(a^3\,e^6-3\,a^2\,c\,d^2\,e^4+3\,a\,c^2\,d^4\,e^2-c^3\,d^6\right)}+\frac{3\,e\,x\,\left(c^2\,d^3+3\,a\,c\,d\,e^2\right)}{2\,\left(a^3\,e^6-3\,a^2\,c\,d^2\,e^4+3\,a\,c^2\,d^4\,e^2-c^3\,d^6\right)}+\frac{3\,c^2\,d^2\,e^2\,x^2}{a^3\,e^6-3\,a^2\,c\,d^2\,e^4+3\,a\,c^2\,d^4\,e^2-c^3\,d^6}}{x\,\left(a^2\,e^3+2\,c\,a\,d^2\,e\right)+x^2\,\left(c^2\,d^3+2\,a\,c\,d\,e^2\right)+a^2\,d\,e^2+c^2\,d^2\,e\,x^3}$$

input
```
int((d + e*x)/(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^3,x)
```

output
```
(6*c*d*e^2*atanh((a^4*e^8 - c^4*d^8 + 2*a*c^3*d^6*e^2 - 2*a^3*c*d^2*e^6)/(
a*e^2 - c*d^2)^4 + (2*c*d*e*x*(a^3*e^6 - c^3*d^6 + 3*a*c^2*d^4*e^2 - 3*a^2
*c*d^2*e^4))/(a*e^2 - c*d^2)^4))/(a*e^2 - c*d^2)^4 - ((2*a^2*e^4 - c^2*d^4
 + 5*a*c*d^2*e^2)/(2*(a^3*e^6 - c^3*d^6 + 3*a*c^2*d^4*e^2 - 3*a^2*c*d^2*e^
4)) + (3*e*x*(c^2*d^3 + 3*a*c*d*e^2))/(2*(a^3*e^6 - c^3*d^6 + 3*a*c^2*d^4*
e^2 - 3*a^2*c*d^2*e^4)) + (3*c^2*d^2*e^2*x^2)/(a^3*e^6 - c^3*d^6 + 3*a*c^2
*d^4*e^2 - 3*a^2*c*d^2*e^4))/(x*(a^2*e^3 + 2*a*c*d^2*e) + x^2*(c^2*d^3 + 2
*a*c*d*e^2) + a^2*d*e^2 + c^2*d^2*e*x^3)
```

Reduce [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 911, normalized size of antiderivative = 6.42

$$\int \frac{d+ex}{(ade+(cd^2+ae^2)x+cdex^2)^3}\,dx = \text{Too large to display}$$

input
```
int((e*x+d)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^3,x)
```

output
```
(12*log(a*e + c*d*x)*a**3*c*d**2*e**6 + 12*log(a*e + c*d*x)*a**3*c*d*e**7*
x + 6*log(a*e + c*d*x)*a**2*c**2*d**4*e**4 + 30*log(a*e + c*d*x)*a**2*c**2
*d**3*e**5*x + 24*log(a*e + c*d*x)*a**2*c**2*d**2*e**6*x**2 + 12*log(a*e +
 c*d*x)*a*c**3*d**5*e**3*x + 24*log(a*e + c*d*x)*a*c**3*d**4*e**4*x**2 + 1
2*log(a*e + c*d*x)*a*c**3*d**3*e**5*x**3 + 6*log(a*e + c*d*x)*c**4*d**6*e*
*2*x**2 + 6*log(a*e + c*d*x)*c**4*d**5*e**3*x**3 - 12*log(d + e*x)*a**3*c*
d**2*e**6 - 12*log(d + e*x)*a**3*c*d*e**7*x - 6*log(d + e*x)*a**2*c**2*d**
4*e**4 - 30*log(d + e*x)*a**2*c**2*d**3*e**5*x - 24*log(d + e*x)*a**2*c**2
*d**2*e**6*x**2 - 12*log(d + e*x)*a*c**3*d**5*e**3*x - 24*log(d + e*x)*a*c
**3*d**4*e**4*x**2 - 12*log(d + e*x)*a*c**3*d**3*e**5*x**3 - 6*log(d + e*x
)*c**4*d**6*e**2*x**2 - 6*log(d + e*x)*c**4*d**5*e**3*x**3 - 4*a**4*e**8 -
 2*a**3*c*d**2*e**6 - 12*a**3*c*d*e**7*x + 3*a**2*c**2*d**4*e**4 + 9*a**2*
c**2*d**3*e**5*x + 4*a*c**3*d**6*e**2 + 6*a*c**3*d**3*e**5*x**3 - c**4*d**
8 + 3*c**4*d**7*e*x - 6*c**4*d**5*e**3*x**3)/(2*(2*a**7*d*e**12 + 2*a**7*e
**13*x - 7*a**6*c*d**3*e**10 - 3*a**6*c*d**2*e**11*x + 4*a**6*c*d*e**12*x*
*2 + 8*a**5*c**2*d**5*e**8 - 6*a**5*c**2*d**4*e**9*x - 12*a**5*c**2*d**3*e
**10*x**2 + 2*a**5*c**2*d**2*e**11*x**3 - 2*a**4*c**3*d**7*e**6 + 14*a**4*
c**3*d**6*e**7*x + 9*a**4*c**3*d**5*e**8*x**2 - 7*a**4*c**3*d**4*e**9*x**3
 - 2*a**3*c**4*d**9*e**4 - 6*a**3*c**4*d**8*e**5*x + 4*a**3*c**4*d**7*e**6
*x**2 + 8*a**3*c**4*d**6*e**7*x**3 + a**2*c**5*d**11*e**2 - 3*a**2*c**5...
```

3.136 $\int \frac{1}{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^3}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 189

$$\int \frac{1}{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^3}\,dx = -\frac{c^2d^2}{2\left(cd^2-ae^2\right)^3(ae+cdx)^2} + \frac{3c^2d^2e}{\left(cd^2-ae^2\right)^4(ae+cdx)} + \frac{e^2}{2\left(cd^2-ae^2\right)^3(d+ex)^2} + \frac{3cde^2}{\left(cd^2-ae^2\right)^4(d+ex)} + \frac{6c^2d^2e^2\log(ae+cdx)}{\left(cd^2-ae^2\right)^5} - \frac{6c^2d^2e^2\log(d+ex)}{\left(cd^2-ae^2\right)^5}$$

output
```
-1/2*c^2*d^2/(-a*e^2+c*d^2)^3/(c*d*x+a*e)^2+3*c^2*d^2*e/(-a*e^2+c*d^2)^4/(
c*d*x+a*e)+1/2*e^2/(-a*e^2+c*d^2)^3/(e*x+d)^2+3*c*d*e^2/(-a*e^2+c*d^2)^4/(
e*x+d)+6*c^2*d^2*e^2*ln(c*d*x+a*e)/(-a*e^2+c*d^2)^5-6*c^2*d^2*e^2*ln(e*x+d
)/(-a*e^2+c*d^2)^5
```

Mathematica [A] (verified)

Time = 0.10 (sec), antiderivative size = 168, normalized size of antiderivative = 0.89

$$\int \frac{1}{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^3}\,dx$$
$$=\frac{\frac{c^2d^2\left(cd^2-ae^2\right)^2}{(ae+cdx)^2}+\frac{6c^2d^2e\left(-cd^2+ae^2\right)}{ae+cdx}-\frac{\left(cd^2e-ae^3\right)^2}{(d+ex)^2}+\frac{6cde^2\left(-cd^2+ae^2\right)}{d+ex}-12c^2d^2e^2\log(ae+cdx)+12c^2d^2e^2\log(d\cdots}{2\left(-cd^2+ae^2\right)^5}$$

input
```
Integrate[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(-3),x]
```

output
```
((c^2*d^2*(c*d^2 - a*e^2)^2)/(a*e + c*d*x)^2 + (6*c^2*d^2*e*(-(c*d^2) + a*
e^2))/(a*e + c*d*x) - (c*d^2*e - a*e^3)^2/(d + e*x)^2 + (6*c*d*e^2*(-(c*d^
2) + a*e^2))/(d + e*x) - 12*c^2*d^2*e^2*Log[a*e + c*d*x] + 12*c^2*d^2*e^2*
Log[d + e*x])/(2*(-(c*d^2) + a*e^2)^5)
```

Rubi [A] (verified)

Time = 0.75 (sec), antiderivative size = 214, normalized size of antiderivative = 1.13, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.074, Rules used = {1084, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^3}\,dx$$
$$\Big\downarrow \text{1084}$$
$$c^3d^3e^3\int\left(-\frac{6}{cd\left(cd^2-ae^2\right)^5(d+ex)}-\frac{3}{c^2d^2\left(cd^2-ae^2\right)^4(d+ex)^2}-\frac{1}{c^3d^3\left(cd^2-ae^2\right)^3(d+ex)^3}+\frac{\cdots}{e\left(cd^2-ae^{\cdots}\right.}\cdots\right.$$
$$\Big\downarrow \text{2009}$$
$$c^3d^3e^3\left(\frac{1}{2c^3d^3e(d+ex)^2\left(cd^2-ae^2\right)^3}+\frac{3}{c^2d^2e(d+ex)\left(cd^2-ae^2\right)^4}+\frac{3}{cde^2\left(cd^2-ae^2\right)^4(ae+cdx)}+\frac{6\log(ae+\cdots}{cde\left(cd^2-\cdots\right.}\cdots\right.$$

input
```
Int[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(-3),x]
```

output
```
c^3*d^3*e^3*(-1/2*1/(c*d*e^3*(c*d^2 - a*e^2)^3*(a*e + c*d*x)^2) + 3/(c*d*e
^2*(c*d^2 - a*e^2)^4*(a*e + c*d*x)) + 1/(2*c^3*d^3*e*(c*d^2 - a*e^2)^3*(d
+ e*x)^2) + 3/(c^2*d^2*e*(c*d^2 - a*e^2)^4*(d + e*x)) + (6*Log[a*e + c*d*x
])/(c*d*e*(c*d^2 - a*e^2)^5) - (6*Log[d + e*x])/(c*d*e*(c*d^2 - a*e^2)^5))
```

Defintions of rubi rules used

rule 1084
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{q = Rt[b^2
- 4*a*c, 2]}, Simp[1/c^p   Int[ExpandIntegrand[(b/2 - q/2 + c*x)^p*(b/2 + q
/2 + c*x)^p, x], x], x] /;  !FractionalPowerFactorQ[q]] /; FreeQ[{a, b, c},
 x] && IntegerQ[p] && NiceSqrtQ[b^2 - 4*a*c]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 1.57 (sec) , antiderivative size = 186, normalized size of antiderivative = 0.98

method	result
default	$\frac{c^2d^2}{2(a\,e^2-c\,d^2)^3(cdx+ae)^2}-\frac{6c^2d^2e^2\ln(cdx+ae)}{(a\,e^2-c\,d^2)^5}+\frac{3c^2d^2e}{(a\,e^2-c\,d^2)^4(cdx+ae)}-\frac{e^2}{2(a\,e^2-c\,d^2)^3(ex+d)^2}+\frac{6c^2d^2e^2\ln(ex+d}{(a\,e^2-c\,d^2)^5}$
risch	$\frac{\frac{6c^3d^3e^3x^3}{a^4e^8-4a^3c\,d^2e^6+6a^2c^2d^4e^4-4a\,c^3d^6e^2+c^4d^8}+\frac{9d^2e^2c^2\left(a\,e^2+c\,d^2\right)x^2}{a^4e^8-4a^3c\,d^2e^6+6a^2c^2d^4e^4-4a\,c^3d^6e^2+c^4d^8}+\frac{2\left(a^2e^4+7ac\,d^2e^2+c^2d^4\right)ec}{a^4e^8-4a^3c\,d^2e^6+6a^2c^2d^4e^4-4a\,c^3}}{(cd\,x^2e+a\,e^2x+c\,d^2x+ade)^2}$
norman	$\frac{\frac{\left(9a\,c^4d^4e^6+9c^5d^6e^4\right)x^2}{d^2c^2e^2\left(a^4e^8-4a^3c\,d^2e^6+6a^2c^2d^4e^4-4a\,c^3d^6e^2+c^4d^8\right)}+\frac{-a^3c^2e^6+7a^2c^3d^2e^4+7a\,c^4d^4e^2-c^5d^6}{2c^2\left(a^4e^8-4a^3c\,d^2e^6+6a^2c^2d^4e^4-4a\,c^3d^6e^2+c^4d^8\right)}+\frac{6c^3}{a^4e^8-4a^3c\,d^2e^6+6a}}{(ex+d)^2(cdx+ae)^2}$
parallelrisch	$\frac{-a^4c^2d^2e^{10}+8a^3c^3d^4e^8-8a\,c^5d^8e^4-12x^3c^6d^7e^5-18x^2c^6d^8e^4-4x\,c^6d^9e^3+12x^3a\,c^5d^5e^7+18x^2a^2c^4d^4e^8+4x\,a^3c^3d^3e^9+24}{}$

input
```
int(1/(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^3,x,method=_RETURNVERBOSE)
```

output
```
1/2*c^2*d^2/(a*e^2-c*d^2)^3/(c*d*x+a*e)^2-6*c^2*d^2/(a*e^2-c*d^2)^5*e^2*ln
(c*d*x+a*e)+3*c^2*d^2/(a*e^2-c*d^2)^4*e/(c*d*x+a*e)-1/2*e^2/(a*e^2-c*d^2)^
3/(e*x+d)^2+6*c^2*d^2/(a*e^2-c*d^2)^5*e^2*ln(e*x+d)+3*e^2/(a*e^2-c*d^2)^4*
c*d/(e*x+d)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 828 vs. $2(185) = 370$.

Time = 0.09 (sec) , antiderivative size = 828, normalized size of antiderivative = 4.38

$$\int \frac{1}{(ade + (cd^2 + ae^2)\,x + cdex^2)^3}\,dx = -\frac{c^4d^8 - 8\,ac^3d^6e^2 + 8\,a^3cd^2e^6 - a^4e^8 - 12\,(c^4d^5e^3 - ac^3d^3e^5)x^3 - 18\,(c^4d^6e^2 - a^2c^2d^2e^6)x^2 - 4}{2\,(a^2c^5d^{12}e^2 - 5\,a^3c^4d^{10}e^4 + 10\,a^4c^3d^8e^6 - 10\,a^5c^2d^6e^8 + 5\,a^6cd^4e^{10} - a^7d^2e^{12} + (c^7d^{12}e^2 - 5\,ac^6d^{10}e^4}$$

input
```
integrate(1/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^3,x, algorithm="fricas")
```

output
```
-1/2*(c^4*d^8 - 8*a*c^3*d^6*e^2 + 8*a^3*c*d^2*e^6 - a^4*e^8 - 12*(c^4*d^5*
e^3 - a*c^3*d^3*e^5)*x^3 - 18*(c^4*d^6*e^2 - a^2*c^2*d^2*e^6)*x^2 - 4*(c^4
*d^7*e + 6*a*c^3*d^5*e^3 - 6*a^2*c^2*d^3*e^5 - a^3*c*d*e^7)*x - 12*(c^4*d^
4*e^4*x^4 + a^2*c^2*d^4*e^4 + 2*(c^4*d^5*e^3 + a*c^3*d^3*e^5)*x^3 + (c^4*d
^6*e^2 + 4*a*c^3*d^4*e^4 + a^2*c^2*d^2*e^6)*x^2 + 2*(a*c^3*d^5*e^3 + a^2*c
^2*d^3*e^5)*x)*log(c*d*x + a*e) + 12*(c^4*d^4*e^4*x^4 + a^2*c^2*d^4*e^4 +
2*(c^4*d^5*e^3 + a*c^3*d^3*e^5)*x^3 + (c^4*d^6*e^2 + 4*a*c^3*d^4*e^4 + a^2
*c^2*d^2*e^6)*x^2 + 2*(a*c^3*d^5*e^3 + a^2*c^2*d^3*e^5)*x)*log(e*x + d))/(
a^2*c^5*d^12*e^2 - 5*a^3*c^4*d^10*e^4 + 10*a^4*c^3*d^8*e^6 - 10*a^5*c^2*d^
6*e^8 + 5*a^6*c*d^4*e^10 - a^7*d^2*e^12 + (c^7*d^12*e^2 - 5*a*c^6*d^10*e^4
 + 10*a^2*c^5*d^8*e^6 - 10*a^3*c^4*d^6*e^8 + 5*a^4*c^3*d^4*e^10 - a^5*c^2*
d^2*e^12)*x^4 + 2*(c^7*d^13*e - 4*a*c^6*d^11*e^3 + 5*a^2*c^5*d^9*e^5 - 5*a
^4*c^3*d^5*e^9 + 4*a^5*c^2*d^3*e^11 - a^6*c*d*e^13)*x^3 + (c^7*d^14 - a*c^
6*d^12*e^2 - 9*a^2*c^5*d^10*e^4 + 25*a^3*c^4*d^8*e^6 - 25*a^4*c^3*d^6*e^8
+ 9*a^5*c^2*d^4*e^10 + a^6*c*d^2*e^12 - a^7*e^14)*x^2 + 2*(a*c^6*d^13*e -
4*a^2*c^5*d^11*e^3 + 5*a^3*c^4*d^9*e^5 - 5*a^5*c^2*d^5*e^9 + 4*a^6*c*d^3*e
^11 - a^7*d*e^13)*x)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1001 vs. $2(173) = 346$.

Time = 1.40 (sec) , antiderivative size = 1001, normalized size of antiderivative = 5.30

$$\int \frac{1}{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^3}\,dx = \text{Too large to display}$$

input
```
integrate(1/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**3,x)
```

output
```
6*c**2*d**2*e**2*log(x + (-6*a**6*c**2*d**2*e**14/(a*e**2 - c*d**2)**5 + 3
6*a**5*c**3*d**4*e**12/(a*e**2 - c*d**2)**5 - 90*a**4*c**4*d**6*e**10/(a*e
**2 - c*d**2)**5 + 120*a**3*c**5*d**8*e**8/(a*e**2 - c*d**2)**5 - 90*a**2*
c**6*d**10*e**6/(a*e**2 - c*d**2)**5 + 36*a*c**7*d**12*e**4/(a*e**2 - c*d*
*2)**5 + 6*a*c**2*d**2*e**4 - 6*c**8*d**14*e**2/(a*e**2 - c*d**2)**5 + 6*c
**3*d**4*e**2)/(12*c**3*d**3*e**3))/(a*e**2 - c*d**2)**5 - 6*c**2*d**2*e**
2*log(x + (6*a**6*c**2*d**2*e**14/(a*e**2 - c*d**2)**5 - 36*a**5*c**3*d**4
*e**12/(a*e**2 - c*d**2)**5 + 90*a**4*c**4*d**6*e**10/(a*e**2 - c*d**2)**5
 - 120*a**3*c**5*d**8*e**8/(a*e**2 - c*d**2)**5 + 90*a**2*c**6*d**10*e**6/
(a*e**2 - c*d**2)**5 - 36*a*c**7*d**12*e**4/(a*e**2 - c*d**2)**5 + 6*a*c**
2*d**2*e**4 + 6*c**8*d**14*e**2/(a*e**2 - c*d**2)**5 + 6*c**3*d**4*e**2)/(
12*c**3*d**3*e**3))/(a*e**2 - c*d**2)**5 + (-a**3*e**6 + 7*a**2*c*d**2*e**
4 + 7*a*c**2*d**4*e**2 - c**3*d**6 + 12*c**3*d**3*e**3*x**3 + x**2*(18*a*c
**2*d**2*e**4 + 18*c**3*d**4*e**2) + x*(4*a**2*c*d*e**5 + 28*a*c**2*d**3*e
**3 + 4*c**3*d**5*e))/(2*a**6*d**2*e**10 - 8*a**5*c*d**4*e**8 + 12*a**4*c*
*2*d**6*e**6 - 8*a**3*c**3*d**8*e**4 + 2*a**2*c**4*d**10*e**2 + x**4*(2*a*
*4*c**2*d**2*e**10 - 8*a**3*c**3*d**4*e**8 + 12*a**2*c**4*d**6*e**6 - 8*a*
c**5*d**8*e**4 + 2*c**6*d**10*e**2) + x**3*(4*a**5*c*d*e**11 - 12*a**4*c**
2*d**3*e**9 + 8*a**3*c**3*d**5*e**7 + 8*a**2*c**4*d**7*e**5 - 12*a*c**5*d*
*9*e**3 + 4*c**6*d**11*e) + x**2*(2*a**6*e**12 - 18*a**4*c**2*d**4*e**8...
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 642 vs. $2(185) = 370$.

Time = 0.04 (sec) , antiderivative size = 642, normalized size of antiderivative = 3.40

$$\int \frac{1}{(ade + (cd^2 + ae^2)x + cdex^2)^3} dx$$
$$= \frac{6c^2d^2e^2 \log (cdx + ae)}{c^5d^{10} - 5ac^4d^8e^2 + 10a^2c^3d^6e^4 - 10a^3c^2d^4e^6 + 5a^4cd^2e^8 - a^5e^{10}}$$
$$- \frac{6c^2d^2e^2 \log (ex + d)}{c^5d^{10} - 5ac^4d^8e^2 + 10a^2c^3d^6e^4 - 10a^3c^2d^4e^6 + 5a^4cd^2e^8 - a^5e^{10}}$$
$$+ \frac{}{2(a^2c^4d^{10}e^2 - 4a^3c^3d^8e^4 + 6a^4c^2d^6e^6 - 4a^5cd^4e^8 + a^6d^2e^{10} + (c^6d^{10}e^2 - 4ac^5d^8e^4 + 6a^2c^4d^6e^6 - 4a}$$

input
```
integrate(1/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^3,x, algorithm="maxima")
```

output
```
6*c^2*d^2*e^2*log(c*d*x + a*e)/(c^5*d^10 - 5*a*c^4*d^8*e^2 + 10*a^2*c^3*d^
6*e^4 - 10*a^3*c^2*d^4*e^6 + 5*a^4*c*d^2*e^8 - a^5*e^10) - 6*c^2*d^2*e^2*l
og(e*x + d)/(c^5*d^10 - 5*a*c^4*d^8*e^2 + 10*a^2*c^3*d^6*e^4 - 10*a^3*c^2*
d^4*e^6 + 5*a^4*c*d^2*e^8 - a^5*e^10) + 1/2*(12*c^3*d^3*e^3*x^3 - c^3*d^6
+ 7*a*c^2*d^4*e^2 + 7*a^2*c*d^2*e^4 - a^3*e^6 + 18*(c^3*d^4*e^2 + a*c^2*d^
2*e^4)*x^2 + 4*(c^3*d^5*e + 7*a*c^2*d^3*e^3 + a^2*c*d*e^5)*x)/(a^2*c^4*d^1
0*e^2 - 4*a^3*c^3*d^8*e^4 + 6*a^4*c^2*d^6*e^6 - 4*a^5*c*d^4*e^8 + a^6*d^2*
e^10 + (c^6*d^10*e^2 - 4*a*c^5*d^8*e^4 + 6*a^2*c^4*d^6*e^6 - 4*a^3*c^3*d^4
*e^8 + a^4*c^2*d^2*e^10)*x^4 + 2*(c^6*d^11*e - 3*a*c^5*d^9*e^3 + 2*a^2*c^4
*d^7*e^5 + 2*a^3*c^3*d^5*e^7 - 3*a^4*c^2*d^3*e^9 + a^5*c*d*e^11)*x^3 + (c^
6*d^12 - 9*a^2*c^4*d^8*e^4 + 16*a^3*c^3*d^6*e^6 - 9*a^4*c^2*d^4*e^8 + a^6*
e^12)*x^2 + 2*(a*c^5*d^11*e - 3*a^2*c^4*d^9*e^3 + 2*a^3*c^3*d^7*e^5 + 2*a^
4*c^2*d^5*e^7 - 3*a^5*c*d^3*e^9 + a^6*d*e^11)*x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 386 vs. $2(185) = 370$.

Time = 0.11 (sec) , antiderivative size = 386, normalized size of antiderivative = 2.04

$$\int \frac{1}{(ade+(cd^2+ae^2)x+cdex^2)^3}\,dx = \frac{6c^3d^3e^2\log(|cdx+ae|)}{c^6d^{11}-5ac^5d^9e^2+10a^2c^4d^7e^4-10a^3c^3d^5e^6+5a^4c^2d^3e^8-a^5cde^{10}} - \frac{6c^2d^2e^3\log(|ex+d|)}{c^5d^{10}e-5ac^4d^8e^3+10a^2c^3d^6e^5-10a^3c^2d^4e^7+5a^4cd^2e^9-a^5e^{11}} + \frac{12c^3d^3e^3x^3+18c^3d^4e^2x^2+18ac^2d^2e^4x^2+4c^3d^5ex+28ac^2d^3e^3x+4a^2cde^5x-c^3d^6+7ac^2d^4e^2+7}{2(c^4d^8-4ac^3d^6e^2+6a^2c^2d^4e^4-4a^3cd^2e^6+a^4e^8)(cdex^2+cd^2x+ae^2x+ade)^2}$$

input
```
integrate(1/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^3,x, algorithm="giac")
```

output
```
6*c^3*d^3*e^2*log(abs(c*d*x + a*e))/(c^6*d^11 - 5*a*c^5*d^9*e^2 + 10*a^2*c
^4*d^7*e^4 - 10*a^3*c^3*d^5*e^6 + 5*a^4*c^2*d^3*e^8 - a^5*c*d*e^10) - 6*c^
2*d^2*e^3*log(abs(e*x + d))/(c^5*d^10*e - 5*a*c^4*d^8*e^3 + 10*a^2*c^3*d^6
*e^5 - 10*a^3*c^2*d^4*e^7 + 5*a^4*c*d^2*e^9 - a^5*e^11) + 1/2*(12*c^3*d^3*
e^3*x^3 + 18*c^3*d^4*e^2*x^2 + 18*a*c^2*d^2*e^4*x^2 + 4*c^3*d^5*e*x + 28*a
*c^2*d^3*e^3*x + 4*a^2*c*d*e^5*x - c^3*d^6 + 7*a*c^2*d^4*e^2 + 7*a^2*c*d^2
*e^4 - a^3*e^6)/((c^4*d^8 - 4*a*c^3*d^6*e^2 + 6*a^2*c^2*d^4*e^4 - 4*a^3*c*
d^2*e^6 + a^4*e^8)*(c*d*e*x^2 + c*d^2*x + a*e^2*x + a*d*e)^2)
```

Mupad [B] (verification not implemented)

Time = 5.83 (sec) , antiderivative size = 616, normalized size of antiderivative = 3.26

$$\int \frac{1}{(ade+(cd^2+ae^2)x+cdex^2)^3}\,dx = \frac{\frac{9\,c\,x^2\,(c^2\,d^4\,e^2+a\,c\,d^2\,e^4)}{a^4\,e^8-4\,a^3\,c\,d^2\,e^6+6\,a^2\,c^2\,d^4\,e^4-4\,a\,c^3\,d^6\,e^2+c^4\,d^8} - \frac{a^3\,e^6-7\,a^2\,c\,d^2\,e^4-7\,a\,c^2\,d^4\,e^2+c^3\,d^6}{2\,(a^4\,e^8-4\,a^3\,c\,d^2\,e^6+6\,a^2\,c^2\,d^4\,e^4-4\,a\,c^3\,d^6\,e^2+c^4\,d^8)} + \frac{6\,c^3\,d^3\,e^3}{a^4\,e^8-4\,a^3\,c\,d^2\,e^6+6\,a^2\,c^2\,d^4}}{x^2\,(a^2\,e^4+4\,a\,c\,d^2\,e^2+c^2\,d^4)+x^3\,(2\,c^2\,d^3\,e+2\,a\,c\,d\,e^3)+x\,(2\,a^2\,d\,e^3+2\,c} - \frac{12\,c^2\,d^2\,e^2\,\mathrm{atanh}\left(\frac{a^5\,e^{10}-3\,a^4\,c\,d^2\,e^8+2\,a^3\,c^2\,d^4\,e^6+2\,a^2\,c^3\,d^6\,e^4-3\,a\,c^4\,d^8\,e^2+c^5\,d^{10}}{(a\,e^2-c\,d^2)^5} + \frac{2\,c\,d\,e\,x\,(a^4\,e^8-4\,a^3\,c\,d^2\,e^6+6\,a^2\,c^2\,d^4\,e^4-4}{(a\,e^2-c\,d^2)^5}\right.}{(a\,e^2-c\,d^2)^5}$$

input
```
int(1/(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^3,x)
```

output
```
((9*c*x^2*(c^2*d^4*e^2 + a*c*d^2*e^4))/(a^4*e^8 + c^4*d^8 - 4*a*c^3*d^6*e^
2 - 4*a^3*c*d^2*e^6 + 6*a^2*c^2*d^4*e^4) - (a^3*e^6 + c^3*d^6 - 7*a*c^2*d^
4*e^2 - 7*a^2*c*d^2*e^4)/(2*(a^4*e^8 + c^4*d^8 - 4*a*c^3*d^6*e^2 - 4*a^3*c
*d^2*e^6 + 6*a^2*c^2*d^4*e^4)) + (6*c^3*d^3*e^3*x^3)/(a^4*e^8 + c^4*d^8 -
4*a*c^3*d^6*e^2 - 4*a^3*c*d^2*e^6 + 6*a^2*c^2*d^4*e^4) + (2*c*d*e*x*(a^2*e
^4 + c^2*d^4 + 7*a*c*d^2*e^2))/(a^4*e^8 + c^4*d^8 - 4*a*c^3*d^6*e^2 - 4*a^
3*c*d^2*e^6 + 6*a^2*c^2*d^4*e^4))/(x^2*(a^2*e^4 + c^2*d^4 + 4*a*c*d^2*e^2)
 + x^3*(2*c^2*d^3*e + 2*a*c*d*e^3) + x*(2*a^2*d*e^3 + 2*a*c*d^3*e) + a^2*d
^2*e^2 + c^2*d^2*e^2*x^4) - (12*c^2*d^2*e^2*atanh((a^5*e^10 + c^5*d^10 - 3
*a*c^4*d^8*e^2 - 3*a^4*c*d^2*e^8 + 2*a^2*c^3*d^6*e^4 + 2*a^3*c^2*d^4*e^6)/
(a*e^2 - c*d^2)^5 + (2*c*d*e*x*(a^4*e^8 + c^4*d^8 - 4*a*c^3*d^6*e^2 - 4*a^
3*c*d^2*e^6 + 6*a^2*c^2*d^4*e^4))/(a*e^2 - c*d^2)^5))/(a*e^2 - c*d^2)^5
```

Reduce [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 1309, normalized size of antiderivative = 6.93

$$\int \frac{1}{(ade + (cd^2 + ae^2)x + cdex^2)^3} \, dx = \text{Too large to display}$$

input
```
int(1/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^3,x)
```

output
```
( - 12*log(a*e + c*d*x)*a**3*c**2*d**4*e**6 - 24*log(a*e + c*d*x)*a**3*c**
2*d**3*e**7*x - 12*log(a*e + c*d*x)*a**3*c**2*d**2*e**8*x**2 - 12*log(a*e
+ c*d*x)*a**2*c**3*d**6*e**4 - 48*log(a*e + c*d*x)*a**2*c**3*d**5*e**5*x -
 60*log(a*e + c*d*x)*a**2*c**3*d**4*e**6*x**2 - 24*log(a*e + c*d*x)*a**2*c
**3*d**3*e**7*x**3 - 24*log(a*e + c*d*x)*a*c**4*d**7*e**3*x - 60*log(a*e +
 c*d*x)*a*c**4*d**6*e**4*x**2 - 48*log(a*e + c*d*x)*a*c**4*d**5*e**5*x**3
- 12*log(a*e + c*d*x)*a*c**4*d**4*e**6*x**4 - 12*log(a*e + c*d*x)*c**5*d**
8*e**2*x**2 - 24*log(a*e + c*d*x)*c**5*d**7*e**3*x**3 - 12*log(a*e + c*d*x
)*c**5*d**6*e**4*x**4 + 12*log(d + e*x)*a**3*c**2*d**4*e**6 + 24*log(d + e
*x)*a**3*c**2*d**3*e**7*x + 12*log(d + e*x)*a**3*c**2*d**2*e**8*x**2 + 12*
log(d + e*x)*a**2*c**3*d**6*e**4 + 48*log(d + e*x)*a**2*c**3*d**5*e**5*x +
 60*log(d + e*x)*a**2*c**3*d**4*e**6*x**2 + 24*log(d + e*x)*a**2*c**3*d**3
*e**7*x**3 + 24*log(d + e*x)*a*c**4*d**7*e**3*x + 60*log(d + e*x)*a*c**4*d
**6*e**4*x**2 + 48*log(d + e*x)*a*c**4*d**5*e**5*x**3 + 12*log(d + e*x)*a*
c**4*d**4*e**6*x**4 + 12*log(d + e*x)*c**5*d**8*e**2*x**2 + 24*log(d + e*x
)*c**5*d**7*e**3*x**3 + 12*log(d + e*x)*c**5*d**6*e**4*x**4 - a**5*e**10 +
 7*a**4*c*d**2*e**8 + 4*a**4*c*d*e**9*x + 2*a**3*c**2*d**4*e**6 + 16*a**3*
c**2*d**3*e**7*x + 12*a**3*c**2*d**2*e**8*x**2 - 2*a**2*c**3*d**6*e**4 - 7
*a*c**4*d**8*e**2 - 16*a*c**4*d**7*e**3*x - 6*a*c**4*d**4*e**6*x**4 + c**5
*d**10 - 4*c**5*d**9*e*x - 12*c**5*d**8*e**2*x**2 + 6*c**5*d**6*e**4*x*...
```

3.137 $\int \frac{1}{(d+ex)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^3}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 223

$$\int \frac{1}{(d+ex)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^3}\,dx = -\frac{c^3d^3}{2\left(cd^2-ae^2\right)^4(ae+cdx)^2}+\frac{4c^3d^3e}{\left(cd^2-ae^2\right)^5(ae+cdx)}+\frac{e^2}{3\left(cd^2-ae^2\right)^3(d+ex)^3}+\frac{3cde^2}{2\left(cd^2-ae^2\right)^4(d+ex)^2}+\frac{6c^2d^2e^2}{\left(cd^2-ae^2\right)^5(d+ex)}+\frac{10c^3d^3e^2\log(ae+cdx)}{\left(cd^2-ae^2\right)^6}-\frac{10c^3d^3e^2\log(d+ex)}{\left(cd^2-ae^2\right)^6}$$

output
```
-1/2*c^3*d^3/(-a*e^2+c*d^2)^4/(c*d*x+a*e)^2+4*c^3*d^3*e/(-a*e^2+c*d^2)^5/(
c*d*x+a*e)+1/3*e^2/(-a*e^2+c*d^2)^3/(e*x+d)^3+3/2*c*d*e^2/(-a*e^2+c*d^2)^4
/(e*x+d)^2+6*c^2*d^2*e^2/(-a*e^2+c*d^2)^5/(e*x+d)+10*c^3*d^3*e^2*ln(c*d*x+
a*e)/(-a*e^2+c*d^2)^6-10*c^3*d^3*e^2*ln(e*x+d)/(-a*e^2+c*d^2)^6
```

Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 201, normalized size of antiderivative = 0.90

$$\int \frac{1}{(d+ex)\left(ade+(cd^2+ae^2)x+cdex^2\right)^3}\,dx = \frac{-\frac{3c^3d^3(cd^2-ae^2)^2}{(ae+cdx)^2}+\frac{24c^3d^3e(cd^2-ae^2)}{ae+cdx}-\frac{2e^2(-cd^2+ae^2)^3}{(d+ex)^3}+\frac{9cd(cd^2e-ae^3)^2}{(d+ex)^2}+\frac{36c^2d^2e^2(cd^2-ae^2)}{d+ex}+60c^3d^3e^2\log(ae+c}{6\left(cd^2-ae^2\right)^6}$$

input
```
Integrate[1/((d + e*x)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^3),x]
```

output
```
((-3*c^3*d^3*(c*d^2 - a*e^2)^2)/(a*e + c*d*x)^2 + (24*c^3*d^3*e*(c*d^2 - a
*e^2))/(a*e + c*d*x) - (2*e^2*(-(c*d^2) + a*e^2)^3)/(d + e*x)^3 + (9*c*d*(
c*d^2*e - a*e^3)^2)/(d + e*x)^2 + (36*c^2*d^2*e^2*(c*d^2 - a*e^2))/(d + e*
x) + 60*c^3*d^3*e^2*Log[a*e + c*d*x] - 60*c^3*d^3*e^2*Log[d + e*x])/(6*(c*
d^2 - a*e^2)^6)
```

Rubi [A] (verified)

Time = 0.77 (sec) , antiderivative size = 223, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.057$, Rules used = {1121, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(d+ex)\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^3}\,dx$$

$$\Big\downarrow \text{1121}$$

$$\int \left(\frac{10c^4d^4e^2}{(cd^2-ae^2)^6(ae+cdx)}-\frac{4c^4d^4e}{(cd^2-ae^2)^5(ae+cdx)^2}+\frac{c^4d^4}{(cd^2-ae^2)^4(ae+cdx)^3}-\frac{10c^3d^3e^3}{(d+ex)(cd^2-ae^2)^6}-\right.$$

$$\Big\downarrow \text{2009}$$

$$\frac{4c^3d^3e}{(cd^2-ae^2)^5(ae+cdx)} - \frac{c^3d^3}{2(cd^2-ae^2)^4(ae+cdx)^2} + \frac{10c^3d^3e^2\log(ae+cdx)}{(cd^2-ae^2)^6} - \frac{10c^3d^3e^2\log(d+ex)}{(cd^2-ae^2)^6} + \frac{6c^2d^2e^2}{(d+ex)(cd^2-ae^2)^5} + \frac{3cde^2}{2(d+ex)^2(cd^2-ae^2)^4} + \frac{e^2}{3(d+ex)^3(cd^2-ae^2)^3}$$

input
```
Int[1/((d + e*x)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^3),x]
```

output
```
-1/2*(c^3*d^3)/((c*d^2 - a*e^2)^4*(a*e + c*d*x)^2) + (4*c^3*d^3*e)/((c*d^2
 - a*e^2)^5*(a*e + c*d*x)) + e^2/(3*(c*d^2 - a*e^2)^3*(d + e*x)^3) + (3*c*
d*e^2)/(2*(c*d^2 - a*e^2)^4*(d + e*x)^2) + (6*c^2*d^2*e^2)/((c*d^2 - a*e^2
)^5*(d + e*x)) + (10*c^3*d^3*e^2*Log[a*e + c*d*x])/(c*d^2 - a*e^2)^6 - (10
*c^3*d^3*e^2*Log[d + e*x])/(c*d^2 - a*e^2)^6
```

Defintions of rubi rules used

rule 1121
```
Int[((d_) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Int[ExpandIntegrand[(d + e*x)^(m + p)*(a/d + (c/e)*x)^p, x], x]
/; FreeQ[{a, b, c, d, e, m, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && (Int
egerQ[p] || (GtQ[a, 0] && GtQ[d, 0] && LtQ[c, 0]))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 1.81 (sec) , antiderivative size = 218, normalized size of antiderivative = 0.98

method	result
default	$-\frac{c^{3} d^{3}}{2\left(a \,e^{2}-c \,d^{2}\right)^{4} \left(c d x +a e \right)^{2}}+\frac{10 c^{3} d^{3} e^{2} \ln \left(c d x +a e \right)}{\left(a \,e^{2}-c \,d^{2}\right)^{6}}-\frac{4 c^{3} d^{3} e}{\left(a \,e^{2}-c \,d^{2}\right)^{5} \left(c d x +a e \right)}-\frac{e^{2}}{3\left(a \,e^{2}-c \,d^{2}\right)^{3} \left(e x +d \right)^{3}}-\frac{10 c^{3} d^{3} e^{2} \ln (}{\left(a \,e^{2}-c \,d\right.}$
risch	$\frac{-\frac{10 c^{4} d^{4} e^{4} x^{4}}{a^{5} e^{10}-5 a^{4} c \,d^{2} e^{8}+10 a^{3} c^{2} d^{4} e^{6}-10 a^{2} c^{3} d^{6} e^{4}+5 a \,c^{4} d^{8} e^{2}-c^{5} d^{10}}-\frac{5 c^{3} d^{3} e^{3} \left(3 a \,e^{2}+5 c \,d^{2}\right) x^{3}}{a^{5} e^{10}-5 a^{4} c \,d^{2} e^{8}+10 a^{3} c^{2} d^{4} e^{6}-10 a^{2} c^{3} d^{6} e^{4}+5 a \,c^{4} d^{8} e^{2}-c^{5} d^{10}}-}{}$
norman	$\frac{-\frac{10 c^{4} d^{4} e^{4} x^{4}}{a^{5} e^{10}-5 a^{4} c \,d^{2} e^{8}+10 a^{3} c^{2} d^{4} e^{6}-10 a^{2} c^{3} d^{6} e^{4}+5 a \,c^{4} d^{8} e^{2}-c^{5} d^{10}}+\frac{\left(-15 a \,c^{5} d^{5} e^{7}-25 c^{6} d^{7} e^{5}\right) x^{3}}{d^{2} c^{2} e^{2} \left(a^{5} e^{10}-5 a^{4} c \,d^{2} e^{8}+10 a^{3} c^{2} d^{4} e^{6}-10 a^{2} c^{3} d^{6} e^{4}+5 a \,c^{4} d^{8} e^{2}-c\right.}}{}$
parallelrisch	$-\frac{-60 x^{4} c^{7} d^{8} e^{7}-150 x^{3} c^{7} d^{9} e^{6}-110 x^{2} c^{7} d^{10} e^{5}-15 x \,c^{7} d^{11} e^{4}+3 e^{3} d^{12} c^{7}+60 \ln \left(e x +d \right) x^{3} a^{2} c^{5} d^{5} e^{10}+360 \ln \left(e x +d \right) x^{3} a \,c^{6} d^{7} e^{8}}{}$

input
```
int(1/(e*x+d)/(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^3,x,method=_RETURNVERBOSE)
```

output
```
-1/2*c^3*d^3/(a*e^2-c*d^2)^4/(c*d*x+a*e)^2+10*c^3*d^3/(a*e^2-c*d^2)^6*e^2*
ln(c*d*x+a*e)-4*c^3*d^3/(a*e^2-c*d^2)^5*e/(c*d*x+a*e)-1/3*e^2/(a*e^2-c*d^2
)^3/(e*x+d)^3-10*c^3*d^3/(a*e^2-c*d^2)^6*e^2*ln(e*x+d)-6*e^2/(a*e^2-c*d^2)
^5*c^2*d^2/(e*x+d)+3/2*e^2/(a*e^2-c*d^2)^4*c*d/(e*x+d)^2
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1222 vs. 2(217) = 434.

Time = 0.11 (sec) , antiderivative size = 1222, normalized size of antiderivative = 5.48

$$\int \frac{1}{(d+ex)\left(ade+(cd^2+ae^2)x+cdex^2\right)^3}\,dx = \text{Too large to display}$$

input
```
integrate(1/(e*x+d)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^3,x, algorithm="fric
as")
```

output
```
-1/6*(3*c^5*d^10 - 30*a*c^4*d^8*e^2 - 20*a^2*c^3*d^6*e^4 + 60*a^3*c^2*d^4*
e^6 - 15*a^4*c*d^2*e^8 + 2*a^5*e^10 - 60*(c^5*d^6*e^4 - a*c^4*d^4*e^6)*x^4
 - 30*(5*c^5*d^7*e^3 - 2*a*c^4*d^5*e^5 - 3*a^2*c^3*d^3*e^7)*x^3 - 10*(11*c
^5*d^8*e^2 + 12*a*c^4*d^6*e^4 - 21*a^2*c^3*d^4*e^6 - 2*a^3*c^2*d^2*e^8)*x^
2 - 5*(3*c^5*d^9*e + 32*a*c^4*d^7*e^3 - 24*a^2*c^3*d^5*e^5 - 12*a^3*c^2*d^
3*e^7 + a^4*c*d*e^9)*x - 60*(c^5*d^5*e^5*x^5 + a^2*c^3*d^6*e^4 + (3*c^5*d^
6*e^4 + 2*a*c^4*d^4*e^6)*x^4 + (3*c^5*d^7*e^3 + 6*a*c^4*d^5*e^5 + a^2*c^3*
d^3*e^7)*x^3 + (c^5*d^8*e^2 + 6*a*c^4*d^6*e^4 + 3*a^2*c^3*d^4*e^6)*x^2 + (
2*a*c^4*d^7*e^3 + 3*a^2*c^3*d^5*e^5)*x)*log(c*d*x + a*e) + 60*(c^5*d^5*e^5
*x^5 + a^2*c^3*d^6*e^4 + (3*c^5*d^6*e^4 + 2*a*c^4*d^4*e^6)*x^4 + (3*c^5*d^
7*e^3 + 6*a*c^4*d^5*e^5 + a^2*c^3*d^3*e^7)*x^3 + (c^5*d^8*e^2 + 6*a*c^4*d^
6*e^4 + 3*a^2*c^3*d^4*e^6)*x^2 + (2*a*c^4*d^7*e^3 + 3*a^2*c^3*d^5*e^5)*x)*
log(e*x + d))/(a^2*c^6*d^15*e^2 - 6*a^3*c^5*d^13*e^4 + 15*a^4*c^4*d^11*e^6
 - 20*a^5*c^3*d^9*e^8 + 15*a^6*c^2*d^7*e^10 - 6*a^7*c*d^5*e^12 + a^8*d^3*e
^14 + (c^8*d^14*e^3 - 6*a*c^7*d^12*e^5 + 15*a^2*c^6*d^10*e^7 - 20*a^3*c^5*
d^8*e^9 + 15*a^4*c^4*d^6*e^11 - 6*a^5*c^3*d^4*e^13 + a^6*c^2*d^2*e^15)*x^5
 + (3*c^8*d^15*e^2 - 16*a*c^7*d^13*e^4 + 33*a^2*c^6*d^11*e^6 - 30*a^3*c^5*
d^9*e^8 + 5*a^4*c^4*d^7*e^10 + 12*a^5*c^3*d^5*e^12 - 9*a^6*c^2*d^3*e^14 +
2*a^7*c*d*e^16)*x^4 + (3*c^8*d^16*e - 12*a*c^7*d^14*e^3 + 10*a^2*c^6*d^12*
e^5 + 24*a^3*c^5*d^10*e^7 - 60*a^4*c^4*d^8*e^9 + 52*a^5*c^3*d^6*e^11 - ...
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1357 vs. $2(206) = 412$.

Time = 2.28 (sec) , antiderivative size = 1357, normalized size of antiderivative = 6.09

$$\int \frac{1}{(d+ex)\left(ade+(cd^2+ae^2)\,x+cdex^2\right)^3}\,dx = \text{Too large to display}$$

input
```
integrate(1/(e*x+d)/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**3,x)
```

output
```
-10*c**3*d**3*e**2*log(x + (-10*a**7*c**3*d**3*e**16/(a*e**2 - c*d**2)**6
+ 70*a**6*c**4*d**5*e**14/(a*e**2 - c*d**2)**6 - 210*a**5*c**5*d**7*e**12/
(a*e**2 - c*d**2)**6 + 350*a**4*c**6*d**9*e**10/(a*e**2 - c*d**2)**6 - 350
*a**3*c**7*d**11*e**8/(a*e**2 - c*d**2)**6 + 210*a**2*c**8*d**13*e**6/(a*e
**2 - c*d**2)**6 - 70*a*c**9*d**15*e**4/(a*e**2 - c*d**2)**6 + 10*a*c**3*d
**3*e**4 + 10*c**10*d**17*e**2/(a*e**2 - c*d**2)**6 + 10*c**4*d**5*e**2)/(
20*c**4*d**4*e**3))/(a*e**2 - c*d**2)**6 + 10*c**3*d**3*e**2*log(x + (10*a
**7*c**3*d**3*e**16/(a*e**2 - c*d**2)**6 - 70*a**6*c**4*d**5*e**14/(a*e**2
 - c*d**2)**6 + 210*a**5*c**5*d**7*e**12/(a*e**2 - c*d**2)**6 - 350*a**4*c
**6*d**9*e**10/(a*e**2 - c*d**2)**6 + 350*a**3*c**7*d**11*e**8/(a*e**2 - c
*d**2)**6 - 210*a**2*c**8*d**13*e**6/(a*e**2 - c*d**2)**6 + 70*a*c**9*d**1
5*e**4/(a*e**2 - c*d**2)**6 + 10*a*c**3*d**3*e**4 - 10*c**10*d**17*e**2/(a
*e**2 - c*d**2)**6 + 10*c**4*d**5*e**2)/(20*c**4*d**4*e**3))/(a*e**2 - c*d
**2)**6 + (-2*a**4*e**8 + 13*a**3*c*d**2*e**6 - 47*a**2*c**2*d**4*e**4 - 2
7*a*c**3*d**6*e**2 + 3*c**4*d**8 - 60*c**4*d**4*e**4*x**4 + x**3*(-90*a*c*
*3*d**3*e**5 - 150*c**4*d**5*e**3) + x**2*(-20*a**2*c**2*d**2*e**6 - 230*a
*c**3*d**4*e**4 - 110*c**4*d**6*e**2) + x*(5*a**3*c*d*e**7 - 55*a**2*c**2*
d**3*e**5 - 175*a*c**3*d**5*e**3 - 15*c**4*d**7*e))/(6*a**7*d**3*e**12 - 3
0*a**6*c*d**5*e**10 + 60*a**5*c**2*d**7*e**8 - 60*a**4*c**3*d**9*e**6 + 30
*a**3*c**4*d**11*e**4 - 6*a**2*c**5*d**13*e**2 + x**5*(6*a**5*c**2*d**2...
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 947 vs. $2(217) = 434$.

Time = 0.08 (sec) , antiderivative size = 947, normalized size of antiderivative = 4.25

$$\int \frac{1}{(d+ex)\left(ade+(cd^2+ae^2)x+cdex^2\right)^3}\,dx = \text{Too large to display}$$

input
```
integrate(1/(e*x+d)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^3,x, algorithm="maxi
ma")
```

output
```
10*c^3*d^3*e^2*log(c*d*x + a*e)/(c^6*d^12 - 6*a*c^5*d^10*e^2 + 15*a^2*c^4*
d^8*e^4 - 20*a^3*c^3*d^6*e^6 + 15*a^4*c^2*d^4*e^8 - 6*a^5*c*d^2*e^10 + a^6
*e^12) - 10*c^3*d^3*e^2*log(e*x + d)/(c^6*d^12 - 6*a*c^5*d^10*e^2 + 15*a^2
*c^4*d^8*e^4 - 20*a^3*c^3*d^6*e^6 + 15*a^4*c^2*d^4*e^8 - 6*a^5*c*d^2*e^10
+ a^6*e^12) + 1/6*(60*c^4*d^4*e^4*x^4 - 3*c^4*d^8 + 27*a*c^3*d^6*e^2 + 47*
a^2*c^2*d^4*e^4 - 13*a^3*c*d^2*e^6 + 2*a^4*e^8 + 30*(5*c^4*d^5*e^3 + 3*a*c
^3*d^3*e^5)*x^3 + 10*(11*c^4*d^6*e^2 + 23*a*c^3*d^4*e^4 + 2*a^2*c^2*d^2*e^
6)*x^2 + 5*(3*c^4*d^7*e + 35*a*c^3*d^5*e^3 + 11*a^2*c^2*d^3*e^5 - a^3*c*d*
e^7)*x)/(a^2*c^5*d^13*e^2 - 5*a^3*c^4*d^11*e^4 + 10*a^4*c^3*d^9*e^6 - 10*a
^5*c^2*d^7*e^8 + 5*a^6*c*d^5*e^10 - a^7*d^3*e^12 + (c^7*d^12*e^3 - 5*a*c^6
*d^10*e^5 + 10*a^2*c^5*d^8*e^7 - 10*a^3*c^4*d^6*e^9 + 5*a^4*c^3*d^4*e^11 -
 a^5*c^2*d^2*e^13)*x^5 + (3*c^7*d^13*e^2 - 13*a*c^6*d^11*e^4 + 20*a^2*c^5*
d^9*e^6 - 10*a^3*c^4*d^7*e^8 - 5*a^4*c^3*d^5*e^10 + 7*a^5*c^2*d^3*e^12 - 2
*a^6*c*d*e^14)*x^4 + (3*c^7*d^14*e - 9*a*c^6*d^12*e^3 + a^2*c^5*d^10*e^5 +
 25*a^3*c^4*d^8*e^7 - 35*a^4*c^3*d^6*e^9 + 17*a^5*c^2*d^4*e^11 - a^6*c*d^2
*e^13 - a^7*e^15)*x^3 + (c^7*d^15 + a*c^6*d^13*e^2 - 17*a^2*c^5*d^11*e^4 +
 35*a^3*c^4*d^9*e^6 - 25*a^4*c^3*d^7*e^8 - a^5*c^2*d^5*e^10 + 9*a^6*c*d^3*
e^12 - 3*a^7*d*e^14)*x^2 + (2*a*c^6*d^14*e - 7*a^2*c^5*d^12*e^3 + 5*a^3*c^
4*d^10*e^5 + 10*a^4*c^3*d^8*e^7 - 20*a^5*c^2*d^6*e^9 + 13*a^6*c*d^4*e^11 -
 3*a^7*d^2*e^13)*x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 503 vs. $2(217) = 434$.

Time = 0.14 (sec) , antiderivative size = 503, normalized size of antiderivative = 2.26

$$\int \frac{1}{(d+ex)\left(ade+(cd^2+ae^2)x+cdex^2\right)^3}\,dx$$
$$= \frac{10\,c^4d^4e^2\log\left(|cdx+ae|\right)}{c^7d^{13}-6\,ac^6d^{11}e^2+15\,a^2c^5d^9e^4-20\,a^3c^4d^7e^6+15\,a^4c^3d^5e^8-6\,a^5c^2d^3e^{10}+a^6cde^{12}}$$
$$-\frac{10\,c^3d^3e^3\log\left(|ex+d|\right)}{c^6d^{12}e-6\,ac^5d^{10}e^3+15\,a^2c^4d^8e^5-20\,a^3c^3d^6e^7+15\,a^4c^2d^4e^9-6\,a^5cd^2e^{11}+a^6e^{13}}$$
$$-\frac{3\,c^5d^{10}-30\,ac^4d^8e^2-20\,a^2c^3d^6e^4+60\,a^3c^2d^4e^6-15\,a^4cd^2e^8+2\,a^5e^{10}-60\,(c^5d^6e^4-ac^4d^4e^6)x^4-3}{}$$

input
```
integrate(1/(e*x+d)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^3,x, algorithm="giac
")
```

output
```
10*c^4*d^4*e^2*log(abs(c*d*x + a*e))/(c^7*d^13 - 6*a*c^6*d^11*e^2 + 15*a^2
*c^5*d^9*e^4 - 20*a^3*c^4*d^7*e^6 + 15*a^4*c^3*d^5*e^8 - 6*a^5*c^2*d^3*e^1
0 + a^6*c*d*e^12) - 10*c^3*d^3*e^3*log(abs(e*x + d))/(c^6*d^12*e - 6*a*c^5
*d^10*e^3 + 15*a^2*c^4*d^8*e^5 - 20*a^3*c^3*d^6*e^7 + 15*a^4*c^2*d^4*e^9 -
 6*a^5*c*d^2*e^11 + a^6*e^13) - 1/6*(3*c^5*d^10 - 30*a*c^4*d^8*e^2 - 20*a^
2*c^3*d^6*e^4 + 60*a^3*c^2*d^4*e^6 - 15*a^4*c*d^2*e^8 + 2*a^5*e^10 - 60*(c
^5*d^6*e^4 - a*c^4*d^4*e^6)*x^4 - 30*(5*c^5*d^7*e^3 - 2*a*c^4*d^5*e^5 - 3*
a^2*c^3*d^3*e^7)*x^3 - 10*(11*c^5*d^8*e^2 + 12*a*c^4*d^6*e^4 - 21*a^2*c^3*
d^4*e^6 - 2*a^3*c^2*d^2*e^8)*x^2 - 5*(3*c^5*d^9*e + 32*a*c^4*d^7*e^3 - 24*
a^2*c^3*d^5*e^5 - 12*a^3*c^2*d^3*e^7 + a^4*c*d*e^9)*x)/((c*d^2 - a*e^2)^6*
(c*d*x + a*e)^2*(e*x + d)^3)
```

Mupad [B] (verification not implemented)

Time = 5.68 (sec) , antiderivative size = 878, normalized size of antiderivative = 3.94

$$\int \frac{1}{(d+ex)\left(ade+(cd^2+ae^2)x+cdex^2\right)^3}\,dx$$
$$= \frac{20\,c^3\,d^3\,e^2\,\mathrm{atanh}\left(\frac{a^6\,e^{12}-4\,a^5\,c\,d^2\,e^{10}+5\,a^4\,c^2\,d^4\,e^8-5\,a^2\,c^4\,d^8\,e^4+4\,a\,c^5\,d^{10}\,e^2-c^6\,d^{12}}{(a\,e^2-c\,d^2)^6}+\frac{2\,c\,d\,e\,x\,(a^5\,e^{10}-5\,a^4\,c\,d^2\,e^8+10\,a^3\,c^2\,d^4\,e^6-1}{(a\,e^2-c\,d^2)^6}\right.}{(a\,e^2-c\,d^2)^6}$$
$$-\frac{\frac{2\,a^4\,e^8-13\,a^3\,c\,d^2\,e^6+47\,a^2\,c^2\,d^4\,e^4+27\,a\,c^3\,d^6\,e^2-3\,c^4\,d^8}{6\,(a^5\,e^{10}-5\,a^4\,c\,d^2\,e^8+10\,a^3\,c^2\,d^4\,e^6-10\,a^2\,c^3\,d^6\,e^4+5\,a\,c^4\,d^8\,e^2-c^5\,d^{10})}+\frac{5\,c^2\,d\,x^3\,(5\,c^2\,d^4\,e^3+3\,a\,c\,d^2\,e^5)}{a^5\,e^{10}-5\,a^4\,c\,d^2\,e^8+10\,a^3\,c^2\,d^4\,e^6-10\,a^2\,c^3\,d^6\,e^4+5\,a\,c^4\,d^8\,e^2-}}{x\,(3\,a^2\,d^2\,e^3+2\,c\,a\,d^4\,e)+x^2\,(3\,a^2\,d\,e}$$

input
```
int(1/((d + e*x)*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^3),x)
```

output
```
(20*c^3*d^3*e^2*atanh((a^6*e^12 - c^6*d^12 + 4*a*c^5*d^10*e^2 - 4*a^5*c*d^
2*e^10 - 5*a^2*c^4*d^8*e^4 + 5*a^4*c^2*d^4*e^8)/(a*e^2 - c*d^2)^6 + (2*c*d
*e*x*(a^5*e^10 - c^5*d^10 + 5*a*c^4*d^8*e^2 - 5*a^4*c*d^2*e^8 - 10*a^2*c^3
*d^6*e^4 + 10*a^3*c^2*d^4*e^6))/(a*e^2 - c*d^2)^6))/(a*e^2 - c*d^2)^6 - ((
2*a^4*e^8 - 3*c^4*d^8 + 27*a*c^3*d^6*e^2 - 13*a^3*c*d^2*e^6 + 47*a^2*c^2*d
^4*e^4)/(6*(a^5*e^10 - c^5*d^10 + 5*a*c^4*d^8*e^2 - 5*a^4*c*d^2*e^8 - 10*a
^2*c^3*d^6*e^4 + 10*a^3*c^2*d^4*e^6)) + (5*c^2*d*x^3*(5*c^2*d^4*e^3 + 3*a*
c*d^2*e^5))/(a^5*e^10 - c^5*d^10 + 5*a*c^4*d^8*e^2 - 5*a^4*c*d^2*e^8 - 10*
a^2*c^3*d^6*e^4 + 10*a^3*c^2*d^4*e^6) + (5*c^2*d^2*x^2*(2*a^2*e^6 + 11*c^2
*d^4*e^2 + 23*a*c*d^2*e^4))/(3*(a^5*e^10 - c^5*d^10 + 5*a*c^4*d^8*e^2 - 5*
a^4*c*d^2*e^8 - 10*a^2*c^3*d^6*e^4 + 10*a^3*c^2*d^4*e^6)) + (10*c^4*d^4*e^
4*x^4)/(a^5*e^10 - c^5*d^10 + 5*a*c^4*d^8*e^2 - 5*a^4*c*d^2*e^8 - 10*a^2*c
^3*d^6*e^4 + 10*a^3*c^2*d^4*e^6) + (5*c*d*e*x*(3*c^3*d^6 - a^3*e^6 + 35*a*
c^2*d^4*e^2 + 11*a^2*c*d^2*e^4))/(6*(a^5*e^10 - c^5*d^10 + 5*a*c^4*d^8*e^2
 - 5*a^4*c*d^2*e^8 - 10*a^2*c^3*d^6*e^4 + 10*a^3*c^2*d^4*e^6)))/(x*(3*a^2*
d^2*e^3 + 2*a*c*d^4*e) + x^2*(c^2*d^5 + 3*a^2*d*e^4 + 6*a*c*d^3*e^2) + x^3
*(a^2*e^5 + 3*c^2*d^4*e + 6*a*c*d^2*e^3) + x^4*(3*c^2*d^3*e^2 + 2*a*c*d*e^
4) + a^2*d^3*e^2 + c^2*d^2*e^3*x^5)
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 1970, normalized size of antiderivative = 8.83

$$\int \frac{1}{(d+ex)\left(ade+(cd^2+ae^2)x+cdex^2\right)^3}\,dx = \text{Too large to display}$$

input
```
int(1/(e*x+d)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^3,x)
```

output

```
(120*log(a*e + c*d*x)*a**3*c**3*d**6*e**6 + 360*log(a*e + c*d*x)*a**3*c**3
*d**5*e**7*x + 360*log(a*e + c*d*x)*a**3*c**3*d**4*e**8*x**2 + 120*log(a*e
 + c*d*x)*a**3*c**3*d**3*e**9*x**3 + 180*log(a*e + c*d*x)*a**2*c**4*d**8*e
**4 + 780*log(a*e + c*d*x)*a**2*c**4*d**7*e**5*x + 1260*log(a*e + c*d*x)*a
**2*c**4*d**6*e**6*x**2 + 900*log(a*e + c*d*x)*a**2*c**4*d**5*e**7*x**3 +
240*log(a*e + c*d*x)*a**2*c**4*d**4*e**8*x**4 + 360*log(a*e + c*d*x)*a*c**
5*d**9*e**3*x + 1200*log(a*e + c*d*x)*a*c**5*d**8*e**4*x**2 + 1440*log(a*e
 + c*d*x)*a*c**5*d**7*e**5*x**3 + 720*log(a*e + c*d*x)*a*c**5*d**6*e**6*x*
*4 + 120*log(a*e + c*d*x)*a*c**5*d**5*e**7*x**5 + 180*log(a*e + c*d*x)*c**
6*d**10*e**2*x**2 + 540*log(a*e + c*d*x)*c**6*d**9*e**3*x**3 + 540*log(a*e
 + c*d*x)*c**6*d**8*e**4*x**4 + 180*log(a*e + c*d*x)*c**6*d**7*e**5*x**5 -
 120*log(d + e*x)*a**3*c**3*d**6*e**6 - 360*log(d + e*x)*a**3*c**3*d**5*e*
*7*x - 360*log(d + e*x)*a**3*c**3*d**4*e**8*x**2 - 120*log(d + e*x)*a**3*c
**3*d**3*e**9*x**3 - 180*log(d + e*x)*a**2*c**4*d**8*e**4 - 780*log(d + e*
x)*a**2*c**4*d**7*e**5*x - 1260*log(d + e*x)*a**2*c**4*d**6*e**6*x**2 - 90
0*log(d + e*x)*a**2*c**4*d**5*e**7*x**3 - 240*log(d + e*x)*a**2*c**4*d**4*
e**8*x**4 - 360*log(d + e*x)*a*c**5*d**9*e**3*x - 1200*log(d + e*x)*a*c**5
*d**8*e**4*x**2 - 1440*log(d + e*x)*a*c**5*d**7*e**5*x**3 - 720*log(d + e*
x)*a*c**5*d**6*e**6*x**4 - 120*log(d + e*x)*a*c**5*d**5*e**7*x**5 - 180*lo
g(d + e*x)*c**6*d**10*e**2*x**2 - 540*log(d + e*x)*c**6*d**9*e**3*x**3 ...
```

3.138 $\int \frac{(d+ex)^{10}}{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^4} dx$

Optimal result

Integrand size = 35, antiderivative size = 217

$$\int \frac{(d+ex)^{10}}{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^4} dx = \frac{e^4(15c^2d^4-24acd^2e^2+10a^2e^4)\,x}{c^6d^6} + \frac{e^5(3cd^2-2ae^2)\,x^2}{c^5d^5} + \frac{e^6x^3}{3c^4d^4} - \frac{\left(cd^2-ae^2\right)^6}{3c^7d^7(ae+cdx)^3} - \frac{3e\left(cd^2-ae^2\right)^5}{c^7d^7(ae+cdx)^2} - \frac{15e^2\left(cd^2-ae^2\right)^4}{c^7d^7(ae+cdx)} + \frac{20e^3\left(cd^2-ae^2\right)^3\log(ae+cdx)}{c^7d^7}$$

output
```
e^4*(10*a^2*e^4-24*a*c*d^2*e^2+15*c^2*d^4)*x/c^6/d^6+e^5*(-2*a*e^2+3*c*d^2
)*x^2/c^5/d^5+1/3*e^6*x^3/c^4/d^4-1/3*(-a*e^2+c*d^2)^6/c^7/d^7/(c*d*x+a*e)
^3-3*e*(-a*e^2+c*d^2)^5/c^7/d^7/(c*d*x+a*e)^2-15*e^2*(-a*e^2+c*d^2)^4/c^7/
d^7/(c*d*x+a*e)+20*e^3*(-a*e^2+c*d^2)^3*ln(c*d*x+a*e)/c^7/d^7
```

Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 335, normalized size of antiderivative = 1.54

$$\int \frac{(d+ex)^{10}}{\left(ade+(cd^2+ae^2)\,x+cdex^2\right)^4}\,dx$$
$$= \frac{-37a^6e^{12}+3a^5cde^{10}(47d-17ex)+3a^4c^2d^2e^8(-65d^2+81dex+13e^2x^2)+a^3c^3d^3e^6(110d^3-405d^2ex}{}$$

input
```
Integrate[(d + e*x)^10/(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^4,x]
```

output
```
(-37*a^6*e^12 + 3*a^5*c*d*e^10*(47*d - 17*e*x) + 3*a^4*c^2*d^2*e^8*(-65*d^
2 + 81*d*e*x + 13*e^2*x^2) + a^3*c^3*d^3*e^6*(110*d^3 - 405*d^2*e*x - 27*d
*e^2*x^2 + 73*e^3*x^3) - 3*a^2*c^4*d^4*e^4*(5*d^4 - 90*d^3*e*x + 45*d^2*e^
2*x^2 + 63*d*e^3*x^3 - 5*e^4*x^4) - 3*a*c^5*d^5*e^2*(d^5 + 15*d^4*e*x - 60
*d^3*e^2*x^2 - 45*d^2*e^3*x^3 + 15*d*e^4*x^4 + e^5*x^5) + c^6*d^6*(-d^6 -
9*d^5*e*x - 45*d^4*e^2*x^2 + 45*d^2*e^4*x^4 + 9*d*e^5*x^5 + e^6*x^6) - 60*
e^3*(-(c*d^2) + a*e^2)^3*(a*e + c*d*x)^3*Log[a*e + c*d*x])/(3*c^7*d^7*(a*e
 + c*d*x)^3)
```

Rubi [A] (verified)

Time = 0.81 (sec) , antiderivative size = 217, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.057, Rules used = {1121, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex)^{10}}{\left(x\,(ae^2+cd^2)+ade+cdex^2\right)^4}\,dx$$
$$\Big\downarrow \; 1121$$
$$\int \left(\frac{10a^2e^8-24acd^2e^6+15c^2d^4e^4}{c^6d^6}+\frac{20\left(cd^2e-ae^3\right)^3}{c^6d^6(ae+cdx)}+\frac{15e^2\left(cd^2-ae^2\right)^4}{c^6d^6(ae+cdx)^2}+\frac{6e\left(cd^2-ae^2\right)^5}{c^6d^6(ae+cdx)^3}+\frac{\left(cd^2-ae^2\right)^6}{c^6d^6(ae+cdx)}\right.$$
$$\Big\downarrow \; 2009$$

$$\frac{e^4x\left(10a^2e^4-24acd^2e^2+15c^2d^4\right)}{c^6d^6}-\frac{15e^2\left(cd^2-ae^2\right)^4}{c^7d^7(ae+cdx)}-\frac{3e\left(cd^2-ae^2\right)^5}{c^7d^7(ae+cdx)^2}-\frac{\left(cd^2-ae^2\right)^6}{3c^7d^7(ae+cdx)^3}+\frac{20e^3\left(cd^2-ae^2\right)^3\log(ae+cdx)}{c^7d^7}+\frac{e^5x^2\left(3cd^2-2ae^2\right)}{c^5d^5}+\frac{e^6x^3}{3c^4d^4}$$

input
```
Int[(d + e*x)^10/(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^4,x]
```

output
```
(e^4*(15*c^2*d^4 - 24*a*c*d^2*e^2 + 10*a^2*e^4)*x)/(c^6*d^6) + (e^5*(3*c*d
^2 - 2*a*e^2)*x^2)/(c^5*d^5) + (e^6*x^3)/(3*c^4*d^4) - (c*d^2 - a*e^2)^6/(
3*c^7*d^7*(a*e + c*d*x)^3) - (3*e*(c*d^2 - a*e^2)^5)/(c^7*d^7*(a*e + c*d*x
)^2) - (15*e^2*(c*d^2 - a*e^2)^4)/(c^7*d^7*(a*e + c*d*x)) + (20*e^3*(c*d^2
 - a*e^2)^3*Log[a*e + c*d*x])/(c^7*d^7)
```

Defintions of rubi rules used

rule 1121
```
Int[((d_) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Int[ExpandIntegrand[(d + e*x)^(m + p)*(a/d + (c/e)*x)^p, x], x]
/; FreeQ[{a, b, c, d, e, m, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && (Int
egerQ[p] || (GtQ[a, 0] && GtQ[d, 0] && LtQ[c, 0]))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 1.60 (sec) , antiderivative size = 399, normalized size of antiderivative = 1.84

method	result
default	$\frac{e^4\left(\frac{1}{3}x^3c^2d^2e^2-2x^2acd\,e^3+3x^2c^2d^3e+10a^2e^4x-24ac\,d^2e^2x+15c^2d^4x\right)}{d^6c^6}-\frac{a^6e^{12}-6a^5d^2e^{10}c+15a^4d^4e^8c^2-20a^3d^6e^6c^3+1\cdots}{3d^7c^7(cdx+ae)^3}$
risch	$\frac{e^6x^3}{3c^4d^4}-\frac{2e^7x^2a}{d^5c^5}+\frac{3e^5x^2}{d^3c^4}+\frac{10e^8a^2x}{d^6c^6}-\frac{24e^6ax}{d^4c^5}+\frac{15e^4x}{d^2c^4}+\frac{\left(-15a^4e^{10}dc+60a^3e^8d^3c^2-90a^2e^6d^5c^3+60a\,c^4d^7e^4-1\cdots\right.}{\cdots}$
norman	$\frac{\frac{e^7\left(5a^2e^4-18ac\,d^2e^2+25c^2d^4\right)x^7}{d^3c^3}-\frac{110a^6e^{12}-285a^5d^2e^{10}c+186a^4d^4e^8c^2+53a^3d^6e^6c^3+15a^2d^8e^4c^4+3a\,d^{10}e^2c^5+d^{12}c^6}{3d^4c^7}+\frac{e^9x^9}{3cd}-\frac{(11\cdots}{\cdots}}{\cdots}$
parallelrisch	$-\frac{270a^5cd\,e^{11}x-810a^4c^2d^3e^9x+810a^3c^3d^5e^7x-270a^2c^4d^7e^5x+3x^5a\,c^5d^5e^7-15x^4a^2c^4d^4e^8+45x^4a\,c^5d^6e^6+110a^6e^{12}+d^{1\cdots}}{\cdots}$

input
```
int((e*x+d)^10/(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^4,x,method=_RETURNVERBOSE
)
```

output
```
e^4/d^6/c^6*(1/3*x^3*c^2*d^2*e^2-2*x^2*a*c*d*e^3+3*x^2*c^2*d^3*e+10*a^2*e^
4*x-24*a*c*d^2*e^2*x+15*c^2*d^4*x)-1/3/d^7/c^7*(a^6*e^12-6*a^5*c*d^2*e^10+
15*a^4*c^2*d^4*e^8-20*a^3*c^3*d^6*e^6+15*a^2*c^4*d^8*e^4-6*a*c^5*d^10*e^2+
c^6*d^12)/(c*d*x+a*e)^3-20*e^3/d^7/c^7*(a^3*e^6-3*a^2*c*d^2*e^4+3*a*c^2*d^
4*e^2-c^3*d^6)*ln(c*d*x+a*e)+3/d^7*e/c^7*(a^5*e^10-5*a^4*c*d^2*e^8+10*a^3*
c^2*d^4*e^6-10*a^2*c^3*d^6*e^4+5*a*c^4*d^8*e^2-c^5*d^10)/(c*d*x+a*e)^2-15*
e^2/c^7/d^7*(a^4*e^8-4*a^3*c*d^2*e^6+6*a^2*c^2*d^4*e^4-4*a*c^3*d^6*e^2+c^4
*d^8)/(c*d*x+a*e)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 644 vs. $2(213) = 426$.

Time = 0.08 (sec) , antiderivative size = 644, normalized size of antiderivative = 2.97

$$\int \frac{(d+ex)^{10}}{(ade+(cd^2+ae^2)x+cdex^2)^4}\,dx = \frac{c^6d^6e^6x^6 - c^6d^{12} - 3\,ac^5d^{10}e^2 - 15\,a^2c^4d^8e^4 + 110\,a^3c^3d^6e^6 - 195\,a^4c^2d^4e^8 + 141\,a^5cd^2e^{10} - 37\,a^6e^{12} + 3}{}$$

input
```
integrate((e*x+d)^10/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^4,x, algorithm="fri
cas")
```

output
```
1/3*(c^6*d^6*e^6*x^6 - c^6*d^12 - 3*a*c^5*d^10*e^2 - 15*a^2*c^4*d^8*e^4 +
110*a^3*c^3*d^6*e^6 - 195*a^4*c^2*d^4*e^8 + 141*a^5*c*d^2*e^10 - 37*a^6*e^
12 + 3*(3*c^6*d^7*e^5 - a*c^5*d^5*e^7)*x^5 + 15*(3*c^6*d^8*e^4 - 3*a*c^5*d
^6*e^6 + a^2*c^4*d^4*e^8)*x^4 + (135*a*c^5*d^7*e^5 - 189*a^2*c^4*d^5*e^7 +
 73*a^3*c^3*d^3*e^9)*x^3 - 3*(15*c^6*d^10*e^2 - 60*a*c^5*d^8*e^4 + 45*a^2*
c^4*d^6*e^6 + 9*a^3*c^3*d^4*e^8 - 13*a^4*c^2*d^2*e^10)*x^2 - 3*(3*c^6*d^11
*e + 15*a*c^5*d^9*e^3 - 90*a^2*c^4*d^7*e^5 + 135*a^3*c^3*d^5*e^7 - 81*a^4*
c^2*d^3*e^9 + 17*a^5*c*d*e^11)*x + 60*(a^3*c^3*d^6*e^6 - 3*a^4*c^2*d^4*e^8
 + 3*a^5*c*d^2*e^10 - a^6*e^12 + (c^6*d^9*e^3 - 3*a*c^5*d^7*e^5 + 3*a^2*c^
4*d^5*e^7 - a^3*c^3*d^3*e^9)*x^3 + 3*(a*c^5*d^8*e^4 - 3*a^2*c^4*d^6*e^6 +
3*a^3*c^3*d^4*e^8 - a^4*c^2*d^2*e^10)*x^2 + 3*(a^2*c^4*d^7*e^5 - 3*a^3*c^3
*d^5*e^7 + 3*a^4*c^2*d^3*e^9 - a^5*c*d*e^11)*x)*log(c*d*x + a*e))/(c^10*d^
10*x^3 + 3*a*c^9*d^9*e*x^2 + 3*a^2*c^8*d^8*e^2*x + a^3*c^7*d^7*e^3)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 425 vs. $2(211) = 422$.

Time = 71.72 (sec) , antiderivative size = 425, normalized size of antiderivative = 1.96

$$\int \frac{(d+ex)^{10}}{(ade+(cd^2+ae^2)x+cdex^2)^4}\,dx = x^2\left(-\frac{2ae^7}{c^5d^5}+\frac{3e^5}{c^4d^3}\right)+x\left(\frac{10a^2e^8}{c^6d^6}-\frac{24ae^6}{c^5d^4}+\frac{15e^4}{c^4d^2}\right) + \frac{-37a^6e^{12}+141a^5cd^2e^{10}-195a^4c^2d^4e^8+110a^3c^3d^6e^6-15a^2c^4d^8e^4-3ac^5d^{10}e^2-c^6d^{12}+x^2(-45a^4c}{3} + \frac{e^6x^3}{3c^4d^4} - \frac{20e^3(ae^2-cd^2)^3\log(ae+cdx)}{c^7d^7}$$

input
```
integrate((e*x+d)**10/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**4,x)
```

output
```
x**2*(-2*a*e**7/(c**5*d**5) + 3*e**5/(c**4*d**3)) + x*(10*a**2*e**8/(c**6*
d**6) - 24*a*e**6/(c**5*d**4) + 15*e**4/(c**4*d**2)) + (-37*a**6*e**12 + 1
41*a**5*c*d**2*e**10 - 195*a**4*c**2*d**4*e**8 + 110*a**3*c**3*d**6*e**6 -
 15*a**2*c**4*d**8*e**4 - 3*a*c**5*d**10*e**2 - c**6*d**12 + x**2*(-45*a**
4*c**2*d**2*e**10 + 180*a**3*c**3*d**4*e**8 - 270*a**2*c**4*d**6*e**6 + 18
0*a*c**5*d**8*e**4 - 45*c**6*d**10*e**2) + x*(-81*a**5*c*d*e**11 + 315*a**
4*c**2*d**3*e**9 - 450*a**3*c**3*d**5*e**7 + 270*a**2*c**4*d**7*e**5 - 45*
a*c**5*d**9*e**3 - 9*c**6*d**11*e))/(3*a**3*c**7*d**7*e**3 + 9*a**2*c**8*d
**8*e**2*x + 9*a*c**9*d**9*e*x**2 + 3*c**10*d**10*x**3) + e**6*x**3/(3*c**
4*d**4) - 20*e**3*(a*e**2 - c*d**2)**3*log(a*e + c*d*x)/(c**7*d**7)
```

Maxima [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 424, normalized size of antiderivative = 1.95

$$\int \frac{(d+ex)^{10}}{(ade+(cd^2+ae^2)\,x+cdex^2)^4}\,dx =$$
$$-\frac{c^6d^{12}+3\,ac^5d^{10}e^2+15\,a^2c^4d^8e^4-110\,a^3c^3d^6e^6+195\,a^4c^2d^4e^8-141\,a^5cd^2e^{10}+37\,a^6e^{12}+45\,(c^6d^{10}e}{3\,(c^{10}d^{10}x}$$
$$+\frac{c^2d^2e^6x^3+3\,(3\,c^2d^3e^5-2\,acde^7)x^2+3\,(15\,c^2d^4e^4-24\,acd^2e^6+10\,a^2e^8)x}{3\,c^6d^6}$$
$$+\frac{20\,(c^3d^6e^3-3\,ac^2d^4e^5+3\,a^2cd^2e^7-a^3e^9)\log\left(cdx+ae\right)}{c^7d^7}$$

input
```
integrate((e*x+d)^10/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^4,x, algorithm="max
ima")
```

output
```
-1/3*(c^6*d^12 + 3*a*c^5*d^10*e^2 + 15*a^2*c^4*d^8*e^4 - 110*a^3*c^3*d^6*e
^6 + 195*a^4*c^2*d^4*e^8 - 141*a^5*c*d^2*e^10 + 37*a^6*e^12 + 45*(c^6*d^10
*e^2 - 4*a*c^5*d^8*e^4 + 6*a^2*c^4*d^6*e^6 - 4*a^3*c^3*d^4*e^8 + a^4*c^2*d
^2*e^10)*x^2 + 9*(c^6*d^11*e + 5*a*c^5*d^9*e^3 - 30*a^2*c^4*d^7*e^5 + 50*a
^3*c^3*d^5*e^7 - 35*a^4*c^2*d^3*e^9 + 9*a^5*c*d*e^11)*x)/(c^10*d^10*x^3 +
3*a*c^9*d^9*e*x^2 + 3*a^2*c^8*d^8*e^2*x + a^3*c^7*d^7*e^3) + 1/3*(c^2*d^2*
e^6*x^3 + 3*(3*c^2*d^3*e^5 - 2*a*c*d*e^7)*x^2 + 3*(15*c^2*d^4*e^4 - 24*a*c
*d^2*e^6 + 10*a^2*e^8)*x)/(c^6*d^6) + 20*(c^3*d^6*e^3 - 3*a*c^2*d^4*e^5 +
3*a^2*c*d^2*e^7 - a^3*e^9)*log(c*d*x + a*e)/(c^7*d^7)
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 398, normalized size of antiderivative = 1.83

$$\int \frac{(d+ex)^{10}}{(ade+(cd^2+ae^2)x+cdex^2)^4}\,dx$$
$$= \frac{20\,(c^3d^6e^3 - 3\,ac^2d^4e^5 + 3\,a^2cd^2e^7 - a^3e^9)\log(|cdx+ae|)}{c^7d^7}$$
$$- \frac{c^6d^{12} + 3\,ac^5d^{10}e^2 + 15\,a^2c^4d^8e^4 - 110\,a^3c^3d^6e^6 + 195\,a^4c^2d^4e^8 - 141\,a^5cd^2e^{10} + 37\,a^6e^{12} + 45\,(c^6d^{10}e^{\cdots}}{\cdots}$$
$$+ \frac{c^8d^8e^6x^3 + 9\,c^8d^9e^5x^2 - 6\,ac^7d^7e^7x^2 + 45\,c^8d^{10}e^4x - 72\,ac^7d^8e^6x + 30\,a^2c^6d^6e^8x}{3\,c^{12}d^{12}}$$

input
```
integrate((e*x+d)^10/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^4,x, algorithm="gia
c")
```

output
```
20*(c^3*d^6*e^3 - 3*a*c^2*d^4*e^5 + 3*a^2*c*d^2*e^7 - a^3*e^9)*log(abs(c*d
*x + a*e))/(c^7*d^7) - 1/3*(c^6*d^12 + 3*a*c^5*d^10*e^2 + 15*a^2*c^4*d^8*e
^4 - 110*a^3*c^3*d^6*e^6 + 195*a^4*c^2*d^4*e^8 - 141*a^5*c*d^2*e^10 + 37*a
^6*e^12 + 45*(c^6*d^10*e^2 - 4*a*c^5*d^8*e^4 + 6*a^2*c^4*d^6*e^6 - 4*a^3*c
^3*d^4*e^8 + a^4*c^2*d^2*e^10)*x^2 + 9*(c^6*d^11*e + 5*a*c^5*d^9*e^3 - 30*
a^2*c^4*d^7*e^5 + 50*a^3*c^3*d^5*e^7 - 35*a^4*c^2*d^3*e^9 + 9*a^5*c*d*e^11
)*x)/((c*d*x + a*e)^3*c^7*d^7) + 1/3*(c^8*d^8*e^6*x^3 + 9*c^8*d^9*e^5*x^2
- 6*a*c^7*d^7*e^7*x^2 + 45*c^8*d^10*e^4*x - 72*a*c^7*d^8*e^6*x + 30*a^2*c^
6*d^6*e^8*x)/(c^12*d^12)
```

Mupad [B] (verification not implemented)

Time = 5.26 (sec) , antiderivative size = 452, normalized size of antiderivative = 2.08

$$\int \frac{(d+ex)^{10}}{(ade+(cd^2+ae^2)x+cdex^2)^4}\,dx$$
$$= x^2\left(\frac{3\,e^5}{c^4\,d^3} - \frac{2\,a\,e^7}{c^5\,d^5}\right) - x\left(\frac{6\,a^2\,e^8}{c^6\,d^6} - \frac{15\,e^4}{c^4\,d^2} + \frac{4\,a\,e\left(\frac{6\,e^5}{c^4\,d^3} - \frac{4\,a\,e^7}{c^5\,d^5}\right)}{c\,d}\right)$$
$$- \frac{x\,(27\,a^5\,e^{11} - 105\,a^4\,c\,d^2\,e^9 + 150\,a^3\,c^2\,d^4\,e^7 - 90\,a^2\,c^3\,d^6\,e^5 + 15\,a\,c^4\,d^8\,e^3 + 3\,c^5\,d^{10}\,e) + x^2\,(15\,a^4\,c\,d\,e^{\cdots}}{a^3\,c^6\,d^6\,e^3\,\cdots}$$
$$- \frac{\ln(a\,e + c\,d\,x)\,(20\,a^3\,e^9 - 60\,a^2\,c\,d^2\,e^7 + 60\,a\,c^2\,d^4\,e^5 - 20\,c^3\,d^6\,e^3)}{c^7\,d^7} + \frac{e^6\,x^3}{3\,c^4\,d^4}$$

input
```
int((d + e*x)^10/(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^4,x)
```

output
```
x^2*((3*e^5)/(c^4*d^3) - (2*a*e^7)/(c^5*d^5)) - x*((6*a^2*e^8)/(c^6*d^6) -
 (15*e^4)/(c^4*d^2) + (4*a*e*((6*e^5)/(c^4*d^3) - (4*a*e^7)/(c^5*d^5)))/(c
*d)) - (x*(27*a^5*e^11 + 3*c^5*d^10*e + 15*a*c^4*d^8*e^3 - 105*a^4*c*d^2*e
^9 - 90*a^2*c^3*d^6*e^5 + 150*a^3*c^2*d^4*e^7) + x^2*(15*c^5*d^9*e^2 - 60*
a*c^4*d^7*e^4 + 90*a^2*c^3*d^5*e^6 - 60*a^3*c^2*d^3*e^8 + 15*a^4*c*d*e^10)
 + (37*a^6*e^12 + c^6*d^12 + 3*a*c^5*d^10*e^2 - 141*a^5*c*d^2*e^10 + 15*a^
2*c^4*d^8*e^4 - 110*a^3*c^3*d^6*e^6 + 195*a^4*c^2*d^4*e^8)/(3*c*d))/(c^9*d
^9*x^3 + a^3*c^6*d^6*e^3 + 3*a*c^8*d^8*e*x^2 + 3*a^2*c^7*d^7*e^2*x) - (log
(a*e + c*d*x)*(20*a^3*e^9 - 20*c^3*d^6*e^3 + 60*a*c^2*d^4*e^5 - 60*a^2*c*d
^2*e^7))/(c^7*d^7) + (e^6*x^3)/(3*c^4*d^4)
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 749, normalized size of antiderivative = 3.45

$$\int \frac{(d+ex)^{10}}{(ade+(cd^2+ae^2)\,x+cdex^2)^4}\,dx$$
$$= \frac{-60\log(cdx+ae)\,a^4c^3d^3e^9x^3+180\log(cdx+ae)\,a^3c^4d^5e^7x^3-180\log(cdx+ae)\,a^2c^5d^7e^5x^3+60\log(c}{}$$

input
```
int((e*x+d)^10/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^4,x)
```

output

```
( - 60*log(a*e + c*d*x)*a**7*e**12 + 180*log(a*e + c*d*x)*a**6*c*d**2*e**1
0 - 180*log(a*e + c*d*x)*a**6*c*d*e**11*x - 180*log(a*e + c*d*x)*a**5*c**2
*d**4*e**8 + 540*log(a*e + c*d*x)*a**5*c**2*d**3*e**9*x - 180*log(a*e + c*
d*x)*a**5*c**2*d**2*e**10*x**2 + 60*log(a*e + c*d*x)*a**4*c**3*d**6*e**6 -
 540*log(a*e + c*d*x)*a**4*c**3*d**5*e**7*x + 540*log(a*e + c*d*x)*a**4*c*
*3*d**4*e**8*x**2 - 60*log(a*e + c*d*x)*a**4*c**3*d**3*e**9*x**3 + 180*log
(a*e + c*d*x)*a**3*c**4*d**7*e**5*x - 540*log(a*e + c*d*x)*a**3*c**4*d**6*
e**6*x**2 + 180*log(a*e + c*d*x)*a**3*c**4*d**5*e**7*x**3 + 180*log(a*e +
c*d*x)*a**2*c**5*d**8*e**4*x**2 - 180*log(a*e + c*d*x)*a**2*c**5*d**7*e**5
*x**3 + 60*log(a*e + c*d*x)*a*c**6*d**9*e**3*x**3 - 50*a**7*e**12 + 150*a*
*6*c*d**2*e**10 - 90*a**6*c*d*e**11*x - 150*a**5*c**2*d**4*e**8 + 270*a**5
*c**2*d**3*e**9*x + 50*a**4*c**3*d**6*e**6 - 270*a**4*c**3*d**5*e**7*x + 6
0*a**4*c**3*d**3*e**9*x**3 + 90*a**3*c**4*d**7*e**5*x - 180*a**3*c**4*d**5
*e**7*x**3 + 15*a**3*c**4*d**4*e**8*x**4 - 3*a**2*c**5*d**10*e**2 + 180*a*
*2*c**5*d**7*e**5*x**3 - 45*a**2*c**5*d**6*e**6*x**4 - 3*a**2*c**5*d**5*e*
*7*x**5 - a*c**6*d**12 - 9*a*c**6*d**11*e*x - 60*a*c**6*d**9*e**3*x**3 + 4
5*a*c**6*d**8*e**4*x**4 + 9*a*c**6*d**7*e**5*x**5 + a*c**6*d**6*e**6*x**6
+ 15*c**7*d**11*e*x**3)/(3*a*c**7*d**7*(a**3*e**3 + 3*a**2*c*d*e**2*x + 3*
a*c**2*d**2*e*x**2 + c**3*d**3*x**3))
```

3.139 $\int \frac{(d+ex)^9}{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^4}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 179

$$\int \frac{(d+ex)^9}{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^4}\,dx=\frac{e^4\left(5cd^2-4ae^2\right)x}{c^5d^5}+\frac{e^5x^2}{2c^4d^4}-\frac{\left(cd^2-ae^2\right)^5}{3c^6d^6(ae+cdx)^3}-\frac{5e\left(cd^2-ae^2\right)^4}{2c^6d^6(ae+cdx)^2}-\frac{10e^2\left(cd^2-ae^2\right)^3}{c^6d^6(ae+cdx)}+\frac{10e^3\left(cd^2-ae^2\right)^2\log(ae+cdx)}{c^6d^6}$$

output

```
e^4*(-4*a*e^2+5*c*d^2)*x/c^5/d^5+1/2*e^5*x^2/c^4/d^4-1/3*(-a*e^2+c*d^2)^5/
c^6/d^6/(c*d*x+a*e)^3-5/2*e*(-a*e^2+c*d^2)^4/c^6/d^6/(c*d*x+a*e)^2-10*e^2*
(-a*e^2+c*d^2)^3/c^6/d^6/(c*d*x+a*e)+10*e^3*(-a*e^2+c*d^2)^2*ln(c*d*x+a*e)
/c^6/d^6
```

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 259, normalized size of antiderivative = 1.45

$$\int \frac{(d+ex)^9}{(ade+(cd^2+ae^2)\,x+cdex^2)^4}\,dx$$
$$= \frac{47a^5e^{10}+a^4cde^8(-130d+81ex)+a^3c^2d^2e^6(110d^2-270dex-9e^2x^2)-a^2c^3d^3e^4(20d^3-270d^2ex+90}{}$$

input
```
Integrate[(d + e*x)^9/(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^4,x]
```

output
```
(47*a^5*e^10 + a^4*c*d*e^8*(-130*d + 81*e*x) + a^3*c^2*d^2*e^6*(110*d^2 -
270*d*e*x - 9*e^2*x^2) - a^2*c^3*d^3*e^4*(20*d^3 - 270*d^2*e*x + 90*d*e^2*
x^2 + 63*e^3*x^3) - 5*a*c^4*d^4*e^2*(d^4 + 12*d^3*e*x - 36*d^2*e^2*x^2 - 1
8*d*e^3*x^3 + 3*e^4*x^4) + c^5*d^5*(-2*d^5 - 15*d^4*e*x - 60*d^3*e^2*x^2 +
 30*d*e^4*x^4 + 3*e^5*x^5) + 60*e^3*(c*d^2 - a*e^2)^2*(a*e + c*d*x)^3*Log[
a*e + c*d*x])/(6*c^6*d^6*(a*e + c*d*x)^3)
```

Rubi [A] (verified)

Time = 0.67 (sec) , antiderivative size = 179, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.057, Rules used = {1121, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex)^9}{(x\,(ae^2+cd^2)+ade+cdex^2)^4}\,dx$$
$$\Big\downarrow \text{1121}$$
$$\int \left(\frac{10e^2\left(cd^2-ae^2\right)^3}{c^5d^5(ae+cdx)^2}+\frac{5e\left(cd^2-ae^2\right)^4}{c^5d^5(ae+cdx)^3}+\frac{\left(cd^2-ae^2\right)^5}{c^5d^5(ae+cdx)^4}+\frac{5cd^2e^4-4ae^6}{c^5d^5}+\frac{10e^3\left(cd^2-ae^2\right)^2}{c^5d^5(ae+cdx)}+\frac{e^5x}{c^4d^4}\right)dx$$
$$\Big\downarrow \text{2009}$$

$$-\frac{10e^2\left(cd^2-ae^2\right)^3}{c^6d^6(ae+cdx)}-\frac{5e\left(cd^2-ae^2\right)^4}{2c^6d^6(ae+cdx)^2}-\frac{\left(cd^2-ae^2\right)^5}{3c^6d^6(ae+cdx)^3}+\frac{10e^3\left(cd^2-ae^2\right)^2\log(ae+cdx)}{c^6d^6}+\frac{e^4x\left(5cd^2-4ae^2\right)}{c^5d^5}+\frac{e^5x^2}{2c^4d^4}$$

input
```
Int[(d + e*x)^9/(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^4,x]
```

output
```
(e^4*(5*c*d^2 - 4*a*e^2)*x)/(c^5*d^5) + (e^5*x^2)/(2*c^4*d^4) - (c*d^2 - a
*e^2)^5/(3*c^6*d^6*(a*e + c*d*x)^3) - (5*e*(c*d^2 - a*e^2)^4)/(2*c^6*d^6*(
a*e + c*d*x)^2) - (10*e^2*(c*d^2 - a*e^2)^3)/(c^6*d^6*(a*e + c*d*x)) + (10
*e^3*(c*d^2 - a*e^2)^2*Log[a*e + c*d*x])/(c^6*d^6)
```

Defintions of rubi rules used

rule 1121
```
Int[((d_) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Int[ExpandIntegrand[(d + e*x)^(m + p)*(a/d + (c/e)*x)^p, x], x]
/; FreeQ[{a, b, c, d, e, m, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && (Int
egerQ[p] || (GtQ[a, 0] && GtQ[d, 0] && LtQ[c, 0]))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 1.59 (sec) , antiderivative size = 300, normalized size of antiderivative = 1.68

method	result
default	$-\frac{e^4\left(-\frac{1}{2}cd\,x^2e+4a\,e^2x-5c\,d^2x\right)}{d^5c^5}-\frac{-a^5e^{10}+5a^4c\,d^2e^8-10a^3c^2d^4e^6+10a^2c^3d^6e^4-5a\,c^4d^8e^2+c^5d^{10}}{3d^6c^6\left(cdx+ae\right)^3}+\frac{10e^3\left(a^2e^4-2ac\,d\cdots\right.}{\cdots}$
risch	$\frac{e^5x^2}{2c^4d^4}-\frac{4e^6ax}{d^5c^5}+\frac{5e^4x}{d^3c^4}+\frac{\left(10a^3c\,e^8d-30a^2c^2e^6d^3+30a\,c^3e^4d^5-10c^4e^2d^7\right)x^2+\frac{5e\left(7a^4e^8-20a^3c\,d^2e^6+18a^2c^2d^4e^4-4a\,c^3d\cdots\right.}{2}}{d^5c^5\left(cdx+ae\right)^3}$
parallelrisch	$\frac{110a^5e^{10}-2c^5d^{10}+270a^4cd\,e^9x-540a^3c^2d^3e^7x+270a^2c^3d^5e^5x-15c^5d^9ex-360\ln\left(cdx+ae\right)x\,a^3c^2d^3e^7+180\ln\left(cdx+ae\right)x\,a^2\cdots}{\cdots}$
norman	$\frac{\frac{110a^5e^{10}-175a^4c\,d^2e^8+11a^3c^2d^4e^6-20a^2c^3d^6e^4-5a\,c^4d^8e^2-2c^5d^{10}}{6c^6d^3}+\frac{e^8x^8}{2cd}+\frac{\left(110a^5e^{16}+635a^4c\,d^2e^{14}-664a^3c^2d^4e^{12}-776a^2c^3d^6e\cdots\right.}{6d^6c^6e^3}}{\cdots}$

input
```
int((e*x+d)^9/(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^4,x,method=_RETURNVERBOSE)
```

output
```
-e^4/d^5/c^5*(-1/2*c*d*x^2*e+4*a*e^2*x-5*c*d^2*x)-1/3/d^6/c^6*(-a^5*e^10+5
*a^4*c*d^2*e^8-10*a^3*c^2*d^4*e^6+10*a^2*c^3*d^6*e^4-5*a*c^4*d^8*e^2+c^5*d
^10)/(c*d*x+a*e)^3+10*e^3/d^6/c^6*(a^2*e^4-2*a*c*d^2*e^2+c^2*d^4)*ln(c*d*x
+a*e)-5/2/d^6*e/c^6*(a^4*e^8-4*a^3*c*d^2*e^6+6*a^2*c^2*d^4*e^4-4*a*c^3*d^6
*e^2+c^4*d^8)/(c*d*x+a*e)^2+10*e^2/c^6/d^6*(a^3*e^6-3*a^2*c*d^2*e^4+3*a*c^
2*d^4*e^2-c^3*d^6)/(c*d*x+a*e)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 488 vs. $2(173) = 346$.

Time = 0.09 (sec) , antiderivative size = 488, normalized size of antiderivative = 2.73

$$\int \frac{(d+ex)^9}{(ade+(cd^2+ae^2)x+cdex^2)^4}\,dx$$
$$= \frac{3\,c^5d^5e^5x^5 - 2\,c^5d^{10} - 5\,ac^4d^8e^2 - 20\,a^2c^3d^6e^4 + 110\,a^3c^2d^4e^6 - 130\,a^4cd^2e^8 + 47\,a^5e^{10} + 15\,(2\,c^5d^6e^4 - \cdots}{\cdots}$$

input
```
integrate((e*x+d)^9/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^4,x, algorithm="fric
as")
```

output
```
1/6*(3*c^5*d^5*e^5*x^5 - 2*c^5*d^10 - 5*a*c^4*d^8*e^2 - 20*a^2*c^3*d^6*e^4
 + 110*a^3*c^2*d^4*e^6 - 130*a^4*c*d^2*e^8 + 47*a^5*e^10 + 15*(2*c^5*d^6*e
^4 - a*c^4*d^4*e^6)*x^4 + 9*(10*a*c^4*d^5*e^5 - 7*a^2*c^3*d^3*e^7)*x^3 - 3
*(20*c^5*d^8*e^2 - 60*a*c^4*d^6*e^4 + 30*a^2*c^3*d^4*e^6 + 3*a^3*c^2*d^2*e
^8)*x^2 - 3*(5*c^5*d^9*e + 20*a*c^4*d^7*e^3 - 90*a^2*c^3*d^5*e^5 + 90*a^3*
c^2*d^3*e^7 - 27*a^4*c*d*e^9)*x + 60*(a^3*c^2*d^4*e^6 - 2*a^4*c*d^2*e^8 +
a^5*e^10 + (c^5*d^7*e^3 - 2*a*c^4*d^5*e^5 + a^2*c^3*d^3*e^7)*x^3 + 3*(a*c^
4*d^6*e^4 - 2*a^2*c^3*d^4*e^6 + a^3*c^2*d^2*e^8)*x^2 + 3*(a^2*c^3*d^5*e^5
- 2*a^3*c^2*d^3*e^7 + a^4*c*d*e^9)*x)*log(c*d*x + a*e))/(c^9*d^9*x^3 + 3*a
*c^8*d^8*e*x^2 + 3*a^2*c^7*d^7*e^2*x + a^3*c^6*d^6*e^3)
```

Sympy [A] (verification not implemented)

Time = 25.01 (sec) , antiderivative size = 337, normalized size of antiderivative = 1.88

$$\int \frac{(d+ex)^9}{(ade+(cd^2+ae^2)x+cdex^2)^4}\,dx = x\left(-\frac{4ae^6}{c^5d^5}+\frac{5e^4}{c^4d^3}\right) + \frac{47a^5e^{10}-130a^4cd^2e^8+110a^3c^2d^4e^6-20a^2c^3d^6e^4-5ac^4d^8e^2-2c^5d^{10}+x^2\cdot(60a^3c^2d^2e^8-180a^2c^3d}{6a^3c^6d^6e^3+18a^2c^7d^7e^2x+} + \frac{e^5x^2}{2c^4d^4} + \frac{10e^3(ae^2-cd^2)^2\log(ae+cdx)}{c^6d^6}$$

input
```
integrate((e*x+d)**9/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**4,x)
```

output
```
x*(-4*a*e**6/(c**5*d**5) + 5*e**4/(c**4*d**3)) + (47*a**5*e**10 - 130*a**4
*c*d**2*e**8 + 110*a**3*c**2*d**4*e**6 - 20*a**2*c**3*d**6*e**4 - 5*a*c**4
*d**8*e**2 - 2*c**5*d**10 + x**2*(60*a**3*c**2*d**2*e**8 - 180*a**2*c**3*d
**4*e**6 + 180*a*c**4*d**6*e**4 - 60*c**5*d**8*e**2) + x*(105*a**4*c*d*e**
9 - 300*a**3*c**2*d**3*e**7 + 270*a**2*c**3*d**5*e**5 - 60*a*c**4*d**7*e**
3 - 15*c**5*d**9*e))/(6*a**3*c**6*d**6*e**3 + 18*a**2*c**7*d**7*e**2*x + 1
8*a*c**8*d**8*e*x**2 + 6*c**9*d**9*x**3) + e**5*x**2/(2*c**4*d**4) + 10*e*
*3*(a*e**2 - c*d**2)**2*log(a*e + c*d*x)/(c**6*d**6)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 326, normalized size of antiderivative = 1.82

$$\int \frac{(d+ex)^9}{(ade+(cd^2+ae^2)x+cdex^2)^4}\,dx = -\frac{2\,c^5d^{10}+5\,ac^4d^8e^2+20\,a^2c^3d^6e^4-110\,a^3c^2d^4e^6+130\,a^4cd^2e^8-47\,a^5e^{10}+60\,(c^5d^8e^2-3\,ac^4d^6e^4+3}{6\,(c^9d^9x^3+3\,ac^8d^8ex^2+3\,a^2c^7} + \frac{cde^5x^2+2\,(5\,cd^2e^4-4\,ae^6)x}{2\,c^5d^5} + \frac{10\,(c^2d^4e^3-2\,acd^2e^5+a^2e^7)\log(cdx+ae)}{c^6d^6}$$

input
```
integrate((e*x+d)^9/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^4,x, algorithm="maxi
ma")
```

output
```
-1/6*(2*c^5*d^10 + 5*a*c^4*d^8*e^2 + 20*a^2*c^3*d^6*e^4 - 110*a^3*c^2*d^4*
e^6 + 130*a^4*c*d^2*e^8 - 47*a^5*e^10 + 60*(c^5*d^8*e^2 - 3*a*c^4*d^6*e^4
+ 3*a^2*c^3*d^4*e^6 - a^3*c^2*d^2*e^8)*x^2 + 15*(c^5*d^9*e + 4*a*c^4*d^7*e
^3 - 18*a^2*c^3*d^5*e^5 + 20*a^3*c^2*d^3*e^7 - 7*a^4*c*d*e^9)*x)/(c^9*d^9*
x^3 + 3*a*c^8*d^8*e*x^2 + 3*a^2*c^7*d^7*e^2*x + a^3*c^6*d^6*e^3) + 1/2*(c*
d*e^5*x^2 + 2*(5*c*d^2*e^4 - 4*a*e^6)*x)/(c^5*d^5) + 10*(c^2*d^4*e^3 - 2*a
*c*d^2*e^5 + a^2*e^7)*log(c*d*x + a*e)/(c^6*d^6)
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 299, normalized size of antiderivative = 1.67

$$\int \frac{(d+ex)^9}{(ade+(cd^2+ae^2)x+cdex^2)^4}\,dx = \frac{10\,(c^2d^4e^3-2\,acd^2e^5+a^2e^7)\log(|cdx+ae|)}{c^6d^6} - \frac{2\,c^5d^{10}+5\,ac^4d^8e^2+20\,a^2c^3d^6e^4-110\,a^3c^2d^4e^6+130\,a^4cd^2e^8-47\,a^5e^{10}+60\,(c^5d^8e^2-3\,ac^4d^6e^4+3\cdots}{6\,(cdx+ae)^3c^6\cdots} + \frac{c^4d^4e^5x^2+10\,c^4d^5e^4x-8\,ac^3d^3e^6x}{2\,c^8d^8}$$

input
```
integrate((e*x+d)^9/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^4,x, algorithm="giac
")
```

output
```
10*(c^2*d^4*e^3 - 2*a*c*d^2*e^5 + a^2*e^7)*log(abs(c*d*x + a*e))/(c^6*d^6)
 - 1/6*(2*c^5*d^10 + 5*a*c^4*d^8*e^2 + 20*a^2*c^3*d^6*e^4 - 110*a^3*c^2*d^
4*e^6 + 130*a^4*c*d^2*e^8 - 47*a^5*e^10 + 60*(c^5*d^8*e^2 - 3*a*c^4*d^6*e^
4 + 3*a^2*c^3*d^4*e^6 - a^3*c^2*d^2*e^8)*x^2 + 15*(c^5*d^9*e + 4*a*c^4*d^7
*e^3 - 18*a^2*c^3*d^5*e^5 + 20*a^3*c^2*d^3*e^7 - 7*a^4*c*d*e^9)*x)/((c*d*x
 + a*e)^3*c^6*d^6) + 1/2*(c^4*d^4*e^5*x^2 + 10*c^4*d^5*e^4*x - 8*a*c^3*d^3
*e^6*x)/(c^8*d^8)
```

Mupad [B] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 331, normalized size of antiderivative = 1.85

$$\int \frac{(d+ex)^9}{(ade+(cd^2+ae^2)x+cdex^2)^4}\,dx = x\left(\frac{5\,e^4}{c^4\,d^3}-\frac{4\,a\,e^6}{c^5\,d^5}\right)$$
$$-\frac{x^2\left(-10\,a^3\,c\,d\,e^8+30\,a^2\,c^2\,d^3\,e^6-30\,a\,c^3\,d^5\,e^4+10\,c^4\,d^7\,e^2\right)+x\left(-\frac{35\,a^4\,e^9}{2}+50\,a^3\,c\,d^2\,e^7-45\,a^2\,c^2\,d^4\,e\right.}{a^3\,c^5\,d^5\,e^3+3\,a^2\,c^6\,d^6\,e^2\,x+3\,a\,c}$$
$$+\frac{\ln\left(a\,e+c\,d\,x\right)\,\left(10\,a^2\,e^7-20\,a\,c\,d^2\,e^5+10\,c^2\,d^4\,e^3\right)}{c^6\,d^6}+\frac{e^5\,x^2}{2\,c^4\,d^4}$$

input
```
int((d + e*x)^9/(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^4,x)
```

output
```
x*((5*e^4)/(c^4*d^3) - (4*a*e^6)/(c^5*d^5)) - (x^2*(10*c^4*d^7*e^2 - 30*a*
c^3*d^5*e^4 + 30*a^2*c^2*d^3*e^6 - 10*a^3*c*d*e^8) + x*((5*c^4*d^8*e)/2 -
(35*a^4*e^9)/2 + 10*a*c^3*d^6*e^3 + 50*a^3*c*d^2*e^7 - 45*a^2*c^2*d^4*e^5)
 + (2*c^5*d^10 - 47*a^5*e^10 + 5*a*c^4*d^8*e^2 + 130*a^4*c*d^2*e^8 + 20*a^
2*c^3*d^6*e^4 - 110*a^3*c^2*d^4*e^6)/(6*c*d))/(c^8*d^8*x^3 + a^3*c^5*d^5*e
^3 + 3*a*c^7*d^7*e*x^2 + 3*a^2*c^6*d^6*e^2*x) + (log(a*e + c*d*x)*(10*a^2*
e^7 + 10*c^2*d^4*e^3 - 20*a*c*d^2*e^5))/(c^6*d^6) + (e^5*x^2)/(2*c^4*d^4)
```

Reduce [B] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 556, normalized size of antiderivative = 3.11

$$\int \frac{(d+ex)^9}{(ade+(cd^2+ae^2)x+cdex^2)^4}\,dx$$
$$=\frac{180\log(cdx+ae)\,a^5cd\,e^9x+60\log(cdx+ae)\,a^3c^3d^3e^7x^3-120\log(cdx+ae)\,a^2c^4d^5e^5x^3+60\log(cdx+}{}$$

input
```
int((e*x+d)^9/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^4,x)
```

output

```
(60*log(a*e + c*d*x)*a**6*e**10 - 120*log(a*e + c*d*x)*a**5*c*d**2*e**8 +
180*log(a*e + c*d*x)*a**5*c*d*e**9*x + 60*log(a*e + c*d*x)*a**4*c**2*d**4*
e**6 - 360*log(a*e + c*d*x)*a**4*c**2*d**3*e**7*x + 180*log(a*e + c*d*x)*a
**4*c**2*d**2*e**8*x**2 + 180*log(a*e + c*d*x)*a**3*c**3*d**5*e**5*x - 360
*log(a*e + c*d*x)*a**3*c**3*d**4*e**6*x**2 + 60*log(a*e + c*d*x)*a**3*c**3
*d**3*e**7*x**3 + 180*log(a*e + c*d*x)*a**2*c**4*d**6*e**4*x**2 - 120*log(
a*e + c*d*x)*a**2*c**4*d**5*e**5*x**3 + 60*log(a*e + c*d*x)*a*c**5*d**7*e*
*3*x**3 + 50*a**6*e**10 - 100*a**5*c*d**2*e**8 + 90*a**5*c*d*e**9*x + 50*a
**4*c**2*d**4*e**6 - 180*a**4*c**2*d**3*e**7*x + 90*a**3*c**3*d**5*e**5*x
- 60*a**3*c**3*d**3*e**7*x**3 - 5*a**2*c**4*d**8*e**2 + 120*a**2*c**4*d**5
*e**5*x**3 - 15*a**2*c**4*d**4*e**6*x**4 - 2*a*c**5*d**10 - 15*a*c**5*d**9
*e*x - 60*a*c**5*d**7*e**3*x**3 + 30*a*c**5*d**6*e**4*x**4 + 3*a*c**5*d**5
*e**5*x**5 + 20*c**6*d**9*e*x**3)/(6*a*c**6*d**6*(a**3*e**3 + 3*a**2*c*d*e
**2*x + 3*a*c**2*d**2*e*x**2 + c**3*d**3*x**3))
```

3.140 $\int \frac{(d+ex)^8}{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^4}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 146

$$\int \frac{(d+ex)^8}{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^4}\,dx = \frac{e^4x}{c^4d^4}-\frac{\left(cd^2-ae^2\right)^4}{3c^5d^5(ae+cdx)^3}-\frac{2e\left(cd^2-ae^2\right)^3}{c^5d^5(ae+cdx)^2} -\frac{6e^2\left(cd^2-ae^2\right)^2}{c^5d^5(ae+cdx)}+\frac{4e^3\left(cd^2-ae^2\right)\log(ae+cdx)}{c^5d^5}$$

output
```
e^4*x/c^4/d^4-1/3*(-a*e^2+c*d^2)^4/c^5/d^5/(c*d*x+a*e)^3-2*e*(-a*e^2+c*d^2
)^3/c^5/d^5/(c*d*x+a*e)^2-6*e^2*(-a*e^2+c*d^2)^2/c^5/d^5/(c*d*x+a*e)+4*e^3
*(-a*e^2+c*d^2)*ln(c*d*x+a*e)/c^5/d^5
```

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 194, normalized size of antiderivative = 1.33

$$\int \frac{(d+ex)^8}{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^4}\,dx = \frac{-13a^4e^8+a^3cde^6(22d-27ex)-3a^2c^2d^2e^4\left(2d^2-18dex+3e^2x^2\right)+ac^3d^3e^2\left(-2d^3-18d^2ex+36de^2x^2\right.}{3c^5d^5(ae+c}$$

input
```
Integrate[(d + e*x)^8/(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^4,x]
```

output
```
(-13*a^4*e^8 + a^3*c*d*e^6*(22*d - 27*e*x) - 3*a^2*c^2*d^2*e^4*(2*d^2 - 18
*d*e*x + 3*e^2*x^2) + a*c^3*d^3*e^2*(-2*d^3 - 18*d^2*e*x + 36*d*e^2*x^2 +
9*e^3*x^3) - c^4*(d^8 + 6*d^7*e*x + 18*d^6*e^2*x^2 - 3*d^4*e^4*x^4) - 12*e
^3*(-(c*d^2) + a*e^2)*(a*e + c*d*x)^3*Log[a*e + c*d*x])/(3*c^5*d^5*(a*e +
c*d*x)^3)
```

Rubi [A] (verified)

Time = 0.58 (sec) , antiderivative size = 146, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.057$, Rules used = {1121, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex)^8}{\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^4}dx$$

$$\Big\downarrow \text{1121}$$

$$\int \left(\frac{6\left(cd^2e-ae^3\right)^2}{c^4d^4(ae+cdx)^2}+\frac{4e\left(cd^2-ae^2\right)^3}{c^4d^4(ae+cdx)^3}+\frac{\left(cd^2-ae^2\right)^4}{c^4d^4(ae+cdx)^4}+\frac{4\left(cd^2e^3-ae^5\right)}{c^4d^4(ae+cdx)}+\frac{e^4}{c^4d^4}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{6e^2\left(cd^2-ae^2\right)^2}{c^5d^5(ae+cdx)}-\frac{2e\left(cd^2-ae^2\right)^3}{c^5d^5(ae+cdx)^2}-\frac{\left(cd^2-ae^2\right)^4}{3c^5d^5(ae+cdx)^3}+\frac{4e^3\left(cd^2-ae^2\right)\log(ae+cdx)}{c^5d^5}+\frac{e^4x}{c^4d^4}$$

input
```
Int[(d + e*x)^8/(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^4,x]
```

output
```
(e^4*x)/(c^4*d^4) - (c*d^2 - a*e^2)^4/(3*c^5*d^5*(a*e + c*d*x)^3) - (2*e*(
c*d^2 - a*e^2)^3)/(c^5*d^5*(a*e + c*d*x)^2) - (6*e^2*(c*d^2 - a*e^2)^2)/(c
^5*d^5*(a*e + c*d*x)) + (4*e^3*(c*d^2 - a*e^2)*Log[a*e + c*d*x])/(c^5*d^5)
```

Defintions of rubi rules used

rule 1121
```
Int[((d_) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Int[ExpandIntegrand[(d + e*x)^(m + p)*(a/d + (c/e)*x)^p, x], x]
/; FreeQ[{a, b, c, d, e, m, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && (Int
egerQ[p] || (GtQ[a, 0] && GtQ[d, 0] && LtQ[c, 0]))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 1.53 (sec) , antiderivative size = 216, normalized size of antiderivative = 1.48

method	result
risch	$\frac{e^{4} x}{c^{4} d^{4}}+\frac{\left(-6 a^{2} e^{6} c d+12 a \,e^{4} c^{2} d^{3}-6 d^{5} c^{3} e^{2}\right) x^{2}-2 e \left(5 e^{6} a^{3}-9 d^{2} e^{4} a^{2} c+3 d^{4} e^{2} a \,c^{2}+d^{6} c^{3}\right) x-\frac{13 a^{4} e^{8}-22 a^{3} c \,d^{2} e^{6}+6 a^{2} c^{2} d^{4} e^{4}+2 a}{3 c d}}{d^{4} c^{4} \left(c d x+a e\right)^{3}}$
default	$\frac{e^{4} x}{c^{4} d^{4}}-\frac{a^{4} e^{8}-4 a^{3} c \,d^{2} e^{6}+6 a^{2} c^{2} d^{4} e^{4}-4 a \,c^{3} d^{6} e^{2}+c^{4} d^{8}}{3 d^{5} c^{5} \left(c d x+a e\right)^{3}}-\frac{4 e^{3} \left(a \,e^{2}-c \,d^{2}\right) \ln \left(c d x+a e\right)}{d^{5} c^{5}}+\frac{2 e \left(e^{6} a^{3}-3 d^{2} e^{4} a^{2} c+3 d^{4} e^{2} a \,c^{2}-\right.}{d^{5} c^{5} \left(c d x+a e\right)^{2}}$
parallelrisch	$-\frac{22 a^{4} e^{8}-22 a^{3} c \,d^{2} e^{6}+6 a^{2} c^{2} d^{4} e^{4}+2 a \,c^{3} d^{6} e^{2}+c^{4} d^{8}-12 \ln \left(c d x+a e\right) a^{3} c \,d^{2} e^{6}-54 a^{2} c^{2} d^{3} e^{5} x+18 a \,c^{3} d^{5} e^{3} x-12 \ln \left(c d x+a e\right) x^{3}}{}$
norman	$\frac{\frac{e^{7} x^{7}}{c d}-\frac{22 a^{4} e^{8}-13 a^{3} c \,d^{2} e^{6}+6 a^{2} c^{2} d^{4} e^{4}+2 a \,c^{3} d^{6} e^{2}+c^{4} d^{8}}{3 c^{5} d^{2}}-\frac{\left(22 a^{4} e^{14}+149 a^{3} c \,d^{2} e^{12}+33 a^{2} c^{2} d^{4} e^{10}+29 a \,c^{3} d^{6} e^{8}+82 c^{4} d^{8} e^{6}\right) x^{3}}{3 d^{5} c^{5} e^{3}}-\left(22 a^{4}\right.}{}$

input
```
int((e*x+d)^8/(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^4,x,method=_RETURNVERBOSE)
```

output
```
e^4*x/c^4/d^4+((-6*a^2*c*d*e^6+12*a*c^2*d^3*e^4-6*c^3*d^5*e^2)*x^2-2*e*(5*
a^3*e^6-9*a^2*c*d^2*e^4+3*a*c^2*d^4*e^2+c^3*d^6)*x-1/3*(13*a^4*e^8-22*a^3*
c*d^2*e^6+6*a^2*c^2*d^4*e^4+2*a*c^3*d^6*e^2+c^4*d^8)/c/d)/d^4/c^4/(c*d*x+a
*e)^3-4*e^5/d^5/c^5*ln(c*d*x+a*e)*a+4*e^3/d^3/c^4*ln(c*d*x+a*e)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 347 vs. $2(144) = 288$.

Time = 0.08 (sec) , antiderivative size = 347, normalized size of antiderivative = 2.38

$$\int \frac{(d+ex)^8}{(ade+(cd^2+ae^2)x+cdex^2)^4}\,dx$$
$$= \frac{3\,c^4d^4e^4x^4 + 9\,ac^3d^3e^5x^3 - c^4d^8 - 2\,ac^3d^6e^2 - 6\,a^2c^2d^4e^4 + 22\,a^3cd^2e^6 - 13\,a^4e^8 - 9\,(2\,c^4d^6e^2 - 4\,ac^3d^4e}{}$$

input
```
integrate((e*x+d)^8/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^4,x, algorithm="fric
as")
```

output
```
1/3*(3*c^4*d^4*e^4*x^4 + 9*a*c^3*d^3*e^5*x^3 - c^4*d^8 - 2*a*c^3*d^6*e^2 -
 6*a^2*c^2*d^4*e^4 + 22*a^3*c*d^2*e^6 - 13*a^4*e^8 - 9*(2*c^4*d^6*e^2 - 4*
a*c^3*d^4*e^4 + a^2*c^2*d^2*e^6)*x^2 - 3*(2*c^4*d^7*e + 6*a*c^3*d^5*e^3 -
18*a^2*c^2*d^3*e^5 + 9*a^3*c*d*e^7)*x + 12*(a^3*c*d^2*e^6 - a^4*e^8 + (c^4
*d^5*e^3 - a*c^3*d^3*e^5)*x^3 + 3*(a*c^3*d^4*e^4 - a^2*c^2*d^2*e^6)*x^2 +
3*(a^2*c^2*d^3*e^5 - a^3*c*d*e^7)*x)*log(c*d*x + a*e))/(c^8*d^8*x^3 + 3*a*
c^7*d^7*e*x^2 + 3*a^2*c^6*d^6*e^2*x + a^3*c^5*d^5*e^3)
```

Sympy [A] (verification not implemented)

Time = 6.34 (sec) , antiderivative size = 257, normalized size of antiderivative = 1.76

$$\int \frac{(d+ex)^8}{(ade+(cd^2+ae^2)x+cdex^2)^4}\,dx$$
$$= \frac{-13a^4e^8 + 22a^3cd^2e^6 - 6a^2c^2d^4e^4 - 2ac^3d^6e^2 - c^4d^8 + x^2(-18a^2c^2d^2e^6 + 36ac^3d^4e^4 - 18c^4d^6e^2) + x(-}{3a^3c^5d^5e^3 + 9a^2c^6d^6e^2x + 9ac^7d^7ex^2 + 3c^8d^8x^3}$$
$$+ \frac{e^4x}{c^4d^4} - \frac{4e^3(ae^2-cd^2)\log(ae+cdx)}{c^5d^5}$$

input
```
integrate((e*x+d)**8/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**4,x)
```

output
```
(-13*a**4*e**8 + 22*a**3*c*d**2*e**6 - 6*a**2*c**2*d**4*e**4 - 2*a*c**3*d*
*6*e**2 - c**4*d**8 + x**2*(-18*a**2*c**2*d**2*e**6 + 36*a*c**3*d**4*e**4
- 18*c**4*d**6*e**2) + x*(-30*a**3*c*d*e**7 + 54*a**2*c**2*d**3*e**5 - 18*
a*c**3*d**5*e**3 - 6*c**4*d**7*e))/(3*a**3*c**5*d**5*e**3 + 9*a**2*c**6*d*
*6*e**2*x + 9*a*c**7*d**7*e*x**2 + 3*c**8*d**8*x**3) + e**4*x/(c**4*d**4)
- 4*e**3*(a*e**2 - c*d**2)*log(a*e + c*d*x)/(c**5*d**5)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 243, normalized size of antiderivative = 1.66

$$\int \frac{(d+ex)^8}{(ade+(cd^2+ae^2)x+cdex^2)^4}\,dx = -\frac{c^4d^8+2\,ac^3d^6e^2+6\,a^2c^2d^4e^4-22\,a^3cd^2e^6+13\,a^4e^8+18\,(c^4d^6e^2-2\,ac^3d^4e^4+a^2c^2d^2e^6)x^2+6\,(c^4d^7e\cdots}{3\,(c^8d^8x^3+3\,ac^7d^7ex^2+3\,a^2c^6d^6e^2x+a^3c^5d^5e^3)} + \frac{e^4x}{c^4d^4} + \frac{4\,(cd^2e^3-ae^5)\log(cdx+ae)}{c^5d^5}$$

input
```
integrate((e*x+d)^8/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^4,x, algorithm="maxi
ma")
```

output
```
-1/3*(c^4*d^8 + 2*a*c^3*d^6*e^2 + 6*a^2*c^2*d^4*e^4 - 22*a^3*c*d^2*e^6 + 1
3*a^4*e^8 + 18*(c^4*d^6*e^2 - 2*a*c^3*d^4*e^4 + a^2*c^2*d^2*e^6)*x^2 + 6*(
c^4*d^7*e + 3*a*c^3*d^5*e^3 - 9*a^2*c^2*d^3*e^5 + 5*a^3*c*d*e^7)*x)/(c^8*d
^8*x^3 + 3*a*c^7*d^7*e*x^2 + 3*a^2*c^6*d^6*e^2*x + a^3*c^5*d^5*e^3) + e^4*
x/(c^4*d^4) + 4*(c*d^2*e^3 - a*e^5)*log(c*d*x + a*e)/(c^5*d^5)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 206, normalized size of antiderivative = 1.41

$$\int \frac{(d+ex)^8}{(ade+(cd^2+ae^2)x+cdex^2)^4}\,dx = \frac{e^4x}{c^4d^4} + \frac{4\,(cd^2e^3-ae^5)\log(|cdx+ae|)}{c^5d^5} - \frac{c^4d^8+2\,ac^3d^6e^2+6\,a^2c^2d^4e^4-22\,a^3cd^2e^6+13\,a^4e^8+18\,(c^4d^6e^2-2\,ac^3d^4e^4+a^2c^2d^2e^6)x^2+6\,(c^4d^7e\cdots}{3\,(cdx+ae)^3c^5d^5}$$

input
```
integrate((e*x+d)^8/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^4,x, algorithm="giac
")
```

output
```
e^4*x/(c^4*d^4) + 4*(c*d^2*e^3 - a*e^5)*log(abs(c*d*x + a*e))/(c^5*d^5) -
1/3*(c^4*d^8 + 2*a*c^3*d^6*e^2 + 6*a^2*c^2*d^4*e^4 - 22*a^3*c*d^2*e^6 + 13
*a^4*e^8 + 18*(c^4*d^6*e^2 - 2*a*c^3*d^4*e^4 + a^2*c^2*d^2*e^6)*x^2 + 6*(c
^4*d^7*e + 3*a*c^3*d^5*e^3 - 9*a^2*c^2*d^3*e^5 + 5*a^3*c*d*e^7)*x)/((c*d*x
 + a*e)^3*c^5*d^5)
```

Mupad [B] (verification not implemented)

Time = 5.28 (sec) , antiderivative size = 246, normalized size of antiderivative = 1.68

$$\int \frac{(d+ex)^8}{(ade+(cd^2+ae^2)\,x+cdex^2)^4}\,dx = \frac{e^4\,x}{c^4\,d^4} - \frac{x\,(10\,a^3\,e^7-18\,a^2\,c\,d^2\,e^5+6\,a\,c^2\,d^4\,e^3+2\,c^3\,d^6\,e)+x^2\,(6\,a^2\,c\,d\,e^6-12\,a\,c^2\,d^3\,e^4+6\,c^3\,d^5\,e^2)+\frac{13\,a^4\,e^8-}{}}{a^3\,c^4\,d^4\,e^3+3\,a^2\,c^5\,d^5\,e^2\,x+3\,a\,c^6\,d^6\,e\,x^2+c^7\,d^7\,x^3} - \frac{\ln(a\,e+c\,d\,x)\,(4\,a\,e^5-4\,c\,d^2\,e^3)}{c^5\,d^5}$$

input
```
int((d + e*x)^8/(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^4,x)
```

output
```
(e^4*x)/(c^4*d^4) - (x*(10*a^3*e^7 + 2*c^3*d^6*e + 6*a*c^2*d^4*e^3 - 18*a^
2*c*d^2*e^5) + x^2*(6*c^3*d^5*e^2 - 12*a*c^2*d^3*e^4 + 6*a^2*c*d*e^6) + (1
3*a^4*e^8 + c^4*d^8 + 2*a*c^3*d^6*e^2 - 22*a^3*c*d^2*e^6 + 6*a^2*c^2*d^4*e
^4)/(3*c*d))/(c^7*d^7*x^3 + a^3*c^4*d^4*e^3 + 3*a*c^6*d^6*e*x^2 + 3*a^2*c^
5*d^5*e^2*x) - (log(a*e + c*d*x)*(4*a*e^5 - 4*c*d^2*e^3))/(c^5*d^5)
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 379, normalized size of antiderivative = 2.60

$$\int \frac{(d+ex)^8}{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^4}\,dx$$
$$= \frac{-12\log(cdx+ae)\,a^5e^8+12\log(cdx+ae)\,a^4c\,d^2e^6-36\log(cdx+ae)\,a^4cd\,e^7x+36\log(cdx+ae)\,a^3c^2d}{}$$

input
```
int((e*x+d)^8/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^4,x)
```

output
```
( - 12*log(a*e + c*d*x)*a**5*e**8 + 12*log(a*e + c*d*x)*a**4*c*d**2*e**6 -
 36*log(a*e + c*d*x)*a**4*c*d*e**7*x + 36*log(a*e + c*d*x)*a**3*c**2*d**3*
e**5*x - 36*log(a*e + c*d*x)*a**3*c**2*d**2*e**6*x**2 + 36*log(a*e + c*d*x
)*a**2*c**3*d**4*e**4*x**2 - 12*log(a*e + c*d*x)*a**2*c**3*d**3*e**5*x**3
+ 12*log(a*e + c*d*x)*a*c**4*d**5*e**3*x**3 - 10*a**5*e**8 + 10*a**4*c*d**
2*e**6 - 18*a**4*c*d*e**7*x + 18*a**3*c**2*d**3*e**5*x - 2*a**2*c**3*d**6*
e**2 + 12*a**2*c**3*d**3*e**5*x**3 - a*c**4*d**8 - 6*a*c**4*d**7*e*x - 12*
a*c**4*d**5*e**3*x**3 + 3*a*c**4*d**4*e**4*x**4 + 6*c**5*d**7*e*x**3)/(3*a
*c**5*d**5*(a**3*e**3 + 3*a**2*c*d*e**2*x + 3*a*c**2*d**2*e*x**2 + c**3*d*
*3*x**3))
```

3.141 $\int \frac{(d+ex)^7}{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^4}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 122

$$\int \frac{(d+ex)^7}{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^4}\,dx = -\frac{\left(cd^2-ae^2\right)^3}{3c^4d^4(ae+cdx)^3}-\frac{3e\left(cd^2-ae^2\right)^2}{2c^4d^4(ae+cdx)^2}-\frac{3e^2\left(cd^2-ae^2\right)}{c^4d^4(ae+cdx)}+\frac{e^3\log(ae+cdx)}{c^4d^4}$$

output
```
-1/3*(-a*e^2+c*d^2)^3/c^4/d^4/(c*d*x+a*e)^3-3/2*e*(-a*e^2+c*d^2)^2/c^4/d^4
/(c*d*x+a*e)^2-3*e^2*(-a*e^2+c*d^2)/c^4/d^4/(c*d*x+a*e)+e^3*ln(c*d*x+a*e)/
c^4/d^4
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 99, normalized size of antiderivative = 0.81

$$\int \frac{(d+ex)^7}{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^4}\,dx = \frac{-\frac{\left(cd^2-ae^2\right)\left(11a^2e^4+acde^2(5d+27ex)+c^2d^2\left(2d^2+9dex+18e^2x^2\right)\right)}{(ae+cdx)^3}+6e^3\log(ae+cdx)}{6c^4d^4}$$

input
```
Integrate[(d + e*x)^7/(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^4,x]
```

output
```
(-(((c*d^2 - a*e^2)*(11*a^2*e^4 + a*c*d*e^2*(5*d + 27*e*x) + c^2*d^2*(2*d^
2 + 9*d*e*x + 18*e^2*x^2)))/(a*e + c*d*x)^3) + 6*e^3*Log[a*e + c*d*x])/(6*
c^4*d^4)
```

Rubi [A] (verified)

Time = 0.49 (sec) , antiderivative size = 122, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.057$, Rules used = {1121, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex)^7}{(x(ae^2+cd^2)+ade+cdex^2)^4}dx$$

$$\Big\downarrow \text{1121}$$

$$\int \left(\frac{e^3}{c^3d^3(ae+cdx)}+\frac{3e(cd^2-ae^2)^2}{c^3d^3(ae+cdx)^3}+\frac{(cd^2-ae^2)^3}{c^3d^3(ae+cdx)^4}+\frac{3(cd^2e^2-ae^4)}{c^3d^3(ae+cdx)^2}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{e^3\log(ae+cdx)}{c^4d^4}-\frac{3e^2(cd^2-ae^2)}{c^4d^4(ae+cdx)}-\frac{3e(cd^2-ae^2)^2}{2c^4d^4(ae+cdx)^2}-\frac{(cd^2-ae^2)^3}{3c^4d^4(ae+cdx)^3}$$

input
```
Int[(d + e*x)^7/(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^4,x]
```

output
```
-1/3*(c*d^2 - a*e^2)^3/(c^4*d^4*(a*e + c*d*x)^3) - (3*e*(c*d^2 - a*e^2)^2)
/(2*c^4*d^4*(a*e + c*d*x)^2) - (3*e^2*(c*d^2 - a*e^2))/(c^4*d^4*(a*e + c*d
*x)) + (e^3*Log[a*e + c*d*x])/(c^4*d^4)
```

Defintions of rubi rules used

rule 1121
```
Int[((d_) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Int[ExpandIntegrand[(d + e*x)^(m + p)*(a/d + (c/e)*x)^p, x], x]
/; FreeQ[{a, b, c, d, e, m, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && (Int
egerQ[p] || (GtQ[a, 0] && GtQ[d, 0] && LtQ[c, 0]))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 1.49 (sec) , antiderivative size = 145, normalized size of antiderivative = 1.19

method	result
risch	$\frac{\frac{3e^2(a e^2-c d^2)x^2}{c^2d^2}+\frac{3e(3a^2e^4-2ac d^2e^2-c^2d^4)x}{2d^3c^3}+\frac{11e^6a^3-6d^2e^4a^2c-3d^4e^2a c^2-2d^6c^3}{6d^4c^4}}{(cdx+ae)^3}+\frac{e^3\ln(cdx+ae)}{c^4d^4}$
default	$-\frac{-e^6a^3+3d^2e^4a^2c-3d^4e^2a c^2+d^6c^3}{3d^4c^4(cdx+ae)^3}+\frac{e^3\ln(cdx+ae)}{c^4d^4}-\frac{3e(a^2e^4-2ac d^2e^2+c^2d^4)}{2d^4c^4(cdx+ae)^2}+\frac{3e^2(a e^2-c d^2)}{c^4d^4(cdx+ae)}$
parallelrisch	$\frac{6\ln(cdx+ae)x^3c^3d^3e^3+18\ln(cdx+ae)x^2a c^2d^2e^4+18\ln(cdx+ae)x a^2cd e^5+18x^2a c^2d^2e^4-18c^3d^4e^2x^2+6\ln(cdx+ae)a^3e^6+}{6d^4c^4(cdx+ae)^3}$
norman	$\frac{\frac{11e^6a^3-6d^2e^4a^2c-3d^4e^2a c^2-2d^6c^3}{6c^4d}+\frac{(11a^3e^{12}+75a^2c d^2e^{10}-3d^4a c^2e^8-83d^6e^6c^3)x^3}{6d^4c^4e^3}+\frac{(11a^3e^{10}+21a^2c d^2e^8-15d^4a c^2e^6-17c^3d^6}{2d^3c^4e^2}}{(ex+d)^3(cdx+ae)^3}$

input
```
int((e*x+d)^7/(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^4,x,method=_RETURNVERBOSE)
```

output
```
(3*e^2*(a*e^2-c*d^2)/c^2/d^2*x^2+3/2*e*(3*a^2*e^4-2*a*c*d^2*e^2-c^2*d^4)/d
^3/c^3*x+1/6*(11*a^3*e^6-6*a^2*c*d^2*e^4-3*a*c^2*d^4*e^2-2*c^3*d^6)/d^4/c^
4)/(c*d*x+a*e)^3+e^3*ln(c*d*x+a*e)/c^4/d^4
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 217, normalized size of antiderivative = 1.78

$$\int \frac{(d+ex)^7}{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^4}\,dx = -\frac{2\,c^3d^6+3\,ac^2d^4e^2+6\,a^2cd^2e^4-11\,a^3e^6+18\,(c^3d^4e^2-ac^2d^2e^4)x^2+9\,(c^3d^5e+2\,ac^2d^3e^3-3\,a^2cde^5)x}{6\,(c^7d^7x^3+3\,ac^6d^6ex^2+3\,a^2c^5d^5e^2x+a^3c^4}$$

input
```
integrate((e*x+d)^7/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^4,x, algorithm="fric
as")
```

output
```
-1/6*(2*c^3*d^6 + 3*a*c^2*d^4*e^2 + 6*a^2*c*d^2*e^4 - 11*a^3*e^6 + 18*(c^3
*d^4*e^2 - a*c^2*d^2*e^4)*x^2 + 9*(c^3*d^5*e + 2*a*c^2*d^3*e^3 - 3*a^2*c*d
*e^5)*x - 6*(c^3*d^3*e^3*x^3 + 3*a*c^2*d^2*e^4*x^2 + 3*a^2*c*d*e^5*x + a^3
*e^6)*log(c*d*x + a*e))/(c^7*d^7*x^3 + 3*a*c^6*d^6*e*x^2 + 3*a^2*c^5*d^5*e
^2*x + a^3*c^4*d^4*e^3)
```

Sympy [A] (verification not implemented)

Time = 1.41 (sec) , antiderivative size = 189, normalized size of antiderivative = 1.55

$$\int \frac{(d+ex)^7}{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^4}\,dx = \frac{11a^3e^6-6a^2cd^2e^4-3ac^2d^4e^2-2c^3d^6+x^2\cdot\left(18ac^2d^2e^4-18c^3d^4e^2\right)+x\left(27a^2cde^5-18ac^2d^3e^3-9c^3d^5e\right)}{6a^3c^4d^4e^3+18a^2c^5d^5e^2x+18ac^6d^6ex^2+6c^7d^7x^3} + \frac{e^3\log\left(ae+cdx\right)}{c^4d^4}$$

input
```
integrate((e*x+d)**7/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**4,x)
```

output
```
(11*a**3*e**6 - 6*a**2*c*d**2*e**4 - 3*a*c**2*d**4*e**2 - 2*c**3*d**6 + x*
*2*(18*a*c**2*d**2*e**4 - 18*c**3*d**4*e**2) + x*(27*a**2*c*d*e**5 - 18*a*
c**2*d**3*e**3 - 9*c**3*d**5*e))/(6*a**3*c**4*d**4*e**3 + 18*a**2*c**5*d**
5*e**2*x + 18*a*c**6*d**6*e*x**2 + 6*c**7*d**7*x**3) + e**3*log(a*e + c*d*
x)/(c**4*d**4)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 179, normalized size of antiderivative = 1.47

$$\int \frac{(d+ex)^7}{\left(ade+(cd^2+ae^2)\,x+cdex^2\right)^4}\,dx = -\frac{2\,c^3d^6+3\,ac^2d^4e^2+6\,a^2cd^2e^4-11\,a^3e^6+18\,(c^3d^4e^2-ac^2d^2e^4)x^2+9\,(c^3d^5e+2\,ac^2d^3e^3-3\,a^2cde^5)x}{6\,(c^7d^7x^3+3\,ac^6d^6ex^2+3\,a^2c^5d^5e^2x+a^3c^4d^4e^3)} + \frac{e^3\log\left(cdx+ae\right)}{c^4d^4}$$

input
```
integrate((e*x+d)^7/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^4,x, algorithm="maxi
ma")
```

output
```
-1/6*(2*c^3*d^6 + 3*a*c^2*d^4*e^2 + 6*a^2*c*d^2*e^4 - 11*a^3*e^6 + 18*(c^3
*d^4*e^2 - a*c^2*d^2*e^4)*x^2 + 9*(c^3*d^5*e + 2*a*c^2*d^3*e^3 - 3*a^2*c*d
*e^5)*x)/(c^7*d^7*x^3 + 3*a*c^6*d^6*e*x^2 + 3*a^2*c^5*d^5*e^2*x + a^3*c^4*
d^4*e^3) + e^3*log(c*d*x + a*e)/(c^4*d^4)
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 142, normalized size of antiderivative = 1.16

$$\int \frac{(d+ex)^7}{\left(ade+(cd^2+ae^2)\,x+cdex^2\right)^4}\,dx = \frac{e^3\log\left(|cdx+ae|\right)}{c^4d^4} - \frac{18\,(c^2d^3e^2-acde^4)x^2+9\,(c^2d^4e+2\,acd^2e^3-3\,a^2e^5)x+\frac{2\,c^3d^6+3\,ac^2d^4e^2+6\,a^2cd^2e^4-11\,a^3e^6}{cd}}{6\,(cdx+ae)^3c^3d^3}$$

input
```
integrate((e*x+d)^7/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^4,x, algorithm="giac
")
```

output
```
e^3*log(abs(c*d*x + a*e))/(c^4*d^4) - 1/6*(18*(c^2*d^3*e^2 - a*c*d*e^4)*x^
2 + 9*(c^2*d^4*e + 2*a*c*d^2*e^3 - 3*a^2*e^5)*x + (2*c^3*d^6 + 3*a*c^2*d^4
*e^2 + 6*a^2*c*d^2*e^4 - 11*a^3*e^6)/(c*d))/((c*d*x + a*e)^3*c^3*d^3)
```

Mupad [B] (verification not implemented)

Time = 5.28 (sec) , antiderivative size = 178, normalized size of antiderivative = 1.46

$$\int \frac{(d+ex)^7}{(ade+(cd^2+ae^2)x+cdex^2)^4}\,dx$$
$$= \frac{e^3\,\ln(a\,e+c\,d\,x)}{c^4\,d^4} - \frac{\frac{-11\,a^3\,e^6+6\,a^2\,c\,d^2\,e^4+3\,a\,c^2\,d^4\,e^2+2\,c^3\,d^6}{6\,c^4\,d^4}+\frac{3\,x\,(-3\,a^2\,e^5+2\,a\,c\,d^2\,e^3+c^2\,d^4\,e)}{2\,c^3\,d^3}-\frac{3\,e^2\,x^2\,(a\,e^2-c\,d^2)}{c^2\,d^2}}{a^3\,e^3+3\,a^2\,c\,d\,e^2\,x+3\,a\,c^2\,d^2\,e\,x^2+c^3\,d^3\,x^3}$$

input
```
int((d + e*x)^7/(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^4,x)
```

output
```
(e^3*log(a*e + c*d*x))/(c^4*d^4) - ((2*c^3*d^6 - 11*a^3*e^6 + 3*a*c^2*d^4*
e^2 + 6*a^2*c*d^2*e^4)/(6*c^4*d^4) + (3*x*(c^2*d^4*e - 3*a^2*e^5 + 2*a*c*d
^2*e^3))/(2*c^3*d^3) - (3*e^2*x^2*(a*e^2 - c*d^2))/(c^2*d^2))/(a^3*e^3 + c
^3*d^3*x^3 + 3*a^2*c*d*e^2*x + 3*a*c^2*d^2*e*x^2)
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 223, normalized size of antiderivative = 1.83

$$\int \frac{(d+ex)^7}{(ade+(cd^2+ae^2)x+cdex^2)^4}\,dx$$
$$= \frac{6\log(cdx+ae)\,a^4e^6+18\log(cdx+ae)\,a^3cd\,e^5x+18\log(cdx+ae)\,a^2c^2d^2e^4x^2+6\log(cdx+ae)\,a\,c^3d^3e}{6a\,c^4d^4\,(c^3d^3x^3+3a\,c^2d^2e\,x^2+3a}$$

input
```
int((e*x+d)^7/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^4,x)
```

output
```
(6*log(a*e + c*d*x)*a**4*e**6 + 18*log(a*e + c*d*x)*a**3*c*d*e**5*x + 18*l
og(a*e + c*d*x)*a**2*c**2*d**2*e**4*x**2 + 6*log(a*e + c*d*x)*a*c**3*d**3*
e**3*x**3 + 5*a**4*e**6 + 9*a**3*c*d*e**5*x - 3*a**2*c**2*d**4*e**2 - 2*a*
c**3*d**6 - 9*a*c**3*d**5*e*x - 6*a*c**3*d**3*e**3*x**3 + 6*c**4*d**5*e*x*
*3)/(6*a*c**4*d**4*(a**3*e**3 + 3*a**2*c*d*e**2*x + 3*a*c**2*d**2*e*x**2 +
 c**3*d**3*x**3))
```

3.142 $\int \frac{(d+ex)^6}{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^4}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 35

$$\int \frac{(d+ex)^6}{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^4}\,dx=-\frac{(d+ex)^3}{3\left(cd^2-ae^2\right)(ae+cdx)^3}$$

output
```
-1/3*(e*x+d)^3/(-a*e^2+c*d^2)/(c*d*x+a*e)^3
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.86

$$\int \frac{(d+ex)^6}{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^4}\,dx = -\frac{a^2e^4+acde^2(d+3ex)+c^2d^2(d^2+3dex+3e^2x^2)}{3c^3d^3(ae+cdx)^3}$$

input
```
Integrate[(d + e*x)^6/(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^4,x]
```

output
```
-1/3*(a^2*e^4 + a*c*d*e^2*(d + 3*e*x) + c^2*d^2*(d^2 + 3*d*e*x + 3*e^2*x^2
))/(c^3*d^3*(a*e + c*d*x)^3)
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.057$, Rules used = {1120, 48}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex)^6}{(x(ae^2+cd^2)+ade+cdex^2)^4}dx$$
$$\Big\downarrow \text{ 1120}$$
$$\int \frac{(d+ex)^2}{(ae+cdx)^4}dx$$
$$\Big\downarrow \text{ 48}$$
$$-\frac{(d+ex)^3}{3(cd^2-ae^2)(ae+cdx)^3}$$

input
```
Int[(d + e*x)^6/(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^4,x]
```

output
```
-1/3*(d + e*x)^3/((c*d^2 - a*e^2)*(a*e + c*d*x)^3)
```

Defintions of rubi rules used

rule 48
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && EqQ[m + n + 2, 0] && NeQ[m, -1]
```

rule 1120
```
Int[((d_) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Int[(d + e*x)^(m + p)*(a/d + (c/e)*x)^p, x] /; FreeQ[{a, b, c, d
, e, m}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[p] && (EqQ[m + p,
0] || EqQ[m + 2*p + 2, 0])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 75 vs. $2(33) = 66$.

Time = 1.51 (sec) , antiderivative size = 76, normalized size of antiderivative = 2.17

method	result
gosper	$-\frac{3x^{2}c^{2}d^{2}e^{2}+3xacd\,e^{3}+3x\,c^{2}d^{3}e+a^{2}e^{4}+ac\,d^{2}e^{2}+c^{2}d^{4}}{3d^{3}c^{3}(cdx+ae)^{3}}$
parallelrisch	$\frac{-3x^{2}c^{2}d^{2}e^{2}-3xacd\,e^{3}-3x\,c^{2}d^{3}e-a^{2}e^{4}-ac\,d^{2}e^{2}-c^{2}d^{4}}{3d^{3}c^{3}(cdx+ae)^{3}}$
risch	$\frac{-\frac{e^{2}x^{2}}{cd}-\frac{e\left(a\,e^{2}+c\,d^{2}\right)x}{c^{2}d^{2}}-\frac{a^{2}e^{4}+ac\,d^{2}e^{2}+c^{2}d^{4}}{3d^{3}c^{3}}}{(cdx+ae)^{3}}$
default	$-\frac{a^{2}e^{4}-2ac\,d^{2}e^{2}+c^{2}d^{4}}{3c^{3}d^{3}(cdx+ae)^{3}}+\frac{e\left(a\,e^{2}-c\,d^{2}\right)}{d^{3}c^{3}(cdx+ae)^{2}}-\frac{e^{2}}{c^{3}d^{3}(cdx+ae)}$
orering	$-\frac{\left(3x^{2}c^{2}d^{2}e^{2}+3xacd\,e^{3}+3x\,c^{2}d^{3}e+a^{2}e^{4}+ac\,d^{2}e^{2}+c^{2}d^{4}\right)(cdx+ae)(ex+d)^{4}}{3d^{3}c^{3}\left(ade+\left(a\,e^{2}+c\,d^{2}\right)x+cd\,x^{2}e\right)^{4}}$
norman	$\frac{-\frac{e^{5}x^{5}}{dc}+\frac{\left(-a^{2}e^{6}-2ac\,d^{2}e^{4}-2c^{2}d^{4}e^{2}\right)x}{c^{3}de}+\frac{\left(-a\,e^{8}-4c\,e^{6}d^{2}\right)x^{4}}{e^{2}c^{2}d^{2}}+\frac{\left(-e^{8}a^{2}-4ac\,d^{2}e^{6}-5c^{2}e^{4}d^{4}\right)x^{2}}{c^{3}d^{2}e^{2}}+\frac{-a^{2}e^{4}-ac\,d^{2}e^{2}-c^{2}d^{4}}{3c^{3}}+\left(-a^{2}e^{10}-\right.}{(ex+d)^{3}(cdx+ae)^{3}}$

input
```
int((e*x+d)^6/(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^4,x,method=_RETURNVERBOSE)
```

output
```
-1/3*(3*c^2*d^2*e^2*x^2+3*a*c*d*e^3*x+3*c^2*d^3*e*x+a^2*e^4+a*c*d^2*e^2+c^
2*d^4)/d^3/c^3/(c*d*x+a*e)^3
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 113 vs. $2(33) = 66$.

Time = 0.08 (sec) , antiderivative size = 113, normalized size of antiderivative = 3.23

$$\int \frac{(d+ex)^6}{\left(ade+(cd^2+ae^2)\,x+cdex^2\right)^4}\,dx = -\frac{3\,c^2d^2e^2x^2+c^2d^4+acd^2e^2+a^2e^4+3\,(c^2d^3e+acde^3)x}{3\,(c^6d^6x^3+3\,ac^5d^5ex^2+3\,a^2c^4d^4e^2x+a^3c^3d^3e^3)}$$

input
```
integrate((e*x+d)^6/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^4,x, algorithm="fric
as")
```

output
```
-1/3*(3*c^2*d^2*e^2*x^2 + c^2*d^4 + a*c*d^2*e^2 + a^2*e^4 + 3*(c^2*d^3*e +
 a*c*d*e^3)*x)/(c^6*d^6*x^3 + 3*a*c^5*d^5*e*x^2 + 3*a^2*c^4*d^4*e^2*x + a^
3*c^3*d^3*e^3)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 121 vs. $2(29) = 58$.

Time = 0.41 (sec) , antiderivative size = 121, normalized size of antiderivative = 3.46

$$\int \frac{(d+ex)^6}{(ade+(cd^2+ae^2)x+cdex^2)^4}\,dx = \frac{-a^2e^4-acd^2e^2-c^2d^4-3c^2d^2e^2x^2+x(-3acde^3-3c^2d^3e)}{3a^3c^3d^3e^3+9a^2c^4d^4e^2x+9ac^5d^5ex^2+3c^6d^6x^3}$$

input
```
integrate((e*x+d)**6/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**4,x)
```

output
```
(-a**2*e**4 - a*c*d**2*e**2 - c**2*d**4 - 3*c**2*d**2*e**2*x**2 + x*(-3*a*
c*d*e**3 - 3*c**2*d**3*e))/(3*a**3*c**3*d**3*e**3 + 9*a**2*c**4*d**4*e**2*
x + 9*a*c**5*d**5*e*x**2 + 3*c**6*d**6*x**3)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 113 vs. $2(33) = 66$.

Time = 0.04 (sec) , antiderivative size = 113, normalized size of antiderivative = 3.23

$$\int \frac{(d+ex)^6}{(ade+(cd^2+ae^2)x+cdex^2)^4}\,dx = -\frac{3\,c^2d^2e^2x^2+c^2d^4+acd^2e^2+a^2e^4+3\,(c^2d^3e+acde^3)x}{3\,(c^6d^6x^3+3\,ac^5d^5ex^2+3\,a^2c^4d^4e^2x+a^3c^3d^3e^3)}$$

input
```
integrate((e*x+d)^6/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^4,x, algorithm="maxi
ma")
```

output
```
-1/3*(3*c^2*d^2*e^2*x^2 + c^2*d^4 + a*c*d^2*e^2 + a^2*e^4 + 3*(c^2*d^3*e +
 a*c*d*e^3)*x)/(c^6*d^6*x^3 + 3*a*c^5*d^5*e*x^2 + 3*a^2*c^4*d^4*e^2*x + a^
3*c^3*d^3*e^3)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 75 vs. 2(33) = 66.

Time = 0.14 (sec) , antiderivative size = 75, normalized size of antiderivative = 2.14

$$\int \frac{(d+ex)^6}{(ade+(cd^2+ae^2)x+cdex^2)^4}\,dx = -\frac{3\,c^2d^2e^2x^2+3\,c^2d^3ex+3\,acde^3x+c^2d^4+acd^2e^2+a^2e^4}{3\,(cdx+ae)^3c^3d^3}$$

input
```
integrate((e*x+d)^6/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^4,x, algorithm="giac
")
```

output
```
-1/3*(3*c^2*d^2*e^2*x^2 + 3*c^2*d^3*e*x + 3*a*c*d*e^3*x + c^2*d^4 + a*c*d^
2*e^2 + a^2*e^4)/((c*d*x + a*e)^3*c^3*d^3)
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 81, normalized size of antiderivative = 2.31

$$\int \frac{(d+ex)^6}{(ade+(cd^2+ae^2)x+cdex^2)^4}\,dx = -\frac{\frac{d}{3\,c}+e\left(\frac{x}{c}-\frac{x^3}{3\,a}\right)+\frac{a\,e^2}{3\,c^2\,d}}{a^3\,e^3+3\,a^2\,c\,d\,e^2\,x+3\,a\,c^2\,d^2\,e\,x^2+c^3\,d^3\,x^3}$$

input
```
int((d + e*x)^6/(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^4,x)
```

output
```
-(d/(3*c) + e*(x/c - x^3/(3*a)) + (a*e^2)/(3*c^2*d))/(a^3*e^3 + c^3*d^3*x^
3 + 3*a^2*c*d*e^2*x + 3*a*c^2*d^2*e*x^2)
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 87, normalized size of antiderivative = 2.49

$$\int \frac{(d+ex)^6}{(ade+(cd^2+ae^2)\,x+cdex^2)^4}\,dx = \frac{c^2de\,x^3-3acdex-a^2e^2-ac\,d^2}{3a\,c^2d\,(c^3d^3x^3+3a\,c^2d^2e\,x^2+3a^2cd\,e^2x+a^3e^3)}$$

input
```
int((e*x+d)^6/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^4,x)
```

output
```
( - a**2*e**2 - a*c*d**2 - 3*a*c*d*e*x + c**2*d*e*x**3)/(3*a*c**2*d*(a**3*
e**3 + 3*a**2*c*d*e**2*x + 3*a*c**2*d**2*e*x**2 + c**3*d**3*x**3))
```

3.143 $\int \frac{(d+ex)^5}{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^4}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 54

$$\int \frac{(d+ex)^5}{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^4}\,dx=-\frac{cd^2-ae^2}{3c^2d^2(ae+cdx)^3}-\frac{e}{2c^2d^2(ae+cdx)^2}$$

output
```
-1/3*(-a*e^2+c*d^2)/c^2/d^2/(c*d*x+a*e)^3-1/2*e/c^2/d^2/(c*d*x+a*e)^2
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.69

$$\int \frac{(d+ex)^5}{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^4}\,dx=-\frac{ae^2+cd(2d+3ex)}{6c^2d^2(ae+cdx)^3}$$

input
```
Integrate[(d + e*x)^5/(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^4,x]
```

output
```
-1/6*(a*e^2 + c*d*(2*d + 3*e*x))/(c^2*d^2*(a*e + c*d*x)^3)
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.057$, Rules used = {1121, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex)^5}{(x(ae^2+cd^2)+ade+cdex^2)^4}dx$$

$$\Big\downarrow \text{1121}$$

$$\int \left(\frac{cd^2-ae^2}{cd(ae+cdx)^4}+\frac{e}{cd(ae+cdx)^3}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{cd^2-ae^2}{3c^2d^2(ae+cdx)^3}-\frac{e}{2c^2d^2(ae+cdx)^2}$$

input
```
Int[(d + e*x)^5/(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^4,x]
```

output
```
-1/3*(c*d^2 - a*e^2)/(c^2*d^2*(a*e + c*d*x)^3) - e/(2*c^2*d^2*(a*e + c*d*x
)^2)
```

Defintions of rubi rules used

rule 1121
```
Int[((d_) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Int[ExpandIntegrand[(d + e*x)^(m + p)*(a/d + (c/e)*x)^p, x], x]
/; FreeQ[{a, b, c, d, e, m, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && (Int
egerQ[p] || (GtQ[a, 0] && GtQ[d, 0] && LtQ[c, 0]))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 1.51 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.69

method	result	size
gosper	$-\frac{3cdxe+a\,e^{2}+2c\,d^{2}}{6c^{2}d^{2}\left(cdx+ae\right)^{3}}$	37
risch	$\frac{-\frac{ex}{2cd}-\frac{a\,e^{2}+2c\,d^{2}}{6c^{2}d^{2}}}{\left(cdx+ae\right)^{3}}$	43
parallelrisch	$\frac{-3d^{2}e\,c^{2}x-ad\,e^{2}c-2c^{2}d^{3}}{6d^{3}c^{3}\left(cdx+ae\right)^{3}}$	46
default	$-\frac{-a\,e^{2}+c\,d^{2}}{3c^{2}d^{2}\left(cdx+ae\right)^{3}}-\frac{e}{2c^{2}d^{2}\left(cdx+ae\right)^{2}}$	51
orering	$-\frac{\left(3cdxe+a\,e^{2}+2c\,d^{2}\right)\left(cdx+ae\right)\left(ex+d\right)^{4}}{6c^{2}d^{2}\left(ade+\left(a\,e^{2}+c\,d^{2}\right)x+cd\,x^{2}e\right)^{4}}$	69
norman	$\frac{\frac{-ad\,e^{2}c-2c^{2}d^{3}}{6c^{3}}+\frac{\left(-acd\,e^{4}-3c^{2}d^{3}e^{2}\right)x}{2c^{3}de}-\frac{e^{4}x^{4}}{2dc}+\frac{\left(-acd\,e^{6}-5e^{4}d^{3}c^{2}\right)x^{2}}{2c^{3}d^{2}e^{2}}+\frac{\left(-acd\,e^{8}-11e^{6}d^{3}c^{2}\right)x^{3}}{6e^{3}d^{3}c^{3}}}{\left(ex+d\right)^{3}\left(cdx+ae\right)^{3}}$	156

input
```
int((e*x+d)^5/(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^4,x,method=_RETURNVERBOSE)
```

output
```
-1/6/c^2/d^2*(3*c*d*e*x+a*e^2+2*c*d^2)/(c*d*x+a*e)^3
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.37

$$\int \frac{(d+ex)^5}{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^4}\,dx = -\frac{3\,cdex+2\,cd^2+ae^2}{6\,\left(c^5d^5x^3+3\,ac^4d^4ex^2+3\,a^2c^3d^3e^2x+a^3c^2d^2e^3\right)}$$

input
```
integrate((e*x+d)^5/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^4,x, algorithm="fric
as")
```

output
```
-1/6*(3*c*d*e*x + 2*c*d^2 + a*e^2)/(c^5*d^5*x^3 + 3*a*c^4*d^4*e*x^2 + 3*a^
2*c^3*d^3*e^2*x + a^3*c^2*d^2*e^3)
```

Sympy [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.48

$$\int \frac{(d+ex)^5}{(ade+(cd^2+ae^2)x+cdex^2)^4}\,dx = \frac{-ae^2-2cd^2-3cdex}{6a^3c^2d^2e^3+18a^2c^3d^3e^2x+18ac^4d^4ex^2+6c^5d^5x^3}$$

input
```
integrate((e*x+d)**5/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**4,x)
```

output
```
(-a*e**2 - 2*c*d**2 - 3*c*d*e*x)/(6*a**3*c**2*d**2*e**3 + 18*a**2*c**3*d**
3*e**2*x + 18*a*c**4*d**4*e*x**2 + 6*c**5*d**5*x**3)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.37

$$\int \frac{(d+ex)^5}{(ade+(cd^2+ae^2)x+cdex^2)^4}\,dx = -\frac{3\,cdex+2\,cd^2+ae^2}{6\,(c^5d^5x^3+3\,ac^4d^4ex^2+3\,a^2c^3d^3e^2x+a^3c^2d^2e^3)}$$

input
```
integrate((e*x+d)^5/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^4,x, algorithm="maxi
ma")
```

output
```
-1/6*(3*c*d*e*x + 2*c*d^2 + a*e^2)/(c^5*d^5*x^3 + 3*a*c^4*d^4*e*x^2 + 3*a^
2*c^3*d^3*e^2*x + a^3*c^2*d^2*e^3)
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.67

$$\int \frac{(d+ex)^5}{(ade+(cd^2+ae^2)x+cdex^2)^4}\,dx = -\frac{3\,cdex+2\,cd^2+ae^2}{6\,(cdx+ae)^3c^2d^2}$$

input
```
integrate((e*x+d)^5/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^4,x, algorithm="giac
")
```

output
```
-1/6*(3*c*d*e*x + 2*c*d^2 + a*e^2)/((c*d*x + a*e)^3*c^2*d^2)
```

Mupad [B] (verification not implemented)

Time = 5.20 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.43

$$\int \frac{(d+ex)^5}{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^4}\,dx=-\frac{\frac{2\,c\,d^2+a\,e^2}{6\,c^2\,d^2}+\frac{e\,x}{2\,c\,d}}{a^3\,e^3+3\,a^2\,c\,d\,e^2\,x+3\,a\,c^2\,d^2\,e\,x^2+c^3\,d^3\,x^3}$$

input
```
int((d + e*x)^5/(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^4,x)
```

output
```
-((a*e^2 + 2*c*d^2)/(6*c^2*d^2) + (e*x)/(2*c*d))/(a^3*e^3 + c^3*d^3*x^3 +
3*a^2*c*d*e^2*x + 3*a*c^2*d^2*e*x^2)
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.31

$$\int \frac{(d+ex)^5}{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^4}\,dx=\frac{-3cdex-a\,e^2-2c\,d^2}{6c^2d^2\left(c^3d^3x^3+3a\,c^2d^2e\,x^2+3a^2cd\,e^2x+a^3e^3\right)}$$

input
```
int((e*x+d)^5/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^4,x)
```

output
```
( - a*e**2 - 2*c*d**2 - 3*c*d*e*x)/(6*c**2*d**2*(a**3*e**3 + 3*a**2*c*d*e*
*2*x + 3*a*c**2*d**2*e*x**2 + c**3*d**3*x**3))
```

3.144 $\int \frac{(d+ex)^4}{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^4}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 20

$$\int \frac{(d+ex)^4}{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^4}\,dx=-\frac{1}{3cd(ae+cdx)^3}$$

output
```
-1/3/c/d/(c*d*x+a*e)^3
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{(d+ex)^4}{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^4}\,dx=-\frac{1}{3cd(ae+cdx)^3}$$

input
```
Integrate[(d + e*x)^4/(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^4,x]
```

output
```
-1/3*1/(c*d*(a*e + c*d*x)^3)
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.057, Rules used = {1120, 17}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex)^4}{(x(ae^2+cd^2)+ade+cdex^2)^4}dx$$
$$\Big\downarrow \text{ 1120}$$
$$\int \frac{1}{(ae+cdx)^4}dx$$
$$\Big\downarrow \text{ 17}$$
$$-\frac{1}{3cd(ae+cdx)^3}$$

input
```
Int[(d + e*x)^4/(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^4,x]
```

output
```
-1/3*1/(c*d*(a*e + c*d*x)^3)
```

Defintions of rubi rules used

rule 17
```
Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1
)/(b*(m + 1))), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]
```

rule 1120
```
Int[((d_) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Int[(d + e*x)^(m + p)*(a/d + (c/e)*x)^p, x] /; FreeQ[{a, b, c, d
, e, m}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[p] && (EqQ[m + p,
0] || EqQ[m + 2*p + 2, 0])
```

Maple [A] (verified)

Time = 1.51 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.95

method	result	size
gosper	$-\frac{1}{3cd(cdx+ae)^{3}}$	19
default	$-\frac{1}{3cd(cdx+ae)^{3}}$	19
risch	$-\frac{1}{3cd(cdx+ae)^{3}}$	19
parallelrisch	$-\frac{1}{3cd(cdx+ae)^{3}}$	19
orering	$-\frac{(cdx+ae)(ex+d)^{4}}{3cd(ade+(a\,e^{2}+c\,d^{2})x+cd\,x^{2}e)^{4}}$	51
norman	$\frac{-\frac{edx}{c}-\frac{e^{2}x^{2}}{c}-\frac{d^{2}}{3c}-\frac{e^{3}x^{3}}{3cd}}{(ex+d)^{3}(cdx+ae)^{3}}$	61

input
```
int((e*x+d)^4/(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^4,x,method=_RETURNVERBOSE)
```

output
```
-1/3/c/d/(c*d*x+a*e)^3
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 52 vs. 2(18) = 36.

Time = 0.08 (sec) , antiderivative size = 52, normalized size of antiderivative = 2.60

$$\int \frac{(d+ex)^4}{(ade+(cd^2+ae^2)\,x+cdex^2)^4}\,dx = -\frac{1}{3\,(c^4d^4x^3+3\,ac^3d^3ex^2+3\,a^2c^2d^2e^2x+a^3cde^3)}$$

input
```
integrate((e*x+d)^4/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^4,x, algorithm="fric
as")
```

output
```
-1/3/(c^4*d^4*x^3 + 3*a*c^3*d^3*e*x^2 + 3*a^2*c^2*d^2*e^2*x + a^3*c*d*e^3)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 58 vs. $2(17) = 34$.

Time = 0.16 (sec) , antiderivative size = 58, normalized size of antiderivative = 2.90

$$\int \frac{(d+ex)^4}{(ade+(cd^2+ae^2)x+cdex^2)^4}\,dx = -\frac{1}{3a^3cde^3+9a^2c^2d^2e^2x+9ac^3d^3ex^2+3c^4d^4x^3}$$

input
```
integrate((e*x+d)**4/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**4,x)
```

output
```
-1/(3*a**3*c*d*e**3 + 9*a**2*c**2*d**2*e**2*x + 9*a*c**3*d**3*e*x**2 + 3*c
**4*d**4*x**3)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 52 vs. $2(18) = 36$.

Time = 0.03 (sec) , antiderivative size = 52, normalized size of antiderivative = 2.60

$$\int \frac{(d+ex)^4}{(ade+(cd^2+ae^2)x+cdex^2)^4}\,dx = -\frac{1}{3\,(c^4d^4x^3+3\,ac^3d^3ex^2+3\,a^2c^2d^2e^2x+a^3cde^3)}$$

input
```
integrate((e*x+d)^4/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^4,x, algorithm="maxi
ma")
```

output
```
-1/3/(c^4*d^4*x^3 + 3*a*c^3*d^3*e*x^2 + 3*a^2*c^2*d^2*e^2*x + a^3*c*d*e^3)
```

Giac [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.90

$$\int \frac{(d+ex)^4}{(ade+(cd^2+ae^2)x+cdex^2)^4}\,dx = -\frac{1}{3\,(cdx+ae)^3cd}$$

input
```
integrate((e*x+d)^4/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^4,x, algorithm="giac
")
```

output
```
-1/3/((c*d*x + a*e)^3*c*d)
```

Mupad [B] (verification not implemented)

Time = 5.37 (sec) , antiderivative size = 54, normalized size of antiderivative = 2.70

$$\int \frac{(d+ex)^4}{(ade+(cd^2+ae^2)x+cdex^2)^4}\,dx = -\frac{1}{3\,a^3\,c\,d\,e^3+9\,a^2\,c^2\,d^2\,e^2\,x+9\,a\,c^3\,d^3\,e\,x^2+3\,c^4\,d^4\,x^3}$$

input
```
int((d + e*x)^4/(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^4,x)
```

output
```
-1/(3*c^4*d^4*x^3 + 3*a^3*c*d*e^3 + 9*a*c^3*d^3*e*x^2 + 9*a^2*c^2*d^2*e^2*
x)
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 52, normalized size of antiderivative = 2.60

$$\int \frac{(d+ex)^4}{(ade+(cd^2+ae^2)x+cdex^2)^4}\,dx = -\frac{1}{3cd\,(c^3d^3x^3+3a\,c^2d^2e\,x^2+3a^2cd\,e^2x+a^3e^3)}$$

input
```
int((e*x+d)^4/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^4,x)
```

output
```
( - 1)/(3*c*d*(a**3*e**3 + 3*a**2*c*d*e**2*x + 3*a*c**2*d**2*e*x**2 + c**3
*d**3*x**3))
```

3.145 $\int \frac{(d+ex)^3}{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^4}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 139

$$\int \frac{(d+ex)^3}{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^4}\,dx = -\frac{1}{3\left(cd^2-ae^2\right)(ae+cdx)^3} + \frac{e}{2\left(cd^2-ae^2\right)^2(ae+cdx)^2} - \frac{e^2}{\left(cd^2-ae^2\right)^3(ae+cdx)} - \frac{e^3\log(ae+cdx)}{\left(cd^2-ae^2\right)^4} + \frac{e^3\log(d+ex)}{\left(cd^2-ae^2\right)^4}$$

output
```
-1/3/(-a*e^2+c*d^2)/(c*d*x+a*e)^3+1/2*e/(-a*e^2+c*d^2)^2/(c*d*x+a*e)^2-e^2
/(-a*e^2+c*d^2)^3/(c*d*x+a*e)-e^3*ln(c*d*x+a*e)/(-a*e^2+c*d^2)^4+e^3*ln(e*
x+d)/(-a*e^2+c*d^2)^4
```

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 117, normalized size of antiderivative = 0.84

$$\int \frac{(d+ex)^3}{(ade+(cd^2+ae^2)x+cdex^2)^4}\,dx = -\frac{\frac{(cd^2-ae^2)(11a^2e^4+acde^2(-7d+15ex)+c^2d^2(2d^2-3dex+6e^2x^2))}{(ae+cdx)^3}+6e^3\log(ae+cdx)-6e^3\log(d+ex)}{6(cd^2-ae^2)^4}$$

input
```
Integrate[(d + e*x)^3/(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^4,x]
```

output
```
-1/6*(((c*d^2 - a*e^2)*(11*a^2*e^4 + a*c*d*e^2*(-7*d + 15*e*x) + c^2*d^2*(
2*d^2 - 3*d*e*x + 6*e^2*x^2)))/(a*e + c*d*x)^3 + 6*e^3*Log[a*e + c*d*x] -
6*e^3*Log[d + e*x])/(c*d^2 - a*e^2)^4
```

Rubi [A] (verified)

Time = 0.51 (sec) , antiderivative size = 139, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.057$, Rules used = {1121, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex)^3}{(x(ae^2+cd^2)+ade+cdex^2)^4}\,dx$$

$$\Big\downarrow \text{1121}$$

$$\int \left(\frac{cde^2}{(cd^2-ae^2)^3(ae+cdx)^2}-\frac{cde}{(cd^2-ae^2)^2(ae+cdx)^3}+\frac{cd}{(cd^2-ae^2)(ae+cdx)^4}+\frac{e^4}{(d+ex)(cd^2-ae^2)^4}-\right.$$

$$\Big\downarrow \text{2009}$$

$$-\frac{e^2}{(cd^2-ae^2)^3(ae+cdx)}+\frac{e}{2(cd^2-ae^2)^2(ae+cdx)^2}-\frac{1}{3(cd^2-ae^2)(ae+cdx)^3}-\frac{e^3\log(ae+cdx)}{(cd^2-ae^2)^4}+\frac{e^3\log(d+ex)}{(cd^2-ae^2)^4}$$

input
```
Int[(d + e*x)^3/(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^4,x]
```

output
```
-1/3*1/((c*d^2 - a*e^2)*(a*e + c*d*x)^3) + e/(2*(c*d^2 - a*e^2)^2*(a*e + c
*d*x)^2) - e^2/((c*d^2 - a*e^2)^3*(a*e + c*d*x)) - (e^3*Log[a*e + c*d*x])/
(c*d^2 - a*e^2)^4 + (e^3*Log[d + e*x])/(c*d^2 - a*e^2)^4
```

Defintions of rubi rules used

rule 1121
```
Int[((d_) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Int[ExpandIntegrand[(d + e*x)^(m + p)*(a/d + (c/e)*x)^p, x], x]
/; FreeQ[{a, b, c, d, e, m, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && (Int
egerQ[p] || (GtQ[a, 0] && GtQ[d, 0] && LtQ[c, 0]))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 1.63 (sec) , antiderivative size = 135, normalized size of antiderivative = 0.97

method	result
default	$\frac{1}{3(a e^2-c d^2)(cdx+ae)^3}+\frac{e}{2(a e^2-c d^2)^2(cdx+ae)^2}+\frac{e^2}{(a e^2-c d^2)^3(cdx+ae)}-\frac{e^3\ln(cdx+ae)}{(a e^2-c d^2)^4}+\frac{e^3\ln(ex+d)}{(a e^2-c d^2)^4}$
risch	$\frac{\frac{e^2c^2d^2x^2}{e^6a^3-3d^2e^4a^2c+3d^4e^2a c^2-d^6c^3}+\frac{cd\left(5a e^2-c d^2\right)ex}{2e^6a^3-6d^2e^4a^2c+6d^4e^2a c^2-2d^6c^3}+\frac{11a^2e^4-7ac d^2e^2+2c^2d^4}{6e^6a^3-18d^2e^4a^2c+18d^4e^2a c^2-6d^6c^3}}{(cdx+ae)^3}+\frac{}{a^4e^8-4a^3c d^2 \cdots}$
parallelrisch	$\frac{12x^2a^2c^4d^5e^4-3x^2a c^5d^7e^2+6\ln(ex+d)a^5cd e^8-6\ln(cdx+ae)a^5cd e^8-5x^3a^2c^4d^4e^5+6x^3a c^5d^6e^3-9x^2a^3c^3d^3e^6-x^3c^6d^8e \cdots}{}$
norman	$\frac{\frac{d^2c^2e^5x^5}{e^6a^3-3d^2e^4a^2c+3d^4e^2a c^2-d^6c^3}+\frac{11a^2c^3d^3e^4-7a c^4d^5e^2+2c^5d^7}{6c^3\left(e^6a^3-3d^2e^4a^2c+3d^4e^2a c^2-d^6c^3\right)}+\frac{\left(5a c^3d^3e^8+5e^6d^5c^4\right)x^4}{2d^2c^2e^2\left(e^6a^3-3d^2e^4a^2c+3d^4e^2a c^2-d^6c^3\right)}+\frac{\left(11a \cdots\right.}{2edc^3\left(e \cdots\right.}}{(ex+d)^3(cdx \cdots}$

input
```
int((e*x+d)^3/(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^4,x,method=_RETURNVERBOSE)
```

output
```
1/3/(a*e^2-c*d^2)/(c*d*x+a*e)^3+1/2*e/(a*e^2-c*d^2)^2/(c*d*x+a*e)^2+e^2/(a
*e^2-c*d^2)^3/(c*d*x+a*e)-e^3/(a*e^2-c*d^2)^4*ln(c*d*x+a*e)+e^3/(a*e^2-c*d
^2)^4*ln(e*x+d)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 484 vs. $2(135) = 270$.

Time = 0.10 (sec) , antiderivative size = 484, normalized size of antiderivative = 3.48

$$\int \frac{(d+ex)^3}{(ade+(cd^2+ae^2)x+cdex^2)^4}\,dx = -\frac{2\,c^3d^6 - 9\,ac^2d^4e^2 + 18\,a^2cd^2e^4 - 11\,a^3e^6 + 6\,(c^3d^4e^2 - ac^2d^2e^4)x^2 - 3\,(c^3d^5e - 6\,ac^2d^3e^3}{6\,(a^3c^4d^8e^3 - 4\,a^4c^3d^6e^5 + 6\,a^5c^2d^4e^7 - 4\,a^6cd^2e^9 + a^7e^{11} + (c^7d^{11} - 4\,ac^6d^9e^2 + 6\,a^2c^5d^7e^4 - 4\,a^3c^4d^5}$$

input
```
integrate((e*x+d)^3/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^4,x, algorithm="fric
as")
```

output
```
-1/6*(2*c^3*d^6 - 9*a*c^2*d^4*e^2 + 18*a^2*c*d^2*e^4 - 11*a^3*e^6 + 6*(c^3
*d^4*e^2 - a*c^2*d^2*e^4)*x^2 - 3*(c^3*d^5*e - 6*a*c^2*d^3*e^3 + 5*a^2*c*d
*e^5)*x + 6*(c^3*d^3*e^3*x^3 + 3*a*c^2*d^2*e^4*x^2 + 3*a^2*c*d*e^5*x + a^3
*e^6)*log(c*d*x + a*e) - 6*(c^3*d^3*e^3*x^3 + 3*a*c^2*d^2*e^4*x^2 + 3*a^2*
c*d*e^5*x + a^3*e^6)*log(e*x + d))/(a^3*c^4*d^8*e^3 - 4*a^4*c^3*d^6*e^5 +
6*a^5*c^2*d^4*e^7 - 4*a^6*c*d^2*e^9 + a^7*e^11 + (c^7*d^11 - 4*a*c^6*d^9*e
^2 + 6*a^2*c^5*d^7*e^4 - 4*a^3*c^4*d^5*e^6 + a^4*c^3*d^3*e^8)*x^3 + 3*(a*c
^6*d^10*e - 4*a^2*c^5*d^8*e^3 + 6*a^3*c^4*d^6*e^5 - 4*a^4*c^3*d^4*e^7 + a^
5*c^2*d^2*e^9)*x^2 + 3*(a^2*c^5*d^9*e^2 - 4*a^3*c^4*d^7*e^4 + 6*a^4*c^3*d^
5*e^6 - 4*a^5*c^2*d^3*e^8 + a^6*c*d*e^10)*x)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 668 vs. $2(119) = 238$.

Time = 0.88 (sec) , antiderivative size = 668, normalized size of antiderivative = 4.81

$$\int \frac{(d+ex)^3}{(ade+(cd^2+ae^2)\,x+cdex^2)^4}\,dx$$
$$= \frac{e^3\log\left(x+\frac{-\frac{a^5e^{13}}{(ae^2-cd^2)^4}+\frac{5a^4cd^2e^{11}}{(ae^2-cd^2)^4}-\frac{10a^3c^2d^4e^9}{(ae^2-cd^2)^4}+\frac{10a^2c^3d^6e^7}{(ae^2-cd^2)^4}-\frac{5ac^4d^8e^5}{(ae^2-cd^2)^4}+ae^5+\frac{c^5d^{10}e^3}{(ae^2-cd^2)^4}+cd^2e^3}{2cde^4}\right)}{(ae^2-cd^2)^4}$$
$$-\frac{e^3\log\left(x+\frac{\frac{a^5e^{13}}{(ae^2-cd^2)^4}-\frac{5a^4cd^2e^{11}}{(ae^2-cd^2)^4}+\frac{10a^3c^2d^4e^9}{(ae^2-cd^2)^4}-\frac{10a^2c^3d^6e^7}{(ae^2-cd^2)^4}+\frac{5ac^4d^8e^5}{(ae^2-cd^2)^4}+ae^5-\frac{c^5d^{10}e^3}{(ae^2-cd^2)^4}+cd^2e^3}{2cde^4}\right)}{(ae^2-cd^2)^4}$$
$$+\frac{11a^2e^4-7acd^2e^2+2c^2d^4}{6a^6e^9-18a^5cd^2e^7+18a^4c^2d^4e^5-6a^3c^3d^6e^3+x^3\cdot(6a^3c^3d^3e^6-18a^2c^4d^5e^4+18ac^5d^7e^2-6c^6d^9)+x}$$

input
```
integrate((e*x+d)**3/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**4,x)
```

output
```
e**3*log(x + (-a**5*e**13/(a*e**2 - c*d**2)**4 + 5*a**4*c*d**2*e**11/(a*e*
*2 - c*d**2)**4 - 10*a**3*c**2*d**4*e**9/(a*e**2 - c*d**2)**4 + 10*a**2*c*
*3*d**6*e**7/(a*e**2 - c*d**2)**4 - 5*a*c**4*d**8*e**5/(a*e**2 - c*d**2)**
4 + a*e**5 + c**5*d**10*e**3/(a*e**2 - c*d**2)**4 + c*d**2*e**3)/(2*c*d*e*
*4))/(a*e**2 - c*d**2)**4 - e**3*log(x + (a**5*e**13/(a*e**2 - c*d**2)**4
- 5*a**4*c*d**2*e**11/(a*e**2 - c*d**2)**4 + 10*a**3*c**2*d**4*e**9/(a*e**
2 - c*d**2)**4 - 10*a**2*c**3*d**6*e**7/(a*e**2 - c*d**2)**4 + 5*a*c**4*d*
*8*e**5/(a*e**2 - c*d**2)**4 + a*e**5 - c**5*d**10*e**3/(a*e**2 - c*d**2)*
*4 + c*d**2*e**3)/(2*c*d*e**4))/(a*e**2 - c*d**2)**4 + (11*a**2*e**4 - 7*a
*c*d**2*e**2 + 2*c**2*d**4 + 6*c**2*d**2*e**2*x**2 + x*(15*a*c*d*e**3 - 3*
c**2*d**3*e))/(6*a**6*e**9 - 18*a**5*c*d**2*e**7 + 18*a**4*c**2*d**4*e**5
- 6*a**3*c**3*d**6*e**3 + x**3*(6*a**3*c**3*d**3*e**6 - 18*a**2*c**4*d**5*
e**4 + 18*a*c**5*d**7*e**2 - 6*c**6*d**9) + x**2*(18*a**4*c**2*d**2*e**7 -
 54*a**3*c**3*d**4*e**5 + 54*a**2*c**4*d**6*e**3 - 18*a*c**5*d**8*e) + x*(
18*a**5*c*d*e**8 - 54*a**4*c**2*d**3*e**6 + 54*a**3*c**3*d**5*e**4 - 18*a*
*2*c**4*d**7*e**2))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 412 vs. $2(135) = 270$.

Time = 0.04 (sec) , antiderivative size = 412, normalized size of antiderivative = 2.96

$$\int \frac{(d+ex)^3}{(ade+(cd^2+ae^2)x+cdex^2)^4}\,dx$$
$$= -\frac{e^3\log(cdx+ae)}{c^4d^8-4\,ac^3d^6e^2+6\,a^2c^2d^4e^4-4\,a^3cd^2e^6+a^4e^8}$$
$$+\frac{e^3\log(ex+d)}{c^4d^8-4\,ac^3d^6e^2+6\,a^2c^2d^4e^4-4\,a^3cd^2e^6+a^4e^8}$$
$$-\frac{6\,c^2d^2e^2x^2+2\,c^2d^4-7\,acd^2e^2+11}{6\,(a^3c^3d^6e^3-3\,a^4c^2d^4e^5+3\,a^5cd^2e^7-a^6e^9+(c^6d^9-3\,ac^5d^7e^2+3\,a^2c^4d^5e^4-a^3c^3d^3e^6)x^3+3\,(ac^5d^8}$$

input
```
integrate((e*x+d)^3/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^4,x, algorithm="maxi
ma")
```

output
```
-e^3*log(c*d*x + a*e)/(c^4*d^8 - 4*a*c^3*d^6*e^2 + 6*a^2*c^2*d^4*e^4 - 4*a
^3*c*d^2*e^6 + a^4*e^8) + e^3*log(e*x + d)/(c^4*d^8 - 4*a*c^3*d^6*e^2 + 6*
a^2*c^2*d^4*e^4 - 4*a^3*c*d^2*e^6 + a^4*e^8) - 1/6*(6*c^2*d^2*e^2*x^2 + 2*
c^2*d^4 - 7*a*c*d^2*e^2 + 11*a^2*e^4 - 3*(c^2*d^3*e - 5*a*c*d*e^3)*x)/(a^3
*c^3*d^6*e^3 - 3*a^4*c^2*d^4*e^5 + 3*a^5*c*d^2*e^7 - a^6*e^9 + (c^6*d^9 -
3*a*c^5*d^7*e^2 + 3*a^2*c^4*d^5*e^4 - a^3*c^3*d^3*e^6)*x^3 + 3*(a*c^5*d^8*
e - 3*a^2*c^4*d^6*e^3 + 3*a^3*c^3*d^4*e^5 - a^4*c^2*d^2*e^7)*x^2 + 3*(a^2*
c^4*d^7*e^2 - 3*a^3*c^3*d^5*e^4 + 3*a^4*c^2*d^3*e^6 - a^5*c*d*e^8)*x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 273 vs. $2(135) = 270$.

Time = 0.12 (sec) , antiderivative size = 273, normalized size of antiderivative = 1.96

$$\int \frac{(d+ex)^3}{(ade+(cd^2+ae^2)\,x+cdex^2)^4}\,dx$$
$$= -\frac{cde^3\log(|cdx+ae|)}{c^5d^9-4\,ac^4d^7e^2+6\,a^2c^3d^5e^4-4\,a^3c^2d^3e^6+a^4cde^8} + \frac{e^4\log(|ex+d|)}{c^4d^8e-4\,ac^3d^6e^3+6\,a^2c^2d^4e^5-4\,a^3cd^2e^7+a^4e^9} - \frac{2\,c^3d^6-9\,ac^2d^4e^2+18\,a^2cd^2e^4-11\,a^3e^6+6\,(c^3d^4e^2-ac^2d^2e^4)x^2-3\,(c^3d^5e-6\,ac^2d^3e^3+5\,a^2cde^5)x}{6\,(cd^2-ae^2)^4(cdx+ae)^3}$$

input
```
integrate((e*x+d)^3/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^4,x, algorithm="giac
")
```

output
```
-c*d*e^3*log(abs(c*d*x + a*e))/(c^5*d^9 - 4*a*c^4*d^7*e^2 + 6*a^2*c^3*d^5*
e^4 - 4*a^3*c^2*d^3*e^6 + a^4*c*d*e^8) + e^4*log(abs(e*x + d))/(c^4*d^8*e
- 4*a*c^3*d^6*e^3 + 6*a^2*c^2*d^4*e^5 - 4*a^3*c*d^2*e^7 + a^4*e^9) - 1/6*(
2*c^3*d^6 - 9*a*c^2*d^4*e^2 + 18*a^2*c*d^2*e^4 - 11*a^3*e^6 + 6*(c^3*d^4*e
^2 - a*c^2*d^2*e^4)*x^2 - 3*(c^3*d^5*e - 6*a*c^2*d^3*e^3 + 5*a^2*c*d*e^5)*
x)/((c*d^2 - a*e^2)^4*(c*d*x + a*e)^3)
```

Mupad [B] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 372, normalized size of antiderivative = 2.68

$$\int \frac{(d+ex)^3}{(ade+(cd^2+ae^2)\,x+cdex^2)^4}\,dx$$
$$= \frac{\frac{11\,a^2\,e^4-7\,a\,c\,d^2\,e^2+2\,c^2\,d^4}{6\,(a^3\,e^6-3\,a^2\,c\,d^2\,e^4+3\,a\,c^2\,d^4\,e^2-c^3\,d^6)} - \frac{e\,x\,(c^2\,d^3-5\,a\,c\,d\,e^2)}{2\,(a^3\,e^6-3\,a^2\,c\,d^2\,e^4+3\,a\,c^2\,d^4\,e^2-c^3\,d^6)} + \frac{c^2\,d^2\,e^2\,x^2}{a^3\,e^6-3\,a^2\,c\,d^2\,e^4+3\,a\,c^2\,d^4\,e^2-c^3\,d^6}}{a^3\,e^3+3\,a^2\,c\,d\,e^2\,x+3\,a\,c^2\,d^2\,e\,x^2+c^3\,d^3\,x^3} - \frac{2\,e^3\,\mathrm{atanh}\left(\frac{a^4\,e^8-2\,a^3\,c\,d^2\,e^6+2\,a\,c^3\,d^6\,e^2-c^4\,d^8}{(a\,e^2-c\,d^2)^4} + \frac{2\,c\,d\,e\,x\,(a^3\,e^6-3\,a^2\,c\,d^2\,e^4+3\,a\,c^2\,d^4\,e^2-c^3\,d^6)}{(a\,e^2-c\,d^2)^4}\right)}{(a\,e^2-c\,d^2)^4}$$

input
```
int((d + e*x)^3/(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^4,x)
```

output
```
((11*a^2*e^4 + 2*c^2*d^4 - 7*a*c*d^2*e^2)/(6*(a^3*e^6 - c^3*d^6 + 3*a*c^2*
d^4*e^2 - 3*a^2*c*d^2*e^4)) - (e*x*(c^2*d^3 - 5*a*c*d*e^2))/(2*(a^3*e^6 -
c^3*d^6 + 3*a*c^2*d^4*e^2 - 3*a^2*c*d^2*e^4)) + (c^2*d^2*e^2*x^2)/(a^3*e^6
 - c^3*d^6 + 3*a*c^2*d^4*e^2 - 3*a^2*c*d^2*e^4))/(a^3*e^3 + c^3*d^3*x^3 +
3*a^2*c*d*e^2*x + 3*a*c^2*d^2*e*x^2) - (2*e^3*atanh((a^4*e^8 - c^4*d^8 + 2
*a*c^3*d^6*e^2 - 2*a^3*c*d^2*e^6)/(a*e^2 - c*d^2)^4 + (2*c*d*e*x*(a^3*e^6
- c^3*d^6 + 3*a*c^2*d^4*e^2 - 3*a^2*c*d^2*e^4))/(a*e^2 - c*d^2)^4))/(a*e^2
 - c*d^2)^4
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 565, normalized size of antiderivative = 4.06

$$\int \frac{(d+ex)^3}{(ade+(cd^2+ae^2)x+cdex^2)^4}\,dx$$
$$= \frac{-6\log(cdx+ae)\,a^4e^6-18\log(cdx+ae)\,a^3cd\,e^5x-18\log(cdx+ae)\,a^2c^2d^2e^4x^2-6\log(cdx+ae)\,a\,c^3d}{6a\left(a^4c^3d^3e^8x^3-4a^3c^4d^5e^6x^3+6a^2c^5d^7e^4x^3-4a\,c^6d^9e^2x^3+c^7d^{11}x^3+3a^5c^2d^2e^9\right.}$$

input
```
int((e*x+d)^3/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^4,x)
```

output
```
( - 6*log(a*e + c*d*x)*a**4*e**6 - 18*log(a*e + c*d*x)*a**3*c*d*e**5*x - 1
8*log(a*e + c*d*x)*a**2*c**2*d**2*e**4*x**2 - 6*log(a*e + c*d*x)*a*c**3*d*
*3*e**3*x**3 + 6*log(d + e*x)*a**4*e**6 + 18*log(d + e*x)*a**3*c*d*e**5*x
+ 18*log(d + e*x)*a**2*c**2*d**2*e**4*x**2 + 6*log(d + e*x)*a*c**3*d**3*e*
*3*x**3 + 9*a**4*e**6 - 16*a**3*c*d**2*e**4 + 9*a**3*c*d*e**5*x + 9*a**2*c
**2*d**4*e**2 - 12*a**2*c**2*d**3*e**3*x - 2*a*c**3*d**6 + 3*a*c**3*d**5*e
*x - 2*a*c**3*d**3*e**3*x**3 + 2*c**4*d**5*e*x**3)/(6*a*(a**7*e**11 - 4*a*
*6*c*d**2*e**9 + 3*a**6*c*d*e**10*x + 6*a**5*c**2*d**4*e**7 - 12*a**5*c**2
*d**3*e**8*x + 3*a**5*c**2*d**2*e**9*x**2 - 4*a**4*c**3*d**6*e**5 + 18*a**
4*c**3*d**5*e**6*x - 12*a**4*c**3*d**4*e**7*x**2 + a**4*c**3*d**3*e**8*x**
3 + a**3*c**4*d**8*e**3 - 12*a**3*c**4*d**7*e**4*x + 18*a**3*c**4*d**6*e**
5*x**2 - 4*a**3*c**4*d**5*e**6*x**3 + 3*a**2*c**5*d**9*e**2*x - 12*a**2*c*
*5*d**8*e**3*x**2 + 6*a**2*c**5*d**7*e**4*x**3 + 3*a*c**6*d**10*e*x**2 - 4
*a*c**6*d**9*e**2*x**3 + c**7*d**11*x**3))
```

3.146 $\int \frac{(d+ex)^2}{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^4}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 173

$$\int \frac{(d+ex)^2}{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^4}\,dx=-\frac{cd}{3\left(cd^2-ae^2\right)^2(ae+cdx)^3}+\frac{cde}{\left(cd^2-ae^2\right)^3(ae+cdx)^2}-\frac{3cde^2}{\left(cd^2-ae^2\right)^4(ae+cdx)}-\frac{e^3}{\left(cd^2-ae^2\right)^4(d+ex)}-\frac{4cde^3\log(ae+cdx)}{\left(cd^2-ae^2\right)^5}+\frac{4cde^3\log(d+ex)}{\left(cd^2-ae^2\right)^5}$$

output
```
-1/3*c*d/(-a*e^2+c*d^2)^2/(c*d*x+a*e)^3+c*d*e/(-a*e^2+c*d^2)^3/(c*d*x+a*e)
^2-3*c*d*e^2/(-a*e^2+c*d^2)^4/(c*d*x+a*e)-e^3/(-a*e^2+c*d^2)^4/(e*x+d)-4*c
*d*e^3*ln(c*d*x+a*e)/(-a*e^2+c*d^2)^5+4*c*d*e^3*ln(e*x+d)/(-a*e^2+c*d^2)^5
```

Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 157, normalized size of antiderivative = 0.91

$$\int \frac{(d+ex)^2}{(ade+(cd^2+ae^2)x+cdex^2)^4}\,dx = \frac{\frac{cd(cd^2-ae^2)^3}{(ae+cdx)^3}-\frac{3cde(cd^2-ae^2)^2}{(ae+cdx)^2}+\frac{9cde^2(cd^2-ae^2)}{ae+cdx}+\frac{3cd^2e^3-3ae^5}{d+ex}+12cde^3\log(ae+cdx)-12cde^3\log(d+ex)}{3(-cd^2+ae^2)^5}$$

input
```
Integrate[(d + e*x)^2/(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^4,x]
```

output
```
((c*d*(c*d^2 - a*e^2)^3)/(a*e + c*d*x)^3 - (3*c*d*e*(c*d^2 - a*e^2)^2)/(a*
e + c*d*x)^2 + (9*c*d*e^2*(c*d^2 - a*e^2))/(a*e + c*d*x) + (3*c*d^2*e^3 -
3*a*e^5)/(d + e*x) + 12*c*d*e^3*Log[a*e + c*d*x] - 12*c*d*e^3*Log[d + e*x]
)/(3*(-(c*d^2) + a*e^2)^5)
```

Rubi [A] (verified)

Time = 0.62 (sec) , antiderivative size = 173, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.057$, Rules used = {1121, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex)^2}{(x(ae^2+cd^2)+ade+cdex^2)^4}\,dx$$

$$\Big\downarrow \text{1121}$$

$$\int \left(\frac{3c^2d^2e^2}{(cd^2-ae^2)^4(ae+cdx)^2}-\frac{2c^2d^2e}{(cd^2-ae^2)^3(ae+cdx)^3}+\frac{c^2d^2}{(cd^2-ae^2)^2(ae+cdx)^4}-\frac{4c^2d^2e^3}{(cd^2-ae^2)^5(ae+cdx)}\right.$$

$$\Big\downarrow \text{2009}$$

$$-\frac{3cde^2}{(cd^2-ae^2)^4(ae+cdx)}+\frac{cde}{(cd^2-ae^2)^3(ae+cdx)^2}-\frac{cd}{3(cd^2-ae^2)^2(ae+cdx)^3}-\frac{e^3}{(d+ex)(cd^2-ae^2)^4}-\frac{4cde^3\log(ae+cdx)}{(cd^2-ae^2)^5}+\frac{4cde^3\log(d+ex)}{(cd^2-ae^2)^5}$$

input
```
Int[(d + e*x)^2/(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^4,x]
```

output
```
-1/3*(c*d)/((c*d^2 - a*e^2)^2*(a*e + c*d*x)^3) + (c*d*e)/((c*d^2 - a*e^2)^
3*(a*e + c*d*x)^2) - (3*c*d*e^2)/((c*d^2 - a*e^2)^4*(a*e + c*d*x)) - e^3/(
(c*d^2 - a*e^2)^4*(d + e*x)) - (4*c*d*e^3*Log[a*e + c*d*x])/(c*d^2 - a*e^2
)^5 + (4*c*d*e^3*Log[d + e*x])/(c*d^2 - a*e^2)^5
```

Defintions of rubi rules used

rule 1121
```
Int[((d_) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Int[ExpandIntegrand[(d + e*x)^(m + p)*(a/d + (c/e)*x)^p, x], x]
/; FreeQ[{a, b, c, d, e, m, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && (Int
egerQ[p] || (GtQ[a, 0] && GtQ[d, 0] && LtQ[c, 0]))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 1.69 (sec) , antiderivative size = 173, normalized size of antiderivative = 1.00

method	result
default	$-\frac{cd}{3(a e^2-c d^2)^2(cdx+ae)^3}+\frac{4cd e^3\ln(cdx+ae)}{(a e^2-c d^2)^5}-\frac{3cd e^2}{(a e^2-c d^2)^4(cdx+ae)}-\frac{cde}{(a e^2-c d^2)^3(cdx+ae)^2}-\frac{e^3}{(a e^2-c d^2)^4(e}$
risch	$\frac{-\frac{4c^3d^3e^3x^3}{a^4e^8-4a^3c d^2e^6+6a^2c^2d^4e^4-4a c^3d^6e^2+c^4d^8}-\frac{2c^2d^2\left(5a e^2+c d^2\right)e^2x^2}{a^4e^8-4a^3c d^2e^6+6a^2c^2d^4e^4-4a c^3d^6e^2+c^4d^8}-\frac{2\left(11a^2e^4+8ac d^2e^2-c^2d}{3\left(a^4e^8-4a^3c d^2e^6+6a^2c^2d^4e^4-4\right.}}{(cdx+ae)^2(cd x^2e+a e^2x+c d^2x+ade)}$
parallelrisch	$-\frac{-12\ln(cdx+ae)x^4c^7d^7e^5+12\ln(ex+d)x^3c^7d^8e^4-12\ln(cdx+ae)x^3c^7d^8e^4+12\ln(ex+d)a^3c^4d^5e^7-12\ln(cdx+ae)a^3c^4d^5e}{}$
norman	$\frac{-\frac{4d^3c^3e^5x^5}{a^4e^8-4a^3c d^2e^6+6a^2c^2d^4e^4-4a c^3d^6e^2+c^4d^8}+\frac{\left(-10a c^4d^4e^8-10d^6e^6c^5\right)x^4}{d^2c^2e^2\left(a^4e^8-4a^3c d^2e^6+6a^2c^2d^4e^4-4a c^3d^6e^2+c^4d^8\right)}+\frac{\left(-a^3c^3d^2e^{10}-19a^2c^4\right.}{e^2d^2c^3\left(a^4e^8-4a^3c d^2e^6\right.}}{}$

input
```
int((e*x+d)^2/(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^4,x,method=_RETURNVERBOSE)
```

output
```
-1/3*c*d/(a*e^2-c*d^2)^2/(c*d*x+a*e)^3+4*c*d/(a*e^2-c*d^2)^5*e^3*ln(c*d*x+
a*e)-3*c*d/(a*e^2-c*d^2)^4*e^2/(c*d*x+a*e)-c*d/(a*e^2-c*d^2)^3*e/(c*d*x+a*
e)^2-e^3/(a*e^2-c*d^2)^4/(e*x+d)-4*c*d/(a*e^2-c*d^2)^5*e^3*ln(e*x+d)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 837 vs. $2(171) = 342$.

Time = 0.10 (sec) , antiderivative size = 837, normalized size of antiderivative = 4.84

$$\int \frac{(d+ex)^2}{(ade+(cd^2+ae^2)\,x+cdex^2)^4}\,dx = \text{Too large to display}$$

input
```
integrate((e*x+d)^2/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^4,x, algorithm="fric
as")
```

output
```
-1/3*(c^4*d^8 - 6*a*c^3*d^6*e^2 + 18*a^2*c^2*d^4*e^4 - 10*a^3*c*d^2*e^6 -
3*a^4*e^8 + 12*(c^4*d^5*e^3 - a*c^3*d^3*e^5)*x^3 + 6*(c^4*d^6*e^2 + 4*a*c^
3*d^4*e^4 - 5*a^2*c^2*d^2*e^6)*x^2 - 2*(c^4*d^7*e - 9*a*c^3*d^5*e^3 - 3*a^
2*c^2*d^3*e^5 + 11*a^3*c*d*e^7)*x + 12*(c^4*d^4*e^4*x^4 + a^3*c*d^2*e^6 +
(c^4*d^5*e^3 + 3*a*c^3*d^3*e^5)*x^3 + 3*(a*c^3*d^4*e^4 + a^2*c^2*d^2*e^6)*
x^2 + (3*a^2*c^2*d^3*e^5 + a^3*c*d*e^7)*x)*log(c*d*x + a*e) - 12*(c^4*d^4*
e^4*x^4 + a^3*c*d^2*e^6 + (c^4*d^5*e^3 + 3*a*c^3*d^3*e^5)*x^3 + 3*(a*c^3*d
^4*e^4 + a^2*c^2*d^2*e^6)*x^2 + (3*a^2*c^2*d^3*e^5 + a^3*c*d*e^7)*x)*log(e
*x + d))/(a^3*c^5*d^11*e^3 - 5*a^4*c^4*d^9*e^5 + 10*a^5*c^3*d^7*e^7 - 10*a
^6*c^2*d^5*e^9 + 5*a^7*c*d^3*e^11 - a^8*d*e^13 + (c^8*d^13*e - 5*a*c^7*d^1
1*e^3 + 10*a^2*c^6*d^9*e^5 - 10*a^3*c^5*d^7*e^7 + 5*a^4*c^4*d^5*e^9 - a^5*
c^3*d^3*e^11)*x^4 + (c^8*d^14 - 2*a*c^7*d^12*e^2 - 5*a^2*c^6*d^10*e^4 + 20
*a^3*c^5*d^8*e^6 - 25*a^4*c^4*d^6*e^8 + 14*a^5*c^3*d^4*e^10 - 3*a^6*c^2*d^
2*e^12)*x^3 + 3*(a*c^7*d^13*e - 4*a^2*c^6*d^11*e^3 + 5*a^3*c^5*d^9*e^5 - 5
*a^5*c^3*d^5*e^9 + 4*a^6*c^2*d^3*e^11 - a^7*c*d*e^13)*x^2 + (3*a^2*c^6*d^1
2*e^2 - 14*a^3*c^5*d^10*e^4 + 25*a^4*c^4*d^8*e^6 - 20*a^5*c^3*d^6*e^8 + 5*
a^6*c^2*d^4*e^10 + 2*a^7*c*d^2*e^12 - a^8*e^14)*x)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1006 vs. $2(160) = 320$.

Time = 1.45 (sec) , antiderivative size = 1006, normalized size of antiderivative = 5.82

$$\int \frac{(d+ex)^2}{(ade+(cd^2+ae^2)\,x+cdex^2)^4}\,dx = \text{Too large to display}$$

input
```
integrate((e*x+d)**2/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**4,x)
```

output
```
-4*c*d*e**3*log(x + (-4*a**6*c*d*e**15/(a*e**2 - c*d**2)**5 + 24*a**5*c**2
*d**3*e**13/(a*e**2 - c*d**2)**5 - 60*a**4*c**3*d**5*e**11/(a*e**2 - c*d**
2)**5 + 80*a**3*c**4*d**7*e**9/(a*e**2 - c*d**2)**5 - 60*a**2*c**5*d**9*e*
*7/(a*e**2 - c*d**2)**5 + 24*a*c**6*d**11*e**5/(a*e**2 - c*d**2)**5 + 4*a*
c*d*e**5 - 4*c**7*d**13*e**3/(a*e**2 - c*d**2)**5 + 4*c**2*d**3*e**3)/(8*c
**2*d**2*e**4))/(a*e**2 - c*d**2)**5 + 4*c*d*e**3*log(x + (4*a**6*c*d*e**1
5/(a*e**2 - c*d**2)**5 - 24*a**5*c**2*d**3*e**13/(a*e**2 - c*d**2)**5 + 60
*a**4*c**3*d**5*e**11/(a*e**2 - c*d**2)**5 - 80*a**3*c**4*d**7*e**9/(a*e**
2 - c*d**2)**5 + 60*a**2*c**5*d**9*e**7/(a*e**2 - c*d**2)**5 - 24*a*c**6*d
**11*e**5/(a*e**2 - c*d**2)**5 + 4*a*c*d*e**5 + 4*c**7*d**13*e**3/(a*e**2
- c*d**2)**5 + 4*c**2*d**3*e**3)/(8*c**2*d**2*e**4))/(a*e**2 - c*d**2)**5
+ (-3*a**3*e**6 - 13*a**2*c*d**2*e**4 + 5*a*c**2*d**4*e**2 - c**3*d**6 - 1
2*c**3*d**3*e**3*x**3 + x**2*(-30*a*c**2*d**2*e**4 - 6*c**3*d**4*e**2) + x
*(-22*a**2*c*d*e**5 - 16*a*c**2*d**3*e**3 + 2*c**3*d**5*e))/(3*a**7*d*e**1
1 - 12*a**6*c*d**3*e**9 + 18*a**5*c**2*d**5*e**7 - 12*a**4*c**3*d**7*e**5
+ 3*a**3*c**4*d**9*e**3 + x**4*(3*a**4*c**3*d**3*e**9 - 12*a**3*c**4*d**5*
e**7 + 18*a**2*c**5*d**7*e**5 - 12*a*c**6*d**9*e**3 + 3*c**7*d**11*e) + x*
*3*(9*a**5*c**2*d**2*e**10 - 33*a**4*c**3*d**4*e**8 + 42*a**3*c**4*d**6*e*
*6 - 18*a**2*c**5*d**8*e**4 - 3*a*c**6*d**10*e**2 + 3*c**7*d**12) + x**2*(
9*a**6*c*d*e**11 - 27*a**5*c**2*d**3*e**9 + 18*a**4*c**3*d**5*e**7 + 18...
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 656 vs. $2(171) = 342$.

Time = 0.05 (sec) , antiderivative size = 656, normalized size of antiderivative = 3.79

$$\int \frac{(d+ex)^2}{(ade+(cd^2+ae^2)x+cdex^2)^4}\,dx$$
$$= -\frac{4\,cde^3\log(cdx+ae)}{c^5d^{10}-5\,ac^4d^8e^2+10\,a^2c^3d^6e^4-10\,a^3c^2d^4e^6+5\,a^4cd^2e^8-a^5e^{10}}$$
$$+\frac{4\,cde^3\log(ex+d)}{c^5d^{10}-5\,ac^4d^8e^2+10\,a^2c^3d^6e^4-10\,a^3c^2d^4e^6+5\,a^4cd^2e^8-a^5e^{10}}$$
$$-\frac{}{3\,(a^3c^4d^9e^3-4\,a^4c^3d^7e^5+6\,a^5c^2d^5e^7-4\,a^6cd^3e^9+a^7de^{11}+(c^7d^{11}e-4\,ac^6d^9e^3+6\,a^2c^5d^7e^5-4\,a^3c^4}$$

input
```
integrate((e*x+d)^2/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^4,x, algorithm="maxi
ma")
```

output
```
-4*c*d*e^3*log(c*d*x + a*e)/(c^5*d^10 - 5*a*c^4*d^8*e^2 + 10*a^2*c^3*d^6*e
^4 - 10*a^3*c^2*d^4*e^6 + 5*a^4*c*d^2*e^8 - a^5*e^10) + 4*c*d*e^3*log(e*x
+ d)/(c^5*d^10 - 5*a*c^4*d^8*e^2 + 10*a^2*c^3*d^6*e^4 - 10*a^3*c^2*d^4*e^6
 + 5*a^4*c*d^2*e^8 - a^5*e^10) - 1/3*(12*c^3*d^3*e^3*x^3 + c^3*d^6 - 5*a*c
^2*d^4*e^2 + 13*a^2*c*d^2*e^4 + 3*a^3*e^6 + 6*(c^3*d^4*e^2 + 5*a*c^2*d^2*e
^4)*x^2 - 2*(c^3*d^5*e - 8*a*c^2*d^3*e^3 - 11*a^2*c*d*e^5)*x)/(a^3*c^4*d^9
*e^3 - 4*a^4*c^3*d^7*e^5 + 6*a^5*c^2*d^5*e^7 - 4*a^6*c*d^3*e^9 + a^7*d*e^1
1 + (c^7*d^11*e - 4*a*c^6*d^9*e^3 + 6*a^2*c^5*d^7*e^5 - 4*a^3*c^4*d^5*e^7
+ a^4*c^3*d^3*e^9)*x^4 + (c^7*d^12 - a*c^6*d^10*e^2 - 6*a^2*c^5*d^8*e^4 +
14*a^3*c^4*d^6*e^6 - 11*a^4*c^3*d^4*e^8 + 3*a^5*c^2*d^2*e^10)*x^3 + 3*(a*c
^6*d^11*e - 3*a^2*c^5*d^9*e^3 + 2*a^3*c^4*d^7*e^5 + 2*a^4*c^3*d^5*e^7 - 3*
a^5*c^2*d^3*e^9 + a^6*c*d*e^11)*x^2 + (3*a^2*c^5*d^10*e^2 - 11*a^3*c^4*d^8
*e^4 + 14*a^4*c^3*d^6*e^6 - 6*a^5*c^2*d^4*e^8 - a^6*c*d^2*e^10 + a^7*e^12)
*x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 386 vs. $2(171) = 342$.

Time = 0.14 (sec) , antiderivative size = 386, normalized size of antiderivative = 2.23

$$\int \frac{(d+ex)^2}{(ade+(cd^2+ae^2)x+cdex^2)^4}\,dx$$
$$= -\frac{4\,c^2d^2e^3\log\left(|cdx+ae|\right)}{c^6d^{11}-5\,ac^5d^9e^2+10\,a^2c^4d^7e^4-10\,a^3c^3d^5e^6+5\,a^4c^2d^3e^8-a^5cde^{10}} + \frac{4\,cde^4\log\left(|ex+d|\right)}{c^5d^{10}e-5\,ac^4d^8e^3+10\,a^2c^3d^6e^5-10\,a^3c^2d^4e^7+5\,a^4cd^2e^9-a^5e^{11}} - \frac{c^4d^8-6\,ac^3d^6e^2+18\,a^2c^2d^4e^4-10\,a^3cd^2e^6-3\,a^4e^8+12\,(c^4d^5e^3-ac^3d^3e^5)x^3+6\,(c^4d^6e^2+4\,ac^3d^4e}{3\,(cd^2-ae^2)^5(cdx+ae)^3(ex+d)}$$

input

```
integrate((e*x+d)^2/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^4,x, algorithm="giac
")
```

output

```
-4*c^2*d^2*e^3*log(abs(c*d*x + a*e))/(c^6*d^11 - 5*a*c^5*d^9*e^2 + 10*a^2*
c^4*d^7*e^4 - 10*a^3*c^3*d^5*e^6 + 5*a^4*c^2*d^3*e^8 - a^5*c*d*e^10) + 4*c
*d*e^4*log(abs(e*x + d))/(c^5*d^10*e - 5*a*c^4*d^8*e^3 + 10*a^2*c^3*d^6*e^
5 - 10*a^3*c^2*d^4*e^7 + 5*a^4*c*d^2*e^9 - a^5*e^11) - 1/3*(c^4*d^8 - 6*a*
c^3*d^6*e^2 + 18*a^2*c^2*d^4*e^4 - 10*a^3*c*d^2*e^6 - 3*a^4*e^8 + 12*(c^4*
d^5*e^3 - a*c^3*d^3*e^5)*x^3 + 6*(c^4*d^6*e^2 + 4*a*c^3*d^4*e^4 - 5*a^2*c^
2*d^2*e^6)*x^2 - 2*(c^4*d^7*e - 9*a*c^3*d^5*e^3 - 3*a^2*c^2*d^3*e^5 + 11*a
^3*c*d*e^7)*x)/((c*d^2 - a*e^2)^5*(c*d*x + a*e)^3*(e*x + d))
```

Mupad [B] (verification not implemented)

Time = 5.61 (sec) , antiderivative size = 617, normalized size of antiderivative = 3.57

$$\int \frac{(d+ex)^2}{(ade+(cd^2+ae^2)x+cdex^2)^4}\,dx$$
$$= \frac{8\,c\,d\,e^3\,\mathrm{atanh}\left(\frac{a^5\,e^{10}-3\,a^4\,c\,d^2\,e^8+2\,a^3\,c^2\,d^4\,e^6+2\,a^2\,c^3\,d^6\,e^4-3\,a\,c^4\,d^8\,e^2+c^5\,d^{10}}{(a\,e^2-c\,d^2)^5} + \frac{2\,c\,d\,e\,x\,(a^4\,e^8-4\,a^3\,c\,d^2\,e^6+6\,a^2\,c^2\,d^4\,e^4-4\,a\,c^3\,d^6\,e}{(a\,e^2-c\,d^2)^5}\right.}{(a\,e^2-c\,d^2)^5} - \frac{\frac{3\,a^3\,e^6+13\,a^2\,c\,d^2\,e^4-5\,a\,c^2\,d^4\,e^2+c^3\,d^6}{3\,(a^4\,e^8-4\,a^3\,c\,d^2\,e^6+6\,a^2\,c^2\,d^4\,e^4-4\,a\,c^3\,d^6\,e^2+c^4\,d^8)} + \frac{2\,e\,x\,(11\,a^2\,c\,d\,e^4+8\,a\,c^2\,d^3\,e^2-c^3\,d^5)}{3\,(a^4\,e^8-4\,a^3\,c\,d^2\,e^6+6\,a^2\,c^2\,d^4\,e^4-4\,a\,c^3\,d^6\,e^2+c^4\,d^8)} + \frac{2\,e^2\,x^2\,(c}{a^4\,e^8-4\,a^3\,c\,d^2\,e^6+6}}{x\,(a^3\,e^4+3\,c\,a^2\,d^2\,e^2)+x^3\,(c^3\,d^4+3\,a\,c^2\,d^2\,e^2)+x^2\,(3\,a^2\,c\,d\,e^3+3}$$

input
```
int((d + e*x)^2/(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^4,x)
```

output
```
(8*c*d*e^3*atanh((a^5*e^10 + c^5*d^10 - 3*a*c^4*d^8*e^2 - 3*a^4*c*d^2*e^8
+ 2*a^2*c^3*d^6*e^4 + 2*a^3*c^2*d^4*e^6)/(a*e^2 - c*d^2)^5 + (2*c*d*e*x*(a
^4*e^8 + c^4*d^8 - 4*a*c^3*d^6*e^2 - 4*a^3*c*d^2*e^6 + 6*a^2*c^2*d^4*e^4))
/(a*e^2 - c*d^2)^5))/(a*e^2 - c*d^2)^5 - ((3*a^3*e^6 + c^3*d^6 - 5*a*c^2*d
^4*e^2 + 13*a^2*c*d^2*e^4)/(3*(a^4*e^8 + c^4*d^8 - 4*a*c^3*d^6*e^2 - 4*a^3
*c*d^2*e^6 + 6*a^2*c^2*d^4*e^4)) + (2*e*x*(8*a*c^2*d^3*e^2 - c^3*d^5 + 11*
a^2*c*d*e^4))/(3*(a^4*e^8 + c^4*d^8 - 4*a*c^3*d^6*e^2 - 4*a^3*c*d^2*e^6 +
6*a^2*c^2*d^4*e^4)) + (2*e^2*x^2*(c^3*d^4 + 5*a*c^2*d^2*e^2))/(a^4*e^8 + c
^4*d^8 - 4*a*c^3*d^6*e^2 - 4*a^3*c*d^2*e^6 + 6*a^2*c^2*d^4*e^4) + (4*c^3*d
^3*e^3*x^3)/(a^4*e^8 + c^4*d^8 - 4*a*c^3*d^6*e^2 - 4*a^3*c*d^2*e^6 + 6*a^2
*c^2*d^4*e^4))/(x*(a^3*e^4 + 3*a^2*c*d^2*e^2) + x^3*(c^3*d^4 + 3*a*c^2*d^2
*e^2) + x^2*(3*a*c^2*d^3*e + 3*a^2*c*d*e^3) + a^3*d*e^3 + c^3*d^3*e*x^4)
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 1362, normalized size of antiderivative = 7.87

$$\int \frac{(d+ex)^2}{(ade+(cd^2+ae^2)x+cdex^2)^4}\,dx = \text{Too large to display}$$

input
```
int((e*x+d)^2/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^4,x)
```

output
```
(36*log(a*e + c*d*x)*a**4*c*d**2*e**8 + 36*log(a*e + c*d*x)*a**4*c*d*e**9*
x + 12*log(a*e + c*d*x)*a**3*c**2*d**4*e**6 + 120*log(a*e + c*d*x)*a**3*c*
*2*d**3*e**7*x + 108*log(a*e + c*d*x)*a**3*c**2*d**2*e**8*x**2 + 36*log(a*
e + c*d*x)*a**2*c**3*d**5*e**5*x + 144*log(a*e + c*d*x)*a**2*c**3*d**4*e**
6*x**2 + 108*log(a*e + c*d*x)*a**2*c**3*d**3*e**7*x**3 + 36*log(a*e + c*d*
x)*a*c**4*d**6*e**4*x**2 + 72*log(a*e + c*d*x)*a*c**4*d**5*e**5*x**3 + 36*
log(a*e + c*d*x)*a*c**4*d**4*e**6*x**4 + 12*log(a*e + c*d*x)*c**5*d**7*e**
3*x**3 + 12*log(a*e + c*d*x)*c**5*d**6*e**4*x**4 - 36*log(d + e*x)*a**4*c*
d**2*e**8 - 36*log(d + e*x)*a**4*c*d*e**9*x - 12*log(d + e*x)*a**3*c**2*d*
*4*e**6 - 120*log(d + e*x)*a**3*c**2*d**3*e**7*x - 108*log(d + e*x)*a**3*c
**2*d**2*e**8*x**2 - 36*log(d + e*x)*a**2*c**3*d**5*e**5*x - 144*log(d + e
*x)*a**2*c**3*d**4*e**6*x**2 - 108*log(d + e*x)*a**2*c**3*d**3*e**7*x**3 -
 36*log(d + e*x)*a*c**4*d**6*e**4*x**2 - 72*log(d + e*x)*a*c**4*d**5*e**5*
x**3 - 36*log(d + e*x)*a*c**4*d**4*e**6*x**4 - 12*log(d + e*x)*c**5*d**7*e
**3*x**3 - 12*log(d + e*x)*c**5*d**6*e**4*x**4 - 9*a**5*e**10 - 21*a**4*c*
d**2*e**8 - 54*a**4*c*d*e**9*x + 32*a**3*c**2*d**4*e**6 + 20*a**3*c**2*d**
3*e**7*x - 54*a**3*c**2*d**2*e**8*x**2 + 24*a**2*c**3*d**5*e**5*x + 42*a**
2*c**3*d**4*e**6*x**2 - 3*a*c**4*d**8*e**2 + 12*a*c**4*d**7*e**3*x + 6*a*c
**4*d**6*e**4*x**2 + 12*a*c**4*d**4*e**6*x**4 + c**5*d**10 - 2*c**5*d**9*e
*x + 6*c**5*d**8*e**2*x**2 - 12*c**5*d**6*e**4*x**4)/(3*(3*a**9*d*e**15...
```

3.147 $\int \frac{d+ex}{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^4}\,dx$

Optimal result

Integrand size = 33, antiderivative size = 226

$$\begin{aligned}\int \frac{d+ex}{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^4}\,dx = &-\frac{c^2d^2}{3\left(cd^2-ae^2\right)^3(ae+cdx)^3}\\&+\frac{3c^2d^2e}{2\left(cd^2-ae^2\right)^4(ae+cdx)^2}\\&-\frac{6c^2d^2e^2}{\left(cd^2-ae^2\right)^5(ae+cdx)}\\&-\frac{e^3}{2\left(cd^2-ae^2\right)^4(d+ex)^2}\\&-\frac{4cde^3}{\left(cd^2-ae^2\right)^5(d+ex)}\\&-\frac{10c^2d^2e^3\log(ae+cdx)}{\left(cd^2-ae^2\right)^6}+\frac{10c^2d^2e^3\log(d+ex)}{\left(cd^2-ae^2\right)^6}\end{aligned}$$

output
```
-1/3*c^2*d^2/(-a*e^2+c*d^2)^3/(c*d*x+a*e)^3+3/2*c^2*d^2*e/(-a*e^2+c*d^2)^4
/(c*d*x+a*e)^2-6*c^2*d^2*e^2/(-a*e^2+c*d^2)^5/(c*d*x+a*e)-1/2*e^3/(-a*e^2+
c*d^2)^4/(e*x+d)^2-4*c*d*e^3/(-a*e^2+c*d^2)^5/(e*x+d)-10*c^2*d^2*e^3*ln(c*
d*x+a*e)/(-a*e^2+c*d^2)^6+10*c^2*d^2*e^3*ln(e*x+d)/(-a*e^2+c*d^2)^6
```

Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 206, normalized size of antiderivative = 0.91

$$\int \frac{d+ex}{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^4}\,dx$$
$$=\frac{\frac{2c^2d^2\left(-cd^2+ae^2\right)^3}{(ae+cdx)^3}+\frac{9c^2d^2e\left(cd^2-ae^2\right)^2}{(ae+cdx)^2}+\frac{36c^2d^2e^2\left(-cd^2+ae^2\right)}{ae+cdx}-\frac{3e^3\left(cd^2-ae^2\right)^2}{(d+ex)^2}+\frac{24cde^3\left(-cd^2+ae^2\right)}{d+ex}-60c^2d^2e^3\log(ae+}{6\left(cd^2-ae^2\right)^6}$$

input
```
Integrate[(d + e*x)/(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^4,x]
```

output
```
((2*c^2*d^2*(-(c*d^2) + a*e^2)^3)/(a*e + c*d*x)^3 + (9*c^2*d^2*e*(c*d^2 -
a*e^2)^2)/(a*e + c*d*x)^2 + (36*c^2*d^2*e^2*(-(c*d^2) + a*e^2))/(a*e + c*d
*x) - (3*e^3*(c*d^2 - a*e^2)^2)/(d + e*x)^2 + (24*c*d*e^3*(-(c*d^2) + a*e^
2))/(d + e*x) - 60*c^2*d^2*e^3*Log[a*e + c*d*x] + 60*c^2*d^2*e^3*Log[d + e
*x])/(6*(c*d^2 - a*e^2)^6)
```

Rubi [A] (verified)

Time = 0.73 (sec) , antiderivative size = 226, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.061, Rules used = {1121, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{d+ex}{\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^4}\,dx$$
$$\Big\downarrow \text{ 1121}$$
$$\int \left(\frac{6c^3d^3e^2}{\left(cd^2-ae^2\right)^5(ae+cdx)^2}-\frac{3c^3d^3e}{\left(cd^2-ae^2\right)^4(ae+cdx)^3}+\frac{c^3d^3}{\left(cd^2-ae^2\right)^3(ae+cdx)^4}-\frac{10c^3d^3e^3}{\left(cd^2-ae^2\right)^6(ae+cdx)}\right.$$
$$\Big\downarrow \text{ 2009}$$

$$-\frac{6c^2d^2e^2}{(cd^2-ae^2)^5(ae+cdx)}+\frac{3c^2d^2e}{2(cd^2-ae^2)^4(ae+cdx)^2}-\frac{c^2d^2}{3(cd^2-ae^2)^3(ae+cdx)^3}-\frac{10c^2d^2e^3\log(ae+cdx)}{(cd^2-ae^2)^6}+\frac{10c^2d^2e^3\log(d+ex)}{(cd^2-ae^2)^6}-\frac{4cde^3}{(d+ex)(cd^2-ae^2)^5}-\frac{e^3}{2(d+ex)^2(cd^2-ae^2)^4}$$

input
```
Int[(d + e*x)/(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^4,x]
```

output
```
-1/3*(c^2*d^2)/((c*d^2 - a*e^2)^3*(a*e + c*d*x)^3) + (3*c^2*d^2*e)/(2*(c*d
^2 - a*e^2)^4*(a*e + c*d*x)^2) - (6*c^2*d^2*e^2)/((c*d^2 - a*e^2)^5*(a*e +
 c*d*x)) - e^3/(2*(c*d^2 - a*e^2)^4*(d + e*x)^2) - (4*c*d*e^3)/((c*d^2 - a
*e^2)^5*(d + e*x)) - (10*c^2*d^2*e^3*Log[a*e + c*d*x])/(c*d^2 - a*e^2)^6 +
 (10*c^2*d^2*e^3*Log[d + e*x])/(c*d^2 - a*e^2)^6
```

Defintions of rubi rules used

rule 1121
```
Int[((d_) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Int[ExpandIntegrand[(d + e*x)^(m + p)*(a/d + (c/e)*x)^p, x], x]
/; FreeQ[{a, b, c, d, e, m, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && (Int
egerQ[p] || (GtQ[a, 0] && GtQ[d, 0] && LtQ[c, 0]))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 1.67 (sec) , antiderivative size = 221, normalized size of antiderivative = 0.98

method	result
default	$\frac{c^2 d^2}{3(a e^2-c d^2)^3 (cdx+ae)^3}-\frac{10c^2 d^2 e^3 \ln(cdx+ae)}{(a e^2-c d^2)^6}+\frac{6c^2 d^2 e^2}{(a e^2-c d^2)^5 (cdx+ae)}+\frac{3c^2 d^2 e}{2(a e^2-c d^2)^4 (cdx+ae)^2}-\frac{e^3}{2(a e^2-c d^2)\ldots}$
risch	$\frac{\frac{10c^4 d^4 e^4 x^4}{a^5 e^{10}-5a^4 c d^2 e^8+10a^3 c^2 d^4 e^6-10a^2 c^3 d^6 e^4+5a c^4 d^8 e^2-c^5 d^{10}}+\frac{5d^3 e^3 c^3 (5a e^2+3c d^2) x^3}{a^5 e^{10}-5a^4 c d^2 e^8+10a^3 c^2 d^4 e^6-10a^2 c^3 d^6 e^4+5a c^4 d^8 e^2-c^5 d^{10}}+\frac{\ldots}{3(\ldots}}{\ldots}$
norman	$\frac{\frac{10d^4 c^4 e^5 x^5}{a^5 e^{10}-5a^4 c d^2 e^8+10a^3 c^2 d^4 e^6-10a^2 c^3 d^6 e^4+5a c^4 d^8 e^2-c^5 d^{10}}+\frac{(25a c^5 d^5 e^8+25c^6 d^7 e^6) x^4}{d^2 c^2 e^2 (a^5 e^{10}-5a^4 c d^2 e^8+10a^3 c^2 d^4 e^6-10a^2 c^3 d^6 e^4+5a c^4 d^8 e^2-c^5\ldots}}{\ldots}$
parallelrisch	$\frac{120\ln(ex+d) x^4 c^8 d^9 e^6-120\ln(cdx+ae) x^4 c^8 d^9 e^6+60\ln(ex+d) x^3 c^8 d^{10} e^5-60\ln(cdx+ae) x^3 c^8 d^{10} e^5+60\ln(ex+d) a^3 c^5 d^7 e^8-\ldots}{\ldots}$

input
```
int((e*x+d)/(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^4,x,method=_RETURNVERBOSE)
```

output
```
1/3*c^2*d^2/(a*e^2-c*d^2)^3/(c*d*x+a*e)^3-10*c^2*d^2/(a*e^2-c*d^2)^6*e^3*l
n(c*d*x+a*e)+6*c^2*d^2/(a*e^2-c*d^2)^5*e^2/(c*d*x+a*e)+3/2*c^2*d^2/(a*e^2-
c*d^2)^4*e/(c*d*x+a*e)^2-1/2*e^3/(a*e^2-c*d^2)^4/(e*x+d)^2+10*c^2*d^2/(a*e
^2-c*d^2)^6*e^3*ln(e*x+d)+4*e^3/(a*e^2-c*d^2)^5*c*d/(e*x+d)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1242 vs. 2(220) = 440.

Time = 0.11 (sec) , antiderivative size = 1242, normalized size of antiderivative = 5.50

$$\int \frac{d+ex}{(ade+(cd^2+ae^2)x+cdex^2)^4} dx = \text{Too large to display}$$

input
```
integrate((e*x+d)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^4,x, algorithm="fricas
")
```

output
```
-1/6*(2*c^5*d^10 - 15*a*c^4*d^8*e^2 + 60*a^2*c^3*d^6*e^4 - 20*a^3*c^2*d^4*
e^6 - 30*a^4*c*d^2*e^8 + 3*a^5*e^10 + 60*(c^5*d^6*e^4 - a*c^4*d^4*e^6)*x^4
 + 30*(3*c^5*d^7*e^3 + 2*a*c^4*d^5*e^5 - 5*a^2*c^3*d^3*e^7)*x^3 + 10*(2*c^
5*d^8*e^2 + 21*a*c^4*d^6*e^4 - 12*a^2*c^3*d^4*e^6 - 11*a^3*c^2*d^2*e^8)*x^
2 - 5*(c^5*d^9*e - 12*a*c^4*d^7*e^3 - 24*a^2*c^3*d^5*e^5 + 32*a^3*c^2*d^3*
e^7 + 3*a^4*c*d*e^9)*x + 60*(c^5*d^5*e^5*x^5 + a^3*c^2*d^4*e^6 + (2*c^5*d^
6*e^4 + 3*a*c^4*d^4*e^6)*x^4 + (c^5*d^7*e^3 + 6*a*c^4*d^5*e^5 + 3*a^2*c^3*
d^3*e^7)*x^3 + (3*a*c^4*d^6*e^4 + 6*a^2*c^3*d^4*e^6 + a^3*c^2*d^2*e^8)*x^2
 + (3*a^2*c^3*d^5*e^5 + 2*a^3*c^2*d^3*e^7)*x)*log(c*d*x + a*e) - 60*(c^5*d
^5*e^5*x^5 + a^3*c^2*d^4*e^6 + (2*c^5*d^6*e^4 + 3*a*c^4*d^4*e^6)*x^4 + (c^
5*d^7*e^3 + 6*a*c^4*d^5*e^5 + 3*a^2*c^3*d^3*e^7)*x^3 + (3*a*c^4*d^6*e^4 +
6*a^2*c^3*d^4*e^6 + a^3*c^2*d^2*e^8)*x^2 + (3*a^2*c^3*d^5*e^5 + 2*a^3*c^2*
d^3*e^7)*x)*log(e*x + d))/(a^3*c^6*d^14*e^3 - 6*a^4*c^5*d^12*e^5 + 15*a^5*
c^4*d^10*e^7 - 20*a^6*c^3*d^8*e^9 + 15*a^7*c^2*d^6*e^11 - 6*a^8*c*d^4*e^13
 + a^9*d^2*e^15 + (c^9*d^15*e^2 - 6*a*c^8*d^13*e^4 + 15*a^2*c^7*d^11*e^6 -
 20*a^3*c^6*d^9*e^8 + 15*a^4*c^5*d^7*e^10 - 6*a^5*c^4*d^5*e^12 + a^6*c^3*d
^3*e^14)*x^5 + (2*c^9*d^16*e - 9*a*c^8*d^14*e^3 + 12*a^2*c^7*d^12*e^5 + 5*
a^3*c^6*d^10*e^7 - 30*a^4*c^5*d^8*e^9 + 33*a^5*c^4*d^6*e^11 - 16*a^6*c^3*d
^4*e^13 + 3*a^7*c^2*d^2*e^15)*x^4 + (c^9*d^17 - 18*a^2*c^7*d^13*e^4 + 52*a
^3*c^6*d^11*e^6 - 60*a^4*c^5*d^9*e^8 + 24*a^5*c^4*d^7*e^10 + 10*a^6*c^3...
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1363 vs. $2(209) = 418$.

Time = 2.49 (sec) , antiderivative size = 1363, normalized size of antiderivative = 6.03

$$\int \frac{d+ex}{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^4}\,dx = \text{Too large to display}$$

input
```
integrate((e*x+d)/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**4,x)
```

output
```
10*c**2*d**2*e**3*log(x + (-10*a**7*c**2*d**2*e**17/(a*e**2 - c*d**2)**6 +
 70*a**6*c**3*d**4*e**15/(a*e**2 - c*d**2)**6 - 210*a**5*c**4*d**6*e**13/(
a*e**2 - c*d**2)**6 + 350*a**4*c**5*d**8*e**11/(a*e**2 - c*d**2)**6 - 350*
a**3*c**6*d**10*e**9/(a*e**2 - c*d**2)**6 + 210*a**2*c**7*d**12*e**7/(a*e*
*2 - c*d**2)**6 - 70*a*c**8*d**14*e**5/(a*e**2 - c*d**2)**6 + 10*a*c**2*d*
*2*e**5 + 10*c**9*d**16*e**3/(a*e**2 - c*d**2)**6 + 10*c**3*d**4*e**3)/(20
*c**3*d**3*e**4))/(a*e**2 - c*d**2)**6 - 10*c**2*d**2*e**3*log(x + (10*a**
7*c**2*d**2*e**17/(a*e**2 - c*d**2)**6 - 70*a**6*c**3*d**4*e**15/(a*e**2 -
 c*d**2)**6 + 210*a**5*c**4*d**6*e**13/(a*e**2 - c*d**2)**6 - 350*a**4*c**
5*d**8*e**11/(a*e**2 - c*d**2)**6 + 350*a**3*c**6*d**10*e**9/(a*e**2 - c*d
**2)**6 - 210*a**2*c**7*d**12*e**7/(a*e**2 - c*d**2)**6 + 70*a*c**8*d**14*
e**5/(a*e**2 - c*d**2)**6 + 10*a*c**2*d**2*e**5 - 10*c**9*d**16*e**3/(a*e*
*2 - c*d**2)**6 + 10*c**3*d**4*e**3)/(20*c**3*d**3*e**4))/(a*e**2 - c*d**2
)**6 + (-3*a**4*e**8 + 27*a**3*c*d**2*e**6 + 47*a**2*c**2*d**4*e**4 - 13*a
*c**3*d**6*e**2 + 2*c**4*d**8 + 60*c**4*d**4*e**4*x**4 + x**3*(150*a*c**3*
d**3*e**5 + 90*c**4*d**5*e**3) + x**2*(110*a**2*c**2*d**2*e**6 + 230*a*c**
3*d**4*e**4 + 20*c**4*d**6*e**2) + x*(15*a**3*c*d*e**7 + 175*a**2*c**2*d**
3*e**5 + 55*a*c**3*d**5*e**3 - 5*c**4*d**7*e))/(6*a**8*d**2*e**13 - 30*a**
7*c*d**4*e**11 + 60*a**6*c**2*d**6*e**9 - 60*a**5*c**3*d**8*e**7 + 30*a**4
*c**4*d**10*e**5 - 6*a**3*c**5*d**12*e**3 + x**5*(6*a**5*c**3*d**3*e**1...
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 956 vs. 2(220) = 440.

Time = 0.08 (sec) , antiderivative size = 956, normalized size of antiderivative = 4.23

$$\int \frac{d+ex}{(ade+(cd^2+ae^2)\,x+cdex^2)^4}\,dx = \text{Too large to display}$$

input
```
integrate((e*x+d)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^4,x, algorithm="maxima
")
```

output
```
-10*c^2*d^2*e^3*log(c*d*x + a*e)/(c^6*d^12 - 6*a*c^5*d^10*e^2 + 15*a^2*c^4
*d^8*e^4 - 20*a^3*c^3*d^6*e^6 + 15*a^4*c^2*d^4*e^8 - 6*a^5*c*d^2*e^10 + a^
6*e^12) + 10*c^2*d^2*e^3*log(e*x + d)/(c^6*d^12 - 6*a*c^5*d^10*e^2 + 15*a^
2*c^4*d^8*e^4 - 20*a^3*c^3*d^6*e^6 + 15*a^4*c^2*d^4*e^8 - 6*a^5*c*d^2*e^10
 + a^6*e^12) - 1/6*(60*c^4*d^4*e^4*x^4 + 2*c^4*d^8 - 13*a*c^3*d^6*e^2 + 47
*a^2*c^2*d^4*e^4 + 27*a^3*c*d^2*e^6 - 3*a^4*e^8 + 30*(3*c^4*d^5*e^3 + 5*a*
c^3*d^3*e^5)*x^3 + 10*(2*c^4*d^6*e^2 + 23*a*c^3*d^4*e^4 + 11*a^2*c^2*d^2*e
^6)*x^2 - 5*(c^4*d^7*e - 11*a*c^3*d^5*e^3 - 35*a^2*c^2*d^3*e^5 - 3*a^3*c*d
*e^7)*x)/(a^3*c^5*d^12*e^3 - 5*a^4*c^4*d^10*e^5 + 10*a^5*c^3*d^8*e^7 - 10*
a^6*c^2*d^6*e^9 + 5*a^7*c*d^4*e^11 - a^8*d^2*e^13 + (c^8*d^13*e^2 - 5*a*c^
7*d^11*e^4 + 10*a^2*c^6*d^9*e^6 - 10*a^3*c^5*d^7*e^8 + 5*a^4*c^4*d^5*e^10
- a^5*c^3*d^3*e^12)*x^5 + (2*c^8*d^14*e - 7*a*c^7*d^12*e^3 + 5*a^2*c^6*d^1
0*e^5 + 10*a^3*c^5*d^8*e^7 - 20*a^4*c^4*d^6*e^9 + 13*a^5*c^3*d^4*e^11 - 3*
a^6*c^2*d^2*e^13)*x^4 + (c^8*d^15 + a*c^7*d^13*e^2 - 17*a^2*c^6*d^11*e^4 +
 35*a^3*c^5*d^9*e^6 - 25*a^4*c^4*d^7*e^8 - a^5*c^3*d^5*e^10 + 9*a^6*c^2*d^
3*e^12 - 3*a^7*c*d*e^14)*x^3 + (3*a*c^7*d^14*e - 9*a^2*c^6*d^12*e^3 + a^3*
c^5*d^10*e^5 + 25*a^4*c^4*d^8*e^7 - 35*a^5*c^3*d^6*e^9 + 17*a^6*c^2*d^4*e^
11 - a^7*c*d^2*e^13 - a^8*e^15)*x^2 + (3*a^2*c^6*d^13*e^2 - 13*a^3*c^5*d^1
1*e^4 + 20*a^4*c^4*d^9*e^6 - 10*a^5*c^3*d^7*e^8 - 5*a^6*c^2*d^5*e^10 + 7*a
^7*c*d^3*e^12 - 2*a^8*d*e^14)*x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 503 vs. $2(220) = 440$.

Time = 0.14 (sec) , antiderivative size = 503, normalized size of antiderivative = 2.23

$$\int \frac{d+ex}{(ade+(cd^2+ae^2)x+cdex^2)^4}\,dx =$$

$$-\frac{10\,c^3d^3e^3\log\left(|cdx+ae|\right)}{c^7d^{13}-6\,ac^6d^{11}e^2+15\,a^2c^5d^9e^4-20\,a^3c^4d^7e^6+15\,a^4c^3d^5e^8-6\,a^5c^2d^3e^{10}+a^6cde^{12}}$$
$$+\frac{10\,c^2d^2e^4\log\left(|ex+d|\right)}{c^6d^{12}e-6\,ac^5d^{10}e^3+15\,a^2c^4d^8e^5-20\,a^3c^3d^6e^7+15\,a^4c^2d^4e^9-6\,a^5cd^2e^{11}+a^6e^{13}}$$
$$-\frac{2\,c^5d^{10}-15\,ac^4d^8e^2+60\,a^2c^3d^6e^4-20\,a^3c^2d^4e^6-30\,a^4cd^2e^8+3\,a^5e^{10}+60\,(c^5d^6e^4-ac^4d^4e^6)x^4+3}{}$$

input
```
integrate((e*x+d)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^4,x, algorithm="giac")
```

output
```
-10*c^3*d^3*e^3*log(abs(c*d*x + a*e))/(c^7*d^13 - 6*a*c^6*d^11*e^2 + 15*a^
2*c^5*d^9*e^4 - 20*a^3*c^4*d^7*e^6 + 15*a^4*c^3*d^5*e^8 - 6*a^5*c^2*d^3*e^
10 + a^6*c*d*e^12) + 10*c^2*d^2*e^4*log(abs(e*x + d))/(c^6*d^12*e - 6*a*c^
5*d^10*e^3 + 15*a^2*c^4*d^8*e^5 - 20*a^3*c^3*d^6*e^7 + 15*a^4*c^2*d^4*e^9
- 6*a^5*c*d^2*e^11 + a^6*e^13) - 1/6*(2*c^5*d^10 - 15*a*c^4*d^8*e^2 + 60*a
^2*c^3*d^6*e^4 - 20*a^3*c^2*d^4*e^6 - 30*a^4*c*d^2*e^8 + 3*a^5*e^10 + 60*(
c^5*d^6*e^4 - a*c^4*d^4*e^6)*x^4 + 30*(3*c^5*d^7*e^3 + 2*a*c^4*d^5*e^5 - 5
*a^2*c^3*d^3*e^7)*x^3 + 10*(2*c^5*d^8*e^2 + 21*a*c^4*d^6*e^4 - 12*a^2*c^3*
d^4*e^6 - 11*a^3*c^2*d^2*e^8)*x^2 - 5*(c^5*d^9*e - 12*a*c^4*d^7*e^3 - 24*a
^2*c^3*d^5*e^5 + 32*a^3*c^2*d^3*e^7 + 3*a^4*c*d*e^9)*x)/((c*d^2 - a*e^2)^6
*(c*d*x + a*e)^3*(e*x + d)^2)
```

Mupad [B] (verification not implemented)

Time = 5.64 (sec) , antiderivative size = 891, normalized size of antiderivative = 3.94

$$\int \frac{d+ex}{\left(ade+(cd^2+ae^2)\,x+cdex^2\right)^4}\,dx$$
$$= \frac{\frac{-3\,a^4\,e^8+27\,a^3\,c\,d^2\,e^6+47\,a^2\,c^2\,d^4\,e^4-13\,a\,c^3\,d^6\,e^2+2\,c^4\,d^8}{6\,\left(a^5\,e^{10}-5\,a^4\,c\,d^2\,e^8+10\,a^3\,c^2\,d^4\,e^6-10\,a^2\,c^3\,d^6\,e^4+5\,a\,c^4\,d^8\,e^2-c^5\,d^{10}\right)}+\frac{5\,e^2\,x^2\,\left(11\,a^2\,c^2\,d^2\,e^4+23\,a\,c^3\,d^4\,e^2+2\,c^4\,d^6\right)}{3\,\left(a^5\,e^{10}-5\,a^4\,c\,d^2\,e^8+10\,a^3\,c^2\,d^4\,e^6-10\,a^2\,c^3\,d^6\,e^4+5\,a\,c^4\,d^8\,e^2-\right.}}{x^2\,\left(a^3\,e^5+6\,a^2\,c\,d^2\,e^3+3\,a\,c^2\,d^4\,e\right)+x^3\,\left(3\right.}$$
$$-\frac{20\,c^2\,d^2\,e^3\,\mathrm{atanh}\left(\frac{a^6\,e^{12}-4\,a^5\,c\,d^2\,e^{10}+5\,a^4\,c^2\,d^4\,e^8-5\,a^2\,c^4\,d^8\,e^4+4\,a\,c^5\,d^{10}\,e^2-c^6\,d^{12}}{{\left(a\,e^2-c\,d^2\right)}^6}+\frac{2\,c\,d\,e\,x\,\left(a^5\,e^{10}-5\,a^4\,c\,d^2\,e^8+10\,a^3\,c^2\,d^4\,e^6\right.}{\left(a\,e^2-c\right.}\right.}{{\left(a\,e^2-c\,d^2\right)}^6}$$

input
```
int((d + e*x)/(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^4,x)
```

output
```
((2*c^4*d^8 - 3*a^4*e^8 - 13*a*c^3*d^6*e^2 + 27*a^3*c*d^2*e^6 + 47*a^2*c^2
*d^4*e^4)/(6*(a^5*e^10 - c^5*d^10 + 5*a*c^4*d^8*e^2 - 5*a^4*c*d^2*e^8 - 10
*a^2*c^3*d^6*e^4 + 10*a^3*c^2*d^4*e^6)) + (5*e^2*x^2*(2*c^4*d^6 + 23*a*c^3
*d^4*e^2 + 11*a^2*c^2*d^2*e^4))/(3*(a^5*e^10 - c^5*d^10 + 5*a*c^4*d^8*e^2
- 5*a^4*c*d^2*e^8 - 10*a^2*c^3*d^6*e^4 + 10*a^3*c^2*d^4*e^6)) + (5*d*e*x*(
3*a^3*c*e^6 - c^4*d^6 + 11*a*c^3*d^4*e^2 + 35*a^2*c^2*d^2*e^4))/(6*(a^5*e^
10 - c^5*d^10 + 5*a*c^4*d^8*e^2 - 5*a^4*c*d^2*e^8 - 10*a^2*c^3*d^6*e^4 + 1
0*a^3*c^2*d^4*e^6)) + (5*c*e*x^3*(3*c^3*d^5*e^2 + 5*a*c^2*d^3*e^4))/(a^5*e
^10 - c^5*d^10 + 5*a*c^4*d^8*e^2 - 5*a^4*c*d^2*e^8 - 10*a^2*c^3*d^6*e^4 +
10*a^3*c^2*d^4*e^6) + (10*c^4*d^4*e^4*x^4)/(a^5*e^10 - c^5*d^10 + 5*a*c^4*
d^8*e^2 - 5*a^4*c*d^2*e^8 - 10*a^2*c^3*d^6*e^4 + 10*a^3*c^2*d^4*e^6))/(x^2
*(a^3*e^5 + 6*a^2*c*d^2*e^3 + 3*a*c^2*d^4*e) + x^3*(c^3*d^5 + 6*a*c^2*d^3*
e^2 + 3*a^2*c*d*e^4) + x*(2*a^3*d*e^4 + 3*a^2*c*d^3*e^2) + x^4*(2*c^3*d^4*
e + 3*a*c^2*d^2*e^3) + a^3*d^2*e^3 + c^3*d^3*e^2*x^5) - (20*c^2*d^2*e^3*at
anh((a^6*e^12 - c^6*d^12 + 4*a*c^5*d^10*e^2 - 4*a^5*c*d^2*e^10 - 5*a^2*c^4
*d^8*e^4 + 5*a^4*c^2*d^4*e^8)/(a*e^2 - c*d^2)^6 + (2*c*d*e*x*(a^5*e^10 - c
^5*d^10 + 5*a*c^4*d^8*e^2 - 5*a^4*c*d^2*e^8 - 10*a^2*c^3*d^6*e^4 + 10*a^3*
c^2*d^4*e^6))/(a*e^2 - c*d^2)^6))/(a*e^2 - c*d^2)^6
```

Reduce [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 1990, normalized size of antiderivative = 8.81

$$\int \frac{d+ex}{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^4}\,dx = \text{Too large to display}$$

input
```
int((e*x+d)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^4,x)
```

output

```
( - 180*log(a*e + c*d*x)*a**4*c**2*d**4*e**8 - 360*log(a*e + c*d*x)*a**4*c
**2*d**3*e**9*x - 180*log(a*e + c*d*x)*a**4*c**2*d**2*e**10*x**2 - 120*log
(a*e + c*d*x)*a**3*c**3*d**6*e**6 - 780*log(a*e + c*d*x)*a**3*c**3*d**5*e*
*7*x - 1200*log(a*e + c*d*x)*a**3*c**3*d**4*e**8*x**2 - 540*log(a*e + c*d*
x)*a**3*c**3*d**3*e**9*x**3 - 360*log(a*e + c*d*x)*a**2*c**4*d**7*e**5*x -
 1260*log(a*e + c*d*x)*a**2*c**4*d**6*e**6*x**2 - 1440*log(a*e + c*d*x)*a*
*2*c**4*d**5*e**7*x**3 - 540*log(a*e + c*d*x)*a**2*c**4*d**4*e**8*x**4 - 3
60*log(a*e + c*d*x)*a*c**5*d**8*e**4*x**2 - 900*log(a*e + c*d*x)*a*c**5*d*
*7*e**5*x**3 - 720*log(a*e + c*d*x)*a*c**5*d**6*e**6*x**4 - 180*log(a*e +
c*d*x)*a*c**5*d**5*e**7*x**5 - 120*log(a*e + c*d*x)*c**6*d**9*e**3*x**3 -
240*log(a*e + c*d*x)*c**6*d**8*e**4*x**4 - 120*log(a*e + c*d*x)*c**6*d**7*
e**5*x**5 + 180*log(d + e*x)*a**4*c**2*d**4*e**8 + 360*log(d + e*x)*a**4*c
**2*d**3*e**9*x + 180*log(d + e*x)*a**4*c**2*d**2*e**10*x**2 + 120*log(d +
 e*x)*a**3*c**3*d**6*e**6 + 780*log(d + e*x)*a**3*c**3*d**5*e**7*x + 1200*
log(d + e*x)*a**3*c**3*d**4*e**8*x**2 + 540*log(d + e*x)*a**3*c**3*d**3*e*
*9*x**3 + 360*log(d + e*x)*a**2*c**4*d**7*e**5*x + 1260*log(d + e*x)*a**2*
c**4*d**6*e**6*x**2 + 1440*log(d + e*x)*a**2*c**4*d**5*e**7*x**3 + 540*log
(d + e*x)*a**2*c**4*d**4*e**8*x**4 + 360*log(d + e*x)*a*c**5*d**8*e**4*x**
2 + 900*log(d + e*x)*a*c**5*d**7*e**5*x**3 + 720*log(d + e*x)*a*c**5*d**6*
e**6*x**4 + 180*log(d + e*x)*a*c**5*d**5*e**7*x**5 + 120*log(d + e*x)*c...
```

3.148 $\int \frac{1}{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^4}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 256

$$\int \frac{1}{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^4}\,dx = -\frac{c^3d^3}{3\left(cd^2-ae^2\right)^4(ae+cdx)^3} + \frac{2c^3d^3e}{\left(cd^2-ae^2\right)^5(ae+cdx)^2} - \frac{10c^3d^3e^2}{\left(cd^2-ae^2\right)^6(ae+cdx)} - \frac{e^3}{3\left(cd^2-ae^2\right)^4(d+ex)^3} - \frac{2cde^3}{\left(cd^2-ae^2\right)^5(d+ex)^2} - \frac{10c^2d^2e^3}{\left(cd^2-ae^2\right)^6(d+ex)} - \frac{20c^3d^3e^3\log(ae+cdx)}{\left(cd^2-ae^2\right)^7} + \frac{20c^3d^3e^3\log(d+ex)}{\left(cd^2-ae^2\right)^7}$$

output
```
-1/3*c^3*d^3/(-a*e^2+c*d^2)^4/(c*d*x+a*e)^3+2*c^3*d^3*e/(-a*e^2+c*d^2)^5/(
c*d*x+a*e)^2-10*c^3*d^3*e^2/(-a*e^2+c*d^2)^6/(c*d*x+a*e)-1/3*e^3/(-a*e^2+c
*d^2)^4/(e*x+d)^3-2*c*d*e^3/(-a*e^2+c*d^2)^5/(e*x+d)^2-10*c^2*d^2*e^3/(-a*
e^2+c*d^2)^6/(e*x+d)-20*c^3*d^3*e^3*ln(c*d*x+a*e)/(-a*e^2+c*d^2)^7+20*c^3*
d^3*e^3*ln(e*x+d)/(-a*e^2+c*d^2)^7
```

Mathematica [A] (verified)

Time = 0.24 (sec) , antiderivative size = 234, normalized size of antiderivative = 0.91

$$\int \frac{1}{(ade + (cd^2 + ae^2)x + cdex^2)^4} dx = \frac{\frac{c^3d^3(cd^2-ae^2)^3}{(ae+cdx)^3} - \frac{6c^3d^3e(cd^2-ae^2)^2}{(ae+cdx)^2} + \frac{30c^3d^3e^2(cd^2-ae^2)}{ae+cdx} + \frac{(cd^2e-ae^3)^3}{(d+ex)^3} + \frac{6cde^3(cd^2-ae^2)^2}{(d+ex)^2} + \frac{30c^2d^2e^3(cd^2-ae^2)}{d+ex} + 60c^3d^3}{3(-cd^2+ae^2)^7}$$

input
```
Integrate[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(-4),x]
```

output
```
((c^3*d^3*(c*d^2 - a*e^2)^3)/(a*e + c*d*x)^3 - (6*c^3*d^3*e*(c*d^2 - a*e^2
)^2)/(a*e + c*d*x)^2 + (30*c^3*d^3*e^2*(c*d^2 - a*e^2))/(a*e + c*d*x) + (c
*d^2*e - a*e^3)^3/(d + e*x)^3 + (6*c*d*e^3*(c*d^2 - a*e^2)^2)/(d + e*x)^2
+ (30*c^2*d^2*e^3*(c*d^2 - a*e^2))/(d + e*x) + 60*c^3*d^3*e^3*Log[a*e + c*
d*x] - 60*c^3*d^3*e^3*Log[d + e*x])/(3*(-(c*d^2) + a*e^2)^7)
```

Rubi [A] (verified)

Time = 0.94 (sec) , antiderivative size = 281, normalized size of antiderivative = 1.10, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.074, Rules used = {1084, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(x(ae^2+cd^2)+ade+cdex^2)^4} dx$$

$$\Big\downarrow \text{ 1084}$$

$$c^4d^4e^4 \int \left(\frac{20}{cd(cd^2-ae^2)^7(d+ex)} + \frac{10}{c^2d^2(cd^2-ae^2)^6(d+ex)^2} + \frac{4}{c^3d^3(cd^2-ae^2)^5(d+ex)^3} + \frac{}{c^4d^4(cd^2-a}\right.$$

$$\Big\downarrow \text{ 2009}$$

$$c^4d^4e^4\left(-\frac{1}{3c^4d^4e(d+ex)^3(cd^2-ae^2)^4} - \frac{2}{c^3d^3e(d+ex)^2(cd^2-ae^2)^5} - \frac{10}{c^2d^2e(d+ex)(cd^2-ae^2)^6} - \frac{}{cde^2(cd^2}\right.$$

input
```
Int[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(-4),x]
```

output
```
c^4*d^4*e^4*(-1/3*1/(c*d*e^4*(c*d^2 - a*e^2)^4*(a*e + c*d*x)^3) + 2/(c*d*e
^3*(c*d^2 - a*e^2)^5*(a*e + c*d*x)^2) - 10/(c*d*e^2*(c*d^2 - a*e^2)^6*(a*e
 + c*d*x)) - 1/(3*c^4*d^4*e*(c*d^2 - a*e^2)^4*(d + e*x)^3) - 2/(c^3*d^3*e*
(c*d^2 - a*e^2)^5*(d + e*x)^2) - 10/(c^2*d^2*e*(c*d^2 - a*e^2)^6*(d + e*x)
) - (20*Log[a*e + c*d*x])/(c*d*e*(c*d^2 - a*e^2)^7) + (20*Log[d + e*x])/(c
*d*e*(c*d^2 - a*e^2)^7))
```

Defintions of rubi rules used

rule 1084
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{q = Rt[b^2
- 4*a*c, 2]}, Simp[1/c^p   Int[ExpandIntegrand[(b/2 - q/2 + c*x)^p*(b/2 + q
/2 + c*x)^p, x], x], x] /;  !FractionalPowerFactorQ[q]] /; FreeQ[{a, b, c},
 x] && IntegerQ[p] && NiceSqrtQ[b^2 - 4*a*c]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 1.64 (sec) , antiderivative size = 253, normalized size of antiderivative = 0.99

method	result
default	$-\frac{c^{3} d^{3}}{3\left(a\,e^{2}-c\,d^{2}\right)^{4}\left(c d x+a e\right)^{3}}+\frac{20 c^{3} d^{3} e^{3} \ln \left(c d x+a e\right)}{\left(a\,e^{2}-c\,d^{2}\right)^{7}}-\frac{10 c^{3} d^{3} e^{2}}{\left(a\,e^{2}-c\,d^{2}\right)^{6}\left(c d x+a e\right)}-\frac{2 c^{3} d^{3} e}{\left(a\,e^{2}-c\,d^{2}\right)^{5}\left(c d x+a e\right)^{2}}-\frac{e^{3}}{3\left(a\,e^{2}-c\,d^{2}\right.}$
risch	Expression too large to display
norman	Expression too large to display
parallelrisch	Expression too large to display

input
```
int(1/(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^4,x,method=_RETURNVERBOSE)
```

output
```
-1/3*c^3*d^3/(a*e^2-c*d^2)^4/(c*d*x+a*e)^3+20*c^3*d^3/(a*e^2-c*d^2)^7*e^3*
ln(c*d*x+a*e)-10*c^3*d^3/(a*e^2-c*d^2)^6*e^2/(c*d*x+a*e)-2*c^3*d^3/(a*e^2-
c*d^2)^5*e/(c*d*x+a*e)^2-1/3*e^3/(a*e^2-c*d^2)^4/(e*x+d)^3-20*c^3*d^3/(a*e
^2-c*d^2)^7*e^3*ln(e*x+d)-10*e^3/(a*e^2-c*d^2)^6*c^2*d^2/(e*x+d)+2*e^3/(a*
e^2-c*d^2)^5*c*d/(e*x+d)^2
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1618 vs. $2(252) = 504$.

Time = 0.13 (sec) , antiderivative size = 1618, normalized size of antiderivative = 6.32

$$\int \frac{1}{\left(ade + (cd^2 + ae^2)\,x + cdex^2\right)^4}\,dx = \text{Too large to display}$$

input
```
integrate(1/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^4,x, algorithm="fricas")
```

output
```
-1/3*(c^6*d^12 - 9*a*c^5*d^10*e^2 + 45*a^2*c^4*d^8*e^4 - 45*a^4*c^2*d^4*e^
8 + 9*a^5*c*d^2*e^10 - a^6*e^12 + 60*(c^6*d^7*e^5 - a*c^5*d^5*e^7)*x^5 + 1
50*(c^6*d^8*e^4 - a^2*c^4*d^4*e^8)*x^4 + 10*(11*c^6*d^9*e^3 + 27*a*c^5*d^7
*e^5 - 27*a^2*c^4*d^5*e^7 - 11*a^3*c^3*d^3*e^9)*x^3 + 15*(c^6*d^10*e^2 + 1
8*a*c^5*d^8*e^4 - 18*a^3*c^3*d^4*e^8 - a^4*c^2*d^2*e^10)*x^2 - 3*(c^6*d^11
*e - 15*a*c^5*d^9*e^3 - 60*a^2*c^4*d^7*e^5 + 60*a^3*c^3*d^5*e^7 + 15*a^4*c
^2*d^3*e^9 - a^5*c*d*e^11)*x + 60*(c^6*d^6*e^6*x^6 + a^3*c^3*d^6*e^6 + 3*(
c^6*d^7*e^5 + a*c^5*d^5*e^7)*x^5 + 3*(c^6*d^8*e^4 + 3*a*c^5*d^6*e^6 + a^2*
c^4*d^4*e^8)*x^4 + (c^6*d^9*e^3 + 9*a*c^5*d^7*e^5 + 9*a^2*c^4*d^5*e^7 + a^
3*c^3*d^3*e^9)*x^3 + 3*(a*c^5*d^8*e^4 + 3*a^2*c^4*d^6*e^6 + a^3*c^3*d^4*e^
8)*x^2 + 3*(a^2*c^4*d^7*e^5 + a^3*c^3*d^5*e^7)*x)*log(c*d*x + a*e) - 60*(c
^6*d^6*e^6*x^6 + a^3*c^3*d^6*e^6 + 3*(c^6*d^7*e^5 + a*c^5*d^5*e^7)*x^5 + 3
*(c^6*d^8*e^4 + 3*a*c^5*d^6*e^6 + a^2*c^4*d^4*e^8)*x^4 + (c^6*d^9*e^3 + 9*
a*c^5*d^7*e^5 + 9*a^2*c^4*d^5*e^7 + a^3*c^3*d^3*e^9)*x^3 + 3*(a*c^5*d^8*e^
4 + 3*a^2*c^4*d^6*e^6 + a^3*c^3*d^4*e^8)*x^2 + 3*(a^2*c^4*d^7*e^5 + a^3*c^
3*d^5*e^7)*x)*log(e*x + d))/(a^3*c^7*d^17*e^3 - 7*a^4*c^6*d^15*e^5 + 21*a^
5*c^5*d^13*e^7 - 35*a^6*c^4*d^11*e^9 + 35*a^7*c^3*d^9*e^11 - 21*a^8*c^2*d^
7*e^13 + 7*a^9*c*d^5*e^15 - a^10*d^3*e^17 + (c^10*d^17*e^3 - 7*a*c^9*d^15*
e^5 + 21*a^2*c^8*d^13*e^7 - 35*a^3*c^7*d^11*e^9 + 35*a^4*c^6*d^9*e^11 - 21
*a^5*c^5*d^7*e^13 + 7*a^6*c^4*d^5*e^15 - a^7*c^3*d^3*e^17)*x^6 + 3*(c^1...
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1748 vs. $2(238) = 476$.

Time = 3.61 (sec) , antiderivative size = 1748, normalized size of antiderivative = 6.83

$$\int \frac{1}{(ade + (cd^2 + ae^2)x + cdex^2)^4} \, dx = \text{Too large to display}$$

input
```
integrate(1/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**4,x)
```

output
```
-20*c**3*d**3*e**3*log(x + (-20*a**8*c**3*d**3*e**19/(a*e**2 - c*d**2)**7
+ 160*a**7*c**4*d**5*e**17/(a*e**2 - c*d**2)**7 - 560*a**6*c**5*d**7*e**15
/(a*e**2 - c*d**2)**7 + 1120*a**5*c**6*d**9*e**13/(a*e**2 - c*d**2)**7 - 1
400*a**4*c**7*d**11*e**11/(a*e**2 - c*d**2)**7 + 1120*a**3*c**8*d**13*e**9
/(a*e**2 - c*d**2)**7 - 560*a**2*c**9*d**15*e**7/(a*e**2 - c*d**2)**7 + 16
0*a*c**10*d**17*e**5/(a*e**2 - c*d**2)**7 + 20*a*c**3*d**3*e**5 - 20*c**11
*d**19*e**3/(a*e**2 - c*d**2)**7 + 20*c**4*d**5*e**3)/(40*c**4*d**4*e**4))
/(a*e**2 - c*d**2)**7 + 20*c**3*d**3*e**3*log(x + (20*a**8*c**3*d**3*e**19
/(a*e**2 - c*d**2)**7 - 160*a**7*c**4*d**5*e**17/(a*e**2 - c*d**2)**7 + 56
0*a**6*c**5*d**7*e**15/(a*e**2 - c*d**2)**7 - 1120*a**5*c**6*d**9*e**13/(a
*e**2 - c*d**2)**7 + 1400*a**4*c**7*d**11*e**11/(a*e**2 - c*d**2)**7 - 112
0*a**3*c**8*d**13*e**9/(a*e**2 - c*d**2)**7 + 560*a**2*c**9*d**15*e**7/(a*
e**2 - c*d**2)**7 - 160*a*c**10*d**17*e**5/(a*e**2 - c*d**2)**7 + 20*a*c**
3*d**3*e**5 + 20*c**11*d**19*e**3/(a*e**2 - c*d**2)**7 + 20*c**4*d**5*e**3
)/(40*c**4*d**4*e**4))/(a*e**2 - c*d**2)**7 + (-a**5*e**10 + 8*a**4*c*d**2
*e**8 - 37*a**3*c**2*d**4*e**6 - 37*a**2*c**3*d**6*e**4 + 8*a*c**4*d**8*e*
*2 - c**5*d**10 - 60*c**5*d**5*e**5*x**5 + x**4*(-150*a*c**4*d**4*e**6 - 1
50*c**5*d**6*e**4) + x**3*(-110*a**2*c**3*d**3*e**7 - 380*a*c**4*d**5*e**5
 - 110*c**5*d**7*e**3) + x**2*(-15*a**3*c**2*d**2*e**8 - 285*a**2*c**3*d**
4*e**6 - 285*a*c**4*d**6*e**4 - 15*c**5*d**8*e**2) + x*(3*a**4*c*d*e**9...
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1278 vs. $2(252) = 504$.

Time = 0.07 (sec) , antiderivative size = 1278, normalized size of antiderivative = 4.99

$$\int \frac{1}{(ade + (cd^2 + ae^2)\,x + cdex^2)^4}\,dx = \text{Too large to display}$$

input
```
integrate(1/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^4,x, algorithm="maxima")
```

output
```
-20*c^3*d^3*e^3*log(c*d*x + a*e)/(c^7*d^14 - 7*a*c^6*d^12*e^2 + 21*a^2*c^5
*d^10*e^4 - 35*a^3*c^4*d^8*e^6 + 35*a^4*c^3*d^6*e^8 - 21*a^5*c^2*d^4*e^10
+ 7*a^6*c*d^2*e^12 - a^7*e^14) + 20*c^3*d^3*e^3*log(e*x + d)/(c^7*d^14 - 7
*a*c^6*d^12*e^2 + 21*a^2*c^5*d^10*e^4 - 35*a^3*c^4*d^8*e^6 + 35*a^4*c^3*d^
6*e^8 - 21*a^5*c^2*d^4*e^10 + 7*a^6*c*d^2*e^12 - a^7*e^14) - 1/3*(60*c^5*d
^5*e^5*x^5 + c^5*d^10 - 8*a*c^4*d^8*e^2 + 37*a^2*c^3*d^6*e^4 + 37*a^3*c^2*
d^4*e^6 - 8*a^4*c*d^2*e^8 + a^5*e^10 + 150*(c^5*d^6*e^4 + a*c^4*d^4*e^6)*x
^4 + 10*(11*c^5*d^7*e^3 + 38*a*c^4*d^5*e^5 + 11*a^2*c^3*d^3*e^7)*x^3 + 15*
(c^5*d^8*e^2 + 19*a*c^4*d^6*e^4 + 19*a^2*c^3*d^4*e^6 + a^3*c^2*d^2*e^8)*x^
2 - 3*(c^5*d^9*e - 14*a*c^4*d^7*e^3 - 74*a^2*c^3*d^5*e^5 - 14*a^3*c^2*d^3*
e^7 + a^4*c*d*e^9)*x)/(a^3*c^6*d^15*e^3 - 6*a^4*c^5*d^13*e^5 + 15*a^5*c^4*
d^11*e^7 - 20*a^6*c^3*d^9*e^9 + 15*a^7*c^2*d^7*e^11 - 6*a^8*c*d^5*e^13 + a
^9*d^3*e^15 + (c^9*d^15*e^3 - 6*a*c^8*d^13*e^5 + 15*a^2*c^7*d^11*e^7 - 20*
a^3*c^6*d^9*e^9 + 15*a^4*c^5*d^7*e^11 - 6*a^5*c^4*d^5*e^13 + a^6*c^3*d^3*e
^15)*x^6 + 3*(c^9*d^16*e^2 - 5*a*c^8*d^14*e^4 + 9*a^2*c^7*d^12*e^6 - 5*a^3
*c^6*d^10*e^8 - 5*a^4*c^5*d^8*e^10 + 9*a^5*c^4*d^6*e^12 - 5*a^6*c^3*d^4*e^
14 + a^7*c^2*d^2*e^16)*x^5 + 3*(c^9*d^17*e - 3*a*c^8*d^15*e^3 - 2*a^2*c^7*
d^13*e^5 + 19*a^3*c^6*d^11*e^7 - 30*a^4*c^5*d^9*e^9 + 19*a^5*c^4*d^7*e^11
- 2*a^6*c^3*d^5*e^13 - 3*a^7*c^2*d^3*e^15 + a^8*c*d*e^17)*x^4 + (c^9*d^18
+ 3*a*c^8*d^16*e^2 - 30*a^2*c^7*d^14*e^4 + 62*a^3*c^6*d^12*e^6 - 36*a^4...
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 635 vs. $2(252) = 504$.

Time = 0.16 (sec) , antiderivative size = 635, normalized size of antiderivative = 2.48

$$\int \frac{1}{(ade + (cd^2 + ae^2)x + cdex^2)^4} \, dx =$$

$$-\frac{20\,c^4d^4e^3\log\left(|cdx + ae|\right)}{c^8d^{15} - 7\,ac^7d^{13}e^2 + 21\,a^2c^6d^{11}e^4 - 35\,a^3c^5d^9e^6 + 35\,a^4c^4d^7e^8 - 21\,a^5c^3d^5e^{10} + 7\,a^6c^2d^3e^{12} - a^7cde^{14}} + \frac{20\,c^3d^3e^4\log\left(|ex + d|\right)}{c^7d^{14}e - 7\,ac^6d^{12}e^3 + 21\,a^2c^5d^{10}e^5 - 35\,a^3c^4d^8e^7 + 35\,a^4c^3d^6e^9 - 21\,a^5c^2d^4e^{11} + 7\,a^6cd^2e^{13} - a^7e^{15}} - \frac{60\,c^5d^5e^5x^5 + 150\,c^5d^6e^4x^4 + 150\,ac^4d^4e^6x^4 + 110\,c^5d^7e^3x^3 + 380\,ac^4d^5e^5x^3 + 110\,a^2c^3d^3e^7x^3 + 15\,c^5d}{3\,(c^6d^{12} - }$$

input
```
integrate(1/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^4,x, algorithm="giac")
```

output
```
-20*c^4*d^4*e^3*log(abs(c*d*x + a*e))/(c^8*d^15 - 7*a*c^7*d^13*e^2 + 21*a^
2*c^6*d^11*e^4 - 35*a^3*c^5*d^9*e^6 + 35*a^4*c^4*d^7*e^8 - 21*a^5*c^3*d^5*
e^10 + 7*a^6*c^2*d^3*e^12 - a^7*c*d*e^14) + 20*c^3*d^3*e^4*log(abs(e*x + d
))/(c^7*d^14*e - 7*a*c^6*d^12*e^3 + 21*a^2*c^5*d^10*e^5 - 35*a^3*c^4*d^8*e
^7 + 35*a^4*c^3*d^6*e^9 - 21*a^5*c^2*d^4*e^11 + 7*a^6*c*d^2*e^13 - a^7*e^1
5) - 1/3*(60*c^5*d^5*e^5*x^5 + 150*c^5*d^6*e^4*x^4 + 150*a*c^4*d^4*e^6*x^4
 + 110*c^5*d^7*e^3*x^3 + 380*a*c^4*d^5*e^5*x^3 + 110*a^2*c^3*d^3*e^7*x^3 +
 15*c^5*d^8*e^2*x^2 + 285*a*c^4*d^6*e^4*x^2 + 285*a^2*c^3*d^4*e^6*x^2 + 15
*a^3*c^2*d^2*e^8*x^2 - 3*c^5*d^9*e*x + 42*a*c^4*d^7*e^3*x + 222*a^2*c^3*d^
5*e^5*x + 42*a^3*c^2*d^3*e^7*x - 3*a^4*c*d*e^9*x + c^5*d^10 - 8*a*c^4*d^8*
e^2 + 37*a^2*c^3*d^6*e^4 + 37*a^3*c^2*d^4*e^6 - 8*a^4*c*d^2*e^8 + a^5*e^10
)/((c^6*d^12 - 6*a*c^5*d^10*e^2 + 15*a^2*c^4*d^8*e^4 - 20*a^3*c^3*d^6*e^6
+ 15*a^4*c^2*d^4*e^8 - 6*a^5*c*d^2*e^10 + a^6*e^12)*(c*d*e*x^2 + c*d^2*x +
 a*e^2*x + a*d*e)^3)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(ade + (cd^2 + ae^2)x + cdex^2)^4} dx = \text{Too large to display}$$

input
```
int(1/(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^4,x)
```

output
```
piecewise(0 < (a*e^2 + c*d^2)^2 - 4*a*c*d^2*e^2, - (20*c^3*d^3*e^3*log(((a
*e^2)/2 + (c*d^2)/2 - ((a*e^2 + c*d^2)^2/4 - a*c*d^2*e^2)^(1/2) + c*d*e*x)
/((a*e^2)/2 + (c*d^2)/2 + ((a*e^2 + c*d^2)^2/4 - a*c*d^2*e^2)^(1/2) + c*d*
e*x)))/((a*e^2 + c*d^2)^2 - 4*a*c*d^2*e^2)^(7/2) - (20*((a*e^2)/2 + (c*d^2
)/2 + c*d*e*x)*((c*d*e)/(30*((a*e^2 + c*d^2)^2 - 4*a*c*d^2*e^2)*(x*(a*e^2
+ c*d^2) + a*d*e + c*d*e*x^2)^3) - (c^2*d^2*e^2)/(6*((a*e^2 + c*d^2)^2 - 4
*a*c*d^2*e^2)^2*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^2) + (c^3*d^3*e^3)
/(((a*e^2 + c*d^2)^2 - 4*a*c*d^2*e^2)^3*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e
*x^2))))/(c*d*e), (a*e^2 + c*d^2)^2 - 4*a*c*d^2*e^2 < 0, - (20*((a*e^2)/2
+ (c*d^2)/2 + c*d*e*x)*((c*d*e)/(30*((a*e^2 + c*d^2)^2 - 4*a*c*d^2*e^2)*(x
*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^3) - (c^2*d^2*e^2)/(6*((a*e^2 + c*d^
2)^2 - 4*a*c*d^2*e^2)^2*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^2) + (c^3*
d^3*e^3)/(((a*e^2 + c*d^2)^2 - 4*a*c*d^2*e^2)^3*(x*(a*e^2 + c*d^2) + a*d*e
 + c*d*e*x^2))))/(c*d*e) - (20*c^3*d^3*e^3*atan(((a*e^2)/2 + (c*d^2)/2 + c
*d*e*x)/(- (a*e^2 + c*d^2)^2/4 + a*c*d^2*e^2)^(1/2)))/((- (a*e^2 + c*d^2)^
2/4 + a*c*d^2*e^2)^(1/2)*((a*e^2 + c*d^2)^2 - 4*a*c*d^2*e^2)^3), ~in((a*e^
2 + c*d^2)^2 - 4*a*c*d^2*e^2, 'real') | (a*e^2 + c*d^2)^2 == 4*a*c*d^2*e^2
, int(1/(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^4, x))
```

Reduce [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 2480, normalized size of antiderivative = 9.69

$$\int \frac{1}{(ade + (cd^2 + ae^2)x + cdex^2)^4} dx = \text{Too large to display}$$

input
```
int(1/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^4,x)
```

output

```
(60*log(a*e + c*d*x)*a**4*c**3*d**6*e**8 + 180*log(a*e + c*d*x)*a**4*c**3*
d**5*e**9*x + 180*log(a*e + c*d*x)*a**4*c**3*d**4*e**10*x**2 + 60*log(a*e
+ c*d*x)*a**4*c**3*d**3*e**11*x**3 + 60*log(a*e + c*d*x)*a**3*c**4*d**8*e*
*6 + 360*log(a*e + c*d*x)*a**3*c**4*d**7*e**7*x + 720*log(a*e + c*d*x)*a**
3*c**4*d**6*e**8*x**2 + 600*log(a*e + c*d*x)*a**3*c**4*d**5*e**9*x**3 + 18
0*log(a*e + c*d*x)*a**3*c**4*d**4*e**10*x**4 + 180*log(a*e + c*d*x)*a**2*c
**5*d**9*e**5*x + 720*log(a*e + c*d*x)*a**2*c**5*d**8*e**6*x**2 + 1080*log
(a*e + c*d*x)*a**2*c**5*d**7*e**7*x**3 + 720*log(a*e + c*d*x)*a**2*c**5*d*
*6*e**8*x**4 + 180*log(a*e + c*d*x)*a**2*c**5*d**5*e**9*x**5 + 180*log(a*e
 + c*d*x)*a*c**6*d**10*e**4*x**2 + 600*log(a*e + c*d*x)*a*c**6*d**9*e**5*x
**3 + 720*log(a*e + c*d*x)*a*c**6*d**8*e**6*x**4 + 360*log(a*e + c*d*x)*a*
c**6*d**7*e**7*x**5 + 60*log(a*e + c*d*x)*a*c**6*d**6*e**8*x**6 + 60*log(a
*e + c*d*x)*c**7*d**11*e**3*x**3 + 180*log(a*e + c*d*x)*c**7*d**10*e**4*x*
*4 + 180*log(a*e + c*d*x)*c**7*d**9*e**5*x**5 + 60*log(a*e + c*d*x)*c**7*d
**8*e**6*x**6 - 60*log(d + e*x)*a**4*c**3*d**6*e**8 - 180*log(d + e*x)*a**
4*c**3*d**5*e**9*x - 180*log(d + e*x)*a**4*c**3*d**4*e**10*x**2 - 60*log(d
 + e*x)*a**4*c**3*d**3*e**11*x**3 - 60*log(d + e*x)*a**3*c**4*d**8*e**6 -
360*log(d + e*x)*a**3*c**4*d**7*e**7*x - 720*log(d + e*x)*a**3*c**4*d**6*e
**8*x**2 - 600*log(d + e*x)*a**3*c**4*d**5*e**9*x**3 - 180*log(d + e*x)*a*
*3*c**4*d**4*e**10*x**4 - 180*log(d + e*x)*a**2*c**5*d**9*e**5*x - 720*...
```

3.149 $\int (d+ex)^{3/2} \left(ade+\left(cd^2+ae^2\right) x+cdex^2\right) \, dx$

Optimal result

Integrand size = 35, antiderivative size = 43

$$\int (d+ex)^{3/2} \left(ade+\left(cd^2+ae^2\right) x+cdex^2\right) \, dx = \frac{2}{7}\left(a-\frac{cd^2}{e^2}\right)(d+ex)^{7/2} + \frac{2cd(d+ex)^{9/2}}{9e^2}$$

output
```
2/7*(a-c*d^2/e^2)*(e*x+d)^(7/2)+2/9*c*d*(e*x+d)^(9/2)/e^2
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.79

$$\int (d+ex)^{3/2} \left(ade+\left(cd^2+ae^2\right) x+cdex^2\right) \, dx = \frac{2(d+ex)^{7/2}\left(9ae^2+cd(-2d+7ex)\right)}{63e^2}$$

input
```
Integrate[(d + e*x)^(3/2)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2),x]
```

output
```
(2*(d + e*x)^(7/2)*(9*a*e^2 + c*d*(-2*d + 7*e*x)))/(63*e^2)
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.057$, Rules used = {1121, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (d+ex)^{3/2}\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)dx$$

$$\Big\downarrow \ 1121$$

$$\int \left(\frac{(d+ex)^{5/2}\left(ae^2-cd^2\right)}{e}+\frac{cd(d+ex)^{7/2}}{e}\right)dx$$

$$\Big\downarrow \ 2009$$

$$\frac{2}{7}(d+ex)^{7/2}\left(a-\frac{cd^2}{e^2}\right)+\frac{2cd(d+ex)^{9/2}}{9e^2}$$

input
```
Int[(d + e*x)^(3/2)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2),x]
```

output
```
(2*(a - (c*d^2)/e^2)*(d + e*x)^(7/2))/7 + (2*c*d*(d + e*x)^(9/2))/(9*e^2)
```

Defintions of rubi rules used

rule 1121
```
Int[((d_) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Int[ExpandIntegrand[(d + e*x)^(m + p)*(a/d + (c/e)*x)^p, x], x]
/; FreeQ[{a, b, c, d, e, m, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && (Int
egerQ[p] || (GtQ[a, 0] && GtQ[d, 0] && LtQ[c, 0]))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 1.59 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.74

method	result	size
gosper	$\frac{2(ex+d)^{\frac{7}{2}}\left(7cdxe+9a\,e^{2}-2c\,d^{2}\right)}{63e^{2}}$	32
pseudoelliptic	$\frac{2(ex+d)^{\frac{7}{2}}\left(7cdxe+9a\,e^{2}-2c\,d^{2}\right)}{63e^{2}}$	32
derivativedivides	$\frac{\frac{2cd(ex+d)^{\frac{9}{2}}}{9}+\frac{2\left(a\,e^{2}-c\,d^{2}\right)(ex+d)^{\frac{7}{2}}}{7}}{e^{2}}$	39
default	$\frac{\frac{2cd(ex+d)^{\frac{9}{2}}}{9}+\frac{2\left(a\,e^{2}-c\,d^{2}\right)(ex+d)^{\frac{7}{2}}}{7}}{e^{2}}$	39
orering	$\frac{2\left(7cdxe+9a\,e^{2}-2c\,d^{2}\right)(ex+d)^{\frac{5}{2}}\left(ade+\left(a\,e^{2}+c\,d^{2}\right)x+cd\,x^{2}e\right)}{63e^{2}(cdx+ae)}$	67
trager	$\frac{2\left(7cd\,e^{4}x^{4}+9a\,e^{5}x^{3}+19c\,d^{2}e^{3}x^{3}+27ad\,e^{4}x^{2}+15c\,d^{3}e^{2}x^{2}+27a\,e^{3}d^{2}x+c\,d^{4}ex+9a\,e^{2}d^{3}-2c\,d^{5}\right)\sqrt{ex+d}}{63e^{2}}$	99
risch	$\frac{2\left(7cd\,e^{4}x^{4}+9a\,e^{5}x^{3}+19c\,d^{2}e^{3}x^{3}+27ad\,e^{4}x^{2}+15c\,d^{3}e^{2}x^{2}+27a\,e^{3}d^{2}x+c\,d^{4}ex+9a\,e^{2}d^{3}-2c\,d^{5}\right)\sqrt{ex+d}}{63e^{2}}$	99

input
```
int((e*x+d)^(3/2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e),x,method=_RETURNVERBOS
E)
```

output
```
2/63*(e*x+d)^(7/2)*(7*c*d*e*x+9*a*e^2-2*c*d^2)/e^2
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 98 vs. 2(35) = 70.

Time = 0.07 (sec) , antiderivative size = 98, normalized size of antiderivative = 2.28

$$\int (d+ex)^{3/2}\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)\,dx = \frac{2\,(7\,cde^4x^4-2\,cd^5+9\,ad^3e^2+(19\,cd^2e^3+9\,ae^5)x^3+3\,(5\,cd^3e^2+9\,ade^4)x^2+(cd^4e+27\,ad}{63\,e^2}$$

input
```
integrate((e*x+d)^(3/2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2),x, algorithm="fr
icas")
```

output
```
2/63*(7*c*d*e^4*x^4 - 2*c*d^5 + 9*a*d^3*e^2 + (19*c*d^2*e^3 + 9*a*e^5)*x^3
 + 3*(5*c*d^3*e^2 + 9*a*d*e^4)*x^2 + (c*d^4*e + 27*a*d^2*e^3)*x)*sqrt(e*x
+ d)/e^2
```

Sympy [A] (verification not implemented)

Time = 0.85 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.23

$$\int (d+ex)^{3/2} \left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)\,dx=\begin{cases}\frac{2\left(\frac{cd(d+ex)^{\frac{9}{2}}}{9e}+\frac{(d+ex)^{\frac{7}{2}}\left(ae^2-cd^2\right)}{7e}\right)}{e} & \text{for } e\neq 0\\ \frac{cd^{\frac{7}{2}}x^2}{2} & \text{otherwise}\end{cases}$$

input
```
integrate((e*x+d)**(3/2)*(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2),x)
```

output
```
Piecewise((2*(c*d*(d + e*x)**(9/2)/(9*e) + (d + e*x)**(7/2)*(a*e**2 - c*d*
*2)/(7*e))/e, Ne(e, 0)), (c*d**(7/2)*x**2/2, True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.88

$$\int (d+ex)^{3/2} \left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)\,dx=\frac{2\left(7\,(ex+d)^{\frac{9}{2}}cd-9\,(cd^2-ae^2)(ex+d)^{\frac{7}{2}}\right)}{63\,e^2}$$

input
```
integrate((e*x+d)^(3/2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2),x, algorithm="ma
xima")
```

output
```
2/63*(7*(e*x + d)^(9/2)*c*d - 9*(c*d^2 - a*e^2)*(e*x + d)^(7/2))/e^2
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 313 vs. $2(35) = 70$.

Time = 0.11 (sec) , antiderivative size = 313, normalized size of antiderivative = 7.28

$$\int (d+ex)^{3/2}\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)\,dx = \frac{2\left(315\sqrt{ex+d}ad^3e+\frac{105\left((ex+d)^{\frac{3}{2}}-3\sqrt{ex+d}d\right)cd^4}{e}+315\left((ex+d)^{\frac{3}{2}}-3\sqrt{ex+d}d\right)ad^2e+\frac{63\left(3\right.}{}\right.}{}$$

input
```
integrate((e*x+d)^(3/2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2),x, algorithm="gi
ac")
```

output
```
2/315*(315*sqrt(e*x + d)*a*d^3*e + 105*((e*x + d)^(3/2) - 3*sqrt(e*x + d)*
d)*c*d^4/e + 315*((e*x + d)^(3/2) - 3*sqrt(e*x + d)*d)*a*d^2*e + 63*(3*(e*
x + d)^(5/2) - 10*(e*x + d)^(3/2)*d + 15*sqrt(e*x + d)*d^2)*c*d^3/e + 63*(
3*(e*x + d)^(5/2) - 10*(e*x + d)^(3/2)*d + 15*sqrt(e*x + d)*d^2)*a*d*e + 2
7*(5*(e*x + d)^(7/2) - 21*(e*x + d)^(5/2)*d + 35*(e*x + d)^(3/2)*d^2 - 35*
sqrt(e*x + d)*d^3)*c*d^2/e + 9*(5*(e*x + d)^(7/2) - 21*(e*x + d)^(5/2)*d +
 35*(e*x + d)^(3/2)*d^2 - 35*sqrt(e*x + d)*d^3)*a*e + (35*(e*x + d)^(9/2)
- 180*(e*x + d)^(7/2)*d + 378*(e*x + d)^(5/2)*d^2 - 420*(e*x + d)^(3/2)*d^
3 + 315*sqrt(e*x + d)*d^4)*c*d/e)/e
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.79

$$\int (d+ex)^{3/2}\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)\,dx = \frac{2\,{\left(d+e\,x\right)}^{7/2}\,\left(9\,a\,e^2-9\,c\,d^2+7\,c\,d\,\left(d+e\,x\right)\right)}{63\,e^2}$$

input
```
int((d + e*x)^(3/2)*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2),x)
```

output
```
(2*(d + e*x)^(7/2)*(9*a*e^2 - 9*c*d^2 + 7*c*d*(d + e*x)))/(63*e^2)
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 97, normalized size of antiderivative = 2.26

$$\int (d+ex)^{3/2} \left(ade+\left(cd^2+ae^2\right)x +cdex^2\right)\,dx = \frac{2\sqrt{ex+d}\,(7cd\,e^4x^4+9a\,e^5x^3+19c\,d^2e^3x^3+27ad\,e^4x^2+15c\,d^3e^2x^2+27a\,d^2e^3x+c\,d^4ex+}{63e^2}$$

input
```
int((e*x+d)^(3/2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2),x)
```

output
```
(2*sqrt(d + e*x)*(9*a*d**3*e**2 + 27*a*d**2*e**3*x + 27*a*d*e**4*x**2 + 9*
a*e**5*x**3 - 2*c*d**5 + c*d**4*e*x + 15*c*d**3*e**2*x**2 + 19*c*d**2*e**3
*x**3 + 7*c*d*e**4*x**4))/(63*e**2)
```

3.150 $\int \sqrt{d+ex}\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)\,dx$

Optimal result

Integrand size = 35, antiderivative size = 43

$$\int \sqrt{d+ex}\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)\,dx=\frac{2}{5}\left(a-\frac{cd^2}{e^2}\right)(d+ex)^{5/2}+\frac{2cd(d+ex)^{7/2}}{7e^2}$$

output
```
2/5*(a-c*d^2/e^2)*(e*x+d)^(5/2)+2/7*c*d*(e*x+d)^(7/2)/e^2
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.79

$$\int \sqrt{d+ex}\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)\,dx=\frac{2(d+ex)^{5/2}\left(7ae^2+cd(-2d+5ex)\right)}{35e^2}$$

input
```
Integrate[Sqrt[d + e*x]*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2),x]
```

output
```
(2*(d + e*x)^(5/2)*(7*a*e^2 + c*d*(-2*d + 5*e*x)))/(35*e^2)
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.057$, Rules used = {1121, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{d+ex}\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right) \, dx$$

$$\Big\downarrow \text{ 1121}$$

$$\int \left(\frac{(d+ex)^{3/2}\left(ae^2-cd^2\right)}{e}+\frac{cd(d+ex)^{5/2}}{e}\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{2}{5}(d+ex)^{5/2}\left(a-\frac{cd^2}{e^2}\right)+\frac{2cd(d+ex)^{7/2}}{7e^2}$$

input
```
Int[Sqrt[d + e*x]*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2),x]
```

output
```
(2*(a - (c*d^2)/e^2)*(d + e*x)^(5/2))/5 + (2*c*d*(d + e*x)^(7/2))/(7*e^2)
```

Defintions of rubi rules used

rule 1121
```
Int[((d_) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Int[ExpandIntegrand[(d + e*x)^(m + p)*(a/d + (c/e)*x)^p, x], x]
/; FreeQ[{a, b, c, d, e, m, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && (Int
egerQ[p] || (GtQ[a, 0] && GtQ[d, 0] && LtQ[c, 0]))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 1.57 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.74

method	result	size
gosper	$\frac{2(ex+d)^{\frac{5}{2}}(5cdxe+7a\,e^{2}-2c\,d^{2})}{35e^{2}}$	32
pseudoelliptic	$\frac{2(ex+d)^{\frac{5}{2}}(5cdxe+7a\,e^{2}-2c\,d^{2})}{35e^{2}}$	32
derivativedivides	$\frac{\frac{2cd(ex+d)^{\frac{7}{2}}}{7}+\frac{2\left(a\,e^{2}-c\,d^{2}\right)(ex+d)^{\frac{5}{2}}}{5}}{e^{2}}$	39
default	$\frac{\frac{2cd(ex+d)^{\frac{7}{2}}}{7}+\frac{2\left(a\,e^{2}-c\,d^{2}\right)(ex+d)^{\frac{5}{2}}}{5}}{e^{2}}$	39
orering	$\frac{2(5cdxe+7a\,e^{2}-2c\,d^{2})(ex+d)^{\frac{3}{2}}(ade+(a\,e^{2}+c\,d^{2})x+cd\,x^{2}e)}{35e^{2}(cdx+ae)}$	67
trager	$\frac{2(5d\,e^{3}c\,x^{3}+7e^{4}a\,x^{2}+8d^{2}e^{2}c\,x^{2}+14ad\,e^{3}x+c\,d^{3}ex+7a\,d^{2}e^{2}-2c\,d^{4})\sqrt{ex+d}}{35e^{2}}$	75
risch	$\frac{2(5d\,e^{3}c\,x^{3}+7e^{4}a\,x^{2}+8d^{2}e^{2}c\,x^{2}+14ad\,e^{3}x+c\,d^{3}ex+7a\,d^{2}e^{2}-2c\,d^{4})\sqrt{ex+d}}{35e^{2}}$	75

input
```
int((e*x+d)^(1/2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e),x,method=_RETURNVERBOS
E)
```

output
```
2/35*(e*x+d)^(5/2)*(5*c*d*e*x+7*a*e^2-2*c*d^2)/e^2
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 74 vs. $2(35) = 70$.

Time = 0.07 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.72

$$\int \sqrt{d+ex}\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)\,dx = \frac{2\,(5\,cde^3x^3-2\,cd^4+7\,ad^2e^2+(8\,cd^2e^2+7\,ae^4)x^2+(cd^3e+14\,ade^3)x)\sqrt{ex+d}}{35\,e^2}$$

input
```
integrate((e*x+d)^(1/2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2),x, algorithm="fr
icas")
```

output
```
2/35*(5*c*d*e^3*x^3 - 2*c*d^4 + 7*a*d^2*e^2 + (8*c*d^2*e^2 + 7*a*e^4)*x^2
+ (c*d^3*e + 14*a*d*e^3)*x)*sqrt(e*x + d)/e^2
```

Sympy [A] (verification not implemented)

Time = 0.84 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.23

$$\int \sqrt{d+ex}\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)\,dx=\begin{cases}\frac{2\left(\frac{cd(d+ex)^{\frac{7}{2}}}{7e}+\frac{(d+ex)^{\frac{5}{2}}\left(ae^2-cd^2\right)}{5e}\right)}{e} & \text{for } e\neq 0\\ \frac{cd^{\frac{5}{2}}x^2}{2} & \text{otherwise}\end{cases}$$

input
```
integrate((e*x+d)**(1/2)*(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2),x)
```

output
```
Piecewise((2*(c*d*(d + e*x)**(7/2)/(7*e) + (d + e*x)**(5/2)*(a*e**2 - c*d*
*2)/(5*e))/e, Ne(e, 0)), (c*d**(5/2)*x**2/2, True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.88

$$\int \sqrt{d+ex}\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)\,dx=\frac{2\left(5\,(ex+d)^{\frac{7}{2}}cd-7\,(cd^2-ae^2)(ex+d)^{\frac{5}{2}}\right)}{35\,e^2}$$

input
```
integrate((e*x+d)^(1/2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2),x, algorithm="ma
xima")
```

output
```
2/35*(5*(e*x + d)^(7/2)*c*d - 7*(c*d^2 - a*e^2)*(e*x + d)^(5/2))/e^2
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 198 vs. $2(35) = 70$.

Time = 0.16 (sec) , antiderivative size = 198, normalized size of antiderivative = 4.60

$$\int \sqrt{d+ex}\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)\,dx$$
$$= \frac{2\left(105\sqrt{ex+d}ad^2e+\frac{35\left((ex+d)^{\frac{3}{2}}-3\sqrt{ex+d}d\right)cd^3}{e}+70\left((ex+d)^{\frac{3}{2}}-3\sqrt{ex+d}d\right)ade+\frac{14\left(3(ex+d)^{\frac{5}{2}}-10(ex+d\right.}{e}}{}$$

input
```
integrate((e*x+d)^(1/2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2),x, algorithm="gi
ac")
```

output
```
2/105*(105*sqrt(e*x + d)*a*d^2*e + 35*((e*x + d)^(3/2) - 3*sqrt(e*x + d)*d
)*c*d^3/e + 70*((e*x + d)^(3/2) - 3*sqrt(e*x + d)*d)*a*d*e + 14*(3*(e*x +
d)^(5/2) - 10*(e*x + d)^(3/2)*d + 15*sqrt(e*x + d)*d^2)*c*d^2/e + 7*(3*(e*
x + d)^(5/2) - 10*(e*x + d)^(3/2)*d + 15*sqrt(e*x + d)*d^2)*a*e + 3*(5*(e*
x + d)^(7/2) - 21*(e*x + d)^(5/2)*d + 35*(e*x + d)^(3/2)*d^2 - 35*sqrt(e*x
 + d)*d^3)*c*d/e)/e
```

Mupad [B] (verification not implemented)

Time = 5.19 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.79

$$\int \sqrt{d+ex}\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)\,dx$$
$$= \frac{2\,{\left(d+e\,x\right)}^{5/2}\,\left(7\,a\,e^2-7\,c\,d^2+5\,c\,d\,\left(d+e\,x\right)\right)}{35\,e^2}$$

input
```
int((d + e*x)^(1/2)*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2),x)
```

output
```
(2*(d + e*x)^(5/2)*(7*a*e^2 - 7*c*d^2 + 5*c*d*(d + e*x)))/(35*e^2)
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.70

$$\int \sqrt{d+ex}\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)\,dx$$
$$=\frac{2\sqrt{ex+d}\,(5cd\,e^3x^3+7a\,e^4x^2+8c\,d^2e^2x^2+14ad\,e^3x+c\,d^3ex+7a\,d^2e^2-2c\,d^4)}{35e^2}$$

input
```
int((e*x+d)^(1/2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2),x)
```

output
```
(2*sqrt(d + e*x)*(7*a*d**2*e**2 + 14*a*d*e**3*x + 7*a*e**4*x**2 - 2*c*d**4
 + c*d**3*e*x + 8*c*d**2*e**2*x**2 + 5*c*d*e**3*x**3))/(35*e**2)
```

3.151 $\int \frac{ade+(cd^2+ae^2)x+cdex^2}{\sqrt{d+ex}}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 43

$$\int \frac{ade+(cd^2+ae^2)\,x+cdex^2}{\sqrt{d+ex}}\,dx=\frac{2}{3}\left(a-\frac{cd^2}{e^2}\right)(d+ex)^{3/2}+\frac{2cd(d+ex)^{5/2}}{5e^2}$$

output
```
2/3*(a-c*d^2/e^2)*(e*x+d)^(3/2)+2/5*c*d*(e*x+d)^(5/2)/e^2
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.79

$$\int \frac{ade+(cd^2+ae^2)\,x+cdex^2}{\sqrt{d+ex}}\,dx=\frac{2(d+ex)^{3/2}\,(5ae^2+cd(-2d+3ex))}{15e^2}$$

input
```
Integrate[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)/Sqrt[d + e*x],x]
```

output
```
(2*(d + e*x)^(3/2)*(5*a*e^2 + c*d*(-2*d + 3*e*x)))/(15*e^2)
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.057$, Rules used = {1121, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x(ae^2+cd^2)+ade+cdex^2}{\sqrt{d+ex}}dx$$

$$\Big\downarrow \text{1121}$$

$$\int \left(\frac{\sqrt{d+ex}(ae^2-cd^2)}{e}+\frac{cd(d+ex)^{3/2}}{e}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{2}{3}(d+ex)^{3/2}\left(a-\frac{cd^2}{e^2}\right)+\frac{2cd(d+ex)^{5/2}}{5e^2}$$

input
```
Int[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)/Sqrt[d + e*x],x]
```

output
```
(2*(a - (c*d^2)/e^2)*(d + e*x)^(3/2))/3 + (2*c*d*(d + e*x)^(5/2))/(5*e^2)
```

Defintions of rubi rules used

rule 1121
```
Int[((d_) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Int[ExpandIntegrand[(d + e*x)^(m + p)*(a/d + (c/e)*x)^p, x], x]
/; FreeQ[{a, b, c, d, e, m, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && (Int
egerQ[p] || (GtQ[a, 0] && GtQ[d, 0] && LtQ[c, 0]))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 1.45 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.74

method	result	size
gosper	$\frac{2(ex+d)^{\frac{3}{2}}\left(3cdxe+5a\,e^{2}-2c\,d^{2}\right)}{15e^{2}}$	32
pseudoelliptic	$\frac{2(ex+d)^{\frac{3}{2}}\left(3cdxe+5a\,e^{2}-2c\,d^{2}\right)}{15e^{2}}$	32
derivativedivides	$\frac{\frac{2cd(ex+d)^{\frac{5}{2}}}{5}+\frac{2\left(a\,e^{2}-c\,d^{2}\right)(ex+d)^{\frac{3}{2}}}{3}}{e^{2}}$	39
default	$\frac{\frac{2cd(ex+d)^{\frac{5}{2}}}{5}+\frac{2\left(a\,e^{2}-c\,d^{2}\right)(ex+d)^{\frac{3}{2}}}{3}}{e^{2}}$	39
trager	$\frac{2\left(3cd\,e^{2}x^{2}+5a\,e^{3}x+c\,d^{2}ex+5ad\,e^{2}-2c\,d^{3}\right)\sqrt{ex+d}}{15e^{2}}$	51
risch	$\frac{2\left(3cd\,e^{2}x^{2}+5a\,e^{3}x+c\,d^{2}ex+5ad\,e^{2}-2c\,d^{3}\right)\sqrt{ex+d}}{15e^{2}}$	51
orering	$\frac{2\left(3cdxe+5a\,e^{2}-2c\,d^{2}\right)\sqrt{ex+d}\,\left(ade+\left(a\,e^{2}+c\,d^{2}\right)x+cd\,x^{2}e\right)}{15e^{2}(cdx+ae)}$	67

input
```
int((a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)/(e*x+d)^(1/2),x,method=_RETURNVERBOS
E)
```

output
```
2/15*(e*x+d)^(3/2)*(3*c*d*e*x+5*a*e^2-2*c*d^2)/e^2
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.19

$$\int \frac{ade+(cd^2+ae^2)\,x+cdex^2}{\sqrt{d+ex}}\,dx = \frac{2\,(3\,cde^2x^2-2\,cd^3+5\,ade^2+(cd^2e+5\,ae^3)x)\sqrt{ex+d}}{15\,e^2}$$

input
```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)/(e*x+d)^(1/2),x, algorithm="fr
icas")
```

output
```
2/15*(3*c*d*e^2*x^2 - 2*c*d^3 + 5*a*d*e^2 + (c*d^2*e + 5*a*e^3)*x)*sqrt(e*
x + d)/e^2
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 104 vs. $2(42) = 84$.

Time = 0.63 (sec) , antiderivative size = 104, normalized size of antiderivative = 2.42

$$\int \frac{ade + (cd^2 + ae^2)\,x + cdex^2}{\sqrt{d+ex}}\,dx = \begin{cases} \frac{2ade\sqrt{d+ex} + \frac{2cd\left(d^2\sqrt{d+ex} - \frac{2d(d+ex)^{\frac{3}{2}}}{3} + \frac{(d+ex)^{\frac{5}{2}}}{5}\right)}{e} + \frac{2\left(ae^2+cd^2\right)\left(-d\sqrt{d+ex} + \frac{(d+ex)^{\frac{3}{2}}}{3}\right)}{e}}{e} & \text{for } e \neq 0 \\ \frac{cd^{\frac{3}{2}}x^2}{2} & \text{otherwise} \end{cases}$$

input
```
integrate((a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)/(e*x+d)**(1/2),x)
```

output
```
Piecewise(((2*a*d*e*sqrt(d + e*x) + 2*c*d*(d**2*sqrt(d + e*x) - 2*d*(d + e
*x)**(3/2)/3 + (d + e*x)**(5/2)/5)/e + 2*(a*e**2 + c*d**2)*(-d*sqrt(d + e*
x) + (d + e*x)**(3/2)/3)/e)/e, Ne(e, 0)), (c*d**(3/2)*x**2/2, True))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 90 vs. $2(35) = 70$.

Time = 0.03 (sec) , antiderivative size = 90, normalized size of antiderivative = 2.09

$$\int \frac{ade + (cd^2 + ae^2)\,x + cdex^2}{\sqrt{d+ex}}\,dx = \frac{2\left(15\sqrt{ex+d}ade + \frac{\left(3\,(ex+d)^{\frac{5}{2}} - 10\,(ex+d)^{\frac{3}{2}}d + 15\,\sqrt{ex+d}d^2\right)cd}{e} + \frac{5\,(cd^2+ae^2)\left((ex+d)^{\frac{3}{2}} - 3\,\sqrt{ex+d}d\right)}{e}\right)}{15\,e}$$

input
```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)/(e*x+d)^(1/2),x, algorithm="ma
xima")
```

output
```
2/15*(15*sqrt(e*x + d)*a*d*e + (3*(e*x + d)^(5/2) - 10*(e*x + d)^(3/2)*d +
 15*sqrt(e*x + d)*d^2)*c*d/e + 5*(c*d^2 + a*e^2)*((e*x + d)^(3/2) - 3*sqrt
(e*x + d)*d)/e)/e
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 105 vs. $2(35) = 70$.

Time = 0.12 (sec) , antiderivative size = 105, normalized size of antiderivative = 2.44

$$\int \frac{ade + (cd^2 + ae^2)x + cdex^2}{\sqrt{d+ex}}\,dx = \frac{2\left(15\sqrt{ex+d}ade + \frac{5\left((ex+d)^{\frac{3}{2}} - 3\sqrt{ex+d}d\right)cd^2}{e} + 5\left((ex+d)^{\frac{3}{2}} - 3\sqrt{ex+d}d\right)ae + \frac{\left(3(ex+d)^{\frac{5}{2}} - 10(ex+d)^{\frac{3}{2}}d + 15\sqrt{}\right.}{e}\right.}{15e}$$

input
```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)/(e*x+d)^(1/2),x, algorithm="gi
ac")
```

output
```
2/15*(15*sqrt(e*x + d)*a*d*e + 5*((e*x + d)^(3/2) - 3*sqrt(e*x + d)*d)*c*d
^2/e + 5*((e*x + d)^(3/2) - 3*sqrt(e*x + d)*d)*a*e + (3*(e*x + d)^(5/2) -
10*(e*x + d)^(3/2)*d + 15*sqrt(e*x + d)*d^2)*c*d/e)/e
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.79

$$\int \frac{ade + (cd^2 + ae^2)x + cdex^2}{\sqrt{d+ex}}\,dx = \frac{2\,(d+e\,x)^{3/2}\,(5\,a\,e^2 - 5\,c\,d^2 + 3\,c\,d\,(d+e\,x))}{15\,e^2}$$

input
```
int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)/(d + e*x)^(1/2),x)
```

output
```
(2*(d + e*x)^(3/2)*(5*a*e^2 - 5*c*d^2 + 3*c*d*(d + e*x)))/(15*e^2)
```

Reduce [B] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.14

$$\int \frac{ade + (cd^2 + ae^2)\,x + cdex^2}{\sqrt{d + ex}}\,dx$$
$$= \frac{2\sqrt{ex + d}\,(3cd\,e^2x^2 + 5a\,e^3x + c\,d^2ex + 5ad\,e^2 - 2c\,d^3)}{15e^2}$$

input
```
int((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)/(e*x+d)^(1/2),x)
```

output
```
(2*sqrt(d + e*x)*(5*a*d*e**2 + 5*a*e**3*x - 2*c*d**3 + c*d**2*e*x + 3*c*d*
e**2*x**2))/(15*e**2)
```

3.152 $\int \frac{ade+(cd^2+ae^2)x+cdex^2}{(d+ex)^{3/2}} dx$

Optimal result

Integrand size = 35, antiderivative size = 41

$$\int \frac{ade+(cd^2+ae^2)x+cdex^2}{(d+ex)^{3/2}} dx = 2\left(a-\frac{cd^2}{e^2}\right)\sqrt{d+ex}+\frac{2cd(d+ex)^{3/2}}{3e^2}$$

output
```
2*(a-c*d^2/e^2)*(e*x+d)^(1/2)+2/3*c*d*(e*x+d)^(3/2)/e^2
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.80

$$\int \frac{ade+(cd^2+ae^2)x+cdex^2}{(d+ex)^{3/2}} dx = \frac{2\sqrt{d+ex}(3ae^2+cd(-2d+ex))}{3e^2}$$

input
```
Integrate[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)/(d + e*x)^(3/2),x]
```

output
```
(2*Sqrt[d + e*x]*(3*a*e^2 + c*d*(-2*d + e*x)))/(3*e^2)
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.057$, Rules used = {1121, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x(ae^2+cd^2)+ade+cdex^2}{(d+ex)^{3/2}}\,dx$$

$$\Big\downarrow \ 1121$$

$$\int \left(\frac{ae^2-cd^2}{e\sqrt{d+ex}}+\frac{cd\sqrt{d+ex}}{e}\right)dx$$

$$\Big\downarrow \ 2009$$

$$2\sqrt{d+ex}\left(a-\frac{cd^2}{e^2}\right)+\frac{2cd(d+ex)^{3/2}}{3e^2}$$

input
```
Int[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)/(d + e*x)^(3/2),x]
```

output
```
2*(a - (c*d^2)/e^2)*Sqrt[d + e*x] + (2*c*d*(d + e*x)^(3/2))/(3*e^2)
```

Defintions of rubi rules used

rule 1121
```
Int[((d_) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Int[ExpandIntegrand[(d + e*x)^(m + p)*(a/d + (c/e)*x)^p, x], x]
/; FreeQ[{a, b, c, d, e, m, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && (Int
egerQ[p] || (GtQ[a, 0] && GtQ[d, 0] && LtQ[c, 0]))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.71 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.76

method	result	size
gosper	$\frac{2\sqrt{ex+d}\,(cdxe+3a\,e^{2}-2c\,d^{2})}{3e^{2}}$	31
trager	$\frac{2\sqrt{ex+d}\,(cdxe+3a\,e^{2}-2c\,d^{2})}{3e^{2}}$	31
risch	$\frac{2\sqrt{ex+d}\,(cdxe+3a\,e^{2}-2c\,d^{2})}{3e^{2}}$	31
pseudoelliptic	$\frac{2\sqrt{ex+d}\,(cdxe+3a\,e^{2}-2c\,d^{2})}{3e^{2}}$	31
derivativedivides	$\frac{\frac{2cd(ex+d)^{\frac{3}{2}}}{3}+2a\,e^{2}\sqrt{ex+d}-2c\,d^{2}\sqrt{ex+d}}{e^{2}}$	43
default	$\frac{\frac{2cd(ex+d)^{\frac{3}{2}}}{3}+2a\,e^{2}\sqrt{ex+d}-2c\,d^{2}\sqrt{ex+d}}{e^{2}}$	43
orering	$\frac{2(cdxe+3a\,e^{2}-2c\,d^{2})\,(ade+(a\,e^{2}+c\,d^{2})x+cd\,x^{2}e)}{3e^{2}\sqrt{ex+d}\,(cdx+ae)}$	66

input
```
int((a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)/(e*x+d)^(3/2),x,method=_RETURNVERBOS
E)
```

output
```
2/3*(e*x+d)^(1/2)*(c*d*e*x+3*a*e^2-2*c*d^2)/e^2
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.73

$$\int \frac{ade+(cd^2+ae^2)\,x+cdex^2}{(d+ex)^{3/2}}\,dx=\frac{2\,(cdex-2\,cd^2+3\,ae^2)\sqrt{ex+d}}{3\,e^2}$$

input
```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)/(e*x+d)^(3/2),x, algorithm="fr
icas")
```

output
```
2/3*(c*d*e*x - 2*c*d^2 + 3*a*e^2)*sqrt(e*x + d)/e^2
```

Sympy [A] (verification not implemented)

Time = 0.77 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.24

$$\int \frac{ade+(cd^2+ae^2)\,x+cdex^2}{(d+ex)^{3/2}}\,dx = \begin{cases} \frac{2\left(\frac{cd(d+ex)^{\frac{3}{2}}}{3e}+\frac{\sqrt{d+ex}\,(ae^2-cd^2)}{e}\right)}{e} & \text{for } e\neq 0 \\ \frac{c\sqrt{d}x^2}{2} & \text{otherwise} \end{cases}$$

input
```
integrate((a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)/(e*x+d)**(3/2),x)
```

output
```
Piecewise((2*(c*d*(d + e*x)**(3/2)/(3*e) + sqrt(d + e*x)*(a*e**2 - c*d**2)
/e)/e, Ne(e, 0)), (c*sqrt(d)*x**2/2, True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.90

$$\int \frac{ade+(cd^2+ae^2)\,x+cdex^2}{(d+ex)^{3/2}}\,dx = \frac{2\left((ex+d)^{\frac{3}{2}}cd-3\,(cd^2-ae^2)\sqrt{ex+d}\right)}{3\,e^2}$$

input
```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)/(e*x+d)^(3/2),x, algorithm="ma
xima")
```

output
```
2/3*((e*x + d)^(3/2)*c*d - 3*(c*d^2 - a*e^2)*sqrt(e*x + d))/e^2
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.00

$$\int \frac{ade+(cd^2+ae^2)\,x+cdex^2}{(d+ex)^{3/2}}\,dx = \frac{2\left(3\sqrt{ex+d}ae+\frac{\left((ex+d)^{\frac{3}{2}}-3\sqrt{ex+d}d\right)cd}{e}\right)}{3\,e}$$

input
```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)/(e*x+d)^(3/2),x, algorithm="gi
ac")
```

output
```
2/3*(3*sqrt(e*x + d)*a*e + ((e*x + d)^(3/2) - 3*sqrt(e*x + d)*d)*c*d/e)/e
```

Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.80

$$\int \frac{ade + (cd^2 + ae^2)\,x + cdex^2}{(d + ex)^{3/2}}\,dx = \frac{2\,\sqrt{d + e\,x}\,(3\,a\,e^2 - 3\,c\,d^2 + c\,d\,(d + e\,x))}{3\,e^2}$$

input
```
int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)/(d + e*x)^(3/2),x)
```

output
```
(2*(d + e*x)^(1/2)*(3*a*e^2 - 3*c*d^2 + c*d*(d + e*x)))/(3*e^2)
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.71

$$\int \frac{ade + (cd^2 + ae^2)\,x + cdex^2}{(d + ex)^{3/2}}\,dx = \frac{2\sqrt{ex + d}\,(cdex + 3a\,e^2 - 2c\,d^2)}{3e^2}$$

input
```
int((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)/(e*x+d)^(3/2),x)
```

output
```
(2*sqrt(d + e*x)*(3*a*e**2 - 2*c*d**2 + c*d*e*x))/(3*e**2)
```

3.153 $\int \frac{ade+(cd^2+ae^2)x+cdex^2}{(d+ex)^{5/2}}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 39

$$\int \frac{ade+(cd^2+ae^2)\,x+cdex^2}{(d+ex)^{5/2}}\,dx = -\frac{2\left(a-\frac{cd^2}{e^2}\right)}{\sqrt{d+ex}}+\frac{2cd\sqrt{d+ex}}{e^2}$$

output
```
(-2*a+2*c*d^2/e^2)/(e*x+d)^(1/2)+2*c*d*(e*x+d)^(1/2)/e^2
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.79

$$\int \frac{ade+(cd^2+ae^2)\,x+cdex^2}{(d+ex)^{5/2}}\,dx = \frac{-2ae^2+2cd(2d+ex)}{e^2\sqrt{d+ex}}$$

input
```
Integrate[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)/(d + e*x)^(5/2),x]
```

output
```
(-2*a*e^2 + 2*c*d*(2*d + e*x))/(e^2*Sqrt[d + e*x])
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.057, Rules used = {1121, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x(ae^2+cd^2)+ade+cdex^2}{(d+ex)^{5/2}}\,dx$$

$$\Big\downarrow \text{1121}$$

$$\int \left(\frac{ae^2-cd^2}{e(d+ex)^{3/2}}+\frac{cd}{e\sqrt{d+ex}}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{2cd\sqrt{d+ex}}{e^2}-\frac{2\left(a-\frac{cd^2}{e^2}\right)}{\sqrt{d+ex}}$$

input
```
Int[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)/(d + e*x)^(5/2),x]
```

output
```
(-2*(a - (c*d^2)/e^2))/Sqrt[d + e*x] + (2*c*d*Sqrt[d + e*x])/e^2
```

Defintions of rubi rules used

rule 1121
```
Int[((d_) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Int[ExpandIntegrand[(d + e*x)^(m + p)*(a/d + (c/e)*x)^p, x], x]
/; FreeQ[{a, b, c, d, e, m, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && (Int
egerQ[p] || (GtQ[a, 0] && GtQ[d, 0] && LtQ[c, 0]))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.70 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.79

method	result	size
gosper	$-\frac{2(-cdxe+a\,e^2-2c\,d^2)}{\sqrt{ex+d}\,e^2}$	31
trager	$-\frac{2(-cdxe+a\,e^2-2c\,d^2)}{\sqrt{ex+d}\,e^2}$	31
pseudoelliptic	$-\frac{2(-cdxe+a\,e^2-2c\,d^2)}{\sqrt{ex+d}\,e^2}$	31
derivativedivides	$\frac{2cd\sqrt{ex+d}-\frac{2\left(a\,e^2-c\,d^2\right)}{\sqrt{ex+d}}}{e^2}$	38
default	$\frac{2cd\sqrt{ex+d}-\frac{2\left(a\,e^2-c\,d^2\right)}{\sqrt{ex+d}}}{e^2}$	38
risch	$\frac{2cd\sqrt{ex+d}}{e^2}-\frac{2(a\,e^2-c\,d^2)}{e^2\sqrt{ex+d}}$	40
orering	$-\frac{2(-cdxe+a\,e^2-2c\,d^2)\,(ade+(a\,e^2+c\,d^2)x+cd\,x^2e)}{e^2(ex+d)^{\frac{3}{2}}\,(cdx+ae)}$	66

input
```
int((a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)/(e*x+d)^(5/2),x,method=_RETURNVERBOS
E)
```

output
```
-2/(e*x+d)^(1/2)*(-c*d*e*x+a*e^2-2*c*d^2)/e^2
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.03

$$\int \frac{ade+(cd^2+ae^2)\,x+cdex^2}{(d+ex)^{5/2}}\,dx=\frac{2\,(cdex+2\,cd^2-ae^2)\sqrt{ex+d}}{e^3x+de^2}$$

input
```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)/(e*x+d)^(5/2),x, algorithm="fr
icas")
```

output
```
2*(c*d*e*x + 2*c*d^2 - a*e^2)*sqrt(e*x + d)/(e^3*x + d*e^2)
```

Sympy [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.49

$$\int \frac{ade + (cd^2 + ae^2)\,x + cdex^2}{(d + ex)^{5/2}}\,dx = \begin{cases} -\frac{2a}{\sqrt{d+ex}} + \frac{4cd^2}{e^2\sqrt{d+ex}} + \frac{2cdx}{e\sqrt{d+ex}} & \text{for } e \neq 0 \\ \frac{cx^2}{2\sqrt{d}} & \text{otherwise} \end{cases}$$

input
```
integrate((a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)/(e*x+d)**(5/2),x)
```

output
```
Piecewise((-2*a/sqrt(d + e*x) + 4*c*d**2/(e**2*sqrt(d + e*x)) + 2*c*d*x/(e
*sqrt(d + e*x)), Ne(e, 0)), (c*x**2/(2*sqrt(d)), True))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.08

$$\int \frac{ade + (cd^2 + ae^2)\,x + cdex^2}{(d + ex)^{5/2}}\,dx = \frac{2\left(\frac{\sqrt{ex+d}cd}{e} + \frac{cd^2-ae^2}{\sqrt{ex+d}e}\right)}{e}$$

input
```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)/(e*x+d)^(5/2),x, algorithm="ma
xima")
```

output
```
2*(sqrt(e*x + d)*c*d/e + (c*d^2 - a*e^2)/(sqrt(e*x + d)*e))/e
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.00

$$\int \frac{ade + (cd^2 + ae^2)\,x + cdex^2}{(d + ex)^{5/2}}\,dx = \frac{2\sqrt{ex+d}cd}{e^2} + \frac{2\,(cd^2 - ae^2)}{\sqrt{ex+d}e^2}$$

input
```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)/(e*x+d)^(5/2),x, algorithm="gi
ac")
```

output
```
2*sqrt(e*x + d)*c*d/e^2 + 2*(c*d^2 - a*e^2)/(sqrt(e*x + d)*e^2)
```

Mupad [B] (verification not implemented)

Time = 5.23 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.77

$$\int \frac{ade + (cd^2 + ae^2) x + cdex^2}{(d + ex)^{5/2}} \, dx = \frac{4 \, c \, d^2 + 2 \, c \, x \, d \, e - 2 \, a \, e^2}{e^2 \, \sqrt{d + e \, x}}$$

input
```
int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)/(d + e*x)^(5/2),x)
```

output
```
(4*c*d^2 - 2*a*e^2 + 2*c*d*e*x)/(e^2*(d + e*x)^(1/2))
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.79

$$\int \frac{ade + (cd^2 + ae^2) x + cdex^2}{(d + ex)^{5/2}} \, dx = \frac{2cdex - 2a \, e^2 + 4c \, d^2}{\sqrt{ex + d} \, e^2}$$

input
```
int((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)/(e*x+d)^(5/2),x)
```

output
```
(2*( - a*e**2 + 2*c*d**2 + c*d*e*x))/(sqrt(d + e*x)*e**2)
```

3.154 $\int \frac{ade+(cd^2+ae^2)x+cdex^2}{(d+ex)^{7/2}}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 41

$$\int \frac{ade+(cd^2+ae^2)\,x+cdex^2}{(d+ex)^{7/2}}\,dx=-\frac{2\left(a-\frac{cd^2}{e^2}\right)}{3(d+ex)^{3/2}}-\frac{2cd}{e^2\sqrt{d+ex}}$$

output
```
1/3*(-2*a+2*c*d^2/e^2)/(e*x+d)^(3/2)-2*c*d/e^2/(e*x+d)^(1/2)
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.80

$$\int \frac{ade+(cd^2+ae^2)\,x+cdex^2}{(d+ex)^{7/2}}\,dx=-\frac{2(ae^2+cd(2d+3ex))}{3e^2(d+ex)^{3/2}}$$

input
```
Integrate[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)/(d + e*x)^(7/2),x]
```

output
```
(-2*(a*e^2 + c*d*(2*d + 3*e*x)))/(3*e^2*(d + e*x)^(3/2))
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.057$, Rules used = {1121, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x(ae^2+cd^2)+ade+cdex^2}{(d+ex)^{7/2}}dx$$

$$\Big\downarrow \text{1121}$$

$$\int \left(\frac{ae^2-cd^2}{e(d+ex)^{5/2}}+\frac{cd}{e(d+ex)^{3/2}}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{2\left(a-\frac{cd^2}{e^2}\right)}{3(d+ex)^{3/2}}-\frac{2cd}{e^2\sqrt{d+ex}}$$

input
```
Int[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)/(d + e*x)^(7/2),x]
```

output
```
(-2*(a - (c*d^2)/e^2))/(3*(d + e*x)^(3/2)) - (2*c*d)/(e^2*Sqrt[d + e*x])
```

Defintions of rubi rules used

rule 1121
```
Int[((d_) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Int[ExpandIntegrand[(d + e*x)^(m + p)*(a/d + (c/e)*x)^p, x], x]
/; FreeQ[{a, b, c, d, e, m, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && (Int
egerQ[p] || (GtQ[a, 0] && GtQ[d, 0] && LtQ[c, 0]))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.68 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.76

method	result	size
gosper	$-\frac{2\left(3cdxe+a\,e^{2}+2c\,d^{2}\right)}{3\left(ex+d\right)^{\frac{3}{2}}e^{2}}$	31
trager	$-\frac{2\left(3cdxe+a\,e^{2}+2c\,d^{2}\right)}{3\left(ex+d\right)^{\frac{3}{2}}e^{2}}$	31
pseudoelliptic	$-\frac{2\left(3cdxe+a\,e^{2}+2c\,d^{2}\right)}{3\left(ex+d\right)^{\frac{3}{2}}e^{2}}$	31
derivativedivides	$\frac{-\frac{2cd}{\sqrt{ex+d}}-\frac{2\left(a\,e^{2}-c\,d^{2}\right)}{3\left(ex+d\right)^{\frac{3}{2}}}}{e^{2}}$	39
default	$\frac{-\frac{2cd}{\sqrt{ex+d}}-\frac{2\left(a\,e^{2}-c\,d^{2}\right)}{3\left(ex+d\right)^{\frac{3}{2}}}}{e^{2}}$	39
orering	$-\frac{2\left(3cdxe+a\,e^{2}+2c\,d^{2}\right)\left(ade+\left(a\,e^{2}+c\,d^{2}\right)x+cd\,x^{2}e\right)}{3e^{2}\left(ex+d\right)^{\frac{5}{2}}\left(cdx+ae\right)}$	66

input
```
int((a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)/(e*x+d)^(7/2),x,method=_RETURNVERBOS
E)
```

output
```
-2/3/(e*x+d)^(3/2)*(3*c*d*e*x+a*e^2+2*c*d^2)/e^2
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.24

$$\int \frac{ade+(cd^2+ae^2)\,x+cdex^2}{(d+ex)^{7/2}}\,dx=-\frac{2\,(3\,cdex+2\,cd^2+ae^2)\sqrt{ex+d}}{3\,(e^4x^2+2\,de^3x+d^2e^2)}$$

input
```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)/(e*x+d)^(7/2),x, algorithm="fr
icas")
```

output
```
-2/3*(3*c*d*e*x + 2*c*d^2 + a*e^2)*sqrt(e*x + d)/(e^4*x^2 + 2*d*e^3*x + d^
2*e^2)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 126 vs. $2(41) = 82$.

Time = 0.43 (sec) , antiderivative size = 126, normalized size of antiderivative = 3.07

$$\int \frac{ade + (cd^2 + ae^2)\,x + cdex^2}{(d + ex)^{7/2}}\,dx = \begin{cases} -\frac{2ae^2}{3de^2\sqrt{d+ex}+3e^3x\sqrt{d+ex}} - \frac{4cd^2}{3de^2\sqrt{d+ex}+3e^3x\sqrt{d+ex}} - \frac{6cdex}{3de^2\sqrt{d+ex}+3e^3x\sqrt{d+ex}} \\ \frac{cx^2}{2d^{\frac{3}{2}}} \end{cases}$$

input
```
integrate((a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)/(e*x+d)**(7/2),x)
```

output
```
Piecewise((-2*a*e**2/(3*d*e**2*sqrt(d + e*x) + 3*e**3*x*sqrt(d + e*x)) - 4
*c*d**2/(3*d*e**2*sqrt(d + e*x) + 3*e**3*x*sqrt(d + e*x)) - 6*c*d*e*x/(3*d
*e**2*sqrt(d + e*x) + 3*e**3*x*sqrt(d + e*x)), Ne(e, 0)), (c*x**2/(2*d**(3
/2)), True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.80

$$\int \frac{ade + (cd^2 + ae^2)\,x + cdex^2}{(d + ex)^{7/2}}\,dx = -\frac{2\,(3\,(ex + d)cd - cd^2 + ae^2)}{3\,(ex + d)^{\frac{3}{2}}e^2}$$

input
```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)/(e*x+d)^(7/2),x, algorithm="ma
xima")
```

output
```
-2/3*(3*(e*x + d)*c*d - c*d^2 + a*e^2)/((e*x + d)^(3/2)*e^2)
```

Giac [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.80

$$\int \frac{ade+(cd^2+ae^2)\,x+cdex^2}{(d+ex)^{7/2}}\,dx=-\frac{2\,(3\,(ex+d)cd-cd^2+ae^2)}{3\,(ex+d)^{\frac{3}{2}}e^2}$$

input
```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)/(e*x+d)^(7/2),x, algorithm="gi
ac")
```

output
```
-2/3*(3*(e*x + d)*c*d - c*d^2 + a*e^2)/((e*x + d)^(3/2)*e^2)
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.83

$$\int \frac{ade+(cd^2+ae^2)\,x+cdex^2}{(d+ex)^{7/2}}\,dx=-\frac{2\,a\,e^2-2\,c\,d^2+6\,c\,d\,(d+e\,x)}{3\,e^2\,(d+e\,x)^{3/2}}$$

input
```
int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)/(d + e*x)^(7/2),x)
```

output
```
-(2*a*e^2 - 2*c*d^2 + 6*c*d*(d + e*x))/(3*e^2*(d + e*x)^(3/2))
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.95

$$\int \frac{ade+(cd^2+ae^2)\,x+cdex^2}{(d+ex)^{7/2}}\,dx=\frac{-2cdex-\frac{2}{3}a\,e^2-\frac{4}{3}c\,d^2}{\sqrt{ex+d}\,e^2\,(ex+d)}$$

input
```
int((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)/(e*x+d)^(7/2),x)
```

output
```
(2*( - a*e**2 - 2*c*d**2 - 3*c*d*e*x))/(3*sqrt(d + e*x)*e**2*(d + e*x))
```

3.155 $\int \frac{ade+(cd^2+ae^2)x+cdex^2}{(d+ex)^{9/2}}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 43

$$\int \frac{ade+(cd^2+ae^2)\,x+cdex^2}{(d+ex)^{9/2}}\,dx=-\frac{2\left(a-\frac{cd^2}{e^2}\right)}{5(d+ex)^{5/2}}-\frac{2cd}{3e^2(d+ex)^{3/2}}$$

output
```
1/5*(-2*a+2*c*d^2/e^2)/(e*x+d)^(5/2)-2/3*c*d/e^2/(e*x+d)^(3/2)
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.79

$$\int \frac{ade+(cd^2+ae^2)\,x+cdex^2}{(d+ex)^{9/2}}\,dx=-\frac{2(3ae^2+cd(2d+5ex))}{15e^2(d+ex)^{5/2}}$$

input
```
Integrate[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)/(d + e*x)^(9/2),x]
```

output
```
(-2*(3*a*e^2 + c*d*(2*d + 5*e*x)))/(15*e^2*(d + e*x)^(5/2))
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.057, Rules used = {1121, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x(ae^2+cd^2)+ade+cdex^2}{(d+ex)^{9/2}}\,dx$$

$$\Big\downarrow \; 1121$$

$$\int \left(\frac{ae^2-cd^2}{e(d+ex)^{7/2}}+\frac{cd}{e(d+ex)^{5/2}}\right)dx$$

$$\Big\downarrow \; 2009$$

$$-\frac{2\left(a-\frac{cd^2}{e^2}\right)}{5(d+ex)^{5/2}}-\frac{2cd}{3e^2(d+ex)^{3/2}}$$

input
```
Int[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)/(d + e*x)^(9/2),x]
```

output
```
(-2*(a - (c*d^2)/e^2))/(5*(d + e*x)^(5/2)) - (2*c*d)/(3*e^2*(d + e*x)^(3/2
))
```

Defintions of rubi rules used

rule 1121
```
Int[((d_) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Int[ExpandIntegrand[(d + e*x)^(m + p)*(a/d + (c/e)*x)^p, x], x]
/; FreeQ[{a, b, c, d, e, m, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && (Int
egerQ[p] || (GtQ[a, 0] && GtQ[d, 0] && LtQ[c, 0]))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.81 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.74

method	result	size
gosper	$-\frac{2\left(5cdxe+3a\,e^{2}+2c\,d^{2}\right)}{15\left(ex+d\right)^{\frac{5}{2}}e^{2}}$	32
trager	$-\frac{2\left(5cdxe+3a\,e^{2}+2c\,d^{2}\right)}{15\left(ex+d\right)^{\frac{5}{2}}e^{2}}$	32
pseudoelliptic	$\frac{-\frac{2}{3}cdxe-\frac{2}{5}a\,e^{2}-\frac{4}{15}c\,d^{2}}{\left(ex+d\right)^{\frac{5}{2}}e^{2}}$	32
derivativedivides	$\frac{-\frac{2\left(a\,e^{2}-c\,d^{2}\right)}{5\left(ex+d\right)^{\frac{5}{2}}}-\frac{2cd}{3\left(ex+d\right)^{\frac{3}{2}}}}{e^{2}}$	39
default	$\frac{-\frac{2\left(a\,e^{2}-c\,d^{2}\right)}{5\left(ex+d\right)^{\frac{5}{2}}}-\frac{2cd}{3\left(ex+d\right)^{\frac{3}{2}}}}{e^{2}}$	39
orering	$-\frac{2\left(5cdxe+3a\,e^{2}+2c\,d^{2}\right)\left(ade+\left(a\,e^{2}+c\,d^{2}\right)x+cd\,x^{2}e\right)}{15e^{2}\left(ex+d\right)^{\frac{7}{2}}\left(cdx+ae\right)}$	67

input
```
int((a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)/(e*x+d)^(9/2),x,method=_RETURNVERBOS
E)
```

output
```
-2/15/(e*x+d)^(5/2)*(5*c*d*e*x+3*a*e^2+2*c*d^2)/e^2
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.47

$$\int \frac{ade+(cd^2+ae^2)\,x+cdex^2}{(d+ex)^{9/2}}\,dx=-\frac{2\,(5\,cdex+2\,cd^2+3\,ae^2)\sqrt{ex+d}}{15\,(e^5x^3+3\,de^4x^2+3\,d^2e^3x+d^3e^2)}$$

input
```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)/(e*x+d)^(9/2),x, algorithm="fr
icas")
```

output
```
-2/15*(5*c*d*e*x + 2*c*d^2 + 3*a*e^2)*sqrt(e*x + d)/(e^5*x^3 + 3*d*e^4*x^2
 + 3*d^2*e^3*x + d^3*e^2)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 187 vs. $2(42) = 84$.

Time = 0.70 (sec) , antiderivative size = 187, normalized size of antiderivative = 4.35

$$\int \frac{ade + (cd^2 + ae^2)\,x + cdex^2}{(d + ex)^{9/2}}\,dx = \begin{cases} -\frac{6ae^2}{15d^2e^2\sqrt{d+ex}+30de^3x\sqrt{d+ex}+15e^4x^2\sqrt{d+ex}} - \frac{4cd^2}{15d^2e^2\sqrt{d+ex}+30de^3x\sqrt{d+ex}+15e^4 \cdots} \\ \frac{cx^2}{2d^{\frac{5}{2}}} \end{cases}$$

input
```
integrate((a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)/(e*x+d)**(9/2),x)
```

output
```
Piecewise((-6*a*e**2/(15*d**2*e**2*sqrt(d + e*x) + 30*d*e**3*x*sqrt(d + e*
x) + 15*e**4*x**2*sqrt(d + e*x)) - 4*c*d**2/(15*d**2*e**2*sqrt(d + e*x) +
30*d*e**3*x*sqrt(d + e*x) + 15*e**4*x**2*sqrt(d + e*x)) - 10*c*d*e*x/(15*d
**2*e**2*sqrt(d + e*x) + 30*d*e**3*x*sqrt(d + e*x) + 15*e**4*x**2*sqrt(d +
 e*x)), Ne(e, 0)), (c*x**2/(2*d**(5/2)), True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.79

$$\int \frac{ade + (cd^2 + ae^2)\,x + cdex^2}{(d + ex)^{9/2}}\,dx = -\frac{2\,(5\,(ex + d)cd - 3\,cd^2 + 3\,ae^2)}{15\,(ex + d)^{\frac{5}{2}}e^2}$$

input
```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)/(e*x+d)^(9/2),x, algorithm="ma
xima")
```

output
```
-2/15*(5*(e*x + d)*c*d - 3*c*d^2 + 3*a*e^2)/((e*x + d)^(5/2)*e^2)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.79

$$\int \frac{ade+(cd^2+ae^2)\,x+cdex^2}{(d+ex)^{9/2}}\,dx = -\frac{2\,(5\,(ex+d)cd-3\,cd^2+3\,ae^2)}{15\,(ex+d)^{\frac{5}{2}}e^2}$$

input
```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)/(e*x+d)^(9/2),x, algorithm="gi
ac")
```

output
```
-2/15*(5*(e*x + d)*c*d - 3*c*d^2 + 3*a*e^2)/((e*x + d)^(5/2)*e^2)
```

Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.79

$$\int \frac{ade+(cd^2+ae^2)\,x+cdex^2}{(d+ex)^{9/2}}\,dx = -\frac{6\,a\,e^2-6\,c\,d^2+10\,c\,d\,(d+e\,x)}{15\,e^2\,(d+e\,x)^{5/2}}$$

input
```
int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)/(d + e*x)^(9/2),x)
```

output
```
-(6*a*e^2 - 6*c*d^2 + 10*c*d*(d + e*x))/(15*e^2*(d + e*x)^(5/2))
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.16

$$\int \frac{ade+(cd^2+ae^2)\,x+cdex^2}{(d+ex)^{9/2}}\,dx = \frac{-\frac{2}{3}cdex-\frac{2}{5}a\,e^2-\frac{4}{15}c\,d^2}{\sqrt{ex+d}\,e^2\,(e^2x^2+2dex+d^2)}$$

input
```
int((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)/(e*x+d)^(9/2),x)
```

output
```
(2*( - 3*a*e**2 - 2*c*d**2 - 5*c*d*e*x))/(15*sqrt(d + e*x)*e**2*(d**2 + 2*
d*e*x + e**2*x**2))
```

3.156 $\int \frac{ade+(cd^2+ae^2)x+cdex^2}{(d+ex)^{11/2}}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 43

$$\int \frac{ade+(cd^2+ae^2)\,x+cdex^2}{(d+ex)^{11/2}}\,dx=-\frac{2\left(a-\frac{cd^2}{e^2}\right)}{7(d+ex)^{7/2}}-\frac{2cd}{5e^2(d+ex)^{5/2}}$$

output
```
1/7*(-2*a+2*c*d^2/e^2)/(e*x+d)^(7/2)-2/5*c*d/e^2/(e*x+d)^(5/2)
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.79

$$\int \frac{ade+(cd^2+ae^2)\,x+cdex^2}{(d+ex)^{11/2}}\,dx=-\frac{2(5ae^2+cd(2d+7ex))}{35e^2(d+ex)^{7/2}}$$

input
```
Integrate[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)/(d + e*x)^(11/2),x]
```

output
```
(-2*(5*a*e^2 + c*d*(2*d + 7*e*x)))/(35*e^2*(d + e*x)^(7/2))
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.057$, Rules used = {1121, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x(ae^2+cd^2)+ade+cdex^2}{(d+ex)^{11/2}}dx$$

$$\Big\downarrow \text{1121}$$

$$\int \left(\frac{ae^2-cd^2}{e(d+ex)^{9/2}}+\frac{cd}{e(d+ex)^{7/2}}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{2\left(a-\frac{cd^2}{e^2}\right)}{7(d+ex)^{7/2}}-\frac{2cd}{5e^2(d+ex)^{5/2}}$$

input
```
Int[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)/(d + e*x)^(11/2),x]
```

output
```
(-2*(a - (c*d^2)/e^2))/(7*(d + e*x)^(7/2)) - (2*c*d)/(5*e^2*(d + e*x)^(5/2
))
```

Defintions of rubi rules used

rule 1121
```
Int[((d_) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Int[ExpandIntegrand[(d + e*x)^(m + p)*(a/d + (c/e)*x)^p, x], x]
/; FreeQ[{a, b, c, d, e, m, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && (Int
egerQ[p] || (GtQ[a, 0] && GtQ[d, 0] && LtQ[c, 0]))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.68 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.74

method	result	size
gosper	$-\frac{2(7cdxe+5a\,e^2+2c\,d^2)}{35(ex+d)^{\frac{7}{2}}e^2}$	32
trager	$-\frac{2(7cdxe+5a\,e^2+2c\,d^2)}{35(ex+d)^{\frac{7}{2}}e^2}$	32
pseudoelliptic	$\frac{-\frac{2}{5}cdxe-\frac{2}{7}a\,e^2-\frac{4}{35}c\,d^2}{(ex+d)^{\frac{7}{2}}e^2}$	32
derivativedivides	$\frac{-\frac{2cd}{5(ex+d)^{\frac{5}{2}}}-\frac{2\left(a\,e^2-c\,d^2\right)}{7(ex+d)^{\frac{7}{2}}}}{e^2}$	39
default	$\frac{-\frac{2cd}{5(ex+d)^{\frac{5}{2}}}-\frac{2\left(a\,e^2-c\,d^2\right)}{7(ex+d)^{\frac{7}{2}}}}{e^2}$	39
orering	$-\frac{2(7cdxe+5a\,e^2+2c\,d^2)\left(ade+\left(a\,e^2+c\,d^2\right)x+cd\,x^2e\right)}{35e^2(ex+d)^{\frac{9}{2}}(cdx+ae)}$	67

input
```
int((a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)/(e*x+d)^(11/2),x,method=_RETURNVERBO
SE)
```

output
```
-2/35/(e*x+d)^(7/2)*(7*c*d*e*x+5*a*e^2+2*c*d^2)/e^2
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 74 vs. $2(35) = 70$.

Time = 0.08 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.72

$$\int \frac{ade+(cd^2+ae^2)\,x+cdex^2}{(d+ex)^{11/2}}\,dx = -\frac{2\,(7\,cdex+2\,cd^2+5\,ae^2)\sqrt{ex+d}}{35\,(e^6x^4+4\,de^5x^3+6\,d^2e^4x^2+4\,d^3e^3x+d^4e^2)}$$

input
```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)/(e*x+d)^(11/2),x, algorithm="f
ricas")
```

output
```
-2/35*(7*c*d*e*x + 2*c*d^2 + 5*a*e^2)*sqrt(e*x + d)/(e^6*x^4 + 4*d*e^5*x^3
 + 6*d^2*e^4*x^2 + 4*d^3*e^3*x + d^4*e^2)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 248 vs. $2(42) = 84$.

Time = 1.36 (sec) , antiderivative size = 248, normalized size of antiderivative = 5.77

$$\int \frac{ade + (cd^2 + ae^2)\,x + cdex^2}{(d + ex)^{11/2}}\,dx = \begin{cases} -\frac{10ae^2}{35d^3e^2\sqrt{d+ex}+105d^2e^3x\sqrt{d+ex}+105de^4x^2\sqrt{d+ex}+35e^5x^3\sqrt{d+ex}} - \frac{}{35d^3e^2\sqrt{d+ex}+} \\ \frac{cx^2}{2d^{\frac{7}{2}}} \end{cases}$$

input
```
integrate((a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)/(e*x+d)**(11/2),x)
```

output
```
Piecewise((-10*a*e**2/(35*d**3*e**2*sqrt(d + e*x) + 105*d**2*e**3*x*sqrt(d
 + e*x) + 105*d*e**4*x**2*sqrt(d + e*x) + 35*e**5*x**3*sqrt(d + e*x)) - 4*
c*d**2/(35*d**3*e**2*sqrt(d + e*x) + 105*d**2*e**3*x*sqrt(d + e*x) + 105*d
*e**4*x**2*sqrt(d + e*x) + 35*e**5*x**3*sqrt(d + e*x)) - 14*c*d*e*x/(35*d*
*3*e**2*sqrt(d + e*x) + 105*d**2*e**3*x*sqrt(d + e*x) + 105*d*e**4*x**2*sq
rt(d + e*x) + 35*e**5*x**3*sqrt(d + e*x)), Ne(e, 0)), (c*x**2/(2*d**(7/2))
, True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.79

$$\int \frac{ade + (cd^2 + ae^2)\,x + cdex^2}{(d + ex)^{11/2}}\,dx = -\frac{2\,(7\,(ex + d)cd - 5\,cd^2 + 5\,ae^2)}{35\,(ex + d)^{\frac{7}{2}}e^2}$$

input
```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)/(e*x+d)^(11/2),x, algorithm="m
axima")
```

output
```
-2/35*(7*(e*x + d)*c*d - 5*c*d^2 + 5*a*e^2)/((e*x + d)^(7/2)*e^2)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.79

$$\int \frac{ade+(cd^2+ae^2)\,x+cdex^2}{(d+ex)^{11/2}}\,dx=-\frac{2\,(7\,(ex+d)cd-5\,cd^2+5\,ae^2)}{35\,(ex+d)^{\frac{7}{2}}e^2}$$

input
```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)/(e*x+d)^(11/2),x, algorithm="g
iac")
```

output
```
-2/35*(7*(e*x + d)*c*d - 5*c*d^2 + 5*a*e^2)/((e*x + d)^(7/2)*e^2)
```

Mupad [B] (verification not implemented)

Time = 5.35 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.72

$$\int \frac{ade+(cd^2+ae^2)\,x+cdex^2}{(d+ex)^{11/2}}\,dx=-\frac{4\,c\,d^2+14\,c\,x\,d\,e+10\,a\,e^2}{35\,e^2\,(d+e\,x)^{7/2}}$$

input
```
int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)/(d + e*x)^(11/2),x)
```

output
```
-(10*a*e^2 + 4*c*d^2 + 14*c*d*e*x)/(35*e^2*(d + e*x)^(7/2))
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.42

$$\int \frac{ade+(cd^2+ae^2)\,x+cdex^2}{(d+ex)^{11/2}}\,dx=\frac{-\frac{2}{5}cdex-\frac{2}{7}a\,e^2-\frac{4}{35}c\,d^2}{\sqrt{ex+d}\,e^2\,(e^3x^3+3d\,e^2x^2+3d^2ex+d^3)}$$

input
```
int((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)/(e*x+d)^(11/2),x)
```

output
```
(2*( - 5*a*e**2 - 2*c*d**2 - 7*c*d*e*x))/(35*sqrt(d + e*x)*e**2*(d**3 + 3*
d**2*e*x + 3*d*e**2*x**2 + e**3*x**3))
```

3.157 $\int \sqrt{d+ex}\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^2\,dx$

Optimal result

Integrand size = 37, antiderivative size = 83

$$\int \sqrt{d+ex}\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^2\,dx = \frac{2\left(cd^2-ae^2\right)^2(d+ex)^{7/2}}{7e^3}-\frac{4cd\left(cd^2-ae^2\right)(d+ex)^{9/2}}{9e^3}+\frac{2c^2d^2(d+ex)^{11/2}}{11e^3}$$

output
```
2/7*(-a*e^2+c*d^2)^2*(e*x+d)^(7/2)/e^3-4/9*c*d*(-a*e^2+c*d^2)*(e*x+d)^(9/2
)/e^3+2/11*c^2*d^2*(e*x+d)^(11/2)/e^3
```

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.81

$$\int \sqrt{d+ex}\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^2\,dx = \frac{2(d+ex)^{7/2}\left(99a^2e^4-22acde^2(2d-7ex)+c^2d^2\left(8d^2-28dex+63e^2x^2\right)\right)}{693e^3}$$

input
```
Integrate[Sqrt[d + e*x]*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^2,x]
```

output
```
(2*(d + e*x)^(7/2)*(99*a^2*e^4 - 22*a*c*d*e^2*(2*d - 7*e*x) + c^2*d^2*(8*d
^2 - 28*d*e*x + 63*e^2*x^2)))/(693*e^3)
```

Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.054$, Rules used = {1121, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{d+ex}\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^2 dx$$

$$\Big\downarrow \text{ 1121}$$

$$\int \left(-\frac{2cd(d+ex)^{7/2}\left(cd^2-ae^2\right)}{e^2}+\frac{(d+ex)^{5/2}\left(ae^2-cd^2\right)^2}{e^2}+\frac{c^2d^2(d+ex)^{9/2}}{e^2}\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$-\frac{4cd(d+ex)^{9/2}\left(cd^2-ae^2\right)}{9e^3}+\frac{2(d+ex)^{7/2}\left(cd^2-ae^2\right)^2}{7e^3}+\frac{2c^2d^2(d+ex)^{11/2}}{11e^3}$$

input
```
Int[Sqrt[d + e*x]*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^2,x]
```

output
```
(2*(c*d^2 - a*e^2)^2*(d + e*x)^(7/2))/(7*e^3) - (4*c*d*(c*d^2 - a*e^2)*(d
+ e*x)^(9/2))/(9*e^3) + (2*c^2*d^2*(d + e*x)^(11/2))/(11*e^3)
```

Defintions of rubi rules used

rule 1121
```
Int[((d_) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Int[ExpandIntegrand[(d + e*x)^(m + p)*(a/d + (c/e)*x)^p, x], x]
/; FreeQ[{a, b, c, d, e, m, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && (Int
egerQ[p] || (GtQ[a, 0] && GtQ[d, 0] && LtQ[c, 0]))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 2.31 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.78

method	result
pseudoelliptic	$\frac{2(ex+d)^{\frac{7}{2}}\left(a^2e^4+\frac{14xacd\,e^3}{9}-\frac{4\left(-\frac{63c\,x^2}{44}+a\right)c\,d^2e^2}{9}-\frac{28x\,c^2d^3e}{99}+\frac{8c^2d^4}{99}\right)}{7e^3}$
derivativedivides	$\frac{\frac{2c^2d^2(ex+d)^{\frac{11}{2}}}{11}+\frac{4cd\left(a\,e^2-c\,d^2\right)(ex+d)^{\frac{9}{2}}}{9}+\frac{2\left(a\,e^2-c\,d^2\right)^2(ex+d)^{\frac{7}{2}}}{7}}{e^3}$
default	$\frac{\frac{2c^2d^2(ex+d)^{\frac{11}{2}}}{11}+\frac{4cd\left(a\,e^2-c\,d^2\right)(ex+d)^{\frac{9}{2}}}{9}+\frac{2\left(a\,e^2-c\,d^2\right)^2(ex+d)^{\frac{7}{2}}}{7}}{e^3}$
gosper	$\frac{2(ex+d)^{\frac{7}{2}}\left(63x^2c^2d^2e^2+154xacd\,e^3-28x\,c^2d^3e+99a^2e^4-44ac\,d^2e^2+8c^2d^4\right)}{693e^3}$
orering	$\frac{2\left(63x^2c^2d^2e^2+154xacd\,e^3-28x\,c^2d^3e+99a^2e^4-44ac\,d^2e^2+8c^2d^4\right)(ex+d)^{\frac{3}{2}}\left(ade+\left(a\,e^2+c\,d^2\right)x+cd\,x^2e\right)^2}{693e^3(cdx+ae)^2}$
trager	$\frac{2\left(63c^2d^2e^5x^5+154acd\,e^6x^4+161e^4d^3c^2x^4+99a^2e^7x^3+418ac\,d^2e^5x^3+113c^2d^4e^3x^3+297a^2d\,e^6x^2+330ac\,d^3e^4x^2+3c\right.}{693e^3}$
risch	$\frac{2\left(63c^2d^2e^5x^5+154acd\,e^6x^4+161e^4d^3c^2x^4+99a^2e^7x^3+418ac\,d^2e^5x^3+113c^2d^4e^3x^3+297a^2d\,e^6x^2+330ac\,d^3e^4x^2+3c\right.}{693e^3}$

input
```
int((e*x+d)^(1/2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^2,x,method=_RETURNVERB
OSE)
```

output
```
2/7*(e*x+d)^(7/2)*(a^2*e^4+14/9*x*a*c*d*e^3-4/9*(-63/44*c*x^2+a)*c*d^2*e^2
-28/99*x*c^2*d^3*e+8/99*c^2*d^4)/e^3
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 184 vs. $2(71) = 142$.

Time = 0.08 (sec) , antiderivative size = 184, normalized size of antiderivative = 2.22

$$\int \sqrt{d+ex}\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^2\,dx$$
$$=\frac{2\,(63\,c^2d^2e^5x^5+8\,c^2d^7-44\,acd^5e^2+99\,a^2d^3e^4+7\,(23\,c^2d^3e^4+22\,acde^6)x^4+(113\,c^2d^4e^3+418\,acd^2e^5}{693\,e}$$

input
```
integrate((e*x+d)^(1/2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^2,x, algorithm="
fricas")
```

output
```
2/693*(63*c^2*d^2*e^5*x^5 + 8*c^2*d^7 - 44*a*c*d^5*e^2 + 99*a^2*d^3*e^4 +
7*(23*c^2*d^3*e^4 + 22*a*c*d*e^6)*x^4 + (113*c^2*d^4*e^3 + 418*a*c*d^2*e^5
 + 99*a^2*e^7)*x^3 + 3*(c^2*d^5*e^2 + 110*a*c*d^3*e^4 + 99*a^2*d*e^6)*x^2
- (4*c^2*d^6*e - 22*a*c*d^4*e^3 - 297*a^2*d^2*e^5)*x)*sqrt(e*x + d)/e^3
```

Sympy [A] (verification not implemented)

Time = 0.88 (sec) , antiderivative size = 110, normalized size of antiderivative = 1.33

$$\int \sqrt{d+ex}\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^2\,dx$$
$$=\begin{cases}\frac{2\left(\frac{c^2d^2(d+ex)^{\frac{11}{2}}}{11e^2}+\frac{(d+ex)^{\frac{9}{2}}\cdot\left(2acde^2-2c^2d^3\right)}{9e^2}+\frac{(d+ex)^{\frac{7}{2}}\left(a^2e^4-2acd^2e^2+c^2d^4\right)}{7e^2}\right)}{e} & \text{for } e\neq 0\\ \frac{c^2d^{\frac{9}{2}}x^3}{3} & \text{otherwise}\end{cases}$$

input
```
integrate((e*x+d)**(1/2)*(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**2,x)
```

output
```
Piecewise((2*(c**2*d**2*(d + e*x)**(11/2)/(11*e**2) + (d + e*x)**(9/2)*(2*
a*c*d*e**2 - 2*c**2*d**3)/(9*e**2) + (d + e*x)**(7/2)*(a**2*e**4 - 2*a*c*d
**2*e**2 + c**2*d**4)/(7*e**2))/e, Ne(e, 0)), (c**2*d**(9/2)*x**3/3, True)
)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.96

$$\int \sqrt{d+ex}\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^2\,dx$$
$$=\frac{2\left(63\,(ex+d)^{\frac{11}{2}}c^2d^2-154\,(c^2d^3-acde^2)(ex+d)^{\frac{9}{2}}+99\,(c^2d^4-2\,acd^2e^2+a^2e^4)(ex+d)^{\frac{7}{2}}\right)}{693\,e^3}$$

input
```
integrate((e*x+d)^(1/2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^2,x, algorithm="
maxima")
```

output
```
2/693*(63*(e*x + d)^(11/2)*c^2*d^2 - 154*(c^2*d^3 - a*c*d*e^2)*(e*x + d)^(
9/2) + 99*(c^2*d^4 - 2*a*c*d^2*e^2 + a^2*e^4)*(e*x + d)^(7/2))/e^3
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 566 vs. 2(71) = 142.

Time = 0.18 (sec) , antiderivative size = 566, normalized size of antiderivative = 6.82

$$\int \sqrt{d+ex}\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^2\,dx=\text{Too large to display}$$

input
```
integrate((e*x+d)^(1/2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^2,x, algorithm="
giac")
```

output
```
2/3465*(3465*sqrt(e*x + d)*a^2*d^3*e^2 + 2310*((e*x + d)^(3/2) - 3*sqrt(e*
x + d)*d)*a*c*d^4 + 3465*((e*x + d)^(3/2) - 3*sqrt(e*x + d)*d)*a^2*d^2*e^2
 + 1386*(3*(e*x + d)^(5/2) - 10*(e*x + d)^(3/2)*d + 15*sqrt(e*x + d)*d^2)*
a*c*d^3 + 231*(3*(e*x + d)^(5/2) - 10*(e*x + d)^(3/2)*d + 15*sqrt(e*x + d)
*d^2)*c^2*d^5/e^2 + 693*(3*(e*x + d)^(5/2) - 10*(e*x + d)^(3/2)*d + 15*sqr
t(e*x + d)*d^2)*a^2*d*e^2 + 594*(5*(e*x + d)^(7/2) - 21*(e*x + d)^(5/2)*d
+ 35*(e*x + d)^(3/2)*d^2 - 35*sqrt(e*x + d)*d^3)*a*c*d^2 + 297*(5*(e*x + d
)^(7/2) - 21*(e*x + d)^(5/2)*d + 35*(e*x + d)^(3/2)*d^2 - 35*sqrt(e*x + d)
*d^3)*c^2*d^4/e^2 + 99*(5*(e*x + d)^(7/2) - 21*(e*x + d)^(5/2)*d + 35*(e*x
 + d)^(3/2)*d^2 - 35*sqrt(e*x + d)*d^3)*a^2*e^2 + 22*(35*(e*x + d)^(9/2) -
 180*(e*x + d)^(7/2)*d + 378*(e*x + d)^(5/2)*d^2 - 420*(e*x + d)^(3/2)*d^3
 + 315*sqrt(e*x + d)*d^4)*a*c*d + 33*(35*(e*x + d)^(9/2) - 180*(e*x + d)^(
7/2)*d + 378*(e*x + d)^(5/2)*d^2 - 420*(e*x + d)^(3/2)*d^3 + 315*sqrt(e*x
+ d)*d^4)*c^2*d^3/e^2 + 5*(63*(e*x + d)^(11/2) - 385*(e*x + d)^(9/2)*d + 9
90*(e*x + d)^(7/2)*d^2 - 1386*(e*x + d)^(5/2)*d^3 + 1155*(e*x + d)^(3/2)*d
^4 - 693*sqrt(e*x + d)*d^5)*c^2*d^2/e^2)/e
```

Mupad [B] (verification not implemented)

Time = 5.35 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.96

$$\int \sqrt{d+ex}\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^2\,dx$$
$$=\frac{2\,{\left(d+e\,x\right)}^{7/2}\,\left(99\,a^2\,e^4+99\,c^2\,d^4+63\,c^2\,d^2\,{\left(d+e\,x\right)}^2-154\,c^2\,d^3\,\left(d+e\,x\right)-198\,a\,c\,d^2\,e^2+154\,a\,c\,d\,e^2\,(a$$
$$693\,e^3$$

input
```
int((d + e*x)^(1/2)*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^2,x)
```

output
```
(2*(d + e*x)^(7/2)*(99*a^2*e^4 + 99*c^2*d^4 + 63*c^2*d^2*(d + e*x)^2 - 154
*c^2*d^3*(d + e*x) - 198*a*c*d^2*e^2 + 154*a*c*d*e^2*(d + e*x)))/(693*e^3)
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 190, normalized size of antiderivative = 2.29

$$\int \sqrt{d+ex}\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^2\,dx$$
$$=\frac{2\sqrt{ex+d}\,(63c^2d^2e^5x^5+154acd\,e^6x^4+161c^2d^3e^4x^4+99a^2e^7x^3+418ac\,d^2e^5x^3+113c^2d^4e^3x^3+297a^2d}{69}$$

input
```
int((e*x+d)^(1/2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^2,x)
```

output
```
(2*sqrt(d + e*x)*(99*a**2*d**3*e**4 + 297*a**2*d**2*e**5*x + 297*a**2*d*e*
*6*x**2 + 99*a**2*e**7*x**3 - 44*a*c*d**5*e**2 + 22*a*c*d**4*e**3*x + 330*
a*c*d**3*e**4*x**2 + 418*a*c*d**2*e**5*x**3 + 154*a*c*d*e**6*x**4 + 8*c**2
*d**7 - 4*c**2*d**6*e*x + 3*c**2*d**5*e**2*x**2 + 113*c**2*d**4*e**3*x**3
+ 161*c**2*d**3*e**4*x**4 + 63*c**2*d**2*e**5*x**5))/(693*e**3)
```

3.158 $\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^2}{\sqrt{d+ex}}\,dx$

Optimal result

Integrand size = 37, antiderivative size = 83

$$\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^2}{\sqrt{d+ex}}\,dx = \frac{2\left(cd^2-ae^2\right)^2(d+ex)^{5/2}}{5e^3} - \frac{4cd\left(cd^2-ae^2\right)(d+ex)^{7/2}}{7e^3} + \frac{2c^2d^2(d+ex)^{9/2}}{9e^3}$$

output
```
2/5*(-a*e^2+c*d^2)^2*(e*x+d)^(5/2)/e^3-4/7*c*d*(-a*e^2+c*d^2)*(e*x+d)^(7/2
)/e^3+2/9*c^2*d^2*(e*x+d)^(9/2)/e^3
```

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.81

$$\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^2}{\sqrt{d+ex}}\,dx = \frac{2(d+ex)^{5/2}\left(63a^2e^4+18acde^2(-2d+5ex)+c^2d^2\left(8d^2-20dex+35e^2x^2\right)\right)}{315e^3}$$

input
```
Integrate[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^2/Sqrt[d + e*x],x]
```

output
```
(2*(d + e*x)^(5/2)*(63*a^2*e^4 + 18*a*c*d*e^2*(-2*d + 5*e*x) + c^2*d^2*(8*
d^2 - 20*d*e*x + 35*e^2*x^2)))/(315*e^3)
```

Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.054, Rules used = {1121, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(x(ae^2+cd^2)+ade+cdex^2)^2}{\sqrt{d+ex}}dx$$

$$\Big\downarrow \text{1121}$$

$$\int \left(-\frac{2cd(d+ex)^{5/2}(cd^2-ae^2)}{e^2}+\frac{(d+ex)^{3/2}(ae^2-cd^2)^2}{e^2}+\frac{c^2d^2(d+ex)^{7/2}}{e^2}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{4cd(d+ex)^{7/2}(cd^2-ae^2)}{7e^3}+\frac{2(d+ex)^{5/2}(cd^2-ae^2)^2}{5e^3}+\frac{2c^2d^2(d+ex)^{9/2}}{9e^3}$$

input
```
Int[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^2/Sqrt[d + e*x],x]
```

output
```
(2*(c*d^2 - a*e^2)^2*(d + e*x)^(5/2))/(5*e^3) - (4*c*d*(c*d^2 - a*e^2)*(d
+ e*x)^(7/2))/(7*e^3) + (2*c^2*d^2*(d + e*x)^(9/2))/(9*e^3)
```

Defintions of rubi rules used

rule 1121
```
Int[((d_) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Int[ExpandIntegrand[(d + e*x)^(m + p)*(a/d + (c/e)*x)^p, x], x]
/; FreeQ[{a, b, c, d, e, m, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && (Int
egerQ[p] || (GtQ[a, 0] && GtQ[d, 0] && LtQ[c, 0]))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 2.18 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.78

method	result
pseudoelliptic	$\frac{2(ex+d)^{\frac{5}{2}}\left(a^2e^4+\frac{10xacd\,e^3}{7}-\frac{4\left(-\frac{35c\,x^2}{36}+a\right)c\,d^2e^2}{7}-\frac{20x\,c^2d^3e}{63}+\frac{8c^2d^4}{63}\right)}{5e^3}$
derivativedivides	$\frac{\frac{2c^2d^2(ex+d)^{\frac{9}{2}}}{9}+\frac{4cd\left(a\,e^2-c\,d^2\right)(ex+d)^{\frac{7}{2}}}{7}+\frac{2\left(a\,e^2-c\,d^2\right)^2(ex+d)^{\frac{5}{2}}}{5}}{e^3}$
default	$\frac{\frac{2c^2d^2(ex+d)^{\frac{9}{2}}}{9}+\frac{4cd\left(a\,e^2-c\,d^2\right)(ex+d)^{\frac{7}{2}}}{7}+\frac{2\left(a\,e^2-c\,d^2\right)^2(ex+d)^{\frac{5}{2}}}{5}}{e^3}$
gosper	$\frac{2(ex+d)^{\frac{5}{2}}\left(35x^2c^2d^2e^2+90xacd\,e^3-20x\,c^2d^3e+63a^2e^4-36ac\,d^2e^2+8c^2d^4\right)}{315e^3}$
orering	$\frac{2\left(35x^2c^2d^2e^2+90xacd\,e^3-20x\,c^2d^3e+63a^2e^4-36ac\,d^2e^2+8c^2d^4\right)\sqrt{ex+d}\,\left(ade+\left(a\,e^2+c\,d^2\right)x+cd\,x^2e\right)^2}{315e^3(cdx+ae)^2}$
trager	$\frac{2(35d^2e^4c^2x^4+90acd\,e^5x^3+50c^2d^3e^3x^3+63a^2e^6x^2+144ac\,d^2e^4x^2+3c^2d^4e^2x^2+126a^2d\,e^5x+18ac\,d^3e^3x-4c^2d^5ex+6}{315e^3}$
risch	$\frac{2(35d^2e^4c^2x^4+90acd\,e^5x^3+50c^2d^3e^3x^3+63a^2e^6x^2+144ac\,d^2e^4x^2+3c^2d^4e^2x^2+126a^2d\,e^5x+18ac\,d^3e^3x-4c^2d^5ex+6}{315e^3}$

input
```
int((a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^2/(e*x+d)^(1/2),x,method=_RETURNVERB
OSE)
```

output
```
2/5*(e*x+d)^(5/2)*(a^2*e^4+10/7*x*a*c*d*e^3-4/7*(-35/36*c*x^2+a)*c*d^2*e^2
-20/63*x*c^2*d^3*e+8/63*c^2*d^4)/e^3
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 147 vs. $2(71) = 142$.

Time = 0.08 (sec) , antiderivative size = 147, normalized size of antiderivative = 1.77

$$\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^2}{\sqrt{d+ex}}\,dx$$
$$= \frac{2\,(35\,c^2d^2e^4x^4+8\,c^2d^6-36\,acd^4e^2+63\,a^2d^2e^4+10\,(5\,c^2d^3e^3+9\,acde^5)x^3+3\,(c^2d^4e^2+48\,acd^2e^4+21}{315\,e^3}$$

input
```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^2/(e*x+d)^(1/2),x, algorithm="
fricas")
```

output
```
2/315*(35*c^2*d^2*e^4*x^4 + 8*c^2*d^6 - 36*a*c*d^4*e^2 + 63*a^2*d^2*e^4 +
10*(5*c^2*d^3*e^3 + 9*a*c*d*e^5)*x^3 + 3*(c^2*d^4*e^2 + 48*a*c*d^2*e^4 + 2
1*a^2*e^6)*x^2 - 2*(2*c^2*d^5*e - 9*a*c*d^3*e^3 - 63*a^2*d*e^5)*x)*sqrt(e*
x + d)/e^3
```

Sympy [A] (verification not implemented)

Time = 0.90 (sec) , antiderivative size = 110, normalized size of antiderivative = 1.33

$$\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^2}{\sqrt{d+ex}}\,dx$$
$$= \begin{cases} \frac{2\left(\frac{c^2d^2(d+ex)^{\frac{9}{2}}}{9e^2}+\frac{(d+ex)^{\frac{7}{2}}\cdot\left(2acde^2-2c^2d^3\right)}{7e^2}+\frac{(d+ex)^{\frac{5}{2}}\left(a^2e^4-2acd^2e^2+c^2d^4\right)}{5e^2}\right)}{e} & \text{for } e\neq 0 \\ \frac{c^2d^{\frac{7}{2}}x^3}{3} & \text{otherwise} \end{cases}$$

input
```
integrate((a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**2/(e*x+d)**(1/2),x)
```

output
```
Piecewise((2*(c**2*d**2*(d + e*x)**(9/2)/(9*e**2) + (d + e*x)**(7/2)*(2*a*
c*d*e**2 - 2*c**2*d**3)/(7*e**2) + (d + e*x)**(5/2)*(a**2*e**4 - 2*a*c*d**
2*e**2 + c**2*d**4)/(5*e**2))/e, Ne(e, 0)), (c**2*d**(7/2)*x**3/3, True))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 280 vs. $2(71) = 142$.

Time = 0.03 (sec) , antiderivative size = 280, normalized size of antiderivative = 3.37

$$\int \frac{(ade + (cd^2 + ae^2)x + cdex^2)^2}{\sqrt{d + ex}}\,dx$$
$$= \frac{2\left(315\,\sqrt{ex+d}a^2d^2e^2 + 42\left(\frac{\left(3\,(ex+d)^{\frac{5}{2}} - 10\,(ex+d)^{\frac{3}{2}}d + 15\,\sqrt{ex+d}d^2\right)cd}{e} + \frac{5\,(cd^2+ae^2)\left((ex+d)^{\frac{3}{2}} - 3\,\sqrt{ex+d}d\right)}{e}\right)ade + \frac{(35\cdots}{\cdots}\right.}{\cdots}$$

input
```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^2/(e*x+d)^(1/2),x, algorithm="
maxima")
```

output
```
2/315*(315*sqrt(e*x + d)*a^2*d^2*e^2 + 42*((3*(e*x + d)^(5/2) - 10*(e*x +
d)^(3/2)*d + 15*sqrt(e*x + d)*d^2)*c*d/e + 5*(c*d^2 + a*e^2)*((e*x + d)^(3
/2) - 3*sqrt(e*x + d)*d)/e)*a*d*e + (35*(e*x + d)^(9/2) - 180*(e*x + d)^(7
/2)*d + 378*(e*x + d)^(5/2)*d^2 - 420*(e*x + d)^(3/2)*d^3 + 315*sqrt(e*x +
 d)*d^4)*c^2*d^2/e^2 + 18*(5*(e*x + d)^(7/2) - 21*(e*x + d)^(5/2)*d + 35*(
e*x + d)^(3/2)*d^2 - 35*sqrt(e*x + d)*d^3)*(c*d^2 + a*e^2)*c*d/e^2 + 21*(c
*d^2 + a*e^2)^2*(3*(e*x + d)^(5/2) - 10*(e*x + d)^(3/2)*d + 15*sqrt(e*x +
d)*d^2)/e^2)/e
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 368 vs. $2(71) = 142$.

Time = 0.15 (sec) , antiderivative size = 368, normalized size of antiderivative = 4.43

$$\int \frac{(ade + (cd^2 + ae^2)x + cdex^2)^2}{\sqrt{d + ex}}\,dx$$
$$= \frac{2\left(315\,\sqrt{ex+d}a^2d^2e^2 + 210\left((ex+d)^{\frac{3}{2}} - 3\,\sqrt{ex+d}d\right)acd^3 + 210\left((ex+d)^{\frac{3}{2}} - 3\,\sqrt{ex+d}d\right)a^2de^2 + 8\cdots\right.}{\cdots}$$

input
```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^2/(e*x+d)^(1/2),x, algorithm="
giac")
```

output
```
2/315*(315*sqrt(e*x + d)*a^2*d^2*e^2 + 210*((e*x + d)^(3/2) - 3*sqrt(e*x +
 d)*d)*a*c*d^3 + 210*((e*x + d)^(3/2) - 3*sqrt(e*x + d)*d)*a^2*d*e^2 + 84*
(3*(e*x + d)^(5/2) - 10*(e*x + d)^(3/2)*d + 15*sqrt(e*x + d)*d^2)*a*c*d^2
+ 21*(3*(e*x + d)^(5/2) - 10*(e*x + d)^(3/2)*d + 15*sqrt(e*x + d)*d^2)*c^2
*d^4/e^2 + 21*(3*(e*x + d)^(5/2) - 10*(e*x + d)^(3/2)*d + 15*sqrt(e*x + d)
*d^2)*a^2*e^2 + 18*(5*(e*x + d)^(7/2) - 21*(e*x + d)^(5/2)*d + 35*(e*x + d
)^(3/2)*d^2 - 35*sqrt(e*x + d)*d^3)*a*c*d + 18*(5*(e*x + d)^(7/2) - 21*(e*
x + d)^(5/2)*d + 35*(e*x + d)^(3/2)*d^2 - 35*sqrt(e*x + d)*d^3)*c^2*d^3/e^
2 + (35*(e*x + d)^(9/2) - 180*(e*x + d)^(7/2)*d + 378*(e*x + d)^(5/2)*d^2
- 420*(e*x + d)^(3/2)*d^3 + 315*sqrt(e*x + d)*d^4)*c^2*d^2/e^2)/e
```

Mupad [B] (verification not implemented)

Time = 5.18 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.96

$$\int \frac{(ade+(cd^2+ae^2)x+cdex^2)^2}{\sqrt{d+ex}}\,dx$$
$$= \frac{2\,(d+e\,x)^{5/2}\,\left(63\,a^2\,e^4+63\,c^2\,d^4+35\,c^2\,d^2\,(d+e\,x)^2-90\,c^2\,d^3\,(d+e\,x)-126\,a\,c\,d^2\,e^2+90\,a\,c\,d\,e^2\,(d+\right.}{315\,e^3}$$

input
```
int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^2/(d + e*x)^(1/2),x)
```

output
```
(2*(d + e*x)^(5/2)*(63*a^2*e^4 + 63*c^2*d^4 + 35*c^2*d^2*(d + e*x)^2 - 90*
c^2*d^3*(d + e*x) - 126*a*c*d^2*e^2 + 90*a*c*d*e^2*(d + e*x)))/(315*e^3)
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 149, normalized size of antiderivative = 1.80

$$\int \frac{(ade+(cd^2+ae^2)x+cdex^2)^2}{\sqrt{d+ex}}\,dx$$
$$= \frac{2\sqrt{ex+d}\,(35c^2d^2e^4x^4+90acd\,e^5x^3+50c^2d^3e^3x^3+63a^2e^6x^2+144ac\,d^2e^4x^2+3c^2d^4e^2x^2+126a^2d\,e^5x}{315e^3}$$

input
```
int((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^2/(e*x+d)^(1/2),x)
```

output
```
(2*sqrt(d + e*x)*(63*a**2*d**2*e**4 + 126*a**2*d*e**5*x + 63*a**2*e**6*x**
2 - 36*a*c*d**4*e**2 + 18*a*c*d**3*e**3*x + 144*a*c*d**2*e**4*x**2 + 90*a*
c*d*e**5*x**3 + 8*c**2*d**6 - 4*c**2*d**5*e*x + 3*c**2*d**4*e**2*x**2 + 50
*c**2*d**3*e**3*x**3 + 35*c**2*d**2*e**4*x**4))/(315*e**3)
```

3.159 $\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^2}{(d+ex)^{3/2}}\,dx$

Optimal result

Integrand size = 37, antiderivative size = 83

$$\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^2}{(d+ex)^{3/2}}\,dx=\frac{2\left(cd^2-ae^2\right)^2(d+ex)^{3/2}}{3e^3}-\frac{4cd\left(cd^2-ae^2\right)(d+ex)^{5/2}}{5e^3}+\frac{2c^2d^2(d+ex)^{7/2}}{7e^3}$$

output
```
2/3*(-a*e^2+c*d^2)^2*(e*x+d)^(3/2)/e^3-4/5*c*d*(-a*e^2+c*d^2)*(e*x+d)^(5/2
)/e^3+2/7*c^2*d^2*(e*x+d)^(7/2)/e^3
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.81

$$\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^2}{(d+ex)^{3/2}}\,dx=\frac{2(d+ex)^{3/2}\left(35a^2e^4+14acde^2(-2d+3ex)+c^2d^2\left(8d^2-12dex+\right.\right.}{105e^3}$$

input
```
Integrate[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^2/(d + e*x)^(3/2),x]
```

output
```
(2*(d + e*x)^(3/2)*(35*a^2*e^4 + 14*a*c*d*e^2*(-2*d + 3*e*x) + c^2*d^2*(8*
d^2 - 12*d*e*x + 15*e^2*x^2)))/(105*e^3)
```

Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.054, Rules used = {1121, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^2}{(d+ex)^{3/2}}dx$$

$$\Big\downarrow \text{1121}$$

$$\int \left(-\frac{2cd(d+ex)^{3/2}\left(cd^2-ae^2\right)}{e^2}+\frac{\sqrt{d+ex}\left(ae^2-cd^2\right)^2}{e^2}+\frac{c^2d^2(d+ex)^{5/2}}{e^2}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{4cd(d+ex)^{5/2}\left(cd^2-ae^2\right)}{5e^3}+\frac{2(d+ex)^{3/2}\left(cd^2-ae^2\right)^2}{3e^3}+\frac{2c^2d^2(d+ex)^{7/2}}{7e^3}$$

input
```
Int[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^2/(d + e*x)^(3/2),x]
```

output
```
(2*(c*d^2 - a*e^2)^2*(d + e*x)^(3/2))/(3*e^3) - (4*c*d*(c*d^2 - a*e^2)*(d
+ e*x)^(5/2))/(5*e^3) + (2*c^2*d^2*(d + e*x)^(7/2))/(7*e^3)
```

Defintions of rubi rules used

rule 1121
```
Int[((d_) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Int[ExpandIntegrand[(d + e*x)^(m + p)*(a/d + (c/e)*x)^p, x], x]
/; FreeQ[{a, b, c, d, e, m, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && (Int
egerQ[p] || (GtQ[a, 0] && GtQ[d, 0] && LtQ[c, 0]))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 2.20 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.78

method	result	siz
pseudoelliptic	$\frac{2(ex+d)^{\frac{3}{2}}\left(a^2e^4+\frac{6xacd\,e^3}{5}-\frac{4c\,d^2\left(-\frac{15c\,x^2}{28}+a\right)e^2}{5}-\frac{12x\,c^2d^3e}{35}+\frac{8c^2d^4}{35}\right)}{3e^3}$	65
derivativedivides	$\frac{\frac{2c^2d^2(ex+d)^{\frac{7}{2}}}{7}+\frac{4cd\left(a\,e^2-c\,d^2\right)(ex+d)^{\frac{5}{2}}}{5}+\frac{2\left(a\,e^2-c\,d^2\right)^2(ex+d)^{\frac{3}{2}}}{3}}{e^3}$	68
default	$\frac{\frac{2c^2d^2(ex+d)^{\frac{7}{2}}}{7}+\frac{4cd\left(a\,e^2-c\,d^2\right)(ex+d)^{\frac{5}{2}}}{5}+\frac{2\left(a\,e^2-c\,d^2\right)^2(ex+d)^{\frac{3}{2}}}{3}}{e^3}$	68
gosper	$\frac{2(ex+d)^{\frac{3}{2}}\left(15x^2c^2d^2e^2+42xacd\,e^3-12x\,c^2d^3e+35a^2e^4-28ac\,d^2e^2+8c^2d^4\right)}{105e^3}$	73
trager	$\frac{2\left(15c^2d^2e^3x^3+42ac\,e^4d\,x^2+3c^2d^3e^2x^2+35a^2e^5x+14ac\,d^2e^3x-4c^2d^4ex+35a^2d\,e^4-28ac\,d^3e^2+8c^2d^5\right)\sqrt{ex+d}}{105e^3}$	11
risch	$\frac{2\left(15c^2d^2e^3x^3+42ac\,e^4d\,x^2+3c^2d^3e^2x^2+35a^2e^5x+14ac\,d^2e^3x-4c^2d^4ex+35a^2d\,e^4-28ac\,d^3e^2+8c^2d^5\right)\sqrt{ex+d}}{105e^3}$	11
orering	$\frac{2\left(15x^2c^2d^2e^2+42xacd\,e^3-12x\,c^2d^3e+35a^2e^4-28ac\,d^2e^2+8c^2d^4\right)\left(ade+\left(a\,e^2+c\,d^2\right)x+cd\,x^2e\right)^2}{105e^3(cdx+ae)^2\sqrt{ex+d}}$	11

input
```
int((a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^2/(e*x+d)^(3/2),x,method=_RETURNVERB
OSE)
```

output
```
2/3*(e*x+d)^(3/2)*(a^2*e^4+6/5*x*a*c*d*e^3-4/5*c*d^2*(-15/28*c*x^2+a)*e^2-
12/35*x*c^2*d^3*e+8/35*c^2*d^4)/e^3
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 109, normalized size of antiderivative = 1.31

$$\int \frac{(ade + (cd^2 + ae^2)x + cdex^2)^2}{(d + ex)^{3/2}}\,dx = \frac{2\,(15\,c^2d^2e^3x^3 + 8\,c^2d^5 - 28\,acd^3e^2 + 35\,a^2de^4 + 3\,(c^2d^3e^2 + 14\,aca}{105\,e^3}$$

input
```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^2/(e*x+d)^(3/2),x, algorithm="
fricas")
```

output
```
2/105*(15*c^2*d^2*e^3*x^3 + 8*c^2*d^5 - 28*a*c*d^3*e^2 + 35*a^2*d*e^4 + 3*
(c^2*d^3*e^2 + 14*a*c*d*e^4)*x^2 - (4*c^2*d^4*e - 14*a*c*d^2*e^3 - 35*a^2*
e^5)*x)*sqrt(e*x + d)/e^3
```

Sympy [A] (verification not implemented)

Time = 0.88 (sec) , antiderivative size = 110, normalized size of antiderivative = 1.33

$$\int \frac{(ade + (cd^2 + ae^2)x + cdex^2)^2}{(d + ex)^{3/2}}\,dx = \begin{cases} \frac{2\left(\frac{c^2d^2(d+ex)^{\frac{7}{2}}}{7e^2} + \frac{(d+ex)^{\frac{5}{2}}\cdot\left(2acde^2 - 2c^2d^3\right)}{5e^2} + \frac{(d+ex)^{\frac{3}{2}}\left(a^2e^4 - 2acd^2e^2 + c^2d^4\right)}{3e^2}\right)}{e} & \text{for } e \\ \frac{c^2d^{\frac{5}{2}}x^3}{3} & \text{other} \end{cases}$$

input
```
integrate((a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**2/(e*x+d)**(3/2),x)
```

output
```
Piecewise((2*(c**2*d**2*(d + e*x)**(7/2)/(7*e**2) + (d + e*x)**(5/2)*(2*a*
c*d*e**2 - 2*c**2*d**3)/(5*e**2) + (d + e*x)**(3/2)*(a**2*e**4 - 2*a*c*d**
2*e**2 + c**2*d**4)/(3*e**2))/e, Ne(e, 0)), (c**2*d**(5/2)*x**3/3, True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.96

$$\int \frac{(ade + (cd^2 + ae^2)x + cdex^2)^2}{(d + ex)^{3/2}}\,dx = \frac{2\left(15\,(ex+d)^{\frac{7}{2}}c^2d^2 - 42\,(c^2d^3 - acde^2)(ex+d)^{\frac{5}{2}} + 35\,(c^2d^4 - 2\,a\right.}{105\,e^3}$$

input
```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^2/(e*x+d)^(3/2),x, algorithm="
maxima")
```

output
```
2/105*(15*(e*x + d)^(7/2)*c^2*d^2 - 42*(c^2*d^3 - a*c*d*e^2)*(e*x + d)^(5/
2) + 35*(c^2*d^4 - 2*a*c*d^2*e^2 + a^2*e^4)*(e*x + d)^(3/2))/e^3
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 208 vs. 2(71) = 142.

Time = 0.17 (sec) , antiderivative size = 208, normalized size of antiderivative = 2.51

$$\int \frac{(ade + (cd^2 + ae^2)x + cdex^2)^2}{(d + ex)^{3/2}}\,dx = \frac{2\left(105\,\sqrt{ex+d}a^2de^2 + 70\left((ex+d)^{\frac{3}{2}} - 3\,\sqrt{ex+d}d\right)acd^2 + 35\left(\right.\right.}{}$$

input
```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^2/(e*x+d)^(3/2),x, algorithm="
giac")
```

output
```
2/105*(105*sqrt(e*x + d)*a^2*d*e^2 + 70*((e*x + d)^(3/2) - 3*sqrt(e*x + d)
*d)*a*c*d^2 + 35*((e*x + d)^(3/2) - 3*sqrt(e*x + d)*d)*a^2*e^2 + 14*(3*(e*
x + d)^(5/2) - 10*(e*x + d)^(3/2)*d + 15*sqrt(e*x + d)*d^2)*a*c*d + 7*(3*(
e*x + d)^(5/2) - 10*(e*x + d)^(3/2)*d + 15*sqrt(e*x + d)*d^2)*c^2*d^3/e^2
+ 3*(5*(e*x + d)^(7/2) - 21*(e*x + d)^(5/2)*d + 35*(e*x + d)^(3/2)*d^2 - 3
5*sqrt(e*x + d)*d^3)*c^2*d^2/e^2)/e
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.96

$$\int \frac{(ade + (cd^2 + ae^2)\, x + cdex^2)^2}{(d + ex)^{3/2}}\, dx = \frac{2\, (d + e\, x)^{3/2}\, \left(35\, a^2\, e^4 + 35\, c^2\, d^4 + 15\, c^2\, d^2\, (d + e\, x)^2 - 42\, c^2\, d^3\, (d \right.}{105\, e^3}$$

input
```
int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^2/(d + e*x)^(3/2),x)
```

output
```
(2*(d + e*x)^(3/2)*(35*a^2*e^4 + 35*c^2*d^4 + 15*c^2*d^2*(d + e*x)^2 - 42*
c^2*d^3*(d + e*x) - 70*a*c*d^2*e^2 + 42*a*c*d*e^2*(d + e*x)))/(105*e^3)
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.30

$$\int \frac{(ade + (cd^2 + ae^2)\, x + cdex^2)^2}{(d + ex)^{3/2}}\, dx = \frac{2\sqrt{ex + d}\, \left(15c^2d^2e^3x^3 + 42acd\, e^4x^2 + 3c^2d^3e^2x^2 + 35a^2e^5x + 14ac \right.}{105e^3}$$

input
```
int((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^2/(e*x+d)^(3/2),x)
```

output
```
(2*sqrt(d + e*x)*(35*a**2*d*e**4 + 35*a**2*e**5*x - 28*a*c*d**3*e**2 + 14*
a*c*d**2*e**3*x + 42*a*c*d*e**4*x**2 + 8*c**2*d**5 - 4*c**2*d**4*e*x + 3*c
**2*d**3*e**2*x**2 + 15*c**2*d**2*e**3*x**3))/(105*e**3)
```

3.160 $\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^2}{(d+ex)^{5/2}}\,dx$

Optimal result

Integrand size = 37, antiderivative size = 81

$$\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^2}{(d+ex)^{5/2}}\,dx=\frac{2\left(cd^2-ae^2\right)^2\sqrt{d+ex}}{e^3}-\frac{4cd\left(cd^2-ae^2\right)(d+ex)^{3/2}}{3e^3}+\frac{2c^2d^2(d+ex)^{5/2}}{5e^3}$$

output
```
2*(-a*e^2+c*d^2)^2*(e*x+d)^(1/2)/e^3-4/3*c*d*(-a*e^2+c*d^2)*(e*x+d)^(3/2)/
e^3+2/5*c^2*d^2*(e*x+d)^(5/2)/e^3
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.81

$$\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^2}{(d+ex)^{5/2}}\,dx=\frac{2\sqrt{d+ex}\left(15a^2e^4+10acde^2(-2d+ex)+c^2d^2\left(8d^2-4dex+3e^2x\right.\right.}{15e^3}$$

input
```
Integrate[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^2/(d + e*x)^(5/2),x]
```

output
```
(2*Sqrt[d + e*x]*(15*a^2*e^4 + 10*a*c*d*e^2*(-2*d + e*x) + c^2*d^2*(8*d^2
- 4*d*e*x + 3*e^2*x^2)))/(15*e^3)
```

Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.054$, Rules used = {1121, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^2}{(d+ex)^{5/2}}dx$$

$$\Big\downarrow \text{ 1121}$$

$$\int \left(-\frac{2cd\sqrt{d+ex}\left(cd^2-ae^2\right)}{e^2}+\frac{\left(ae^2-cd^2\right)^2}{e^2\sqrt{d+ex}}+\frac{c^2d^2(d+ex)^{3/2}}{e^2}\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$-\frac{4cd(d+ex)^{3/2}\left(cd^2-ae^2\right)}{3e^3}+\frac{2\sqrt{d+ex}\left(cd^2-ae^2\right)^2}{e^3}+\frac{2c^2d^2(d+ex)^{5/2}}{5e^3}$$

input
```
Int[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^2/(d + e*x)^(5/2),x]
```

output
```
(2*(c*d^2 - a*e^2)^2*Sqrt[d + e*x])/e^3 - (4*c*d*(c*d^2 - a*e^2)*(d + e*x)
^(3/2))/(3*e^3) + (2*c^2*d^2*(d + e*x)^(5/2))/(5*e^3)
```

Defintions of rubi rules used

rule 1121
```
Int[((d_) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Int[ExpandIntegrand[(d + e*x)^(m + p)*(a/d + (c/e)*x)^p, x], x]
/; FreeQ[{a, b, c, d, e, m, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && (Int
egerQ[p] || (GtQ[a, 0] && GtQ[d, 0] && LtQ[c, 0]))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 1.39 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.80

method	result	size
pseudoelliptic	$\frac{2\sqrt{ex+d}\left(a^2e^4+\frac{2xacd\,e^3}{3}-\frac{4c\left(-\frac{3c\,x^2}{20}+a\right)d^2e^2}{3}-\frac{4x\,c^2d^3e}{15}+\frac{8c^2d^4}{15}\right)}{e^3}$	65
derivativedivides	$\frac{\frac{2c^2d^2(ex+d)^{\frac{5}{2}}}{5}+\frac{4\left(a\,e^2-c\,d^2\right)cd(ex+d)^{\frac{3}{2}}}{3}+2\left(a\,e^2-c\,d^2\right)^2\sqrt{ex+d}}{e^3}$	67
default	$\frac{\frac{2c^2d^2(ex+d)^{\frac{5}{2}}}{5}+\frac{4\left(a\,e^2-c\,d^2\right)cd(ex+d)^{\frac{3}{2}}}{3}+2\left(a\,e^2-c\,d^2\right)^2\sqrt{ex+d}}{e^3}$	67
gosper	$\frac{2\sqrt{ex+d}\,(3x^2c^2d^2e^2+10xacd\,e^3-4x\,c^2d^3e+15a^2e^4-20ac\,d^2e^2+8c^2d^4)}{15e^3}$	73
trager	$\frac{2\sqrt{ex+d}\,(3x^2c^2d^2e^2+10xacd\,e^3-4x\,c^2d^3e+15a^2e^4-20ac\,d^2e^2+8c^2d^4)}{15e^3}$	73
risch	$\frac{2\sqrt{ex+d}\,(3x^2c^2d^2e^2+10xacd\,e^3-4x\,c^2d^3e+15a^2e^4-20ac\,d^2e^2+8c^2d^4)}{15e^3}$	73
orering	$\frac{2(3x^2c^2d^2e^2+10xacd\,e^3-4x\,c^2d^3e+15a^2e^4-20ac\,d^2e^2+8c^2d^4)\left(ade+\left(a\,e^2+c\,d^2\right)x+cd\,x^2e\right)^2}{15e^3(cdx+ae)^2(ex+d)^{\frac{3}{2}}}$	110

input
```
int((a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^2/(e*x+d)^(5/2),x,method=_RETURNVERB
OSE)
```

output
```
2*(e*x+d)^(1/2)*(a^2*e^4+2/3*x*a*c*d*e^3-4/3*c*(-3/20*c*x^2+a)*d^2*e^2-4/1
5*x*c^2*d^3*e+8/15*c^2*d^4)/e^3
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.91

$$\int \frac{(ade + (cd^2 + ae^2)x + cdex^2)^2}{(d + ex)^{5/2}}\,dx = \frac{2\,(3\,c^2d^2e^2x^2 + 8\,c^2d^4 - 20\,acd^2e^2 + 15\,a^2e^4 - 2\,(2\,c^2d^3e - 5\,acde^3)}{15\,e^3}$$

input
```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^2/(e*x+d)^(5/2),x, algorithm="
fricas")
```

output
```
2/15*(3*c^2*d^2*e^2*x^2 + 8*c^2*d^4 - 20*a*c*d^2*e^2 + 15*a^2*e^4 - 2*(2*c
^2*d^3*e - 5*a*c*d*e^3)*x)*sqrt(e*x + d)/e^3
```

Sympy [A] (verification not implemented)

Time = 0.88 (sec) , antiderivative size = 109, normalized size of antiderivative = 1.35

$$\int \frac{(ade + (cd^2 + ae^2)x + cdex^2)^2}{(d + ex)^{5/2}}\,dx = \begin{cases} \frac{2\left(\frac{c^2d^2(d+ex)^{\frac{5}{2}}}{5e^2} + \frac{(d+ex)^{\frac{3}{2}}\cdot\left(2acde^2 - 2c^2d^3\right)}{3e^2} + \frac{\sqrt{d+ex}\left(a^2e^4 - 2acd^2e^2 + c^2d^4\right)}{e^2}\right)}{e} & \text{for } e \neq \\ \frac{c^2d^{\frac{3}{2}}x^3}{3} & \text{otherw} \end{cases}$$

input
```
integrate((a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**2/(e*x+d)**(5/2),x)
```

output
```
Piecewise((2*(c**2*d**2*(d + e*x)**(5/2)/(5*e**2) + (d + e*x)**(3/2)*(2*a*
c*d*e**2 - 2*c**2*d**3)/(3*e**2) + sqrt(d + e*x)*(a**2*e**4 - 2*a*c*d**2*e
**2 + c**2*d**4)/e**2)/e, Ne(e, 0)), (c**2*d**(3/2)*x**3/3, True))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.99

$$\int \frac{(ade+(cd^2+ae^2)\,x+cdex^2)^2}{(d+ex)^{5/2}}\,dx = \frac{2\left(3\,(ex+d)^{\frac{5}{2}}c^2d^2-10\,(c^2d^3-acde^2)(ex+d)^{\frac{3}{2}}+15\,(c^2d^4-2\,ac\right.}{15\,e^3}$$

input
```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^2/(e*x+d)^(5/2),x, algorithm="
maxima")
```

output
```
2/15*(3*(e*x + d)^(5/2)*c^2*d^2 - 10*(c^2*d^3 - a*c*d*e^2)*(e*x + d)^(3/2)
 + 15*(c^2*d^4 - 2*a*c*d^2*e^2 + a^2*e^4)*sqrt(e*x + d))/e^3
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.06

$$\int \frac{(ade+(cd^2+ae^2)\,x+cdex^2)^2}{(d+ex)^{5/2}}\,dx = \frac{2\left(15\sqrt{ex+d}\,a^2e^2+10\left((ex+d)^{\frac{3}{2}}-3\sqrt{ex+d}\,d\right)acd+\frac{\left(3\,(ex+d)^{\frac{5}{2}}\right.}{}\right.}{15\,e}$$

input
```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^2/(e*x+d)^(5/2),x, algorithm="
giac")
```

output
```
2/15*(15*sqrt(e*x + d)*a^2*e^2 + 10*((e*x + d)^(3/2) - 3*sqrt(e*x + d)*d)*
a*c*d + (3*(e*x + d)^(5/2) - 10*(e*x + d)^(3/2)*d + 15*sqrt(e*x + d)*d^2)*
c^2*d^2/e^2)/e
```

Mupad [B] (verification not implemented)

Time = 5.09 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.99

$$\int \frac{\left(ade+(cd^2+ae^2)\,x+cdex^2\right)^2}{(d+ex)^{5/2}}\,dx = \frac{2\,\sqrt{d+e\,x}\,\left(15\,a^2\,e^4+15\,c^2\,d^4+3\,c^2\,d^2\,{\left(d+e\,x\right)}^2-10\,c^2\,d^3\,\left(d+e\right.}{15\,e^3}$$

input
```
int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^2/(d + e*x)^(5/2),x)
```

output
```
(2*(d + e*x)^(1/2)*(15*a^2*e^4 + 15*c^2*d^4 + 3*c^2*d^2*(d + e*x)^2 - 10*c
^2*d^3*(d + e*x) - 30*a*c*d^2*e^2 + 10*a*c*d*e^2*(d + e*x)))/(15*e^3)
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.88

$$\int \frac{\left(ade+(cd^2+ae^2)\,x+cdex^2\right)^2}{(d+ex)^{5/2}}\,dx = \frac{2\sqrt{ex+d}\,(3c^2d^2e^2x^2+10acd\,e^3x-4c^2d^3ex+15a^2e^4-20ac\,d^2e^2}{15e^3}$$

input
```
int((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^2/(e*x+d)^(5/2),x)
```

output
```
(2*sqrt(d + e*x)*(15*a**2*e**4 - 20*a*c*d**2*e**2 + 10*a*c*d*e**3*x + 8*c*
*2*d**4 - 4*c**2*d**3*e*x + 3*c**2*d**2*e**2*x**2))/(15*e**3)
```

3.161 $\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^2}{(d+ex)^{7/2}}\,dx$

Optimal result

Integrand size = 37, antiderivative size = 79

$$\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^2}{(d+ex)^{7/2}}\,dx=-\frac{2\left(cd^2-ae^2\right)^2}{e^3\sqrt{d+ex}}-\frac{4cd\left(cd^2-ae^2\right)\sqrt{d+ex}}{e^3}+\frac{2c^2d^2(d+ex)^{3/2}}{3e^3}$$

output
```
-2*(-a*e^2+c*d^2)^2/e^3/(e*x+d)^(1/2)-4*c*d*(-a*e^2+c*d^2)*(e*x+d)^(1/2)/e
^3+2/3*c^2*d^2*(e*x+d)^(3/2)/e^3
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.82

$$\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^2}{(d+ex)^{7/2}}\,dx=\frac{2\left(-3a^2e^4+6acde^2(2d+ex)+c^2d^2\left(-8d^2-4dex+e^2x^2\right)\right)}{3e^3\sqrt{d+ex}}$$

input
```
Integrate[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^2/(d + e*x)^(7/2),x]
```

output
```
(2*(-3*a^2*e^4 + 6*a*c*d*e^2*(2*d + e*x) + c^2*d^2*(-8*d^2 - 4*d*e*x + e^2
*x^2)))/(3*e^3*Sqrt[d + e*x])
```

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.054$, Rules used = {1121, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^2}{(d+ex)^{7/2}}\,dx$$

$$\Big\downarrow \text{ 1121}$$

$$\int \left(-\frac{2cd\left(cd^2-ae^2\right)}{e^2\sqrt{d+ex}}+\frac{\left(ae^2-cd^2\right)^2}{e^2(d+ex)^{3/2}}+\frac{c^2d^2\sqrt{d+ex}}{e^2}\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$-\frac{4cd\sqrt{d+ex}\left(cd^2-ae^2\right)}{e^3}-\frac{2\left(cd^2-ae^2\right)^2}{e^3\sqrt{d+ex}}+\frac{2c^2d^2(d+ex)^{3/2}}{3e^3}$$

input
```
Int[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^2/(d + e*x)^(7/2),x]
```

output
```
(-2*(c*d^2 - a*e^2)^2)/(e^3*Sqrt[d + e*x]) - (4*c*d*(c*d^2 - a*e^2)*Sqrt[d
 + e*x])/e^3 + (2*c^2*d^2*(d + e*x)^(3/2))/(3*e^3)
```

Defintions of rubi rules used

rule 1121
```
Int[((d_) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Int[ExpandIntegrand[(d + e*x)^(m + p)*(a/d + (c/e)*x)^p, x], x]
/; FreeQ[{a, b, c, d, e, m, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && (Int
egerQ[p] || (GtQ[a, 0] && GtQ[d, 0] && LtQ[c, 0]))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 1.40 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.82

method	result	size
pseudoelliptic	$-\frac{2\left(a^{2} e^{4}-2 x a c d\, e^{3}-4\left(\frac{c\, x^{2}}{12}+a\right) c\, d^{2} e^{2}+\frac{4 x\, c^{2} d^{3} e}{3}+\frac{8 c^{2} d^{4}}{3}\right)}{\sqrt{e x+d}\, e^{3}}$	65
risch	$\frac{2 c d \left(c d x e+6 a\, e^{2}-5 c\, d^{2}\right) \sqrt{e x+d}}{3 e^{3}}-\frac{2 \left(a^{2} e^{4}-2 a c\, d^{2} e^{2}+c^{2} d^{4}\right)}{e^{3} \sqrt{e x+d}}$	71
gosper	$-\frac{2 \left(-x^{2} c^{2} d^{2} e^{2}-6 x a c d\, e^{3}+4 x\, c^{2} d^{3} e+3 a^{2} e^{4}-12 a c\, d^{2} e^{2}+8 c^{2} d^{4}\right)}{3 \sqrt{e x+d}\, e^{3}}$	73
trager	$-\frac{2 \left(-x^{2} c^{2} d^{2} e^{2}-6 x a c d\, e^{3}+4 x\, c^{2} d^{3} e+3 a^{2} e^{4}-12 a c\, d^{2} e^{2}+8 c^{2} d^{4}\right)}{3 \sqrt{e x+d}\, e^{3}}$	73
derivativedivides	$\frac{\frac{2 c^{2} d^{2} \left(e x+d\right)^{\frac{3}{2}}}{3}+4 a c d\, e^{2} \sqrt{e x+d}-4 c^{2} d^{3} \sqrt{e x+d}-\frac{2 \left(a^{2} e^{4}-2 a c\, d^{2} e^{2}+c^{2} d^{4}\right)}{\sqrt{e x+d}}}{e^{3}}$	86
default	$\frac{\frac{2 c^{2} d^{2} \left(e x+d\right)^{\frac{3}{2}}}{3}+4 a c d\, e^{2} \sqrt{e x+d}-4 c^{2} d^{3} \sqrt{e x+d}-\frac{2 \left(a^{2} e^{4}-2 a c\, d^{2} e^{2}+c^{2} d^{4}\right)}{\sqrt{e x+d}}}{e^{3}}$	86
orering	$-\frac{2 \left(-x^{2} c^{2} d^{2} e^{2}-6 x a c d\, e^{3}+4 x\, c^{2} d^{3} e+3 a^{2} e^{4}-12 a c\, d^{2} e^{2}+8 c^{2} d^{4}\right) \left(a d e+\left(a\, e^{2}+c\, d^{2}\right) x+c d\, x^{2} e\right)^{2}}{3 e^{3} \left(c d x+a e\right)^{2} \left(e x+d\right)^{\frac{5}{2}}}$	110

input
```
int((a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^2/(e*x+d)^(7/2),x,method=_RETURNVERB
OSE)
```

output
```
-2/(e*x+d)^(1/2)*(a^2*e^4-2*x*a*c*d*e^3-4*(1/12*c*x^2+a)*c*d^2*e^2+4/3*x*c
^2*d^3*e+8/3*c^2*d^4)/e^3
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.05

$$\int \frac{(ade + (cd^2 + ae^2)x + cdex^2)^2}{(d + ex)^{7/2}} dx = \frac{2(c^2d^2e^2x^2 - 8c^2d^4 + 12acd^2e^2 - 3a^2e^4 - 2(2c^2d^3e - 3acde^3)x)}{3(e^4x + de^3)}$$

input
```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^2/(e*x+d)^(7/2),x, algorithm="
fricas")
```

output
```
2/3*(c^2*d^2*e^2*x^2 - 8*c^2*d^4 + 12*a*c*d^2*e^2 - 3*a^2*e^4 - 2*(2*c^2*d
^3*e - 3*a*c*d*e^3)*x)*sqrt(e*x + d)/(e^4*x + d*e^3)
```

Sympy [A] (verification not implemented)

Time = 0.45 (sec) , antiderivative size = 133, normalized size of antiderivative = 1.68

$$\int \frac{(ade + (cd^2 + ae^2)x + cdex^2)^2}{(d + ex)^{7/2}} dx = \begin{cases} -\frac{2a^2e}{\sqrt{d+ex}} + \frac{8acd^2}{e\sqrt{d+ex}} + \frac{4acdx}{\sqrt{d+ex}} - \frac{16c^2d^4}{3e^3\sqrt{d+ex}} - \frac{8c^2d^3x}{3e^2\sqrt{d+ex}} + \frac{2c^2d^2x^2}{3e\sqrt{d+ex}} & \text{for} \\ \frac{c^2\sqrt{d}x^3}{3} & \text{oth} \end{cases}$$

input
```
integrate((a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**2/(e*x+d)**(7/2),x)
```

output
```
Piecewise((-2*a**2*e/sqrt(d + e*x) + 8*a*c*d**2/(e*sqrt(d + e*x)) + 4*a*c*
d*x/sqrt(d + e*x) - 16*c**2*d**4/(3*e**3*sqrt(d + e*x)) - 8*c**2*d**3*x/(3
*e**2*sqrt(d + e*x)) + 2*c**2*d**2*x**2/(3*e*sqrt(d + e*x)), Ne(e, 0)), (c
**2*sqrt(d)*x**3/3, True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.10

$$\int \frac{\left(ade + \left(cd^2 + ae^2\right)x + cdex^2\right)^2}{(d+ex)^{7/2}}\,dx = \frac{2\left(\frac{(ex+d)^{\frac{3}{2}}c^2d^2 - 6\left(c^2d^3 - acde^2\right)\sqrt{ex+d}}{e^2} - \frac{3\left(c^2d^4 - 2\,acd^2e^2 + a^2e^4\right)}{\sqrt{ex+d}e^2}\right)}{3\,e}$$

input
```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^2/(e*x+d)^(7/2),x, algorithm="
maxima")
```

output
```
2/3*(((e*x + d)^(3/2)*c^2*d^2 - 6*(c^2*d^3 - a*c*d*e^2)*sqrt(e*x + d))/e^2
 - 3*(c^2*d^4 - 2*a*c*d^2*e^2 + a^2*e^4)/(sqrt(e*x + d)*e^2))/e
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.19

$$\int \frac{\left(ade + \left(cd^2 + ae^2\right)x + cdex^2\right)^2}{(d+ex)^{7/2}}\,dx = -\frac{2\left(c^2d^4 - 2\,acd^2e^2 + a^2e^4\right)}{\sqrt{ex+d}e^3} + \frac{2\left((ex+d)^{\frac{3}{2}}c^2d^2e^6 - 6\sqrt{ex+d}c^2d^3e^6 + 6\sqrt{ex+d}acde^8\right)}{3\,e^9}$$

input
```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^2/(e*x+d)^(7/2),x, algorithm="
giac")
```

output
```
-2*(c^2*d^4 - 2*a*c*d^2*e^2 + a^2*e^4)/(sqrt(e*x + d)*e^3) + 2/3*((e*x + d
)^(3/2)*c^2*d^2*e^6 - 6*sqrt(e*x + d)*c^2*d^3*e^6 + 6*sqrt(e*x + d)*a*c*d*
e^8)/e^9
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.01

$$\int \frac{(ade + (cd^2 + ae^2)x + cdex^2)^2}{(d + ex)^{7/2}}\,dx = -\frac{6\,a^2\,e^4 + 6\,c^2\,d^4 - 2\,c^2\,d^2\,(d + e\,x)^2 + 12\,c^2\,d^3\,(d + e\,x) - 12\,a\,c\,d^2\,e^2 - 12\,a\,c\,d\,e^2\,(d + e\,x)}{3\,e^3\,\sqrt{d + e\,x}}$$

input
```
int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^2/(d + e*x)^(7/2),x)
```

output
```
-(6*a^2*e^4 + 6*c^2*d^4 - 2*c^2*d^2*(d + e*x)^2 + 12*c^2*d^3*(d + e*x) - 1
2*a*c*d^2*e^2 - 12*a*c*d*e^2*(d + e*x))/(3*e^3*(d + e*x)^(1/2))
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.91

$$\int \frac{(ade + (cd^2 + ae^2)x + cdex^2)^2}{(d + ex)^{7/2}}\,dx = \frac{\frac{2}{3}c^2d^2e^2x^2 + 4acd\,e^3x - \frac{8}{3}c^2d^3ex - 2a^2e^4 + 8ac\,d^2e^2 - \frac{16}{3}c^2d^4}{\sqrt{ex + d}\,e^3}$$

input
```
int((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^2/(e*x+d)^(7/2),x)
```

output
```
(2*( - 3*a**2*e**4 + 12*a*c*d**2*e**2 + 6*a*c*d*e**3*x - 8*c**2*d**4 - 4*c
**2*d**3*e*x + c**2*d**2*e**2*x**2))/(3*sqrt(d + e*x)*e**3)
```

3.162 $\int \frac{(ade+(cd^2+ae^2)x+cdex^2)^2}{(d+ex)^{9/2}}\,dx$

Optimal result

Integrand size = 37, antiderivative size = 79

$$\int \frac{(ade+(cd^2+ae^2)x+cdex^2)^2}{(d+ex)^{9/2}}\,dx = -\frac{2(cd^2-ae^2)^2}{3e^3(d+ex)^{3/2}}+\frac{4cd(cd^2-ae^2)}{e^3\sqrt{d+ex}}+\frac{2c^2d^2\sqrt{d+ex}}{e^3}$$

output
```
-2/3*(-a*e^2+c*d^2)^2/e^3/(e*x+d)^(3/2)+4*c*d*(-a*e^2+c*d^2)/e^3/(e*x+d)^(
1/2)+2*c^2*d^2*(e*x+d)^(1/2)/e^3
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.86

$$\int \frac{(ade+(cd^2+ae^2)x+cdex^2)^2}{(d+ex)^{9/2}}\,dx = \frac{-2a^2e^4-4acde^2(2d+3ex)+2c^2d^2(8d^2+12dex+3e^2x^2)}{3e^3(d+ex)^{3/2}}$$

input
```
Integrate[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^2/(d + e*x)^(9/2),x]
```

output
```
(-2*a^2*e^4 - 4*a*c*d*e^2*(2*d + 3*e*x) + 2*c^2*d^2*(8*d^2 + 12*d*e*x + 3*
e^2*x^2))/(3*e^3*(d + e*x)^(3/2))
```

Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.054$, Rules used = {1121, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(x(ae^2+cd^2)+ade+cdex^2)^2}{(d+ex)^{9/2}}\,dx$$

$$\Big\downarrow \text{1121}$$

$$\int \left(-\frac{2cd(cd^2-ae^2)}{e^2(d+ex)^{3/2}}+\frac{(ae^2-cd^2)^2}{e^2(d+ex)^{5/2}}+\frac{c^2d^2}{e^2\sqrt{d+ex}}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{4cd(cd^2-ae^2)}{e^3\sqrt{d+ex}}-\frac{2(cd^2-ae^2)^2}{3e^3(d+ex)^{3/2}}+\frac{2c^2d^2\sqrt{d+ex}}{e^3}$$

input
```
Int[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^2/(d + e*x)^(9/2),x]
```

output
```
(-2*(c*d^2 - a*e^2)^2)/(3*e^3*(d + e*x)^(3/2)) + (4*c*d*(c*d^2 - a*e^2))/(
e^3*Sqrt[d + e*x]) + (2*c^2*d^2*Sqrt[d + e*x])/e^3
```

Defintions of rubi rules used

rule 1121
```
Int[((d_) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Int[ExpandIntegrand[(d + e*x)^(m + p)*(a/d + (c/e)*x)^p, x], x]
/; FreeQ[{a, b, c, d, e, m, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && (Int
egerQ[p] || (GtQ[a, 0] && GtQ[d, 0] && LtQ[c, 0]))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 1.46 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.78

method	result	size
risch	$\frac{2c^{2}d^{2}\sqrt{ex+d}}{e^{3}}-\frac{2\left(6cdxe+a\,e^{2}+5c\,d^{2}\right)\left(a\,e^{2}-c\,d^{2}\right)}{3e^{3}\left(ex+d\right)^{\frac{3}{2}}}$	62
pseudoelliptic	$-\frac{2\left(a^{2}e^{4}+6xacd\,e^{3}+4\left(-\frac{3c\,x^{2}}{4}+a\right)c\,d^{2}e^{2}-12x\,c^{2}d^{3}e-8c^{2}d^{4}\right)}{3\left(ex+d\right)^{\frac{3}{2}}e^{3}}$	65
gosper	$-\frac{2\left(-3x^{2}c^{2}d^{2}e^{2}+6xacd\,e^{3}-12x\,c^{2}d^{3}e+a^{2}e^{4}+4ac\,d^{2}e^{2}-8c^{2}d^{4}\right)}{3\left(ex+d\right)^{\frac{3}{2}}e^{3}}$	72
trager	$-\frac{2\left(-3x^{2}c^{2}d^{2}e^{2}+6xacd\,e^{3}-12x\,c^{2}d^{3}e+a^{2}e^{4}+4ac\,d^{2}e^{2}-8c^{2}d^{4}\right)}{3\left(ex+d\right)^{\frac{3}{2}}e^{3}}$	72
derivativedivides	$\frac{2c^{2}d^{2}\sqrt{ex+d}-\frac{4cd\left(a\,e^{2}-c\,d^{2}\right)}{\sqrt{ex+d}}-\frac{2\left(a^{2}e^{4}-2ac\,d^{2}e^{2}+c^{2}d^{4}\right)}{3\left(ex+d\right)^{\frac{3}{2}}}}{e^{3}}$	78
default	$\frac{2c^{2}d^{2}\sqrt{ex+d}-\frac{4cd\left(a\,e^{2}-c\,d^{2}\right)}{\sqrt{ex+d}}-\frac{2\left(a^{2}e^{4}-2ac\,d^{2}e^{2}+c^{2}d^{4}\right)}{3\left(ex+d\right)^{\frac{3}{2}}}}{e^{3}}$	78
orering	$-\frac{2\left(-3x^{2}c^{2}d^{2}e^{2}+6xacd\,e^{3}-12x\,c^{2}d^{3}e+a^{2}e^{4}+4ac\,d^{2}e^{2}-8c^{2}d^{4}\right)\left(ade+\left(a\,e^{2}+c\,d^{2}\right)x+cd\,x^{2}e\right)^{2}}{3e^{3}\left(cdx+ae\right)^{2}\left(ex+d\right)^{\frac{7}{2}}}$	109

input
```
int((a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^2/(e*x+d)^(9/2),x,method=_RETURNVERB
OSE)
```

output
```
2*c^2*d^2*(e*x+d)^(1/2)/e^3-2/3*(6*c*d*e*x+a*e^2+5*c*d^2)*(a*e^2-c*d^2)/e^
3/(e*x+d)^(3/2)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.20

$$\int \frac{\left(ade+(cd^2+ae^2)\,x+cdex^2\right)^2}{(d+ex)^{9/2}}\,dx=\frac{2\left(3\,c^2d^2e^2x^2+8\,c^2d^4-4\,acd^2e^2-a^2e^4+6\left(2\,c^2d^3e-acde^3\right)x\right)\sqrt{e}}{3\left(e^5x^2+2\,de^4x+d^2e^3\right)}$$

input
```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^2/(e*x+d)^(9/2),x, algorithm="
fricas")
```

output
```
2/3*(3*c^2*d^2*e^2*x^2 + 8*c^2*d^4 - 4*a*c*d^2*e^2 - a^2*e^4 + 6*(2*c^2*d^
3*e - a*c*d*e^3)*x)*sqrt(e*x + d)/(e^5*x^2 + 2*d*e^4*x + d^2*e^3)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 264 vs. $2(73) = 146$.

Time = 0.71 (sec) , antiderivative size = 264, normalized size of antiderivative = 3.34

$$\int \frac{(ade + (cd^2 + ae^2)x + cdex^2)^2}{(d+ex)^{9/2}}\,dx = \begin{cases} -\frac{2a^2e^4}{3de^3\sqrt{d+ex}+3e^4x\sqrt{d+ex}} - \frac{8acd^2e^2}{3de^3\sqrt{d+ex}+3e^4x\sqrt{d+ex}} - \frac{12acde^3x}{3de^3\sqrt{d+ex}+3e^4x\sqrt{d+ex}} \\ \frac{c^2x^3}{3\sqrt{d}} \end{cases}$$

input
```
integrate((a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**2/(e*x+d)**(9/2),x)
```

output
```
Piecewise((-2*a**2*e**4/(3*d*e**3*sqrt(d + e*x) + 3*e**4*x*sqrt(d + e*x))
- 8*a*c*d**2*e**2/(3*d*e**3*sqrt(d + e*x) + 3*e**4*x*sqrt(d + e*x)) - 12*a
*c*d*e**3*x/(3*d*e**3*sqrt(d + e*x) + 3*e**4*x*sqrt(d + e*x)) + 16*c**2*d*
*4/(3*d*e**3*sqrt(d + e*x) + 3*e**4*x*sqrt(d + e*x)) + 24*c**2*d**3*e*x/(3
*d*e**3*sqrt(d + e*x) + 3*e**4*x*sqrt(d + e*x)) + 6*c**2*d**2*e**2*x**2/(3
*d*e**3*sqrt(d + e*x) + 3*e**4*x*sqrt(d + e*x)), Ne(e, 0)), (c**2*x**3/(3*
sqrt(d)), True))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 84, normalized size of antiderivative = 1.06

$$\int \frac{(ade + (cd^2 + ae^2)x + cdex^2)^2}{(d+ex)^{9/2}}\,dx = \frac{2\left(\frac{3\sqrt{ex+d}c^2d^2}{e^2} - \frac{c^2d^4 - 2acd^2e^2 + a^2e^4 - 6(c^2d^3 - acde^2)(ex+d)}{(ex+d)^{\frac{3}{2}}e^2}\right)}{3e}$$

input
```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^2/(e*x+d)^(9/2),x, algorithm="
maxima")
```

output
```
2/3*(3*sqrt(e*x + d)*c^2*d^2/e^2 - (c^2*d^4 - 2*a*c*d^2*e^2 + a^2*e^4 - 6*
(c^2*d^3 - a*c*d*e^2)*(e*x + d))/((e*x + d)^(3/2)*e^2))/e
```

Giac [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 84, normalized size of antiderivative = 1.06

$$\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^2}{(d+ex)^{9/2}}\,dx = \frac{2\sqrt{ex+d}c^2d^2}{e^3} + \frac{2\left(6\,(ex+d)c^2d^3-c^2d^4-6\,(ex+d)acde^2+2\,acd^2e^2-a^2e^4\right)}{3\,(ex+d)^{\frac{3}{2}}e^3}$$

input
```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^2/(e*x+d)^(9/2),x, algorithm="
giac")
```

output
```
2*sqrt(e*x + d)*c^2*d^2/e^3 + 2/3*(6*(e*x + d)*c^2*d^3 - c^2*d^4 - 6*(e*x
+ d)*a*c*d*e^2 + 2*a*c*d^2*e^2 - a^2*e^4)/((e*x + d)^(3/2)*e^3)
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.01

$$\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^2}{(d+ex)^{9/2}}\,dx = -\frac{2\,a^2\,e^4+2\,c^2\,d^4-6\,c^2\,d^2\,{\left(d+e\,x\right)}^2-12\,c^2\,d^3\,\left(d+e\,x\right)-4\,a\,c\,d^2\,e^2+12\,a\,c\,d\,e^2\,\left(d+e\,x\right)}{3\,e^3\,{\left(d+e\,x\right)}^{3/2}}$$

input
```
int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^2/(d + e*x)^(9/2),x)
```

output
```
-(2*a^2*e^4 + 2*c^2*d^4 - 6*c^2*d^2*(d + e*x)^2 - 12*c^2*d^3*(d + e*x) - 4
*a*c*d^2*e^2 + 12*a*c*d*e^2*(d + e*x))/(3*e^3*(d + e*x)^(3/2))
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.01

$$\int \frac{(ade+(cd^2+ae^2)x+cdex^2)^2}{(d+ex)^{9/2}}\,dx = \frac{2c^2d^2e^2x^2-4acd\,e^3x+8c^2d^3ex-\frac{2}{3}a^2e^4-\frac{8}{3}ac\,d^2e^2+\frac{16}{3}c^2d^4}{\sqrt{ex+d}\,e^3\,(ex+d)}$$

input
```
int((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^2/(e*x+d)^(9/2),x)
```

output
```
(2*( - a**2*e**4 - 4*a*c*d**2*e**2 - 6*a*c*d*e**3*x + 8*c**2*d**4 + 12*c**
2*d**3*e*x + 3*c**2*d**2*e**2*x**2))/(3*sqrt(d + e*x)*e**3*(d + e*x))
```

3.163 $\int \frac{(ade+(cd^2+ae^2)x+cdex^2)^2}{(d+ex)^{11/2}}\,dx$

Optimal result

Integrand size = 37, antiderivative size = 81

$$\int \frac{(ade+(cd^2+ae^2)x+cdex^2)^2}{(d+ex)^{11/2}}\,dx = -\frac{2(cd^2-ae^2)^2}{5e^3(d+ex)^{5/2}}+\frac{4cd(cd^2-ae^2)}{3e^3(d+ex)^{3/2}}-\frac{2c^2d^2}{e^3\sqrt{d+ex}}$$

output
```
-2/5*(-a*e^2+c*d^2)^2/e^3/(e*x+d)^(5/2)+4/3*c*d*(-a*e^2+c*d^2)/e^3/(e*x+d)
^(3/2)-2*c^2*d^2/e^3/(e*x+d)^(1/2)
```

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.83

$$\int \frac{(ade+(cd^2+ae^2)x+cdex^2)^2}{(d+ex)^{11/2}}\,dx = -\frac{2(3a^2e^4+2acde^2(2d+5ex)+c^2d^2(8d^2+20dex+15e^2x^2))}{15e^3(d+ex)^{5/2}}$$

input
```
Integrate[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^2/(d + e*x)^(11/2),x]
```

output
```
(-2*(3*a^2*e^4 + 2*a*c*d*e^2*(2*d + 5*e*x) + c^2*d^2*(8*d^2 + 20*d*e*x + 1
5*e^2*x^2)))/(15*e^3*(d + e*x)^(5/2))
```

Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.054$, Rules used = {1121, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^2}{(d+ex)^{11/2}}\,dx$$

$$\Big\downarrow \text{1121}$$

$$\int \left(-\frac{2cd\left(cd^2-ae^2\right)}{e^2(d+ex)^{5/2}}+\frac{\left(ae^2-cd^2\right)^2}{e^2(d+ex)^{7/2}}+\frac{c^2d^2}{e^2(d+ex)^{3/2}}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{4cd\left(cd^2-ae^2\right)}{3e^3(d+ex)^{3/2}}-\frac{2\left(cd^2-ae^2\right)^2}{5e^3(d+ex)^{5/2}}-\frac{2c^2d^2}{e^3\sqrt{d+ex}}$$

input
```
Int[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^2/(d + e*x)^(11/2),x]
```

output
```
(-2*(c*d^2 - a*e^2)^2)/(5*e^3*(d + e*x)^(5/2)) + (4*c*d*(c*d^2 - a*e^2))/(
3*e^3*(d + e*x)^(3/2)) - (2*c^2*d^2)/(e^3*Sqrt[d + e*x])
```

Defintions of rubi rules used

rule 1121
```
Int[((d_) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Int[ExpandIntegrand[(d + e*x)^(m + p)*(a/d + (c/e)*x)^p, x], x]
/; FreeQ[{a, b, c, d, e, m, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && (Int
egerQ[p] || (GtQ[a, 0] && GtQ[d, 0] && LtQ[c, 0]))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 1.39 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.80

method	result	size
pseudoelliptic	$-\frac{2\left(a^{2} e^{4}+\frac{10 x a c d \,e^{3}}{3}+\frac{4\left(\frac{15 c \,x^{2}}{4}+a \right) c \,d^{2} e^{2}}{3}+\frac{20 x \,c^{2} d^{3} e}{3}+\frac{8 c^{2} d^{4}}{3}\right)}{5 \left(e x +d \right)^{\frac{5}{2}} e^{3}}$	65
gosper	$-\frac{2 \left(15 x^{2} c^{2} d^{2} e^{2}+10 x a c d \,e^{3}+20 x \,c^{2} d^{3} e +3 a^{2} e^{4}+4 a c \,d^{2} e^{2}+8 c^{2} d^{4}\right)}{15 \left(e x +d \right)^{\frac{5}{2}} e^{3}}$	73
trager	$-\frac{2 \left(15 x^{2} c^{2} d^{2} e^{2}+10 x a c d \,e^{3}+20 x \,c^{2} d^{3} e +3 a^{2} e^{4}+4 a c \,d^{2} e^{2}+8 c^{2} d^{4}\right)}{15 \left(e x +d \right)^{\frac{5}{2}} e^{3}}$	73
derivativedivides	$\frac{-\frac{2 \left(a^{2} e^{4}-2 a c \,d^{2} e^{2}+c^{2} d^{4}\right)}{5 \left(e x +d \right)^{\frac{5}{2}}}-\frac{2 c^{2} d^{2}}{\sqrt{e x +d}}-\frac{4 c d \left(a \,e^{2}-c \,d^{2}\right)}{3 \left(e x +d \right)^{\frac{3}{2}}}}{e^{3}}$	79
default	$\frac{-\frac{2 \left(a^{2} e^{4}-2 a c \,d^{2} e^{2}+c^{2} d^{4}\right)}{5 \left(e x +d \right)^{\frac{5}{2}}}-\frac{2 c^{2} d^{2}}{\sqrt{e x +d}}-\frac{4 c d \left(a \,e^{2}-c \,d^{2}\right)}{3 \left(e x +d \right)^{\frac{3}{2}}}}{e^{3}}$	79
orering	$-\frac{2 \left(15 x^{2} c^{2} d^{2} e^{2}+10 x a c d \,e^{3}+20 x \,c^{2} d^{3} e +3 a^{2} e^{4}+4 a c \,d^{2} e^{2}+8 c^{2} d^{4}\right) \left(a d e +\left(a \,e^{2}+c \,d^{2}\right) x +c d \,x^{2} e \right)^{2}}{15 e^{3} \left(c d x +a e \right)^{2} \left(e x +d \right)^{\frac{9}{2}}}$	110

input
```
int((a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^2/(e*x+d)^(11/2),x,method=_RETURNVER
BOSE)
```

output
```
-2/5/(e*x+d)^(5/2)*(a^2*e^4+10/3*x*a*c*d*e^3+4/3*(15/4*c*x^2+a)*c*d^2*e^2+
20/3*x*c^2*d^3*e+8/3*c^2*d^4)/e^3
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.30

$$\int \frac{(ade + (cd^2 + ae^2)x + cdex^2)^2}{(d + ex)^{11/2}}\,dx = -\frac{2\,(15\,c^2d^2e^2x^2 + 8\,c^2d^4 + 4\,acd^2e^2 + 3\,a^2e^4 + 10\,(2\,c^2d^3e + acde^3)x)\sqrt{ex + d}}{15\,(e^6x^3 + 3\,de^5x^2 + 3\,d^2e^4x + d^3e^3)}$$

input
```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^2/(e*x+d)^(11/2),x, algorithm=
"fricas")
```

output
```
-2/15*(15*c^2*d^2*e^2*x^2 + 8*c^2*d^4 + 4*a*c*d^2*e^2 + 3*a^2*e^4 + 10*(2*
c^2*d^3*e + a*c*d*e^3)*x)*sqrt(e*x + d)/(e^6*x^3 + 3*d*e^5*x^2 + 3*d^2*e^4
*x + d^3*e^3)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 388 vs. $2(75) = 150$.

Time = 0.96 (sec) , antiderivative size = 388, normalized size of antiderivative = 4.79

$$\int \frac{(ade + (cd^2 + ae^2)x + cdex^2)^2}{(d + ex)^{11/2}}\,dx = \begin{cases} -\frac{6a^2e^4}{15d^2e^3\sqrt{d+ex}+30de^4x\sqrt{d+ex}+15e^5x^2\sqrt{d+ex}} - \frac{8acd^2e^2}{15d^2e^3\sqrt{d+ex}+30de^4x\sqrt{d+ex}+1} \\ \frac{c^2x^3}{3d^{\frac{3}{2}}} \end{cases}$$

input
```
integrate((a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**2/(e*x+d)**(11/2),x)
```

output
```
Piecewise((-6*a**2*e**4/(15*d**2*e**3*sqrt(d + e*x) + 30*d*e**4*x*sqrt(d +
 e*x) + 15*e**5*x**2*sqrt(d + e*x)) - 8*a*c*d**2*e**2/(15*d**2*e**3*sqrt(d
 + e*x) + 30*d*e**4*x*sqrt(d + e*x) + 15*e**5*x**2*sqrt(d + e*x)) - 20*a*c
*d*e**3*x/(15*d**2*e**3*sqrt(d + e*x) + 30*d*e**4*x*sqrt(d + e*x) + 15*e**
5*x**2*sqrt(d + e*x)) - 16*c**2*d**4/(15*d**2*e**3*sqrt(d + e*x) + 30*d*e*
*4*x*sqrt(d + e*x) + 15*e**5*x**2*sqrt(d + e*x)) - 40*c**2*d**3*e*x/(15*d*
*2*e**3*sqrt(d + e*x) + 30*d*e**4*x*sqrt(d + e*x) + 15*e**5*x**2*sqrt(d +
e*x)) - 30*c**2*d**2*e**2*x**2/(15*d**2*e**3*sqrt(d + e*x) + 30*d*e**4*x*s
qrt(d + e*x) + 15*e**5*x**2*sqrt(d + e*x)), Ne(e, 0)), (c**2*x**3/(3*d**(3
/2)), True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.95

$$\int \frac{(ade + (cd^2 + ae^2)x + cdex^2)^2}{(d+ex)^{11/2}}\,dx = -\frac{2\left(15\,(ex+d)^2c^2d^2 + 3\,c^2d^4 - 6\,acd^2e^2 + 3\,a^2e^4 - 10\,(c^2d^3 - acde^2)(ex+d)\right)}{15\,(ex+d)^{\frac{5}{2}}e^3}$$

input
```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^2/(e*x+d)^(11/2),x, algorithm=
"maxima")
```

output
```
-2/15*(15*(e*x + d)^2*c^2*d^2 + 3*c^2*d^4 - 6*a*c*d^2*e^2 + 3*a^2*e^4 - 10
*(c^2*d^3 - a*c*d*e^2)*(e*x + d))/((e*x + d)^(5/2)*e^3)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.99

$$\int \frac{(ade + (cd^2 + ae^2)x + cdex^2)^2}{(d+ex)^{11/2}}\,dx = -\frac{2\left(15\,(ex+d)^2c^2d^2 - 10\,(ex+d)c^2d^3 + 3\,c^2d^4 + 10\,(ex+d)acde^2 - 6\,acd^2e^2 + 3\,a^2e^4\right)}{15\,(ex+d)^{\frac{5}{2}}e^3}$$

input
```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^2/(e*x+d)^(11/2),x, algorithm=
"giac")
```

output
```
-2/15*(15*(e*x + d)^2*c^2*d^2 - 10*(e*x + d)*c^2*d^3 + 3*c^2*d^4 + 10*(e*x
 + d)*a*c*d*e^2 - 6*a*c*d^2*e^2 + 3*a^2*e^4)/((e*x + d)^(5/2)*e^3)
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.96

$$\int \frac{(ade + (cd^2 + ae^2)x + cdex^2)^2}{(d+ex)^{11/2}}\,dx = -\frac{\frac{2\,a^2\,e^4}{5} + \frac{2\,c^2\,d^4}{5} - \left(\frac{4\,c^2\,d^3}{3} - \frac{4\,a\,c\,d\,e^2}{3}\right)(d+e\,x) + 2\,c^2\,d^2\,(d+e\,x)^2 - \frac{4\,a\,c\,d^2\,e^2}{5}}{e^3\,(d+e\,x)^{5/2}}$$

input
```
int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^2/(d + e*x)^(11/2),x)
```

output
```
-((2*a^2*e^4)/5 + (2*c^2*d^4)/5 - ((4*c^2*d^3)/3 - (4*a*c*d*e^2)/3)*(d + e
*x) + 2*c^2*d^2*(d + e*x)^2 - (4*a*c*d^2*e^2)/5)/(e^3*(d + e*x)^(5/2))
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.12

$$\int \frac{(ade + (cd^2 + ae^2)x + cdex^2)^2}{(d+ex)^{11/2}}\,dx = \frac{-2c^2d^2e^2x^2 - \frac{4}{3}acd\,e^3x - \frac{8}{3}c^2d^3ex - \frac{2}{5}a^2e^4 - \frac{8}{15}ac\,d^2e^2 - \frac{16}{15}c^2d^4}{\sqrt{ex+d}\,e^3\,(e^2x^2 + 2dex + d^2)}$$

input
```
int((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^2/(e*x+d)^(11/2),x)
```

output
```
(2*( - 3*a**2*e**4 - 4*a*c*d**2*e**2 - 10*a*c*d*e**3*x - 8*c**2*d**4 - 20*
c**2*d**3*e*x - 15*c**2*d**2*e**2*x**2))/(15*sqrt(d + e*x)*e**3*(d**2 + 2*
d*e*x + e**2*x**2))
```

3.164 $\int \frac{(ade+(cd^2+ae^2)x+cdex^2)^2}{(d+ex)^{13/2}}\,dx$

Optimal result

Integrand size = 37, antiderivative size = 83

$$\int \frac{(ade+(cd^2+ae^2)\,x+cdex^2)^2}{(d+ex)^{13/2}}\,dx=-\frac{2(cd^2-ae^2)^2}{7e^3(d+ex)^{7/2}}+\frac{4cd(cd^2-ae^2)}{5e^3(d+ex)^{5/2}}-\frac{2c^2d^2}{3e^3(d+ex)^{3/2}}$$

output
```
-2/7*(-a*e^2+c*d^2)^2/e^3/(e*x+d)^(7/2)+4/5*c*d*(-a*e^2+c*d^2)/e^3/(e*x+d)
^(5/2)-2/3*c^2*d^2/e^3/(e*x+d)^(3/2)
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.81

$$\int \frac{(ade+(cd^2+ae^2)\,x+cdex^2)^2}{(d+ex)^{13/2}}\,dx = -\frac{2(15a^2e^4+6acde^2(2d+7ex)+c^2d^2(8d^2+28dex+35e^2x^2))}{105e^3(d+ex)^{7/2}}$$

input
```
Integrate[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^2/(d + e*x)^(13/2),x]
```

output
```
(-2*(15*a^2*e^4 + 6*a*c*d*e^2*(2*d + 7*e*x) + c^2*d^2*(8*d^2 + 28*d*e*x +
35*e^2*x^2)))/(105*e^3*(d + e*x)^(7/2))
```

Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.054$, Rules used = {1121, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^2}{(d+ex)^{13/2}}dx$$

$$\Big\downarrow \text{1121}$$

$$\int \left(-\frac{2cd\left(cd^2-ae^2\right)}{e^2(d+ex)^{7/2}}+\frac{\left(ae^2-cd^2\right)^2}{e^2(d+ex)^{9/2}}+\frac{c^2d^2}{e^2(d+ex)^{5/2}}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{4cd\left(cd^2-ae^2\right)}{5e^3(d+ex)^{5/2}}-\frac{2\left(cd^2-ae^2\right)^2}{7e^3(d+ex)^{7/2}}-\frac{2c^2d^2}{3e^3(d+ex)^{3/2}}$$

input
```
Int[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^2/(d + e*x)^(13/2),x]
```

output
```
(-2*(c*d^2 - a*e^2)^2)/(7*e^3*(d + e*x)^(7/2)) + (4*c*d*(c*d^2 - a*e^2))/(
5*e^3*(d + e*x)^(5/2)) - (2*c^2*d^2)/(3*e^3*(d + e*x)^(3/2))
```

Defintions of rubi rules used

rule 1121
```
Int[((d_) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Int[ExpandIntegrand[(d + e*x)^(m + p)*(a/d + (c/e)*x)^p, x], x]
/; FreeQ[{a, b, c, d, e, m, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && (Int
egerQ[p] || (GtQ[a, 0] && GtQ[d, 0] && LtQ[c, 0]))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 1.37 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.78

method	result	size
pseudoelliptic	$-\frac{2\left(a^{2} e^{4}+\frac{14 x a c d\, e^{3}}{5}+\frac{4 c\, d^{2}\left(\frac{35 c\, x^{2}}{12}+a\right) e^{2}}{5}+\frac{28 x\, c^{2} d^{3} e}{15}+\frac{8 c^{2} d^{4}}{15}\right)}{7 \left(e x +d \right)^{\frac{7}{2}} e^{3}}$	65
gosper	$-\frac{2 \left(35 x^{2} c^{2} d^{2} e^{2}+42 x a c d\, e^{3}+28 x\, c^{2} d^{3} e +15 a^{2} e^{4}+12 a c\, d^{2} e^{2}+8 c^{2} d^{4}\right)}{105 \left(e x +d \right)^{\frac{7}{2}} e^{3}}$	73
trager	$-\frac{2 \left(35 x^{2} c^{2} d^{2} e^{2}+42 x a c d\, e^{3}+28 x\, c^{2} d^{3} e +15 a^{2} e^{4}+12 a c\, d^{2} e^{2}+8 c^{2} d^{4}\right)}{105 \left(e x +d \right)^{\frac{7}{2}} e^{3}}$	73
derivativedivides	$\frac{-\frac{4 c d \left(a\, e^{2}-c\, d^{2}\right)}{5 \left(e x +d \right)^{\frac{5}{2}}}-\frac{2 \left(a^{2} e^{4}-2 a c\, d^{2} e^{2}+c^{2} d^{4}\right)}{7 \left(e x +d \right)^{\frac{7}{2}}}-\frac{2 c^{2} d^{2}}{3 \left(e x +d \right)^{\frac{3}{2}}}}{e^{3}}$	79
default	$\frac{-\frac{4 c d \left(a\, e^{2}-c\, d^{2}\right)}{5 \left(e x +d \right)^{\frac{5}{2}}}-\frac{2 \left(a^{2} e^{4}-2 a c\, d^{2} e^{2}+c^{2} d^{4}\right)}{7 \left(e x +d \right)^{\frac{7}{2}}}-\frac{2 c^{2} d^{2}}{3 \left(e x +d \right)^{\frac{3}{2}}}}{e^{3}}$	79
orering	$-\frac{2 \left(35 x^{2} c^{2} d^{2} e^{2}+42 x a c d\, e^{3}+28 x\, c^{2} d^{3} e +15 a^{2} e^{4}+12 a c\, d^{2} e^{2}+8 c^{2} d^{4}\right) \left(a d e +\left(a\, e^{2}+c\, d^{2}\right) x +c d\, x^{2} e \right)^{2}}{105 e^{3} \left(c d x +a e \right)^{2} \left(e x +d \right)^{\frac{11}{2}}}$	110

input
```
int((a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^2/(e*x+d)^(13/2),x,method=_RETURNVER
BOSE)
```

output
```
-2/7/(e*x+d)^(7/2)*(a^2*e^4+14/5*x*a*c*d*e^3+4/5*c*d^2*(35/12*c*x^2+a)*e^2
+28/15*x*c^2*d^3*e+8/15*c^2*d^4)/e^3
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.41

$$\int \frac{(ade + (cd^2 + ae^2)x + cdex^2)^2}{(d + ex)^{13/2}} dx = -\frac{2\left(35\,c^2d^2e^2x^2 + 8\,c^2d^4 + 12\,acd^2e^2 + 15\,a^2e^4 + 14\,(2\,c^2d^3e + 3\,acde^3)x\right)\sqrt{ex + d}}{105\,(e^7x^4 + 4\,de^6x^3 + 6\,d^2e^5x^2 + 4\,d^3e^4x + d^4e^3)}$$

input
```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^2/(e*x+d)^(13/2),x, algorithm=
"fricas")
```

output
```
-2/105*(35*c^2*d^2*e^2*x^2 + 8*c^2*d^4 + 12*a*c*d^2*e^2 + 15*a^2*e^4 + 14*
(2*c^2*d^3*e + 3*a*c*d*e^3)*x)*sqrt(e*x + d)/(e^7*x^4 + 4*d*e^6*x^3 + 6*d^
2*e^5*x^2 + 4*d^3*e^4*x + d^4*e^3)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 510 vs. 2(76) = 152.

Time = 1.39 (sec) , antiderivative size = 510, normalized size of antiderivative = 6.14

$$\int \frac{(ade + (cd^2 + ae^2)x + cdex^2)^2}{(d + ex)^{13/2}} dx = \begin{cases} -\frac{30a^2e^4}{105d^3e^3\sqrt{d+ex}+315d^2e^4x\sqrt{d+ex}+315de^5x^2\sqrt{d+ex}+105e^6x^3\sqrt{d+ex}} - \frac{}{105d^3e^3\sqrt{\cdot}} \\ \frac{c^2x^3}{3d^{\frac{5}{2}}} \end{cases}$$

input
```
integrate((a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**2/(e*x+d)**(13/2),x)
```

output
```
Piecewise((-30*a**2*e**4/(105*d**3*e**3*sqrt(d + e*x) + 315*d**2*e**4*x*sq
rt(d + e*x) + 315*d*e**5*x**2*sqrt(d + e*x) + 105*e**6*x**3*sqrt(d + e*x))
 - 24*a*c*d**2*e**2/(105*d**3*e**3*sqrt(d + e*x) + 315*d**2*e**4*x*sqrt(d
+ e*x) + 315*d*e**5*x**2*sqrt(d + e*x) + 105*e**6*x**3*sqrt(d + e*x)) - 84
*a*c*d*e**3*x/(105*d**3*e**3*sqrt(d + e*x) + 315*d**2*e**4*x*sqrt(d + e*x)
 + 315*d*e**5*x**2*sqrt(d + e*x) + 105*e**6*x**3*sqrt(d + e*x)) - 16*c**2*
d**4/(105*d**3*e**3*sqrt(d + e*x) + 315*d**2*e**4*x*sqrt(d + e*x) + 315*d*
e**5*x**2*sqrt(d + e*x) + 105*e**6*x**3*sqrt(d + e*x)) - 56*c**2*d**3*e*x/
(105*d**3*e**3*sqrt(d + e*x) + 315*d**2*e**4*x*sqrt(d + e*x) + 315*d*e**5*
x**2*sqrt(d + e*x) + 105*e**6*x**3*sqrt(d + e*x)) - 70*c**2*d**2*e**2*x**2
/(105*d**3*e**3*sqrt(d + e*x) + 315*d**2*e**4*x*sqrt(d + e*x) + 315*d*e**5
*x**2*sqrt(d + e*x) + 105*e**6*x**3*sqrt(d + e*x)), Ne(e, 0)), (c**2*x**3/
(3*d**(5/2)), True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.93

$$\int \frac{(ade + (cd^2 + ae^2)\,x + cdex^2)^2}{(d+ex)^{13/2}}\,dx =$$
$$-\frac{2\left(35\,(ex+d)^2c^2d^2 + 15\,c^2d^4 - 30\,acd^2e^2 + 15\,a^2e^4 - 42\,(c^2d^3 - acde^2)(ex+d)\right)}{105\,(ex+d)^{\frac{7}{2}}e^3}$$

input
```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^2/(e*x+d)^(13/2),x, algorithm=
"maxima")
```

output
```
-2/105*(35*(e*x + d)^2*c^2*d^2 + 15*c^2*d^4 - 30*a*c*d^2*e^2 + 15*a^2*e^4
- 42*(c^2*d^3 - a*c*d*e^2)*(e*x + d))/((e*x + d)^(7/2)*e^3)
```

Giac [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.96

$$\int \frac{(ade+(cd^2+ae^2)x+cdex^2)^2}{(d+ex)^{13/2}}\,dx = -\frac{2\left(35\,(ex+d)^2c^2d^2-42\,(ex+d)c^2d^3+15\,c^2d^4+42\,(ex+d)acde^2-30\,acd^2e^2+15\,a^2e^4\right)}{105\,(ex+d)^{\frac{7}{2}}e^3}$$

input
```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^2/(e*x+d)^(13/2),x, algorithm=
"giac")
```

output
```
-2/105*(35*(e*x + d)^2*c^2*d^2 - 42*(e*x + d)*c^2*d^3 + 15*c^2*d^4 + 42*(e
*x + d)*a*c*d*e^2 - 30*a*c*d^2*e^2 + 15*a^2*e^4)/((e*x + d)^(7/2)*e^3)
```

Mupad [B] (verification not implemented)

Time = 5.15 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.94

$$\int \frac{(ade+(cd^2+ae^2)x+cdex^2)^2}{(d+ex)^{13/2}}\,dx = -\frac{\frac{2\,a^2\,e^4}{7}+\frac{2\,c^2\,d^4}{7}-\left(\frac{4\,c^2\,d^3}{5}-\frac{4\,a\,c\,d\,e^2}{5}\right)\,(d+e\,x)+\frac{2\,c^2\,d^2\,(d+e\,x)^2}{3}-\frac{4\,a\,c\,d^2\,e^2}{7}}{e^3\,(d+e\,x)^{7/2}}$$

input
```
int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^2/(d + e*x)^(13/2),x)
```

output
```
-((2*a^2*e^4)/7 + (2*c^2*d^4)/7 - ((4*c^2*d^3)/5 - (4*a*c*d*e^2)/5)*(d + e
*x) + (2*c^2*d^2*(d + e*x)^2)/3 - (4*a*c*d^2*e^2)/7)/(e^3*(d + e*x)^(7/2))
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 102, normalized size of antiderivative = 1.23

$$\int \frac{(ade+(cd^2+ae^2)x+cdex^2)^2}{(d+ex)^{13/2}}\,dx = \frac{-\frac{2}{3}c^2d^2e^2x^2-\frac{4}{5}acd\,e^3x-\frac{8}{15}c^2d^3ex-\frac{2}{7}a^2e^4-\frac{8}{35}ac\,d^2e^2-\frac{16}{105}c^2d^4}{\sqrt{ex+d}\,e^3\left(e^3x^3+3d\,e^2x^2+3d^2ex+d^3\right)}$$

input

```
int((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^2/(e*x+d)^(13/2),x)
```

output

```
(2*( - 15*a**2*e**4 - 12*a*c*d**2*e**2 - 42*a*c*d*e**3*x - 8*c**2*d**4 - 2
8*c**2*d**3*e*x - 35*c**2*d**2*e**2*x**2))/(105*sqrt(d + e*x)*e**3*(d**3 +
 3*d**2*e*x + 3*d*e**2*x**2 + e**3*x**3))
```

3.165 $\int \sqrt{d+ex}\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^3\,dx$

Optimal result

Integrand size = 37, antiderivative size = 119

$$\int \sqrt{d+ex}\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^3\,dx = -\frac{2\left(cd^2-ae^2\right)^3(d+ex)^{9/2}}{9e^4}+\frac{6cd\left(cd^2-ae^2\right)^2(d+ex)^{11/2}}{11e^4} - \frac{6c^2d^2\left(cd^2-ae^2\right)(d+ex)^{13/2}}{13e^4}+\frac{2c^3d^3(d+ex)^{15/2}}{15e^4}$$

output
```
-2/9*(-a*e^2+c*d^2)^3*(e*x+d)^(9/2)/e^4+6/11*c*d*(-a*e^2+c*d^2)^2*(e*x+d)^
(11/2)/e^4-6/13*c^2*d^2*(-a*e^2+c*d^2)*(e*x+d)^(13/2)/e^4+2/15*c^3*d^3*(e*
x+d)^(15/2)/e^4
```

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 111, normalized size of antiderivative = 0.93

$$\int \sqrt{d+ex}\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^3\,dx = \frac{2(d+ex)^{9/2}\left(715a^3e^6-195a^2cde^4(2d-9ex)+15ac^2d^2e^2\left(8d^2-36dex+99e^2x^2\right)+c^3d^3\left(-16d^3+72d^2\right.\right.}{6435e^4}$$

input
```
Integrate[Sqrt[d + e*x]*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^3,x]
```

output
```
(2*(d + e*x)^(9/2)*(715*a^3*e^6 - 195*a^2*c*d*e^4*(2*d - 9*e*x) + 15*a*c^2
*d^2*e^2*(8*d^2 - 36*d*e*x + 99*e^2*x^2) + c^3*d^3*(-16*d^3 + 72*d^2*e*x -
 198*d*e^2*x^2 + 429*e^3*x^3)))/(6435*e^4)
```

Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.054, Rules used = {1121, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{d+ex}\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^3 dx$$

$$\Big\downarrow \text{1121}$$

$$\int \left(-\frac{3c^2d^2(d+ex)^{11/2}\left(cd^2-ae^2\right)}{e^3}+\frac{3cd(d+ex)^{9/2}\left(cd^2-ae^2\right)^2}{e^3}+\frac{(d+ex)^{7/2}\left(ae^2-cd^2\right)^3}{e^3}+\frac{c^3d^3(d+ex)^1}{e^3}\right.$$

$$\Big\downarrow \text{2009}$$

$$-\frac{6c^2d^2(d+ex)^{13/2}\left(cd^2-ae^2\right)}{13e^4}+\frac{6cd(d+ex)^{11/2}\left(cd^2-ae^2\right)^2}{11e^4}-\frac{2(d+ex)^{9/2}\left(cd^2-ae^2\right)^3}{9e^4}+\frac{2c^3d^3(d+ex)^{15/2}}{15e^4}$$

input
```
Int[Sqrt[d + e*x]*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^3,x]
```

output
```
(-2*(c*d^2 - a*e^2)^3*(d + e*x)^(9/2))/(9*e^4) + (6*c*d*(c*d^2 - a*e^2)^2*
(d + e*x)^(11/2))/(11*e^4) - (6*c^2*d^2*(c*d^2 - a*e^2)*(d + e*x)^(13/2))/
(13*e^4) + (2*c^3*d^3*(d + e*x)^(15/2))/(15*e^4)
```

Defintions of rubi rules used

rule 1121
```
Int[((d_) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Int[ExpandIntegrand[(d + e*x)^(m + p)*(a/d + (c/e)*x)^p, x], x]
/; FreeQ[{a, b, c, d, e, m, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && (Int
egerQ[p] || (GtQ[a, 0] && GtQ[d, 0] && LtQ[c, 0]))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 2.56 (sec) , antiderivative size = 97, normalized size of antiderivative = 0.82

method	result
derivativedivides	$\frac{\frac{2c^3d^3(ex+d)^{\frac{15}{2}}}{15}+\frac{6\left(a\,e^2-c\,d^2\right)c^2d^2(ex+d)^{\frac{13}{2}}}{13}+\frac{6\left(a\,e^2-c\,d^2\right)^2cd(ex+d)^{\frac{11}{2}}}{11}+\frac{2\left(a\,e^2-c\,d^2\right)^3(ex+d)^{\frac{9}{2}}}{9}}{e^4}$
default	$\frac{\frac{2c^3d^3(ex+d)^{\frac{15}{2}}}{15}+\frac{6\left(a\,e^2-c\,d^2\right)c^2d^2(ex+d)^{\frac{13}{2}}}{13}+\frac{6\left(a\,e^2-c\,d^2\right)^2cd(ex+d)^{\frac{11}{2}}}{11}+\frac{2\left(a\,e^2-c\,d^2\right)^3(ex+d)^{\frac{9}{2}}}{9}}{e^4}$
pseudoelliptic	$\frac{2(ex+d)^{\frac{9}{2}}\left(e^6a^3+\frac{27x\,a^2cd\,e^5}{11}-\frac{6ca\,d^2\left(-\frac{99c\,x^2}{26}+a\right)e^4}{11}-\frac{108x\,c^2d^3\left(-\frac{143c\,x^2}{180}+a\right)e^3}{143}+\frac{24c^2d^4\left(-\frac{33c\,x^2}{20}+a\right)e^2}{143}+\frac{72c^3d^5ex}{715}\right.}{9e^4}$
gosper	$\frac{2(ex+d)^{\frac{9}{2}}\left(429c^3d^3e^3x^3+1485x^2a\,c^2d^2e^4-198c^3d^4e^2x^2+1755x\,a^2cd\,e^5-540xa\,c^2d^3e^3+72c^3d^5ex+715e^6a^3-390d^2e\right.}{6435e^4}$
orering	$\frac{2\left(429c^3d^3e^3x^3+1485x^2a\,c^2d^2e^4-198c^3d^4e^2x^2+1755x\,a^2cd\,e^5-540xa\,c^2d^3e^3+72c^3d^5ex+715e^6a^3-390d^2e^4a^2c+120\right.}{6435e^4(cdx+ae)^3}$
trager	$\frac{2\left(429c^3d^3e^7x^7+1485a\,c^2d^2e^8x^6+1518c^3d^4e^6x^6+1755a^2cd\,e^9x^5+5400a\,c^2d^3e^7x^5+1854c^3d^5e^5x^5+715a^3e^{10}x^4+663\right.}{}$
risch	$\frac{2\left(429c^3d^3e^7x^7+1485a\,c^2d^2e^8x^6+1518c^3d^4e^6x^6+1755a^2cd\,e^9x^5+5400a\,c^2d^3e^7x^5+1854c^3d^5e^5x^5+715a^3e^{10}x^4+663\right.}{}$

input
```
int((e*x+d)^(1/2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^3,x,method=_RETURNVERB
OSE)
```

output
```
2/e^4*(1/15*c^3*d^3*(e*x+d)^(15/2)+3/13*(a*e^2-c*d^2)*c^2*d^2*(e*x+d)^(13/
2)+3/11*(a*e^2-c*d^2)^2*c*d*(e*x+d)^(11/2)+1/9*(a*e^2-c*d^2)^3*(e*x+d)^(9/
2))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 335 vs. $2(103) = 206$.

Time = 0.08 (sec) , antiderivative size = 335, normalized size of antiderivative = 2.82

$$\int \sqrt{d+ex}\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^3\,dx = \frac{2\left(429\,c^3d^3e^7x^7-16\,c^3d^{10}+120\,ac^2d^8e^2-390\,a^2cd^6e^4+715\,a^3d^4e^6+33\,(46\,c^3d^4e^6+45\,ac^2d^2e^8)x^6+9\right.}{}$$

input
```
integrate((e*x+d)^(1/2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^3,x, algorithm="
fricas")
```

output
```
2/6435*(429*c^3*d^3*e^7*x^7 - 16*c^3*d^10 + 120*a*c^2*d^8*e^2 - 390*a^2*c*
d^6*e^4 + 715*a^3*d^4*e^6 + 33*(46*c^3*d^4*e^6 + 45*a*c^2*d^2*e^8)*x^6 + 9
*(206*c^3*d^5*e^5 + 600*a*c^2*d^3*e^7 + 195*a^2*c*d*e^9)*x^5 + 5*(160*c^3*
d^6*e^4 + 1374*a*c^2*d^4*e^6 + 1326*a^2*c*d^2*e^8 + 143*a^3*e^10)*x^4 + 5*
(c^3*d^7*e^3 + 636*a*c^2*d^5*e^5 + 1794*a^2*c*d^3*e^7 + 572*a^3*d*e^9)*x^3
 - 3*(2*c^3*d^8*e^2 - 15*a*c^2*d^6*e^4 - 1560*a^2*c*d^4*e^6 - 1430*a^3*d^2
*e^8)*x^2 + (8*c^3*d^9*e - 60*a*c^2*d^7*e^3 + 195*a^2*c*d^5*e^5 + 2860*a^3
*d^3*e^7)*x)*sqrt(e*x + d)/e^4
```

Sympy [A] (verification not implemented)

Time = 0.98 (sec) , antiderivative size = 178, normalized size of antiderivative = 1.50

$$\int \sqrt{d+ex}\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^3\,dx = \begin{cases} \frac{2\left(\frac{c^3d^3(d+ex)^{\frac{15}{2}}}{15e^3}+\frac{(d+ex)^{\frac{13}{2}}\cdot\left(3ac^2d^2e^2-3c^3d^4\right)}{13e^3}+\frac{(d+ex)^{\frac{11}{2}}\cdot\left(3a^2cde^4-6ac^2d^3e^2+3c^3d^5\right)}{11e^3}+\frac{(d+ex)^{\frac{9}{2}}\left(a^3e^6-3a^2cd^2e^4+3ac^2d^4e^2-c^3d^6\right)}{9e^3}\right)}{e} & \text{for} \\ \frac{c^3d^{\frac{13}{2}}x^4}{4} & \text{oth} \end{cases}$$

input
```
integrate((e*x+d)**(1/2)*(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**3,x)
```

output
```
Piecewise((2*(c**3*d**3*(d + e*x)**(15/2)/(15*e**3) + (d + e*x)**(13/2)*(3
*a*c**2*d**2*e**2 - 3*c**3*d**4)/(13*e**3) + (d + e*x)**(11/2)*(3*a**2*c*d
*e**4 - 6*a*c**2*d**3*e**2 + 3*c**3*d**5)/(11*e**3) + (d + e*x)**(9/2)*(a*
*3*e**6 - 3*a**2*c*d**2*e**4 + 3*a*c**2*d**4*e**2 - c**3*d**6)/(9*e**3))/e
, Ne(e, 0)), (c**3*d**(13/2)*x**4/4, True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 137, normalized size of antiderivative = 1.15

$$\int \sqrt{d+ex}\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^3\,dx = \frac{2\left(429\,(ex+d)^{\frac{15}{2}}c^3d^3-1485\,(c^3d^4-ac^2d^2e^2)(ex+d)^{\frac{13}{2}}+1755\,(c^3d^5-2\,ac^2d^3e^2+a^2cde^4)(ex+d)^{\frac{11}{2}}-\right.}{6435\,e^4}$$

input
```
integrate((e*x+d)^(1/2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^3,x, algorithm="
maxima")
```

output
```
2/6435*(429*(e*x + d)^(15/2)*c^3*d^3 - 1485*(c^3*d^4 - a*c^2*d^2*e^2)*(e*x
 + d)^(13/2) + 1755*(c^3*d^5 - 2*a*c^2*d^3*e^2 + a^2*c*d*e^4)*(e*x + d)^(1
1/2) - 715*(c^3*d^6 - 3*a*c^2*d^4*e^2 + 3*a^2*c*d^2*e^4 - a^3*e^6)*(e*x +
d)^(9/2))/e^4
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1214 vs. $2(103) = 206$.

Time = 0.16 (sec) , antiderivative size = 1214, normalized size of antiderivative = 10.20

$$\int \sqrt{d+ex}\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^3\,dx = \text{Too large to display}$$

input
```
integrate((e*x+d)^(1/2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^3,x, algorithm="
giac")
```

output
```
2/45045*(45045*sqrt(e*x + d)*a^3*d^4*e^3 + 45045*((e*x + d)^(3/2) - 3*sqrt
(e*x + d)*d)*a^2*c*d^5*e + 60060*((e*x + d)^(3/2) - 3*sqrt(e*x + d)*d)*a^3
*d^3*e^3 + 9009*(3*(e*x + d)^(5/2) - 10*(e*x + d)^(3/2)*d + 15*sqrt(e*x +
d)*d^2)*a*c^2*d^6/e + 36036*(3*(e*x + d)^(5/2) - 10*(e*x + d)^(3/2)*d + 15
*sqrt(e*x + d)*d^2)*a^2*c*d^4*e + 18018*(3*(e*x + d)^(5/2) - 10*(e*x + d)^
(3/2)*d + 15*sqrt(e*x + d)*d^2)*a^3*d^2*e^3 + 1287*(5*(e*x + d)^(7/2) - 21
*(e*x + d)^(5/2)*d + 35*(e*x + d)^(3/2)*d^2 - 35*sqrt(e*x + d)*d^3)*c^3*d^
7/e^3 + 15444*(5*(e*x + d)^(7/2) - 21*(e*x + d)^(5/2)*d + 35*(e*x + d)^(3/
2)*d^2 - 35*sqrt(e*x + d)*d^3)*a*c^2*d^5/e + 23166*(5*(e*x + d)^(7/2) - 21
*(e*x + d)^(5/2)*d + 35*(e*x + d)^(3/2)*d^2 - 35*sqrt(e*x + d)*d^3)*a^2*c*
d^3*e + 5148*(5*(e*x + d)^(7/2) - 21*(e*x + d)^(5/2)*d + 35*(e*x + d)^(3/2
)*d^2 - 35*sqrt(e*x + d)*d^3)*a^3*d*e^3 + 572*(35*(e*x + d)^(9/2) - 180*(e
*x + d)^(7/2)*d + 378*(e*x + d)^(5/2)*d^2 - 420*(e*x + d)^(3/2)*d^3 + 315*
sqrt(e*x + d)*d^4)*c^3*d^6/e^3 + 2574*(35*(e*x + d)^(9/2) - 180*(e*x + d)^
(7/2)*d + 378*(e*x + d)^(5/2)*d^2 - 420*(e*x + d)^(3/2)*d^3 + 315*sqrt(e*x
 + d)*d^4)*a*c^2*d^4/e + 1716*(35*(e*x + d)^(9/2) - 180*(e*x + d)^(7/2)*d
+ 378*(e*x + d)^(5/2)*d^2 - 420*(e*x + d)^(3/2)*d^3 + 315*sqrt(e*x + d)*d^
4)*a^2*c*d^2*e + 143*(35*(e*x + d)^(9/2) - 180*(e*x + d)^(7/2)*d + 378*(e*
x + d)^(5/2)*d^2 - 420*(e*x + d)^(3/2)*d^3 + 315*sqrt(e*x + d)*d^4)*a^3*e^
3 + 390*(63*(e*x + d)^(11/2) - 385*(e*x + d)^(9/2)*d + 990*(e*x + d)^(7...
```

Mupad [B] (verification not implemented)

Time = 5.16 (sec) , antiderivative size = 106, normalized size of antiderivative = 0.89

$$\int \sqrt{d+ex}\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^3\,dx = \frac{2\,{\left(a\,e^2-c\,d^2\right)}^3\,{\left(d+e\,x\right)}^{9/2}}{9\,e^4}-\frac{\left(6\,c^3\,d^4-6\,a\,c^2\,d^2\,e^2\right)\,{\left(d+e\,x\right)}^{13/2}}{13\,e^4}+\frac{2\,c^3\,d^3\,{\left(d+e\,x\right)}^{15/2}}{15\,e^4}+\frac{6\,c\,d\,{\left(a\,e^2-c\,d^2\right)}^2\,{\left(d+e\,x\right)}^{11/2}}{11\,e^4}$$

input
```
int((d + e*x)^(1/2)*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^3,x)
```

output
```
(2*(a*e^2 - c*d^2)^3*(d + e*x)^(9/2))/(9*e^4) - ((6*c^3*d^4 - 6*a*c^2*d^2*
e^2)*(d + e*x)^(13/2))/(13*e^4) + (2*c^3*d^3*(d + e*x)^(15/2))/(15*e^4) +
(6*c*d*(a*e^2 - c*d^2)^2*(d + e*x)^(11/2))/(11*e^4)
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 357, normalized size of antiderivative = 3.00

$$\int \sqrt{d+ex}\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^3\,dx$$
$$=\frac{2\sqrt{ex+d}\,(429c^3d^3e^7x^7+1485a\,c^2d^2e^8x^6+1518c^3d^4e^6x^6+1755a^2cd\,e^9x^5+5400a\,c^2d^3e^7x^5+1854c^3d^5e^5 \cdots}{}$$

input
```
int((e*x+d)^(1/2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^3,x)
```

output
```
(2*sqrt(d + e*x)*(715*a**3*d**4*e**6 + 2860*a**3*d**3*e**7*x + 4290*a**3*d
**2*e**8*x**2 + 2860*a**3*d*e**9*x**3 + 715*a**3*e**10*x**4 - 390*a**2*c*d
**6*e**4 + 195*a**2*c*d**5*e**5*x + 4680*a**2*c*d**4*e**6*x**2 + 8970*a**2
*c*d**3*e**7*x**3 + 6630*a**2*c*d**2*e**8*x**4 + 1755*a**2*c*d*e**9*x**5 +
 120*a*c**2*d**8*e**2 - 60*a*c**2*d**7*e**3*x + 45*a*c**2*d**6*e**4*x**2 +
 3180*a*c**2*d**5*e**5*x**3 + 6870*a*c**2*d**4*e**6*x**4 + 5400*a*c**2*d**
3*e**7*x**5 + 1485*a*c**2*d**2*e**8*x**6 - 16*c**3*d**10 + 8*c**3*d**9*e*x
 - 6*c**3*d**8*e**2*x**2 + 5*c**3*d**7*e**3*x**3 + 800*c**3*d**6*e**4*x**4
 + 1854*c**3*d**5*e**5*x**5 + 1518*c**3*d**4*e**6*x**6 + 429*c**3*d**3*e**
7*x**7))/(6435*e**4)
```

3.166 $\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^3}{\sqrt{d+ex}}\,dx$

Optimal result

Integrand size = 37, antiderivative size = 119

$$\int \frac{(ade+(cd^2+ae^2)\,x+cdex^2)^3}{\sqrt{d+ex}}\,dx=-\frac{2(cd^2-ae^2)^3\,(d+ex)^{7/2}}{7e^4}+\frac{2cd(cd^2-ae^2)^2\,(d+ex)^{9/2}}{3e^4}-\frac{6c^2d^2(cd^2-ae^2)\,(d+ex)^{11/2}}{11e^4}+\frac{2c^3d^3(d+ex)^{13/2}}{13e^4}$$

output
```
-2/7*(-a*e^2+c*d^2)^3*(e*x+d)^(7/2)/e^4+2/3*c*d*(-a*e^2+c*d^2)^2*(e*x+d)^(
9/2)/e^4-6/11*c^2*d^2*(-a*e^2+c*d^2)*(e*x+d)^(11/2)/e^4+2/13*c^3*d^3*(e*x+
d)^(13/2)/e^4
```

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 111, normalized size of antiderivative = 0.93

$$\int \frac{(ade + (cd^2 + ae^2)x + cdex^2)^3}{\sqrt{d+ex}}\,dx$$
$$= \frac{2(d+ex)^{7/2}(429a^3e^6 - 143a^2cde^4(2d-7ex) + 13ac^2d^2e^2(8d^2 - 28dex + 63e^2x^2) + c^3d^3(-16d^3 + 56d^2}{3003e^4}$$

input
```
Integrate[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^3/Sqrt[d + e*x],x]
```

output
```
(2*(d + e*x)^(7/2)*(429*a^3*e^6 - 143*a^2*c*d*e^4*(2*d - 7*e*x) + 13*a*c^2
*d^2*e^2*(8*d^2 - 28*d*e*x + 63*e^2*x^2) + c^3*d^3*(-16*d^3 + 56*d^2*e*x -
 126*d*e^2*x^2 + 231*e^3*x^3)))/(3003*e^4)
```

Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.054$, Rules used = {1121, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(x(ae^2+cd^2) + ade + cdex^2)^3}{\sqrt{d+ex}}\,dx$$

$$\Big\downarrow \text{1121}$$

$$\int \left(-\frac{3c^2d^2(d+ex)^{9/2}(cd^2-ae^2)}{e^3} + \frac{3cd(d+ex)^{7/2}(cd^2-ae^2)^2}{e^3} + \frac{(d+ex)^{5/2}(ae^2-cd^2)^3}{e^3} + \frac{c^3d^3(d+ex)^{11}}{e^3}\right.$$

$$\Big\downarrow \text{2009}$$

$$-\frac{6c^2d^2(d+ex)^{11/2}(cd^2-ae^2)}{11e^4} + \frac{2cd(d+ex)^{9/2}(cd^2-ae^2)^2}{3e^4} - \frac{2(d+ex)^{7/2}(cd^2-ae^2)^3}{7e^4} + \frac{2c^3d^3(d+ex)^{13/2}}{13e^4}$$

input
```
Int[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^3/Sqrt[d + e*x],x]
```

output
```
(-2*(c*d^2 - a*e^2)^3*(d + e*x)^(7/2))/(7*e^4) + (2*c*d*(c*d^2 - a*e^2)^2*
(d + e*x)^(9/2))/(3*e^4) - (6*c^2*d^2*(c*d^2 - a*e^2)*(d + e*x)^(11/2))/(1
1*e^4) + (2*c^3*d^3*(d + e*x)^(13/2))/(13*e^4)
```

Defintions of rubi rules used

rule 1121
```
Int[((d_) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Int[ExpandIntegrand[(d + e*x)^(m + p)*(a/d + (c/e)*x)^p, x], x]
/; FreeQ[{a, b, c, d, e, m, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && (Int
egerQ[p] || (GtQ[a, 0] && GtQ[d, 0] && LtQ[c, 0]))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 2.38 (sec) , antiderivative size = 97, normalized size of antiderivative = 0.82

method	result
derivativedivides	$\frac{\frac{2c^3d^3(ex+d)^{\frac{13}{2}}}{13}+\frac{6\left(a\,e^2-c\,d^2\right)c^2d^2(ex+d)^{\frac{11}{2}}}{11}+\frac{2\left(a\,e^2-c\,d^2\right)^2cd(ex+d)^{\frac{9}{2}}}{3}+\frac{2\left(a\,e^2-c\,d^2\right)^3(ex+d)^{\frac{7}{2}}}{7}}{e^4}$
default	$\frac{\frac{2c^3d^3(ex+d)^{\frac{13}{2}}}{13}+\frac{6\left(a\,e^2-c\,d^2\right)c^2d^2(ex+d)^{\frac{11}{2}}}{11}+\frac{2\left(a\,e^2-c\,d^2\right)^2cd(ex+d)^{\frac{9}{2}}}{3}+\frac{2\left(a\,e^2-c\,d^2\right)^3(ex+d)^{\frac{7}{2}}}{7}}{e^4}$
pseudoelliptic	$\frac{2(ex+d)^{\frac{7}{2}}\left(e^6a^3+\frac{7x\,a^2cd\,e^5}{3}-\frac{2\left(-\frac{63c\,x^2}{22}+a\right)ca\,d^2e^4}{3}-\frac{28x\,c^2d^3\left(-\frac{33c\,x^2}{52}+a\right)e^3}{33}+\frac{8c^2d^4\left(-\frac{63c\,x^2}{52}+a\right)e^2}{33}+\frac{56c^3d^5ex}{429}-\cdots\right)}{7e^4}$
gosper	$\frac{2(ex+d)^{\frac{7}{2}}\left(231c^3d^3e^3x^3+819x^2a\,c^2d^2e^4-126c^3d^4e^2x^2+1001x\,a^2cd\,e^5-364xa\,c^2d^3e^3+56c^3d^5ex+429e^6a^3-286d^2e^4\cdots\right)}{3003e^4}$
orering	$\frac{2\left(231c^3d^3e^3x^3+819x^2a\,c^2d^2e^4-126c^3d^4e^2x^2+1001x\,a^2cd\,e^5-364xa\,c^2d^3e^3+56c^3d^5ex+429e^6a^3-286d^2e^4a^2c+104\cdots\right)}{3003e^4(cdx+ae)^3}$
trager	$\frac{2\left(231c^3d^3e^6x^6+819a\,c^2d^2e^7x^5+567c^3d^4e^5x^5+1001a^2cd\,e^8x^4+2093a\,c^2d^3e^6x^4+371c^3d^5e^4x^4+429a^3e^9x^3+2717a^2c\cdots\right)}{\cdots}$
risch	$\frac{2\left(231c^3d^3e^6x^6+819a\,c^2d^2e^7x^5+567c^3d^4e^5x^5+1001a^2cd\,e^8x^4+2093a\,c^2d^3e^6x^4+371c^3d^5e^4x^4+429a^3e^9x^3+2717a^2c\cdots\right)}{\cdots}$

input
```
int((a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^3/(e*x+d)^(1/2),x,method=_RETURNVERB
OSE)
```

output
```
2/e^4*(1/13*c^3*d^3*(e*x+d)^(13/2)+3/11*(a*e^2-c*d^2)*c^2*d^2*(e*x+d)^(11/
2)+1/3*(a*e^2-c*d^2)^2*c*d*(e*x+d)^(9/2)+1/7*(a*e^2-c*d^2)^3*(e*x+d)^(7/2)
)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 283 vs. $2(103) = 206$.

Time = 0.08 (sec) , antiderivative size = 283, normalized size of antiderivative = 2.38

$$\int \frac{(ade + (cd^2 + ae^2)x + cdex^2)^3}{\sqrt{d + ex}} dx = \frac{2 (231 c^3 d^3 e^6 x^6 - 16 c^3 d^9 + 104 ac^2 d^7 e^2 - 286 a^2 cd^5 e^4 + 429 a^3 d^3 e^6 + 63 (9 c^3 d^4 e^5 + 13 ac^2 d^2 e^7) x^5 + 7 (5}{}$$

input
```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^3/(e*x+d)^(1/2),x, algorithm="
fricas")
```

output
```
2/3003*(231*c^3*d^3*e^6*x^6 - 16*c^3*d^9 + 104*a*c^2*d^7*e^2 - 286*a^2*c*d
^5*e^4 + 429*a^3*d^3*e^6 + 63*(9*c^3*d^4*e^5 + 13*a*c^2*d^2*e^7)*x^5 + 7*(
53*c^3*d^5*e^4 + 299*a*c^2*d^3*e^6 + 143*a^2*c*d*e^8)*x^4 + (5*c^3*d^6*e^3
 + 1469*a*c^2*d^4*e^5 + 2717*a^2*c*d^2*e^7 + 429*a^3*e^9)*x^3 - 3*(2*c^3*d
^7*e^2 - 13*a*c^2*d^5*e^4 - 715*a^2*c*d^3*e^6 - 429*a^3*d*e^8)*x^2 + (8*c^
3*d^8*e - 52*a*c^2*d^6*e^3 + 143*a^2*c*d^4*e^5 + 1287*a^3*d^2*e^7)*x)*sqrt
(e*x + d)/e^4
```

Sympy [A] (verification not implemented)

Time = 1.01 (sec) , antiderivative size = 178, normalized size of antiderivative = 1.50

$$\int \frac{(ade+(cd^2+ae^2)x+cdex^2)^3}{\sqrt{d+ex}}\,dx$$
$$= \begin{cases} \frac{2\left(\frac{c^3d^3(d+ex)^{\frac{13}{2}}}{13e^3}+\frac{(d+ex)^{\frac{11}{2}}\cdot(3ac^2d^2e^2-3c^3d^4)}{11e^3}+\frac{(d+ex)^{\frac{9}{2}}\cdot(3a^2cde^4-6ac^2d^3e^2+3c^3d^5)}{9e^3}+\frac{(d+ex)^{\frac{7}{2}}\left(a^3e^6-3a^2cd^2e^4+3ac^2d^4e^2-c^3d^6\right)}{7e^3}\right)}{e} & \text{for } \\ \frac{c^3d^{\frac{11}{2}}x^4}{4} & \text{oth} \end{cases}$$

input
```
integrate((a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**3/(e*x+d)**(1/2),x)
```

output
```
Piecewise((2*(c**3*d**3*(d + e*x)**(13/2)/(13*e**3) + (d + e*x)**(11/2)*(3
*a*c**2*d**2*e**2 - 3*c**3*d**4)/(11*e**3) + (d + e*x)**(9/2)*(3*a**2*c*d*
e**4 - 6*a*c**2*d**3*e**2 + 3*c**3*d**5)/(9*e**3) + (d + e*x)**(7/2)*(a**3
*e**6 - 3*a**2*c*d**2*e**4 + 3*a*c**2*d**4*e**2 - c**3*d**6)/(7*e**3))/e,
Ne(e, 0)), (c**3*d**(11/2)*x**4/4, True))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 611 vs. 2(103) = 206.

Time = 0.04 (sec) , antiderivative size = 611, normalized size of antiderivative = 5.13

$$\int \frac{(ade+(cd^2+ae^2)x+cdex^2)^3}{\sqrt{d+ex}}\,dx = \text{Too large to display}$$

input
```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^3/(e*x+d)^(1/2),x, algorithm="
maxima")
```

output
```
2/15015*(15015*sqrt(e*x + d)*a^3*d^3*e^3 + 3003*((3*(e*x + d)^(5/2) - 10*(
e*x + d)^(3/2)*d + 15*sqrt(e*x + d)*d^2)*c*d/e + 5*(c*d^2 + a*e^2)*((e*x +
 d)^(3/2) - 3*sqrt(e*x + d)*d)/e)*a^2*d^2*e^2 + 5*(231*(e*x + d)^(13/2) -
1638*(e*x + d)^(11/2)*d + 5005*(e*x + d)^(9/2)*d^2 - 8580*(e*x + d)^(7/2)*
d^3 + 9009*(e*x + d)^(5/2)*d^4 - 6006*(e*x + d)^(3/2)*d^5 + 3003*sqrt(e*x
+ d)*d^6)*c^3*d^3/e^3 + 143*((35*(e*x + d)^(9/2) - 180*(e*x + d)^(7/2)*d +
 378*(e*x + d)^(5/2)*d^2 - 420*(e*x + d)^(3/2)*d^3 + 315*sqrt(e*x + d)*d^4
)*c^2*d^2/e^2 + 18*(5*(e*x + d)^(7/2) - 21*(e*x + d)^(5/2)*d + 35*(e*x + d
)^(3/2)*d^2 - 35*sqrt(e*x + d)*d^3)*(c*d^2 + a*e^2)*c*d/e^2 + 21*(c*d^2 +
a*e^2)^2*(3*(e*x + d)^(5/2) - 10*(e*x + d)^(3/2)*d + 15*sqrt(e*x + d)*d^2)
/e^2)*a*d*e + 65*(63*(e*x + d)^(11/2) - 385*(e*x + d)^(9/2)*d + 990*(e*x +
 d)^(7/2)*d^2 - 1386*(e*x + d)^(5/2)*d^3 + 1155*(e*x + d)^(3/2)*d^4 - 693*
sqrt(e*x + d)*d^5)*(c*d^2 + a*e^2)*c^2*d^2/e^3 + 143*(35*(e*x + d)^(9/2) -
 180*(e*x + d)^(7/2)*d + 378*(e*x + d)^(5/2)*d^2 - 420*(e*x + d)^(3/2)*d^3
 + 315*sqrt(e*x + d)*d^4)*(c*d^2 + a*e^2)^2*c*d/e^3 + 429*(5*(e*x + d)^(7/
2) - 21*(e*x + d)^(5/2)*d + 35*(e*x + d)^(3/2)*d^2 - 35*sqrt(e*x + d)*d^3)
*(c*d^2 + a*e^2)^3/e^3)/e
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 874 vs. $2(103) = 206$.

Time = 0.18 (sec) , antiderivative size = 874, normalized size of antiderivative = 7.34

$$\int \frac{(ade + (cd^2 + ae^2)x + cdex^2)^3}{\sqrt{d + ex}}\,dx = \text{Too large to display}$$

input
```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^3/(e*x+d)^(1/2),x, algorithm="
giac")
```

output
```
2/15015*(15015*sqrt(e*x + d)*a^3*d^3*e^3 + 15015*((e*x + d)^(3/2) - 3*sqrt
(e*x + d)*d)*a^2*c*d^4*e + 15015*((e*x + d)^(3/2) - 3*sqrt(e*x + d)*d)*a^3
*d^2*e^3 + 3003*(3*(e*x + d)^(5/2) - 10*(e*x + d)^(3/2)*d + 15*sqrt(e*x +
d)*d^2)*a*c^2*d^5/e + 9009*(3*(e*x + d)^(5/2) - 10*(e*x + d)^(3/2)*d + 15*
sqrt(e*x + d)*d^2)*a^2*c*d^3*e + 3003*(3*(e*x + d)^(5/2) - 10*(e*x + d)^(3
/2)*d + 15*sqrt(e*x + d)*d^2)*a^3*d*e^3 + 429*(5*(e*x + d)^(7/2) - 21*(e*x
 + d)^(5/2)*d + 35*(e*x + d)^(3/2)*d^2 - 35*sqrt(e*x + d)*d^3)*c^3*d^6/e^3
 + 3861*(5*(e*x + d)^(7/2) - 21*(e*x + d)^(5/2)*d + 35*(e*x + d)^(3/2)*d^2
 - 35*sqrt(e*x + d)*d^3)*a*c^2*d^4/e + 3861*(5*(e*x + d)^(7/2) - 21*(e*x +
 d)^(5/2)*d + 35*(e*x + d)^(3/2)*d^2 - 35*sqrt(e*x + d)*d^3)*a^2*c*d^2*e +
 429*(5*(e*x + d)^(7/2) - 21*(e*x + d)^(5/2)*d + 35*(e*x + d)^(3/2)*d^2 -
35*sqrt(e*x + d)*d^3)*a^3*e^3 + 143*(35*(e*x + d)^(9/2) - 180*(e*x + d)^(7
/2)*d + 378*(e*x + d)^(5/2)*d^2 - 420*(e*x + d)^(3/2)*d^3 + 315*sqrt(e*x +
 d)*d^4)*c^3*d^5/e^3 + 429*(35*(e*x + d)^(9/2) - 180*(e*x + d)^(7/2)*d + 3
78*(e*x + d)^(5/2)*d^2 - 420*(e*x + d)^(3/2)*d^3 + 315*sqrt(e*x + d)*d^4)*
a*c^2*d^3/e + 143*(35*(e*x + d)^(9/2) - 180*(e*x + d)^(7/2)*d + 378*(e*x +
 d)^(5/2)*d^2 - 420*(e*x + d)^(3/2)*d^3 + 315*sqrt(e*x + d)*d^4)*a^2*c*d*e
 + 65*(63*(e*x + d)^(11/2) - 385*(e*x + d)^(9/2)*d + 990*(e*x + d)^(7/2)*d
^2 - 1386*(e*x + d)^(5/2)*d^3 + 1155*(e*x + d)^(3/2)*d^4 - 693*sqrt(e*x +
d)*d^5)*c^3*d^4/e^3 + 65*(63*(e*x + d)^(11/2) - 385*(e*x + d)^(9/2)*d +...
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 106, normalized size of antiderivative = 0.89

$$\int \frac{\left(ade+(cd^2+ae^2)\,x+cdex^2\right)^3}{\sqrt{d+ex}}\,dx = \frac{2\,\left(a\,e^2-c\,d^2\right)^3\,\left(d+e\,x\right)^{7/2}}{7\,e^4} - \frac{\left(6\,c^3\,d^4-6\,a\,c^2\,d^2\,e^2\right)\,\left(d+e\,x\right)^{11/2}}{11\,e^4} + \frac{2\,c^3\,d^3\,\left(d+e\,x\right)^{13/2}}{13\,e^4} + \frac{2\,c\,d\,\left(a\,e^2-c\,d^2\right)^2\,\left(d+e\,x\right)^{9/2}}{3\,e^4}$$

input
```
int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^3/(d + e*x)^(1/2),x)
```

output
```
(2*(a*e^2 - c*d^2)^3*(d + e*x)^(7/2))/(7*e^4) - ((6*c^3*d^4 - 6*a*c^2*d^2*
e^2)*(d + e*x)^(11/2))/(11*e^4) + (2*c^3*d^3*(d + e*x)^(13/2))/(13*e^4) +
(2*c*d*(a*e^2 - c*d^2)^2*(d + e*x)^(9/2))/(3*e^4)
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 299, normalized size of antiderivative = 2.51

$$\int \frac{(ade + (cd^2 + ae^2)x + cdex^2)^3}{\sqrt{d+ex}}\,dx$$
$$= \frac{2\sqrt{ex+d}\,(231c^3d^3e^6x^6 + 819a\,c^2d^2e^7x^5 + 567c^3d^4e^5x^5 + 1001a^2cd\,e^8x^4 + 2093a\,c^2d^3e^6x^4 + 371c^3d^5e^4x^4}{}$$

input
```
int((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^3/(e*x+d)^(1/2),x)
```

output
```
(2*sqrt(d + e*x)*(429*a**3*d**3*e**6 + 1287*a**3*d**2*e**7*x + 1287*a**3*d
*e**8*x**2 + 429*a**3*e**9*x**3 - 286*a**2*c*d**5*e**4 + 143*a**2*c*d**4*e
**5*x + 2145*a**2*c*d**3*e**6*x**2 + 2717*a**2*c*d**2*e**7*x**3 + 1001*a**
2*c*d*e**8*x**4 + 104*a*c**2*d**7*e**2 - 52*a*c**2*d**6*e**3*x + 39*a*c**2
*d**5*e**4*x**2 + 1469*a*c**2*d**4*e**5*x**3 + 2093*a*c**2*d**3*e**6*x**4
+ 819*a*c**2*d**2*e**7*x**5 - 16*c**3*d**9 + 8*c**3*d**8*e*x - 6*c**3*d**7
*e**2*x**2 + 5*c**3*d**6*e**3*x**3 + 371*c**3*d**5*e**4*x**4 + 567*c**3*d*
*4*e**5*x**5 + 231*c**3*d**3*e**6*x**6))/(3003*e**4)
```

3.167 $\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^3}{(d+ex)^{3/2}}\,dx$

Optimal result

Integrand size = 37, antiderivative size = 119

$$\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^3}{(d+ex)^{3/2}}\,dx=-\frac{2\left(cd^2-ae^2\right)^3(d+ex)^{5/2}}{5e^4}+\frac{6cd\left(cd^2-ae^2\right)^2(d+ex)^{7/2}}{7e^4}-\frac{2c^2d^2\left(cd^2-ae^2\right)(d+ex)^{9/2}}{3e^4}+\frac{2c^3d^3(d+ex)^{11/2}}{11e^4}$$

output
```
-2/5*(-a*e^2+c*d^2)^3*(e*x+d)^(5/2)/e^4+6/7*c*d*(-a*e^2+c*d^2)^2*(e*x+d)^(
7/2)/e^4-2/3*c^2*d^2*(-a*e^2+c*d^2)*(e*x+d)^(9/2)/e^4+2/11*c^3*d^3*(e*x+d)
^(11/2)/e^4
```

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 111, normalized size of antiderivative = 0.93

$$\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^3}{(d+ex)^{3/2}}\,dx=\frac{2(d+ex)^{5/2}\left(231a^3e^6-99a^2cde^4(2d-5ex)+11ac^2d^2e^2(8d^2-20\right.}{1155e}$$

input
```
Integrate[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^3/(d + e*x)^(3/2),x]
```

output
```
(2*(d + e*x)^(5/2)*(231*a^3*e^6 - 99*a^2*c*d*e^4*(2*d - 5*e*x) + 11*a*c^2*
d^2*e^2*(8*d^2 - 20*d*e*x + 35*e^2*x^2) + c^3*d^3*(-16*d^3 + 40*d^2*e*x -
70*d*e^2*x^2 + 105*e^3*x^3)))/(1155*e^4)
```

Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.054$, Rules used = {1121, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(x(ae^2+cd^2)+ade+cdex^2)^3}{(d+ex)^{3/2}}\,dx$$

$$\Big\downarrow \text{1121}$$

$$\int \left(-\frac{3c^2d^2(d+ex)^{7/2}(cd^2-ae^2)}{e^3}+\frac{3cd(d+ex)^{5/2}(cd^2-ae^2)^2}{e^3}+\frac{(d+ex)^{3/2}(ae^2-cd^2)^3}{e^3}+\frac{c^3d^3(d+ex)^{9/}}{e^3}\right.$$

$$\Big\downarrow \text{2009}$$

$$-\frac{2c^2d^2(d+ex)^{9/2}(cd^2-ae^2)}{3e^4}+\frac{6cd(d+ex)^{7/2}(cd^2-ae^2)^2}{7e^4}-\frac{2(d+ex)^{5/2}(cd^2-ae^2)^3}{5e^4}+\frac{2c^3d^3(d+ex)^{11/2}}{11e^4}$$

input
```
Int[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^3/(d + e*x)^(3/2),x]
```

output
```
(-2*(c*d^2 - a*e^2)^3*(d + e*x)^(5/2))/(5*e^4) + (6*c*d*(c*d^2 - a*e^2)^2*
(d + e*x)^(7/2))/(7*e^4) - (2*c^2*d^2*(c*d^2 - a*e^2)*(d + e*x)^(9/2))/(3*
e^4) + (2*c^3*d^3*(d + e*x)^(11/2))/(11*e^4)
```

Defintions of rubi rules used

rule 1121
```
Int[((d_) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Int[ExpandIntegrand[(d + e*x)^(m + p)*(a/d + (c/e)*x)^p, x], x]
/; FreeQ[{a, b, c, d, e, m, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && (Int
egerQ[p] || (GtQ[a, 0] && GtQ[d, 0] && LtQ[c, 0]))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 2.40 (sec) , antiderivative size = 97, normalized size of antiderivative = 0.82

method	result
derivativedivides	$\frac{\frac{2c^3d^3(ex+d)^{\frac{11}{2}}}{11}+\frac{2\left(a\,e^2-c\,d^2\right)c^2d^2(ex+d)^{\frac{9}{2}}}{3}+\frac{6\left(a\,e^2-c\,d^2\right)^2cd(ex+d)^{\frac{7}{2}}}{7}+\frac{2\left(a\,e^2-c\,d^2\right)^3(ex+d)^{\frac{5}{2}}}{5}}{e^4}$
default	$\frac{\frac{2c^3d^3(ex+d)^{\frac{11}{2}}}{11}+\frac{2\left(a\,e^2-c\,d^2\right)c^2d^2(ex+d)^{\frac{9}{2}}}{3}+\frac{6\left(a\,e^2-c\,d^2\right)^2cd(ex+d)^{\frac{7}{2}}}{7}+\frac{2\left(a\,e^2-c\,d^2\right)^3(ex+d)^{\frac{5}{2}}}{5}}{e^4}$
pseudoelliptic	$\frac{2(ex+d)^{\frac{5}{2}}\left(e^6a^3+\frac{15x\,a^2cd\,e^5}{7}-\frac{6\left(-\frac{35c\,x^2}{18}+a\right)ca\,d^2e^4}{7}-\frac{20\left(-\frac{21c\,x^2}{44}+a\right)x\,c^2d^3e^3}{21}+\frac{8c^2\left(-\frac{35c\,x^2}{44}+a\right)d^4e^2}{21}+\frac{40c^3d^5ex}{231}-1\cdots\right.}{5e^4}$
gosper	$\frac{2(ex+d)^{\frac{5}{2}}\left(105c^3d^3e^3x^3+385x^2a\,c^2d^2e^4-70c^3d^4e^2x^2+495x\,a^2cd\,e^5-220xa\,c^2d^3e^3+40c^3d^5ex+231e^6a^3-198d^2e^4a^2\cdots\right.}{1155e^4}$
orering	$\frac{2\left(105c^3d^3e^3x^3+385x^2a\,c^2d^2e^4-70c^3d^4e^2x^2+495x\,a^2cd\,e^5-220xa\,c^2d^3e^3+40c^3d^5ex+231e^6a^3-198d^2e^4a^2c+88d^4e^2\cdots\right.}{1155e^4(cdx+ae)^3\sqrt{ex+d}}$
trager	$2\left(105d^3c^3e^5x^5+385a\,e^6c^2d^2x^4+140d^4c^3e^4x^4+495a^2e^7cd\,x^3+550a\,e^5c^2d^3x^3+5d^5c^3e^3x^3+231a^3e^8x^2+792a^2c\,d^2e^6x\cdots\right.$
risch	$2\left(105d^3c^3e^5x^5+385a\,e^6c^2d^2x^4+140d^4c^3e^4x^4+495a^2e^7cd\,x^3+550a\,e^5c^2d^3x^3+5d^5c^3e^3x^3+231a^3e^8x^2+792a^2c\,d^2e^6x\cdots\right.$

input
```
int((a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^3/(e*x+d)^(3/2),x,method=_RETURNVERB
OSE)
```

output
```
2/e^4*(1/11*c^3*d^3*(e*x+d)^(11/2)+1/3*(a*e^2-c*d^2)*c^2*d^2*(e*x+d)^(9/2)
+3/7*(a*e^2-c*d^2)^2*c*d*(e*x+d)^(7/2)+1/5*(a*e^2-c*d^2)^3*(e*x+d)^(5/2))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 231 vs. $2(103) = 206$.

Time = 0.09 (sec) , antiderivative size = 231, normalized size of antiderivative = 1.94

$$\int \frac{(ade + (cd^2 + ae^2)x + cdex^2)^3}{(d+ex)^{3/2}}\,dx = \frac{2\,(105\,c^3d^3e^5x^5 - 16\,c^3d^8 + 88\,ac^2d^6e^2 - 198\,a^2cd^4e^4 + 231\,a^3d^2e^6}{}$$

input
```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^3/(e*x+d)^(3/2),x, algorithm="
fricas")
```

output
```
2/1155*(105*c^3*d^3*e^5*x^5 - 16*c^3*d^8 + 88*a*c^2*d^6*e^2 - 198*a^2*c*d^
4*e^4 + 231*a^3*d^2*e^6 + 35*(4*c^3*d^4*e^4 + 11*a*c^2*d^2*e^6)*x^4 + 5*(c
^3*d^5*e^3 + 110*a*c^2*d^3*e^5 + 99*a^2*c*d*e^7)*x^3 - 3*(2*c^3*d^6*e^2 -
11*a*c^2*d^4*e^4 - 264*a^2*c*d^2*e^6 - 77*a^3*e^8)*x^2 + (8*c^3*d^7*e - 44
*a*c^2*d^5*e^3 + 99*a^2*c*d^3*e^5 + 462*a^3*d*e^7)*x)*sqrt(e*x + d)/e^4
```

Sympy [A] (verification not implemented)

Time = 1.13 (sec) , antiderivative size = 178, normalized size of antiderivative = 1.50

$$\int \frac{(ade + (cd^2 + ae^2)x + cdex^2)^3}{(d+ex)^{3/2}}\,dx = \begin{cases} \frac{2\left(\frac{c^3d^3(d+ex)^{\frac{11}{2}}}{11e^3} + \frac{(d+ex)^{\frac{9}{2}}\cdot\left(3ac^2d^2e^2 - 3c^3d^4\right)}{9e^3} + \frac{(d+ex)^{\frac{7}{2}}\cdot\left(3a^2cde^4 - 6ac^2d^3e^2 + 3c^3d^5\right)}{7e^3} + \cdots\right.}{e} \\ \frac{c^3d^{\frac{9}{2}}x^4}{4} \end{cases}$$

input
```
integrate((a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**3/(e*x+d)**(3/2),x)
```

output
```
Piecewise((2*(c**3*d**3*(d + e*x)**(11/2)/(11*e**3) + (d + e*x)**(9/2)*(3*
a*c**2*d**2*e**2 - 3*c**3*d**4)/(9*e**3) + (d + e*x)**(7/2)*(3*a**2*c*d*e*
*4 - 6*a*c**2*d**3*e**2 + 3*c**3*d**5)/(7*e**3) + (d + e*x)**(5/2)*(a**3*e
**6 - 3*a**2*c*d**2*e**4 + 3*a*c**2*d**4*e**2 - c**3*d**6)/(5*e**3))/e, Ne
(e, 0)), (c**3*d**(9/2)*x**4/4, True))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 137, normalized size of antiderivative = 1.15

$$\int \frac{(ade + (cd^2 + ae^2)x + cdex^2)^3}{(d + ex)^{3/2}}\, dx = \frac{2\left(105\,(ex + d)^{\frac{11}{2}}c^3d^3 - 385\,(c^3d^4 - ac^2d^2e^2)(ex + d)^{\frac{9}{2}} + 495\,(c^3d^5\right.}{}$$

input
```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^3/(e*x+d)^(3/2),x, algorithm="
maxima")
```

output
```
2/1155*(105*(e*x + d)^(11/2)*c^3*d^3 - 385*(c^3*d^4 - a*c^2*d^2*e^2)*(e*x
+ d)^(9/2) + 495*(c^3*d^5 - 2*a*c^2*d^3*e^2 + a^2*c*d*e^4)*(e*x + d)^(7/2)
 - 231*(c^3*d^6 - 3*a*c^2*d^4*e^2 + 3*a^2*c*d^2*e^4 - a^3*e^6)*(e*x + d)^(
5/2))/e^4
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 582 vs. 2(103) = 206.

Time = 0.13 (sec) , antiderivative size = 582, normalized size of antiderivative = 4.89

$$\int \frac{(ade + (cd^2 + ae^2)x + cdex^2)^3}{(d + ex)^{3/2}}\, dx = \text{Too large to display}$$

input
```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^3/(e*x+d)^(3/2),x, algorithm="
giac")
```

output
```
2/3465*(3465*sqrt(e*x + d)*a^3*d^2*e^3 + 3465*((e*x + d)^(3/2) - 3*sqrt(e*
x + d)*d)*a^2*c*d^3*e + 2310*((e*x + d)^(3/2) - 3*sqrt(e*x + d)*d)*a^3*d*e
^3 + 693*(3*(e*x + d)^(5/2) - 10*(e*x + d)^(3/2)*d + 15*sqrt(e*x + d)*d^2)
*a*c^2*d^4/e + 1386*(3*(e*x + d)^(5/2) - 10*(e*x + d)^(3/2)*d + 15*sqrt(e*
x + d)*d^2)*a^2*c*d^2*e + 231*(3*(e*x + d)^(5/2) - 10*(e*x + d)^(3/2)*d +
15*sqrt(e*x + d)*d^2)*a^3*e^3 + 99*(5*(e*x + d)^(7/2) - 21*(e*x + d)^(5/2)
*d + 35*(e*x + d)^(3/2)*d^2 - 35*sqrt(e*x + d)*d^3)*c^3*d^5/e^3 + 594*(5*(
e*x + d)^(7/2) - 21*(e*x + d)^(5/2)*d + 35*(e*x + d)^(3/2)*d^2 - 35*sqrt(e
*x + d)*d^3)*a*c^2*d^3/e + 297*(5*(e*x + d)^(7/2) - 21*(e*x + d)^(5/2)*d +
 35*(e*x + d)^(3/2)*d^2 - 35*sqrt(e*x + d)*d^3)*a^2*c*d*e + 22*(35*(e*x +
d)^(9/2) - 180*(e*x + d)^(7/2)*d + 378*(e*x + d)^(5/2)*d^2 - 420*(e*x + d)
^(3/2)*d^3 + 315*sqrt(e*x + d)*d^4)*c^3*d^4/e^3 + 33*(35*(e*x + d)^(9/2) -
 180*(e*x + d)^(7/2)*d + 378*(e*x + d)^(5/2)*d^2 - 420*(e*x + d)^(3/2)*d^3
 + 315*sqrt(e*x + d)*d^4)*a*c^2*d^2/e + 5*(63*(e*x + d)^(11/2) - 385*(e*x
+ d)^(9/2)*d + 990*(e*x + d)^(7/2)*d^2 - 1386*(e*x + d)^(5/2)*d^3 + 1155*(
e*x + d)^(3/2)*d^4 - 693*sqrt(e*x + d)*d^5)*c^3*d^3/e^3)/e
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 106, normalized size of antiderivative = 0.89

$$\int \frac{(ade + (cd^2 + ae^2)x + cdex^2)^3}{(d+ex)^{3/2}}\,dx = \frac{2\,(a\,e^2 - c\,d^2)^3\,(d+e\,x)^{5/2}}{5\,e^4} - \frac{(6\,c^3\,d^4 - 6\,a\,c^2\,d^2\,e^2)\,(d+e\,x)^{9/2}}{9\,e^4} + \frac{2\,c^3\,d^3\,(d+e\,x)^{11/2}}{11\,e^4} + \frac{6\,c\,d\,(a\,e^2 - c\,d^2)^2\,(d+e\,x)^{7/2}}{7\,e^4}$$

input
```
int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^3/(d + e*x)^(3/2),x)
```

output
```
(2*(a*e^2 - c*d^2)^3*(d + e*x)^(5/2))/(5*e^4) - ((6*c^3*d^4 - 6*a*c^2*d^2*
e^2)*(d + e*x)^(9/2))/(9*e^4) + (2*c^3*d^3*(d + e*x)^(11/2))/(11*e^4) + (6
*c*d*(a*e^2 - c*d^2)^2*(d + e*x)^(7/2))/(7*e^4)
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 241, normalized size of antiderivative = 2.03

$$\int \frac{(ade+(cd^2+ae^2)x+cdex^2)^3}{(d+ex)^{3/2}}\,dx = \frac{2\sqrt{ex+d}\,(105c^3d^3e^5x^5+385a\,c^2d^2e^6x^4+140c^3d^4e^4x^4+495a^2cd$$

input
```
int((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^3/(e*x+d)^(3/2),x)
```

output
```
(2*sqrt(d + e*x)*(231*a**3*d**2*e**6 + 462*a**3*d*e**7*x + 231*a**3*e**8*x
**2 - 198*a**2*c*d**4*e**4 + 99*a**2*c*d**3*e**5*x + 792*a**2*c*d**2*e**6*
x**2 + 495*a**2*c*d*e**7*x**3 + 88*a*c**2*d**6*e**2 - 44*a*c**2*d**5*e**3*
x + 33*a*c**2*d**4*e**4*x**2 + 550*a*c**2*d**3*e**5*x**3 + 385*a*c**2*d**2
*e**6*x**4 - 16*c**3*d**8 + 8*c**3*d**7*e*x - 6*c**3*d**6*e**2*x**2 + 5*c*
*3*d**5*e**3*x**3 + 140*c**3*d**4*e**4*x**4 + 105*c**3*d**3*e**5*x**5))/(1
155*e**4)
```

3.168 $\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^3}{(d+ex)^{5/2}}\,dx$

Optimal result

Integrand size = 37, antiderivative size = 119

$$\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^3}{(d+ex)^{5/2}}\,dx=-\frac{2(cd^2-ae^2)^3\,(d+ex)^{3/2}}{3e^4}+\frac{6cd(cd^2-ae^2)^2\,(d+ex)^{5/2}}{5e^4}-\frac{6c^2d^2(cd^2-ae^2)\,(d+ex)^{7/2}}{7e^4}+\frac{2c^3d^3(d+ex)^{9/2}}{9e^4}$$

output
```
-2/3*(-a*e^2+c*d^2)^3*(e*x+d)^(3/2)/e^4+6/5*c*d*(-a*e^2+c*d^2)^2*(e*x+d)^(
5/2)/e^4-6/7*c^2*d^2*(-a*e^2+c*d^2)*(e*x+d)^(7/2)/e^4+2/9*c^3*d^3*(e*x+d)^
(9/2)/e^4
```

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 111, normalized size of antiderivative = 0.93

$$\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^3}{(d+ex)^{5/2}}\,dx=\frac{2(d+ex)^{3/2}\left(105a^3e^6-63a^2cde^4(2d-3ex)+9ac^2d^2e^2(8d^2-12d\right.}{315e^4}$$

input
```
Integrate[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^3/(d + e*x)^(5/2),x]
```

output
```
(2*(d + e*x)^(3/2)*(105*a^3*e^6 - 63*a^2*c*d*e^4*(2*d - 3*e*x) + 9*a*c^2*d
^2*e^2*(8*d^2 - 12*d*e*x + 15*e^2*x^2) + c^3*d^3*(-16*d^3 + 24*d^2*e*x - 3
0*d*e^2*x^2 + 35*e^3*x^3)))/(315*e^4)
```

Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.054$, Rules used = {1121, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^3}{(d+ex)^{5/2}}dx$$

$$\Big\downarrow \text{1121}$$

$$\int \left(-\frac{3c^2d^2(d+ex)^{5/2}\left(cd^2-ae^2\right)}{e^3}+\frac{3cd(d+ex)^{3/2}\left(cd^2-ae^2\right)^2}{e^3}+\frac{\sqrt{d+ex}\left(ae^2-cd^2\right)^3}{e^3}+\frac{c^3d^3(d+ex)^{7/2}}{e^3}\right)$$

$$\Big\downarrow \text{2009}$$

$$-\frac{6c^2d^2(d+ex)^{7/2}\left(cd^2-ae^2\right)}{7e^4}+\frac{6cd(d+ex)^{5/2}\left(cd^2-ae^2\right)^2}{5e^4}-\frac{2(d+ex)^{3/2}\left(cd^2-ae^2\right)^3}{3e^4}+\frac{2c^3d^3(d+ex)^{9/2}}{9e^4}$$

input
```
Int[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^3/(d + e*x)^(5/2),x]
```

output
```
(-2*(c*d^2 - a*e^2)^3*(d + e*x)^(3/2))/(3*e^4) + (6*c*d*(c*d^2 - a*e^2)^2*
(d + e*x)^(5/2))/(5*e^4) - (6*c^2*d^2*(c*d^2 - a*e^2)*(d + e*x)^(7/2))/(7*
e^4) + (2*c^3*d^3*(d + e*x)^(9/2))/(9*e^4)
```

Defintions of rubi rules used

rule 1121
```
Int[((d_) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Int[ExpandIntegrand[(d + e*x)^(m + p)*(a/d + (c/e)*x)^p, x], x]
/; FreeQ[{a, b, c, d, e, m, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && (Int
egerQ[p] || (GtQ[a, 0] && GtQ[d, 0] && LtQ[c, 0]))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 2.33 (sec) , antiderivative size = 97, normalized size of antiderivative = 0.82

method	result
derivativedivides	$\frac{\frac{2c^3d^3(ex+d)^{\frac{9}{2}}}{9}+\frac{6\left(a\,e^2-c\,d^2\right)c^2d^2(ex+d)^{\frac{7}{2}}}{7}+\frac{6\left(a\,e^2-c\,d^2\right)^2cd(ex+d)^{\frac{5}{2}}}{5}+\frac{2\left(a\,e^2-c\,d^2\right)^3(ex+d)^{\frac{3}{2}}}{3}}{e^4}$
default	$\frac{\frac{2c^3d^3(ex+d)^{\frac{9}{2}}}{9}+\frac{6\left(a\,e^2-c\,d^2\right)c^2d^2(ex+d)^{\frac{7}{2}}}{7}+\frac{6\left(a\,e^2-c\,d^2\right)^2cd(ex+d)^{\frac{5}{2}}}{5}+\frac{2\left(a\,e^2-c\,d^2\right)^3(ex+d)^{\frac{3}{2}}}{3}}{e^4}$
pseudoelliptic	$\frac{2(ex+d)^{\frac{3}{2}}\left(e^6a^3+\frac{9x\,a^2cd\,e^5}{5}-\frac{6\left(-\frac{15c\,x^2}{14}+a\right)ca\,d^2e^4}{5}-\frac{36\left(-\frac{35c\,x^2}{108}+a\right)x\,c^2d^3e^3}{35}+\frac{24c^2\left(-\frac{5c\,x^2}{12}+a\right)d^4e^2}{35}+\frac{8c^3d^5ex}{35}-\frac{16d\cdots}{10\cdots}\cdots\right.}{3e^4}$
gosper	$\frac{2(ex+d)^{\frac{3}{2}}\left(35c^3d^3e^3x^3+135x^2a\,c^2d^2e^4-30c^3d^4e^2x^2+189x\,a^2cd\,e^5-108xa\,c^2d^3e^3+24c^3d^5ex+105e^6a^3-126d^2e^4a^2c\cdots\right.}{315e^4}$
orering	$\frac{2\left(35c^3d^3e^3x^3+135x^2a\,c^2d^2e^4-30c^3d^4e^2x^2+189x\,a^2cd\,e^5-108xa\,c^2d^3e^3+24c^3d^5ex+105e^6a^3-126d^2e^4a^2c+72d^4e^2\cdots\right.}{315e^4(cdx+ae)^3(ex+d)^{\frac{3}{2}}}$
trager	$\frac{2\left(35c^3d^3e^4x^4+135a\,c^2d^2e^5x^3+5c^3d^4e^3x^3+189a^2cd\,e^6x^2+27a\,c^2d^3e^4x^2-6c^3d^5e^2x^2+105a^3e^7x+63a^2c\,d^2e^5x-36a\,c\cdots\right.}{315e^4}$
risch	$\frac{2\left(35c^3d^3e^4x^4+135a\,c^2d^2e^5x^3+5c^3d^4e^3x^3+189a^2cd\,e^6x^2+27a\,c^2d^3e^4x^2-6c^3d^5e^2x^2+105a^3e^7x+63a^2c\,d^2e^5x-36a\,c\cdots\right.}{315e^4}$

input
```
int((a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^3/(e*x+d)^(5/2),x,method=_RETURNVERB
OSE)
```

output
```
2/e^4*(1/9*c^3*d^3*(e*x+d)^(9/2)+3/7*(a*e^2-c*d^2)*c^2*d^2*(e*x+d)^(7/2)+3
/5*(a*e^2-c*d^2)^2*c*d*(e*x+d)^(5/2)+1/3*(a*e^2-c*d^2)^3*(e*x+d)^(3/2))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 179, normalized size of antiderivative = 1.50

$$\int \frac{(ade+(cd^2+ae^2)\,x+cdex^2)^3}{(d+ex)^{5/2}}\,dx = \frac{2\,(35\,c^3d^3e^4x^4-16\,c^3d^7+72\,ac^2d^5e^2-126\,a^2cd^3e^4+105\,a^3de^6+$$

input
```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^3/(e*x+d)^(5/2),x, algorithm="
fricas")
```

output
```
2/315*(35*c^3*d^3*e^4*x^4 - 16*c^3*d^7 + 72*a*c^2*d^5*e^2 - 126*a^2*c*d^3*
e^4 + 105*a^3*d*e^6 + 5*(c^3*d^4*e^3 + 27*a*c^2*d^2*e^5)*x^3 - 3*(2*c^3*d^
5*e^2 - 9*a*c^2*d^3*e^4 - 63*a^2*c*d*e^6)*x^2 + (8*c^3*d^6*e - 36*a*c^2*d^
4*e^3 + 63*a^2*c*d^2*e^5 + 105*a^3*e^7)*x)*sqrt(e*x + d)/e^4
```

Sympy [A] (verification not implemented)

Time = 1.08 (sec) , antiderivative size = 178, normalized size of antiderivative = 1.50

$$\int \frac{(ade+(cd^2+ae^2)\,x+cdex^2)^3}{(d+ex)^{5/2}}\,dx = \begin{cases} \frac{2\left(\frac{c^3d^3(d+ex)^{\frac{9}{2}}}{9e^3}+\frac{(d+ex)^{\frac{7}{2}}\cdot\left(3ac^2d^2e^2-3c^3d^4\right)}{7e^3}+\frac{(d+ex)^{\frac{5}{2}}\cdot\left(3a^2cde^4-6ac^2d^3e^2+3c^3d^5\right)}{5e^3}+\frac{(d-}{}\right.}{e} \\ \frac{c^3d^{\frac{7}{2}}x^4}{4} \end{cases}$$

input
```
integrate((a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**3/(e*x+d)**(5/2),x)
```

output
```
Piecewise((2*(c**3*d**3*(d + e*x)**(9/2)/(9*e**3) + (d + e*x)**(7/2)*(3*a*
c**2*d**2*e**2 - 3*c**3*d**4)/(7*e**3) + (d + e*x)**(5/2)*(3*a**2*c*d*e**4
 - 6*a*c**2*d**3*e**2 + 3*c**3*d**5)/(5*e**3) + (d + e*x)**(3/2)*(a**3*e**
6 - 3*a**2*c*d**2*e**4 + 3*a*c**2*d**4*e**2 - c**3*d**6)/(3*e**3))/e, Ne(e
, 0)), (c**3*d**(7/2)*x**4/4, True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 137, normalized size of antiderivative = 1.15

$$\int \frac{(ade + (cd^2 + ae^2)x + cdex^2)^3}{(d + ex)^{5/2}}\,dx = \frac{2\left(35\,(ex + d)^{\frac{9}{2}}c^3d^3 - 135\,(c^3d^4 - ac^2d^2e^2)(ex + d)^{\frac{7}{2}} + 189\,(c^3d^5 -\right.}{}$$

input
```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^3/(e*x+d)^(5/2),x, algorithm="
maxima")
```

output
```
2/315*(35*(e*x + d)^(9/2)*c^3*d^3 - 135*(c^3*d^4 - a*c^2*d^2*e^2)*(e*x + d
)^(7/2) + 189*(c^3*d^5 - 2*a*c^2*d^3*e^2 + a^2*c*d*e^4)*(e*x + d)^(5/2) -
105*(c^3*d^6 - 3*a*c^2*d^4*e^2 + 3*a^2*c*d^2*e^4 - a^3*e^6)*(e*x + d)^(3/2
))/e^4
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 337 vs. 2(103) = 206.

Time = 0.15 (sec) , antiderivative size = 337, normalized size of antiderivative = 2.83

$$\int \frac{(ade + (cd^2 + ae^2)x + cdex^2)^3}{(d + ex)^{5/2}}\,dx = \frac{2\left(315\,\sqrt{ex + d}a^3de^3 + 315\left((ex + d)^{\frac{3}{2}} - 3\,\sqrt{ex + d}d\right)a^2cd^2e + 10\right.}{}$$

input
```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^3/(e*x+d)^(5/2),x, algorithm="
giac")
```

output
```
2/315*(315*sqrt(e*x + d)*a^3*d*e^3 + 315*((e*x + d)^(3/2) - 3*sqrt(e*x + d
)*d)*a^2*c*d^2*e + 105*((e*x + d)^(3/2) - 3*sqrt(e*x + d)*d)*a^3*e^3 + 63*
(3*(e*x + d)^(5/2) - 10*(e*x + d)^(3/2)*d + 15*sqrt(e*x + d)*d^2)*a*c^2*d^
3/e + 63*(3*(e*x + d)^(5/2) - 10*(e*x + d)^(3/2)*d + 15*sqrt(e*x + d)*d^2)
*a^2*c*d*e + 9*(5*(e*x + d)^(7/2) - 21*(e*x + d)^(5/2)*d + 35*(e*x + d)^(3
/2)*d^2 - 35*sqrt(e*x + d)*d^3)*c^3*d^4/e^3 + 27*(5*(e*x + d)^(7/2) - 21*(
e*x + d)^(5/2)*d + 35*(e*x + d)^(3/2)*d^2 - 35*sqrt(e*x + d)*d^3)*a*c^2*d^
2/e + (35*(e*x + d)^(9/2) - 180*(e*x + d)^(7/2)*d + 378*(e*x + d)^(5/2)*d^
2 - 420*(e*x + d)^(3/2)*d^3 + 315*sqrt(e*x + d)*d^4)*c^3*d^3/e^3)/e
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 106, normalized size of antiderivative = 0.89

$$\int \frac{(ade+(cd^2+ae^2)x+cdex^2)^3}{(d+ex)^{5/2}}\,dx = \frac{2\,(a\,e^2-c\,d^2)^3\,(d+e\,x)^{3/2}}{3\,e^4} - \frac{(6\,c^3\,d^4-6\,a\,c^2\,d^2\,e^2)\,(d+e\,x)^{7/2}}{7\,e^4} + \frac{2\,c^3\,d^3\,(d+e\,x)^{9/2}}{9\,e^4} + \frac{6\,c\,d\,(a\,e^2-c\,d^2)^2\,(d+e\,x)^{5/2}}{5\,e^4}$$

input
```
int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^3/(d + e*x)^(5/2),x)
```

output
```
(2*(a*e^2 - c*d^2)^3*(d + e*x)^(3/2))/(3*e^4) - ((6*c^3*d^4 - 6*a*c^2*d^2*
e^2)*(d + e*x)^(7/2))/(7*e^4) + (2*c^3*d^3*(d + e*x)^(9/2))/(9*e^4) + (6*c
*d*(a*e^2 - c*d^2)^2*(d + e*x)^(5/2))/(5*e^4)
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 183, normalized size of antiderivative = 1.54

$$\int \frac{(ade+(cd^2+ae^2)x+cdex^2)^3}{(d+ex)^{5/2}}\,dx = \frac{2\sqrt{ex+d}\,(35c^3d^3e^4x^4+135a\,c^2d^2e^5x^3+5c^3d^4e^3x^3+189a^2cd\,e^6x^2}{}$$

input
```
int((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^3/(e*x+d)^(5/2),x)
```

output
```
(2*sqrt(d + e*x)*(105*a**3*d*e**6 + 105*a**3*e**7*x - 126*a**2*c*d**3*e**4
 + 63*a**2*c*d**2*e**5*x + 189*a**2*c*d*e**6*x**2 + 72*a*c**2*d**5*e**2 -
36*a*c**2*d**4*e**3*x + 27*a*c**2*d**3*e**4*x**2 + 135*a*c**2*d**2*e**5*x*
*3 - 16*c**3*d**7 + 8*c**3*d**6*e*x - 6*c**3*d**5*e**2*x**2 + 5*c**3*d**4*
e**3*x**3 + 35*c**3*d**3*e**4*x**4))/(315*e**4)
```

3.169 $\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^3}{(d+ex)^{7/2}}\,dx$

Optimal result

Integrand size = 37, antiderivative size = 115

$$\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^3}{(d+ex)^{7/2}}\,dx=-\frac{2\left(cd^2-ae^2\right)^3\sqrt{d+ex}}{e^4}+\frac{2cd\left(cd^2-ae^2\right)^2(d+ex)^{3/2}}{e^4}-\frac{6c^2d^2\left(cd^2-ae^2\right)(d+ex)^{5/2}}{5e^4}+\frac{2c^3d^3(d+ex)^{7/2}}{7e^4}$$

output
```
-2*(-a*e^2+c*d^2)^3*(e*x+d)^(1/2)/e^4+2*c*d*(-a*e^2+c*d^2)^2*(e*x+d)^(3/2)
/e^4-6/5*c^2*d^2*(-a*e^2+c*d^2)*(e*x+d)^(5/2)/e^4+2/7*c^3*d^3*(e*x+d)^(7/2
)/e^4
```

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 110, normalized size of antiderivative = 0.96

$$\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^3}{(d+ex)^{7/2}}\,dx=\frac{2\sqrt{d+ex}(35a^3e^6+35a^2cde^4(-2d+ex)+7ac^2d^2e^2(8d^2-4dex+}{35e^4}$$

input
```
Integrate[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^3/(d + e*x)^(7/2),x]
```

output
```
(2*Sqrt[d + e*x]*(35*a^3*e^6 + 35*a^2*c*d*e^4*(-2*d + e*x) + 7*a*c^2*d^2*e
^2*(8*d^2 - 4*d*e*x + 3*e^2*x^2) + c^3*d^3*(-16*d^3 + 8*d^2*e*x - 6*d*e^2*
x^2 + 5*e^3*x^3)))/(35*e^4)
```

Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 115, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.054$, Rules used = {1121, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^3}{(d+ex)^{7/2}}dx$$

$$\Big\downarrow \text{1121}$$

$$\int \left(-\frac{3c^2d^2(d+ex)^{3/2}\left(cd^2-ae^2\right)}{e^3}+\frac{3cd\sqrt{d+ex}\left(cd^2-ae^2\right)^2}{e^3}+\frac{\left(ae^2-cd^2\right)^3}{e^3\sqrt{d+ex}}+\frac{c^3d^3(d+ex)^{5/2}}{e^3}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{6c^2d^2(d+ex)^{5/2}\left(cd^2-ae^2\right)}{5e^4}+\frac{2cd(d+ex)^{3/2}\left(cd^2-ae^2\right)^2}{e^4}-\frac{2\sqrt{d+ex}\left(cd^2-ae^2\right)^3}{e^4}+\frac{2c^3d^3(d+ex)^{7/2}}{7e^4}$$

input
```
Int[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^3/(d + e*x)^(7/2),x]
```

output
```
(-2*(c*d^2 - a*e^2)^3*Sqrt[d + e*x])/e^4 + (2*c*d*(c*d^2 - a*e^2)^2*(d + e
*x)^(3/2))/e^4 - (6*c^2*d^2*(c*d^2 - a*e^2)*(d + e*x)^(5/2))/(5*e^4) + (2*
c^3*d^3*(d + e*x)^(7/2))/(7*e^4)
```

Defintions of rubi rules used

rule 1121
```
Int[((d_) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Int[ExpandIntegrand[(d + e*x)^(m + p)*(a/d + (c/e)*x)^p, x], x]
/; FreeQ[{a, b, c, d, e, m, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && (Int
egerQ[p] || (GtQ[a, 0] && GtQ[d, 0] && LtQ[c, 0]))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 1.47 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.83

method	result
derivativedivides	$\frac{\frac{2c^3d^3(ex+d)^{\frac{7}{2}}}{7}+\frac{6\left(a\,e^2-c\,d^2\right)c^2d^2(ex+d)^{\frac{5}{2}}}{5}+2\left(a\,e^2-c\,d^2\right)^2cd(ex+d)^{\frac{3}{2}}+2\left(a\,e^2-c\,d^2\right)^3\sqrt{ex+d}}{e^4}$
default	$\frac{\frac{2c^3d^3(ex+d)^{\frac{7}{2}}}{7}+\frac{6\left(a\,e^2-c\,d^2\right)c^2d^2(ex+d)^{\frac{5}{2}}}{5}+2\left(a\,e^2-c\,d^2\right)^2cd(ex+d)^{\frac{3}{2}}+2\left(a\,e^2-c\,d^2\right)^3\sqrt{ex+d}}{e^4}$
pseudoelliptic	$\frac{2\sqrt{ex+d}\left(-\frac{16\left(-\frac{5}{16}e^3x^3+\frac{3}{8}d\,e^2x^2-\frac{1}{2}d^2ex+d^3\right)d^3c^3}{35}+\frac{8e^2\left(\frac{3}{8}e^2x^2-\frac{1}{2}dex+d^2\right)a\,d^2c^2}{5}-2e^4\left(-\frac{ex}{2}+d\right)a^2dc+e^6a^3\right)}{e^4}$
gosper	$\frac{2\sqrt{ex+d}\,(5c^3d^3e^3x^3+21x^2a\,c^2d^2e^4-6c^3d^4e^2x^2+35x\,a^2cd\,e^5-28xa\,c^2d^3e^3+8c^3d^5ex+35e^6a^3-70d^2e^4a^2c+56d^4e^2a\,c}{35e^4}$
trager	$\frac{2\sqrt{ex+d}\,(5c^3d^3e^3x^3+21x^2a\,c^2d^2e^4-6c^3d^4e^2x^2+35x\,a^2cd\,e^5-28xa\,c^2d^3e^3+8c^3d^5ex+35e^6a^3-70d^2e^4a^2c+56d^4e^2a\,c}{35e^4}$
risch	$\frac{2\sqrt{ex+d}\,(5c^3d^3e^3x^3+21x^2a\,c^2d^2e^4-6c^3d^4e^2x^2+35x\,a^2cd\,e^5-28xa\,c^2d^3e^3+8c^3d^5ex+35e^6a^3-70d^2e^4a^2c+56d^4e^2a\,c}{35e^4}$
orering	$\frac{2(5c^3d^3e^3x^3+21x^2a\,c^2d^2e^4-6c^3d^4e^2x^2+35x\,a^2cd\,e^5-28xa\,c^2d^3e^3+8c^3d^5ex+35e^6a^3-70d^2e^4a^2c+56d^4e^2a\,c^2-16d^6}{35e^4(cdx+ae)^3(ex+d)^{\frac{5}{2}}}$

input
```
int((a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^3/(e*x+d)^(7/2),x,method=_RETURNVERB
OSE)
```

output
```
2/e^4*(1/7*c^3*d^3*(e*x+d)^(7/2)+3/5*(a*e^2-c*d^2)*c^2*d^2*(e*x+d)^(5/2)+(
a*e^2-c*d^2)^2*c*d*(e*x+d)^(3/2)+(a*e^2-c*d^2)^3*(e*x+d)^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 130, normalized size of antiderivative = 1.13

$$\int \frac{(ade+(cd^2+ae^2)x+cdex^2)^3}{(d+ex)^{7/2}}\,dx = \frac{2\,(5\,c^3d^3e^3x^3-16\,c^3d^6+56\,ac^2d^4e^2-70\,a^2cd^2e^4+35\,a^3e^6-3\,(2\,c}{3}$$

input
```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^3/(e*x+d)^(7/2),x, algorithm="
fricas")
```

output
```
2/35*(5*c^3*d^3*e^3*x^3 - 16*c^3*d^6 + 56*a*c^2*d^4*e^2 - 70*a^2*c*d^2*e^4
 + 35*a^3*e^6 - 3*(2*c^3*d^4*e^2 - 7*a*c^2*d^2*e^4)*x^2 + (8*c^3*d^5*e - 2
8*a*c^2*d^3*e^3 + 35*a^2*c*d*e^5)*x)*sqrt(e*x + d)/e^4
```

Sympy [A] (verification not implemented)

Time = 1.10 (sec) , antiderivative size = 177, normalized size of antiderivative = 1.54

$$\int \frac{(ade+(cd^2+ae^2)x+cdex^2)^3}{(d+ex)^{7/2}}\,dx = \begin{cases} \frac{2\left(\frac{c^3d^3(d+ex)^{\frac{7}{2}}}{7e^3}+\frac{(d+ex)^{\frac{5}{2}}\cdot\left(3ac^2d^2e^2-3c^3d^4\right)}{5e^3}+\frac{(d+ex)^{\frac{3}{2}}\cdot\left(3a^2cde^4-6ac^2d^3e^2+3c^3d^5\right)}{3e^3}+\frac{\sqrt{a}}{}\right.}{e} \\ \frac{c^3d^{\frac{5}{2}}x^4}{4} \end{cases}$$

input
```
integrate((a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**3/(e*x+d)**(7/2),x)
```

output
```
Piecewise((2*(c**3*d**3*(d + e*x)**(7/2)/(7*e**3) + (d + e*x)**(5/2)*(3*a*
c**2*d**2*e**2 - 3*c**3*d**4)/(5*e**3) + (d + e*x)**(3/2)*(3*a**2*c*d*e**4
 - 6*a*c**2*d**3*e**2 + 3*c**3*d**5)/(3*e**3) + sqrt(d + e*x)*(a**3*e**6 -
 3*a**2*c*d**2*e**4 + 3*a*c**2*d**4*e**2 - c**3*d**6)/e**3)/e, Ne(e, 0)),
(c**3*d**(5/2)*x**4/4, True))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 137, normalized size of antiderivative = 1.19

$$\int \frac{(ade + (cd^2 + ae^2)x + cdex^2)^3}{(d+ex)^{7/2}}\,dx = \frac{2\left(5(ex+d)^{\frac{7}{2}}c^3d^3 - 21(c^3d^4 - ac^2d^2e^2)(ex+d)^{\frac{5}{2}} + 35(c^3d^5 - 2\,c\right.}{}$$

input
```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^3/(e*x+d)^(7/2),x, algorithm="
maxima")
```

output
```
2/35*(5*(e*x + d)^(7/2)*c^3*d^3 - 21*(c^3*d^4 - a*c^2*d^2*e^2)*(e*x + d)^(
5/2) + 35*(c^3*d^5 - 2*a*c^2*d^3*e^2 + a^2*c*d*e^4)*(e*x + d)^(3/2) - 35*(
c^3*d^6 - 3*a*c^2*d^4*e^2 + 3*a^2*c*d^2*e^4 - a^3*e^6)*sqrt(e*x + d))/e^4
```

Giac [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 145, normalized size of antiderivative = 1.26

$$\int \frac{(ade + (cd^2 + ae^2)x + cdex^2)^3}{(d+ex)^{7/2}}\,dx = \frac{2\left(35\sqrt{ex+d}a^3e^3 + 35\left((ex+d)^{\frac{3}{2}} - 3\sqrt{ex+d}d\right)a^2cde + \frac{7\left(3(ex\right.}{}\right.}{}$$

input
```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^3/(e*x+d)^(7/2),x, algorithm="
giac")
```

output
```
2/35*(35*sqrt(e*x + d)*a^3*e^3 + 35*((e*x + d)^(3/2) - 3*sqrt(e*x + d)*d)*
a^2*c*d*e + 7*(3*(e*x + d)^(5/2) - 10*(e*x + d)^(3/2)*d + 15*sqrt(e*x + d)
*d^2)*a*c^2*d^2/e + (5*(e*x + d)^(7/2) - 21*(e*x + d)^(5/2)*d + 35*(e*x +
d)^(3/2)*d^2 - 35*sqrt(e*x + d)*d^3)*c^3*d^3/e^3)/e
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 106, normalized size of antiderivative = 0.92

$$\int \frac{(ade+(cd^2+ae^2)x+cdex^2)^3}{(d+ex)^{7/2}}\,dx = \frac{2\,(a\,e^2-c\,d^2)^3\,\sqrt{d+e\,x}}{e^4} - \frac{\left(6\,c^3\,d^4-6\,a\,c^2\,d^2\,e^2\right)\,(d+e\,x)^{5/2}}{5\,e^4} + \frac{2\,c^3\,d^3\,(d+e\,x)^{7/2}}{7\,e^4} + \frac{2\,c\,d\,(a\,e^2-c\,d^2)^2\,(d+e\,x)^{3/2}}{e^4}$$

input
```
int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^3/(d + e*x)^(7/2),x)
```

output
```
(2*(a*e^2 - c*d^2)^3*(d + e*x)^(1/2))/e^4 - ((6*c^3*d^4 - 6*a*c^2*d^2*e^2)
*(d + e*x)^(5/2))/(5*e^4) + (2*c^3*d^3*(d + e*x)^(7/2))/(7*e^4) + (2*c*d*(
a*e^2 - c*d^2)^2*(d + e*x)^(3/2))/e^4
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 129, normalized size of antiderivative = 1.12

$$\int \frac{(ade+(cd^2+ae^2)x+cdex^2)^3}{(d+ex)^{7/2}}\,dx = \frac{2\sqrt{ex+d}\,(5c^3d^3e^3x^3+21a\,c^2d^2e^4x^2-6c^3d^4e^2x^2+35a^2cd\,e^5x-2}{35}$$

input
```
int((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^3/(e*x+d)^(7/2),x)
```

output
```
(2*sqrt(d + e*x)*(35*a**3*e**6 - 70*a**2*c*d**2*e**4 + 35*a**2*c*d*e**5*x
+ 56*a*c**2*d**4*e**2 - 28*a*c**2*d**3*e**3*x + 21*a*c**2*d**2*e**4*x**2 -
 16*c**3*d**6 + 8*c**3*d**5*e*x - 6*c**3*d**4*e**2*x**2 + 5*c**3*d**3*e**3
*x**3))/(35*e**4)
```

3.170 $\int \frac{(ade+(cd^2+ae^2)x+cdex^2)^3}{(d+ex)^{9/2}}\,dx$

Optimal result

Integrand size = 37, antiderivative size = 113

$$\int \frac{(ade+(cd^2+ae^2)x+cdex^2)^3}{(d+ex)^{9/2}}\,dx = \frac{2(cd^2-ae^2)^3}{e^4\sqrt{d+ex}} + \frac{6cd(cd^2-ae^2)^2\sqrt{d+ex}}{e^4} - \frac{2c^2d^2(cd^2-ae^2)(d+ex)^{3/2}}{e^4} + \frac{2c^3d^3(d+ex)^{5/2}}{5e^4}$$

output
```
2*(-a*e^2+c*d^2)^3/e^4/(e*x+d)^(1/2)+6*c*d*(-a*e^2+c*d^2)^2*(e*x+d)^(1/2)/
e^4-2*c^2*d^2*(-a*e^2+c*d^2)*(e*x+d)^(3/2)/e^4+2/5*c^3*d^3*(e*x+d)^(5/2)/e
^4
```

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 109, normalized size of antiderivative = 0.96

$$\int \frac{(ade+(cd^2+ae^2)x+cdex^2)^3}{(d+ex)^{9/2}}\,dx = \frac{2(-5a^3e^6+15a^2cde^4(2d+ex)-5ac^2d^2e^2(8d^2+4dex-e^2x^2)+c}{5e^4\sqrt{d+ex}}$$

input
```
Integrate[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^3/(d + e*x)^(9/2),x]
```

output
```
(2*(-5*a^3*e^6 + 15*a^2*c*d*e^4*(2*d + e*x) - 5*a*c^2*d^2*e^2*(8*d^2 + 4*d
*e*x - e^2*x^2) + c^3*d^3*(16*d^3 + 8*d^2*e*x - 2*d*e^2*x^2 + e^3*x^3)))/(
5*e^4*Sqrt[d + e*x])
```

Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.054$, Rules used = {1121, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^3}{(d+ex)^{9/2}}dx$$

$$\Big\downarrow \text{1121}$$

$$\int \left(-\frac{3c^2d^2\sqrt{d+ex}\left(cd^2-ae^2\right)}{e^3}+\frac{3cd\left(cd^2-ae^2\right)^2}{e^3\sqrt{d+ex}}+\frac{\left(ae^2-cd^2\right)^3}{e^3(d+ex)^{3/2}}+\frac{c^3d^3(d+ex)^{3/2}}{e^3}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{2c^2d^2(d+ex)^{3/2}\left(cd^2-ae^2\right)}{e^4}+\frac{6cd\sqrt{d+ex}\left(cd^2-ae^2\right)^2}{e^4}+\frac{2\left(cd^2-ae^2\right)^3}{e^4\sqrt{d+ex}}+\frac{2c^3d^3(d+ex)^{5/2}}{5e^4}$$

input
```
Int[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^3/(d + e*x)^(9/2),x]
```

output
```
(2*(c*d^2 - a*e^2)^3)/(e^4*Sqrt[d + e*x]) + (6*c*d*(c*d^2 - a*e^2)^2*Sqrt[
d + e*x])/e^4 - (2*c^2*d^2*(c*d^2 - a*e^2)*(d + e*x)^(3/2))/e^4 + (2*c^3*d
^3*(d + e*x)^(5/2))/(5*e^4)
```

Defintions of rubi rules used

rule 1121
```
Int[((d_) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Int[ExpandIntegrand[(d + e*x)^(m + p)*(a/d + (c/e)*x)^p, x], x]
/; FreeQ[{a, b, c, d, e, m, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && (Int
egerQ[p] || (GtQ[a, 0] && GtQ[d, 0] && LtQ[c, 0]))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 1.53 (sec) , antiderivative size = 105, normalized size of antiderivative = 0.93

method	result
pseudoelliptic	$\frac{\frac{2\left(d^3e^3x^3-2d^4e^2x^2+8d^5ex+16d^6\right)c^3}{5}-16e^2\left(-\frac{1}{8}e^2x^2+\frac{1}{2}dex+d^2\right)a\,d^2c^2+12e^4\left(\frac{ex}{2}+d\right)a^2dc-2e^6a^3}{\sqrt{ex+d}\,e^4}$
risch	$\frac{2cd\left(x^2c^2d^2e^2+5xacd\,e^3-3x\,c^2d^3e+15a^2e^4-25ac\,d^2e^2+11c^2d^4\right)\sqrt{ex+d}}{5e^4}-\frac{2\left(e^6a^3-3d^2e^4a^2c+3d^4e^2a\,c^2-d^6c^3\right)}{e^4\sqrt{ex+d}}$
gosper	$-\frac{2(-c^3d^3e^3x^3-5x^2a\,c^2d^2e^4+2c^3d^4e^2x^2-15x\,a^2cd\,e^5+20xa\,c^2d^3e^3-8c^3d^5ex+5e^6a^3-30d^2e^4a^2c+40d^4e^2a\,c^2-16d}{5\sqrt{ex+d}\,e^4}$
trager	$-\frac{2(-c^3d^3e^3x^3-5x^2a\,c^2d^2e^4+2c^3d^4e^2x^2-15x\,a^2cd\,e^5+20xa\,c^2d^3e^3-8c^3d^5ex+5e^6a^3-30d^2e^4a^2c+40d^4e^2a\,c^2-16d}{5\sqrt{ex+d}\,e^4}$
derivativedivides	$\frac{\frac{2c^3d^3(ex+d)^{\frac{5}{2}}}{5}+2a\,c^2d^2e^2(ex+d)^{\frac{3}{2}}-2c^3d^4(ex+d)^{\frac{3}{2}}+6a^2cd\,e^4\sqrt{ex+d}-12a\,c^2d^3e^2\sqrt{ex+d}+6c^3d^5\sqrt{ex+d}-\frac{2\left(e^6a^3-3d^2\right.}{}}{e^4}$
default	$\frac{\frac{2c^3d^3(ex+d)^{\frac{5}{2}}}{5}+2a\,c^2d^2e^2(ex+d)^{\frac{3}{2}}-2c^3d^4(ex+d)^{\frac{3}{2}}+6a^2cd\,e^4\sqrt{ex+d}-12a\,c^2d^3e^2\sqrt{ex+d}+6c^3d^5\sqrt{ex+d}-\frac{2\left(e^6a^3-3d^2\right.}{}}{e^4}$
orering	$-\frac{2(-c^3d^3e^3x^3-5x^2a\,c^2d^2e^4+2c^3d^4e^2x^2-15x\,a^2cd\,e^5+20xa\,c^2d^3e^3-8c^3d^5ex+5e^6a^3-30d^2e^4a^2c+40d^4e^2a\,c^2-16d}{5e^4(cdx+ae)^3(ex+d)^{\frac{7}{2}}}$

input
```
int((a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^3/(e*x+d)^(9/2),x,method=_RETURNVERB
OSE)
```

output
```
2/5*((d^3*e^3*x^3-2*d^4*e^2*x^2+8*d^5*e*x+16*d^6)*c^3-40*e^2*(-1/8*e^2*x^2
+1/2*d*e*x+d^2)*a*d^2*c^2+30*e^4*(1/2*e*x+d)*a^2*d*c-5*e^6*a^3)/(e*x+d)^(1
/2)/e^4
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 139, normalized size of antiderivative = 1.23

$$\int \frac{(ade+(cd^2+ae^2)x+cdex^2)^3}{(d+ex)^{9/2}}\,dx = \frac{2\,(c^3d^3e^3x^3+16\,c^3d^6-40\,ac^2d^4e^2+30\,a^2cd^2e^4-5\,a^3e^6-(2\,c^3d^4e \cdots}{5\,(e^5x+ \cdots}$$

input
```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^3/(e*x+d)^(9/2),x, algorithm="
fricas")
```

output
```
2/5*(c^3*d^3*e^3*x^3 + 16*c^3*d^6 - 40*a*c^2*d^4*e^2 + 30*a^2*c*d^2*e^4 -
5*a^3*e^6 - (2*c^3*d^4*e^2 - 5*a*c^2*d^2*e^4)*x^2 + (8*c^3*d^5*e - 20*a*c^
2*d^3*e^3 + 15*a^2*c*d*e^5)*x)*sqrt(e*x + d)/(e^5*x + d*e^4)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 230 vs. $2(105) = 210$.

Time = 0.78 (sec) , antiderivative size = 230, normalized size of antiderivative = 2.04

$$\int \frac{(ade+(cd^2+ae^2)x+cdex^2)^3}{(d+ex)^{9/2}}\,dx = \begin{cases} -\frac{2a^3e^2}{\sqrt{d+ex}}+\frac{12a^2cd^2}{\sqrt{d+ex}}+\frac{6a^2cdex}{\sqrt{d+ex}}-\frac{16ac^2d^4}{e^2\sqrt{d+ex}}-\frac{8ac^2d^3x}{e\sqrt{d+ex}}+\frac{2ac^2d^2x^2}{\sqrt{d+ex}}+\frac{32c^3 \cdots}{5e^4\sqrt{ \cdots}} \cdots \\ \frac{c^3d^{\frac{3}{2}}x^4}{4} \end{cases}$$

input
```
integrate((a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**3/(e*x+d)**(9/2),x)
```

output
```
Piecewise((-2*a**3*e**2/sqrt(d + e*x) + 12*a**2*c*d**2/sqrt(d + e*x) + 6*a
**2*c*d*e*x/sqrt(d + e*x) - 16*a*c**2*d**4/(e**2*sqrt(d + e*x)) - 8*a*c**2
*d**3*x/(e*sqrt(d + e*x)) + 2*a*c**2*d**2*x**2/sqrt(d + e*x) + 32*c**3*d**
6/(5*e**4*sqrt(d + e*x)) + 16*c**3*d**5*x/(5*e**3*sqrt(d + e*x)) - 4*c**3*
d**4*x**2/(5*e**2*sqrt(d + e*x)) + 2*c**3*d**3*x**3/(5*e*sqrt(d + e*x)), N
e(e, 0)), (c**3*d**(3/2)*x**4/4, True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 144, normalized size of antiderivative = 1.27

$$\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^3}{(d+ex)^{9/2}}\,dx = \frac{2\left(\frac{(ex+d)^{\frac{5}{2}}c^3d^3-5\left(c^3d^4-ac^2d^2e^2\right)(ex+d)^{\frac{3}{2}}+15\left(c^3d^5-2\,ac^2d^3e^2+a^2cde^4\right)\sqrt{ex+d}}{e^3}+\right.}{5\,e}$$

input
```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^3/(e*x+d)^(9/2),x, algorithm="
maxima")
```

output
```
2/5*(((e*x + d)^(5/2)*c^3*d^3 - 5*(c^3*d^4 - a*c^2*d^2*e^2)*(e*x + d)^(3/2
) + 15*(c^3*d^5 - 2*a*c^2*d^3*e^2 + a^2*c*d*e^4)*sqrt(e*x + d))/e^3 + 5*(c
^3*d^6 - 3*a*c^2*d^4*e^2 + 3*a^2*c*d^2*e^4 - a^3*e^6)/(sqrt(e*x + d)*e^3))
/e
```

Giac [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 167, normalized size of antiderivative = 1.48

$$\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^3}{(d+ex)^{9/2}}\,dx = \frac{2\left(c^3d^6-3\,ac^2d^4e^2+3\,a^2cd^2e^4-a^3e^6\right)}{\sqrt{ex+d}e^4}$$
$$+\frac{2\left((ex+d)^{\frac{5}{2}}c^3d^3e^{16}-5\,(ex+d)^{\frac{3}{2}}c^3d^4e^{16}+15\sqrt{ex+d}c^3d^5e^{16}+5\,(ex+d)^{\frac{3}{2}}ac^2d^2e^{18}-30\sqrt{ex+d}ac^2d^3\right.}{5\,e^{20}}$$

input
```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^3/(e*x+d)^(9/2),x, algorithm="
giac")
```

output
```
2*(c^3*d^6 - 3*a*c^2*d^4*e^2 + 3*a^2*c*d^2*e^4 - a^3*e^6)/(sqrt(e*x + d)*e
^4) + 2/5*((e*x + d)^(5/2)*c^3*d^3*e^16 - 5*(e*x + d)^(3/2)*c^3*d^4*e^16 +
 15*sqrt(e*x + d)*c^3*d^5*e^16 + 5*(e*x + d)^(3/2)*a*c^2*d^2*e^18 - 30*sqr
t(e*x + d)*a*c^2*d^3*e^18 + 15*sqrt(e*x + d)*a^2*c*d*e^20)/e^20
```

Mupad [B] (verification not implemented)

Time = 5.16 (sec) , antiderivative size = 133, normalized size of antiderivative = 1.18

$$\int \frac{(ade + (cd^2 + ae^2)x + cdex^2)^3}{(d + ex)^{9/2}} dx = \frac{2c^3 d^3 (d + ex)^{5/2}}{5e^4} - \frac{(6c^3 d^4 - 6ac^2 d^2 e^2)(d + ex)^{3/2}}{3e^4} - \frac{2a^3 e^6 - 6a^2 c d^2 e^4 + 6ac^2 d^4 e^2 - 2c^3 d^6}{e^4 \sqrt{d + ex}} + \frac{6cd(ae^2 - cd^2)^2 \sqrt{d + ex}}{e^4}$$

input
```
int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^3/(d + e*x)^(9/2),x)
```

output
```
(2*c^3*d^3*(d + e*x)^(5/2))/(5*e^4) - ((6*c^3*d^4 - 6*a*c^2*d^2*e^2)*(d +
e*x)^(3/2))/(3*e^4) - (2*a^3*e^6 - 2*c^3*d^6 + 6*a*c^2*d^4*e^2 - 6*a^2*c*d
^2*e^4)/(e^4*(d + e*x)^(1/2)) + (6*c*d*(a*e^2 - c*d^2)^2*(d + e*x)^(1/2))/
e^4
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 130, normalized size of antiderivative = 1.15

$$\int \frac{(ade + (cd^2 + ae^2)x + cdex^2)^3}{(d + ex)^{9/2}} dx = \frac{\frac{2}{5}c^3d^3e^3x^3 + 2ac^2d^2e^4x^2 - \frac{4}{5}c^3d^4e^2x^2 + 6a^2cde^5x - 8ac^2d^3e^3x + 1}{\sqrt{ex + d}\, e^4}$$

input
```
int((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^3/(e*x+d)^(9/2),x)
```

output
```
(2*( - 5*a**3*e**6 + 30*a**2*c*d**2*e**4 + 15*a**2*c*d*e**5*x - 40*a*c**2*
d**4*e**2 - 20*a*c**2*d**3*e**3*x + 5*a*c**2*d**2*e**4*x**2 + 16*c**3*d**6
 + 8*c**3*d**5*e*x - 2*c**3*d**4*e**2*x**2 + c**3*d**3*e**3*x**3))/(5*sqrt
(d + e*x)*e**4)
```

3.171 $\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^3}{(d+ex)^{11/2}}\,dx$

Optimal result

Integrand size = 37, antiderivative size = 115

$$\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^3}{(d+ex)^{11/2}}\,dx=\frac{2\left(cd^2-ae^2\right)^3}{3e^4(d+ex)^{3/2}}-\frac{6cd\left(cd^2-ae^2\right)^2}{e^4\sqrt{d+ex}}-\frac{6c^2d^2\left(cd^2-ae^2\right)\sqrt{d+ex}}{e^4}+\frac{2c^3d^3(d+ex)^{3/2}}{3e^4}$$

output
```
2/3*(-a*e^2+c*d^2)^3/e^4/(e*x+d)^(3/2)-6*c*d*(-a*e^2+c*d^2)^2/e^4/(e*x+d)^
(1/2)-6*c^2*d^2*(-a*e^2+c*d^2)*(e*x+d)^(1/2)/e^4+2/3*c^3*d^3*(e*x+d)^(3/2)
/e^4
```

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 110, normalized size of antiderivative = 0.96

$$\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^3}{(d+ex)^{11/2}}\,dx=-\frac{2\left(a^3e^6+3a^2cde^4(2d+3ex)-3ac^2d^2e^2\left(8d^2+12dex+3e^2x^2\right)+c^3d^3\left(16d^3+24d^2ex+6de^2x^2-e^3x^3\right)\right)}{3e^4(d+ex)^{3/2}}$$

input
```
Integrate[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^3/(d + e*x)^(11/2),x]
```

output
```
(-2*(a^3*e^6 + 3*a^2*c*d*e^4*(2*d + 3*e*x) - 3*a*c^2*d^2*e^2*(8*d^2 + 12*d
*e*x + 3*e^2*x^2) + c^3*d^3*(16*d^3 + 24*d^2*e*x + 6*d*e^2*x^2 - e^3*x^3))
)/(3*e^4*(d + e*x)^(3/2))
```

Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 115, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.054$, Rules used = {1121, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^3}{(d+ex)^{11/2}}\,dx$$

$$\Big\downarrow \text{1121}$$

$$\int \left(-\frac{3c^2d^2\left(cd^2-ae^2\right)}{e^3\sqrt{d+ex}}+\frac{3cd\left(cd^2-ae^2\right)^2}{e^3(d+ex)^{3/2}}+\frac{\left(ae^2-cd^2\right)^3}{e^3(d+ex)^{5/2}}+\frac{c^3d^3\sqrt{d+ex}}{e^3}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{6c^2d^2\sqrt{d+ex}\left(cd^2-ae^2\right)}{e^4}-\frac{6cd\left(cd^2-ae^2\right)^2}{e^4\sqrt{d+ex}}+\frac{2\left(cd^2-ae^2\right)^3}{3e^4(d+ex)^{3/2}}+\frac{2c^3d^3(d+ex)^{3/2}}{3e^4}$$

input
```
Int[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^3/(d + e*x)^(11/2),x]
```

output
```
(2*(c*d^2 - a*e^2)^3)/(3*e^4*(d + e*x)^(3/2)) - (6*c*d*(c*d^2 - a*e^2)^2)/
(e^4*Sqrt[d + e*x]) - (6*c^2*d^2*(c*d^2 - a*e^2)*Sqrt[d + e*x])/e^4 + (2*c
^3*d^3*(d + e*x)^(3/2))/(3*e^4)
```

Defintions of rubi rules used

rule 1121
```
Int[((d_) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Int[ExpandIntegrand[(d + e*x)^(m + p)*(a/d + (c/e)*x)^p, x], x]
/; FreeQ[{a, b, c, d, e, m, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && (Int
egerQ[p] || (GtQ[a, 0] && GtQ[d, 0] && LtQ[c, 0]))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 1.54 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.72

method	result
pseudoelliptic	$-\frac{2(-cdxe+a e^2-2c d^2)\left(a^2e^4+10xacd e^3+8\left(\frac{c x^2}{8}+a\right)c d^2e^2-8x c^2d^3e-8c^2d^4\right)}{3(ex+d)^{\frac{3}{2}}e^4}$
risch	$\frac{2c^2d^2(cdxe+9a e^2-8c d^2)\sqrt{ex+d}}{3e^4}-\frac{2(9cdxe+a e^2+8c d^2)(a^2e^4-2ac d^2e^2+c^2d^4)}{3e^4(ex+d)^{\frac{3}{2}}}$
gosper	$-\frac{2(-c^3d^3e^3x^3-9x^2a c^2d^2e^4+6c^3d^4e^2x^2+9x a^2cd e^5-36xa c^2d^3e^3+24c^3d^5ex+e^6a^3+6d^2e^4a^2c-24d^4e^2a c^2+16d^6c}{3(ex+d)^{\frac{3}{2}}e^4}$
trager	$-\frac{2(-c^3d^3e^3x^3-9x^2a c^2d^2e^4+6c^3d^4e^2x^2+9x a^2cd e^5-36xa c^2d^3e^3+24c^3d^5ex+e^6a^3+6d^2e^4a^2c-24d^4e^2a c^2+16d^6c}{3(ex+d)^{\frac{3}{2}}e^4}$
derivativedivides	$\frac{\frac{2c^3d^3(ex+d)^{\frac{3}{2}}}{3}+6a c^2d^2e^2\sqrt{ex+d}-6c^3d^4\sqrt{ex+d}-\frac{6cd\left(a^2e^4-2ac d^2e^2+c^2d^4\right)}{\sqrt{ex+d}}-\frac{2\left(e^6a^3-3d^2e^4a^2c+3d^4e^2a c^2-d^6c^3\right)}{3(ex+d)^{\frac{3}{2}}}}{e^4}$
default	$\frac{\frac{2c^3d^3(ex+d)^{\frac{3}{2}}}{3}+6a c^2d^2e^2\sqrt{ex+d}-6c^3d^4\sqrt{ex+d}-\frac{6cd\left(a^2e^4-2ac d^2e^2+c^2d^4\right)}{\sqrt{ex+d}}-\frac{2\left(e^6a^3-3d^2e^4a^2c+3d^4e^2a c^2-d^6c^3\right)}{3(ex+d)^{\frac{3}{2}}}}{e^4}$
orering	$-\frac{2(-c^3d^3e^3x^3-9x^2a c^2d^2e^4+6c^3d^4e^2x^2+9x a^2cd e^5-36xa c^2d^3e^3+24c^3d^5ex+e^6a^3+6d^2e^4a^2c-24d^4e^2a c^2+16d^6c}{3e^4(cdx+ae)^3(ex+d)^{\frac{9}{2}}}$

input
```
int((a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^3/(e*x+d)^(11/2),x,method=_RETURNVER
BOSE)
```

output
```
-2/3/(e*x+d)^(3/2)*(-c*d*e*x+a*e^2-2*c*d^2)*(a^2*e^4+10*x*a*c*d*e^3+8*(1/8
*c*x^2+a)*c*d^2*e^2-8*x*c^2*d^3*e-8*c^2*d^4)/e^4
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 151, normalized size of antiderivative = 1.31

$$\int \frac{(ade+(cd^2+ae^2)x+cdex^2)^3}{(d+ex)^{11/2}}\,dx = \frac{2\left(c^3d^3e^3x^3-16\,c^3d^6+24\,ac^2d^4e^2-6\,a^2cd^2e^4-a^3e^6-3\left(2\,c^3d^4e^2\ldots\right.\right.}{3\left(e^6x^2+2\,de\ldots\right.}$$

input
```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^3/(e*x+d)^(11/2),x, algorithm=
"fricas")
```

output
```
2/3*(c^3*d^3*e^3*x^3 - 16*c^3*d^6 + 24*a*c^2*d^4*e^2 - 6*a^2*c*d^2*e^4 - a
^3*e^6 - 3*(2*c^3*d^4*e^2 - 3*a*c^2*d^2*e^4)*x^2 - 3*(8*c^3*d^5*e - 12*a*c
^2*d^3*e^3 + 3*a^2*c*d*e^5)*x)*sqrt(e*x + d)/(e^6*x^2 + 2*d*e^5*x + d^2*e^
4)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 450 vs. 2(107) = 214.

Time = 1.08 (sec) , antiderivative size = 450, normalized size of antiderivative = 3.91

$$\int \frac{(ade+(cd^2+ae^2)x+cdex^2)^3}{(d+ex)^{11/2}}\,dx = \begin{cases} -\frac{2a^3e^6}{3de^4\sqrt{d+ex}+3e^5x\sqrt{d+ex}} - \frac{12a^2cd^2e^4}{3de^4\sqrt{d+ex}+3e^5x\sqrt{d+ex}} - \frac{18a^2cde^5x}{3de^4\sqrt{d+ex}+3e^5x\sqrt{d+\ldots}} \ldots \\ \frac{c^3\sqrt{d}x^4}{4} \end{cases}$$

input
```
integrate((a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**3/(e*x+d)**(11/2),x)
```

output
```
Piecewise((-2*a**3*e**6/(3*d*e**4*sqrt(d + e*x) + 3*e**5*x*sqrt(d + e*x))
- 12*a**2*c*d**2*e**4/(3*d*e**4*sqrt(d + e*x) + 3*e**5*x*sqrt(d + e*x)) -
18*a**2*c*d*e**5*x/(3*d*e**4*sqrt(d + e*x) + 3*e**5*x*sqrt(d + e*x)) + 48*
a*c**2*d**4*e**2/(3*d*e**4*sqrt(d + e*x) + 3*e**5*x*sqrt(d + e*x)) + 72*a*
c**2*d**3*e**3*x/(3*d*e**4*sqrt(d + e*x) + 3*e**5*x*sqrt(d + e*x)) + 18*a*
c**2*d**2*e**4*x**2/(3*d*e**4*sqrt(d + e*x) + 3*e**5*x*sqrt(d + e*x)) - 32
*c**3*d**6/(3*d*e**4*sqrt(d + e*x) + 3*e**5*x*sqrt(d + e*x)) - 48*c**3*d**
5*e*x/(3*d*e**4*sqrt(d + e*x) + 3*e**5*x*sqrt(d + e*x)) - 12*c**3*d**4*e**
2*x**2/(3*d*e**4*sqrt(d + e*x) + 3*e**5*x*sqrt(d + e*x)) + 2*c**3*d**3*e**
3*x**3/(3*d*e**4*sqrt(d + e*x) + 3*e**5*x*sqrt(d + e*x)), Ne(e, 0)), (c**3
*sqrt(d)*x**4/4, True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 141, normalized size of antiderivative = 1.23

$$\int \frac{(ade+(cd^2+ae^2)x+cdex^2)^3}{(d+ex)^{11/2}}\,dx = \frac{2\left(\frac{(ex+d)^{\frac{3}{2}}c^3d^3-9\,(c^3d^4-ac^2d^2e^2)\sqrt{ex+d}}{e^3}+\frac{c^3d^6-3\,ac^2d^4e^2+3\,a^2cd^2e^4-a^3e^6-9\,(c^3\cdots}{(ex+d)^{\frac{3}{2}}e^3}\right)}{3\,e}$$

input
```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^3/(e*x+d)^(11/2),x, algorithm=
"maxima")
```

output
```
2/3*(((e*x + d)^(3/2)*c^3*d^3 - 9*(c^3*d^4 - a*c^2*d^2*e^2)*sqrt(e*x + d))
/e^3 + (c^3*d^6 - 3*a*c^2*d^4*e^2 + 3*a^2*c*d^2*e^4 - a^3*e^6 - 9*(c^3*d^5
 - 2*a*c^2*d^3*e^2 + a^2*c*d*e^4)*(e*x + d))/((e*x + d)^(3/2)*e^3))/e
```

Giac [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 158, normalized size of antiderivative = 1.37

$$\int \frac{(ade+(cd^2+ae^2)x+cdex^2)^3}{(d+ex)^{11/2}}\,dx = -\frac{2\,(9\,(ex+d)c^3d^5-c^3d^6-18\,(ex+d)ac^2d^3e^2+3\,ac^2d^4e^2+9\,(ex+d)a^2cde^4-3\,a^2cd^2e^4+a^3e^6)}{3\,(ex+d)^{\frac{3}{2}}e^4} + \frac{2\left((ex+d)^{\frac{3}{2}}c^3d^3e^8-9\,\sqrt{ex+d}c^3d^4e^8+9\,\sqrt{ex+d}ac^2d^2e^{10}\right)}{3\,e^{12}}$$

input
```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^3/(e*x+d)^(11/2),x, algorithm=
"giac")
```

output
```
-2/3*(9*(e*x + d)*c^3*d^5 - c^3*d^6 - 18*(e*x + d)*a*c^2*d^3*e^2 + 3*a*c^2
*d^4*e^2 + 9*(e*x + d)*a^2*c*d*e^4 - 3*a^2*c*d^2*e^4 + a^3*e^6)/((e*x + d)
^(3/2)*e^4) + 2/3*((e*x + d)^(3/2)*c^3*d^3*e^8 - 9*sqrt(e*x + d)*c^3*d^4*e
^8 + 9*sqrt(e*x + d)*a*c^2*d^2*e^10)/e^12
```

Mupad [B] (verification not implemented)

Time = 5.41 (sec) , antiderivative size = 147, normalized size of antiderivative = 1.28

$$\int \frac{(ade + (cd^2 + ae^2)x + cdex^2)^3}{(d+ex)^{11/2}}\,dx = -\frac{2a^3e^6 - 2c^3d^6 - 2c^3d^3(d+ex)^3 + 18c^3d^4(d+ex)^2 + 18c^3d^5(d+ex) + 6ac^2d^4e^2 - 6a^2cd^2e^4 - 3}{3e^4(d+ex)^{3/2}}$$

input
```
int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^3/(d + e*x)^(11/2),x)
```

output
```
-(2*a^3*e^6 - 2*c^3*d^6 - 2*c^3*d^3*(d + e*x)^3 + 18*c^3*d^4*(d + e*x)^2 +
 18*c^3*d^5*(d + e*x) + 6*a*c^2*d^4*e^2 - 6*a^2*c*d^2*e^4 - 36*a*c^2*d^3*e
^2*(d + e*x) - 18*a*c^2*d^2*e^2*(d + e*x)^2 + 18*a^2*c*d*e^4*(d + e*x))/(3
*e^4*(d + e*x)^(3/2))
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 137, normalized size of antiderivative = 1.19

$$\int \frac{(ade + (cd^2 + ae^2)x + cdex^2)^3}{(d+ex)^{11/2}}\,dx = \frac{\frac{2}{3}c^3d^3e^3x^3 + 6ac^2d^2e^4x^2 - 4c^3d^4e^2x^2 - 6a^2cde^5x + 24ac^2d^3e^3x -}{\sqrt{ex+d}\,e^4(ex+}$$

input
```
int((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^3/(e*x+d)^(11/2),x)
```

output
```
(2*( - a**3*e**6 - 6*a**2*c*d**2*e**4 - 9*a**2*c*d*e**5*x + 24*a*c**2*d**4
*e**2 + 36*a*c**2*d**3*e**3*x + 9*a*c**2*d**2*e**4*x**2 - 16*c**3*d**6 - 2
4*c**3*d**5*e*x - 6*c**3*d**4*e**2*x**2 + c**3*d**3*e**3*x**3))/(3*sqrt(d
+ e*x)*e**4*(d + e*x))
```

3.172 $\int \frac{(ade+(cd^2+ae^2)x+cdex^2)^3}{(d+ex)^{13/2}}\,dx$

Optimal result

Integrand size = 37, antiderivative size = 113

$$\int \frac{(ade+(cd^2+ae^2)x+cdex^2)^3}{(d+ex)^{13/2}}\,dx = \frac{2(cd^2-ae^2)^3}{5e^4(d+ex)^{5/2}} - \frac{2cd(cd^2-ae^2)^2}{e^4(d+ex)^{3/2}} + \frac{6c^2d^2(cd^2-ae^2)}{e^4\sqrt{d+ex}} + \frac{2c^3d^3\sqrt{d+ex}}{e^4}$$

output
```
2/5*(-a*e^2+c*d^2)^3/e^4/(e*x+d)^(5/2)-2*c*d*(-a*e^2+c*d^2)^2/e^4/(e*x+d)^
(3/2)+6*c^2*d^2*(-a*e^2+c*d^2)/e^4/(e*x+d)^(1/2)+2*c^3*d^3*(e*x+d)^(1/2)/e
^4
```

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 109, normalized size of antiderivative = 0.96

$$\int \frac{(ade+(cd^2+ae^2)x+cdex^2)^3}{(d+ex)^{13/2}}\,dx = -\frac{2(a^3e^6+a^2cde^4(2d+5ex)+ac^2d^2e^2(8d^2+20dex+15e^2x^2)-c^3d^3(16d^3+40d^2ex+30de^2x^2+5e^3x^3))}{5e^4(d+ex)^{5/2}}$$

input
```
Integrate[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^3/(d + e*x)^(13/2),x]
```

output
```
(-2*(a^3*e^6 + a^2*c*d*e^4*(2*d + 5*e*x) + a*c^2*d^2*e^2*(8*d^2 + 20*d*e*x
 + 15*e^2*x^2) - c^3*d^3*(16*d^3 + 40*d^2*e*x + 30*d*e^2*x^2 + 5*e^3*x^3))
)/(5*e^4*(d + e*x)^(5/2))
```

Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.054$, Rules used = {1121, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^3}{(d+ex)^{13/2}}dx$$

$$\Big\downarrow \text{ 1121}$$

$$\int \left(-\frac{3c^2d^2\left(cd^2-ae^2\right)}{e^3(d+ex)^{3/2}}+\frac{3cd\left(cd^2-ae^2\right)^2}{e^3(d+ex)^{5/2}}+\frac{\left(ae^2-cd^2\right)^3}{e^3(d+ex)^{7/2}}+\frac{c^3d^3}{e^3\sqrt{d+ex}}\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{6c^2d^2\left(cd^2-ae^2\right)}{e^4\sqrt{d+ex}}-\frac{2cd\left(cd^2-ae^2\right)^2}{e^4(d+ex)^{3/2}}+\frac{2\left(cd^2-ae^2\right)^3}{5e^4(d+ex)^{5/2}}+\frac{2c^3d^3\sqrt{d+ex}}{e^4}$$

input
```
Int[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^3/(d + e*x)^(13/2),x]
```

output
```
(2*(c*d^2 - a*e^2)^3)/(5*e^4*(d + e*x)^(5/2)) - (2*c*d*(c*d^2 - a*e^2)^2)/
(e^4*(d + e*x)^(3/2)) + (6*c^2*d^2*(c*d^2 - a*e^2))/(e^4*Sqrt[d + e*x]) +
(2*c^3*d^3*Sqrt[d + e*x])/e^4
```

Defintions of rubi rules used

rule 1121
```
Int[((d_) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Int[ExpandIntegrand[(d + e*x)^(m + p)*(a/d + (c/e)*x)^p, x], x]
/; FreeQ[{a, b, c, d, e, m, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && (Int
egerQ[p] || (GtQ[a, 0] && GtQ[d, 0] && LtQ[c, 0]))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 1.46 (sec) , antiderivative size = 106, normalized size of antiderivative = 0.94

method	result
pseudoelliptic	$\frac{\frac{2\left(5d^{3}e^{3}x^{3}+30d^{4}e^{2}x^{2}+40d^{5}ex+16d^{6}\right)c^{3}}{5}-\frac{16e^{2}\left(\frac{15}{8}e^{2}x^{2}+\frac{5}{2}dex+d^{2}\right)a\,d^{2}c^{2}}{5}-\frac{4e^{4}\left(\frac{5ex}{2}+d\right)a^{2}dc}{5}-\frac{2e^{6}a^{3}}{5}}{(ex+d)^{\frac{5}{2}}e^{4}}$
risch	$\frac{2c^{3}d^{3}\sqrt{ex+d}}{e^{4}}-\frac{2(15x^{2}c^{2}d^{2}e^{2}+5xacd\,e^{3}+25x\,c^{2}d^{3}e+a^{2}e^{4}+3ac\,d^{2}e^{2}+11c^{2}d^{4})(a\,e^{2}-c\,d^{2})}{5e^{4}\sqrt{ex+d}\,(e^{2}x^{2}+2dex+d^{2})}$
gosper	$-\frac{2(-5c^{3}d^{3}e^{3}x^{3}+15x^{2}a\,c^{2}d^{2}e^{4}-30c^{3}d^{4}e^{2}x^{2}+5x\,a^{2}cd\,e^{5}+20xa\,c^{2}d^{3}e^{3}-40c^{3}d^{5}ex+e^{6}a^{3}+2d^{2}e^{4}a^{2}c+8d^{4}e^{2}a\,c^{2}-16d}{5(ex+d)^{\frac{5}{2}}e^{4}}$
trager	$-\frac{2(-5c^{3}d^{3}e^{3}x^{3}+15x^{2}a\,c^{2}d^{2}e^{4}-30c^{3}d^{4}e^{2}x^{2}+5x\,a^{2}cd\,e^{5}+20xa\,c^{2}d^{3}e^{3}-40c^{3}d^{5}ex+e^{6}a^{3}+2d^{2}e^{4}a^{2}c+8d^{4}e^{2}a\,c^{2}-16d}{5(ex+d)^{\frac{5}{2}}e^{4}}$
derivativedivides	$\frac{2c^{3}d^{3}\sqrt{ex+d}-\frac{2\left(e^{6}a^{3}-3d^{2}e^{4}a^{2}c+3d^{4}e^{2}a\,c^{2}-d^{6}c^{3}\right)}{5(ex+d)^{\frac{5}{2}}}-\frac{6c^{2}d^{2}\left(a\,e^{2}-c\,d^{2}\right)}{\sqrt{ex+d}}-\frac{2cd\left(a^{2}e^{4}-2ac\,d^{2}e^{2}+c^{2}d^{4}\right)}{(ex+d)^{\frac{3}{2}}}}{e^{4}}$
default	$\frac{2c^{3}d^{3}\sqrt{ex+d}-\frac{2\left(e^{6}a^{3}-3d^{2}e^{4}a^{2}c+3d^{4}e^{2}a\,c^{2}-d^{6}c^{3}\right)}{5(ex+d)^{\frac{5}{2}}}-\frac{6c^{2}d^{2}\left(a\,e^{2}-c\,d^{2}\right)}{\sqrt{ex+d}}-\frac{2cd\left(a^{2}e^{4}-2ac\,d^{2}e^{2}+c^{2}d^{4}\right)}{(ex+d)^{\frac{3}{2}}}}{e^{4}}$
orering	$-\frac{2(-5c^{3}d^{3}e^{3}x^{3}+15x^{2}a\,c^{2}d^{2}e^{4}-30c^{3}d^{4}e^{2}x^{2}+5x\,a^{2}cd\,e^{5}+20xa\,c^{2}d^{3}e^{3}-40c^{3}d^{5}ex+e^{6}a^{3}+2d^{2}e^{4}a^{2}c+8d^{4}e^{2}a\,c^{2}-16d}{5e^{4}(cdx+ae)^{3}(ex+d)^{\frac{11}{2}}}$

input
```
int((a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^3/(e*x+d)^(13/2),x,method=_RETURNVER
BOSE)
```

output
```
2/5*((5*d^3*e^3*x^3+30*d^4*e^2*x^2+40*d^5*e*x+16*d^6)*c^3-8*e^2*(15/8*e^2*
x^2+5/2*d*e*x+d^2)*a*d^2*c^2-2*e^4*(5/2*e*x+d)*a^2*d*c-e^6*a^3)/(e*x+d)^(5
/2)/e^4
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 163, normalized size of antiderivative = 1.44

$$\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^3}{(d+ex)^{13/2}}\,dx = \frac{2\left(5\,c^3d^3e^3x^3+16\,c^3d^6-8\,ac^2d^4e^2-2\,a^2cd^2e^4-a^3e^6+15\left(2\,c^3d^4 \cdots\right.\right.}{5\left(e^7x^3+3\,de^6x^2+3 \cdots\right.}$$

input
```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^3/(e*x+d)^(13/2),x, algorithm=
"fricas")
```

output
```
2/5*(5*c^3*d^3*e^3*x^3 + 16*c^3*d^6 - 8*a*c^2*d^4*e^2 - 2*a^2*c*d^2*e^4 -
a^3*e^6 + 15*(2*c^3*d^4*e^2 - a*c^2*d^2*e^4)*x^2 + 5*(8*c^3*d^5*e - 4*a*c^
2*d^3*e^3 - a^2*c*d*e^5)*x)*sqrt(e*x + d)/(e^7*x^3 + 3*d*e^6*x^2 + 3*d^2*e
^5*x + d^3*e^4)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 654 vs. 2(105) = 210.

Time = 1.45 (sec) , antiderivative size = 654, normalized size of antiderivative = 5.79

$$\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^3}{(d+ex)^{13/2}}\,dx = \begin{cases} -\frac{2a^3e^6}{5d^2e^4\sqrt{d+ex}+10de^5x\sqrt{d+ex}+5e^6x^2\sqrt{d+ex}} - \frac{4a^2cd^2e^4}{5d^2e^4\sqrt{d+ex}+10de^5x\sqrt{d+ex}+5e^6x \cdots} \\ \frac{c^3x^4}{4\sqrt{d}} \end{cases}$$

input
```
integrate((a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**3/(e*x+d)**(13/2),x)
```

output
```
Piecewise((-2*a**3*e**6/(5*d**2*e**4*sqrt(d + e*x) + 10*d*e**5*x*sqrt(d +
e*x) + 5*e**6*x**2*sqrt(d + e*x)) - 4*a**2*c*d**2*e**4/(5*d**2*e**4*sqrt(d
 + e*x) + 10*d*e**5*x*sqrt(d + e*x) + 5*e**6*x**2*sqrt(d + e*x)) - 10*a**2
*c*d*e**5*x/(5*d**2*e**4*sqrt(d + e*x) + 10*d*e**5*x*sqrt(d + e*x) + 5*e**
6*x**2*sqrt(d + e*x)) - 16*a*c**2*d**4*e**2/(5*d**2*e**4*sqrt(d + e*x) + 1
0*d*e**5*x*sqrt(d + e*x) + 5*e**6*x**2*sqrt(d + e*x)) - 40*a*c**2*d**3*e**
3*x/(5*d**2*e**4*sqrt(d + e*x) + 10*d*e**5*x*sqrt(d + e*x) + 5*e**6*x**2*s
qrt(d + e*x)) - 30*a*c**2*d**2*e**4*x**2/(5*d**2*e**4*sqrt(d + e*x) + 10*d
*e**5*x*sqrt(d + e*x) + 5*e**6*x**2*sqrt(d + e*x)) + 32*c**3*d**6/(5*d**2*
e**4*sqrt(d + e*x) + 10*d*e**5*x*sqrt(d + e*x) + 5*e**6*x**2*sqrt(d + e*x)
) + 80*c**3*d**5*e*x/(5*d**2*e**4*sqrt(d + e*x) + 10*d*e**5*x*sqrt(d + e*x
) + 5*e**6*x**2*sqrt(d + e*x)) + 60*c**3*d**4*e**2*x**2/(5*d**2*e**4*sqrt(
d + e*x) + 10*d*e**5*x*sqrt(d + e*x) + 5*e**6*x**2*sqrt(d + e*x)) + 10*c**
3*d**3*e**3*x**3/(5*d**2*e**4*sqrt(d + e*x) + 10*d*e**5*x*sqrt(d + e*x) +
5*e**6*x**2*sqrt(d + e*x)), Ne(e, 0)), (c**3*x**4/(4*sqrt(d)), True))
```

Maxima [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 140, normalized size of antiderivative = 1.24

$$\int \frac{\left(ade + \left(cd^2 + ae^2\right)x + cdex^2\right)^3}{(d+ex)^{13/2}}\,dx = \frac{2\left(\frac{5\sqrt{ex+d}c^3d^3}{e^3} + \frac{c^3d^6 - 3\,ac^2d^4e^2 + 3\,a^2cd^2e^4 - a^3e^6 + 15\,\left(c^3d^4 - ac^2d^2e^2\right)(ex+d)^2 - 5\,(}{(ex+d)^{\frac{5}{2}}e^3}\right.}{5\,e}$$

input
```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^3/(e*x+d)^(13/2),x, algorithm=
"maxima")
```

output
```
2/5*(5*sqrt(e*x + d)*c^3*d^3/e^3 + (c^3*d^6 - 3*a*c^2*d^4*e^2 + 3*a^2*c*d^
2*e^4 - a^3*e^6 + 15*(c^3*d^4 - a*c^2*d^2*e^2)*(e*x + d)^2 - 5*(c^3*d^5 -
2*a*c^2*d^3*e^2 + a^2*c*d*e^4)*(e*x + d))/((e*x + d)^(5/2)*e^3))/e
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 150, normalized size of antiderivative = 1.33

$$\int \frac{(ade + (cd^2 + ae^2)x + cdex^2)^3}{(d+ex)^{13/2}}\,dx = \frac{2\sqrt{ex+d}c^3d^3}{e^4} + \frac{2\left(15\,(ex+d)^2c^3d^4 - 5\,(ex+d)c^3d^5 + c^3d^6 - 15\,(ex+d)^2ac^2d^2e^2 + 10\,(ex+d)ac^2d^3e^2 - 3\,ac^2d^4e^2 - 5\,(\right.}{5\,(ex+d)^{\frac{5}{2}}e^4}$$

input
```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^3/(e*x+d)^(13/2),x, algorithm=
"giac")
```

output
```
2*sqrt(e*x + d)*c^3*d^3/e^4 + 2/5*(15*(e*x + d)^2*c^3*d^4 - 5*(e*x + d)*c^
3*d^5 + c^3*d^6 - 15*(e*x + d)^2*a*c^2*d^2*e^2 + 10*(e*x + d)*a*c^2*d^3*e^
2 - 3*a*c^2*d^4*e^2 - 5*(e*x + d)*a^2*c*d*e^4 + 3*a^2*c*d^2*e^4 - a^3*e^6)
/((e*x + d)^(5/2)*e^4)
```

Mupad [B] (verification not implemented)

Time = 5.27 (sec) , antiderivative size = 129, normalized size of antiderivative = 1.14

$$\int \frac{(ade + (cd^2 + ae^2)x + cdex^2)^3}{(d+ex)^{13/2}}\,dx = -\frac{2\left(a^3\,e^6 + 2\,a^2\,c\,d^2\,e^4 + 5\,a^2\,c\,d\,e^5\,x + 8\,a\,c^2\,d^4\,e^2 + 20\,a\,c^2\,d^3\,e^3\,x + 15\,a\,c^2\,d^2\,e^4\,x^2 - 16\,c^3\,d^6 - 40\,c^3\,d^5\,e\,x\right.}{5\,e^4\,(d+e\,x)^{5/2}}$$

input
```
int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^3/(d + e*x)^(13/2),x)
```

output
```
-(2*(a^3*e^6 - 16*c^3*d^6 + 8*a*c^2*d^4*e^2 + 2*a^2*c*d^2*e^4 - 30*c^3*d^4
*e^2*x^2 - 5*c^3*d^3*e^3*x^3 - 40*c^3*d^5*e*x + 5*a^2*c*d*e^5*x + 20*a*c^2
*d^3*e^3*x + 15*a*c^2*d^2*e^4*x^2))/(5*e^4*(d + e*x)^(5/2))
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 149, normalized size of antiderivative = 1.32

$$\int \frac{(ade + (cd^2 + ae^2)x + cdex^2)^3}{(d + ex)^{13/2}}\,dx = \frac{2c^3d^3e^3x^3 - 6a\,c^2d^2e^4x^2 + 12c^3d^4e^2x^2 - 2a^2cd\,e^5x - 8a\,c^2d^3e^3x +}{\sqrt{ex+d}\,e^4\,(e^2x^2 + 2d}$$

input
```
int((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^3/(e*x+d)^(13/2),x)
```

output
```
(2*( - a**3*e**6 - 2*a**2*c*d**2*e**4 - 5*a**2*c*d*e**5*x - 8*a*c**2*d**4*
e**2 - 20*a*c**2*d**3*e**3*x - 15*a*c**2*d**2*e**4*x**2 + 16*c**3*d**6 + 4
0*c**3*d**5*e*x + 30*c**3*d**4*e**2*x**2 + 5*c**3*d**3*e**3*x**3))/(5*sqrt
(d + e*x)*e**4*(d**2 + 2*d*e*x + e**2*x**2))
```

3.173 $\int \frac{(d+ex)^{9/2}}{ade+(cd^2+ae^2)x+cdex^2}\,dx$

Optimal result

Integrand size = 37, antiderivative size = 180

$$\int \frac{(d+ex)^{9/2}}{ade+(cd^2+ae^2)\,x+cdex^2}\,dx = \frac{2(cd^2-ae^2)^3\sqrt{d+ex}}{c^4d^4} + \frac{2(cd^2-ae^2)^2\,(d+ex)^{3/2}}{3c^3d^3} + \frac{2(cd^2-ae^2)\,(d+ex)^{5/2}}{5c^2d^2} + \frac{2(d+ex)^{7/2}}{7cd} - \frac{2(cd^2-ae^2)^{7/2}\operatorname{arctanh}\left(\frac{\sqrt{c}\sqrt{d}\sqrt{d+ex}}{\sqrt{cd^2-ae^2}}\right)}{c^{9/2}d^{9/2}}$$

output
```
2*(-a*e^2+c*d^2)^3*(e*x+d)^(1/2)/c^4/d^4+2/3*(-a*e^2+c*d^2)^2*(e*x+d)^(3/2
)/c^3/d^3+2/5*(-a*e^2+c*d^2)*(e*x+d)^(5/2)/c^2/d^2+2/7*(e*x+d)^(7/2)/c/d-2
*(-a*e^2+c*d^2)^(7/2)*arctanh(c^(1/2)*d^(1/2)*(e*x+d)^(1/2)/(-a*e^2+c*d^2)
^(1/2))/c^(9/2)/d^(9/2)
```

Mathematica [A] (verified)

Time = 0.21 (sec) , antiderivative size = 179, normalized size of antiderivative = 0.99

$$\int \frac{(d+ex)^{9/2}}{ade+(cd^2+ae^2)\,x+cdex^2}\,dx=\frac{2\sqrt{d+ex}(-105a^3e^6+35a^2cde^4(10d+ex)-7ac^2d^2e^2(58d^2+16dex}{105c^4d^4}$$
$$+\frac{2(-cd^2+ae^2)^{7/2}\arctan\left(\frac{\sqrt{c}\sqrt{d}\sqrt{d+ex}}{\sqrt{-cd^2+ae^2}}\right)}{c^{9/2}d^{9/2}}$$

input
```
Integrate[(d + e*x)^(9/2)/(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2),x]
```

output
```
(2*Sqrt[d + e*x]*(-105*a^3*e^6 + 35*a^2*c*d*e^4*(10*d + e*x) - 7*a*c^2*d^2
*e^2*(58*d^2 + 16*d*e*x + 3*e^2*x^2) + c^3*d^3*(176*d^3 + 122*d^2*e*x + 66
*d*e^2*x^2 + 15*e^3*x^3)))/(105*c^4*d^4) + (2*(-(c*d^2) + a*e^2)^(7/2)*Arc
Tan[(Sqrt[c]*Sqrt[d]*Sqrt[d + e*x])/Sqrt[-(c*d^2) + a*e^2]])/(c^(9/2)*d^(9
/2))
```

Rubi [A] (verified)

Time = 0.53 (sec) , antiderivative size = 207, normalized size of antiderivative = 1.15, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.189, Rules used = {1121, 60, 60, 60, 60, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex)^{9/2}}{x\,(ae^2+cd^2)+ade+cdex^2}\,dx$$
$$\Big\downarrow \text{1121}$$
$$\int \frac{(d+ex)^{7/2}}{ae+cdx}dx$$
$$\Big\downarrow \text{60}$$
$$\frac{\left(d^2-\frac{ae^2}{c}\right)\int \frac{(d+ex)^{5/2}}{ae+cdx}dx}{d}+\frac{2(d+ex)^{7/2}}{7cd}$$

$$\Big\downarrow \; 60$$

$$\frac{\left(d^2-\frac{ae^2}{c}\right)\left(\frac{\left(d^2-\frac{ae^2}{c}\right)\int \frac{(d+ex)^{3/2}}{ae+cdx}dx}{d}+\frac{2(d+ex)^{5/2}}{5cd}\right)}{d}+\frac{2(d+ex)^{7/2}}{7cd}$$

$$\Big\downarrow \; 60$$

$$\frac{\left(d^2-\frac{ae^2}{c}\right)\left(\frac{\left(d^2-\frac{ae^2}{c}\right)\left(\frac{\left(d^2-\frac{ae^2}{c}\right)\int \frac{\sqrt{d+ex}}{ae+cdx}dx}{d}+\frac{2(d+ex)^{3/2}}{3cd}\right)}{d}+\frac{2(d+ex)^{5/2}}{5cd}\right)}{d}+\frac{2(d+ex)^{7/2}}{7cd}$$

$$\Big\downarrow \; 60$$

$$\frac{\left(d^2-\frac{ae^2}{c}\right)\left(\frac{\left(d^2-\frac{ae^2}{c}\right)\left(\frac{\left(d^2-\frac{ae^2}{c}\right)\left(\frac{\left(d^2-\frac{ae^2}{c}\right)\int \frac{1}{(ae+cdx)\sqrt{d+ex}}dx}{d}+\frac{2\sqrt{d+ex}}{cd}\right)}{d}+\frac{2(d+ex)^{3/2}}{3cd}\right)}{d}+\frac{2(d+ex)^{5/2}}{5cd}\right)}{d}+\frac{2(d+ex)^{7/2}}{7cd}$$

$$\Big\downarrow \; 73$$

$$\frac{\left(d^2-\frac{ae^2}{c}\right)\left(\frac{\left(d^2-\frac{ae^2}{c}\right)\left(\frac{\left(d^2-\frac{ae^2}{c}\right)\left(\frac{2\left(d^2-\frac{ae^2}{c}\right)\int \frac{1}{-\frac{cd^2}{e}+\frac{c(d+ex)d}{e}+ae}d\sqrt{d+ex}}{de}+\frac{2\sqrt{d+ex}}{cd}\right)}{d}+\frac{2(d+ex)^{3/2}}{3cd}\right)}{d}+\frac{2(d+ex)^{5/2}}{5cd}\right)}{d}+\frac{2(d+ex)^{7/2}}{7cd}$$

$$\Big\downarrow 221$$

$$\frac{\left(d^2-\frac{ae^2}{c}\right)\left(\frac{\left(d^2-\frac{ae^2}{c}\right)\left(\frac{\left(d^2-\frac{ae^2}{c}\right)\left(\frac{2\sqrt{d+ex}}{cd}-\frac{2\left(d^2-\frac{ae^2}{c}\right)\text{arctanh}\left(\frac{\sqrt{c}\sqrt{d}\sqrt{d+ex}}{\sqrt{cd^2-ae^2}}\right)}{\sqrt{c}d^{3/2}\sqrt{cd^2-ae^2}}\right)}{d}+\frac{2(d+ex)^{3/2}}{3cd}\right)}{d}+\frac{2(d+ex)^{5/2}}{5cd}\right)}{d}+\frac{2(d+ex)^{7/2}}{7cd}$$

input
```
Int[(d + e*x)^(9/2)/(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2),x]
```

output
```
(2*(d + e*x)^(7/2))/(7*c*d) + ((d^2 - (a*e^2)/c)*((2*(d + e*x)^(5/2))/(5*c
*d) + ((d^2 - (a*e^2)/c)*((2*(d + e*x)^(3/2))/(3*c*d) + ((d^2 - (a*e^2)/c)
*((2*Sqrt[d + e*x])/(c*d) - (2*(d^2 - (a*e^2)/c)*ArcTanh[(Sqrt[c]*Sqrt[d]*
Sqrt[d + e*x])/Sqrt[c*d^2 - a*e^2]])/(Sqrt[c]*d^(3/2)*Sqrt[c*d^2 - a*e^2])
))/d))/d))/d
```

Defintions of rubi rules used

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 1121
```
Int[((d_) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Int[ExpandIntegrand[(d + e*x)^(m + p)*(a/d + (c/e)*x)^p, x], x]
/; FreeQ[{a, b, c, d, e, m, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && (Int
egerQ[p] || (GtQ[a, 0] && GtQ[d, 0] && LtQ[c, 0]))
```

Maple [A] (verified)

Time = 4.47 (sec) , antiderivative size = 185, normalized size of antiderivative = 1.03

method	result
pseudoelliptic	$\frac{2\left(a e^2-c d^2\right)^4 \arctan\left(\frac{cd\sqrt{ex+d}}{\sqrt{cd\left(a e^2-c d^2\right)}}\right)-2\sqrt{ex+d}\,\sqrt{cd(a e^2-c d^2)}\left(-\frac{176d^3\left(\frac{15}{176}e^3x^3+\frac{3}{8}d e^2x^2+\frac{61}{88}d^2ex+d^3\right)c^3}{105}+\frac{58(\cdots}{\cdots}\cdots\right.}{d^4c^4\sqrt{cd(a e^2-c d^2)}\cdots}$
derivativedivides	$-\frac{2\left(-\frac{c^3d^3(ex+d)^{\frac{7}{2}}}{7}+\frac{\left(a e^2-c d^2\right)c^2d^2(ex+d)^{\frac{5}{2}}}{5}-\frac{\left(a^2e^4-2ac d^2e^2+c^2d^4\right)(ex+d)^{\frac{3}{2}}cd}{3}+\left(a e^2-c d^2\right)\left(a^2e^4-2ac d^2e^2+c^2d^4\cdots\right.\right.}{c^4d^4\cdots}$
default	$-\frac{2\left(-\frac{c^3d^3(ex+d)^{\frac{7}{2}}}{7}+\frac{\left(a e^2-c d^2\right)c^2d^2(ex+d)^{\frac{5}{2}}}{5}-\frac{\left(a^2e^4-2ac d^2e^2+c^2d^4\right)(ex+d)^{\frac{3}{2}}cd}{3}+\left(a e^2-c d^2\right)\left(a^2e^4-2ac d^2e^2+c^2d^4\cdots\right.\right.}{c^4d^4\cdots}$
risch	$-\frac{2\left(-15c^3d^3e^3x^3+21x^2a c^2d^2e^4-66c^3d^4e^2x^2-35x a^2cd e^5+112xa c^2d^3e^3-122c^3d^5ex+105e^6a^3-350d^2e^4a^2c+406\cdots\right.}{105d^4c^4}$

input
```
int((e*x+d)^(9/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e),x,method=_RETURNVERBOS
E)
```

output
```
2*((a*e^2-c*d^2)^4*arctan(c*d*(e*x+d)^(1/2)/(c*d*(a*e^2-c*d^2))^(1/2))-(e*
x+d)^(1/2)*(c*d*(a*e^2-c*d^2))^(1/2)*(-176/105*d^3*(15/176*e^3*x^3+3/8*d*e
^2*x^2+61/88*d^2*e*x+d^3)*c^3+58/15*(3/58*e^2*x^2+8/29*d*e*x+d^2)*e^2*a*d^
2*c^2-10/3*e^4*(1/10*e*x+d)*a^2*d*c+e^6*a^3))/(c*d*(a*e^2-c*d^2))^(1/2)/d^
4/c^4
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 510, normalized size of antiderivative = 2.83

$$\int \frac{(d+ex)^{9/2}}{ade+(cd^2+ae^2)x+cdex^2}\,dx = \left[-\frac{105\,(c^3d^6-3\,ac^2d^4e^2+3\,a^2cd^2e^4-a^3e^6)\sqrt{\frac{cd^2-ae^2}{cd}}\log\left(\frac{cdex+2\,cd^2-\cdots}{\cdots}\right)\cdots}{\cdots}\right.$$

$$-\frac{2\left(105\,(c^3d^6-3\,ac^2d^4e^2+3\,a^2cd^2e^4-a^3e^6)\sqrt{-\frac{cd^2-ae^2}{cd}}\arctan\left(-\frac{\sqrt{ex+d}cd\sqrt{-\frac{cd^2-ae^2}{cd}}}{cd^2-ae^2}\right)-(15\,c^3d^3e^3x^3+1\cdots\right.}{\cdots}$$

input
```
integrate((e*x+d)^(9/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2),x, algorithm="fr
icas")
```

output
```
[-1/105*(105*(c^3*d^6 - 3*a*c^2*d^4*e^2 + 3*a^2*c*d^2*e^4 - a^3*e^6)*sqrt(
(c*d^2 - a*e^2)/(c*d))*log((c*d*e*x + 2*c*d^2 - a*e^2 + 2*sqrt(e*x + d)*c*
d*sqrt((c*d^2 - a*e^2)/(c*d)))/(c*d*x + a*e)) - 2*(15*c^3*d^3*e^3*x^3 + 17
6*c^3*d^6 - 406*a*c^2*d^4*e^2 + 350*a^2*c*d^2*e^4 - 105*a^3*e^6 + 3*(22*c^
3*d^4*e^2 - 7*a*c^2*d^2*e^4)*x^2 + (122*c^3*d^5*e - 112*a*c^2*d^3*e^3 + 35
*a^2*c*d*e^5)*x)*sqrt(e*x + d))/(c^4*d^4), -2/105*(105*(c^3*d^6 - 3*a*c^2*
d^4*e^2 + 3*a^2*c*d^2*e^4 - a^3*e^6)*sqrt(-(c*d^2 - a*e^2)/(c*d))*arctan(-
sqrt(e*x + d)*c*d*sqrt(-(c*d^2 - a*e^2)/(c*d))/(c*d^2 - a*e^2)) - (15*c^3*
d^3*e^3*x^3 + 176*c^3*d^6 - 406*a*c^2*d^4*e^2 + 350*a^2*c*d^2*e^4 - 105*a^
3*e^6 + 3*(22*c^3*d^4*e^2 - 7*a*c^2*d^2*e^4)*x^2 + (122*c^3*d^5*e - 112*a*
c^2*d^3*e^3 + 35*a^2*c*d*e^5)*x)*sqrt(e*x + d))/(c^4*d^4)]
```

Sympy [A] (verification not implemented)

Time = 161.60 (sec) , antiderivative size = 230, normalized size of antiderivative = 1.28

$$\int \frac{(d+ex)^{9/2}}{ade+(cd^2+ae^2)x+cdex^2}\,dx = \begin{cases} \frac{2\left(\frac{e(d+ex)^{\frac{7}{2}}}{7cd}+\frac{(d+ex)^{\frac{5}{2}}(-ae^3+cd^2e)}{5c^2d^2}+\frac{(d+ex)^{\frac{3}{2}}(a^2e^5-2acd^2e^3+c^2d^4e)}{3c^3d^3}+\frac{\sqrt{d+ex}(-a^3e^7+3a^2c\cdots}{c\cdots}\cdots\right)}{e} \\ \frac{d^{\frac{5}{2}}\log(x)}{c} \end{cases}$$

input
```
integrate((e*x+d)**(9/2)/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2),x)
```

output
```
Piecewise((2*(e*(d + e*x)**(7/2)/(7*c*d) + (d + e*x)**(5/2)*(-a*e**3 + c*d
**2*e)/(5*c**2*d**2) + (d + e*x)**(3/2)*(a**2*e**5 - 2*a*c*d**2*e**3 + c**
2*d**4*e)/(3*c**3*d**3) + sqrt(d + e*x)*(-a**3*e**7 + 3*a**2*c*d**2*e**5 -
 3*a*c**2*d**4*e**3 + c**3*d**6*e)/(c**4*d**4) + e*(a*e**2 - c*d**2)**4*at
an(sqrt(d + e*x)/sqrt((a*e**2 - c*d**2)/(c*d)))/(c**5*d**5*sqrt((a*e**2 -
c*d**2)/(c*d))))/e, Ne(e, 0)), (d**(5/2)*log(x)/c, True))
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(d+ex)^{9/2}}{ade+(cd^2+ae^2)x+cdex^2}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((e*x+d)^(9/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2),x, algorithm="ma
xima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(a*e^2-c*d^2>0)', see `assume?` f
or more de
```

Giac [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 298, normalized size of antiderivative = 1.66

$$\int \frac{(d+ex)^{9/2}}{ade+(cd^2+ae^2)x+cdex^2}\,dx = \frac{2\left(c^4d^8-4\,ac^3d^6e^2+6\,a^2c^2d^4e^4-4\,a^3cd^2e^6+a^4e^8\right)\arctan\left(\frac{\sqrt{ex+dc}}{\sqrt{-c^2d^3+a}}\right.}{\sqrt{-c^2d^3+acde^2}c^4d^4} + \frac{2\left(15\,(ex+d)^{\frac{7}{2}}c^6d^6+21\,(ex+d)^{\frac{5}{2}}c^6d^7+35\,(ex+d)^{\frac{3}{2}}c^6d^8+105\,\sqrt{ex+d}c^6d^9-21\,(ex+d)^{\frac{5}{2}}ac^5d^5e^2-\right.}{}$$

input
```
integrate((e*x+d)^(9/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2),x, algorithm="gi
ac")
```

output
```
2*(c^4*d^8 - 4*a*c^3*d^6*e^2 + 6*a^2*c^2*d^4*e^4 - 4*a^3*c*d^2*e^6 + a^4*e
^8)*arctan(sqrt(e*x + d)*c*d/sqrt(-c^2*d^3 + a*c*d*e^2))/(sqrt(-c^2*d^3 +
a*c*d*e^2)*c^4*d^4) + 2/105*(15*(e*x + d)^(7/2)*c^6*d^6 + 21*(e*x + d)^(5/
2)*c^6*d^7 + 35*(e*x + d)^(3/2)*c^6*d^8 + 105*sqrt(e*x + d)*c^6*d^9 - 21*(
e*x + d)^(5/2)*a*c^5*d^5*e^2 - 70*(e*x + d)^(3/2)*a*c^5*d^6*e^2 - 315*sqrt
(e*x + d)*a*c^5*d^7*e^2 + 35*(e*x + d)^(3/2)*a^2*c^4*d^4*e^4 + 315*sqrt(e*
x + d)*a^2*c^4*d^5*e^4 - 105*sqrt(e*x + d)*a^3*c^3*d^3*e^6)/(c^7*d^7)
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 207, normalized size of antiderivative = 1.15

$$\int \frac{(d+ex)^{9/2}}{ade+(cd^2+ae^2)\,x+cdex^2}\,dx = \frac{2\,(d+e\,x)^{7/2}}{7\,c\,d} + \frac{2\,(a\,e^2-c\,d^2)^2\,(d+e\,x)^{3/2}}{3\,c^3\,d^3} - \frac{2\,(a\,e^2-c\,d^2)^3\,\sqrt{d+e\,x}}{c^4\,d^4} + \frac{2\,\mathrm{atan}\left(\frac{\sqrt{c}\,\sqrt{d}\,(a\,e^2-c\,d^2)^{7/2}\,\sqrt{d+e\,x}}{a^4\,e^8-4\,a^3\,c\,d^2\,e^6+6\,a^2\,c^2\,d^4\,e^4-4\,a\,c^3\,d^6\,e^2+c^4\,d^8}\right)\,(a\,e^2-c\,d^2)^{7/2}}{c^{9/2}\,d^{9/2}} - \frac{2\,(a\,e^2-c\,d^2)\,(d+e\,x)^{5/2}}{5\,c^2\,d^2}$$

input
```
int((d + e*x)^(9/2)/(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2),x)
```

output
```
(2*(d + e*x)^(7/2))/(7*c*d) + (2*(a*e^2 - c*d^2)^2*(d + e*x)^(3/2))/(3*c^3
*d^3) - (2*(a*e^2 - c*d^2)^3*(d + e*x)^(1/2))/(c^4*d^4) + (2*atan((c^(1/2)
*d^(1/2)*(a*e^2 - c*d^2)^(7/2)*(d + e*x)^(1/2))/(a^4*e^8 + c^4*d^8 - 4*a*c
^3*d^6*e^2 - 4*a^3*c*d^2*e^6 + 6*a^2*c^2*d^4*e^4))*(a*e^2 - c*d^2)^(7/2))/
(c^(9/2)*d^(9/2)) - (2*(a*e^2 - c*d^2)*(d + e*x)^(5/2))/(5*c^2*d^2)
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 434, normalized size of antiderivative = 2.41

$$\int \frac{(d+ex)^{9/2}}{ade+(cd^2+ae^2)\,x+cdex^2}\,dx = \frac{2\sqrt{d}\,\sqrt{c}\,\sqrt{a\,e^2-c\,d^2}\;atan\left(\frac{\sqrt{ex+d}\,cd}{\sqrt{d}\,\sqrt{c}\,\sqrt{a\,e^2-c\,d^2}}\right)a^3e^6-6\sqrt{d}\,\sqrt{c}\,\sqrt{a\,e^2-c\,d}}{}$$

input
```
int((e*x+d)^(9/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2),x)
```

output
```
(2*(105*sqrt(d)*sqrt(c)*sqrt(a*e**2 - c*d**2)*atan((sqrt(d + e*x)*c*d)/(sq
rt(d)*sqrt(c)*sqrt(a*e**2 - c*d**2)))*a**3*e**6 - 315*sqrt(d)*sqrt(c)*sqrt
(a*e**2 - c*d**2)*atan((sqrt(d + e*x)*c*d)/(sqrt(d)*sqrt(c)*sqrt(a*e**2 -
c*d**2)))*a**2*c*d**2*e**4 + 315*sqrt(d)*sqrt(c)*sqrt(a*e**2 - c*d**2)*ata
n((sqrt(d + e*x)*c*d)/(sqrt(d)*sqrt(c)*sqrt(a*e**2 - c*d**2)))*a*c**2*d**4
*e**2 - 105*sqrt(d)*sqrt(c)*sqrt(a*e**2 - c*d**2)*atan((sqrt(d + e*x)*c*d)
/(sqrt(d)*sqrt(c)*sqrt(a*e**2 - c*d**2)))*c**3*d**6 - 105*sqrt(d + e*x)*a*
*3*c*d*e**6 + 350*sqrt(d + e*x)*a**2*c**2*d**3*e**4 + 35*sqrt(d + e*x)*a**
2*c**2*d**2*e**5*x - 406*sqrt(d + e*x)*a*c**3*d**5*e**2 - 112*sqrt(d + e*x
)*a*c**3*d**4*e**3*x - 21*sqrt(d + e*x)*a*c**3*d**3*e**4*x**2 + 176*sqrt(d
 + e*x)*c**4*d**7 + 122*sqrt(d + e*x)*c**4*d**6*e*x + 66*sqrt(d + e*x)*c**
4*d**5*e**2*x**2 + 15*sqrt(d + e*x)*c**4*d**4*e**3*x**3))/(105*c**5*d**5)
```

3.174 $\int \frac{(d+ex)^{7/2}}{ade+(cd^2+ae^2)x+cdex^2}\,dx$

Optimal result

Integrand size = 37, antiderivative size = 147

$$\int \frac{(d+ex)^{7/2}}{ade+(cd^2+ae^2)x+cdex^2}\,dx=\frac{2(cd^2-ae^2)^2\sqrt{d+ex}}{c^3d^3}+\frac{2(cd^2-ae^2)(d+ex)^{3/2}}{3c^2d^2}+\frac{2(d+ex)^{5/2}}{5cd}-\frac{2(cd^2-ae^2)^{5/2}\,\text{arctanh}\left(\frac{\sqrt{c}\sqrt{d}\sqrt{d+ex}}{\sqrt{cd^2-ae^2}}\right)}{c^{7/2}d^{7/2}}$$

output
```
2*(-a*e^2+c*d^2)^2*(e*x+d)^(1/2)/c^3/d^3+2/3*(-a*e^2+c*d^2)*(e*x+d)^(3/2)/
c^2/d^2+2/5*(e*x+d)^(5/2)/c/d-2*(-a*e^2+c*d^2)^(5/2)*arctanh(c^(1/2)*d^(1/
2)*(e*x+d)^(1/2)/(-a*e^2+c*d^2)^(1/2))/c^(7/2)/d^(7/2)
```

Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 135, normalized size of antiderivative = 0.92

$$\int \frac{(d+ex)^{7/2}}{ade+(cd^2+ae^2)x+cdex^2}\,dx=\frac{2\sqrt{d+ex}(15a^2e^4-5acde^2(7d+ex)+c^2d^2(23d^2+11dex+3e^2x^2))}{15c^3d^3}-\frac{2(-cd^2+ae^2)^{5/2}\arctan\left(\frac{\sqrt{c}\sqrt{d}\sqrt{d+ex}}{\sqrt{-cd^2+ae^2}}\right)}{c^{7/2}d^{7/2}}$$

input
```
Integrate[(d + e*x)^(7/2)/(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2),x]
```

output
```
(2*Sqrt[d + e*x]*(15*a^2*e^4 - 5*a*c*d*e^2*(7*d + e*x) + c^2*d^2*(23*d^2 +
 11*d*e*x + 3*e^2*x^2)))/(15*c^3*d^3) - (2*(-(c*d^2) + a*e^2)^(5/2)*ArcTan
[(Sqrt[c]*Sqrt[d]*Sqrt[d + e*x])/Sqrt[-(c*d^2) + a*e^2]])/(c^(7/2)*d^(7/2)
)
```

Rubi [A] (verified)

Time = 0.46 (sec) , antiderivative size = 170, normalized size of antiderivative = 1.16, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.162$, Rules used = {1121, 60, 60, 60, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex)^{7/2}}{x\left(ae^2+cd^2\right)+ade+cdex^2}dx$$

$$\Big\downarrow \text{1121}$$

$$\int \frac{(d+ex)^{5/2}}{ae+cdx}dx$$

$$\Big\downarrow \text{60}$$

$$\frac{\left(d^2-\frac{ae^2}{c}\right)\int \frac{(d+ex)^{3/2}}{ae+cdx}dx}{d}+\frac{2(d+ex)^{5/2}}{5cd}$$

$$\Big\downarrow \text{60}$$

$$\frac{\left(d^2-\frac{ae^2}{c}\right)\left(\frac{\left(d^2-\frac{ae^2}{c}\right)\int \frac{\sqrt{d+ex}}{ae+cdx}dx}{d}+\frac{2(d+ex)^{3/2}}{3cd}\right)}{d}+\frac{2(d+ex)^{5/2}}{5cd}$$

$$\Big\downarrow \text{60}$$

$$\frac{\left(d^2-\frac{ae^2}{c}\right)\left(\frac{\left(d^2-\frac{ae^2}{c}\right)\left(\frac{\left(d^2-\frac{ae^2}{c}\right)\int \frac{1}{(ae+cdx)\sqrt{d+ex}}dx}{d}+\frac{2\sqrt{d+ex}}{cd}\right)}{d}+\frac{2(d+ex)^{3/2}}{3cd}\right)}{d}+\frac{2(d+ex)^{5/2}}{5cd}$$

$$\Big\downarrow \text{73}$$

$$\frac{\left(d^2-\frac{ae^2}{c}\right)\left(\frac{\left(d^2-\frac{ae^2}{c}\right)\left(\frac{2\left(d^2-\frac{ae^2}{c}\right)\int\frac{1}{-\frac{cd^2}{e}+\frac{c(d+ex)d}{e}+ae}d\sqrt{d+ex}}{de}+\frac{2\sqrt{d+ex}}{cd}\right)}{d}+\frac{2(d+ex)^{3/2}}{3cd}\right)}{d}+\frac{2(d+ex)^{5/2}}{5cd}$$

$$\Big\downarrow \text{221}$$

$$\frac{\left(d^2-\frac{ae^2}{c}\right)\left(\frac{\left(d^2-\frac{ae^2}{c}\right)\left(\frac{2\sqrt{d+ex}}{cd}-\frac{2\left(d^2-\frac{ae^2}{c}\right)\text{arctanh}\left(\frac{\sqrt{c}\sqrt{d}\sqrt{d+ex}}{\sqrt{cd^2-ae^2}}\right)}{\sqrt{c}d^{3/2}\sqrt{cd^2-ae^2}}\right)}{d}+\frac{2(d+ex)^{3/2}}{3cd}\right)}{d}+\frac{2(d+ex)^{5/2}}{5cd}$$

input

```
Int[(d + e*x)^(7/2)/(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2),x]
```

output

```
(2*(d + e*x)^(5/2))/(5*c*d) + ((d^2 - (a*e^2)/c)*((2*(d + e*x)^(3/2))/(3*c
*d) + ((d^2 - (a*e^2)/c)*((2*Sqrt[d + e*x])/(c*d) - (2*(d^2 - (a*e^2)/c)*A
rcTanh[(Sqrt[c]*Sqrt[d]*Sqrt[d + e*x])/Sqrt[c*d^2 - a*e^2]])/(Sqrt[c]*d^(3
/2)*Sqrt[c*d^2 - a*e^2])))/d))/d
```

Defintions of rubi rules used

rule 60

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 1121
```
Int[((d_) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Int[ExpandIntegrand[(d + e*x)^(m + p)*(a/d + (c/e)*x)^p, x], x]
/; FreeQ[{a, b, c, d, e, m, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && (Int
egerQ[p] || (GtQ[a, 0] && GtQ[d, 0] && LtQ[c, 0]))
```

Maple [A] (verified)

Time = 2.01 (sec) , antiderivative size = 146, normalized size of antiderivative = 0.99

method	result
pseudoelliptic	$-\frac{2\left(\left(a\,e^{2}-c\,d^{2}\right)^{3}\arctan\left(\frac{cd\sqrt{ex+d}}{\sqrt{cd\left(a\,e^{2}-c\,d^{2}\right)}}\right)-\sqrt{ex+d}\,\sqrt{cd(a\,e^{2}-c\,d^{2})}\left(\frac{\left(d^{2}e^{2}x^{2}+\frac{11}{3}d^{3}ex+\frac{23}{3}d^{4}\right)c^{2}}{5}-\frac{7e^{2}\left(d+\frac{ex}{7}\right)adc}{3}\right.\right.}{\sqrt{cd(a\,e^{2}-c\,d^{2})}\,d^{3}c^{3}}$
risch	$\frac{2\left(3x^{2}c^{2}d^{2}e^{2}-5xacd\,e^{3}+11x\,c^{2}d^{3}e+15a^{2}e^{4}-35ac\,d^{2}e^{2}+23c^{2}d^{4}\right)\sqrt{ex+d}}{15d^{3}c^{3}}-\frac{2\left(e^{6}a^{3}-3d^{2}e^{4}a^{2}c+3d^{4}e^{2}a\,c^{2}-d^{6}c^{3}\right)\text{arc}}{d^{3}c^{3}\sqrt{cd(a\,e^{2}-c\,d}}$
derivativedivides	$\frac{\frac{2c^{2}d^{2}(ex+d)^{\frac{5}{2}}}{5}-\frac{2acd\,e^{2}(ex+d)^{\frac{3}{2}}}{3}+\frac{2c^{2}d^{3}(ex+d)^{\frac{3}{2}}}{3}+2a^{2}e^{4}\sqrt{ex+d}-4ac\,d^{2}e^{2}\sqrt{ex+d}+2c^{2}d^{4}\sqrt{ex+d}}{d^{3}c^{3}}+\frac{2\left(-e^{6}a^{3}+3d^{2}e^{4}c\right.}{}$
default	$\frac{\frac{2c^{2}d^{2}(ex+d)^{\frac{5}{2}}}{5}-\frac{2acd\,e^{2}(ex+d)^{\frac{3}{2}}}{3}+\frac{2c^{2}d^{3}(ex+d)^{\frac{3}{2}}}{3}+2a^{2}e^{4}\sqrt{ex+d}-4ac\,d^{2}e^{2}\sqrt{ex+d}+2c^{2}d^{4}\sqrt{ex+d}}{d^{3}c^{3}}+\frac{2\left(-e^{6}a^{3}+3d^{2}e^{4}c\right.}{}$

input
```
int((e*x+d)^(7/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e),x,method=_RETURNVERBOS
E)
```

output
```
-2*((a*e^2-c*d^2)^3*arctan(c*d*(e*x+d)^(1/2)/(c*d*(a*e^2-c*d^2))^(1/2))-(e
*x+d)^(1/2)*(c*d*(a*e^2-c*d^2))^(1/2)*(1/5*(d^2*e^2*x^2+11/3*d^3*e*x+23/3*
d^4)*c^2-7/3*e^2*(d+1/7*e*x)*a*d*c+a^2*e^4))/(c*d*(a*e^2-c*d^2))^(1/2)/d^3
/c^3
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 366, normalized size of antiderivative = 2.49

$$\int \frac{(d+ex)^{7/2}}{ade+(cd^2+ae^2)\,x+cdex^2}\,dx = \left[\frac{15\,(c^2d^4-2\,acd^2e^2+a^2e^4)\sqrt{\frac{cd^2-ae^2}{cd}}\log\left(\frac{cdex+2\,cd^2-ae^2-2\,\sqrt{ex+d}cd\sqrt{\frac{cd^2}{}}}{cdx+ae}\right.}{}\right.$$

$$-\frac{2\left(15\,(c^2d^4-2\,acd^2e^2+a^2e^4)\sqrt{-\frac{cd^2-ae^2}{cd}}\arctan\left(-\frac{\sqrt{ex+d}cd\sqrt{-\frac{cd^2-ae^2}{cd}}}{cd^2-ae^2}\right)-(3\,c^2d^2e^2x^2+23\,c^2d^4-35\,acd\right.}{15\,c^3d^3}$$

input
```
integrate((e*x+d)^(7/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2),x, algorithm="fr
icas")
```

output
```
[1/15*(15*(c^2*d^4 - 2*a*c*d^2*e^2 + a^2*e^4)*sqrt((c*d^2 - a*e^2)/(c*d))*
log((c*d*e*x + 2*c*d^2 - a*e^2 - 2*sqrt(e*x + d)*c*d*sqrt((c*d^2 - a*e^2)/
(c*d)))/(c*d*x + a*e)) + 2*(3*c^2*d^2*e^2*x^2 + 23*c^2*d^4 - 35*a*c*d^2*e^
2 + 15*a^2*e^4 + (11*c^2*d^3*e - 5*a*c*d*e^3)*x)*sqrt(e*x + d))/(c^3*d^3),
 -2/15*(15*(c^2*d^4 - 2*a*c*d^2*e^2 + a^2*e^4)*sqrt(-(c*d^2 - a*e^2)/(c*d)
)*arctan(-sqrt(e*x + d)*c*d*sqrt(-(c*d^2 - a*e^2)/(c*d))/(c*d^2 - a*e^2))
- (3*c^2*d^2*e^2*x^2 + 23*c^2*d^4 - 35*a*c*d^2*e^2 + 15*a^2*e^4 + (11*c^2*
d^3*e - 5*a*c*d*e^3)*x)*sqrt(e*x + d))/(c^3*d^3)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(d+ex)^{7/2}}{ade+(cd^2+ae^2)\,x+cdex^2}\,dx = \text{Timed out}$$

input
```
integrate((e*x+d)**(7/2)/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2),x)
```

output
```
Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(d+ex)^{7/2}}{ade+(cd^2+ae^2)\,x+cdex^2}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((e*x+d)^(7/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2),x, algorithm="ma
xima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(a*e^2-c*d^2>0)', see `assume?` f
or more de
```

Giac [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 209, normalized size of antiderivative = 1.42

$$\int \frac{(d+ex)^{7/2}}{ade+(cd^2+ae^2)\,x+cdex^2}\,dx = \frac{2\,(c^3d^6-3\,ac^2d^4e^2+3\,a^2cd^2e^4-a^3e^6)\arctan\left(\frac{\sqrt{ex+d}cd}{\sqrt{-c^2d^3+acde^2}}\right)}{\sqrt{-c^2d^3+acde^2}c^3d^3} + \frac{2\left(3\,(ex+d)^{\frac{5}{2}}c^4d^4+5\,(ex+d)^{\frac{3}{2}}c^4d^5+15\,\sqrt{ex+d}c^4d^6-5\,(ex+d)^{\frac{3}{2}}ac^3d^3e^2-30\,\sqrt{ex+d}ac^3d^4e^2+15\right.}{15\,c^5d^5}$$

input
```
integrate((e*x+d)^(7/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2),x, algorithm="gi
ac")
```

output
```
2*(c^3*d^6 - 3*a*c^2*d^4*e^2 + 3*a^2*c*d^2*e^4 - a^3*e^6)*arctan(sqrt(e*x
+ d)*c*d/sqrt(-c^2*d^3 + a*c*d*e^2))/(sqrt(-c^2*d^3 + a*c*d*e^2)*c^3*d^3)
+ 2/15*(3*(e*x + d)^(5/2)*c^4*d^4 + 5*(e*x + d)^(3/2)*c^4*d^5 + 15*sqrt(e*
x + d)*c^4*d^6 - 5*(e*x + d)^(3/2)*a*c^3*d^3*e^2 - 30*sqrt(e*x + d)*a*c^3*
d^4*e^2 + 15*sqrt(e*x + d)*a^2*c^2*d^2*e^4)/(c^5*d^5)
```

Mupad [B] (verification not implemented)

Time = 5.24 (sec) , antiderivative size = 165, normalized size of antiderivative = 1.12

$$\int \frac{(d+ex)^{7/2}}{ade+(cd^2+ae^2)\,x+cdex^2}\,dx=\frac{2\,{\left(d+e\,x\right)}^{5/2}}{5\,c\,d}+\frac{2\,{\left(a\,e^2-c\,d^2\right)}^2\,\sqrt{d+e\,x}}{c^3\,d^3}-\frac{2\,\left(a\,e^2-c\,d^2\right)\,{\left(d+e\,x\right)}^{3/2}}{3\,c^2\,d^2}-\frac{2\,\mathrm{atan}\left(\frac{\sqrt{c}\,\sqrt{d}\,{\left(a\,e^2-c\,d^2\right)}^{5/2}\,\sqrt{d+e\,x}}{a^3\,e^6-3\,a^2\,c\,d^2\,e^4+3\,a\,c^2\,d^4\,e^2-c^3\,d^6}\right)\,{\left(a\,e^2-c\,d^2\right)}^{5/2}}{c^{7/2}\,d^{7/2}}$$

input
```
int((d + e*x)^(7/2)/(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2),x)
```

output
```
(2*(d + e*x)^(5/2))/(5*c*d) + (2*(a*e^2 - c*d^2)^2*(d + e*x)^(1/2))/(c^3*d
^3) - (2*(a*e^2 - c*d^2)*(d + e*x)^(3/2))/(3*c^2*d^2) - (2*atan((c^(1/2)*d
^(1/2)*(a*e^2 - c*d^2)^(5/2)*(d + e*x)^(1/2))/(a^3*e^6 - c^3*d^6 + 3*a*c^2
*d^4*e^2 - 3*a^2*c*d^2*e^4))*(a*e^2 - c*d^2)^(5/2))/(c^(7/2)*d^(7/2))
```

Reduce [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 288, normalized size of antiderivative = 1.96

$$\int \frac{(d+ex)^{7/2}}{ade+(cd^2+ae^2)\,x+cdex^2}\,dx=\frac{-2\sqrt{d}\,\sqrt{c}\,\sqrt{a\,e^2-c\,d^2}\,atan\left(\frac{\sqrt{ex+d}\,cd}{\sqrt{d}\,\sqrt{c}\,\sqrt{a\,e^2-c\,d^2}}\right)a^2e^4+4\sqrt{d}\,\sqrt{c}\,\sqrt{a\,e^2-c\,d^2}\cdots}{\cdots}$$

input
```
int((e*x+d)^(7/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2),x)
```

output
```
(2*( - 15*sqrt(d)*sqrt(c)*sqrt(a*e**2 - c*d**2)*atan((sqrt(d + e*x)*c*d)/(
sqrt(d)*sqrt(c)*sqrt(a*e**2 - c*d**2)))*a**2*e**4 + 30*sqrt(d)*sqrt(c)*sqr
t(a*e**2 - c*d**2)*atan((sqrt(d + e*x)*c*d)/(sqrt(d)*sqrt(c)*sqrt(a*e**2 -
 c*d**2)))*a*c*d**2*e**2 - 15*sqrt(d)*sqrt(c)*sqrt(a*e**2 - c*d**2)*atan((
sqrt(d + e*x)*c*d)/(sqrt(d)*sqrt(c)*sqrt(a*e**2 - c*d**2)))*c**2*d**4 + 15
*sqrt(d + e*x)*a**2*c*d*e**4 - 35*sqrt(d + e*x)*a*c**2*d**3*e**2 - 5*sqrt(
d + e*x)*a*c**2*d**2*e**3*x + 23*sqrt(d + e*x)*c**3*d**5 + 11*sqrt(d + e*x
)*c**3*d**4*e*x + 3*sqrt(d + e*x)*c**3*d**3*e**2*x**2))/(15*c**4*d**4)
```

3.175 $\int \frac{(d+ex)^{5/2}}{ade+(cd^2+ae^2)x+cdex^2}\,dx$

Optimal result

Integrand size = 37, antiderivative size = 114

$$\int \frac{(d+ex)^{5/2}}{ade+(cd^2+ae^2)\,x+cdex^2}\,dx=\frac{2(cd^2-ae^2)\sqrt{d+ex}}{c^2d^2}+\frac{2(d+ex)^{3/2}}{3cd}-\frac{2(cd^2-ae^2)^{3/2}\operatorname{arctanh}\left(\frac{\sqrt{c}\sqrt{d}\sqrt{d+ex}}{\sqrt{cd^2-ae^2}}\right)}{c^{5/2}d^{5/2}}$$

output
```
2*(-a*e^2+c*d^2)*(e*x+d)^(1/2)/c^2/d^2+2/3*(e*x+d)^(3/2)/c/d-2*(-a*e^2+c*d
^2)^(3/2)*arctanh(c^(1/2)*d^(1/2)*(e*x+d)^(1/2)/(-a*e^2+c*d^2)^(1/2))/c^(5
/2)/d^(5/2)
```

Mathematica [A] (verified)

Time = 0.19 (sec) , antiderivative size = 102, normalized size of antiderivative = 0.89

$$\int \frac{(d+ex)^{5/2}}{ade+(cd^2+ae^2)\,x+cdex^2}\,dx=\frac{2\sqrt{d+ex}(-3ae^2+cd(4d+ex))}{3c^2d^2}+\frac{2(-cd^2+ae^2)^{3/2}\arctan\left(\frac{\sqrt{c}\sqrt{d}\sqrt{d+ex}}{\sqrt{-cd^2+ae^2}}\right)}{c^{5/2}d^{5/2}}$$

input
```
Integrate[(d + e*x)^(5/2)/(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2),x]
```

output
```
(2*Sqrt[d + e*x]*(-3*a*e^2 + c*d*(4*d + e*x)))/(3*c^2*d^2) + (2*(-(c*d^2)
+ a*e^2)^(3/2)*ArcTan[(Sqrt[c]*Sqrt[d]*Sqrt[d + e*x])/Sqrt[-(c*d^2) + a*e^
2]])/(c^(5/2)*d^(5/2))
```

Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 133, normalized size of antiderivative = 1.17, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.135$, Rules used = {1121, 60, 60, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex)^{5/2}}{x(ae^2+cd^2)+ade+cdex^2}dx$$

$$\Big\downarrow \text{1121}$$

$$\int \frac{(d+ex)^{3/2}}{ae+cdx}dx$$

$$\Big\downarrow \text{60}$$

$$\frac{\left(d^2-\frac{ae^2}{c}\right)\int \frac{\sqrt{d+ex}}{ae+cdx}dx}{d}+\frac{2(d+ex)^{3/2}}{3cd}$$

$$\Big\downarrow \text{60}$$

$$\frac{\left(d^2-\frac{ae^2}{c}\right)\left(\frac{\left(d^2-\frac{ae^2}{c}\right)\int \frac{1}{(ae+cdx)\sqrt{d+ex}}dx}{d}+\frac{2\sqrt{d+ex}}{cd}\right)}{d}+\frac{2(d+ex)^{3/2}}{3cd}$$

$$\Big\downarrow \text{73}$$

$$\frac{\left(d^2-\frac{ae^2}{c}\right)\left(\frac{2\left(d^2-\frac{ae^2}{c}\right)\int \frac{1}{-\frac{cd^2}{e}+\frac{c(d+ex)d}{e}+ae}d\sqrt{d+ex}}{de}+\frac{2\sqrt{d+ex}}{cd}\right)}{d}+\frac{2(d+ex)^{3/2}}{3cd}$$

$$\Big\downarrow \text{221}$$

$$\frac{\left(d^2-\frac{ae^2}{c}\right)\left(\frac{2\sqrt{d+ex}}{cd}-\frac{2\left(d^2-\frac{ae^2}{c}\right)\operatorname{arctanh}\left(\frac{\sqrt{c}\sqrt{d}\sqrt{d+ex}}{\sqrt{cd^2-ae^2}}\right)}{\sqrt{c}d^{3/2}\sqrt{cd^2-ae^2}}\right)}{d}+\frac{2(d+ex)^{3/2}}{3cd}$$

input
```
Int[(d + e*x)^(5/2)/(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2),x]
```

output
```
(2*(d + e*x)^(3/2))/(3*c*d) + ((d^2 - (a*e^2)/c)*((2*Sqrt[d + e*x])/(c*d)
- (2*(d^2 - (a*e^2)/c)*ArcTanh[(Sqrt[c]*Sqrt[d]*Sqrt[d + e*x])/Sqrt[c*d^2
- a*e^2]])/(Sqrt[c]*d^(3/2)*Sqrt[c*d^2 - a*e^2])))/d
```

Defintions of rubi rules used

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 1121
```
Int[((d_) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Int[ExpandIntegrand[(d + e*x)^(m + p)*(a/d + (c/e)*x)^p, x], x]
/; FreeQ[{a, b, c, d, e, m, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && (Int
egerQ[p] || (GtQ[a, 0] && GtQ[d, 0] && LtQ[c, 0]))
```

Maple [A] (verified)

Time = 1.99 (sec) , antiderivative size = 112, normalized size of antiderivative = 0.98

method	result	size
pseudoelliptic	$-\frac{2\left(-\left(a\,e^{2}-c\,d^{2}\right)^{2} \arctan\left(\frac{c d \sqrt{e x+d}}{\sqrt{c d \left(a\,e^{2}-c\,d^{2}\right)}}\right)+\sqrt{c d \left(a\,e^{2}-c\,d^{2}\right)}\,\sqrt{e x+d}\,\left(-\frac{4 \left(\frac{e x}{4}+d\right) d c}{3}+a\,e^{2}\right)\right)}{\sqrt{c d \left(a\,e^{2}-c\,d^{2}\right)}\,c^{2} d^{2}}$	112
risch	$-\frac{2\left(-c d x e+3 a\,e^{2}-4 c\,d^{2}\right) \sqrt{e x+d}}{3 c^{2} d^{2}}+\frac{2\left(a^{2} e^{4}-2 a c\,d^{2} e^{2}+c^{2} d^{4}\right) \arctan\left(\frac{c d \sqrt{e x+d}}{\sqrt{c d \left(a\,e^{2}-c\,d^{2}\right)}}\right)}{c^{2} d^{2} \sqrt{c d \left(a\,e^{2}-c\,d^{2}\right)}}$	114
derivativedivides	$-\frac{2\left(-\frac{c d \left(e x+d\right)^{\frac{3}{2}}}{3}+a\,e^{2} \sqrt{e x+d}-c\,d^{2} \sqrt{e x+d}\right)}{c^{2} d^{2}}+\frac{2\left(a^{2} e^{4}-2 a c\,d^{2} e^{2}+c^{2} d^{4}\right) \arctan\left(\frac{c d \sqrt{e x+d}}{\sqrt{c d \left(a\,e^{2}-c\,d^{2}\right)}}\right)}{c^{2} d^{2} \sqrt{c d \left(a\,e^{2}-c\,d^{2}\right)}}$	125
default	$-\frac{2\left(-\frac{c d \left(e x+d\right)^{\frac{3}{2}}}{3}+a\,e^{2} \sqrt{e x+d}-c\,d^{2} \sqrt{e x+d}\right)}{c^{2} d^{2}}+\frac{2\left(a^{2} e^{4}-2 a c\,d^{2} e^{2}+c^{2} d^{4}\right) \arctan\left(\frac{c d \sqrt{e x+d}}{\sqrt{c d \left(a\,e^{2}-c\,d^{2}\right)}}\right)}{c^{2} d^{2} \sqrt{c d \left(a\,e^{2}-c\,d^{2}\right)}}$	125

input
```
int((e*x+d)^(5/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e),x,method=_RETURNVERBOS
E)
```

output
```
-2/(c*d*(a*e^2-c*d^2))^(1/2)*(-(a*e^2-c*d^2)^2*arctan(c*d*(e*x+d)^(1/2)/(c
*d*(a*e^2-c*d^2))^(1/2))+(c*d*(a*e^2-c*d^2))^(1/2)*(e*x+d)^(1/2)*(-4/3*(1/
4*e*x+d)*d*c+a*e^2))/c^2/d^2
```

Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 254, normalized size of antiderivative = 2.23

$$\int \frac{(d+ex)^{5/2}}{ade+(cd^2+ae^2)\,x+cdex^2}\,dx = \left[-\frac{3\,(cd^2-ae^2)\sqrt{\frac{cd^2-ae^2}{cd}}\log\left(\frac{cdex+2\,cd^2-ae^2+2\,\sqrt{ex+d}cd\sqrt{\frac{cd^2-ae^2}{cd}}}{cdx+ae}\right)-2\,(c}{3\,c^2d^2}\right.$$

$$\left.-\frac{2\left(3\,(cd^2-ae^2)\sqrt{-\frac{cd^2-ae^2}{cd}}\arctan\left(-\frac{\sqrt{ex+d}cd\sqrt{-\frac{cd^2-ae^2}{cd}}}{cd^2-ae^2}\right)-(cdex+4\,cd^2-3\,ae^2)\sqrt{ex+d}\right)}{3\,c^2d^2}\right]$$

input
```
integrate((e*x+d)^(5/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2),x, algorithm="fr
icas")
```

output
```
[-1/3*(3*(c*d^2 - a*e^2)*sqrt((c*d^2 - a*e^2)/(c*d))*log((c*d*e*x + 2*c*d^
2 - a*e^2 + 2*sqrt(e*x + d)*c*d*sqrt((c*d^2 - a*e^2)/(c*d)))/(c*d*x + a*e)
) - 2*(c*d*e*x + 4*c*d^2 - 3*a*e^2)*sqrt(e*x + d))/(c^2*d^2), -2/3*(3*(c*d
^2 - a*e^2)*sqrt(-(c*d^2 - a*e^2)/(c*d))*arctan(-sqrt(e*x + d)*c*d*sqrt(-(
c*d^2 - a*e^2)/(c*d))/(c*d^2 - a*e^2)) - (c*d*e*x + 4*c*d^2 - 3*a*e^2)*sqr
t(e*x + d))/(c^2*d^2)]
```

Sympy [A] (verification not implemented)

Time = 29.12 (sec) , antiderivative size = 122, normalized size of antiderivative = 1.07

$$\int \frac{(d+ex)^{5/2}}{ade+(cd^2+ae^2)\,x+cdex^2}\,dx = \begin{cases} \frac{2\left(\frac{e(d+ex)^{\frac{3}{2}}}{3cd}+\frac{\sqrt{d+ex}\left(-ae^3+cd^2e\right)}{c^2d^2}+\frac{e\left(ae^2-cd^2\right)^2\operatorname{atan}\left(\frac{\sqrt{d+ex}}{\sqrt{\frac{ae^2-cd^2}{cd}}}\right)}{c^3d^3\sqrt{\frac{ae^2-cd^2}{cd}}}\right)}{e} & \text{for}\ e\neq 0\\ \frac{\sqrt{d}\log{(x)}}{c} & \text{otherwise} \end{cases}$$

input
```
integrate((e*x+d)**(5/2)/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2),x)
```

output
```
Piecewise((2*(e*(d + e*x)**(3/2)/(3*c*d) + sqrt(d + e*x)*(-a*e**3 + c*d**2
*e)/(c**2*d**2) + e*(a*e**2 - c*d**2)**2*atan(sqrt(d + e*x)/sqrt((a*e**2 -
 c*d**2)/(c*d)))/(c**3*d**3*sqrt((a*e**2 - c*d**2)/(c*d))))/e, Ne(e, 0)),
(sqrt(d)*log(x)/c, True))
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(d+ex)^{5/2}}{ade+(cd^2+ae^2)\,x+cdex^2}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((e*x+d)^(5/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2),x, algorithm="ma
xima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(a*e^2-c*d^2>0)', see `assume?` f
or more de
```

Giac [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 134, normalized size of antiderivative = 1.18

$$\int \frac{(d+ex)^{5/2}}{ade+(cd^2+ae^2)\,x+cdex^2}\,dx = \frac{2\,(c^2d^4-2\,acd^2e^2+a^2e^4)\arctan\left(\frac{\sqrt{ex+d}cd}{\sqrt{-c^2d^3+acde^2}}\right)}{\sqrt{-c^2d^3+acde^2}c^2d^2} + \frac{2\left((ex+d)^{\frac{3}{2}}c^2d^2+3\sqrt{ex+d}c^2d^3-3\sqrt{ex+d}acde^2\right)}{3\,c^3d^3}$$

input
```
integrate((e*x+d)^(5/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2),x, algorithm="gi
ac")
```

output
```
2*(c^2*d^4 - 2*a*c*d^2*e^2 + a^2*e^4)*arctan(sqrt(e*x + d)*c*d/sqrt(-c^2*d
^3 + a*c*d*e^2))/(sqrt(-c^2*d^3 + a*c*d*e^2)*c^2*d^2) + 2/3*((e*x + d)^(3/
2)*c^2*d^2 + 3*sqrt(e*x + d)*c^2*d^3 - 3*sqrt(e*x + d)*a*c*d*e^2)/(c^3*d^3
)
```

Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.06

$$\int \frac{(d+ex)^{5/2}}{ade+(cd^2+ae^2)\,x+cdex^2}\,dx = \frac{2\,(d+e\,x)^{3/2}}{3\,c\,d} + \frac{2\,\mathrm{atan}\left(\frac{\sqrt{c}\,\sqrt{d}\,\left(a\,e^2-c\,d^2\right)^{3/2}\,\sqrt{d+e\,x}}{a^2\,e^4-2\,a\,c\,d^2\,e^2+c^2\,d^4}\right)\,\left(a\,e^2-c\,d^2\right)^{3/2}}{c^{5/2}\,d^{5/2}} - \frac{2\,\left(a\,e^2-c\,d^2\right)\,\sqrt{d+e\,x}}{c^2\,d^2}$$

input
```
int((d + e*x)^(5/2)/(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2),x)
```

output
```
(2*(d + e*x)^(3/2))/(3*c*d) + (2*atan((c^(1/2)*d^(1/2)*(a*e^2 - c*d^2)^(3/
2)*(d + e*x)^(1/2))/(a^2*e^4 + c^2*d^4 - 2*a*c*d^2*e^2))*(a*e^2 - c*d^2)^(
3/2))/(c^(5/2)*d^(5/2)) - (2*(a*e^2 - c*d^2)*(d + e*x)^(1/2))/(c^2*d^2)
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 164, normalized size of antiderivative = 1.44

$$\int \frac{(d+ex)^{5/2}}{ade+(cd^2+ae^2)\,x+cdex^2}\,dx = \frac{2\sqrt{d}\,\sqrt{c}\,\sqrt{a\,e^2-c\,d^2}\;atan\left(\frac{\sqrt{ex+d}\,cd}{\sqrt{d}\,\sqrt{c}\,\sqrt{a\,e^2-c\,d^2}}\right)a\,e^2-2\sqrt{d}\,\sqrt{c}\,\sqrt{a\,e^2-c\,d}}{}$$

input
```
int((e*x+d)^(5/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2),x)
```

output
```
(2*(3*sqrt(d)*sqrt(c)*sqrt(a*e**2 - c*d**2)*atan((sqrt(d + e*x)*c*d)/(sqrt
(d)*sqrt(c)*sqrt(a*e**2 - c*d**2)))*a*e**2 - 3*sqrt(d)*sqrt(c)*sqrt(a*e**2
 - c*d**2)*atan((sqrt(d + e*x)*c*d)/(sqrt(d)*sqrt(c)*sqrt(a*e**2 - c*d**2)
))*c*d**2 - 3*sqrt(d + e*x)*a*c*d*e**2 + 4*sqrt(d + e*x)*c**2*d**3 + sqrt(
d + e*x)*c**2*d**2*e*x))/(3*c**3*d**3)
```

3.176 $\int \frac{(d+ex)^{3/2}}{ade+(cd^2+ae^2)x+cdex^2}\,dx$

Optimal result

Integrand size = 37, antiderivative size = 83

$$\int \frac{(d+ex)^{3/2}}{ade+(cd^2+ae^2)x+cdex^2}\,dx=\frac{2\sqrt{d+ex}}{cd}-\frac{2\sqrt{cd^2-ae^2}\,\mathrm{arctanh}\left(\frac{\sqrt{c}\sqrt{d}\sqrt{d+ex}}{\sqrt{cd^2-ae^2}}\right)}{c^{3/2}d^{3/2}}$$

output
```
2*(e*x+d)^(1/2)/c/d-2*(-a*e^2+c*d^2)^(1/2)*arctanh(c^(1/2)*d^(1/2)*(e*x+d)
^(1/2)/(-a*e^2+c*d^2)^(1/2))/c^(3/2)/d^(3/2)
```

Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.00

$$\int \frac{(d+ex)^{3/2}}{ade+(cd^2+ae^2)x+cdex^2}\,dx=\frac{2\sqrt{d+ex}}{cd}-\frac{2\sqrt{-cd^2+ae^2}\arctan\left(\frac{\sqrt{c}\sqrt{d}\sqrt{d+ex}}{\sqrt{-cd^2+ae^2}}\right)}{c^{3/2}d^{3/2}}$$

input
```
Integrate[(d + e*x)^(3/2)/(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2),x]
```

output
```
(2*Sqrt[d + e*x])/(c*d) - (2*Sqrt[-(c*d^2) + a*e^2]*ArcTan[(Sqrt[c]*Sqrt[d
]*Sqrt[d + e*x])/Sqrt[-(c*d^2) + a*e^2]])/(c^(3/2)*d^(3/2))
```

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 96, normalized size of antiderivative = 1.16, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.108, Rules used = {1121, 60, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex)^{3/2}}{x(ae^2+cd^2)+ade+cdex^2}dx$$

$$\Big\downarrow \text{1121}$$

$$\int \frac{\sqrt{d+ex}}{ae+cdx}dx$$

$$\Big\downarrow \text{60}$$

$$\frac{\left(d^2-\frac{ae^2}{c}\right)\int \frac{1}{(ae+cdx)\sqrt{d+ex}}dx}{d}+\frac{2\sqrt{d+ex}}{cd}$$

$$\Big\downarrow \text{73}$$

$$\frac{2\left(d^2-\frac{ae^2}{c}\right)\int \frac{1}{-\frac{cd^2}{e}+\frac{c(d+ex)d}{e}+ae}d\sqrt{d+ex}}{de}+\frac{2\sqrt{d+ex}}{cd}$$

$$\Big\downarrow \text{221}$$

$$\frac{2\sqrt{d+ex}}{cd}-\frac{2\left(d^2-\frac{ae^2}{c}\right)\text{arctanh}\left(\frac{\sqrt{c}\sqrt{d}\sqrt{d+ex}}{\sqrt{cd^2-ae^2}}\right)}{\sqrt{c}d^{3/2}\sqrt{cd^2-ae^2}}$$

input
```
Int[(d + e*x)^(3/2)/(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2),x]
```

output
```
(2*Sqrt[d + e*x])/(c*d) - (2*(d^2 - (a*e^2)/c)*ArcTanh[(Sqrt[c]*Sqrt[d]*Sq
rt[d + e*x])/Sqrt[c*d^2 - a*e^2]])/(Sqrt[c]*d^(3/2)*Sqrt[c*d^2 - a*e^2])
```

Defintions of rubi rules used

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 1121
```
Int[((d_) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Int[ExpandIntegrand[(d + e*x)^(m + p)*(a/d + (c/e)*x)^p, x], x]
/; FreeQ[{a, b, c, d, e, m, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && (Int
egerQ[p] || (GtQ[a, 0] && GtQ[d, 0] && LtQ[c, 0]))
```

Maple [A] (verified)

Time = 1.96 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.92

method	result	size
pseudoelliptic	$\frac{2\sqrt{ex+d}-\frac{2\left(a\,e^{2}-c\,d^{2}\right)\arctan\left(\frac{cd\sqrt{ex+d}}{\sqrt{cd\left(a\,e^{2}-c\,d^{2}\right)}}\right)}{\sqrt{cd\left(a\,e^{2}-c\,d^{2}\right)}}}{cd}$	76
risch	$\frac{2\_O1\sqrt{ex+d}}{d}-\frac{2(a\,e^{2}-c\,d^{2})\arctan\left(\frac{\sqrt{ex+d}\,d}{\sqrt{\left(\_O1a\,e^{2}-d^{2}\right)d}}\right)}{c^{2}d\sqrt{\left(\_O1a\,e^{2}-d^{2}\right)d}}$	77
derivativedivides	$\frac{2\sqrt{ex+d}}{cd}+\frac{2(-a\,e^{2}+c\,d^{2})\arctan\left(\frac{cd\sqrt{ex+d}}{\sqrt{cd\left(a\,e^{2}-c\,d^{2}\right)}}\right)}{cd\sqrt{cd(a\,e^{2}-c\,d^{2})}}$	82
default	$\frac{2\sqrt{ex+d}}{cd}+\frac{2(-a\,e^{2}+c\,d^{2})\arctan\left(\frac{cd\sqrt{ex+d}}{\sqrt{cd\left(a\,e^{2}-c\,d^{2}\right)}}\right)}{cd\sqrt{cd(a\,e^{2}-c\,d^{2})}}$	82

input
```
int((e*x+d)^(3/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e),x,method=_RETURNVERBOS
E)
```

output
```
2/c/d*((e*x+d)^(1/2)-(a*e^2-c*d^2)*arctan(c*d*(e*x+d)^(1/2)/(c*d*(a*e^2-c*
d^2))^(1/2))/(c*d*(a*e^2-c*d^2))^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 191, normalized size of antiderivative = 2.30

$$\int \frac{(d+ex)^{3/2}}{ade+(cd^2+ae^2)\,x+cdex^2}\,dx=\left[\frac{\sqrt{\frac{cd^2-ae^2}{cd}}\log\left(\frac{cdex+2\,cd^2-ae^2-2\,\sqrt{ex+d}cd\sqrt{\frac{cd^2-ae^2}{cd}}}{cdx+ae}\right)+2\,\sqrt{ex+d}}{cd},\right.$$
$$\left.-\frac{2\left(\sqrt{-\frac{cd^2-ae^2}{cd}}\arctan\left(-\frac{\sqrt{ex+d}cd\sqrt{-\frac{cd^2-ae^2}{cd}}}{cd^2-ae^2}\right)-\sqrt{ex+d}\right)}{cd}\right]$$

input
```
integrate((e*x+d)^(3/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2),x, algorithm="fr
icas")
```

output
```
[(sqrt((c*d^2 - a*e^2)/(c*d))*log((c*d*e*x + 2*c*d^2 - a*e^2 - 2*sqrt(e*x
+ d)*c*d*sqrt((c*d^2 - a*e^2)/(c*d)))/(c*d*x + a*e)) + 2*sqrt(e*x + d))/(c
*d), -2*(sqrt(-(c*d^2 - a*e^2)/(c*d))*arctan(-sqrt(e*x + d)*c*d*sqrt(-(c*d
^2 - a*e^2)/(c*d))/(c*d^2 - a*e^2)) - sqrt(e*x + d))/(c*d)]
```

Sympy [A] (verification not implemented)

Time = 1.54 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.08

$$\int \frac{(d+ex)^{3/2}}{ade+(cd^2+ae^2)\,x+cdex^2}\,dx = \begin{cases} \frac{2\left(\frac{e\sqrt{d+ex}}{cd}-\frac{e\left(ae^2-cd^2\right)\operatorname{atan}\left(\frac{\sqrt{d+ex}}{\sqrt{\frac{ae^2-cd^2}{cd}}}\right)}{c^2d^2\sqrt{\frac{ae^2-cd^2}{cd}}}\right)}{e} & \text{for } e\neq 0 \\ \frac{\log(x)}{c\sqrt{d}} & \text{otherwise} \end{cases}$$

input
```
integrate((e*x+d)**(3/2)/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2),x)
```

output
```
Piecewise((2*(e*sqrt(d + e*x)/(c*d) - e*(a*e**2 - c*d**2)*atan(sqrt(d + e*
x)/sqrt((a*e**2 - c*d**2)/(c*d)))/(c**2*d**2*sqrt((a*e**2 - c*d**2)/(c*d))
))/e, Ne(e, 0)), (log(x)/(c*sqrt(d)), True))
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(d+ex)^{3/2}}{ade+(cd^2+ae^2)\,x+cdex^2}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((e*x+d)^(3/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2),x, algorithm="ma
xima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(a*e^2-c*d^2>0)', see `assume?` f
or more de
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.00

$$\int \frac{(d+ex)^{3/2}}{ade+(cd^2+ae^2)\,x+cdex^2}\,dx = \frac{2\,(cd^2-ae^2)\arctan\left(\frac{\sqrt{ex+d}cd}{\sqrt{-c^2d^3+acde^2}}\right)}{\sqrt{-c^2d^3+acde^2}cd}+\frac{2\sqrt{ex+d}}{cd}$$

input
```
integrate((e*x+d)^(3/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2),x, algorithm="gi
ac")
```

output
```
2*(c*d^2 - a*e^2)*arctan(sqrt(e*x + d)*c*d/sqrt(-c^2*d^3 + a*c*d*e^2))/(sq
rt(-c^2*d^3 + a*c*d*e^2)*c*d) + 2*sqrt(e*x + d)/(c*d)
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.81

$$\int \frac{(d+ex)^{3/2}}{ade+(cd^2+ae^2)\,x+cdex^2}\,dx = \frac{2\sqrt{d+e\,x}}{c\,d}-\frac{2\,\mathrm{atan}\left(\frac{\sqrt{c}\,\sqrt{d}\,\sqrt{d+e\,x}}{\sqrt{a\,e^2-c\,d^2}}\right)\,\sqrt{a\,e^2-c\,d^2}}{c^{3/2}\,d^{3/2}}$$

input
```
int((d + e*x)^(3/2)/(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2),x)
```

output
```
(2*(d + e*x)^(1/2))/(c*d) - (2*atan((c^(1/2)*d^(1/2)*(d + e*x)^(1/2))/(a*e
^2 - c*d^2)^(1/2))*(a*e^2 - c*d^2)^(1/2))/(c^(3/2)*d^(3/2))
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.84

$$\int \frac{(d+ex)^{3/2}}{ade+(cd^2+ae^2)\,x+cdex^2}\,dx = \frac{-2\sqrt{d}\,\sqrt{c}\,\sqrt{a\,e^2-c\,d^2}\,atan\left(\frac{\sqrt{ex+d}\,cd}{\sqrt{d}\,\sqrt{c}\,\sqrt{a\,e^2-c\,d^2}}\right)+2\sqrt{ex+d}\,cd}{c^2d^2}$$

input
```
int((e*x+d)^(3/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2),x)
```

output
```
(2*( - sqrt(d)*sqrt(c)*sqrt(a*e**2 - c*d**2)*atan((sqrt(d + e*x)*c*d)/(sqr
t(d)*sqrt(c)*sqrt(a*e**2 - c*d**2))) + sqrt(d + e*x)*c*d))/(c**2*d**2)
```

3.177 $\int \frac{\sqrt{d+ex}}{ade+(cd^2+ae^2)x+cdex^2}\,dx$

Optimal result

Integrand size = 37, antiderivative size = 65

$$\int \frac{\sqrt{d+ex}}{ade+(cd^2+ae^2)\,x+cdex^2}\,dx=-\frac{2\text{arctanh}\left(\frac{\sqrt{c}\sqrt{d}\sqrt{d+ex}}{\sqrt{cd^2-ae^2}}\right)}{\sqrt{c}\sqrt{d}\sqrt{cd^2-ae^2}}$$

output
```
-2*arctanh(c^(1/2)*d^(1/2)*(e*x+d)^(1/2)/(-a*e^2+c*d^2)^(1/2))/c^(1/2)/d^(
1/2)/(-a*e^2+c*d^2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt{d+ex}}{ade+(cd^2+ae^2)\,x+cdex^2}\,dx=\frac{2\arctan\left(\frac{\sqrt{c}\sqrt{d}\sqrt{d+ex}}{\sqrt{-cd^2+ae^2}}\right)}{\sqrt{c}\sqrt{d}\sqrt{-cd^2+ae^2}}$$

input
```
Integrate[Sqrt[d + e*x]/(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2),x]
```

output
```
(2*ArcTan[(Sqrt[c]*Sqrt[d]*Sqrt[d + e*x])/Sqrt[-(c*d^2) + a*e^2]])/(Sqrt[c
]*Sqrt[d]*Sqrt[-(c*d^2) + a*e^2])
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.081$, Rules used = {1121, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{d+ex}}{x(ae^2+cd^2)+ade+cdex^2}dx$$

$$\Big\downarrow \text{1121}$$

$$\int \frac{1}{\sqrt{d+ex}(ae+cdx)}dx$$

$$\Big\downarrow \text{73}$$

$$\frac{2\int \frac{1}{-\frac{cd^2}{e}+\frac{c(d+ex)d}{e}+ae}d\sqrt{d+ex}}{e}$$

$$\Big\downarrow \text{221}$$

$$-\frac{2\text{arctanh}\left(\frac{\sqrt{c}\sqrt{d}\sqrt{d+ex}}{\sqrt{cd^2-ae^2}}\right)}{\sqrt{c}\sqrt{d}\sqrt{cd^2-ae^2}}$$

input
```
Int[Sqrt[d + e*x]/(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2),x]
```

output
```
(-2*ArcTanh[(Sqrt[c]*Sqrt[d]*Sqrt[d + e*x])/Sqrt[c*d^2 - a*e^2]])/(Sqrt[c]
*Sqrt[d]*Sqrt[c*d^2 - a*e^2])
```

Defintions of rubi rules used

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 1121
```
Int[((d_) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Int[ExpandIntegrand[(d + e*x)^(m + p)*(a/d + (c/e)*x)^p, x], x]
/; FreeQ[{a, b, c, d, e, m, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && (Int
egerQ[p] || (GtQ[a, 0] && GtQ[d, 0] && LtQ[c, 0]))
```

Maple [A] (verified)

Time = 1.78 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.74

method	result	size
derivativedivides	$\frac{2 \arctan\left(\frac{c d \sqrt{e x+d}}{\sqrt{c d \left(a \,e^{2}-c \,d^{2}\right)}}\right)}{\sqrt{c d \left(a \,e^{2}-c \,d^{2}\right)}}$	48
default	$\frac{2 \arctan\left(\frac{c d \sqrt{e x+d}}{\sqrt{c d \left(a \,e^{2}-c \,d^{2}\right)}}\right)}{\sqrt{c d \left(a \,e^{2}-c \,d^{2}\right)}}$	48
pseudoelliptic	$\frac{2 \arctan\left(\frac{c d \sqrt{e x+d}}{\sqrt{c d \left(a \,e^{2}-c \,d^{2}\right)}}\right)}{\sqrt{c d \left(a \,e^{2}-c \,d^{2}\right)}}$	48

input
```
int((e*x+d)^(1/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e),x,method=_RETURNVERBOS
E)
```

output
```
2/(c*d*(a*e^2-c*d^2))^(1/2)*arctan(c*d*(e*x+d)^(1/2)/(c*d*(a*e^2-c*d^2))^(
1/2))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 155, normalized size of antiderivative = 2.38

$$\int \frac{\sqrt{d+ex}}{ade+(cd^2+ae^2)\,x+cdex^2}\,dx$$
$$=\left[\frac{\log\left(\frac{cdex+2\,cd^2-ae^2-2\,\sqrt{c^2d^3-acde^2}\sqrt{ex+d}}{cdx+ae}\right)}{\sqrt{c^2d^3-acde^2}},\ \frac{2\,\sqrt{-c^2d^3+acde^2}\arctan\left(\frac{\sqrt{-c^2d^3+acde^2}\sqrt{ex+d}}{cdex+cd^2}\right)}{c^2d^3-acde^2}\right]$$

input
```
integrate((e*x+d)^(1/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2),x, algorithm="fr
icas")
```

output
```
[log((c*d*e*x + 2*c*d^2 - a*e^2 - 2*sqrt(c^2*d^3 - a*c*d*e^2)*sqrt(e*x + d
))/(c*d*x + a*e))/sqrt(c^2*d^3 - a*c*d*e^2), 2*sqrt(-c^2*d^3 + a*c*d*e^2)*
arctan(sqrt(-c^2*d^3 + a*c*d*e^2)*sqrt(e*x + d)/(c*d*e*x + c*d^2))/(c^2*d^
3 - a*c*d*e^2)]
```

Sympy [A] (verification not implemented)

Time = 1.30 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.89

$$\int \frac{\sqrt{d+ex}}{ade+(cd^2+ae^2)\,x+cdex^2}\,dx=\begin{cases}\frac{2\,\mathrm{atan}\left(\frac{\sqrt{d+ex}}{\sqrt{\frac{ae^2-cd^2}{cd}}}\right)}{cd\sqrt{\frac{ae^2-cd^2}{cd}}} & \text{for } e\neq 0\\ \frac{\log(x)}{cd^{\frac{3}{2}}} & \text{otherwise}\end{cases}$$

input
```
integrate((e*x+d)**(1/2)/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2),x)
```

output
```
Piecewise((2*atan(sqrt(d + e*x)/sqrt((a*e**2 - c*d**2)/(c*d)))/(c*d*sqrt((
a*e**2 - c*d**2)/(c*d))), Ne(e, 0)), (log(x)/(c*d**(3/2)), True))
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\sqrt{d+ex}}{ade+(cd^2+ae^2)x+cdex^2}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((e*x+d)^(1/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2),x, algorithm="ma
xima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(a*e^2-c*d^2>0)', see `assume?` f
or more de
```

Giac [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.75

$$\int \frac{\sqrt{d+ex}}{ade+(cd^2+ae^2)x+cdex^2}\,dx = \frac{2\arctan\left(\frac{\sqrt{ex+d}cd}{\sqrt{-c^2d^3+acde^2}}\right)}{\sqrt{-c^2d^3+acde^2}}$$

input
```
integrate((e*x+d)^(1/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2),x, algorithm="gi
ac")
```

output
```
2*arctan(sqrt(e*x + d)*c*d/sqrt(-c^2*d^3 + a*c*d*e^2))/sqrt(-c^2*d^3 + a*c
*d*e^2)
```

Mupad [B] (verification not implemented)

Time = 5.26 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.75

$$\int \frac{\sqrt{d+ex}}{ade+(cd^2+ae^2)\,x+cdex^2}\,dx=\frac{2\,\mathrm{atan}\left(\frac{c\,d\,\sqrt{d+e\,x}}{\sqrt{a\,c\,d\,e^2-c^2\,d^3}}\right)}{\sqrt{a\,c\,d\,e^2-c^2\,d^3}}$$

input
```
int((d + e*x)^(1/2)/(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2),x)
```

output
```
(2*atan((c*d*(d + e*x)^(1/2))/(a*c*d*e^2 - c^2*d^3)^(1/2)))/(a*c*d*e^2 - c
^2*d^3)^(1/2)
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.11

$$\int \frac{\sqrt{d+ex}}{ade+(cd^2+ae^2)\,x+cdex^2}\,dx=\frac{2\sqrt{d}\,\sqrt{c}\,\sqrt{a\,e^2-c\,d^2}\,atan\left(\frac{\sqrt{ex+d}\,cd}{\sqrt{d}\,\sqrt{c}\,\sqrt{a\,e^2-c\,d^2}}\right)}{cd\,(a\,e^2-c\,d^2)}$$

input
```
int((e*x+d)^(1/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2),x)
```

output
```
(2*sqrt(d)*sqrt(c)*sqrt(a*e**2 - c*d**2)*atan((sqrt(d + e*x)*c*d)/(sqrt(d)
*sqrt(c)*sqrt(a*e**2 - c*d**2))))/(c*d*(a*e**2 - c*d**2))
```

3.178 $\int \frac{1}{\sqrt{d+ex}\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)}\,dx$

Optimal result

Integrand size = 37, antiderivative size = 91

$$\int \frac{1}{\sqrt{d+ex}\,(ade+(cd^2+ae^2)\,x+cdex^2)}\,dx = \frac{2}{(cd^2-ae^2)\,\sqrt{d+ex}}-\frac{2\sqrt{c}\sqrt{d}\operatorname{arctanh}\left(\frac{\sqrt{c}\sqrt{d}\sqrt{d+ex}}{\sqrt{cd^2-ae^2}}\right)}{(cd^2-ae^2)^{3/2}}$$

output
```
2/(-a*e^2+c*d^2)/(e*x+d)^(1/2)-2*c^(1/2)*d^(1/2)*arctanh(c^(1/2)*d^(1/2)*(
e*x+d)^(1/2)/(-a*e^2+c*d^2)^(1/2))/(-a*e^2+c*d^2)^(3/2)
```

Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.00

$$\int \frac{1}{\sqrt{d+ex}\,(ade+(cd^2+ae^2)\,x+cdex^2)}\,dx = \frac{2}{(cd^2-ae^2)\,\sqrt{d+ex}}-\frac{2\sqrt{c}\sqrt{d}\arctan\left(\frac{\sqrt{c}\sqrt{d}\sqrt{d+ex}}{\sqrt{-cd^2+ae^2}}\right)}{(-cd^2+ae^2)^{3/2}}$$

input
```
Integrate[1/(Sqrt[d + e*x]*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)),x]
```

output
```
2/((c*d^2 - a*e^2)*Sqrt[d + e*x]) - (2*Sqrt[c]*Sqrt[d]*ArcTan[(Sqrt[c]*Sqr
t[d]*Sqrt[d + e*x])/Sqrt[-(c*d^2) + a*e^2]])/(-(c*d^2) + a*e^2)^(3/2)
```

Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.108$, Rules used = {1121, 61, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{d+ex}\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)}dx$$

$$\Big\downarrow \text{1121}$$

$$\int \frac{1}{(d+ex)^{3/2}(ae+cdx)}dx$$

$$\Big\downarrow \text{61}$$

$$\frac{cd\int \frac{1}{(ae+cdx)\sqrt{d+ex}}dx}{cd^2-ae^2}+\frac{2}{\sqrt{d+ex}\left(cd^2-ae^2\right)}$$

$$\Big\downarrow \text{73}$$

$$\frac{2cd\int \frac{1}{-\frac{cd^2}{e}+\frac{c(d+ex)d}{e}+ae}d\sqrt{d+ex}}{e\left(cd^2-ae^2\right)}+\frac{2}{\sqrt{d+ex}\left(cd^2-ae^2\right)}$$

$$\Big\downarrow \text{221}$$

$$\frac{2}{\sqrt{d+ex}\left(cd^2-ae^2\right)}-\frac{2\sqrt{c}\sqrt{d}\,\text{arctanh}\left(\frac{\sqrt{c}\sqrt{d}\sqrt{d+ex}}{\sqrt{cd^2-ae^2}}\right)}{\left(cd^2-ae^2\right)^{3/2}}$$

input
```
Int[1/(Sqrt[d + e*x]*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)),x]
```

output
```
2/((c*d^2 - a*e^2)*Sqrt[d + e*x]) - (2*Sqrt[c]*Sqrt[d]*ArcTanh[(Sqrt[c]*Sq
rt[d]*Sqrt[d + e*x])/Sqrt[c*d^2 - a*e^2]])/(c*d^2 - a*e^2)^(3/2)
```

Defintions of rubi rules used

rule 61
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && LtQ[m, -1] &&  !(LtQ[n, -1] && (EqQ[a, 0
] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && IntLinearQ[a, b, c, d
, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 1121
```
Int[((d_) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Int[ExpandIntegrand[(d + e*x)^(m + p)*(a/d + (c/e)*x)^p, x], x]
/; FreeQ[{a, b, c, d, e, m, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && (Int
egerQ[p] || (GtQ[a, 0] && GtQ[d, 0] && LtQ[c, 0]))
```

Maple [A] (verified)

Time = 1.80 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.84

method	result	size
pseudoelliptic	$\frac{-\frac{2}{\sqrt{ex+d}}-\frac{2cd\arctan\left(\frac{cd\sqrt{ex+d}}{\sqrt{cd\left(a\,e^2-c\,d^2\right)}}\right)}{\sqrt{cd\left(a\,e^2-c\,d^2\right)}}}{a\,e^2-c\,d^2}$	76
derivativedivides	$-\frac{2}{\left(a\,e^2-c\,d^2\right)\sqrt{ex+d}}-\frac{2cd\arctan\left(\frac{cd\sqrt{ex+d}}{\sqrt{cd\left(a\,e^2-c\,d^2\right)}}\right)}{\left(a\,e^2-c\,d^2\right)\sqrt{cd\left(a\,e^2-c\,d^2\right)}}$	88
default	$-\frac{2}{\left(a\,e^2-c\,d^2\right)\sqrt{ex+d}}-\frac{2cd\arctan\left(\frac{cd\sqrt{ex+d}}{\sqrt{cd\left(a\,e^2-c\,d^2\right)}}\right)}{\left(a\,e^2-c\,d^2\right)\sqrt{cd\left(a\,e^2-c\,d^2\right)}}$	88

input
```
int(1/(e*x+d)^(1/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e),x,method=_RETURNVERB
OSE)
```

output
```
2/(a*e^2-c*d^2)*(-1/(e*x+d)^(1/2)-c*d/(c*d*(a*e^2-c*d^2))^(1/2)*arctan(c*d
*(e*x+d)^(1/2)/(c*d*(a*e^2-c*d^2))^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 233, normalized size of antiderivative = 2.56

$$\int \frac{1}{\sqrt{d+ex}\,(ade+(cd^2+ae^2)\,x+cdex^2)}\,dx$$
$$=\left[-\frac{(ex+d)\sqrt{\frac{cd}{cd^2-ae^2}}\log\left(\frac{cdex+2\,cd^2-ae^2+2\,(cd^2-ae^2)\sqrt{ex+d}\sqrt{\frac{cd}{cd^2-ae^2}}}{cdx+ae}\right)-2\,\sqrt{ex+d}}{cd^3-ade^2+(cd^2e-ae^3)x},\frac{2\left((ex+d)\sqrt{-\frac{cd}{cd^2-ae^2}}\,\mathrm{a}\right.}{cd^3-}\right.$$

input
```
integrate(1/(e*x+d)^(1/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2),x, algorithm="
fricas")
```

output
```
[-((e*x + d)*sqrt(c*d/(c*d^2 - a*e^2))*log((c*d*e*x + 2*c*d^2 - a*e^2 + 2*
(c*d^2 - a*e^2)*sqrt(e*x + d)*sqrt(c*d/(c*d^2 - a*e^2)))/(c*d*x + a*e)) -
2*sqrt(e*x + d))/(c*d^3 - a*d*e^2 + (c*d^2*e - a*e^3)*x), 2*((e*x + d)*sqr
t(-c*d/(c*d^2 - a*e^2))*arctan(sqrt(e*x + d)*sqrt(-c*d/(c*d^2 - a*e^2))) +
 sqrt(e*x + d))/(c*d^3 - a*d*e^2 + (c*d^2*e - a*e^3)*x)]
```

Sympy [A] (verification not implemented)

Time = 2.02 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.01

$$\int \frac{1}{\sqrt{d+ex}\,(ade+(cd^2+ae^2)\,x+cdex^2)}\,dx = \begin{cases} \frac{2\left(-\frac{e}{\sqrt{d+ex}\left(ae^2-cd^2\right)}-\frac{e\,\mathrm{atan}\left(\frac{\sqrt{d+ex}}{\sqrt{\frac{ae^2-cd^2}{cd}}}\right)}{\sqrt{\frac{ae^2-cd^2}{cd}}\left(ae^2-cd^2\right)}\right)}{e} & \text{for } e\neq 0 \\ \frac{\log\left(x\right)}{cd^{\frac{5}{2}}} & \text{otherwise} \end{cases}$$

input
```
integrate(1/(e*x+d)**(1/2)/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2),x)
```

output
```
Piecewise((2*(-e/(sqrt(d + e*x)*(a*e**2 - c*d**2)) - e*atan(sqrt(d + e*x)/
sqrt((a*e**2 - c*d**2)/(c*d)))/(sqrt((a*e**2 - c*d**2)/(c*d))*(a*e**2 - c*
d**2)))/e, Ne(e, 0)), (log(x)/(c*d**(5/2)), True))
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{\sqrt{d+ex}\,(ade+(cd^2+ae^2)\,x+cdex^2)}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(1/(e*x+d)^(1/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2),x, algorithm="
maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(a*e^2-c*d^2>0)', see `assume?` f
or more de
```

Giac [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.98

$$\int \frac{1}{\sqrt{d+ex}\,(ade+(cd^2+ae^2)\,x+cdex^2)}\,dx = \frac{2\,cd\arctan\left(\frac{\sqrt{ex+d}cd}{\sqrt{-c^2d^3+acde^2}}\right)}{\sqrt{-c^2d^3+acde^2}(cd^2-ae^2)}+\frac{2}{(cd^2-ae^2)\sqrt{ex+d}}$$

input
```
integrate(1/(e*x+d)^(1/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2),x, algorithm="
giac")
```

output
```
2*c*d*arctan(sqrt(e*x + d)*c*d/sqrt(-c^2*d^3 + a*c*d*e^2))/(sqrt(-c^2*d^3
+ a*c*d*e^2)*(c*d^2 - a*e^2)) + 2/((c*d^2 - a*e^2)*sqrt(e*x + d))
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.82

$$\int \frac{1}{\sqrt{d+ex}\,(ade+(cd^2+ae^2)\,x+cdex^2)}\,dx = -\frac{2}{(a\,e^2-c\,d^2)\,\sqrt{d+e\,x}}-\frac{2\,\sqrt{c}\,\sqrt{d}\,\mathrm{atan}\left(\frac{\sqrt{c}\,\sqrt{d}\,\sqrt{d+e\,x}}{\sqrt{a\,e^2-c\,d^2}}\right)}{(a\,e^2-c\,d^2)^{3/2}}$$

input
```
int(1/((d + e*x)^(1/2)*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)),x)
```

output
```
- 2/((a*e^2 - c*d^2)*(d + e*x)^(1/2)) - (2*c^(1/2)*d^(1/2)*atan((c^(1/2)*d
^(1/2)*(d + e*x)^(1/2))/(a*e^2 - c*d^2)^(1/2)))/(a*e^2 - c*d^2)^(3/2)
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.18

$$\int \frac{1}{\sqrt{d+ex}\,(ade+(cd^2+ae^2)\,x+cdex^2)}\,dx$$
$$= \frac{-2\sqrt{d}\,\sqrt{c}\,\sqrt{ex+d}\,\sqrt{a\,e^2-c\,d^2}\,atan\left(\frac{\sqrt{ex+d}\,cd}{\sqrt{d}\,\sqrt{c}\,\sqrt{a\,e^2-c\,d^2}}\right)-2a\,e^2+2c\,d^2}{\sqrt{ex+d}\,\left(a^2e^4-2ac\,d^2e^2+c^2d^4\right)}$$

input
```
int(1/(e*x+d)^(1/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2),x)
```

output
```
(2*( - sqrt(d)*sqrt(c)*sqrt(d + e*x)*sqrt(a*e**2 - c*d**2)*atan((sqrt(d +
e*x)*c*d)/(sqrt(d)*sqrt(c)*sqrt(a*e**2 - c*d**2))) - a*e**2 + c*d**2))/(sq
rt(d + e*x)*(a**2*e**4 - 2*a*c*d**2*e**2 + c**2*d**4))
```

3.179 $\int \frac{1}{(d+ex)^{3/2}\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)}\,dx$

Optimal result

Integrand size = 37, antiderivative size = 120

$$\int \frac{1}{(d+ex)^{3/2}\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)}\,dx = \frac{2}{3\left(cd^2-ae^2\right)(d+ex)^{3/2}} + \frac{2cd}{\left(cd^2-ae^2\right)^2\sqrt{d+ex}} - \frac{2c^{3/2}d^{3/2}\text{arctanh}\left(\frac{\sqrt{c}\sqrt{d}\sqrt{d+ex}}{\sqrt{cd^2-ae^2}}\right)}{\left(cd^2-ae^2\right)^{5/2}}$$

output
```
2/3/(-a*e^2+c*d^2)/(e*x+d)^(3/2)+2*c*d/(-a*e^2+c*d^2)^2/(e*x+d)^(1/2)-2*c^
(3/2)*d^(3/2)*arctanh(c^(1/2)*d^(1/2)*(e*x+d)^(1/2)/(-a*e^2+c*d^2)^(1/2))/
(-a*e^2+c*d^2)^(5/2)
```

Mathematica [A] (verified)

Time = 0.26 (sec) , antiderivative size = 111, normalized size of antiderivative = 0.92

$$\int \frac{1}{(d+ex)^{3/2}\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)}\,dx = \frac{2(-ae^2+cd(4d+3ex))}{3\left(cd^2-ae^2\right)^2(d+ex)^{3/2}} + \frac{2c^{3/2}d^{3/2}\arctan\left(\frac{\sqrt{c}\sqrt{d}\sqrt{d+ex}}{\sqrt{-cd^2+ae^2}}\right)}{\left(-cd^2+ae^2\right)^{5/2}}$$

input
```
Integrate[1/((d + e*x)^(3/2)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)),x]
```

output
```
(2*(-(a*e^2) + c*d*(4*d + 3*e*x)))/(3*(c*d^2 - a*e^2)^2*(d + e*x)^(3/2)) +
 (2*c^(3/2)*d^(3/2)*ArcTan[(Sqrt[c]*Sqrt[d]*Sqrt[d + e*x])/Sqrt[-(c*d^2) +
 a*e^2]])/(-(c*d^2) + a*e^2)^(5/2)
```

Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 136, normalized size of antiderivative = 1.13, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.135$, Rules used = {1121, 61, 61, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(d+ex)^{3/2}\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)}dx$$

$$\Big\downarrow \text{ 1121}$$

$$\int \frac{1}{(d+ex)^{5/2}(ae+cdx)}dx$$

$$\Big\downarrow \text{ 61}$$

$$\frac{cd\int \frac{1}{(ae+cdx)(d+ex)^{3/2}}dx}{cd^2-ae^2}+\frac{2}{3(d+ex)^{3/2}\left(cd^2-ae^2\right)}$$

$$\Big\downarrow \text{ 61}$$

$$\frac{cd\left(\frac{cd\int \frac{1}{(ae+cdx)\sqrt{d+ex}}dx}{cd^2-ae^2}+\frac{2}{\sqrt{d+ex}(cd^2-ae^2)}\right)}{cd^2-ae^2}+\frac{2}{3(d+ex)^{3/2}\left(cd^2-ae^2\right)}$$

$$\Big\downarrow \text{ 73}$$

$$\frac{cd\left(\frac{2cd\int \frac{1}{-\frac{cd^2}{e}+\frac{c(d+ex)d}{e}+ae}d\sqrt{d+ex}}{e(cd^2-ae^2)}+\frac{2}{\sqrt{d+ex}(cd^2-ae^2)}\right)}{cd^2-ae^2}+\frac{2}{3(d+ex)^{3/2}\left(cd^2-ae^2\right)}$$

$$\Big\downarrow \text{ 221}$$

$$\frac{cd\left(\frac{2}{\sqrt{d+ex}(cd^2-ae^2)}-\frac{2\sqrt{c}\sqrt{d}\operatorname{arctanh}\left(\frac{\sqrt{c}\sqrt{d}\sqrt{d+ex}}{\sqrt{cd^2-ae^2}}\right)}{(cd^2-ae^2)^{3/2}}\right)}{cd^2-ae^2}+\frac{2}{3(d+ex)^{3/2}(cd^2-ae^2)}$$

input
```
Int[1/((d + e*x)^(3/2)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)),x]
```

output
```
2/(3*(c*d^2 - a*e^2)*(d + e*x)^(3/2)) + (c*d*(2/((c*d^2 - a*e^2)*Sqrt[d +
e*x]) - (2*Sqrt[c]*Sqrt[d]*ArcTanh[(Sqrt[c]*Sqrt[d]*Sqrt[d + e*x])/Sqrt[c*
d^2 - a*e^2]])/(c*d^2 - a*e^2)^(3/2)))/(c*d^2 - a*e^2)
```

Defintions of rubi rules used

rule 61
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && LtQ[m, -1] &&  !(LtQ[n, -1] && (EqQ[a, 0
] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && IntLinearQ[a, b, c, d
, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 1121
```
Int[((d_) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Int[ExpandIntegrand[(d + e*x)^(m + p)*(a/d + (c/e)*x)^p, x], x]
/; FreeQ[{a, b, c, d, e, m, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && (Int
egerQ[p] || (GtQ[a, 0] && GtQ[d, 0] && LtQ[c, 0]))
```

Maple [A] (verified)

Time = 1.94 (sec) , antiderivative size = 117, normalized size of antiderivative = 0.98

method	result	size
derivativedivides	$\frac{2c^2d^2\arctan\left(\frac{cd\sqrt{ex+d}}{\sqrt{cd\left(a\,e^2-c\,d^2\right)}}\right)}{\left(a\,e^2-c\,d^2\right)^2\sqrt{cd(a\,e^2-c\,d^2)}}-\frac{2}{3(a\,e^2-c\,d^2)(ex+d)^{\frac{3}{2}}}+\frac{2cd}{\left(a\,e^2-c\,d^2\right)^2\sqrt{ex+d}}$	117
default	$\frac{2c^2d^2\arctan\left(\frac{cd\sqrt{ex+d}}{\sqrt{cd\left(a\,e^2-c\,d^2\right)}}\right)}{\left(a\,e^2-c\,d^2\right)^2\sqrt{cd(a\,e^2-c\,d^2)}}-\frac{2}{3(a\,e^2-c\,d^2)(ex+d)^{\frac{3}{2}}}+\frac{2cd}{\left(a\,e^2-c\,d^2\right)^2\sqrt{ex+d}}$	117
pseudoelliptic	$\frac{2c^2d^2\arctan\left(\frac{cd\sqrt{ex+d}}{\sqrt{cd\left(a\,e^2-c\,d^2\right)}}\right)}{\left(a\,e^2-c\,d^2\right)^2\sqrt{cd(a\,e^2-c\,d^2)}}-\frac{2}{(ex+d)^{\frac{3}{2}}\left(3a\,e^2-3c\,d^2\right)}+\frac{2cd}{\left(a\,e^2-c\,d^2\right)^2\sqrt{ex+d}}$	118

input
```
int(1/(e*x+d)^(3/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e),x,method=_RETURNVERB
OSE)
```

output
```
2*c^2*d^2/(a*e^2-c*d^2)^2/(c*d*(a*e^2-c*d^2))^(1/2)*arctan(c*d*(e*x+d)^(1/
2)/(c*d*(a*e^2-c*d^2))^(1/2))-2/3/(a*e^2-c*d^2)/(e*x+d)^(3/2)+2/(a*e^2-c*d
^2)^2*c*d/(e*x+d)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 434, normalized size of antiderivative = 3.62

$$\int \frac{1}{(d+ex)^{3/2}\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)}\,dx=\left[\frac{3\left(cde^2x^2+2\,cd^2ex+cd^3\right)\sqrt{\frac{cd}{cd^2-ae^2}}\log\left(\frac{cdex+2\,cd^2-ae^2\cdots}{\cdots}\right.}{3\left(c^2d^6-2\,acd^4e^2+a^2d^2e^4+\left(c^2d^4e^2-2\,a\cdots\right.\right.}\right.$$

input
```
integrate(1/(e*x+d)^(3/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2),x, algorithm="
fricas")
```

output
```
[1/3*(3*(c*d*e^2*x^2 + 2*c*d^2*e*x + c*d^3)*sqrt(c*d/(c*d^2 - a*e^2))*log(
(c*d*e*x + 2*c*d^2 - a*e^2 - 2*(c*d^2 - a*e^2)*sqrt(e*x + d)*sqrt(c*d/(c*d
^2 - a*e^2)))/(c*d*x + a*e)) + 2*(3*c*d*e*x + 4*c*d^2 - a*e^2)*sqrt(e*x +
d))/(c^2*d^6 - 2*a*c*d^4*e^2 + a^2*d^2*e^4 + (c^2*d^4*e^2 - 2*a*c*d^2*e^4
+ a^2*e^6)*x^2 + 2*(c^2*d^5*e - 2*a*c*d^3*e^3 + a^2*d*e^5)*x), 2/3*(3*(c*d
*e^2*x^2 + 2*c*d^2*e*x + c*d^3)*sqrt(-c*d/(c*d^2 - a*e^2))*arctan(sqrt(e*x
 + d)*sqrt(-c*d/(c*d^2 - a*e^2))) + (3*c*d*e*x + 4*c*d^2 - a*e^2)*sqrt(e*x
 + d))/(c^2*d^6 - 2*a*c*d^4*e^2 + a^2*d^2*e^4 + (c^2*d^4*e^2 - 2*a*c*d^2*e
^4 + a^2*e^6)*x^2 + 2*(c^2*d^5*e - 2*a*c*d^3*e^3 + a^2*d*e^5)*x)]
```

Sympy [A] (verification not implemented)

Time = 2.00 (sec) , antiderivative size = 124, normalized size of antiderivative = 1.03

$$\int \frac{1}{(d+ex)^{3/2}\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)}\,dx = \begin{cases} \frac{2\left(\frac{cde}{\sqrt{d+ex}\left(ae^2-cd^2\right)^2}+\frac{cde\operatorname{atan}\left(\frac{\sqrt{d+ex}}{\sqrt{\frac{ae^2-cd^2}{cd}}}\right)}{\sqrt{\frac{ae^2-cd^2}{cd}}\left(ae^2-cd^2\right)^2}-\frac{e}{3(d+ex)^{\frac{3}{2}}\left(ae^2-cd^2\right)}\right)}{e} \\ \frac{\log(x)}{cd^{\frac{7}{2}}} \end{cases}$$

input
```
integrate(1/(e*x+d)**(3/2)/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2),x)
```

output
```
Piecewise((2*(c*d*e/(sqrt(d + e*x)*(a*e**2 - c*d**2)**2) + c*d*e*atan(sqrt
(d + e*x)/sqrt((a*e**2 - c*d**2)/(c*d)))/(sqrt((a*e**2 - c*d**2)/(c*d))*(a
*e**2 - c*d**2)**2) - e/(3*(d + e*x)**(3/2)*(a*e**2 - c*d**2)))/e, Ne(e, 0
)), (log(x)/(c*d**(7/2)), True))
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{(d+ex)^{3/2}\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(1/(e*x+d)^(3/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2),x, algorithm="
maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(a*e^2-c*d^2>0)', see `assume?` f
or more de
```

Giac [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 140, normalized size of antiderivative = 1.17

$$\int \frac{1}{(d+ex)^{3/2}\,(ade+(cd^2+ae^2)\,x+cdex^2)}\,dx = \frac{2\,c^2d^2\arctan\left(\frac{\sqrt{ex+d}cd}{\sqrt{-c^2d^3+acde^2}}\right)}{(c^2d^4-2\,acd^2e^2+a^2e^4)\sqrt{-c^2d^3+acde^2}} + \frac{2\,(3\,(ex+d)cd+cd^2-ae^2)}{3\,(c^2d^4-2\,acd^2e^2+a^2e^4)(ex+d)^{\frac{3}{2}}}$$

input
```
integrate(1/(e*x+d)^(3/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2),x, algorithm="
giac")
```

output
```
2*c^2*d^2*arctan(sqrt(e*x + d)*c*d/sqrt(-c^2*d^3 + a*c*d*e^2))/((c^2*d^4 -
 2*a*c*d^2*e^2 + a^2*e^4)*sqrt(-c^2*d^3 + a*c*d*e^2)) + 2/3*(3*(e*x + d)*c
*d + c*d^2 - a*e^2)/((c^2*d^4 - 2*a*c*d^2*e^2 + a^2*e^4)*(e*x + d)^(3/2))
```

Mupad [B] (verification not implemented)

Time = 5.24 (sec) , antiderivative size = 127, normalized size of antiderivative = 1.06

$$\int \frac{1}{(d+ex)^{3/2}\,(ade+(cd^2+ae^2)\,x+cdex^2)}\,dx = \frac{2\,c^{3/2}\,d^{3/2}\,\mathrm{atan}\left(\frac{\sqrt{c}\,\sqrt{d}\,\sqrt{d+e\,x}\,\left(a^2\,e^4-2\,a\,c\,d^2\,e^2+c^2\,d^4\right)}{{\left(a\,e^2-c\,d^2\right)}^{5/2}}\right)}{{\left(a\,e^2-c\,d^2\right)}^{5/2}} - \frac{\frac{2}{3\,\left(a\,e^2-c\,d^2\right)}-\frac{2\,c\,d\,\left(d+e\,x\right)}{{\left(a\,e^2-c\,d^2\right)}^2}}{{\left(d+e\,x\right)}^{3/2}}$$

input
```
int(1/((d + e*x)^(3/2)*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)),x)
```

output
```
(2*c^(3/2)*d^(3/2)*atan((c^(1/2)*d^(1/2)*(d + e*x)^(1/2)*(a^2*e^4 + c^2*d^
4 - 2*a*c*d^2*e^2))/(a*e^2 - c*d^2)^(5/2)))/(a*e^2 - c*d^2)^(5/2) - (2/(3*
(a*e^2 - c*d^2)) - (2*c*d*(d + e*x))/(a*e^2 - c*d^2)^2)/(d + e*x)^(3/2)
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 267, normalized size of antiderivative = 2.22

$$\int \frac{1}{(d+ex)^{3/2}\,(ade+(cd^2+ae^2)\,x+cdex^2)}\,dx = \frac{2\sqrt{d}\,\sqrt{c}\,\sqrt{ex+d}\,\sqrt{a\,e^2-c\,d^2}\,atan\left(\frac{\sqrt{ex+d}\,cd}{\sqrt{d}\,\sqrt{c}\,\sqrt{a\,e^2-c\,d^2}}\right)c\,d^2}{\sqrt{ex+d}\,\left(a^3e^7x-3a\right.}$$

input
```
int(1/(e*x+d)^(3/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2),x)
```

output
```
(2*(3*sqrt(d)*sqrt(c)*sqrt(d + e*x)*sqrt(a*e**2 - c*d**2)*atan((sqrt(d + e
*x)*c*d)/(sqrt(d)*sqrt(c)*sqrt(a*e**2 - c*d**2)))*c*d**2 + 3*sqrt(d)*sqrt(
c)*sqrt(d + e*x)*sqrt(a*e**2 - c*d**2)*atan((sqrt(d + e*x)*c*d)/(sqrt(d)*s
qrt(c)*sqrt(a*e**2 - c*d**2)))*c*d*e*x - a**2*e**4 + 5*a*c*d**2*e**2 + 3*a
*c*d*e**3*x - 4*c**2*d**4 - 3*c**2*d**3*e*x))/(3*sqrt(d + e*x)*(a**3*d*e**
6 + a**3*e**7*x - 3*a**2*c*d**3*e**4 - 3*a**2*c*d**2*e**5*x + 3*a*c**2*d**
5*e**2 + 3*a*c**2*d**4*e**3*x - c**3*d**7 - c**3*d**6*e*x))
```

3.180 $\int \frac{1}{(d+ex)^{5/2}\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)}\,dx$

Optimal result

Integrand size = 37, antiderivative size = 153

$$\int \frac{1}{(d+ex)^{5/2}\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)}\,dx=\frac{2}{5\left(cd^2-ae^2\right)(d+ex)^{5/2}}+\frac{2cd}{3\left(cd^2-ae^2\right)^2(d+ex)^{3/2}}+\frac{2c^2d^2}{\left(cd^2-ae^2\right)^3\sqrt{d+ex}}-\frac{2c^{5/2}d^{5/2}\text{arctanh}\left(\frac{\sqrt{c}\sqrt{d}\sqrt{d+ex}}{\sqrt{cd^2-ae^2}}\right)}{\left(cd^2-ae^2\right)^{7/2}}$$

output
```
2/5/(-a*e^2+c*d^2)/(e*x+d)^(5/2)+2/3*c*d/(-a*e^2+c*d^2)^2/(e*x+d)^(3/2)+2*
c^2*d^2/(-a*e^2+c*d^2)^3/(e*x+d)^(1/2)-2*c^(5/2)*d^(5/2)*arctanh(c^(1/2)*d
^(1/2)*(e*x+d)^(1/2)/(-a*e^2+c*d^2)^(1/2))/(-a*e^2+c*d^2)^(7/2)
```

Mathematica [A] (verified)

Time = 0.28 (sec) , antiderivative size = 145, normalized size of antiderivative = 0.95

$$\int \frac{1}{(d+ex)^{5/2}\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)}\,dx=\frac{6a^2e^4-2acde^2(11d+5ex)+2c^2d^2(23d^2+35dex+15e}{15\left(cd^2-ae^2\right)^3(d+ex)^{5/2}}-\frac{2c^{5/2}d^{5/2}\arctan\left(\frac{\sqrt{c}\sqrt{d}\sqrt{d+ex}}{\sqrt{-cd^2+ae^2}}\right)}{\left(-cd^2+ae^2\right)^{7/2}}$$

input
```
Integrate[1/((d + e*x)^(5/2)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)),x]
```

output
```
(6*a^2*e^4 - 2*a*c*d*e^2*(11*d + 5*e*x) + 2*c^2*d^2*(23*d^2 + 35*d*e*x + 1
5*e^2*x^2))/(15*(c*d^2 - a*e^2)^3*(d + e*x)^(5/2)) - (2*c^(5/2)*d^(5/2)*Ar
cTan[(Sqrt[c]*Sqrt[d]*Sqrt[d + e*x])/Sqrt[-(c*d^2) + a*e^2]])/(-(c*d^2) +
a*e^2)^(7/2)
```

Rubi [A] (verified)

Time = 0.48 (sec) , antiderivative size = 181, normalized size of antiderivative = 1.18, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.162$, Rules used = {1121, 61, 61, 61, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(d+ex)^{5/2} \left(x \left(ae^2+cd^2\right)+ade+cdex^2\right)} \, dx$$

$$\Big\downarrow \text{1121}$$

$$\int \frac{1}{(d+ex)^{7/2}(ae+cdx)}dx$$

$$\Big\downarrow \text{61}$$

$$\frac{cd\int \frac{1}{(ae+cdx)(d+ex)^{5/2}}dx}{cd^2-ae^2}+\frac{2}{5(d+ex)^{5/2} \left(cd^2-ae^2\right)}$$

$$\Big\downarrow \text{61}$$

$$\frac{cd\left(\frac{cd\int \frac{1}{(ae+cdx)(d+ex)^{3/2}}dx}{cd^2-ae^2}+\frac{2}{3(d+ex)^{3/2}(cd^2-ae^2)}\right)}{cd^2-ae^2}+\frac{2}{5(d+ex)^{5/2} \left(cd^2-ae^2\right)}$$

$$\Big\downarrow \text{61}$$

$$\frac{cd\left(\frac{cd\left(\frac{cd\int \frac{1}{(ae+cdx)\sqrt{d+ex}}dx}{cd^2-ae^2}+\frac{2}{\sqrt{d+ex}\left(cd^2-ae^2\right)}\right)}{cd^2-ae^2}+\frac{2}{3(d+ex)^{3/2}(cd^2-ae^2)}\right)}{cd^2-ae^2}+\frac{2}{5(d+ex)^{5/2} \left(cd^2-ae^2\right)}$$

$$\Big\downarrow \; 73$$

$$\frac{cd\left(\frac{cd\left(\frac{2cd\int \frac{1}{-\frac{cd^2}{e}+\frac{c(d+ex)d}{e}+ae}d\sqrt{d+ex}}{e(cd^2-ae^2)}+\frac{2}{\sqrt{d+ex}(cd^2-ae^2)}\right)}{cd^2-ae^2}+\frac{2}{3(d+ex)^{3/2}(cd^2-ae^2)}\right)}{cd^2-ae^2}+\frac{2}{5(d+ex)^{5/2}(cd^2-ae^2)}$$

$$\Big\downarrow \; 221$$

$$\frac{cd\left(\frac{cd\left(\frac{2}{\sqrt{d+ex}(cd^2-ae^2)}-\frac{2\sqrt{c}\sqrt{d}\text{arctanh}\left(\frac{\sqrt{c}\sqrt{d}\sqrt{d+ex}}{\sqrt{cd^2-ae^2}}\right)}{(cd^2-ae^2)^{3/2}}\right)}{cd^2-ae^2}+\frac{2}{3(d+ex)^{3/2}(cd^2-ae^2)}\right)}{cd^2-ae^2}+\frac{2}{5(d+ex)^{5/2}(cd^2-ae^2)}$$

input
```
Int[1/((d + e*x)^(5/2)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)),x]
```

output
```
2/(5*(c*d^2 - a*e^2)*(d + e*x)^(5/2)) + (c*d*(2/(3*(c*d^2 - a*e^2)*(d + e*
x)^(3/2)) + (c*d*(2/((c*d^2 - a*e^2)*Sqrt[d + e*x]) - (2*Sqrt[c]*Sqrt[d]*A
rcTanh[(Sqrt[c]*Sqrt[d]*Sqrt[d + e*x])/Sqrt[c*d^2 - a*e^2]])/(c*d^2 - a*e^
2)^(3/2)))/(c*d^2 - a*e^2)))/(c*d^2 - a*e^2)
```

Defintions of rubi rules used

rule 61
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && LtQ[m, -1] &&  !(LtQ[n, -1] && (EqQ[a, 0
] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && IntLinearQ[a, b, c, d
, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 1121
```
Int[((d_) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Int[ExpandIntegrand[(d + e*x)^(m + p)*(a/d + (c/e)*x)^p, x], x]
/; FreeQ[{a, b, c, d, e, m, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && (Int
egerQ[p] || (GtQ[a, 0] && GtQ[d, 0] && LtQ[c, 0]))
```

Maple [A] (verified)

Time = 1.92 (sec) , antiderivative size = 146, normalized size of antiderivative = 0.95

method	result
derivativedivides	$-\frac{2}{5\left(a \,e^{2}-c \,d^{2}\right) \left(e x +d \right)^{\frac{5}{2}}}-\frac{2 c^{2} d^{2}}{\left(a \,e^{2}-c \,d^{2}\right)^{3} \sqrt{e x +d}}+\frac{2 c d}{3 \left(a \,e^{2}-c \,d^{2}\right)^{2} \left(e x +d \right)^{\frac{3}{2}}}-\frac{2 c^{3} d^{3} \arctan \left(\frac{c d \sqrt{e x +d}}{\sqrt{c d \left(a \,e^{2}-c \,d^{2}\right)}}\right)}{\left(a \,e^{2}-c \,d^{2}\right)^{3} \sqrt{c d \left(a \,e^{2}-c \,d^{2}\right)}}$
default	$-\frac{2}{5\left(a \,e^{2}-c \,d^{2}\right) \left(e x +d \right)^{\frac{5}{2}}}-\frac{2 c^{2} d^{2}}{\left(a \,e^{2}-c \,d^{2}\right)^{3} \sqrt{e x +d}}+\frac{2 c d}{3 \left(a \,e^{2}-c \,d^{2}\right)^{2} \left(e x +d \right)^{\frac{3}{2}}}-\frac{2 c^{3} d^{3} \arctan \left(\frac{c d \sqrt{e x +d}}{\sqrt{c d \left(a \,e^{2}-c \,d^{2}\right)}}\right)}{\left(a \,e^{2}-c \,d^{2}\right)^{3} \sqrt{c d \left(a \,e^{2}-c \,d^{2}\right)}}$
pseudoelliptic	$-\frac{2 \left(5 c^{3} d^{3} \arctan \left(\frac{c d \sqrt{e x +d}}{\sqrt{c d \left(a \,e^{2}-c \,d^{2}\right)}}\right) \left(e x +d \right)^{\frac{5}{2}}+\sqrt{c d \left(a \,e^{2}-c \,d^{2}\right)}\, \left(\left(5 d^{2} e^{2} x^{2}+\frac{35}{3} d^{3} e x +\frac{23}{3} d^{4}\right) c^{2}-\frac{11 e^{2} \left(\frac{5 e x}{11}+d \right) a d c}{3}+c \right.\right.}{5 \left(e x +d \right)^{\frac{5}{2}} \sqrt{c d \left(a \,e^{2}-c \,d^{2}\right)}\, \left(a \,e^{2}-c \,d^{2}\right)^{3}}$

input
```
int(1/(e*x+d)^(5/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e),x,method=_RETURNVERB
OSE)
```

output
```
-2/5/(a*e^2-c*d^2)/(e*x+d)^(5/2)-2/(a*e^2-c*d^2)^3*c^2*d^2/(e*x+d)^(1/2)+2
/3/(a*e^2-c*d^2)^2*c*d/(e*x+d)^(3/2)-2*c^3*d^3/(a*e^2-c*d^2)^3/(c*d*(a*e^2
-c*d^2))^(1/2)*arctan(c*d*(e*x+d)^(1/2)/(c*d*(a*e^2-c*d^2))^(1/2))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 357 vs. $2(129) = 258$.

Time = 0.10 (sec) , antiderivative size = 756, normalized size of antiderivative = 4.94

$$\int \frac{1}{(d+ex)^{5/2}\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)}\,dx = \left[-\frac{15\left(c^2d^2e^3x^3+3\,c^2d^3e^2x^2+3\,c^2d^4ex+c^2d^5\right)\sqrt{\frac{cd}{cd^2-}}}{15\left(c^3d^9-3\,ac^2d^7e^2+3\,a^2cd^5e^4-a^3d^3e^6+\left(c^3d^6e^3\right.\right.}\right.$$

input
```
integrate(1/(e*x+d)^(5/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2),x, algorithm="
fricas")
```

output
```
[-1/15*(15*(c^2*d^2*e^3*x^3 + 3*c^2*d^3*e^2*x^2 + 3*c^2*d^4*e*x + c^2*d^5)
*sqrt(c*d/(c*d^2 - a*e^2))*log((c*d*e*x + 2*c*d^2 - a*e^2 + 2*(c*d^2 - a*e
^2)*sqrt(e*x + d)*sqrt(c*d/(c*d^2 - a*e^2)))/(c*d*x + a*e)) - 2*(15*c^2*d^
2*e^2*x^2 + 23*c^2*d^4 - 11*a*c*d^2*e^2 + 3*a^2*e^4 + 5*(7*c^2*d^3*e - a*c
*d*e^3)*x)*sqrt(e*x + d))/(c^3*d^9 - 3*a*c^2*d^7*e^2 + 3*a^2*c*d^5*e^4 - a
^3*d^3*e^6 + (c^3*d^6*e^3 - 3*a*c^2*d^4*e^5 + 3*a^2*c*d^2*e^7 - a^3*e^9)*x
^3 + 3*(c^3*d^7*e^2 - 3*a*c^2*d^5*e^4 + 3*a^2*c*d^3*e^6 - a^3*d*e^8)*x^2 +
 3*(c^3*d^8*e - 3*a*c^2*d^6*e^3 + 3*a^2*c*d^4*e^5 - a^3*d^2*e^7)*x), 2/15*
(15*(c^2*d^2*e^3*x^3 + 3*c^2*d^3*e^2*x^2 + 3*c^2*d^4*e*x + c^2*d^5)*sqrt(-
c*d/(c*d^2 - a*e^2))*arctan(sqrt(e*x + d)*sqrt(-c*d/(c*d^2 - a*e^2))) + (1
5*c^2*d^2*e^2*x^2 + 23*c^2*d^4 - 11*a*c*d^2*e^2 + 3*a^2*e^4 + 5*(7*c^2*d^3
*e - a*c*d*e^3)*x)*sqrt(e*x + d))/(c^3*d^9 - 3*a*c^2*d^7*e^2 + 3*a^2*c*d^5
*e^4 - a^3*d^3*e^6 + (c^3*d^6*e^3 - 3*a*c^2*d^4*e^5 + 3*a^2*c*d^2*e^7 - a^
3*e^9)*x^3 + 3*(c^3*d^7*e^2 - 3*a*c^2*d^5*e^4 + 3*a^2*c*d^3*e^6 - a^3*d*e^
8)*x^2 + 3*(c^3*d^8*e - 3*a*c^2*d^6*e^3 + 3*a^2*c*d^4*e^5 - a^3*d^2*e^7)*x
)]
```

Sympy [A] (verification not implemented)

Time = 13.14 (sec) , antiderivative size = 160, normalized size of antiderivative = 1.05

$$\int \frac{1}{(d+ex)^{5/2}\left(ade+(cd^2+ae^2)x+cdex^2\right)}\,dx = \begin{cases} \frac{2\left(-\frac{c^2d^2e}{\sqrt{d+ex}\left(ae^2-cd^2\right)^3}-\frac{c^2d^2e\,\mathrm{atan}\left(\frac{\sqrt{d+ex}}{\sqrt{\frac{ae^2-cd^2}{cd}}}\right)}{\sqrt{\frac{ae^2-cd^2}{cd}}\left(ae^2-cd^2\right)^3}+\frac{cde}{3(d+ex)^{\frac{3}{2}}\left(ae^2-cd^2\right)^2}\right.}{e} \\ \frac{\log(x)}{cd^{\frac{9}{2}}} \end{cases}$$

input
```
integrate(1/(e*x+d)**(5/2)/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2),x)
```

output
```
Piecewise((2*(-c**2*d**2*e/(sqrt(d + e*x)*(a*e**2 - c*d**2)**3) - c**2*d**
2*e*atan(sqrt(d + e*x)/sqrt((a*e**2 - c*d**2)/(c*d)))/(sqrt((a*e**2 - c*d*
*2)/(c*d))*(a*e**2 - c*d**2)**3) + c*d*e/(3*(d + e*x)**(3/2)*(a*e**2 - c*d
**2)**2) - e/(5*(d + e*x)**(5/2)*(a*e**2 - c*d**2)))/e, Ne(e, 0)), (log(x)
/(c*d**(9/2)), True))
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{(d+ex)^{5/2}\left(ade+(cd^2+ae^2)x+cdex^2\right)}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(1/(e*x+d)^(5/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2),x, algorithm="
maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(a*e^2-c*d^2>0)', see `assume?` f
or more de
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 217, normalized size of antiderivative = 1.42

$$\int \frac{1}{(d+ex)^{5/2}\left(ade+(cd^2+ae^2)x+cdex^2\right)}\,dx = \frac{2\,c^3d^3\arctan\left(\frac{\sqrt{ex+d}cd}{\sqrt{-c^2d^3+acde^2}}\right)}{(c^3d^6-3\,ac^2d^4e^2+3\,a^2cd^2e^4-a^3e^6)\sqrt{-c^2d^3+acde^2}} + \frac{2\left(15\,(ex+d)^2c^2d^2+5\,(ex+d)c^2d^3+3\,c^2d^4-5\,(ex+d)acde^2-6\,acd^2e^2+3\,a^2e^4\right)}{15\,(c^3d^6-3\,ac^2d^4e^2+3\,a^2cd^2e^4-a^3e^6)(ex+d)^{\frac{5}{2}}}$$

input
```
integrate(1/(e*x+d)^(5/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2),x, algorithm="
giac")
```

output
```
2*c^3*d^3*arctan(sqrt(e*x + d)*c*d/sqrt(-c^2*d^3 + a*c*d*e^2))/((c^3*d^6 -
 3*a*c^2*d^4*e^2 + 3*a^2*c*d^2*e^4 - a^3*e^6)*sqrt(-c^2*d^3 + a*c*d*e^2))
+ 2/15*(15*(e*x + d)^2*c^2*d^2 + 5*(e*x + d)*c^2*d^3 + 3*c^2*d^4 - 5*(e*x
+ d)*a*c*d*e^2 - 6*a*c*d^2*e^2 + 3*a^2*e^4)/((c^3*d^6 - 3*a*c^2*d^4*e^2 +
3*a^2*c*d^2*e^4 - a^3*e^6)*(e*x + d)^(5/2))
```

Mupad [B] (verification not implemented)

Time = 5.33 (sec) , antiderivative size = 171, normalized size of antiderivative = 1.12

$$\int \frac{1}{(d+ex)^{5/2}\left(ade+(cd^2+ae^2)x+cdex^2\right)}\,dx = -\frac{\frac{2}{5\,(a\,e^2-c\,d^2)}+\frac{2\,c^2\,d^2\,(d+e\,x)^2}{(a\,e^2-c\,d^2)^3}-\frac{2\,c\,d\,(d+e\,x)}{3\,(a\,e^2-c\,d^2)^2}}{(d+e\,x)^{5/2}} - \frac{2\,c^{5/2}\,d^{5/2}\,\mathrm{atan}\left(\frac{\sqrt{c}\,\sqrt{d}\,\sqrt{d+e\,x}\,(a^3\,e^6-3\,a^2\,c\,d^2\,e^4+3\,a\,c^2\,d^4\,e^2-c^3\,d^6)}{(a\,e^2-c\,d^2)^{7/2}}\right)}{(a\,e^2-c\,d^2)^{7/2}}$$

input
```
int(1/((d + e*x)^(5/2)*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)),x)
```

output
```
- (2/(5*(a*e^2 - c*d^2)) + (2*c^2*d^2*(d + e*x)^2)/(a*e^2 - c*d^2)^3 - (2*
c*d*(d + e*x))/(3*(a*e^2 - c*d^2)^2))/(d + e*x)^(5/2) - (2*c^(5/2)*d^(5/2)
*atan((c^(1/2)*d^(1/2)*(d + e*x)^(1/2)*(a^3*e^6 - c^3*d^6 + 3*a*c^2*d^4*e^
2 - 3*a^2*c*d^2*e^4))/(a*e^2 - c*d^2)^(7/2)))/(a*e^2 - c*d^2)^(7/2)
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 503, normalized size of antiderivative = 3.29

$$\int \frac{1}{(d+ex)^{5/2}\left(ade+(cd^2+ae^2)\,x+cdex^2\right)}\,dx = \frac{-2\sqrt{d}\,\sqrt{c}\,\sqrt{ex+d}\,\sqrt{a\,e^2-c\,d^2}\;atan\left(\frac{\sqrt{ex+d}\,cd}{\sqrt{d}\,\sqrt{c}\,\sqrt{a\,e^2-c\,d^2}}\right)c^2}{\sqrt{ex+d}\,(}$$

input
```
int(1/(e*x+d)^(5/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2),x)
```

output
```
(2*( - 15*sqrt(d)*sqrt(c)*sqrt(d + e*x)*sqrt(a*e**2 - c*d**2)*atan((sqrt(d
 + e*x)*c*d)/(sqrt(d)*sqrt(c)*sqrt(a*e**2 - c*d**2)))*c**2*d**4 - 30*sqrt(
d)*sqrt(c)*sqrt(d + e*x)*sqrt(a*e**2 - c*d**2)*atan((sqrt(d + e*x)*c*d)/(s
qrt(d)*sqrt(c)*sqrt(a*e**2 - c*d**2)))*c**2*d**3*e*x - 15*sqrt(d)*sqrt(c)*
sqrt(d + e*x)*sqrt(a*e**2 - c*d**2)*atan((sqrt(d + e*x)*c*d)/(sqrt(d)*sqrt
(c)*sqrt(a*e**2 - c*d**2)))*c**2*d**2*e**2*x**2 - 3*a**3*e**6 + 14*a**2*c*
d**2*e**4 + 5*a**2*c*d*e**5*x - 34*a*c**2*d**4*e**2 - 40*a*c**2*d**3*e**3*
x - 15*a*c**2*d**2*e**4*x**2 + 23*c**3*d**6 + 35*c**3*d**5*e*x + 15*c**3*d
**4*e**2*x**2))/(15*sqrt(d + e*x)*(a**4*d**2*e**8 + 2*a**4*d*e**9*x + a**4
*e**10*x**2 - 4*a**3*c*d**4*e**6 - 8*a**3*c*d**3*e**7*x - 4*a**3*c*d**2*e*
*8*x**2 + 6*a**2*c**2*d**6*e**4 + 12*a**2*c**2*d**5*e**5*x + 6*a**2*c**2*d
**4*e**6*x**2 - 4*a*c**3*d**8*e**2 - 8*a*c**3*d**7*e**3*x - 4*a*c**3*d**6*
e**4*x**2 + c**4*d**10 + 2*c**4*d**9*e*x + c**4*d**8*e**2*x**2))
```

3.181 $\int \frac{1}{(d+ex)^{7/2}(ade+(cd^2+ae^2)x+cdex^2)}\,dx$

Optimal result

Integrand size = 37, antiderivative size = 186

$$\int \frac{1}{(d+ex)^{7/2}\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)}\,dx=\frac{2}{7\left(cd^2-ae^2\right)(d+ex)^{7/2}}+\frac{2cd}{5\left(cd^2-ae^2\right)^2(d+ex)^{5/2}}+\frac{2c^2d^2}{3\left(cd^2-ae^2\right)^3(d+ex)^{3/2}}+\frac{2c^3d^3}{\left(cd^2-ae^2\right)^4\sqrt{d+ex}}-\frac{2c^{7/2}d^{7/2}\operatorname{arctanh}\left(\frac{\sqrt{c}\sqrt{d}\sqrt{d+ex}}{\sqrt{cd^2-ae^2}}\right)}{\left(cd^2-ae^2\right)^{9/2}}$$

output
```
2/7/(-a*e^2+c*d^2)/(e*x+d)^(7/2)+2/5*c*d/(-a*e^2+c*d^2)^2/(e*x+d)^(5/2)+2/
3*c^2*d^2/(-a*e^2+c*d^2)^3/(e*x+d)^(3/2)+2*c^3*d^3/(-a*e^2+c*d^2)^4/(e*x+d
)^(1/2)-2*c^(7/2)*d^(7/2)*arctanh(c^(1/2)*d^(1/2)*(e*x+d)^(1/2)/(-a*e^2+c*
d^2)^(1/2))/(-a*e^2+c*d^2)^(9/2)
```

Mathematica [A] (verified)

Time = 0.30 (sec) , antiderivative size = 189, normalized size of antiderivative = 1.02

$$\int \frac{1}{(d+ex)^{7/2}\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)}\,dx = \frac{-30a^3e^6+6a^2cde^4(22d+7ex)-2ac^2d^2e^2(122d^2+112}{105\left(cd^2-\right.} + \frac{2c^{7/2}d^{7/2}\arctan\left(\frac{\sqrt{c}\sqrt{d}\sqrt{d+ex}}{\sqrt{-cd^2+ae^2}}\right)}{\left(-cd^2+ae^2\right)^{9/2}}$$

input
```
Integrate[1/((d + e*x)^(7/2)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)),x]
```

output
```
(-30*a^3*e^6 + 6*a^2*c*d*e^4*(22*d + 7*e*x) - 2*a*c^2*d^2*e^2*(122*d^2 + 1
12*d*e*x + 35*e^2*x^2) + 2*c^3*d^3*(176*d^3 + 406*d^2*e*x + 350*d*e^2*x^2
+ 105*e^3*x^3))/(105*(c*d^2 - a*e^2)^4*(d + e*x)^(7/2)) + (2*c^(7/2)*d^(7/
2)*ArcTan[(Sqrt[c]*Sqrt[d]*Sqrt[d + e*x])/Sqrt[-(c*d^2) + a*e^2]])/(-(c*d^
2) + a*e^2)^(9/2)
```

Rubi [A] (verified)

Time = 0.51 (sec) , antiderivative size = 226, normalized size of antiderivative = 1.22, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.189$, Rules used = {1121, 61, 61, 61, 61, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(d+ex)^{7/2}\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)}dx$$

$$\Big\downarrow \text{ 1121}$$

$$\int \frac{1}{(d+ex)^{9/2}(ae+cdx)}dx$$

$$\Big\downarrow \text{ 61}$$

$$\frac{cd\int \frac{1}{(ae+cdx)(d+ex)^{7/2}}dx}{cd^2-ae^2}+\frac{2}{7(d+ex)^{7/2}\left(cd^2-ae^2\right)}$$

$$\Big\downarrow \text{61}$$

$$\frac{cd\left(\frac{cd\int \frac{1}{(ae+cdx)(d+ex)^{5/2}}dx}{cd^2-ae^2}+\frac{2}{5(d+ex)^{5/2}(cd^2-ae^2)}\right)}{cd^2-ae^2}+\frac{2}{7(d+ex)^{7/2}\left(cd^2-ae^2\right)}$$

$$\Big\downarrow \text{61}$$

$$\frac{cd\left(\frac{cd\left(\frac{cd\int \frac{1}{(ae+cdx)(d+ex)^{3/2}}dx}{cd^2-ae^2}+\frac{2}{3(d+ex)^{3/2}\left(cd^2-ae^2\right)}\right)}{cd^2-ae^2}+\frac{2}{5(d+ex)^{5/2}(cd^2-ae^2)}\right)}{cd^2-ae^2}+\frac{2}{7(d+ex)^{7/2}\left(cd^2-ae^2\right)}$$

$$\Big\downarrow \text{61}$$

$$\frac{cd\left(\frac{cd\left(\frac{cd\left(\frac{cd\int \frac{1}{(ae+cdx)\sqrt{d+ex}}dx}{cd^2-ae^2}+\frac{2}{\sqrt{d+ex}\left(cd^2-ae^2\right)}\right)}{cd^2-ae^2}+\frac{2}{3(d+ex)^{3/2}\left(cd^2-ae^2\right)}\right)}{cd^2-ae^2}+\frac{2}{5(d+ex)^{5/2}(cd^2-ae^2)}\right)}{cd^2-ae^2}+\frac{2}{7(d+ex)^{7/2}\left(cd^2-ae^2\right)}$$

$$\Big\downarrow \text{73}$$

$$\frac{cd\left(\frac{cd\left(\frac{cd\left(\frac{2cd\int \frac{1}{-\frac{cd^2}{e}+\frac{c(d+ex)d}{e}+ae}d\sqrt{d+ex}}{e\left(cd^2-ae^2\right)}+\frac{2}{\sqrt{d+ex}\left(cd^2-ae^2\right)}\right)}{cd^2-ae^2}+\frac{2}{3(d+ex)^{3/2}\left(cd^2-ae^2\right)}\right)}{cd^2-ae^2}+\frac{2}{5(d+ex)^{5/2}(cd^2-ae^2)}\right)}{cd^2-ae^2}+\frac{2}{7(d+ex)^{7/2}\left(cd^2-ae^2\right)}$$

$$\Big\downarrow \text{221}$$

$$\frac{cd\left(\frac{cd\left(\frac{cd\left(\frac{2}{\sqrt{d+ex}\left(cd^2-ae^2\right)}-\frac{2\sqrt{c}\sqrt{d}\text{arctanh}\left(\frac{\sqrt{c}\sqrt{d}\sqrt{d+ex}}{\sqrt{cd^2-ae^2}}\right)}{\left(cd^2-ae^2\right)^{3/2}}\right)}{cd^2-ae^2}+\frac{2}{3(d+ex)^{3/2}\left(cd^2-ae^2\right)}\right)}{cd^2-ae^2}+\frac{2}{5(d+ex)^{5/2}(cd^2-ae^2)}\right)}{cd^2-ae^2}+\frac{2}{7(d+ex)^{7/2}\left(cd^2-ae^2\right)}$$

input
```
Int[1/((d + e*x)^(7/2)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)),x]
```

output
```
2/(7*(c*d^2 - a*e^2)*(d + e*x)^(7/2)) + (c*d*(2/(5*(c*d^2 - a*e^2)*(d + e*
x)^(5/2)) + (c*d*(2/(3*(c*d^2 - a*e^2)*(d + e*x)^(3/2)) + (c*d*(2/((c*d^2
- a*e^2)*Sqrt[d + e*x]) - (2*Sqrt[c]*Sqrt[d]*ArcTanh[(Sqrt[c]*Sqrt[d]*Sqrt
[d + e*x])/Sqrt[c*d^2 - a*e^2]])/(c*d^2 - a*e^2)^(3/2)))/(c*d^2 - a*e^2)))
/(c*d^2 - a*e^2)))/(c*d^2 - a*e^2)
```

Defintions of rubi rules used

rule 61
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && LtQ[m, -1] &&  !(LtQ[n, -1] && (EqQ[a, 0
] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && IntLinearQ[a, b, c, d
, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 1121
```
Int[((d_) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Int[ExpandIntegrand[(d + e*x)^(m + p)*(a/d + (c/e)*x)^p, x], x]
/; FreeQ[{a, b, c, d, e, m, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && (Int
egerQ[p] || (GtQ[a, 0] && GtQ[d, 0] && LtQ[c, 0]))
```

Maple [A] (verified)

Time = 1.92 (sec) , antiderivative size = 175, normalized size of antiderivative = 0.94

method	result
derivativedivides	$-\frac{2}{7(a e^2-c d^2)(ex+d)^{\frac{7}{2}}}-\frac{2c^2d^2}{3(a e^2-c d^2)^3(ex+d)^{\frac{3}{2}}}+\frac{2c^3d^3}{(a e^2-c d^2)^4\sqrt{ex+d}}+\frac{2cd}{5(a e^2-c d^2)^2(ex+d)^{\frac{5}{2}}}+\frac{2c^4d^4 \text{ar}\cdots}{(a e^2-\cdots}$
default	$-\frac{2}{7(a e^2-c d^2)(ex+d)^{\frac{7}{2}}}-\frac{2c^2d^2}{3(a e^2-c d^2)^3(ex+d)^{\frac{3}{2}}}+\frac{2c^3d^3}{(a e^2-c d^2)^4\sqrt{ex+d}}+\frac{2cd}{5(a e^2-c d^2)^2(ex+d)^{\frac{5}{2}}}+\frac{2c^4d^4 \text{ar}\cdots}{(a e^2-\cdots}$
pseudoelliptic	$-\frac{2\left(-7c^4d^4\arctan\left(\frac{cd\sqrt{ex+d}}{\sqrt{cd\left(a e^2-c d^2\right)}}\right)(ex+d)^{\frac{7}{2}}+\left(\left(-7d^3e^3x^3-\frac{70}{3}d^4e^2x^2-\frac{406}{15}d^5ex-\frac{176}{15}d^6\right)c^3+\frac{122e^2a d^2\left(\frac{35}{122}e^2x^{2}\cdots\right.}{15}\cdots\right.\right.}{7(ex+d)^{\frac{7}{2}}\sqrt{cd(a e^2-c d^2)}\,(a e^2-c d^2)^4}$

input
```
int(1/(e*x+d)^(7/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e),x,method=_RETURNVERB
OSE)
```

output
```
-2/7/(a*e^2-c*d^2)/(e*x+d)^(7/2)-2/3/(a*e^2-c*d^2)^3*c^2*d^2/(e*x+d)^(3/2)
+2/(a*e^2-c*d^2)^4*c^3*d^3/(e*x+d)^(1/2)+2/5/(a*e^2-c*d^2)^2*c*d/(e*x+d)^(
5/2)+2*c^4*d^4/(a*e^2-c*d^2)^4/(c*d*(a*e^2-c*d^2))^(1/2)*arctan(c*d*(e*x+d
)^(1/2)/(c*d*(a*e^2-c*d^2))^(1/2))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 544 vs. $2(158) = 316$.

Time = 0.12 (sec) , antiderivative size = 1130, normalized size of antiderivative = 6.08

$$\int \frac{1}{(d+ex)^{7/2}\left(ade+(cd^2+ae^2)x+cdex^2\right)}\,dx = \text{Too large to display}$$

input
```
integrate(1/(e*x+d)^(7/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2),x, algorithm="
fricas")
```

output
```
[1/105*(105*(c^3*d^3*e^4*x^4 + 4*c^3*d^4*e^3*x^3 + 6*c^3*d^5*e^2*x^2 + 4*c
^3*d^6*e*x + c^3*d^7)*sqrt(c*d/(c*d^2 - a*e^2))*log((c*d*e*x + 2*c*d^2 - a
*e^2 - 2*(c*d^2 - a*e^2)*sqrt(e*x + d)*sqrt(c*d/(c*d^2 - a*e^2)))/(c*d*x +
 a*e)) + 2*(105*c^3*d^3*e^3*x^3 + 176*c^3*d^6 - 122*a*c^2*d^4*e^2 + 66*a^2
*c*d^2*e^4 - 15*a^3*e^6 + 35*(10*c^3*d^4*e^2 - a*c^2*d^2*e^4)*x^2 + 7*(58*
c^3*d^5*e - 16*a*c^2*d^3*e^3 + 3*a^2*c*d*e^5)*x)*sqrt(e*x + d))/(c^4*d^12
- 4*a*c^3*d^10*e^2 + 6*a^2*c^2*d^8*e^4 - 4*a^3*c*d^6*e^6 + a^4*d^4*e^8 + (
c^4*d^8*e^4 - 4*a*c^3*d^6*e^6 + 6*a^2*c^2*d^4*e^8 - 4*a^3*c*d^2*e^10 + a^4
*e^12)*x^4 + 4*(c^4*d^9*e^3 - 4*a*c^3*d^7*e^5 + 6*a^2*c^2*d^5*e^7 - 4*a^3*
c*d^3*e^9 + a^4*d*e^11)*x^3 + 6*(c^4*d^10*e^2 - 4*a*c^3*d^8*e^4 + 6*a^2*c^
2*d^6*e^6 - 4*a^3*c*d^4*e^8 + a^4*d^2*e^10)*x^2 + 4*(c^4*d^11*e - 4*a*c^3*
d^9*e^3 + 6*a^2*c^2*d^7*e^5 - 4*a^3*c*d^5*e^7 + a^4*d^3*e^9)*x), 2/105*(10
5*(c^3*d^3*e^4*x^4 + 4*c^3*d^4*e^3*x^3 + 6*c^3*d^5*e^2*x^2 + 4*c^3*d^6*e*x
 + c^3*d^7)*sqrt(-c*d/(c*d^2 - a*e^2))*arctan(sqrt(e*x + d)*sqrt(-c*d/(c*d
^2 - a*e^2))) + (105*c^3*d^3*e^3*x^3 + 176*c^3*d^6 - 122*a*c^2*d^4*e^2 + 6
6*a^2*c*d^2*e^4 - 15*a^3*e^6 + 35*(10*c^3*d^4*e^2 - a*c^2*d^2*e^4)*x^2 + 7
*(58*c^3*d^5*e - 16*a*c^2*d^3*e^3 + 3*a^2*c*d*e^5)*x)*sqrt(e*x + d))/(c^4*
d^12 - 4*a*c^3*d^10*e^2 + 6*a^2*c^2*d^8*e^4 - 4*a^3*c*d^6*e^6 + a^4*d^4*e^
8 + (c^4*d^8*e^4 - 4*a*c^3*d^6*e^6 + 6*a^2*c^2*d^4*e^8 - 4*a^3*c*d^2*e^10
+ a^4*e^12)*x^4 + 4*(c^4*d^9*e^3 - 4*a*c^3*d^7*e^5 + 6*a^2*c^2*d^5*e^7 ...
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{(d+ex)^{7/2}\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)}\,dx=\text{Timed out}$$

input
```
integrate(1/(e*x+d)**(7/2)/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2),x)
```

output
```
Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{(d+ex)^{7/2}\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)}\,dx=\text{Exception raised: ValueError}$$

input
```
integrate(1/(e*x+d)^(7/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2),x, algorithm="
maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(a*e^2-c*d^2>0)', see `assume?` f
or more de
```

Giac [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 310, normalized size of antiderivative = 1.67

$$\int \frac{1}{(d+ex)^{7/2}\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)}\,dx=\frac{2\,c^4d^4\arctan\left(\frac{\sqrt{ex+d}cd}{\sqrt{-c^2d^3+acde^2}}\right)}{\left(c^4d^8-4\,ac^3d^6e^2+6\,a^2c^2d^4e^4-4\,a^3cd^2e^6+a^4e^8\right)\sqrt{-c^2}}$$
$$+\frac{2\left(105\,(ex+d)^3c^3d^3+35\,(ex+d)^2c^3d^4+21\,(ex+d)c^3d^5+15\,c^3d^6-35\,(ex+d)^2ac^2d^2e^2-42\,(ex+d)c}{105\left(c^4d^8-4\,ac^3d^6e^2+6\,a^2c^2d^4e^4-4\,a^3cd^2e^6+a^4e\right.}$$

input
```
integrate(1/(e*x+d)^(7/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2),x, algorithm="
giac")
```

output
```
2*c^4*d^4*arctan(sqrt(e*x + d)*c*d/sqrt(-c^2*d^3 + a*c*d*e^2))/((c^4*d^8 -
 4*a*c^3*d^6*e^2 + 6*a^2*c^2*d^4*e^4 - 4*a^3*c*d^2*e^6 + a^4*e^8)*sqrt(-c^
2*d^3 + a*c*d*e^2)) + 2/105*(105*(e*x + d)^3*c^3*d^3 + 35*(e*x + d)^2*c^3*
d^4 + 21*(e*x + d)*c^3*d^5 + 15*c^3*d^6 - 35*(e*x + d)^2*a*c^2*d^2*e^2 - 4
2*(e*x + d)*a*c^2*d^3*e^2 - 45*a*c^2*d^4*e^2 + 21*(e*x + d)*a^2*c*d*e^4 +
45*a^2*c*d^2*e^4 - 15*a^3*e^6)/((c^4*d^8 - 4*a*c^3*d^6*e^2 + 6*a^2*c^2*d^4
*e^4 - 4*a^3*c*d^2*e^6 + a^4*e^8)*(e*x + d)^(7/2))
```

Mupad [B] (verification not implemented)

Time = 5.34 (sec) , antiderivative size = 213, normalized size of antiderivative = 1.15

$$\int \frac{1}{(d+ex)^{7/2}\,(ade+(cd^2+ae^2)\,x+cdex^2)}\,dx = \frac{2\,c^{7/2}\,d^{7/2}\,\mathrm{atan}\left(\frac{\sqrt{c}\,\sqrt{d}\,\sqrt{d+e\,x}\,\left(a^4\,e^8-4\,a^3\,c\,d^2\,e^6+6\,a^2\,c^2\,d^4\,e^4-4\,a\,c^3\right.}{{\left(a\,e^2-c\,d^2\right)}^{9/2}}\right.}{{\left(a\,e^2-c\,d^2\right)}^{9/2}}$$

$$-\frac{\frac{2}{7\,(a\,e^2-c\,d^2)}+\frac{2\,c^2\,d^2\,{(d+e\,x)}^2}{3\,{(a\,e^2-c\,d^2)}^3}-\frac{2\,c^3\,d^3\,{(d+e\,x)}^3}{{(a\,e^2-c\,d^2)}^4}-\frac{2\,c\,d\,(d+e\,x)}{5\,{(a\,e^2-c\,d^2)}^2}}{{(d+e\,x)}^{7/2}}$$

input
```
int(1/((d + e*x)^(7/2)*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)),x)
```

output
```
(2*c^(7/2)*d^(7/2)*atan((c^(1/2)*d^(1/2)*(d + e*x)^(1/2)*(a^4*e^8 + c^4*d^
8 - 4*a*c^3*d^6*e^2 - 4*a^3*c*d^2*e^6 + 6*a^2*c^2*d^4*e^4))/(a*e^2 - c*d^2
)^(9/2)))/(a*e^2 - c*d^2)^(9/2) - (2/(7*(a*e^2 - c*d^2)) + (2*c^2*d^2*(d +
 e*x)^2)/(3*(a*e^2 - c*d^2)^3) - (2*c^3*d^3*(d + e*x)^3)/(a*e^2 - c*d^2)^4
 - (2*c*d*(d + e*x))/(5*(a*e^2 - c*d^2)^2))/(d + e*x)^(7/2)
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 788, normalized size of antiderivative = 4.24

$$\int \frac{1}{(d+ex)^{7/2}\,(ade+(cd^2+ae^2)\,x+cdex^2)}\,dx = \frac{2\sqrt{d}\,\sqrt{c}\,\sqrt{ex+d}\,\sqrt{a\,e^2-c\,d^2}\,atan\left(\frac{\sqrt{ex+d}\,cd}{\sqrt{d}\,\sqrt{c}\,\sqrt{a\,e^2-c\,d^2}}\right)c^3d^6}{\sqrt{ex+d}\,(a^5e^{13}x^3-}$$

input
```
int(1/(e*x+d)^(7/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2),x)
```

output
```
(2*(105*sqrt(d)*sqrt(c)*sqrt(d + e*x)*sqrt(a*e**2 - c*d**2)*atan((sqrt(d +
 e*x)*c*d)/(sqrt(d)*sqrt(c)*sqrt(a*e**2 - c*d**2)))*c**3*d**6 + 315*sqrt(d
)*sqrt(c)*sqrt(d + e*x)*sqrt(a*e**2 - c*d**2)*atan((sqrt(d + e*x)*c*d)/(sq
rt(d)*sqrt(c)*sqrt(a*e**2 - c*d**2)))*c**3*d**5*e*x + 315*sqrt(d)*sqrt(c)*
sqrt(d + e*x)*sqrt(a*e**2 - c*d**2)*atan((sqrt(d + e*x)*c*d)/(sqrt(d)*sqrt
(c)*sqrt(a*e**2 - c*d**2)))*c**3*d**4*e**2*x**2 + 105*sqrt(d)*sqrt(c)*sqrt
(d + e*x)*sqrt(a*e**2 - c*d**2)*atan((sqrt(d + e*x)*c*d)/(sqrt(d)*sqrt(c)*
sqrt(a*e**2 - c*d**2)))*c**3*d**3*e**3*x**3 - 15*a**4*e**8 + 81*a**3*c*d**
2*e**6 + 21*a**3*c*d*e**7*x - 188*a**2*c**2*d**4*e**4 - 133*a**2*c**2*d**3
*e**5*x - 35*a**2*c**2*d**2*e**6*x**2 + 298*a*c**3*d**6*e**2 + 518*a*c**3*
d**5*e**3*x + 385*a*c**3*d**4*e**4*x**2 + 105*a*c**3*d**3*e**5*x**3 - 176*
c**4*d**8 - 406*c**4*d**7*e*x - 350*c**4*d**6*e**2*x**2 - 105*c**4*d**5*e*
*3*x**3))/(105*sqrt(d + e*x)*(a**5*d**3*e**10 + 3*a**5*d**2*e**11*x + 3*a*
*5*d*e**12*x**2 + a**5*e**13*x**3 - 5*a**4*c*d**5*e**8 - 15*a**4*c*d**4*e*
*9*x - 15*a**4*c*d**3*e**10*x**2 - 5*a**4*c*d**2*e**11*x**3 + 10*a**3*c**2
*d**7*e**6 + 30*a**3*c**2*d**6*e**7*x + 30*a**3*c**2*d**5*e**8*x**2 + 10*a
**3*c**2*d**4*e**9*x**3 - 10*a**2*c**3*d**9*e**4 - 30*a**2*c**3*d**8*e**5*
x - 30*a**2*c**3*d**7*e**6*x**2 - 10*a**2*c**3*d**6*e**7*x**3 + 5*a*c**4*d
**11*e**2 + 15*a*c**4*d**10*e**3*x + 15*a*c**4*d**9*e**4*x**2 + 5*a*c**4*d
**8*e**5*x**3 - c**5*d**13 - 3*c**5*d**12*e*x - 3*c**5*d**11*e**2*x**2 ...
```

3.182 $\int \frac{(d+ex)^{13/2}}{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^2}\,dx$

Optimal result

Integrand size = 37, antiderivative size = 210

$$\int \frac{(d+ex)^{13/2}}{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^2}\,dx = \frac{9e(cd^2-ae^2)^3\sqrt{d+ex}}{c^5d^5} + \frac{3e(cd^2-ae^2)^2(d+ex)^{3/2}}{c^4d^4} + \frac{9e(cd^2-ae^2)(d+ex)^{5/2}}{5c^3d^3} + \frac{9e(d+ex)^{7/2}}{7c^2d^2} - \frac{(d+ex)^{9/2}}{cd(ae+cdx)} - \frac{9e(cd^2-ae^2)^{7/2}\operatorname{arctanh}\left(\frac{\sqrt{c}\sqrt{d}\sqrt{d+ex}}{\sqrt{cd^2-ae^2}}\right)}{c^{11/2}d^{11/2}}$$

output
```
9*e*(-a*e^2+c*d^2)^3*(e*x+d)^(1/2)/c^5/d^5+3*e*(-a*e^2+c*d^2)^2*(e*x+d)^(3
/2)/c^4/d^4+9/5*e*(-a*e^2+c*d^2)*(e*x+d)^(5/2)/c^3/d^3+9/7*e*(e*x+d)^(7/2)
/c^2/d^2-(e*x+d)^(9/2)/c/d/(c*d*x+a*e)-9*e*(-a*e^2+c*d^2)^(7/2)*arctanh(c^
(1/2)*d^(1/2)*(e*x+d)^(1/2)/(-a*e^2+c*d^2)^(1/2))/c^(11/2)/d^(11/2)
```

Mathematica [A] (verified)

Time = 0.53 (sec) , antiderivative size = 246, normalized size of antiderivative = 1.17

$$\int \frac{(d+ex)^{13/2}}{(ade+(cd^2+ae^2)x+cdex^2)^2}\,dx =$$
$$-\frac{\sqrt{d+ex}(315a^4e^8-210a^3cde^6(5d-ex)+42a^2c^2d^2e^4(29d^2-17dex-e^2x^2)+6ac^3d^3e^2(-88d^3+142d^2\cdots}{35c^5d^5(ae+cdx)}$$
$$+\frac{9e(-cd^2+ae^2)^{7/2}\arctan\left(\frac{\sqrt{c}\sqrt{d}\sqrt{d+ex}}{\sqrt{-cd^2+ae^2}}\right)}{c^{11/2}d^{11/2}}$$

input
```
Integrate[(d + e*x)^(13/2)/(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^2,x]
```

output
```
-1/35*(Sqrt[d + e*x]*(315*a^4*e^8 - 210*a^3*c*d*e^6*(5*d - e*x) + 42*a^2*c
^2*d^2*e^4*(29*d^2 - 17*d*e*x - e^2*x^2) + 6*a*c^3*d^3*e^2*(-88*d^3 + 142*
d^2*e*x + 23*d*e^2*x^2 + 3*e^3*x^3) + c^4*d^4*(35*d^4 - 388*d^3*e*x - 156*
d^2*e^2*x^2 - 58*d*e^3*x^3 - 10*e^4*x^4)))/(c^5*d^5*(a*e + c*d*x)) + (9*e*
(-(c*d^2) + a*e^2)^(7/2)*ArcTan[(Sqrt[c]*Sqrt[d]*Sqrt[d + e*x])/Sqrt[-(c*d
^2) + a*e^2]])/(c^(11/2)*d^(11/2))
```

Rubi [A] (verified)

Time = 0.51 (sec) , antiderivative size = 246, normalized size of antiderivative = 1.17, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.216$, Rules used = {1121, 51, 60, 60, 60, 60, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex)^{13/2}}{(x(ae^2+cd^2)+ade+cdex^2)^2}dx$$
$$\Big\downarrow \text{1121}$$
$$\int \frac{(d+ex)^{9/2}}{(ae+cdx)^2}dx$$
$$\Big\downarrow \text{51}$$

$$\frac{9e\int\frac{(d+ex)^{7/2}}{ae+cdx}dx}{2cd}-\frac{(d+ex)^{9/2}}{cd(ae+cdx)}$$

$$\Big\downarrow \text{60}$$

$$\frac{9e\left(\frac{\left(d^2-\frac{ae^2}{c}\right)\int\frac{(d+ex)^{5/2}}{ae+cdx}dx}{d}+\frac{2(d+ex)^{7/2}}{7cd}\right)}{2cd}-\frac{(d+ex)^{9/2}}{cd(ae+cdx)}$$

$$\Big\downarrow \text{60}$$

$$\frac{9e\left(\frac{\left(d^2-\frac{ae^2}{c}\right)\left(\frac{\left(d^2-\frac{ae^2}{c}\right)\int\frac{(d+ex)^{3/2}}{ae+cdx}dx}{d}+\frac{2(d+ex)^{5/2}}{5cd}\right)}{d}+\frac{2(d+ex)^{7/2}}{7cd}\right)}{2cd}-\frac{(d+ex)^{9/2}}{cd(ae+cdx)}$$

$$\Big\downarrow \text{60}$$

$$\frac{9e\left(\frac{\left(d^2-\frac{ae^2}{c}\right)\left(\frac{\left(d^2-\frac{ae^2}{c}\right)\left(\frac{\left(d^2-\frac{ae^2}{c}\right)\int\frac{\sqrt{d+ex}}{ae+cdx}dx}{d}+\frac{2(d+ex)^{3/2}}{3cd}\right)}{d}+\frac{2(d+ex)^{5/2}}{5cd}\right)}{d}+\frac{2(d+ex)^{7/2}}{7cd}\right)}{2cd}-\frac{(d+ex)^{9/2}}{cd(ae+cdx)}$$

$$\Big\downarrow \text{60}$$

$$\frac{9e\left(\frac{\left(d^2-\frac{ae^2}{c}\right)\left(\frac{\left(d^2-\frac{ae^2}{c}\right)\left(\frac{\left(d^2-\frac{ae^2}{c}\right)\left(\frac{\left(d^2-\frac{ae^2}{c}\right)\int\frac{1}{(ae+cdx)\sqrt{d+ex}}dx}{d}+\frac{2\sqrt{d+ex}}{cd}\right)}{d}+\frac{2(d+ex)^{3/2}}{3cd}\right)}{d}+\frac{2(d+ex)^{5/2}}{5cd}\right)}{d}+\frac{2(d+ex)^{7/2}}{7cd}\right)}{2cd}-\frac{(d+ex)^{9/2}}{cd(ae+cdx)}$$

$$\Big\downarrow \; 73$$

$$\frac{9e\left(\frac{\left(d^2-\frac{ae^2}{c}\right)\left(\frac{\left(d^2-\frac{ae^2}{c}\right)\left(\frac{\left(d^2-\frac{ae^2}{c}\right)\left(\frac{2\left(d^2-\frac{ae^2}{c}\right)\int \frac{1}{-\frac{cd^2}{e}+\frac{c(d+ex)d}{e}+ae}d\sqrt{d+ex}}{de}+\frac{2\sqrt{d+ex}}{cd}\right)}{d}+\frac{2(d+ex)^{3/2}}{3cd}\right)}{d}+\frac{2(d+ex)^{5/2}}{5cd}\right)}{d}+\frac{2(d+ex)^{7/2}}{7cd}\right)}{2cd}$$

$$\frac{(d+ex)^{9/2}}{cd(ae+cdx)}$$

$$\Big\downarrow \text{221}$$

$$\frac{9e\left(\frac{\left(d^2-\frac{ae^2}{c}\right)\left(\frac{\left(d^2-\frac{ae^2}{c}\right)\left(\frac{\left(d^2-\frac{ae^2}{c}\right)\left(\frac{2\sqrt{d+ex}}{cd}-\frac{2\left(d^2-\frac{ae^2}{c}\right)\text{arctanh}\left(\frac{\sqrt{c}\sqrt{d}\sqrt{d+ex}}{\sqrt{cd^2-ae^2}}\right)}{\sqrt{c}d^{3/2}\sqrt{cd^2-ae^2}}\right)}{d}+\frac{2(d+ex)^{3/2}}{3cd}\right)}{d}+\frac{2(d+ex)^{5/2}}{5cd}\right)}{d}+\frac{2(d+ex)^{7/2}}{7cd}\right)}{2cd}-\frac{(d+ex)^{9/2}}{cd(ae+cdx)}$$

input
```
Int[(d + e*x)^(13/2)/(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^2,x]
```

output
```
-((d + e*x)^(9/2)/(c*d*(a*e + c*d*x))) + (9*e*((2*(d + e*x)^(7/2))/(7*c*d)
 + ((d^2 - (a*e^2)/c)*((2*(d + e*x)^(5/2))/(5*c*d) + ((d^2 - (a*e^2)/c)*((
2*(d + e*x)^(3/2))/(3*c*d) + ((d^2 - (a*e^2)/c)*((2*Sqrt[d + e*x])/(c*d) -
 (2*(d^2 - (a*e^2)/c)*ArcTanh[(Sqrt[c]*Sqrt[d]*Sqrt[d + e*x])/Sqrt[c*d^2 -
 a*e^2]])/(Sqrt[c]*d^(3/2)*Sqrt[c*d^2 - a*e^2])))/d))/d))/d))/(2*c*d)
```

Defintions of rubi rules used

rule 51
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Simp[d*(n/(b*(m + 1)))
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d, n}, x
] && ILtQ[m, -1] && FractionQ[n] && GtQ[n, 0]
```

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 1121
```
Int[((d_) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Int[ExpandIntegrand[(d + e*x)^(m + p)*(a/d + (c/e)*x)^p, x], x]
/; FreeQ[{a, b, c, d, e, m, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && (Int
egerQ[p] || (GtQ[a, 0] && GtQ[d, 0] && LtQ[c, 0]))
```

Maple [A] (verified)

Time = 4.24 (sec) , antiderivative size = 257, normalized size of antiderivative = 1.22

method	result
pseudoelliptic	$\frac{9e(a\,e^2-c\,d^2)^4(cdx+ae)\arctan\left(\frac{cd\sqrt{ex+d}}{\sqrt{cd(a\,e^2-c\,d^2)}}\right)-9\sqrt{ex+d}\,\sqrt{cd(a\,e^2-c\,d^2)}\left(\left(\frac{\left(-\frac{2}{7}e^4x^4-\frac{58}{35}d\,e^3x^3-\frac{156}{35}d^2e^2x^2-\frac{388}{35}\right.}{9}\right.\right.}{d^5c^5(cdx+}$
risch	$-\frac{2e(-5c^3d^3e^3x^3+14x^2a\,c^2d^2e^4-29c^3d^4e^2x^2-35x\,a^2cd\,e^5+98xa\,c^2d^3e^3-78c^3d^5ex+140e^6a^3-455d^2e^4a^2c+504d^4e}{35d^5c^5}$
derivativedivides	$2e\left(-\frac{-\frac{c^3d^3(ex+d)^{\frac{7}{2}}}{7}+\frac{2(ex+d)^{\frac{5}{2}}a\,c^2d^2e^2}{5}-\frac{2(ex+d)^{\frac{5}{2}}c^3d^4}{5}-(ex+d)^{\frac{3}{2}}a^2cd\,e^4+2(ex+d)^{\frac{3}{2}}a\,c^2d^3e^2-(ex+d)^{\frac{3}{2}}c^3d^5+4}{c^5d^5}\right.$
default	$2e\left(-\frac{-\frac{c^3d^3(ex+d)^{\frac{7}{2}}}{7}+\frac{2(ex+d)^{\frac{5}{2}}a\,c^2d^2e^2}{5}-\frac{2(ex+d)^{\frac{5}{2}}c^3d^4}{5}-(ex+d)^{\frac{3}{2}}a^2cd\,e^4+2(ex+d)^{\frac{3}{2}}a\,c^2d^3e^2-(ex+d)^{\frac{3}{2}}c^3d^5+4}{c^5d^5}\right.$

input

```
int((e*x+d)^(13/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^2,x,method=_RETURNVER
BOSE)
```

output

```
9*(e*(a*e^2-c*d^2)^4*(c*d*x+a*e)*arctan(c*d*(e*x+d)^(1/2)/(c*d*(a*e^2-c*d^
2))^(1/2))-(e*x+d)^(1/2)*(c*d*(a*e^2-c*d^2))^(1/2)*(1/9*(-2/7*e^4*x^4-58/3
5*d*e^3*x^3-156/35*d^2*e^2*x^2-388/35*d^3*e*x+d^4)*d^4*c^4-176/105*e^2*(-3
/88*e^3*x^3-23/88*d*e^2*x^2-71/44*d^2*e*x+d^3)*a*d^3*c^3+58/15*e^4*a^2*d^2
*(-1/29*e^2*x^2-17/29*d*e*x+d^2)*c^2-10/3*e^6*a^3*(-1/5*e*x+d)*d*c+a^4*e^8
))/(c*d*(a*e^2-c*d^2))^(1/2)/d^5/c^5/(c*d*x+a*e)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 387 vs. $2(182) = 364$.

Time = 0.10 (sec) , antiderivative size = 788, normalized size of antiderivative = 3.75

$$\int \frac{(d+ex)^{13/2}}{(ade+(cd^2+ae^2)x+cdex^2)^2}\,dx = \left[-\frac{315\,(ac^3d^6e^2-3\,a^2c^2d^4e^4+3\,a^3cd^2e^6-a^4e^8+(c^4d^7e-3\,ac^3d\right.$$

$$-\frac{315\,(ac^3d^6e^2-3\,a^2c^2d^4e^4+3\,a^3cd^2e^6-a^4e^8+(c^4d^7e-3\,ac^3d^5e^3+3\,a^2c^2d^3e^5-a^3cde^7)x)\sqrt{-\frac{cd^2-ae^2}{cd}}\,\mathrm{ar}}{}$$

input
```
integrate((e*x+d)^(13/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^2,x, algorithm=
"fricas")
```

output
```
[-1/70*(315*(a*c^3*d^6*e^2 - 3*a^2*c^2*d^4*e^4 + 3*a^3*c*d^2*e^6 - a^4*e^8
 + (c^4*d^7*e - 3*a*c^3*d^5*e^3 + 3*a^2*c^2*d^3*e^5 - a^3*c*d*e^7)*x)*sqrt
((c*d^2 - a*e^2)/(c*d))*log((c*d*e*x + 2*c*d^2 - a*e^2 + 2*sqrt(e*x + d)*c
*d*sqrt((c*d^2 - a*e^2)/(c*d)))/(c*d*x + a*e)) - 2*(10*c^4*d^4*e^4*x^4 - 3
5*c^4*d^8 + 528*a*c^3*d^6*e^2 - 1218*a^2*c^2*d^4*e^4 + 1050*a^3*c*d^2*e^6
- 315*a^4*e^8 + 2*(29*c^4*d^5*e^3 - 9*a*c^3*d^3*e^5)*x^3 + 6*(26*c^4*d^6*e
^2 - 23*a*c^3*d^4*e^4 + 7*a^2*c^2*d^2*e^6)*x^2 + 2*(194*c^4*d^7*e - 426*a*
c^3*d^5*e^3 + 357*a^2*c^2*d^3*e^5 - 105*a^3*c*d*e^7)*x)*sqrt(e*x + d))/(c^
6*d^6*x + a*c^5*d^5*e), -1/35*(315*(a*c^3*d^6*e^2 - 3*a^2*c^2*d^4*e^4 + 3*
a^3*c*d^2*e^6 - a^4*e^8 + (c^4*d^7*e - 3*a*c^3*d^5*e^3 + 3*a^2*c^2*d^3*e^5
 - a^3*c*d*e^7)*x)*sqrt(-(c*d^2 - a*e^2)/(c*d))*arctan(-sqrt(e*x + d)*c*d*
sqrt(-(c*d^2 - a*e^2)/(c*d))/(c*d^2 - a*e^2)) - (10*c^4*d^4*e^4*x^4 - 35*c
^4*d^8 + 528*a*c^3*d^6*e^2 - 1218*a^2*c^2*d^4*e^4 + 1050*a^3*c*d^2*e^6 - 3
15*a^4*e^8 + 2*(29*c^4*d^5*e^3 - 9*a*c^3*d^3*e^5)*x^3 + 6*(26*c^4*d^6*e^2
- 23*a*c^3*d^4*e^4 + 7*a^2*c^2*d^2*e^6)*x^2 + 2*(194*c^4*d^7*e - 426*a*c^3
*d^5*e^3 + 357*a^2*c^2*d^3*e^5 - 105*a^3*c*d*e^7)*x)*sqrt(e*x + d))/(c^6*d
^6*x + a*c^5*d^5*e)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(d+ex)^{13/2}}{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^2}\,dx=\text{Timed out}$$

input
```
integrate((e*x+d)**(13/2)/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**2,x)
```

output
```
Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(d+ex)^{13/2}}{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^2}\,dx=\text{Exception raised: ValueError}$$

input
```
integrate((e*x+d)^(13/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^2,x, algorithm=
"maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(a*e^2-c*d^2>0)', see `assume?` f
or more de
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 422 vs. $2(182) = 364$.

Time = 0.18 (sec) , antiderivative size = 422, normalized size of antiderivative = 2.01

$$\int \frac{(d+ex)^{13/2}}{(ade+(cd^2+ae^2)x+cdex^2)^2}\,dx = \frac{9\,(c^4d^8e-4\,ac^3d^6e^3+6\,a^2c^2d^4e^5-4\,a^3cd^2e^7+a^4e^9)\arctan\left(\frac{\sqrt{e}}{\sqrt{-c^2}}\right.}{\sqrt{-c^2d^3+acde^2}c^5d^5}$$
$$-\frac{\sqrt{ex+d}c^4d^8e-4\sqrt{ex+d}ac^3d^6e^3+6\sqrt{ex+d}a^2c^2d^4e^5-4\sqrt{ex+d}a^3cd^2e^7+\sqrt{ex+d}a^4e^9}{((ex+d)cd-cd^2+ae^2)c^5d^5}$$
$$+\frac{2\left(5\,(ex+d)^{\frac{7}{2}}c^{12}d^{12}e+14\,(ex+d)^{\frac{5}{2}}c^{12}d^{13}e+35\,(ex+d)^{\frac{3}{2}}c^{12}d^{14}e+140\sqrt{ex+d}c^{12}d^{15}e-14\,(ex+d)^{\frac{5}{2}}c\right.}{}$$

input
```
integrate((e*x+d)^(13/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^2,x, algorithm=
"giac")
```

output
```
9*(c^4*d^8*e - 4*a*c^3*d^6*e^3 + 6*a^2*c^2*d^4*e^5 - 4*a^3*c*d^2*e^7 + a^4
*e^9)*arctan(sqrt(e*x + d)*c*d/sqrt(-c^2*d^3 + a*c*d*e^2))/(sqrt(-c^2*d^3
+ a*c*d*e^2)*c^5*d^5) - (sqrt(e*x + d)*c^4*d^8*e - 4*sqrt(e*x + d)*a*c^3*d
^6*e^3 + 6*sqrt(e*x + d)*a^2*c^2*d^4*e^5 - 4*sqrt(e*x + d)*a^3*c*d^2*e^7 +
 sqrt(e*x + d)*a^4*e^9)/(((e*x + d)*c*d - c*d^2 + a*e^2)*c^5*d^5) + 2/35*(
5*(e*x + d)^(7/2)*c^12*d^12*e + 14*(e*x + d)^(5/2)*c^12*d^13*e + 35*(e*x +
 d)^(3/2)*c^12*d^14*e + 140*sqrt(e*x + d)*c^12*d^15*e - 14*(e*x + d)^(5/2)
*a*c^11*d^11*e^3 - 70*(e*x + d)^(3/2)*a*c^11*d^12*e^3 - 420*sqrt(e*x + d)*
a*c^11*d^13*e^3 + 35*(e*x + d)^(3/2)*a^2*c^10*d^10*e^5 + 420*sqrt(e*x + d)
*a^2*c^10*d^11*e^5 - 140*sqrt(e*x + d)*a^3*c^9*d^9*e^7)/(c^14*d^14)
```

Mupad [B] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 443, normalized size of antiderivative = 2.11

$$\int \frac{(d+ex)^{13/2}}{(ade+(cd^2+ae^2)\,x+cdex^2)^2}\,dx = \frac{2\,e\,(d+e\,x)^{7/2}}{7\,c^2\,d^2}$$
$$-\left(\frac{(2\,c^2\,d^3-2\,a\,c\,d\,e^2)\,\left(\frac{2\,e\,(a\,e^2-c\,d^2)^2}{c^4\,d^4}-\frac{2\,e\,(2\,c^2\,d^3-2\,a\,c\,d\,e^2)^2}{c^6\,d^6}\right)}{c^2\,d^2}+\frac{2\,e\,(a\,e^2-c\,d^2)^2\,(2\,c^2\,d^3-2\,a\,c\,d\,e^2)}{c^6\,d^6}\right)\sqrt{d+}$$
$$-\frac{\sqrt{d+e\,x}\,(a^4\,e^9-4\,a^3\,c\,d^2\,e^7+6\,a^2\,c^2\,d^4\,e^5-4\,a\,c^3\,d^6\,e^3+c^4\,d^8\,e)}{c^6\,d^6\,(d+e\,x)-c^6\,d^7+a\,c^5\,d^5\,e^2}$$
$$-\left(\frac{2\,e\,(a\,e^2-c\,d^2)^2}{3\,c^4\,d^4}-\frac{2\,e\,(2\,c^2\,d^3-2\,a\,c\,d\,e^2)^2}{3\,c^6\,d^6}\right)(d+e\,x)^{3/2}$$
$$+\frac{2\,e\,(2\,c^2\,d^3-2\,a\,c\,d\,e^2)\,(d+e\,x)^{5/2}}{5\,c^4\,d^4}$$
$$+\frac{9\,e\,\mathrm{atan}\left(\frac{\sqrt{c}\,\sqrt{d}\,e\,(a\,e^2-c\,d^2)^{7/2}\,\sqrt{d+e\,x}}{a^4\,e^9-4\,a^3\,c\,d^2\,e^7+6\,a^2\,c^2\,d^4\,e^5-4\,a\,c^3\,d^6\,e^3+c^4\,d^8\,e}\right)(a\,e^2-c\,d^2)^{7/2}}{c^{11/2}\,d^{11/2}}$$

input
```
int((d + e*x)^(13/2)/(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^2,x)
```

output
```
(2*e*(d + e*x)^(7/2))/(7*c^2*d^2) - (((2*c^2*d^3 - 2*a*c*d*e^2)*((2*e*(a*e
^2 - c*d^2)^2)/(c^4*d^4) - (2*e*(2*c^2*d^3 - 2*a*c*d*e^2)^2)/(c^6*d^6)))/(
c^2*d^2) + (2*e*(a*e^2 - c*d^2)^2*(2*c^2*d^3 - 2*a*c*d*e^2))/(c^6*d^6))*(d
 + e*x)^(1/2) - ((d + e*x)^(1/2)*(a^4*e^9 + c^4*d^8*e - 4*a*c^3*d^6*e^3 -
4*a^3*c*d^2*e^7 + 6*a^2*c^2*d^4*e^5))/(c^6*d^6*(d + e*x) - c^6*d^7 + a*c^5
*d^5*e^2) - ((2*e*(a*e^2 - c*d^2)^2)/(3*c^4*d^4) - (2*e*(2*c^2*d^3 - 2*a*c
*d*e^2)^2)/(3*c^6*d^6))*(d + e*x)^(3/2) + (2*e*(2*c^2*d^3 - 2*a*c*d*e^2)*(
d + e*x)^(5/2))/(5*c^4*d^4) + (9*e*atan((c^(1/2)*d^(1/2)*e*(a*e^2 - c*d^2)
^(7/2)*(d + e*x)^(1/2))/(a^4*e^9 + c^4*d^8*e - 4*a*c^3*d^6*e^3 - 4*a^3*c*d
^2*e^7 + 6*a^2*c^2*d^4*e^5))*(a*e^2 - c*d^2)^(7/2))/(c^(11/2)*d^(11/2))
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 804, normalized size of antiderivative = 3.83

$$\int \frac{(d+ex)^{13/2}}{(ade+(cd^2+ae^2)\,x+cdex^2)^2}\,dx = \text{Too large to display}$$

input
```
int((e*x+d)^(13/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^2,x)
```

output
```
(315*sqrt(d)*sqrt(c)*sqrt(a*e**2 - c*d**2)*atan((sqrt(d + e*x)*c*d)/(sqrt(
d)*sqrt(c)*sqrt(a*e**2 - c*d**2)))*a**4*e**8 - 945*sqrt(d)*sqrt(c)*sqrt(a*
e**2 - c*d**2)*atan((sqrt(d + e*x)*c*d)/(sqrt(d)*sqrt(c)*sqrt(a*e**2 - c*d
**2)))*a**3*c*d**2*e**6 + 315*sqrt(d)*sqrt(c)*sqrt(a*e**2 - c*d**2)*atan((
sqrt(d + e*x)*c*d)/(sqrt(d)*sqrt(c)*sqrt(a*e**2 - c*d**2)))*a**3*c*d*e**7*
x + 945*sqrt(d)*sqrt(c)*sqrt(a*e**2 - c*d**2)*atan((sqrt(d + e*x)*c*d)/(sq
rt(d)*sqrt(c)*sqrt(a*e**2 - c*d**2)))*a**2*c**2*d**4*e**4 - 945*sqrt(d)*sq
rt(c)*sqrt(a*e**2 - c*d**2)*atan((sqrt(d + e*x)*c*d)/(sqrt(d)*sqrt(c)*sqrt
(a*e**2 - c*d**2)))*a**2*c**2*d**3*e**5*x - 315*sqrt(d)*sqrt(c)*sqrt(a*e**
2 - c*d**2)*atan((sqrt(d + e*x)*c*d)/(sqrt(d)*sqrt(c)*sqrt(a*e**2 - c*d**2
)))*a*c**3*d**6*e**2 + 945*sqrt(d)*sqrt(c)*sqrt(a*e**2 - c*d**2)*atan((sqr
t(d + e*x)*c*d)/(sqrt(d)*sqrt(c)*sqrt(a*e**2 - c*d**2)))*a*c**3*d**5*e**3*
x - 315*sqrt(d)*sqrt(c)*sqrt(a*e**2 - c*d**2)*atan((sqrt(d + e*x)*c*d)/(sq
rt(d)*sqrt(c)*sqrt(a*e**2 - c*d**2)))*c**4*d**7*e*x - 315*sqrt(d + e*x)*a*
*4*c*d*e**8 + 1050*sqrt(d + e*x)*a**3*c**2*d**3*e**6 - 210*sqrt(d + e*x)*a
**3*c**2*d**2*e**7*x - 1218*sqrt(d + e*x)*a**2*c**3*d**5*e**4 + 714*sqrt(d
 + e*x)*a**2*c**3*d**4*e**5*x + 42*sqrt(d + e*x)*a**2*c**3*d**3*e**6*x**2
+ 528*sqrt(d + e*x)*a*c**4*d**7*e**2 - 852*sqrt(d + e*x)*a*c**4*d**6*e**3*
x - 138*sqrt(d + e*x)*a*c**4*d**5*e**4*x**2 - 18*sqrt(d + e*x)*a*c**4*d**4
*e**5*x**3 - 35*sqrt(d + e*x)*c**5*d**9 + 388*sqrt(d + e*x)*c**5*d**8*e...
```

3.183 $\int \frac{(d+ex)^{11/2}}{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^2}\,dx$

Optimal result

Integrand size = 37, antiderivative size = 178

$$\begin{aligned}\int \frac{(d+ex)^{11/2}}{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^2}\,dx &= \frac{7e\left(cd^2-ae^2\right)^2\sqrt{d+ex}}{c^4d^4} \\ &+ \frac{7e\left(cd^2-ae^2\right)(d+ex)^{3/2}}{3c^3d^3} + \frac{7e(d+ex)^{5/2}}{5c^2d^2} \\ &- \frac{(d+ex)^{7/2}}{cd(ae+cdx)} - \frac{7e\left(cd^2-ae^2\right)^{5/2}\operatorname{arctanh}\left(\frac{\sqrt{c}\sqrt{d}\sqrt{d+ex}}{\sqrt{cd^2-ae^2}}\right)}{c^{9/2}d^{9/2}}\end{aligned}$$

output
```
7*e*(-a*e^2+c*d^2)^2*(e*x+d)^(1/2)/c^4/d^4+7/3*e*(-a*e^2+c*d^2)*(e*x+d)^(3
/2)/c^3/d^3+7/5*e*(e*x+d)^(5/2)/c^2/d^2-(e*x+d)^(7/2)/c/d/(c*d*x+a*e)-7*e*
(-a*e^2+c*d^2)^(5/2)*arctanh(c^(1/2)*d^(1/2)*(e*x+d)^(1/2)/(-a*e^2+c*d^2)^
(1/2))/c^(9/2)/d^(9/2)
```

Mathematica [A] (verified)

Time = 0.40 (sec) , antiderivative size = 191, normalized size of antiderivative = 1.07

$$\int \frac{(d+ex)^{11/2}}{(ade+(cd^2+ae^2)\,x+cdex^2)^2}\,dx = \frac{\sqrt{d+ex}(105a^3e^6-35a^2cde^4(7d-2ex)+7ac^2d^2e^2(23d^2-24dex}{15c^4d^4(ae+c} - \frac{7e(-cd^2+ae^2)^{5/2}\arctan\left(\frac{\sqrt{c}\sqrt{d}\sqrt{d+ex}}{\sqrt{-cd^2+ae^2}}\right)}{c^{9/2}d^{9/2}}$$

input
```
Integrate[(d + e*x)^(11/2)/(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^2,x]
```

output
```
(Sqrt[d + e*x]*(105*a^3*e^6 - 35*a^2*c*d*e^4*(7*d - 2*e*x) + 7*a*c^2*d^2*e
^2*(23*d^2 - 24*d*e*x - 2*e^2*x^2) + c^3*d^3*(-15*d^3 + 116*d^2*e*x + 32*d
*e^2*x^2 + 6*e^3*x^3)))/(15*c^4*d^4*(a*e + c*d*x)) - (7*e*(-(c*d^2) + a*e^
2)^(5/2)*ArcTan[(Sqrt[c]*Sqrt[d]*Sqrt[d + e*x])/Sqrt[-(c*d^2) + a*e^2]])/(
c^(9/2)*d^(9/2))
```

Rubi [A] (verified)

Time = 0.48 (sec) , antiderivative size = 209, normalized size of antiderivative = 1.17, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.189$, Rules used = {1121, 51, 60, 60, 60, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex)^{11/2}}{(x\,(ae^2+cd^2)+ade+cdex^2)^2}\,dx$$

$$\Big\downarrow \text{ 1121}$$

$$\int \frac{(d+ex)^{7/2}}{(ae+cdx)^2}dx$$

$$\Big\downarrow \text{ 51}$$

$$\frac{7e\int \frac{(d+ex)^{5/2}}{ae+cdx}dx}{2cd} - \frac{(d+ex)^{7/2}}{cd(ae+cdx)}$$

$$\Big\downarrow \ 60$$

$$\frac{7e\left(\frac{\left(d^2-\frac{ae^2}{c}\right)\int \frac{(d+ex)^{3/2}}{ae+cdx}dx}{d}+\frac{2(d+ex)^{5/2}}{5cd}\right)}{2cd}-\frac{(d+ex)^{7/2}}{cd(ae+cdx)}$$

$$\Big\downarrow \ 60$$

$$\frac{7e\left(\frac{\left(d^2-\frac{ae^2}{c}\right)\left(\frac{\left(d^2-\frac{ae^2}{c}\right)\int \frac{\sqrt{d+ex}}{ae+cdx}dx}{d}+\frac{2(d+ex)^{3/2}}{3cd}\right)}{d}+\frac{2(d+ex)^{5/2}}{5cd}\right)}{2cd}-\frac{(d+ex)^{7/2}}{cd(ae+cdx)}$$

$$\Big\downarrow \ 60$$

$$\frac{7e\left(\frac{\left(d^2-\frac{ae^2}{c}\right)\left(\frac{\left(d^2-\frac{ae^2}{c}\right)\left(\frac{\left(d^2-\frac{ae^2}{c}\right)\int \frac{1}{(ae+cdx)\sqrt{d+ex}}dx}{d}+\frac{2\sqrt{d+ex}}{cd}\right)}{d}+\frac{2(d+ex)^{3/2}}{3cd}\right)}{d}+\frac{2(d+ex)^{5/2}}{5cd}\right)}{2cd}-\frac{(d+ex)^{7/2}}{cd(ae+cdx)}$$

$$\Big\downarrow \ 73$$

$$\frac{7e\left(\frac{\left(d^2-\frac{ae^2}{c}\right)\left(\frac{\left(d^2-\frac{ae^2}{c}\right)\left(\frac{2\left(d^2-\frac{ae^2}{c}\right)\int \frac{1}{-\frac{cd^2}{e}+\frac{c(d+ex)d}{e}+ae}d\sqrt{d+ex}}{de}+\frac{2\sqrt{d+ex}}{cd}\right)}{d}+\frac{2(d+ex)^{3/2}}{3cd}\right)}{d}+\frac{2(d+ex)^{5/2}}{5cd}\right)}{2cd}-\frac{(d+ex)^{7/2}}{cd(ae+cdx)}$$

$$\Big\downarrow \text{221}$$

$$\frac{7e\left(\frac{\left(d^2-\frac{ae^2}{c}\right)\left(\frac{\left(d^2-\frac{ae^2}{c}\right)\left(\frac{2\sqrt{d+ex}}{cd}-\frac{2\left(d^2-\frac{ae^2}{c}\right)\operatorname{arctanh}\left(\frac{\sqrt{c}\sqrt{d}\sqrt{d+ex}}{\sqrt{cd^2-ae^2}}\right)}{\sqrt{c}d^{3/2}\sqrt{cd^2-ae^2}}\right)}{d}+\frac{2(d+ex)^{3/2}}{3cd}\right)}{d}+\frac{2(d+ex)^{5/2}}{5cd}\right)}{2cd}-\frac{(d+ex)^{7/2}}{cd(ae+cdx)}$$

input
```
Int[(d + e*x)^(11/2)/(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^2,x]
```

output
```
-((d + e*x)^(7/2)/(c*d*(a*e + c*d*x))) + (7*e*((2*(d + e*x)^(5/2))/(5*c*d)
 + ((d^2 - (a*e^2)/c)*((2*(d + e*x)^(3/2))/(3*c*d) + ((d^2 - (a*e^2)/c)*((
2*Sqrt[d + e*x])/(c*d) - (2*(d^2 - (a*e^2)/c)*ArcTanh[(Sqrt[c]*Sqrt[d]*Sqr
t[d + e*x])/Sqrt[c*d^2 - a*e^2]])/(Sqrt[c]*d^(3/2)*Sqrt[c*d^2 - a*e^2])))/
d))/d))/(2*c*d)
```

Defintions of rubi rules used

rule 51
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Simp[d*(n/(b*(m + 1)))
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d, n}, x
] && ILtQ[m, -1] && FractionQ[n] && GtQ[n, 0]
```

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 1121
```
Int[((d_) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Int[ExpandIntegrand[(d + e*x)^(m + p)*(a/d + (c/e)*x)^p, x], x]
/; FreeQ[{a, b, c, d, e, m, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && (Int
egerQ[p] || (GtQ[a, 0] && GtQ[d, 0] && LtQ[c, 0]))
```

Maple [A] (verified)

Time = 4.12 (sec) , antiderivative size = 204, normalized size of antiderivative = 1.15

method	result
pseudoelliptic	$-\frac{7\left(e\left(a e^2-c d^2\right)^3(cdx+ae)\arctan\left(\frac{cd\sqrt{ex+d}}{\sqrt{cd\left(a e^2-c d^2\right)}}\right)-\left(-\frac{\left(-\frac{2}{5}e^3x^3-\frac{32}{15}d e^2x^2-\frac{116}{15}d^2ex+d^3\right)d^3c^3}{7}+\frac{23e^2\left(-\frac{2}{23}e^2x^2-\right.}{}\right.}{\sqrt{cd(a e^2-c d^2)}\,d^4c^4(cdx+ae)}$
risch	$\frac{2e\left(3x^2c^2d^2e^2-10xacd e^3+16x c^2d^3e+45a^2e^4-100ac d^2e^2+58c^2d^4\right)\sqrt{ex+d}}{15d^4c^4}-\frac{\left(2e^6a^3-6d^2e^4a^2c+6d^4e^2a c^2-2d^6c^3\right.}{}$
derivativedivides	$2e\left(\frac{\frac{c^2d^2(ex+d)^{\frac{5}{2}}}{5}-\frac{2acd\,e^2(ex+d)^{\frac{3}{2}}}{3}+\frac{2c^2d^3(ex+d)^{\frac{3}{2}}}{3}+3a^2e^4\sqrt{ex+d}-6ac\,d^2e^2\sqrt{ex+d}+3c^2d^4\sqrt{ex+d}}{c^4d^4}-\frac{\left(-\frac{1}{2}e^6a^3+\frac{3}{2}\right.}{}\right.$
default	$2e\left(\frac{\frac{c^2d^2(ex+d)^{\frac{5}{2}}}{5}-\frac{2acd\,e^2(ex+d)^{\frac{3}{2}}}{3}+\frac{2c^2d^3(ex+d)^{\frac{3}{2}}}{3}+3a^2e^4\sqrt{ex+d}-6ac\,d^2e^2\sqrt{ex+d}+3c^2d^4\sqrt{ex+d}}{c^4d^4}-\frac{\left(-\frac{1}{2}e^6a^3+\frac{3}{2}\right.}{}\right.$

input
```
int((e*x+d)^(11/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^2,x,method=_RETURNVER
BOSE)
```

output
```
-7/(c*d*(a*e^2-c*d^2))^(1/2)*(e*(a*e^2-c*d^2)^3*(c*d*x+a*e)*arctan(c*d*(e*
x+d)^(1/2)/(c*d*(a*e^2-c*d^2))^(1/2))-(-1/7*(-2/5*e^3*x^3-32/15*d*e^2*x^2-
116/15*d^2*e*x+d^3)*d^3*c^3+23/15*e^2*(-2/23*e^2*x^2-24/23*d*e*x+d^2)*a*d^
2*c^2-7/3*e^4*(-2/7*e*x+d)*a^2*d*c+e^6*a^3)*(e*x+d)^(1/2)*(c*d*(a*e^2-c*d^
2))^(1/2))/d^4/c^4/(c*d*x+a*e)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 586, normalized size of antiderivative = 3.29

$$\int \frac{(d+ex)^{11/2}}{(ade+(cd^2+ae^2)\,x+cdex^2)^2}\,dx = \left[\frac{105\,(ac^2d^4e^2-2\,a^2cd^2e^4+a^3e^6+(c^3d^5e-2\,ac^2d^3e^3+a^2cde^5)x)\right.$$

$$-\frac{105\,(ac^2d^4e^2-2\,a^2cd^2e^4+a^3e^6+(c^3d^5e-2\,ac^2d^3e^3+a^2cde^5)x)\sqrt{-\frac{cd^2-ae^2}{cd}}\arctan\left(-\frac{\sqrt{ex+d}cd\sqrt{-\frac{cd^2-ae^2}{cd}}}{cd^2-ae^2}\right.}{}$$

input
```
integrate((e*x+d)^(11/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^2,x, algorithm=
"fricas")
```

output
```
[1/30*(105*(a*c^2*d^4*e^2 - 2*a^2*c*d^2*e^4 + a^3*e^6 + (c^3*d^5*e - 2*a*c
^2*d^3*e^3 + a^2*c*d*e^5)*x)*sqrt((c*d^2 - a*e^2)/(c*d))*log((c*d*e*x + 2*
c*d^2 - a*e^2 - 2*sqrt(e*x + d)*c*d*sqrt((c*d^2 - a*e^2)/(c*d)))/(c*d*x +
a*e)) + 2*(6*c^3*d^3*e^3*x^3 - 15*c^3*d^6 + 161*a*c^2*d^4*e^2 - 245*a^2*c*
d^2*e^4 + 105*a^3*e^6 + 2*(16*c^3*d^4*e^2 - 7*a*c^2*d^2*e^4)*x^2 + 2*(58*c
^3*d^5*e - 84*a*c^2*d^3*e^3 + 35*a^2*c*d*e^5)*x)*sqrt(e*x + d))/(c^5*d^5*x
 + a*c^4*d^4*e), -1/15*(105*(a*c^2*d^4*e^2 - 2*a^2*c*d^2*e^4 + a^3*e^6 + (
c^3*d^5*e - 2*a*c^2*d^3*e^3 + a^2*c*d*e^5)*x)*sqrt(-(c*d^2 - a*e^2)/(c*d))
*arctan(-sqrt(e*x + d)*c*d*sqrt(-(c*d^2 - a*e^2)/(c*d))/(c*d^2 - a*e^2)) -
 (6*c^3*d^3*e^3*x^3 - 15*c^3*d^6 + 161*a*c^2*d^4*e^2 - 245*a^2*c*d^2*e^4 +
 105*a^3*e^6 + 2*(16*c^3*d^4*e^2 - 7*a*c^2*d^2*e^4)*x^2 + 2*(58*c^3*d^5*e
- 84*a*c^2*d^3*e^3 + 35*a^2*c*d*e^5)*x)*sqrt(e*x + d))/(c^5*d^5*x + a*c^4*
d^4*e)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(d+ex)^{11/2}}{(ade+(cd^2+ae^2)x+cdex^2)^2}\,dx = \text{Timed out}$$

input
```
integrate((e*x+d)**(11/2)/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**2,x)
```

output
```
Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(d+ex)^{11/2}}{(ade+(cd^2+ae^2)x+cdex^2)^2}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((e*x+d)^(11/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^2,x, algorithm=
"maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(a*e^2-c*d^2>0)', see `assume?` f
or more de
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 312 vs. $2(152) = 304$.

Time = 0.16 (sec) , antiderivative size = 312, normalized size of antiderivative = 1.75

$$\int \frac{(d+ex)^{11/2}}{(ade+(cd^2+ae^2)x+cdex^2)^2}\,dx = \frac{7\,(c^3d^6e-3\,ac^2d^4e^3+3\,a^2cd^2e^5-a^3e^7)\arctan\left(\frac{\sqrt{ex+d}cd}{\sqrt{-c^2d^3+acde^2}}\right)}{\sqrt{-c^2d^3+acde^2}c^4d^4}$$
$$-\frac{\sqrt{ex+d}c^3d^6e-3\sqrt{ex+d}ac^2d^4e^3+3\sqrt{ex+d}a^2cd^2e^5-\sqrt{ex+d}a^3e^7}{((ex+d)cd-cd^2+ae^2)c^4d^4}$$
$$+\frac{2\left(3\,(ex+d)^{\frac{5}{2}}c^8d^8e+10\,(ex+d)^{\frac{3}{2}}c^8d^9e+45\sqrt{ex+d}c^8d^{10}e-10\,(ex+d)^{\frac{3}{2}}ac^7d^7e^3-90\sqrt{ex+d}ac^7d^8e\right.}{15\,c^{10}d^{10}}$$

input
```
integrate((e*x+d)^(11/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^2,x, algorithm=
"giac")
```

output
```
7*(c^3*d^6*e - 3*a*c^2*d^4*e^3 + 3*a^2*c*d^2*e^5 - a^3*e^7)*arctan(sqrt(e*
x + d)*c*d/sqrt(-c^2*d^3 + a*c*d*e^2))/(sqrt(-c^2*d^3 + a*c*d*e^2)*c^4*d^4
) - (sqrt(e*x + d)*c^3*d^6*e - 3*sqrt(e*x + d)*a*c^2*d^4*e^3 + 3*sqrt(e*x
+ d)*a^2*c*d^2*e^5 - sqrt(e*x + d)*a^3*e^7)/(((e*x + d)*c*d - c*d^2 + a*e^
2)*c^4*d^4) + 2/15*(3*(e*x + d)^(5/2)*c^8*d^8*e + 10*(e*x + d)^(3/2)*c^8*d
^9*e + 45*sqrt(e*x + d)*c^8*d^10*e - 10*(e*x + d)^(3/2)*a*c^7*d^7*e^3 - 90
*sqrt(e*x + d)*a*c^7*d^8*e^3 + 45*sqrt(e*x + d)*a^2*c^6*d^6*e^5)/(c^10*d^1
0)
```

Mupad [B] (verification not implemented)

Time = 5.45 (sec) , antiderivative size = 290, normalized size of antiderivative = 1.63

$$\int \frac{(d+ex)^{11/2}}{(ade+(cd^2+ae^2)x+cdex^2)^2}\,dx = \frac{\sqrt{d+e\,x}\,(a^3\,e^7-3\,a^2\,c\,d^2\,e^5+3\,a\,c^2\,d^4\,e^3-c^3\,d^6\,e)}{c^5\,d^5\,(d+e\,x)-c^5\,d^6+a\,c^4\,d^4\,e^2}$$
$$-\left(\frac{2\,e\,(a\,e^2-c\,d^2)^2}{c^4\,d^4}-\frac{2\,e\,(2\,c^2\,d^3-2\,a\,c\,d\,e^2)^2}{c^6\,d^6}\right)\sqrt{d+e\,x}$$
$$+\frac{2\,e\,(d+e\,x)^{5/2}}{5\,c^2\,d^2}+\frac{2\,e\,(2\,c^2\,d^3-2\,a\,c\,d\,e^2)\,(d+e\,x)^{3/2}}{3\,c^4\,d^4}$$
$$-\frac{7\,e\,\mathrm{atan}\left(\frac{\sqrt{c}\,\sqrt{d}\,e\,(a\,e^2-c\,d^2)^{5/2}\,\sqrt{d+e\,x}}{a^3\,e^7-3\,a^2\,c\,d^2\,e^5+3\,a\,c^2\,d^4\,e^3-c^3\,d^6\,e}\right)\,(a\,e^2-c\,d^2)^{5/2}}{c^{9/2}\,d^{9/2}}$$

input
```
int((d + e*x)^(11/2)/(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^2,x)
```

output
```
((d + e*x)^(1/2)*(a^3*e^7 - c^3*d^6*e + 3*a*c^2*d^4*e^3 - 3*a^2*c*d^2*e^5)
)/(c^5*d^5*(d + e*x) - c^5*d^6 + a*c^4*d^4*e^2) - ((2*e*(a*e^2 - c*d^2)^2)
/(c^4*d^4) - (2*e*(2*c^2*d^3 - 2*a*c*d*e^2)^2)/(c^6*d^6))*(d + e*x)^(1/2)
+ (2*e*(d + e*x)^(5/2))/(5*c^2*d^2) + (2*e*(2*c^2*d^3 - 2*a*c*d*e^2)*(d +
e*x)^(3/2))/(3*c^4*d^4) - (7*e*atan((c^(1/2)*d^(1/2)*e*(a*e^2 - c*d^2)^(5/
2)*(d + e*x)^(1/2))/(a^3*e^7 - c^3*d^6*e + 3*a*c^2*d^4*e^3 - 3*a^2*c*d^2*e
^5))*(a*e^2 - c*d^2)^(5/2))/(c^(9/2)*d^(9/2))
```

Reduce [B] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 570, normalized size of antiderivative = 3.20

$$\int \frac{(d+ex)^{11/2}}{\left(ade+(cd^2+ae^2)x+cdex^2\right)^2}\,dx = \frac{-105\sqrt{d}\,\sqrt{c}\,\sqrt{a\,e^2-c\,d^2}\,atan\left(\frac{\sqrt{ex+d}\,cd}{\sqrt{d}\,\sqrt{c}\,\sqrt{a\,e^2-c\,d^2}}\right)a^3e^6+210\sqrt{d}\,\sqrt{c}\,\sqrt{}$$

input
```
int((e*x+d)^(11/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^2,x)
```

output
```
( - 105*sqrt(d)*sqrt(c)*sqrt(a*e**2 - c*d**2)*atan((sqrt(d + e*x)*c*d)/(sq
rt(d)*sqrt(c)*sqrt(a*e**2 - c*d**2)))*a**3*e**6 + 210*sqrt(d)*sqrt(c)*sqrt
(a*e**2 - c*d**2)*atan((sqrt(d + e*x)*c*d)/(sqrt(d)*sqrt(c)*sqrt(a*e**2 -
c*d**2)))*a**2*c*d**2*e**4 - 105*sqrt(d)*sqrt(c)*sqrt(a*e**2 - c*d**2)*ata
n((sqrt(d + e*x)*c*d)/(sqrt(d)*sqrt(c)*sqrt(a*e**2 - c*d**2)))*a**2*c*d*e*
*5*x - 105*sqrt(d)*sqrt(c)*sqrt(a*e**2 - c*d**2)*atan((sqrt(d + e*x)*c*d)/
(sqrt(d)*sqrt(c)*sqrt(a*e**2 - c*d**2)))*a*c**2*d**4*e**2 + 210*sqrt(d)*sq
rt(c)*sqrt(a*e**2 - c*d**2)*atan((sqrt(d + e*x)*c*d)/(sqrt(d)*sqrt(c)*sqrt
(a*e**2 - c*d**2)))*a*c**2*d**3*e**3*x - 105*sqrt(d)*sqrt(c)*sqrt(a*e**2 -
 c*d**2)*atan((sqrt(d + e*x)*c*d)/(sqrt(d)*sqrt(c)*sqrt(a*e**2 - c*d**2)))
*c**3*d**5*e*x + 105*sqrt(d + e*x)*a**3*c*d*e**6 - 245*sqrt(d + e*x)*a**2*
c**2*d**3*e**4 + 70*sqrt(d + e*x)*a**2*c**2*d**2*e**5*x + 161*sqrt(d + e*x
)*a*c**3*d**5*e**2 - 168*sqrt(d + e*x)*a*c**3*d**4*e**3*x - 14*sqrt(d + e*
x)*a*c**3*d**3*e**4*x**2 - 15*sqrt(d + e*x)*c**4*d**7 + 116*sqrt(d + e*x)*
c**4*d**6*e*x + 32*sqrt(d + e*x)*c**4*d**5*e**2*x**2 + 6*sqrt(d + e*x)*c**
4*d**4*e**3*x**3)/(15*c**5*d**5*(a*e + c*d*x))
```

3.184 $\int \frac{(d+ex)^{9/2}}{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^2}\,dx$

Optimal result

Integrand size = 37, antiderivative size = 144

$$\int \frac{(d+ex)^{9/2}}{(ade+(cd^2+ae^2)\,x+cdex^2)^2}\,dx = \frac{5e(cd^2-ae^2)\sqrt{d+ex}}{c^3d^3} + \frac{5e(d+ex)^{3/2}}{3c^2d^2} - \frac{(d+ex)^{5/2}}{cd(ae+cdx)} - \frac{5e(cd^2-ae^2)^{3/2}\operatorname{arctanh}\left(\frac{\sqrt{c}\sqrt{d}\sqrt{d+ex}}{\sqrt{cd^2-ae^2}}\right)}{c^{7/2}d^{7/2}}$$

output
```
5*e*(-a*e^2+c*d^2)*(e*x+d)^(1/2)/c^3/d^3+5/3*e*(e*x+d)^(3/2)/c^2/d^2-(e*x+
d)^(5/2)/c/d/(c*d*x+a*e)-5*e*(-a*e^2+c*d^2)^(3/2)*arctanh(c^(1/2)*d^(1/2)*
(e*x+d)^(1/2)/(-a*e^2+c*d^2)^(1/2))/c^(7/2)/d^(7/2)
```

Mathematica [A] (verified)

Time = 0.36 (sec) , antiderivative size = 146, normalized size of antiderivative = 1.01

$$\int \frac{(d+ex)^{9/2}}{(ade+(cd^2+ae^2)\,x+cdex^2)^2}\,dx = -\frac{\sqrt{d+ex}(15a^2e^4+10acde^2(-2d+ex)+c^2d^2(3d^2-14dex-2e^2x^2))}{3c^3d^3(ae+cdx)} + \frac{5e(-cd^2+ae^2)^{3/2}\arctan\left(\frac{\sqrt{c}\sqrt{d}\sqrt{d+ex}}{\sqrt{-cd^2+ae^2}}\right)}{c^{7/2}d^{7/2}}$$

input
```
Integrate[(d + e*x)^(9/2)/(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^2,x]
```

output
```
-1/3*(Sqrt[d + e*x]*(15*a^2*e^4 + 10*a*c*d*e^2*(-2*d + e*x) + c^2*d^2*(3*d
^2 - 14*d*e*x - 2*e^2*x^2)))/(c^3*d^3*(a*e + c*d*x)) + (5*e*(-(c*d^2) + a*
e^2)^(3/2)*ArcTan[(Sqrt[c]*Sqrt[d]*Sqrt[d + e*x])/Sqrt[-(c*d^2) + a*e^2]])
/(c^(7/2)*d^(7/2))
```

Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 172, normalized size of antiderivative = 1.19, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.162$, Rules used = {1121, 51, 60, 60, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex)^{9/2}}{(x(ae^2+cd^2)+ade+cdex^2)^2}dx$$

$$\Big\downarrow \text{ 1121}$$

$$\int \frac{(d+ex)^{5/2}}{(ae+cdx)^2}dx$$

$$\Big\downarrow \text{ 51}$$

$$\frac{5e\int \frac{(d+ex)^{3/2}}{ae+cdx}dx}{2cd}-\frac{(d+ex)^{5/2}}{cd(ae+cdx)}$$

$$\Big\downarrow \text{ 60}$$

$$\frac{5e\left(\frac{\left(d^2-\frac{ae^2}{c}\right)\int \frac{\sqrt{d+ex}}{ae+cdx}dx}{d}+\frac{2(d+ex)^{3/2}}{3cd}\right)}{2cd}-\frac{(d+ex)^{5/2}}{cd(ae+cdx)}$$

$$\Big\downarrow \text{ 60}$$

$$\frac{5e\left(\frac{\left(d^2-\frac{ae^2}{c}\right)\left(\frac{\left(d^2-\frac{ae^2}{c}\right)\int\frac{1}{(ae+cdx)\sqrt{d+ex}}dx}{d}+\frac{2\sqrt{d+ex}}{cd}\right)}{d}+\frac{2(d+ex)^{3/2}}{3cd}\right)}{2cd}-\frac{(d+ex)^{5/2}}{cd(ae+cdx)}$$

$\Big\downarrow$ 73

$$\frac{5e\left(\frac{\left(d^2-\frac{ae^2}{c}\right)\left(\frac{2\left(d^2-\frac{ae^2}{c}\right)\int\frac{1}{-\frac{cd^2}{e}+\frac{c(d+ex)d}{e}+ae}d\sqrt{d+ex}}{de}+\frac{2\sqrt{d+ex}}{cd}\right)}{d}+\frac{2(d+ex)^{3/2}}{3cd}\right)}{2cd}-\frac{(d+ex)^{5/2}}{cd(ae+cdx)}$$

$\Big\downarrow$ 221

$$\frac{5e\left(\frac{\left(d^2-\frac{ae^2}{c}\right)\left(\frac{2\sqrt{d+ex}}{cd}-\frac{2\left(d^2-\frac{ae^2}{c}\right)\text{arctanh}\left(\frac{\sqrt{c}\sqrt{d}\sqrt{d+ex}}{\sqrt{cd^2-ae^2}}\right)}{\sqrt{c}d^{3/2}\sqrt{cd^2-ae^2}}\right)}{d}+\frac{2(d+ex)^{3/2}}{3cd}\right)}{2cd}-\frac{(d+ex)^{5/2}}{cd(ae+cdx)}$$

input
```
Int[(d + e*x)^(9/2)/(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^2,x]
```

output
```
-((d + e*x)^(5/2)/(c*d*(a*e + c*d*x))) + (5*e*((2*(d + e*x)^(3/2))/(3*c*d)
 + ((d^2 - (a*e^2)/c)*((2*Sqrt[d + e*x])/(c*d) - (2*(d^2 - (a*e^2)/c)*ArcT
anh[(Sqrt[c]*Sqrt[d]*Sqrt[d + e*x])/Sqrt[c*d^2 - a*e^2]])/(Sqrt[c]*d^(3/2)
*Sqrt[c*d^2 - a*e^2])))/d))/(2*c*d)
```

Defintions of rubi rules used

rule 51
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Simp[d*(n/(b*(m + 1)))
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d, n}, x
] && ILtQ[m, -1] && FractionQ[n] && GtQ[n, 0]
```

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 1121
```
Int[((d_) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Int[ExpandIntegrand[(d + e*x)^(m + p)*(a/d + (c/e)*x)^p, x], x]
/; FreeQ[{a, b, c, d, e, m, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && (Int
egerQ[p] || (GtQ[a, 0] && GtQ[d, 0] && LtQ[c, 0]))
```

Maple [A] (verified)

Time = 4.16 (sec) , antiderivative size = 151, normalized size of antiderivative = 1.05

method	result
risch	$-\frac{2e(-cdxe+6a\,e^2-7c\,d^2)\sqrt{ex+d}}{3d^3c^3}+\frac{(2a^2e^4-4ac\,d^2e^2+2c^2d^4)e\left(-\frac{\sqrt{ex+d}}{2\left(cd(ex+d)+a\,e^2-c\,d^2\right)}+\frac{5\arctan\left(\frac{cd\sqrt{ex+d}}{\sqrt{cd\left(a\,e^2-c\,d^2\right)}}\right)}{2\sqrt{cd\left(a\,e^2-c\,d^2\right)}}\right)}{c^3d^3}$
pseudoelliptic	$-\frac{5\left(-e\left(a\,e^2-c\,d^2\right)^2(cdx+ae)\arctan\left(\frac{cd\sqrt{ex+d}}{\sqrt{cd\left(a\,e^2-c\,d^2\right)}}\right)\right)+\sqrt{ex+d}\left(\frac{\left(-\frac{2}{3}e^2x^2-\frac{14}{3}dex+d^2\right)d^2c^2}{5}-\frac{4e^2\left(-\frac{ex}{2}+d\right)adc}{3}+a\right.}{\sqrt{cd(a\,e^2-c\,d^2)}\,d^3c^3(cdx+ae)}$
derivativedivides	$2e\left(-\frac{-\frac{cd(ex+d)^{\frac{3}{2}}}{3}+2a\,e^2\sqrt{ex+d}-2c\,d^2\sqrt{ex+d}}{c^3d^3}+\frac{\frac{\left(-\frac{1}{2}a^2e^4+ac\,d^2e^2-\frac{1}{2}c^2d^4\right)\sqrt{ex+d}}{cd(ex+d)+a\,e^2-c\,d^2}+\frac{5\left(a^2e^4-2ac\,d^2e^2+c^2d^4\right)\text{arcta}}{2\sqrt{cd\left(a\,e^2-\right.}}}{c^3d^3}\right.$
default	$2e\left(-\frac{-\frac{cd(ex+d)^{\frac{3}{2}}}{3}+2a\,e^2\sqrt{ex+d}-2c\,d^2\sqrt{ex+d}}{c^3d^3}+\frac{\frac{\left(-\frac{1}{2}a^2e^4+ac\,d^2e^2-\frac{1}{2}c^2d^4\right)\sqrt{ex+d}}{cd(ex+d)+a\,e^2-c\,d^2}+\frac{5\left(a^2e^4-2ac\,d^2e^2+c^2d^4\right)\text{arcta}}{2\sqrt{cd\left(a\,e^2-\right.}}}{c^3d^3}\right.$

input
```
int((e*x+d)^(9/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^2,x,method=_RETURNVERB
OSE)
```

output
```
-2/3*e*(-c*d*e*x+6*a*e^2-7*c*d^2)*(e*x+d)^(1/2)/d^3/c^3+1/c^3/d^3*(2*a^2*e
^4-4*a*c*d^2*e^2+2*c^2*d^4)*e*(-1/2*(e*x+d)^(1/2)/(c*d*(e*x+d)+a*e^2-c*d^2
)+5/2/(c*d*(a*e^2-c*d^2))^(1/2)*arctan(c*d*(e*x+d)^(1/2)/(c*d*(a*e^2-c*d^2
))^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 420, normalized size of antiderivative = 2.92

$$\int \frac{(d+ex)^{9/2}}{\left(ade+(cd^2+ae^2)x+cdex^2\right)^2}\,dx = \left[-\frac{15\left(acd^2e^2-a^2e^4+(c^2d^3e-acde^3)x\right)\sqrt{\frac{cd^2-ae^2}{cd}}\log\left(\frac{cdex+2\,cd^2-\cdots}{\cdots}\right)\cdots}{\cdots}\right.$$

$$-\frac{15\left(acd^2e^2-a^2e^4+(c^2d^3e-acde^3)x\right)\sqrt{-\frac{cd^2-ae^2}{cd}}\arctan\left(-\frac{\sqrt{ex+d}cd\sqrt{-\frac{cd^2-ae^2}{cd}}}{cd^2-ae^2}\right)-\left(2\,c^2d^2e^2x^2-3\,c^2d^4+\cdots\right.}{3\left(c^4d^4x+ac^3d^3e\right)}$$

input
```
integrate((e*x+d)^(9/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^2,x, algorithm="
fricas")
```

output
```
[-1/6*(15*(a*c*d^2*e^2 - a^2*e^4 + (c^2*d^3*e - a*c*d*e^3)*x)*sqrt((c*d^2
- a*e^2)/(c*d))*log((c*d*e*x + 2*c*d^2 - a*e^2 + 2*sqrt(e*x + d)*c*d*sqrt(
(c*d^2 - a*e^2)/(c*d)))/(c*d*x + a*e)) - 2*(2*c^2*d^2*e^2*x^2 - 3*c^2*d^4
+ 20*a*c*d^2*e^2 - 15*a^2*e^4 + 2*(7*c^2*d^3*e - 5*a*c*d*e^3)*x)*sqrt(e*x
+ d))/(c^4*d^4*x + a*c^3*d^3*e), -1/3*(15*(a*c*d^2*e^2 - a^2*e^4 + (c^2*d^
3*e - a*c*d*e^3)*x)*sqrt(-(c*d^2 - a*e^2)/(c*d))*arctan(-sqrt(e*x + d)*c*d
*sqrt(-(c*d^2 - a*e^2)/(c*d))/(c*d^2 - a*e^2)) - (2*c^2*d^2*e^2*x^2 - 3*c^
2*d^4 + 20*a*c*d^2*e^2 - 15*a^2*e^4 + 2*(7*c^2*d^3*e - 5*a*c*d*e^3)*x)*sqr
t(e*x + d))/(c^4*d^4*x + a*c^3*d^3*e)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(d+ex)^{9/2}}{\left(ade+(cd^2+ae^2)x+cdex^2\right)^2}\,dx = \text{Timed out}$$

input
```
integrate((e*x+d)**(9/2)/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**2,x)
```

output
```
Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(d+ex)^{9/2}}{(ade+(cd^2+ae^2)\,x+cdex^2)^2}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((e*x+d)^(9/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^2,x, algorithm="
maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(a*e^2-c*d^2>0)', see `assume?` f
or more de
```

Giac [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 218, normalized size of antiderivative = 1.51

$$\int \frac{(d+ex)^{9/2}}{(ade+(cd^2+ae^2)\,x+cdex^2)^2}\,dx = \frac{5\,(c^2d^4e-2\,acd^2e^3+a^2e^5)\arctan\left(\frac{\sqrt{ex+d}cd}{\sqrt{-c^2d^3+acde^2}}\right)}{\sqrt{-c^2d^3+acde^2}c^3d^3} - \frac{\sqrt{ex+d}c^2d^4e-2\sqrt{ex+d}acd^2e^3+\sqrt{ex+d}a^2e^5}{((ex+d)cd-cd^2+ae^2)c^3d^3} + \frac{2\left((ex+d)^{\frac{3}{2}}c^4d^4e+6\sqrt{ex+d}c^4d^5e-6\sqrt{ex+d}ac^3d^3e^3\right)}{3\,c^6d^6}$$

input
```
integrate((e*x+d)^(9/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^2,x, algorithm="
giac")
```

output
```
5*(c^2*d^4*e - 2*a*c*d^2*e^3 + a^2*e^5)*arctan(sqrt(e*x + d)*c*d/sqrt(-c^2
*d^3 + a*c*d*e^2))/(sqrt(-c^2*d^3 + a*c*d*e^2)*c^3*d^3) - (sqrt(e*x + d)*c
^2*d^4*e - 2*sqrt(e*x + d)*a*c*d^2*e^3 + sqrt(e*x + d)*a^2*e^5)/(((e*x + d
)*c*d - c*d^2 + a*e^2)*c^3*d^3) + 2/3*((e*x + d)^(3/2)*c^4*d^4*e + 6*sqrt(
e*x + d)*c^4*d^5*e - 6*sqrt(e*x + d)*a*c^3*d^3*e^3)/(c^6*d^6)
```

Mupad [B] (verification not implemented)

Time = 5.30 (sec) , antiderivative size = 200, normalized size of antiderivative = 1.39

$$\int \frac{(d+ex)^{9/2}}{\left(ade+(cd^2+ae^2)\,x+cdex^2\right)^2}\,dx = \frac{2\,e\,{(d+e\,x)}^{3/2}}{3\,c^2\,d^2} - \frac{\sqrt{d+e\,x}\,\left(a^2\,e^5-2\,a\,c\,d^2\,e^3+c^2\,d^4\,e\right)}{c^4\,d^4\,\left(d+e\,x\right)-c^4\,d^5+a\,c^3\,d^3\,e^2} + \frac{2\,e\,\left(2\,c^2\,d^3-2\,a\,c\,d\,e^2\right)\,\sqrt{d+e\,x}}{c^4\,d^4} + \frac{5\,e\,\mathrm{atan}\left(\frac{\sqrt{c}\,\sqrt{d}\,e\,{\left(a\,e^2-c\,d^2\right)}^{3/2}\,\sqrt{d+e\,x}}{a^2\,e^5-2\,a\,c\,d^2\,e^3+c^2\,d^4\,e}\right)\,{\left(a\,e^2-c\,d^2\right)}^{3/2}}{c^{7/2}\,d^{7/2}}$$

input
```
int((d + e*x)^(9/2)/(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^2,x)
```

output
```
(2*e*(d + e*x)^(3/2))/(3*c^2*d^2) - ((d + e*x)^(1/2)*(a^2*e^5 + c^2*d^4*e
 - 2*a*c*d^2*e^3))/(c^4*d^4*(d + e*x) - c^4*d^5 + a*c^3*d^3*e^2) + (2*e*(2*
c^2*d^3 - 2*a*c*d*e^2)*(d + e*x)^(1/2))/(c^4*d^4) + (5*e*atan((c^(1/2)*d^(
1/2)*e*(a*e^2 - c*d^2)^(3/2)*(d + e*x)^(1/2))/(a^2*e^5 + c^2*d^4*e - 2*a*c
*d^2*e^3))*(a*e^2 - c*d^2)^(3/2))/(c^(7/2)*d^(7/2))
```

Reduce [B] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 359, normalized size of antiderivative = 2.49

$$\int \frac{(d+ex)^{9/2}}{\left(ade+(cd^2+ae^2)\,x+cdex^2\right)^2}\,dx = \frac{15\sqrt{d}\,\sqrt{c}\,\sqrt{a\,e^2-c\,d^2}\,atan\left(\frac{\sqrt{ex+d}\,cd}{\sqrt{d}\,\sqrt{c}\,\sqrt{a\,e^2-c\,d^2}}\right)a^2e^4-15\sqrt{d}\,\sqrt{c}\,\sqrt{a\,e^2}$$

input
```
int((e*x+d)^(9/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^2,x)
```

output
```
(15*sqrt(d)*sqrt(c)*sqrt(a*e**2 - c*d**2)*atan((sqrt(d + e*x)*c*d)/(sqrt(d
)*sqrt(c)*sqrt(a*e**2 - c*d**2)))*a**2*e**4 - 15*sqrt(d)*sqrt(c)*sqrt(a*e*
*2 - c*d**2)*atan((sqrt(d + e*x)*c*d)/(sqrt(d)*sqrt(c)*sqrt(a*e**2 - c*d**
2)))*a*c*d**2*e**2 + 15*sqrt(d)*sqrt(c)*sqrt(a*e**2 - c*d**2)*atan((sqrt(d
 + e*x)*c*d)/(sqrt(d)*sqrt(c)*sqrt(a*e**2 - c*d**2)))*a*c*d*e**3*x - 15*sq
rt(d)*sqrt(c)*sqrt(a*e**2 - c*d**2)*atan((sqrt(d + e*x)*c*d)/(sqrt(d)*sqrt
(c)*sqrt(a*e**2 - c*d**2)))*c**2*d**3*e*x - 15*sqrt(d + e*x)*a**2*c*d*e**4
 + 20*sqrt(d + e*x)*a*c**2*d**3*e**2 - 10*sqrt(d + e*x)*a*c**2*d**2*e**3*x
 - 3*sqrt(d + e*x)*c**3*d**5 + 14*sqrt(d + e*x)*c**3*d**4*e*x + 2*sqrt(d +
 e*x)*c**3*d**3*e**2*x**2)/(3*c**4*d**4*(a*e + c*d*x))
```

3.185 $\int \frac{(d+ex)^{7/2}}{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^2}\,dx$

Optimal result

Integrand size = 37, antiderivative size = 112

$$\int \frac{(d+ex)^{7/2}}{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^2}\,dx=\frac{3e\sqrt{d+ex}}{c^2d^2}-\frac{(d+ex)^{3/2}}{cd(ae+cdx)}-\frac{3e\sqrt{cd^2-ae^2}\,\text{arctanh}\left(\frac{\sqrt{c}\sqrt{d}\sqrt{d+ex}}{\sqrt{cd^2-ae^2}}\right)}{c^{5/2}d^{5/2}}$$

output
```
3*e*(e*x+d)^(1/2)/c^2/d^2-(e*x+d)^(3/2)/c/d/(c*d*x+a*e)-3*e*(-a*e^2+c*d^2)
^(1/2)*arctanh(c^(1/2)*d^(1/2)*(e*x+d)^(1/2)/(-a*e^2+c*d^2)^(1/2))/c^(5/2)
/d^(5/2)
```

Mathematica [A] (verified)

Time = 0.32 (sec) , antiderivative size = 110, normalized size of antiderivative = 0.98

$$\int \frac{(d+ex)^{7/2}}{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^2}\,dx=\frac{\sqrt{d+ex}(3ae^2-cd(d-2ex))}{c^2d^2(ae+cdx)}-\frac{3e\sqrt{-cd^2+ae^2}\arctan\left(\frac{\sqrt{c}\sqrt{d}\sqrt{d+ex}}{\sqrt{-cd^2+ae^2}}\right)}{c^{5/2}d^{5/2}}$$

input
```
Integrate[(d + e*x)^(7/2)/(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^2,x]
```

output
```
(Sqrt[d + e*x]*(3*a*e^2 - c*d*(d - 2*e*x)))/(c^2*d^2*(a*e + c*d*x)) - (3*e
*Sqrt[-(c*d^2) + a*e^2]*ArcTan[(Sqrt[c]*Sqrt[d]*Sqrt[d + e*x])/Sqrt[-(c*d^
2) + a*e^2]])/(c^(5/2)*d^(5/2))
```

Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 135, normalized size of antiderivative = 1.21, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.135$, Rules used = {1121, 51, 60, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex)^{7/2}}{(x(ae^2+cd^2)+ade+cdex^2)^2}dx$$

$$\Big\downarrow \text{1121}$$

$$\int \frac{(d+ex)^{3/2}}{(ae+cdx)^2}dx$$

$$\Big\downarrow \text{51}$$

$$\frac{3e\int \frac{\sqrt{d+ex}}{ae+cdx}dx}{2cd}-\frac{(d+ex)^{3/2}}{cd(ae+cdx)}$$

$$\Big\downarrow \text{60}$$

$$\frac{3e\left(\frac{\left(d^2-\frac{ae^2}{c}\right)\int \frac{1}{(ae+cdx)\sqrt{d+ex}}dx}{d}+\frac{2\sqrt{d+ex}}{cd}\right)}{2cd}-\frac{(d+ex)^{3/2}}{cd(ae+cdx)}$$

$$\Big\downarrow \text{73}$$

$$\frac{3e\left(\frac{2\left(d^2-\frac{ae^2}{c}\right)\int \frac{1}{-\frac{cd^2}{e}+\frac{c(d+ex)d}{e}+ae}d\sqrt{d+ex}}{de}+\frac{2\sqrt{d+ex}}{cd}\right)}{2cd}-\frac{(d+ex)^{3/2}}{cd(ae+cdx)}$$

$$\Big\downarrow \text{221}$$

$$\frac{3e\left(\frac{2\sqrt{d+ex}}{cd}-\frac{2\left(d^2-\frac{ae^2}{c}\right)\text{arctanh}\left(\frac{\sqrt{c}\sqrt{d}\sqrt{d+ex}}{\sqrt{cd^2-ae^2}}\right)}{\sqrt{c}d^{3/2}\sqrt{cd^2-ae^2}}\right)}{2cd}-\frac{(d+ex)^{3/2}}{cd(ae+cdx)}$$

input
```
Int[(d + e*x)^(7/2)/(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^2,x]
```

output
```
-((d + e*x)^(3/2)/(c*d*(a*e + c*d*x))) + (3*e*((2*Sqrt[d + e*x])/(c*d) - (
2*(d^2 - (a*e^2)/c)*ArcTanh[(Sqrt[c]*Sqrt[d]*Sqrt[d + e*x])/Sqrt[c*d^2 - a
*e^2]])/(Sqrt[c]*d^(3/2)*Sqrt[c*d^2 - a*e^2])))/(2*c*d)
```

Defintions of rubi rules used

rule 51
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Simp[d*(n/(b*(m + 1)))
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d, n}, x
] && ILtQ[m, -1] && FractionQ[n] && GtQ[n, 0]
```

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 1121
```
Int[((d_) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Int[ExpandIntegrand[(d + e*x)^(m + p)*(a/d + (c/e)*x)^p, x], x]
/; FreeQ[{a, b, c, d, e, m, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && (Int
egerQ[p] || (GtQ[a, 0] && GtQ[d, 0] && LtQ[c, 0]))
```

Maple [A] (verified)

Time = 4.85 (sec) , antiderivative size = 129, normalized size of antiderivative = 1.15

method	result	size
pseudoelliptic	$\frac{3\sqrt{ex+d}\,\sqrt{cd(a\,e^2-c\,d^2)}\left(-\frac{d(-2ex+d)c}{3}+a\,e^2\right)-3e\left(a\,e^2-c\,d^2\right)(cdx+ae)\arctan\left(\frac{cd\sqrt{ex+d}}{\sqrt{cd\left(a\,e^2-c\,d^2\right)}}\right)}{\sqrt{cd(a\,e^2-c\,d^2)}\,c^2d^2(cdx+ae)}$	129
derivativedivides	$2e\left(\frac{\sqrt{ex+d}}{c^2d^2}-\frac{\frac{\left(-\frac{a\,e^2}{2}+\frac{c\,d^2}{2}\right)\sqrt{ex+d}}{cd(ex+d)+a\,e^2-c\,d^2}+\frac{3\left(a\,e^2-c\,d^2\right)\arctan\left(\frac{cd\sqrt{ex+d}}{\sqrt{cd\left(a\,e^2-c\,d^2\right)}}\right)}{2\sqrt{cd\left(a\,e^2-c\,d^2\right)}}}{c^2d^2}\right)$	130
default	$2e\left(\frac{\sqrt{ex+d}}{c^2d^2}-\frac{\frac{\left(-\frac{a\,e^2}{2}+\frac{c\,d^2}{2}\right)\sqrt{ex+d}}{cd(ex+d)+a\,e^2-c\,d^2}+\frac{3\left(a\,e^2-c\,d^2\right)\arctan\left(\frac{cd\sqrt{ex+d}}{\sqrt{cd\left(a\,e^2-c\,d^2\right)}}\right)}{2\sqrt{cd\left(a\,e^2-c\,d^2\right)}}}{c^2d^2}\right)$	130
risch	Expression too large to display	3814

input
```
int((e*x+d)^(7/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^2,x,method=_RETURNVERB
OSE)
```

output
```
(3*(e*x+d)^(1/2)*(c*d*(a*e^2-c*d^2))^(1/2)*(-1/3*d*(-2*e*x+d)*c+a*e^2)-3*e
*(a*e^2-c*d^2)*(c*d*x+a*e)*arctan(c*d*(e*x+d)^(1/2)/(c*d*(a*e^2-c*d^2))^(1
/2)))/(c*d*(a*e^2-c*d^2))^(1/2)/c^2/d^2/(c*d*x+a*e)
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 282, normalized size of antiderivative = 2.52

$$\int \frac{(d+ex)^{7/2}}{(ade+(cd^2+ae^2)x+cdex^2)^2}\,dx = \left[\frac{3\,(cdex+ae^2)\sqrt{\frac{cd^2-ae^2}{cd}}\log\left(\frac{cdex+2\,cd^2-ae^2-2\,\sqrt{ex+d}cd\sqrt{\frac{cd^2-ae^2}{cd}}}{cdx+ae}\right)+2}{2\,(c^3d^3x+ac^2d^2e)}\right.$$

$$\left.-\frac{3\,(cdex+ae^2)\sqrt{-\frac{cd^2-ae^2}{cd}}\arctan\left(-\frac{\sqrt{ex+d}cd\sqrt{-\frac{cd^2-ae^2}{cd}}}{cd^2-ae^2}\right)-(2\,cdex-cd^2+3\,ae^2)\sqrt{ex+d}}{c^3d^3x+ac^2d^2e}\right]$$

input
```
integrate((e*x+d)^(7/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^2,x, algorithm="
fricas")
```

output
```
[1/2*(3*(c*d*e*x + a*e^2)*sqrt((c*d^2 - a*e^2)/(c*d))*log((c*d*e*x + 2*c*d
^2 - a*e^2 - 2*sqrt(e*x + d)*c*d*sqrt((c*d^2 - a*e^2)/(c*d)))/(c*d*x + a*e
)) + 2*(2*c*d*e*x - c*d^2 + 3*a*e^2)*sqrt(e*x + d))/(c^3*d^3*x + a*c^2*d^2
*e), -(3*(c*d*e*x + a*e^2)*sqrt(-(c*d^2 - a*e^2)/(c*d))*arctan(-sqrt(e*x +
 d)*c*d*sqrt(-(c*d^2 - a*e^2)/(c*d))/(c*d^2 - a*e^2)) - (2*c*d*e*x - c*d^2
 + 3*a*e^2)*sqrt(e*x + d))/(c^3*d^3*x + a*c^2*d^2*e)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(d+ex)^{7/2}}{(ade+(cd^2+ae^2)x+cdex^2)^2}\,dx = \text{Timed out}$$

input
```
integrate((e*x+d)**(7/2)/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**2,x)
```

output
```
Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(d+ex)^{7/2}}{(ade+(cd^2+ae^2)\,x+cdex^2)^2}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((e*x+d)^(7/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^2,x, algorithm="
maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(a*e^2-c*d^2>0)', see `assume?` f
or more de
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 142, normalized size of antiderivative = 1.27

$$\int \frac{(d+ex)^{7/2}}{(ade+(cd^2+ae^2)\,x+cdex^2)^2}\,dx = \frac{2\,\sqrt{ex+d}e}{c^2d^2} + \frac{3\,(cd^2e-ae^3)\arctan\left(\frac{\sqrt{ex+d}cd}{\sqrt{-c^2d^3+acde^2}}\right)}{\sqrt{-c^2d^3+acde^2}c^2d^2} - \frac{\sqrt{ex+d}cd^2e-\sqrt{ex+d}ae^3}{((ex+d)cd-cd^2+ae^2)c^2d^2}$$

input
```
integrate((e*x+d)^(7/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^2,x, algorithm="
giac")
```

output
```
2*sqrt(e*x + d)*e/(c^2*d^2) + 3*(c*d^2*e - a*e^3)*arctan(sqrt(e*x + d)*c*d
/sqrt(-c^2*d^3 + a*c*d*e^2))/(sqrt(-c^2*d^3 + a*c*d*e^2)*c^2*d^2) - (sqrt(
e*x + d)*c*d^2*e - sqrt(e*x + d)*a*e^3)/(((e*x + d)*c*d - c*d^2 + a*e^2)*c
^2*d^2)
```

Mupad [B] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 140, normalized size of antiderivative = 1.25

$$\int \frac{(d+ex)^{7/2}}{(ade+(cd^2+ae^2)\,x+cdex^2)^2}\,dx = \frac{(a\,e^3-c\,d^2\,e)\,\sqrt{d+e\,x}}{c^3\,d^3\,(d+e\,x)-c^3\,d^4+a\,c^2\,d^2\,e^2} + \frac{2\,e\,\sqrt{d+e\,x}}{c^2\,d^2} - \frac{3\,e\,\mathrm{atan}\left(\frac{\sqrt{c}\,\sqrt{d}\,e\,\sqrt{a\,e^2-c\,d^2}\,\sqrt{d+e\,x}}{a\,e^3-c\,d^2\,e}\right)\,\sqrt{a\,e^2-c\,d^2}}{c^{5/2}\,d^{5/2}}$$

input
```
int((d + e*x)^(7/2)/(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^2,x)
```

output
```
((a*e^3 - c*d^2*e)*(d + e*x)^(1/2))/(c^3*d^3*(d + e*x) - c^3*d^4 + a*c^2*d
^2*e^2) + (2*e*(d + e*x)^(1/2))/(c^2*d^2) - (3*e*atan((c^(1/2)*d^(1/2)*e*(
a*e^2 - c*d^2)^(1/2)*(d + e*x)^(1/2))/(a*e^3 - c*d^2*e))*(a*e^2 - c*d^2)^(
1/2))/(c^(5/2)*d^(5/2))
```

Reduce [B] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 174, normalized size of antiderivative = 1.55

$$\int \frac{(d+ex)^{7/2}}{(ade+(cd^2+ae^2)\,x+cdex^2)^2}\,dx = \frac{-3\sqrt{d}\,\sqrt{c}\,\sqrt{a\,e^2-c\,d^2}\,atan\left(\frac{\sqrt{ex+d}\,cd}{\sqrt{d}\,\sqrt{c}\,\sqrt{a\,e^2-c\,d^2}}\right)a\,e^2-3\sqrt{d}\,\sqrt{c}\,\sqrt{a\,e^2}}{}$$

input
```
int((e*x+d)^(7/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^2,x)
```

output
```
( - 3*sqrt(d)*sqrt(c)*sqrt(a*e**2 - c*d**2)*atan((sqrt(d + e*x)*c*d)/(sqrt
(d)*sqrt(c)*sqrt(a*e**2 - c*d**2)))*a*e**2 - 3*sqrt(d)*sqrt(c)*sqrt(a*e**2
 - c*d**2)*atan((sqrt(d + e*x)*c*d)/(sqrt(d)*sqrt(c)*sqrt(a*e**2 - c*d**2)
))*c*d*e*x + 3*sqrt(d + e*x)*a*c*d*e**2 - sqrt(d + e*x)*c**2*d**3 + 2*sqrt
(d + e*x)*c**2*d**2*e*x)/(c**3*d**3*(a*e + c*d*x))
```

3.186 $\int \frac{(d+ex)^{5/2}}{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^2}\,dx$

Optimal result

Integrand size = 37, antiderivative size = 94

$$\int \frac{(d+ex)^{5/2}}{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^2}\,dx=-\frac{\sqrt{d+ex}}{cd(ae+cdx)}-\frac{e\text{arctanh}\left(\frac{\sqrt{c}\sqrt{d}\sqrt{d+ex}}{\sqrt{cd^2-ae^2}}\right)}{c^{3/2}d^{3/2}\sqrt{cd^2-ae^2}}$$

output
```
-(e*x+d)^(1/2)/c/d/(c*d*x+a*e)-e*arctanh(c^(1/2)*d^(1/2)*(e*x+d)^(1/2)/(-a
*e^2+c*d^2)^(1/2))/c^(3/2)/d^(3/2)/(-a*e^2+c*d^2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.21 (sec) , antiderivative size = 93, normalized size of antiderivative = 0.99

$$\int \frac{(d+ex)^{5/2}}{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^2}\,dx=-\frac{\sqrt{d+ex}}{acde+c^2d^2x}+\frac{e\arctan\left(\frac{\sqrt{c}\sqrt{d}\sqrt{d+ex}}{\sqrt{-cd^2+ae^2}}\right)}{c^{3/2}d^{3/2}\sqrt{-cd^2+ae^2}}$$

input
```
Integrate[(d + e*x)^(5/2)/(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^2,x]
```

output
```
-(Sqrt[d + e*x]/(a*c*d*e + c^2*d^2*x)) + (e*ArcTan[(Sqrt[c]*Sqrt[d]*Sqrt[d
 + e*x])/Sqrt[-(c*d^2) + a*e^2]])/(c^(3/2)*d^(3/2)*Sqrt[-(c*d^2) + a*e^2])
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.108, Rules used = {1121, 51, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex)^{5/2}}{(x(ae^2+cd^2)+ade+cdex^2)^2}dx$$

$$\Big\downarrow \ 1121$$

$$\int \frac{\sqrt{d+ex}}{(ae+cdx)^2}dx$$

$$\Big\downarrow \ 51$$

$$\frac{e\int \frac{1}{(ae+cdx)\sqrt{d+ex}}dx}{2cd}-\frac{\sqrt{d+ex}}{cd(ae+cdx)}$$

$$\Big\downarrow \ 73$$

$$\frac{\int \frac{1}{-\frac{cd^2}{e}+\frac{c(d+ex)d}{e}+ae}d\sqrt{d+ex}}{cd}-\frac{\sqrt{d+ex}}{cd(ae+cdx)}$$

$$\Big\downarrow \ 221$$

$$-\frac{e\text{arctanh}\left(\frac{\sqrt{c}\sqrt{d}\sqrt{d+ex}}{\sqrt{cd^2-ae^2}}\right)}{c^{3/2}d^{3/2}\sqrt{cd^2-ae^2}}-\frac{\sqrt{d+ex}}{cd(ae+cdx)}$$

input
```
Int[(d + e*x)^(5/2)/(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^2,x]
```

output
```
-(Sqrt[d + e*x]/(c*d*(a*e + c*d*x))) - (e*ArcTanh[(Sqrt[c]*Sqrt[d]*Sqrt[d
+ e*x])/Sqrt[c*d^2 - a*e^2]])/(c^(3/2)*d^(3/2)*Sqrt[c*d^2 - a*e^2])
```

Defintions of rubi rules used

rule 51
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Simp[d*(n/(b*(m + 1)))
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d, n}, x
] && ILtQ[m, -1] && FractionQ[n] && GtQ[n, 0]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 1121
```
Int[((d_) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Int[ExpandIntegrand[(d + e*x)^(m + p)*(a/d + (c/e)*x)^p, x], x]
/; FreeQ[{a, b, c, d, e, m, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && (Int
egerQ[p] || (GtQ[a, 0] && GtQ[d, 0] && LtQ[c, 0]))
```

Maple [A] (verified)

Time = 4.04 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.80

method	result	size
pseudoelliptic	$\frac{-\frac{\sqrt{ex+d}}{cdx+ae}+\frac{e\arctan\left(\frac{cd\sqrt{ex+d}}{\sqrt{cd\left(a\,e^{2}-c\,d^{2}\right)}}\right)}{\sqrt{cd\left(a\,e^{2}-c\,d^{2}\right)}}}{cd}$	75
derivativedivides	$2e\left(-\frac{\sqrt{ex+d}}{2cd(cd(ex+d)+a\,e^{2}-c\,d^{2})}+\frac{\arctan\left(\frac{cd\sqrt{ex+d}}{\sqrt{cd\left(a\,e^{2}-c\,d^{2}\right)}}\right)}{2cd\sqrt{cd(a\,e^{2}-c\,d^{2})}}\right)$	95
default	$2e\left(-\frac{\sqrt{ex+d}}{2cd(cd(ex+d)+a\,e^{2}-c\,d^{2})}+\frac{\arctan\left(\frac{cd\sqrt{ex+d}}{\sqrt{cd\left(a\,e^{2}-c\,d^{2}\right)}}\right)}{2cd\sqrt{cd(a\,e^{2}-c\,d^{2})}}\right)$	95

input
```
int((e*x+d)^(5/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^2,x,method=_RETURNVERB
OSE)
```

output
```
1/c/d*(-(e*x+d)^(1/2)/(c*d*x+a*e)+e/(c*d*(a*e^2-c*d^2))^(1/2)*arctan(c*d*(
e*x+d)^(1/2)/(c*d*(a*e^2-c*d^2))^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 309, normalized size of antiderivative = 3.29

$$\int \frac{(d+ex)^{5/2}}{\left(ade+(cd^2+ae^2)\,x+cdex^2\right)^2}\,dx=\left[\frac{\sqrt{c^2d^3-acde^2}(cdex+ae^2)\log\left(\frac{cdex+2\,cd^2-ae^2-2\,\sqrt{c^2d^3-acde^2}\sqrt{ex+d}}{cdx+ae}\right)}{2\,(ac^3d^4e-a^2c^2d^2e^3+(c^4d^5-ac^3d^3e^2)}\right.$$

input
```
integrate((e*x+d)^(5/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^2,x, algorithm="
fricas")
```

output
```
[1/2*(sqrt(c^2*d^3 - a*c*d*e^2)*(c*d*e*x + a*e^2)*log((c*d*e*x + 2*c*d^2 -
 a*e^2 - 2*sqrt(c^2*d^3 - a*c*d*e^2)*sqrt(e*x + d))/(c*d*x + a*e)) - 2*(c^
2*d^3 - a*c*d*e^2)*sqrt(e*x + d))/(a*c^3*d^4*e - a^2*c^2*d^2*e^3 + (c^4*d^
5 - a*c^3*d^3*e^2)*x), (sqrt(-c^2*d^3 + a*c*d*e^2)*(c*d*e*x + a*e^2)*arcta
n(sqrt(-c^2*d^3 + a*c*d*e^2)*sqrt(e*x + d)/(c*d*e*x + c*d^2)) - (c^2*d^3 -
 a*c*d*e^2)*sqrt(e*x + d))/(a*c^3*d^4*e - a^2*c^2*d^2*e^3 + (c^4*d^5 - a*c
^3*d^3*e^2)*x)]
```

Sympy [F]

$$\int \frac{(d+ex)^{5/2}}{(ade+(cd^2+ae^2)\,x+cdex^2)^2}\,dx=\int \frac{\sqrt{d+ex}}{(ae+cdx)^2}\,dx$$

input
```
integrate((e*x+d)**(5/2)/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**2,x)
```

output
```
Integral(sqrt(d + e*x)/(a*e + c*d*x)**2, x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(d+ex)^{5/2}}{(ade+(cd^2+ae^2)\,x+cdex^2)^2}\,dx=\text{Exception raised: ValueError}$$

input
```
integrate((e*x+d)^(5/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^2,x, algorithm="
maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(a*e^2-c*d^2>0)', see `assume?` f
or more de
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.00

$$\int \frac{(d+ex)^{5/2}}{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^2}\,dx = \frac{e\arctan\left(\frac{\sqrt{ex+d}cd}{\sqrt{-c^2d^3+acde^2}}\right)}{\sqrt{-c^2d^3+acde^2}cd} - \frac{\sqrt{ex+d}e}{((ex+d)cd-cd^2+ae^2)cd}$$

input
```
integrate((e*x+d)^(5/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^2,x, algorithm="
giac")
```

output
```
e*arctan(sqrt(e*x + d)*c*d/sqrt(-c^2*d^3 + a*c*d*e^2))/(sqrt(-c^2*d^3 + a*
c*d*e^2)*c*d) - sqrt(e*x + d)*e/(((e*x + d)*c*d - c*d^2 + a*e^2)*c*d)
```

Mupad [B] (verification not implemented)

Time = 5.19 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.86

$$\int \frac{(d+ex)^{5/2}}{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^2}\,dx = \frac{e\,\mathrm{atan}\left(\frac{\sqrt{c}\,\sqrt{d}\,\sqrt{d+e\,x}}{\sqrt{a\,e^2-c\,d^2}}\right)}{c^{3/2}\,d^{3/2}\,\sqrt{a\,e^2-c\,d^2}} - \frac{e\,\sqrt{d+e\,x}}{x\,c^2\,d^2\,e+a\,c\,d\,e^2}$$

input
```
int((d + e*x)^(5/2)/(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^2,x)
```

output
```
(e*atan((c^(1/2)*d^(1/2)*(d + e*x)^(1/2))/(a*e^2 - c*d^2)^(1/2)))/(c^(3/2)
*d^(3/2)*(a*e^2 - c*d^2)^(1/2)) - (e*(d + e*x)^(1/2))/(a*c*d*e^2 + c^2*d^2
*e*x)
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 180, normalized size of antiderivative = 1.91

$$\int \frac{(d+ex)^{5/2}}{\left(ade+(cd^2+ae^2)\,x+cdex^2\right)^2}\,dx = \frac{\sqrt{d}\,\sqrt{c}\,\sqrt{a\,e^2-c\,d^2}\;atan\left(\frac{\sqrt{ex+d}\,cd}{\sqrt{d}\,\sqrt{c}\,\sqrt{a\,e^2-c\,d^2}}\right)a\,e^2+\sqrt{d}\,\sqrt{c}\,\sqrt{a\,e^2-c\,d}}{c^2d^2\left(acd\,e^2x-c^2d^3x\right.}$$

input
```
int((e*x+d)^(5/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^2,x)
```

output
```
(sqrt(d)*sqrt(c)*sqrt(a*e**2 - c*d**2)*atan((sqrt(d + e*x)*c*d)/(sqrt(d)*s
qrt(c)*sqrt(a*e**2 - c*d**2)))*a*e**2 + sqrt(d)*sqrt(c)*sqrt(a*e**2 - c*d*
*2)*atan((sqrt(d + e*x)*c*d)/(sqrt(d)*sqrt(c)*sqrt(a*e**2 - c*d**2)))*c*d*
e*x - sqrt(d + e*x)*a*c*d*e**2 + sqrt(d + e*x)*c**2*d**3)/(c**2*d**2*(a**2
*e**3 - a*c*d**2*e + a*c*d*e**2*x - c**2*d**3*x))
```

3.187 $\int \frac{(d+ex)^{3/2}}{(ade+(cd^2+ae^2)x+cdex^2)^2} dx$

Optimal result

Integrand size = 37, antiderivative size = 101

$$\int \frac{(d+ex)^{3/2}}{(ade+(cd^2+ae^2)x+cdex^2)^2} dx = -\frac{\sqrt{d+ex}}{(cd^2-ae^2)(ae+cdx)}+\frac{e\text{arctanh}\left(\frac{\sqrt{c}\sqrt{d}\sqrt{d+ex}}{\sqrt{cd^2-ae^2}}\right)}{\sqrt{c}\sqrt{d}(cd^2-ae^2)^{3/2}}$$

output
```
-(e*x+d)^(1/2)/(-a*e^2+c*d^2)/(c*d*x+a*e)+e*arctanh(c^(1/2)*d^(1/2)*(e*x+d
)^(1/2)/(-a*e^2+c*d^2)^(1/2))/c^(1/2)/d^(1/2)/(-a*e^2+c*d^2)^(3/2)
```

Mathematica [A] (verified)

Time = 0.20 (sec) , antiderivative size = 100, normalized size of antiderivative = 0.99

$$\int \frac{(d+ex)^{3/2}}{(ade+(cd^2+ae^2)x+cdex^2)^2} dx = \frac{\sqrt{d+ex}}{(-cd^2+ae^2)(ae+cdx)}+\frac{e\arctan\left(\frac{\sqrt{c}\sqrt{d}\sqrt{d+ex}}{\sqrt{-cd^2+ae^2}}\right)}{\sqrt{c}\sqrt{d}(-cd^2+ae^2)^{3/2}}$$

input
```
Integrate[(d + e*x)^(3/2)/(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^2,x]
```

output
```
Sqrt[d + e*x]/((-(c*d^2) + a*e^2)*(a*e + c*d*x)) + (e*ArcTan[(Sqrt[c]*Sqrt
[d]*Sqrt[d + e*x])/Sqrt[-(c*d^2) + a*e^2]])/(Sqrt[c]*Sqrt[d]*(-(c*d^2) + a
*e^2)^(3/2))
```

Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 101, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.108$, Rules used = {1121, 52, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex)^{3/2}}{\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^2}dx$$

$$\Big\downarrow \text{1121}$$

$$\int \frac{1}{\sqrt{d+ex}(ae+cdx)^2}dx$$

$$\Big\downarrow \text{52}$$

$$-\frac{e\int \frac{1}{(ae+cdx)\sqrt{d+ex}}dx}{2\left(cd^2-ae^2\right)}-\frac{\sqrt{d+ex}}{\left(cd^2-ae^2\right)(ae+cdx)}$$

$$\Big\downarrow \text{73}$$

$$-\frac{\int \frac{1}{-\frac{cd^2}{e}+\frac{c(d+ex)d}{e}+ae}d\sqrt{d+ex}}{cd^2-ae^2}-\frac{\sqrt{d+ex}}{\left(cd^2-ae^2\right)(ae+cdx)}$$

$$\Big\downarrow \text{221}$$

$$\frac{e\text{arctanh}\left(\frac{\sqrt{c}\sqrt{d}\sqrt{d+ex}}{\sqrt{cd^2-ae^2}}\right)}{\sqrt{c}\sqrt{d}\left(cd^2-ae^2\right)^{3/2}}-\frac{\sqrt{d+ex}}{\left(cd^2-ae^2\right)(ae+cdx)}$$

input
```
Int[(d + e*x)^(3/2)/(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^2,x]
```

output
```
-(Sqrt[d + e*x]/((c*d^2 - a*e^2)*(a*e + c*d*x))) + (e*ArcTanh[(Sqrt[c]*Sqr
t[d]*Sqrt[d + e*x])/Sqrt[c*d^2 - a*e^2]])/(Sqrt[c]*Sqrt[d]*(c*d^2 - a*e^2)
^(3/2))
```

Defintions of rubi rules used

rule 52
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && ILtQ[m, -1] && FractionQ[n] && LtQ[n, 0]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 1121
```
Int[((d_) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Int[ExpandIntegrand[(d + e*x)^(m + p)*(a/d + (c/e)*x)^p, x], x]
/; FreeQ[{a, b, c, d, e, m, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && (Int
egerQ[p] || (GtQ[a, 0] && GtQ[d, 0] && LtQ[c, 0]))
```

Maple [A] (verified)

Time = 1.86 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.81

method	result	size
pseudoelliptic	$\frac{\frac{\sqrt{ex+d}}{cdx+ae}+\frac{e\arctan\left(\frac{cd\sqrt{ex+d}}{\sqrt{cd\left(a\,e^{2}-c\,d^{2}\right)}}\right)}{\sqrt{cd\left(a\,e^{2}-c\,d^{2}\right)}}}{a\,e^{2}-c\,d^{2}}$	82
derivativedivides	$2e\left(\frac{\sqrt{ex+d}}{2(a\,e^{2}-c\,d^{2})(cd(ex+d)+a\,e^{2}-c\,d^{2})}+\frac{\arctan\left(\frac{cd\sqrt{ex+d}}{\sqrt{cd\left(a\,e^{2}-c\,d^{2}\right)}}\right)}{2(a\,e^{2}-c\,d^{2})\sqrt{cd(a\,e^{2}-c\,d^{2})}}\right)$	111
default	$2e\left(\frac{\sqrt{ex+d}}{2(a\,e^{2}-c\,d^{2})(cd(ex+d)+a\,e^{2}-c\,d^{2})}+\frac{\arctan\left(\frac{cd\sqrt{ex+d}}{\sqrt{cd\left(a\,e^{2}-c\,d^{2}\right)}}\right)}{2(a\,e^{2}-c\,d^{2})\sqrt{cd(a\,e^{2}-c\,d^{2})}}\right)$	111

input
```
int((e*x+d)^(3/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^2,x,method=_RETURNVERB
OSE)
```

output
```
1/(a*e^2-c*d^2)*((e*x+d)^(1/2)/(c*d*x+a*e)+e/(c*d*(a*e^2-c*d^2))^(1/2)*arc
tan(c*d*(e*x+d)^(1/2)/(c*d*(a*e^2-c*d^2))^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 353, normalized size of antiderivative = 3.50

$$\int \frac{(d+ex)^{3/2}}{(ade+(cd^2+ae^2)\,x+cdex^2)^2}\,dx = \left[-\frac{\sqrt{c^2d^3-acde^2}(cdex+ae^2)\log\left(\frac{cdex+2\,cd^2-ae^2-2\,\sqrt{c^2d^3-acde^2}\sqrt{ex+c}}{cdx+ae}\right.}{2\,(ac^3d^5e-2\,a^2c^2d^3e^3+a^3cde^5+(c^4d^6-2\,ac^3d^4}\right.$$

$$\left.-\frac{\sqrt{-c^2d^3+acde^2}(cdex+ae^2)\arctan\left(\frac{\sqrt{-c^2d^3+acde^2}\sqrt{ex+d}}{cdex+cd^2}\right)+(c^2d^3-acde^2)\sqrt{ex+d}}{ac^3d^5e-2\,a^2c^2d^3e^3+a^3cde^5+(c^4d^6-2\,ac^3d^4e^2+a^2c^2d^2e^4)x}\right]$$

input
```
integrate((e*x+d)^(3/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^2,x, algorithm="
fricas")
```

output
```
[-1/2*(sqrt(c^2*d^3 - a*c*d*e^2)*(c*d*e*x + a*e^2)*log((c*d*e*x + 2*c*d^2
- a*e^2 - 2*sqrt(c^2*d^3 - a*c*d*e^2)*sqrt(e*x + d))/(c*d*x + a*e)) + 2*(c
^2*d^3 - a*c*d*e^2)*sqrt(e*x + d))/(a*c^3*d^5*e - 2*a^2*c^2*d^3*e^3 + a^3*
c*d*e^5 + (c^4*d^6 - 2*a*c^3*d^4*e^2 + a^2*c^2*d^2*e^4)*x), -(sqrt(-c^2*d^
3 + a*c*d*e^2)*(c*d*e*x + a*e^2)*arctan(sqrt(-c^2*d^3 + a*c*d*e^2)*sqrt(e*
x + d)/(c*d*e*x + c*d^2)) + (c^2*d^3 - a*c*d*e^2)*sqrt(e*x + d))/(a*c^3*d^
5*e - 2*a^2*c^2*d^3*e^3 + a^3*c*d*e^5 + (c^4*d^6 - 2*a*c^3*d^4*e^2 + a^2*c
^2*d^2*e^4)*x)]
```

Sympy [F]

$$\int \frac{(d+ex)^{3/2}}{(ade+(cd^2+ae^2)\,x+cdex^2)^2}\,dx = \int \frac{1}{\sqrt{d+ex}\,(ae+cdx)^2}\,dx$$

input
```
integrate((e*x+d)**(3/2)/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**2,x)
```

output
```
Integral(1/(sqrt(d + e*x)*(a*e + c*d*x)**2), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(d+ex)^{3/2}}{(ade+(cd^2+ae^2)\,x+cdex^2)^2}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((e*x+d)^(3/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^2,x, algorithm="
maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(a*e^2-c*d^2>0)', see `assume?` f
or more de
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 111, normalized size of antiderivative = 1.10

$$\int \frac{(d+ex)^{3/2}}{(ade+(cd^2+ae^2)\,x+cdex^2)^2}\,dx = -\frac{e\arctan\left(\frac{\sqrt{ex+d}cd}{\sqrt{-c^2d^3+acde^2}}\right)}{\sqrt{-c^2d^3+acde^2}(cd^2-ae^2)} - \frac{\sqrt{ex+d}e}{((ex+d)cd-cd^2+ae^2)(cd^2-ae^2)}$$

input
```
integrate((e*x+d)^(3/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^2,x, algorithm="
giac")
```

output
```
-e*arctan(sqrt(e*x + d)*c*d/sqrt(-c^2*d^3 + a*c*d*e^2))/(sqrt(-c^2*d^3 + a
*c*d*e^2)*(c*d^2 - a*e^2)) - sqrt(e*x + d)*e/(((e*x + d)*c*d - c*d^2 + a*e
^2)*(c*d^2 - a*e^2))
```

Mupad [B] (verification not implemented)

Time = 5.24 (sec) , antiderivative size = 97, normalized size of antiderivative = 0.96

$$\int \frac{(d+ex)^{3/2}}{(ade+(cd^2+ae^2)\,x+cdex^2)^2}\,dx = \frac{e\,\mathrm{atan}\left(\frac{c\,d\,\sqrt{d+e\,x}}{\sqrt{c\,d}\,\sqrt{a\,e^2-c\,d^2}}\right)}{\sqrt{c\,d}\,{\left(a\,e^2-c\,d^2\right)}^{3/2}} + \frac{e\,\sqrt{d+e\,x}}{\left(a\,e^2-c\,d^2\right)\,\left(a\,e^2-c\,d^2+c\,d\,\left(d+e\,x\right)\right)}$$

input
```
int((d + e*x)^(3/2)/(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^2,x)
```

output
```
(e*atan((c*d*(d + e*x)^(1/2))/((c*d)^(1/2)*(a*e^2 - c*d^2)^(1/2))))/((c*d)
^(1/2)*(a*e^2 - c*d^2)^(3/2)) + (e*(d + e*x)^(1/2))/((a*e^2 - c*d^2)*(a*e^
2 - c*d^2 + c*d*(d + e*x)))
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 207, normalized size of antiderivative = 2.05

$$\int \frac{(d+ex)^{3/2}}{\left(ade+(cd^2+ae^2)x+cdex^2\right)^2}\,dx = \frac{\sqrt{d}\,\sqrt{c}\,\sqrt{a\,e^2-c\,d^2}\,atan\left(\frac{\sqrt{ex+d}\,cd}{\sqrt{d}\,\sqrt{c}\,\sqrt{a\,e^2-c\,d^2}}\right)a\,e^2+\sqrt{d}\,\sqrt{c}\,\sqrt{a\,e^2-c\,d}}{cd\left(a^2cd\,e^4x-2a\,c^2d^3e^2x+c^3d^5x\right.}$$

input
```
int((e*x+d)^(3/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^2,x)
```

output
```
(sqrt(d)*sqrt(c)*sqrt(a*e**2 - c*d**2)*atan((sqrt(d + e*x)*c*d)/(sqrt(d)*s
qrt(c)*sqrt(a*e**2 - c*d**2)))*a*e**2 + sqrt(d)*sqrt(c)*sqrt(a*e**2 - c*d*
*2)*atan((sqrt(d + e*x)*c*d)/(sqrt(d)*sqrt(c)*sqrt(a*e**2 - c*d**2)))*c*d*
e*x + sqrt(d + e*x)*a*c*d*e**2 - sqrt(d + e*x)*c**2*d**3)/(c*d*(a**3*e**5
- 2*a**2*c*d**2*e**3 + a**2*c*d*e**4*x + a*c**2*d**4*e - 2*a*c**2*d**3*e**
2*x + c**3*d**5*x))
```

3.188 $\int \frac{\sqrt{d+ex}}{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^2}\,dx$

Optimal result

Integrand size = 37, antiderivative size = 128

$$\int \frac{\sqrt{d+ex}}{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^2}\,dx = -\frac{3e}{\left(cd^2-ae^2\right)^2\sqrt{d+ex}} - \frac{1}{\left(cd^2-ae^2\right)(ae+cdx)\sqrt{d+ex}} + \frac{3\sqrt{c}\sqrt{d}e\,\text{arctanh}\left(\frac{\sqrt{c}\sqrt{d}\sqrt{d+ex}}{\sqrt{cd^2-ae^2}}\right)}{\left(cd^2-ae^2\right)^{5/2}}$$

output
```
-3*e/(-a*e^2+c*d^2)^2/(e*x+d)^(1/2)-1/(-a*e^2+c*d^2)/(c*d*x+a*e)/(e*x+d)^(
1/2)+3*c^(1/2)*d^(1/2)*e*arctanh(c^(1/2)*d^(1/2)*(e*x+d)^(1/2)/(-a*e^2+c*d
^2)^(1/2))/(-a*e^2+c*d^2)^(5/2)
```

Mathematica [A] (verified)

Time = 0.42 (sec) , antiderivative size = 118, normalized size of antiderivative = 0.92

$$\int \frac{\sqrt{d+ex}}{\left(ade+(cd^2+ae^2)\,x+cdex^2\right)^2}\,dx = -\frac{2ae^2+cd(d+3ex)}{\left(cd^2-ae^2\right)^2(ae+cdx)\sqrt{d+ex}} - \frac{3\sqrt{c}\sqrt{d}e\arctan\left(\frac{\sqrt{c}\sqrt{d}\sqrt{d+ex}}{\sqrt{-cd^2+ae^2}}\right)}{\left(-cd^2+ae^2\right)^{5/2}}$$

input
```
Integrate[Sqrt[d + e*x]/(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^2,x]
```

output
```
-((2*a*e^2 + c*d*(d + 3*e*x))/((c*d^2 - a*e^2)^2*(a*e + c*d*x)*Sqrt[d + e*
x])) - (3*Sqrt[c]*Sqrt[d]*e*ArcTan[(Sqrt[c]*Sqrt[d]*Sqrt[d + e*x])/Sqrt[-(
c*d^2) + a*e^2]])/(-(c*d^2) + a*e^2)^(5/2)
```

Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 146, normalized size of antiderivative = 1.14, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.135$, Rules used = {1121, 52, 61, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{d+ex}}{\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^2}dx$$

$$\Big\downarrow \text{ 1121}$$

$$\int \frac{1}{(d+ex)^{3/2}(ae+cdx)^2}dx$$

$$\Big\downarrow \text{ 52}$$

$$-\frac{3e\int \frac{1}{(ae+cdx)(d+ex)^{3/2}}dx}{2\left(cd^2-ae^2\right)} - \frac{1}{\sqrt{d+ex}\left(cd^2-ae^2\right)(ae+cdx)}$$

$$\Big\downarrow \text{ 61}$$

$$-\frac{3e\left(\frac{cd\int \frac{1}{(ae+cdx)\sqrt{d+ex}}dx}{cd^2-ae^2}+\frac{2}{\sqrt{d+ex}(cd^2-ae^2)}\right)}{2\left(cd^2-ae^2\right)}-\frac{1}{\sqrt{d+ex}\left(cd^2-ae^2\right)\left(ae+cdx\right)}$$

$$\Big\downarrow \text{73}$$

$$-\frac{3e\left(\frac{2cd\int \frac{1}{-\frac{cd^2}{e}+\frac{c(d+ex)d}{e}+ae}d\sqrt{d+ex}}{e(cd^2-ae^2)}+\frac{2}{\sqrt{d+ex}(cd^2-ae^2)}\right)}{2\left(cd^2-ae^2\right)}-\frac{1}{\sqrt{d+ex}\left(cd^2-ae^2\right)\left(ae+cdx\right)}$$

$$\Big\downarrow \text{221}$$

$$-\frac{3e\left(\frac{2}{\sqrt{d+ex}(cd^2-ae^2)}-\frac{2\sqrt{c}\sqrt{d}\text{arctanh}\left(\frac{\sqrt{c}\sqrt{d}\sqrt{d+ex}}{\sqrt{cd^2-ae^2}}\right)}{(cd^2-ae^2)^{3/2}}\right)}{2\left(cd^2-ae^2\right)}-\frac{1}{\sqrt{d+ex}\left(cd^2-ae^2\right)\left(ae+cdx\right)}$$

input

```
Int[Sqrt[d + e*x]/(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^2,x]
```

output

```
-(1/((c*d^2 - a*e^2)*(a*e + c*d*x)*Sqrt[d + e*x])) - (3*e*(2/((c*d^2 - a*e
^2)*Sqrt[d + e*x]) - (2*Sqrt[c]*Sqrt[d]*ArcTanh[(Sqrt[c]*Sqrt[d]*Sqrt[d +
e*x])/Sqrt[c*d^2 - a*e^2]])/(c*d^2 - a*e^2)^(3/2)))/(2*(c*d^2 - a*e^2))
```

Defintions of rubi rules used

rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && ILtQ[m, -1] && FractionQ[n] && LtQ[n, 0]
```

rule 61

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && LtQ[m, -1] &&  !(LtQ[n, -1] && (EqQ[a, 0
] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && IntLinearQ[a, b, c, d
, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 1121
```
Int[((d_) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Int[ExpandIntegrand[(d + e*x)^(m + p)*(a/d + (c/e)*x)^p, x], x]
/; FreeQ[{a, b, c, d, e, m, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && (Int
egerQ[p] || (GtQ[a, 0] && GtQ[d, 0] && LtQ[c, 0]))
```

Maple [A] (verified)

Time = 1.95 (sec) , antiderivative size = 100, normalized size of antiderivative = 0.78

method	result	size
pseudoelliptic	$\frac{e\left(-\frac{2}{\sqrt{ex+d}}-\frac{cd\sqrt{ex+d}}{e(cdx+ae)}-\frac{3cd\arctan\left(\frac{cd\sqrt{ex+d}}{\sqrt{cd\left(a\,e^{2}-c\,d^{2}\right)}}\right)}{\sqrt{cd\left(a\,e^{2}-c\,d^{2}\right)}}\right)}{\left(a\,e^{2}-c\,d^{2}\right)^{2}}$	100
derivativedivides	$2e\left(-\frac{cd\left(\frac{\sqrt{ex+d}}{2cd(ex+d)+2a\,e^{2}-2c\,d^{2}}+\frac{3\arctan\left(\frac{cd\sqrt{ex+d}}{\sqrt{cd\left(a\,e^{2}-c\,d^{2}\right)}}\right)}{2\sqrt{cd\left(a\,e^{2}-c\,d^{2}\right)}}\right)}{\left(a\,e^{2}-c\,d^{2}\right)^{2}}-\frac{1}{\left(a\,e^{2}-c\,d^{2}\right)^{2}\sqrt{ex+d}}\right)$	125
default	$2e\left(-\frac{cd\left(\frac{\sqrt{ex+d}}{2cd(ex+d)+2a\,e^{2}-2c\,d^{2}}+\frac{3\arctan\left(\frac{cd\sqrt{ex+d}}{\sqrt{cd\left(a\,e^{2}-c\,d^{2}\right)}}\right)}{2\sqrt{cd\left(a\,e^{2}-c\,d^{2}\right)}}\right)}{\left(a\,e^{2}-c\,d^{2}\right)^{2}}-\frac{1}{\left(a\,e^{2}-c\,d^{2}\right)^{2}\sqrt{ex+d}}\right)$	125

input
```
int((e*x+d)^(1/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^2,x,method=_RETURNVERB
OSE)
```

output
```
e/(a*e^2-c*d^2)^2*(-2/(e*x+d)^(1/2)-c*d*(e*x+d)^(1/2)/e/(c*d*x+a*e)-3*c*d/
(c*d*(a*e^2-c*d^2))^(1/2)*arctan(c*d*(e*x+d)^(1/2)/(c*d*(a*e^2-c*d^2))^(1/
2)))
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 476, normalized size of antiderivative = 3.72

$$\int \frac{\sqrt{d+ex}}{\left(ade+(cd^2+ae^2)\,x+cdex^2\right)^2}\,dx$$

$$= \left[\frac{3\left(cde^2x^2+ade^2+(cd^2e+ae^3)x\right)\sqrt{\frac{cd}{cd^2-ae^2}}\log\left(\frac{cdex+2\,cd^2-ae^2+2\,(cd^2-ae^2)\sqrt{ex+d}\sqrt{\frac{cd}{cd^2-ae^2}}}{cdx+ae}\right)-2\,(3\,cdex+c}{2\,(ac^2d^5e-2\,a^2cd^3e^3+a^3de^5+(c^3d^5e-2\,ac^2d^3e^3+a^2cde^5)x^2+(c^3d^6-ac^2d^4e^2-a^2cd^2e^4}\right.$$

$$-\frac{3\left(cde^2x^2+ade^2+(cd^2e+ae^3)x\right)\sqrt{-\frac{cd}{cd^2-ae^2}}\arctan\left(\sqrt{ex+d}\sqrt{-\frac{cd}{cd^2-ae^2}}\right)+(3\,cdex+cd^2+2\,ae^2)\sqrt{}}{ac^2d^5e-2\,a^2cd^3e^3+a^3de^5+(c^3d^5e-2\,ac^2d^3e^3+a^2cde^5)x^2+(c^3d^6-ac^2d^4e^2-a^2cd^2e^4+a^3e^6}$$

input
```
integrate((e*x+d)^(1/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^2,x, algorithm="
fricas")
```

output
```
[1/2*(3*(c*d*e^2*x^2 + a*d*e^2 + (c*d^2*e + a*e^3)*x)*sqrt(c*d/(c*d^2 - a*
e^2))*log((c*d*e*x + 2*c*d^2 - a*e^2 + 2*(c*d^2 - a*e^2)*sqrt(e*x + d)*sqr
t(c*d/(c*d^2 - a*e^2)))/(c*d*x + a*e)) - 2*(3*c*d*e*x + c*d^2 + 2*a*e^2)*s
qrt(e*x + d))/(a*c^2*d^5*e - 2*a^2*c*d^3*e^3 + a^3*d*e^5 + (c^3*d^5*e - 2*
a*c^2*d^3*e^3 + a^2*c*d*e^5)*x^2 + (c^3*d^6 - a*c^2*d^4*e^2 - a^2*c*d^2*e^
4 + a^3*e^6)*x), -(3*(c*d*e^2*x^2 + a*d*e^2 + (c*d^2*e + a*e^3)*x)*sqrt(-c
*d/(c*d^2 - a*e^2))*arctan(sqrt(e*x + d)*sqrt(-c*d/(c*d^2 - a*e^2))) + (3*
c*d*e*x + c*d^2 + 2*a*e^2)*sqrt(e*x + d))/(a*c^2*d^5*e - 2*a^2*c*d^3*e^3 +
 a^3*d*e^5 + (c^3*d^5*e - 2*a*c^2*d^3*e^3 + a^2*c*d*e^5)*x^2 + (c^3*d^6 -
a*c^2*d^4*e^2 - a^2*c*d^2*e^4 + a^3*e^6)*x)]
```

Sympy [F]

$$\int \frac{\sqrt{d+ex}}{\left(ade+(cd^2+ae^2)\,x+cdex^2\right)^2}\,dx = \int \frac{1}{(d+ex)^{\frac{3}{2}}\,(ae+cdx)^2}\,dx$$

input
```
integrate((e*x+d)**(1/2)/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**2,x)
```

output
```
Integral(1/((d + e*x)**(3/2)*(a*e + c*d*x)**2), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\sqrt{d+ex}}{(ade+(cd^2+ae^2)x+cdex^2)^2}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((e*x+d)^(1/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^2,x, algorithm="
maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(a*e^2-c*d^2>0)', see `assume?` f
or more de
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 171, normalized size of antiderivative = 1.34

$$\int \frac{\sqrt{d+ex}}{(ade+(cd^2+ae^2)x+cdex^2)^2}\,dx = -\frac{3\,cde\arctan\left(\frac{\sqrt{ex+d}cd}{\sqrt{-c^2d^3+acde^2}}\right)}{(c^2d^4-2\,acd^2e^2+a^2e^4)\sqrt{-c^2d^3+acde^2}} - \frac{3\,(ex+d)cde-2\,cd^2e+2\,ae^3}{(c^2d^4-2\,acd^2e^2+a^2e^4)\left((ex+d)^{\frac{3}{2}}cd-\sqrt{ex+d}cd^2+\sqrt{ex+d}ae^2\right)}$$

input
```
integrate((e*x+d)^(1/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^2,x, algorithm="
giac")
```

output
```
-3*c*d*e*arctan(sqrt(e*x + d)*c*d/sqrt(-c^2*d^3 + a*c*d*e^2))/((c^2*d^4 -
2*a*c*d^2*e^2 + a^2*e^4)*sqrt(-c^2*d^3 + a*c*d*e^2)) - (3*(e*x + d)*c*d*e
- 2*c*d^2*e + 2*a*e^3)/((c^2*d^4 - 2*a*c*d^2*e^2 + a^2*e^4)*((e*x + d)^(3/
2)*c*d - sqrt(e*x + d)*c*d^2 + sqrt(e*x + d)*a*e^2))
```

Mupad [B] (verification not implemented)

Time = 5.24 (sec) , antiderivative size = 155, normalized size of antiderivative = 1.21

$$\int \frac{\sqrt{d+ex}}{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^2}\,dx = -\frac{\frac{2\,e}{a\,e^2-c\,d^2}+\frac{3\,c\,d\,e\,(d+e\,x)}{\left(a\,e^2-c\,d^2\right)^2}}{\left(a\,e^2-c\,d^2\right)\,\sqrt{d+e\,x}+c\,d\,(d+e\,x)^{3/2}} - \frac{3\,\sqrt{c}\,\sqrt{d}\,e\,\mathrm{atan}\left(\frac{\sqrt{c}\,\sqrt{d}\,\sqrt{d+e\,x}\,\left(a^2\,e^4-2\,a\,c\,d^2\,e^2+c^2\,d^4\right)}{\left(a\,e^2-c\,d^2\right)^{5/2}}\right)}{\left(a\,e^2-c\,d^2\right)^{5/2}}$$

input
```
int((d + e*x)^(1/2)/(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^2,x)
```

output
```
- ((2*e)/(a*e^2 - c*d^2) + (3*c*d*e*(d + e*x))/(a*e^2 - c*d^2)^2)/((a*e^2
- c*d^2)*(d + e*x)^(1/2) + c*d*(d + e*x)^(3/2)) - (3*c^(1/2)*d^(1/2)*e*ata
n((c^(1/2)*d^(1/2)*(d + e*x)^(1/2)*(a^2*e^4 + c^2*d^4 - 2*a*c*d^2*e^2))/(a
*e^2 - c*d^2)^(5/2)))/(a*e^2 - c*d^2)^(5/2)
```

Reduce [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 270, normalized size of antiderivative = 2.11

$$\int \frac{\sqrt{d+ex}}{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^2}\,dx = \frac{-3\sqrt{d}\,\sqrt{c}\,\sqrt{ex+d}\,\sqrt{a\,e^2-c\,d^2}\,atan\left(\frac{\sqrt{ex+d}\,cd}{\sqrt{d}\,\sqrt{c}\,\sqrt{a\,e^2-c\,d^2}}\right)a\,e^2-3\sqrt{d}\,\sqrt{c}\,\sqrt{ex+d}\,\sqrt{a\,e^2-c\,d^2}\,atan\left(\frac{\sqrt{ex+}}{\sqrt{d}\,\sqrt{c}\,\sqrt{a}}\right.}{\sqrt{ex+d}\,\left(a^3cd\,e^6x-3a^2c^2d^3e^4x+3a\,c^3d^5e^2x-c^4d^7x+a^4e^7-3a^3c\,d\right.}$$

input
```
int((e*x+d)^(1/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^2,x)
```

output
```
( - 3*sqrt(d)*sqrt(c)*sqrt(d + e*x)*sqrt(a*e**2 - c*d**2)*atan((sqrt(d + e
*x)*c*d)/(sqrt(d)*sqrt(c)*sqrt(a*e**2 - c*d**2)))*a*e**2 - 3*sqrt(d)*sqrt(
c)*sqrt(d + e*x)*sqrt(a*e**2 - c*d**2)*atan((sqrt(d + e*x)*c*d)/(sqrt(d)*s
qrt(c)*sqrt(a*e**2 - c*d**2)))*c*d*e*x - 2*a**2*e**4 + a*c*d**2*e**2 - 3*a
*c*d*e**3*x + c**2*d**4 + 3*c**2*d**3*e*x)/(sqrt(d + e*x)*(a**4*e**7 - 3*a
**3*c*d**2*e**5 + a**3*c*d*e**6*x + 3*a**2*c**2*d**4*e**3 - 3*a**2*c**2*d*
*3*e**4*x - a*c**3*d**6*e + 3*a*c**3*d**5*e**2*x - c**4*d**7*x))
```

3.189 $\int \frac{1}{\sqrt{d+ex}\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^2}\,dx$

Optimal result

Integrand size = 37, antiderivative size = 158

$$\int \frac{1}{\sqrt{d+ex}\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^2}\,dx = -\frac{5e}{3\left(cd^2-ae^2\right)^2(d+ex)^{3/2}}-\frac{1}{\left(cd^2-ae^2\right)(ae+cdx)(d+ex)^{3/2}} - \frac{5cde}{\left(cd^2-ae^2\right)^3\sqrt{d+ex}}+\frac{5c^{3/2}d^{3/2}e\,\text{arctanh}\left(\frac{\sqrt{c}\sqrt{d}\sqrt{d+ex}}{\sqrt{cd^2-ae^2}}\right)}{\left(cd^2-ae^2\right)^{7/2}}$$

output
```
-5/3*e/(-a*e^2+c*d^2)^2/(e*x+d)^(3/2)-1/(-a*e^2+c*d^2)/(c*d*x+a*e)/(e*x+d)
^(3/2)-5*c*d*e/(-a*e^2+c*d^2)^3/(e*x+d)^(1/2)+5*c^(3/2)*d^(3/2)*e*arctanh(
c^(1/2)*d^(1/2)*(e*x+d)^(1/2)/(-a*e^2+c*d^2)^(1/2))/(-a*e^2+c*d^2)^(7/2)
```

Mathematica [A] (verified)

Time = 0.38 (sec) , antiderivative size = 156, normalized size of antiderivative = 0.99

$$\int \frac{1}{\sqrt{d+ex}\,(ade+(cd^2+ae^2)\,x+cdex^2)^2}\,dx = \frac{2a^2e^4-2acde^2(7d+5ex)-c^2d^2(3d^2+20dex+15e^2x^2)}{3\,(cd^2-ae^2)^3\,(ae+cdx)(d+ex)^{3/2}} + \frac{5c^{3/2}d^{3/2}e\arctan\left(\frac{\sqrt{c}\sqrt{d}\sqrt{d+ex}}{\sqrt{-cd^2+ae^2}}\right)}{(-cd^2+ae^2)^{7/2}}$$

input
```
Integrate[1/(Sqrt[d + e*x]*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^2),x]
```

output
```
(2*a^2*e^4 - 2*a*c*d*e^2*(7*d + 5*e*x) - c^2*d^2*(3*d^2 + 20*d*e*x + 15*e^
2*x^2))/(3*(c*d^2 - a*e^2)^3*(a*e + c*d*x)*(d + e*x)^(3/2)) + (5*c^(3/2)*d
^(3/2)*e*ArcTan[(Sqrt[c]*Sqrt[d]*Sqrt[d + e*x])/Sqrt[-(c*d^2) + a*e^2]])/(
-(c*d^2) + a*e^2)^(7/2)
```

Rubi [A] (verified)

Time = 0.47 (sec) , antiderivative size = 191, normalized size of antiderivative = 1.21, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.162$, Rules used = {1121, 52, 61, 61, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{d+ex}\,(x\,(ae^2+cd^2)+ade+cdex^2)^2}\,dx$$

$$\Big\downarrow \text{1121}$$

$$\int \frac{1}{(d+ex)^{5/2}(ae+cdx)^2}dx$$

$$\Big\downarrow \text{52}$$

$$-\frac{5e\int \frac{1}{(ae+cdx)(d+ex)^{5/2}}dx}{2\,(cd^2-ae^2)} - \frac{1}{(d+ex)^{3/2}\,(cd^2-ae^2)\,(ae+cdx)}$$

$$\Big\downarrow \text{61}$$

$$-\frac{5e\left(\frac{cd\int \frac{1}{(ae+cdx)(d+ex)^{3/2}}dx}{cd^2-ae^2}+\frac{2}{3(d+ex)^{3/2}(cd^2-ae^2)}\right)}{2\left(cd^2-ae^2\right)}-\frac{1}{(d+ex)^{3/2}\left(cd^2-ae^2\right)(ae+cdx)}$$

$$\Big\downarrow \text{61}$$

$$-\frac{5e\left(\frac{cd\left(\frac{cd\int \frac{1}{(ae+cdx)\sqrt{d+ex}}dx}{cd^2-ae^2}+\frac{2}{\sqrt{d+ex}\left(cd^2-ae^2\right)}\right)}{cd^2-ae^2}+\frac{2}{3(d+ex)^{3/2}(cd^2-ae^2)}\right)}{2\left(cd^2-ae^2\right)}-\frac{1}{(d+ex)^{3/2}\left(cd^2-ae^2\right)(ae+cdx)}$$

$$\Big\downarrow \text{73}$$

$$-\frac{5e\left(\frac{cd\left(\frac{2cd\int \frac{1}{-\frac{cd^2}{e}+\frac{c(d+ex)d}{e}+ae}d\sqrt{d+ex}}{e\left(cd^2-ae^2\right)}+\frac{2}{\sqrt{d+ex}\left(cd^2-ae^2\right)}\right)}{cd^2-ae^2}+\frac{2}{3(d+ex)^{3/2}(cd^2-ae^2)}\right)}{2\left(cd^2-ae^2\right)}-\frac{1}{(d+ex)^{3/2}\left(cd^2-ae^2\right)(ae+cdx)}$$

$$\Big\downarrow \text{221}$$

$$-\frac{5e\left(\frac{cd\left(\frac{2}{\sqrt{d+ex}\left(cd^2-ae^2\right)}-\frac{2\sqrt{c}\sqrt{d}\text{arctanh}\left(\frac{\sqrt{c}\sqrt{d}\sqrt{d+ex}}{\sqrt{cd^2-ae^2}}\right)}{\left(cd^2-ae^2\right)^{3/2}}\right)}{cd^2-ae^2}+\frac{2}{3(d+ex)^{3/2}(cd^2-ae^2)}\right)}{2\left(cd^2-ae^2\right)}-\frac{1}{(d+ex)^{3/2}\left(cd^2-ae^2\right)(ae+cdx)}$$

input
```
Int[1/(Sqrt[d + e*x]*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^2),x]
```

output
```
-(1/((c*d^2 - a*e^2)*(a*e + c*d*x)*(d + e*x)^(3/2))) - (5*e*(2/(3*(c*d^2 -
 a*e^2)*(d + e*x)^(3/2)) + (c*d*(2/((c*d^2 - a*e^2)*Sqrt[d + e*x]) - (2*Sq
rt[c]*Sqrt[d]*ArcTanh[(Sqrt[c]*Sqrt[d]*Sqrt[d + e*x])/Sqrt[c*d^2 - a*e^2]]
)/(c*d^2 - a*e^2)^(3/2)))/(c*d^2 - a*e^2)))/(2*(c*d^2 - a*e^2))
```

Defintions of rubi rules used

rule 52
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && ILtQ[m, -1] && FractionQ[n] && LtQ[n, 0]
```

rule 61
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && LtQ[m, -1] &&  !(LtQ[n, -1] && (EqQ[a, 0
] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && IntLinearQ[a, b, c, d
, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 1121
```
Int[((d_) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Int[ExpandIntegrand[(d + e*x)^(m + p)*(a/d + (c/e)*x)^p, x], x]
/; FreeQ[{a, b, c, d, e, m, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && (Int
egerQ[p] || (GtQ[a, 0] && GtQ[d, 0] && LtQ[c, 0]))
```

Maple [A] (verified)

Time = 2.07 (sec) , antiderivative size = 153, normalized size of antiderivative = 0.97

method	result
derivativedivides	$2e\left(-\frac{1}{3(a\,e^2-c\,d^2)^2(ex+d)^{\frac{3}{2}}}+\frac{2cd}{(a\,e^2-c\,d^2)^3\sqrt{ex+d}}+\frac{c^2d^2\left(\frac{\sqrt{ex+d}}{2cd(ex+d)+2a\,e^2-2c\,d^2}+\frac{5\arctan\left(\frac{cd\sqrt{ex+d}}{\sqrt{cd\left(a\,e^2-c\,d^2\right)}}\right)}{2\sqrt{cd\left(a\,e^2-c\,d^2\right)}}\right)}{(a\,e^2-c\,d^2)^3}\right)$
default	$2e\left(-\frac{1}{3(a\,e^2-c\,d^2)^2(ex+d)^{\frac{3}{2}}}+\frac{2cd}{(a\,e^2-c\,d^2)^3\sqrt{ex+d}}+\frac{c^2d^2\left(\frac{\sqrt{ex+d}}{2cd(ex+d)+2a\,e^2-2c\,d^2}+\frac{5\arctan\left(\frac{cd\sqrt{ex+d}}{\sqrt{cd\left(a\,e^2-c\,d^2\right)}}\right)}{2\sqrt{cd\left(a\,e^2-c\,d^2\right)}}\right)}{(a\,e^2-c\,d^2)^3}\right)$
pseudoelliptic	$e\left(-\frac{2}{3(a\,e^2-c\,d^2)^2(ex+d)^{\frac{3}{2}}}+\frac{4cd}{(a\,e^2-c\,d^2)^3\sqrt{ex+d}}+\frac{\sqrt{ex+d}\,c^2d^2}{e(cdx+ae)(a\,e^2-c\,d^2)^3}+\frac{5\arctan\left(\frac{cd\sqrt{ex+d}}{\sqrt{cd\left(a\,e^2-c\,d^2\right)}}\right)c^2}{\sqrt{cd(a\,e^2-c\,d^2)}\,(a\,e^2-c\,d^2)^3}\right)$

input
```
int(1/(e*x+d)^(1/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^2,x,method=_RETURNVE
RBOSE)
```

output
```
2*e*(-1/3/(a*e^2-c*d^2)^2/(e*x+d)^(3/2)+2/(a*e^2-c*d^2)^3*c*d/(e*x+d)^(1/2
)+1/(a*e^2-c*d^2)^3*c^2*d^2*(1/2*(e*x+d)^(1/2)/(c*d*(e*x+d)+a*e^2-c*d^2)+5
/2/(c*d*(a*e^2-c*d^2))^(1/2)*arctan(c*d*(e*x+d)^(1/2)/(c*d*(a*e^2-c*d^2))^
(1/2))))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 406 vs. $2(136) = 272$.

Time = 0.11 (sec) , antiderivative size = 854, normalized size of antiderivative = 5.41

$$\int \frac{1}{\sqrt{d+ex}\,(ade+(cd^2+ae^2)\,x+cdex^2)^2}\,dx = \text{Too large to display}$$

input
```
integrate(1/(e*x+d)^(1/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^2,x, algorithm
="fricas")
```

output
```
[-1/6*(15*(c^2*d^2*e^3*x^3 + a*c*d^3*e^2 + (2*c^2*d^3*e^2 + a*c*d*e^4)*x^2
 + (c^2*d^4*e + 2*a*c*d^2*e^3)*x)*sqrt(c*d/(c*d^2 - a*e^2))*log((c*d*e*x +
 2*c*d^2 - a*e^2 - 2*(c*d^2 - a*e^2)*sqrt(e*x + d)*sqrt(c*d/(c*d^2 - a*e^2
)))/(c*d*x + a*e)) + 2*(15*c^2*d^2*e^2*x^2 + 3*c^2*d^4 + 14*a*c*d^2*e^2 -
2*a^2*e^4 + 10*(2*c^2*d^3*e + a*c*d*e^3)*x)*sqrt(e*x + d))/(a*c^3*d^8*e -
3*a^2*c^2*d^6*e^3 + 3*a^3*c*d^4*e^5 - a^4*d^2*e^7 + (c^4*d^7*e^2 - 3*a*c^3
*d^5*e^4 + 3*a^2*c^2*d^3*e^6 - a^3*c*d*e^8)*x^3 + (2*c^4*d^8*e - 5*a*c^3*d
^6*e^3 + 3*a^2*c^2*d^4*e^5 + a^3*c*d^2*e^7 - a^4*e^9)*x^2 + (c^4*d^9 - a*c
^3*d^7*e^2 - 3*a^2*c^2*d^5*e^4 + 5*a^3*c*d^3*e^6 - 2*a^4*d*e^8)*x), -1/3*(
15*(c^2*d^2*e^3*x^3 + a*c*d^3*e^2 + (2*c^2*d^3*e^2 + a*c*d*e^4)*x^2 + (c^2
*d^4*e + 2*a*c*d^2*e^3)*x)*sqrt(-c*d/(c*d^2 - a*e^2))*arctan(sqrt(e*x + d)
*sqrt(-c*d/(c*d^2 - a*e^2))) + (15*c^2*d^2*e^2*x^2 + 3*c^2*d^4 + 14*a*c*d^
2*e^2 - 2*a^2*e^4 + 10*(2*c^2*d^3*e + a*c*d*e^3)*x)*sqrt(e*x + d))/(a*c^3*
d^8*e - 3*a^2*c^2*d^6*e^3 + 3*a^3*c*d^4*e^5 - a^4*d^2*e^7 + (c^4*d^7*e^2 -
 3*a*c^3*d^5*e^4 + 3*a^2*c^2*d^3*e^6 - a^3*c*d*e^8)*x^3 + (2*c^4*d^8*e - 5
*a*c^3*d^6*e^3 + 3*a^2*c^2*d^4*e^5 + a^3*c*d^2*e^7 - a^4*e^9)*x^2 + (c^4*d
^9 - a*c^3*d^7*e^2 - 3*a^2*c^2*d^5*e^4 + 5*a^3*c*d^3*e^6 - 2*a^4*d*e^8)*x)
]
```

Sympy [F]

$$\int \frac{1}{\sqrt{d+ex}\,(ade+(cd^2+ae^2)\,x+cdex^2)^2}\,dx = \int \frac{1}{(d+ex)^{\frac{5}{2}}\,(ae+cdx)^2}\,dx$$

input
```
integrate(1/(e*x+d)**(1/2)/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**2,x)
```

output
```
Integral(1/((d + e*x)**(5/2)*(a*e + c*d*x)**2), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{\sqrt{d+ex}\,(ade+(cd^2+ae^2)\,x+cdex^2)^2}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(1/(e*x+d)^(1/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^2,x, algorithm
="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(a*e^2-c*d^2>0)', see `assume?` f
or more de
```

Giac [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 253, normalized size of antiderivative = 1.60

$$\begin{aligned}
&\int \frac{1}{\sqrt{d+ex}\,(ade+(cd^2+ae^2)\,x+cdex^2)^2}\,dx \\
&= -\frac{5\,c^2d^2e\arctan\left(\frac{\sqrt{ex+d}cd}{\sqrt{-c^2d^3+acde^2}}\right)}{(c^3d^6-3\,ac^2d^4e^2+3\,a^2cd^2e^4-a^3e^6)\sqrt{-c^2d^3+acde^2}} \\
&\quad -\frac{\sqrt{ex+d}c^2d^2e}{(c^3d^6-3\,ac^2d^4e^2+3\,a^2cd^2e^4-a^3e^6)((ex+d)cd-cd^2+ae^2)} \\
&\quad -\frac{2\,(6\,(ex+d)cde+cd^2e-ae^3)}{3\,(c^3d^6-3\,ac^2d^4e^2+3\,a^2cd^2e^4-a^3e^6)(ex+d)^{\frac{3}{2}}}
\end{aligned}$$

input
```
integrate(1/(e*x+d)^(1/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^2,x, algorithm
="giac")
```

output
```
-5*c^2*d^2*e*arctan(sqrt(e*x + d)*c*d/sqrt(-c^2*d^3 + a*c*d*e^2))/((c^3*d^
6 - 3*a*c^2*d^4*e^2 + 3*a^2*c*d^2*e^4 - a^3*e^6)*sqrt(-c^2*d^3 + a*c*d*e^2
)) - sqrt(e*x + d)*c^2*d^2*e/((c^3*d^6 - 3*a*c^2*d^4*e^2 + 3*a^2*c*d^2*e^4
 - a^3*e^6)*((e*x + d)*c*d - c*d^2 + a*e^2)) - 2/3*(6*(e*x + d)*c*d*e + c*
d^2*e - a*e^3)/((c^3*d^6 - 3*a*c^2*d^4*e^2 + 3*a^2*c*d^2*e^4 - a^3*e^6)*(e
*x + d)^(3/2))
```

Mupad [B] (verification not implemented)

Time = 5.39 (sec) , antiderivative size = 200, normalized size of antiderivative = 1.27

$$\int \frac{1}{\sqrt{d+ex}\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^2}\,dx = \frac{\frac{10\,c\,d\,e\,(d+e\,x)}{3\,{\left(a\,e^2-c\,d^2\right)}^2}-\frac{2\,e}{3\,\left(a\,e^2-c\,d^2\right)}+\frac{5\,c^2\,d^2\,e\,{(d+e\,x)}^2}{{\left(a\,e^2-c\,d^2\right)}^3}}{\left(a\,e^2-c\,d^2\right)\,{\left(d+e\,x\right)}^{3/2}+c\,d\,{\left(d+e\,x\right)}^{5/2}} + \frac{5\,c^{3/2}\,d^{3/2}\,e\,\mathrm{atan}\left(\frac{\sqrt{c}\,\sqrt{d}\,\sqrt{d+e\,x}\,\left(a^3\,e^6-3\,a^2\,c\,d^2\,e^4+3\,a\,c^2\,d^4\,e^2-c^3\,d^6\right)}{{\left(a\,e^2-c\,d^2\right)}^{7/2}}\right)}{{\left(a\,e^2-c\,d^2\right)}^{7/2}}$$

input
```
int(1/((d + e*x)^(1/2)*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^2),x)
```

output
```
((10*c*d*e*(d + e*x))/(3*(a*e^2 - c*d^2)^2) - (2*e)/(3*(a*e^2 - c*d^2)) +
(5*c^2*d^2*e*(d + e*x)^2)/(a*e^2 - c*d^2)^3)/((a*e^2 - c*d^2)*(d + e*x)^(3
/2) + c*d*(d + e*x)^(5/2)) + (5*c^(3/2)*d^(3/2)*e*atan((c^(1/2)*d^(1/2)*(d
 + e*x)^(1/2)*(a^3*e^6 - c^3*d^6 + 3*a*c^2*d^4*e^2 - 3*a^2*c*d^2*e^4))/(a*
e^2 - c*d^2)^(7/2)))/(a*e^2 - c*d^2)^(7/2)
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 585, normalized size of antiderivative = 3.70

$$\int \frac{1}{\sqrt{d+ex}\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^2}\,dx = \frac{15\sqrt{d}\,\sqrt{c}\,\sqrt{ex+d}\,\sqrt{a\,e^2-c\,d^2}\;atan\left(\frac{\sqrt{ex+d}\,cd}{\sqrt{d}\,\sqrt{c}\,\sqrt{a\,e^2-c\,d^2}}\right)ac\,d^2e^2+15\sqrt{d}\,\sqrt{c}\,\sqrt{ex+d}\,\sqrt{a\,e^2-c\,d^2}\;atan\left(\frac{\sqrt{}}{\sqrt{d}\,\sqrt{}}\right.}{3\sqrt{ex+d}\,\left(a^4cd\,e^9x^2-4a\right.}$$

input
```
int(1/(e*x+d)^(1/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^2,x)
```

output
```
(15*sqrt(d)*sqrt(c)*sqrt(d + e*x)*sqrt(a*e**2 - c*d**2)*atan((sqrt(d + e*x
)*c*d)/(sqrt(d)*sqrt(c)*sqrt(a*e**2 - c*d**2)))*a*c*d**2*e**2 + 15*sqrt(d)
*sqrt(c)*sqrt(d + e*x)*sqrt(a*e**2 - c*d**2)*atan((sqrt(d + e*x)*c*d)/(sqr
t(d)*sqrt(c)*sqrt(a*e**2 - c*d**2)))*a*c*d*e**3*x + 15*sqrt(d)*sqrt(c)*sqr
t(d + e*x)*sqrt(a*e**2 - c*d**2)*atan((sqrt(d + e*x)*c*d)/(sqrt(d)*sqrt(c)
*sqrt(a*e**2 - c*d**2)))*c**2*d**3*e*x + 15*sqrt(d)*sqrt(c)*sqrt(d + e*x)*
sqrt(a*e**2 - c*d**2)*atan((sqrt(d + e*x)*c*d)/(sqrt(d)*sqrt(c)*sqrt(a*e**
2 - c*d**2)))*c**2*d**2*e**2*x**2 - 2*a**3*e**6 + 16*a**2*c*d**2*e**4 + 10
*a**2*c*d*e**5*x - 11*a*c**2*d**4*e**2 + 10*a*c**2*d**3*e**3*x + 15*a*c**2
*d**2*e**4*x**2 - 3*c**3*d**6 - 20*c**3*d**5*e*x - 15*c**3*d**4*e**2*x**2)
/(3*sqrt(d + e*x)*(a**5*d*e**9 + a**5*e**10*x - 4*a**4*c*d**3*e**7 - 3*a**
4*c*d**2*e**8*x + a**4*c*d*e**9*x**2 + 6*a**3*c**2*d**5*e**5 + 2*a**3*c**2
*d**4*e**6*x - 4*a**3*c**2*d**3*e**7*x**2 - 4*a**2*c**3*d**7*e**3 + 2*a**2
*c**3*d**6*e**4*x + 6*a**2*c**3*d**5*e**5*x**2 + a*c**4*d**9*e - 3*a*c**4*
d**8*e**2*x - 4*a*c**4*d**7*e**3*x**2 + c**5*d**10*x + c**5*d**9*e*x**2))
```

3.190 $\int \frac{1}{(d+ex)^{3/2}(ade+(cd^2+ae^2)x+cdex^2)^2}\,dx$

Optimal result

Integrand size = 37, antiderivative size = 192

$$\int \frac{1}{(d+ex)^{3/2}\left(ade+(cd^2+ae^2)x+cdex^2\right)^2}\,dx = -\frac{7e}{5\left(cd^2-ae^2\right)^2(d+ex)^{5/2}} - \frac{1}{(cd^2-ae^2)(ae+cdx)(d+ex)^{5/2}} - \frac{7cde}{3\left(cd^2-ae^2\right)^3(d+ex)^{3/2}} - \frac{7c^2d^2e}{\left(cd^2-ae^2\right)^4\sqrt{d+ex}} + \frac{7c^{5/2}d^{5/2}e\operatorname{arctanh}\left(\frac{\sqrt{c}\sqrt{d}\sqrt{d+ex}}{\sqrt{cd^2-ae^2}}\right)}{\left(cd^2-ae^2\right)^{9/2}}$$

output
```
-7/5*e/(-a*e^2+c*d^2)^2/(e*x+d)^(5/2)-1/(-a*e^2+c*d^2)/(c*d*x+a*e)/(e*x+d)
^(5/2)-7/3*c*d*e/(-a*e^2+c*d^2)^3/(e*x+d)^(3/2)-7*c^2*d^2*e/(-a*e^2+c*d^2)
^4/(e*x+d)^(1/2)+7*c^(5/2)*d^(5/2)*e*arctanh(c^(1/2)*d^(1/2)*(e*x+d)^(1/2)
/(-a*e^2+c*d^2)^(1/2))/(-a*e^2+c*d^2)^(9/2)
```

Mathematica [A] (verified)

Time = 0.50 (sec) , antiderivative size = 200, normalized size of antiderivative = 1.04

$$\int \frac{1}{(d+ex)^{3/2}\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^2}\,dx = \frac{-6a^3e^6+2a^2cde^4(16d+7ex)-2ac^2d^2e^2(58d^2+84de}{15\left(cd^2-ae^2\right)^4(}$$

$$-\frac{7c^{5/2}d^{5/2}e\arctan\left(\frac{\sqrt{c}\sqrt{d}\sqrt{d+ex}}{\sqrt{-cd^2+ae^2}}\right)}{\left(-cd^2+ae^2\right)^{9/2}}$$

input
```
Integrate[1/((d + e*x)^(3/2)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^2),x]
```

output
```
(-6*a^3*e^6 + 2*a^2*c*d*e^4*(16*d + 7*e*x) - 2*a*c^2*d^2*e^2*(58*d^2 + 84*
d*e*x + 35*e^2*x^2) - c^3*d^3*(15*d^3 + 161*d^2*e*x + 245*d*e^2*x^2 + 105*
e^3*x^3))/(15*(c*d^2 - a*e^2)^4*(a*e + c*d*x)*(d + e*x)^(5/2)) - (7*c^(5/2
)*d^(5/2)*e*ArcTan[(Sqrt[c]*Sqrt[d]*Sqrt[d + e*x])/Sqrt[-(c*d^2) + a*e^2]]
)/(-(c*d^2) + a*e^2)^(9/2)
```

Rubi [A] (verified)

Time = 0.51 (sec) , antiderivative size = 236, normalized size of antiderivative = 1.23, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.189$, Rules used = {1121, 52, 61, 61, 61, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(d+ex)^{3/2}\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^2}dx$$

$$\Big\downarrow \text{ 1121}$$

$$\int \frac{1}{(d+ex)^{7/2}(ae+cdx)^2}dx$$

$$\Big\downarrow \text{ 52}$$

$$-\frac{7e\int \frac{1}{(ae+cdx)(d+ex)^{7/2}}dx}{2\left(cd^2-ae^2\right)}-\frac{1}{(d+ex)^{5/2}\left(cd^2-ae^2\right)(ae+cdx)}$$

$$\Big\downarrow \ 61$$

$$-\frac{7e\left(\frac{cd\int \frac{1}{(ae+cdx)(d+ex)^{5/2}}dx}{cd^2-ae^2}+\frac{2}{5(d+ex)^{5/2}(cd^2-ae^2)}\right)}{2\left(cd^2-ae^2\right)}-\frac{1}{(d+ex)^{5/2}\left(cd^2-ae^2\right)(ae+cdx)}$$

$$\Big\downarrow \ 61$$

$$-\frac{7e\left(\frac{cd\left(\frac{cd\int \frac{1}{(ae+cdx)(d+ex)^{3/2}}dx}{cd^2-ae^2}+\frac{2}{3(d+ex)^{3/2}\left(cd^2-ae^2\right)}\right)}{cd^2-ae^2}+\frac{2}{5(d+ex)^{5/2}(cd^2-ae^2)}\right)}{2\left(cd^2-ae^2\right)}-\frac{1}{(d+ex)^{5/2}\left(cd^2-ae^2\right)(ae+cdx)}$$

$$\Big\downarrow \ 61$$

$$-\frac{7e\left(\frac{cd\left(\frac{cd\left(\frac{cd\int \frac{1}{(ae+cdx)\sqrt{d+ex}}dx}{cd^2-ae^2}+\frac{2}{\sqrt{d+ex}\left(cd^2-ae^2\right)}\right)}{cd^2-ae^2}+\frac{2}{3(d+ex)^{3/2}\left(cd^2-ae^2\right)}\right)}{cd^2-ae^2}+\frac{2}{5(d+ex)^{5/2}(cd^2-ae^2)}\right)}{2\left(cd^2-ae^2\right)}-\frac{1}{(d+ex)^{5/2}\left(cd^2-ae^2\right)(ae+cdx)}$$

$$\Big\downarrow \ 73$$

$$-\frac{7e\left(\frac{cd\left(\frac{cd\left(\frac{2cd\int \frac{1}{-\frac{cd^2}{e}+\frac{c(d+ex)d}{e}+ae}d\sqrt{d+ex}}{e\left(cd^2-ae^2\right)}+\frac{2}{\sqrt{d+ex}\left(cd^2-ae^2\right)}\right)}{cd^2-ae^2}+\frac{2}{3(d+ex)^{3/2}\left(cd^2-ae^2\right)}\right)}{cd^2-ae^2}+\frac{2}{5(d+ex)^{5/2}(cd^2-ae^2)}\right)}{2\left(cd^2-ae^2\right)}-\frac{1}{(d+ex)^{5/2}\left(cd^2-ae^2\right)(ae+cdx)}$$

$$\Big\downarrow \ 221$$

$$-\frac{7e\left(\frac{cd\left(\frac{cd\left(\frac{2}{\sqrt{d+ex}(cd^2-ae^2)}-\frac{2\sqrt{c}\sqrt{d}\operatorname{arctanh}\left(\frac{\sqrt{c}\sqrt{d}\sqrt{d+ex}}{\sqrt{cd^2-ae^2}}\right)}{(cd^2-ae^2)^{3/2}}\right)}{cd^2-ae^2}+\frac{2}{3(d+ex)^{3/2}(cd^2-ae^2)}\right)}{cd^2-ae^2}+\frac{2}{5(d+ex)^{5/2}(cd^2-ae^2)}\right)}{2(cd^2-ae^2)}-\frac{1}{(d+ex)^{5/2}(cd^2-ae^2)(ae+cdx)}$$

input
```
Int[1/((d + e*x)^(3/2)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^2),x]
```

output
```
-(1/((c*d^2 - a*e^2)*(a*e + c*d*x)*(d + e*x)^(5/2))) - (7*e*(2/(5*(c*d^2 -
 a*e^2)*(d + e*x)^(5/2)) + (c*d*(2/(3*(c*d^2 - a*e^2)*(d + e*x)^(3/2)) + (
c*d*(2/((c*d^2 - a*e^2)*Sqrt[d + e*x]) - (2*Sqrt[c]*Sqrt[d]*ArcTanh[(Sqrt[
c]*Sqrt[d]*Sqrt[d + e*x])/Sqrt[c*d^2 - a*e^2]])/(c*d^2 - a*e^2)^(3/2)))/(c
*d^2 - a*e^2)))/(c*d^2 - a*e^2)))/(2*(c*d^2 - a*e^2))
```

Defintions of rubi rules used

rule 52
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && ILtQ[m, -1] && FractionQ[n] && LtQ[n, 0]
```

rule 61
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && LtQ[m, -1] &&  !(LtQ[n, -1] && (EqQ[a, 0
] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && IntLinearQ[a, b, c, d
, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 1121
```
Int[((d_) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Int[ExpandIntegrand[(d + e*x)^(m + p)*(a/d + (c/e)*x)^p, x], x]
/; FreeQ[{a, b, c, d, e, m, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && (Int
egerQ[p] || (GtQ[a, 0] && GtQ[d, 0] && LtQ[c, 0]))
```

Maple [A] (verified)

Time = 2.02 (sec) , antiderivative size = 183, normalized size of antiderivative = 0.95

method	result
derivativedivides	$2e\left(-\frac{c^3d^3\left(\frac{\sqrt{ex+d}}{2cd(ex+d)+2a e^2-2c d^2}+\frac{7\arctan\left(\frac{cd\sqrt{ex+d}}{\sqrt{cd\left(a e^2-c d^2\right)}}\right)}{2\sqrt{cd\left(a e^2-c d^2\right)}}\right)}{(a e^2-c d^2)^4}-\frac{1}{5(a e^2-c d^2)^2(ex+d)^{\frac{5}{2}}}-\frac{3c^2d^2}{(a e^2-c d^2)^4\sqrt{ex \cdots}}\cdots$
default	$2e\left(-\frac{c^3d^3\left(\frac{\sqrt{ex+d}}{2cd(ex+d)+2a e^2-2c d^2}+\frac{7\arctan\left(\frac{cd\sqrt{ex+d}}{\sqrt{cd\left(a e^2-c d^2\right)}}\right)}{2\sqrt{cd\left(a e^2-c d^2\right)}}\right)}{(a e^2-c d^2)^4}-\frac{1}{5(a e^2-c d^2)^2(ex+d)^{\frac{5}{2}}}-\frac{3c^2d^2}{(a e^2-c d^2)^4\sqrt{ex \cdots}}\cdots$
pseudoelliptic	$-\frac{2\left(\frac{35c^3d^3e(ex+d)^{\frac{5}{2}}(cdx+ae)\arctan\left(\frac{cd\sqrt{ex+d}}{\sqrt{cd\left(a e^2-c d^2\right)}}\right)}{2}+\left(\frac{5\left(7e^3x^3+\frac{49}{3}d e^2x^2+\frac{161}{15}d^2ex+d^3\right)d^3c^3}{2}+\frac{58e^2a d^2\left(\frac{35}{58}e^2x^2+\cdots\right.}{3}\cdots\right.\right.}{5(ex+d)^{\frac{5}{2}}\sqrt{cd(a e^2-c d^2)}\,(a e^2-c d^2)^4(cdx+ae)}$

input
```
int(1/(e*x+d)^(3/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^2,x,method=_RETURNVE
RBOSE)
```

output
```
2*e*(-1/(a*e^2-c*d^2)^4*c^3*d^3*(1/2*(e*x+d)^(1/2)/(c*d*(e*x+d)+a*e^2-c*d^
2)+7/2/(c*d*(a*e^2-c*d^2))^(1/2)*arctan(c*d*(e*x+d)^(1/2)/(c*d*(a*e^2-c*d^
2))^(1/2)))-1/5/(a*e^2-c*d^2)^2/(e*x+d)^(5/2)-3/(a*e^2-c*d^2)^4*c^2*d^2/(e
*x+d)^(1/2)+2/3/(a*e^2-c*d^2)^3*c*d/(e*x+d)^(3/2))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 631 vs. $2(166) = 332$.

Time = 0.17 (sec) , antiderivative size = 1304, normalized size of antiderivative = 6.79

$$\int \frac{1}{(d+ex)^{3/2}\left(ade+(cd^2+ae^2)x+cdex^2\right)^2}\,dx = \text{Too large to display}$$

input

```
integrate(1/(e*x+d)^(3/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^2,x, algorithm
="fricas")
```

output

```
[1/30*(105*(c^3*d^3*e^4*x^4 + a*c^2*d^5*e^2 + (3*c^3*d^4*e^3 + a*c^2*d^2*e
^5)*x^3 + 3*(c^3*d^5*e^2 + a*c^2*d^3*e^4)*x^2 + (c^3*d^6*e + 3*a*c^2*d^4*e
^3)*x)*sqrt(c*d/(c*d^2 - a*e^2))*log((c*d*e*x + 2*c*d^2 - a*e^2 + 2*(c*d^2
 - a*e^2)*sqrt(e*x + d)*sqrt(c*d/(c*d^2 - a*e^2)))/(c*d*x + a*e)) - 2*(105
*c^3*d^3*e^3*x^3 + 15*c^3*d^6 + 116*a*c^2*d^4*e^2 - 32*a^2*c*d^2*e^4 + 6*a
^3*e^6 + 35*(7*c^3*d^4*e^2 + 2*a*c^2*d^2*e^4)*x^2 + 7*(23*c^3*d^5*e + 24*a
*c^2*d^3*e^3 - 2*a^2*c*d*e^5)*x)*sqrt(e*x + d))/(a*c^4*d^11*e - 4*a^2*c^3*
d^9*e^3 + 6*a^3*c^2*d^7*e^5 - 4*a^4*c*d^5*e^7 + a^5*d^3*e^9 + (c^5*d^9*e^3
 - 4*a*c^4*d^7*e^5 + 6*a^2*c^3*d^5*e^7 - 4*a^3*c^2*d^3*e^9 + a^4*c*d*e^11)
*x^4 + (3*c^5*d^10*e^2 - 11*a*c^4*d^8*e^4 + 14*a^2*c^3*d^6*e^6 - 6*a^3*c^2
*d^4*e^8 - a^4*c*d^2*e^10 + a^5*e^12)*x^3 + 3*(c^5*d^11*e - 3*a*c^4*d^9*e^
3 + 2*a^2*c^3*d^7*e^5 + 2*a^3*c^2*d^5*e^7 - 3*a^4*c*d^3*e^9 + a^5*d*e^11)*
x^2 + (c^5*d^12 - a*c^4*d^10*e^2 - 6*a^2*c^3*d^8*e^4 + 14*a^3*c^2*d^6*e^6
- 11*a^4*c*d^4*e^8 + 3*a^5*d^2*e^10)*x), -1/15*(105*(c^3*d^3*e^4*x^4 + a*c
^2*d^5*e^2 + (3*c^3*d^4*e^3 + a*c^2*d^2*e^5)*x^3 + 3*(c^3*d^5*e^2 + a*c^2*
d^3*e^4)*x^2 + (c^3*d^6*e + 3*a*c^2*d^4*e^3)*x)*sqrt(-c*d/(c*d^2 - a*e^2))
*arctan(sqrt(e*x + d)*sqrt(-c*d/(c*d^2 - a*e^2))) + (105*c^3*d^3*e^3*x^3 +
 15*c^3*d^6 + 116*a*c^2*d^4*e^2 - 32*a^2*c*d^2*e^4 + 6*a^3*e^6 + 35*(7*c^3
*d^4*e^2 + 2*a*c^2*d^2*e^4)*x^2 + 7*(23*c^3*d^5*e + 24*a*c^2*d^3*e^3 - 2*a
^2*c*d*e^5)*x)*sqrt(e*x + d))/(a*c^4*d^11*e - 4*a^2*c^3*d^9*e^3 + 6*a^3...
```

Sympy [F]

$$\int \frac{1}{(d+ex)^{3/2}\left(ade+(cd^2+ae^2)\,x+cdex^2\right)^2}\,dx=\int \frac{1}{(d+ex)^{\frac{7}{2}}\,(ae+cdx)^2}\,dx$$

input
```
integrate(1/(e*x+d)**(3/2)/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**2,x)
```

output
```
Integral(1/((d + e*x)**(7/2)*(a*e + c*d*x)**2), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{(d+ex)^{3/2}\left(ade+(cd^2+ae^2)\,x+cdex^2\right)^2}\,dx=\text{Exception raised: ValueError}$$

input
```
integrate(1/(e*x+d)^(3/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^2,x, algorithm
="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(a*e^2-c*d^2>0)', see `assume?` f
or more de
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 340 vs. $2(166) = 332$.

Time = 0.15 (sec) , antiderivative size = 340, normalized size of antiderivative = 1.77

$$\int \frac{1}{(d+ex)^{3/2} \left(ade+(cd^2+ae^2)\, x+cdex^2\right)^2}\, dx = -\frac{7\, c^3 d^3 e \arctan\left(\frac{\sqrt{ex+d}cd}{\sqrt{-c^2d^3+acde^2}}\right)}{(c^4d^8-4\,ac^3d^6e^2+6\,a^2c^2d^4e^4-4\,a^3cd^2e^6+a^4e^8)\sqrt{-c^2d^3+acde^2}} - \frac{\sqrt{ex+d}c^3d^3e}{(c^4d^8-4\,ac^3d^6e^2+6\,a^2c^2d^4e^4-4\,a^3cd^2e^6+a^4e^8)((ex+d)cd-cd^2+ae^2)} - \frac{2\left(45\,(ex+d)^2c^2d^2e+10\,(ex+d)c^2d^3e+3\,c^2d^4e-10\,(ex+d)acde^3-6\,acd^2e^3+3\,a^2e^5\right)}{15\,(c^4d^8-4\,ac^3d^6e^2+6\,a^2c^2d^4e^4-4\,a^3cd^2e^6+a^4e^8)(ex+d)^{\frac{5}{2}}}$$

input
```
integrate(1/(e*x+d)^(3/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^2,x, algorithm
="giac")
```

output
```
-7*c^3*d^3*e*arctan(sqrt(e*x + d)*c*d/sqrt(-c^2*d^3 + a*c*d*e^2))/((c^4*d^
8 - 4*a*c^3*d^6*e^2 + 6*a^2*c^2*d^4*e^4 - 4*a^3*c*d^2*e^6 + a^4*e^8)*sqrt(
-c^2*d^3 + a*c*d*e^2)) - sqrt(e*x + d)*c^3*d^3*e/((c^4*d^8 - 4*a*c^3*d^6*e
^2 + 6*a^2*c^2*d^4*e^4 - 4*a^3*c*d^2*e^6 + a^4*e^8)*((e*x + d)*c*d - c*d^2
 + a*e^2)) - 2/15*(45*(e*x + d)^2*c^2*d^2*e + 10*(e*x + d)*c^2*d^3*e + 3*c
^2*d^4*e - 10*(e*x + d)*a*c*d*e^3 - 6*a*c*d^2*e^3 + 3*a^2*e^5)/((c^4*d^8 -
 4*a*c^3*d^6*e^2 + 6*a^2*c^2*d^4*e^4 - 4*a^3*c*d^2*e^6 + a^4*e^8)*(e*x + d
)^(5/2))
```

Mupad [B] (verification not implemented)

Time = 5.29 (sec) , antiderivative size = 244, normalized size of antiderivative = 1.27

$$\int \frac{1}{(d+ex)^{3/2} \left(ade+(cd^2+ae^2)\, x+cdex^2\right)^2}\, dx = -\frac{\frac{2\,e}{5\,(a\,e^2-c\,d^2)}-\frac{14\,c\,d\,e\,(d+e\,x)}{15\,{(a\,e^2-c\,d^2)}^2}+\frac{14\,c^2\,d^2\,e\,{(d+e\,x)}^2}{3\,{(a\,e^2-c\,d^2)}^3}+\frac{7\,c^3\,d^3\,e\,{(d+e\,x)}^3}{{(a\,e^2-c\,d^2)}^4}}{\left(a\,e^2-c\,d^2\right)\,{\left(d+e\,x\right)}^{5/2}+c\,d\,{\left(d+e\,x\right)}^{7/2}} - \frac{7\,c^{5/2}\,d^{5/2}\,e\,\mathrm{atan}\left(\frac{\sqrt{c}\,\sqrt{d}\,\sqrt{d+e\,x}\,\left(a^4\,e^8-4\,a^3\,c\,d^2\,e^6+6\,a^2\,c^2\,d^4\,e^4-4\,a\,c^3\,d^6\,e^2+c^4\,d^8\right)}{{\left(a\,e^2-c\,d^2\right)}^{9/2}}\right)}{{\left(a\,e^2-c\,d^2\right)}^{9/2}}$$

input
```
int(1/((d + e*x)^(3/2)*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^2),x)
```

output
```
- ((2*e)/(5*(a*e^2 - c*d^2)) - (14*c*d*e*(d + e*x))/(15*(a*e^2 - c*d^2)^2)
 + (14*c^2*d^2*e*(d + e*x)^2)/(3*(a*e^2 - c*d^2)^3) + (7*c^3*d^3*e*(d + e*
x)^3)/(a*e^2 - c*d^2)^4)/((a*e^2 - c*d^2)*(d + e*x)^(5/2) + c*d*(d + e*x)^
(7/2)) - (7*c^(5/2)*d^(5/2)*e*atan((c^(1/2)*d^(1/2)*(d + e*x)^(1/2)*(a^4*e
^8 + c^4*d^8 - 4*a*c^3*d^6*e^2 - 4*a^3*c*d^2*e^6 + 6*a^2*c^2*d^4*e^4))/(a*
e^2 - c*d^2)^(9/2)))/(a*e^2 - c*d^2)^(9/2)
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 933, normalized size of antiderivative = 4.86

$$\int \frac{1}{(d+ex)^{3/2}\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^2}\,dx = \text{Too large to display}$$

input
```
int(1/(e*x+d)^(3/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^2,x)
```

output
```
( - 105*sqrt(d)*sqrt(c)*sqrt(d + e*x)*sqrt(a*e**2 - c*d**2)*atan((sqrt(d +
 e*x)*c*d)/(sqrt(d)*sqrt(c)*sqrt(a*e**2 - c*d**2)))*a*c**2*d**4*e**2 - 210
*sqrt(d)*sqrt(c)*sqrt(d + e*x)*sqrt(a*e**2 - c*d**2)*atan((sqrt(d + e*x)*c
*d)/(sqrt(d)*sqrt(c)*sqrt(a*e**2 - c*d**2)))*a*c**2*d**3*e**3*x - 105*sqrt
(d)*sqrt(c)*sqrt(d + e*x)*sqrt(a*e**2 - c*d**2)*atan((sqrt(d + e*x)*c*d)/(
sqrt(d)*sqrt(c)*sqrt(a*e**2 - c*d**2)))*a*c**2*d**2*e**4*x**2 - 105*sqrt(d
)*sqrt(c)*sqrt(d + e*x)*sqrt(a*e**2 - c*d**2)*atan((sqrt(d + e*x)*c*d)/(sq
rt(d)*sqrt(c)*sqrt(a*e**2 - c*d**2)))*c**3*d**5*e*x - 210*sqrt(d)*sqrt(c)*
sqrt(d + e*x)*sqrt(a*e**2 - c*d**2)*atan((sqrt(d + e*x)*c*d)/(sqrt(d)*sqrt
(c)*sqrt(a*e**2 - c*d**2)))*c**3*d**4*e**2*x**2 - 105*sqrt(d)*sqrt(c)*sqrt
(d + e*x)*sqrt(a*e**2 - c*d**2)*atan((sqrt(d + e*x)*c*d)/(sqrt(d)*sqrt(c)*
sqrt(a*e**2 - c*d**2)))*c**3*d**3*e**3*x**3 - 6*a**4*e**8 + 38*a**3*c*d**2
*e**6 + 14*a**3*c*d*e**7*x - 148*a**2*c**2*d**4*e**4 - 182*a**2*c**2*d**3*
e**5*x - 70*a**2*c**2*d**2*e**6*x**2 + 101*a*c**3*d**6*e**2 + 7*a*c**3*d**
5*e**3*x - 175*a*c**3*d**4*e**4*x**2 - 105*a*c**3*d**3*e**5*x**3 + 15*c**4
*d**8 + 161*c**4*d**7*e*x + 245*c**4*d**6*e**2*x**2 + 105*c**4*d**5*e**3*x
**3)/(15*sqrt(d + e*x)*(a**6*d**2*e**11 + 2*a**6*d*e**12*x + a**6*e**13*x*
*2 - 5*a**5*c*d**4*e**9 - 9*a**5*c*d**3*e**10*x - 3*a**5*c*d**2*e**11*x**2
 + a**5*c*d*e**12*x**3 + 10*a**4*c**2*d**6*e**7 + 15*a**4*c**2*d**5*e**8*x
 - 5*a**4*c**2*d**3*e**10*x**3 - 10*a**3*c**3*d**8*e**5 - 10*a**3*c**3*...
```

3.191 $\int \frac{(d+ex)^{15/2}}{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^3}\,dx$

Optimal result

Integrand size = 37, antiderivative size = 222

$$\int \frac{(d+ex)^{15/2}}{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^3}\,dx = \frac{63e^2\left(cd^2-ae^2\right)^2\sqrt{d+ex}}{4c^5d^5} + \frac{21e^2\left(cd^2-ae^2\right)(d+ex)^{3/2}}{4c^4d^4} + \frac{63e^2(d+ex)^{5/2}}{20c^3d^3} - \frac{9e(d+ex)^{7/2}}{4c^2d^2(ae+cdx)} - \frac{(d+ex)^{9/2}}{2cd(ae+cdx)^2} - \frac{63e^2\left(cd^2-ae^2\right)^{5/2}\operatorname{arctanh}\left(\frac{\sqrt{c}\sqrt{d}\sqrt{d+ex}}{\sqrt{cd^2-ae^2}}\right)}{4c^{11/2}d^{11/2}}$$

output
```
63/4*e^2*(-a*e^2+c*d^2)^2*(e*x+d)^(1/2)/c^5/d^5+21/4*e^2*(-a*e^2+c*d^2)*(e
*x+d)^(3/2)/c^4/d^4+63/20*e^2*(e*x+d)^(5/2)/c^3/d^3-9/4*e*(e*x+d)^(7/2)/c^
2/d^2/(c*d*x+a*e)-1/2*(e*x+d)^(9/2)/c/d/(c*d*x+a*e)^2-63/4*e^2*(-a*e^2+c*d
^2)^(5/2)*arctanh(c^(1/2)*d^(1/2)*(e*x+d)^(1/2)/(-a*e^2+c*d^2)^(1/2))/c^(1
1/2)/d^(11/2)
```

Mathematica [A] (verified)

Time = 0.84 (sec) , antiderivative size = 250, normalized size of antiderivative = 1.13

$$\int \frac{(d+ex)^{15/2}}{(ade+(cd^2+ae^2)\,x+cdex^2)^3}\,dx =$$
$$-\frac{\sqrt{d+ex}(-315a^4e^8+105a^3cde^6(7d-5ex)-21a^2c^2d^2e^4(23d^2-59dex+8e^2x^2)+3ac^3d^3e^2(15d^3-277}{20c^5d^5(ae+cdx)^2}$$
$$-\frac{63e^2(-cd^2+ae^2)^{5/2}\arctan\left(\frac{\sqrt{c}\sqrt{d}\sqrt{d+ex}}{\sqrt{-cd^2+ae^2}}\right)}{4c^{11/2}d^{11/2}}$$

input
```
Integrate[(d + e*x)^(15/2)/(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^3,x]
```

output
```
-1/20*(Sqrt[d + e*x]*(-315*a^4*e^8 + 105*a^3*c*d*e^6*(7*d - 5*e*x) - 21*a^
2*c^2*d^2*e^4*(23*d^2 - 59*d*e*x + 8*e^2*x^2) + 3*a*c^3*d^3*e^2*(15*d^3 -
277*d^2*e*x + 136*d*e^2*x^2 + 8*e^3*x^3) + c^4*d^4*(10*d^4 + 85*d^3*e*x -
288*d^2*e^2*x^2 - 56*d*e^3*x^3 - 8*e^4*x^4)))/(c^5*d^5*(a*e + c*d*x)^2) -
(63*e^2*(-(c*d^2) + a*e^2)^(5/2)*ArcTan[(Sqrt[c]*Sqrt[d]*Sqrt[d + e*x])/Sq
rt[-(c*d^2) + a*e^2]])/(4*c^(11/2)*d^(11/2))
```

Rubi [A] (verified)

Time = 0.52 (sec) , antiderivative size = 250, normalized size of antiderivative = 1.13, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.216$, Rules used = {1121, 51, 51, 60, 60, 60, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex)^{15/2}}{(x(ae^2+cd^2)+ade+cdex^2)^3}dx$$
$$\Big\downarrow \text{ 1121}$$
$$\int \frac{(d+ex)^{9/2}}{(ae+cdx)^3}dx$$
$$\Big\downarrow \text{ 51}$$

$$\frac{9e\int \frac{(d+ex)^{7/2}}{(ae+cdx)^2}dx}{4cd}-\frac{(d+ex)^{9/2}}{2cd(ae+cdx)^2}$$

$$\Big\downarrow \; 51$$

$$\frac{9e\left(\frac{7e\int \frac{(d+ex)^{5/2}}{ae+cdx}dx}{2cd}-\frac{(d+ex)^{7/2}}{cd(ae+cdx)}\right)}{4cd}-\frac{(d+ex)^{9/2}}{2cd(ae+cdx)^2}$$

$$\Big\downarrow \; 60$$

$$\frac{9e\left(\frac{7e\left(\frac{\left(d^2-\frac{ae^2}{c}\right)\int \frac{(d+ex)^{3/2}}{ae+cdx}dx}{d}+\frac{2(d+ex)^{5/2}}{5cd}\right)}{2cd}-\frac{(d+ex)^{7/2}}{cd(ae+cdx)}\right)}{4cd}-\frac{(d+ex)^{9/2}}{2cd(ae+cdx)^2}$$

$$\Big\downarrow \; 60$$

$$\frac{9e\left(\frac{7e\left(\frac{\left(d^2-\frac{ae^2}{c}\right)\left(\frac{\left(d^2-\frac{ae^2}{c}\right)\int \frac{\sqrt{d+ex}}{ae+cdx}dx}{d}+\frac{2(d+ex)^{3/2}}{3cd}\right)}{d}+\frac{2(d+ex)^{5/2}}{5cd}\right)}{2cd}-\frac{(d+ex)^{7/2}}{cd(ae+cdx)}\right)}{4cd}-\frac{(d+ex)^{9/2}}{2cd(ae+cdx)^2}$$

$$\Big\downarrow \; 60$$

$$\frac{9e\left(\frac{7e\left(\frac{\left(d^2-\frac{ae^2}{c}\right)\left(\frac{\left(d^2-\frac{ae^2}{c}\right)\left(\frac{\left(d^2-\frac{ae^2}{c}\right)\int\frac{1}{(ae+cdx)\sqrt{d+ex}}dx}{d}+\frac{2\sqrt{d+ex}}{cd}\right)}{d}+\frac{2(d+ex)^{3/2}}{3cd}\right)}{d}+\frac{2(d+ex)^{5/2}}{5cd}\right)}{2cd}-\frac{(d+ex)^{7/2}}{cd(ae+cdx)}\right)}{4cd}-\frac{(d+ex)^{9/2}}{2cd(ae+cdx)^2}$$

$$\Big\downarrow \; 73$$

$$\frac{9e\left(\frac{7e\left(\frac{\left(d^2-\frac{ae^2}{c}\right)\left(\frac{\left(d^2-\frac{ae^2}{c}\right)\left(\frac{2\left(d^2-\frac{ae^2}{c}\right)\int \frac{1}{-\frac{cd^2}{e}+\frac{c(d+ex)d}{e}+ae}d\sqrt{d+ex}}{de}+\frac{2\sqrt{d+ex}}{cd}\right)}{d}+\frac{2(d+ex)^{3/2}}{3cd}\right)}{d}+\frac{2(d+ex)^{5/2}}{5cd}\right)}{2cd}-\frac{(d+ex)^{7/2}}{cd(ae+cdx)}\right)}{4cd}-\frac{(d+ex)^{9/2}}{2cd(ae+cdx)^2}$$

$$\Big\downarrow \; 221$$

$$\frac{9e\left(\frac{7e\left(\frac{\left(d^2-\frac{ae^2}{c}\right)\left(\frac{\left(d^2-\frac{ae^2}{c}\right)\left(\frac{2\sqrt{d+ex}}{cd}-\frac{2\left(d^2-\frac{ae^2}{c}\right)\text{arctanh}\left(\frac{\sqrt{c}\sqrt{d}\sqrt{d+ex}}{\sqrt{cd^2-ae^2}}\right)}{\sqrt{c}d^{3/2}\sqrt{cd^2-ae^2}}\right)}{d}+\frac{2(d+ex)^{3/2}}{3cd}\right)}{d}+\frac{2(d+ex)^{5/2}}{5cd}\right)}{2cd}-\frac{(d+ex)^{7/2}}{cd(ae+cdx)}\right)}{4cd}-\frac{(d+ex)^{9/2}}{2cd(ae+cdx)^2}$$

input
```
Int[(d + e*x)^(15/2)/(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^3,x]
```

output
```
-1/2*(d + e*x)^(9/2)/(c*d*(a*e + c*d*x)^2) + (9*e*(-((d + e*x)^(7/2)/(c*d*
(a*e + c*d*x))) + (7*e*((2*(d + e*x)^(5/2))/(5*c*d) + ((d^2 - (a*e^2)/c)*(
(2*(d + e*x)^(3/2))/(3*c*d) + ((d^2 - (a*e^2)/c)*((2*Sqrt[d + e*x])/(c*d)
- (2*(d^2 - (a*e^2)/c)*ArcTanh[(Sqrt[c]*Sqrt[d]*Sqrt[d + e*x])/Sqrt[c*d^2
- a*e^2]])/(Sqrt[c]*d^(3/2)*Sqrt[c*d^2 - a*e^2])))/d))/d))/(2*c*d)))/(4*c*
d)
```

Defintions of rubi rules used

rule 51
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Simp[d*(n/(b*(m + 1)))
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d, n}, x
] && ILtQ[m, -1] && FractionQ[n] && GtQ[n, 0]
```

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 1121
```
Int[((d_) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Int[ExpandIntegrand[(d + e*x)^(m + p)*(a/d + (c/e)*x)^p, x], x]
/; FreeQ[{a, b, c, d, e, m, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && (Int
egerQ[p] || (GtQ[a, 0] && GtQ[d, 0] && LtQ[c, 0]))
```

Maple [A] (verified)

Time = 16.65 (sec) , antiderivative size = 235, normalized size of antiderivative = 1.06

method	result
risch	$\frac{2e^2\left(x^2c^2d^2e^2-5xacd\,e^3+7x\,c^2d^3e+30a^2e^4-65ac\,d^2e^2+36c^2d^4\right)\sqrt{ex+d}}{5d^5c^5}-\frac{\left(2e^6a^3-6d^2e^4a^2c+6d^4e^2a\,c^2-2d^6c^3\right)e^2 \cdots}{\cdots}$
pseudoelliptic	$-\frac{63\left(e^2\left(a\,e^2-c\,d^2\right)^3(cdx+ae)^2\arctan\left(\frac{cd\sqrt{ex+d}}{\sqrt{cd\left(a\,e^2-c\,d^2\right)}}\right)-\sqrt{ex+d}\,\sqrt{cd(a\,e^2-c\,d^2)}\left(-\frac{2\left(-\frac{4}{5}e^4x^4-\frac{28}{5}d\,e^3x^3-\frac{144}{5}d^2 \cdots\right.}{63} \cdots\right.\right.}{4\sqrt{cd(a \cdots}}$
derivativedivides	$2e^2\left(\frac{\frac{c^2d^2(ex+d)^{\frac{5}{2}}}{5}-acd\,e^2(ex+d)^{\frac{3}{2}}+c^2d^3(ex+d)^{\frac{3}{2}}+6a^2e^4\sqrt{ex+d}-12ac\,d^2e^2\sqrt{ex+d}+6c^2d^4\sqrt{ex+d}}{c^5d^5}-\frac{\left(-\frac{17}{8}d\,e^6c \cdots\right.}{\cdots} \cdots\right.$
default	$2e^2\left(\frac{\frac{c^2d^2(ex+d)^{\frac{5}{2}}}{5}-acd\,e^2(ex+d)^{\frac{3}{2}}+c^2d^3(ex+d)^{\frac{3}{2}}+6a^2e^4\sqrt{ex+d}-12ac\,d^2e^2\sqrt{ex+d}+6c^2d^4\sqrt{ex+d}}{c^5d^5}-\frac{\left(-\frac{17}{8}d\,e^6c \cdots\right.}{\cdots} \cdots\right.$

input
```
int((e*x+d)^(15/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^3,x,method=_RETURNVER
BOSE)
```

output
```
2/5*e^2*(c^2*d^2*e^2*x^2-5*a*c*d*e^3*x+7*c^2*d^3*e*x+30*a^2*e^4-65*a*c*d^2
*e^2+36*c^2*d^4)*(e*x+d)^(1/2)/d^5/c^5-1/d^5/c^5*(2*a^3*e^6-6*a^2*c*d^2*e^
4+6*a*c^2*d^4*e^2-2*c^3*d^6)*e^2*((-17/8*c*d*(e*x+d)^(3/2)+(-15/8*a*e^2+15
/8*c*d^2)*(e*x+d)^(1/2))/(c*d*(e*x+d)+a*e^2-c*d^2)^2+63/8/(c*d*(a*e^2-c*d^
2))^(1/2)*arctan(c*d*(e*x+d)^(1/2)/(c*d*(a*e^2-c*d^2))^(1/2)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 422 vs. $2(186) = 372$.

Time = 0.15 (sec) , antiderivative size = 858, normalized size of antiderivative = 3.86

$$\int \frac{(d+ex)^{15/2}}{(ade+(cd^2+ae^2)\,x+cdex^2)^3}\,dx = \text{Too large to display}$$

input
```
integrate((e*x+d)^(15/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^3,x, algorithm=
"fricas")
```

output
```
[1/40*(315*(a^2*c^2*d^4*e^4 - 2*a^3*c*d^2*e^6 + a^4*e^8 + (c^4*d^6*e^2 - 2
*a*c^3*d^4*e^4 + a^2*c^2*d^2*e^6)*x^2 + 2*(a*c^3*d^5*e^3 - 2*a^2*c^2*d^3*e
^5 + a^3*c*d*e^7)*x)*sqrt((c*d^2 - a*e^2)/(c*d))*log((c*d*e*x + 2*c*d^2 -
a*e^2 - 2*sqrt(e*x + d)*c*d*sqrt((c*d^2 - a*e^2)/(c*d)))/(c*d*x + a*e)) +
2*(8*c^4*d^4*e^4*x^4 - 10*c^4*d^8 - 45*a*c^3*d^6*e^2 + 483*a^2*c^2*d^4*e^4
 - 735*a^3*c*d^2*e^6 + 315*a^4*e^8 + 8*(7*c^4*d^5*e^3 - 3*a*c^3*d^3*e^5)*x
^3 + 24*(12*c^4*d^6*e^2 - 17*a*c^3*d^4*e^4 + 7*a^2*c^2*d^2*e^6)*x^2 - (85*
c^4*d^7*e - 831*a*c^3*d^5*e^3 + 1239*a^2*c^2*d^3*e^5 - 525*a^3*c*d*e^7)*x)
*sqrt(e*x + d))/(c^7*d^7*x^2 + 2*a*c^6*d^6*e*x + a^2*c^5*d^5*e^2), -1/20*(
315*(a^2*c^2*d^4*e^4 - 2*a^3*c*d^2*e^6 + a^4*e^8 + (c^4*d^6*e^2 - 2*a*c^3*
d^4*e^4 + a^2*c^2*d^2*e^6)*x^2 + 2*(a*c^3*d^5*e^3 - 2*a^2*c^2*d^3*e^5 + a^
3*c*d*e^7)*x)*sqrt(-(c*d^2 - a*e^2)/(c*d))*arctan(-sqrt(e*x + d)*c*d*sqrt(
-(c*d^2 - a*e^2)/(c*d))/(c*d^2 - a*e^2)) - (8*c^4*d^4*e^4*x^4 - 10*c^4*d^8
 - 45*a*c^3*d^6*e^2 + 483*a^2*c^2*d^4*e^4 - 735*a^3*c*d^2*e^6 + 315*a^4*e^
8 + 8*(7*c^4*d^5*e^3 - 3*a*c^3*d^3*e^5)*x^3 + 24*(12*c^4*d^6*e^2 - 17*a*c^
3*d^4*e^4 + 7*a^2*c^2*d^2*e^6)*x^2 - (85*c^4*d^7*e - 831*a*c^3*d^5*e^3 + 1
239*a^2*c^2*d^3*e^5 - 525*a^3*c*d*e^7)*x)*sqrt(e*x + d))/(c^7*d^7*x^2 + 2*
a*c^6*d^6*e*x + a^2*c^5*d^5*e^2)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(d+ex)^{15/2}}{(ade+(cd^2+ae^2)\,x+cdex^2)^3}\,dx = \text{Timed out}$$

input
```
integrate((e*x+d)**(15/2)/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**3,x)
```

output
```
Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(d+ex)^{15/2}}{(ade+(cd^2+ae^2)x+cdex^2)^3}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((e*x+d)^(15/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^3,x, algorithm=
"maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(a*e^2-c*d^2>0)', see `assume?` f
or more de
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 418 vs. $2(186) = 372$.

Time = 0.19 (sec) , antiderivative size = 418, normalized size of antiderivative = 1.88

$$\int \frac{(d+ex)^{15/2}}{(ade+(cd^2+ae^2)x+cdex^2)^3}\,dx = \frac{63\,(c^3d^6e^2-3\,ac^2d^4e^4+3\,a^2cd^2e^6-a^3e^8)\arctan\left(\frac{\sqrt{ex+d}cd}{\sqrt{-c^2d^3+acde^2}}\right)}{4\,\sqrt{-c^2d^3+acde^2}c^5d^5}$$
$$-\frac{17\,(ex+d)^{\frac{3}{2}}c^4d^7e^2-15\,\sqrt{ex+d}c^4d^8e^2-51\,(ex+d)^{\frac{3}{2}}ac^3d^5e^4+60\,\sqrt{ex+d}ac^3d^6e^4+51\,(ex+d)^{\frac{3}{2}}a^2c^2}{4\,((ex+d)cd-cd^2+}$$
$$+\frac{2\left((ex+d)^{\frac{5}{2}}c^{12}d^{12}e^2+5\,(ex+d)^{\frac{3}{2}}c^{12}d^{13}e^2+30\,\sqrt{ex+d}c^{12}d^{14}e^2-5\,(ex+d)^{\frac{3}{2}}ac^{11}d^{11}e^4-60\,\sqrt{ex+d}ac\right.}{5\,c^{15}d^{15}}$$

input
```
integrate((e*x+d)^(15/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^3,x, algorithm=
"giac")
```

output
```
63/4*(c^3*d^6*e^2 - 3*a*c^2*d^4*e^4 + 3*a^2*c*d^2*e^6 - a^3*e^8)*arctan(sq
rt(e*x + d)*c*d/sqrt(-c^2*d^3 + a*c*d*e^2))/(sqrt(-c^2*d^3 + a*c*d*e^2)*c^
5*d^5) - 1/4*(17*(e*x + d)^(3/2)*c^4*d^7*e^2 - 15*sqrt(e*x + d)*c^4*d^8*e^
2 - 51*(e*x + d)^(3/2)*a*c^3*d^5*e^4 + 60*sqrt(e*x + d)*a*c^3*d^6*e^4 + 51
*(e*x + d)^(3/2)*a^2*c^2*d^3*e^6 - 90*sqrt(e*x + d)*a^2*c^2*d^4*e^6 - 17*(
e*x + d)^(3/2)*a^3*c*d*e^8 + 60*sqrt(e*x + d)*a^3*c*d^2*e^8 - 15*sqrt(e*x
+ d)*a^4*e^10)/(((e*x + d)*c*d - c*d^2 + a*e^2)^2*c^5*d^5) + 2/5*((e*x + d
)^(5/2)*c^12*d^12*e^2 + 5*(e*x + d)^(3/2)*c^12*d^13*e^2 + 30*sqrt(e*x + d)
*c^12*d^14*e^2 - 5*(e*x + d)^(3/2)*a*c^11*d^11*e^4 - 60*sqrt(e*x + d)*a*c^
11*d^12*e^4 + 30*sqrt(e*x + d)*a^2*c^10*d^10*e^6)/(c^15*d^15)
```

Mupad [B] (verification not implemented)

Time = 5.46 (sec) , antiderivative size = 430, normalized size of antiderivative = 1.94

$$\int \frac{(d+ex)^{15/2}}{(ade+(cd^2+ae^2)\,x+cdex^2)^3}\,dx = \frac{\sqrt{d+e\,x}\,\left(\frac{15\,a^4\,e^{10}}{4}-15\,a^3\,c\,d^2\,e^8+\frac{45\,a^2\,c^2\,d^4\,e^6}{2}-15\,a\,c^3\,d^6\,e^4+\frac{15\,c^4 \cdots}{4}\right.}{c^7\,d^9-\left(2\,c^7\,d^8-2\,a\,c^6\,d^6\,e^2\right)\,(d+e\,x)\cdots}$$
$$+\left(\frac{2\,e^2\,\left(3\,c^3\,d^4-3\,a\,c^2\,d^2\,e^2\right)^2}{c^9\,d^9}-\frac{6\,e^2\,\left(a\,e^2-c\,d^2\right)^2}{c^5\,d^5}\right)\sqrt{d+e\,x}+\frac{2\,e^2\,(d+e\,x)^{5/2}}{5\,c^3\,d^3}$$
$$-\frac{63\,e^2\,\mathrm{atan}\left(\frac{\sqrt{c}\,\sqrt{d}\,e^2\,\left(a\,e^2-c\,d^2\right)^{5/2}\,\sqrt{d+e\,x}}{a^3\,e^8-3\,a^2\,c\,d^2\,e^6+3\,a\,c^2\,d^4\,e^4-c^3\,d^6\,e^2}\right)\,\left(a\,e^2-c\,d^2\right)^{5/2}}{4\,c^{11/2}\,d^{11/2}}$$
$$+\frac{2\,e^2\,\left(3\,c^3\,d^4-3\,a\,c^2\,d^2\,e^2\right)\,(d+e\,x)^{3/2}}{3\,c^6\,d^6}$$

input
```
int((d + e*x)^(15/2)/(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^3,x)
```

output
```
((d + e*x)^(1/2)*((15*a^4*e^10)/4 + (15*c^4*d^8*e^2)/4 - 15*a*c^3*d^6*e^4
- 15*a^3*c*d^2*e^8 + (45*a^2*c^2*d^4*e^6)/2) - (d + e*x)^(3/2)*((17*c^4*d^
7*e^2)/4 - (51*a*c^3*d^5*e^4)/4 + (51*a^2*c^2*d^3*e^6)/4 - (17*a^3*c*d*e^8
)/4))/(c^7*d^9 - (2*c^7*d^8 - 2*a*c^6*d^6*e^2)*(d + e*x) + c^7*d^7*(d + e*
x)^2 - 2*a*c^6*d^7*e^2 + a^2*c^5*d^5*e^4) + ((2*e^2*(3*c^3*d^4 - 3*a*c^2*d
^2*e^2)^2)/(c^9*d^9) - (6*e^2*(a*e^2 - c*d^2)^2)/(c^5*d^5))*(d + e*x)^(1/2
) + (2*e^2*(d + e*x)^(5/2))/(5*c^3*d^3) - (63*e^2*atan((c^(1/2)*d^(1/2)*e^
2*(a*e^2 - c*d^2)^(5/2)*(d + e*x)^(1/2))/(a^3*e^8 - c^3*d^6*e^2 + 3*a*c^2*
d^4*e^4 - 3*a^2*c*d^2*e^6))*(a*e^2 - c*d^2)^(5/2))/(4*c^(11/2)*d^(11/2)) +
 (2*e^2*(3*c^3*d^4 - 3*a*c^2*d^2*e^2)*(d + e*x)^(3/2))/(3*c^6*d^6)
```

Reduce [B] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 895, normalized size of antiderivative = 4.03

$$\int \frac{(d+ex)^{15/2}}{(ade+(cd^2+ae^2)x+cdex^2)^3}\,dx = \text{Too large to display}$$

input
```
int((e*x+d)^(15/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^3,x)
```

output
```
( - 315*sqrt(d)*sqrt(c)*sqrt(a*e**2 - c*d**2)*atan((sqrt(d + e*x)*c*d)/(sq
rt(d)*sqrt(c)*sqrt(a*e**2 - c*d**2)))*a**4*e**8 + 630*sqrt(d)*sqrt(c)*sqrt
(a*e**2 - c*d**2)*atan((sqrt(d + e*x)*c*d)/(sqrt(d)*sqrt(c)*sqrt(a*e**2 -
c*d**2)))*a**3*c*d**2*e**6 - 630*sqrt(d)*sqrt(c)*sqrt(a*e**2 - c*d**2)*ata
n((sqrt(d + e*x)*c*d)/(sqrt(d)*sqrt(c)*sqrt(a*e**2 - c*d**2)))*a**3*c*d*e*
*7*x - 315*sqrt(d)*sqrt(c)*sqrt(a*e**2 - c*d**2)*atan((sqrt(d + e*x)*c*d)/
(sqrt(d)*sqrt(c)*sqrt(a*e**2 - c*d**2)))*a**2*c**2*d**4*e**4 + 1260*sqrt(d
)*sqrt(c)*sqrt(a*e**2 - c*d**2)*atan((sqrt(d + e*x)*c*d)/(sqrt(d)*sqrt(c)*
sqrt(a*e**2 - c*d**2)))*a**2*c**2*d**3*e**5*x - 315*sqrt(d)*sqrt(c)*sqrt(a
*e**2 - c*d**2)*atan((sqrt(d + e*x)*c*d)/(sqrt(d)*sqrt(c)*sqrt(a*e**2 - c*
d**2)))*a**2*c**2*d**2*e**6*x**2 - 630*sqrt(d)*sqrt(c)*sqrt(a*e**2 - c*d**
2)*atan((sqrt(d + e*x)*c*d)/(sqrt(d)*sqrt(c)*sqrt(a*e**2 - c*d**2)))*a*c**
3*d**5*e**3*x + 630*sqrt(d)*sqrt(c)*sqrt(a*e**2 - c*d**2)*atan((sqrt(d + e
*x)*c*d)/(sqrt(d)*sqrt(c)*sqrt(a*e**2 - c*d**2)))*a*c**3*d**4*e**4*x**2 -
315*sqrt(d)*sqrt(c)*sqrt(a*e**2 - c*d**2)*atan((sqrt(d + e*x)*c*d)/(sqrt(d
)*sqrt(c)*sqrt(a*e**2 - c*d**2)))*c**4*d**6*e**2*x**2 + 315*sqrt(d + e*x)*
a**4*c*d*e**8 - 735*sqrt(d + e*x)*a**3*c**2*d**3*e**6 + 525*sqrt(d + e*x)*
a**3*c**2*d**2*e**7*x + 483*sqrt(d + e*x)*a**2*c**3*d**5*e**4 - 1239*sqrt(
d + e*x)*a**2*c**3*d**4*e**5*x + 168*sqrt(d + e*x)*a**2*c**3*d**3*e**6*x**
2 - 45*sqrt(d + e*x)*a*c**4*d**7*e**2 + 831*sqrt(d + e*x)*a*c**4*d**6*e...
```

3.192 $\int \frac{(d+ex)^{13/2}}{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^3} dx$

Optimal result

Integrand size = 37, antiderivative size = 186

$$\int \frac{(d+ex)^{13/2}}{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^3} dx = \frac{35e^2(cd^2-ae^2)\sqrt{d+ex}}{4c^4d^4}+\frac{35e^2(d+ex)^{3/2}}{12c^3d^3} - \frac{7e(d+ex)^{5/2}}{4c^2d^2(ae+cdx)}-\frac{(d+ex)^{7/2}}{2cd(ae+cdx)^2}-\frac{35e^2(cd^2-ae^2)^{3/2}\operatorname{arctanh}\left(\frac{\sqrt{c}\sqrt{d}\sqrt{d+ex}}{\sqrt{cd^2-ae^2}}\right)}{4c^{9/2}d^{9/2}}$$

output
```
35/4*e^2*(-a*e^2+c*d^2)*(e*x+d)^(1/2)/c^4/d^4+35/12*e^2*(e*x+d)^(3/2)/c^3/
d^3-7/4*e*(e*x+d)^(5/2)/c^2/d^2/(c*d*x+a*e)-1/2*(e*x+d)^(7/2)/c/d/(c*d*x+a
*e)^2-35/4*e^2*(-a*e^2+c*d^2)^(3/2)*arctanh(c^(1/2)*d^(1/2)*(e*x+d)^(1/2)/
(-a*e^2+c*d^2)^(1/2))/c^(9/2)/d^(9/2)
```

Mathematica [A] (verified)

Time = 0.62 (sec) , antiderivative size = 207, normalized size of antiderivative = 1.11

$$\int \frac{(d+ex)^{13/2}}{(ade+(cd^2+ae^2)\,x+cdex^2)^3}\,dx =$$
$$-\frac{\sqrt{d+ex}(105a^3e^6-35a^2cde^4(4d-5ex)+7ac^2d^2e^2(3d^2-34dex+8e^2x^2)+c^3d^3(6d^3+39d^2ex-80de^2}{12c^4d^4(ae+cdx)^2}$$
$$+\frac{35(cd^2e-ae^3)^2\arctan\left(\frac{\sqrt{c}\sqrt{d}\sqrt{d+ex}}{\sqrt{-cd^2+ae^2}}\right)}{4c^{9/2}d^{9/2}\sqrt{-cd^2+ae^2}}$$

input
```
Integrate[(d + e*x)^(13/2)/(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^3,x]
```

output
```
-1/12*(Sqrt[d + e*x]*(105*a^3*e^6 - 35*a^2*c*d*e^4*(4*d - 5*e*x) + 7*a*c^2
*d^2*e^2*(3*d^2 - 34*d*e*x + 8*e^2*x^2) + c^3*d^3*(6*d^3 + 39*d^2*e*x - 80
*d*e^2*x^2 - 8*e^3*x^3)))/(c^4*d^4*(a*e + c*d*x)^2) + (35*(c*d^2*e - a*e^3
)^2*ArcTan[(Sqrt[c]*Sqrt[d]*Sqrt[d + e*x])/Sqrt[-(c*d^2) + a*e^2]])/(4*c^(
9/2)*d^(9/2)*Sqrt[-(c*d^2) + a*e^2])
```

Rubi [A] (verified)

Time = 0.46 (sec) , antiderivative size = 213, normalized size of antiderivative = 1.15, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.189$, Rules used = {1121, 51, 51, 60, 60, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex)^{13/2}}{(x\,(ae^2+cd^2)+ade+cdex^2)^3}\,dx$$
$$\Big\downarrow \text{1121}$$
$$\int \frac{(d+ex)^{7/2}}{(ae+cdx)^3}dx$$
$$\Big\downarrow \text{51}$$

$$\frac{7e\int \frac{(d+ex)^{5/2}}{(ae+cdx)^2}dx}{4cd}-\frac{(d+ex)^{7/2}}{2cd(ae+cdx)^2}$$

$$\Big\downarrow \ 51$$

$$\frac{7e\left(\frac{5e\int \frac{(d+ex)^{3/2}}{ae+cdx}dx}{2cd}-\frac{(d+ex)^{5/2}}{cd(ae+cdx)}\right)}{4cd}-\frac{(d+ex)^{7/2}}{2cd(ae+cdx)^2}$$

$$\Big\downarrow \ 60$$

$$\frac{7e\left(\frac{5e\left(\frac{\left(d^2-\frac{ae^2}{c}\right)\int \frac{\sqrt{d+ex}}{ae+cdx}dx}{d}+\frac{2(d+ex)^{3/2}}{3cd}\right)}{2cd}-\frac{(d+ex)^{5/2}}{cd(ae+cdx)}\right)}{4cd}-\frac{(d+ex)^{7/2}}{2cd(ae+cdx)^2}$$

$$\Big\downarrow \ 60$$

$$\frac{7e\left(\frac{5e\left(\frac{\left(d^2-\frac{ae^2}{c}\right)\left(\frac{\left(d^2-\frac{ae^2}{c}\right)\int \frac{1}{(ae+cdx)\sqrt{d+ex}}dx}{d}+\frac{2\sqrt{d+ex}}{cd}\right)}{d}+\frac{2(d+ex)^{3/2}}{3cd}\right)}{2cd}-\frac{(d+ex)^{5/2}}{cd(ae+cdx)}\right)}{4cd}-\frac{(d+ex)^{7/2}}{2cd(ae+cdx)^2}$$

$$\Big\downarrow \ 73$$

$$\frac{7e\left(\frac{5e\left(\frac{\left(d^2-\frac{ae^2}{c}\right)\left(\frac{2\left(d^2-\frac{ae^2}{c}\right)\int\frac{1}{-\frac{cd^2}{e}+\frac{c(d+ex)d}{e}+ae}d\sqrt{d+ex}}{de}+\frac{2\sqrt{d+ex}}{cd}\right)}{d}+\frac{2(d+ex)^{3/2}}{3cd}\right)}{2cd}-\frac{(d+ex)^{5/2}}{cd(ae+cdx)}\right)}{4cd}-\frac{(d+ex)^{7/2}}{2cd(ae+cdx)^2}$$

$$\Big\downarrow \; 221$$

$$\frac{7e\left(\frac{5e\left(\frac{\left(d^2-\frac{ae^2}{c}\right)\left(\frac{2\sqrt{d+ex}}{cd}-\frac{2\left(d^2-\frac{ae^2}{c}\right)\text{arctanh}\left(\frac{\sqrt{c}\sqrt{d}\sqrt{d+ex}}{\sqrt{cd^2-ae^2}}\right)}{\sqrt{c}d^{3/2}\sqrt{cd^2-ae^2}}\right)}{d}+\frac{2(d+ex)^{3/2}}{3cd}\right)}{2cd}-\frac{(d+ex)^{5/2}}{cd(ae+cdx)}\right)}{4cd}-\frac{(d+ex)^{7/2}}{2cd(ae+cdx)^2}$$

input
```
Int[(d + e*x)^(13/2)/(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^3,x]
```

output
```
-1/2*(d + e*x)^(7/2)/(c*d*(a*e + c*d*x)^2) + (7*e*(-((d + e*x)^(5/2)/(c*d*
(a*e + c*d*x))) + (5*e*((2*(d + e*x)^(3/2))/(3*c*d) + ((d^2 - (a*e^2)/c)*(
(2*Sqrt[d + e*x])/(c*d) - (2*(d^2 - (a*e^2)/c)*ArcTanh[(Sqrt[c]*Sqrt[d]*Sq
rt[d + e*x])/Sqrt[c*d^2 - a*e^2]])/(Sqrt[c]*d^(3/2)*Sqrt[c*d^2 - a*e^2])))
/d))/(2*c*d)))/(4*c*d)
```

Defintions of rubi rules used

rule 51
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Simp[d*(n/(b*(m + 1)))
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d, n}, x
] && ILtQ[m, -1] && FractionQ[n] && GtQ[n, 0]
```

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 1121
```
Int[((d_) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Int[ExpandIntegrand[(d + e*x)^(m + p)*(a/d + (c/e)*x)^p, x], x]
/; FreeQ[{a, b, c, d, e, m, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && (Int
egerQ[p] || (GtQ[a, 0] && GtQ[d, 0] && LtQ[c, 0]))
```

Maple [A] (verified)

Time = 15.89 (sec) , antiderivative size = 180, normalized size of antiderivative = 0.97

method	result
risch	$-\frac{2e^2\left(-cdxe+9a\,e^2-10c\,d^2\right)\sqrt{ex+d}}{3d^4c^4}+\frac{\left(2a^2e^4-4ac\,d^2e^2+2c^2d^4\right)e^2\left(\frac{-\frac{13cd(ex+d)^{\frac{3}{2}}}{8}+\left(-\frac{11a\,e^2}{8}+\frac{11c\,d^2}{8}\right)\sqrt{ex+d}}{\left(cd(ex+d)+a\,e^2-c\,d^2\right)^2}+\frac{35}{\ldots}\right.}{d^4c^4}$
pseudoelliptic	$\frac{\frac{35e^2\left(a\,e^2-c\,d^2\right)^2(cdx+ae)^2\arctan\left(\frac{cd\sqrt{ex+d}}{\sqrt{cd\left(a\,e^2-c\,d^2\right)}}\right)}{4}-35\sqrt{ex+d}\left(\frac{2d^3\left(-\frac{4}{3}e^3x^3-\frac{40}{3}d\,e^2x^2+\frac{13}{2}d^2ex+d^3\right)c^3}{35}+\frac{e^2a\,d^2\left(\frac{8}{3}e^2x^2\ldots\right.}{4}\right.}{d^4c^4(cdx+ae)^2\sqrt{cd(a\,e^2-c\,d^2)}}$
derivativedivides	$2e^2\left(-\frac{-\frac{cd(ex+d)^{\frac{3}{2}}}{3}+3a\,e^2\sqrt{ex+d}-3c\,d^2\sqrt{ex+d}}{c^4d^4}+\frac{\frac{\left(-\frac{13}{8}d\,e^4a^2c+\frac{13}{4}d^3e^2a\,c^2-\frac{13}{8}d^5c^3\right)(ex+d)^{\frac{3}{2}}+\left(-\frac{11}{8}e^6a^3+\frac{33}{8}d^2e\ldots\right.}{\left(cd(ex+d)+a\,e^2-c\,d^2\right)^2}}{\ldots}\right.$
default	$2e^2\left(-\frac{-\frac{cd(ex+d)^{\frac{3}{2}}}{3}+3a\,e^2\sqrt{ex+d}-3c\,d^2\sqrt{ex+d}}{c^4d^4}+\frac{\frac{\left(-\frac{13}{8}d\,e^4a^2c+\frac{13}{4}d^3e^2a\,c^2-\frac{13}{8}d^5c^3\right)(ex+d)^{\frac{3}{2}}+\left(-\frac{11}{8}e^6a^3+\frac{33}{8}d^2e\ldots\right.}{\left(cd(ex+d)+a\,e^2-c\,d^2\right)^2}}{\ldots}\right.$

input
```
int((e*x+d)^(13/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^3,x,method=_RETURNVER
BOSE)
```

output
```
-2/3*e^2*(-c*d*e*x+9*a*e^2-10*c*d^2)*(e*x+d)^(1/2)/d^4/c^4+1/d^4/c^4*(2*a^
2*e^4-4*a*c*d^2*e^2+2*c^2*d^4)*e^2*((-13/8*c*d*(e*x+d)^(3/2)+(-11/8*a*e^2+
11/8*c*d^2)*(e*x+d)^(1/2))/(c*d*(e*x+d)+a*e^2-c*d^2)^2+35/8/(c*d*(a*e^2-c*
d^2))^(1/2)*arctan(c*d*(e*x+d)^(1/2)/(c*d*(a*e^2-c*d^2))^(1/2)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 312 vs. $2(154) = 308$.

Time = 0.12 (sec) , antiderivative size = 638, normalized size of antiderivative = 3.43

$$\int \frac{(d+ex)^{13/2}}{\left(ade+(cd^2+ae^2)\,x+cdex^2\right)^3}\,dx = \Bigg[-\frac{105\,(a^2cd^2e^4 - a^3e^6 + (c^3d^4e^2 - ac^2d^2e^4)x^2 + 2\,(ac^2d^3e^3 - a^2c\cdots}{\cdots}$$

$$-\frac{105\,(a^2cd^2e^4 - a^3e^6 + (c^3d^4e^2 - ac^2d^2e^4)x^2 + 2\,(ac^2d^3e^3 - a^2cde^5)x)\sqrt{-\frac{cd^2-ae^2}{cd}}\arctan\left(-\frac{\sqrt{ex+d}\,cd\sqrt{-\frac{cd^2-\cdots}{c\cdots}}}{cd^2-ae^2}\cdots\right.}{\cdots}$$

input
```
integrate((e*x+d)^(13/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^3,x, algorithm=
"fricas")
```

output
```
[-1/24*(105*(a^2*c*d^2*e^4 - a^3*e^6 + (c^3*d^4*e^2 - a*c^2*d^2*e^4)*x^2 +
 2*(a*c^2*d^3*e^3 - a^2*c*d*e^5)*x)*sqrt((c*d^2 - a*e^2)/(c*d))*log((c*d*e
*x + 2*c*d^2 - a*e^2 + 2*sqrt(e*x + d)*c*d*sqrt((c*d^2 - a*e^2)/(c*d)))/(c
*d*x + a*e)) - 2*(8*c^3*d^3*e^3*x^3 - 6*c^3*d^6 - 21*a*c^2*d^4*e^2 + 140*a
^2*c*d^2*e^4 - 105*a^3*e^6 + 8*(10*c^3*d^4*e^2 - 7*a*c^2*d^2*e^4)*x^2 - (3
9*c^3*d^5*e - 238*a*c^2*d^3*e^3 + 175*a^2*c*d*e^5)*x)*sqrt(e*x + d))/(c^6*
d^6*x^2 + 2*a*c^5*d^5*e*x + a^2*c^4*d^4*e^2), -1/12*(105*(a^2*c*d^2*e^4 -
a^3*e^6 + (c^3*d^4*e^2 - a*c^2*d^2*e^4)*x^2 + 2*(a*c^2*d^3*e^3 - a^2*c*d*e
^5)*x)*sqrt(-(c*d^2 - a*e^2)/(c*d))*arctan(-sqrt(e*x + d)*c*d*sqrt(-(c*d^2
 - a*e^2)/(c*d))/(c*d^2 - a*e^2)) - (8*c^3*d^3*e^3*x^3 - 6*c^3*d^6 - 21*a*
c^2*d^4*e^2 + 140*a^2*c*d^2*e^4 - 105*a^3*e^6 + 8*(10*c^3*d^4*e^2 - 7*a*c^
2*d^2*e^4)*x^2 - (39*c^3*d^5*e - 238*a*c^2*d^3*e^3 + 175*a^2*c*d*e^5)*x)*s
qrt(e*x + d))/(c^6*d^6*x^2 + 2*a*c^5*d^5*e*x + a^2*c^4*d^4*e^2)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(d+ex)^{13/2}}{\left(ade+(cd^2+ae^2)\,x+cdex^2\right)^3}\,dx=\text{Timed out}$$

input
```
integrate((e*x+d)**(13/2)/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**3,x)
```

output
```
Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(d+ex)^{13/2}}{\left(ade+(cd^2+ae^2)\,x+cdex^2\right)^3}\,dx=\text{Exception raised: ValueError}$$

input
```
integrate((e*x+d)^(13/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^3,x, algorithm=
"maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(a*e^2-c*d^2>0)', see `assume?` f
or more de
```

Giac [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 303, normalized size of antiderivative = 1.63

$$\int \frac{(d+ex)^{13/2}}{\left(ade+(cd^2+ae^2)\,x+cdex^2\right)^3}\,dx=\frac{35\,(c^2d^4e^2-2\,acd^2e^4+a^2e^6)\arctan\left(\frac{\sqrt{ex+d}cd}{\sqrt{-c^2d^3+acde^2}}\right)}{4\sqrt{-c^2d^3+acde^2}c^4d^4}-\frac{13\,(ex+d)^{\frac{3}{2}}c^3d^5e^2-11\sqrt{ex+d}c^3d^6e^2-26\,(ex+d)^{\frac{3}{2}}ac^2d^3e^4+33\sqrt{ex+d}ac^2d^4e^4+13\,(ex+d)^{\frac{3}{2}}a^2cd}{4\left((ex+d)cd-cd^2+ae^2\right)^2c^4d^4}+\frac{2\left((ex+d)^{\frac{3}{2}}c^6d^6e^2+9\sqrt{ex+d}c^6d^7e^2-9\sqrt{ex+d}ac^5d^5e^4\right)}{3\,c^9d^9}$$

input
```
integrate((e*x+d)^(13/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^3,x, algorithm=
"giac")
```

output
```
35/4*(c^2*d^4*e^2 - 2*a*c*d^2*e^4 + a^2*e^6)*arctan(sqrt(e*x + d)*c*d/sqrt
(-c^2*d^3 + a*c*d*e^2))/(sqrt(-c^2*d^3 + a*c*d*e^2)*c^4*d^4) - 1/4*(13*(e*
x + d)^(3/2)*c^3*d^5*e^2 - 11*sqrt(e*x + d)*c^3*d^6*e^2 - 26*(e*x + d)^(3/
2)*a*c^2*d^3*e^4 + 33*sqrt(e*x + d)*a*c^2*d^4*e^4 + 13*(e*x + d)^(3/2)*a^2
*c*d*e^6 - 33*sqrt(e*x + d)*a^2*c*d^2*e^6 + 11*sqrt(e*x + d)*a^3*e^8)/(((e
*x + d)*c*d - c*d^2 + a*e^2)^2*c^4*d^4) + 2/3*((e*x + d)^(3/2)*c^6*d^6*e^2
 + 9*sqrt(e*x + d)*c^6*d^7*e^2 - 9*sqrt(e*x + d)*a*c^5*d^5*e^4)/(c^9*d^9)
```

Mupad [B] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 319, normalized size of antiderivative = 1.72

$$\int \frac{(d+ex)^{13/2}}{(ade+(cd^2+ae^2)x+cdex^2)^3}\,dx = \frac{2\,e^2\,(d+e\,x)^{3/2}}{3\,c^3\,d^3} - \frac{(d+e\,x)^{3/2}\left(\frac{13\,a^2\,c\,d\,e^6}{4}-\frac{13\,a\,c^2\,d^3\,e^4}{2}+\frac{13\,c^3\,d^5\,e^2}{4}\right)+\sqrt{d+e\,x}\left(\frac{11\,a^3\,e^8}{4}-\frac{33\,a^2\,c\,d^2\,e^6}{4}+\frac{33\,a\,c^2\,d^4\,e^4}{4}-\frac{11\,c^3\,d^6\,e^2}{4}\right)}{c^6\,d^8-\left(2\,c^6\,d^7-2\,a\,c^5\,d^5\,e^2\right)\,(d+e\,x)+c^6\,d^6\,(d+e\,x)^2-2\,a\,c^5\,d^6\,e^2+a^2\,c^4\,d^4\,e^4} + \frac{2\,e^2\,\left(3\,c^3\,d^4-3\,a\,c^2\,d^2\,e^2\right)\,\sqrt{d+e\,x}}{c^6\,d^6} + \frac{35\,e^2\,\mathrm{atan}\left(\frac{\sqrt{c}\,\sqrt{d}\,e^2\,\left(a\,e^2-c\,d^2\right)^{3/2}\,\sqrt{d+e\,x}}{a^2\,e^6-2\,a\,c\,d^2\,e^4+c^2\,d^4\,e^2}\right)\,\left(a\,e^2-c\,d^2\right)^{3/2}}{4\,c^{9/2}\,d^{9/2}}$$

input
```
int((d + e*x)^(13/2)/(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^3,x)
```

output
```
(2*e^2*(d + e*x)^(3/2))/(3*c^3*d^3) - ((d + e*x)^(3/2)*((13*c^3*d^5*e^2)/4
 - (13*a*c^2*d^3*e^4)/2 + (13*a^2*c*d*e^6)/4) + (d + e*x)^(1/2)*((11*a^3*e
^8)/4 - (11*c^3*d^6*e^2)/4 + (33*a*c^2*d^4*e^4)/4 - (33*a^2*c*d^2*e^6)/4))
/(c^6*d^8 - (2*c^6*d^7 - 2*a*c^5*d^5*e^2)*(d + e*x) + c^6*d^6*(d + e*x)^2
 - 2*a*c^5*d^6*e^2 + a^2*c^4*d^4*e^4) + (2*e^2*(3*c^3*d^4 - 3*a*c^2*d^2*e^2
)*(d + e*x)^(1/2))/(c^6*d^6) + (35*e^2*atan((c^(1/2)*d^(1/2)*e^2*(a*e^2 -
c*d^2)^(3/2)*(d + e*x)^(1/2))/(a^2*e^6 + c^2*d^4*e^2 - 2*a*c*d^2*e^4))*(a*
e^2 - c*d^2)^(3/2))/(4*c^(9/2)*d^(9/2))
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 594, normalized size of antiderivative = 3.19

$$\int \frac{(d+ex)^{13/2}}{\left(ade+(cd^2+ae^2)\,x+cdex^2\right)^3}\,dx = \frac{105\sqrt{d}\,\sqrt{c}\,\sqrt{a\,e^2-c\,d^2}\,atan\left(\frac{\sqrt{ex+d}\,cd}{\sqrt{d}\,\sqrt{c}\,\sqrt{a\,e^2-c\,d^2}}\right)a^3e^6-105\sqrt{d}\,\sqrt{c}\,\sqrt{a}$$

input
```
int((e*x+d)^(13/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^3,x)
```

output
```
(105*sqrt(d)*sqrt(c)*sqrt(a*e**2 - c*d**2)*atan((sqrt(d + e*x)*c*d)/(sqrt(
d)*sqrt(c)*sqrt(a*e**2 - c*d**2)))*a**3*e**6 - 105*sqrt(d)*sqrt(c)*sqrt(a*
e**2 - c*d**2)*atan((sqrt(d + e*x)*c*d)/(sqrt(d)*sqrt(c)*sqrt(a*e**2 - c*d
**2)))*a**2*c*d**2*e**4 + 210*sqrt(d)*sqrt(c)*sqrt(a*e**2 - c*d**2)*atan((
sqrt(d + e*x)*c*d)/(sqrt(d)*sqrt(c)*sqrt(a*e**2 - c*d**2)))*a**2*c*d*e**5*
x - 210*sqrt(d)*sqrt(c)*sqrt(a*e**2 - c*d**2)*atan((sqrt(d + e*x)*c*d)/(sq
rt(d)*sqrt(c)*sqrt(a*e**2 - c*d**2)))*a*c**2*d**3*e**3*x + 105*sqrt(d)*sqr
t(c)*sqrt(a*e**2 - c*d**2)*atan((sqrt(d + e*x)*c*d)/(sqrt(d)*sqrt(c)*sqrt(
a*e**2 - c*d**2)))*a*c**2*d**2*e**4*x**2 - 105*sqrt(d)*sqrt(c)*sqrt(a*e**2
 - c*d**2)*atan((sqrt(d + e*x)*c*d)/(sqrt(d)*sqrt(c)*sqrt(a*e**2 - c*d**2)
))*c**3*d**4*e**2*x**2 - 105*sqrt(d + e*x)*a**3*c*d*e**6 + 140*sqrt(d + e*
x)*a**2*c**2*d**3*e**4 - 175*sqrt(d + e*x)*a**2*c**2*d**2*e**5*x - 21*sqrt
(d + e*x)*a*c**3*d**5*e**2 + 238*sqrt(d + e*x)*a*c**3*d**4*e**3*x - 56*sqr
t(d + e*x)*a*c**3*d**3*e**4*x**2 - 6*sqrt(d + e*x)*c**4*d**7 - 39*sqrt(d +
 e*x)*c**4*d**6*e*x + 80*sqrt(d + e*x)*c**4*d**5*e**2*x**2 + 8*sqrt(d + e*
x)*c**4*d**4*e**3*x**3)/(12*c**5*d**5*(a**2*e**2 + 2*a*c*d*e*x + c**2*d**2
*x**2))
```

3.193 $\int \frac{(d+ex)^{11/2}}{(ade+(cd^2+ae^2)x+cdex^2)^3}\,dx$

Optimal result

Integrand size = 37, antiderivative size = 152

$$\int \frac{(d+ex)^{11/2}}{(ade+(cd^2+ae^2)\,x+cdex^2)^3}\,dx = \frac{15e^2\sqrt{d+ex}}{4c^3d^3} - \frac{5e(d+ex)^{3/2}}{4c^2d^2(ae+cdx)} - \frac{(d+ex)^{5/2}}{2cd(ae+cdx)^2} - \frac{15e^2\sqrt{cd^2-ae^2}\,\text{arctanh}\left(\frac{\sqrt{c}\sqrt{d}\sqrt{d+ex}}{\sqrt{cd^2-ae^2}}\right)}{4c^{7/2}d^{7/2}}$$

output

```
15/4*e^2*(e*x+d)^(1/2)/c^3/d^3-5/4*e*(e*x+d)^(3/2)/c^2/d^2/(c*d*x+a*e)-1/2
*(e*x+d)^(5/2)/c/d/(c*d*x+a*e)^2-15/4*e^2*(-a*e^2+c*d^2)^(1/2)*arctanh(c^(
1/2)*d^(1/2)*(e*x+d)^(1/2)/(-a*e^2+c*d^2)^(1/2))/c^(7/2)/d^(7/2)
```

Mathematica [A] (verified)

Time = 0.58 (sec) , antiderivative size = 149, normalized size of antiderivative = 0.98

$$\int \frac{(d+ex)^{11/2}}{(ade+(cd^2+ae^2)\,x+cdex^2)^3}\,dx = -\frac{\sqrt{d+ex}(-15a^2e^4+5acde^2(d-5ex)+c^2d^2(2d^2+9dex-8e^2x^2))}{4c^3d^3(ae+cdx)^2} - \frac{15e^2\sqrt{-cd^2+ae^2}\arctan\left(\frac{\sqrt{c}\sqrt{d}\sqrt{d+ex}}{\sqrt{-cd^2+ae^2}}\right)}{4c^{7/2}d^{7/2}}$$

input
```
Integrate[(d + e*x)^(11/2)/(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^3,x]
```

output
```
-1/4*(Sqrt[d + e*x]*(-15*a^2*e^4 + 5*a*c*d*e^2*(d - 5*e*x) + c^2*d^2*(2*d^
2 + 9*d*e*x - 8*e^2*x^2)))/(c^3*d^3*(a*e + c*d*x)^2) - (15*e^2*Sqrt[-(c*d^
2) + a*e^2]*ArcTan[(Sqrt[c]*Sqrt[d]*Sqrt[d + e*x])/Sqrt[-(c*d^2) + a*e^2]]
)/(4*c^(7/2)*d^(7/2))
```

Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 176, normalized size of antiderivative = 1.16, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.162$, Rules used = {1121, 51, 51, 60, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex)^{11/2}}{(x(ae^2+cd^2)+ade+cdex^2)^3}dx$$

$$\Big\downarrow \text{1121}$$

$$\int \frac{(d+ex)^{5/2}}{(ae+cdx)^3}dx$$

$$\Big\downarrow \text{51}$$

$$\frac{5e\int \frac{(d+ex)^{3/2}}{(ae+cdx)^2}dx}{4cd}-\frac{(d+ex)^{5/2}}{2cd(ae+cdx)^2}$$

$$\Big\downarrow \text{51}$$

$$\frac{5e\left(\frac{3e\int \frac{\sqrt{d+ex}}{ae+cdx}dx}{2cd}-\frac{(d+ex)^{3/2}}{cd(ae+cdx)}\right)}{4cd}-\frac{(d+ex)^{5/2}}{2cd(ae+cdx)^2}$$

$$\Big\downarrow \text{60}$$

$$\frac{5e\left(\frac{3e\left(\frac{\left(d^2-\frac{ae^2}{c}\right)\int \frac{1}{(ae+cdx)\sqrt{d+ex}}dx}{d}+\frac{2\sqrt{d+ex}}{cd}\right)}{2cd}-\frac{(d+ex)^{3/2}}{cd(ae+cdx)}\right)}{4cd}-\frac{(d+ex)^{5/2}}{2cd(ae+cdx)^2}$$

$$\Big\downarrow 73$$

$$\frac{5e\left(\frac{3e\left(\frac{2\left(d^2-\frac{ae^2}{c}\right)\int \frac{1}{-\frac{cd^2}{e}+\frac{c(d+ex)d}{e}+ae}d\sqrt{d+ex}}{de}+\frac{2\sqrt{d+ex}}{cd}\right)}{2cd}-\frac{(d+ex)^{3/2}}{cd(ae+cdx)}\right)}{4cd}-\frac{(d+ex)^{5/2}}{2cd(ae+cdx)^2}$$

$$\Big\downarrow 221$$

$$\frac{5e\left(\frac{3e\left(\frac{2\sqrt{d+ex}}{cd}-\frac{2\left(d^2-\frac{ae^2}{c}\right)\text{arctanh}\left(\frac{\sqrt{c}\sqrt{d}\sqrt{d+ex}}{\sqrt{cd^2-ae^2}}\right)}{\sqrt{c}d^{3/2}\sqrt{cd^2-ae^2}}\right)}{2cd}-\frac{(d+ex)^{3/2}}{cd(ae+cdx)}\right)}{4cd}-\frac{(d+ex)^{5/2}}{2cd(ae+cdx)^2}$$

input
```
Int[(d + e*x)^(11/2)/(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^3,x]
```

output
```
-1/2*(d + e*x)^(5/2)/(c*d*(a*e + c*d*x)^2) + (5*e*(-((d + e*x)^(3/2)/(c*d*
(a*e + c*d*x))) + (3*e*((2*Sqrt[d + e*x])/(c*d) - (2*(d^2 - (a*e^2)/c)*Arc
Tanh[(Sqrt[c]*Sqrt[d]*Sqrt[d + e*x])/Sqrt[c*d^2 - a*e^2]])/(Sqrt[c]*d^(3/2
)*Sqrt[c*d^2 - a*e^2])))/(2*c*d)))/(4*c*d)
```

Defintions of rubi rules used

rule 51
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Simp[d*(n/(b*(m + 1)))
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d, n}, x
] && ILtQ[m, -1] && FractionQ[n] && GtQ[n, 0]
```

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 73

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 1121

```
Int[((d_) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Int[ExpandIntegrand[(d + e*x)^(m + p)*(a/d + (c/e)*x)^p, x], x]
/; FreeQ[{a, b, c, d, e, m, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && (Int
egerQ[p] || (GtQ[a, 0] && GtQ[d, 0] && LtQ[c, 0]))
```

Maple [A] (verified)

Time = 18.31 (sec) , antiderivative size = 164, normalized size of antiderivative = 1.08

method	result
pseudoelliptic	$\frac{-\frac{15e^2\left(a\,e^2-c\,d^2\right)(cdx+ae)^2\arctan\left(\frac{cd\sqrt{ex+d}}{\sqrt{cd\left(a\,e^2-c\,d^2\right)}}\right)}{4}+\frac{15\sqrt{ex+d}\,\sqrt{cd\left(a\,e^2-c\,d^2\right)}\left(-\frac{2\left(-4e^2x^2+\frac{9}{2}dex+d^2\right)d^2c^2}{15}-\frac{ad\,e^2(-5}{3}\right.}{4}}{d^3c^3(cdx+ae)^2\sqrt{cd(a\,e^2-c\,d^2)}}$
derivativedivides	$2e^2\left(\frac{\sqrt{ex+d}}{c^3d^3}-\frac{\frac{\left(-\frac{9}{8}ad\,e^2c+\frac{9}{8}c^2d^3\right)(ex+d)^{\frac{3}{2}}+\left(-\frac{7}{8}a^2e^4+\frac{7}{4}ac\,d^2e^2-\frac{7}{8}c^2d^4\right)\sqrt{ex+d}}{\left(cd(ex+d)+a\,e^2-c\,d^2\right)^2}+\frac{15\left(a\,e^2-c\,d^2\right)\arctan\left(\frac{cd\sqrt{ex+d}}{\sqrt{cd\left(a\,e^2-c\right.}}\right.}{8\sqrt{cd\left(a\,e^2-c\,d^2\right)}}}{c^3d^3}\right.$
default	$2e^2\left(\frac{\sqrt{ex+d}}{c^3d^3}-\frac{\frac{\left(-\frac{9}{8}ad\,e^2c+\frac{9}{8}c^2d^3\right)(ex+d)^{\frac{3}{2}}+\left(-\frac{7}{8}a^2e^4+\frac{7}{4}ac\,d^2e^2-\frac{7}{8}c^2d^4\right)\sqrt{ex+d}}{\left(cd(ex+d)+a\,e^2-c\,d^2\right)^2}+\frac{15\left(a\,e^2-c\,d^2\right)\arctan\left(\frac{cd\sqrt{ex+d}}{\sqrt{cd\left(a\,e^2-c\right.}}\right.}{8\sqrt{cd\left(a\,e^2-c\,d^2\right)}}}{c^3d^3}\right.$
risch	$\frac{2e^2\_\_O1\sqrt{ex+d}}{d^3}-\frac{(a\,e^2-c\,d^2)e^2\left(\sum_{\_R=\mathrm{RootOf}\left(c^2d^3\_Z^6+\left(3ac\,d^2e^2-3c^2d^4\right)\_Z^4+\left(3a^2d\,e^4-6ac\,d^3e^2+3c^2d^5\right)\_Z^2+a^3\right.}\right.}{}$

input
```
int((e*x+d)^(11/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^3,x,method=_RETURNVER
BOSE)
```

output
```
15/4*(-e^2*(a*e^2-c*d^2)*(c*d*x+a*e)^2*arctan(c*d*(e*x+d)^(1/2)/(c*d*(a*e^
2-c*d^2))^(1/2))+(e*x+d)^(1/2)*(c*d*(a*e^2-c*d^2))^(1/2)*(-2/15*(-4*e^2*x^
2+9/2*d*e*x+d^2)*d^2*c^2-1/3*a*d*e^2*(-5*e*x+d)*c+a^2*e^4))/(c*d*(a*e^2-c*
d^2))^(1/2)/d^3/c^3/(c*d*x+a*e)^2
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 440, normalized size of antiderivative = 2.89

$$\int \frac{(d+ex)^{11/2}}{(ade+(cd^2+ae^2)x+cdex^2)^3}\,dx = \left[\frac{15\,(c^2d^2e^2x^2+2\,acde^3x+a^2e^4)\sqrt{\frac{cd^2-ae^2}{cd}}\log\left(\frac{cdex+2\,cd^2-ae^2-2\,\sqrt{ex}}{cdx+ae}\right.}{8\,(}\right.$$

$$-\frac{15\,(c^2d^2e^2x^2+2\,acde^3x+a^2e^4)\sqrt{-\frac{cd^2-ae^2}{cd}}\arctan\left(-\frac{\sqrt{ex+d}cd\sqrt{-\frac{cd^2-ae^2}{cd}}}{cd^2-ae^2}\right)-(8\,c^2d^2e^2x^2-2\,c^2d^4-5\,acd^2}{4\,(c^5d^5x^2+2\,ac^4d^4ex+a^2c^3d^3e^2)}$$

input
```
integrate((e*x+d)^(11/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^3,x, algorithm=
"fricas")
```

output
```
[1/8*(15*(c^2*d^2*e^2*x^2 + 2*a*c*d*e^3*x + a^2*e^4)*sqrt((c*d^2 - a*e^2)/
(c*d))*log((c*d*e*x + 2*c*d^2 - a*e^2 - 2*sqrt(e*x + d)*c*d*sqrt((c*d^2 -
a*e^2)/(c*d)))/(c*d*x + a*e)) + 2*(8*c^2*d^2*e^2*x^2 - 2*c^2*d^4 - 5*a*c*d
^2*e^2 + 15*a^2*e^4 - (9*c^2*d^3*e - 25*a*c*d*e^3)*x)*sqrt(e*x + d))/(c^5*
d^5*x^2 + 2*a*c^4*d^4*e*x + a^2*c^3*d^3*e^2), -1/4*(15*(c^2*d^2*e^2*x^2 +
2*a*c*d*e^3*x + a^2*e^4)*sqrt(-(c*d^2 - a*e^2)/(c*d))*arctan(-sqrt(e*x + d
)*c*d*sqrt(-(c*d^2 - a*e^2)/(c*d))/(c*d^2 - a*e^2)) - (8*c^2*d^2*e^2*x^2 -
 2*c^2*d^4 - 5*a*c*d^2*e^2 + 15*a^2*e^4 - (9*c^2*d^3*e - 25*a*c*d*e^3)*x)*
sqrt(e*x + d))/(c^5*d^5*x^2 + 2*a*c^4*d^4*e*x + a^2*c^3*d^3*e^2)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(d+ex)^{11/2}}{(ade+(cd^2+ae^2)x+cdex^2)^3}\,dx = \text{Timed out}$$

input
```
integrate((e*x+d)**(11/2)/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**3,x)
```

output
```
Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(d+ex)^{11/2}}{\left(ade+(cd^2+ae^2)\,x+cdex^2\right)^3}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((e*x+d)^(11/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^3,x, algorithm=
"maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(a*e^2-c*d^2>0)', see `assume?` f
or more de
```

Giac [A] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 203, normalized size of antiderivative = 1.34

$$\int \frac{(d+ex)^{11/2}}{\left(ade+(cd^2+ae^2)\,x+cdex^2\right)^3}\,dx = \frac{2\sqrt{ex+d}e^2}{c^3d^3} + \frac{15\,(cd^2e^2-ae^4)\arctan\left(\frac{\sqrt{ex+d}cd}{\sqrt{-c^2d^3+acde^2}}\right)}{4\sqrt{-c^2d^3+acde^2}c^3d^3} - \frac{9\,(ex+d)^{\frac{3}{2}}c^2d^3e^2-7\sqrt{ex+d}c^2d^4e^2-9\,(ex+d)^{\frac{3}{2}}acde^4+14\sqrt{ex+d}acd^2e^4-7\sqrt{ex+d}a^2e^6}{4\left((ex+d)cd-cd^2+ae^2\right)^2c^3d^3}$$

input
```
integrate((e*x+d)^(11/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^3,x, algorithm=
"giac")
```

output
```
2*sqrt(e*x + d)*e^2/(c^3*d^3) + 15/4*(c*d^2*e^2 - a*e^4)*arctan(sqrt(e*x +
 d)*c*d/sqrt(-c^2*d^3 + a*c*d*e^2))/(sqrt(-c^2*d^3 + a*c*d*e^2)*c^3*d^3) -
 1/4*(9*(e*x + d)^(3/2)*c^2*d^3*e^2 - 7*sqrt(e*x + d)*c^2*d^4*e^2 - 9*(e*x
 + d)^(3/2)*a*c*d*e^4 + 14*sqrt(e*x + d)*a*c*d^2*e^4 - 7*sqrt(e*x + d)*a^2
*e^6)/(((e*x + d)*c*d - c*d^2 + a*e^2)^2*c^3*d^3)
```

Mupad [B] (verification not implemented)

Time = 5.34 (sec) , antiderivative size = 240, normalized size of antiderivative = 1.58

$$\int \frac{(d+ex)^{11/2}}{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^3}\,dx = \frac{2\,e^2\,\sqrt{d+e\,x}}{c^3\,d^3} - \frac{\left(\frac{9\,c^2\,d^3\,e^2}{4}-\frac{9\,a\,c\,d\,e^4}{4}\right)\,{\left(d+e\,x\right)}^{3/2}-\sqrt{d+e\,x}\,\left(\frac{7\,a^2\,e^6}{4}-\frac{7\,a\,c\,d^2\,e^4}{2}+\frac{7\,c^2\,d^4\,e^2}{4}\right)}{c^5\,d^7-\left(2\,c^5\,d^6-2\,a\,c^4\,d^4\,e^2\right)\,\left(d+e\,x\right)+c^5\,d^5\,{\left(d+e\,x\right)}^2-2\,a\,c^4\,d^5\,e^2+a^2\,c^3\,d^3\,e^4} - \frac{15\,e^2\,\mathrm{atan}\left(\frac{\sqrt{c}\,\sqrt{d}\,e^2\,\sqrt{a\,e^2-c\,d^2}\,\sqrt{d+e\,x}}{a\,e^4-c\,d^2\,e^2}\right)\,\sqrt{a\,e^2-c\,d^2}}{4\,c^{7/2}\,d^{7/2}}$$

input
```
int((d + e*x)^(11/2)/(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^3,x)
```

output
```
(2*e^2*(d + e*x)^(1/2))/(c^3*d^3) - (((9*c^2*d^3*e^2)/4 - (9*a*c*d*e^4)/4)
*(d + e*x)^(3/2) - (d + e*x)^(1/2)*((7*a^2*e^6)/4 + (7*c^2*d^4*e^2)/4 - (7
*a*c*d^2*e^4)/2))/(c^5*d^7 - (2*c^5*d^6 - 2*a*c^4*d^4*e^2)*(d + e*x) + c^5
*d^5*(d + e*x)^2 - 2*a*c^4*d^5*e^2 + a^2*c^3*d^3*e^4) - (15*e^2*atan((c^(1
/2)*d^(1/2)*e^2*(a*e^2 - c*d^2)^(1/2)*(d + e*x)^(1/2))/(a*e^4 - c*d^2*e^2)
)*(a*e^2 - c*d^2)^(1/2))/(4*c^(7/2)*d^(7/2))
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 320, normalized size of antiderivative = 2.11

$$\int \frac{(d+ex)^{11/2}}{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^3}\,dx = \frac{-15\sqrt{d}\,\sqrt{c}\,\sqrt{a\,e^2-c\,d^2}\,atan\left(\frac{\sqrt{ex+d}\,cd}{\sqrt{d}\,\sqrt{c}\,\sqrt{a\,e^2-c\,d^2}}\right)a^2e^4-30\sqrt{d}\,\sqrt{c}\,\sqrt{a}}{}$$

input
```
int((e*x+d)^(11/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^3,x)
```

output
```
( - 15*sqrt(d)*sqrt(c)*sqrt(a*e**2 - c*d**2)*atan((sqrt(d + e*x)*c*d)/(sqr
t(d)*sqrt(c)*sqrt(a*e**2 - c*d**2)))*a**2*e**4 - 30*sqrt(d)*sqrt(c)*sqrt(a
*e**2 - c*d**2)*atan((sqrt(d + e*x)*c*d)/(sqrt(d)*sqrt(c)*sqrt(a*e**2 - c*
d**2)))*a*c*d*e**3*x - 15*sqrt(d)*sqrt(c)*sqrt(a*e**2 - c*d**2)*atan((sqrt
(d + e*x)*c*d)/(sqrt(d)*sqrt(c)*sqrt(a*e**2 - c*d**2)))*c**2*d**2*e**2*x**
2 + 15*sqrt(d + e*x)*a**2*c*d*e**4 - 5*sqrt(d + e*x)*a*c**2*d**3*e**2 + 25
*sqrt(d + e*x)*a*c**2*d**2*e**3*x - 2*sqrt(d + e*x)*c**3*d**5 - 9*sqrt(d +
 e*x)*c**3*d**4*e*x + 8*sqrt(d + e*x)*c**3*d**3*e**2*x**2)/(4*c**4*d**4*(a
**2*e**2 + 2*a*c*d*e*x + c**2*d**2*x**2))
```

3.194 $\int \frac{(d+ex)^{9/2}}{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^3} dx$

Optimal result

Integrand size = 37, antiderivative size = 130

$$\int \frac{(d+ex)^{9/2}}{(ade+(cd^2+ae^2)\,x+cdex^2)^3}\,dx = -\frac{3e\sqrt{d+ex}}{4c^2d^2(ae+cdx)} - \frac{(d+ex)^{3/2}}{2cd(ae+cdx)^2} - \frac{3e^2\text{arctanh}\left(\frac{\sqrt{c}\sqrt{d}\sqrt{d+ex}}{\sqrt{cd^2-ae^2}}\right)}{4c^{5/2}d^{5/2}\sqrt{cd^2-ae^2}}$$

output
```
-3/4*e*(e*x+d)^(1/2)/c^2/d^2/(c*d*x+a*e)-1/2*(e*x+d)^(3/2)/c/d/(c*d*x+a*e)
^2-3/4*e^2*arctanh(c^(1/2)*d^(1/2)*(e*x+d)^(1/2)/(-a*e^2+c*d^2)^(1/2))/c^(
5/2)/d^(5/2)/(-a*e^2+c*d^2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.44 (sec) , antiderivative size = 118, normalized size of antiderivative = 0.91

$$\int \frac{(d+ex)^{9/2}}{(ade+(cd^2+ae^2)\,x+cdex^2)^3}\,dx = -\frac{\sqrt{d+ex}(3ae^2+cd(2d+5ex))}{4c^2d^2(ae+cdx)^2} + \frac{3e^2\arctan\left(\frac{\sqrt{c}\sqrt{d}\sqrt{d+ex}}{\sqrt{-cd^2+ae^2}}\right)}{4c^{5/2}d^{5/2}\sqrt{-cd^2+ae^2}}$$

input
```
Integrate[(d + e*x)^(9/2)/(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^3,x]
```

output
```
-1/4*(Sqrt[d + e*x]*(3*a*e^2 + c*d*(2*d + 5*e*x)))/(c^2*d^2*(a*e + c*d*x)^
2) + (3*e^2*ArcTan[(Sqrt[c]*Sqrt[d]*Sqrt[d + e*x])/Sqrt[-(c*d^2) + a*e^2]]
)/(4*c^(5/2)*d^(5/2)*Sqrt[-(c*d^2) + a*e^2])
```

Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 135, normalized size of antiderivative = 1.04, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.135$, Rules used = {1121, 51, 51, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex)^{9/2}}{(x(ae^2+cd^2)+ade+cdex^2)^3}dx$$

$$\Big\downarrow \text{1121}$$

$$\int \frac{(d+ex)^{3/2}}{(ae+cdx)^3}dx$$

$$\Big\downarrow \text{51}$$

$$\frac{3e\int \frac{\sqrt{d+ex}}{(ae+cdx)^2}dx}{4cd}-\frac{(d+ex)^{3/2}}{2cd(ae+cdx)^2}$$

$$\Big\downarrow \text{51}$$

$$\frac{3e\left(\frac{e\int \frac{1}{(ae+cdx)\sqrt{d+ex}}dx}{2cd}-\frac{\sqrt{d+ex}}{cd(ae+cdx)}\right)}{4cd}-\frac{(d+ex)^{3/2}}{2cd(ae+cdx)^2}$$

$$\Big\downarrow \text{73}$$

$$\frac{3e\left(\frac{\int \frac{1}{-\frac{cd^2}{e}+\frac{c(d+ex)d}{e}+ae}d\sqrt{d+ex}}{cd}-\frac{\sqrt{d+ex}}{cd(ae+cdx)}\right)}{4cd}-\frac{(d+ex)^{3/2}}{2cd(ae+cdx)^2}$$

$$\Big\downarrow \text{221}$$

$$\frac{3e\left(-\frac{e\text{arctanh}\left(\frac{\sqrt{c}\sqrt{d}\sqrt{d+ex}}{\sqrt{cd^2-ae^2}}\right)}{c^{3/2}d^{3/2}\sqrt{cd^2-ae^2}}-\frac{\sqrt{d+ex}}{cd(ae+cdx)}\right)}{4cd}-\frac{(d+ex)^{3/2}}{2cd(ae+cdx)^2}$$

input
```
Int[(d + e*x)^(9/2)/(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^3,x]
```

output
```
-1/2*(d + e*x)^(3/2)/(c*d*(a*e + c*d*x)^2) + (3*e*(-(Sqrt[d + e*x]/(c*d*(a
*e + c*d*x))) - (e*ArcTanh[(Sqrt[c]*Sqrt[d]*Sqrt[d + e*x])/Sqrt[c*d^2 - a*
e^2]])/(c^(3/2)*d^(3/2)*Sqrt[c*d^2 - a*e^2])))/(4*c*d)
```

Defintions of rubi rules used

rule 51
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Simp[d*(n/(b*(m + 1)))
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d, n}, x
] && ILtQ[m, -1] && FractionQ[n] && GtQ[n, 0]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 1121
```
Int[((d_) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Int[ExpandIntegrand[(d + e*x)^(m + p)*(a/d + (c/e)*x)^p, x], x]
/; FreeQ[{a, b, c, d, e, m, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && (Int
egerQ[p] || (GtQ[a, 0] && GtQ[d, 0] && LtQ[c, 0]))
```

Maple [A] (verified)

Time = 15.54 (sec) , antiderivative size = 101, normalized size of antiderivative = 0.78

method	result	size
pseudoelliptic	$\frac{e^2\left(-\frac{\sqrt{ex+d}\,\left(5cdxe+3a\,e^2+2c\,d^2\right)}{e^2(cdx+ae)^2}+\frac{3\arctan\left(\frac{cd\sqrt{ex+d}}{\sqrt{cd\left(a\,e^2-c\,d^2\right)}}\right)}{\sqrt{cd\left(a\,e^2-c\,d^2\right)}}\right)}{4c^2d^2}$	101
derivativedivides	$2e^2\left(\frac{-\frac{5(ex+d)^{\frac{3}{2}}}{8cd}-\frac{3\left(a\,e^2-c\,d^2\right)\sqrt{ex+d}}{8c^2d^2}}{\left(cd(ex+d)+a\,e^2-c\,d^2\right)^2}+\frac{3\arctan\left(\frac{cd\sqrt{ex+d}}{\sqrt{cd\left(a\,e^2-c\,d^2\right)}}\right)}{8c^2d^2\sqrt{cd(a\,e^2-c\,d^2)}}\right)$	126
default	$2e^2\left(\frac{-\frac{5(ex+d)^{\frac{3}{2}}}{8cd}-\frac{3\left(a\,e^2-c\,d^2\right)\sqrt{ex+d}}{8c^2d^2}}{\left(cd(ex+d)+a\,e^2-c\,d^2\right)^2}+\frac{3\arctan\left(\frac{cd\sqrt{ex+d}}{\sqrt{cd\left(a\,e^2-c\,d^2\right)}}\right)}{8c^2d^2\sqrt{cd(a\,e^2-c\,d^2)}}\right)$	126

input
```
int((e*x+d)^(9/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^3,x,method=_RETURNVERB
OSE)
```

output
```
1/4*e^2/c^2/d^2*(-(e*x+d)^(1/2)*(5*c*d*e*x+3*a*e^2+2*c*d^2)/e^2/(c*d*x+a*e
)^2+3/(c*d*(a*e^2-c*d^2))^(1/2)*arctan(c*d*(e*x+d)^(1/2)/(c*d*(a*e^2-c*d^2
))^(1/2)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 237 vs. $2(106) = 212$.

Time = 0.11 (sec) , antiderivative size = 492, normalized size of antiderivative = 3.78

$$\int \frac{(d+ex)^{9/2}}{\left(ade+(cd^2+ae^2)\,x+cdex^2\right)^3}\,dx = \left[\frac{3\left(c^2d^2e^2x^2+2\,acde^3x+a^2e^4\right)\sqrt{c^2d^3-acde^2}\log\left(\frac{cdex+2\,cd^2-ae^2-2}{cd}\right.}{8\left(a^2c^4d^5e^2-a^3c^3d^3e^4+(\right.}\right.$$

input
```
integrate((e*x+d)^(9/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^3,x, algorithm="
fricas")
```

output
```
[1/8*(3*(c^2*d^2*e^2*x^2 + 2*a*c*d*e^3*x + a^2*e^4)*sqrt(c^2*d^3 - a*c*d*e
^2)*log((c*d*e*x + 2*c*d^2 - a*e^2 - 2*sqrt(c^2*d^3 - a*c*d*e^2)*sqrt(e*x
+ d))/(c*d*x + a*e)) - 2*(2*c^3*d^5 + a*c^2*d^3*e^2 - 3*a^2*c*d*e^4 + 5*(c
^3*d^4*e - a*c^2*d^2*e^3)*x)*sqrt(e*x + d))/(a^2*c^4*d^5*e^2 - a^3*c^3*d^3
*e^4 + (c^6*d^7 - a*c^5*d^5*e^2)*x^2 + 2*(a*c^5*d^6*e - a^2*c^4*d^4*e^3)*x
), 1/4*(3*(c^2*d^2*e^2*x^2 + 2*a*c*d*e^3*x + a^2*e^4)*sqrt(-c^2*d^3 + a*c*
d*e^2)*arctan(sqrt(-c^2*d^3 + a*c*d*e^2)*sqrt(e*x + d)/(c*d*e*x + c*d^2))
- (2*c^3*d^5 + a*c^2*d^3*e^2 - 3*a^2*c*d*e^4 + 5*(c^3*d^4*e - a*c^2*d^2*e^
3)*x)*sqrt(e*x + d))/(a^2*c^4*d^5*e^2 - a^3*c^3*d^3*e^4 + (c^6*d^7 - a*c^5
*d^5*e^2)*x^2 + 2*(a*c^5*d^6*e - a^2*c^4*d^4*e^3)*x)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(d+ex)^{9/2}}{(ade+(cd^2+ae^2)\,x+cdex^2)^3}\,dx = \text{Timed out}$$

input
```
integrate((e*x+d)**(9/2)/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**3,x)
```

output
```
Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(d+ex)^{9/2}}{(ade+(cd^2+ae^2)\,x+cdex^2)^3}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((e*x+d)^(9/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^3,x, algorithm="
maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(a*e^2-c*d^2>0)', see `assume?` f
or more de
```

Giac [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 133, normalized size of antiderivative = 1.02

$$\int \frac{(d+ex)^{9/2}}{(ade+(cd^2+ae^2)x+cdex^2)^3}\,dx = \frac{3\,e^2\arctan\left(\frac{\sqrt{ex+d}cd}{\sqrt{-c^2d^3+acde^2}}\right)}{4\sqrt{-c^2d^3+acde^2}c^2d^2} - \frac{5\,(ex+d)^{\frac{3}{2}}cde^2-3\sqrt{ex+d}cd^2e^2+3\sqrt{ex+d}ae^4}{4\left((ex+d)cd-cd^2+ae^2\right)^2c^2d^2}$$

input
```
integrate((e*x+d)^(9/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^3,x, algorithm="
giac")
```

output
```
3/4*e^2*arctan(sqrt(e*x + d)*c*d/sqrt(-c^2*d^3 + a*c*d*e^2))/(sqrt(-c^2*d^
3 + a*c*d*e^2)*c^2*d^2) - 1/4*(5*(e*x + d)^(3/2)*c*d*e^2 - 3*sqrt(e*x + d)
*c*d^2*e^2 + 3*sqrt(e*x + d)*a*e^4)/((((e*x + d)*c*d - c*d^2 + a*e^2)^2*c^2
*d^2)
```

Mupad [B] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 171, normalized size of antiderivative = 1.32

$$\int \frac{(d+ex)^{9/2}}{(ade+(cd^2+ae^2)x+cdex^2)^3}\,dx = \frac{3\,e^2\,\mathrm{atan}\left(\frac{\sqrt{c}\,\sqrt{d}\,\sqrt{d+e\,x}}{\sqrt{a\,e^2-c\,d^2}}\right)}{4\,c^{5/2}\,d^{5/2}\,\sqrt{a\,e^2-c\,d^2}} - \frac{\frac{5\,e^2\,(d+e\,x)^{3/2}}{4\,c\,d}+\frac{3\,e^2\,(a\,e^2-c\,d^2)\,\sqrt{d+e\,x}}{4\,c^2\,d^2}}{a^2\,e^4+c^2\,d^4-(2\,c^2\,d^3-2\,a\,c\,d\,e^2)\,(d+e\,x)+c^2\,d^2\,(d+e\,x)^2-2\,a\,c\,d^2\,e^2}$$

input
```
int((d + e*x)^(9/2)/(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^3,x)
```

output
```
(3*e^2*atan((c^(1/2)*d^(1/2)*(d + e*x)^(1/2))/(a*e^2 - c*d^2)^(1/2)))/(4*c
^(5/2)*d^(5/2)*(a*e^2 - c*d^2)^(1/2)) - ((5*e^2*(d + e*x)^(3/2))/(4*c*d) +
 (3*e^2*(a*e^2 - c*d^2)*(d + e*x)^(1/2))/(4*c^2*d^2))/(a^2*e^4 + c^2*d^4 -
 (2*c^2*d^3 - 2*a*c*d*e^2)*(d + e*x) + c^2*d^2*(d + e*x)^2 - 2*a*c*d^2*e^2
)
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 341, normalized size of antiderivative = 2.62

$$\int \frac{(d+ex)^{9/2}}{(ade+(cd^2+ae^2)x+cdex^2)^3}\,dx = \frac{3\sqrt{d}\,\sqrt{c}\,\sqrt{a\,e^2-c\,d^2}\,atan\left(\frac{\sqrt{ex+d}\,cd}{\sqrt{d}\,\sqrt{c}\,\sqrt{a\,e^2-c\,d^2}}\right)a^2e^4+6\sqrt{d}\,\sqrt{c}\,\sqrt{a\,e^2-}}{}$$

input
```
int((e*x+d)^(9/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^3,x)
```

output
```
(3*sqrt(d)*sqrt(c)*sqrt(a*e**2 - c*d**2)*atan((sqrt(d + e*x)*c*d)/(sqrt(d)
*sqrt(c)*sqrt(a*e**2 - c*d**2)))*a**2*e**4 + 6*sqrt(d)*sqrt(c)*sqrt(a*e**2
 - c*d**2)*atan((sqrt(d + e*x)*c*d)/(sqrt(d)*sqrt(c)*sqrt(a*e**2 - c*d**2)
))*a*c*d*e**3*x + 3*sqrt(d)*sqrt(c)*sqrt(a*e**2 - c*d**2)*atan((sqrt(d + e
*x)*c*d)/(sqrt(d)*sqrt(c)*sqrt(a*e**2 - c*d**2)))*c**2*d**2*e**2*x**2 - 3*
sqrt(d + e*x)*a**2*c*d*e**4 + sqrt(d + e*x)*a*c**2*d**3*e**2 - 5*sqrt(d +
e*x)*a*c**2*d**2*e**3*x + 2*sqrt(d + e*x)*c**3*d**5 + 5*sqrt(d + e*x)*c**3
*d**4*e*x)/(4*c**3*d**3*(a**3*e**4 - a**2*c*d**2*e**2 + 2*a**2*c*d*e**3*x
- 2*a*c**2*d**3*e*x + a*c**2*d**2*e**2*x**2 - c**3*d**4*x**2))
```

3.195 $\int \frac{(d+ex)^{7/2}}{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^3}\,dx$

Optimal result

Integrand size = 37, antiderivative size = 144

$$\int \frac{(d+ex)^{7/2}}{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^3}\,dx=-\frac{\sqrt{d+ex}}{2cd(ae+cdx)^2}-\frac{e\sqrt{d+ex}}{4cd\left(cd^2-ae^2\right)(ae+cdx)}+\frac{e^2\text{arctanh}\left(\frac{\sqrt{c}\sqrt{d}\sqrt{d+ex}}{\sqrt{cd^2-ae^2}}\right)}{4c^{3/2}d^{3/2}\left(cd^2-ae^2\right)^{3/2}}$$

output

```
-1/2*(e*x+d)^(1/2)/c/d/(c*d*x+a*e)^2-1/4*e*(e*x+d)^(1/2)/c/d/(-a*e^2+c*d^2
)/(c*d*x+a*e)+1/4*e^2*arctanh(c^(1/2)*d^(1/2)*(e*x+d)^(1/2)/(-a*e^2+c*d^2)
^(1/2))/c^(3/2)/d^(3/2)/(-a*e^2+c*d^2)^(3/2)
```

Mathematica [A] (verified)

Time = 0.51 (sec) , antiderivative size = 131, normalized size of antiderivative = 0.91

$$\int \frac{(d+ex)^{7/2}}{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^3}\,dx=\frac{\sqrt{d+ex}(ae^2-cd(2d+ex))}{4cd\left(cd^2-ae^2\right)(ae+cdx)^2}+\frac{e^2\arctan\left(\frac{\sqrt{c}\sqrt{d}\sqrt{d+ex}}{\sqrt{-cd^2+ae^2}}\right)}{4c^{3/2}d^{3/2}\left(-cd^2+ae^2\right)^{3/2}}$$

input
```
Integrate[(d + e*x)^(7/2)/(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^3,x]
```

output
```
(Sqrt[d + e*x]*(a*e^2 - c*d*(2*d + e*x)))/(4*c*d*(c*d^2 - a*e^2)*(a*e + c*
d*x)^2) + (e^2*ArcTan[(Sqrt[c]*Sqrt[d]*Sqrt[d + e*x])/Sqrt[-(c*d^2) + a*e^
2]])/(4*c^(3/2)*d^(3/2)*(-(c*d^2) + a*e^2)^(3/2))
```

Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 142, normalized size of antiderivative = 0.99, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.135$, Rules used = {1121, 51, 52, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex)^{7/2}}{(x(ae^2+cd^2)+ade+cdex^2)^3}dx$$

$$\Big\downarrow \text{ 1121}$$

$$\int \frac{\sqrt{d+ex}}{(ae+cdx)^3}dx$$

$$\Big\downarrow \text{ 51}$$

$$\frac{e\int \frac{1}{(ae+cdx)^2\sqrt{d+ex}}dx}{4cd}-\frac{\sqrt{d+ex}}{2cd(ae+cdx)^2}$$

$$\Big\downarrow \text{ 52}$$

$$\frac{e\left(-\frac{e\int \frac{1}{(ae+cdx)\sqrt{d+ex}}dx}{2(cd^2-ae^2)}-\frac{\sqrt{d+ex}}{(cd^2-ae^2)(ae+cdx)}\right)}{4cd}-\frac{\sqrt{d+ex}}{2cd(ae+cdx)^2}$$

$$\Big\downarrow \text{ 73}$$

$$\frac{e\left(-\frac{\int \frac{1}{-\frac{cd^2}{e}+\frac{c(d+ex)d}{e}+ae}d\sqrt{d+ex}}{cd^2-ae^2}-\frac{\sqrt{d+ex}}{(cd^2-ae^2)(ae+cdx)}\right)}{4cd}-\frac{\sqrt{d+ex}}{2cd(ae+cdx)^2}$$

$$\Big\downarrow \text{ 221}$$

$$\frac{e\left(\frac{e\,\text{arctanh}\left(\frac{\sqrt{c}\sqrt{d}\sqrt{d+ex}}{\sqrt{cd^2-ae^2}}\right)}{\sqrt{c}\sqrt{d}(cd^2-ae^2)^{3/2}}-\frac{\sqrt{d+ex}}{(cd^2-ae^2)(ae+cdx)}\right)}{4cd}-\frac{\sqrt{d+ex}}{2cd(ae+cdx)^2}$$

input
```
Int[(d + e*x)^(7/2)/(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^3,x]
```

output
```
-1/2*Sqrt[d + e*x]/(c*d*(a*e + c*d*x)^2) + (e*(-(Sqrt[d + e*x]/((c*d^2 - a
*e^2)*(a*e + c*d*x))) + (e*ArcTanh[(Sqrt[c]*Sqrt[d]*Sqrt[d + e*x])/Sqrt[c*
d^2 - a*e^2]])/(Sqrt[c]*Sqrt[d]*(c*d^2 - a*e^2)^(3/2))))/(4*c*d)
```

Defintions of rubi rules used

rule 51
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Simp[d*(n/(b*(m + 1)))
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d, n}, x
] && ILtQ[m, -1] && FractionQ[n] && GtQ[n, 0]
```

rule 52
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && ILtQ[m, -1] && FractionQ[n] && LtQ[n, 0]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 1121
```
Int[((d_) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Int[ExpandIntegrand[(d + e*x)^(m + p)*(a/d + (c/e)*x)^p, x], x]
/; FreeQ[{a, b, c, d, e, m, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && (Int
egerQ[p] || (GtQ[a, 0] && GtQ[d, 0] && LtQ[c, 0]))
```

Maple [A] (verified)

Time = 15.66 (sec) , antiderivative size = 136, normalized size of antiderivative = 0.94

method	result	size
derivativedivides	$2e^2\left(\frac{\frac{(ex+d)^{\frac{3}{2}}}{8a\,e^2-8c\,d^2}-\frac{\sqrt{ex+d}}{8cd}}{\left(cd(ex+d)+a\,e^2-c\,d^2\right)^2}+\frac{\arctan\left(\frac{cd\sqrt{ex+d}}{\sqrt{cd\left(a\,e^2-c\,d^2\right)}}\right)}{8\left(a\,e^2-c\,d^2\right)cd\sqrt{cd\left(a\,e^2-c\,d^2\right)}}\right)$	136
default	$2e^2\left(\frac{\frac{(ex+d)^{\frac{3}{2}}}{8a\,e^2-8c\,d^2}-\frac{\sqrt{ex+d}}{8cd}}{\left(cd(ex+d)+a\,e^2-c\,d^2\right)^2}+\frac{\arctan\left(\frac{cd\sqrt{ex+d}}{\sqrt{cd\left(a\,e^2-c\,d^2\right)}}\right)}{8\left(a\,e^2-c\,d^2\right)cd\sqrt{cd\left(a\,e^2-c\,d^2\right)}}\right)$	136
pseudoelliptic	$\frac{\arctan\left(\frac{cd\sqrt{ex+d}}{\sqrt{cd\left(a\,e^2-c\,d^2\right)}}\right)e^2(cdx+ae)^2-\sqrt{ex+d}\left(\left(-dex-2d^2\right)c+a\,e^2\right)\sqrt{cd\left(a\,e^2-c\,d^2\right)}}{4\sqrt{cd\left(a\,e^2-c\,d^2\right)}\,cd\left(a\,e^2-c\,d^2\right)(cdx+ae)^2}$	138

input
```
int((e*x+d)^(7/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^3,x,method=_RETURNVERB
OSE)
```

output
```
2*e^2*((1/8/(a*e^2-c*d^2)*(e*x+d)^(3/2)-1/8*(e*x+d)^(1/2)/c/d)/(c*d*(e*x+d
)+a*e^2-c*d^2)^2+1/8/(a*e^2-c*d^2)/c/d/(c*d*(a*e^2-c*d^2))^(1/2)*arctan(c*
d*(e*x+d)^(1/2)/(c*d*(a*e^2-c*d^2))^(1/2)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 273 vs. $2(120) = 240$.

Time = 0.11 (sec) , antiderivative size = 565, normalized size of antiderivative = 3.92

$$\int \frac{(d+ex)^{7/2}}{\left(ade+(cd^2+ae^2)x+cdex^2\right)^3}\,dx = \Bigg[-\frac{(c^2d^2e^2x^2+2\,acde^3x+a^2e^4)\sqrt{c^2d^3-acde^2}\log\left(\frac{cdex+2\,cd^2-ae^2-\cdots}{cd\cdots}\right.}{8\,(a^2c^4d^6e^2-2\,a^3c^3d^4e^4+a^4c^2d^2e^6+(c^6d^8-\cdots}$$

$$-\frac{(c^2d^2e^2x^2+2\,acde^3x+a^2e^4)\sqrt{-c^2d^3+acde^2}\arctan\left(\frac{\sqrt{-c^2d^3+acde^2}\sqrt{ex+d}}{cdex+cd^2}\right)+(2\,c^3d^5-3\,ac^2d^3e^2+a^2cde^4\cdots}{4\,(a^2c^4d^6e^2-2\,a^3c^3d^4e^4+a^4c^2d^2e^6+(c^6d^8-2\,ac^5d^6e^2+a^2c^4d^4e^4)x^2+2\,(ac^5d^7e-2\,a^2c^4\cdots}$$

input
```
integrate((e*x+d)^(7/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^3,x, algorithm="
fricas")
```

output
```
[-1/8*((c^2*d^2*e^2*x^2 + 2*a*c*d*e^3*x + a^2*e^4)*sqrt(c^2*d^3 - a*c*d*e^
2)*log((c*d*e*x + 2*c*d^2 - a*e^2 - 2*sqrt(c^2*d^3 - a*c*d*e^2)*sqrt(e*x +
 d))/(c*d*x + a*e)) + 2*(2*c^3*d^5 - 3*a*c^2*d^3*e^2 + a^2*c*d*e^4 + (c^3*
d^4*e - a*c^2*d^2*e^3)*x)*sqrt(e*x + d))/(a^2*c^4*d^6*e^2 - 2*a^3*c^3*d^4*
e^4 + a^4*c^2*d^2*e^6 + (c^6*d^8 - 2*a*c^5*d^6*e^2 + a^2*c^4*d^4*e^4)*x^2
+ 2*(a*c^5*d^7*e - 2*a^2*c^4*d^5*e^3 + a^3*c^3*d^3*e^5)*x), -1/4*((c^2*d^2
*e^2*x^2 + 2*a*c*d*e^3*x + a^2*e^4)*sqrt(-c^2*d^3 + a*c*d*e^2)*arctan(sqrt
(-c^2*d^3 + a*c*d*e^2)*sqrt(e*x + d)/(c*d*e*x + c*d^2)) + (2*c^3*d^5 - 3*a
*c^2*d^3*e^2 + a^2*c*d*e^4 + (c^3*d^4*e - a*c^2*d^2*e^3)*x)*sqrt(e*x + d))
/(a^2*c^4*d^6*e^2 - 2*a^3*c^3*d^4*e^4 + a^4*c^2*d^2*e^6 + (c^6*d^8 - 2*a*c
^5*d^6*e^2 + a^2*c^4*d^4*e^4)*x^2 + 2*(a*c^5*d^7*e - 2*a^2*c^4*d^5*e^3 + a
^3*c^3*d^3*e^5)*x)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(d+ex)^{7/2}}{\left(ade+(cd^2+ae^2)x+cdex^2\right)^3}\,dx = \text{Timed out}$$

input
```
integrate((e*x+d)**(7/2)/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**3,x)
```

output
```
Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(d+ex)^{7/2}}{(ade+(cd^2+ae^2)\,x+cdex^2)^3}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((e*x+d)^(7/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^3,x, algorithm="
maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(a*e^2-c*d^2>0)', see `assume?` f
or more de
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 155, normalized size of antiderivative = 1.08

$$\int \frac{(d+ex)^{7/2}}{(ade+(cd^2+ae^2)\,x+cdex^2)^3}\,dx = -\frac{e^2\arctan\left(\frac{\sqrt{ex+d}cd}{\sqrt{-c^2d^3+acde^2}}\right)}{4\,(c^2d^3-acde^2)\sqrt{-c^2d^3+acde^2}} - \frac{(ex+d)^{\frac{3}{2}}cde^2+\sqrt{ex+d}cd^2e^2-\sqrt{ex+d}ae^4}{4\,(c^2d^3-acde^2)((ex+d)cd-cd^2+ae^2)^2}$$

input
```
integrate((e*x+d)^(7/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^3,x, algorithm="
giac")
```

output
```
-1/4*e^2*arctan(sqrt(e*x + d)*c*d/sqrt(-c^2*d^3 + a*c*d*e^2))/((c^2*d^3 -
a*c*d*e^2)*sqrt(-c^2*d^3 + a*c*d*e^2)) - 1/4*((e*x + d)^(3/2)*c*d*e^2 + sq
rt(e*x + d)*c*d^2*e^2 - sqrt(e*x + d)*a*e^4)/((c^2*d^3 - a*c*d*e^2)*((e*x
+ d)*c*d - c*d^2 + a*e^2)^2)
```

Mupad [B] (verification not implemented)

Time = 5.27 (sec) , antiderivative size = 166, normalized size of antiderivative = 1.15

$$\int \frac{(d+ex)^{7/2}}{(ade+(cd^2+ae^2)\,x+cdex^2)^3}\,dx = \frac{\frac{e^2\,(d+e\,x)^{3/2}}{4\,(a\,e^2-c\,d^2)}-\frac{e^2\,\sqrt{d+e\,x}}{4\,c\,d}}{a^2\,e^4+c^2\,d^4-(2\,c^2\,d^3-2\,a\,c\,d\,e^2)\,(d+e\,x)+c^2\,d^2\,(d+e\,x)^2-2}$$
$$+\frac{e^2\,\mathrm{atan}\left(\frac{\sqrt{c}\,\sqrt{d}\,\sqrt{d+e\,x}}{\sqrt{a\,e^2-c\,d^2}}\right)}{4\,c^{3/2}\,d^{3/2}\,(a\,e^2-c\,d^2)^{3/2}}$$

input
```
int((d + e*x)^(7/2)/(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^3,x)
```

output
```
((e^2*(d + e*x)^(3/2))/(4*(a*e^2 - c*d^2)) - (e^2*(d + e*x)^(1/2))/(4*c*d)
)/(a^2*e^4 + c^2*d^4 - (2*c^2*d^3 - 2*a*c*d*e^2)*(d + e*x) + c^2*d^2*(d +
e*x)^2 - 2*a*c*d^2*e^2) + (e^2*atan((c^(1/2)*d^(1/2)*(d + e*x)^(1/2))/(a*e
^2 - c*d^2)^(1/2)))/(4*c^(3/2)*d^(3/2)*(a*e^2 - c*d^2)^(3/2))
```

Reduce [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 383, normalized size of antiderivative = 2.66

$$\int \frac{(d+ex)^{7/2}}{(ade+(cd^2+ae^2)\,x+cdex^2)^3}\,dx = \frac{\sqrt{d}\,\sqrt{c}\,\sqrt{a\,e^2-c\,d^2}\,atan\left(\frac{\sqrt{ex+d}\,cd}{\sqrt{d}\,\sqrt{c}\,\sqrt{a\,e^2-c\,d^2}}\right)a^2e^4+2\sqrt{d}\,\sqrt{c}\,\sqrt{a\,e^2-c}}{}$$

input
```
int((e*x+d)^(7/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^3,x)
```

output
```
(sqrt(d)*sqrt(c)*sqrt(a*e**2 - c*d**2)*atan((sqrt(d + e*x)*c*d)/(sqrt(d)*s
qrt(c)*sqrt(a*e**2 - c*d**2)))*a**2*e**4 + 2*sqrt(d)*sqrt(c)*sqrt(a*e**2 -
 c*d**2)*atan((sqrt(d + e*x)*c*d)/(sqrt(d)*sqrt(c)*sqrt(a*e**2 - c*d**2)))
*a*c*d*e**3*x + sqrt(d)*sqrt(c)*sqrt(a*e**2 - c*d**2)*atan((sqrt(d + e*x)*
c*d)/(sqrt(d)*sqrt(c)*sqrt(a*e**2 - c*d**2)))*c**2*d**2*e**2*x**2 - sqrt(d
 + e*x)*a**2*c*d*e**4 + 3*sqrt(d + e*x)*a*c**2*d**3*e**2 + sqrt(d + e*x)*a
*c**2*d**2*e**3*x - 2*sqrt(d + e*x)*c**3*d**5 - sqrt(d + e*x)*c**3*d**4*e*
x)/(4*c**2*d**2*(a**4*e**6 - 2*a**3*c*d**2*e**4 + 2*a**3*c*d*e**5*x + a**2
*c**2*d**4*e**2 - 4*a**2*c**2*d**3*e**3*x + a**2*c**2*d**2*e**4*x**2 + 2*a
*c**3*d**5*e*x - 2*a*c**3*d**4*e**2*x**2 + c**4*d**6*x**2))
```

3.196 $\int \frac{(d+ex)^{5/2}}{(ade+(cd^2+ae^2)x+cdex^2)^3} dx$

Optimal result

Integrand size = 37, antiderivative size = 146

$$\int \frac{(d+ex)^{5/2}}{(ade+(cd^2+ae^2)x+cdex^2)^3} dx = -\frac{\sqrt{d+ex}}{2(cd^2-ae^2)(ae+cdx)^2} + \frac{3e\sqrt{d+ex}}{4(cd^2-ae^2)^2(ae+cdx)} - \frac{3e^2\text{arctanh}\left(\frac{\sqrt{c}\sqrt{d}\sqrt{d+ex}}{\sqrt{cd^2-ae^2}}\right)}{4\sqrt{c}\sqrt{d}(cd^2-ae^2)^{5/2}}$$

output
```
-1/2*(e*x+d)^(1/2)/(-a*e^2+c*d^2)/(c*d*x+a*e)^2+3/4*e*(e*x+d)^(1/2)/(-a*e^
2+c*d^2)^2/(c*d*x+a*e)-3/4*e^2*arctanh(c^(1/2)*d^(1/2)*(e*x+d)^(1/2)/(-a*e
^2+c*d^2)^(1/2))/c^(1/2)/d^(1/2)/(-a*e^2+c*d^2)^(5/2)
```

Mathematica [A] (verified)

Time = 0.33 (sec) , antiderivative size = 125, normalized size of antiderivative = 0.86

$$\int \frac{(d+ex)^{5/2}}{(ade+(cd^2+ae^2)x+cdex^2)^3} dx = \frac{1}{4}\left(\frac{\sqrt{d+ex}(5ae^2+cd(-2d+3ex))}{(cd^2-ae^2)^2(ae+cdx)^2} + \frac{3e^2\arctan\left(\frac{\sqrt{c}\sqrt{d}\sqrt{d+ex}}{\sqrt{-cd^2+ae^2}}\right)}{\sqrt{c}\sqrt{d}(-cd^2+ae^2)^{5/2}}\right)$$

input
```
Integrate[(d + e*x)^(5/2)/(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^3,x]
```

output
```
((Sqrt[d + e*x]*(5*a*e^2 + c*d*(-2*d + 3*e*x)))/((c*d^2 - a*e^2)^2*(a*e +
c*d*x)^2) + (3*e^2*ArcTan[(Sqrt[c]*Sqrt[d]*Sqrt[d + e*x])/Sqrt[-(c*d^2) +
a*e^2]])/(Sqrt[c]*Sqrt[d]*(-(c*d^2) + a*e^2)^(5/2)))/4
```

Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 158, normalized size of antiderivative = 1.08, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.135$, Rules used = {1121, 52, 52, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex)^{5/2}}{(x(ae^2+cd^2)+ade+cdex^2)^3}dx$$

$$\Big\downarrow \text{ 1121}$$

$$\int \frac{1}{\sqrt{d+ex}(ae+cdx)^3}dx$$

$$\Big\downarrow \text{ 52}$$

$$-\frac{3e\int \frac{1}{(ae+cdx)^2\sqrt{d+ex}}dx}{4(cd^2-ae^2)}-\frac{\sqrt{d+ex}}{2(cd^2-ae^2)(ae+cdx)^2}$$

$$\Big\downarrow \text{ 52}$$

$$-\frac{3e\left(-\frac{e\int \frac{1}{(ae+cdx)\sqrt{d+ex}}dx}{2(cd^2-ae^2)}-\frac{\sqrt{d+ex}}{(cd^2-ae^2)(ae+cdx)}\right)}{4(cd^2-ae^2)}-\frac{\sqrt{d+ex}}{2(cd^2-ae^2)(ae+cdx)^2}$$

$$\Big\downarrow \text{ 73}$$

$$-\frac{3e\left(-\frac{\int \frac{1}{-\frac{cd^2}{e}+\frac{c(d+ex)d}{e}+ae}d\sqrt{d+ex}}{cd^2-ae^2}-\frac{\sqrt{d+ex}}{(cd^2-ae^2)(ae+cdx)}\right)}{4(cd^2-ae^2)}-\frac{\sqrt{d+ex}}{2(cd^2-ae^2)(ae+cdx)^2}$$

$$\Big\downarrow \text{ 221}$$

$$-\frac{3e\left(\frac{e\,\text{arctanh}\left(\frac{\sqrt{c}\sqrt{d}\sqrt{d+ex}}{\sqrt{cd^2-ae^2}}\right)}{\sqrt{c}\sqrt{d}(cd^2-ae^2)^{3/2}}-\frac{\sqrt{d+ex}}{(cd^2-ae^2)(ae+cdx)}\right)}{4\left(cd^2-ae^2\right)}-\frac{\sqrt{d+ex}}{2\left(cd^2-ae^2\right)(ae+cdx)^2}$$

input
```
Int[(d + e*x)^(5/2)/(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^3,x]
```

output
```
-1/2*Sqrt[d + e*x]/((c*d^2 - a*e^2)*(a*e + c*d*x)^2) - (3*e*(-(Sqrt[d + e*
x]/((c*d^2 - a*e^2)*(a*e + c*d*x))) + (e*ArcTanh[(Sqrt[c]*Sqrt[d]*Sqrt[d +
 e*x])/Sqrt[c*d^2 - a*e^2]])/(Sqrt[c]*Sqrt[d]*(c*d^2 - a*e^2)^(3/2))))/(4*
(c*d^2 - a*e^2))
```

Defintions of rubi rules used

rule 52
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && ILtQ[m, -1] && FractionQ[n] && LtQ[n, 0]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 1121
```
Int[((d_) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Int[ExpandIntegrand[(d + e*x)^(m + p)*(a/d + (c/e)*x)^p, x], x]
/; FreeQ[{a, b, c, d, e, m, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && (Int
egerQ[p] || (GtQ[a, 0] && GtQ[d, 0] && LtQ[c, 0]))
```

Maple [A] (verified)

Time = 15.50 (sec) , antiderivative size = 129, normalized size of antiderivative = 0.88

method	result
pseudoelliptic	$\frac{\frac{3 \arctan\left(\frac{c d \sqrt{e x+d}}{\sqrt{c d \left(a \,e^{2}-c \,d^{2}\right)}}\right) e^{2} (c d x+a e)^{2}}{4}+\frac{5 \sqrt{e x+d}\, \left(-\frac{2 \left(-\frac{3 e x}{2}+d \right) d c}{5}+a \,e^{2}\right) \sqrt{c d \left(a \,e^{2}-c \,d^{2}\right)}}{4}}{(a \,e^{2}-c \,d^{2})^{2} (c d x+a e)^{2} \sqrt{c d (a \,e^{2}-c \,d^{2})}}$
derivativedivides	$2 e^{2} \left(\frac{\sqrt{e x+d}}{4 (a \,e^{2}-c \,d^{2}) (c d (e x+d)+a \,e^{2}-c \,d^{2})^{2}}+\frac{\frac{3 \sqrt{e x+d}}{8 \left(a \,e^{2}-c \,d^{2}\right) \left(c d (e x+d)+a \,e^{2}-c \,d^{2}\right)}+\frac{3 \arctan\left(\frac{c d \sqrt{e x+d}}{\sqrt{c d \left(a \,e^{2}-c \,d^{2}\right)}}\right)}{8 \left(a \,e^{2}-c \,d^{2}\right) \sqrt{c d \left(a \,e^{2}-c \,d^{2}\right)}}}{a \,e^{2}-c \,d^{2}}\right)$
default	$2 e^{2} \left(\frac{\sqrt{e x+d}}{4 (a \,e^{2}-c \,d^{2}) (c d (e x+d)+a \,e^{2}-c \,d^{2})^{2}}+\frac{\frac{3 \sqrt{e x+d}}{8 \left(a \,e^{2}-c \,d^{2}\right) \left(c d (e x+d)+a \,e^{2}-c \,d^{2}\right)}+\frac{3 \arctan\left(\frac{c d \sqrt{e x+d}}{\sqrt{c d \left(a \,e^{2}-c \,d^{2}\right)}}\right)}{8 \left(a \,e^{2}-c \,d^{2}\right) \sqrt{c d \left(a \,e^{2}-c \,d^{2}\right)}}}{a \,e^{2}-c \,d^{2}}\right)$

input
```
int((e*x+d)^(5/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^3,x,method=_RETURNVERB
OSE)
```

output
```
5/4/(c*d*(a*e^2-c*d^2))^(1/2)*(3/5*arctan(c*d*(e*x+d)^(1/2)/(c*d*(a*e^2-c*
d^2))^(1/2))*e^2*(c*d*x+a*e)^2+(e*x+d)^(1/2)*(-2/5*(-3/2*e*x+d)*d*c+a*e^2)
*(c*d*(a*e^2-c*d^2))^(1/2))/(a*e^2-c*d^2)^2/(c*d*x+a*e)^2
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 318 vs. 2(122) = 244.

Time = 0.11 (sec) , antiderivative size = 654, normalized size of antiderivative = 4.48

$$\int \frac{(d+ex)^{5/2}}{\left(ade+(cd^2+ae^2)\,x+cdex^2\right)^3}\,dx = \left[\frac{3\left(c^2d^2e^2x^2+2\,acde^3x+a^2e^4\right)\sqrt{c^2d^3-acde^2}\,\log\left(\frac{cdex+2\,cd^2-ae^2-2\cdots}{cd\cdots}\right)\cdots}{8\left(a^2c^4d^7e^2-3\,a^3c^3d^5e^4+3\,a^4c^2d^3e^6-a^5cde^8+(c^6d^9-3\,ac^5d\cdots\right.}\right.$$

input
```
integrate((e*x+d)^(5/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^3,x, algorithm="
fricas")
```

output
```
[1/8*(3*(c^2*d^2*e^2*x^2 + 2*a*c*d*e^3*x + a^2*e^4)*sqrt(c^2*d^3 - a*c*d*e
^2)*log((c*d*e*x + 2*c*d^2 - a*e^2 - 2*sqrt(c^2*d^3 - a*c*d*e^2)*sqrt(e*x
+ d))/(c*d*x + a*e)) - 2*(2*c^3*d^5 - 7*a*c^2*d^3*e^2 + 5*a^2*c*d*e^4 - 3*
(c^3*d^4*e - a*c^2*d^2*e^3)*x)*sqrt(e*x + d))/(a^2*c^4*d^7*e^2 - 3*a^3*c^3
*d^5*e^4 + 3*a^4*c^2*d^3*e^6 - a^5*c*d*e^8 + (c^6*d^9 - 3*a*c^5*d^7*e^2 +
3*a^2*c^4*d^5*e^4 - a^3*c^3*d^3*e^6)*x^2 + 2*(a*c^5*d^8*e - 3*a^2*c^4*d^6*
e^3 + 3*a^3*c^3*d^4*e^5 - a^4*c^2*d^2*e^7)*x), 1/4*(3*(c^2*d^2*e^2*x^2 + 2
*a*c*d*e^3*x + a^2*e^4)*sqrt(-c^2*d^3 + a*c*d*e^2)*arctan(sqrt(-c^2*d^3 +
a*c*d*e^2)*sqrt(e*x + d)/(c*d*e*x + c*d^2)) - (2*c^3*d^5 - 7*a*c^2*d^3*e^2
 + 5*a^2*c*d*e^4 - 3*(c^3*d^4*e - a*c^2*d^2*e^3)*x)*sqrt(e*x + d))/(a^2*c^
4*d^7*e^2 - 3*a^3*c^3*d^5*e^4 + 3*a^4*c^2*d^3*e^6 - a^5*c*d*e^8 + (c^6*d^9
 - 3*a*c^5*d^7*e^2 + 3*a^2*c^4*d^5*e^4 - a^3*c^3*d^3*e^6)*x^2 + 2*(a*c^5*d
^8*e - 3*a^2*c^4*d^6*e^3 + 3*a^3*c^3*d^4*e^5 - a^4*c^2*d^2*e^7)*x)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(d+ex)^{5/2}}{(ade+(cd^2+ae^2)\,x+cdex^2)^3}\,dx = \text{Timed out}$$

input
```
integrate((e*x+d)**(5/2)/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**3,x)
```

output
```
Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(d+ex)^{5/2}}{(ade+(cd^2+ae^2)\,x+cdex^2)^3}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((e*x+d)^(5/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^3,x, algorithm="
maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(a*e^2-c*d^2>0)', see `assume?` f
or more de
```

Giac [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 175, normalized size of antiderivative = 1.20

$$\int \frac{(d+ex)^{5/2}}{(ade+(cd^2+ae^2)\,x+cdex^2)^3}\,dx = \frac{3\,e^2\arctan\left(\frac{\sqrt{ex+d}cd}{\sqrt{-c^2d^3+acde^2}}\right)}{4\,(c^2d^4-2\,acd^2e^2+a^2e^4)\sqrt{-c^2d^3+acde^2}} + \frac{3\,(ex+d)^{\frac{3}{2}}cde^2-5\,\sqrt{ex+d}cd^2e^2+5\,\sqrt{ex+d}ae^4}{4\,(c^2d^4-2\,acd^2e^2+a^2e^4)((ex+d)cd-cd^2+ae^2)^2}$$

input
```
integrate((e*x+d)^(5/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^3,x, algorithm="
giac")
```

output
```
3/4*e^2*arctan(sqrt(e*x + d)*c*d/sqrt(-c^2*d^3 + a*c*d*e^2))/((c^2*d^4 - 2
*a*c*d^2*e^2 + a^2*e^4)*sqrt(-c^2*d^3 + a*c*d*e^2)) + 1/4*(3*(e*x + d)^(3/
2)*c*d*e^2 - 5*sqrt(e*x + d)*c*d^2*e^2 + 5*sqrt(e*x + d)*a*e^4)/((c^2*d^4
- 2*a*c*d^2*e^2 + a^2*e^4)*((e*x + d)*c*d - c*d^2 + a*e^2)^2)
```

Mupad [B] (verification not implemented)

Time = 5.28 (sec) , antiderivative size = 177, normalized size of antiderivative = 1.21

$$\int \frac{(d+ex)^{5/2}}{(ade+(cd^2+ae^2)\,x+cdex^2)^3}\,dx = \frac{\frac{5\,e^2\,\sqrt{d+e\,x}}{4\,(a\,e^2-c\,d^2)}+\frac{3\,c\,d\,e^2\,(d+e\,x)^{3/2}}{4\,(a\,e^2-c\,d^2)^2}}{a^2\,e^4+c^2\,d^4-(2\,c^2\,d^3-2\,a\,c\,d\,e^2)\,(d+e\,x)+c^2\,d^2\,(d+e\,x)^2-2} + \frac{3\,e^2\,\mathrm{atan}\left(\frac{c\,d\,\sqrt{d+e\,x}}{\sqrt{c\,d}\,\sqrt{a\,e^2-c\,d^2}}\right)}{4\,\sqrt{c\,d}\,(a\,e^2-c\,d^2)^{5/2}}$$

input
```
int((d + e*x)^(5/2)/(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^3,x)
```

output
```
((5*e^2*(d + e*x)^(1/2))/(4*(a*e^2 - c*d^2)) + (3*c*d*e^2*(d + e*x)^(3/2))
/(4*(a*e^2 - c*d^2)^2))/(a^2*e^4 + c^2*d^4 - (2*c^2*d^3 - 2*a*c*d*e^2)*(d
+ e*x) + c^2*d^2*(d + e*x)^2 - 2*a*c*d^2*e^2) + (3*e^2*atan((c*d*(d + e*x)
^(1/2))/((c*d)^(1/2)*(a*e^2 - c*d^2)^(1/2))))/(4*(c*d)^(1/2)*(a*e^2 - c*d^
2)^(5/2))
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 434, normalized size of antiderivative = 2.97

$$\int \frac{(d+ex)^{5/2}}{(ade+(cd^2+ae^2)x+cdex^2)^3}\,dx = \frac{3\sqrt{d}\,\sqrt{c}\,\sqrt{a\,e^2-c\,d^2}\,atan\left(\frac{\sqrt{ex+d}\,cd}{\sqrt{d}\,\sqrt{c}\,\sqrt{a\,e^2-c\,d^2}}\right)a^2e^4+6\sqrt{d}\,\sqrt{c}\,\sqrt{a\,e^2-}{4cd\,(a^3c^2d^2e^6x}$$

input
```
int((e*x+d)^(5/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^3,x)
```

output
```
(3*sqrt(d)*sqrt(c)*sqrt(a*e**2 - c*d**2)*atan((sqrt(d + e*x)*c*d)/(sqrt(d)
*sqrt(c)*sqrt(a*e**2 - c*d**2)))*a**2*e**4 + 6*sqrt(d)*sqrt(c)*sqrt(a*e**2
 - c*d**2)*atan((sqrt(d + e*x)*c*d)/(sqrt(d)*sqrt(c)*sqrt(a*e**2 - c*d**2)
))*a*c*d*e**3*x + 3*sqrt(d)*sqrt(c)*sqrt(a*e**2 - c*d**2)*atan((sqrt(d + e
*x)*c*d)/(sqrt(d)*sqrt(c)*sqrt(a*e**2 - c*d**2)))*c**2*d**2*e**2*x**2 + 5*
sqrt(d + e*x)*a**2*c*d*e**4 - 7*sqrt(d + e*x)*a*c**2*d**3*e**2 + 3*sqrt(d
+ e*x)*a*c**2*d**2*e**3*x + 2*sqrt(d + e*x)*c**3*d**5 - 3*sqrt(d + e*x)*c*
*3*d**4*e*x)/(4*c*d*(a**5*e**8 - 3*a**4*c*d**2*e**6 + 2*a**4*c*d*e**7*x +
3*a**3*c**2*d**4*e**4 - 6*a**3*c**2*d**3*e**5*x + a**3*c**2*d**2*e**6*x**2
 - a**2*c**3*d**6*e**2 + 6*a**2*c**3*d**5*e**3*x - 3*a**2*c**3*d**4*e**4*x
**2 - 2*a*c**4*d**7*e*x + 3*a*c**4*d**6*e**2*x**2 - c**5*d**8*x**2))
```

3.197 $\int \frac{(d+ex)^{3/2}}{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^3}\,dx$

Optimal result

Integrand size = 37, antiderivative size = 176

$$\int \frac{(d+ex)^{3/2}}{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^3}\,dx=\frac{15e^2}{4\left(cd^2-ae^2\right)^3\sqrt{d+ex}}-\frac{1}{2\left(cd^2-ae^2\right)(ae+cdx)^2\sqrt{d+ex}}+\frac{5e}{4\left(cd^2-ae^2\right)^2(ae+cdx)\sqrt{d+ex}}-\frac{15\sqrt{c}\sqrt{d}e^2\operatorname{arctanh}\left(\frac{\sqrt{c}\sqrt{d}\sqrt{d+ex}}{\sqrt{cd^2-ae^2}}\right)}{4\left(cd^2-ae^2\right)^{7/2}}$$

output
```
15/4*e^2/(-a*e^2+c*d^2)^3/(e*x+d)^(1/2)-1/2/(-a*e^2+c*d^2)/(c*d*x+a*e)^2/(
e*x+d)^(1/2)+5/4*e/(-a*e^2+c*d^2)^2/(c*d*x+a*e)/(e*x+d)^(1/2)-15/4*c^(1/2)
*d^(1/2)*e^2*arctanh(c^(1/2)*d^(1/2)*(e*x+d)^(1/2)/(-a*e^2+c*d^2)^(1/2))/(
-a*e^2+c*d^2)^(7/2)
```

Mathematica [A] (verified)

Time = 0.72 (sec) , antiderivative size = 157, normalized size of antiderivative = 0.89

$$\int \frac{(d+ex)^{3/2}}{(ade+(cd^2+ae^2)x+cdex^2)^3}\,dx=\frac{1}{4}\left(\frac{8a^2e^4+acde^2(9d+25ex)+c^2d^2(-2d^2+5dex+15e^2x^2)}{(cd^2-ae^2)^3(ae+cdx)^2\sqrt{d+ex}}-\frac{15\sqrt{c}\sqrt{d}e^2\arctan\left(\frac{\sqrt{c}\sqrt{d}\sqrt{d+ex}}{\sqrt{-cd^2+ae^2}}\right)}{(-cd^2+ae^2)^{7/2}}\right)$$

input
```
Integrate[(d + e*x)^(3/2)/(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^3,x]
```

output
```
((8*a^2*e^4 + a*c*d*e^2*(9*d + 25*e*x) + c^2*d^2*(-2*d^2 + 5*d*e*x + 15*e^
2*x^2))/((c*d^2 - a*e^2)^3*(a*e + c*d*x)^2*Sqrt[d + e*x]) - (15*Sqrt[c]*Sq
rt[d]*e^2*ArcTan[(Sqrt[c]*Sqrt[d]*Sqrt[d + e*x])/Sqrt[-(c*d^2) + a*e^2]])/
(-(c*d^2) + a*e^2)^(7/2))/4
```

Rubi [A] (verified)

Time = 0.47 (sec) , antiderivative size = 203, normalized size of antiderivative = 1.15, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.162, Rules used = {1121, 52, 52, 61, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex)^{3/2}}{(x(ae^2+cd^2)+ade+cdex^2)^3}dx$$

$$\Big\downarrow \text{ 1121}$$

$$\int \frac{1}{(d+ex)^{3/2}(ae+cdx)^3}dx$$

$$\Big\downarrow \text{ 52}$$

$$-\frac{5e\int \frac{1}{(ae+cdx)^2(d+ex)^{3/2}}dx}{4(cd^2-ae^2)}-\frac{1}{2\sqrt{d+ex}(cd^2-ae^2)(ae+cdx)^2}$$

$$\Big\downarrow \ 52$$

$$-\frac{5e\left(-\frac{3e\int \frac{1}{(ae+cdx)(d+ex)^{3/2}}dx}{2(cd^2-ae^2)}-\frac{1}{\sqrt{d+ex}(cd^2-ae^2)(ae+cdx)}\right)}{4\left(cd^2-ae^2\right)}-\frac{1}{2\sqrt{d+ex}\left(cd^2-ae^2\right)(ae+cdx)^2}$$

$$\Big\downarrow \ 61$$

$$-\frac{5e\left(-\frac{3e\left(\frac{cd\int \frac{1}{(ae+cdx)\sqrt{d+ex}}dx}{cd^2-ae^2}+\frac{2}{\sqrt{d+ex}\left(cd^2-ae^2\right)}\right)}{2(cd^2-ae^2)}-\frac{1}{\sqrt{d+ex}(cd^2-ae^2)(ae+cdx)}\right)}{4\left(cd^2-ae^2\right)}-\frac{1}{2\sqrt{d+ex}\left(cd^2-ae^2\right)(ae+cdx)^2}$$

$$\Big\downarrow \ 73$$

$$-\frac{5e\left(-\frac{3e\left(\frac{2cd\int \frac{1}{-\frac{cd^2}{e}+\frac{c(d+ex)d}{e}+ae}d\sqrt{d+ex}}{e\left(cd^2-ae^2\right)}+\frac{2}{\sqrt{d+ex}\left(cd^2-ae^2\right)}\right)}{2(cd^2-ae^2)}-\frac{1}{\sqrt{d+ex}(cd^2-ae^2)(ae+cdx)}\right)}{4\left(cd^2-ae^2\right)}-\frac{1}{2\sqrt{d+ex}\left(cd^2-ae^2\right)(ae+cdx)^2}$$

$$\Big\downarrow \ 221$$

$$-\frac{5e\left(-\frac{3e\left(\frac{2}{\sqrt{d+ex}\left(cd^2-ae^2\right)}-\frac{2\sqrt{c}\sqrt{d}\text{arctanh}\left(\frac{\sqrt{c}\sqrt{d}\sqrt{d+ex}}{\sqrt{cd^2-ae^2}}\right)}{\left(cd^2-ae^2\right)^{3/2}}\right)}{2(cd^2-ae^2)}-\frac{1}{\sqrt{d+ex}(cd^2-ae^2)(ae+cdx)}\right)}{4\left(cd^2-ae^2\right)}-\frac{1}{2\sqrt{d+ex}\left(cd^2-ae^2\right)(ae+cdx)^2}$$

input
```
Int[(d + e*x)^(3/2)/(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^3,x]
```

output
```
-1/2*1/((c*d^2 - a*e^2)*(a*e + c*d*x)^2*Sqrt[d + e*x]) - (5*e*(-(1/((c*d^2
 - a*e^2)*(a*e + c*d*x)*Sqrt[d + e*x])) - (3*e*(2/((c*d^2 - a*e^2)*Sqrt[d
+ e*x]) - (2*Sqrt[c]*Sqrt[d]*ArcTanh[(Sqrt[c]*Sqrt[d]*Sqrt[d + e*x])/Sqrt[
c*d^2 - a*e^2]])/(c*d^2 - a*e^2)^(3/2)))/(2*(c*d^2 - a*e^2))))/(4*(c*d^2 -
 a*e^2))
```

Defintions of rubi rules used

rule 52
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && ILtQ[m, -1] && FractionQ[n] && LtQ[n, 0]
```

rule 61
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && LtQ[m, -1] &&  !(LtQ[n, -1] && (EqQ[a, 0
] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && IntLinearQ[a, b, c, d
, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 1121
```
Int[((d_) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Int[ExpandIntegrand[(d + e*x)^(m + p)*(a/d + (c/e)*x)^p, x], x]
/; FreeQ[{a, b, c, d, e, m, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && (Int
egerQ[p] || (GtQ[a, 0] && GtQ[d, 0] && LtQ[c, 0]))
```

Maple [A] (verified)

Time = 2.00 (sec) , antiderivative size = 152, normalized size of antiderivative = 0.86

method	result
derivativedivides	$2e^{2}\left(-\frac{1}{\left(a\,e^{2}-c\,d^{2}\right)^{3}\sqrt{ex+d}}-\frac{cd\left(\frac{\frac{7cd(ex+d)^{\frac{3}{2}}}{8}+\left(\frac{9a\,e^{2}}{8}-\frac{9c\,d^{2}}{8}\right)\sqrt{ex+d}}{\left(cd(ex+d)+a\,e^{2}-c\,d^{2}\right)^{2}}+\frac{15\arctan\left(\frac{cd\sqrt{ex+d}}{\sqrt{cd\left(a\,e^{2}-c\,d^{2}\right)}}\right)}{8\sqrt{cd\left(a\,e^{2}-c\,d^{2}\right)}}\right)}{\left(a\,e^{2}-c\,d^{2}\right)^{3}}\right)$
default	$2e^{2}\left(-\frac{1}{\left(a\,e^{2}-c\,d^{2}\right)^{3}\sqrt{ex+d}}-\frac{cd\left(\frac{\frac{7cd(ex+d)^{\frac{3}{2}}}{8}+\left(\frac{9a\,e^{2}}{8}-\frac{9c\,d^{2}}{8}\right)\sqrt{ex+d}}{\left(cd(ex+d)+a\,e^{2}-c\,d^{2}\right)^{2}}+\frac{15\arctan\left(\frac{cd\sqrt{ex+d}}{\sqrt{cd\left(a\,e^{2}-c\,d^{2}\right)}}\right)}{8\sqrt{cd\left(a\,e^{2}-c\,d^{2}\right)}}\right)}{\left(a\,e^{2}-c\,d^{2}\right)^{3}}\right)$
pseudoelliptic	$-\frac{15\left(cd\,e^{2}\sqrt{ex+d}\,(cdx+ae)^{2}\arctan\left(\frac{cd\sqrt{ex+d}}{\sqrt{cd\left(a\,e^{2}-c\,d^{2}\right)}}\right)+\frac{8\left(\left(\frac{15}{8}d^{2}e^{2}x^{2}+\frac{5}{8}d^{3}ex-\frac{1}{4}d^{4}\right)c^{2}+\frac{9e^{2}a\left(\frac{25ex}{9}+d\right)dc}{8}+a^{2}e^{4}\right)\sqrt{c}}{15}\right.}{4\sqrt{ex+d}\,\sqrt{cd(a\,e^{2}-c\,d^{2})}\,(a\,e^{2}-c\,d^{2})^{3}(cdx+ae)^{2}}$

input

```
int((e*x+d)^(3/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^3,x,method=_RETURNVERB
OSE)
```

output

```
2*e^2*(-1/(a*e^2-c*d^2)^3/(e*x+d)^(1/2)-1/(a*e^2-c*d^2)^3*c*d*((7/8*c*d*(e
*x+d)^(3/2)+(9/8*a*e^2-9/8*c*d^2)*(e*x+d)^(1/2))/(c*d*(e*x+d)+a*e^2-c*d^2)
^2+15/8/(c*d*(a*e^2-c*d^2))^(1/2)*arctan(c*d*(e*x+d)^(1/2)/(c*d*(a*e^2-c*d
^2))^(1/2)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 414 vs. $2(148) = 296$.

Time = 0.13 (sec) , antiderivative size = 870, normalized size of antiderivative = 4.94

$$\int \frac{(d+ex)^{3/2}}{(ade+(cd^2+ae^2)x+cdex^2)^3}\,dx = \text{Too large to display}$$

input
```
integrate((e*x+d)^(3/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^3,x, algorithm="
fricas")
```

output
```
[-1/8*(15*(c^2*d^2*e^3*x^3 + a^2*d*e^4 + (c^2*d^3*e^2 + 2*a*c*d*e^4)*x^2 +
 (2*a*c*d^2*e^3 + a^2*e^5)*x)*sqrt(c*d/(c*d^2 - a*e^2))*log((c*d*e*x + 2*c
*d^2 - a*e^2 + 2*(c*d^2 - a*e^2)*sqrt(e*x + d)*sqrt(c*d/(c*d^2 - a*e^2)))/
(c*d*x + a*e)) - 2*(15*c^2*d^2*e^2*x^2 - 2*c^2*d^4 + 9*a*c*d^2*e^2 + 8*a^2
*e^4 + 5*(c^2*d^3*e + 5*a*c*d*e^3)*x)*sqrt(e*x + d))/(a^2*c^3*d^7*e^2 - 3*
a^3*c^2*d^5*e^4 + 3*a^4*c*d^3*e^6 - a^5*d*e^8 + (c^5*d^8*e - 3*a*c^4*d^6*e
^3 + 3*a^2*c^3*d^4*e^5 - a^3*c^2*d^2*e^7)*x^3 + (c^5*d^9 - a*c^4*d^7*e^2 -
 3*a^2*c^3*d^5*e^4 + 5*a^3*c^2*d^3*e^6 - 2*a^4*c*d*e^8)*x^2 + (2*a*c^4*d^8
*e - 5*a^2*c^3*d^6*e^3 + 3*a^3*c^2*d^4*e^5 + a^4*c*d^2*e^7 - a^5*e^9)*x),
1/4*(15*(c^2*d^2*e^3*x^3 + a^2*d*e^4 + (c^2*d^3*e^2 + 2*a*c*d*e^4)*x^2 + (
2*a*c*d^2*e^3 + a^2*e^5)*x)*sqrt(-c*d/(c*d^2 - a*e^2))*arctan(sqrt(e*x + d
)*sqrt(-c*d/(c*d^2 - a*e^2))) + (15*c^2*d^2*e^2*x^2 - 2*c^2*d^4 + 9*a*c*d^
2*e^2 + 8*a^2*e^4 + 5*(c^2*d^3*e + 5*a*c*d*e^3)*x)*sqrt(e*x + d))/(a^2*c^3
*d^7*e^2 - 3*a^3*c^2*d^5*e^4 + 3*a^4*c*d^3*e^6 - a^5*d*e^8 + (c^5*d^8*e -
3*a*c^4*d^6*e^3 + 3*a^2*c^3*d^4*e^5 - a^3*c^2*d^2*e^7)*x^3 + (c^5*d^9 - a*
c^4*d^7*e^2 - 3*a^2*c^3*d^5*e^4 + 5*a^3*c^2*d^3*e^6 - 2*a^4*c*d*e^8)*x^2 +
 (2*a*c^4*d^8*e - 5*a^2*c^3*d^6*e^3 + 3*a^3*c^2*d^4*e^5 + a^4*c*d^2*e^7 -
a^5*e^9)*x)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(d+ex)^{3/2}}{(ade+(cd^2+ae^2)x+cdex^2)^3}\,dx = \text{Timed out}$$

input
```
integrate((e*x+d)**(3/2)/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**3,x)
```

output
```
Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(d+ex)^{3/2}}{(ade+(cd^2+ae^2)\,x+cdex^2)^3}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((e*x+d)^(3/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^3,x, algorithm="
maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(a*e^2-c*d^2>0)', see `assume?` f
or more de
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 269, normalized size of antiderivative = 1.53

$$\int \frac{(d+ex)^{3/2}}{(ade+(cd^2+ae^2)\,x+cdex^2)^3}\,dx = \frac{15\,cde^2\arctan\left(\frac{\sqrt{ex+d}cd}{\sqrt{-c^2d^3+acde^2}}\right)}{4\,(c^3d^6-3\,ac^2d^4e^2+3\,a^2cd^2e^4-a^3e^6)\sqrt{-c^2d^3+acde^2}}$$
$$+\frac{2\,e^2}{(c^3d^6-3\,ac^2d^4e^2+3\,a^2cd^2e^4-a^3e^6)\sqrt{ex+d}}$$
$$+\frac{7\,(ex+d)^{\frac{3}{2}}c^2d^2e^2-9\,\sqrt{ex+d}c^2d^3e^2+9\,\sqrt{ex+d}acde^4}{4\,(c^3d^6-3\,ac^2d^4e^2+3\,a^2cd^2e^4-a^3e^6)((ex+d)cd-cd^2+ae^2)^2}$$

input
```
integrate((e*x+d)^(3/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^3,x, algorithm="
giac")
```

output
```
15/4*c*d*e^2*arctan(sqrt(e*x + d)*c*d/sqrt(-c^2*d^3 + a*c*d*e^2))/((c^3*d^
6 - 3*a*c^2*d^4*e^2 + 3*a^2*c*d^2*e^4 - a^3*e^6)*sqrt(-c^2*d^3 + a*c*d*e^2
)) + 2*e^2/((c^3*d^6 - 3*a*c^2*d^4*e^2 + 3*a^2*c*d^2*e^4 - a^3*e^6)*sqrt(e
*x + d)) + 1/4*(7*(e*x + d)^(3/2)*c^2*d^2*e^2 - 9*sqrt(e*x + d)*c^2*d^3*e^
2 + 9*sqrt(e*x + d)*a*c*d*e^4)/((c^3*d^6 - 3*a*c^2*d^4*e^2 + 3*a^2*c*d^2*e
^4 - a^3*e^6)*((e*x + d)*c*d - c*d^2 + a*e^2)^2)
```

Mupad [B] (verification not implemented)

Time = 5.29 (sec) , antiderivative size = 251, normalized size of antiderivative = 1.43

$$\int \frac{(d+ex)^{3/2}}{(ade+(cd^2+ae^2)\,x+cdex^2)^3}\,dx = -\frac{\frac{2\,e^2}{a\,e^2-c\,d^2}+\frac{25\,c\,d\,e^2\,(d+e\,x)}{4\,{(a\,e^2-c\,d^2)}^2}+\frac{15\,c^2\,d^2\,e^2\,{(d+e\,x)}^2}{4\,{(a\,e^2-c\,d^2)}^3}}{\sqrt{d+e\,x}\,(a^2\,e^4-2\,a\,c\,d^2\,e^2+c^2\,d^4)-(2\,c^2\,d^3-2\,a\,c\,d\,e^2)\,{(d+e\,x)}^{3/2}+c^2\,d^2\,{(d+e\,x)}^{5/2}} - \frac{15\,\sqrt{c}\,\sqrt{d}\,e^2\,\mathrm{atan}\left(\frac{\sqrt{c}\,\sqrt{d}\,\sqrt{d+e\,x}\,(a^3\,e^6-3\,a^2\,c\,d^2\,e^4+3\,a\,c^2\,d^4\,e^2-c^3\,d^6)}{{(a\,e^2-c\,d^2)}^{7/2}}\right)}{4\,{(a\,e^2-c\,d^2)}^{7/2}}$$

input
```
int((d + e*x)^(3/2)/(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^3,x)
```

output
```
- ((2*e^2)/(a*e^2 - c*d^2) + (25*c*d*e^2*(d + e*x))/(4*(a*e^2 - c*d^2)^2)
+ (15*c^2*d^2*e^2*(d + e*x)^2)/(4*(a*e^2 - c*d^2)^3))/((d + e*x)^(1/2)*(a^
2*e^4 + c^2*d^4 - 2*a*c*d^2*e^2) - (2*c^2*d^3 - 2*a*c*d*e^2)*(d + e*x)^(3/
2) + c^2*d^2*(d + e*x)^(5/2)) - (15*c^(1/2)*d^(1/2)*e^2*atan((c^(1/2)*d^(1
/2)*(d + e*x)^(1/2)*(a^3*e^6 - c^3*d^6 + 3*a*c^2*d^4*e^2 - 3*a^2*c*d^2*e^4
))/(a*e^2 - c*d^2)^(7/2)))/(4*(a*e^2 - c*d^2)^(7/2))
```

Reduce [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 518, normalized size of antiderivative = 2.94

$$\int \frac{(d+ex)^{3/2}}{(ade+(cd^2+ae^2)x+cdex^2)^3}\,dx = \frac{-15\sqrt{d}\,\sqrt{c}\,\sqrt{ex+d}\,\sqrt{a\,e^2-c\,d^2}\,atan\left(\frac{\sqrt{ex+d}\,cd}{\sqrt{d}\,\sqrt{c}\,\sqrt{a\,e^2-c\,d^2}}\right)a^2e^4-30\sqrt{}}{4\sqrt{ex+d}\,(a^4c^2d^2e^{}}$$

input
```
int((e*x+d)^(3/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^3,x)
```

output
```
( - 15*sqrt(d)*sqrt(c)*sqrt(d + e*x)*sqrt(a*e**2 - c*d**2)*atan((sqrt(d +
e*x)*c*d)/(sqrt(d)*sqrt(c)*sqrt(a*e**2 - c*d**2)))*a**2*e**4 - 30*sqrt(d)*
sqrt(c)*sqrt(d + e*x)*sqrt(a*e**2 - c*d**2)*atan((sqrt(d + e*x)*c*d)/(sqrt
(d)*sqrt(c)*sqrt(a*e**2 - c*d**2)))*a*c*d*e**3*x - 15*sqrt(d)*sqrt(c)*sqrt
(d + e*x)*sqrt(a*e**2 - c*d**2)*atan((sqrt(d + e*x)*c*d)/(sqrt(d)*sqrt(c)*
sqrt(a*e**2 - c*d**2)))*c**2*d**2*e**2*x**2 - 8*a**3*e**6 - a**2*c*d**2*e*
*4 - 25*a**2*c*d*e**5*x + 11*a*c**2*d**4*e**2 + 20*a*c**2*d**3*e**3*x - 15
*a*c**2*d**2*e**4*x**2 - 2*c**3*d**6 + 5*c**3*d**5*e*x + 15*c**3*d**4*e**2
*x**2)/(4*sqrt(d + e*x)*(a**6*e**10 - 4*a**5*c*d**2*e**8 + 2*a**5*c*d*e**9
*x + 6*a**4*c**2*d**4*e**6 - 8*a**4*c**2*d**3*e**7*x + a**4*c**2*d**2*e**8
*x**2 - 4*a**3*c**3*d**6*e**4 + 12*a**3*c**3*d**5*e**5*x - 4*a**3*c**3*d**
4*e**6*x**2 + a**2*c**4*d**8*e**2 - 8*a**2*c**4*d**7*e**3*x + 6*a**2*c**4*
d**6*e**4*x**2 + 2*a*c**5*d**9*e*x - 4*a*c**5*d**8*e**2*x**2 + c**6*d**10*
x**2))
```

3.198 $\int \frac{\sqrt{d+ex}}{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^3}\,dx$

Optimal result

Integrand size = 37, antiderivative size = 208

$$\int \frac{\sqrt{d+ex}}{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^3}\,dx = \frac{35e^2}{12\left(cd^2-ae^2\right)^3(d+ex)^{3/2}} - \frac{1}{2\left(cd^2-ae^2\right)(ae+cdx)^2(d+ex)^{3/2}} + \frac{7e}{4\left(cd^2-ae^2\right)^2(ae+cdx)(d+ex)^{3/2}} + \frac{35cde^2}{4\left(cd^2-ae^2\right)^4\sqrt{d+ex}} - \frac{35c^{3/2}d^{3/2}e^2\operatorname{arctanh}\left(\frac{\sqrt{c}\sqrt{d}\sqrt{d+ex}}{\sqrt{cd^2-ae^2}}\right)}{4\left(cd^2-ae^2\right)^{9/2}}$$

output
```
35/12*e^2/(-a*e^2+c*d^2)^3/(e*x+d)^(3/2)-1/2/(-a*e^2+c*d^2)/(c*d*x+a*e)^2/
(e*x+d)^(3/2)+7/4*e/(-a*e^2+c*d^2)^2/(c*d*x+a*e)/(e*x+d)^(3/2)+35/4*c*d*e^
2/(-a*e^2+c*d^2)^4/(e*x+d)^(1/2)-35/4*c^(3/2)*d^(3/2)*e^2*arctanh(c^(1/2)*
d^(1/2)*(e*x+d)^(1/2)/(-a*e^2+c*d^2)^(1/2))/(-a*e^2+c*d^2)^(9/2)
```

Mathematica [A] (verified)

Time = 0.71 (sec) , antiderivative size = 202, normalized size of antiderivative = 0.97

$$\int \frac{\sqrt{d+ex}}{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^3}\,dx$$
$$= \frac{-8a^3e^6+8a^2cde^4(10d+7ex)+ac^2d^2e^2(39d^2+238dex+175e^2x^2)+c^3d^3(-6d^3+21d^2ex+140de^2x^2}{12\left(cd^2-ae^2\right)^4(ae+cdx)^2(d+ex)^{3/2}}$$
$$+\frac{35c^{3/2}d^{3/2}e^2\arctan\left(\frac{\sqrt{c}\sqrt{d}\sqrt{d+ex}}{\sqrt{-cd^2+ae^2}}\right)}{4\left(-cd^2+ae^2\right)^{9/2}}$$

input
```
Integrate[Sqrt[d + e*x]/(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^3,x]
```

output
```
(-8*a^3*e^6 + 8*a^2*c*d*e^4*(10*d + 7*e*x) + a*c^2*d^2*e^2*(39*d^2 + 238*d
*e*x + 175*e^2*x^2) + c^3*d^3*(-6*d^3 + 21*d^2*e*x + 140*d*e^2*x^2 + 105*e
^3*x^3))/(12*(c*d^2 - a*e^2)^4*(a*e + c*d*x)^2*(d + e*x)^(3/2)) + (35*c^(3
/2)*d^(3/2)*e^2*ArcTan[(Sqrt[c]*Sqrt[d]*Sqrt[d + e*x])/Sqrt[-(c*d^2) + a*e
^2]])/(4*(-(c*d^2) + a*e^2)^(9/2))
```

Rubi [A] (verified)

Time = 0.51 (sec) , antiderivative size = 248, normalized size of antiderivative = 1.19, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.189, Rules used = {1121, 52, 52, 61, 61, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{d+ex}}{\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^3}dx$$
$$\Big\downarrow \; 1121$$
$$\int \frac{1}{(d+ex)^{5/2}(ae+cdx)^3}dx$$
$$\Big\downarrow \; 52$$

$$-\frac{7e\int \frac{1}{(ae+cdx)^2(d+ex)^{5/2}}dx}{4\left(cd^2-ae^2\right)}-\frac{1}{2(d+ex)^{3/2}\left(cd^2-ae^2\right)(ae+cdx)^2}$$

$\downarrow$ 52

$$-\frac{7e\left(-\frac{5e\int \frac{1}{(ae+cdx)(d+ex)^{5/2}}dx}{2(cd^2-ae^2)}-\frac{1}{(d+ex)^{3/2}(cd^2-ae^2)(ae+cdx)}\right)}{4\left(cd^2-ae^2\right)}-\frac{1}{2(d+ex)^{3/2}\left(cd^2-ae^2\right)(ae+cdx)^2}$$

$\downarrow$ 61

$$-\frac{7e\left(-\frac{5e\left(\frac{cd\int \frac{1}{(ae+cdx)(d+ex)^{3/2}}dx}{cd^2-ae^2}+\frac{2}{3(d+ex)^{3/2}\left(cd^2-ae^2\right)}\right)}{2(cd^2-ae^2)}-\frac{1}{(d+ex)^{3/2}(cd^2-ae^2)(ae+cdx)}\right)}{4\left(cd^2-ae^2\right)}-\frac{1}{2(d+ex)^{3/2}\left(cd^2-ae^2\right)(ae+cdx)^2}$$

$\downarrow$ 61

$$-\frac{7e\left(-\frac{5e\left(\frac{cd\left(\frac{cd\int \frac{1}{(ae+cdx)\sqrt{d+ex}}dx}{cd^2-ae^2}+\frac{2}{\sqrt{d+ex}\left(cd^2-ae^2\right)}\right)}{cd^2-ae^2}+\frac{2}{3(d+ex)^{3/2}\left(cd^2-ae^2\right)}\right)}{2(cd^2-ae^2)}-\frac{1}{(d+ex)^{3/2}(cd^2-ae^2)(ae+cdx)}\right)}{4\left(cd^2-ae^2\right)}-\frac{1}{2(d+ex)^{3/2}\left(cd^2-ae^2\right)(ae+cdx)^2}$$

$\downarrow$ 73

$$-\frac{7e\left(-\frac{5e\left(\frac{cd\left(\frac{2cd\int \frac{1}{-\frac{cd^2}{e}+\frac{c(d+ex)d}{e}+ae}d\sqrt{d+ex}}{e\left(cd^2-ae^2\right)}+\frac{2}{\sqrt{d+ex}\left(cd^2-ae^2\right)}\right)}{cd^2-ae^2}+\frac{2}{3(d+ex)^{3/2}\left(cd^2-ae^2\right)}\right)}{2(cd^2-ae^2)}-\frac{1}{(d+ex)^{3/2}(cd^2-ae^2)(ae+cdx)}\right)}{4\left(cd^2-ae^2\right)}-\frac{1}{2(d+ex)^{3/2}\left(cd^2-ae^2\right)(ae+cdx)^2}$$

$\Big\downarrow$ 221

$$-\frac{7e\left(-\frac{5e\left(\frac{cd\left(\frac{2}{\sqrt{d+ex}\left(cd^2-ae^2\right)}-\frac{2\sqrt{c}\sqrt{d}\text{arctanh}\left(\frac{\sqrt{c}\sqrt{d}\sqrt{d+ex}}{\sqrt{cd^2-ae^2}}\right)}{\left(cd^2-ae^2\right)^{3/2}}\right)}{cd^2-ae^2}+\frac{2}{3(d+ex)^{3/2}\left(cd^2-ae^2\right)}\right)}{2(cd^2-ae^2)}-\frac{1}{(d+ex)^{3/2}(cd^2-ae^2)(ae+cdx)}\right)}{4\left(cd^2-ae^2\right)}-\frac{1}{2(d+ex)^{3/2}\left(cd^2-ae^2\right)(ae+cdx)^2}$$

input
```
Int[Sqrt[d + e*x]/(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^3,x]
```

output
```
-1/2*1/((c*d^2 - a*e^2)*(a*e + c*d*x)^2*(d + e*x)^(3/2)) - (7*e*(-(1/((c*d
^2 - a*e^2)*(a*e + c*d*x)*(d + e*x)^(3/2))) - (5*e*(2/(3*(c*d^2 - a*e^2)*(
d + e*x)^(3/2)) + (c*d*(2/((c*d^2 - a*e^2)*Sqrt[d + e*x]) - (2*Sqrt[c]*Sqr
t[d]*ArcTanh[(Sqrt[c]*Sqrt[d]*Sqrt[d + e*x])/Sqrt[c*d^2 - a*e^2]])/(c*d^2
- a*e^2)^(3/2)))/(c*d^2 - a*e^2)))/(2*(c*d^2 - a*e^2))))/(4*(c*d^2 - a*e^2
))
```

Defintions of rubi rules used

rule 52
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && ILtQ[m, -1] && FractionQ[n] && LtQ[n, 0]
```

rule 61
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && LtQ[m, -1] &&  !(LtQ[n, -1] && (EqQ[a, 0
] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && IntLinearQ[a, b, c, d
, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 1121
```
Int[((d_) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Int[ExpandIntegrand[(d + e*x)^(m + p)*(a/d + (c/e)*x)^p, x], x]
/; FreeQ[{a, b, c, d, e, m, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && (Int
egerQ[p] || (GtQ[a, 0] && GtQ[d, 0] && LtQ[c, 0]))
```

Maple [A] (verified)

Time = 1.98 (sec) , antiderivative size = 180, normalized size of antiderivative = 0.87

method	result
derivativedivides	$2e^{2}\left(-\frac{1}{3(a\,e^{2}-c\,d^{2})^{3}(ex+d)^{\frac{3}{2}}}+\frac{3cd}{(a\,e^{2}-c\,d^{2})^{4}\sqrt{ex+d}}+\frac{c^{2}d^{2}\left(\frac{\frac{11cd(ex+d)^{\frac{3}{2}}}{8}+\left(\frac{13a\,e^{2}}{8}-\frac{13c\,d^{2}}{8}\right)\sqrt{ex+d}}{\left(cd(ex+d)+a\,e^{2}-c\,d^{2}\right)^{2}}+\frac{35\arctan\cdots}{8\sqrt{\cdots}}\cdots\right.}{(a\,e^{2}-c\,d^{2})^{4}}\cdots$
default	$2e^{2}\left(-\frac{1}{3(a\,e^{2}-c\,d^{2})^{3}(ex+d)^{\frac{3}{2}}}+\frac{3cd}{(a\,e^{2}-c\,d^{2})^{4}\sqrt{ex+d}}+\frac{c^{2}d^{2}\left(\frac{\frac{11cd(ex+d)^{\frac{3}{2}}}{8}+\left(\frac{13a\,e^{2}}{8}-\frac{13c\,d^{2}}{8}\right)\sqrt{ex+d}}{\left(cd(ex+d)+a\,e^{2}-c\,d^{2}\right)^{2}}+\frac{35\arctan\cdots}{8\sqrt{\cdots}}\cdots\right.}{(a\,e^{2}-c\,d^{2})^{4}}\cdots$
pseudoelliptic	$-\frac{2\left(-\frac{105c^{2}d^{2}e^{2}(ex+d)^{\frac{3}{2}}(cdx+ae)^{2}\arctan\left(\frac{cd\sqrt{ex+d}}{\sqrt{cd\left(a\,e^{2}-c\,d^{2}\right)}}\right)}{8}+\left(\frac{3\left(-\frac{35}{2}e^{3}x^{3}-\frac{70}{3}d\,e^{2}x^{2}-\frac{7}{2}d^{2}ex+d^{3}\right)d^{3}c^{3}}{4}-\frac{39e^{2}\left(\frac{175}{39}e^{2}\cdots\right.}{\cdots}\cdots\right.\right.}{3(ex+d)^{\frac{3}{2}}\sqrt{cd(a\,e^{2}-c\,d^{2})}\,(a\,e^{2}-c\,d^{2})^{4}(cdx+ae)\cdots}$

input
```
int((e*x+d)^(1/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^3,x,method=_RETURNVERB
OSE)
```

output
```
2*e^2*(-1/3/(a*e^2-c*d^2)^3/(e*x+d)^(3/2)+3/(a*e^2-c*d^2)^4*c*d/(e*x+d)^(1
/2)+c^2/(a*e^2-c*d^2)^4*d^2*((11/8*c*d*(e*x+d)^(3/2)+(13/8*a*e^2-13/8*c*d^
2)*(e*x+d)^(1/2))/(c*d*(e*x+d)+a*e^2-c*d^2)^2+35/8/(c*d*(a*e^2-c*d^2))^(1/
2)*arctan(c*d*(e*x+d)^(1/2)/(c*d*(a*e^2-c*d^2))^(1/2))))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 642 vs. $2(176) = 352$.

Time = 0.20 (sec) , antiderivative size = 1326, normalized size of antiderivative = 6.38

$$\int \frac{\sqrt{d+ex}}{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^3}\,dx = \text{Too large to display}$$

input
```
integrate((e*x+d)^(1/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^3,x, algorithm="
fricas")
```

output
```
[1/24*(105*(c^3*d^3*e^4*x^4 + a^2*c*d^3*e^4 + 2*(c^3*d^4*e^3 + a*c^2*d^2*e
^5)*x^3 + (c^3*d^5*e^2 + 4*a*c^2*d^3*e^4 + a^2*c*d*e^6)*x^2 + 2*(a*c^2*d^4
*e^3 + a^2*c*d^2*e^5)*x)*sqrt(c*d/(c*d^2 - a*e^2))*log((c*d*e*x + 2*c*d^2
- a*e^2 - 2*(c*d^2 - a*e^2)*sqrt(e*x + d)*sqrt(c*d/(c*d^2 - a*e^2)))/(c*d*
x + a*e)) + 2*(105*c^3*d^3*e^3*x^3 - 6*c^3*d^6 + 39*a*c^2*d^4*e^2 + 80*a^2
*c*d^2*e^4 - 8*a^3*e^6 + 35*(4*c^3*d^4*e^2 + 5*a*c^2*d^2*e^4)*x^2 + 7*(3*c
^3*d^5*e + 34*a*c^2*d^3*e^3 + 8*a^2*c*d*e^5)*x)*sqrt(e*x + d))/(a^2*c^4*d^
10*e^2 - 4*a^3*c^3*d^8*e^4 + 6*a^4*c^2*d^6*e^6 - 4*a^5*c*d^4*e^8 + a^6*d^2
*e^10 + (c^6*d^10*e^2 - 4*a*c^5*d^8*e^4 + 6*a^2*c^4*d^6*e^6 - 4*a^3*c^3*d^
4*e^8 + a^4*c^2*d^2*e^10)*x^4 + 2*(c^6*d^11*e - 3*a*c^5*d^9*e^3 + 2*a^2*c^
4*d^7*e^5 + 2*a^3*c^3*d^5*e^7 - 3*a^4*c^2*d^3*e^9 + a^5*c*d*e^11)*x^3 + (c
^6*d^12 - 9*a^2*c^4*d^8*e^4 + 16*a^3*c^3*d^6*e^6 - 9*a^4*c^2*d^4*e^8 + a^6
*e^12)*x^2 + 2*(a*c^5*d^11*e - 3*a^2*c^4*d^9*e^3 + 2*a^3*c^3*d^7*e^5 + 2*a
^4*c^2*d^5*e^7 - 3*a^5*c*d^3*e^9 + a^6*d*e^11)*x), 1/12*(105*(c^3*d^3*e^4*
x^4 + a^2*c*d^3*e^4 + 2*(c^3*d^4*e^3 + a*c^2*d^2*e^5)*x^3 + (c^3*d^5*e^2 +
 4*a*c^2*d^3*e^4 + a^2*c*d*e^6)*x^2 + 2*(a*c^2*d^4*e^3 + a^2*c*d^2*e^5)*x)
*sqrt(-c*d/(c*d^2 - a*e^2))*arctan(sqrt(e*x + d)*sqrt(-c*d/(c*d^2 - a*e^2)
)) + (105*c^3*d^3*e^3*x^3 - 6*c^3*d^6 + 39*a*c^2*d^4*e^2 + 80*a^2*c*d^2*e^
4 - 8*a^3*e^6 + 35*(4*c^3*d^4*e^2 + 5*a*c^2*d^2*e^4)*x^2 + 7*(3*c^3*d^5*e
+ 34*a*c^2*d^3*e^3 + 8*a^2*c*d*e^5)*x)*sqrt(e*x + d))/(a^2*c^4*d^10*e^2...
```

Sympy [F]

$$\int \frac{\sqrt{d+ex}}{(ade+(cd^2+ae^2)\,x+cdex^2)^3}\,dx = \int \frac{1}{(d+ex)^{\frac{5}{2}}\,(ae+cdx)^3}\,dx$$

input
```
integrate((e*x+d)**(1/2)/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**3,x)
```

output
```
Integral(1/((d + e*x)**(5/2)*(a*e + c*d*x)**3), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\sqrt{d+ex}}{(ade+(cd^2+ae^2)\,x+cdex^2)^3}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((e*x+d)^(1/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^3,x, algorithm="
maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(a*e^2-c*d^2>0)', see `assume?` f
or more de
```

Giac [A] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 340, normalized size of antiderivative = 1.63

$$\int \frac{\sqrt{d+ex}}{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^3}\,dx = \frac{35\,c^2d^2e^2\arctan\left(\frac{\sqrt{ex+d}cd}{\sqrt{-c^2d^3+acde^2}}\right)}{4\left(c^4d^8-4\,ac^3d^6e^2+6\,a^2c^2d^4e^4-4\,a^3cd^2e^6+a^4e^8\right)\sqrt{-c^2d^3+acde^2}} + \frac{2\left(9\left(ex+d\right)cde^2+cd^2e^2-ae^4\right)}{3\left(c^4d^8-4\,ac^3d^6e^2+6\,a^2c^2d^4e^4-4\,a^3cd^2e^6+a^4e^8\right)\left(ex+d\right)^{\frac{3}{2}}} + \frac{11\left(ex+d\right)^{\frac{3}{2}}c^3d^3e^2-13\sqrt{ex+d}c^3d^4e^2+13\sqrt{ex+d}ac^2d^2e^4}{4\left(c^4d^8-4\,ac^3d^6e^2+6\,a^2c^2d^4e^4-4\,a^3cd^2e^6+a^4e^8\right)\left(\left(ex+d\right)cd-cd^2+ae^2\right)^2}$$

input
```
integrate((e*x+d)^(1/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^3,x, algorithm="
giac")
```

output
```
35/4*c^2*d^2*e^2*arctan(sqrt(e*x + d)*c*d/sqrt(-c^2*d^3 + a*c*d*e^2))/((c^
4*d^8 - 4*a*c^3*d^6*e^2 + 6*a^2*c^2*d^4*e^4 - 4*a^3*c*d^2*e^6 + a^4*e^8)*s
qrt(-c^2*d^3 + a*c*d*e^2)) + 2/3*(9*(e*x + d)*c*d*e^2 + c*d^2*e^2 - a*e^4)
/((c^4*d^8 - 4*a*c^3*d^6*e^2 + 6*a^2*c^2*d^4*e^4 - 4*a^3*c*d^2*e^6 + a^4*e
^8)*(e*x + d)^(3/2)) + 1/4*(11*(e*x + d)^(3/2)*c^3*d^3*e^2 - 13*sqrt(e*x +
 d)*c^3*d^4*e^2 + 13*sqrt(e*x + d)*a*c^2*d^2*e^4)/((c^4*d^8 - 4*a*c^3*d^6*
e^2 + 6*a^2*c^2*d^4*e^4 - 4*a^3*c*d^2*e^6 + a^4*e^8)*((e*x + d)*c*d - c*d^
2 + a*e^2)^2)
```

Mupad [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 296, normalized size of antiderivative = 1.42

$$\int \frac{\sqrt{d+ex}}{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^3}\,dx = \frac{\frac{14\,c\,d\,e^2\,(d+e\,x)}{3\,{\left(a\,e^2-c\,d^2\right)}^2}-\frac{2\,e^2}{3\,\left(a\,e^2-c\,d^2\right)}+\frac{175\,c^2\,d^2\,e^2\,{(d+e\,x)}^2}{12\,{\left(a\,e^2-c\,d^2\right)}^3}+\frac{35\,c^3\,d^3\,e^2\,{(d+e\,x)}^3}{4\,{\left(a\,e^2-c\,d^2\right)}^4}}{{(d+e\,x)}^{3/2}\,\left(a^2\,e^4-2\,a\,c\,d^2\,e^2+c^2\,d^4\right)-\left(2\,c^2\,d^3-2\,a\,c\,d\,e^2\right)\,{(d+e\,x)}^{5/2}+c^2\,d^2\,{(d+e\,x)}^{7/2}} + \frac{35\,c^{3/2}\,d^{3/2}\,e^2\,\mathrm{atan}\left(\frac{\sqrt{c}\,\sqrt{d}\,\sqrt{d+e\,x}\,\left(a^4\,e^8-4\,a^3\,c\,d^2\,e^6+6\,a^2\,c^2\,d^4\,e^4-4\,a\,c^3\,d^6\,e^2+c^4\,d^8\right)}{{\left(a\,e^2-c\,d^2\right)}^{9/2}}\right)}{4\,{\left(a\,e^2-c\,d^2\right)}^{9/2}}$$

input
```
int((d + e*x)^(1/2)/(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^3,x)
```

output
```
((14*c*d*e^2*(d + e*x))/(3*(a*e^2 - c*d^2)^2) - (2*e^2)/(3*(a*e^2 - c*d^2)
) + (175*c^2*d^2*e^2*(d + e*x)^2)/(12*(a*e^2 - c*d^2)^3) + (35*c^3*d^3*e^2
*(d + e*x)^3)/(4*(a*e^2 - c*d^2)^4))/((d + e*x)^(3/2)*(a^2*e^4 + c^2*d^4 -
 2*a*c*d^2*e^2) - (2*c^2*d^3 - 2*a*c*d*e^2)*(d + e*x)^(5/2) + c^2*d^2*(d +
 e*x)^(7/2)) + (35*c^(3/2)*d^(3/2)*e^2*atan((c^(1/2)*d^(1/2)*(d + e*x)^(1/
2)*(a^4*e^8 + c^4*d^8 - 4*a*c^3*d^6*e^2 - 4*a^3*c*d^2*e^6 + 6*a^2*c^2*d^4*
e^4))/(a*e^2 - c*d^2)^(9/2)))/(4*(a*e^2 - c*d^2)^(9/2))
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 944, normalized size of antiderivative = 4.54

$$\int \frac{\sqrt{d+ex}}{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^3}\,dx = \text{Too large to display}$$

input
```
int((e*x+d)^(1/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^3,x)
```

output
```
(105*sqrt(d)*sqrt(c)*sqrt(d + e*x)*sqrt(a*e**2 - c*d**2)*atan((sqrt(d + e*
x)*c*d)/(sqrt(d)*sqrt(c)*sqrt(a*e**2 - c*d**2)))*a**2*c*d**2*e**4 + 105*sq
rt(d)*sqrt(c)*sqrt(d + e*x)*sqrt(a*e**2 - c*d**2)*atan((sqrt(d + e*x)*c*d)
/(sqrt(d)*sqrt(c)*sqrt(a*e**2 - c*d**2)))*a**2*c*d*e**5*x + 210*sqrt(d)*sq
rt(c)*sqrt(d + e*x)*sqrt(a*e**2 - c*d**2)*atan((sqrt(d + e*x)*c*d)/(sqrt(d
)*sqrt(c)*sqrt(a*e**2 - c*d**2)))*a*c**2*d**3*e**3*x + 210*sqrt(d)*sqrt(c)
*sqrt(d + e*x)*sqrt(a*e**2 - c*d**2)*atan((sqrt(d + e*x)*c*d)/(sqrt(d)*sqr
t(c)*sqrt(a*e**2 - c*d**2)))*a*c**2*d**2*e**4*x**2 + 105*sqrt(d)*sqrt(c)*s
qrt(d + e*x)*sqrt(a*e**2 - c*d**2)*atan((sqrt(d + e*x)*c*d)/(sqrt(d)*sqrt(
c)*sqrt(a*e**2 - c*d**2)))*c**3*d**4*e**2*x**2 + 105*sqrt(d)*sqrt(c)*sqrt(
d + e*x)*sqrt(a*e**2 - c*d**2)*atan((sqrt(d + e*x)*c*d)/(sqrt(d)*sqrt(c)*s
qrt(a*e**2 - c*d**2)))*c**3*d**3*e**3*x**3 - 8*a**4*e**8 + 88*a**3*c*d**2*
e**6 + 56*a**3*c*d*e**7*x - 41*a**2*c**2*d**4*e**4 + 182*a**2*c**2*d**3*e*
*5*x + 175*a**2*c**2*d**2*e**6*x**2 - 45*a*c**3*d**6*e**2 - 217*a*c**3*d**
5*e**3*x - 35*a*c**3*d**4*e**4*x**2 + 105*a*c**3*d**3*e**5*x**3 + 6*c**4*d
**8 - 21*c**4*d**7*e*x - 140*c**4*d**6*e**2*x**2 - 105*c**4*d**5*e**3*x**3
)/(12*sqrt(d + e*x)*(a**7*d*e**12 + a**7*e**13*x - 5*a**6*c*d**3*e**10 - 3
*a**6*c*d**2*e**11*x + 2*a**6*c*d*e**12*x**2 + 10*a**5*c**2*d**5*e**8 - 9*
a**5*c**2*d**3*e**10*x**2 + a**5*c**2*d**2*e**11*x**3 - 10*a**4*c**3*d**7*
e**6 + 10*a**4*c**3*d**6*e**7*x + 15*a**4*c**3*d**5*e**8*x**2 - 5*a**4*...
```

3.199 $\int \frac{1}{\sqrt{d+ex}(ade+(cd^2+ae^2)x+cdex^2)^3}\,dx$

Optimal result

Integrand size = 37, antiderivative size = 244

$$\int \frac{1}{\sqrt{d+ex}\,(ade+(cd^2+ae^2)\,x+cdex^2)^3}\,dx = \frac{63e^2}{20\,(cd^2-ae^2)^3\,(d+ex)^{5/2}} - \frac{1}{2\,(cd^2-ae^2)\,(ae+cdx)^2(d+ex)^{5/2}} + \frac{9e}{4\,(cd^2-ae^2)^2\,(ae+cdx)(d+ex)^{5/2}} + \frac{21cde^2}{4\,(cd^2-ae^2)^4\,(d+ex)^{3/2}} + \frac{63c^2d^2e^2}{4\,(cd^2-ae^2)^5\,\sqrt{d+ex}} - \frac{63c^{5/2}d^{5/2}e^2\text{arctanh}\left(\frac{\sqrt{c}\sqrt{d}\sqrt{d+ex}}{\sqrt{cd^2-ae^2}}\right)}{4\,(cd^2-ae^2)^{11/2}}$$

output
```
63/20*e^2/(-a*e^2+c*d^2)^3/(e*x+d)^(5/2)-1/2/(-a*e^2+c*d^2)/(c*d*x+a*e)^2/
(e*x+d)^(5/2)+9/4*e/(-a*e^2+c*d^2)^2/(c*d*x+a*e)/(e*x+d)^(5/2)+21/4*c*d*e^
2/(-a*e^2+c*d^2)^4/(e*x+d)^(3/2)+63/4*c^2*d^2*e^2/(-a*e^2+c*d^2)^5/(e*x+d)
^(1/2)-63/4*c^(5/2)*d^(5/2)*e^2*arctanh(c^(1/2)*d^(1/2)*(e*x+d)^(1/2)/(-a*
e^2+c*d^2)^(1/2))/(-a*e^2+c*d^2)^(11/2)
```

Mathematica [A] (verified)

Time = 1.13 (sec) , antiderivative size = 257, normalized size of antiderivative = 1.05

$$\int \frac{1}{\sqrt{d+ex}\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^3}\,dx$$
$$= \frac{8a^4e^8-8a^3cde^6(7d+3ex)+24a^2c^2d^2e^4(12d^2+17dex+7e^2x^2)+ac^3d^3e^2(85d^3+831d^2ex+1239de^2x}{20\left(cd^2-ae^2\right)^5(ae+cdx)^2(d+ex}$$
$$-\frac{63c^{5/2}d^{5/2}e^2\arctan\left(\frac{\sqrt{c}\sqrt{d}\sqrt{d+ex}}{\sqrt{-cd^2+ae^2}}\right)}{4\left(-cd^2+ae^2\right)^{11/2}}$$

input
```
Integrate[1/(Sqrt[d + e*x]*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^3),x]
```

output
```
(8*a^4*e^8 - 8*a^3*c*d*e^6*(7*d + 3*e*x) + 24*a^2*c^2*d^2*e^4*(12*d^2 + 17
*d*e*x + 7*e^2*x^2) + a*c^3*d^3*e^2*(85*d^3 + 831*d^2*e*x + 1239*d*e^2*x^2
 + 525*e^3*x^3) + c^4*d^4*(-10*d^4 + 45*d^3*e*x + 483*d^2*e^2*x^2 + 735*d*
e^3*x^3 + 315*e^4*x^4))/(20*(c*d^2 - a*e^2)^5*(a*e + c*d*x)^2*(d + e*x)^(5
/2)) - (63*c^(5/2)*d^(5/2)*e^2*ArcTan[(Sqrt[c]*Sqrt[d]*Sqrt[d + e*x])/Sqrt
[-(c*d^2) + a*e^2]])/(4*(-(c*d^2) + a*e^2)^(11/2))
```

Rubi [A] (verified)

Time = 0.57 (sec) , antiderivative size = 293, normalized size of antiderivative = 1.20, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.216, Rules used = {1121, 52, 52, 61, 61, 61, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{d+ex}\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^3}dx$$

$$\Big\downarrow \text{1121}$$

$$\int \frac{1}{(d+ex)^{7/2}(ae+cdx)^3}dx$$

$$\Big\downarrow \text{52}$$

$$-\frac{9e\int \frac{1}{(ae+cdx)^2(d+ex)^{7/2}}dx}{4\left(cd^2-ae^2\right)}-\frac{1}{2(d+ex)^{5/2}\left(cd^2-ae^2\right)(ae+cdx)^2}$$

$$\Big\downarrow \; 52$$

$$-\frac{9e\left(-\frac{7e\int \frac{1}{(ae+cdx)(d+ex)^{7/2}}dx}{2(cd^2-ae^2)}-\frac{1}{(d+ex)^{5/2}(cd^2-ae^2)(ae+cdx)}\right)}{4\left(cd^2-ae^2\right)}-\frac{1}{2(d+ex)^{5/2}\left(cd^2-ae^2\right)(ae+cdx)^2}$$

$$\Big\downarrow \; 61$$

$$-\frac{9e\left(-\frac{7e\left(\frac{cd\int \frac{1}{(ae+cdx)(d+ex)^{5/2}}dx}{cd^2-ae^2}+\frac{2}{5(d+ex)^{5/2}\left(cd^2-ae^2\right)}\right)}{2(cd^2-ae^2)}-\frac{1}{(d+ex)^{5/2}(cd^2-ae^2)(ae+cdx)}\right)}{4\left(cd^2-ae^2\right)}-\frac{1}{2(d+ex)^{5/2}\left(cd^2-ae^2\right)(ae+cdx)^2}$$

$$\Big\downarrow \; 61$$

$$-\frac{9e\left(-\frac{7e\left(\frac{cd\left(\frac{cd\int \frac{1}{(ae+cdx)(d+ex)^{3/2}}dx}{cd^2-ae^2}+\frac{2}{3(d+ex)^{3/2}\left(cd^2-ae^2\right)}\right)}{cd^2-ae^2}+\frac{2}{5(d+ex)^{5/2}\left(cd^2-ae^2\right)}\right)}{2(cd^2-ae^2)}-\frac{1}{(d+ex)^{5/2}(cd^2-ae^2)(ae+cdx)}\right)}{4\left(cd^2-ae^2\right)}-\frac{1}{2(d+ex)^{5/2}\left(cd^2-ae^2\right)(ae+cdx)^2}$$

$$\Big\downarrow \; 61$$

$$-\frac{9e\left(-\frac{7e\left(\frac{cd\left(\frac{cd\left(\frac{cd\int\frac{1}{(ae+cdx)\sqrt{d+ex}}dx}{cd^2-ae^2}+\frac{2}{\sqrt{d+ex}\left(cd^2-ae^2\right)}\right)}{cd^2-ae^2}+\frac{2}{3(d+ex)^{3/2}\left(cd^2-ae^2\right)}\right)}{cd^2-ae^2}+\frac{2}{5(d+ex)^{5/2}\left(cd^2-ae^2\right)}\right)}{2(cd^2-ae^2)}-\frac{1}{(d+ex)^{5/2}(cd^2-ae^2)(a}\right.}{4\left(cd^2-ae^2\right)}$$

$$\frac{1}{2(d+ex)^{5/2}\left(cd^2-ae^2\right)(ae+cdx)^2}$$

$$\Big\downarrow \; 73$$

$$-\frac{9e\left(-\frac{7e\left(\frac{cd\left(\frac{cd\left(\frac{2cd\int\frac{1}{-\frac{cd^2}{e}+\frac{c(d+ex)d}{e}+ae}d\sqrt{d+ex}}{e\left(cd^2-ae^2\right)}+\frac{2}{\sqrt{d+ex}\left(cd^2-ae^2\right)}\right)}{cd^2-ae^2}+\frac{2}{3(d+ex)^{3/2}\left(cd^2-ae^2\right)}\right)}{cd^2-ae^2}+\frac{2}{5(d+ex)^{5/2}\left(cd^2-ae^2\right)}\right)}{2(cd^2-ae^2)}-\frac{}{(d+ex)^{5/2}(}\right.}{4\left(cd^2-ae^2\right)}$$

$$\frac{1}{2(d+ex)^{5/2}\left(cd^2-ae^2\right)(ae+cdx)^2}$$

$$\Big\downarrow \; 221$$

$$-\frac{9e\left(-\frac{7e\left(\frac{cd\left(\frac{cd\left(\frac{2}{\sqrt{d+ex}\left(cd^2-ae^2\right)}-\frac{2\sqrt{c}\sqrt{d}\operatorname{arctanh}\left(\frac{\sqrt{c}\sqrt{d}\sqrt{d+ex}}{\sqrt{cd^2-ae^2}}\right)}{\left(cd^2-ae^2\right)^{3/2}}\right)}{cd^2-ae^2}+\frac{2}{3(d+ex)^{3/2}\left(cd^2-ae^2\right)}\right)}{cd^2-ae^2}+\frac{2}{5(d+ex)^{5/2}\left(cd^2-ae^2\right)}\right)}{2(cd^2-ae^2)}-\frac{\cdots}{(d+ex)^{5/2}(c\cdots}\cdots\right)}{4\left(cd^2-ae^2\right)}$$

$$\frac{1}{2(d+ex)^{5/2}\left(cd^2-ae^2\right)(ae+cdx)^2}$$

input
```
Int[1/(Sqrt[d + e*x]*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^3),x]
```

output
```
-1/2*1/((c*d^2 - a*e^2)*(a*e + c*d*x)^2*(d + e*x)^(5/2)) - (9*e*(-(1/((c*d
^2 - a*e^2)*(a*e + c*d*x)*(d + e*x)^(5/2))) - (7*e*(2/(5*(c*d^2 - a*e^2)*(
d + e*x)^(5/2)) + (c*d*(2/(3*(c*d^2 - a*e^2)*(d + e*x)^(3/2)) + (c*d*(2/((
c*d^2 - a*e^2)*Sqrt[d + e*x]) - (2*Sqrt[c]*Sqrt[d]*ArcTanh[(Sqrt[c]*Sqrt[d
]*Sqrt[d + e*x])/Sqrt[c*d^2 - a*e^2]])/(c*d^2 - a*e^2)^(3/2)))/(c*d^2 - a*
e^2)))/(c*d^2 - a*e^2)))/(2*(c*d^2 - a*e^2))))/(4*(c*d^2 - a*e^2))
```

Defintions of rubi rules used

rule 52
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && ILtQ[m, -1] && FractionQ[n] && LtQ[n, 0]
```

rule 61
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && LtQ[m, -1] &&  !(LtQ[n, -1] && (EqQ[a, 0
] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && IntLinearQ[a, b, c, d
, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 1121
```
Int[((d_) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Int[ExpandIntegrand[(d + e*x)^(m + p)*(a/d + (c/e)*x)^p, x], x]
/; FreeQ[{a, b, c, d, e, m, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && (Int
egerQ[p] || (GtQ[a, 0] && GtQ[d, 0] && LtQ[c, 0]))
```

Maple [A] (verified)

Time = 2.02 (sec) , antiderivative size = 209, normalized size of antiderivative = 0.86

method	result
derivativedivides	$2e^2\left(-\frac{1}{5(a\,e^2-c\,d^2)^3(ex+d)^{\frac{5}{2}}}-\frac{6c^2d^2}{(a\,e^2-c\,d^2)^5\sqrt{ex+d}}+\frac{cd}{(a\,e^2-c\,d^2)^4(ex+d)^{\frac{3}{2}}}-\frac{c^3d^3\left(\frac{\frac{15cd(ex+d)^{\frac{3}{2}}}{8}+\left(\frac{17a\,e^2}{8}\cdots\right.}{\left(cd(ex+d)+a\,e^{\cdots}\right.}\cdots\right.}{\cdots}\right.$
default	$2e^2\left(-\frac{1}{5(a\,e^2-c\,d^2)^3(ex+d)^{\frac{5}{2}}}-\frac{6c^2d^2}{(a\,e^2-c\,d^2)^5\sqrt{ex+d}}+\frac{cd}{(a\,e^2-c\,d^2)^4(ex+d)^{\frac{3}{2}}}-\frac{c^3d^3\left(\frac{\frac{15cd(ex+d)^{\frac{3}{2}}}{8}+\left(\frac{17a\,e^2}{8}\cdots\right.}{\left(cd(ex+d)+a\,e^{\cdots}\right.}\cdots\right.}{\cdots}\right.$
pseudoelliptic	$-\frac{2\left(\frac{315c^3d^3e^2(ex+d)^{\frac{5}{2}}(cdx+ae)^2\arctan\left(\frac{cd\sqrt{ex+d}}{\sqrt{cd\left(a\,e^2-c\,d^2\right)}}\right)}{8}+\left(-\frac{5\left(-\frac{63}{2}e^4x^4-\frac{147}{2}d\,e^3x^3-\frac{483}{10}d^2e^2x^2-\frac{9}{2}d^3ex+d^4\right)d^4c^4}{4}+\cdots\right.\right.}{5(ex+d)^{\frac{5}{2}}\sqrt{cd(a\,e\cdots}}$

input
```
int(1/(e*x+d)^(1/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^3,x,method=_RETURNVE
RBOSE)
```

output
```
2*e^2*(-1/5/(a*e^2-c*d^2)^3/(e*x+d)^(5/2)-6/(a*e^2-c*d^2)^5*c^2*d^2/(e*x+d
)^(1/2)+1/(a*e^2-c*d^2)^4*c*d/(e*x+d)^(3/2)-1/(a*e^2-c*d^2)^5*c^3*d^3*((15
/8*c*d*(e*x+d)^(3/2)+(17/8*a*e^2-17/8*c*d^2)*(e*x+d)^(1/2))/(c*d*(e*x+d)+a
*e^2-c*d^2)^2+63/8/(c*d*(a*e^2-c*d^2))^(1/2)*arctan(c*d*(e*x+d)^(1/2)/(c*d
*(a*e^2-c*d^2))^(1/2))))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 969 vs. $2(208) = 416$.

Time = 0.37 (sec) , antiderivative size = 1980, normalized size of antiderivative = 8.11

$$\int \frac{1}{\sqrt{d+ex}\,(ade+(cd^2+ae^2)\,x+cdex^2)^3}\,dx = \text{Too large to display}$$

input
```
integrate(1/(e*x+d)^(1/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^3,x, algorithm
="fricas")
```

output
```
[-1/40*(315*(c^4*d^4*e^5*x^5 + a^2*c^2*d^5*e^4 + (3*c^4*d^5*e^4 + 2*a*c^3*
d^3*e^6)*x^4 + (3*c^4*d^6*e^3 + 6*a*c^3*d^4*e^5 + a^2*c^2*d^2*e^7)*x^3 + (
c^4*d^7*e^2 + 6*a*c^3*d^5*e^4 + 3*a^2*c^2*d^3*e^6)*x^2 + (2*a*c^3*d^6*e^3
+ 3*a^2*c^2*d^4*e^5)*x)*sqrt(c*d/(c*d^2 - a*e^2))*log((c*d*e*x + 2*c*d^2 -
 a*e^2 + 2*(c*d^2 - a*e^2)*sqrt(e*x + d)*sqrt(c*d/(c*d^2 - a*e^2)))/(c*d*x
 + a*e)) - 2*(315*c^4*d^4*e^4*x^4 - 10*c^4*d^8 + 85*a*c^3*d^6*e^2 + 288*a^
2*c^2*d^4*e^4 - 56*a^3*c*d^2*e^6 + 8*a^4*e^8 + 105*(7*c^4*d^5*e^3 + 5*a*c^
3*d^3*e^5)*x^3 + 21*(23*c^4*d^6*e^2 + 59*a*c^3*d^4*e^4 + 8*a^2*c^2*d^2*e^6
)*x^2 + 3*(15*c^4*d^7*e + 277*a*c^3*d^5*e^3 + 136*a^2*c^2*d^3*e^5 - 8*a^3*
c*d*e^7)*x)*sqrt(e*x + d))/(a^2*c^5*d^13*e^2 - 5*a^3*c^4*d^11*e^4 + 10*a^4
*c^3*d^9*e^6 - 10*a^5*c^2*d^7*e^8 + 5*a^6*c*d^5*e^10 - a^7*d^3*e^12 + (c^7
*d^12*e^3 - 5*a*c^6*d^10*e^5 + 10*a^2*c^5*d^8*e^7 - 10*a^3*c^4*d^6*e^9 + 5
*a^4*c^3*d^4*e^11 - a^5*c^2*d^2*e^13)*x^5 + (3*c^7*d^13*e^2 - 13*a*c^6*d^1
1*e^4 + 20*a^2*c^5*d^9*e^6 - 10*a^3*c^4*d^7*e^8 - 5*a^4*c^3*d^5*e^10 + 7*a
^5*c^2*d^3*e^12 - 2*a^6*c*d*e^14)*x^4 + (3*c^7*d^14*e - 9*a*c^6*d^12*e^3 +
 a^2*c^5*d^10*e^5 + 25*a^3*c^4*d^8*e^7 - 35*a^4*c^3*d^6*e^9 + 17*a^5*c^2*d
^4*e^11 - a^6*c*d^2*e^13 - a^7*e^15)*x^3 + (c^7*d^15 + a*c^6*d^13*e^2 - 17
*a^2*c^5*d^11*e^4 + 35*a^3*c^4*d^9*e^6 - 25*a^4*c^3*d^7*e^8 - a^5*c^2*d^5*
e^10 + 9*a^6*c*d^3*e^12 - 3*a^7*d*e^14)*x^2 + (2*a*c^6*d^14*e - 7*a^2*c^5*
d^12*e^3 + 5*a^3*c^4*d^10*e^5 + 10*a^4*c^3*d^8*e^7 - 20*a^5*c^2*d^6*e^9...
```

Sympy [F]

$$\int \frac{1}{\sqrt{d+ex}\,(ade+(cd^2+ae^2)\,x+cdex^2)^3}\,dx = \int \frac{1}{(d+ex)^{\frac{7}{2}}\,(ae+cdx)^3}\,dx$$

input
```
integrate(1/(e*x+d)**(1/2)/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**3,x)
```

output
```
Integral(1/((d + e*x)**(7/2)*(a*e + c*d*x)**3), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{\sqrt{d+ex}\,(ade+(cd^2+ae^2)\,x+cdex^2)^3}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(1/(e*x+d)^(1/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^3,x, algorithm
="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(a*e^2-c*d^2>0)', see `assume?` f
or more de
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 433 vs. $2(208) = 416$.

Time = 0.19 (sec) , antiderivative size = 433, normalized size of antiderivative = 1.77

$$\int \frac{1}{\sqrt{d+ex}\,(ade+(cd^2+ae^2)\,x+cdex^2)^3}\,dx$$
$$= \frac{63\,c^3d^3e^2\arctan\left(\frac{\sqrt{ex+d}cd}{\sqrt{-c^2d^3+acde^2}}\right)}{4\,(c^5d^{10}-5\,ac^4d^8e^2+10\,a^2c^3d^6e^4-10\,a^3c^2d^4e^6+5\,a^4cd^2e^8-a^5e^{10})\sqrt{-c^2d^3+acde^2}}$$
$$+\frac{15\,(ex+d)^{\frac{3}{2}}c^4d^4e^2-17\,\sqrt{ex+d}c^4d^5e^2+17\,\sqrt{ex+d}ac^3d^3e^4}{4\,(c^5d^{10}-5\,ac^4d^8e^2+10\,a^2c^3d^6e^4-10\,a^3c^2d^4e^6+5\,a^4cd^2e^8-a^5e^{10})((ex+d)cd-cd^2+ae^2)^2}$$
$$+\frac{2\,(30\,(ex+d)^2c^2d^2e^2+5\,(ex+d)c^2d^3e^2+c^2d^4e^2-5\,(ex+d)acde^4-2\,acd^2e^4+a^2e^6)}{5\,(c^5d^{10}-5\,ac^4d^8e^2+10\,a^2c^3d^6e^4-10\,a^3c^2d^4e^6+5\,a^4cd^2e^8-a^5e^{10})(ex+d)^{\frac{5}{2}}}$$

input
```
integrate(1/(e*x+d)^(1/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^3,x, algorithm
="giac")
```

output
```
63/4*c^3*d^3*e^2*arctan(sqrt(e*x + d)*c*d/sqrt(-c^2*d^3 + a*c*d*e^2))/((c^
5*d^10 - 5*a*c^4*d^8*e^2 + 10*a^2*c^3*d^6*e^4 - 10*a^3*c^2*d^4*e^6 + 5*a^4
*c*d^2*e^8 - a^5*e^10)*sqrt(-c^2*d^3 + a*c*d*e^2)) + 1/4*(15*(e*x + d)^(3/
2)*c^4*d^4*e^2 - 17*sqrt(e*x + d)*c^4*d^5*e^2 + 17*sqrt(e*x + d)*a*c^3*d^3
*e^4)/((c^5*d^10 - 5*a*c^4*d^8*e^2 + 10*a^2*c^3*d^6*e^4 - 10*a^3*c^2*d^4*e
^6 + 5*a^4*c*d^2*e^8 - a^5*e^10)*((e*x + d)*c*d - c*d^2 + a*e^2)^2) + 2/5*
(30*(e*x + d)^2*c^2*d^2*e^2 + 5*(e*x + d)*c^2*d^3*e^2 + c^2*d^4*e^2 - 5*(e
*x + d)*a*c*d*e^4 - 2*a*c*d^2*e^4 + a^2*e^6)/((c^5*d^10 - 5*a*c^4*d^8*e^2
+ 10*a^2*c^3*d^6*e^4 - 10*a^3*c^2*d^4*e^6 + 5*a^4*c*d^2*e^8 - a^5*e^10)*(e
*x + d)^(5/2))
```

Mupad [B] (verification not implemented)

Time = 5.41 (sec) , antiderivative size = 344, normalized size of antiderivative = 1.41

$$\int \frac{1}{\sqrt{d+ex}\,(ade+(cd^2+ae^2)\,x+cdex^2)^3}\,dx =$$
$$-\frac{\frac{2\,e^2}{5\,(a\,e^2-c\,d^2)}-\frac{6\,c\,d\,e^2\,(d+e\,x)}{5\,{(a\,e^2-c\,d^2)}^2}+\frac{42\,c^2\,d^2\,e^2\,{(d+e\,x)}^2}{5\,{(a\,e^2-c\,d^2)}^3}+\frac{105\,c^3\,d^3\,e^2\,{(d+e\,x)}^3}{4\,{(a\,e^2-c\,d^2)}^4}+\frac{63\,c^4\,d^4\,e^2\,{(d+e\,x)}^4}{4\,{(a\,e^2-c\,d^2)}^5}}{{(d+e\,x)}^{5/2}\,\left(a^2\,e^4-2\,a\,c\,d^2\,e^2+c^2\,d^4\right)-\left(2\,c^2\,d^3-2\,a\,c\,d\,e^2\right)\,{(d+e\,x)}^{7/2}+c^2\,d^2\,{(d+e\,x)}^{9/2}}$$
$$-\frac{63\,c^{5/2}\,d^{5/2}\,e^2\,\mathrm{atan}\left(\frac{\sqrt{c}\,\sqrt{d}\,\sqrt{d+e\,x}\,\left(a^5\,e^{10}-5\,a^4\,c\,d^2\,e^8+10\,a^3\,c^2\,d^4\,e^6-10\,a^2\,c^3\,d^6\,e^4+5\,a\,c^4\,d^8\,e^2-c^5\,d^{10}\right)}{{(a\,e^2-c\,d^2)}^{11/2}}\right)}{4\,{(a\,e^2-c\,d^2)}^{11/2}}$$

input
```
int(1/((d + e*x)^(1/2)*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^3),x)
```

output
```
- ((2*e^2)/(5*(a*e^2 - c*d^2)) - (6*c*d*e^2*(d + e*x))/(5*(a*e^2 - c*d^2)^
2) + (42*c^2*d^2*e^2*(d + e*x)^2)/(5*(a*e^2 - c*d^2)^3) + (105*c^3*d^3*e^2
*(d + e*x)^3)/(4*(a*e^2 - c*d^2)^4) + (63*c^4*d^4*e^2*(d + e*x)^4)/(4*(a*e
^2 - c*d^2)^5))/((d + e*x)^(5/2)*(a^2*e^4 + c^2*d^4 - 2*a*c*d^2*e^2) - (2*
c^2*d^3 - 2*a*c*d*e^2)*(d + e*x)^(7/2) + c^2*d^2*(d + e*x)^(9/2)) - (63*c^
(5/2)*d^(5/2)*e^2*atan((c^(1/2)*d^(1/2)*(d + e*x)^(1/2)*(a^5*e^10 - c^5*d^
10 + 5*a*c^4*d^8*e^2 - 5*a^4*c*d^2*e^8 - 10*a^2*c^3*d^6*e^4 + 10*a^3*c^2*d
^4*e^6))/(a*e^2 - c*d^2)^(11/2)))/(4*(a*e^2 - c*d^2)^(11/2))
```

Reduce [B] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 1491, normalized size of antiderivative = 6.11

$$\int \frac{1}{\sqrt{d+ex}\,(ade+(cd^2+ae^2)\,x+cdex^2)^3}\,dx = \text{Too large to display}$$

input
```
int(1/(e*x+d)^(1/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^3,x)
```

output

```
( - 315*sqrt(d)*sqrt(c)*sqrt(d + e*x)*sqrt(a*e**2 - c*d**2)*atan((sqrt(d +
 e*x)*c*d)/(sqrt(d)*sqrt(c)*sqrt(a*e**2 - c*d**2)))*a**2*c**2*d**4*e**4 -
630*sqrt(d)*sqrt(c)*sqrt(d + e*x)*sqrt(a*e**2 - c*d**2)*atan((sqrt(d + e*x
)*c*d)/(sqrt(d)*sqrt(c)*sqrt(a*e**2 - c*d**2)))*a**2*c**2*d**3*e**5*x - 31
5*sqrt(d)*sqrt(c)*sqrt(d + e*x)*sqrt(a*e**2 - c*d**2)*atan((sqrt(d + e*x)*
c*d)/(sqrt(d)*sqrt(c)*sqrt(a*e**2 - c*d**2)))*a**2*c**2*d**2*e**6*x**2 - 6
30*sqrt(d)*sqrt(c)*sqrt(d + e*x)*sqrt(a*e**2 - c*d**2)*atan((sqrt(d + e*x)
*c*d)/(sqrt(d)*sqrt(c)*sqrt(a*e**2 - c*d**2)))*a*c**3*d**5*e**3*x - 1260*s
qrt(d)*sqrt(c)*sqrt(d + e*x)*sqrt(a*e**2 - c*d**2)*atan((sqrt(d + e*x)*c*d
)/(sqrt(d)*sqrt(c)*sqrt(a*e**2 - c*d**2)))*a*c**3*d**4*e**4*x**2 - 630*sqr
t(d)*sqrt(c)*sqrt(d + e*x)*sqrt(a*e**2 - c*d**2)*atan((sqrt(d + e*x)*c*d)/
(sqrt(d)*sqrt(c)*sqrt(a*e**2 - c*d**2)))*a*c**3*d**3*e**5*x**3 - 315*sqrt(
d)*sqrt(c)*sqrt(d + e*x)*sqrt(a*e**2 - c*d**2)*atan((sqrt(d + e*x)*c*d)/(s
qrt(d)*sqrt(c)*sqrt(a*e**2 - c*d**2)))*c**4*d**6*e**2*x**2 - 630*sqrt(d)*s
qrt(c)*sqrt(d + e*x)*sqrt(a*e**2 - c*d**2)*atan((sqrt(d + e*x)*c*d)/(sqrt(
d)*sqrt(c)*sqrt(a*e**2 - c*d**2)))*c**4*d**5*e**3*x**3 - 315*sqrt(d)*sqrt(
c)*sqrt(d + e*x)*sqrt(a*e**2 - c*d**2)*atan((sqrt(d + e*x)*c*d)/(sqrt(d)*s
qrt(c)*sqrt(a*e**2 - c*d**2)))*c**4*d**4*e**4*x**4 - 8*a**5*e**10 + 64*a**
4*c*d**2*e**8 + 24*a**4*c*d*e**9*x - 344*a**3*c**2*d**4*e**6 - 432*a**3*c*
*2*d**3*e**7*x - 168*a**3*c**2*d**2*e**8*x**2 + 203*a**2*c**3*d**6*e**4...
```

3.200 $\int (d+ex)^3 \sqrt{ade+(cd^2+ae^2)x+cdex^2}\,dx$

Optimal result

Integrand size = 37, antiderivative size = 358

$$\begin{aligned}
&\int (d+ex)^3\sqrt{ade+(cd^2+ae^2)x+cdex^2}\,dx\\
&=\frac{7(cd^2-ae^2)^4\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{128c^4d^4e}\\
&\quad+\frac{7(cd^2-ae^2)^2(ade+(cd^2+ae^2)x+cdex^2)^{3/2}}{48c^3d^3}\\
&\quad+\frac{7(cd^2-ae^2)^3(ade+(cd^2+ae^2)x+cdex^2)^{3/2}}{64c^4d^4(d+ex)}\\
&\quad+\frac{7(cd^2-ae^2)(d+ex)(ade+(cd^2+ae^2)x+cdex^2)^{3/2}}{40c^2d^2}\\
&\quad+\frac{(d+ex)^2(ade+(cd^2+ae^2)x+cdex^2)^{3/2}}{5cd}\\
&\quad-\frac{7(cd^2-ae^2)^5\operatorname{arctanh}\left(\frac{\sqrt{e}\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{\sqrt{c}\sqrt{d}(d+ex)}\right)}{128c^{9/2}d^{9/2}e^{3/2}}
\end{aligned}$$

output
```
7/128*(-a*e^2+c*d^2)^4*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/c^4/d^4/e+7
/48*(-a*e^2+c*d^2)^2*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/c^3/d^3+7/64*
(-a*e^2+c*d^2)^3*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/c^4/d^4/(e*x+d)+7
/40*(-a*e^2+c*d^2)*(e*x+d)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/c^2/d^2
+1/5*(e*x+d)^2*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/c/d-7/128*(-a*e^2+c
*d^2)^5*arctanh(e^(1/2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/c^(1/2)/d^
(1/2)/(e*x+d))/c^(9/2)/d^(9/2)/e^(3/2)
```

Mathematica [A] (verified)

Time = 0.77 (sec) , antiderivative size = 278, normalized size of antiderivative = 0.78

$$\int (d+ex)^3\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}\,dx$$
$$= \frac{\sqrt{(ae+cdx)(d+ex)}\left(\sqrt{c}\sqrt{d}\sqrt{e}(-105a^4e^8+70a^3cde^6(7d+ex)-14a^2c^2d^2e^4(64d^2+23dex+4e^2x^2)+\right.}{}$$

input
```
Integrate[(d + e*x)^3*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2],x]
```

output
```
(Sqrt[(a*e + c*d*x)*(d + e*x)]*(Sqrt[c]*Sqrt[d]*Sqrt[e]*(-105*a^4*e^8 + 70
*a^3*c*d*e^6*(7*d + e*x) - 14*a^2*c^2*d^2*e^4*(64*d^2 + 23*d*e*x + 4*e^2*x
^2) + 2*a*c^3*d^3*e^2*(395*d^3 + 289*d^2*e*x + 128*d*e^2*x^2 + 24*e^3*x^3)
 + c^4*d^4*(105*d^4 + 1210*d^3*e*x + 2104*d^2*e^2*x^2 + 1488*d*e^3*x^3 + 3
84*e^4*x^4)) - (105*(c*d^2 - a*e^2)^5*ArcTanh[(Sqrt[c]*Sqrt[d]*Sqrt[d + e*
x])/(Sqrt[e]*Sqrt[a*e + c*d*x])])/(Sqrt[a*e + c*d*x]*Sqrt[d + e*x])))/(192
0*c^(9/2)*d^(9/2)*e^(3/2))
```

Rubi [A] (verified)

Time = 0.87 (sec) , antiderivative size = 351, normalized size of antiderivative = 0.98, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.162$, Rules used = {1134, 1134, 1160, 1087, 1092, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (d+ex)^3 \sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}\,dx$$

$$\Big\downarrow \text{1134}$$

$$\frac{7\left(d^2-\frac{ae^2}{c}\right)\int (d+ex)^2\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}dx}{10d}+\frac{(d+ex)^2\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{5cd}$$

$$\Big\downarrow \text{1134}$$

$$\frac{7\left(d^2-\frac{ae^2}{c}\right)\left(\frac{5\left(d^2-\frac{ae^2}{c}\right)\int (d+ex)\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}dx}{8d}+\frac{(d+ex)\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{4cd}\right)}{10d}+\frac{(d+ex)^2\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{5cd}$$

$$\Big\downarrow \text{1160}$$

$$\frac{7\left(d^2-\frac{ae^2}{c}\right)\left(\frac{5\left(d^2-\frac{ae^2}{c}\right)\left(\frac{\left(d^2-\frac{ae^2}{c}\right)\int \sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}dx}{2d}+\frac{\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{3cd}\right)}{8d}+\frac{(d+ex)\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{4cd}\right)}{10d}+\frac{(d+ex)^2\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{5cd}$$

$$\Big\downarrow \text{1087}$$

$$\frac{7\left(d^2-\frac{ae^2}{c}\right)\left(\frac{5\left(d^2-\frac{ae^2}{c}\right)\left(\frac{\left(d^2-\frac{ae^2}{c}\right)\left(\frac{\left(ae^2+cd^2+2cdex\right)\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{4cde}-\frac{\left(cd^2-ae^2\right)^2\int\frac{1}{\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}dx}{8cde}\right)}{2d}+\frac{\left(x\left(ae^2+\right.\right.}{}\right)}{8d}}{10d}$$

$$\frac{(d+ex)^2\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{5cd}$$

$\downarrow$ 1092

$$\frac{7\left(d^2-\frac{ae^2}{c}\right)\left(\frac{5\left(d^2-\frac{ae^2}{c}\right)\left(\frac{\left(d^2-\frac{ae^2}{c}\right)\left(\frac{\left(ae^2+cd^2+2cdex\right)\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{4cde}-\frac{\left(cd^2-ae^2\right)^2\int\frac{1}{4cde-\frac{\left(cd^2+2cexd+ae^2\right)^2}{cdex^2+\left(cd^2+ae^2\right)x+ade}}d\frac{cd^2-}{\sqrt{cdex^2+}}}{4cde}\right.}{2d}}{8d}\right.}{10d}$$

$$\frac{(d+ex)^2\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{5cd}$$

$\downarrow$ 219

$$\frac{7\left(d^2-\frac{ae^2}{c}\right)\left(\frac{5\left(d^2-\frac{ae^2}{c}\right)\left(\frac{\left(d^2-\frac{ae^2}{c}\right)\left(\frac{\left(ae^2+cd^2+2cdex\right)\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{4cde}-\frac{\left(cd^2-ae^2\right)^2\text{arctanh}\left(\frac{ae^2+cd^2+2cdex}{2\sqrt{c}\sqrt{d}\sqrt{e}\sqrt{x\left(ae^2+cd^2\right)+ade+cd}}\right.}{8c^{3/2}d^{3/2}e^{3/2}}\right.}{2d}}{8d}}{10d}$$

$$\frac{(d+ex)^2\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{5cd}$$

input
```
Int[(d + e*x)^3*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2],x]
```

output
```
((d + e*x)^2*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2))/(5*c*d) + (7*(
d^2 - (a*e^2)/c)*(((d + e*x)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2)
)/(4*c*d) + (5*(d^2 - (a*e^2)/c)*((a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^
(3/2)/(3*c*d) + ((d^2 - (a*e^2)/c)*(((c*d^2 + a*e^2 + 2*c*d*e*x)*Sqrt[a*d*
e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(4*c*d*e) - ((c*d^2 - a*e^2)^2*ArcTanh
[(c*d^2 + a*e^2 + 2*c*d*e*x)/(2*Sqrt[c]*Sqrt[d]*Sqrt[e]*Sqrt[a*d*e + (c*d^
2 + a*e^2)*x + c*d*e*x^2])])/(8*c^(3/2)*d^(3/2)*e^(3/2))))/(2*d)))/(8*d)))
/(10*d)
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1087
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^p/(2*c*(2*p + 1))), x] - Simp[p*((b^2 - 4*a*c)/(2*c*(2*
p + 1)))   Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] &&
GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[3*p])
```

rule 1092
```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[I
nt[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a
, b, c}, x]
```

rule 1134
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[e*(d + e*x)^(m - 1)*((a + b*x + c*x^2)^(p + 1)/(c*(m + 2*p +
 1))), x] + Simp[(m + p)*((2*c*d - b*e)/(c*(m + 2*p + 1)))   Int[(d + e*x)^
(m - 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[
c*d^2 - b*d*e + a*e^2, 0] && GtQ[m, 1] && NeQ[m + 2*p + 1, 0] && IntegerQ[2
*p]
```

rule 1160
```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[e*((a + b*x + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Simp[(2*c*d - b
*e)/(2*c)   Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
 && NeQ[p, -1]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1542 vs. $2(322) = 644$.

Time = 1.95 (sec) , antiderivative size = 1543, normalized size of antiderivative = 4.31

method	result	size
default	Expression too large to display	1543

input
```
int((e*x+d)^3*(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(1/2),x,method=_RETURNVERB
OSE)
```

output
```
d^3*(1/4*(2*c*d*e*x+a*e^2+c*d^2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(1/2)/c
/d/e+1/8*(4*a*c*d^2*e^2-(a*e^2+c*d^2)^2)/d/e/c*ln((1/2*a*e^2+1/2*c*d^2+c*d
*x*e)/(d*e*c)^(1/2)+(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(1/2))/(d*e*c)^(1/2)
)+e^3*(1/5*x^2*(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(3/2)/d/e/c-7/10*(a*e^2+c
*d^2)/d/e/c*(1/4*x*(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(3/2)/d/e/c-5/8*(a*e^
2+c*d^2)/d/e/c*(1/3*(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(3/2)/d/e/c-1/2*(a*e
^2+c*d^2)/d/e/c*(1/4*(2*c*d*e*x+a*e^2+c*d^2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*x^
2*e)^(1/2)/c/d/e+1/8*(4*a*c*d^2*e^2-(a*e^2+c*d^2)^2)/d/e/c*ln((1/2*a*e^2+1
/2*c*d^2+c*d*x*e)/(d*e*c)^(1/2)+(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(1/2))/(
d*e*c)^(1/2)))-1/4*a/c*(1/4*(2*c*d*e*x+a*e^2+c*d^2)*(a*d*e+(a*e^2+c*d^2)*x
+c*d*x^2*e)^(1/2)/c/d/e+1/8*(4*a*c*d^2*e^2-(a*e^2+c*d^2)^2)/d/e/c*ln((1/2*
a*e^2+1/2*c*d^2+c*d*x*e)/(d*e*c)^(1/2)+(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(
1/2))/(d*e*c)^(1/2)))-2/5*a/c*(1/3*(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(3/2)
/d/e/c-1/2*(a*e^2+c*d^2)/d/e/c*(1/4*(2*c*d*e*x+a*e^2+c*d^2)*(a*d*e+(a*e^2+
c*d^2)*x+c*d*x^2*e)^(1/2)/c/d/e+1/8*(4*a*c*d^2*e^2-(a*e^2+c*d^2)^2)/d/e/c*
ln((1/2*a*e^2+1/2*c*d^2+c*d*x*e)/(d*e*c)^(1/2)+(a*d*e+(a*e^2+c*d^2)*x+c*d*
x^2*e)^(1/2))/(d*e*c)^(1/2))))+3*d*e^2*(1/4*x*(a*d*e+(a*e^2+c*d^2)*x+c*d*x
^2*e)^(3/2)/d/e/c-5/8*(a*e^2+c*d^2)/d/e/c*(1/3*(a*d*e+(a*e^2+c*d^2)*x+c*d*
x^2*e)^(3/2)/d/e/c-1/2*(a*e^2+c*d^2)/d/e/c*(1/4*(2*c*d*e*x+a*e^2+c*d^2)*(a
*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(1/2)/c/d/e+1/8*(4*a*c*d^2*e^2-(a*e^2+c...
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 844, normalized size of antiderivative = 2.36

$$\int (d+ex)^3\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}\,dx = \text{Too large to display}$$

input
```
integrate((e*x+d)^3*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x, algorithm="
fricas")
```

output
```
[-1/7680*(105*(c^5*d^10 - 5*a*c^4*d^8*e^2 + 10*a^2*c^3*d^6*e^4 - 10*a^3*c^
2*d^4*e^6 + 5*a^4*c*d^2*e^8 - a^5*e^10)*sqrt(c*d*e)*log(8*c^2*d^2*e^2*x^2
+ c^2*d^4 + 6*a*c*d^2*e^2 + a^2*e^4 + 4*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 +
a*e^2)*x)*(2*c*d*e*x + c*d^2 + a*e^2)*sqrt(c*d*e) + 8*(c^2*d^3*e + a*c*d*e
^3)*x) - 4*(384*c^5*d^5*e^5*x^4 + 105*c^5*d^9*e + 790*a*c^4*d^7*e^3 - 896*
a^2*c^3*d^5*e^5 + 490*a^3*c^2*d^3*e^7 - 105*a^4*c*d*e^9 + 48*(31*c^5*d^6*e
^4 + a*c^4*d^4*e^6)*x^3 + 8*(263*c^5*d^7*e^3 + 32*a*c^4*d^5*e^5 - 7*a^2*c^
3*d^3*e^7)*x^2 + 2*(605*c^5*d^8*e^2 + 289*a*c^4*d^6*e^4 - 161*a^2*c^3*d^4*
e^6 + 35*a^3*c^2*d^2*e^8)*x)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x))/
(c^5*d^5*e^2), 1/3840*(105*(c^5*d^10 - 5*a*c^4*d^8*e^2 + 10*a^2*c^3*d^6*e^
4 - 10*a^3*c^2*d^4*e^6 + 5*a^4*c*d^2*e^8 - a^5*e^10)*sqrt(-c*d*e)*arctan(1
/2*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(2*c*d*e*x + c*d^2 + a*e^2)
*sqrt(-c*d*e)/(c^2*d^2*e^2*x^2 + a*c*d^2*e^2 + (c^2*d^3*e + a*c*d*e^3)*x))
 + 2*(384*c^5*d^5*e^5*x^4 + 105*c^5*d^9*e + 790*a*c^4*d^7*e^3 - 896*a^2*c^
3*d^5*e^5 + 490*a^3*c^2*d^3*e^7 - 105*a^4*c*d*e^9 + 48*(31*c^5*d^6*e^4 + a
*c^4*d^4*e^6)*x^3 + 8*(263*c^5*d^7*e^3 + 32*a*c^4*d^5*e^5 - 7*a^2*c^3*d^3*
e^7)*x^2 + 2*(605*c^5*d^8*e^2 + 289*a*c^4*d^6*e^4 - 161*a^2*c^3*d^4*e^6 +
35*a^3*c^2*d^2*e^8)*x)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x))/(c^5*d
^5*e^2)]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1646 vs. $2(338) = 676$.

Time = 0.95 (sec) , antiderivative size = 1646, normalized size of antiderivative = 4.60

$$\int (d+ex)^3\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}\,dx = \text{Too large to display}$$

input
```
integrate((e*x+d)**3*(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(1/2),x)
```

output
```
Piecewise((sqrt(a*d*e + c*d*e*x**2 + x*(a*e**2 + c*d**2))*(e**3*x**4/5 + x
**3*(a*e**5 + 4*c*d**2*e**3 - e**3*(9*a*e**2/2 + 9*c*d**2/2)/5)/(4*c*d*e)
+ x**2*(16*a*d*e**4/5 + 6*c*d**3*e**2 - (7*a*e**2/2 + 7*c*d**2/2)*(a*e**5
+ 4*c*d**2*e**3 - e**3*(9*a*e**2/2 + 9*c*d**2/2)/5)/(4*c*d*e))/(3*c*d*e) +
 x*(6*a*d**2*e**3 - 3*a*(a*e**5 + 4*c*d**2*e**3 - e**3*(9*a*e**2/2 + 9*c*d
**2/2)/5)/(4*c) + 4*c*d**4*e - (5*a*e**2/2 + 5*c*d**2/2)*(16*a*d*e**4/5 +
6*c*d**3*e**2 - (7*a*e**2/2 + 7*c*d**2/2)*(a*e**5 + 4*c*d**2*e**3 - e**3*(
9*a*e**2/2 + 9*c*d**2/2)/5)/(4*c*d*e))/(3*c*d*e))/(2*c*d*e) + (4*a*d**3*e*
*2 - 2*a*(16*a*d*e**4/5 + 6*c*d**3*e**2 - (7*a*e**2/2 + 7*c*d**2/2)*(a*e**
5 + 4*c*d**2*e**3 - e**3*(9*a*e**2/2 + 9*c*d**2/2)/5)/(4*c*d*e))/(3*c) + c
*d**5 - (3*a*e**2/2 + 3*c*d**2/2)*(6*a*d**2*e**3 - 3*a*(a*e**5 + 4*c*d**2*
e**3 - e**3*(9*a*e**2/2 + 9*c*d**2/2)/5)/(4*c) + 4*c*d**4*e - (5*a*e**2/2
+ 5*c*d**2/2)*(16*a*d*e**4/5 + 6*c*d**3*e**2 - (7*a*e**2/2 + 7*c*d**2/2)*(
a*e**5 + 4*c*d**2*e**3 - e**3*(9*a*e**2/2 + 9*c*d**2/2)/5)/(4*c*d*e))/(3*c
*d*e))/(2*c*d*e))/(c*d*e)) + (a*d**4*e - a*(6*a*d**2*e**3 - 3*a*(a*e**5 +
4*c*d**2*e**3 - e**3*(9*a*e**2/2 + 9*c*d**2/2)/5)/(4*c) + 4*c*d**4*e - (5*
a*e**2/2 + 5*c*d**2/2)*(16*a*d*e**4/5 + 6*c*d**3*e**2 - (7*a*e**2/2 + 7*c*
d**2/2)*(a*e**5 + 4*c*d**2*e**3 - e**3*(9*a*e**2/2 + 9*c*d**2/2)/5)/(4*c*d
*e))/(3*c*d*e))/(2*c) - (a*e**2 + c*d**2)*(4*a*d**3*e**2 - 2*a*(16*a*d*e**
4/5 + 6*c*d**3*e**2 - (7*a*e**2/2 + 7*c*d**2/2)*(a*e**5 + 4*c*d**2*e**3...
```

Maxima [F(-2)]

Exception generated.

$$\int (d+ex)^3\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((e*x+d)^3*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x, algorithm="
maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

Giac [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 400, normalized size of antiderivative = 1.12

$$\int (d+ex)^3\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}\,dx$$
$$=\frac{1}{1920}\sqrt{cdex^2+cd^2x+ae^2x+ade}\left(2\left(4\left(6\left(8\,e^3x+\frac{31\,c^4d^5e^6+ac^3d^3e^8}{c^4d^4e^4}\right)x+\frac{263\,c^4d^6e^5+32\,ac^3d^4e^7}{c^4d^4e^4}\right.\right.\right.$$
$$+\frac{7\,(c^5d^{10}-5\,ac^4d^8e^2+10\,a^2c^3d^6e^4-10\,a^3c^2d^4e^6+5\,a^4cd^2e^8-a^5e^{10})\log\left(\left|-cd^2-ae^2-2\,\sqrt{cde}\left(\sqrt{cd}\right.\right.\right.}{256\,\sqrt{cde}c^4d^4e}$$

input
```
integrate((e*x+d)^3*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x, algorithm="
giac")
```

output
```
1/1920*sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*x + a*d*e)*(2*(4*(6*(8*e^3*x + (31
*c^4*d^5*e^6 + a*c^3*d^3*e^8)/(c^4*d^4*e^4))*x + (263*c^4*d^6*e^5 + 32*a*c
^3*d^4*e^7 - 7*a^2*c^2*d^2*e^9)/(c^4*d^4*e^4))*x + (605*c^4*d^7*e^4 + 289*
a*c^3*d^5*e^6 - 161*a^2*c^2*d^3*e^8 + 35*a^3*c*d*e^10)/(c^4*d^4*e^4))*x +
(105*c^4*d^8*e^3 + 790*a*c^3*d^6*e^5 - 896*a^2*c^2*d^4*e^7 + 490*a^3*c*d^2
*e^9 - 105*a^4*e^11)/(c^4*d^4*e^4)) + 7/256*(c^5*d^10 - 5*a*c^4*d^8*e^2 +
10*a^2*c^3*d^6*e^4 - 10*a^3*c^2*d^4*e^6 + 5*a^4*c*d^2*e^8 - a^5*e^10)*log(
abs(-c*d^2 - a*e^2 - 2*sqrt(c*d*e)*(sqrt(c*d*e)*x - sqrt(c*d*e*x^2 + c*d^2
*x + a*e^2*x + a*d*e))))/(sqrt(c*d*e)*c^4*d^4*e)
```

Mupad [F(-1)]

Timed out.

$$\int (d+ex)^3\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}\,dx$$
$$=\int (d+e\,x)^3\,\sqrt{c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e}\,dx$$

input
```
int((d + e*x)^3*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2),x)
```

output
```
int((d + e*x)^3*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 790, normalized size of antiderivative = 2.21

$$\int (d+ex)^3\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}\,dx=\text{Too large to display}$$

input
```
int((e*x+d)^3*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x)
```

output
```
( - 105*sqrt(d + e*x)*sqrt(a*e + c*d*x)*a**4*c*d*e**9 + 490*sqrt(d + e*x)*
sqrt(a*e + c*d*x)*a**3*c**2*d**3*e**7 + 70*sqrt(d + e*x)*sqrt(a*e + c*d*x)
*a**3*c**2*d**2*e**8*x - 896*sqrt(d + e*x)*sqrt(a*e + c*d*x)*a**2*c**3*d**
5*e**5 - 322*sqrt(d + e*x)*sqrt(a*e + c*d*x)*a**2*c**3*d**4*e**6*x - 56*sq
rt(d + e*x)*sqrt(a*e + c*d*x)*a**2*c**3*d**3*e**7*x**2 + 790*sqrt(d + e*x)
*sqrt(a*e + c*d*x)*a*c**4*d**7*e**3 + 578*sqrt(d + e*x)*sqrt(a*e + c*d*x)*
a*c**4*d**6*e**4*x + 256*sqrt(d + e*x)*sqrt(a*e + c*d*x)*a*c**4*d**5*e**5*
x**2 + 48*sqrt(d + e*x)*sqrt(a*e + c*d*x)*a*c**4*d**4*e**6*x**3 + 105*sqrt
(d + e*x)*sqrt(a*e + c*d*x)*c**5*d**9*e + 1210*sqrt(d + e*x)*sqrt(a*e + c*
d*x)*c**5*d**8*e**2*x + 2104*sqrt(d + e*x)*sqrt(a*e + c*d*x)*c**5*d**7*e**
3*x**2 + 1488*sqrt(d + e*x)*sqrt(a*e + c*d*x)*c**5*d**6*e**4*x**3 + 384*sq
rt(d + e*x)*sqrt(a*e + c*d*x)*c**5*d**5*e**5*x**4 + 105*sqrt(e)*sqrt(d)*sq
rt(c)*log((sqrt(e)*sqrt(a*e + c*d*x) + sqrt(d)*sqrt(c)*sqrt(d + e*x))/sqrt
(a*e**2 - c*d**2))*a**5*e**10 - 525*sqrt(e)*sqrt(d)*sqrt(c)*log((sqrt(e)*s
qrt(a*e + c*d*x) + sqrt(d)*sqrt(c)*sqrt(d + e*x))/sqrt(a*e**2 - c*d**2))*a
**4*c*d**2*e**8 + 1050*sqrt(e)*sqrt(d)*sqrt(c)*log((sqrt(e)*sqrt(a*e + c*d
*x) + sqrt(d)*sqrt(c)*sqrt(d + e*x))/sqrt(a*e**2 - c*d**2))*a**3*c**2*d**4
*e**6 - 1050*sqrt(e)*sqrt(d)*sqrt(c)*log((sqrt(e)*sqrt(a*e + c*d*x) + sqrt
(d)*sqrt(c)*sqrt(d + e*x))/sqrt(a*e**2 - c*d**2))*a**2*c**3*d**6*e**4 + 52
5*sqrt(e)*sqrt(d)*sqrt(c)*log((sqrt(e)*sqrt(a*e + c*d*x) + sqrt(d)*sqrt...
```

3.201 $\int (d+ex)^2 \sqrt{ade+(cd^2+ae^2)x+cdex^2}\,dx$

Optimal result

Integrand size = 37, antiderivative size = 298

$$\begin{aligned}\int (d+ex)^2\sqrt{ade+(cd^2+ae^2)x+cdex^2}\,dx &= \frac{5(cd^2-ae^2)^3\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{64c^3d^3e} \\ &+ \frac{5(cd^2-ae^2)(ade+(cd^2+ae^2)x+cdex^2)^{3/2}}{24c^2d^2} \\ &+ \frac{5(cd^2-ae^2)^2(ade+(cd^2+ae^2)x+cdex^2)^{3/2}}{32c^3d^3(d+ex)} \\ &+ \frac{(d+ex)(ade+(cd^2+ae^2)x+cdex^2)^{3/2}}{4cd} \\ &- \frac{5(cd^2-ae^2)^4\operatorname{arctanh}\left(\frac{\sqrt{e}\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{\sqrt{c}\sqrt{d}(d+ex)}\right)}{64c^{7/2}d^{7/2}e^{3/2}}\end{aligned}$$

output
```
5/64*(-a*e^2+c*d^2)^3*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/c^3/d^3/e+5/
24*(-a*e^2+c*d^2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/c^2/d^2+5/32*(-a
*e^2+c*d^2)^2*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/c^3/d^3/(e*x+d)+1/4*
(e*x+d)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/c/d-5/64*(-a*e^2+c*d^2)^4*
arctanh(e^(1/2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/c^(1/2)/d^(1/2)/(e
*x+d))/c^(7/2)/d^(7/2)/e^(3/2)
```

Mathematica [A] (verified)

Time = 0.53 (sec) , antiderivative size = 223, normalized size of antiderivative = 0.75

$$\int (d+ex)^2\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}\,dx$$
$$=\frac{\sqrt{(ae+cdx)(d+ex)}\left(\sqrt{c}\sqrt{d}\sqrt{e}(15a^3e^6-5a^2cde^4(11d+2ex)+ac^2d^2e^2(73d^2+36dex+8e^2x^2)+c^3d^3\right.}{192c^{7/2}d^{7/2}e^{3/2}}$$

input
```
Integrate[(d + e*x)^2*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2],x]
```

output
```
(Sqrt[(a*e + c*d*x)*(d + e*x)]*(Sqrt[c]*Sqrt[d]*Sqrt[e]*(15*a^3*e^6 - 5*a^
2*c*d*e^4*(11*d + 2*e*x) + a*c^2*d^2*e^2*(73*d^2 + 36*d*e*x + 8*e^2*x^2) +
 c^3*d^3*(15*d^3 + 118*d^2*e*x + 136*d*e^2*x^2 + 48*e^3*x^3)) - (15*(c*d^2
 - a*e^2)^4*ArcTanh[(Sqrt[c]*Sqrt[d]*Sqrt[d + e*x])/(Sqrt[e]*Sqrt[a*e + c*
d*x])])/(Sqrt[a*e + c*d*x]*Sqrt[d + e*x])))/(192*c^(7/2)*d^(7/2)*e^(3/2))
```

Rubi [A] (verified)

Time = 0.68 (sec) , antiderivative size = 284, normalized size of antiderivative = 0.95, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.135$, Rules used = {1134, 1160, 1087, 1092, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (d+ex)^2\sqrt{x\,(ae^2+cd^2)+ade+cdex^2}\,dx$$

$$\Big\downarrow \; 1134$$

$$\frac{5\left(d^2-\frac{ae^2}{c}\right)\int (d+ex)\sqrt{cdex^2+(cd^2+ae^2)\,x+ade}dx}{8d}+\frac{(d+ex)\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{4cd}$$

$$\Big\downarrow \; 1160$$

$$\frac{5\left(d^2-\frac{ae^2}{c}\right)\left(\frac{\left(d^2-\frac{ae^2}{c}\right)\int\sqrt{cdex^2+(cd^2+ae^2)x+ade}dx}{2d}+\frac{\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{3cd}\right)}{8d}+\frac{(d+ex)\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{4cd}$$

$\downarrow$ 1087

$$\frac{5\left(d^2-\frac{ae^2}{c}\right)\left(\frac{\left(d^2-\frac{ae^2}{c}\right)\left(\frac{\left(ae^2+cd^2+2cdex\right)\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{4cde}-\frac{\left(cd^2-ae^2\right)^2\int\frac{1}{\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}dx}{8cde}\right)}{2d}+\frac{\left(x\left(ae^2+cd^2\right)+ade\cdots\right.}{3cd}\right.}{8d}$$

$$\frac{(d+ex)\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{4cd}$$

$\downarrow$ 1092

$$\frac{5\left(d^2-\frac{ae^2}{c}\right)\left(\frac{\left(d^2-\frac{ae^2}{c}\right)\left(\frac{\left(ae^2+cd^2+2cdex\right)\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{4cde}-\frac{\left(cd^2-ae^2\right)^2\int\frac{1}{4cde-\frac{\left(cd^2+2cexd+ae^2\right)^2}{cdex^2+\left(cd^2+ae^2\right)x+ade}}d\frac{cd^2+2cexd+ae^2}{\sqrt{cdex^2+\left(cd^2+ae^2\right)x+a\cdots}}}{4cde}\cdots\right.}{2d}\right.}{8d}$$

$$\frac{(d+ex)\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{4cd}$$

$\downarrow$ 219

$$\frac{5\left(d^2-\frac{ae^2}{c}\right)\left(\frac{\left(d^2-\frac{ae^2}{c}\right)\left(\frac{\left(ae^2+cd^2+2cdex\right)\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{4cde}-\frac{\left(cd^2-ae^2\right)^2\operatorname{arctanh}\left(\frac{ae^2+cd^2+2cdex}{2\sqrt{c}\sqrt{d}\sqrt{e}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}\right)}{8c^{3/2}d^{3/2}e^{3/2}}\right)}{2d}+\cdots\right)}{8d}$$

$$\frac{(d+ex)\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{4cd}$$

input
```
Int[(d + e*x)^2*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2],x]
```

output
```
((d + e*x)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2))/(4*c*d) + (5*(d^
2 - (a*e^2)/c)*((a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2)/(3*c*d) + ((
d^2 - (a*e^2)/c)*(((c*d^2 + a*e^2 + 2*c*d*e*x)*Sqrt[a*d*e + (c*d^2 + a*e^2
)*x + c*d*e*x^2])/(4*c*d*e) - ((c*d^2 - a*e^2)^2*ArcTanh[(c*d^2 + a*e^2 +
2*c*d*e*x)/(2*Sqrt[c]*Sqrt[d]*Sqrt[e]*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d
*e*x^2])])/(8*c^(3/2)*d^(3/2)*e^(3/2))))/(2*d)))/(8*d)
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1087
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^p/(2*c*(2*p + 1))), x] - Simp[p*((b^2 - 4*a*c)/(2*c*(2*
p + 1)))   Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] &&
GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[3*p])
```

rule 1092
```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[I
nt[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a
, b, c}, x]
```

rule 1134
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[e*(d + e*x)^(m - 1)*((a + b*x + c*x^2)^(p + 1)/(c*(m + 2*p +
 1))), x] + Simp[(m + p)*((2*c*d - b*e)/(c*(m + 2*p + 1)))   Int[(d + e*x)^
(m - 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[
c*d^2 - b*d*e + a*e^2, 0] && GtQ[m, 1] && NeQ[m + 2*p + 1, 0] && IntegerQ[2
*p]
```

rule 1160
```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[e*((a + b*x + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Simp[(2*c*d - b
*e)/(2*c)   Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
 && NeQ[p, -1]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 815 vs. $2(266) = 532$.

Time = 1.69 (sec) , antiderivative size = 816, normalized size of antiderivative = 2.74

method	result
default	$d^2\left(\frac{(2cdxe+a\,e^2+c\,d^2)\sqrt{ade+(a\,e^2+c\,d^2)x+cd\,x^2e}}{4cde}+\frac{\left(4ac\,d^2e^2-(a\,e^2+c\,d^2)^2\right)\ln\left(\frac{\frac{1}{2}a\,e^2+\frac{1}{2}c\,d^2+cdxe}{\sqrt{dec}}+\sqrt{ade+(a\,e^2+c\,d^2)x+}\right.}{8dec\sqrt{dec}}\right.$

input
```
int((e*x+d)^2*(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(1/2),x,method=_RETURNVERB
OSE)
```

output
```
d^2*(1/4*(2*c*d*e*x+a*e^2+c*d^2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(1/2)/c
/d/e+1/8*(4*a*c*d^2*e^2-(a*e^2+c*d^2)^2)/d/e/c*ln((1/2*a*e^2+1/2*c*d^2+c*d
*x*e)/(d*e*c)^(1/2)+(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(1/2))/(d*e*c)^(1/2)
)+e^2*(1/4*x*(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(3/2)/d/e/c-5/8*(a*e^2+c*d^
2)/d/e/c*(1/3*(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(3/2)/d/e/c-1/2*(a*e^2+c*d
^2)/d/e/c*(1/4*(2*c*d*e*x+a*e^2+c*d^2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(
1/2)/c/d/e+1/8*(4*a*c*d^2*e^2-(a*e^2+c*d^2)^2)/d/e/c*ln((1/2*a*e^2+1/2*c*d
^2+c*d*x*e)/(d*e*c)^(1/2)+(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(1/2))/(d*e*c)
^(1/2)))-1/4*a/c*(1/4*(2*c*d*e*x+a*e^2+c*d^2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*x
^2*e)^(1/2)/c/d/e+1/8*(4*a*c*d^2*e^2-(a*e^2+c*d^2)^2)/d/e/c*ln((1/2*a*e^2+
1/2*c*d^2+c*d*x*e)/(d*e*c)^(1/2)+(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(1/2))/
(d*e*c)^(1/2)))+2*d*e*(1/3*(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(3/2)/d/e/c-1
/2*(a*e^2+c*d^2)/d/e/c*(1/4*(2*c*d*e*x+a*e^2+c*d^2)*(a*d*e+(a*e^2+c*d^2)*x
+c*d*x^2*e)^(1/2)/c/d/e+1/8*(4*a*c*d^2*e^2-(a*e^2+c*d^2)^2)/d/e/c*ln((1/2*
a*e^2+1/2*c*d^2+c*d*x*e)/(d*e*c)^(1/2)+(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(
1/2))/(d*e*c)^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 672, normalized size of antiderivative = 2.26

$$\int (d+ex)^2\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}\,dx$$
$$= \left[\frac{15\,(c^4d^8-4\,ac^3d^6e^2+6\,a^2c^2d^4e^4-4\,a^3cd^2e^6+a^4e^8)\sqrt{cde}\log\Big(8\,c^2d^2e^2x^2+c^2d^4+6\,acd^2e^2+a^2e^4-\cdots}{}\right.$$

input
```
integrate((e*x+d)^2*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x, algorithm="
fricas")
```

output
```
[1/768*(15*(c^4*d^8 - 4*a*c^3*d^6*e^2 + 6*a^2*c^2*d^4*e^4 - 4*a^3*c*d^2*e^
6 + a^4*e^8)*sqrt(c*d*e)*log(8*c^2*d^2*e^2*x^2 + c^2*d^4 + 6*a*c*d^2*e^2 +
 a^2*e^4 - 4*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(2*c*d*e*x + c*d^
2 + a*e^2)*sqrt(c*d*e) + 8*(c^2*d^3*e + a*c*d*e^3)*x) + 4*(48*c^4*d^4*e^4*
x^3 + 15*c^4*d^7*e + 73*a*c^3*d^5*e^3 - 55*a^2*c^2*d^3*e^5 + 15*a^3*c*d*e^
7 + 8*(17*c^4*d^5*e^3 + a*c^3*d^3*e^5)*x^2 + 2*(59*c^4*d^6*e^2 + 18*a*c^3*
d^4*e^4 - 5*a^2*c^2*d^2*e^6)*x)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x
))/(c^4*d^4*e^2), 1/384*(15*(c^4*d^8 - 4*a*c^3*d^6*e^2 + 6*a^2*c^2*d^4*e^4
 - 4*a^3*c*d^2*e^6 + a^4*e^8)*sqrt(-c*d*e)*arctan(1/2*sqrt(c*d*e*x^2 + a*d
*e + (c*d^2 + a*e^2)*x)*(2*c*d*e*x + c*d^2 + a*e^2)*sqrt(-c*d*e)/(c^2*d^2*
e^2*x^2 + a*c*d^2*e^2 + (c^2*d^3*e + a*c*d*e^3)*x)) + 2*(48*c^4*d^4*e^4*x^
3 + 15*c^4*d^7*e + 73*a*c^3*d^5*e^3 - 55*a^2*c^2*d^3*e^5 + 15*a^3*c*d*e^7
+ 8*(17*c^4*d^5*e^3 + a*c^3*d^3*e^5)*x^2 + 2*(59*c^4*d^6*e^2 + 18*a*c^3*d^
4*e^4 - 5*a^2*c^2*d^2*e^6)*x)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x))
/(c^4*d^4*e^2)]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 989 vs. $2(282) = 564$.

Time = 0.83 (sec) , antiderivative size = 989, normalized size of antiderivative = 3.32

$$\int (d+ex)^2\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}\,dx = \text{Too large to display}$$

input
```
integrate((e*x+d)**2*(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(1/2),x)
```

output
```
Piecewise((sqrt(a*d*e + c*d*e*x**2 + x*(a*e**2 + c*d**2))*(e**2*x**3/4 + x
**2*(a*e**4 + 3*c*d**2*e**2 - e**2*(7*a*e**2/2 + 7*c*d**2/2)/4)/(3*c*d*e)
+ x*(9*a*d*e**3/4 + 3*c*d**3*e - (5*a*e**2/2 + 5*c*d**2/2)*(a*e**4 + 3*c*d
**2*e**2 - e**2*(7*a*e**2/2 + 7*c*d**2/2)/4)/(3*c*d*e))/(2*c*d*e) + (3*a*d
**2*e**2 - 2*a*(a*e**4 + 3*c*d**2*e**2 - e**2*(7*a*e**2/2 + 7*c*d**2/2)/4)
/(3*c) + c*d**4 - (3*a*e**2/2 + 3*c*d**2/2)*(9*a*d*e**3/4 + 3*c*d**3*e - (
5*a*e**2/2 + 5*c*d**2/2)*(a*e**4 + 3*c*d**2*e**2 - e**2*(7*a*e**2/2 + 7*c*
d**2/2)/4)/(3*c*d*e))/(2*c*d*e))/(c*d*e)) + (a*d**3*e - a*(9*a*d*e**3/4 +
3*c*d**3*e - (5*a*e**2/2 + 5*c*d**2/2)*(a*e**4 + 3*c*d**2*e**2 - e**2*(7*a
*e**2/2 + 7*c*d**2/2)/4)/(3*c*d*e))/(2*c) - (a*e**2 + c*d**2)*(3*a*d**2*e*
*2 - 2*a*(a*e**4 + 3*c*d**2*e**2 - e**2*(7*a*e**2/2 + 7*c*d**2/2)/4)/(3*c)
 + c*d**4 - (3*a*e**2/2 + 3*c*d**2/2)*(9*a*d*e**3/4 + 3*c*d**3*e - (5*a*e*
*2/2 + 5*c*d**2/2)*(a*e**4 + 3*c*d**2*e**2 - e**2*(7*a*e**2/2 + 7*c*d**2/2
)/4)/(3*c*d*e))/(2*c*d*e))/(2*c*d*e))*Piecewise((log(a*e**2 + c*d**2 + 2*c
*d*e*x + 2*sqrt(c*d*e)*sqrt(a*d*e + c*d*e*x**2 + x*(a*e**2 + c*d**2)))/sqr
t(c*d*e), Ne(a*d*e - (a*e**2 + c*d**2)**2/(4*c*d*e), 0)), ((x - (-a*e**2 -
 c*d**2)/(2*c*d*e))*log(x - (-a*e**2 - c*d**2)/(2*c*d*e))/sqrt(c*d*e*(x -
(-a*e**2 - c*d**2)/(2*c*d*e))**2), True)), Ne(c*d*e, 0)), (2*(c**2*d**6*(a
*d*e + x*(a*e**2 + c*d**2))**(3/2)/(3*(a**2*e**4 + 2*a*c*d**2*e**2 + c**2*
d**4)) + 2*c*d**3*e*(a*d*e + x*(a*e**2 + c*d**2))**(5/2)/(5*(a**2*e**4 ...
```

Maxima [F(-2)]

Exception generated.

$$\int (d+ex)^2 \sqrt{ade + (cd^2+ae^2)\,x + cdex^2}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((e*x+d)^2*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x, algorithm="
maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

Giac [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 305, normalized size of antiderivative = 1.02

$$\int (d+ex)^2\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}\,dx$$
$$=\frac{1}{192}\sqrt{cdex^2+cd^2x+ae^2x+ade}\left(2\left(4\left(6\,e^2x+\frac{17\,c^3d^4e^4+ac^2d^2e^6}{c^3d^3e^3}\right)x+\frac{59\,c^3d^5e^3+18\,ac^2d^3e^5-5\,a}{c^3d^3e^3}\right.\right.$$
$$+\frac{5\left(c^4d^8-4\,ac^3d^6e^2+6\,a^2c^2d^4e^4-4\,a^3cd^2e^6+a^4e^8\right)\log\left(\left|-cd^2-ae^2-2\,\sqrt{cde}\left(\sqrt{cde}x-\sqrt{cdex^2+c}\right.\right.\right.}{128\,\sqrt{cde}c^3d^3e}$$

input
```
integrate((e*x+d)^2*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x, algorithm="
giac")
```

output
```
1/192*sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*x + a*d*e)*(2*(4*(6*e^2*x + (17*c^3
*d^4*e^4 + a*c^2*d^2*e^6)/(c^3*d^3*e^3))*x + (59*c^3*d^5*e^3 + 18*a*c^2*d^
3*e^5 - 5*a^2*c*d*e^7)/(c^3*d^3*e^3))*x + (15*c^3*d^6*e^2 + 73*a*c^2*d^4*e
^4 - 55*a^2*c*d^2*e^6 + 15*a^3*e^8)/(c^3*d^3*e^3)) + 5/128*(c^4*d^8 - 4*a*
c^3*d^6*e^2 + 6*a^2*c^2*d^4*e^4 - 4*a^3*c*d^2*e^6 + a^4*e^8)*log(abs(-c*d^
2 - a*e^2 - 2*sqrt(c*d*e)*(sqrt(c*d*e)*x - sqrt(c*d*e*x^2 + c*d^2*x + a*e^
2*x + a*d*e))))/(sqrt(c*d*e)*c^3*d^3*e)
```

Mupad [F(-1)]

Timed out.

$$\int (d+ex)^2\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}\,dx$$
$$=\int {\left(d+e\,x\right)}^2\,\sqrt{c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e}\,dx$$

input
```
int((d + e*x)^2*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2),x)
```

output
```
int((d + e*x)^2*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 579, normalized size of antiderivative = 1.94

$$\int (d+ex)^2 \sqrt{ade+(cd^2+ae^2)\,x+cdex^2}\,dx$$

$$= \frac{15\sqrt{ex+d}\,\sqrt{cdx+ae}\,a^3cd\,e^7 - 55\sqrt{ex+d}\,\sqrt{cdx+ae}\,a^2c^2d^3e^5 - 10\sqrt{ex+d}\,\sqrt{cdx+ae}\,a^2c^2d^2e^6x + 73}{}$$

input
```
int((e*x+d)^2*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x)
```

output
```
(15*sqrt(d + e*x)*sqrt(a*e + c*d*x)*a**3*c*d*e**7 - 55*sqrt(d + e*x)*sqrt(
a*e + c*d*x)*a**2*c**2*d**3*e**5 - 10*sqrt(d + e*x)*sqrt(a*e + c*d*x)*a**2
*c**2*d**2*e**6*x + 73*sqrt(d + e*x)*sqrt(a*e + c*d*x)*a*c**3*d**5*e**3 +
36*sqrt(d + e*x)*sqrt(a*e + c*d*x)*a*c**3*d**4*e**4*x + 8*sqrt(d + e*x)*sq
rt(a*e + c*d*x)*a*c**3*d**3*e**5*x**2 + 15*sqrt(d + e*x)*sqrt(a*e + c*d*x)
*c**4*d**7*e + 118*sqrt(d + e*x)*sqrt(a*e + c*d*x)*c**4*d**6*e**2*x + 136*
sqrt(d + e*x)*sqrt(a*e + c*d*x)*c**4*d**5*e**3*x**2 + 48*sqrt(d + e*x)*sqr
t(a*e + c*d*x)*c**4*d**4*e**4*x**3 - 15*sqrt(e)*sqrt(d)*sqrt(c)*log((sqrt(
e)*sqrt(a*e + c*d*x) + sqrt(d)*sqrt(c)*sqrt(d + e*x))/sqrt(a*e**2 - c*d**2
))*a**4*e**8 + 60*sqrt(e)*sqrt(d)*sqrt(c)*log((sqrt(e)*sqrt(a*e + c*d*x) +
 sqrt(d)*sqrt(c)*sqrt(d + e*x))/sqrt(a*e**2 - c*d**2))*a**3*c*d**2*e**6 -
90*sqrt(e)*sqrt(d)*sqrt(c)*log((sqrt(e)*sqrt(a*e + c*d*x) + sqrt(d)*sqrt(c
)*sqrt(d + e*x))/sqrt(a*e**2 - c*d**2))*a**2*c**2*d**4*e**4 + 60*sqrt(e)*s
qrt(d)*sqrt(c)*log((sqrt(e)*sqrt(a*e + c*d*x) + sqrt(d)*sqrt(c)*sqrt(d + e
*x))/sqrt(a*e**2 - c*d**2))*a*c**3*d**6*e**2 - 15*sqrt(e)*sqrt(d)*sqrt(c)*
log((sqrt(e)*sqrt(a*e + c*d*x) + sqrt(d)*sqrt(c)*sqrt(d + e*x))/sqrt(a*e**
2 - c*d**2))*c**4*d**8)/(192*c**4*d**4*e**2)
```

3.202 $\int (d+ex)\sqrt{ade+(cd^2+ae^2)x+cdex^2}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 240

$$\begin{aligned}\int (d+ex)\sqrt{ade+(cd^2+ae^2)x+cdex^2}\,dx &= \frac{(cd^2-ae^2)^2\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{8c^2d^2e}+\frac{(ade+(cd^2+ae^2)x+cdex^2)^{3/2}}{3cd}\\ &\quad+\frac{(cd^2-ae^2)(ade+(cd^2+ae^2)x+cdex^2)^{3/2}}{4c^2d^2(d+ex)}\\ &\quad-\frac{(cd^2-ae^2)^3\operatorname{arctanh}\left(\frac{\sqrt{e}\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{\sqrt{c}\sqrt{d}(d+ex)}\right)}{8c^{5/2}d^{5/2}e^{3/2}}\end{aligned}$$

output
```
1/8*(-a*e^2+c*d^2)^2*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/c^2/d^2/e+1/3
*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/c/d+1/4*(-a*e^2+c*d^2)*(a*d*e+(a*
e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/c^2/d^2/(e*x+d)-1/8*(-a*e^2+c*d^2)^3*arctanh
(e^(1/2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/c^(1/2)/d^(1/2)/(e*x+d))/
c^(5/2)/d^(5/2)/e^(3/2)
```

Mathematica [A] (verified)

Time = 0.37 (sec) , antiderivative size = 179, normalized size of antiderivative = 0.75

$$\int (d+ex)\sqrt{ade+(cd^2+ae^2)x+cdex^2}\,dx$$

$$= \frac{\sqrt{(ae+cdx)(d+ex)}\left(\sqrt{c}\sqrt{d}\sqrt{e}(-3a^2e^4+2acde^2(4d+ex)+c^2d^2(3d^2+14dex+8e^2x^2)) - \frac{3(cd^2-ae^2)^3 \cdots}{\sqrt{c}\cdots}\right.}{24c^{5/2}d^{5/2}e^{3/2}}$$

input
```
Integrate[(d + e*x)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2],x]
```

output
```
(Sqrt[(a*e + c*d*x)*(d + e*x)]*(Sqrt[c]*Sqrt[d]*Sqrt[e]*(-3*a^2*e^4 + 2*a*
c*d*e^2*(4*d + e*x) + c^2*d^2*(3*d^2 + 14*d*e*x + 8*e^2*x^2)) - (3*(c*d^2
- a*e^2)^3*ArcTanh[(Sqrt[c]*Sqrt[d]*Sqrt[d + e*x])/(Sqrt[e]*Sqrt[a*e + c*d
*x])])/(Sqrt[a*e + c*d*x]*Sqrt[d + e*x])))/(24*c^(5/2)*d^(5/2)*e^(3/2))
```

Rubi [A] (verified)

Time = 0.52 (sec) , antiderivative size = 219, normalized size of antiderivative = 0.91, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.114$, Rules used = {1160, 1087, 1092, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (d+ex)\sqrt{x(ae^2+cd^2)+ade+cdex^2}\,dx$$

$$\Big\downarrow \; 1160$$

$$\frac{\left(d^2-\frac{ae^2}{c}\right)\int \sqrt{cdex^2+(cd^2+ae^2)x+ade}dx}{2d}+\frac{\left(x(ae^2+cd^2)+ade+cdex^2\right)^{3/2}}{3cd}$$

$$\Big\downarrow \; 1087$$

$$\frac{\left(d^2-\frac{ae^2}{c}\right)\left(\frac{(ae^2+cd^2+2cdex)\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{4cde}-\frac{(cd^2-ae^2)^2\int\frac{1}{\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}dx}{8cde}\right)}{2d}+\frac{\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{3cd}$$

$$\Big\downarrow \text{1092}$$

$$\frac{\left(d^2-\frac{ae^2}{c}\right)\left(\frac{(ae^2+cd^2+2cdex)\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{4cde}-\frac{(cd^2-ae^2)^2\int\frac{1}{4cde-\frac{\left(cd^2+2cexd+ae^2\right)^2}{cdex^2+\left(cd^2+ae^2\right)x+ade}}d\frac{cd^2+2cexd+ae^2}{\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}}{4cde}\right)}{2d}+\frac{\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{3cd}$$

$$\Big\downarrow \text{219}$$

$$\frac{\left(d^2-\frac{ae^2}{c}\right)\left(\frac{(ae^2+cd^2+2cdex)\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{4cde}-\frac{(cd^2-ae^2)^2\text{arctanh}\left(\frac{ae^2+cd^2+2cdex}{2\sqrt{c}\sqrt{d}\sqrt{e}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}\right)}{8c^{3/2}d^{3/2}e^{3/2}}\right)}{2d}+\frac{\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{3cd}$$

input
```
Int[(d + e*x)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2],x]
```

output
```
(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2)/(3*c*d) + ((d^2 - (a*e^2)/c)
*(((c*d^2 + a*e^2 + 2*c*d*e*x)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]
)/(4*c*d*e) - ((c*d^2 - a*e^2)^2*ArcTanh[(c*d^2 + a*e^2 + 2*c*d*e*x)/(2*Sq
rt[c]*Sqrt[d]*Sqrt[e]*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])])/(8*c^
(3/2)*d^(3/2)*e^(3/2))))/(2*d)
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1087
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^p/(2*c*(2*p + 1))), x] - Simp[p*((b^2 - 4*a*c)/(2*c*(2*
p + 1)))   Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] &&
GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[3*p])
```

rule 1092
```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[I
nt[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a
, b, c}, x]
```

rule 1160
```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[e*((a + b*x + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Simp[(2*c*d - b
*e)/(2*c)   Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
 && NeQ[p, -1]
```

Maple [A] (verified)

Time = 1.48 (sec) , antiderivative size = 373, normalized size of antiderivative = 1.55

method	result
default	$d\left(\frac{\left(2cdxe+a\,e^{2}+c\,d^{2}\right)\sqrt{ade+\left(a\,e^{2}+c\,d^{2}\right)x+cd\,x^{2}e}}{4cde}+\frac{\left(4ac\,d^{2}e^{2}-\left(a\,e^{2}+c\,d^{2}\right)^{2}\right)\ln\left(\frac{\frac{1}{2}a\,e^{2}+\frac{1}{2}c\,d^{2}+cdxe}{\sqrt{dec}}+\sqrt{ade+\left(a\,e^{2}+c\,d^{2}\right)x+c\right.}{8dec\sqrt{dec}}\right.$

input
```
int((e*x+d)*(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(1/2),x,method=_RETURNVERBOS
E)
```

output
```
d*(1/4*(2*c*d*e*x+a*e^2+c*d^2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(1/2)/c/d
/e+1/8*(4*a*c*d^2*e^2-(a*e^2+c*d^2)^2)/d/e/c*ln((1/2*a*e^2+1/2*c*d^2+c*d*x
*e)/(d*e*c)^(1/2)+(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(1/2))/(d*e*c)^(1/2))+
e*(1/3*(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(3/2)/d/e/c-1/2*(a*e^2+c*d^2)/d/e
/c*(1/4*(2*c*d*e*x+a*e^2+c*d^2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(1/2)/c/
d/e+1/8*(4*a*c*d^2*e^2-(a*e^2+c*d^2)^2)/d/e/c*ln((1/2*a*e^2+1/2*c*d^2+c*d*
x*e)/(d*e*c)^(1/2)+(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(1/2))/(d*e*c)^(1/2))
)
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 532, normalized size of antiderivative = 2.22

$$\int (d+ex)\sqrt{ade+(cd^2+ae^2)x+cdex^2}\,dx$$
$$= \left[-\frac{3\left(c^3d^6-3\,ac^2d^4e^2+3\,a^2cd^2e^4-a^3e^6\right)\sqrt{cde}\log\left(8\,c^2d^2e^2x^2+c^2d^4+6\,acd^2e^2+a^2e^4+4\,\sqrt{cdex^2+}\right.}{}\right.$$

input
```
integrate((e*x+d)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x, algorithm="fr
icas")
```

output
```
[-1/96*(3*(c^3*d^6 - 3*a*c^2*d^4*e^2 + 3*a^2*c*d^2*e^4 - a^3*e^6)*sqrt(c*d
*e)*log(8*c^2*d^2*e^2*x^2 + c^2*d^4 + 6*a*c*d^2*e^2 + a^2*e^4 + 4*sqrt(c*d
*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(2*c*d*e*x + c*d^2 + a*e^2)*sqrt(c*d*e
) + 8*(c^2*d^3*e + a*c*d*e^3)*x) - 4*(8*c^3*d^3*e^3*x^2 + 3*c^3*d^5*e + 8*
a*c^2*d^3*e^3 - 3*a^2*c*d*e^5 + 2*(7*c^3*d^4*e^2 + a*c^2*d^2*e^4)*x)*sqrt(
c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x))/(c^3*d^3*e^2), 1/48*(3*(c^3*d^6 -
3*a*c^2*d^4*e^2 + 3*a^2*c*d^2*e^4 - a^3*e^6)*sqrt(-c*d*e)*arctan(1/2*sqrt(
c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(2*c*d*e*x + c*d^2 + a*e^2)*sqrt(-c
*d*e)/(c^2*d^2*e^2*x^2 + a*c*d^2*e^2 + (c^2*d^3*e + a*c*d*e^3)*x)) + 2*(8*
c^3*d^3*e^3*x^2 + 3*c^3*d^5*e + 8*a*c^2*d^3*e^3 - 3*a^2*c*d*e^5 + 2*(7*c^3
*d^4*e^2 + a*c^2*d^2*e^4)*x)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x))/
(c^3*d^3*e^2)]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 571 vs. $2(218) = 436$.

Time = 1.19 (sec) , antiderivative size = 571, normalized size of antiderivative = 2.38

$$\int (d+ex)\sqrt{ade+(cd^2+ae^2)x+cdex^2}\,dx$$

$$= \begin{cases} \left(\left(\frac{ex^2}{3} + \frac{x\left(ae^3+2cd^2e-\frac{e\left(\frac{5ae^2}{2}+\frac{5cd^2}{2}\right)}{3}\right)}{2cde} + \frac{\frac{4ade^2}{3}+cd^3-\frac{\left(\frac{3ae^2}{2}+\frac{3cd^2}{2}\right)\left(ae^3+2cd^2e-\frac{e\left(\frac{5ae^2}{2}+\frac{5cd^2}{2}\right)}{3}\right)}{2cde}}{cde}\right)\sqrt{ade+cdex^2+x(\right. & \\ \frac{2\left(\frac{cd^3\left(ade+x\left(ae^2+cd^2\right)\right)^{\frac{3}{2}}}{3\left(ae^2+cd^2\right)}+\frac{e\left(ade+x\left(ae^2+cd^2\right)\right)^{\frac{5}{2}}}{5\left(ae^2+cd^2\right)}\right)}{ae^2+cd^2} & \\ \sqrt{ade}\left(dx+\frac{ex^2}{2}\right) & \end{cases}$$

input
```
integrate((e*x+d)*(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(1/2),x)
```

output
```
Piecewise(((e*x**2/3 + x*(a*e**3 + 2*c*d**2*e - e*(5*a*e**2/2 + 5*c*d**2/2
)/3)/(2*c*d*e) + (4*a*d*e**2/3 + c*d**3 - (3*a*e**2/2 + 3*c*d**2/2)*(a*e**
3 + 2*c*d**2*e - e*(5*a*e**2/2 + 5*c*d**2/2)/3)/(2*c*d*e))/(c*d*e))*sqrt(a
*d*e + c*d*e*x**2 + x*(a*e**2 + c*d**2)) + (a*d**2*e - a*(a*e**3 + 2*c*d**
2*e - e*(5*a*e**2/2 + 5*c*d**2/2)/3)/(2*c) - (a*e**2 + c*d**2)*(4*a*d*e**2
/3 + c*d**3 - (3*a*e**2/2 + 3*c*d**2/2)*(a*e**3 + 2*c*d**2*e - e*(5*a*e**2
/2 + 5*c*d**2/2)/3)/(2*c*d*e))/(2*c*d*e))*Piecewise((log(a*e**2 + c*d**2 +
 2*c*d*e*x + 2*sqrt(c*d*e)*sqrt(a*d*e + c*d*e*x**2 + x*(a*e**2 + c*d**2)))
/sqrt(c*d*e), Ne(a*d*e - (a*e**2 + c*d**2)**2/(4*c*d*e), 0)), ((x - (-a*e*
*2 - c*d**2)/(2*c*d*e))*log(x - (-a*e**2 - c*d**2)/(2*c*d*e))/sqrt(c*d*e*(
x - (-a*e**2 - c*d**2)/(2*c*d*e))**2), True)), Ne(c*d*e, 0)), (2*(c*d**3*(
a*d*e + x*(a*e**2 + c*d**2))**(3/2)/(3*(a*e**2 + c*d**2)) + e*(a*d*e + x*(
a*e**2 + c*d**2))**(5/2)/(5*(a*e**2 + c*d**2)))/(a*e**2 + c*d**2), Ne(a*e*
*2 + c*d**2, 0)), (sqrt(a*d*e)*(d*x + e*x**2/2), True))
```

Maxima [F(-2)]

Exception generated.

$$\int (d+ex)\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((e*x+d)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x, algorithm="ma
xima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

Giac [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 222, normalized size of antiderivative = 0.92

$$\int (d+ex)\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}\,dx = \frac{1}{24}\sqrt{cdex^2+cd^2x+ae^2x+ade}\left(2\left(4\,ex+\frac{7\,c^2d^3e^2+acde^4}{c^2d^2e^2}\right)x+\frac{3\,c^2d^4e+8\,acd^2e^3-3\,a^2e^5}{c^2d^2e^2}\right) + \frac{(c^3d^6-3\,ac^2d^4e^2+3\,a^2cd^2e^4-a^3e^6)\log\left(\left|-cd^2-ae^2-2\,\sqrt{cde}\left(\sqrt{cde}x-\sqrt{cdex^2+cd^2x+ae^2x+ade}\right)\right|\right)}{16\,\sqrt{cde}c^2d^2e}$$

input
```
integrate((e*x+d)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x, algorithm="gi
ac")
```

output
```
1/24*sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*x + a*d*e)*(2*(4*e*x + (7*c^2*d^3*e^
2 + a*c*d*e^4)/(c^2*d^2*e^2))*x + (3*c^2*d^4*e + 8*a*c*d^2*e^3 - 3*a^2*e^5
)/(c^2*d^2*e^2)) + 1/16*(c^3*d^6 - 3*a*c^2*d^4*e^2 + 3*a^2*c*d^2*e^4 - a^3
*e^6)*log(abs(-c*d^2 - a*e^2 - 2*sqrt(c*d*e)*(sqrt(c*d*e)*x - sqrt(c*d*e*x
^2 + c*d^2*x + a*e^2*x + a*d*e))))/(sqrt(c*d*e)*c^2*d^2*e)
```

Mupad [B] (verification not implemented)

Time = 5.84 (sec) , antiderivative size = 307, normalized size of antiderivative = 1.28

$$\int (d+ex)\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}\,dx$$
$$= d\left(\frac{x}{2}+\frac{c\,d^2+a\,e^2}{4\,c\,d\,e}\right)\sqrt{c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e}$$
$$-\frac{d\,\ln\left(2\,\sqrt{\left(a\,e+c\,d\,x\right)\,\left(d+e\,x\right)}\,\sqrt{c\,d\,e}+a\,e^2+c\,d^2+2\,c\,d\,e\,x\right)\,\left(\frac{\left(c\,d^2+a\,e^2\right)^2}{4}-a\,c\,d^2\,e^2\right)}{2\,{\left(c\,d\,e\right)}^{3/2}}$$
$$+\frac{e\,\ln\left(2\,\sqrt{\left(a\,e+c\,d\,x\right)\,\left(d+e\,x\right)}\,\sqrt{c\,d\,e}+a\,e^2+c\,d^2+2\,c\,d\,e\,x\right)\,\left({\left(c\,d^2+a\,e^2\right)}^3-4\,a\,c\,d^2\,e^2\,\left(c\,d^2+a\,e^2\right.\right.}{16\,{\left(c\,d\,e\right)}^{5/2}}$$
$$+\frac{\sqrt{c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e}\,\left(8\,c\,d\,e\,\left(c\,d\,e\,x^2+a\,d\,e\right)-3\,{\left(c\,d^2+a\,e^2\right)}^2+2\,c\,d\,e\,x\,\left(c\,d^2+a\,e^2\right)\right)}{24\,c^2\,d^2\,e}$$

input
```
int((d + e*x)*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2),x)
```

output
```
d*(x/2 + (a*e^2 + c*d^2)/(4*c*d*e))*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2
)^(1/2) - (d*log(2*((a*e + c*d*x)*(d + e*x))^(1/2)*(c*d*e)^(1/2) + a*e^2 +
 c*d^2 + 2*c*d*e*x)*((a*e^2 + c*d^2)^2/4 - a*c*d^2*e^2))/(2*(c*d*e)^(3/2))
 + (e*log(2*((a*e + c*d*x)*(d + e*x))^(1/2)*(c*d*e)^(1/2) + a*e^2 + c*d^2
+ 2*c*d*e*x)*((a*e^2 + c*d^2)^3 - 4*a*c*d^2*e^2*(a*e^2 + c*d^2)))/(16*(c*d
*e)^(5/2)) + ((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2)*(8*c*d*e*(a*d*
e + c*d*e*x^2) - 3*(a*e^2 + c*d^2)^2 + 2*c*d*e*x*(a*e^2 + c*d^2)))/(24*c^2
*d^2*e)
```

Reduce [B] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 400, normalized size of antiderivative = 1.67

$$\int (d+ex)\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}\,dx$$
$$=\frac{-3\sqrt{ex+d}\,\sqrt{cdx+ae}\,a^2cd\,e^5+8\sqrt{ex+d}\,\sqrt{cdx+ae}\,a\,c^2d^3e^3+2\sqrt{ex+d}\,\sqrt{cdx+ae}\,a\,c^2d^2e^4x+3\sqrt{ex}}{}$$

input
```
int((e*x+d)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x)
```

output

```
( - 3*sqrt(d + e*x)*sqrt(a*e + c*d*x)*a**2*c*d*e**5 + 8*sqrt(d + e*x)*sqrt
(a*e + c*d*x)*a*c**2*d**3*e**3 + 2*sqrt(d + e*x)*sqrt(a*e + c*d*x)*a*c**2*
d**2*e**4*x + 3*sqrt(d + e*x)*sqrt(a*e + c*d*x)*c**3*d**5*e + 14*sqrt(d +
e*x)*sqrt(a*e + c*d*x)*c**3*d**4*e**2*x + 8*sqrt(d + e*x)*sqrt(a*e + c*d*x
)*c**3*d**3*e**3*x**2 + 3*sqrt(e)*sqrt(d)*sqrt(c)*log((sqrt(e)*sqrt(a*e +
c*d*x) + sqrt(d)*sqrt(c)*sqrt(d + e*x))/sqrt(a*e**2 - c*d**2))*a**3*e**6 -
 9*sqrt(e)*sqrt(d)*sqrt(c)*log((sqrt(e)*sqrt(a*e + c*d*x) + sqrt(d)*sqrt(c
)*sqrt(d + e*x))/sqrt(a*e**2 - c*d**2))*a**2*c*d**2*e**4 + 9*sqrt(e)*sqrt(
d)*sqrt(c)*log((sqrt(e)*sqrt(a*e + c*d*x) + sqrt(d)*sqrt(c)*sqrt(d + e*x))
/sqrt(a*e**2 - c*d**2))*a*c**2*d**4*e**2 - 3*sqrt(e)*sqrt(d)*sqrt(c)*log((
sqrt(e)*sqrt(a*e + c*d*x) + sqrt(d)*sqrt(c)*sqrt(d + e*x))/sqrt(a*e**2 - c
*d**2))*c**3*d**6)/(24*c**3*d**3*e**2)
```

3.203 $\int \sqrt{ade+(cd^2+ae^2)x+cdex^2}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 144

$$\int \sqrt{ade+(cd^2+ae^2)x+cdex^2}\,dx = \frac{(cd^2+ae^2+2cdex)\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{4cde} - \frac{(cd^2-ae^2)^2\operatorname{arctanh}\left(\frac{\sqrt{c}\sqrt{d}(d+ex)}{\sqrt{e}\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\right)}{4c^{3/2}d^{3/2}e^{3/2}}$$

output

```
1/4*(2*c*d*e*x+a*e^2+c*d^2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/c/d/e-
1/4*(-a*e^2+c*d^2)^2*arctanh(c^(1/2)*d^(1/2)*(e*x+d)/e^(1/2)/(a*d*e+(a*e^2
+c*d^2)*x+c*d*e*x^2)^(1/2))/c^(3/2)/d^(3/2)/e^(3/2)
```

Mathematica [A] (verified)

Time = 0.43 (sec) , antiderivative size = 144, normalized size of antiderivative = 1.00

$$\int \sqrt{ade+(cd^2+ae^2)x+cdex^2}\,dx = \frac{\sqrt{(ae+cdx)(d+ex)}\left(\sqrt{c}\sqrt{d}\sqrt{e}(ae^2+cd(d+2ex)) - \frac{(cd^2-ae^2)^2\operatorname{arctanh}\left(\frac{\sqrt{c}\sqrt{d}\sqrt{d+ex}}{\sqrt{e}\sqrt{ae+cdx}}\right)}{\sqrt{ae+cdx}\sqrt{d+ex}}\right)}{4c^{3/2}d^{3/2}e^{3/2}}$$

input
```
Integrate[Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2],x]
```

output
```
(Sqrt[(a*e + c*d*x)*(d + e*x)]*(Sqrt[c]*Sqrt[d]*Sqrt[e]*(a*e^2 + c*d*(d +
2*e*x)) - ((c*d^2 - a*e^2)^2*ArcTanh[(Sqrt[c]*Sqrt[d]*Sqrt[d + e*x])/(Sqrt
[e]*Sqrt[a*e + c*d*x])])/(Sqrt[a*e + c*d*x]*Sqrt[d + e*x])))/(4*c^(3/2)*d^
(3/2)*e^(3/2))
```

Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 159, normalized size of antiderivative = 1.10, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.103$, Rules used = {1087, 1092, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}dx$$

$$\Big\downarrow \text{1087}$$

$$\frac{\left(ae^2+cd^2+2cdex\right)\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{4cde}-\frac{\left(cd^2-ae^2\right)^2\int \frac{1}{\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}dx}{8cde}$$

$$\Big\downarrow \text{1092}$$

$$\frac{\left(ae^2+cd^2+2cdex\right)\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{4cde}-\frac{\left(cd^2-ae^2\right)^2\int \frac{1}{4cde-\frac{\left(cd^2+2cexd+ae^2\right)^2}{cdex^2+\left(cd^2+ae^2\right)x+ade}}d\frac{cd^2+2cexd+ae^2}{\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}}{4cde}$$

$$\Big\downarrow \text{219}$$

$$\frac{\left(ae^2+cd^2+2cdex\right)\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{4cde}-\frac{\left(cd^2-ae^2\right)^2\text{arctanh}\left(\frac{ae^2+cd^2+2cdex}{2\sqrt{c}\sqrt{d}\sqrt{e}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}\right)}{8c^{3/2}d^{3/2}e^{3/2}}$$

input
```
Int[Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2],x]
```

output
```
((c*d^2 + a*e^2 + 2*c*d*e*x)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/
(4*c*d*e) - ((c*d^2 - a*e^2)^2*ArcTanh[(c*d^2 + a*e^2 + 2*c*d*e*x)/(2*Sqrt
[c]*Sqrt[d]*Sqrt[e]*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])])/(8*c^(3
/2)*d^(3/2)*e^(3/2))
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1087
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^p/(2*c*(2*p + 1))), x] - Simp[p*((b^2 - 4*a*c)/(2*c*(2*
p + 1)))   Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] &&
GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[3*p])
```

rule 1092
```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[I
nt[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a
, b, c}, x]
```

Maple [A] (verified)

Time = 1.39 (sec) , antiderivative size = 154, normalized size of antiderivative = 1.07

method	result
default	$\frac{(2cdxe+a e^2+c d^2)\sqrt{ade+(a e^2+c d^2)x+cd x^2 e}}{4cde}+\frac{(4ac d^2 e^2-(a e^2+c d^2)^2)\ln\left(\frac{\frac{1}{2}a e^2+\frac{1}{2}c d^2+cdxe}{\sqrt{dec}}+\sqrt{ade+(a e^2+c d^2)x+cd x^2}\right.}{8dec\sqrt{dec}}$

input
```
int((a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/4*(2*c*d*e*x+a*e^2+c*d^2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(1/2)/c/d/e+
1/8*(4*a*c*d^2*e^2-(a*e^2+c*d^2)^2)/d/e/c*ln((1/2*a*e^2+1/2*c*d^2+c*d*x*e)
/(d*e*c)^(1/2)+(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(1/2))/(d*e*c)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 412, normalized size of antiderivative = 2.86

$$\int \sqrt{ade + (cd^2 + ae^2)\,x + cdex^2}\,dx$$
$$= \left[\frac{(c^2d^4 - 2\,acd^2e^2 + a^2e^4)\sqrt{cde}\log\left(8\,c^2d^2e^2x^2 + c^2d^4 + 6\,acd^2e^2 + a^2e^4 - 4\,\sqrt{cdex^2 + ade + (cd^2 + ae^2}\right.}{}\right.$$

input
```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x, algorithm="fricas")
```

output
```
[1/16*((c^2*d^4 - 2*a*c*d^2*e^2 + a^2*e^4)*sqrt(c*d*e)*log(8*c^2*d^2*e^2*x
^2 + c^2*d^4 + 6*a*c*d^2*e^2 + a^2*e^4 - 4*sqrt(c*d*e*x^2 + a*d*e + (c*d^2
 + a*e^2)*x)*(2*c*d*e*x + c*d^2 + a*e^2)*sqrt(c*d*e) + 8*(c^2*d^3*e + a*c*
d*e^3)*x) + 4*(2*c^2*d^2*e^2*x + c^2*d^3*e + a*c*d*e^3)*sqrt(c*d*e*x^2 + a
*d*e + (c*d^2 + a*e^2)*x))/(c^2*d^2*e^2), 1/8*((c^2*d^4 - 2*a*c*d^2*e^2 +
a^2*e^4)*sqrt(-c*d*e)*arctan(1/2*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*
x)*(2*c*d*e*x + c*d^2 + a*e^2)*sqrt(-c*d*e)/(c^2*d^2*e^2*x^2 + a*c*d^2*e^2
 + (c^2*d^3*e + a*c*d*e^3)*x)) + 2*(2*c^2*d^2*e^2*x + c^2*d^3*e + a*c*d*e^
3)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x))/(c^2*d^2*e^2)]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 306 vs. $2(134) = 268$.

Time = 0.40 (sec) , antiderivative size = 306, normalized size of antiderivative = 2.12

$$\int \sqrt{ade + (cd^2 + ae^2)\,x + cdex^2}\,dx$$
$$= \begin{cases} \left(\frac{x}{2} + \frac{\frac{ae^2}{4} + \frac{cd^2}{4}}{cde}\right)\sqrt{ade + cdex^2 + x\,(ae^2 + cd^2)} + \left(\frac{ade}{2} - \frac{\left(\frac{ae^2}{4} + \frac{cd^2}{4}\right)(ae^2 + cd^2)}{2cde}\right)\left(\left(\begin{cases} \frac{\log\left(ae^2 + cd^2 + 2cdex + 2\sqrt{cd}\right.}{\sqrt{c}} \\ \frac{\left(x - \frac{-ae^2 - cd^2}{2cde}\right)\log\left(x - \frac{-ae}{2}\right.}{\sqrt{cde\left(x - \frac{-ae^2 - cd^2}{2cde}\right)}} \end{cases}\right.\right. \\ \frac{2\left(ade + x\left(ae^2 + cd^2\right)\right)^{\frac{3}{2}}}{3(ae^2 + cd^2)} \\ x\sqrt{ade} \end{cases}$$

input
```
integrate((a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(1/2),x)
```

output
```
Piecewise(((x/2 + (a*e**2/4 + c*d**2/4)/(c*d*e))*sqrt(a*d*e + c*d*e*x**2 +
 x*(a*e**2 + c*d**2)) + (a*d*e/2 - (a*e**2/4 + c*d**2/4)*(a*e**2 + c*d**2)
/(2*c*d*e))*Piecewise((log(a*e**2 + c*d**2 + 2*c*d*e*x + 2*sqrt(c*d*e)*sqr
t(a*d*e + c*d*e*x**2 + x*(a*e**2 + c*d**2)))/sqrt(c*d*e), Ne(a*d*e - (a*e*
*2 + c*d**2)**2/(4*c*d*e), 0)), ((x - (-a*e**2 - c*d**2)/(2*c*d*e))*log(x
- (-a*e**2 - c*d**2)/(2*c*d*e))/sqrt(c*d*e*(x - (-a*e**2 - c*d**2)/(2*c*d*
e))**2), True)), Ne(c*d*e, 0)), (2*(a*d*e + x*(a*e**2 + c*d**2))**(3/2)/(3
*(a*e**2 + c*d**2)), Ne(a*e**2 + c*d**2, 0)), (x*sqrt(a*d*e), True))
```

Maxima [F(-2)]

Exception generated.

$$\int \sqrt{ade + (cd^2 + ae^2)\,x + cdex^2}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 156, normalized size of antiderivative = 1.08

$$\int \sqrt{ade + (cd^2 + ae^2)\,x + cdex^2}\,dx = \frac{1}{4}\sqrt{cdex^2 + cd^2x + ae^2x + ade}\left(2\,x + \frac{cd^2 + ae^2}{cde}\right) + \frac{(c^2d^4 - 2\,acd^2e^2 + a^2e^4)\log\left(\left|-cd^2 - ae^2 - 2\sqrt{cde}\left(\sqrt{cde}x - \sqrt{cdex^2 + cd^2x + ae^2x + ade}\right)\right|\right)}{8\sqrt{cde}cde}$$

input
```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x, algorithm="giac")
```

output
```
1/4*sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*x + a*d*e)*(2*x + (c*d^2 + a*e^2)/(c*
d*e)) + 1/8*(c^2*d^4 - 2*a*c*d^2*e^2 + a^2*e^4)*log(abs(-c*d^2 - a*e^2 - 2
*sqrt(c*d*e)*(sqrt(c*d*e)*x - sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*x + a*d*e))
))/(sqrt(c*d*e)*c*d*e)
```

Mupad [B] (verification not implemented)

Time = 5.43 (sec) , antiderivative size = 131, normalized size of antiderivative = 0.91

$$\int \sqrt{ade + (cd^2 + ae^2)x + cdex^2}\,dx$$
$$= \left(\frac{x}{2} + \frac{c\,d^2 + a\,e^2}{4\,c\,d\,e}\right)\sqrt{c\,d\,e\,x^2 + (c\,d^2 + a\,e^2)\,x + a\,d\,e}$$
$$- \frac{\ln\left(2\sqrt{(a\,e + c\,d\,x)\,(d + e\,x)}\sqrt{c\,d\,e} + a\,e^2 + c\,d^2 + 2\,c\,d\,e\,x\right)\left(\frac{(c\,d^2 + a\,e^2)^2}{4} - a\,c\,d^2\,e^2\right)}{2\,(c\,d\,e)^{3/2}}$$

input
```
int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2),x)
```

output
```
(x/2 + (a*e^2 + c*d^2)/(4*c*d*e))*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^
(1/2) - (log(2*((a*e + c*d*x)*(d + e*x))^(1/2)*(c*d*e)^(1/2) + a*e^2 + c*d
^2 + 2*c*d*e*x)*((a*e^2 + c*d^2)^2/4 - a*c*d^2*e^2))/(2*(c*d*e)^(3/2))
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 251, normalized size of antiderivative = 1.74

$$\int \sqrt{ade + (cd^2 + ae^2)x + cdex^2}\,dx$$
$$= \frac{\sqrt{ex + d}\sqrt{cdx + ae}\,acd\,e^3 + \sqrt{ex + d}\sqrt{cdx + ae}\,c^2d^3e + 2\sqrt{ex + d}\sqrt{cdx + ae}\,c^2d^2e^2x - \sqrt{e}\sqrt{d}\sqrt{c}\log(}{}$$

input
```
int((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x)
```

output

```
(sqrt(d + e*x)*sqrt(a*e + c*d*x)*a*c*d*e**3 + sqrt(d + e*x)*sqrt(a*e + c*d
*x)*c**2*d**3*e + 2*sqrt(d + e*x)*sqrt(a*e + c*d*x)*c**2*d**2*e**2*x - sqr
t(e)*sqrt(d)*sqrt(c)*log((sqrt(e)*sqrt(a*e + c*d*x) + sqrt(d)*sqrt(c)*sqrt
(d + e*x))/sqrt(a*e**2 - c*d**2))*a**2*e**4 + 2*sqrt(e)*sqrt(d)*sqrt(c)*lo
g((sqrt(e)*sqrt(a*e + c*d*x) + sqrt(d)*sqrt(c)*sqrt(d + e*x))/sqrt(a*e**2
- c*d**2))*a*c*d**2*e**2 - sqrt(e)*sqrt(d)*sqrt(c)*log((sqrt(e)*sqrt(a*e +
 c*d*x) + sqrt(d)*sqrt(c)*sqrt(d + e*x))/sqrt(a*e**2 - c*d**2))*c**2*d**4)
/(4*c**2*d**2*e**2)
```

3.204 $\int \frac{\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{d+ex} \, dx$

Optimal result

Integrand size = 37, antiderivative size = 116

$$\int \frac{\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{d+ex}\,dx=\frac{\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{e}-\frac{(cd^2-ae^2)\,\text{arctanh}\left(\frac{\sqrt{e}\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{\sqrt{c}\sqrt{d}(d+ex)}\right)}{\sqrt{c}\sqrt{d}e^{3/2}}$$

output
```
(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/e-(-a*e^2+c*d^2)*arctanh(e^(1/2)*(
a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/c^(1/2)/d^(1/2)/(e*x+d))/c^(1/2)/d^
(1/2)/e^(3/2)
```

Mathematica [A] (verified)

Time = 0.22 (sec) , antiderivative size = 112, normalized size of antiderivative = 0.97

$$\int \frac{\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{d+ex}\,dx=\frac{\sqrt{(ae+cdx)(d+ex)}\left(\sqrt{e}+\frac{(-cd^2+ae^2)\text{arctanh}\left(\frac{\sqrt{c}\sqrt{d}\sqrt{d+ex}}{\sqrt{e}\sqrt{ae+cdx}}\right)}{\sqrt{c}\sqrt{d}\sqrt{ae+cdx}\sqrt{d+ex}}\right)}{e^{3/2}}$$

input
```
Integrate[Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]/(d + e*x),x]
```

output
```
(Sqrt[(a*e + c*d*x)*(d + e*x)]*(Sqrt[e] + ((-(c*d^2) + a*e^2)*ArcTanh[(Sqr
t[c]*Sqrt[d]*Sqrt[d + e*x])/(Sqrt[e]*Sqrt[a*e + c*d*x])])/(Sqrt[c]*Sqrt[d]
*Sqrt[a*e + c*d*x]*Sqrt[d + e*x])))/e^(3/2)
```

Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 131, normalized size of antiderivative = 1.13, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.081$, Rules used = {1131, 1092, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{d+ex}dx$$

$$\Big\downarrow \text{1131}$$

$$\frac{\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{e}-\frac{\left(cd^2-ae^2\right)\int \frac{1}{\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}dx}{2e}$$

$$\Big\downarrow \text{1092}$$

$$\frac{\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{e}-\frac{\left(cd^2-ae^2\right)\int \frac{1}{4cde-\frac{\left(cd^2+2cexd+ae^2\right)^2}{cdex^2+\left(cd^2+ae^2\right)x+ade}}d\frac{cd^2+2cexd+ae^2}{\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}}{e}$$

$$\Big\downarrow \text{219}$$

$$\frac{\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{e}-\frac{\left(cd^2-ae^2\right)\text{arctanh}\left(\frac{ae^2+cd^2+2cdex}{2\sqrt{c}\sqrt{d}\sqrt{e}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}\right)}{2\sqrt{c}\sqrt{d}e^{3/2}}$$

input
```
Int[Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]/(d + e*x),x]
```

output
```
Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]/e - ((c*d^2 - a*e^2)*ArcTanh[(
c*d^2 + a*e^2 + 2*c*d*e*x)/(2*Sqrt[c]*Sqrt[d]*Sqrt[e]*Sqrt[a*d*e + (c*d^2
+ a*e^2)*x + c*d*e*x^2])])/(2*Sqrt[c]*Sqrt[d]*e^(3/2))
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1092
```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[I
nt[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a
, b, c}, x]
```

rule 1131
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(d + e*x)^(m + 1)*((a + b*x + c*x^2)^p/(e*(m + 2*p + 1))), x
] - Simp[p*((2*c*d - b*e)/(e^2*(m + 2*p + 1)))   Int[(d + e*x)^(m + 1)*(a +
 b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c*d^2 - b
*d*e + a*e^2, 0] && GtQ[p, 0] && (LeQ[-2, m, 0] || EqQ[m + p + 1, 0]) && Ne
Q[m + 2*p + 1, 0] && IntegerQ[2*p]
```

Maple [A] (verified)

Time = 1.77 (sec) , antiderivative size = 131, normalized size of antiderivative = 1.13

method	result	size
default	$\frac{\sqrt{dec\left(x+\frac{d}{e}\right)^{2}+\left(a\,e^{2}-c\,d^{2}\right)\left(x+\frac{d}{e}\right)}+\frac{\left(a\,e^{2}-c\,d^{2}\right)\ln\left(\frac{\frac{a\,e^{2}}{2}-\frac{c\,d^{2}}{2}+dec\left(x+\frac{d}{e}\right)}{\sqrt{dec}}+\sqrt{dec\left(x+\frac{d}{e}\right)^{2}+\left(a\,e^{2}-c\,d^{2}\right)\left(x+\frac{d}{e}\right)}\right)}{2\sqrt{dec}}}{e}$	131

input
```
int((a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(1/2)/(e*x+d),x,method=_RETURNVERBOS
E)
```

output
```
1/e*((d*e*c*(x+d/e)^2+(a*e^2-c*d^2)*(x+d/e))^(1/2)+1/2*(a*e^2-c*d^2)*ln((1
/2*a*e^2-1/2*c*d^2+d*e*c*(x+d/e))/(d*e*c)^(1/2)+(d*e*c*(x+d/e)^2+(a*e^2-c*
d^2)*(x+d/e))^(1/2))/(d*e*c)^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 337, normalized size of antiderivative = 2.91

$$\int \frac{\sqrt{ade + (cd^2 + ae^2)x + cdex^2}}{d + ex} dx$$
$$= \left[\frac{4\sqrt{cdex^2 + ade + (cd^2 + ae^2)x}cde - (cd^2 - ae^2)\sqrt{cde}\log\left(8c^2d^2e^2x^2 + c^2d^4 + 6acd^2e^2 + a^2e^4 + 4\sqrt{\right.}{4cde^2} \right.$$

input
```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(e*x+d),x, algorithm="fr
icas")
```

output
```
[1/4*(4*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*c*d*e - (c*d^2 - a*e^2
)*sqrt(c*d*e)*log(8*c^2*d^2*e^2*x^2 + c^2*d^4 + 6*a*c*d^2*e^2 + a^2*e^4 +
4*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(2*c*d*e*x + c*d^2 + a*e^2)*
sqrt(c*d*e) + 8*(c^2*d^3*e + a*c*d*e^3)*x))/(c*d*e^2), 1/2*(2*sqrt(c*d*e*x
^2 + a*d*e + (c*d^2 + a*e^2)*x)*c*d*e + (c*d^2 - a*e^2)*sqrt(-c*d*e)*arcta
n(1/2*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(2*c*d*e*x + c*d^2 + a*e
^2)*sqrt(-c*d*e)/(c^2*d^2*e^2*x^2 + a*c*d^2*e^2 + (c^2*d^3*e + a*c*d*e^3)*
x))/(c*d*e^2)]
```

Sympy [F]

$$\int \frac{\sqrt{ade + (cd^2 + ae^2)x + cdex^2}}{d + ex} dx = \int \frac{\sqrt{(d + ex)(ae + cdx)}}{d + ex} dx$$

input
```
integrate((a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(1/2)/(e*x+d),x)
```

output
```
Integral(sqrt((d + e*x)*(a*e + c*d*x))/(d + e*x), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\sqrt{ade + (cd^2 + ae^2)\,x + cdex^2}}{d + ex}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(e*x+d),x, algorithm="ma
xima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e*(a*e^2-c*d^2)>0)', see `assume
?` for mor
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 114, normalized size of antiderivative = 0.98

$$\int \frac{\sqrt{ade + (cd^2 + ae^2)\,x + cdex^2}}{d + ex}\,dx = \frac{(cd^2 - ae^2)\log\left(\left|-cd^2 - ae^2 - 2\sqrt{cde}\left(\sqrt{cde}x - \sqrt{cdex^2 + cd^2x + ae^2x + ade}\right)\right|\right)}{2\sqrt{cde}e} + \frac{\sqrt{cdex^2 + cd^2x + ae^2x + ade}}{e}$$

input
```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(e*x+d),x, algorithm="gi
ac")
```

output
```
1/2*(c*d^2 - a*e^2)*log(abs(-c*d^2 - a*e^2 - 2*sqrt(c*d*e)*(sqrt(c*d*e)*x
- sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*x + a*d*e))))/(sqrt(c*d*e)*e) + sqrt(c*
d*e*x^2 + c*d^2*x + a*e^2*x + a*d*e)/e
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{ade + (cd^2 + ae^2) x + cdex^2}}{d + ex} dx = \int \frac{\sqrt{c\,d\,e\,x^2 + (c\,d^2 + a\,e^2)\ x + a\,d\,e}}{d + e\,x} dx$$

input
```
int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2)/(d + e*x),x)
```

output
```
int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2)/(d + e*x), x)
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 135, normalized size of antiderivative = 1.16

$$\int \frac{\sqrt{ade + (cd^2 + ae^2) x + cdex^2}}{d + ex} dx = \frac{\sqrt{ex+d}\,\sqrt{cdx+ae}\,cde + \sqrt{e}\,\sqrt{d}\,\sqrt{c}\log\left(\frac{\sqrt{e}\,\sqrt{cdx+ae}+\sqrt{d}\,\sqrt{c}\,\sqrt{ex+d}}{\sqrt{a\,e^2-c\,d^2}}\right) a\,e^2 - \sqrt{e}\,\sqrt{d}\,\sqrt{c}\log\left(\frac{\sqrt{e}\,\sqrt{cdx+ae}+\sqrt{d}\,\sqrt{c}\,\sqrt{ex+d}}{\sqrt{a\,e^2-c\,d^2}}\right) c\,d^2}{cd\,e^2}$$

input
```
int((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(e*x+d),x)
```

output
```
(sqrt(d + e*x)*sqrt(a*e + c*d*x)*c*d*e + sqrt(e)*sqrt(d)*sqrt(c)*log((sqrt
(e)*sqrt(a*e + c*d*x) + sqrt(d)*sqrt(c)*sqrt(d + e*x))/sqrt(a*e**2 - c*d**
2))*a*e**2 - sqrt(e)*sqrt(d)*sqrt(c)*log((sqrt(e)*sqrt(a*e + c*d*x) + sqrt
(d)*sqrt(c)*sqrt(d + e*x))/sqrt(a*e**2 - c*d**2))*c*d**2)/(c*d*e**2)
```

3.205 $\int \frac{\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{(d+ex)^2}\,dx$

Optimal result

Integrand size = 37, antiderivative size = 112

$$\int \frac{\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{(d+ex)^2}\,dx = -\frac{2\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{e(d+ex)} + \frac{2\sqrt{c}\sqrt{d}\operatorname{arctanh}\left(\frac{\sqrt{e}\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{\sqrt{c}\sqrt{d}(d+ex)}\right)}{e^{3/2}}$$

output
```
-2*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/e/(e*x+d)+2*c^(1/2)*d^(1/2)*arc
tanh(e^(1/2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/c^(1/2)/d^(1/2)/(e*x+
d))/e^(3/2)
```

Mathematica [A] (verified)

Time = 0.16 (sec) , antiderivative size = 110, normalized size of antiderivative = 0.98

$$\int \frac{\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{(d+ex)^2}\,dx = \frac{2\sqrt{(ae+cdx)(d+ex)}\left(-\frac{\sqrt{e}}{d+ex}+\frac{\sqrt{c}\sqrt{d}\operatorname{arctanh}\left(\frac{\sqrt{c}\sqrt{d}\sqrt{d+ex}}{\sqrt{e}\sqrt{ae+cdx}}\right)}{\sqrt{ae+cdx}\sqrt{d+ex}}\right)}{e^{3/2}}$$

input
```
Integrate[Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]/(d + e*x)^2,x]
```

output
```
(2*Sqrt[(a*e + c*d*x)*(d + e*x)]*(-(Sqrt[e]/(d + e*x)) + (Sqrt[c]*Sqrt[d]*
ArcTanh[(Sqrt[c]*Sqrt[d]*Sqrt[d + e*x])/(Sqrt[e]*Sqrt[a*e + c*d*x])])/(Sqr
t[a*e + c*d*x]*Sqrt[d + e*x])))/e^(3/2)
```

Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 124, normalized size of antiderivative = 1.11, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.135$, Rules used = {1125, 25, 27, 1092, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{(d+ex)^2}dx$$

$$\Big\downarrow \text{1125}$$

$$-\frac{\int -\frac{cde}{\sqrt{cdex^2+(cd^2+ae^2)x+ade}}dx}{e^2}-\frac{2\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{e(d+ex)}$$

$$\Big\downarrow \text{25}$$

$$\frac{\int \frac{cde}{\sqrt{cdex^2+(cd^2+ae^2)x+ade}}dx}{e^2}-\frac{2\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{e(d+ex)}$$

$$\Big\downarrow \text{27}$$

$$\frac{cd\int \frac{1}{\sqrt{cdex^2+(cd^2+ae^2)x+ade}}dx}{e}-\frac{2\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{e(d+ex)}$$

$$\Big\downarrow \text{1092}$$

$$\frac{2cd\int \frac{1}{4cde-\frac{\left(cd^2+2cexd+ae^2\right)^2}{cdex^2+\left(cd^2+ae^2\right)x+ade}}d\frac{cd^2+2cexd+ae^2}{\sqrt{cdex^2+(cd^2+ae^2)x+ade}}}{e}-\frac{2\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{e(d+ex)}$$

$$\Big\downarrow \text{219}$$

$$\frac{\sqrt{c}\sqrt{d}\operatorname{arctanh}\left(\frac{ae^2+cd^2+2cdex}{2\sqrt{c}\sqrt{d}\sqrt{e}\sqrt{x(ae^2+cd^2)+ade+cdex^2}}\right)}{e^{3/2}}-\frac{2\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{e(d+ex)}$$

input
```
Int[Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]/(d + e*x)^2,x]
```

output
```
(-2*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(e*(d + e*x)) + (Sqrt[c]*
Sqrt[d]*ArcTanh[(c*d^2 + a*e^2 + 2*c*d*e*x)/(2*Sqrt[c]*Sqrt[d]*Sqrt[e]*Sqr
t[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])])/e^(3/2)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1092
```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[I
nt[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a
, b, c}, x]
```

rule 1125
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_
Symbol] :> Simp[-2*e^(2*m + 3)*(Sqrt[a + b*x + c*x^2]/((-2*c*d + b*e)^(m +
2)*(d + e*x))), x] - Simp[e^(2*m + 2)   Int[(1/Sqrt[a + b*x + c*x^2])*Expan
dToSum[((-2*c*d + b*e)^(-m - 1) - ((-c)*d + b*e + c*e*x)^(-m - 1))/(d + e*x
), x], x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0]
 && ILtQ[m, 0] && EqQ[m + p, -3/2]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 211 vs. $2(96) = 192$.

Time = 1.87 (sec) , antiderivative size = 212, normalized size of antiderivative = 1.89

method	result
default	$\frac{-\frac{2\left(dec\left(x+\frac{d}{e}\right)^{2}+\left(a\,e^{2}-c\,d^{2}\right)\left(x+\frac{d}{e}\right)\right)^{\frac{3}{2}}}{\left(a\,e^{2}-c\,d^{2}\right)\left(x+\frac{d}{e}\right)^{2}}+\frac{2dec\left(\sqrt{dec\left(x+\frac{d}{e}\right)^{2}+\left(a\,e^{2}-c\,d^{2}\right)\left(x+\frac{d}{e}\right)}+\frac{\left(a\,e^{2}-c\,d^{2}\right)\ln\left(\frac{\frac{a\,e^{2}}{2}-\frac{c\,d^{2}}{2}+dec\left(x+\frac{d}{e}\right)}{\sqrt{dec}}+\sqrt{dec\left(x+\frac{d}{e}\right)}\right.}{2\sqrt{dec}}\right.}{a\,e^{2}-c\,d^{2}}}{e^{2}}$

input
```
int((a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(1/2)/(e*x+d)^2,x,method=_RETURNVERB
OSE)
```

output
```
1/e^2*(-2/(a*e^2-c*d^2)/(x+d/e)^2*(d*e*c*(x+d/e)^2+(a*e^2-c*d^2)*(x+d/e))^
(3/2)+2*d*e*c/(a*e^2-c*d^2)*((d*e*c*(x+d/e)^2+(a*e^2-c*d^2)*(x+d/e))^(1/2)
+1/2*(a*e^2-c*d^2)*ln((1/2*a*e^2-1/2*c*d^2+d*e*c*(x+d/e))/(d*e*c)^(1/2)+(d
*e*c*(x+d/e)^2+(a*e^2-c*d^2)*(x+d/e))^(1/2))/(d*e*c)^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 326, normalized size of antiderivative = 2.91

$$\int \frac{\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{(d+ex)^2}\,dx$$

$$= \left[\frac{(ex+d)\sqrt{\frac{cd}{e}}\log\left(8\,c^2d^2e^2x^2+c^2d^4+6\,acd^2e^2+a^2e^4+4\,(2\,cde^2x+cd^2e+ae^3)\sqrt{cdex^2+ade+(cd^2\right.}{2\,(e^2x+de)}\right.$$

$$\left.-\frac{(ex+d)\sqrt{-\frac{cd}{e}}\arctan\left(\frac{\sqrt{cdex^2+ade+(cd^2+ae^2)x}\,(2\,cdex+cd^2+ae^2)\sqrt{-\frac{cd}{e}}}{2\,(c^2d^2ex^2+acd^2e+(c^2d^3+acde^2)x)}\right)+2\,\sqrt{cdex^2+ade+(cd^2+ae^2)x}}{e^2x+de}\right]$$

input
```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(e*x+d)^2,x, algorithm="
fricas")
```

output
```
[1/2*((e*x + d)*sqrt(c*d/e)*log(8*c^2*d^2*e^2*x^2 + c^2*d^4 + 6*a*c*d^2*e^
2 + a^2*e^4 + 4*(2*c*d*e^2*x + c*d^2*e + a*e^3)*sqrt(c*d*e*x^2 + a*d*e + (
c*d^2 + a*e^2)*x)*sqrt(c*d/e) + 8*(c^2*d^3*e + a*c*d*e^3)*x) - 4*sqrt(c*d*
e*x^2 + a*d*e + (c*d^2 + a*e^2)*x))/(e^2*x + d*e), -((e*x + d)*sqrt(-c*d/e
)*arctan(1/2*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(2*c*d*e*x + c*d^
2 + a*e^2)*sqrt(-c*d/e)/(c^2*d^2*e*x^2 + a*c*d^2*e + (c^2*d^3 + a*c*d*e^2)
*x)) + 2*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x))/(e^2*x + d*e)]
```

Sympy [F]

$$\int \frac{\sqrt{ade + (cd^2 + ae^2)\,x + cdex^2}}{(d + ex)^2}\,dx = \int \frac{\sqrt{(d + ex)\,(ae + cdx)}}{(d + ex)^2}\,dx$$

input
```
integrate((a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(1/2)/(e*x+d)**2,x)
```

output
```
Integral(sqrt((d + e*x)*(a*e + c*d*x))/(d + e*x)**2, x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\sqrt{ade + (cd^2 + ae^2)\,x + cdex^2}}{(d + ex)^2}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(e*x+d)^2,x, algorithm="
maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e*(a*e^2-c*d^2)>0)', see `assume
?` for mor
```

Giac [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 185, normalized size of antiderivative = 1.65

$$\int \frac{\sqrt{ade + (cd^2 + ae^2)x + cdex^2}}{(d+ex)^2}\,dx =$$
$$-2\left(\frac{cd\arctan\left(\frac{\sqrt{cde-\frac{cd^2e}{ex+d}+\frac{ae^3}{ex+d}}}{\sqrt{-cde}}\right)\operatorname{sgn}\left(\frac{1}{ex+d}\right)\operatorname{sgn}(e)}{\sqrt{-cde}e|e|} + \frac{\sqrt{cde-\frac{cd^2e}{ex+d}+\frac{ae^3}{ex+d}}\operatorname{sgn}\left(\frac{1}{ex+d}\right)\operatorname{sgn}(e)}{e^2|e|} - \frac{\left(cde\operatorname{arc}\right.}{}\right.$$

input
```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(e*x+d)^2,x, algorithm="
giac")
```

output
```
-2*(c*d*arctan(sqrt(c*d*e - c*d^2*e/(e*x + d) + a*e^3/(e*x + d))/sqrt(-c*d
*e))*sgn(1/(e*x + d))*sgn(e)/(sqrt(-c*d*e)*e*abs(e)) + sqrt(c*d*e - c*d^2*
e/(e*x + d) + a*e^3/(e*x + d))*sgn(1/(e*x + d))*sgn(e)/(e^2*abs(e)) - (c*d
*e*arctan(sqrt(c*d*e)/sqrt(-c*d*e)) + sqrt(c*d*e)*sqrt(-c*d*e))*sgn(1/(e*x
 + d))*sgn(e)/(sqrt(-c*d*e)*e^2*abs(e)))*abs(e)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{ade + (cd^2 + ae^2)x + cdex^2}}{(d+ex)^2}\,dx = \int \frac{\sqrt{c\,d\,e\,x^2 + (c\,d^2 + a\,e^2)\,x + a\,d\,e}}{(d+e\,x)^2}\,dx$$

input
```
int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2)/(d + e*x)^2,x)
```

output
```
int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2)/(d + e*x)^2, x)
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 149, normalized size of antiderivative = 1.33

$$\int \frac{\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{(d+ex)^2}\,dx$$
$$= \frac{-2\sqrt{ex+d}\,\sqrt{cdx+ae}\,e+2\sqrt{e}\,\sqrt{d}\,\sqrt{c}\log\left(\frac{\sqrt{e}\,\sqrt{cdx+ae}+\sqrt{d}\,\sqrt{c}\,\sqrt{ex+d}}{\sqrt{a\,e^2-c\,d^2}}\right)d+2\sqrt{e}\,\sqrt{d}\,\sqrt{c}\log\left(\frac{\sqrt{e}\,\sqrt{cdx+ae}+\sqrt{d}\,\sqrt{c}}{\sqrt{a\,e^2-c\,d^2}}\right.}{e^2\,(ex+d)}$$

input
```
int((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(e*x+d)^2,x)
```

output
```
(2*( - sqrt(d + e*x)*sqrt(a*e + c*d*x)*e + sqrt(e)*sqrt(d)*sqrt(c)*log((sq
rt(e)*sqrt(a*e + c*d*x) + sqrt(d)*sqrt(c)*sqrt(d + e*x))/sqrt(a*e**2 - c*d
**2))*d + sqrt(e)*sqrt(d)*sqrt(c)*log((sqrt(e)*sqrt(a*e + c*d*x) + sqrt(d)
*sqrt(c)*sqrt(d + e*x))/sqrt(a*e**2 - c*d**2))*e*x - sqrt(e)*sqrt(d)*sqrt(
c)*d - sqrt(e)*sqrt(d)*sqrt(c)*e*x))/(e**2*(d + e*x))
```

3.206 $\int \frac{\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{(d+ex)^3} dx$

Optimal result

Integrand size = 37, antiderivative size = 54

$$\int \frac{\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{(d+ex)^3}\,dx=\frac{2(ade+(cd^2+ae^2)\,x+cdex^2)^{3/2}}{3\,(cd^2-ae^2)\,(d+ex)^3}$$

output
```
2/3*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/(-a*e^2+c*d^2)/(e*x+d)^3
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.80

$$\int \frac{\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{(d+ex)^3}\,dx=\frac{2((ae+cdx)(d+ex))^{3/2}}{3\,(cd^2-ae^2)\,(d+ex)^3}$$

input
```
Integrate[Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]/(d + e*x)^3,x]
```

output
```
(2*((a*e + c*d*x)*(d + e*x))^(3/2))/(3*(c*d^2 - a*e^2)*(d + e*x)^3)
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.027, Rules used = {1123}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{(d+ex)^3}dx$$

$$\Big\downarrow \text{1123}$$

$$\frac{2\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{3(d+ex)^3\left(cd^2-ae^2\right)}$$

input
```
Int[Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]/(d + e*x)^3,x]
```

output
```
(2*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2))/(3*(c*d^2 - a*e^2)*(d +
e*x)^3)
```

Defintions of rubi rules used

rule 1123
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[e*(d + e*x)^m*((a + b*x + c*x^2)^(p + 1)/((p + 1)*(2*c*d - b
*e))), x] /; FreeQ[{a, b, c, d, e, m, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2,
0] && EqQ[m + 2*p + 2, 0]
```

Maple [A] (verified)

Time = 2.20 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.07

method	result	size
gosper	$-\frac{2(cdx+ae)\sqrt{cd\,x^{2}e+a\,e^{2}x+c\,d^{2}x+ade}}{3(ex+d)^{2}(a\,e^{2}-c\,d^{2})}$	58
trager	$-\frac{2(cdx+ae)\sqrt{cd\,x^{2}e+a\,e^{2}x+c\,d^{2}x+ade}}{3(ex+d)^{2}(a\,e^{2}-c\,d^{2})}$	58
orering	$-\frac{2(cdx+ae)\sqrt{ade+(a\,e^{2}+c\,d^{2})x+cd\,x^{2}e}}{3(ex+d)^{2}(a\,e^{2}-c\,d^{2})}$	59
default	$-\frac{2\left(dec\left(x+\frac{d}{e}\right)^{2}+(a\,e^{2}-c\,d^{2})\left(x+\frac{d}{e}\right)\right)^{\frac{3}{2}}}{3e^{3}(a\,e^{2}-c\,d^{2})\left(x+\frac{d}{e}\right)^{3}}$	65

input
```
int((a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(1/2)/(e*x+d)^3,x,method=_RETURNVERB
OSE)
```

output
```
-2/3/(e*x+d)^2*(c*d*x+a*e)/(a*e^2-c*d^2)*(c*d*e*x^2+a*e^2*x+c*d^2*x+a*d*e)
^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.67

$$\int \frac{\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{(d+ex)^3}\,dx = \frac{2\,\sqrt{cdex^2+ade+(cd^2+ae^2)x}(cdx+ae)}{3\,(cd^4-ad^2e^2+(cd^2e^2-ae^4)x^2+2\,(cd^3e-ade^3)x)}$$

input
```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(e*x+d)^3,x, algorithm="
fricas")
```

output
```
2/3*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(c*d*x + a*e)/(c*d^4 - a*d
^2*e^2 + (c*d^2*e^2 - a*e^4)*x^2 + 2*(c*d^3*e - a*d*e^3)*x)
```

Sympy [F]

$$\int \frac{\sqrt{ade + (cd^2 + ae^2)\,x + cdex^2}}{(d+ex)^3}\,dx = \int \frac{\sqrt{(d+ex)\,(ae+cdx)}}{(d+ex)^3}\,dx$$

input
```
integrate((a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(1/2)/(e*x+d)**3,x)
```

output
```
Integral(sqrt((d + e*x)*(a*e + c*d*x))/(d + e*x)**3, x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\sqrt{ade + (cd^2 + ae^2)\,x + cdex^2}}{(d+ex)^3}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(e*x+d)^3,x, algorithm="
maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e*(a*e^2-c*d^2)>0)', see `assume
?` for mor
```

Giac [F(-2)]

Exception generated.

$$\int \frac{\sqrt{ade + (cd^2 + ae^2)\,x + cdex^2}}{(d+ex)^3}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(e*x+d)^3,x, algorithm="
giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Unable to divide, perhaps due to ro
unding error%%%{%%%{1,[0,0,2]%%%},[4]%%%}+%%%{%%{[%%%{-4,[0,1,1]%%%},0]:[1
,0,%%%{-1
```

Mupad [B] (verification not implemented)

Time = 5.54 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.07

$$\int \frac{\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{(d+ex)^3}\,dx=-\frac{2\,(a\,e+c\,d\,x)\,\sqrt{c\,d\,e\,x^2+(c\,d^2+a\,e^2)\,x+a\,d\,e}}{3\,(a\,e^2-c\,d^2)\,(d+e\,x)^2}$$

input
```
int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2)/(d + e*x)^3,x)
```

output
```
-(2*(a*e + c*d*x)*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2))/(3*(a*e^2
 - c*d^2)*(d + e*x)^2)
```

Reduce [B] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 145, normalized size of antiderivative = 2.69

$$\int \frac{\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{(d+ex)^3}\,dx$$
$$=\frac{-\frac{2\sqrt{ex+d}\,\sqrt{cdx+ae}\,a\,e^3}{3}-\frac{2\sqrt{ex+d}\,\sqrt{cdx+ae}\,cd\,e^2x}{3}-\frac{2\sqrt{e}\,\sqrt{d}\,\sqrt{c}\,c\,d^3}{3}-\frac{4\sqrt{e}\,\sqrt{d}\,\sqrt{c}\,c\,d^2ex}{3}-\frac{2\sqrt{e}\,\sqrt{d}\,\sqrt{c}\,cd\,e^2x^2}{3}}{e^2\,(a\,e^4x^2-c\,d^2e^2x^2+2ad\,e^3x-2c\,d^3ex+a\,d^2e^2-c\,d^4)}$$

input
```
int((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(e*x+d)^3,x)
```

output
```
(2*( - sqrt(d + e*x)*sqrt(a*e + c*d*x)*a*e**3 - sqrt(d + e*x)*sqrt(a*e + c
*d*x)*c*d*e**2*x - sqrt(e)*sqrt(d)*sqrt(c)*c*d**3 - 2*sqrt(e)*sqrt(d)*sqrt
(c)*c*d**2*e*x - sqrt(e)*sqrt(d)*sqrt(c)*c*d*e**2*x**2))/(3*e**2*(a*d**2*e
**2 + 2*a*d*e**3*x + a*e**4*x**2 - c*d**4 - 2*c*d**3*e*x - c*d**2*e**2*x**
2))
```

3.207 $\int \frac{\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{(d+ex)^4} dx$

Optimal result

Integrand size = 37, antiderivative size = 111

$$\int \frac{\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{(d+ex)^4} dx = \frac{2(ade+(cd^2+ae^2)x+cdex^2)^{3/2}}{5(cd^2-ae^2)(d+ex)^4} + \frac{4cd(ade+(cd^2+ae^2)x+cdex^2)^{3/2}}{15(cd^2-ae^2)^2(d+ex)^3}$$

output
```
2/5*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/(-a*e^2+c*d^2)/(e*x+d)^4+4/15*
c*d*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/(-a*e^2+c*d^2)^2/(e*x+d)^3
```

Mathematica [A] (verified)

Time = 0.24 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.73

$$\int \frac{\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{(d+ex)^4} dx = \frac{2\sqrt{(ae+cdx)(d+ex)}(-3a^2e^3+acde(5d-ex)+c^2d^2x(5d+2ex))}{15(cd^2-ae^2)^2(d+ex)^3}$$

input
```
Integrate[Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]/(d + e*x)^4,x]
```

output
```
(2*Sqrt[(a*e + c*d*x)*(d + e*x)]*(-3*a^2*e^3 + a*c*d*e*(5*d - e*x) + c^2*d
^2*x*(5*d + 2*e*x)))/(15*(c*d^2 - a*e^2)^2*(d + e*x)^3)
```

Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 111, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.054$, Rules used = {1129, 1123}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{(d+ex)^4}dx$$

$$\Big\downarrow \text{1129}$$

$$\frac{2cd\int \frac{\sqrt{cdex^2+(cd^2+ae^2)x+ade}}{(d+ex)^3}dx}{5\left(cd^2-ae^2\right)}+\frac{2\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{5(d+ex)^4\left(cd^2-ae^2\right)}$$

$$\Big\downarrow \text{1123}$$

$$\frac{4cd\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{15(d+ex)^3\left(cd^2-ae^2\right)^2}+\frac{2\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{5(d+ex)^4\left(cd^2-ae^2\right)}$$

input
```
Int[Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]/(d + e*x)^4,x]
```

output
```
(2*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2))/(5*(c*d^2 - a*e^2)*(d +
e*x)^4) + (4*c*d*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2))/(15*(c*d^2
 - a*e^2)^2*(d + e*x)^3)
```

Defintions of rubi rules used

rule 1123
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[e*(d + e*x)^m*((a + b*x + c*x^2)^(p + 1)/((p + 1)*(2*c*d - b
*e))), x] /; FreeQ[{a, b, c, d, e, m, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2,
0] && EqQ[m + 2*p + 2, 0]
```

rule 1129
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(-e)*(d + e*x)^m*((a + b*x + c*x^2)^(p + 1)/((m + p + 1)*(2*
c*d - b*e))), x] + Simp[c*(Simplify[m + 2*p + 2]/((m + p + 1)*(2*c*d - b*e)
))   Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d
, e, m, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && ILtQ[Simplify[m + 2*p +
2], 0]
```

Maple [A] (verified)

Time = 2.44 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.81

method	result	size
gosper	$-\frac{2(cdx+ae)\left(-2cdxe+3a\,e^{2}-5c\,d^{2}\right)\sqrt{cd\,x^{2}e+a\,e^{2}x+c\,d^{2}x+ade}}{15(ex+d)^{3}\left(a^{2}e^{4}-2ac\,d^{2}e^{2}+c^{2}d^{4}\right)}$	90
orering	$-\frac{2\left(-2cdxe+3a\,e^{2}-5c\,d^{2}\right)(cdx+ae)\sqrt{ade+\left(a\,e^{2}+c\,d^{2}\right)x+cd\,x^{2}e}}{15\left(a^{2}e^{4}-2ac\,d^{2}e^{2}+c^{2}d^{4}\right)(ex+d)^{3}}$	91
trager	$-\frac{2\left(-2d^{2}e\,c^{2}x^{2}+acd\,e^{2}x-5c^{2}d^{3}x+3a^{2}e^{3}-5d^{2}eac\right)\sqrt{cd\,x^{2}e+a\,e^{2}x+c\,d^{2}x+ade}}{15\left(a^{2}e^{4}-2ac\,d^{2}e^{2}+c^{2}d^{4}\right)(ex+d)^{3}}$	109
default	$\frac{-\frac{2\left(dec\left(x+\frac{d}{e}\right)^{2}+\left(a\,e^{2}-c\,d^{2}\right)\left(x+\frac{d}{e}\right)\right)^{\frac{3}{2}}}{5\left(a\,e^{2}-c\,d^{2}\right)\left(x+\frac{d}{e}\right)^{4}}+\frac{4dec\left(dec\left(x+\frac{d}{e}\right)^{2}+\left(a\,e^{2}-c\,d^{2}\right)\left(x+\frac{d}{e}\right)\right)^{\frac{3}{2}}}{15\left(a\,e^{2}-c\,d^{2}\right)^{2}\left(x+\frac{d}{e}\right)^{3}}}{e^{4}}$	131

input
```
int((a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(1/2)/(e*x+d)^4,x,method=_RETURNVERB
OSE)
```

output
```
-2/15*(c*d*x+a*e)*(-2*c*d*e*x+3*a*e^2-5*c*d^2)*(c*d*e*x^2+a*e^2*x+c*d^2*x+
a*d*e)^(1/2)/(e*x+d)^3/(a^2*e^4-2*a*c*d^2*e^2+c^2*d^4)
```

Fricas [A] (verification not implemented)

Time = 0.43 (sec) , antiderivative size = 205, normalized size of antiderivative = 1.85

$$\int \frac{\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{(d+ex)^4}\,dx$$
$$= \frac{2\,(2\,c^2d^2ex^2+5\,acd^2e-3\,a^2e^3+(5\,c^2d^3-acde^2)x)\sqrt{cdex^2+ade+(cd^2+ae^2)}}{15\,(c^2d^7-2\,acd^5e^2+a^2d^3e^4+(c^2d^4e^3-2\,acd^2e^5+a^2e^7)x^3+3\,(c^2d^5e^2-2\,acd^3e^4+a^2de^6)x^2+3\,(c^2d^6}$$

input
```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(e*x+d)^4,x, algorithm="
fricas")
```

output
```
2/15*(2*c^2*d^2*e*x^2 + 5*a*c*d^2*e - 3*a^2*e^3 + (5*c^2*d^3 - a*c*d*e^2)*
x)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)/(c^2*d^7 - 2*a*c*d^5*e^2 +
a^2*d^3*e^4 + (c^2*d^4*e^3 - 2*a*c*d^2*e^5 + a^2*e^7)*x^3 + 3*(c^2*d^5*e^2
 - 2*a*c*d^3*e^4 + a^2*d*e^6)*x^2 + 3*(c^2*d^6*e - 2*a*c*d^4*e^3 + a^2*d^2
*e^5)*x)
```

Sympy [F]

$$\int \frac{\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{(d+ex)^4}\,dx = \int \frac{\sqrt{(d+ex)\,(ae+cdx)}}{(d+ex)^4}\,dx$$

input
```
integrate((a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(1/2)/(e*x+d)**4,x)
```

output
```
Integral(sqrt((d + e*x)*(a*e + c*d*x))/(d + e*x)**4, x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\sqrt{ade + (cd^2 + ae^2)x + cdex^2}}{(d + ex)^4}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(e*x+d)^4,x, algorithm="
maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e*(a*e^2-c*d^2)>0)', see `assume
?` for mor
```

Giac [F(-2)]

Exception generated.

$$\int \frac{\sqrt{ade + (cd^2 + ae^2)x + cdex^2}}{(d + ex)^4}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(e*x+d)^4,x, algorithm="
giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Unable to divide, perhaps due to ro
unding error%%%{%%%{1,[0,0,3]%%%},[6]%%%}+%%%{%%{[%%%{-6,[0,1,2]%%%},0]:[1
,0,%%%{-1
```

Mupad [B] (verification not implemented)

Time = 5.66 (sec) , antiderivative size = 562, normalized size of antiderivative = 5.06

$$\int \frac{\sqrt{ade + (cd^2 + ae^2)x + cdex^2}}{(d+ex)^4}\,dx$$
$$= \frac{\left(\frac{4c^2d^3}{5(ae^2-cd^2)(3ae^3-3cd^2e)} - \frac{4acde^2}{5(ae^2-cd^2)(3ae^3-3cd^2e)}\right)\sqrt{cdex^2 + (cd^2+ae^2)\,x + ade}}{(d+ex)^2}$$
$$- \frac{\left(\frac{2ae^2}{5ae^3-5cd^2e} - \frac{2cd^2}{5ae^3-5cd^2e}\right)\sqrt{cdex^2 + (cd^2+ae^2)\,x + ade}}{(d+ex)^3}$$
$$+ \frac{\left(\frac{4c^3d^4+4ac^2d^2e^2}{15e(ae^2-cd^2)^3} - \frac{8c^3d^4}{15e(ae^2-cd^2)^3}\right)\sqrt{cdex^2 + (cd^2+ae^2)\,x + ade}}{d+ex}$$
$$+ \frac{\left(\frac{2c^2d^3+2acde^2}{5(ae^2-cd^2)(3ae^3-3cd^2e)} - \frac{4c^2d^3}{5(ae^2-cd^2)(3ae^3-3cd^2e)}\right)\sqrt{cdex^2 + (cd^2+ae^2)\,x + ade}}{(d+ex)^2}$$
$$+ \frac{\left(\frac{8c^3d^4}{15e(ae^2-cd^2)^3} - \frac{8ac^2d^2e}{15(ae^2-cd^2)^3}\right)\sqrt{cdex^2 + (cd^2+ae^2)\,x + ade}}{d+ex}$$
$$+ \frac{8c^2d^2\sqrt{cdex^2 + (cd^2+ae^2)\,x + ade}}{15e(ae^2-cd^2)^2(d+ex)}$$

input
```
int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2)/(d + e*x)^4,x)
```

output
```
(((4*c^2*d^3)/(5*(a*e^2 - c*d^2)*(3*a*e^3 - 3*c*d^2*e)) - (4*a*c*d*e^2)/(5
*(a*e^2 - c*d^2)*(3*a*e^3 - 3*c*d^2*e)))*(x*(a*e^2 + c*d^2) + a*d*e + c*d*
e*x^2)^(1/2))/(d + e*x)^2 - (((2*a*e^2)/(5*a*e^3 - 5*c*d^2*e) - (2*c*d^2)/
(5*a*e^3 - 5*c*d^2*e))*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2))/(d +
 e*x)^3 + (((4*c^3*d^4 + 4*a*c^2*d^2*e^2)/(15*e*(a*e^2 - c*d^2)^3) - (8*c^
3*d^4)/(15*e*(a*e^2 - c*d^2)^3))*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(
1/2))/(d + e*x) + (((2*c^2*d^3 + 2*a*c*d*e^2)/(5*(a*e^2 - c*d^2)*(3*a*e^3
- 3*c*d^2*e)) - (4*c^2*d^3)/(5*(a*e^2 - c*d^2)*(3*a*e^3 - 3*c*d^2*e)))*(x*
(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2))/(d + e*x)^2 + (((8*c^3*d^4)/(1
5*e*(a*e^2 - c*d^2)^3) - (8*a*c^2*d^2*e)/(15*(a*e^2 - c*d^2)^3))*(x*(a*e^2
 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2))/(d + e*x) + (8*c^2*d^2*(x*(a*e^2 + c
*d^2) + a*d*e + c*d*e*x^2)^(1/2))/(15*e*(a*e^2 - c*d^2)^2*(d + e*x))
```

Reduce [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 342, normalized size of antiderivative = 3.08

$$\int \frac{\sqrt{ade + (cd^2 + ae^2)\,x + cdex^2}}{(d + ex)^4}\,dx$$

$$= \frac{-\frac{2\sqrt{ex+d}\,\sqrt{cdx+ae}\,a^2e^5}{5} + \frac{2\sqrt{ex+d}\,\sqrt{cdx+ae}\,ac\,d^2e^3}{3} - \frac{2\sqrt{ex+d}\,\sqrt{cdx+ae}\,acd\,e^4x}{15} + \frac{2\sqrt{ex+d}\,\sqrt{cdx+ae}\,c^2d^3e^2x}{3} + \frac{4\sqrt{ex+d}\,\sqrt{cdx+a}}{15}}{e^2\left(a^2e^7x^3 - 2ac\,d^2e^5x^3 + c^2d^4e^3x^3 + 3a^2d\,e^6x^2 - 6ac\,d^3e^4x^2 + 3c^2d^5e^2x^2 + 3a^2d^2\right.}$$

input
```
int((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(e*x+d)^4,x)
```

output
```
(2*( - 3*sqrt(d + e*x)*sqrt(a*e + c*d*x)*a**2*e**5 + 5*sqrt(d + e*x)*sqrt(
a*e + c*d*x)*a*c*d**2*e**3 - sqrt(d + e*x)*sqrt(a*e + c*d*x)*a*c*d*e**4*x
+ 5*sqrt(d + e*x)*sqrt(a*e + c*d*x)*c**2*d**3*e**2*x + 2*sqrt(d + e*x)*sqr
t(a*e + c*d*x)*c**2*d**2*e**3*x**2 - 2*sqrt(e)*sqrt(d)*sqrt(c)*c**2*d**5 -
 6*sqrt(e)*sqrt(d)*sqrt(c)*c**2*d**4*e*x - 6*sqrt(e)*sqrt(d)*sqrt(c)*c**2*
d**3*e**2*x**2 - 2*sqrt(e)*sqrt(d)*sqrt(c)*c**2*d**2*e**3*x**3))/(15*e**2*
(a**2*d**3*e**4 + 3*a**2*d**2*e**5*x + 3*a**2*d*e**6*x**2 + a**2*e**7*x**3
 - 2*a*c*d**5*e**2 - 6*a*c*d**4*e**3*x - 6*a*c*d**3*e**4*x**2 - 2*a*c*d**2
*e**5*x**3 + c**2*d**7 + 3*c**2*d**6*e*x + 3*c**2*d**5*e**2*x**2 + c**2*d*
*4*e**3*x**3))
```

3.208 $\int \frac{\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{(d+ex)^5} dx$

Optimal result

Integrand size = 37, antiderivative size = 171

$$\int \frac{\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{(d+ex)^5} dx = \frac{2(ade+(cd^2+ae^2)x+cdex^2)^{3/2}}{7(cd^2-ae^2)(d+ex)^5} + \frac{8cd(ade+(cd^2+ae^2)x+cdex^2)^{3/2}}{35(cd^2-ae^2)^2(d+ex)^4} + \frac{16c^2d^2(ade+(cd^2+ae^2)x+cdex^2)^{3/2}}{105(cd^2-ae^2)^3(d+ex)^3}$$

output
```
2/7*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/(-a*e^2+c*d^2)/(e*x+d)^5+8/35*
c*d*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/(-a*e^2+c*d^2)^2/(e*x+d)^4+16/
105*c^2*d^2*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/(-a*e^2+c*d^2)^3/(e*x+
d)^3
```

Mathematica [A] (verified)

Time = 1.56 (sec) , antiderivative size = 124, normalized size of antiderivative = 0.73

$$\int \frac{\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{(d+ex)^5}\,dx$$
$$=\frac{2\sqrt{(ae+cdx)(d+ex)}(15a^3e^5+3a^2cde^3(-14d+ex)+ac^2d^2e(35d^2-14dex-4e^2x^2)+c^3d^3x(35d^2+}{105\,(cd^2-ae^2)^3\,(d+ex)^4}$$

input
```
Integrate[Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]/(d + e*x)^5,x]
```

output
```
(2*Sqrt[(a*e + c*d*x)*(d + e*x)]*(15*a^3*e^5 + 3*a^2*c*d*e^3*(-14*d + e*x)
 + a*c^2*d^2*e*(35*d^2 - 14*d*e*x - 4*e^2*x^2) + c^3*d^3*x*(35*d^2 + 28*d*
e*x + 8*e^2*x^2)))/(105*(c*d^2 - a*e^2)^3*(d + e*x)^4)
```

Rubi [A] (verified)

Time = 0.53 (sec) , antiderivative size = 186, normalized size of antiderivative = 1.09, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.081, Rules used = {1129, 1129, 1123}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{x\,(ae^2+cd^2)+ade+cdex^2}}{(d+ex)^5}\,dx$$

$$\Big\downarrow \text{1129}$$

$$\frac{4cd\int \frac{\sqrt{cdex^2+(cd^2+ae^2)x+ade}}{(d+ex)^4}dx}{7\,(cd^2-ae^2)}+\frac{2\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{7(d+ex)^5\,(cd^2-ae^2)}$$

$$\Big\downarrow \text{1129}$$

$$\frac{4cd\left(\frac{2cd\int\frac{\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}{(d+ex)^3}dx}{5\left(cd^2-ae^2\right)}+\frac{2\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{5(d+ex)^4\left(cd^2-ae^2\right)}\right)}{7\left(cd^2-ae^2\right)}+\frac{2\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{7(d+ex)^5\left(cd^2-ae^2\right)}$$

$$\Big\downarrow \text{1123}$$

$$\frac{2\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{7(d+ex)^5\left(cd^2-ae^2\right)}+\frac{4cd\left(\frac{4cd\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{15(d+ex)^3\left(cd^2-ae^2\right)^2}+\frac{2\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{5(d+ex)^4\left(cd^2-ae^2\right)}\right)}{7\left(cd^2-ae^2\right)}$$

input
```
Int[Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]/(d + e*x)^5,x]
```

output
```
(2*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2))/(7*(c*d^2 - a*e^2)*(d +
e*x)^5) + (4*c*d*((2*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2))/(5*(c*
d^2 - a*e^2)*(d + e*x)^4) + (4*c*d*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)
^(3/2))/(15*(c*d^2 - a*e^2)^2*(d + e*x)^3)))/(7*(c*d^2 - a*e^2))
```

Defintions of rubi rules used

rule 1123
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[e*(d + e*x)^m*((a + b*x + c*x^2)^(p + 1)/((p + 1)*(2*c*d - b
*e))), x] /; FreeQ[{a, b, c, d, e, m, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2,
0] && EqQ[m + 2*p + 2, 0]
```

rule 1129
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(-e)*(d + e*x)^m*((a + b*x + c*x^2)^(p + 1)/((m + p + 1)*(2*
c*d - b*e))), x] + Simp[c*(Simplify[m + 2*p + 2]/((m + p + 1)*(2*c*d - b*e)
))   Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d
, e, m, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && ILtQ[Simplify[m + 2*p +
2], 0]
```

Maple [A] (verified)

Time = 2.97 (sec) , antiderivative size = 146, normalized size of antiderivative = 0.85

method	result
gosper	$-\frac{2(cdx+ae)\left(8x^2c^2d^2e^2-12xacd\,e^3+28x\,c^2d^3e+15a^2e^4-42ac\,d^2e^2+35c^2d^4\right)\sqrt{cd\,x^2e+a\,e^2x+c\,d^2x+ade}}{105(ex+d)^4\left(e^6a^3-3d^2e^4a^2c+3d^4e^2a\,c^2-d^6c^3\right)}$
orering	$-\frac{2\left(8x^2c^2d^2e^2-12xacd\,e^3+28x\,c^2d^3e+15a^2e^4-42ac\,d^2e^2+35c^2d^4\right)(cdx+ae)\sqrt{ade+(a\,e^2+c\,d^2)x+cd\,x^2e}}{105\left(e^6a^3-3d^2e^4a^2c+3d^4e^2a\,c^2-d^6c^3\right)(ex+d)^4}$
trager	$-\frac{2\left(8e^2d^3c^3x^3-4e^3a\,c^2d^2x^2+28d^4e\,c^3x^2+3d\,e^4a^2cx-14d^3e^2a\,c^2x+35d^5c^3x+15a^3e^5-42d^2e^3a^2c+35a\,c^2d^4e\right)\sqrt{cd\,x^2e+a\,e^2x+c}}{105\left(e^6a^3-3d^2e^4a^2c+3d^4e^2a\,c^2-d^6c^3\right)(ex+d)^4}$
default	$\frac{-\frac{2\left(dec\left(x+\frac{d}{e}\right)^2+\left(a\,e^2-c\,d^2\right)\left(x+\frac{d}{e}\right)\right)^{\frac{3}{2}}}{7\left(a\,e^2-c\,d^2\right)\left(x+\frac{d}{e}\right)^5}-\frac{4dec\left(-\frac{2\left(dec\left(x+\frac{d}{e}\right)^2+\left(a\,e^2-c\,d^2\right)\left(x+\frac{d}{e}\right)\right)^{\frac{3}{2}}}{5\left(a\,e^2-c\,d^2\right)\left(x+\frac{d}{e}\right)^4}+\frac{4dec\left(dec\left(x+\frac{d}{e}\right)^2+\left(a\,e^2-c\,d^2\right)\left(x+\frac{d}{e}\right)\right)^{\frac{3}{2}}}{15\left(a\,e^2-c\,d^2\right)^2\left(x+\frac{d}{e}\right)^3}\right)}{7\left(a\,e^2-c\,d^2\right)}}{e^5}$

input
```
int((a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(1/2)/(e*x+d)^5,x,method=_RETURNVERB
OSE)
```

output
```
-2/105*(c*d*x+a*e)*(8*c^2*d^2*e^2*x^2-12*a*c*d*e^3*x+28*c^2*d^3*e*x+15*a^2
*e^4-42*a*c*d^2*e^2+35*c^2*d^4)*(c*d*e*x^2+a*e^2*x+c*d^2*x+a*d*e)^(1/2)/(e
*x+d)^4/(a^3*e^6-3*a^2*c*d^2*e^4+3*a*c^2*d^4*e^2-c^3*d^6)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 373 vs. $2(159) = 318$.

Time = 1.29 (sec) , antiderivative size = 373, normalized size of antiderivative = 2.18

$$\int \frac{\sqrt{ade + (cd^2 + ae^2)\,x + cdex^2}}{(d + ex)^5}\,dx$$
$$= \frac{2\,(8\,c^3d^3e^2x^3 + 35\,ac^2d^4e - 42\,a^2cd^2e^3 + 15\,a^3e^5 + 4\,(7\,c^3d^4}{105\,(c^3d^{10} - 3\,ac^2d^8e^2 + 3\,a^2cd^6e^4 - a^3d^4e^6 + (c^3d^6e^4 - 3\,ac^2d^4e^6 + 3\,a^2cd^2e^8 - a^3e^{10})x^4 + 4\,(c^3d^7e^3 - 3}$$

input
```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(e*x+d)^5,x, algorithm="
fricas")
```

output
```
2/105*(8*c^3*d^3*e^2*x^3 + 35*a*c^2*d^4*e - 42*a^2*c*d^2*e^3 + 15*a^3*e^5
+ 4*(7*c^3*d^4*e - a*c^2*d^2*e^3)*x^2 + (35*c^3*d^5 - 14*a*c^2*d^3*e^2 + 3
*a^2*c*d*e^4)*x)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)/(c^3*d^10 - 3
*a*c^2*d^8*e^2 + 3*a^2*c*d^6*e^4 - a^3*d^4*e^6 + (c^3*d^6*e^4 - 3*a*c^2*d^
4*e^6 + 3*a^2*c*d^2*e^8 - a^3*e^10)*x^4 + 4*(c^3*d^7*e^3 - 3*a*c^2*d^5*e^5
 + 3*a^2*c*d^3*e^7 - a^3*d*e^9)*x^3 + 6*(c^3*d^8*e^2 - 3*a*c^2*d^6*e^4 + 3
*a^2*c*d^4*e^6 - a^3*d^2*e^8)*x^2 + 4*(c^3*d^9*e - 3*a*c^2*d^7*e^3 + 3*a^2
*c*d^5*e^5 - a^3*d^3*e^7)*x)
```

Sympy [F]

$$\int \frac{\sqrt{ade + (cd^2 + ae^2)\,x + cdex^2}}{(d+ex)^5}\,dx = \int \frac{\sqrt{(d+ex)\,(ae+cdx)}}{(d+ex)^5}\,dx$$

input
```
integrate((a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(1/2)/(e*x+d)**5,x)
```

output
```
Integral(sqrt((d + e*x)*(a*e + c*d*x))/(d + e*x)**5, x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\sqrt{ade + (cd^2 + ae^2)\,x + cdex^2}}{(d+ex)^5}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(e*x+d)^5,x, algorithm="
maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e*(a*e^2-c*d^2)>0)', see `assume
?` for mor
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 698 vs. $2(159) = 318$.

Time = 0.19 (sec) , antiderivative size = 698, normalized size of antiderivative = 4.08

$$\int \frac{\sqrt{ade + (cd^2 + ae^2)\,x + cdex^2}}{(d + ex)^5}\,dx = \text{Too large to display}$$

input
```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(e*x+d)^5,x, algorithm="
giac")
```

output
```
-2/105*(8*sqrt(c*d*e)*c^3*d^3*sgn(1/(e*x + d))*sgn(e)/(c^3*d^6*e^2*abs(e)
- 3*a*c^2*d^4*e^4*abs(e) + 3*a^2*c*d^2*e^6*abs(e) - a^3*e^8*abs(e)) + (3*(
35*sqrt(c*d*e - c*d^2*e/(e*x + d) + a*e^3/(e*x + d))*c^3*d^3*e^3 - 35*(c*d
*e - c*d^2*e/(e*x + d) + a*e^3/(e*x + d))^(3/2)*c^2*d^2*e^2 + 21*(c*d*e -
c*d^2*e/(e*x + d) + a*e^3/(e*x + d))^(5/2)*c*d*e - 5*(c*d*e - c*d^2*e/(e*x
 + d) + a*e^3/(e*x + d))^(7/2))*c*d^2*e*sgn(1/(e*x + d))*sgn(e)/(c^3*d^6*e
^6 - 3*a*c^2*d^4*e^8 + 3*a^2*c*d^2*e^10 - a^3*e^12) - 3*(35*sqrt(c*d*e - c
*d^2*e/(e*x + d) + a*e^3/(e*x + d))*c^3*d^3*e^3 - 35*(c*d*e - c*d^2*e/(e*x
 + d) + a*e^3/(e*x + d))^(3/2)*c^2*d^2*e^2 + 21*(c*d*e - c*d^2*e/(e*x + d)
 + a*e^3/(e*x + d))^(5/2)*c*d*e - 5*(c*d*e - c*d^2*e/(e*x + d) + a*e^3/(e*
x + d))^(7/2))*a*e^3*sgn(1/(e*x + d))*sgn(e)/(c^3*d^6*e^6 - 3*a*c^2*d^4*e^
8 + 3*a^2*c*d^2*e^10 - a^3*e^12) - 7*(15*sqrt(c*d*e - c*d^2*e/(e*x + d) +
a*e^3/(e*x + d))*c^2*d^2*e^2 - 10*(c*d*e - c*d^2*e/(e*x + d) + a*e^3/(e*x
+ d))^(3/2)*c*d*e + 3*(c*d*e - c*d^2*e/(e*x + d) + a*e^3/(e*x + d))^(5/2))
*c*d*sgn(1/(e*x + d))*sgn(e)/(c*d^2*e^2 - a*e^4)^2)/(c*d^2*abs(e) - a*e^2*
abs(e)))*abs(e)
```

Mupad [B] (verification not implemented)

Time = 6.04 (sec) , antiderivative size = 877, normalized size of antiderivative = 5.13

$$\int \frac{\sqrt{ade + (cd^2 + ae^2)\,x + cdex^2}}{(d + ex)^5}\,dx = \text{Too large to display}$$

input
```
int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2)/(d + e*x)^5,x)
```

output
```
(((4*c^2*d^3)/(7*(a*e^2 - c*d^2)*(5*a*e^3 - 5*c*d^2*e)) - (4*a*c*d*e^2)/(7
*(a*e^2 - c*d^2)*(5*a*e^3 - 5*c*d^2*e)))*(x*(a*e^2 + c*d^2) + a*d*e + c*d*
e*x^2)^(1/2))/(d + e*x)^3 - (((2*a*e^2)/(7*a*e^3 - 7*c*d^2*e) - (2*c*d^2)/
(7*a*e^3 - 7*c*d^2*e))*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2))/(d +
 e*x)^4 + (((4*c^3*d^4 + 4*a*c^2*d^2*e^2)/(35*(a*e^2 - c*d^2)^2*(3*a*e^3 -
 3*c*d^2*e)) - (8*c^3*d^4)/(35*(a*e^2 - c*d^2)^2*(3*a*e^3 - 3*c*d^2*e)))*(
x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2))/(d + e*x)^2 + (((8*c^4*d^5 +
 8*a*c^3*d^3*e^2)/(105*e*(a*e^2 - c*d^2)^4) - (16*c^4*d^5)/(105*e*(a*e^2 -
 c*d^2)^4))*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2))/(d + e*x) + (((
2*c^2*d^3 + 2*a*c*d*e^2)/(7*(a*e^2 - c*d^2)*(5*a*e^3 - 5*c*d^2*e)) - (4*c^
2*d^3)/(7*(a*e^2 - c*d^2)*(5*a*e^3 - 5*c*d^2*e)))*(x*(a*e^2 + c*d^2) + a*d
*e + c*d*e*x^2)^(1/2))/(d + e*x)^3 + (((8*c^3*d^4)/(35*(a*e^2 - c*d^2)^2*(
3*a*e^3 - 3*c*d^2*e)) - (8*a*c^2*d^2*e^2)/(35*(a*e^2 - c*d^2)^2*(3*a*e^3 -
 3*c*d^2*e)))*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2))/(d + e*x)^2 +
 (((16*c^4*d^5)/(105*e*(a*e^2 - c*d^2)^4) - (16*a*c^3*d^3*e)/(105*(a*e^2 -
 c*d^2)^4))*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2))/(d + e*x) + (12
*c^2*d^2*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2))/(35*(a*e^2 - c*d^2
)*(3*a*e^3 - 3*c*d^2*e)*(d + e*x)^2) - (8*c^3*d^3*(x*(a*e^2 + c*d^2) + a*d
*e + c*d*e*x^2)^(1/2))/(105*e*(a*e^2 - c*d^2)^3*(d + e*x))
```

Reduce [B] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 603, normalized size of antiderivative = 3.53

$$\int \frac{\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{(d+ex)^5}\,dx$$
$$= \frac{-\frac{2\sqrt{ex+d}\,\sqrt{cdx+ae}\,a^3e^7}{7}+\frac{4\sqrt{ex+d}\,\sqrt{cdx+ae}\,a^2c\,d^2e^5}{5}-\frac{2\sqrt{ex+d}\,\sqrt{cdx+ae}\,a^2cd\,e^6x}{35}-\frac{2\sqrt{ex+d}\,\sqrt{cdx+ae}\,a\,c^2d^4e^3}{3}+\frac{4\sqrt{ex+d}\,\sqrt{cdx}}{1}}{e^2\left(a^3e^{10}x^4-3a^2c\,d^2e^8x^4+3a\,c^2d^4e^6x^4-c^3d^6e^4x^4+4a^3d\,e^9x^3-12a^2c\,d^3e^7x^3+\right.}$$

input
```
int((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(e*x+d)^5,x)
```

output

```
(2*( - 15*sqrt(d + e*x)*sqrt(a*e + c*d*x)*a**3*e**7 + 42*sqrt(d + e*x)*sqr
t(a*e + c*d*x)*a**2*c*d**2*e**5 - 3*sqrt(d + e*x)*sqrt(a*e + c*d*x)*a**2*c
*d*e**6*x - 35*sqrt(d + e*x)*sqrt(a*e + c*d*x)*a*c**2*d**4*e**3 + 14*sqrt(
d + e*x)*sqrt(a*e + c*d*x)*a*c**2*d**3*e**4*x + 4*sqrt(d + e*x)*sqrt(a*e +
 c*d*x)*a*c**2*d**2*e**5*x**2 - 35*sqrt(d + e*x)*sqrt(a*e + c*d*x)*c**3*d*
*5*e**2*x - 28*sqrt(d + e*x)*sqrt(a*e + c*d*x)*c**3*d**4*e**3*x**2 - 8*sqr
t(d + e*x)*sqrt(a*e + c*d*x)*c**3*d**3*e**4*x**3 + 8*sqrt(e)*sqrt(d)*sqrt(
c)*c**3*d**7 + 32*sqrt(e)*sqrt(d)*sqrt(c)*c**3*d**6*e*x + 48*sqrt(e)*sqrt(
d)*sqrt(c)*c**3*d**5*e**2*x**2 + 32*sqrt(e)*sqrt(d)*sqrt(c)*c**3*d**4*e**3
*x**3 + 8*sqrt(e)*sqrt(d)*sqrt(c)*c**3*d**3*e**4*x**4))/(105*e**2*(a**3*d*
*4*e**6 + 4*a**3*d**3*e**7*x + 6*a**3*d**2*e**8*x**2 + 4*a**3*d*e**9*x**3
+ a**3*e**10*x**4 - 3*a**2*c*d**6*e**4 - 12*a**2*c*d**5*e**5*x - 18*a**2*c
*d**4*e**6*x**2 - 12*a**2*c*d**3*e**7*x**3 - 3*a**2*c*d**2*e**8*x**4 + 3*a
*c**2*d**8*e**2 + 12*a*c**2*d**7*e**3*x + 18*a*c**2*d**6*e**4*x**2 + 12*a*
c**2*d**5*e**5*x**3 + 3*a*c**2*d**4*e**6*x**4 - c**3*d**10 - 4*c**3*d**9*e
*x - 6*c**3*d**8*e**2*x**2 - 4*c**3*d**7*e**3*x**3 - c**3*d**6*e**4*x**4))
```

3.209 $\int \frac{\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{(d+ex)^6} dx$

Optimal result

Integrand size = 37, antiderivative size = 231

$$\int \frac{\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{(d+ex)^6} dx = \frac{2(ade+(cd^2+ae^2)x+cdex^2)^{3/2}}{9(cd^2-ae^2)(d+ex)^6} + \frac{4cd(ade+(cd^2+ae^2)x+cdex^2)^{3/2}}{21(cd^2-ae^2)^2(d+ex)^5} + \frac{16c^2d^2(ade+(cd^2+ae^2)x+cdex^2)^{3/2}}{105(cd^2-ae^2)^3(d+ex)^4} + \frac{32c^3d^3(ade+(cd^2+ae^2)x+cdex^2)^{3/2}}{315(cd^2-ae^2)^4(d+ex)^3}$$

output
```
2/9*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/(-a*e^2+c*d^2)/(e*x+d)^6+4/21*
c*d*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/(-a*e^2+c*d^2)^2/(e*x+d)^5+16/
105*c^2*d^2*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/(-a*e^2+c*d^2)^3/(e*x+
d)^4+32/315*c^3*d^3*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/(-a*e^2+c*d^2)
^4/(e*x+d)^3
```

Mathematica [A] (verified)

Time = 10.06 (sec) , antiderivative size = 180, normalized size of antiderivative = 0.78

$$\int \frac{\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{(d+ex)^6}\,dx = \frac{2\sqrt{(ae+cdx)(d+ex)}(-35a^4e^7+5a^3cde^5(27d-ex)+3a^2c^2d^2e^3(-63d^2+9dex+2e^2x^2)+ac^3d^3e(105}{315\,(cd^2-ae^2)^4\,(d+ex}$$

input
```
Integrate[Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]/(d + e*x)^6,x]
```

output
```
(2*Sqrt[(a*e + c*d*x)*(d + e*x)]*(-35*a^4*e^7 + 5*a^3*c*d*e^5*(27*d - e*x)
 + 3*a^2*c^2*d^2*e^3*(-63*d^2 + 9*d*e*x + 2*e^2*x^2) + a*c^3*d^3*e*(105*d^
3 - 63*d^2*e*x - 36*d*e^2*x^2 - 8*e^3*x^3) + c^4*d^4*x*(105*d^3 + 126*d^2*
e*x + 72*d*e^2*x^2 + 16*e^3*x^3)))/(315*(c*d^2 - a*e^2)^4*(d + e*x)^5)
```

Rubi [A] (verified)

Time = 0.68 (sec) , antiderivative size = 261, normalized size of antiderivative = 1.13, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.108, Rules used = {1129, 1129, 1129, 1123}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{x\,(ae^2+cd^2)+ade+cdex^2}}{(d+ex)^6}\,dx$$

$$\Big\downarrow \text{ 1129}$$

$$\frac{2cd\int \frac{\sqrt{cdex^2+(cd^2+ae^2)x+ade}}{(d+ex)^5}dx}{3\,(cd^2-ae^2)}+\frac{2\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{9(d+ex)^6\,(cd^2-ae^2)}$$

$$\Big\downarrow \text{ 1129}$$

$$\frac{2cd\left(\frac{4cd\int \frac{\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}{(d+ex)^4}dx}{7(cd^2-ae^2)}+\frac{2\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{7(d+ex)^5(cd^2-ae^2)}\right)}{3\left(cd^2-ae^2\right)}+\frac{2\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{9(d+ex)^6\left(cd^2-ae^2\right)}$$

$$\Big\downarrow \; 1129$$

$$\frac{2cd\left(\frac{4cd\left(\frac{2cd\int \frac{\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}{(d+ex)^3}dx}{5\left(cd^2-ae^2\right)}+\frac{2\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{5(d+ex)^4\left(cd^2-ae^2\right)}\right)}{7(cd^2-ae^2)}+\frac{2\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{7(d+ex)^5(cd^2-ae^2)}\right)}{3\left(cd^2-ae^2\right)}+\frac{2\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{9(d+ex)^6\left(cd^2-ae^2\right)}$$

$$\Big\downarrow \; 1123$$

$$\frac{2\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{9(d+ex)^6\left(cd^2-ae^2\right)}+\frac{2cd\left(\frac{2\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{7(d+ex)^5(cd^2-ae^2)}+\frac{4cd\left(\frac{4cd\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{15(d+ex)^3\left(cd^2-ae^2\right)^2}+\frac{2\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{5(d+ex)^4\left(cd^2-ae^2\right)}\right)}{7(cd^2-ae^2)}\right)}{3\left(cd^2-ae^2\right)}$$

input

```
Int[Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]/(d + e*x)^6,x]
```

output

```
(2*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2))/(9*(c*d^2 - a*e^2)*(d +
e*x)^6) + (2*c*d*((2*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2))/(7*(c*
d^2 - a*e^2)*(d + e*x)^5) + (4*c*d*((2*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*
x^2)^(3/2))/(5*(c*d^2 - a*e^2)*(d + e*x)^4) + (4*c*d*(a*d*e + (c*d^2 + a*e
^2)*x + c*d*e*x^2)^(3/2))/(15*(c*d^2 - a*e^2)^2*(d + e*x)^3)))/(7*(c*d^2 -
 a*e^2))))/(3*(c*d^2 - a*e^2))
```

Defintions of rubi rules used

rule 1123
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[e*(d + e*x)^m*((a + b*x + c*x^2)^(p + 1)/((p + 1)*(2*c*d - b
*e))), x] /; FreeQ[{a, b, c, d, e, m, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2,
0] && EqQ[m + 2*p + 2, 0]
```

rule 1129
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(-e)*(d + e*x)^m*((a + b*x + c*x^2)^(p + 1)/((m + p + 1)*(2*
c*d - b*e))), x] + Simp[c*(Simplify[m + 2*p + 2]/((m + p + 1)*(2*c*d - b*e)
))   Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d
, e, m, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && ILtQ[Simplify[m + 2*p +
2], 0]
```

Maple [A] (verified)

Time = 3.63 (sec) , antiderivative size = 217, normalized size of antiderivative = 0.94

method	result
gosper	$-\frac{2(cdx+ae)\left(-16c^3d^3e^3x^3+24x^2ac^2d^2e^4-72c^3d^4e^2x^2-30x\,a^2cd\,e^5+108xa\,c^2d^3e^3-126c^3d^5ex+35e^6a^3-135d^2e^4a^2c+189d^4e^2c\right.}{315(ex+d)^5\left(a^4e^8-4a^3c\,d^2e^6+6a^2c^2d^4e^4-4a\,c^3d^6e^2+c^4d^8\right)}$
orering	$-\frac{2\left(-16c^3d^3e^3x^3+24x^2a\,c^2d^2e^4-72c^3d^4e^2x^2-30x\,a^2cd\,e^5+108xa\,c^2d^3e^3-126c^3d^5ex+35e^6a^3-135d^2e^4a^2c+189d^4e^2a\,c^2-105d\right.}{315\left(a^4e^8-4a^3c\,d^2e^6+6a^2c^2d^4e^4-4a\,c^3d^6e^2+c^4d^8\right)(ex+d)^5}$
trager	$-\frac{2\left(-16c^4d^4e^3x^4+8a\,c^3d^3e^4x^3-72c^4d^5e^2x^3-6a^2c^2d^2e^5x^2+36a\,c^3d^4e^3x^2-126c^4d^6e\,x^2+5d\,e^6c\,a^3x-27d^3e^4a^2c^2x+63d^5e^2a\,c^3x\right.}{315\left(a^4e^8-4a^3c\,d^2e^6+6a^2c^2d^4e^4-4a\,c^3d^6e^2+c^4d\right.}$
default	$\frac{-\frac{2\left(dec\left(x+\frac{d}{e}\right)^2+\left(a\,e^2-c\,d^2\right)\left(x+\frac{d}{e}\right)\right)^{\frac{3}{2}}}{9\left(a\,e^2-c\,d^2\right)\left(x+\frac{d}{e}\right)^6}-\frac{2dec\left(-\frac{2\left(dec\left(x+\frac{d}{e}\right)^2+\left(a\,e^2-c\,d^2\right)\left(x+\frac{d}{e}\right)\right)^{\frac{3}{2}}}{7\left(a\,e^2-c\,d^2\right)\left(x+\frac{d}{e}\right)^5}-\frac{4dec\left(-\frac{2\left(dec\left(x+\frac{d}{e}\right)^2+\left(a\,e^2-c\,d^2\right)\left(x+\frac{d}{e}\right)\right)^{\frac{3}{2}}}{5\left(a\,e^2-c\,d^2\right)\left(x+\frac{d}{e}\right)^4}\right.}{7\left(a\,e^2-\right.}\right.}{3\left(a\,e^2-c\,d^2\right)}}{e^6}$

input
```
int((a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(1/2)/(e*x+d)^6,x,method=_RETURNVERB
OSE)
```

output
```
-2/315*(c*d*x+a*e)*(-16*c^3*d^3*e^3*x^3+24*a*c^2*d^2*e^4*x^2-72*c^3*d^4*e^
2*x^2-30*a^2*c*d*e^5*x+108*a*c^2*d^3*e^3*x-126*c^3*d^5*e*x+35*a^3*e^6-135*
a^2*c*d^2*e^4+189*a*c^2*d^4*e^2-105*c^3*d^6)*(c*d*e*x^2+a*e^2*x+c*d^2*x+a*
d*e)^(1/2)/(e*x+d)^5/(a^4*e^8-4*a^3*c*d^2*e^6+6*a^2*c^2*d^4*e^4-4*a*c^3*d^
6*e^2+c^4*d^8)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 572 vs. $2(215) = 430$.

Time = 4.76 (sec) , antiderivative size = 572, normalized size of antiderivative = 2.48

$$\int \frac{\sqrt{ade + (cd^2 + ae^2)x + cdex^2}}{(d + ex)^6}\,dx$$
$$= \frac{2\,(16\,c^4d^4e^3x^4 + 105\,ac^3d^6e \cdots}{315\,(c^4d^{13} - 4\,ac^3d^{11}e^2 + 6\,a^2c^2d^9e^4 - 4\,a^3cd^7e^6 + a^4d^5e^8 + (c^4d^8e^5 - 4\,ac^3d^6e^7 + 6\,a^2c^2d^4e^9 - 4\,a^3cd^2e^{1} \cdots}$$

input
```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(e*x+d)^6,x, algorithm="
fricas")
```

output
```
2/315*(16*c^4*d^4*e^3*x^4 + 105*a*c^3*d^6*e - 189*a^2*c^2*d^4*e^3 + 135*a^
3*c*d^2*e^5 - 35*a^4*e^7 + 8*(9*c^4*d^5*e^2 - a*c^3*d^3*e^4)*x^3 + 6*(21*c
^4*d^6*e - 6*a*c^3*d^4*e^3 + a^2*c^2*d^2*e^5)*x^2 + (105*c^4*d^7 - 63*a*c^
3*d^5*e^2 + 27*a^2*c^2*d^3*e^4 - 5*a^3*c*d*e^6)*x)*sqrt(c*d*e*x^2 + a*d*e
+ (c*d^2 + a*e^2)*x)/(c^4*d^13 - 4*a*c^3*d^11*e^2 + 6*a^2*c^2*d^9*e^4 - 4*
a^3*c*d^7*e^6 + a^4*d^5*e^8 + (c^4*d^8*e^5 - 4*a*c^3*d^6*e^7 + 6*a^2*c^2*d
^4*e^9 - 4*a^3*c*d^2*e^11 + a^4*e^13)*x^5 + 5*(c^4*d^9*e^4 - 4*a*c^3*d^7*e
^6 + 6*a^2*c^2*d^5*e^8 - 4*a^3*c*d^3*e^10 + a^4*d*e^12)*x^4 + 10*(c^4*d^10
*e^3 - 4*a*c^3*d^8*e^5 + 6*a^2*c^2*d^6*e^7 - 4*a^3*c*d^4*e^9 + a^4*d^2*e^1
1)*x^3 + 10*(c^4*d^11*e^2 - 4*a*c^3*d^9*e^4 + 6*a^2*c^2*d^7*e^6 - 4*a^3*c*
d^5*e^8 + a^4*d^3*e^10)*x^2 + 5*(c^4*d^12*e - 4*a*c^3*d^10*e^3 + 6*a^2*c^2
*d^8*e^5 - 4*a^3*c*d^6*e^7 + a^4*d^4*e^9)*x)
```

Sympy [F]

$$\int \frac{\sqrt{ade + (cd^2 + ae^2)\,x + cdex^2}}{(d + ex)^6}\,dx = \int \frac{\sqrt{(d + ex)\,(ae + cdx)}}{(d + ex)^6}\,dx$$

input
```
integrate((a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(1/2)/(e*x+d)**6,x)
```

output
```
Integral(sqrt((d + e*x)*(a*e + c*d*x))/(d + e*x)**6, x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\sqrt{ade + (cd^2 + ae^2)\,x + cdex^2}}{(d + ex)^6}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(e*x+d)^6,x, algorithm="
maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e*(a*e^2-c*d^2)>0)', see `assume
?` for mor
```

Giac [F(-2)]

Exception generated.

$$\int \frac{\sqrt{ade + (cd^2 + ae^2)\,x + cdex^2}}{(d + ex)^6}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(e*x+d)^6,x, algorithm="
giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Unable to divide, perhaps due to ro
unding error%%%{%%%{1,[0,0,5]%%%},[10]%%%}+%%%{%%{[%%%{-10,[0,1,4]%%%},0]:
[1,0,%%%{
```

Mupad [B] (verification not implemented)

Time = 6.70 (sec) , antiderivative size = 1192, normalized size of antiderivative = 5.16

$$\int \frac{\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{(d+ex)^6}\,dx = \text{Too large to display}$$

input
```
int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2)/(d + e*x)^6,x)
```

output
```
(((4*c^2*d^3)/(9*(a*e^2 - c*d^2)*(7*a*e^3 - 7*c*d^2*e)) - (4*a*c*d*e^2)/(9
*(a*e^2 - c*d^2)*(7*a*e^3 - 7*c*d^2*e)))*(x*(a*e^2 + c*d^2) + a*d*e + c*d*
e*x^2)^(1/2))/(d + e*x)^4 - (((2*a*e^2)/(9*a*e^3 - 9*c*d^2*e) - (2*c*d^2)/
(9*a*e^3 - 9*c*d^2*e))*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2))/(d +
 e*x)^5 + (((4*c^3*d^4 + 4*a*c^2*d^2*e^2)/(63*(a*e^2 - c*d^2)^2*(5*a*e^3 -
 5*c*d^2*e)) - (8*c^3*d^4)/(63*(a*e^2 - c*d^2)^2*(5*a*e^3 - 5*c*d^2*e)))*(
x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2))/(d + e*x)^3 + (((8*c^4*d^5 +
 8*a*c^3*d^3*e^2)/(315*(a*e^2 - c*d^2)^3*(3*a*e^3 - 3*c*d^2*e)) - (16*c^4*
d^5)/(315*(a*e^2 - c*d^2)^3*(3*a*e^3 - 3*c*d^2*e)))*(x*(a*e^2 + c*d^2) + a
*d*e + c*d*e*x^2)^(1/2))/(d + e*x)^2 + (((16*c^5*d^6 + 16*a*c^4*d^4*e^2)/(
945*e*(a*e^2 - c*d^2)^5) - (32*c^5*d^6)/(945*e*(a*e^2 - c*d^2)^5))*(x*(a*e
^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2))/(d + e*x) + (((2*c^2*d^3 + 2*a*c*d
*e^2)/(9*(a*e^2 - c*d^2)*(7*a*e^3 - 7*c*d^2*e)) - (4*c^2*d^3)/(9*(a*e^2 -
c*d^2)*(7*a*e^3 - 7*c*d^2*e)))*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/
2))/(d + e*x)^4 + (((8*c^3*d^4)/(63*(a*e^2 - c*d^2)^2*(5*a*e^3 - 5*c*d^2*e
)) - (8*a*c^2*d^2*e^2)/(63*(a*e^2 - c*d^2)^2*(5*a*e^3 - 5*c*d^2*e)))*(x*(a
*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2))/(d + e*x)^3 + (((16*c^4*d^5)/(31
5*(a*e^2 - c*d^2)^3*(3*a*e^3 - 3*c*d^2*e)) - (16*a*c^3*d^3*e^2)/(315*(a*e^
2 - c*d^2)^3*(3*a*e^3 - 3*c*d^2*e)))*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^
2)^(1/2))/(d + e*x)^2 + (((32*c^5*d^6)/(945*e*(a*e^2 - c*d^2)^5) - (32*...
```

Reduce [B] (verification not implemented)

Time = 0.41 (sec) , antiderivative size = 926, normalized size of antiderivative = 4.01

$$\int \frac{\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{(d+ex)^6}\,dx = \text{Too large to display}$$

input
```
int((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(e*x+d)^6,x)
```

output
```
(2*( - 35*sqrt(d + e*x)*sqrt(a*e + c*d*x)*a**4*e**9 + 135*sqrt(d + e*x)*sq
rt(a*e + c*d*x)*a**3*c*d**2*e**7 - 5*sqrt(d + e*x)*sqrt(a*e + c*d*x)*a**3*
c*d*e**8*x - 189*sqrt(d + e*x)*sqrt(a*e + c*d*x)*a**2*c**2*d**4*e**5 + 27*
sqrt(d + e*x)*sqrt(a*e + c*d*x)*a**2*c**2*d**3*e**6*x + 6*sqrt(d + e*x)*sq
rt(a*e + c*d*x)*a**2*c**2*d**2*e**7*x**2 + 105*sqrt(d + e*x)*sqrt(a*e + c*
d*x)*a*c**3*d**6*e**3 - 63*sqrt(d + e*x)*sqrt(a*e + c*d*x)*a*c**3*d**5*e**
4*x - 36*sqrt(d + e*x)*sqrt(a*e + c*d*x)*a*c**3*d**4*e**5*x**2 - 8*sqrt(d
+ e*x)*sqrt(a*e + c*d*x)*a*c**3*d**3*e**6*x**3 + 105*sqrt(d + e*x)*sqrt(a*
e + c*d*x)*c**4*d**7*e**2*x + 126*sqrt(d + e*x)*sqrt(a*e + c*d*x)*c**4*d**
6*e**3*x**2 + 72*sqrt(d + e*x)*sqrt(a*e + c*d*x)*c**4*d**5*e**4*x**3 + 16*
sqrt(d + e*x)*sqrt(a*e + c*d*x)*c**4*d**4*e**5*x**4 - 16*sqrt(e)*sqrt(d)*s
qrt(c)*c**4*d**9 - 80*sqrt(e)*sqrt(d)*sqrt(c)*c**4*d**8*e*x - 160*sqrt(e)*
sqrt(d)*sqrt(c)*c**4*d**7*e**2*x**2 - 160*sqrt(e)*sqrt(d)*sqrt(c)*c**4*d**
6*e**3*x**3 - 80*sqrt(e)*sqrt(d)*sqrt(c)*c**4*d**5*e**4*x**4 - 16*sqrt(e)*
sqrt(d)*sqrt(c)*c**4*d**4*e**5*x**5))/(315*e**2*(a**4*d**5*e**8 + 5*a**4*d
**4*e**9*x + 10*a**4*d**3*e**10*x**2 + 10*a**4*d**2*e**11*x**3 + 5*a**4*d*
e**12*x**4 + a**4*e**13*x**5 - 4*a**3*c*d**7*e**6 - 20*a**3*c*d**6*e**7*x
- 40*a**3*c*d**5*e**8*x**2 - 40*a**3*c*d**4*e**9*x**3 - 20*a**3*c*d**3*e**
10*x**4 - 4*a**3*c*d**2*e**11*x**5 + 6*a**2*c**2*d**9*e**4 + 30*a**2*c**2*
d**8*e**5*x + 60*a**2*c**2*d**7*e**6*x**2 + 60*a**2*c**2*d**6*e**7*x**3...
```

3.210 $\int (d+ex)^2 \left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2} dx$

Optimal result

Integrand size = 37, antiderivative size = 421

$$\int (d+ex)^2 \left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2} dx = -\frac{7(cd^2-ae^2)^5\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{512c^4d^4e^2} + \frac{7(cd^2-ae^2)^4\left(ade+(cd^2+ae^2)x+cdex^2\right)^{3/2}}{768c^4d^4e(d+ex)} + \frac{7(cd^2-ae^2)\left(ade+(cd^2+ae^2)x+cdex^2\right)^{5/2}}{60c^2d^2} + \frac{7(cd^2-ae^2)^3\left(ade+(cd^2+ae^2)x+cdex^2\right)^{5/2}}{192c^4d^4(d+ex)^2} + \frac{7(cd^2-ae^2)^2\left(ade+(cd^2+ae^2)x+cdex^2\right)^{5/2}}{96c^3d^3(d+ex)} + \frac{(d+ex)\left(ade+(cd^2+ae^2)x+cdex^2\right)^{5/2}}{6cd} + \frac{7(cd^2-ae^2)^6\operatorname{arctanh}\left(\frac{\sqrt{e}\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{\sqrt{c}\sqrt{d}(d+ex)}\right)}{512c^{9/2}d^{9/2}e^{5/2}}$$

output
```
-7/512*(-a*e^2+c*d^2)^5*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/c^4/d^4/e^
2+7/768*(-a*e^2+c*d^2)^4*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/c^4/d^4/e
/(e*x+d)+7/60*(-a*e^2+c*d^2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/c^2/d
^2+7/192*(-a*e^2+c*d^2)^3*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/c^4/d^4/
(e*x+d)^2+7/96*(-a*e^2+c*d^2)^2*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/c^
3/d^3/(e*x+d)+1/6*(e*x+d)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/c/d+7/51
2*(-a*e^2+c*d^2)^6*arctanh(e^(1/2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)
/c^(1/2)/d^(1/2)/(e*x+d))/c^(9/2)/d^(9/2)/e^(5/2)
```

Mathematica [A] (verified)

Time = 0.93 (sec) , antiderivative size = 361, normalized size of antiderivative = 0.86

$$\int (d+ex)^2 \left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2} dx = \frac{((ae+cdx)(d+ex))^{3/2}\left(\frac{\sqrt{c}\sqrt{d}\sqrt{e}(-105a^5e^{10}+35a^4cde^8(17d+2ex)-14a^3c^2d^2e^6(99d^2+28dex+4e^2x^2)+6a^2c^3\ldots}{\ldots}\right.}{\ldots}$$

input
```
Integrate[(d + e*x)^2*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2),x]
```

output
```
(((a*e + c*d*x)*(d + e*x))^(3/2)*((Sqrt[c]*Sqrt[d]*Sqrt[e]*(-105*a^5*e^10
+ 35*a^4*c*d*e^8*(17*d + 2*e*x) - 14*a^3*c^2*d^2*e^6*(99*d^2 + 28*d*e*x +
4*e^2*x^2) + 6*a^2*c^3*d^3*e^4*(281*d^3 + 150*d^2*e*x + 52*d*e^2*x^2 + 8*e
^3*x^3) + a*c^4*d^4*e^2*(595*d^4 + 5752*d^3*e*x + 9528*d^2*e^2*x^2 + 6560*
d*e^3*x^3 + 1664*e^4*x^4) + c^5*d^5*(-105*d^5 + 70*d^4*e*x + 3016*d^3*e^2*
x^2 + 6192*d^2*e^3*x^3 + 4736*d*e^4*x^4 + 1280*e^5*x^5)))/((a*e + c*d*x)*(
d + e*x)) + (105*(c*d^2 - a*e^2)^6*ArcTanh[(Sqrt[c]*Sqrt[d]*Sqrt[d + e*x])
/(Sqrt[e]*Sqrt[a*e + c*d*x])])/((a*e + c*d*x)^(3/2)*(d + e*x)^(3/2))))/(76
80*c^(9/2)*d^(9/2)*e^(5/2))
```

Rubi [A] (verified)

Time = 0.83 (sec) , antiderivative size = 371, normalized size of antiderivative = 0.88, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.162$, Rules used = {1134, 1160, 1087, 1087, 1092, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (d+ex)^2 \left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2} dx$$

$$\Big\downarrow \text{1134}$$

$$\frac{7\left(d^2-\frac{ae^2}{c}\right)\int (d+ex)\left(cdex^2+\left(cd^2+ae^2\right)x+ade\right)^{3/2}dx}{12d}+\frac{(d+ex)\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/2}}{6cd}$$

$$\Big\downarrow \text{1160}$$

$$\frac{7\left(d^2-\frac{ae^2}{c}\right)\left(\frac{\left(d^2-\frac{ae^2}{c}\right)\int \left(cdex^2+\left(cd^2+ae^2\right)x+ade\right)^{3/2}dx}{2d}+\frac{\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/2}}{5cd}\right)}{12d}+\frac{(d+ex)\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/2}}{6cd}$$

$$\Big\downarrow \text{1087}$$

$$\frac{7\left(d^2-\frac{ae^2}{c}\right)\left(\frac{\left(d^2-\frac{ae^2}{c}\right)\left(\frac{\left(ae^2+cd^2+2cdex\right)\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{8cde}-\frac{3\left(cd^2-ae^2\right)^2\int \sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}dx}{16cde}\right)}{2d}+\frac{\left(x\left(ae^2+cd^2\right)+\ldots}{5\ldots}\right.}{12d}+\frac{(d+ex)\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/2}}{6cd}$$

$$\Big\downarrow \text{1087}$$

$$\frac{7\left(d^2-\frac{ae^2}{c}\right)\left(\frac{\left(d^2-\frac{ae^2}{c}\right)\left(\frac{\left(ae^2+cd^2+2cdex\right)\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{8cde}-\frac{3\left(cd^2-ae^2\right)^2\left(\frac{\left(ae^2+cd^2+2cdex\right)\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{4cde}-\frac{\left(cd^2- \cdots\right.}{\cdots}\right)}{16cde}\cdots\right)}{2d}\cdots\right)}{12d}\cdots$$

$$\frac{(d+ex)\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/2}}{6cd}$$

$\downarrow$ 1092

$$\frac{7\left(d^2-\frac{ae^2}{c}\right)\left(\frac{\left(d^2-\frac{ae^2}{c}\right)\left(\frac{\left(ae^2+cd^2+2cdex\right)\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{8cde}-\frac{3\left(cd^2-ae^2\right)^2\left(\frac{\left(ae^2+cd^2+2cdex\right)\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{4cde}-\frac{\left(cd^2- \cdots\right.}{\cdots}\right)}{1\cdots}\cdots\right)}{2d}\cdots\right)}{12d}\cdots$$

$$\frac{(d+ex)\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/2}}{6cd}$$

$\downarrow$ 219

$$\frac{7\left(d^2-\frac{ae^2}{c}\right)\left(\frac{\left(d^2-\frac{ae^2}{c}\right)\left(\frac{\left(ae^2+cd^2+2cdex\right)\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{8cde}-\frac{3\left(cd^2-ae^2\right)^2\left(\frac{\left(ae^2+cd^2+2cdex\right)\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{4cde}-\frac{\left(cd^2-\cdots\right.}{\cdots}\right)}{16cde}\right)}{2d}\cdots\right)}{12d}$$

$$\frac{(d+ex)\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/2}}{6cd}$$

input

```
Int[(d + e*x)^2*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2),x]
```

output

```
((d + e*x)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2))/(6*c*d) + (7*(d^
2 - (a*e^2)/c)*((a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2)/(5*c*d) + ((
d^2 - (a*e^2)/c)*(((c*d^2 + a*e^2 + 2*c*d*e*x)*(a*d*e + (c*d^2 + a*e^2)*x
+ c*d*e*x^2)^(3/2))/(8*c*d*e) - (3*(c*d^2 - a*e^2)^2*(((c*d^2 + a*e^2 + 2*
c*d*e*x)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(4*c*d*e) - ((c*d^2
- a*e^2)^2*ArcTanh[(c*d^2 + a*e^2 + 2*c*d*e*x)/(2*Sqrt[c]*Sqrt[d]*Sqrt[e]*
Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])])/(8*c^(3/2)*d^(3/2)*e^(3/2))
))/(16*c*d*e)))/(2*d)))/(12*d)
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1087
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^p/(2*c*(2*p + 1))), x] - Simp[p*((b^2 - 4*a*c)/(2*c*(2*
p + 1)))   Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] &&
GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[3*p])
```

rule 1092
```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[I
nt[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a
, b, c}, x]
```

rule 1134
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[e*(d + e*x)^(m - 1)*((a + b*x + c*x^2)^(p + 1)/(c*(m + 2*p +
 1))), x] + Simp[(m + p)*((2*c*d - b*e)/(c*(m + 2*p + 1)))   Int[(d + e*x)^
(m - 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[
c*d^2 - b*d*e + a*e^2, 0] && GtQ[m, 1] && NeQ[m + 2*p + 1, 0] && IntegerQ[2
*p]
```

rule 1160
```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[e*((a + b*x + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Simp[(2*c*d - b
*e)/(2*c)   Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
 && NeQ[p, -1]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1187 vs. $2(381) = 762$.

Time = 1.79 (sec) , antiderivative size = 1188, normalized size of antiderivative = 2.82

method	result	size
default	Expression too large to display	1188

input
```
int((e*x+d)^2*(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(3/2),x,method=_RETURNVERB
OSE)
```

output
```
d^2*(1/8*(2*c*d*e*x+a*e^2+c*d^2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(3/2)/c
/d/e+3/16*(4*a*c*d^2*e^2-(a*e^2+c*d^2)^2)/d/e/c*(1/4*(2*c*d*e*x+a*e^2+c*d^
2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(1/2)/c/d/e+1/8*(4*a*c*d^2*e^2-(a*e^2
+c*d^2)^2)/d/e/c*ln((1/2*a*e^2+1/2*c*d^2+c*d*x*e)/(d*e*c)^(1/2)+(a*d*e+(a*
e^2+c*d^2)*x+c*d*x^2*e)^(1/2))/(d*e*c)^(1/2)))+e^2*(1/6*x*(a*d*e+(a*e^2+c*
d^2)*x+c*d*x^2*e)^(5/2)/d/e/c-7/12*(a*e^2+c*d^2)/d/e/c*(1/5*(a*d*e+(a*e^2+
c*d^2)*x+c*d*x^2*e)^(5/2)/d/e/c-1/2*(a*e^2+c*d^2)/d/e/c*(1/8*(2*c*d*e*x+a*
e^2+c*d^2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(3/2)/c/d/e+3/16*(4*a*c*d^2*e
^2-(a*e^2+c*d^2)^2)/d/e/c*(1/4*(2*c*d*e*x+a*e^2+c*d^2)*(a*d*e+(a*e^2+c*d^2
)*x+c*d*x^2*e)^(1/2)/c/d/e+1/8*(4*a*c*d^2*e^2-(a*e^2+c*d^2)^2)/d/e/c*ln((1
/2*a*e^2+1/2*c*d^2+c*d*x*e)/(d*e*c)^(1/2)+(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e
)^(1/2))/(d*e*c)^(1/2))))-1/6*a/c*(1/8*(2*c*d*e*x+a*e^2+c*d^2)*(a*d*e+(a*e
^2+c*d^2)*x+c*d*x^2*e)^(3/2)/c/d/e+3/16*(4*a*c*d^2*e^2-(a*e^2+c*d^2)^2)/d/
e/c*(1/4*(2*c*d*e*x+a*e^2+c*d^2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(1/2)/c
/d/e+1/8*(4*a*c*d^2*e^2-(a*e^2+c*d^2)^2)/d/e/c*ln((1/2*a*e^2+1/2*c*d^2+c*d
*x*e)/(d*e*c)^(1/2)+(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(1/2))/(d*e*c)^(1/2)
)))+2*d*e*(1/5*(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(5/2)/d/e/c-1/2*(a*e^2+c*
d^2)/d/e/c*(1/8*(2*c*d*e*x+a*e^2+c*d^2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^
(3/2)/c/d/e+3/16*(4*a*c*d^2*e^2-(a*e^2+c*d^2)^2)/d/e/c*(1/4*(2*c*d*e*x+a*e
^2+c*d^2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(1/2)/c/d/e+1/8*(4*a*c*d^2*...
```

Fricas [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 1042, normalized size of antiderivative = 2.48

$$\int (d+ex)^2 \left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2} dx = \text{Too large to display}$$

input
```
integrate((e*x+d)^2*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),x, algorithm="
fricas")
```

output
```
[1/30720*(105*(c^6*d^12 - 6*a*c^5*d^10*e^2 + 15*a^2*c^4*d^8*e^4 - 20*a^3*c
^3*d^6*e^6 + 15*a^4*c^2*d^4*e^8 - 6*a^5*c*d^2*e^10 + a^6*e^12)*sqrt(c*d*e)
*log(8*c^2*d^2*e^2*x^2 + c^2*d^4 + 6*a*c*d^2*e^2 + a^2*e^4 + 4*sqrt(c*d*e*
x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(2*c*d*e*x + c*d^2 + a*e^2)*sqrt(c*d*e) +
 8*(c^2*d^3*e + a*c*d*e^3)*x) + 4*(1280*c^6*d^6*e^6*x^5 - 105*c^6*d^11*e +
 595*a*c^5*d^9*e^3 + 1686*a^2*c^4*d^7*e^5 - 1386*a^3*c^3*d^5*e^7 + 595*a^4
*c^2*d^3*e^9 - 105*a^5*c*d*e^11 + 128*(37*c^6*d^7*e^5 + 13*a*c^5*d^5*e^7)*
x^4 + 16*(387*c^6*d^8*e^4 + 410*a*c^5*d^6*e^6 + 3*a^2*c^4*d^4*e^8)*x^3 + 8
*(377*c^6*d^9*e^3 + 1191*a*c^5*d^7*e^5 + 39*a^2*c^4*d^5*e^7 - 7*a^3*c^3*d^
3*e^9)*x^2 + 2*(35*c^6*d^10*e^2 + 2876*a*c^5*d^8*e^4 + 450*a^2*c^4*d^6*e^6
 - 196*a^3*c^3*d^4*e^8 + 35*a^4*c^2*d^2*e^10)*x)*sqrt(c*d*e*x^2 + a*d*e +
(c*d^2 + a*e^2)*x))/(c^5*d^5*e^3), -1/15360*(105*(c^6*d^12 - 6*a*c^5*d^10*
e^2 + 15*a^2*c^4*d^8*e^4 - 20*a^3*c^3*d^6*e^6 + 15*a^4*c^2*d^4*e^8 - 6*a^5
*c*d^2*e^10 + a^6*e^12)*sqrt(-c*d*e)*arctan(1/2*sqrt(c*d*e*x^2 + a*d*e + (
c*d^2 + a*e^2)*x)*(2*c*d*e*x + c*d^2 + a*e^2)*sqrt(-c*d*e)/(c^2*d^2*e^2*x^
2 + a*c*d^2*e^2 + (c^2*d^3*e + a*c*d*e^3)*x)) - 2*(1280*c^6*d^6*e^6*x^5 -
105*c^6*d^11*e + 595*a*c^5*d^9*e^3 + 1686*a^2*c^4*d^7*e^5 - 1386*a^3*c^3*d
^5*e^7 + 595*a^4*c^2*d^3*e^9 - 105*a^5*c*d*e^11 + 128*(37*c^6*d^7*e^5 + 13
*a*c^5*d^5*e^7)*x^4 + 16*(387*c^6*d^8*e^4 + 410*a*c^5*d^6*e^6 + 3*a^2*c^4*
d^4*e^8)*x^3 + 8*(377*c^6*d^9*e^3 + 1191*a*c^5*d^7*e^5 + 39*a^2*c^4*d^5...
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2909 vs. $2(400) = 800$.

Time = 1.42 (sec) , antiderivative size = 2909, normalized size of antiderivative = 6.91

$$\int (d+ex)^2\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}\,dx=\text{Too large to display}$$

input
```
integrate((e*x+d)**2*(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(3/2),x)
```

output
```
Piecewise((sqrt(a*d*e + c*d*e*x**2 + x*(a*e**2 + c*d**2))*(c*d*e**3*x**5/6
 + x**4*(2*a*c*d*e**5 + 4*c**2*d**3*e**3 - c*d*e**3*(11*a*e**2/2 + 11*c*d*
*2/2)/6)/(5*c*d*e) + x**3*(a**2*e**6 + 43*a*c*d**2*e**4/6 + 6*c**2*d**4*e*
*2 - (9*a*e**2/2 + 9*c*d**2/2)*(2*a*c*d*e**5 + 4*c**2*d**3*e**3 - c*d*e**3
*(11*a*e**2/2 + 11*c*d**2/2)/6)/(5*c*d*e))/(4*c*d*e) + x**2*(4*a**2*d*e**5
 + 12*a*c*d**3*e**3 - 4*a*(2*a*c*d*e**5 + 4*c**2*d**3*e**3 - c*d*e**3*(11*
a*e**2/2 + 11*c*d**2/2)/6)/(5*c) + 4*c**2*d**5*e - (7*a*e**2/2 + 7*c*d**2/
2)*(a**2*e**6 + 43*a*c*d**2*e**4/6 + 6*c**2*d**4*e**2 - (9*a*e**2/2 + 9*c*
d**2/2)*(2*a*c*d*e**5 + 4*c**2*d**3*e**3 - c*d*e**3*(11*a*e**2/2 + 11*c*d*
*2/2)/6)/(5*c*d*e))/(4*c*d*e))/(3*c*d*e) + x*(6*a**2*d**2*e**4 + 8*a*c*d**
4*e**2 - 3*a*(a**2*e**6 + 43*a*c*d**2*e**4/6 + 6*c**2*d**4*e**2 - (9*a*e**
2/2 + 9*c*d**2/2)*(2*a*c*d*e**5 + 4*c**2*d**3*e**3 - c*d*e**3*(11*a*e**2/2
 + 11*c*d**2/2)/6)/(5*c*d*e))/(4*c) + c**2*d**6 - (5*a*e**2/2 + 5*c*d**2/2
)*(4*a**2*d*e**5 + 12*a*c*d**3*e**3 - 4*a*(2*a*c*d*e**5 + 4*c**2*d**3*e**3
 - c*d*e**3*(11*a*e**2/2 + 11*c*d**2/2)/6)/(5*c) + 4*c**2*d**5*e - (7*a*e*
*2/2 + 7*c*d**2/2)*(a**2*e**6 + 43*a*c*d**2*e**4/6 + 6*c**2*d**4*e**2 - (9
*a*e**2/2 + 9*c*d**2/2)*(2*a*c*d*e**5 + 4*c**2*d**3*e**3 - c*d*e**3*(11*a*
e**2/2 + 11*c*d**2/2)/6)/(5*c*d*e))/(4*c*d*e))/(3*c*d*e))/(2*c*d*e) + (4*a
**2*d**3*e**3 + 2*a*c*d**5*e - 2*a*(4*a**2*d*e**5 + 12*a*c*d**3*e**3 - 4*a
*(2*a*c*d*e**5 + 4*c**2*d**3*e**3 - c*d*e**3*(11*a*e**2/2 + 11*c*d**2/2...
```

Maxima [F(-2)]

Exception generated.

$$\int (d+ex)^2 \left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2} dx = \text{Exception raised: ValueError}$$

input
```
integrate((e*x+d)^2*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),x, algorithm="
maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

Giac [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 519, normalized size of antiderivative = 1.23

$$\int (d+ex)^2 \left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}\,dx=\frac{1}{7680}\sqrt{cdex^2+cd^2x+ae^2x+ade}\left(2\left(4\left(2\left(8\left(10\,cde^3x+\frac{37\,c^6d^7e^7+13\,ac^5d^5e^9}{c^5d^5e^5}\right)x+\right.\right.\right.\right.\right.$$
$$-\frac{7\left(c^6d^{12}-6\,ac^5d^{10}e^2+15\,a^2c^4d^8e^4-20\,a^3c^3d^6e^6+15\,a^4c^2d^4e^8-6\,a^5cd^2e^{10}+a^6e^{12}\right)\log\left(\left|-cd^2-ae^2-\right.\right.}{1024\sqrt{cde}c^4d^4e^2}$$

input
```
integrate((e*x+d)^2*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),x, algorithm="
giac")
```

output
```
1/7680*sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*x + a*d*e)*(2*(4*(2*(8*(10*c*d*e^3
*x + (37*c^6*d^7*e^7 + 13*a*c^5*d^5*e^9)/(c^5*d^5*e^5))*x + (387*c^6*d^8*e
^6 + 410*a*c^5*d^6*e^8 + 3*a^2*c^4*d^4*e^10)/(c^5*d^5*e^5))*x + (377*c^6*d
^9*e^5 + 1191*a*c^5*d^7*e^7 + 39*a^2*c^4*d^5*e^9 - 7*a^3*c^3*d^3*e^11)/(c^
5*d^5*e^5))*x + (35*c^6*d^10*e^4 + 2876*a*c^5*d^8*e^6 + 450*a^2*c^4*d^6*e^
8 - 196*a^3*c^3*d^4*e^10 + 35*a^4*c^2*d^2*e^12)/(c^5*d^5*e^5))*x - (105*c^
6*d^11*e^3 - 595*a*c^5*d^9*e^5 - 1686*a^2*c^4*d^7*e^7 + 1386*a^3*c^3*d^5*e
^9 - 595*a^4*c^2*d^3*e^11 + 105*a^5*c*d*e^13)/(c^5*d^5*e^5)) - 7/1024*(c^6
*d^12 - 6*a*c^5*d^10*e^2 + 15*a^2*c^4*d^8*e^4 - 20*a^3*c^3*d^6*e^6 + 15*a^
4*c^2*d^4*e^8 - 6*a^5*c*d^2*e^10 + a^6*e^12)*log(abs(-c*d^2 - a*e^2 - 2*sq
rt(c*d*e)*(sqrt(c*d*e)*x - sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*x + a*d*e))))/
(sqrt(c*d*e)*c^4*d^4*e^2)
```

Mupad [F(-1)]

Timed out.

$$\int (d+ex)^2 \left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}\,dx=\int (d+e\,x)^2\,\left(c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e\right)^{3/2}\,dx$$

input
```
int((d + e*x)^2*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(3/2),x)
```

output
```
int((d + e*x)^2*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(3/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 1033, normalized size of antiderivative = 2.45

$$\int (d+ex)^2 \left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}\,dx = \text{Too large to display}$$

input
```
int((e*x+d)^2*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),x)
```

output
```
( - 105*sqrt(d + e*x)*sqrt(a*e + c*d*x)*a**5*c*d*e**11 + 595*sqrt(d + e*x)
*sqrt(a*e + c*d*x)*a**4*c**2*d**3*e**9 + 70*sqrt(d + e*x)*sqrt(a*e + c*d*x
)*a**4*c**2*d**2*e**10*x - 1386*sqrt(d + e*x)*sqrt(a*e + c*d*x)*a**3*c**3*
d**5*e**7 - 392*sqrt(d + e*x)*sqrt(a*e + c*d*x)*a**3*c**3*d**4*e**8*x - 56
*sqrt(d + e*x)*sqrt(a*e + c*d*x)*a**3*c**3*d**3*e**9*x**2 + 1686*sqrt(d +
e*x)*sqrt(a*e + c*d*x)*a**2*c**4*d**7*e**5 + 900*sqrt(d + e*x)*sqrt(a*e +
c*d*x)*a**2*c**4*d**6*e**6*x + 312*sqrt(d + e*x)*sqrt(a*e + c*d*x)*a**2*c*
*4*d**5*e**7*x**2 + 48*sqrt(d + e*x)*sqrt(a*e + c*d*x)*a**2*c**4*d**4*e**8
*x**3 + 595*sqrt(d + e*x)*sqrt(a*e + c*d*x)*a*c**5*d**9*e**3 + 5752*sqrt(d
 + e*x)*sqrt(a*e + c*d*x)*a*c**5*d**8*e**4*x + 9528*sqrt(d + e*x)*sqrt(a*e
 + c*d*x)*a*c**5*d**7*e**5*x**2 + 6560*sqrt(d + e*x)*sqrt(a*e + c*d*x)*a*c
**5*d**6*e**6*x**3 + 1664*sqrt(d + e*x)*sqrt(a*e + c*d*x)*a*c**5*d**5*e**7
*x**4 - 105*sqrt(d + e*x)*sqrt(a*e + c*d*x)*c**6*d**11*e + 70*sqrt(d + e*x
)*sqrt(a*e + c*d*x)*c**6*d**10*e**2*x + 3016*sqrt(d + e*x)*sqrt(a*e + c*d*
x)*c**6*d**9*e**3*x**2 + 6192*sqrt(d + e*x)*sqrt(a*e + c*d*x)*c**6*d**8*e*
*4*x**3 + 4736*sqrt(d + e*x)*sqrt(a*e + c*d*x)*c**6*d**7*e**5*x**4 + 1280*
sqrt(d + e*x)*sqrt(a*e + c*d*x)*c**6*d**6*e**6*x**5 + 105*sqrt(e)*sqrt(d)*
sqrt(c)*log((sqrt(e)*sqrt(a*e + c*d*x) + sqrt(d)*sqrt(c)*sqrt(d + e*x))/sq
rt(a*e**2 - c*d**2))*a**6*e**12 - 630*sqrt(e)*sqrt(d)*sqrt(c)*log((sqrt(e)
*sqrt(a*e + c*d*x) + sqrt(d)*sqrt(c)*sqrt(d + e*x))/sqrt(a*e**2 - c*d**...
```

3.211 $\int (d+ex)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 363

$$\begin{aligned}\int (d+ex)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}\,dx = & -\frac{3(cd^2-ae^2)^4\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{128c^3d^3e^2} \\ & +\frac{(cd^2-ae^2)^3\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{64c^3d^3e(d+ex)} \\ & +\frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{5cd} \\ & +\frac{(cd^2-ae^2)^2\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{16c^3d^3(d+ex)^2} \\ & +\frac{(cd^2-ae^2)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{8c^2d^2(d+ex)} \\ & +\frac{3(cd^2-ae^2)^5\operatorname{arctanh}\left(\frac{\sqrt{e}\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{\sqrt{c}\sqrt{d}(d+ex)}\right)}{128c^{7/2}d^{7/2}e^{5/2}}\end{aligned}$$

output
```
-3/128*(-a*e^2+c*d^2)^4*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/c^3/d^3/e^
2+1/64*(-a*e^2+c*d^2)^3*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/c^3/d^3/e/
(e*x+d)+1/5*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/c/d+1/16*(-a*e^2+c*d^2
)^2*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/c^3/d^3/(e*x+d)^2+1/8*(-a*e^2+
c*d^2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/c^2/d^2/(e*x+d)+3/128*(-a*e
^2+c*d^2)^5*arctanh(e^(1/2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/c^(1/2
)/d^(1/2)/(e*x+d))/c^(7/2)/d^(7/2)/e^(5/2)
```

Mathematica [A] (verified)

Time = 0.80 (sec) , antiderivative size = 295, normalized size of antiderivative = 0.81

$$\int (d+ex)\left(ade+\left(cd^2+ae^2\right)x +cdex^2\right)^{3/2}\,dx = \frac{((ae+cdx)(d+ex))^{3/2}\left(\frac{\sqrt{c}\sqrt{d}\sqrt{e}(15a^4e^8-10a^3cde^6(7d+ex)+2a^2c^2d^2e^4(64d^2+23dex+4e^2x^2)+2ac^3d^3e^2(35a \cdots}{(ae+c \cdots}\right.}{6 \cdots}$$

input
```
Integrate[(d + e*x)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2),x]
```

output
```
(((a*e + c*d*x)*(d + e*x))^(3/2)*((Sqrt[c]*Sqrt[d]*Sqrt[e]*(15*a^4*e^8 - 1
0*a^3*c*d*e^6*(7*d + e*x) + 2*a^2*c^2*d^2*e^4*(64*d^2 + 23*d*e*x + 4*e^2*x
^2) + 2*a*c^3*d^3*e^2*(35*d^3 + 233*d^2*e*x + 256*d*e^2*x^2 + 88*e^3*x^3)
+ c^4*d^4*(-15*d^4 + 10*d^3*e*x + 248*d^2*e^2*x^2 + 336*d*e^3*x^3 + 128*e^
4*x^4)))/((a*e + c*d*x)*(d + e*x)) + (15*(c*d^2 - a*e^2)^5*ArcTanh[(Sqrt[c
]*Sqrt[d]*Sqrt[d + e*x])/(Sqrt[e]*Sqrt[a*e + c*d*x])])/((a*e + c*d*x)^(3/2
)*(d + e*x)^(3/2))))/(640*c^(7/2)*d^(7/2)*e^(5/2))
```

Rubi [A] (verified)

Time = 0.65 (sec) , antiderivative size = 306, normalized size of antiderivative = 0.84, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {1160, 1087, 1087, 1092, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (d+ex)\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}dx$$

$$\Big\downarrow \text{ 1160}$$

$$\frac{\left(d^2-\frac{ae^2}{c}\right)\int \left(cdex^2+\left(cd^2+ae^2\right)x+ade\right)^{3/2}dx}{2d}+\frac{\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/2}}{5cd}$$

$$\Big\downarrow \text{ 1087}$$

$$\frac{\left(d^2-\frac{ae^2}{c}\right)\left(\frac{\left(ae^2+cd^2+2cdex\right)\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{8cde}-\frac{3\left(cd^2-ae^2\right)^2\int \sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}dx}{16cde}\right)}{2d}+\frac{\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/2}}{5cd}$$

$$\Big\downarrow \text{ 1087}$$

$$\frac{\left(d^2-\frac{ae^2}{c}\right)\left(\frac{\left(ae^2+cd^2+2cdex\right)\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{8cde}-\frac{3\left(cd^2-ae^2\right)^2\left(\frac{\left(ae^2+cd^2+2cdex\right)\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{4cde}-\frac{\left(cd^2-ae^2\right)^2\int}{}\right.}{16cde}\right.}{2d}$$
$$\frac{\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/2}}{5cd}$$

$$\Big\downarrow \text{ 1092}$$

$$\frac{\left(d^2-\frac{ae^2}{c}\right)\left(\frac{\left(ae^2+cd^2+2cdex\right)\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{8cde}-\frac{3\left(cd^2-ae^2\right)^2\left(\frac{\left(ae^2+cd^2+2cdex\right)\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{4cde}-\frac{\left(cd^2-ae^2\right)^2\int}{}\right.}{16cde}\right.}{2d}$$
$$\frac{\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/2}}{5cd}$$

$$\Big\downarrow \text{ 219}$$

$$\frac{\left(d^2-\frac{ae^2}{c}\right)\left(\frac{(ae^2+cd^2+2cdex)\left(x(ae^2+cd^2)+ade+cdex^2\right)^{3/2}}{8cde}-\frac{3(cd^2-ae^2)^2\left(\frac{(ae^2+cd^2+2cdex)\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{4cde}-\frac{(cd^2-ae^2)^2 \mathrm{a}}{}\right.}{16cde}\right.}{2d}$$

$$\frac{\left(x(ae^2+cd^2)+ade+cdex^2\right)^{5/2}}{5cd}$$

input
```
Int[(d + e*x)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2),x]
```

output
```
(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2)/(5*c*d) + ((d^2 - (a*e^2)/c)
*(((c*d^2 + a*e^2 + 2*c*d*e*x)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/
2))/(8*c*d*e) - (3*(c*d^2 - a*e^2)^2*(((c*d^2 + a*e^2 + 2*c*d*e*x)*Sqrt[a*
d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(4*c*d*e) - ((c*d^2 - a*e^2)^2*ArcTa
nh[(c*d^2 + a*e^2 + 2*c*d*e*x)/(2*Sqrt[c]*Sqrt[d]*Sqrt[e]*Sqrt[a*d*e + (c*
d^2 + a*e^2)*x + c*d*e*x^2])])/(8*c^(3/2)*d^(3/2)*e^(3/2))))/(16*c*d*e)))/
(2*d)
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1087
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^p/(2*c*(2*p + 1))), x] - Simp[p*((b^2 - 4*a*c)/(2*c*(2*
p + 1)))   Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] &&
GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[3*p])
```

rule 1092
```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[I
nt[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a
, b, c}, x]
```

rule 1160
```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[e*((a + b*x + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Simp[(2*c*d - b
*e)/(2*c)   Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
 && NeQ[p, -1]
```

Maple [A] (verified)

Time = 1.50 (sec) , antiderivative size = 559, normalized size of antiderivative = 1.54

method	result
default	$d\left(\frac{\left(2cdxe+a\,e^{2}+c\,d^{2}\right)\left(ade+\left(a\,e^{2}+c\,d^{2}\right)x+cd\,x^{2}e\right)^{\frac{3}{2}}}{8cde}+\frac{3\left(4ac\,d^{2}e^{2}-\left(a\,e^{2}+c\,d^{2}\right)^{2}\right)\left(\frac{\left(2cdxe+a\,e^{2}+c\,d^{2}\right)\sqrt{ade+\left(a\,e^{2}+c\,d^{2}\right)x+cd\,x}}{4cde}\right.}{}\right.$

input
```
int((e*x+d)*(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(3/2),x,method=_RETURNVERBOS
E)
```

output
```
d*(1/8*(2*c*d*e*x+a*e^2+c*d^2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(3/2)/c/d
/e+3/16*(4*a*c*d^2*e^2-(a*e^2+c*d^2)^2)/d/e/c*(1/4*(2*c*d*e*x+a*e^2+c*d^2)
*(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(1/2)/c/d/e+1/8*(4*a*c*d^2*e^2-(a*e^2+c
*d^2)^2)/d/e/c*ln((1/2*a*e^2+1/2*c*d^2+c*d*x*e)/(d*e*c)^(1/2)+(a*d*e+(a*e^
2+c*d^2)*x+c*d*x^2*e)^(1/2))/(d*e*c)^(1/2)))+e*(1/5*(a*d*e+(a*e^2+c*d^2)*x
+c*d*x^2*e)^(5/2)/d/e/c-1/2*(a*e^2+c*d^2)/d/e/c*(1/8*(2*c*d*e*x+a*e^2+c*d^
2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(3/2)/c/d/e+3/16*(4*a*c*d^2*e^2-(a*e^
2+c*d^2)^2)/d/e/c*(1/4*(2*c*d*e*x+a*e^2+c*d^2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*
x^2*e)^(1/2)/c/d/e+1/8*(4*a*c*d^2*e^2-(a*e^2+c*d^2)^2)/d/e/c*ln((1/2*a*e^2
+1/2*c*d^2+c*d*x*e)/(d*e*c)^(1/2)+(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(1/2))
/(d*e*c)^(1/2))))
```

Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 844, normalized size of antiderivative = 2.33

$$\int (d+ex)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}\,dx=\text{Too large to display}$$

input
```
integrate((e*x+d)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),x, algorithm="fr
icas")
```

output
```
[-1/2560*(15*(c^5*d^10 - 5*a*c^4*d^8*e^2 + 10*a^2*c^3*d^6*e^4 - 10*a^3*c^2
*d^4*e^6 + 5*a^4*c*d^2*e^8 - a^5*e^10)*sqrt(c*d*e)*log(8*c^2*d^2*e^2*x^2 +
 c^2*d^4 + 6*a*c*d^2*e^2 + a^2*e^4 - 4*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a
*e^2)*x)*(2*c*d*e*x + c*d^2 + a*e^2)*sqrt(c*d*e) + 8*(c^2*d^3*e + a*c*d*e^
3)*x) - 4*(128*c^5*d^5*e^5*x^4 - 15*c^5*d^9*e + 70*a*c^4*d^7*e^3 + 128*a^2
*c^3*d^5*e^5 - 70*a^3*c^2*d^3*e^7 + 15*a^4*c*d*e^9 + 16*(21*c^5*d^6*e^4 +
11*a*c^4*d^4*e^6)*x^3 + 8*(31*c^5*d^7*e^3 + 64*a*c^4*d^5*e^5 + a^2*c^3*d^3
*e^7)*x^2 + 2*(5*c^5*d^8*e^2 + 233*a*c^4*d^6*e^4 + 23*a^2*c^3*d^4*e^6 - 5*
a^3*c^2*d^2*e^8)*x)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x))/(c^4*d^4*
e^3), -1/1280*(15*(c^5*d^10 - 5*a*c^4*d^8*e^2 + 10*a^2*c^3*d^6*e^4 - 10*a^
3*c^2*d^4*e^6 + 5*a^4*c*d^2*e^8 - a^5*e^10)*sqrt(-c*d*e)*arctan(1/2*sqrt(c
*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(2*c*d*e*x + c*d^2 + a*e^2)*sqrt(-c*
d*e)/(c^2*d^2*e^2*x^2 + a*c*d^2*e^2 + (c^2*d^3*e + a*c*d*e^3)*x)) - 2*(128
*c^5*d^5*e^5*x^4 - 15*c^5*d^9*e + 70*a*c^4*d^7*e^3 + 128*a^2*c^3*d^5*e^5 -
 70*a^3*c^2*d^3*e^7 + 15*a^4*c*d*e^9 + 16*(21*c^5*d^6*e^4 + 11*a*c^4*d^4*e
^6)*x^3 + 8*(31*c^5*d^7*e^3 + 64*a*c^4*d^5*e^5 + a^2*c^3*d^3*e^7)*x^2 + 2*
(5*c^5*d^8*e^2 + 233*a*c^4*d^6*e^4 + 23*a^2*c^3*d^4*e^6 - 5*a^3*c^2*d^2*e^
8)*x)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x))/(c^4*d^4*e^3)]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1681 vs. $2(335) = 670$.

Time = 1.27 (sec) , antiderivative size = 1681, normalized size of antiderivative = 4.63

$$\int (d+ex)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}\,dx = \text{Too large to display}$$

input
```
integrate((e*x+d)*(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(3/2),x)
```

output
```
Piecewise((sqrt(a*d*e + c*d*e*x**2 + x*(a*e**2 + c*d**2))*(c*d*e**2*x**4/5
 + x**3*(2*a*c*d*e**4 + 3*c**2*d**3*e**2 - c*d*e**2*(9*a*e**2/2 + 9*c*d**2
/2)/5)/(4*c*d*e) + x**2*(a**2*e**5 + 26*a*c*d**2*e**3/5 + 3*c**2*d**4*e -
(7*a*e**2/2 + 7*c*d**2/2)*(2*a*c*d*e**4 + 3*c**2*d**3*e**2 - c*d*e**2*(9*a
*e**2/2 + 9*c*d**2/2)/5)/(4*c*d*e))/(3*c*d*e) + x*(3*a**2*d*e**4 + 6*a*c*d
**3*e**2 - 3*a*(2*a*c*d*e**4 + 3*c**2*d**3*e**2 - c*d*e**2*(9*a*e**2/2 + 9
*c*d**2/2)/5)/(4*c) + c**2*d**5 - (5*a*e**2/2 + 5*c*d**2/2)*(a**2*e**5 + 2
6*a*c*d**2*e**3/5 + 3*c**2*d**4*e - (7*a*e**2/2 + 7*c*d**2/2)*(2*a*c*d*e**
4 + 3*c**2*d**3*e**2 - c*d*e**2*(9*a*e**2/2 + 9*c*d**2/2)/5)/(4*c*d*e))/(3
*c*d*e))/(2*c*d*e) + (3*a**2*d**2*e**3 + 2*a*c*d**4*e - 2*a*(a**2*e**5 + 2
6*a*c*d**2*e**3/5 + 3*c**2*d**4*e - (7*a*e**2/2 + 7*c*d**2/2)*(2*a*c*d*e**
4 + 3*c**2*d**3*e**2 - c*d*e**2*(9*a*e**2/2 + 9*c*d**2/2)/5)/(4*c*d*e))/(3
*c) - (3*a*e**2/2 + 3*c*d**2/2)*(3*a**2*d*e**4 + 6*a*c*d**3*e**2 - 3*a*(2*
a*c*d*e**4 + 3*c**2*d**3*e**2 - c*d*e**2*(9*a*e**2/2 + 9*c*d**2/2)/5)/(4*c
) + c**2*d**5 - (5*a*e**2/2 + 5*c*d**2/2)*(a**2*e**5 + 26*a*c*d**2*e**3/5
+ 3*c**2*d**4*e - (7*a*e**2/2 + 7*c*d**2/2)*(2*a*c*d*e**4 + 3*c**2*d**3*e*
*2 - c*d*e**2*(9*a*e**2/2 + 9*c*d**2/2)/5)/(4*c*d*e))/(3*c*d*e))/(2*c*d*e)
)/(c*d*e)) + (a**2*d**3*e**2 - a*(3*a**2*d*e**4 + 6*a*c*d**3*e**2 - 3*a*(2
*a*c*d*e**4 + 3*c**2*d**3*e**2 - c*d*e**2*(9*a*e**2/2 + 9*c*d**2/2)/5)/(4*
c) + c**2*d**5 - (5*a*e**2/2 + 5*c*d**2/2)*(a**2*e**5 + 26*a*c*d**2*e**...
```

Maxima [F(-2)]

Exception generated.

$$\int (d+ex) \left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2} dx = \text{Exception raised: ValueError}$$

input
```
integrate((e*x+d)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),x, algorithm="ma
xima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

Giac [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 411, normalized size of antiderivative = 1.13

$$\int (d+ex)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}\,dx=\frac{1}{640}\sqrt{cdex^2+cd^2x+ae^2x+ade}\left(2\left(4\left(2\left(8\,cde^2x+\frac{21\,c^5d^6e^5+11\,ac^4d^4e^7}{c^4d^4e^4}\right)x+\frac{31\,c^5\cdots}{\cdots}\right.\right.\right.$$
$$-\frac{3\left(c^5d^{10}-5\,ac^4d^8e^2+10\,a^2c^3d^6e^4-10\,a^3c^2d^4e^6+5\,a^4cd^2e^8-a^5e^{10}\right)\log\left(\left|-cd^2-ae^2-2\sqrt{cde}\left(\sqrt{cde}x\cdots\right.\right.\right.}{256\sqrt{cde}c^3d^3e^2}$$

input
```
integrate((e*x+d)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),x, algorithm="gi
ac")
```

output
```
1/640*sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*x + a*d*e)*(2*(4*(2*(8*c*d*e^2*x +
(21*c^5*d^6*e^5 + 11*a*c^4*d^4*e^7)/(c^4*d^4*e^4))*x + (31*c^5*d^7*e^4 + 6
4*a*c^4*d^5*e^6 + a^2*c^3*d^3*e^8)/(c^4*d^4*e^4))*x + (5*c^5*d^8*e^3 + 233
*a*c^4*d^6*e^5 + 23*a^2*c^3*d^4*e^7 - 5*a^3*c^2*d^2*e^9)/(c^4*d^4*e^4))*x
- (15*c^5*d^9*e^2 - 70*a*c^4*d^7*e^4 - 128*a^2*c^3*d^5*e^6 + 70*a^3*c^2*d^
3*e^8 - 15*a^4*c*d*e^10)/(c^4*d^4*e^4)) - 3/256*(c^5*d^10 - 5*a*c^4*d^8*e^
2 + 10*a^2*c^3*d^6*e^4 - 10*a^3*c^2*d^4*e^6 + 5*a^4*c*d^2*e^8 - a^5*e^10)*
log(abs(-c*d^2 - a*e^2 - 2*sqrt(c*d*e)*(sqrt(c*d*e)*x - sqrt(c*d*e*x^2 + c
*d^2*x + a*e^2*x + a*d*e))))/(sqrt(c*d*e)*c^3*d^3*e^2)
```

Mupad [B] (verification not implemented)

Time = 5.88 (sec) , antiderivative size = 646, normalized size of antiderivative = 1.78

$$\int (d+ex)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}\,dx=\frac{\left(c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e\right)^{5/2}}{5\,c\,d}$$
$$+\frac{\left(\frac{c\,d^2}{2}+c\,x\,d\,e+\frac{a\,e^2}{2}\right)\,\left(c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e\right)^{3/2}}{4\,c\,e}$$
$$-\frac{\left(\frac{3\,\left(c\,d^2+a\,e^2\right)^2}{4}-3\,a\,c\,d^2\,e^2\right)\,\left(\left(\frac{x}{2}+\frac{c\,d^2+a\,e^2}{4\,c\,d\,e}\right)\,\sqrt{c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e}-\frac{\ln\left(2\,\sqrt{\left(a\,e+c\,d\,x\right)\,\left(d+e\,x\right)}\,\sqrt{c\,d\right.}}{}\right.}{4\,c\,e}$$
$$-\frac{\left(c\,d^2+a\,e^2\right)\,\left(\frac{x\,\left(c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e\right)^{3/2}}{4}-\frac{3\,a\,d\,e\,\left(\ln\left(2\,\sqrt{\left(a\,e+c\,d\,x\right)\,\left(d+e\,x\right)}\,\sqrt{c\,d\,e}+a\,e^2+c\,d^2+2\,c\,d\,e\,x\right)\,\left(\frac{\left(c\,d^2+a\,e^2\right)^2}{8\,\left(c\,d\,e\right)^{3/2}}-\frac{}{2}\right.\right.}{4}\right.}{}$$

input
```
int((d + e*x)*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(3/2),x)
```

output
```
(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(5/2)/(5*c*d) + (((a*e^2)/2 + (c*d
^2)/2 + c*d*e*x)*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(3/2))/(4*c*e) -
(((3*(a*e^2 + c*d^2)^2)/4 - 3*a*c*d^2*e^2)*((x/2 + (a*e^2 + c*d^2)/(4*c*d*
e))*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2) - (log(2*((a*e + c*d*x)*
(d + e*x))^(1/2)*(c*d*e)^(1/2) + a*e^2 + c*d^2 + 2*c*d*e*x)*((a*e^2 + c*d^
2)^2/4 - a*c*d^2*e^2))/(2*(c*d*e)^(3/2))))/(4*c*e) - ((a*e^2 + c*d^2)*((x*
(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(3/2))/4 - (3*a*d*e*(log(2*((a*e +
 c*d*x)*(d + e*x))^(1/2)*(c*d*e)^(1/2) + a*e^2 + c*d^2 + 2*c*d*e*x)*((a*e^
2 + c*d^2)^2/(8*(c*d*e)^(3/2)) - (a*d*e)/(2*(c*d*e)^(1/2))) - ((a*e^2 + c*
d^2 + 2*c*d*e*x)*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2))/(4*c*d*e))
)/4 + ((a*e^2 + c*d^2)*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(3/2))/(8*c
*d*e) + (3*(a*e^2 + c*d^2)^2*(log(2*((a*e + c*d*x)*(d + e*x))^(1/2)*(c*d*e
)^(1/2) + a*e^2 + c*d^2 + 2*c*d*e*x)*((a*e^2 + c*d^2)^2/(8*(c*d*e)^(3/2))
- (a*d*e)/(2*(c*d*e)^(1/2))) - ((a*e^2 + c*d^2 + 2*c*d*e*x)*(x*(a*e^2 + c*
d^2) + a*d*e + c*d*e*x^2)^(1/2))/(4*c*d*e)))/(16*c*d*e)))/(2*c*d)
```

Reduce [B] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 790, normalized size of antiderivative = 2.18

$$\int (d+ex)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}\,dx=\text{Too large to display}$$

input
```
int((e*x+d)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),x)
```

output
```
(15*sqrt(d + e*x)*sqrt(a*e + c*d*x)*a**4*c*d*e**9 - 70*sqrt(d + e*x)*sqrt(
a*e + c*d*x)*a**3*c**2*d**3*e**7 - 10*sqrt(d + e*x)*sqrt(a*e + c*d*x)*a**3
*c**2*d**2*e**8*x + 128*sqrt(d + e*x)*sqrt(a*e + c*d*x)*a**2*c**3*d**5*e**
5 + 46*sqrt(d + e*x)*sqrt(a*e + c*d*x)*a**2*c**3*d**4*e**6*x + 8*sqrt(d +
e*x)*sqrt(a*e + c*d*x)*a**2*c**3*d**3*e**7*x**2 + 70*sqrt(d + e*x)*sqrt(a*
e + c*d*x)*a*c**4*d**7*e**3 + 466*sqrt(d + e*x)*sqrt(a*e + c*d*x)*a*c**4*d
**6*e**4*x + 512*sqrt(d + e*x)*sqrt(a*e + c*d*x)*a*c**4*d**5*e**5*x**2 + 1
76*sqrt(d + e*x)*sqrt(a*e + c*d*x)*a*c**4*d**4*e**6*x**3 - 15*sqrt(d + e*x
)*sqrt(a*e + c*d*x)*c**5*d**9*e + 10*sqrt(d + e*x)*sqrt(a*e + c*d*x)*c**5*
d**8*e**2*x + 248*sqrt(d + e*x)*sqrt(a*e + c*d*x)*c**5*d**7*e**3*x**2 + 33
6*sqrt(d + e*x)*sqrt(a*e + c*d*x)*c**5*d**6*e**4*x**3 + 128*sqrt(d + e*x)*
sqrt(a*e + c*d*x)*c**5*d**5*e**5*x**4 - 15*sqrt(e)*sqrt(d)*sqrt(c)*log((sq
rt(e)*sqrt(a*e + c*d*x) + sqrt(d)*sqrt(c)*sqrt(d + e*x))/sqrt(a*e**2 - c*d
**2))*a**5*e**10 + 75*sqrt(e)*sqrt(d)*sqrt(c)*log((sqrt(e)*sqrt(a*e + c*d*
x) + sqrt(d)*sqrt(c)*sqrt(d + e*x))/sqrt(a*e**2 - c*d**2))*a**4*c*d**2*e**
8 - 150*sqrt(e)*sqrt(d)*sqrt(c)*log((sqrt(e)*sqrt(a*e + c*d*x) + sqrt(d)*s
qrt(c)*sqrt(d + e*x))/sqrt(a*e**2 - c*d**2))*a**3*c**2*d**4*e**6 + 150*sqr
t(e)*sqrt(d)*sqrt(c)*log((sqrt(e)*sqrt(a*e + c*d*x) + sqrt(d)*sqrt(c)*sqrt
(d + e*x))/sqrt(a*e**2 - c*d**2))*a**2*c**3*d**6*e**4 - 75*sqrt(e)*sqrt(d)
*sqrt(c)*log((sqrt(e)*sqrt(a*e + c*d*x) + sqrt(d)*sqrt(c)*sqrt(d + e*x)...
```

3.212 $\int \left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2} dx$

Optimal result

Integrand size = 29, antiderivative size = 217

$$\int \left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2} dx = -\frac{3(cd^2-ae^2)^2\left(cd^2+ae^2+2cdex\right)\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{64c^2d^2e^2} + \frac{\left(cd^2+ae^2+2cdex\right)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{8cde} + \frac{3(cd^2-ae^2)^4\operatorname{arctanh}\left(\frac{\sqrt{c}\sqrt{d}(d+ex)}{\sqrt{e}\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\right)}{64c^{5/2}d^{5/2}e^{5/2}}$$

output
```
-3/64*(-a*e^2+c*d^2)^2*(2*c*d*e*x+a*e^2+c*d^2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*
e*x^2)^(1/2)/c^2/d^2/e^2+1/8*(2*c*d*e*x+a*e^2+c*d^2)*(a*d*e+(a*e^2+c*d^2)*
x+c*d*e*x^2)^(3/2)/c/d/e+3/64*(-a*e^2+c*d^2)^4*arctanh(c^(1/2)*d^(1/2)*(e*
x+d)/e^(1/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2))/c^(5/2)/d^(5/2)/e^(5
/2)
```

Mathematica [A] (verified)

Time = 0.67 (sec) , antiderivative size = 239, normalized size of antiderivative = 1.10

$$\int \left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}\,dx=\frac{((ae+cdx)(d+ex))^{3/2}\left(\frac{\sqrt{c}\sqrt{d}\sqrt{e}(-3a^3e^6+a^2cde^4(11d+2ex)+ac^2d^2e^2(11d^2+44dex+24e^2x^2)+c^3d^3(-3d^3+2}{(ae+cdx)(d+ex)}\right.}{64c^{5/2}d^{5/2}e^{5/2}}$$

input
```
Integrate[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2),x]
```

output
```
(((a*e + c*d*x)*(d + e*x))^(3/2)*((Sqrt[c]*Sqrt[d]*Sqrt[e]*(-3*a^3*e^6 + a
^2*c*d*e^4*(11*d + 2*e*x) + a*c^2*d^2*e^2*(11*d^2 + 44*d*e*x + 24*e^2*x^2)
 + c^3*d^3*(-3*d^3 + 2*d^2*e*x + 24*d*e^2*x^2 + 16*e^3*x^3)))/((a*e + c*d*
x)*(d + e*x)) + (3*(c*d^2 - a*e^2)^4*ArcTanh[(Sqrt[c]*Sqrt[d]*Sqrt[d + e*x
])/(Sqrt[e]*Sqrt[a*e + c*d*x])])/((a*e + c*d*x)^(3/2)*(d + e*x)^(3/2))))/(
64*c^(5/2)*d^(5/2)*e^(5/2))
```

Rubi [A] (verified)

Time = 0.53 (sec) , antiderivative size = 246, normalized size of antiderivative = 1.13, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.138$, Rules used = {1087, 1087, 1092, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}dx$$

$$\Big\downarrow \text{ 1087}$$

$$\frac{\left(ae^2+cd^2+2cdex\right)\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{8cde}-\frac{3\left(cd^2-ae^2\right)^2\int \sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}dx}{16cde}$$

$$\Big\downarrow \text{ 1087}$$

$$\frac{\left(ae^2+cd^2+2cdex\right)\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{8cde}-\frac{3\left(cd^2-ae^2\right)^2\left(\frac{\left(ae^2+cd^2+2cdex\right)\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{4cde}-\frac{\left(cd^2-ae^2\right)^2\int\frac{1}{\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}dx}{8cde}\right)}{16cde}$$

$$\Big\downarrow \ 1092$$

$$\frac{\left(ae^2+cd^2+2cdex\right)\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{8cde}-\frac{3\left(cd^2-ae^2\right)^2\left(\frac{\left(ae^2+cd^2+2cdex\right)\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{4cde}-\frac{\left(cd^2-ae^2\right)^2\int\frac{1}{4cde-\frac{\left(cd^2+2cexd+ae^2\right)^2}{cdex^2+\left(cd^2+ae^2\right)x+ade}}d\frac{cd^2+2cexd+ae^2}{\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}}{4cde}\right)}{16cde}$$

$$\Big\downarrow \ 219$$

$$\frac{\left(ae^2+cd^2+2cdex\right)\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{8cde}-\frac{3\left(cd^2-ae^2\right)^2\left(\frac{\left(ae^2+cd^2+2cdex\right)\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{4cde}-\frac{\left(cd^2-ae^2\right)^2\text{arctanh}\left(\frac{ae^2+cd^2+2cdex}{2\sqrt{c}\sqrt{d}\sqrt{e}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}\right)}{8c^{3/2}d^{3/2}e^{3/2}}\right)}{16cde}$$

input
```
Int[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2),x]
```

output
```
((c*d^2 + a*e^2 + 2*c*d*e*x)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2)
)/(8*c*d*e) - (3*(c*d^2 - a*e^2)^2*(((c*d^2 + a*e^2 + 2*c*d*e*x)*Sqrt[a*d*
e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(4*c*d*e) - ((c*d^2 - a*e^2)^2*ArcTanh
[(c*d^2 + a*e^2 + 2*c*d*e*x)/(2*Sqrt[c]*Sqrt[d]*Sqrt[e]*Sqrt[a*d*e + (c*d^
2 + a*e^2)*x + c*d*e*x^2])])/(8*c^(3/2)*d^(3/2)*e^(3/2))))/(16*c*d*e)
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1087
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^p/(2*c*(2*p + 1))), x] - Simp[p*((b^2 - 4*a*c)/(2*c*(2*
p + 1)))   Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] &&
GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[3*p])
```

rule 1092
```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[I
nt[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a
, b, c}, x]
```

Maple [A] (verified)

Time = 1.46 (sec) , antiderivative size = 247, normalized size of antiderivative = 1.14

method	result
default	$\frac{(2cdxe+a e^2+c d^2)(ade+(a e^2+c d^2)x+cd x^2 e)^{\frac{3}{2}}}{8cde}+\frac{3\left(4ac d^2e^2-(a e^2+c d^2)^2\right)\left(\frac{(2cdxe+a e^2+c d^2)\sqrt{ade+(a e^2+c d^2)x+cd x^2 e}}{4cde}+\cdots\right.}{16\cdots}$

input
```
int((a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/8*(2*c*d*e*x+a*e^2+c*d^2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(3/2)/c/d/e+
3/16*(4*a*c*d^2*e^2-(a*e^2+c*d^2)^2)/d/e/c*(1/4*(2*c*d*e*x+a*e^2+c*d^2)*(a
*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(1/2)/c/d/e+1/8*(4*a*c*d^2*e^2-(a*e^2+c*d^
2)^2)/d/e/c*ln((1/2*a*e^2+1/2*c*d^2+c*d*x*e)/(d*e*c)^(1/2)+(a*d*e+(a*e^2+c
*d^2)*x+c*d*x^2*e)^(1/2))/(d*e*c)^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 666, normalized size of antiderivative = 3.07

$$\int \left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}\,dx = \left[\frac{3\left(c^4d^8-4\,ac^3d^6e^2+6\,a^2c^2d^4e^4-4\,a^3cd^2e^6+a^4e^8\right)\sqrt{cde}\log\left(8\,c^2d^2e^2x^2+c^2d^4+6\,acd\right.}{} \right.$$

$$-\frac{3\left(c^4d^8-4\,ac^3d^6e^2+6\,a^2c^2d^4e^4-4\,a^3cd^2e^6+a^4e^8\right)\sqrt{-cde}\arctan\left(\frac{\sqrt{cdex^2+ade+(cd^2+ae^2)x}\,(2\,cdex+cd^2+ae^2)\sqrt{-}}{2\,(c^2d^2e^2x^2+acd^2e^2+(c^2d^3e+acde^3)x)}\right.}{}$$

input
```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),x, algorithm="fricas")
```

output
```
[1/256*(3*(c^4*d^8 - 4*a*c^3*d^6*e^2 + 6*a^2*c^2*d^4*e^4 - 4*a^3*c*d^2*e^6
 + a^4*e^8)*sqrt(c*d*e)*log(8*c^2*d^2*e^2*x^2 + c^2*d^4 + 6*a*c*d^2*e^2 +
a^2*e^4 + 4*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(2*c*d*e*x + c*d^2
 + a*e^2)*sqrt(c*d*e) + 8*(c^2*d^3*e + a*c*d*e^3)*x) + 4*(16*c^4*d^4*e^4*x
^3 - 3*c^4*d^7*e + 11*a*c^3*d^5*e^3 + 11*a^2*c^2*d^3*e^5 - 3*a^3*c*d*e^7 +
 24*(c^4*d^5*e^3 + a*c^3*d^3*e^5)*x^2 + 2*(c^4*d^6*e^2 + 22*a*c^3*d^4*e^4
+ a^2*c^2*d^2*e^6)*x)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x))/(c^3*d^
3*e^3), -1/128*(3*(c^4*d^8 - 4*a*c^3*d^6*e^2 + 6*a^2*c^2*d^4*e^4 - 4*a^3*c
*d^2*e^6 + a^4*e^8)*sqrt(-c*d*e)*arctan(1/2*sqrt(c*d*e*x^2 + a*d*e + (c*d^
2 + a*e^2)*x)*(2*c*d*e*x + c*d^2 + a*e^2)*sqrt(-c*d*e)/(c^2*d^2*e^2*x^2 +
a*c*d^2*e^2 + (c^2*d^3*e + a*c*d*e^3)*x)) - 2*(16*c^4*d^4*e^4*x^3 - 3*c^4*
d^7*e + 11*a*c^3*d^5*e^3 + 11*a^2*c^2*d^3*e^5 - 3*a^3*c*d*e^7 + 24*(c^4*d^
5*e^3 + a*c^3*d^3*e^5)*x^2 + 2*(c^4*d^6*e^2 + 22*a*c^3*d^4*e^4 + a^2*c^2*d
^2*e^6)*x)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x))/(c^3*d^3*e^3)]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 442 vs. $2(209) = 418$.

Time = 1.62 (sec) , antiderivative size = 1853, normalized size of antiderivative = 8.54

$$\int \left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}\,dx = \text{Too large to display}$$

input
```
integrate((a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(3/2),x)
```

output
```
a*d*e*Piecewise(((x/2 + (a*e**2/4 + c*d**2/4)/(c*d*e))*sqrt(a*d*e + c*d*e*
x**2 + x*(a*e**2 + c*d**2)) + (a*d*e/2 - (a*e**2/4 + c*d**2/4)*(a*e**2 + c
*d**2)/(2*c*d*e))*Piecewise((log(a*e**2 + c*d**2 + 2*c*d*e*x + 2*sqrt(c*d*
e)*sqrt(a*d*e + c*d*e*x**2 + x*(a*e**2 + c*d**2)))/sqrt(c*d*e), Ne(a*d*e -
 (a*e**2 + c*d**2)**2/(4*c*d*e), 0)), ((x - (-a*e**2 - c*d**2)/(2*c*d*e))*
log(x - (-a*e**2 - c*d**2)/(2*c*d*e))/sqrt(c*d*e*(x - (-a*e**2 - c*d**2)/(
2*c*d*e))**2), True)), Ne(c*d*e, 0)), (2*(a*d*e + x*(a*e**2 + c*d**2))**(3
/2)/(3*(a*e**2 + c*d**2)), Ne(a*e**2 + c*d**2, 0)), (x*sqrt(a*d*e), True))
 + a*e**2*Piecewise(((-a*(a*e**2/6 + c*d**2/6)/(2*c) - (a*e**2 + c*d**2)*(
a*d*e/3 - (a*e**2/6 + c*d**2/6)*(3*a*e**2/2 + 3*c*d**2/2)/(2*c*d*e))/(2*c*
d*e))*Piecewise((log(a*e**2 + c*d**2 + 2*c*d*e*x + 2*sqrt(c*d*e)*sqrt(a*d*
e + c*d*e*x**2 + x*(a*e**2 + c*d**2)))/sqrt(c*d*e), Ne(a*d*e - (a*e**2 + c
*d**2)**2/(4*c*d*e), 0)), ((x - (-a*e**2 - c*d**2)/(2*c*d*e))*log(x - (-a*
e**2 - c*d**2)/(2*c*d*e))/sqrt(c*d*e*(x - (-a*e**2 - c*d**2)/(2*c*d*e))**2
), True)) + (x**2/3 + x*(a*e**2/6 + c*d**2/6)/(2*c*d*e) + (a*d*e/3 - (a*e*
*2/6 + c*d**2/6)*(3*a*e**2/2 + 3*c*d**2/2)/(2*c*d*e))/(c*d*e))*sqrt(a*d*e
+ c*d*e*x**2 + x*(a*e**2 + c*d**2)), Ne(c*d*e, 0)), (2*(-a*d*e*(a*d*e + x*
(a*e**2 + c*d**2))**(3/2)/3 + (a*d*e + x*(a*e**2 + c*d**2))**(5/2)/5)/(a*e
**2 + c*d**2)**2, Ne(a*e**2 + c*d**2, 0)), (x**2*sqrt(a*d*e)/2, True)) + c
*d**2*Piecewise(((-a*(a*e**2/6 + c*d**2/6)/(2*c) - (a*e**2 + c*d**2)*(a...
```

Maxima [F(-2)]

Exception generated.

$$\int \left(ade + \left(cd^2 + ae^2\right)x + cdex^2\right)^{3/2} dx = \text{Exception raised: ValueError}$$

input
```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

Giac [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 310, normalized size of antiderivative = 1.43

$$\int \left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}\,dx=\frac{1}{64}\sqrt{cdex^2+cd^2x+ae^2x+ade}\left(2\left(4\left(2\,cdex+\frac{3\left(c^4d^5e^3+ac^3d^3e^5\right)}{c^3d^3e^3}\right)x+\frac{c^4d^6e^2+22\,c\ldots}{\ldots}\right.\right.$$
$$-\frac{3\left(c^4d^8-4\,ac^3d^6e^2+6\,a^2c^2d^4e^4-4\,a^3cd^2e^6+a^4e^8\right)\log\left(\left|-cd^2-ae^2-2\sqrt{cde}\left(\sqrt{cde}x-\sqrt{cdex^2+cd^2\ldots}\right.\right.\right.}{128\sqrt{cde}c^2d^2e^2}$$

input
```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),x, algorithm="giac")
```

output
```
1/64*sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*x + a*d*e)*(2*(4*(2*c*d*e*x + 3*(c^4
*d^5*e^3 + a*c^3*d^3*e^5)/(c^3*d^3*e^3))*x + (c^4*d^6*e^2 + 22*a*c^3*d^4*e
^4 + a^2*c^2*d^2*e^6)/(c^3*d^3*e^3))*x - (3*c^4*d^7*e - 11*a*c^3*d^5*e^3 -
 11*a^2*c^2*d^3*e^5 + 3*a^3*c*d*e^7)/(c^3*d^3*e^3)) - 3/128*(c^4*d^8 - 4*a
*c^3*d^6*e^2 + 6*a^2*c^2*d^4*e^4 - 4*a^3*c*d^2*e^6 + a^4*e^8)*log(abs(-c*d
^2 - a*e^2 - 2*sqrt(c*d*e)*(sqrt(c*d*e)*x - sqrt(c*d*e*x^2 + c*d^2*x + a*e
^2*x + a*d*e))))/(sqrt(c*d*e)*c^2*d^2*e^2)
```

Mupad [B] (verification not implemented)

Time = 5.50 (sec) , antiderivative size = 225, normalized size of antiderivative = 1.04

$$\int \left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}\,dx=\frac{\left(\frac{c\,d^2}{2}+c\,x\,d\,e+\frac{a\,e^2}{2}\right)\,\left(c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e\right)^{3/2}}{4\,c\,d\,e}$$
$$-\frac{\left(\frac{3\,\left(c\,d^2+a\,e^2\right)^2}{4}-3\,a\,c\,d^2\,e^2\right)\,\left(\left(\frac{x}{2}+\frac{c\,d^2+a\,e^2}{4\,c\,d\,e}\right)\,\sqrt{c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e}-\frac{\ln\left(2\,\sqrt{\left(a\,e+c\,d\,x\right)\,\left(d+e\,x\right)}\,\sqrt{c\,d\ldots}\right.}{\ldots}\right.}{4\,c\,d\,e}$$

input
```
int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(3/2),x)
```

output
```
(((a*e^2)/2 + (c*d^2)/2 + c*d*e*x)*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)
^(3/2))/(4*c*d*e) - (((3*(a*e^2 + c*d^2)^2)/4 - 3*a*c*d^2*e^2)*((x/2 + (a*
e^2 + c*d^2)/(4*c*d*e))*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2) - (l
og(2*((a*e + c*d*x)*(d + e*x))^(1/2)*(c*d*e)^(1/2) + a*e^2 + c*d^2 + 2*c*d
*e*x)*((a*e^2 + c*d^2)^2/4 - a*c*d^2*e^2))/(2*(c*d*e)^(3/2))))/(4*c*d*e)
```

Reduce [B] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 579, normalized size of antiderivative = 2.67

$$\int \left(ade + \left(cd^2 + ae^2\right)x + cdex^2\right)^{3/2} dx = \frac{-3\sqrt{ex+d}\,\sqrt{cdx+ae}\,a^3cd\,e^7 + 11\sqrt{ex+d}\,\sqrt{cdx+ae}\,a^2c^2d^3e^5 + 2\sqrt{ex+d}\,\sqrt{cdx+ae}\,a}{}$$

input
```
int((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),x)
```

output
```
( - 3*sqrt(d + e*x)*sqrt(a*e + c*d*x)*a**3*c*d*e**7 + 11*sqrt(d + e*x)*sqr
t(a*e + c*d*x)*a**2*c**2*d**3*e**5 + 2*sqrt(d + e*x)*sqrt(a*e + c*d*x)*a**
2*c**2*d**2*e**6*x + 11*sqrt(d + e*x)*sqrt(a*e + c*d*x)*a*c**3*d**5*e**3 +
 44*sqrt(d + e*x)*sqrt(a*e + c*d*x)*a*c**3*d**4*e**4*x + 24*sqrt(d + e*x)*
sqrt(a*e + c*d*x)*a*c**3*d**3*e**5*x**2 - 3*sqrt(d + e*x)*sqrt(a*e + c*d*x
)*c**4*d**7*e + 2*sqrt(d + e*x)*sqrt(a*e + c*d*x)*c**4*d**6*e**2*x + 24*sq
rt(d + e*x)*sqrt(a*e + c*d*x)*c**4*d**5*e**3*x**2 + 16*sqrt(d + e*x)*sqrt(
a*e + c*d*x)*c**4*d**4*e**4*x**3 + 3*sqrt(e)*sqrt(d)*sqrt(c)*log((sqrt(e)*
sqrt(a*e + c*d*x) + sqrt(d)*sqrt(c)*sqrt(d + e*x))/sqrt(a*e**2 - c*d**2))*
a**4*e**8 - 12*sqrt(e)*sqrt(d)*sqrt(c)*log((sqrt(e)*sqrt(a*e + c*d*x) + sq
rt(d)*sqrt(c)*sqrt(d + e*x))/sqrt(a*e**2 - c*d**2))*a**3*c*d**2*e**6 + 18*
sqrt(e)*sqrt(d)*sqrt(c)*log((sqrt(e)*sqrt(a*e + c*d*x) + sqrt(d)*sqrt(c)*s
qrt(d + e*x))/sqrt(a*e**2 - c*d**2))*a**2*c**2*d**4*e**4 - 12*sqrt(e)*sqrt
(d)*sqrt(c)*log((sqrt(e)*sqrt(a*e + c*d*x) + sqrt(d)*sqrt(c)*sqrt(d + e*x)
)/sqrt(a*e**2 - c*d**2))*a*c**3*d**6*e**2 + 3*sqrt(e)*sqrt(d)*sqrt(c)*log(
(sqrt(e)*sqrt(a*e + c*d*x) + sqrt(d)*sqrt(c)*sqrt(d + e*x))/sqrt(a*e**2 -
c*d**2))*c**4*d**8)/(64*c**3*d**3*e**3)
```

3.213 $\int \frac{(ade+(cd^2+ae^2)x+cdex^2)^{3/2}}{d+ex} dx$

Optimal result

Integrand size = 37, antiderivative size = 245

$$\begin{aligned}\int \frac{(ade+(cd^2+ae^2)x+cdex^2)^{3/2}}{d+ex} dx = & -\frac{(cd^2-ae^2)^2\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{8cde^2} \\ & + \frac{\left(\frac{d}{e}-\frac{ae}{cd}\right)(ade+(cd^2+ae^2)x+cdex^2)^{3/2}}{12(d+ex)} + \frac{(ade+(cd^2+ae^2)x+cdex^2)^{5/2}}{3cd(d+ex)^2} \\ & + \frac{(cd^2-ae^2)^3\operatorname{arctanh}\left(\frac{\sqrt{e}\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{\sqrt{c}\sqrt{d}(d+ex)}\right)}{8c^{3/2}d^{3/2}e^{5/2}}\end{aligned}$$

output

```
-1/8*(-a*e^2+c*d^2)^2*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/c/d/e^2+(d/e
-a*e/c/d)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/(12*e*x+12*d)+1/3*(a*d*e
+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/c/d/(e*x+d)^2+1/8*(-a*e^2+c*d^2)^3*arcta
nh(e^(1/2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/c^(1/2)/d^(1/2)/(e*x+d)
)/c^(3/2)/d^(3/2)/e^(5/2)
```

Mathematica [A] (verified)

Time = 0.41 (sec) , antiderivative size = 180, normalized size of antiderivative = 0.73

$$\int \frac{(ade+(cd^2+ae^2)x+cdex^2)^{3/2}}{d+ex}\,dx = \frac{\sqrt{(ae+cdx)(d+ex)}\left(\sqrt{c}\sqrt{d}\sqrt{e}(3a^2e^4+2acde^2(4d+7ex)+c^2\right.}{24c^{3/2}d^{3/}}$$

input
```
Integrate[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2)/(d + e*x),x]
```

output
```
(Sqrt[(a*e + c*d*x)*(d + e*x)]*(Sqrt[c]*Sqrt[d]*Sqrt[e]*(3*a^2*e^4 + 2*a*c
*d*e^2*(4*d + 7*e*x) + c^2*d^2*(-3*d^2 + 2*d*e*x + 8*e^2*x^2)) + (3*(c*d^2
 - a*e^2)^3*ArcTanh[(Sqrt[c]*Sqrt[d]*Sqrt[d + e*x])/(Sqrt[e]*Sqrt[a*e + c*
d*x])])/(Sqrt[a*e + c*d*x]*Sqrt[d + e*x])))/(24*c^(3/2)*d^(3/2)*e^(5/2))
```

Rubi [A] (verified)

Time = 0.51 (sec) , antiderivative size = 215, normalized size of antiderivative = 0.88, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.108, Rules used = {1131, 1087, 1092, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{d+ex}dx$$

$$\Big\downarrow \text{1131}$$

$$\frac{\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{3e}-\frac{\left(cd^2-ae^2\right)\int\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}dx}{2e}$$

$$\Big\downarrow \text{1087}$$

$$\frac{\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{3e}-\frac{\left(cd^2-ae^2\right)\left(\frac{\left(ae^2+cd^2+2cdex\right)\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{4cde}-\frac{\left(cd^2-ae^2\right)^2\int\frac{1}{\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}dx}{8cde}\right)}{2e}$$

$$\Big\downarrow \text{1092}$$

$$\frac{\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{3e}-\frac{\left(cd^2-ae^2\right)\left(\frac{\left(ae^2+cd^2+2cdex\right)\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{4cde}-\frac{\left(cd^2-ae^2\right)^2\int \frac{1}{4cde-\frac{\left(cd^2+2cexd+ae^2\right)^2}{cdex^2+\left(cd^2+ae^2\right)x+ade}}d\frac{cd^2+2cexd+ae^2}{\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}}{4cde}\right)}{2e}$$

$$\Big\downarrow \text{219}$$

$$\frac{\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{3e}-\frac{\left(cd^2-ae^2\right)\left(\frac{\left(ae^2+cd^2+2cdex\right)\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{4cde}-\frac{\left(cd^2-ae^2\right)^2\text{arctanh}\left(\frac{ae^2+cd^2+2cdex}{2\sqrt{c}\sqrt{d}\sqrt{e}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}\right)}{8c^{3/2}d^{3/2}e^{3/2}}\right)}{2e}$$

input
```
Int[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2)/(d + e*x),x]
```

output
```
(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2)/(3*e) - ((c*d^2 - a*e^2)*(((
c*d^2 + a*e^2 + 2*c*d*e*x)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(4
*c*d*e) - ((c*d^2 - a*e^2)^2*ArcTanh[(c*d^2 + a*e^2 + 2*c*d*e*x)/(2*Sqrt[c
]*Sqrt[d]*Sqrt[e]*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])])/(8*c^(3/2
)*d^(3/2)*e^(3/2))))/(2*e)
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1087
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^p/(2*c*(2*p + 1))), x] - Simp[p*((b^2 - 4*a*c)/(2*c*(2*
p + 1)))   Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] &&
GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[3*p])
```

rule 1092
```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[I
nt[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a
, b, c}, x]
```

rule 1131
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(d + e*x)^(m + 1)*((a + b*x + c*x^2)^p/(e*(m + 2*p + 1))), x
] - Simp[p*((2*c*d - b*e)/(e^2*(m + 2*p + 1)))   Int[(d + e*x)^(m + 1)*(a +
 b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c*d^2 - b
*d*e + a*e^2, 0] && GtQ[p, 0] && (LeQ[-2, m, 0] || EqQ[m + p + 1, 0]) && Ne
Q[m + 2*p + 1, 0] && IntegerQ[2*p]
```

Maple [A] (verified)

Time = 1.78 (sec) , antiderivative size = 230, normalized size of antiderivative = 0.94

method	result
default	$\frac{\frac{\left(dec\left(x+\frac{d}{e}\right)^2+\left(a e^2-c d^2\right)\left(x+\frac{d}{e}\right)\right)^{\frac{3}{2}}}{3}+\frac{\left(a e^2-c d^2\right)\left(\frac{\left(2dec\left(x+\frac{d}{e}\right)+a e^2-c d^2\right)\sqrt{dec\left(x+\frac{d}{e}\right)^2+\left(a e^2-c d^2\right)\left(x+\frac{d}{e}\right)}}{4dec}-\frac{\left(a e^2-c d^2\right)^2\ln\left(\frac{\frac{a e^2}{2}-\cdots}{\cdots}\right.}{\cdots}\right.}{2}}{e}$

input
```
int((a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(3/2)/(e*x+d),x,method=_RETURNVERBOS
E)
```

output
```
1/e*(1/3*(d*e*c*(x+d/e)^2+(a*e^2-c*d^2)*(x+d/e))^(3/2)+1/2*(a*e^2-c*d^2)*(
1/4*(2*d*e*c*(x+d/e)+a*e^2-c*d^2)/d/e/c*(d*e*c*(x+d/e)^2+(a*e^2-c*d^2)*(x+
d/e))^(1/2)-1/8*(a*e^2-c*d^2)^2/d/e/c*ln((1/2*a*e^2-1/2*c*d^2+d*e*c*(x+d/e
))/(d*e*c)^(1/2)+(d*e*c*(x+d/e)^2+(a*e^2-c*d^2)*(x+d/e))^(1/2))/(d*e*c)^(1
/2)))
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 532, normalized size of antiderivative = 2.17

$$\int \frac{(ade + (cd^2 + ae^2)\,x + cdex^2)^{3/2}}{d + ex}\,dx = \left[-\frac{3\,(c^3d^6 - 3\,ac^2d^4e^2 + 3\,a^2cd^2e^4 - a^3e^6)\sqrt{cde}\,\log\left(8\,c^2d^2e^2x^2 + \right.}{} \right.$$

$$-\frac{3\,(c^3d^6 - 3\,ac^2d^4e^2 + 3\,a^2cd^2e^4 - a^3e^6)\sqrt{-cde}\,\arctan\left(\frac{\sqrt{cdex^2+ade+(cd^2+ae^2)x}\,(2\,cdex+cd^2+ae^2)\sqrt{-cde}}{2\,(c^2d^2e^2x^2+acd^2e^2+(c^2d^3e+acde^3)x)}\right) - 2\,(8\,c^3}{48\,c^2d^2e^3}$$

input
```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/(e*x+d),x, algorithm="fr
icas")
```

output
```
[-1/96*(3*(c^3*d^6 - 3*a*c^2*d^4*e^2 + 3*a^2*c*d^2*e^4 - a^3*e^6)*sqrt(c*d
*e)*log(8*c^2*d^2*e^2*x^2 + c^2*d^4 + 6*a*c*d^2*e^2 + a^2*e^4 - 4*sqrt(c*d
*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(2*c*d*e*x + c*d^2 + a*e^2)*sqrt(c*d*e
) + 8*(c^2*d^3*e + a*c*d*e^3)*x) - 4*(8*c^3*d^3*e^3*x^2 - 3*c^3*d^5*e + 8*
a*c^2*d^3*e^3 + 3*a^2*c*d*e^5 + 2*(c^3*d^4*e^2 + 7*a*c^2*d^2*e^4)*x)*sqrt(
c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x))/(c^2*d^2*e^3), -1/48*(3*(c^3*d^6 -
 3*a*c^2*d^4*e^2 + 3*a^2*c*d^2*e^4 - a^3*e^6)*sqrt(-c*d*e)*arctan(1/2*sqrt
(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(2*c*d*e*x + c*d^2 + a*e^2)*sqrt(-
c*d*e)/(c^2*d^2*e^2*x^2 + a*c*d^2*e^2 + (c^2*d^3*e + a*c*d*e^3)*x)) - 2*(8
*c^3*d^3*e^3*x^2 - 3*c^3*d^5*e + 8*a*c^2*d^3*e^3 + 3*a^2*c*d*e^5 + 2*(c^3*
d^4*e^2 + 7*a*c^2*d^2*e^4)*x)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x))
/(c^2*d^2*e^3)]
```

Sympy [A] (verification not implemented)

Time = 1.65 (sec) , antiderivative size = 751, normalized size of antiderivative = 3.07

$$\int \frac{(ade + (cd^2 + ae^2)x + cdex^2)^{3/2}}{d + ex} dx = \text{Too large to display}$$

input
```
integrate((a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(3/2)/(e*x+d),x)
```

output
```
a*e*Piecewise(((x/2 + (a*e**2/4 + c*d**2/4)/(c*d*e))*sqrt(a*d*e + c*d*e*x*
*2 + x*(a*e**2 + c*d**2)) + (a*d*e/2 - (a*e**2/4 + c*d**2/4)*(a*e**2 + c*d
**2)/(2*c*d*e))*Piecewise((log(a*e**2 + c*d**2 + 2*c*d*e*x + 2*sqrt(c*d*e)
*sqrt(a*d*e + c*d*e*x**2 + x*(a*e**2 + c*d**2)))/sqrt(c*d*e), Ne(a*d*e - (
a*e**2 + c*d**2)**2/(4*c*d*e), 0)), ((x - (-a*e**2 - c*d**2)/(2*c*d*e))*lo
g(x - (-a*e**2 - c*d**2)/(2*c*d*e))/sqrt(c*d*e*(x - (-a*e**2 - c*d**2)/(2*
c*d*e))**2), True)), Ne(c*d*e, 0)), (2*(a*d*e + x*(a*e**2 + c*d**2))**(3/2
)/(3*(a*e**2 + c*d**2)), Ne(a*e**2 + c*d**2, 0)), (x*sqrt(a*d*e), True)) +
 c*d*Piecewise(((-a*(a*e**2/6 + c*d**2/6)/(2*c) - (a*e**2 + c*d**2)*(a*d*e
/3 - (a*e**2/6 + c*d**2/6)*(3*a*e**2/2 + 3*c*d**2/2)/(2*c*d*e))/(2*c*d*e))
*Piecewise((log(a*e**2 + c*d**2 + 2*c*d*e*x + 2*sqrt(c*d*e)*sqrt(a*d*e + c
*d*e*x**2 + x*(a*e**2 + c*d**2)))/sqrt(c*d*e), Ne(a*d*e - (a*e**2 + c*d**2
)**2/(4*c*d*e), 0)), ((x - (-a*e**2 - c*d**2)/(2*c*d*e))*log(x - (-a*e**2
- c*d**2)/(2*c*d*e))/sqrt(c*d*e*(x - (-a*e**2 - c*d**2)/(2*c*d*e))**2), Tr
ue)) + (x**2/3 + x*(a*e**2/6 + c*d**2/6)/(2*c*d*e) + (a*d*e/3 - (a*e**2/6
+ c*d**2/6)*(3*a*e**2/2 + 3*c*d**2/2)/(2*c*d*e))/(c*d*e))*sqrt(a*d*e + c*d
*e*x**2 + x*(a*e**2 + c*d**2)), Ne(c*d*e, 0)), (2*(-a*d*e*(a*d*e + x*(a*e*
*2 + c*d**2))**(3/2)/3 + (a*d*e + x*(a*e**2 + c*d**2))**(5/2)/5)/(a*e**2 +
 c*d**2)**2, Ne(a*e**2 + c*d**2, 0)), (x**2*sqrt(a*d*e)/2, True))
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(ade + (cd^2 + ae^2)x + cdex^2)^{3/2}}{d + ex} dx = \text{Exception raised: ValueError}$$

input
```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/(e*x+d),x, algorithm="ma
xima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

Giac [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 229, normalized size of antiderivative = 0.93

$$\int \frac{\left(ade+(cd^2+ae^2)\,x+cdex^2\right)^{3/2}}{d+ex}\,dx=\frac{1}{24}\sqrt{cdex^2+cd^2x+ae^2x+ade}\left(2\left(4\,cdx+\frac{c^3d^4e+7\,ac^2d^2e^3}{c^2d^2e^2}\right)\right.$$
$$-\frac{\left(c^3d^6-3\,ac^2d^4e^2+3\,a^2cd^2e^4-a^3e^6\right)\log\left(\left|-cd^2-ae^2-2\,\sqrt{cde}\left(\sqrt{cde}x-\sqrt{cdex^2+cd^2x+ae^2x+ade}\right.\right.\right.}{16\,\sqrt{cde}cde^2}$$

input
```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/(e*x+d),x, algorithm="gi
ac")
```

output
```
1/24*sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*x + a*d*e)*(2*(4*c*d*x + (c^3*d^4*e
+ 7*a*c^2*d^2*e^3)/(c^2*d^2*e^2))*x - (3*c^3*d^5 - 8*a*c^2*d^3*e^2 - 3*a^2
*c*d*e^4)/(c^2*d^2*e^2)) - 1/16*(c^3*d^6 - 3*a*c^2*d^4*e^2 + 3*a^2*c*d^2*e
^4 - a^3*e^6)*log(abs(-c*d^2 - a*e^2 - 2*sqrt(c*d*e)*(sqrt(c*d*e)*x - sqrt
(c*d*e*x^2 + c*d^2*x + a*e^2*x + a*d*e))))/(sqrt(c*d*e)*c*d*e^2)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\left(ade+(cd^2+ae^2)\,x+cdex^2\right)^{3/2}}{d+ex}\,dx=\int \frac{{\left(c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e\right)}^{3/2}}{d+e\,x}\,dx$$

input
```
int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(3/2)/(d + e*x),x)
```

output
```
int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(3/2)/(d + e*x), x)
```

Reduce [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 400, normalized size of antiderivative = 1.63

$$\int \frac{(ade + (cd^2 + ae^2)x + cdex^2)^{3/2}}{d + ex} dx = \frac{3\sqrt{ex + d}\,\sqrt{cdx + ae}\,a^2cd\,e^5 + 8\sqrt{ex + d}\,\sqrt{cdx + ae}\,a\,c^2d^3e^3 + 1}{}$$

input
```
int((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/(e*x+d),x)
```

output
```
(3*sqrt(d + e*x)*sqrt(a*e + c*d*x)*a**2*c*d*e**5 + 8*sqrt(d + e*x)*sqrt(a*
e + c*d*x)*a*c**2*d**3*e**3 + 14*sqrt(d + e*x)*sqrt(a*e + c*d*x)*a*c**2*d*
*2*e**4*x - 3*sqrt(d + e*x)*sqrt(a*e + c*d*x)*c**3*d**5*e + 2*sqrt(d + e*x
)*sqrt(a*e + c*d*x)*c**3*d**4*e**2*x + 8*sqrt(d + e*x)*sqrt(a*e + c*d*x)*c
**3*d**3*e**3*x**2 - 3*sqrt(e)*sqrt(d)*sqrt(c)*log((sqrt(e)*sqrt(a*e + c*d
*x) + sqrt(d)*sqrt(c)*sqrt(d + e*x))/sqrt(a*e**2 - c*d**2))*a**3*e**6 + 9*
sqrt(e)*sqrt(d)*sqrt(c)*log((sqrt(e)*sqrt(a*e + c*d*x) + sqrt(d)*sqrt(c)*s
qrt(d + e*x))/sqrt(a*e**2 - c*d**2))*a**2*c*d**2*e**4 - 9*sqrt(e)*sqrt(d)*
sqrt(c)*log((sqrt(e)*sqrt(a*e + c*d*x) + sqrt(d)*sqrt(c)*sqrt(d + e*x))/sq
rt(a*e**2 - c*d**2))*a*c**2*d**4*e**2 + 3*sqrt(e)*sqrt(d)*sqrt(c)*log((sqr
t(e)*sqrt(a*e + c*d*x) + sqrt(d)*sqrt(c)*sqrt(d + e*x))/sqrt(a*e**2 - c*d*
*2))*c**3*d**6)/(24*c**2*d**2*e**3)
```

3.214 $\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{(d+ex)^2}\,dx$

Optimal result

Integrand size = 37, antiderivative size = 174

$$\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{(d+ex)^2}\,dx=\frac{3}{4}\left(a-\frac{cd^2}{e^2}\right)\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}+\frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{2e(d+ex)}+\frac{3\left(cd^2-ae^2\right)^2\operatorname{arctanh}\left(\frac{\sqrt{e}\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{\sqrt{c}\sqrt{d}(d+ex)}\right)}{4\sqrt{c}\sqrt{d}e^{5/2}}$$

output
```
3/4*(a-c*d^2/e^2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)+1/2*(a*d*e+(a*e^
2+c*d^2)*x+c*d*e*x^2)^(3/2)/e/(e*x+d)+3/4*(-a*e^2+c*d^2)^2*arctanh(e^(1/2)
*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/c^(1/2)/d^(1/2)/(e*x+d))/c^(1/2)/
d^(1/2)/e^(5/2)
```

Mathematica [A] (verified)

Time = 0.39 (sec), antiderivative size = 150, normalized size of antiderivative = 0.86

$$\int \frac{(ade+(cd^2+ae^2)x+cdex^2)^{3/2}}{(d+ex)^2}\,dx = \frac{\sqrt{e}(ae+cdx)(d+ex)\left(5ae^2+cd(-3d+2ex)\right)+\frac{3(cd^2-ae^2)^2\sqrt{ae+c}}{}}{4e^{5/2}\sqrt{(ae+cdx)(d+ex)}}$$

input
```
Integrate[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2)/(d + e*x)^2,x]
```

output
```
(Sqrt[e]*(a*e + c*d*x)*(d + e*x)*(5*a*e^2 + c*d*(-3*d + 2*e*x)) + (3*(c*d^
2 - a*e^2)^2*Sqrt[a*e + c*d*x]*Sqrt[d + e*x]*ArcTanh[(Sqrt[c]*Sqrt[d]*Sqrt
[d + e*x])/(Sqrt[e]*Sqrt[a*e + c*d*x])])/(Sqrt[c]*Sqrt[d]))/(4*e^(5/2)*Sqr
t[(a*e + c*d*x)*(d + e*x)])
```

Rubi [A] (verified)

Time = 0.51 (sec), antiderivative size = 194, normalized size of antiderivative = 1.11, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.108$, Rules used = {1131, 1131, 1092, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{(d+ex)^2}dx$$

$$\Big\downarrow \; 1131$$

$$\frac{\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{2e(d+ex)}-\frac{3\left(cd^2-ae^2\right)\int \frac{\sqrt{cdex^2+(cd^2+ae^2)x+ade}}{d+ex}dx}{4e}$$

$$\Big\downarrow \; 1131$$

$$\frac{\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{2e(d+ex)}-\frac{3\left(cd^2-ae^2\right)\left(\frac{\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{e}-\frac{(cd^2-ae^2)\int \frac{1}{\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}dx}{2e}\right)}{4e}$$

$$\Big\downarrow \text{1092}$$

$$\frac{\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{2e(d+ex)}-\frac{3\left(cd^2-ae^2\right)\left(\frac{\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{e}-\frac{\left(cd^2-ae^2\right)\int\frac{1}{4cde-\frac{\left(cd^2+2cexd+ae^2\right)^2}{cdex^2+\left(cd^2+ae^2\right)x+ade}}d\frac{cd^2+2cexd+ae^2}{\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}}{e}\right)}{4e}$$

$$\Big\downarrow \text{219}$$

$$\frac{\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{2e(d+ex)}-\frac{3\left(cd^2-ae^2\right)\left(\frac{\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{e}-\frac{\left(cd^2-ae^2\right)\text{arctanh}\left(\frac{ae^2+cd^2+2cdex}{2\sqrt{c}\sqrt{d}\sqrt{e}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}\right)}{2\sqrt{c}\sqrt{d}e^{3/2}}\right)}{4e}$$

input
```
Int[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2)/(d + e*x)^2,x]
```

output
```
(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2)/(2*e*(d + e*x)) - (3*(c*d^2
- a*e^2)*(Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]/e - ((c*d^2 - a*e^2)
*ArcTanh[(c*d^2 + a*e^2 + 2*c*d*e*x)/(2*Sqrt[c]*Sqrt[d]*Sqrt[e]*Sqrt[a*d*e
 + (c*d^2 + a*e^2)*x + c*d*e*x^2])])/(2*Sqrt[c]*Sqrt[d]*e^(3/2))))/(4*e)
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1092
```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[I
nt[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a
, b, c}, x]
```

rule 1131
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(d + e*x)^(m + 1)*((a + b*x + c*x^2)^p/(e*(m + 2*p + 1))), x
] - Simp[p*((2*c*d - b*e)/(e^2*(m + 2*p + 1)))   Int[(d + e*x)^(m + 1)*(a +
 b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c*d^2 - b
*d*e + a*e^2, 0] && GtQ[p, 0] && (LeQ[-2, m, 0] || EqQ[m + p + 1, 0]) && Ne
Q[m + 2*p + 1, 0] && IntegerQ[2*p]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 310 vs. $2(150) = 300$.

Time = 1.90 (sec) , antiderivative size = 311, normalized size of antiderivative = 1.79

method	result
default	$\frac{\frac{2\left(dec\left(x+\frac{d}{e}\right)^2+\left(a\,e^2-c\,d^2\right)\left(x+\frac{d}{e}\right)\right)^{\frac{5}{2}}}{\left(a\,e^2-c\,d^2\right)\left(x+\frac{d}{e}\right)^2}-\frac{6dec\left(\frac{\left(dec\left(x+\frac{d}{e}\right)^2+\left(a\,e^2-c\,d^2\right)\left(x+\frac{d}{e}\right)\right)^{\frac{3}{2}}}{3}+\frac{\left(a\,e^2-c\,d^2\right)\left(\frac{\left(2dec\left(x+\frac{d}{e}\right)+a\,e^2-c\,d^2\right)\sqrt{dec\left(x+\frac{d}{e}\right)^2 \cdots}}{4dec}\cdots\right)}{\cdots}\right)}{\cdots}}{e^2}$

input
```
int((a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(3/2)/(e*x+d)^2,x,method=_RETURNVERB
OSE)
```

output
```
1/e^2*(2/(a*e^2-c*d^2)/(x+d/e)^2*(d*e*c*(x+d/e)^2+(a*e^2-c*d^2)*(x+d/e))^(
5/2)-6*d*e*c/(a*e^2-c*d^2)*(1/3*(d*e*c*(x+d/e)^2+(a*e^2-c*d^2)*(x+d/e))^(3
/2)+1/2*(a*e^2-c*d^2)*(1/4*(2*d*e*c*(x+d/e)+a*e^2-c*d^2)/d/e/c*(d*e*c*(x+d
/e)^2+(a*e^2-c*d^2)*(x+d/e))^(1/2)-1/8*(a*e^2-c*d^2)^2/d/e/c*ln((1/2*a*e^2
-1/2*c*d^2+d*e*c*(x+d/e))/(d*e*c)^(1/2)+(d*e*c*(x+d/e)^2+(a*e^2-c*d^2)*(x+
d/e))^(1/2))/(d*e*c)^(1/2))))
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 418, normalized size of antiderivative = 2.40

$$\int \frac{(ade+(cd^2+ae^2)\,x+cdex^2)^{3/2}}{(d+ex)^2}\,dx = \left[\frac{3\,(c^2d^4-2\,acd^2e^2+a^2e^4)\sqrt{cde}\log\left(8\,c^2d^2e^2x^2+c^2d^4+6\,acd^2e^2 \cdots\right.}{\cdots}\right.$$

$$-\frac{3\,(c^2d^4-2\,acd^2e^2+a^2e^4)\sqrt{-cde}\arctan\left(\frac{\sqrt{cdex^2+ade+(cd^2+ae^2)x}\,(2\,cdex+cd^2+ae^2)\sqrt{-cde}}{2\,(c^2d^2e^2x^2+acd^2e^2+(c^2d^3e+acde^3)x)}\right)-2\,(2\,c^2d^2e^2x-3\,c^2 \cdots}{8\,cde^3}$$

input
```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/(e*x+d)^2,x, algorithm="
fricas")
```

output
```
[1/16*(3*(c^2*d^4 - 2*a*c*d^2*e^2 + a^2*e^4)*sqrt(c*d*e)*log(8*c^2*d^2*e^2
*x^2 + c^2*d^4 + 6*a*c*d^2*e^2 + a^2*e^4 + 4*sqrt(c*d*e*x^2 + a*d*e + (c*d
^2 + a*e^2)*x)*(2*c*d*e*x + c*d^2 + a*e^2)*sqrt(c*d*e) + 8*(c^2*d^3*e + a*
c*d*e^3)*x) + 4*(2*c^2*d^2*e^2*x - 3*c^2*d^3*e + 5*a*c*d*e^3)*sqrt(c*d*e*x
^2 + a*d*e + (c*d^2 + a*e^2)*x))/(c*d*e^3), -1/8*(3*(c^2*d^4 - 2*a*c*d^2*e
^2 + a^2*e^4)*sqrt(-c*d*e)*arctan(1/2*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*
e^2)*x)*(2*c*d*e*x + c*d^2 + a*e^2)*sqrt(-c*d*e)/(c^2*d^2*e^2*x^2 + a*c*d^
2*e^2 + (c^2*d^3*e + a*c*d*e^3)*x)) - 2*(2*c^2*d^2*e^2*x - 3*c^2*d^3*e + 5
*a*c*d*e^3)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x))/(c*d*e^3)]
```

Sympy [F]

$$\int \frac{(ade+(cd^2+ae^2)\,x+cdex^2)^{3/2}}{(d+ex)^2}\,dx = \int \frac{((d+ex)\,(ae+cdx))^{\frac{3}{2}}}{(d+ex)^2}\,dx$$

input
```
integrate((a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(3/2)/(e*x+d)**2,x)
```

output
```
Integral(((d + e*x)*(a*e + c*d*x))**(3/2)/(d + e*x)**2, x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(ade + (cd^2 + ae^2)x + cdex^2)^{3/2}}{(d + ex)^2} dx = \text{Exception raised: ValueError}$$

input
```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/(e*x+d)^2,x, algorithm="
maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e*(a*e^2-c*d^2)>0)', see `assume
?` for mor
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 471 vs. 2(150) = 300.

Time = 0.29 (sec) , antiderivative size = 471, normalized size of antiderivative = 2.71

$$\int \frac{(ade + (cd^2 + ae^2)x + cdex^2)^{3/2}}{(d + ex)^2} dx =$$

$$-\frac{1}{4}\left(\frac{3\left(c^2d^4\mathrm{sgn}\left(\frac{1}{ex+d}\right)\mathrm{sgn}(e) - 2\,acd^2e^2\mathrm{sgn}\left(\frac{1}{ex+d}\right)\mathrm{sgn}(e) + a^2e^4\mathrm{sgn}\left(\frac{1}{ex+d}\right)\mathrm{sgn}(e)\right)\arctan\left(\frac{\sqrt{cde-\frac{cd^2e}{ex+d}+\frac{ae^3}{ex+}}}{\sqrt{-cde}}\right.}{\sqrt{-cde}e^2|e|}\right.$$

input
```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/(e*x+d)^2,x, algorithm="
giac")
```

output
```
-1/4*(3*(c^2*d^4*sgn(1/(e*x + d))*sgn(e) - 2*a*c*d^2*e^2*sgn(1/(e*x + d))*
sgn(e) + a^2*e^4*sgn(1/(e*x + d))*sgn(e))*arctan(sqrt(c*d*e - c*d^2*e/(e*x
 + d) + a*e^3/(e*x + d))/sqrt(-c*d*e))/(sqrt(-c*d*e)*e^2*abs(e)) + (3*sqrt
(c*d*e - c*d^2*e/(e*x + d) + a*e^3/(e*x + d))*c^3*d^5*e*sgn(1/(e*x + d))*s
gn(e) - 6*sqrt(c*d*e - c*d^2*e/(e*x + d) + a*e^3/(e*x + d))*a*c^2*d^3*e^3*
sgn(1/(e*x + d))*sgn(e) + 3*sqrt(c*d*e - c*d^2*e/(e*x + d) + a*e^3/(e*x +
d))*a^2*c*d*e^5*sgn(1/(e*x + d))*sgn(e) - 5*(c*d*e - c*d^2*e/(e*x + d) + a
*e^3/(e*x + d))^(3/2)*c^2*d^4*sgn(1/(e*x + d))*sgn(e) + 10*(c*d*e - c*d^2*
e/(e*x + d) + a*e^3/(e*x + d))^(3/2)*a*c*d^2*e^2*sgn(1/(e*x + d))*sgn(e) -
 5*(c*d*e - c*d^2*e/(e*x + d) + a*e^3/(e*x + d))^(3/2)*a^2*e^4*sgn(1/(e*x
+ d))*sgn(e))/((c*d^2*e/(e*x + d) - a*e^3/(e*x + d))^2*e^2*abs(e)))*abs(e)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(ade + (cd^2 + ae^2)x + cdex^2)^{3/2}}{(d+ex)^2}\,dx = \int \frac{\left(c\,d\,e\,x^2 + \left(c\,d^2 + a\,e^2\right)\,x + a\,d\,e\right)^{3/2}}{\left(d + e\,x\right)^2}\,dx$$

input
```
int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(3/2)/(d + e*x)^2,x)
```

output
```
int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(3/2)/(d + e*x)^2, x)
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 253, normalized size of antiderivative = 1.45

$$\int \frac{(ade + (cd^2 + ae^2)x + cdex^2)^{3/2}}{(d+ex)^2}\,dx = \frac{5\sqrt{ex+d}\,\sqrt{cdx+ae}\,acd\,e^3 - 3\sqrt{ex+d}\,\sqrt{cdx+ae}\,c^2d^3e + 2\sqrt{e}$$

input
```
int((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/(e*x+d)^2,x)
```

output
```
(5*sqrt(d + e*x)*sqrt(a*e + c*d*x)*a*c*d*e**3 - 3*sqrt(d + e*x)*sqrt(a*e +
 c*d*x)*c**2*d**3*e + 2*sqrt(d + e*x)*sqrt(a*e + c*d*x)*c**2*d**2*e**2*x +
 3*sqrt(e)*sqrt(d)*sqrt(c)*log((sqrt(e)*sqrt(a*e + c*d*x) + sqrt(d)*sqrt(c
)*sqrt(d + e*x))/sqrt(a*e**2 - c*d**2))*a**2*e**4 - 6*sqrt(e)*sqrt(d)*sqrt
(c)*log((sqrt(e)*sqrt(a*e + c*d*x) + sqrt(d)*sqrt(c)*sqrt(d + e*x))/sqrt(a
*e**2 - c*d**2))*a*c*d**2*e**2 + 3*sqrt(e)*sqrt(d)*sqrt(c)*log((sqrt(e)*sq
rt(a*e + c*d*x) + sqrt(d)*sqrt(c)*sqrt(d + e*x))/sqrt(a*e**2 - c*d**2))*c*
*2*d**4)/(4*c*d*e**3)
```

3.215 $\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{(d+ex)^3}\,dx$

Optimal result

Integrand size = 37, antiderivative size = 160

$$\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{(d+ex)^3}\,dx = \frac{3cd\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{e^2} - \frac{2\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{e(d+ex)^2} - \frac{3\sqrt{c}\sqrt{d}\left(cd^2-ae^2\right)\operatorname{arctanh}\left(\frac{\sqrt{e}\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{\sqrt{c}\sqrt{d}(d+ex)}\right)}{e^{5/2}}$$

output
```
3*c*d*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/e^2-2*(a*d*e+(a*e^2+c*d^2)*x
+c*d*e*x^2)^(3/2)/e/(e*x+d)^2-3*c^(1/2)*d^(1/2)*(-a*e^2+c*d^2)*arctanh(e^(
1/2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/c^(1/2)/d^(1/2)/(e*x+d))/e^(5
/2)
```

Mathematica [A] (verified)

Time = 0.38 (sec) , antiderivative size = 148, normalized size of antiderivative = 0.92

$$\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{(d+ex)^3}\,dx = \frac{((ae+cdx)(d+ex))^{3/2}\left(\frac{\sqrt{e}(-2ae^2+cd(3d+ex))}{(ae+cdx)(d+ex)^2}-\frac{3\sqrt{c}\sqrt{d}(cd^2-ae^2)\text{arct}}{(ae+cdx)^{3/2}(}\right.}{e^{5/2}}$$

input
```
Integrate[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2)/(d + e*x)^3,x]
```

output
```
(((a*e + c*d*x)*(d + e*x))^(3/2)*((Sqrt[e]*(-2*a*e^2 + c*d*(3*d + e*x)))/(
(a*e + c*d*x)*(d + e*x)^2) - (3*Sqrt[c]*Sqrt[d]*(c*d^2 - a*e^2)*ArcTanh[(S
qrt[c]*Sqrt[d]*Sqrt[d + e*x])/(Sqrt[e]*Sqrt[a*e + c*d*x])])/((a*e + c*d*x)
^(3/2)*(d + e*x)^(3/2))))/e^(5/2)
```

Rubi [A] (verified)

Time = 0.57 (sec) , antiderivative size = 186, normalized size of antiderivative = 1.16, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.135$, Rules used = {1125, 27, 1160, 1092, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{(d+ex)^3}dx$$

$$\Big\downarrow \ 1125$$

$$-\frac{\int \frac{cde^2\left(cd^2-cexd-2ae^2\right)}{\sqrt{cdex^2+(cd^2+ae^2)x+ade}}dx}{e^4}-\frac{2\left(a-\frac{cd^2}{e^2}\right)\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{d+ex}$$

$$\Big\downarrow \ 27$$

$$-\frac{cd\int \frac{cd^2-cexd-2ae^2}{\sqrt{cdex^2+(cd^2+ae^2)x+ade}}dx}{e^2}-\frac{2\left(a-\frac{cd^2}{e^2}\right)\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{d+ex}$$

$$\Big\downarrow \ 1160$$

$$-\frac{cd\left(\frac{3}{2}\left(cd^2-ae^2\right)\int \frac{1}{\sqrt{cdex^2+(cd^2+ae^2)x+ade}}dx-\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}\right)}{e^2}-\frac{2\left(a-\frac{cd^2}{e^2}\right)\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{d+ex}$$

$$\Big\downarrow \text{1092}$$

$$-\frac{cd\left(3\left(cd^2-ae^2\right)\int \frac{1}{4cde-\frac{\left(cd^2+2cexd+ae^2\right)^2}{cdex^2+\left(cd^2+ae^2\right)x+ade}}d\frac{cd^2+2cexd+ae^2}{\sqrt{cdex^2+(cd^2+ae^2)x+ade}}-\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}\right)}{e^2}-\frac{2\left(a-\frac{cd^2}{e^2}\right)\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{d+ex}$$

$$\Big\downarrow \text{219}$$

$$-\frac{cd\left(\frac{3\left(cd^2-ae^2\right)\text{arctanh}\left(\frac{ae^2+cd^2+2cdex}{2\sqrt{c}\sqrt{d}\sqrt{e}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}\right)}{2\sqrt{c}\sqrt{d}\sqrt{e}}-\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}\right)}{e^2}-\frac{2\left(a-\frac{cd^2}{e^2}\right)\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{d+ex}$$

input
```
Int[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2)/(d + e*x)^3,x]
```

output
```
(-2*(a - (c*d^2)/e^2)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(d + e*
x) - (c*d*(-Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2] + (3*(c*d^2 - a*e^
2)*ArcTanh[(c*d^2 + a*e^2 + 2*c*d*e*x)/(2*Sqrt[c]*Sqrt[d]*Sqrt[e]*Sqrt[a*d
*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])])/(2*Sqrt[c]*Sqrt[d]*Sqrt[e])))/e^2
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1092
```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[I
nt[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a
, b, c}, x]
```

rule 1125
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_
Symbol] :> Simp[-2*e^(2*m + 3)*(Sqrt[a + b*x + c*x^2]/((-2*c*d + b*e)^(m +
2)*(d + e*x))), x] - Simp[e^(2*m + 2)   Int[(1/Sqrt[a + b*x + c*x^2])*Expan
dToSum[((-2*c*d + b*e)^(-m - 1) - ((-c)*d + b*e + c*e*x)^(-m - 1))/(d + e*x
), x], x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0]
 && ILtQ[m, 0] && EqQ[m + p, -3/2]
```

rule 1160
```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[e*((a + b*x + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Simp[(2*c*d - b
*e)/(2*c)   Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
 && NeQ[p, -1]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 391 vs. 2(142) = 284.

Time = 2.10 (sec) , antiderivative size = 392, normalized size of antiderivative = 2.45

method	result
default	$\frac{-\frac{2\left(dec\left(x+\frac{d}{e}\right)^2+\left(a\,e^2-c\,d^2\right)\left(x+\frac{d}{e}\right)\right)^{\frac{5}{2}}}{\left(a\,e^2-c\,d^2\right)\left(x+\frac{d}{e}\right)^3}+\frac{4dec\left(\frac{2\left(dec\left(x+\frac{d}{e}\right)^2+\left(a\,e^2-c\,d^2\right)\left(x+\frac{d}{e}\right)\right)^{\frac{5}{2}}}{\left(a\,e^2-c\,d^2\right)\left(x+\frac{d}{e}\right)^2}-\frac{6dec\left(\frac{\left(dec\left(x+\frac{d}{e}\right)^2+\left(a\,e^2-c\,d^2\right)\left(x+\frac{d}{e}\right)\right)^{\frac{3}{2}}}{3}+\frac{(a}{}\right.}{}\right.}{}}{}$

input
```
int((a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(3/2)/(e*x+d)^3,x,method=_RETURNVERB
OSE)
```

output
```
1/e^3*(-2/(a*e^2-c*d^2)/(x+d/e)^3*(d*e*c*(x+d/e)^2+(a*e^2-c*d^2)*(x+d/e))^
(5/2)+4*d*e*c/(a*e^2-c*d^2)*(2/(a*e^2-c*d^2)/(x+d/e)^2*(d*e*c*(x+d/e)^2+(a
*e^2-c*d^2)*(x+d/e))^(5/2)-6*d*e*c/(a*e^2-c*d^2)*(1/3*(d*e*c*(x+d/e)^2+(a*
e^2-c*d^2)*(x+d/e))^(3/2)+1/2*(a*e^2-c*d^2)*(1/4*(2*d*e*c*(x+d/e)+a*e^2-c*
d^2)/d/e/c*(d*e*c*(x+d/e)^2+(a*e^2-c*d^2)*(x+d/e))^(1/2)-1/8*(a*e^2-c*d^2)
^2/d/e/c*ln((1/2*a*e^2-1/2*c*d^2+d*e*c*(x+d/e))/(d*e*c)^(1/2)+(d*e*c*(x+d/
e)^2+(a*e^2-c*d^2)*(x+d/e))^(1/2))/(d*e*c)^(1/2)))))
```

Fricas [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 414, normalized size of antiderivative = 2.59

$$\int \frac{(ade+(cd^2+ae^2)x+cdex^2)^{3/2}}{(d+ex)^3}\,dx = \left[-\frac{3\,(cd^3-ade^2+(cd^2e-ae^3)x)\sqrt{\frac{cd}{e}}\log\left(8\,c^2d^2e^2x^2+c^2d^4+\cdots\right.}{\cdots}\right.$$

input
```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/(e*x+d)^3,x, algorithm="
fricas")
```

output
```
[-1/4*(3*(c*d^3 - a*d*e^2 + (c*d^2*e - a*e^3)*x)*sqrt(c*d/e)*log(8*c^2*d^2
*e^2*x^2 + c^2*d^4 + 6*a*c*d^2*e^2 + a^2*e^4 + 4*(2*c*d*e^2*x + c*d^2*e +
a*e^3)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(c*d/e) + 8*(c^2*d^
3*e + a*c*d*e^3)*x) - 4*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(c*d*e
*x + 3*c*d^2 - 2*a*e^2))/(e^3*x + d*e^2), 1/2*(3*(c*d^3 - a*d*e^2 + (c*d^2
*e - a*e^3)*x)*sqrt(-c*d/e)*arctan(1/2*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a
*e^2)*x)*(2*c*d*e*x + c*d^2 + a*e^2)*sqrt(-c*d/e)/(c^2*d^2*e*x^2 + a*c*d^2
*e + (c^2*d^3 + a*c*d*e^2)*x)) + 2*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2
)*x)*(c*d*e*x + 3*c*d^2 - 2*a*e^2))/(e^3*x + d*e^2)]
```

Sympy [F]

$$\int \frac{(ade+(cd^2+ae^2)x+cdex^2)^{3/2}}{(d+ex)^3}\,dx = \int \frac{((d+ex)\,(ae+cdx))^{\frac{3}{2}}}{(d+ex)^3}\,dx$$

input
```
integrate((a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(3/2)/(e*x+d)**3,x)
```

output
```
Integral(((d + e*x)*(a*e + c*d*x))**(3/2)/(d + e*x)**3, x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(ade+(cd^2+ae^2)\,x+cdex^2)^{3/2}}{(d+ex)^3}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/(e*x+d)^3,x, algorithm="
maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e*(a*e^2-c*d^2)>0)', see `assume
?` for mor
```

Giac [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 224, normalized size of antiderivative = 1.40

$$\int \frac{(ade+(cd^2+ae^2)\,x+cdex^2)^{3/2}}{(d+ex)^3}\,dx = \frac{\sqrt{cdex^2+cd^2x+ae^2x+ade}cd}{e^2} + \frac{3\,(c^2d^3-acde^2)\log\left(\left|-cd^2-ae^2-2\sqrt{cde}\left(\sqrt{cde}x-\sqrt{cdex^2+cd^2x+ae^2x+ade}\right)\right|\right)}{2\sqrt{cde}e^2} + \frac{2\left(\sqrt{cde}c^2d^4-2\sqrt{cde}acd^2e^2+\sqrt{cde}a^2e^4\right)}{\sqrt{cde}\left(\left(\sqrt{cde}x-\sqrt{cdex^2+cd^2x+ae^2x+ade}\right)e+\sqrt{cde}d\right)e^2}$$

input
```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/(e*x+d)^3,x, algorithm="
giac")
```

output
```
sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*x + a*d*e)*c*d/e^2 + 3/2*(c^2*d^3 - a*c*d
*e^2)*log(abs(-c*d^2 - a*e^2 - 2*sqrt(c*d*e)*(sqrt(c*d*e)*x - sqrt(c*d*e*x
^2 + c*d^2*x + a*e^2*x + a*d*e))))/(sqrt(c*d*e)*e^2) + 2*(sqrt(c*d*e)*c^2*
d^4 - 2*sqrt(c*d*e)*a*c*d^2*e^2 + sqrt(c*d*e)*a^2*e^4)/(sqrt(c*d*e)*((sqrt
(c*d*e)*x - sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*x + a*d*e))*e + sqrt(c*d*e)*d
)*e^2)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(ade + (cd^2 + ae^2) x + cdex^2)^{3/2}}{(d + ex)^3} dx = \int \frac{(c\,d\,e\,x^2 + (c\,d^2 + a\,e^2)\,x + a\,d\,e)^{3/2}}{(d + e\,x)^3} dx$$

input
```
int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(3/2)/(d + e*x)^3,x)
```

output
```
int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(3/2)/(d + e*x)^3, x)
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 347, normalized size of antiderivative = 2.17

$$\int \frac{(ade + (cd^2 + ae^2) x + cdex^2)^{3/2}}{(d + ex)^3} dx = \frac{-8\sqrt{ex + d}\,\sqrt{cdx + ae}\,a\,e^3 + 12\sqrt{ex + d}\,\sqrt{cdx + ae}\,c\,d^2e + 4\sqrt{e}$$

input
```
int((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/(e*x+d)^3,x)
```

output
```
( - 8*sqrt(d + e*x)*sqrt(a*e + c*d*x)*a*e**3 + 12*sqrt(d + e*x)*sqrt(a*e +
 c*d*x)*c*d**2*e + 4*sqrt(d + e*x)*sqrt(a*e + c*d*x)*c*d*e**2*x + 12*sqrt(
e)*sqrt(d)*sqrt(c)*log((sqrt(e)*sqrt(a*e + c*d*x) + sqrt(d)*sqrt(c)*sqrt(d
 + e*x))/sqrt(a*e**2 - c*d**2))*a*d*e**2 + 12*sqrt(e)*sqrt(d)*sqrt(c)*log(
(sqrt(e)*sqrt(a*e + c*d*x) + sqrt(d)*sqrt(c)*sqrt(d + e*x))/sqrt(a*e**2 -
c*d**2))*a*e**3*x - 12*sqrt(e)*sqrt(d)*sqrt(c)*log((sqrt(e)*sqrt(a*e + c*d
*x) + sqrt(d)*sqrt(c)*sqrt(d + e*x))/sqrt(a*e**2 - c*d**2))*c*d**3 - 12*sq
rt(e)*sqrt(d)*sqrt(c)*log((sqrt(e)*sqrt(a*e + c*d*x) + sqrt(d)*sqrt(c)*sqr
t(d + e*x))/sqrt(a*e**2 - c*d**2))*c*d**2*e*x - 9*sqrt(e)*sqrt(d)*sqrt(c)*
a*d*e**2 - 9*sqrt(e)*sqrt(d)*sqrt(c)*a*e**3*x + 9*sqrt(e)*sqrt(d)*sqrt(c)*
c*d**3 + 9*sqrt(e)*sqrt(d)*sqrt(c)*c*d**2*e*x)/(4*e**3*(d + e*x))
```

3.216 $\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{(d+ex)^4}\,dx$

Optimal result

Integrand size = 37, antiderivative size = 157

$$\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{(d+ex)^4}\,dx=-\frac{2cd\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{e^2(d+ex)}-\frac{2\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{3e(d+ex)^3}+\frac{2c^{3/2}d^{3/2}\text{arctanh}\left(\frac{\sqrt{e}\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{\sqrt{c}\sqrt{d}(d+ex)}\right)}{e^{5/2}}$$

output
```
-2*c*d*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/e^2/(e*x+d)-2/3*(a*d*e+(a*e
^2+c*d^2)*x+c*d*e*x^2)^(3/2)/e/(e*x+d)^3+2*c^(3/2)*d^(3/2)*arctanh(e^(1/2)
*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/c^(1/2)/d^(1/2)/(e*x+d))/e^(5/2)
```

Mathematica [A] (verified)

Time = 0.31 (sec) , antiderivative size = 130, normalized size of antiderivative = 0.83

$$\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{(d+ex)^4}\,dx=\frac{2\sqrt{(ae+cdx)(d+ex)}\left(-\frac{\sqrt{e}\left(ae^2+cd(3d+4ex)\right)}{(d+ex)^2}+\frac{3c^{3/2}d^{3/2}\text{arctanh}\left(\frac{\sqrt{c}\cdots}{\sqrt{e}\cdots}\right)}{\sqrt{ae+cdx}\sqrt{d+e\cdots}}\right.}{3e^{5/2}}$$

input
```
Integrate[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2)/(d + e*x)^4,x]
```

output
```
(2*Sqrt[(a*e + c*d*x)*(d + e*x)]*(-((Sqrt[e]*(a*e^2 + c*d*(3*d + 4*e*x)))/
(d + e*x)^2) + (3*c^(3/2)*d^(3/2)*ArcTanh[(Sqrt[c]*Sqrt[d]*Sqrt[d + e*x])/
(Sqrt[e]*Sqrt[a*e + c*d*x])])/(Sqrt[a*e + c*d*x]*Sqrt[d + e*x])))/(3*e^(5/
2))
```

Rubi [A] (verified)

Time = 0.51 (sec) , antiderivative size = 174, normalized size of antiderivative = 1.11, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.162$, Rules used = {1130, 1125, 25, 27, 1092, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{(d+ex)^4}dx$$

$$\Big\downarrow \text{1130}$$

$$\frac{cd\int \frac{\sqrt{cdex^2+(cd^2+ae^2)x+ade}}{(d+ex)^2}dx}{e}-\frac{2\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{3e(d+ex)^3}$$

$$\Big\downarrow \text{1125}$$

$$\frac{cd\left(-\frac{\int -\frac{cde}{\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}dx}{e^2}-\frac{2\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{e(d+ex)}\right)}{e}-\frac{2\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{3e(d+ex)^3}$$

$$\Big\downarrow \text{25}$$

$$\frac{cd\left(\frac{\int \frac{cde}{\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}dx}{e^2}-\frac{2\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{e(d+ex)}\right)}{e}-\frac{2\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{3e(d+ex)^3}$$

$$\Big\downarrow \text{27}$$

$$\frac{cd\left(\frac{cd\int \frac{1}{\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}dx}{e}-\frac{2\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{e(d+ex)}\right)}{e}-\frac{2\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{3e(d+ex)^3}$$

$\Big\downarrow$ 1092

$$\frac{cd\left(\frac{2cd\int \frac{1}{4cde-\frac{\left(cd^2+2cexd+ae^2\right)^2}{cdex^2+\left(cd^2+ae^2\right)x+ade}}d\frac{cd^2+2cexd+ae^2}{\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}}{e}-\frac{2\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{e(d+ex)}\right)}{e}-\frac{2\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{3e(d+ex)^3}$$

$\Big\downarrow$ 219

$$\frac{cd\left(\frac{\sqrt{c}\sqrt{d}\text{arctanh}\left(\frac{ae^2+cd^2+2cdex}{2\sqrt{c}\sqrt{d}\sqrt{e}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}\right)}{e^{3/2}}-\frac{2\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{e(d+ex)}\right)}{e}-\frac{2\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{3e(d+ex)^3}$$

input
```
Int[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2)/(d + e*x)^4,x]
```

output
```
(-2*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2))/(3*e*(d + e*x)^3) + (c*
d*((-2*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(e*(d + e*x)) + (Sqrt[
c]*Sqrt[d]*ArcTanh[(c*d^2 + a*e^2 + 2*c*d*e*x)/(2*Sqrt[c]*Sqrt[d]*Sqrt[e]*
Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])])/e^(3/2)))/e
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1092
```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[I
nt[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a
, b, c}, x]
```

rule 1125
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_
Symbol] :> Simp[-2*e^(2*m + 3)*(Sqrt[a + b*x + c*x^2]/((-2*c*d + b*e)^(m +
2)*(d + e*x))), x] - Simp[e^(2*m + 2)   Int[(1/Sqrt[a + b*x + c*x^2])*Expan
dToSum[((-2*c*d + b*e)^(-m - 1) - ((-c)*d + b*e + c*e*x)^(-m - 1))/(d + e*x
), x], x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0]
 && ILtQ[m, 0] && EqQ[m + p, -3/2]
```

rule 1130
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(d + e*x)^(m + 1)*((a + b*x + c*x^2)^p/(e*(m + p + 1))), x]
- Simp[c*(p/(e^2*(m + p + 1)))   Int[(d + e*x)^(m + 2)*(a + b*x + c*x^2)^(p
 - 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0]
 && GtQ[p, 0] && (LtQ[m, -2] || EqQ[m + 2*p + 1, 0]) && NeQ[m + p + 1, 0] &
& IntegerQ[2*p]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 472 vs. 2(137) = 274.

Time = 2.43 (sec) , antiderivative size = 473, normalized size of antiderivative = 3.01

method	result
default	$-\frac{2\left(dec\left(x+\frac{d}{e}\right)^2+\left(a\,e^2-c\,d^2\right)\left(x+\frac{d}{e}\right)\right)^{\frac{5}{2}}}{3\left(a\,e^2-c\,d^2\right)\left(x+\frac{d}{e}\right)^4}+\frac{2dec\left(-\frac{2\left(dec\left(x+\frac{d}{e}\right)^2+\left(a\,e^2-c\,d^2\right)\left(x+\frac{d}{e}\right)\right)^{\frac{5}{2}}}{\left(a\,e^2-c\,d^2\right)\left(x+\frac{d}{e}\right)^3}+\frac{4dec\left(\frac{2\left(dec\left(x+\frac{d}{e}\right)^2+\left(a\,e^2-c\,d^2\right)\left(x+\frac{d}{e}\right)\right)^{\frac{5}{2}}}{\left(a\,e^2-c\,d^2\right)\left(x+\frac{d}{e}\right)^2}-\cdots\right.}{\cdots}\right.}{\cdots}$

input
```
int((a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(3/2)/(e*x+d)^4,x,method=_RETURNVERB
OSE)
```

output
```
1/e^4*(-2/3/(a*e^2-c*d^2)/(x+d/e)^4*(d*e*c*(x+d/e)^2+(a*e^2-c*d^2)*(x+d/e)
)^(5/2)+2/3*d*e*c/(a*e^2-c*d^2)*(-2/(a*e^2-c*d^2)/(x+d/e)^3*(d*e*c*(x+d/e)
^2+(a*e^2-c*d^2)*(x+d/e))^(5/2)+4*d*e*c/(a*e^2-c*d^2)*(2/(a*e^2-c*d^2)/(x+
d/e)^2*(d*e*c*(x+d/e)^2+(a*e^2-c*d^2)*(x+d/e))^(5/2)-6*d*e*c/(a*e^2-c*d^2)
*(1/3*(d*e*c*(x+d/e)^2+(a*e^2-c*d^2)*(x+d/e))^(3/2)+1/2*(a*e^2-c*d^2)*(1/4
*(2*d*e*c*(x+d/e)+a*e^2-c*d^2)/d/e/c*(d*e*c*(x+d/e)^2+(a*e^2-c*d^2)*(x+d/e
))^(1/2)-1/8*(a*e^2-c*d^2)^2/d/e/c*ln((1/2*a*e^2-1/2*c*d^2+d*e*c*(x+d/e))/
(d*e*c)^(1/2)+(d*e*c*(x+d/e)^2+(a*e^2-c*d^2)*(x+d/e))^(1/2))/(d*e*c)^(1/2)
)))))
```

Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 426, normalized size of antiderivative = 2.71

$$\int \frac{(ade+(cd^2+ae^2)x+cdex^2)^{3/2}}{(d+ex)^4}\,dx = \left[\frac{3\,(cde^2x^2+2\,cd^2ex+cd^3)\sqrt{\frac{cd}{e}}\log\left(8\,c^2d^2e^2x^2+c^2d^4+6\,acd^2e\right.}{}\right.$$

$$-\frac{3\,(cde^2x^2+2\,cd^2ex+cd^3)\sqrt{-\frac{cd}{e}}\arctan\left(\frac{\sqrt{cdex^2+ade+(cd^2+ae^2)x}\,(2\,cdex+cd^2+ae^2)\sqrt{-\frac{cd}{e}}}{2\,(c^2d^2ex^2+acd^2e+(c^2d^3+acde^2)x)}\right)+2\,\sqrt{cdex^2+ade+}}{3\,(e^4x^2+2\,de^3x+d^2e^2)}$$

input
```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/(e*x+d)^4,x, algorithm="
fricas")
```

output
```
[1/6*(3*(c*d*e^2*x^2 + 2*c*d^2*e*x + c*d^3)*sqrt(c*d/e)*log(8*c^2*d^2*e^2*
x^2 + c^2*d^4 + 6*a*c*d^2*e^2 + a^2*e^4 + 4*(2*c*d*e^2*x + c*d^2*e + a*e^3
)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(c*d/e) + 8*(c^2*d^3*e +
 a*c*d*e^3)*x) - 4*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(4*c*d*e*x
+ 3*c*d^2 + a*e^2))/(e^4*x^2 + 2*d*e^3*x + d^2*e^2), -1/3*(3*(c*d*e^2*x^2
+ 2*c*d^2*e*x + c*d^3)*sqrt(-c*d/e)*arctan(1/2*sqrt(c*d*e*x^2 + a*d*e + (c
*d^2 + a*e^2)*x)*(2*c*d*e*x + c*d^2 + a*e^2)*sqrt(-c*d/e)/(c^2*d^2*e*x^2 +
 a*c*d^2*e + (c^2*d^3 + a*c*d*e^2)*x)) + 2*sqrt(c*d*e*x^2 + a*d*e + (c*d^2
 + a*e^2)*x)*(4*c*d*e*x + 3*c*d^2 + a*e^2))/(e^4*x^2 + 2*d*e^3*x + d^2*e^2
)]
```

Sympy [F]

$$\int \frac{(ade + (cd^2 + ae^2)x + cdex^2)^{3/2}}{(d+ex)^4}\,dx = \int \frac{((d+ex)(ae+cdx))^{\frac{3}{2}}}{(d+ex)^4}\,dx$$

input
```
integrate((a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(3/2)/(e*x+d)**4,x)
```

output
```
Integral(((d + e*x)*(a*e + c*d*x))**(3/2)/(d + e*x)**4, x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(ade + (cd^2 + ae^2)x + cdex^2)^{3/2}}{(d+ex)^4}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/(e*x+d)^4,x, algorithm="
maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e*(a*e^2-c*d^2)>0)', see `assume
?` for mor
```

Giac [F(-2)]

Exception generated.

$$\int \frac{(ade + (cd^2 + ae^2)x + cdex^2)^{3/2}}{(d+ex)^4}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/(e*x+d)^4,x, algorithm="
giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Unable to divide, perhaps due to ro
unding error%%%{%%{[%%%{1,[0,0,10]%%%},0]:[1,0,%%%{-1,[1,1,1]%%%}]%%},[4,4
]%%%}+%%%
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(ade+(cd^2+ae^2)\,x+cdex^2)^{3/2}}{(d+ex)^4}\,dx=\int \frac{{\left(c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e\right)}^{3/2}}{{\left(d+e\,x\right)}^4}\,dx$$

input
```
int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(3/2)/(d + e*x)^4,x)
```

output
```
int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(3/2)/(d + e*x)^4, x)
```

Reduce [B] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 255, normalized size of antiderivative = 1.62

$$\int \frac{(ade+(cd^2+ae^2)\,x+cdex^2)^{3/2}}{(d+ex)^4}\,dx=\frac{-\frac{2\sqrt{ex+d}\,\sqrt{cdx+ae}\,a\,e^3}{3}-2\sqrt{ex+d}\,\sqrt{cdx+ae}\,c\,d^2e-\frac{8\sqrt{ex+d}\,\sqrt{cdx+ae}}{3}}{}$$

input
```
int((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/(e*x+d)^4,x)
```

output
```
(2*( - sqrt(d + e*x)*sqrt(a*e + c*d*x)*a*e**3 - 3*sqrt(d + e*x)*sqrt(a*e +
 c*d*x)*c*d**2*e - 4*sqrt(d + e*x)*sqrt(a*e + c*d*x)*c*d*e**2*x + 3*sqrt(e
)*sqrt(d)*sqrt(c)*log((sqrt(e)*sqrt(a*e + c*d*x) + sqrt(d)*sqrt(c)*sqrt(d
+ e*x))/sqrt(a*e**2 - c*d**2))*c*d**3 + 6*sqrt(e)*sqrt(d)*sqrt(c)*log((sqr
t(e)*sqrt(a*e + c*d*x) + sqrt(d)*sqrt(c)*sqrt(d + e*x))/sqrt(a*e**2 - c*d*
*2))*c*d**2*e*x + 3*sqrt(e)*sqrt(d)*sqrt(c)*log((sqrt(e)*sqrt(a*e + c*d*x)
 + sqrt(d)*sqrt(c)*sqrt(d + e*x))/sqrt(a*e**2 - c*d**2))*c*d*e**2*x**2))/(
3*e**3*(d**2 + 2*d*e*x + e**2*x**2))
```

3.217 $\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{(d+ex)^5} dx$

Optimal result

Integrand size = 37, antiderivative size = 54

$$\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{(d+ex)^5}\,dx=\frac{2\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{5\left(cd^2-ae^2\right)(d+ex)^5}$$

output
```
2/5*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(-a*e^2+c*d^2)/(e*x+d)^5
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.80

$$\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{(d+ex)^5}\,dx=\frac{2((ae+cdx)(d+ex))^{5/2}}{5\left(cd^2-ae^2\right)(d+ex)^5}$$

input
```
Integrate[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2)/(d + e*x)^5,x]
```

output
```
(2*((a*e + c*d*x)*(d + e*x))^(5/2))/(5*(c*d^2 - a*e^2)*(d + e*x)^5)
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.027, Rules used = {1123}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{(d+ex)^5}dx$$

$$\Big\downarrow \ 1123$$

$$\frac{2\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/2}}{5(d+ex)^5\left(cd^2-ae^2\right)}$$

input
```
Int[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2)/(d + e*x)^5,x]
```

output
```
(2*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2))/(5*(c*d^2 - a*e^2)*(d +
e*x)^5)
```

Defintions of rubi rules used

rule 1123
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[e*(d + e*x)^m*((a + b*x + c*x^2)^(p + 1)/((p + 1)*(2*c*d - b
*e))), x] /; FreeQ[{a, b, c, d, e, m, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2,
0] && EqQ[m + 2*p + 2, 0]
```

Maple [A] (verified)

Time = 2.94 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.07

method	result	size
gosper	$-\frac{2(cdx+ae)\left(cd\,x^{2}e+a\,e^{2}x+c\,d^{2}x+ade\right)^{\frac{3}{2}}}{5(ex+d)^{4}\left(a\,e^{2}-c\,d^{2}\right)}$	58
orering	$-\frac{2(cdx+ae)\left(ade+\left(a\,e^{2}+c\,d^{2}\right)x+cd\,x^{2}e\right)^{\frac{3}{2}}}{5(ex+d)^{4}\left(a\,e^{2}-c\,d^{2}\right)}$	59
default	$-\frac{2\left(dec\left(x+\frac{d}{e}\right)^{2}+\left(a\,e^{2}-c\,d^{2}\right)\left(x+\frac{d}{e}\right)\right)^{\frac{5}{2}}}{5e^{5}\left(a\,e^{2}-c\,d^{2}\right)\left(x+\frac{d}{e}\right)^{5}}$	65
trager	$-\frac{2\left(d^{2}c^{2}x^{2}+2acdex+e^{2}a^{2}\right)\sqrt{cd\,x^{2}e+a\,e^{2}x+c\,d^{2}x+ade}}{5(ex+d)^{3}\left(a\,e^{2}-c\,d^{2}\right)}$	75

input
```
int((a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(3/2)/(e*x+d)^5,x,method=_RETURNVERB
OSE)
```

output
```
-2/5/(e*x+d)^4*(c*d*x+a*e)/(a*e^2-c*d^2)*(c*d*e*x^2+a*e^2*x+c*d^2*x+a*d*e)
^(3/2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 130 vs. 2(50) = 100.

Time = 0.59 (sec) , antiderivative size = 130, normalized size of antiderivative = 2.41

$$\int \frac{(ade+(cd^2+ae^2)\,x+cdex^2)^{3/2}}{(d+ex)^5}\,dx = \frac{2\,(c^2d^2x^2+2\,acdex+a^2e^2)\sqrt{cdex^2+ade+(cd^2+ae^2)}}{5\,(cd^5-ad^3e^2+(cd^2e^3-ae^5)x^3+3\,(cd^3e^2-ade^4)x^2+3\,(cd^4e}$$

input
```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/(e*x+d)^5,x, algorithm="
fricas")
```

output
```
2/5*(c^2*d^2*x^2 + 2*a*c*d*e*x + a^2*e^2)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2
+ a*e^2)*x)/(c*d^5 - a*d^3*e^2 + (c*d^2*e^3 - a*e^5)*x^3 + 3*(c*d^3*e^2 -
a*d*e^4)*x^2 + 3*(c*d^4*e - a*d^2*e^3)*x)
```

Sympy [F]

$$\int \frac{(ade + (cd^2 + ae^2)x + cdex^2)^{3/2}}{(d+ex)^5} \, dx = \int \frac{((d+ex)(ae+cdx))^{\frac{3}{2}}}{(d+ex)^5} \, dx$$

input
```
integrate((a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(3/2)/(e*x+d)**5,x)
```

output
```
Integral(((d + e*x)*(a*e + c*d*x))**(3/2)/(d + e*x)**5, x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(ade + (cd^2 + ae^2)x + cdex^2)^{3/2}}{(d+ex)^5} \, dx = \text{Exception raised: ValueError}$$

input
```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/(e*x+d)^5,x, algorithm="
maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e*(a*e^2-c*d^2)>0)', see `assume
?` for mor
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 806 vs. $2(50) = 100$.

Time = 0.26 (sec) , antiderivative size = 806, normalized size of antiderivative = 14.93

$$\int \frac{(ade + (cd^2 + ae^2)x + cdex^2)^{3/2}}{(d+ex)^5} \, dx = \text{Too large to display}$$

input
```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/(e*x+d)^5,x, algorithm="
giac")
```

output
```
-2/15*(3*sqrt(c*d*e)*c^2*d^2*sgn(1/(e*x + d))*sgn(e)/(c*d^2*e^3*abs(e) - a
*e^5*abs(e)) - ((15*sqrt(c*d*e - c*d^2*e/(e*x + d) + a*e^3/(e*x + d))*c^2*
d^2*e^2 - 10*(c*d*e - c*d^2*e/(e*x + d) + a*e^3/(e*x + d))^(3/2)*c*d*e + 3
*(c*d*e - c*d^2*e/(e*x + d) + a*e^3/(e*x + d))^(5/2))*c^2*d^4*e^2*sgn(1/(e
*x + d))*sgn(e)/(c*d^2*e^2 - a*e^4)^2 - 2*(15*sqrt(c*d*e - c*d^2*e/(e*x +
d) + a*e^3/(e*x + d))*c^2*d^2*e^2 - 10*(c*d*e - c*d^2*e/(e*x + d) + a*e^3/
(e*x + d))^(3/2)*c*d*e + 3*(c*d*e - c*d^2*e/(e*x + d) + a*e^3/(e*x + d))^(
5/2))*a*c*d^2*e^4*sgn(1/(e*x + d))*sgn(e)/(c*d^2*e^2 - a*e^4)^2 + (15*sqrt
(c*d*e - c*d^2*e/(e*x + d) + a*e^3/(e*x + d))*c^2*d^2*e^2 - 10*(c*d*e - c*
d^2*e/(e*x + d) + a*e^3/(e*x + d))^(3/2)*c*d*e + 3*(c*d*e - c*d^2*e/(e*x +
 d) + a*e^3/(e*x + d))^(5/2))*a^2*e^6*sgn(1/(e*x + d))*sgn(e)/(c*d^2*e^2 -
 a*e^4)^2 - 10*(3*sqrt(c*d*e - c*d^2*e/(e*x + d) + a*e^3/(e*x + d))*c*d*e
- (c*d*e - c*d^2*e/(e*x + d) + a*e^3/(e*x + d))^(3/2))*c^2*d^3*e*sgn(1/(e*
x + d))*sgn(e)/(c*d^2*e^2 - a*e^4) + 10*(3*sqrt(c*d*e - c*d^2*e/(e*x + d)
+ a*e^3/(e*x + d))*c*d*e - (c*d*e - c*d^2*e/(e*x + d) + a*e^3/(e*x + d))^(
3/2))*a*c*d*e^3*sgn(1/(e*x + d))*sgn(e)/(c*d^2*e^2 - a*e^4) + 15*sqrt(c*d*
e - c*d^2*e/(e*x + d) + a*e^3/(e*x + d))*c^2*d^2*sgn(1/(e*x + d))*sgn(e))/
(c*d^2*e^3*abs(e) - a*e^5*abs(e)))*abs(e)
```

Mupad [B] (verification not implemented)

Time = 6.10 (sec) , antiderivative size = 901, normalized size of antiderivative = 16.69

$$\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{(d+ex)^5}\,dx = \text{Too large to display}$$

input
```
int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(3/2)/(d + e*x)^5,x)
```

output
```
(((d*((4*c^3*d^4)/(5*(a*e^2 - c*d^2)*(3*a*e^3 - 3*c*d^2*e)) - (2*c^2*d^2*(
5*a*e^2 - c*d^2))/(5*(a*e^2 - c*d^2)*(3*a*e^3 - 3*c*d^2*e))))/e + (2*a*c^2
*d^3*e^2 - 2*c^3*d^5 + 4*a^2*c*d*e^4)/(5*e*(a*e^2 - c*d^2)*(3*a*e^3 - 3*c*
d^2*e)))*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2))/(d + e*x)^2 - (((d
*((40*c^4*d^5 - 56*a*c^3*d^3*e^2)/(15*e*(a*e^2 - c*d^2)^3) + (8*c^4*d^5)/(
15*e*(a*e^2 - c*d^2)^3)))/e + (8*a*c^2*d^2*(6*a*e^2 - 5*c*d^2))/(15*(a*e^2
 - c*d^2)^3))*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2))/(d + e*x) - (
((2*a^2*e^3)/(5*a*e^3 - 5*c*d^2*e) + (d*((2*c^2*d^3)/(5*a*e^3 - 5*c*d^2*e)
 - (4*a*c*d*e^2)/(5*a*e^3 - 5*c*d^2*e)))/e)*(x*(a*e^2 + c*d^2) + a*d*e + c
*d*e*x^2)^(1/2))/(d + e*x)^3 - (((10*c^3*d^4 - 22*a*c^2*d^2*e^2)/(15*e^2*(
a*e^2 - c*d^2)^2) + (4*c^3*d^4)/(5*e^2*(a*e^2 - c*d^2)^2))*(x*(a*e^2 + c*d
^2) + a*d*e + c*d*e*x^2)^(1/2))/(d + e*x) - (((d*((12*c^3*d^4 - 20*a*c^2*d
^2*e^2)/(5*(a*e^2 - c*d^2)*(3*a*e^3 - 3*c*d^2*e)) + (4*c^3*d^4)/(5*(a*e^2
- c*d^2)*(3*a*e^3 - 3*c*d^2*e))))/e + (4*a*c*d*e*(4*a*e^2 - 3*c*d^2))/(5*(
a*e^2 - c*d^2)*(3*a*e^3 - 3*c*d^2*e)))*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*
x^2)^(1/2))/(d + e*x)^2 + (((d*((8*c^4*d^5)/(15*e*(a*e^2 - c*d^2)^3) - (4*
c^3*d^3*(11*a*e^2 - 7*c*d^2))/(15*e*(a*e^2 - c*d^2)^3)))/e + (4*a*c^3*d^4*
e^2 - 16*c^4*d^6 + 20*a^2*c^2*d^2*e^4)/(15*e^2*(a*e^2 - c*d^2)^3))*(x*(a*e
^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2))/(d + e*x)
```

Reduce [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 229, normalized size of antiderivative = 4.24

$$\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{(d+ex)^5}\,dx=\frac{-\frac{2\sqrt{ex+d}\,\sqrt{cdx+ae}\,a^2e^5}{5}-\frac{4\sqrt{ex+d}\,\sqrt{cdx+ae}\,acd\,e^4x}{5}-\frac{2\sqrt{ex+d}\,\sqrt{cdx+ae}\,c^2d^2e^3}{5}}{e^3\left(a\,e^5x^3-c\,d^2e^3x^3+3ad\,e^4x^2-3c\right.}$$

input
```
int((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/(e*x+d)^5,x)
```

output
```
(2*( - sqrt(d + e*x)*sqrt(a*e + c*d*x)*a**2*e**5 - 2*sqrt(d + e*x)*sqrt(a*
e + c*d*x)*a*c*d*e**4*x - sqrt(d + e*x)*sqrt(a*e + c*d*x)*c**2*d**2*e**3*x
**2 - sqrt(e)*sqrt(d)*sqrt(c)*c**2*d**5 - 3*sqrt(e)*sqrt(d)*sqrt(c)*c**2*d
**4*e*x - 3*sqrt(e)*sqrt(d)*sqrt(c)*c**2*d**3*e**2*x**2 - sqrt(e)*sqrt(d)*
sqrt(c)*c**2*d**2*e**3*x**3))/(5*e**3*(a*d**3*e**2 + 3*a*d**2*e**3*x + 3*a
*d*e**4*x**2 + a*e**5*x**3 - c*d**5 - 3*c*d**4*e*x - 3*c*d**3*e**2*x**2 -
c*d**2*e**3*x**3))
```

3.218 $\int \frac{(ade+(cd^2+ae^2)x+cdex^2)^{3/2}}{(d+ex)^6}\,dx$

Optimal result

Integrand size = 37, antiderivative size = 111

$$\int \frac{(ade+(cd^2+ae^2)x+cdex^2)^{3/2}}{(d+ex)^6}\,dx = \frac{2(ade+(cd^2+ae^2)x+cdex^2)^{5/2}}{7(cd^2-ae^2)(d+ex)^6} + \frac{4cd(ade+(cd^2+ae^2)x+cdex^2)^{5/2}}{35(cd^2-ae^2)^2(d+ex)^5}$$

output
```
2/7*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(-a*e^2+c*d^2)/(e*x+d)^6+4/35*
c*d*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(-a*e^2+c*d^2)^2/(e*x+d)^5
```

Mathematica [A] (verified)

Time = 0.21 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.55

$$\int \frac{(ade+(cd^2+ae^2)x+cdex^2)^{3/2}}{(d+ex)^6}\,dx = \frac{2((ae+cdx)(d+ex))^{5/2}(-5ae^2+cd(7d+2ex))}{35(cd^2-ae^2)^2(d+ex)^6}$$

input
```
Integrate[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2)/(d + e*x)^6,x]
```

output
```
(2*((a*e + c*d*x)*(d + e*x))^(5/2)*(-5*a*e^2 + c*d*(7*d + 2*e*x)))/(35*(c*
d^2 - a*e^2)^2*(d + e*x)^6)
```

Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 111, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.054, Rules used = {1129, 1123}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{(d+ex)^6}dx$$

$$\Big\downarrow \text{1129}$$

$$\frac{2cd\int \frac{\left(cdex^2+\left(cd^2+ae^2\right)x+ade\right)^{3/2}}{(d+ex)^5}dx}{7\left(cd^2-ae^2\right)}+\frac{2\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/2}}{7(d+ex)^6\left(cd^2-ae^2\right)}$$

$$\Big\downarrow \text{1123}$$

$$\frac{4cd\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/2}}{35(d+ex)^5\left(cd^2-ae^2\right)^2}+\frac{2\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/2}}{7(d+ex)^6\left(cd^2-ae^2\right)}$$

input
```
Int[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2)/(d + e*x)^6,x]
```

output
```
(2*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2))/(7*(c*d^2 - a*e^2)*(d +
e*x)^6) + (4*c*d*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2))/(35*(c*d^2
 - a*e^2)^2*(d + e*x)^5)
```

Defintions of rubi rules used

rule 1123
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[e*(d + e*x)^m*((a + b*x + c*x^2)^(p + 1)/((p + 1)*(2*c*d - b
*e))), x] /; FreeQ[{a, b, c, d, e, m, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2,
0] && EqQ[m + 2*p + 2, 0]
```

rule 1129
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(-e)*(d + e*x)^m*((a + b*x + c*x^2)^(p + 1)/((m + p + 1)*(2*
c*d - b*e))), x] + Simp[c*(Simplify[m + 2*p + 2]/((m + p + 1)*(2*c*d - b*e)
))   Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d
, e, m, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && ILtQ[Simplify[m + 2*p +
2], 0]
```

Maple [A] (verified)

Time = 3.57 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.81

method	result	size
gosper	$-\frac{2(cdx+ae)\left(-2cdxe+5a\,e^{2}-7c\,d^{2}\right)\left(cd\,x^{2}e+a\,e^{2}x+c\,d^{2}x+ade\right)^{\frac{3}{2}}}{35(ex+d)^{5}\left(a^{2}e^{4}-2ac\,d^{2}e^{2}+c^{2}d^{4}\right)}$	90
orering	$-\frac{2\left(-2cdxe+5a\,e^{2}-7c\,d^{2}\right)(cdx+ae)\left(ade+\left(a\,e^{2}+c\,d^{2}\right)x+cd\,x^{2}e\right)^{\frac{3}{2}}}{35\left(a^{2}e^{4}-2ac\,d^{2}e^{2}+c^{2}d^{4}\right)(ex+d)^{5}}$	91
default	$\frac{-\frac{2\left(dec\left(x+\frac{d}{e}\right)^{2}+\left(a\,e^{2}-c\,d^{2}\right)\left(x+\frac{d}{e}\right)\right)^{\frac{5}{2}}}{7\left(a\,e^{2}-c\,d^{2}\right)\left(x+\frac{d}{e}\right)^{6}}+\frac{4dec\left(dec\left(x+\frac{d}{e}\right)^{2}+\left(a\,e^{2}-c\,d^{2}\right)\left(x+\frac{d}{e}\right)\right)^{\frac{5}{2}}}{35\left(a\,e^{2}-c\,d^{2}\right)^{2}\left(x+\frac{d}{e}\right)^{5}}}{e^{6}}$	131
trager	$-\frac{2\left(-2c^{3}d^{3}e\,x^{3}+a\,c^{2}d^{2}e^{2}x^{2}-7c^{3}d^{4}x^{2}+8a^{2}cd\,e^{3}x-14a\,c^{2}d^{3}ex+5e^{4}a^{3}-7d^{2}e^{2}a^{2}c\right)\sqrt{cd\,x^{2}e+a\,e^{2}x+c\,d^{2}x+ade}}{35\left(a^{2}e^{4}-2ac\,d^{2}e^{2}+c^{2}d^{4}\right)(ex+d)^{4}}$	143

input
```
int((a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(3/2)/(e*x+d)^6,x,method=_RETURNVERB
OSE)
```

output
```
-2/35*(c*d*x+a*e)*(-2*c*d*e*x+5*a*e^2-7*c*d^2)*(c*d*e*x^2+a*e^2*x+c*d^2*x+
a*d*e)^(3/2)/(e*x+d)^5/(a^2*e^4-2*a*c*d^2*e^2+c^2*d^4)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 275 vs. $2(103) = 206$.

Time = 1.77 (sec) , antiderivative size = 275, normalized size of antiderivative = 2.48

$$\int \frac{(ade + (cd^2 + ae^2)\,x + cdex^2)^{3/2}}{(d+ex)^6}\,dx = \frac{2\,(2\,c^3d^3ex^3 + 7\,a^2cd^2e^2 - 5\,a^3e^4 + (7\,c^3}{35\,(c^2d^8 - 2\,acd^6e^2 + a^2d^4e^4 + (c^2d^4e^4 - 2\,acd^2e^6 + a^2e^8)x^4 + 4\,(}$$

input
```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/(e*x+d)^6,x, algorithm="
fricas")
```

output
```
2/35*(2*c^3*d^3*e*x^3 + 7*a^2*c*d^2*e^2 - 5*a^3*e^4 + (7*c^3*d^4 - a*c^2*d
^2*e^2)*x^2 + 2*(7*a*c^2*d^3*e - 4*a^2*c*d*e^3)*x)*sqrt(c*d*e*x^2 + a*d*e
+ (c*d^2 + a*e^2)*x)/(c^2*d^8 - 2*a*c*d^6*e^2 + a^2*d^4*e^4 + (c^2*d^4*e^4
 - 2*a*c*d^2*e^6 + a^2*e^8)*x^4 + 4*(c^2*d^5*e^3 - 2*a*c*d^3*e^5 + a^2*d*e
^7)*x^3 + 6*(c^2*d^6*e^2 - 2*a*c*d^4*e^4 + a^2*d^2*e^6)*x^2 + 4*(c^2*d^7*e
 - 2*a*c*d^5*e^3 + a^2*d^3*e^5)*x)
```

Sympy [F]

$$\int \frac{(ade + (cd^2 + ae^2)\,x + cdex^2)^{3/2}}{(d+ex)^6}\,dx = \int \frac{((d+ex)\,(ae+cdx))^{\frac{3}{2}}}{(d+ex)^6}\,dx$$

input
```
integrate((a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(3/2)/(e*x+d)**6,x)
```

output
```
Integral(((d + e*x)*(a*e + c*d*x))**(3/2)/(d + e*x)**6, x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{(d+ex)^6}\,dx=\text{Exception raised: ValueError}$$

input
```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/(e*x+d)^6,x, algorithm="
maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e*(a*e^2-c*d^2)>0)', see `assume
?` for mor
```

Giac [F(-2)]

Exception generated.

$$\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{(d+ex)^6}\,dx=\text{Exception raised: TypeError}$$

input
```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/(e*x+d)^6,x, algorithm="
giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Unable to divide, perhaps due to ro
unding error%%%{%%%{1,[0,0,4]%%%},[8]%%%}+%%%{%%{[%%%{-8,[0,1,3]%%%},0]:[1
,0,%%%{-1
```

Mupad [B] (verification not implemented)

Time = 6.79 (sec) , antiderivative size = 1477, normalized size of antiderivative = 13.31

$$\int \frac{(ade+(cd^2+ae^2)\,x+cdex^2)^{3/2}}{(d+ex)^6}\,dx = \text{Too large to display}$$

input
```
int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(3/2)/(d + e*x)^6,x)
```

output
```
(((d*((4*c^3*d^4)/(7*(a*e^2 - c*d^2)*(5*a*e^3 - 5*c*d^2*e)) - (2*c^2*d^2*(
5*a*e^2 - c*d^2))/(7*(a*e^2 - c*d^2)*(5*a*e^3 - 5*c*d^2*e))))/e + (2*a*c^2
*d^3*e^2 - 2*c^3*d^5 + 4*a^2*c*d*e^4)/(7*e*(a*e^2 - c*d^2)*(5*a*e^3 - 5*c*
d^2*e)))*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2))/(d + e*x)^3 - (((1
4*c^3*d^4 - 34*a*c^2*d^2*e^2)/(35*e*(a*e^2 - c*d^2)*(3*a*e^3 - 3*c*d^2*e))
 + (4*c^3*d^4)/(7*e*(a*e^2 - c*d^2)*(3*a*e^3 - 3*c*d^2*e)))*(x*(a*e^2 + c*
d^2) + a*d*e + c*d*e*x^2)^(1/2))/(d + e*x)^2 - (((d*((56*c^4*d^5 - 72*a*c^
3*d^3*e^2)/(35*(a*e^2 - c*d^2)^2*(3*a*e^3 - 3*c*d^2*e)) + (8*c^4*d^5)/(35*
(a*e^2 - c*d^2)^2*(3*a*e^3 - 3*c*d^2*e))))/e + (8*a*c^2*d^2*e*(8*a*e^2 - 7
*c*d^2))/(35*(a*e^2 - c*d^2)^2*(3*a*e^3 - 3*c*d^2*e)))*(x*(a*e^2 + c*d^2)
+ a*d*e + c*d*e*x^2)^(1/2))/(d + e*x)^2 + (((d*((16*c^5*d^6)/(105*e*(a*e^2
 - c*d^2)^4) - (8*c^4*d^4*(19*a*e^2 - 15*c*d^2))/(105*e*(a*e^2 - c*d^2)^4)
))/e + (8*c^3*d^3*(9*a^2*e^4 - 8*c^2*d^4 + a*c*d^2*e^2))/(105*e^2*(a*e^2 -
 c*d^2)^4))*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2))/(d + e*x) - (((
2*a^2*e^3)/(7*a*e^3 - 7*c*d^2*e) + (d*((2*c^2*d^3)/(7*a*e^3 - 7*c*d^2*e) -
 (4*a*c*d*e^2)/(7*a*e^3 - 7*c*d^2*e)))/e)*(x*(a*e^2 + c*d^2) + a*d*e + c*d
*e*x^2)^(1/2))/(d + e*x)^4 - (((28*c^4*d^5 - 36*a*c^3*d^3*e^2)/(35*e^2*(a*
e^2 - c*d^2)^3) + (8*c^4*d^5)/(35*e^2*(a*e^2 - c*d^2)^3))*(x*(a*e^2 + c*d^
2) + a*d*e + c*d*e*x^2)^(1/2))/(d + e*x) - (((d*((16*c^5*d^6)/(105*e*(a*e^
2 - c*d^2)^4) - (16*c^4*d^4*(11*a*e^2 - 9*c*d^2))/(105*e*(a*e^2 - c*d^2...
```

Reduce [B] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 467, normalized size of antiderivative = 4.21

$$\int \frac{(ade+(cd^2+ae^2)\,x+cdex^2)^{3/2}}{(d+ex)^6}\,dx = \frac{-\frac{2\sqrt{ex+d}\,\sqrt{cdx+ae}\,a^3e^7}{7}+\frac{2\sqrt{ex+d}\,\sqrt{cdx+ae}\,a^2c\,d^2e^5}{5}-\frac{16\sqrt{ex+d}\,\sqrt{cdx+ae}\,a^2cd}{35}}{e^3\,(a^2e^8x^4-2ac\,d^2e^6x^4+}$$

input
```
int((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/(e*x+d)^6,x)
```

output
```
(2*( - 5*sqrt(d + e*x)*sqrt(a*e + c*d*x)*a**3*e**7 + 7*sqrt(d + e*x)*sqrt(
a*e + c*d*x)*a**2*c*d**2*e**5 - 8*sqrt(d + e*x)*sqrt(a*e + c*d*x)*a**2*c*d
*e**6*x + 14*sqrt(d + e*x)*sqrt(a*e + c*d*x)*a*c**2*d**3*e**4*x - sqrt(d +
 e*x)*sqrt(a*e + c*d*x)*a*c**2*d**2*e**5*x**2 + 7*sqrt(d + e*x)*sqrt(a*e +
 c*d*x)*c**3*d**4*e**3*x**2 + 2*sqrt(d + e*x)*sqrt(a*e + c*d*x)*c**3*d**3*
e**4*x**3 - 2*sqrt(e)*sqrt(d)*sqrt(c)*c**3*d**7 - 8*sqrt(e)*sqrt(d)*sqrt(c
)*c**3*d**6*e*x - 12*sqrt(e)*sqrt(d)*sqrt(c)*c**3*d**5*e**2*x**2 - 8*sqrt(
e)*sqrt(d)*sqrt(c)*c**3*d**4*e**3*x**3 - 2*sqrt(e)*sqrt(d)*sqrt(c)*c**3*d*
*3*e**4*x**4))/(35*e**3*(a**2*d**4*e**4 + 4*a**2*d**3*e**5*x + 6*a**2*d**2
*e**6*x**2 + 4*a**2*d*e**7*x**3 + a**2*e**8*x**4 - 2*a*c*d**6*e**2 - 8*a*c
*d**5*e**3*x - 12*a*c*d**4*e**4*x**2 - 8*a*c*d**3*e**5*x**3 - 2*a*c*d**2*e
**6*x**4 + c**2*d**8 + 4*c**2*d**7*e*x + 6*c**2*d**6*e**2*x**2 + 4*c**2*d*
*5*e**3*x**3 + c**2*d**4*e**4*x**4))
```

3.219 $\int \frac{(ade+(cd^2+ae^2)x+cdex^2)^{3/2}}{(d+ex)^7} dx$

Optimal result

Integrand size = 37, antiderivative size = 171

$$\int \frac{(ade+(cd^2+ae^2)x+cdex^2)^{3/2}}{(d+ex)^7} dx = \frac{2(ade+(cd^2+ae^2)x+cdex^2)^{5/2}}{9(cd^2-ae^2)(d+ex)^7} + \frac{8cd(ade+(cd^2+ae^2)x+cdex^2)^{5/2}}{63(cd^2-ae^2)^2(d+ex)^6} + \frac{16c^2d^2(ade+(cd^2+ae^2)x+cdex^2)^{5/2}}{315(cd^2-ae^2)^3(d+ex)^5}$$

output
```
2/9*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(-a*e^2+c*d^2)/(e*x+d)^7+8/63*
c*d*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(-a*e^2+c*d^2)^2/(e*x+d)^6+16/
315*c^2*d^2*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(-a*e^2+c*d^2)^3/(e*x+
d)^5
```

Mathematica [A] (verified)

Time = 0.50 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.55

$$\int \frac{(ade+(cd^2+ae^2)x+cdex^2)^{3/2}}{(d+ex)^7} dx = \frac{2((ae+cdx)(d+ex))^{5/2}(35a^2e^4-10acde^2(9d+2ex)+c^2d^2(63}{315(cd^2-ae^2)^3(d+ex)^7}$$

input
```
Integrate[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2)/(d + e*x)^7,x]
```

output
```
(2*((a*e + c*d*x)*(d + e*x))^(5/2)*(35*a^2*e^4 - 10*a*c*d*e^2*(9*d + 2*e*x
) + c^2*d^2*(63*d^2 + 36*d*e*x + 8*e^2*x^2)))/(315*(c*d^2 - a*e^2)^3*(d +
e*x)^7)
```

Rubi [A] (verified)

Time = 0.54 (sec) , antiderivative size = 186, normalized size of antiderivative = 1.09, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.081$, Rules used = {1129, 1129, 1123}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{(d+ex)^7}dx$$

$$\Big\downarrow \text{1129}$$

$$\frac{4cd\int \frac{\left(cdex^2+\left(cd^2+ae^2\right)x+ade\right)^{3/2}}{(d+ex)^6}dx}{9\left(cd^2-ae^2\right)}+\frac{2\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/2}}{9(d+ex)^7\left(cd^2-ae^2\right)}$$

$$\Big\downarrow \text{1129}$$

$$\frac{4cd\left(\frac{2cd\int \frac{\left(cdex^2+\left(cd^2+ae^2\right)x+ade\right)^{3/2}}{(d+ex)^5}dx}{7\left(cd^2-ae^2\right)}+\frac{2\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/2}}{7(d+ex)^6\left(cd^2-ae^2\right)}\right)}{9\left(cd^2-ae^2\right)}+\frac{2\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/2}}{9(d+ex)^7\left(cd^2-ae^2\right)}$$

$$\Big\downarrow \text{1123}$$

$$\frac{2\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/2}}{9(d+ex)^7\left(cd^2-ae^2\right)}+\frac{4cd\left(\frac{4cd\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/2}}{35(d+ex)^5\left(cd^2-ae^2\right)^2}+\frac{2\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/2}}{7(d+ex)^6\left(cd^2-ae^2\right)}\right)}{9\left(cd^2-ae^2\right)}$$

input
```
Int[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2)/(d + e*x)^7,x]
```

output
```
(2*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2))/(9*(c*d^2 - a*e^2)*(d +
e*x)^7) + (4*c*d*((2*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2))/(7*(c*
d^2 - a*e^2)*(d + e*x)^6) + (4*c*d*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)
^(5/2))/(35*(c*d^2 - a*e^2)^2*(d + e*x)^5)))/(9*(c*d^2 - a*e^2))
```

Defintions of rubi rules used

rule 1123
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[e*(d + e*x)^m*((a + b*x + c*x^2)^(p + 1)/((p + 1)*(2*c*d - b
*e))), x] /; FreeQ[{a, b, c, d, e, m, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2,
0] && EqQ[m + 2*p + 2, 0]
```

rule 1129
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(-e)*(d + e*x)^m*((a + b*x + c*x^2)^(p + 1)/((m + p + 1)*(2*
c*d - b*e))), x] + Simp[c*(Simplify[m + 2*p + 2]/((m + p + 1)*(2*c*d - b*e)
))   Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d
, e, m, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && ILtQ[Simplify[m + 2*p +
2], 0]
```

Maple [A] (verified)

Time = 4.60 (sec) , antiderivative size = 146, normalized size of antiderivative = 0.85

method	result
gosper	$-\frac{2(cdx+ae)\left(8x^{2}c^{2}d^{2}e^{2}-20xacd\,e^{3}+36x\,c^{2}d^{3}e+35a^{2}e^{4}-90ac\,d^{2}e^{2}+63c^{2}d^{4}\right)\left(cd\,x^{2}e+a\,e^{2}x+c\,d^{2}x+ade\right)^{\frac{3}{2}}}{315(ex+d)^{6}\left(e^{6}a^{3}-3d^{2}e^{4}a^{2}c+3d^{4}e^{2}a\,c^{2}-d^{6}c^{3}\right)}$
orering	$-\frac{2\left(8x^{2}c^{2}d^{2}e^{2}-20xacd\,e^{3}+36x\,c^{2}d^{3}e+35a^{2}e^{4}-90ac\,d^{2}e^{2}+63c^{2}d^{4}\right)(cdx+ae)\left(ade+\left(a\,e^{2}+c\,d^{2}\right)x+cd\,x^{2}e\right)^{\frac{3}{2}}}{315\left(e^{6}a^{3}-3d^{2}e^{4}a^{2}c+3d^{4}e^{2}a\,c^{2}-d^{6}c^{3}\right)(ex+d)^{6}}$
default	$\frac{-\frac{2\left(dec\left(x+\frac{d}{e}\right)^{2}+\left(a\,e^{2}-c\,d^{2}\right)\left(x+\frac{d}{e}\right)\right)^{\frac{5}{2}}}{9\left(a\,e^{2}-c\,d^{2}\right)\left(x+\frac{d}{e}\right)^{7}}-\frac{4dec\left(-\frac{2\left(dec\left(x+\frac{d}{e}\right)^{2}+\left(a\,e^{2}-c\,d^{2}\right)\left(x+\frac{d}{e}\right)\right)^{\frac{5}{2}}}{7\left(a\,e^{2}-c\,d^{2}\right)\left(x+\frac{d}{e}\right)^{6}}+\frac{4dec\left(dec\left(x+\frac{d}{e}\right)^{2}+\left(a\,e^{2}-c\,d^{2}\right)\left(x+\frac{d}{e}\right)\right)^{\frac{5}{2}}}{35\left(a\,e^{2}-c\,d^{2}\right)^{2}\left(x+\frac{d}{e}\right)^{5}}\right)}{9\left(a\,e^{2}-c\,d^{2}\right)}}{e^{7}}$
trager	$-\frac{2\left(8c^{4}d^{4}e^{2}x^{4}-4a\,c^{3}d^{3}e^{3}x^{3}+36c^{4}d^{5}e\,x^{3}+3a^{2}c^{2}d^{2}e^{4}x^{2}-18a\,c^{3}d^{4}e^{2}x^{2}+63c^{4}d^{6}x^{2}+50d\,e^{5}c\,a^{3}x-144a^{2}c^{2}d^{3}e^{3}x+126a\,c^{3}d^{5}ex+35\right.}{315\left(e^{6}a^{3}-3d^{2}e^{4}a^{2}c+3d^{4}e^{2}a\,c^{2}-d^{6}c^{3}\right)(ex+d)^{5}}$

input
```
int((a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(3/2)/(e*x+d)^7,x,method=_RETURNVERB
OSE)
```

output
```
-2/315*(c*d*x+a*e)*(8*c^2*d^2*e^2*x^2-20*a*c*d*e^3*x+36*c^2*d^3*e*x+35*a^2
*e^4-90*a*c*d^2*e^2+63*c^2*d^4)*(c*d*e*x^2+a*e^2*x+c*d^2*x+a*d*e)^(3/2)/(e
*x+d)^6/(a^3*e^6-3*a^2*c*d^2*e^4+3*a*c^2*d^4*e^2-c^3*d^6)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 472 vs. $2(159) = 318$.

Time = 6.15 (sec) , antiderivative size = 472, normalized size of antiderivative = 2.76

$$\int \frac{(ade + (cd^2 + ae^2)x + cdex^2)^{3/2}}{(d + ex)^7}\,dx = \frac{2\,(8\,c^4d^4e^2x^4 + 63}{315\,(c^3d^{11} - 3\,ac^2d^9e^2 + 3\,a^2cd^7e^4 - a^3d^5e^6 + (c^3d^6e^5 - 3\,ac^2d^4e}$$

input
```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/(e*x+d)^7,x, algorithm="
fricas")
```

output
```
2/315*(8*c^4*d^4*e^2*x^4 + 63*a^2*c^2*d^4*e^2 - 90*a^3*c*d^2*e^4 + 35*a^4*
e^6 + 4*(9*c^4*d^5*e - a*c^3*d^3*e^3)*x^3 + 3*(21*c^4*d^6 - 6*a*c^3*d^4*e^
2 + a^2*c^2*d^2*e^4)*x^2 + 2*(63*a*c^3*d^5*e - 72*a^2*c^2*d^3*e^3 + 25*a^3
*c*d*e^5)*x)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)/(c^3*d^11 - 3*a*c
^2*d^9*e^2 + 3*a^2*c*d^7*e^4 - a^3*d^5*e^6 + (c^3*d^6*e^5 - 3*a*c^2*d^4*e^
7 + 3*a^2*c*d^2*e^9 - a^3*e^11)*x^5 + 5*(c^3*d^7*e^4 - 3*a*c^2*d^5*e^6 + 3
*a^2*c*d^3*e^8 - a^3*d*e^10)*x^4 + 10*(c^3*d^8*e^3 - 3*a*c^2*d^6*e^5 + 3*a
^2*c*d^4*e^7 - a^3*d^2*e^9)*x^3 + 10*(c^3*d^9*e^2 - 3*a*c^2*d^7*e^4 + 3*a^
2*c*d^5*e^6 - a^3*d^3*e^8)*x^2 + 5*(c^3*d^10*e - 3*a*c^2*d^8*e^3 + 3*a^2*c
*d^6*e^5 - a^3*d^4*e^7)*x)
```

Sympy [F]

$$\int \frac{(ade + (cd^2 + ae^2)x + cdex^2)^{3/2}}{(d+ex)^7} dx = \int \frac{((d+ex)(ae+cdx))^{\frac{3}{2}}}{(d+ex)^7} dx$$

input
```
integrate((a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(3/2)/(e*x+d)**7,x)
```

output
```
Integral(((d + e*x)*(a*e + c*d*x))**(3/2)/(d + e*x)**7, x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(ade + (cd^2 + ae^2)x + cdex^2)^{3/2}}{(d+ex)^7} dx = \text{Exception raised: ValueError}$$

input
```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/(e*x+d)^7,x, algorithm="
maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e*(a*e^2-c*d^2)>0)', see `assume
?` for mor
```

Giac [F(-2)]

Exception generated.

$$\int \frac{(ade + (cd^2 + ae^2)x + cdex^2)^{3/2}}{(d+ex)^7} dx = \text{Exception raised: TypeError}$$

input
```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/(e*x+d)^7,x, algorithm="
giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Unable to divide, perhaps due to ro
unding error%%%{%%%{1,[0,0,5]%%%},[10]%%%}+%%%{%%{[%%%{-10,[0,1,4]%%%},0]:
[1,0,%%%{
```

Mupad [B] (verification not implemented)

Time = 7.29 (sec) , antiderivative size = 2067, normalized size of antiderivative = 12.09

$$\int \frac{(ade+(cd^2+ae^2)x+cdex^2)^{3/2}}{(d+ex)^7}\,dx = \text{Too large to display}$$

input
```
int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(3/2)/(d + e*x)^7,x)
```

output
```
(((d*((4*c^3*d^4)/(9*(a*e^2 - c*d^2)*(7*a*e^3 - 7*c*d^2*e)) - (2*c^2*d^2*(
5*a*e^2 - c*d^2))/(9*(a*e^2 - c*d^2)*(7*a*e^3 - 7*c*d^2*e))))/e + (2*a*c^2
*d^3*e^2 - 2*c^3*d^5 + 4*a^2*c*d*e^4)/(9*e*(a*e^2 - c*d^2)*(7*a*e^3 - 7*c*
d^2*e)))*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2))/(d + e*x)^4 - (((1
48*c^4*d^5 - 188*a*c^3*d^3*e^2)/(315*e*(a*e^2 - c*d^2)^2*(3*a*e^3 - 3*c*d^
2*e)) + (8*c^4*d^5)/(63*e*(a*e^2 - c*d^2)^2*(3*a*e^3 - 3*c*d^2*e)))*(x*(a*
e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2))/(d + e*x)^2 - (((d*((16*c^5*d^6)/
(315*(a*e^2 - c*d^2)^3*(3*a*e^3 - 3*c*d^2*e)) - (32*c^4*d^4*(7*a*e^2 - 6*c
*d^2))/(315*(a*e^2 - c*d^2)^3*(3*a*e^3 - 3*c*d^2*e))))/e + (16*a*c^3*d^3*e
*(13*a*e^2 - 12*c*d^2))/(315*(a*e^2 - c*d^2)^3*(3*a*e^3 - 3*c*d^2*e)))*(x*
(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2))/(d + e*x)^2 - (((18*c^3*d^4 -
46*a*c^2*d^2*e^2)/(63*e*(a*e^2 - c*d^2)*(5*a*e^3 - 5*c*d^2*e)) + (4*c^3*d^
4)/(9*e*(a*e^2 - c*d^2)*(5*a*e^3 - 5*c*d^2*e)))*(x*(a*e^2 + c*d^2) + a*d*e
 + c*d*e*x^2)^(1/2))/(d + e*x)^3 - (((16*c^5*d^6)/(315*e^2*(a*e^2 - c*d^2)
^4) - (8*c^4*d^4*(47*a*e^2 - 41*c*d^2))/(945*e^2*(a*e^2 - c*d^2)^4))*(x*(a
*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2))/(d + e*x) - (((d*((72*c^4*d^5 -
88*a*c^3*d^3*e^2)/(63*(a*e^2 - c*d^2)^2*(5*a*e^3 - 5*c*d^2*e)) + (8*c^4*d^
5)/(63*(a*e^2 - c*d^2)^2*(5*a*e^3 - 5*c*d^2*e))))/e + (8*a*c^2*d^2*e*(10*a
*e^2 - 9*c*d^2))/(63*(a*e^2 - c*d^2)^2*(5*a*e^3 - 5*c*d^2*e)))*(x*(a*e^2 +
 c*d^2) + a*d*e + c*d*e*x^2)^(1/2))/(d + e*x)^3 + (((d*((32*c^6*d^7)/(9...
```

Reduce [B] (verification not implemented)

Time = 0.42 (sec) , antiderivative size = 777, normalized size of antiderivative = 4.54

$$\int \frac{(ade + (cd^2 + ae^2)x + cdex^2)^{3/2}}{(d+ex)^7} dx = \frac{-\frac{2\sqrt{ex+d}\sqrt{cdx+ae}\,a^4e^9}{9} + \frac{4\sqrt{ex+d}\sqrt{cdx+ae}\,a^3c\,d^2e^7}{7} - \frac{20\sqrt{ex+d}\sqrt{cdx+ae}\,a^3cd}{63}}{e^3\,(a^3e^{11}x^5 - 3a^2c\,d^2e^9x}$$

input
```
int((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/(e*x+d)^7,x)
```

output
```
(2*( - 35*sqrt(d + e*x)*sqrt(a*e + c*d*x)*a**4*e**9 + 90*sqrt(d + e*x)*sqr
t(a*e + c*d*x)*a**3*c*d**2*e**7 - 50*sqrt(d + e*x)*sqrt(a*e + c*d*x)*a**3*
c*d*e**8*x - 63*sqrt(d + e*x)*sqrt(a*e + c*d*x)*a**2*c**2*d**4*e**5 + 144*
sqrt(d + e*x)*sqrt(a*e + c*d*x)*a**2*c**2*d**3*e**6*x - 3*sqrt(d + e*x)*sq
rt(a*e + c*d*x)*a**2*c**2*d**2*e**7*x**2 - 126*sqrt(d + e*x)*sqrt(a*e + c*
d*x)*a*c**3*d**5*e**4*x + 18*sqrt(d + e*x)*sqrt(a*e + c*d*x)*a*c**3*d**4*e
**5*x**2 + 4*sqrt(d + e*x)*sqrt(a*e + c*d*x)*a*c**3*d**3*e**6*x**3 - 63*sq
rt(d + e*x)*sqrt(a*e + c*d*x)*c**4*d**6*e**3*x**2 - 36*sqrt(d + e*x)*sqrt(
a*e + c*d*x)*c**4*d**5*e**4*x**3 - 8*sqrt(d + e*x)*sqrt(a*e + c*d*x)*c**4*
d**4*e**5*x**4 + 8*sqrt(e)*sqrt(d)*sqrt(c)*c**4*d**9 + 40*sqrt(e)*sqrt(d)*
sqrt(c)*c**4*d**8*e*x + 80*sqrt(e)*sqrt(d)*sqrt(c)*c**4*d**7*e**2*x**2 + 8
0*sqrt(e)*sqrt(d)*sqrt(c)*c**4*d**6*e**3*x**3 + 40*sqrt(e)*sqrt(d)*sqrt(c)
*c**4*d**5*e**4*x**4 + 8*sqrt(e)*sqrt(d)*sqrt(c)*c**4*d**4*e**5*x**5))/(31
5*e**3*(a**3*d**5*e**6 + 5*a**3*d**4*e**7*x + 10*a**3*d**3*e**8*x**2 + 10*
a**3*d**2*e**9*x**3 + 5*a**3*d*e**10*x**4 + a**3*e**11*x**5 - 3*a**2*c*d**
7*e**4 - 15*a**2*c*d**6*e**5*x - 30*a**2*c*d**5*e**6*x**2 - 30*a**2*c*d**4
*e**7*x**3 - 15*a**2*c*d**3*e**8*x**4 - 3*a**2*c*d**2*e**9*x**5 + 3*a*c**2
*d**9*e**2 + 15*a*c**2*d**8*e**3*x + 30*a*c**2*d**7*e**4*x**2 + 30*a*c**2*
d**6*e**5*x**3 + 15*a*c**2*d**5*e**6*x**4 + 3*a*c**2*d**4*e**7*x**5 - c**3
*d**11 - 5*c**3*d**10*e*x - 10*c**3*d**9*e**2*x**2 - 10*c**3*d**8*e**3*...
```

3.220 $\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{(d+ex)^8}\,dx$

Optimal result

Integrand size = 37, antiderivative size = 231

$$\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{(d+ex)^8}\,dx=\frac{2\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{11\left(cd^2-ae^2\right)(d+ex)^8}+\frac{4cd\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{33\left(cd^2-ae^2\right)^2(d+ex)^7}+\frac{16c^2d^2\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{231\left(cd^2-ae^2\right)^3(d+ex)^6}+\frac{32c^3d^3\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{1155\left(cd^2-ae^2\right)^4(d+ex)^5}$$

output
```
2/11*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(-a*e^2+c*d^2)/(e*x+d)^8+4/33
*c*d*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(-a*e^2+c*d^2)^2/(e*x+d)^7+16
/231*c^2*d^2*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(-a*e^2+c*d^2)^3/(e*x
+d)^6+32/1155*c^3*d^3*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(-a*e^2+c*d^
2)^4/(e*x+d)^5
```

Mathematica [A] (verified)

Time = 1.44 (sec) , antiderivative size = 138, normalized size of antiderivative = 0.60

$$\int \frac{(ade + (cd^2 + ae^2)\,x + cdex^2)^{3/2}}{(d+ex)^8}\,dx = \frac{2((ae+cdx)(d+ex))^{5/2}\left(-105a^3e^6 + 35a^2cde^4(11d+2ex) - 5a\right.}{1155\,(c}$$

input
```
Integrate[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2)/(d + e*x)^8,x]
```

output
```
(2*((a*e + c*d*x)*(d + e*x))^(5/2)*(-105*a^3*e^6 + 35*a^2*c*d*e^4*(11*d +
2*e*x) - 5*a*c^2*d^2*e^2*(99*d^2 + 44*d*e*x + 8*e^2*x^2) + c^3*d^3*(231*d^
3 + 198*d^2*e*x + 88*d*e^2*x^2 + 16*e^3*x^3)))/(1155*(c*d^2 - a*e^2)^4*(d
+ e*x)^8)
```

Rubi [A] (verified)

Time = 0.66 (sec) , antiderivative size = 261, normalized size of antiderivative = 1.13, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.108$, Rules used = {1129, 1129, 1129, 1123}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{(d+ex)^8}dx$$

$$\Big\downarrow \text{1129}$$

$$\frac{6cd\int \frac{\left(cdex^2+\left(cd^2+ae^2\right)x+ade\right)^{3/2}}{(d+ex)^7}dx}{11\left(cd^2-ae^2\right)}+\frac{2\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/2}}{11(d+ex)^8\left(cd^2-ae^2\right)}$$

$$\Big\downarrow \text{1129}$$

$$\frac{6cd\left(\frac{4cd\int \frac{\left(cdex^2+\left(cd^2+ae^2\right)x+ade\right)^{3/2}}{(d+ex)^6}dx}{9\left(cd^2-ae^2\right)}+\frac{2\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/2}}{9(d+ex)^7\left(cd^2-ae^2\right)}\right)}{11\left(cd^2-ae^2\right)}+\frac{2\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/2}}{11(d+ex)^8\left(cd^2-ae^2\right)}$$

$$\Big\downarrow 1129$$

$$\frac{6cd\left(\frac{4cd\left(\frac{2cd\int\frac{\left(cdex^2+\left(cd^2+ae^2\right)x+ade\right)^{3/2}}{(d+ex)^5}dx}{7\left(cd^2-ae^2\right)}+\frac{2\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/2}}{7(d+ex)^6\left(cd^2-ae^2\right)}\right)}{9(cd^2-ae^2)}+\frac{2\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/2}}{9(d+ex)^7(cd^2-ae^2)}\right)}{11\left(cd^2-ae^2\right)}+\frac{2\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/2}}{11(d+ex)^8\left(cd^2-ae^2\right)}$$

$$\Big\downarrow 1123$$

$$\frac{2\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/2}}{11(d+ex)^8\left(cd^2-ae^2\right)}+\frac{6cd\left(\frac{2\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/2}}{9(d+ex)^7(cd^2-ae^2)}+\frac{4cd\left(\frac{4cd\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/2}}{35(d+ex)^5\left(cd^2-ae^2\right)^2}+\frac{2\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/2}}{7(d+ex)^6\left(cd^2-ae^2\right)}\right)}{9(cd^2-ae^2)}\right)}{11\left(cd^2-ae^2\right)}$$

input
```
Int[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2)/(d + e*x)^8,x]
```

output
```
(2*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2))/(11*(c*d^2 - a*e^2)*(d +
 e*x)^8) + (6*c*d*((2*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2))/(9*(c
*d^2 - a*e^2)*(d + e*x)^7) + (4*c*d*((2*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e
*x^2)^(5/2))/(7*(c*d^2 - a*e^2)*(d + e*x)^6) + (4*c*d*(a*d*e + (c*d^2 + a*
e^2)*x + c*d*e*x^2)^(5/2))/(35*(c*d^2 - a*e^2)^2*(d + e*x)^5)))/(9*(c*d^2
- a*e^2))))/(11*(c*d^2 - a*e^2))
```

Defintions of rubi rules used

rule 1123
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[e*(d + e*x)^m*((a + b*x + c*x^2)^(p + 1)/((p + 1)*(2*c*d - b
*e))), x] /; FreeQ[{a, b, c, d, e, m, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2,
0] && EqQ[m + 2*p + 2, 0]
```

rule 1129
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(-e)*(d + e*x)^m*((a + b*x + c*x^2)^(p + 1)/((m + p + 1)*(2*
c*d - b*e))), x] + Simp[c*(Simplify[m + 2*p + 2]/((m + p + 1)*(2*c*d - b*e)
))   Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d
, e, m, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && ILtQ[Simplify[m + 2*p +
2], 0]
```

Maple [A] (verified)

Time = 5.60 (sec) , antiderivative size = 217, normalized size of antiderivative = 0.94

method	result
gosper	$-\frac{2(cdx+ae)\left(-16c^3d^3e^3x^3+40x^2a\,c^2d^2e^4-88c^3d^4e^2x^2-70x\,a^2cd\,e^5+220xa\,c^2d^3e^3-198c^3d^5ex+105e^6a^3-385d^2e^4a^2c+495d^4e^2\cdots\right.}{1155(ex+d)^7\left(a^4e^8-4a^3c\,d^2e^6+6a^2c^2d^4e^4-4a\,c^3d^6e^2+c^4d^8\right)}$
orering	$-\frac{2\left(-16c^3d^3e^3x^3+40x^2a\,c^2d^2e^4-88c^3d^4e^2x^2-70x\,a^2cd\,e^5+220xa\,c^2d^3e^3-198c^3d^5ex+105e^6a^3-385d^2e^4a^2c+495d^4e^2a\,c^2-231\cdots\right.}{1155\left(a^4e^8-4a^3c\,d^2e^6+6a^2c^2d^4e^4-4a\,c^3d^6e^2+c^4d^8\right)(ex+d)^7}$
default	$\frac{-\frac{2\left(dec\left(x+\frac{d}{e}\right)^2+\left(a\,e^2-c\,d^2\right)\left(x+\frac{d}{e}\right)\right)^{\frac{5}{2}}}{11\left(a\,e^2-c\,d^2\right)\left(x+\frac{d}{e}\right)^8}-\frac{6dec\left(-\frac{2\left(dec\left(x+\frac{d}{e}\right)^2+\left(a\,e^2-c\,d^2\right)\left(x+\frac{d}{e}\right)\right)^{\frac{5}{2}}}{9\left(a\,e^2-c\,d^2\right)\left(x+\frac{d}{e}\right)^7}-\frac{4dec\left(-\frac{2\left(dec\left(x+\frac{d}{e}\right)^2+\left(a\,e^2-c\,d^2\right)\left(x+\frac{d}{e}\right)\right)^{\frac{5}{2}}}{7\left(a\,e^2-c\,d^2\right)\left(x+\frac{d}{e}\right)^6}\cdots\right)}{9\left(a\,e^2-\cdots\right)}\right)}{11\left(a\,e^2-c\,d^2\right)}\cdots}{e^8}$
trager	$-\frac{2\left(-16c^5d^5e^3x^5+8a\,c^4d^4e^4x^4-88c^5d^6e^2x^4-6a^2c^3d^3e^5x^3+44a\,c^4d^5e^3x^3-198c^5d^7e\,x^3+5a^3c^2d^2e^6x^2-33a^2c^3d^4e^4x^2+99a\,c^4d^6\cdots\right.}{1155\left(a^4e^8-4a^3\cdots\right.}$

input
```
int((a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(3/2)/(e*x+d)^8,x,method=_RETURNVERB
OSE)
```

output
```
-2/1155*(c*d*x+a*e)*(-16*c^3*d^3*e^3*x^3+40*a*c^2*d^2*e^4*x^2-88*c^3*d^4*e
^2*x^2-70*a^2*c*d*e^5*x+220*a*c^2*d^3*e^3*x-198*c^3*d^5*e*x+105*a^3*e^6-38
5*a^2*c*d^2*e^4+495*a*c^2*d^4*e^2-231*c^3*d^6)*(c*d*e*x^2+a*e^2*x+c*d^2*x+
a*d*e)^(3/2)/(e*x+d)^7/(a^4*e^8-4*a^3*c*d^2*e^6+6*a^2*c^2*d^4*e^4-4*a*c^3*
d^6*e^2+c^4*d^8)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 699 vs. $2(215) = 430$.

Time = 15.09 (sec) , antiderivative size = 699, normalized size of antiderivative = 3.03

$$\int \frac{(ade + (cd^2 + ae^2)x + cdex^2)^{3/2}}{(d + ex)^8} dx = \frac{}{1155 (c^4d^{14} - 4ac^3d^{12}e^2 + 6a^2c^2d^{10}e^4 - 4a^3cd^8e^6 + a^4d^6e^8 + (c^4}$$

input
```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/(e*x+d)^8,x, algorithm="
fricas")
```

output
```
2/1155*(16*c^5*d^5*e^3*x^5 + 231*a^2*c^3*d^6*e^2 - 495*a^3*c^2*d^4*e^4 + 3
85*a^4*c*d^2*e^6 - 105*a^5*e^8 + 8*(11*c^5*d^6*e^2 - a*c^4*d^4*e^4)*x^4 +
2*(99*c^5*d^7*e - 22*a*c^4*d^5*e^3 + 3*a^2*c^3*d^3*e^5)*x^3 + (231*c^5*d^8
 - 99*a*c^4*d^6*e^2 + 33*a^2*c^3*d^4*e^4 - 5*a^3*c^2*d^2*e^6)*x^2 + 2*(231
*a*c^4*d^7*e - 396*a^2*c^3*d^5*e^3 + 275*a^3*c^2*d^3*e^5 - 70*a^4*c*d*e^7)
*x)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)/(c^4*d^14 - 4*a*c^3*d^12*e
^2 + 6*a^2*c^2*d^10*e^4 - 4*a^3*c*d^8*e^6 + a^4*d^6*e^8 + (c^4*d^8*e^6 - 4
*a*c^3*d^6*e^8 + 6*a^2*c^2*d^4*e^10 - 4*a^3*c*d^2*e^12 + a^4*e^14)*x^6 + 6
*(c^4*d^9*e^5 - 4*a*c^3*d^7*e^7 + 6*a^2*c^2*d^5*e^9 - 4*a^3*c*d^3*e^11 + a
^4*d*e^13)*x^5 + 15*(c^4*d^10*e^4 - 4*a*c^3*d^8*e^6 + 6*a^2*c^2*d^6*e^8 -
4*a^3*c*d^4*e^10 + a^4*d^2*e^12)*x^4 + 20*(c^4*d^11*e^3 - 4*a*c^3*d^9*e^5
+ 6*a^2*c^2*d^7*e^7 - 4*a^3*c*d^5*e^9 + a^4*d^3*e^11)*x^3 + 15*(c^4*d^12*e
^2 - 4*a*c^3*d^10*e^4 + 6*a^2*c^2*d^8*e^6 - 4*a^3*c*d^6*e^8 + a^4*d^4*e^10
)*x^2 + 6*(c^4*d^13*e - 4*a*c^3*d^11*e^3 + 6*a^2*c^2*d^9*e^5 - 4*a^3*c*d^7
*e^7 + a^4*d^5*e^9)*x)
```

Sympy [F]

$$\int \frac{(ade + (cd^2 + ae^2)x + cdex^2)^{3/2}}{(d + ex)^8}\,dx = \int \frac{((d + ex)(ae + cdx))^{\frac{3}{2}}}{(d + ex)^8}\,dx$$

input
```
integrate((a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(3/2)/(e*x+d)**8,x)
```

output
```
Integral(((d + e*x)*(a*e + c*d*x))**(3/2)/(d + e*x)**8, x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(ade + (cd^2 + ae^2)x + cdex^2)^{3/2}}{(d + ex)^8}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/(e*x+d)^8,x, algorithm="
maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e*(a*e^2-c*d^2)>0)', see `assume
?` for mor
```

Giac [F(-2)]

Exception generated.

$$\int \frac{(ade + (cd^2 + ae^2)x + cdex^2)^{3/2}}{(d + ex)^8}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/(e*x+d)^8,x, algorithm="
giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Unable to divide, perhaps due to ro
unding error%%%{%%%{1,[0,0,6]%%%},[12]%%%}+%%%{%%{[%%%{-12,[0,1,5]%%%},0]:
[1,0,%%%{
```

Mupad [B] (verification not implemented)

Time = 7.98 (sec) , antiderivative size = 2657, normalized size of antiderivative = 11.50

$$\int \frac{\left(ade+(cd^2+ae^2)\,x+cdex^2\right)^{3/2}}{(d+ex)^8}\,dx = \text{Too large to display}$$

input
```
int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(3/2)/(d + e*x)^8,x)
```

output
```
(((d*((4*c^3*d^4)/(11*(a*e^2 - c*d^2)*(9*a*e^3 - 9*c*d^2*e)) - (2*c^2*d^2*
(5*a*e^2 - c*d^2))/(11*(a*e^2 - c*d^2)*(9*a*e^3 - 9*c*d^2*e))))/e + (2*a*c
^2*d^3*e^2 - 2*c^3*d^5 + 4*a^2*c*d*e^4)/(11*e*(a*e^2 - c*d^2)*(9*a*e^3 - 9
*c*d^2*e)))*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2))/(d + e*x)^5 - (
((228*c^4*d^5 - 284*a*c^3*d^3*e^2)/(693*e*(a*e^2 - c*d^2)^2*(5*a*e^3 - 5*c
*d^2*e)) + (8*c^4*d^5)/(99*e*(a*e^2 - c*d^2)^2*(5*a*e^3 - 5*c*d^2*e)))*(x*
(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2))/(d + e*x)^3 - (((d*((16*c^5*d^
6)/(693*(a*e^2 - c*d^2)^3*(5*a*e^3 - 5*c*d^2*e)) - (16*c^4*d^4*(17*a*e^2 -
 15*c*d^2))/(693*(a*e^2 - c*d^2)^3*(5*a*e^3 - 5*c*d^2*e))))/e + (16*a*c^3*
d^3*e*(16*a*e^2 - 15*c*d^2))/(693*(a*e^2 - c*d^2)^3*(5*a*e^3 - 5*c*d^2*e))
)*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2))/(d + e*x)^3 - (((d*((32*c
^6*d^7)/(3465*(a*e^2 - c*d^2)^4*(3*a*e^3 - 3*c*d^2*e)) - (64*c^5*d^5*(10*a
*e^2 - 9*c*d^2))/(3465*(a*e^2 - c*d^2)^4*(3*a*e^3 - 3*c*d^2*e))))/e + (32*
a*c^4*d^4*e*(19*a*e^2 - 18*c*d^2))/(3465*(a*e^2 - c*d^2)^4*(3*a*e^3 - 3*c*
d^2*e)))*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2))/(d + e*x)^2 - (((2
2*c^3*d^4 - 58*a*c^2*d^2*e^2)/(99*e*(a*e^2 - c*d^2)*(7*a*e^3 - 7*c*d^2*e))
 + (4*c^3*d^4)/(11*e*(a*e^2 - c*d^2)*(7*a*e^3 - 7*c*d^2*e)))*(x*(a*e^2 + c
*d^2) + a*d*e + c*d*e*x^2)^(1/2))/(d + e*x)^4 - (((32*c^6*d^7)/(3465*e^2*(
a*e^2 - c*d^2)^5) - (16*c^5*d^5*(71*a*e^2 - 65*c*d^2))/(10395*e^2*(a*e^2 -
 c*d^2)^5))*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2))/(d + e*x) - ...
```

Reduce [B] (verification not implemented)

Time = 0.61 (sec) , antiderivative size = 1149, normalized size of antiderivative = 4.97

$$\int \frac{(ade+(cd^2+ae^2)x+cdex^2)^{3/2}}{(d+ex)^8}\,dx = \text{Too large to display}$$

input
```
int((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/(e*x+d)^8,x)
```

output
```
(2*( - 105*sqrt(d + e*x)*sqrt(a*e + c*d*x)*a**5*e**11 + 385*sqrt(d + e*x)*
sqrt(a*e + c*d*x)*a**4*c*d**2*e**9 - 140*sqrt(d + e*x)*sqrt(a*e + c*d*x)*a
**4*c*d*e**10*x - 495*sqrt(d + e*x)*sqrt(a*e + c*d*x)*a**3*c**2*d**4*e**7
+ 550*sqrt(d + e*x)*sqrt(a*e + c*d*x)*a**3*c**2*d**3*e**8*x - 5*sqrt(d + e
*x)*sqrt(a*e + c*d*x)*a**3*c**2*d**2*e**9*x**2 + 231*sqrt(d + e*x)*sqrt(a*
e + c*d*x)*a**2*c**3*d**6*e**5 - 792*sqrt(d + e*x)*sqrt(a*e + c*d*x)*a**2*
c**3*d**5*e**6*x + 33*sqrt(d + e*x)*sqrt(a*e + c*d*x)*a**2*c**3*d**4*e**7*
x**2 + 6*sqrt(d + e*x)*sqrt(a*e + c*d*x)*a**2*c**3*d**3*e**8*x**3 + 462*sq
rt(d + e*x)*sqrt(a*e + c*d*x)*a*c**4*d**7*e**4*x - 99*sqrt(d + e*x)*sqrt(a
*e + c*d*x)*a*c**4*d**6*e**5*x**2 - 44*sqrt(d + e*x)*sqrt(a*e + c*d*x)*a*c
**4*d**5*e**6*x**3 - 8*sqrt(d + e*x)*sqrt(a*e + c*d*x)*a*c**4*d**4*e**7*x*
*4 + 231*sqrt(d + e*x)*sqrt(a*e + c*d*x)*c**5*d**8*e**3*x**2 + 198*sqrt(d
+ e*x)*sqrt(a*e + c*d*x)*c**5*d**7*e**4*x**3 + 88*sqrt(d + e*x)*sqrt(a*e +
 c*d*x)*c**5*d**6*e**5*x**4 + 16*sqrt(d + e*x)*sqrt(a*e + c*d*x)*c**5*d**5
*e**6*x**5 - 16*sqrt(e)*sqrt(d)*sqrt(c)*c**5*d**11 - 96*sqrt(e)*sqrt(d)*sq
rt(c)*c**5*d**10*e*x - 240*sqrt(e)*sqrt(d)*sqrt(c)*c**5*d**9*e**2*x**2 - 3
20*sqrt(e)*sqrt(d)*sqrt(c)*c**5*d**8*e**3*x**3 - 240*sqrt(e)*sqrt(d)*sqrt(
c)*c**5*d**7*e**4*x**4 - 96*sqrt(e)*sqrt(d)*sqrt(c)*c**5*d**6*e**5*x**5 -
16*sqrt(e)*sqrt(d)*sqrt(c)*c**5*d**5*e**6*x**6))/(1155*e**3*(a**4*d**6*e**
8 + 6*a**4*d**5*e**9*x + 15*a**4*d**4*e**10*x**2 + 20*a**4*d**3*e**11*x...
```

3.221 $\int (d+ex)^2 \left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2} dx$

Optimal result

Integrand size = 37, antiderivative size = 544

$$\begin{aligned}
&\int (d+ex)^2 \left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2} dx = \frac{45\left(cd^2-ae^2\right)^7\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{16384c^5d^5e^3} \\
&-\frac{15\left(cd^2-ae^2\right)^6\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{8192c^5d^5e^2(d+ex)} \\
&+\frac{3\left(cd^2-ae^2\right)^5\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{2048c^5d^5e(d+ex)^2} \\
&+\frac{9\left(cd^2-ae^2\right)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{7/2}}{112c^2d^2} \\
&+\frac{9\left(cd^2-ae^2\right)^4\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{7/2}}{1024c^5d^5(d+ex)^3} \\
&+\frac{3\left(cd^2-ae^2\right)^3\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{7/2}}{128c^4d^4(d+ex)^2} \\
&+\frac{3\left(cd^2-ae^2\right)^2\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{7/2}}{64c^3d^3(d+ex)} \\
&+\frac{(d+ex)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{7/2}}{8cd} \\
&-\frac{45\left(cd^2-ae^2\right)^8\operatorname{arctanh}\left(\frac{\sqrt{e}\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{\sqrt{c}\sqrt{d}(d+ex)}\right)}{16384c^{11/2}d^{11/2}e^{7/2}}
\end{aligned}$$

output
```
45/16384*(-a*e^2+c*d^2)^7*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/c^5/d^5/
e^3-15/8192*(-a*e^2+c*d^2)^6*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/c^5/d
^5/e^2/(e*x+d)+3/2048*(-a*e^2+c*d^2)^5*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(
5/2)/c^5/d^5/e/(e*x+d)^2+9/112*(-a*e^2+c*d^2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e
*x^2)^(7/2)/c^2/d^2+9/1024*(-a*e^2+c*d^2)^4*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x
^2)^(7/2)/c^5/d^5/(e*x+d)^3+3/128*(-a*e^2+c*d^2)^3*(a*d*e+(a*e^2+c*d^2)*x+
c*d*e*x^2)^(7/2)/c^4/d^4/(e*x+d)^2+3/64*(-a*e^2+c*d^2)^2*(a*d*e+(a*e^2+c*d
^2)*x+c*d*e*x^2)^(7/2)/c^3/d^3/(e*x+d)+1/8*(e*x+d)*(a*d*e+(a*e^2+c*d^2)*x+
c*d*e*x^2)^(7/2)/c/d-45/16384*(-a*e^2+c*d^2)^8*arctanh(e^(1/2)*(a*d*e+(a*e
^2+c*d^2)*x+c*d*e*x^2)^(1/2)/c^(1/2)/d^(1/2)/(e*x+d))/c^(11/2)/d^(11/2)/e^
(7/2)
```

Mathematica [A] (verified)

Time = 1.57 (sec) , antiderivative size = 526, normalized size of antiderivative = 0.97

$$\int (d+ex)^2 \left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}\,dx = \frac{((ae+cdx)(d+ex))^{5/2}\left(\frac{\sqrt{c}\sqrt{d}\sqrt{e}(315a^7e^{14}-105a^6cde^{12}(23d+2ex)+21a^5c^2d^2e^{10}(383d^2+76dex+8e^2x^2)-3a^4\cdots}{\cdots}\right.}{\cdots}$$

input
```
Integrate[(d + e*x)^2*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2),x]
```

output
```
(((a*e + c*d*x)*(d + e*x))^(5/2)*((Sqrt[c]*Sqrt[d]*Sqrt[e]*(315*a^7*e^14 -
 105*a^6*c*d*e^12*(23*d + 2*e*x) + 21*a^5*c^2*d^2*e^10*(383*d^2 + 76*d*e*x
 + 8*e^2*x^2) - 3*a^4*c^3*d^3*e^8*(5053*d^3 + 1754*d^2*e*x + 424*d*e^2*x^2
 + 48*e^3*x^3) + a^3*c^4*d^4*e^6*(17609*d^4 + 9800*d^3*e*x + 4176*d^2*e^2*
x^2 + 1088*d*e^3*x^3 + 128*e^4*x^4) + 3*a^2*c^5*d^5*e^4*(2681*d^5 + 31014*
d^4*e*x + 66928*d^3*e^2*x^2 + 68320*d^2*e^3*x^3 + 34432*d*e^4*x^4 + 6912*e
^5*x^5) + 3*a*c^6*d^6*e^2*(-805*d^6 + 532*d^5*e*x + 32344*d^4*e^2*x^2 + 87
744*d^3*e^3*x^3 + 99968*d^2*e^4*x^4 + 53760*d*e^5*x^5 + 11264*e^6*x^6) + c
^7*d^7*(315*d^7 - 210*d^6*e*x + 168*d^5*e^2*x^2 + 32624*d^4*e^3*x^3 + 9843
2*d^3*e^4*x^4 + 119040*d^2*e^5*x^5 + 66560*d*e^6*x^6 + 14336*e^7*x^7)))/((
a*e + c*d*x)^2*(d + e*x)^2) - (315*(c*d^2 - a*e^2)^8*ArcTanh[(Sqrt[c]*Sqrt
[d]*Sqrt[d + e*x])/(Sqrt[e]*Sqrt[a*e + c*d*x])])/((a*e + c*d*x)^(5/2)*(d +
 e*x)^(5/2))))/(114688*c^(11/2)*d^(11/2)*e^(7/2))
```

Rubi [A] (verified)

Time = 0.97 (sec) , antiderivative size = 458, normalized size of antiderivative = 0.84, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.189, Rules used = {1134, 1160, 1087, 1087, 1087, 1092, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (d+ex)^2 \left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/2} dx$$

$$\Big\downarrow \text{1134}$$

$$\frac{9\left(d^2-\frac{ae^2}{c}\right)\int (d+ex)\left(cdex^2+\left(cd^2+ae^2\right)x+ade\right)^{5/2}dx}{16d}+\frac{(d+ex)\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{7/2}}{8cd}$$

$$\Big\downarrow \text{1160}$$

$$\frac{9\left(d^2-\frac{ae^2}{c}\right)\left(\frac{\left(d^2-\frac{ae^2}{c}\right)\int \left(cdex^2+\left(cd^2+ae^2\right)x+ade\right)^{5/2}dx}{2d}+\frac{\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{7/2}}{7cd}\right)}{16d}+\frac{(d+ex)\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{7/2}}{8cd}$$

$$\Big\downarrow \text{1087}$$

$$\frac{9\left(d^2-\frac{ae^2}{c}\right)\left(\frac{\left(d^2-\frac{ae^2}{c}\right)\left(\frac{\left(ae^2+cd^2+2cdex\right)\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/2}}{12cde}-\frac{5\left(cd^2-ae^2\right)^2\int \left(cdex^2+\left(cd^2+ae^2\right)x+ade\right)^{3/2}dx}{24cde}\right)}{2d}+\frac{\left(x\left(ae^2+cd^2\right)\right.}{}\right.}{16d}$$

$$\frac{(d+ex)\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{7/2}}{8cd}$$

$$\Big\downarrow \text{1087}$$

$$9\left(d^2-\frac{ae^2}{c}\right)\left(\frac{\left(d^2-\frac{ae^2}{c}\right)\left(\frac{\left(ae^2+cd^2+2cdex\right)\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/2}}{12cde}-\frac{5\left(cd^2-ae^2\right)^2\left(\frac{\left(ae^2+cd^2+2cdex\right)\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{8cde}-\frac{3}{}\right.}{24cde}\right.}{2d}\right.}{16d}$$

$$\frac{(d+ex)\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{7/2}}{8cd}$$

$\downarrow$ 1087

$$9\left(d^2-\frac{ae^2}{c}\right)\left(\frac{\left(d^2-\frac{ae^2}{c}\right)\left(\frac{\left(ae^2+cd^2+2cdex\right)\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/2}}{12cde}-\frac{5\left(cd^2-ae^2\right)^2\left(\frac{\left(ae^2+cd^2+2cdex\right)\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{8cde}-\frac{3}{}\right.}{}\right.}{2d}\right.}{16}$$

$$\frac{(d+ex)\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{7/2}}{8cd}$$

$\downarrow$ 1092

$$9\left(d^2-\frac{ae^2}{c}\right)\left(\frac{\left(d^2-\frac{ae^2}{c}\right)\left(\frac{\left(ae^2+cd^2+2cdex\right)\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/2}}{12cde}-\frac{5\left(cd^2-ae^2\right)^2\left(\frac{\left(ae^2+cd^2+2cdex\right)\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{8cde}-\cdots\right.}{\cdots}\right.}{\cdots}\right.$$

$$\frac{(d+ex)\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{7/2}}{8cd}$$

$\downarrow$ 219

$$3$$

$$5\left(cd^2-ae^2\right)^2\left(\frac{\left(ae^2+cd^2+2cdex\right)\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{8cde}-\right.$$

$$\left(d^2-\frac{ae^2}{c}\right)\left(\frac{\left(ae^2+cd^2+2cdex\right)\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/2}}{12cde}-\right.$$

$$9\left(d^2-\frac{ae^2}{c}\right)\qquad 2a$$

$$\frac{(d+ex)\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{7/2}}{8cd}$$

input
```
Int[(d + e*x)^2*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2),x]
```

output
```
((d + e*x)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(7/2))/(8*c*d) + (9*(d^
2 - (a*e^2)/c)*((a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(7/2)/(7*c*d) + ((
d^2 - (a*e^2)/c)*(((c*d^2 + a*e^2 + 2*c*d*e*x)*(a*d*e + (c*d^2 + a*e^2)*x
+ c*d*e*x^2)^(5/2))/(12*c*d*e) - (5*(c*d^2 - a*e^2)^2*(((c*d^2 + a*e^2 + 2
*c*d*e*x)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2))/(8*c*d*e) - (3*(c
*d^2 - a*e^2)^2*(((c*d^2 + a*e^2 + 2*c*d*e*x)*Sqrt[a*d*e + (c*d^2 + a*e^2)
*x + c*d*e*x^2])/(4*c*d*e) - ((c*d^2 - a*e^2)^2*ArcTanh[(c*d^2 + a*e^2 + 2
*c*d*e*x)/(2*Sqrt[c]*Sqrt[d]*Sqrt[e]*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*
e*x^2])])/(8*c^(3/2)*d^(3/2)*e^(3/2))))/(16*c*d*e)))/(24*c*d*e)))/(2*d)))/
(16*d)
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1087
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^p/(2*c*(2*p + 1))), x] - Simp[p*((b^2 - 4*a*c)/(2*c*(2*
p + 1)))   Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] &&
GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[3*p])
```

rule 1092
```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[I
nt[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a
, b, c}, x]
```

rule 1134
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[e*(d + e*x)^(m - 1)*((a + b*x + c*x^2)^(p + 1)/(c*(m + 2*p +
 1))), x] + Simp[(m + p)*((2*c*d - b*e)/(c*(m + 2*p + 1)))   Int[(d + e*x)^
(m - 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[
c*d^2 - b*d*e + a*e^2, 0] && GtQ[m, 1] && NeQ[m + 2*p + 1, 0] && IntegerQ[2
*p]
```

rule 1160
```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[e*((a + b*x + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Simp[(2*c*d - b
*e)/(2*c)   Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
 && NeQ[p, -1]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1559 vs. $2(496) = 992$.

Time = 1.70 (sec) , antiderivative size = 1560, normalized size of antiderivative = 2.87

method	result	size
default	Expression too large to display	1560

input
```
int((e*x+d)^2*(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(5/2),x,method=_RETURNVERB
OSE)
```

output
```
d^2*(1/12*(2*c*d*e*x+a*e^2+c*d^2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(5/2)/
c/d/e+5/24*(4*a*c*d^2*e^2-(a*e^2+c*d^2)^2)/d/e/c*(1/8*(2*c*d*e*x+a*e^2+c*d
^2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(3/2)/c/d/e+3/16*(4*a*c*d^2*e^2-(a*e
^2+c*d^2)^2)/d/e/c*(1/4*(2*c*d*e*x+a*e^2+c*d^2)*(a*d*e+(a*e^2+c*d^2)*x+c*d
*x^2*e)^(1/2)/c/d/e+1/8*(4*a*c*d^2*e^2-(a*e^2+c*d^2)^2)/d/e/c*ln((1/2*a*e^
2+1/2*c*d^2+c*d*x*e)/(d*e*c)^(1/2)+(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(1/2)
)/(d*e*c)^(1/2))))+e^2*(1/8*x*(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(7/2)/d/e/
c-9/16*(a*e^2+c*d^2)/d/e/c*(1/7*(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(7/2)/d/
e/c-1/2*(a*e^2+c*d^2)/d/e/c*(1/12*(2*c*d*e*x+a*e^2+c*d^2)*(a*d*e+(a*e^2+c*
d^2)*x+c*d*x^2*e)^(5/2)/c/d/e+5/24*(4*a*c*d^2*e^2-(a*e^2+c*d^2)^2)/d/e/c*(
1/8*(2*c*d*e*x+a*e^2+c*d^2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(3/2)/c/d/e+
3/16*(4*a*c*d^2*e^2-(a*e^2+c*d^2)^2)/d/e/c*(1/4*(2*c*d*e*x+a*e^2+c*d^2)*(a
*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(1/2)/c/d/e+1/8*(4*a*c*d^2*e^2-(a*e^2+c*d^
2)^2)/d/e/c*ln((1/2*a*e^2+1/2*c*d^2+c*d*x*e)/(d*e*c)^(1/2)+(a*d*e+(a*e^2+c
*d^2)*x+c*d*x^2*e)^(1/2))/(d*e*c)^(1/2)))))-1/8*a/c*(1/12*(2*c*d*e*x+a*e^2
+c*d^2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(5/2)/c/d/e+5/24*(4*a*c*d^2*e^2-
(a*e^2+c*d^2)^2)/d/e/c*(1/8*(2*c*d*e*x+a*e^2+c*d^2)*(a*d*e+(a*e^2+c*d^2)*x
+c*d*x^2*e)^(3/2)/c/d/e+3/16*(4*a*c*d^2*e^2-(a*e^2+c*d^2)^2)/d/e/c*(1/4*(2
*c*d*e*x+a*e^2+c*d^2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(1/2)/c/d/e+1/8*(4
*a*c*d^2*e^2-(a*e^2+c*d^2)^2)/d/e/c*ln((1/2*a*e^2+1/2*c*d^2+c*d*x*e)/(d...
```

Fricas [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 1520, normalized size of antiderivative = 2.79

$$\int (d+ex)^2 \left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2} dx = \text{Too large to display}$$

input
```
integrate((e*x+d)^2*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2),x, algorithm="
fricas")
```

output
```
[1/458752*(315*(c^8*d^16 - 8*a*c^7*d^14*e^2 + 28*a^2*c^6*d^12*e^4 - 56*a^3
*c^5*d^10*e^6 + 70*a^4*c^4*d^8*e^8 - 56*a^5*c^3*d^6*e^10 + 28*a^6*c^2*d^4*
e^12 - 8*a^7*c*d^2*e^14 + a^8*e^16)*sqrt(c*d*e)*log(8*c^2*d^2*e^2*x^2 + c^
2*d^4 + 6*a*c*d^2*e^2 + a^2*e^4 - 4*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^
2)*x)*(2*c*d*e*x + c*d^2 + a*e^2)*sqrt(c*d*e) + 8*(c^2*d^3*e + a*c*d*e^3)*
x) + 4*(14336*c^8*d^8*e^8*x^7 + 315*c^8*d^15*e - 2415*a*c^7*d^13*e^3 + 804
3*a^2*c^6*d^11*e^5 + 17609*a^3*c^5*d^9*e^7 - 15159*a^4*c^4*d^7*e^9 + 8043*
a^5*c^3*d^5*e^11 - 2415*a^6*c^2*d^3*e^13 + 315*a^7*c*d*e^15 + 1024*(65*c^8
*d^9*e^7 + 33*a*c^7*d^7*e^9)*x^6 + 768*(155*c^8*d^10*e^6 + 210*a*c^7*d^8*e
^8 + 27*a^2*c^6*d^6*e^10)*x^5 + 128*(769*c^8*d^11*e^5 + 2343*a*c^7*d^9*e^7
 + 807*a^2*c^6*d^7*e^9 + a^3*c^5*d^5*e^11)*x^4 + 16*(2039*c^8*d^12*e^4 + 1
6452*a*c^7*d^10*e^6 + 12810*a^2*c^6*d^8*e^8 + 68*a^3*c^5*d^6*e^10 - 9*a^4*
c^4*d^4*e^12)*x^3 + 24*(7*c^8*d^13*e^3 + 4043*a*c^7*d^11*e^5 + 8366*a^2*c^
6*d^9*e^7 + 174*a^3*c^5*d^7*e^9 - 53*a^4*c^4*d^5*e^11 + 7*a^5*c^3*d^3*e^13
)*x^2 - 2*(105*c^8*d^14*e^2 - 798*a*c^7*d^12*e^4 - 46521*a^2*c^6*d^10*e^6
- 4900*a^3*c^5*d^8*e^8 + 2631*a^4*c^4*d^6*e^10 - 798*a^5*c^3*d^4*e^12 + 10
5*a^6*c^2*d^2*e^14)*x)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x))/(c^6*d
^6*e^4), 1/229376*(315*(c^8*d^16 - 8*a*c^7*d^14*e^2 + 28*a^2*c^6*d^12*e^4
- 56*a^3*c^5*d^10*e^6 + 70*a^4*c^4*d^8*e^8 - 56*a^5*c^3*d^6*e^10 + 28*a^6*
c^2*d^4*e^12 - 8*a^7*c*d^2*e^14 + a^8*e^16)*sqrt(-c*d*e)*arctan(1/2*sqr...
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 8918 vs. $2(518) = 1036$.

Time = 6.60 (sec) , antiderivative size = 8918, normalized size of antiderivative = 16.39

$$\int (d+ex)^2 \left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2} dx = \text{Too large to display}$$

input
```
integrate((e*x+d)**2*(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(5/2),x)
```

output
```
Piecewise((sqrt(a*d*e + c*d*e*x**2 + x*(a*e**2 + c*d**2))*(c**2*d**2*e**4*
x**7/8 + x**6*(3*a*c**2*d**2*e**6 + 5*c**3*d**4*e**4 - c**2*d**2*e**4*(15*
a*e**2/2 + 15*c*d**2/2)/8)/(7*c*d*e) + x**5*(3*a**2*c*d*e**7 + 113*a*c**2*
d**3*e**5/8 + 10*c**3*d**5*e**3 - (13*a*e**2/2 + 13*c*d**2/2)*(3*a*c**2*d*
*2*e**6 + 5*c**3*d**4*e**4 - c**2*d**2*e**4*(15*a*e**2/2 + 15*c*d**2/2)/8)
/(7*c*d*e))/(6*c*d*e) + x**4*(a**3*e**8 + 15*a**2*c*d**2*e**6 + 30*a*c**2*
d**4*e**4 - 6*a*(3*a*c**2*d**2*e**6 + 5*c**3*d**4*e**4 - c**2*d**2*e**4*(1
5*a*e**2/2 + 15*c*d**2/2)/8)/(7*c) + 10*c**3*d**6*e**2 - (11*a*e**2/2 + 11
*c*d**2/2)*(3*a**2*c*d*e**7 + 113*a*c**2*d**3*e**5/8 + 10*c**3*d**5*e**3 -
 (13*a*e**2/2 + 13*c*d**2/2)*(3*a*c**2*d**2*e**6 + 5*c**3*d**4*e**4 - c**2
*d**2*e**4*(15*a*e**2/2 + 15*c*d**2/2)/8)/(7*c*d*e))/(6*c*d*e))/(5*c*d*e)
+ x**3*(5*a**3*d*e**7 + 30*a**2*c*d**3*e**5 + 30*a*c**2*d**5*e**3 - 5*a*(3
*a**2*c*d*e**7 + 113*a*c**2*d**3*e**5/8 + 10*c**3*d**5*e**3 - (13*a*e**2/2
 + 13*c*d**2/2)*(3*a*c**2*d**2*e**6 + 5*c**3*d**4*e**4 - c**2*d**2*e**4*(1
5*a*e**2/2 + 15*c*d**2/2)/8)/(7*c*d*e))/(6*c) + 5*c**3*d**7*e - (9*a*e**2/
2 + 9*c*d**2/2)*(a**3*e**8 + 15*a**2*c*d**2*e**6 + 30*a*c**2*d**4*e**4 - 6
*a*(3*a*c**2*d**2*e**6 + 5*c**3*d**4*e**4 - c**2*d**2*e**4*(15*a*e**2/2 +
15*c*d**2/2)/8)/(7*c) + 10*c**3*d**6*e**2 - (11*a*e**2/2 + 11*c*d**2/2)*(3
*a**2*c*d*e**7 + 113*a*c**2*d**3*e**5/8 + 10*c**3*d**5*e**3 - (13*a*e**2/2
 + 13*c*d**2/2)*(3*a*c**2*d**2*e**6 + 5*c**3*d**4*e**4 - c**2*d**2*e**4...
```

Maxima [F(-2)]

Exception generated.

$$\int (d+ex)^2 \left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2} dx = \text{Exception raised: ValueError}$$

input
```
integrate((e*x+d)^2*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2),x, algorithm="
maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

Giac [A] (verification not implemented)

Time = 0.36 (sec) , antiderivative size = 786, normalized size of antiderivative = 1.44

$$\int (d+ex)^2 \left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2} dx = \text{Too large to display}$$

input
```
integrate((e*x+d)^2*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2),x, algorithm="
giac")
```

output
```
1/114688*sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*x + a*d*e)*(2*(4*(2*(8*(2*(4*(14
*c^2*d^2*e^4*x + (65*c^9*d^10*e^10 + 33*a*c^8*d^8*e^12)/(c^7*d^7*e^7))*x +
 3*(155*c^9*d^11*e^9 + 210*a*c^8*d^9*e^11 + 27*a^2*c^7*d^7*e^13)/(c^7*d^7*
e^7))*x + (769*c^9*d^12*e^8 + 2343*a*c^8*d^10*e^10 + 807*a^2*c^7*d^8*e^12
+ a^3*c^6*d^6*e^14)/(c^7*d^7*e^7))*x + (2039*c^9*d^13*e^7 + 16452*a*c^8*d^
11*e^9 + 12810*a^2*c^7*d^9*e^11 + 68*a^3*c^6*d^7*e^13 - 9*a^4*c^5*d^5*e^15
)/(c^7*d^7*e^7))*x + 3*(7*c^9*d^14*e^6 + 4043*a*c^8*d^12*e^8 + 8366*a^2*c^
7*d^10*e^10 + 174*a^3*c^6*d^8*e^12 - 53*a^4*c^5*d^6*e^14 + 7*a^5*c^4*d^4*e
^16)/(c^7*d^7*e^7))*x - (105*c^9*d^15*e^5 - 798*a*c^8*d^13*e^7 - 46521*a^2
*c^7*d^11*e^9 - 4900*a^3*c^6*d^9*e^11 + 2631*a^4*c^5*d^7*e^13 - 798*a^5*c^
4*d^5*e^15 + 105*a^6*c^3*d^3*e^17)/(c^7*d^7*e^7))*x + (315*c^9*d^16*e^4 -
2415*a*c^8*d^14*e^6 + 8043*a^2*c^7*d^12*e^8 + 17609*a^3*c^6*d^10*e^10 - 15
159*a^4*c^5*d^8*e^12 + 8043*a^5*c^4*d^6*e^14 - 2415*a^6*c^3*d^4*e^16 + 315
*a^7*c^2*d^2*e^18)/(c^7*d^7*e^7)) + 45/32768*(c^8*d^16 - 8*a*c^7*d^14*e^2
+ 28*a^2*c^6*d^12*e^4 - 56*a^3*c^5*d^10*e^6 + 70*a^4*c^4*d^8*e^8 - 56*a^5*
c^3*d^6*e^10 + 28*a^6*c^2*d^4*e^12 - 8*a^7*c*d^2*e^14 + a^8*e^16)*log(abs(
-c*d^2 - a*e^2 - 2*sqrt(c*d*e)*(sqrt(c*d*e)*x - sqrt(c*d*e*x^2 + c*d^2*x +
 a*e^2*x + a*d*e))))/(sqrt(c*d*e)*c^5*d^5*e^3)
```

Mupad [F(-1)]

Timed out.

$$\int (d+ex)^2 \left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2} dx = \int (d+e\,x)^2\,\left(c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e\right)^{5/2} \,dx$$

input
```
int((d + e*x)^2*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(5/2),x)
```

output
```
int((d + e*x)^2*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(5/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.47 (sec) , antiderivative size = 1615, normalized size of antiderivative = 2.97

$$\int (d+ex)^2 \left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2} dx = \text{Too large to display}$$

input
```
int((e*x+d)^2*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2),x)
```

output
```
(315*sqrt(d + e*x)*sqrt(a*e + c*d*x)*a**7*c*d*e**15 - 2415*sqrt(d + e*x)*s
qrt(a*e + c*d*x)*a**6*c**2*d**3*e**13 - 210*sqrt(d + e*x)*sqrt(a*e + c*d*x
)*a**6*c**2*d**2*e**14*x + 8043*sqrt(d + e*x)*sqrt(a*e + c*d*x)*a**5*c**3*
d**5*e**11 + 1596*sqrt(d + e*x)*sqrt(a*e + c*d*x)*a**5*c**3*d**4*e**12*x +
 168*sqrt(d + e*x)*sqrt(a*e + c*d*x)*a**5*c**3*d**3*e**13*x**2 - 15159*sqr
t(d + e*x)*sqrt(a*e + c*d*x)*a**4*c**4*d**7*e**9 - 5262*sqrt(d + e*x)*sqrt
(a*e + c*d*x)*a**4*c**4*d**6*e**10*x - 1272*sqrt(d + e*x)*sqrt(a*e + c*d*x
)*a**4*c**4*d**5*e**11*x**2 - 144*sqrt(d + e*x)*sqrt(a*e + c*d*x)*a**4*c**
4*d**4*e**12*x**3 + 17609*sqrt(d + e*x)*sqrt(a*e + c*d*x)*a**3*c**5*d**9*e
**7 + 9800*sqrt(d + e*x)*sqrt(a*e + c*d*x)*a**3*c**5*d**8*e**8*x + 4176*sq
rt(d + e*x)*sqrt(a*e + c*d*x)*a**3*c**5*d**7*e**9*x**2 + 1088*sqrt(d + e*x
)*sqrt(a*e + c*d*x)*a**3*c**5*d**6*e**10*x**3 + 128*sqrt(d + e*x)*sqrt(a*e
 + c*d*x)*a**3*c**5*d**5*e**11*x**4 + 8043*sqrt(d + e*x)*sqrt(a*e + c*d*x)
*a**2*c**6*d**11*e**5 + 93042*sqrt(d + e*x)*sqrt(a*e + c*d*x)*a**2*c**6*d*
*10*e**6*x + 200784*sqrt(d + e*x)*sqrt(a*e + c*d*x)*a**2*c**6*d**9*e**7*x*
*2 + 204960*sqrt(d + e*x)*sqrt(a*e + c*d*x)*a**2*c**6*d**8*e**8*x**3 + 103
296*sqrt(d + e*x)*sqrt(a*e + c*d*x)*a**2*c**6*d**7*e**9*x**4 + 20736*sqrt(
d + e*x)*sqrt(a*e + c*d*x)*a**2*c**6*d**6*e**10*x**5 - 2415*sqrt(d + e*x)*
sqrt(a*e + c*d*x)*a*c**7*d**13*e**3 + 1596*sqrt(d + e*x)*sqrt(a*e + c*d*x)
*a*c**7*d**12*e**4*x + 97032*sqrt(d + e*x)*sqrt(a*e + c*d*x)*a*c**7*d**...
```

3.222 $\int (d+ex)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 486

$$\begin{aligned}
&\int (d+ex)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}\,dx = \frac{5\left(cd^2-ae^2\right)^6\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{1024c^4d^4e^3} \\
&-\frac{5\left(cd^2-ae^2\right)^5\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{1536c^4d^4e^2(d+ex)} \\
&+\frac{\left(cd^2-ae^2\right)^4\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{384c^4d^4e(d+ex)^2} \\
&+\frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{7/2}}{7cd} \\
&+\frac{\left(cd^2-ae^2\right)^3\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{7/2}}{64c^4d^4(d+ex)^3} \\
&+\frac{\left(cd^2-ae^2\right)^2\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{7/2}}{24c^3d^3(d+ex)^2} \\
&+\frac{\left(cd^2-ae^2\right)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{7/2}}{12c^2d^2(d+ex)} \\
&-\frac{5\left(cd^2-ae^2\right)^7\operatorname{arctanh}\left(\frac{\sqrt{e}\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{\sqrt{c}\sqrt{d}(d+ex)}\right)}{1024c^{9/2}d^{9/2}e^{7/2}}
\end{aligned}$$

output
```
5/1024*(-a*e^2+c*d^2)^6*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/c^4/d^4/e^
3-5/1536*(-a*e^2+c*d^2)^5*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/c^4/d^4/
e^2/(e*x+d)+1/384*(-a*e^2+c*d^2)^4*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)
/c^4/d^4/e/(e*x+d)^2+1/7*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(7/2)/c/d+1/64*
(-a*e^2+c*d^2)^3*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(7/2)/c^4/d^4/(e*x+d)^3
+1/24*(-a*e^2+c*d^2)^2*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(7/2)/c^3/d^3/(e*
x+d)^2+1/12*(-a*e^2+c*d^2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(7/2)/c^2/d^2
/(e*x+d)-5/1024*(-a*e^2+c*d^2)^7*arctanh(e^(1/2)*(a*d*e+(a*e^2+c*d^2)*x+c*
d*e*x^2)^(1/2)/c^(1/2)/d^(1/2)/(e*x+d))/c^(9/2)/d^(9/2)/e^(7/2)
```

Mathematica [A] (verified)

Time = 1.31 (sec) , antiderivative size = 437, normalized size of antiderivative = 0.90

$$\int (d+ex)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}\,dx=\frac{((ae+cdx)(d+ex))^{5/2}\left(\frac{\sqrt{c}\sqrt{d}\sqrt{e}\left(-105a^6e^{12}+70a^5cde^{10}(10d+ex)-7a^4c^2d^2e^8\left(283d^2+66dex+8e^2x^2\right)+4a^3c^3\cdots\right.}{\cdots}\right.}{\cdots}$$

input
```
Integrate[(d + e*x)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2),x]
```

output
```
(((a*e + c*d*x)*(d + e*x))^(5/2)*((Sqrt[c]*Sqrt[d]*Sqrt[e]*(-105*a^6*e^12
+ 70*a^5*c*d*e^10*(10*d + e*x) - 7*a^4*c^2*d^2*e^8*(283*d^2 + 66*d*e*x + 8
*e^2*x^2) + 4*a^3*c^3*d^3*e^6*(768*d^3 + 323*d^2*e*x + 92*d*e^2*x^2 + 12*e
^3*x^3) + a^2*c^4*d^4*e^4*(1981*d^4 + 17140*d^3*e*x + 27648*d^2*e^2*x^2 +
18800*d*e^3*x^3 + 4736*e^4*x^4) + 2*a*c^5*d^5*e^2*(-350*d^5 + 231*d^4*e*x
+ 9032*d^3*e^2*x^2 + 18248*d^2*e^3*x^3 + 13824*d*e^4*x^4 + 3712*e^5*x^5) +
 c^6*d^6*(105*d^6 - 70*d^5*e*x + 56*d^4*e^2*x^2 + 6096*d^3*e^3*x^3 + 13696
*d^2*e^4*x^4 + 11008*d*e^5*x^5 + 3072*e^6*x^6)))/((a*e + c*d*x)^2*(d + e*x
)^2) - (105*(c*d^2 - a*e^2)^7*ArcTanh[(Sqrt[c]*Sqrt[d]*Sqrt[d + e*x])/(Sqr
t[e]*Sqrt[a*e + c*d*x])])/((a*e + c*d*x)^(5/2)*(d + e*x)^(5/2))))/(21504*c
^(9/2)*d^(9/2)*e^(7/2))
```

Rubi [A] (verified)

Time = 0.85 (sec) , antiderivative size = 393, normalized size of antiderivative = 0.81, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.171$, Rules used = {1160, 1087, 1087, 1087, 1092, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (d+ex)\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/2}dx$$

$$\Big\downarrow \text{1160}$$

$$\frac{\left(d^2-\frac{ae^2}{c}\right)\int \left(cdex^2+\left(cd^2+ae^2\right)x+ade\right)^{5/2}dx}{2d}+\frac{\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{7/2}}{7cd}$$

$$\Big\downarrow \text{1087}$$

$$\frac{\left(d^2-\frac{ae^2}{c}\right)\left(\frac{\left(ae^2+cd^2+2cdex\right)\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/2}}{12cde}-\frac{5\left(cd^2-ae^2\right)^2\int \left(cdex^2+\left(cd^2+ae^2\right)x+ade\right)^{3/2}dx}{24cde}\right)}{2d}+\frac{\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{7/2}}{7cd}$$

$$\Big\downarrow \text{1087}$$

$$\frac{\left(d^2-\frac{ae^2}{c}\right)\left(\frac{\left(ae^2+cd^2+2cdex\right)\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/2}}{12cde}-\frac{5\left(cd^2-ae^2\right)^2\left(\frac{\left(ae^2+cd^2+2cdex\right)\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{8cde}-\frac{3\left(cd^2-ae\right.}{}\right.}{24cde}\right.}{2d}$$

$$\frac{\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{7/2}}{7cd}$$

$$\Big\downarrow \text{1087}$$

$$\left(d^2-\frac{ae^2}{c}\right)\left(\frac{(ae^2+cd^2+2cdex)\left(x(ae^2+cd^2)+ade+cdex^2\right)^{5/2}}{12cde}-\frac{5\left(cd^2-ae^2\right)^2\left(\frac{\left(ae^2+cd^2+2cdex\right)\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{8cde}-\frac{3\left(cd^2-ae\right.}{}\right.}{}\right.}{2d}$$

$$\frac{\left(x(ae^2+cd^2)+ade+cdex^2\right)^{7/2}}{7cd}$$

$\downarrow$ 1092

$$\left(d^2-\frac{ae^2}{c}\right)\left(\frac{(ae^2+cd^2+2cdex)\left(x(ae^2+cd^2)+ade+cdex^2\right)^{5/2}}{12cde}-\frac{5\left(cd^2-ae^2\right)^2\left(\frac{\left(ae^2+cd^2+2cdex\right)\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{8cde}-\frac{3\left(cd^2-ae\right.}{}\right.}{}\right.}{2d}$$

$$\frac{\left(x(ae^2+cd^2)+ade+cdex^2\right)^{7/2}}{7cd}$$

$\downarrow$ 219

$$\frac{\left(d^2-\frac{ae^2}{c}\right)\left(\frac{(ae^2+cd^2+2cdex)\left(x(ae^2+cd^2)+ade+cdex^2\right)^{5/2}}{12cde}-\frac{5(cd^2-ae^2)^2\left(\frac{(ae^2+cd^2+2cdex)\left(x(ae^2+cd^2)+ade+cdex^2\right)^{3/2}}{8cde}-\frac{3(cd^2-ae^2)\cdots}{\cdots}\right)}{\cdots}\right)}{2d}$$

$$\frac{\left(x(ae^2+cd^2)+ade+cdex^2\right)^{7/2}}{7cd}$$

input
```
Int[(d + e*x)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2),x]
```

output
```
(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(7/2)/(7*c*d) + ((d^2 - (a*e^2)/c)
*(((c*d^2 + a*e^2 + 2*c*d*e*x)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/
2))/(12*c*d*e) - (5*(c*d^2 - a*e^2)^2*(((c*d^2 + a*e^2 + 2*c*d*e*x)*(a*d*e
 + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2))/(8*c*d*e) - (3*(c*d^2 - a*e^2)^2*
(((c*d^2 + a*e^2 + 2*c*d*e*x)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])
/(4*c*d*e) - ((c*d^2 - a*e^2)^2*ArcTanh[(c*d^2 + a*e^2 + 2*c*d*e*x)/(2*Sqr
t[c]*Sqrt[d]*Sqrt[e]*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])])/(8*c^(
3/2)*d^(3/2)*e^(3/2))))/(16*c*d*e)))/(24*c*d*e)))/(2*d)
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1087
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^p/(2*c*(2*p + 1))), x] - Simp[p*((b^2 - 4*a*c)/(2*c*(2*
p + 1)))   Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] &&
GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[3*p])
```

rule 1092
```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[I
nt[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a
, b, c}, x]
```

rule 1160
```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[e*((a + b*x + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Simp[(2*c*d - b
*e)/(2*c)   Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
 && NeQ[p, -1]
```

Maple [A] (verified)

Time = 1.50 (sec) , antiderivative size = 745, normalized size of antiderivative = 1.53

method	result
default	$d\left(\frac{\left(2cdxe+a\,e^{2}+c\,d^{2}\right)\left(ade+\left(a\,e^{2}+c\,d^{2}\right)x+cd\,x^{2}e\right)^{\frac{5}{2}}}{12cde}+\frac{5\left(4ac\,d^{2}e^{2}-\left(a\,e^{2}+c\,d^{2}\right)^{2}\right)\left(\frac{\left(2cdxe+a\,e^{2}+c\,d^{2}\right)\left(ade+\left(a\,e^{2}+c\,d^{2}\right)x+cd\,x^{2}\right.}{8cde}\right.}{}\right.$

input

```
int((e*x+d)*(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(5/2),x,method=_RETURNVERBOS
E)
```

output
```
d*(1/12*(2*c*d*e*x+a*e^2+c*d^2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(5/2)/c/
d/e+5/24*(4*a*c*d^2*e^2-(a*e^2+c*d^2)^2)/d/e/c*(1/8*(2*c*d*e*x+a*e^2+c*d^2
)*(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(3/2)/c/d/e+3/16*(4*a*c*d^2*e^2-(a*e^2
+c*d^2)^2)/d/e/c*(1/4*(2*c*d*e*x+a*e^2+c*d^2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*x
^2*e)^(1/2)/c/d/e+1/8*(4*a*c*d^2*e^2-(a*e^2+c*d^2)^2)/d/e/c*ln((1/2*a*e^2+
1/2*c*d^2+c*d*x*e)/(d*e*c)^(1/2)+(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(1/2))/
(d*e*c)^(1/2))))+e*(1/7*(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(7/2)/d/e/c-1/2*
(a*e^2+c*d^2)/d/e/c*(1/12*(2*c*d*e*x+a*e^2+c*d^2)*(a*d*e+(a*e^2+c*d^2)*x+c
*d*x^2*e)^(5/2)/c/d/e+5/24*(4*a*c*d^2*e^2-(a*e^2+c*d^2)^2)/d/e/c*(1/8*(2*c
*d*e*x+a*e^2+c*d^2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(3/2)/c/d/e+3/16*(4*
a*c*d^2*e^2-(a*e^2+c*d^2)^2)/d/e/c*(1/4*(2*c*d*e*x+a*e^2+c*d^2)*(a*d*e+(a*
e^2+c*d^2)*x+c*d*x^2*e)^(1/2)/c/d/e+1/8*(4*a*c*d^2*e^2-(a*e^2+c*d^2)^2)/d/
e/c*ln((1/2*a*e^2+1/2*c*d^2+c*d*x*e)/(d*e*c)^(1/2)+(a*d*e+(a*e^2+c*d^2)*x+
c*d*x^2*e)^(1/2))/(d*e*c)^(1/2)))))
```

Fricas [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 1270, normalized size of antiderivative = 2.61

$$\int (d+ex)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}\,dx=\text{Too large to display}$$

input
```
integrate((e*x+d)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2),x, algorithm="fr
icas")
```

output
```
[-1/86016*(105*(c^7*d^14 - 7*a*c^6*d^12*e^2 + 21*a^2*c^5*d^10*e^4 - 35*a^3
*c^4*d^8*e^6 + 35*a^4*c^3*d^6*e^8 - 21*a^5*c^2*d^4*e^10 + 7*a^6*c*d^2*e^12
 - a^7*e^14)*sqrt(c*d*e)*log(8*c^2*d^2*e^2*x^2 + c^2*d^4 + 6*a*c*d^2*e^2 +
 a^2*e^4 + 4*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(2*c*d*e*x + c*d^
2 + a*e^2)*sqrt(c*d*e) + 8*(c^2*d^3*e + a*c*d*e^3)*x) - 4*(3072*c^7*d^7*e^
7*x^6 + 105*c^7*d^13*e - 700*a*c^6*d^11*e^3 + 1981*a^2*c^5*d^9*e^5 + 3072*
a^3*c^4*d^7*e^7 - 1981*a^4*c^3*d^5*e^9 + 700*a^5*c^2*d^3*e^11 - 105*a^6*c*
d*e^13 + 256*(43*c^7*d^8*e^6 + 29*a*c^6*d^6*e^8)*x^5 + 128*(107*c^7*d^9*e^
5 + 216*a*c^6*d^7*e^7 + 37*a^2*c^5*d^5*e^9)*x^4 + 16*(381*c^7*d^10*e^4 + 2
281*a*c^6*d^8*e^6 + 1175*a^2*c^5*d^6*e^8 + 3*a^3*c^4*d^4*e^10)*x^3 + 8*(7*
c^7*d^11*e^3 + 2258*a*c^6*d^9*e^5 + 3456*a^2*c^5*d^7*e^7 + 46*a^3*c^4*d^5*
e^9 - 7*a^4*c^3*d^3*e^11)*x^2 - 2*(35*c^7*d^12*e^2 - 231*a*c^6*d^10*e^4 -
8570*a^2*c^5*d^8*e^6 - 646*a^3*c^4*d^6*e^8 + 231*a^4*c^3*d^4*e^10 - 35*a^5
*c^2*d^2*e^12)*x)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x))/(c^5*d^5*e^
4), 1/43008*(105*(c^7*d^14 - 7*a*c^6*d^12*e^2 + 21*a^2*c^5*d^10*e^4 - 35*a
^3*c^4*d^8*e^6 + 35*a^4*c^3*d^6*e^8 - 21*a^5*c^2*d^4*e^10 + 7*a^6*c*d^2*e^
12 - a^7*e^14)*sqrt(-c*d*e)*arctan(1/2*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a
*e^2)*x)*(2*c*d*e*x + c*d^2 + a*e^2)*sqrt(-c*d*e)/(c^2*d^2*e^2*x^2 + a*c*d
^2*e^2 + (c^2*d^3*e + a*c*d*e^3)*x)) + 2*(3072*c^7*d^7*e^7*x^6 + 105*c^7*d
^13*e - 700*a*c^6*d^11*e^3 + 1981*a^2*c^5*d^9*e^5 + 3072*a^3*c^4*d^7*e^...
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 5268 vs. $2(452) = 904$.

Time = 4.51 (sec) , antiderivative size = 5268, normalized size of antiderivative = 10.84

$$\int (d+ex)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}\,dx = \text{Too large to display}$$

input
```
integrate((e*x+d)*(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(5/2),x)
```

output
```
Piecewise((sqrt(a*d*e + c*d*e*x**2 + x*(a*e**2 + c*d**2))*(c**2*d**2*e**3*
x**6/7 + x**5*(3*a*c**2*d**2*e**5 + 4*c**3*d**4*e**3 - c**2*d**2*e**3*(13*
a*e**2/2 + 13*c*d**2/2)/7)/(6*c*d*e) + x**4*(3*a**2*c*d*e**6 + 78*a*c**2*d
**3*e**4/7 + 6*c**3*d**5*e**2 - (11*a*e**2/2 + 11*c*d**2/2)*(3*a*c**2*d**2
*e**5 + 4*c**3*d**4*e**3 - c**2*d**2*e**3*(13*a*e**2/2 + 13*c*d**2/2)/7)/(
6*c*d*e))/(5*c*d*e) + x**3*(a**3*e**7 + 12*a**2*c*d**2*e**5 + 18*a*c**2*d*
*4*e**3 - 5*a*(3*a*c**2*d**2*e**5 + 4*c**3*d**4*e**3 - c**2*d**2*e**3*(13*
a*e**2/2 + 13*c*d**2/2)/7)/(6*c) + 4*c**3*d**6*e - (9*a*e**2/2 + 9*c*d**2/
2)*(3*a**2*c*d*e**6 + 78*a*c**2*d**3*e**4/7 + 6*c**3*d**5*e**2 - (11*a*e**
2/2 + 11*c*d**2/2)*(3*a*c**2*d**2*e**5 + 4*c**3*d**4*e**3 - c**2*d**2*e**3
*(13*a*e**2/2 + 13*c*d**2/2)/7)/(6*c*d*e))/(5*c*d*e))/(4*c*d*e) + x**2*(4*
a**3*d*e**6 + 18*a**2*c*d**3*e**4 + 12*a*c**2*d**5*e**2 - 4*a*(3*a**2*c*d*
e**6 + 78*a*c**2*d**3*e**4/7 + 6*c**3*d**5*e**2 - (11*a*e**2/2 + 11*c*d**2
/2)*(3*a*c**2*d**2*e**5 + 4*c**3*d**4*e**3 - c**2*d**2*e**3*(13*a*e**2/2 +
 13*c*d**2/2)/7)/(6*c*d*e))/(5*c) + c**3*d**7 - (7*a*e**2/2 + 7*c*d**2/2)*
(a**3*e**7 + 12*a**2*c*d**2*e**5 + 18*a*c**2*d**4*e**3 - 5*a*(3*a*c**2*d**
2*e**5 + 4*c**3*d**4*e**3 - c**2*d**2*e**3*(13*a*e**2/2 + 13*c*d**2/2)/7)/
(6*c) + 4*c**3*d**6*e - (9*a*e**2/2 + 9*c*d**2/2)*(3*a**2*c*d*e**6 + 78*a*
c**2*d**3*e**4/7 + 6*c**3*d**5*e**2 - (11*a*e**2/2 + 11*c*d**2/2)*(3*a*c**
2*d**2*e**5 + 4*c**3*d**4*e**3 - c**2*d**2*e**3*(13*a*e**2/2 + 13*c*d**...
```

Maxima [F(-2)]

Exception generated.

$$\int (d+ex) \left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2} dx = \text{Exception raised: ValueError}$$

input
```
integrate((e*x+d)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2),x, algorithm="ma
xima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

Giac [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 650, normalized size of antiderivative = 1.34

$$\int (d+ex)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}\,dx=\frac{1}{21504}\sqrt{cdex^2+cd^2x+ae^2x+ade}\left(2\left(4\left(2\left(8\left(2\left(12\,c^2d^2e^3x+\frac{43\,c^8d^9e^8+29\,ac^7d^7e}{c^6d^6e^6}\right.\right.\right.\right.\right.\right.$$
$$+\frac{5\left(c^7d^{14}-7\,ac^6d^{12}e^2+21\,a^2c^5d^{10}e^4-35\,a^3c^4d^8e^6+35\,a^4c^3d^6e^8-21\,a^5c^2d^4e^{10}+7\,a^6cd^2e^{12}-a^7e^{14}\right)\log}{2048\,\sqrt{cde}c^4d^4e^3}$$

input
```
integrate((e*x+d)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2),x, algorithm="gi
ac")
```

output
```
1/21504*sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*x + a*d*e)*(2*(4*(2*(8*(2*(12*c^2
*d^2*e^3*x + (43*c^8*d^9*e^8 + 29*a*c^7*d^7*e^10)/(c^6*d^6*e^6))*x + (107*
c^8*d^10*e^7 + 216*a*c^7*d^8*e^9 + 37*a^2*c^6*d^6*e^11)/(c^6*d^6*e^6))*x +
 (381*c^8*d^11*e^6 + 2281*a*c^7*d^9*e^8 + 1175*a^2*c^6*d^7*e^10 + 3*a^3*c^
5*d^5*e^12)/(c^6*d^6*e^6))*x + (7*c^8*d^12*e^5 + 2258*a*c^7*d^10*e^7 + 345
6*a^2*c^6*d^8*e^9 + 46*a^3*c^5*d^6*e^11 - 7*a^4*c^4*d^4*e^13)/(c^6*d^6*e^6
))*x - (35*c^8*d^13*e^4 - 231*a*c^7*d^11*e^6 - 8570*a^2*c^6*d^9*e^8 - 646*
a^3*c^5*d^7*e^10 + 231*a^4*c^4*d^5*e^12 - 35*a^5*c^3*d^3*e^14)/(c^6*d^6*e^
6))*x + (105*c^8*d^14*e^3 - 700*a*c^7*d^12*e^5 + 1981*a^2*c^6*d^10*e^7 + 3
072*a^3*c^5*d^8*e^9 - 1981*a^4*c^4*d^6*e^11 + 700*a^5*c^3*d^4*e^13 - 105*a
^6*c^2*d^2*e^15)/(c^6*d^6*e^6)) + 5/2048*(c^7*d^14 - 7*a*c^6*d^12*e^2 + 21
*a^2*c^5*d^10*e^4 - 35*a^3*c^4*d^8*e^6 + 35*a^4*c^3*d^6*e^8 - 21*a^5*c^2*d
^4*e^10 + 7*a^6*c*d^2*e^12 - a^7*e^14)*log(abs(-c*d^2 - a*e^2 - 2*sqrt(c*d
*e)*(sqrt(c*d*e)*x - sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*x + a*d*e))))/(sqrt(
c*d*e)*c^4*d^4*e^3)
```

Mupad [F(-1)]

Timed out.

$$\int (d+ex)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}\,dx=\int (d+e\,x)\,\left(c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e\right)^{5/2}\,dx$$

input
```
int((d + e*x)*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(5/2),x)
```

output
```
int((d + e*x)*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(5/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.40 (sec) , antiderivative size = 1308, normalized size of antiderivative = 2.69

$$\int (d+ex)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}\,dx=\text{Too large to display}$$

input
```
int((e*x+d)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2),x)
```

output
```
( - 105*sqrt(d + e*x)*sqrt(a*e + c*d*x)*a**6*c*d*e**13 + 700*sqrt(d + e*x)
*sqrt(a*e + c*d*x)*a**5*c**2*d**3*e**11 + 70*sqrt(d + e*x)*sqrt(a*e + c*d*
x)*a**5*c**2*d**2*e**12*x - 1981*sqrt(d + e*x)*sqrt(a*e + c*d*x)*a**4*c**3
*d**5*e**9 - 462*sqrt(d + e*x)*sqrt(a*e + c*d*x)*a**4*c**3*d**4*e**10*x -
56*sqrt(d + e*x)*sqrt(a*e + c*d*x)*a**4*c**3*d**3*e**11*x**2 + 3072*sqrt(d
 + e*x)*sqrt(a*e + c*d*x)*a**3*c**4*d**7*e**7 + 1292*sqrt(d + e*x)*sqrt(a*
e + c*d*x)*a**3*c**4*d**6*e**8*x + 368*sqrt(d + e*x)*sqrt(a*e + c*d*x)*a**
3*c**4*d**5*e**9*x**2 + 48*sqrt(d + e*x)*sqrt(a*e + c*d*x)*a**3*c**4*d**4*
e**10*x**3 + 1981*sqrt(d + e*x)*sqrt(a*e + c*d*x)*a**2*c**5*d**9*e**5 + 17
140*sqrt(d + e*x)*sqrt(a*e + c*d*x)*a**2*c**5*d**8*e**6*x + 27648*sqrt(d +
 e*x)*sqrt(a*e + c*d*x)*a**2*c**5*d**7*e**7*x**2 + 18800*sqrt(d + e*x)*sqr
t(a*e + c*d*x)*a**2*c**5*d**6*e**8*x**3 + 4736*sqrt(d + e*x)*sqrt(a*e + c*
d*x)*a**2*c**5*d**5*e**9*x**4 - 700*sqrt(d + e*x)*sqrt(a*e + c*d*x)*a*c**6
*d**11*e**3 + 462*sqrt(d + e*x)*sqrt(a*e + c*d*x)*a*c**6*d**10*e**4*x + 18
064*sqrt(d + e*x)*sqrt(a*e + c*d*x)*a*c**6*d**9*e**5*x**2 + 36496*sqrt(d +
 e*x)*sqrt(a*e + c*d*x)*a*c**6*d**8*e**6*x**3 + 27648*sqrt(d + e*x)*sqrt(a
*e + c*d*x)*a*c**6*d**7*e**7*x**4 + 7424*sqrt(d + e*x)*sqrt(a*e + c*d*x)*a
*c**6*d**6*e**8*x**5 + 105*sqrt(d + e*x)*sqrt(a*e + c*d*x)*c**7*d**13*e -
70*sqrt(d + e*x)*sqrt(a*e + c*d*x)*c**7*d**12*e**2*x + 56*sqrt(d + e*x)*sq
rt(a*e + c*d*x)*c**7*d**11*e**3*x**2 + 6096*sqrt(d + e*x)*sqrt(a*e + c*...
```

3.223 $\int \left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 290

$$\int \left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}\,dx=\frac{5\left(cd^2-ae^2\right)^4\left(cd^2+ae^2+2cdex\right)\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{512c^3d^3e^3}-\frac{5\left(cd^2-ae^2\right)^2\left(cd^2+ae^2+2cdex\right)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{192c^2d^2e^2}+\frac{\left(cd^2+ae^2+2cdex\right)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{12cde}-\frac{5\left(cd^2-ae^2\right)^6\operatorname{arctanh}\left(\frac{\sqrt{c}\sqrt{d}(d+ex)}{\sqrt{e}\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\right)}{512c^{7/2}d^{7/2}e^{7/2}}$$

output
```
5/512*(-a*e^2+c*d^2)^4*(2*c*d*e*x+a*e^2+c*d^2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*
e*x^2)^(1/2)/c^3/d^3/e^3-5/192*(-a*e^2+c*d^2)^2*(2*c*d*e*x+a*e^2+c*d^2)*(a
*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/c^2/d^2/e^2+1/12*(2*c*d*e*x+a*e^2+c*
d^2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/c/d/e-5/512*(-a*e^2+c*d^2)^6*
arctanh(c^(1/2)*d^(1/2)*(e*x+d)/e^(1/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^
(1/2))/c^(7/2)/d^(7/2)/e^(7/2)
```

Mathematica [A] (verified)

Time = 1.08 (sec) , antiderivative size = 361, normalized size of antiderivative = 1.24

$$\int \left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}\,dx=\frac{((ae+cdx)(d+ex))^{5/2}\left(\frac{\sqrt{c}\sqrt{d}\sqrt{e}(15a^5e^{10}-5a^4cde^8(17d+2ex)+2a^3c^2d^2e^6(99d^2+28dex+4e^2x^2)+6a^2c^3d^3e^4(\right.}{}$$

input
```
Integrate[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2),x]
```

output
```
(((a*e + c*d*x)*(d + e*x))^(5/2)*((Sqrt[c]*Sqrt[d]*Sqrt[e]*(15*a^5*e^10 -
5*a^4*c*d*e^8*(17*d + 2*e*x) + 2*a^3*c^2*d^2*e^6*(99*d^2 + 28*d*e*x + 4*e^
2*x^2) + 6*a^2*c^3*d^3*e^4*(33*d^3 + 198*d^2*e*x + 212*d*e^2*x^2 + 72*e^3*
x^3) + a*c^4*d^4*e^2*(-85*d^4 + 56*d^3*e*x + 1272*d^2*e^2*x^2 + 1696*d*e^3
*x^3 + 640*e^4*x^4) + c^5*d^5*(15*d^5 - 10*d^4*e*x + 8*d^3*e^2*x^2 + 432*d
^2*e^3*x^3 + 640*d*e^4*x^4 + 256*e^5*x^5)))/((a*e + c*d*x)^2*(d + e*x)^2)
- (15*(c*d^2 - a*e^2)^6*ArcTanh[(Sqrt[c]*Sqrt[d]*Sqrt[d + e*x])/(Sqrt[e]*S
qrt[a*e + c*d*x])])/((a*e + c*d*x)^(5/2)*(d + e*x)^(5/2))))/(1536*c^(7/2)*
d^(7/2)*e^(7/2))
```

Rubi [A] (verified)

Time = 0.67 (sec) , antiderivative size = 333, normalized size of antiderivative = 1.15, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.172, Rules used = {1087, 1087, 1087, 1092, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/2}dx$$

$$\Big\downarrow \text{1087}$$

$$\frac{(ae^2+cd^2+2cdex)\left(x(ae^2+cd^2)+ade+cdex^2\right)^{5/2}}{12cde}-\frac{5(cd^2-ae^2)^2\int\left(cdex^2+(cd^2+ae^2)x+ade\right)^{3/2}dx}{24cde}$$

$\downarrow$ 1087

$$\frac{(ae^2+cd^2+2cdex)\left(x(ae^2+cd^2)+ade+cdex^2\right)^{5/2}}{12cde}-\frac{5(cd^2-ae^2)^2\left(\frac{(ae^2+cd^2+2cdex)\left(x(ae^2+cd^2)+ade+cdex^2\right)^{3/2}}{8cde}-\frac{3(cd^2-ae^2)^2\int\sqrt{cdex^2+(cd^2+ae^2)x+ade}dx}{16cde}\right)}{24cde}$$

$\downarrow$ 1087

$$\frac{(ae^2+cd^2+2cdex)\left(x(ae^2+cd^2)+ade+cdex^2\right)^{5/2}}{12cde}-\frac{5(cd^2-ae^2)^2\left(\frac{(ae^2+cd^2+2cdex)\left(x(ae^2+cd^2)+ade+cdex^2\right)^{3/2}}{8cde}-\frac{3(cd^2-ae^2)^2\left(\frac{(ae^2+cd^2+2cdex)\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{4cde}-\frac{(cd^2-ae^2}{}\right.}{16cde}\right.}{24cde}$$

$\downarrow$ 1092

$$\frac{(ae^2+cd^2+2cdex)\left(x(ae^2+cd^2)+ade+cdex^2\right)^{5/2}}{12cde}-\frac{5(cd^2-ae^2)^2\left(\frac{(ae^2+cd^2+2cdex)\left(x(ae^2+cd^2)+ade+cdex^2\right)^{3/2}}{8cde}-\frac{3(cd^2-ae^2)^2\left(\frac{(ae^2+cd^2+2cdex)\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{4cde}-\frac{(cd^2-ae^2}{}\right.}{16cde}\right.}{24cde}$$

$\downarrow$ 219

$$\frac{\left(ae^2+cd^2+2cdex\right)\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/2}}{12cde}-$$

$$\frac{5\left(cd^2-ae^2\right)^2\left(\frac{\left(ae^2+cd^2+2cdex\right)\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{8cde}-\frac{3\left(cd^2-ae^2\right)^2\left(\frac{\left(ae^2+cd^2+2cdex\right)\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{4cde}-\frac{\left(cd^2-ae^2\right.}{}\right.}{16cde}\right.}{24cde}$$

input
```
Int[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2),x]
```

output
```
((c*d^2 + a*e^2 + 2*c*d*e*x)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2)
)/(12*c*d*e) - (5*(c*d^2 - a*e^2)^2*(((c*d^2 + a*e^2 + 2*c*d*e*x)*(a*d*e +
 (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2))/(8*c*d*e) - (3*(c*d^2 - a*e^2)^2*((
(c*d^2 + a*e^2 + 2*c*d*e*x)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(
4*c*d*e) - ((c*d^2 - a*e^2)^2*ArcTanh[(c*d^2 + a*e^2 + 2*c*d*e*x)/(2*Sqrt[
c]*Sqrt[d]*Sqrt[e]*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])])/(8*c^(3/
2)*d^(3/2)*e^(3/2))))/(16*c*d*e)))/(24*c*d*e)
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1087
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^p/(2*c*(2*p + 1))), x] - Simp[p*((b^2 - 4*a*c)/(2*c*(2*
p + 1)))   Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] &&
GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[3*p])
```

rule 1092
```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[I
nt[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a
, b, c}, x]
```

Maple [A] (verified)

Time = 1.39 (sec) , antiderivative size = 340, normalized size of antiderivative = 1.17

method	result
default	$\frac{(2cdxe+a\,e^2+c\,d^2)\left(ade+\left(a\,e^2+c\,d^2\right)x+cd\,x^2e\right)^{\frac{5}{2}}}{12cde}+\frac{5\left(4ac\,d^2e^2-\left(a\,e^2+c\,d^2\right)^2\right)\left(\frac{\left(2cdxe+a\,e^2+c\,d^2\right)\left(ade+\left(a\,e^2+c\,d^2\right)x+cd\,x^2e\right)^{\frac{3}{2}}}{8cde}\right.}{}$

input
```
int((a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
1/12*(2*c*d*e*x+a*e^2+c*d^2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(5/2)/c/d/e
+5/24*(4*a*c*d^2*e^2-(a*e^2+c*d^2)^2)/d/e/c*(1/8*(2*c*d*e*x+a*e^2+c*d^2)*(
a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(3/2)/c/d/e+3/16*(4*a*c*d^2*e^2-(a*e^2+c*
d^2)^2)/d/e/c*(1/4*(2*c*d*e*x+a*e^2+c*d^2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*
e)^(1/2)/c/d/e+1/8*(4*a*c*d^2*e^2-(a*e^2+c*d^2)^2)/d/e/c*ln((1/2*a*e^2+1/2
*c*d^2+c*d*x*e)/(d*e*c)^(1/2)+(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(1/2))/(d*
e*c)^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 1034, normalized size of antiderivative = 3.57

$$\int \left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}\,dx=\text{Too large to display}$$

input
```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2),x, algorithm="fricas")
```

output
```
[1/6144*(15*(c^6*d^12 - 6*a*c^5*d^10*e^2 + 15*a^2*c^4*d^8*e^4 - 20*a^3*c^3
*d^6*e^6 + 15*a^4*c^2*d^4*e^8 - 6*a^5*c*d^2*e^10 + a^6*e^12)*sqrt(c*d*e)*l
og(8*c^2*d^2*e^2*x^2 + c^2*d^4 + 6*a*c*d^2*e^2 + a^2*e^4 - 4*sqrt(c*d*e*x^
2 + a*d*e + (c*d^2 + a*e^2)*x)*(2*c*d*e*x + c*d^2 + a*e^2)*sqrt(c*d*e) + 8
*(c^2*d^3*e + a*c*d*e^3)*x) + 4*(256*c^6*d^6*e^6*x^5 + 15*c^6*d^11*e - 85*
a*c^5*d^9*e^3 + 198*a^2*c^4*d^7*e^5 + 198*a^3*c^3*d^5*e^7 - 85*a^4*c^2*d^3
*e^9 + 15*a^5*c*d*e^11 + 640*(c^6*d^7*e^5 + a*c^5*d^5*e^7)*x^4 + 16*(27*c^
6*d^8*e^4 + 106*a*c^5*d^6*e^6 + 27*a^2*c^4*d^4*e^8)*x^3 + 8*(c^6*d^9*e^3 +
 159*a*c^5*d^7*e^5 + 159*a^2*c^4*d^5*e^7 + a^3*c^3*d^3*e^9)*x^2 - 2*(5*c^6
*d^10*e^2 - 28*a*c^5*d^8*e^4 - 594*a^2*c^4*d^6*e^6 - 28*a^3*c^3*d^4*e^8 +
5*a^4*c^2*d^2*e^10)*x)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x))/(c^4*d
^4*e^4), 1/3072*(15*(c^6*d^12 - 6*a*c^5*d^10*e^2 + 15*a^2*c^4*d^8*e^4 - 20
*a^3*c^3*d^6*e^6 + 15*a^4*c^2*d^4*e^8 - 6*a^5*c*d^2*e^10 + a^6*e^12)*sqrt(
-c*d*e)*arctan(1/2*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(2*c*d*e*x
+ c*d^2 + a*e^2)*sqrt(-c*d*e)/(c^2*d^2*e^2*x^2 + a*c*d^2*e^2 + (c^2*d^3*e
+ a*c*d*e^3)*x)) + 2*(256*c^6*d^6*e^6*x^5 + 15*c^6*d^11*e - 85*a*c^5*d^9*e
^3 + 198*a^2*c^4*d^7*e^5 + 198*a^3*c^3*d^5*e^7 - 85*a^4*c^2*d^3*e^9 + 15*a
^5*c*d*e^11 + 640*(c^6*d^7*e^5 + a*c^5*d^5*e^7)*x^4 + 16*(27*c^6*d^8*e^4 +
 106*a*c^5*d^6*e^6 + 27*a^2*c^4*d^4*e^8)*x^3 + 8*(c^6*d^9*e^3 + 159*a*c^5*
d^7*e^5 + 159*a^2*c^4*d^5*e^7 + a^3*c^3*d^3*e^9)*x^2 - 2*(5*c^6*d^10*e^...
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 654 vs. $2(282) = 564$.

Time = 7.84 (sec) , antiderivative size = 6698, normalized size of antiderivative = 23.10

$$\int \left(ade + \left(cd^2 + ae^2\right)x + cdex^2\right)^{5/2} dx = \text{Too large to display}$$

input
```
integrate((a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(5/2),x)
```

output
```
a**2*d**2*e**2*Piecewise(((x/2 + (a*e**2/4 + c*d**2/4)/(c*d*e))*sqrt(a*d*e
 + c*d*e*x**2 + x*(a*e**2 + c*d**2)) + (a*d*e/2 - (a*e**2/4 + c*d**2/4)*(a
*e**2 + c*d**2)/(2*c*d*e))*Piecewise((log(a*e**2 + c*d**2 + 2*c*d*e*x + 2*
sqrt(c*d*e)*sqrt(a*d*e + c*d*e*x**2 + x*(a*e**2 + c*d**2)))/sqrt(c*d*e), N
e(a*d*e - (a*e**2 + c*d**2)**2/(4*c*d*e), 0)), ((x - (-a*e**2 - c*d**2)/(2
*c*d*e))*log(x - (-a*e**2 - c*d**2)/(2*c*d*e))/sqrt(c*d*e*(x - (-a*e**2 -
c*d**2)/(2*c*d*e))**2), True)), Ne(c*d*e, 0)), (2*(a*d*e + x*(a*e**2 + c*d
**2))**(3/2)/(3*(a*e**2 + c*d**2)), Ne(a*e**2 + c*d**2, 0)), (x*sqrt(a*d*e
), True)) + 2*a**2*d*e**3*Piecewise(((-a*(a*e**2/6 + c*d**2/6)/(2*c) - (a*
e**2 + c*d**2)*(a*d*e/3 - (a*e**2/6 + c*d**2/6)*(3*a*e**2/2 + 3*c*d**2/2)/
(2*c*d*e))/(2*c*d*e))*Piecewise((log(a*e**2 + c*d**2 + 2*c*d*e*x + 2*sqrt(
c*d*e)*sqrt(a*d*e + c*d*e*x**2 + x*(a*e**2 + c*d**2)))/sqrt(c*d*e), Ne(a*d
*e - (a*e**2 + c*d**2)**2/(4*c*d*e), 0)), ((x - (-a*e**2 - c*d**2)/(2*c*d*
e))*log(x - (-a*e**2 - c*d**2)/(2*c*d*e))/sqrt(c*d*e*(x - (-a*e**2 - c*d**
2)/(2*c*d*e))**2), True)) + (x**2/3 + x*(a*e**2/6 + c*d**2/6)/(2*c*d*e) +
(a*d*e/3 - (a*e**2/6 + c*d**2/6)*(3*a*e**2/2 + 3*c*d**2/2)/(2*c*d*e))/(c*d
*e))*sqrt(a*d*e + c*d*e*x**2 + x*(a*e**2 + c*d**2)), Ne(c*d*e, 0)), (2*(-a
*d*e*(a*d*e + x*(a*e**2 + c*d**2))**(3/2)/3 + (a*d*e + x*(a*e**2 + c*d**2)
)**(5/2)/5)/(a*e**2 + c*d**2)**2, Ne(a*e**2 + c*d**2, 0)), (x**2*sqrt(a*d*
e)/2, True)) + a**2*e**4*Piecewise(((-a*(a*d*e/4 - (a*e**2/8 + c*d**2/8...
```

Maxima [F(-2)]

Exception generated.

$$\int \left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}\,dx=\text{Exception raised: ValueError}$$

input
```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

Giac [A] (verification not implemented)

Time = 0.50 (sec) , antiderivative size = 524, normalized size of antiderivative = 1.81

$$\int \left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}\,dx=\frac{1}{1536}\sqrt{cdex^2+cd^2x+ae^2x+ade}\left(2\left(4\left(2\left(8\left(2\,c^2d^2e^2x+\frac{5\left(c^7d^8e^6+ac^6d^6e^8\right)}{c^5d^5e^5}\right)x+\right.\right.\right.\right.\right.$$
$$+\frac{5\left(c^6d^{12}-6\,ac^5d^{10}e^2+15\,a^2c^4d^8e^4-20\,a^3c^3d^6e^6+15\,a^4c^2d^4e^8-6\,a^5cd^2e^{10}+a^6e^{12}\right)\log\left(\left|-cd^2-ae^2-\right.\right.}{1024\sqrt{cde}c^3d^3e^3}$$

input
```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2),x, algorithm="giac")
```

output
```
1/1536*sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*x + a*d*e)*(2*(4*(2*(8*(2*c^2*d^2*
e^2*x + 5*(c^7*d^8*e^6 + a*c^6*d^6*e^8)/(c^5*d^5*e^5))*x + (27*c^7*d^9*e^5
 + 106*a*c^6*d^7*e^7 + 27*a^2*c^5*d^5*e^9)/(c^5*d^5*e^5))*x + (c^7*d^10*e^
4 + 159*a*c^6*d^8*e^6 + 159*a^2*c^5*d^6*e^8 + a^3*c^4*d^4*e^10)/(c^5*d^5*e
^5))*x - (5*c^7*d^11*e^3 - 28*a*c^6*d^9*e^5 - 594*a^2*c^5*d^7*e^7 - 28*a^3
*c^4*d^5*e^9 + 5*a^4*c^3*d^3*e^11)/(c^5*d^5*e^5))*x + (15*c^7*d^12*e^2 - 8
5*a*c^6*d^10*e^4 + 198*a^2*c^5*d^8*e^6 + 198*a^3*c^4*d^6*e^8 - 85*a^4*c^3*
d^4*e^10 + 15*a^5*c^2*d^2*e^12)/(c^5*d^5*e^5)) + 5/1024*(c^6*d^12 - 6*a*c^
5*d^10*e^2 + 15*a^2*c^4*d^8*e^4 - 20*a^3*c^3*d^6*e^6 + 15*a^4*c^2*d^4*e^8
- 6*a^5*c*d^2*e^10 + a^6*e^12)*log(abs(-c*d^2 - a*e^2 - 2*sqrt(c*d*e)*(sqr
t(c*d*e)*x - sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*x + a*d*e))))/(sqrt(c*d*e)*c
^3*d^3*e^3)
```

Mupad [B] (verification not implemented)

Time = 5.47 (sec) , antiderivative size = 319, normalized size of antiderivative = 1.10

$$\int \left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}\,dx=\frac{\left(\frac{c\,d^2}{2}+c\,x\,d\,e+\frac{a\,e^2}{2}\right)\left(c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)x+a\,d\,e\right)^{5/2}}{6\,c\,d\,e}-\frac{\left(\frac{5\left(c\,d^2+a\,e^2\right)^2}{4}-5\,a\,c\,d^2\,e^2\right)\left(\frac{\left(\frac{c\,d^2}{2}+c\,x\,d\,e+\frac{a\,e^2}{2}\right)\left(c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)x+a\,d\,e\right)^{3/2}}{4\,c\,d\,e}-\frac{\left(\frac{3\left(c\,d^2+a\,e^2\right)^2}{4}-3\,a\,c\,d^2\,e^2\right)\left(\left(\frac{x}{2}+\frac{c\,d^2+a}{4\,c\,d}\right.\right.}{}\right.}{6\,c\,d\,e}$$

input
```
int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(5/2),x)
```

output
```
(((a*e^2)/2 + (c*d^2)/2 + c*d*e*x)*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)
^(5/2))/(6*c*d*e) - (((5*(a*e^2 + c*d^2)^2)/4 - 5*a*c*d^2*e^2)*((((a*e^2)/
2 + (c*d^2)/2 + c*d*e*x)*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(3/2))/(4
*c*d*e) - (((3*(a*e^2 + c*d^2)^2)/4 - 3*a*c*d^2*e^2)*((x/2 + (a*e^2 + c*d^
2)/(4*c*d*e))*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2) - (log(2*((a*e
 + c*d*x)*(d + e*x))^(1/2)*(c*d*e)^(1/2) + a*e^2 + c*d^2 + 2*c*d*e*x)*((a*
e^2 + c*d^2)^2/4 - a*c*d^2*e^2))/(2*(c*d*e)^(3/2))))/(4*c*d*e)))/(6*c*d*e)
```

Reduce [B] (verification not implemented)

Time = 0.37 (sec) , antiderivative size = 1033, normalized size of antiderivative = 3.56

$$\int \left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}\,dx=\text{Too large to display}$$

input
```
int((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2),x)
```

output

```
(15*sqrt(d + e*x)*sqrt(a*e + c*d*x)*a**5*c*d*e**11 - 85*sqrt(d + e*x)*sqrt
(a*e + c*d*x)*a**4*c**2*d**3*e**9 - 10*sqrt(d + e*x)*sqrt(a*e + c*d*x)*a**
4*c**2*d**2*e**10*x + 198*sqrt(d + e*x)*sqrt(a*e + c*d*x)*a**3*c**3*d**5*e
**7 + 56*sqrt(d + e*x)*sqrt(a*e + c*d*x)*a**3*c**3*d**4*e**8*x + 8*sqrt(d
+ e*x)*sqrt(a*e + c*d*x)*a**3*c**3*d**3*e**9*x**2 + 198*sqrt(d + e*x)*sqrt
(a*e + c*d*x)*a**2*c**4*d**7*e**5 + 1188*sqrt(d + e*x)*sqrt(a*e + c*d*x)*a
**2*c**4*d**6*e**6*x + 1272*sqrt(d + e*x)*sqrt(a*e + c*d*x)*a**2*c**4*d**5
*e**7*x**2 + 432*sqrt(d + e*x)*sqrt(a*e + c*d*x)*a**2*c**4*d**4*e**8*x**3
- 85*sqrt(d + e*x)*sqrt(a*e + c*d*x)*a*c**5*d**9*e**3 + 56*sqrt(d + e*x)*s
qrt(a*e + c*d*x)*a*c**5*d**8*e**4*x + 1272*sqrt(d + e*x)*sqrt(a*e + c*d*x)
*a*c**5*d**7*e**5*x**2 + 1696*sqrt(d + e*x)*sqrt(a*e + c*d*x)*a*c**5*d**6*
e**6*x**3 + 640*sqrt(d + e*x)*sqrt(a*e + c*d*x)*a*c**5*d**5*e**7*x**4 + 15
*sqrt(d + e*x)*sqrt(a*e + c*d*x)*c**6*d**11*e - 10*sqrt(d + e*x)*sqrt(a*e
+ c*d*x)*c**6*d**10*e**2*x + 8*sqrt(d + e*x)*sqrt(a*e + c*d*x)*c**6*d**9*e
**3*x**2 + 432*sqrt(d + e*x)*sqrt(a*e + c*d*x)*c**6*d**8*e**4*x**3 + 640*s
qrt(d + e*x)*sqrt(a*e + c*d*x)*c**6*d**7*e**5*x**4 + 256*sqrt(d + e*x)*sqr
t(a*e + c*d*x)*c**6*d**6*e**6*x**5 - 15*sqrt(e)*sqrt(d)*sqrt(c)*log((sqrt(
e)*sqrt(a*e + c*d*x) + sqrt(d)*sqrt(c)*sqrt(d + e*x))/sqrt(a*e**2 - c*d**2
))*a**6*e**12 + 90*sqrt(e)*sqrt(d)*sqrt(c)*log((sqrt(e)*sqrt(a*e + c*d*x)
+ sqrt(d)*sqrt(c)*sqrt(d + e*x))/sqrt(a*e**2 - c*d**2))*a**5*c*d**2*e**...
```

3.224 $\int \frac{(ade+(cd^2+ae^2)x+cdex^2)^{5/2}}{d+ex} dx$

Optimal result

Integrand size = 37, antiderivative size = 373

$$\begin{aligned}\int \frac{(ade+(cd^2+ae^2)x+cdex^2)^{5/2}}{d+ex} dx &= \frac{3(cd^2-ae^2)^4\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{128c^2d^2e^3} \\ &- \frac{(cd^2-ae^2)^3(ade+(cd^2+ae^2)x+cdex^2)^{3/2}}{64c^2d^2e^2(d+ex)} \\ &+ \frac{(cd^2-ae^2)^2(ade+(cd^2+ae^2)x+cdex^2)^{5/2}}{80c^2d^2e(d+ex)^2} \\ &+ \frac{3(cd^2-ae^2)(ade+(cd^2+ae^2)x+cdex^2)^{7/2}}{40c^2d^2(d+ex)^3} + \frac{(ade+(cd^2+ae^2)x+cdex^2)^{7/2}}{5cd(d+ex)^2} \\ &- \frac{3(cd^2-ae^2)^5\operatorname{arctanh}\left(\frac{\sqrt{e}\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{\sqrt{c}\sqrt{d}(d+ex)}\right)}{128c^{5/2}d^{5/2}e^{7/2}}\end{aligned}$$

output
```
3/128*(-a*e^2+c*d^2)^4*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/c^2/d^2/e^3
-1/64*(-a*e^2+c*d^2)^3*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/c^2/d^2/e^2
/(e*x+d)+1/80*(-a*e^2+c*d^2)^2*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/c^2
/d^2/e/(e*x+d)^2+3/40*(-a*e^2+c*d^2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(7/
2)/c^2/d^2/(e*x+d)^3+1/5*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(7/2)/c/d/(e*x+
d)^2-3/128*(-a*e^2+c*d^2)^5*arctanh(e^(1/2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x
^2)^(1/2)/c^(1/2)/d^(1/2)/(e*x+d))/c^(5/2)/d^(5/2)/e^(7/2)
```

Mathematica [A] (verified)

Time = 0.85 (sec) , antiderivative size = 295, normalized size of antiderivative = 0.79

$$\int \frac{(ade + (cd^2 + ae^2)x + cdex^2)^{5/2}}{d + ex} \, dx = \frac{((ae + cdx)(d + ex))^{3/2} \left(\frac{\sqrt{c}\sqrt{d}\sqrt{e}(-15a^4e^8 + 10a^3cde^6(7d + ex) + 2a^2c^2d^2e^4(6}{} \right.}{}$$

input
```
Integrate[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2)/(d + e*x),x]
```

output
```
(((a*e + c*d*x)*(d + e*x))^(3/2)*((Sqrt[c]*Sqrt[d]*Sqrt[e]*(-15*a^4*e^8 +
10*a^3*c*d*e^6*(7*d + e*x) + 2*a^2*c^2*d^2*e^4*(64*d^2 + 233*d*e*x + 124*e
^2*x^2) + 2*a*c^3*d^3*e^2*(-35*d^3 + 23*d^2*e*x + 256*d*e^2*x^2 + 168*e^3*
x^3) + c^4*d^4*(15*d^4 - 10*d^3*e*x + 8*d^2*e^2*x^2 + 176*d*e^3*x^3 + 128*
e^4*x^4)))/((a*e + c*d*x)*(d + e*x)) - (15*(c*d^2 - a*e^2)^5*ArcTanh[(Sqrt
[c]*Sqrt[d]*Sqrt[d + e*x])/(Sqrt[e]*Sqrt[a*e + c*d*x])])/((a*e + c*d*x)^(3
/2)*(d + e*x)^(3/2))))/(640*c^(5/2)*d^(5/2)*e^(7/2))
```

Rubi [A] (verified)

Time = 0.66 (sec) , antiderivative size = 302, normalized size of antiderivative = 0.81, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.135$, Rules used = {1131, 1087, 1087, 1092, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(x(ae^2 + cd^2) + ade + cdex^2)^{5/2}}{d + ex} \, dx$$

$$\Big\downarrow \text{ 1131}$$

$$\frac{(x(ae^2 + cd^2) + ade + cdex^2)^{5/2}}{5e} - \frac{(cd^2 - ae^2) \int (cdex^2 + (cd^2 + ae^2)x + ade)^{3/2} dx}{2e}$$

$$\Big\downarrow \text{ 1087}$$

$$\frac{\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/2}}{5e}-\frac{\left(cd^2-ae^2\right)\left(\frac{\left(ae^2+cd^2+2cdex\right)\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{8cde}-\frac{3\left(cd^2-ae^2\right)^2\int\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}dx}{16cde}\right)}{2e}$$

$\downarrow$ 1087

$$\frac{\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/2}}{5e}-\frac{\left(cd^2-ae^2\right)\left(\frac{\left(ae^2+cd^2+2cdex\right)\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{8cde}-\frac{3\left(cd^2-ae^2\right)^2\left(\frac{\left(ae^2+cd^2+2cdex\right)\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{4cde}-\frac{\left(cd^2-ae^2\right)^2 \ldots}{\ldots}\right.}{16cde}\right.}{2e}$$

$\downarrow$ 1092

$$\frac{\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/2}}{5e}-\frac{\left(cd^2-ae^2\right)\left(\frac{\left(ae^2+cd^2+2cdex\right)\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{8cde}-\frac{3\left(cd^2-ae^2\right)^2\left(\frac{\left(ae^2+cd^2+2cdex\right)\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{4cde}-\frac{\left(cd^2-ae^2\right)^2 \ldots}{\ldots}\right.}{16cde}\right.}{2e}$$

$\downarrow$ 219

$$\frac{\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/2}}{5e}-\frac{\left(cd^2-ae^2\right)\left(\frac{\left(ae^2+cd^2+2cdex\right)\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{8cde}-\frac{3\left(cd^2-ae^2\right)^2\left(\frac{\left(ae^2+cd^2+2cdex\right)\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{4cde}-\frac{\left(cd^2-ae^2\right)^2 \ldots}{\ldots}\right.}{16cde}\right.}{2e}$$

input
```
Int[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2)/(d + e*x),x]
```

output
```
(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2)/(5*e) - ((c*d^2 - a*e^2)*(((
c*d^2 + a*e^2 + 2*c*d*e*x)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2))/
(8*c*d*e) - (3*(c*d^2 - a*e^2)^2*(((c*d^2 + a*e^2 + 2*c*d*e*x)*Sqrt[a*d*e
+ (c*d^2 + a*e^2)*x + c*d*e*x^2])/(4*c*d*e) - ((c*d^2 - a*e^2)^2*ArcTanh[(
c*d^2 + a*e^2 + 2*c*d*e*x)/(2*Sqrt[c]*Sqrt[d]*Sqrt[e]*Sqrt[a*d*e + (c*d^2
+ a*e^2)*x + c*d*e*x^2])])/(8*c^(3/2)*d^(3/2)*e^(3/2))))/(16*c*d*e)))/(2*e
)
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1087
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^p/(2*c*(2*p + 1))), x] - Simp[p*((b^2 - 4*a*c)/(2*c*(2*
p + 1)))   Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] &&
GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[3*p])
```

rule 1092
```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[I
nt[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a
, b, c}, x]
```

rule 1131
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(d + e*x)^(m + 1)*((a + b*x + c*x^2)^p/(e*(m + 2*p + 1))), x
] - Simp[p*((2*c*d - b*e)/(e^2*(m + 2*p + 1)))   Int[(d + e*x)^(m + 1)*(a +
 b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c*d^2 - b
*d*e + a*e^2, 0] && GtQ[p, 0] && (LeQ[-2, m, 0] || EqQ[m + p + 1, 0]) && Ne
Q[m + 2*p + 1, 0] && IntegerQ[2*p]
```

Maple [A] (verified)

Time = 1.72 (sec) , antiderivative size = 327, normalized size of antiderivative = 0.88

method	result
default	$\frac{\left(dec\left(x+\frac{d}{e}\right)^2+\left(a e^2-c d^2\right)\left(x+\frac{d}{e}\right)\right)^{\frac{5}{2}}}{5}+\frac{\left(a e^2-c d^2\right)\left(\frac{\left(2dec\left(x+\frac{d}{e}\right)+a e^2-c d^2\right)\left(dec\left(x+\frac{d}{e}\right)^2+\left(a e^2-c d^2\right)\left(x+\frac{d}{e}\right)\right)^{\frac{3}{2}}}{8dec}-\frac{3\left(a e^2-c d^2\right)^2\left(\frac{(2de \cdots}{\cdots}\right.}{\cdots}\right.}{\cdots}$

input
```
int((a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(5/2)/(e*x+d),x,method=_RETURNVERBOS
E)
```

output
```
1/e*(1/5*(d*e*c*(x+d/e)^2+(a*e^2-c*d^2)*(x+d/e))^(5/2)+1/2*(a*e^2-c*d^2)*(
1/8*(2*d*e*c*(x+d/e)+a*e^2-c*d^2)/d/e/c*(d*e*c*(x+d/e)^2+(a*e^2-c*d^2)*(x+
d/e))^(3/2)-3/16*(a*e^2-c*d^2)^2/d/e/c*(1/4*(2*d*e*c*(x+d/e)+a*e^2-c*d^2)/
d/e/c*(d*e*c*(x+d/e)^2+(a*e^2-c*d^2)*(x+d/e))^(1/2)-1/8*(a*e^2-c*d^2)^2/d/
e/c*ln((1/2*a*e^2-1/2*c*d^2+d*e*c*(x+d/e))/(d*e*c)^(1/2)+(d*e*c*(x+d/e)^2+
(a*e^2-c*d^2)*(x+d/e))^(1/2))/(d*e*c)^(1/2))))
```

Fricas [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 844, normalized size of antiderivative = 2.26

$$\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{d+ex}\,dx=\text{Too large to display}$$

input
```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(e*x+d),x, algorithm="fr
icas")
```

output
```
[-1/2560*(15*(c^5*d^10 - 5*a*c^4*d^8*e^2 + 10*a^2*c^3*d^6*e^4 - 10*a^3*c^2
*d^4*e^6 + 5*a^4*c*d^2*e^8 - a^5*e^10)*sqrt(c*d*e)*log(8*c^2*d^2*e^2*x^2 +
 c^2*d^4 + 6*a*c*d^2*e^2 + a^2*e^4 + 4*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a
*e^2)*x)*(2*c*d*e*x + c*d^2 + a*e^2)*sqrt(c*d*e) + 8*(c^2*d^3*e + a*c*d*e^
3)*x) - 4*(128*c^5*d^5*e^5*x^4 + 15*c^5*d^9*e - 70*a*c^4*d^7*e^3 + 128*a^2
*c^3*d^5*e^5 + 70*a^3*c^2*d^3*e^7 - 15*a^4*c*d*e^9 + 16*(11*c^5*d^6*e^4 +
21*a*c^4*d^4*e^6)*x^3 + 8*(c^5*d^7*e^3 + 64*a*c^4*d^5*e^5 + 31*a^2*c^3*d^3
*e^7)*x^2 - 2*(5*c^5*d^8*e^2 - 23*a*c^4*d^6*e^4 - 233*a^2*c^3*d^4*e^6 - 5*
a^3*c^2*d^2*e^8)*x)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x))/(c^3*d^3*
e^4), 1/1280*(15*(c^5*d^10 - 5*a*c^4*d^8*e^2 + 10*a^2*c^3*d^6*e^4 - 10*a^3
*c^2*d^4*e^6 + 5*a^4*c*d^2*e^8 - a^5*e^10)*sqrt(-c*d*e)*arctan(1/2*sqrt(c*
d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(2*c*d*e*x + c*d^2 + a*e^2)*sqrt(-c*d
*e)/(c^2*d^2*e^2*x^2 + a*c*d^2*e^2 + (c^2*d^3*e + a*c*d*e^3)*x)) + 2*(128*
c^5*d^5*e^5*x^4 + 15*c^5*d^9*e - 70*a*c^4*d^7*e^3 + 128*a^2*c^3*d^5*e^5 +
70*a^3*c^2*d^3*e^7 - 15*a^4*c*d*e^9 + 16*(11*c^5*d^6*e^4 + 21*a*c^4*d^4*e^
6)*x^3 + 8*(c^5*d^7*e^3 + 64*a*c^4*d^5*e^5 + 31*a^2*c^3*d^3*e^7)*x^2 - 2*(
5*c^5*d^8*e^2 - 23*a*c^4*d^6*e^4 - 233*a^2*c^3*d^4*e^6 - 5*a^3*c^2*d^2*e^8
)*x)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x))/(c^3*d^3*e^4)]
```

Sympy [A] (verification not implemented)

Time = 35.97 (sec) , antiderivative size = 3516, normalized size of antiderivative = 9.43

$$\int \frac{\left(ade + (cd^2 + ae^2)\,x + cdex^2\right)^{5/2}}{d + ex}\,dx = \text{Too large to display}$$

input
```
integrate((a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(5/2)/(e*x+d),x)
```

output
```
a**2*d*e**2*Piecewise(((x/2 + (a*e**2/4 + c*d**2/4)/(c*d*e))*sqrt(a*d*e +
c*d*e*x**2 + x*(a*e**2 + c*d**2)) + (a*d*e/2 - (a*e**2/4 + c*d**2/4)*(a*e*
*2 + c*d**2)/(2*c*d*e))*Piecewise((log(a*e**2 + c*d**2 + 2*c*d*e*x + 2*sqr
t(c*d*e)*sqrt(a*d*e + c*d*e*x**2 + x*(a*e**2 + c*d**2)))/sqrt(c*d*e), Ne(a
*d*e - (a*e**2 + c*d**2)**2/(4*c*d*e), 0)), ((x - (-a*e**2 - c*d**2)/(2*c*
d*e))*log(x - (-a*e**2 - c*d**2)/(2*c*d*e))/sqrt(c*d*e*(x - (-a*e**2 - c*d
**2)/(2*c*d*e))**2), True)), Ne(c*d*e, 0)), (2*(a*d*e + x*(a*e**2 + c*d**2
))**(3/2)/(3*(a*e**2 + c*d**2)), Ne(a*e**2 + c*d**2, 0)), (x*sqrt(a*d*e),
True)) + a**2*e**3*Piecewise(((-a*(a*e**2/6 + c*d**2/6)/(2*c) - (a*e**2 +
c*d**2)*(a*d*e/3 - (a*e**2/6 + c*d**2/6)*(3*a*e**2/2 + 3*c*d**2/2)/(2*c*d*
e))/(2*c*d*e))*Piecewise((log(a*e**2 + c*d**2 + 2*c*d*e*x + 2*sqrt(c*d*e)*
sqrt(a*d*e + c*d*e*x**2 + x*(a*e**2 + c*d**2)))/sqrt(c*d*e), Ne(a*d*e - (a
*e**2 + c*d**2)**2/(4*c*d*e), 0)), ((x - (-a*e**2 - c*d**2)/(2*c*d*e))*log
(x - (-a*e**2 - c*d**2)/(2*c*d*e))/sqrt(c*d*e*(x - (-a*e**2 - c*d**2)/(2*c
*d*e))**2), True)) + (x**2/3 + x*(a*e**2/6 + c*d**2/6)/(2*c*d*e) + (a*d*e/
3 - (a*e**2/6 + c*d**2/6)*(3*a*e**2/2 + 3*c*d**2/2)/(2*c*d*e))/(c*d*e))*sq
rt(a*d*e + c*d*e*x**2 + x*(a*e**2 + c*d**2)), Ne(c*d*e, 0)), (2*(-a*d*e*(a
*d*e + x*(a*e**2 + c*d**2))**(3/2)/3 + (a*d*e + x*(a*e**2 + c*d**2))**(5/2
)/5)/(a*e**2 + c*d**2)**2, Ne(a*e**2 + c*d**2, 0)), (x**2*sqrt(a*d*e)/2, T
rue)) + 2*a*c*d**2*e*Piecewise(((-a*(a*e**2/6 + c*d**2/6)/(2*c) - (a*e*...
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(ade + (cd^2 + ae^2)\, x + cdex^2)^{5/2}}{d + ex}\, dx = \text{Exception raised: ValueError}$$

input
```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(e*x+d),x, algorithm="ma
xima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

Giac [A] (verification not implemented)

Time = 0.51 (sec) , antiderivative size = 415, normalized size of antiderivative = 1.11

$$\int \frac{(ade+(cd^2+ae^2)x+cdex^2)^{5/2}}{d+ex}\,dx = \frac{1}{640}\sqrt{cdex^2+cd^2x+ae^2x+ade}\left(2\left(4\left(2\left(8\,c^2d^2ex+\frac{11\,c^6d^7e}{}\right.\right.\right.\right.$$
$$+\frac{3\,(c^5d^{10}-5\,ac^4d^8e^2+10\,a^2c^3d^6e^4-10\,a^3c^2d^4e^6+5\,a^4cd^2e^8-a^5e^{10})\log\left(\left|-cd^2-ae^2-2\sqrt{cde}\left(\sqrt{cde}x\right.\right.\right.}{256\sqrt{cde}c^2d^2e^3}$$

input
```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(e*x+d),x, algorithm="gi
ac")
```

output
```
1/640*sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*x + a*d*e)*(2*(4*(2*(8*c^2*d^2*e*x
+ (11*c^6*d^7*e^4 + 21*a*c^5*d^5*e^6)/(c^4*d^4*e^4))*x + (c^6*d^8*e^3 + 64
*a*c^5*d^6*e^5 + 31*a^2*c^4*d^4*e^7)/(c^4*d^4*e^4))*x - (5*c^6*d^9*e^2 - 2
3*a*c^5*d^7*e^4 - 233*a^2*c^4*d^5*e^6 - 5*a^3*c^3*d^3*e^8)/(c^4*d^4*e^4))*
x + (15*c^6*d^10*e - 70*a*c^5*d^8*e^3 + 128*a^2*c^4*d^6*e^5 + 70*a^3*c^3*d
^4*e^7 - 15*a^4*c^2*d^2*e^9)/(c^4*d^4*e^4)) + 3/256*(c^5*d^10 - 5*a*c^4*d^
8*e^2 + 10*a^2*c^3*d^6*e^4 - 10*a^3*c^2*d^4*e^6 + 5*a^4*c*d^2*e^8 - a^5*e^
10)*log(abs(-c*d^2 - a*e^2 - 2*sqrt(c*d*e)*(sqrt(c*d*e)*x - sqrt(c*d*e*x^2
 + c*d^2*x + a*e^2*x + a*d*e))))/(sqrt(c*d*e)*c^2*d^2*e^3)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(ade+(cd^2+ae^2)x+cdex^2)^{5/2}}{d+ex}\,dx = \int \frac{\left(c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e\right)^{5/2}}{d+e\,x}\,dx$$

input
```
int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(5/2)/(d + e*x),x)
```

output
```
int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(5/2)/(d + e*x), x)
```

Reduce [B] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 790, normalized size of antiderivative = 2.12

$$\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{d+ex}\,dx=\text{Too large to display}$$

input
```
int((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(e*x+d),x)
```

output
```
( - 15*sqrt(d + e*x)*sqrt(a*e + c*d*x)*a**4*c*d*e**9 + 70*sqrt(d + e*x)*sq
rt(a*e + c*d*x)*a**3*c**2*d**3*e**7 + 10*sqrt(d + e*x)*sqrt(a*e + c*d*x)*a
**3*c**2*d**2*e**8*x + 128*sqrt(d + e*x)*sqrt(a*e + c*d*x)*a**2*c**3*d**5*
e**5 + 466*sqrt(d + e*x)*sqrt(a*e + c*d*x)*a**2*c**3*d**4*e**6*x + 248*sqr
t(d + e*x)*sqrt(a*e + c*d*x)*a**2*c**3*d**3*e**7*x**2 - 70*sqrt(d + e*x)*s
qrt(a*e + c*d*x)*a*c**4*d**7*e**3 + 46*sqrt(d + e*x)*sqrt(a*e + c*d*x)*a*c
**4*d**6*e**4*x + 512*sqrt(d + e*x)*sqrt(a*e + c*d*x)*a*c**4*d**5*e**5*x**
2 + 336*sqrt(d + e*x)*sqrt(a*e + c*d*x)*a*c**4*d**4*e**6*x**3 + 15*sqrt(d
+ e*x)*sqrt(a*e + c*d*x)*c**5*d**9*e - 10*sqrt(d + e*x)*sqrt(a*e + c*d*x)*
c**5*d**8*e**2*x + 8*sqrt(d + e*x)*sqrt(a*e + c*d*x)*c**5*d**7*e**3*x**2 +
 176*sqrt(d + e*x)*sqrt(a*e + c*d*x)*c**5*d**6*e**4*x**3 + 128*sqrt(d + e*
x)*sqrt(a*e + c*d*x)*c**5*d**5*e**5*x**4 + 15*sqrt(e)*sqrt(d)*sqrt(c)*log(
(sqrt(e)*sqrt(a*e + c*d*x) + sqrt(d)*sqrt(c)*sqrt(d + e*x))/sqrt(a*e**2 -
c*d**2))*a**5*e**10 - 75*sqrt(e)*sqrt(d)*sqrt(c)*log((sqrt(e)*sqrt(a*e + c
*d*x) + sqrt(d)*sqrt(c)*sqrt(d + e*x))/sqrt(a*e**2 - c*d**2))*a**4*c*d**2*
e**8 + 150*sqrt(e)*sqrt(d)*sqrt(c)*log((sqrt(e)*sqrt(a*e + c*d*x) + sqrt(d
)*sqrt(c)*sqrt(d + e*x))/sqrt(a*e**2 - c*d**2))*a**3*c**2*d**4*e**6 - 150*
sqrt(e)*sqrt(d)*sqrt(c)*log((sqrt(e)*sqrt(a*e + c*d*x) + sqrt(d)*sqrt(c)*s
qrt(d + e*x))/sqrt(a*e**2 - c*d**2))*a**2*c**3*d**6*e**4 + 75*sqrt(e)*sqrt
(d)*sqrt(c)*log((sqrt(e)*sqrt(a*e + c*d*x) + sqrt(d)*sqrt(c)*sqrt(d + e...
```

3.225 $\int \frac{(ade+(cd^2+ae^2)x+cdex^2)^{5/2}}{(d+ex)^2} dx$

Optimal result

Integrand size = 37, antiderivative size = 308

$$\int \frac{(ade+(cd^2+ae^2)x+cdex^2)^{5/2}}{(d+ex)^2} dx = \frac{5(cd^2-ae^2)^3\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{64cde^3} - \frac{5(cd^2-ae^2)^2(ade+(cd^2+ae^2)x+cdex^2)^{3/2}}{96cde^2(d+ex)} + \frac{\left(\frac{d}{e}-\frac{ae}{cd}\right)(ade+(cd^2+ae^2)x+cdex^2)^{5/2}}{24(d+ex)^2} + \frac{(ade+(cd^2+ae^2)x+cdex^2)^{7/2}}{4cd(d+ex)^3} - \frac{5(cd^2-ae^2)^4\operatorname{arctanh}\left(\frac{\sqrt{e}\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{\sqrt{c}\sqrt{d}(d+ex)}\right)}{64c^{3/2}d^{3/2}e^{7/2}}$$

output
```
5/64*(-a*e^2+c*d^2)^3*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/c/d/e^3-5/96
*(-a*e^2+c*d^2)^2*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/c/d/e^2/(e*x+d)+
1/24*(d/e-a*e/c/d)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(e*x+d)^2+1/4*(
a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(7/2)/c/d/(e*x+d)^3-5/64*(-a*e^2+c*d^2)^4
*arctanh(e^(1/2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/c^(1/2)/d^(1/2)/(
e*x+d))/c^(3/2)/d^(3/2)/e^(7/2)
```

Mathematica [A] (verified)

Time = 0.55 (sec) , antiderivative size = 222, normalized size of antiderivative = 0.72

$$\int \frac{(ade + (cd^2 + ae^2)x + cdex^2)^{5/2}}{(d+ex)^2}\,dx = \frac{\sqrt{(ae+cdx)(d+ex)}\left(\sqrt{c}\sqrt{d}\sqrt{e}(15a^3e^6 + a^2cde^4(73d + 118ex)\right.}{}$$

input
```
Integrate[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2)/(d + e*x)^2,x]
```

output
```
(Sqrt[(a*e + c*d*x)*(d + e*x)]*(Sqrt[c]*Sqrt[d]*Sqrt[e]*(15*a^3*e^6 + a^2*
c*d*e^4*(73*d + 118*e*x) + a*c^2*d^2*e^2*(-55*d^2 + 36*d*e*x + 136*e^2*x^2
) + c^3*d^3*(15*d^3 - 10*d^2*e*x + 8*d*e^2*x^2 + 48*e^3*x^3)) - (15*(c*d^2
 - a*e^2)^4*ArcTanh[(Sqrt[c]*Sqrt[d]*Sqrt[d + e*x])/(Sqrt[e]*Sqrt[a*e + c*
d*x])])/(Sqrt[a*e + c*d*x]*Sqrt[d + e*x])))/(192*c^(3/2)*d^(3/2)*e^(7/2))
```

Rubi [A] (verified)

Time = 0.68 (sec) , antiderivative size = 279, normalized size of antiderivative = 0.91, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.162$, Rules used = {1127, 1134, 1160, 1087, 1092, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/2}}{(d+ex)^2}dx$$

$$\Big\downarrow \text{ 1127}$$

$$\int (ae+cdx)^2\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}dx$$

$$\Big\downarrow \text{ 1134}$$

$$\frac{(ae+cdx)\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{4e}-\frac{5\left(cd^2-ae^2\right)\int (ae+cdx)\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}dx}{8e}$$

$$\downarrow 1160$$

$$\frac{(ae+cdx)\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{4e}-\frac{5\left(cd^2-ae^2\right)\left(\frac{\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{3e}-\frac{\left(cd^2-ae^2\right)\int\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}dx}{2e}\right)}{8e}$$

$$\downarrow 1087$$

$$\frac{(ae+cdx)\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{4e}-\frac{5\left(cd^2-ae^2\right)\left(\frac{\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{3e}-\frac{\left(cd^2-ae^2\right)\left(\frac{\left(ae^2+cd^2+2cdex\right)\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{4cde}-\frac{\left(cd^2-ae^2\right)^2\int\frac{1}{\sqrt{cdex^2+\left(cd^2+ae^2 \cdots\right.}}}{8cde}\cdots\right.}{2e}\cdots\right.}{8e}$$

$$\downarrow 1092$$

$$\frac{(ae+cdx)\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{4e}-\frac{5\left(cd^2-ae^2\right)\left(\frac{\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{3e}-\frac{\left(cd^2-ae^2\right)\left(\frac{\left(ae^2+cd^2+2cdex\right)\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{4cde}-\frac{\left(cd^2-ae^2\right)^2\int\frac{1}{4cde-\frac{\left(cd^2+2cex \cdots\right.}{cdex^2+\left(cd^2 \cdots\right.}}}{\cdots}\cdots\right.}{2e}\cdots\right.}{8e}$$

$$\downarrow 219$$

$$\frac{(ae+cdx)\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{4e} - \frac{5\left(cd^2-ae^2\right)\left(\frac{\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{3e} - \frac{\left(cd^2-ae^2\right)\left(\frac{\left(ae^2+cd^2+2cdex\right)\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{4cde} - \frac{\left(cd^2-ae^2\right)^2\operatorname{arctanh}\left(\frac{}{2\sqrt{c}\sqrt{d}\sqrt{}}\right)}{8c^{3/2}d^{3/2}}\right)}{2e}\right)}{8e}$$

input
```
Int[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2)/(d + e*x)^2,x]
```

output
```
((a*e + c*d*x)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2))/(4*e) - (5*(
c*d^2 - a*e^2)*((a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2)/(3*e) - ((c*
d^2 - a*e^2)*(((c*d^2 + a*e^2 + 2*c*d*e*x)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x
+ c*d*e*x^2])/(4*c*d*e) - ((c*d^2 - a*e^2)^2*ArcTanh[(c*d^2 + a*e^2 + 2*c*
d*e*x)/(2*Sqrt[c]*Sqrt[d]*Sqrt[e]*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x
^2])])/(8*c^(3/2)*d^(3/2)*e^(3/2))))/(2*e)))/(8*e)
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1087
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^p/(2*c*(2*p + 1))), x] - Simp[p*((b^2 - 4*a*c)/(2*c*(2*
p + 1)))   Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] &&
GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[3*p])
```

rule 1092
```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[I
nt[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a
, b, c}, x]
```

rule 1127
```
Int[((d_) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Sy
mbol] :> Int[(a + b*x + c*x^2)^(m + p)/(a/d + c*(x/e))^m, x] /; FreeQ[{a, b
, c, d, e}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[m] && RationalQ
[p] && (LtQ[0, -m, p] || LtQ[p, -m, 0]) && NeQ[m, 2] && NeQ[m, -1]
```

rule 1134
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[e*(d + e*x)^(m - 1)*((a + b*x + c*x^2)^(p + 1)/(c*(m + 2*p +
 1))), x] + Simp[(m + p)*((2*c*d - b*e)/(c*(m + 2*p + 1)))   Int[(d + e*x)^
(m - 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[
c*d^2 - b*d*e + a*e^2, 0] && GtQ[m, 1] && NeQ[m + 2*p + 1, 0] && IntegerQ[2
*p]
```

rule 1160
```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[e*((a + b*x + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Simp[(2*c*d - b
*e)/(2*c)   Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
 && NeQ[p, -1]
```

Maple [A] (verified)

Time = 1.86 (sec) , antiderivative size = 408, normalized size of antiderivative = 1.32

method	result
default	$\frac{\frac{2\left(dec\left(x+\frac{d}{e}\right)^{2}+\left(a\,e^{2}-c\,d^{2}\right)\left(x+\frac{d}{e}\right)\right)^{\frac{7}{2}}}{3\left(a\,e^{2}-c\,d^{2}\right)\left(x+\frac{d}{e}\right)^{2}}-\frac{10dec\left(\frac{\left(dec\left(x+\frac{d}{e}\right)^{2}+\left(a\,e^{2}-c\,d^{2}\right)\left(x+\frac{d}{e}\right)\right)^{\frac{5}{2}}}{5}+\frac{\left(a\,e^{2}-c\,d^{2}\right)\left(\frac{\left(2dec\left(x+\frac{d}{e}\right)+a\,e^{2}-c\,d^{2}\right)\left(dec\left(x+\frac{d}{e}\right)\cdots\right.}{8dec}\cdots\right)}{\cdots}\right)}{\cdots}}{\cdots}$

input
```
int((a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(5/2)/(e*x+d)^2,x,method=_RETURNVERB
OSE)
```

output
```
1/e^2*(2/3/(a*e^2-c*d^2)/(x+d/e)^2*(d*e*c*(x+d/e)^2+(a*e^2-c*d^2)*(x+d/e))
^(7/2)-10/3*d*e*c/(a*e^2-c*d^2)*(1/5*(d*e*c*(x+d/e)^2+(a*e^2-c*d^2)*(x+d/e
))^(5/2)+1/2*(a*e^2-c*d^2)*(1/8*(2*d*e*c*(x+d/e)+a*e^2-c*d^2)/d/e/c*(d*e*c
*(x+d/e)^2+(a*e^2-c*d^2)*(x+d/e))^(3/2)-3/16*(a*e^2-c*d^2)^2/d/e/c*(1/4*(2
*d*e*c*(x+d/e)+a*e^2-c*d^2)/d/e/c*(d*e*c*(x+d/e)^2+(a*e^2-c*d^2)*(x+d/e))^
(1/2)-1/8*(a*e^2-c*d^2)^2/d/e/c*ln((1/2*a*e^2-1/2*c*d^2+d*e*c*(x+d/e))/(d*
e*c)^(1/2)+(d*e*c*(x+d/e)^2+(a*e^2-c*d^2)*(x+d/e))^(1/2))/(d*e*c)^(1/2))))
)
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 672, normalized size of antiderivative = 2.18

$$\int \frac{(ade + (cd^2 + ae^2)x + cdex^2)^{5/2}}{(d + ex)^2} dx = \left[\frac{15 (c^4d^8 - 4ac^3d^6e^2 + 6a^2c^2d^4e^4 - 4a^3cd^2e^6 + a^4e^8)\sqrt{cde} \log \left(\right.}{} \right.$$

input
```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(e*x+d)^2,x, algorithm="
fricas")
```

output
```
[1/768*(15*(c^4*d^8 - 4*a*c^3*d^6*e^2 + 6*a^2*c^2*d^4*e^4 - 4*a^3*c*d^2*e^
6 + a^4*e^8)*sqrt(c*d*e)*log(8*c^2*d^2*e^2*x^2 + c^2*d^4 + 6*a*c*d^2*e^2 +
 a^2*e^4 - 4*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(2*c*d*e*x + c*d^
2 + a*e^2)*sqrt(c*d*e) + 8*(c^2*d^3*e + a*c*d*e^3)*x) + 4*(48*c^4*d^4*e^4*
x^3 + 15*c^4*d^7*e - 55*a*c^3*d^5*e^3 + 73*a^2*c^2*d^3*e^5 + 15*a^3*c*d*e^
7 + 8*(c^4*d^5*e^3 + 17*a*c^3*d^3*e^5)*x^2 - 2*(5*c^4*d^6*e^2 - 18*a*c^3*d
^4*e^4 - 59*a^2*c^2*d^2*e^6)*x)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x
))/(c^2*d^2*e^4), 1/384*(15*(c^4*d^8 - 4*a*c^3*d^6*e^2 + 6*a^2*c^2*d^4*e^4
 - 4*a^3*c*d^2*e^6 + a^4*e^8)*sqrt(-c*d*e)*arctan(1/2*sqrt(c*d*e*x^2 + a*d
*e + (c*d^2 + a*e^2)*x)*(2*c*d*e*x + c*d^2 + a*e^2)*sqrt(-c*d*e)/(c^2*d^2*
e^2*x^2 + a*c*d^2*e^2 + (c^2*d^3*e + a*c*d*e^3)*x)) + 2*(48*c^4*d^4*e^4*x^
3 + 15*c^4*d^7*e - 55*a*c^3*d^5*e^3 + 73*a^2*c^2*d^3*e^5 + 15*a^3*c*d*e^7
+ 8*(c^4*d^5*e^3 + 17*a*c^3*d^3*e^5)*x^2 - 2*(5*c^4*d^6*e^2 - 18*a*c^3*d^4
*e^4 - 59*a^2*c^2*d^2*e^6)*x)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x))
/(c^2*d^2*e^4)]
```

Sympy [A] (verification not implemented)

Time = 137.23 (sec) , antiderivative size = 1416, normalized size of antiderivative = 4.60

$$\int \frac{\left(ade + (cd^2 + ae^2)\,x + cdex^2\right)^{5/2}}{(d+ex)^2}\,dx = \text{Too large to display}$$

input
```
integrate((a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(5/2)/(e*x+d)**2,x)
```

output
```
a**2*e**2*Piecewise(((x/2 + (a*e**2/4 + c*d**2/4)/(c*d*e))*sqrt(a*d*e + c*
d*e*x**2 + x*(a*e**2 + c*d**2)) + (a*d*e/2 - (a*e**2/4 + c*d**2/4)*(a*e**2
 + c*d**2)/(2*c*d*e))*Piecewise((log(a*e**2 + c*d**2 + 2*c*d*e*x + 2*sqrt(
c*d*e)*sqrt(a*d*e + c*d*e*x**2 + x*(a*e**2 + c*d**2)))/sqrt(c*d*e), Ne(a*d
*e - (a*e**2 + c*d**2)**2/(4*c*d*e), 0)), ((x - (-a*e**2 - c*d**2)/(2*c*d*
e))*log(x - (-a*e**2 - c*d**2)/(2*c*d*e))/sqrt(c*d*e*(x - (-a*e**2 - c*d**
2)/(2*c*d*e))**2), True)), Ne(c*d*e, 0)), (2*(a*d*e + x*(a*e**2 + c*d**2))
**(3/2)/(3*(a*e**2 + c*d**2)), Ne(a*e**2 + c*d**2, 0)), (x*sqrt(a*d*e), Tr
ue)) + 2*a*c*d*e*Piecewise(((-a*(a*e**2/6 + c*d**2/6)/(2*c) - (a*e**2 + c*
d**2)*(a*d*e/3 - (a*e**2/6 + c*d**2/6)*(3*a*e**2/2 + 3*c*d**2/2)/(2*c*d*e)
)/(2*c*d*e))*Piecewise((log(a*e**2 + c*d**2 + 2*c*d*e*x + 2*sqrt(c*d*e)*sq
rt(a*d*e + c*d*e*x**2 + x*(a*e**2 + c*d**2)))/sqrt(c*d*e), Ne(a*d*e - (a*e
**2 + c*d**2)**2/(4*c*d*e), 0)), ((x - (-a*e**2 - c*d**2)/(2*c*d*e))*log(x
 - (-a*e**2 - c*d**2)/(2*c*d*e))/sqrt(c*d*e*(x - (-a*e**2 - c*d**2)/(2*c*d
*e))**2), True)) + (x**2/3 + x*(a*e**2/6 + c*d**2/6)/(2*c*d*e) + (a*d*e/3
- (a*e**2/6 + c*d**2/6)*(3*a*e**2/2 + 3*c*d**2/2)/(2*c*d*e))/(c*d*e))*sqrt
(a*d*e + c*d*e*x**2 + x*(a*e**2 + c*d**2)), Ne(c*d*e, 0)), (2*(-a*d*e*(a*d
*e + x*(a*e**2 + c*d**2))**(3/2)/3 + (a*d*e + x*(a*e**2 + c*d**2))**(5/2)/
5)/(a*e**2 + c*d**2)**2, Ne(a*e**2 + c*d**2, 0)), (x**2*sqrt(a*d*e)/2, Tru
e)) + c**2*d**2*Piecewise(((-a*(a*d*e/4 - (a*e**2/8 + c*d**2/8)*(5*a*e*...
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(ade + (cd^2 + ae^2)x + cdex^2)^{5/2}}{(d+ex)^2} dx = \text{Exception raised: ValueError}$$

input
```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(e*x+d)^2,x, algorithm="
maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1319 vs. $2(276) = 552$.

Time = 0.89 (sec) , antiderivative size = 1319, normalized size of antiderivative = 4.28

$$\int \frac{(ade + (cd^2 + ae^2)x + cdex^2)^{5/2}}{(d + ex)^2} dx = \text{Too large to display}$$

input
```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(e*x+d)^2,x, algorithm="
giac")
```

output
```
1/192*(15*(c^4*d^8*sgn(1/(e*x + d))*sgn(e) - 4*a*c^3*d^6*e^2*sgn(1/(e*x +
d))*sgn(e) + 6*a^2*c^2*d^4*e^4*sgn(1/(e*x + d))*sgn(e) - 4*a^3*c*d^2*e^6*s
gn(1/(e*x + d))*sgn(e) + a^4*e^8*sgn(1/(e*x + d))*sgn(e))*arctan(sqrt(c*d*
e - c*d^2*e/(e*x + d) + a*e^3/(e*x + d))/sqrt(-c*d*e))/(sqrt(-c*d*e)*c*d*e
^3*abs(e)) + (15*sqrt(c*d*e - c*d^2*e/(e*x + d) + a*e^3/(e*x + d))*c^7*d^1
1*e^3*sgn(1/(e*x + d))*sgn(e) - 60*sqrt(c*d*e - c*d^2*e/(e*x + d) + a*e^3/
(e*x + d))*a*c^6*d^9*e^5*sgn(1/(e*x + d))*sgn(e) + 90*sqrt(c*d*e - c*d^2*e
/(e*x + d) + a*e^3/(e*x + d))*a^2*c^5*d^7*e^7*sgn(1/(e*x + d))*sgn(e) - 60
*sqrt(c*d*e - c*d^2*e/(e*x + d) + a*e^3/(e*x + d))*a^3*c^4*d^5*e^9*sgn(1/(
e*x + d))*sgn(e) + 15*sqrt(c*d*e - c*d^2*e/(e*x + d) + a*e^3/(e*x + d))*a^
4*c^3*d^3*e^11*sgn(1/(e*x + d))*sgn(e) - 55*(c*d*e - c*d^2*e/(e*x + d) + a
*e^3/(e*x + d))^(3/2)*c^6*d^10*e^2*sgn(1/(e*x + d))*sgn(e) + 220*(c*d*e -
c*d^2*e/(e*x + d) + a*e^3/(e*x + d))^(3/2)*a*c^5*d^8*e^4*sgn(1/(e*x + d))*
sgn(e) - 330*(c*d*e - c*d^2*e/(e*x + d) + a*e^3/(e*x + d))^(3/2)*a^2*c^4*d
^6*e^6*sgn(1/(e*x + d))*sgn(e) + 220*(c*d*e - c*d^2*e/(e*x + d) + a*e^3/(e
*x + d))^(3/2)*a^3*c^3*d^4*e^8*sgn(1/(e*x + d))*sgn(e) - 55*(c*d*e - c*d^2
*e/(e*x + d) + a*e^3/(e*x + d))^(3/2)*a^4*c^2*d^2*e^10*sgn(1/(e*x + d))*sg
n(e) + 73*(c*d*e - c*d^2*e/(e*x + d) + a*e^3/(e*x + d))^(5/2)*c^5*d^9*e*sg
n(1/(e*x + d))*sgn(e) - 292*(c*d*e - c*d^2*e/(e*x + d) + a*e^3/(e*x + d))^
(5/2)*a*c^4*d^7*e^3*sgn(1/(e*x + d))*sgn(e) + 438*(c*d*e - c*d^2*e/(e*x...
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(ade+(cd^2+ae^2)x+cdex^2)^{5/2}}{(d+ex)^2}\,dx = \int \frac{\left(c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e\right)^{5/2}}{\left(d+e\,x\right)^2}\,dx$$

input
```
int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(5/2)/(d + e*x)^2,x)
```

output
```
int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(5/2)/(d + e*x)^2, x)
```

Reduce [B] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 579, normalized size of antiderivative = 1.88

$$\int \frac{(ade+(cd^2+ae^2)x+cdex^2)^{5/2}}{(d+ex)^2}\,dx = \frac{15\sqrt{ex+d}\,\sqrt{cdx+ae}\,a^3cd\,e^7+73\sqrt{ex+d}\,\sqrt{cdx+ae}\,a^2c^2d^3e^5}{}$$

input
```
int((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(e*x+d)^2,x)
```

output
```
(15*sqrt(d + e*x)*sqrt(a*e + c*d*x)*a**3*c*d*e**7 + 73*sqrt(d + e*x)*sqrt(
a*e + c*d*x)*a**2*c**2*d**3*e**5 + 118*sqrt(d + e*x)*sqrt(a*e + c*d*x)*a**
2*c**2*d**2*e**6*x - 55*sqrt(d + e*x)*sqrt(a*e + c*d*x)*a*c**3*d**5*e**3 +
 36*sqrt(d + e*x)*sqrt(a*e + c*d*x)*a*c**3*d**4*e**4*x + 136*sqrt(d + e*x)
*sqrt(a*e + c*d*x)*a*c**3*d**3*e**5*x**2 + 15*sqrt(d + e*x)*sqrt(a*e + c*d
*x)*c**4*d**7*e - 10*sqrt(d + e*x)*sqrt(a*e + c*d*x)*c**4*d**6*e**2*x + 8*
sqrt(d + e*x)*sqrt(a*e + c*d*x)*c**4*d**5*e**3*x**2 + 48*sqrt(d + e*x)*sqr
t(a*e + c*d*x)*c**4*d**4*e**4*x**3 - 15*sqrt(e)*sqrt(d)*sqrt(c)*log((sqrt(
e)*sqrt(a*e + c*d*x) + sqrt(d)*sqrt(c)*sqrt(d + e*x))/sqrt(a*e**2 - c*d**2
))*a**4*e**8 + 60*sqrt(e)*sqrt(d)*sqrt(c)*log((sqrt(e)*sqrt(a*e + c*d*x) +
 sqrt(d)*sqrt(c)*sqrt(d + e*x))/sqrt(a*e**2 - c*d**2))*a**3*c*d**2*e**6 -
90*sqrt(e)*sqrt(d)*sqrt(c)*log((sqrt(e)*sqrt(a*e + c*d*x) + sqrt(d)*sqrt(c
)*sqrt(d + e*x))/sqrt(a*e**2 - c*d**2))*a**2*c**2*d**4*e**4 + 60*sqrt(e)*s
qrt(d)*sqrt(c)*log((sqrt(e)*sqrt(a*e + c*d*x) + sqrt(d)*sqrt(c)*sqrt(d + e
*x))/sqrt(a*e**2 - c*d**2))*a*c**3*d**6*e**2 - 15*sqrt(e)*sqrt(d)*sqrt(c)*
log((sqrt(e)*sqrt(a*e + c*d*x) + sqrt(d)*sqrt(c)*sqrt(d + e*x))/sqrt(a*e**
2 - c*d**2))*c**4*d**8)/(192*c**2*d**2*e**4)
```

3.226 $\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{(d+ex)^3}\,dx$

Optimal result

Integrand size = 37, antiderivative size = 231

$$\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{(d+ex)^3}\,dx=\frac{5\left(cd^2-ae^2\right)^2\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{8e^3}+\frac{5\left(a-\frac{cd^2}{e^2}\right)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{12(d+ex)}+\frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{3e(d+ex)^2}-\frac{5\left(cd^2-ae^2\right)^3\operatorname{arctanh}\left(\frac{\sqrt{e}\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{\sqrt{c}\sqrt{d}(d+ex)}\right)}{8\sqrt{c}\sqrt{d}e^{7/2}}$$

output
```
5/8*(-a*e^2+c*d^2)^2*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/e^3+5*(a-c*d^
2/e^2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/(12*e*x+12*d)+1/3*(a*d*e+(a
*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/e/(e*x+d)^2-5/8*(-a*e^2+c*d^2)^3*arctanh(e^
(1/2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/c^(1/2)/d^(1/2)/(e*x+d))/c^(
1/2)/d^(1/2)/e^(7/2)
```

Mathematica [A] (verified)

Time = 0.55 (sec) , antiderivative size = 183, normalized size of antiderivative = 0.79

$$\int \frac{(ade+(cd^2+ae^2)x+cdex^2)^{5/2}}{(d+ex)^3}\,dx = \frac{\sqrt{e}(ae+cdx)(d+ex)\left(33a^2e^4+2acde^2(-20d+13ex)+c^2d^2(15\right.}{24e^{7/2}\sqrt{(ae}}$$

input
```
Integrate[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2)/(d + e*x)^3,x]
```

output
```
(Sqrt[e]*(a*e + c*d*x)*(d + e*x)*(33*a^2*e^4 + 2*a*c*d*e^2*(-20*d + 13*e*x
) + c^2*d^2*(15*d^2 - 10*d*e*x + 8*e^2*x^2)) - (15*(c*d^2 - a*e^2)^3*Sqrt[
a*e + c*d*x]*Sqrt[d + e*x]*ArcTanh[(Sqrt[c]*Sqrt[d]*Sqrt[d + e*x])/(Sqrt[e
]*Sqrt[a*e + c*d*x])])/(Sqrt[c]*Sqrt[d]))/(24*e^(7/2)*Sqrt[(a*e + c*d*x)*(
d + e*x)])
```

Rubi [A] (verified)

Time = 0.62 (sec) , antiderivative size = 264, normalized size of antiderivative = 1.14, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.135$, Rules used = {1130, 1131, 1087, 1092, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/2}}{(d+ex)^3}dx$$

$$\Big\downarrow \text{ 1130}$$

$$\frac{2\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/2}}{e(d+ex)^2}-\frac{5cd\int \frac{\left(cdex^2+\left(cd^2+ae^2\right)x+ade\right)^{3/2}}{d+ex}dx}{e}$$

$$\Big\downarrow \text{ 1131}$$

$$\frac{2\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/2}}{e(d+ex)^2}-\frac{5cd\left(\frac{\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{3e}-\frac{\left(cd^2-ae^2\right)\int\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}dx}{2e}\right)}{e}$$

$\Big\downarrow$ 1087

$$\frac{2\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/2}}{e(d+ex)^2}-\frac{5cd\left(\frac{\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{3e}-\frac{\left(cd^2-ae^2\right)\left(\frac{\left(ae^2+cd^2+2cdex\right)\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{4cde}-\frac{\left(cd^2-ae^2\right)^2\int\frac{1}{\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}dx}{8cde}\right)}{2e}\right)}{e}$$

$\Big\downarrow$ 1092

$$\frac{2\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/2}}{e(d+ex)^2}-\frac{5cd\left(\frac{\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{3e}-\frac{\left(cd^2-ae^2\right)\left(\frac{\left(ae^2+cd^2+2cdex\right)\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{4cde}-\frac{\left(cd^2-ae^2\right)^2\int\frac{1}{4cde-\frac{\left(cd^2+2cexd+ae^2\right)^2}{cdex^2+\left(cd^2+ae^2\right)x+ade}}d}{4cde}\right)}{2e}\right)}{e}$$

$\Big\downarrow$ 219

$$\frac{2\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/2}}{e(d+ex)^2}-\frac{5cd\left(\frac{\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{3e}-\frac{\left(cd^2-ae^2\right)\left(\frac{\left(ae^2+cd^2+2cdex\right)\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{4cde}-\frac{\left(cd^2-ae^2\right)^2\text{arctanh}\left(\frac{ae^2+cd^2+2c}{2\sqrt{c}\sqrt{d}\sqrt{e}\sqrt{x\left(ae^2+cd^2\right.}}\right.}{8c^{3/2}d^{3/2}e^{3/2}}\right.}{2e}\right.}{e}$$

input
```
Int[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2)/(d + e*x)^3,x]
```

output
```
(2*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2))/(e*(d + e*x)^2) - (5*c*d
*((a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2)/(3*e) - ((c*d^2 - a*e^2)*(
((c*d^2 + a*e^2 + 2*c*d*e*x)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/
(4*c*d*e) - ((c*d^2 - a*e^2)^2*ArcTanh[(c*d^2 + a*e^2 + 2*c*d*e*x)/(2*Sqrt
[c]*Sqrt[d]*Sqrt[e]*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])])/(8*c^(3
/2)*d^(3/2)*e^(3/2))))/(2*e)))/e
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1087
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^p/(2*c*(2*p + 1))), x] - Simp[p*((b^2 - 4*a*c)/(2*c*(2*
p + 1)))   Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] &&
GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[3*p])
```

rule 1092
```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[I
nt[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a
, b, c}, x]
```

rule 1130
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(d + e*x)^(m + 1)*((a + b*x + c*x^2)^p/(e*(m + p + 1))), x]
- Simp[c*(p/(e^2*(m + p + 1)))   Int[(d + e*x)^(m + 2)*(a + b*x + c*x^2)^(p
 - 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0]
 && GtQ[p, 0] && (LtQ[m, -2] || EqQ[m + 2*p + 1, 0]) && NeQ[m + p + 1, 0] &
& IntegerQ[2*p]
```

rule 1131
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(d + e*x)^(m + 1)*((a + b*x + c*x^2)^p/(e*(m + 2*p + 1))), x
] - Simp[p*((2*c*d - b*e)/(e^2*(m + 2*p + 1)))   Int[(d + e*x)^(m + 1)*(a +
 b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c*d^2 - b
*d*e + a*e^2, 0] && GtQ[p, 0] && (LeQ[-2, m, 0] || EqQ[m + p + 1, 0]) && Ne
Q[m + 2*p + 1, 0] && IntegerQ[2*p]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 488 vs. $2(206) = 412$.

Time = 2.14 (sec) , antiderivative size = 489, normalized size of antiderivative = 2.12

method	result
default	$\frac{\frac{2\left(dec\left(x+\frac{d}{e}\right)^2+\left(a e^2-c d^2\right)\left(x+\frac{d}{e}\right)\right)^{\frac{7}{2}}}{\left(a e^2-c d^2\right)\left(x+\frac{d}{e}\right)^3}-\frac{8dec\left(\frac{2\left(dec\left(x+\frac{d}{e}\right)^2+\left(a e^2-c d^2\right)\left(x+\frac{d}{e}\right)\right)^{\frac{7}{2}}}{3\left(a e^2-c d^2\right)\left(x+\frac{d}{e}\right)^2}-\frac{10dec\left(\frac{\left(dec\left(x+\frac{d}{e}\right)^2+\left(a e^2-c d^2\right)\left(x+\frac{d}{e}\right)\right)^{\frac{5}{2}}}{5}+\cdots\right.}{\cdots}\right.}{\cdots}}{\cdots}$

input
```
int((a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(5/2)/(e*x+d)^3,x,method=_RETURNVERB
OSE)
```

output
```
1/e^3*(2/(a*e^2-c*d^2)/(x+d/e)^3*(d*e*c*(x+d/e)^2+(a*e^2-c*d^2)*(x+d/e))^(
7/2)-8*d*e*c/(a*e^2-c*d^2)*(2/3/(a*e^2-c*d^2)/(x+d/e)^2*(d*e*c*(x+d/e)^2+(
a*e^2-c*d^2)*(x+d/e))^(7/2)-10/3*d*e*c/(a*e^2-c*d^2)*(1/5*(d*e*c*(x+d/e)^2
+(a*e^2-c*d^2)*(x+d/e))^(5/2)+1/2*(a*e^2-c*d^2)*(1/8*(2*d*e*c*(x+d/e)+a*e^
2-c*d^2)/d/e/c*(d*e*c*(x+d/e)^2+(a*e^2-c*d^2)*(x+d/e))^(3/2)-3/16*(a*e^2-c
*d^2)^2/d/e/c*(1/4*(2*d*e*c*(x+d/e)+a*e^2-c*d^2)/d/e/c*(d*e*c*(x+d/e)^2+(a
*e^2-c*d^2)*(x+d/e))^(1/2)-1/8*(a*e^2-c*d^2)^2/d/e/c*ln((1/2*a*e^2-1/2*c*d
^2+d*e*c*(x+d/e))/(d*e*c)^(1/2)+(d*e*c*(x+d/e)^2+(a*e^2-c*d^2)*(x+d/e))^(1
/2))/(d*e*c)^(1/2))))))
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 534, normalized size of antiderivative = 2.31

$$\int \frac{(ade + (cd^2 + ae^2)x + cdex^2)^{5/2}}{(d+ex)^3}\,dx = \left[-\frac{15\,(c^3d^6 - 3\,ac^2d^4e^2 + 3\,a^2cd^2e^4 - a^3e^6)\sqrt{cde}\log\left(8\,c^2d^2e^2x^2\right.}{}\right.$$

input
```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(e*x+d)^3,x, algorithm="
fricas")
```

output
```
[-1/96*(15*(c^3*d^6 - 3*a*c^2*d^4*e^2 + 3*a^2*c*d^2*e^4 - a^3*e^6)*sqrt(c*
d*e)*log(8*c^2*d^2*e^2*x^2 + c^2*d^4 + 6*a*c*d^2*e^2 + a^2*e^4 + 4*sqrt(c*
d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(2*c*d*e*x + c*d^2 + a*e^2)*sqrt(c*d*
e) + 8*(c^2*d^3*e + a*c*d*e^3)*x) - 4*(8*c^3*d^3*e^3*x^2 + 15*c^3*d^5*e -
40*a*c^2*d^3*e^3 + 33*a^2*c*d*e^5 - 2*(5*c^3*d^4*e^2 - 13*a*c^2*d^2*e^4)*x
)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x))/(c*d*e^4), 1/48*(15*(c^3*d^
6 - 3*a*c^2*d^4*e^2 + 3*a^2*c*d^2*e^4 - a^3*e^6)*sqrt(-c*d*e)*arctan(1/2*s
qrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(2*c*d*e*x + c*d^2 + a*e^2)*sqr
t(-c*d*e)/(c^2*d^2*e^2*x^2 + a*c*d^2*e^2 + (c^2*d^3*e + a*c*d*e^3)*x)) + 2
*(8*c^3*d^3*e^3*x^2 + 15*c^3*d^5*e - 40*a*c^2*d^3*e^3 + 33*a^2*c*d*e^5 - 2
*(5*c^3*d^4*e^2 - 13*a*c^2*d^2*e^4)*x)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a
*e^2)*x))/(c*d*e^4)]
```

Sympy [F]

$$\int \frac{(ade+(cd^2+ae^2)x+cdex^2)^{5/2}}{(d+ex)^3}\,dx=\int \frac{((d+ex)(ae+cdx))^{\frac{5}{2}}}{(d+ex)^3}\,dx$$

input
```
integrate((a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(5/2)/(e*x+d)**3,x)
```

output
```
Integral(((d + e*x)*(a*e + c*d*x))**(5/2)/(d + e*x)**3, x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(ade+(cd^2+ae^2)x+cdex^2)^{5/2}}{(d+ex)^3}\,dx=\text{Exception raised: ValueError}$$

input
```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(e*x+d)^3,x, algorithm="
maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e*(a*e^2-c*d^2)>0)', see `assume
?` for mor
```

Giac [A] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 235, normalized size of antiderivative = 1.02

$$\int \frac{(ade+(cd^2+ae^2)x+cdex^2)^{5/2}}{(d+ex)^3}\,dx=\frac{1}{24}\sqrt{cdex^2+cd^2x+ae^2x+ade}\left(2\left(\frac{4c^2d^2x}{e}-\frac{5c^4d^5e-13ac^3d\cdots}{c^2d^2e^3}\right.\right.$$
$$+\frac{5(c^3d^6-3ac^2d^4e^2+3a^2cd^2e^4-a^3e^6)\log\left(\left|-cd^2-ae^2-2\sqrt{cde}\left(\sqrt{cde}x-\sqrt{cdex^2+cd^2x+ae^2x+a\cdots}\right.\right.\right.}{16\sqrt{cde}e^3}$$

input
```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(e*x+d)^3,x, algorithm="
giac")
```

output
```
1/24*sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*x + a*d*e)*(2*(4*c^2*d^2*x/e - (5*c^
4*d^5*e - 13*a*c^3*d^3*e^3)/(c^2*d^2*e^3))*x + (15*c^4*d^6 - 40*a*c^3*d^4*
e^2 + 33*a^2*c^2*d^2*e^4)/(c^2*d^2*e^3)) + 5/16*(c^3*d^6 - 3*a*c^2*d^4*e^2
 + 3*a^2*c*d^2*e^4 - a^3*e^6)*log(abs(-c*d^2 - a*e^2 - 2*sqrt(c*d*e)*(sqrt
(c*d*e)*x - sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*x + a*d*e))))/(sqrt(c*d*e)*e^
3)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(ade+(cd^2+ae^2)\,x+cdex^2)^{5/2}}{(d+ex)^3}\,dx=\int \frac{{\left(c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e\right)}^{5/2}}{{\left(d+e\,x\right)}^3}\,dx$$

input
```
int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(5/2)/(d + e*x)^3,x)
```

output
```
int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(5/2)/(d + e*x)^3, x)
```

Reduce [B] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 400, normalized size of antiderivative = 1.73

$$\int \frac{(ade+(cd^2+ae^2)\,x+cdex^2)^{5/2}}{(d+ex)^3}\,dx=\frac{33\sqrt{ex+d}\,\sqrt{cdx+ae}\,a^2cd\,e^5-40\sqrt{ex+d}\,\sqrt{cdx+ae}\,a\,c^2d^3e^3+}{}$$

input
```
int((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(e*x+d)^3,x)
```

output

```
(33*sqrt(d + e*x)*sqrt(a*e + c*d*x)*a**2*c*d*e**5 - 40*sqrt(d + e*x)*sqrt(
a*e + c*d*x)*a*c**2*d**3*e**3 + 26*sqrt(d + e*x)*sqrt(a*e + c*d*x)*a*c**2*
d**2*e**4*x + 15*sqrt(d + e*x)*sqrt(a*e + c*d*x)*c**3*d**5*e - 10*sqrt(d +
 e*x)*sqrt(a*e + c*d*x)*c**3*d**4*e**2*x + 8*sqrt(d + e*x)*sqrt(a*e + c*d*
x)*c**3*d**3*e**3*x**2 + 15*sqrt(e)*sqrt(d)*sqrt(c)*log((sqrt(e)*sqrt(a*e
+ c*d*x) + sqrt(d)*sqrt(c)*sqrt(d + e*x))/sqrt(a*e**2 - c*d**2))*a**3*e**6
 - 45*sqrt(e)*sqrt(d)*sqrt(c)*log((sqrt(e)*sqrt(a*e + c*d*x) + sqrt(d)*sqr
t(c)*sqrt(d + e*x))/sqrt(a*e**2 - c*d**2))*a**2*c*d**2*e**4 + 45*sqrt(e)*s
qrt(d)*sqrt(c)*log((sqrt(e)*sqrt(a*e + c*d*x) + sqrt(d)*sqrt(c)*sqrt(d + e
*x))/sqrt(a*e**2 - c*d**2))*a*c**2*d**4*e**2 - 15*sqrt(e)*sqrt(d)*sqrt(c)*
log((sqrt(e)*sqrt(a*e + c*d*x) + sqrt(d)*sqrt(c)*sqrt(d + e*x))/sqrt(a*e**
2 - c*d**2))*c**3*d**6)/(24*c*d*e**4)
```

3.227 $\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{(d+ex)^4} \, dx$

Optimal result

Integrand size = 37, antiderivative size = 222

$$\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{(d+ex)^4} \, dx = \frac{15cd\left(a-\frac{cd^2}{e^2}\right)\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{4e} + \frac{5cd\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{2e^2(d+ex)} - \frac{2\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{e(d+ex)^3} + \frac{15\sqrt{c}\sqrt{d}\left(cd^2-ae^2\right)^2 \operatorname{arctanh}\left(\frac{\sqrt{e}\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{\sqrt{c}\sqrt{d}(d+ex)}\right)}{4e^{7/2}}$$

output
```
15/4*c*d*(a-c*d^2/e^2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/e+5/2*c*d*(
a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/e^2/(e*x+d)-2*(a*d*e+(a*e^2+c*d^2)*
x+c*d*e*x^2)^(5/2)/e/(e*x+d)^3+15/4*c^(1/2)*d^(1/2)*(-a*e^2+c*d^2)^2*arcta
nh(e^(1/2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/c^(1/2)/d^(1/2)/(e*x+d)
)/e^(7/2)
```

Mathematica [A] (verified)

Time = 0.49 (sec) , antiderivative size = 186, normalized size of antiderivative = 0.84

$$\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{(d+ex)^4}\,dx=\frac{((ae+cdx)(d+ex))^{5/2}\left(\frac{\sqrt{e}\left(-8a^2e^4+acde^2(25d+9ex)+c^2d^2\left(-15d^2-5dex+\right.\right.}{(ae+cdx)^2(d+ex)^3}\right.}{4e^{7/2}}$$

input
```
Integrate[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2)/(d + e*x)^4,x]
```

output
```
(((a*e + c*d*x)*(d + e*x))^(5/2)*((Sqrt[e]*(-8*a^2*e^4 + a*c*d*e^2*(25*d +
 9*e*x) + c^2*d^2*(-15*d^2 - 5*d*e*x + 2*e^2*x^2)))/((a*e + c*d*x)^2*(d +
e*x)^3) + (15*Sqrt[c]*Sqrt[d]*(c*d^2 - a*e^2)^2*ArcTanh[(Sqrt[c]*Sqrt[d]*S
qrt[d + e*x])/(Sqrt[e]*Sqrt[a*e + c*d*x])])/((a*e + c*d*x)^(5/2)*(d + e*x)
^(5/2))))/(4*e^(7/2))
```

Rubi [A] (verified)

Time = 0.96 (sec) , antiderivative size = 257, normalized size of antiderivative = 1.16, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.189, Rules used = {1125, 25, 2192, 27, 1160, 1092, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/2}}{(d+ex)^4}dx$$

$$\Big\downarrow \text{ 1125}$$

$$-\frac{\int -\frac{c^3d^3x^2e^5-c^2d^2\left(cd^2-3ae^2\right)xe^4+cd\left(c^2d^4-3ace^2d^2+3a^2e^4\right)e^3}{\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}dx}{e^6}-\frac{2\left(cd^2-ae^2\right)^2\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{e^3(d+ex)}$$

$$\Big\downarrow \text{ 25}$$

$$\frac{\frac{\int \frac{c^3d^3x^2e^5-c^2d^2(cd^2-3ae^2)xe^4+cd(c^2d^4-3ace^2d^2+3a^2e^4)e^3}{\sqrt{cdex^2+(cd^2+ae^2)x+ade}}dx}{e^6}-\frac{2(cd^2-ae^2)^2\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{e^3(d+ex)}}{}$$

$$\Big\downarrow \; 2192$$

$$\frac{\frac{\int \frac{c^2d^2e^4\left(2\left(2cd^2-3ae^2\right)\left(cd^2-2ae^2\right)-cde\left(7cd^2-9ae^2\right)x\right)}{2\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}dx}{2cde}+\frac{1}{2}c^2d^2e^4x\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{e^6}-\frac{2(cd^2-ae^2)^2\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{e^3(d+ex)}$$

$$\Big\downarrow \; 27$$

$$\frac{\frac{1}{4}cde^3\int \frac{2(2cd^2-3ae^2)(cd^2-2ae^2)-cde(7cd^2-9ae^2)x}{\sqrt{cdex^2+(cd^2+ae^2)x+ade}}dx+\frac{1}{2}c^2d^2e^4x\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{e^6}-\frac{2(cd^2-ae^2)^2\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{e^3(d+ex)}$$

$$\Big\downarrow \; 1160$$

$$\frac{\frac{1}{4}cde^3\left(\frac{15}{2}(cd^2-ae^2)^2\int \frac{1}{\sqrt{cdex^2+(cd^2+ae^2)x+ade}}dx-(7cd^2-9ae^2)\sqrt{x(ae^2+cd^2)+ade+cdex^2}\right)+\frac{1}{2}c^2d^2e^4x\sqrt{}}{e^6}$$

$$\frac{2(cd^2-ae^2)^2\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{e^3(d+ex)}$$

$$\Big\downarrow \; 1092$$

$$\frac{\frac{1}{4}cde^3\left(15(cd^2-ae^2)^2\int \frac{1}{4cde-\frac{\left(cd^2+2cexd+ae^2\right)^2}{cdex^2+\left(cd^2+ae^2\right)x+ade}}d\frac{cd^2+2cexd+ae^2}{\sqrt{cdex^2+(cd^2+ae^2)x+ade}}-(7cd^2-9ae^2)\sqrt{x(ae^2+cd^2)+ade}\right.}{e^6}$$

$$\frac{2(cd^2-ae^2)^2\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{e^3(d+ex)}$$

$$\Big\downarrow \; 219$$

$$\frac{\frac{1}{4}cde^3\left(\frac{15(cd^2-ae^2)^2\text{arctanh}\left(\frac{ae^2+cd^2+2cdex}{2\sqrt{c}\sqrt{d}\sqrt{e}\sqrt{x(ae^2+cd^2)+ade+cdex^2}}\right)}{2\sqrt{c}\sqrt{d}\sqrt{e}}-(7cd^2-9ae^2)\sqrt{x(ae^2+cd^2)+ade+cdex^2}\right)+\frac{1}{2}c}{e^6}$$

$$\frac{2(cd^2-ae^2)^2\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{e^3(d+ex)}$$

input
```
Int[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2)/(d + e*x)^4,x]
```

output
```
(-2*(c*d^2 - a*e^2)^2*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(e^3*(d
 + e*x)) + ((c^2*d^2*e^4*x*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/2
+ (c*d*e^3*(-((7*c*d^2 - 9*a*e^2)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x
^2]) + (15*(c*d^2 - a*e^2)^2*ArcTanh[(c*d^2 + a*e^2 + 2*c*d*e*x)/(2*Sqrt[c
]*Sqrt[d]*Sqrt[e]*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])])/(2*Sqrt[c
]*Sqrt[d]*Sqrt[e])))/4)/e^6
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1092
```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[I
nt[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a
, b, c}, x]
```

rule 1125
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_
Symbol] :> Simp[-2*e^(2*m + 3)*(Sqrt[a + b*x + c*x^2]/((-2*c*d + b*e)^(m +
2)*(d + e*x))), x] - Simp[e^(2*m + 2)   Int[(1/Sqrt[a + b*x + c*x^2])*Expan
dToSum[((-2*c*d + b*e)^(-m - 1) - ((-c)*d + b*e + c*e*x)^(-m - 1))/(d + e*x
), x], x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0]
 && ILtQ[m, 0] && EqQ[m + p, -3/2]
```

rule 1160
```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[e*((a + b*x + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Simp[(2*c*d - b
*e)/(2*c)   Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
 && NeQ[p, -1]
```

rule 2192
```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{q =
Expon[Pq, x], e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[e*x^(q - 1)*((a + b*x +
 c*x^2)^(p + 1)/(c*(q + 2*p + 1))), x] + Simp[1/(c*(q + 2*p + 1))   Int[(a
+ b*x + c*x^2)^p*ExpandToSum[c*(q + 2*p + 1)*Pq - a*e*(q - 1)*x^(q - 2) - b
*e*(q + p)*x^(q - 1) - c*e*(q + 2*p + 1)*x^q, x], x], x]] /; FreeQ[{a, b, c
, p}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] &&  !LeQ[p, -1]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 569 vs. $2(196) = 392$.

Time = 2.52 (sec) , antiderivative size = 570, normalized size of antiderivative = 2.57

method	result
	$6dec\left(\frac{2\left(dec\left(x+\frac{d}{e}\right)^2+\left(a\,e^2-c\,d^2\right)\left(x+\frac{d}{e}\right)\right)^{\frac{7}{2}}}{\left(a\,e^2-c\,d^2\right)\left(x+\frac{d}{e}\right)^3}-\frac{8dec\left(\frac{2\left(dec\left(x+\frac{d}{e}\right)^2+\left(a\,e^2-c\,d^2\right)\left(x+\frac{d}{e}\right)\right)^{\frac{7}{2}}}{3\left(a\,e^2-c\,d^2\right)\left(x+\frac{d}{e}\right)^2}-\cdots\right)}{\cdots}\right.$ 1

input
```
int((a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(5/2)/(e*x+d)^4,x,method=_RETURNVERB
OSE)
```

output
```
1/e^4*(-2/(a*e^2-c*d^2)/(x+d/e)^4*(d*e*c*(x+d/e)^2+(a*e^2-c*d^2)*(x+d/e))^
(7/2)+6*d*e*c/(a*e^2-c*d^2)*(2/(a*e^2-c*d^2)/(x+d/e)^3*(d*e*c*(x+d/e)^2+(a
*e^2-c*d^2)*(x+d/e))^(7/2)-8*d*e*c/(a*e^2-c*d^2)*(2/3/(a*e^2-c*d^2)/(x+d/e
)^2*(d*e*c*(x+d/e)^2+(a*e^2-c*d^2)*(x+d/e))^(7/2)-10/3*d*e*c/(a*e^2-c*d^2)
*(1/5*(d*e*c*(x+d/e)^2+(a*e^2-c*d^2)*(x+d/e))^(5/2)+1/2*(a*e^2-c*d^2)*(1/8
*(2*d*e*c*(x+d/e)+a*e^2-c*d^2)/d/e/c*(d*e*c*(x+d/e)^2+(a*e^2-c*d^2)*(x+d/e
))^(3/2)-3/16*(a*e^2-c*d^2)^2/d/e/c*(1/4*(2*d*e*c*(x+d/e)+a*e^2-c*d^2)/d/e
/c*(d*e*c*(x+d/e)^2+(a*e^2-c*d^2)*(x+d/e))^(1/2)-1/8*(a*e^2-c*d^2)^2/d/e/c
*ln((1/2*a*e^2-1/2*c*d^2+d*e*c*(x+d/e))/(d*e*c)^(1/2)+(d*e*c*(x+d/e)^2+(a*
e^2-c*d^2)*(x+d/e))^(1/2))/(d*e*c)^(1/2)))))))
```

Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 554, normalized size of antiderivative = 2.50

$$\int \frac{(ade + (cd^2 + ae^2)x + cdex^2)^{5/2}}{(d+ex)^4}\,dx = \left[\frac{15\,(c^2d^5 - 2\,acd^3e^2 + a^2de^4 + (c^2d^4e - 2\,acd^2e^3 + a^2e^5)x)\sqrt{\frac{cd}{e}}\,\cdots}{\cdots}\right.$$

$$-\frac{15\,(c^2d^5 - 2\,acd^3e^2 + a^2de^4 + (c^2d^4e - 2\,acd^2e^3 + a^2e^5)x)\sqrt{-\frac{cd}{e}}\arctan\left(\frac{\sqrt{cdex^2+ade+(cd^2+ae^2)x}\,(2\,cdex+cd^2+\cdots}{2\,(c^2d^2ex^2+acd^2e+(c^2d^3+acde^2)\cdots}\right.}{8\,(e\cdots}$$

input
```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(e*x+d)^4,x, algorithm="
fricas")
```

output
```
[1/16*(15*(c^2*d^5 - 2*a*c*d^3*e^2 + a^2*d*e^4 + (c^2*d^4*e - 2*a*c*d^2*e^
3 + a^2*e^5)*x)*sqrt(c*d/e)*log(8*c^2*d^2*e^2*x^2 + c^2*d^4 + 6*a*c*d^2*e^
2 + a^2*e^4 + 4*(2*c*d*e^2*x + c*d^2*e + a*e^3)*sqrt(c*d*e*x^2 + a*d*e + (
c*d^2 + a*e^2)*x)*sqrt(c*d/e) + 8*(c^2*d^3*e + a*c*d*e^3)*x) + 4*(2*c^2*d^
2*e^2*x^2 - 15*c^2*d^4 + 25*a*c*d^2*e^2 - 8*a^2*e^4 - (5*c^2*d^3*e - 9*a*c
*d*e^3)*x)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x))/(e^4*x + d*e^3), -
1/8*(15*(c^2*d^5 - 2*a*c*d^3*e^2 + a^2*d*e^4 + (c^2*d^4*e - 2*a*c*d^2*e^3
+ a^2*e^5)*x)*sqrt(-c*d/e)*arctan(1/2*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*
e^2)*x)*(2*c*d*e*x + c*d^2 + a*e^2)*sqrt(-c*d/e)/(c^2*d^2*e*x^2 + a*c*d^2*
e + (c^2*d^3 + a*c*d*e^2)*x)) - 2*(2*c^2*d^2*e^2*x^2 - 15*c^2*d^4 + 25*a*c
*d^2*e^2 - 8*a^2*e^4 - (5*c^2*d^3*e - 9*a*c*d*e^3)*x)*sqrt(c*d*e*x^2 + a*d
*e + (c*d^2 + a*e^2)*x))/(e^4*x + d*e^3)]
```

Sympy [F]

$$\int \frac{(ade + (cd^2 + ae^2)x + cdex^2)^{5/2}}{(d+ex)^4}\,dx = \int \frac{((d+ex)(ae+cdx))^{\frac{5}{2}}}{(d+ex)^4}\,dx$$

input
```
integrate((a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(5/2)/(e*x+d)**4,x)
```

output
```
Integral(((d + e*x)*(a*e + c*d*x))**(5/2)/(d + e*x)**4, x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(ade + (cd^2 + ae^2)x + cdex^2)^{5/2}}{(d+ex)^4}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(e*x+d)^4,x, algorithm="
maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e*(a*e^2-c*d^2)>0)', see `assume
?` for mor
```

Giac [A] (verification not implemented)

Time = 0.48 (sec) , antiderivative size = 302, normalized size of antiderivative = 1.36

$$\int \frac{(ade+(cd^2+ae^2)x+cdex^2)^{5/2}}{(d+ex)^4}\,dx = \frac{1}{4}\sqrt{cdex^2+cd^2x+ae^2x+ade}\left(\frac{2\,c^2d^2x}{e^2}-\frac{7\,c^3d^4e^5-9\,ac^2d^2e^7}{cde^8}\right)$$
$$-\frac{15\,(c^3d^5-2\,ac^2d^3e^2+a^2cde^4)\log\left(\left|-cd^2-ae^2-2\,\sqrt{cde}\left(\sqrt{cde}x-\sqrt{cdex^2+cd^2x+ae^2x+ade}\right)\right|\right)}{8\,\sqrt{cde}e^3}$$
$$-\frac{2\left(\sqrt{cde}c^3d^6-3\,\sqrt{cde}ac^2d^4e^2+3\,\sqrt{cde}a^2cd^2e^4-\sqrt{cde}a^3e^6\right)}{\sqrt{cde}\left(\left(\sqrt{cde}x-\sqrt{cdex^2+cd^2x+ae^2x+ade}\right)e+\sqrt{cde}d\right)e^3}$$

input
```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(e*x+d)^4,x, algorithm="
giac")
```

output
```
1/4*sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*x + a*d*e)*(2*c^2*d^2*x/e^2 - (7*c^3*
d^4*e^5 - 9*a*c^2*d^2*e^7)/(c*d*e^8)) - 15/8*(c^3*d^5 - 2*a*c^2*d^3*e^2 +
a^2*c*d*e^4)*log(abs(-c*d^2 - a*e^2 - 2*sqrt(c*d*e)*(sqrt(c*d*e)*x - sqrt(
c*d*e*x^2 + c*d^2*x + a*e^2*x + a*d*e))))/(sqrt(c*d*e)*e^3) - 2*(sqrt(c*d*
e)*c^3*d^6 - 3*sqrt(c*d*e)*a*c^2*d^4*e^2 + 3*sqrt(c*d*e)*a^2*c*d^2*e^4 - s
qrt(c*d*e)*a^3*e^6)/(sqrt(c*d*e)*((sqrt(c*d*e)*x - sqrt(c*d*e*x^2 + c*d^2*
x + a*e^2*x + a*d*e))*e + sqrt(c*d*e)*d)*e^3)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(ade+(cd^2+ae^2)x+cdex^2)^{5/2}}{(d+ex)^4}\,dx=\int \frac{\left(c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e\right)^{5/2}}{\left(d+e\,x\right)^4}\,dx$$

input
```
int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(5/2)/(d + e*x)^4,x)
```

output
```
int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(5/2)/(d + e*x)^4, x)
```

Reduce [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 597, normalized size of antiderivative = 2.69

$$\int \frac{(ade+(cd^2+ae^2)x+cdex^2)^{5/2}}{(d+ex)^4}\,dx=\frac{-8\sqrt{ex+d}\,\sqrt{cdx+ae}\,a^2e^5+25\sqrt{ex+d}\,\sqrt{cdx+ae}\,ac\,d^2e^3+9\cdots}{}$$

input
```
int((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(e*x+d)^4,x)
```

output

```
( - 8*sqrt(d + e*x)*sqrt(a*e + c*d*x)*a**2*e**5 + 25*sqrt(d + e*x)*sqrt(a*
e + c*d*x)*a*c*d**2*e**3 + 9*sqrt(d + e*x)*sqrt(a*e + c*d*x)*a*c*d*e**4*x
- 15*sqrt(d + e*x)*sqrt(a*e + c*d*x)*c**2*d**4*e - 5*sqrt(d + e*x)*sqrt(a*
e + c*d*x)*c**2*d**3*e**2*x + 2*sqrt(d + e*x)*sqrt(a*e + c*d*x)*c**2*d**2*
e**3*x**2 + 15*sqrt(e)*sqrt(d)*sqrt(c)*log((sqrt(e)*sqrt(a*e + c*d*x) + sq
rt(d)*sqrt(c)*sqrt(d + e*x))/sqrt(a*e**2 - c*d**2))*a**2*d*e**4 + 15*sqrt(
e)*sqrt(d)*sqrt(c)*log((sqrt(e)*sqrt(a*e + c*d*x) + sqrt(d)*sqrt(c)*sqrt(d
 + e*x))/sqrt(a*e**2 - c*d**2))*a**2*e**5*x - 30*sqrt(e)*sqrt(d)*sqrt(c)*l
og((sqrt(e)*sqrt(a*e + c*d*x) + sqrt(d)*sqrt(c)*sqrt(d + e*x))/sqrt(a*e**2
 - c*d**2))*a*c*d**3*e**2 - 30*sqrt(e)*sqrt(d)*sqrt(c)*log((sqrt(e)*sqrt(a
*e + c*d*x) + sqrt(d)*sqrt(c)*sqrt(d + e*x))/sqrt(a*e**2 - c*d**2))*a*c*d*
*2*e**3*x + 15*sqrt(e)*sqrt(d)*sqrt(c)*log((sqrt(e)*sqrt(a*e + c*d*x) + sq
rt(d)*sqrt(c)*sqrt(d + e*x))/sqrt(a*e**2 - c*d**2))*c**2*d**5 + 15*sqrt(e)
*sqrt(d)*sqrt(c)*log((sqrt(e)*sqrt(a*e + c*d*x) + sqrt(d)*sqrt(c)*sqrt(d +
 e*x))/sqrt(a*e**2 - c*d**2))*c**2*d**4*e*x - 10*sqrt(e)*sqrt(d)*sqrt(c)*a
**2*d*e**4 - 10*sqrt(e)*sqrt(d)*sqrt(c)*a**2*e**5*x + 20*sqrt(e)*sqrt(d)*s
qrt(c)*a*c*d**3*e**2 + 20*sqrt(e)*sqrt(d)*sqrt(c)*a*c*d**2*e**3*x - 10*sqr
t(e)*sqrt(d)*sqrt(c)*c**2*d**5 - 10*sqrt(e)*sqrt(d)*sqrt(c)*c**2*d**4*e*x)
/(4*e**4*(d + e*x))
```

3.228 $\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{(d+ex)^5}\,dx$

Optimal result

Integrand size = 37, antiderivative size = 211

$$\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{(d+ex)^5}\,dx=\frac{5c^2d^2\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{e^3}-\frac{10cd\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{3e^2(d+ex)^2}-\frac{2\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{3e(d+ex)^4}-\frac{5c^{3/2}d^{3/2}\left(cd^2-ae^2\right)\operatorname{arctanh}\left(\frac{\sqrt{e}\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{\sqrt{c}\sqrt{d}(d+ex)}\right)}{e^{7/2}}$$

output
```
5*c^2*d^2*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/e^3-10/3*c*d*(a*d*e+(a*e
^2+c*d^2)*x+c*d*e*x^2)^(3/2)/e^2/(e*x+d)^2-2/3*(a*d*e+(a*e^2+c*d^2)*x+c*d*
e*x^2)^(5/2)/e/(e*x+d)^4-5*c^(3/2)*d^(3/2)*(-a*e^2+c*d^2)*arctanh(e^(1/2)*
(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/c^(1/2)/d^(1/2)/(e*x+d))/e^(7/2)
```

Mathematica [A] (verified)

Time = 0.46 (sec) , antiderivative size = 185, normalized size of antiderivative = 0.88

$$\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{(d+ex)^5}\,dx=\frac{((ae+cdx)(d+ex))^{5/2}\left(\frac{\sqrt{e}\left(-2a^2e^4-2acde^2(5d+7ex)+c^2d^2\left(15d^2+20dex+3\right.\right.}{(ae+cdx)^2(d+ex)^4}\right.}{3e^{7/2}}$$

input
```
Integrate[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2)/(d + e*x)^5,x]
```

output
```
(((a*e + c*d*x)*(d + e*x))^(5/2)*((Sqrt[e]*(-2*a^2*e^4 - 2*a*c*d*e^2*(5*d
+ 7*e*x) + c^2*d^2*(15*d^2 + 20*d*e*x + 3*e^2*x^2)))/((a*e + c*d*x)^2*(d +
 e*x)^4) - (15*c^(3/2)*d^(3/2)*(c*d^2 - a*e^2)*ArcTanh[(Sqrt[c]*Sqrt[d]*Sq
rt[d + e*x])/(Sqrt[e]*Sqrt[a*e + c*d*x])])/((a*e + c*d*x)^(5/2)*(d + e*x)^
(5/2))))/(3*e^(7/2))
```

Rubi [A] (verified)

Time = 0.68 (sec) , antiderivative size = 239, normalized size of antiderivative = 1.13, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.162, Rules used = {1130, 1125, 27, 1160, 1092, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/2}}{(d+ex)^5}dx$$

$$\Big\downarrow \text{1130}$$

$$\frac{5cd\int \frac{\left(cdex^2+\left(cd^2+ae^2\right)x+ade\right)^{3/2}}{(d+ex)^3}dx}{3e}-\frac{2\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/2}}{3e(d+ex)^4}$$

$$\Big\downarrow \text{1125}$$

$$\frac{5cd\left(-\frac{\int \frac{cde^2\left(cd^2-cexd-2ae^2\right)}{\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}dx}{e^4}-\frac{2\left(a-\frac{cd^2}{e^2}\right)\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{d+ex}\right)}{3e}-\frac{2\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/2}}{3e(d+ex)^4}$$

$$\Big\downarrow \; 27$$

$$\frac{5cd\left(-\frac{cd\int \frac{cd^2-cexd-2ae^2}{\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}dx}{e^2}-\frac{2\left(a-\frac{cd^2}{e^2}\right)\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{d+ex}\right)}{3e}-\frac{2\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/2}}{3e(d+ex)^4}$$

$$\Big\downarrow \; 1160$$

$$\frac{5cd\left(-\frac{cd\left(\frac{3}{2}\left(cd^2-ae^2\right)\int \frac{1}{\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}dx-\sqrt{x(ae^2+cd^2)+ade+cdex^2}\right)}{e^2}-\frac{2\left(a-\frac{cd^2}{e^2}\right)\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{d+ex}\right)}{3e}-\frac{2\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/2}}{3e(d+ex)^4}$$

$$\Big\downarrow \; 1092$$

$$\frac{5cd\left(-\frac{cd\left(3\left(cd^2-ae^2\right)\int \frac{1}{4cde-\frac{\left(cd^2+2cexd+ae^2\right)^2}{cdex^2+\left(cd^2+ae^2\right)x+ade}}d\frac{cd^2+2cexd+ae^2}{\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}-\sqrt{x(ae^2+cd^2)+ade+cdex^2}\right)}{e^2}-\frac{2\left(a-\frac{cd^2}{e^2}\right)\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{d+ex}\right)}{3e}-\frac{2\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/2}}{3e(d+ex)^4}$$

$$\Big\downarrow \; 219$$

$$\frac{5cd\left(-\frac{cd\left(\frac{3\left(cd^2-ae^2\right)\text{arctanh}\left(\frac{ae^2+cd^2+2cdex}{2\sqrt{c}\sqrt{d}\sqrt{e}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}\right)}{2\sqrt{c}\sqrt{d}\sqrt{e}}-\sqrt{x(ae^2+cd^2)+ade+cdex^2}\right)}{e^2}-\frac{2\left(a-\frac{cd^2}{e^2}\right)\sqrt{x(ae^2+cd^2)+ade+cd}}{d+ex}\right.}{3e}$$

$$\frac{2\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/2}}{3e(d+ex)^4}$$

input
```
Int[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2)/(d + e*x)^5,x]
```

output
```
(-2*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2))/(3*e*(d + e*x)^4) + (5*
c*d*((-2*(a - (c*d^2)/e^2)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(d
 + e*x) - (c*d*(-Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2] + (3*(c*d^2 -
 a*e^2)*ArcTanh[(c*d^2 + a*e^2 + 2*c*d*e*x)/(2*Sqrt[c]*Sqrt[d]*Sqrt[e]*Sqr
t[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])])/(2*Sqrt[c]*Sqrt[d]*Sqrt[e])))/
e^2))/(3*e)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1092
```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[I
nt[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a
, b, c}, x]
```

rule 1125
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_
Symbol] :> Simp[-2*e^(2*m + 3)*(Sqrt[a + b*x + c*x^2]/((-2*c*d + b*e)^(m +
2)*(d + e*x))), x] - Simp[e^(2*m + 2)   Int[(1/Sqrt[a + b*x + c*x^2])*Expan
dToSum[((-2*c*d + b*e)^(-m - 1) - ((-c)*d + b*e + c*e*x)^(-m - 1))/(d + e*x
), x], x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0]
 && ILtQ[m, 0] && EqQ[m + p, -3/2]
```

rule 1130
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(d + e*x)^(m + 1)*((a + b*x + c*x^2)^p/(e*(m + p + 1))), x]
- Simp[c*(p/(e^2*(m + p + 1)))   Int[(d + e*x)^(m + 2)*(a + b*x + c*x^2)^(p
 - 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0]
 && GtQ[p, 0] && (LtQ[m, -2] || EqQ[m + 2*p + 1, 0]) && NeQ[m + p + 1, 0] &
& IntegerQ[2*p]
```

rule 1160
```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[e*((a + b*x + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Simp[(2*c*d - b
*e)/(2*c)   Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
 && NeQ[p, -1]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 650 vs. $2(187) = 374$.

Time = 3.19 (sec) , antiderivative size = 651, normalized size of antiderivative = 3.09

method	result
	$$6dec\left(\frac{2\left(dec\left(x+\frac{d}{e}\right)^{2}+\left(a\,e^{2}-c\,d^{2}\right)\left(x+\frac{d}{e}\right)\right)^{\frac{7}{2}}}{\left(a\,e^{2}-c\,d^{2}\right)\left(x+\frac{d}{e}\right)^{3}}-\right.$$

input
```
int((a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(5/2)/(e*x+d)^5,x,method=_RETURNVERB
OSE)
```

output
```
1/e^5*(-2/3/(a*e^2-c*d^2)/(x+d/e)^5*(d*e*c*(x+d/e)^2+(a*e^2-c*d^2)*(x+d/e)
)^(7/2)+4/3*d*e*c/(a*e^2-c*d^2)*(-2/(a*e^2-c*d^2)/(x+d/e)^4*(d*e*c*(x+d/e)
^2+(a*e^2-c*d^2)*(x+d/e))^(7/2)+6*d*e*c/(a*e^2-c*d^2)*(2/(a*e^2-c*d^2)/(x+
d/e)^3*(d*e*c*(x+d/e)^2+(a*e^2-c*d^2)*(x+d/e))^(7/2)-8*d*e*c/(a*e^2-c*d^2)
*(2/3/(a*e^2-c*d^2)/(x+d/e)^2*(d*e*c*(x+d/e)^2+(a*e^2-c*d^2)*(x+d/e))^(7/2
)-10/3*d*e*c/(a*e^2-c*d^2)*(1/5*(d*e*c*(x+d/e)^2+(a*e^2-c*d^2)*(x+d/e))^(5
/2)+1/2*(a*e^2-c*d^2)*(1/8*(2*d*e*c*(x+d/e)+a*e^2-c*d^2)/d/e/c*(d*e*c*(x+d
/e)^2+(a*e^2-c*d^2)*(x+d/e))^(3/2)-3/16*(a*e^2-c*d^2)^2/d/e/c*(1/4*(2*d*e*
c*(x+d/e)+a*e^2-c*d^2)/d/e/c*(d*e*c*(x+d/e)^2+(a*e^2-c*d^2)*(x+d/e))^(1/2)
-1/8*(a*e^2-c*d^2)^2/d/e/c*ln((1/2*a*e^2-1/2*c*d^2+d*e*c*(x+d/e))/(d*e*c)^
(1/2)+(d*e*c*(x+d/e)^2+(a*e^2-c*d^2)*(x+d/e))^(1/2))/(d*e*c)^(1/2)))))))
```

Fricas [A] (verification not implemented)

Time = 0.35 (sec) , antiderivative size = 594, normalized size of antiderivative = 2.82

$$\int \frac{(ade + (cd^2 + ae^2)\,x + cdex^2)^{5/2}}{(d+ex)^5}\,dx = \left[-\frac{15\,(c^2d^5 - acd^3e^2 + (c^2d^3e^2 - acde^4)x^2 + 2\,(c^2d^4e - acd^2e^3)x}{}\right.$$

input
```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(e*x+d)^5,x, algorithm="
fricas")
```

output
```
[-1/12*(15*(c^2*d^5 - a*c*d^3*e^2 + (c^2*d^3*e^2 - a*c*d*e^4)*x^2 + 2*(c^2
*d^4*e - a*c*d^2*e^3)*x)*sqrt(c*d/e)*log(8*c^2*d^2*e^2*x^2 + c^2*d^4 + 6*a
*c*d^2*e^2 + a^2*e^4 + 4*(2*c*d*e^2*x + c*d^2*e + a*e^3)*sqrt(c*d*e*x^2 +
a*d*e + (c*d^2 + a*e^2)*x)*sqrt(c*d/e) + 8*(c^2*d^3*e + a*c*d*e^3)*x) - 4*
(3*c^2*d^2*e^2*x^2 + 15*c^2*d^4 - 10*a*c*d^2*e^2 - 2*a^2*e^4 + 2*(10*c^2*d
^3*e - 7*a*c*d*e^3)*x)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x))/(e^5*x
^2 + 2*d*e^4*x + d^2*e^3), 1/6*(15*(c^2*d^5 - a*c*d^3*e^2 + (c^2*d^3*e^2 -
 a*c*d*e^4)*x^2 + 2*(c^2*d^4*e - a*c*d^2*e^3)*x)*sqrt(-c*d/e)*arctan(1/2*s
qrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(2*c*d*e*x + c*d^2 + a*e^2)*sqr
t(-c*d/e)/(c^2*d^2*e*x^2 + a*c*d^2*e + (c^2*d^3 + a*c*d*e^2)*x)) + 2*(3*c^
2*d^2*e^2*x^2 + 15*c^2*d^4 - 10*a*c*d^2*e^2 - 2*a^2*e^4 + 2*(10*c^2*d^3*e
- 7*a*c*d*e^3)*x)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x))/(e^5*x^2 +
2*d*e^4*x + d^2*e^3)]
```

Sympy [F]

$$\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{(d+ex)^5}\,dx=\int \frac{\left((d+ex)\,(ae+cdx)\right)^{\frac{5}{2}}}{(d+ex)^5}\,dx$$

input
```
integrate((a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(5/2)/(e*x+d)**5,x)
```

output
```
Integral(((d + e*x)*(a*e + c*d*x))**(5/2)/(d + e*x)**5, x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{(d+ex)^5}\,dx=\text{Exception raised: ValueError}$$

input
```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(e*x+d)^5,x, algorithm="
maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e*(a*e^2-c*d^2)>0)', see `assume
?` for mor
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 462 vs. $2(187) = 374$.

Time = 0.63 (sec) , antiderivative size = 462, normalized size of antiderivative = 2.19

$$\int \frac{(ade + (cd^2 + ae^2)x + cdex^2)^{5/2}}{(d + ex)^5} dx = \frac{1}{3}\left(\frac{15\left(c^3d^4\mathrm{sgn}\left(\frac{1}{ex+d}\right)\mathrm{sgn}(e) - ac^2d^2e^2\mathrm{sgn}\left(\frac{1}{ex+d}\right)\mathrm{sgn}(e)\right)\arctan}{\sqrt{-cde}e^3|e|}\right.$$

input
```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(e*x+d)^5,x, algorithm="
giac")
```

output
```
1/3*(15*(c^3*d^4*sgn(1/(e*x + d))*sgn(e) - a*c^2*d^2*e^2*sgn(1/(e*x + d))*
sgn(e))*arctan(sqrt(c*d*e - c*d^2*e/(e*x + d) + a*e^3/(e*x + d))/sqrt(-c*d
*e))/(sqrt(-c*d*e)*e^3*abs(e)) + 3*(sqrt(c*d*e - c*d^2*e/(e*x + d) + a*e^3
/(e*x + d))*c^3*d^4*sgn(1/(e*x + d))*sgn(e) - sqrt(c*d*e - c*d^2*e/(e*x +
d) + a*e^3/(e*x + d))*a*c^2*d^2*e^2*sgn(1/(e*x + d))*sgn(e))/((c*d^2*e/(e*
x + d) - a*e^3/(e*x + d))*e^3*abs(e)) + 2*(6*sqrt(c*d*e - c*d^2*e/(e*x + d
) + a*e^3/(e*x + d))*c^2*d^3*e^13*sgn(1/(e*x + d))*sgn(e) - 6*sqrt(c*d*e -
 c*d^2*e/(e*x + d) + a*e^3/(e*x + d))*a*c*d*e^15*sgn(1/(e*x + d))*sgn(e) +
 (c*d*e - c*d^2*e/(e*x + d) + a*e^3/(e*x + d))^(3/2)*c*d^2*e^12*sgn(1/(e*x
 + d))*sgn(e) - (c*d*e - c*d^2*e/(e*x + d) + a*e^3/(e*x + d))^(3/2)*a*e^14
*sgn(1/(e*x + d))*sgn(e))/(e^17*abs(e)))*abs(e)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(ade + (cd^2 + ae^2)x + cdex^2)^{5/2}}{(d + ex)^5} dx = \int \frac{(c\,d\,e\,x^2 + (c\,d^2 + a\,e^2)\,x + a\,d\,e)^{5/2}}{(d + e\,x)^5} dx$$

input
```
int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(5/2)/(d + e*x)^5,x)
```

output
```
int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(5/2)/(d + e*x)^5, x)
```

Reduce [B] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 622, normalized size of antiderivative = 2.95

$$\int \frac{(ade + (cd^2 + ae^2)x + cdex^2)^{5/2}}{(d + ex)^5} dx = \frac{-4\sqrt{ex + d}\sqrt{cdx + ae}\,a^2e^5 - 20\sqrt{ex + d}\sqrt{cdx + ae}\,ac\,d^2e^3 - 28}{}$$

input
```
int((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(e*x+d)^5,x)
```

output
```
( - 4*sqrt(d + e*x)*sqrt(a*e + c*d*x)*a**2*e**5 - 20*sqrt(d + e*x)*sqrt(a*
e + c*d*x)*a*c*d**2*e**3 - 28*sqrt(d + e*x)*sqrt(a*e + c*d*x)*a*c*d*e**4*x
 + 30*sqrt(d + e*x)*sqrt(a*e + c*d*x)*c**2*d**4*e + 40*sqrt(d + e*x)*sqrt(
a*e + c*d*x)*c**2*d**3*e**2*x + 6*sqrt(d + e*x)*sqrt(a*e + c*d*x)*c**2*d**
2*e**3*x**2 + 30*sqrt(e)*sqrt(d)*sqrt(c)*log((sqrt(e)*sqrt(a*e + c*d*x) +
sqrt(d)*sqrt(c)*sqrt(d + e*x))/sqrt(a*e**2 - c*d**2))*a*c*d**3*e**2 + 60*s
qrt(e)*sqrt(d)*sqrt(c)*log((sqrt(e)*sqrt(a*e + c*d*x) + sqrt(d)*sqrt(c)*sq
rt(d + e*x))/sqrt(a*e**2 - c*d**2))*a*c*d**2*e**3*x + 30*sqrt(e)*sqrt(d)*s
qrt(c)*log((sqrt(e)*sqrt(a*e + c*d*x) + sqrt(d)*sqrt(c)*sqrt(d + e*x))/sqr
t(a*e**2 - c*d**2))*a*c*d*e**4*x**2 - 30*sqrt(e)*sqrt(d)*sqrt(c)*log((sqrt
(e)*sqrt(a*e + c*d*x) + sqrt(d)*sqrt(c)*sqrt(d + e*x))/sqrt(a*e**2 - c*d**
2))*c**2*d**5 - 60*sqrt(e)*sqrt(d)*sqrt(c)*log((sqrt(e)*sqrt(a*e + c*d*x)
+ sqrt(d)*sqrt(c)*sqrt(d + e*x))/sqrt(a*e**2 - c*d**2))*c**2*d**4*e*x - 30
*sqrt(e)*sqrt(d)*sqrt(c)*log((sqrt(e)*sqrt(a*e + c*d*x) + sqrt(d)*sqrt(c)*
sqrt(d + e*x))/sqrt(a*e**2 - c*d**2))*c**2*d**3*e**2*x**2 + 5*sqrt(e)*sqrt
(d)*sqrt(c)*a*c*d**3*e**2 + 10*sqrt(e)*sqrt(d)*sqrt(c)*a*c*d**2*e**3*x + 5
*sqrt(e)*sqrt(d)*sqrt(c)*a*c*d*e**4*x**2 - 5*sqrt(e)*sqrt(d)*sqrt(c)*c**2*
d**5 - 10*sqrt(e)*sqrt(d)*sqrt(c)*c**2*d**4*e*x - 5*sqrt(e)*sqrt(d)*sqrt(c
)*c**2*d**3*e**2*x**2)/(6*e**4*(d**2 + 2*d*e*x + e**2*x**2))
```

3.229 $\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{(d+ex)^6}\,dx$

Optimal result

Integrand size = 37, antiderivative size = 206

$$\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{(d+ex)^6}\,dx=-\frac{2c^2d^2\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{e^3(d+ex)}-\frac{2cd\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{3e^2(d+ex)^3}-\frac{2\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{5e(d+ex)^5}+\frac{2c^{5/2}d^{5/2}\operatorname{arctanh}\left(\frac{\sqrt{e}\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{\sqrt{c}\sqrt{d}(d+ex)}\right)}{e^{7/2}}$$

output
```
-2*c^2*d^2*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/e^3/(e*x+d)-2/3*c*d*(a*
d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/e^2/(e*x+d)^3-2/5*(a*d*e+(a*e^2+c*d^2
)*x+c*d*e*x^2)^(5/2)/e/(e*x+d)^5+2*c^(5/2)*d^(5/2)*arctanh(e^(1/2)*(a*d*e+
(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/c^(1/2)/d^(1/2)/(e*x+d))/e^(7/2)
```

Mathematica [A] (verified)

Time = 0.48 (sec) , antiderivative size = 173, normalized size of antiderivative = 0.84

$$\int \frac{(ade+(cd^2+ae^2)x+cdex^2)^{5/2}}{(d+ex)^6}\,dx=\frac{2((ae+cdx)(d+ex))^{5/2}\left(-\frac{\sqrt{e}(3a^2e^4+acde^2(5d+11ex)+c^2d^2(15d^2+35dex\cdot}{(ae+cdx)^2(d+ex)^5}\right.}{15e^{7/2}}$$

input
```
Integrate[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2)/(d + e*x)^6,x]
```

output
```
(2*((a*e + c*d*x)*(d + e*x))^(5/2)*(-((Sqrt[e]*(3*a^2*e^4 + a*c*d*e^2*(5*d
 + 11*e*x) + c^2*d^2*(15*d^2 + 35*d*e*x + 23*e^2*x^2)))/((a*e + c*d*x)^2*(
d + e*x)^5)) + (15*c^(5/2)*d^(5/2)*ArcTanh[(Sqrt[c]*Sqrt[d]*Sqrt[d + e*x])
/(Sqrt[e]*Sqrt[a*e + c*d*x])])/((a*e + c*d*x)^(5/2)*(d + e*x)^(5/2))))/(15
*e^(7/2))
```

Rubi [A] (verified)

Time = 0.60 (sec) , antiderivative size = 224, normalized size of antiderivative = 1.09, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.189, Rules used = {1130, 1130, 1125, 25, 27, 1092, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/2}}{(d+ex)^6}dx$$

$$\Big\downarrow \text{ 1130}$$

$$\frac{cd\int \frac{\left(cdex^2+\left(cd^2+ae^2\right)x+ade\right)^{3/2}}{(d+ex)^4}dx}{e}-\frac{2\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/2}}{5e(d+ex)^5}$$

$$\Big\downarrow \text{ 1130}$$

$$\frac{cd\left(\frac{cd\int \frac{\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}{(d+ex)^2}dx}{e}-\frac{2\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{3e(d+ex)^3}\right)}{e}-\frac{2\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/2}}{5e(d+ex)^5}$$

$$\Big\downarrow \text{ 1125}$$

$$\frac{cd\left(\frac{cd\left(-\frac{\int -\frac{cde}{\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}dx}{e^2}-\frac{2\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{e(d+ex)}\right)}{e}-\frac{2\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{3e(d+ex)^3}\right)}{e}-\frac{2\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/2}}{5e(d+ex)^5}$$

$$\Big\downarrow \text{ 25}$$

$$\frac{cd\left(\frac{cd\left(\frac{\int \frac{cde}{\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}dx}{e^2}-\frac{2\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{e(d+ex)}\right)}{e}-\frac{2\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{3e(d+ex)^3}\right)}{e}-\frac{2\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/2}}{5e(d+ex)^5}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{cd\left(\frac{cd\left(\frac{cd\int \frac{1}{\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}dx}{e}-\frac{2\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{e(d+ex)}\right)}{e}-\frac{2\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{3e(d+ex)^3}\right)}{e}-\frac{2\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/2}}{5e(d+ex)^5}$$

$$\Big\downarrow \text{ 1092}$$

$$\frac{cd\left(\frac{cd\left(\frac{2cd\int \frac{1}{4cde-\frac{\left(cd^2+2cexd+ae^2\right)^2}{cdex^2+\left(cd^2+ae^2\right)x+ade}}d\frac{cd^2+2cexd+ae^2}{\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}}{e}-\frac{2\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{e(d+ex)}\right)}{e}-\frac{2\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{3e(d+ex)^3}\right)}{e}-\frac{2\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/2}}{5e(d+ex)^5}$$

$$\Big\downarrow \; 219$$

$$\frac{cd\left(\frac{cd\left(\frac{\sqrt{c}\sqrt{d}\,\text{arctanh}\left(\frac{ae^2+cd^2+2cdex}{2\sqrt{c}\sqrt{d}\sqrt{e}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}\right)}{e^{3/2}}-\frac{2\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{e(d+ex)}\right)}{e}-\frac{2\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{3e(d+ex)^3}\right)}{e}-\frac{2\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/2}}{5e(d+ex)^5}$$

input
```
Int[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2)/(d + e*x)^6,x]
```

output
```
(-2*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2))/(5*e*(d + e*x)^5) + (c*
d*((-2*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2))/(3*e*(d + e*x)^3) +
(c*d*((-2*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(e*(d + e*x)) + (Sq
rt[c]*Sqrt[d]*ArcTanh[(c*d^2 + a*e^2 + 2*c*d*e*x)/(2*Sqrt[c]*Sqrt[d]*Sqrt[
e]*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])])/e^(3/2)))/e))/e
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1092
```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[I
nt[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a
, b, c}, x]
```

rule 1125
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_
Symbol] :> Simp[-2*e^(2*m + 3)*(Sqrt[a + b*x + c*x^2]/((-2*c*d + b*e)^(m +
2)*(d + e*x))), x] - Simp[e^(2*m + 2)   Int[(1/Sqrt[a + b*x + c*x^2])*Expan
dToSum[((-2*c*d + b*e)^(-m - 1) - ((-c)*d + b*e + c*e*x)^(-m - 1))/(d + e*x
), x], x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0]
 && ILtQ[m, 0] && EqQ[m + p, -3/2]
```

rule 1130
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(d + e*x)^(m + 1)*((a + b*x + c*x^2)^p/(e*(m + p + 1))), x]
- Simp[c*(p/(e^2*(m + p + 1)))   Int[(d + e*x)^(m + 2)*(a + b*x + c*x^2)^(p
 - 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0]
 && GtQ[p, 0] && (LtQ[m, -2] || EqQ[m + 2*p + 1, 0]) && NeQ[m + p + 1, 0] &
& IntegerQ[2*p]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 731 vs. $2(182) = 364$.

Time = 4.16 (sec) , antiderivative size = 732, normalized size of antiderivative = 3.55

method	result

input
```
int((a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(5/2)/(e*x+d)^6,x,method=_RETURNVERB
OSE)
```

output
```
1/e^6*(-2/5/(a*e^2-c*d^2)/(x+d/e)^6*(d*e*c*(x+d/e)^2+(a*e^2-c*d^2)*(x+d/e)
)^(7/2)+2/5*d*e*c/(a*e^2-c*d^2)*(-2/3/(a*e^2-c*d^2)/(x+d/e)^5*(d*e*c*(x+d/
e)^2+(a*e^2-c*d^2)*(x+d/e))^(7/2)+4/3*d*e*c/(a*e^2-c*d^2)*(-2/(a*e^2-c*d^2
)/(x+d/e)^4*(d*e*c*(x+d/e)^2+(a*e^2-c*d^2)*(x+d/e))^(7/2)+6*d*e*c/(a*e^2-c
*d^2)*(2/(a*e^2-c*d^2)/(x+d/e)^3*(d*e*c*(x+d/e)^2+(a*e^2-c*d^2)*(x+d/e))^(
7/2)-8*d*e*c/(a*e^2-c*d^2)*(2/3/(a*e^2-c*d^2)/(x+d/e)^2*(d*e*c*(x+d/e)^2+(
a*e^2-c*d^2)*(x+d/e))^(7/2)-10/3*d*e*c/(a*e^2-c*d^2)*(1/5*(d*e*c*(x+d/e)^2
+(a*e^2-c*d^2)*(x+d/e))^(5/2)+1/2*(a*e^2-c*d^2)*(1/8*(2*d*e*c*(x+d/e)+a*e^
2-c*d^2)/d/e/c*(d*e*c*(x+d/e)^2+(a*e^2-c*d^2)*(x+d/e))^(3/2)-3/16*(a*e^2-c
*d^2)^2/d/e/c*(1/4*(2*d*e*c*(x+d/e)+a*e^2-c*d^2)/d/e/c*(d*e*c*(x+d/e)^2+(a
*e^2-c*d^2)*(x+d/e))^(1/2)-1/8*(a*e^2-c*d^2)^2/d/e/c*ln((1/2*a*e^2-1/2*c*d
^2+d*e*c*(x+d/e))/(d*e*c)^(1/2)+(d*e*c*(x+d/e)^2+(a*e^2-c*d^2)*(x+d/e))^(1
/2))/(d*e*c)^(1/2)))))))))
```

Fricas [A] (verification not implemented)

Time = 0.96 (sec) , antiderivative size = 578, normalized size of antiderivative = 2.81

$$\int \frac{(ade + (cd^2 + ae^2)\,x + cdex^2)^{5/2}}{(d + ex)^6}\,dx = \left[\frac{15\,(c^2d^2e^3x^3 + 3\,c^2d^3e^2x^2 + 3\,c^2d^4ex + c^2d^5)\sqrt{\frac{cd}{e}}\log\left(8\,c^2d^2e^2x\right.}{}\right.$$

$$-\frac{15\,(c^2d^2e^3x^3 + 3\,c^2d^3e^2x^2 + 3\,c^2d^4ex + c^2d^5)\sqrt{-\frac{cd}{e}}\arctan\left(\frac{\sqrt{cdex^2+ade+(cd^2+ae^2)x}\,(2\,cdex+cd^2+ae^2)\sqrt{-\frac{cd}{e}}}{2\,(c^2d^2ex^2+acd^2e+(c^2d^3+acde^2)x)}\right) + 2}{15\,(e^6x^3 + 3\,de^5x^2 +}$$

input
```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(e*x+d)^6,x, algorithm="
fricas")
```

output
```
[1/30*(15*(c^2*d^2*e^3*x^3 + 3*c^2*d^3*e^2*x^2 + 3*c^2*d^4*e*x + c^2*d^5)*
sqrt(c*d/e)*log(8*c^2*d^2*e^2*x^2 + c^2*d^4 + 6*a*c*d^2*e^2 + a^2*e^4 + 4*
(2*c*d*e^2*x + c*d^2*e + a*e^3)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x
)*sqrt(c*d/e) + 8*(c^2*d^3*e + a*c*d*e^3)*x) - 4*(23*c^2*d^2*e^2*x^2 + 15*
c^2*d^4 + 5*a*c*d^2*e^2 + 3*a^2*e^4 + (35*c^2*d^3*e + 11*a*c*d*e^3)*x)*sqr
t(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x))/(e^6*x^3 + 3*d*e^5*x^2 + 3*d^2*e
^4*x + d^3*e^3), -1/15*(15*(c^2*d^2*e^3*x^3 + 3*c^2*d^3*e^2*x^2 + 3*c^2*d^
4*e*x + c^2*d^5)*sqrt(-c*d/e)*arctan(1/2*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 +
 a*e^2)*x)*(2*c*d*e*x + c*d^2 + a*e^2)*sqrt(-c*d/e)/(c^2*d^2*e*x^2 + a*c*d
^2*e + (c^2*d^3 + a*c*d*e^2)*x)) + 2*(23*c^2*d^2*e^2*x^2 + 15*c^2*d^4 + 5*
a*c*d^2*e^2 + 3*a^2*e^4 + (35*c^2*d^3*e + 11*a*c*d*e^3)*x)*sqrt(c*d*e*x^2
+ a*d*e + (c*d^2 + a*e^2)*x))/(e^6*x^3 + 3*d*e^5*x^2 + 3*d^2*e^4*x + d^3*e
^3)]
```

Sympy [F]

$$\int \frac{(ade + (cd^2 + ae^2)x + cdex^2)^{5/2}}{(d+ex)^6}\,dx = \int \frac{((d+ex)(ae+cdx))^{\frac{5}{2}}}{(d+ex)^6}\,dx$$

input
```
integrate((a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(5/2)/(e*x+d)**6,x)
```

output
```
Integral(((d + e*x)*(a*e + c*d*x))**(5/2)/(d + e*x)**6, x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(ade + (cd^2 + ae^2)x + cdex^2)^{5/2}}{(d+ex)^6}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(e*x+d)^6,x, algorithm="
maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e*(a*e^2-c*d^2)>0)', see `assume
?` for mor
```

Giac [F(-2)]

Exception generated.

$$\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{(d+ex)^6}\,dx=\text{Exception raised: TypeError}$$

input
```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(e*x+d)^6,x, algorithm="
giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Unable to divide, perhaps due to ro
unding error%%%{%%{[%%%{1,[0,0,15]%%%},0]:[1,0,%%%{-1,[1,1,1]%%%}]%%},[6,6
]%%%}+%%%
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{(d+ex)^6}\,dx=\int \frac{{\left(c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e\right)}^{5/2}}{{\left(d+e\,x\right)}^6}\,dx$$

input
```
int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(5/2)/(d + e*x)^6,x)
```

output
```
int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(5/2)/(d + e*x)^6, x)
```

Reduce [B] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 491, normalized size of antiderivative = 2.38

$$\int \frac{(ade + (cd^2 + ae^2)x + cdex^2)^{5/2}}{(d + ex)^6} dx = \frac{-\frac{2\sqrt{ex+d}\sqrt{cdx+ae}\,a^2e^5}{5} - \frac{2\sqrt{ex+d}\sqrt{cdx+ae}\,ac\,d^2e^3}{3} - \frac{22\sqrt{ex+d}\sqrt{cdx+ae}\,acd\,e^4}{15}}{}$$

input
```
int((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(e*x+d)^6,x)
```

output
```
(2*( - 3*sqrt(d + e*x)*sqrt(a*e + c*d*x)*a**2*e**5 - 5*sqrt(d + e*x)*sqrt(
a*e + c*d*x)*a*c*d**2*e**3 - 11*sqrt(d + e*x)*sqrt(a*e + c*d*x)*a*c*d*e**4
*x - 15*sqrt(d + e*x)*sqrt(a*e + c*d*x)*c**2*d**4*e - 35*sqrt(d + e*x)*sqr
t(a*e + c*d*x)*c**2*d**3*e**2*x - 23*sqrt(d + e*x)*sqrt(a*e + c*d*x)*c**2*
d**2*e**3*x**2 + 15*sqrt(e)*sqrt(d)*sqrt(c)*log((sqrt(e)*sqrt(a*e + c*d*x)
 + sqrt(d)*sqrt(c)*sqrt(d + e*x))/sqrt(a*e**2 - c*d**2))*c**2*d**5 + 45*sq
rt(e)*sqrt(d)*sqrt(c)*log((sqrt(e)*sqrt(a*e + c*d*x) + sqrt(d)*sqrt(c)*sqr
t(d + e*x))/sqrt(a*e**2 - c*d**2))*c**2*d**4*e*x + 45*sqrt(e)*sqrt(d)*sqrt
(c)*log((sqrt(e)*sqrt(a*e + c*d*x) + sqrt(d)*sqrt(c)*sqrt(d + e*x))/sqrt(a
*e**2 - c*d**2))*c**2*d**3*e**2*x**2 + 15*sqrt(e)*sqrt(d)*sqrt(c)*log((sqr
t(e)*sqrt(a*e + c*d*x) + sqrt(d)*sqrt(c)*sqrt(d + e*x))/sqrt(a*e**2 - c*d*
*2))*c**2*d**2*e**3*x**3 + 5*sqrt(e)*sqrt(d)*sqrt(c)*c**2*d**5 + 15*sqrt(e
)*sqrt(d)*sqrt(c)*c**2*d**4*e*x + 15*sqrt(e)*sqrt(d)*sqrt(c)*c**2*d**3*e**
2*x**2 + 5*sqrt(e)*sqrt(d)*sqrt(c)*c**2*d**2*e**3*x**3))/(15*e**4*(d**3 +
3*d**2*e*x + 3*d*e**2*x**2 + e**3*x**3))
```

3.230 $\int \frac{\left(ade+(cd^2+ae^2)x+cdex^2\right)^{5/2}}{(d+ex)^7}\,dx$

Optimal result

Integrand size = 37, antiderivative size = 54

$$\int \frac{\left(ade+(cd^2+ae^2)\,x+cdex^2\right)^{5/2}}{(d+ex)^7}\,dx=\frac{2\left(ade+(cd^2+ae^2)\,x+cdex^2\right)^{7/2}}{7\,(cd^2-ae^2)\,(d+ex)^7}$$

output
```
2/7*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(7/2)/(-a*e^2+c*d^2)/(e*x+d)^7
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.80

$$\int \frac{\left(ade+(cd^2+ae^2)\,x+cdex^2\right)^{5/2}}{(d+ex)^7}\,dx=\frac{2((ae+cdx)(d+ex))^{7/2}}{7\,(cd^2-ae^2)\,(d+ex)^7}$$

input
```
Integrate[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2)/(d + e*x)^7,x]
```

output
```
(2*((a*e + c*d*x)*(d + e*x))^(7/2))/(7*(c*d^2 - a*e^2)*(d + e*x)^7)
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.027, Rules used = {1123}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/2}}{(d+ex)^7}\,dx$$

$$\Big\downarrow \ 1123$$

$$\frac{2\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{7/2}}{7(d+ex)^7\left(cd^2-ae^2\right)}$$

input
```
Int[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2)/(d + e*x)^7,x]
```

output
```
(2*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(7/2))/(7*(c*d^2 - a*e^2)*(d +
e*x)^7)
```

Defintions of rubi rules used

rule 1123
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[e*(d + e*x)^m*((a + b*x + c*x^2)^(p + 1)/((p + 1)*(2*c*d - b
*e))), x] /; FreeQ[{a, b, c, d, e, m, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2,
0] && EqQ[m + 2*p + 2, 0]
```

Maple [A] (verified)

Time = 5.63 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.07

method	result	size
gosper	$-\frac{2(cdx+ae)\left(cd\,x^2e+a\,e^2x+c\,d^2x+ade\right)^{\frac{5}{2}}}{7(ex+d)^6\left(a\,e^2-c\,d^2\right)}$	58
orering	$-\frac{2(cdx+ae)\left(ade+\left(a\,e^2+c\,d^2\right)x+cd\,x^2e\right)^{\frac{5}{2}}}{7(ex+d)^6\left(a\,e^2-c\,d^2\right)}$	59
default	$-\frac{2\left(dec\left(x+\frac{d}{e}\right)^2+\left(a\,e^2-c\,d^2\right)\left(x+\frac{d}{e}\right)\right)^{\frac{7}{2}}}{7e^7\left(a\,e^2-c\,d^2\right)\left(x+\frac{d}{e}\right)^7}$	65
trager	$-\frac{2\left(d^3c^3x^3+3a\,c^2d^2e\,x^2+3a^2cd\,e^2x+e^3a^3\right)\sqrt{cd\,x^2e+a\,e^2x+c\,d^2x+ade}}{7(ex+d)^4\left(a\,e^2-c\,d^2\right)}$	92

input
```
int((a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(5/2)/(e*x+d)^7,x,method=_RETURNVERB
OSE)
```

output
```
-2/7/(e*x+d)^6*(c*d*x+a*e)/(a*e^2-c*d^2)*(c*d*e*x^2+a*e^2*x+c*d^2*x+a*d*e)
^(5/2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 170 vs. 2(50) = 100.

Time = 2.14 (sec) , antiderivative size = 170, normalized size of antiderivative = 3.15

$$\int \frac{(ade+(cd^2+ae^2)\,x+cdex^2)^{5/2}}{(d+ex)^7}\,dx = \frac{2\,(c^3d^3x^3+3\,ac^2d^2ex^2+3\,a^2cde^2x+a^3e^3)\sqrt{cdex^2}}{7\,(cd^6-ad^4e^2+(cd^2e^4-ae^6)x^4+4\,(cd^3e^3-ade^5)x^3+6\,(cd^4e^2}$$

input
```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(e*x+d)^7,x, algorithm="
fricas")
```

output
```
2/7*(c^3*d^3*x^3 + 3*a*c^2*d^2*e*x^2 + 3*a^2*c*d*e^2*x + a^3*e^3)*sqrt(c*d
*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)/(c*d^6 - a*d^4*e^2 + (c*d^2*e^4 - a*e^
6)*x^4 + 4*(c*d^3*e^3 - a*d*e^5)*x^3 + 6*(c*d^4*e^2 - a*d^2*e^4)*x^2 + 4*(
c*d^5*e - a*d^3*e^3)*x)
```

Sympy [F]

$$\int \frac{(ade + (cd^2 + ae^2)x + cdex^2)^{5/2}}{(d + ex)^7} dx = \int \frac{((d + ex)(ae + cdx))^{\frac{5}{2}}}{(d + ex)^7} dx$$

input
```
integrate((a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(5/2)/(e*x+d)**7,x)
```

output
```
Integral(((d + e*x)*(a*e + c*d*x))**(5/2)/(d + e*x)**7, x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(ade + (cd^2 + ae^2)x + cdex^2)^{5/2}}{(d + ex)^7} dx = \text{Exception raised: ValueError}$$

input
```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(e*x+d)^7,x, algorithm="
maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e*(a*e^2-c*d^2)>0)', see `assume
?` for mor
```

Giac [F(-2)]

Exception generated.

$$\int \frac{(ade + (cd^2 + ae^2)x + cdex^2)^{5/2}}{(d + ex)^7} dx = \text{Exception raised: TypeError}$$

input
```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(e*x+d)^7,x, algorithm="
giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Unable to divide, perhaps due to ro
unding error%%%{%%%{1,[0,0,4]%%%},[8]%%%}+%%%{%%{[%%%{-8,[0,1,3]%%%},0]:[1
,0,%%%{-1
```

Mupad [B] (verification not implemented)

Time = 7.17 (sec) , antiderivative size = 2156, normalized size of antiderivative = 39.93

$$\int \frac{(ade + (cd^2 + ae^2)x + cdex^2)^{5/2}}{(d + ex)^7} dx = \text{Too large to display}$$

input
```
int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(5/2)/(d + e*x)^7,x)
```

output
```
(((d*((d*((16*c^6*d^7)/(105*e*(a*e^2 - c*d^2)^4) - (16*c^5*d^5*(4*a*e^2 -
3*c*d^2))/(35*e*(a*e^2 - c*d^2)^4)))/e + (16*c^4*d^4*(56*a^2*e^4 + 35*c^2*
d^4 - 88*a*c*d^2*e^2))/(105*e^2*(a*e^2 - c*d^2)^4)))/e - (16*a*c^3*d^3*(45
*a^2*e^4 + 35*c^2*d^4 - 79*a*c*d^2*e^2))/(105*e*(a*e^2 - c*d^2)^4))*(x*(a*
e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2))/(d + e*x) - (((2*a^3*e^4)/(7*a*e^
3 - 7*c*d^2*e) - (d*((d*((2*c^3*d^4)/(7*a*e^3 - 7*c*d^2*e) - (6*a*c^2*d^2*
e^2)/(7*a*e^3 - 7*c*d^2*e)))/e + (6*a^2*c*d*e^3)/(7*a*e^3 - 7*c*d^2*e)))/e
)*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2))/(d + e*x)^4 + (((d*((d*((
4*c^4*d^5)/(7*(a*e^2 - c*d^2)*(5*a*e^3 - 5*c*d^2*e)) - (4*c^3*d^3*(7*a*e^2
 - 4*c*d^2))/(7*(a*e^2 - c*d^2)*(5*a*e^3 - 5*c*d^2*e))))/e + (16*c^4*d^6 -
 64*a*c^3*d^4*e^2 + 60*a^2*c^2*d^2*e^4)/(7*e*(a*e^2 - c*d^2)*(5*a*e^3 - 5*
c*d^2*e))))/e - (4*a*c*d*(3*a*e^2 - 2*c*d^2)^2)/(7*(a*e^2 - c*d^2)*(5*a*e^
3 - 5*c*d^2*e)))*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2))/(d + e*x)^
3 + (((d*((4*c^4*d^5)/(7*e*(a*e^2 - c*d^2)*(3*a*e^3 - 3*c*d^2*e)) - (2*c^3
*d^3*(7*a*e^2 - 3*c*d^2))/(7*e*(a*e^2 - c*d^2)*(3*a*e^3 - 3*c*d^2*e))))/e
+ (16*c^4*d^6 - 62*a*c^3*d^4*e^2 + 66*a^2*c^2*d^2*e^4)/(35*e^2*(a*e^2 - c*
d^2)*(3*a*e^3 - 3*c*d^2*e)))*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2)
)/(d + e*x)^2 + (((216*c^6*d^9 - 280*a*c^5*d^7*e^2 - 208*a^2*c^4*d^5*e^4 +
 288*a^3*c^3*d^3*e^6)/(105*e^3*(a*e^2 - c*d^2)^4) - (d*((d*(((16*c^6*d^7)/(
105*e*(a*e^2 - c*d^2)^4) - (8*c^5*d^5*(7*a*e^2 - 5*c*d^2))/(35*e*(a*e^2...
```

Reduce [B] (verification not implemented)

Time = 0.47 (sec) , antiderivative size = 305, normalized size of antiderivative = 5.65

$$\int \frac{(ade + (cd^2 + ae^2)\,x + cdex^2)^{5/2}}{(d + ex)^7}\,dx = \frac{-\frac{2\sqrt{ex+d}\,\sqrt{cdx+ae}\,a^3e^7}{7} - \frac{6\sqrt{ex+d}\,\sqrt{cdx+ae}\,a^2cd\,e^6x}{7} - \frac{6\sqrt{ex+d}\,\sqrt{cdx+ae}\,a\,c^2d^2}{7}}{e^4\,(a\,e^6x^4 - c\,d^2e^4x^4 + 4ad}$$

input
```
int((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(e*x+d)^7,x)
```

output
```
(2*( - sqrt(d + e*x)*sqrt(a*e + c*d*x)*a**3*e**7 - 3*sqrt(d + e*x)*sqrt(a*
e + c*d*x)*a**2*c*d*e**6*x - 3*sqrt(d + e*x)*sqrt(a*e + c*d*x)*a*c**2*d**2
*e**5*x**2 - sqrt(d + e*x)*sqrt(a*e + c*d*x)*c**3*d**3*e**4*x**3 - sqrt(e)
*sqrt(d)*sqrt(c)*c**3*d**7 - 4*sqrt(e)*sqrt(d)*sqrt(c)*c**3*d**6*e*x - 6*s
qrt(e)*sqrt(d)*sqrt(c)*c**3*d**5*e**2*x**2 - 4*sqrt(e)*sqrt(d)*sqrt(c)*c**
3*d**4*e**3*x**3 - sqrt(e)*sqrt(d)*sqrt(c)*c**3*d**3*e**4*x**4))/(7*e**4*(
a*d**4*e**2 + 4*a*d**3*e**3*x + 6*a*d**2*e**4*x**2 + 4*a*d*e**5*x**3 + a*e
**6*x**4 - c*d**6 - 4*c*d**5*e*x - 6*c*d**4*e**2*x**2 - 4*c*d**3*e**3*x**3
 - c*d**2*e**4*x**4))
```

3.231 $\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{(d+ex)^8} dx$

Optimal result

Integrand size = 37, antiderivative size = 111

$$\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{(d+ex)^8} dx = \frac{2\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{7/2}}{9\left(cd^2-ae^2\right)(d+ex)^8} + \frac{4cd\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{7/2}}{63\left(cd^2-ae^2\right)^2(d+ex)^7}$$

output
```
2/9*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(7/2)/(-a*e^2+c*d^2)/(e*x+d)^8+4/63*
c*d*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(7/2)/(-a*e^2+c*d^2)^2/(e*x+d)^7
```

Mathematica [A] (verified)

Time = 0.30 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.55

$$\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{(d+ex)^8} dx = \frac{2((ae+cdx)(d+ex))^{7/2}\left(-7ae^2+cd(9d+2ex)\right)}{63\left(cd^2-ae^2\right)^2(d+ex)^8}$$

input
```
Integrate[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2)/(d + e*x)^8,x]
```

output
```
(2*((a*e + c*d*x)*(d + e*x))^(7/2)*(-7*a*e^2 + c*d*(9*d + 2*e*x)))/(63*(c*
d^2 - a*e^2)^2*(d + e*x)^8)
```

Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 111, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.054$, Rules used = {1129, 1123}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/2}}{(d+ex)^8}dx$$

$$\Big\downarrow \text{ 1129}$$

$$\frac{2cd\int \frac{\left(cdex^2+\left(cd^2+ae^2\right)x+ade\right)^{5/2}}{(d+ex)^7}dx}{9\left(cd^2-ae^2\right)}+\frac{2\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{7/2}}{9(d+ex)^8\left(cd^2-ae^2\right)}$$

$$\Big\downarrow \text{ 1123}$$

$$\frac{4cd\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{7/2}}{63(d+ex)^7\left(cd^2-ae^2\right)^2}+\frac{2\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{7/2}}{9(d+ex)^8\left(cd^2-ae^2\right)}$$

input
```
Int[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2)/(d + e*x)^8,x]
```

output
```
(2*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(7/2))/(9*(c*d^2 - a*e^2)*(d +
e*x)^8) + (4*c*d*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(7/2))/(63*(c*d^2
 - a*e^2)^2*(d + e*x)^7)
```

Defintions of rubi rules used

rule 1123
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[e*(d + e*x)^m*((a + b*x + c*x^2)^(p + 1)/((p + 1)*(2*c*d - b
*e))), x] /; FreeQ[{a, b, c, d, e, m, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2,
0] && EqQ[m + 2*p + 2, 0]
```

rule 1129
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(-e)*(d + e*x)^m*((a + b*x + c*x^2)^(p + 1)/((m + p + 1)*(2*
c*d - b*e))), x] + Simp[c*(Simplify[m + 2*p + 2]/((m + p + 1)*(2*c*d - b*e)
))   Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d
, e, m, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && ILtQ[Simplify[m + 2*p +
2], 0]
```

Maple [A] (verified)

Time = 7.37 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.81

method	result
gosper	$-\frac{2(cdx+ae)\left(-2cdxe+7a\,e^{2}-9c\,d^{2}\right)\left(cd\,x^{2}e+a\,e^{2}x+c\,d^{2}x+ade\right)^{\frac{5}{2}}}{63(ex+d)^{7}\left(a^{2}e^{4}-2ac\,d^{2}e^{2}+c^{2}d^{4}\right)}$
orering	$-\frac{2\left(-2cdxe+7a\,e^{2}-9c\,d^{2}\right)(cdx+ae)\left(ade+\left(a\,e^{2}+c\,d^{2}\right)x+cd\,x^{2}e\right)^{\frac{5}{2}}}{63\left(a^{2}e^{4}-2ac\,d^{2}e^{2}+c^{2}d^{4}\right)(ex+d)^{7}}$
default	$\frac{-\frac{2\left(dec\left(x+\frac{d}{e}\right)^{2}+\left(a\,e^{2}-c\,d^{2}\right)\left(x+\frac{d}{e}\right)\right)^{\frac{7}{2}}}{9\left(a\,e^{2}-c\,d^{2}\right)\left(x+\frac{d}{e}\right)^{8}}+\frac{4dec\left(dec\left(x+\frac{d}{e}\right)^{2}+\left(a\,e^{2}-c\,d^{2}\right)\left(x+\frac{d}{e}\right)\right)^{\frac{7}{2}}}{63\left(a\,e^{2}-c\,d^{2}\right)^{2}\left(x+\frac{d}{e}\right)^{7}}}{e^{8}}$
trager	$-\frac{2\left(-2c^{4}d^{4}e\,x^{4}+a\,c^{3}d^{3}e^{2}x^{3}-9c^{4}d^{5}x^{3}+15a^{2}c^{2}d^{2}e^{3}x^{2}-27a\,c^{3}d^{4}e\,x^{2}+19a^{3}cd\,e^{4}x-27a^{2}c^{2}d^{3}e^{2}x+7a^{4}e^{5}-9a^{3}c\,d^{2}e^{3}\right)\sqrt{cd\,x^{2}e+a\,e\cdots}}{63\left(a^{2}e^{4}-2ac\,d^{2}e^{2}+c^{2}d^{4}\right)(ex+d)^{5}}$

input
```
int((a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(5/2)/(e*x+d)^8,x,method=_RETURNVERB
OSE)
```

output
```
-2/63*(c*d*x+a*e)*(-2*c*d*e*x+7*a*e^2-9*c*d^2)*(c*d*e*x^2+a*e^2*x+c*d^2*x+
a*d*e)^(5/2)/(e*x+d)^7/(a^2*e^4-2*a*c*d^2*e^2+c^2*d^4)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 344 vs. 2(103) = 206.

Time = 7.88 (sec) , antiderivative size = 344, normalized size of antiderivative = 3.10

$$\int \frac{(ade+(cd^2+ae^2)\,x+cdex^2)^{5/2}}{(d+ex)^8}\,dx = \frac{2\,(2\,c^4d^4ex^4+9\,a^3cd^2e^3-7\,a^4e^5+(9}{63\,(c^2d^9-2\,acd^7e^2+a^2d^5e^4+(c^2d^4e^5-2\,acd^2e^7+a^2e^9)x^5+5\,(}$$

input
```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(e*x+d)^8,x, algorithm="
fricas")
```

output
```
2/63*(2*c^4*d^4*e*x^4 + 9*a^3*c*d^2*e^3 - 7*a^4*e^5 + (9*c^4*d^5 - a*c^3*d
^3*e^2)*x^3 + 3*(9*a*c^3*d^4*e - 5*a^2*c^2*d^2*e^3)*x^2 + (27*a^2*c^2*d^3*
e^2 - 19*a^3*c*d*e^4)*x)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)/(c^2*
d^9 - 2*a*c*d^7*e^2 + a^2*d^5*e^4 + (c^2*d^4*e^5 - 2*a*c*d^2*e^7 + a^2*e^9
)*x^5 + 5*(c^2*d^5*e^4 - 2*a*c*d^3*e^6 + a^2*d*e^8)*x^4 + 10*(c^2*d^6*e^3
- 2*a*c*d^4*e^5 + a^2*d^2*e^7)*x^3 + 10*(c^2*d^7*e^2 - 2*a*c*d^5*e^4 + a^2
*d^3*e^6)*x^2 + 5*(c^2*d^8*e - 2*a*c*d^6*e^3 + a^2*d^4*e^5)*x)
```

Sympy [F]

$$\int \frac{(ade+(cd^2+ae^2)\,x+cdex^2)^{5/2}}{(d+ex)^8}\,dx = \int \frac{((d+ex)\,(ae+cdx))^{\frac{5}{2}}}{(d+ex)^8}\,dx$$

input
```
integrate((a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(5/2)/(e*x+d)**8,x)
```

output
```
Integral(((d + e*x)*(a*e + c*d*x))**(5/2)/(d + e*x)**8, x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(ade + (cd^2 + ae^2)x + cdex^2)^{5/2}}{(d+ex)^8}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(e*x+d)^8,x, algorithm="
maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e*(a*e^2-c*d^2)>0)', see `assume
?` for mor
```

Giac [F(-2)]

Exception generated.

$$\int \frac{(ade + (cd^2 + ae^2)x + cdex^2)^{5/2}}{(d+ex)^8}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(e*x+d)^8,x, algorithm="
giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Unable to divide, perhaps due to ro
unding error%%%{%%%{1,[0,0,5]%%%},[10]%%%}+%%%{%%{[%%%{-10,[0,1,4]%%%},0]:
[1,0,%%%{
```

Mupad [B] (verification not implemented)

Time = 8.06 (sec) , antiderivative size = 3125, normalized size of antiderivative = 28.15

$$\int \frac{(ade + (cd^2 + ae^2)\,x + cdex^2)^{5/2}}{(d + ex)^8}\,dx = \text{Too large to display}$$

input
```
int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(5/2)/(d + e*x)^8,x)
```

output
```
(((d*((8*c^5*d^6)/(63*e*(a*e^2 - c*d^2)^2*(3*a*e^3 - 3*c*d^2*e)) - (4*c^4*
d^4*(17*a*e^2 - 13*c*d^2))/(63*e*(a*e^2 - c*d^2)^2*(3*a*e^3 - 3*c*d^2*e)))
)/e + (4*c^3*d^3*(148*a^2*e^4 + 73*c^2*d^4 - 211*a*c*d^2*e^2))/(315*e^2*(a
*e^2 - c*d^2)^2*(3*a*e^3 - 3*c*d^2*e)))*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e
*x^2)^(1/2))/(d + e*x)^2 - (((2*a^3*e^4)/(9*a*e^3 - 9*c*d^2*e) - (d*((d*((
2*c^3*d^4)/(9*a*e^3 - 9*c*d^2*e) - (6*a*c^2*d^2*e^2)/(9*a*e^3 - 9*c*d^2*e)
))/e + (6*a^2*c*d*e^3)/(9*a*e^3 - 9*c*d^2*e)))/e)*(x*(a*e^2 + c*d^2) + a*d
*e + c*d*e*x^2)^(1/2))/(d + e*x)^5 + (((2*c^4*d^5 + 26*a*c^3*d^3*e^2)/(63*
e^2*(a*e^2 - c*d^2)*(3*a*e^3 - 3*c*d^2*e)) - (4*c^4*d^5)/(9*e^2*(a*e^2 - c
*d^2)*(3*a*e^3 - 3*c*d^2*e)))*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2
))/(d + e*x)^2 + (((d*((d*((32*c^7*d^8)/(945*e*(a*e^2 - c*d^2)^5) - (32*c^
6*d^6*(17*a*e^2 - 14*c*d^2))/(945*e*(a*e^2 - c*d^2)^5)))/e + (32*c^5*d^5*(
116*a^2*e^4 + 85*c^2*d^4 - 198*a*c*d^2*e^2))/(945*e^2*(a*e^2 - c*d^2)^5)))
/e - (32*a*c^4*d^4*(100*a^2*e^4 + 85*c^2*d^4 - 184*a*c*d^2*e^2))/(945*e*(a
*e^2 - c*d^2)^5))*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2))/(d + e*x)
 + (((d*((d*((4*c^4*d^5)/(9*(a*e^2 - c*d^2)*(7*a*e^3 - 7*c*d^2*e)) - (4*c^
3*d^3*(8*a*e^2 - 5*c*d^2))/(9*(a*e^2 - c*d^2)*(7*a*e^3 - 7*c*d^2*e))))/e +
 (20*c^4*d^6 - 80*a*c^3*d^4*e^2 + 72*a^2*c^2*d^2*e^4)/(9*e*(a*e^2 - c*d^2)
*(7*a*e^3 - 7*c*d^2*e))))/e - (4*a*c*d*(11*a^2*e^4 + 5*c^2*d^4 - 15*a*c*d^
2*e^2))/(9*(a*e^2 - c*d^2)*(7*a*e^3 - 7*c*d^2*e)))*(x*(a*e^2 + c*d^2) +...
```

Reduce [B] (verification not implemented)

Time = 0.39 (sec) , antiderivative size = 592, normalized size of antiderivative = 5.33

$$\int \frac{(ade + (cd^2 + ae^2)\,x + cdex^2)^{5/2}}{(d + ex)^8}\,dx = \frac{-\frac{2\sqrt{ex+d}\,\sqrt{cdx+ae}\,a^4e^9}{9} + \frac{2\sqrt{ex+d}\,\sqrt{cdx+ae}\,a^3c\,d^2e^7}{7} - \frac{38\sqrt{ex+d}\,\sqrt{cdx+ae}\,a^3cd}{63}}{e^4\,(a^2e^9x^5 -}$$

input
```
int((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(e*x+d)^8,x)
```

output
```
(2*( - 7*sqrt(d + e*x)*sqrt(a*e + c*d*x)*a**4*e**9 + 9*sqrt(d + e*x)*sqrt(
a*e + c*d*x)*a**3*c*d**2*e**7 - 19*sqrt(d + e*x)*sqrt(a*e + c*d*x)*a**3*c*
d*e**8*x + 27*sqrt(d + e*x)*sqrt(a*e + c*d*x)*a**2*c**2*d**3*e**6*x - 15*s
qrt(d + e*x)*sqrt(a*e + c*d*x)*a**2*c**2*d**2*e**7*x**2 + 27*sqrt(d + e*x)
*sqrt(a*e + c*d*x)*a*c**3*d**4*e**5*x**2 - sqrt(d + e*x)*sqrt(a*e + c*d*x)
*a*c**3*d**3*e**6*x**3 + 9*sqrt(d + e*x)*sqrt(a*e + c*d*x)*c**4*d**5*e**4*
x**3 + 2*sqrt(d + e*x)*sqrt(a*e + c*d*x)*c**4*d**4*e**5*x**4 - 2*sqrt(e)*s
qrt(d)*sqrt(c)*c**4*d**9 - 10*sqrt(e)*sqrt(d)*sqrt(c)*c**4*d**8*e*x - 20*s
qrt(e)*sqrt(d)*sqrt(c)*c**4*d**7*e**2*x**2 - 20*sqrt(e)*sqrt(d)*sqrt(c)*c*
*4*d**6*e**3*x**3 - 10*sqrt(e)*sqrt(d)*sqrt(c)*c**4*d**5*e**4*x**4 - 2*sqr
t(e)*sqrt(d)*sqrt(c)*c**4*d**4*e**5*x**5))/(63*e**4*(a**2*d**5*e**4 + 5*a*
*2*d**4*e**5*x + 10*a**2*d**3*e**6*x**2 + 10*a**2*d**2*e**7*x**3 + 5*a**2*
d*e**8*x**4 + a**2*e**9*x**5 - 2*a*c*d**7*e**2 - 10*a*c*d**6*e**3*x - 20*a
*c*d**5*e**4*x**2 - 20*a*c*d**4*e**5*x**3 - 10*a*c*d**3*e**6*x**4 - 2*a*c*
d**2*e**7*x**5 + c**2*d**9 + 5*c**2*d**8*e*x + 10*c**2*d**7*e**2*x**2 + 10
*c**2*d**6*e**3*x**3 + 5*c**2*d**5*e**4*x**4 + c**2*d**4*e**5*x**5))
```

3.232 $\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{(d+ex)^9}\,dx$

Optimal result

Integrand size = 37, antiderivative size = 171

$$\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{(d+ex)^9}\,dx=\frac{2\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{7/2}}{11\left(cd^2-ae^2\right)(d+ex)^9}+\frac{8cd\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{7/2}}{99\left(cd^2-ae^2\right)^2(d+ex)^8}+\frac{16c^2d^2\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{7/2}}{693\left(cd^2-ae^2\right)^3(d+ex)^7}$$

output
```
2/11*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(7/2)/(-a*e^2+c*d^2)/(e*x+d)^9+8/99
*c*d*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(7/2)/(-a*e^2+c*d^2)^2/(e*x+d)^8+16
/693*c^2*d^2*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(7/2)/(-a*e^2+c*d^2)^3/(e*x
+d)^7
```

Mathematica [A] (verified)

Time = 0.75 (sec) , antiderivative size = 104, normalized size of antiderivative = 0.61

$$\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{(d+ex)^9}\,dx=\frac{2(ae+cdx)^3\sqrt{(ae+cdx)(d+ex)}\left(63a^2e^4-14acde^2(11d+2ex)\right.}{693\left(cd^2-ae^2\right)^3(d+ex)^6}$$

input
```
Integrate[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2)/(d + e*x)^9,x]
```

output
```
(2*(a*e + c*d*x)^3*Sqrt[(a*e + c*d*x)*(d + e*x)]*(63*a^2*e^4 - 14*a*c*d*e^
2*(11*d + 2*e*x) + c^2*d^2*(99*d^2 + 44*d*e*x + 8*e^2*x^2)))/(693*(c*d^2 -
 a*e^2)^3*(d + e*x)^6)
```

Rubi [A] (verified)

Time = 0.52 (sec) , antiderivative size = 186, normalized size of antiderivative = 1.09, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.081$, Rules used = {1129, 1129, 1123}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/2}}{(d+ex)^9}dx$$

$$\Big\downarrow \text{1129}$$

$$\frac{4cd\int \frac{\left(cdex^2+\left(cd^2+ae^2\right)x+ade\right)^{5/2}}{(d+ex)^8}dx}{11\left(cd^2-ae^2\right)}+\frac{2\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{7/2}}{11(d+ex)^9\left(cd^2-ae^2\right)}$$

$$\Big\downarrow \text{1129}$$

$$\frac{4cd\left(\frac{2cd\int \frac{\left(cdex^2+\left(cd^2+ae^2\right)x+ade\right)^{5/2}}{(d+ex)^7}dx}{9\left(cd^2-ae^2\right)}+\frac{2\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{7/2}}{9(d+ex)^8\left(cd^2-ae^2\right)}\right)}{11\left(cd^2-ae^2\right)}+\frac{2\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{7/2}}{11(d+ex)^9\left(cd^2-ae^2\right)}$$

$$\Big\downarrow \text{1123}$$

$$\frac{2\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{7/2}}{11(d+ex)^9\left(cd^2-ae^2\right)}+\frac{4cd\left(\frac{4cd\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{7/2}}{63(d+ex)^7\left(cd^2-ae^2\right)^2}+\frac{2\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{7/2}}{9(d+ex)^8\left(cd^2-ae^2\right)}\right)}{11\left(cd^2-ae^2\right)}$$

input
```
Int[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2)/(d + e*x)^9,x]
```

output
```
(2*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(7/2))/(11*(c*d^2 - a*e^2)*(d +
 e*x)^9) + (4*c*d*((2*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(7/2))/(9*(c
*d^2 - a*e^2)*(d + e*x)^8) + (4*c*d*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2
)^(7/2))/(63*(c*d^2 - a*e^2)^2*(d + e*x)^7)))/(11*(c*d^2 - a*e^2))
```

Defintions of rubi rules used

rule 1123
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[e*(d + e*x)^m*((a + b*x + c*x^2)^(p + 1)/((p + 1)*(2*c*d - b
*e))), x] /; FreeQ[{a, b, c, d, e, m, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2,
0] && EqQ[m + 2*p + 2, 0]
```

rule 1129
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(-e)*(d + e*x)^m*((a + b*x + c*x^2)^(p + 1)/((m + p + 1)*(2*
c*d - b*e))), x] + Simp[c*(Simplify[m + 2*p + 2]/((m + p + 1)*(2*c*d - b*e)
))   Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d
, e, m, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && ILtQ[Simplify[m + 2*p +
2], 0]
```

Maple [A] (verified)

Time = 12.99 (sec) , antiderivative size = 146, normalized size of antiderivative = 0.85

method	result
gosper	$-\frac{2(cdx+ae)\left(8x^2c^2d^2e^2-28xacd\,e^3+44x\,c^2d^3e+63a^2e^4-154ac\,d^2e^2+99c^2d^4\right)\left(cd\,x^2e+a\,e^2x+c\,d^2x+ade\right)^{\frac{5}{2}}}{693(ex+d)^8\left(e^6a^3-3d^2e^4a^2c+3d^4e^2a\,c^2-d^6c^3\right)}$
orering	$-\frac{2\left(8x^2c^2d^2e^2-28xacd\,e^3+44x\,c^2d^3e+63a^2e^4-154ac\,d^2e^2+99c^2d^4\right)(cdx+ae)\left(ade+\left(a\,e^2+c\,d^2\right)x+cd\,x^2e\right)^{\frac{5}{2}}}{693\left(e^6a^3-3d^2e^4a^2c+3d^4e^2a\,c^2-d^6c^3\right)(ex+d)^8}$
default	$\frac{-\frac{2\left(dec\left(x+\frac{d}{e}\right)^2+\left(a\,e^2-c\,d^2\right)\left(x+\frac{d}{e}\right)\right)^{\frac{7}{2}}}{11\left(a\,e^2-c\,d^2\right)\left(x+\frac{d}{e}\right)^9}-\frac{4dec\left(-\frac{2\left(dec\left(x+\frac{d}{e}\right)^2+\left(a\,e^2-c\,d^2\right)\left(x+\frac{d}{e}\right)\right)^{\frac{7}{2}}}{9\left(a\,e^2-c\,d^2\right)\left(x+\frac{d}{e}\right)^8}+\frac{4dec\left(dec\left(x+\frac{d}{e}\right)^2+\left(a\,e^2-c\,d^2\right)\left(x+\frac{d}{e}\right)\right)^{\frac{7}{2}}}{63\left(a\,e^2-c\,d^2\right)^2\left(x+\frac{d}{e}\right)^7}\right)}{11\left(a\,e^2-c\,d^2\right)}}{e^9}$
trager	$-\frac{2\left(8c^5d^5e^2x^5-4a\,c^4d^4e^3x^4+44c^5d^6e\,x^4+3a^2c^3d^3e^4x^3-22a\,c^4d^5e^2x^3+99c^5d^7x^3+113a^3c^2d^2e^5x^2-330a^2c^3d^4e^3x^2+297a\,c^4d^6e \cdots\right.}{693\left(e^6a^3-3d^2e^4a^2c+3d^4e^2a\,c \cdots\right.}$

input
```
int((a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(5/2)/(e*x+d)^9,x,method=_RETURNVERB
OSE)
```

output
```
-2/693*(c*d*x+a*e)*(8*c^2*d^2*e^2*x^2-28*a*c*d*e^3*x+44*c^2*d^3*e*x+63*a^2
*e^4-154*a*c*d^2*e^2+99*c^2*d^4)*(c*d*e*x^2+a*e^2*x+c*d^2*x+a*d*e)^(5/2)/(
e*x+d)^8/(a^3*e^6-3*a^2*c*d^2*e^4+3*a*c^2*d^4*e^2-c^3*d^6)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 569 vs. $2(159) = 318$.

Time = 18.33 (sec) , antiderivative size = 569, normalized size of antiderivative = 3.33

$$\int \frac{(ade + (cd^2 + ae^2)x + cdex^2)^{5/2}}{(d + ex)^9} dx = \frac{2(8c^5d^5e^2x^5 + 99}{693(c^3d^{12} - 3ac^2d^{10}e^2 + 3a^2cd^8e^4 - a^3d^6e^6 + (c^3d^6e^6 - 3ac^2d^4}$$

input
```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(e*x+d)^9,x, algorithm="
fricas")
```

output
```
2/693*(8*c^5*d^5*e^2*x^5 + 99*a^3*c^2*d^4*e^3 - 154*a^4*c*d^2*e^5 + 63*a^5
*e^7 + 4*(11*c^5*d^6*e - a*c^4*d^4*e^3)*x^4 + (99*c^5*d^7 - 22*a*c^4*d^5*e
^2 + 3*a^2*c^3*d^3*e^4)*x^3 + (297*a*c^4*d^6*e - 330*a^2*c^3*d^4*e^3 + 113
*a^3*c^2*d^2*e^5)*x^2 + (297*a^2*c^3*d^5*e^2 - 418*a^3*c^2*d^3*e^4 + 161*a
^4*c*d*e^6)*x)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)/(c^3*d^12 - 3*a
*c^2*d^10*e^2 + 3*a^2*c*d^8*e^4 - a^3*d^6*e^6 + (c^3*d^6*e^6 - 3*a*c^2*d^4
*e^8 + 3*a^2*c*d^2*e^10 - a^3*e^12)*x^6 + 6*(c^3*d^7*e^5 - 3*a*c^2*d^5*e^7
 + 3*a^2*c*d^3*e^9 - a^3*d*e^11)*x^5 + 15*(c^3*d^8*e^4 - 3*a*c^2*d^6*e^6 +
 3*a^2*c*d^4*e^8 - a^3*d^2*e^10)*x^4 + 20*(c^3*d^9*e^3 - 3*a*c^2*d^7*e^5 +
 3*a^2*c*d^5*e^7 - a^3*d^3*e^9)*x^3 + 15*(c^3*d^10*e^2 - 3*a*c^2*d^8*e^4 +
 3*a^2*c*d^6*e^6 - a^3*d^4*e^8)*x^2 + 6*(c^3*d^11*e - 3*a*c^2*d^9*e^3 + 3*
a^2*c*d^7*e^5 - a^3*d^5*e^7)*x)
```

Sympy [F]

$$\int \frac{(ade + (cd^2 + ae^2)x + cdex^2)^{5/2}}{(d+ex)^9} dx = \int \frac{((d+ex)(ae+cdx))^{\frac{5}{2}}}{(d+ex)^9} dx$$

input
```
integrate((a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(5/2)/(e*x+d)**9,x)
```

output
```
Integral(((d + e*x)*(a*e + c*d*x))**(5/2)/(d + e*x)**9, x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(ade + (cd^2 + ae^2)x + cdex^2)^{5/2}}{(d+ex)^9} dx = \text{Exception raised: ValueError}$$

input
```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(e*x+d)^9,x, algorithm="
maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e*(a*e^2-c*d^2)>0)', see `assume
?` for mor
```

Giac [F(-2)]

Exception generated.

$$\int \frac{(ade + (cd^2 + ae^2)x + cdex^2)^{5/2}}{(d+ex)^9} dx = \text{Exception raised: TypeError}$$

input
```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(e*x+d)^9,x, algorithm="
giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Unable to divide, perhaps due to ro
unding error%%%{%%%{1,[0,0,6]%%%},[12]%%%}+%%%{%%{[%%%{-12,[0,1,5]%%%},0]:
[1,0,%%%{
```

Mupad [B] (verification not implemented)

Time = 9.32 (sec) , antiderivative size = 4096, normalized size of antiderivative = 23.95

$$\int \frac{(ade + (cd^2 + ae^2)\,x + cdex^2)^{5/2}}{(d+ex)^9}\,dx = \text{Too large to display}$$

input
```
int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(5/2)/(d + e*x)^9,x)
```

output
```
(((d*((8*c^5*d^6)/(99*e*(a*e^2 - c*d^2)^2*(5*a*e^3 - 5*c*d^2*e)) - (4*c^4*
d^4*(19*a*e^2 - 15*c*d^2))/(99*e*(a*e^2 - c*d^2)^2*(5*a*e^3 - 5*c*d^2*e)))
)/e + (4*c^3*d^3*(33*a^2*e^4 + 16*c^2*d^4 - 47*a*c*d^2*e^2))/(99*e^2*(a*e^
2 - c*d^2)^2*(5*a*e^3 - 5*c*d^2*e)))*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^
2)^(1/2))/(d + e*x)^3 - (((2*a^3*e^4)/(11*a*e^3 - 11*c*d^2*e) - (d*((d*((2
*c^3*d^4)/(11*a*e^3 - 11*c*d^2*e) - (6*a*c^2*d^2*e^2)/(11*a*e^3 - 11*c*d^2
*e)))/e + (6*a^2*c*d*e^3)/(11*a*e^3 - 11*c*d^2*e)))/e)*(x*(a*e^2 + c*d^2)
+ a*d*e + c*d*e*x^2)^(1/2))/(d + e*x)^6 + (((d*((16*c^6*d^7)/(693*e*(a*e^2
 - c*d^2)^3*(3*a*e^3 - 3*c*d^2*e)) - (8*c^5*d^5*(29*a*e^2 - 25*c*d^2))/(69
3*e*(a*e^2 - c*d^2)^3*(3*a*e^3 - 3*c*d^2*e))))/e + (8*c^4*d^4*(466*a^2*e^4
 + 331*c^2*d^4 - 787*a*c*d^2*e^2))/(3465*e^2*(a*e^2 - c*d^2)^3*(3*a*e^3 -
3*c*d^2*e)))*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2))/(d + e*x)^2 +
(((6*c^4*d^5 + 22*a*c^3*d^3*e^2)/(77*e^2*(a*e^2 - c*d^2)*(5*a*e^3 - 5*c*d^
2*e)) - (4*c^4*d^5)/(11*e^2*(a*e^2 - c*d^2)*(5*a*e^3 - 5*c*d^2*e)))*(x*(a*
e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2))/(d + e*x)^3 + (((188*c^5*d^6 - 14
8*a*c^4*d^4*e^2)/(495*e^2*(a*e^2 - c*d^2)^2*(3*a*e^3 - 3*c*d^2*e)) - (8*c^
5*d^6)/(99*e^2*(a*e^2 - c*d^2)^2*(3*a*e^3 - 3*c*d^2*e)))*(x*(a*e^2 + c*d^2
) + a*d*e + c*d*e*x^2)^(1/2))/(d + e*x)^2 + (((d*((d*((64*c^8*d^9)/(10395*
e*(a*e^2 - c*d^2)^6) - (64*c^7*d^7*(23*a*e^2 - 20*c*d^2))/(10395*e*(a*e^2
- c*d^2)^6)))/e + (64*c^6*d^6*(218*a^2*e^4 + 175*c^2*d^4 - 390*a*c*d^2*...
```

Reduce [B] (verification not implemented)

Time = 0.67 (sec) , antiderivative size = 951, normalized size of antiderivative = 5.56

$$\int \frac{(ade + (cd^2 + ae^2)x + cdex^2)^{5/2}}{(d + ex)^9} dx = \text{Too large to display}$$

input
```
int((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(e*x+d)^9,x)
```

output
```
(2*( - 63*sqrt(d + e*x)*sqrt(a*e + c*d*x)*a**5*e**11 + 154*sqrt(d + e*x)*s
qrt(a*e + c*d*x)*a**4*c*d**2*e**9 - 161*sqrt(d + e*x)*sqrt(a*e + c*d*x)*a*
*4*c*d*e**10*x - 99*sqrt(d + e*x)*sqrt(a*e + c*d*x)*a**3*c**2*d**4*e**7 +
418*sqrt(d + e*x)*sqrt(a*e + c*d*x)*a**3*c**2*d**3*e**8*x - 113*sqrt(d + e
*x)*sqrt(a*e + c*d*x)*a**3*c**2*d**2*e**9*x**2 - 297*sqrt(d + e*x)*sqrt(a*
e + c*d*x)*a**2*c**3*d**5*e**6*x + 330*sqrt(d + e*x)*sqrt(a*e + c*d*x)*a**
2*c**3*d**4*e**7*x**2 - 3*sqrt(d + e*x)*sqrt(a*e + c*d*x)*a**2*c**3*d**3*e
**8*x**3 - 297*sqrt(d + e*x)*sqrt(a*e + c*d*x)*a*c**4*d**6*e**5*x**2 + 22*
sqrt(d + e*x)*sqrt(a*e + c*d*x)*a*c**4*d**5*e**6*x**3 + 4*sqrt(d + e*x)*sq
rt(a*e + c*d*x)*a*c**4*d**4*e**7*x**4 - 99*sqrt(d + e*x)*sqrt(a*e + c*d*x)
*c**5*d**7*e**4*x**3 - 44*sqrt(d + e*x)*sqrt(a*e + c*d*x)*c**5*d**6*e**5*x
**4 - 8*sqrt(d + e*x)*sqrt(a*e + c*d*x)*c**5*d**5*e**6*x**5 + 8*sqrt(e)*sq
rt(d)*sqrt(c)*c**5*d**11 + 48*sqrt(e)*sqrt(d)*sqrt(c)*c**5*d**10*e*x + 120
*sqrt(e)*sqrt(d)*sqrt(c)*c**5*d**9*e**2*x**2 + 160*sqrt(e)*sqrt(d)*sqrt(c)
*c**5*d**8*e**3*x**3 + 120*sqrt(e)*sqrt(d)*sqrt(c)*c**5*d**7*e**4*x**4 + 4
8*sqrt(e)*sqrt(d)*sqrt(c)*c**5*d**6*e**5*x**5 + 8*sqrt(e)*sqrt(d)*sqrt(c)*
c**5*d**5*e**6*x**6))/(693*e**4*(a**3*d**6*e**6 + 6*a**3*d**5*e**7*x + 15*
a**3*d**4*e**8*x**2 + 20*a**3*d**3*e**9*x**3 + 15*a**3*d**2*e**10*x**4 + 6
*a**3*d*e**11*x**5 + a**3*e**12*x**6 - 3*a**2*c*d**8*e**4 - 18*a**2*c*d**7
*e**5*x - 45*a**2*c*d**6*e**6*x**2 - 60*a**2*c*d**5*e**7*x**3 - 45*a**2...
```

3.233 $\int \frac{(ade+(cd^2+ae^2)x+cdex^2)^{5/2}}{(d+ex)^{10}}\,dx$

Optimal result

Integrand size = 37, antiderivative size = 231

$$\int \frac{(ade+(cd^2+ae^2)x+cdex^2)^{5/2}}{(d+ex)^{10}}\,dx = \frac{2(ade+(cd^2+ae^2)x+cdex^2)^{7/2}}{13(cd^2-ae^2)(d+ex)^{10}} + \frac{12cd(ade+(cd^2+ae^2)x+cdex^2)^{7/2}}{143(cd^2-ae^2)^2(d+ex)^9} + \frac{16c^2d^2(ade+(cd^2+ae^2)x+cdex^2)^{7/2}}{429(cd^2-ae^2)^3(d+ex)^8} + \frac{32c^3d^3(ade+(cd^2+ae^2)x+cdex^2)^{7/2}}{3003(cd^2-ae^2)^4(d+ex)^7}$$

output
```
2/13*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(7/2)/(-a*e^2+c*d^2)/(e*x+d)^10+12/
143*c*d*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(7/2)/(-a*e^2+c*d^2)^2/(e*x+d)^9
+16/429*c^2*d^2*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(7/2)/(-a*e^2+c*d^2)^3/(
e*x+d)^8+32/3003*c^3*d^3*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(7/2)/(-a*e^2+c
*d^2)^4/(e*x+d)^7
```

Mathematica [A] (verified)

Time = 1.88 (sec) , antiderivative size = 148, normalized size of antiderivative = 0.64

$$\int \frac{(ade+(cd^2+ae^2)\,x+cdex^2)^{5/2}}{(d+ex)^{10}}\,dx = \frac{2(ae+cdx)^3\sqrt{(ae+cdx)(d+ex)}(-231a^3e^6+63a^2cde^4(13d+2}{3}$$

input
```
Integrate[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2)/(d + e*x)^10,x]
```

output
```
(2*(a*e + c*d*x)^3*Sqrt[(a*e + c*d*x)*(d + e*x)]*(-231*a^3*e^6 + 63*a^2*c*
d*e^4*(13*d + 2*e*x) - 7*a*c^2*d^2*e^2*(143*d^2 + 52*d*e*x + 8*e^2*x^2) +
c^3*d^3*(429*d^3 + 286*d^2*e*x + 104*d*e^2*x^2 + 16*e^3*x^3)))/(3003*(c*d^
2 - a*e^2)^4*(d + e*x)^7)
```

Rubi [A] (verified)

Time = 0.66 (sec) , antiderivative size = 261, normalized size of antiderivative = 1.13, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.108, Rules used = {1129, 1129, 1129, 1123}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/2}}{(d+ex)^{10}}dx$$

$$\Big\downarrow \text{1129}$$

$$\frac{6cd\int \frac{\left(cdex^2+(cd^2+ae^2)x+ade\right)^{5/2}}{(d+ex)^9}dx}{13\left(cd^2-ae^2\right)}+\frac{2\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{7/2}}{13(d+ex)^{10}\left(cd^2-ae^2\right)}$$

$$\Big\downarrow \text{1129}$$

$$\frac{6cd\left(\frac{4cd\int \frac{\left(cdex^2+\left(cd^2+ae^2\right)x+ade\right)^{5/2}}{(d+ex)^8}dx}{11(cd^2-ae^2)}+\frac{2\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{7/2}}{11(d+ex)^9(cd^2-ae^2)}\right)}{13\left(cd^2-ae^2\right)}+\frac{2\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{7/2}}{13(d+ex)^{10}\left(cd^2-ae^2\right)}$$

$$\Big\downarrow \text{1129}$$

$$\frac{6cd\left(\frac{4cd\left(\frac{2cd\int \frac{\left(cdex^2+\left(cd^2+ae^2\right)x+ade\right)^{5/2}}{(d+ex)^7}dx}{9\left(cd^2-ae^2\right)}+\frac{2\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{7/2}}{9(d+ex)^8\left(cd^2-ae^2\right)}\right)}{11(cd^2-ae^2)}+\frac{2\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{7/2}}{11(d+ex)^9(cd^2-ae^2)}\right)}{13\left(cd^2-ae^2\right)}+\frac{2\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{7/2}}{13(d+ex)^{10}\left(cd^2-ae^2\right)}$$

$$\Big\downarrow \text{1123}$$

$$\frac{2\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{7/2}}{13(d+ex)^{10}\left(cd^2-ae^2\right)}+\frac{6cd\left(\frac{2\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{7/2}}{11(d+ex)^9(cd^2-ae^2)}+\frac{4cd\left(\frac{4cd\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{7/2}}{63(d+ex)^7\left(cd^2-ae^2\right)^2}+\frac{2\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{7/2}}{9(d+ex)^8\left(cd^2-ae^2\right)}\right)}{11(cd^2-ae^2)}\right)}{13\left(cd^2-ae^2\right)}$$

input
```
Int[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2)/(d + e*x)^10,x]
```

output
```
(2*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(7/2))/(13*(c*d^2 - a*e^2)*(d +
 e*x)^10) + (6*c*d*((2*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(7/2))/(11*
(c*d^2 - a*e^2)*(d + e*x)^9) + (4*c*d*((2*(a*d*e + (c*d^2 + a*e^2)*x + c*d
*e*x^2)^(7/2))/(9*(c*d^2 - a*e^2)*(d + e*x)^8) + (4*c*d*(a*d*e + (c*d^2 +
a*e^2)*x + c*d*e*x^2)^(7/2))/(63*(c*d^2 - a*e^2)^2*(d + e*x)^7)))/(11*(c*d
^2 - a*e^2))))/(13*(c*d^2 - a*e^2))
```

Defintions of rubi rules used

rule 1123
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[e*(d + e*x)^m*((a + b*x + c*x^2)^(p + 1)/((p + 1)*(2*c*d - b
*e))), x] /; FreeQ[{a, b, c, d, e, m, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2,
0] && EqQ[m + 2*p + 2, 0]
```

rule 1129
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(-e)*(d + e*x)^m*((a + b*x + c*x^2)^(p + 1)/((m + p + 1)*(2*
c*d - b*e))), x] + Simp[c*(Simplify[m + 2*p + 2]/((m + p + 1)*(2*c*d - b*e)
))   Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d
, e, m, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && ILtQ[Simplify[m + 2*p +
2], 0]
```

Maple [A] (verified)

Time = 16.93 (sec) , antiderivative size = 217, normalized size of antiderivative = 0.94

method	result
gosper	$-\frac{2(cdx+ae)(-16c^3d^3e^3x^3+56x^2a\,c^2d^2e^4-104c^3d^4e^2x^2-126x\,a^2cd\,e^5+364xa\,c^2d^3e^3-286c^3d^5ex+231e^6a^3-819d^2e^4a^2c+1001c}{3003(ex+d)^9(a^4e^8-4a^3c\,d^2e^6+6a^2c^2d^4e^4-4a\,c^3d^6e^2+c^4d^8)}$
orering	$-\frac{2(-16c^3d^3e^3x^3+56x^2a\,c^2d^2e^4-104c^3d^4e^2x^2-126x\,a^2cd\,e^5+364xa\,c^2d^3e^3-286c^3d^5ex+231e^6a^3-819d^2e^4a^2c+1001d^4e^2a\,c^2-}{3003(a^4e^8-4a^3c\,d^2e^6+6a^2c^2d^4e^4-4a\,c^3d^6e^2+c^4d^8)(ex+d)^9}$
default	$\frac{-\frac{2\left(dec\left(x+\frac{d}{e}\right)^2+\left(a\,e^2-c\,d^2\right)\left(x+\frac{d}{e}\right)\right)^{\frac{7}{2}}}{13\left(a\,e^2-c\,d^2\right)\left(x+\frac{d}{e}\right)^{10}}-\frac{6dec\left(-\frac{2\left(dec\left(x+\frac{d}{e}\right)^2+\left(a\,e^2-c\,d^2\right)\left(x+\frac{d}{e}\right)\right)^{\frac{7}{2}}}{11\left(a\,e^2-c\,d^2\right)\left(x+\frac{d}{e}\right)^9}-\frac{4dec\left(-\frac{2\left(dec\left(x+\frac{d}{e}\right)^2+\left(a\,e^2-c\,d^2\right)\left(x+\frac{d}{e}\right)\right)^{\frac{7}{2}}}{9\left(a\,e^2-c\,d^2\right)\left(x+\frac{d}{e}\right)^8}\right.}{11\left(a\,e^2\right.}\right.}{13\left(a\,e^2-c\,d^2\right)}}{e^{10}}$
trager	$-\frac{2(-16c^6d^6e^3x^6+8a\,c^5d^5e^4x^5-104c^6d^7e^2x^5-6a^2c^4d^4e^5x^4+52a\,c^5d^6e^3x^4-286c^6d^8e\,x^4+5a^3c^3d^3e^6x^3-39a^2c^4d^5e^4x^3+143a\,c^5}{}$

input
```
int((a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(5/2)/(e*x+d)^10,x,method=_RETURNVER
BOSE)
```

output
```
-2/3003*(c*d*x+a*e)*(-16*c^3*d^3*e^3*x^3+56*a*c^2*d^2*e^4*x^2-104*c^3*d^4*
e^2*x^2-126*a^2*c*d*e^5*x+364*a*c^2*d^3*e^3*x-286*c^3*d^5*e*x+231*a^3*e^6-
819*a^2*c*d^2*e^4+1001*a*c^2*d^4*e^2-429*c^3*d^6)*(c*d*e*x^2+a*e^2*x+c*d^2
*x+a*d*e)^(5/2)/(e*x+d)^9/(a^4*e^8-4*a^3*c*d^2*e^6+6*a^2*c^2*d^4*e^4-4*a*c
^3*d^6*e^2+c^4*d^8)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 823 vs. $2(215) = 430$.

Time = 40.70 (sec) , antiderivative size = 823, normalized size of antiderivative = 3.56

$$\int \frac{(ade + (cd^2 + ae^2)x + cdex^2)^{5/2}}{(d + ex)^{10}} dx = \text{Too large to display}$$

input
```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(e*x+d)^10,x, algorithm=
"fricas")
```

output
```
2/3003*(16*c^6*d^6*e^3*x^6 + 429*a^3*c^3*d^6*e^3 - 1001*a^4*c^2*d^4*e^5 +
819*a^5*c*d^2*e^7 - 231*a^6*e^9 + 8*(13*c^6*d^7*e^2 - a*c^5*d^5*e^4)*x^5 +
 2*(143*c^6*d^8*e - 26*a*c^5*d^6*e^3 + 3*a^2*c^4*d^4*e^5)*x^4 + (429*c^6*d
^9 - 143*a*c^5*d^7*e^2 + 39*a^2*c^4*d^5*e^4 - 5*a^3*c^3*d^3*e^6)*x^3 + (12
87*a*c^5*d^8*e - 2145*a^2*c^4*d^6*e^3 + 1469*a^3*c^3*d^4*e^5 - 371*a^4*c^2
*d^2*e^7)*x^2 + (1287*a^2*c^4*d^7*e^2 - 2717*a^3*c^3*d^5*e^4 + 2093*a^4*c^
2*d^3*e^6 - 567*a^5*c*d*e^8)*x)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x
)/(c^4*d^15 - 4*a*c^3*d^13*e^2 + 6*a^2*c^2*d^11*e^4 - 4*a^3*c*d^9*e^6 + a^
4*d^7*e^8 + (c^4*d^8*e^7 - 4*a*c^3*d^6*e^9 + 6*a^2*c^2*d^4*e^11 - 4*a^3*c*
d^2*e^13 + a^4*e^15)*x^7 + 7*(c^4*d^9*e^6 - 4*a*c^3*d^7*e^8 + 6*a^2*c^2*d^
5*e^10 - 4*a^3*c*d^3*e^12 + a^4*d*e^14)*x^6 + 21*(c^4*d^10*e^5 - 4*a*c^3*d
^8*e^7 + 6*a^2*c^2*d^6*e^9 - 4*a^3*c*d^4*e^11 + a^4*d^2*e^13)*x^5 + 35*(c^
4*d^11*e^4 - 4*a*c^3*d^9*e^6 + 6*a^2*c^2*d^7*e^8 - 4*a^3*c*d^5*e^10 + a^4*
d^3*e^12)*x^4 + 35*(c^4*d^12*e^3 - 4*a*c^3*d^10*e^5 + 6*a^2*c^2*d^8*e^7 -
4*a^3*c*d^6*e^9 + a^4*d^4*e^11)*x^3 + 21*(c^4*d^13*e^2 - 4*a*c^3*d^11*e^4
+ 6*a^2*c^2*d^9*e^6 - 4*a^3*c*d^7*e^8 + a^4*d^5*e^10)*x^2 + 7*(c^4*d^14*e
- 4*a*c^3*d^12*e^3 + 6*a^2*c^2*d^10*e^5 - 4*a^3*c*d^8*e^7 + a^4*d^6*e^9)*x
)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\left(ade + (cd^2 + ae^2)\,x + cdex^2\right)^{5/2}}{(d+ex)^{10}}\,dx = \text{Timed out}$$

input
```
integrate((a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(5/2)/(e*x+d)**10,x)
```

output
```
Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\left(ade + (cd^2 + ae^2)\,x + cdex^2\right)^{5/2}}{(d+ex)^{10}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(e*x+d)^10,x, algorithm=
"maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e*(a*e^2-c*d^2)>0)', see `assume
?` for mor
```

Giac [F(-2)]

Exception generated.

$$\int \frac{\left(ade + (cd^2 + ae^2)\,x + cdex^2\right)^{5/2}}{(d+ex)^{10}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(e*x+d)^10,x, algorithm=
"giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Unable to divide, perhaps due to ro
unding error%%%{%%%{1,[0,0,7]%%%},[14]%%%}+%%%{%%{[%%%{-14,[0,1,6]%%%},0]:
[1,0,%%%{
```

Mupad [B] (verification not implemented)

Time = 10.94 (sec) , antiderivative size = 5069, normalized size of antiderivative = 21.94

$$\int \frac{(ade + (cd^2 + ae^2)x + cdex^2)^{5/2}}{(d+ex)^{10}}\,dx = \text{Too large to display}$$

input
```
int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(5/2)/(d + e*x)^10,x)
```

output
```
(((d*((8*c^5*d^6)/(143*e*(a*e^2 - c*d^2)^2*(7*a*e^3 - 7*c*d^2*e)) - (4*c^4
*d^4*(21*a*e^2 - 17*c*d^2))/(143*e*(a*e^2 - c*d^2)^2*(7*a*e^3 - 7*c*d^2*e)
)))/e + (4*c^3*d^3*(110*a^2*e^4 + 53*c^2*d^4 - 157*a*c*d^2*e^2))/(429*e^2*
(a*e^2 - c*d^2)^2*(7*a*e^3 - 7*c*d^2*e)))*(x*(a*e^2 + c*d^2) + a*d*e + c*d
*e*x^2)^(1/2))/(d + e*x)^4 - (((2*a^3*e^4)/(13*a*e^3 - 13*c*d^2*e) - (d*((
d*((2*c^3*d^4)/(13*a*e^3 - 13*c*d^2*e) - (6*a*c^2*d^2*e^2)/(13*a*e^3 - 13*
c*d^2*e)))/e + (6*a^2*c*d*e^3)/(13*a*e^3 - 13*c*d^2*e)))/e)*(x*(a*e^2 + c*
d^2) + a*d*e + c*d*e*x^2)^(1/2))/(d + e*x)^7 + (((d*((16*c^6*d^7)/(1287*e*
(a*e^2 - c*d^2)^3*(5*a*e^3 - 5*c*d^2*e)) - (8*c^5*d^5*(33*a*e^2 - 29*c*d^2
))/(1287*e*(a*e^2 - c*d^2)^3*(5*a*e^3 - 5*c*d^2*e))))/e + (8*c^4*d^4*(112*
a^2*e^4 + 81*c^2*d^4 - 191*a*c*d^2*e^2))/(1287*e^2*(a*e^2 - c*d^2)^3*(5*a*
e^3 - 5*c*d^2*e)))*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2))/(d + e*x
)^3 + (((d*((32*c^7*d^8)/(9009*e*(a*e^2 - c*d^2)^4*(3*a*e^3 - 3*c*d^2*e))
- (16*c^6*d^6*(43*a*e^2 - 39*c*d^2))/(9009*e*(a*e^2 - c*d^2)^4*(3*a*e^3 -
3*c*d^2*e))))/e + (16*c^5*d^5*(1089*a^2*e^4 + 884*c^2*d^4 - 1963*a*c*d^2*e
^2))/(45045*e^2*(a*e^2 - c*d^2)^4*(3*a*e^3 - 3*c*d^2*e)))*(x*(a*e^2 + c*d^
2) + a*d*e + c*d*e*x^2)^(1/2))/(d + e*x)^2 + (((38*c^4*d^5 + 94*a*c^3*d^3*
e^2)/(429*e^2*(a*e^2 - c*d^2)*(7*a*e^3 - 7*c*d^2*e)) - (4*c^4*d^5)/(13*e^2
*(a*e^2 - c*d^2)*(7*a*e^3 - 7*c*d^2*e)))*(x*(a*e^2 + c*d^2) + a*d*e + c*d*
e*x^2)^(1/2))/(d + e*x)^4 + (((348*c^5*d^6 - 292*a*c^4*d^4*e^2)/(1001*e...
```

Reduce [B] (verification not implemented)

Time = 1.21 (sec) , antiderivative size = 1372, normalized size of antiderivative = 5.94

$$\int \frac{(ade + (cd^2 + ae^2)\, x + cdex^2)^{5/2}}{(d + ex)^{10}}\, dx = \text{Too large to display}$$

input
```
int((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(e*x+d)^10,x)
```

output
```
(2*( - 231*sqrt(d + e*x)*sqrt(a*e + c*d*x)*a**6*e**13 + 819*sqrt(d + e*x)*
sqrt(a*e + c*d*x)*a**5*c*d**2*e**11 - 567*sqrt(d + e*x)*sqrt(a*e + c*d*x)*
a**5*c*d*e**12*x - 1001*sqrt(d + e*x)*sqrt(a*e + c*d*x)*a**4*c**2*d**4*e**
9 + 2093*sqrt(d + e*x)*sqrt(a*e + c*d*x)*a**4*c**2*d**3*e**10*x - 371*sqrt
(d + e*x)*sqrt(a*e + c*d*x)*a**4*c**2*d**2*e**11*x**2 + 429*sqrt(d + e*x)*
sqrt(a*e + c*d*x)*a**3*c**3*d**6*e**7 - 2717*sqrt(d + e*x)*sqrt(a*e + c*d*
x)*a**3*c**3*d**5*e**8*x + 1469*sqrt(d + e*x)*sqrt(a*e + c*d*x)*a**3*c**3*
d**4*e**9*x**2 - 5*sqrt(d + e*x)*sqrt(a*e + c*d*x)*a**3*c**3*d**3*e**10*x*
*3 + 1287*sqrt(d + e*x)*sqrt(a*e + c*d*x)*a**2*c**4*d**7*e**6*x - 2145*sqr
t(d + e*x)*sqrt(a*e + c*d*x)*a**2*c**4*d**6*e**7*x**2 + 39*sqrt(d + e*x)*s
qrt(a*e + c*d*x)*a**2*c**4*d**5*e**8*x**3 + 6*sqrt(d + e*x)*sqrt(a*e + c*d
*x)*a**2*c**4*d**4*e**9*x**4 + 1287*sqrt(d + e*x)*sqrt(a*e + c*d*x)*a*c**5
*d**8*e**5*x**2 - 143*sqrt(d + e*x)*sqrt(a*e + c*d*x)*a*c**5*d**7*e**6*x**
3 - 52*sqrt(d + e*x)*sqrt(a*e + c*d*x)*a*c**5*d**6*e**7*x**4 - 8*sqrt(d +
e*x)*sqrt(a*e + c*d*x)*a*c**5*d**5*e**8*x**5 + 429*sqrt(d + e*x)*sqrt(a*e
+ c*d*x)*c**6*d**9*e**4*x**3 + 286*sqrt(d + e*x)*sqrt(a*e + c*d*x)*c**6*d*
*8*e**5*x**4 + 104*sqrt(d + e*x)*sqrt(a*e + c*d*x)*c**6*d**7*e**6*x**5 + 1
6*sqrt(d + e*x)*sqrt(a*e + c*d*x)*c**6*d**6*e**7*x**6 - 16*sqrt(e)*sqrt(d)
*sqrt(c)*c**6*d**13 - 112*sqrt(e)*sqrt(d)*sqrt(c)*c**6*d**12*e*x - 336*sqr
t(e)*sqrt(d)*sqrt(c)*c**6*d**11*e**2*x**2 - 560*sqrt(e)*sqrt(d)*sqrt(c)...
```

3.234 $\int (d+ex) \left(ade + (cd^2 + ae^2) x + cdex^2\right)^{7/2} dx$

Optimal result

Integrand size = 35, antiderivative size = 609

$$\int (d+ex)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{7/2}\,dx = -\frac{35(cd^2-ae^2)^8\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{32768c^5d^5e^4} + \frac{35(cd^2-ae^2)^7\left(ade+(cd^2+ae^2)x+cdex^2\right)^{3/2}}{49152c^5d^5e^3(d+ex)} - \frac{7(cd^2-ae^2)^6\left(ade+(cd^2+ae^2)x+cdex^2\right)^{5/2}}{12288c^5d^5e^2(d+ex)^2} + \frac{(cd^2-ae^2)^5\left(ade+(cd^2+ae^2)x+cdex^2\right)^{7/2}}{2048c^5d^5e(d+ex)^3} + \frac{\left(ade+(cd^2+ae^2)x+cdex^2\right)^{9/2}}{9cd} + \frac{(cd^2-ae^2)^4\left(ade+(cd^2+ae^2)x+cdex^2\right)^{9/2}}{256c^5d^5(d+ex)^4} + \frac{5(cd^2-ae^2)^3\left(ade+(cd^2+ae^2)x+cdex^2\right)^{9/2}}{384c^4d^4(d+ex)^3} + \frac{(cd^2-ae^2)^2\left(ade+(cd^2+ae^2)x+cdex^2\right)^{9/2}}{32c^3d^3(d+ex)^2} + \frac{(cd^2-ae^2)\left(ade+(cd^2+ae^2)x+cdex^2\right)^{9/2}}{16c^2d^2(d+ex)} + \frac{35(cd^2-ae^2)^9\operatorname{arctanh}\left(\frac{\sqrt{e}\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{\sqrt{c}\sqrt{d}(d+ex)}\right)}{32768c^{11/2}d^{11/2}e^{9/2}}$$

output
```
-35/32768*(-a*e^2+c*d^2)^8*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/c^5/d^5
/e^4+35/49152*(-a*e^2+c*d^2)^7*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/c^5
/d^5/e^3/(e*x+d)-7/12288*(-a*e^2+c*d^2)^6*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2
)^(5/2)/c^5/d^5/e^2/(e*x+d)^2+1/2048*(-a*e^2+c*d^2)^5*(a*d*e+(a*e^2+c*d^2)
*x+c*d*e*x^2)^(7/2)/c^5/d^5/e/(e*x+d)^3+1/9*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x
^2)^(9/2)/c/d+1/256*(-a*e^2+c*d^2)^4*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(9/
2)/c^5/d^5/(e*x+d)^4+5/384*(-a*e^2+c*d^2)^3*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x
^2)^(9/2)/c^4/d^4/(e*x+d)^3+1/32*(-a*e^2+c*d^2)^2*(a*d*e+(a*e^2+c*d^2)*x+c
*d*e*x^2)^(9/2)/c^3/d^3/(e*x+d)^2+1/16*(-a*e^2+c*d^2)*(a*d*e+(a*e^2+c*d^2)
*x+c*d*e*x^2)^(9/2)/c^2/d^2/(e*x+d)+35/32768*(-a*e^2+c*d^2)^9*arctanh(e^(1
/2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/c^(1/2)/d^(1/2)/(e*x+d))/c^(11
/2)/d^(11/2)/e^(9/2)
```

Mathematica [A] (verified)

Time = 2.08 (sec) , antiderivative size = 625, normalized size of antiderivative = 1.03

$$\int (d+ex)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{7/2}\,dx=\frac{((ae+cdx)(d+ex))^{7/2}\left(\frac{\sqrt{c}\sqrt{d}\sqrt{e}(315a^8e^{16}-210a^7cde^{14}(13d+ex)+42a^6c^2d^2e^{12}(249d^2+43dex+4e^2x^2)-18a^5}{}\right.}{}$$

input
```
Integrate[(d + e*x)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(7/2),x]
```

output
```
(((a*e + c*d*x)*(d + e*x))^(7/2)*((Sqrt[c]*Sqrt[d]*Sqrt[e]*(315*a^8*e^16 -
 210*a^7*c*d*e^14*(13*d + e*x) + 42*a^6*c^2*d^2*e^12*(249*d^2 + 43*d*e*x +
 4*e^2*x^2) - 18*a^5*c^3*d^3*e^10*(1289*d^3 + 381*d^2*e*x + 80*d*e^2*x^2 +
 8*e^3*x^3) + 2*a^4*c^4*d^4*e^8*(16384*d^4 + 7531*d^3*e*x + 2724*d^2*e^2*x
^2 + 616*d*e^3*x^3 + 64*e^4*x^4) + 2*a^3*c^5*d^5*e^6*(11601*d^5 + 123541*d
^4*e*x + 262144*d^3*e^2*x^2 + 265776*d^2*e^3*x^3 + 133504*d*e^4*x^4 + 2675
2*e^5*x^5) + 6*a^2*c^6*d^6*e^4*(-1743*d^6 + 1143*d^5*e*x + 64628*d^4*e^2*x
^2 + 173552*d^3*e^3*x^3 + 196608*d^2*e^4*x^4 + 105344*d*e^5*x^5 + 22016*e^
6*x^6) + 2*a*c^7*d^7*e^2*(1365*d^7 - 903*d^6*e*x + 720*d^5*e^2*x^2 + 13045
6*d^4*e^3*x^3 + 390784*d^3*e^4*x^4 + 470400*d^2*e^5*x^5 + 262144*d*e^6*x^6
 + 56320*e^7*x^7) + c^8*d^8*(-315*d^8 + 210*d^7*e*x - 168*d^6*e^2*x^2 + 14
4*d^5*e^3*x^3 + 65408*d^4*e^4*x^4 + 208640*d^3*e^5*x^5 + 261120*d^2*e^6*x^
6 + 149504*d*e^7*x^7 + 32768*e^8*x^8)))/((a*e + c*d*x)^3*(d + e*x)^3) + (3
15*(c*d^2 - a*e^2)^9*ArcTanh[(Sqrt[c]*Sqrt[d]*Sqrt[d + e*x])/(Sqrt[e]*Sqrt
[a*e + c*d*x])])/((a*e + c*d*x)^(7/2)*(d + e*x)^(7/2))))/(294912*c^(11/2)*
d^(11/2)*e^(9/2))
```

Rubi [A] (verified)

Time = 1.03 (sec) , antiderivative size = 480, normalized size of antiderivative = 0.79, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {1160, 1087, 1087, 1087, 1087, 1092, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (d+ex)\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{7/2}dx$$

$$\Big\downarrow \text{1160}$$

$$\frac{\left(d^2-\frac{ae^2}{c}\right)\int \left(cdex^2+\left(cd^2+ae^2\right)x+ade\right)^{7/2}dx}{2d}+\frac{\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{9/2}}{9cd}$$

$$\Big\downarrow \text{1087}$$

$$\frac{\left(d^2-\frac{ae^2}{c}\right)\left(\frac{\left(ae^2+cd^2+2cdex\right)\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{7/2}}{16cde}-\frac{7\left(cd^2-ae^2\right)^2\int \left(cdex^2+\left(cd^2+ae^2\right)x+ade\right)^{5/2}dx}{32cde}\right)}{2d}+\frac{\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{9/2}}{9cd}$$

$$\Big\downarrow \text{1087}$$

$$\frac{\left(d^2-\frac{ae^2}{c}\right)\left(\frac{\left(ae^2+cd^2+2cdex\right)\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{7/2}}{16cde}-\frac{7\left(cd^2-ae^2\right)^2\left(\frac{\left(ae^2+cd^2+2cdex\right)\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/2}}{12cde}-\frac{5\left(cd^2-ae\right.}{}\right.}{32cde}\right.}{2d}$$

$$\frac{\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{9/2}}{9cd}$$

$$\Big\downarrow \text{1087}$$

$$\frac{\left(d^2-\frac{ae^2}{c}\right)\left(\frac{\left(ae^2+cd^2+2cdex\right)\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{7/2}}{16cde}-\frac{7\left(cd^2-ae^2\right)^2\left(\frac{\left(ae^2+cd^2+2cdex\right)\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/2}}{12cde}-\frac{5\left(cd^2-ae\right.}{}\right.}{}\right.}{2d}$$

$$\frac{\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{9/2}}{9cd}$$

$$\Big\downarrow \text{1087}$$

$$\left(d^2-\frac{ae^2}{c}\right)\left(\frac{(ae^2+cd^2+2cdex)\left(x(ae^2+cd^2)+ade+cdex^2\right)^{7/2}}{16cde}-\frac{7(cd^2-ae^2)^2\left(\frac{(ae^2+cd^2+2cdex)\left(x(ae^2+cd^2)+ade+cdex^2\right)^{5/2}}{12cde}-\frac{5(cd^2-ae\cdots}{\cdots}\right.}{\cdots}\right.$$

$$\frac{\left(x(ae^2+cd^2)+ade+cdex^2\right)^{9/2}}{9cd}$$

$\downarrow$ 1092

$$5\left(cd^2-ae\right.$$

$$7\left(cd^2-ae^2\right)^2\left(\frac{\left(ae^2+cd^2+2cdex\right)\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/2}}{12cde}-\right.$$

$$\left(d^2-\frac{ae^2}{c}\right)\left(\frac{\left(ae^2+cd^2+2cdex\right)\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{7/2}}{16cde}-\right.$$

$$\frac{\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{9/2}}{9cd}$$

$$\Big\downarrow \; 219$$

$$5\left(cd^2-ae\right.$$

$$7\left(cd^2-ae^2\right)^2\left(\frac{\left(ae^2+cd^2+2cdex\right)\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/2}}{12cde}-\right.$$

$$\left(d^2-\frac{ae^2}{c}\right)\left(\frac{\left(ae^2+cd^2+2cdex\right)\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{7/2}}{16cde}-\right.$$

$$\frac{\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{9/2}}{9cd}$$

input

```
Int[(d + e*x)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(7/2),x]
```

output
```
(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(9/2)/(9*c*d) + ((d^2 - (a*e^2)/c)
*(((c*d^2 + a*e^2 + 2*c*d*e*x)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(7/
2))/(16*c*d*e) - (7*(c*d^2 - a*e^2)^2*(((c*d^2 + a*e^2 + 2*c*d*e*x)*(a*d*e
 + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2))/(12*c*d*e) - (5*(c*d^2 - a*e^2)^2
*(((c*d^2 + a*e^2 + 2*c*d*e*x)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/
2))/(8*c*d*e) - (3*(c*d^2 - a*e^2)^2*(((c*d^2 + a*e^2 + 2*c*d*e*x)*Sqrt[a*
d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(4*c*d*e) - ((c*d^2 - a*e^2)^2*ArcTa
nh[(c*d^2 + a*e^2 + 2*c*d*e*x)/(2*Sqrt[c]*Sqrt[d]*Sqrt[e]*Sqrt[a*d*e + (c*
d^2 + a*e^2)*x + c*d*e*x^2])])/(8*c^(3/2)*d^(3/2)*e^(3/2))))/(16*c*d*e)))/
(24*c*d*e)))/(32*c*d*e)))/(2*d)
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1087
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^p/(2*c*(2*p + 1))), x] - Simp[p*((b^2 - 4*a*c)/(2*c*(2*
p + 1)))   Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] &&
GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[3*p])
```

rule 1092
```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[I
nt[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a
, b, c}, x]
```

rule 1160
```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[e*((a + b*x + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Simp[(2*c*d - b
*e)/(2*c)   Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
 && NeQ[p, -1]
```

Maple [A] (verified)

Time = 1.54 (sec) , antiderivative size = 931, normalized size of antiderivative = 1.53

method	result
default	$d\left(\frac{(2cdxe+a\,e^2+c\,d^2)\,(ade+(a\,e^2+c\,d^2)x+cd\,x^2e)^{\frac{7}{2}}}{16cde}+\frac{7\left(4ac\,d^2e^2-(a\,e^2+c\,d^2)^2\right)\left(\frac{\left(2cdxe+a\,e^2+c\,d^2\right)\left(ade+\left(a\,e^2+c\,d^2\right)x+cd\,x^2 \right.}{12cde}\right.}{}\right.$

input
```
int((e*x+d)*(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(7/2),x,method=_RETURNVERBOS
E)
```

output
```
d*(1/16*(2*c*d*e*x+a*e^2+c*d^2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(7/2)/c/
d/e+7/32*(4*a*c*d^2*e^2-(a*e^2+c*d^2)^2)/d/e/c*(1/12*(2*c*d*e*x+a*e^2+c*d^
2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(5/2)/c/d/e+5/24*(4*a*c*d^2*e^2-(a*e^
2+c*d^2)^2)/d/e/c*(1/8*(2*c*d*e*x+a*e^2+c*d^2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*
x^2*e)^(3/2)/c/d/e+3/16*(4*a*c*d^2*e^2-(a*e^2+c*d^2)^2)/d/e/c*(1/4*(2*c*d*
e*x+a*e^2+c*d^2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(1/2)/c/d/e+1/8*(4*a*c*
d^2*e^2-(a*e^2+c*d^2)^2)/d/e/c*ln((1/2*a*e^2+1/2*c*d^2+c*d*x*e)/(d*e*c)^(1
/2)+(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(1/2))/(d*e*c)^(1/2)))))+e*(1/9*(a*d
*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(9/2)/d/e/c-1/2*(a*e^2+c*d^2)/d/e/c*(1/16*(2
*c*d*e*x+a*e^2+c*d^2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(7/2)/c/d/e+7/32*(
4*a*c*d^2*e^2-(a*e^2+c*d^2)^2)/d/e/c*(1/12*(2*c*d*e*x+a*e^2+c*d^2)*(a*d*e+
(a*e^2+c*d^2)*x+c*d*x^2*e)^(5/2)/c/d/e+5/24*(4*a*c*d^2*e^2-(a*e^2+c*d^2)^2
)/d/e/c*(1/8*(2*c*d*e*x+a*e^2+c*d^2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(3/
2)/c/d/e+3/16*(4*a*c*d^2*e^2-(a*e^2+c*d^2)^2)/d/e/c*(1/4*(2*c*d*e*x+a*e^2+
c*d^2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(1/2)/c/d/e+1/8*(4*a*c*d^2*e^2-(a
*e^2+c*d^2)^2)/d/e/c*ln((1/2*a*e^2+1/2*c*d^2+c*d*x*e)/(d*e*c)^(1/2)+(a*d*e
+(a*e^2+c*d^2)*x+c*d*x^2*e)^(1/2))/(d*e*c)^(1/2))))))
```

Fricas [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 1804, normalized size of antiderivative = 2.96

$$\int (d+ex) \left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{7/2}\,dx = \text{Too large to display}$$

input
```
integrate((e*x+d)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(7/2),x, algorithm="fr
icas")
```

output
```
[-1/1179648*(315*(c^9*d^18 - 9*a*c^8*d^16*e^2 + 36*a^2*c^7*d^14*e^4 - 84*a
^3*c^6*d^12*e^6 + 126*a^4*c^5*d^10*e^8 - 126*a^5*c^4*d^8*e^10 + 84*a^6*c^3
*d^6*e^12 - 36*a^7*c^2*d^4*e^14 + 9*a^8*c*d^2*e^16 - a^9*e^18)*sqrt(c*d*e)
*log(8*c^2*d^2*e^2*x^2 + c^2*d^4 + 6*a*c*d^2*e^2 + a^2*e^4 - 4*sqrt(c*d*e*
x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(2*c*d*e*x + c*d^2 + a*e^2)*sqrt(c*d*e) +
 8*(c^2*d^3*e + a*c*d*e^3)*x) - 4*(32768*c^9*d^9*e^9*x^8 - 315*c^9*d^17*e
+ 2730*a*c^8*d^15*e^3 - 10458*a^2*c^7*d^13*e^5 + 23202*a^3*c^6*d^11*e^7 +
32768*a^4*c^5*d^9*e^9 - 23202*a^5*c^4*d^7*e^11 + 10458*a^6*c^3*d^5*e^13 -
2730*a^7*c^2*d^3*e^15 + 315*a^8*c*d*e^17 + 2048*(73*c^9*d^10*e^8 + 55*a*c^
8*d^8*e^10)*x^7 + 1024*(255*c^9*d^11*e^7 + 512*a*c^8*d^9*e^9 + 129*a^2*c^7
*d^7*e^11)*x^6 + 256*(815*c^9*d^12*e^6 + 3675*a*c^8*d^10*e^8 + 2469*a^2*c^
7*d^8*e^10 + 209*a^3*c^6*d^6*e^12)*x^5 + 128*(511*c^9*d^13*e^5 + 6106*a*c^
8*d^11*e^7 + 9216*a^2*c^7*d^9*e^9 + 2086*a^3*c^6*d^7*e^11 + a^4*c^5*d^5*e^
13)*x^4 + 16*(9*c^9*d^14*e^4 + 16307*a*c^8*d^12*e^6 + 65082*a^2*c^7*d^10*e
^8 + 33222*a^3*c^6*d^8*e^10 + 77*a^4*c^5*d^6*e^12 - 9*a^5*c^4*d^4*e^14)*x^
3 - 8*(21*c^9*d^15*e^3 - 180*a*c^8*d^13*e^5 - 48471*a^2*c^7*d^11*e^7 - 655
36*a^3*c^6*d^9*e^9 - 681*a^4*c^5*d^7*e^11 + 180*a^5*c^4*d^5*e^13 - 21*a^6*
c^3*d^3*e^15)*x^2 + 2*(105*c^9*d^16*e^2 - 903*a*c^8*d^14*e^4 + 3429*a^2*c^
7*d^12*e^6 + 123541*a^3*c^6*d^10*e^8 + 7531*a^4*c^5*d^8*e^10 - 3429*a^5*c^
4*d^6*e^12 + 903*a^6*c^3*d^4*e^14 - 105*a^7*c^2*d^2*e^16)*x)*sqrt(c*d*e...
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 15410 vs. 2(571) = 1142.

Time = 19.71 (sec) , antiderivative size = 15410, normalized size of antiderivative = 25.30

$$\int (d+ex)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{7/2}\,dx = \text{Too large to display}$$

input
```
integrate((e*x+d)*(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(7/2),x)
```

output
```
Piecewise((sqrt(a*d*e + c*d*e*x**2 + x*(a*e**2 + c*d**2))*(c**3*d**3*e**4*
x**8/9 + x**7*(4*a*c**3*d**3*e**6 + 5*c**4*d**5*e**4 - c**3*d**3*e**4*(17*
a*e**2/2 + 17*c*d**2/2)/9)/(8*c*d*e) + x**6*(6*a**2*c**2*d**2*e**7 + 172*a
*c**3*d**4*e**5/9 + 10*c**4*d**6*e**3 - (15*a*e**2/2 + 15*c*d**2/2)*(4*a*c
**3*d**3*e**6 + 5*c**4*d**5*e**4 - c**3*d**3*e**4*(17*a*e**2/2 + 17*c*d**2
/2)/9)/(8*c*d*e))/(7*c*d*e) + x**5*(4*a**3*c*d*e**8 + 30*a**2*c**2*d**3*e*
*6 + 40*a*c**3*d**5*e**4 - 7*a*(4*a*c**3*d**3*e**6 + 5*c**4*d**5*e**4 - c*
*3*d**3*e**4*(17*a*e**2/2 + 17*c*d**2/2)/9)/(8*c) + 10*c**4*d**7*e**2 - (1
3*a*e**2/2 + 13*c*d**2/2)*(6*a**2*c**2*d**2*e**7 + 172*a*c**3*d**4*e**5/9
+ 10*c**4*d**6*e**3 - (15*a*e**2/2 + 15*c*d**2/2)*(4*a*c**3*d**3*e**6 + 5*
c**4*d**5*e**4 - c**3*d**3*e**4*(17*a*e**2/2 + 17*c*d**2/2)/9)/(8*c*d*e))/
(7*c*d*e))/(6*c*d*e) + x**4*(a**4*e**9 + 20*a**3*c*d**2*e**7 + 60*a**2*c**
2*d**4*e**5 + 40*a*c**3*d**6*e**3 - 6*a*(6*a**2*c**2*d**2*e**7 + 172*a*c**
3*d**4*e**5/9 + 10*c**4*d**6*e**3 - (15*a*e**2/2 + 15*c*d**2/2)*(4*a*c**3*
d**3*e**6 + 5*c**4*d**5*e**4 - c**3*d**3*e**4*(17*a*e**2/2 + 17*c*d**2/2)/
9)/(8*c*d*e))/(7*c) + 5*c**4*d**8*e - (11*a*e**2/2 + 11*c*d**2/2)*(4*a**3*
c*d*e**8 + 30*a**2*c**2*d**3*e**6 + 40*a*c**3*d**5*e**4 - 7*a*(4*a*c**3*d*
*3*e**6 + 5*c**4*d**5*e**4 - c**3*d**3*e**4*(17*a*e**2/2 + 17*c*d**2/2)/9)
/(8*c) + 10*c**4*d**7*e**2 - (13*a*e**2/2 + 13*c*d**2/2)*(6*a**2*c**2*d**2
*e**7 + 172*a*c**3*d**4*e**5/9 + 10*c**4*d**6*e**3 - (15*a*e**2/2 + 15*...
```

Maxima [F(-2)]

Exception generated.

$$\int (d+ex)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{7/2}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((e*x+d)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(7/2),x, algorithm="ma
xima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

Giac [A] (verification not implemented)

Time = 0.36 (sec) , antiderivative size = 936, normalized size of antiderivative = 1.54

$$\int (d+ex)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{7/2}\,dx=\text{Too large to display}$$

input
```
integrate((e*x+d)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(7/2),x, algorithm="gi
ac")
```

output
```
1/294912*sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*x + a*d*e)*(2*(4*(2*(8*(2*(4*(2*
(16*c^3*d^3*e^4*x + (73*c^11*d^12*e^11 + 55*a*c^10*d^10*e^13)/(c^8*d^8*e^8
))*x + (255*c^11*d^13*e^10 + 512*a*c^10*d^11*e^12 + 129*a^2*c^9*d^9*e^14)/
(c^8*d^8*e^8))*x + (815*c^11*d^14*e^9 + 3675*a*c^10*d^12*e^11 + 2469*a^2*c
^9*d^10*e^13 + 209*a^3*c^8*d^8*e^15)/(c^8*d^8*e^8))*x + (511*c^11*d^15*e^8
 + 6106*a*c^10*d^13*e^10 + 9216*a^2*c^9*d^11*e^12 + 2086*a^3*c^8*d^9*e^14
+ a^4*c^7*d^7*e^16)/(c^8*d^8*e^8))*x + (9*c^11*d^16*e^7 + 16307*a*c^10*d^1
4*e^9 + 65082*a^2*c^9*d^12*e^11 + 33222*a^3*c^8*d^10*e^13 + 77*a^4*c^7*d^8
*e^15 - 9*a^5*c^6*d^6*e^17)/(c^8*d^8*e^8))*x - (21*c^11*d^17*e^6 - 180*a*c
^10*d^15*e^8 - 48471*a^2*c^9*d^13*e^10 - 65536*a^3*c^8*d^11*e^12 - 681*a^4
*c^7*d^9*e^14 + 180*a^5*c^6*d^7*e^16 - 21*a^6*c^5*d^5*e^18)/(c^8*d^8*e^8))
*x + (105*c^11*d^18*e^5 - 903*a*c^10*d^16*e^7 + 3429*a^2*c^9*d^14*e^9 + 12
3541*a^3*c^8*d^12*e^11 + 7531*a^4*c^7*d^10*e^13 - 3429*a^5*c^6*d^8*e^15 +
903*a^6*c^5*d^6*e^17 - 105*a^7*c^4*d^4*e^19)/(c^8*d^8*e^8))*x - (315*c^11*
d^19*e^4 - 2730*a*c^10*d^17*e^6 + 10458*a^2*c^9*d^15*e^8 - 23202*a^3*c^8*d
^13*e^10 - 32768*a^4*c^7*d^11*e^12 + 23202*a^5*c^6*d^9*e^14 - 10458*a^6*c^
5*d^7*e^16 + 2730*a^7*c^4*d^5*e^18 - 315*a^8*c^3*d^3*e^20)/(c^8*d^8*e^8))
- 35/65536*(c^9*d^18 - 9*a*c^8*d^16*e^2 + 36*a^2*c^7*d^14*e^4 - 84*a^3*c^6
*d^12*e^6 + 126*a^4*c^5*d^10*e^8 - 126*a^5*c^4*d^8*e^10 + 84*a^6*c^3*d^6*e
^12 - 36*a^7*c^2*d^4*e^14 + 9*a^8*c*d^2*e^16 - a^9*e^18)*log(abs(-c*d^2...
```

Mupad [F(-1)]

Timed out.

$$\int (d+ex)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{7/2}\,dx=\int (d+e\,x)\,\left(c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e\right)^{7/2}\,dx$$

input
```
int((d + e*x)*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(7/2),x)
```

output
```
int((d + e*x)*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(7/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.63 (sec) , antiderivative size = 1954, normalized size of antiderivative = 3.21

$$\int (d+ex)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{7/2}\,dx = \text{Too large to display}$$

input
```
int((e*x+d)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(7/2),x)
```

output
```
(315*sqrt(d + e*x)*sqrt(a*e + c*d*x)*a**8*c*d*e**17 - 2730*sqrt(d + e*x)*s
qrt(a*e + c*d*x)*a**7*c**2*d**3*e**15 - 210*sqrt(d + e*x)*sqrt(a*e + c*d*x
)*a**7*c**2*d**2*e**16*x + 10458*sqrt(d + e*x)*sqrt(a*e + c*d*x)*a**6*c**3
*d**5*e**13 + 1806*sqrt(d + e*x)*sqrt(a*e + c*d*x)*a**6*c**3*d**4*e**14*x
+ 168*sqrt(d + e*x)*sqrt(a*e + c*d*x)*a**6*c**3*d**3*e**15*x**2 - 23202*sq
rt(d + e*x)*sqrt(a*e + c*d*x)*a**5*c**4*d**7*e**11 - 6858*sqrt(d + e*x)*sq
rt(a*e + c*d*x)*a**5*c**4*d**6*e**12*x - 1440*sqrt(d + e*x)*sqrt(a*e + c*d
*x)*a**5*c**4*d**5*e**13*x**2 - 144*sqrt(d + e*x)*sqrt(a*e + c*d*x)*a**5*c
**4*d**4*e**14*x**3 + 32768*sqrt(d + e*x)*sqrt(a*e + c*d*x)*a**4*c**5*d**9
*e**9 + 15062*sqrt(d + e*x)*sqrt(a*e + c*d*x)*a**4*c**5*d**8*e**10*x + 544
8*sqrt(d + e*x)*sqrt(a*e + c*d*x)*a**4*c**5*d**7*e**11*x**2 + 1232*sqrt(d
+ e*x)*sqrt(a*e + c*d*x)*a**4*c**5*d**6*e**12*x**3 + 128*sqrt(d + e*x)*sqr
t(a*e + c*d*x)*a**4*c**5*d**5*e**13*x**4 + 23202*sqrt(d + e*x)*sqrt(a*e +
c*d*x)*a**3*c**6*d**11*e**7 + 247082*sqrt(d + e*x)*sqrt(a*e + c*d*x)*a**3*
c**6*d**10*e**8*x + 524288*sqrt(d + e*x)*sqrt(a*e + c*d*x)*a**3*c**6*d**9*
e**9*x**2 + 531552*sqrt(d + e*x)*sqrt(a*e + c*d*x)*a**3*c**6*d**8*e**10*x*
*3 + 267008*sqrt(d + e*x)*sqrt(a*e + c*d*x)*a**3*c**6*d**7*e**11*x**4 + 53
504*sqrt(d + e*x)*sqrt(a*e + c*d*x)*a**3*c**6*d**6*e**12*x**5 - 10458*sqrt
(d + e*x)*sqrt(a*e + c*d*x)*a**2*c**7*d**13*e**5 + 6858*sqrt(d + e*x)*sqrt
(a*e + c*d*x)*a**2*c**7*d**12*e**6*x + 387768*sqrt(d + e*x)*sqrt(a*e + ...
```

3.235 $\int \left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{7/2}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 363

$$\begin{aligned}\int \left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{7/2}\,dx =& -\frac{35(cd^2-ae^2)^6\,(cd^2+ae^2+2cdex)\,\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{16384c^4d^4e^4}\\&+\frac{35(cd^2-ae^2)^4\,(cd^2+ae^2+2cdex)\,(ade+(cd^2+ae^2)\,x+cdex^2)^{3/2}}{6144c^3d^3e^3}\\&-\frac{7(cd^2-ae^2)^2\,(cd^2+ae^2+2cdex)\,(ade+(cd^2+ae^2)\,x+cdex^2)^{5/2}}{384c^2d^2e^2}\\&+\frac{(cd^2+ae^2+2cdex)\,(ade+(cd^2+ae^2)\,x+cdex^2)^{7/2}}{16cde}\\&+\frac{35(cd^2-ae^2)^8\,\mathrm{arctanh}\left(\frac{\sqrt{c}\sqrt{d}(d+ex)}{\sqrt{e}\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\right)}{16384c^{9/2}d^{9/2}e^{9/2}}\end{aligned}$$

output

```
-35/16384*(-a*e^2+c*d^2)^6*(2*c*d*e*x+a*e^2+c*d^2)*(a*d*e+(a*e^2+c*d^2)*x+
c*d*e*x^2)^(1/2)/c^4/d^4/e^4+35/6144*(-a*e^2+c*d^2)^4*(2*c*d*e*x+a*e^2+c*d
^2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/c^3/d^3/e^3-7/384*(-a*e^2+c*d^
2)^2*(2*c*d*e*x+a*e^2+c*d^2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/c^2/d
^2/e^2+1/16*(2*c*d*e*x+a*e^2+c*d^2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(7/2
)/c/d/e+35/16384*(-a*e^2+c*d^2)^8*arctanh(c^(1/2)*d^(1/2)*(e*x+d)/e^(1/2)/
(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2))/c^(9/2)/d^(9/2)/e^(9/2)
```

Mathematica [A] (verified)

Time = 1.72 (sec) , antiderivative size = 523, normalized size of antiderivative = 1.44

$$\int \left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{7/2}\,dx=\frac{((ae+cdx)(d+ex))^{7/2}\left(\frac{\sqrt{c}\sqrt{d}\sqrt{e}(-105a^7e^{14}+35a^6cde^{12}(23d+2ex)-7a^5c^2d^2e^{10}(383d^2+76dex+8e^2x^2)+a^4c^3}{}\right.}{}$$

input
```
Integrate[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(7/2),x]
```

output
```
(((a*e + c*d*x)*(d + e*x))^(7/2)*((Sqrt[c]*Sqrt[d]*Sqrt[e]*(-105*a^7*e^14
+ 35*a^6*c*d*e^12*(23*d + 2*e*x) - 7*a^5*c^2*d^2*e^10*(383*d^2 + 76*d*e*x
+ 8*e^2*x^2) + a^4*c^3*d^3*e^8*(5053*d^3 + 1754*d^2*e*x + 424*d*e^2*x^2 +
48*e^3*x^3) + a^3*c^4*d^4*e^6*(5053*d^4 + 40424*d^3*e*x + 64144*d^2*e^2*x^
2 + 43328*d*e^3*x^3 + 10880*e^4*x^4) + a^2*c^5*d^5*e^4*(-2681*d^5 + 1754*d
^4*e*x + 64144*d^3*e^2*x^2 + 128288*d^2*e^3*x^3 + 96640*d*e^4*x^4 + 25856*
e^5*x^5) + a*c^6*d^6*e^2*(805*d^6 - 532*d^5*e*x + 424*d^4*e^2*x^2 + 43328*
d^3*e^3*x^3 + 96640*d^2*e^4*x^4 + 77312*d*e^5*x^5 + 21504*e^6*x^6) + c^7*d
^7*(-105*d^7 + 70*d^6*e*x - 56*d^5*e^2*x^2 + 48*d^4*e^3*x^3 + 10880*d^3*e^
4*x^4 + 25856*d^2*e^5*x^5 + 21504*d*e^6*x^6 + 6144*e^7*x^7)))/((a*e + c*d*
x)^3*(d + e*x)^3) + (105*(c*d^2 - a*e^2)^8*ArcTanh[(Sqrt[c]*Sqrt[d]*Sqrt[d
 + e*x])/(Sqrt[e]*Sqrt[a*e + c*d*x])])/((a*e + c*d*x)^(7/2)*(d + e*x)^(7/2
))))/(49152*c^(9/2)*d^(9/2)*e^(9/2))
```

Rubi [A] (verified)

Time = 0.82 (sec) , antiderivative size = 420, normalized size of antiderivative = 1.16, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.207, Rules used = {1087, 1087, 1087, 1087, 1092, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{7/2}dx$$

$\Big\downarrow$ 1087

$$\frac{(ae^2+cd^2+2cdex)\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{7/2}}{16cde}-\frac{7\left(cd^2-ae^2\right)^2\int\left(cdex^2+\left(cd^2+ae^2\right)x+ade\right)^{5/2}dx}{32cde}$$

$\Big\downarrow$ 1087

$$\frac{(ae^2+cd^2+2cdex)\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{7/2}}{16cde}-\frac{7\left(cd^2-ae^2\right)^2\left(\frac{(ae^2+cd^2+2cdex)\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/2}}{12cde}-\frac{5\left(cd^2-ae^2\right)^2\int\left(cdex^2+\left(cd^2+ae^2\right)x+ade\right)^{3/2}dx}{24cde}\right)}{32cde}$$

$\Big\downarrow$ 1087

$$\frac{(ae^2+cd^2+2cdex)\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{7/2}}{16cde}-\frac{7\left(cd^2-ae^2\right)^2\left(\frac{(ae^2+cd^2+2cdex)\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/2}}{12cde}-\frac{5\left(cd^2-ae^2\right)^2\left(\frac{(ae^2+cd^2+2cdex)\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{8cde}-\frac{3\left(cd^2\cdots\right.}{\cdots}\right.}{24cde}\right.}{32cde}$$

$\Big\downarrow$ 1087

$$\frac{(ae^2+cd^2+2cdex)\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{7/2}}{16cde}-\frac{7\left(cd^2-ae^2\right)^2\left(\frac{(ae^2+cd^2+2cdex)\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/2}}{12cde}-\frac{5\left(cd^2-ae^2\right)^2\left(\frac{(ae^2+cd^2+2cdex)\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{8cde}-\frac{3\left(cd^2\cdots\right.}{\cdots}\right.}{\cdots}\right.}{32cde}$$

$\Big\downarrow$ 1092

$$\frac{\left(ae^2+cd^2+2cdex\right)\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{7/2}}{16cde} -$$

$$7\left(cd^2-ae^2\right)^2 \left(\frac{\left(ae^2+cd^2+2cdex\right)\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/2}}{12cde} - \frac{5\left(cd^2-ae^2\right)^2 \left(\frac{\left(ae^2+cd^2+2cdex\right)\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{8cde} - \frac{3\left(cd^2 \cdots\right.}{\cdots} \right)}{\cdots} \right.$$

$$\overline{ \quad 32cde}$$

$\downarrow$ 219

$$\frac{\left(ae^2+cd^2+2cdex\right)\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{7/2}}{16cde} -$$

$$7\left(cd^2-ae^2\right)^2 \left(\frac{\left(ae^2+cd^2+2cdex\right)\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/2}}{12cde} - \frac{5\left(cd^2-ae^2\right)^2 \left(\frac{\left(ae^2+cd^2+2cdex\right)\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{8cde} - \frac{3\left(cd^2 \cdots\right.}{\cdots} \right)}{\cdots} \right.$$

$$\overline{ \quad 32cde}$$

input
```
Int[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(7/2),x]
```

output
```
((c*d^2 + a*e^2 + 2*c*d*e*x)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(7/2)
)/(16*c*d*e) - (7*(c*d^2 - a*e^2)^2*(((c*d^2 + a*e^2 + 2*c*d*e*x)*(a*d*e +
 (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2))/(12*c*d*e) - (5*(c*d^2 - a*e^2)^2*(
((c*d^2 + a*e^2 + 2*c*d*e*x)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2)
)/(8*c*d*e) - (3*(c*d^2 - a*e^2)^2*(((c*d^2 + a*e^2 + 2*c*d*e*x)*Sqrt[a*d*
e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(4*c*d*e) - ((c*d^2 - a*e^2)^2*ArcTanh
[(c*d^2 + a*e^2 + 2*c*d*e*x)/(2*Sqrt[c]*Sqrt[d]*Sqrt[e]*Sqrt[a*d*e + (c*d^
2 + a*e^2)*x + c*d*e*x^2])])/(8*c^(3/2)*d^(3/2)*e^(3/2))))/(16*c*d*e)))/(2
4*c*d*e)))/(32*c*d*e)
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1087
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^p/(2*c*(2*p + 1))), x] - Simp[p*((b^2 - 4*a*c)/(2*c*(2*
p + 1)))   Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] &&
GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[3*p])
```

rule 1092
```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[I
nt[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a
, b, c}, x]
```

Maple [A] (verified)

Time = 1.40 (sec) , antiderivative size = 433, normalized size of antiderivative = 1.19

method	result
default	$\frac{(2cdxe+a\,e^2+c\,d^2)\left(ade+(a\,e^2+c\,d^2)x+cd\,x^2e\right)^{\frac{7}{2}}}{16cde}+\frac{7\left(4ac\,d^2e^2-(a\,e^2+c\,d^2)^2\right)\left(\frac{(2cdxe+a\,e^2+c\,d^2)\left(ade+(a\,e^2+c\,d^2)x+cd\,x^2e\right)^{\frac{5}{2}}}{12cde}\right.}{}$

input
```
int((a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(7/2),x,method=_RETURNVERBOSE)
```

output
```
1/16*(2*c*d*e*x+a*e^2+c*d^2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(7/2)/c/d/e
+7/32*(4*a*c*d^2*e^2-(a*e^2+c*d^2)^2)/d/e/c*(1/12*(2*c*d*e*x+a*e^2+c*d^2)*
(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(5/2)/c/d/e+5/24*(4*a*c*d^2*e^2-(a*e^2+c
*d^2)^2)/d/e/c*(1/8*(2*c*d*e*x+a*e^2+c*d^2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2
*e)^(3/2)/c/d/e+3/16*(4*a*c*d^2*e^2-(a*e^2+c*d^2)^2)/d/e/c*(1/4*(2*c*d*e*x
+a*e^2+c*d^2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(1/2)/c/d/e+1/8*(4*a*c*d^2
*e^2-(a*e^2+c*d^2)^2)/d/e/c*ln((1/2*a*e^2+1/2*c*d^2+c*d*x*e)/(d*e*c)^(1/2)
+(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(1/2))/(d*e*c)^(1/2))))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 752 vs. 2(331) = 662.

Time = 0.17 (sec) , antiderivative size = 1518, normalized size of antiderivative = 4.18

$$\int \left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{7/2}\,dx=\text{Too large to display}$$

input
```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(7/2),x, algorithm="fricas")
```

output
```
[1/196608*(105*(c^8*d^16 - 8*a*c^7*d^14*e^2 + 28*a^2*c^6*d^12*e^4 - 56*a^3
*c^5*d^10*e^6 + 70*a^4*c^4*d^8*e^8 - 56*a^5*c^3*d^6*e^10 + 28*a^6*c^2*d^4*
e^12 - 8*a^7*c*d^2*e^14 + a^8*e^16)*sqrt(c*d*e)*log(8*c^2*d^2*e^2*x^2 + c^
2*d^4 + 6*a*c*d^2*e^2 + a^2*e^4 + 4*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^
2)*x)*(2*c*d*e*x + c*d^2 + a*e^2)*sqrt(c*d*e) + 8*(c^2*d^3*e + a*c*d*e^3)*
x) + 4*(6144*c^8*d^8*e^8*x^7 - 105*c^8*d^15*e + 805*a*c^7*d^13*e^3 - 2681*
a^2*c^6*d^11*e^5 + 5053*a^3*c^5*d^9*e^7 + 5053*a^4*c^4*d^7*e^9 - 2681*a^5*
c^3*d^5*e^11 + 805*a^6*c^2*d^3*e^13 - 105*a^7*c*d*e^15 + 21504*(c^8*d^9*e^
7 + a*c^7*d^7*e^9)*x^6 + 256*(101*c^8*d^10*e^6 + 302*a*c^7*d^8*e^8 + 101*a
^2*c^6*d^6*e^10)*x^5 + 640*(17*c^8*d^11*e^5 + 151*a*c^7*d^9*e^7 + 151*a^2*
c^6*d^7*e^9 + 17*a^3*c^5*d^5*e^11)*x^4 + 16*(3*c^8*d^12*e^4 + 2708*a*c^7*d
^10*e^6 + 8018*a^2*c^6*d^8*e^8 + 2708*a^3*c^5*d^6*e^10 + 3*a^4*c^4*d^4*e^1
2)*x^3 - 8*(7*c^8*d^13*e^3 - 53*a*c^7*d^11*e^5 - 8018*a^2*c^6*d^9*e^7 - 80
18*a^3*c^5*d^7*e^9 - 53*a^4*c^4*d^5*e^11 + 7*a^5*c^3*d^3*e^13)*x^2 + 2*(35
*c^8*d^14*e^2 - 266*a*c^7*d^12*e^4 + 877*a^2*c^6*d^10*e^6 + 20212*a^3*c^5*
d^8*e^8 + 877*a^4*c^4*d^6*e^10 - 266*a^5*c^3*d^4*e^12 + 35*a^6*c^2*d^2*e^1
4)*x)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x))/(c^5*d^5*e^5), -1/98304
*(105*(c^8*d^16 - 8*a*c^7*d^14*e^2 + 28*a^2*c^6*d^12*e^4 - 56*a^3*c^5*d^10
*e^6 + 70*a^4*c^4*d^8*e^8 - 56*a^5*c^3*d^6*e^10 + 28*a^6*c^2*d^4*e^12 - 8*
a^7*c*d^2*e^14 + a^8*e^16)*sqrt(-c*d*e)*arctan(1/2*sqrt(c*d*e*x^2 + a*d...
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 988 vs. $2(355) = 710$.

Time = 49.50 (sec) , antiderivative size = 20264, normalized size of antiderivative = 55.82

$$\int \left(ade + \left(cd^2 + ae^2\right)x + cdex^2\right)^{7/2} dx = \text{Too large to display}$$

input
```
integrate((a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(7/2),x)
```

output
```
a**3*d**3*e**3*Piecewise(((x/2 + (a*e**2/4 + c*d**2/4)/(c*d*e))*sqrt(a*d*e
 + c*d*e*x**2 + x*(a*e**2 + c*d**2)) + (a*d*e/2 - (a*e**2/4 + c*d**2/4)*(a
*e**2 + c*d**2)/(2*c*d*e))*Piecewise((log(a*e**2 + c*d**2 + 2*c*d*e*x + 2*
sqrt(c*d*e)*sqrt(a*d*e + c*d*e*x**2 + x*(a*e**2 + c*d**2)))/sqrt(c*d*e), N
e(a*d*e - (a*e**2 + c*d**2)**2/(4*c*d*e), 0)), ((x - (-a*e**2 - c*d**2)/(2
*c*d*e))*log(x - (-a*e**2 - c*d**2)/(2*c*d*e))/sqrt(c*d*e*(x - (-a*e**2 -
c*d**2)/(2*c*d*e))**2), True)), Ne(c*d*e, 0)), (2*(a*d*e + x*(a*e**2 + c*d
**2))**(3/2)/(3*(a*e**2 + c*d**2)), Ne(a*e**2 + c*d**2, 0)), (x*sqrt(a*d*e
), True)) + 3*a**3*d**2*e**4*Piecewise(((-a*(a*e**2/6 + c*d**2/6)/(2*c) -
(a*e**2 + c*d**2)*(a*d*e/3 - (a*e**2/6 + c*d**2/6)*(3*a*e**2/2 + 3*c*d**2/
2)/(2*c*d*e))/(2*c*d*e))*Piecewise((log(a*e**2 + c*d**2 + 2*c*d*e*x + 2*sq
rt(c*d*e)*sqrt(a*d*e + c*d*e*x**2 + x*(a*e**2 + c*d**2)))/sqrt(c*d*e), Ne(
a*d*e - (a*e**2 + c*d**2)**2/(4*c*d*e), 0)), ((x - (-a*e**2 - c*d**2)/(2*c
*d*e))*log(x - (-a*e**2 - c*d**2)/(2*c*d*e))/sqrt(c*d*e*(x - (-a*e**2 - c*
d**2)/(2*c*d*e))**2), True)) + (x**2/3 + x*(a*e**2/6 + c*d**2/6)/(2*c*d*e)
 + (a*d*e/3 - (a*e**2/6 + c*d**2/6)*(3*a*e**2/2 + 3*c*d**2/2)/(2*c*d*e))/(
c*d*e))*sqrt(a*d*e + c*d*e*x**2 + x*(a*e**2 + c*d**2)), Ne(c*d*e, 0)), (2*
(-a*d*e*(a*d*e + x*(a*e**2 + c*d**2))**(3/2)/3 + (a*d*e + x*(a*e**2 + c*d*
*2))**(5/2)/5)/(a*e**2 + c*d**2)**2, Ne(a*e**2 + c*d**2, 0)), (x**2*sqrt(a
*d*e)/2, True)) + 3*a**3*d*e**5*Piecewise(((-a*(a*d*e/4 - (a*e**2/8 + c...
```

Maxima [F(-2)]

Exception generated.

$$\int \left(ade + \left(cd^2 + ae^2\right)x + cdex^2\right)^{7/2} dx = \text{Exception raised: ValueError}$$

input
```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(7/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 786 vs. $2(331) = 662$.

Time = 0.49 (sec) , antiderivative size = 786, normalized size of antiderivative = 2.17

$$\int \left(ade + \left(cd^2 + ae^2\right)x + cdex^2\right)^{7/2} dx = \text{Too large to display}$$

input
```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(7/2),x, algorithm="giac")
```

output
```
1/49152*sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*x + a*d*e)*(2*(4*(2*(8*(2*(12*(2*
c^3*d^3*e^3*x + 7*(c^10*d^11*e^9 + a*c^9*d^9*e^11)/(c^7*d^7*e^7))*x + (101
*c^10*d^12*e^8 + 302*a*c^9*d^10*e^10 + 101*a^2*c^8*d^8*e^12)/(c^7*d^7*e^7)
)*x + 5*(17*c^10*d^13*e^7 + 151*a*c^9*d^11*e^9 + 151*a^2*c^8*d^9*e^11 + 17
*a^3*c^7*d^7*e^13)/(c^7*d^7*e^7))*x + (3*c^10*d^14*e^6 + 2708*a*c^9*d^12*e
^8 + 8018*a^2*c^8*d^10*e^10 + 2708*a^3*c^7*d^8*e^12 + 3*a^4*c^6*d^6*e^14)/
(c^7*d^7*e^7))*x - (7*c^10*d^15*e^5 - 53*a*c^9*d^13*e^7 - 8018*a^2*c^8*d^1
1*e^9 - 8018*a^3*c^7*d^9*e^11 - 53*a^4*c^6*d^7*e^13 + 7*a^5*c^5*d^5*e^15)/
(c^7*d^7*e^7))*x + (35*c^10*d^16*e^4 - 266*a*c^9*d^14*e^6 + 877*a^2*c^8*d^
12*e^8 + 20212*a^3*c^7*d^10*e^10 + 877*a^4*c^6*d^8*e^12 - 266*a^5*c^5*d^6*
e^14 + 35*a^6*c^4*d^4*e^16)/(c^7*d^7*e^7))*x - (105*c^10*d^17*e^3 - 805*a*
c^9*d^15*e^5 + 2681*a^2*c^8*d^13*e^7 - 5053*a^3*c^7*d^11*e^9 - 5053*a^4*c^
6*d^9*e^11 + 2681*a^5*c^5*d^7*e^13 - 805*a^6*c^4*d^5*e^15 + 105*a^7*c^3*d^
3*e^17)/(c^7*d^7*e^7)) - 35/32768*(c^8*d^16 - 8*a*c^7*d^14*e^2 + 28*a^2*c^
6*d^12*e^4 - 56*a^3*c^5*d^10*e^6 + 70*a^4*c^4*d^8*e^8 - 56*a^5*c^3*d^6*e^1
0 + 28*a^6*c^2*d^4*e^12 - 8*a^7*c*d^2*e^14 + a^8*e^16)*log(abs(-c*d^2 - a*
e^2 - 2*sqrt(c*d*e)*(sqrt(c*d*e)*x - sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*x +
a*d*e))))/(sqrt(c*d*e)*c^4*d^4*e^4)
```

Mupad [B] (verification not implemented)

Time = 5.43 (sec) , antiderivative size = 413, normalized size of antiderivative = 1.14

$$\int \left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{7/2}\,dx=\frac{\left(\frac{c\,d^2}{2}+c\,x\,d\,e+\frac{a\,e^2}{2}\right)\,\left(c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e\right)^{7/2}}{8\,c\,d\,e}-\frac{\left(\frac{7\,\left(c\,d^2+a\,e^2\right)^2}{4}-7\,a\,c\,d^2\,e^2\right)\,\left(\frac{\left(\frac{c\,d^2}{2}+c\,x\,d\,e+\frac{a\,e^2}{2}\right)\,\left(c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e\right)^{5/2}}{6\,c\,d\,e}-\frac{\left(\frac{5\,\left(c\,d^2+a\,e^2\right)^2}{4}-5\,a\,c\,d^2\,e^2\right)\,\left(\frac{\frac{c\,d^2}{2}+c\,x\,d}{}\cdots\right)}{}\cdots\right)}{}$$

input
```
int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(7/2),x)
```

output
```
(((a*e^2)/2 + (c*d^2)/2 + c*d*e*x)*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)
^(7/2))/(8*c*d*e) - (((7*(a*e^2 + c*d^2)^2)/4 - 7*a*c*d^2*e^2)*((((a*e^2)/
2 + (c*d^2)/2 + c*d*e*x)*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(5/2))/(6
*c*d*e) - (((5*(a*e^2 + c*d^2)^2)/4 - 5*a*c*d^2*e^2)*((((a*e^2)/2 + (c*d^2
)/2 + c*d*e*x)*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(3/2))/(4*c*d*e) -
(((3*(a*e^2 + c*d^2)^2)/4 - 3*a*c*d^2*e^2)*((x/2 + (a*e^2 + c*d^2)/(4*c*d*
e))*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2) - (log(2*((a*e + c*d*x)*
(d + e*x))^(1/2)*(c*d*e)^(1/2) + a*e^2 + c*d^2 + 2*c*d*e*x)*((a*e^2 + c*d^
2)^2/4 - a*c*d^2*e^2))/(2*(c*d*e)^(3/2))))/(4*c*d*e)))/(6*c*d*e)))/(8*c*d*
e)
```

Reduce [B] (verification not implemented)

Time = 0.41 (sec) , antiderivative size = 1615, normalized size of antiderivative = 4.45

$$\int \left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{7/2} dx = \text{Too large to display}$$

input
```
int((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(7/2),x)
```

output
```
( - 105*sqrt(d + e*x)*sqrt(a*e + c*d*x)*a**7*c*d*e**15 + 805*sqrt(d + e*x)
*sqrt(a*e + c*d*x)*a**6*c**2*d**3*e**13 + 70*sqrt(d + e*x)*sqrt(a*e + c*d*
x)*a**6*c**2*d**2*e**14*x - 2681*sqrt(d + e*x)*sqrt(a*e + c*d*x)*a**5*c**3
*d**5*e**11 - 532*sqrt(d + e*x)*sqrt(a*e + c*d*x)*a**5*c**3*d**4*e**12*x -
 56*sqrt(d + e*x)*sqrt(a*e + c*d*x)*a**5*c**3*d**3*e**13*x**2 + 5053*sqrt(
d + e*x)*sqrt(a*e + c*d*x)*a**4*c**4*d**7*e**9 + 1754*sqrt(d + e*x)*sqrt(a
*e + c*d*x)*a**4*c**4*d**6*e**10*x + 424*sqrt(d + e*x)*sqrt(a*e + c*d*x)*a
**4*c**4*d**5*e**11*x**2 + 48*sqrt(d + e*x)*sqrt(a*e + c*d*x)*a**4*c**4*d*
*4*e**12*x**3 + 5053*sqrt(d + e*x)*sqrt(a*e + c*d*x)*a**3*c**5*d**9*e**7 +
 40424*sqrt(d + e*x)*sqrt(a*e + c*d*x)*a**3*c**5*d**8*e**8*x + 64144*sqrt(
d + e*x)*sqrt(a*e + c*d*x)*a**3*c**5*d**7*e**9*x**2 + 43328*sqrt(d + e*x)*
sqrt(a*e + c*d*x)*a**3*c**5*d**6*e**10*x**3 + 10880*sqrt(d + e*x)*sqrt(a*e
 + c*d*x)*a**3*c**5*d**5*e**11*x**4 - 2681*sqrt(d + e*x)*sqrt(a*e + c*d*x)
*a**2*c**6*d**11*e**5 + 1754*sqrt(d + e*x)*sqrt(a*e + c*d*x)*a**2*c**6*d**
10*e**6*x + 64144*sqrt(d + e*x)*sqrt(a*e + c*d*x)*a**2*c**6*d**9*e**7*x**2
 + 128288*sqrt(d + e*x)*sqrt(a*e + c*d*x)*a**2*c**6*d**8*e**8*x**3 + 96640
*sqrt(d + e*x)*sqrt(a*e + c*d*x)*a**2*c**6*d**7*e**9*x**4 + 25856*sqrt(d +
 e*x)*sqrt(a*e + c*d*x)*a**2*c**6*d**6*e**10*x**5 + 805*sqrt(d + e*x)*sqrt
(a*e + c*d*x)*a*c**7*d**13*e**3 - 532*sqrt(d + e*x)*sqrt(a*e + c*d*x)*a*c*
*7*d**12*e**4*x + 424*sqrt(d + e*x)*sqrt(a*e + c*d*x)*a*c**7*d**11*e**5...
```

3.236 $\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{7/2}}{d+ex}\,dx$

Optimal result

Integrand size = 37, antiderivative size = 496

$$\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{7/2}}{d+ex}\,dx = -\frac{5(cd^2-ae^2)^6\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{1024c^3d^3e^4} + \frac{5(cd^2-ae^2)^5\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{1536c^3d^3e^3(d+ex)} - \frac{(cd^2-ae^2)^4\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{384c^3d^3e^2(d+ex)^2} + \frac{(cd^2-ae^2)^3\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{7/2}}{448c^3d^3e(d+ex)^3} + \frac{(cd^2-ae^2)^2\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{9/2}}{56c^3d^3(d+ex)^4} + \frac{5(cd^2-ae^2)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{9/2}}{84c^2d^2(d+ex)^3} + \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{9/2}}{7cd(d+ex)^2} + \frac{5(cd^2-ae^2)^7\operatorname{arctanh}\left(\frac{\sqrt{e}\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{\sqrt{c}\sqrt{d}(d+ex)}\right)}{1024c^{7/2}d^{7/2}e^{9/2}}$$

output
```
-5/1024*(-a*e^2+c*d^2)^6*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/c^3/d^3/e
^4+5/1536*(-a*e^2+c*d^2)^5*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/c^3/d^3
/e^3/(e*x+d)-1/384*(-a*e^2+c*d^2)^4*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2
)/c^3/d^3/e^2/(e*x+d)^2+1/448*(-a*e^2+c*d^2)^3*(a*d*e+(a*e^2+c*d^2)*x+c*d*
e*x^2)^(7/2)/c^3/d^3/e/(e*x+d)^3+1/56*(-a*e^2+c*d^2)^2*(a*d*e+(a*e^2+c*d^2
)*x+c*d*e*x^2)^(9/2)/c^3/d^3/(e*x+d)^4+5/84*(-a*e^2+c*d^2)*(a*d*e+(a*e^2+c
*d^2)*x+c*d*e*x^2)^(9/2)/c^2/d^2/(e*x+d)^3+1/7*(a*d*e+(a*e^2+c*d^2)*x+c*d*
e*x^2)^(9/2)/c/d/(e*x+d)^2+5/1024*(-a*e^2+c*d^2)^7*arctanh(e^(1/2)*(a*d*e+
(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/c^(1/2)/d^(1/2)/(e*x+d))/c^(7/2)/d^(7/2)/
e^(9/2)
```

Mathematica [A] (verified)

Time = 1.31 (sec) , antiderivative size = 437, normalized size of antiderivative = 0.88

$$\int \frac{(ade+(cd^2+ae^2)x+cdex^2)^{7/2}}{d+ex}\,dx = \frac{((ae+cdx)(d+ex))^{5/2}\left(\frac{\sqrt{c}\sqrt{d}\sqrt{e}(105a^6e^{12}-70a^5cde^{10}(10d+ex)+7a^4c^2d^2e^8}{}\right.}{}$$

input
```
Integrate[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(7/2)/(d + e*x),x]
```

output
```
(((a*e + c*d*x)*(d + e*x))^(5/2)*((Sqrt[c]*Sqrt[d]*Sqrt[e]*(105*a^6*e^12 -
 70*a^5*c*d*e^10*(10*d + e*x) + 7*a^4*c^2*d^2*e^8*(283*d^2 + 66*d*e*x + 8*
e^2*x^2) + 4*a^3*c^3*d^3*e^6*(768*d^3 + 4285*d^2*e*x + 4516*d*e^2*x^2 + 15
24*e^3*x^3) + a^2*c^4*d^4*e^4*(-1981*d^4 + 1292*d^3*e*x + 27648*d^2*e^2*x^
2 + 36496*d*e^3*x^3 + 13696*e^4*x^4) + 2*a*c^5*d^5*e^2*(350*d^5 - 231*d^4*
e*x + 184*d^3*e^2*x^2 + 9400*d^2*e^3*x^3 + 13824*d*e^4*x^4 + 5504*e^5*x^5)
 + c^6*d^6*(-105*d^6 + 70*d^5*e*x - 56*d^4*e^2*x^2 + 48*d^3*e^3*x^3 + 4736
*d^2*e^4*x^4 + 7424*d*e^5*x^5 + 3072*e^6*x^6)))/((a*e + c*d*x)^2*(d + e*x)
^2) + (105*(c*d^2 - a*e^2)^7*ArcTanh[(Sqrt[c]*Sqrt[d]*Sqrt[d + e*x])/(Sqrt
[e]*Sqrt[a*e + c*d*x])])/((a*e + c*d*x)^(5/2)*(d + e*x)^(5/2))))/(21504*c^
(7/2)*d^(7/2)*e^(9/2))
```

Rubi [A] (verified)

Time = 0.82 (sec) , antiderivative size = 389, normalized size of antiderivative = 0.78, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.162$, Rules used = {1131, 1087, 1087, 1087, 1092, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{7/2}}{d+ex}dx$$

$$\Big\downarrow \text{1131}$$

$$\frac{\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{7/2}}{7e}-\frac{\left(cd^2-ae^2\right)\int \left(cdex^2+\left(cd^2+ae^2\right)x+ade\right)^{5/2}dx}{2e}$$

$$\Big\downarrow \text{1087}$$

$$\frac{\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{7/2}}{7e}-\frac{\left(cd^2-ae^2\right)\left(\frac{\left(ae^2+cd^2+2cdex\right)\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/2}}{12cde}-\frac{5\left(cd^2-ae^2\right)^2\int \left(cdex^2+\left(cd^2+ae^2\right)x+ade\right)^{3/2}dx}{24cde}\right)}{2e}$$

$$\Big\downarrow \text{1087}$$

$$\frac{\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{7/2}}{7e}-\frac{\left(cd^2-ae^2\right)\left(\frac{\left(ae^2+cd^2+2cdex\right)\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/2}}{12cde}-\frac{5\left(cd^2-ae^2\right)^2\left(\frac{\left(ae^2+cd^2+2cdex\right)\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{8cde}-\frac{3\left(cd^2-ae\right.}{}\right.}{24cde}\right.}{2e}$$

$$\Big\downarrow \text{1087}$$

$$\frac{\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{7/2}}{7e}-\frac{\left(cd^2-ae^2\right)\left(\frac{\left(ae^2+cd^2+2cdex\right)\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/2}}{12cde}-\frac{5\left(cd^2-ae^2\right)^2\left(\frac{\left(ae^2+cd^2+2cdex\right)\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{8cde}-\frac{3\left(cd^2-ae \cdots\right.}{\cdots}\right.}{\cdots}\right.}{2e \cdots}$$

$\downarrow$ 1092

$$\frac{\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{7/2}}{7e}-\frac{\left(cd^2-ae^2\right)\left(\frac{\left(ae^2+cd^2+2cdex\right)\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/2}}{12cde}-\frac{5\left(cd^2-ae^2\right)^2\left(\frac{\left(ae^2+cd^2+2cdex\right)\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{8cde}-\frac{3\left(cd^2-ae \cdots\right.}{\cdots}\right.}{\cdots}\right.}{2e \cdots}$$

$\downarrow$ 219

$$\frac{\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{7/2}}{7e}-$$

$$\frac{\left(cd^2-ae^2\right)\left(\frac{\left(ae^2+cd^2+2cdex\right)\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/2}}{12cde}-\frac{5\left(cd^2-ae^2\right)^2\left(\frac{\left(ae^2+cd^2+2cdex\right)\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{8cde}-\frac{3\left(cd^2-ae\right.}{}\right.}{}\right.}{2e}$$

input
```
Int[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(7/2)/(d + e*x),x]
```

output
```
(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(7/2)/(7*e) - ((c*d^2 - a*e^2)*(((
c*d^2 + a*e^2 + 2*c*d*e*x)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2))/
(12*c*d*e) - (5*(c*d^2 - a*e^2)^2*(((c*d^2 + a*e^2 + 2*c*d*e*x)*(a*d*e + (
c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2))/(8*c*d*e) - (3*(c*d^2 - a*e^2)^2*(((c
*d^2 + a*e^2 + 2*c*d*e*x)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(4*
c*d*e) - ((c*d^2 - a*e^2)^2*ArcTanh[(c*d^2 + a*e^2 + 2*c*d*e*x)/(2*Sqrt[c]
*Sqrt[d]*Sqrt[e]*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])])/(8*c^(3/2)
*d^(3/2)*e^(3/2))))/(16*c*d*e)))/(24*c*d*e)))/(2*e)
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1087
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^p/(2*c*(2*p + 1))), x] - Simp[p*((b^2 - 4*a*c)/(2*c*(2*
p + 1)))   Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] &&
GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[3*p])
```

rule 1092
```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[I
nt[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a
, b, c}, x]
```

rule 1131
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(d + e*x)^(m + 1)*((a + b*x + c*x^2)^p/(e*(m + 2*p + 1))), x
] - Simp[p*((2*c*d - b*e)/(e^2*(m + 2*p + 1)))   Int[(d + e*x)^(m + 1)*(a +
 b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c*d^2 - b
*d*e + a*e^2, 0] && GtQ[p, 0] && (LeQ[-2, m, 0] || EqQ[m + p + 1, 0]) && Ne
Q[m + 2*p + 1, 0] && IntegerQ[2*p]
```

Maple [A] (verified)

Time = 1.72 (sec) , antiderivative size = 424, normalized size of antiderivative = 0.85

method	result
default	$\frac{\left(dec\left(x+\frac{d}{e}\right)^{2}+\left(a\,e^{2}-c\,d^{2}\right)\left(x+\frac{d}{e}\right)\right)^{\frac{7}{2}}}{7}+\frac{\left(a\,e^{2}-c\,d^{2}\right)\left(\frac{\left(2dec\left(x+\frac{d}{e}\right)+a\,e^{2}-c\,d^{2}\right)\left(dec\left(x+\frac{d}{e}\right)^{2}+\left(a\,e^{2}-c\,d^{2}\right)\left(x+\frac{d}{e}\right)\right)^{\frac{5}{2}}}{12dec}-\frac{5\left(a\,e^{2}-c\,d^{2}\right)^{2}\left(\frac{(2de\cdots}{\cdots}\cdots\right.}{\cdots}\right.}{\cdots}$

input
```
int((a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(7/2)/(e*x+d),x,method=_RETURNVERBOS
E)
```

output
```
1/e*(1/7*(d*e*c*(x+d/e)^2+(a*e^2-c*d^2)*(x+d/e))^(7/2)+1/2*(a*e^2-c*d^2)*(
1/12*(2*d*e*c*(x+d/e)+a*e^2-c*d^2)/d/e/c*(d*e*c*(x+d/e)^2+(a*e^2-c*d^2)*(x
+d/e))^(5/2)-5/24*(a*e^2-c*d^2)^2/d/e/c*(1/8*(2*d*e*c*(x+d/e)+a*e^2-c*d^2)
/d/e/c*(d*e*c*(x+d/e)^2+(a*e^2-c*d^2)*(x+d/e))^(3/2)-3/16*(a*e^2-c*d^2)^2/
d/e/c*(1/4*(2*d*e*c*(x+d/e)+a*e^2-c*d^2)/d/e/c*(d*e*c*(x+d/e)^2+(a*e^2-c*d
^2)*(x+d/e))^(1/2)-1/8*(a*e^2-c*d^2)^2/d/e/c*ln((1/2*a*e^2-1/2*c*d^2+d*e*c
*(x+d/e))/(d*e*c)^(1/2)+(d*e*c*(x+d/e)^2+(a*e^2-c*d^2)*(x+d/e))^(1/2))/(d*
e*c)^(1/2)))))
```

Fricas [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 1270, normalized size of antiderivative = 2.56

$$\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{7/2}}{d+ex}\,dx=\text{Too large to display}$$

input
```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(7/2)/(e*x+d),x, algorithm="fr
icas")
```

output
```
[-1/86016*(105*(c^7*d^14 - 7*a*c^6*d^12*e^2 + 21*a^2*c^5*d^10*e^4 - 35*a^3
*c^4*d^8*e^6 + 35*a^4*c^3*d^6*e^8 - 21*a^5*c^2*d^4*e^10 + 7*a^6*c*d^2*e^12
 - a^7*e^14)*sqrt(c*d*e)*log(8*c^2*d^2*e^2*x^2 + c^2*d^4 + 6*a*c*d^2*e^2 +
 a^2*e^4 - 4*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(2*c*d*e*x + c*d^
2 + a*e^2)*sqrt(c*d*e) + 8*(c^2*d^3*e + a*c*d*e^3)*x) - 4*(3072*c^7*d^7*e^
7*x^6 - 105*c^7*d^13*e + 700*a*c^6*d^11*e^3 - 1981*a^2*c^5*d^9*e^5 + 3072*
a^3*c^4*d^7*e^7 + 1981*a^4*c^3*d^5*e^9 - 700*a^5*c^2*d^3*e^11 + 105*a^6*c*
d*e^13 + 256*(29*c^7*d^8*e^6 + 43*a*c^6*d^6*e^8)*x^5 + 128*(37*c^7*d^9*e^5
 + 216*a*c^6*d^7*e^7 + 107*a^2*c^5*d^5*e^9)*x^4 + 16*(3*c^7*d^10*e^4 + 117
5*a*c^6*d^8*e^6 + 2281*a^2*c^5*d^6*e^8 + 381*a^3*c^4*d^4*e^10)*x^3 - 8*(7*
c^7*d^11*e^3 - 46*a*c^6*d^9*e^5 - 3456*a^2*c^5*d^7*e^7 - 2258*a^3*c^4*d^5*
e^9 - 7*a^4*c^3*d^3*e^11)*x^2 + 2*(35*c^7*d^12*e^2 - 231*a*c^6*d^10*e^4 +
646*a^2*c^5*d^8*e^6 + 8570*a^3*c^4*d^6*e^8 + 231*a^4*c^3*d^4*e^10 - 35*a^5
*c^2*d^2*e^12)*x)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x))/(c^4*d^4*e^
5), -1/43008*(105*(c^7*d^14 - 7*a*c^6*d^12*e^2 + 21*a^2*c^5*d^10*e^4 - 35*
a^3*c^4*d^8*e^6 + 35*a^4*c^3*d^6*e^8 - 21*a^5*c^2*d^4*e^10 + 7*a^6*c*d^2*e
^12 - a^7*e^14)*sqrt(-c*d*e)*arctan(1/2*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 +
a*e^2)*x)*(2*c*d*e*x + c*d^2 + a*e^2)*sqrt(-c*d*e)/(c^2*d^2*e^2*x^2 + a*c*
d^2*e^2 + (c^2*d^3*e + a*c*d*e^3)*x)) - 2*(3072*c^7*d^7*e^7*x^6 - 105*c^7*
d^13*e + 700*a*c^6*d^11*e^3 - 1981*a^2*c^5*d^9*e^5 + 3072*a^3*c^4*d^7*e...
```

Sympy [A] (verification not implemented)

Time = 17.79 (sec) , antiderivative size = 11608, normalized size of antiderivative = 23.40

$$\int \frac{(ade + (cd^2 + ae^2)x + cdex^2)^{7/2}}{d + ex} dx = \text{Too large to display}$$

input
```
integrate((a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(7/2)/(e*x+d),x)
```

output
```
a**3*d**2*e**3*Piecewise(((x/2 + (a*e**2/4 + c*d**2/4)/(c*d*e))*sqrt(a*d*e
 + c*d*e*x**2 + x*(a*e**2 + c*d**2)) + (a*d*e/2 - (a*e**2/4 + c*d**2/4)*(a
*e**2 + c*d**2)/(2*c*d*e))*Piecewise((log(a*e**2 + c*d**2 + 2*c*d*e*x + 2*
sqrt(c*d*e)*sqrt(a*d*e + c*d*e*x**2 + x*(a*e**2 + c*d**2)))/sqrt(c*d*e), N
e(a*d*e - (a*e**2 + c*d**2)**2/(4*c*d*e), 0)), ((x - (-a*e**2 - c*d**2)/(2
*c*d*e))*log(x - (-a*e**2 - c*d**2)/(2*c*d*e))/sqrt(c*d*e*(x - (-a*e**2 -
c*d**2)/(2*c*d*e))**2), True)), Ne(c*d*e, 0)), (2*(a*d*e + x*(a*e**2 + c*d
**2))**(3/2)/(3*(a*e**2 + c*d**2)), Ne(a*e**2 + c*d**2, 0)), (x*sqrt(a*d*e
), True)) + 2*a**3*d*e**4*Piecewise(((-a*(a*e**2/6 + c*d**2/6)/(2*c) - (a*
e**2 + c*d**2)*(a*d*e/3 - (a*e**2/6 + c*d**2/6)*(3*a*e**2/2 + 3*c*d**2/2)/
(2*c*d*e))/(2*c*d*e))*Piecewise((log(a*e**2 + c*d**2 + 2*c*d*e*x + 2*sqrt(
c*d*e)*sqrt(a*d*e + c*d*e*x**2 + x*(a*e**2 + c*d**2)))/sqrt(c*d*e), Ne(a*d
*e - (a*e**2 + c*d**2)**2/(4*c*d*e), 0)), ((x - (-a*e**2 - c*d**2)/(2*c*d*
e))*log(x - (-a*e**2 - c*d**2)/(2*c*d*e))/sqrt(c*d*e*(x - (-a*e**2 - c*d**
2)/(2*c*d*e))**2), True)) + (x**2/3 + x*(a*e**2/6 + c*d**2/6)/(2*c*d*e) +
(a*d*e/3 - (a*e**2/6 + c*d**2/6)*(3*a*e**2/2 + 3*c*d**2/2)/(2*c*d*e))/(c*d
*e))*sqrt(a*d*e + c*d*e*x**2 + x*(a*e**2 + c*d**2)), Ne(c*d*e, 0)), (2*(-a
*d*e*(a*d*e + x*(a*e**2 + c*d**2))**(3/2)/3 + (a*d*e + x*(a*e**2 + c*d**2)
)**(5/2)/5)/(a*e**2 + c*d**2)**2, Ne(a*e**2 + c*d**2, 0)), (x**2*sqrt(a*d*
e)/2, True)) + a**3*e**5*Piecewise(((-a*(a*d*e/4 - (a*e**2/8 + c*d**2/8...
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\left(ade + (cd^2 + ae^2)\,x + cdex^2\right)^{7/2}}{d + ex}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(7/2)/(e*x+d),x, algorithm="ma
xima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

Giac [A] (verification not implemented)

Time = 0.45 (sec) , antiderivative size = 651, normalized size of antiderivative = 1.31

$$\int \frac{(ade + (cd^2 + ae^2)x + cdex^2)^{7/2}}{d + ex} dx = \frac{1}{21504}\sqrt{cdex^2 + cd^2x + ae^2x + ade}\left(2\left(4\left(2\left(8\left(2\left(12\,c^3d^3e^2x\right.\right.\right.\right.\right.\right.$$
$$-\frac{5\,(c^7d^{14} - 7\,ac^6d^{12}e^2 + 21\,a^2c^5d^{10}e^4 - 35\,a^3c^4d^8e^6 + 35\,a^4c^3d^6e^8 - 21\,a^5c^2d^4e^{10} + 7\,a^6cd^2e^{12} - a^7e^{14})\log}{2048\sqrt{cde}c^3d^3e^4}$$

input
```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(7/2)/(e*x+d),x, algorithm="gi
ac")
```

output
```
1/21504*sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*x + a*d*e)*(2*(4*(2*(8*(2*(12*c^3
*d^3*e^2*x + (29*c^9*d^10*e^7 + 43*a*c^8*d^8*e^9)/(c^6*d^6*e^6))*x + (37*c
^9*d^11*e^6 + 216*a*c^8*d^9*e^8 + 107*a^2*c^7*d^7*e^10)/(c^6*d^6*e^6))*x +
 (3*c^9*d^12*e^5 + 1175*a*c^8*d^10*e^7 + 2281*a^2*c^7*d^8*e^9 + 381*a^3*c^
6*d^6*e^11)/(c^6*d^6*e^6))*x - (7*c^9*d^13*e^4 - 46*a*c^8*d^11*e^6 - 3456*
a^2*c^7*d^9*e^8 - 2258*a^3*c^6*d^7*e^10 - 7*a^4*c^5*d^5*e^12)/(c^6*d^6*e^6
))*x + (35*c^9*d^14*e^3 - 231*a*c^8*d^12*e^5 + 646*a^2*c^7*d^10*e^7 + 8570
*a^3*c^6*d^8*e^9 + 231*a^4*c^5*d^6*e^11 - 35*a^5*c^4*d^4*e^13)/(c^6*d^6*e^
6))*x - (105*c^9*d^15*e^2 - 700*a*c^8*d^13*e^4 + 1981*a^2*c^7*d^11*e^6 - 3
072*a^3*c^6*d^9*e^8 - 1981*a^4*c^5*d^7*e^10 + 700*a^5*c^4*d^5*e^12 - 105*a
^6*c^3*d^3*e^14)/(c^6*d^6*e^6)) - 5/2048*(c^7*d^14 - 7*a*c^6*d^12*e^2 + 21
*a^2*c^5*d^10*e^4 - 35*a^3*c^4*d^8*e^6 + 35*a^4*c^3*d^6*e^8 - 21*a^5*c^2*d
^4*e^10 + 7*a^6*c*d^2*e^12 - a^7*e^14)*log(abs(-c*d^2 - a*e^2 - 2*sqrt(c*d
*e)*(sqrt(c*d*e)*x - sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*x + a*d*e))))/(sqrt(
c*d*e)*c^3*d^3*e^4)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(ade + (cd^2 + ae^2)x + cdex^2)^{7/2}}{d + ex} dx = \int \frac{(c\,d\,e\,x^2 + (c\,d^2 + a\,e^2)\,x + a\,d\,e)^{7/2}}{d + e\,x} dx$$

input
```
int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(7/2)/(d + e*x),x)
```

output
```
int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(7/2)/(d + e*x), x)
```

Reduce [B] (verification not implemented)

Time = 0.39 (sec) , antiderivative size = 1308, normalized size of antiderivative = 2.64

$$\int \frac{(ade + (cd^2 + ae^2)x + cdex^2)^{7/2}}{d + ex} dx = \text{Too large to display}$$

input
```
int((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(7/2)/(e*x+d),x)
```

output
```
(105*sqrt(d + e*x)*sqrt(a*e + c*d*x)*a**6*c*d*e**13 - 700*sqrt(d + e*x)*sq
rt(a*e + c*d*x)*a**5*c**2*d**3*e**11 - 70*sqrt(d + e*x)*sqrt(a*e + c*d*x)*
a**5*c**2*d**2*e**12*x + 1981*sqrt(d + e*x)*sqrt(a*e + c*d*x)*a**4*c**3*d*
*5*e**9 + 462*sqrt(d + e*x)*sqrt(a*e + c*d*x)*a**4*c**3*d**4*e**10*x + 56*
sqrt(d + e*x)*sqrt(a*e + c*d*x)*a**4*c**3*d**3*e**11*x**2 + 3072*sqrt(d +
e*x)*sqrt(a*e + c*d*x)*a**3*c**4*d**7*e**7 + 17140*sqrt(d + e*x)*sqrt(a*e
+ c*d*x)*a**3*c**4*d**6*e**8*x + 18064*sqrt(d + e*x)*sqrt(a*e + c*d*x)*a**
3*c**4*d**5*e**9*x**2 + 6096*sqrt(d + e*x)*sqrt(a*e + c*d*x)*a**3*c**4*d**
4*e**10*x**3 - 1981*sqrt(d + e*x)*sqrt(a*e + c*d*x)*a**2*c**5*d**9*e**5 +
1292*sqrt(d + e*x)*sqrt(a*e + c*d*x)*a**2*c**5*d**8*e**6*x + 27648*sqrt(d
+ e*x)*sqrt(a*e + c*d*x)*a**2*c**5*d**7*e**7*x**2 + 36496*sqrt(d + e*x)*sq
rt(a*e + c*d*x)*a**2*c**5*d**6*e**8*x**3 + 13696*sqrt(d + e*x)*sqrt(a*e +
c*d*x)*a**2*c**5*d**5*e**9*x**4 + 700*sqrt(d + e*x)*sqrt(a*e + c*d*x)*a*c*
*6*d**11*e**3 - 462*sqrt(d + e*x)*sqrt(a*e + c*d*x)*a*c**6*d**10*e**4*x +
368*sqrt(d + e*x)*sqrt(a*e + c*d*x)*a*c**6*d**9*e**5*x**2 + 18800*sqrt(d +
 e*x)*sqrt(a*e + c*d*x)*a*c**6*d**8*e**6*x**3 + 27648*sqrt(d + e*x)*sqrt(a
*e + c*d*x)*a*c**6*d**7*e**7*x**4 + 11008*sqrt(d + e*x)*sqrt(a*e + c*d*x)*
a*c**6*d**6*e**8*x**5 - 105*sqrt(d + e*x)*sqrt(a*e + c*d*x)*c**7*d**13*e +
 70*sqrt(d + e*x)*sqrt(a*e + c*d*x)*c**7*d**12*e**2*x - 56*sqrt(d + e*x)*s
qrt(a*e + c*d*x)*c**7*d**11*e**3*x**2 + 48*sqrt(d + e*x)*sqrt(a*e + c*d...
```

3.237 $\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{7/2}}{(d+ex)^2}\,dx$

Optimal result

Integrand size = 37, antiderivative size = 436

$$\begin{aligned}\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{7/2}}{(d+ex)^2}\,dx =& -\frac{7\left(cd^2-ae^2\right)^5\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{512c^2d^2e^4} \\ &+ \frac{7\left(cd^2-ae^2\right)^4\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{768c^2d^2e^3(d+ex)} \\ &- \frac{7\left(cd^2-ae^2\right)^3\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{960c^2d^2e^2(d+ex)^2} \\ &+ \frac{\left(cd^2-ae^2\right)^2\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{7/2}}{160c^2d^2e(d+ex)^3} \\ &+ \frac{\left(cd^2-ae^2\right)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{9/2}}{20c^2d^2(d+ex)^4} \\ &+ \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{9/2}}{6cd(d+ex)^3} \\ &+ \frac{7\left(cd^2-ae^2\right)^6\operatorname{arctanh}\left(\frac{\sqrt{e}\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{\sqrt{c}\sqrt{d}(d+ex)}\right)}{512c^{5/2}d^{5/2}e^{9/2}}\end{aligned}$$

output
```
-7/512*(-a*e^2+c*d^2)^5*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/c^2/d^2/e^
4+7/768*(-a*e^2+c*d^2)^4*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/c^2/d^2/e
^3/(e*x+d)-7/960*(-a*e^2+c*d^2)^3*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/
c^2/d^2/e^2/(e*x+d)^2+1/160*(-a*e^2+c*d^2)^2*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*
x^2)^(7/2)/c^2/d^2/e/(e*x+d)^3+1/20*(-a*e^2+c*d^2)*(a*d*e+(a*e^2+c*d^2)*x+
c*d*e*x^2)^(9/2)/c^2/d^2/(e*x+d)^4+1/6*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(
9/2)/c/d/(e*x+d)^3+7/512*(-a*e^2+c*d^2)^6*arctanh(e^(1/2)*(a*d*e+(a*e^2+c*
d^2)*x+c*d*e*x^2)^(1/2)/c^(1/2)/d^(1/2)/(e*x+d))/c^(5/2)/d^(5/2)/e^(9/2)
```

Mathematica [A] (verified)

Time = 1.06 (sec) , antiderivative size = 361, normalized size of antiderivative = 0.83

$$\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{7/2}}{(d+ex)^2}\,dx=\frac{((ae+cdx)(d+ex))^{3/2}\left(\frac{\sqrt{c}\sqrt{d}\sqrt{e}\left(-105a^5e^{10}+35a^4cde^8(17d+2ex)+2a^3c^2d^2\cdots\right.}{\cdots}\right.}{\cdots}$$

input
```
Integrate[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(7/2)/(d + e*x)^2,x]
```

output
```
(((a*e + c*d*x)*(d + e*x))^(3/2)*((Sqrt[c]*Sqrt[d]*Sqrt[e]*(-105*a^5*e^10
+ 35*a^4*c*d*e^8*(17*d + 2*e*x) + 2*a^3*c^2*d^2*e^6*(843*d^2 + 2876*d*e*x
+ 1508*e^2*x^2) + 6*a^2*c^3*d^3*e^4*(-231*d^3 + 150*d^2*e*x + 1588*d*e^2*x
^2 + 1032*e^3*x^3) + a*c^4*d^4*e^2*(595*d^4 - 392*d^3*e*x + 312*d^2*e^2*x^
2 + 6560*d*e^3*x^3 + 4736*e^4*x^4) + c^5*d^5*(-105*d^5 + 70*d^4*e*x - 56*d
^3*e^2*x^2 + 48*d^2*e^3*x^3 + 1664*d*e^4*x^4 + 1280*e^5*x^5)))/((a*e + c*d
*x)*(d + e*x)) + (105*(c*d^2 - a*e^2)^6*ArcTanh[(Sqrt[c]*Sqrt[d]*Sqrt[d +
e*x])/(Sqrt[e]*Sqrt[a*e + c*d*x])])/((a*e + c*d*x)^(3/2)*(d + e*x)^(3/2)))
)/(7680*c^(5/2)*d^(5/2)*e^(9/2))
```

Rubi [A] (verified)

Time = 0.83 (sec) , antiderivative size = 366, normalized size of antiderivative = 0.84, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.189, Rules used = {1127, 1134, 1160, 1087, 1087, 1092, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{7/2}}{(d+ex)^2}dx$$

$$\Big\downarrow \text{ 1127}$$

$$\int (ae+cdx)^2\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}dx$$

$$\Big\downarrow \text{ 1134}$$

$$\frac{(ae+cdx)\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/2}}{6e}-\frac{7\left(cd^2-ae^2\right)\int (ae+cdx)\left(cdex^2+\left(cd^2+ae^2\right)x+ade\right)^{3/2}dx}{12e}$$

$$\Big\downarrow \text{ 1160}$$

$$\frac{(ae+cdx)\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/2}}{6e}-\frac{7\left(cd^2-ae^2\right)\left(\frac{\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/2}}{5e}-\frac{\left(cd^2-ae^2\right)\int \left(cdex^2+\left(cd^2+ae^2\right)x+ade\right)^{3/2}dx}{2e}\right)}{12e}$$

$$\Big\downarrow \text{ 1087}$$

$$\frac{(ae+cdx)\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/2}}{6e}-\frac{7\left(cd^2-ae^2\right)\left(\frac{\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/2}}{5e}-\frac{\left(cd^2-ae^2\right)\left(\frac{\left(ae^2+cd^2+2cdex\right)\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{8cde}-\frac{3\left(cd^2-ae^2\right)^2\int \sqrt{cdex^2+\left(cd^2\right.}}{16cde}\right.}{2e}\right.}{12e}$$

$$\Big\downarrow \text{ 1087}$$

$$\frac{(ae+cdx)\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/2}}{6e}-$$

$$\frac{7\left(cd^2-ae^2\right)\left(\frac{\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/2}}{5e}-\frac{\left(cd^2-ae^2\right)\left(\frac{\left(ae^2+cd^2+2cdex\right)\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{8cde}-\frac{3\left(cd^2-ae^2\right)^2\left(\frac{\left(ae^2+cd^2+2cd\ldots\right.}{\ldots}\right.}{\ldots}\right)}{2e}\ldots\right)}{12e}$$

$\downarrow$ 1092

$$\frac{(ae+cdx)\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/2}}{6e}-$$

$$\frac{7\left(cd^2-ae^2\right)\left(\frac{\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/2}}{5e}-\frac{\left(cd^2-ae^2\right)\left(\frac{\left(ae^2+cd^2+2cdex\right)\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{8cde}-\frac{3\left(cd^2-ae^2\right)^2\left(\frac{\left(ae^2+cd^2+2cd\ldots\right.}{\ldots}\right.}{\ldots}\right)}{\ldots}\right)}{12e}$$

$\downarrow$ 219

$$\frac{(ae+cdx)\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/2}}{6e}-$$

$$\frac{7\left(cd^2-ae^2\right)\left(\frac{\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/2}}{5e}-\frac{\left(cd^2-ae^2\right)\left(\frac{\left(ae^2+cd^2+2cdex\right)\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{8cde}-\frac{3\left(cd^2-ae^2\right)^2\left(\frac{\left(ae^2+cd^2+2cd\right.}{}\right.}{}\right.}{}\right.}{12e}$$

input
```
Int[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(7/2)/(d + e*x)^2,x]
```

output
```
((a*e + c*d*x)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2))/(6*e) - (7*(
c*d^2 - a*e^2)*((a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2)/(5*e) - ((c*
d^2 - a*e^2)*(((c*d^2 + a*e^2 + 2*c*d*e*x)*(a*d*e + (c*d^2 + a*e^2)*x + c*
d*e*x^2)^(3/2))/(8*c*d*e) - (3*(c*d^2 - a*e^2)^2*(((c*d^2 + a*e^2 + 2*c*d*
e*x)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(4*c*d*e) - ((c*d^2 - a*
e^2)^2*ArcTanh[(c*d^2 + a*e^2 + 2*c*d*e*x)/(2*Sqrt[c]*Sqrt[d]*Sqrt[e]*Sqrt
[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])])/(8*c^(3/2)*d^(3/2)*e^(3/2))))/(
16*c*d*e)))/(2*e)))/(12*e)
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1087
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^p/(2*c*(2*p + 1))), x] - Simp[p*((b^2 - 4*a*c)/(2*c*(2*
p + 1)))   Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] &&
GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[3*p])
```

rule 1092
```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[I
nt[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a
, b, c}, x]
```

rule 1127
```
Int[((d_) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Sy
mbol] :> Int[(a + b*x + c*x^2)^(m + p)/(a/d + c*(x/e))^m, x] /; FreeQ[{a, b
, c, d, e}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[m] && RationalQ
[p] && (LtQ[0, -m, p] || LtQ[p, -m, 0]) && NeQ[m, 2] && NeQ[m, -1]
```

rule 1134
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[e*(d + e*x)^(m - 1)*((a + b*x + c*x^2)^(p + 1)/(c*(m + 2*p +
 1))), x] + Simp[(m + p)*((2*c*d - b*e)/(c*(m + 2*p + 1)))   Int[(d + e*x)^
(m - 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[
c*d^2 - b*d*e + a*e^2, 0] && GtQ[m, 1] && NeQ[m + 2*p + 1, 0] && IntegerQ[2
*p]
```

rule 1160
```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[e*((a + b*x + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Simp[(2*c*d - b
*e)/(2*c)   Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
 && NeQ[p, -1]
```

Maple [A] (verified)

Time = 1.85 (sec) , antiderivative size = 505, normalized size of antiderivative = 1.16

method	result
default	$\frac{\frac{2\left(dec\left(x+\frac{d}{e}\right)^{2}+\left(a\,e^{2}-c\,d^{2}\right)\left(x+\frac{d}{e}\right)\right)^{\frac{9}{2}}}{5\left(a\,e^{2}-c\,d^{2}\right)\left(x+\frac{d}{e}\right)^{2}}-\frac{14dec\left(\frac{\left(dec\left(x+\frac{d}{e}\right)^{2}+\left(a\,e^{2}-c\,d^{2}\right)\left(x+\frac{d}{e}\right)\right)^{\frac{7}{2}}}{7}+\frac{\left(a\,e^{2}-c\,d^{2}\right)\left(\frac{\left(2dec\left(x+\frac{d}{e}\right)+a\,e^{2}-c\,d^{2}\right)\left(dec\left(x+\frac{d}{e}\right)\right.}{12dec}\right.}{}\right.}{}}{}$

input
```
int((a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(7/2)/(e*x+d)^2,x,method=_RETURNVERB
OSE)
```

output
```
1/e^2*(2/5/(a*e^2-c*d^2)/(x+d/e)^2*(d*e*c*(x+d/e)^2+(a*e^2-c*d^2)*(x+d/e))
^(9/2)-14/5*d*e*c/(a*e^2-c*d^2)*(1/7*(d*e*c*(x+d/e)^2+(a*e^2-c*d^2)*(x+d/e
))^(7/2)+1/2*(a*e^2-c*d^2)*(1/12*(2*d*e*c*(x+d/e)+a*e^2-c*d^2)/d/e/c*(d*e*
c*(x+d/e)^2+(a*e^2-c*d^2)*(x+d/e))^(5/2)-5/24*(a*e^2-c*d^2)^2/d/e/c*(1/8*(
2*d*e*c*(x+d/e)+a*e^2-c*d^2)/d/e/c*(d*e*c*(x+d/e)^2+(a*e^2-c*d^2)*(x+d/e))
^(3/2)-3/16*(a*e^2-c*d^2)^2/d/e/c*(1/4*(2*d*e*c*(x+d/e)+a*e^2-c*d^2)/d/e/c
*(d*e*c*(x+d/e)^2+(a*e^2-c*d^2)*(x+d/e))^(1/2)-1/8*(a*e^2-c*d^2)^2/d/e/c*l
n((1/2*a*e^2-1/2*c*d^2+d*e*c*(x+d/e))/(d*e*c)^(1/2)+(d*e*c*(x+d/e)^2+(a*e^
2-c*d^2)*(x+d/e))^(1/2))/(d*e*c)^(1/2))))))
```

Fricas [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 1042, normalized size of antiderivative = 2.39

$$\int \frac{(ade + (cd^2 + ae^2)\,x + cdex^2)^{7/2}}{(d + ex)^2}\,dx = \text{Too large to display}$$

input
```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(7/2)/(e*x+d)^2,x, algorithm="
fricas")
```

output
```
[1/30720*(105*(c^6*d^12 - 6*a*c^5*d^10*e^2 + 15*a^2*c^4*d^8*e^4 - 20*a^3*c
^3*d^6*e^6 + 15*a^4*c^2*d^4*e^8 - 6*a^5*c*d^2*e^10 + a^6*e^12)*sqrt(c*d*e)
*log(8*c^2*d^2*e^2*x^2 + c^2*d^4 + 6*a*c*d^2*e^2 + a^2*e^4 + 4*sqrt(c*d*e*
x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(2*c*d*e*x + c*d^2 + a*e^2)*sqrt(c*d*e) +
 8*(c^2*d^3*e + a*c*d*e^3)*x) + 4*(1280*c^6*d^6*e^6*x^5 - 105*c^6*d^11*e +
 595*a*c^5*d^9*e^3 - 1386*a^2*c^4*d^7*e^5 + 1686*a^3*c^3*d^5*e^7 + 595*a^4
*c^2*d^3*e^9 - 105*a^5*c*d*e^11 + 128*(13*c^6*d^7*e^5 + 37*a*c^5*d^5*e^7)*
x^4 + 16*(3*c^6*d^8*e^4 + 410*a*c^5*d^6*e^6 + 387*a^2*c^4*d^4*e^8)*x^3 - 8
*(7*c^6*d^9*e^3 - 39*a*c^5*d^7*e^5 - 1191*a^2*c^4*d^5*e^7 - 377*a^3*c^3*d^
3*e^9)*x^2 + 2*(35*c^6*d^10*e^2 - 196*a*c^5*d^8*e^4 + 450*a^2*c^4*d^6*e^6
+ 2876*a^3*c^3*d^4*e^8 + 35*a^4*c^2*d^2*e^10)*x)*sqrt(c*d*e*x^2 + a*d*e +
(c*d^2 + a*e^2)*x))/(c^3*d^3*e^5), -1/15360*(105*(c^6*d^12 - 6*a*c^5*d^10*
e^2 + 15*a^2*c^4*d^8*e^4 - 20*a^3*c^3*d^6*e^6 + 15*a^4*c^2*d^4*e^8 - 6*a^5
*c*d^2*e^10 + a^6*e^12)*sqrt(-c*d*e)*arctan(1/2*sqrt(c*d*e*x^2 + a*d*e + (
c*d^2 + a*e^2)*x)*(2*c*d*e*x + c*d^2 + a*e^2)*sqrt(-c*d*e)/(c^2*d^2*e^2*x^
2 + a*c*d^2*e^2 + (c^2*d^3*e + a*c*d*e^3)*x)) - 2*(1280*c^6*d^6*e^6*x^5 -
105*c^6*d^11*e + 595*a*c^5*d^9*e^3 - 1386*a^2*c^4*d^7*e^5 + 1686*a^3*c^3*d
^5*e^7 + 595*a^4*c^2*d^3*e^9 - 105*a^5*c*d*e^11 + 128*(13*c^6*d^7*e^5 + 37
*a*c^5*d^5*e^7)*x^4 + 16*(3*c^6*d^8*e^4 + 410*a*c^5*d^6*e^6 + 387*a^2*c^4*
d^4*e^8)*x^3 - 8*(7*c^6*d^9*e^3 - 39*a*c^5*d^7*e^5 - 1191*a^2*c^4*d^5*e...
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{7/2}}{(d+ex)^2}\,dx=\text{Timed out}$$

input
```
integrate((a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(7/2)/(e*x+d)**2,x)
```

output
```
Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(ade + (cd^2 + ae^2)x + cdex^2)^{7/2}}{(d + ex)^2} dx = \text{Exception raised: ValueError}$$

input
```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(7/2)/(e*x+d)^2,x, algorithm="
maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2611 vs. 2(396) = 792.

Time = 1.22 (sec) , antiderivative size = 2611, normalized size of antiderivative = 5.99

$$\int \frac{(ade + (cd^2 + ae^2)x + cdex^2)^{7/2}}{(d + ex)^2} dx = \text{Too large to display}$$

input
```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(7/2)/(e*x+d)^2,x, algorithm="
giac")
```

output
```
-1/7680*(105*(c^6*d^12*sgn(1/(e*x + d))*sgn(e) - 6*a*c^5*d^10*e^2*sgn(1/(e
*x + d))*sgn(e) + 15*a^2*c^4*d^8*e^4*sgn(1/(e*x + d))*sgn(e) - 20*a^3*c^3*
d^6*e^6*sgn(1/(e*x + d))*sgn(e) + 15*a^4*c^2*d^4*e^8*sgn(1/(e*x + d))*sgn(
e) - 6*a^5*c*d^2*e^10*sgn(1/(e*x + d))*sgn(e) + a^6*e^12*sgn(1/(e*x + d))*
sgn(e))*arctan(sqrt(c*d*e - c*d^2*e/(e*x + d) + a*e^3/(e*x + d))/sqrt(-c*d
*e))/(sqrt(-c*d*e)*c^2*d^2*e^4*abs(e)) + (105*sqrt(c*d*e - c*d^2*e/(e*x +
d) + a*e^3/(e*x + d))*c^11*d^17*e^5*sgn(1/(e*x + d))*sgn(e) - 630*sqrt(c*d
*e - c*d^2*e/(e*x + d) + a*e^3/(e*x + d))*a*c^10*d^15*e^7*sgn(1/(e*x + d))
*sgn(e) + 1575*sqrt(c*d*e - c*d^2*e/(e*x + d) + a*e^3/(e*x + d))*a^2*c^9*d
^13*e^9*sgn(1/(e*x + d))*sgn(e) - 2100*sqrt(c*d*e - c*d^2*e/(e*x + d) + a*
e^3/(e*x + d))*a^3*c^8*d^11*e^11*sgn(1/(e*x + d))*sgn(e) + 1575*sqrt(c*d*e
 - c*d^2*e/(e*x + d) + a*e^3/(e*x + d))*a^4*c^7*d^9*e^13*sgn(1/(e*x + d))*
sgn(e) - 630*sqrt(c*d*e - c*d^2*e/(e*x + d) + a*e^3/(e*x + d))*a^5*c^6*d^7
*e^15*sgn(1/(e*x + d))*sgn(e) + 105*sqrt(c*d*e - c*d^2*e/(e*x + d) + a*e^3
/(e*x + d))*a^6*c^5*d^5*e^17*sgn(1/(e*x + d))*sgn(e) - 595*(c*d*e - c*d^2*
e/(e*x + d) + a*e^3/(e*x + d))^(3/2)*c^10*d^16*e^4*sgn(1/(e*x + d))*sgn(e)
 + 3570*(c*d*e - c*d^2*e/(e*x + d) + a*e^3/(e*x + d))^(3/2)*a*c^9*d^14*e^6
*sgn(1/(e*x + d))*sgn(e) - 8925*(c*d*e - c*d^2*e/(e*x + d) + a*e^3/(e*x +
d))^(3/2)*a^2*c^8*d^12*e^8*sgn(1/(e*x + d))*sgn(e) + 11900*(c*d*e - c*d^2*
e/(e*x + d) + a*e^3/(e*x + d))^(3/2)*a^3*c^7*d^10*e^10*sgn(1/(e*x + d))...
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(ade + (cd^2 + ae^2)x + cdex^2)^{7/2}}{(d + ex)^2} dx = \int \frac{(c\,d\,e\,x^2 + (c\,d^2 + a\,e^2)\,x + a\,d\,e)^{7/2}}{(d + e\,x)^2} dx$$

input
```
int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(7/2)/(d + e*x)^2,x)
```

output
```
int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(7/2)/(d + e*x)^2, x)
```

Reduce [B] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 1033, normalized size of antiderivative = 2.37

$$\int \frac{(ade + (cd^2 + ae^2)\,x + cdex^2)^{7/2}}{(d + ex)^2}\,dx = \text{Too large to display}$$

input

```
int((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(7/2)/(e*x+d)^2,x)
```

output

```
( - 105*sqrt(d + e*x)*sqrt(a*e + c*d*x)*a**5*c*d*e**11 + 595*sqrt(d + e*x)
*sqrt(a*e + c*d*x)*a**4*c**2*d**3*e**9 + 70*sqrt(d + e*x)*sqrt(a*e + c*d*x
)*a**4*c**2*d**2*e**10*x + 1686*sqrt(d + e*x)*sqrt(a*e + c*d*x)*a**3*c**3*
d**5*e**7 + 5752*sqrt(d + e*x)*sqrt(a*e + c*d*x)*a**3*c**3*d**4*e**8*x + 3
016*sqrt(d + e*x)*sqrt(a*e + c*d*x)*a**3*c**3*d**3*e**9*x**2 - 1386*sqrt(d
 + e*x)*sqrt(a*e + c*d*x)*a**2*c**4*d**7*e**5 + 900*sqrt(d + e*x)*sqrt(a*e
 + c*d*x)*a**2*c**4*d**6*e**6*x + 9528*sqrt(d + e*x)*sqrt(a*e + c*d*x)*a**
2*c**4*d**5*e**7*x**2 + 6192*sqrt(d + e*x)*sqrt(a*e + c*d*x)*a**2*c**4*d**
4*e**8*x**3 + 595*sqrt(d + e*x)*sqrt(a*e + c*d*x)*a*c**5*d**9*e**3 - 392*s
qrt(d + e*x)*sqrt(a*e + c*d*x)*a*c**5*d**8*e**4*x + 312*sqrt(d + e*x)*sqrt
(a*e + c*d*x)*a*c**5*d**7*e**5*x**2 + 6560*sqrt(d + e*x)*sqrt(a*e + c*d*x)
*a*c**5*d**6*e**6*x**3 + 4736*sqrt(d + e*x)*sqrt(a*e + c*d*x)*a*c**5*d**5*
e**7*x**4 - 105*sqrt(d + e*x)*sqrt(a*e + c*d*x)*c**6*d**11*e + 70*sqrt(d +
 e*x)*sqrt(a*e + c*d*x)*c**6*d**10*e**2*x - 56*sqrt(d + e*x)*sqrt(a*e + c*
d*x)*c**6*d**9*e**3*x**2 + 48*sqrt(d + e*x)*sqrt(a*e + c*d*x)*c**6*d**8*e*
*4*x**3 + 1664*sqrt(d + e*x)*sqrt(a*e + c*d*x)*c**6*d**7*e**5*x**4 + 1280*
sqrt(d + e*x)*sqrt(a*e + c*d*x)*c**6*d**6*e**6*x**5 + 105*sqrt(e)*sqrt(d)*
sqrt(c)*log((sqrt(e)*sqrt(a*e + c*d*x) + sqrt(d)*sqrt(c)*sqrt(d + e*x))/sq
rt(a*e**2 - c*d**2))*a**6*e**12 - 630*sqrt(e)*sqrt(d)*sqrt(c)*log((sqrt(e)
*sqrt(a*e + c*d*x) + sqrt(d)*sqrt(c)*sqrt(d + e*x))/sqrt(a*e**2 - c*d**...
```

3.238 $\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{7/2}}{(d+ex)^3}\,dx$

Optimal result

Integrand size = 37, antiderivative size = 371

$$\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{7/2}}{(d+ex)^3}\,dx = -\frac{7\left(cd^2-ae^2\right)^4\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{128cde^4} + \frac{7\left(cd^2-ae^2\right)^3\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{192cde^3(d+ex)} - \frac{7\left(cd^2-ae^2\right)^2\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{240cde^2(d+ex)^2} + \frac{\left(\frac{d}{e}-\frac{ae}{cd}\right)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{7/2}}{40(d+ex)^3} + \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{9/2}}{5cd(d+ex)^4} + \frac{7\left(cd^2-ae^2\right)^5\operatorname{arctanh}\left(\frac{\sqrt{e}\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{\sqrt{c}\sqrt{d}(d+ex)}\right)}{128c^{3/2}d^{3/2}e^{9/2}}$$

output
```
-7/128*(-a*e^2+c*d^2)^4*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/c/d/e^4+7/
192*(-a*e^2+c*d^2)^3*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/c/d/e^3/(e*x+
d)-7/240*(-a*e^2+c*d^2)^2*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/c/d/e^2/
(e*x+d)^2+1/40*(d/e-a*e/c/d)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(7/2)/(e*x+
d)^3+1/5*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(9/2)/c/d/(e*x+d)^4+7/128*(-a*e
^2+c*d^2)^5*arctanh(e^(1/2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/c^(1/2
)/d^(1/2)/(e*x+d))/c^(3/2)/d^(3/2)/e^(9/2)
```

Mathematica [A] (verified)

Time = 0.79 (sec) , antiderivative size = 279, normalized size of antiderivative = 0.75

$$\int \frac{(ade + (cd^2 + ae^2)x + cdex^2)^{7/2}}{(d+ex)^3}\,dx = \frac{\sqrt{(ae+cdx)(d+ex)}\left(\sqrt{c}\sqrt{d}\sqrt{e}(105a^4e^8 + 10a^3cde^6(79d + 121e\right.}{}$$

input
```
Integrate[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(7/2)/(d + e*x)^3,x]
```

output
```
(Sqrt[(a*e + c*d*x)*(d + e*x)]*(Sqrt[c]*Sqrt[d]*Sqrt[e]*(105*a^4*e^8 + 10*
a^3*c*d*e^6*(79*d + 121*e*x) + 2*a^2*c^2*d^2*e^4*(-448*d^2 + 289*d*e*x + 1
052*e^2*x^2) + 2*a*c^3*d^3*e^2*(245*d^3 - 161*d^2*e*x + 128*d*e^2*x^2 + 74
4*e^3*x^3) + c^4*d^4*(-105*d^4 + 70*d^3*e*x - 56*d^2*e^2*x^2 + 48*d*e^3*x^
3 + 384*e^4*x^4)) + (105*(c*d^2 - a*e^2)^5*ArcTanh[(Sqrt[c]*Sqrt[d]*Sqrt[d
 + e*x])/(Sqrt[e]*Sqrt[a*e + c*d*x])])/(Sqrt[a*e + c*d*x]*Sqrt[d + e*x])))
/(1920*c^(3/2)*d^(3/2)*e^(9/2))
```

Rubi [A] (verified)

Time = 0.83 (sec) , antiderivative size = 345, normalized size of antiderivative = 0.93, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.189$, Rules used = {1127, 1134, 1134, 1160, 1087, 1092, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{7/2}}{(d+ex)^3}dx$$

$$\Big\downarrow \text{1127}$$

$$\int (ae+cdx)^3\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}dx$$

$$\Big\downarrow \text{1134}$$

$$\frac{(ae+cdx)^2\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{5e}-\frac{7\left(cd^2-ae^2\right)\int (ae+cdx)^2\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}dx}{10e}$$

$$\Big\downarrow \text{1134}$$

$$\frac{(ae+cdx)^2\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{5e}-\frac{7\left(cd^2-ae^2\right)\left(\frac{(ae+cdx)\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{4e}-\frac{5\left(cd^2-ae^2\right)\int (ae+cdx)\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}dx}{8e}\right)}{10e}$$

$$\Big\downarrow \text{1160}$$

$$\frac{(ae+cdx)^2\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{5e}-\frac{7\left(cd^2-ae^2\right)\left(\frac{(ae+cdx)\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{4e}-\frac{5\left(cd^2-ae^2\right)\left(\frac{\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{3e}-\frac{\left(cd^2-ae^2\right)\int \sqrt{cdex^2+\left(cd^2+ae^2\right)x+}}{2e}\right.}{8e}\right.}{10e}$$

$$\Big\downarrow \text{1087}$$

$$\frac{(ae+cdx)^2\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{5e}-$$

$$\frac{7\left(cd^2-ae^2\right)\left(\frac{(ae+cdx)\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{4e}-\frac{5\left(cd^2-ae^2\right)\left(\frac{\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{3e}-\frac{\left(cd^2-ae^2\right)\left(\frac{\left(ae^2+cd^2+2cdex\right)\sqrt{x\left(a\right.}}{4cd}\right.}{}\right.}{8e}\right.}{10e}$$

$\downarrow$ 1092

$$\frac{(ae+cdx)^2\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{5e}-$$

$$\frac{7\left(cd^2-ae^2\right)\left(\frac{(ae+cdx)\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{4e}-\frac{5\left(cd^2-ae^2\right)\left(\frac{\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{3e}-\frac{\left(cd^2-ae^2\right)\left(\frac{\left(ae^2+cd^2+2cdex\right)\sqrt{x\left(a\right.}}{4cd}\right.}{}\right.}{}\right.}{10e}$$

$\downarrow$ 219

$$\frac{(ae+cdx)^2\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{5e}-\frac{7\left(cd^2-ae^2\right)\left(\frac{(ae+cdx)\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{4e}-\frac{5\left(cd^2-ae^2\right)\left(\frac{\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{3e}-\frac{\left(cd^2-ae^2\right)\left(\frac{\left(ae^2+cd^2+2cdex\right)\sqrt{x\left(c\dots\right.}}{4cd\dots}\dots\right)}{\dots}\right)}{\dots}\right)}{10e}$$

input
```
Int[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(7/2)/(d + e*x)^3,x]
```

output
```
((a*e + c*d*x)^2*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2))/(5*e) - (7
*(c*d^2 - a*e^2)*(((a*e + c*d*x)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(
3/2))/(4*e) - (5*(c*d^2 - a*e^2)*((a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^
(3/2)/(3*e) - ((c*d^2 - a*e^2)*(((c*d^2 + a*e^2 + 2*c*d*e*x)*Sqrt[a*d*e +
(c*d^2 + a*e^2)*x + c*d*e*x^2])/(4*c*d*e) - ((c*d^2 - a*e^2)^2*ArcTanh[(c*
d^2 + a*e^2 + 2*c*d*e*x)/(2*Sqrt[c]*Sqrt[d]*Sqrt[e]*Sqrt[a*d*e + (c*d^2 +
a*e^2)*x + c*d*e*x^2])])/(8*c^(3/2)*d^(3/2)*e^(3/2))))/(2*e)))/(8*e)))/(10
*e)
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1087
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^p/(2*c*(2*p + 1))), x] - Simp[p*((b^2 - 4*a*c)/(2*c*(2*
p + 1)))   Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] &&
GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[3*p])
```

rule 1092
```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[I
nt[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a
, b, c}, x]
```

rule 1127
```
Int[((d_) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Sy
mbol] :> Int[(a + b*x + c*x^2)^(m + p)/(a/d + c*(x/e))^m, x] /; FreeQ[{a, b
, c, d, e}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[m] && RationalQ
[p] && (LtQ[0, -m, p] || LtQ[p, -m, 0]) && NeQ[m, 2] && NeQ[m, -1]
```

rule 1134
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[e*(d + e*x)^(m - 1)*((a + b*x + c*x^2)^(p + 1)/(c*(m + 2*p +
 1))), x] + Simp[(m + p)*((2*c*d - b*e)/(c*(m + 2*p + 1)))   Int[(d + e*x)^
(m - 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[
c*d^2 - b*d*e + a*e^2, 0] && GtQ[m, 1] && NeQ[m + 2*p + 1, 0] && IntegerQ[2
*p]
```

rule 1160
```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[e*((a + b*x + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Simp[(2*c*d - b
*e)/(2*c)   Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
 && NeQ[p, -1]
```

Maple [A] (verified)

Time = 2.16 (sec) , antiderivative size = 586, normalized size of antiderivative = 1.58

method	result
	$4dec\left(\frac{2\left(dec\left(x+\frac{d}{e}\right)^2+\left(a\,e^2-c\,d^2\right)\left(x+\frac{d}{e}\right)\right)^{\frac{9}{2}}}{5\left(a\,e^2-c\,d^2\right)\left(x+\frac{d}{e}\right)^2}-\frac{14dec\left(\frac{\left(dec\left(x+\frac{d}{e}\right)^2+\left(a\,e^2-c\,d^2\right)\left(x+\frac{d}{e}\right)\right)^{\frac{7}{2}}}{7}+\right.}{}\right.$ $\left(a\,\right.$

input
```
int((a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(7/2)/(e*x+d)^3,x,method=_RETURNVERB
OSE)
```

output
```
1/e^3*(2/3/(a*e^2-c*d^2)/(x+d/e)^3*(d*e*c*(x+d/e)^2+(a*e^2-c*d^2)*(x+d/e))
^(9/2)-4*d*e*c/(a*e^2-c*d^2)*(2/5/(a*e^2-c*d^2)/(x+d/e)^2*(d*e*c*(x+d/e)^2
+(a*e^2-c*d^2)*(x+d/e))^(9/2)-14/5*d*e*c/(a*e^2-c*d^2)*(1/7*(d*e*c*(x+d/e)
^2+(a*e^2-c*d^2)*(x+d/e))^(7/2)+1/2*(a*e^2-c*d^2)*(1/12*(2*d*e*c*(x+d/e)+a
*e^2-c*d^2)/d/e/c*(d*e*c*(x+d/e)^2+(a*e^2-c*d^2)*(x+d/e))^(5/2)-5/24*(a*e^
2-c*d^2)^2/d/e/c*(1/8*(2*d*e*c*(x+d/e)+a*e^2-c*d^2)/d/e/c*(d*e*c*(x+d/e)^2
+(a*e^2-c*d^2)*(x+d/e))^(3/2)-3/16*(a*e^2-c*d^2)^2/d/e/c*(1/4*(2*d*e*c*(x+
d/e)+a*e^2-c*d^2)/d/e/c*(d*e*c*(x+d/e)^2+(a*e^2-c*d^2)*(x+d/e))^(1/2)-1/8*
(a*e^2-c*d^2)^2/d/e/c*ln((1/2*a*e^2-1/2*c*d^2+d*e*c*(x+d/e))/(d*e*c)^(1/2)
+(d*e*c*(x+d/e)^2+(a*e^2-c*d^2)*(x+d/e))^(1/2))/(d*e*c)^(1/2))))))
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 844, normalized size of antiderivative = 2.27

$$\int \frac{(ade + (cd^2 + ae^2)\,x + cdex^2)^{7/2}}{(d+ex)^3}\,dx = \text{Too large to display}$$

input
```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(7/2)/(e*x+d)^3,x, algorithm="
fricas")
```

output
```
[-1/7680*(105*(c^5*d^10 - 5*a*c^4*d^8*e^2 + 10*a^2*c^3*d^6*e^4 - 10*a^3*c^
2*d^4*e^6 + 5*a^4*c*d^2*e^8 - a^5*e^10)*sqrt(c*d*e)*log(8*c^2*d^2*e^2*x^2
+ c^2*d^4 + 6*a*c*d^2*e^2 + a^2*e^4 - 4*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 +
a*e^2)*x)*(2*c*d*e*x + c*d^2 + a*e^2)*sqrt(c*d*e) + 8*(c^2*d^3*e + a*c*d*e
^3)*x) - 4*(384*c^5*d^5*e^5*x^4 - 105*c^5*d^9*e + 490*a*c^4*d^7*e^3 - 896*
a^2*c^3*d^5*e^5 + 790*a^3*c^2*d^3*e^7 + 105*a^4*c*d*e^9 + 48*(c^5*d^6*e^4
+ 31*a*c^4*d^4*e^6)*x^3 - 8*(7*c^5*d^7*e^3 - 32*a*c^4*d^5*e^5 - 263*a^2*c^
3*d^3*e^7)*x^2 + 2*(35*c^5*d^8*e^2 - 161*a*c^4*d^6*e^4 + 289*a^2*c^3*d^4*e
^6 + 605*a^3*c^2*d^2*e^8)*x)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x))/
(c^2*d^2*e^5), -1/3840*(105*(c^5*d^10 - 5*a*c^4*d^8*e^2 + 10*a^2*c^3*d^6*e
^4 - 10*a^3*c^2*d^4*e^6 + 5*a^4*c*d^2*e^8 - a^5*e^10)*sqrt(-c*d*e)*arctan(
1/2*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(2*c*d*e*x + c*d^2 + a*e^2
)*sqrt(-c*d*e)/(c^2*d^2*e^2*x^2 + a*c*d^2*e^2 + (c^2*d^3*e + a*c*d*e^3)*x)
) - 2*(384*c^5*d^5*e^5*x^4 - 105*c^5*d^9*e + 490*a*c^4*d^7*e^3 - 896*a^2*c
^3*d^5*e^5 + 790*a^3*c^2*d^3*e^7 + 105*a^4*c*d*e^9 + 48*(c^5*d^6*e^4 + 31*
a*c^4*d^4*e^6)*x^3 - 8*(7*c^5*d^7*e^3 - 32*a*c^4*d^5*e^5 - 263*a^2*c^3*d^3
*e^7)*x^2 + 2*(35*c^5*d^8*e^2 - 161*a*c^4*d^6*e^4 + 289*a^2*c^3*d^4*e^6 +
605*a^3*c^2*d^2*e^8)*x)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x))/(c^2*
d^2*e^5)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(ade + (cd^2 + ae^2)\,x + cdex^2)^{7/2}}{(d + ex)^3}\,dx = \text{Timed out}$$

input
```
integrate((a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(7/2)/(e*x+d)**3,x)
```

output
```
Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(ade + (cd^2 + ae^2)x + cdex^2)^{7/2}}{(d+ex)^3}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(7/2)/(e*x+d)^3,x, algorithm="
maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

Giac [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 412, normalized size of antiderivative = 1.11

$$\int \frac{(ade + (cd^2 + ae^2)x + cdex^2)^{7/2}}{(d+ex)^3}\,dx = \frac{1}{1920}\sqrt{cdex^2 + cd^2x + ae^2x + ade}\left(2\left(4\left(6\left(8\,c^3d^3x + \frac{c^7d^8e^3 + }{c^{}}\right.\right.\right.\right. \\ - \frac{7\,(c^5d^{10} - 5\,ac^4d^8e^2 + 10\,a^2c^3d^6e^4 - 10\,a^3c^2d^4e^6 + 5\,a^4cd^2e^8 - a^5e^{10})\log\left(\left|-cd^2 - ae^2 - 2\sqrt{cde}\left(\sqrt{cdex}\right.\right.\right.}{256\sqrt{cde}cde^4}$$

input
```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(7/2)/(e*x+d)^3,x, algorithm="
giac")
```

output
```
1/1920*sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*x + a*d*e)*(2*(4*(6*(8*c^3*d^3*x +
 (c^7*d^8*e^3 + 31*a*c^6*d^6*e^5)/(c^4*d^4*e^4))*x - (7*c^7*d^9*e^2 - 32*a
*c^6*d^7*e^4 - 263*a^2*c^5*d^5*e^6)/(c^4*d^4*e^4))*x + (35*c^7*d^10*e - 16
1*a*c^6*d^8*e^3 + 289*a^2*c^5*d^6*e^5 + 605*a^3*c^4*d^4*e^7)/(c^4*d^4*e^4)
)*x - (105*c^7*d^11 - 490*a*c^6*d^9*e^2 + 896*a^2*c^5*d^7*e^4 - 790*a^3*c^
4*d^5*e^6 - 105*a^4*c^3*d^3*e^8)/(c^4*d^4*e^4)) - 7/256*(c^5*d^10 - 5*a*c^
4*d^8*e^2 + 10*a^2*c^3*d^6*e^4 - 10*a^3*c^2*d^4*e^6 + 5*a^4*c*d^2*e^8 - a^
5*e^10)*log(abs(-c*d^2 - a*e^2 - 2*sqrt(c*d*e)*(sqrt(c*d*e)*x - sqrt(c*d*e
*x^2 + c*d^2*x + a*e^2*x + a*d*e))))/(sqrt(c*d*e)*c*d*e^4)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(ade + (cd^2 + ae^2)x + cdex^2)^{7/2}}{(d + ex)^3} dx = \int \frac{(c\,d\,e\,x^2 + (c\,d^2 + a\,e^2)\,x + a\,d\,e)^{7/2}}{(d + e\,x)^3} dx$$

input
```
int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(7/2)/(d + e*x)^3,x)
```

output
```
int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(7/2)/(d + e*x)^3, x)
```

Reduce [B] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 790, normalized size of antiderivative = 2.13

$$\int \frac{(ade + (cd^2 + ae^2)x + cdex^2)^{7/2}}{(d + ex)^3} dx = \text{Too large to display}$$

input
```
int((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(7/2)/(e*x+d)^3,x)
```

output

```
(105*sqrt(d + e*x)*sqrt(a*e + c*d*x)*a**4*c*d*e**9 + 790*sqrt(d + e*x)*sqr
t(a*e + c*d*x)*a**3*c**2*d**3*e**7 + 1210*sqrt(d + e*x)*sqrt(a*e + c*d*x)*
a**3*c**2*d**2*e**8*x - 896*sqrt(d + e*x)*sqrt(a*e + c*d*x)*a**2*c**3*d**5
*e**5 + 578*sqrt(d + e*x)*sqrt(a*e + c*d*x)*a**2*c**3*d**4*e**6*x + 2104*s
qrt(d + e*x)*sqrt(a*e + c*d*x)*a**2*c**3*d**3*e**7*x**2 + 490*sqrt(d + e*x
)*sqrt(a*e + c*d*x)*a*c**4*d**7*e**3 - 322*sqrt(d + e*x)*sqrt(a*e + c*d*x)
*a*c**4*d**6*e**4*x + 256*sqrt(d + e*x)*sqrt(a*e + c*d*x)*a*c**4*d**5*e**5
*x**2 + 1488*sqrt(d + e*x)*sqrt(a*e + c*d*x)*a*c**4*d**4*e**6*x**3 - 105*s
qrt(d + e*x)*sqrt(a*e + c*d*x)*c**5*d**9*e + 70*sqrt(d + e*x)*sqrt(a*e + c
*d*x)*c**5*d**8*e**2*x - 56*sqrt(d + e*x)*sqrt(a*e + c*d*x)*c**5*d**7*e**3
*x**2 + 48*sqrt(d + e*x)*sqrt(a*e + c*d*x)*c**5*d**6*e**4*x**3 + 384*sqrt(
d + e*x)*sqrt(a*e + c*d*x)*c**5*d**5*e**5*x**4 - 105*sqrt(e)*sqrt(d)*sqrt(
c)*log((sqrt(e)*sqrt(a*e + c*d*x) + sqrt(d)*sqrt(c)*sqrt(d + e*x))/sqrt(a*
e**2 - c*d**2))*a**5*e**10 + 525*sqrt(e)*sqrt(d)*sqrt(c)*log((sqrt(e)*sqrt
(a*e + c*d*x) + sqrt(d)*sqrt(c)*sqrt(d + e*x))/sqrt(a*e**2 - c*d**2))*a**4
*c*d**2*e**8 - 1050*sqrt(e)*sqrt(d)*sqrt(c)*log((sqrt(e)*sqrt(a*e + c*d*x)
 + sqrt(d)*sqrt(c)*sqrt(d + e*x))/sqrt(a*e**2 - c*d**2))*a**3*c**2*d**4*e*
*6 + 1050*sqrt(e)*sqrt(d)*sqrt(c)*log((sqrt(e)*sqrt(a*e + c*d*x) + sqrt(d)
*sqrt(c)*sqrt(d + e*x))/sqrt(a*e**2 - c*d**2))*a**2*c**3*d**6*e**4 - 525*s
qrt(e)*sqrt(d)*sqrt(c)*log((sqrt(e)*sqrt(a*e + c*d*x) + sqrt(d)*sqrt(c)...
```

3.239 $\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{7/2}}{(d+ex)^4}\,dx$

Optimal result

Integrand size = 37, antiderivative size = 288

$$\begin{aligned}\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{7/2}}{(d+ex)^4}\,dx = &-\frac{35\left(cd^2-ae^2\right)^3\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{64e^4}\\ &+\frac{35\left(cd^2-ae^2\right)^2\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{96e^3(d+ex)}\\ &+\frac{7\left(a-\frac{cd^2}{e^2}\right)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{24(d+ex)^2}+\frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{7/2}}{4e(d+ex)^3}\\ &+\frac{35\left(cd^2-ae^2\right)^4\operatorname{arctanh}\left(\frac{\sqrt{e}\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{\sqrt{c}\sqrt{d}(d+ex)}\right)}{64\sqrt{c}\sqrt{d}e^{9/2}}\end{aligned}$$

output
```
-35/64*(-a*e^2+c*d^2)^3*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/e^4+35/96*
(-a*e^2+c*d^2)^2*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/e^3/(e*x+d)+7/24*
(a-c*d^2/e^2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(e*x+d)^2+1/4*(a*d*e
+(a*e^2+c*d^2)*x+c*d*e*x^2)^(7/2)/e/(e*x+d)^3+35/64*(-a*e^2+c*d^2)^4*arcta
nh(e^(1/2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/c^(1/2)/d^(1/2)/(e*x+d)
)/c^(1/2)/d^(1/2)/e^(9/2)
```

Mathematica [A] (verified)

Time = 0.67 (sec) , antiderivative size = 225, normalized size of antiderivative = 0.78

$$\int \frac{(ade+(cd^2+ae^2)\,x+cdex^2)^{7/2}}{(d+ex)^4}\,dx = \frac{\sqrt{e}(ae+cdx)(d+ex)\,(279a^3e^6+a^2cde^4(-511d+326ex)+ac^2}{}$$

input
```
Integrate[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(7/2)/(d + e*x)^4,x]
```

output
```
(Sqrt[e]*(a*e + c*d*x)*(d + e*x)*(279*a^3*e^6 + a^2*c*d*e^4*(-511*d + 326*
e*x) + a*c^2*d^2*e^2*(385*d^2 - 252*d*e*x + 200*e^2*x^2) + c^3*d^3*(-105*d
^3 + 70*d^2*e*x - 56*d*e^2*x^2 + 48*e^3*x^3)) + (105*(c*d^2 - a*e^2)^4*Sqr
t[a*e + c*d*x]*Sqrt[d + e*x]*ArcTanh[(Sqrt[c]*Sqrt[d]*Sqrt[d + e*x])/(Sqrt
[e]*Sqrt[a*e + c*d*x])])/(Sqrt[c]*Sqrt[d]))/(192*e^(9/2)*Sqrt[(a*e + c*d*x
)*(d + e*x)])
```

Rubi [A] (verified)

Time = 0.80 (sec) , antiderivative size = 328, normalized size of antiderivative = 1.14, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.189$, Rules used = {1130, 1127, 1134, 1160, 1087, 1092, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{7/2}}{(d+ex)^4}dx$$

$$\Big\downarrow \text{1130}$$

$$\frac{2\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{7/2}}{e(d+ex)^3}-\frac{7cd\int \frac{\left(cdex^2+(cd^2+ae^2)x+ade\right)^{5/2}}{(d+ex)^2}dx}{e}$$

$$\Big\downarrow \text{1127}$$

$$\frac{2\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{7/2}}{e(d+ex)^3}-\frac{7cd\int (ae+cdx)^2\sqrt{cdex^2+(cd^2+ae^2)\,x+ade}dx}{e}$$

$$\Big\downarrow \text{1134}$$

$$\frac{2\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{7/2}}{e(d+ex)^3}-\frac{7cd\left(\frac{(ae+cdx)\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{4e}-\frac{5\left(cd^2-ae^2\right)\int(ae+cdx)\sqrt{cdex^2+(cd^2+ae^2)x+ade}dx}{8e}\right)}{e}$$

$$\Big\downarrow \text{1160}$$

$$\frac{2\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{7/2}}{e(d+ex)^3}-\frac{7cd\left(\frac{(ae+cdx)\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{4e}-\frac{5\left(cd^2-ae^2\right)\left(\frac{\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{3e}-\frac{\left(cd^2-ae^2\right)\int\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}dx}{2e}\right)}{8e}\right)}{e}$$

$$\Big\downarrow \text{1087}$$

$$\frac{2\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{7/2}}{e(d+ex)^3}-\frac{7cd\left(\frac{(ae+cdx)\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{4e}-\frac{5\left(cd^2-ae^2\right)\left(\frac{\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{3e}-\frac{\left(cd^2-ae^2\right)\left(\frac{\left(ae^2+cd^2+2cdex\right)\sqrt{x\left(ae^2+cd^2\right)+ade}}{4cde}\right.}{}\right)}{8e}\right)}{e}$$

$$\Big\downarrow \text{1092}$$

$$\frac{2\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{7/2}}{e(d+ex)^3}-$$

$$\frac{7cd\left(\frac{(ae+cdx)\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{4e}-\frac{5\left(cd^2-ae^2\right)\left(\frac{\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{3e}-\frac{\left(cd^2-ae^2\right)\left(\frac{\left(ae^2+cd^2+2cdex\right)\sqrt{x\left(ae^2+cd^2\right)+ade \cdots}}{4cde}\cdots\right.}{\cdots}\right.}{8e \cdots}\right.}{e \cdots}$$

$\downarrow$ 219

$$\frac{2\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{7/2}}{e(d+ex)^3}-$$

$$\frac{7cd\left(\frac{(ae+cdx)\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{4e}-\frac{5\left(cd^2-ae^2\right)\left(\frac{\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{3e}-\frac{\left(cd^2-ae^2\right)\left(\frac{\left(ae^2+cd^2+2cdex\right)\sqrt{x\left(ae^2+cd^2\right)+ade \cdots}}{4cde}\cdots\right.}{\cdots}\right.}{8e \cdots}\right.}{e \cdots}$$

input
```
Int[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(7/2)/(d + e*x)^4,x]
```

output
```
(2*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(7/2))/(e*(d + e*x)^3) - (7*c*d
*(((a*e + c*d*x)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2))/(4*e) - (5
*(c*d^2 - a*e^2)*((a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2)/(3*e) - ((
c*d^2 - a*e^2)*(((c*d^2 + a*e^2 + 2*c*d*e*x)*Sqrt[a*d*e + (c*d^2 + a*e^2)*
x + c*d*e*x^2])/(4*c*d*e) - ((c*d^2 - a*e^2)^2*ArcTanh[(c*d^2 + a*e^2 + 2*
c*d*e*x)/(2*Sqrt[c]*Sqrt[d]*Sqrt[e]*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e
*x^2])])/(8*c^(3/2)*d^(3/2)*e^(3/2))))/(2*e)))/(8*e)))/e
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1087
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^p/(2*c*(2*p + 1))), x] - Simp[p*((b^2 - 4*a*c)/(2*c*(2*
p + 1)))   Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] &&
GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[3*p])
```

rule 1092
```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[I
nt[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a
, b, c}, x]
```

rule 1127
```
Int[((d_) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Sy
mbol] :> Int[(a + b*x + c*x^2)^(m + p)/(a/d + c*(x/e))^m, x] /; FreeQ[{a, b
, c, d, e}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[m] && RationalQ
[p] && (LtQ[0, -m, p] || LtQ[p, -m, 0]) && NeQ[m, 2] && NeQ[m, -1]
```

rule 1130
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(d + e*x)^(m + 1)*((a + b*x + c*x^2)^p/(e*(m + p + 1))), x]
- Simp[c*(p/(e^2*(m + p + 1)))   Int[(d + e*x)^(m + 2)*(a + b*x + c*x^2)^(p
 - 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0]
 && GtQ[p, 0] && (LtQ[m, -2] || EqQ[m + 2*p + 1, 0]) && NeQ[m + p + 1, 0] &
& IntegerQ[2*p]
```

rule 1134
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[e*(d + e*x)^(m - 1)*((a + b*x + c*x^2)^(p + 1)/(c*(m + 2*p +
 1))), x] + Simp[(m + p)*((2*c*d - b*e)/(c*(m + 2*p + 1)))   Int[(d + e*x)^
(m - 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[
c*d^2 - b*d*e + a*e^2, 0] && GtQ[m, 1] && NeQ[m + 2*p + 1, 0] && IntegerQ[2
*p]
```

rule 1160
```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[e*((a + b*x + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Simp[(2*c*d - b
*e)/(2*c)   Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
 && NeQ[p, -1]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 666 vs. $2(256) = 512$.

Time = 2.49 (sec) , antiderivative size = 667, normalized size of antiderivative = 2.32

method	result
	$4dec\left(\frac{2\left(dec\left(x+\frac{d}{e}\right)^{2}+\left(a\,e^{2}-c\,d^{2}\right)\left(x+\frac{d}{e}\right)\right)^{\frac{9}{2}}}{5\left(a\,e^{2}-c\,d^{2}\right)\left(x+\frac{d}{e}\right)^{2}}-\right.$ 14

input
```
int((a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(7/2)/(e*x+d)^4,x,method=_RETURNVERB
OSE)
```

output
```
1/e^4*(2/(a*e^2-c*d^2)/(x+d/e)^4*(d*e*c*(x+d/e)^2+(a*e^2-c*d^2)*(x+d/e))^(
9/2)-10*d*e*c/(a*e^2-c*d^2)*(2/3/(a*e^2-c*d^2)/(x+d/e)^3*(d*e*c*(x+d/e)^2+
(a*e^2-c*d^2)*(x+d/e))^(9/2)-4*d*e*c/(a*e^2-c*d^2)*(2/5/(a*e^2-c*d^2)/(x+d
/e)^2*(d*e*c*(x+d/e)^2+(a*e^2-c*d^2)*(x+d/e))^(9/2)-14/5*d*e*c/(a*e^2-c*d^
2)*(1/7*(d*e*c*(x+d/e)^2+(a*e^2-c*d^2)*(x+d/e))^(7/2)+1/2*(a*e^2-c*d^2)*(1
/12*(2*d*e*c*(x+d/e)+a*e^2-c*d^2)/d/e/c*(d*e*c*(x+d/e)^2+(a*e^2-c*d^2)*(x+
d/e))^(5/2)-5/24*(a*e^2-c*d^2)^2/d/e/c*(1/8*(2*d*e*c*(x+d/e)+a*e^2-c*d^2)/
d/e/c*(d*e*c*(x+d/e)^2+(a*e^2-c*d^2)*(x+d/e))^(3/2)-3/16*(a*e^2-c*d^2)^2/d
/e/c*(1/4*(2*d*e*c*(x+d/e)+a*e^2-c*d^2)/d/e/c*(d*e*c*(x+d/e)^2+(a*e^2-c*d^
2)*(x+d/e))^(1/2)-1/8*(a*e^2-c*d^2)^2/d/e/c*ln((1/2*a*e^2-1/2*c*d^2+d*e*c*
(x+d/e))/(d*e*c)^(1/2)+(d*e*c*(x+d/e)^2+(a*e^2-c*d^2)*(x+d/e))^(1/2))/(d*e
*c)^(1/2)))))))
```

Fricas [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 674, normalized size of antiderivative = 2.34

$$\int \frac{(ade+(cd^2+ae^2)x+cdex^2)^{7/2}}{(d+ex)^4}\,dx = \left[\frac{105\,(c^4d^8-4\,ac^3d^6e^2+6\,a^2c^2d^4e^4-4\,a^3cd^2e^6+a^4e^8)\sqrt{cde}\log\right.$$

$$-\frac{105\,(c^4d^8-4\,ac^3d^6e^2+6\,a^2c^2d^4e^4-4\,a^3cd^2e^6+a^4e^8)\sqrt{-cde}\arctan\left(\frac{\sqrt{cdex^2+ade+(cd^2+ae^2)x}\,(2\,cdex+cd^2+ae^2)}{2\,(c^2d^2e^2x^2+acd^2e^2+(c^2d^3e+acde^3)x)}\right.}{}$$

input
```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(7/2)/(e*x+d)^4,x, algorithm="
fricas")
```

output
```
[1/768*(105*(c^4*d^8 - 4*a*c^3*d^6*e^2 + 6*a^2*c^2*d^4*e^4 - 4*a^3*c*d^2*e
^6 + a^4*e^8)*sqrt(c*d*e)*log(8*c^2*d^2*e^2*x^2 + c^2*d^4 + 6*a*c*d^2*e^2
+ a^2*e^4 + 4*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(2*c*d*e*x + c*d
^2 + a*e^2)*sqrt(c*d*e) + 8*(c^2*d^3*e + a*c*d*e^3)*x) + 4*(48*c^4*d^4*e^4
*x^3 - 105*c^4*d^7*e + 385*a*c^3*d^5*e^3 - 511*a^2*c^2*d^3*e^5 + 279*a^3*c
*d*e^7 - 8*(7*c^4*d^5*e^3 - 25*a*c^3*d^3*e^5)*x^2 + 2*(35*c^4*d^6*e^2 - 12
6*a*c^3*d^4*e^4 + 163*a^2*c^2*d^2*e^6)*x)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2
+ a*e^2)*x))/(c*d*e^5), -1/384*(105*(c^4*d^8 - 4*a*c^3*d^6*e^2 + 6*a^2*c^2
*d^4*e^4 - 4*a^3*c*d^2*e^6 + a^4*e^8)*sqrt(-c*d*e)*arctan(1/2*sqrt(c*d*e*x
^2 + a*d*e + (c*d^2 + a*e^2)*x)*(2*c*d*e*x + c*d^2 + a*e^2)*sqrt(-c*d*e)/(
c^2*d^2*e^2*x^2 + a*c*d^2*e^2 + (c^2*d^3*e + a*c*d*e^3)*x)) - 2*(48*c^4*d^
4*e^4*x^3 - 105*c^4*d^7*e + 385*a*c^3*d^5*e^3 - 511*a^2*c^2*d^3*e^5 + 279*
a^3*c*d*e^7 - 8*(7*c^4*d^5*e^3 - 25*a*c^3*d^3*e^5)*x^2 + 2*(35*c^4*d^6*e^2
 - 126*a*c^3*d^4*e^4 + 163*a^2*c^2*d^2*e^6)*x)*sqrt(c*d*e*x^2 + a*d*e + (c
*d^2 + a*e^2)*x))/(c*d*e^5)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(ade + (cd^2 + ae^2)\,x + cdex^2)^{7/2}}{(d+ex)^4}\,dx = \text{Timed out}$$

input
```
integrate((a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(7/2)/(e*x+d)**4,x)
```

output
```
Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(ade + (cd^2 + ae^2)\,x + cdex^2)^{7/2}}{(d+ex)^4}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(7/2)/(e*x+d)^4,x, algorithm="
maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e*(a*e^2-c*d^2)>0)', see `assume
?` for mor
```

Giac [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 315, normalized size of antiderivative = 1.09

$$\int \frac{(ade+(cd^2+ae^2)x+cdex^2)^{7/2}}{(d+ex)^4}\,dx = \frac{1}{192}\sqrt{cdex^2+cd^2x+ae^2x+ade}\left(2\left(4\left(\frac{6c^3d^3x}{e}-\frac{7c^6d^7e^2-2}{c^3d^3e}\right.\right.\right.$$
$$-\frac{35\,(c^4d^8-4\,ac^3d^6e^2+6\,a^2c^2d^4e^4-4\,a^3cd^2e^6+a^4e^8)\log\left(\left|-cd^2-ae^2-2\sqrt{cde}\left(\sqrt{cde}x-\sqrt{cdex^2+cd}\right.\right.\right.}{128\sqrt{cde}e^4}$$

input
```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(7/2)/(e*x+d)^4,x, algorithm="
giac")
```

output
```
1/192*sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*x + a*d*e)*(2*(4*(6*c^3*d^3*x/e - (
7*c^6*d^7*e^2 - 25*a*c^5*d^5*e^4)/(c^3*d^3*e^4))*x + (35*c^6*d^8*e - 126*a
*c^5*d^6*e^3 + 163*a^2*c^4*d^4*e^5)/(c^3*d^3*e^4))*x - (105*c^6*d^9 - 385*
a*c^5*d^7*e^2 + 511*a^2*c^4*d^5*e^4 - 279*a^3*c^3*d^3*e^6)/(c^3*d^3*e^4))
- 35/128*(c^4*d^8 - 4*a*c^3*d^6*e^2 + 6*a^2*c^2*d^4*e^4 - 4*a^3*c*d^2*e^6
+ a^4*e^8)*log(abs(-c*d^2 - a*e^2 - 2*sqrt(c*d*e)*(sqrt(c*d*e)*x - sqrt(c*
d*e*x^2 + c*d^2*x + a*e^2*x + a*d*e))))/(sqrt(c*d*e)*e^4)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(ade+(cd^2+ae^2)x+cdex^2)^{7/2}}{(d+ex)^4}\,dx = \int \frac{\left(c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e\right)^{7/2}}{\left(d+e\,x\right)^4}\,dx$$

input
```
int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(7/2)/(d + e*x)^4,x)
```

output
```
int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(7/2)/(d + e*x)^4, x)
```

Reduce [B] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 579, normalized size of antiderivative = 2.01

$$\int \frac{(ade + (cd^2 + ae^2)x + cdex^2)^{7/2}}{(d+ex)^4}\,dx = \frac{279\sqrt{ex+d}\,\sqrt{cdx+ae}\,a^3cd\,e^7 - 511\sqrt{ex+d}\,\sqrt{cdx+ae}\,a^2c^2d^3e}{}$$

input
```
int((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(7/2)/(e*x+d)^4,x)
```

output
```
(279*sqrt(d + e*x)*sqrt(a*e + c*d*x)*a**3*c*d*e**7 - 511*sqrt(d + e*x)*sqr
t(a*e + c*d*x)*a**2*c**2*d**3*e**5 + 326*sqrt(d + e*x)*sqrt(a*e + c*d*x)*a
**2*c**2*d**2*e**6*x + 385*sqrt(d + e*x)*sqrt(a*e + c*d*x)*a*c**3*d**5*e**
3 - 252*sqrt(d + e*x)*sqrt(a*e + c*d*x)*a*c**3*d**4*e**4*x + 200*sqrt(d +
e*x)*sqrt(a*e + c*d*x)*a*c**3*d**3*e**5*x**2 - 105*sqrt(d + e*x)*sqrt(a*e
+ c*d*x)*c**4*d**7*e + 70*sqrt(d + e*x)*sqrt(a*e + c*d*x)*c**4*d**6*e**2*x
 - 56*sqrt(d + e*x)*sqrt(a*e + c*d*x)*c**4*d**5*e**3*x**2 + 48*sqrt(d + e*
x)*sqrt(a*e + c*d*x)*c**4*d**4*e**4*x**3 + 105*sqrt(e)*sqrt(d)*sqrt(c)*log
((sqrt(e)*sqrt(a*e + c*d*x) + sqrt(d)*sqrt(c)*sqrt(d + e*x))/sqrt(a*e**2 -
 c*d**2))*a**4*e**8 - 420*sqrt(e)*sqrt(d)*sqrt(c)*log((sqrt(e)*sqrt(a*e +
c*d*x) + sqrt(d)*sqrt(c)*sqrt(d + e*x))/sqrt(a*e**2 - c*d**2))*a**3*c*d**2
*e**6 + 630*sqrt(e)*sqrt(d)*sqrt(c)*log((sqrt(e)*sqrt(a*e + c*d*x) + sqrt(
d)*sqrt(c)*sqrt(d + e*x))/sqrt(a*e**2 - c*d**2))*a**2*c**2*d**4*e**4 - 420
*sqrt(e)*sqrt(d)*sqrt(c)*log((sqrt(e)*sqrt(a*e + c*d*x) + sqrt(d)*sqrt(c)*
sqrt(d + e*x))/sqrt(a*e**2 - c*d**2))*a*c**3*d**6*e**2 + 105*sqrt(e)*sqrt(
d)*sqrt(c)*log((sqrt(e)*sqrt(a*e + c*d*x) + sqrt(d)*sqrt(c)*sqrt(d + e*x))
/sqrt(a*e**2 - c*d**2))*c**4*d**8)/(192*c*d*e**5)
```

3.240 $\int \frac{(ade+(cd^2+ae^2)x+cdex^2)^{7/2}}{(d+ex)^5} dx$

Optimal result

Integrand size = 37, antiderivative size = 281

$$\int \frac{(ade+(cd^2+ae^2)x+cdex^2)^{7/2}}{(d+ex)^5} dx = \frac{35cd(cd^2-ae^2)^2\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{8e^4} + \frac{35cd\left(a-\frac{cd^2}{e^2}\right)(ade+(cd^2+ae^2)x+cdex^2)^{3/2}}{12e(d+ex)} + \frac{7cd(ade+(cd^2+ae^2)x+cdex^2)^{5/2}}{3e^2(d+ex)^2} - \frac{2(ade+(cd^2+ae^2)x+cdex^2)^{7/2}}{e(d+ex)^4} - \frac{35\sqrt{c}\sqrt{d}(cd^2-ae^2)^3\operatorname{arctanh}\left(\frac{\sqrt{e}\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{\sqrt{c}\sqrt{d}(d+ex)}\right)}{8e^{9/2}}$$

output
```
35/8*c*d*(-a*e^2+c*d^2)^2*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/e^4+35/1
2*c*d*(a-c*d^2/e^2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/e/(e*x+d)+7/3*
c*d*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/e^2/(e*x+d)^2-2*(a*d*e+(a*e^2+
c*d^2)*x+c*d*e*x^2)^(7/2)/e/(e*x+d)^4-35/8*c^(1/2)*d^(1/2)*(-a*e^2+c*d^2)^
3*arctanh(e^(1/2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/c^(1/2)/d^(1/2)/
(e*x+d))/e^(9/2)
```

Mathematica [A] (verified)

Time = 0.55 (sec) , antiderivative size = 221, normalized size of antiderivative = 0.79

$$\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{7/2}}{(d+ex)^5}\,dx=\frac{\sqrt{(ae+cdx)(d+ex)}\left(\frac{\sqrt{e}\left(-48a^3e^6+3a^2cde^4(77d+29ex)-2ac^2d^2e^2\left(140d^2+49\right.\right.}{d+}\right.}{}$$

input
```
Integrate[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(7/2)/(d + e*x)^5,x]
```

output
```
(Sqrt[(a*e + c*d*x)*(d + e*x)]*((Sqrt[e]*(-48*a^3*e^6 + 3*a^2*c*d*e^4*(77*
d + 29*e*x) - 2*a*c^2*d^2*e^2*(140*d^2 + 49*d*e*x - 19*e^2*x^2) + c^3*d^3*
(105*d^3 + 35*d^2*e*x - 14*d*e^2*x^2 + 8*e^3*x^3)))/(d + e*x) - (105*Sqrt[
c]*Sqrt[d]*(c*d^2 - a*e^2)^3*ArcTanh[(Sqrt[c]*Sqrt[d]*Sqrt[d + e*x])/(Sqrt
[e]*Sqrt[a*e + c*d*x])])/(Sqrt[a*e + c*d*x]*Sqrt[d + e*x])))/(24*e^(9/2))
```

Rubi [A] (verified)

Time = 1.56 (sec) , antiderivative size = 348, normalized size of antiderivative = 1.24, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.216, Rules used = {1125, 2192, 27, 2192, 27, 1160, 1092, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{7/2}}{(d+ex)^5}\,dx$$

$$\Big\downarrow \text{1125}$$

$$\frac{2\left(cd^2-ae^2\right)^3\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{e^4(d+ex)}-\frac{\int \frac{-c^4d^4x^3e^7+c^3d^3\left(cd^2-4ae^2\right)x^2e^6-c^2d^2\left(c^2d^4-4ace^2d^2+6a^2e^4\right)xe^5+cd\left(cd^2-2ae^2\right)\left(c^2d^4-2ace^2d^2+2a^2e^4\right)e^4}{\sqrt{cdex^2+(cd^2+ae^2)x+ade}}dx}{e^8}$$

$$\Big\downarrow \text{2192}$$

$$\frac{2\left(cd^2-ae^2\right)^3\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{e^4(d+ex)} - \frac{\frac{\int \frac{c^4d^4\left(11cd^2-19ae^2\right)x^2e^7-2c^3d^3\left(3c^2d^4-14ace^2d^2+18a^2e^4\right)xe^6+6c^2d^2\left(cd^2-2ae^2\right)\left(c^2d^4-2ace^2d^2+2a^2e^4\right)e^5}{2\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}dx}{3cde} - \frac{1}{3}c^3d^3e^6x^2\sqrt{x\left(ae^2+cd^2\right)}}{e^8}$$

$\downarrow$ 27

$$\frac{2\left(cd^2-ae^2\right)^3\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{e^4(d+ex)} - \frac{\frac{\int \frac{c^4d^4\left(11cd^2-19ae^2\right)x^2e^7-2c^3d^3\left(3c^2d^4-14ace^2d^2+18a^2e^4\right)xe^6+6c^2d^2\left(cd^2-2ae^2\right)\left(c^2d^4-2ace^2d^2+2a^2e^4\right)e^5}{\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}dx}{6cde} - \frac{1}{3}c^3d^3e^6x^2\sqrt{x\left(ae^2+cd^2\right)}}{e^8}$$

$\downarrow$ 2192

$$\frac{2\left(cd^2-ae^2\right)^3\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{e^4(d+ex)} - \frac{\frac{\frac{\int \frac{c^3d^3e^6\left(2\left(12c^3d^6-59ac^2e^2d^4+91a^2ce^4d^2-48a^3e^6\right)-cde\left(57c^2d^4-136ace^2d^2+87a^2e^4\right)x\right)}{2\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}dx}{2cde}+\frac{1}{2}c^3d^3e^6x\left(11cd^2-19ae^2\right)\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{6cde} -}{e^8}$$

$\downarrow$ 27

$$\frac{2\left(cd^2-ae^2\right)^3\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{e^4(d+ex)} - \frac{\frac{\frac{1}{4}c^2d^2e^5\int \frac{2\left(12c^3d^6-59ac^2e^2d^4+91a^2ce^4d^2-48a^3e^6\right)-cde\left(57c^2d^4-136ace^2d^2+87a^2e^4\right)x}{\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}dx+\frac{1}{2}c^3d^3e^6x\left(11cd^2-19ae^2\right)\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{6cde} -}{e^8}$$

$\downarrow$ 1160

$$\frac{2\left(cd^2-ae^2\right)^3\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{e^4(d+ex)} - \frac{\frac{\frac{1}{4}c^2d^2e^5\left(\frac{105}{2}\left(cd^2-ae^2\right)^3\int \frac{1}{\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}dx-\left(87a^2e^4-136acd^2e^2+57c^2d^4\right)\sqrt{x(ae^2+cd^2)+ade+cdex^2}\right)+\frac{1}{2}c^3d^3e^6x\left(11cd^2-19ae^2\right.}{6cde}}{e^8}$$

$\downarrow$ 1092

$$\frac{2\left(cd^2-ae^2\right)^3\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{e^4(d+ex)}-$$

$$\frac{\frac{\frac{1}{4}c^2d^2e^5\left(105\left(cd^2-ae^2\right)^3\int\frac{1}{4cde-\frac{\left(cd^2+2cexd+ae^2\right)^2}{cdex^2+\left(cd^2+ae^2\right)x+ade}}d\frac{cd^2+2cexd+ae^2}{\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}-\left(87a^2e^4-136acd^2e^2+57c^2d^4\right)\sqrt{x(ae^2+cd^2)+ade+cdex^2}\right.}{6cde}}{e^8}$$

$$\Big\downarrow \text{219}$$

$$\frac{2\left(cd^2-ae^2\right)^3\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{e^4(d+ex)}-$$

$$\frac{\frac{\frac{1}{4}c^2d^2e^5\left(\frac{105\left(cd^2-ae^2\right)^3\text{arctanh}\left(\frac{ae^2+cd^2+2cdex}{2\sqrt{c}\sqrt{d}\sqrt{e}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}\right)}{2\sqrt{c}\sqrt{d}\sqrt{e}}-\left(87a^2e^4-136acd^2e^2+57c^2d^4\right)\sqrt{x(ae^2+cd^2)+ade+cdex^2}\right)+\frac{1}{2}c^3d^3e}{6cde}}{e^8}$$

input

```
Int[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(7/2)/(d + e*x)^5,x]
```

output

```
(2*(c*d^2 - a*e^2)^3*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(e^4*(d
+ e*x)) - (-1/3*(c^3*d^3*e^6*x^2*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^
2]) + ((c^3*d^3*e^6*(11*c*d^2 - 19*a*e^2)*x*Sqrt[a*d*e + (c*d^2 + a*e^2)*x
 + c*d*e*x^2])/2 + (c^2*d^2*e^5*(-((57*c^2*d^4 - 136*a*c*d^2*e^2 + 87*a^2*
e^4)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]) + (105*(c*d^2 - a*e^2)^3
*ArcTanh[(c*d^2 + a*e^2 + 2*c*d*e*x)/(2*Sqrt[c]*Sqrt[d]*Sqrt[e]*Sqrt[a*d*e
 + (c*d^2 + a*e^2)*x + c*d*e*x^2])])/(2*Sqrt[c]*Sqrt[d]*Sqrt[e])))/4)/(6*c
*d*e))/e^8
```

Defintions of rubi rules used

rule 27

```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1092
```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[I
nt[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a
, b, c}, x]
```

rule 1125
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_
Symbol] :> Simp[-2*e^(2*m + 3)*(Sqrt[a + b*x + c*x^2]/((-2*c*d + b*e)^(m +
2)*(d + e*x))), x] - Simp[e^(2*m + 2)   Int[(1/Sqrt[a + b*x + c*x^2])*Expan
dToSum[((-2*c*d + b*e)^(-m - 1) - ((-c)*d + b*e + c*e*x)^(-m - 1))/(d + e*x
), x], x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0]
 && ILtQ[m, 0] && EqQ[m + p, -3/2]
```

rule 1160
```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[e*((a + b*x + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Simp[(2*c*d - b
*e)/(2*c)   Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
 && NeQ[p, -1]
```

rule 2192
```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{q =
Expon[Pq, x], e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[e*x^(q - 1)*((a + b*x +
 c*x^2)^(p + 1)/(c*(q + 2*p + 1))), x] + Simp[1/(c*(q + 2*p + 1))   Int[(a
+ b*x + c*x^2)^p*ExpandToSum[c*(q + 2*p + 1)*Pq - a*e*(q - 1)*x^(q - 2) - b
*e*(q + p)*x^(q - 1) - c*e*(q + 2*p + 1)*x^q, x], x], x]] /; FreeQ[{a, b, c
, p}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] &&  !LeQ[p, -1]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 747 vs. $2(251) = 502$.

Time = 3.15 (sec) , antiderivative size = 748, normalized size of antiderivative = 2.66

method	result

input
```
int((a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(7/2)/(e*x+d)^5,x,method=_RETURNVERB
OSE)
```

output
```
1/e^5*(-2/(a*e^2-c*d^2)/(x+d/e)^5*(d*e*c*(x+d/e)^2+(a*e^2-c*d^2)*(x+d/e))^
(9/2)+8*d*e*c/(a*e^2-c*d^2)*(2/(a*e^2-c*d^2)/(x+d/e)^4*(d*e*c*(x+d/e)^2+(a
*e^2-c*d^2)*(x+d/e))^(9/2)-10*d*e*c/(a*e^2-c*d^2)*(2/3/(a*e^2-c*d^2)/(x+d/
e)^3*(d*e*c*(x+d/e)^2+(a*e^2-c*d^2)*(x+d/e))^(9/2)-4*d*e*c/(a*e^2-c*d^2)*(
2/5/(a*e^2-c*d^2)/(x+d/e)^2*(d*e*c*(x+d/e)^2+(a*e^2-c*d^2)*(x+d/e))^(9/2)-
14/5*d*e*c/(a*e^2-c*d^2)*(1/7*(d*e*c*(x+d/e)^2+(a*e^2-c*d^2)*(x+d/e))^(7/2
)+1/2*(a*e^2-c*d^2)*(1/12*(2*d*e*c*(x+d/e)+a*e^2-c*d^2)/d/e/c*(d*e*c*(x+d/
e)^2+(a*e^2-c*d^2)*(x+d/e))^(5/2)-5/24*(a*e^2-c*d^2)^2/d/e/c*(1/8*(2*d*e*c
*(x+d/e)+a*e^2-c*d^2)/d/e/c*(d*e*c*(x+d/e)^2+(a*e^2-c*d^2)*(x+d/e))^(3/2)-
3/16*(a*e^2-c*d^2)^2/d/e/c*(1/4*(2*d*e*c*(x+d/e)+a*e^2-c*d^2)/d/e/c*(d*e*c
*(x+d/e)^2+(a*e^2-c*d^2)*(x+d/e))^(1/2)-1/8*(a*e^2-c*d^2)^2/d/e/c*ln((1/2*
a*e^2-1/2*c*d^2+d*e*c*(x+d/e))/(d*e*c)^(1/2)+(d*e*c*(x+d/e)^2+(a*e^2-c*d^2
)*(x+d/e))^(1/2))/(d*e*c)^(1/2))))))))
```

Fricas [A] (verification not implemented)

Time = 0.43 (sec) , antiderivative size = 726, normalized size of antiderivative = 2.58

$$\int \frac{(ade + (cd^2 + ae^2)x + cdex^2)^{7/2}}{(d + ex)^5}\,dx = \left[-\frac{105\,(c^3d^7 - 3\,ac^2d^5e^2 + 3\,a^2cd^3e^4 - a^3de^6 + (c^3d^6e - 3\,ac^2d^4e}{}\right.$$

input
```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(7/2)/(e*x+d)^5,x, algorithm="
fricas")
```

output
```
[-1/96*(105*(c^3*d^7 - 3*a*c^2*d^5*e^2 + 3*a^2*c*d^3*e^4 - a^3*d*e^6 + (c^
3*d^6*e - 3*a*c^2*d^4*e^3 + 3*a^2*c*d^2*e^5 - a^3*e^7)*x)*sqrt(c*d/e)*log(
8*c^2*d^2*e^2*x^2 + c^2*d^4 + 6*a*c*d^2*e^2 + a^2*e^4 + 4*(2*c*d*e^2*x + c
*d^2*e + a*e^3)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(c*d/e) +
8*(c^2*d^3*e + a*c*d*e^3)*x) - 4*(8*c^3*d^3*e^3*x^3 + 105*c^3*d^6 - 280*a*
c^2*d^4*e^2 + 231*a^2*c*d^2*e^4 - 48*a^3*e^6 - 2*(7*c^3*d^4*e^2 - 19*a*c^2
*d^2*e^4)*x^2 + (35*c^3*d^5*e - 98*a*c^2*d^3*e^3 + 87*a^2*c*d*e^5)*x)*sqrt
(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x))/(e^5*x + d*e^4), 1/48*(105*(c^3*d
^7 - 3*a*c^2*d^5*e^2 + 3*a^2*c*d^3*e^4 - a^3*d*e^6 + (c^3*d^6*e - 3*a*c^2*
d^4*e^3 + 3*a^2*c*d^2*e^5 - a^3*e^7)*x)*sqrt(-c*d/e)*arctan(1/2*sqrt(c*d*e
*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(2*c*d*e*x + c*d^2 + a*e^2)*sqrt(-c*d/e)
/(c^2*d^2*e*x^2 + a*c*d^2*e + (c^2*d^3 + a*c*d*e^2)*x)) + 2*(8*c^3*d^3*e^3
*x^3 + 105*c^3*d^6 - 280*a*c^2*d^4*e^2 + 231*a^2*c*d^2*e^4 - 48*a^3*e^6 -
2*(7*c^3*d^4*e^2 - 19*a*c^2*d^2*e^4)*x^2 + (35*c^3*d^5*e - 98*a*c^2*d^3*e^
3 + 87*a^2*c*d*e^5)*x)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x))/(e^5*x
 + d*e^4)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(ade + (cd^2 + ae^2)\,x + cdex^2)^{7/2}}{(d+ex)^5}\,dx = \text{Timed out}$$

input
```
integrate((a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(7/2)/(e*x+d)**5,x)
```

output
```
Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(ade + (cd^2 + ae^2)\,x + cdex^2)^{7/2}}{(d+ex)^5}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(7/2)/(e*x+d)^5,x, algorithm="
maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e*(a*e^2-c*d^2)>0)', see `assume
?` for mor
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1066 vs. $2(251) = 502$.

Time = 0.59 (sec) , antiderivative size = 1066, normalized size of antiderivative = 3.79

$$\int \frac{(ade + (cd^2 + ae^2)x + cdex^2)^{7/2}}{(d + ex)^5} dx = \text{Too large to display}$$

input
```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(7/2)/(e*x+d)^5,x, algorithm="
giac")
```

output
```
1/24*(105*(c^4*d^7*sgn(1/(e*x + d))*sgn(e) - 3*a*c^3*d^5*e^2*sgn(1/(e*x +
d))*sgn(e) + 3*a^2*c^2*d^3*e^4*sgn(1/(e*x + d))*sgn(e) - a^3*c*d*e^6*sgn(1
/(e*x + d))*sgn(e))*arctan(sqrt(c*d*e - c*d^2*e/(e*x + d) + a*e^3/(e*x + d
))/sqrt(-c*d*e))/(sqrt(-c*d*e)*e^4*abs(e)) + 48*(sqrt(c*d*e - c*d^2*e/(e*x
 + d) + a*e^3/(e*x + d))*c^3*d^6*sgn(1/(e*x + d))*sgn(e) - 3*sqrt(c*d*e -
c*d^2*e/(e*x + d) + a*e^3/(e*x + d))*a*c^2*d^4*e^2*sgn(1/(e*x + d))*sgn(e)
 + 3*sqrt(c*d*e - c*d^2*e/(e*x + d) + a*e^3/(e*x + d))*a^2*c*d^2*e^4*sgn(1
/(e*x + d))*sgn(e) - sqrt(c*d*e - c*d^2*e/(e*x + d) + a*e^3/(e*x + d))*a^3
*e^6*sgn(1/(e*x + d))*sgn(e))/(e^5*abs(e)) + (57*sqrt(c*d*e - c*d^2*e/(e*x
 + d) + a*e^3/(e*x + d))*c^6*d^9*e^2*sgn(1/(e*x + d))*sgn(e) - 171*sqrt(c*
d*e - c*d^2*e/(e*x + d) + a*e^3/(e*x + d))*a*c^5*d^7*e^4*sgn(1/(e*x + d))*
sgn(e) + 171*sqrt(c*d*e - c*d^2*e/(e*x + d) + a*e^3/(e*x + d))*a^2*c^4*d^5
*e^6*sgn(1/(e*x + d))*sgn(e) - 57*sqrt(c*d*e - c*d^2*e/(e*x + d) + a*e^3/(
e*x + d))*a^3*c^3*d^3*e^8*sgn(1/(e*x + d))*sgn(e) - 136*(c*d*e - c*d^2*e/(
e*x + d) + a*e^3/(e*x + d))^(3/2)*c^5*d^8*e*sgn(1/(e*x + d))*sgn(e) + 408*
(c*d*e - c*d^2*e/(e*x + d) + a*e^3/(e*x + d))^(3/2)*a*c^4*d^6*e^3*sgn(1/(e
*x + d))*sgn(e) - 408*(c*d*e - c*d^2*e/(e*x + d) + a*e^3/(e*x + d))^(3/2)*
a^2*c^3*d^4*e^5*sgn(1/(e*x + d))*sgn(e) + 136*(c*d*e - c*d^2*e/(e*x + d) +
 a*e^3/(e*x + d))^(3/2)*a^3*c^2*d^2*e^7*sgn(1/(e*x + d))*sgn(e) + 87*(c*d*
e - c*d^2*e/(e*x + d) + a*e^3/(e*x + d))^(5/2)*c^4*d^7*sgn(1/(e*x + d))...
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(ade + (cd^2 + ae^2)\,x + cdex^2)^{7/2}}{(d + ex)^5}\,dx = \int \frac{\left(c\,d\,e\,x^2 + \left(c\,d^2 + a\,e^2\right)\,x + a\,d\,e\right)^{7/2}}{\left(d + e\,x\right)^5}\,dx$$

input
```
int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(7/2)/(d + e*x)^5,x)
```

output
```
int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(7/2)/(d + e*x)^5, x)
```

Reduce [B] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 879, normalized size of antiderivative = 3.13

$$\int \frac{(ade + (cd^2 + ae^2)\,x + cdex^2)^{7/2}}{(d + ex)^5}\,dx = \text{Too large to display}$$

input
```
int((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(7/2)/(e*x+d)^5,x)
```

output
```
( - 384*sqrt(d + e*x)*sqrt(a*e + c*d*x)*a**3*e**7 + 1848*sqrt(d + e*x)*sqr
t(a*e + c*d*x)*a**2*c*d**2*e**5 + 696*sqrt(d + e*x)*sqrt(a*e + c*d*x)*a**2
*c*d*e**6*x - 2240*sqrt(d + e*x)*sqrt(a*e + c*d*x)*a*c**2*d**4*e**3 - 784*
sqrt(d + e*x)*sqrt(a*e + c*d*x)*a*c**2*d**3*e**4*x + 304*sqrt(d + e*x)*sqr
t(a*e + c*d*x)*a*c**2*d**2*e**5*x**2 + 840*sqrt(d + e*x)*sqrt(a*e + c*d*x)
*c**3*d**6*e + 280*sqrt(d + e*x)*sqrt(a*e + c*d*x)*c**3*d**5*e**2*x - 112*
sqrt(d + e*x)*sqrt(a*e + c*d*x)*c**3*d**4*e**3*x**2 + 64*sqrt(d + e*x)*sqr
t(a*e + c*d*x)*c**3*d**3*e**4*x**3 + 840*sqrt(e)*sqrt(d)*sqrt(c)*log((sqrt
(e)*sqrt(a*e + c*d*x) + sqrt(d)*sqrt(c)*sqrt(d + e*x))/sqrt(a*e**2 - c*d**
2))*a**3*d*e**6 + 840*sqrt(e)*sqrt(d)*sqrt(c)*log((sqrt(e)*sqrt(a*e + c*d*
x) + sqrt(d)*sqrt(c)*sqrt(d + e*x))/sqrt(a*e**2 - c*d**2))*a**3*e**7*x - 2
520*sqrt(e)*sqrt(d)*sqrt(c)*log((sqrt(e)*sqrt(a*e + c*d*x) + sqrt(d)*sqrt(
c)*sqrt(d + e*x))/sqrt(a*e**2 - c*d**2))*a**2*c*d**3*e**4 - 2520*sqrt(e)*s
qrt(d)*sqrt(c)*log((sqrt(e)*sqrt(a*e + c*d*x) + sqrt(d)*sqrt(c)*sqrt(d + e
*x))/sqrt(a*e**2 - c*d**2))*a**2*c*d**2*e**5*x + 2520*sqrt(e)*sqrt(d)*sqrt
(c)*log((sqrt(e)*sqrt(a*e + c*d*x) + sqrt(d)*sqrt(c)*sqrt(d + e*x))/sqrt(a
*e**2 - c*d**2))*a*c**2*d**5*e**2 + 2520*sqrt(e)*sqrt(d)*sqrt(c)*log((sqrt
(e)*sqrt(a*e + c*d*x) + sqrt(d)*sqrt(c)*sqrt(d + e*x))/sqrt(a*e**2 - c*d**
2))*a*c**2*d**4*e**3*x - 840*sqrt(e)*sqrt(d)*sqrt(c)*log((sqrt(e)*sqrt(a*e
 + c*d*x) + sqrt(d)*sqrt(c)*sqrt(d + e*x))/sqrt(a*e**2 - c*d**2))*c**3*...
```

3.241 $\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{7/2}}{(d+ex)^6}\,dx$

Optimal result

Integrand size = 37, antiderivative size = 277

$$\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{7/2}}{(d+ex)^6}\,dx=\frac{35c^2d^2\left(a-\frac{cd^2}{e^2}\right)\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{4e^2}+\frac{35c^2d^2\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{6e^3(d+ex)}-\frac{14cd\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{3e^2(d+ex)^3}-\frac{2\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{7/2}}{3e(d+ex)^5}+\frac{35c^{3/2}d^{3/2}\left(cd^2-ae^2\right)^2\operatorname{arctanh}\left(\frac{\sqrt{e}\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{\sqrt{c}\sqrt{d}(d+ex)}\right)}{4e^{9/2}}$$

output
```
35/4*c^2*d^2*(a-c*d^2/e^2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/e^2+35/
6*c^2*d^2*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/e^3/(e*x+d)-14/3*c*d*(a*
d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/e^2/(e*x+d)^3-2/3*(a*d*e+(a*e^2+c*d^2
)*x+c*d*e*x^2)^(7/2)/e/(e*x+d)^5+35/4*c^(3/2)*d^(3/2)*(-a*e^2+c*d^2)^2*arc
tanh(e^(1/2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/c^(1/2)/d^(1/2)/(e*x+
d))/e^(9/2)
```

Mathematica [A] (verified)

Time = 0.48 (sec) , antiderivative size = 222, normalized size of antiderivative = 0.80

$$\int \frac{(ade+(cd^2+ae^2)\,x+cdex^2)^{7/2}}{(d+ex)^6}\,dx = \frac{\sqrt{(ae+cdx)(d+ex)}\Bigg(-\frac{\sqrt{e}(8a^3e^6+8a^2cde^4(7d+10ex)-ac^2d^2e^2(175d^2+238d}{(d+}}{}$$

input
```
Integrate[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(7/2)/(d + e*x)^6,x]
```

output
```
(Sqrt[(a*e + c*d*x)*(d + e*x)]*(-((Sqrt[e]*(8*a^3*e^6 + 8*a^2*c*d*e^4*(7*d
 + 10*e*x) - a*c^2*d^2*e^2*(175*d^2 + 238*d*e*x + 39*e^2*x^2) + c^3*d^3*(1
05*d^3 + 140*d^2*e*x + 21*d*e^2*x^2 - 6*e^3*x^3)))/(d + e*x)^2) + (105*c^(
3/2)*d^(3/2)*(c*d^2 - a*e^2)^2*ArcTanh[(Sqrt[c]*Sqrt[d]*Sqrt[d + e*x])/(Sq
rt[e]*Sqrt[a*e + c*d*x])])/(Sqrt[a*e + c*d*x]*Sqrt[d + e*x])))/(12*e^(9/2)
)
```

Rubi [A] (verified)

Time = 1.07 (sec) , antiderivative size = 310, normalized size of antiderivative = 1.12, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.216$, Rules used = {1130, 1125, 25, 2192, 27, 1160, 1092, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{7/2}}{(d+ex)^6}dx$$

$$\Big\downarrow \text{1130}$$

$$\frac{7cd\int \frac{\left(cdex^2+(cd^2+ae^2)x+ade\right)^{5/2}}{(d+ex)^4}dx}{3e}-\frac{2\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{7/2}}{3e(d+ex)^5}$$

$$\Big\downarrow \text{1125}$$

$$\frac{7cd\left(-\frac{\int -\frac{c^3d^3x^2e^5-c^2d^2\left(cd^2-3ae^2\right)xe^4+cd\left(c^2d^4-3ace^2d^2+3a^2e^4\right)e^3}{\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}dx}{e^6}-\frac{2(cd^2-ae^2)^2\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{e^3(d+ex)}\right)}{3e}-\frac{2\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{7/2}}{3e(d+ex)^5}$$

$$\Big\downarrow \; 25$$

$$\frac{7cd\left(\frac{\int \frac{c^3d^3x^2e^5-c^2d^2\left(cd^2-3ae^2\right)xe^4+cd\left(c^2d^4-3ace^2d^2+3a^2e^4\right)e^3}{\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}dx}{e^6}-\frac{2(cd^2-ae^2)^2\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{e^3(d+ex)}\right)}{3e}-\frac{2\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{7/2}}{3e(d+ex)^5}$$

$$\Big\downarrow \; 2192$$

$$\frac{7cd\left(\frac{\frac{\int \frac{c^2d^2e^4\left(2\left(2cd^2-3ae^2\right)\left(cd^2-2ae^2\right)-cde\left(7cd^2-9ae^2\right)x\right)}{2\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}dx}{2cde}+\frac{1}{2}c^2d^2e^4x\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{e^6}-\frac{2(cd^2-ae^2)^2\sqrt{x(ae^2+cd^2)+ade+cd}}{e^3(d+ex)}\right.}{3e}-\frac{2\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{7/2}}{3e(d+ex)^5}$$

$$\Big\downarrow \; 27$$

$$\frac{7cd\left(\frac{\frac{1}{4}cde^3\int \frac{2\left(2cd^2-3ae^2\right)\left(cd^2-2ae^2\right)-cde\left(7cd^2-9ae^2\right)x}{\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}dx+\frac{1}{2}c^2d^2e^4x\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{e^6}-\frac{2(cd^2-ae^2)^2\sqrt{x(ae^2+cd^2)+ade+cdex}}{e^3(d+ex)}\right.}{3e}-\frac{2\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{7/2}}{3e(d+ex)^5}$$

$$\Big\downarrow \; 1160$$

$$\frac{7cd\left(\frac{\frac{1}{4}cde^3\left(\frac{15}{2}\left(cd^2-ae^2\right)^2\int\frac{1}{\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}dx-\left(7cd^2-9ae^2\right)\sqrt{x(ae^2+cd^2)+ade+cdex^2}\right)+\frac{1}{2}c^2d^2e^4x\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{e^6}\right.}{3e}$$

$$\frac{2\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{7/2}}{3e(d+ex)^5}$$

$\downarrow$ 1092

$$\frac{7cd\left(\frac{\frac{1}{4}cde^3\left(15\left(cd^2-ae^2\right)^2\int\frac{1}{4cde-\frac{\left(cd^2+2cexd+ae^2\right)^2}{cdex^2+\left(cd^2+ae^2\right)x+ade}}d\frac{cd^2+2cexd+ae^2}{\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}-\left(7cd^2-9ae^2\right)\sqrt{x(ae^2+cd^2)+ade+cdex^2}\right)+\frac{1}{2}c^2d^2e^4}{e^6}\right.}{3e}$$

$$\frac{2\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{7/2}}{3e(d+ex)^5}$$

$\downarrow$ 219

$$\frac{7cd\left(\frac{\frac{1}{4}cde^3\left(\frac{15\left(cd^2-ae^2\right)^2\text{arctanh}\left(\frac{ae^2+cd^2+2cdex}{2\sqrt{c}\sqrt{d}\sqrt{e}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}\right)}{2\sqrt{c}\sqrt{d}\sqrt{e}}-\left(7cd^2-9ae^2\right)\sqrt{x(ae^2+cd^2)+ade+cdex^2}\right)+\frac{1}{2}c^2d^2e^4x\sqrt{x(ae^2+c}}{e^6}\right.}{3e}$$

$$\frac{2\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{7/2}}{3e(d+ex)^5}$$

input
```
Int[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(7/2)/(d + e*x)^6,x]
```

output
```
(-2*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(7/2))/(3*e*(d + e*x)^5) + (7*
c*d*((-2*(c*d^2 - a*e^2)^2*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(e
^3*(d + e*x)) + ((c^2*d^2*e^4*x*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2
])/2 + (c*d*e^3*(-((7*c*d^2 - 9*a*e^2)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*
d*e*x^2]) + (15*(c*d^2 - a*e^2)^2*ArcTanh[(c*d^2 + a*e^2 + 2*c*d*e*x)/(2*S
qrt[c]*Sqrt[d]*Sqrt[e]*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])])/(2*S
qrt[c]*Sqrt[d]*Sqrt[e])))/4)/e^6))/(3*e)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1092
```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[I
nt[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a
, b, c}, x]
```

rule 1125
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_
Symbol] :> Simp[-2*e^(2*m + 3)*(Sqrt[a + b*x + c*x^2]/((-2*c*d + b*e)^(m +
2)*(d + e*x))), x] - Simp[e^(2*m + 2)   Int[(1/Sqrt[a + b*x + c*x^2])*Expan
dToSum[((-2*c*d + b*e)^(-m - 1) - ((-c)*d + b*e + c*e*x)^(-m - 1))/(d + e*x
), x], x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0]
 && ILtQ[m, 0] && EqQ[m + p, -3/2]
```

rule 1130
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(d + e*x)^(m + 1)*((a + b*x + c*x^2)^p/(e*(m + p + 1))), x]
- Simp[c*(p/(e^2*(m + p + 1)))   Int[(d + e*x)^(m + 2)*(a + b*x + c*x^2)^(p
 - 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0]
 && GtQ[p, 0] && (LtQ[m, -2] || EqQ[m + 2*p + 1, 0]) && NeQ[m + p + 1, 0] &
& IntegerQ[2*p]
```

rule 1160
```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[e*((a + b*x + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Simp[(2*c*d - b
*e)/(2*c)   Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
 && NeQ[p, -1]
```

rule 2192
```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{q =
Expon[Pq, x], e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[e*x^(q - 1)*((a + b*x +
 c*x^2)^(p + 1)/(c*(q + 2*p + 1))), x] + Simp[1/(c*(q + 2*p + 1))   Int[(a
+ b*x + c*x^2)^p*ExpandToSum[c*(q + 2*p + 1)*Pq - a*e*(q - 1)*x^(q - 2) - b
*e*(q + p)*x^(q - 1) - c*e*(q + 2*p + 1)*x^q, x], x], x]] /; FreeQ[{a, b, c
, p}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] &&  !LeQ[p, -1]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 828 vs. $2(245) = 490$.

Time = 4.04 (sec) , antiderivative size = 829, normalized size of antiderivative = 2.99

method	result

input
```
int((a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(7/2)/(e*x+d)^6,x,method=_RETURNVERB
OSE)
```

output
```
1/e^6*(-2/3/(a*e^2-c*d^2)/(x+d/e)^6*(d*e*c*(x+d/e)^2+(a*e^2-c*d^2)*(x+d/e)
)^(9/2)+2*d*e*c/(a*e^2-c*d^2)*(-2/(a*e^2-c*d^2)/(x+d/e)^5*(d*e*c*(x+d/e)^2
+(a*e^2-c*d^2)*(x+d/e))^(9/2)+8*d*e*c/(a*e^2-c*d^2)*(2/(a*e^2-c*d^2)/(x+d/
e)^4*(d*e*c*(x+d/e)^2+(a*e^2-c*d^2)*(x+d/e))^(9/2)-10*d*e*c/(a*e^2-c*d^2)*
(2/3/(a*e^2-c*d^2)/(x+d/e)^3*(d*e*c*(x+d/e)^2+(a*e^2-c*d^2)*(x+d/e))^(9/2)
-4*d*e*c/(a*e^2-c*d^2)*(2/5/(a*e^2-c*d^2)/(x+d/e)^2*(d*e*c*(x+d/e)^2+(a*e^
2-c*d^2)*(x+d/e))^(9/2)-14/5*d*e*c/(a*e^2-c*d^2)*(1/7*(d*e*c*(x+d/e)^2+(a*
e^2-c*d^2)*(x+d/e))^(7/2)+1/2*(a*e^2-c*d^2)*(1/12*(2*d*e*c*(x+d/e)+a*e^2-c
*d^2)/d/e/c*(d*e*c*(x+d/e)^2+(a*e^2-c*d^2)*(x+d/e))^(5/2)-5/24*(a*e^2-c*d^
2)^2/d/e/c*(1/8*(2*d*e*c*(x+d/e)+a*e^2-c*d^2)/d/e/c*(d*e*c*(x+d/e)^2+(a*e^
2-c*d^2)*(x+d/e))^(3/2)-3/16*(a*e^2-c*d^2)^2/d/e/c*(1/4*(2*d*e*c*(x+d/e)+a
*e^2-c*d^2)/d/e/c*(d*e*c*(x+d/e)^2+(a*e^2-c*d^2)*(x+d/e))^(1/2)-1/8*(a*e^2
-c*d^2)^2/d/e/c*ln((1/2*a*e^2-1/2*c*d^2+d*e*c*(x+d/e))/(d*e*c)^(1/2)+(d*e*
c*(x+d/e)^2+(a*e^2-c*d^2)*(x+d/e))^(1/2))/(d*e*c)^(1/2)))))))))
```

Fricas [A] (verification not implemented)

Time = 0.77 (sec) , antiderivative size = 786, normalized size of antiderivative = 2.84

$$\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{7/2}}{(d+ex)^6}\,dx = \text{Too large to display}$$

input
```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(7/2)/(e*x+d)^6,x, algorithm="
fricas")
```

output
```
[1/48*(105*(c^3*d^7 - 2*a*c^2*d^5*e^2 + a^2*c*d^3*e^4 + (c^3*d^5*e^2 - 2*a
*c^2*d^3*e^4 + a^2*c*d*e^6)*x^2 + 2*(c^3*d^6*e - 2*a*c^2*d^4*e^3 + a^2*c*d
^2*e^5)*x)*sqrt(c*d/e)*log(8*c^2*d^2*e^2*x^2 + c^2*d^4 + 6*a*c*d^2*e^2 + a
^2*e^4 + 4*(2*c*d*e^2*x + c*d^2*e + a*e^3)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2
 + a*e^2)*x)*sqrt(c*d/e) + 8*(c^2*d^3*e + a*c*d*e^3)*x) + 4*(6*c^3*d^3*e^3
*x^3 - 105*c^3*d^6 + 175*a*c^2*d^4*e^2 - 56*a^2*c*d^2*e^4 - 8*a^3*e^6 - 3*
(7*c^3*d^4*e^2 - 13*a*c^2*d^2*e^4)*x^2 - 2*(70*c^3*d^5*e - 119*a*c^2*d^3*e
^3 + 40*a^2*c*d*e^5)*x)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x))/(e^6*
x^2 + 2*d*e^5*x + d^2*e^4), -1/24*(105*(c^3*d^7 - 2*a*c^2*d^5*e^2 + a^2*c*
d^3*e^4 + (c^3*d^5*e^2 - 2*a*c^2*d^3*e^4 + a^2*c*d*e^6)*x^2 + 2*(c^3*d^6*e
 - 2*a*c^2*d^4*e^3 + a^2*c*d^2*e^5)*x)*sqrt(-c*d/e)*arctan(1/2*sqrt(c*d*e*
x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(2*c*d*e*x + c*d^2 + a*e^2)*sqrt(-c*d/e)/
(c^2*d^2*e*x^2 + a*c*d^2*e + (c^2*d^3 + a*c*d*e^2)*x)) - 2*(6*c^3*d^3*e^3*
x^3 - 105*c^3*d^6 + 175*a*c^2*d^4*e^2 - 56*a^2*c*d^2*e^4 - 8*a^3*e^6 - 3*(
7*c^3*d^4*e^2 - 13*a*c^2*d^2*e^4)*x^2 - 2*(70*c^3*d^5*e - 119*a*c^2*d^3*e^
3 + 40*a^2*c*d*e^5)*x)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x))/(e^6*x
^2 + 2*d*e^5*x + d^2*e^4)]
```

Sympy [F]

$$\int \frac{(ade + (cd^2 + ae^2)\,x + cdex^2)^{7/2}}{(d+ex)^6}\,dx = \int \frac{((d+ex)\,(ae+cdx))^{\frac{7}{2}}}{(d+ex)^6}\,dx$$

input
```
integrate((a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(7/2)/(e*x+d)**6,x)
```

output
```
Integral(((d + e*x)*(a*e + c*d*x))**(7/2)/(d + e*x)**6, x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(ade + (cd^2 + ae^2)\,x + cdex^2)^{7/2}}{(d+ex)^6}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(7/2)/(e*x+d)^6,x, algorithm="
maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e*(a*e^2-c*d^2)>0)', see `assume
?` for mor
```

Giac [F(-2)]

Exception generated.

$$\int \frac{(ade+(cd^2+ae^2)\,x+cdex^2)^{7/2}}{(d+ex)^6}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(7/2)/(e*x+d)^6,x, algorithm="
giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(ade+(cd^2+ae^2)\,x+cdex^2)^{7/2}}{(d+ex)^6}\,dx = \int \frac{{\left(c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e\right)}^{7/2}}{{\left(d+e\,x\right)}^6}\,dx$$

input
```
int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(7/2)/(d + e*x)^6,x)
```

output
```
int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(7/2)/(d + e*x)^6, x)
```

Reduce [B] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 991, normalized size of antiderivative = 3.58

$$\int \frac{\left(ade+(cd^2+ae^2)\,x+cdex^2\right)^{7/2}}{(d+ex)^6}\,dx = \text{Too large to display}$$

input

```
int((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(7/2)/(e*x+d)^6,x)
```

output

```
( - 64*sqrt(d + e*x)*sqrt(a*e + c*d*x)*a**3*e**7 - 448*sqrt(d + e*x)*sqrt(
a*e + c*d*x)*a**2*c*d**2*e**5 - 640*sqrt(d + e*x)*sqrt(a*e + c*d*x)*a**2*c
*d*e**6*x + 1400*sqrt(d + e*x)*sqrt(a*e + c*d*x)*a*c**2*d**4*e**3 + 1904*s
qrt(d + e*x)*sqrt(a*e + c*d*x)*a*c**2*d**3*e**4*x + 312*sqrt(d + e*x)*sqrt
(a*e + c*d*x)*a*c**2*d**2*e**5*x**2 - 840*sqrt(d + e*x)*sqrt(a*e + c*d*x)*
c**3*d**6*e - 1120*sqrt(d + e*x)*sqrt(a*e + c*d*x)*c**3*d**5*e**2*x - 168*
sqrt(d + e*x)*sqrt(a*e + c*d*x)*c**3*d**4*e**3*x**2 + 48*sqrt(d + e*x)*sqr
t(a*e + c*d*x)*c**3*d**3*e**4*x**3 + 840*sqrt(e)*sqrt(d)*sqrt(c)*log((sqrt
(e)*sqrt(a*e + c*d*x) + sqrt(d)*sqrt(c)*sqrt(d + e*x))/sqrt(a*e**2 - c*d**
2))*a**2*c*d**3*e**4 + 1680*sqrt(e)*sqrt(d)*sqrt(c)*log((sqrt(e)*sqrt(a*e
+ c*d*x) + sqrt(d)*sqrt(c)*sqrt(d + e*x))/sqrt(a*e**2 - c*d**2))*a**2*c*d*
*2*e**5*x + 840*sqrt(e)*sqrt(d)*sqrt(c)*log((sqrt(e)*sqrt(a*e + c*d*x) + s
qrt(d)*sqrt(c)*sqrt(d + e*x))/sqrt(a*e**2 - c*d**2))*a**2*c*d*e**6*x**2 -
1680*sqrt(e)*sqrt(d)*sqrt(c)*log((sqrt(e)*sqrt(a*e + c*d*x) + sqrt(d)*sqrt
(c)*sqrt(d + e*x))/sqrt(a*e**2 - c*d**2))*a*c**2*d**5*e**2 - 3360*sqrt(e)*
sqrt(d)*sqrt(c)*log((sqrt(e)*sqrt(a*e + c*d*x) + sqrt(d)*sqrt(c)*sqrt(d +
e*x))/sqrt(a*e**2 - c*d**2))*a*c**2*d**4*e**3*x - 1680*sqrt(e)*sqrt(d)*sqr
t(c)*log((sqrt(e)*sqrt(a*e + c*d*x) + sqrt(d)*sqrt(c)*sqrt(d + e*x))/sqrt(
a*e**2 - c*d**2))*a*c**2*d**3*e**4*x**2 + 840*sqrt(e)*sqrt(d)*sqrt(c)*log(
(sqrt(e)*sqrt(a*e + c*d*x) + sqrt(d)*sqrt(c)*sqrt(d + e*x))/sqrt(a*e**2...
```

3.242 $\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{7/2}}{(d+ex)^7}\,dx$

Optimal result

Integrand size = 37, antiderivative size = 260

$$\begin{aligned}\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{7/2}}{(d+ex)^7}\,dx &= \frac{7c^3d^3\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{e^4} \\ &- \frac{14c^2d^2\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{3e^3(d+ex)^2} \\ &- \frac{14cd\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{15e^2(d+ex)^4} - \frac{2\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{7/2}}{5e(d+ex)^6} \\ &- \frac{7c^{5/2}d^{5/2}\left(cd^2-ae^2\right)\operatorname{arctanh}\left(\frac{\sqrt{e}\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{\sqrt{c}\sqrt{d}(d+ex)}\right)}{e^{9/2}}\end{aligned}$$

output
```
7*c^3*d^3*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/e^4-14/3*c^2*d^2*(a*d*e+
(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/e^3/(e*x+d)^2-14/15*c*d*(a*d*e+(a*e^2+c*d
^2)*x+c*d*e*x^2)^(5/2)/e^2/(e*x+d)^4-2/5*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)
^(7/2)/e/(e*x+d)^6-7*c^(5/2)*d^(5/2)*(-a*e^2+c*d^2)*arctanh(e^(1/2)*(a*d*e
+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/c^(1/2)/d^(1/2)/(e*x+d))/e^(9/2)
```

Mathematica [A] (verified)

Time = 0.49 (sec) , antiderivative size = 219, normalized size of antiderivative = 0.84

$$\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{7/2}}{(d+ex)^7}\,dx=\frac{\sqrt{(ae+cdx)(d+ex)}\left(\frac{\sqrt{e}\left(-6a^3e^6-2a^2cde^4(7d+16ex)-2ac^2d^2e^2(35d^2+84de\cdots\right.}{(d+\cdots}\right.}{\cdots}$$

input
```
Integrate[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(7/2)/(d + e*x)^7,x]
```

output
```
(Sqrt[(a*e + c*d*x)*(d + e*x)]*((Sqrt[e]*(-6*a^3*e^6 - 2*a^2*c*d*e^4*(7*d
+ 16*e*x) - 2*a*c^2*d^2*e^2*(35*d^2 + 84*d*e*x + 58*e^2*x^2) + c^3*d^3*(10
5*d^3 + 245*d^2*e*x + 161*d*e^2*x^2 + 15*e^3*x^3)))/(d + e*x)^3 - (105*c^(
5/2)*d^(5/2)*(c*d^2 - a*e^2)*ArcTanh[(Sqrt[c]*Sqrt[d]*Sqrt[d + e*x])/(Sqrt
[e]*Sqrt[a*e + c*d*x])])/(Sqrt[a*e + c*d*x]*Sqrt[d + e*x])))/(15*e^(9/2))
```

Rubi [A] (verified)

Time = 0.78 (sec) , antiderivative size = 292, normalized size of antiderivative = 1.12, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.189, Rules used = {1130, 1130, 1125, 27, 1160, 1092, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{7/2}}{(d+ex)^7}dx$$

$$\Big\downarrow \text{1130}$$

$$\frac{7cd\int \frac{\left(cdex^2+\left(cd^2+ae^2\right)x+ade\right)^{5/2}}{(d+ex)^5}dx}{5e}-\frac{2\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{7/2}}{5e(d+ex)^6}$$

$$\Big\downarrow \text{1130}$$

$$\frac{7cd\left(\frac{5cd\int \frac{\left(cdex^2+\left(cd^2+ae^2\right)x+ade\right)^{3/2}}{(d+ex)^3}dx}{3e}-\frac{2\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/2}}{3e(d+ex)^4}\right)}{5e}-\frac{2\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{7/2}}{5e(d+ex)^6}$$

$$\Big\downarrow \ 1125$$

$$\frac{7cd\left(\frac{5cd\left(-\frac{\int \frac{cde^2\left(cd^2-cexd-2ae^2\right)}{\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}dx}{e^4}-\frac{2\left(a-\frac{cd^2}{e^2}\right)\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{d+ex}\right)}{3e}-\frac{2\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/2}}{3e(d+ex)^4}\right)}{5e}-\frac{2\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{7/2}}{5e(d+ex)^6}$$

$$\Big\downarrow \ 27$$

$$\frac{7cd\left(\frac{5cd\left(-\frac{cd\int \frac{cd^2-cexd-2ae^2}{\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}dx}{e^2}-\frac{2\left(a-\frac{cd^2}{e^2}\right)\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{d+ex}\right)}{3e}-\frac{2\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/2}}{3e(d+ex)^4}\right)}{5e}-\frac{2\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{7/2}}{5e(d+ex)^6}$$

$$\Big\downarrow \ 1160$$

$$\frac{7cd\left(\frac{5cd\left(-\frac{cd\left(\frac{3}{2}\left(cd^2-ae^2\right)\int\frac{1}{\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}dx-\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}\right)}{e^2}-\frac{2\left(a-\frac{cd^2}{e^2}\right)\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{d+ex}\right)}{3e}-\frac{2(x(ae^2+c}{3}\right)}{5e}$$

$$\frac{2\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{7/2}}{5e(d+ex)^6}$$

$\downarrow$ 1092

$$\frac{7cd\left(\frac{5cd\left(-\frac{cd\left(3\left(cd^2-ae^2\right)\int\frac{1}{4cde-\frac{\left(cd^2+2cexd+ae^2\right)^2}{cdex^2+\left(cd^2+ae^2\right)x+ade}}d\frac{cd^2+2cexd+ae^2}{\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}-\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}\right)}{e^2}-\frac{2\left(a-\frac{cd^2}{e^2}\right)\sqrt{x\left(ae^2+cd^2\right)+}}{d+ex}\right)}{3e}\right)}{5e}$$

$$\frac{2\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{7/2}}{5e(d+ex)^6}$$

$\downarrow$ 219

$$\frac{7cd\left(\frac{5cd\left(-\frac{cd\left(\frac{3\left(cd^2-ae^2\right)\operatorname{arctanh}\left(\frac{ae^2+cd^2+2cdex}{2\sqrt{c}\sqrt{d}\sqrt{e}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}\right)}{2\sqrt{c}\sqrt{d}\sqrt{e}}-\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}\right)}{e^2}-\frac{2\left(a-\frac{cd^2}{e^2}\right)\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{d+ex}\right)}{3e}\right)}{5e}$$

$$\frac{2\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{7/2}}{5e(d+ex)^6}$$

input
```
Int[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(7/2)/(d + e*x)^7,x]
```

output
```
(-2*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(7/2))/(5*e*(d + e*x)^6) + (7*
c*d*((-2*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2))/(3*e*(d + e*x)^4)
+ (5*c*d*((-2*(a - (c*d^2)/e^2)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2
])/(d + e*x) - (c*d*(-Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2] + (3*(c*
d^2 - a*e^2)*ArcTanh[(c*d^2 + a*e^2 + 2*c*d*e*x)/(2*Sqrt[c]*Sqrt[d]*Sqrt[e
]*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])])/(2*Sqrt[c]*Sqrt[d]*Sqrt[e
])))/e^2))/(3*e)))/(5*e)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1092
```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[I
nt[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a
, b, c}, x]
```

rule 1125
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_
Symbol] :> Simp[-2*e^(2*m + 3)*(Sqrt[a + b*x + c*x^2]/((-2*c*d + b*e)^(m +
2)*(d + e*x))), x] - Simp[e^(2*m + 2)   Int[(1/Sqrt[a + b*x + c*x^2])*Expan
dToSum[((-2*c*d + b*e)^(-m - 1) - ((-c)*d + b*e + c*e*x)^(-m - 1))/(d + e*x
), x], x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0]
 && ILtQ[m, 0] && EqQ[m + p, -3/2]
```

rule 1130
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(d + e*x)^(m + 1)*((a + b*x + c*x^2)^p/(e*(m + p + 1))), x]
- Simp[c*(p/(e^2*(m + p + 1)))   Int[(d + e*x)^(m + 2)*(a + b*x + c*x^2)^(p
 - 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0]
 && GtQ[p, 0] && (LtQ[m, -2] || EqQ[m + 2*p + 1, 0]) && NeQ[m + p + 1, 0] &
& IntegerQ[2*p]
```

rule 1160
```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[e*((a + b*x + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Simp[(2*c*d - b
*e)/(2*c)   Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
 && NeQ[p, -1]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 909 vs. $2(232) = 464$.

Time = 5.47 (sec) , antiderivative size = 910, normalized size of antiderivative = 3.50

method	result

input
```
int((a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(7/2)/(e*x+d)^7,x,method=_RETURNVERB
OSE)
```

output
```
1/e^7*(-2/5/(a*e^2-c*d^2)/(x+d/e)^7*(d*e*c*(x+d/e)^2+(a*e^2-c*d^2)*(x+d/e)
)^(9/2)+4/5*d*e*c/(a*e^2-c*d^2)*(-2/3/(a*e^2-c*d^2)/(x+d/e)^6*(d*e*c*(x+d/
e)^2+(a*e^2-c*d^2)*(x+d/e))^(9/2)+2*d*e*c/(a*e^2-c*d^2)*(-2/(a*e^2-c*d^2)/
(x+d/e)^5*(d*e*c*(x+d/e)^2+(a*e^2-c*d^2)*(x+d/e))^(9/2)+8*d*e*c/(a*e^2-c*d
^2)*(2/(a*e^2-c*d^2)/(x+d/e)^4*(d*e*c*(x+d/e)^2+(a*e^2-c*d^2)*(x+d/e))^(9/
2)-10*d*e*c/(a*e^2-c*d^2)*(2/3/(a*e^2-c*d^2)/(x+d/e)^3*(d*e*c*(x+d/e)^2+(a
*e^2-c*d^2)*(x+d/e))^(9/2)-4*d*e*c/(a*e^2-c*d^2)*(2/5/(a*e^2-c*d^2)/(x+d/e
)^2*(d*e*c*(x+d/e)^2+(a*e^2-c*d^2)*(x+d/e))^(9/2)-14/5*d*e*c/(a*e^2-c*d^2)
*(1/7*(d*e*c*(x+d/e)^2+(a*e^2-c*d^2)*(x+d/e))^(7/2)+1/2*(a*e^2-c*d^2)*(1/1
2*(2*d*e*c*(x+d/e)+a*e^2-c*d^2)/d/e/c*(d*e*c*(x+d/e)^2+(a*e^2-c*d^2)*(x+d/
e))^(5/2)-5/24*(a*e^2-c*d^2)^2/d/e/c*(1/8*(2*d*e*c*(x+d/e)+a*e^2-c*d^2)/d/
e/c*(d*e*c*(x+d/e)^2+(a*e^2-c*d^2)*(x+d/e))^(3/2)-3/16*(a*e^2-c*d^2)^2/d/e
/c*(1/4*(2*d*e*c*(x+d/e)+a*e^2-c*d^2)/d/e/c*(d*e*c*(x+d/e)^2+(a*e^2-c*d^2)
*(x+d/e))^(1/2)-1/8*(a*e^2-c*d^2)^2/d/e/c*ln((1/2*a*e^2-1/2*c*d^2+d*e*c*(x
+d/e))/(d*e*c)^(1/2)+(d*e*c*(x+d/e)^2+(a*e^2-c*d^2)*(x+d/e))^(1/2))/(d*e*c
)^(1/2))))))))))
```

Fricas [A] (verification not implemented)

Time = 1.16 (sec) , antiderivative size = 798, normalized size of antiderivative = 3.07

$$\int \frac{\left(ade + \left(cd^2 + ae^2\right)x + cdex^2\right)^{7/2}}{(d+ex)^7}\,dx = \text{Too large to display}$$

input
```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(7/2)/(e*x+d)^7,x, algorithm="
fricas")
```

output
```
[-1/60*(105*(c^3*d^7 - a*c^2*d^5*e^2 + (c^3*d^4*e^3 - a*c^2*d^2*e^5)*x^3 +
 3*(c^3*d^5*e^2 - a*c^2*d^3*e^4)*x^2 + 3*(c^3*d^6*e - a*c^2*d^4*e^3)*x)*sq
rt(c*d/e)*log(8*c^2*d^2*e^2*x^2 + c^2*d^4 + 6*a*c*d^2*e^2 + a^2*e^4 + 4*(2
*c*d*e^2*x + c*d^2*e + a*e^3)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*
sqrt(c*d/e) + 8*(c^2*d^3*e + a*c*d*e^3)*x) - 4*(15*c^3*d^3*e^3*x^3 + 105*c
^3*d^6 - 70*a*c^2*d^4*e^2 - 14*a^2*c*d^2*e^4 - 6*a^3*e^6 + (161*c^3*d^4*e^
2 - 116*a*c^2*d^2*e^4)*x^2 + (245*c^3*d^5*e - 168*a*c^2*d^3*e^3 - 32*a^2*c
*d*e^5)*x)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x))/(e^7*x^3 + 3*d*e^6
*x^2 + 3*d^2*e^5*x + d^3*e^4), 1/30*(105*(c^3*d^7 - a*c^2*d^5*e^2 + (c^3*d
^4*e^3 - a*c^2*d^2*e^5)*x^3 + 3*(c^3*d^5*e^2 - a*c^2*d^3*e^4)*x^2 + 3*(c^3
*d^6*e - a*c^2*d^4*e^3)*x)*sqrt(-c*d/e)*arctan(1/2*sqrt(c*d*e*x^2 + a*d*e
+ (c*d^2 + a*e^2)*x)*(2*c*d*e*x + c*d^2 + a*e^2)*sqrt(-c*d/e)/(c^2*d^2*e*x
^2 + a*c*d^2*e + (c^2*d^3 + a*c*d*e^2)*x)) + 2*(15*c^3*d^3*e^3*x^3 + 105*c
^3*d^6 - 70*a*c^2*d^4*e^2 - 14*a^2*c*d^2*e^4 - 6*a^3*e^6 + (161*c^3*d^4*e^
2 - 116*a*c^2*d^2*e^4)*x^2 + (245*c^3*d^5*e - 168*a*c^2*d^3*e^3 - 32*a^2*c
*d*e^5)*x)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x))/(e^7*x^3 + 3*d*e^6
*x^2 + 3*d^2*e^5*x + d^3*e^4)]
```

Sympy [F]

$$\int \frac{(ade + (cd^2 + ae^2)x + cdex^2)^{7/2}}{(d+ex)^7}\,dx = \int \frac{((d+ex)(ae+cdx))^{\frac{7}{2}}}{(d+ex)^7}\,dx$$

input
```
integrate((a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(7/2)/(e*x+d)**7,x)
```

output
```
Integral(((d + e*x)*(a*e + c*d*x))**(7/2)/(d + e*x)**7, x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(ade + (cd^2 + ae^2)x + cdex^2)^{7/2}}{(d+ex)^7}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(7/2)/(e*x+d)^7,x, algorithm="
maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e*(a*e^2-c*d^2)>0)', see `assume
?` for mor
```

Giac [F(-2)]

Exception generated.

$$\int \frac{(ade + (cd^2 + ae^2) x + cdex^2)^{7/2}}{(d + ex)^7} dx = \text{Exception raised: TypeError}$$

input
```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(7/2)/(e*x+d)^7,x, algorithm="
giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(ade + (cd^2 + ae^2) x + cdex^2)^{7/2}}{(d + ex)^7} dx = \int \frac{(c\,d\,e\,x^2 + (c\,d^2 + a\,e^2)\,x + a\,d\,e)^{7/2}}{(d + e\,x)^7} dx$$

input
```
int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(7/2)/(d + e*x)^7,x)
```

output
```
int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(7/2)/(d + e*x)^7, x)
```

Reduce [B] (verification not implemented)

Time = 0.35 (sec) , antiderivative size = 931, normalized size of antiderivative = 3.58

$$\int \frac{(ade + (cd^2 + ae^2)x + cdex^2)^{7/2}}{(d + ex)^7} dx = \text{Too large to display}$$

input
```
int((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(7/2)/(e*x+d)^7,x)
```

output
```
( - 24*sqrt(d + e*x)*sqrt(a*e + c*d*x)*a**3*e**7 - 56*sqrt(d + e*x)*sqrt(a
*e + c*d*x)*a**2*c*d**2*e**5 - 128*sqrt(d + e*x)*sqrt(a*e + c*d*x)*a**2*c*
d*e**6*x - 280*sqrt(d + e*x)*sqrt(a*e + c*d*x)*a*c**2*d**4*e**3 - 672*sqrt
(d + e*x)*sqrt(a*e + c*d*x)*a*c**2*d**3*e**4*x - 464*sqrt(d + e*x)*sqrt(a*
e + c*d*x)*a*c**2*d**2*e**5*x**2 + 420*sqrt(d + e*x)*sqrt(a*e + c*d*x)*c**
3*d**6*e + 980*sqrt(d + e*x)*sqrt(a*e + c*d*x)*c**3*d**5*e**2*x + 644*sqrt
(d + e*x)*sqrt(a*e + c*d*x)*c**3*d**4*e**3*x**2 + 60*sqrt(d + e*x)*sqrt(a*
e + c*d*x)*c**3*d**3*e**4*x**3 + 420*sqrt(e)*sqrt(d)*sqrt(c)*log((sqrt(e)*
sqrt(a*e + c*d*x) + sqrt(d)*sqrt(c)*sqrt(d + e*x))/sqrt(a*e**2 - c*d**2))*
a*c**2*d**5*e**2 + 1260*sqrt(e)*sqrt(d)*sqrt(c)*log((sqrt(e)*sqrt(a*e + c*
d*x) + sqrt(d)*sqrt(c)*sqrt(d + e*x))/sqrt(a*e**2 - c*d**2))*a*c**2*d**4*e
**3*x + 1260*sqrt(e)*sqrt(d)*sqrt(c)*log((sqrt(e)*sqrt(a*e + c*d*x) + sqrt
(d)*sqrt(c)*sqrt(d + e*x))/sqrt(a*e**2 - c*d**2))*a*c**2*d**3*e**4*x**2 +
420*sqrt(e)*sqrt(d)*sqrt(c)*log((sqrt(e)*sqrt(a*e + c*d*x) + sqrt(d)*sqrt(
c)*sqrt(d + e*x))/sqrt(a*e**2 - c*d**2))*a*c**2*d**2*e**5*x**3 - 420*sqrt(
e)*sqrt(d)*sqrt(c)*log((sqrt(e)*sqrt(a*e + c*d*x) + sqrt(d)*sqrt(c)*sqrt(d
 + e*x))/sqrt(a*e**2 - c*d**2))*c**3*d**7 - 1260*sqrt(e)*sqrt(d)*sqrt(c)*l
og((sqrt(e)*sqrt(a*e + c*d*x) + sqrt(d)*sqrt(c)*sqrt(d + e*x))/sqrt(a*e**2
 - c*d**2))*c**3*d**6*e*x - 1260*sqrt(e)*sqrt(d)*sqrt(c)*log((sqrt(e)*sqrt
(a*e + c*d*x) + sqrt(d)*sqrt(c)*sqrt(d + e*x))/sqrt(a*e**2 - c*d**2))*c...
```

3.243 $\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{7/2}}{(d+ex)^8}\,dx$

Optimal result

Integrand size = 37, antiderivative size = 255

$$\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{7/2}}{(d+ex)^8}\,dx=-\frac{2c^3d^3\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{e^4(d+ex)}-\frac{2c^2d^2\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{3e^3(d+ex)^3}-\frac{2cd\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{5e^2(d+ex)^5}-\frac{2\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{7/2}}{7e(d+ex)^7}+\frac{2c^{7/2}d^{7/2}\text{arctanh}\left(\frac{\sqrt{e}\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{\sqrt{c}\sqrt{d}(d+ex)}\right)}{e^{9/2}}$$

output
```
-2*c^3*d^3*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/e^4/(e*x+d)-2/3*c^2*d^2
*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/e^3/(e*x+d)^3-2/5*c*d*(a*d*e+(a*e
^2+c*d^2)*x+c*d*e*x^2)^(5/2)/e^2/(e*x+d)^5-2/7*(a*d*e+(a*e^2+c*d^2)*x+c*d*
e*x^2)^(7/2)/e/(e*x+d)^7+2*c^(7/2)*d^(7/2)*arctanh(e^(1/2)*(a*d*e+(a*e^2+c
*d^2)*x+c*d*e*x^2)^(1/2)/c^(1/2)/d^(1/2)/(e*x+d))/e^(9/2)
```

Mathematica [A] (verified)

Time = 0.39 (sec) , antiderivative size = 207, normalized size of antiderivative = 0.81

$$\int \frac{(ade+(cd^2+ae^2)x+cdex^2)^{7/2}}{(d+ex)^8}\,dx = \frac{2\sqrt{(ae+cdx)(d+ex)}\left(-\frac{\sqrt{e}(15a^3e^6+3a^2cde^4(7d+22ex)+ac^2d^2e^2(35d^2+112\ldots}{\ldots}\right.}{\ldots}$$

input
```
Integrate[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(7/2)/(d + e*x)^8,x]
```

output
```
(2*Sqrt[(a*e + c*d*x)*(d + e*x)]*(-((Sqrt[e]*(15*a^3*e^6 + 3*a^2*c*d*e^4*(
7*d + 22*e*x) + a*c^2*d^2*e^2*(35*d^2 + 112*d*e*x + 122*e^2*x^2) + c^3*d^3
*(105*d^3 + 350*d^2*e*x + 406*d*e^2*x^2 + 176*e^3*x^3)))/(d + e*x)^4) + (1
05*c^(7/2)*d^(7/2)*ArcTanh[(Sqrt[c]*Sqrt[d]*Sqrt[d + e*x])/(Sqrt[e]*Sqrt[a
*e + c*d*x])])/(Sqrt[a*e + c*d*x]*Sqrt[d + e*x])))/(105*e^(9/2))
```

Rubi [A] (verified)

Time = 0.76 (sec) , antiderivative size = 274, normalized size of antiderivative = 1.07, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.216, Rules used = {1130, 1130, 1130, 1125, 25, 27, 1092, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(x(ae^2+cd^2)+ade+cdex^2)^{7/2}}{(d+ex)^8}dx$$

$$\Big\downarrow \text{1130}$$

$$\frac{cd\int \frac{(cdex^2+(cd^2+ae^2)x+ade)^{5/2}}{(d+ex)^6}dx}{e}-\frac{2(x(ae^2+cd^2)+ade+cdex^2)^{7/2}}{7e(d+ex)^7}$$

$$\Big\downarrow \text{1130}$$

$$\frac{cd\left(\frac{cd\int \frac{\left(cdex^2+\left(cd^2+ae^2\right)x+ade\right)^{3/2}}{(d+ex)^4}dx}{e}-\frac{2\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/2}}{5e(d+ex)^5}\right)}{e}-\frac{2\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{7/2}}{7e(d+ex)^7}$$

$\downarrow$ 1130

$$\frac{cd\left(\frac{cd\left(\frac{cd\int \frac{\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}{(d+ex)^2}dx}{e}-\frac{2\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{3e(d+ex)^3}\right)}{e}-\frac{2\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/2}}{5e(d+ex)^5}\right)}{e}-\frac{2\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{7/2}}{7e(d+ex)^7}$$

$\downarrow$ 1125

$$\frac{cd\left(\frac{cd\left(\frac{cd\left(-\frac{\int -\frac{cde}{\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}dx}{e^2}-\frac{2\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{e(d+ex)}\right)}{e}-\frac{2\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{3e(d+ex)^3}\right)}{e}-\frac{2\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/2}}{5e(d+ex)^5}\right)}{e}-\frac{2\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{7/2}}{7e(d+ex)^7}$$

$\downarrow$ 25

$$\frac{cd\left(\frac{cd\left(\frac{cd\left(\frac{\int \frac{cde}{\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}dx}{e^2}-\frac{2\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{e(d+ex)}\right)}{e}-\frac{2\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{3e(d+ex)^3}\right)}{e}-\frac{2(x(ae^2+cd^2)+ade+cdex^2)^{5/2}}{5e(d+ex)^5}\right.}{e}$$

$$\frac{2\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{7/2}}{7e(d+ex)^7}$$

$\downarrow$ 27

$$\frac{cd\left(\frac{cd\left(\frac{cd\left(\frac{cd\int \frac{1}{\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}dx}{e}-\frac{2\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{e(d+ex)}\right)}{e}-\frac{2\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{3e(d+ex)^3}\right)}{e}-\frac{2(x(ae^2+cd^2)+ade+cdex^2)^{5}}{5e(d+ex)^5}\right.}{e}$$

$$\frac{2\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{7/2}}{7e(d+ex)^7}$$

$\downarrow$ 1092

$$\frac{cd\left(\frac{cd\left(\frac{cd\left(\frac{2cd\int\frac{1}{4cde-\frac{\left(cd^2+2cexd+ae^2\right)^2}{cdex^2+\left(cd^2+ae^2\right)x+ade}}d\frac{cd^2+2cexd+ae^2}{\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}}{e}-\frac{2\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{e(d+ex)}\right)}{e}-\frac{2\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{3e(d+ex)^3}\right)}{e}-\cdots\right)}{e}$$

$$\frac{2\left(x(ae^2+cd^2)+ade+cdex^2\right)^{7/2}}{7e(d+ex)^7}$$

$$\Big\downarrow \text{219}$$

$$\frac{cd\left(\frac{cd\left(\frac{cd\left(\frac{\sqrt{c}\sqrt{d}\,\text{arctanh}\left(\frac{ae^2+cd^2+2cdex}{2\sqrt{c}\sqrt{d}\sqrt{e}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}\right)}{e^{3/2}}-\frac{2\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{e(d+ex)}\right)}{e}-\frac{2\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{3e(d+ex)^3}\right)}{e}-\frac{2(x(ae^{\cdots}}{\cdots}\right)}{e}$$

$$\frac{2\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{7/2}}{7e(d+ex)^7}$$

input
```
Int[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(7/2)/(d + e*x)^8,x]
```

output
```
(-2*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(7/2))/(7*e*(d + e*x)^7) + (c*
d*((-2*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2))/(5*e*(d + e*x)^5) +
(c*d*((-2*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2))/(3*e*(d + e*x)^3)
 + (c*d*((-2*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(e*(d + e*x)) +
(Sqrt[c]*Sqrt[d]*ArcTanh[(c*d^2 + a*e^2 + 2*c*d*e*x)/(2*Sqrt[c]*Sqrt[d]*Sq
rt[e]*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]])])/e^(3/2)))/e))/e))/e
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1092
```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[I
nt[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a
, b, c}, x]
```

rule 1125
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_
Symbol] :> Simp[-2*e^(2*m + 3)*(Sqrt[a + b*x + c*x^2]/((-2*c*d + b*e)^(m +
2)*(d + e*x))), x] - Simp[e^(2*m + 2)   Int[(1/Sqrt[a + b*x + c*x^2])*Expan
dToSum[((-2*c*d + b*e)^(-m - 1) - ((-c)*d + b*e + c*e*x)^(-m - 1))/(d + e*x
), x], x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0]
 && ILtQ[m, 0] && EqQ[m + p, -3/2]
```

rule 1130
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(d + e*x)^(m + 1)*((a + b*x + c*x^2)^p/(e*(m + p + 1))), x]
- Simp[c*(p/(e^2*(m + p + 1)))   Int[(d + e*x)^(m + 2)*(a + b*x + c*x^2)^(p
 - 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0]
 && GtQ[p, 0] && (LtQ[m, -2] || EqQ[m + 2*p + 1, 0]) && NeQ[m + p + 1, 0] &
& IntegerQ[2*p]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 990 vs. $2(227) = 454$.

Time = 7.24 (sec) , antiderivative size = 991, normalized size of antiderivative = 3.89

method	result	size
default	Expression too large to display	991

input
```
int((a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(7/2)/(e*x+d)^8,x,method=_RETURNVERB
OSE)
```

output
```
1/e^8*(-2/7/(a*e^2-c*d^2)/(x+d/e)^8*(d*e*c*(x+d/e)^2+(a*e^2-c*d^2)*(x+d/e)
)^(9/2)+2/7*d*e*c/(a*e^2-c*d^2)*(-2/5/(a*e^2-c*d^2)/(x+d/e)^7*(d*e*c*(x+d/
e)^2+(a*e^2-c*d^2)*(x+d/e))^(9/2)+4/5*d*e*c/(a*e^2-c*d^2)*(-2/3/(a*e^2-c*d
^2)/(x+d/e)^6*(d*e*c*(x+d/e)^2+(a*e^2-c*d^2)*(x+d/e))^(9/2)+2*d*e*c/(a*e^2
-c*d^2)*(-2/(a*e^2-c*d^2)/(x+d/e)^5*(d*e*c*(x+d/e)^2+(a*e^2-c*d^2)*(x+d/e)
)^(9/2)+8*d*e*c/(a*e^2-c*d^2)*(2/(a*e^2-c*d^2)/(x+d/e)^4*(d*e*c*(x+d/e)^2+
(a*e^2-c*d^2)*(x+d/e))^(9/2)-10*d*e*c/(a*e^2-c*d^2)*(2/3/(a*e^2-c*d^2)/(x+
d/e)^3*(d*e*c*(x+d/e)^2+(a*e^2-c*d^2)*(x+d/e))^(9/2)-4*d*e*c/(a*e^2-c*d^2)
*(2/5/(a*e^2-c*d^2)/(x+d/e)^2*(d*e*c*(x+d/e)^2+(a*e^2-c*d^2)*(x+d/e))^(9/2
)-14/5*d*e*c/(a*e^2-c*d^2)*(1/7*(d*e*c*(x+d/e)^2+(a*e^2-c*d^2)*(x+d/e))^(7
/2)+1/2*(a*e^2-c*d^2)*(1/12*(2*d*e*c*(x+d/e)+a*e^2-c*d^2)/d/e/c*(d*e*c*(x+
d/e)^2+(a*e^2-c*d^2)*(x+d/e))^(5/2)-5/24*(a*e^2-c*d^2)^2/d/e/c*(1/8*(2*d*e
*c*(x+d/e)+a*e^2-c*d^2)/d/e/c*(d*e*c*(x+d/e)^2+(a*e^2-c*d^2)*(x+d/e))^(3/2
)-3/16*(a*e^2-c*d^2)^2/d/e/c*(1/4*(2*d*e*c*(x+d/e)+a*e^2-c*d^2)/d/e/c*(d*e
*c*(x+d/e)^2+(a*e^2-c*d^2)*(x+d/e))^(1/2)-1/8*(a*e^2-c*d^2)^2/d/e/c*ln((1/
2*a*e^2-1/2*c*d^2+d*e*c*(x+d/e))/(d*e*c)^(1/2)+(d*e*c*(x+d/e)^2+(a*e^2-c*d
^2)*(x+d/e))^(1/2))/(d*e*c)^(1/2))))))))))
```

Fricas [A] (verification not implemented)

Time = 3.53 (sec) , antiderivative size = 744, normalized size of antiderivative = 2.92

$$\int \frac{(ade+(cd^2+ae^2)\,x+cdex^2)^{7/2}}{(d+ex)^8}\,dx = \left[\frac{105\,(c^3d^3e^4x^4+4\,c^3d^4e^3x^3+6\,c^3d^5e^2x^2+4\,c^3d^6ex+c^3d^7)\sqrt{\frac{cd}{e}}}{}\right.$$

$$-\frac{105\,(c^3d^3e^4x^4+4\,c^3d^4e^3x^3+6\,c^3d^5e^2x^2+4\,c^3d^6ex+c^3d^7)\sqrt{-\frac{cd}{e}}\arctan\left(\frac{\sqrt{cdex^2+ade+(cd^2+ae^2)x}\,(2\,cdex+cd^2+}{2\,(c^2d^2ex^2+acd^2e+(c^2d^3+acde^2}\right.}{}$$

input
```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(7/2)/(e*x+d)^8,x, algorithm="
fricas")
```

output
```
[1/210*(105*(c^3*d^3*e^4*x^4 + 4*c^3*d^4*e^3*x^3 + 6*c^3*d^5*e^2*x^2 + 4*c
^3*d^6*e*x + c^3*d^7)*sqrt(c*d/e)*log(8*c^2*d^2*e^2*x^2 + c^2*d^4 + 6*a*c*
d^2*e^2 + a^2*e^4 + 4*(2*c*d*e^2*x + c*d^2*e + a*e^3)*sqrt(c*d*e*x^2 + a*d
*e + (c*d^2 + a*e^2)*x)*sqrt(c*d/e) + 8*(c^2*d^3*e + a*c*d*e^3)*x) - 4*(17
6*c^3*d^3*e^3*x^3 + 105*c^3*d^6 + 35*a*c^2*d^4*e^2 + 21*a^2*c*d^2*e^4 + 15
*a^3*e^6 + 2*(203*c^3*d^4*e^2 + 61*a*c^2*d^2*e^4)*x^2 + 2*(175*c^3*d^5*e +
 56*a*c^2*d^3*e^3 + 33*a^2*c*d*e^5)*x)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a
*e^2)*x))/(e^8*x^4 + 4*d*e^7*x^3 + 6*d^2*e^6*x^2 + 4*d^3*e^5*x + d^4*e^4),
 -1/105*(105*(c^3*d^3*e^4*x^4 + 4*c^3*d^4*e^3*x^3 + 6*c^3*d^5*e^2*x^2 + 4*
c^3*d^6*e*x + c^3*d^7)*sqrt(-c*d/e)*arctan(1/2*sqrt(c*d*e*x^2 + a*d*e + (c
*d^2 + a*e^2)*x)*(2*c*d*e*x + c*d^2 + a*e^2)*sqrt(-c*d/e)/(c^2*d^2*e*x^2 +
 a*c*d^2*e + (c^2*d^3 + a*c*d*e^2)*x)) + 2*(176*c^3*d^3*e^3*x^3 + 105*c^3*
d^6 + 35*a*c^2*d^4*e^2 + 21*a^2*c*d^2*e^4 + 15*a^3*e^6 + 2*(203*c^3*d^4*e^
2 + 61*a*c^2*d^2*e^4)*x^2 + 2*(175*c^3*d^5*e + 56*a*c^2*d^3*e^3 + 33*a^2*c
*d*e^5)*x)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x))/(e^8*x^4 + 4*d*e^7
*x^3 + 6*d^2*e^6*x^2 + 4*d^3*e^5*x + d^4*e^4)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(ade + (cd^2 + ae^2)\,x + cdex^2)^{7/2}}{(d + ex)^8}\,dx = \text{Timed out}$$

input
```
integrate((a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(7/2)/(e*x+d)**8,x)
```

output
```
Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(ade + (cd^2 + ae^2)\,x + cdex^2)^{7/2}}{(d + ex)^8}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(7/2)/(e*x+d)^8,x, algorithm="
maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e*(a*e^2-c*d^2)>0)', see `assume
?` for mor
```

Giac [F(-2)]

Exception generated.

$$\int \frac{(ade + (cd^2 + ae^2)x + cdex^2)^{7/2}}{(d + ex)^8} dx = \text{Exception raised: TypeError}$$

input
```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(7/2)/(e*x+d)^8,x, algorithm="
giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Unable to divide, perhaps due to ro
unding error%%%{%%{[%%%{1,[0,0,20]%%%},0]:[1,0,%%%{-1,[1,1,1]%%%}]%%},[8,8
]%%%}+%%%
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(ade + (cd^2 + ae^2)x + cdex^2)^{7/2}}{(d + ex)^8} dx = \int \frac{(c\,d\,e\,x^2 + (c\,d^2 + a\,e^2)\,x + a\,d\,e)^{7/2}}{(d + e\,x)^8} dx$$

input
```
int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(7/2)/(d + e*x)^8,x)
```

output
```
int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(7/2)/(d + e*x)^8, x)
```

Reduce [B] (verification not implemented)

Time = 0.72 (sec) , antiderivative size = 701, normalized size of antiderivative = 2.75

$$\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{7/2}}{(d+ex)^8}\,dx=\text{Too large to display}$$

input
```
int((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(7/2)/(e*x+d)^8,x)
```

output
```
(2*( - 15*sqrt(d + e*x)*sqrt(a*e + c*d*x)*a**3*e**7 - 21*sqrt(d + e*x)*sqr
t(a*e + c*d*x)*a**2*c*d**2*e**5 - 66*sqrt(d + e*x)*sqrt(a*e + c*d*x)*a**2*
c*d*e**6*x - 35*sqrt(d + e*x)*sqrt(a*e + c*d*x)*a*c**2*d**4*e**3 - 112*sqr
t(d + e*x)*sqrt(a*e + c*d*x)*a*c**2*d**3*e**4*x - 122*sqrt(d + e*x)*sqrt(a
*e + c*d*x)*a*c**2*d**2*e**5*x**2 - 105*sqrt(d + e*x)*sqrt(a*e + c*d*x)*c*
*3*d**6*e - 350*sqrt(d + e*x)*sqrt(a*e + c*d*x)*c**3*d**5*e**2*x - 406*sqr
t(d + e*x)*sqrt(a*e + c*d*x)*c**3*d**4*e**3*x**2 - 176*sqrt(d + e*x)*sqrt(
a*e + c*d*x)*c**3*d**3*e**4*x**3 + 105*sqrt(e)*sqrt(d)*sqrt(c)*log((sqrt(e
)*sqrt(a*e + c*d*x) + sqrt(d)*sqrt(c)*sqrt(d + e*x))/sqrt(a*e**2 - c*d**2)
)*c**3*d**7 + 420*sqrt(e)*sqrt(d)*sqrt(c)*log((sqrt(e)*sqrt(a*e + c*d*x) +
 sqrt(d)*sqrt(c)*sqrt(d + e*x))/sqrt(a*e**2 - c*d**2))*c**3*d**6*e*x + 630
*sqrt(e)*sqrt(d)*sqrt(c)*log((sqrt(e)*sqrt(a*e + c*d*x) + sqrt(d)*sqrt(c)*
sqrt(d + e*x))/sqrt(a*e**2 - c*d**2))*c**3*d**5*e**2*x**2 + 420*sqrt(e)*sq
rt(d)*sqrt(c)*log((sqrt(e)*sqrt(a*e + c*d*x) + sqrt(d)*sqrt(c)*sqrt(d + e*
x))/sqrt(a*e**2 - c*d**2))*c**3*d**4*e**3*x**3 + 105*sqrt(e)*sqrt(d)*sqrt(
c)*log((sqrt(e)*sqrt(a*e + c*d*x) + sqrt(d)*sqrt(c)*sqrt(d + e*x))/sqrt(a*
e**2 - c*d**2))*c**3*d**3*e**4*x**4 + 56*sqrt(e)*sqrt(d)*sqrt(c)*c**3*d**7
 + 224*sqrt(e)*sqrt(d)*sqrt(c)*c**3*d**6*e*x + 336*sqrt(e)*sqrt(d)*sqrt(c)
*c**3*d**5*e**2*x**2 + 224*sqrt(e)*sqrt(d)*sqrt(c)*c**3*d**4*e**3*x**3 + 5
6*sqrt(e)*sqrt(d)*sqrt(c)*c**3*d**3*e**4*x**4))/(105*e**5*(d**4 + 4*d**...
```

3.244 $\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{7/2}}{(d+ex)^9}\,dx$

Optimal result

Integrand size = 37, antiderivative size = 54

$$\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{7/2}}{(d+ex)^9}\,dx=\frac{2\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{9/2}}{9\left(cd^2-ae^2\right)(d+ex)^9}$$

output
```
2/9*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(9/2)/(-a*e^2+c*d^2)/(e*x+d)^9
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.98

$$\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{7/2}}{(d+ex)^9}\,dx=\frac{2(ae+cdx)^4\sqrt{(ae+cdx)(d+ex)}}{9\left(cd^2-ae^2\right)(d+ex)^5}$$

input
```
Integrate[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(7/2)/(d + e*x)^9,x]
```

output
```
(2*(a*e + c*d*x)^4*Sqrt[(a*e + c*d*x)*(d + e*x)])/(9*(c*d^2 - a*e^2)*(d + 
e*x)^5)
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.027, Rules used = {1123}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{7/2}}{(d+ex)^9}dx$$

$$\Big\downarrow \; 1123$$

$$\frac{2\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{9/2}}{9(d+ex)^9\left(cd^2-ae^2\right)}$$

input
```
Int[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(7/2)/(d + e*x)^9,x]
```

output
```
(2*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(9/2))/(9*(c*d^2 - a*e^2)*(d +
e*x)^9)
```

Defintions of rubi rules used

rule 1123
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[e*(d + e*x)^m*((a + b*x + c*x^2)^(p + 1)/((p + 1)*(2*c*d - b
*e))), x] /; FreeQ[{a, b, c, d, e, m, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2,
0] && EqQ[m + 2*p + 2, 0]
```

Maple [A] (verified)

Time = 8.90 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.07

method	result	size
gosper	$-\frac{2(cdx+ae)(cd\,x^2e+a\,e^2x+c\,d^2x+ade)^{\frac{7}{2}}}{9(ex+d)^8(a\,e^2-c\,d^2)}$	58
orering	$-\frac{2(cdx+ae)(ade+(a\,e^2+c\,d^2)x+cd\,x^2e)^{\frac{7}{2}}}{9(a\,e^2-c\,d^2)(ex+d)^8}$	59
default	$-\frac{2\left(dec\left(x+\frac{d}{e}\right)^2+(a\,e^2-c\,d^2)\left(x+\frac{d}{e}\right)\right)^{\frac{9}{2}}}{9e^9(a\,e^2-c\,d^2)\left(x+\frac{d}{e}\right)^9}$	65
trager	$-\frac{2(c^4x^4d^4+4a\,c^3d^3e\,x^3+6a^2c^2d^2e^2x^2+4a^3cd\,e^3x+a^4e^4)\sqrt{cd\,x^2e+a\,e^2x+c\,d^2x+ade}}{9(ex+d)^5(a\,e^2-c\,d^2)}$	109

input
```
int((a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(7/2)/(e*x+d)^9,x,method=_RETURNVERB
OSE)
```

output
```
-2/9/(e*x+d)^8*(c*d*x+a*e)/(a*e^2-c*d^2)*(c*d*e*x^2+a*e^2*x+c*d^2*x+a*d*e)
^(7/2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 210 vs. 2(50) = 100.

Time = 8.26 (sec) , antiderivative size = 210, normalized size of antiderivative = 3.89

$$\int \frac{(ade+(cd^2+ae^2)\,x+cdex^2)^{7/2}}{(d+ex)^9}\,dx = \frac{2\,(c^4d^4x^4+4\,ac^3d^3ex^3+6\,a^2c^2d^2e^2x^2+4\,a^3cde}{9\,(cd^7-ad^5e^2+(cd^2e^5-ae^7)x^5+5\,(cd^3e^4-ade^6)x^4+10\,(cd^4e}$$

input
```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(7/2)/(e*x+d)^9,x, algorithm="
fricas")
```

output
```
2/9*(c^4*d^4*x^4 + 4*a*c^3*d^3*e*x^3 + 6*a^2*c^2*d^2*e^2*x^2 + 4*a^3*c*d*e
^3*x + a^4*e^4)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)/(c*d^7 - a*d^5
*e^2 + (c*d^2*e^5 - a*e^7)*x^5 + 5*(c*d^3*e^4 - a*d*e^6)*x^4 + 10*(c*d^4*e
^3 - a*d^2*e^5)*x^3 + 10*(c*d^5*e^2 - a*d^3*e^4)*x^2 + 5*(c*d^6*e - a*d^4*
e^3)*x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(ade + (cd^2 + ae^2)\,x + cdex^2)^{7/2}}{(d + ex)^9}\,dx = \text{Timed out}$$

input
```
integrate((a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(7/2)/(e*x+d)**9,x)
```

output
```
Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(ade + (cd^2 + ae^2)\,x + cdex^2)^{7/2}}{(d + ex)^9}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(7/2)/(e*x+d)^9,x, algorithm="
maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e*(a*e^2-c*d^2)>0)', see `assume
?` for mor
```

Giac [F(-2)]

Exception generated.

$$\int \frac{(ade + (cd^2 + ae^2)x + cdex^2)^{7/2}}{(d+ex)^9} dx = \text{Exception raised: TypeError}$$

input
```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(7/2)/(e*x+d)^9,x, algorithm="
giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Unable to divide, perhaps due to ro
unding error%%%{%%%{1,[0,0,5]%%%},[10]%%%}+%%%{%%{[%%%{-10,[0,1,4]%%%},0]:
[1,0,%%%{
```

Mupad [B] (verification not implemented)

Time = 9.57 (sec) , antiderivative size = 4399, normalized size of antiderivative = 81.46

$$\int \frac{(ade + (cd^2 + ae^2)x + cdex^2)^{7/2}}{(d+ex)^9} dx = \text{Too large to display}$$

input
```
int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(7/2)/(d + e*x)^9,x)
```

output
```
(((d*((d*((d*((4*c^5*d^6)/(9*(a*e^2 - c*d^2)*(7*a*e^3 - 7*c*d^2*e)) - (2*c
^4*d^4*(9*a*e^2 - c*d^2))/(9*(a*e^2 - c*d^2)*(7*a*e^3 - 7*c*d^2*e))))/e +
(2*c^3*d^3*(16*a^2*e^4 + c^2*d^4 - 5*a*c*d^2*e^2))/(9*e*(a*e^2 - c*d^2)*(7
*a*e^3 - 7*c*d^2*e))))/e - (2*c^2*d^2*(14*a^3*e^6 - c^3*d^6 + 5*a*c^2*d^4*
e^2 - 10*a^2*c*d^2*e^4))/(9*e^2*(a*e^2 - c*d^2)*(7*a*e^3 - 7*c*d^2*e))))/e
 - (2*c^5*d^9 - 6*a*c^4*d^7*e^2 + 4*a^2*c^3*d^5*e^4 + 4*a^3*c^2*d^3*e^6 -
8*a^4*c*d*e^8)/(9*e^3*(a*e^2 - c*d^2)*(7*a*e^3 - 7*c*d^2*e)))*(x*(a*e^2 +
c*d^2) + a*d*e + c*d*e*x^2)^(1/2))/(d + e*x)^4 - (((d*((d*((d*((4*c^5*d^6)
/(9*(a*e^2 - c*d^2)*(7*a*e^3 - 7*c*d^2*e)) - (4*c^4*d^4*(9*a*e^2 - 5*c*d^2
))/(9*(a*e^2 - c*d^2)*(7*a*e^3 - 7*c*d^2*e))))/e + (4*c^3*d^3*(26*a^2*e^4
+ 5*c^2*d^4 - 25*a*c*d^2*e^2))/(9*e*(a*e^2 - c*d^2)*(7*a*e^3 - 7*c*d^2*e))
))/e + (20*c^5*d^8 - 100*a*c^4*d^6*e^2 + 200*a^2*c^3*d^4*e^4 - 136*a^3*c^2
*d^2*e^6)/(9*e^2*(a*e^2 - c*d^2)*(7*a*e^3 - 7*c*d^2*e))))/e + (4*a*c*d*(16
*a^3*e^6 - 5*c^3*d^6 + 20*a*c^2*d^4*e^2 - 30*a^2*c*d^2*e^4))/(9*e*(a*e^2 -
 c*d^2)*(7*a*e^3 - 7*c*d^2*e)))*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1
/2))/(d + e*x)^4 - (((d*((d*((d*((32*c^8*d^9)/(945*e*(a*e^2 - c*d^2)^5) -
(64*c^7*d^7*(9*a*e^2 - 7*c*d^2))/(945*e*(a*e^2 - c*d^2)^5)))/e + (32*c^6*d
^6*(133*a^2*e^4 + 85*c^2*d^4 - 212*a*c*d^2*e^2))/(945*e^2*(a*e^2 - c*d^2)^
5)))/e - (64*c^5*d^5*(263*a^3*e^6 - 155*c^3*d^6 + 550*a*c^2*d^4*e^2 - 656*
a^2*c*d^2*e^4))/(945*e^3*(a*e^2 - c*d^2)^5)))/e + (32*a*c^4*d^4*(410*a^...
```

Reduce [B] (verification not implemented)

Time = 2.54 (sec) , antiderivative size = 381, normalized size of antiderivative = 7.06

$$\int \frac{(ade + (cd^2 + ae^2)x + cdex^2)^{7/2}}{(d+ex)^9}\,dx = \frac{-\frac{2\sqrt{ex+d}\,\sqrt{cdx+ae}\,a^4e^9}{9} - \frac{8\sqrt{ex+d}\,\sqrt{cdx+ae}\,a^3cd\,e^8x}{9} - \frac{4\sqrt{ex+d}\,\sqrt{cdx+ae}\,a^2c^2d}{3}}{e^5\left(a\,e^7x^5 - c\,d^2e^5\right.}$$

input
```
int((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(7/2)/(e*x+d)^9,x)
```

output
```
(2*( - sqrt(d + e*x)*sqrt(a*e + c*d*x)*a**4*e**9 - 4*sqrt(d + e*x)*sqrt(a*
e + c*d*x)*a**3*c*d*e**8*x - 6*sqrt(d + e*x)*sqrt(a*e + c*d*x)*a**2*c**2*d
**2*e**7*x**2 - 4*sqrt(d + e*x)*sqrt(a*e + c*d*x)*a*c**3*d**3*e**6*x**3 -
sqrt(d + e*x)*sqrt(a*e + c*d*x)*c**4*d**4*e**5*x**4 - sqrt(e)*sqrt(d)*sqrt
(c)*c**4*d**9 - 5*sqrt(e)*sqrt(d)*sqrt(c)*c**4*d**8*e*x - 10*sqrt(e)*sqrt(
d)*sqrt(c)*c**4*d**7*e**2*x**2 - 10*sqrt(e)*sqrt(d)*sqrt(c)*c**4*d**6*e**3
*x**3 - 5*sqrt(e)*sqrt(d)*sqrt(c)*c**4*d**5*e**4*x**4 - sqrt(e)*sqrt(d)*sq
rt(c)*c**4*d**4*e**5*x**5))/(9*e**5*(a*d**5*e**2 + 5*a*d**4*e**3*x + 10*a*
d**3*e**4*x**2 + 10*a*d**2*e**5*x**3 + 5*a*d*e**6*x**4 + a*e**7*x**5 - c*d
**7 - 5*c*d**6*e*x - 10*c*d**5*e**2*x**2 - 10*c*d**4*e**3*x**3 - 5*c*d**3*
e**4*x**4 - c*d**2*e**5*x**5))
```

3.245 $\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{7/2}}{(d+ex)^{10}}\,dx$

Optimal result

Integrand size = 37, antiderivative size = 111

$$\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{7/2}}{(d+ex)^{10}}\,dx=\frac{2\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{9/2}}{11\left(cd^2-ae^2\right)(d+ex)^{10}}+\frac{4cd\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{9/2}}{99\left(cd^2-ae^2\right)^2(d+ex)^9}$$

output
```
2/11*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(9/2)/(-a*e^2+c*d^2)/(e*x+d)^10+4/9
9*c*d*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(9/2)/(-a*e^2+c*d^2)^2/(e*x+d)^9
```

Mathematica [A] (verified)

Time = 0.42 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.65

$$\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{7/2}}{(d+ex)^{10}}\,dx=\frac{2(ae+cdx)^4\sqrt{(ae+cdx)(d+ex)}\left(11cd^2-9ae^2+2cdex\right)}{99\left(cd^2-ae^2\right)^2(d+ex)^6}$$

input
```
Integrate[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(7/2)/(d + e*x)^10,x]
```

output
```
(2*(a*e + c*d*x)^4*Sqrt[(a*e + c*d*x)*(d + e*x)]*(11*c*d^2 - 9*a*e^2 + 2*c
*d*e*x))/(99*(c*d^2 - a*e^2)^2*(d + e*x)^6)
```

Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 111, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.054$, Rules used = {1129, 1123}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{7/2}}{(d+ex)^{10}}dx$$

$$\Big\downarrow \text{1129}$$

$$\frac{2cd\int \frac{\left(cdex^2+\left(cd^2+ae^2\right)x+ade\right)^{7/2}}{(d+ex)^9}dx}{11\left(cd^2-ae^2\right)}+\frac{2\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{9/2}}{11(d+ex)^{10}\left(cd^2-ae^2\right)}$$

$$\Big\downarrow \text{1123}$$

$$\frac{4cd\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{9/2}}{99(d+ex)^9\left(cd^2-ae^2\right)^2}+\frac{2\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{9/2}}{11(d+ex)^{10}\left(cd^2-ae^2\right)}$$

input
```
Int[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(7/2)/(d + e*x)^10,x]
```

output
```
(2*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(9/2))/(11*(c*d^2 - a*e^2)*(d +
 e*x)^10) + (4*c*d*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(9/2))/(99*(c*d
^2 - a*e^2)^2*(d + e*x)^9)
```

Defintions of rubi rules used

rule 1123
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[e*(d + e*x)^m*((a + b*x + c*x^2)^(p + 1)/((p + 1)*(2*c*d - b
*e))), x] /; FreeQ[{a, b, c, d, e, m, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2,
0] && EqQ[m + 2*p + 2, 0]
```

rule 1129
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(-e)*(d + e*x)^m*((a + b*x + c*x^2)^(p + 1)/((m + p + 1)*(2*
c*d - b*e))), x] + Simp[c*(Simplify[m + 2*p + 2]/((m + p + 1)*(2*c*d - b*e)
))   Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d
, e, m, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && ILtQ[Simplify[m + 2*p +
2], 0]
```

Maple [A] (verified)

Time = 10.61 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.81

method	result
gosper	$-\frac{2(cdx+ae)\left(-2cdxe+9a\,e^{2}-11c\,d^{2}\right)\left(cd\,x^{2}e+a\,e^{2}x+c\,d^{2}x+ade\right)^{\frac{7}{2}}}{99(ex+d)^{9}\left(a^{2}e^{4}-2ac\,d^{2}e^{2}+c^{2}d^{4}\right)}$
orering	$-\frac{2\left(-2cdxe+9a\,e^{2}-11c\,d^{2}\right)(cdx+ae)\left(ade+\left(a\,e^{2}+c\,d^{2}\right)x+cd\,x^{2}e\right)^{\frac{7}{2}}}{99\left(a^{2}e^{4}-2ac\,d^{2}e^{2}+c^{2}d^{4}\right)(ex+d)^{9}}$
default	$\frac{-\frac{2\left(dec\left(x+\frac{d}{e}\right)^{2}+\left(a\,e^{2}-c\,d^{2}\right)\left(x+\frac{d}{e}\right)\right)^{\frac{9}{2}}}{11\left(a\,e^{2}-c\,d^{2}\right)\left(x+\frac{d}{e}\right)^{10}}+\frac{4dec\left(dec\left(x+\frac{d}{e}\right)^{2}+\left(a\,e^{2}-c\,d^{2}\right)\left(x+\frac{d}{e}\right)\right)^{\frac{9}{2}}}{99\left(a\,e^{2}-c\,d^{2}\right)^{2}\left(x+\frac{d}{e}\right)^{9}}}{e^{10}}$
trager	$-\frac{2\left(-2c^{5}d^{5}e\,x^{5}+a\,c^{4}d^{4}e^{2}x^{4}-11c^{5}d^{6}x^{4}+24a^{2}c^{3}d^{3}e^{3}x^{3}-44a\,c^{4}d^{5}e\,x^{3}+46a^{3}c^{2}d^{2}e^{4}x^{2}-66a^{2}c^{3}d^{4}e^{2}x^{2}+34a^{4}cd\,e^{5}x-44a^{3}c^{2}d^{3}e^{3}x\right.}{99\left(a^{2}e^{4}-2ac\,d^{2}e^{2}+c^{2}d^{4}\right)(ex+d)^{6}}$

input
```
int((a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(7/2)/(e*x+d)^10,x,method=_RETURNVER
BOSE)
```

output
```
-2/99*(c*d*x+a*e)*(-2*c*d*e*x+9*a*e^2-11*c*d^2)*(c*d*e*x^2+a*e^2*x+c*d^2*x
+a*d*e)^(7/2)/(e*x+d)^9/(a^2*e^4-2*a*c*d^2*e^2+c^2*d^4)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 415 vs. $2(103) = 206$.

Time = 23.34 (sec) , antiderivative size = 415, normalized size of antiderivative = 3.74

$$\int \frac{(ade + (cd^2 + ae^2)x + cdex^2)^{7/2}}{(d+ex)^{10}}\,dx = \frac{2\,(2\,c^5d^5ex^5 + 11\,a^4cd^2e^4 - 9\,a^5e^6 + (}{99\,(c^2d^{10} - 2\,acd^8e^2 + a^2d^6e^4 + (c^2d^4e^6 - 2\,acd^2e^8 + a^2e^{10})x^6 + (}$$

input
```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(7/2)/(e*x+d)^10,x, algorithm=
"fricas")
```

output
```
2/99*(2*c^5*d^5*e*x^5 + 11*a^4*c*d^2*e^4 - 9*a^5*e^6 + (11*c^5*d^6 - a*c^4
*d^4*e^2)*x^4 + 4*(11*a*c^4*d^5*e - 6*a^2*c^3*d^3*e^3)*x^3 + 2*(33*a^2*c^3
*d^4*e^2 - 23*a^3*c^2*d^2*e^4)*x^2 + 2*(22*a^3*c^2*d^3*e^3 - 17*a^4*c*d*e^
5)*x)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)/(c^2*d^10 - 2*a*c*d^8*e^
2 + a^2*d^6*e^4 + (c^2*d^4*e^6 - 2*a*c*d^2*e^8 + a^2*e^10)*x^6 + 6*(c^2*d^
5*e^5 - 2*a*c*d^3*e^7 + a^2*d*e^9)*x^5 + 15*(c^2*d^6*e^4 - 2*a*c*d^4*e^6 +
 a^2*d^2*e^8)*x^4 + 20*(c^2*d^7*e^3 - 2*a*c*d^5*e^5 + a^2*d^3*e^7)*x^3 + 1
5*(c^2*d^8*e^2 - 2*a*c*d^6*e^4 + a^2*d^4*e^6)*x^2 + 6*(c^2*d^9*e - 2*a*c*d
^7*e^3 + a^2*d^5*e^5)*x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(ade + (cd^2 + ae^2)x + cdex^2)^{7/2}}{(d+ex)^{10}}\,dx = \text{Timed out}$$

input
```
integrate((a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(7/2)/(e*x+d)**10,x)
```

output
```
Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(ade + (cd^2 + ae^2)x + cdex^2)^{7/2}}{(d + ex)^{10}} dx = \text{Exception raised: ValueError}$$

input
```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(7/2)/(e*x+d)^10,x, algorithm=
"maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e*(a*e^2-c*d^2)>0)', see `assume
?` for mor
```

Giac [F(-2)]

Exception generated.

$$\int \frac{(ade + (cd^2 + ae^2)x + cdex^2)^{7/2}}{(d + ex)^{10}} dx = \text{Exception raised: TypeError}$$

input
```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(7/2)/(e*x+d)^10,x, algorithm=
"giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Unable to divide, perhaps due to ro
unding error%%%{%%%{1,[0,0,6]%%%},[12]%%%}+%%%{%%{[%%%{-12,[0,1,5]%%%},0]:
[1,0,%%%{
```

Mupad [B] (verification not implemented)

Time = 11.11 (sec) , antiderivative size = 5860, normalized size of antiderivative = 52.79

$$\int \frac{\left(ade+(cd^2+ae^2)\,x+cdex^2\right)^{7/2}}{(d+ex)^{10}}\,dx = \text{Too large to display}$$

input
```
int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(7/2)/(d + e*x)^10,x)
```

output
```
(((d*((8*c^6*d^7)/(99*e^2*(a*e^2 - c*d^2)^2*(3*a*e^3 - 3*c*d^2*e)) - (4*c^
5*d^5*(21*a*e^2 - 17*c*d^2))/(99*e^2*(a*e^2 - c*d^2)^2*(3*a*e^3 - 3*c*d^2*
e))))/e + (4*c^4*d^4*(66*a^2*e^4 - 67*c^2*d^4 + 15*a*c*d^2*e^2))/(693*e^3*
(a*e^2 - c*d^2)^2*(3*a*e^3 - 3*c*d^2*e)))*(x*(a*e^2 + c*d^2) + a*d*e + c*d
*e*x^2)^(1/2))/(d + e*x)^2 - (((286*c^5*d^6 - 706*a*c^4*d^4*e^2)/(1155*e^3
*(a*e^2 - c*d^2)*(3*a*e^3 - 3*c*d^2*e)) + (4*c^5*d^6)/(11*e^3*(a*e^2 - c*d
^2)*(3*a*e^3 - 3*c*d^2*e)))*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2))
/(d + e*x)^2 - (((d*((d*((d*((64*c^9*d^10)/(10395*e*(a*e^2 - c*d^2)^6) - (
256*c^8*d^8*(6*a*e^2 - 5*c*d^2))/(10395*e*(a*e^2 - c*d^2)^6)))/e + (64*c^7
*d^7*(241*a^2*e^4 + 175*c^2*d^4 - 410*a*c*d^2*e^2))/(10395*e^2*(a*e^2 - c*
d^2)^6)))/e - (128*c^6*d^6*(662*a^3*e^6 - 455*c^3*d^6 + 1540*a*c^2*d^4*e^2
 - 1745*a^2*c*d^2*e^4))/(10395*e^3*(a*e^2 - c*d^2)^6)))/e + (64*a*c^5*d^5*
(1106*a^3*e^6 - 910*c^3*d^6 + 2905*a*c^2*d^4*e^2 - 3100*a^2*c*d^2*e^4))/(1
0395*e^2*(a*e^2 - c*d^2)^6))*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2)
)/(d + e*x) - (((d*((d*((d*((4*c^5*d^6)/(11*(a*e^2 - c*d^2)*(9*a*e^3 - 9*c
*d^2*e)) - (8*c^4*d^4*(5*a*e^2 - 3*c*d^2))/(11*(a*e^2 - c*d^2)*(9*a*e^3 -
9*c*d^2*e))))/e + (24*c^3*d^3*(5*a^2*e^4 + c^2*d^4 - 5*a*c*d^2*e^2))/(11*e
*(a*e^2 - c*d^2)*(9*a*e^3 - 9*c*d^2*e))))/e + (24*c^5*d^8 - 120*a*c^4*d^6*
e^2 + 240*a^2*c^3*d^4*e^4 - 160*a^3*c^2*d^2*e^6)/(11*e^2*(a*e^2 - c*d^2)*(
9*a*e^3 - 9*c*d^2*e))))/e + (4*a*c*d*(19*a^3*e^6 - 6*c^3*d^6 + 24*a*c^2...
```

Reduce [B] (verification not implemented)

Time = 0.69 (sec) , antiderivative size = 717, normalized size of antiderivative = 6.46

$$\int \frac{\left(ade+(cd^2+ae^2)\,x+cdex^2\right)^{7/2}}{(d+ex)^{10}}\,dx = \frac{-\frac{2\sqrt{ex+d}\,\sqrt{cdx+ae}\,a^5e^{11}}{11}+\frac{2\sqrt{ex+d}\,\sqrt{cdx+ae}\,a^4c\,d^2e^9}{9}-\frac{68\sqrt{ex+d}\,\sqrt{cdx+ae}\,a^4c}{99}}{}$$

input
```
int((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(7/2)/(e*x+d)^10,x)
```

output

```
(2*( - 9*sqrt(d + e*x)*sqrt(a*e + c*d*x)*a**5*e**11 + 11*sqrt(d + e*x)*sqr
t(a*e + c*d*x)*a**4*c*d**2*e**9 - 34*sqrt(d + e*x)*sqrt(a*e + c*d*x)*a**4*
c*d*e**10*x + 44*sqrt(d + e*x)*sqrt(a*e + c*d*x)*a**3*c**2*d**3*e**8*x - 4
6*sqrt(d + e*x)*sqrt(a*e + c*d*x)*a**3*c**2*d**2*e**9*x**2 + 66*sqrt(d + e
*x)*sqrt(a*e + c*d*x)*a**2*c**3*d**4*e**7*x**2 - 24*sqrt(d + e*x)*sqrt(a*e
 + c*d*x)*a**2*c**3*d**3*e**8*x**3 + 44*sqrt(d + e*x)*sqrt(a*e + c*d*x)*a*
c**4*d**5*e**6*x**3 - sqrt(d + e*x)*sqrt(a*e + c*d*x)*a*c**4*d**4*e**7*x**
4 + 11*sqrt(d + e*x)*sqrt(a*e + c*d*x)*c**5*d**6*e**5*x**4 + 2*sqrt(d + e*
x)*sqrt(a*e + c*d*x)*c**5*d**5*e**6*x**5 - 2*sqrt(e)*sqrt(d)*sqrt(c)*c**5*
d**11 - 12*sqrt(e)*sqrt(d)*sqrt(c)*c**5*d**10*e*x - 30*sqrt(e)*sqrt(d)*sqr
t(c)*c**5*d**9*e**2*x**2 - 40*sqrt(e)*sqrt(d)*sqrt(c)*c**5*d**8*e**3*x**3
- 30*sqrt(e)*sqrt(d)*sqrt(c)*c**5*d**7*e**4*x**4 - 12*sqrt(e)*sqrt(d)*sqrt
(c)*c**5*d**6*e**5*x**5 - 2*sqrt(e)*sqrt(d)*sqrt(c)*c**5*d**5*e**6*x**6))/
(99*e**5*(a**2*d**6*e**4 + 6*a**2*d**5*e**5*x + 15*a**2*d**4*e**6*x**2 + 2
0*a**2*d**3*e**7*x**3 + 15*a**2*d**2*e**8*x**4 + 6*a**2*d*e**9*x**5 + a**2
*e**10*x**6 - 2*a*c*d**8*e**2 - 12*a*c*d**7*e**3*x - 30*a*c*d**6*e**4*x**2
 - 40*a*c*d**5*e**5*x**3 - 30*a*c*d**4*e**6*x**4 - 12*a*c*d**3*e**7*x**5 -
 2*a*c*d**2*e**8*x**6 + c**2*d**10 + 6*c**2*d**9*e*x + 15*c**2*d**8*e**2*x
**2 + 20*c**2*d**7*e**3*x**3 + 15*c**2*d**6*e**4*x**4 + 6*c**2*d**5*e**5*x
**5 + c**2*d**4*e**6*x**6))
```

3.246 $\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{7/2}}{(d+ex)^{11}}\,dx$

Optimal result

Integrand size = 37, antiderivative size = 171

$$\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{7/2}}{(d+ex)^{11}}\,dx = \frac{2\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{9/2}}{13\left(cd^2-ae^2\right)(d+ex)^{11}} + \frac{8cd\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{9/2}}{143\left(cd^2-ae^2\right)^2(d+ex)^{10}} + \frac{16c^2d^2\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{9/2}}{1287\left(cd^2-ae^2\right)^3(d+ex)^9}$$

output
```
2/13*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(9/2)/(-a*e^2+c*d^2)/(e*x+d)^11+8/1
43*c*d*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(9/2)/(-a*e^2+c*d^2)^2/(e*x+d)^10
+16/1287*c^2*d^2*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(9/2)/(-a*e^2+c*d^2)^3/
(e*x+d)^9
```

Mathematica [A] (verified)

Time = 1.08 (sec) , antiderivative size = 104, normalized size of antiderivative = 0.61

$$\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{7/2}}{(d+ex)^{11}}\,dx = \frac{2(ae+cdx)^4\sqrt{(ae+cdx)(d+ex)}\left(99a^2e^4-18acde^2(13d+2ex)\right.}{1287\left(cd^2-ae^2\right)^3(d+ex)^7}$$

input
```
Integrate[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(7/2)/(d + e*x)^11,x]
```

output
```
(2*(a*e + c*d*x)^4*Sqrt[(a*e + c*d*x)*(d + e*x)]*(99*a^2*e^4 - 18*a*c*d*e^
2*(13*d + 2*e*x) + c^2*d^2*(143*d^2 + 52*d*e*x + 8*e^2*x^2)))/(1287*(c*d^2
 - a*e^2)^3*(d + e*x)^7)
```

Rubi [A] (verified)

Time = 0.53 (sec) , antiderivative size = 186, normalized size of antiderivative = 1.09, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.081$, Rules used = {1129, 1129, 1123}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{7/2}}{(d+ex)^{11}}dx$$

$$\Big\downarrow \text{1129}$$

$$\frac{4cd\int \frac{\left(cdex^2+\left(cd^2+ae^2\right)x+ade\right)^{7/2}}{(d+ex)^{10}}dx}{13\left(cd^2-ae^2\right)}+\frac{2\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{9/2}}{13(d+ex)^{11}\left(cd^2-ae^2\right)}$$

$$\Big\downarrow \text{1129}$$

$$\frac{4cd\left(\frac{2cd\int \frac{\left(cdex^2+\left(cd^2+ae^2\right)x+ade\right)^{7/2}}{(d+ex)^9}dx}{11\left(cd^2-ae^2\right)}+\frac{2\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{9/2}}{11(d+ex)^{10}\left(cd^2-ae^2\right)}\right)}{13\left(cd^2-ae^2\right)}+\frac{2\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{9/2}}{13(d+ex)^{11}\left(cd^2-ae^2\right)}$$

$$\Big\downarrow \text{1123}$$

$$\frac{2\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{9/2}}{13(d+ex)^{11}\left(cd^2-ae^2\right)}+\frac{4cd\left(\frac{4cd\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{9/2}}{99(d+ex)^9\left(cd^2-ae^2\right)^2}+\frac{2\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{9/2}}{11(d+ex)^{10}\left(cd^2-ae^2\right)}\right)}{13\left(cd^2-ae^2\right)}$$

input
```
Int[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(7/2)/(d + e*x)^11,x]
```

output

```
(2*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(9/2))/(13*(c*d^2 - a*e^2)*(d +
 e*x)^11) + (4*c*d*((2*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(9/2))/(11*
(c*d^2 - a*e^2)*(d + e*x)^10) + (4*c*d*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*
x^2)^(9/2))/(99*(c*d^2 - a*e^2)^2*(d + e*x)^9)))/(13*(c*d^2 - a*e^2))
```

Defintions of rubi rules used

rule 1123

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[e*(d + e*x)^m*((a + b*x + c*x^2)^(p + 1)/((p + 1)*(2*c*d - b
*e))), x] /; FreeQ[{a, b, c, d, e, m, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2,
0] && EqQ[m + 2*p + 2, 0]
```

rule 1129

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(-e)*(d + e*x)^m*((a + b*x + c*x^2)^(p + 1)/((m + p + 1)*(2*
c*d - b*e))), x] + Simp[c*(Simplify[m + 2*p + 2]/((m + p + 1)*(2*c*d - b*e)
))   Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d
, e, m, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && ILtQ[Simplify[m + 2*p +
2], 0]
```

Maple [A] (verified)

Time = 13.00 (sec) , antiderivative size = 146, normalized size of antiderivative = 0.85

method	result
gosper	$-\frac{2(cdx+ae)\left(8x^{2}c^{2}d^{2}e^{2}-36xacd\,e^{3}+52x\,c^{2}d^{3}e+99a^{2}e^{4}-234ac\,d^{2}e^{2}+143c^{2}d^{4}\right)\left(cd\,x^{2}e+a\,e^{2}x+c\,d^{2}x+ade\right)^{\frac{7}{2}}}{1287(ex+d)^{10}\left(e^{6}a^{3}-3d^{2}e^{4}a^{2}c+3d^{4}e^{2}a\,c^{2}-d^{6}c^{3}\right)}$
orering	$-\frac{2\left(8x^{2}c^{2}d^{2}e^{2}-36xacd\,e^{3}+52x\,c^{2}d^{3}e+99a^{2}e^{4}-234ac\,d^{2}e^{2}+143c^{2}d^{4}\right)(cdx+ae)\left(ade+\left(a\,e^{2}+c\,d^{2}\right)x+cd\,x^{2}e\right)^{\frac{7}{2}}}{1287\left(e^{6}a^{3}-3d^{2}e^{4}a^{2}c+3d^{4}e^{2}a\,c^{2}-d^{6}c^{3}\right)(ex+d)^{10}}$
default	$\frac{-\frac{2\left(dec\left(x+\frac{d}{e}\right)^{2}+\left(a\,e^{2}-c\,d^{2}\right)\left(x+\frac{d}{e}\right)\right)^{\frac{9}{2}}}{13\left(a\,e^{2}-c\,d^{2}\right)\left(x+\frac{d}{e}\right)^{11}}-\frac{4dec\left(-\frac{2\left(dec\left(x+\frac{d}{e}\right)^{2}+\left(a\,e^{2}-c\,d^{2}\right)\left(x+\frac{d}{e}\right)\right)^{\frac{9}{2}}}{11\left(a\,e^{2}-c\,d^{2}\right)\left(x+\frac{d}{e}\right)^{10}}+\frac{4dec\left(dec\left(x+\frac{d}{e}\right)^{2}+\left(a\,e^{2}-c\,d^{2}\right)\left(x+\frac{d}{e}\right)\right)^{\frac{9}{2}}}{99\left(a\,e^{2}-c\,d^{2}\right)^{2}\left(x+\frac{d}{e}\right)^{9}}\right)}{13\left(a\,e^{2}-c\,d^{2}\right)}}{e^{11}}$
trager	$-\frac{2\left(8c^{6}d^{6}e^{2}x^{6}-4a\,c^{5}d^{5}e^{3}x^{5}+52c^{6}d^{7}e\,x^{5}+3a^{2}c^{4}d^{4}e^{4}x^{4}-26a\,c^{5}d^{6}e^{2}x^{4}+143c^{6}d^{8}x^{4}+212a^{3}c^{3}d^{3}e^{5}x^{3}-624a^{2}c^{4}d^{5}e^{3}x^{3}+572a\,c^{5}d^{7}\right.}{1287}$

input
```
int((a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(7/2)/(e*x+d)^11,x,method=_RETURNVER
BOSE)
```

output
```
-2/1287*(c*d*x+a*e)*(8*c^2*d^2*e^2*x^2-36*a*c*d*e^3*x+52*c^2*d^3*e*x+99*a^
2*e^4-234*a*c*d^2*e^2+143*c^2*d^4)*(c*d*e*x^2+a*e^2*x+c*d^2*x+a*d*e)^(7/2)
/(e*x+d)^10/(a^3*e^6-3*a^2*c*d^2*e^4+3*a*c^2*d^4*e^2-c^3*d^6)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 670 vs. $2(159) = 318$.

Time = 49.49 (sec) , antiderivative size = 670, normalized size of antiderivative = 3.92

$$\int \frac{(ade + (cd^2 + ae^2)x + cdex^2)^{7/2}}{(d+ex)^{11}}\,dx = \frac{2\,(8\,c^6d^6e^2x^6 + 143}{1287\,(c^3d^{13} - 3\,ac^2d^{11}e^2 + 3\,a^2cd^9e^4 - a^3d^7e^6 + (c^3d^6e^7 - 3\,ac^2d}$$

input
```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(7/2)/(e*x+d)^11,x, algorithm=
"fricas")
```

output
```
2/1287*(8*c^6*d^6*e^2*x^6 + 143*a^4*c^2*d^4*e^4 - 234*a^5*c*d^2*e^6 + 99*a
^6*e^8 + 4*(13*c^6*d^7*e - a*c^5*d^5*e^3)*x^5 + (143*c^6*d^8 - 26*a*c^5*d^
6*e^2 + 3*a^2*c^4*d^4*e^4)*x^4 + 4*(143*a*c^5*d^7*e - 156*a^2*c^4*d^5*e^3
+ 53*a^3*c^3*d^3*e^5)*x^3 + 2*(429*a^2*c^4*d^6*e^2 - 598*a^3*c^3*d^4*e^4 +
 229*a^4*c^2*d^2*e^6)*x^2 + 4*(143*a^3*c^3*d^5*e^3 - 221*a^4*c^2*d^3*e^5 +
 90*a^5*c*d*e^7)*x)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)/(c^3*d^13
- 3*a*c^2*d^11*e^2 + 3*a^2*c*d^9*e^4 - a^3*d^7*e^6 + (c^3*d^6*e^7 - 3*a*c^
2*d^4*e^9 + 3*a^2*c*d^2*e^11 - a^3*e^13)*x^7 + 7*(c^3*d^7*e^6 - 3*a*c^2*d^
5*e^8 + 3*a^2*c*d^3*e^10 - a^3*d*e^12)*x^6 + 21*(c^3*d^8*e^5 - 3*a*c^2*d^6
*e^7 + 3*a^2*c*d^4*e^9 - a^3*d^2*e^11)*x^5 + 35*(c^3*d^9*e^4 - 3*a*c^2*d^7
*e^6 + 3*a^2*c*d^5*e^8 - a^3*d^3*e^10)*x^4 + 35*(c^3*d^10*e^3 - 3*a*c^2*d^
8*e^5 + 3*a^2*c*d^6*e^7 - a^3*d^4*e^9)*x^3 + 21*(c^3*d^11*e^2 - 3*a*c^2*d^
9*e^4 + 3*a^2*c*d^7*e^6 - a^3*d^5*e^8)*x^2 + 7*(c^3*d^12*e - 3*a*c^2*d^10*
e^3 + 3*a^2*c*d^8*e^5 - a^3*d^6*e^7)*x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(ade+(cd^2+ae^2)\,x+cdex^2)^{7/2}}{(d+ex)^{11}}\,dx=\text{Timed out}$$

input
```
integrate((a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(7/2)/(e*x+d)**11,x)
```

output
```
Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(ade+(cd^2+ae^2)\,x+cdex^2)^{7/2}}{(d+ex)^{11}}\,dx=\text{Exception raised: ValueError}$$

input
```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(7/2)/(e*x+d)^11,x, algorithm=
"maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e*(a*e^2-c*d^2)>0)', see `assume
?` for mor
```

Giac [F(-2)]

Exception generated.

$$\int \frac{(ade+(cd^2+ae^2)\,x+cdex^2)^{7/2}}{(d+ex)^{11}}\,dx=\text{Exception raised: TypeError}$$

input
```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(7/2)/(e*x+d)^11,x, algorithm=
"giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Unable to divide, perhaps due to ro
unding error%%%{%%%{1,[0,0,7]%%%},[14]%%%}+%%%{%%{[%%%{-14,[0,1,6]%%%},0]:
[1,0,%%%{
```

Mupad [B] (verification not implemented)

Time = 14.71 (sec) , antiderivative size = 7337, normalized size of antiderivative = 42.91

$$\int \frac{\left(ade + (cd^2 + ae^2)\,x + cdex^2\right)^{7/2}}{(d+ex)^{11}}\,dx = \text{Too large to display}$$

input
```
int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(7/2)/(d + e*x)^11,x)
```

output
```
(((d*((8*c^6*d^7)/(143*e^2*(a*e^2 - c*d^2)^2*(5*a*e^3 - 5*c*d^2*e)) - (4*c
^5*d^5*(23*a*e^2 - 19*c*d^2))/(143*e^2*(a*e^2 - c*d^2)^2*(5*a*e^3 - 5*c*d^
2*e))))/e + (4*c^4*d^4*(127*a^2*e^4 - 314*c^2*d^4 + 229*a*c*d^2*e^2))/(300
3*e^3*(a*e^2 - c*d^2)^2*(5*a*e^3 - 5*c*d^2*e)))*(x*(a*e^2 + c*d^2) + a*d*e
 + c*d*e*x^2)^(1/2))/(d + e*x)^3 + (((d*((16*c^7*d^8)/(1287*e^2*(a*e^2 - c
*d^2)^3*(3*a*e^3 - 3*c*d^2*e)) - (8*c^6*d^6*(35*a*e^2 - 31*c*d^2))/(1287*e
^2*(a*e^2 - c*d^2)^3*(3*a*e^3 - 3*c*d^2*e))))/e - (8*c^5*d^5*(641*a^2*e^4
+ 872*c^2*d^4 - 1527*a*c*d^2*e^2))/(9009*e^3*(a*e^2 - c*d^2)^3*(3*a*e^3 -
3*c*d^2*e)))*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2))/(d + e*x)^2 -
(((26*c^5*d^6 - 70*a*c^4*d^4*e^2)/(143*e^3*(a*e^2 - c*d^2)*(5*a*e^3 - 5*c*
d^2*e)) + (4*c^5*d^6)/(13*e^3*(a*e^2 - c*d^2)*(5*a*e^3 - 5*c*d^2*e)))*(x*(
a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2))/(d + e*x)^3 - (((6788*c^6*d^7 -
 7628*a*c^5*d^5*e^2)/(15015*e^3*(a*e^2 - c*d^2)^2*(3*a*e^3 - 3*c*d^2*e)) +
 (8*c^6*d^7)/(143*e^3*(a*e^2 - c*d^2)^2*(3*a*e^3 - 3*c*d^2*e)))*(x*(a*e^2
+ c*d^2) + a*d*e + c*d*e*x^2)^(1/2))/(d + e*x)^2 - (((d*((d*((d*((128*c^10
*d^11)/(135135*e*(a*e^2 - c*d^2)^7) - (128*c^9*d^9*(31*a*e^2 - 27*c*d^2))/
(135135*e*(a*e^2 - c*d^2)^7)))/e + (128*c^8*d^8*(409*a^2*e^4 + 322*c^2*d^4
 - 725*a*c*d^2*e^2))/(135135*e^2*(a*e^2 - c*d^2)^7)))/e - (128*c^7*d^7*(30
11*a^3*e^6 - 2282*c^3*d^6 + 7490*a*c^2*d^4*e^2 - 8215*a^2*c*d^2*e^4))/(135
135*e^3*(a*e^2 - c*d^2)^7)))/e + (128*a*c^6*d^6*(2632*a^3*e^6 - 2282*c^...
```

Reduce [B] (verification not implemented)

Time = 1.61 (sec) , antiderivative size = 1125, normalized size of antiderivative = 6.58

$$\int \frac{(ade + (cd^2 + ae^2)x + cdex^2)^{7/2}}{(d + ex)^{11}} dx = \text{Too large to display}$$

input
```
int((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(7/2)/(e*x+d)^11,x)
```

output
```
(2*( - 99*sqrt(d + e*x)*sqrt(a*e + c*d*x)*a**6*e**13 + 234*sqrt(d + e*x)*s
qrt(a*e + c*d*x)*a**5*c*d**2*e**11 - 360*sqrt(d + e*x)*sqrt(a*e + c*d*x)*a
**5*c*d*e**12*x - 143*sqrt(d + e*x)*sqrt(a*e + c*d*x)*a**4*c**2*d**4*e**9
+ 884*sqrt(d + e*x)*sqrt(a*e + c*d*x)*a**4*c**2*d**3*e**10*x - 458*sqrt(d
+ e*x)*sqrt(a*e + c*d*x)*a**4*c**2*d**2*e**11*x**2 - 572*sqrt(d + e*x)*sqr
t(a*e + c*d*x)*a**3*c**3*d**5*e**8*x + 1196*sqrt(d + e*x)*sqrt(a*e + c*d*x
)*a**3*c**3*d**4*e**9*x**2 - 212*sqrt(d + e*x)*sqrt(a*e + c*d*x)*a**3*c**3
*d**3*e**10*x**3 - 858*sqrt(d + e*x)*sqrt(a*e + c*d*x)*a**2*c**4*d**6*e**7
*x**2 + 624*sqrt(d + e*x)*sqrt(a*e + c*d*x)*a**2*c**4*d**5*e**8*x**3 - 3*s
qrt(d + e*x)*sqrt(a*e + c*d*x)*a**2*c**4*d**4*e**9*x**4 - 572*sqrt(d + e*x
)*sqrt(a*e + c*d*x)*a*c**5*d**7*e**6*x**3 + 26*sqrt(d + e*x)*sqrt(a*e + c*
d*x)*a*c**5*d**6*e**7*x**4 + 4*sqrt(d + e*x)*sqrt(a*e + c*d*x)*a*c**5*d**5
*e**8*x**5 - 143*sqrt(d + e*x)*sqrt(a*e + c*d*x)*c**6*d**8*e**5*x**4 - 52*
sqrt(d + e*x)*sqrt(a*e + c*d*x)*c**6*d**7*e**6*x**5 - 8*sqrt(d + e*x)*sqrt
(a*e + c*d*x)*c**6*d**6*e**7*x**6 + 8*sqrt(e)*sqrt(d)*sqrt(c)*c**6*d**13 +
 56*sqrt(e)*sqrt(d)*sqrt(c)*c**6*d**12*e*x + 168*sqrt(e)*sqrt(d)*sqrt(c)*c
**6*d**11*e**2*x**2 + 280*sqrt(e)*sqrt(d)*sqrt(c)*c**6*d**10*e**3*x**3 + 2
80*sqrt(e)*sqrt(d)*sqrt(c)*c**6*d**9*e**4*x**4 + 168*sqrt(e)*sqrt(d)*sqrt(
c)*c**6*d**8*e**5*x**5 + 56*sqrt(e)*sqrt(d)*sqrt(c)*c**6*d**7*e**6*x**6 +
8*sqrt(e)*sqrt(d)*sqrt(c)*c**6*d**6*e**7*x**7))/(1287*e**5*(a**3*d**7*e...
```

3.247 $\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{7/2}}{(d+ex)^{12}}\,dx$

Optimal result

Integrand size = 37, antiderivative size = 231

$$\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{7/2}}{(d+ex)^{12}}\,dx = \frac{2\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{9/2}}{15\left(cd^2-ae^2\right)(d+ex)^{12}} + \frac{4cd\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{9/2}}{65\left(cd^2-ae^2\right)^2(d+ex)^{11}} + \frac{16c^2d^2\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{9/2}}{715\left(cd^2-ae^2\right)^3(d+ex)^{10}} + \frac{32c^3d^3\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{9/2}}{6435\left(cd^2-ae^2\right)^4(d+ex)^9}$$

output
```
2/15*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(9/2)/(-a*e^2+c*d^2)/(e*x+d)^12+4/6
5*c*d*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(9/2)/(-a*e^2+c*d^2)^2/(e*x+d)^11+
16/715*c^2*d^2*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(9/2)/(-a*e^2+c*d^2)^3/(e
*x+d)^10+32/6435*c^3*d^3*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(9/2)/(-a*e^2+c
*d^2)^4/(e*x+d)^9
```

Mathematica [A] (verified)

Time = 2.42 (sec) , antiderivative size = 148, normalized size of antiderivative = 0.64

$$\int \frac{(ade + (cd^2 + ae^2)x + cdex^2)^{7/2}}{(d + ex)^{12}} dx = \frac{2(ae + cdx)^4 \sqrt{(ae + cdx)(d + ex)}(-429a^3e^6 + 99a^2cde^4(15d + 2$$

input
```
Integrate[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(7/2)/(d + e*x)^12,x]
```

output
```
(2*(a*e + c*d*x)^4*Sqrt[(a*e + c*d*x)*(d + e*x)]*(-429*a^3*e^6 + 99*a^2*c*
d*e^4*(15*d + 2*e*x) - 9*a*c^2*d^2*e^2*(195*d^2 + 60*d*e*x + 8*e^2*x^2) +
c^3*d^3*(715*d^3 + 390*d^2*e*x + 120*d*e^2*x^2 + 16*e^3*x^3)))/(6435*(c*d^
2 - a*e^2)^4*(d + e*x)^8)
```

Rubi [A] (verified)

Time = 0.70 (sec) , antiderivative size = 261, normalized size of antiderivative = 1.13, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.108$, Rules used = {1129, 1129, 1129, 1123}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(x\left(ae^2 + cd^2\right) + ade + cdex^2\right)^{7/2}}{(d + ex)^{12}} dx$$

$$\Big\downarrow \text{1129}$$

$$\frac{2cd \int \frac{\left(cdex^2+(cd^2+ae^2)x+ade\right)^{7/2}}{(d+ex)^{11}}dx}{5\left(cd^2 - ae^2\right)} + \frac{2\left(x\left(ae^2 + cd^2\right) + ade + cdex^2\right)^{9/2}}{15(d + ex)^{12}\left(cd^2 - ae^2\right)}$$

$$\Big\downarrow \text{1129}$$

$$\frac{2cd\left(\frac{4cd \int \frac{\left(cdex^2+\left(cd^2+ae^2\right)x+ade\right)^{7/2}}{(d+ex)^{10}}dx}{13(cd^2-ae^2)} + \frac{2\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{9/2}}{13(d+ex)^{11}(cd^2-ae^2)}\right)}{5\left(cd^2 - ae^2\right)} + \frac{2\left(x\left(ae^2 + cd^2\right) + ade + cdex^2\right)^{9/2}}{15(d + ex)^{12}\left(cd^2 - ae^2\right)}$$

$$\Big\downarrow \text{1129}$$

$$\frac{2cd\left(\frac{4cd\left(\frac{2cd\int \frac{\left(cdex^2+\left(cd^2+ae^2\right)x+ade\right)^{7/2}}{(d+ex)^9}dx}{11\left(cd^2-ae^2\right)}+\frac{2\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{9/2}}{11(d+ex)^{10}\left(cd^2-ae^2\right)}\right)}{13(cd^2-ae^2)}+\frac{2(x(ae^2+cd^2)+ade+cdex^2)^{9/2}}{13(d+ex)^{11}(cd^2-ae^2)}\right)}{5\left(cd^2-ae^2\right)}+\frac{2\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{9/2}}{15(d+ex)^{12}\left(cd^2-ae^2\right)}$$

$$\Big\downarrow \text{1123}$$

$$\frac{2\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{9/2}}{15(d+ex)^{12}\left(cd^2-ae^2\right)}+\frac{2cd\left(\frac{2(x(ae^2+cd^2)+ade+cdex^2)^{9/2}}{13(d+ex)^{11}(cd^2-ae^2)}+\frac{4cd\left(\frac{4cd\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{9/2}}{99(d+ex)^9\left(cd^2-ae^2\right)^2}+\frac{2\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{9/2}}{11(d+ex)^{10}\left(cd^2-ae^2\right)}\right)}{13(cd^2-ae^2)}\right)}{5\left(cd^2-ae^2\right)}$$

input

```
Int[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(7/2)/(d + e*x)^12,x]
```

output

```
(2*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(9/2))/(15*(c*d^2 - a*e^2)*(d +
 e*x)^12) + (2*c*d*((2*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(9/2))/(13*
(c*d^2 - a*e^2)*(d + e*x)^11) + (4*c*d*((2*(a*d*e + (c*d^2 + a*e^2)*x + c*
d*e*x^2)^(9/2))/(11*(c*d^2 - a*e^2)*(d + e*x)^10) + (4*c*d*(a*d*e + (c*d^2
 + a*e^2)*x + c*d*e*x^2)^(9/2))/(99*(c*d^2 - a*e^2)^2*(d + e*x)^9)))/(13*(
c*d^2 - a*e^2))))/(5*(c*d^2 - a*e^2))
```

Defintions of rubi rules used

rule 1123
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[e*(d + e*x)^m*((a + b*x + c*x^2)^(p + 1)/((p + 1)*(2*c*d - b
*e))), x] /; FreeQ[{a, b, c, d, e, m, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2,
0] && EqQ[m + 2*p + 2, 0]
```

rule 1129
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(-e)*(d + e*x)^m*((a + b*x + c*x^2)^(p + 1)/((m + p + 1)*(2*
c*d - b*e))), x] + Simp[c*(Simplify[m + 2*p + 2]/((m + p + 1)*(2*c*d - b*e)
))   Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d
, e, m, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && ILtQ[Simplify[m + 2*p +
2], 0]
```

Maple [A] (verified)

Time = 18.61 (sec) , antiderivative size = 217, normalized size of antiderivative = 0.94

method	result
gosper	$-\frac{2(cdx+ae)\left(-16c^3d^3e^3x^3+72x^2a\,c^2d^2e^4-120c^3d^4e^2x^2-198x\,a^2cd\,e^5+540xa\,c^2d^3e^3-390c^3d^5ex+429e^6a^3-1485d^2e^4a^2c+1755\cdots\right.}{6435(ex+d)^{11}\left(a^4e^8-4a^3c\,d^2e^6+6a^2c^2d^4e^4-4a\,c^3d^6e^2+c^4d^8\right)\cdots}$
orering	$-\frac{2\left(-16c^3d^3e^3x^3+72x^2a\,c^2d^2e^4-120c^3d^4e^2x^2-198x\,a^2cd\,e^5+540xa\,c^2d^3e^3-390c^3d^5ex+429e^6a^3-1485d^2e^4a^2c+1755d^4e^2a\,c^2\cdots\right.}{6435\left(a^4e^8-4a^3c\,d^2e^6+6a^2c^2d^4e^4-4a\,c^3d^6e^2+c^4d^8\right)(ex+d)^{11}\cdots}$
default	$\frac{-\frac{2\left(dec\left(x+\frac{d}{e}\right)^2+\left(a\,e^2-c\,d^2\right)\left(x+\frac{d}{e}\right)\right)^{\frac{9}{2}}}{15\left(a\,e^2-c\,d^2\right)\left(x+\frac{d}{e}\right)^{12}}-\frac{2dec\left(-\frac{2\left(dec\left(x+\frac{d}{e}\right)^2+\left(a\,e^2-c\,d^2\right)\left(x+\frac{d}{e}\right)\right)^{\frac{9}{2}}}{13\left(a\,e^2-c\,d^2\right)\left(x+\frac{d}{e}\right)^{11}}-\frac{4dec\left(-\frac{2\left(dec\left(x+\frac{d}{e}\right)^2+\left(a\,e^2-c\,d^2\right)\left(x+\frac{d}{e}\right)\right)^{\frac{9}{2}}}{11\left(a\,e^2-c\,d^2\right)\left(x+\frac{d}{e}\right)^{10}}\cdots\right.}{13\left(a\,e^2\cdots\right.}\cdots\right.}{5\left(a\,e^2-c\,d^2\right)}\cdots}{e^{12}}$
trager	$-\frac{2\left(-16c^7d^7e^3x^7+8a\,c^6d^6e^4x^6-120c^7d^8e^2x^6-6a^2c^5d^5e^5x^5+60a\,c^6d^7e^3x^5-390c^7d^9e\,x^5+5a^3c^4d^4e^6x^4-45a^2c^5d^6e^4x^4+195a\,c^6\cdots\right.}{\cdots}$

input
```
int((a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(7/2)/(e*x+d)^12,x,method=_RETURNVER
BOSE)
```

output
```
-2/6435*(c*d*x+a*e)*(-16*c^3*d^3*e^3*x^3+72*a*c^2*d^2*e^4*x^2-120*c^3*d^4*
e^2*x^2-198*a^2*c*d*e^5*x+540*a*c^2*d^3*e^3*x-390*c^3*d^5*e*x+429*a^3*e^6-
1485*a^2*c*d^2*e^4+1755*a*c^2*d^4*e^2-715*c^3*d^6)*(c*d*e*x^2+a*e^2*x+c*d^
2*x+a*d*e)^(7/2)/(e*x+d)^11/(a^4*e^8-4*a^3*c*d^2*e^6+6*a^2*c^2*d^4*e^4-4*a
*c^3*d^6*e^2+c^4*d^8)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 951 vs. $2(215) = 430$.

Time = 97.84 (sec) , antiderivative size = 951, normalized size of antiderivative = 4.12

$$\int \frac{(ade + (cd^2 + ae^2)x + cdex^2)^{7/2}}{(d + ex)^{12}} \, dx = \text{Too large to display}$$

input
```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(7/2)/(e*x+d)^12,x, algorithm=
"fricas")
```

output
```
2/6435*(16*c^7*d^7*e^3*x^7 + 715*a^4*c^3*d^6*e^4 - 1755*a^5*c^2*d^4*e^6 +
1485*a^6*c*d^2*e^8 - 429*a^7*e^10 + 8*(15*c^7*d^8*e^2 - a*c^6*d^6*e^4)*x^6
 + 6*(65*c^7*d^9*e - 10*a*c^6*d^7*e^3 + a^2*c^5*d^5*e^5)*x^5 + 5*(143*c^7*
d^10 - 39*a*c^6*d^8*e^2 + 9*a^2*c^5*d^6*e^4 - a^3*c^4*d^4*e^6)*x^4 + 20*(1
43*a*c^6*d^9*e - 234*a^2*c^5*d^7*e^3 + 159*a^3*c^4*d^5*e^5 - 40*a^4*c^3*d^
3*e^7)*x^3 + 6*(715*a^2*c^5*d^8*e^2 - 1495*a^3*c^4*d^6*e^4 + 1145*a^4*c^3*
d^4*e^6 - 309*a^5*c^2*d^2*e^8)*x^2 + 2*(1430*a^3*c^4*d^7*e^3 - 3315*a^4*c^
3*d^5*e^5 + 2700*a^5*c^2*d^3*e^7 - 759*a^6*c*d*e^9)*x)*sqrt(c*d*e*x^2 + a*
d*e + (c*d^2 + a*e^2)*x)/(c^4*d^16 - 4*a*c^3*d^14*e^2 + 6*a^2*c^2*d^12*e^4
 - 4*a^3*c*d^10*e^6 + a^4*d^8*e^8 + (c^4*d^8*e^8 - 4*a*c^3*d^6*e^10 + 6*a^
2*c^2*d^4*e^12 - 4*a^3*c*d^2*e^14 + a^4*e^16)*x^8 + 8*(c^4*d^9*e^7 - 4*a*c
^3*d^7*e^9 + 6*a^2*c^2*d^5*e^11 - 4*a^3*c*d^3*e^13 + a^4*d*e^15)*x^7 + 28*
(c^4*d^10*e^6 - 4*a*c^3*d^8*e^8 + 6*a^2*c^2*d^6*e^10 - 4*a^3*c*d^4*e^12 +
a^4*d^2*e^14)*x^6 + 56*(c^4*d^11*e^5 - 4*a*c^3*d^9*e^7 + 6*a^2*c^2*d^7*e^9
 - 4*a^3*c*d^5*e^11 + a^4*d^3*e^13)*x^5 + 70*(c^4*d^12*e^4 - 4*a*c^3*d^10*
e^6 + 6*a^2*c^2*d^8*e^8 - 4*a^3*c*d^6*e^10 + a^4*d^4*e^12)*x^4 + 56*(c^4*d
^13*e^3 - 4*a*c^3*d^11*e^5 + 6*a^2*c^2*d^9*e^7 - 4*a^3*c*d^7*e^9 + a^4*d^5
*e^11)*x^3 + 28*(c^4*d^14*e^2 - 4*a*c^3*d^12*e^4 + 6*a^2*c^2*d^10*e^6 - 4*
a^3*c*d^8*e^8 + a^4*d^6*e^10)*x^2 + 8*(c^4*d^15*e - 4*a*c^3*d^13*e^3 + 6*a
^2*c^2*d^11*e^5 - 4*a^3*c*d^9*e^7 + a^4*d^7*e^9)*x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\left(ade + (cd^2 + ae^2)\,x + cdex^2\right)^{7/2}}{(d+ex)^{12}}\,dx = \text{Timed out}$$

input
```
integrate((a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(7/2)/(e*x+d)**12,x)
```

output
```
Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\left(ade + (cd^2 + ae^2)\,x + cdex^2\right)^{7/2}}{(d+ex)^{12}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(7/2)/(e*x+d)^12,x, algorithm=
"maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e*(a*e^2-c*d^2)>0)', see `assume
?` for mor
```

Giac [F(-2)]

Exception generated.

$$\int \frac{\left(ade + (cd^2 + ae^2)\,x + cdex^2\right)^{7/2}}{(d+ex)^{12}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(7/2)/(e*x+d)^12,x, algorithm=
"giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Unable to divide, perhaps due to ro
unding error%%%{%%%{1,[0,0,8]%%%},[16]%%%}+%%%{%%{[%%%{-16,[0,1,7]%%%},0]:
[1,0,%%%{
```

Mupad [B] (verification not implemented)

Time = 15.52 (sec) , antiderivative size = 8811, normalized size of antiderivative = 38.14

$$\int \frac{(ade + (cd^2 + ae^2)\,x + cdex^2)^{7/2}}{(d+ex)^{12}}\,dx = \text{Too large to display}$$

input
```
int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(7/2)/(d + e*x)^12,x)
```

output
```
(((d*((8*c^6*d^7)/(195*e^2*(a*e^2 - c*d^2)^2*(7*a*e^3 - 7*c*d^2*e)) - (4*c
^5*d^5*(25*a*e^2 - 21*c*d^2))/(195*e^2*(a*e^2 - c*d^2)^2*(7*a*e^3 - 7*c*d^
2*e))))/e + (4*c^4*d^4*(376*a^2*e^4 - 1901*c^2*d^4 + 1723*a*c*d^2*e^2))/(1
9305*e^3*(a*e^2 - c*d^2)^2*(7*a*e^3 - 7*c*d^2*e)))*(x*(a*e^2 + c*d^2) + a*
d*e + c*d*e*x^2)^(1/2))/(d + e*x)^4 + (((d*((16*c^7*d^8)/(2145*e^2*(a*e^2
- c*d^2)^3*(5*a*e^3 - 5*c*d^2*e)) - (8*c^6*d^6*(39*a*e^2 - 35*c*d^2))/(214
5*e^2*(a*e^2 - c*d^2)^3*(5*a*e^3 - 5*c*d^2*e))))/e - (8*c^5*d^5*(1011*a^2*
e^4 + 1270*c^2*d^4 - 2295*a*c*d^2*e^2))/(15015*e^3*(a*e^2 - c*d^2)^3*(5*a*
e^3 - 5*c*d^2*e)))*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2))/(d + e*x
)^3 + (((d*((32*c^8*d^9)/(19305*e^2*(a*e^2 - c*d^2)^4*(3*a*e^3 - 3*c*d^2*e
)) - (16*c^7*d^7*(51*a*e^2 - 47*c*d^2))/(19305*e^2*(a*e^2 - c*d^2)^4*(3*a*
e^3 - 3*c*d^2*e))))/e - (16*c^6*d^6*(3259*a^2*e^4 + 3504*c^2*d^4 - 6773*a*
c*d^2*e^2))/(96525*e^3*(a*e^2 - c*d^2)^4*(3*a*e^3 - 3*c*d^2*e)))*(x*(a*e^2
 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2))/(d + e*x)^2 - (((950*c^5*d^6 - 2666*
a*c^4*d^4*e^2)/(6435*e^3*(a*e^2 - c*d^2)*(7*a*e^3 - 7*c*d^2*e)) + (4*c^5*d
^6)/(15*e^3*(a*e^2 - c*d^2)*(7*a*e^3 - 7*c*d^2*e)))*(x*(a*e^2 + c*d^2) + a
*d*e + c*d*e*x^2)^(1/2))/(d + e*x)^4 - (((18244*c^6*d^7 - 20092*a*c^5*d^5*
e^2)/(45045*e^3*(a*e^2 - c*d^2)^2*(5*a*e^3 - 5*c*d^2*e)) + (8*c^6*d^7)/(19
5*e^3*(a*e^2 - c*d^2)^2*(5*a*e^3 - 5*c*d^2*e)))*(x*(a*e^2 + c*d^2) + a*d*e
 + c*d*e*x^2)^(1/2))/(d + e*x)^3 - (((d*((d*((d*((256*c^11*d^12)/(20270...
```

Reduce [B] (verification not implemented)

Time = 2.31 (sec) , antiderivative size = 1595, normalized size of antiderivative = 6.90

$$\int \frac{(ade + (cd^2 + ae^2) x + cdex^2)^{7/2}}{(d + ex)^{12}} \, dx = \text{Too large to display}$$

input
```
int((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(7/2)/(e*x+d)^12,x)
```

output
```
(2*( - 429*sqrt(d + e*x)*sqrt(a*e + c*d*x)*a**7*e**15 + 1485*sqrt(d + e*x)
*sqrt(a*e + c*d*x)*a**6*c*d**2*e**13 - 1518*sqrt(d + e*x)*sqrt(a*e + c*d*x
)*a**6*c*d*e**14*x - 1755*sqrt(d + e*x)*sqrt(a*e + c*d*x)*a**5*c**2*d**4*e
**11 + 5400*sqrt(d + e*x)*sqrt(a*e + c*d*x)*a**5*c**2*d**3*e**12*x - 1854*
sqrt(d + e*x)*sqrt(a*e + c*d*x)*a**5*c**2*d**2*e**13*x**2 + 715*sqrt(d + e
*x)*sqrt(a*e + c*d*x)*a**4*c**3*d**6*e**9 - 6630*sqrt(d + e*x)*sqrt(a*e +
c*d*x)*a**4*c**3*d**5*e**10*x + 6870*sqrt(d + e*x)*sqrt(a*e + c*d*x)*a**4*
c**3*d**4*e**11*x**2 - 800*sqrt(d + e*x)*sqrt(a*e + c*d*x)*a**4*c**3*d**3*
e**12*x**3 + 2860*sqrt(d + e*x)*sqrt(a*e + c*d*x)*a**3*c**4*d**7*e**8*x -
8970*sqrt(d + e*x)*sqrt(a*e + c*d*x)*a**3*c**4*d**6*e**9*x**2 + 3180*sqrt(
d + e*x)*sqrt(a*e + c*d*x)*a**3*c**4*d**5*e**10*x**3 - 5*sqrt(d + e*x)*sqr
t(a*e + c*d*x)*a**3*c**4*d**4*e**11*x**4 + 4290*sqrt(d + e*x)*sqrt(a*e + c
*d*x)*a**2*c**5*d**8*e**7*x**2 - 4680*sqrt(d + e*x)*sqrt(a*e + c*d*x)*a**2
*c**5*d**7*e**8*x**3 + 45*sqrt(d + e*x)*sqrt(a*e + c*d*x)*a**2*c**5*d**6*e
**9*x**4 + 6*sqrt(d + e*x)*sqrt(a*e + c*d*x)*a**2*c**5*d**5*e**10*x**5 + 2
860*sqrt(d + e*x)*sqrt(a*e + c*d*x)*a*c**6*d**9*e**6*x**3 - 195*sqrt(d + e
*x)*sqrt(a*e + c*d*x)*a*c**6*d**8*e**7*x**4 - 60*sqrt(d + e*x)*sqrt(a*e +
c*d*x)*a*c**6*d**7*e**8*x**5 - 8*sqrt(d + e*x)*sqrt(a*e + c*d*x)*a*c**6*d*
*6*e**9*x**6 + 715*sqrt(d + e*x)*sqrt(a*e + c*d*x)*c**7*d**10*e**5*x**4 +
390*sqrt(d + e*x)*sqrt(a*e + c*d*x)*c**7*d**9*e**6*x**5 + 120*sqrt(d + ...
```

3.248 $\int \frac{(d+ex)^3}{\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\,dx$

Optimal result

Integrand size = 37, antiderivative size = 240

$$\begin{aligned}\int \frac{(d+ex)^3}{\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}\,dx &= \frac{5(cd^2-ae^2)^2\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{8c^3d^3}\\ &\quad+\frac{5(cd^2-ae^2)\,(d+ex)\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{12c^2d^2}\\ &\quad+\frac{(d+ex)^2\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{3cd}\\ &\quad+\frac{5(cd^2-ae^2)^3\operatorname{arctanh}\left(\frac{\sqrt{c}\sqrt{d}(d+ex)}{\sqrt{e}\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\right)}{8c^{7/2}d^{7/2}\sqrt{e}}\end{aligned}$$

output
```
5/8*(-a*e^2+c*d^2)^2*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/c^3/d^3+5/12*
(-a*e^2+c*d^2)*(e*x+d)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/c^2/d^2+1/3
*(e*x+d)^2*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/c/d+5/8*(-a*e^2+c*d^2)^
3*arctanh(c^(1/2)*d^(1/2)*(e*x+d)/e^(1/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2
)^(1/2))/c^(7/2)/d^(7/2)/e^(1/2)
```

Mathematica [A] (verified)

Time = 0.36 (sec) , antiderivative size = 187, normalized size of antiderivative = 0.78

$$\int \frac{(d+ex)^3}{\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}\,dx$$
$$= \frac{\sqrt{c}\sqrt{d}(ae+cdx)(d+ex)\left(15a^2e^4-10acde^2(4d+ex)+c^2d^2(33d^2+26dex+8e^2x^2)\right)+\frac{15(cd^2-ae^2)^3\sqrt{ae+}}{}}{24c^{7/2}d^{7/2}\sqrt{(ae+cdx)(d+ex)}}$$

input
```
Integrate[(d + e*x)^3/Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2],x]
```

output
```
(Sqrt[c]*Sqrt[d]*(a*e + c*d*x)*(d + e*x)*(15*a^2*e^4 - 10*a*c*d*e^2*(4*d +
 e*x) + c^2*d^2*(33*d^2 + 26*d*e*x + 8*e^2*x^2)) + (15*(c*d^2 - a*e^2)^3*S
qrt[a*e + c*d*x]*Sqrt[d + e*x]*ArcTanh[(Sqrt[c]*Sqrt[d]*Sqrt[d + e*x])/(Sq
rt[e]*Sqrt[a*e + c*d*x])])/Sqrt[e])/(24*c^(7/2)*d^(7/2)*Sqrt[(a*e + c*d*x)
*(d + e*x)])
```

Rubi [A] (verified)

Time = 0.68 (sec) , antiderivative size = 267, normalized size of antiderivative = 1.11, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.135$, Rules used = {1134, 1134, 1160, 1092, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex)^3}{\sqrt{x\,(ae^2+cd^2)+ade+cdex^2}}\,dx$$
$$\Big\downarrow \text{1134}$$
$$\frac{5\left(d^2-\frac{ae^2}{c}\right)\int \frac{(d+ex)^2}{\sqrt{cdex^2+(cd^2+ae^2)x+ade}}dx}{6d}+\frac{(d+ex)^2\sqrt{x\,(ae^2+cd^2)+ade+cdex^2}}{3cd}$$
$$\Big\downarrow \text{1134}$$

$$\frac{5\left(d^2-\frac{ae^2}{c}\right)\left(\frac{3\left(d^2-\frac{ae^2}{c}\right)\int \frac{d+ex}{\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}dx}{4d}+\frac{(d+ex)\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{2cd}\right)}{6d}+\frac{(d+ex)^2\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{3cd}$$

$\downarrow$ 1160

$$\frac{5\left(d^2-\frac{ae^2}{c}\right)\left(\frac{3\left(d^2-\frac{ae^2}{c}\right)\left(\frac{\left(d^2-\frac{ae^2}{c}\right)\int \frac{1}{\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}dx}{2d}+\frac{\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{cd}\right)}{4d}+\frac{(d+ex)\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{2cd}\right)}{6d}+\frac{(d+ex)^2\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{3cd}$$

$\downarrow$ 1092

$$\frac{5\left(d^2-\frac{ae^2}{c}\right)\left(\frac{3\left(d^2-\frac{ae^2}{c}\right)\left(\frac{\left(d^2-\frac{ae^2}{c}\right)\int \frac{1}{4cde-\frac{\left(cd^2+2cexd+ae^2\right)^2}{cdex^2+\left(cd^2+ae^2\right)x+ade}}d\frac{cd^2+2cexd+ae^2}{\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}}{d}+\frac{\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{cd}\right)}{4d}+\frac{(d+ex)\cdots}{}\right)}{6d}+\frac{(d+ex)^2\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{3cd}$$

$\downarrow$ 219

$$\frac{5\left(d^2-\frac{ae^2}{c}\right)\left(\frac{3\left(d^2-\frac{ae^2}{c}\right)\left(\frac{\left(d^2-\frac{ae^2}{c}\right)\operatorname{arctanh}\left(\frac{ae^2+cd^2+2cdex}{2\sqrt{c}\sqrt{d}\sqrt{e}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}\right)}{2\sqrt{c}d^{3/2}\sqrt{e}}+\frac{\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{cd}\right)}{4d}+\frac{(d+ex)\sqrt{x(ae^2+c}}{2c}\right.}{6d}$$

$$\frac{(d+ex)^2\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{3cd}$$

input
```
Int[(d + e*x)^3/Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2],x]
```

output
```
((d + e*x)^2*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(3*c*d) + (5*(d^
2 - (a*e^2)/c)*(((d + e*x)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(2
*c*d) + (3*(d^2 - (a*e^2)/c)*(Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]/
(c*d) + ((d^2 - (a*e^2)/c)*ArcTanh[(c*d^2 + a*e^2 + 2*c*d*e*x)/(2*Sqrt[c]*
Sqrt[d]*Sqrt[e]*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])])/(2*Sqrt[c]*
d^(3/2)*Sqrt[e])))/(4*d)))/(6*d)
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1092
```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[I
nt[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a
, b, c}, x]
```

rule 1134
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[e*(d + e*x)^(m - 1)*((a + b*x + c*x^2)^(p + 1)/(c*(m + 2*p +
 1))), x] + Simp[(m + p)*((2*c*d - b*e)/(c*(m + 2*p + 1)))   Int[(d + e*x)^
(m - 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[
c*d^2 - b*d*e + a*e^2, 0] && GtQ[m, 1] && NeQ[m + 2*p + 1, 0] && IntegerQ[2
*p]
```

rule 1160
```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[e*((a + b*x + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Simp[(2*c*d - b
*e)/(2*c)   Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
 && NeQ[p, -1]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 887 vs. $2(212) = 424$.

Time = 1.31 (sec) , antiderivative size = 888, normalized size of antiderivative = 3.70

method	result
default	$\frac{d^3 \ln\left(\frac{\frac{1}{2} a e^2+\frac{1}{2} c d^2+cdxe}{\sqrt{dec}}+\sqrt{ade+(a e^2+c d^2)x+cd x^2 e}\right)}{\sqrt{dec}}+e^3\left(\frac{x^2\sqrt{ade+(a e^2+c d^2)x+cd x^2 e}}{3dec}-\frac{5(a e^2+c d^2)\left(\frac{x\sqrt{ade+(a e^2 \cdots}}{\cdots}\right.}{\cdots}\right.$

input
```
int((e*x+d)^3/(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(1/2),x,method=_RETURNVERB
OSE)
```

output
```
d^3*ln((1/2*a*e^2+1/2*c*d^2+c*d*x*e)/(d*e*c)^(1/2)+(a*d*e+(a*e^2+c*d^2)*x+
c*d*x^2*e)^(1/2))/(d*e*c)^(1/2)+e^3*(1/3*x^2/d/e/c*(a*d*e+(a*e^2+c*d^2)*x+
c*d*x^2*e)^(1/2)-5/6*(a*e^2+c*d^2)/d/e/c*(1/2*x/d/e/c*(a*d*e+(a*e^2+c*d^2)
*x+c*d*x^2*e)^(1/2)-3/4*(a*e^2+c*d^2)/d/e/c*(1/d/e/c*(a*d*e+(a*e^2+c*d^2)*
x+c*d*x^2*e)^(1/2)-1/2*(a*e^2+c*d^2)/d/e/c*ln((1/2*a*e^2+1/2*c*d^2+c*d*x*e
)/(d*e*c)^(1/2)+(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(1/2))/(d*e*c)^(1/2))-1/
2*a/c*ln((1/2*a*e^2+1/2*c*d^2+c*d*x*e)/(d*e*c)^(1/2)+(a*d*e+(a*e^2+c*d^2)*
x+c*d*x^2*e)^(1/2))/(d*e*c)^(1/2))-2/3*a/c*(1/d/e/c*(a*d*e+(a*e^2+c*d^2)*x
+c*d*x^2*e)^(1/2)-1/2*(a*e^2+c*d^2)/d/e/c*ln((1/2*a*e^2+1/2*c*d^2+c*d*x*e)
/(d*e*c)^(1/2)+(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(1/2))/(d*e*c)^(1/2)))+3*
d*e^2*(1/2*x/d/e/c*(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(1/2)-3/4*(a*e^2+c*d^
2)/d/e/c*(1/d/e/c*(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(1/2)-1/2*(a*e^2+c*d^2
)/d/e/c*ln((1/2*a*e^2+1/2*c*d^2+c*d*x*e)/(d*e*c)^(1/2)+(a*d*e+(a*e^2+c*d^2
)*x+c*d*x^2*e)^(1/2))/(d*e*c)^(1/2))-1/2*a/c*ln((1/2*a*e^2+1/2*c*d^2+c*d*x
*e)/(d*e*c)^(1/2)+(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(1/2))/(d*e*c)^(1/2))+
3*d^2*e*(1/d/e/c*(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(1/2)-1/2*(a*e^2+c*d^2)
/d/e/c*ln((1/2*a*e^2+1/2*c*d^2+c*d*x*e)/(d*e*c)^(1/2)+(a*d*e+(a*e^2+c*d^2)
*x+c*d*x^2*e)^(1/2))/(d*e*c)^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 534, normalized size of antiderivative = 2.22

$$\int \frac{(d+ex)^3}{\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}\,dx$$

$$= \left[-\frac{15\,(c^3d^6-3\,ac^2d^4e^2+3\,a^2cd^2e^4-a^3e^6)\sqrt{cde}\log\left(8\,c^2d^2e^2x^2+c^2d^4+6\,acd^2e^2+a^2e^4-4\,\sqrt{cdex^2+\cdots}\right.}{\cdots}\right.$$

$$-\frac{15\,(c^3d^6-3\,ac^2d^4e^2+3\,a^2cd^2e^4-a^3e^6)\sqrt{-cde}\arctan\left(\frac{\sqrt{cdex^2+ade+(cd^2+ae^2)x}\,(2\,cdex+cd^2+ae^2)\sqrt{-cde}}{2\,(c^2d^2e^2x^2+acd^2e^2+(c^2d^3e+acde^3)x)}\right)-2\cdots}{4\cdots}$$

input
```
integrate((e*x+d)^3/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x, algorithm="
fricas")
```

output
```
[-1/96*(15*(c^3*d^6 - 3*a*c^2*d^4*e^2 + 3*a^2*c*d^2*e^4 - a^3*e^6)*sqrt(c*
d*e)*log(8*c^2*d^2*e^2*x^2 + c^2*d^4 + 6*a*c*d^2*e^2 + a^2*e^4 - 4*sqrt(c*
d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(2*c*d*e*x + c*d^2 + a*e^2)*sqrt(c*d*
e) + 8*(c^2*d^3*e + a*c*d*e^3)*x) - 4*(8*c^3*d^3*e^3*x^2 + 33*c^3*d^5*e -
40*a*c^2*d^3*e^3 + 15*a^2*c*d*e^5 + 2*(13*c^3*d^4*e^2 - 5*a*c^2*d^2*e^4)*x
)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x))/(c^4*d^4*e), -1/48*(15*(c^3
*d^6 - 3*a*c^2*d^4*e^2 + 3*a^2*c*d^2*e^4 - a^3*e^6)*sqrt(-c*d*e)*arctan(1/
2*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(2*c*d*e*x + c*d^2 + a*e^2)*
sqrt(-c*d*e)/(c^2*d^2*e^2*x^2 + a*c*d^2*e^2 + (c^2*d^3*e + a*c*d*e^3)*x))
- 2*(8*c^3*d^3*e^3*x^2 + 33*c^3*d^5*e - 40*a*c^2*d^3*e^3 + 15*a^2*c*d*e^5
+ 2*(13*c^3*d^4*e^2 - 5*a*c^2*d^2*e^4)*x)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2
+ a*e^2)*x))/(c^4*d^4*e)]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 782 vs. $2(230) = 460$.

Time = 0.89 (sec) , antiderivative size = 782, normalized size of antiderivative = 3.26

$$\int \frac{(d+ex)^3}{\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}\,dx = \text{Too large to display}$$

input
```
integrate((e*x+d)**3/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(1/2),x)
```

output
```
Piecewise(((-a*(3*d*e**2 - e**2*(5*a*e**2/2 + 5*c*d**2/2)/(3*c*d))/(2*c) +
 d**3 - (a*e**2 + c*d**2)*(-2*a*e**3/(3*c) + 3*d**2*e - (3*a*e**2/2 + 3*c*
d**2/2)*(3*d*e**2 - e**2*(5*a*e**2/2 + 5*c*d**2/2)/(3*c*d))/(2*c*d*e))/(2*
c*d*e))*Piecewise((log(a*e**2 + c*d**2 + 2*c*d*e*x + 2*sqrt(c*d*e)*sqrt(a*
d*e + c*d*e*x**2 + x*(a*e**2 + c*d**2)))/sqrt(c*d*e), Ne(a*d*e - (a*e**2 +
 c*d**2)**2/(4*c*d*e), 0)), ((x - (-a*e**2 - c*d**2)/(2*c*d*e))*log(x - (-
a*e**2 - c*d**2)/(2*c*d*e))/sqrt(c*d*e*(x - (-a*e**2 - c*d**2)/(2*c*d*e))*
*2), True)) + sqrt(a*d*e + c*d*e*x**2 + x*(a*e**2 + c*d**2))*(e**2*x**2/(3
*c*d) + x*(3*d*e**2 - e**2*(5*a*e**2/2 + 5*c*d**2/2)/(3*c*d))/(2*c*d*e) +
(-2*a*e**3/(3*c) + 3*d**2*e - (3*a*e**2/2 + 3*c*d**2/2)*(3*d*e**2 - e**2*(
5*a*e**2/2 + 5*c*d**2/2)/(3*c*d))/(2*c*d*e))/(c*d*e)), Ne(c*d*e, 0)), (2*(
c**3*d**9*sqrt(a*d*e + x*(a*e**2 + c*d**2))/(a**3*e**6 + 3*a**2*c*d**2*e**
4 + 3*a*c**2*d**4*e**2 + c**3*d**6) + c**2*d**6*e*(a*d*e + x*(a*e**2 + c*d
**2))**(3/2)/(a**3*e**6 + 3*a**2*c*d**2*e**4 + 3*a*c**2*d**4*e**2 + c**3*d
**6) + 3*c*d**3*e**2*(a*d*e + x*(a*e**2 + c*d**2))**(5/2)/(5*(a**3*e**6 +
3*a**2*c*d**2*e**4 + 3*a*c**2*d**4*e**2 + c**3*d**6)) + e**3*(a*d*e + x*(a
*e**2 + c*d**2))**(7/2)/(7*(a**3*e**6 + 3*a**2*c*d**2*e**4 + 3*a*c**2*d**4
*e**2 + c**3*d**6)))/(a*e**2 + c*d**2), Ne(a*e**2 + c*d**2, 0)), (Piecewis
e((d**3*x, Eq(e, 0)), ((d + e*x)**4/(4*e), True))/sqrt(a*d*e), True))
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(d+ex)^3}{\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((e*x+d)^3/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x, algorithm="
maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(a*e^2-c*d^2>0)', see `assume?` f
or more de
```

Giac [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 230, normalized size of antiderivative = 0.96

$$\int \frac{(d+ex)^3}{\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\,dx$$
$$=\frac{1}{24}\sqrt{cdex^2+cd^2x+ae^2x+ade}\left(2\left(\frac{4e^2x}{cd}+\frac{13c^2d^3e^3-5acde^5}{c^3d^3e^2}\right)x+\frac{33c^2d^4e^2-40acd^2e^4+15a^2e^6}{c^3d^3e^2}\right)$$
$$-\frac{5(c^3d^6-3ac^2d^4e^2+3a^2cd^2e^4-a^3e^6)\log\left(\left|-cd^2-ae^2-2\sqrt{cde}\left(\sqrt{cde}x-\sqrt{cdex^2+cd^2x+ae^2x+ade}\right)\right|\right)}{16\sqrt{cde}c^3d^3}$$

input
```
integrate((e*x+d)^3/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x, algorithm="
giac")
```

output
```
1/24*sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*x + a*d*e)*(2*(4*e^2*x/(c*d) + (13*c
^2*d^3*e^3 - 5*a*c*d*e^5)/(c^3*d^3*e^2))*x + (33*c^2*d^4*e^2 - 40*a*c*d^2*
e^4 + 15*a^2*e^6)/(c^3*d^3*e^2)) - 5/16*(c^3*d^6 - 3*a*c^2*d^4*e^2 + 3*a^2
*c*d^2*e^4 - a^3*e^6)*log(abs(-c*d^2 - a*e^2 - 2*sqrt(c*d*e)*(sqrt(c*d*e)*
x - sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*x + a*d*e))))/(sqrt(c*d*e)*c^3*d^3)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d+ex)^3}{\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\,dx=\int \frac{(d+e\,x)^3}{\sqrt{c\,d\,e\,x^2+(c\,d^2+a\,e^2)\,x+a\,d\,e}}\,dx$$

input
```
int((d + e*x)^3/(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2),x)
```

output
```
int((d + e*x)^3/(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 400, normalized size of antiderivative = 1.67

$$\int \frac{(d+ex)^3}{\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}\,dx$$
$$= \frac{15\sqrt{ex+d}\,\sqrt{cdx+ae}\,a^2cd\,e^5-40\sqrt{ex+d}\,\sqrt{cdx+ae}\,a\,c^2d^3e^3-10\sqrt{ex+d}\,\sqrt{cdx+ae}\,a\,c^2d^2e^4x+33\sqrt{}}{}$$

input
```
int((e*x+d)^3/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x)
```

output
```
(15*sqrt(d + e*x)*sqrt(a*e + c*d*x)*a**2*c*d*e**5 - 40*sqrt(d + e*x)*sqrt(
a*e + c*d*x)*a*c**2*d**3*e**3 - 10*sqrt(d + e*x)*sqrt(a*e + c*d*x)*a*c**2*
d**2*e**4*x + 33*sqrt(d + e*x)*sqrt(a*e + c*d*x)*c**3*d**5*e + 26*sqrt(d +
 e*x)*sqrt(a*e + c*d*x)*c**3*d**4*e**2*x + 8*sqrt(d + e*x)*sqrt(a*e + c*d*
x)*c**3*d**3*e**3*x**2 - 15*sqrt(e)*sqrt(d)*sqrt(c)*log((sqrt(e)*sqrt(a*e
+ c*d*x) + sqrt(d)*sqrt(c)*sqrt(d + e*x))/sqrt(a*e**2 - c*d**2))*a**3*e**6
 + 45*sqrt(e)*sqrt(d)*sqrt(c)*log((sqrt(e)*sqrt(a*e + c*d*x) + sqrt(d)*sqr
t(c)*sqrt(d + e*x))/sqrt(a*e**2 - c*d**2))*a**2*c*d**2*e**4 - 45*sqrt(e)*s
qrt(d)*sqrt(c)*log((sqrt(e)*sqrt(a*e + c*d*x) + sqrt(d)*sqrt(c)*sqrt(d + e
*x))/sqrt(a*e**2 - c*d**2))*a*c**2*d**4*e**2 + 15*sqrt(e)*sqrt(d)*sqrt(c)*
log((sqrt(e)*sqrt(a*e + c*d*x) + sqrt(d)*sqrt(c)*sqrt(d + e*x))/sqrt(a*e**
2 - c*d**2))*c**3*d**6)/(24*c**4*d**4*e)
```

3.249 $\int \frac{(d+ex)^2}{\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\,dx$

Optimal result

Integrand size = 37, antiderivative size = 180

$$\int \frac{(d+ex)^2}{\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}\,dx = \frac{3(cd^2-ae^2)\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{4c^2d^2} + \frac{(d+ex)\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{2cd} + \frac{3(cd^2-ae^2)^2\operatorname{arctanh}\left(\frac{\sqrt{c}\sqrt{d}(d+ex)}{\sqrt{e}\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\right)}{4c^{5/2}d^{5/2}\sqrt{e}}$$

output

```
3/4*(-a*e^2+c*d^2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/c^2/d^2+1/2*(e*
x+d)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/c/d+3/4*(-a*e^2+c*d^2)^2*arct
anh(c^(1/2)*d^(1/2)*(e*x+d)/e^(1/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2
))/c^(5/2)/d^(5/2)/e^(1/2)
```

Mathematica [A] (verified)

Time = 0.30 (sec) , antiderivative size = 155, normalized size of antiderivative = 0.86

$$\int \frac{(d+ex)^2}{\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}\,dx$$
$$= \frac{\sqrt{c}\sqrt{d}(ae+cdx)(d+ex)\left(-3ae^2+cd(5d+2ex)\right)+\frac{3\left(cd^2-ae^2\right)^2\sqrt{ae+cdx}\sqrt{d+ex}\operatorname{arctanh}\left(\frac{\sqrt{c}\sqrt{d}\sqrt{d+ex}}{\sqrt{e}\sqrt{ae+cdx}}\right)}{\sqrt{e}}}{4c^{5/2}d^{5/2}\sqrt{(ae+cdx)(d+ex)}}$$

input
```
Integrate[(d + e*x)^2/Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2],x]
```

output
```
(Sqrt[c]*Sqrt[d]*(a*e + c*d*x)*(d + e*x)*(-3*a*e^2 + c*d*(5*d + 2*e*x)) +
(3*(c*d^2 - a*e^2)^2*Sqrt[a*e + c*d*x]*Sqrt[d + e*x]*ArcTanh[(Sqrt[c]*Sqrt
[d]*Sqrt[d + e*x])/(Sqrt[e]*Sqrt[a*e + c*d*x])]])/Sqrt[e])/(4*c^(5/2)*d^(5/
2)*Sqrt[(a*e + c*d*x)*(d + e*x)])
```

Rubi [A] (verified)

Time = 0.52 (sec) , antiderivative size = 200, normalized size of antiderivative = 1.11, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.108$, Rules used = {1134, 1160, 1092, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex)^2}{\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}dx$$
$$\Big\downarrow \text{1134}$$
$$\frac{3\left(d^2-\frac{ae^2}{c}\right)\int \frac{d+ex}{\sqrt{cdex^2+(cd^2+ae^2)x+ade}}dx}{4d}+\frac{(d+ex)\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{2cd}$$
$$\Big\downarrow \text{1160}$$

$$\frac{3\left(d^2-\frac{ae^2}{c}\right)\left(\frac{\left(d^2-\frac{ae^2}{c}\right)\int \frac{1}{\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}dx}{2d}+\frac{\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{cd}\right)}{4d}+\frac{(d+ex)\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{2cd}$$

$$\Big\downarrow \ 1092$$

$$\frac{3\left(d^2-\frac{ae^2}{c}\right)\left(\frac{\left(d^2-\frac{ae^2}{c}\right)\int \frac{1}{4cde-\frac{\left(cd^2+2cexd+ae^2\right)^2}{cdex^2+\left(cd^2+ae^2\right)x+ade}}d\frac{cd^2+2cexd+ae^2}{\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}}{d}+\frac{\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{cd}\right)}{4d}+\frac{(d+ex)\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{2cd}$$

$$\Big\downarrow \ 219$$

$$\frac{3\left(d^2-\frac{ae^2}{c}\right)\left(\frac{\left(d^2-\frac{ae^2}{c}\right)\text{arctanh}\left(\frac{ae^2+cd^2+2cdex}{2\sqrt{c}\sqrt{d}\sqrt{e}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}\right)}{2\sqrt{c}d^{3/2}\sqrt{e}}+\frac{\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{cd}\right)}{4d}+\frac{(d+ex)\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{2cd}$$

input
```
Int[(d + e*x)^2/Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2],x]
```

output
```
((d + e*x)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(2*c*d) + (3*(d^2
- (a*e^2)/c)*(Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]/(c*d) + ((d^2 -
(a*e^2)/c)*ArcTanh[(c*d^2 + a*e^2 + 2*c*d*e*x)/(2*Sqrt[c]*Sqrt[d]*Sqrt[e]*
Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])])/(2*Sqrt[c]*d^(3/2)*Sqrt[e])
))/(4*d)
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1092
```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[I
nt[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a
, b, c}, x]
```

rule 1134
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[e*(d + e*x)^(m - 1)*((a + b*x + c*x^2)^(p + 1)/(c*(m + 2*p +
 1))), x] + Simp[(m + p)*((2*c*d - b*e)/(c*(m + 2*p + 1)))   Int[(d + e*x)^
(m - 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[
c*d^2 - b*d*e + a*e^2, 0] && GtQ[m, 1] && NeQ[m + 2*p + 1, 0] && IntegerQ[2
*p]
```

rule 1160
```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[e*((a + b*x + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Simp[(2*c*d - b
*e)/(2*c)   Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
 && NeQ[p, -1]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 441 vs. $2(156) = 312$.

Time = 1.13 (sec) , antiderivative size = 442, normalized size of antiderivative = 2.46

method	result
default	$\frac{d^{2} \ln\left(\frac{\frac{1}{2} a\,e^{2}+\frac{1}{2} c\,d^{2}+cdxe}{\sqrt{dec}}+\sqrt{ade+(a\,e^{2}+c\,d^{2})x+cd\,x^{2} e}\right)}{\sqrt{dec}}+e^{2}\left(\frac{x\sqrt{ade+(a\,e^{2}+c\,d^{2})x+cd\,x^{2} e}}{2dec}-\frac{3\left(a\,e^{2}+c\,d^{2}\right)\left(\frac{\sqrt{ade+\left(a\,e^{2}+\right.}}{de}\right.}{}\right.$

input
```
int((e*x+d)^2/(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(1/2),x,method=_RETURNVERB
OSE)
```

output
```
d^2*ln((1/2*a*e^2+1/2*c*d^2+c*d*x*e)/(d*e*c)^(1/2)+(a*d*e+(a*e^2+c*d^2)*x+
c*d*x^2*e)^(1/2))/(d*e*c)^(1/2)+e^2*(1/2*x/d/e/c*(a*d*e+(a*e^2+c*d^2)*x+c*
d*x^2*e)^(1/2)-3/4*(a*e^2+c*d^2)/d/e/c*(1/d/e/c*(a*d*e+(a*e^2+c*d^2)*x+c*d
*x^2*e)^(1/2)-1/2*(a*e^2+c*d^2)/d/e/c*ln((1/2*a*e^2+1/2*c*d^2+c*d*x*e)/(d*
e*c)^(1/2)+(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(1/2))/(d*e*c)^(1/2))-1/2*a/c
*ln((1/2*a*e^2+1/2*c*d^2+c*d*x*e)/(d*e*c)^(1/2)+(a*d*e+(a*e^2+c*d^2)*x+c*d
*x^2*e)^(1/2))/(d*e*c)^(1/2))+2*d*e*(1/d/e/c*(a*d*e+(a*e^2+c*d^2)*x+c*d*x^
2*e)^(1/2)-1/2*(a*e^2+c*d^2)/d/e/c*ln((1/2*a*e^2+1/2*c*d^2+c*d*x*e)/(d*e*c
)^(1/2)+(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(1/2))/(d*e*c)^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 418, normalized size of antiderivative = 2.32

$$\int \frac{(d+ex)^2}{\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}\,dx$$

$$= \left[\frac{3\,(c^2d^4-2\,acd^2e^2+a^2e^4)\sqrt{cde}\log\Big(8\,c^2d^2e^2x^2+c^2d^4+6\,acd^2e^2+a^2e^4+4\,\sqrt{cdex^2+ade+(cd^2+a}}{}\right.$$

$$-\frac{3\,(c^2d^4-2\,acd^2e^2+a^2e^4)\sqrt{-cde}\arctan\left(\frac{\sqrt{cdex^2+ade+(cd^2+ae^2)x}\,(2\,cdex+cd^2+ae^2)\sqrt{-cde}}{2\,(c^2d^2e^2x^2+acd^2e^2+(c^2d^3e+acde^3)x)}\right)-2\,(2\,c^2d^2e^2x+5}{8\,c^3d^3e}$$

input
```
integrate((e*x+d)^2/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x, algorithm="
fricas")
```

output
```
[1/16*(3*(c^2*d^4 - 2*a*c*d^2*e^2 + a^2*e^4)*sqrt(c*d*e)*log(8*c^2*d^2*e^2
*x^2 + c^2*d^4 + 6*a*c*d^2*e^2 + a^2*e^4 + 4*sqrt(c*d*e*x^2 + a*d*e + (c*d
^2 + a*e^2)*x)*(2*c*d*e*x + c*d^2 + a*e^2)*sqrt(c*d*e) + 8*(c^2*d^3*e + a*
c*d*e^3)*x) + 4*(2*c^2*d^2*e^2*x + 5*c^2*d^3*e - 3*a*c*d*e^3)*sqrt(c*d*e*x
^2 + a*d*e + (c*d^2 + a*e^2)*x))/(c^3*d^3*e), -1/8*(3*(c^2*d^4 - 2*a*c*d^2
*e^2 + a^2*e^4)*sqrt(-c*d*e)*arctan(1/2*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 +
a*e^2)*x)*(2*c*d*e*x + c*d^2 + a*e^2)*sqrt(-c*d*e)/(c^2*d^2*e^2*x^2 + a*c*
d^2*e^2 + (c^2*d^3*e + a*c*d*e^3)*x)) - 2*(2*c^2*d^2*e^2*x + 5*c^2*d^3*e -
 3*a*c*d*e^3)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x))/(c^3*d^3*e)]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 502 vs. $2(173) = 346$.

Time = 0.77 (sec) , antiderivative size = 502, normalized size of antiderivative = 2.79

$$\int \frac{(d+ex)^2}{\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}\,dx$$

$$= \begin{cases} \left(\frac{ex}{2cd} + \frac{2de - \frac{e\left(\frac{3ae^2}{2}+\frac{3cd^2}{2}\right)}{2cd}}{cde}\right)\sqrt{ade+cdex^2+x\,(ae^2+cd^2)} + \left(-\frac{ae^2}{2c} + d^2 - \frac{(ae^2+cd^2)\left(2de - \frac{e\left(\frac{3ae^2}{2}+\frac{3cd^2}{2}\right)}{2cd}\right)}{2cde} \right. \\ \frac{2\left(\frac{c^2d^6\sqrt{ade+x(ae^2+cd^2)}}{a^2e^4+2acd^2e^2+c^2d^4} + \frac{2cd^3e\left(ade+x\left(ae^2+cd^2\right)\right)^{\frac{3}{2}}}{3\left(a^2e^4+2acd^2e^2+c^2d^4\right)} + \frac{e^2\left(ade+x\left(ae^2+cd^2\right)\right)^{\frac{5}{2}}}{5\left(a^2e^4+2acd^2e^2+c^2d^4\right)}\right)}{ae^2+cd^2} \\ \frac{\begin{cases} d^2x & \text{for } e=0 \\ \frac{(d+ex)^3}{3e} & \text{otherwise} \end{cases}}{\sqrt{ade}} \end{cases}$$

input
```
integrate((e*x+d)**2/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(1/2),x)
```

output

```
Piecewise(((e*x/(2*c*d) + (2*d*e - e*(3*a*e**2/2 + 3*c*d**2/2)/(2*c*d))/(c
*d*e))*sqrt(a*d*e + c*d*e*x**2 + x*(a*e**2 + c*d**2)) + (-a*e**2/(2*c) + d
**2 - (a*e**2 + c*d**2)*(2*d*e - e*(3*a*e**2/2 + 3*c*d**2/2)/(2*c*d))/(2*c
*d*e))*Piecewise((log(a*e**2 + c*d**2 + 2*c*d*e*x + 2*sqrt(c*d*e)*sqrt(a*d
*e + c*d*e*x**2 + x*(a*e**2 + c*d**2)))/sqrt(c*d*e), Ne(a*d*e - (a*e**2 +
c*d**2)**2/(4*c*d*e), 0)), ((x - (-a*e**2 - c*d**2)/(2*c*d*e))*log(x - (-a
*e**2 - c*d**2)/(2*c*d*e))/sqrt(c*d*e*(x - (-a*e**2 - c*d**2)/(2*c*d*e))**
2), True)), Ne(c*d*e, 0)), (2*(c**2*d**6*sqrt(a*d*e + x*(a*e**2 + c*d**2))
/(a**2*e**4 + 2*a*c*d**2*e**2 + c**2*d**4) + 2*c*d**3*e*(a*d*e + x*(a*e**2
 + c*d**2))**(3/2)/(3*(a**2*e**4 + 2*a*c*d**2*e**2 + c**2*d**4)) + e**2*(a
*d*e + x*(a*e**2 + c*d**2))**(5/2)/(5*(a**2*e**4 + 2*a*c*d**2*e**2 + c**2*
d**4)))/(a*e**2 + c*d**2), Ne(a*e**2 + c*d**2, 0)), (Piecewise((d**2*x, Eq
(e, 0)), ((d + e*x)**3/(3*e), True))/sqrt(a*d*e), True))
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(d+ex)^2}{\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}\,dx = \text{Exception raised: ValueError}$$

input

```
integrate((e*x+d)^2/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x, algorithm="
maxima")
```

output

```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(a*e^2-c*d^2>0)', see `assume?` f
or more de
```

Giac [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 163, normalized size of antiderivative = 0.91

$$\int \frac{(d+ex)^2}{\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}\,dx$$
$$=\frac{1}{4}\sqrt{cdex^2+cd^2x+ae^2x+ade}\left(\frac{2\,ex}{cd}+\frac{5\,cd^2e-3\,ae^3}{c^2d^2e}\right)$$
$$-\frac{3\,(c^2d^4-2\,acd^2e^2+a^2e^4)\log\left(\left|-cd^2-ae^2-2\,\sqrt{cde}\left(\sqrt{cde}x-\sqrt{cdex^2+cd^2x+ae^2x+ade}\right)\right|\right)}{8\,\sqrt{cde}c^2d^2}$$

input
```
integrate((e*x+d)^2/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x, algorithm="
giac")
```

output
```
1/4*sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*x + a*d*e)*(2*e*x/(c*d) + (5*c*d^2*e
- 3*a*e^3)/(c^2*d^2*e)) - 3/8*(c^2*d^4 - 2*a*c*d^2*e^2 + a^2*e^4)*log(abs(
-c*d^2 - a*e^2 - 2*sqrt(c*d*e)*(sqrt(c*d*e)*x - sqrt(c*d*e*x^2 + c*d^2*x +
 a*e^2*x + a*d*e))))/(sqrt(c*d*e)*c^2*d^2)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d+ex)^2}{\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}\,dx=\int \frac{{\left(d+e\,x\right)}^2}{\sqrt{c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e}}\,dx$$

input
```
int((d + e*x)^2/(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2),x)
```

output
```
int((d + e*x)^2/(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 253, normalized size of antiderivative = 1.41

$$\int \frac{(d+ex)^2}{\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}\,dx$$
$$= \frac{-3\sqrt{ex+d}\,\sqrt{cdx+ae}\,acd\,e^3+5\sqrt{ex+d}\,\sqrt{cdx+ae}\,c^2d^3e+2\sqrt{ex+d}\,\sqrt{cdx+ae}\,c^2d^2e^2x+3\sqrt{e}\,\sqrt{d}\,\sqrt{}}{}$$

input
```
int((e*x+d)^2/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x)
```

output
```
( - 3*sqrt(d + e*x)*sqrt(a*e + c*d*x)*a*c*d*e**3 + 5*sqrt(d + e*x)*sqrt(a*
e + c*d*x)*c**2*d**3*e + 2*sqrt(d + e*x)*sqrt(a*e + c*d*x)*c**2*d**2*e**2*
x + 3*sqrt(e)*sqrt(d)*sqrt(c)*log((sqrt(e)*sqrt(a*e + c*d*x) + sqrt(d)*sqr
t(c)*sqrt(d + e*x))/sqrt(a*e**2 - c*d**2))*a**2*e**4 - 6*sqrt(e)*sqrt(d)*s
qrt(c)*log((sqrt(e)*sqrt(a*e + c*d*x) + sqrt(d)*sqrt(c)*sqrt(d + e*x))/sqr
t(a*e**2 - c*d**2))*a*c*d**2*e**2 + 3*sqrt(e)*sqrt(d)*sqrt(c)*log((sqrt(e)
*sqrt(a*e + c*d*x) + sqrt(d)*sqrt(c)*sqrt(d + e*x))/sqrt(a*e**2 - c*d**2))
*c**2*d**4)/(4*c**3*d**3*e)
```

3.250 $\int \frac{d+ex}{\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 116

$$\int \frac{d+ex}{\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\,dx=\frac{\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{cd}+\frac{(cd^2-ae^2)\,\text{arctanh}\left(\frac{\sqrt{c}\sqrt{d}(d+ex)}{\sqrt{e}\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\right)}{c^{3/2}d^{3/2}\sqrt{e}}$$

output
```
(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/c/d+(-a*e^2+c*d^2)*arctanh(c^(1/2)
*d^(1/2)*(e*x+d)/e^(1/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2))/c^(3/2)/
d^(3/2)/e^(1/2)
```

Mathematica [A] (verified)

Time = 0.28 (sec) , antiderivative size = 136, normalized size of antiderivative = 1.17

$$\int \frac{d+ex}{\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\,dx=\frac{\sqrt{c}\sqrt{d}\sqrt{e}(ae+cdx)(d+ex)+(cd^2-ae^2)\sqrt{ae+cdx}\sqrt{d+ex}\,\text{arctanh}\left(\frac{\sqrt{c}\sqrt{d}\sqrt{d+ex}}{\sqrt{e}\sqrt{ae+cdx}}\right)}{c^{3/2}d^{3/2}\sqrt{e}\sqrt{(ae+cdx)(d+ex)}}$$

input
```
Integrate[(d + e*x)/Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2],x]
```

output
```
(Sqrt[c]*Sqrt[d]*Sqrt[e]*(a*e + c*d*x)*(d + e*x) + (c*d^2 - a*e^2)*Sqrt[a*
e + c*d*x]*Sqrt[d + e*x]*ArcTanh[(Sqrt[c]*Sqrt[d]*Sqrt[d + e*x])/(Sqrt[e]*
Sqrt[a*e + c*d*x])])/(c^(3/2)*d^(3/2)*Sqrt[e]*Sqrt[(a*e + c*d*x)*(d + e*x)
])
```

Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 135, normalized size of antiderivative = 1.16, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.086$, Rules used = {1160, 1092, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{d+ex}{\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}dx$$

$$\Big\downarrow \text{1160}$$

$$\frac{\left(d^2-\frac{ae^2}{c}\right)\int \frac{1}{\sqrt{cdex^2+(cd^2+ae^2)x+ade}}dx}{2d}+\frac{\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{cd}$$

$$\Big\downarrow \text{1092}$$

$$\frac{\left(d^2-\frac{ae^2}{c}\right)\int \frac{1}{4cde-\frac{\left(cd^2+2cexd+ae^2\right)^2}{cdex^2+\left(cd^2+ae^2\right)x+ade}}d\frac{cd^2+2cexd+ae^2}{\sqrt{cdex^2+(cd^2+ae^2)x+ade}}}{d}+\frac{\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{cd}$$

$$\Big\downarrow \text{219}$$

$$\frac{\left(d^2-\frac{ae^2}{c}\right)\text{arctanh}\left(\frac{ae^2+cd^2+2cdex}{2\sqrt{c}\sqrt{d}\sqrt{e}\sqrt{x(ae^2+cd^2)+ade+cdex^2}}\right)}{2\sqrt{c}d^{3/2}\sqrt{e}}+\frac{\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{cd}$$

input
```
Int[(d + e*x)/Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2],x]
```

output
```
Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]/(c*d) + ((d^2 - (a*e^2)/c)*Arc
Tanh[(c*d^2 + a*e^2 + 2*c*d*e*x)/(2*Sqrt[c]*Sqrt[d]*Sqrt[e]*Sqrt[a*d*e + (
c*d^2 + a*e^2)*x + c*d*e*x^2])])/(2*Sqrt[c]*d^(3/2)*Sqrt[e])
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1092
```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[I
nt[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a
, b, c}, x]
```

rule 1160
```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[e*((a + b*x + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Simp[(2*c*d - b
*e)/(2*c)   Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
 && NeQ[p, -1]
```

Maple [A] (verified)

Time = 0.99 (sec) , antiderivative size = 186, normalized size of antiderivative = 1.60

method	result
default	$\frac{d \ln\left(\frac{\frac{1}{2} a e^2+\frac{1}{2} c d^2+c d x e}{\sqrt{d e c}}+\sqrt{a d e+\left(a e^2+c d^2\right) x+c d x^2 e}\right)}{\sqrt{d e c}}+e\left(\frac{\sqrt{a d e+\left(a e^2+c d^2\right) x+c d x^2 e}}{d e c}-\frac{\left(a e^2+c d^2\right) \ln\left(\frac{\frac{1}{2} a e^2+\frac{1}{2} c d^2+c d x}{\sqrt{d e c}}\right.}{2 d e}\right.$

input
```
int((e*x+d)/(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(1/2),x,method=_RETURNVERBOS
E)
```

output
```
d*ln((1/2*a*e^2+1/2*c*d^2+c*d*x*e)/(d*e*c)^(1/2)+(a*d*e+(a*e^2+c*d^2)*x+c*
d*x^2*e)^(1/2))/(d*e*c)^(1/2)+e*(1/d/e/c*(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)
^(1/2)-1/2*(a*e^2+c*d^2)/d/e/c*ln((1/2*a*e^2+1/2*c*d^2+c*d*x*e)/(d*e*c)^(1
/2)+(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(1/2))/(d*e*c)^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 338, normalized size of antiderivative = 2.91

$$\int \frac{d+ex}{\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}\,dx$$
$$= \left[\frac{4\sqrt{cdex^2+ade+(cd^2+ae^2)x}cde-(cd^2-ae^2)\sqrt{cde}\log\left(8\,c^2d^2e^2x^2+c^2d^4+6\,acd^2e^2+a^2e^4-4\,\sqrt{}\right.}{4\,c^2d^2e}\right.$$

input
```
integrate((e*x+d)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x, algorithm="fr
icas")
```

output
```
[1/4*(4*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*c*d*e - (c*d^2 - a*e^2
)*sqrt(c*d*e)*log(8*c^2*d^2*e^2*x^2 + c^2*d^4 + 6*a*c*d^2*e^2 + a^2*e^4 -
4*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(2*c*d*e*x + c*d^2 + a*e^2)*
sqrt(c*d*e) + 8*(c^2*d^3*e + a*c*d*e^3)*x))/(c^2*d^2*e), 1/2*(2*sqrt(c*d*e
*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*c*d*e - (c*d^2 - a*e^2)*sqrt(-c*d*e)*arc
tan(1/2*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(2*c*d*e*x + c*d^2 + a
*e^2)*sqrt(-c*d*e)/(c^2*d^2*e^2*x^2 + a*c*d^2*e^2 + (c^2*d^3*e + a*c*d*e^3
)*x)))/(c^2*d^2*e)]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 335 vs. $2(107) = 214$.

Time = 0.68 (sec) , antiderivative size = 335, normalized size of antiderivative = 2.89

$$\int \frac{d+ex}{\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}\,dx$$
$$= \begin{cases} \left(d-\frac{ae^2+cd^2}{2cd}\right)\left(\begin{cases} \frac{\log\left(ae^2+cd^2+2cdex+2\sqrt{cde}\sqrt{ade+cdex^2+x(ae^2+cd^2)}\right)}{\sqrt{cde}} & \text{for } ade-\frac{(ae^2+cd^2)^2}{4cde}\neq 0 \\ \frac{\left(x-\frac{-ae^2-cd^2}{2cde}\right)\log\left(x-\frac{-ae^2-cd^2}{2cde}\right)}{\sqrt{cde\left(x-\frac{-ae^2-cd^2}{2cde}\right)^2}} & \text{otherwise} \end{cases}\right)+\frac{\sqrt{ade+cdex^2 \ldots}}{\ldots} \\ \frac{2d\sqrt{ade+x(ae^2+cd^2)}+\frac{2e\left(-ade\sqrt{ade+x\left(ae^2+cd^2\right)}+\frac{\left(ade+x\left(ae^2+cd^2\right)\right)^{\frac{3}{2}}}{3}\right)}{ae^2+cd^2}}{ae^2+cd^2} \\ \frac{dx+\frac{ex^2}{2}}{\sqrt{ade}} \end{cases}$$

input
```
integrate((e*x+d)/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(1/2),x)
```

output
```
Piecewise(((d - (a*e**2 + c*d**2)/(2*c*d))*Piecewise((log(a*e**2 + c*d**2
+ 2*c*d*e*x + 2*sqrt(c*d*e)*sqrt(a*d*e + c*d*e*x**2 + x*(a*e**2 + c*d**2))
)/sqrt(c*d*e), Ne(a*d*e - (a*e**2 + c*d**2)**2/(4*c*d*e), 0)), ((x - (-a*e
**2 - c*d**2)/(2*c*d*e))*log(x - (-a*e**2 - c*d**2)/(2*c*d*e))/sqrt(c*d*e*
(x - (-a*e**2 - c*d**2)/(2*c*d*e))**2), True)) + sqrt(a*d*e + c*d*e*x**2 +
 x*(a*e**2 + c*d**2))/(c*d), Ne(c*d*e, 0)), ((2*d*sqrt(a*d*e + x*(a*e**2 +
 c*d**2)) + 2*e*(-a*d*e*sqrt(a*d*e + x*(a*e**2 + c*d**2)) + (a*d*e + x*(a*
e**2 + c*d**2))**(3/2)/3)/(a*e**2 + c*d**2))/(a*e**2 + c*d**2), Ne(a*e**2
+ c*d**2, 0)), ((d*x + e*x**2/2)/sqrt(a*d*e), True))
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{d+ex}{\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((e*x+d)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x, algorithm="ma
xima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(a*e^2-c*d^2>0)', see `assume?` f
or more de
```

Giac [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 120, normalized size of antiderivative = 1.03

$$\int \frac{d+ex}{\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}\,dx = -\frac{(cd^2-ae^2)\log\left(\left|-cd^2-ae^2-2\sqrt{cde}\left(\sqrt{cde}x-\sqrt{cdex^2+cd^2x+ae^2x+ade}\right)\right|\right)}{2\sqrt{cde}cd} + \frac{\sqrt{cdex^2+cd^2x+ae^2x+ade}}{cd}$$

input
```
integrate((e*x+d)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x, algorithm="gi
ac")
```

output
```
-1/2*(c*d^2 - a*e^2)*log(abs(-c*d^2 - a*e^2 - 2*sqrt(c*d*e)*(sqrt(c*d*e)*x
 - sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*x + a*d*e))))/(sqrt(c*d*e)*c*d) + sqrt
(c*d*e*x^2 + c*d^2*x + a*e^2*x + a*d*e)/(c*d)
```

Mupad [B] (verification not implemented)

Time = 5.90 (sec) , antiderivative size = 144, normalized size of antiderivative = 1.24

$$\int \frac{d+ex}{\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}\,dx = \frac{\sqrt{c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e}}{c\,d} - \frac{a\,e^3\,\ln\left(2\,\sqrt{\left(a\,e+c\,d\,x\right)\,\left(d+e\,x\right)}\,\sqrt{c\,d\,e}+a\,e^2+c\,d^2+2\,c\,d\,e\,x\right)}{2\,{\left(c\,d\,e\right)}^{3/2}} + \frac{c\,d^2\,e\,\ln\left(2\,\sqrt{\left(a\,e+c\,d\,x\right)\,\left(d+e\,x\right)}\,\sqrt{c\,d\,e}+a\,e^2+c\,d^2+2\,c\,d\,e\,x\right)}{2\,{\left(c\,d\,e\right)}^{3/2}}$$

input
```
int((d + e*x)/(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2),x)
```

output
```
(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2)/(c*d) - (a*e^3*log(2*((a*e +
 c*d*x)*(d + e*x))^(1/2)*(c*d*e)^(1/2) + a*e^2 + c*d^2 + 2*c*d*e*x))/(2*(c
*d*e)^(3/2)) + (c*d^2*e*log(2*((a*e + c*d*x)*(d + e*x))^(1/2)*(c*d*e)^(1/2
) + a*e^2 + c*d^2 + 2*c*d*e*x))/(2*(c*d*e)^(3/2))
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 135, normalized size of antiderivative = 1.16

$$\int \frac{d+ex}{\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\,dx$$
$$= \frac{\sqrt{ex+d}\,\sqrt{cdx+ae}\,cde-\sqrt{e}\,\sqrt{d}\,\sqrt{c}\log\left(\frac{\sqrt{e}\,\sqrt{cdx+ae}+\sqrt{d}\,\sqrt{c}\,\sqrt{ex+d}}{\sqrt{a\,e^2-c\,d^2}}\right)a\,e^2+\sqrt{e}\,\sqrt{d}\,\sqrt{c}\log\left(\frac{\sqrt{e}\,\sqrt{cdx+ae}+\sqrt{d}\,\sqrt{c}\,\sqrt{}}{\sqrt{a\,e^2-c\,d^2}}\right.}{c^2d^2e}$$

input
```
int((e*x+d)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x)
```

output
```
(sqrt(d + e*x)*sqrt(a*e + c*d*x)*c*d*e - sqrt(e)*sqrt(d)*sqrt(c)*log((sqrt
(e)*sqrt(a*e + c*d*x) + sqrt(d)*sqrt(c)*sqrt(d + e*x))/sqrt(a*e**2 - c*d**
2))*a*e**2 + sqrt(e)*sqrt(d)*sqrt(c)*log((sqrt(e)*sqrt(a*e + c*d*x) + sqrt
(d)*sqrt(c)*sqrt(d + e*x))/sqrt(a*e**2 - c*d**2))*c*d**2)/(c**2*d**2*e)
```

3.251 $\int \frac{1}{\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 68

$$\int \frac{1}{\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}\,dx=\frac{2\text{arctanh}\left(\frac{\sqrt{c}\sqrt{d}(d+ex)}{\sqrt{e}\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\right)}{\sqrt{c}\sqrt{d}\sqrt{e}}$$

output
```
2*arctanh(c^(1/2)*d^(1/2)*(e*x+d)/e^(1/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2
)^(1/2))/c^(1/2)/d^(1/2)/e^(1/2)
```

Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.38

$$\int \frac{1}{\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}\,dx=\frac{2\sqrt{ae+cdx}\sqrt{d+ex}\text{arctanh}\left(\frac{\sqrt{c}\sqrt{d}\sqrt{d+ex}}{\sqrt{e}\sqrt{ae+cdx}}\right)}{\sqrt{c}\sqrt{d}\sqrt{e}\sqrt{(ae+cdx)(d+ex)}}$$

input
```
Integrate[1/Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2],x]
```

output
```
(2*Sqrt[a*e + c*d*x]*Sqrt[d + e*x]*ArcTanh[(Sqrt[c]*Sqrt[d]*Sqrt[d + e*x])
/(Sqrt[e]*Sqrt[a*e + c*d*x])])/(Sqrt[c]*Sqrt[d]*Sqrt[e]*Sqrt[(a*e + c*d*x)
*(d + e*x)])
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.21, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.069$, Rules used = {1092, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}dx$$

$$\Big\downarrow \text{ 1092}$$

$$2\int \frac{1}{4cde-\frac{\left(cd^2+2cexd+ae^2\right)^2}{cdex^2+\left(cd^2+ae^2\right)x+ade}}d\frac{cd^2+2cexd+ae^2}{\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}$$

$$\Big\downarrow \text{ 219}$$

$$\frac{\text{arctanh}\left(\frac{ae^2+cd^2+2cdex}{2\sqrt{c}\sqrt{d}\sqrt{e}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}\right)}{\sqrt{c}\sqrt{d}\sqrt{e}}$$

input
```
Int[1/Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2],x]
```

output
```
ArcTanh[(c*d^2 + a*e^2 + 2*c*d*e*x)/(2*Sqrt[c]*Sqrt[d]*Sqrt[e]*Sqrt[a*d*e
+ (c*d^2 + a*e^2)*x + c*d*e*x^2])]/(Sqrt[c]*Sqrt[d]*Sqrt[e])
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1092
```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[I
nt[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a
, b, c}, x]
```

Maple [A] (verified)

Time = 0.91 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.91

method	result	size
default	$\frac{\ln\left(\frac{\frac{1}{2} a\, e^{2}+\frac{1}{2} c\, d^{2}+c d x e}{\sqrt{d e c}}+\sqrt{a d e+\left(a\, e^{2}+c\, d^{2}\right) x+c d\, x^{2} e}\right)}{\sqrt{d e c}}$	62

input
```
int(1/(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
ln((1/2*a*e^2+1/2*c*d^2+c*d*x*e)/(d*e*c)^(1/2)+(a*d*e+(a*e^2+c*d^2)*x+c*d*
x^2*e)^(1/2))/(d*e*c)^(1/2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 115 vs. 2(54) = 108.

Time = 0.10 (sec) , antiderivative size = 244, normalized size of antiderivative = 3.59

$$\int \frac{1}{\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}\,dx$$
$$= \left[\frac{\sqrt{cde}\log\left(8\,c^2d^2e^2x^2+c^2d^4+6\,acd^2e^2+a^2e^4+4\,\sqrt{cdex^2+ade+(cd^2+ae^2)x}(2\,cdex+cd^2+ae^2)\sqrt{}\right.}{2\,cde}\right.$$
$$\left. -\frac{\sqrt{-cde}\arctan\left(\frac{\sqrt{cdex^2+ade+(cd^2+ae^2)x}(2\,cdex+cd^2+ae^2)\sqrt{-cde}}{2\,(c^2d^2e^2x^2+acd^2e^2+(c^2d^3e+acde^3)x)}\right)}{cde}\right]$$

input
```
integrate(1/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x, algorithm="fricas")
```

output
```
[1/2*sqrt(c*d*e)*log(8*c^2*d^2*e^2*x^2 + c^2*d^4 + 6*a*c*d^2*e^2 + a^2*e^4
 + 4*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(2*c*d*e*x + c*d^2 + a*e^
2)*sqrt(c*d*e) + 8*(c^2*d^3*e + a*c*d*e^3)*x)/(c*d*e), -sqrt(-c*d*e)*arcta
n(1/2*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(2*c*d*e*x + c*d^2 + a*e
^2)*sqrt(-c*d*e)/(c^2*d^2*e^2*x^2 + a*c*d^2*e^2 + (c^2*d^3*e + a*c*d*e^3)*
x))/(c*d*e)]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 218 vs. $2(66) = 132$.

Time = 0.50 (sec) , antiderivative size = 218, normalized size of antiderivative = 3.21

$$\int \frac{1}{\sqrt{ade + (cd^2 + ae^2)\,x + cdex^2}}\,dx = \begin{cases} \frac{\log\left(ae^2+cd^2+2cdex+2\sqrt{cde}\sqrt{ade+cdex^2+x(ae^2+cd^2)}\right)}{\sqrt{cde}} & \text{for } cde \neq 0 \wedge ade - \frac{(ae^2+cd^2)^2}{4cde} \neq 0 \\ \frac{\left(x-\frac{-ae^2-cd^2}{2cde}\right)\log\left(x-\frac{-ae^2-cd^2}{2cde}\right)}{\sqrt{cde\left(x-\frac{-ae^2-cd^2}{2cde}\right)^2}} & \text{for } cde \neq 0 \\ \frac{2\sqrt{ade+x(ae^2+cd^2)}}{ae^2+cd^2} & \text{for } ae^2+cd^2 \neq 0 \\ \frac{x}{\sqrt{ade}} & \text{otherwise} \end{cases}$$

input
```
integrate(1/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(1/2),x)
```

output
```
Piecewise((log(a*e**2 + c*d**2 + 2*c*d*e*x + 2*sqrt(c*d*e)*sqrt(a*d*e + c*
d*e*x**2 + x*(a*e**2 + c*d**2)))/sqrt(c*d*e), Ne(c*d*e, 0) & Ne(a*d*e - (a
*e**2 + c*d**2)**2/(4*c*d*e), 0)), ((x - (-a*e**2 - c*d**2)/(2*c*d*e))*log
(x - (-a*e**2 - c*d**2)/(2*c*d*e))/sqrt(c*d*e*(x - (-a*e**2 - c*d**2)/(2*c
*d*e))**2), Ne(c*d*e, 0)), (2*sqrt(a*d*e + x*(a*e**2 + c*d**2))/(a*e**2 +
c*d**2), Ne(a*e**2 + c*d**2, 0)), (x/sqrt(a*d*e), True))
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{\sqrt{ade + (cd^2 + ae^2)x + cdex^2}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(1/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(a*e^2-c*d^2>0)', see `assume?` f
or more de
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 156 vs. $2(54) = 108$.

Time = 0.16 (sec) , antiderivative size = 156, normalized size of antiderivative = 2.29

$$\begin{aligned}&\int \frac{1}{\sqrt{ade + (cd^2 + ae^2)x + cdex^2}}\,dx \\ &= \frac{1}{4}\sqrt{cdex^2 + cd^2x + ae^2x + ade}\left(2x + \frac{cd^2 + ae^2}{cde}\right) \\ &\quad + \frac{(c^2d^4 - 2acd^2e^2 + a^2e^4)\log\left(\left|-cd^2 - ae^2 - 2\sqrt{cde}\left(\sqrt{cde}x - \sqrt{cdex^2 + cd^2x + ae^2x + ade}\right)\right|\right)}{8\sqrt{cde}cde}\end{aligned}$$

input
```
integrate(1/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x, algorithm="giac")
```

output
```
1/4*sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*x + a*d*e)*(2*x + (c*d^2 + a*e^2)/(c*
d*e)) + 1/8*(c^2*d^4 - 2*a*c*d^2*e^2 + a^2*e^4)*log(abs(-c*d^2 - a*e^2 - 2
*sqrt(c*d*e)*(sqrt(c*d*e)*x - sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*x + a*d*e))
))/(sqrt(c*d*e)*c*d*e)
```

Mupad [B] (verification not implemented)

Time = 5.85 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.72

$$\int \frac{1}{\sqrt{ade + (cd^2 + ae^2)\,x + cdex^2}}\,dx = \frac{\ln\left(2\sqrt{(a\,e + c\,d\,x)\,(d + e\,x)}\,\sqrt{c\,d\,e} + a\,e^2 + c\,d^2 + 2\,c\,d\,e\,x\right)}{\sqrt{c\,d\,e}}$$

input
```
int(1/(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2),x)
```

output
```
log(2*((a*e + c*d*x)*(d + e*x))^(1/2)*(c*d*e)^(1/2) + a*e^2 + c*d^2 + 2*c*
d*e*x)/(c*d*e)^(1/2)
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.85

$$\int \frac{1}{\sqrt{ade + (cd^2 + ae^2)\,x + cdex^2}}\,dx = \frac{2\sqrt{e}\,\sqrt{d}\,\sqrt{c}\log\left(\frac{\sqrt{e}\,\sqrt{cdx+ae}+\sqrt{d}\,\sqrt{c}\,\sqrt{ex+d}}{\sqrt{a\,e^2-c\,d^2}}\right)}{cde}$$

input
```
int(1/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x)
```

output
```
(2*sqrt(e)*sqrt(d)*sqrt(c)*log((sqrt(e)*sqrt(a*e + c*d*x) + sqrt(d)*sqrt(c
)*sqrt(d + e*x))/sqrt(a*e**2 - c*d**2)))/(c*d*e)
```

3.252 $\int \frac{1}{(d+ex)\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\,dx$

Optimal result

Integrand size = 37, antiderivative size = 52

$$\int \frac{1}{(d+ex)\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}\,dx=\frac{2\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{(cd^2-ae^2)\,(d+ex)}$$

output
```
2*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(-a*e^2+c*d^2)/(e*x+d)
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.81

$$\int \frac{1}{(d+ex)\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}\,dx=\frac{2(ae+cdx)}{(cd^2-ae^2)\sqrt{(ae+cdx)(d+ex)}}$$

input
```
Integrate[1/((d + e*x)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]),x]
```

output
```
(2*(a*e + c*d*x))/((c*d^2 - a*e^2)*Sqrt[(a*e + c*d*x)*(d + e*x)])
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.027$, Rules used = {1123}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(d+ex)\sqrt{x(ae^2+cd^2)+ade+cdex^2}}\,dx$$

$$\Big\downarrow \text{1123}$$

$$\frac{2\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{(d+ex)(cd^2-ae^2)}$$

input
```
Int[1/((d + e*x)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]),x]
```

output
```
(2*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/((c*d^2 - a*e^2)*(d + e*x)
)
```

Defintions of rubi rules used

rule 1123
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[e*(d + e*x)^m*((a + b*x + c*x^2)^(p + 1)/((p + 1)*(2*c*d - b
*e))), x] /; FreeQ[{a, b, c, d, e, m, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2,
0] && EqQ[m + 2*p + 2, 0]
```

Maple [A] (verified)

Time = 1.12 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.96

method	result	size
trager	$-\frac{2\sqrt{cd\,x^{2}e+a\,e^{2}x+c\,d^{2}x+ade}}{\left(a\,e^{2}-c\,d^{2}\right)\left(ex+d\right)}$	50
gosper	$-\frac{2\left(cdx+ae\right)}{\left(a\,e^{2}-c\,d^{2}\right)\sqrt{cd\,x^{2}e+a\,e^{2}x+c\,d^{2}x+ade}}$	51
orering	$-\frac{2\left(cdx+ae\right)}{\left(a\,e^{2}-c\,d^{2}\right)\sqrt{ade+\left(a\,e^{2}+c\,d^{2}\right)x+cd\,x^{2}e}}$	52
default	$-\frac{2\sqrt{dec\left(x+\frac{d}{e}\right)^{2}+\left(a\,e^{2}-c\,d^{2}\right)\left(x+\frac{d}{e}\right)}}{e\left(a\,e^{2}-c\,d^{2}\right)\left(x+\frac{d}{e}\right)}$	65

input
```
int(1/(e*x+d)/(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(1/2),x,method=_RETURNVERB
OSE)
```

output
```
-2/(a*e^2-c*d^2)/(e*x+d)*(c*d*e*x^2+a*e^2*x+c*d^2*x+a*d*e)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.13

$$\int \frac{1}{(d+ex)\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}\,dx = \frac{2\,\sqrt{cdex^2+ade+(cd^2+ae^2)x}}{cd^3-ade^2+(cd^2e-ae^3)x}$$

input
```
integrate(1/(e*x+d)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x, algorithm="
fricas")
```

output
```
2*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)/(c*d^3 - a*d*e^2 + (c*d^2*e
- a*e^3)*x)
```

Sympy [F]

$$\int \frac{1}{(d+ex)\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\,dx = \int \frac{1}{\sqrt{(d+ex)(ae+cdx)}\,(d+ex)}\,dx$$

input
```
integrate(1/(e*x+d)/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(1/2),x)
```

output
```
Integral(1/(sqrt((d + e*x)*(a*e + c*d*x))*(d + e*x)), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{(d+ex)\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(1/(e*x+d)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x, algorithm="
maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e*(a*e^2-c*d^2)>0)', see `assume
?` for mor
```

Giac [F(-2)]

Exception generated.

$$\int \frac{1}{(d+ex)\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(1/(e*x+d)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x, algorithm="
giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Unable to divide, perhaps due to ro
unding error%%%{%%%{1,[0,0,1]%%%},[2]%%%}+%%%{%%{[%%%{-2,[0,1,0]%%%},0]:[1
,0,%%%{-1
```

Mupad [B] (verification not implemented)

Time = 5.61 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.96

$$\int \frac{1}{(d+ex)\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}\,dx = -\frac{2\,\sqrt{c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e}}{\left(a\,e^2-c\,d^2\right)\,\left(d+e\,x\right)}$$

input
```
int(1/((d + e*x)*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2)),x)
```

output
```
-(2*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2))/((a*e^2 - c*d^2)*(d + e
*x))
```

Reduce [B] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.37

$$\int \frac{1}{(d+ex)\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}\,dx = \frac{-2\sqrt{ex+d}\,\sqrt{cdx+ae}\,e-2\sqrt{e}\,\sqrt{d}\,\sqrt{c}\,d-2\sqrt{e}\,\sqrt{d}\,\sqrt{c}\,ex}{e\left(a\,e^3x-c\,d^2ex+ad\,e^2-c\,d^3\right)}$$

input
```
int(1/(e*x+d)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x)
```

output
```
( - 2*(sqrt(d + e*x)*sqrt(a*e + c*d*x)*e + sqrt(e)*sqrt(d)*sqrt(c)*d + sqr
t(e)*sqrt(d)*sqrt(c)*e*x))/(e*(a*d*e**2 + a*e**3*x - c*d**3 - c*d**2*e*x))
```

3.253 $\int \frac{1}{(d+ex)^2\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\,dx$

Optimal result

Integrand size = 37, antiderivative size = 111

$$\int \frac{1}{(d+ex)^2\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\,dx = \frac{2\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{3(cd^2-ae^2)(d+ex)^2}+\frac{4cd\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{3(cd^2-ae^2)^2(d+ex)}$$

output
```
2/3*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(-a*e^2+c*d^2)/(e*x+d)^2+4/3*c
*d*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(-a*e^2+c*d^2)^2/(e*x+d)
```

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.55

$$\int \frac{1}{(d+ex)^2\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\,dx = \frac{2\sqrt{(ae+cdx)(d+ex)}(-ae^2+cd(3d+2ex))}{3(cd^2-ae^2)^2(d+ex)^2}$$

input
```
Integrate[1/((d + e*x)^2*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]),x]
```

output
```
(2*Sqrt[(a*e + c*d*x)*(d + e*x)]*(-(a*e^2) + c*d*(3*d + 2*e*x)))/(3*(c*d^2
 - a*e^2)^2*(d + e*x)^2)
```

Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 111, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.054$, Rules used = {1129, 1123}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(d+ex)^2\sqrt{x(ae^2+cd^2)+ade+cdex^2}}dx$$

$$\Big\downarrow \text{1129}$$

$$\frac{2cd\int \frac{1}{(d+ex)\sqrt{cdex^2+(cd^2+ae^2)x+ade}}dx}{3(cd^2-ae^2)}+\frac{2\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{3(d+ex)^2(cd^2-ae^2)}$$

$$\Big\downarrow \text{1123}$$

$$\frac{4cd\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{3(d+ex)(cd^2-ae^2)^2}+\frac{2\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{3(d+ex)^2(cd^2-ae^2)}$$

input
```
Int[1/((d + e*x)^2*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]),x]
```

output
```
(2*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(3*(c*d^2 - a*e^2)*(d + e*
x)^2) + (4*c*d*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(3*(c*d^2 - a*
e^2)^2*(d + e*x))
```

Defintions of rubi rules used

rule 1123
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[e*(d + e*x)^m*((a + b*x + c*x^2)^(p + 1)/((p + 1)*(2*c*d - b
*e))), x] /; FreeQ[{a, b, c, d, e, m, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2,
0] && EqQ[m + 2*p + 2, 0]
```

rule 1129
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(-e)*(d + e*x)^m*((a + b*x + c*x^2)^(p + 1)/((m + p + 1)*(2*
c*d - b*e))), x] + Simp[c*(Simplify[m + 2*p + 2]/((m + p + 1)*(2*c*d - b*e)
))   Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d
, e, m, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && ILtQ[Simplify[m + 2*p +
2], 0]
```

Maple [A] (verified)

Time = 1.18 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.73

method	result	size
trager	$-\frac{2\left(-2cdxe+a\,e^{2}-3c\,d^{2}\right)\sqrt{cd\,x^{2}e+a\,e^{2}x+c\,d^{2}x+ade}}{3\left(a^{2}e^{4}-2ac\,d^{2}e^{2}+c^{2}d^{4}\right)\left(ex+d\right)^{2}}$	81
gosper	$-\frac{2\left(cdx+ae\right)\left(-2cdxe+a\,e^{2}-3c\,d^{2}\right)}{3\left(ex+d\right)\left(a^{2}e^{4}-2ac\,d^{2}e^{2}+c^{2}d^{4}\right)\sqrt{cd\,x^{2}e+a\,e^{2}x+c\,d^{2}x+ade}}$	89
orering	$-\frac{2\left(-2cdxe+a\,e^{2}-3c\,d^{2}\right)\left(cdx+ae\right)}{3\left(a^{2}e^{4}-2ac\,d^{2}e^{2}+c^{2}d^{4}\right)\left(ex+d\right)\sqrt{ade+\left(a\,e^{2}+c\,d^{2}\right)x+cd\,x^{2}e}}$	90
default	$\frac{-\frac{2\sqrt{dec\left(x+\frac{d}{e}\right)^{2}+\left(a\,e^{2}-c\,d^{2}\right)\left(x+\frac{d}{e}\right)}}{3\left(a\,e^{2}-c\,d^{2}\right)\left(x+\frac{d}{e}\right)^{2}}+\frac{4dec\sqrt{dec\left(x+\frac{d}{e}\right)^{2}+\left(a\,e^{2}-c\,d^{2}\right)\left(x+\frac{d}{e}\right)}}{3\left(a\,e^{2}-c\,d^{2}\right)^{2}\left(x+\frac{d}{e}\right)}}{e^{2}}$	131

input
```
int(1/(e*x+d)^2/(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(1/2),x,method=_RETURNVE
RBOSE)
```

output
```
-2/3*(-2*c*d*e*x+a*e^2-3*c*d^2)/(a^2*e^4-2*a*c*d^2*e^2+c^2*d^4)/(e*x+d)^2*
(c*d*e*x^2+a*e^2*x+c*d^2*x+a*d*e)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 140, normalized size of antiderivative = 1.26

$$\int \frac{1}{(d+ex)^2\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}\,dx = \frac{2\,\sqrt{cdex^2+ade+(cd^2+ae^2)x}(2\,cdex+3\,cd^2-ae^2)}{3\,(c^2d^6-2\,acd^4e^2+a^2d^2e^4+(c^2d^4e^2-2\,acd^2e^4+a^2e^6)x^2+2\,(c^2d^5e-2\,acd^3e^3+a^2de^5)x)}$$

input
```
integrate(1/(e*x+d)^2/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x, algorithm
="fricas")
```

output
```
2/3*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(2*c*d*e*x + 3*c*d^2 - a*e
^2)/(c^2*d^6 - 2*a*c*d^4*e^2 + a^2*d^2*e^4 + (c^2*d^4*e^2 - 2*a*c*d^2*e^4
+ a^2*e^6)*x^2 + 2*(c^2*d^5*e - 2*a*c*d^3*e^3 + a^2*d*e^5)*x)
```

Sympy [F]

$$\int \frac{1}{(d+ex)^2\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}\,dx = \int \frac{1}{\sqrt{(d+ex)\,(ae+cdx)}\,(d+ex)^2}\,dx$$

input
```
integrate(1/(e*x+d)**2/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(1/2),x)
```

output
```
Integral(1/(sqrt((d + e*x)*(a*e + c*d*x))*(d + e*x)**2), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{(d+ex)^2\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(1/(e*x+d)^2/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x, algorithm
="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e*(a*e^2-c*d^2)>0)', see `assume
?` for mor
```

Giac [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 176, normalized size of antiderivative = 1.59

$$\int \frac{1}{(d+ex)^2\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}\,dx$$
$$= -\frac{4\sqrt{cde}cd\mathrm{sgn}\left(\frac{1}{ex+d}\right)\mathrm{sgn}(e)}{3\left(c^2d^4e-2\,acd^2e^3+a^2e^5\right)}$$
$$+\frac{2\left(3\sqrt{cde-\frac{cd^2e}{ex+d}+\frac{ae^3}{ex+d}}cde-\left(cde-\frac{cd^2e}{ex+d}+\frac{ae^3}{ex+d}\right)^{\frac{3}{2}}\right)}{3\left(cd^2e^2-ae^4\right)\left(cd^2\mathrm{sgn}\left(\frac{1}{ex+d}\right)\mathrm{sgn}\left(e\right)-ae^2\mathrm{sgn}\left(\frac{1}{ex+d}\right)\mathrm{sgn}\left(e\right)\right)}$$

input
```
integrate(1/(e*x+d)^2/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x, algorithm
="giac")
```

output
```
-4/3*sqrt(c*d*e)*c*d*sgn(1/(e*x + d))*sgn(e)/(c^2*d^4*e - 2*a*c*d^2*e^3 +
a^2*e^5) + 2/3*(3*sqrt(c*d*e - c*d^2*e/(e*x + d) + a*e^3/(e*x + d))*c*d*e
- (c*d*e - c*d^2*e/(e*x + d) + a*e^3/(e*x + d))^(3/2))/((c*d^2*e^2 - a*e^4
)*(c*d^2*sgn(1/(e*x + d))*sgn(e) - a*e^2*sgn(1/(e*x + d))*sgn(e)))
```

Mupad [B] (verification not implemented)

Time = 5.73 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.62

$$\int \frac{1}{(d+ex)^2\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}\,dx$$
$$=\frac{2\left(3\,c\,d^2+2\,c\,x\,d\,e-a\,e^2\right)\,\sqrt{c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e}}{3\,{\left(a\,e^2-c\,d^2\right)}^2\,{\left(d+e\,x\right)}^2}$$

input
```
int(1/((d + e*x)^2*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2)),x)
```

output
```
(2*(3*c*d^2 - a*e^2 + 2*c*d*e*x)*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(
1/2))/(3*(a*e^2 - c*d^2)^2*(d + e*x)^2)
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 211, normalized size of antiderivative = 1.90

$$\int \frac{1}{(d+ex)^2\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}\,dx$$
$$= \frac{-\frac{2\sqrt{ex+d}\,\sqrt{cdx+ae}\,a\,e^3}{3}+2\sqrt{ex+d}\,\sqrt{cdx+ae}\,c\,d^2e+\frac{4\sqrt{ex+d}\,\sqrt{cdx+ae}\,cd\,e^2x}{3}-\frac{4\sqrt{e}\,\sqrt{d}\,\sqrt{c}\,c\,d^3}{3}-\frac{8\sqrt{e}\,\sqrt{d}\,\sqrt{c}\,c\,d^2ex}{3}-}{e\,(a^2e^6x^2-2ac\,d^2e^4x^2+c^2d^4e^2x^2+2a^2d\,e^5x-4ac\,d^3e^3x+2c^2d^5ex+a^2d^2e^4-2ac\,d^4e^2+c^2}$$

input
```
int(1/(e*x+d)^2/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x)
```

output
```
(2*( - sqrt(d + e*x)*sqrt(a*e + c*d*x)*a*e**3 + 3*sqrt(d + e*x)*sqrt(a*e +
 c*d*x)*c*d**2*e + 2*sqrt(d + e*x)*sqrt(a*e + c*d*x)*c*d*e**2*x - 2*sqrt(e
)*sqrt(d)*sqrt(c)*c*d**3 - 4*sqrt(e)*sqrt(d)*sqrt(c)*c*d**2*e*x - 2*sqrt(e
)*sqrt(d)*sqrt(c)*c*d*e**2*x**2))/(3*e*(a**2*d**2*e**4 + 2*a**2*d*e**5*x +
 a**2*e**6*x**2 - 2*a*c*d**4*e**2 - 4*a*c*d**3*e**3*x - 2*a*c*d**2*e**4*x*
*2 + c**2*d**6 + 2*c**2*d**5*e*x + c**2*d**4*e**2*x**2))
```

3.254 $\int \frac{1}{(d+ex)^3\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}\,dx$

Optimal result

Integrand size = 37, antiderivative size = 171

$$\int \frac{1}{(d+ex)^3\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}\,dx = \frac{2\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{5\,(cd^2-ae^2)\,(d+ex)^3}+\frac{8cd\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{15\,(cd^2-ae^2)^2\,(d+ex)^2}+\frac{16c^2d^2\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{15\,(cd^2-ae^2)^3\,(d+ex)}$$

output
```
2/5*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(-a*e^2+c*d^2)/(e*x+d)^3+8/15*
c*d*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(-a*e^2+c*d^2)^2/(e*x+d)^2+16/
15*c^2*d^2*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(-a*e^2+c*d^2)^3/(e*x+d
)
```

Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.55

$$\int \frac{1}{(d+ex)^3\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}\,dx$$
$$=\frac{2\sqrt{(ae+cdx)(d+ex)}(3a^2e^4-2acde^2(5d+2ex)+c^2d^2(15d^2+20dex+8e^2x^2))}{15\,(cd^2-ae^2)^3\,(d+ex)^3}$$

input
```
Integrate[1/((d + e*x)^3*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]),x]
```

output
```
(2*Sqrt[(a*e + c*d*x)*(d + e*x)]*(3*a^2*e^4 - 2*a*c*d*e^2*(5*d + 2*e*x) +
c^2*d^2*(15*d^2 + 20*d*e*x + 8*e^2*x^2)))/(15*(c*d^2 - a*e^2)^3*(d + e*x)^
3)
```

Rubi [A] (verified)

Time = 0.53 (sec) , antiderivative size = 186, normalized size of antiderivative = 1.09, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.081, Rules used = {1129, 1129, 1123}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(d+ex)^3\sqrt{x\,(ae^2+cd^2)+ade+cdex^2}}\,dx$$
$$\Big\downarrow \text{1129}$$
$$\frac{4cd\int \frac{1}{(d+ex)^2\sqrt{cdex^2+(cd^2+ae^2)x+ade}}dx}{5\,(cd^2-ae^2)}+\frac{2\sqrt{x\,(ae^2+cd^2)+ade+cdex^2}}{5(d+ex)^3\,(cd^2-ae^2)}$$
$$\Big\downarrow \text{1129}$$

$$\frac{4cd\left(\frac{2cd\int \frac{1}{(d+ex)\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}dx}{3(cd^2-ae^2)}+\frac{2\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{3(d+ex)^2(cd^2-ae^2)}\right)}{5\left(cd^2-ae^2\right)}+\frac{2\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{5(d+ex)^3\left(cd^2-ae^2\right)}$$

$$\Big\downarrow \text{ 1123}$$

$$\frac{4cd\left(\frac{4cd\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{3(d+ex)(cd^2-ae^2)^2}+\frac{2\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{3(d+ex)^2(cd^2-ae^2)}\right)}{5\left(cd^2-ae^2\right)}+\frac{2\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{5(d+ex)^3\left(cd^2-ae^2\right)}$$

input
```
Int[1/((d + e*x)^3*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]),x]
```

output
```
(2*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(5*(c*d^2 - a*e^2)*(d + e*
x)^3) + (4*c*d*((2*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(3*(c*d^2
- a*e^2)*(d + e*x)^2) + (4*c*d*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]
)/(3*(c*d^2 - a*e^2)^2*(d + e*x))))/(5*(c*d^2 - a*e^2))
```

Defintions of rubi rules used

rule 1123
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[e*(d + e*x)^m*((a + b*x + c*x^2)^(p + 1)/((p + 1)*(2*c*d - b
*e))), x] /; FreeQ[{a, b, c, d, e, m, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2,
0] && EqQ[m + 2*p + 2, 0]
```

rule 1129
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(-e)*(d + e*x)^m*((a + b*x + c*x^2)^(p + 1)/((m + p + 1)*(2*
c*d - b*e))), x] + Simp[c*(Simplify[m + 2*p + 2]/((m + p + 1)*(2*c*d - b*e)
))   Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d
, e, m, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && ILtQ[Simplify[m + 2*p +
2], 0]
```

Maple [A] (verified)

Time = 1.33 (sec) , antiderivative size = 138, normalized size of antiderivative = 0.81

method	result	size
trager	$-\frac{2(8x^2c^2d^2e^2-4xacd\,e^3+20x\,c^2d^3e+3a^2e^4-10ac\,d^2e^2+15c^2d^4)\sqrt{cd\,x^2e+a\,e^2x+c\,d^2x+ade}}{15(e^6a^3-3d^2e^4a^2c+3d^4e^2a\,c^2-d^6c^3)(ex+d)^3}$	138
gosper	$-\frac{2(cdx+ae)(8x^2c^2d^2e^2-4xacd\,e^3+20x\,c^2d^3e+3a^2e^4-10ac\,d^2e^2+15c^2d^4)}{15(ex+d)^2(e^6a^3-3d^2e^4a^2c+3d^4e^2a\,c^2-d^6c^3)\sqrt{cd\,x^2e+a\,e^2x+c\,d^2x+ade}}$	146
orering	$-\frac{2(8x^2c^2d^2e^2-4xacd\,e^3+20x\,c^2d^3e+3a^2e^4-10ac\,d^2e^2+15c^2d^4)(cdx+ae)}{15(e^6a^3-3d^2e^4a^2c+3d^4e^2a\,c^2-d^6c^3)(ex+d)^2\sqrt{ade+(a\,e^2+c\,d^2)x+cd\,x^2e}}$	147
default	$\frac{-\frac{2\sqrt{dec\left(x+\frac{d}{e}\right)^2+\left(a\,e^2-c\,d^2\right)\left(x+\frac{d}{e}\right)}}{5\left(a\,e^2-c\,d^2\right)\left(x+\frac{d}{e}\right)^3}-\frac{4dec\left(-\frac{2\sqrt{dec\left(x+\frac{d}{e}\right)^2+\left(a\,e^2-c\,d^2\right)\left(x+\frac{d}{e}\right)}}{3\left(a\,e^2-c\,d^2\right)\left(x+\frac{d}{e}\right)^2}+\frac{4dec\sqrt{dec\left(x+\frac{d}{e}\right)^2+\left(a\,e^2-c\,d^2\right)\left(x+\frac{d}{e}\right)}}{3\left(a\,e^2-c\,d^2\right)^2\left(x+\frac{d}{e}\right)}\right)}{5\left(a\,e^2-c\,d^2\right)}}{e^3}$	212

input
```
int(1/(e*x+d)^3/(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(1/2),x,method=_RETURNVE
RBOSE)
```

output
```
-2/15*(8*c^2*d^2*e^2*x^2-4*a*c*d*e^3*x+20*c^2*d^3*e*x+3*a^2*e^4-10*a*c*d^2
*e^2+15*c^2*d^4)/(a^3*e^6-3*a^2*c*d^2*e^4+3*a*c^2*d^4*e^2-c^3*d^6)/(e*x+d)
^3*(c*d*e*x^2+a*e^2*x+c*d^2*x+a*d*e)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 1.29 (sec) , antiderivative size = 279, normalized size of antiderivative = 1.63

$$\int \frac{1}{(d+ex)^3\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}\,dx$$
$$= \frac{2\,(8\,c^2d^2e^2x^2+15\,c^2d^4-10\,acd^2e^2+3\,a^2e^4+4\,(5\,c^2d^3e-acd}{15\,(c^3d^9-3\,ac^2d^7e^2+3\,a^2cd^5e^4-a^3d^3e^6+(c^3d^6e^3-3\,ac^2d^4e^5+3\,a^2cd^2e^7-a^3e^9)x^3+3\,(c^3d^7e^2-3\,ac}$$

input
```
integrate(1/(e*x+d)^3/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x, algorithm
="fricas")
```

output
```
2/15*(8*c^2*d^2*e^2*x^2 + 15*c^2*d^4 - 10*a*c*d^2*e^2 + 3*a^2*e^4 + 4*(5*c
^2*d^3*e - a*c*d*e^3)*x)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)/(c^3*
d^9 - 3*a*c^2*d^7*e^2 + 3*a^2*c*d^5*e^4 - a^3*d^3*e^6 + (c^3*d^6*e^3 - 3*a
*c^2*d^4*e^5 + 3*a^2*c*d^2*e^7 - a^3*e^9)*x^3 + 3*(c^3*d^7*e^2 - 3*a*c^2*d
^5*e^4 + 3*a^2*c*d^3*e^6 - a^3*d*e^8)*x^2 + 3*(c^3*d^8*e - 3*a*c^2*d^6*e^3
 + 3*a^2*c*d^4*e^5 - a^3*d^2*e^7)*x)
```

Sympy [F]

$$\int \frac{1}{(d+ex)^3\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}\,dx = \int \frac{1}{\sqrt{(d+ex)\,(ae+cdx)}\,(d+ex)^3}\,dx$$

input
```
integrate(1/(e*x+d)**3/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(1/2),x)
```

output
```
Integral(1/(sqrt((d + e*x)*(a*e + c*d*x))*(d + e*x)**3), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{(d+ex)^3\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(1/(e*x+d)^3/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x, algorithm
="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e*(a*e^2-c*d^2)>0)', see `assume
?` for mor
```

Giac [F(-2)]

Exception generated.

$$\int \frac{1}{(d+ex)^3\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(1/(e*x+d)^3/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x, algorithm
="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Unable to divide, perhaps due to ro
unding error%%%{%%%{1,[0,0,3]%%%},[6]%%%}+%%%{%%{[%%%{-6,[0,1,2]%%%},0]:[1
,0,%%%{-1
```

Mupad [B] (verification not implemented)

Time = 6.17 (sec) , antiderivative size = 110, normalized size of antiderivative = 0.64

$$\int \frac{1}{(d+ex)^3\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}\,dx = -\frac{2\sqrt{c\,d\,e\,x^2+(c\,d^2+a\,e^2)\,x+a\,d\,e}\,(3\,a^2\,e^4-10\,a\,c\,d^2\,e^2-4\,a\,c\,d\,e^3\,x+15\,c^2\,d^4+20\,c^2\,d^3\,e\,x+8\,c^2\,d}{15\,(a\,e^2-c\,d^2)^3\,(d+e\,x)^3}$$

input
```
int(1/((d + e*x)^3*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2)),x)
```

output
```
-(2*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2)*(3*a^2*e^4 + 15*c^2*d^4
+ 8*c^2*d^2*e^2*x^2 - 10*a*c*d^2*e^2 + 20*c^2*d^3*e*x - 4*a*c*d*e^3*x))/(1
5*(a*e^2 - c*d^2)^3*(d + e*x)^3)
```

Reduce [B] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 431, normalized size of antiderivative = 2.52

$$\int \frac{1}{(d+ex)^3\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}\,dx$$
$$= \frac{-\frac{2\sqrt{ex+d}\,\sqrt{cdx+ae}\,a^2e^5}{5}+\frac{4\sqrt{ex+d}\,\sqrt{cdx+ae}\,ac\,d^2e^3}{3}+\frac{8\sqrt{ex+d}\,\sqrt{cdx+ae}\,acd\,e^4x}{15}-2\sqrt{ex+d}\,\sqrt{cdx+ae}\,c^2d^4e-\frac{8\sqrt{ex+d}}{}}{e\,(a^3e^9x^3-3a^2c\,d^2e^7x^3+3a\,c^2d^4e^5x^3-c^3d^6e^3x^3+3a^3d\,e^8x^2-9a^2c\,d^3e^6x^2+9a\,c^2d^5e^4x^2-}$$

input
```
int(1/(e*x+d)^3/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x)
```

output
```
(2*( - 3*sqrt(d + e*x)*sqrt(a*e + c*d*x)*a**2*e**5 + 10*sqrt(d + e*x)*sqrt
(a*e + c*d*x)*a*c*d**2*e**3 + 4*sqrt(d + e*x)*sqrt(a*e + c*d*x)*a*c*d*e**4
*x - 15*sqrt(d + e*x)*sqrt(a*e + c*d*x)*c**2*d**4*e - 20*sqrt(d + e*x)*sqr
t(a*e + c*d*x)*c**2*d**3*e**2*x - 8*sqrt(d + e*x)*sqrt(a*e + c*d*x)*c**2*d
**2*e**3*x**2 + 8*sqrt(e)*sqrt(d)*sqrt(c)*c**2*d**5 + 24*sqrt(e)*sqrt(d)*s
qrt(c)*c**2*d**4*e*x + 24*sqrt(e)*sqrt(d)*sqrt(c)*c**2*d**3*e**2*x**2 + 8*
sqrt(e)*sqrt(d)*sqrt(c)*c**2*d**2*e**3*x**3))/(15*e*(a**3*d**3*e**6 + 3*a*
*3*d**2*e**7*x + 3*a**3*d*e**8*x**2 + a**3*e**9*x**3 - 3*a**2*c*d**5*e**4
- 9*a**2*c*d**4*e**5*x - 9*a**2*c*d**3*e**6*x**2 - 3*a**2*c*d**2*e**7*x**3
 + 3*a*c**2*d**7*e**2 + 9*a*c**2*d**6*e**3*x + 9*a*c**2*d**5*e**4*x**2 + 3
*a*c**2*d**4*e**5*x**3 - c**3*d**9 - 3*c**3*d**8*e*x - 3*c**3*d**7*e**2*x*
*2 - c**3*d**6*e**3*x**3))
```

3.255 $\int \frac{1}{(d+ex)^4\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\,dx$

Optimal result

Integrand size = 37, antiderivative size = 231

$$\begin{aligned}\int \frac{1}{(d+ex)^4\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\,dx &= \frac{2\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{7(cd^2-ae^2)(d+ex)^4}+\frac{12cd\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{35(cd^2-ae^2)^2(d+ex)^3}\\ &\quad+\frac{16c^2d^2\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{35(cd^2-ae^2)^3(d+ex)^2}+\frac{32c^3d^3\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{35(cd^2-ae^2)^4(d+ex)}\end{aligned}$$

output
```
2/7*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(-a*e^2+c*d^2)/(e*x+d)^4+12/35
*c*d*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(-a*e^2+c*d^2)^2/(e*x+d)^3+16
/35*c^2*d^2*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(-a*e^2+c*d^2)^3/(e*x+
d)^2+32/35*c^3*d^3*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(-a*e^2+c*d^2)^
4/(e*x+d)
```

Mathematica [A] (verified)

Time = 0.24 (sec) , antiderivative size = 138, normalized size of antiderivative = 0.60

$$\int \frac{1}{(d+ex)^4\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}\,dx$$
$$= \frac{2\sqrt{(ae+cdx)(d+ex)}(-5a^3e^6+3a^2cde^4(7d+2ex)-ac^2d^2e^2(35d^2+28dex+8e^2x^2)+c^3d^3(35d^3+70}{35\,(cd^2-ae^2)^4\,(d+ex)^4}$$

input
```
Integrate[1/((d + e*x)^4*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]),x]
```

output
```
(2*Sqrt[(a*e + c*d*x)*(d + e*x)]*(-5*a^3*e^6 + 3*a^2*c*d*e^4*(7*d + 2*e*x)
 - a*c^2*d^2*e^2*(35*d^2 + 28*d*e*x + 8*e^2*x^2) + c^3*d^3*(35*d^3 + 70*d^
2*e*x + 56*d*e^2*x^2 + 16*e^3*x^3)))/(35*(c*d^2 - a*e^2)^4*(d + e*x)^4)
```

Rubi [A] (verified)

Time = 0.67 (sec) , antiderivative size = 261, normalized size of antiderivative = 1.13, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.108, Rules used = {1129, 1129, 1129, 1123}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(d+ex)^4\sqrt{x\,(ae^2+cd^2)+ade+cdex^2}}\,dx$$

$$\Big\downarrow \ 1129$$

$$\frac{6cd\int \frac{1}{(d+ex)^3\sqrt{cdex^2+(cd^2+ae^2)x+ade}}dx}{7\,(cd^2-ae^2)}+\frac{2\sqrt{x\,(ae^2+cd^2)+ade+cdex^2}}{7(d+ex)^4\,(cd^2-ae^2)}$$

$$\Big\downarrow \ 1129$$

$$\frac{6cd\left(\frac{4cd\int \frac{1}{(d+ex)^2\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}dx}{5(cd^2-ae^2)}+\frac{2\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{5(d+ex)^3(cd^2-ae^2)}\right)}{7\left(cd^2-ae^2\right)}+\frac{2\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{7(d+ex)^4\left(cd^2-ae^2\right)}$$

$$\Big\downarrow \ 1129$$

$$\frac{6cd\left(\frac{4cd\left(\frac{2cd\int \frac{1}{(d+ex)\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}dx}{3\left(cd^2-ae^2\right)}+\frac{2\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{3(d+ex)^2\left(cd^2-ae^2\right)}\right)}{5(cd^2-ae^2)}+\frac{2\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{5(d+ex)^3(cd^2-ae^2)}\right)}{7\left(cd^2-ae^2\right)}+\frac{2\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{7(d+ex)^4\left(cd^2-ae^2\right)}$$

$$\Big\downarrow \ 1123$$

$$\frac{6cd\left(\frac{4cd\left(\frac{4cd\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{3(d+ex)\left(cd^2-ae^2\right)^2}+\frac{2\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{3(d+ex)^2\left(cd^2-ae^2\right)}\right)}{5(cd^2-ae^2)}+\frac{2\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{5(d+ex)^3(cd^2-ae^2)}\right)}{7\left(cd^2-ae^2\right)}+\frac{2\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{7(d+ex)^4\left(cd^2-ae^2\right)}$$

input
```
Int[1/((d + e*x)^4*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]),x]
```

output
```
(2*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(7*(c*d^2 - a*e^2)*(d + e*
x)^4) + (6*c*d*((2*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(5*(c*d^2
- a*e^2)*(d + e*x)^3) + (4*c*d*((2*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*
x^2])/(3*(c*d^2 - a*e^2)*(d + e*x)^2) + (4*c*d*Sqrt[a*d*e + (c*d^2 + a*e^2
)*x + c*d*e*x^2])/(3*(c*d^2 - a*e^2)^2*(d + e*x))))/(5*(c*d^2 - a*e^2))))/
(7*(c*d^2 - a*e^2))
```

Defintions of rubi rules used

rule 1123
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[e*(d + e*x)^m*((a + b*x + c*x^2)^(p + 1)/((p + 1)*(2*c*d - b
*e))), x] /; FreeQ[{a, b, c, d, e, m, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2,
0] && EqQ[m + 2*p + 2, 0]
```

rule 1129
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(-e)*(d + e*x)^m*((a + b*x + c*x^2)^(p + 1)/((m + p + 1)*(2*
c*d - b*e))), x] + Simp[c*(Simplify[m + 2*p + 2]/((m + p + 1)*(2*c*d - b*e)
))   Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d
, e, m, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && ILtQ[Simplify[m + 2*p +
2], 0]
```

Maple [A] (verified)

Time = 1.46 (sec) , antiderivative size = 209, normalized size of antiderivative = 0.90

method	result
trager	$-\frac{2\left(-16c^3d^3e^3x^3+8x^2a\,c^2d^2e^4-56c^3d^4e^2x^2-6x\,a^2cd\,e^5+28xa\,c^2d^3e^3-70c^3d^5ex+5e^6a^3-21d^2e^4a^2c+35d^4e^2a\,c^2-35d^6c^3\right)\sqrt{cd}}{35(a^4e^8-4a^3c\,d^2e^6+6a^2c^2d^4e^4-4a\,c^3d^6e^2+c^4d^8)(ex+d)^4}$
gosper	$-\frac{2(cdx+ae)\left(-16c^3d^3e^3x^3+8x^2a\,c^2d^2e^4-56c^3d^4e^2x^2-6x\,a^2cd\,e^5+28xa\,c^2d^3e^3-70c^3d^5ex+5e^6a^3-21d^2e^4a^2c+35d^4e^2a\,c^2-35a\right.}{35(ex+d)^3(a^4e^8-4a^3c\,d^2e^6+6a^2c^2d^4e^4-4a\,c^3d^6e^2+c^4d^8)\sqrt{cd\,x^2e+a\,e^2x+c\,d^2x+ade}}$
orering	$-\frac{2\left(-16c^3d^3e^3x^3+8x^2a\,c^2d^2e^4-56c^3d^4e^2x^2-6x\,a^2cd\,e^5+28xa\,c^2d^3e^3-70c^3d^5ex+5e^6a^3-21d^2e^4a^2c+35d^4e^2a\,c^2-35d^6c^3\right)(cdx\right.}{35(a^4e^8-4a^3c\,d^2e^6+6a^2c^2d^4e^4-4a\,c^3d^6e^2+c^4d^8)(ex+d)^3\sqrt{ade+(a\,e^2+c\,d^2)x+cd\,x^2e}}$
default	$\frac{-\frac{2\sqrt{dec\left(x+\frac{d}{e}\right)^2+\left(a\,e^2-c\,d^2\right)\left(x+\frac{d}{e}\right)}}{7\left(a\,e^2-c\,d^2\right)\left(x+\frac{d}{e}\right)^4}-\frac{6dec\left(-\frac{2\sqrt{dec\left(x+\frac{d}{e}\right)^2+\left(a\,e^2-c\,d^2\right)\left(x+\frac{d}{e}\right)}}{5\left(a\,e^2-c\,d^2\right)\left(x+\frac{d}{e}\right)^3}-\frac{4dec\left(-\frac{2\sqrt{dec\left(x+\frac{d}{e}\right)^2+\left(a\,e^2-c\,d^2\right)\left(x+\frac{d}{e}\right)}}{3\left(a\,e^2-c\,d^2\right)\left(x+\frac{d}{e}\right)^2}+\frac{4dec\sqrt{c}\cdots}{\cdots}\right)}{5\left(a\,e^2-c\,d^2\right)}\right)}{7\left(a\,e^2-c\,d^2\right)}}{e^4}$

input
```
int(1/(e*x+d)^4/(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(1/2),x,method=_RETURNVE
RBOSE)
```

output
```
-2/35*(-16*c^3*d^3*e^3*x^3+8*a*c^2*d^2*e^4*x^2-56*c^3*d^4*e^2*x^2-6*a^2*c*
d*e^5*x+28*a*c^2*d^3*e^3*x-70*c^3*d^5*e*x+5*a^3*e^6-21*a^2*c*d^2*e^4+35*a*
c^2*d^4*e^2-35*c^3*d^6)/(a^4*e^8-4*a^3*c*d^2*e^6+6*a^2*c^2*d^4*e^4-4*a*c^3
*d^6*e^2+c^4*d^8)/(e*x+d)^4*(c*d*e*x^2+a*e^2*x+c*d^2*x+a*d*e)^(1/2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 452 vs. $2(215) = 430$.

Time = 2.97 (sec) , antiderivative size = 452, normalized size of antiderivative = 1.96

$$\int \frac{1}{(d+ex)^4\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}\,dx$$

$$= \frac{2\,(16\,c^3d^3e^3x^3+35\,c^3d^6-35\,ac^2}{35\,(c^4d^{12}-4\,ac^3d^{10}e^2+6\,a^2c^2d^8e^4-4\,a^3cd^6e^6+a^4d^4e^8+(c^4d^8e^4-4\,ac^3d^6e^6+6\,a^2c^2d^4e^8-4\,a^3cd^2e^{10}}$$

input
```
integrate(1/(e*x+d)^4/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x, algorithm
="fricas")
```

output
```
2/35*(16*c^3*d^3*e^3*x^3 + 35*c^3*d^6 - 35*a*c^2*d^4*e^2 + 21*a^2*c*d^2*e^
4 - 5*a^3*e^6 + 8*(7*c^3*d^4*e^2 - a*c^2*d^2*e^4)*x^2 + 2*(35*c^3*d^5*e -
14*a*c^2*d^3*e^3 + 3*a^2*c*d*e^5)*x)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e
^2)*x)/(c^4*d^12 - 4*a*c^3*d^10*e^2 + 6*a^2*c^2*d^8*e^4 - 4*a^3*c*d^6*e^6
+ a^4*d^4*e^8 + (c^4*d^8*e^4 - 4*a*c^3*d^6*e^6 + 6*a^2*c^2*d^4*e^8 - 4*a^3
*c*d^2*e^10 + a^4*e^12)*x^4 + 4*(c^4*d^9*e^3 - 4*a*c^3*d^7*e^5 + 6*a^2*c^2
*d^5*e^7 - 4*a^3*c*d^3*e^9 + a^4*d*e^11)*x^3 + 6*(c^4*d^10*e^2 - 4*a*c^3*d
^8*e^4 + 6*a^2*c^2*d^6*e^6 - 4*a^3*c*d^4*e^8 + a^4*d^2*e^10)*x^2 + 4*(c^4*
d^11*e - 4*a*c^3*d^9*e^3 + 6*a^2*c^2*d^7*e^5 - 4*a^3*c*d^5*e^7 + a^4*d^3*e
^9)*x)
```

Sympy [F]

$$\int \frac{1}{(d+ex)^4\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}\,dx = \int \frac{1}{\sqrt{(d+ex)\,(ae+cdx)}\,(d+ex)^4}\,dx$$

input
```
integrate(1/(e*x+d)**4/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(1/2),x)
```

output
```
Integral(1/(sqrt((d + e*x)*(a*e + c*d*x))*(d + e*x)**4), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{(d+ex)^4\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(1/(e*x+d)^4/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x, algorithm
="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e*(a*e^2-c*d^2)>0)', see `assume
?` for mor
```

Giac [F(-2)]

Exception generated.

$$\int \frac{1}{(d+ex)^4\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(1/(e*x+d)^4/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x, algorithm
="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Unable to divide, perhaps due to ro
unding error%%%{%%%{1,[0,0,4]%%%},[8]%%%}+%%%{%%{[%%%{-8,[0,1,3]%%%},0]:[1
,0,%%%{-1
```

Mupad [B] (verification not implemented)

Time = 5.88 (sec) , antiderivative size = 252, normalized size of antiderivative = 1.09

$$\int \frac{1}{(d+ex)^4\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}\,dx$$
$$=\frac{32\,c^3\,d^3\,\sqrt{c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e}}{35\,\left(a\,e^2-c\,d^2\right)^4\,\left(d+e\,x\right)}-\frac{2\,e\,\sqrt{c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e}}{\left(7\,a\,e^3-7\,c\,d^2\,e\right)\,\left(d+e\,x\right)^4}$$
$$-\frac{48\,c^2\,d^2\,e\,\sqrt{c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e}}{35\,\left(a\,e^2-c\,d^2\right)^2\,\left(3\,a\,e^3-3\,c\,d^2\,e\right)\,\left(d+e\,x\right)^2}$$
$$+\frac{12\,c\,d\,e\,\sqrt{c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e}}{7\,\left(a\,e^2-c\,d^2\right)\,\left(5\,a\,e^3-5\,c\,d^2\,e\right)\,\left(d+e\,x\right)^3}$$

input
```
int(1/((d + e*x)^4*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2)),x)
```

output
```
(32*c^3*d^3*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2))/(35*(a*e^2 - c*
d^2)^4*(d + e*x)) - (2*e*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2))/((
7*a*e^3 - 7*c*d^2*e)*(d + e*x)^4) - (48*c^2*d^2*e*(x*(a*e^2 + c*d^2) + a*d
*e + c*d*e*x^2)^(1/2))/(35*(a*e^2 - c*d^2)^2*(3*a*e^3 - 3*c*d^2*e)*(d + e*
x)^2) + (12*c*d*e*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2))/(7*(a*e^2
 - c*d^2)*(5*a*e^3 - 5*c*d^2*e)*(d + e*x)^3)
```

Reduce [B] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 705, normalized size of antiderivative = 3.05

$$\int \frac{1}{(d+ex)^4\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}\,dx$$
$$=\frac{-\frac{2\sqrt{ex+d}\,\sqrt{cdx+ae}\,a^3e^7}{7}+\frac{6\sqrt{ex+d}\,\sqrt{cdx+ae}\,a^2c\,d^2e^5}{5}+\frac{12\sqrt{ex+d}\,\sqrt{cdx+ae}\,a^2cd\,e^6x}{35}-2\sqrt{ex+d}\,\sqrt{cdx+ae}\,a\,c^2d^4e^3-\frac{8}{}}{e\left(a^4e^{12}x^4-4a^3c\,d^2e^{10}x^4+6a^2c^2d^4e^8x^4-4a\,c^3d^6e^6x^4+c^4d^8e^4x^4+4a^4d\,e^{11}x^3-16a^3c\,d^3e^9x^3\right.}$$

input
```
int(1/(e*x+d)^4/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x)
```

output
```
(2*( - 5*sqrt(d + e*x)*sqrt(a*e + c*d*x)*a**3*e**7 + 21*sqrt(d + e*x)*sqrt
(a*e + c*d*x)*a**2*c*d**2*e**5 + 6*sqrt(d + e*x)*sqrt(a*e + c*d*x)*a**2*c*
d*e**6*x - 35*sqrt(d + e*x)*sqrt(a*e + c*d*x)*a*c**2*d**4*e**3 - 28*sqrt(d
 + e*x)*sqrt(a*e + c*d*x)*a*c**2*d**3*e**4*x - 8*sqrt(d + e*x)*sqrt(a*e +
c*d*x)*a*c**2*d**2*e**5*x**2 + 35*sqrt(d + e*x)*sqrt(a*e + c*d*x)*c**3*d**
6*e + 70*sqrt(d + e*x)*sqrt(a*e + c*d*x)*c**3*d**5*e**2*x + 56*sqrt(d + e*
x)*sqrt(a*e + c*d*x)*c**3*d**4*e**3*x**2 + 16*sqrt(d + e*x)*sqrt(a*e + c*d
*x)*c**3*d**3*e**4*x**3 - 16*sqrt(e)*sqrt(d)*sqrt(c)*c**3*d**7 - 64*sqrt(e
)*sqrt(d)*sqrt(c)*c**3*d**6*e*x - 96*sqrt(e)*sqrt(d)*sqrt(c)*c**3*d**5*e**
2*x**2 - 64*sqrt(e)*sqrt(d)*sqrt(c)*c**3*d**4*e**3*x**3 - 16*sqrt(e)*sqrt(
d)*sqrt(c)*c**3*d**3*e**4*x**4))/(35*e*(a**4*d**4*e**8 + 4*a**4*d**3*e**9*
x + 6*a**4*d**2*e**10*x**2 + 4*a**4*d*e**11*x**3 + a**4*e**12*x**4 - 4*a**
3*c*d**6*e**6 - 16*a**3*c*d**5*e**7*x - 24*a**3*c*d**4*e**8*x**2 - 16*a**3
*c*d**3*e**9*x**3 - 4*a**3*c*d**2*e**10*x**4 + 6*a**2*c**2*d**8*e**4 + 24*
a**2*c**2*d**7*e**5*x + 36*a**2*c**2*d**6*e**6*x**2 + 24*a**2*c**2*d**5*e*
*7*x**3 + 6*a**2*c**2*d**4*e**8*x**4 - 4*a*c**3*d**10*e**2 - 16*a*c**3*d**
9*e**3*x - 24*a*c**3*d**8*e**4*x**2 - 16*a*c**3*d**7*e**5*x**3 - 4*a*c**3*
d**6*e**6*x**4 + c**4*d**12 + 4*c**4*d**11*e*x + 6*c**4*d**10*e**2*x**2 +
4*c**4*d**9*e**3*x**3 + c**4*d**8*e**4*x**4))
```

3.256 $\int \frac{(d+ex)^5}{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}\,dx$

Optimal result

Integrand size = 37, antiderivative size = 287

$$\int \frac{(d+ex)^5}{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}\,dx=-\frac{2(d+ex)^4}{cd\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}+\frac{35e\left(cd^2-ae^2\right)^2\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{8c^4d^4}+\frac{35e\left(cd^2-ae^2\right)(d+ex)\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{12c^3d^3}+\frac{7e(d+ex)^2\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{3c^2d^2}+\frac{35\sqrt{e}\left(cd^2-ae^2\right)^3\operatorname{arctanh}\left(\frac{\sqrt{c}\sqrt{d}(d+ex)}{\sqrt{e}\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\right)}{8c^{9/2}d^{9/2}}$$

output
```
-2*(e*x+d)^4/c/d/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)+35/8*e*(-a*e^2+c*
d^2)^2*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/c^4/d^4+35/12*e*(-a*e^2+c*d
^2)*(e*x+d)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/c^3/d^3+7/3*e*(e*x+d)^
2*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/c^2/d^2+35/8*e^(1/2)*(-a*e^2+c*d
^2)^3*arctanh(c^(1/2)*d^(1/2)*(e*x+d)/e^(1/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e
*x^2)^(1/2))/c^(9/2)/d^(9/2)
```

Mathematica [A] (verified)

Time = 0.58 (sec) , antiderivative size = 234, normalized size of antiderivative = 0.82

$$\int \frac{(d+ex)^5}{(ade+(cd^2+ae^2)x+cdex^2)^{3/2}}\,dx = \frac{\sqrt{c}\sqrt{d}(ae+cdx)(d+ex)^2\left(105a^3e^6-35a^2cde^4(8d-ex)+7ac^2\right.}{}$$

input
```
Integrate[(d + e*x)^5/(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2),x]
```

output
```
(Sqrt[c]*Sqrt[d]*(a*e + c*d*x)*(d + e*x)^2*(105*a^3*e^6 - 35*a^2*c*d*e^4*(
8*d - e*x) + 7*a*c^2*d^2*e^2*(33*d^2 - 14*d*e*x - 2*e^2*x^2) + c^3*d^3*(-4
8*d^3 + 87*d^2*e*x + 38*d*e^2*x^2 + 8*e^3*x^3)) + 105*Sqrt[e]*(c*d^2 - a*e
^2)^3*(a*e + c*d*x)^(3/2)*(d + e*x)^(3/2)*ArcTanh[(Sqrt[c]*Sqrt[d]*Sqrt[d
+ e*x])/(Sqrt[e]*Sqrt[a*e + c*d*x])])/(24*c^(9/2)*d^(9/2)*((a*e + c*d*x)*(
d + e*x))^(3/2))
```

Rubi [A] (verified)

Time = 1.51 (sec) , antiderivative size = 349, normalized size of antiderivative = 1.22, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.216$, Rules used = {1124, 2192, 27, 2192, 27, 1160, 1092, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex)^5}{(x(ae^2+cd^2)+ade+cdex^2)^{3/2}}dx$$

$$\Big\downarrow \text{1124}$$

$$\frac{\int \frac{c^3d^3x^3e^6+c^2d^2(4cd^2-ae^2)x^2e^5+cd(6c^2d^4-4ace^2d^2+a^2e^4)xe^4+(2cd^2-ae^2)(2c^2d^4-2ace^2d^2+a^2e^4)e^3}{\sqrt{cdex^2+(cd^2+ae^2)x+ade}}dx}{c^4d^4e^2}-\frac{2(d+ex)(cd^2-ae^2)^3}{c^4d^4\sqrt{x(ae^2+cd^2)+ade+cdex^2}}$$

$$\Big\downarrow \text{2192}$$

$$\frac{\frac{\int \frac{c^3d^3\left(19cd^2-11ae^2\right)x^2e^6+2c^2d^2\left(18c^2d^4-14ace^2d^2+3a^2e^4\right)xe^5+6cd\left(2cd^2-ae^2\right)\left(2c^2d^4-2ace^2d^2+a^2e^4\right)e^4}{2\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}dx}{3cde}+\frac{1}{3}c^2d^2e^5x^2\sqrt{x\left(ae^2+cd^2\right)+}}{c^4d^4e^2}$$

$$\frac{2(d+ex)\left(cd^2-ae^2\right)^3}{c^4d^4\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}$$

$\Big\downarrow$ 27

$$\frac{\frac{\int \frac{c^3d^3\left(19cd^2-11ae^2\right)x^2e^6+2c^2d^2\left(18c^2d^4-14ace^2d^2+3a^2e^4\right)xe^5+6cd\left(2cd^2-ae^2\right)\left(2c^2d^4-2ace^2d^2+a^2e^4\right)e^4}{\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}dx}{6cde}+\frac{1}{3}c^2d^2e^5x^2\sqrt{x\left(ae^2+cd^2\right)+}}{c^4d^4e^2}$$

$$\frac{2(d+ex)\left(cd^2-ae^2\right)^3}{c^4d^4\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}$$

$\Big\downarrow$ 2192

$$\frac{\frac{\frac{\int \frac{c^2d^2e^5\left(2\left(48c^3d^6-91ac^2e^2d^4+59a^2ce^4d^2-12a^3e^6\right)+cde\left(87c^2d^4-136ace^2d^2+57a^2e^4\right)x\right)}{2\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}dx}{2cde}+\frac{1}{2}c^2d^2e^5x\left(19cd^2-11ae^2\right)\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{6cde}-}{c^4d^4e^2}$$

$$\frac{2(d+ex)\left(cd^2-ae^2\right)^3}{c^4d^4\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}$$

$\Big\downarrow$ 27

$$\frac{\frac{\frac{1}{4}cde^4\int \frac{2\left(48c^3d^6-91ac^2e^2d^4+59a^2ce^4d^2-12a^3e^6\right)+cde\left(87c^2d^4-136ace^2d^2+57a^2e^4\right)x}{\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}dx+\frac{1}{2}c^2d^2e^5x\left(19cd^2-11ae^2\right)\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{6cde}+}{c^4d^4e^2}$$

$$\frac{2(d+ex)\left(cd^2-ae^2\right)^3}{c^4d^4\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}$$

$\Big\downarrow$ 1160

$$\frac{\frac{\frac{1}{4}cde^4\left(\frac{105}{2}\left(cd^2-ae^2\right)^3\int \frac{1}{\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}dx+\left(57a^2e^4-136acd^2e^2+87c^2d^4\right)\sqrt{x(ae^2+cd^2)+ade+cdex^2}\right)+\frac{1}{2}c^2d^2e^5x\left(19cd^2-11ae^2\right)}{6cde}}{c^4d^4e^2}$$

$$\frac{2(d+ex)\left(cd^2-ae^2\right)^3}{c^4d^4\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}$$

$\Big\downarrow$ 1092

$$\frac{\frac{\frac{1}{4}cde^4\left(105\left(cd^2-ae^2\right)^3\int\frac{1}{4cde-\frac{\left(cd^2+2cexd+ae^2\right)^2}{cdex^2+\left(cd^2+ae^2\right)x+ade}}d\frac{cd^2+2cexd+ae^2}{\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}+\left(57a^2e^4-136acd^2e^2+87c^2d^4\right)\sqrt{x(ae^2+cd^2)+ade+cdex^2}\right)}{6cde}}{c^4d^4e^2}$$

$$\frac{2(d+ex)\left(cd^2-ae^2\right)^3}{c^4d^4\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}$$

$$\Big\downarrow \text{219}$$

$$\frac{\frac{\frac{1}{4}cde^4\left(\left(57a^2e^4-136acd^2e^2+87c^2d^4\right)\sqrt{x(ae^2+cd^2)+ade+cdex^2}+\frac{105\left(cd^2-ae^2\right)^3\text{arctanh}\left(\frac{ae^2+cd^2+2cdex}{2\sqrt{c}\sqrt{d}\sqrt{e}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}\right)}{2\sqrt{c}\sqrt{d}\sqrt{e}}\right)}{6cde}+\frac{1}{2}c^2d^2e^5x}{c^4d^4e^2}$$

$$\frac{2(d+ex)\left(cd^2-ae^2\right)^3}{c^4d^4\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}$$

input
```
Int[(d + e*x)^5/(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2),x]
```

output
```
(-2*(c*d^2 - a*e^2)^3*(d + e*x))/(c^4*d^4*Sqrt[a*d*e + (c*d^2 + a*e^2)*x +
 c*d*e*x^2]) + ((c^2*d^2*e^5*x^2*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^
2])/3 + ((c^2*d^2*e^5*(19*c*d^2 - 11*a*e^2)*x*Sqrt[a*d*e + (c*d^2 + a*e^2)
*x + c*d*e*x^2])/2 + (c*d*e^4*((87*c^2*d^4 - 136*a*c*d^2*e^2 + 57*a^2*e^4)
*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2] + (105*(c*d^2 - a*e^2)^3*ArcT
anh[(c*d^2 + a*e^2 + 2*c*d*e*x)/(2*Sqrt[c]*Sqrt[d]*Sqrt[e]*Sqrt[a*d*e + (c
*d^2 + a*e^2)*x + c*d*e*x^2])])/(2*Sqrt[c]*Sqrt[d]*Sqrt[e])))/4)/(6*c*d*e)
)/(c^4*d^4*e^2)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1092
```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[I
nt[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a
, b, c}, x]
```

rule 1124
```
Int[((d_.) + (e_.)*(x_))^(m_.)/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(3/2), x
_Symbol] :> Simp[-2*e*(2*c*d - b*e)^(m - 2)*((d + e*x)/(c^(m - 1)*Sqrt[a +
b*x + c*x^2])), x] + Simp[e^2/c^(m - 1)   Int[(1/Sqrt[a + b*x + c*x^2])*Exp
andToSum[((2*c*d - b*e)^(m - 1) - c^(m - 1)*(d + e*x)^(m - 1))/(c*d - b*e -
 c*e*x), x], x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c*d^2 - b*d*e + a*e
^2, 0] && IGtQ[m, 0]
```

rule 1160
```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[e*((a + b*x + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Simp[(2*c*d - b
*e)/(2*c)   Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
 && NeQ[p, -1]
```

rule 2192
```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{q =
Expon[Pq, x], e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[e*x^(q - 1)*((a + b*x +
 c*x^2)^(p + 1)/(c*(q + 2*p + 1))), x] + Simp[1/(c*(q + 2*p + 1))   Int[(a
+ b*x + c*x^2)^p*ExpandToSum[c*(q + 2*p + 1)*Pq - a*e*(q - 1)*x^(q - 2) - b
*e*(q + p)*x^(q - 1) - c*e*(q + 2*p + 1)*x^q, x], x], x]] /; FreeQ[{a, b, c
, p}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] &&  !LeQ[p, -1]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 3116 vs. $2(257) = 514$.

Time = 1.71 (sec) , antiderivative size = 3117, normalized size of antiderivative = 10.86

method	result	size
default	Expression too large to display	3117

input
```
int((e*x+d)^5/(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(3/2),x,method=_RETURNVERB
OSE)
```

output
```
2*d^5*(2*c*d*e*x+a*e^2+c*d^2)/(4*a*c*d^2*e^2-(a*e^2+c*d^2)^2)/(a*d*e+(a*e^
2+c*d^2)*x+c*d*x^2*e)^(1/2)+e^5*(1/3*x^4/d/e/c/(a*d*e+(a*e^2+c*d^2)*x+c*d*
x^2*e)^(1/2)-7/6*(a*e^2+c*d^2)/d/e/c*(1/2*x^3/d/e/c/(a*d*e+(a*e^2+c*d^2)*x
+c*d*x^2*e)^(1/2)-5/4*(a*e^2+c*d^2)/d/e/c*(x^2/d/e/c/(a*d*e+(a*e^2+c*d^2)*
x+c*d*x^2*e)^(1/2)-3/2*(a*e^2+c*d^2)/d/e/c*(-x/d/e/c/(a*d*e+(a*e^2+c*d^2)*
x+c*d*x^2*e)^(1/2)-1/2*(a*e^2+c*d^2)/d/e/c*(-1/d/e/c/(a*d*e+(a*e^2+c*d^2)*
x+c*d*x^2*e)^(1/2)-(a*e^2+c*d^2)/d/e/c*(2*c*d*e*x+a*e^2+c*d^2)/(4*a*c*d^2*
e^2-(a*e^2+c*d^2)^2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(1/2))+1/d/e/c*ln((
1/2*a*e^2+1/2*c*d^2+c*d*x*e)/(d*e*c)^(1/2)+(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*
e)^(1/2))/(d*e*c)^(1/2))-2*a/c*(-1/d/e/c/(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)
^(1/2)-(a*e^2+c*d^2)/d/e/c*(2*c*d*e*x+a*e^2+c*d^2)/(4*a*c*d^2*e^2-(a*e^2+c
*d^2)^2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(1/2)))-3/2*a/c*(-x/d/e/c/(a*d*
e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(1/2)-1/2*(a*e^2+c*d^2)/d/e/c*(-1/d/e/c/(a*d*
e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(1/2)-(a*e^2+c*d^2)/d/e/c*(2*c*d*e*x+a*e^2+c*
d^2)/(4*a*c*d^2*e^2-(a*e^2+c*d^2)^2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(1/
2))+1/d/e/c*ln((1/2*a*e^2+1/2*c*d^2+c*d*x*e)/(d*e*c)^(1/2)+(a*d*e+(a*e^2+c
*d^2)*x+c*d*x^2*e)^(1/2))/(d*e*c)^(1/2)))-4/3*a/c*(x^2/d/e/c/(a*d*e+(a*e^2
+c*d^2)*x+c*d*x^2*e)^(1/2)-3/2*(a*e^2+c*d^2)/d/e/c*(-x/d/e/c/(a*d*e+(a*e^2
+c*d^2)*x+c*d*x^2*e)^(1/2)-1/2*(a*e^2+c*d^2)/d/e/c*(-1/d/e/c/(a*d*e+(a*e^2
+c*d^2)*x+c*d*x^2*e)^(1/2)-(a*e^2+c*d^2)/d/e/c*(2*c*d*e*x+a*e^2+c*d^2)/...
```

Fricas [A] (verification not implemented)

Time = 0.64 (sec) , antiderivative size = 759, normalized size of antiderivative = 2.64

$$\int \frac{(d+ex)^5}{(ade+(cd^2+ae^2)x+cdex^2)^{3/2}}\,dx = \left[-\frac{105\,(ac^3d^6e-3\,a^2c^2d^4e^3+3\,a^3cd^2e^5-a^4e^7+(c^4d^7-3\,ac^3d^5\cdots}{\cdots}\right.$$

$$-\frac{105\,(ac^3d^6e-3\,a^2c^2d^4e^3+3\,a^3cd^2e^5-a^4e^7+(c^4d^7-3\,ac^3d^5e^2+3\,a^2c^2d^3e^4-a^3cde^6)x)\sqrt{-\frac{e}{cd}}\arctan\left(\cdots\right.}{\cdots}$$

input
```
integrate((e*x+d)^5/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),x, algorithm="
fricas")
```

output
```
[-1/96*(105*(a*c^3*d^6*e - 3*a^2*c^2*d^4*e^3 + 3*a^3*c*d^2*e^5 - a^4*e^7 +
 (c^4*d^7 - 3*a*c^3*d^5*e^2 + 3*a^2*c^2*d^3*e^4 - a^3*c*d*e^6)*x)*sqrt(e/(
c*d))*log(8*c^2*d^2*e^2*x^2 + c^2*d^4 + 6*a*c*d^2*e^2 + a^2*e^4 + 8*(c^2*d
^3*e + a*c*d*e^3)*x - 4*(2*c^2*d^2*e*x + c^2*d^3 + a*c*d*e^2)*sqrt(c*d*e*x
^2 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(e/(c*d))) - 4*(8*c^3*d^3*e^3*x^3 - 48
*c^3*d^6 + 231*a*c^2*d^4*e^2 - 280*a^2*c*d^2*e^4 + 105*a^3*e^6 + 2*(19*c^3
*d^4*e^2 - 7*a*c^2*d^2*e^4)*x^2 + (87*c^3*d^5*e - 98*a*c^2*d^3*e^3 + 35*a^
2*c*d*e^5)*x)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x))/(c^5*d^5*x + a*
c^4*d^4*e), -1/48*(105*(a*c^3*d^6*e - 3*a^2*c^2*d^4*e^3 + 3*a^3*c*d^2*e^5
- a^4*e^7 + (c^4*d^7 - 3*a*c^3*d^5*e^2 + 3*a^2*c^2*d^3*e^4 - a^3*c*d*e^6)*
x)*sqrt(-e/(c*d))*arctan(1/2*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(
2*c*d*e*x + c*d^2 + a*e^2)*sqrt(-e/(c*d))/(c*d*e^2*x^2 + a*d*e^2 + (c*d^2*
e + a*e^3)*x)) - 2*(8*c^3*d^3*e^3*x^3 - 48*c^3*d^6 + 231*a*c^2*d^4*e^2 - 2
80*a^2*c*d^2*e^4 + 105*a^3*e^6 + 2*(19*c^3*d^4*e^2 - 7*a*c^2*d^2*e^4)*x^2
+ (87*c^3*d^5*e - 98*a*c^2*d^3*e^3 + 35*a^2*c*d*e^5)*x)*sqrt(c*d*e*x^2 + a
*d*e + (c*d^2 + a*e^2)*x))/(c^5*d^5*x + a*c^4*d^4*e)]
```

Sympy [F]

$$\int \frac{(d+ex)^5}{(ade+(cd^2+ae^2)\,x+cdex^2)^{3/2}}\,dx=\int \frac{(d+ex)^5}{((d+ex)\,(ae+cdx))^{\frac{3}{2}}}\,dx$$

input
```
integrate((e*x+d)**5/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(3/2),x)
```

output
```
Integral((d + e*x)**5/((d + e*x)*(a*e + c*d*x))**(3/2), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(d+ex)^5}{(ade+(cd^2+ae^2)\,x+cdex^2)^{3/2}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((e*x+d)^5/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),x, algorithm="
maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(a*e^2-c*d^2>0)', see `assume?` f
or more de
```

Giac [F(-2)]

Exception generated.

$$\int \frac{(d+ex)^5}{(ade+(cd^2+ae^2)\,x+cdex^2)^{3/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((e*x+d)^5/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),x, algorithm="
giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Unable to divide, perhaps due to ro
unding error%%%{%%{[%%%{16,[5,5,4]%%%},0]:[1,0,%%%{-1,[1,1,1]%%%}]%%},[2,2
]%%%}+%%%
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d+ex)^5}{(ade+(cd^2+ae^2)\,x+cdex^2)^{3/2}}\,dx = \int \frac{(d+e\,x)^5}{(c\,d\,e\,x^2+(c\,d^2+a\,e^2)\,x+a\,d\,e)^{3/2}}\,dx$$

input
```
int((d + e*x)^5/(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(3/2),x)
```

output
```
int((d + e*x)^5/(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(3/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 569, normalized size of antiderivative = 1.98

$$\int \frac{(d+ex)^5}{(ade+(cd^2+ae^2)\,x+cdex^2)^{3/2}}\,dx = \frac{-840\sqrt{e}\,\sqrt{d}\,\sqrt{c}\,\sqrt{cdx+ae}\,\log\left(\frac{\sqrt{e}\,\sqrt{cdx+ae}+\sqrt{d}\,\sqrt{c}\,\sqrt{ex+d}}{\sqrt{a\,e^2-c\,d^2}}\right)a^3e^6+25$$

input
```
int((e*x+d)^5/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),x)
```

output
```
( - 840*sqrt(e)*sqrt(d)*sqrt(c)*sqrt(a*e + c*d*x)*log((sqrt(e)*sqrt(a*e +
c*d*x) + sqrt(d)*sqrt(c)*sqrt(d + e*x))/sqrt(a*e**2 - c*d**2))*a**3*e**6 +
 2520*sqrt(e)*sqrt(d)*sqrt(c)*sqrt(a*e + c*d*x)*log((sqrt(e)*sqrt(a*e + c*
d*x) + sqrt(d)*sqrt(c)*sqrt(d + e*x))/sqrt(a*e**2 - c*d**2))*a**2*c*d**2*e
**4 - 2520*sqrt(e)*sqrt(d)*sqrt(c)*sqrt(a*e + c*d*x)*log((sqrt(e)*sqrt(a*e
 + c*d*x) + sqrt(d)*sqrt(c)*sqrt(d + e*x))/sqrt(a*e**2 - c*d**2))*a*c**2*d
**4*e**2 + 840*sqrt(e)*sqrt(d)*sqrt(c)*sqrt(a*e + c*d*x)*log((sqrt(e)*sqrt
(a*e + c*d*x) + sqrt(d)*sqrt(c)*sqrt(d + e*x))/sqrt(a*e**2 - c*d**2))*c**3
*d**6 + 525*sqrt(e)*sqrt(d)*sqrt(c)*sqrt(a*e + c*d*x)*a**3*e**6 - 1575*sqr
t(e)*sqrt(d)*sqrt(c)*sqrt(a*e + c*d*x)*a**2*c*d**2*e**4 + 1575*sqrt(e)*sqr
t(d)*sqrt(c)*sqrt(a*e + c*d*x)*a*c**2*d**4*e**2 - 525*sqrt(e)*sqrt(d)*sqrt
(c)*sqrt(a*e + c*d*x)*c**3*d**6 + 840*sqrt(d + e*x)*a**3*c*d*e**6 - 2240*s
qrt(d + e*x)*a**2*c**2*d**3*e**4 + 280*sqrt(d + e*x)*a**2*c**2*d**2*e**5*x
 + 1848*sqrt(d + e*x)*a*c**3*d**5*e**2 - 784*sqrt(d + e*x)*a*c**3*d**4*e**
3*x - 112*sqrt(d + e*x)*a*c**3*d**3*e**4*x**2 - 384*sqrt(d + e*x)*c**4*d**
7 + 696*sqrt(d + e*x)*c**4*d**6*e*x + 304*sqrt(d + e*x)*c**4*d**5*e**2*x**
2 + 64*sqrt(d + e*x)*c**4*d**4*e**3*x**3)/(192*sqrt(a*e + c*d*x)*c**5*d**5
)
```

3.257 $\int \frac{(d+ex)^4}{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}\,dx$

Optimal result

Integrand size = 37, antiderivative size = 226

$$\int \frac{(d+ex)^4}{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}\,dx = -\frac{2(d+ex)^3}{cd\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}} + \frac{15e(cd^2-ae^2)\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{4c^3d^3} + \frac{5e(d+ex)\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{2c^2d^2} + \frac{15\sqrt{e}(cd^2-ae^2)^2\operatorname{arctanh}\left(\frac{\sqrt{c}\sqrt{d}(d+ex)}{\sqrt{e}\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\right)}{4c^{7/2}d^{7/2}}$$

output
```
-2*(e*x+d)^3/c/d/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)+15/4*e*(-a*e^2+c*
d^2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/c^3/d^3+5/2*e*(e*x+d)*(a*d*e+
(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/c^2/d^2+15/4*e^(1/2)*(-a*e^2+c*d^2)^2*arc
tanh(c^(1/2)*d^(1/2)*(e*x+d)/e^(1/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/
2))/c^(7/2)/d^(7/2)
```

Mathematica [A] (verified)

Time = 0.48 (sec) , antiderivative size = 180, normalized size of antiderivative = 0.80

$$\int \frac{(d+ex)^4}{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}\,dx = \frac{-\sqrt{c}\sqrt{d}(d+ex)\left(15a^2e^4+5acde^2(-5d+ex)+c^2d^2(8d^2-9dex\right.}{4c^{7/2}d^{7/2}\sqrt{}}$$

input
```
Integrate[(d + e*x)^4/(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2),x]
```

output
```
(-(Sqrt[c]*Sqrt[d]*(d + e*x)*(15*a^2*e^4 + 5*a*c*d*e^2*(-5*d + e*x) + c^2*
d^2*(8*d^2 - 9*d*e*x - 2*e^2*x^2))) + 15*Sqrt[e]*(c*d^2 - a*e^2)^2*Sqrt[a*
e + c*d*x]*Sqrt[d + e*x]*ArcTanh[(Sqrt[c]*Sqrt[d]*Sqrt[d + e*x])/(Sqrt[e]*
Sqrt[a*e + c*d*x])])/(4*c^(7/2)*d^(7/2)*Sqrt[(a*e + c*d*x)*(d + e*x)])
```

Rubi [A] (verified)

Time = 0.90 (sec) , antiderivative size = 257, normalized size of antiderivative = 1.14, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.162$, Rules used = {1124, 2192, 27, 1160, 1092, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex)^4}{\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}dx$$

$$\Big\downarrow \text{1124}$$

$$\frac{\int \frac{c^2d^2x^2e^4+cd\left(3cd^2-ae^2\right)xe^3+\left(3c^2d^4-3ace^2d^2+a^2e^4\right)e^2}{\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}dx}{c^3d^3e}-\frac{2(d+ex)\left(cd^2-ae^2\right)^2}{c^3d^3\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}$$

$$\Big\downarrow \text{2192}$$

$$\frac{\frac{\int \frac{cde^3\left(2\left(3cd^2-2ae^2\right)\left(2cd^2-ae^2\right)+cde\left(9cd^2-7ae^2\right)x\right)}{2\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}dx}{2cde}+\frac{1}{2}cde^3x\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{c^3d^3e}-\frac{2(d+ex)\left(cd^2-ae^2\right)^2}{c^3d^3\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}$$

$$\Big\downarrow \; 27$$

$$\frac{\frac{1}{4}e^2\int \frac{2(3cd^2-2ae^2)(2cd^2-ae^2)+cde(9cd^2-7ae^2)x}{\sqrt{cdex^2+(cd^2+ae^2)x+ade}}dx+\frac{1}{2}cde^3x\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{c^3d^3e}-\frac{2(d+ex)(cd^2-ae^2)^2}{c^3d^3\sqrt{x(ae^2+cd^2)+ade+cdex^2}}$$

$$\Big\downarrow \; 1160$$

$$\frac{\frac{1}{4}e^2\left(\frac{15}{2}(cd^2-ae^2)^2\int \frac{1}{\sqrt{cdex^2+(cd^2+ae^2)x+ade}}dx+(9cd^2-7ae^2)\sqrt{x(ae^2+cd^2)+ade+cdex^2}\right)+\frac{1}{2}cde^3x\sqrt{x(a}}{c^3d^3e}$$
$$\frac{2(d+ex)(cd^2-ae^2)^2}{c^3d^3\sqrt{x(ae^2+cd^2)+ade+cdex^2}}$$

$$\Big\downarrow \; 1092$$

$$\frac{\frac{1}{4}e^2\left(15(cd^2-ae^2)^2\int \frac{1}{4cde-\frac{(cd^2+2cexd+ae^2)^2}{cdex^2+(cd^2+ae^2)x+ade}}d\frac{cd^2+2cexd+ae^2}{\sqrt{cdex^2+(cd^2+ae^2)x+ade}}+(9cd^2-7ae^2)\sqrt{x(ae^2+cd^2)+ade+}\right.}{c^3d^3e}$$
$$\frac{2(d+ex)(cd^2-ae^2)^2}{c^3d^3\sqrt{x(ae^2+cd^2)+ade+cdex^2}}$$

$$\Big\downarrow \; 219$$

$$\frac{\frac{1}{4}e^2\left(\frac{15(cd^2-ae^2)^2\text{arctanh}\left(\frac{ae^2+cd^2+2cdex}{2\sqrt{c}\sqrt{d}\sqrt{e}\sqrt{x(ae^2+cd^2)+ade+cdex^2}}\right)}{2\sqrt{c}\sqrt{d}\sqrt{e}}+(9cd^2-7ae^2)\sqrt{x(ae^2+cd^2)+ade+cdex^2}\right)+\frac{1}{2}cde}{c^3d^3e}$$
$$\frac{2(d+ex)(cd^2-ae^2)^2}{c^3d^3\sqrt{x(ae^2+cd^2)+ade+cdex^2}}$$

input
```
Int[(d + e*x)^4/(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2),x]
```

output
```
(-2*(c*d^2 - a*e^2)^2*(d + e*x))/(c^3*d^3*Sqrt[a*d*e + (c*d^2 + a*e^2)*x +
 c*d*e*x^2]) + ((c*d*e^3*x*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/2
+ (e^2*((9*c*d^2 - 7*a*e^2)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2] +
(15*(c*d^2 - a*e^2)^2*ArcTanh[(c*d^2 + a*e^2 + 2*c*d*e*x)/(2*Sqrt[c]*Sqrt[
d]*Sqrt[e]*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])])/(2*Sqrt[c]*Sqrt[
d]*Sqrt[e])))/4)/(c^3*d^3*e)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1092
```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[I
nt[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a
, b, c}, x]
```

rule 1124
```
Int[((d_.) + (e_.)*(x_))^(m_.)/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(3/2), x
_Symbol] :> Simp[-2*e*(2*c*d - b*e)^(m - 2)*((d + e*x)/(c^(m - 1)*Sqrt[a +
b*x + c*x^2])), x] + Simp[e^2/c^(m - 1)   Int[(1/Sqrt[a + b*x + c*x^2])*Exp
andToSum[((2*c*d - b*e)^(m - 1) - c^(m - 1)*(d + e*x)^(m - 1))/(c*d - b*e -
 c*e*x), x], x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c*d^2 - b*d*e + a*e
^2, 0] && IGtQ[m, 0]
```

rule 1160
```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[e*((a + b*x + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Simp[(2*c*d - b
*e)/(2*c)   Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
 && NeQ[p, -1]
```

rule 2192
```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{q =
Expon[Pq, x], e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[e*x^(q - 1)*((a + b*x +
 c*x^2)^(p + 1)/(c*(q + 2*p + 1))), x] + Simp[1/(c*(q + 2*p + 1))   Int[(a
+ b*x + c*x^2)^p*ExpandToSum[c*(q + 2*p + 1)*Pq - a*e*(q - 1)*x^(q - 2) - b
*e*(q + p)*x^(q - 1) - c*e*(q + 2*p + 1)*x^q, x], x], x]] /; FreeQ[{a, b, c
, p}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] &&  !LeQ[p, -1]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1769 vs. $2(200) = 400$.

Time = 1.43 (sec) , antiderivative size = 1770, normalized size of antiderivative = 7.83

method	result	size
default	Expression too large to display	1770

input
```
int((e*x+d)^4/(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(3/2),x,method=_RETURNVERB
OSE)
```

output
```
2*d^4*(2*c*d*e*x+a*e^2+c*d^2)/(4*a*c*d^2*e^2-(a*e^2+c*d^2)^2)/(a*d*e+(a*e^
2+c*d^2)*x+c*d*x^2*e)^(1/2)+e^4*(1/2*x^3/d/e/c/(a*d*e+(a*e^2+c*d^2)*x+c*d*
x^2*e)^(1/2)-5/4*(a*e^2+c*d^2)/d/e/c*(x^2/d/e/c/(a*d*e+(a*e^2+c*d^2)*x+c*d
*x^2*e)^(1/2)-3/2*(a*e^2+c*d^2)/d/e/c*(-x/d/e/c/(a*d*e+(a*e^2+c*d^2)*x+c*d
*x^2*e)^(1/2)-1/2*(a*e^2+c*d^2)/d/e/c*(-1/d/e/c/(a*d*e+(a*e^2+c*d^2)*x+c*d
*x^2*e)^(1/2)-(a*e^2+c*d^2)/d/e/c*(2*c*d*e*x+a*e^2+c*d^2)/(4*a*c*d^2*e^2-(
a*e^2+c*d^2)^2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(1/2))+1/d/e/c*ln((1/2*a
*e^2+1/2*c*d^2+c*d*x*e)/(d*e*c)^(1/2)+(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(1
/2))/(d*e*c)^(1/2))-2*a/c*(-1/d/e/c/(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(1/2
)-(a*e^2+c*d^2)/d/e/c*(2*c*d*e*x+a*e^2+c*d^2)/(4*a*c*d^2*e^2-(a*e^2+c*d^2)
^2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(1/2)))-3/2*a/c*(-x/d/e/c/(a*d*e+(a*
e^2+c*d^2)*x+c*d*x^2*e)^(1/2)-1/2*(a*e^2+c*d^2)/d/e/c*(-1/d/e/c/(a*d*e+(a*
e^2+c*d^2)*x+c*d*x^2*e)^(1/2)-(a*e^2+c*d^2)/d/e/c*(2*c*d*e*x+a*e^2+c*d^2)/
(4*a*c*d^2*e^2-(a*e^2+c*d^2)^2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(1/2))+1
/d/e/c*ln((1/2*a*e^2+1/2*c*d^2+c*d*x*e)/(d*e*c)^(1/2)+(a*d*e+(a*e^2+c*d^2)
*x+c*d*x^2*e)^(1/2))/(d*e*c)^(1/2)))+4*d*e^3*(x^2/d/e/c/(a*d*e+(a*e^2+c*d^
2)*x+c*d*x^2*e)^(1/2)-3/2*(a*e^2+c*d^2)/d/e/c*(-x/d/e/c/(a*d*e+(a*e^2+c*d^
2)*x+c*d*x^2*e)^(1/2)-1/2*(a*e^2+c*d^2)/d/e/c*(-1/d/e/c/(a*d*e+(a*e^2+c*d^
2)*x+c*d*x^2*e)^(1/2)-(a*e^2+c*d^2)/d/e/c*(2*c*d*e*x+a*e^2+c*d^2)/(4*a*c*d
^2*e^2-(a*e^2+c*d^2)^2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(1/2))+1/d/e/...
```

Fricas [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 585, normalized size of antiderivative = 2.59

$$\int \frac{(d+ex)^4}{(ade+(cd^2+ae^2)x+cdex^2)^{3/2}}\,dx = \left[\frac{15\,(ac^2d^4e-2\,a^2cd^2e^3+a^3e^5+(c^3d^5-2\,ac^2d^3e^2+a^2cde^4)x)\sqrt{\cdots}}{\cdots}\right.$$

$$-\frac{15\,(ac^2d^4e-2\,a^2cd^2e^3+a^3e^5+(c^3d^5-2\,ac^2d^3e^2+a^2cde^4)x)\sqrt{-\frac{e}{cd}}\arctan\left(\frac{\sqrt{cdex^2+ade+(cd^2+ae^2)x}\,(2\,cdex+\cdots}{2\,(cde^2x^2+ade^2+(cd^2e+\cdots}\right.}{8\,(c^{\cdots}}$$

input
```
integrate((e*x+d)^4/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),x, algorithm="
fricas")
```

output
```
[1/16*(15*(a*c^2*d^4*e - 2*a^2*c*d^2*e^3 + a^3*e^5 + (c^3*d^5 - 2*a*c^2*d^
3*e^2 + a^2*c*d*e^4)*x)*sqrt(e/(c*d))*log(8*c^2*d^2*e^2*x^2 + c^2*d^4 + 6*
a*c*d^2*e^2 + a^2*e^4 + 8*(c^2*d^3*e + a*c*d*e^3)*x + 4*(2*c^2*d^2*e*x + c
^2*d^3 + a*c*d*e^2)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(e/(c*
d))) + 4*(2*c^2*d^2*e^2*x^2 - 8*c^2*d^4 + 25*a*c*d^2*e^2 - 15*a^2*e^4 + (9
*c^2*d^3*e - 5*a*c*d*e^3)*x)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x))/
(c^4*d^4*x + a*c^3*d^3*e), -1/8*(15*(a*c^2*d^4*e - 2*a^2*c*d^2*e^3 + a^3*e
^5 + (c^3*d^5 - 2*a*c^2*d^3*e^2 + a^2*c*d*e^4)*x)*sqrt(-e/(c*d))*arctan(1/
2*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(2*c*d*e*x + c*d^2 + a*e^2)*
sqrt(-e/(c*d))/(c*d*e^2*x^2 + a*d*e^2 + (c*d^2*e + a*e^3)*x)) - 2*(2*c^2*d
^2*e^2*x^2 - 8*c^2*d^4 + 25*a*c*d^2*e^2 - 15*a^2*e^4 + (9*c^2*d^3*e - 5*a*
c*d*e^3)*x)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x))/(c^4*d^4*x + a*c^
3*d^3*e)]
```

Sympy [F]

$$\int \frac{(d+ex)^4}{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}\,dx=\int \frac{(d+ex)^4}{\left((d+ex)\left(ae+cdx\right)\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate((e*x+d)**4/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(3/2),x)
```

output
```
Integral((d + e*x)**4/((d + e*x)*(a*e + c*d*x))**(3/2), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(d+ex)^4}{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}\,dx=\text{Exception raised: ValueError}$$

input
```
integrate((e*x+d)^4/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),x, algorithm="
maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(a*e^2-c*d^2>0)', see `assume?` f
or more de
```

Giac [F(-2)]

Exception generated.

$$\int \frac{(d+ex)^4}{(ade+(cd^2+ae^2)x+cdex^2)^{3/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((e*x+d)^4/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),x, algorithm="
giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Unable to divide, perhaps due to ro
unding error%%%{%%{[%%%{8,[4,4,4]%%%},0]:[1,0,%%%{-1,[1,1,1]%%%}]%%},[2,2]
%%%}+%%%{
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d+ex)^4}{(ade+(cd^2+ae^2)x+cdex^2)^{3/2}}\,dx = \int \frac{(d+e\,x)^4}{(c\,d\,e\,x^2+(c\,d^2+a\,e^2)\,x+a\,d\,e)^{3/2}}\,dx$$

input
```
int((d + e*x)^4/(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(3/2),x)
```

output
```
int((d + e*x)^4/(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(3/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 388, normalized size of antiderivative = 1.72

$$\int \frac{(d+ex)^4}{(ade+(cd^2+ae^2)\,x+cdex^2)^{3/2}}\,dx = \frac{15\sqrt{e}\,\sqrt{d}\,\sqrt{c}\,\sqrt{cdx+ae}\log\left(\frac{\sqrt{e}\,\sqrt{cdx+ae}+\sqrt{d}\,\sqrt{c}\,\sqrt{ex+d}}{\sqrt{a\,e^2-c\,d^2}}\right)a^2e^4-30\sqrt{e}$$

input
```
int((e*x+d)^4/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),x)
```

output
```
(15*sqrt(e)*sqrt(d)*sqrt(c)*sqrt(a*e + c*d*x)*log((sqrt(e)*sqrt(a*e + c*d*
x) + sqrt(d)*sqrt(c)*sqrt(d + e*x))/sqrt(a*e**2 - c*d**2))*a**2*e**4 - 30*
sqrt(e)*sqrt(d)*sqrt(c)*sqrt(a*e + c*d*x)*log((sqrt(e)*sqrt(a*e + c*d*x) +
 sqrt(d)*sqrt(c)*sqrt(d + e*x))/sqrt(a*e**2 - c*d**2))*a*c*d**2*e**2 + 15*
sqrt(e)*sqrt(d)*sqrt(c)*sqrt(a*e + c*d*x)*log((sqrt(e)*sqrt(a*e + c*d*x) +
 sqrt(d)*sqrt(c)*sqrt(d + e*x))/sqrt(a*e**2 - c*d**2))*c**2*d**4 - 10*sqrt
(e)*sqrt(d)*sqrt(c)*sqrt(a*e + c*d*x)*a**2*e**4 + 20*sqrt(e)*sqrt(d)*sqrt(
c)*sqrt(a*e + c*d*x)*a*c*d**2*e**2 - 10*sqrt(e)*sqrt(d)*sqrt(c)*sqrt(a*e +
 c*d*x)*c**2*d**4 - 15*sqrt(d + e*x)*a**2*c*d*e**4 + 25*sqrt(d + e*x)*a*c*
*2*d**3*e**2 - 5*sqrt(d + e*x)*a*c**2*d**2*e**3*x - 8*sqrt(d + e*x)*c**3*d
**5 + 9*sqrt(d + e*x)*c**3*d**4*e*x + 2*sqrt(d + e*x)*c**3*d**3*e**2*x**2)
/(4*sqrt(a*e + c*d*x)*c**4*d**4)
```

3.258 $\int \frac{(d+ex)^3}{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}\,dx$

Optimal result

Integrand size = 37, antiderivative size = 163

$$\int \frac{(d+ex)^3}{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}\,dx = -\frac{2(d+ex)^2}{cd\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}+\frac{3e\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{c^2d^2}+\frac{3\sqrt{e}\left(cd^2-ae^2\right)\operatorname{arctanh}\left(\frac{\sqrt{c}\sqrt{d}(d+ex)}{\sqrt{e}\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\right)}{c^{5/2}d^{5/2}}$$

output
```
-2*(e*x+d)^2/c/d/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)+3*e*(a*d*e+(a*e^2
+c*d^2)*x+c*d*e*x^2)^(1/2)/c^2/d^2+3*e^(1/2)*(-a*e^2+c*d^2)*arctanh(c^(1/2
)*d^(1/2)*(e*x+d)/e^(1/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2))/c^(5/2)
/d^(5/2)
```

Mathematica [A] (verified)

Time = 0.26 (sec) , antiderivative size = 143, normalized size of antiderivative = 0.88

$$\int \frac{(d+ex)^3}{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}\,dx = \frac{-\sqrt{c}\sqrt{d}(d+ex)\left(-3ae^2+cd(2d-ex)\right)+3\sqrt{e}\left(cd^2-ae^2\right)\sqrt{ae -}}{c^{5/2}d^{5/2}\sqrt{(ae+cdx)(d+ex)}}$$

input
```
Integrate[(d + e*x)^3/(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2),x]
```

output
```
(-(Sqrt[c]*Sqrt[d]*(d + e*x)*(-3*a*e^2 + c*d*(2*d - e*x))) + 3*Sqrt[e]*(c*
d^2 - a*e^2)*Sqrt[a*e + c*d*x]*Sqrt[d + e*x]*ArcTanh[(Sqrt[c]*Sqrt[d]*Sqrt
[d + e*x])/(Sqrt[e]*Sqrt[a*e + c*d*x])])/(c^(5/2)*d^(5/2)*Sqrt[(a*e + c*d*
x)*(d + e*x)])
```

Rubi [A] (verified)

Time = 0.57 (sec) , antiderivative size = 190, normalized size of antiderivative = 1.17, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.135, Rules used = {1124, 27, 1160, 1092, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex)^3}{\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}dx$$

$$\Big\downarrow \text{1124}$$

$$\frac{\int \frac{e\left(2cd^2+cexd-ae^2\right)}{\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}dx}{c^2d^2}-\frac{2(d+ex)\left(cd^2-ae^2\right)}{c^2d^2\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}$$

$$\Big\downarrow \text{27}$$

$$\frac{e\int \frac{2cd^2+cexd-ae^2}{\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}dx}{c^2d^2}-\frac{2(d+ex)\left(cd^2-ae^2\right)}{c^2d^2\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}$$

$$\Big\downarrow \text{1160}$$

$$\frac{e\left(\frac{3}{2}\left(cd^2-ae^2\right)\int \frac{1}{\sqrt{cdex^2+(cd^2+ae^2)x+ade}}dx+\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}\right)}{c^2d^2}-\frac{2(d+ex)\left(cd^2-ae^2\right)}{c^2d^2\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}$$

$\Big\downarrow$ 1092

$$\frac{e\left(3\left(cd^2-ae^2\right)\int \frac{1}{4cde-\frac{\left(cd^2+2cexd+ae^2\right)^2}{cdex^2+\left(cd^2+ae^2\right)x+ade}}d\frac{cd^2+2cexd+ae^2}{\sqrt{cdex^2+(cd^2+ae^2)x+ade}}+\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}\right)}{c^2d^2}-\frac{2(d+ex)\left(cd^2-ae^2\right)}{c^2d^2\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}$$

$\Big\downarrow$ 219

$$\frac{e\left(\frac{3\left(cd^2-ae^2\right)\text{arctanh}\left(\frac{ae^2+cd^2+2cdex}{2\sqrt{c}\sqrt{d}\sqrt{e}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}\right)}{2\sqrt{c}\sqrt{d}\sqrt{e}}+\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}\right)}{c^2d^2}-\frac{2(d+ex)\left(cd^2-ae^2\right)}{c^2d^2\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}$$

input
```
Int[(d + e*x)^3/(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2),x]
```

output
```
(-2*(c*d^2 - a*e^2)*(d + e*x))/(c^2*d^2*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c
*d*e*x^2]) + (e*(Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2] + (3*(c*d^2 -
 a*e^2)*ArcTanh[(c*d^2 + a*e^2 + 2*c*d*e*x)/(2*Sqrt[c]*Sqrt[d]*Sqrt[e]*Sqr
t[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])])/(2*Sqrt[c]*Sqrt[d]*Sqrt[e])))/
(c^2*d^2)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1092
```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[I
nt[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a
, b, c}, x]
```

rule 1124
```
Int[((d_.) + (e_.)*(x_))^(m_.)/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(3/2), x
_Symbol] :> Simp[-2*e*(2*c*d - b*e)^(m - 2)*((d + e*x)/(c^(m - 1)*Sqrt[a +
b*x + c*x^2])), x] + Simp[e^2/c^(m - 1)   Int[(1/Sqrt[a + b*x + c*x^2])*Exp
andToSum[((2*c*d - b*e)^(m - 1) - c^(m - 1)*(d + e*x)^(m - 1))/(c*d - b*e -
 c*e*x), x], x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c*d^2 - b*d*e + a*e
^2, 0] && IGtQ[m, 0]
```

rule 1160
```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[e*((a + b*x + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Simp[(2*c*d - b
*e)/(2*c)   Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
 && NeQ[p, -1]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 959 vs. $2(145) = 290$.

Time = 1.23 (sec) , antiderivative size = 960, normalized size of antiderivative = 5.89

method	result
default	$\frac{2d^3(2cdxe+a\,e^2+c\,d^2)}{\left(4ac\,d^2e^2-(a\,e^2+c\,d^2)^2\right)\sqrt{ade+(a\,e^2+c\,d^2)x+cd\,x^2e}}+e^3\left(\frac{x^2}{dec\sqrt{ade+(a\,e^2+c\,d^2)x+cd\,x^2e}}-\frac{3(a\,e^2+c\,d^2)\left(-\frac{}{dec\sqrt{ade+(a}}\right.}{}\right.$

input
```
int((e*x+d)^3/(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(3/2),x,method=_RETURNVERB
OSE)
```

output
```
2*d^3*(2*c*d*e*x+a*e^2+c*d^2)/(4*a*c*d^2*e^2-(a*e^2+c*d^2)^2)/(a*d*e+(a*e^
2+c*d^2)*x+c*d*x^2*e)^(1/2)+e^3*(x^2/d/e/c/(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*
e)^(1/2)-3/2*(a*e^2+c*d^2)/d/e/c*(-x/d/e/c/(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*
e)^(1/2)-1/2*(a*e^2+c*d^2)/d/e/c*(-1/d/e/c/(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*
e)^(1/2)-(a*e^2+c*d^2)/d/e/c*(2*c*d*e*x+a*e^2+c*d^2)/(4*a*c*d^2*e^2-(a*e^2
+c*d^2)^2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(1/2))+1/d/e/c*ln((1/2*a*e^2+
1/2*c*d^2+c*d*x*e)/(d*e*c)^(1/2)+(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(1/2))/
(d*e*c)^(1/2))-2*a/c*(-1/d/e/c/(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(1/2)-(a*
e^2+c*d^2)/d/e/c*(2*c*d*e*x+a*e^2+c*d^2)/(4*a*c*d^2*e^2-(a*e^2+c*d^2)^2)/(
a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(1/2)))+3*d*e^2*(-x/d/e/c/(a*d*e+(a*e^2+c
*d^2)*x+c*d*x^2*e)^(1/2)-1/2*(a*e^2+c*d^2)/d/e/c*(-1/d/e/c/(a*d*e+(a*e^2+c
*d^2)*x+c*d*x^2*e)^(1/2)-(a*e^2+c*d^2)/d/e/c*(2*c*d*e*x+a*e^2+c*d^2)/(4*a*
c*d^2*e^2-(a*e^2+c*d^2)^2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(1/2))+1/d/e/
c*ln((1/2*a*e^2+1/2*c*d^2+c*d*x*e)/(d*e*c)^(1/2)+(a*d*e+(a*e^2+c*d^2)*x+c*
d*x^2*e)^(1/2))/(d*e*c)^(1/2))+3*d^2*e*(-1/d/e/c/(a*d*e+(a*e^2+c*d^2)*x+c*
d*x^2*e)^(1/2)-(a*e^2+c*d^2)/d/e/c*(2*c*d*e*x+a*e^2+c*d^2)/(4*a*c*d^2*e^2-
(a*e^2+c*d^2)^2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 447, normalized size of antiderivative = 2.74

$$\int \frac{(d+ex)^3}{(ade+(cd^2+ae^2)x+cdex^2)^{3/2}}\,dx = \left[-\frac{3\left(acd^2e-a^2e^3+(c^2d^3-acde^2)x\right)\sqrt{\frac{e}{cd}}\log\left(8c^2d^2e^2x^2+c^2d\right.}{}\right.$$

$$-\frac{3\left(acd^2e-a^2e^3+(c^2d^3-acde^2)x\right)\sqrt{-\frac{e}{cd}}\arctan\left(\frac{\sqrt{cdex^2+ade+(cd^2+ae^2)x}\,(2\,cdex+cd^2+ae^2)\sqrt{-\frac{e}{cd}}}{2\,(cde^2x^2+ade^2+(cd^2e+ae^3)x)}\right)-2\sqrt{cdex^2}}{2\,(c^3d^3x+ac^2d^2e)}$$

input
```
integrate((e*x+d)^3/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),x, algorithm="
fricas")
```

output
```
[-1/4*(3*(a*c*d^2*e - a^2*e^3 + (c^2*d^3 - a*c*d*e^2)*x)*sqrt(e/(c*d))*log
(8*c^2*d^2*e^2*x^2 + c^2*d^4 + 6*a*c*d^2*e^2 + a^2*e^4 + 8*(c^2*d^3*e + a*
c*d*e^3)*x - 4*(2*c^2*d^2*e*x + c^2*d^3 + a*c*d*e^2)*sqrt(c*d*e*x^2 + a*d*
e + (c*d^2 + a*e^2)*x)*sqrt(e/(c*d))) - 4*sqrt(c*d*e*x^2 + a*d*e + (c*d^2
+ a*e^2)*x)*(c*d*e*x - 2*c*d^2 + 3*a*e^2))/(c^3*d^3*x + a*c^2*d^2*e), -1/2
*(3*(a*c*d^2*e - a^2*e^3 + (c^2*d^3 - a*c*d*e^2)*x)*sqrt(-e/(c*d))*arctan(
1/2*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(2*c*d*e*x + c*d^2 + a*e^2
)*sqrt(-e/(c*d))/(c*d*e^2*x^2 + a*d*e^2 + (c*d^2*e + a*e^3)*x)) - 2*sqrt(c
*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(c*d*e*x - 2*c*d^2 + 3*a*e^2))/(c^3*
d^3*x + a*c^2*d^2*e)]
```

Sympy [F]

$$\int \frac{(d+ex)^3}{(ade+(cd^2+ae^2)x+cdex^2)^{3/2}}\,dx = \int \frac{(d+ex)^3}{((d+ex)(ae+cdx))^{\frac{3}{2}}}\,dx$$

input
```
integrate((e*x+d)**3/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(3/2),x)
```

output
```
Integral((d + e*x)**3/((d + e*x)*(a*e + c*d*x))**(3/2), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(d+ex)^3}{\left(ade+(cd^2+ae^2)\,x+cdex^2\right)^{3/2}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((e*x+d)^3/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),x, algorithm="
maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(a*e^2-c*d^2>0)', see `assume?` f
or more de
```

Giac [F(-2)]

Exception generated.

$$\int \frac{(d+ex)^3}{\left(ade+(cd^2+ae^2)\,x+cdex^2\right)^{3/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((e*x+d)^3/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),x, algorithm="
giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Unable to divide, perhaps due to ro
unding error%%%{%%{[%%%{2,[3,3,4]%%%},0]:[1,0,%%%{-1,[1,1,1]%%%}]%%},[2,2]
%%%}+%%%{
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d+ex)^3}{(ade+(cd^2+ae^2)x+cdex^2)^{3/2}}\,dx = \int \frac{(d+e\,x)^3}{(c\,d\,e\,x^2+(c\,d^2+a\,e^2)\,x+a\,d\,e)^{3/2}}\,dx$$

input
```
int((d + e*x)^3/(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(3/2),x)
```

output
```
int((d + e*x)^3/(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(3/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 230, normalized size of antiderivative = 1.41

$$\int \frac{(d+ex)^3}{(ade+(cd^2+ae^2)x+cdex^2)^{3/2}}\,dx = \frac{-12\sqrt{e}\,\sqrt{d}\,\sqrt{c}\,\sqrt{cdx+ae}\log\left(\frac{\sqrt{e}\,\sqrt{cdx+ae}+\sqrt{d}\,\sqrt{c}\,\sqrt{ex+d}}{\sqrt{a\,e^2-c\,d^2}}\right)a\,e^2+12\sqrt{}}{}$$

input
```
int((e*x+d)^3/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),x)
```

output
```
( - 12*sqrt(e)*sqrt(d)*sqrt(c)*sqrt(a*e + c*d*x)*log((sqrt(e)*sqrt(a*e + c
*d*x) + sqrt(d)*sqrt(c)*sqrt(d + e*x))/sqrt(a*e**2 - c*d**2))*a*e**2 + 12*
sqrt(e)*sqrt(d)*sqrt(c)*sqrt(a*e + c*d*x)*log((sqrt(e)*sqrt(a*e + c*d*x) +
 sqrt(d)*sqrt(c)*sqrt(d + e*x))/sqrt(a*e**2 - c*d**2))*c*d**2 + 9*sqrt(e)*
sqrt(d)*sqrt(c)*sqrt(a*e + c*d*x)*a*e**2 - 9*sqrt(e)*sqrt(d)*sqrt(c)*sqrt(
a*e + c*d*x)*c*d**2 + 12*sqrt(d + e*x)*a*c*d*e**2 - 8*sqrt(d + e*x)*c**2*d
**3 + 4*sqrt(d + e*x)*c**2*d**2*e*x)/(4*sqrt(a*e + c*d*x)*c**3*d**3)
```

3.259 $\int \frac{(d+ex)^2}{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}\,dx$

Optimal result

Integrand size = 37, antiderivative size = 111

$$\int \frac{(d+ex)^2}{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}\,dx=-\frac{2(d+ex)}{cd\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}+\frac{2\sqrt{e}\operatorname{arctanh}\left(\frac{\sqrt{c}\sqrt{d}(d+ex)}{\sqrt{e}\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}\right)}{c^{3/2}d^{3/2}}$$

output
```
(-2*e*x-2*d)/c/d/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)+2*e^(1/2)*arctanh
(c^(1/2)*d^(1/2)*(e*x+d)/e^(1/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2))/
c^(3/2)/d^(3/2)
```

Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.02

$$\int \frac{(d+ex)^2}{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}\,dx=\frac{-2\sqrt{c}\sqrt{d}(d+ex)+2\sqrt{e}\sqrt{ae+cdx}\sqrt{d+ex}\operatorname{arctanh}\left(\frac{\sqrt{c}\sqrt{d}\sqrt{d+ex}}{\sqrt{e}\sqrt{ae+cdx}}\right)}{c^{3/2}d^{3/2}\sqrt{(ae+cdx)(d+ex)}}$$

input
```
Integrate[(d + e*x)^2/(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2),x]
```

output
```
(-2*Sqrt[c]*Sqrt[d]*(d + e*x) + 2*Sqrt[e]*Sqrt[a*e + c*d*x]*Sqrt[d + e*x]*
ArcTanh[(Sqrt[c]*Sqrt[d]*Sqrt[d + e*x])/(Sqrt[e]*Sqrt[a*e + c*d*x])])/(c^(
3/2)*d^(3/2)*Sqrt[(a*e + c*d*x)*(d + e*x)])
```

Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 125, normalized size of antiderivative = 1.13, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.081$, Rules used = {1124, 1092, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex)^2}{(x(ae^2+cd^2)+ade+cdex^2)^{3/2}}dx$$

$$\Big\downarrow \text{1124}$$

$$\frac{e\int \frac{1}{\sqrt{cdex^2+(cd^2+ae^2)x+ade}}dx}{cd}-\frac{2(d+ex)}{cd\sqrt{x(ae^2+cd^2)+ade+cdex^2}}$$

$$\Big\downarrow \text{1092}$$

$$\frac{2e\int \frac{1}{4cde-\frac{\left(cd^2+2cexd+ae^2\right)^2}{cdex^2+\left(cd^2+ae^2\right)x+ade}}d\frac{cd^2+2cexd+ae^2}{\sqrt{cdex^2+(cd^2+ae^2)x+ade}}}{cd}-\frac{2(d+ex)}{cd\sqrt{x(ae^2+cd^2)+ade+cdex^2}}$$

$$\Big\downarrow \text{219}$$

$$\frac{\sqrt{e}\text{arctanh}\left(\frac{ae^2+cd^2+2cdex}{2\sqrt{c}\sqrt{d}\sqrt{e}\sqrt{x(ae^2+cd^2)+ade+cdex^2}}\right)}{c^{3/2}d^{3/2}}-\frac{2(d+ex)}{cd\sqrt{x(ae^2+cd^2)+ade+cdex^2}}$$

input
```
Int[(d + e*x)^2/(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2),x]
```

output
```
(-2*(d + e*x))/(c*d*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]) + (Sqrt[e
]*ArcTanh[(c*d^2 + a*e^2 + 2*c*d*e*x)/(2*Sqrt[c]*Sqrt[d]*Sqrt[e]*Sqrt[a*d*
e + (c*d^2 + a*e^2)*x + c*d*e*x^2])])/(c^(3/2)*d^(3/2))
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1092
```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[I
nt[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a
, b, c}, x]
```

rule 1124
```
Int[((d_.) + (e_.)*(x_))^(m_.)/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(3/2), x
_Symbol] :> Simp[-2*e*(2*c*d - b*e)^(m - 2)*((d + e*x)/(c^(m - 1)*Sqrt[a +
b*x + c*x^2])), x] + Simp[e^2/c^(m - 1)   Int[(1/Sqrt[a + b*x + c*x^2])*Exp
andToSum[((2*c*d - b*e)^(m - 1) - c^(m - 1)*(d + e*x)^(m - 1))/(c*d - b*e -
 c*e*x), x], x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c*d^2 - b*d*e + a*e
^2, 0] && IGtQ[m, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 484 vs. $2(97) = 194$.

Time = 1.09 (sec) , antiderivative size = 485, normalized size of antiderivative = 4.37

method	result
default	$\frac{2d^{2}\left(2cdxe+a\,e^{2}+c\,d^{2}\right)}{\left(4ac\,d^{2}e^{2}-\left(a\,e^{2}+c\,d^{2}\right)^{2}\right)\sqrt{ade+\left(a\,e^{2}+c\,d^{2}\right)x+cd\,x^{2}e}}+e^{2}\left(-\frac{x}{dec\sqrt{ade+\left(a\,e^{2}+c\,d^{2}\right)x+cd\,x^{2}e}}-\frac{\left(a\,e^{2}+c\,d^{2}\right)\left(-\frac{}{dec\sqrt{ade+(}}\right.}{}\right.$

input
```
int((e*x+d)^2/(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(3/2),x,method=_RETURNVERB
OSE)
```

output
```
2*d^2*(2*c*d*e*x+a*e^2+c*d^2)/(4*a*c*d^2*e^2-(a*e^2+c*d^2)^2)/(a*d*e+(a*e^
2+c*d^2)*x+c*d*x^2*e)^(1/2)+e^2*(-x/d/e/c/(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e
)^(1/2)-1/2*(a*e^2+c*d^2)/d/e/c*(-1/d/e/c/(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e
)^(1/2)-(a*e^2+c*d^2)/d/e/c*(2*c*d*e*x+a*e^2+c*d^2)/(4*a*c*d^2*e^2-(a*e^2+
c*d^2)^2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(1/2))+1/d/e/c*ln((1/2*a*e^2+1
/2*c*d^2+c*d*x*e)/(d*e*c)^(1/2)+(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(1/2))/(
d*e*c)^(1/2))+2*d*e*(-1/d/e/c/(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(1/2)-(a*e
^2+c*d^2)/d/e/c*(2*c*d*e*x+a*e^2+c*d^2)/(4*a*c*d^2*e^2-(a*e^2+c*d^2)^2)/(a
*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 349, normalized size of antiderivative = 3.14

$$\int \frac{(d+ex)^2}{\left(ade+(cd^2+ae^2)\,x+cdex^2\right)^{3/2}}\,dx = \left[\frac{(cdx+ae)\sqrt{\frac{e}{cd}}\log\left(8\,c^2d^2e^2x^2+c^2d^4+6\,acd^2e^2+a^2e^4+8\,(c^2\right.}{}, -\frac{(cdx+ae)\sqrt{-\frac{e}{cd}}\arctan\left(\frac{\sqrt{cdex^2+ade+(cd^2+ae^2)x}\,(2\,cdex+cd^2+ae^2)\sqrt{-\frac{e}{cd}}}{2\,(cde^2x^2+ade^2+(cd^2e+ae^3)x)}\right)+2\,\sqrt{cdex^2+ade+(cd^2+ae^2)x}}{c^2d^2x+acde}\right]$$

input
```
integrate((e*x+d)^2/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),x, algorithm="
fricas")
```

output
```
[1/2*((c*d*x + a*e)*sqrt(e/(c*d))*log(8*c^2*d^2*e^2*x^2 + c^2*d^4 + 6*a*c*
d^2*e^2 + a^2*e^4 + 8*(c^2*d^3*e + a*c*d*e^3)*x + 4*(2*c^2*d^2*e*x + c^2*d
^3 + a*c*d*e^2)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(e/(c*d)))
 - 4*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x))/(c^2*d^2*x + a*c*d*e), -
((c*d*x + a*e)*sqrt(-e/(c*d))*arctan(1/2*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 +
 a*e^2)*x)*(2*c*d*e*x + c*d^2 + a*e^2)*sqrt(-e/(c*d))/(c*d*e^2*x^2 + a*d*e
^2 + (c*d^2*e + a*e^3)*x)) + 2*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)
)/(c^2*d^2*x + a*c*d*e)]
```

Sympy [F]

$$\int \frac{(d+ex)^2}{(ade+(cd^2+ae^2)\,x+cdex^2)^{3/2}}\,dx=\int \frac{(d+ex)^2}{((d+ex)\,(ae+cdx))^{\frac{3}{2}}}\,dx$$

input
```
integrate((e*x+d)**2/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(3/2),x)
```

output
```
Integral((d + e*x)**2/((d + e*x)*(a*e + c*d*x))**(3/2), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(d+ex)^2}{(ade+(cd^2+ae^2)\,x+cdex^2)^{3/2}}\,dx=\text{Exception raised: ValueError}$$

input
```
integrate((e*x+d)^2/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),x, algorithm="
maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(a*e^2-c*d^2>0)', see `assume?` f
or more de
```

Giac [F(-2)]

Exception generated.

$$\int \frac{(d+ex)^2}{(ade+(cd^2+ae^2)\,x+cdex^2)^{3/2}}\,dx=\text{Exception raised: TypeError}$$

input
```
integrate((e*x+d)^2/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),x, algorithm="
giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Unable to divide, perhaps due to ro
unding error%%%{%%{[%%%{1,[1,1,4]%%%},0]:[1,0,%%%{-1,[1,1,1]%%%}]%%},[2,2]
%%%}+%%%{
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d+ex)^2}{(ade+(cd^2+ae^2)x+cdex^2)^{3/2}}\,dx = \int \frac{(d+e\,x)^2}{\left(c\,d\,e\,x^2+(c\,d^2+a\,e^2)\,x+a\,d\,e\right)^{3/2}}\,dx$$

input
```
int((d + e*x)^2/(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(3/2),x)
```

output
```
int((d + e*x)^2/(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(3/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 104, normalized size of antiderivative = 0.94

$$\int \frac{(d+ex)^2}{(ade+(cd^2+ae^2)x+cdex^2)^{3/2}}\,dx = \frac{2\sqrt{e}\,\sqrt{d}\,\sqrt{c}\,\sqrt{cdx+ae}\log\left(\frac{\sqrt{e}\,\sqrt{cdx+ae}+\sqrt{d}\,\sqrt{c}\,\sqrt{ex+d}}{\sqrt{a\,e^2-c\,d^2}}\right)-2\sqrt{e}\,\sqrt{d}\,\sqrt{c}}{\sqrt{cdx+ae}\,c^2d^2}$$

input
```
int((e*x+d)^2/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),x)
```

output
```
(2*(sqrt(e)*sqrt(d)*sqrt(c)*sqrt(a*e + c*d*x)*log((sqrt(e)*sqrt(a*e + c*d*
x) + sqrt(d)*sqrt(c)*sqrt(d + e*x))/sqrt(a*e**2 - c*d**2)) - sqrt(e)*sqrt(
d)*sqrt(c)*sqrt(a*e + c*d*x) - sqrt(d + e*x)*c*d))/(sqrt(a*e + c*d*x)*c**2
*d**2)
```

3.260 $\int \frac{d+ex}{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 50

$$\int \frac{d+ex}{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}\,dx=-\frac{2(d+ex)}{\left(cd^2-ae^2\right)\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}$$

output
```
(-2*e*x-2*d)/(-a*e^2+c*d^2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.78

$$\int \frac{d+ex}{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}\,dx=-\frac{2(d+ex)}{\left(cd^2-ae^2\right)\sqrt{(ae+cdx)(d+ex)}}$$

input
```
Integrate[(d + e*x)/(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2),x]
```

output
```
(-2*(d + e*x))/((c*d^2 - a*e^2)*Sqrt[(a*e + c*d*x)*(d + e*x)])
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.057$, Rules used = {1124, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{d+ex}{\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}dx$$

$$\Big\downarrow \text{ 1124}$$

$$e^2\int 0dx-\frac{2(d+ex)}{\left(cd^2-ae^2\right)\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}$$

$$\Big\downarrow \text{ 24}$$

$$-\frac{2(d+ex)}{\left(cd^2-ae^2\right)\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}$$

input
```
Int[(d + e*x)/(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2),x]
```

output
```
(-2*(d + e*x))/((c*d^2 - a*e^2)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2
])
```

Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 1124
```
Int[((d_.) + (e_.)*(x_))^(m_.)/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(3/2), x
_Symbol] :> Simp[-2*e*(2*c*d - b*e)^(m - 2)*((d + e*x)/(c^(m - 1)*Sqrt[a +
b*x + c*x^2])), x] + Simp[e^2/c^(m - 1)   Int[(1/Sqrt[a + b*x + c*x^2])*Exp
andToSum[((2*c*d - b*e)^(m - 1) - c^(m - 1)*(d + e*x)^(m - 1))/(c*d - b*e -
 c*e*x), x], x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c*d^2 - b*d*e + a*e
^2, 0] && IGtQ[m, 0]
```

Maple [A] (verified)

Time = 0.98 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.06

method	result
trager	$\frac{2\sqrt{cd\,x^2e+a\,e^2x+c\,d^2x+ade}}{(a\,e^2-c\,d^2)(cdx+ae)}$
gosper	$\frac{2(ex+d)^2(cdx+ae)}{(a\,e^2-c\,d^2)(cd\,x^2e+a\,e^2x+c\,d^2x+ade)^{\frac{3}{2}}}$
orering	$\frac{2(ex+d)^2(cdx+ae)}{(a\,e^2-c\,d^2)(ade+(a\,e^2+c\,d^2)x+cd\,x^2e)^{\frac{3}{2}}}$
default	$\frac{2d(2cdxe+a\,e^2+c\,d^2)}{\left(4ac\,d^2e^2-(a\,e^2+c\,d^2)^2\right)\sqrt{ade+(a\,e^2+c\,d^2)x+cd\,x^2e}}+e\left(-\frac{1}{dec\sqrt{ade+(a\,e^2+c\,d^2)x+cd\,x^2e}}-\frac{(a\,e^2+c\,d^2)(2c}{dec\left(4ac\,d^2e^2-(a\,e^2+c\,d^2)^2\right.}\right.$

input
```
int((e*x+d)/(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(3/2),x,method=_RETURNVERBOS
E)
```

output
```
2/(a*e^2-c*d^2)/(c*d*x+a*e)*(c*d*e*x^2+a*e^2*x+c*d^2*x+a*d*e)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.30

$$\int \frac{d+ex}{(ade+(cd^2+ae^2)\,x+cdex^2)^{3/2}}\,dx=-\frac{2\,\sqrt{cdex^2+ade+(cd^2+ae^2)x}}{acd^2e-a^2e^3+(c^2d^3-acde^2)x}$$

input
```
integrate((e*x+d)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),x, algorithm="fr
icas")
```

output
```
-2*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)/(a*c*d^2*e - a^2*e^3 + (c^2
*d^3 - a*c*d*e^2)*x)
```

Sympy [F]

$$\int \frac{d+ex}{(ade+(cd^2+ae^2)x+cdex^2)^{3/2}}\,dx = \int \frac{d+ex}{((d+ex)(ae+cdx))^{\frac{3}{2}}}\,dx$$

input
```
integrate((e*x+d)/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(3/2),x)
```

output
```
Integral((d + e*x)/((d + e*x)*(a*e + c*d*x))**(3/2), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{d+ex}{(ade+(cd^2+ae^2)x+cdex^2)^{3/2}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((e*x+d)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),x, algorithm="ma
xima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(a*e^2-c*d^2>0)', see `assume?` f
or more de
```

Giac [F(-2)]

Exception generated.

$$\int \frac{d+ex}{(ade+(cd^2+ae^2)x+cdex^2)^{3/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((e*x+d)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),x, algorithm="gi
ac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Unable to divide, perhaps due to ro
unding error%%%{%%%{1,[1,1,0]%%%},[2,0]%%%}+%%%{%%{[%%%{-2,[0,0,1]%%%},0]:
[1,0,%%%{
```

Mupad [B] (verification not implemented)

Time = 5.30 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.06

$$\int \frac{d+ex}{(ade+(cd^2+ae^2)x+cdex^2)^{3/2}}\,dx = \frac{2\sqrt{c\,d\,e\,x^2+(c\,d^2+a\,e^2)\,x+a\,d\,e}}{(a\,e+c\,d\,x)\,(a\,e^2-c\,d^2)}$$

input
```
int((d + e*x)/(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(3/2),x)
```

output
```
(2*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2))/((a*e + c*d*x)*(a*e^2 -
c*d^2))
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.18

$$\int \frac{d+ex}{(ade+(cd^2+ae^2)x+cdex^2)^{3/2}}\,dx = \frac{2\sqrt{e}\,\sqrt{d}\,\sqrt{c}\,\sqrt{cdx+ae}+2\sqrt{ex+d}\,cd}{\sqrt{cdx+ae}\,cd\,(a\,e^2-c\,d^2)}$$

input
```
int((e*x+d)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),x)
```

output
```
(2*(sqrt(e)*sqrt(d)*sqrt(c)*sqrt(a*e + c*d*x) + sqrt(d + e*x)*c*d))/(sqrt(
a*e + c*d*x)*c*d*(a*e**2 - c*d**2))
```

3.261 $\int \frac{1}{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 62

$$\int \frac{1}{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}\,dx=-\frac{2\left(cd^2+ae^2+2cdex\right)}{\left(cd^2-ae^2\right)^2\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}$$

output
```
(-4*c*d*e*x-2*a*e^2-2*c*d^2)/(-a*e^2+c*d^2)^2/(a*d*e+(a*e^2+c*d^2)*x+c*d*e
*x^2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.79

$$\int \frac{1}{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}\,dx=-\frac{2\left(ae^2+cd(d+2ex)\right)}{\left(cd^2-ae^2\right)^2\sqrt{(ae+cdx)(d+ex)}}$$

input
```
Integrate[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(-3/2),x]
```

output
```
(-2*(a*e^2 + c*d*(d + 2*e*x)))/((c*d^2 - a*e^2)^2*Sqrt[(a*e + c*d*x)*(d +
e*x)])
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.034$, Rules used = $\{1088\}$

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(x(ae^2+cd^2)+ade+cdex^2)^{3/2}}\,dx$$

$$\Big\downarrow \text{1088}$$

$$-\frac{2(ae^2+cd^2+2cdex)}{(cd^2-ae^2)^2\sqrt{x(ae^2+cd^2)+ade+cdex^2}}$$

input
```
Int[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(-3/2),x]
```

output
```
(-2*(c*d^2 + a*e^2 + 2*c*d*e*x))/((c*d^2 - a*e^2)^2*Sqrt[a*d*e + (c*d^2 +
a*e^2)*x + c*d*e*x^2])
```

Defintions of rubi rules used

rule 1088
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-3/2), x_Symbol] :> Simp[-2*((b +
2*c*x)/((b^2 - 4*a*c)*Sqrt[a + b*x + c*x^2])), x] /; FreeQ[{a, b, c}, x] &&
 NeQ[b^2 - 4*a*c, 0]
```

Maple [A] (verified)

Time = 0.92 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.97

method	result	size
trager	$-\frac{2\left(2cdxe+a\,e^{2}+c\,d^{2}\right)}{\left(a\,e^{2}-c\,d^{2}\right)^{2}\sqrt{cd\,x^{2}e+a\,e^{2}x+c\,d^{2}x+ade}}$	60
default	$\frac{4cdxe+2a\,e^{2}+2c\,d^{2}}{\left(4ac\,d^{2}e^{2}-\left(a\,e^{2}+c\,d^{2}\right)^{2}\right)\sqrt{ade+\left(a\,e^{2}+c\,d^{2}\right)x+cd\,x^{2}e}}$	75
gosper	$-\frac{2(ex+d)(cdx+ae)\left(2cdxe+a\,e^{2}+c\,d^{2}\right)}{\left(a^{2}e^{4}-2ac\,d^{2}e^{2}+c^{2}d^{4}\right)\left(cd\,x^{2}e+a\,e^{2}x+c\,d^{2}x+ade\right)^{\frac{3}{2}}}$	86
orering	$-\frac{2\left(2cdxe+a\,e^{2}+c\,d^{2}\right)(ex+d)(cdx+ae)}{\left(a^{2}e^{4}-2ac\,d^{2}e^{2}+c^{2}d^{4}\right)\left(ade+\left(a\,e^{2}+c\,d^{2}\right)x+cd\,x^{2}e\right)^{\frac{3}{2}}}$	87

input
```
int(1/(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-2*(2*c*d*e*x+a*e^2+c*d^2)/(a*e^2-c*d^2)^2/(c*d*e*x^2+a*e^2*x+c*d^2*x+a*d*
e)^(1/2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 153 vs. $2(60) = 120$.

Time = 0.45 (sec) , antiderivative size = 153, normalized size of antiderivative = 2.47

$$\int \frac{1}{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}\,dx =$$
$$-\frac{2\sqrt{cdex^2+ade+\left(cd^2+ae^2\right)x}\left(2\,cdex+cd^2+ae^2\right)}{ac^2d^5e-2\,a^2cd^3e^3+a^3de^5+\left(c^3d^5e-2\,ac^2d^3e^3+a^2cde^5\right)x^2+\left(c^3d^6-ac^2d^4e^2-a^2cd^2e^4+a^3e^6\right)x}$$

input
```
integrate(1/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),x, algorithm="fricas")
```

output
```
-2*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(2*c*d*e*x + c*d^2 + a*e^2)
/(a*c^2*d^5*e - 2*a^2*c*d^3*e^3 + a^3*d*e^5 + (c^3*d^5*e - 2*a*c^2*d^3*e^3
 + a^2*c*d*e^5)*x^2 + (c^3*d^6 - a*c^2*d^4*e^2 - a^2*c*d^2*e^4 + a^3*e^6)*
x)
```

Sympy [F]

$$\int \frac{1}{(ade+(cd^2+ae^2)\,x+cdex^2)^{3/2}}\,dx = \int \frac{1}{(ade+cdex^2+x\,(ae^2+cd^2))^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(3/2),x)
```

output
```
Integral((a*d*e + c*d*e*x**2 + x*(a*e**2 + c*d**2))**(-3/2), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{(ade+(cd^2+ae^2)\,x+cdex^2)^{3/2}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(1/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(a*e^2-c*d^2>0)', see `assume?` f
or more de
```

Giac [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 101, normalized size of antiderivative = 1.63

$$\int \frac{1}{(ade+(cd^2+ae^2)\,x+cdex^2)^{3/2}}\,dx = -\frac{2\left(\frac{2\,cdex}{c^2d^4-2\,acd^2e^2+a^2e^4}+\frac{cd^2+ae^2}{c^2d^4-2\,acd^2e^2+a^2e^4}\right)}{\sqrt{cdex^2+cd^2x+ae^2x+ade}}$$

input
```
integrate(1/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),x, algorithm="giac")
```

output
```
-2*(2*c*d*e*x/(c^2*d^4 - 2*a*c*d^2*e^2 + a^2*e^4) + (c*d^2 + a*e^2)/(c^2*d
^4 - 2*a*c*d^2*e^2 + a^2*e^4))/sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*x + a*d*e)
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.21

$$\int \frac{1}{(ade + (cd^2 + ae^2)x + cdex^2)^{3/2}}\,dx = -\frac{\frac{c\,d^2}{2} + c\,x\,d\,e + \frac{a\,e^2}{2}}{\left(\frac{(c\,d^2+a\,e^2)^2}{4} - a\,c\,d^2\,e^2\right)\sqrt{c\,d\,e\,x^2 + (c\,d^2 + a\,e^2)\,x + a\,d\,e}}$$

input
```
int(1/(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(3/2),x)
```

output
```
-((a*e^2)/2 + (c*d^2)/2 + c*d*e*x)/((((a*e^2 + c*d^2)^2/4 - a*c*d^2*e^2)*(x
*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2))
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 143, normalized size of antiderivative = 2.31

$$\int \frac{1}{(ade + (cd^2 + ae^2)x + cdex^2)^{3/2}}\,dx = \frac{-4\sqrt{e}\,\sqrt{d}\,\sqrt{c}\,\sqrt{cdx + ae}\,d - 4\sqrt{e}\,\sqrt{d}\,\sqrt{c}\,\sqrt{cdx + ae}\,ex - 2\sqrt{ex -}}{\sqrt{cdx + ae}\,(a^2e^5x - 2ac\,d^2e^3x + c^2d^4ex + a^2e}$$

input
```
int(1/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),x)
```

output
```
(2*( - 2*sqrt(e)*sqrt(d)*sqrt(c)*sqrt(a*e + c*d*x)*d - 2*sqrt(e)*sqrt(d)*s
qrt(c)*sqrt(a*e + c*d*x)*e*x - sqrt(d + e*x)*a*e**2 - sqrt(d + e*x)*c*d**2
 - 2*sqrt(d + e*x)*c*d*e*x))/(sqrt(a*e + c*d*x)*(a**2*d*e**4 + a**2*e**5*x
 - 2*a*c*d**3*e**2 - 2*a*c*d**2*e**3*x + c**2*d**5 + c**2*d**4*e*x))
```

3.262 $\int \frac{1}{(d+ex)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}\,dx$

Optimal result

Integrand size = 37, antiderivative size = 165

$$\int \frac{1}{(d+ex)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}\,dx = -\frac{2}{\left(cd^2-ae^2\right)(d+ex)\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}} - \frac{8e\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{3\left(cd^2-ae^2\right)^2(d+ex)^2} - \frac{16cde\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{3\left(cd^2-ae^2\right)^3(d+ex)}$$

output
```
-2/(-a*e^2+c*d^2)/(e*x+d)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)-8/3*e*(a
*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(-a*e^2+c*d^2)^2/(e*x+d)^2-16/3*c*d*
e*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(-a*e^2+c*d^2)^3/(e*x+d)
```

Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.58

$$\int \frac{1}{(d+ex)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}\,dx = \frac{2a^2e^4-4acde^2(3d+2ex)-2c^2d^2(3d^2+12dex+8e^2x^2}{3\left(cd^2-ae^2\right)^3(d+ex)\sqrt{(ae+cdx)(d+ex)}}$$

input
```
Integrate[1/((d + e*x)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2)),x]
```

output
```
(2*a^2*e^4 - 4*a*c*d*e^2*(3*d + 2*e*x) - 2*c^2*d^2*(3*d^2 + 12*d*e*x + 8*e
^2*x^2))/(3*(c*d^2 - a*e^2)^3*(d + e*x)*Sqrt[(a*e + c*d*x)*(d + e*x)])
```

Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 121, normalized size of antiderivative = 0.73, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.054$, Rules used = {1129, 1088}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(d+ex)\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}dx$$

$$\Big\downarrow \text{1129}$$

$$\frac{4cd\int \frac{1}{\left(cdex^2+\left(cd^2+ae^2\right)x+ade\right)^{3/2}}dx}{3\left(cd^2-ae^2\right)}+\frac{2}{3(d+ex)\left(cd^2-ae^2\right)\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}$$

$$\Big\downarrow \text{1088}$$

$$\frac{2}{3(d+ex)\left(cd^2-ae^2\right)\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}-\frac{8cd\left(ae^2+cd^2+2cdex\right)}{3\left(cd^2-ae^2\right)^3\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}$$

input
```
Int[1/((d + e*x)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2)),x]
```

output
```
2/(3*(c*d^2 - a*e^2)*(d + e*x)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]
) - (8*c*d*(c*d^2 + a*e^2 + 2*c*d*e*x))/(3*(c*d^2 - a*e^2)^3*Sqrt[a*d*e +
(c*d^2 + a*e^2)*x + c*d*e*x^2])
```

Defintions of rubi rules used

rule 1088
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-3/2), x_Symbol] :> Simp[-2*((b + 
2*c*x)/((b^2 - 4*a*c)*Sqrt[a + b*x + c*x^2])), x] /; FreeQ[{a, b, c}, x] && 
 NeQ[b^2 - 4*a*c, 0]
```

rule 1129
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(-e)*(d + e*x)^m*((a + b*x + c*x^2)^(p + 1)/((m + p + 1)*(2*
c*d - b*e))), x] + Simp[c*(Simplify[m + 2*p + 2]/((m + p + 1)*(2*c*d - b*e)
))   Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d
, e, m, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && ILtQ[Simplify[m + 2*p + 
2], 0]
```

Maple [A] (verified)

Time = 1.17 (sec) , antiderivative size = 138, normalized size of antiderivative = 0.84

method	result	size
gosper	$-\frac{2(cdx+ae)\left(-8x^{2}c^{2}d^{2}e^{2}-4xacd\,e^{3}-12x\,c^{2}d^{3}e+a^{2}e^{4}-6ac\,d^{2}e^{2}-3c^{2}d^{4}\right)}{3(e^{6}a^{3}-3d^{2}e^{4}a^{2}c+3d^{4}e^{2}a\,c^{2}-d^{6}c^{3})(cd\,x^{2}e+a\,e^{2}x+c\,d^{2}x+ade)^{\frac{3}{2}}}$	138
orering	$-\frac{2\left(-8x^{2}c^{2}d^{2}e^{2}-4xacd\,e^{3}-12x\,c^{2}d^{3}e+a^{2}e^{4}-6ac\,d^{2}e^{2}-3c^{2}d^{4}\right)(cdx+ae)}{3(e^{6}a^{3}-3d^{2}e^{4}a^{2}c+3d^{4}e^{2}a\,c^{2}-d^{6}c^{3})(ade+(a\,e^{2}+c\,d^{2})x+cd\,x^{2}e)^{\frac{3}{2}}}$	139
default	$\frac{-\frac{2}{3\left(a\,e^{2}-c\,d^{2}\right)\left(x+\frac{d}{e}\right)\sqrt{dec\left(x+\frac{d}{e}\right)^{2}+\left(a\,e^{2}-c\,d^{2}\right)\left(x+\frac{d}{e}\right)}}+\frac{8dec\left(2dec\left(x+\frac{d}{e}\right)+a\,e^{2}-c\,d^{2}\right)}{3\left(a\,e^{2}-c\,d^{2}\right)^{3}\sqrt{dec\left(x+\frac{d}{e}\right)^{2}+\left(a\,e^{2}-c\,d^{2}\right)\left(x+\frac{d}{e}\right)}}}{e}$	146
trager	$-\frac{2\left(-8x^{2}c^{2}d^{2}e^{2}-4xacd\,e^{3}-12x\,c^{2}d^{3}e+a^{2}e^{4}-6ac\,d^{2}e^{2}-3c^{2}d^{4}\right)\sqrt{cd\,x^{2}e+a\,e^{2}x+c\,d^{2}x+ade}}{3(a^{2}e^{4}-2ac\,d^{2}e^{2}+c^{2}d^{4})(ex+d)^{2}(a\,e^{2}-c\,d^{2})(cdx+ae)}$	146

input
```
int(1/(e*x+d)/(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(3/2),x,method=_RETURNVERB
OSE)
```

output
```
-2/3*(c*d*x+a*e)*(-8*c^2*d^2*e^2*x^2-4*a*c*d*e^3*x-12*c^2*d^3*e*x+a^2*e^4-
6*a*c*d^2*e^2-3*c^2*d^4)/(a^3*e^6-3*a^2*c*d^2*e^4+3*a*c^2*d^4*e^2-c^3*d^6)
/(c*d*e*x^2+a*e^2*x+c*d^2*x+a*d*e)^(3/2)
```

Fricas [A] (verification not implemented)

Time = 1.26 (sec) , antiderivative size = 306, normalized size of antiderivative = 1.85

$$\int \frac{1}{(d+ex)\left(ade+(cd^2+ae^2)x+cdex^2\right)^{3/2}}\,dx =$$

$$-\frac{2\left(8c^2d^2e^2x^2+3c^2d^4+6acd^2e^2-a^2e^4+4(3c^2d\right.}{3\left(ac^3d^8e-3a^2c^2d^6e^3+3a^3cd^4e^5-a^4d^2e^7+(c^4d^7e^2-3ac^3d^5e^4+3a^2c^2d^3e^6-a^3cde^8)x^3+(2c^4d^8e-\right.}$$

input
```
integrate(1/(e*x+d)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),x, algorithm="
fricas")
```

output
```
-2/3*(8*c^2*d^2*e^2*x^2 + 3*c^2*d^4 + 6*a*c*d^2*e^2 - a^2*e^4 + 4*(3*c^2*d
^3*e + a*c*d*e^3)*x)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)/(a*c^3*d^
8*e - 3*a^2*c^2*d^6*e^3 + 3*a^3*c*d^4*e^5 - a^4*d^2*e^7 + (c^4*d^7*e^2 - 3
*a*c^3*d^5*e^4 + 3*a^2*c^2*d^3*e^6 - a^3*c*d*e^8)*x^3 + (2*c^4*d^8*e - 5*a
*c^3*d^6*e^3 + 3*a^2*c^2*d^4*e^5 + a^3*c*d^2*e^7 - a^4*e^9)*x^2 + (c^4*d^9
 - a*c^3*d^7*e^2 - 3*a^2*c^2*d^5*e^4 + 5*a^3*c*d^3*e^6 - 2*a^4*d*e^8)*x)
```

Sympy [F]

$$\int \frac{1}{(d+ex)\left(ade+(cd^2+ae^2)x+cdex^2\right)^{3/2}}\,dx = \int \frac{1}{\left((d+ex)(ae+cdx)\right)^{\frac{3}{2}}(d+ex)}\,dx$$

input
```
integrate(1/(e*x+d)/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(3/2),x)
```

output
```
Integral(1/(((d + e*x)*(a*e + c*d*x))**(3/2)*(d + e*x)), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{(d+ex)\left(ade+(cd^2+ae^2)\,x+cdex^2\right)^{3/2}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(1/(e*x+d)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),x, algorithm="
maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e*(a*e^2-c*d^2)>0)', see `assume
?` for mor
```

Giac [F]

$$\int \frac{1}{(d+ex)\left(ade+(cd^2+ae^2)\,x+cdex^2\right)^{3/2}}\,dx = \int \frac{1}{(cdex^2+ade+(cd^2+ae^2)x)^{\frac{3}{2}}(ex+d)}\,dx$$

input
```
integrate(1/(e*x+d)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),x, algorithm="
giac")
```

output
```
integrate(1/((c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^(3/2)*(e*x + d)), x)
```

Mupad [B] (verification not implemented)

Time = 5.59 (sec) , antiderivative size = 120, normalized size of antiderivative = 0.73

$$\int \frac{1}{(d+ex)\left(ade+(cd^2+ae^2)\,x+cdex^2\right)^{3/2}}\,dx = \frac{2\sqrt{c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e}\,\left(-a^2\,e^4+6\,a\,c\,d^2\,e^2}{3\,\left(a\,e+c\,d\,x\right)\,\left(a\,e^2-\right.}$$

input
```
int(1/((d + e*x)*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(3/2)),x)
```

output
```
(2*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2)*(3*c^2*d^4 - a^2*e^4 + 8*
c^2*d^2*e^2*x^2 + 6*a*c*d^2*e^2 + 12*c^2*d^3*e*x + 4*a*c*d*e^3*x))/(3*(a*e
 + c*d*x)*(a*e^2 - c*d^2)^3*(d + e*x)^2)
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 323, normalized size of antiderivative = 1.96

$$\int \frac{1}{(d+ex)\left(ade+(cd^2+ae^2)x+cdex^2\right)^{3/2}}\,dx = \frac{-\frac{16\sqrt{e}\,\sqrt{d}\,\sqrt{c}\,\sqrt{cdx+ae}\,c\,d^3}{3}-\frac{32\sqrt{e}\,\sqrt{d}\,\sqrt{c}\,\sqrt{cdx+ae}\,c\,d^2ex}{3}-\frac{16\sqrt{e}\,\sqrt{d}}{}}{\sqrt{cdx+ae}\,\left(a^3e^8x^2-3a^2c\,d^2e^6x^2+3a\,c^2d^4\right.}$$

input
```
int(1/(e*x+d)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),x)
```

output
```
(2*( - 8*sqrt(e)*sqrt(d)*sqrt(c)*sqrt(a*e + c*d*x)*c*d**3 - 16*sqrt(e)*sqr
t(d)*sqrt(c)*sqrt(a*e + c*d*x)*c*d**2*e*x - 8*sqrt(e)*sqrt(d)*sqrt(c)*sqrt
(a*e + c*d*x)*c*d*e**2*x**2 - sqrt(d + e*x)*a**2*e**4 + 6*sqrt(d + e*x)*a*
c*d**2*e**2 + 4*sqrt(d + e*x)*a*c*d*e**3*x + 3*sqrt(d + e*x)*c**2*d**4 + 1
2*sqrt(d + e*x)*c**2*d**3*e*x + 8*sqrt(d + e*x)*c**2*d**2*e**2*x**2))/(3*s
qrt(a*e + c*d*x)*(a**3*d**2*e**6 + 2*a**3*d*e**7*x + a**3*e**8*x**2 - 3*a*
*2*c*d**4*e**4 - 6*a**2*c*d**3*e**5*x - 3*a**2*c*d**2*e**6*x**2 + 3*a*c**2
*d**6*e**2 + 6*a*c**2*d**5*e**3*x + 3*a*c**2*d**4*e**4*x**2 - c**3*d**8 -
2*c**3*d**7*e*x - c**3*d**6*e**2*x**2))
```

3.263 $\int \frac{1}{(d+ex)^2 \left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}\,dx$

Optimal result

Integrand size = 37, antiderivative size = 226

$$\int \frac{1}{(d+ex)^2 \left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}\,dx = -\frac{2}{\left(cd^2-ae^2\right)(d+ex)^2\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}} - \frac{12e\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{5\left(cd^2-ae^2\right)^2(d+ex)^3} - \frac{16cde\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{5\left(cd^2-ae^2\right)^3(d+ex)^2} - \frac{32c^2d^2e\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{5\left(cd^2-ae^2\right)^4(d+ex)}$$

output
```
-2/(-a*e^2+c*d^2)/(e*x+d)^2/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)-12/5*e
*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(-a*e^2+c*d^2)^2/(e*x+d)^3-16/5*c
*d*e*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(-a*e^2+c*d^2)^3/(e*x+d)^2-32
/5*c^2*d^2*e*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(-a*e^2+c*d^2)^4/(e*x
+d)
```

Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 136, normalized size of antiderivative = 0.60

$$\int \frac{1}{(d+ex)^2 (ade+(cd^2+ae^2)x+cdex^2)^{3/2}} dx = -\frac{2(a^3e^6-a^2cde^4(5d+2ex)+ac^2d^2e^2(15d^2+20dex+8e^2x^2)+c^3d^3(5d^3+30d^2ex+40de^2x^2+16e^3x^3))}{5(cd^2-ae^2)^4 (d+ex)^2\sqrt{(ae+cdx)(d+ex)}}$$

input
```
Integrate[1/((d + e*x)^2*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2)),x]
```

output
```
(-2*(a^3*e^6 - a^2*c*d*e^4*(5*d + 2*e*x) + a*c^2*d^2*e^2*(15*d^2 + 20*d*e*
x + 8*e^2*x^2) + c^3*d^3*(5*d^3 + 30*d^2*e*x + 40*d*e^2*x^2 + 16*e^3*x^3))
)/(5*(c*d^2 - a*e^2)^4*(d + e*x)^2*Sqrt[(a*e + c*d*x)*(d + e*x)])
```

Rubi [A] (verified)

Time = 0.51 (sec) , antiderivative size = 196, normalized size of antiderivative = 0.87, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.081$, Rules used = {1129, 1129, 1088}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(d+ex)^2 (x(ae^2+cd^2)+ade+cdex^2)^{3/2}} dx$$

$$\Big\downarrow \text{ 1129}$$

$$\frac{6cd\int \frac{1}{(d+ex)(cdex^2+(cd^2+ae^2)x+ade)^{3/2}}dx}{5(cd^2-ae^2)}+\frac{2}{5(d+ex)^2(cd^2-ae^2)\sqrt{x(ae^2+cd^2)+ade+cdex^2}}$$

$$\Big\downarrow \text{ 1129}$$

$$\frac{6cd\left(\frac{4cd\int \frac{1}{\left(cdex^2+\left(cd^2+ae^2\right)x+ade\right)^{3/2}}dx}{3(cd^2-ae^2)}+\frac{2}{3(d+ex)(cd^2-ae^2)\sqrt{x(ae^2+cd^2)+ade+cdex^2}}\right)}{5\left(cd^2-ae^2\right)}+\frac{2}{5(d+ex)^2\left(cd^2-ae^2\right)\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}$$

$$\Big\downarrow \text{1088}$$

$$\frac{6cd\left(\frac{2}{3(d+ex)(cd^2-ae^2)\sqrt{x(ae^2+cd^2)+ade+cdex^2}}-\frac{8cd(ae^2+cd^2+2cdex)}{3(cd^2-ae^2)^3\sqrt{x(ae^2+cd^2)+ade+cdex^2}}\right)}{5\left(cd^2-ae^2\right)}+\frac{2}{5(d+ex)^2\left(cd^2-ae^2\right)\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}$$

input
```
Int[1/((d + e*x)^2*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2)),x]
```

output
```
2/(5*(c*d^2 - a*e^2)*(d + e*x)^2*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^
2]) + (6*c*d*(2/(3*(c*d^2 - a*e^2)*(d + e*x)*Sqrt[a*d*e + (c*d^2 + a*e^2)*
x + c*d*e*x^2]) - (8*c*d*(c*d^2 + a*e^2 + 2*c*d*e*x))/(3*(c*d^2 - a*e^2)^3
*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])))/(5*(c*d^2 - a*e^2))
```

Defintions of rubi rules used

rule 1088
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-3/2), x_Symbol] :> Simp[-2*((b +
2*c*x)/((b^2 - 4*a*c)*Sqrt[a + b*x + c*x^2])), x] /; FreeQ[{a, b, c}, x] &&
 NeQ[b^2 - 4*a*c, 0]
```

rule 1129
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(-e)*(d + e*x)^m*((a + b*x + c*x^2)^(p + 1)/((m + p + 1)*(2*
c*d - b*e))), x] + Simp[c*(Simplify[m + 2*p + 2]/((m + p + 1)*(2*c*d - b*e)
))   Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d
, e, m, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && ILtQ[Simplify[m + 2*p +
2], 0]
```

Maple [A] (verified)

Time = 1.24 (sec) , antiderivative size = 216, normalized size of antiderivative = 0.96

method	result
gosper	$-\frac{2(cdx+ae)(16c^3d^3e^3x^3+8x^2ac^2d^2e^4+40c^3d^4e^2x^2-2xa^2cde^5+20xac^2d^3e^3+30c^3d^5ex+e^6a^3-5d^2e^4a^2c+15d^4e^2ac^2+5d^6c^3)}{5(ex+d)(a^4e^8-4a^3cd^2e^6+6a^2c^2d^4e^4-4ac^3d^6e^2+c^4d^8)(cdx^2e+ae^2x+cd^2x+ade)^{\frac{3}{2}}}$
orering	$-\frac{2(16c^3d^3e^3x^3+8x^2ac^2d^2e^4+40c^3d^4e^2x^2-2xa^2cde^5+20xac^2d^3e^3+30c^3d^5ex+e^6a^3-5d^2e^4a^2c+15d^4e^2ac^2+5d^6c^3)(cdx+ae)}{5(a^4e^8-4a^3cd^2e^6+6a^2c^2d^4e^4-4ac^3d^6e^2+c^4d^8)(ex+d)(ade+(ae^2+cd^2)x+cdx^2e)^{\frac{3}{2}}}$
trager	$-\frac{2(16c^3d^3e^3x^3+8x^2ac^2d^2e^4+40c^3d^4e^2x^2-2xa^2cde^5+20xac^2d^3e^3+30c^3d^5ex+e^6a^3-5d^2e^4a^2c+15d^4e^2ac^2+5d^6c^3)\sqrt{cdx^2e+}}{5(cdx+ae)(ae^2-cd^2)(e^6a^3-3d^2e^4a^2c+3d^4e^2ac^2-d^6c^3)(ex+d)^3}$
default	$\frac{-\frac{2}{5(ae^2-cd^2)(x+\frac{d}{e})^2\sqrt{dec(x+\frac{d}{e})^2+(ae^2-cd^2)(x+\frac{d}{e})}}-\frac{6dec\left(-\frac{2}{3(ae^2-cd^2)(x+\frac{d}{e})\sqrt{dec(x+\frac{d}{e})^2+(ae^2-cd^2)(x+\frac{d}{e})}}+\frac{8dec}{3(ae^2-cd^2)^3}\right.}{5(ae^2-cd^2)}}{e^2}$

input
```
int(1/(e*x+d)^2/(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(3/2),x,method=_RETURNVE
RBOSE)
```

output
```
-2/5*(c*d*x+a*e)*(16*c^3*d^3*e^3*x^3+8*a*c^2*d^2*e^4*x^2+40*c^3*d^4*e^2*x^
2-2*a^2*c*d*e^5*x+20*a*c^2*d^3*e^3*x+30*c^3*d^5*e*x+a^3*e^6-5*a^2*c*d^2*e^
4+15*a*c^2*d^4*e^2+5*c^3*d^6)/(e*x+d)/(a^4*e^8-4*a^3*c*d^2*e^6+6*a^2*c^2*d
^4*e^4-4*a*c^3*d^6*e^2+c^4*d^8)/(c*d*e*x^2+a*e^2*x+c*d^2*x+a*d*e)^(3/2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 494 vs. 2(212) = 424.

Time = 3.58 (sec) , antiderivative size = 494, normalized size of antiderivative = 2.19

$$\int \frac{1}{(d+ex)^2\,(ade+(cd^2+ae^2)\,x+cdex^2)^{3/2}}\,dx =$$

$$-\frac{2\,(16\,c^3 d}{5\,(ac^4d^{11}e-4\,a^2c^3d^9e^3+6\,a^3c^2d^7e^5-4\,a^4cd^5e^7+a^5d^3e^9+(c^5d^9e^3-4\,ac^4d^7e^5+6\,a^2c^3d^5e^7-4\,a^3c^2d^3e}$$

input
```
integrate(1/(e*x+d)^2/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),x, algorithm
="fricas")
```

output
```
-2/5*(16*c^3*d^3*e^3*x^3 + 5*c^3*d^6 + 15*a*c^2*d^4*e^2 - 5*a^2*c*d^2*e^4
+ a^3*e^6 + 8*(5*c^3*d^4*e^2 + a*c^2*d^2*e^4)*x^2 + 2*(15*c^3*d^5*e + 10*a
*c^2*d^3*e^3 - a^2*c*d*e^5)*x)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)
/(a*c^4*d^11*e - 4*a^2*c^3*d^9*e^3 + 6*a^3*c^2*d^7*e^5 - 4*a^4*c*d^5*e^7 +
 a^5*d^3*e^9 + (c^5*d^9*e^3 - 4*a*c^4*d^7*e^5 + 6*a^2*c^3*d^5*e^7 - 4*a^3*
c^2*d^3*e^9 + a^4*c*d*e^11)*x^4 + (3*c^5*d^10*e^2 - 11*a*c^4*d^8*e^4 + 14*
a^2*c^3*d^6*e^6 - 6*a^3*c^2*d^4*e^8 - a^4*c*d^2*e^10 + a^5*e^12)*x^3 + 3*(
c^5*d^11*e - 3*a*c^4*d^9*e^3 + 2*a^2*c^3*d^7*e^5 + 2*a^3*c^2*d^5*e^7 - 3*a
^4*c*d^3*e^9 + a^5*d*e^11)*x^2 + (c^5*d^12 - a*c^4*d^10*e^2 - 6*a^2*c^3*d^
8*e^4 + 14*a^3*c^2*d^6*e^6 - 11*a^4*c*d^4*e^8 + 3*a^5*d^2*e^10)*x)
```

Sympy [F]

$$\int \frac{1}{(d+ex)^2\left(ade+(cd^2+ae^2)x+cdex^2\right)^{3/2}}\,dx=\int \frac{1}{\left((d+ex)(ae+cdx)\right)^{\frac{3}{2}}(d+ex)^2}\,dx$$

input
```
integrate(1/(e*x+d)**2/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(3/2),x)
```

output
```
Integral(1/(((d + e*x)*(a*e + c*d*x))**(3/2)*(d + e*x)**2), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{(d+ex)^2\left(ade+(cd^2+ae^2)x+cdex^2\right)^{3/2}}\,dx=\text{Exception raised: ValueError}$$

input
```
integrate(1/(e*x+d)^2/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),x, algorithm
="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e*(a*e^2-c*d^2)>0)', see `assume
?` for mor
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 3930 vs. $2(212) = 424$.

Time = 0.29 (sec) , antiderivative size = 3930, normalized size of antiderivative = 17.39

$$\int \frac{1}{(d+ex)^2 (ade+(cd^2+ae^2) x+cdex^2)^{3/2}} \, dx = \text{Too large to display}$$

input
```
integrate(1/(e*x+d)^2/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),x, algorithm
="giac")
```

output
```
2/5*(16*c^3*d^3*e*abs(e)*sgn(1/(e*x + d))*sgn(e)/(sqrt(c*d*e)*c^4*d^8 - 4*
sqrt(c*d*e)*a*c^3*d^6*e^2 + 6*sqrt(c*d*e)*a^2*c^2*d^4*e^4 - 4*sqrt(c*d*e)*
a^3*c*d^2*e^6 + sqrt(c*d*e)*a^4*e^8) - (5*c^3*d^3/((c^4*d^8*e^2*sgn(1/(e*x
 + d))*sgn(e) - 4*a*c^3*d^6*e^4*sgn(1/(e*x + d))*sgn(e) + 6*a^2*c^2*d^4*e^
6*sgn(1/(e*x + d))*sgn(e) - 4*a^3*c*d^2*e^8*sgn(1/(e*x + d))*sgn(e) + a^4*
e^10*sgn(1/(e*x + d))*sgn(e))*sqrt(c*d*e - c*d^2*e/(e*x + d) + a*e^3/(e*x
+ d))) + (15*sqrt(c*d*e - c*d^2*e/(e*x + d) + a*e^3/(e*x + d))*c^18*d^34*e
^22*sgn(1/(e*x + d))^4*sgn(e)^4 - 240*sqrt(c*d*e - c*d^2*e/(e*x + d) + a*e
^3/(e*x + d))*a*c^17*d^32*e^24*sgn(1/(e*x + d))^4*sgn(e)^4 + 1800*sqrt(c*d
*e - c*d^2*e/(e*x + d) + a*e^3/(e*x + d))*a^2*c^16*d^30*e^26*sgn(1/(e*x +
d))^4*sgn(e)^4 - 8400*sqrt(c*d*e - c*d^2*e/(e*x + d) + a*e^3/(e*x + d))*a^
3*c^15*d^28*e^28*sgn(1/(e*x + d))^4*sgn(e)^4 + 27300*sqrt(c*d*e - c*d^2*e/
(e*x + d) + a*e^3/(e*x + d))*a^4*c^14*d^26*e^30*sgn(1/(e*x + d))^4*sgn(e)^
4 - 65520*sqrt(c*d*e - c*d^2*e/(e*x + d) + a*e^3/(e*x + d))*a^5*c^13*d^24*
e^32*sgn(1/(e*x + d))^4*sgn(e)^4 + 120120*sqrt(c*d*e - c*d^2*e/(e*x + d) +
 a*e^3/(e*x + d))*a^6*c^12*d^22*e^34*sgn(1/(e*x + d))^4*sgn(e)^4 - 171600*
sqrt(c*d*e - c*d^2*e/(e*x + d) + a*e^3/(e*x + d))*a^7*c^11*d^20*e^36*sgn(1
/(e*x + d))^4*sgn(e)^4 + 193050*sqrt(c*d*e - c*d^2*e/(e*x + d) + a*e^3/(e*
x + d))*a^8*c^10*d^18*e^38*sgn(1/(e*x + d))^4*sgn(e)^4 - 171600*sqrt(c*d*e
 - c*d^2*e/(e*x + d) + a*e^3/(e*x + d))*a^9*c^9*d^16*e^40*sgn(1/(e*x + ...
```

Mupad [B] (verification not implemented)

Time = 5.71 (sec) , antiderivative size = 1005, normalized size of antiderivative = 4.45

$$\int \frac{1}{(d+ex)^2 (ade+(cd^2+ae^2)x+cdex^2)^{3/2}} \, dx = \text{Too large to display}$$

input
```
int(1/((d + e*x)^2*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(3/2)),x)
```

output
```
(((16*c^3*d^4*e)/(15*(a*e^2 - c*d^2)^5) - (8*c^2*d^2*e*(a*e^2 + c*d^2))/(1
5*(a*e^2 - c*d^2)^5))*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2))/(d +
e*x) - (((e^2*(10*c^2*d^3 - 18*a*c*d*e^2))/(5*(a*e^2 - c*d^2)^2*(3*a^2*e^5
 + 3*c^2*d^4*e - 6*a*c*d^2*e^3)) + (8*c^2*d^3*e^2)/(5*(a*e^2 - c*d^2)^2*(3
*a^2*e^5 + 3*c^2*d^4*e - 6*a*c*d^2*e^3)))*(x*(a*e^2 + c*d^2) + a*d*e + c*d
*e*x^2)^(1/2))/(d + e*x)^2 - (2*e^2*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2
)^(1/2))/((d + e*x)^3*(5*a^2*e^5 + 5*c^2*d^4*e - 10*a*c*d^2*e^3)) - ((x*((
32*a*c^5*d^6*e^4)/(15*(a*e^2 - c*d^2)^4*(c^3*d^5*e - 2*a*c^2*d^3*e^3 + a^2
*c*d*e^5)) - ((a*e^2 + c*d^2)*((16*c^5*d^5*e^3*(a*e^2 + c*d^2))/(15*(a*e^2
 - c*d^2)^4*(c^3*d^5*e - 2*a*c^2*d^3*e^3 + a^2*c*d*e^5)) + (8*c^5*d^5*e^3*
(3*a*e^2 - 11*c*d^2))/(15*(a*e^2 - c*d^2)^4*(c^3*d^5*e - 2*a*c^2*d^3*e^3 +
 a^2*c*d*e^5))))/(c*d*e) + (2*c^2*d^2*e^2*(30*c^4*d^6 - 104*a*c^3*d^4*e^2
+ 58*a^2*c^2*d^2*e^4))/(15*(a*e^2 - c*d^2)^4*(c^3*d^5*e - 2*a*c^2*d^3*e^3
+ a^2*c*d*e^5)) + (4*c^4*d^4*e^2*(a*e^2 + c*d^2)*(3*a*e^2 - 11*c*d^2))/(15
*(a*e^2 - c*d^2)^4*(c^3*d^5*e - 2*a*c^2*d^3*e^3 + a^2*c*d*e^5))) - (a*((16
*c^5*d^5*e^3*(a*e^2 + c*d^2))/(15*(a*e^2 - c*d^2)^4*(c^3*d^5*e - 2*a*c^2*d
^3*e^3 + a^2*c*d*e^5)) + (8*c^5*d^5*e^3*(3*a*e^2 - 11*c*d^2))/(15*(a*e^2 -
 c*d^2)^4*(c^3*d^5*e - 2*a*c^2*d^3*e^3 + a^2*c*d*e^5))))/c + (c*d*e*(a*e^2
 + c*d^2)*(30*c^4*d^6 - 104*a*c^3*d^4*e^2 + 58*a^2*c^2*d^2*e^4))/(15*(a*e^
2 - c*d^2)^4*(c^3*d^5*e - 2*a*c^2*d^3*e^3 + a^2*c*d*e^5)))*(x*(a*e^2 + ...
```

Reduce [B] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 561, normalized size of antiderivative = 2.48

$$\int \frac{1}{(d+ex)^2 (ade+(cd^2+ae^2)x+cdex^2)^{3/2}} \, dx = \frac{\frac{32\sqrt{e}\,\sqrt{d}\,\sqrt{c}\,\sqrt{cdx+ae}\,c^2d^5}{5} + \frac{96\sqrt{e}\,\sqrt{d}\,\sqrt{c}\,\sqrt{cdx+ae}\,c^2d^4ex}{5} + \frac{96\sqrt{e}\,\sqrt{c}}{}}{\sqrt{cdx+ae}\,(a^4e^{11}x^3 - 4a^3c\,d^2e^9x^3 + 6a^2c^2d^4}$$

input
```
int(1/(e*x+d)^2/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),x)
```

output

```
(2*(16*sqrt(e)*sqrt(d)*sqrt(c)*sqrt(a*e + c*d*x)*c**2*d**5 + 48*sqrt(e)*sq
rt(d)*sqrt(c)*sqrt(a*e + c*d*x)*c**2*d**4*e*x + 48*sqrt(e)*sqrt(d)*sqrt(c)
*sqrt(a*e + c*d*x)*c**2*d**3*e**2*x**2 + 16*sqrt(e)*sqrt(d)*sqrt(c)*sqrt(a
*e + c*d*x)*c**2*d**2*e**3*x**3 - sqrt(d + e*x)*a**3*e**6 + 5*sqrt(d + e*x
)*a**2*c*d**2*e**4 + 2*sqrt(d + e*x)*a**2*c*d*e**5*x - 15*sqrt(d + e*x)*a*
c**2*d**4*e**2 - 20*sqrt(d + e*x)*a*c**2*d**3*e**3*x - 8*sqrt(d + e*x)*a*c
**2*d**2*e**4*x**2 - 5*sqrt(d + e*x)*c**3*d**6 - 30*sqrt(d + e*x)*c**3*d**
5*e*x - 40*sqrt(d + e*x)*c**3*d**4*e**2*x**2 - 16*sqrt(d + e*x)*c**3*d**3*
e**3*x**3))/(5*sqrt(a*e + c*d*x)*(a**4*d**3*e**8 + 3*a**4*d**2*e**9*x + 3*
a**4*d*e**10*x**2 + a**4*e**11*x**3 - 4*a**3*c*d**5*e**6 - 12*a**3*c*d**4*
e**7*x - 12*a**3*c*d**3*e**8*x**2 - 4*a**3*c*d**2*e**9*x**3 + 6*a**2*c**2*
d**7*e**4 + 18*a**2*c**2*d**6*e**5*x + 18*a**2*c**2*d**5*e**6*x**2 + 6*a**
2*c**2*d**4*e**7*x**3 - 4*a*c**3*d**9*e**2 - 12*a*c**3*d**8*e**3*x - 12*a*
c**3*d**7*e**4*x**2 - 4*a*c**3*d**6*e**5*x**3 + c**4*d**11 + 3*c**4*d**10*
e*x + 3*c**4*d**9*e**2*x**2 + c**4*d**8*e**3*x**3))
```

3.264 $\int \frac{1}{(d+ex)^3\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}\,dx$

Optimal result

Integrand size = 37, antiderivative size = 287

$$\int \frac{1}{(d+ex)^3\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}\,dx = -\frac{2}{\left(cd^2-ae^2\right)(d+ex)^3\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}} - \frac{16e\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{7\left(cd^2-ae^2\right)^2(d+ex)^4} - \frac{96cde\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{35\left(cd^2-ae^2\right)^3(d+ex)^3} - \frac{128c^2d^2e\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{35\left(cd^2-ae^2\right)^4(d+ex)^2} - \frac{256c^3d^3e\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{35\left(cd^2-ae^2\right)^5(d+ex)}$$

output
```
-2/(-a*e^2+c*d^2)/(e*x+d)^3/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)-16/7*e
*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(-a*e^2+c*d^2)^2/(e*x+d)^4-96/35*
c*d*e*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(-a*e^2+c*d^2)^3/(e*x+d)^3-1
28/35*c^2*d^2*e*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(-a*e^2+c*d^2)^4/(
e*x+d)^2-256/35*c^3*d^3*e*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(-a*e^2+
c*d^2)^5/(e*x+d)
```

Mathematica [A] (verified)

Time = 0.28 (sec) , antiderivative size = 193, normalized size of antiderivative = 0.67

$$\int \frac{1}{(d+ex)^3\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}\,dx =$$
$$-\frac{2(-5a^4e^8+4a^3cde^6(7d+2ex)-2a^2c^2d^2e^4(35d^2+28dex+8e^2x^2)+4ac^3d^3e^2(35d^3+70d^2ex+56de^2x}{35\left(cd^2-ae^2\right)^5(d+ex)^3\sqrt{(ae+cdx)(}}$$

input
```
Integrate[1/((d + e*x)^3*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2)),x]
```

output
```
(-2*(-5*a^4*e^8 + 4*a^3*c*d*e^6*(7*d + 2*e*x) - 2*a^2*c^2*d^2*e^4*(35*d^2
+ 28*d*e*x + 8*e^2*x^2) + 4*a*c^3*d^3*e^2*(35*d^3 + 70*d^2*e*x + 56*d*e^2*
x^2 + 16*e^3*x^3) + c^4*d^4*(35*d^4 + 280*d^3*e*x + 560*d^2*e^2*x^2 + 448*
d*e^3*x^3 + 128*e^4*x^4)))/(35*(c*d^2 - a*e^2)^5*(d + e*x)^3*Sqrt[(a*e + c
*d*x)*(d + e*x)])
```

Rubi [A] (verified)

Time = 0.65 (sec) , antiderivative size = 271, normalized size of antiderivative = 0.94, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.108$, Rules used = {1129, 1129, 1129, 1088}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(d+ex)^3\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}dx$$

$$\Big\downarrow \text{1129}$$

$$\frac{8cd\int \frac{1}{(d+ex)^2(cdex^2+(cd^2+ae^2)x+ade)^{3/2}}dx}{7\left(cd^2-ae^2\right)}+\frac{2}{7(d+ex)^3\left(cd^2-ae^2\right)\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}$$

$$\Big\downarrow \text{1129}$$

$$\frac{8cd\left(\frac{6cd\int \frac{1}{(d+ex)\left(cdex^2+\left(cd^2+ae^2\right)x+ade\right)^{3/2}}dx}{5(cd^2-ae^2)}+\frac{2}{5(d+ex)^2(cd^2-ae^2)\sqrt{x(ae^2+cd^2)+ade+cdex^2}}\right)}{7\left(cd^2-ae^2\right)}+\frac{2}{7(d+ex)^3\left(cd^2-ae^2\right)\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}$$

$\Big\downarrow$ 1129

$$\frac{8cd\left(\frac{6cd\left(\frac{4cd\int \frac{1}{\left(cdex^2+\left(cd^2+ae^2\right)x+ade\right)^{3/2}}dx}{3\left(cd^2-ae^2\right)}+\frac{2}{3(d+ex)\left(cd^2-ae^2\right)\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}\right)}{5(cd^2-ae^2)}+\frac{2}{5(d+ex)^2(cd^2-ae^2)\sqrt{x(ae^2+cd^2)+ade+cdex^2}}\right)}{7\left(cd^2-ae^2\right)}+\frac{2}{7(d+ex)^3\left(cd^2-ae^2\right)\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}$$

$\Big\downarrow$ 1088

$$\frac{8cd\left(\frac{6cd\left(\frac{2}{3(d+ex)\left(cd^2-ae^2\right)\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}-\frac{8cd\left(ae^2+cd^2+2cdex\right)}{3\left(cd^2-ae^2\right)^3\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}\right)}{5(cd^2-ae^2)}+\frac{2}{5(d+ex)^2(cd^2-ae^2)\sqrt{x(ae^2+cd^2)+ade+cdex^2}}\right)}{7\left(cd^2-ae^2\right)}+\frac{2}{7(d+ex)^3\left(cd^2-ae^2\right)\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}$$

input
```
Int[1/((d + e*x)^3*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2)),x]
```

output
```
2/(7*(c*d^2 - a*e^2)*(d + e*x)^3*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^
2]) + (8*c*d*(2/(5*(c*d^2 - a*e^2)*(d + e*x)^2*Sqrt[a*d*e + (c*d^2 + a*e^2
)*x + c*d*e*x^2]) + (6*c*d*(2/(3*(c*d^2 - a*e^2)*(d + e*x)*Sqrt[a*d*e + (c
*d^2 + a*e^2)*x + c*d*e*x^2]) - (8*c*d*(c*d^2 + a*e^2 + 2*c*d*e*x))/(3*(c*
d^2 - a*e^2)^3*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])))/(5*(c*d^2 -
a*e^2))))/(7*(c*d^2 - a*e^2))
```

Defintions of rubi rules used

rule 1088
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-3/2), x_Symbol] :> Simp[-2*((b +
2*c*x)/((b^2 - 4*a*c)*Sqrt[a + b*x + c*x^2])), x] /; FreeQ[{a, b, c}, x] &&
 NeQ[b^2 - 4*a*c, 0]
```

rule 1129
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(-e)*(d + e*x)^m*((a + b*x + c*x^2)^(p + 1)/((m + p + 1)*(2*
c*d - b*e))), x] + Simp[c*(Simplify[m + 2*p + 2]/((m + p + 1)*(2*c*d - b*e)
))   Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d
, e, m, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && ILtQ[Simplify[m + 2*p +
2], 0]
```

Maple [A] (verified)

Time = 1.42 (sec) , antiderivative size = 307, normalized size of antiderivative = 1.07

method	result
gosper	$-\frac{2(cdx+ae)(-128c^4d^4e^4x^4-64a\,c^3d^3e^5x^3-448c^4d^5e^3x^3+16a^2c^2d^2e^6x^2-224a\,c^3d^4e^4x^2-560c^4d^6e^2x^2-8a^3cd\,e^7x+56a^2c^2d^3e^5}{35(ex+d)^2(a^5e^{10}-5a^4c\,d^2e^8+10a^3c^2d^4e^6-10a^2c^3d^6e^4+5a\,c^4d^8e^2-c^5d}$
default	$\frac{-\frac{2}{7\left(a\,e^2-c\,d^2\right)\left(x+\frac{d}{e}\right)^3\sqrt{dec\left(x+\frac{d}{e}\right)^2+\left(a\,e^2-c\,d^2\right)\left(x+\frac{d}{e}\right)}}-\frac{8dec\left(-\frac{2}{5\left(a\,e^2-c\,d^2\right)\left(x+\frac{d}{e}\right)^2\sqrt{dec\left(x+\frac{d}{e}\right)^2+\left(a\,e^2-c\,d^2\right)\left(x+\frac{d}{e}\right)}}-\frac{6dec\left(-\frac{}{3(a}\right.}{}\right.}{}}{e^3}$
trager	$-\frac{2(-128c^4d^4e^4x^4-64a\,c^3d^3e^5x^3-448c^4d^5e^3x^3+16a^2c^2d^2e^6x^2-224a\,c^3d^4e^4x^2-560c^4d^6e^2x^2-8a^3cd\,e^7x+56a^2c^2d^3e^5x-280a\,c}{35(cdx+ae)(a\,e^2-c\,d^2)(a^4e^8-4a^3c\,d^2e^6+6a^2c^2d^4e}$
orering	$-\frac{2(-128c^4d^4e^4x^4-64a\,c^3d^3e^5x^3-448c^4d^5e^3x^3+16a^2c^2d^2e^6x^2-224a\,c^3d^4e^4x^2-560c^4d^6e^2x^2-8a^3cd\,e^7x+56a^2c^2d^3e^5x-280a\,c}{35(a^5e^{10}-5a^4c\,d^2e^8+10a^3c^2d^4e^6-10a^2c^3d^6e^4+5a\,c^4d^8e^2-c^5d^{10})(ex+d}$

input
```
int(1/(e*x+d)^3/(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(3/2),x,method=_RETURNVE
RBOSE)
```

output
```
-2/35*(c*d*x+a*e)*(-128*c^4*d^4*e^4*x^4-64*a*c^3*d^3*e^5*x^3-448*c^4*d^5*e
^3*x^3+16*a^2*c^2*d^2*e^6*x^2-224*a*c^3*d^4*e^4*x^2-560*c^4*d^6*e^2*x^2-8*
a^3*c*d*e^7*x+56*a^2*c^2*d^3*e^5*x-280*a*c^3*d^5*e^3*x-280*c^4*d^7*e*x+5*a
^4*e^8-28*a^3*c*d^2*e^6+70*a^2*c^2*d^4*e^4-140*a*c^3*d^6*e^2-35*c^4*d^8)/(
e*x+d)^2/(a^5*e^10-5*a^4*c*d^2*e^8+10*a^3*c^2*d^4*e^6-10*a^2*c^3*d^6*e^4+5
*a*c^4*d^8*e^2-c^5*d^10)/(c*d*e*x^2+a*e^2*x+c*d^2*x+a*d*e)^(3/2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 733 vs. $2(269) = 538$.

Time = 9.13 (sec) , antiderivative size = 733, normalized size of antiderivative = 2.55

$$\int \frac{1}{(d+ex)^3 \left(ade+(cd^2+ae^2)x+cdex^2\right)^{3/2}}\,dx =$$

$$-\frac{}{35\left(ac^5d^{14}e-5\,a^2c^4d^{12}e^3+10\,a^3c^3d^{10}e^5-10\,a^4c^2d^8e^7+5\,a^5cd^6e^9-a^6d^4e^{11}+\left(c^6d^{11}e^4-5\,ac^5d^9e^6+10\right.\right.}$$

input
```
integrate(1/(e*x+d)^3/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),x, algorithm
="fricas")
```

output
```
-2/35*(128*c^4*d^4*e^4*x^4 + 35*c^4*d^8 + 140*a*c^3*d^6*e^2 - 70*a^2*c^2*d
^4*e^4 + 28*a^3*c*d^2*e^6 - 5*a^4*e^8 + 64*(7*c^4*d^5*e^3 + a*c^3*d^3*e^5)
*x^3 + 16*(35*c^4*d^6*e^2 + 14*a*c^3*d^4*e^4 - a^2*c^2*d^2*e^6)*x^2 + 8*(3
5*c^4*d^7*e + 35*a*c^3*d^5*e^3 - 7*a^2*c^2*d^3*e^5 + a^3*c*d*e^7)*x)*sqrt(
c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)/(a*c^5*d^14*e - 5*a^2*c^4*d^12*e^3
+ 10*a^3*c^3*d^10*e^5 - 10*a^4*c^2*d^8*e^7 + 5*a^5*c*d^6*e^9 - a^6*d^4*e^1
1 + (c^6*d^11*e^4 - 5*a*c^5*d^9*e^6 + 10*a^2*c^4*d^7*e^8 - 10*a^3*c^3*d^5*
e^10 + 5*a^4*c^2*d^3*e^12 - a^5*c*d*e^14)*x^5 + (4*c^6*d^12*e^3 - 19*a*c^5
*d^10*e^5 + 35*a^2*c^4*d^8*e^7 - 30*a^3*c^3*d^6*e^9 + 10*a^4*c^2*d^4*e^11
+ a^5*c*d^2*e^13 - a^6*e^15)*x^4 + 2*(3*c^6*d^13*e^2 - 13*a*c^5*d^11*e^4 +
 20*a^2*c^4*d^9*e^6 - 10*a^3*c^3*d^7*e^8 - 5*a^4*c^2*d^5*e^10 + 7*a^5*c*d^
3*e^12 - 2*a^6*d*e^14)*x^3 + 2*(2*c^6*d^14*e - 7*a*c^5*d^12*e^3 + 5*a^2*c^
4*d^10*e^5 + 10*a^3*c^3*d^8*e^7 - 20*a^4*c^2*d^6*e^9 + 13*a^5*c*d^4*e^11 -
 3*a^6*d^2*e^13)*x^2 + (c^6*d^15 - a*c^5*d^13*e^2 - 10*a^2*c^4*d^11*e^4 +
30*a^3*c^3*d^9*e^6 - 35*a^4*c^2*d^7*e^8 + 19*a^5*c*d^5*e^10 - 4*a^6*d^3*e^
12)*x)
```

Sympy [F]

$$\int \frac{1}{(d+ex)^3 (ade+(cd^2+ae^2)x+cdex^2)^{3/2}} dx = \int \frac{1}{((d+ex)(ae+cdx))^{\frac{3}{2}} (d+ex)^3} dx$$

input
```
integrate(1/(e*x+d)**3/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(3/2),x)
```

output
```
Integral(1/(((d + e*x)*(a*e + c*d*x))**(3/2)*(d + e*x)**3), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{(d+ex)^3 (ade+(cd^2+ae^2)x+cdex^2)^{3/2}} dx = \text{Exception raised: ValueError}$$

input
```
integrate(1/(e*x+d)^3/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),x, algorithm
="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e*(a*e^2-c*d^2)>0)', see `assume
?` for mor
```

Giac [F]

$$\int \frac{1}{(d+ex)^3 (ade+(cd^2+ae^2)x+cdex^2)^{3/2}} dx = \int \frac{1}{(cdex^2+ade+(cd^2+ae^2)x)^{\frac{3}{2}}(ex+d)^3} dx$$

input
```
integrate(1/(e*x+d)^3/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),x, algorithm
="giac")
```

output
```
integrate(1/((c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^(3/2)*(e*x + d)^3), x
)
```

Mupad [B] (verification not implemented)

Time = 6.51 (sec) , antiderivative size = 2121, normalized size of antiderivative = 7.39

$$\int \frac{1}{(d+ex)^3\left(ade+(cd^2+ae^2)x+cdex^2\right)^{3/2}}\,dx = \text{Too large to display}$$

input
```
int(1/((d + e*x)^3*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(3/2)),x)
```

output
```
(((d*((24*c^4*d^5*e^3)/(35*(a*e^2 - c*d^2)^4*(3*a^2*e^5 + 3*c^2*d^4*e - 6*
a*c*d^2*e^3)) + (4*c^3*d^3*e^3*(17*a*e^2 - 29*c*d^2))/(35*(a*e^2 - c*d^2)^
4*(3*a^2*e^5 + 3*c^2*d^4*e - 6*a*c*d^2*e^3))))/e - (e^2*(70*c^4*d^6 - 256*
a*c^3*d^4*e^2 + 162*a^2*c^2*d^2*e^4))/(35*(a*e^2 - c*d^2)^4*(3*a^2*e^5 + 3
*c^2*d^4*e - 6*a*c*d^2*e^3)))*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2
))/(d + e*x)^2 - (((e^2*(14*c^2*d^3 - 26*a*c*d*e^2))/(7*(a*e^2 - c*d^2)^2*
(5*a^2*e^5 + 5*c^2*d^4*e - 10*a*c*d^2*e^3)) + (12*c^2*d^3*e^2)/(7*(a*e^2 -
 c*d^2)^2*(5*a^2*e^5 + 5*c^2*d^4*e - 10*a*c*d^2*e^3)))*(x*(a*e^2 + c*d^2)
+ a*d*e + c*d*e*x^2)^(1/2))/(d + e*x)^3 - (((d*((16*c^5*d^6*e^2)/(35*(a*e^
2 - c*d^2)^7) + (16*c^4*d^4*e^2*(7*a*e^2 - 13*c*d^2))/(105*(a*e^2 - c*d^2)
^7)))/e + (4*c^3*d^3*e*(23*c^2*d^4 - 17*a^2*e^4 + 6*a*c*d^2*e^2))/(105*(a*
e^2 - c*d^2)^7))*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2))/(d + e*x)
+ (((24*c^3*d^4*e^2)/(35*(a*e^2 - c*d^2)^4*(3*a*e^3 - 3*c*d^2*e)) - (12*c^
2*d^2*e^2*(a*e^2 + c*d^2))/(35*(a*e^2 - c*d^2)^4*(3*a*e^3 - 3*c*d^2*e)))*(
x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2))/(d + e*x)^2 - (2*e^2*(x*(a*e
^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2))/((d + e*x)^4*(7*a^2*e^5 + 7*c^2*d^
4*e - 14*a*c*d^2*e^3)) - (((a*(((a*e^2 + c*d^2)*((16*c^7*d^7*e^4*(a*e^2 +
c*d^2))/(35*(a*e^2 - c*d^2)^6*(c^3*d^5*e - 2*a*c^2*d^3*e^3 + a^2*c*d*e^5))
 + (32*c^7*d^7*e^4*(4*a*e^2 - 13*c*d^2))/(105*(a*e^2 - c*d^2)^6*(c^3*d^5*e
 - 2*a*c^2*d^3*e^3 + a^2*c*d*e^5))))/(c*d*e) + (16*c^6*d^6*e^3*(27*a^2*...
```

Reduce [B] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 852, normalized size of antiderivative = 2.97

$$\int \frac{1}{(d+ex)^3 (ade+(cd^2+ae^2)x+cdex^2)^{3/2}}\,dx = \text{Too large to display}$$

input
```
int(1/(e*x+d)^3/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),x)
```

output
```
(2*( - 128*sqrt(e)*sqrt(d)*sqrt(c)*sqrt(a*e + c*d*x)*c**3*d**7 - 512*sqrt(
e)*sqrt(d)*sqrt(c)*sqrt(a*e + c*d*x)*c**3*d**6*e*x - 768*sqrt(e)*sqrt(d)*s
qrt(c)*sqrt(a*e + c*d*x)*c**3*d**5*e**2*x**2 - 512*sqrt(e)*sqrt(d)*sqrt(c)
*sqrt(a*e + c*d*x)*c**3*d**4*e**3*x**3 - 128*sqrt(e)*sqrt(d)*sqrt(c)*sqrt(
a*e + c*d*x)*c**3*d**3*e**4*x**4 - 5*sqrt(d + e*x)*a**4*e**8 + 28*sqrt(d +
 e*x)*a**3*c*d**2*e**6 + 8*sqrt(d + e*x)*a**3*c*d*e**7*x - 70*sqrt(d + e*x
)*a**2*c**2*d**4*e**4 - 56*sqrt(d + e*x)*a**2*c**2*d**3*e**5*x - 16*sqrt(d
 + e*x)*a**2*c**2*d**2*e**6*x**2 + 140*sqrt(d + e*x)*a*c**3*d**6*e**2 + 28
0*sqrt(d + e*x)*a*c**3*d**5*e**3*x + 224*sqrt(d + e*x)*a*c**3*d**4*e**4*x*
*2 + 64*sqrt(d + e*x)*a*c**3*d**3*e**5*x**3 + 35*sqrt(d + e*x)*c**4*d**8 +
 280*sqrt(d + e*x)*c**4*d**7*e*x + 560*sqrt(d + e*x)*c**4*d**6*e**2*x**2 +
 448*sqrt(d + e*x)*c**4*d**5*e**3*x**3 + 128*sqrt(d + e*x)*c**4*d**4*e**4*
x**4))/(35*sqrt(a*e + c*d*x)*(a**5*d**4*e**10 + 4*a**5*d**3*e**11*x + 6*a*
*5*d**2*e**12*x**2 + 4*a**5*d*e**13*x**3 + a**5*e**14*x**4 - 5*a**4*c*d**6
*e**8 - 20*a**4*c*d**5*e**9*x - 30*a**4*c*d**4*e**10*x**2 - 20*a**4*c*d**3
*e**11*x**3 - 5*a**4*c*d**2*e**12*x**4 + 10*a**3*c**2*d**8*e**6 + 40*a**3*
c**2*d**7*e**7*x + 60*a**3*c**2*d**6*e**8*x**2 + 40*a**3*c**2*d**5*e**9*x*
*3 + 10*a**3*c**2*d**4*e**10*x**4 - 10*a**2*c**3*d**10*e**4 - 40*a**2*c**3
*d**9*e**5*x - 60*a**2*c**3*d**8*e**6*x**2 - 40*a**2*c**3*d**7*e**7*x**3 -
 10*a**2*c**3*d**6*e**8*x**4 + 5*a*c**4*d**12*e**2 + 20*a*c**4*d**11*e*...
```

3.265 $\int \frac{(d+ex)^6}{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}\,dx$

Optimal result

Integrand size = 37, antiderivative size = 279

$$\int \frac{(d+ex)^6}{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}\,dx = -\frac{2(d+ex)^5}{3cd\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}} - \frac{14e(d+ex)^3}{3c^2d^2\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}} + \frac{35e^2(cd^2-ae^2)\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{4c^4d^4} + \frac{35e^2(d+ex)\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{6c^3d^3} + \frac{35e^{3/2}\left(cd^2-ae^2\right)^2\operatorname{arctanh}\left(\frac{\sqrt{c}\sqrt{d}(d+ex)}{\sqrt{e}\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\right)}{4c^{9/2}d^{9/2}}$$

output
```
-2/3*(e*x+d)^5/c/d/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)-14/3*e*(e*x+d)^
3/c^2/d^2/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)+35/4*e^2*(-a*e^2+c*d^2)*
(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/c^4/d^4+35/6*e^2*(e*x+d)*(a*d*e+(a
*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/c^3/d^3+35/4*e^(3/2)*(-a*e^2+c*d^2)^2*arcta
nh(c^(1/2)*d^(1/2)*(e*x+d)/e^(1/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)
)/c^(9/2)/d^(9/2)
```

Mathematica [A] (verified)

Time = 0.61 (sec) , antiderivative size = 230, normalized size of antiderivative = 0.82

$$\int \frac{(d+ex)^6}{(ade+(cd^2+ae^2)x+cdex^2)^{5/2}}\,dx = \frac{\sqrt{(ae+cdx)(d+ex)}\left(-\frac{\sqrt{c}\sqrt{d}(105a^3e^6-35a^2cde^4(5d-4ex)+7ac^2d^2e^2(8d^2-\cdots}{(ae+\cdots}\right.}{}$$

input
```
Integrate[(d + e*x)^6/(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2),x]
```

output
```
(Sqrt[(a*e + c*d*x)*(d + e*x)]*(-((Sqrt[c]*Sqrt[d]*(105*a^3*e^6 - 35*a^2*c
*d*e^4*(5*d - 4*e*x) + 7*a*c^2*d^2*e^2*(8*d^2 - 34*d*e*x + 3*e^2*x^2) + c^
3*d^3*(8*d^3 + 80*d^2*e*x - 39*d*e^2*x^2 - 6*e^3*x^3)))/(a*e + c*d*x)^2) +
 (105*e^(3/2)*(c*d^2 - a*e^2)^2*ArcTanh[(Sqrt[c]*Sqrt[d]*Sqrt[d + e*x])/(S
qrt[e]*Sqrt[a*e + c*d*x])])/(Sqrt[a*e + c*d*x]*Sqrt[d + e*x])))/(12*c^(9/2
)*d^(9/2))
```

Rubi [A] (verified)

Time = 0.97 (sec) , antiderivative size = 315, normalized size of antiderivative = 1.13, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.189$, Rules used = {1133, 1124, 2192, 27, 1160, 1092, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex)^6}{(x(ae^2+cd^2)+ade+cdex^2)^{5/2}}\,dx$$

$$\Big\downarrow \text{ 1133}$$

$$\frac{7e\int \frac{(d+ex)^4}{(cdex^2+(cd^2+ae^2)x+ade)^{3/2}}dx}{3cd}-\frac{2(d+ex)^5}{3cd\,(x(ae^2+cd^2)+ade+cdex^2)^{3/2}}$$

$$\Big\downarrow \text{ 1124}$$

$$\frac{7e\left(\frac{\int \frac{c^2d^2x^2e^4+cd\left(3cd^2-ae^2\right)xe^3+\left(3c^2d^4-3ace^2d^2+a^2e^4\right)e^2}{\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}dx}{c^3d^3e}-\frac{2(d+ex)\left(cd^2-ae^2\right)^2}{c^3d^3\sqrt{x(ae^2+cd^2)+ade+cdex^2}}\right)}{3cd}-\frac{2(d+ex)^5}{3cd\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}$$

$$\Big\downarrow \; 2192$$

$$\frac{7e\left(\frac{\frac{\int \frac{cde^3\left(2\left(3cd^2-2ae^2\right)\left(2cd^2-ae^2\right)+cde\left(9cd^2-7ae^2\right)x\right)}{2\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}dx}{2cde}+\frac{1}{2}cde^3x\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{c^3d^3e}-\frac{2(d+ex)\left(cd^2-ae^2\right)^2}{c^3d^3\sqrt{x(ae^2+cd^2)+ade+cdex^2}}\right)}{3cd}-\frac{2(d+ex)^5}{3cd\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}$$

$$\Big\downarrow \; 27$$

$$\frac{7e\left(\frac{\frac{1}{4}e^2\int \frac{2\left(3cd^2-2ae^2\right)\left(2cd^2-ae^2\right)+cde\left(9cd^2-7ae^2\right)x}{\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}dx+\frac{1}{2}cde^3x\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{c^3d^3e}-\frac{2(d+ex)\left(cd^2-ae^2\right)^2}{c^3d^3\sqrt{x(ae^2+cd^2)+ade+cdex^2}}\right)}{3cd}-\frac{2(d+ex)^5}{3cd\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}$$

$$\Big\downarrow \; 1160$$

$$\frac{7e\left(\frac{\frac{1}{4}e^2\left(\frac{15}{2}\left(cd^2-ae^2\right)^2\int \frac{1}{\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}dx+\left(9cd^2-7ae^2\right)\sqrt{x(ae^2+cd^2)+ade+cdex^2}\right)+\frac{1}{2}cde^3x\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{c^3d^3e}-\frac{}{c^3}\right.}{3cd}$$

$$-\frac{2(d+ex)^5}{3cd\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}$$

$$\Big\downarrow \; 1092$$

$$7e\left(\frac{\frac{1}{4}e^2\left(15\left(cd^2-ae^2\right)^2\int \frac{1}{4cde-\frac{\left(cd^2+2cexd+ae^2\right)^2}{cdex^2+\left(cd^2+ae^2\right)x+ade}}d\frac{cd^2+2cexd+ae^2}{\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}+\left(9cd^2-7ae^2\right)\sqrt{x(ae^2+cd^2)+ade+cdex^2}\right)+\frac{1}{2}cde^3x\sqrt{x(c}}{c^3d^3e}\right.$$

$$3cd$$

$$\frac{2(d+ex)^5}{3cd\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}$$

$$\Big\downarrow \; 219$$

$$7e\left(\frac{\frac{1}{4}e^2\left(\frac{15\left(cd^2-ae^2\right)^2\text{arctanh}\left(\frac{ae^2+cd^2+2cdex}{2\sqrt{c}\sqrt{d}\sqrt{e}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}\right)}{2\sqrt{c}\sqrt{d}\sqrt{e}}+\left(9cd^2-7ae^2\right)\sqrt{x(ae^2+cd^2)+ade+cdex^2}\right)+\frac{1}{2}cde^3x\sqrt{x(ae^2+cd^2)+c}}{c^3d^3e}\right.$$

$$3cd$$

$$\frac{2(d+ex)^5}{3cd\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}$$

input
```
Int[(d + e*x)^6/(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2),x]
```

output
```
(-2*(d + e*x)^5)/(3*c*d*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2)) + (
7*e*((-2*(c*d^2 - a*e^2)^2*(d + e*x))/(c^3*d^3*Sqrt[a*d*e + (c*d^2 + a*e^2
)*x + c*d*e*x^2]) + ((c*d*e^3*x*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2
])/2 + (e^2*((9*c*d^2 - 7*a*e^2)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^
2] + (15*(c*d^2 - a*e^2)^2*ArcTanh[(c*d^2 + a*e^2 + 2*c*d*e*x)/(2*Sqrt[c]*
Sqrt[d]*Sqrt[e]*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])])/(2*Sqrt[c]*
Sqrt[d]*Sqrt[e])))/4)/(c^3*d^3*e)))/(3*c*d)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1092
```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[I
nt[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a
, b, c}, x]
```

rule 1124
```
Int[((d_.) + (e_.)*(x_))^(m_.)/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(3/2), x
_Symbol] :> Simp[-2*e*(2*c*d - b*e)^(m - 2)*((d + e*x)/(c^(m - 1)*Sqrt[a +
b*x + c*x^2])), x] + Simp[e^2/c^(m - 1)   Int[(1/Sqrt[a + b*x + c*x^2])*Exp
andToSum[((2*c*d - b*e)^(m - 1) - c^(m - 1)*(d + e*x)^(m - 1))/(c*d - b*e -
 c*e*x), x], x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c*d^2 - b*d*e + a*e
^2, 0] && IGtQ[m, 0]
```

rule 1133
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[e*(d + e*x)^(m - 1)*((a + b*x + c*x^2)^(p + 1)/(c*(p + 1))),
 x] - Simp[e^2*((m + p)/(c*(p + 1)))   Int[(d + e*x)^(m - 2)*(a + b*x + c*x
^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c*d^2 - b*d*e + a*e
^2, 0] && LtQ[p, -1] && GtQ[m, 1] && IntegerQ[2*p]
```

rule 1160
```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[e*((a + b*x + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Simp[(2*c*d - b
*e)/(2*c)   Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
 && NeQ[p, -1]
```

rule 2192
```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{q =
Expon[Pq, x], e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[e*x^(q - 1)*((a + b*x +
 c*x^2)^(p + 1)/(c*(q + 2*p + 1))), x] + Simp[1/(c*(q + 2*p + 1))   Int[(a
+ b*x + c*x^2)^p*ExpandToSum[c*(q + 2*p + 1)*Pq - a*e*(q - 1)*x^(q - 2) - b
*e*(q + p)*x^(q - 1) - c*e*(q + 2*p + 1)*x^q, x], x], x]] /; FreeQ[{a, b, c
, p}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] &&  !LeQ[p, -1]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 7419 vs. $2(247) = 494$.

Time = 2.02 (sec) , antiderivative size = 7420, normalized size of antiderivative = 26.59

method	result	size
default	Expression too large to display	7420

input
```
int((e*x+d)^6/(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(5/2),x,method=_RETURNVERB
OSE)
```

output
```
result too large to display
```

Fricas [A] (verification not implemented)

Time = 1.58 (sec) , antiderivative size = 833, normalized size of antiderivative = 2.99

$$\int \frac{(d+ex)^6}{(ade+(cd^2+ae^2)\,x+cdex^2)^{5/2}}\,dx = \text{Too large to display}$$

input
```
integrate((e*x+d)^6/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2),x, algorithm="
fricas")
```

output
```
[1/48*(105*(a^2*c^2*d^4*e^3 - 2*a^3*c*d^2*e^5 + a^4*e^7 + (c^4*d^6*e - 2*a
*c^3*d^4*e^3 + a^2*c^2*d^2*e^5)*x^2 + 2*(a*c^3*d^5*e^2 - 2*a^2*c^2*d^3*e^4
 + a^3*c*d*e^6)*x)*sqrt(e/(c*d))*log(8*c^2*d^2*e^2*x^2 + c^2*d^4 + 6*a*c*d
^2*e^2 + a^2*e^4 + 8*(c^2*d^3*e + a*c*d*e^3)*x + 4*(2*c^2*d^2*e*x + c^2*d^
3 + a*c*d*e^2)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(e/(c*d)))
+ 4*(6*c^3*d^3*e^3*x^3 - 8*c^3*d^6 - 56*a*c^2*d^4*e^2 + 175*a^2*c*d^2*e^4
- 105*a^3*e^6 + 3*(13*c^3*d^4*e^2 - 7*a*c^2*d^2*e^4)*x^2 - 2*(40*c^3*d^5*e
 - 119*a*c^2*d^3*e^3 + 70*a^2*c*d*e^5)*x)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2
+ a*e^2)*x))/(c^6*d^6*x^2 + 2*a*c^5*d^5*e*x + a^2*c^4*d^4*e^2), -1/24*(105
*(a^2*c^2*d^4*e^3 - 2*a^3*c*d^2*e^5 + a^4*e^7 + (c^4*d^6*e - 2*a*c^3*d^4*e
^3 + a^2*c^2*d^2*e^5)*x^2 + 2*(a*c^3*d^5*e^2 - 2*a^2*c^2*d^3*e^4 + a^3*c*d
*e^6)*x)*sqrt(-e/(c*d))*arctan(1/2*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2
)*x)*(2*c*d*e*x + c*d^2 + a*e^2)*sqrt(-e/(c*d))/(c*d*e^2*x^2 + a*d*e^2 + (
c*d^2*e + a*e^3)*x)) - 2*(6*c^3*d^3*e^3*x^3 - 8*c^3*d^6 - 56*a*c^2*d^4*e^2
 + 175*a^2*c*d^2*e^4 - 105*a^3*e^6 + 3*(13*c^3*d^4*e^2 - 7*a*c^2*d^2*e^4)*
x^2 - 2*(40*c^3*d^5*e - 119*a*c^2*d^3*e^3 + 70*a^2*c*d*e^5)*x)*sqrt(c*d*e*
x^2 + a*d*e + (c*d^2 + a*e^2)*x))/(c^6*d^6*x^2 + 2*a*c^5*d^5*e*x + a^2*c^4
*d^4*e^2)]
```

Sympy [F]

$$\int \frac{(d+ex)^6}{(ade+(cd^2+ae^2)\,x+cdex^2)^{5/2}}\,dx=\int \frac{(d+ex)^6}{((d+ex)\,(ae+cdx))^{\frac{5}{2}}}\,dx$$

input
```
integrate((e*x+d)**6/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(5/2),x)
```

output
```
Integral((d + e*x)**6/((d + e*x)*(a*e + c*d*x))**(5/2), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(d+ex)^6}{\left(ade+(cd^2+ae^2)\,x+cdex^2\right)^{5/2}}\,dx=\text{Exception raised: ValueError}$$

input
```
integrate((e*x+d)^6/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2),x, algorithm="
maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(a*e^2-c*d^2>0)', see `assume?` f
or more de
```

Giac [F(-2)]

Exception generated.

$$\int \frac{(d+ex)^6}{\left(ade+(cd^2+ae^2)\,x+cdex^2\right)^{5/2}}\,dx=\text{Exception raised: TypeError}$$

input
```
integrate((e*x+d)^6/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2),x, algorithm="
giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Unable to divide, perhaps due to ro
unding error%%%{%%{[%%%{8,[6,6,8]%%%},0]:[1,0,%%%{-1,[1,1,1]%%%}]%%},[4,4]
%%%}+%%%{
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d+ex)^6}{(ade+(cd^2+ae^2)x+cdex^2)^{5/2}}\,dx = \int \frac{(d+e\,x)^6}{(c\,d\,e\,x^2+(c\,d^2+a\,e^2)\,x+a\,d\,e)^{5/2}}\,dx$$

input
```
int((d + e*x)^6/(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(5/2),x)
```

output
```
int((d + e*x)^6/(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(5/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 773, normalized size of antiderivative = 2.77

$$\int \frac{(d+ex)^6}{(ade+(cd^2+ae^2)x+cdex^2)^{5/2}}\,dx = \text{Too large to display}$$

input
```
int((e*x+d)^6/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2),x)
```

output

```
(840*sqrt(e)*sqrt(d)*sqrt(c)*sqrt(a*e + c*d*x)*log((sqrt(e)*sqrt(a*e + c*d
*x) + sqrt(d)*sqrt(c)*sqrt(d + e*x))/sqrt(a*e**2 - c*d**2))*a**3*e**6 - 16
80*sqrt(e)*sqrt(d)*sqrt(c)*sqrt(a*e + c*d*x)*log((sqrt(e)*sqrt(a*e + c*d*x
) + sqrt(d)*sqrt(c)*sqrt(d + e*x))/sqrt(a*e**2 - c*d**2))*a**2*c*d**2*e**4
 + 840*sqrt(e)*sqrt(d)*sqrt(c)*sqrt(a*e + c*d*x)*log((sqrt(e)*sqrt(a*e + c
*d*x) + sqrt(d)*sqrt(c)*sqrt(d + e*x))/sqrt(a*e**2 - c*d**2))*a**2*c*d*e**
5*x + 840*sqrt(e)*sqrt(d)*sqrt(c)*sqrt(a*e + c*d*x)*log((sqrt(e)*sqrt(a*e
+ c*d*x) + sqrt(d)*sqrt(c)*sqrt(d + e*x))/sqrt(a*e**2 - c*d**2))*a*c**2*d*
*4*e**2 - 1680*sqrt(e)*sqrt(d)*sqrt(c)*sqrt(a*e + c*d*x)*log((sqrt(e)*sqrt
(a*e + c*d*x) + sqrt(d)*sqrt(c)*sqrt(d + e*x))/sqrt(a*e**2 - c*d**2))*a*c*
*2*d**3*e**3*x + 840*sqrt(e)*sqrt(d)*sqrt(c)*sqrt(a*e + c*d*x)*log((sqrt(e
)*sqrt(a*e + c*d*x) + sqrt(d)*sqrt(c)*sqrt(d + e*x))/sqrt(a*e**2 - c*d**2)
)*c**3*d**5*e*x + 175*sqrt(e)*sqrt(d)*sqrt(c)*sqrt(a*e + c*d*x)*a**3*e**6
- 350*sqrt(e)*sqrt(d)*sqrt(c)*sqrt(a*e + c*d*x)*a**2*c*d**2*e**4 + 175*sqr
t(e)*sqrt(d)*sqrt(c)*sqrt(a*e + c*d*x)*a**2*c*d*e**5*x + 175*sqrt(e)*sqrt(
d)*sqrt(c)*sqrt(a*e + c*d*x)*a*c**2*d**4*e**2 - 350*sqrt(e)*sqrt(d)*sqrt(c
)*sqrt(a*e + c*d*x)*a*c**2*d**3*e**3*x + 175*sqrt(e)*sqrt(d)*sqrt(c)*sqrt(
a*e + c*d*x)*c**3*d**5*e*x - 840*sqrt(d + e*x)*a**3*c*d*e**6 + 1400*sqrt(d
 + e*x)*a**2*c**2*d**3*e**4 - 1120*sqrt(d + e*x)*a**2*c**2*d**2*e**5*x - 4
48*sqrt(d + e*x)*a*c**3*d**5*e**2 + 1904*sqrt(d + e*x)*a*c**3*d**4*e**3...
```

3.266 $\int \frac{(d+ex)^5}{(ade+(cd^2+ae^2)x+cdex^2)^{5/2}}\,dx$

Optimal result

Integrand size = 37, antiderivative size = 214

$$\int \frac{(d+ex)^5}{(ade+(cd^2+ae^2)\,x+cdex^2)^{5/2}}\,dx = -\frac{2(d+ex)^4}{3cd\,(ade+(cd^2+ae^2)\,x+cdex^2)^{3/2}} - \frac{10e(d+ex)^2}{3c^2d^2\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}} + \frac{5e^2\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{c^3d^3} + \frac{5e^{3/2}(cd^2-ae^2)\,\text{arctanh}\left(\frac{\sqrt{c}\sqrt{d}(d+ex)}{\sqrt{e}\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\right)}{c^{7/2}d^{7/2}}$$

output

```
-2/3*(e*x+d)^4/c/d/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)-10/3*e*(e*x+d)^
2/c^2/d^2/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)+5*e^2*(a*d*e+(a*e^2+c*d^
2)*x+c*d*e*x^2)^(1/2)/c^3/d^3+5*e^(3/2)*(-a*e^2+c*d^2)*arctanh(c^(1/2)*d^(
1/2)*(e*x+d)/e^(1/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2))/c^(7/2)/d^(7
/2)
```

Mathematica [A] (verified)

Time = 0.46 (sec) , antiderivative size = 189, normalized size of antiderivative = 0.88

$$\int \frac{(d+ex)^5}{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}\,dx = \frac{((ae+cdx)(d+ex))^{5/2}\left(-\frac{\sqrt{c}\sqrt{d}\left(-15a^2e^4+10acde^2(d-2ex)+c^2d^2(2d^2+14c\right.}{(ae+cdx)^4(d+ex)^2}\right.}{3c^{7/2}d^{7/2}}$$

input
```
Integrate[(d + e*x)^5/(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2),x]
```

output
```
(((a*e + c*d*x)*(d + e*x))^(5/2)*(-((Sqrt[c]*Sqrt[d]*(-15*a^2*e^4 + 10*a*c
*d*e^2*(d - 2*e*x) + c^2*d^2*(2*d^2 + 14*d*e*x - 3*e^2*x^2)))/((a*e + c*d*
x)^4*(d + e*x)^2)) + (15*e^(3/2)*(c*d^2 - a*e^2)*ArcTanh[(Sqrt[c]*Sqrt[d]*
Sqrt[d + e*x])/(Sqrt[e]*Sqrt[a*e + c*d*x])])/((a*e + c*d*x)^(5/2)*(d + e*x
)^(5/2))))/(3*c^(7/2)*d^(7/2))
```

Rubi [A] (verified)

Time = 0.67 (sec) , antiderivative size = 248, normalized size of antiderivative = 1.16, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.162$, Rules used = {1133, 1124, 27, 1160, 1092, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex)^5}{\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/2}}dx$$

$$\Big\downarrow \text{ 1133}$$

$$\frac{5e\int \frac{(d+ex)^3}{\left(cdex^2+\left(cd^2+ae^2\right)x+ade\right)^{3/2}}dx}{3cd}-\frac{2(d+ex)^4}{3cd\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}$$

$$\Big\downarrow \text{ 1124}$$

$$\frac{5e\left(\frac{\int \frac{e\left(2cd^2+cexd-ae^2\right)}{\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}dx}{c^2d^2}-\frac{2(d+ex)\left(cd^2-ae^2\right)}{c^2d^2\sqrt{x(ae^2+cd^2)+ade+cdex^2}}\right)}{3cd}-\frac{2(d+ex)^4}{3cd\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}$$

$\Big\downarrow$ 27

$$\frac{5e\left(\frac{e\int \frac{2cd^2+cexd-ae^2}{\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}dx}{c^2d^2}-\frac{2(d+ex)\left(cd^2-ae^2\right)}{c^2d^2\sqrt{x(ae^2+cd^2)+ade+cdex^2}}\right)}{3cd}-\frac{2(d+ex)^4}{3cd\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}$$

$\Big\downarrow$ 1160

$$\frac{5e\left(\frac{e\left(\frac{3}{2}\left(cd^2-ae^2\right)\int \frac{1}{\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}dx+\sqrt{x(ae^2+cd^2)+ade+cdex^2}\right)}{c^2d^2}-\frac{2(d+ex)\left(cd^2-ae^2\right)}{c^2d^2\sqrt{x(ae^2+cd^2)+ade+cdex^2}}\right)}{3cd}-\frac{2(d+ex)^4}{3cd\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}$$

$\Big\downarrow$ 1092

$$\frac{5e\left(\frac{e\left(3\left(cd^2-ae^2\right)\int \frac{1}{4cde-\frac{\left(cd^2+2cexd+ae^2\right)^2}{cdex^2+\left(cd^2+ae^2\right)x+ade}}d\frac{cd^2+2cexd+ae^2}{\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}+\sqrt{x(ae^2+cd^2)+ade+cdex^2}\right)}{c^2d^2}-\frac{2(d+ex)\left(cd^2-ae^2\right)}{c^2d^2\sqrt{x(ae^2+cd^2)+ade+cdex^2}}\right)}{3cd}-\frac{2(d+ex)^4}{3cd\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}$$

$\Big\downarrow$ 219

$$\frac{5e\left(\frac{e\left(\frac{3(cd^2-ae^2)\operatorname{arctanh}\left(\frac{ae^2+cd^2+2cdex}{2\sqrt{c}\sqrt{d}\sqrt{e}\sqrt{x(ae^2+cd^2)+ade+cdex^2}}\right)}{2\sqrt{c}\sqrt{d}\sqrt{e}}+\sqrt{x(ae^2+cd^2)+ade+cdex^2}\right)}{c^2d^2}-\frac{2(d+ex)(cd^2-ae^2)}{c^2d^2\sqrt{x(ae^2+cd^2)+ade+cdex^2}}\right)}{3cd}-\frac{2(d+ex)^4}{3cd\,(x\,(ae^2+cd^2)+ade+cdex^2)^{3/2}}$$

input
```
Int[(d + e*x)^5/(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2),x]
```

output
```
(-2*(d + e*x)^4)/(3*c*d*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2)) + (
5*e*((-2*(c*d^2 - a*e^2)*(d + e*x))/(c^2*d^2*Sqrt[a*d*e + (c*d^2 + a*e^2)*
x + c*d*e*x^2]) + (e*(Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2] + (3*(c*
d^2 - a*e^2)*ArcTanh[(c*d^2 + a*e^2 + 2*c*d*e*x)/(2*Sqrt[c]*Sqrt[d]*Sqrt[e
]*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])])/(2*Sqrt[c]*Sqrt[d]*Sqrt[e
])))/(c^2*d^2)))/(3*c*d)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1092
```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[I
nt[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a
, b, c}, x]
```

rule 1124
```
Int[((d_.) + (e_.)*(x_))^(m_.)/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(3/2), x
_Symbol] :> Simp[-2*e*(2*c*d - b*e)^(m - 2)*((d + e*x)/(c^(m - 1)*Sqrt[a +
b*x + c*x^2])), x] + Simp[e^2/c^(m - 1)   Int[(1/Sqrt[a + b*x + c*x^2])*Exp
andToSum[((2*c*d - b*e)^(m - 1) - c^(m - 1)*(d + e*x)^(m - 1))/(c*d - b*e -
 c*e*x), x], x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c*d^2 - b*d*e + a*e
^2, 0] && IGtQ[m, 0]
```

rule 1133
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[e*(d + e*x)^(m - 1)*((a + b*x + c*x^2)^(p + 1)/(c*(p + 1))),
 x] - Simp[e^2*((m + p)/(c*(p + 1)))   Int[(d + e*x)^(m - 2)*(a + b*x + c*x
^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c*d^2 - b*d*e + a*e
^2, 0] && LtQ[p, -1] && GtQ[m, 1] && IntegerQ[2*p]
```

rule 1160
```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[e*((a + b*x + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Simp[(2*c*d - b
*e)/(2*c)   Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
 && NeQ[p, -1]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 4446 vs. $2(190) = 380$.

Time = 1.75 (sec) , antiderivative size = 4447, normalized size of antiderivative = 20.78

method	result	size
default	Expression too large to display	4447

input
```
int((e*x+d)^5/(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(5/2),x,method=_RETURNVERB
OSE)
```

output
```
d^5*(2/3*(2*c*d*e*x+a*e^2+c*d^2)/(4*a*c*d^2*e^2-(a*e^2+c*d^2)^2)/(a*d*e+(a
*e^2+c*d^2)*x+c*d*x^2*e)^(3/2)+16/3*d*e*c/(4*a*c*d^2*e^2-(a*e^2+c*d^2)^2)^
2*(2*c*d*e*x+a*e^2+c*d^2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(1/2))+e^5*(x^
4/d/e/c/(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(3/2)-5/2*(a*e^2+c*d^2)/d/e/c*(-
1/3*x^3/d/e/c/(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(3/2)-1/2*(a*e^2+c*d^2)/d/
e/c*(-x^2/d/e/c/(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(3/2)+1/2*(a*e^2+c*d^2)/
d/e/c*(-1/2*x/d/e/c/(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(3/2)-1/4*(a*e^2+c*d
^2)/d/e/c*(-1/3/d/e/c/(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(3/2)-1/2*(a*e^2+c
*d^2)/d/e/c*(2/3*(2*c*d*e*x+a*e^2+c*d^2)/(4*a*c*d^2*e^2-(a*e^2+c*d^2)^2)/(
a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(3/2)+16/3*d*e*c/(4*a*c*d^2*e^2-(a*e^2+c*
d^2)^2)^2*(2*c*d*e*x+a*e^2+c*d^2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(1/2))
)+1/2*a/c*(2/3*(2*c*d*e*x+a*e^2+c*d^2)/(4*a*c*d^2*e^2-(a*e^2+c*d^2)^2)/(a*
d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(3/2)+16/3*d*e*c/(4*a*c*d^2*e^2-(a*e^2+c*d^
2)^2)^2*(2*c*d*e*x+a*e^2+c*d^2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(1/2)))+
2*a/c*(-1/3/d/e/c/(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(3/2)-1/2*(a*e^2+c*d^2
)/d/e/c*(2/3*(2*c*d*e*x+a*e^2+c*d^2)/(4*a*c*d^2*e^2-(a*e^2+c*d^2)^2)/(a*d*
e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(3/2)+16/3*d*e*c/(4*a*c*d^2*e^2-(a*e^2+c*d^2)
^2)^2*(2*c*d*e*x+a*e^2+c*d^2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(1/2))))+1
/d/e/c*(-x/d/e/c/(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(1/2)-1/2*(a*e^2+c*d^2)
/d/e/c*(-1/d/e/c/(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(1/2)-(a*e^2+c*d^2)/...
```

Fricas [A] (verification not implemented)

Time = 0.69 (sec) , antiderivative size = 641, normalized size of antiderivative = 3.00

$$\int \frac{(d+ex)^5}{\left(ade+(cd^2+ae^2)\,x+cdex^2\right)^{5/2}}\,dx = \left[-\frac{15\left(a^2cd^2e^3-a^3e^5+(c^3d^4e-ac^2d^2e^3)x^2+2\left(ac^2d^3e^2-a^2cd\right.\right.}{}\right.$$

$$-\frac{15\left(a^2cd^2e^3-a^3e^5+(c^3d^4e-ac^2d^2e^3)x^2+2\left(ac^2d^3e^2-a^2cde^4\right)x\right)\sqrt{-\frac{e}{cd}}\arctan\left(\frac{\sqrt{cdex^2+ade+(cd^2+ae^2)x}\,(2}{2\left(cde^2x^2+ade^2+(\right.}\right.}{6\left(c^5d^5x\right.}$$

input
```
integrate((e*x+d)^5/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2),x, algorithm="
fricas")
```

output
```
[-1/12*(15*(a^2*c*d^2*e^3 - a^3*e^5 + (c^3*d^4*e - a*c^2*d^2*e^3)*x^2 + 2*
(a*c^2*d^3*e^2 - a^2*c*d*e^4)*x)*sqrt(e/(c*d))*log(8*c^2*d^2*e^2*x^2 + c^2
*d^4 + 6*a*c*d^2*e^2 + a^2*e^4 + 8*(c^2*d^3*e + a*c*d*e^3)*x - 4*(2*c^2*d^
2*e*x + c^2*d^3 + a*c*d*e^2)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*s
qrt(e/(c*d))) - 4*(3*c^2*d^2*e^2*x^2 - 2*c^2*d^4 - 10*a*c*d^2*e^2 + 15*a^2
*e^4 - 2*(7*c^2*d^3*e - 10*a*c*d*e^3)*x)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 +
 a*e^2)*x))/(c^5*d^5*x^2 + 2*a*c^4*d^4*e*x + a^2*c^3*d^3*e^2), -1/6*(15*(a
^2*c*d^2*e^3 - a^3*e^5 + (c^3*d^4*e - a*c^2*d^2*e^3)*x^2 + 2*(a*c^2*d^3*e^
2 - a^2*c*d*e^4)*x)*sqrt(-e/(c*d))*arctan(1/2*sqrt(c*d*e*x^2 + a*d*e + (c*
d^2 + a*e^2)*x)*(2*c*d*e*x + c*d^2 + a*e^2)*sqrt(-e/(c*d))/(c*d*e^2*x^2 +
a*d*e^2 + (c*d^2*e + a*e^3)*x)) - 2*(3*c^2*d^2*e^2*x^2 - 2*c^2*d^4 - 10*a*
c*d^2*e^2 + 15*a^2*e^4 - 2*(7*c^2*d^3*e - 10*a*c*d*e^3)*x)*sqrt(c*d*e*x^2
+ a*d*e + (c*d^2 + a*e^2)*x))/(c^5*d^5*x^2 + 2*a*c^4*d^4*e*x + a^2*c^3*d^3
*e^2)]
```

Sympy [F]

$$\int \frac{(d+ex)^5}{(ade+(cd^2+ae^2)x+cdex^2)^{5/2}}\,dx = \int \frac{(d+ex)^5}{((d+ex)(ae+cdx))^{\frac{5}{2}}}\,dx$$

input
```
integrate((e*x+d)**5/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(5/2),x)
```

output
```
Integral((d + e*x)**5/((d + e*x)*(a*e + c*d*x))**(5/2), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(d+ex)^5}{(ade+(cd^2+ae^2)x+cdex^2)^{5/2}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((e*x+d)^5/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2),x, algorithm="
maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(a*e^2-c*d^2>0)', see `assume?` f
or more de
```

Giac [F(-2)]

Exception generated.

$$\int \frac{(d+ex)^5}{(ade+(cd^2+ae^2)x+cdex^2)^{5/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((e*x+d)^5/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2),x, algorithm="
giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Unable to divide, perhaps due to ro
unding error%%%{%%{[%%%{2,[5,5,8]%%%},0]:[1,0,%%%{-1,[1,1,1]%%%}]%%},[4,4]
%%%}+%%%{
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d+ex)^5}{(ade+(cd^2+ae^2)x+cdex^2)^{5/2}}\,dx = \int \frac{(d+e\,x)^5}{(c\,d\,e\,x^2+(c\,d^2+a\,e^2)\ x+a\,d\,e)^{5/2}}\,dx$$

input
```
int((d + e*x)^5/(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(5/2),x)
```

output
```
int((d + e*x)^5/(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(5/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 491, normalized size of antiderivative = 2.29

$$\int \frac{(d+ex)^5}{\left(ade+(cd^2+ae^2)\,x+cdex^2\right)^{5/2}}\,dx = \frac{-30\sqrt{e}\,\sqrt{d}\,\sqrt{c}\,\sqrt{cdx+ae}\log\left(\frac{\sqrt{e}\,\sqrt{cdx+ae}+\sqrt{d}\,\sqrt{c}\,\sqrt{ex+d}}{\sqrt{a\,e^2-c\,d^2}}\right)a^2e^4+30\sqrt{}}{}$$

input
```
int((e*x+d)^5/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2),x)
```

output
```
( - 30*sqrt(e)*sqrt(d)*sqrt(c)*sqrt(a*e + c*d*x)*log((sqrt(e)*sqrt(a*e + c
*d*x) + sqrt(d)*sqrt(c)*sqrt(d + e*x))/sqrt(a*e**2 - c*d**2))*a**2*e**4 +
30*sqrt(e)*sqrt(d)*sqrt(c)*sqrt(a*e + c*d*x)*log((sqrt(e)*sqrt(a*e + c*d*x
) + sqrt(d)*sqrt(c)*sqrt(d + e*x))/sqrt(a*e**2 - c*d**2))*a*c*d**2*e**2 -
30*sqrt(e)*sqrt(d)*sqrt(c)*sqrt(a*e + c*d*x)*log((sqrt(e)*sqrt(a*e + c*d*x
) + sqrt(d)*sqrt(c)*sqrt(d + e*x))/sqrt(a*e**2 - c*d**2))*a*c*d*e**3*x + 3
0*sqrt(e)*sqrt(d)*sqrt(c)*sqrt(a*e + c*d*x)*log((sqrt(e)*sqrt(a*e + c*d*x)
 + sqrt(d)*sqrt(c)*sqrt(d + e*x))/sqrt(a*e**2 - c*d**2))*c**2*d**3*e*x - 5
*sqrt(e)*sqrt(d)*sqrt(c)*sqrt(a*e + c*d*x)*a**2*e**4 + 5*sqrt(e)*sqrt(d)*s
qrt(c)*sqrt(a*e + c*d*x)*a*c*d**2*e**2 - 5*sqrt(e)*sqrt(d)*sqrt(c)*sqrt(a*
e + c*d*x)*a*c*d*e**3*x + 5*sqrt(e)*sqrt(d)*sqrt(c)*sqrt(a*e + c*d*x)*c**2
*d**3*e*x + 30*sqrt(d + e*x)*a**2*c*d*e**4 - 20*sqrt(d + e*x)*a*c**2*d**3*
e**2 + 40*sqrt(d + e*x)*a*c**2*d**2*e**3*x - 4*sqrt(d + e*x)*c**3*d**5 - 2
8*sqrt(d + e*x)*c**3*d**4*e*x + 6*sqrt(d + e*x)*c**3*d**3*e**2*x**2)/(6*sq
rt(a*e + c*d*x)*c**4*d**4*(a*e + c*d*x))
```

3.267 $\int \frac{(d+ex)^4}{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}\,dx$

Optimal result

Integrand size = 37, antiderivative size = 158

$$\int \frac{(d+ex)^4}{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}\,dx = -\frac{2(d+ex)^3}{3cd\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}} - \frac{2e(d+ex)}{c^2d^2\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}} + \frac{2e^{3/2}\operatorname{arctanh}\left(\frac{\sqrt{c}\sqrt{d}(d+ex)}{\sqrt{e}\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\right)}{c^{5/2}d^{5/2}}$$

output

```
-2/3*(e*x+d)^3/c/d/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)-2*e*(e*x+d)/c^2
/d^2/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)+2*e^(3/2)*arctanh(c^(1/2)*d^(
1/2)*(e*x+d)/e^(1/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2))/c^(5/2)/d^(5
/2)
```

Mathematica [A] (verified)

Time = 0.26 (sec) , antiderivative size = 137, normalized size of antiderivative = 0.87

$$\int \frac{(d+ex)^4}{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}\,dx = \frac{2\sqrt{(ae+cdx)(d+ex)}\left(-\frac{\sqrt{c}\sqrt{d}(3ae^2+cd(d+4ex))}{(ae+cdx)^2} + \frac{3e^{3/2}\operatorname{arctanh}\left(\frac{\sqrt{c}\sqrt{}}{\sqrt{e}\sqrt{}}\right.}{\sqrt{ae+cdx}\sqrt{d+}}\right.}{3c^{5/2}d^{5/2}}$$

input
```
Integrate[(d + e*x)^4/(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2),x]
```

output
```
(2*Sqrt[(a*e + c*d*x)*(d + e*x)]*(-((Sqrt[c]*Sqrt[d]*(3*a*e^2 + c*d*(d + 4
*e*x)))/(a*e + c*d*x)^2) + (3*e^(3/2)*ArcTanh[(Sqrt[c]*Sqrt[d]*Sqrt[d + e*
x])/(Sqrt[e]*Sqrt[a*e + c*d*x])])/(Sqrt[a*e + c*d*x]*Sqrt[d + e*x])))/(3*c
^(5/2)*d^(5/2))
```

Rubi [A] (verified)

Time = 0.50 (sec) , antiderivative size = 180, normalized size of antiderivative = 1.14, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.108$, Rules used = {1133, 1124, 1092, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex)^4}{\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/2}}dx$$

$$\Big\downarrow \text{1133}$$

$$\frac{e\int \frac{(d+ex)^2}{\left(cdex^2+\left(cd^2+ae^2\right)x+ade\right)^{3/2}}dx}{cd}-\frac{2(d+ex)^3}{3cd\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}$$

$$\Big\downarrow \text{1124}$$

$$\frac{e\left(\frac{e\int \frac{1}{\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}dx}{cd}-\frac{2(d+ex)}{cd\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}\right)}{cd}-\frac{2(d+ex)^3}{3cd\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}$$

$$\Big\downarrow \text{1092}$$

$$\frac{e\left(\frac{2e\int \frac{1}{4cde-\frac{\left(cd^2+2cexd+ae^2\right)^2}{cdex^2+\left(cd^2+ae^2\right)x+ade}}d\frac{cd^2+2cexd+ae^2}{\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}}{cd}-\frac{2(d+ex)}{cd\sqrt{x(ae^2+cd^2)+ade+cdex^2}}\right)}{cd}-\frac{2(d+ex)^3}{3cd\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}$$

$$\Big\downarrow \text{ 219}$$

$$\frac{e\left(\frac{\sqrt{e}\,\text{arctanh}\left(\frac{ae^2+cd^2+2cdex}{2\sqrt{c}\sqrt{d}\sqrt{e}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}\right)}{c^{3/2}d^{3/2}}-\frac{2(d+ex)}{cd\sqrt{x(ae^2+cd^2)+ade+cdex^2}}\right)}{cd}-\frac{2(d+ex)^3}{3cd\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}$$

input

```
Int[(d + e*x)^4/(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2),x]
```

output

```
(-2*(d + e*x)^3)/(3*c*d*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2)) + (
e*((-2*(d + e*x))/(c*d*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]) + (Sqr
t[e]*ArcTanh[(c*d^2 + a*e^2 + 2*c*d*e*x)/(2*Sqrt[c]*Sqrt[d]*Sqrt[e]*Sqrt[a
*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])])/(c^(3/2)*d^(3/2))))/(c*d)
```

Defintions of rubi rules used

rule 219

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1092

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[I
nt[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a
, b, c}, x]
```

rule 1124
```
Int[((d_.) + (e_.)*(x_))^(m_.)/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(3/2), x
_Symbol] :> Simp[-2*e*(2*c*d - b*e)^(m - 2)*((d + e*x)/(c^(m - 1)*Sqrt[a +
b*x + c*x^2])), x] + Simp[e^2/c^(m - 1)   Int[(1/Sqrt[a + b*x + c*x^2])*Exp
andToSum[((2*c*d - b*e)^(m - 1) - c^(m - 1)*(d + e*x)^(m - 1))/(c*d - b*e -
 c*e*x), x], x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c*d^2 - b*d*e + a*e
^2, 0] && IGtQ[m, 0]
```

rule 1133
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[e*(d + e*x)^(m - 1)*((a + b*x + c*x^2)^(p + 1)/(c*(p + 1))),
 x] - Simp[e^2*((m + p)/(c*(p + 1)))   Int[(d + e*x)^(m - 2)*(a + b*x + c*x
^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c*d^2 - b*d*e + a*e
^2, 0] && LtQ[p, -1] && GtQ[m, 1] && IntegerQ[2*p]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 2598 vs. $2(138) = 276$.

Time = 1.38 (sec) , antiderivative size = 2599, normalized size of antiderivative = 16.45

method	result	size
default	Expression too large to display	2599

input
```
int((e*x+d)^4/(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(5/2),x,method=_RETURNVERB
OSE)
```

output
```
d^4*(2/3*(2*c*d*e*x+a*e^2+c*d^2)/(4*a*c*d^2*e^2-(a*e^2+c*d^2)^2)/(a*d*e+(a
*e^2+c*d^2)*x+c*d*x^2*e)^(3/2)+16/3*d*e*c/(4*a*c*d^2*e^2-(a*e^2+c*d^2)^2)^
2*(2*c*d*e*x+a*e^2+c*d^2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(1/2))+e^4*(-1
/3*x^3/d/e/c/(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(3/2)-1/2*(a*e^2+c*d^2)/d/e
/c*(-x^2/d/e/c/(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(3/2)+1/2*(a*e^2+c*d^2)/d
/e/c*(-1/2*x/d/e/c/(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(3/2)-1/4*(a*e^2+c*d^
2)/d/e/c*(-1/3/d/e/c/(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(3/2)-1/2*(a*e^2+c*
d^2)/d/e/c*(2/3*(2*c*d*e*x+a*e^2+c*d^2)/(4*a*c*d^2*e^2-(a*e^2+c*d^2)^2)/(a
*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(3/2)+16/3*d*e*c/(4*a*c*d^2*e^2-(a*e^2+c*d
^2)^2)^2*(2*c*d*e*x+a*e^2+c*d^2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(1/2)))
+1/2*a/c*(2/3*(2*c*d*e*x+a*e^2+c*d^2)/(4*a*c*d^2*e^2-(a*e^2+c*d^2)^2)/(a*d
*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(3/2)+16/3*d*e*c/(4*a*c*d^2*e^2-(a*e^2+c*d^2
)^2)^2*(2*c*d*e*x+a*e^2+c*d^2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(1/2)))+2
*a/c*(-1/3/d/e/c/(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(3/2)-1/2*(a*e^2+c*d^2)
/d/e/c*(2/3*(2*c*d*e*x+a*e^2+c*d^2)/(4*a*c*d^2*e^2-(a*e^2+c*d^2)^2)/(a*d*e
+(a*e^2+c*d^2)*x+c*d*x^2*e)^(3/2)+16/3*d*e*c/(4*a*c*d^2*e^2-(a*e^2+c*d^2)^
2)^2*(2*c*d*e*x+a*e^2+c*d^2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(1/2))))+1/
d/e/c*(-x/d/e/c/(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(1/2)-1/2*(a*e^2+c*d^2)/
d/e/c*(-1/d/e/c/(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(1/2)-(a*e^2+c*d^2)/d/e/
c*(2*c*d*e*x+a*e^2+c*d^2)/(4*a*c*d^2*e^2-(a*e^2+c*d^2)^2)/(a*d*e+(a*e^2...
```

Fricas [A] (verification not implemented)

Time = 0.50 (sec) , antiderivative size = 469, normalized size of antiderivative = 2.97

$$\int \frac{(d+ex)^4}{(ade+(cd^2+ae^2)x+cdex^2)^{5/2}}\,dx = \left[\frac{3\left(c^2d^2ex^2+2\,acde^2x+a^2e^3\right)\sqrt{\frac{e}{cd}}\log\left(8\,c^2d^2e^2x^2+c^2d^4+6\,ac\right.}{}\right.$$

$$-\frac{3\left(c^2d^2ex^2+2\,acde^2x+a^2e^3\right)\sqrt{-\frac{e}{cd}}\arctan\left(\frac{\sqrt{cdex^2+ade+(cd^2+ae^2)x}\,(2\,cdex+cd^2+ae^2)\sqrt{-\frac{e}{cd}}}{2\,(cde^2x^2+ade^2+(cd^2e+ae^3)x)}\right)+2\sqrt{cdex^2+ade}}{3\left(c^4d^4x^2+2\,ac^3d^3ex+a^2c^2d^2e^2\right)}$$

input
```
integrate((e*x+d)^4/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2),x, algorithm="
fricas")
```

output
```
[1/6*(3*(c^2*d^2*e*x^2 + 2*a*c*d*e^2*x + a^2*e^3)*sqrt(e/(c*d))*log(8*c^2*
d^2*e^2*x^2 + c^2*d^4 + 6*a*c*d^2*e^2 + a^2*e^4 + 8*(c^2*d^3*e + a*c*d*e^3
)*x + 4*(2*c^2*d^2*e*x + c^2*d^3 + a*c*d*e^2)*sqrt(c*d*e*x^2 + a*d*e + (c*
d^2 + a*e^2)*x)*sqrt(e/(c*d))) - 4*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2
)*x)*(4*c*d*e*x + c*d^2 + 3*a*e^2))/(c^4*d^4*x^2 + 2*a*c^3*d^3*e*x + a^2*c
^2*d^2*e^2), -1/3*(3*(c^2*d^2*e*x^2 + 2*a*c*d*e^2*x + a^2*e^3)*sqrt(-e/(c*
d))*arctan(1/2*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(2*c*d*e*x + c*
d^2 + a*e^2)*sqrt(-e/(c*d))/(c*d*e^2*x^2 + a*d*e^2 + (c*d^2*e + a*e^3)*x))
 + 2*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(4*c*d*e*x + c*d^2 + 3*a*
e^2))/(c^4*d^4*x^2 + 2*a*c^3*d^3*e*x + a^2*c^2*d^2*e^2)]
```

Sympy [F]

$$\int \frac{(d+ex)^4}{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}\,dx=\int \frac{(d+ex)^4}{\left((d+ex)\left(ae+cdx\right)\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate((e*x+d)**4/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(5/2),x)
```

output
```
Integral((d + e*x)**4/((d + e*x)*(a*e + c*d*x))**(5/2), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(d+ex)^4}{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}\,dx=\text{Exception raised: ValueError}$$

input
```
integrate((e*x+d)^4/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2),x, algorithm="
maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(a*e^2-c*d^2>0)', see `assume?` f
or more de
```

Giac [F(-2)]

Exception generated.

$$\int \frac{(d+ex)^4}{(ade+(cd^2+ae^2)x+cdex^2)^{5/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((e*x+d)^4/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2),x, algorithm="
giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Unable to divide, perhaps due to ro
unding error%%%{%%{[%%%{1,[2,2,8]%%%},0]:[1,0,%%%{-1,[1,1,1]%%%}]%%},[4,4]
%%%}+%%%{
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d+ex)^4}{(ade+(cd^2+ae^2)x+cdex^2)^{5/2}}\,dx = \int \frac{(d+e\,x)^4}{(c\,d\,e\,x^2+(c\,d^2+a\,e^2)\,x+a\,d\,e)^{5/2}}\,dx$$

input
```
int((d + e*x)^4/(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(5/2),x)
```

output
```
int((d + e*x)^4/(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(5/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 198, normalized size of antiderivative = 1.25

$$\int \frac{(d+ex)^4}{(ade+(cd^2+ae^2)x+cdex^2)^{5/2}}\,dx = \frac{2\sqrt{e}\,\sqrt{d}\,\sqrt{c}\,\sqrt{cdx+ae}\log\left(\frac{\sqrt{e}\,\sqrt{cdx+ae}+\sqrt{d}\,\sqrt{c}\,\sqrt{ex+d}}{\sqrt{a\,e^2-c\,d^2}}\right)a\,e^2+2\sqrt{e}\,\sqrt{}$$

input
```
int((e*x+d)^4/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2),x)
```

output

```
(2*(3*sqrt(e)*sqrt(d)*sqrt(c)*sqrt(a*e + c*d*x)*log((sqrt(e)*sqrt(a*e + c*
d*x) + sqrt(d)*sqrt(c)*sqrt(d + e*x))/sqrt(a*e**2 - c*d**2))*a*e**2 + 3*sq
rt(e)*sqrt(d)*sqrt(c)*sqrt(a*e + c*d*x)*log((sqrt(e)*sqrt(a*e + c*d*x) + s
qrt(d)*sqrt(c)*sqrt(d + e*x))/sqrt(a*e**2 - c*d**2))*c*d*e*x - 3*sqrt(d +
e*x)*a*c*d*e**2 - sqrt(d + e*x)*c**2*d**3 - 4*sqrt(d + e*x)*c**2*d**2*e*x)
)/(3*sqrt(a*e + c*d*x)*c**3*d**3*(a*e + c*d*x))
```

3.268 $\int \frac{(d+ex)^3}{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}\,dx$

Optimal result

Integrand size = 37, antiderivative size = 54

$$\int \frac{(d+ex)^3}{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}\,dx = -\frac{2(d+ex)^3}{3\left(cd^2-ae^2\right)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}$$

output
```
-2/3*(e*x+d)^3/(-a*e^2+c*d^2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.85

$$\int \frac{(d+ex)^3}{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}\,dx = -\frac{2((ae+cdx)(d+ex))^{3/2}}{3\left(cd^2-ae^2\right)(ae+cdx)^3}$$

input
```
Integrate[(d + e*x)^3/(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2),x]
```

output
```
(-2*((a*e + c*d*x)*(d + e*x))^(3/2))/(3*(c*d^2 - a*e^2)*(a*e + c*d*x)^3)
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.027$, Rules used = $\{1123\}$

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex)^3}{\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/2}}dx$$

$$\Big\downarrow \text{1123}$$

$$-\frac{2(d+ex)^3}{3\left(cd^2-ae^2\right)\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}$$

input
```
Int[(d + e*x)^3/(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2),x]
```

output
```
(-2*(d + e*x)^3)/(3*(c*d^2 - a*e^2)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2
)^(3/2))
```

Defintions of rubi rules used

rule 1123
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[e*(d + e*x)^m*((a + b*x + c*x^2)^(p + 1)/((p + 1)*(2*c*d - b
*e))), x] /; FreeQ[{a, b, c, d, e, m, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2,
0] && EqQ[m + 2*p + 2, 0]
```

Maple [A] (verified)

Time = 1.25 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.07

method	result	size
gosper	$\frac{2(ex+d)^4(cdx+ae)}{3(a\,e^2-c\,d^2)(cd\,x^2e+a\,e^2x+c\,d^2x+ade)^{\frac{5}{2}}}$	58
trager	$\frac{2(ex+d)\sqrt{cd\,x^2e+a\,e^2x+c\,d^2x+ade}}{3(cdx+ae)^2(a\,e^2-c\,d^2)}$	58
orering	$\frac{2(ex+d)^4(cdx+ae)}{3(a\,e^2-c\,d^2)(ade+(a\,e^2+c\,d^2)x+cd\,x^2e)^{\frac{5}{2}}}$	59
default	Expression too large to display	1536

input
```
int((e*x+d)^3/(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(5/2),x,method=_RETURNVERB
OSE)
```

output
```
2/3*(e*x+d)^4*(c*d*x+a*e)/(a*e^2-c*d^2)/(c*d*e*x^2+a*e^2*x+c*d^2*x+a*d*e)^
(5/2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 103 vs. 2(50) = 100.

Time = 0.36 (sec) , antiderivative size = 103, normalized size of antiderivative = 1.91

$$\int \frac{(d+ex)^3}{(ade+(cd^2+ae^2)\,x+cdex^2)^{5/2}}\,dx = -\frac{2\,\sqrt{cdex^2+ade+(cd^2+ae^2)x}(ex+d)}{3\,(a^2cd^2e^2-a^3e^4+(c^3d^4-ac^2d^2e^2)x^2+2\,(ac^2d^3e-a^2cde^3)x)}$$

input
```
integrate((e*x+d)^3/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2),x, algorithm="
fricas")
```

output
```
-2/3*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(e*x + d)/(a^2*c*d^2*e^2
- a^3*e^4 + (c^3*d^4 - a*c^2*d^2*e^2)*x^2 + 2*(a*c^2*d^3*e - a^2*c*d*e^3)*
x)
```

Sympy [F]

$$\int \frac{(d+ex)^3}{(ade+(cd^2+ae^2)x+cdex^2)^{5/2}}\,dx = \int \frac{(d+ex)^3}{((d+ex)(ae+cdx))^{\frac{5}{2}}}\,dx$$

input
```
integrate((e*x+d)**3/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(5/2),x)
```

output
```
Integral((d + e*x)**3/((d + e*x)*(a*e + c*d*x))**(5/2), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(d+ex)^3}{(ade+(cd^2+ae^2)x+cdex^2)^{5/2}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((e*x+d)^3/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2),x, algorithm="
maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(a*e^2-c*d^2>0)', see `assume?` f
or more de
```

Giac [F(-2)]

Exception generated.

$$\int \frac{(d+ex)^3}{(ade+(cd^2+ae^2)x+cdex^2)^{5/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((e*x+d)^3/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2),x, algorithm="
giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Unable to divide, perhaps due to ro
unding error%%%{%%%{1,[2,2,0]%%%},[4,0]%%%}+%%%{%%{[%%%{-4,[1,1,1]%%%},0]:
[1,0,%%%{
```

Mupad [B] (verification not implemented)

Time = 5.71 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.07

$$\int \frac{(d+ex)^3}{\left(ade+(cd^2+ae^2)\,x+cdex^2\right)^{5/2}}\,dx=\frac{2\,\left(d+e\,x\right)\,\sqrt{c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e}}{3\,{\left(a\,e+c\,d\,x\right)}^2\,\left(a\,e^2-c\,d^2\right)}$$

input
```
int((d + e*x)^3/(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(5/2),x)
```

output
```
(2*(d + e*x)*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2))/(3*(a*e + c*d*
x)^2*(a*e^2 - c*d^2))
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 123, normalized size of antiderivative = 2.28

$$\int \frac{(d+ex)^3}{\left(ade+(cd^2+ae^2)\,x+cdex^2\right)^{5/2}}\,dx=\frac{\frac{2\sqrt{e}\,\sqrt{d}\,\sqrt{c}\,\sqrt{cdx+ae}\,a\,e^2}{3}+\frac{2\sqrt{e}\,\sqrt{d}\,\sqrt{c}\,\sqrt{cdx+ae}\,cdex}{3}+\frac{2\sqrt{ex+d}\,c^2d^3}{3}+\frac{2\sqrt{ex+d}}{3}}{\sqrt{cdx+ae}\,c^2d^2\left(acd\,e^2x-c^2d^3x+a^2e^3-ac\,d^2e\right)}$$

input
```
int((e*x+d)^3/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2),x)
```

output
```
(2*(sqrt(e)*sqrt(d)*sqrt(c)*sqrt(a*e + c*d*x)*a*e**2 + sqrt(e)*sqrt(d)*sqr
t(c)*sqrt(a*e + c*d*x)*c*d*e*x + sqrt(d + e*x)*c**2*d**3 + sqrt(d + e*x)*c
**2*d**2*e*x))/(3*sqrt(a*e + c*d*x)*c**2*d**2*(a**2*e**3 - a*c*d**2*e + a*
c*d*e**2*x - c**2*d**3*x))
```

3.269 $\int \frac{(d+ex)^2}{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}\,dx$

Optimal result

Integrand size = 37, antiderivative size = 108

$$\int \frac{(d+ex)^2}{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}\,dx = -\frac{2(d+ex)^2}{3\left(cd^2-ae^2\right)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}+\frac{4e(d+ex)}{3\left(cd^2-ae^2\right)^2\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}$$

output
```
-2/3*(e*x+d)^2/(-a*e^2+c*d^2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)+4/3*
e*(e*x+d)/(-a*e^2+c*d^2)^2/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.55

$$\int \frac{(d+ex)^2}{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}\,dx = -\frac{2(d+ex)^2\left(-3ae^2+cd(d-2ex)\right)}{3\left(cd^2-ae^2\right)^2((ae+cdx)(d+ex))^{3/2}}$$

input
```
Integrate[(d + e*x)^2/(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2),x]
```

output
```
(-2*(d + e*x)^2*(-3*a*e^2 + c*d*(d - 2*e*x)))/(3*(c*d^2 - a*e^2)^2*((a*e +
 c*d*x)*(d + e*x))^(3/2))
```

Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.07, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.054$, Rules used = {1126, 1088}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex)^2}{(x(ae^2+cd^2)+ade+cdex^2)^{5/2}}dx$$

$$\Big\downarrow \ 1126$$

$$-\frac{e\int \frac{1}{(cdex^2+(cd^2+ae^2)x+ade)^{3/2}}dx}{3cd}-\frac{2(d+ex)}{3cd\,(x(ae^2+cd^2)+ade+cdex^2)^{3/2}}$$

$$\Big\downarrow \ 1088$$

$$\frac{2e(ae^2+cd^2+2cdex)}{3cd\,(cd^2-ae^2)^2\sqrt{x(ae^2+cd^2)+ade+cdex^2}}-\frac{2(d+ex)}{3cd\,(x(ae^2+cd^2)+ade+cdex^2)^{3/2}}$$

input
```
Int[(d + e*x)^2/(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2),x]
```

output
```
(-2*(d + e*x))/(3*c*d*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2)) + (2*
e*(c*d^2 + a*e^2 + 2*c*d*e*x))/(3*c*d*(c*d^2 - a*e^2)^2*Sqrt[a*d*e + (c*d^
2 + a*e^2)*x + c*d*e*x^2])
```

Defintions of rubi rules used

rule 1088
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-3/2), x_Symbol] :> Simp[-2*((b +
2*c*x)/((b^2 - 4*a*c)*Sqrt[a + b*x + c*x^2])), x] /; FreeQ[{a, b, c}, x] &&
 NeQ[b^2 - 4*a*c, 0]
```

rule 1126
```
Int[((d_.) + (e_.)*(x_))^2*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symb
ol] :> Simp[e*(d + e*x)*((a + b*x + c*x^2)^(p + 1)/(c*(p + 1))), x] - Simp[
e^2*((p + 2)/(c*(p + 1)))   Int[(a + b*x + c*x^2)^(p + 1), x], x] /; FreeQ[
{a, b, c, d, e, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[p, -1]
```

Maple [A] (verified)

Time = 1.12 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.79

method	result
trager	$\frac{2(2cdxe+3a\,e^2-c\,d^2)\sqrt{cd\,x^2e+a\,e^2x+c\,d^2x+ade}}{3(a^2e^4-2ac\,d^2e^2+c^2d^4)(cdx+ae)^2}$
gosper	$\frac{2(ex+d)^3(cdx+ae)(2cdxe+3a\,e^2-c\,d^2)}{3(a^2e^4-2ac\,d^2e^2+c^2d^4)(cd\,x^2e+a\,e^2x+c\,d^2x+ade)^{\frac{5}{2}}}$
orering	$\frac{2(2cdxe+3a\,e^2-c\,d^2)(ex+d)^3(cdx+ae)}{3(a^2e^4-2ac\,d^2e^2+c^2d^4)(ade+(a\,e^2+c\,d^2)x+cd\,x^2e)^{\frac{5}{2}}}$
default	$d^2\left(\frac{\frac{4}{3}cdxe+\frac{2}{3}a\,e^2+\frac{2}{3}c\,d^2}{\left(4ac\,d^2e^2-(a\,e^2+c\,d^2)^2\right)(ade+(a\,e^2+c\,d^2)x+cd\,x^2e)^{\frac{3}{2}}}+\frac{16dec(2cdxe+a\,e^2+c\,d^2)}{3\left(4ac\,d^2e^2-(a\,e^2+c\,d^2)^2\right)^2\sqrt{ade+(a\,e^2+c\,d^2)x+cd\,x^2e}}\right)+e$

input
```
int((e*x+d)^2/(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(5/2),x,method=_RETURNVERB
OSE)
```

output
```
2/3*(2*c*d*e*x+3*a*e^2-c*d^2)/(a^2*e^4-2*a*c*d^2*e^2+c^2*d^4)/(c*d*x+a*e)^
2*(c*d*e*x^2+a*e^2*x+c*d^2*x+a*d*e)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.48 (sec) , antiderivative size = 156, normalized size of antiderivative = 1.44

$$\int \frac{(d+ex)^2}{(ade+(cd^2+ae^2)\,x+cdex^2)^{5/2}}\,dx = \frac{2\,\sqrt{cdex^2+ade+(cd^2+ae^2)x}(2\,cdex -}{3\,(a^2c^2d^4e^2-2\,a^3cd^2e^4+a^4e^6+(c^4d^6-2\,ac^3d^4e^2+a^2c^2d^2e^4)x^2}$$

input
```
integrate((e*x+d)^2/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2),x, algorithm="
fricas")
```

output
```
2/3*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(2*c*d*e*x - c*d^2 + 3*a*e
^2)/(a^2*c^2*d^4*e^2 - 2*a^3*c*d^2*e^4 + a^4*e^6 + (c^4*d^6 - 2*a*c^3*d^4*
e^2 + a^2*c^2*d^2*e^4)*x^2 + 2*(a*c^3*d^5*e - 2*a^2*c^2*d^3*e^3 + a^3*c*d*
e^5)*x)
```

Sympy [F]

$$\int \frac{(d+ex)^2}{(ade+(cd^2+ae^2)\,x+cdex^2)^{5/2}}\,dx = \int \frac{(d+ex)^2}{((d+ex)\,(ae+cdx))^{\frac{5}{2}}}\,dx$$

input
```
integrate((e*x+d)**2/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(5/2),x)
```

output
```
Integral((d + e*x)**2/((d + e*x)*(a*e + c*d*x))**(5/2), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(d+ex)^2}{(ade+(cd^2+ae^2)\,x+cdex^2)^{5/2}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((e*x+d)^2/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2),x, algorithm="
maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(a*e^2-c*d^2>0)', see `assume?` f
or more de
```

Giac [F(-2)]

Exception generated.

$$\int \frac{(d+ex)^2}{(ade+(cd^2+ae^2)\,x+cdex^2)^{5/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((e*x+d)^2/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2),x, algorithm="
giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Unable to divide, perhaps due to ro
unding error%%%{%%%{1,[2,2,0]%%%},[4,0]%%%}+%%%{%%{[%%%{-4,[1,1,1]%%%},0]:
[1,0,%%%{
```

Mupad [B] (verification not implemented)

Time = 5.57 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.67

$$\int \frac{(d+ex)^2}{(ade+(cd^2+ae^2)\,x+cdex^2)^{5/2}}\,dx = \frac{2\left(-c\,d^2+2\,c\,x\,d\,e+3\,a\,e^2\right)\,\sqrt{c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e}}{3\,{\left(a\,e+c\,d\,x\right)}^2\,{\left(a\,e^2-c\,d^2\right)}^2}$$

input
```
int((d + e*x)^2/(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(5/2),x)
```

output
```
(2*(3*a*e^2 - c*d^2 + 2*c*d*e*x)*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(
1/2))/(3*(a*e + c*d*x)^2*(a*e^2 - c*d^2)^2)
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 168, normalized size of antiderivative = 1.56

$$\int \frac{(d+ex)^2}{\left(ade+(cd^2+ae^2)\,x+cdex^2\right)^{5/2}}\,dx = \frac{-\frac{4\sqrt{e}\,\sqrt{d}\,\sqrt{c}\,\sqrt{cdx+ae}\,a\,e^2}{3} - \frac{4\sqrt{e}\,\sqrt{d}\,\sqrt{c}\,\sqrt{cdx+ae}\,cdex}{3} + 2\sqrt{ex+d}\,acd\,e^2 -}{\sqrt{cdx+ae}\,cd\,(a^2cd\,e^4x - 2a\,c^2d^3e^2x + c^3d^5x + a^3e^5 - 2a}$$

input
```
int((e*x+d)^2/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2),x)
```

output
```
(2*( - 2*sqrt(e)*sqrt(d)*sqrt(c)*sqrt(a*e + c*d*x)*a*e**2 - 2*sqrt(e)*sqrt
(d)*sqrt(c)*sqrt(a*e + c*d*x)*c*d*e*x + 3*sqrt(d + e*x)*a*c*d*e**2 - sqrt(
d + e*x)*c**2*d**3 + 2*sqrt(d + e*x)*c**2*d**2*e*x))/(3*sqrt(a*e + c*d*x)*
c*d*(a**3*e**5 - 2*a**2*c*d**2*e**3 + a**2*c*d*e**4*x + a*c**2*d**4*e - 2*
a*c**2*d**3*e**2*x + c**3*d**5*x))
```

3.270 $\int \frac{d+ex}{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 162

$$\int \frac{d+ex}{(ade+(cd^2+ae^2)\,x+cdex^2)^{5/2}}\,dx = \frac{2(d+ex)}{(cd^2-ae^2)\,(ade+(cd^2+ae^2)\,x+cdex^2)^{3/2}} - \frac{8cd(d+ex)^2}{3\,(cd^2-ae^2)^2\,(ade+(cd^2+ae^2)\,x+cdex^2)^{3/2}} + \frac{16cde(d+ex)}{3\,(cd^2-ae^2)^3\,\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}$$

output

```
2*(e*x+d)/(-a*e^2+c*d^2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)-8/3*c*d*(
e*x+d)^2/(-a*e^2+c*d^2)^2/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)+16/3*c*d
*e*(e*x+d)/(-a*e^2+c*d^2)^3/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.56

$$\int \frac{d+ex}{(ade+(cd^2+ae^2)\,x+cdex^2)^{5/2}}\,dx = \frac{2(d+ex)\,(3a^2e^4+6acde^2(d+2ex)+c^2d^2(-d^2+4dex+8e^2x^2)}{3\,(cd^2-ae^2)^3\,((ae+cdx)(d+ex))^{3/2}}$$

input
```
Integrate[(d + e*x)/(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2),x]
```

output
```
(2*(d + e*x)*(3*a^2*e^4 + 6*a*c*d*e^2*(d + 2*e*x) + c^2*d^2*(-d^2 + 4*d*e*
x + 8*e^2*x^2)))/(3*(c*d^2 - a*e^2)^3*((a*e + c*d*x)*(d + e*x))^(3/2))
```

Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 118, normalized size of antiderivative = 0.73, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.057$, Rules used = {1159, 1088}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{d+ex}{(x\,(ae^2+cd^2)+ade+cdex^2)^{5/2}}\,dx$$

$$\Big\downarrow \text{ 1159}$$

$$-\frac{4e\int \frac{1}{(cdex^2+(cd^2+ae^2)x+ade)^{3/2}}dx}{3\,(cd^2-ae^2)}-\frac{2(d+ex)}{3\,(cd^2-ae^2)\,(x\,(ae^2+cd^2)+ade+cdex^2)^{3/2}}$$

$$\Big\downarrow \text{ 1088}$$

$$\frac{8e(ae^2+cd^2+2cdex)}{3\,(cd^2-ae^2)^3\,\sqrt{x\,(ae^2+cd^2)+ade+cdex^2}}-\frac{2(d+ex)}{3\,(cd^2-ae^2)\,(x\,(ae^2+cd^2)+ade+cdex^2)^{3/2}}$$

input
```
Int[(d + e*x)/(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2),x]
```

output
```
(-2*(d + e*x))/(3*(c*d^2 - a*e^2)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^
(3/2)) + (8*e*(c*d^2 + a*e^2 + 2*c*d*e*x))/(3*(c*d^2 - a*e^2)^3*Sqrt[a*d*e
 + (c*d^2 + a*e^2)*x + c*d*e*x^2])
```

Defintions of rubi rules used

rule 1088
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-3/2), x_Symbol] :> Simp[-2*((b +
2*c*x)/((b^2 - 4*a*c)*Sqrt[a + b*x + c*x^2])), x] /; FreeQ[{a, b, c}, x] &&
 NeQ[b^2 - 4*a*c, 0]
```

rule 1159
```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[((b*d - 2*a*e + (2*c*d - b*e)*x)/((p + 1)*(b^2 - 4*a*c)))*(a + b*
x + c*x^2)^(p + 1), x] - Simp[(2*p + 3)*((2*c*d - b*e)/((p + 1)*(b^2 - 4*a*
c)))   Int[(a + b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] &
& LtQ[p, -1] && NeQ[p, -3/2]
```

Maple [A] (verified)

Time = 1.01 (sec) , antiderivative size = 146, normalized size of antiderivative = 0.90

method	result
gosper	$-\frac{2(ex+d)^{2}(cdx+ae)\left(8x^{2}c^{2}d^{2}e^{2}+12xacd\,e^{3}+4x\,c^{2}d^{3}e+3a^{2}e^{4}+6ac\,d^{2}e^{2}-c^{2}d^{4}\right)}{3\left(e^{6}a^{3}-3d^{2}e^{4}a^{2}c+3d^{4}e^{2}a\,c^{2}-d^{6}c^{3}\right)\left(cd\,x^{2}e+a\,e^{2}x+c\,d^{2}x+ade\right)^{\frac{5}{2}}}$
trager	$-\frac{2\left(8x^{2}c^{2}d^{2}e^{2}+12xacd\,e^{3}+4x\,c^{2}d^{3}e+3a^{2}e^{4}+6ac\,d^{2}e^{2}-c^{2}d^{4}\right)\sqrt{cd\,x^{2}e+a\,e^{2}x+c\,d^{2}x+ade}}{3\left(a^{2}e^{4}-2ac\,d^{2}e^{2}+c^{2}d^{4}\right)(cdx+ae)^{2}\left(a\,e^{2}-c\,d^{2}\right)(ex+d)}$
orering	$-\frac{2\left(8x^{2}c^{2}d^{2}e^{2}+12xacd\,e^{3}+4x\,c^{2}d^{3}e+3a^{2}e^{4}+6ac\,d^{2}e^{2}-c^{2}d^{4}\right)(ex+d)^{2}(cdx+ae)}{3\left(e^{6}a^{3}-3d^{2}e^{4}a^{2}c+3d^{4}e^{2}a\,c^{2}-d^{6}c^{3}\right)\left(ade+\left(a\,e^{2}+c\,d^{2}\right)x+cd\,x^{2}e\right)^{\frac{5}{2}}}$
default	$d\left(\frac{\frac{4}{3}cdxe+\frac{2}{3}a\,e^{2}+\frac{2}{3}c\,d^{2}}{\left(4ac\,d^{2}e^{2}-\left(a\,e^{2}+c\,d^{2}\right)^{2}\right)\left(ade+\left(a\,e^{2}+c\,d^{2}\right)x+cd\,x^{2}e\right)^{\frac{3}{2}}}+\frac{16dec\left(2cdxe+a\,e^{2}+c\,d^{2}\right)}{3\left(4ac\,d^{2}e^{2}-\left(a\,e^{2}+c\,d^{2}\right)^{2}\right)^{2}\sqrt{ade+\left(a\,e^{2}+c\,d^{2}\right)x+cd\,x^{2}e}}\right)+e$

input
```
int((e*x+d)/(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(5/2),x,method=_RETURNVERBOS
E)
```

output
```
-2/3*(e*x+d)^2*(c*d*x+a*e)*(8*c^2*d^2*e^2*x^2+12*a*c*d*e^3*x+4*c^2*d^3*e*x
+3*a^2*e^4+6*a*c*d^2*e^2-c^2*d^4)/(a^3*e^6-3*a^2*c*d^2*e^4+3*a*c^2*d^4*e^2
-c^3*d^6)/(c*d*e*x^2+a*e^2*x+c*d^2*x+a*d*e)^(5/2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 316 vs. $2(152) = 304$.

Time = 1.69 (sec) , antiderivative size = 316, normalized size of antiderivative = 1.95

$$\int \frac{d+ex}{(ade+(cd^2+ae^2)x+cdex^2)^{5/2}}\,dx = \frac{2\,(8\,c^2}{3\,(a^2c^3d^7e^2-3\,a^3c^2d^5e^4+3\,a^4cd^3e^6-a^5de^8+(c^5d^8e-3\,ac^4d^6e}$$

input
```
integrate((e*x+d)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2),x, algorithm="fr
icas")
```

output
```
2/3*(8*c^2*d^2*e^2*x^2 - c^2*d^4 + 6*a*c*d^2*e^2 + 3*a^2*e^4 + 4*(c^2*d^3*
e + 3*a*c*d*e^3)*x)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)/(a^2*c^3*d
^7*e^2 - 3*a^3*c^2*d^5*e^4 + 3*a^4*c*d^3*e^6 - a^5*d*e^8 + (c^5*d^8*e - 3*
a*c^4*d^6*e^3 + 3*a^2*c^3*d^4*e^5 - a^3*c^2*d^2*e^7)*x^3 + (c^5*d^9 - a*c^
4*d^7*e^2 - 3*a^2*c^3*d^5*e^4 + 5*a^3*c^2*d^3*e^6 - 2*a^4*c*d*e^8)*x^2 + (
2*a*c^4*d^8*e - 5*a^2*c^3*d^6*e^3 + 3*a^3*c^2*d^4*e^5 + a^4*c*d^2*e^7 - a^
5*e^9)*x)
```

Sympy [F]

$$\int \frac{d+ex}{(ade+(cd^2+ae^2)x+cdex^2)^{5/2}}\,dx = \int \frac{d+ex}{((d+ex)(ae+cdx))^{\frac{5}{2}}}\,dx$$

input
```
integrate((e*x+d)/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(5/2),x)
```

output
```
Integral((d + e*x)/((d + e*x)*(a*e + c*d*x))**(5/2), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{d+ex}{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((e*x+d)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2),x, algorithm="ma
xima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(a*e^2-c*d^2>0)', see `assume?` f
or more de
```

Giac [F]

$$\int \frac{d+ex}{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}\,dx = \int \frac{ex+d}{\left(cdex^2+ade+\left(cd^2+ae^2\right)x\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate((e*x+d)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2),x, algorithm="gi
ac")
```

output
```
integrate((e*x + d)/(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^(5/2), x)
```

Mupad [B] (verification not implemented)

Time = 5.57 (sec) , antiderivative size = 120, normalized size of antiderivative = 0.74

$$\int \frac{d+ex}{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}\,dx =$$
$$-\frac{2\sqrt{c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e}\,\left(3\,a^2\,e^4+6\,a\,c\,d^2\,e^2+12\,a\,c\,d\,e^3\,x-c^2\,d^4+4\,c^2\,d^3\,e\,x+8\,c^2\,d^2\,e^2\,x^2\right)}{3\,{\left(a\,e+c\,d\,x\right)}^2\,{\left(a\,e^2-c\,d^2\right)}^3\,\left(d+e\,x\right)}$$

input
```
int((d + e*x)/(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(5/2),x)
```

output
```
-(2*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2)*(3*a^2*e^4 - c^2*d^4 + 8
*c^2*d^2*e^2*x^2 + 6*a*c*d^2*e^2 + 4*c^2*d^3*e*x + 12*a*c*d*e^3*x))/(3*(a*
e + c*d*x)^2*(a*e^2 - c*d^2)^3*(d + e*x))
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 346, normalized size of antiderivative = 2.14

$$\int \frac{d+ex}{(ade+(cd^2+ae^2)x+cdex^2)^{5/2}}\,dx = \frac{\frac{16\sqrt{e}\,\sqrt{d}\,\sqrt{c}\,\sqrt{cdx+ae}\,ad\,e^2}{3}+\frac{16\sqrt{e}\,\sqrt{d}\,\sqrt{c}\,\sqrt{cdx+ae}\,a\,e^3x}{3}+\frac{16\sqrt{e}\,\sqrt{d}\,\sqrt{c}\,\sqrt{cdx+ae}}{3}}{\sqrt{cdx+ae}\,(a^3cd\,e^7x^2-3a^2c^2d^3e^5x^2+3}$$

input
```
int((e*x+d)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2),x)
```

output
```
(2*(8*sqrt(e)*sqrt(d)*sqrt(c)*sqrt(a*e + c*d*x)*a*d*e**2 + 8*sqrt(e)*sqrt(
d)*sqrt(c)*sqrt(a*e + c*d*x)*a*e**3*x + 8*sqrt(e)*sqrt(d)*sqrt(c)*sqrt(a*e
 + c*d*x)*c*d**2*e*x + 8*sqrt(e)*sqrt(d)*sqrt(c)*sqrt(a*e + c*d*x)*c*d*e**
2*x**2 - 3*sqrt(d + e*x)*a**2*e**4 - 6*sqrt(d + e*x)*a*c*d**2*e**2 - 12*sq
rt(d + e*x)*a*c*d*e**3*x + sqrt(d + e*x)*c**2*d**4 - 4*sqrt(d + e*x)*c**2*
d**3*e*x - 8*sqrt(d + e*x)*c**2*d**2*e**2*x**2))/(3*sqrt(a*e + c*d*x)*(a**
4*d*e**7 + a**4*e**8*x - 3*a**3*c*d**3*e**5 - 2*a**3*c*d**2*e**6*x + a**3*
c*d*e**7*x**2 + 3*a**2*c**2*d**5*e**3 - 3*a**2*c**2*d**3*e**5*x**2 - a*c**
3*d**7*e + 2*a*c**3*d**6*e**2*x + 3*a*c**3*d**5*e**3*x**2 - c**4*d**8*x -
c**4*d**7*e*x**2))
```

3.271 $\int \frac{1}{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 132

$$\int \frac{1}{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}\,dx = -\frac{2(cd^2+ae^2+2cdex)}{3\left(cd^2-ae^2\right)^2\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}} + \frac{16cde(cd^2+ae^2+2cdex)}{3\left(cd^2-ae^2\right)^4\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}$$

output
```
1/3*(-4*c*d*e*x-2*a*e^2-2*c*d^2)/(-a*e^2+c*d^2)^2/(a*d*e+(a*e^2+c*d^2)*x+c
*d*e*x^2)^(3/2)+16/3*c*d*e*(2*c*d*e*x+a*e^2+c*d^2)/(-a*e^2+c*d^2)^4/(a*d*e
+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.19 (sec) , antiderivative size = 132, normalized size of antiderivative = 1.00

$$\int \frac{1}{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}\,dx = \frac{-2a^3e^6+6a^2cde^4(3d+2ex)+6ac^2d^2e^2(3d^2+12dex+8e^2x^2)+}{3\left(cd^2-ae^2\right)^4((ae+cdx)(d+}$$

input
```
Integrate[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(-5/2),x]
```

output
```
(-2*a^3*e^6 + 6*a^2*c*d*e^4*(3*d + 2*e*x) + 6*a*c^2*d^2*e^2*(3*d^2 + 12*d*
e*x + 8*e^2*x^2) + 2*c^3*d^3*(-d^3 + 6*d^2*e*x + 24*d*e^2*x^2 + 16*e^3*x^3
))/(3*(c*d^2 - a*e^2)^4*((a*e + c*d*x)*(d + e*x))^(3/2))
```

Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 132, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.069, Rules used = {1089, 1088}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/2}}dx$$

$$\Big\downarrow \text{1089}$$

$$-\frac{8cde\int \frac{1}{(cdex^2+(cd^2+ae^2)x+ade)^{3/2}}dx}{3\left(cd^2-ae^2\right)^2}-\frac{2\left(ae^2+cd^2+2cdex\right)}{3\left(cd^2-ae^2\right)^2\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}$$

$$\Big\downarrow \text{1088}$$

$$\frac{16cde\left(ae^2+cd^2+2cdex\right)}{3\left(cd^2-ae^2\right)^4\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}-\frac{2\left(ae^2+cd^2+2cdex\right)}{3\left(cd^2-ae^2\right)^2\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}$$

input
```
Int[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(-5/2),x]
```

output
```
(-2*(c*d^2 + a*e^2 + 2*c*d*e*x))/(3*(c*d^2 - a*e^2)^2*(a*d*e + (c*d^2 + a*
e^2)*x + c*d*e*x^2)^(3/2)) + (16*c*d*e*(c*d^2 + a*e^2 + 2*c*d*e*x))/(3*(c*
d^2 - a*e^2)^4*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])
```

Defintions of rubi rules used

rule 1088
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-3/2), x_Symbol] :> Simp[-2*((b +
2*c*x)/((b^2 - 4*a*c)*Sqrt[a + b*x + c*x^2])), x] /; FreeQ[{a, b, c}, x] &&
 NeQ[b^2 - 4*a*c, 0]
```

rule 1089
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^(p + 1)/((p + 1)*(b^2 - 4*a*c))), x] - Simp[2*c*((2*p +
 3)/((p + 1)*(b^2 - 4*a*c)))   Int[(a + b*x + c*x^2)^(p + 1), x], x] /; Fre
eQ[{a, b, c}, x] && LtQ[p, -1] && (IntegerQ[4*p] || IntegerQ[3*p])
```

Maple [A] (verified)

Time = 0.94 (sec) , antiderivative size = 153, normalized size of antiderivative = 1.16

method	result
default	$\frac{\frac{4}{3} c d x e+\frac{2}{3} a e^{2}+\frac{2}{3} c d^{2}}{\left(4 a c d^{2} e^{2}-\left(a e^{2}+c d^{2}\right)^{2}\right) \left(a d e+\left(a e^{2}+c d^{2}\right) x+c d x^{2} e\right)^{\frac{3}{2}}}+\frac{16 d e c \left(2 c d x e+a e^{2}+c d^{2}\right)}{3 \left(4 a c d^{2} e^{2}-\left(a e^{2}+c d^{2}\right)^{2}\right)^{2} \sqrt{a d e+\left(a e^{2}+c d^{2}\right) x+c d x^{2} e}}$
trager	$-\frac{2 \left(-16 c^{3} d^{3} e^{3} x^{3}-24 x^{2} a c^{2} d^{2} e^{4}-24 c^{3} d^{4} e^{2} x^{2}-6 x a^{2} c d e^{5}-36 x a c^{2} d^{3} e^{3}-6 c^{3} d^{5} e x+e^{6} a^{3}-9 d^{2} e^{4} a^{2} c-9 d^{4} e^{2} a c^{2}+d^{6} c^{3}\right)}{3 \left(e^{6} a^{3}-3 d^{2} e^{4} a^{2} c+3 d^{4} e^{2} a c^{2}-d^{6} c^{3}\right) \left(c d x^{2} e+a e^{2} x+c d^{2} x+a d e\right)^{\frac{3}{2}} \left(a e^{2}-c d^{2}\right)}$
gosper	$-\frac{2 \left(e x+d\right) \left(c d x+a e\right) \left(-16 c^{3} d^{3} e^{3} x^{3}-24 x^{2} a c^{2} d^{2} e^{4}-24 c^{3} d^{4} e^{2} x^{2}-6 x a^{2} c d e^{5}-36 x a c^{2} d^{3} e^{3}-6 c^{3} d^{5} e x+e^{6} a^{3}-9 d^{2} e^{4} a^{2} c-9 d^{4} e^{2} a c^{2}+\right.}{3 \left(a^{4} e^{8}-4 a^{3} c d^{2} e^{6}+6 a^{2} c^{2} d^{4} e^{4}-4 a c^{3} d^{6} e^{2}+c^{4} d^{8}\right) \left(c d x^{2} e+a e^{2} x+c d^{2} x+a d e\right)^{\frac{5}{2}}}$
orering	$-\frac{2 \left(-16 c^{3} d^{3} e^{3} x^{3}-24 x^{2} a c^{2} d^{2} e^{4}-24 c^{3} d^{4} e^{2} x^{2}-6 x a^{2} c d e^{5}-36 x a c^{2} d^{3} e^{3}-6 c^{3} d^{5} e x+e^{6} a^{3}-9 d^{2} e^{4} a^{2} c-9 d^{4} e^{2} a c^{2}+d^{6} c^{3}\right) \left(e x+d\right) \left(c d\right.}{3 \left(a^{4} e^{8}-4 a^{3} c d^{2} e^{6}+6 a^{2} c^{2} d^{4} e^{4}-4 a c^{3} d^{6} e^{2}+c^{4} d^{8}\right) \left(a d e+\left(a e^{2}+c d^{2}\right) x+c d x^{2} e\right)^{\frac{5}{2}}}$

input
```
int(1/(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
2/3*(2*c*d*e*x+a*e^2+c*d^2)/(4*a*c*d^2*e^2-(a*e^2+c*d^2)^2)/(a*d*e+(a*e^2+
c*d^2)*x+c*d*x^2*e)^(3/2)+16/3*d*e*c/(4*a*c*d^2*e^2-(a*e^2+c*d^2)^2)^2*(2*
c*d*e*x+a*e^2+c*d^2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(1/2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 491 vs. $2(124) = 248$.

Time = 4.84 (sec) , antiderivative size = 491, normalized size of antiderivative = 3.72

$$\int \frac{1}{(ade + (cd^2 + ae^2)x + cdex^2)^{5/2}} dx = \frac{}{3 (a^2c^4d^{10}e^2 - 4 a^3c^3d^8e^4 + 6 a^4c^2d^6e^6 - 4 a^5cd^4e^8 + a^6d^2e^{10} + (c}$$

input
```
integrate(1/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2),x, algorithm="fricas")
```

output
```
2/3*(16*c^3*d^3*e^3*x^3 - c^3*d^6 + 9*a*c^2*d^4*e^2 + 9*a^2*c*d^2*e^4 - a^
3*e^6 + 24*(c^3*d^4*e^2 + a*c^2*d^2*e^4)*x^2 + 6*(c^3*d^5*e + 6*a*c^2*d^3*
e^3 + a^2*c*d*e^5)*x)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)/(a^2*c^4
*d^10*e^2 - 4*a^3*c^3*d^8*e^4 + 6*a^4*c^2*d^6*e^6 - 4*a^5*c*d^4*e^8 + a^6*
d^2*e^10 + (c^6*d^10*e^2 - 4*a*c^5*d^8*e^4 + 6*a^2*c^4*d^6*e^6 - 4*a^3*c^3
*d^4*e^8 + a^4*c^2*d^2*e^10)*x^4 + 2*(c^6*d^11*e - 3*a*c^5*d^9*e^3 + 2*a^2
*c^4*d^7*e^5 + 2*a^3*c^3*d^5*e^7 - 3*a^4*c^2*d^3*e^9 + a^5*c*d*e^11)*x^3 +
 (c^6*d^12 - 9*a^2*c^4*d^8*e^4 + 16*a^3*c^3*d^6*e^6 - 9*a^4*c^2*d^4*e^8 +
a^6*e^12)*x^2 + 2*(a*c^5*d^11*e - 3*a^2*c^4*d^9*e^3 + 2*a^3*c^3*d^7*e^5 +
2*a^4*c^2*d^5*e^7 - 3*a^5*c*d^3*e^9 + a^6*d*e^11)*x)
```

Sympy [F]

$$\int \frac{1}{(ade + (cd^2 + ae^2)x + cdex^2)^{5/2}} dx = \int \frac{1}{(ade + cdex^2 + x(ae^2 + cd^2))^{\frac{5}{2}}} dx$$

input
```
integrate(1/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(5/2),x)
```

output
```
Integral((a*d*e + c*d*e*x**2 + x*(a*e**2 + c*d**2))**(-5/2), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(1/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(a*e^2-c*d^2>0)', see `assume?` f
or more de
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 366 vs. $2(124) = 248$.

Time = 0.19 (sec) , antiderivative size = 366, normalized size of antiderivative = 2.77

$$\int \frac{1}{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}\,dx = \frac{2\left(2\left(4\left(\frac{2\,c^3d^3e^3x}{c^4d^8-4\,ac^3d^6e^2+6\,a^2c^2d^4e^4-4\,a^3cd^2e^6+a^4e^8}+\frac{3\,(c^3d^4e^2+c}{c^4d^8-4\,ac^3d^6e^2+6\,a^2c^2a}\right.\right.\right.}{}$$

input
```
integrate(1/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2),x, algorithm="giac")
```

output
```
2/3*(2*(4*(2*c^3*d^3*e^3*x/(c^4*d^8 - 4*a*c^3*d^6*e^2 + 6*a^2*c^2*d^4*e^4
- 4*a^3*c*d^2*e^6 + a^4*e^8) + 3*(c^3*d^4*e^2 + a*c^2*d^2*e^4)/(c^4*d^8 -
4*a*c^3*d^6*e^2 + 6*a^2*c^2*d^4*e^4 - 4*a^3*c*d^2*e^6 + a^4*e^8))*x + 3*(c
^3*d^5*e + 6*a*c^2*d^3*e^3 + a^2*c*d*e^5)/(c^4*d^8 - 4*a*c^3*d^6*e^2 + 6*a
^2*c^2*d^4*e^4 - 4*a^3*c*d^2*e^6 + a^4*e^8))*x - (c^3*d^6 - 9*a*c^2*d^4*e^
2 - 9*a^2*c*d^2*e^4 + a^3*e^6)/(c^4*d^8 - 4*a*c^3*d^6*e^2 + 6*a^2*c^2*d^4*
e^4 - 4*a^3*c*d^2*e^6 + a^4*e^8))/(c*d*e*x^2 + c*d^2*x + a*e^2*x + a*d*e)^
(3/2)
```

Mupad [B] (verification not implemented)

Time = 5.36 (sec) , antiderivative size = 131, normalized size of antiderivative = 0.99

$$\int \frac{1}{(ade+(cd^2+ae^2)x+cdex^2)^{5/2}}\,dx = \frac{(2\,c\,d^2+4\,c\,x\,d\,e+2\,a\,e^2)\,\left(8\,c^2\,d^2\,e^2\,x^2-(c\,d^2+a\,e^2)^2+12\,a\,c\,d\right.}{3\,\left((c\,d^2+a\,e^2)^2-4\,a\,c\,d^2\,e^2\right)^2\,(c\,d\,e\,x^2+(c\,d^2+a\,e}$$

input
```
int(1/(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(5/2),x)
```

output
```
((2*a*e^2 + 2*c*d^2 + 4*c*d*e*x)*(8*c^2*d^2*e^2*x^2 - (a*e^2 + c*d^2)^2 +
12*a*c*d^2*e^2 + 8*c*d*e*x*(a*e^2 + c*d^2)))/(3*((a*e^2 + c*d^2)^2 - 4*a*c
*d^2*e^2)^2*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(3/2))
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 647, normalized size of antiderivative = 4.90

$$\int \frac{1}{(ade+(cd^2+ae^2)x+cdex^2)^{5/2}}\,dx = \frac{-\frac{32\sqrt{e}\,\sqrt{d}\,\sqrt{c}\,\sqrt{cdx+ae}\,ac\,d^3e^2}{3}-\frac{64\sqrt{e}\,\sqrt{d}\,\sqrt{c}\,\sqrt{cdx+ae}\,ac\,d^2e^3x}{3}-\frac{32\sqrt{e}\,\sqrt{d}\,\sqrt{c}\,\sqrt{}}{}}{\sqrt{cdx+ae}\,\left(a^4cd\,e^{10}x^3-4a^3c^2d^3e^{8}\right.}$$

input
```
int(1/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2),x)
```

output
```
(2*( - 16*sqrt(e)*sqrt(d)*sqrt(c)*sqrt(a*e + c*d*x)*a*c*d**3*e**2 - 32*sqr
t(e)*sqrt(d)*sqrt(c)*sqrt(a*e + c*d*x)*a*c*d**2*e**3*x - 16*sqrt(e)*sqrt(d
)*sqrt(c)*sqrt(a*e + c*d*x)*a*c*d*e**4*x**2 - 16*sqrt(e)*sqrt(d)*sqrt(c)*s
qrt(a*e + c*d*x)*c**2*d**4*e*x - 32*sqrt(e)*sqrt(d)*sqrt(c)*sqrt(a*e + c*d
*x)*c**2*d**3*e**2*x**2 - 16*sqrt(e)*sqrt(d)*sqrt(c)*sqrt(a*e + c*d*x)*c**
2*d**2*e**3*x**3 - sqrt(d + e*x)*a**3*e**6 + 9*sqrt(d + e*x)*a**2*c*d**2*e
**4 + 6*sqrt(d + e*x)*a**2*c*d*e**5*x + 9*sqrt(d + e*x)*a*c**2*d**4*e**2 +
 36*sqrt(d + e*x)*a*c**2*d**3*e**3*x + 24*sqrt(d + e*x)*a*c**2*d**2*e**4*x
**2 - sqrt(d + e*x)*c**3*d**6 + 6*sqrt(d + e*x)*c**3*d**5*e*x + 24*sqrt(d
+ e*x)*c**3*d**4*e**2*x**2 + 16*sqrt(d + e*x)*c**3*d**3*e**3*x**3))/(3*sqr
t(a*e + c*d*x)*(a**5*d**2*e**9 + 2*a**5*d*e**10*x + a**5*e**11*x**2 - 4*a*
*4*c*d**4*e**7 - 7*a**4*c*d**3*e**8*x - 2*a**4*c*d**2*e**9*x**2 + a**4*c*d
*e**10*x**3 + 6*a**3*c**2*d**6*e**5 + 8*a**3*c**2*d**5*e**6*x - 2*a**3*c**
2*d**4*e**7*x**2 - 4*a**3*c**2*d**3*e**8*x**3 - 4*a**2*c**3*d**8*e**3 - 2*
a**2*c**3*d**7*e**4*x + 8*a**2*c**3*d**6*e**5*x**2 + 6*a**2*c**3*d**5*e**6
*x**3 + a*c**4*d**10*e - 2*a*c**4*d**9*e**2*x - 7*a*c**4*d**8*e**3*x**2 -
4*a*c**4*d**7*e**4*x**3 + c**5*d**11*x + 2*c**5*d**10*e*x**2 + c**5*d**9*e
**2*x**3))
```

3.272 $\int \frac{1}{(d+ex)(ade+(cd^2+ae^2)x+cdex^2)^{5/2}}\,dx$

Optimal result

Integrand size = 37, antiderivative size = 289

$$\int \frac{1}{(d+ex)(ade+(cd^2+ae^2)x+cdex^2)^{5/2}}\,dx = -\frac{2}{3(cd^2-ae^2)(d+ex)(ade+(cd^2+ae^2)x+cdex^2)^{3/2}} + \frac{16e}{3(cd^2-ae^2)^2(d+ex)^2\sqrt{ade+(cd^2+ae^2)x+cdex^2}} + \frac{32e^2\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{5(cd^2-ae^2)^3(d+ex)^3} + \frac{128cde^2\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{15(cd^2-ae^2)^4(d+ex)^2} + \frac{256c^2d^2e^2\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{15(cd^2-ae^2)^5(d+ex)}$$

output
```
-2/3/(-a*e^2+c*d^2)/(e*x+d)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)+16/3*e
/(-a*e^2+c*d^2)^2/(e*x+d)^2/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)+32/5*e
^2*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(-a*e^2+c*d^2)^3/(e*x+d)^3+128/
15*c*d*e^2*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(-a*e^2+c*d^2)^4/(e*x+d
)^2+256/15*c^2*d^2*e^2*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(-a*e^2+c*d
^2)^5/(e*x+d)
```

Mathematica [A] (verified)

Time = 0.28 (sec) , antiderivative size = 193, normalized size of antiderivative = 0.67

$$\int \frac{1}{(d+ex)\left(ade+(cd^2+ae^2)\,x+cdex^2\right)^{5/2}}\,dx = \frac{2(3a^4e^8-4a^3cde^6(5d+2ex)+6a^2c^2d^2e^4(15d^2+20dex}{}$$

input
```
Integrate[1/((d + e*x)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2)),x]
```

output
```
(2*(3*a^4*e^8 - 4*a^3*c*d*e^6*(5*d + 2*e*x) + 6*a^2*c^2*d^2*e^4*(15*d^2 +
20*d*e*x + 8*e^2*x^2) + 12*a*c^3*d^3*e^2*(5*d^3 + 30*d^2*e*x + 40*d*e^2*x^
2 + 16*e^3*x^3) + c^4*d^4*(-5*d^4 + 40*d^3*e*x + 240*d^2*e^2*x^2 + 320*d*e
^3*x^3 + 128*e^4*x^4)))/(15*(c*d^2 - a*e^2)^5*(d + e*x)*((a*e + c*d*x)*(d
+ e*x))^(3/2))
```

Rubi [A] (verified)

Time = 0.48 (sec) , antiderivative size = 207, normalized size of antiderivative = 0.72, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.081$, Rules used = {1129, 1089, 1088}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(d+ex)\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/2}}dx$$

$$\Big\downarrow \; 1129$$

$$\frac{8cd\int \frac{1}{(cdex^2+(cd^2+ae^2)x+ade)^{5/2}}dx}{5\left(cd^2-ae^2\right)}+\frac{2}{5(d+ex)\left(cd^2-ae^2\right)\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}$$

$$\Big\downarrow \; 1089$$

$$\frac{8cd\left(-\frac{8cde\int \frac{1}{\left(cdex^2+\left(cd^2+ae^2\right)x+ade\right)^{3/2}}dx}{3(cd^2-ae^2)^2}-\frac{2(ae^2+cd^2+2cdex)}{3(cd^2-ae^2)^2(x(ae^2+cd^2)+ade+cdex^2)^{3/2}}\right)}{5\left(cd^2-ae^2\right)}+\frac{2}{5(d+ex)\left(cd^2-ae^2\right)\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}$$

$$\Big\downarrow \text{1088}$$

$$\frac{8cd\left(\frac{16cde(ae^2+cd^2+2cdex)}{3(cd^2-ae^2)^4\sqrt{x(ae^2+cd^2)+ade+cdex^2}}-\frac{2(ae^2+cd^2+2cdex)}{3(cd^2-ae^2)^2(x(ae^2+cd^2)+ade+cdex^2)^{3/2}}\right)}{5\left(cd^2-ae^2\right)}+\frac{2}{5(d+ex)\left(cd^2-ae^2\right)\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}$$

input
```
Int[1/((d + e*x)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2)),x]
```

output
```
2/(5*(c*d^2 - a*e^2)*(d + e*x)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/
2)) + (8*c*d*((-2*(c*d^2 + a*e^2 + 2*c*d*e*x))/(3*(c*d^2 - a*e^2)^2*(a*d*e
 + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2)) + (16*c*d*e*(c*d^2 + a*e^2 + 2*c*
d*e*x))/(3*(c*d^2 - a*e^2)^4*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]))
)/(5*(c*d^2 - a*e^2))
```

Defintions of rubi rules used

rule 1088
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-3/2), x_Symbol] :> Simp[-2*((b +
2*c*x)/((b^2 - 4*a*c)*Sqrt[a + b*x + c*x^2])), x] /; FreeQ[{a, b, c}, x] &&
 NeQ[b^2 - 4*a*c, 0]
```

rule 1089
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^(p + 1)/((p + 1)*(b^2 - 4*a*c))), x] - Simp[2*c*((2*p +
 3)/((p + 1)*(b^2 - 4*a*c)))   Int[(a + b*x + c*x^2)^(p + 1), x], x] /; Fre
eQ[{a, b, c}, x] && LtQ[p, -1] && (IntegerQ[4*p] || IntegerQ[3*p])
```

rule 1129
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(-e)*(d + e*x)^m*((a + b*x + c*x^2)^(p + 1)/((m + p + 1)*(2*
c*d - b*e))), x] + Simp[c*(Simplify[m + 2*p + 2]/((m + p + 1)*(2*c*d - b*e)
))   Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d
, e, m, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && ILtQ[Simplify[m + 2*p +
2], 0]
```

Maple [A] (verified)

Time = 1.20 (sec) , antiderivative size = 242, normalized size of antiderivative = 0.84

method	result
default	$\frac{-\frac{2}{5\left(a e^2-c d^2\right)\left(x+\frac{d}{e}\right)\left(dec\left(x+\frac{d}{e}\right)^2+\left(a e^2-c d^2\right)\left(x+\frac{d}{e}\right)\right)^{\frac{3}{2}}}-\frac{8dec\left(-\frac{2\left(2dec\left(x+\frac{d}{e}\right)+a e^2-c d^2\right)}{3\left(a e^2-c d^2\right)^2\left(dec\left(x+\frac{d}{e}\right)^2+\left(a e^2-c d^2\right)\left(x+\frac{d}{e}\right)\right)^{\frac{3}{2}}}+\frac{16dec}{3\left(a e^2-c d^2\right)^4}\right.}{5\left(a e^2-c d^2\right)}}{e}$
gosper	$-\frac{2(cdx+ae)(128c^4d^4e^4x^4+192a c^3d^3e^5x^3+320c^4d^5e^3x^3+48a^2c^2d^2e^6x^2+480a c^3d^4e^4x^2+240c^4d^6e^2x^2-8a^3cd e^7x+120a^2c^2d^3e}{15(a^5e^{10}-5a^4c d^2e^8+10a^3c^2d^4e^6-10a^2c^3d^6e^4+5a c^4d^8e^2-c^5d^{10})(cd}$
orering	$-\frac{2(128c^4d^4e^4x^4+192a c^3d^3e^5x^3+320c^4d^5e^3x^3+48a^2c^2d^2e^6x^2+480a c^3d^4e^4x^2+240c^4d^6e^2x^2-8a^3cd e^7x+120a^2c^2d^3e^5x+360a}{15(a^5e^{10}-5a^4c d^2e^8+10a^3c^2d^4e^6-10a^2c^3d^6e^4+5a c^4d^8e^2-c^5d^{10})(ad}$
trager	$-\frac{2(128c^4d^4e^4x^4+192a c^3d^3e^5x^3+320c^4d^5e^3x^3+48a^2c^2d^2e^6x^2+480a c^3d^4e^4x^2+240c^4d^6e^2x^2-8a^3cd e^7x+120a^2c^2d^3e^5x+360a}{15(a^4e^8-4a^3c d^2e^6+6a^2c^2d^4e^4-4a c^3d^6e^2+c^4d^8)(cd}$

input
```
int(1/(e*x+d)/(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(5/2),x,method=_RETURNVERB
OSE)
```

output
```
1/e*(-2/5/(a*e^2-c*d^2)/(x+d/e)/(d*e*c*(x+d/e)^2+(a*e^2-c*d^2)*(x+d/e))^(3
/2)-8/5*d*e*c/(a*e^2-c*d^2)*(-2/3*(2*d*e*c*(x+d/e)+a*e^2-c*d^2)/(a*e^2-c*d
^2)^2/(d*e*c*(x+d/e)^2+(a*e^2-c*d^2)*(x+d/e))^(3/2)+16/3*d*e*c/(a*e^2-c*d^
2)^4*(2*d*e*c*(x+d/e)+a*e^2-c*d^2)/(d*e*c*(x+d/e)^2+(a*e^2-c*d^2)*(x+d/e))
^(1/2)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 769 vs. 2(269) = 538.

Time = 14.14 (sec) , antiderivative size = 769, normalized size of antiderivative = 2.66

$$\int \frac{1}{(d+ex)\left(ade+(cd^2+ae^2)x+cdex^2\right)^{5/2}}\,dx = \frac{}{15\left(a^2c^5d^{13}e^2-5\,a^3c^4d^{11}e^4+10\,a^4c^3d^9e^6-10\,a^5c^2d^7e^8\right.}$$

input
```
integrate(1/(e*x+d)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2),x, algorithm="
fricas")
```

output
```
2/15*(128*c^4*d^4*e^4*x^4 - 5*c^4*d^8 + 60*a*c^3*d^6*e^2 + 90*a^2*c^2*d^4*
e^4 - 20*a^3*c*d^2*e^6 + 3*a^4*e^8 + 64*(5*c^4*d^5*e^3 + 3*a*c^3*d^3*e^5)*
x^3 + 48*(5*c^4*d^6*e^2 + 10*a*c^3*d^4*e^4 + a^2*c^2*d^2*e^6)*x^2 + 8*(5*c
^4*d^7*e + 45*a*c^3*d^5*e^3 + 15*a^2*c^2*d^3*e^5 - a^3*c*d*e^7)*x)*sqrt(c*
d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)/(a^2*c^5*d^13*e^2 - 5*a^3*c^4*d^11*e^
4 + 10*a^4*c^3*d^9*e^6 - 10*a^5*c^2*d^7*e^8 + 5*a^6*c*d^5*e^10 - a^7*d^3*e
^12 + (c^7*d^12*e^3 - 5*a*c^6*d^10*e^5 + 10*a^2*c^5*d^8*e^7 - 10*a^3*c^4*d
^6*e^9 + 5*a^4*c^3*d^4*e^11 - a^5*c^2*d^2*e^13)*x^5 + (3*c^7*d^13*e^2 - 13
*a*c^6*d^11*e^4 + 20*a^2*c^5*d^9*e^6 - 10*a^3*c^4*d^7*e^8 - 5*a^4*c^3*d^5*
e^10 + 7*a^5*c^2*d^3*e^12 - 2*a^6*c*d*e^14)*x^4 + (3*c^7*d^14*e - 9*a*c^6*
d^12*e^3 + a^2*c^5*d^10*e^5 + 25*a^3*c^4*d^8*e^7 - 35*a^4*c^3*d^6*e^9 + 17
*a^5*c^2*d^4*e^11 - a^6*c*d^2*e^13 - a^7*e^15)*x^3 + (c^7*d^15 + a*c^6*d^1
3*e^2 - 17*a^2*c^5*d^11*e^4 + 35*a^3*c^4*d^9*e^6 - 25*a^4*c^3*d^7*e^8 - a^
5*c^2*d^5*e^10 + 9*a^6*c*d^3*e^12 - 3*a^7*d*e^14)*x^2 + (2*a*c^6*d^14*e -
7*a^2*c^5*d^12*e^3 + 5*a^3*c^4*d^10*e^5 + 10*a^4*c^3*d^8*e^7 - 20*a^5*c^2*
d^6*e^9 + 13*a^6*c*d^4*e^11 - 3*a^7*d^2*e^13)*x)
```

Sympy [F]

$$\int \frac{1}{(d+ex)\left(ade+(cd^2+ae^2)x+cdex^2\right)^{5/2}}\,dx = \int \frac{1}{\left((d+ex)(ae+cdx)\right)^{\frac{5}{2}}(d+ex)}\,dx$$

input
```
integrate(1/(e*x+d)/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(5/2),x)
```

output
```
Integral(1/(((d + e*x)*(a*e + c*d*x))**(5/2)*(d + e*x)), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{(d+ex)\left(ade+(cd^2+ae^2)x+cdex^2\right)^{5/2}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(1/(e*x+d)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2),x, algorithm="
maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e*(a*e^2-c*d^2)>0)', see `assume
?` for mor
```

Giac [F]

$$\int \frac{1}{(d+ex)\left(ade+(cd^2+ae^2)x+cdex^2\right)^{5/2}}\,dx = \int \frac{1}{(cdex^2+ade+(cd^2+ae^2)x)^{\frac{5}{2}}(ex+d)}\,dx$$

input
```
integrate(1/(e*x+d)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2),x, algorithm="
giac")
```

output
```
integrate(1/((c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^(5/2)*(e*x + d)), x)
```

Mupad [B] (verification not implemented)

Time = 5.85 (sec) , antiderivative size = 253, normalized size of antiderivative = 0.88

$$\int \frac{1}{(d+ex)\left(ade+(cd^2+ae^2)x+cdex^2\right)^{5/2}}\,dx =$$
$$-\frac{2\sqrt{c\,d\,e\,x^2+(c\,d^2+a\,e^2)\,x+a\,d\,e}\,(3\,a^4\,e^8-20\,a^3\,c\,d^2\,e^6-8\,a^3\,c\,d\,e^7\,x+90\,a^2\,c^2\,d^4\,e^4+120\,a^2\,c^2\,d^3\,e^5\,x}{}$$

input
```
int(1/((d + e*x)*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(5/2)),x)
```

output
```
-(2*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2)*(3*a^4*e^8 - 5*c^4*d^8 +
 60*a*c^3*d^6*e^2 - 20*a^3*c*d^2*e^6 + 90*a^2*c^2*d^4*e^4 + 240*c^4*d^6*e^
2*x^2 + 320*c^4*d^5*e^3*x^3 + 128*c^4*d^4*e^4*x^4 + 40*c^4*d^7*e*x - 8*a^3
*c*d*e^7*x + 48*a^2*c^2*d^2*e^6*x^2 + 360*a*c^3*d^5*e^3*x + 120*a^2*c^2*d^
3*e^5*x + 480*a*c^3*d^4*e^4*x^2 + 192*a*c^3*d^3*e^5*x^3))/(15*(a*e + c*d*x
)^2*(a*e^2 - c*d^2)^5*(d + e*x)^3)
```

Reduce [B] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 977, normalized size of antiderivative = 3.38

$$\int \frac{1}{(d+ex)\left(ade+(cd^2+ae^2)\,x+cdex^2\right)^{5/2}}\,dx = \text{Too large to display}$$

input

```
int(1/(e*x+d)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2),x)
```

output

```
(2*(128*sqrt(e)*sqrt(d)*sqrt(c)*sqrt(a*e + c*d*x)*a*c**2*d**5*e**2 + 384*s
qrt(e)*sqrt(d)*sqrt(c)*sqrt(a*e + c*d*x)*a*c**2*d**4*e**3*x + 384*sqrt(e)*
sqrt(d)*sqrt(c)*sqrt(a*e + c*d*x)*a*c**2*d**3*e**4*x**2 + 128*sqrt(e)*sqrt
(d)*sqrt(c)*sqrt(a*e + c*d*x)*a*c**2*d**2*e**5*x**3 + 128*sqrt(e)*sqrt(d)*
sqrt(c)*sqrt(a*e + c*d*x)*c**3*d**6*e*x + 384*sqrt(e)*sqrt(d)*sqrt(c)*sqrt
(a*e + c*d*x)*c**3*d**5*e**2*x**2 + 384*sqrt(e)*sqrt(d)*sqrt(c)*sqrt(a*e +
 c*d*x)*c**3*d**4*e**3*x**3 + 128*sqrt(e)*sqrt(d)*sqrt(c)*sqrt(a*e + c*d*x
)*c**3*d**3*e**4*x**4 - 3*sqrt(d + e*x)*a**4*e**8 + 20*sqrt(d + e*x)*a**3*
c*d**2*e**6 + 8*sqrt(d + e*x)*a**3*c*d*e**7*x - 90*sqrt(d + e*x)*a**2*c**2
*d**4*e**4 - 120*sqrt(d + e*x)*a**2*c**2*d**3*e**5*x - 48*sqrt(d + e*x)*a*
*2*c**2*d**2*e**6*x**2 - 60*sqrt(d + e*x)*a*c**3*d**6*e**2 - 360*sqrt(d +
e*x)*a*c**3*d**5*e**3*x - 480*sqrt(d + e*x)*a*c**3*d**4*e**4*x**2 - 192*sq
rt(d + e*x)*a*c**3*d**3*e**5*x**3 + 5*sqrt(d + e*x)*c**4*d**8 - 40*sqrt(d
+ e*x)*c**4*d**7*e*x - 240*sqrt(d + e*x)*c**4*d**6*e**2*x**2 - 320*sqrt(d
+ e*x)*c**4*d**5*e**3*x**3 - 128*sqrt(d + e*x)*c**4*d**4*e**4*x**4))/(15*s
qrt(a*e + c*d*x)*(a**6*d**3*e**11 + 3*a**6*d**2*e**12*x + 3*a**6*d*e**13*x
**2 + a**6*e**14*x**3 - 5*a**5*c*d**5*e**9 - 14*a**5*c*d**4*e**10*x - 12*a
**5*c*d**3*e**11*x**2 - 2*a**5*c*d**2*e**12*x**3 + a**5*c*d*e**13*x**4 + 1
0*a**4*c**2*d**7*e**7 + 25*a**4*c**2*d**6*e**8*x + 15*a**4*c**2*d**5*e**9*
x**2 - 5*a**4*c**2*d**4*e**10*x**3 - 5*a**4*c**2*d**3*e**11*x**4 - 10*a...
```

3.273 $\int \frac{1}{(d+ex)^2\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}\,dx$

Optimal result

Integrand size = 37, antiderivative size = 352

$$\begin{aligned}
&\int \frac{1}{(d+ex)^2\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}\,dx = \\
&-\frac{2}{3\left(cd^2-ae^2\right)(d+ex)^2\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}} \\
&+\frac{20e}{3\left(cd^2-ae^2\right)^2(d+ex)^3\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}} \\
&+\frac{160e^2\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{21\left(cd^2-ae^2\right)^3(d+ex)^4}+\frac{64cde^2\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{7\left(cd^2-ae^2\right)^4(d+ex)^3} \\
&+\frac{256c^2d^2e^2\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{21\left(cd^2-ae^2\right)^5(d+ex)^2} \\
&+\frac{512c^3d^3e^2\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{21\left(cd^2-ae^2\right)^6(d+ex)}
\end{aligned}$$

output
```
-2/3/(-a*e^2+c*d^2)/(e*x+d)^2/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)+20/3
*e/(-a*e^2+c*d^2)^2/(e*x+d)^3/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)+160/
21*e^2*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(-a*e^2+c*d^2)^3/(e*x+d)^4+
64/7*c*d*e^2*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(-a*e^2+c*d^2)^4/(e*x
+d)^3+256/21*c^2*d^2*e^2*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(-a*e^2+c
*d^2)^5/(e*x+d)^2+512/21*c^3*d^3*e^2*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/
2)/(-a*e^2+c*d^2)^6/(e*x+d)
```

Mathematica [A] (verified)

Time = 0.44 (sec) , antiderivative size = 259, normalized size of antiderivative = 0.74

$$\int \frac{1}{(d+ex)^2 \left(ade+(cd^2+ae^2)\,x+cdex^2\right)^{5/2}}\,dx = \frac{2(-3a^5e^{10}+3a^4cde^8(7d+2ex)-2a^3c^2d^2e^6(35d^2+28}{}$$

input
```
Integrate[1/((d + e*x)^2*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2)),x]
```

output
```
(2*(-3*a^5*e^10 + 3*a^4*c*d*e^8*(7*d + 2*e*x) - 2*a^3*c^2*d^2*e^6*(35*d^2
+ 28*d*e*x + 8*e^2*x^2) + 6*a^2*c^3*d^3*e^4*(35*d^3 + 70*d^2*e*x + 56*d*e^
2*x^2 + 16*e^3*x^3) + 3*a*c^4*d^4*e^2*(35*d^4 + 280*d^3*e*x + 560*d^2*e^2*
x^2 + 448*d*e^3*x^3 + 128*e^4*x^4) + c^5*d^5*(-7*d^5 + 70*d^4*e*x + 560*d^
3*e^2*x^2 + 1120*d^2*e^3*x^3 + 896*d*e^4*x^4 + 256*e^5*x^5)))/(21*(c*d^2 -
 a*e^2)^6*(d + e*x)^2*((a*e + c*d*x)*(d + e*x))^(3/2))
```

Rubi [A] (verified)

Time = 0.63 (sec) , antiderivative size = 282, normalized size of antiderivative = 0.80, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.108$, Rules used = {1129, 1129, 1089, 1088}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(d+ex)^2 \left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/2}}\,dx$$

$\Big\downarrow$ 1129

$$\frac{10cd\int \frac{1}{(d+ex)(cdex^2+(cd^2+ae^2)x+ade)^{5/2}}dx}{7\left(cd^2-ae^2\right)}+\frac{2}{7(d+ex)^2\left(cd^2-ae^2\right)\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}$$

$\Big\downarrow$ 1129

$$\frac{10cd\left(\frac{8cd\int \frac{1}{\left(cdex^2+\left(cd^2+ae^2\right)x+ade\right)^{5/2}}dx}{5(cd^2-ae^2)}+\frac{2}{5(d+ex)(cd^2-ae^2)(x(ae^2+cd^2)+ade+cdex^2)^{3/2}}\right)}{7\left(cd^2-ae^2\right)}+\frac{2}{7(d+ex)^2\left(cd^2-ae^2\right)\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}$$

$\Big\downarrow$ 1089

$$\frac{10cd\left(\frac{8cd\left(-\frac{8cde\int \frac{1}{\left(cdex^2+\left(cd^2+ae^2\right)x+ade\right)^{3/2}}dx}{3\left(cd^2-ae^2\right)^2}-\frac{2\left(ae^2+cd^2+2cdex\right)}{3\left(cd^2-ae^2\right)^2\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}\right)}{5(cd^2-ae^2)}+\frac{2}{5(d+ex)(cd^2-ae^2)(x(ae^2+cd^2)+ade+c}\right.}{7\left(cd^2-ae^2\right)}$$

$$\frac{2}{7(d+ex)^2\left(cd^2-ae^2\right)\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}$$

$\Big\downarrow$ 1088

$$\frac{10cd\left(\frac{8cd\left(\frac{16cde\left(ae^2+cd^2+2cdex\right)}{3\left(cd^2-ae^2\right)^4\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}-\frac{2\left(ae^2+cd^2+2cdex\right)}{3\left(cd^2-ae^2\right)^2\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}\right)}{5(cd^2-ae^2)}+\frac{2}{5(d+ex)(cd^2-ae^2)(x(ae^2+cd^2)+ade+cde}\right.}{7\left(cd^2-ae^2\right)}$$

$$\frac{2}{7(d+ex)^2\left(cd^2-ae^2\right)\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}$$

input
```
Int[1/((d + e*x)^2*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2)),x]
```

output
```
2/(7*(c*d^2 - a*e^2)*(d + e*x)^2*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(
3/2)) + (10*c*d*(2/(5*(c*d^2 - a*e^2)*(d + e*x)*(a*d*e + (c*d^2 + a*e^2)*x
 + c*d*e*x^2)^(3/2)) + (8*c*d*((-2*(c*d^2 + a*e^2 + 2*c*d*e*x))/(3*(c*d^2
- a*e^2)^2*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2)) + (16*c*d*e*(c*d
^2 + a*e^2 + 2*c*d*e*x))/(3*(c*d^2 - a*e^2)^4*Sqrt[a*d*e + (c*d^2 + a*e^2)
*x + c*d*e*x^2])))/(5*(c*d^2 - a*e^2))))/(7*(c*d^2 - a*e^2))
```

Defintions of rubi rules used

rule 1088
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-3/2), x_Symbol] :> Simp[-2*((b +
2*c*x)/((b^2 - 4*a*c)*Sqrt[a + b*x + c*x^2])), x] /; FreeQ[{a, b, c}, x] &&
 NeQ[b^2 - 4*a*c, 0]
```

rule 1089
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^(p + 1)/((p + 1)*(b^2 - 4*a*c))), x] - Simp[2*c*((2*p +
 3)/((p + 1)*(b^2 - 4*a*c)))   Int[(a + b*x + c*x^2)^(p + 1), x], x] /; Fre
eQ[{a, b, c}, x] && LtQ[p, -1] && (IntegerQ[4*p] || IntegerQ[3*p])
```

rule 1129
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(-e)*(d + e*x)^m*((a + b*x + c*x^2)^(p + 1)/((m + p + 1)*(2*
c*d - b*e))), x] + Simp[c*(Simplify[m + 2*p + 2]/((m + p + 1)*(2*c*d - b*e)
))   Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d
, e, m, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && ILtQ[Simplify[m + 2*p +
2], 0]
```

Maple [A] (verified)

Time = 1.26 (sec) , antiderivative size = 323, normalized size of antiderivative = 0.92

method	result
default	$\frac{-\frac{2}{7\left(a e^2-c d^2\right)\left(x+\frac{d}{e}\right)^2\left(dec\left(x+\frac{d}{e}\right)^2+\left(a e^2-c d^2\right)\left(x+\frac{d}{e}\right)\right)^{\frac{3}{2}}}-\frac{10dec\left(-\frac{2}{5\left(a e^2-c d^2\right)\left(x+\frac{d}{e}\right)\left(dec\left(x+\frac{d}{e}\right)^2+\left(a e^2-c d^2\right)\left(x+\frac{d}{e}\right)\right)^{\frac{3}{2}}}-\frac{8dec\left(\cdots\right.}{\cdots}\right.}{\cdots}}{e^2}$
gosper	$-\frac{2(cdx+ae)\left(-256x^5e^5d^5c^5-384x^4a\,c^4d^4e^6-896x^4c^5d^6e^4-96x^3a^2c^3d^3e^7-1344x^3a\,c^4d^5e^5-1120x^3c^5d^7e^3+16x^2a^3c^2d^2e^8-336\cdots\right.}{21(ex+d)\left(a^6e^{12}-6a^5d^2e^{10}c+\cdots\right.}$
orering	$-\frac{2\left(-256x^5e^5d^5c^5-384x^4a\,c^4d^4e^6-896x^4c^5d^6e^4-96x^3a^2c^3d^3e^7-1344x^3a\,c^4d^5e^5-1120x^3c^5d^7e^3+16x^2a^3c^2d^2e^8-336x^2a^2c^3d^4\cdots\right.}{21\left(a^6e^{12}-6a^5d^2e^{10}c+15a^4d^4e\cdots\right.}$
trager	$-\frac{2\left(-256x^5e^5d^5c^5-384x^4a\,c^4d^4e^6-896x^4c^5d^6e^4-96x^3a^2c^3d^3e^7-1344x^3a\,c^4d^5e^5-1120x^3c^5d^7e^3+16x^2a^3c^2d^2e^8-336x^2a^2c^3d^4\cdots\right.}{21\left(a^5e^{10}-\cdots\right.}$

input
```
int(1/(e*x+d)^2/(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(5/2),x,method=_RETURNVE
RBOSE)
```

output
```
1/e^2*(-2/7/(a*e^2-c*d^2)/(x+d/e)^2/(d*e*c*(x+d/e)^2+(a*e^2-c*d^2)*(x+d/e)
)^(3/2)-10/7*d*e*c/(a*e^2-c*d^2)*(-2/5/(a*e^2-c*d^2)/(x+d/e)/(d*e*c*(x+d/e
)^2+(a*e^2-c*d^2)*(x+d/e))^(3/2)-8/5*d*e*c/(a*e^2-c*d^2)*(-2/3*(2*d*e*c*(x
+d/e)+a*e^2-c*d^2)/(a*e^2-c*d^2)^2/(d*e*c*(x+d/e)^2+(a*e^2-c*d^2)*(x+d/e))
^(3/2)+16/3*d*e*c/(a*e^2-c*d^2)^4*(2*d*e*c*(x+d/e)+a*e^2-c*d^2)/(d*e*c*(x+
d/e)^2+(a*e^2-c*d^2)*(x+d/e))^(1/2))))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1058 vs. 2(328) = 656.

Time = 33.17 (sec) , antiderivative size = 1058, normalized size of antiderivative = 3.01

$$\int \frac{1}{(d+ex)^2\left(ade+(cd^2+ae^2)\,x+cdex^2\right)^{5/2}}\,dx = \text{Too large to display}$$

input
```
integrate(1/(e*x+d)^2/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2),x, algorithm
="fricas")
```

output
```
2/21*(256*c^5*d^5*e^5*x^5 - 7*c^5*d^10 + 105*a*c^4*d^8*e^2 + 210*a^2*c^3*d
^6*e^4 - 70*a^3*c^2*d^4*e^6 + 21*a^4*c*d^2*e^8 - 3*a^5*e^10 + 128*(7*c^5*d
^6*e^4 + 3*a*c^4*d^4*e^6)*x^4 + 32*(35*c^5*d^7*e^3 + 42*a*c^4*d^5*e^5 + 3*
a^2*c^3*d^3*e^7)*x^3 + 16*(35*c^5*d^8*e^2 + 105*a*c^4*d^6*e^4 + 21*a^2*c^3
*d^4*e^6 - a^3*c^2*d^2*e^8)*x^2 + 2*(35*c^5*d^9*e + 420*a*c^4*d^7*e^3 + 21
0*a^2*c^3*d^5*e^5 - 28*a^3*c^2*d^3*e^7 + 3*a^4*c*d*e^9)*x)*sqrt(c*d*e*x^2
+ a*d*e + (c*d^2 + a*e^2)*x)/(a^2*c^6*d^16*e^2 - 6*a^3*c^5*d^14*e^4 + 15*a
^4*c^4*d^12*e^6 - 20*a^5*c^3*d^10*e^8 + 15*a^6*c^2*d^8*e^10 - 6*a^7*c*d^6*
e^12 + a^8*d^4*e^14 + (c^8*d^14*e^4 - 6*a*c^7*d^12*e^6 + 15*a^2*c^6*d^10*e
^8 - 20*a^3*c^5*d^8*e^10 + 15*a^4*c^4*d^6*e^12 - 6*a^5*c^3*d^4*e^14 + a^6*
c^2*d^2*e^16)*x^6 + 2*(2*c^8*d^15*e^3 - 11*a*c^7*d^13*e^5 + 24*a^2*c^6*d^1
1*e^7 - 25*a^3*c^5*d^9*e^9 + 10*a^4*c^4*d^7*e^11 + 3*a^5*c^3*d^5*e^13 - 4*
a^6*c^2*d^3*e^15 + a^7*c*d*e^17)*x^5 + (6*c^8*d^16*e^2 - 28*a*c^7*d^14*e^4
 + 43*a^2*c^6*d^12*e^6 - 6*a^3*c^5*d^10*e^8 - 55*a^4*c^4*d^8*e^10 + 64*a^5
*c^3*d^6*e^12 - 27*a^6*c^2*d^4*e^14 + 2*a^7*c*d^2*e^16 + a^8*e^18)*x^4 + 4
*(c^8*d^17*e - 3*a*c^7*d^15*e^3 - 2*a^2*c^6*d^13*e^5 + 19*a^3*c^5*d^11*e^7
 - 30*a^4*c^4*d^9*e^9 + 19*a^5*c^3*d^7*e^11 - 2*a^6*c^2*d^5*e^13 - 3*a^7*c
*d^3*e^15 + a^8*d*e^17)*x^3 + (c^8*d^18 + 2*a*c^7*d^16*e^2 - 27*a^2*c^6*d^
14*e^4 + 64*a^3*c^5*d^12*e^6 - 55*a^4*c^4*d^10*e^8 - 6*a^5*c^3*d^8*e^10 +
43*a^6*c^2*d^6*e^12 - 28*a^7*c*d^4*e^14 + 6*a^8*d^2*e^16)*x^2 + 2*(a*c^...
```

Sympy [F]

$$\int \frac{1}{(d+ex)^2\left(ade+(cd^2+ae^2)\,x+cdex^2\right)^{5/2}}\,dx = \int \frac{1}{\left((d+ex)\,(ae+cdx)\right)^{\frac{5}{2}}\,(d+ex)^2}\,dx$$

input
```
integrate(1/(e*x+d)**2/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(5/2),x)
```

output
```
Integral(1/(((d + e*x)*(a*e + c*d*x))**(5/2)*(d + e*x)**2), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{(d+ex)^2 (ade + (cd^2 + ae^2) x + cdex^2)^{5/2}} \, dx = \text{Exception raised: ValueError}$$

input
```
integrate(1/(e*x+d)^2/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2),x, algorithm
="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e*(a*e^2-c*d^2)>0)', see `assume
?` for mor
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 10592 vs. $2(328) = 656$.

Time $= 0.43$ (sec) , antiderivative size $= 10592$, normalized size of antiderivative $= 30.09$

$$\int \frac{1}{(d+ex)^2 (ade + (cd^2 + ae^2) x + cdex^2)^{5/2}} \, dx = \text{Too large to display}$$

input
```
integrate(1/(e*x+d)^2/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2),x, algorithm
="giac")
```

output
```
-2/21*(256*c^4*d^4*e^2*abs(e)*sgn(1/(e*x + d))*sgn(e)/(sqrt(c*d*e)*c^6*d^1
2 - 6*sqrt(c*d*e)*a*c^5*d^10*e^2 + 15*sqrt(c*d*e)*a^2*c^4*d^8*e^4 - 20*sqr
t(c*d*e)*a^3*c^3*d^6*e^6 + 15*sqrt(c*d*e)*a^4*c^2*d^4*e^8 - 6*sqrt(c*d*e)*
a^5*c*d^2*e^10 + sqrt(c*d*e)*a^6*e^12) - e^6*((210*sqrt(c*d*e - c*d^2*e/(e
*x + d) + a*e^3/(e*x + d))*c^39*d^75*e^51*sgn(1/(e*x + d))^6*sgn(e)^6 - 75
60*sqrt(c*d*e - c*d^2*e/(e*x + d) + a*e^3/(e*x + d))*a*c^38*d^73*e^53*sgn(
1/(e*x + d))^6*sgn(e)^6 + 132300*sqrt(c*d*e - c*d^2*e/(e*x + d) + a*e^3/(e
*x + d))*a^2*c^37*d^71*e^55*sgn(1/(e*x + d))^6*sgn(e)^6 - 1499400*sqrt(c*d
*e - c*d^2*e/(e*x + d) + a*e^3/(e*x + d))*a^3*c^36*d^69*e^57*sgn(1/(e*x +
d))^6*sgn(e)^6 + 12370050*sqrt(c*d*e - c*d^2*e/(e*x + d) + a*e^3/(e*x + d)
)*a^4*c^35*d^67*e^59*sgn(1/(e*x + d))^6*sgn(e)^6 - 79168320*sqrt(c*d*e - c
*d^2*e/(e*x + d) + a*e^3/(e*x + d))*a^5*c^34*d^65*e^61*sgn(1/(e*x + d))^6*
sgn(e)^6 + 409036320*sqrt(c*d*e - c*d^2*e/(e*x + d) + a*e^3/(e*x + d))*a^6
*c^33*d^63*e^63*sgn(1/(e*x + d))^6*sgn(e)^6 - 1753012800*sqrt(c*d*e - c*d^
2*e/(e*x + d) + a*e^3/(e*x + d))*a^7*c^32*d^61*e^65*sgn(1/(e*x + d))^6*sgn
(e)^6 + 6354671400*sqrt(c*d*e - c*d^2*e/(e*x + d) + a*e^3/(e*x + d))*a^8*c
^31*d^59*e^67*sgn(1/(e*x + d))^6*sgn(e)^6 - 19770088800*sqrt(c*d*e - c*d^2
*e/(e*x + d) + a*e^3/(e*x + d))*a^9*c^30*d^57*e^69*sgn(1/(e*x + d))^6*sgn(
e)^6 + 53379239760*sqrt(c*d*e - c*d^2*e/(e*x + d) + a*e^3/(e*x + d))*a^10*
c^29*d^55*e^71*sgn(1/(e*x + d))^6*sgn(e)^6 - 126169112160*sqrt(c*d*e - ...
```

Mupad [B] (verification not implemented)

Time = 6.68 (sec) , antiderivative size = 3654, normalized size of antiderivative = 10.38

$$\int \frac{1}{(d+ex)^2 \left(ade+(cd^2+ae^2)\,x+cdex^2\right)^{5/2}}\,dx = \text{Too large to display}$$

input
```
int(1/((d + e*x)^2*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(5/2)),x)
```

output
```
(((d*((12*c^3*d^4*e^4)/(7*(a*e^2 - c*d^2)^3*(5*a^3*e^7 - 5*c^3*d^6*e + 15*
a*c^2*d^4*e^3 - 15*a^2*c*d^2*e^5)) - (2*c^2*d^2*e^4*(19*a*e^2 - 7*c*d^2))/
(7*(a*e^2 - c*d^2)^3*(5*a^3*e^7 - 5*c^3*d^6*e + 15*a*c^2*d^4*e^3 - 15*a^2*
c*d^2*e^5))))/e + (e^3*(14*c^3*d^5 - 42*a*c^2*d^3*e^2 + 40*a^2*c*d*e^4))/(
7*(a*e^2 - c*d^2)^3*(5*a^3*e^7 - 5*c^3*d^6*e + 15*a*c^2*d^4*e^3 - 15*a^2*c
*d^2*e^5)))*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2))/(d + e*x)^3 - (
((d*((24*c^4*d^5*e^4)/(35*(a*e^2 - c*d^2)^6*(3*a*e^3 - 3*c*d^2*e)) - (8*c^
3*d^3*e^4*(11*a*e^2 - 5*c*d^2))/(35*(a*e^2 - c*d^2)^6*(3*a*e^3 - 3*c*d^2*e
))))/e + (2*c^2*d^2*e^3*(19*a^2*e^4 - 13*c^2*d^4 + 6*a*c*d^2*e^2))/(35*(a*
e^2 - c*d^2)^6*(3*a*e^3 - 3*c*d^2*e)))*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*
x^2)^(1/2))/(d + e*x)^2 + (((24*c^4*d^5*e^2)/(35*(a*e^2 - c*d^2)^7) - (4*c
^3*d^3*e^2*(47*a*e^2 - 29*c*d^2))/(105*(a*e^2 - c*d^2)^7))*(x*(a*e^2 + c*d
^2) + a*d*e + c*d*e*x^2)^(1/2))/(d + e*x) - (2*e^3*(x*(a*e^2 + c*d^2) + a*
d*e + c*d*e*x^2)^(1/2))/((d + e*x)^4*(7*a^3*e^7 - 7*c^3*d^6*e + 21*a*c^2*d
^4*e^3 - 21*a^2*c*d^2*e^5)) - ((x*((a*(((a*e^2 + c*d^2)*((8*c^7*d^7*e^5*(a
*e^2 + c*d^2))/(35*(a*e^2 - c*d^2)^6*(c^3*d^5*e - 2*a*c^2*d^3*e^3 + a^2*c*
d*e^5)) - (16*c^7*d^7*e^5*(17*a*e^2 - 5*c*d^2))/(105*(a*e^2 - c*d^2)^6*(c^
3*d^5*e - 2*a*c^2*d^3*e^3 + a^2*c*d*e^5))))/(c*d*e) + (4*c^6*d^6*e^4*(13*a
^2*e^4 - 31*c^2*d^4 + 42*a*c*d^2*e^2))/(35*(a*e^2 - c*d^2)^6*(c^3*d^5*e -
2*a*c^2*d^3*e^3 + a^2*c*d*e^5)) - (16*a*c^7*d^8*e^6)/(35*(a*e^2 - c*d^2...
```

Reduce [B] (verification not implemented)

Time = 0.45 (sec) , antiderivative size = 1384, normalized size of antiderivative = 3.93

$$\int \frac{1}{(d+ex)^2 (ade+(cd^2+ae^2)x+cdex^2)^{5/2}}\,dx = \text{Too large to display}$$

input
```
int(1/(e*x+d)^2/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2),x)
```

output

```
(2*( - 256*sqrt(e)*sqrt(d)*sqrt(c)*sqrt(a*e + c*d*x)*a*c**3*d**7*e**2 - 10
24*sqrt(e)*sqrt(d)*sqrt(c)*sqrt(a*e + c*d*x)*a*c**3*d**6*e**3*x - 1536*sqr
t(e)*sqrt(d)*sqrt(c)*sqrt(a*e + c*d*x)*a*c**3*d**5*e**4*x**2 - 1024*sqrt(e
)*sqrt(d)*sqrt(c)*sqrt(a*e + c*d*x)*a*c**3*d**4*e**5*x**3 - 256*sqrt(e)*sq
rt(d)*sqrt(c)*sqrt(a*e + c*d*x)*a*c**3*d**3*e**6*x**4 - 256*sqrt(e)*sqrt(d
)*sqrt(c)*sqrt(a*e + c*d*x)*c**4*d**8*e*x - 1024*sqrt(e)*sqrt(d)*sqrt(c)*s
qrt(a*e + c*d*x)*c**4*d**7*e**2*x**2 - 1536*sqrt(e)*sqrt(d)*sqrt(c)*sqrt(a
*e + c*d*x)*c**4*d**6*e**3*x**3 - 1024*sqrt(e)*sqrt(d)*sqrt(c)*sqrt(a*e +
c*d*x)*c**4*d**5*e**4*x**4 - 256*sqrt(e)*sqrt(d)*sqrt(c)*sqrt(a*e + c*d*x)
*c**4*d**4*e**5*x**5 - 3*sqrt(d + e*x)*a**5*e**10 + 21*sqrt(d + e*x)*a**4*
c*d**2*e**8 + 6*sqrt(d + e*x)*a**4*c*d*e**9*x - 70*sqrt(d + e*x)*a**3*c**2
*d**4*e**6 - 56*sqrt(d + e*x)*a**3*c**2*d**3*e**7*x - 16*sqrt(d + e*x)*a**
3*c**2*d**2*e**8*x**2 + 210*sqrt(d + e*x)*a**2*c**3*d**6*e**4 + 420*sqrt(d
 + e*x)*a**2*c**3*d**5*e**5*x + 336*sqrt(d + e*x)*a**2*c**3*d**4*e**6*x**2
 + 96*sqrt(d + e*x)*a**2*c**3*d**3*e**7*x**3 + 105*sqrt(d + e*x)*a*c**4*d*
*8*e**2 + 840*sqrt(d + e*x)*a*c**4*d**7*e**3*x + 1680*sqrt(d + e*x)*a*c**4
*d**6*e**4*x**2 + 1344*sqrt(d + e*x)*a*c**4*d**5*e**5*x**3 + 384*sqrt(d +
e*x)*a*c**4*d**4*e**6*x**4 - 7*sqrt(d + e*x)*c**5*d**10 + 70*sqrt(d + e*x)
*c**5*d**9*e*x + 560*sqrt(d + e*x)*c**5*d**8*e**2*x**2 + 1120*sqrt(d + e*x
)*c**5*d**7*e**3*x**3 + 896*sqrt(d + e*x)*c**5*d**6*e**4*x**4 + 256*sqr...
```

3.274 $\int \frac{1}{(d+ex)^3\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}\,dx$

Optimal result

Integrand size = 37, antiderivative size = 413

$$\begin{aligned}
\int \frac{1}{(d+ex)^3\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}\,dx = & -\frac{2}{3\left(cd^2-ae^2\right)(d+ex)^3\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}} \\
& +\frac{8e}{\left(cd^2-ae^2\right)^2(d+ex)^4\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}} \\
& +\frac{80e^2\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{9\left(cd^2-ae^2\right)^3(d+ex)^5}+\frac{640cde^2\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{63\left(cd^2-ae^2\right)^4(d+ex)^4} \\
& +\frac{256c^2d^2e^2\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{21\left(cd^2-ae^2\right)^5(d+ex)^3} \\
& +\frac{1024c^3d^3e^2\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{63\left(cd^2-ae^2\right)^6(d+ex)^2} \\
& +\frac{2048c^4d^4e^2\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{63\left(cd^2-ae^2\right)^7(d+ex)}
\end{aligned}$$

output
```
-2/3/(-a*e^2+c*d^2)/(e*x+d)^3/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)+8*e/
(-a*e^2+c*d^2)^2/(e*x+d)^4/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)+80/9*e^
2*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(-a*e^2+c*d^2)^3/(e*x+d)^5+640/6
3*c*d*e^2*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(-a*e^2+c*d^2)^4/(e*x+d)
^4+256/21*c^2*d^2*e^2*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(-a*e^2+c*d^
2)^5/(e*x+d)^3+1024/63*c^3*d^3*e^2*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)
/(-a*e^2+c*d^2)^6/(e*x+d)^2+2048/63*c^4*d^4*e^2*(a*d*e+(a*e^2+c*d^2)*x+c*d
*e*x^2)^(1/2)/(-a*e^2+c*d^2)^7/(e*x+d)
```

Mathematica [A] (verified)

Time = 0.49 (sec) , antiderivative size = 336, normalized size of antiderivative = 0.81

$$\int \frac{1}{(d+ex)^3\left(ade+(cd^2+ae^2)x+cdex^2\right)^{5/2}}\,dx = \frac{2(7a^6e^{12}-6a^5cde^{10}(9d+2ex)+3a^4c^2d^2e^8(63d^2+36a$$

input
```
Integrate[1/((d + e*x)^3*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2)),x]
```

output
```
(2*(7*a^6*e^12 - 6*a^5*c*d*e^10*(9*d + 2*e*x) + 3*a^4*c^2*d^2*e^8*(63*d^2
+ 36*d*e*x + 8*e^2*x^2) - 4*a^3*c^3*d^3*e^6*(105*d^3 + 126*d^2*e*x + 72*d*
e^2*x^2 + 16*e^3*x^3) + 3*a^2*c^4*d^4*e^4*(315*d^4 + 840*d^3*e*x + 1008*d^
2*e^2*x^2 + 576*d*e^3*x^3 + 128*e^4*x^4) + 6*a*c^5*d^5*e^2*(63*d^5 + 630*d
^4*e*x + 1680*d^3*e^2*x^2 + 2016*d^2*e^3*x^3 + 1152*d*e^4*x^4 + 256*e^5*x^
5) + c^6*d^6*(-21*d^6 + 252*d^5*e*x + 2520*d^4*e^2*x^2 + 6720*d^3*e^3*x^3
+ 8064*d^2*e^4*x^4 + 4608*d*e^5*x^5 + 1024*e^6*x^6)))/(63*(c*d^2 - a*e^2)^
7*(d + e*x)^3*((a*e + c*d*x)*(d + e*x))^(3/2))
```

Rubi [A] (verified)

Time = 0.78 (sec) , antiderivative size = 357, normalized size of antiderivative = 0.86, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.135$, Rules used = {1129, 1129, 1129, 1089, 1088}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(d+ex)^3\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/2}}\,dx$$

$$\Big\downarrow \; 1129$$

$$\frac{4cd\int \frac{1}{(d+ex)^2(cdex^2+(cd^2+ae^2)x+ade)^{5/2}}dx}{3\left(cd^2-ae^2\right)}+\frac{2}{9(d+ex)^3\left(cd^2-ae^2\right)\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}$$

$$\Big\downarrow \; 1129$$

$$\frac{4cd\left(\frac{10cd\int \frac{1}{(d+ex)\left(cdex^2+\left(cd^2+ae^2\right)x+ade\right)^{5/2}}dx}{7(cd^2-ae^2)}+\frac{2}{7(d+ex)^2(cd^2-ae^2)(x(ae^2+cd^2)+ade+cdex^2)^{3/2}}\right)}{3\left(cd^2-ae^2\right)}+\frac{2}{9(d+ex)^3\left(cd^2-ae^2\right)\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}$$

$$\Big\downarrow \; 1129$$

$$\frac{4cd\left(\frac{10cd\left(\frac{8cd\int \frac{1}{\left(cdex^2+\left(cd^2+ae^2\right)x+ade\right)^{5/2}}dx}{5\left(cd^2-ae^2\right)}+\frac{2}{5(d+ex)\left(cd^2-ae^2\right)\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}\right)}{7(cd^2-ae^2)}+\frac{2}{7(d+ex)^2(cd^2-ae^2)(x(ae^2+cd^2)+ade}\right.}{3\left(cd^2-ae^2\right)}$$

$$\frac{2}{9(d+ex)^3\left(cd^2-ae^2\right)\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}$$

$$\Big\downarrow \; 1089$$

$$\frac{4cd\left(\frac{10cd\left(\frac{8cd\left(-\frac{8cde\int \frac{1}{\left(cdex^2+\left(cd^2+ae^2\right)x+ade\right)^{3/2}}dx}{3\left(cd^2-ae^2\right)^2}-\frac{2\left(ae^2+cd^2+2cdex\right)}{3\left(cd^2-ae^2\right)^2\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}\right)}{5\left(cd^2-ae^2\right)}+\frac{2}{5(d+ex)\left(cd^2-ae^2\right)\left(x\left(ae^2+cd^2\right)+ade+c\cdots\right.}\right)}{7(cd^2-ae^2)}}{3\left(cd^2-ae^2\right)}$$

$$\frac{2}{9(d+ex)^3\left(cd^2-ae^2\right)\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}$$

$\big\downarrow$ 1088

$$\frac{4cd\left(\frac{10cd\left(\frac{8cd\left(\frac{16cde\left(ae^2+cd^2+2cdex\right)}{3\left(cd^2-ae^2\right)^4\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}-\frac{2\left(ae^2+cd^2+2cdex\right)}{3\left(cd^2-ae^2\right)^2\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}\right)}{5\left(cd^2-ae^2\right)}+\frac{2}{5(d+ex)\left(cd^2-ae^2\right)\left(x\left(ae^2+cd^2\right)+ade+cde\cdots\right.}\right)}{7(cd^2-ae^2)}}{3\left(cd^2-ae^2\right)}$$

$$\frac{2}{9(d+ex)^3\left(cd^2-ae^2\right)\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}$$

input

```
Int[1/((d + e*x)^3*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2)),x]
```

output

```
2/(9*(c*d^2 - a*e^2)*(d + e*x)^3*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(
3/2)) + (4*c*d*(2/(7*(c*d^2 - a*e^2)*(d + e*x)^2*(a*d*e + (c*d^2 + a*e^2)*
x + c*d*e*x^2)^(3/2)) + (10*c*d*(2/(5*(c*d^2 - a*e^2)*(d + e*x)*(a*d*e + (
c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2)) + (8*c*d*((-2*(c*d^2 + a*e^2 + 2*c*d*
e*x))/(3*(c*d^2 - a*e^2)^2*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2))
+ (16*c*d*e*(c*d^2 + a*e^2 + 2*c*d*e*x))/(3*(c*d^2 - a*e^2)^4*Sqrt[a*d*e +
 (c*d^2 + a*e^2)*x + c*d*e*x^2])))/(5*(c*d^2 - a*e^2))))/(7*(c*d^2 - a*e^2
))))/(3*(c*d^2 - a*e^2))
```

Defintions of rubi rules used

rule 1088
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-3/2), x_Symbol] :> Simp[-2*((b +
2*c*x)/((b^2 - 4*a*c)*Sqrt[a + b*x + c*x^2])), x] /; FreeQ[{a, b, c}, x] &&
 NeQ[b^2 - 4*a*c, 0]
```

rule 1089
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^(p + 1)/((p + 1)*(b^2 - 4*a*c))), x] - Simp[2*c*((2*p +
 3)/((p + 1)*(b^2 - 4*a*c)))   Int[(a + b*x + c*x^2)^(p + 1), x], x] /; Fre
eQ[{a, b, c}, x] && LtQ[p, -1] && (IntegerQ[4*p] || IntegerQ[3*p])
```

rule 1129
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(-e)*(d + e*x)^m*((a + b*x + c*x^2)^(p + 1)/((m + p + 1)*(2*
c*d - b*e))), x] + Simp[c*(Simplify[m + 2*p + 2]/((m + p + 1)*(2*c*d - b*e)
))   Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d
, e, m, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && ILtQ[Simplify[m + 2*p +
2], 0]
```

Maple [A] (verified)

Time = 1.44 (sec) , antiderivative size = 404, normalized size of antiderivative = 0.98

method	result
default	$-\frac{2}{9\left(a\,e^{2}-c\,d^{2}\right)\left(x+\frac{d}{e}\right)^{3}\left(dec\left(x+\frac{d}{e}\right)^{2}+\left(a\,e^{2}-c\,d^{2}\right)\left(x+\frac{d}{e}\right)\right)^{\frac{3}{2}}}-\frac{4dec\left(-\frac{2}{7\left(a\,e^{2}-c\,d^{2}\right)\left(x+\frac{d}{e}\right)^{2}\left(dec\left(x+\frac{d}{e}\right)^{2}+\left(a\,e^{2}-c\,d^{2}\right)\left(x+\frac{d}{e}\right)\right)^{\frac{3}{2}}}-\frac{10dec \ldots}{\ldots}\right)}{\ldots}$
gosper	$-\frac{2(cdx+ae)(1024x^6e^6c^6d^6+1536x^5a\,c^5d^5e^7+4608x^5c^6d^7e^5+384x^4a^2c^4d^4e^8+6912x^4a\,c^5d^6e^6+8064x^4c^6d^8e^4-64x^3a^3c^3d^3e^9+\ldots}{\ldots}$
trager	$-\frac{2(1024x^6e^6c^6d^6+1536x^5a\,c^5d^5e^7+4608x^5c^6d^7e^5+384x^4a^2c^4d^4e^8+6912x^4a\,c^5d^6e^6+8064x^4c^6d^8e^4-64x^3a^3c^3d^3e^9+1728x^3a^2\ldots}{\ldots}$
orering	$-\frac{2(1024x^6e^6c^6d^6+1536x^5a\,c^5d^5e^7+4608x^5c^6d^7e^5+384x^4a^2c^4d^4e^8+6912x^4a\,c^5d^6e^6+8064x^4c^6d^8e^4-64x^3a^3c^3d^3e^9+1728x^3a^2\ldots}{\ldots}$

input
```
int(1/(e*x+d)^3/(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(5/2),x,method=_RETURNVE
RBOSE)
```

output
```
1/e^3*(-2/9/(a*e^2-c*d^2)/(x+d/e)^3/(d*e*c*(x+d/e)^2+(a*e^2-c*d^2)*(x+d/e)
)^(3/2)-4/3*d*e*c/(a*e^2-c*d^2)*(-2/7/(a*e^2-c*d^2)/(x+d/e)^2/(d*e*c*(x+d/
e)^2+(a*e^2-c*d^2)*(x+d/e))^(3/2)-10/7*d*e*c/(a*e^2-c*d^2)*(-2/5/(a*e^2-c*
d^2)/(x+d/e)/(d*e*c*(x+d/e)^2+(a*e^2-c*d^2)*(x+d/e))^(3/2)-8/5*d*e*c/(a*e^
2-c*d^2)*(-2/3*(2*d*e*c*(x+d/e)+a*e^2-c*d^2)/(a*e^2-c*d^2)^2/(d*e*c*(x+d/e
)^2+(a*e^2-c*d^2)*(x+d/e))^(3/2)+16/3*d*e*c/(a*e^2-c*d^2)^4*(2*d*e*c*(x+d/
e)+a*e^2-c*d^2)/(d*e*c*(x+d/e)^2+(a*e^2-c*d^2)*(x+d/e))^(1/2)))))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1392 vs. $2(387) = 774$.

Time = 63.14 (sec) , antiderivative size = 1392, normalized size of antiderivative = 3.37

$$\int \frac{1}{(d+ex)^3 \left(ade+(cd^2+ae^2) x+cdex^2\right)^{5/2}} \, dx = \text{Too large to display}$$

input
```
integrate(1/(e*x+d)^3/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2),x, algorithm
="fricas")
```

output
```
2/63*(1024*c^6*d^6*e^6*x^6 - 21*c^6*d^12 + 378*a*c^5*d^10*e^2 + 945*a^2*c^
4*d^8*e^4 - 420*a^3*c^3*d^6*e^6 + 189*a^4*c^2*d^4*e^8 - 54*a^5*c*d^2*e^10
+ 7*a^6*e^12 + 1536*(3*c^6*d^7*e^5 + a*c^5*d^5*e^7)*x^5 + 384*(21*c^6*d^8*
e^4 + 18*a*c^5*d^6*e^6 + a^2*c^4*d^4*e^8)*x^4 + 64*(105*c^6*d^9*e^3 + 189*
a*c^5*d^7*e^5 + 27*a^2*c^4*d^5*e^7 - a^3*c^3*d^3*e^9)*x^3 + 24*(105*c^6*d^
10*e^2 + 420*a*c^5*d^8*e^4 + 126*a^2*c^4*d^6*e^6 - 12*a^3*c^3*d^4*e^8 + a^
4*c^2*d^2*e^10)*x^2 + 12*(21*c^6*d^11*e + 315*a*c^5*d^9*e^3 + 210*a^2*c^4*
d^7*e^5 - 42*a^3*c^3*d^5*e^7 + 9*a^4*c^2*d^3*e^9 - a^5*c*d*e^11)*x)*sqrt(c
*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)/(a^2*c^7*d^19*e^2 - 7*a^3*c^6*d^17*e
^4 + 21*a^4*c^5*d^15*e^6 - 35*a^5*c^4*d^13*e^8 + 35*a^6*c^3*d^11*e^10 - 21
*a^7*c^2*d^9*e^12 + 7*a^8*c*d^7*e^14 - a^9*d^5*e^16 + (c^9*d^16*e^5 - 7*a*
c^8*d^14*e^7 + 21*a^2*c^7*d^12*e^9 - 35*a^3*c^6*d^10*e^11 + 35*a^4*c^5*d^8
*e^13 - 21*a^5*c^4*d^6*e^15 + 7*a^6*c^3*d^4*e^17 - a^7*c^2*d^2*e^19)*x^7 +
 (5*c^9*d^17*e^4 - 33*a*c^8*d^15*e^6 + 91*a^2*c^7*d^13*e^8 - 133*a^3*c^6*d
^11*e^10 + 105*a^4*c^5*d^9*e^12 - 35*a^5*c^4*d^7*e^14 - 7*a^6*c^3*d^5*e^16
 + 9*a^7*c^2*d^3*e^18 - 2*a^8*c*d*e^20)*x^6 + (10*c^9*d^18*e^3 - 60*a*c^8*
d^16*e^5 + 141*a^2*c^7*d^14*e^7 - 147*a^3*c^6*d^12*e^9 + 21*a^4*c^5*d^10*e
^11 + 105*a^5*c^4*d^8*e^13 - 105*a^6*c^3*d^6*e^15 + 39*a^7*c^2*d^4*e^17 -
3*a^8*c*d^2*e^19 - a^9*e^21)*x^5 + 5*(2*c^9*d^19*e^2 - 10*a*c^8*d^17*e^4 +
 15*a^2*c^7*d^15*e^6 + 7*a^3*c^6*d^13*e^8 - 49*a^4*c^5*d^11*e^10 + 63*a...
```

Sympy [F]

$$\int \frac{1}{(d+ex)^3 (ade+(cd^2+ae^2)x+cdex^2)^{5/2}}\,dx = \int \frac{1}{((d+ex)(ae+cdx))^{\frac{5}{2}}(d+ex)^3}\,dx$$

input
```
integrate(1/(e*x+d)**3/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(5/2),x)
```

output
```
Integral(1/(((d + e*x)*(a*e + c*d*x))**(5/2)*(d + e*x)**3), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{(d+ex)^3 (ade+(cd^2+ae^2)x+cdex^2)^{5/2}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(1/(e*x+d)^3/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2),x, algorithm
="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e*(a*e^2-c*d^2)>0)', see `assume
?` for mor
```

Giac [F]

$$\int \frac{1}{(d+ex)^3 (ade+(cd^2+ae^2)x+cdex^2)^{5/2}}\,dx = \int \frac{1}{(cdex^2+ade+(cd^2+ae^2)x)^{\frac{5}{2}}(ex+d)^3}\,dx$$

input
```
integrate(1/(e*x+d)^3/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2),x, algorithm
="giac")
```

output
```
integrate(1/((c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^(5/2)*(e*x + d)^3), x
)
```

Mupad [B] (verification not implemented)

Time = 8.62 (sec) , antiderivative size = 10949, normalized size of antiderivative = 26.51

$$\int \frac{1}{(d+ex)^3 \left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}\,dx = \text{Too large to display}$$

input
```
int(1/((d + e*x)^3*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(5/2)),x)
```

output
```
(((d*((d*((64*c^7*d^8*e^4)/(315*(a*e^2 - c*d^2)^10) - (8*c^6*d^6*e^4*(49*a
*e^2 - 25*c*d^2))/(315*(a*e^2 - c*d^2)^10)))/e + (8*c^5*d^5*e^3*(23*a^2*e^
4 - 51*c^2*d^4 + 52*a*c*d^2*e^2))/(315*(a*e^2 - c*d^2)^10)))/e + (2*c^4*d^
4*e^2*(439*a^3*e^6 - 367*c^3*d^6 + 1305*a*c^2*d^4*e^2 - 1409*a^2*c*d^2*e^4
))/(315*(a*e^2 - c*d^2)^10))*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2)
)/(d + e*x) - (((d*((32*c^4*d^5*e^4)/(63*(a*e^2 - c*d^2)^6*(5*a*e^3 - 5*c*
d^2*e)) - (4*c^3*d^3*e^4*(29*a*e^2 - 13*c*d^2))/(63*(a*e^2 - c*d^2)^6*(5*a
*e^3 - 5*c*d^2*e))))/e + (2*c^2*d^2*e^3*(25*a^2*e^4 - 17*c^2*d^4 + 8*a*c*d
^2*e^2))/(63*(a*e^2 - c*d^2)^6*(5*a*e^3 - 5*c*d^2*e)))*(x*(a*e^2 + c*d^2)
+ a*d*e + c*d*e*x^2)^(1/2))/(d + e*x)^3 + (((d*((16*c^3*d^4*e^4)/(9*(a*e^2
 - c*d^2)^3*(7*a^3*e^7 - 7*c^3*d^6*e + 21*a*c^2*d^4*e^3 - 21*a^2*c*d^2*e^5
)) - (2*c^2*d^2*e^4*(25*a*e^2 - 9*c*d^2))/(9*(a*e^2 - c*d^2)^3*(7*a^3*e^7
- 7*c^3*d^6*e + 21*a*c^2*d^4*e^3 - 21*a^2*c*d^2*e^5))))/e + (e^3*(18*c^3*d
^5 - 54*a*c^2*d^3*e^2 + 52*a^2*c*d*e^4))/(9*(a*e^2 - c*d^2)^3*(7*a^3*e^7 -
 7*c^3*d^6*e + 21*a*c^2*d^4*e^3 - 21*a^2*c*d^2*e^5)))*(x*(a*e^2 + c*d^2) +
 a*d*e + c*d*e*x^2)^(1/2))/(d + e*x)^4 + (((d*((d*((d*((32*c^6*d^7*e^6)/(6
3*(a*e^2 - c*d^2)^6*(5*a^3*e^7 - 5*c^3*d^6*e + 15*a*c^2*d^4*e^3 - 15*a^2*c
*d^2*e^5)) - (4*c^5*d^5*e^6*(45*a*e^2 - 13*c*d^2))/(63*(a*e^2 - c*d^2)^6*(
5*a^3*e^7 - 5*c^3*d^6*e + 15*a*c^2*d^4*e^3 - 15*a^2*c*d^2*e^5))))/e + (2*c
^4*d^4*e^5*(a^2*e^4 - 173*c^2*d^4 + 268*a*c*d^2*e^2))/(63*(a*e^2 - c*d^...
```

Reduce [B] (verification not implemented)

Time = 0.78 (sec) , antiderivative size = 1820, normalized size of antiderivative = 4.41

$$\int \frac{1}{(d+ex)^3 \left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}\,dx = \text{Too large to display}$$

input
```
int(1/(e*x+d)^3/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2),x)
```

output
```
(2*(1024*sqrt(e)*sqrt(d)*sqrt(c)*sqrt(a*e + c*d*x)*a*c**4*d**9*e**2 + 5120
*sqrt(e)*sqrt(d)*sqrt(c)*sqrt(a*e + c*d*x)*a*c**4*d**8*e**3*x + 10240*sqrt
(e)*sqrt(d)*sqrt(c)*sqrt(a*e + c*d*x)*a*c**4*d**7*e**4*x**2 + 10240*sqrt(e
)*sqrt(d)*sqrt(c)*sqrt(a*e + c*d*x)*a*c**4*d**6*e**5*x**3 + 5120*sqrt(e)*s
qrt(d)*sqrt(c)*sqrt(a*e + c*d*x)*a*c**4*d**5*e**6*x**4 + 1024*sqrt(e)*sqrt
(d)*sqrt(c)*sqrt(a*e + c*d*x)*a*c**4*d**4*e**7*x**5 + 1024*sqrt(e)*sqrt(d)
*sqrt(c)*sqrt(a*e + c*d*x)*c**5*d**10*e*x + 5120*sqrt(e)*sqrt(d)*sqrt(c)*s
qrt(a*e + c*d*x)*c**5*d**9*e**2*x**2 + 10240*sqrt(e)*sqrt(d)*sqrt(c)*sqrt(
a*e + c*d*x)*c**5*d**8*e**3*x**3 + 10240*sqrt(e)*sqrt(d)*sqrt(c)*sqrt(a*e
+ c*d*x)*c**5*d**7*e**4*x**4 + 5120*sqrt(e)*sqrt(d)*sqrt(c)*sqrt(a*e + c*d
*x)*c**5*d**6*e**5*x**5 + 1024*sqrt(e)*sqrt(d)*sqrt(c)*sqrt(a*e + c*d*x)*c
**5*d**5*e**6*x**6 - 7*sqrt(d + e*x)*a**6*e**12 + 54*sqrt(d + e*x)*a**5*c*
d**2*e**10 + 12*sqrt(d + e*x)*a**5*c*d*e**11*x - 189*sqrt(d + e*x)*a**4*c*
*2*d**4*e**8 - 108*sqrt(d + e*x)*a**4*c**2*d**3*e**9*x - 24*sqrt(d + e*x)*
a**4*c**2*d**2*e**10*x**2 + 420*sqrt(d + e*x)*a**3*c**3*d**6*e**6 + 504*sq
rt(d + e*x)*a**3*c**3*d**5*e**7*x + 288*sqrt(d + e*x)*a**3*c**3*d**4*e**8*
x**2 + 64*sqrt(d + e*x)*a**3*c**3*d**3*e**9*x**3 - 945*sqrt(d + e*x)*a**2*
c**4*d**8*e**4 - 2520*sqrt(d + e*x)*a**2*c**4*d**7*e**5*x - 3024*sqrt(d +
e*x)*a**2*c**4*d**6*e**6*x**2 - 1728*sqrt(d + e*x)*a**2*c**4*d**5*e**7*x**
3 - 384*sqrt(d + e*x)*a**2*c**4*d**4*e**8*x**4 - 378*sqrt(d + e*x)*a*c*...
```

3.275 $\int \frac{1+x}{(2+3x+x^2)^{3/2}} \, dx$

Optimal result

Integrand size = 16, antiderivative size = 17

$$\int \frac{1+x}{(2+3x+x^2)^{3/2}} \, dx = \frac{2(1+x)}{\sqrt{2+3x+x^2}}$$

output
```
2*(1+x)/(x^2+3*x+2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.12

$$\int \frac{1+x}{(2+3x+x^2)^{3/2}} \, dx = \frac{2\sqrt{2+3x+x^2}}{2+x}$$

input
```
Integrate[(1 + x)/(2 + 3*x + x^2)^(3/2),x]
```

output
```
(2*Sqrt[2 + 3*x + x^2])/(2 + x)
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {1124, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x+1}{(x^2+3x+2)^{3/2}}\,dx$$

$$\Big\downarrow \text{ 1124}$$

$$\int 0dx+\frac{2(x+1)}{\sqrt{x^2+3x+2}}$$

$$\Big\downarrow \text{ 24}$$

$$\frac{2(x+1)}{\sqrt{x^2+3x+2}}$$

input
```
Int[(1 + x)/(2 + 3*x + x^2)^(3/2),x]
```

output
```
(2*(1 + x))/Sqrt[2 + 3*x + x^2]
```

Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 1124
```
Int[((d_.) + (e_.)*(x_))^(m_.)/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(3/2), x
_Symbol] :> Simp[-2*e*(2*c*d - b*e)^(m - 2)*((d + e*x)/(c^(m - 1)*Sqrt[a +
b*x + c*x^2])), x] + Simp[e^2/c^(m - 1)   Int[(1/Sqrt[a + b*x + c*x^2])*Exp
andToSum[((2*c*d - b*e)^(m - 1) - c^(m - 1)*(d + e*x)^(m - 1))/(c*d - b*e -
 c*e*x), x], x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c*d^2 - b*d*e + a*e
^2, 0] && IGtQ[m, 0]
```

Maple [A] (verified)

Time = 0.66 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.94

method	result	size
risch	$\frac{2+2x}{\sqrt{x^2+3x+2}}$	16
trager	$\frac{2\sqrt{x^2+3x+2}}{2+x}$	18
gosper	$\frac{2(2+x)(x+1)^2}{(x^2+3x+2)^{\frac{3}{2}}}$	21
orering	$\frac{2(2+x)(x+1)^2}{(x^2+3x+2)^{\frac{3}{2}}}$	21
default	$\frac{2x+3}{\sqrt{x^2+3x+2}}-\frac{1}{\sqrt{x^2+3x+2}}$	30

input
```
int((x+1)/(x^2+3*x+2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
2*(x+1)/(x^2+3*x+2)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.18

$$\int \frac{1+x}{(2+3x+x^2)^{3/2}}\,dx=\frac{2\left(x+\sqrt{x^2+3\,x+2}+2\right)}{x+2}$$

input
```
integrate((1+x)/(x^2+3*x+2)^(3/2),x, algorithm="fricas")
```

output
```
2*(x + sqrt(x^2 + 3*x + 2) + 2)/(x + 2)
```

Sympy [F]

$$\int \frac{1+x}{(2+3x+x^2)^{3/2}}\,dx = \int \frac{x+1}{((x+1)(x+2))^{\frac{3}{2}}}\,dx$$

input
```
integrate((1+x)/(x**2+3*x+2)**(3/2),x)
```

output
```
Integral((x + 1)/((x + 1)*(x + 2))**(3/2), x)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.53

$$\int \frac{1+x}{(2+3x+x^2)^{3/2}}\,dx = \frac{2\,x}{\sqrt{x^2+3\,x+2}} + \frac{2}{\sqrt{x^2+3\,x+2}}$$

input
```
integrate((1+x)/(x^2+3*x+2)^(3/2),x, algorithm="maxima")
```

output
```
2*x/sqrt(x^2 + 3*x + 2) + 2/sqrt(x^2 + 3*x + 2)
```

Giac [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.12

$$\int \frac{1+x}{(2+3x+x^2)^{3/2}}\,dx = \frac{2}{x-\sqrt{x^2+3\,x+2}+2}$$

input
```
integrate((1+x)/(x^2+3*x+2)^(3/2),x, algorithm="giac")
```

output
```
2/(x - sqrt(x^2 + 3*x + 2) + 2)
```

Mupad [B] (verification not implemented)

Time = 5.26 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00

$$\int \frac{1+x}{(2+3x+x^2)^{3/2}}\,dx = \frac{2\,\sqrt{x^2+3\,x+2}}{x+2}$$

input
```
int((x + 1)/(3*x + x^2 + 2)^(3/2),x)
```

output
```
(2*(3*x + x^2 + 2)^(1/2))/(x + 2)
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.12

$$\int \frac{1+x}{(2+3x+x^2)^{3/2}}\,dx = \frac{2\sqrt{x^2+3x+2}+2x+4}{x+2}$$

input
```
int((1+x)/(x^2+3*x+2)^(3/2),x)
```

output
```
(2*(sqrt(x**2 + 3*x + 2) + x + 2))/(x + 2)
```

3.276 $\int \frac{1}{(d+ex)\sqrt{\frac{-cd^2+bde}{e^2}+bx+cx^2}}\,dx$

Optimal result

Integrand size = 36, antiderivative size = 48

$$\int \frac{1}{(d+ex)\sqrt{\frac{-cd^2+bde}{e^2}+bx+cx^2}}\,dx = \frac{2e\sqrt{-\frac{d(cd-be)}{e^2}+bx+cx^2}}{(2cd-be)(d+ex)}$$

output
```
2*e*(-d*(-b*e+c*d)/e^2+b*x+c*x^2)^(1/2)/(-b*e+2*c*d)/(e*x+d)
```

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.94

$$\int \frac{1}{(d+ex)\sqrt{\frac{-cd^2+bde}{e^2}+bx+cx^2}}\,dx = -\frac{2e\sqrt{\frac{(d+ex)(-cd+be+cex)}{e^2}}}{(-2cd+be)(d+ex)}$$

input
```
Integrate[1/((d + e*x)*Sqrt[(-(c*d^2) + b*d*e)/e^2 + b*x + c*x^2]),x]
```

output
```
(-2*e*Sqrt[((d + e*x)*(-(c*d) + b*e + c*e*x))/e^2])/((-2*c*d + b*e)*(d + e
*x))
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.028, Rules used = {1123}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(d+ex)\sqrt{\frac{bde-cd^2}{e^2}+bx+cx^2}}dx$$

$$\Big\downarrow \text{ 1123}$$

$$\frac{2e\sqrt{-\frac{d(cd-be)}{e^2}+bx+cx^2}}{(d+ex)(2cd-be)}$$

input
```
Int[1/((d + e*x)*Sqrt[(-(c*d^2) + b*d*e)/e^2 + b*x + c*x^2]),x]
```

output
```
(2*e*Sqrt[-((d*(c*d - b*e))/e^2) + b*x + c*x^2])/((2*c*d - b*e)*(d + e*x))
```

Defintions of rubi rules used

rule 1123
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[e*(d + e*x)^m*((a + b*x + c*x^2)^(p + 1)/((p + 1)*(2*c*d - b
*e))), x] /; FreeQ[{a, b, c, d, e, m, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2,
0] && EqQ[m + 2*p + 2, 0]
```

Maple [A] (verified)

Time = 1.28 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.12

method	result	size
orering	$-\frac{2(cex+be-cd)}{e(be-2cd)\sqrt{\frac{bde-c\,d^{2}}{e^{2}}+bx+c\,x^{2}}}$	54
default	$-\frac{2\sqrt{c\left(x+\frac{d}{e}\right)^{2}+\frac{(be-2cd)\left(x+\frac{d}{e}\right)}{e}}}{(be-2cd)\left(x+\frac{d}{e}\right)}$	55
trager	$-\frac{2e\sqrt{-\frac{-x^{2}c\,e^{2}-xb\,e^{2}-bde+c\,d^{2}}{e^{2}}}}{(be-2cd)(ex+d)}$	55
gosper	$-\frac{2(cex+be-cd)}{e(be-2cd)\sqrt{\frac{x^{2}c\,e^{2}+xb\,e^{2}+bde-c\,d^{2}}{e^{2}}}}$	59

input
```
int(1/(e*x+d)/((b*d*e-c*d^2)/e^2+b*x+c*x^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-2*(c*e*x+b*e-c*d)/e/(b*e-2*c*d)/((b*d*e-c*d^2)/e^2+b*x+c*x^2)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.29

$$\int \frac{1}{(d+ex)\sqrt{\frac{-cd^2+bde}{e^2}+bx+cx^2}}\,dx = \frac{2\,e\sqrt{\frac{ce^2x^2+be^2x-cd^2+bde}{e^2}}}{2\,cd^2-bde+(2\,cde-be^2)x}$$

input
```
integrate(1/(e*x+d)/((b*d*e-c*d^2)/e^2+b*x+c*x^2)^(1/2),x, algorithm="fric
as")
```

output
```
2*e*sqrt((c*e^2*x^2 + b*e^2*x - c*d^2 + b*d*e)/e^2)/(2*c*d^2 - b*d*e + (2*
c*d*e - b*e^2)*x)
```

Sympy [F]

$$\int \frac{1}{(d+ex)\sqrt{\frac{-cd^2+bde}{e^2}+bx+cx^2}}\,dx = \int \frac{1}{\sqrt{\left(\frac{d}{e}+x\right)\left(b-\frac{cd}{e}+cx\right)}\,(d+ex)}\,dx$$

input
```
integrate(1/(e*x+d)/((b*d*e-c*d**2)/e**2+b*x+c*x**2)**(1/2),x)
```

output
```
Integral(1/(sqrt((d/e + x)*(b - c*d/e + c*x))*(d + e*x)), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{(d+ex)\sqrt{\frac{-cd^2+bde}{e^2}+bx+cx^2}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(1/(e*x+d)/((b*d*e-c*d^2)/e^2+b*x+c*x^2)^(1/2),x, algorithm="maxi
ma")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(b*e-2*c*d>0)', see `assume?` for
 more deta
```

Giac [F]

$$\int \frac{1}{(d+ex)\sqrt{\frac{-cd^2+bde}{e^2}+bx+cx^2}}\,dx = \int \frac{1}{\sqrt{cx^2+bx-\frac{cd^2-bde}{e^2}}\,(ex+d)}\,dx$$

input
```
integrate(1/(e*x+d)/((b*d*e-c*d^2)/e^2+b*x+c*x^2)^(1/2),x, algorithm="giac
")
```

output
```
undef
```

Mupad [B] (verification not implemented)

Time = 5.27 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.98

$$\int \frac{1}{(d+ex)\sqrt{\frac{-cd^2+bde}{e^2}+bx+cx^2}}\,dx = -\frac{2\,e\,\sqrt{b\,x-\frac{c\,d^2-b\,d\,e}{e^2}+c\,x^2}}{(b\,e-2\,c\,d)\,(d+e\,x)}$$

input
```
int(1/((d + e*x)*(b*x - (c*d^2 - b*d*e)/e^2 + c*x^2)^(1/2)),x)
```

output
```
-(2*e*(b*x - (c*d^2 - b*d*e)/e^2 + c*x^2)^(1/2))/((b*e - 2*c*d)*(d + e*x))
```

Reduce [B] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.23

$$\int \frac{1}{(d+ex)\sqrt{\frac{-cd^2+bde}{e^2}+bx+cx^2}}\,dx = \frac{-2\sqrt{ex+d}\,\sqrt{cex+be-cd}-2\sqrt{c}\,d-2\sqrt{c}\,ex}{b\,e^2x-2cdex+bde-2c\,d^2}$$

input
```
int(1/(e*x+d)/((b*d*e-c*d^2)/e^2+b*x+c*x^2)^(1/2),x)
```

output
```
( - 2*(sqrt(d + e*x)*sqrt(b*e - c*d + c*e*x) + sqrt(c)*d + sqrt(c)*e*x))/(
b*d*e + b*e**2*x - 2*c*d**2 - 2*c*d*e*x)
```

3.277 $\int \frac{-1+x}{\sqrt{3-4x+x^2}} \, dx$

Optimal result

Integrand size = 16, antiderivative size = 34

$$\int \frac{-1+x}{\sqrt{3-4x+x^2}} \, dx = \sqrt{3-4x+x^2} - 2\text{arctanh}\left(\frac{1-x}{\sqrt{3-4x+x^2}}\right)$$

output
```
(x^2-4*x+3)^(1/2)-2*arctanh((1-x)/(x^2-4*x+3)^(1/2))
```

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.00

$$\int \frac{-1+x}{\sqrt{3-4x+x^2}} \, dx = \sqrt{3-4x+x^2} + 2\text{arctanh}\left(\frac{\sqrt{3-4x+x^2}}{-3+x}\right)$$

input
```
Integrate[(-1 + x)/Sqrt[3 - 4*x + x^2],x]
```

output
```
Sqrt[3 - 4*x + x^2] + 2*ArcTanh[Sqrt[3 - 4*x + x^2]/(-3 + x)]
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.188$, Rules used = {1160, 1092, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x-1}{\sqrt{x^2-4x+3}}dx$$

$$\Big\downarrow \text{1160}$$

$$\int \frac{1}{\sqrt{x^2-4x+3}}dx+\sqrt{x^2-4x+3}$$

$$\Big\downarrow \text{1092}$$

$$2\int \frac{1}{4-\frac{4(2-x)^2}{x^2-4x+3}}d\left(-\frac{2(2-x)}{\sqrt{x^2-4x+3}}\right)+\sqrt{x^2-4x+3}$$

$$\Big\downarrow \text{219}$$

$$\sqrt{x^2-4x+3}-\text{arctanh}\left(\frac{2-x}{\sqrt{x^2-4x+3}}\right)$$

input
```
Int[(-1 + x)/Sqrt[3 - 4*x + x^2],x]
```

output
```
Sqrt[3 - 4*x + x^2] - ArcTanh[(2 - x)/Sqrt[3 - 4*x + x^2]]
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1092
```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[I
nt[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a
, b, c}, x]
```

rule 1160
```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[e*((a + b*x + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Simp[(2*c*d - b
*e)/(2*c)   Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
 && NeQ[p, -1]
```

Maple [A] (verified)

Time = 0.67 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.76

method	result	size
default	$\sqrt{x^2-4x+3}+\ln\left(x-2+\sqrt{x^2-4x+3}\right)$	26
risch	$\sqrt{x^2-4x+3}+\ln\left(x-2+\sqrt{x^2-4x+3}\right)$	26
trager	$\sqrt{x^2-4x+3}-\ln\left(\sqrt{x^2-4x+3}+2-x\right)$	30

input
```
int((x-1)/(x^2-4*x+3)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
(x^2-4*x+3)^(1/2)+ln(x-2+(x^2-4*x+3)^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.85

$$\int \frac{-1+x}{\sqrt{3-4x+x^2}}\,dx=\sqrt{x^2-4\,x+3}-\log\left(-x+\sqrt{x^2-4\,x+3}+2\right)$$

input
```
integrate((x-1)/(x^2-4*x+3)^(1/2),x, algorithm="fricas")
```

output
```
sqrt(x^2 - 4*x + 3) - log(-x + sqrt(x^2 - 4*x + 3) + 2)
```

Sympy [A] (verification not implemented)

Time = 0.35 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.91

$$\int \frac{-1+x}{\sqrt{3-4x+x^2}}\,dx = \sqrt{x^2-4x+3} + \log\left(2x + 2\sqrt{x^2-4x+3} - 4\right)$$

input
```
integrate((x-1)/(x**2-4*x+3)**(1/2),x)
```

output
```
sqrt(x**2 - 4*x + 3) + log(2*x + 2*sqrt(x**2 - 4*x + 3) - 4)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.85

$$\int \frac{-1+x}{\sqrt{3-4x+x^2}}\,dx = \sqrt{x^2-4\,x+3} + \log\left(2\,x + 2\,\sqrt{x^2-4\,x+3} - 4\right)$$

input
```
integrate((x-1)/(x^2-4*x+3)^(1/2),x, algorithm="maxima")
```

output
```
sqrt(x^2 - 4*x + 3) + log(2*x + 2*sqrt(x^2 - 4*x + 3) - 4)
```

Giac [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.88

$$\int \frac{-1+x}{\sqrt{3-4x+x^2}}\,dx = \sqrt{x^2-4\,x+3} - \log\left(\left|-x + \sqrt{x^2-4\,x+3} + 2\right|\right)$$

input
```
integrate((x-1)/(x^2-4*x+3)^(1/2),x, algorithm="giac")
```

output
```
sqrt(x^2 - 4*x + 3) - log(abs(-x + sqrt(x^2 - 4*x + 3) + 2))
```

Mupad [B] (verification not implemented)

Time = 5.45 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.74

$$\int \frac{-1+x}{\sqrt{3-4x+x^2}}\,dx = \ln\left(x+\sqrt{x^2-4\,x+3}-2\right)+\sqrt{x^2-4\,x+3}$$

input
```
int((x - 1)/(x^2 - 4*x + 3)^(1/2),x)
```

output
```
log(x + (x^2 - 4*x + 3)^(1/2) - 2) + (x^2 - 4*x + 3)^(1/2)
```

Reduce [B] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.68

$$\int \frac{-1+x}{\sqrt{3-4x+x^2}}\,dx = \sqrt{x^2-4x+3}+\log\left(\sqrt{x^2-4x+3}+x-2\right)$$

input
```
int((x-1)/(x^2-4*x+3)^(1/2),x)
```

output
```
sqrt(x**2 - 4*x + 3) + log(sqrt(x**2 - 4*x + 3) + x - 2)
```

3.278 $\int (d+ex)^{7/2}\sqrt{ade+(cd^2+ae^2)x+cdex^2}\,dx$

Optimal result

Integrand size = 39, antiderivative size = 305

$$\int (d+ex)^{7/2}\sqrt{ade+(cd^2+ae^2)x+cdex^2}\,dx=\frac{2(cd^2-ae^2)^4(ade+(cd^2+ae^2)x+cdex^2)^{3/2}}{3c^5d^5(d+ex)^{3/2}}+\frac{8e(cd^2-ae^2)^3(ade+(cd^2+ae^2)x+cdex^2)^{5/2}}{5c^5d^5(d+ex)^{5/2}}+\frac{12e^2(cd^2-ae^2)^2(ade+(cd^2+ae^2)x+cdex^2)^{7/2}}{7c^5d^5(d+ex)^{7/2}}+\frac{8e^3(cd^2-ae^2)(ade+(cd^2+ae^2)x+cdex^2)^{9/2}}{9c^5d^5(d+ex)^{9/2}}+\frac{2e^4(ade+(cd^2+ae^2)x+cdex^2)^{11/2}}{11c^5d^5(d+ex)^{11/2}}$$

output
```
2/3*(-a*e^2+c*d^2)^4*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/c^5/d^5/(e*x+
d)^(3/2)+8/5*e*(-a*e^2+c*d^2)^3*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/c^
5/d^5/(e*x+d)^(5/2)+12/7*e^2*(-a*e^2+c*d^2)^2*(a*d*e+(a*e^2+c*d^2)*x+c*d*e
*x^2)^(7/2)/c^5/d^5/(e*x+d)^(7/2)+8/9*e^3*(-a*e^2+c*d^2)*(a*d*e+(a*e^2+c*d
^2)*x+c*d*e*x^2)^(9/2)/c^5/d^5/(e*x+d)^(9/2)+2/11*e^4*(a*d*e+(a*e^2+c*d^2)
*x+c*d*e*x^2)^(11/2)/c^5/d^5/(e*x+d)^(11/2)
```

Mathematica [A] (verified)

Time = 0.16 (sec) , antiderivative size = 187, normalized size of antiderivative = 0.61

$$\int (d+ex)^{7/2}\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}\,dx=\frac{2((ae+cdx)(d+ex))^{3/2}\,(128a^4e^8-64a^3cde^6(11d+3ex)+48\cdots}{\cdots}$$

input
```
Integrate[(d + e*x)^(7/2)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2],x]
```

output
```
(2*((a*e + c*d*x)*(d + e*x))^(3/2)*(128*a^4*e^8 - 64*a^3*c*d*e^6*(11*d + 3
*e*x) + 48*a^2*c^2*d^2*e^4*(33*d^2 + 22*d*e*x + 5*e^2*x^2) - 8*a*c^3*d^3*e
^2*(231*d^3 + 297*d^2*e*x + 165*d*e^2*x^2 + 35*e^3*x^3) + c^4*d^4*(1155*d^
4 + 2772*d^3*e*x + 2970*d^2*e^2*x^2 + 1540*d*e^3*x^3 + 315*e^4*x^4)))/(346
5*c^5*d^5*(d + e*x)^(3/2))
```

Rubi [A] (verified)

Time = 0.82 (sec) , antiderivative size = 317, normalized size of antiderivative = 1.04, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.128, Rules used = {1128, 1128, 1128, 1128, 1122}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (d+ex)^{7/2}\sqrt{x\,(ae^2+cd^2)+ade+cdex^2}\,dx$$

$$\Big\downarrow \ 1128$$

$$\frac{8\left(d^2-\frac{ae^2}{c}\right)\int (d+ex)^{5/2}\sqrt{cdex^2+(cd^2+ae^2)\,x+ade}dx}{11d}+\frac{2(d+ex)^{5/2}\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{11cd}$$

$$\Big\downarrow \ 1128$$

$$\frac{8\left(d^2-\frac{ae^2}{c}\right)\left(\frac{2\left(d^2-\frac{ae^2}{c}\right)\int (d+ex)^{3/2}\sqrt{cdex^2+(cd^2+ae^2)x+ade}dx}{3d}+\frac{2(d+ex)^{3/2}\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{9cd}\right)}{11d}+\frac{2(d+ex)^{5/2}\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{11cd}$$

$\downarrow$ 1128

$$\frac{8\left(d^2-\frac{ae^2}{c}\right)\left(\frac{2\left(d^2-\frac{ae^2}{c}\right)\left(\frac{4\left(d^2-\frac{ae^2}{c}\right)\int \sqrt{d+ex}\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}dx}{7d}+\frac{2\sqrt{d+ex}\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{7cd}\right)}{3d}+\frac{2(d+ex)^{3/2}(x(ae^2}{}\right.}{11d}$$
$$\frac{2(d+ex)^{5/2}\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{11cd}$$

$\downarrow$ 1128

$$\frac{8\left(d^2-\frac{ae^2}{c}\right)\left(\frac{2\left(d^2-\frac{ae^2}{c}\right)\left(\frac{4\left(d^2-\frac{ae^2}{c}\right)\left(\frac{2\left(d^2-\frac{ae^2}{c}\right)\int \frac{\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}{\sqrt{d+ex}}dx}{5d}+\frac{2\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{5cd\sqrt{d+ex}}\right)}{7d}+\frac{2\sqrt{d+ex}\left(x\left(ae^2+cd^2\right.\right.}{7cd}\right.}{3d}\right.}{11d}$$
$$\frac{2(d+ex)^{5/2}\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{11cd}$$

$\downarrow$ 1122

$$\frac{2(d+ex)^{5/2}\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{11cd}+$$

$$\frac{8\left(d^2-\frac{ae^2}{c}\right)\left(\frac{2(d+ex)^{3/2}\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{9cd}+\frac{2\left(d^2-\frac{ae^2}{c}\right)\left(\frac{2\sqrt{d+ex}\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{7cd}+\frac{4\left(d^2-\frac{ae^2}{c}\right)\left(\frac{2\left(x\left(ae^2+cd^2\right)\cdots\right.}{5c\cdots}\cdots\right.}{\cdots}\right.}{3d}\cdots\right.}{11d}\cdots$$

input
```
Int[(d + e*x)^(7/2)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2],x]
```

output
```
(2*(d + e*x)^(5/2)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2))/(11*c*d)
 + (8*(d^2 - (a*e^2)/c)*((2*(d + e*x)^(3/2)*(a*d*e + (c*d^2 + a*e^2)*x + c
*d*e*x^2)^(3/2))/(9*c*d) + (2*(d^2 - (a*e^2)/c)*((2*Sqrt[d + e*x]*(a*d*e +
 (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2))/(7*c*d) + (4*(d^2 - (a*e^2)/c)*((4*
(d^2 - (a*e^2)/c)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2))/(15*c*d^2
*(d + e*x)^(3/2)) + (2*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2))/(5*c
*d*Sqrt[d + e*x])))/(7*d)))/(3*d)))/(11*d)
```

Defintions of rubi rules used

rule 1122
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[e*(d + e*x)^(m - 1)*((a + b*x + c*x^2)^(p + 1)/(c*(p + 1))),
 x] /; FreeQ[{a, b, c, d, e, m, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&
EqQ[m + p, 0]
```

rule 1128
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[e*(d + e*x)^(m - 1)*((a + b*x + c*x^2)^(p + 1)/(c*(m + 2*p +
 1))), x] + Simp[Simplify[m + p]*((2*c*d - b*e)/(c*(m + 2*p + 1)))   Int[(d
 + e*x)^(m - 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p},
 x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && IGtQ[Simplify[m + p], 0]
```

Maple [A] (verified)

Time = 1.19 (sec) , antiderivative size = 233, normalized size of antiderivative = 0.76

method	result
default	$\frac{2(cdx+ae)(315c^4d^4e^4x^4-280a\,c^3d^3e^5x^3+1540c^4d^5e^3x^3+240a^2c^2d^2e^6x^2-1320a\,c^3d^4e^4x^2+2970c^4d^6e^2x^2-192a^3cd\,e^7x+1056a^2}{3465d^5c^5\sqrt{}}$
gosper	$\frac{2(cdx+ae)(315c^4d^4e^4x^4-280a\,c^3d^3e^5x^3+1540c^4d^5e^3x^3+240a^2c^2d^2e^6x^2-1320a\,c^3d^4e^4x^2+2970c^4d^6e^2x^2-192a^3cd\,e^7x+1056a^2}{3465d^5}$
orering	$\frac{2(315c^4d^4e^4x^4-280a\,c^3d^3e^5x^3+1540c^4d^5e^3x^3+240a^2c^2d^2e^6x^2-1320a\,c^3d^4e^4x^2+2970c^4d^6e^2x^2-192a^3cd\,e^7x+1056a^2c^2d^3e^5x-}{3465d^5}$

input
```
int((e*x+d)^(7/2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(1/2),x,method=_RETURN
VERBOSE)
```

output
```
2/3465*(c*d*x+a*e)*(315*c^4*d^4*e^4*x^4-280*a*c^3*d^3*e^5*x^3+1540*c^4*d^5
*e^3*x^3+240*a^2*c^2*d^2*e^6*x^2-1320*a*c^3*d^4*e^4*x^2+2970*c^4*d^6*e^2*x
^2-192*a^3*c*d*e^7*x+1056*a^2*c^2*d^3*e^5*x-2376*a*c^3*d^5*e^3*x+2772*c^4*
d^7*e*x+128*a^4*e^8-704*a^3*c*d^2*e^6+1584*a^2*c^2*d^4*e^4-1848*a*c^3*d^6*
e^2+1155*c^4*d^8)*((e*x+d)*(c*d*x+a*e))^(1/2)/d^5/c^5/(e*x+d)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 315, normalized size of antiderivative = 1.03

$$\int (d+ex)^{7/2}\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}\,dx = \frac{2\,(315\,c^5d^5e^4x^5+1155\,ac^4d^8e-1848\,a^2c^3d^6e^3+1584\,a^3c^2d^4e^5}{}$$

input
```
integrate((e*x+d)^(7/2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x, algorit
hm="fricas")
```

output
```
2/3465*(315*c^5*d^5*e^4*x^5 + 1155*a*c^4*d^8*e - 1848*a^2*c^3*d^6*e^3 + 15
84*a^3*c^2*d^4*e^5 - 704*a^4*c*d^2*e^7 + 128*a^5*e^9 + 35*(44*c^5*d^6*e^3
+ a*c^4*d^4*e^5)*x^4 + 10*(297*c^5*d^7*e^2 + 22*a*c^4*d^5*e^4 - 4*a^2*c^3*
d^3*e^6)*x^3 + 6*(462*c^5*d^8*e + 99*a*c^4*d^6*e^3 - 44*a^2*c^3*d^4*e^5 +
8*a^3*c^2*d^2*e^7)*x^2 + (1155*c^5*d^9 + 924*a*c^4*d^7*e^2 - 792*a^2*c^3*d
^5*e^4 + 352*a^3*c^2*d^3*e^6 - 64*a^4*c*d*e^8)*x)*sqrt(c*d*e*x^2 + a*d*e +
 (c*d^2 + a*e^2)*x)*sqrt(e*x + d)/(c^5*d^5*e*x + c^5*d^6)
```

Sympy [F(-1)]

Timed out.

$$\int (d+ex)^{7/2}\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}\,dx=\text{Timed out}$$

input
```
integrate((e*x+d)**(7/2)*(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(1/2),x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 296, normalized size of antiderivative = 0.97

$$\int (d+ex)^{7/2}\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}\,dx=\frac{2\,(315\,c^5d^5e^4x^5+1155\,ac^4d^8e-1848\,a^2c^3d^6e^3+1584\,a^3c^2d^4e^5}{}$$

input
```
integrate((e*x+d)^(7/2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x, algorit
hm="maxima")
```

output
```
2/3465*(315*c^5*d^5*e^4*x^5 + 1155*a*c^4*d^8*e - 1848*a^2*c^3*d^6*e^3 + 15
84*a^3*c^2*d^4*e^5 - 704*a^4*c*d^2*e^7 + 128*a^5*e^9 + 35*(44*c^5*d^6*e^3
+ a*c^4*d^4*e^5)*x^4 + 10*(297*c^5*d^7*e^2 + 22*a*c^4*d^5*e^4 - 4*a^2*c^3*
d^3*e^6)*x^3 + 6*(462*c^5*d^8*e + 99*a*c^4*d^6*e^3 - 44*a^2*c^3*d^4*e^5 +
8*a^3*c^2*d^2*e^7)*x^2 + (1155*c^5*d^9 + 924*a*c^4*d^7*e^2 - 792*a^2*c^3*d
^5*e^4 + 352*a^3*c^2*d^3*e^6 - 64*a^4*c*d*e^8)*x)*sqrt(c*d*x + a*e)*(e*x +
 d)/(c^5*d^5*e*x + c^5*d^6)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 646 vs. $2(275) = 550$.

Time = 0.17 (sec) , antiderivative size = 646, normalized size of antiderivative = 2.12

$$\int (d+ex)^{7/2}\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}\,dx = \text{Too large to display}$$

input
```
integrate((e*x+d)^(7/2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x, algorit
hm="giac")
```

output
```
2/3465*(3465*sqrt(c*d*x + a*e)*a*d^4*e - 1155*(3*sqrt(c*d*x + a*e)*a*e - (
c*d*x + a*e)^(3/2))*d^4 - 4620*(3*sqrt(c*d*x + a*e)*a*e - (c*d*x + a*e)^(3
/2))*a*d^2*e^2/c + 924*(15*sqrt(c*d*x + a*e)*a^2*e^2 - 10*(c*d*x + a*e)^(3
/2)*a*e + 3*(c*d*x + a*e)^(5/2))*d^2*e/c + 1386*(15*sqrt(c*d*x + a*e)*a^2*
e^2 - 10*(c*d*x + a*e)^(3/2)*a*e + 3*(c*d*x + a*e)^(5/2))*a*e^3/c^2 - 594*
(35*sqrt(c*d*x + a*e)*a^3*e^3 - 35*(c*d*x + a*e)^(3/2)*a^2*e^2 + 21*(c*d*x
 + a*e)^(5/2)*a*e - 5*(c*d*x + a*e)^(7/2))*e^2/c^2 - 396*(35*sqrt(c*d*x +
a*e)*a^3*e^3 - 35*(c*d*x + a*e)^(3/2)*a^2*e^2 + 21*(c*d*x + a*e)^(5/2)*a*e
 - 5*(c*d*x + a*e)^(7/2))*a*e^4/(c^3*d^2) + 44*(315*sqrt(c*d*x + a*e)*a^4*
e^4 - 420*(c*d*x + a*e)^(3/2)*a^3*e^3 + 378*(c*d*x + a*e)^(5/2)*a^2*e^2 -
180*(c*d*x + a*e)^(7/2)*a*e + 35*(c*d*x + a*e)^(9/2))*e^3/(c^3*d^2) + 11*(
315*sqrt(c*d*x + a*e)*a^4*e^4 - 420*(c*d*x + a*e)^(3/2)*a^3*e^3 + 378*(c*d
*x + a*e)^(5/2)*a^2*e^2 - 180*(c*d*x + a*e)^(7/2)*a*e + 35*(c*d*x + a*e)^(
9/2))*a*e^5/(c^4*d^4) - 5*(693*sqrt(c*d*x + a*e)*a^5*e^5 - 1155*(c*d*x + a
*e)^(3/2)*a^4*e^4 + 1386*(c*d*x + a*e)^(5/2)*a^3*e^3 - 990*(c*d*x + a*e)^(
7/2)*a^2*e^2 + 385*(c*d*x + a*e)^(9/2)*a*e - 63*(c*d*x + a*e)^(11/2))*e^4/
(c^4*d^4))/(c*d)
```

Mupad [B] (verification not implemented)

Time = 5.59 (sec) , antiderivative size = 346, normalized size of antiderivative = 1.13

$$\int (d+ex)^{7/2}\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}\,dx = \frac{\sqrt{c\,d\,e\,x^2+(c\,d^2+a\,e^2)\,x+a\,d\,e}\,\left(\frac{2\,e^3\,x^5\,\sqrt{d+e\,x}}{11}+\frac{4\,x^2\,\sqrt{d+e\,x}\,(8\,a^3}{}\right.}{}$$

input
```
int((d + e*x)^(7/2)*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2),x)
```

output
```
((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2)*((2*e^3*x^5*(d + e*x)^(1/2)
)/11 + (4*x^2*(d + e*x)^(1/2)*(8*a^3*e^6 + 462*c^3*d^6 + 99*a*c^2*d^4*e^2
- 44*a^2*c*d^2*e^4))/(1155*c^3*d^3) + ((d + e*x)^(1/2)*(256*a^5*e^9 - 1408
*a^4*c*d^2*e^7 - 3696*a^2*c^3*d^6*e^3 + 3168*a^3*c^2*d^4*e^5 + 2310*a*c^4*
d^8*e))/(3465*c^5*d^5*e) + (2*e^2*x^4*(a*e^2 + 44*c*d^2)*(d + e*x)^(1/2))/
(99*c*d) + (x*(d + e*x)^(1/2)*(2310*c^5*d^9 + 1848*a*c^4*d^7*e^2 - 1584*a^
2*c^3*d^5*e^4 + 704*a^3*c^2*d^3*e^6 - 128*a^4*c*d*e^8))/(3465*c^5*d^5*e) +
 (4*e*x^3*(d + e*x)^(1/2)*(297*c^2*d^4 - 4*a^2*e^4 + 22*a*c*d^2*e^2))/(693
*c^2*d^2)))/(x + d/e)
```

Reduce [B] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 289, normalized size of antiderivative = 0.95

$$\int (d+ex)^{7/2}\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}\,dx = \frac{2\sqrt{cdx+ae}\,(315c^5d^5e^4x^5+35a\,c^4d^4e^5x^4+1540c^5d^6e^3x^4-40}{}$$

input
```
int((e*x+d)^(7/2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x)
```

output
```
(2*sqrt(a*e + c*d*x)*(128*a**5*e**9 - 704*a**4*c*d**2*e**7 - 64*a**4*c*d*e
**8*x + 1584*a**3*c**2*d**4*e**5 + 352*a**3*c**2*d**3*e**6*x + 48*a**3*c**
2*d**2*e**7*x**2 - 1848*a**2*c**3*d**6*e**3 - 792*a**2*c**3*d**5*e**4*x -
264*a**2*c**3*d**4*e**5*x**2 - 40*a**2*c**3*d**3*e**6*x**3 + 1155*a*c**4*d
**8*e + 924*a*c**4*d**7*e**2*x + 594*a*c**4*d**6*e**3*x**2 + 220*a*c**4*d*
*5*e**4*x**3 + 35*a*c**4*d**4*e**5*x**4 + 1155*c**5*d**9*x + 2772*c**5*d**
8*e*x**2 + 2970*c**5*d**7*e**2*x**3 + 1540*c**5*d**6*e**3*x**4 + 315*c**5*
d**5*e**4*x**5))/(3465*c**5*d**5)
```

3.279 $\int (d+ex)^{5/2}\sqrt{ade+(cd^2+ae^2)x+cdex^2}\,dx$

Optimal result

Integrand size = 39, antiderivative size = 240

$$\int (d+ex)^{5/2}\sqrt{ade+(cd^2+ae^2)x+cdex^2}\,dx=\frac{2(cd^2-ae^2)^3\left(ade+(cd^2+ae^2)x+cdex^2\right)^{3/2}}{3c^4d^4(d+ex)^{3/2}}+\frac{6e(cd^2-ae^2)^2\left(ade+(cd^2+ae^2)x+cdex^2\right)^{5/2}}{5c^4d^4(d+ex)^{5/2}}+\frac{6e^2(cd^2-ae^2)\left(ade+(cd^2+ae^2)x+cdex^2\right)^{7/2}}{7c^4d^4(d+ex)^{7/2}}+\frac{2e^3\left(ade+(cd^2+ae^2)x+cdex^2\right)^{9/2}}{9c^4d^4(d+ex)^{9/2}}$$

output
```
2/3*(-a*e^2+c*d^2)^3*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/c^4/d^4/(e*x+
d)^(3/2)+6/5*e*(-a*e^2+c*d^2)^2*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/c^
4/d^4/(e*x+d)^(5/2)+6/7*e^2*(-a*e^2+c*d^2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^
2)^(7/2)/c^4/d^4/(e*x+d)^(7/2)+2/9*e^3*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(
9/2)/c^4/d^4/(e*x+d)^(9/2)
```

Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 131, normalized size of antiderivative = 0.55

$$\int (d+ex)^{5/2}\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}\,dx=\frac{2((ae+cdx)(d+ex))^{3/2}\left(-16a^3e^6+24a^2cde^4(3d+ex)-6ac\right.}{31}$$

input
```
Integrate[(d + e*x)^(5/2)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2],x]
```

output
```
(2*((a*e + c*d*x)*(d + e*x))^(3/2)*(-16*a^3*e^6 + 24*a^2*c*d*e^4*(3*d + e*
x) - 6*a*c^2*d^2*e^2*(21*d^2 + 18*d*e*x + 5*e^2*x^2) + c^3*d^3*(105*d^3 +
189*d^2*e*x + 135*d*e^2*x^2 + 35*e^3*x^3)))/(315*c^4*d^4*(d + e*x)^(3/2))
```

Rubi [A] (verified)

Time = 0.69 (sec) , antiderivative size = 248, normalized size of antiderivative = 1.03, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.103, Rules used = {1128, 1128, 1128, 1122}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (d+ex)^{5/2}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}\,dx$$

$$\Big\downarrow \text{1128}$$

$$\frac{2\left(d^2-\frac{ae^2}{c}\right)\int (d+ex)^{3/2}\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}dx}{3d}+\frac{2(d+ex)^{3/2}\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{9cd}$$

$$\Big\downarrow \text{1128}$$

$$\frac{2\left(d^2-\frac{ae^2}{c}\right)\left(\frac{4\left(d^2-\frac{ae^2}{c}\right)\int\sqrt{d+ex}\sqrt{cdex^2+(cd^2+ae^2)x+ade}dx}{7d}+\frac{2\sqrt{d+ex}\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{7cd}\right)}{3d}+\frac{2(d+ex)^{3/2}\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{9cd}$$

$$\Big\downarrow \text{1128}$$

$$\frac{2\left(d^2-\frac{ae^2}{c}\right)\left(\frac{4\left(d^2-\frac{ae^2}{c}\right)\left(\frac{2\left(d^2-\frac{ae^2}{c}\right)\int\frac{\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}{\sqrt{d+ex}}dx}{5d}+\frac{2\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{5cd\sqrt{d+ex}}\right)}{7d}+\frac{2\sqrt{d+ex}\left(x\left(ae^2+cd^2\right)+ade+c\right.}{7cd}}{3d}$$
$$\frac{2(d+ex)^{3/2}\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{9cd}$$

$$\Big\downarrow \text{1122}$$

$$\frac{2(d+ex)^{3/2}\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{9cd}+$$
$$\frac{2\left(d^2-\frac{ae^2}{c}\right)\left(\frac{2\sqrt{d+ex}\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{7cd}+\frac{4\left(d^2-\frac{ae^2}{c}\right)\left(\frac{2\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{5cd\sqrt{d+ex}}+\frac{4\left(d^2-\frac{ae^2}{c}\right)\left(x\left(ae^2+cd^2\right)+ade+cdex\right.}{15cd^2(d+ex)^{3/2}}\right.}{7d}\right.}{3d}$$

input
```
Int[(d + e*x)^(5/2)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2],x]
```

output
```
(2*(d + e*x)^(3/2)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2))/(9*c*d)
+ (2*(d^2 - (a*e^2)/c)*((2*Sqrt[d + e*x]*(a*d*e + (c*d^2 + a*e^2)*x + c*d*
e*x^2)^(3/2))/(7*c*d) + (4*(d^2 - (a*e^2)/c)*((4*(d^2 - (a*e^2)/c)*(a*d*e
+ (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2))/(15*c*d^2*(d + e*x)^(3/2)) + (2*(a
*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2))/(5*c*d*Sqrt[d + e*x])))/(7*d)
))/(3*d)
```

Defintions of rubi rules used

rule 1122
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[e*(d + e*x)^(m - 1)*((a + b*x + c*x^2)^(p + 1)/(c*(p + 1))),
 x] /; FreeQ[{a, b, c, d, e, m, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&
EqQ[m + p, 0]
```

rule 1128
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[e*(d + e*x)^(m - 1)*((a + b*x + c*x^2)^(p + 1)/(c*(m + 2*p +
 1))), x] + Simp[Simplify[m + p]*((2*c*d - b*e)/(c*(m + 2*p + 1)))   Int[(d
 + e*x)^(m - 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p},
 x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && IGtQ[Simplify[m + p], 0]
```

Maple [A] (verified)

Time = 1.18 (sec) , antiderivative size = 158, normalized size of antiderivative = 0.66

method	result
default	$-\frac{2(cdx+ae)\left(-35c^3d^3e^3x^3+30x^2a\,c^2d^2e^4-135c^3d^4e^2x^2-24x\,a^2cd\,e^5+108xa\,c^2d^3e^3-189c^3d^5ex+16e^6a^3-72d^2e^4a^2c+126d^4e^2c\cdots\right.}{315d^4c^4\sqrt{ex+d}}$
gosper	$-\frac{2(cdx+ae)\left(-35c^3d^3e^3x^3+30x^2a\,c^2d^2e^4-135c^3d^4e^2x^2-24x\,a^2cd\,e^5+108xa\,c^2d^3e^3-189c^3d^5ex+16e^6a^3-72d^2e^4a^2c+126d^4e^2c\cdots\right.}{315d^4c^4\sqrt{ex+d}}$
orering	$-\frac{2\left(-35c^3d^3e^3x^3+30x^2a\,c^2d^2e^4-135c^3d^4e^2x^2-24x\,a^2cd\,e^5+108xa\,c^2d^3e^3-189c^3d^5ex+16e^6a^3-72d^2e^4a^2c+126d^4e^2a\,c^2-105d\cdots\right.}{315d^4c^4\sqrt{ex+d}}$

input
```
int((e*x+d)^(5/2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(1/2),x,method=_RETURN
VERBOSE)
```

output
```
-2/315*(c*d*x+a*e)*(-35*c^3*d^3*e^3*x^3+30*a*c^2*d^2*e^4*x^2-135*c^3*d^4*e
^2*x^2-24*a^2*c*d*e^5*x+108*a*c^2*d^3*e^3*x-189*c^3*d^5*e*x+16*a^3*e^6-72*
a^2*c*d^2*e^4+126*a*c^2*d^4*e^2-105*c^3*d^6)*((e*x+d)*(c*d*x+a*e))^(1/2)/d
^4/c^4/(e*x+d)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 230, normalized size of antiderivative = 0.96

$$\int (d+ex)^{5/2}\sqrt{ade+(cd^2+ae^2)x+cdex^2}\,dx=\frac{2\left(35\,c^4d^4e^3x^4+105\,ac^3d^6e-126\,a^2c^2d^4e^3+72\,a^3cd^2e^5-16\,a\right.}{}$$

input
```
integrate((e*x+d)^(5/2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x, algorit
hm="fricas")
```

output
```
2/315*(35*c^4*d^4*e^3*x^4 + 105*a*c^3*d^6*e - 126*a^2*c^2*d^4*e^3 + 72*a^3
*c*d^2*e^5 - 16*a^4*e^7 + 5*(27*c^4*d^5*e^2 + a*c^3*d^3*e^4)*x^3 + 3*(63*c
^4*d^6*e + 9*a*c^3*d^4*e^3 - 2*a^2*c^2*d^2*e^5)*x^2 + (105*c^4*d^7 + 63*a*
c^3*d^5*e^2 - 36*a^2*c^2*d^3*e^4 + 8*a^3*c*d*e^6)*x)*sqrt(c*d*e*x^2 + a*d*
e + (c*d^2 + a*e^2)*x)*sqrt(e*x + d)/(c^4*d^4*e*x + c^4*d^5)
```

Sympy [F(-1)]

Timed out.

$$\int (d+ex)^{5/2}\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}\,dx=\text{Timed out}$$

input
```
integrate((e*x+d)**(5/2)*(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(1/2),x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 211, normalized size of antiderivative = 0.88

$$\int (d+ex)^{5/2}\sqrt{ade+(cd^2+ae^2)x+cdex^2}\,dx = \frac{2\,(35\,c^4d^4e^3x^4+105\,ac^3d^6e-126\,a^2c^2d^4e^3+72\,a^3cd^2e^5-16\,a$$

input
```
integrate((e*x+d)^(5/2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x, algorit
hm="maxima")
```

output
```
2/315*(35*c^4*d^4*e^3*x^4 + 105*a*c^3*d^6*e - 126*a^2*c^2*d^4*e^3 + 72*a^3
*c*d^2*e^5 - 16*a^4*e^7 + 5*(27*c^4*d^5*e^2 + a*c^3*d^3*e^4)*x^3 + 3*(63*c
^4*d^6*e + 9*a*c^3*d^4*e^3 - 2*a^2*c^2*d^2*e^5)*x^2 + (105*c^4*d^7 + 63*a*
c^3*d^5*e^2 - 36*a^2*c^2*d^3*e^4 + 8*a^3*c*d*e^6)*x)*sqrt(c*d*x + a*e)*(e*
x + d)/(c^4*d^4*e*x + c^4*d^5)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 444 vs. 2(216) = 432.

Time = 0.19 (sec) , antiderivative size = 444, normalized size of antiderivative = 1.85

$$\int (d+ex)^{5/2}\sqrt{ade+(cd^2+ae^2)x+cdex^2}\,dx = \frac{2\left(315\sqrt{cdx+ae}ad^3e-105\left(3\sqrt{cdx+ae}ae-(cdx+ae)^{\frac{3}{2}}\right)\right.$$

input
```
integrate((e*x+d)^(5/2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x, algorit
hm="giac")
```

output
```
2/315*(315*sqrt(c*d*x + a*e)*a*d^3*e - 105*(3*sqrt(c*d*x + a*e)*a*e - (c*d
*x + a*e)^(3/2))*d^3 - 315*(3*sqrt(c*d*x + a*e)*a*e - (c*d*x + a*e)^(3/2))
*a*d*e^2/c + 63*(15*sqrt(c*d*x + a*e)*a^2*e^2 - 10*(c*d*x + a*e)^(3/2)*a*e
 + 3*(c*d*x + a*e)^(5/2))*d*e/c + 63*(15*sqrt(c*d*x + a*e)*a^2*e^2 - 10*(c
*d*x + a*e)^(3/2)*a*e + 3*(c*d*x + a*e)^(5/2))*a*e^3/(c^2*d) - 27*(35*sqrt
(c*d*x + a*e)*a^3*e^3 - 35*(c*d*x + a*e)^(3/2)*a^2*e^2 + 21*(c*d*x + a*e)^
(5/2)*a*e - 5*(c*d*x + a*e)^(7/2))*e^2/(c^2*d) - 9*(35*sqrt(c*d*x + a*e)*a
^3*e^3 - 35*(c*d*x + a*e)^(3/2)*a^2*e^2 + 21*(c*d*x + a*e)^(5/2)*a*e - 5*(
c*d*x + a*e)^(7/2))*a*e^4/(c^3*d^3) + (315*sqrt(c*d*x + a*e)*a^4*e^4 - 420
*(c*d*x + a*e)^(3/2)*a^3*e^3 + 378*(c*d*x + a*e)^(5/2)*a^2*e^2 - 180*(c*d*
x + a*e)^(7/2)*a*e + 35*(c*d*x + a*e)^(9/2))*e^3/(c^3*d^3))/(c*d)
```

Mupad [B] (verification not implemented)

Time = 5.59 (sec) , antiderivative size = 256, normalized size of antiderivative = 1.07

$$\int (d+ex)^{5/2}\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}\,dx=\frac{\sqrt{c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e}\,\left(\frac{2\,e^2\,x^4\,\sqrt{d+e\,x}}{9}-\frac{\sqrt{d+e\,x}\,\left(32\,a^4\,e^7\right.}{}\right.}{}$$

input
```
int((d + e*x)^(5/2)*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2),x)
```

output
```
((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2)*((2*e^2*x^4*(d + e*x)^(1/2)
)/9 - ((d + e*x)^(1/2)*(32*a^4*e^7 - 144*a^3*c*d^2*e^5 + 252*a^2*c^2*d^4*e
^3 - 210*a*c^3*d^6*e))/(315*c^4*d^4*e) + (2*x^2*(d + e*x)^(1/2)*(63*c^2*d^
4 - 2*a^2*e^4 + 9*a*c*d^2*e^2))/(105*c^2*d^2) + (x*(d + e*x)^(1/2)*(210*c^
4*d^7 + 126*a*c^3*d^5*e^2 - 72*a^2*c^2*d^3*e^4 + 16*a^3*c*d*e^6))/(315*c^4
*d^4*e) + (2*e*x^3*(a*e^2 + 27*c*d^2)*(d + e*x)^(1/2))/(63*c*d)))/(x + d/e
)
```

Reduce [B] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 197, normalized size of antiderivative = 0.82

$$\int (d+ex)^{5/2}\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}\,dx = \frac{2\sqrt{cdx+ae}\,(35c^4d^4e^3x^4+5a\,c^3d^3e^4x^3+135c^4d^5e^2x^3-6a^2c^2d$$

input
```
int((e*x+d)^(5/2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x)
```

output
```
(2*sqrt(a*e + c*d*x)*( - 16*a**4*e**7 + 72*a**3*c*d**2*e**5 + 8*a**3*c*d*e
**6*x - 126*a**2*c**2*d**4*e**3 - 36*a**2*c**2*d**3*e**4*x - 6*a**2*c**2*d
**2*e**5*x**2 + 105*a*c**3*d**6*e + 63*a*c**3*d**5*e**2*x + 27*a*c**3*d**4
*e**3*x**2 + 5*a*c**3*d**3*e**4*x**3 + 105*c**4*d**7*x + 189*c**4*d**6*e*x
**2 + 135*c**4*d**5*e**2*x**3 + 35*c**4*d**4*e**3*x**4))/(315*c**4*d**4)
```

3.280 $\int (d+ex)^{3/2}\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}\,dx$

Optimal result

Integrand size = 39, antiderivative size = 175

$$\int (d+ex)^{3/2}\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}\,dx = \frac{2(cd^2-ae^2)^2\left(ade+(cd^2+ae^2)\,x+cdex^2\right)^{3/2}}{3c^3d^3(d+ex)^{3/2}} + \frac{4e(cd^2-ae^2)\left(ade+(cd^2+ae^2)\,x+cdex^2\right)^{5/2}}{5c^3d^3(d+ex)^{5/2}} + \frac{2e^2\left(ade+(cd^2+ae^2)\,x+cdex^2\right)^{7/2}}{7c^3d^3(d+ex)^{7/2}}$$

output
```
2/3*(-a*e^2+c*d^2)^2*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/c^3/d^3/(e*x+
d)^(3/2)+4/5*e*(-a*e^2+c*d^2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/c^3/
d^3/(e*x+d)^(5/2)+2/7*e^2*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(7/2)/c^3/d^3/
(e*x+d)^(7/2)
```

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.50

$$\int (d+ex)^{3/2}\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}\,dx = \frac{2((ae+cdx)(d+ex))^{3/2}\left(8a^2e^4-4acde^2(7d+3ex)+c^2d^2(35\right.}{105c^3d^3(d+ex)^{3/2}}$$

input
```
Integrate[(d + e*x)^(3/2)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2],x]
```

output
```
(2*((a*e + c*d*x)*(d + e*x))^(3/2)*(8*a^2*e^4 - 4*a*c*d*e^2*(7*d + 3*e*x)
+ c^2*d^2*(35*d^2 + 42*d*e*x + 15*e^2*x^2)))/(105*c^3*d^3*(d + e*x)^(3/2))
```

Rubi [A] (verified)

Time = 0.54 (sec) , antiderivative size = 179, normalized size of antiderivative = 1.02, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {1128, 1128, 1122}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (d+ex)^{3/2}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}\,dx$$

$$\Big\downarrow \; 1128$$

$$\frac{4\left(d^2-\frac{ae^2}{c}\right)\int \sqrt{d+ex}\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}dx}{7d}+\frac{2\sqrt{d+ex}\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{7cd}$$

$$\Big\downarrow \; 1128$$

$$\frac{4\left(d^2-\frac{ae^2}{c}\right)\left(\frac{2\left(d^2-\frac{ae^2}{c}\right)\int\frac{\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}{\sqrt{d+ex}}dx}{5d}+\frac{2\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{5cd\sqrt{d+ex}}\right)}{7d}+\frac{2\sqrt{d+ex}\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{7cd}$$

$$\Big\downarrow \; 1122$$

$$\frac{2\sqrt{d+ex}\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{7cd}+\frac{4\left(d^2-\frac{ae^2}{c}\right)\left(\frac{2\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{5cd\sqrt{d+ex}}+\frac{4\left(d^2-\frac{ae^2}{c}\right)\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{15cd^2(d+ex)^{3/2}}\right)}{7d}$$

input
```
Int[(d + e*x)^(3/2)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2],x]
```

output
```
(2*Sqrt[d + e*x]*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2))/(7*c*d) +
(4*(d^2 - (a*e^2)/c)*((4*(d^2 - (a*e^2)/c)*(a*d*e + (c*d^2 + a*e^2)*x + c*
d*e*x^2)^(3/2))/(15*c*d^2*(d + e*x)^(3/2)) + (2*(a*d*e + (c*d^2 + a*e^2)*x
 + c*d*e*x^2)^(3/2))/(5*c*d*Sqrt[d + e*x])))/(7*d)
```

Defintions of rubi rules used

rule 1122
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[e*(d + e*x)^(m - 1)*((a + b*x + c*x^2)^(p + 1)/(c*(p + 1))),
 x] /; FreeQ[{a, b, c, d, e, m, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&
EqQ[m + p, 0]
```

rule 1128
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[e*(d + e*x)^(m - 1)*((a + b*x + c*x^2)^(p + 1)/(c*(m + 2*p +
 1))), x] + Simp[Simplify[m + p]*((2*c*d - b*e)/(c*(m + 2*p + 1)))   Int[(d
 + e*x)^(m - 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p},
 x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && IGtQ[Simplify[m + p], 0]
```

Maple [A] (verified)

Time = 1.25 (sec) , antiderivative size = 100, normalized size of antiderivative = 0.57

method	result	size
default	$\frac{2(cdx+ae)(15x^2c^2d^2e^2-12xacd\,e^3+42x\,c^2d^3e+8a^2e^4-28ac\,d^2e^2+35c^2d^4)\sqrt{(ex+d)(cdx+ae)}}{105d^3c^3\sqrt{ex+d}}$	100
gosper	$\frac{2(cdx+ae)(15x^2c^2d^2e^2-12xacd\,e^3+42x\,c^2d^3e+8a^2e^4-28ac\,d^2e^2+35c^2d^4)\sqrt{cd\,x^2e+a\,e^2x+c\,d^2x+ade}}{105d^3c^3\sqrt{ex+d}}$	110
orering	$\frac{2(15x^2c^2d^2e^2-12xacd\,e^3+42x\,c^2d^3e+8a^2e^4-28ac\,d^2e^2+35c^2d^4)(cdx+ae)\sqrt{ade+(a\,e^2+c\,d^2)x+cd\,x^2e}}{105d^3c^3\sqrt{ex+d}}$	111

input
```
int((e*x+d)^(3/2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(1/2),x,method=_RETURN
VERBOSE)
```

output
```
2/105*(c*d*x+a*e)*(15*c^2*d^2*e^2*x^2-12*a*c*d*e^3*x+42*c^2*d^3*e*x+8*a^2*
e^4-28*a*c*d^2*e^2+35*c^2*d^4)*((e*x+d)*(c*d*x+a*e))^(1/2)/d^3/c^3/(e*x+d)
^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 159, normalized size of antiderivative = 0.91

$$\int (d+ex)^{3/2}\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}\,dx = \frac{2\,(15\,c^3d^3e^2x^3+35\,ac^2d^4e-28\,a^2cd^2e^3+8\,a^3e^5+3\,(14\,c^3d^4e\,-$$

input
```
integrate((e*x+d)^(3/2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x, algorit
hm="fricas")
```

output
```
2/105*(15*c^3*d^3*e^2*x^3 + 35*a*c^2*d^4*e - 28*a^2*c*d^2*e^3 + 8*a^3*e^5
+ 3*(14*c^3*d^4*e + a*c^2*d^2*e^3)*x^2 + (35*c^3*d^5 + 14*a*c^2*d^3*e^2 -
4*a^2*c*d*e^4)*x)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(e*x + d
)/(c^3*d^3*e*x + c^3*d^4)
```

Sympy [F]

$$\int (d+ex)^{3/2}\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}\,dx=\int \sqrt{(d+ex)\,(ae+cdx)}(d+ex)^{\frac{3}{2}}\,dx$$

input
```
integrate((e*x+d)**(3/2)*(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(1/2),x)
```

output
```
Integral(sqrt((d + e*x)*(a*e + c*d*x))*(d + e*x)**(3/2), x)
```

Maxima [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 140, normalized size of antiderivative = 0.80

$$\int (d+ex)^{3/2}\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}\,dx=\frac{2\,(15\,c^3d^3e^2x^3+35\,ac^2d^4e-28\,a^2cd^2e^3+8\,a^3e^5+3\,(14\,c^3d^4e}{105\,(c}$$

input
```
integrate((e*x+d)^(3/2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x, algorit
hm="maxima")
```

output
```
2/105*(15*c^3*d^3*e^2*x^3 + 35*a*c^2*d^4*e - 28*a^2*c*d^2*e^3 + 8*a^3*e^5
+ 3*(14*c^3*d^4*e + a*c^2*d^2*e^3)*x^2 + (35*c^3*d^5 + 14*a*c^2*d^3*e^2 -
4*a^2*c*d*e^4)*x)*sqrt(c*d*x + a*e)*(e*x + d)/(c^3*d^3*e*x + c^3*d^4)
```

Giac [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 276, normalized size of antiderivative = 1.58

$$\int (d+ex)^{3/2}\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}\,dx=\frac{2\left(105\,\sqrt{cdx+ae}ad^2e-35\left(3\,\sqrt{cdx+ae}ae-(cdx+ae)^{\frac{3}{2}}\right)d}{}$$

input
```
integrate((e*x+d)^(3/2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x, algorit
hm="giac")
```

output
```
2/105*(105*sqrt(c*d*x + a*e)*a*d^2*e - 35*(3*sqrt(c*d*x + a*e)*a*e - (c*d*
x + a*e)^(3/2))*d^2 - 70*(3*sqrt(c*d*x + a*e)*a*e - (c*d*x + a*e)^(3/2))*a
*e^2/c + 14*(15*sqrt(c*d*x + a*e)*a^2*e^2 - 10*(c*d*x + a*e)^(3/2)*a*e + 3
*(c*d*x + a*e)^(5/2))*e/c + 7*(15*sqrt(c*d*x + a*e)*a^2*e^2 - 10*(c*d*x +
a*e)^(3/2)*a*e + 3*(c*d*x + a*e)^(5/2))*a*e^3/(c^2*d^2) - 3*(35*sqrt(c*d*x
 + a*e)*a^3*e^3 - 35*(c*d*x + a*e)^(3/2)*a^2*e^2 + 21*(c*d*x + a*e)^(5/2)*
a*e - 5*(c*d*x + a*e)^(7/2))*e^2/(c^2*d^2))/(c*d)
```

Mupad [B] (verification not implemented)

Time = 5.33 (sec) , antiderivative size = 180, normalized size of antiderivative = 1.03

$$\int (d+ex)^{3/2}\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}\,dx=\frac{\sqrt{c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e}\,\left(\frac{2\,e\,x^3\,\sqrt{d+e\,x}}{7}+\frac{\sqrt{d+e\,x}\,\left(16\,a^3\,e^5-\right.}{1}\right.}{}$$

input
```
int((d + e*x)^(3/2)*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2),x)
```

output
```
((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2)*((2*e*x^3*(d + e*x)^(1/2))/
7 + ((d + e*x)^(1/2)*(16*a^3*e^5 - 56*a^2*c*d^2*e^3 + 70*a*c^2*d^4*e))/(10
5*c^3*d^3*e) + (2*x^2*(a*e^2 + 14*c*d^2)*(d + e*x)^(1/2))/(35*c*d) + (x*(d
 + e*x)^(1/2)*(70*c^3*d^5 + 28*a*c^2*d^3*e^2 - 8*a^2*c*d*e^4))/(105*c^3*d^
3*e)))/(x + d/e)
```

Reduce [B] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 122, normalized size of antiderivative = 0.70

$$\int (d+ex)^{3/2}\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}\,dx = \frac{2\sqrt{cdx+ae}\,(15c^3d^3e^2x^3+3a\,c^2d^2e^3x^2+42c^3d^4e\,x^2-4a^2cd\,e^4}{105c}$$

input
```
int((e*x+d)^(3/2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x)
```

output
```
(2*sqrt(a*e + c*d*x)*(8*a**3*e**5 - 28*a**2*c*d**2*e**3 - 4*a**2*c*d*e**4*
x + 35*a*c**2*d**4*e + 14*a*c**2*d**3*e**2*x + 3*a*c**2*d**2*e**3*x**2 + 3
5*c**3*d**5*x + 42*c**3*d**4*e*x**2 + 15*c**3*d**3*e**2*x**3))/(105*c**3*d
**3)
```

3.281 $\int \sqrt{d+ex}\sqrt{ade+(cd^2+ae^2)x+cdex^2}\,dx$

Optimal result

Integrand size = 39, antiderivative size = 110

$$\int \sqrt{d+ex}\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}\,dx = \frac{2(cd^2-ae^2)\,(ade+(cd^2+ae^2)\,x+cdex^2)^{3/2}}{3c^2d^2(d+ex)^{3/2}}+\frac{2e(ade+(cd^2+ae^2)\,x+cdex^2)^{5/2}}{5c^2d^2(d+ex)^{5/2}}$$

output
```
2/3*(-a*e^2+c*d^2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/c^2/d^2/(e*x+d)
^(3/2)+2/5*e*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/c^2/d^2/(e*x+d)^(5/2)
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.50

$$\int \sqrt{d+ex}\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}\,dx = \frac{2((ae+cdx)(d+ex))^{3/2}\,(-2ae^2+cd(5d+3ex))}{15c^2d^2(d+ex)^{3/2}}$$

input
```
Integrate[Sqrt[d + e*x]*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2],x]
```

output
```
(2*((a*e + c*d*x)*(d + e*x))^(3/2)*(-2*a*e^2 + c*d*(5*d + 3*e*x)))/(15*c^2
*d^2*(d + e*x)^(3/2))
```

Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 110, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.051$, Rules used = {1128, 1122}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{d+ex}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}\,dx$$

$$\Big\downarrow \ 1128$$

$$\frac{2\left(d^2-\frac{ae^2}{c}\right)\int \frac{\sqrt{cdex^2+(cd^2+ae^2)x+ade}}{\sqrt{d+ex}}dx}{5d}+\frac{2\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{5cd\sqrt{d+ex}}$$

$$\Big\downarrow \ 1122$$

$$\frac{2\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{5cd\sqrt{d+ex}}+\frac{4\left(d^2-\frac{ae^2}{c}\right)\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{15cd^2(d+ex)^{3/2}}$$

input
```
Int[Sqrt[d + e*x]*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2],x]
```

output
```
(4*(d^2 - (a*e^2)/c)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2))/(15*c*
d^2*(d + e*x)^(3/2)) + (2*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2))/(
5*c*d*Sqrt[d + e*x])
```

Defintions of rubi rules used

rule 1122
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[e*(d + e*x)^(m - 1)*((a + b*x + c*x^2)^(p + 1)/(c*(p + 1))),
 x] /; FreeQ[{a, b, c, d, e, m, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&
EqQ[m + p, 0]
```

rule 1128
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[e*(d + e*x)^(m - 1)*((a + b*x + c*x^2)^(p + 1)/(c*(m + 2*p +
 1))), x] + Simp[Simplify[m + p]*((2*c*d - b*e)/(c*(m + 2*p + 1)))   Int[(d
 + e*x)^(m - 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p},
 x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && IGtQ[Simplify[m + p], 0]
```

Maple [A] (verified)

Time = 1.12 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.54

method	result	size
default	$-\frac{2(cdx+ae)\left(-3cdxe+2a\,e^{2}-5c\,d^{2}\right)\sqrt{(ex+d)(cdx+ae)}}{15c^{2}d^{2}\sqrt{ex+d}}$	59
gosper	$-\frac{2(cdx+ae)\left(-3cdxe+2a\,e^{2}-5c\,d^{2}\right)\sqrt{cd\,x^{2}e+a\,e^{2}x+c\,d^{2}x+ade}}{15c^{2}d^{2}\sqrt{ex+d}}$	69
orering	$-\frac{2\left(-3cdxe+2a\,e^{2}-5c\,d^{2}\right)(cdx+ae)\sqrt{ade+\left(a\,e^{2}+c\,d^{2}\right)x+cd\,x^{2}e}}{15c^{2}d^{2}\sqrt{ex+d}}$	70

input
```
int((e*x+d)^(1/2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(1/2),x,method=_RETURN
VERBOSE)
```

output
```
-2/15*(c*d*x+a*e)*(-3*c*d*e*x+2*a*e^2-5*c*d^2)*((e*x+d)*(c*d*x+a*e))^(1/2)
/c^2/d^2/(e*x+d)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 102, normalized size of antiderivative = 0.93

$$\int \sqrt{d+ex}\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}\,dx = \frac{2\,(3\,c^2d^2ex^2+5\,acd^2e-2\,a^2e^3+(5\,c^2d^3+acde^2)x)\sqrt{cdex^2+ade+(cd^2+ae^2)x}\sqrt{ex+d}}{15\,(c^2d^2ex+c^2d^3)}$$

input
```
integrate((e*x+d)^(1/2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x, algorit
hm="fricas")
```

output
```
2/15*(3*c^2*d^2*e*x^2 + 5*a*c*d^2*e - 2*a^2*e^3 + (5*c^2*d^3 + a*c*d*e^2)*
x)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(e*x + d)/(c^2*d^2*e*x
+ c^2*d^3)
```

Sympy [F]

$$\int \sqrt{d+ex}\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}\,dx = \int \sqrt{(d+ex)\,(ae+cdx)}\sqrt{d+ex}\,dx$$

input
```
integrate((e*x+d)**(1/2)*(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(1/2),x)
```

output
```
Integral(sqrt((d + e*x)*(a*e + c*d*x))*sqrt(d + e*x), x)
```

Maxima [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.75

$$\int \sqrt{d+ex}\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}\,dx = \frac{2\,(3\,c^2d^2ex^2+5\,acd^2e-2\,a^2e^3+(5\,c^2d^3+acde^2)x)\sqrt{cdx+ae}(ex+d)}{15\,(c^2d^2ex+c^2d^3)}$$

input
```
integrate((e*x+d)^(1/2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x, algorit
hm="maxima")
```

output
```
2/15*(3*c^2*d^2*e*x^2 + 5*a*c*d^2*e - 2*a^2*e^3 + (5*c^2*d^3 + a*c*d*e^2)*
x)*sqrt(c*d*x + a*e)*(e*x + d)/(c^2*d^2*e*x + c^2*d^3)
```

Giac [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 146, normalized size of antiderivative = 1.33

$$\int \sqrt{d+ex}\sqrt{ade+(cd^2+ae^2)x+cdex^2}\,dx$$
$$= \frac{2\left(15\sqrt{cdx+ae}ade-5\left(3\sqrt{cdx+ae}ae-(cdx+ae)^{\frac{3}{2}}\right)d-\frac{5\left(3\sqrt{cdx+ae}ae-(cdx+ae)^{\frac{3}{2}}\right)ae^2}{cd}+\frac{\left(15\sqrt{cdx+ae}a^2e^2\right.}{}\right.}{15\,cd}$$

input
```
integrate((e*x+d)^(1/2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x, algorit
hm="giac")
```

output
```
2/15*(15*sqrt(c*d*x + a*e)*a*d*e - 5*(3*sqrt(c*d*x + a*e)*a*e - (c*d*x + a
*e)^(3/2))*d - 5*(3*sqrt(c*d*x + a*e)*a*e - (c*d*x + a*e)^(3/2))*a*e^2/(c*
d) + (15*sqrt(c*d*x + a*e)*a^2*e^2 - 10*(c*d*x + a*e)^(3/2)*a*e + 3*(c*d*x
 + a*e)^(5/2))*e/(c*d))/(c*d)
```

Mupad [B] (verification not implemented)

Time = 5.29 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.10

$$\int \sqrt{d+ex}\sqrt{ade+(cd^2+ae^2)x+cdex^2}\,dx$$
$$= \frac{\sqrt{c\,d\,e\,x^2+(c\,d^2+a\,e^2)\,x+a\,d\,e}\left(\frac{2\,x^2\,\sqrt{d+e\,x}}{5}-\frac{(4\,a^2\,e^3-10\,a\,c\,d^2\,e)\,\sqrt{d+e\,x}}{15\,c^2\,d^2\,e}+\frac{x\,(10\,c^2\,d^3+2\,a\,c\,d\,e^2)\,\sqrt{d+e\,x}}{15\,c^2\,d^2\,e}\right)}{x+\frac{d}{e}}$$

input
```
int((d + e*x)^(1/2)*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2),x)
```

output
```
((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2)*((2*x^2*(d + e*x)^(1/2))/5
- ((4*a^2*e^3 - 10*a*c*d^2*e)*(d + e*x)^(1/2))/(15*c^2*d^2*e) + (x*(10*c^2
*d^3 + 2*a*c*d*e^2)*(d + e*x)^(1/2))/(15*c^2*d^2*e)))/(x + d/e)
```

Reduce [B] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.57

$$\int \sqrt{d+ex}\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}\,dx = \frac{2\sqrt{cdx+ae}\,(3c^2d^2e\,x^2+acd\,e^2x+5c^2d^3x-2a^2e^3+5ac\,d^2e)}{15c^2d^2}$$

input
```
int((e*x+d)^(1/2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x)
```

output
```
(2*sqrt(a*e + c*d*x)*( - 2*a**2*e**3 + 5*a*c*d**2*e + a*c*d*e**2*x + 5*c**
2*d**3*x + 3*c**2*d**2*e*x**2))/(15*c**2*d**2)
```

3.282 $\int \frac{\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{\sqrt{d+ex}}\,dx$

Optimal result

Integrand size = 39, antiderivative size = 48

$$\int \frac{\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{\sqrt{d+ex}}\,dx=\frac{2\left(ade+(cd^2+ae^2)\,x+cdex^2\right)^{3/2}}{3cd(d+ex)^{3/2}}$$

output
```
2/3*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/c/d/(e*x+d)^(3/2)
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.77

$$\int \frac{\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{\sqrt{d+ex}}\,dx=\frac{2((ae+cdx)(d+ex))^{3/2}}{3cd(d+ex)^{3/2}}$$

input
```
Integrate[Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]/Sqrt[d + e*x],x]
```

output
```
(2*((a*e + c*d*x)*(d + e*x))^(3/2))/(3*c*d*(d + e*x)^(3/2))
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.026$, Rules used = {1122}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{\sqrt{d+ex}}dx$$

$$\Big\downarrow \text{1122}$$

$$\frac{2\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{3cd(d+ex)^{3/2}}$$

input
```
Int[Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]/Sqrt[d + e*x],x]
```

output
```
(2*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2))/(3*c*d*(d + e*x)^(3/2))
```

Defintions of rubi rules used

rule 1122
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[e*(d + e*x)^(m - 1)*((a + b*x + c*x^2)^(p + 1)/(c*(p + 1))),
 x] /; FreeQ[{a, b, c, d, e, m, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&
EqQ[m + p, 0]
```

Maple [A] (verified)

Time = 1.05 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.83

method	result	size
default	$\frac{2(cdx+ae)\sqrt{(ex+d)(cdx+ae)}}{3cd\sqrt{ex+d}}$	40
gosper	$\frac{2(cdx+ae)\sqrt{cd\,x^{2}e+a\,e^{2}x+c\,d^{2}x+ade}}{3cd\sqrt{ex+d}}$	50
orering	$\frac{2(cdx+ae)\sqrt{ade+(a\,e^{2}+c\,d^{2})x+cd\,x^{2}e}}{3cd\sqrt{ex+d}}$	51

input
```
int((a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(1/2)/(e*x+d)^(1/2),x,method=_RETURN
VERBOSE)
```

output
```
2/3*(c*d*x+a*e)*((e*x+d)*(c*d*x+a*e))^(1/2)/c/d/(e*x+d)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.19

$$\int \frac{\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{\sqrt{d+ex}}\,dx = \frac{2\,\sqrt{cdex^2+ade+(cd^2+ae^2)x}(cdx+ae)\sqrt{ex+d}}{3\,(cdex+cd^2)}$$

input
```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(e*x+d)^(1/2),x, algorit
hm="fricas")
```

output
```
2/3*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(c*d*x + a*e)*sqrt(e*x + d
)/(c*d*e*x + c*d^2)
```

Sympy [F]

$$\int \frac{\sqrt{ade + (cd^2 + ae^2)\,x + cdex^2}}{\sqrt{d + ex}}\,dx = \int \frac{\sqrt{(d + ex)\,(ae + cdx)}}{\sqrt{d + ex}}\,dx$$

input
```
integrate((a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(1/2)/(e*x+d)**(1/2),x)
```

output
```
Integral(sqrt((d + e*x)*(a*e + c*d*x))/sqrt(d + e*x), x)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.38

$$\int \frac{\sqrt{ade + (cd^2 + ae^2)\,x + cdex^2}}{\sqrt{d + ex}}\,dx = \frac{2\,(cdx + ae)^{\frac{3}{2}}}{3\,cd}$$

input
```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(e*x+d)^(1/2),x, algorit
hm="maxima")
```

output
```
2/3*(c*d*x + a*e)^(3/2)/(c*d)
```

Giac [A] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.38

$$\int \frac{\sqrt{ade + (cd^2 + ae^2)\,x + cdex^2}}{\sqrt{d + ex}}\,dx = \frac{2\,(cdx + ae)^{\frac{3}{2}}}{3\,cd}$$

input
```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(e*x+d)^(1/2),x, algorit
hm="giac")
```

output
```
2/3*(c*d*x + a*e)^(3/2)/(c*d)
```

Mupad [B] (verification not implemented)

Time = 5.30 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.02

$$\int \frac{\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{\sqrt{d+ex}}\,dx = \frac{\left(\frac{2x}{3}+\frac{2\,a\,e}{3\,c\,d}\right)\sqrt{c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e}}{\sqrt{d+e\,x}}$$

input
```
int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2)/(d + e*x)^(1/2),x)
```

output
```
(((2*x)/3 + (2*a*e)/(3*c*d))*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2)
)/(d + e*x)^(1/2)
```

Reduce [B] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.52

$$\int \frac{\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{\sqrt{d+ex}}\,dx = \frac{2\sqrt{cdx+ae}\,(cdx+ae)}{3cd}$$

input
```
int((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(e*x+d)^(1/2),x)
```

output
```
(2*sqrt(a*e + c*d*x)*(a*e + c*d*x))/(3*c*d)
```

3.283 $\int \frac{\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{(d+ex)^{3/2}}\,dx$

Optimal result

Integrand size = 39, antiderivative size = 128

$$\int \frac{\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{(d+ex)^{3/2}}\,dx=\frac{2\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{e\sqrt{d+ex}}-\frac{2\sqrt{cd^2-ae^2}\arctan\left(\frac{\sqrt{e}\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{\sqrt{cd^2-ae^2}\sqrt{d+ex}}\right)}{e^{3/2}}$$

output
```
2*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/e/(e*x+d)^(1/2)-2*(-a*e^2+c*d^2)
^(1/2)*arctan(e^(1/2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(-a*e^2+c*d^
2)^(1/2)/(e*x+d)^(1/2))/e^(3/2)
```

Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 118, normalized size of antiderivative = 0.92

$$\int \frac{\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{(d+ex)^{3/2}}\,dx=\frac{2\sqrt{ae+cdx}\sqrt{d+ex}\left(\sqrt{e}\sqrt{ae+cdx}-\sqrt{cd^2-ae^2}\arctan\left(\frac{\sqrt{e}\sqrt{ae+c}}{\sqrt{cd^2-a}}\right.\right.}{e^{3/2}\sqrt{(ae+cdx)(d+ex)}}$$

input
```
Integrate[Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]/(d + e*x)^(3/2),x]
```

output
```
(2*Sqrt[a*e + c*d*x]*Sqrt[d + e*x]*(Sqrt[e]*Sqrt[a*e + c*d*x] - Sqrt[c*d^2
 - a*e^2]*ArcTan[(Sqrt[e]*Sqrt[a*e + c*d*x])/Sqrt[c*d^2 - a*e^2]]))/(e^(3/
2)*Sqrt[(a*e + c*d*x)*(d + e*x)])
```

Rubi [A] (verified)

Time = 0.48 (sec) , antiderivative size = 128, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {1131, 1136, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{(d+ex)^{3/2}}dx$$

$$\Big\downarrow \text{1131}$$

$$\frac{2\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{e\sqrt{d+ex}}-\frac{(cd^2-ae^2)\int \frac{1}{\sqrt{d+ex}\sqrt{cdex^2+(cd^2+ae^2)x+ade}}dx}{e}$$

$$\Big\downarrow \text{1136}$$

$$\frac{2\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{e\sqrt{d+ex}}-2(cd^2-ae^2)\int \frac{1}{\frac{(cdex^2+(cd^2+ae^2)x+ade)e^2}{d+ex}+(cd^2-ae^2)e}d\frac{\sqrt{cdex^2+(cd^2+ae^2)x+ade}}{\sqrt{d+ex}}$$

$$\Big\downarrow \text{218}$$

$$\frac{2\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{e\sqrt{d+ex}}-\frac{2\sqrt{cd^2-ae^2}\arctan\left(\frac{\sqrt{e}\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{\sqrt{d+ex}\sqrt{cd^2-ae^2}}\right)}{e^{3/2}}$$

input
```
Int[Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]/(d + e*x)^(3/2),x]
```

output
```
(2*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(e*Sqrt[d + e*x]) - (2*Sqr
t[c*d^2 - a*e^2]*ArcTan[(Sqrt[e]*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^
2])/(Sqrt[c*d^2 - a*e^2]*Sqrt[d + e*x])])/e^(3/2)
```

Defintions of rubi rules used

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 1131
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(d + e*x)^(m + 1)*((a + b*x + c*x^2)^p/(e*(m + 2*p + 1))), x
] - Simp[p*((2*c*d - b*e)/(e^2*(m + 2*p + 1)))   Int[(d + e*x)^(m + 1)*(a +
 b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c*d^2 - b
*d*e + a*e^2, 0] && GtQ[p, 0] && (LeQ[-2, m, 0] || EqQ[m + p + 1, 0]) && Ne
Q[m + 2*p + 1, 0] && IntegerQ[2*p]
```

rule 1136
```
Int[1/(Sqrt[(d_.) + (e_.)*(x_)]*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x
_Symbol] :> Simp[2*e   Subst[Int[1/(2*c*d - b*e + e^2*x^2), x], x, Sqrt[a +
 b*x + c*x^2]/Sqrt[d + e*x]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c*d^2
- b*d*e + a*e^2, 0]
```

Maple [A] (verified)

Time = 1.08 (sec) , antiderivative size = 153, normalized size of antiderivative = 1.20

method	result	size
default	$-\frac{2\sqrt{(ex+d)(cdx+ae)}\left(\operatorname{arctanh}\left(\frac{e\sqrt{cdx+ae}}{\sqrt{\left(a\,e^{2}-c\,d^{2}\right)e}}\right)a\,e^{2}-\operatorname{arctanh}\left(\frac{e\sqrt{cdx+ae}}{\sqrt{\left(a\,e^{2}-c\,d^{2}\right)e}}\right)c\,d^{2}-\sqrt{cdx+ae}\,\sqrt{\left(a\,e^{2}-c\,d^{2}\right)e}\right)}{\sqrt{ex+d}\,\sqrt{cdx+ae}\,e\sqrt{\left(a\,e^{2}-c\,d^{2}\right)e}}$	153

input
```
int((a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(1/2)/(e*x+d)^(3/2),x,method=_RETURN
VERBOSE)
```

output
```
-2*((e*x+d)*(c*d*x+a*e))^(1/2)*(arctanh(e*(c*d*x+a*e)^(1/2)/((a*e^2-c*d^2)
*e)^(1/2))*a*e^2-arctanh(e*(c*d*x+a*e)^(1/2)/((a*e^2-c*d^2)*e)^(1/2))*c*d^
2-(c*d*x+a*e)^(1/2)*((a*e^2-c*d^2)*e)^(1/2))/(e*x+d)^(1/2)/(c*d*x+a*e)^(1/
2)/e/((a*e^2-c*d^2)*e)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 341, normalized size of antiderivative = 2.66

$$\int \frac{\sqrt{ade + (cd^2 + ae^2)x + cdex^2}}{(d + ex)^{3/2}}\,dx = \left[\frac{(ex+d)\sqrt{-\frac{cd^2-ae^2}{e}}\log\left(-\frac{cde^2x^2+2\,ae^3x-cd^3+2\,ade^2-2\,\sqrt{cdex^2+ade+(cd^2}}{e^2x^2+2\,dex+d^2}\right.}{e^2x}\right.$$

input
```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(e*x+d)^(3/2),x, algorit
hm="fricas")
```

output
```
[((e*x + d)*sqrt(-(c*d^2 - a*e^2)/e)*log(-(c*d*e^2*x^2 + 2*a*e^3*x - c*d^3
 + 2*a*d*e^2 - 2*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(e*x + d)
*e*sqrt(-(c*d^2 - a*e^2)/e))/(e^2*x^2 + 2*d*e*x + d^2)) + 2*sqrt(c*d*e*x^2
 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(e*x + d))/(e^2*x + d*e), 2*((e*x + d)*s
qrt((c*d^2 - a*e^2)/e)*arctan(-sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)
*sqrt(e*x + d)*e*sqrt((c*d^2 - a*e^2)/e)/(c*d^3 - a*d*e^2 + (c*d^2*e - a*e
^3)*x)) + sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(e*x + d))/(e^2*
x + d*e)]
```

Sympy [F]

$$\int \frac{\sqrt{ade + (cd^2 + ae^2)x + cdex^2}}{(d + ex)^{3/2}}\,dx = \int \frac{\sqrt{(d+ex)(ae+cdx)}}{(d+ex)^{\frac{3}{2}}}\,dx$$

input
```
integrate((a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(1/2)/(e*x+d)**(3/2),x)
```

output
```
Integral(sqrt((d + e*x)*(a*e + c*d*x))/(d + e*x)**(3/2), x)
```

Maxima [F]

$$\int \frac{\sqrt{ade + (cd^2 + ae^2)\,x + cdex^2}}{(d + ex)^{3/2}}\,dx = \int \frac{\sqrt{cdex^2 + ade + (cd^2 + ae^2)x}}{(ex + d)^{\frac{3}{2}}}\,dx$$

input
```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(e*x+d)^(3/2),x, algorit
hm="maxima")
```

output
```
integrate(sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)/(e*x + d)^(3/2), x)
```

Giac [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 106, normalized size of antiderivative = 0.83

$$\int \frac{\sqrt{ade + (cd^2 + ae^2)\,x + cdex^2}}{(d + ex)^{3/2}}\,dx = -\frac{2\left(\frac{(cd^2 - ae^2)\arctan\left(\frac{\sqrt{(ex+d)cde - cd^2e + ae^3}}{\sqrt{cd^2e - ae^3}}\right)}{\sqrt{cd^2e - ae^3}} - \frac{\sqrt{(ex+d)cde - cd^2e + ae^3}}{e}\right)|e|}{e^2}$$

input
```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(e*x+d)^(3/2),x, algorit
hm="giac")
```

output
```
-2*((c*d^2 - a*e^2)*arctan(sqrt((e*x + d)*c*d*e - c*d^2*e + a*e^3)/sqrt(c*
d^2*e - a*e^3))/sqrt(c*d^2*e - a*e^3) - sqrt((e*x + d)*c*d*e - c*d^2*e + a
*e^3)/e)*abs(e)/e^2
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{ade + (cd^2 + ae^2)x + cdex^2}}{(d + ex)^{3/2}} dx = \int \frac{\sqrt{c\,d\,e\,x^2 + (c\,d^2 + a\,e^2)\,x + a\,d\,e}}{(d + e\,x)^{3/2}} dx$$

input
```
int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2)/(d + e*x)^(3/2),x)
```

output
```
int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2)/(d + e*x)^(3/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.51

$$\int \frac{\sqrt{ade + (cd^2 + ae^2)x + cdex^2}}{(d + ex)^{3/2}} dx = \frac{-2\sqrt{e}\,\sqrt{-a\,e^2 + c\,d^2}\,atan\left(\frac{\sqrt{cdx+ae}\,e}{\sqrt{e}\,\sqrt{-a\,e^2+c\,d^2}}\right) + 2\sqrt{cdx + ae}\,e}{e^2}$$

input
```
int((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(e*x+d)^(3/2),x)
```

output
```
(2*( - sqrt(e)*sqrt( - a*e**2 + c*d**2)*atan((sqrt(a*e + c*d*x)*e)/(sqrt(e
)*sqrt( - a*e**2 + c*d**2))) + sqrt(a*e + c*d*x)*e))/e**2
```

3.284 $\int \frac{\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{(d+ex)^{5/2}}\,dx$

Optimal result

Integrand size = 39, antiderivative size = 129

$$\int \frac{\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{(d+ex)^{5/2}}\,dx = -\frac{\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{e(d+ex)^{3/2}}+\frac{cd\arctan\left(\frac{\sqrt{e}\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{\sqrt{cd^2-ae^2}\sqrt{d+ex}}\right)}{e^{3/2}\sqrt{cd^2-ae^2}}$$

output
```
-(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/e/(e*x+d)^(3/2)+c*d*arctan(e^(1/2
)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(-a*e^2+c*d^2)^(1/2)/(e*x+d)^(1/
2))/e^(3/2)/(-a*e^2+c*d^2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.25 (sec) , antiderivative size = 112, normalized size of antiderivative = 0.87

$$\int \frac{\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{(d+ex)^{5/2}}\,dx = \frac{\sqrt{(ae+cdx)(d+ex)}\left(-\sqrt{e}+\frac{cd(d+ex)\arctan\left(\frac{\sqrt{e}\sqrt{ae+cdx}}{\sqrt{cd^2-ae^2}}\right)}{\sqrt{cd^2-ae^2}\sqrt{ae+cdx}}\right)}{e^{3/2}(d+ex)^{3/2}}$$

input
```
Integrate[Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]/(d + e*x)^(5/2),x]
```

output
```
(Sqrt[(a*e + c*d*x)*(d + e*x)]*(-Sqrt[e] + (c*d*(d + e*x)*ArcTan[(Sqrt[e]*
Sqrt[a*e + c*d*x])/Sqrt[c*d^2 - a*e^2]])/(Sqrt[c*d^2 - a*e^2]*Sqrt[a*e + c
*d*x])))/(e^(3/2)*(d + e*x)^(3/2))
```

Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 129, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {1130, 1136, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{x (ae^2+cd^2)+ade+cdex^2}}{(d+ex)^{5/2}} \, dx$$

$$\Big\downarrow \text{ 1130}$$

$$\frac{cd \int \frac{1}{\sqrt{d+ex}\sqrt{cdex^2+(cd^2+ae^2)x+ade}}dx}{2e}-\frac{\sqrt{x (ae^2+cd^2)+ade+cdex^2}}{e(d+ex)^{3/2}}$$

$$\Big\downarrow \text{ 1136}$$

$$cd \int \frac{1}{\frac{(cdex^2+(cd^2+ae^2)x+ade)e^2}{d+ex}+(cd^2-ae^2)\,e}d\frac{\sqrt{cdex^2+(cd^2+ae^2)\,x+ade}}{\sqrt{d+ex}}-\frac{\sqrt{x (ae^2+cd^2)+ade+cdex^2}}{e(d+ex)^{3/2}}$$

$$\Big\downarrow \text{ 218}$$

$$\frac{cd \arctan\left(\frac{\sqrt{e}\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{\sqrt{d+ex}\sqrt{cd^2-ae^2}}\right)}{e^{3/2}\sqrt{cd^2-ae^2}}-\frac{\sqrt{x (ae^2+cd^2)+ade+cdex^2}}{e(d+ex)^{3/2}}$$

input
```
Int[Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]/(d + e*x)^(5/2),x]
```

output
```
-(Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]/(e*(d + e*x)^(3/2))) + (c*d*
ArcTan[(Sqrt[e]*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(Sqrt[c*d^2 -
 a*e^2]*Sqrt[d + e*x])])/(e^(3/2)*Sqrt[c*d^2 - a*e^2])
```

Defintions of rubi rules used

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 1130
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(d + e*x)^(m + 1)*((a + b*x + c*x^2)^p/(e*(m + p + 1))), x]
- Simp[c*(p/(e^2*(m + p + 1)))   Int[(d + e*x)^(m + 2)*(a + b*x + c*x^2)^(p
 - 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0]
 && GtQ[p, 0] && (LtQ[m, -2] || EqQ[m + 2*p + 1, 0]) && NeQ[m + p + 1, 0] &
& IntegerQ[2*p]
```

rule 1136
```
Int[1/(Sqrt[(d_.) + (e_.)*(x_)]*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x
_Symbol] :> Simp[2*e   Subst[Int[1/(2*c*d - b*e + e^2*x^2), x], x, Sqrt[a +
 b*x + c*x^2]/Sqrt[d + e*x]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c*d^2
- b*d*e + a*e^2, 0]
```

Maple [A] (verified)

Time = 1.08 (sec) , antiderivative size = 153, normalized size of antiderivative = 1.19

method	result	size
default	$\frac{\left(-\operatorname{arctanh}\left(\frac{e\sqrt{cdx+ae}}{\sqrt{\left(a\,e^{2}-c\,d^{2}\right)e}}\right)cdex-\operatorname{arctanh}\left(\frac{e\sqrt{cdx+ae}}{\sqrt{\left(a\,e^{2}-c\,d^{2}\right)e}}\right)c\,d^{2}-\sqrt{cdx+ae}\,\sqrt{\left(a\,e^{2}-c\,d^{2}\right)e}\right)\sqrt{\left(ex+d\right)\left(cdx+ae\right)}}{\left(ex+d\right)^{\frac{3}{2}}\sqrt{cdx+ae}\,e\sqrt{\left(a\,e^{2}-c\,d^{2}\right)e}}$	153

input
```
int((a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(1/2)/(e*x+d)^(5/2),x,method=_RETURN
VERBOSE)
```

output
```
(-arctanh(e*(c*d*x+a*e)^(1/2)/((a*e^2-c*d^2)*e)^(1/2))*c*d*e*x-arctanh(e*(
c*d*x+a*e)^(1/2)/((a*e^2-c*d^2)*e)^(1/2))*c*d^2-(c*d*x+a*e)^(1/2)*((a*e^2-
c*d^2)*e)^(1/2))*((e*x+d)*(c*d*x+a*e))^(1/2)/(e*x+d)^(3/2)/(c*d*x+a*e)^(1/
2)/e/((a*e^2-c*d^2)*e)^(1/2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 230 vs. $2(113) = 226$.

Time = 0.10 (sec) , antiderivative size = 482, normalized size of antiderivative = 3.74

$$\int \frac{\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{(d+ex)^{5/2}}\,dx = \Bigg[-\frac{(cde^2x^2+2\,cd^2ex+cd^3)\sqrt{-cd^2e+ae^3}\log\left(-\frac{cde^2x^2+2\,ae^3x-cd^3+2\cdots}{\cdots}\right.}{2\,(cd^4e^2-\cdots}$$

$$-\frac{(cde^2x^2+2\,cd^2ex+cd^3)\sqrt{cd^2e-ae^3}\arctan\left(-\frac{\sqrt{cdex^2+ade+(cd^2+ae^2)x}\sqrt{cd^2e-ae^3}\sqrt{ex+d}}{cd^3-ade^2+(cd^2e-ae^3)x}\right)+\sqrt{cdex^2+ade+\cdots}}{cd^4e^2-ad^2e^4+(cd^2e^4-ae^6)x^2+2\,(cd^3e^3-ade^5)x}$$

input
```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(e*x+d)^(5/2),x, algorit
hm="fricas")
```

output
```
[-1/2*((c*d*e^2*x^2 + 2*c*d^2*e*x + c*d^3)*sqrt(-c*d^2*e + a*e^3)*log(-(c*
d*e^2*x^2 + 2*a*e^3*x - c*d^3 + 2*a*d*e^2 - 2*sqrt(c*d*e*x^2 + a*d*e + (c*
d^2 + a*e^2)*x)*sqrt(-c*d^2*e + a*e^3)*sqrt(e*x + d))/(e^2*x^2 + 2*d*e*x +
 d^2)) + 2*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(c*d^2*e - a*e^3)*s
qrt(e*x + d))/(c*d^4*e^2 - a*d^2*e^4 + (c*d^2*e^4 - a*e^6)*x^2 + 2*(c*d^3*
e^3 - a*d*e^5)*x), -((c*d*e^2*x^2 + 2*c*d^2*e*x + c*d^3)*sqrt(c*d^2*e - a*
e^3)*arctan(-sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(c*d^2*e - a*
e^3)*sqrt(e*x + d)/(c*d^3 - a*d*e^2 + (c*d^2*e - a*e^3)*x)) + sqrt(c*d*e*x
^2 + a*d*e + (c*d^2 + a*e^2)*x)*(c*d^2*e - a*e^3)*sqrt(e*x + d))/(c*d^4*e^
2 - a*d^2*e^4 + (c*d^2*e^4 - a*e^6)*x^2 + 2*(c*d^3*e^3 - a*d*e^5)*x)]
```

Sympy [F]

$$\int \frac{\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{(d+ex)^{5/2}}\,dx = \int \frac{\sqrt{(d+ex)\,(ae+cdx)}}{(d+ex)^{\frac{5}{2}}}\,dx$$

input
```
integrate((a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(1/2)/(e*x+d)**(5/2),x)
```

output
```
Integral(sqrt((d + e*x)*(a*e + c*d*x))/(d + e*x)**(5/2), x)
```

Maxima [F]

$$\int \frac{\sqrt{ade + (cd^2 + ae^2)\,x + cdex^2}}{(d + ex)^{5/2}}\,dx = \int \frac{\sqrt{cdex^2 + ade + (cd^2 + ae^2)x}}{(ex + d)^{\frac{5}{2}}}\,dx$$

input
```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(e*x+d)^(5/2),x, algorit
hm="maxima")
```

output
```
integrate(sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)/(e*x + d)^(5/2), x)
```

Giac [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 111, normalized size of antiderivative = 0.86

$$\int \frac{\sqrt{ade + (cd^2 + ae^2)\,x + cdex^2}}{(d + ex)^{5/2}}\,dx = \frac{cd\left(\frac{\arctan\left(\frac{\sqrt{(ex+d)cde - cd^2e + ae^3}}{\sqrt{cd^2e - ae^3}}\right)}{\sqrt{cd^2e - ae^3}\,e} - \frac{\sqrt{(ex+d)cde - cd^2e + ae^3}}{(ex+d)cde^2}\right)|e|}{e}$$

input
```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(e*x+d)^(5/2),x, algorit
hm="giac")
```

output
```
c*d*(arctan(sqrt((e*x + d)*c*d*e - c*d^2*e + a*e^3)/sqrt(c*d^2*e - a*e^3))
/(sqrt(c*d^2*e - a*e^3)*e) - sqrt((e*x + d)*c*d*e - c*d^2*e + a*e^3)/((e*x
 + d)*c*d*e^2))*abs(e)/e
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{(d+ex)^{5/2}}\,dx=\int \frac{\sqrt{c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e}}{{\left(d+e\,x\right)}^{5/2}} \,dx$$

input
```
int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2)/(d + e*x)^(5/2),x)
```

output
```
int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2)/(d + e*x)^(5/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 168, normalized size of antiderivative = 1.30

$$\int \frac{\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{(d+ex)^{5/2}}\,dx=\frac{-\sqrt{e}\,\sqrt{-a\,e^2+c\,d^2}\,atan\left(\frac{\sqrt{cdx+ae}\,e}{\sqrt{e}\,\sqrt{-a\,e^2+c\,d^2}}\right)c\,d^2-\sqrt{e}\,\sqrt{-a\,e^2+c\,d^2}\,a}{e^2\left(a\,e^3x-c\,d^2ex+\right.}$$

input
```
int((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(e*x+d)^(5/2),x)
```

output
```
( - sqrt(e)*sqrt( - a*e**2 + c*d**2)*atan((sqrt(a*e + c*d*x)*e)/(sqrt(e)*s
qrt( - a*e**2 + c*d**2)))*c*d**2 - sqrt(e)*sqrt( - a*e**2 + c*d**2)*atan((
sqrt(a*e + c*d*x)*e)/(sqrt(e)*sqrt( - a*e**2 + c*d**2)))*c*d*e*x - sqrt(a*
e + c*d*x)*a*e**3 + sqrt(a*e + c*d*x)*c*d**2*e)/(e**2*(a*d*e**2 + a*e**3*x
 - c*d**3 - c*d**2*e*x))
```

3.285 $\int \frac{\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{(d+ex)^{7/2}}\,dx$

Optimal result

Integrand size = 39, antiderivative size = 199

$$\int \frac{\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{(d+ex)^{7/2}}\,dx = -\frac{\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{2e(d+ex)^{5/2}} + \frac{cd\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{4e\,(cd^2-ae^2)\,(d+ex)^{3/2}} + \frac{c^2d^2\arctan\left(\frac{\sqrt{e}\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{\sqrt{cd^2-ae^2}\sqrt{d+ex}}\right)}{4e^{3/2}\,(cd^2-ae^2)^{3/2}}$$

output
```
-1/2*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/e/(e*x+d)^(5/2)+1/4*c*d*(a*d*
e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/e/(-a*e^2+c*d^2)/(e*x+d)^(3/2)+1/4*c^2*
d^2*arctan(e^(1/2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(-a*e^2+c*d^2)^
(1/2)/(e*x+d)^(1/2))/e^(3/2)/(-a*e^2+c*d^2)^(3/2)
```

Mathematica [A] (verified)

Time = 0.45 (sec) , antiderivative size = 165, normalized size of antiderivative = 0.83

$$\int \frac{\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{(d+ex)^{7/2}}\,dx = \frac{\sqrt{(ae+cdx)(d+ex)}\left(\sqrt{e}\sqrt{cd^2-ae^2}\sqrt{ae+cdx}(2ae^2+cd(-d+e\right.}{4e^{3/2}\,(cd^2-ae^2)^{3/2}\,\sqrt{ae+cdx}(d}$$

input
```
Integrate[Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]/(d + e*x)^(7/2),x]
```

output
```
(Sqrt[(a*e + c*d*x)*(d + e*x)]*(Sqrt[e]*Sqrt[c*d^2 - a*e^2]*Sqrt[a*e + c*d
*x]*(2*a*e^2 + c*d*(-d + e*x)) + c^2*d^2*(d + e*x)^2*ArcTan[(Sqrt[e]*Sqrt[
a*e + c*d*x])/Sqrt[c*d^2 - a*e^2]]))/(4*e^(3/2)*(c*d^2 - a*e^2)^(3/2)*Sqrt
[a*e + c*d*x]*(d + e*x)^(5/2))
```

Rubi [A] (verified)

Time = 0.58 (sec) , antiderivative size = 194, normalized size of antiderivative = 0.97, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.103$, Rules used = {1130, 1135, 1136, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{x \left(ae^2+cd^2\right)+ade+cdex^2}}{(d+ex)^{7/2}} \, dx$$

$$\Big\downarrow \ 1130$$

$$\frac{cd \int \frac{1}{(d+ex)^{3/2}\sqrt{cdex^2+(cd^2+ae^2)x+ade}}dx}{4e}-\frac{\sqrt{x \left(ae^2+cd^2\right)+ade+cdex^2}}{2e(d+ex)^{5/2}}$$

$$\Big\downarrow \ 1135$$

$$\frac{cd \left(\frac{cd \int \frac{1}{\sqrt{d+ex}\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}dx}{2(cd^2-ae^2)}+\frac{\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{(d+ex)^{3/2}(cd^2-ae^2)}\right)}{4e}-\frac{\sqrt{x \left(ae^2+cd^2\right)+ade+cdex^2}}{2e(d+ex)^{5/2}}$$

$$\Big\downarrow \ 1136$$

$$\frac{cd\left(\frac{cde\int \frac{1}{\frac{\left(cdex^2+\left(cd^2+ae^2\right)x+ade\right)e^2}{d+ex}+\left(cd^2-ae^2\right)e}d\frac{\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}{\sqrt{d+ex}}}{cd^2-ae^2}+\frac{\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{(d+ex)^{3/2}(cd^2-ae^2)}\right)}{4e}-\frac{\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{2e(d+ex)^{5/2}}$$

$$\Big\downarrow \ 218$$

$$\frac{cd\left(\frac{cd\arctan\left(\frac{\sqrt{e}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{\sqrt{d+ex}\sqrt{cd^2-ae^2}}\right)}{\sqrt{e}(cd^2-ae^2)^{3/2}}+\frac{\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{(d+ex)^{3/2}(cd^2-ae^2)}\right)}{4e}-\frac{\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{2e(d+ex)^{5/2}}$$

input
```
Int[Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]/(d + e*x)^(7/2),x]
```

output
```
-1/2*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]/(e*(d + e*x)^(5/2)) + (c*
d*(Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]/((c*d^2 - a*e^2)*(d + e*x)^
(3/2)) + (c*d*ArcTan[(Sqrt[e]*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])
/(Sqrt[c*d^2 - a*e^2]*Sqrt[d + e*x])])/(Sqrt[e]*(c*d^2 - a*e^2)^(3/2))))/(
4*e)
```

Defintions of rubi rules used

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 1130
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(d + e*x)^(m + 1)*((a + b*x + c*x^2)^p/(e*(m + p + 1))), x]
- Simp[c*(p/(e^2*(m + p + 1)))   Int[(d + e*x)^(m + 2)*(a + b*x + c*x^2)^(p
 - 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0]
 && GtQ[p, 0] && (LtQ[m, -2] || EqQ[m + 2*p + 1, 0]) && NeQ[m + p + 1, 0] &
& IntegerQ[2*p]
```

rule 1135
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(-e)*(d + e*x)^m*((a + b*x + c*x^2)^(p + 1)/((m + p + 1)*(2*
c*d - b*e))), x] + Simp[c*((m + 2*p + 2)/((m + p + 1)*(2*c*d - b*e)))   Int
[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p},
 x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[m, 0] && NeQ[m + p + 1, 0] && I
ntegerQ[2*p]
```

rule 1136
```
Int[1/(Sqrt[(d_.) + (e_.)*(x_)]*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x
_Symbol] :> Simp[2*e   Subst[Int[1/(2*c*d - b*e + e^2*x^2), x], x, Sqrt[a +
 b*x + c*x^2]/Sqrt[d + e*x]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c*d^2
- b*d*e + a*e^2, 0]
```

Maple [A] (verified)

Time = 1.09 (sec) , antiderivative size = 282, normalized size of antiderivative = 1.42

method	result
default	$\frac{\sqrt{(ex+d)(cdx+ae)}\left(\operatorname{arctanh}\left(\frac{e\sqrt{cdx+ae}}{\sqrt{(a\,e^2-c\,d^2)e}}\right)c^2d^2e^2x^2+2\,\operatorname{arctanh}\left(\frac{e\sqrt{cdx+ae}}{\sqrt{(a\,e^2-c\,d^2)e}}\right)c^2d^3ex+\operatorname{arctanh}\left(\frac{e\sqrt{cdx+ae}}{\sqrt{(a\,e^2-c\,d^2)e}}\right)c^2d^4-c}{4(ex+d)^{\frac{5}{2}}\sqrt{cdx+ae}\,(a\,e^2-c\,d^2)e\sqrt{(a\,e^2-}}$

input
```
int((a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(1/2)/(e*x+d)^(7/2),x,method=_RETURN
VERBOSE)
```

output
```
1/4*((e*x+d)*(c*d*x+a*e))^(1/2)*(arctanh(e*(c*d*x+a*e)^(1/2)/((a*e^2-c*d^2
)*e)^(1/2))*c^2*d^2*e^2*x^2+2*arctanh(e*(c*d*x+a*e)^(1/2)/((a*e^2-c*d^2)*e
)^(1/2))*c^2*d^3*e*x+arctanh(e*(c*d*x+a*e)^(1/2)/((a*e^2-c*d^2)*e)^(1/2))*
c^2*d^4-c*d*e*x*(c*d*x+a*e)^(1/2)*((a*e^2-c*d^2)*e)^(1/2)-2*((a*e^2-c*d^2)
*e)^(1/2)*(c*d*x+a*e)^(1/2)*a*e^2+((a*e^2-c*d^2)*e)^(1/2)*(c*d*x+a*e)^(1/2
)*c*d^2)/(e*x+d)^(5/2)/(c*d*x+a*e)^(1/2)/(a*e^2-c*d^2)/e/((a*e^2-c*d^2)*e)
^(1/2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 363 vs. $2(173) = 346$.

Time = 0.13 (sec) , antiderivative size = 748, normalized size of antiderivative = 3.76

$$\int \frac{\sqrt{ade + (cd^2 + ae^2)x + cdex^2}}{(d + ex)^{7/2}}\,dx = \left[\frac{(c^2d^2e^3x^3 + 3\,c^2d^3e^2x^2 + 3\,c^2d^4ex + c^2d^5)\sqrt{-cd^2e + ae^3}\log\left(-\frac{cd}{\ }\right.}{8\,(c^2d^7e^2 - 2\,acd^5e^4 + }\right.$$

$$-\frac{(c^2d^2e^3x^3 + 3\,c^2d^3e^2x^2 + 3\,c^2d^4ex + c^2d^5)\sqrt{cd^2e - ae^3}\arctan\left(-\frac{\sqrt{cdex^2 + ade + (cd^2 + ae^2)x}\sqrt{cd^2e - ae^3}\sqrt{ex + d}}{cd^3 - ade^2 + (cd^2e - ae^3)x}\right) + (}{4\,(c^2d^7e^2 - 2\,acd^5e^4 + a^2d^3e^6 + (c^2d^4e^5 - 2\,acd^2e^7 + a^2e^9)x^3 + 3\,(c^2d^5e^4 - }$$

input
```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(e*x+d)^(7/2),x, algorit
hm="fricas")
```

output
```
[1/8*((c^2*d^2*e^3*x^3 + 3*c^2*d^3*e^2*x^2 + 3*c^2*d^4*e*x + c^2*d^5)*sqrt
(-c*d^2*e + a*e^3)*log(-(c*d*e^2*x^2 + 2*a*e^3*x - c*d^3 + 2*a*d*e^2 + 2*s
qrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(-c*d^2*e + a*e^3)*sqrt(e*x
 + d))/(e^2*x^2 + 2*d*e*x + d^2)) - 2*(c^2*d^4*e - 3*a*c*d^2*e^3 + 2*a^2*e
^5 - (c^2*d^3*e^2 - a*c*d*e^4)*x)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)
*x)*sqrt(e*x + d))/(c^2*d^7*e^2 - 2*a*c*d^5*e^4 + a^2*d^3*e^6 + (c^2*d^4*e
^5 - 2*a*c*d^2*e^7 + a^2*e^9)*x^3 + 3*(c^2*d^5*e^4 - 2*a*c*d^3*e^6 + a^2*d
*e^8)*x^2 + 3*(c^2*d^6*e^3 - 2*a*c*d^4*e^5 + a^2*d^2*e^7)*x), -1/4*((c^2*d
^2*e^3*x^3 + 3*c^2*d^3*e^2*x^2 + 3*c^2*d^4*e*x + c^2*d^5)*sqrt(c*d^2*e - a
*e^3)*arctan(-sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(c*d^2*e - a
*e^3)*sqrt(e*x + d)/(c*d^3 - a*d*e^2 + (c*d^2*e - a*e^3)*x)) + (c^2*d^4*e
- 3*a*c*d^2*e^3 + 2*a^2*e^5 - (c^2*d^3*e^2 - a*c*d*e^4)*x)*sqrt(c*d*e*x^2
+ a*d*e + (c*d^2 + a*e^2)*x)*sqrt(e*x + d))/(c^2*d^7*e^2 - 2*a*c*d^5*e^4 +
 a^2*d^3*e^6 + (c^2*d^4*e^5 - 2*a*c*d^2*e^7 + a^2*e^9)*x^3 + 3*(c^2*d^5*e^
4 - 2*a*c*d^3*e^6 + a^2*d*e^8)*x^2 + 3*(c^2*d^6*e^3 - 2*a*c*d^4*e^5 + a^2*
d^2*e^7)*x)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\sqrt{ade + (cd^2 + ae^2)x + cdex^2}}{(d + ex)^{7/2}}\,dx = \text{Timed out}$$

input
```
integrate((a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(1/2)/(e*x+d)**(7/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{\sqrt{ade + (cd^2 + ae^2)x + cdex^2}}{(d + ex)^{7/2}}\,dx = \int \frac{\sqrt{cdex^2 + ade + (cd^2 + ae^2)x}}{(ex + d)^{\frac{7}{2}}}\,dx$$

input
```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(e*x+d)^(7/2),x, algorit
hm="maxima")
```

output
```
integrate(sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)/(e*x + d)^(7/2), x)
```

Giac [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 228, normalized size of antiderivative = 1.15

$$\int \frac{\sqrt{ade + (cd^2 + ae^2)x + cdex^2}}{(d + ex)^{7/2}}\,dx = \frac{\left(\frac{c^3d^3e\arctan\left(\frac{\sqrt{(ex+d)cde - cd^2e + ae^3}}{\sqrt{cd^2e - ae^3}}\right)}{\sqrt{cd^2e - ae^3}(cd^2 - ae^2)} - \frac{\sqrt{(ex+d)cde - cd^2e + ae^3}c^4d^5e^2 - \sqrt{(ex+d)cd}}{(cd^2 - ae}\right.}{4\,cde^3}$$

input
```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(e*x+d)^(7/2),x, algorit
hm="giac")
```

output
```
1/4*(c^3*d^3*e*arctan(sqrt((e*x + d)*c*d*e - c*d^2*e + a*e^3)/sqrt(c*d^2*e
 - a*e^3))/(sqrt(c*d^2*e - a*e^3)*(c*d^2 - a*e^2)) - (sqrt((e*x + d)*c*d*e
 - c*d^2*e + a*e^3)*c^4*d^5*e^2 - sqrt((e*x + d)*c*d*e - c*d^2*e + a*e^3)*
a*c^3*d^3*e^4 - ((e*x + d)*c*d*e - c*d^2*e + a*e^3)^(3/2)*c^3*d^3*e)/((c*d
^2 - a*e^2)*(e*x + d)^2*c^2*d^2*e^2))*abs(e)/(c*d*e^3)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{ade + (cd^2 + ae^2)\,x + cdex^2}}{(d + ex)^{7/2}}\,dx = \int \frac{\sqrt{c\,d\,e\,x^2 + (c\,d^2 + a\,e^2)\,x + a\,d\,e}}{(d + e\,x)^{7/2}}\,dx$$

input
```
int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2)/(d + e*x)^(7/2),x)
```

output
```
int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2)/(d + e*x)^(7/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 363, normalized size of antiderivative = 1.82

$$\int \frac{\sqrt{ade + (cd^2 + ae^2)\,x + cdex^2}}{(d + ex)^{7/2}}\,dx = \frac{\sqrt{e}\,\sqrt{-a\,e^2 + c\,d^2}\,atan\left(\frac{\sqrt{cdx+ae}\,e}{\sqrt{e}\,\sqrt{-a\,e^2+c\,d^2}}\right)c^2d^4 + 2\sqrt{e}\,\sqrt{-a\,e^2 + c\,d^2}\,at}{}$$

input
```
int((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(e*x+d)^(7/2),x)
```

output

```
(sqrt(e)*sqrt( - a*e**2 + c*d**2)*atan((sqrt(a*e + c*d*x)*e)/(sqrt(e)*sqrt
( - a*e**2 + c*d**2)))*c**2*d**4 + 2*sqrt(e)*sqrt( - a*e**2 + c*d**2)*atan
((sqrt(a*e + c*d*x)*e)/(sqrt(e)*sqrt( - a*e**2 + c*d**2)))*c**2*d**3*e*x +
 sqrt(e)*sqrt( - a*e**2 + c*d**2)*atan((sqrt(a*e + c*d*x)*e)/(sqrt(e)*sqrt
( - a*e**2 + c*d**2)))*c**2*d**2*e**2*x**2 - 2*sqrt(a*e + c*d*x)*a**2*e**5
 + 3*sqrt(a*e + c*d*x)*a*c*d**2*e**3 - sqrt(a*e + c*d*x)*a*c*d*e**4*x - sq
rt(a*e + c*d*x)*c**2*d**4*e + sqrt(a*e + c*d*x)*c**2*d**3*e**2*x)/(4*e**2*
(a**2*d**2*e**4 + 2*a**2*d*e**5*x + a**2*e**6*x**2 - 2*a*c*d**4*e**2 - 4*a
*c*d**3*e**3*x - 2*a*c*d**2*e**4*x**2 + c**2*d**6 + 2*c**2*d**5*e*x + c**2
*d**4*e**2*x**2))
```

3.286 $\int \frac{\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{(d+ex)^{9/2}}\,dx$

Optimal result

Integrand size = 39, antiderivative size = 264

$$\int \frac{\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{(d+ex)^{9/2}}\,dx = -\frac{\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{3e(d+ex)^{7/2}} + \frac{cd\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{12e(cd^2-ae^2)(d+ex)^{5/2}} + \frac{c^2d^2\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{8e(cd^2-ae^2)^2(d+ex)^{3/2}} + \frac{c^3d^3\arctan\left(\frac{\sqrt{e}\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{\sqrt{cd^2-ae^2}\sqrt{d+ex}}\right)}{8e^{3/2}(cd^2-ae^2)^{5/2}}$$

output
```
-1/3*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/e/(e*x+d)^(7/2)+1/12*c*d*(a*d
*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/e/(-a*e^2+c*d^2)/(e*x+d)^(5/2)+1/8*c^2
*d^2*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/e/(-a*e^2+c*d^2)^2/(e*x+d)^(3
/2)+1/8*c^3*d^3*arctan(e^(1/2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(-a
*e^2+c*d^2)^(1/2)/(e*x+d)^(1/2))/e^(3/2)/(-a*e^2+c*d^2)^(5/2)
```

Mathematica [A] (verified)

Time = 0.71 (sec) , antiderivative size = 200, normalized size of antiderivative = 0.76

$$\int \frac{\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{(d+ex)^{9/2}}\,dx = \frac{\sqrt{(ae+cdx)(d+ex)}\left(\sqrt{e}\sqrt{cd^2-ae^2}\sqrt{ae+cdx}(-8a^2e^4+2acde^2(\right.}{24e^{3/2}\,(cd^2-a}$$

input
```
Integrate[Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]/(d + e*x)^(9/2),x]
```

output
```
(Sqrt[(a*e + c*d*x)*(d + e*x)]*(Sqrt[e]*Sqrt[c*d^2 - a*e^2]*Sqrt[a*e + c*d
*x]*(-8*a^2*e^4 + 2*a*c*d*e^2*(7*d - e*x) + c^2*d^2*(-3*d^2 + 8*d*e*x + 3*
e^2*x^2)) + 3*c^3*d^3*(d + e*x)^3*ArcTan[(Sqrt[e]*Sqrt[a*e + c*d*x])/Sqrt[
c*d^2 - a*e^2]]))/(24*e^(3/2)*(c*d^2 - a*e^2)^(5/2)*Sqrt[a*e + c*d*x]*(d +
 e*x)^(7/2))
```

Rubi [A] (verified)

Time = 0.70 (sec) , antiderivative size = 271, normalized size of antiderivative = 1.03, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.128$, Rules used = {1130, 1135, 1135, 1136, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{x\,(ae^2+cd^2)+ade+cdex^2}}{(d+ex)^{9/2}}\,dx$$

$$\Big\downarrow \text{1130}$$

$$\frac{cd\int \frac{1}{(d+ex)^{5/2}\sqrt{cdex^2+(cd^2+ae^2)x+ade}}dx}{6e}-\frac{\sqrt{x\,(ae^2+cd^2)+ade+cdex^2}}{3e(d+ex)^{7/2}}$$

$$\Big\downarrow \text{1135}$$

$$\frac{cd\left(\frac{3cd\int \frac{1}{(d+ex)^{3/2}\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}dx}{4(cd^2-ae^2)}+\frac{\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{2(d+ex)^{5/2}(cd^2-ae^2)}\right)}{6e}-\frac{\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{3e(d+ex)^{7/2}}$$

$\Big\downarrow$ 1135

$$\frac{cd\left(\frac{3cd\left(\frac{cd\int \frac{1}{\sqrt{d+ex}\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}dx}{2\left(cd^2-ae^2\right)}+\frac{\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{(d+ex)^{3/2}\left(cd^2-ae^2\right)}\right)}{4(cd^2-ae^2)}+\frac{\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{2(d+ex)^{5/2}(cd^2-ae^2)}\right)}{6e}-\frac{\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{3e(d+ex)^{7/2}}$$

$\Big\downarrow$ 1136

$$\frac{cd\left(\frac{3cd\left(\frac{cde\int \frac{1}{\frac{\left(cdex^2+\left(cd^2+ae^2\right)x+ade\right)e^2}{d+ex}+\left(cd^2-ae^2\right)e}d\frac{\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}{\sqrt{d+ex}}}{cd^2-ae^2}+\frac{\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{(d+ex)^{3/2}\left(cd^2-ae^2\right)}\right)}{4(cd^2-ae^2)}+\frac{\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{2(d+ex)^{5/2}(cd^2-ae^2)}\right)}{6e}-\frac{\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{3e(d+ex)^{7/2}}$$

$\Big\downarrow$ 218

$$\frac{cd\left(\frac{3cd\left(\frac{cd\arctan\left(\frac{\sqrt{e}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{\sqrt{d+ex}\sqrt{cd^2-ae^2}}\right)}{\sqrt{e}\left(cd^2-ae^2\right)^{3/2}}+\frac{\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{(d+ex)^{3/2}\left(cd^2-ae^2\right)}\right)}{4(cd^2-ae^2)}+\frac{\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{2(d+ex)^{5/2}(cd^2-ae^2)}\right)}{6e}-\frac{\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{3e(d+ex)^{7/2}}$$

input
```
Int[Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]/(d + e*x)^(9/2),x]
```

output
```
-1/3*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]/(e*(d + e*x)^(7/2)) + (c*
d*(Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]/(2*(c*d^2 - a*e^2)*(d + e*x
)^(5/2)) + (3*c*d*(Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]/((c*d^2 - a
*e^2)*(d + e*x)^(3/2)) + (c*d*ArcTan[(Sqrt[e]*Sqrt[a*d*e + (c*d^2 + a*e^2)
*x + c*d*e*x^2])/(Sqrt[c*d^2 - a*e^2]*Sqrt[d + e*x])])/(Sqrt[e]*(c*d^2 - a
*e^2)^(3/2))))/(4*(c*d^2 - a*e^2))))/(6*e)
```

Defintions of rubi rules used

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 1130
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(d + e*x)^(m + 1)*((a + b*x + c*x^2)^p/(e*(m + p + 1))), x]
- Simp[c*(p/(e^2*(m + p + 1)))   Int[(d + e*x)^(m + 2)*(a + b*x + c*x^2)^(p
 - 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0]
 && GtQ[p, 0] && (LtQ[m, -2] || EqQ[m + 2*p + 1, 0]) && NeQ[m + p + 1, 0] &
& IntegerQ[2*p]
```

rule 1135
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(-e)*(d + e*x)^m*((a + b*x + c*x^2)^(p + 1)/((m + p + 1)*(2*
c*d - b*e))), x] + Simp[c*((m + 2*p + 2)/((m + p + 1)*(2*c*d - b*e)))   Int
[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p},
 x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[m, 0] && NeQ[m + p + 1, 0] && I
ntegerQ[2*p]
```

rule 1136
```
Int[1/(Sqrt[(d_.) + (e_.)*(x_)]*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x
_Symbol] :> Simp[2*e   Subst[Int[1/(2*c*d - b*e + e^2*x^2), x], x, Sqrt[a +
 b*x + c*x^2]/Sqrt[d + e*x]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c*d^2
- b*d*e + a*e^2, 0]
```

Maple [A] (verified)

Time = 1.08 (sec) , antiderivative size = 447, normalized size of antiderivative = 1.69

method	result
default	$-\frac{\sqrt{(ex+d)(cdx+ae)}\left(3\,\operatorname{arctanh}\left(\frac{e\sqrt{cdx+ae}}{\sqrt{(a\,e^{2}-c\,d^{2})e}}\right)c^{3}d^{3}e^{3}x^{3}+9\,\operatorname{arctanh}\left(\frac{e\sqrt{cdx+ae}}{\sqrt{(a\,e^{2}-c\,d^{2})e}}\right)c^{3}d^{4}e^{2}x^{2}+9\,\operatorname{arctanh}\left(\frac{e\sqrt{cdx+ae}}{\sqrt{(a\,e^{2}-c\,d^{2})e}}\right.\right.}{}$

input
```
int((a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(1/2)/(e*x+d)^(9/2),x,method=_RETURN
VERBOSE)
```

output
```
-1/24*((e*x+d)*(c*d*x+a*e))^(1/2)*(3*arctanh(e*(c*d*x+a*e)^(1/2)/((a*e^2-c
*d^2)*e)^(1/2))*c^3*d^3*e^3*x^3+9*arctanh(e*(c*d*x+a*e)^(1/2)/((a*e^2-c*d^
2)*e)^(1/2))*c^3*d^4*e^2*x^2+9*arctanh(e*(c*d*x+a*e)^(1/2)/((a*e^2-c*d^2)*
e)^(1/2))*c^3*d^5*e*x+3*arctanh(e*(c*d*x+a*e)^(1/2)/((a*e^2-c*d^2)*e)^(1/2
))*c^3*d^6-3*c^2*d^2*e^2*x^2*(c*d*x+a*e)^(1/2)*((a*e^2-c*d^2)*e)^(1/2)+2*a
*c*d*e^3*x*(c*d*x+a*e)^(1/2)*((a*e^2-c*d^2)*e)^(1/2)-8*c^2*d^3*e*x*(c*d*x+
a*e)^(1/2)*((a*e^2-c*d^2)*e)^(1/2)+8*((a*e^2-c*d^2)*e)^(1/2)*(c*d*x+a*e)^(
1/2)*a^2*e^4-14*((a*e^2-c*d^2)*e)^(1/2)*(c*d*x+a*e)^(1/2)*a*c*d^2*e^2+3*((
a*e^2-c*d^2)*e)^(1/2)*(c*d*x+a*e)^(1/2)*c^2*d^4)/(e*x+d)^(7/2)/((a*e^2-c*d
^2)*e)^(1/2)/e/(a*e^2-c*d^2)^2/(c*d*x+a*e)^(1/2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 546 vs. $2(232) = 464$.

Time = 0.13 (sec) , antiderivative size = 1114, normalized size of antiderivative = 4.22

$$\int \frac{\sqrt{ade + (cd^2 + ae^2)\,x + cdex^2}}{(d + ex)^{9/2}}\,dx = \text{Too large to display}$$

input
```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(e*x+d)^(9/2),x, algorit
hm="fricas")
```

output
```
[-1/48*(3*(c^3*d^3*e^4*x^4 + 4*c^3*d^4*e^3*x^3 + 6*c^3*d^5*e^2*x^2 + 4*c^3
*d^6*e*x + c^3*d^7)*sqrt(-c*d^2*e + a*e^3)*log(-(c*d*e^2*x^2 + 2*a*e^3*x -
 c*d^3 + 2*a*d*e^2 - 2*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(-c
*d^2*e + a*e^3)*sqrt(e*x + d))/(e^2*x^2 + 2*d*e*x + d^2)) + 2*(3*c^3*d^6*e
 - 17*a*c^2*d^4*e^3 + 22*a^2*c*d^2*e^5 - 8*a^3*e^7 - 3*(c^3*d^4*e^3 - a*c^
2*d^2*e^5)*x^2 - 2*(4*c^3*d^5*e^2 - 5*a*c^2*d^3*e^4 + a^2*c*d*e^6)*x)*sqrt
(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(e*x + d))/(c^3*d^10*e^2 - 3*a
*c^2*d^8*e^4 + 3*a^2*c*d^6*e^6 - a^3*d^4*e^8 + (c^3*d^6*e^6 - 3*a*c^2*d^4*
e^8 + 3*a^2*c*d^2*e^10 - a^3*e^12)*x^4 + 4*(c^3*d^7*e^5 - 3*a*c^2*d^5*e^7
+ 3*a^2*c*d^3*e^9 - a^3*d*e^11)*x^3 + 6*(c^3*d^8*e^4 - 3*a*c^2*d^6*e^6 + 3
*a^2*c*d^4*e^8 - a^3*d^2*e^10)*x^2 + 4*(c^3*d^9*e^3 - 3*a*c^2*d^7*e^5 + 3*
a^2*c*d^5*e^7 - a^3*d^3*e^9)*x), -1/24*(3*(c^3*d^3*e^4*x^4 + 4*c^3*d^4*e^3
*x^3 + 6*c^3*d^5*e^2*x^2 + 4*c^3*d^6*e*x + c^3*d^7)*sqrt(c*d^2*e - a*e^3)*
arctan(-sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(c*d^2*e - a*e^3)*
sqrt(e*x + d)/(c*d^3 - a*d*e^2 + (c*d^2*e - a*e^3)*x)) + (3*c^3*d^6*e - 17
*a*c^2*d^4*e^3 + 22*a^2*c*d^2*e^5 - 8*a^3*e^7 - 3*(c^3*d^4*e^3 - a*c^2*d^2
*e^5)*x^2 - 2*(4*c^3*d^5*e^2 - 5*a*c^2*d^3*e^4 + a^2*c*d*e^6)*x)*sqrt(c*d*
e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(e*x + d))/(c^3*d^10*e^2 - 3*a*c^2*
d^8*e^4 + 3*a^2*c*d^6*e^6 - a^3*d^4*e^8 + (c^3*d^6*e^6 - 3*a*c^2*d^4*e^8 +
 3*a^2*c*d^2*e^10 - a^3*e^12)*x^4 + 4*(c^3*d^7*e^5 - 3*a*c^2*d^5*e^7 + ...
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\sqrt{ade + (cd^2 + ae^2)\,x + cdex^2}}{(d + ex)^{9/2}}\,dx = \text{Timed out}$$

input
```
integrate((a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(1/2)/(e*x+d)**(9/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{\sqrt{ade + (cd^2 + ae^2)\,x + cdex^2}}{(d + ex)^{9/2}}\,dx = \int \frac{\sqrt{cdex^2 + ade + (cd^2 + ae^2)x}}{(ex + d)^{\frac{9}{2}}}\,dx$$

input
```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(e*x+d)^(9/2),x, algorit
hm="maxima")
```

output
```
integrate(sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)/(e*x + d)^(9/2), x)
```

Giac [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 337, normalized size of antiderivative = 1.28

$$\int \frac{\sqrt{ade + (cd^2 + ae^2)\,x + cdex^2}}{(d + ex)^{9/2}}\,dx = \frac{1}{24}\,c^3d^3e\left(\frac{3\,\arctan\left(\frac{\sqrt{(ex+d)cde-cd^2e+ae^3}}{\sqrt{cd^2e-ae^3}}\right)}{(c^2d^4e^3 - 2\,acd^2e^5 + a^2e^7)\sqrt{cd^2e - ae^3}} - \frac{3\,\sqrt{(ex + d)c}}{}\right.$$

input
```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(e*x+d)^(9/2),x, algorit
hm="giac")
```

output
```
1/24*c^3*d^3*e*(3*arctan(sqrt((e*x + d)*c*d*e - c*d^2*e + a*e^3)/sqrt(c*d^
2*e - a*e^3))/((c^2*d^4*e^3 - 2*a*c*d^2*e^5 + a^2*e^7)*sqrt(c*d^2*e - a*e^
3)) - (3*sqrt((e*x + d)*c*d*e - c*d^2*e + a*e^3)*c^2*d^4*e^2 - 6*sqrt((e*x
 + d)*c*d*e - c*d^2*e + a*e^3)*a*c*d^2*e^4 + 3*sqrt((e*x + d)*c*d*e - c*d^
2*e + a*e^3)*a^2*e^6 - 8*((e*x + d)*c*d*e - c*d^2*e + a*e^3)^(3/2)*c*d^2*e
 + 8*((e*x + d)*c*d*e - c*d^2*e + a*e^3)^(3/2)*a*e^3 - 3*((e*x + d)*c*d*e
- c*d^2*e + a*e^3)^(5/2))/((c^2*d^4*e^3 - 2*a*c*d^2*e^5 + a^2*e^7)*(e*x +
d)^3*c^3*d^3*e^3))*abs(e)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{ade + (cd^2 + ae^2)\,x + cdex^2}}{(d + ex)^{9/2}}\,dx = \int \frac{\sqrt{c\,d\,e\,x^2 + (c\,d^2 + a\,e^2)\;x + a\,d\,e}}{(d + e\,x)^{9/2}}\,dx$$

input
```
int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2)/(d + e*x)^(9/2),x)
```

output
```
int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2)/(d + e*x)^(9/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 626, normalized size of antiderivative = 2.37

$$\int \frac{\sqrt{ade + (cd^2 + ae^2)\,x + cdex^2}}{(d + ex)^{9/2}}\,dx = \frac{-3\sqrt{e}\,\sqrt{-a\,e^2 + c\,d^2}\;atan\left(\frac{\sqrt{cdx+ae}\,e}{\sqrt{e}\,\sqrt{-a\,e^2+c\,d^2}}\right)c^3d^6 - 9\sqrt{e}\,\sqrt{-a\,e^2 + c\,d}}{}$$

input
```
int((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(e*x+d)^(9/2),x)
```

output
```
( - 3*sqrt(e)*sqrt( - a*e**2 + c*d**2)*atan((sqrt(a*e + c*d*x)*e)/(sqrt(e)
*sqrt( - a*e**2 + c*d**2)))*c**3*d**6 - 9*sqrt(e)*sqrt( - a*e**2 + c*d**2)
*atan((sqrt(a*e + c*d*x)*e)/(sqrt(e)*sqrt( - a*e**2 + c*d**2)))*c**3*d**5*
e*x - 9*sqrt(e)*sqrt( - a*e**2 + c*d**2)*atan((sqrt(a*e + c*d*x)*e)/(sqrt(
e)*sqrt( - a*e**2 + c*d**2)))*c**3*d**4*e**2*x**2 - 3*sqrt(e)*sqrt( - a*e*
*2 + c*d**2)*atan((sqrt(a*e + c*d*x)*e)/(sqrt(e)*sqrt( - a*e**2 + c*d**2))
)*c**3*d**3*e**3*x**3 - 8*sqrt(a*e + c*d*x)*a**3*e**7 + 22*sqrt(a*e + c*d*
x)*a**2*c*d**2*e**5 - 2*sqrt(a*e + c*d*x)*a**2*c*d*e**6*x - 17*sqrt(a*e +
c*d*x)*a*c**2*d**4*e**3 + 10*sqrt(a*e + c*d*x)*a*c**2*d**3*e**4*x + 3*sqrt
(a*e + c*d*x)*a*c**2*d**2*e**5*x**2 + 3*sqrt(a*e + c*d*x)*c**3*d**6*e - 8*
sqrt(a*e + c*d*x)*c**3*d**5*e**2*x - 3*sqrt(a*e + c*d*x)*c**3*d**4*e**3*x*
*2)/(24*e**2*(a**3*d**3*e**6 + 3*a**3*d**2*e**7*x + 3*a**3*d*e**8*x**2 + a
**3*e**9*x**3 - 3*a**2*c*d**5*e**4 - 9*a**2*c*d**4*e**5*x - 9*a**2*c*d**3*
e**6*x**2 - 3*a**2*c*d**2*e**7*x**3 + 3*a*c**2*d**7*e**2 + 9*a*c**2*d**6*e
**3*x + 9*a*c**2*d**5*e**4*x**2 + 3*a*c**2*d**4*e**5*x**3 - c**3*d**9 - 3*
c**3*d**8*e*x - 3*c**3*d**7*e**2*x**2 - c**3*d**6*e**3*x**3))
```

3.287 $\int (d+ex)^{5/2} \left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2} dx$

Optimal result

Integrand size = 39, antiderivative size = 305

$$\int (d+ex)^{5/2} \left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2} dx = \frac{2(cd^2-ae^2)^4 \left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{5c^5d^5(d+ex)^{5/2}} + \frac{8e(cd^2-ae^2)^3 \left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{7/2}}{7c^5d^5(d+ex)^{7/2}} + \frac{4e^2(cd^2-ae^2)^2 \left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{9/2}}{3c^5d^5(d+ex)^{9/2}} + \frac{8e^3(cd^2-ae^2) \left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{11/2}}{11c^5d^5(d+ex)^{11/2}} + \frac{2e^4 \left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{13/2}}{13c^5d^5(d+ex)^{13/2}}$$

output
```
2/5*(-a*e^2+c*d^2)^4*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/c^5/d^5/(e*x+
d)^(5/2)+8/7*e*(-a*e^2+c*d^2)^3*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(7/2)/c^
5/d^5/(e*x+d)^(7/2)+4/3*e^2*(-a*e^2+c*d^2)^2*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*
x^2)^(9/2)/c^5/d^5/(e*x+d)^(9/2)+8/11*e^3*(-a*e^2+c*d^2)*(a*d*e+(a*e^2+c*d
^2)*x+c*d*e*x^2)^(11/2)/c^5/d^5/(e*x+d)^(11/2)+2/13*e^4*(a*d*e+(a*e^2+c*d^
2)*x+c*d*e*x^2)^(13/2)/c^5/d^5/(e*x+d)^(13/2)
```

Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 187, normalized size of antiderivative = 0.61

$$\int (d+ex)^{5/2}\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}\,dx=\frac{2((ae+cdx)(d+ex))^{5/2}\left(128a^4e^8-64a^3cde^6(13d+5ex)+16a^2c^2d^2e^4(143d^2+130dex\right.}{}$$

input
```
Integrate[(d + e*x)^(5/2)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2),x]
```

output
```
(2*((a*e + c*d*x)*(d + e*x))^(5/2)*(128*a^4*e^8 - 64*a^3*c*d*e^6*(13*d + 5
*e*x) + 16*a^2*c^2*d^2*e^4*(143*d^2 + 130*d*e*x + 35*e^2*x^2) - 8*a*c^3*d^
3*e^2*(429*d^3 + 715*d^2*e*x + 455*d*e^2*x^2 + 105*e^3*x^3) + c^4*d^4*(300
3*d^4 + 8580*d^3*e*x + 10010*d^2*e^2*x^2 + 5460*d*e^3*x^3 + 1155*e^4*x^4))
)/(15015*c^5*d^5*(d + e*x)^(5/2))
```

Rubi [A] (verified)

Time = 0.79 (sec) , antiderivative size = 317, normalized size of antiderivative = 1.04, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.128$, Rules used = {1128, 1128, 1128, 1128, 1122}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (d+ex)^{5/2}\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}dx$$

$$\Big\downarrow \text{1128}$$

$$\frac{8\left(d^2-\frac{ae^2}{c}\right)\int (d+ex)^{3/2}\left(cdex^2+\left(cd^2+ae^2\right)x+ade\right)^{3/2}dx}{13d}+\frac{2(d+ex)^{3/2}\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/2}}{13cd}$$

$$\Big\downarrow \text{1128}$$

$$\frac{8\left(d^2-\frac{ae^2}{c}\right)\left(\frac{6\left(d^2-\frac{ae^2}{c}\right)\int \sqrt{d+ex}\left(cdex^2+\left(cd^2+ae^2\right)x+ade\right)^{3/2}dx}{11d}+\frac{2\sqrt{d+ex}\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/2}}{11cd}\right)}{13d}+\frac{2(d+ex)^{3/2}\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/2}}{13cd}$$

$$\Big\downarrow \text{ 1128}$$

$$\frac{8\left(d^2-\frac{ae^2}{c}\right)\left(\frac{6\left(d^2-\frac{ae^2}{c}\right)\left(\frac{4\left(d^2-\frac{ae^2}{c}\right)\int \frac{\left(cdex^2+\left(cd^2+ae^2\right)x+ade\right)^{3/2}}{\sqrt{d+ex}}dx}{9d}+\frac{2\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/2}}{9cd\sqrt{d+ex}}\right)}{11d}+\frac{2\sqrt{d+ex}\left(x\left(ae^2+cd^2\right)+ade\right.}{11cd}\right.}{13d}+\frac{2(d+ex)^{3/2}\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/2}}{13cd}$$

$$\Big\downarrow \text{ 1128}$$

$$\frac{8\left(d^2-\frac{ae^2}{c}\right)\left(\frac{6\left(d^2-\frac{ae^2}{c}\right)\left(\frac{4\left(d^2-\frac{ae^2}{c}\right)\left(\frac{2\left(d^2-\frac{ae^2}{c}\right)\int \frac{\left(cdex^2+\left(cd^2+ae^2\right)x+ade\right)^{3/2}}{(d+ex)^{3/2}}dx}{7d}+\frac{2\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/2}}{7cd(d+ex)^{3/2}}\right)}{9d}+\frac{2\left(x\left(ae^2+cd^2\right)+\right.}{9cd\sqrt{d}}\right.}{11d}\right.}{13d}+\frac{2(d+ex)^{3/2}\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/2}}{13cd}$$

$$\Big\downarrow \text{ 1122}$$

$$\frac{2(d+ex)^{3/2}\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/2}}{13cd}+\frac{8\left(d^2-\frac{ae^2}{c}\right)\left(\frac{2\sqrt{d+ex}\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/2}}{11cd}+\frac{6\left(d^2-\frac{ae^2}{c}\right)\left(\frac{2\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/2}}{9cd\sqrt{d+ex}}+\frac{4\left(d^2-\frac{ae^2}{c}\right)\left(\frac{2\left(x\left(ae^2+cd^2\right)+ade+c\cdots\right.}{7cd(d+ex)^{3/2}}\right.}{}\right.}{11d}\right.}{13d}$$

input
```
Int[(d + e*x)^(5/2)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2),x]
```

output
```
(2*(d + e*x)^(3/2)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2))/(13*c*d)
 + (8*(d^2 - (a*e^2)/c)*((2*Sqrt[d + e*x]*(a*d*e + (c*d^2 + a*e^2)*x + c*d
*e*x^2)^(5/2))/(11*c*d) + (6*(d^2 - (a*e^2)/c)*((2*(a*d*e + (c*d^2 + a*e^2
)*x + c*d*e*x^2)^(5/2))/(9*c*d*Sqrt[d + e*x]) + (4*(d^2 - (a*e^2)/c)*((4*(
d^2 - (a*e^2)/c)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2))/(35*c*d^2*
(d + e*x)^(5/2)) + (2*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2))/(7*c*
d*(d + e*x)^(3/2))))/(9*d)))/(11*d)))/(13*d)
```

Defintions of rubi rules used

rule 1122
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[e*(d + e*x)^(m - 1)*((a + b*x + c*x^2)^(p + 1)/(c*(p + 1))),
 x] /; FreeQ[{a, b, c, d, e, m, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&
EqQ[m + p, 0]
```

rule 1128
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[e*(d + e*x)^(m - 1)*((a + b*x + c*x^2)^(p + 1)/(c*(m + 2*p +
 1))), x] + Simp[Simplify[m + p]*((2*c*d - b*e)/(c*(m + 2*p + 1)))   Int[(d
 + e*x)^(m - 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p},
 x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && IGtQ[Simplify[m + p], 0]
```

Maple [A] (verified)

Time = 1.20 (sec) , antiderivative size = 235, normalized size of antiderivative = 0.77

method	result
default	$\frac{2\sqrt{(ex+d)(cdx+ae)}\,(cdx+ae)^2(1155c^4d^4e^4x^4-840a\,c^3d^3e^5x^3+5460c^4d^5e^3x^3+560a^2c^2d^2e^6x^2-3640a\,c^3d^4e^4x^2+10010c^4d^6e^2x^2}{15015\sqrt{ex}}$
gosper	$\frac{2(cdx+ae)(1155c^4d^4e^4x^4-840a\,c^3d^3e^5x^3+5460c^4d^5e^3x^3+560a^2c^2d^2e^6x^2-3640a\,c^3d^4e^4x^2+10010c^4d^6e^2x^2-320a^3cd\,e^7x+2080}{15015c}$
orering	$\frac{2(1155c^4d^4e^4x^4-840a\,c^3d^3e^5x^3+5460c^4d^5e^3x^3+560a^2c^2d^2e^6x^2-3640a\,c^3d^4e^4x^2+10010c^4d^6e^2x^2-320a^3cd\,e^7x+2080a^2c^2d^3e^5}{15015}$

input
```
int((e*x+d)^(5/2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(3/2),x,method=_RETURN
VERBOSE)
```

output
```
2/15015/(e*x+d)^(1/2)*((e*x+d)*(c*d*x+a*e))^(1/2)*(c*d*x+a*e)^2*(1155*c^4*
d^4*e^4*x^4-840*a*c^3*d^3*e^5*x^3+5460*c^4*d^5*e^3*x^3+560*a^2*c^2*d^2*e^6
*x^2-3640*a*c^3*d^4*e^4*x^2+10010*c^4*d^6*e^2*x^2-320*a^3*c*d*e^7*x+2080*a
^2*c^2*d^3*e^5*x-5720*a*c^3*d^5*e^3*x+8580*c^4*d^7*e*x+128*a^4*e^8-832*a^3
*c*d^2*e^6+2288*a^2*c^2*d^4*e^4-3432*a*c^3*d^6*e^2+3003*c^4*d^8)/d^5/c^5
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 392, normalized size of antiderivative = 1.29

$$\int (d+ex)^{5/2}\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}\,dx = \frac{2\left(1155\,c^6d^6e^4x^6+3003\,a^2c^4d^8e^2-3432\,a^3c^3d^6e^4+2288\,a^4c^2d^4e^6-832\,a^5cd^2e^8+128\,a^6\right.}{}$$

input
```
integrate((e*x+d)^(5/2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),x, algorit
hm="fricas")
```

output
```
2/15015*(1155*c^6*d^6*e^4*x^6 + 3003*a^2*c^4*d^8*e^2 - 3432*a^3*c^3*d^6*e^
4 + 2288*a^4*c^2*d^4*e^6 - 832*a^5*c*d^2*e^8 + 128*a^6*e^10 + 210*(26*c^6*
d^7*e^3 + 7*a*c^5*d^5*e^5)*x^5 + 35*(286*c^6*d^8*e^2 + 208*a*c^5*d^6*e^4 +
 a^2*c^4*d^4*e^6)*x^4 + 20*(429*c^6*d^9*e + 715*a*c^5*d^7*e^3 + 13*a^2*c^4
*d^5*e^5 - 2*a^3*c^3*d^3*e^7)*x^3 + 3*(1001*c^6*d^10 + 4576*a*c^5*d^8*e^2
+ 286*a^2*c^4*d^6*e^4 - 104*a^3*c^3*d^4*e^6 + 16*a^4*c^2*d^2*e^8)*x^2 + 2*
(3003*a*c^5*d^9*e + 858*a^2*c^4*d^7*e^3 - 572*a^3*c^3*d^5*e^5 + 208*a^4*c^
2*d^3*e^7 - 32*a^5*c*d*e^9)*x)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)
*sqrt(e*x + d)/(c^5*d^5*e*x + c^5*d^6)
```

Sympy [F(-1)]

Timed out.

$$\int (d+ex)^{5/2} \left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2} dx = \text{Timed out}$$

input
```
integrate((e*x+d)**(5/2)*(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(3/2),x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 373, normalized size of antiderivative = 1.22

$$\int (d+ex)^{5/2} \left(ade+\left(cd^2+ae^2\right)x +cdex^2\right)^{3/2} dx = \frac{2 \left(1155\, c^6 d^6 e^4 x^6 + 3003\, a^2 c^4 d^8 e^2 - 3432\, a^3 c^3 d^6 e^4 + 2288\, a^4 c^2 d^4 e^6 - 832\, a^5 c d^2 e^8 + 128\, a^6 \right.}{}$$

input
```
integrate((e*x+d)^(5/2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),x, algorit
hm="maxima")
```

output
```
2/15015*(1155*c^6*d^6*e^4*x^6 + 3003*a^2*c^4*d^8*e^2 - 3432*a^3*c^3*d^6*e^
4 + 2288*a^4*c^2*d^4*e^6 - 832*a^5*c*d^2*e^8 + 128*a^6*e^10 + 210*(26*c^6*
d^7*e^3 + 7*a*c^5*d^5*e^5)*x^5 + 35*(286*c^6*d^8*e^2 + 208*a*c^5*d^6*e^4 +
 a^2*c^4*d^4*e^6)*x^4 + 20*(429*c^6*d^9*e + 715*a*c^5*d^7*e^3 + 13*a^2*c^4
*d^5*e^5 - 2*a^3*c^3*d^3*e^7)*x^3 + 3*(1001*c^6*d^10 + 4576*a*c^5*d^8*e^2
+ 286*a^2*c^4*d^6*e^4 - 104*a^3*c^3*d^4*e^6 + 16*a^4*c^2*d^2*e^8)*x^2 + 2*
(3003*a*c^5*d^9*e + 858*a^2*c^4*d^7*e^3 - 572*a^3*c^3*d^5*e^5 + 208*a^4*c^
2*d^3*e^7 - 32*a^5*c*d*e^9)*x)*sqrt(c*d*x + a*e)*(e*x + d)/(c^5*d^5*e*x +
c^5*d^6)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1115 vs. $2(275) = 550$.

Time = 0.18 (sec) , antiderivative size = 1115, normalized size of antiderivative = 3.66

$$\int (d+ex)^{5/2} \left(ade + \left(cd^2+ae^2\right)x + cdex^2\right)^{3/2} \, dx = \text{Too large to display}$$

input
```
integrate((e*x+d)^(5/2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),x, algorit
hm="giac")
```

output
```
2/45045*(45045*sqrt(c*d*x + a*e)*a^2*d^4*e^2 - 30030*(3*sqrt(c*d*x + a*e)*
a*e - (c*d*x + a*e)^(3/2))*a*d^4*e - 60060*(3*sqrt(c*d*x + a*e)*a*e - (c*d
*x + a*e)^(3/2))*a^2*d^2*e^3/c + 3003*(15*sqrt(c*d*x + a*e)*a^2*e^2 - 10*(
c*d*x + a*e)^(3/2)*a*e + 3*(c*d*x + a*e)^(5/2))*d^4 + 24024*(15*sqrt(c*d*x
 + a*e)*a^2*e^2 - 10*(c*d*x + a*e)^(3/2)*a*e + 3*(c*d*x + a*e)^(5/2))*a*d^
2*e^2/c + 18018*(15*sqrt(c*d*x + a*e)*a^2*e^2 - 10*(c*d*x + a*e)^(3/2)*a*e
 + 3*(c*d*x + a*e)^(5/2))*a^2*e^4/c^2 - 5148*(35*sqrt(c*d*x + a*e)*a^3*e^3
 - 35*(c*d*x + a*e)^(3/2)*a^2*e^2 + 21*(c*d*x + a*e)^(5/2)*a*e - 5*(c*d*x
+ a*e)^(7/2))*d^2*e/c - 15444*(35*sqrt(c*d*x + a*e)*a^3*e^3 - 35*(c*d*x +
a*e)^(3/2)*a^2*e^2 + 21*(c*d*x + a*e)^(5/2)*a*e - 5*(c*d*x + a*e)^(7/2))*a
*e^3/c^2 - 5148*(35*sqrt(c*d*x + a*e)*a^3*e^3 - 35*(c*d*x + a*e)^(3/2)*a^2
*e^2 + 21*(c*d*x + a*e)^(5/2)*a*e - 5*(c*d*x + a*e)^(7/2))*a^2*e^5/(c^3*d^
2) + 858*(315*sqrt(c*d*x + a*e)*a^4*e^4 - 420*(c*d*x + a*e)^(3/2)*a^3*e^3
+ 378*(c*d*x + a*e)^(5/2)*a^2*e^2 - 180*(c*d*x + a*e)^(7/2)*a*e + 35*(c*d*
x + a*e)^(9/2))*e^2/c^2 + 1144*(315*sqrt(c*d*x + a*e)*a^4*e^4 - 420*(c*d*x
 + a*e)^(3/2)*a^3*e^3 + 378*(c*d*x + a*e)^(5/2)*a^2*e^2 - 180*(c*d*x + a*e
)^(7/2)*a*e + 35*(c*d*x + a*e)^(9/2))*a*e^4/(c^3*d^2) + 143*(315*sqrt(c*d*
x + a*e)*a^4*e^4 - 420*(c*d*x + a*e)^(3/2)*a^3*e^3 + 378*(c*d*x + a*e)^(5/
2)*a^2*e^2 - 180*(c*d*x + a*e)^(7/2)*a*e + 35*(c*d*x + a*e)^(9/2))*a^2*e^6
/(c^4*d^4) - 260*(693*sqrt(c*d*x + a*e)*a^5*e^5 - 1155*(c*d*x + a*e)^(3...
```

Mupad [B] (verification not implemented)

Time = 5.63 (sec) , antiderivative size = 424, normalized size of antiderivative = 1.39

$$\int (d+ex)^{5/2} \left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2} dx = \frac{\sqrt{c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e}\,\left(\frac{4\,e^2\,x^5\,\left(26\,c\,d^2+7\,a\,e^2\right)\,\sqrt{d+e\,x}}{143}+\frac{2\,c\,d\,e^3\,x^6\,\sqrt{d+e\,x}}{13}+\frac{8\,x^3\,\sqrt{d+e\,x}\,(-\cdots}{\cdots}\right.}{\cdots}$$

input
```
int((d + e*x)^(5/2)*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(3/2),x)
```

output
```
((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2)*((4*e^2*x^5*(7*a*e^2 + 26*c
*d^2)*(d + e*x)^(1/2))/143 + (2*c*d*e^3*x^6*(d + e*x)^(1/2))/13 + (8*x^3*(
d + e*x)^(1/2)*(429*c^3*d^6 - 2*a^3*e^6 + 715*a*c^2*d^4*e^2 + 13*a^2*c*d^2
*e^4))/(3003*c^2*d^2) + ((d + e*x)^(1/2)*(256*a^6*e^10 - 1664*a^5*c*d^2*e^
8 + 6006*a^2*c^4*d^8*e^2 - 6864*a^3*c^3*d^6*e^4 + 4576*a^4*c^2*d^4*e^6))/(
15015*c^5*d^5*e) + (2*e*x^4*(d + e*x)^(1/2)*(a^2*e^4 + 286*c^2*d^4 + 208*a
*c*d^2*e^2))/(429*c*d) + (x^2*(d + e*x)^(1/2)*(6006*c^6*d^10 + 27456*a*c^5
*d^8*e^2 + 1716*a^2*c^4*d^6*e^4 - 624*a^3*c^3*d^4*e^6 + 96*a^4*c^2*d^2*e^8
))/(15015*c^5*d^5*e) + (4*a*x*(d + e*x)^(1/2)*(3003*c^4*d^8 - 32*a^4*e^8 +
 858*a*c^3*d^6*e^2 + 208*a^3*c*d^2*e^6 - 572*a^2*c^2*d^4*e^4))/(15015*c^4*
d^4)))/(x + d/e)
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 374, normalized size of antiderivative = 1.23

$$\int (d+ex)^{5/2}\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}\,dx=\frac{2\sqrt{cdx+ae}\,(1155c^6d^6e^4x^6+1470a\,c^5d^5e^5x^5+5460c^6d^7e^3x^5+35a^2c^4d^4e^6x^4+7280a\,c^5d}{}$$

input
```
int((e*x+d)^(5/2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),x)
```

output
```
(2*sqrt(a*e + c*d*x)*(128*a**6*e**10 - 832*a**5*c*d**2*e**8 - 64*a**5*c*d*
e**9*x + 2288*a**4*c**2*d**4*e**6 + 416*a**4*c**2*d**3*e**7*x + 48*a**4*c*
*2*d**2*e**8*x**2 - 3432*a**3*c**3*d**6*e**4 - 1144*a**3*c**3*d**5*e**5*x
- 312*a**3*c**3*d**4*e**6*x**2 - 40*a**3*c**3*d**3*e**7*x**3 + 3003*a**2*c
**4*d**8*e**2 + 1716*a**2*c**4*d**7*e**3*x + 858*a**2*c**4*d**6*e**4*x**2
+ 260*a**2*c**4*d**5*e**5*x**3 + 35*a**2*c**4*d**4*e**6*x**4 + 6006*a*c**5
*d**9*e*x + 13728*a*c**5*d**8*e**2*x**2 + 14300*a*c**5*d**7*e**3*x**3 + 72
80*a*c**5*d**6*e**4*x**4 + 1470*a*c**5*d**5*e**5*x**5 + 3003*c**6*d**10*x*
*2 + 8580*c**6*d**9*e*x**3 + 10010*c**6*d**8*e**2*x**4 + 5460*c**6*d**7*e*
*3*x**5 + 1155*c**6*d**6*e**4*x**6))/(15015*c**5*d**5)
```

3.288 $\int (d+ex)^{3/2} \left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}\,dx$

Optimal result

Integrand size = 39, antiderivative size = 240

$$\int (d+ex)^{3/2} \left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}\,dx = \frac{2(cd^2-ae^2)^3\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{5c^4d^4(d+ex)^{5/2}} + \frac{6e(cd^2-ae^2)^2\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{7/2}}{7c^4d^4(d+ex)^{7/2}} + \frac{2e^2(cd^2-ae^2)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{9/2}}{3c^4d^4(d+ex)^{9/2}} + \frac{2e^3\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{11/2}}{11c^4d^4(d+ex)^{11/2}}$$

output
```
2/5*(-a*e^2+c*d^2)^3*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/c^4/d^4/(e*x+
d)^(5/2)+6/7*e*(-a*e^2+c*d^2)^2*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(7/2)/c^
4/d^4/(e*x+d)^(7/2)+2/3*e^2*(-a*e^2+c*d^2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^
2)^(9/2)/c^4/d^4/(e*x+d)^(9/2)+2/11*e^3*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^
(11/2)/c^4/d^4/(e*x+d)^(11/2)
```

Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 132, normalized size of antiderivative = 0.55

$$\int (d+ex)^{3/2} \left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2} dx = \frac{2((ae+cdx)(d+ex))^{5/2}\left(-16a^3e^6+8a^2cde^4(11d+5ex)-2ac^2d^2e^2(99d^2+110dex+35\right.}{1155c^4d^4(d+ex)^{5/2}}$$

input
```
Integrate[(d + e*x)^(3/2)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2),x]
```

output
```
(2*((a*e + c*d*x)*(d + e*x))^(5/2)*(-16*a^3*e^6 + 8*a^2*c*d*e^4*(11*d + 5*
e*x) - 2*a*c^2*d^2*e^2*(99*d^2 + 110*d*e*x + 35*e^2*x^2) + c^3*d^3*(231*d^
3 + 495*d^2*e*x + 385*d*e^2*x^2 + 105*e^3*x^3)))/(1155*c^4*d^4*(d + e*x)^(
5/2))
```

Rubi [A] (verified)

Time = 0.68 (sec) , antiderivative size = 248, normalized size of antiderivative = 1.03, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.103$, Rules used = {1128, 1128, 1128, 1122}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (d+ex)^{3/2} \left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2} dx$$

$$\Big\downarrow \text{1128}$$

$$\frac{6\left(d^2-\frac{ae^2}{c}\right)\int \sqrt{d+ex}\left(cdex^2+\left(cd^2+ae^2\right)x+ade\right)^{3/2}dx}{11d}+\frac{2\sqrt{d+ex}\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/2}}{11cd}$$

$$\Big\downarrow \text{1128}$$

$$\frac{6\left(d^2-\frac{ae^2}{c}\right)\left(\frac{4\left(d^2-\frac{ae^2}{c}\right)\int \frac{\left(cdex^2+\left(cd^2+ae^2\right)x+ade\right)^{3/2}}{\sqrt{d+ex}}dx}{9d}+\frac{2(x(ae^2+cd^2)+ade+cdex^2)^{5/2}}{9cd\sqrt{d+ex}}\right)}{11d}+\frac{2\sqrt{d+ex}\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/2}}{11cd}$$

$\Big\downarrow$ 1128

$$\frac{6\left(d^2-\frac{ae^2}{c}\right)\left(\frac{4\left(d^2-\frac{ae^2}{c}\right)\left(\frac{2\left(d^2-\frac{ae^2}{c}\right)\int \frac{\left(cdex^2+\left(cd^2+ae^2\right)x+ade\right)^{3/2}}{(d+ex)^{3/2}}dx}{7d}+\frac{2\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/2}}{7cd(d+ex)^{3/2}}\right)}{9d}+\frac{2(x(ae^2+cd^2)+ade+cdex}{9cd\sqrt{d+ex}}\right.}{11d}$$

$$\frac{2\sqrt{d+ex}\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/2}}{11cd}$$

$\Big\downarrow$ 1122

$$\frac{2\sqrt{d+ex}\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/2}}{11cd}+$$

$$\frac{6\left(d^2-\frac{ae^2}{c}\right)\left(\frac{2(x(ae^2+cd^2)+ade+cdex^2)^{5/2}}{9cd\sqrt{d+ex}}+\frac{4\left(d^2-\frac{ae^2}{c}\right)\left(\frac{2\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/2}}{7cd(d+ex)^{3/2}}+\frac{4\left(d^2-\frac{ae^2}{c}\right)\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/2}}{35cd^2(d+ex)^{5/2}}\right)}{9d}\right.}{11d}$$

input
```
Int[(d + e*x)^(3/2)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2),x]
```

output
```
(2*Sqrt[d + e*x]*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2))/(11*c*d) +
 (6*(d^2 - (a*e^2)/c)*((2*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2))/(
9*c*d*Sqrt[d + e*x]) + (4*(d^2 - (a*e^2)/c)*((4*(d^2 - (a*e^2)/c)*(a*d*e +
 (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2))/(35*c*d^2*(d + e*x)^(5/2)) + (2*(a*
d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2))/(7*c*d*(d + e*x)^(3/2))))/(9*d
)))/(11*d)
```

Defintions of rubi rules used

rule 1122
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[e*(d + e*x)^(m - 1)*((a + b*x + c*x^2)^(p + 1)/(c*(p + 1))),
 x] /; FreeQ[{a, b, c, d, e, m, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&
EqQ[m + p, 0]
```

rule 1128
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[e*(d + e*x)^(m - 1)*((a + b*x + c*x^2)^(p + 1)/(c*(m + 2*p +
 1))), x] + Simp[Simplify[m + p]*((2*c*d - b*e)/(c*(m + 2*p + 1)))   Int[(d
 + e*x)^(m - 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p},
 x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && IGtQ[Simplify[m + p], 0]
```

Maple [A] (verified)

Time = 1.24 (sec) , antiderivative size = 160, normalized size of antiderivative = 0.67

method	result
default	$-\frac{2\sqrt{(ex+d)(cdx+ae)}\,(cdx+ae)^2(-105c^3d^3e^3x^3+70x^2a\,c^2d^2e^4-385c^3d^4e^2x^2-40x\,a^2cd\,e^5+220xa\,c^2d^3e^3-495c^3d^5ex+16e^6a^3-}{1155\sqrt{ex+d}\,d^4c^4}$
gosper	$-\frac{2(cdx+ae)(-105c^3d^3e^3x^3+70x^2a\,c^2d^2e^4-385c^3d^4e^2x^2-40x\,a^2cd\,e^5+220xa\,c^2d^3e^3-495c^3d^5ex+16e^6a^3-88d^2e^4a^2c+198d^4e^2}{1155d^4c^4(ex+d)^{\frac{3}{2}}}$
orering	$-\frac{2(-105c^3d^3e^3x^3+70x^2a\,c^2d^2e^4-385c^3d^4e^2x^2-40x\,a^2cd\,e^5+220xa\,c^2d^3e^3-495c^3d^5ex+16e^6a^3-88d^2e^4a^2c+198d^4e^2a\,c^2-231}{1155d^4c^4(ex+d)^{\frac{3}{2}}}$

input
```
int((e*x+d)^(3/2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(3/2),x,method=_RETURN
VERBOSE)
```

output
```
-2/1155/(e*x+d)^(1/2)*((e*x+d)*(c*d*x+a*e))^(1/2)*(c*d*x+a*e)^2*(-105*c^3*
d^3*e^3*x^3+70*a*c^2*d^2*e^4*x^2-385*c^3*d^4*e^2*x^2-40*a^2*c*d*e^5*x+220*
a*c^2*d^3*e^3*x-495*c^3*d^5*e*x+16*a^3*e^6-88*a^2*c*d^2*e^4+198*a*c^2*d^4*
e^2-231*c^3*d^6)/d^4/c^4
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 292, normalized size of antiderivative = 1.22

$$\int (d+ex)^{3/2} \left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}\,dx=\frac{2\left(105\,c^5d^5e^3x^5+231\,a^2c^3d^6e^2-198\,a^3c^2d^4e^4+88\,a^4cd^2e^6-16\,a^5e^8+35\left(11\,c^5d^6e^2+4\,a\right.\right.}{}$$

input
```
integrate((e*x+d)^(3/2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),x, algorit
hm="fricas")
```

output
```
2/1155*(105*c^5*d^5*e^3*x^5 + 231*a^2*c^3*d^6*e^2 - 198*a^3*c^2*d^4*e^4 +
88*a^4*c*d^2*e^6 - 16*a^5*e^8 + 35*(11*c^5*d^6*e^2 + 4*a*c^4*d^4*e^4)*x^4
+ 5*(99*c^5*d^7*e + 110*a*c^4*d^5*e^3 + a^2*c^3*d^3*e^5)*x^3 + 3*(77*c^5*d
^8 + 264*a*c^4*d^6*e^2 + 11*a^2*c^3*d^4*e^4 - 2*a^3*c^2*d^2*e^6)*x^2 + (46
2*a*c^4*d^7*e + 99*a^2*c^3*d^5*e^3 - 44*a^3*c^2*d^3*e^5 + 8*a^4*c*d*e^7)*x
)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(e*x + d)/(c^4*d^4*e*x +
 c^4*d^5)
```

Sympy [F]

$$\int (d+ex)^{3/2} \left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}\,dx=\int \left((d+ex)\,(ae+cdx)\right)^{\frac{3}{2}}(d+ex)^{\frac{3}{2}}\,dx$$

input
```
integrate((e*x+d)**(3/2)*(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(3/2),x)
```

output
```
Integral(((d + e*x)*(a*e + c*d*x))**(3/2)*(d + e*x)**(3/2), x)
```

Maxima [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 273, normalized size of antiderivative = 1.14

$$\int (d+ex)^{3/2} \left(ade+\left(cd^2+ae^2\right)x +cdex^2\right)^{3/2} dx = \frac{2\,(105\,c^5d^5e^3x^5+231\,a^2c^3d^6e^2-198\,a^3c^2d^4e^4+88\,a^4cd^2e^6-16\,a^5e^8+35\,(11\,c^5d^6e^2+4\,a$$

input
```
integrate((e*x+d)^(3/2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),x, algorit
hm="maxima")
```

output
```
2/1155*(105*c^5*d^5*e^3*x^5 + 231*a^2*c^3*d^6*e^2 - 198*a^3*c^2*d^4*e^4 +
88*a^4*c*d^2*e^6 - 16*a^5*e^8 + 35*(11*c^5*d^6*e^2 + 4*a*c^4*d^4*e^4)*x^4
+ 5*(99*c^5*d^7*e + 110*a*c^4*d^5*e^3 + a^2*c^3*d^3*e^5)*x^3 + 3*(77*c^5*d
^8 + 264*a*c^4*d^6*e^2 + 11*a^2*c^3*d^4*e^4 - 2*a^3*c^2*d^2*e^6)*x^2 + (46
2*a*c^4*d^7*e + 99*a^2*c^3*d^5*e^3 - 44*a^3*c^2*d^3*e^5 + 8*a^4*c*d*e^7)*x
)*sqrt(c*d*x + a*e)*(e*x + d)/(c^4*d^4*e*x + c^4*d^5)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 784 vs. 2(216) = 432.

Time = 0.19 (sec) , antiderivative size = 784, normalized size of antiderivative = 3.27

$$\int (d+ex)^{3/2} \left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2} dx = \text{Too large to display}$$

input
```
integrate((e*x+d)^(3/2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),x, algorit
hm="giac")
```

output
```
2/3465*(3465*sqrt(c*d*x + a*e)*a^2*d^3*e^2 - 2310*(3*sqrt(c*d*x + a*e)*a*e
 - (c*d*x + a*e)^(3/2))*a*d^3*e - 3465*(3*sqrt(c*d*x + a*e)*a*e - (c*d*x +
 a*e)^(3/2))*a^2*d*e^3/c + 231*(15*sqrt(c*d*x + a*e)*a^2*e^2 - 10*(c*d*x +
 a*e)^(3/2)*a*e + 3*(c*d*x + a*e)^(5/2))*d^3 + 1386*(15*sqrt(c*d*x + a*e)*
a^2*e^2 - 10*(c*d*x + a*e)^(3/2)*a*e + 3*(c*d*x + a*e)^(5/2))*a*d*e^2/c +
693*(15*sqrt(c*d*x + a*e)*a^2*e^2 - 10*(c*d*x + a*e)^(3/2)*a*e + 3*(c*d*x
+ a*e)^(5/2))*a^2*e^4/(c^2*d) - 297*(35*sqrt(c*d*x + a*e)*a^3*e^3 - 35*(c*
d*x + a*e)^(3/2)*a^2*e^2 + 21*(c*d*x + a*e)^(5/2)*a*e - 5*(c*d*x + a*e)^(7
/2))*d*e/c - 594*(35*sqrt(c*d*x + a*e)*a^3*e^3 - 35*(c*d*x + a*e)^(3/2)*a^
2*e^2 + 21*(c*d*x + a*e)^(5/2)*a*e - 5*(c*d*x + a*e)^(7/2))*a*e^3/(c^2*d)
- 99*(35*sqrt(c*d*x + a*e)*a^3*e^3 - 35*(c*d*x + a*e)^(3/2)*a^2*e^2 + 21*(
c*d*x + a*e)^(5/2)*a*e - 5*(c*d*x + a*e)^(7/2))*a^2*e^5/(c^3*d^3) + 33*(31
5*sqrt(c*d*x + a*e)*a^4*e^4 - 420*(c*d*x + a*e)^(3/2)*a^3*e^3 + 378*(c*d*x
 + a*e)^(5/2)*a^2*e^2 - 180*(c*d*x + a*e)^(7/2)*a*e + 35*(c*d*x + a*e)^(9/
2))*e^2/(c^2*d) + 22*(315*sqrt(c*d*x + a*e)*a^4*e^4 - 420*(c*d*x + a*e)^(3
/2)*a^3*e^3 + 378*(c*d*x + a*e)^(5/2)*a^2*e^2 - 180*(c*d*x + a*e)^(7/2)*a*
e + 35*(c*d*x + a*e)^(9/2))*a*e^4/(c^3*d^3) - 5*(693*sqrt(c*d*x + a*e)*a^5
*e^5 - 1155*(c*d*x + a*e)^(3/2)*a^4*e^4 + 1386*(c*d*x + a*e)^(5/2)*a^3*e^3
 - 990*(c*d*x + a*e)^(7/2)*a^2*e^2 + 385*(c*d*x + a*e)^(9/2)*a*e - 63*(c*d
*x + a*e)^(11/2))*e^3/(c^3*d^3))/(c*d)
```

Mupad [B] (verification not implemented)

Time = 5.56 (sec) , antiderivative size = 320, normalized size of antiderivative = 1.33

$$\int (d+ex)^{3/2} \left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}\,dx=\frac{\sqrt{c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e}\,\left(\frac{2\,e\,x^4\,\left(11\,c\,d^2+4\,a\,e^2\right)\,\sqrt{d+e\,x}}{33}+\frac{2\,c\,d\,e^2\,x^5\,\sqrt{d+e\,x}}{11}-\frac{\sqrt{d+e\,x}\,\left(32\,a^5\,e\right.}{}\right.}{}$$

input
```
int((d + e*x)^(3/2)*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(3/2),x)
```

output
```
((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2)*((2*e*x^4*(4*a*e^2 + 11*c*d
^2)*(d + e*x)^(1/2))/33 + (2*c*d*e^2*x^5*(d + e*x)^(1/2))/11 - ((d + e*x)^
(1/2)*(32*a^5*e^8 - 176*a^4*c*d^2*e^6 - 462*a^2*c^3*d^6*e^2 + 396*a^3*c^2*
d^4*e^4))/(1155*c^4*d^4*e) + (2*x^3*(d + e*x)^(1/2)*(a^2*e^4 + 99*c^2*d^4
+ 110*a*c*d^2*e^2))/(231*c*d) + (x^2*(d + e*x)^(1/2)*(462*c^5*d^8 + 1584*a
*c^4*d^6*e^2 + 66*a^2*c^3*d^4*e^4 - 12*a^3*c^2*d^2*e^6))/(1155*c^4*d^4*e)
+ (2*a*x*(d + e*x)^(1/2)*(8*a^3*e^6 + 462*c^3*d^6 + 99*a*c^2*d^4*e^2 - 44*
a^2*c*d^2*e^4))/(1155*c^3*d^3)))/(x + d/e)
```

Reduce [B] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 265, normalized size of antiderivative = 1.10

$$\int (d+ex)^{3/2} \left(ade+\left(cd^2+ae^2\right)x +cdex^2\right)^{3/2} dx = \frac{2\sqrt{cdx+ae}\,(105c^5d^5e^3x^5+140a\,c^4d^4e^4x^4+385c^5d^6e^2x^4+5a^2c^3d^3e^5x^3+550a\,c^4d^5e^3x^3}{}$$

input
```
int((e*x+d)^(3/2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),x)
```

output
```
(2*sqrt(a*e + c*d*x)*( - 16*a**5*e**8 + 88*a**4*c*d**2*e**6 + 8*a**4*c*d*e
**7*x - 198*a**3*c**2*d**4*e**4 - 44*a**3*c**2*d**3*e**5*x - 6*a**3*c**2*d
**2*e**6*x**2 + 231*a**2*c**3*d**6*e**2 + 99*a**2*c**3*d**5*e**3*x + 33*a*
*2*c**3*d**4*e**4*x**2 + 5*a**2*c**3*d**3*e**5*x**3 + 462*a*c**4*d**7*e*x
+ 792*a*c**4*d**6*e**2*x**2 + 550*a*c**4*d**5*e**3*x**3 + 140*a*c**4*d**4*
e**4*x**4 + 231*c**5*d**8*x**2 + 495*c**5*d**7*e*x**3 + 385*c**5*d**6*e**2
*x**4 + 105*c**5*d**5*e**3*x**5))/(1155*c**4*d**4)
```

3.289 $\int \sqrt{d+ex}\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}\,dx$

Optimal result

Integrand size = 39, antiderivative size = 175

$$\int \sqrt{d+ex}\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}\,dx=\frac{2\left(cd^2-ae^2\right)^2\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{5c^3d^3(d+ex)^{5/2}}+\frac{4e\left(cd^2-ae^2\right)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{7/2}}{7c^3d^3(d+ex)^{7/2}}+\frac{2e^2\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{9/2}}{9c^3d^3(d+ex)^{9/2}}$$

output
```
2/5*(-a*e^2+c*d^2)^2*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/c^3/d^3/(e*x+
d)^(5/2)+4/7*e*(-a*e^2+c*d^2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(7/2)/c^3/
d^3/(e*x+d)^(7/2)+2/9*e^2*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(9/2)/c^3/d^3/
(e*x+d)^(9/2)
```

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.50

$$\int \sqrt{d+ex}\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}\,dx=\frac{2((ae+cdx)(d+ex))^{5/2}\left(8a^2e^4-4acde^2(9d+5ex)+c^2d^2\left(63d^2+90dex+35e^2x^2\right)\right)}{315c^3d^3(d+ex)^{5/2}}$$

input
```
Integrate[Sqrt[d + e*x]*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2),x]
```

output
```
(2*((a*e + c*d*x)*(d + e*x))^(5/2)*(8*a^2*e^4 - 4*a*c*d*e^2*(9*d + 5*e*x)
+ c^2*d^2*(63*d^2 + 90*d*e*x + 35*e^2*x^2)))/(315*c^3*d^3*(d + e*x)^(5/2))
```

Rubi [A] (verified)

Time = 0.51 (sec) , antiderivative size = 179, normalized size of antiderivative = 1.02, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}=0.077$, Rules used = {1128, 1128, 1122}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{d+ex}\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}dx$$

$$\Big\downarrow \text{1128}$$

$$\frac{4\left(d^2-\frac{ae^2}{c}\right)\int \frac{\left(cdex^2+\left(cd^2+ae^2\right)x+ade\right)^{3/2}}{\sqrt{d+ex}}dx}{9d}+\frac{2\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/2}}{9cd\sqrt{d+ex}}$$

$$\Big\downarrow \text{1128}$$

$$\frac{4\left(d^2-\frac{ae^2}{c}\right)\left(\frac{2\left(d^2-\frac{ae^2}{c}\right)\int \frac{\left(cdex^2+\left(cd^2+ae^2\right)x+ade\right)^{3/2}}{(d+ex)^{3/2}}dx}{7d}+\frac{2\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/2}}{7cd(d+ex)^{3/2}}\right)}{9d}+\frac{2\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/2}}{9cd\sqrt{d+ex}}$$

$$\Big\downarrow \text{1122}$$

$$\frac{2\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/2}}{9cd\sqrt{d+ex}}+\frac{4\left(d^2-\frac{ae^2}{c}\right)\left(\frac{2\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/2}}{7cd(d+ex)^{3/2}}+\frac{4\left(d^2-\frac{ae^2}{c}\right)\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/2}}{35cd^2(d+ex)^{5/2}}\right)}{9d}$$

input
```
Int[Sqrt[d + e*x]*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2),x]
```

output
```
(2*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2))/(9*c*d*Sqrt[d + e*x]) +
(4*(d^2 - (a*e^2)/c)*((4*(d^2 - (a*e^2)/c)*(a*d*e + (c*d^2 + a*e^2)*x + c*
d*e*x^2)^(5/2))/(35*c*d^2*(d + e*x)^(5/2)) + (2*(a*d*e + (c*d^2 + a*e^2)*x
 + c*d*e*x^2)^(5/2))/(7*c*d*(d + e*x)^(3/2))))/(9*d)
```

Defintions of rubi rules used

rule 1122
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[e*(d + e*x)^(m - 1)*((a + b*x + c*x^2)^(p + 1)/(c*(p + 1))),
 x] /; FreeQ[{a, b, c, d, e, m, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&
EqQ[m + p, 0]
```

rule 1128
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[e*(d + e*x)^(m - 1)*((a + b*x + c*x^2)^(p + 1)/(c*(m + 2*p +
 1))), x] + Simp[Simplify[m + p]*((2*c*d - b*e)/(c*(m + 2*p + 1)))   Int[(d
 + e*x)^(m - 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p},
 x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && IGtQ[Simplify[m + p], 0]
```

Maple [A] (verified)

Time = 1.24 (sec) , antiderivative size = 102, normalized size of antiderivative = 0.58

method	result	size
default	$\frac{2\sqrt{(ex+d)(cdx+ae)}\,(cdx+ae)^2\left(35x^2c^2d^2e^2-20xacd\,e^3+90x\,c^2d^3e+8a^2e^4-36ac\,d^2e^2+63c^2d^4\right)}{315\sqrt{ex+d}\,d^3c^3}$	102
gosper	$\frac{2(cdx+ae)\left(35x^2c^2d^2e^2-20xacd\,e^3+90x\,c^2d^3e+8a^2e^4-36ac\,d^2e^2+63c^2d^4\right)\left(cd\,x^2e+a\,e^2x+c\,d^2x+ade\right)^{\frac{3}{2}}}{315d^3c^3(ex+d)^{\frac{3}{2}}}$	110
orering	$\frac{2\left(35x^2c^2d^2e^2-20xacd\,e^3+90x\,c^2d^3e+8a^2e^4-36ac\,d^2e^2+63c^2d^4\right)(cdx+ae)\left(ade+\left(a\,e^2+c\,d^2\right)x+cd\,x^2e\right)^{\frac{3}{2}}}{315d^3c^3(ex+d)^{\frac{3}{2}}}$	111

input
```
int((e*x+d)^(1/2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(3/2),x,method=_RETURN
VERBOSE)
```

output
```
2/315/(e*x+d)^(1/2)*((e*x+d)*(c*d*x+a*e))^(1/2)*(c*d*x+a*e)^2*(35*c^2*d^2*
e^2*x^2-20*a*c*d*e^3*x+90*c^2*d^3*e*x+8*a^2*e^4-36*a*c*d^2*e^2+63*c^2*d^4)
/d^3/c^3
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 208, normalized size of antiderivative = 1.19

$$\int \sqrt{d+ex}\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}\,dx = \frac{2\left(35\,c^4d^4e^2x^4+63\,a^2c^2d^4e^2-36\,a^3cd^2e^4+8\,a^4e^6+10\left(9\,c^4d^5e+5\,ac^3d^3e^3\right)x^3+3\left(21\,c^4\right.\right.}{}$$

input
```
integrate((e*x+d)^(1/2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),x, algorit
hm="fricas")
```

output
```
2/315*(35*c^4*d^4*e^2*x^4 + 63*a^2*c^2*d^4*e^2 - 36*a^3*c*d^2*e^4 + 8*a^4*
e^6 + 10*(9*c^4*d^5*e + 5*a*c^3*d^3*e^3)*x^3 + 3*(21*c^4*d^6 + 48*a*c^3*d^
4*e^2 + a^2*c^2*d^2*e^4)*x^2 + 2*(63*a*c^3*d^5*e + 9*a^2*c^2*d^3*e^3 - 2*a
^3*c*d*e^5)*x)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(e*x + d)/(
c^3*d^3*e*x + c^3*d^4)
```

Sympy [F]

$$\int \sqrt{d+ex}\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}\,dx=\int \left((d+ex)\left(ae+cdx\right)\right)^{\frac{3}{2}}\sqrt{d+ex}\,dx$$

input
```
integrate((e*x+d)**(1/2)*(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(3/2),x)
```

output
```
Integral(((d + e*x)*(a*e + c*d*x))**(3/2)*sqrt(d + e*x), x)
```

Maxima [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 189, normalized size of antiderivative = 1.08

$$\int \sqrt{d+ex}\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}\,dx=\frac{2\left(35\,c^4d^4e^2x^4+63\,a^2c^2d^4e^2-36\,a^3cd^2e^4+8\,a^4e^6+10\,(9\,c^4d^5e+5\,ac^3d^3e^3)x^3+3\,(21\,c^4d\right.}{315\,(c^3d^3}$$

input
```
integrate((e*x+d)^(1/2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),x, algorit
hm="maxima")
```

output
```
2/315*(35*c^4*d^4*e^2*x^4 + 63*a^2*c^2*d^4*e^2 - 36*a^3*c*d^2*e^4 + 8*a^4*
e^6 + 10*(9*c^4*d^5*e + 5*a*c^3*d^3*e^3)*x^3 + 3*(21*c^4*d^6 + 48*a*c^3*d^
4*e^2 + a^2*c^2*d^2*e^4)*x^2 + 2*(63*a*c^3*d^5*e + 9*a^2*c^2*d^3*e^3 - 2*a
^3*c*d*e^5)*x)*sqrt(c*d*x + a*e)*(e*x + d)/(c^3*d^3*e*x + c^3*d^4)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 500 vs. 2(157) = 314.

Time = 0.16 (sec) , antiderivative size = 500, normalized size of antiderivative = 2.86

$$\int \sqrt{d+ex}\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}\,dx=\frac{2\left(315\sqrt{cdx+ae}a^2d^2e^2-210\left(3\sqrt{cdx+ae}ae-(cdx+ae)^{\frac{3}{2}}\right)ad^2e-\frac{210\left(3\sqrt{cdx+ae}ae-(cd\right.}{c}\right.}{}$$

input
```
integrate((e*x+d)^(1/2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),x, algorit
hm="giac")
```

output
```
2/315*(315*sqrt(c*d*x + a*e)*a^2*d^2*e^2 - 210*(3*sqrt(c*d*x + a*e)*a*e -
(c*d*x + a*e)^(3/2))*a*d^2*e - 210*(3*sqrt(c*d*x + a*e)*a*e - (c*d*x + a*e
)^(3/2))*a^2*e^3/c + 21*(15*sqrt(c*d*x + a*e)*a^2*e^2 - 10*(c*d*x + a*e)^(
3/2)*a*e + 3*(c*d*x + a*e)^(5/2))*d^2 + 84*(15*sqrt(c*d*x + a*e)*a^2*e^2 -
 10*(c*d*x + a*e)^(3/2)*a*e + 3*(c*d*x + a*e)^(5/2))*a*e^2/c + 21*(15*sqrt
(c*d*x + a*e)*a^2*e^2 - 10*(c*d*x + a*e)^(3/2)*a*e + 3*(c*d*x + a*e)^(5/2)
)*a^2*e^4/(c^2*d^2) - 18*(35*sqrt(c*d*x + a*e)*a^3*e^3 - 35*(c*d*x + a*e)^
(3/2)*a^2*e^2 + 21*(c*d*x + a*e)^(5/2)*a*e - 5*(c*d*x + a*e)^(7/2))*e/c -
18*(35*sqrt(c*d*x + a*e)*a^3*e^3 - 35*(c*d*x + a*e)^(3/2)*a^2*e^2 + 21*(c*
d*x + a*e)^(5/2)*a*e - 5*(c*d*x + a*e)^(7/2))*a*e^3/(c^2*d^2) + (315*sqrt(
c*d*x + a*e)*a^4*e^4 - 420*(c*d*x + a*e)^(3/2)*a^3*e^3 + 378*(c*d*x + a*e)
^(5/2)*a^2*e^2 - 180*(c*d*x + a*e)^(7/2)*a*e + 35*(c*d*x + a*e)^(9/2))*e^2
/(c^2*d^2))/(c*d)
```

Mupad [B] (verification not implemented)

Time = 5.43 (sec) , antiderivative size = 230, normalized size of antiderivative = 1.31

$$\int \sqrt{d+ex}\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}\,dx=\frac{\sqrt{c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e}\,\left(x^3\,\left(\frac{4\,c\,d^2}{7}+\frac{20\,a\,e^2}{63}\right)\,\sqrt{d+e\,x}+\frac{\sqrt{d+e\,x}\,\left(16\,a^4\,e^6-72\,a^3\,c\,d^2\,e^4\right.}{315\,c^3\,d^3\,e}\right.}{}$$

input
```
int((d + e*x)^(1/2)*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(3/2),x)
```

output
```
((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2)*(x^3*((20*a*e^2)/63 + (4*c*
d^2)/7)*(d + e*x)^(1/2) + ((d + e*x)^(1/2)*(16*a^4*e^6 - 72*a^3*c*d^2*e^4
+ 126*a^2*c^2*d^4*e^2))/(315*c^3*d^3*e) + (2*c*d*e*x^4*(d + e*x)^(1/2))/9
+ (x^2*(d + e*x)^(1/2)*(126*c^4*d^6 + 288*a*c^3*d^4*e^2 + 6*a^2*c^2*d^2*e^
4))/(315*c^3*d^3*e) + (4*a*x*(d + e*x)^(1/2)*(63*c^2*d^4 - 2*a^2*e^4 + 9*a
*c*d^2*e^2))/(315*c^2*d^2)))/(x + d/e)
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 173, normalized size of antiderivative = 0.99

$$\int \sqrt{d+ex}\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}\,dx=\frac{2\sqrt{cdx+ae}\,(35c^4d^4e^2x^4+50a\,c^3d^3e^3x^3+90c^4d^5e\,x^3+3a^2c^2d^2e^4x^2+144a\,c^3d^4e^2x^2+63}{315c^3}$$

input
```
int((e*x+d)^(1/2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),x)
```

output
```
(2*sqrt(a*e + c*d*x)*(8*a**4*e**6 - 36*a**3*c*d**2*e**4 - 4*a**3*c*d*e**5*
x + 63*a**2*c**2*d**4*e**2 + 18*a**2*c**2*d**3*e**3*x + 3*a**2*c**2*d**2*e
**4*x**2 + 126*a*c**3*d**5*e*x + 144*a*c**3*d**4*e**2*x**2 + 50*a*c**3*d**
3*e**3*x**3 + 63*c**4*d**6*x**2 + 90*c**4*d**5*e*x**3 + 35*c**4*d**4*e**2*
x**4))/(315*c**3*d**3)
```

3.290 $\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{\sqrt{d+ex}}\,dx$

Optimal result

Integrand size = 39, antiderivative size = 110

$$\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{\sqrt{d+ex}}\,dx=\frac{2(cd^2-ae^2)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{5c^2d^2(d+ex)^{5/2}}+\frac{2e\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{7/2}}{7c^2d^2(d+ex)^{7/2}}$$

output
```
2/5*(-a*e^2+c*d^2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/c^2/d^2/(e*x+d)
^(5/2)+2/7*e*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(7/2)/c^2/d^2/(e*x+d)^(7/2)
```

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.50

$$\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{\sqrt{d+ex}}\,dx=\frac{2((ae+cdx)(d+ex))^{5/2}\left(-2ae^2+cd(7d+5ex)\right)}{35c^2d^2(d+ex)^{5/2}}$$

input
```
Integrate[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2)/Sqrt[d + e*x],x]
```

output
```
(2*((a*e + c*d*x)*(d + e*x))^(5/2)*(-2*a*e^2 + c*d*(7*d + 5*e*x)))/(35*c^2
*d^2*(d + e*x)^(5/2))
```

Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 110, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.051$, Rules used = {1128, 1122}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{\sqrt{d+ex}}dx$$

$$\Big\downarrow \text{ 1128}$$

$$\frac{2\left(d^2-\frac{ae^2}{c}\right)\int \frac{\left(cdex^2+\left(cd^2+ae^2\right)x+ade\right)^{3/2}}{(d+ex)^{3/2}}dx}{7d}+\frac{2\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/2}}{7cd(d+ex)^{3/2}}$$

$$\Big\downarrow \text{ 1122}$$

$$\frac{2\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/2}}{7cd(d+ex)^{3/2}}+\frac{4\left(d^2-\frac{ae^2}{c}\right)\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/2}}{35cd^2(d+ex)^{5/2}}$$

input
```
Int[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2)/Sqrt[d + e*x],x]
```

output
```
(4*(d^2 - (a*e^2)/c)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2))/(35*c*
d^2*(d + e*x)^(5/2)) + (2*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2))/(
7*c*d*(d + e*x)^(3/2))
```

Defintions of rubi rules used

rule 1122
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[e*(d + e*x)^(m - 1)*((a + b*x + c*x^2)^(p + 1)/(c*(p + 1))),
 x] /; FreeQ[{a, b, c, d, e, m, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&
EqQ[m + p, 0]
```

rule 1128
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[e*(d + e*x)^(m - 1)*((a + b*x + c*x^2)^(p + 1)/(c*(m + 2*p +
 1))), x] + Simp[Simplify[m + p]*((2*c*d - b*e)/(c*(m + 2*p + 1)))   Int[(d
 + e*x)^(m - 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p},
 x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && IGtQ[Simplify[m + p], 0]
```

Maple [A] (verified)

Time = 1.22 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.55

method	result	size
default	$-\frac{2\sqrt{(ex+d)(cdx+ae)}\,(cdx+ae)^{2}\left(-5cdxe+2a\,e^{2}-7c\,d^{2}\right)}{35\sqrt{ex+d}\,c^{2}d^{2}}$	61
gosper	$-\frac{2(cdx+ae)\left(-5cdxe+2a\,e^{2}-7c\,d^{2}\right)\left(cd\,x^{2}e+a\,e^{2}x+c\,d^{2}x+ade\right)^{\frac{3}{2}}}{35c^{2}d^{2}(ex+d)^{\frac{3}{2}}}$	69
orering	$-\frac{2\left(-5cdxe+2a\,e^{2}-7c\,d^{2}\right)(cdx+ae)\left(ade+\left(a\,e^{2}+c\,d^{2}\right)x+cd\,x^{2}e\right)^{\frac{3}{2}}}{35c^{2}d^{2}(ex+d)^{\frac{3}{2}}}$	70

input
```
int((a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(3/2)/(e*x+d)^(1/2),x,method=_RETURN
VERBOSE)
```

output
```
-2/35*((e*x+d)*(c*d*x+a*e))^(1/2)/(e*x+d)^(1/2)*(c*d*x+a*e)^2*(-5*c*d*e*x+
2*a*e^2-7*c*d^2)/c^2/d^2
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 135, normalized size of antiderivative = 1.23

$$\int \frac{\left(ade + (cd^2 + ae^2)\,x + cdex^2\right)^{3/2}}{\sqrt{d+ex}}\,dx = \frac{2\,(5\,c^3d^3ex^3 + 7\,a^2cd^2e^2 - 2\,a^3e^4 + (7\,c^3d^4 + 8\,ac^2d^2e^2)x^2 + (14\,a}{35\,(c^2d^2ex +}$$

input
```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/(e*x+d)^(1/2),x, algorit
hm="fricas")
```

output
```
2/35*(5*c^3*d^3*e*x^3 + 7*a^2*c*d^2*e^2 - 2*a^3*e^4 + (7*c^3*d^4 + 8*a*c^2
*d^2*e^2)*x^2 + (14*a*c^2*d^3*e + a^2*c*d*e^3)*x)*sqrt(c*d*e*x^2 + a*d*e +
 (c*d^2 + a*e^2)*x)*sqrt(e*x + d)/(c^2*d^2*e*x + c^2*d^3)
```

Sympy [F]

$$\int \frac{\left(ade + (cd^2 + ae^2)\,x + cdex^2\right)^{3/2}}{\sqrt{d+ex}}\,dx = \int \frac{\left((d+ex)\,(ae+cdx)\right)^{\frac{3}{2}}}{\sqrt{d+ex}}\,dx$$

input
```
integrate((a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(3/2)/(e*x+d)**(1/2),x)
```

output
```
Integral(((d + e*x)*(a*e + c*d*x))**(3/2)/sqrt(d + e*x), x)
```

Maxima [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 98, normalized size of antiderivative = 0.89

$$\int \frac{\left(ade + (cd^2 + ae^2)\,x + cdex^2\right)^{3/2}}{\sqrt{d+ex}}\,dx = \frac{2\,(5\,c^3d^3ex^3 + 7\,a^2cd^2e^2 - 2\,a^3e^4 + (7\,c^3d^4 + 8\,ac^2d^2e^2)x^2 + (14\,a}{35\,c^2d^2}$$

input
```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/(e*x+d)^(1/2),x, algorit
hm="maxima")
```

output
```
2/35*(5*c^3*d^3*e*x^3 + 7*a^2*c*d^2*e^2 - 2*a^3*e^4 + (7*c^3*d^4 + 8*a*c^2
*d^2*e^2)*x^2 + (14*a*c^2*d^3*e + a^2*c*d*e^3)*x)*sqrt(c*d*x + a*e)/(c^2*d
^2)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 278 vs. 2(98) = 196.

Time = 0.16 (sec) , antiderivative size = 278, normalized size of antiderivative = 2.53

$$\int \frac{(ade + (cd^2 + ae^2)x + cdex^2)^{3/2}}{\sqrt{d+ex}}\,dx = \frac{2\left(105\sqrt{cdx+ae}a^2de^2 - 70\left(3\sqrt{cdx+ae}ae - (cdx+ae)^{\frac{3}{2}}\right)ad\right.}{}$$

input
```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/(e*x+d)^(1/2),x, algorit
hm="giac")
```

output
```
2/105*(105*sqrt(c*d*x + a*e)*a^2*d*e^2 - 70*(3*sqrt(c*d*x + a*e)*a*e - (c*
d*x + a*e)^(3/2))*a*d*e - 35*(3*sqrt(c*d*x + a*e)*a*e - (c*d*x + a*e)^(3/2
))*a^2*e^3/(c*d) + 7*(15*sqrt(c*d*x + a*e)*a^2*e^2 - 10*(c*d*x + a*e)^(3/2
)*a*e + 3*(c*d*x + a*e)^(5/2))*d + 14*(15*sqrt(c*d*x + a*e)*a^2*e^2 - 10*(
c*d*x + a*e)^(3/2)*a*e + 3*(c*d*x + a*e)^(5/2))*a*e^2/(c*d) - 3*(35*sqrt(c
*d*x + a*e)*a^3*e^3 - 35*(c*d*x + a*e)^(3/2)*a^2*e^2 + 21*(c*d*x + a*e)^(5
/2)*a*e - 5*(c*d*x + a*e)^(7/2))*e/(c*d))/(c*d)
```

Mupad [B] (verification not implemented)

Time = 5.40 (sec) , antiderivative size = 128, normalized size of antiderivative = 1.16

$$\int \frac{(ade + (cd^2 + ae^2)x + cdex^2)^{3/2}}{\sqrt{d+ex}}\,dx = \frac{\sqrt{c\,d\,e\,x^2 + (c\,d^2 + a\,e^2)\,x + a\,d\,e}\left(\frac{x^2\,(14\,c^3\,d^4 + 16\,a\,c^2\,d^2\,e^2)}{35\,c^2\,d^2} - \frac{4\,a^3\,e^4 - }{35}\right.}{\sqrt{d+e\,x}}$$

input
```
int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(3/2)/(d + e*x)^(1/2),x)
```

output
```
((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2)*((x^2*(14*c^3*d^4 + 16*a*c^
2*d^2*e^2))/(35*c^2*d^2) - (4*a^3*e^4 - 14*a^2*c*d^2*e^2)/(35*c^2*d^2) + (
2*c*d*e*x^3)/7 + (2*a*e*x*(a*e^2 + 14*c*d^2))/(35*c*d)))/(d + e*x)^(1/2)
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 97, normalized size of antiderivative = 0.88

$$\int \frac{\left(ade + (cd^2 + ae^2)\,x + cdex^2\right)^{3/2}}{\sqrt{d+ex}}\,dx = \frac{2\sqrt{cdx+ae}\,(5c^3d^3e\,x^3 + 8a\,c^2d^2e^2x^2 + 7c^3d^4x^2 + a^2cd\,e^3x + 14a}{35c^2d^2}$$

input
```
int((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/(e*x+d)^(1/2),x)
```

output
```
(2*sqrt(a*e + c*d*x)*( - 2*a**3*e**4 + 7*a**2*c*d**2*e**2 + a**2*c*d*e**3*
x + 14*a*c**2*d**3*e*x + 8*a*c**2*d**2*e**2*x**2 + 7*c**3*d**4*x**2 + 5*c*
*3*d**3*e*x**3))/(35*c**2*d**2)
```

3.291 $\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{(d+ex)^{3/2}}\,dx$

Optimal result

Integrand size = 39, antiderivative size = 48

$$\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{(d+ex)^{3/2}}\,dx=\frac{2\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{5cd(d+ex)^{5/2}}$$

output
```
2/5*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/c/d/(e*x+d)^(5/2)
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.77

$$\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{(d+ex)^{3/2}}\,dx=\frac{2((ae+cdx)(d+ex))^{5/2}}{5cd(d+ex)^{5/2}}$$

input
```
Integrate[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2)/(d + e*x)^(3/2),x]
```

output
```
(2*((a*e + c*d*x)*(d + e*x))^(5/2))/(5*c*d*(d + e*x)^(5/2))
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.026, Rules used = {1122}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{(d+ex)^{3/2}}dx$$

$$\Big\downarrow \text{ 1122}$$

$$\frac{2\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/2}}{5cd(d+ex)^{5/2}}$$

input
```
Int[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2)/(d + e*x)^(3/2),x]
```

output
```
(2*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2))/(5*c*d*(d + e*x)^(5/2))
```

Defintions of rubi rules used

rule 1122
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[e*(d + e*x)^(m - 1)*((a + b*x + c*x^2)^(p + 1)/(c*(p + 1))),
 x] /; FreeQ[{a, b, c, d, e, m, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&
EqQ[m + p, 0]
```

Maple [A] (verified)

Time = 1.05 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.88

method	result	size
default	$\frac{2\sqrt{(ex+d)(cdx+ae)}\,(cdx+ae)^2}{5\sqrt{ex+d}\,cd}$	42
gosper	$\frac{2(cdx+ae)\left(cd\,x^2e+a\,e^2x+c\,d^2x+ade\right)^{\frac{3}{2}}}{5cd(ex+d)^{\frac{3}{2}}}$	50
orering	$\frac{2(cdx+ae)\left(ade+\left(a\,e^2+c\,d^2\right)x+cd\,x^2e\right)^{\frac{3}{2}}}{5cd(ex+d)^{\frac{3}{2}}}$	51

input
```
int((a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(3/2)/(e*x+d)^(3/2),x,method=_RETURN
VERBOSE)
```

output
```
2/5*((e*x+d)*(c*d*x+a*e))^(1/2)/(e*x+d)^(1/2)*(c*d*x+a*e)^2/c/d
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.54

$$\int \frac{(ade+(cd^2+ae^2)\,x+cdex^2)^{3/2}}{(d+ex)^{3/2}}\,dx = \frac{2\,(c^2d^2x^2+2\,acdex+a^2e^2)\,\sqrt{cdex^2+ade+(cd^2+ae^2)x}\sqrt{ex+}}{5\,(cdex+cd^2)}$$

input
```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/(e*x+d)^(3/2),x, algorit
hm="fricas")
```

output
```
2/5*(c^2*d^2*x^2 + 2*a*c*d*e*x + a^2*e^2)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2
+ a*e^2)*x)*sqrt(e*x + d)/(c*d*e*x + c*d^2)
```

Sympy [F]

$$\int \frac{(ade+(cd^2+ae^2)\,x+cdex^2)^{3/2}}{(d+ex)^{3/2}}\,dx = \int \frac{((d+ex)\,(ae+cdx))^{\frac{3}{2}}}{(d+ex)^{\frac{3}{2}}}\,dx$$

input
```
integrate((a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(3/2)/(e*x+d)**(3/2),x)
```

output
```
Integral(((d + e*x)*(a*e + c*d*x))**(3/2)/(d + e*x)**(3/2), x)
```

Maxima [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.90

$$\int \frac{(ade + (cd^2 + ae^2)\,x + cdex^2)^{3/2}}{(d + ex)^{3/2}}\,dx = \frac{2\,(c^2d^2x^2 + 2\,acdex + a^2e^2)\sqrt{cdx + ae}}{5\,cd}$$

input
```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/(e*x+d)^(3/2),x, algorit
hm="maxima")
```

output
```
2/5*(c^2*d^2*x^2 + 2*a*c*d*e*x + a^2*e^2)*sqrt(c*d*x + a*e)/(c*d)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 114 vs. 2(42) = 84.

Time = 0.19 (sec) , antiderivative size = 114, normalized size of antiderivative = 2.38

$$\int \frac{(ade + (cd^2 + ae^2)\,x + cdex^2)^{3/2}}{(d + ex)^{3/2}}\,dx = \frac{2\left(\frac{5\,((ex+d)cde-cd^2e+ae^3)^{\frac{3}{2}}\,a|e|}{cde^2} - \frac{\left(5\,((ex+d)cde-cd^2e+ae^3)^{\frac{3}{2}}\,ae^3-3\,((ex+d)cde\right.}{cde^5}\right.}{15\,e}$$

input
```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/(e*x+d)^(3/2),x, algorit
hm="giac")
```

output
```
2/15*(5*((e*x + d)*c*d*e - c*d^2*e + a*e^3)^(3/2)*a*abs(e)/(c*d*e^2) - (5*
((e*x + d)*c*d*e - c*d^2*e + a*e^3)^(3/2)*a*e^3 - 3*((e*x + d)*c*d*e - c*d
^2*e + a*e^3)^(5/2))*abs(e)/(c*d*e^5))/e
```

Mupad [B] (verification not implemented)

Time = 5.22 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.29

$$\int \frac{(ade + (cd^2 + ae^2)x + cdex^2)^{3/2}}{(d + ex)^{3/2}}\,dx = \frac{\left(\frac{4\,a\,e\,x}{5} + \frac{2\,c\,d\,x^2}{5} + \frac{2\,a^2\,e^2}{5\,c\,d}\right)\,\sqrt{c\,d\,e\,x^2 + (c\,d^2 + a\,e^2)\,x + a\,d\,e}}{\sqrt{d + e\,x}}$$

input
```
int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(3/2)/(d + e*x)^(3/2),x)
```

output
```
(((4*a*e*x)/5 + (2*c*d*x^2)/5 + (2*a^2*e^2)/(5*c*d))*(x*(a*e^2 + c*d^2) +
a*d*e + c*d*e*x^2)^(1/2))/(d + e*x)^(1/2)
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.88

$$\int \frac{(ade + (cd^2 + ae^2)x + cdex^2)^{3/2}}{(d + ex)^{3/2}}\,dx = \frac{2\sqrt{cdx + ae}\,(c^2d^2x^2 + 2acdex + a^2e^2)}{5cd}$$

input
```
int((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/(e*x+d)^(3/2),x)
```

output
```
(2*sqrt(a*e + c*d*x)*(a**2*e**2 + 2*a*c*d*e*x + c**2*d**2*x**2))/(5*c*d)
```

3.292 $\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{(d+ex)^{5/2}}\,dx$

Optimal result

Integrand size = 39, antiderivative size = 181

$$\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{(d+ex)^{5/2}}\,dx=\frac{2\left(a-\frac{cd^2}{e^2}\right)\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{\sqrt{d+ex}}+\frac{2\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{3e(d+ex)^{3/2}}+\frac{2\left(cd^2-ae^2\right)^{3/2}\arctan\left(\frac{\sqrt{e}\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{\sqrt{cd^2-ae^2}\sqrt{d+ex}}\right)}{e^{5/2}}$$

output
```
2*(a-c*d^2/e^2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(e*x+d)^(1/2)+2/3*
(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/e/(e*x+d)^(3/2)+2*(-a*e^2+c*d^2)^(
3/2)*arctan(e^(1/2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(-a*e^2+c*d^2)
^(1/2)/(e*x+d)^(1/2))/e^(5/2)
```

Mathematica [A] (verified)

Time = 0.19 (sec) , antiderivative size = 137, normalized size of antiderivative = 0.76

$$\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{(d+ex)^{5/2}}\,dx=\frac{2\sqrt{ae+cdx}\sqrt{d+ex}\left(\sqrt{e}\sqrt{ae+cdx}(4ae^2+cd(-3d+ex))+3(\right.}{3e^{5/2}\sqrt{(ae+cdx)(d+ex)}}$$

input
```
Integrate[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2)/(d + e*x)^(5/2),x]
```

output
```
(2*Sqrt[a*e + c*d*x]*Sqrt[d + e*x]*(Sqrt[e]*Sqrt[a*e + c*d*x]*(4*a*e^2 + c
*d*(-3*d + e*x)) + 3*(c*d^2 - a*e^2)^(3/2)*ArcTan[(Sqrt[e]*Sqrt[a*e + c*d*
x])/Sqrt[c*d^2 - a*e^2]]))/(3*e^(5/2)*Sqrt[(a*e + c*d*x)*(d + e*x)])
```

Rubi [A] (verified)

Time = 0.58 (sec) , antiderivative size = 191, normalized size of antiderivative = 1.06, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.103$, Rules used = {1131, 1131, 1136, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{(d+ex)^{5/2}}dx$$

$$\Big\downarrow \text{1131}$$

$$\frac{2\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{3e(d+ex)^{3/2}}-\frac{\left(cd^2-ae^2\right)\int \frac{\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}{(d+ex)^{3/2}}dx}{e}$$

$$\Big\downarrow \text{1131}$$

$$\frac{2\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{3e(d+ex)^{3/2}}-\frac{\left(cd^2-ae^2\right)\left(\frac{2\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{e\sqrt{d+ex}}-\frac{\left(cd^2-ae^2\right)\int \frac{1}{\sqrt{d+ex}\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}dx}{e}\right)}{e}$$

$$\Big\downarrow \text{1136}$$

$$\frac{2\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{3e(d+ex)^{3/2}}-\frac{\left(cd^2-ae^2\right)\left(\frac{2\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{e\sqrt{d+ex}}-2\left(cd^2-ae^2\right)\int \frac{1}{\frac{\left(cdex^2+\left(cd^2+ae^2\right)x+ade\right)e^2}{d+ex}+\left(cd^2-ae^2\right)e}d\frac{\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}{\sqrt{d+ex}}\right)}{e}$$

$$\Big\downarrow \text{218}$$

$$\frac{2\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{3e(d+ex)^{3/2}}-\frac{\left(cd^2-ae^2\right)\left(\frac{2\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{e\sqrt{d+ex}}-\frac{2\sqrt{cd^2-ae^2}\arctan\left(\frac{\sqrt{e}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{\sqrt{d+ex}\sqrt{cd^2-ae^2}}\right)}{e^{3/2}}\right)}{e}$$

input
```
Int[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2)/(d + e*x)^(5/2),x]
```

output
```
(2*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2))/(3*e*(d + e*x)^(3/2)) -
((c*d^2 - a*e^2)*((2*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(e*Sqrt[
d + e*x]) - (2*Sqrt[c*d^2 - a*e^2]*ArcTan[(Sqrt[e]*Sqrt[a*d*e + (c*d^2 + a
*e^2)*x + c*d*e*x^2])/(Sqrt[c*d^2 - a*e^2]*Sqrt[d + e*x])])/e^(3/2)))/e
```

Defintions of rubi rules used

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 1131
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(d + e*x)^(m + 1)*((a + b*x + c*x^2)^p/(e*(m + 2*p + 1))), x
] - Simp[p*((2*c*d - b*e)/(e^2*(m + 2*p + 1)))   Int[(d + e*x)^(m + 1)*(a +
 b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c*d^2 - b
*d*e + a*e^2, 0] && GtQ[p, 0] && (LeQ[-2, m, 0] || EqQ[m + p + 1, 0]) && Ne
Q[m + 2*p + 1, 0] && IntegerQ[2*p]
```

rule 1136
```
Int[1/(Sqrt[(d_.) + (e_.)*(x_)]*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x
_Symbol] :> Simp[2*e   Subst[Int[1/(2*c*d - b*e + e^2*x^2), x], x, Sqrt[a +
 b*x + c*x^2]/Sqrt[d + e*x]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c*d^2
- b*d*e + a*e^2, 0]
```

Maple [A] (verified)

Time = 1.09 (sec) , antiderivative size = 265, normalized size of antiderivative = 1.46

method	result
default	$-\frac{2\sqrt{(ex+d)(cdx+ae)}\left(3\,\mathrm{arctanh}\left(\frac{e\sqrt{cdx+ae}}{\sqrt{\left(a\,e^{2}-c\,d^{2}\right)e}}\right)a^{2}e^{4}-6\,\mathrm{arctanh}\left(\frac{e\sqrt{cdx+ae}}{\sqrt{\left(a\,e^{2}-c\,d^{2}\right)e}}\right)ac\,d^{2}e^{2}+3\,\mathrm{arctanh}\left(\frac{e\sqrt{cdx+ae}}{\sqrt{\left(a\,e^{2}-c\,d^{2}\right)e}}\right)c^{2}d}{3\sqrt{ex+d}\,\sqrt{cdx+ae}\,e^{2}\sqrt{(a\,e^{2}-c\,d}}$

input
```
int((a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(3/2)/(e*x+d)^(5/2),x,method=_RETURN
VERBOSE)
```

output
```
-2/3*((e*x+d)*(c*d*x+a*e))^(1/2)*(3*arctanh(e*(c*d*x+a*e)^(1/2)/((a*e^2-c*
d^2)*e)^(1/2))*a^2*e^4-6*arctanh(e*(c*d*x+a*e)^(1/2)/((a*e^2-c*d^2)*e)^(1/
2))*a*c*d^2*e^2+3*arctanh(e*(c*d*x+a*e)^(1/2)/((a*e^2-c*d^2)*e)^(1/2))*c^2
*d^4-c*d*e*x*(c*d*x+a*e)^(1/2)*((a*e^2-c*d^2)*e)^(1/2)-4*((a*e^2-c*d^2)*e)
^(1/2)*(c*d*x+a*e)^(1/2)*a*e^2+3*((a*e^2-c*d^2)*e)^(1/2)*(c*d*x+a*e)^(1/2)
*c*d^2)/(e*x+d)^(1/2)/(c*d*x+a*e)^(1/2)/e^2/((a*e^2-c*d^2)*e)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 431, normalized size of antiderivative = 2.38

$$\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{(d+ex)^{5/2}}\,dx = \left[-\frac{3\left(cd^3-ade^2+\left(cd^2e-ae^3\right)x\right)\sqrt{-\frac{cd^2-ae^2}{e}}\log\left(-\frac{cde^2x^2+2\,ae^3x}{}\right.}{}\right.$$

input
```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/(e*x+d)^(5/2),x, algorit
hm="fricas")
```

output
```
[-1/3*(3*(c*d^3 - a*d*e^2 + (c*d^2*e - a*e^3)*x)*sqrt(-(c*d^2 - a*e^2)/e)*
log(-(c*d*e^2*x^2 + 2*a*e^3*x - c*d^3 + 2*a*d*e^2 - 2*sqrt(c*d*e*x^2 + a*d
*e + (c*d^2 + a*e^2)*x)*sqrt(e*x + d)*e*sqrt(-(c*d^2 - a*e^2)/e))/(e^2*x^2
 + 2*d*e*x + d^2)) - 2*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(c*d*e*
x - 3*c*d^2 + 4*a*e^2)*sqrt(e*x + d))/(e^3*x + d*e^2), -2/3*(3*(c*d^3 - a*
d*e^2 + (c*d^2*e - a*e^3)*x)*sqrt((c*d^2 - a*e^2)/e)*arctan(-sqrt(c*d*e*x^
2 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(e*x + d)*e*sqrt((c*d^2 - a*e^2)/e)/(c*
d^3 - a*d*e^2 + (c*d^2*e - a*e^3)*x)) - sqrt(c*d*e*x^2 + a*d*e + (c*d^2 +
a*e^2)*x)*(c*d*e*x - 3*c*d^2 + 4*a*e^2)*sqrt(e*x + d))/(e^3*x + d*e^2)]
```

Sympy [F]

$$\int \frac{(ade + (cd^2 + ae^2)\, x + cdex^2)^{3/2}}{(d + ex)^{5/2}}\, dx = \int \frac{((d + ex)\,(ae + cdx))^{\frac{3}{2}}}{(d + ex)^{\frac{5}{2}}}\, dx$$

input
```
integrate((a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(3/2)/(e*x+d)**(5/2),x)
```

output
```
Integral(((d + e*x)*(a*e + c*d*x))**(3/2)/(d + e*x)**(5/2), x)
```

Maxima [F]

$$\int \frac{(ade + (cd^2 + ae^2)\, x + cdex^2)^{3/2}}{(d + ex)^{5/2}}\, dx = \int \frac{(cdex^2 + ade + (cd^2 + ae^2)x)^{\frac{3}{2}}}{(ex + d)^{\frac{5}{2}}}\, dx$$

input
```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/(e*x+d)^(5/2),x, algorit
hm="maxima")
```

output
```
integrate((c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^(3/2)/(e*x + d)^(5/2), x
)
```

Giac [A] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 199, normalized size of antiderivative = 1.10

$$\int \frac{(ade+(cd^2+ae^2)x+cdex^2)^{3/2}}{(d+ex)^{5/2}}\,dx = \frac{2\left(\frac{3\,(c^2d^4|e|-2\,acd^2e^2|e|+a^2e^4|e|)\arctan\left(\frac{\sqrt{(ex+d)cde-cd^2e+ae^3}}{\sqrt{cd^2e-ae^3}}\right)}{\sqrt{cd^2e-ae^3}} - \frac{3\,\sqrt{(ex+d)cd}}{}\right.}{3}$$

input
```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/(e*x+d)^(5/2),x, algorit
hm="giac")
```

output
```
2/3*(3*(c^2*d^4*abs(e) - 2*a*c*d^2*e^2*abs(e) + a^2*e^4*abs(e))*arctan(sqr
t((e*x + d)*c*d*e - c*d^2*e + a*e^3)/sqrt(c*d^2*e - a*e^3))/sqrt(c*d^2*e -
 a*e^3) - (3*sqrt((e*x + d)*c*d*e - c*d^2*e + a*e^3)*c*d^2*e^5*abs(e) - 3*
sqrt((e*x + d)*c*d*e - c*d^2*e + a*e^3)*a*e^7*abs(e) - ((e*x + d)*c*d*e -
c*d^2*e + a*e^3)^(3/2)*e^4*abs(e))/e^6)/e^3
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(ade+(cd^2+ae^2)x+cdex^2)^{3/2}}{(d+ex)^{5/2}}\,dx = \int \frac{\left(c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e\right)^{3/2}}{{\left(d+e\,x\right)}^{5/2}}\,dx$$

input
```
int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(3/2)/(d + e*x)^(5/2),x)
```

output
```
int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(3/2)/(d + e*x)^(5/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 157, normalized size of antiderivative = 0.87

$$\int \frac{(ade + (cd^2 + ae^2)x + cdex^2)^{3/2}}{(d + ex)^{5/2}}\,dx = \frac{-2\sqrt{e}\,\sqrt{-a\,e^2 + c\,d^2}\,atan\left(\frac{\sqrt{cdx+ae}\,e}{\sqrt{e}\,\sqrt{-a\,e^2+c\,d^2}}\right)a\,e^2 + 2\sqrt{e}\,\sqrt{-a\,e^2 + c}}{}$$

input
```
int((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/(e*x+d)^(5/2),x)
```

output
```
(2*( - 3*sqrt(e)*sqrt( - a*e**2 + c*d**2)*atan((sqrt(a*e + c*d*x)*e)/(sqrt
(e)*sqrt( - a*e**2 + c*d**2)))*a*e**2 + 3*sqrt(e)*sqrt( - a*e**2 + c*d**2)
*atan((sqrt(a*e + c*d*x)*e)/(sqrt(e)*sqrt( - a*e**2 + c*d**2)))*c*d**2 + 4
*sqrt(a*e + c*d*x)*a*e**3 - 3*sqrt(a*e + c*d*x)*c*d**2*e + sqrt(a*e + c*d*
x)*c*d*e**2*x))/(3*e**3)
```

3.293 $\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{(d+ex)^{7/2}}\,dx$

Optimal result

Integrand size = 39, antiderivative size = 175

$$\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{(d+ex)^{7/2}}\,dx=\frac{3cd\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{e^2\sqrt{d+ex}}-\frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{e(d+ex)^{5/2}}-\frac{3cd\sqrt{cd^2-ae^2}\arctan\left(\frac{\sqrt{e}\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{\sqrt{cd^2-ae^2}\sqrt{d+ex}}\right)}{e^{5/2}}$$

output
```
3*c*d*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/e^2/(e*x+d)^(1/2)-(a*d*e+(a*
e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/e/(e*x+d)^(5/2)-3*c*d*(-a*e^2+c*d^2)^(1/2)*a
rctan(e^(1/2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(-a*e^2+c*d^2)^(1/2)
/(e*x+d)^(1/2))/e^(5/2)
```

Mathematica [A] (verified)

Time = 0.30 (sec) , antiderivative size = 142, normalized size of antiderivative = 0.81

$$\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{(d+ex)^{7/2}}\,dx=\frac{\sqrt{(ae+cdx)(d+ex)}\left(\sqrt{e}\sqrt{ae+cdx}\left(-ae^2+cd(3d+2ex)\right)-3c\right.}{e^{5/2}\sqrt{ae+cdx}(d+ex)^3}$$

input
```
Integrate[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2)/(d + e*x)^(7/2),x]
```

output
```
(Sqrt[(a*e + c*d*x)*(d + e*x)]*(Sqrt[e]*Sqrt[a*e + c*d*x]*(-(a*e^2) + c*d*
(3*d + 2*e*x)) - 3*c*d*Sqrt[c*d^2 - a*e^2]*(d + e*x)*ArcTan[(Sqrt[e]*Sqrt[
a*e + c*d*x])/Sqrt[c*d^2 - a*e^2]]))/(e^(5/2)*Sqrt[a*e + c*d*x]*(d + e*x)^
(3/2))
```

Rubi [A] (verified)

Time = 0.58 (sec) , antiderivative size = 181, normalized size of antiderivative = 1.03, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.103, Rules used = {1130, 1131, 1136, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{(d+ex)^{7/2}}dx$$

$$\Big\downarrow \; 1130$$

$$\frac{3cd\int \frac{\sqrt{cdex^2+(cd^2+ae^2)x+ade}}{(d+ex)^{3/2}}dx}{2e}-\frac{\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{e(d+ex)^{5/2}}$$

$$\Big\downarrow \; 1131$$

$$\frac{3cd\left(\frac{2\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{e\sqrt{d+ex}}-\frac{\left(cd^2-ae^2\right)\int \frac{1}{\sqrt{d+ex}\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}dx}{e}\right)}{2e}-\frac{\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{e(d+ex)^{5/2}}$$

$$\Big\downarrow \; 1136$$

$$\frac{3cd\left(\frac{2\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{e\sqrt{d+ex}}-2\left(cd^2-ae^2\right)\int \frac{1}{\frac{\left(cdex^2+\left(cd^2+ae^2\right)x+ade\right)e^2}{d+ex}+(cd^2-ae^2)e}d\frac{\sqrt{cdex^2+(cd^2+ae^2)x+ade}}{\sqrt{d+ex}}\right)}{2e}-\frac{\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{e(d+ex)^{5/2}}$$

$$\Big\downarrow \; 218$$

$$\frac{3cd\left(\frac{2\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{e\sqrt{d+ex}}-\frac{2\sqrt{cd^2-ae^2}\arctan\left(\frac{\sqrt{e}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{\sqrt{d+ex}\sqrt{cd^2-ae^2}}\right)}{e^{3/2}}\right)}{2e}-\frac{\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{e(d+ex)^{5/2}}$$

input
```
Int[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2)/(d + e*x)^(7/2),x]
```

output
```
-((a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2)/(e*(d + e*x)^(5/2))) + (3*
c*d*((2*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(e*Sqrt[d + e*x]) - (
2*Sqrt[c*d^2 - a*e^2]*ArcTan[(Sqrt[e]*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d
*e*x^2])/(Sqrt[c*d^2 - a*e^2]*Sqrt[d + e*x])])/e^(3/2)))/(2*e)
```

Defintions of rubi rules used

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 1130
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(d + e*x)^(m + 1)*((a + b*x + c*x^2)^p/(e*(m + p + 1))), x]
- Simp[c*(p/(e^2*(m + p + 1)))   Int[(d + e*x)^(m + 2)*(a + b*x + c*x^2)^(p
 - 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0]
 && GtQ[p, 0] && (LtQ[m, -2] || EqQ[m + 2*p + 1, 0]) && NeQ[m + p + 1, 0] &
& IntegerQ[2*p]
```

rule 1131
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(d + e*x)^(m + 1)*((a + b*x + c*x^2)^p/(e*(m + 2*p + 1))), x
] - Simp[p*((2*c*d - b*e)/(e^2*(m + 2*p + 1)))   Int[(d + e*x)^(m + 1)*(a +
 b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c*d^2 - b
*d*e + a*e^2, 0] && GtQ[p, 0] && (LeQ[-2, m, 0] || EqQ[m + p + 1, 0]) && Ne
Q[m + 2*p + 1, 0] && IntegerQ[2*p]
```

rule 1136
```
Int[1/(Sqrt[(d_.) + (e_.)*(x_)]*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x
_Symbol] :> Simp[2*e   Subst[Int[1/(2*c*d - b*e + e^2*x^2), x], x, Sqrt[a +
 b*x + c*x^2]/Sqrt[d + e*x]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c*d^2
- b*d*e + a*e^2, 0]
```

Maple [A] (verified)

Time = 1.07 (sec) , antiderivative size = 304, normalized size of antiderivative = 1.74

method	result
default	$\frac{\left(-3\,\operatorname{arctanh}\left(\frac{e\sqrt{cdx+ae}}{\sqrt{\left(a\,e^{2}-c\,d^{2}\right)e}}\right)acd\,e^{3}x+3\,\operatorname{arctanh}\left(\frac{e\sqrt{cdx+ae}}{\sqrt{\left(a\,e^{2}-c\,d^{2}\right)e}}\right)c^{2}d^{3}ex-3\,\operatorname{arctanh}\left(\frac{e\sqrt{cdx+ae}}{\sqrt{\left(a\,e^{2}-c\,d^{2}\right)e}}\right)ac\,d^{2}e^{2}+3\,\operatorname{arctanh}\left(\right.\right.}{(ex+d)^{\frac{3}{2}}\sqrt{cdx}}$

input
```
int((a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(3/2)/(e*x+d)^(7/2),x,method=_RETURN
VERBOSE)
```

output
```
(-3*arctanh(e*(c*d*x+a*e)^(1/2)/((a*e^2-c*d^2)*e)^(1/2))*a*c*d*e^3*x+3*arc
tanh(e*(c*d*x+a*e)^(1/2)/((a*e^2-c*d^2)*e)^(1/2))*c^2*d^3*e*x-3*arctanh(e*
(c*d*x+a*e)^(1/2)/((a*e^2-c*d^2)*e)^(1/2))*a*c*d^2*e^2+3*arctanh(e*(c*d*x+
a*e)^(1/2)/((a*e^2-c*d^2)*e)^(1/2))*c^2*d^4+2*c*d*e*x*(c*d*x+a*e)^(1/2)*((
a*e^2-c*d^2)*e)^(1/2)-((a*e^2-c*d^2)*e)^(1/2)*(c*d*x+a*e)^(1/2)*a*e^2+3*((
a*e^2-c*d^2)*e)^(1/2)*(c*d*x+a*e)^(1/2)*c*d^2)*((e*x+d)*(c*d*x+a*e))^(1/2)
/(e*x+d)^(3/2)/(c*d*x+a*e)^(1/2)/e^2/((a*e^2-c*d^2)*e)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 443, normalized size of antiderivative = 2.53

$$\int \frac{(ade+(cd^2+ae^2)x+cdex^2)^{3/2}}{(d+ex)^{7/2}}\,dx = \left[\frac{3\,(cde^2x^2+2\,cd^2ex+cd^3)\sqrt{-\frac{cd^2-ae^2}{e}}\log\left(-\frac{cde^2x^2+2\,ae^3x-cd^3+2\,c}{}\right.}{}\right.$$

input
```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/(e*x+d)^(7/2),x, algorit
hm="fricas")
```

output
```
[1/2*(3*(c*d*e^2*x^2 + 2*c*d^2*e*x + c*d^3)*sqrt(-(c*d^2 - a*e^2)/e)*log(-
(c*d*e^2*x^2 + 2*a*e^3*x - c*d^3 + 2*a*d*e^2 - 2*sqrt(c*d*e*x^2 + a*d*e +
(c*d^2 + a*e^2)*x)*sqrt(e*x + d)*e*sqrt(-(c*d^2 - a*e^2)/e))/(e^2*x^2 + 2*
d*e*x + d^2)) + 2*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(2*c*d*e*x +
 3*c*d^2 - a*e^2)*sqrt(e*x + d))/(e^4*x^2 + 2*d*e^3*x + d^2*e^2), (3*(c*d*
e^2*x^2 + 2*c*d^2*e*x + c*d^3)*sqrt((c*d^2 - a*e^2)/e)*arctan(-sqrt(c*d*e*
x^2 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(e*x + d)*e*sqrt((c*d^2 - a*e^2)/e)/(
c*d^3 - a*d*e^2 + (c*d^2*e - a*e^3)*x)) + sqrt(c*d*e*x^2 + a*d*e + (c*d^2
+ a*e^2)*x)*(2*c*d*e*x + 3*c*d^2 - a*e^2)*sqrt(e*x + d))/(e^4*x^2 + 2*d*e^
3*x + d^2*e^2)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(ade+(cd^2+ae^2)x+cdex^2)^{3/2}}{(d+ex)^{7/2}}\,dx = \text{Timed out}$$

input
```
integrate((a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(3/2)/(e*x+d)**(7/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{(ade + (cd^2 + ae^2)\,x + cdex^2)^{3/2}}{(d + ex)^{7/2}}\,dx = \int \frac{(cdex^2 + ade + (cd^2 + ae^2)x)^{\frac{3}{2}}}{(ex + d)^{\frac{7}{2}}}\,dx$$

input
```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/(e*x+d)^(7/2),x, algorit
hm="maxima")
```

output
```
integrate((c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^(3/2)/(e*x + d)^(7/2), x
)
```

Giac [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 198, normalized size of antiderivative = 1.13

$$\int \frac{(ade + (cd^2 + ae^2)\,x + cdex^2)^{3/2}}{(d + ex)^{7/2}}\,dx =$$
$$-\frac{cd\left(\frac{3\,(cd^2|e| - ae^2|e|)\arctan\left(\frac{\sqrt{(ex+d)cde - cd^2e + ae^3}}{\sqrt{cd^2e - ae^3}}\right)}{\sqrt{cd^2e - ae^3}\,e} - \frac{2\sqrt{(ex+d)cde - cd^2e + ae^3}\,|e|}{e^2} - \frac{\sqrt{(ex+d)cde - cd^2e + ae^3}\,cd^2|e| - \sqrt{(ex+d)cde - c}}{(ex+d)cde^2}\right.}{e^2}$$

input
```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/(e*x+d)^(7/2),x, algorit
hm="giac")
```

output
```
-c*d*(3*(c*d^2*abs(e) - a*e^2*abs(e))*arctan(sqrt((e*x + d)*c*d*e - c*d^2*
e + a*e^3)/sqrt(c*d^2*e - a*e^3))/(sqrt(c*d^2*e - a*e^3)*e) - 2*sqrt((e*x
+ d)*c*d*e - c*d^2*e + a*e^3)*abs(e)/e^2 - (sqrt((e*x + d)*c*d*e - c*d^2*e
 + a*e^3)*c*d^2*abs(e) - sqrt((e*x + d)*c*d*e - c*d^2*e + a*e^3)*a*e^2*abs
(e))/((e*x + d)*c*d*e^2))/e^2
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(ade+(cd^2+ae^2)x+cdex^2)^{3/2}}{(d+ex)^{7/2}}\,dx=\int \frac{(c\,d\,e\,x^2+(c\,d^2+a\,e^2)\,x+a\,d\,e)^{3/2}}{(d+e\,x)^{7/2}}\,dx$$

input
```
int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(3/2)/(d + e*x)^(7/2),x)
```

output
```
int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(3/2)/(d + e*x)^(7/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 164, normalized size of antiderivative = 0.94

$$\int \frac{(ade+(cd^2+ae^2)x+cdex^2)^{3/2}}{(d+ex)^{7/2}}\,dx=\frac{-3\sqrt{e}\,\sqrt{-a\,e^2+c\,d^2}\,atan\left(\frac{\sqrt{cdx+ae}\,e}{\sqrt{e}\,\sqrt{-a\,e^2+c\,d^2}}\right)c\,d^2-3\sqrt{e}\,\sqrt{-a\,e^2+c \cdots}}{}$$

input
```
int((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/(e*x+d)^(7/2),x)
```

output
```
( - 3*sqrt(e)*sqrt( - a*e**2 + c*d**2)*atan((sqrt(a*e + c*d*x)*e)/(sqrt(e)
*sqrt( - a*e**2 + c*d**2)))*c*d**2 - 3*sqrt(e)*sqrt( - a*e**2 + c*d**2)*at
an((sqrt(a*e + c*d*x)*e)/(sqrt(e)*sqrt( - a*e**2 + c*d**2)))*c*d*e*x - sqr
t(a*e + c*d*x)*a*e**3 + 3*sqrt(a*e + c*d*x)*c*d**2*e + 2*sqrt(a*e + c*d*x)
*c*d*e**2*x)/(e**3*(d + e*x))
```

3.294 $\int \frac{\left(ade+(cd^2+ae^2)x+cdex^2\right)^{3/2}}{(d+ex)^{9/2}}\,dx$

Optimal result

Integrand size = 39, antiderivative size = 185

$$\int \frac{\left(ade+(cd^2+ae^2)\,x+cdex^2\right)^{3/2}}{(d+ex)^{9/2}}\,dx=-\frac{3cd\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{4e^2(d+ex)^{3/2}}-\frac{\left(ade+(cd^2+ae^2)\,x+cdex^2\right)^{3/2}}{2e(d+ex)^{7/2}}+\frac{3c^2d^2\arctan\left(\frac{\sqrt{e}\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{\sqrt{cd^2-ae^2}\sqrt{d+ex}}\right)}{4e^{5/2}\sqrt{cd^2-ae^2}}$$

output
```
-3/4*c*d*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/e^2/(e*x+d)^(3/2)-1/2*(a*
d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/e/(e*x+d)^(7/2)+3/4*c^2*d^2*arctan(e^
(1/2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(-a*e^2+c*d^2)^(1/2)/(e*x+d)
^(1/2))/e^(5/2)/(-a*e^2+c*d^2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.45 (sec) , antiderivative size = 140, normalized size of antiderivative = 0.76

$$\int \frac{\left(ade+(cd^2+ae^2)\,x+cdex^2\right)^{3/2}}{(d+ex)^{9/2}}\,dx=\frac{\sqrt{(ae+cdx)(d+ex)}\left(-\sqrt{e}(2ae^2+cd(3d+5ex))+\frac{3c^2d^2(d+ex)^2\cdots}{\sqrt{cd^2-\cdots}}\right.}{4e^{5/2}(d+ex)^{5/2}}$$

input
```
Integrate[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2)/(d + e*x)^(9/2),x]
```

output
```
(Sqrt[(a*e + c*d*x)*(d + e*x)]*(-(Sqrt[e]*(2*a*e^2 + c*d*(3*d + 5*e*x))) +
 (3*c^2*d^2*(d + e*x)^2*ArcTan[(Sqrt[e]*Sqrt[a*e + c*d*x])/Sqrt[c*d^2 - a*
e^2]])/(Sqrt[c*d^2 - a*e^2]*Sqrt[a*e + c*d*x])))/(4*e^(5/2)*(d + e*x)^(5/2
))
```

Rubi [A] (verified)

Time = 0.52 (sec) , antiderivative size = 184, normalized size of antiderivative = 0.99, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.103$, Rules used = {1130, 1130, 1136, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{(d+ex)^{9/2}}dx$$

$$\Big\downarrow \text{1130}$$

$$\frac{3cd\int \frac{\sqrt{cdex^2+(cd^2+ae^2)x+ade}}{(d+ex)^{5/2}}dx}{4e}-\frac{\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{2e(d+ex)^{7/2}}$$

$$\Big\downarrow \text{1130}$$

$$\frac{3cd\left(\frac{cd\int \frac{1}{\sqrt{d+ex}\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}dx}{2e}-\frac{\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{e(d+ex)^{3/2}}\right)}{4e}-\frac{\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{2e(d+ex)^{7/2}}$$

$$\Big\downarrow \text{1136}$$

$$\frac{3cd\left(cd\int \frac{1}{\frac{\left(cdex^2+\left(cd^2+ae^2\right)x+ade\right)e^2}{d+ex}+(cd^2-ae^2)e}d\frac{\sqrt{cdex^2+(cd^2+ae^2)x+ade}}{\sqrt{d+ex}}-\frac{\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{e(d+ex)^{3/2}}\right)}{4e}-\frac{\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{2e(d+ex)^{7/2}}$$

$$\Big\downarrow \text{218}$$

$$\frac{3cd\left(\frac{cd\arctan\left(\frac{\sqrt{e}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{\sqrt{d+ex}\sqrt{cd^2-ae^2}}\right)}{e^{3/2}\sqrt{cd^2-ae^2}}-\frac{\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{e(d+ex)^{3/2}}\right)}{4e}-\frac{\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{2e(d+ex)^{7/2}}$$

input
```
Int[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2)/(d + e*x)^(9/2),x]
```

output
```
-1/2*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2)/(e*(d + e*x)^(7/2)) + (
3*c*d*(-(Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]/(e*(d + e*x)^(3/2)))
+ (c*d*ArcTan[(Sqrt[e]*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(Sqrt[
c*d^2 - a*e^2]*Sqrt[d + e*x])])/(e^(3/2)*Sqrt[c*d^2 - a*e^2])))/(4*e)
```

Defintions of rubi rules used

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 1130
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(d + e*x)^(m + 1)*((a + b*x + c*x^2)^p/(e*(m + p + 1))), x]
 - Simp[c*(p/(e^2*(m + p + 1)))   Int[(d + e*x)^(m + 2)*(a + b*x + c*x^2)^(p
 - 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0]
 && GtQ[p, 0] && (LtQ[m, -2] || EqQ[m + 2*p + 1, 0]) && NeQ[m + p + 1, 0] &
& IntegerQ[2*p]
```

rule 1136
```
Int[1/(Sqrt[(d_.) + (e_.)*(x_)]*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x
_Symbol] :> Simp[2*e   Subst[Int[1/(2*c*d - b*e + e^2*x^2), x], x, Sqrt[a +
 b*x + c*x^2]/Sqrt[d + e*x]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c*d^2
- b*d*e + a*e^2, 0]
```

Maple [A] (verified)

Time = 1.06 (sec) , antiderivative size = 271, normalized size of antiderivative = 1.46

method	result
default	$-\frac{\sqrt{(ex+d)(cdx+ae)}\left(3\,\operatorname{arctanh}\left(\frac{e\sqrt{cdx+ae}}{\sqrt{\left(a\,e^{2}-c\,d^{2}\right)e}}\right)c^{2}d^{2}e^{2}x^{2}+6\,\operatorname{arctanh}\left(\frac{e\sqrt{cdx+ae}}{\sqrt{\left(a\,e^{2}-c\,d^{2}\right)e}}\right)c^{2}d^{3}ex+3\,\operatorname{arctanh}\left(\frac{e\sqrt{cdx+ae}}{\sqrt{\left(a\,e^{2}-c\,d^{2}\right)e}}\right)\right.}{4(ex+d)^{\frac{5}{2}}\sqrt{cdx+ae}\,e^{2}\sqrt{(a\,e^{2}-}}$

input
```
int((a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(3/2)/(e*x+d)^(9/2),x,method=_RETURN
VERBOSE)
```

output
```
-1/4*((e*x+d)*(c*d*x+a*e))^(1/2)*(3*arctanh(e*(c*d*x+a*e)^(1/2)/((a*e^2-c*
d^2)*e)^(1/2))*c^2*d^2*e^2*x^2+6*arctanh(e*(c*d*x+a*e)^(1/2)/((a*e^2-c*d^2
)*e)^(1/2))*c^2*d^3*e*x+3*arctanh(e*(c*d*x+a*e)^(1/2)/((a*e^2-c*d^2)*e)^(1
/2))*c^2*d^4+5*c*d*e*x*(c*d*x+a*e)^(1/2)*((a*e^2-c*d^2)*e)^(1/2)+2*((a*e^2
-c*d^2)*e)^(1/2)*(c*d*x+a*e)^(1/2)*a*e^2+3*((a*e^2-c*d^2)*e)^(1/2)*(c*d*x+
a*e)^(1/2)*c*d^2)/(e*x+d)^(5/2)/(c*d*x+a*e)^(1/2)/e^2/((a*e^2-c*d^2)*e)^(1
/2)
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 648, normalized size of antiderivative = 3.50

$$\int \frac{\left(ade+\left(cd^{2}+ae^{2}\right)x+cdex^{2}\right)^{3/2}}{(d+ex)^{9/2}}\,dx=\left[-\frac{3\left(c^{2}d^{2}e^{3}x^{3}+3\,c^{2}d^{3}e^{2}x^{2}+3\,c^{2}d^{4}ex+c^{2}d^{5}\right)\sqrt{-cd^{2}e+ae^{3}}\log}{}\right.$$

input
```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/(e*x+d)^(9/2),x, algorit
hm="fricas")
```

output
```
[-1/8*(3*(c^2*d^2*e^3*x^3 + 3*c^2*d^3*e^2*x^2 + 3*c^2*d^4*e*x + c^2*d^5)*s
qrt(-c*d^2*e + a*e^3)*log(-(c*d*e^2*x^2 + 2*a*e^3*x - c*d^3 + 2*a*d*e^2 -
2*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(-c*d^2*e + a*e^3)*sqrt(
e*x + d))/(e^2*x^2 + 2*d*e*x + d^2)) + 2*(3*c^2*d^4*e - a*c*d^2*e^3 - 2*a^
2*e^5 + 5*(c^2*d^3*e^2 - a*c*d*e^4)*x)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a
*e^2)*x)*sqrt(e*x + d))/(c*d^5*e^3 - a*d^3*e^5 + (c*d^2*e^6 - a*e^8)*x^3 +
 3*(c*d^3*e^5 - a*d*e^7)*x^2 + 3*(c*d^4*e^4 - a*d^2*e^6)*x), -1/4*(3*(c^2*
d^2*e^3*x^3 + 3*c^2*d^3*e^2*x^2 + 3*c^2*d^4*e*x + c^2*d^5)*sqrt(c*d^2*e -
a*e^3)*arctan(-sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(c*d^2*e -
a*e^3)*sqrt(e*x + d)/(c*d^3 - a*d*e^2 + (c*d^2*e - a*e^3)*x)) + (3*c^2*d^4
*e - a*c*d^2*e^3 - 2*a^2*e^5 + 5*(c^2*d^3*e^2 - a*c*d*e^4)*x)*sqrt(c*d*e*x
^2 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(e*x + d))/(c*d^5*e^3 - a*d^3*e^5 + (c
*d^2*e^6 - a*e^8)*x^3 + 3*(c*d^3*e^5 - a*d*e^7)*x^2 + 3*(c*d^4*e^4 - a*d^2
*e^6)*x)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\left(ade + \left(cd^2 + ae^2\right)x + cdex^2\right)^{3/2}}{(d + ex)^{9/2}}\, dx = \text{Timed out}$$

input
```
integrate((a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(3/2)/(e*x+d)**(9/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{\left(ade + \left(cd^2 + ae^2\right)x + cdex^2\right)^{3/2}}{(d + ex)^{9/2}}\, dx = \int \frac{\left(cdex^2 + ade + (cd^2 + ae^2)x\right)^{\frac{3}{2}}}{(ex + d)^{\frac{9}{2}}}\, dx$$

input
```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/(e*x+d)^(9/2),x, algorit
hm="maxima")
```

output
```
integrate((c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^(3/2)/(e*x + d)^(9/2), x
)
```

Giac [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 208, normalized size of antiderivative = 1.12

$$\int \frac{(ade + (cd^2 + ae^2)x + cdex^2)^{3/2}}{(d + ex)^{9/2}}\,dx = \frac{\frac{3c^3d^3e|e|\arctan\left(\frac{\sqrt{(ex+d)cde-cd^2e+ae^3}}{\sqrt{cd^2e-ae^3}}\right)}{\sqrt{cd^2e-ae^3}} - \frac{3\sqrt{(ex+d)cde-cd^2e+ae^3}c^4d^5e^2|e|-3\sqrt{}}{}}{4cde^4}$$

input
```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/(e*x+d)^(9/2),x, algorit
hm="giac")
```

output
```
1/4*(3*c^3*d^3*e*abs(e)*arctan(sqrt((e*x + d)*c*d*e - c*d^2*e + a*e^3)/sqr
t(c*d^2*e - a*e^3))/sqrt(c*d^2*e - a*e^3) - (3*sqrt((e*x + d)*c*d*e - c*d^
2*e + a*e^3)*c^4*d^5*e^2*abs(e) - 3*sqrt((e*x + d)*c*d*e - c*d^2*e + a*e^3
)*a*c^3*d^3*e^4*abs(e) + 5*((e*x + d)*c*d*e - c*d^2*e + a*e^3)^(3/2)*c^3*d
^3*e*abs(e))/((e*x + d)^2*c^2*d^2*e^2))/(c*d*e^4)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(ade + (cd^2 + ae^2)x + cdex^2)^{3/2}}{(d + ex)^{9/2}}\,dx = \int \frac{\left(c\,d\,e\,x^2 + \left(c\,d^2 + a\,e^2\right)\,x + a\,d\,e\right)^{3/2}}{\left(d + e\,x\right)^{9/2}}\,dx$$

input
```
int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(3/2)/(d + e*x)^(9/2),x)
```

output
```
int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(3/2)/(d + e*x)^(9/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 322, normalized size of antiderivative = 1.74

$$\int \frac{(ade + (cd^2 + ae^2)x + cdex^2)^{3/2}}{(d + ex)^{9/2}} \, dx = \frac{-3\sqrt{e}\,\sqrt{-a\,e^2 + c\,d^2}\,atan\left(\frac{\sqrt{cdx+ae}\,e}{\sqrt{e}\,\sqrt{-a\,e^2+c\,d^2}}\right)c^2d^4 - 6\sqrt{e}\,\sqrt{-a\,e^2 + c}$$

input
```
int((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/(e*x+d)^(9/2),x)
```

output
```
( - 3*sqrt(e)*sqrt( - a*e**2 + c*d**2)*atan((sqrt(a*e + c*d*x)*e)/(sqrt(e)
*sqrt( - a*e**2 + c*d**2)))*c**2*d**4 - 6*sqrt(e)*sqrt( - a*e**2 + c*d**2)
*atan((sqrt(a*e + c*d*x)*e)/(sqrt(e)*sqrt( - a*e**2 + c*d**2)))*c**2*d**3*
e*x - 3*sqrt(e)*sqrt( - a*e**2 + c*d**2)*atan((sqrt(a*e + c*d*x)*e)/(sqrt(
e)*sqrt( - a*e**2 + c*d**2)))*c**2*d**2*e**2*x**2 - 2*sqrt(a*e + c*d*x)*a*
*2*e**5 - sqrt(a*e + c*d*x)*a*c*d**2*e**3 - 5*sqrt(a*e + c*d*x)*a*c*d*e**4
*x + 3*sqrt(a*e + c*d*x)*c**2*d**4*e + 5*sqrt(a*e + c*d*x)*c**2*d**3*e**2*
x)/(4*e**3*(a*d**2*e**2 + 2*a*d*e**3*x + a*e**4*x**2 - c*d**4 - 2*c*d**3*e
*x - c*d**2*e**2*x**2))
```

3.295 $\int \frac{(ade+(cd^2+ae^2)x+cdex^2)^{3/2}}{(d+ex)^{11/2}}\,dx$

Optimal result

Integrand size = 39, antiderivative size = 250

$$\int \frac{(ade+(cd^2+ae^2)\,x+cdex^2)^{3/2}}{(d+ex)^{11/2}}\,dx = -\frac{cd\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{4e^2(d+ex)^{5/2}}+\frac{c^2d^2\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{8e^2\,(cd^2-ae^2)\,(d+ex)^{3/2}} -\frac{(ade+(cd^2+ae^2)\,x+cdex^2)^{3/2}}{3e(d+ex)^{9/2}}+\frac{c^3d^3\arctan\left(\frac{\sqrt{e}\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{\sqrt{cd^2-ae^2}\sqrt{d+ex}}\right)}{8e^{5/2}\,(cd^2-ae^2)^{3/2}}$$

output
```
-1/4*c*d*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/e^2/(e*x+d)^(5/2)+1/8*c^2
*d^2*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/e^2/(-a*e^2+c*d^2)/(e*x+d)^(3
/2)-1/3*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/e/(e*x+d)^(9/2)+1/8*c^3*d^
3*arctan(e^(1/2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(-a*e^2+c*d^2)^(1
/2)/(e*x+d)^(1/2))/e^(5/2)/(-a*e^2+c*d^2)^(3/2)
```

Mathematica [A] (verified)

Time = 0.71 (sec) , antiderivative size = 198, normalized size of antiderivative = 0.79

$$\int \frac{(ade+(cd^2+ae^2)x+cdex^2)^{3/2}}{(d+ex)^{11/2}}\,dx = \frac{\sqrt{(ae+cdx)(d+ex)}\left(\sqrt{e}\sqrt{cd^2-ae^2}\sqrt{ae+cdx}(8a^2e^4-2acde^2\right.}{24e^{5/2}(cd^2-}$$

input
```
Integrate[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2)/(d + e*x)^(11/2),x
]
```

output
```
(Sqrt[(a*e + c*d*x)*(d + e*x)]*(Sqrt[e]*Sqrt[c*d^2 - a*e^2]*Sqrt[a*e + c*d
*x]*(8*a^2*e^4 - 2*a*c*d*e^2*(d - 7*e*x) + c^2*d^2*(-3*d^2 - 8*d*e*x + 3*e
^2*x^2)) + 3*c^3*d^3*(d + e*x)^3*ArcTan[(Sqrt[e]*Sqrt[a*e + c*d*x])/Sqrt[c
*d^2 - a*e^2]]))/(24*e^(5/2)*(c*d^2 - a*e^2)^(3/2)*Sqrt[a*e + c*d*x]*(d +
e*x)^(7/2))
```

Rubi [A] (verified)

Time = 0.67 (sec) , antiderivative size = 249, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.128$, Rules used = {1130, 1130, 1135, 1136, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{(d+ex)^{11/2}}dx$$

$$\Big\downarrow \; 1130$$

$$\frac{cd\int \frac{\sqrt{cdex^2+(cd^2+ae^2)x+ade}}{(d+ex)^{7/2}}dx}{2e}-\frac{\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{3e(d+ex)^{9/2}}$$

$$\Big\downarrow \; 1130$$

$$\frac{cd\left(\frac{cd\int \frac{1}{(d+ex)^{3/2}\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}dx}{4e}-\frac{\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{2e(d+ex)^{5/2}}\right)}{2e}-\frac{\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{3e(d+ex)^{9/2}}$$

$$\Big\downarrow \text{ 1135}$$

$$\frac{cd\left(\frac{cd\left(\frac{cd\int \frac{1}{\sqrt{d+ex}\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}dx}{2\left(cd^2-ae^2\right)}+\frac{\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{(d+ex)^{3/2}\left(cd^2-ae^2\right)}\right)}{4e}-\frac{\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{2e(d+ex)^{5/2}}\right)}{2e}-\frac{\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{3e(d+ex)^{9/2}}$$

$$\Big\downarrow \text{ 1136}$$

$$\frac{cd\left(\frac{cd\left(\frac{cde\int \frac{1}{\frac{\left(cdex^2+\left(cd^2+ae^2\right)x+ade\right)e^2}{d+ex}+\left(cd^2-ae^2\right)e}d\frac{\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}{\sqrt{d+ex}}}{cd^2-ae^2}+\frac{\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{(d+ex)^{3/2}\left(cd^2-ae^2\right)}\right)}{4e}-\frac{\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{2e(d+ex)^{5/2}}\right)}{2e}-\frac{\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{3e(d+ex)^{9/2}}$$

$$\Big\downarrow \text{ 218}$$

$$\frac{cd\left(\frac{cd\left(\frac{cd\arctan\left(\frac{\sqrt{e}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{\sqrt{d+ex}\sqrt{cd^2-ae^2}}\right)}{\sqrt{e}\left(cd^2-ae^2\right)^{3/2}}+\frac{\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{(d+ex)^{3/2}\left(cd^2-ae^2\right)}\right)}{4e}-\frac{\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{2e(d+ex)^{5/2}}\right)}{2e}-\frac{\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{3e(d+ex)^{9/2}}$$

input
```
Int[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2)/(d + e*x)^(11/2),x]
```

output
```
-1/3*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2)/(e*(d + e*x)^(9/2)) + (
c*d*(-1/2*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]/(e*(d + e*x)^(5/2))
+ (c*d*(Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]/((c*d^2 - a*e^2)*(d +
e*x)^(3/2)) + (c*d*ArcTan[(Sqrt[e]*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*
x^2])/(Sqrt[c*d^2 - a*e^2]*Sqrt[d + e*x])])/(Sqrt[e]*(c*d^2 - a*e^2)^(3/2)
)))/(4*e)))/(2*e)
```

Defintions of rubi rules used

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 1130
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(d + e*x)^(m + 1)*((a + b*x + c*x^2)^p/(e*(m + p + 1))), x]
- Simp[c*(p/(e^2*(m + p + 1)))   Int[(d + e*x)^(m + 2)*(a + b*x + c*x^2)^(p
 - 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0]
 && GtQ[p, 0] && (LtQ[m, -2] || EqQ[m + 2*p + 1, 0]) && NeQ[m + p + 1, 0] &
& IntegerQ[2*p]
```

rule 1135
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(-e)*(d + e*x)^m*((a + b*x + c*x^2)^(p + 1)/((m + p + 1)*(2*
c*d - b*e))), x] + Simp[c*((m + 2*p + 2)/((m + p + 1)*(2*c*d - b*e)))   Int
[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p},
 x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[m, 0] && NeQ[m + p + 1, 0] && I
ntegerQ[2*p]
```

rule 1136
```
Int[1/(Sqrt[(d_.) + (e_.)*(x_)]*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x
_Symbol] :> Simp[2*e   Subst[Int[1/(2*c*d - b*e + e^2*x^2), x], x, Sqrt[a +
 b*x + c*x^2]/Sqrt[d + e*x]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c*d^2
- b*d*e + a*e^2, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 446 vs. $2(218) = 436$.

Time = 1.07 (sec) , antiderivative size = 447, normalized size of antiderivative = 1.79

method	result
default	$\frac{\sqrt{(ex+d)(cdx+ae)}\left(3\operatorname{arctanh}\left(\frac{e\sqrt{cdx+ae}}{\sqrt{\left(a\,e^{2}-c\,d^{2}\right)e}}\right)c^{3}d^{3}e^{3}x^{3}+9\operatorname{arctanh}\left(\frac{e\sqrt{cdx+ae}}{\sqrt{\left(a\,e^{2}-c\,d^{2}\right)e}}\right)c^{3}d^{4}e^{2}x^{2}+9\operatorname{arctanh}\left(\frac{e\sqrt{cdx+ae}}{\sqrt{\left(a\,e^{2}-c\,d^{2}\right)e}}\right)c}{}$

input
```
int((a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(3/2)/(e*x+d)^(11/2),x,method=_RETUR
NVERBOSE)
```

output
```
1/24*((e*x+d)*(c*d*x+a*e))^(1/2)*(3*arctanh(e*(c*d*x+a*e)^(1/2)/((a*e^2-c*
d^2)*e)^(1/2))*c^3*d^3*e^3*x^3+9*arctanh(e*(c*d*x+a*e)^(1/2)/((a*e^2-c*d^2
)*e)^(1/2))*c^3*d^4*e^2*x^2+9*arctanh(e*(c*d*x+a*e)^(1/2)/((a*e^2-c*d^2)*e
)^(1/2))*c^3*d^5*e*x+3*arctanh(e*(c*d*x+a*e)^(1/2)/((a*e^2-c*d^2)*e)^(1/2)
)*c^3*d^6-3*c^2*d^2*e^2*x^2*(c*d*x+a*e)^(1/2)*((a*e^2-c*d^2)*e)^(1/2)-14*a
*c*d*e^3*x*(c*d*x+a*e)^(1/2)*((a*e^2-c*d^2)*e)^(1/2)+8*c^2*d^3*e*x*(c*d*x+
a*e)^(1/2)*((a*e^2-c*d^2)*e)^(1/2)-8*((a*e^2-c*d^2)*e)^(1/2)*(c*d*x+a*e)^(
1/2)*a^2*e^4+2*((a*e^2-c*d^2)*e)^(1/2)*(c*d*x+a*e)^(1/2)*a*c*d^2*e^2+3*((a
*e^2-c*d^2)*e)^(1/2)*(c*d*x+a*e)^(1/2)*c^2*d^4)/(e*x+d)^(7/2)/(c*d*x+a*e)^
(1/2)/(a*e^2-c*d^2)/e^2/((a*e^2-c*d^2)*e)^(1/2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 472 vs. $2(218) = 436$.

Time = 0.12 (sec) , antiderivative size = 966, normalized size of antiderivative = 3.86

$$\int \frac{(ade + (cd^2 + ae^2)x + cdex^2)^{3/2}}{(d + ex)^{11/2}}\,dx = \text{Too large to display}$$

input
```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/(e*x+d)^(11/2),x, algori
thm="fricas")
```

output
```
[1/48*(3*(c^3*d^3*e^4*x^4 + 4*c^3*d^4*e^3*x^3 + 6*c^3*d^5*e^2*x^2 + 4*c^3*
d^6*e*x + c^3*d^7)*sqrt(-c*d^2*e + a*e^3)*log(-(c*d*e^2*x^2 + 2*a*e^3*x -
c*d^3 + 2*a*d*e^2 + 2*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(-c*
d^2*e + a*e^3)*sqrt(e*x + d))/(e^2*x^2 + 2*d*e*x + d^2)) - 2*(3*c^3*d^6*e
- a*c^2*d^4*e^3 - 10*a^2*c*d^2*e^5 + 8*a^3*e^7 - 3*(c^3*d^4*e^3 - a*c^2*d^
2*e^5)*x^2 + 2*(4*c^3*d^5*e^2 - 11*a*c^2*d^3*e^4 + 7*a^2*c*d*e^6)*x)*sqrt(
c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(e*x + d))/(c^2*d^8*e^3 - 2*a*c
*d^6*e^5 + a^2*d^4*e^7 + (c^2*d^4*e^7 - 2*a*c*d^2*e^9 + a^2*e^11)*x^4 + 4*
(c^2*d^5*e^6 - 2*a*c*d^3*e^8 + a^2*d*e^10)*x^3 + 6*(c^2*d^6*e^5 - 2*a*c*d^
4*e^7 + a^2*d^2*e^9)*x^2 + 4*(c^2*d^7*e^4 - 2*a*c*d^5*e^6 + a^2*d^3*e^8)*x
), -1/24*(3*(c^3*d^3*e^4*x^4 + 4*c^3*d^4*e^3*x^3 + 6*c^3*d^5*e^2*x^2 + 4*c
^3*d^6*e*x + c^3*d^7)*sqrt(c*d^2*e - a*e^3)*arctan(-sqrt(c*d*e*x^2 + a*d*e
 + (c*d^2 + a*e^2)*x)*sqrt(c*d^2*e - a*e^3)*sqrt(e*x + d)/(c*d^3 - a*d*e^2
 + (c*d^2*e - a*e^3)*x)) + (3*c^3*d^6*e - a*c^2*d^4*e^3 - 10*a^2*c*d^2*e^5
 + 8*a^3*e^7 - 3*(c^3*d^4*e^3 - a*c^2*d^2*e^5)*x^2 + 2*(4*c^3*d^5*e^2 - 11
*a*c^2*d^3*e^4 + 7*a^2*c*d*e^6)*x)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2
)*x)*sqrt(e*x + d))/(c^2*d^8*e^3 - 2*a*c*d^6*e^5 + a^2*d^4*e^7 + (c^2*d^4*
e^7 - 2*a*c*d^2*e^9 + a^2*e^11)*x^4 + 4*(c^2*d^5*e^6 - 2*a*c*d^3*e^8 + a^2
*d*e^10)*x^3 + 6*(c^2*d^6*e^5 - 2*a*c*d^4*e^7 + a^2*d^2*e^9)*x^2 + 4*(c^2*
d^7*e^4 - 2*a*c*d^5*e^6 + a^2*d^3*e^8)*x)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(ade + (cd^2 + ae^2)\,x + cdex^2)^{3/2}}{(d + ex)^{11/2}}\,dx = \text{Timed out}$$

input
```
integrate((a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(3/2)/(e*x+d)**(11/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{(ade + (cd^2 + ae^2)\,x + cdex^2)^{3/2}}{(d + ex)^{11/2}}\,dx = \int \frac{(cdex^2 + ade + (cd^2 + ae^2)x)^{\frac{3}{2}}}{(ex + d)^{\frac{11}{2}}}\,dx$$

input
```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/(e*x+d)^(11/2),x, algori
thm="maxima")
```

output
```
integrate((c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^(3/2)/(e*x + d)^(11/2),
x)
```

Giac [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 322, normalized size of antiderivative = 1.29

$$\int \frac{(ade + (cd^2 + ae^2)\,x + cdex^2)^{3/2}}{(d + ex)^{11/2}}\,dx = \frac{1}{24}\,c^3d^3\left(\frac{3\,|e|\arctan\left(\frac{\sqrt{(ex+d)cde-cd^2e+ae^3}}{\sqrt{cd^2e-ae^3}}\right)}{(cd^2e^3 - ae^5)\sqrt{cd^2e - ae^3}} - \frac{3\,\sqrt{(ex + d)cde - \cdots}}{\cdots}\right.$$

input
```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/(e*x+d)^(11/2),x, algori
thm="giac")
```

output
```
1/24*c^3*d^3*(3*abs(e)*arctan(sqrt((e*x + d)*c*d*e - c*d^2*e + a*e^3)/sqrt
(c*d^2*e - a*e^3))/((c*d^2*e^3 - a*e^5)*sqrt(c*d^2*e - a*e^3)) - (3*sqrt((
e*x + d)*c*d*e - c*d^2*e + a*e^3)*c^2*d^4*e^2*abs(e) - 6*sqrt((e*x + d)*c*
d*e - c*d^2*e + a*e^3)*a*c*d^2*e^4*abs(e) + 3*sqrt((e*x + d)*c*d*e - c*d^2
*e + a*e^3)*a^2*e^6*abs(e) + 8*((e*x + d)*c*d*e - c*d^2*e + a*e^3)^(3/2)*c
*d^2*e*abs(e) - 8*((e*x + d)*c*d*e - c*d^2*e + a*e^3)^(3/2)*a*e^3*abs(e) -
 3*((e*x + d)*c*d*e - c*d^2*e + a*e^3)^(5/2)*abs(e))/((c*d^2*e^3 - a*e^5)*
(e*x + d)^3*c^3*d^3*e^3))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(ade + (cd^2 + ae^2)x + cdex^2)^{3/2}}{(d + ex)^{11/2}}\,dx = \int \frac{\left(c\,d\,e\,x^2 + \left(c\,d^2 + a\,e^2\right)\,x + a\,d\,e\right)^{3/2}}{\left(d + e\,x\right)^{11/2}}\,dx$$

input
```
int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(3/2)/(d + e*x)^(11/2),x)
```

output
```
int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(3/2)/(d + e*x)^(11/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 560, normalized size of antiderivative = 2.24

$$\int \frac{(ade + (cd^2 + ae^2)x + cdex^2)^{3/2}}{(d + ex)^{11/2}}\,dx = \frac{3\sqrt{e}\,\sqrt{-a\,e^2 + c\,d^2}\,atan\left(\frac{\sqrt{cdx+ae}\,e}{\sqrt{e}\,\sqrt{-a\,e^2+c\,d^2}}\right)c^3 d^6 + 9\sqrt{e}\,\sqrt{-a\,e^2 + c\,d}$$

input
```
int((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/(e*x+d)^(11/2),x)
```

output

```
(3*sqrt(e)*sqrt( - a*e**2 + c*d**2)*atan((sqrt(a*e + c*d*x)*e)/(sqrt(e)*sq
rt( - a*e**2 + c*d**2)))*c**3*d**6 + 9*sqrt(e)*sqrt( - a*e**2 + c*d**2)*at
an((sqrt(a*e + c*d*x)*e)/(sqrt(e)*sqrt( - a*e**2 + c*d**2)))*c**3*d**5*e*x
 + 9*sqrt(e)*sqrt( - a*e**2 + c*d**2)*atan((sqrt(a*e + c*d*x)*e)/(sqrt(e)*
sqrt( - a*e**2 + c*d**2)))*c**3*d**4*e**2*x**2 + 3*sqrt(e)*sqrt( - a*e**2
+ c*d**2)*atan((sqrt(a*e + c*d*x)*e)/(sqrt(e)*sqrt( - a*e**2 + c*d**2)))*c
**3*d**3*e**3*x**3 - 8*sqrt(a*e + c*d*x)*a**3*e**7 + 10*sqrt(a*e + c*d*x)*
a**2*c*d**2*e**5 - 14*sqrt(a*e + c*d*x)*a**2*c*d*e**6*x + sqrt(a*e + c*d*x
)*a*c**2*d**4*e**3 + 22*sqrt(a*e + c*d*x)*a*c**2*d**3*e**4*x - 3*sqrt(a*e
+ c*d*x)*a*c**2*d**2*e**5*x**2 - 3*sqrt(a*e + c*d*x)*c**3*d**6*e - 8*sqrt(
a*e + c*d*x)*c**3*d**5*e**2*x + 3*sqrt(a*e + c*d*x)*c**3*d**4*e**3*x**2)/(
24*e**3*(a**2*d**3*e**4 + 3*a**2*d**2*e**5*x + 3*a**2*d*e**6*x**2 + a**2*e
**7*x**3 - 2*a*c*d**5*e**2 - 6*a*c*d**4*e**3*x - 6*a*c*d**3*e**4*x**2 - 2*
a*c*d**2*e**5*x**3 + c**2*d**7 + 3*c**2*d**6*e*x + 3*c**2*d**5*e**2*x**2 +
 c**2*d**4*e**3*x**3))
```

3.296 $\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{(d+ex)^{13/2}}\,dx$

Optimal result

Integrand size = 39, antiderivative size = 315

$$\begin{aligned}\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{(d+ex)^{13/2}}\,dx &= -\frac{cd\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{8e^2(d+ex)^{7/2}} \\ &+ \frac{c^2d^2\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{32e^2\left(cd^2-ae^2\right)(d+ex)^{5/2}} + \frac{3c^3d^3\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{64e^2\left(cd^2-ae^2\right)^2(d+ex)^{3/2}} \\ &- \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{4e(d+ex)^{11/2}} + \frac{3c^4d^4\arctan\left(\frac{\sqrt{e}\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{\sqrt{cd^2-ae^2}\sqrt{d+ex}}\right)}{64e^{5/2}\left(cd^2-ae^2\right)^{5/2}}\end{aligned}$$

output
```
-1/8*c*d*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/e^2/(e*x+d)^(7/2)+1/32*c^
2*d^2*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/e^2/(-a*e^2+c*d^2)/(e*x+d)^(
5/2)+3/64*c^3*d^3*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/e^2/(-a*e^2+c*d^
2)^2/(e*x+d)^(3/2)-1/4*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/e/(e*x+d)^(
11/2)+3/64*c^4*d^4*arctan(e^(1/2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/
(-a*e^2+c*d^2)^(1/2)/(e*x+d)^(1/2))/e^(5/2)/(-a*e^2+c*d^2)^(5/2)
```

Mathematica [A] (verified)

Time = 1.13 (sec) , antiderivative size = 242, normalized size of antiderivative = 0.77

$$\int \frac{(ade + (cd^2 + ae^2)x + cdex^2)^{3/2}}{(d+ex)^{13/2}}\,dx = \frac{c^4d^4((ae+cdx)(d+ex))^{3/2}\left(-\frac{\sqrt{e}(16a^3e^6-24a^2cde^4(d-ex)+2ac^2d^2e^2(d^2\cdots}{c^4d^4(cd^2-a\cdots}\right.}{64e^{5/2}(d\cdots}$$

input
```
Integrate[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2)/(d + e*x)^(13/2),x
]
```

output
```
(c^4*d^4*((a*e + c*d*x)*(d + e*x))^(3/2)*(-((Sqrt[e]*(16*a^3*e^6 - 24*a^2*
c*d*e^4*(d - e*x) + 2*a*c^2*d^2*e^2*(d^2 - 22*d*e*x + e^2*x^2) + c^3*d^3*(
3*d^3 + 11*d^2*e*x - 11*d*e^2*x^2 - 3*e^3*x^3)))/(c^4*d^4*(c*d^2 - a*e^2)^
2*(a*e + c*d*x)*(d + e*x)^4)) + (3*ArcTan[(Sqrt[e]*Sqrt[a*e + c*d*x])/Sqrt
[c*d^2 - a*e^2]])/((c*d^2 - a*e^2)^(5/2)*(a*e + c*d*x)^(3/2))))/(64*e^(5/2
)*(d + e*x)^(3/2))
```

Rubi [A] (verified)

Time = 0.86 (sec) , antiderivative size = 326, normalized size of antiderivative = 1.03, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {1130, 1130, 1135, 1135, 1136, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(x(ae^2+cd^2)+ade+cdex^2)^{3/2}}{(d+ex)^{13/2}}dx$$

$$\Big\downarrow \text{1130}$$

$$\frac{3cd\int \frac{\sqrt{cdex^2+(cd^2+ae^2)x+ade}}{(d+ex)^{9/2}}dx}{8e} - \frac{(x(ae^2+cd^2)+ade+cdex^2)^{3/2}}{4e(d+ex)^{11/2}}$$

$$\Big\downarrow \text{1130}$$

$$\frac{3cd\left(\frac{cd\int \frac{1}{(d+ex)^{5/2}\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}dx}{6e}-\frac{\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{3e(d+ex)^{7/2}}\right)}{8e}-\frac{\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{4e(d+ex)^{11/2}}$$

$\big\downarrow$ 1135

$$\frac{3cd\left(\frac{cd\left(\frac{3cd\int \frac{1}{(d+ex)^{3/2}\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}dx}{4\left(cd^2-ae^2\right)}+\frac{\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{2(d+ex)^{5/2}\left(cd^2-ae^2\right)}\right)}{6e}-\frac{\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{3e(d+ex)^{7/2}}\right)}{8e}-\frac{\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{4e(d+ex)^{11/2}}$$

$\big\downarrow$ 1135

$$\frac{3cd\left(\frac{cd\left(\frac{3cd\left(\frac{cd\int \frac{1}{\sqrt{d+ex}\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}dx}{2\left(cd^2-ae^2\right)}+\frac{\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{(d+ex)^{3/2}\left(cd^2-ae^2\right)}\right)}{4\left(cd^2-ae^2\right)}+\frac{\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{2(d+ex)^{5/2}\left(cd^2-ae^2\right)}\right)}{6e}-\frac{\sqrt{x(ae^2+cd^2)+ade+cdex}}{3e(d+ex)^{7/2}}\right)}{8e}$$

$$\frac{\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{4e(d+ex)^{11/2}}$$

$\big\downarrow$ 1136

$$3cd\left(\frac{cd\left(\frac{3cd\left(\frac{cde\int \frac{1}{\frac{\left(cdex^2+\left(cd^2+ae^2\right)x+ade\right)e^2}{d+ex}+\left(cd^2-ae^2\right)e}d\frac{\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}{\sqrt{d+ex}}}{cd^2-ae^2}+\frac{\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{(d+ex)^{3/2}\left(cd^2-ae^2\right)}\right)}{4\left(cd^2-ae^2\right)}+\frac{\sqrt{x\left(ae^2+cd^2\right)+ade+cde}}{2(d+ex)^{5/2}\left(cd^2-ae^2\right)}\right.}{6e}\right.}{8e}$$

$$\frac{\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{4e(d+ex)^{11/2}}$$

$\downarrow$ 218

$$\frac{3cd\left(\frac{cd\left(\frac{3cd\left(\frac{cd\arctan\left(\frac{\sqrt{e}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{\sqrt{d+ex}\sqrt{cd^2-ae^2}}\right)}{\sqrt{e}\left(cd^2-ae^2\right)^{3/2}}+\frac{\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{(d+ex)^{3/2}\left(cd^2-ae^2\right)}\right)}{4\left(cd^2-ae^2\right)}+\frac{\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{2(d+ex)^{5/2}\left(cd^2-ae^2\right)}\right)}{6e}-\frac{\sqrt{x(ae^2+cd^2)+ade+cde}}{3e(d+ex)^{7/2}}\right)}{8e}$$

$$\frac{\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{4e(d+ex)^{11/2}}$$

input

```
Int[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2)/(d + e*x)^(13/2),x]
```

output

```
-1/4*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2)/(e*(d + e*x)^(11/2)) +
(3*c*d*(-1/3*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]/(e*(d + e*x)^(7/2
)) + (c*d*(Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]/(2*(c*d^2 - a*e^2)*
(d + e*x)^(5/2)) + (3*c*d*(Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]/((c
*d^2 - a*e^2)*(d + e*x)^(3/2)) + (c*d*ArcTan[(Sqrt[e]*Sqrt[a*d*e + (c*d^2
+ a*e^2)*x + c*d*e*x^2])/(Sqrt[c*d^2 - a*e^2]*Sqrt[d + e*x])])/(Sqrt[e]*(c
*d^2 - a*e^2)^(3/2))))/(4*(c*d^2 - a*e^2))))/(6*e)))/(8*e)
```

Defintions of rubi rules used

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 1130
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(d + e*x)^(m + 1)*((a + b*x + c*x^2)^p/(e*(m + p + 1))), x]
- Simp[c*(p/(e^2*(m + p + 1)))   Int[(d + e*x)^(m + 2)*(a + b*x + c*x^2)^(p
 - 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0]
 && GtQ[p, 0] && (LtQ[m, -2] || EqQ[m + 2*p + 1, 0]) && NeQ[m + p + 1, 0] &
& IntegerQ[2*p]
```

rule 1135
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(-e)*(d + e*x)^m*((a + b*x + c*x^2)^(p + 1)/((m + p + 1)*(2*
c*d - b*e))), x] + Simp[c*((m + 2*p + 2)/((m + p + 1)*(2*c*d - b*e)))   Int
[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p},
 x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[m, 0] && NeQ[m + p + 1, 0] && I
ntegerQ[2*p]
```

rule 1136
```
Int[1/(Sqrt[(d_.) + (e_.)*(x_)]*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x
_Symbol] :> Simp[2*e   Subst[Int[1/(2*c*d - b*e + e^2*x^2), x], x, Sqrt[a +
 b*x + c*x^2]/Sqrt[d + e*x]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c*d^2
- b*d*e + a*e^2, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 651 vs. $2(277) = 554$.

Time = 1.08 (sec) , antiderivative size = 652, normalized size of antiderivative = 2.07

method	result
default	$-\frac{\sqrt{(ex+d)(cdx+ae)}\left(3\,\mathrm{arctanh}\left(\frac{e\sqrt{cdx+ae}}{\sqrt{\left(a\,e^{2}-c\,d^{2}\right)e}}\right)c^{4}d^{4}e^{4}x^{4}+12\,\mathrm{arctanh}\left(\frac{e\sqrt{cdx+ae}}{\sqrt{\left(a\,e^{2}-c\,d^{2}\right)e}}\right)c^{4}d^{5}e^{3}x^{3}+18\,\mathrm{arctanh}\left(\frac{e\sqrt{cdx+ae}}{\sqrt{\left(a\,e^{2}-c\,d^{2}\right.}}\right.\right.}{}$

input
```
int((a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(3/2)/(e*x+d)^(13/2),x,method=_RETUR
NVERBOSE)
```

output
```
-1/64*((e*x+d)*(c*d*x+a*e))^(1/2)*(3*arctanh(e*(c*d*x+a*e)^(1/2)/((a*e^2-c
*d^2)*e)^(1/2))*c^4*d^4*e^4*x^4+12*arctanh(e*(c*d*x+a*e)^(1/2)/((a*e^2-c*d
^2)*e)^(1/2))*c^4*d^5*e^3*x^3+18*arctanh(e*(c*d*x+a*e)^(1/2)/((a*e^2-c*d^2
)*e)^(1/2))*c^4*d^6*e^2*x^2+12*arctanh(e*(c*d*x+a*e)^(1/2)/((a*e^2-c*d^2)*
e)^(1/2))*c^4*d^7*e*x-3*c^3*d^3*e^3*x^3*(c*d*x+a*e)^(1/2)*((a*e^2-c*d^2)*e
)^(1/2)+3*arctanh(e*(c*d*x+a*e)^(1/2)/((a*e^2-c*d^2)*e)^(1/2))*c^4*d^8+2*a
*c^2*d^2*e^4*x^2*(c*d*x+a*e)^(1/2)*((a*e^2-c*d^2)*e)^(1/2)-11*c^3*d^4*e^2*
x^2*(c*d*x+a*e)^(1/2)*((a*e^2-c*d^2)*e)^(1/2)+24*a^2*c*d*e^5*x*(c*d*x+a*e)
^(1/2)*((a*e^2-c*d^2)*e)^(1/2)-44*a*c^2*d^3*e^3*x*(c*d*x+a*e)^(1/2)*((a*e^
2-c*d^2)*e)^(1/2)+11*c^3*d^5*e*x*(c*d*x+a*e)^(1/2)*((a*e^2-c*d^2)*e)^(1/2)
+16*((a*e^2-c*d^2)*e)^(1/2)*(c*d*x+a*e)^(1/2)*a^3*e^6-24*((a*e^2-c*d^2)*e)
^(1/2)*(c*d*x+a*e)^(1/2)*a^2*c*d^2*e^4+2*((a*e^2-c*d^2)*e)^(1/2)*(c*d*x+a*
e)^(1/2)*a*c^2*d^4*e^2+3*((a*e^2-c*d^2)*e)^(1/2)*(c*d*x+a*e)^(1/2)*c^3*d^6
)/(e*x+d)^(9/2)/((a*e^2-c*d^2)*e)^(1/2)/e^2/(a*e^2-c*d^2)^2/(c*d*x+a*e)^(1
/2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 682 vs. $2(277) = 554$.

Time = 0.15 (sec) , antiderivative size = 1386, normalized size of antiderivative = 4.40

$$\int \frac{(ade + (cd^2 + ae^2)x + cdex^2)^{3/2}}{(d + ex)^{13/2}} dx = \text{Too large to display}$$

input
```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/(e*x+d)^(13/2),x, algori
thm="fricas")
```

output
```
[-1/128*(3*(c^4*d^4*e^5*x^5 + 5*c^4*d^5*e^4*x^4 + 10*c^4*d^6*e^3*x^3 + 10*
c^4*d^7*e^2*x^2 + 5*c^4*d^8*e*x + c^4*d^9)*sqrt(-c*d^2*e + a*e^3)*log(-(c*
d*e^2*x^2 + 2*a*e^3*x - c*d^3 + 2*a*d*e^2 - 2*sqrt(c*d*e*x^2 + a*d*e + (c*
d^2 + a*e^2)*x)*sqrt(-c*d^2*e + a*e^3)*sqrt(e*x + d))/(e^2*x^2 + 2*d*e*x +
 d^2)) + 2*(3*c^4*d^8*e - a*c^3*d^6*e^3 - 26*a^2*c^2*d^4*e^5 + 40*a^3*c*d^
2*e^7 - 16*a^4*e^9 - 3*(c^4*d^5*e^4 - a*c^3*d^3*e^6)*x^3 - (11*c^4*d^6*e^3
 - 13*a*c^3*d^4*e^5 + 2*a^2*c^2*d^2*e^7)*x^2 + (11*c^4*d^7*e^2 - 55*a*c^3*
d^5*e^4 + 68*a^2*c^2*d^3*e^6 - 24*a^3*c*d*e^8)*x)*sqrt(c*d*e*x^2 + a*d*e +
 (c*d^2 + a*e^2)*x)*sqrt(e*x + d))/(c^3*d^11*e^3 - 3*a*c^2*d^9*e^5 + 3*a^2
*c*d^7*e^7 - a^3*d^5*e^9 + (c^3*d^6*e^8 - 3*a*c^2*d^4*e^10 + 3*a^2*c*d^2*e
^12 - a^3*e^14)*x^5 + 5*(c^3*d^7*e^7 - 3*a*c^2*d^5*e^9 + 3*a^2*c*d^3*e^11
- a^3*d*e^13)*x^4 + 10*(c^3*d^8*e^6 - 3*a*c^2*d^6*e^8 + 3*a^2*c*d^4*e^10 -
 a^3*d^2*e^12)*x^3 + 10*(c^3*d^9*e^5 - 3*a*c^2*d^7*e^7 + 3*a^2*c*d^5*e^9 -
 a^3*d^3*e^11)*x^2 + 5*(c^3*d^10*e^4 - 3*a*c^2*d^8*e^6 + 3*a^2*c*d^6*e^8 -
 a^3*d^4*e^10)*x), -1/64*(3*(c^4*d^4*e^5*x^5 + 5*c^4*d^5*e^4*x^4 + 10*c^4*
d^6*e^3*x^3 + 10*c^4*d^7*e^2*x^2 + 5*c^4*d^8*e*x + c^4*d^9)*sqrt(c*d^2*e -
 a*e^3)*arctan(-sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(c*d^2*e -
 a*e^3)*sqrt(e*x + d)/(c*d^3 - a*d*e^2 + (c*d^2*e - a*e^3)*x)) + (3*c^4*d^
8*e - a*c^3*d^6*e^3 - 26*a^2*c^2*d^4*e^5 + 40*a^3*c*d^2*e^7 - 16*a^4*e^9 -
 3*(c^4*d^5*e^4 - a*c^3*d^3*e^6)*x^3 - (11*c^4*d^6*e^3 - 13*a*c^3*d^4*e...
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(ade + (cd^2 + ae^2)x + cdex^2)^{3/2}}{(d + ex)^{13/2}}\,dx = \text{Timed out}$$

input
```
integrate((a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(3/2)/(e*x+d)**(13/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{(d+ex)^{13/2}}\,dx=\int \frac{\left(cdex^2+ade+\left(cd^2+ae^2\right)x\right)^{\frac{3}{2}}}{(ex+d)^{\frac{13}{2}}}\,dx$$

input
```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/(e*x+d)^(13/2),x, algori
thm="maxima")
```

output
```
integrate((c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^(3/2)/(e*x + d)^(13/2),
x)
```

Giac [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 532, normalized size of antiderivative = 1.69

$$\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{(d+ex)^{13/2}}\,dx=\frac{\frac{3\,c^5d^5e|e|\arctan\left(\frac{\sqrt{(ex+d)cde-cd^2e+ae^3}}{\sqrt{cd^2e-ae^3}}\right)}{\left(c^2d^4-2\,acd^2e^2+a^2e^4\right)\sqrt{cd^2e-ae^3}}-\frac{3\,\sqrt{(ex+d)cde-cd^2e+ae^3}\,c^8d^{11}e^4|e|-9\,\cdots}{\cdots}}{\cdots}$$

input
```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/(e*x+d)^(13/2),x, algori
thm="giac")
```

output
```
1/64*(3*c^5*d^5*e*abs(e)*arctan(sqrt((e*x + d)*c*d*e - c*d^2*e + a*e^3)/sq
rt(c*d^2*e - a*e^3))/((c^2*d^4 - 2*a*c*d^2*e^2 + a^2*e^4)*sqrt(c*d^2*e - a
*e^3)) - (3*sqrt((e*x + d)*c*d*e - c*d^2*e + a*e^3)*c^8*d^11*e^4*abs(e) -
9*sqrt((e*x + d)*c*d*e - c*d^2*e + a*e^3)*a*c^7*d^9*e^6*abs(e) + 9*sqrt((e
*x + d)*c*d*e - c*d^2*e + a*e^3)*a^2*c^6*d^7*e^8*abs(e) - 3*sqrt((e*x + d)
*c*d*e - c*d^2*e + a*e^3)*a^3*c^5*d^5*e^10*abs(e) + 11*((e*x + d)*c*d*e -
c*d^2*e + a*e^3)^(3/2)*c^7*d^9*e^3*abs(e) - 22*((e*x + d)*c*d*e - c*d^2*e
+ a*e^3)^(3/2)*a*c^6*d^7*e^5*abs(e) + 11*((e*x + d)*c*d*e - c*d^2*e + a*e^
3)^(3/2)*a^2*c^5*d^5*e^7*abs(e) - 11*((e*x + d)*c*d*e - c*d^2*e + a*e^3)^(
5/2)*c^6*d^7*e^2*abs(e) + 11*((e*x + d)*c*d*e - c*d^2*e + a*e^3)^(5/2)*a*c
^5*d^5*e^4*abs(e) - 3*((e*x + d)*c*d*e - c*d^2*e + a*e^3)^(7/2)*c^5*d^5*e*
abs(e))/((c^2*d^4 - 2*a*c*d^2*e^2 + a^2*e^4)*(e*x + d)^4*c^4*d^4*e^4))/(c*
d*e^4)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(ade + (cd^2 + ae^2)x + cdex^2)^{3/2}}{(d + ex)^{13/2}} dx = \int \frac{(c\,d\,e\,x^2 + (c\,d^2 + a\,e^2)\,x + a\,d\,e)^{3/2}}{(d + e\,x)^{13/2}} dx$$

input
```
int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(3/2)/(d + e*x)^(13/2),x)
```

output
```
int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(3/2)/(d + e*x)^(13/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 864, normalized size of antiderivative = 2.74

$$\int \frac{(ade + (cd^2 + ae^2)x + cdex^2)^{3/2}}{(d + ex)^{13/2}} dx = \frac{-3\sqrt{e}\,\sqrt{-a\,e^2 + c\,d^2}\,atan\left(\frac{\sqrt{cdx+ae}\,e}{\sqrt{e}\,\sqrt{-a\,e^2+c\,d^2}}\right) c^4 d^8 - 12\sqrt{e}\,\sqrt{-a\,e^2 +}}{}$$

input
```
int((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/(e*x+d)^(13/2),x)
```

output

```
( - 3*sqrt(e)*sqrt( - a*e**2 + c*d**2)*atan((sqrt(a*e + c*d*x)*e)/(sqrt(e)
*sqrt( - a*e**2 + c*d**2)))*c**4*d**8 - 12*sqrt(e)*sqrt( - a*e**2 + c*d**2
)*atan((sqrt(a*e + c*d*x)*e)/(sqrt(e)*sqrt( - a*e**2 + c*d**2)))*c**4*d**7
*e*x - 18*sqrt(e)*sqrt( - a*e**2 + c*d**2)*atan((sqrt(a*e + c*d*x)*e)/(sqr
t(e)*sqrt( - a*e**2 + c*d**2)))*c**4*d**6*e**2*x**2 - 12*sqrt(e)*sqrt( - a
*e**2 + c*d**2)*atan((sqrt(a*e + c*d*x)*e)/(sqrt(e)*sqrt( - a*e**2 + c*d**
2)))*c**4*d**5*e**3*x**3 - 3*sqrt(e)*sqrt( - a*e**2 + c*d**2)*atan((sqrt(a
*e + c*d*x)*e)/(sqrt(e)*sqrt( - a*e**2 + c*d**2)))*c**4*d**4*e**4*x**4 - 1
6*sqrt(a*e + c*d*x)*a**4*e**9 + 40*sqrt(a*e + c*d*x)*a**3*c*d**2*e**7 - 24
*sqrt(a*e + c*d*x)*a**3*c*d*e**8*x - 26*sqrt(a*e + c*d*x)*a**2*c**2*d**4*e
**5 + 68*sqrt(a*e + c*d*x)*a**2*c**2*d**3*e**6*x - 2*sqrt(a*e + c*d*x)*a**
2*c**2*d**2*e**7*x**2 - sqrt(a*e + c*d*x)*a*c**3*d**6*e**3 - 55*sqrt(a*e +
 c*d*x)*a*c**3*d**5*e**4*x + 13*sqrt(a*e + c*d*x)*a*c**3*d**4*e**5*x**2 +
3*sqrt(a*e + c*d*x)*a*c**3*d**3*e**6*x**3 + 3*sqrt(a*e + c*d*x)*c**4*d**8*
e + 11*sqrt(a*e + c*d*x)*c**4*d**7*e**2*x - 11*sqrt(a*e + c*d*x)*c**4*d**6
*e**3*x**2 - 3*sqrt(a*e + c*d*x)*c**4*d**5*e**4*x**3)/(64*e**3*(a**3*d**4*
e**6 + 4*a**3*d**3*e**7*x + 6*a**3*d**2*e**8*x**2 + 4*a**3*d*e**9*x**3 + a
**3*e**10*x**4 - 3*a**2*c*d**6*e**4 - 12*a**2*c*d**5*e**5*x - 18*a**2*c*d*
*4*e**6*x**2 - 12*a**2*c*d**3*e**7*x**3 - 3*a**2*c*d**2*e**8*x**4 + 3*a*c*
*2*d**8*e**2 + 12*a*c**2*d**7*e**3*x + 18*a*c**2*d**6*e**4*x**2 + 12*a*...
```

3.297 $\int (d+ex)^{3/2} \left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2} dx$

Optimal result

Integrand size = 39, antiderivative size = 305

$$\int (d+ex)^{3/2} \left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2} dx = \frac{2(cd^2-ae^2)^4 \left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{7/2}}{7c^5d^5(d+ex)^{7/2}} + \frac{8e(cd^2-ae^2)^3 \left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{9/2}}{9c^5d^5(d+ex)^{9/2}} + \frac{12e^2(cd^2-ae^2)^2 \left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{11/2}}{11c^5d^5(d+ex)^{11/2}} + \frac{8e^3(cd^2-ae^2) \left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{13/2}}{13c^5d^5(d+ex)^{13/2}} + \frac{2e^4 \left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{15/2}}{15c^5d^5(d+ex)^{15/2}}$$

output
```
2/7*(-a*e^2+c*d^2)^4*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(7/2)/c^5/d^5/(e*x+
d)^(7/2)+8/9*e*(-a*e^2+c*d^2)^3*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(9/2)/c^
5/d^5/(e*x+d)^(9/2)+12/11*e^2*(-a*e^2+c*d^2)^2*(a*d*e+(a*e^2+c*d^2)*x+c*d*
e*x^2)^(11/2)/c^5/d^5/(e*x+d)^(11/2)+8/13*e^3*(-a*e^2+c*d^2)*(a*d*e+(a*e^2
+c*d^2)*x+c*d*e*x^2)^(13/2)/c^5/d^5/(e*x+d)^(13/2)+2/15*e^4*(a*d*e+(a*e^2+
c*d^2)*x+c*d*e*x^2)^(15/2)/c^5/d^5/(e*x+d)^(15/2)
```

Mathematica [A] (verified)

Time = 0.16 (sec) , antiderivative size = 197, normalized size of antiderivative = 0.65

$$\int (d+ex)^{3/2} \left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2} dx = \frac{2(ae+cdx)^3\sqrt{(ae+cdx)(d+ex)}(128a^4e^8-64a^3cde^6(15d+7ex)+48a^2c^2d^2e^4(65d^2+\ldots}{\ldots}$$

input
```
Integrate[(d + e*x)^(3/2)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2),x]
```

output
```
(2*(a*e + c*d*x)^3*Sqrt[(a*e + c*d*x)*(d + e*x)]*(128*a^4*e^8 - 64*a^3*c*d
*e^6*(15*d + 7*e*x) + 48*a^2*c^2*d^2*e^4*(65*d^2 + 70*d*e*x + 21*e^2*x^2)
- 8*a*c^3*d^3*e^2*(715*d^3 + 1365*d^2*e*x + 945*d*e^2*x^2 + 231*e^3*x^3) +
 c^4*d^4*(6435*d^4 + 20020*d^3*e*x + 24570*d^2*e^2*x^2 + 13860*d*e^3*x^3 +
 3003*e^4*x^4)))/(45045*c^5*d^5*Sqrt[d + e*x])
```

Rubi [A] (verified)

Time = 0.77 (sec) , antiderivative size = 317, normalized size of antiderivative = 1.04, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.128$, Rules used = {1128, 1128, 1128, 1128, 1122}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (d+ex)^{3/2} \left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/2} dx$$

$$\Big\downarrow \text{1128}$$

$$\frac{8\left(d^2-\frac{ae^2}{c}\right)\int \sqrt{d+ex}\left(cdex^2+\left(cd^2+ae^2\right)x+ade\right)^{5/2}dx}{15d}+\frac{2\sqrt{d+ex}\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{7/2}}{15cd}$$

$$\Big\downarrow \text{1128}$$

$$\frac{8\left(d^2-\frac{ae^2}{c}\right)\left(\frac{6\left(d^2-\frac{ae^2}{c}\right)\int\frac{\left(cdex^2+\left(cd^2+ae^2\right)x+ade\right)^{5/2}}{\sqrt{d+ex}}dx}{13d}+\frac{2\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{7/2}}{13cd\sqrt{d+ex}}\right)}{15d}+\frac{2\sqrt{d+ex}\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{7/2}}{15cd}$$

$\downarrow$ 1128

$$\frac{8\left(d^2-\frac{ae^2}{c}\right)\left(\frac{6\left(d^2-\frac{ae^2}{c}\right)\left(\frac{4\left(d^2-\frac{ae^2}{c}\right)\int\frac{\left(cdex^2+\left(cd^2+ae^2\right)x+ade\right)^{5/2}}{(d+ex)^{3/2}}dx}{11d}+\frac{2\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{7/2}}{11cd(d+ex)^{3/2}}\right)}{13d}+\frac{2\left(x\left(ae^2+cd^2\right)+ade+cdex\right.}{13cd\sqrt{d+ex}}\right.}{15d}+\frac{2\sqrt{d+ex}\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{7/2}}{15cd}$$

$\downarrow$ 1128

$$\frac{8\left(d^2-\frac{ae^2}{c}\right)\left(\frac{6\left(d^2-\frac{ae^2}{c}\right)\left(\frac{4\left(d^2-\frac{ae^2}{c}\right)\left(\frac{2\left(d^2-\frac{ae^2}{c}\right)\int\frac{\left(cdex^2+\left(cd^2+ae^2\right)x+ade\right)^{5/2}}{(d+ex)^{5/2}}dx}{9d}+\frac{2\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{7/2}}{9cd(d+ex)^{5/2}}\right)}{11d}+\frac{2\left(x\left(ae^2+cd^2\right)+\right.}{11cd(d+}\right.}{13d}\right.}{15d}+\frac{2\sqrt{d+ex}\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{7/2}}{15cd}$$

$\downarrow$ 1122

$$\frac{2\sqrt{d+ex}\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{7/2}}{15cd}+$$

$$\frac{8\left(d^2-\frac{ae^2}{c}\right)\left(\frac{2\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{7/2}}{13cd\sqrt{d+ex}}+\frac{6\left(d^2-\frac{ae^2}{c}\right)\left(\frac{2\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{7/2}}{11cd(d+ex)^{3/2}}+\frac{4\left(d^2-\frac{ae^2}{c}\right)\left(\frac{2\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{7/}}{9cd(d+ex)^{5/2}}\right.}{}\right.}{13d}\right.}{15d}$$

input
```
Int[(d + e*x)^(3/2)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2),x]
```

output
```
(2*Sqrt[d + e*x]*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(7/2))/(15*c*d) +
 (8*(d^2 - (a*e^2)/c)*((2*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(7/2))/(
13*c*d*Sqrt[d + e*x]) + (6*(d^2 - (a*e^2)/c)*((2*(a*d*e + (c*d^2 + a*e^2)*
x + c*d*e*x^2)^(7/2))/(11*c*d*(d + e*x)^(3/2)) + (4*(d^2 - (a*e^2)/c)*((4*
(d^2 - (a*e^2)/c)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(7/2))/(63*c*d^2
*(d + e*x)^(7/2)) + (2*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(7/2))/(9*c
*d*(d + e*x)^(5/2))))/(11*d)))/(13*d)))/(15*d)
```

Defintions of rubi rules used

rule 1122
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[e*(d + e*x)^(m - 1)*((a + b*x + c*x^2)^(p + 1)/(c*(p + 1))),
 x] /; FreeQ[{a, b, c, d, e, m, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&
EqQ[m + p, 0]
```

rule 1128
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[e*(d + e*x)^(m - 1)*((a + b*x + c*x^2)^(p + 1)/(c*(m + 2*p +
 1))), x] + Simp[Simplify[m + p]*((2*c*d - b*e)/(c*(m + 2*p + 1)))   Int[(d
 + e*x)^(m - 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p},
 x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && IGtQ[Simplify[m + p], 0]
```

Maple [A] (verified)

Time = 1.19 (sec) , antiderivative size = 235, normalized size of antiderivative = 0.77

method	result
default	$\frac{2\sqrt{(ex+d)(cdx+ae)}\,(cdx+ae)^3(3003c^4d^4e^4x^4-1848a\,c^3d^3e^5x^3+13860c^4d^5e^3x^3+1008a^2c^2d^2e^6x^2-7560a\,c^3d^4e^4x^2+24570c^4d^6e}{45045\sqrt{}}$
gosper	$\frac{2(cdx+ae)(3003c^4d^4e^4x^4-1848a\,c^3d^3e^5x^3+13860c^4d^5e^3x^3+1008a^2c^2d^2e^6x^2-7560a\,c^3d^4e^4x^2+24570c^4d^6e^2x^2-448a^3cd\,e^7x+3}{450}$
orering	$\frac{2(3003c^4d^4e^4x^4-1848a\,c^3d^3e^5x^3+13860c^4d^5e^3x^3+1008a^2c^2d^2e^6x^2-7560a\,c^3d^4e^4x^2+24570c^4d^6e^2x^2-448a^3cd\,e^7x+3360a^2c^2d}{450}$

input
```
int((e*x+d)^(3/2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(5/2),x,method=_RETURN
VERBOSE)
```

output
```
2/45045/(e*x+d)^(1/2)*((e*x+d)*(c*d*x+a*e))^(1/2)*(c*d*x+a*e)^3*(3003*c^4*
d^4*e^4*x^4-1848*a*c^3*d^3*e^5*x^3+13860*c^4*d^5*e^3*x^3+1008*a^2*c^2*d^2*
e^6*x^2-7560*a*c^3*d^4*e^4*x^2+24570*c^4*d^6*e^2*x^2-448*a^3*c*d*e^7*x+336
0*a^2*c^2*d^3*e^5*x-10920*a*c^3*d^5*e^3*x+20020*c^4*d^7*e*x+128*a^4*e^8-96
0*a^3*c*d^2*e^6+3120*a^2*c^2*d^4*e^4-5720*a*c^3*d^6*e^2+6435*c^4*d^8)/d^5/
c^5
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 467, normalized size of antiderivative = 1.53

$$\int (d+ex)^{3/2}\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}\,dx = \frac{2\left(3003\,c^7d^7e^4x^7+6435\,a^3c^4d^8e^3-5720\,a^4c^3d^6e^5+3120\,a^5c^2d^4e^7-960\,a^6cd^2e^9+128\,a^7\right.}{}$$

input
```
integrate((e*x+d)^(3/2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2),x, algorit
hm="fricas")
```

output
```
2/45045*(3003*c^7*d^7*e^4*x^7 + 6435*a^3*c^4*d^8*e^3 - 5720*a^4*c^3*d^6*e^
5 + 3120*a^5*c^2*d^4*e^7 - 960*a^6*c*d^2*e^9 + 128*a^7*e^11 + 231*(60*c^7*
d^8*e^3 + 31*a*c^6*d^6*e^5)*x^6 + 63*(390*c^7*d^9*e^2 + 540*a*c^6*d^7*e^4
+ 71*a^2*c^5*d^5*e^6)*x^5 + 35*(572*c^7*d^10*e + 1794*a*c^6*d^8*e^3 + 636*
a^2*c^5*d^6*e^5 + a^3*c^4*d^4*e^7)*x^4 + 5*(1287*c^7*d^11 + 10868*a*c^6*d^
9*e^2 + 8814*a^2*c^5*d^7*e^4 + 60*a^3*c^4*d^5*e^6 - 8*a^4*c^3*d^3*e^8)*x^3
 + 3*(6435*a*c^6*d^10*e + 14300*a^2*c^5*d^8*e^3 + 390*a^3*c^4*d^6*e^5 - 12
0*a^4*c^3*d^4*e^7 + 16*a^5*c^2*d^2*e^9)*x^2 + (19305*a^2*c^5*d^9*e^2 + 286
0*a^3*c^4*d^7*e^4 - 1560*a^4*c^3*d^5*e^6 + 480*a^5*c^2*d^3*e^8 - 64*a^6*c*
d*e^10)*x)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(e*x + d)/(c^5*
d^5*e*x + c^5*d^6)
```

Sympy [F(-1)]

Timed out.

$$\int (d+ex)^{3/2} \left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2} dx = \text{Timed out}$$

input
```
integrate((e*x+d)**(3/2)*(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(5/2),x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 448, normalized size of antiderivative = 1.47

$$\int (d+ex)^{3/2} \left(ade+\left(cd^2+ae^2\right)x +cdex^2\right)^{5/2} dx = \frac{2\left(3003\,c^7d^7e^4x^7+6435\,a^3c^4d^8e^3-5720\,a^4c^3d^6e^5+3120\,a^5c^2d^4e^7-960\,a^6cd^2e^9+128\,a^7\right.}{}$$

input
```
integrate((e*x+d)^(3/2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2),x, algorit
hm="maxima")
```

output
```
2/45045*(3003*c^7*d^7*e^4*x^7 + 6435*a^3*c^4*d^8*e^3 - 5720*a^4*c^3*d^6*e^
5 + 3120*a^5*c^2*d^4*e^7 - 960*a^6*c*d^2*e^9 + 128*a^7*e^11 + 231*(60*c^7*
d^8*e^3 + 31*a*c^6*d^6*e^5)*x^6 + 63*(390*c^7*d^9*e^2 + 540*a*c^6*d^7*e^4
+ 71*a^2*c^5*d^5*e^6)*x^5 + 35*(572*c^7*d^10*e + 1794*a*c^6*d^8*e^3 + 636*
a^2*c^5*d^6*e^5 + a^3*c^4*d^4*e^7)*x^4 + 5*(1287*c^7*d^11 + 10868*a*c^6*d^
9*e^2 + 8814*a^2*c^5*d^7*e^4 + 60*a^3*c^4*d^5*e^6 - 8*a^4*c^3*d^3*e^8)*x^3
 + 3*(6435*a*c^6*d^10*e + 14300*a^2*c^5*d^8*e^3 + 390*a^3*c^4*d^6*e^5 - 12
0*a^4*c^3*d^4*e^7 + 16*a^5*c^2*d^2*e^9)*x^2 + (19305*a^2*c^5*d^9*e^2 + 286
0*a^3*c^4*d^7*e^4 - 1560*a^4*c^3*d^5*e^6 + 480*a^5*c^2*d^3*e^8 - 64*a^6*c*
d*e^10)*x)*sqrt(c*d*x + a*e)*(e*x + d)/(c^5*d^5*e*x + c^5*d^6)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1674 vs. $2(275) = 550$.

Time = 0.18 (sec) , antiderivative size = 1674, normalized size of antiderivative = 5.49

$$\int (d+ex)^{3/2} \left(ade + \left(cd^2 + ae^2\right) x + cdex^2\right)^{5/2} \, dx = \text{Too large to display}$$

input
```
integrate((e*x+d)^(3/2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2),x, algorit
hm="giac")
```

output
```
2/45045*(45045*sqrt(c*d*x + a*e)*a^3*d^4*e^3 - 45045*(3*sqrt(c*d*x + a*e)*
a*e - (c*d*x + a*e)^(3/2))*a^2*d^4*e^2 - 60060*(3*sqrt(c*d*x + a*e)*a*e -
(c*d*x + a*e)^(3/2))*a^3*d^2*e^4/c + 9009*(15*sqrt(c*d*x + a*e)*a^2*e^2 -
10*(c*d*x + a*e)^(3/2)*a*e + 3*(c*d*x + a*e)^(5/2))*a*d^4*e + 36036*(15*sq
rt(c*d*x + a*e)*a^2*e^2 - 10*(c*d*x + a*e)^(3/2)*a*e + 3*(c*d*x + a*e)^(5/
2))*a^2*d^2*e^3/c + 18018*(15*sqrt(c*d*x + a*e)*a^2*e^2 - 10*(c*d*x + a*e)
^(3/2)*a*e + 3*(c*d*x + a*e)^(5/2))*a^3*e^5/c^2 - 1287*(35*sqrt(c*d*x + a*
e)*a^3*e^3 - 35*(c*d*x + a*e)^(3/2)*a^2*e^2 + 21*(c*d*x + a*e)^(5/2)*a*e -
 5*(c*d*x + a*e)^(7/2))*d^4 - 15444*(35*sqrt(c*d*x + a*e)*a^3*e^3 - 35*(c*
d*x + a*e)^(3/2)*a^2*e^2 + 21*(c*d*x + a*e)^(5/2)*a*e - 5*(c*d*x + a*e)^(7
/2))*a*d^2*e^2/c - 23166*(35*sqrt(c*d*x + a*e)*a^3*e^3 - 35*(c*d*x + a*e)^
(3/2)*a^2*e^2 + 21*(c*d*x + a*e)^(5/2)*a*e - 5*(c*d*x + a*e)^(7/2))*a^2*e^
4/c^2 - 5148*(35*sqrt(c*d*x + a*e)*a^3*e^3 - 35*(c*d*x + a*e)^(3/2)*a^2*e^
2 + 21*(c*d*x + a*e)^(5/2)*a*e - 5*(c*d*x + a*e)^(7/2))*a^3*e^6/(c^3*d^2)
+ 572*(315*sqrt(c*d*x + a*e)*a^4*e^4 - 420*(c*d*x + a*e)^(3/2)*a^3*e^3 + 3
78*(c*d*x + a*e)^(5/2)*a^2*e^2 - 180*(c*d*x + a*e)^(7/2)*a*e + 35*(c*d*x +
 a*e)^(9/2))*d^2*e/c + 2574*(315*sqrt(c*d*x + a*e)*a^4*e^4 - 420*(c*d*x +
a*e)^(3/2)*a^3*e^3 + 378*(c*d*x + a*e)^(5/2)*a^2*e^2 - 180*(c*d*x + a*e)^(
7/2)*a*e + 35*(c*d*x + a*e)^(9/2))*a*e^3/c^2 + 1716*(315*sqrt(c*d*x + a*e)
*a^4*e^4 - 420*(c*d*x + a*e)^(3/2)*a^3*e^3 + 378*(c*d*x + a*e)^(5/2)*a^...
```

Mupad [B] (verification not implemented)

Time = 5.77 (sec) , antiderivative size = 501, normalized size of antiderivative = 1.64

$$\int (d+ex)^{3/2} \left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2} dx = \frac{\sqrt{c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e}\,\left(\frac{2\,e\,x^5\,\sqrt{d+e\,x}\,\left(71\,a^2\,e^4+540\,a\,c\,d^2\,e^2+390\,c^2\,d^4\right)}{715}+\frac{2\,x^4\,\sqrt{d+e\,x}\,\left(a^3\,e^6+\dots\right.}{}\right.}{}$$

input
```
int((d + e*x)^(3/2)*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(5/2),x)
```

output
```
((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2)*((2*e*x^5*(d + e*x)^(1/2)*(
71*a^2*e^4 + 390*c^2*d^4 + 540*a*c*d^2*e^2))/715 + (2*x^4*(d + e*x)^(1/2)*
(a^3*e^6 + 572*c^3*d^6 + 1794*a*c^2*d^4*e^2 + 636*a^2*c*d^2*e^4))/(1287*c*
d) + ((d + e*x)^(1/2)*(256*a^7*e^11 - 1920*a^6*c*d^2*e^9 + 12870*a^3*c^4*d
^8*e^3 - 11440*a^4*c^3*d^6*e^5 + 6240*a^5*c^2*d^4*e^7))/(45045*c^5*d^5*e)
+ (2*c^2*d^2*e^3*x^7*(d + e*x)^(1/2))/15 + (2*a*x^2*(d + e*x)^(1/2)*(16*a^
4*e^8 + 6435*c^4*d^8 + 14300*a*c^3*d^6*e^2 - 120*a^3*c*d^2*e^6 + 390*a^2*c
^2*d^4*e^4))/(15015*c^3*d^3) + (x^3*(d + e*x)^(1/2)*(12870*c^7*d^11 + 1086
80*a*c^6*d^9*e^2 + 88140*a^2*c^5*d^7*e^4 + 600*a^3*c^4*d^5*e^6 - 80*a^4*c^
3*d^3*e^8))/(45045*c^5*d^5*e) + (2*c*d*e^2*x^6*(31*a*e^2 + 60*c*d^2)*(d +
e*x)^(1/2))/195 + (2*a^2*e*x*(d + e*x)^(1/2)*(19305*c^4*d^8 - 64*a^4*e^8 +
 2860*a*c^3*d^6*e^2 + 480*a^3*c*d^2*e^6 - 1560*a^2*c^2*d^4*e^4))/(45045*c^
4*d^4)))/(x + d/e)
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 459, normalized size of antiderivative = 1.50

$$\int (d+ex)^{3/2} \left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2} dx = \frac{2\sqrt{cdx+ae}\,(3003c^7d^7e^4x^7+7161a\,c^6d^6e^5x^6+13860c^7d^8e^3x^6+4473a^2c^5d^5e^6x^5+34020}{}$$

input
```
int((e*x+d)^(3/2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2),x)
```

output
```
(2*sqrt(a*e + c*d*x)*(128*a**7*e**11 - 960*a**6*c*d**2*e**9 - 64*a**6*c*d*
e**10*x + 3120*a**5*c**2*d**4*e**7 + 480*a**5*c**2*d**3*e**8*x + 48*a**5*c
**2*d**2*e**9*x**2 - 5720*a**4*c**3*d**6*e**5 - 1560*a**4*c**3*d**5*e**6*x
 - 360*a**4*c**3*d**4*e**7*x**2 - 40*a**4*c**3*d**3*e**8*x**3 + 6435*a**3*
c**4*d**8*e**3 + 2860*a**3*c**4*d**7*e**4*x + 1170*a**3*c**4*d**6*e**5*x**
2 + 300*a**3*c**4*d**5*e**6*x**3 + 35*a**3*c**4*d**4*e**7*x**4 + 19305*a**
2*c**5*d**9*e**2*x + 42900*a**2*c**5*d**8*e**3*x**2 + 44070*a**2*c**5*d**7
*e**4*x**3 + 22260*a**2*c**5*d**6*e**5*x**4 + 4473*a**2*c**5*d**5*e**6*x**
5 + 19305*a*c**6*d**10*e*x**2 + 54340*a*c**6*d**9*e**2*x**3 + 62790*a*c**6
*d**8*e**3*x**4 + 34020*a*c**6*d**7*e**4*x**5 + 7161*a*c**6*d**6*e**5*x**6
 + 6435*c**7*d**11*x**3 + 20020*c**7*d**10*e*x**4 + 24570*c**7*d**9*e**2*x
**5 + 13860*c**7*d**8*e**3*x**6 + 3003*c**7*d**7*e**4*x**7))/(45045*c**5*d
**5)
```

3.298 $\int \sqrt{d+ex}\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}\,dx$

Optimal result

Integrand size = 39, antiderivative size = 240

$$\int \sqrt{d+ex}\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}\,dx=\frac{2\left(cd^2-ae^2\right)^3\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{7/2}}{7c^4d^4(d+ex)^{7/2}}+\frac{2e\left(cd^2-ae^2\right)^2\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{9/2}}{3c^4d^4(d+ex)^{9/2}}+\frac{6e^2\left(cd^2-ae^2\right)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{11/2}}{11c^4d^4(d+ex)^{11/2}}+\frac{2e^3\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{13/2}}{13c^4d^4(d+ex)^{13/2}}$$

output
```
2/7*(-a*e^2+c*d^2)^3*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(7/2)/c^4/d^4/(e*x+
d)^(7/2)+2/3*e*(-a*e^2+c*d^2)^2*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(9/2)/c^
4/d^4/(e*x+d)^(9/2)+6/11*e^2*(-a*e^2+c*d^2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x
^2)^(11/2)/c^4/d^4/(e*x+d)^(11/2)+2/13*e^3*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^
2)^(13/2)/c^4/d^4/(e*x+d)^(13/2)
```

Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 142, normalized size of antiderivative = 0.59

$$\int \sqrt{d+ex}\left(ade+\left(cd^2+ae^2\right)x +cdex^2\right)^{5/2} dx = \frac{2(ae+cdx)^3\sqrt{(ae+cdx)(d+ex)}(-16a^3e^6+8a^2cde^4(13d+7ex)-2ac^2d^2e^2(143d^2+18}{3003c^4d^4\sqrt{d+ex}}$$

input
```
Integrate[Sqrt[d + e*x]*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2),x]
```

output
```
(2*(a*e + c*d*x)^3*Sqrt[(a*e + c*d*x)*(d + e*x)]*(-16*a^3*e^6 + 8*a^2*c*d*
e^4*(13*d + 7*e*x) - 2*a*c^2*d^2*e^2*(143*d^2 + 182*d*e*x + 63*e^2*x^2) +
c^3*d^3*(429*d^3 + 1001*d^2*e*x + 819*d*e^2*x^2 + 231*e^3*x^3)))/(3003*c^4
*d^4*Sqrt[d + e*x])
```

Rubi [A] (verified)

Time = 0.66 (sec) , antiderivative size = 248, normalized size of antiderivative = 1.03, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.103, Rules used = {1128, 1128, 1128, 1122}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{d+ex}\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/2} dx$$

$$\Big\downarrow \ 1128$$

$$\frac{6\left(d^2-\frac{ae^2}{c}\right)\int \frac{\left(cdex^2+\left(cd^2+ae^2\right)x+ade\right)^{5/2}}{\sqrt{d+ex}}dx}{13d}+\frac{2\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{7/2}}{13cd\sqrt{d+ex}}$$

$$\Big\downarrow \ 1128$$

$$\frac{6\left(d^2-\frac{ae^2}{c}\right)\left(\frac{4\left(d^2-\frac{ae^2}{c}\right)\int \frac{\left(cdex^2+\left(cd^2+ae^2\right)x+ade\right)^{5/2}}{(d+ex)^{3/2}}dx}{11d}+\frac{2(x(ae^2+cd^2)+ade+cdex^2)^{7/2}}{11cd(d+ex)^{3/2}}\right)}{13d}+\frac{2(x(ae^2+cd^2)+ade+cdex^2)^{7/2}}{13cd\sqrt{d+ex}}$$

$$\Big\downarrow \text{1128}$$

$$\frac{6\left(d^2-\frac{ae^2}{c}\right)\left(\frac{4\left(d^2-\frac{ae^2}{c}\right)\left(\frac{2\left(d^2-\frac{ae^2}{c}\right)\int \frac{\left(cdex^2+\left(cd^2+ae^2\right)x+ade\right)^{5/2}}{(d+ex)^{5/2}}dx}{9d}+\frac{2\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{7/2}}{9cd(d+ex)^{5/2}}\right)}{11d}+\frac{2(x(ae^2+cd^2)+ade+cdex^2)^{7/2}}{11cd(d+ex)^{3/2}}\right)}{13d}+\frac{2(x(ae^2+cd^2)+ade+cdex^2)^{7/2}}{13cd\sqrt{d+ex}}$$

$$\Big\downarrow \text{1122}$$

$$\frac{2(x(ae^2+cd^2)+ade+cdex^2)^{7/2}}{13cd\sqrt{d+ex}}+\frac{6\left(d^2-\frac{ae^2}{c}\right)\left(\frac{2(x(ae^2+cd^2)+ade+cdex^2)^{7/2}}{11cd(d+ex)^{3/2}}+\frac{4\left(d^2-\frac{ae^2}{c}\right)\left(\frac{2\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{7/2}}{9cd(d+ex)^{5/2}}+\frac{4\left(d^2-\frac{ae^2}{c}\right)\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{7/2}}{63cd^2(d+ex)^{7/2}}\right)}{11d}\right)}{13d}$$

input
```
Int[Sqrt[d + e*x]*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2),x]
```

output
```
(2*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(7/2))/(13*c*d*Sqrt[d + e*x]) +
 (6*(d^2 - (a*e^2)/c)*((2*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(7/2))/(
11*c*d*(d + e*x)^(3/2)) + (4*(d^2 - (a*e^2)/c)*((4*(d^2 - (a*e^2)/c)*(a*d*
e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(7/2))/(63*c*d^2*(d + e*x)^(7/2)) + (2*
(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(7/2))/(9*c*d*(d + e*x)^(5/2))))/(
11*d)))/(13*d)
```

Defintions of rubi rules used

rule 1122
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[e*(d + e*x)^(m - 1)*((a + b*x + c*x^2)^(p + 1)/(c*(p + 1))),
 x] /; FreeQ[{a, b, c, d, e, m, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&
EqQ[m + p, 0]
```

rule 1128
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[e*(d + e*x)^(m - 1)*((a + b*x + c*x^2)^(p + 1)/(c*(m + 2*p +
 1))), x] + Simp[Simplify[m + p]*((2*c*d - b*e)/(c*(m + 2*p + 1)))   Int[(d
 + e*x)^(m - 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p},
 x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && IGtQ[Simplify[m + p], 0]
```

Maple [A] (verified)

Time = 1.20 (sec) , antiderivative size = 160, normalized size of antiderivative = 0.67

method	result
default	$-\frac{2\sqrt{(ex+d)(cdx+ae)}\,(cdx+ae)^3(-231c^3d^3e^3x^3+126x^2a\,c^2d^2e^4-819c^3d^4e^2x^2-56x\,a^2cd\,e^5+364xa\,c^2d^3e^3-1001c^3d^5ex+16e^6a}{3003\sqrt{ex+d}\,d^4c^4}$
gosper	$-\frac{2(cdx+ae)(-231c^3d^3e^3x^3+126x^2a\,c^2d^2e^4-819c^3d^4e^2x^2-56x\,a^2cd\,e^5+364xa\,c^2d^3e^3-1001c^3d^5ex+16e^6a^3-104d^2e^4a^2c+286}{3003d^4c^4(ex+d)^{\frac{5}{2}}}$
orering	$-\frac{2(-231c^3d^3e^3x^3+126x^2a\,c^2d^2e^4-819c^3d^4e^2x^2-56x\,a^2cd\,e^5+364xa\,c^2d^3e^3-1001c^3d^5ex+16e^6a^3-104d^2e^4a^2c+286d^4e^2a\,c^2-}{3003d^4c^4(ex+d)^{\frac{5}{2}}}$

input
```
int((e*x+d)^(1/2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(5/2),x,method=_RETURN
VERBOSE)
```

output
```
-2/3003/(e*x+d)^(1/2)*((e*x+d)*(c*d*x+a*e))^(1/2)*(c*d*x+a*e)^3*(-231*c^3*
d^3*e^3*x^3+126*a*c^2*d^2*e^4*x^2-819*c^3*d^4*e^2*x^2-56*a^2*c*d*e^5*x+364
*a*c^2*d^3*e^3*x-1001*c^3*d^5*e*x+16*a^3*e^6-104*a^2*c*d^2*e^4+286*a*c^2*d
^4*e^2-429*c^3*d^6)/d^4/c^4
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 354, normalized size of antiderivative = 1.48

$$\int \sqrt{d+ex}\left(ade+\left(cd^2+ae^2\right)x +cdex^2\right)^{5/2}\,dx = \frac{2\,(231\,c^6d^6e^3x^6+429\,a^3c^3d^6e^3-286\,a^4c^2d^4e^5+104\,a^5cd^2e^7-16\,a^6e^9+63\,(13\,c^6d^7e^2+9\cdots}{\cdots}$$

input
```
integrate((e*x+d)^(1/2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2),x, algorit
hm="fricas")
```

output
```
2/3003*(231*c^6*d^6*e^3*x^6 + 429*a^3*c^3*d^6*e^3 - 286*a^4*c^2*d^4*e^5 +
104*a^5*c*d^2*e^7 - 16*a^6*e^9 + 63*(13*c^6*d^7*e^2 + 9*a*c^5*d^5*e^4)*x^5
 + 7*(143*c^6*d^8*e + 299*a*c^5*d^6*e^3 + 53*a^2*c^4*d^4*e^5)*x^4 + (429*c
^6*d^9 + 2717*a*c^5*d^7*e^2 + 1469*a^2*c^4*d^5*e^4 + 5*a^3*c^3*d^3*e^6)*x^
3 + 3*(429*a*c^5*d^8*e + 715*a^2*c^4*d^6*e^3 + 13*a^3*c^3*d^4*e^5 - 2*a^4*
c^2*d^2*e^7)*x^2 + (1287*a^2*c^4*d^7*e^2 + 143*a^3*c^3*d^5*e^4 - 52*a^4*c^
2*d^3*e^6 + 8*a^5*c*d*e^8)*x)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*
sqrt(e*x + d)/(c^4*d^4*e*x + c^4*d^5)
```

Sympy [F(-1)]

Timed out.

$$\int \sqrt{d+ex}\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}\,dx = \text{Timed out}$$

input
```
integrate((e*x+d)**(1/2)*(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(5/2),x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 335, normalized size of antiderivative = 1.40

$$\int \sqrt{d+ex}\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}\,dx=\frac{2\left(231\,c^6d^6e^3x^6+429\,a^3c^3d^6e^3-286\,a^4c^2d^4e^5+104\,a^5cd^2e^7-16\,a^6e^9+63\left(13\,c^6d^7e^2+9\right.\right.}{}$$

input
```
integrate((e*x+d)^(1/2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2),x, algorit
hm="maxima")
```

output
```
2/3003*(231*c^6*d^6*e^3*x^6 + 429*a^3*c^3*d^6*e^3 - 286*a^4*c^2*d^4*e^5 +
104*a^5*c*d^2*e^7 - 16*a^6*e^9 + 63*(13*c^6*d^7*e^2 + 9*a*c^5*d^5*e^4)*x^5
 + 7*(143*c^6*d^8*e + 299*a*c^5*d^6*e^3 + 53*a^2*c^4*d^4*e^5)*x^4 + (429*c
^6*d^9 + 2717*a*c^5*d^7*e^2 + 1469*a^2*c^4*d^5*e^4 + 5*a^3*c^3*d^3*e^6)*x^
3 + 3*(429*a*c^5*d^8*e + 715*a^2*c^4*d^6*e^3 + 13*a^3*c^3*d^4*e^5 - 2*a^4*
c^2*d^2*e^7)*x^2 + (1287*a^2*c^4*d^7*e^2 + 143*a^3*c^3*d^5*e^4 - 52*a^4*c^
2*d^3*e^6 + 8*a^5*c*d*e^8)*x)*sqrt(c*d*x + a*e)*(e*x + d)/(c^4*d^4*e*x + c
^4*d^5)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1195 vs. 2(216) = 432.

Time = 0.16 (sec) , antiderivative size = 1195, normalized size of antiderivative = 4.98

$$\int \sqrt{d+ex}\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}\,dx=\text{Too large to display}$$

input
```
integrate((e*x+d)^(1/2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2),x, algorit
hm="giac")
```

output
```
2/15015*(15015*sqrt(c*d*x + a*e)*a^3*d^3*e^3 - 15015*(3*sqrt(c*d*x + a*e)*
a*e - (c*d*x + a*e)^(3/2))*a^2*d^3*e^2 - 15015*(3*sqrt(c*d*x + a*e)*a*e -
(c*d*x + a*e)^(3/2))*a^3*d*e^4/c + 3003*(15*sqrt(c*d*x + a*e)*a^2*e^2 - 10
*(c*d*x + a*e)^(3/2)*a*e + 3*(c*d*x + a*e)^(5/2))*a*d^3*e + 9009*(15*sqrt(
c*d*x + a*e)*a^2*e^2 - 10*(c*d*x + a*e)^(3/2)*a*e + 3*(c*d*x + a*e)^(5/2))
*a^2*d*e^3/c + 3003*(15*sqrt(c*d*x + a*e)*a^2*e^2 - 10*(c*d*x + a*e)^(3/2)
*a*e + 3*(c*d*x + a*e)^(5/2))*a^3*e^5/(c^2*d) - 429*(35*sqrt(c*d*x + a*e)*
a^3*e^3 - 35*(c*d*x + a*e)^(3/2)*a^2*e^2 + 21*(c*d*x + a*e)^(5/2)*a*e - 5*
(c*d*x + a*e)^(7/2))*d^3 - 3861*(35*sqrt(c*d*x + a*e)*a^3*e^3 - 35*(c*d*x
+ a*e)^(3/2)*a^2*e^2 + 21*(c*d*x + a*e)^(5/2)*a*e - 5*(c*d*x + a*e)^(7/2))
*a*d*e^2/c - 3861*(35*sqrt(c*d*x + a*e)*a^3*e^3 - 35*(c*d*x + a*e)^(3/2)*a
^2*e^2 + 21*(c*d*x + a*e)^(5/2)*a*e - 5*(c*d*x + a*e)^(7/2))*a^2*e^4/(c^2*
d) - 429*(35*sqrt(c*d*x + a*e)*a^3*e^3 - 35*(c*d*x + a*e)^(3/2)*a^2*e^2 +
21*(c*d*x + a*e)^(5/2)*a*e - 5*(c*d*x + a*e)^(7/2))*a^3*e^6/(c^3*d^3) + 14
3*(315*sqrt(c*d*x + a*e)*a^4*e^4 - 420*(c*d*x + a*e)^(3/2)*a^3*e^3 + 378*(
c*d*x + a*e)^(5/2)*a^2*e^2 - 180*(c*d*x + a*e)^(7/2)*a*e + 35*(c*d*x + a*e
)^(9/2))*d*e/c + 429*(315*sqrt(c*d*x + a*e)*a^4*e^4 - 420*(c*d*x + a*e)^(3
/2)*a^3*e^3 + 378*(c*d*x + a*e)^(5/2)*a^2*e^2 - 180*(c*d*x + a*e)^(7/2)*a*
e + 35*(c*d*x + a*e)^(9/2))*a*e^3/(c^2*d) + 143*(315*sqrt(c*d*x + a*e)*a^4
*e^4 - 420*(c*d*x + a*e)^(3/2)*a^3*e^3 + 378*(c*d*x + a*e)^(5/2)*a^2*e^...
```

Mupad [B] (verification not implemented)

Time = 5.64 (sec) , antiderivative size = 383, normalized size of antiderivative = 1.60

$$\int \sqrt{d+ex}\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}\,dx=\frac{\sqrt{c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e}\,\left(x^4\,\sqrt{d+e\,x}\,\left(\frac{106\,a^2\,e^4}{429}+\frac{46\,a\,c\,d^2\,e^2}{33}+\frac{2\,c^2\,d^4}{3}\right)-\frac{\sqrt{d+e\,x}\,(32\,\cdots}{\cdots}$$

input
```
int((d + e*x)^(1/2)*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(5/2),x)
```

output
```
((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2)*(x^4*(d + e*x)^(1/2)*((106*
a^2*e^4)/429 + (2*c^2*d^4)/3 + (46*a*c*d^2*e^2)/33) - ((d + e*x)^(1/2)*(32
*a^6*e^9 - 208*a^5*c*d^2*e^7 - 858*a^3*c^3*d^6*e^3 + 572*a^4*c^2*d^4*e^5))
/(3003*c^4*d^4*e) + (2*c^2*d^2*e^2*x^6*(d + e*x)^(1/2))/13 + (x^3*(d + e*x
)^(1/2)*(858*c^6*d^9 + 5434*a*c^5*d^7*e^2 + 2938*a^2*c^4*d^5*e^4 + 10*a^3*
c^3*d^3*e^6))/(3003*c^4*d^4*e) + (6*c*d*e*x^5*(9*a*e^2 + 13*c*d^2)*(d + e*
x)^(1/2))/143 + (2*a*x^2*(d + e*x)^(1/2)*(429*c^3*d^6 - 2*a^3*e^6 + 715*a*
c^2*d^4*e^2 + 13*a^2*c*d^2*e^4))/(1001*c^2*d^2) + (2*a^2*e*x*(d + e*x)^(1/
2)*(8*a^3*e^6 + 1287*c^3*d^6 + 143*a*c^2*d^4*e^2 - 52*a^2*c*d^2*e^4))/(300
3*c^3*d^3)))/(x + d/e)
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 333, normalized size of antiderivative = 1.39

$$\int \sqrt{d+ex}\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}\,dx = \frac{2\sqrt{cdx+ae}\,(231c^6d^6e^3x^6+567a\,c^5d^5e^4x^5+819c^6d^7e^2x^5+371a^2c^4d^4e^5x^4+2093a\,c^5d^6e^3}{}$$

input
```
int((e*x+d)^(1/2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2),x)
```

output
```
(2*sqrt(a*e + c*d*x)*( - 16*a**6*e**9 + 104*a**5*c*d**2*e**7 + 8*a**5*c*d*
e**8*x - 286*a**4*c**2*d**4*e**5 - 52*a**4*c**2*d**3*e**6*x - 6*a**4*c**2*
d**2*e**7*x**2 + 429*a**3*c**3*d**6*e**3 + 143*a**3*c**3*d**5*e**4*x + 39*
a**3*c**3*d**4*e**5*x**2 + 5*a**3*c**3*d**3*e**6*x**3 + 1287*a**2*c**4*d**
7*e**2*x + 2145*a**2*c**4*d**6*e**3*x**2 + 1469*a**2*c**4*d**5*e**4*x**3 +
 371*a**2*c**4*d**4*e**5*x**4 + 1287*a*c**5*d**8*e*x**2 + 2717*a*c**5*d**7
*e**2*x**3 + 2093*a*c**5*d**6*e**3*x**4 + 567*a*c**5*d**5*e**4*x**5 + 429*
c**6*d**9*x**3 + 1001*c**6*d**8*e*x**4 + 819*c**6*d**7*e**2*x**5 + 231*c**
6*d**6*e**3*x**6))/(3003*c**4*d**4)
```

3.299 $\int \frac{(ade+(cd^2+ae^2)x+cdex^2)^{5/2}}{\sqrt{d+ex}}\,dx$

Optimal result

Integrand size = 39, antiderivative size = 175

$$\int \frac{(ade+(cd^2+ae^2)x+cdex^2)^{5/2}}{\sqrt{d+ex}}\,dx = \frac{2(cd^2-ae^2)^2(ade+(cd^2+ae^2)x+cdex^2)^{7/2}}{7c^3d^3(d+ex)^{7/2}} + \frac{4e(cd^2-ae^2)(ade+(cd^2+ae^2)x+cdex^2)^{9/2}}{9c^3d^3(d+ex)^{9/2}} + \frac{2e^2(ade+(cd^2+ae^2)x+cdex^2)^{11/2}}{11c^3d^3(d+ex)^{11/2}}$$

output
```
2/7*(-a*e^2+c*d^2)^2*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(7/2)/c^3/d^3/(e*x+
d)^(7/2)+4/9*e*(-a*e^2+c*d^2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(9/2)/c^3/
d^3/(e*x+d)^(9/2)+2/11*e^2*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(11/2)/c^3/d^
3/(e*x+d)^(11/2)
```

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 98, normalized size of antiderivative = 0.56

$$\int \frac{(ade+(cd^2+ae^2)x+cdex^2)^{5/2}}{\sqrt{d+ex}}\,dx = \frac{2(ae+cdx)^3\sqrt{(ae+cdx)(d+ex)}(8a^2e^4-4acde^2(11d+7ex)+}{693c^3d^3\sqrt{d+ex}}$$

input
```
Integrate[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2)/Sqrt[d + e*x],x]
```

output
```
(2*(a*e + c*d*x)^3*Sqrt[(a*e + c*d*x)*(d + e*x)]*(8*a^2*e^4 - 4*a*c*d*e^2*
(11*d + 7*e*x) + c^2*d^2*(99*d^2 + 154*d*e*x + 63*e^2*x^2)))/(693*c^3*d^3*
Sqrt[d + e*x])
```

Rubi [A] (verified)

Time = 0.52 (sec) , antiderivative size = 179, normalized size of antiderivative = 1.02, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {1128, 1128, 1122}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/2}}{\sqrt{d+ex}}dx$$

$$\Big\downarrow \text{1128}$$

$$\frac{4\left(d^2-\frac{ae^2}{c}\right)\int \frac{\left(cdex^2+\left(cd^2+ae^2\right)x+ade\right)^{5/2}}{(d+ex)^{3/2}}dx}{11d}+\frac{2\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{7/2}}{11cd(d+ex)^{3/2}}$$

$$\Big\downarrow \text{1128}$$

$$\frac{4\left(d^2-\frac{ae^2}{c}\right)\left(\frac{2\left(d^2-\frac{ae^2}{c}\right)\int \frac{\left(cdex^2+\left(cd^2+ae^2\right)x+ade\right)^{5/2}}{(d+ex)^{5/2}}dx}{9d}+\frac{2\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{7/2}}{9cd(d+ex)^{5/2}}\right)}{11d}+\frac{2\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{7/2}}{11cd(d+ex)^{3/2}}$$

$$\Big\downarrow \text{1122}$$

$$\frac{2\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{7/2}}{11cd(d+ex)^{3/2}}+\frac{4\left(d^2-\frac{ae^2}{c}\right)\left(\frac{2\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{7/2}}{9cd(d+ex)^{5/2}}+\frac{4\left(d^2-\frac{ae^2}{c}\right)\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{7/2}}{63cd^2(d+ex)^{7/2}}\right)}{11d}$$

input
```
Int[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2)/Sqrt[d + e*x],x]
```

output
```
(2*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(7/2))/(11*c*d*(d + e*x)^(3/2))
 + (4*(d^2 - (a*e^2)/c)*((4*(d^2 - (a*e^2)/c)*(a*d*e + (c*d^2 + a*e^2)*x +
 c*d*e*x^2)^(7/2))/(63*c*d^2*(d + e*x)^(7/2)) + (2*(a*d*e + (c*d^2 + a*e^2
)*x + c*d*e*x^2)^(7/2))/(9*c*d*(d + e*x)^(5/2))))/(11*d)
```

Defintions of rubi rules used

rule 1122
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[e*(d + e*x)^(m - 1)*((a + b*x + c*x^2)^(p + 1)/(c*(p + 1))),
 x] /; FreeQ[{a, b, c, d, e, m, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&
EqQ[m + p, 0]
```

rule 1128
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[e*(d + e*x)^(m - 1)*((a + b*x + c*x^2)^(p + 1)/(c*(m + 2*p +
 1))), x] + Simp[Simplify[m + p]*((2*c*d - b*e)/(c*(m + 2*p + 1)))   Int[(d
 + e*x)^(m - 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p},
 x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && IGtQ[Simplify[m + p], 0]
```

Maple [A] (verified)

Time = 1.21 (sec) , antiderivative size = 102, normalized size of antiderivative = 0.58

method	result	size
default	$\frac{2\sqrt{(ex+d)(cdx+ae)}\,(cdx+ae)^3\left(63x^2c^2d^2e^2-28xacd\,e^3+154x\,c^2d^3e+8a^2e^4-44ac\,d^2e^2+99c^2d^4\right)}{693\sqrt{ex+d}\,d^3c^3}$	102
gosper	$\frac{2(cdx+ae)\left(63x^2c^2d^2e^2-28xacd\,e^3+154x\,c^2d^3e+8a^2e^4-44ac\,d^2e^2+99c^2d^4\right)\left(cd\,x^2e+a\,e^2x+c\,d^2x+ade\right)^{\frac{5}{2}}}{693d^3c^3(ex+d)^{\frac{5}{2}}}$	110
orering	$\frac{2\left(63x^2c^2d^2e^2-28xacd\,e^3+154x\,c^2d^3e+8a^2e^4-44ac\,d^2e^2+99c^2d^4\right)(cdx+ae)\left(ade+\left(a\,e^2+c\,d^2\right)x+cd\,x^2e\right)^{\frac{5}{2}}}{693d^3c^3(ex+d)^{\frac{5}{2}}}$	111

input
```
int((a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(5/2)/(e*x+d)^(1/2),x,method=_RETURN
VERBOSE)
```

output
```
2/693*((e*x+d)*(c*d*x+a*e))^(1/2)/(e*x+d)^(1/2)*(c*d*x+a*e)^3*(63*c^2*d^2*
e^2*x^2-28*a*c*d*e^3*x+154*c^2*d^3*e*x+8*a^2*e^4-44*a*c*d^2*e^2+99*c^2*d^4
)/d^3/c^3
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 254, normalized size of antiderivative = 1.45

$$\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{\sqrt{d+ex}}\,dx = \frac{2\left(63\,c^5d^5e^2x^5+99\,a^3c^2d^4e^3-44\,a^4cd^2e^5+8\,a^5e^7+7\left(22\,c^5d^6e\right.\right.}{}$$

input
```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(e*x+d)^(1/2),x, algorit
hm="fricas")
```

output
```
2/693*(63*c^5*d^5*e^2*x^5 + 99*a^3*c^2*d^4*e^3 - 44*a^4*c*d^2*e^5 + 8*a^5*
e^7 + 7*(22*c^5*d^6*e + 23*a*c^4*d^4*e^3)*x^4 + (99*c^5*d^7 + 418*a*c^4*d^
5*e^2 + 113*a^2*c^3*d^3*e^4)*x^3 + 3*(99*a*c^4*d^6*e + 110*a^2*c^3*d^4*e^3
 + a^3*c^2*d^2*e^5)*x^2 + (297*a^2*c^3*d^5*e^2 + 22*a^3*c^2*d^3*e^4 - 4*a^
4*c*d*e^6)*x)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(e*x + d)/(c
^3*d^3*e*x + c^3*d^4)
```

Sympy [F]

$$\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{\sqrt{d+ex}}\,dx = \int \frac{\left((d+ex)\left(ae+cdx\right)\right)^{\frac{5}{2}}}{\sqrt{d+ex}}\,dx$$

input
```
integrate((a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(5/2)/(e*x+d)**(1/2),x)
```

output
```
Integral(((d + e*x)*(a*e + c*d*x))**(5/2)/sqrt(d + e*x), x)
```

Maxima [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 217, normalized size of antiderivative = 1.24

$$\int \frac{(ade + (cd^2 + ae^2)\,x + cdex^2)^{5/2}}{\sqrt{d+ex}}\,dx = \frac{2\,(63\,c^5d^5e^2x^5 + 99\,a^3c^2d^4e^3 - 44\,a^4cd^2e^5 + 8\,a^5e^7 + 7\,(22\,c^5d^6e -$$

input
```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(e*x+d)^(1/2),x, algorit
hm="maxima")
```

output
```
2/693*(63*c^5*d^5*e^2*x^5 + 99*a^3*c^2*d^4*e^3 - 44*a^4*c*d^2*e^5 + 8*a^5*
e^7 + 7*(22*c^5*d^6*e + 23*a*c^4*d^4*e^3)*x^4 + (99*c^5*d^7 + 418*a*c^4*d^
5*e^2 + 113*a^2*c^3*d^3*e^4)*x^3 + 3*(99*a*c^4*d^6*e + 110*a^2*c^3*d^4*e^3
 + a^3*c^2*d^2*e^5)*x^2 + (297*a^2*c^3*d^5*e^2 + 22*a^3*c^2*d^3*e^4 - 4*a^
4*c*d*e^6)*x)*sqrt(c*d*x + a*e)/(c^3*d^3)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 780 vs. 2(157) = 314.

Time = 0.18 (sec) , antiderivative size = 780, normalized size of antiderivative = 4.46

$$\int \frac{(ade + (cd^2 + ae^2)\,x + cdex^2)^{5/2}}{\sqrt{d+ex}}\,dx = \text{Too large to display}$$

input
```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(e*x+d)^(1/2),x, algorit
hm="giac")
```

output
```
2/3465*(3465*sqrt(c*d*x + a*e)*a^3*d^2*e^3 - 3465*(3*sqrt(c*d*x + a*e)*a*e
 - (c*d*x + a*e)^(3/2))*a^2*d^2*e^2 - 2310*(3*sqrt(c*d*x + a*e)*a*e - (c*d
*x + a*e)^(3/2))*a^3*e^4/c + 693*(15*sqrt(c*d*x + a*e)*a^2*e^2 - 10*(c*d*x
 + a*e)^(3/2)*a*e + 3*(c*d*x + a*e)^(5/2))*a*d^2*e + 1386*(15*sqrt(c*d*x +
 a*e)*a^2*e^2 - 10*(c*d*x + a*e)^(3/2)*a*e + 3*(c*d*x + a*e)^(5/2))*a^2*e^
3/c + 231*(15*sqrt(c*d*x + a*e)*a^2*e^2 - 10*(c*d*x + a*e)^(3/2)*a*e + 3*(
c*d*x + a*e)^(5/2))*a^3*e^5/(c^2*d^2) - 99*(35*sqrt(c*d*x + a*e)*a^3*e^3 -
 35*(c*d*x + a*e)^(3/2)*a^2*e^2 + 21*(c*d*x + a*e)^(5/2)*a*e - 5*(c*d*x +
a*e)^(7/2))*d^2 - 594*(35*sqrt(c*d*x + a*e)*a^3*e^3 - 35*(c*d*x + a*e)^(3/
2)*a^2*e^2 + 21*(c*d*x + a*e)^(5/2)*a*e - 5*(c*d*x + a*e)^(7/2))*a*e^2/c -
 297*(35*sqrt(c*d*x + a*e)*a^3*e^3 - 35*(c*d*x + a*e)^(3/2)*a^2*e^2 + 21*(
c*d*x + a*e)^(5/2)*a*e - 5*(c*d*x + a*e)^(7/2))*a^2*e^4/(c^2*d^2) + 22*(31
5*sqrt(c*d*x + a*e)*a^4*e^4 - 420*(c*d*x + a*e)^(3/2)*a^3*e^3 + 378*(c*d*x
 + a*e)^(5/2)*a^2*e^2 - 180*(c*d*x + a*e)^(7/2)*a*e + 35*(c*d*x + a*e)^(9/
2))*e/c + 33*(315*sqrt(c*d*x + a*e)*a^4*e^4 - 420*(c*d*x + a*e)^(3/2)*a^3*
e^3 + 378*(c*d*x + a*e)^(5/2)*a^2*e^2 - 180*(c*d*x + a*e)^(7/2)*a*e + 35*(
c*d*x + a*e)^(9/2))*a*e^3/(c^2*d^2) - 5*(693*sqrt(c*d*x + a*e)*a^5*e^5 - 1
155*(c*d*x + a*e)^(3/2)*a^4*e^4 + 1386*(c*d*x + a*e)^(5/2)*a^3*e^3 - 990*(
c*d*x + a*e)^(7/2)*a^2*e^2 + 385*(c*d*x + a*e)^(9/2)*a*e - 63*(c*d*x + a*e
)^(11/2))*e^2/(c^2*d^2))/(c*d)
```

Mupad [B] (verification not implemented)

Time = 5.52 (sec) , antiderivative size = 241, normalized size of antiderivative = 1.38

$$\int \frac{(ade+(cd^2+ae^2)\,x+cdex^2)^{5/2}}{\sqrt{d+ex}}\,dx = \frac{\sqrt{c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e}\,\left(\frac{16\,a^5\,e^7-88\,a^4\,c\,d^2\,e^5+198\,a^3\,c^2\,d^4\,e^3}{693\,c^3\,d^3}\right.}{}$$

input
```
int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(5/2)/(d + e*x)^(1/2),x)
```

output
```
((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2)*((16*a^5*e^7 - 88*a^4*c*d^2
*e^5 + 198*a^3*c^2*d^4*e^3)/(693*c^3*d^3) + (x^3*(198*c^5*d^7 + 836*a*c^4*
d^5*e^2 + 226*a^2*c^3*d^3*e^4))/(693*c^3*d^3) + (2*c^2*d^2*e^2*x^5)/11 + (
2*c*d*e*x^4*(23*a*e^2 + 22*c*d^2))/99 + (2*a*e*x^2*(a^2*e^4 + 99*c^2*d^4 +
 110*a*c*d^2*e^2))/(231*c*d) + (2*a^2*e^2*x*(297*c^2*d^4 - 4*a^2*e^4 + 22*
a*c*d^2*e^2))/(693*c^2*d^2)))/(d + e*x)^(1/2)
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 224, normalized size of antiderivative = 1.28

$$\int \frac{(ade + (cd^2 + ae^2)x + cdex^2)^{5/2}}{\sqrt{d + ex}}\,dx = \frac{2\sqrt{cdx + ae}\,(63c^5d^5e^2x^5 + 161a\,c^4d^4e^3x^4 + 154c^5d^6e\,x^4 + 113a^2c$$

input
```
int((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(e*x+d)^(1/2),x)
```

output
```
(2*sqrt(a*e + c*d*x)*(8*a**5*e**7 - 44*a**4*c*d**2*e**5 - 4*a**4*c*d*e**6*
x + 99*a**3*c**2*d**4*e**3 + 22*a**3*c**2*d**3*e**4*x + 3*a**3*c**2*d**2*e
**5*x**2 + 297*a**2*c**3*d**5*e**2*x + 330*a**2*c**3*d**4*e**3*x**2 + 113*
a**2*c**3*d**3*e**4*x**3 + 297*a*c**4*d**6*e*x**2 + 418*a*c**4*d**5*e**2*x
**3 + 161*a*c**4*d**4*e**3*x**4 + 99*c**5*d**7*x**3 + 154*c**5*d**6*e*x**4
 + 63*c**5*d**5*e**2*x**5))/(693*c**3*d**3)
```

3.300 $\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{(d+ex)^{3/2}}\,dx$

Optimal result

Integrand size = 39, antiderivative size = 110

$$\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{(d+ex)^{3/2}}\,dx=\frac{2(cd^2-ae^2)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{7/2}}{7c^2d^2(d+ex)^{7/2}}+\frac{2e\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{9/2}}{9c^2d^2(d+ex)^{9/2}}$$

output
```
2/7*(-a*e^2+c*d^2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(7/2)/c^2/d^2/(e*x+d)
^(7/2)+2/9*e*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(9/2)/c^2/d^2/(e*x+d)^(9/2)
```

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.59

$$\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{(d+ex)^{3/2}}\,dx=\frac{2(ae+cdx)^3\sqrt{(ae+cdx)(d+ex)}\left(-2ae^2+cd(9d+7ex)\right)}{63c^2d^2\sqrt{d+ex}}$$

input
```
Integrate[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2)/(d + e*x)^(3/2),x]
```

output
```
(2*(a*e + c*d*x)^3*Sqrt[(a*e + c*d*x)*(d + e*x)]*(-2*a*e^2 + c*d*(9*d + 7*
e*x)))/(63*c^2*d^2*Sqrt[d + e*x])
```

Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 110, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.051$, Rules used = $\{1128, 1122\}$

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/2}}{(d+ex)^{3/2}}dx$$

$$\Big\downarrow \text{ 1128}$$

$$\frac{2\left(d^2-\frac{ae^2}{c}\right)\int \frac{\left(cdex^2+\left(cd^2+ae^2\right)x+ade\right)^{5/2}}{(d+ex)^{5/2}}dx}{9d}+\frac{2\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{7/2}}{9cd(d+ex)^{5/2}}$$

$$\Big\downarrow \text{ 1122}$$

$$\frac{2\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{7/2}}{9cd(d+ex)^{5/2}}+\frac{4\left(d^2-\frac{ae^2}{c}\right)\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{7/2}}{63cd^2(d+ex)^{7/2}}$$

input
```
Int[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2)/(d + e*x)^(3/2),x]
```

output
```
(4*(d^2 - (a*e^2)/c)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(7/2))/(63*c*
d^2*(d + e*x)^(7/2)) + (2*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(7/2))/(
9*c*d*(d + e*x)^(5/2))
```

Defintions of rubi rules used

rule 1122
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[e*(d + e*x)^(m - 1)*((a + b*x + c*x^2)^(p + 1)/(c*(p + 1))),
 x] /; FreeQ[{a, b, c, d, e, m, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&
EqQ[m + p, 0]
```

rule 1128
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[e*(d + e*x)^(m - 1)*((a + b*x + c*x^2)^(p + 1)/(c*(m + 2*p +
 1))), x] + Simp[Simplify[m + p]*((2*c*d - b*e)/(c*(m + 2*p + 1)))   Int[(d
 + e*x)^(m - 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p},
 x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && IGtQ[Simplify[m + p], 0]
```

Maple [A] (verified)

Time = 1.18 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.55

method	result	size
default	$-\frac{2\sqrt{(ex+d)(cdx+ae)}\,(cdx+ae)^{3}\left(-7cdxe+2a\,e^{2}-9c\,d^{2}\right)}{63\sqrt{ex+d}\,c^{2}d^{2}}$	61
gosper	$-\frac{2(cdx+ae)\left(-7cdxe+2a\,e^{2}-9c\,d^{2}\right)\left(cd\,x^{2}e+a\,e^{2}x+c\,d^{2}x+ade\right)^{\frac{5}{2}}}{63c^{2}d^{2}(ex+d)^{\frac{5}{2}}}$	69
orering	$-\frac{2\left(-7cdxe+2a\,e^{2}-9c\,d^{2}\right)(cdx+ae)\left(ade+\left(a\,e^{2}+c\,d^{2}\right)x+cd\,x^{2}e\right)^{\frac{5}{2}}}{63c^{2}d^{2}(ex+d)^{\frac{5}{2}}}$	70

input
```
int((a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(5/2)/(e*x+d)^(3/2),x,method=_RETURN
VERBOSE)
```

output
```
-2/63*((e*x+d)*(c*d*x+a*e))^(1/2)/(e*x+d)^(1/2)*(c*d*x+a*e)^3*(-7*c*d*e*x+
2*a*e^2-9*c*d^2)/c^2/d^2
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 169, normalized size of antiderivative = 1.54

$$\int \frac{(ade + (cd^2 + ae^2)x + cdex^2)^{5/2}}{(d + ex)^{3/2}}\,dx = \frac{2\,(7\,c^4d^4ex^4 + 9\,a^3cd^2e^3 - 2\,a^4e^5 + (9\,c^4d^5 + 19\,ac^3d^3e^2)x^3 + 3\,(9$$

input
```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(e*x+d)^(3/2),x, algorit
hm="fricas")
```

output
```
2/63*(7*c^4*d^4*e*x^4 + 9*a^3*c*d^2*e^3 - 2*a^4*e^5 + (9*c^4*d^5 + 19*a*c^
3*d^3*e^2)*x^3 + 3*(9*a*c^3*d^4*e + 5*a^2*c^2*d^2*e^3)*x^2 + (27*a^2*c^2*d
^3*e^2 + a^3*c*d*e^4)*x)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(
e*x + d)/(c^2*d^2*e*x + c^2*d^3)
```

Sympy [F]

$$\int \frac{(ade + (cd^2 + ae^2)x + cdex^2)^{5/2}}{(d + ex)^{3/2}}\,dx = \int \frac{((d + ex)(ae + cdx))^{\frac{5}{2}}}{(d + ex)^{\frac{3}{2}}}\,dx$$

input
```
integrate((a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(5/2)/(e*x+d)**(3/2),x)
```

output
```
Integral(((d + e*x)*(a*e + c*d*x))**(5/2)/(d + e*x)**(3/2), x)
```

Maxima [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 132, normalized size of antiderivative = 1.20

$$\int \frac{(ade + (cd^2 + ae^2)x + cdex^2)^{5/2}}{(d + ex)^{3/2}}\,dx = \frac{2\,(7\,c^4d^4ex^4 + 9\,a^3cd^2e^3 - 2\,a^4e^5 + (9\,c^4d^5 + 19\,ac^3d^3e^2)x^3 + 3\,(9}{63\,c^2}$$

input
```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(e*x+d)^(3/2),x, algorit
hm="maxima")
```

output
```
2/63*(7*c^4*d^4*e*x^4 + 9*a^3*c*d^2*e^3 - 2*a^4*e^5 + (9*c^4*d^5 + 19*a*c^
3*d^3*e^2)*x^3 + 3*(9*a*c^3*d^4*e + 5*a^2*c^2*d^2*e^3)*x^2 + (27*a^2*c^2*d
^3*e^2 + a^3*c*d*e^4)*x)*sqrt(c*d*x + a*e)/(c^2*d^2)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 520 vs. $2(98) = 196$.

Time = 0.22 (sec) , antiderivative size = 520, normalized size of antiderivative = 4.73

$$\int \frac{(ade + (cd^2 + ae^2)x + cdex^2)^{5/2}}{(d + ex)^{3/2}}\,dx = \frac{2\left(\frac{105\,((ex+d)cde-cd^2e+ae^3)^{\frac{3}{2}}a^2|e|}{ce} - \frac{42\left(5\,((ex+d)cde-cd^2e+ae^3)^{\frac{3}{2}}ae^3-3\,((ex\right.}{ce^4}\right.}{}$$

input
```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(e*x+d)^(3/2),x, algorit
hm="giac")
```

output
```
2/315*(105*((e*x + d)*c*d*e - c*d^2*e + a*e^3)^(3/2)*a^2*abs(e)/(c*e) - 42
*(5*((e*x + d)*c*d*e - c*d^2*e + a*e^3)^(3/2)*a*e^3 - 3*((e*x + d)*c*d*e -
 c*d^2*e + a*e^3)^(5/2))*a*abs(e)/(c*e^4) - 21*(5*((e*x + d)*c*d*e - c*d^2
*e + a*e^3)^(3/2)*a*e^3 - 3*((e*x + d)*c*d*e - c*d^2*e + a*e^3)^(5/2))*a^2
*abs(e)/(c^2*d^2*e^2) + 3*(35*((e*x + d)*c*d*e - c*d^2*e + a*e^3)^(3/2)*a^
2*e^6 - 42*((e*x + d)*c*d*e - c*d^2*e + a*e^3)^(5/2)*a*e^3 + 15*((e*x + d)
*c*d*e - c*d^2*e + a*e^3)^(7/2))*abs(e)/(c*e^7) + 6*(35*((e*x + d)*c*d*e -
 c*d^2*e + a*e^3)^(3/2)*a^2*e^6 - 42*((e*x + d)*c*d*e - c*d^2*e + a*e^3)^(
5/2)*a*e^3 + 15*((e*x + d)*c*d*e - c*d^2*e + a*e^3)^(7/2))*a*abs(e)/(c^2*d
^2*e^5) - (105*((e*x + d)*c*d*e - c*d^2*e + a*e^3)^(3/2)*a^3*e^9 - 189*((e
*x + d)*c*d*e - c*d^2*e + a*e^3)^(5/2)*a^2*e^6 + 135*((e*x + d)*c*d*e - c*
d^2*e + a*e^3)^(7/2)*a*e^3 - 35*((e*x + d)*c*d*e - c*d^2*e + a*e^3)^(9/2))
*abs(e)/(c^2*d^2*e^8))/e
```

Mupad [B] (verification not implemented)

Time = 5.41 (sec) , antiderivative size = 156, normalized size of antiderivative = 1.42

$$\int \frac{(ade + (cd^2 + ae^2)x + cdex^2)^{5/2}}{(d + ex)^{3/2}} dx = \frac{\sqrt{c\,d\,e\,x^2 + (c\,d^2 + a\,e^2)\,x + a\,d\,e}\left(\frac{x^3\,(18\,c^4\,d^5 + 38\,a\,c^3\,d^3\,e^2)}{63\,c^2\,d^2} - \frac{4\,a^4\,e^5 - }{63}\right.}{\sqrt{d +}}$$

input
```
int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(5/2)/(d + e*x)^(3/2),x)
```

output
```
((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2)*((x^3*(18*c^4*d^5 + 38*a*c^
3*d^3*e^2))/(63*c^2*d^2) - (4*a^4*e^5 - 18*a^3*c*d^2*e^3)/(63*c^2*d^2) + (
2*c^2*d^2*e*x^4)/9 + (2*a*e*x^2*(5*a*e^2 + 9*c*d^2))/21 + (2*a^2*e^2*x*(a*
e^2 + 27*c*d^2))/(63*c*d)))/(d + e*x)^(1/2)
```

Reduce [B] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 131, normalized size of antiderivative = 1.19

$$\int \frac{(ade + (cd^2 + ae^2)x + cdex^2)^{5/2}}{(d + ex)^{3/2}} dx = \frac{2\sqrt{cdx + ae}\,(7c^4d^4e\,x^4 + 19a\,c^3d^3e^2x^3 + 9c^4d^5x^3 + 15a^2c^2d^2e^3x^2}{63c^2}$$

input
```
int((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(e*x+d)^(3/2),x)
```

output
```
(2*sqrt(a*e + c*d*x)*( - 2*a**4*e**5 + 9*a**3*c*d**2*e**3 + a**3*c*d*e**4*
x + 27*a**2*c**2*d**3*e**2*x + 15*a**2*c**2*d**2*e**3*x**2 + 27*a*c**3*d**
4*e*x**2 + 19*a*c**3*d**3*e**2*x**3 + 9*c**4*d**5*x**3 + 7*c**4*d**4*e*x**
4))/(63*c**2*d**2)
```

3.301 $\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{(d+ex)^{5/2}}\,dx$

Optimal result

Integrand size = 39, antiderivative size = 48

$$\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{(d+ex)^{5/2}}\,dx=\frac{2\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{7/2}}{7cd(d+ex)^{7/2}}$$

output
```
2/7*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(7/2)/c/d/(e*x+d)^(7/2)
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.77

$$\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{(d+ex)^{5/2}}\,dx=\frac{2((ae+cdx)(d+ex))^{7/2}}{7cd(d+ex)^{7/2}}$$

input
```
Integrate[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2)/(d + e*x)^(5/2),x]
```

output
```
(2*((a*e + c*d*x)*(d + e*x))^(7/2))/(7*c*d*(d + e*x)^(7/2))
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.026$, Rules used = {1122}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/2}}{(d+ex)^{5/2}}\,dx$$

$$\Big\downarrow \ 1122$$

$$\frac{2\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{7/2}}{7cd(d+ex)^{7/2}}$$

input
```
Int[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2)/(d + e*x)^(5/2),x]
```

output
```
(2*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(7/2))/(7*c*d*(d + e*x)^(7/2))
```

Defintions of rubi rules used

rule 1122
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[e*(d + e*x)^(m - 1)*((a + b*x + c*x^2)^(p + 1)/(c*(p + 1))),
 x] /; FreeQ[{a, b, c, d, e, m, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&
EqQ[m + p, 0]
```

Maple [A] (verified)

Time = 1.04 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.88

method	result	size
default	$\frac{2\sqrt{(ex+d)(cdx+ae)}\,(cdx+ae)^{3}}{7\sqrt{ex+d}\,cd}$	42
gosper	$\frac{2(cdx+ae)\left(cd\,x^{2}e+a\,e^{2}x+c\,d^{2}x+ade\right)^{\frac{5}{2}}}{7cd(ex+d)^{\frac{5}{2}}}$	50
orering	$\frac{2(cdx+ae)\left(ade+\left(a\,e^{2}+c\,d^{2}\right)x+cd\,x^{2}e\right)^{\frac{5}{2}}}{7cd(ex+d)^{\frac{5}{2}}}$	51

input
```
int((a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(5/2)/(e*x+d)^(5/2),x,method=_RETURN
VERBOSE)
```

output
```
2/7*((e*x+d)*(c*d*x+a*e))^(1/2)/(e*x+d)^(1/2)*(c*d*x+a*e)^3/c/d
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 91 vs. $2(42) = 84$.

Time = 0.09 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.90

$$\int \frac{(ade+(cd^2+ae^2)\,x+cdex^2)^{5/2}}{(d+ex)^{5/2}}\,dx = \frac{2\,(c^3d^3x^3+3\,ac^2d^2ex^2+3\,a^2cde^2x+a^3e^3)\sqrt{cdex^2+ade+(cd^2}}{7\,(cdex+cd^2)}$$

input
```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(e*x+d)^(5/2),x, algorit
hm="fricas")
```

output
```
2/7*(c^3*d^3*x^3 + 3*a*c^2*d^2*e*x^2 + 3*a^2*c*d*e^2*x + a^3*e^3)*sqrt(c*d
*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(e*x + d)/(c*d*e*x + c*d^2)
```

Sympy [F]

$$\int \frac{(ade + (cd^2 + ae^2)x + cdex^2)^{5/2}}{(d + ex)^{5/2}}\, dx = \int \frac{((d + ex)(ae + cdx))^{\frac{5}{2}}}{(d + ex)^{\frac{5}{2}}}\, dx$$

input
```
integrate((a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(5/2)/(e*x+d)**(5/2),x)
```

output
```
Integral(((d + e*x)*(a*e + c*d*x))**(5/2)/(d + e*x)**(5/2), x)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.25

$$\int \frac{(ade + (cd^2 + ae^2)x + cdex^2)^{5/2}}{(d + ex)^{5/2}}\, dx = \frac{2\,(c^3d^3x^3 + 3\,ac^2d^2ex^2 + 3\,a^2cde^2x + a^3e^3)\sqrt{cdx + ae}}{7\,cd}$$

input
```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(e*x+d)^(5/2),x, algorit
hm="maxima")
```

output
```
2/7*(c^3*d^3*x^3 + 3*a*c^2*d^2*e*x^2 + 3*a^2*c*d*e^2*x + a^3*e^3)*sqrt(c*d
*x + a*e)/(c*d)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 218 vs. 2(42) = 84.

Time = 0.19 (sec) , antiderivative size = 218, normalized size of antiderivative = 4.54

$$\int \frac{(ade + (cd^2 + ae^2)x + cdex^2)^{5/2}}{(d + ex)^{5/2}}\, dx = \frac{2\left(\frac{35\,((ex+d)cde-cd^2e+ae^3)^{\frac{3}{2}}\,a^2|e|}{cde} - \frac{14\left(5\,((ex+d)cde-cd^2e+ae^3)^{\frac{3}{2}}\,ae^3-3\,((ex+\cdots\right.}{cde^4}\cdots\right.}{\cdots}$$

input
```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(e*x+d)^(5/2),x, algorit
hm="giac")
```

output
```
2/105*(35*((e*x + d)*c*d*e - c*d^2*e + a*e^3)^(3/2)*a^2*abs(e)/(c*d*e) - 1
4*(5*((e*x + d)*c*d*e - c*d^2*e + a*e^3)^(3/2)*a*e^3 - 3*((e*x + d)*c*d*e
- c*d^2*e + a*e^3)^(5/2))*a*abs(e)/(c*d*e^4) + (35*((e*x + d)*c*d*e - c*d^
2*e + a*e^3)^(3/2)*a^2*e^6 - 42*((e*x + d)*c*d*e - c*d^2*e + a*e^3)^(5/2)*
a*e^3 + 15*((e*x + d)*c*d*e - c*d^2*e + a*e^3)^(7/2))*abs(e)/(c*d*e^7))/e
```

Mupad [B] (verification not implemented)

Time = 5.29 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.65

$$\int \frac{(ade + (cd^2 + ae^2)x + cdex^2)^{5/2}}{(d+ex)^{5/2}}\,dx = \frac{\sqrt{c\,d\,e\,x^2 + (c\,d^2 + a\,e^2)\,x + a\,d\,e}\left(\frac{6\,a^2\,e^2\,x}{7} + \frac{2\,c^2\,d^2\,x^3}{7} + \frac{2\,a^3\,e^3}{7\,c\,d} + 6\right.}{\sqrt{d+e\,x}}$$

input
```
int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(5/2)/(d + e*x)^(5/2),x)
```

output
```
((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2)*((6*a^2*e^2*x)/7 + (2*c^2*d
^2*x^3)/7 + (2*a^3*e^3)/(7*c*d) + (6*a*c*d*e*x^2)/7))/(d + e*x)^(1/2)
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.23

$$\int \frac{(ade + (cd^2 + ae^2)x + cdex^2)^{5/2}}{(d+ex)^{5/2}}\,dx = \frac{2\sqrt{cdx+ae}\,(c^3d^3x^3 + 3a\,c^2d^2e\,x^2 + 3a^2cd\,e^2x + a^3e^3)}{7cd}$$

input
```
int((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(e*x+d)^(5/2),x)
```

output
```
(2*sqrt(a*e + c*d*x)*(a**3*e**3 + 3*a**2*c*d*e**2*x + 3*a*c**2*d**2*e*x**2
 + c**3*d**3*x**3))/(7*c*d)
```

3.302 $\int \frac{\left(ade+(cd^2+ae^2)x+cdex^2\right)^{5/2}}{(d+ex)^{7/2}}\,dx$

Optimal result

Integrand size = 39, antiderivative size = 240

$$\int \frac{\left(ade+(cd^2+ae^2)\,x+cdex^2\right)^{5/2}}{(d+ex)^{7/2}}\,dx=\frac{2(cd^2-ae^2)^2\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{e^3\sqrt{d+ex}}+\frac{2\left(a-\frac{cd^2}{e^2}\right)\left(ade+(cd^2+ae^2)\,x+cdex^2\right)^{3/2}}{3(d+ex)^{3/2}}+\frac{2\left(ade+(cd^2+ae^2)\,x+cdex^2\right)^{5/2}}{5e(d+ex)^{5/2}}-\frac{2(cd^2-ae^2)^{5/2}\arctan\left(\frac{\sqrt{e}\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{\sqrt{cd^2-ae^2}\sqrt{d+ex}}\right)}{e^{7/2}}$$

output
```
2*(-a*e^2+c*d^2)^2*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/e^3/(e*x+d)^(1/
2)+2/3*(a-c*d^2/e^2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/(e*x+d)^(3/2)
+2/5*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/e/(e*x+d)^(5/2)-2*(-a*e^2+c*d
^2)^(5/2)*arctan(e^(1/2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(-a*e^2+c
*d^2)^(1/2)/(e*x+d)^(1/2))/e^(7/2)
```

Mathematica [A] (verified)

Time = 0.23 (sec) , antiderivative size = 170, normalized size of antiderivative = 0.71

$$\int \frac{(ade+(cd^2+ae^2)x+cdex^2)^{5/2}}{(d+ex)^{7/2}}\,dx = \frac{2\sqrt{ae+cdx}\sqrt{d+ex}\left(\sqrt{e}\sqrt{ae+cdx}(23a^2e^4+acde^2(-35d+11e$$
$$15e^{7/2}\sqrt{(}$$

input
```
Integrate[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2)/(d + e*x)^(7/2),x]
```

output
```
(2*Sqrt[a*e + c*d*x]*Sqrt[d + e*x]*(Sqrt[e]*Sqrt[a*e + c*d*x]*(23*a^2*e^4
+ a*c*d*e^2*(-35*d + 11*e*x) + c^2*d^2*(15*d^2 - 5*d*e*x + 3*e^2*x^2)) - 1
5*(c*d^2 - a*e^2)^(5/2)*ArcTan[(Sqrt[e]*Sqrt[a*e + c*d*x])/Sqrt[c*d^2 - a*
e^2]]))/(15*e^(7/2)*Sqrt[(a*e + c*d*x)*(d + e*x)])
```

Rubi [A] (verified)

Time = 0.70 (sec) , antiderivative size = 254, normalized size of antiderivative = 1.06, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.128$, Rules used = {1131, 1131, 1131, 1136, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/2}}{(d+ex)^{7/2}}dx$$

$$\Big\downarrow \text{1131}$$

$$\frac{2\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/2}}{5e(d+ex)^{5/2}}-\frac{\left(cd^2-ae^2\right)\int \frac{\left(cdex^2+\left(cd^2+ae^2\right)x+ade\right)^{3/2}}{(d+ex)^{5/2}}dx}{e}$$

$$\Big\downarrow \text{1131}$$

$$\frac{2\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/2}}{5e(d+ex)^{5/2}}-\frac{\left(cd^2-ae^2\right)\left(\frac{2\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{3e(d+ex)^{3/2}}-\frac{\left(cd^2-ae^2\right)\int\frac{\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}{(d+ex)^{3/2}}dx}{e}\right)}{e}$$

$\big\downarrow$ 1131

$$\frac{2\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/2}}{5e(d+ex)^{5/2}}-\frac{\left(cd^2-ae^2\right)\left(\frac{2\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{3e(d+ex)^{3/2}}-\frac{\left(cd^2-ae^2\right)\left(\frac{2\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{e\sqrt{d+ex}}-\frac{\left(cd^2-ae^2\right)\int\frac{1}{\sqrt{d+ex}\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}dx}{e}\right)}{e}\cdots\right)}{e}$$

$\big\downarrow$ 1136

$$\frac{2\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/2}}{5e(d+ex)^{5/2}}-\frac{\left(cd^2-ae^2\right)\left(\frac{2\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{3e(d+ex)^{3/2}}-\frac{\left(cd^2-ae^2\right)\left(\frac{2\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{e\sqrt{d+ex}}-2\left(cd^2-ae^2\right)\int\frac{1}{\frac{\left(cdex^2+\left(cd^2+ae^2\right)x+ade\right)e^2}{d+ex}+\left(cd^2\cdots\right.}\cdots\right)}{e}\cdots\right)}{e}$$

$\big\downarrow$ 218

$$\frac{2\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/2}}{5e(d+ex)^{5/2}}-\frac{\left(cd^2-ae^2\right)\left(\frac{2\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{3e(d+ex)^{3/2}}-\frac{\left(cd^2-ae^2\right)\left(\frac{2\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{e\sqrt{d+ex}}-\frac{2\sqrt{cd^2-ae^2}\arctan\left(\frac{\sqrt{e}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{\sqrt{d+ex}\sqrt{cd^2-ae^2}}\right)}{e^{3/2}}\cdots\right)}{e}\cdots\right)}{e}$$

input
```
Int[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2)/(d + e*x)^(7/2),x]
```

output
```
(2*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2))/(5*e*(d + e*x)^(5/2)) -
((c*d^2 - a*e^2)*((2*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2))/(3*e*(
d + e*x)^(3/2)) - ((c*d^2 - a*e^2)*((2*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*
d*e*x^2])/(e*Sqrt[d + e*x]) - (2*Sqrt[c*d^2 - a*e^2]*ArcTan[(Sqrt[e]*Sqrt[
a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(Sqrt[c*d^2 - a*e^2]*Sqrt[d + e*x]
)])/e^(3/2)))/e))/e
```

Defintions of rubi rules used

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 1131
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(d + e*x)^(m + 1)*((a + b*x + c*x^2)^p/(e*(m + 2*p + 1))), x
] - Simp[p*((2*c*d - b*e)/(e^2*(m + 2*p + 1)))   Int[(d + e*x)^(m + 1)*(a +
 b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c*d^2 - b
*d*e + a*e^2, 0] && GtQ[p, 0] && (LeQ[-2, m, 0] || EqQ[m + p + 1, 0]) && Ne
Q[m + 2*p + 1, 0] && IntegerQ[2*p]
```

rule 1136
```
Int[1/(Sqrt[(d_.) + (e_.)*(x_)]*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x
_Symbol] :> Simp[2*e   Subst[Int[1/(2*c*d - b*e + e^2*x^2), x], x, Sqrt[a +
 b*x + c*x^2]/Sqrt[d + e*x]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c*d^2
- b*d*e + a*e^2, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 426 vs. $2(212) = 424$.

Time = 1.05 (sec) , antiderivative size = 427, normalized size of antiderivative = 1.78

method	result
default	$-\frac{2\sqrt{(ex+d)(cdx+ae)}\left(15\operatorname{arctanh}\left(\frac{e\sqrt{cdx+ae}}{\sqrt{(a\,e^2-c\,d^2)e}}\right)a^3e^6-45\operatorname{arctanh}\left(\frac{e\sqrt{cdx+ae}}{\sqrt{(a\,e^2-c\,d^2)e}}\right)a^2c\,d^2e^4+45\operatorname{arctanh}\left(\frac{e\sqrt{cdx+ae}}{\sqrt{(a\,e^2-c\,d^2)e}}\right)\right.}{}$

input
```
int((a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(5/2)/(e*x+d)^(7/2),x,method=_RETURN
VERBOSE)
```

output
```
-2/15*((e*x+d)*(c*d*x+a*e))^(1/2)*(15*arctanh(e*(c*d*x+a*e)^(1/2)/((a*e^2-
c*d^2)*e)^(1/2))*a^3*e^6-45*arctanh(e*(c*d*x+a*e)^(1/2)/((a*e^2-c*d^2)*e)^
(1/2))*a^2*c*d^2*e^4+45*arctanh(e*(c*d*x+a*e)^(1/2)/((a*e^2-c*d^2)*e)^(1/2
))*a*c^2*d^4*e^2-15*arctanh(e*(c*d*x+a*e)^(1/2)/((a*e^2-c*d^2)*e)^(1/2))*c
^3*d^6-3*c^2*d^2*e^2*x^2*(c*d*x+a*e)^(1/2)*((a*e^2-c*d^2)*e)^(1/2)-11*a*c*
d*e^3*x*(c*d*x+a*e)^(1/2)*((a*e^2-c*d^2)*e)^(1/2)+5*c^2*d^3*e*x*(c*d*x+a*e
)^(1/2)*((a*e^2-c*d^2)*e)^(1/2)-23*((a*e^2-c*d^2)*e)^(1/2)*(c*d*x+a*e)^(1/
2)*a^2*e^4+35*((a*e^2-c*d^2)*e)^(1/2)*(c*d*x+a*e)^(1/2)*a*c*d^2*e^2-15*((a
*e^2-c*d^2)*e)^(1/2)*(c*d*x+a*e)^(1/2)*c^2*d^4)/(e*x+d)^(1/2)/(c*d*x+a*e)^
(1/2)/e^3/((a*e^2-c*d^2)*e)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 570, normalized size of antiderivative = 2.38

$$\int \frac{(ade + (cd^2 + ae^2)\,x + cdex^2)^{5/2}}{(d + ex)^{7/2}}\,dx = \left[\frac{15\,(c^2d^5 - 2\,acd^3e^2 + a^2de^4 + (c^2d^4e - 2\,acd^2e^3 + a^2e^5)x)\sqrt{-\cdots}}{}\right.$$

input
```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(e*x+d)^(7/2),x, algorit
hm="fricas")
```

output
```
[1/15*(15*(c^2*d^5 - 2*a*c*d^3*e^2 + a^2*d*e^4 + (c^2*d^4*e - 2*a*c*d^2*e^
3 + a^2*e^5)*x)*sqrt(-(c*d^2 - a*e^2)/e)*log(-(c*d*e^2*x^2 + 2*a*e^3*x - c
*d^3 + 2*a*d*e^2 - 2*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(e*x
+ d)*e*sqrt(-(c*d^2 - a*e^2)/e))/(e^2*x^2 + 2*d*e*x + d^2)) + 2*(3*c^2*d^2
*e^2*x^2 + 15*c^2*d^4 - 35*a*c*d^2*e^2 + 23*a^2*e^4 - (5*c^2*d^3*e - 11*a*
c*d*e^3)*x)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(e*x + d))/(e^
4*x + d*e^3), 2/15*(15*(c^2*d^5 - 2*a*c*d^3*e^2 + a^2*d*e^4 + (c^2*d^4*e -
 2*a*c*d^2*e^3 + a^2*e^5)*x)*sqrt((c*d^2 - a*e^2)/e)*arctan(-sqrt(c*d*e*x^
2 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(e*x + d)*e*sqrt((c*d^2 - a*e^2)/e)/(c*
d^3 - a*d*e^2 + (c*d^2*e - a*e^3)*x)) + (3*c^2*d^2*e^2*x^2 + 15*c^2*d^4 -
35*a*c*d^2*e^2 + 23*a^2*e^4 - (5*c^2*d^3*e - 11*a*c*d*e^3)*x)*sqrt(c*d*e*x
^2 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(e*x + d))/(e^4*x + d*e^3)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(ade + (cd^2 + ae^2)\, x + cdex^2)^{5/2}}{(d + ex)^{7/2}}\, dx = \text{Timed out}$$

input
```
integrate((a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(5/2)/(e*x+d)**(7/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{(ade + (cd^2 + ae^2)\, x + cdex^2)^{5/2}}{(d + ex)^{7/2}}\, dx = \int \frac{(cdex^2 + ade + (cd^2 + ae^2)x)^{\frac{5}{2}}}{(ex + d)^{\frac{7}{2}}}\, dx$$

input
```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(e*x+d)^(7/2),x, algorit
hm="maxima")
```

output
```
integrate((c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^(5/2)/(e*x + d)^(7/2), x
)
```

Giac [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 323, normalized size of antiderivative = 1.35

$$\int \frac{(ade + (cd^2 + ae^2)x + cdex^2)^{5/2}}{(d + ex)^{7/2}} dx =$$

$$-\frac{2\left(\frac{15\,(c^3d^6|e|-3\,ac^2d^4e^2|e|+3\,a^2cd^2e^4|e|-a^3e^6|e|)\arctan\left(\frac{\sqrt{(ex+d)cde-cd^2e+ae^3}}{\sqrt{cd^2e-ae^3}}\right)}{\sqrt{cd^2e-ae^3}} - \frac{15\,\sqrt{(ex+d)cde-cd^2e+ae^3}c^2d^4e^{14}|e|-30\,\sqrt{(ex+d)}\cdots}{\cdots}\right)}{\cdots}$$

input
```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(e*x+d)^(7/2),x, algorit
hm="giac")
```

output
```
-2/15*(15*(c^3*d^6*abs(e) - 3*a*c^2*d^4*e^2*abs(e) + 3*a^2*c*d^2*e^4*abs(e
) - a^3*e^6*abs(e))*arctan(sqrt((e*x + d)*c*d*e - c*d^2*e + a*e^3)/sqrt(c*
d^2*e - a*e^3))/sqrt(c*d^2*e - a*e^3) - (15*sqrt((e*x + d)*c*d*e - c*d^2*e
 + a*e^3)*c^2*d^4*e^14*abs(e) - 30*sqrt((e*x + d)*c*d*e - c*d^2*e + a*e^3)
*a*c*d^2*e^16*abs(e) + 15*sqrt((e*x + d)*c*d*e - c*d^2*e + a*e^3)*a^2*e^18
*abs(e) - 5*((e*x + d)*c*d*e - c*d^2*e + a*e^3)^(3/2)*c*d^2*e^13*abs(e) +
5*((e*x + d)*c*d*e - c*d^2*e + a*e^3)^(3/2)*a*e^15*abs(e) + 3*((e*x + d)*c
*d*e - c*d^2*e + a*e^3)^(5/2)*e^12*abs(e))/e^15)/e^4
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(ade + (cd^2 + ae^2)x + cdex^2)^{5/2}}{(d + ex)^{7/2}} dx = \int \frac{(c\,d\,e\,x^2 + (c\,d^2 + a\,e^2)\,x + a\,d\,e)^{5/2}}{(d + e\,x)^{7/2}} dx$$

input
```
int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(5/2)/(d + e*x)^(7/2),x)
```

output
```
int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(5/2)/(d + e*x)^(7/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 286, normalized size of antiderivative = 1.19

$$\int \frac{(ade + (cd^2 + ae^2)x + cdex^2)^{5/2}}{(d + ex)^{7/2}} dx = \frac{-2\sqrt{e}\sqrt{-a e^2 + c d^2}\, atan\left(\frac{\sqrt{cdx+ae}\, e}{\sqrt{e}\sqrt{-a e^2+c d^2}}\right) a^2e^4 + 4\sqrt{e}\sqrt{-a e^2 + c}$$

input
```
int((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(e*x+d)^(7/2),x)
```

output
```
(2*( - 15*sqrt(e)*sqrt( - a*e**2 + c*d**2)*atan((sqrt(a*e + c*d*x)*e)/(sqr
t(e)*sqrt( - a*e**2 + c*d**2)))*a**2*e**4 + 30*sqrt(e)*sqrt( - a*e**2 + c*
d**2)*atan((sqrt(a*e + c*d*x)*e)/(sqrt(e)*sqrt( - a*e**2 + c*d**2)))*a*c*d
**2*e**2 - 15*sqrt(e)*sqrt( - a*e**2 + c*d**2)*atan((sqrt(a*e + c*d*x)*e)/
(sqrt(e)*sqrt( - a*e**2 + c*d**2)))*c**2*d**4 + 23*sqrt(a*e + c*d*x)*a**2*
e**5 - 35*sqrt(a*e + c*d*x)*a*c*d**2*e**3 + 11*sqrt(a*e + c*d*x)*a*c*d*e**
4*x + 15*sqrt(a*e + c*d*x)*c**2*d**4*e - 5*sqrt(a*e + c*d*x)*c**2*d**3*e**
2*x + 3*sqrt(a*e + c*d*x)*c**2*d**2*e**3*x**2))/(15*e**4)
```

3.303 $\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{(d+ex)^{9/2}}\,dx$

Optimal result

Integrand size = 39, antiderivative size = 233

$$\begin{aligned}\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{(d+ex)^{9/2}}\,dx &= \frac{5cd\left(a-\frac{cd^2}{e^2}\right)\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{e\sqrt{d+ex}} \\ &+ \frac{5cd\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{3e^2(d+ex)^{3/2}} - \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{e(d+ex)^{7/2}} \\ &+ \frac{5cd\left(cd^2-ae^2\right)^{3/2}\arctan\left(\frac{\sqrt{e}\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{\sqrt{cd^2-ae^2}\sqrt{d+ex}}\right)}{e^{7/2}}\end{aligned}$$

output
```
5*c*d*(a-c*d^2/e^2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/e/(e*x+d)^(1/2
)+5/3*c*d*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/e^2/(e*x+d)^(3/2)-(a*d*e
+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/e/(e*x+d)^(7/2)+5*c*d*(-a*e^2+c*d^2)^(3/
2)*arctan(e^(1/2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(-a*e^2+c*d^2)^(
1/2)/(e*x+d)^(1/2))/e^(7/2)
```

Mathematica [A] (verified)

Time = 0.39 (sec) , antiderivative size = 178, normalized size of antiderivative = 0.76

$$\int \frac{(ade+(cd^2+ae^2)\,x+cdex^2)^{5/2}}{(d+ex)^{9/2}}\,dx = \frac{\sqrt{(ae+cdx)(d+ex)}\left(\sqrt{e}\sqrt{ae+cdx}(-3a^2e^4+2acde^2(10d+7e\right.}{3e}$$

input
```
Integrate[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2)/(d + e*x)^(9/2),x]
```

output
```
(Sqrt[(a*e + c*d*x)*(d + e*x)]*(Sqrt[e]*Sqrt[a*e + c*d*x]*(-3*a^2*e^4 + 2*
a*c*d*e^2*(10*d + 7*e*x) + c^2*d^2*(-15*d^2 - 10*d*e*x + 2*e^2*x^2)) + 15*
c*d*(c*d^2 - a*e^2)^(3/2)*(d + e*x)*ArcTan[(Sqrt[e]*Sqrt[a*e + c*d*x])/Sqr
t[c*d^2 - a*e^2]]))/(3*e^(7/2)*Sqrt[a*e + c*d*x]*(d + e*x)^(3/2))
```

Rubi [A] (verified)

Time = 0.68 (sec) , antiderivative size = 244, normalized size of antiderivative = 1.05, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.128$, Rules used = {1130, 1131, 1131, 1136, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/2}}{(d+ex)^{9/2}}dx$$

$$\Big\downarrow \textbf{1130}$$

$$\frac{5cd\int \frac{\left(cdex^2+\left(cd^2+ae^2\right)x+ade\right)^{3/2}}{(d+ex)^{5/2}}dx}{2e}-\frac{\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/2}}{e(d+ex)^{7/2}}$$

$$\Big\downarrow \textbf{1131}$$

$$\frac{5cd\left(\frac{2\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{3e(d+ex)^{3/2}}-\frac{\left(cd^2-ae^2\right)\int \frac{\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}{(d+ex)^{3/2}}dx}{e}\right)}{2e}-\frac{\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/2}}{e(d+ex)^{7/2}}$$

$$\Big\downarrow \; 1131$$

$$\frac{5cd\left(\frac{2\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{3e(d+ex)^{3/2}}-\frac{\left(cd^2-ae^2\right)\left(\frac{2\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{e\sqrt{d+ex}}-\frac{\left(cd^2-ae^2\right)\int \frac{1}{\sqrt{d+ex}\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}dx}{e}\right)}{e}\right)}{2e}-\frac{\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/2}}{e(d+ex)^{7/2}}$$

$$\Big\downarrow \; 1136$$

$$\frac{5cd\left(\frac{2\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{3e(d+ex)^{3/2}}-\frac{\left(cd^2-ae^2\right)\left(\frac{2\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{e\sqrt{d+ex}}-2\left(cd^2-ae^2\right)\int \frac{1}{\frac{\left(cdex^2+\left(cd^2+ae^2\right)x+ade\right)e^2}{d+ex}+\left(cd^2-ae^2\right)e}d\sqrt{\cdots}\right.}{e}}{2e}$$

$$-\frac{\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/2}}{e(d+ex)^{7/2}}$$

$$\Big\downarrow \; 218$$

$$\frac{5cd\left(\frac{2\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{3e(d+ex)^{3/2}}-\frac{\left(cd^2-ae^2\right)\left(\frac{2\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{e\sqrt{d+ex}}-\frac{2\sqrt{cd^2-ae^2}\arctan\left(\frac{\sqrt{e}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{\sqrt{d+ex}\sqrt{cd^2-ae^2}}\right)}{e^{3/2}}\right)}{e}\right)}{2e}-\frac{\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/2}}{e(d+ex)^{7/2}}$$

input
```
Int[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2)/(d + e*x)^(9/2),x]
```

output
```
-((a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2)/(e*(d + e*x)^(7/2))) + (5*
c*d*((2*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2))/(3*e*(d + e*x)^(3/2
)) - ((c*d^2 - a*e^2)*((2*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(e*
Sqrt[d + e*x]) - (2*Sqrt[c*d^2 - a*e^2]*ArcTan[(Sqrt[e]*Sqrt[a*d*e + (c*d^
2 + a*e^2)*x + c*d*e*x^2])/(Sqrt[c*d^2 - a*e^2]*Sqrt[d + e*x])])/e^(3/2)))
/e))/(2*e)
```

Defintions of rubi rules used

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 1130
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(d + e*x)^(m + 1)*((a + b*x + c*x^2)^p/(e*(m + p + 1))), x]
- Simp[c*(p/(e^2*(m + p + 1)))   Int[(d + e*x)^(m + 2)*(a + b*x + c*x^2)^(p
 - 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0]
 && GtQ[p, 0] && (LtQ[m, -2] || EqQ[m + 2*p + 1, 0]) && NeQ[m + p + 1, 0] &
& IntegerQ[2*p]
```

rule 1131
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(d + e*x)^(m + 1)*((a + b*x + c*x^2)^p/(e*(m + 2*p + 1))), x
] - Simp[p*((2*c*d - b*e)/(e^2*(m + 2*p + 1)))   Int[(d + e*x)^(m + 1)*(a +
 b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c*d^2 - b
*d*e + a*e^2, 0] && GtQ[p, 0] && (LeQ[-2, m, 0] || EqQ[m + p + 1, 0]) && Ne
Q[m + 2*p + 1, 0] && IntegerQ[2*p]
```

rule 1136
```
Int[1/(Sqrt[(d_.) + (e_.)*(x_)]*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x
_Symbol] :> Simp[2*e   Subst[Int[1/(2*c*d - b*e + e^2*x^2), x], x, Sqrt[a +
 b*x + c*x^2]/Sqrt[d + e*x]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c*d^2
- b*d*e + a*e^2, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 510 vs. 2(207) = 414.

Time = 1.07 (sec) , antiderivative size = 511, normalized size of antiderivative = 2.19

method	result
default	$-\frac{\sqrt{(ex+d)(cdx+ae)}\left(15\,\mathrm{arctanh}\left(\frac{e\sqrt{cdx+ae}}{\sqrt{(a\,e^2-c\,d^2)e}}\right)a^2cd\,e^5x-30\,\mathrm{arctanh}\left(\frac{e\sqrt{cdx+ae}}{\sqrt{(a\,e^2-c\,d^2)e}}\right)a\,c^2d^3e^3x+15\,\mathrm{arctanh}\left(\frac{e\sqrt{cdx+ae}}{\sqrt{(a\,e^2-c\,d^2)e}}\right.\right.}{}$

input
```
int((a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(5/2)/(e*x+d)^(9/2),x,method=_RETURN
VERBOSE)
```

output
```
-1/3*((e*x+d)*(c*d*x+a*e))^(1/2)*(15*arctanh(e*(c*d*x+a*e)^(1/2)/((a*e^2-c
*d^2)*e)^(1/2))*a^2*c*d*e^5*x-30*arctanh(e*(c*d*x+a*e)^(1/2)/((a*e^2-c*d^2
)*e)^(1/2))*a*c^2*d^3*e^3*x+15*arctanh(e*(c*d*x+a*e)^(1/2)/((a*e^2-c*d^2)*
e)^(1/2))*c^3*d^5*e*x+15*arctanh(e*(c*d*x+a*e)^(1/2)/((a*e^2-c*d^2)*e)^(1/
2))*a^2*c*d^2*e^4-30*arctanh(e*(c*d*x+a*e)^(1/2)/((a*e^2-c*d^2)*e)^(1/2))*
a*c^2*d^4*e^2+15*arctanh(e*(c*d*x+a*e)^(1/2)/((a*e^2-c*d^2)*e)^(1/2))*c^3*
d^6-2*c^2*d^2*e^2*x^2*(c*d*x+a*e)^(1/2)*((a*e^2-c*d^2)*e)^(1/2)-14*a*c*d*e
^3*x*(c*d*x+a*e)^(1/2)*((a*e^2-c*d^2)*e)^(1/2)+10*c^2*d^3*e*x*(c*d*x+a*e)^
(1/2)*((a*e^2-c*d^2)*e)^(1/2)+3*((a*e^2-c*d^2)*e)^(1/2)*(c*d*x+a*e)^(1/2)*
a^2*e^4-20*((a*e^2-c*d^2)*e)^(1/2)*(c*d*x+a*e)^(1/2)*a*c*d^2*e^2+15*((a*e^
2-c*d^2)*e)^(1/2)*(c*d*x+a*e)^(1/2)*c^2*d^4)/(e*x+d)^(3/2)/(c*d*x+a*e)^(1/
2)/e^3/((a*e^2-c*d^2)*e)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 611, normalized size of antiderivative = 2.62

$$\int \frac{(ade+(cd^2+ae^2)x+cdex^2)^{5/2}}{(d+ex)^{9/2}}\,dx = \left[-\frac{15\,(c^2d^5-acd^3e^2+(c^2d^3e^2-acde^4)x^2+2\,(c^2d^4e-acd^2e^3)x}{}\right.$$

input
```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(e*x+d)^(9/2),x, algorit
hm="fricas")
```

output
```
[-1/6*(15*(c^2*d^5 - a*c*d^3*e^2 + (c^2*d^3*e^2 - a*c*d*e^4)*x^2 + 2*(c^2*
d^4*e - a*c*d^2*e^3)*x)*sqrt(-(c*d^2 - a*e^2)/e)*log(-(c*d*e^2*x^2 + 2*a*e
^3*x - c*d^3 + 2*a*d*e^2 - 2*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*s
qrt(e*x + d)*e*sqrt(-(c*d^2 - a*e^2)/e))/(e^2*x^2 + 2*d*e*x + d^2)) - 2*(2
*c^2*d^2*e^2*x^2 - 15*c^2*d^4 + 20*a*c*d^2*e^2 - 3*a^2*e^4 - 2*(5*c^2*d^3*
e - 7*a*c*d*e^3)*x)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(e*x +
 d))/(e^5*x^2 + 2*d*e^4*x + d^2*e^3), -1/3*(15*(c^2*d^5 - a*c*d^3*e^2 + (c
^2*d^3*e^2 - a*c*d*e^4)*x^2 + 2*(c^2*d^4*e - a*c*d^2*e^3)*x)*sqrt((c*d^2 -
 a*e^2)/e)*arctan(-sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(e*x +
d)*e*sqrt((c*d^2 - a*e^2)/e)/(c*d^3 - a*d*e^2 + (c*d^2*e - a*e^3)*x)) - (2
*c^2*d^2*e^2*x^2 - 15*c^2*d^4 + 20*a*c*d^2*e^2 - 3*a^2*e^4 - 2*(5*c^2*d^3*
e - 7*a*c*d*e^3)*x)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(e*x +
 d))/(e^5*x^2 + 2*d*e^4*x + d^2*e^3)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(ade + (cd^2 + ae^2)\,x + cdex^2)^{5/2}}{(d + ex)^{9/2}}\,dx = \text{Timed out}$$

input
```
integrate((a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(5/2)/(e*x+d)**(9/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{(ade + (cd^2 + ae^2)\,x + cdex^2)^{5/2}}{(d + ex)^{9/2}}\,dx = \int \frac{(cdex^2 + ade + (cd^2 + ae^2)x)^{\frac{5}{2}}}{(ex + d)^{\frac{9}{2}}}\,dx$$

input
```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(e*x+d)^(9/2),x, algorit
hm="maxima")
```

output
```
integrate((c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^(5/2)/(e*x + d)^(9/2), x
)
```

Giac [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 325, normalized size of antiderivative = 1.39

$$\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{(d+ex)^{9/2}}\,dx = \frac{cd\left(\frac{15\,(c^2d^4|e|-2\,acd^2e^2|e|+a^2e^4|e|)\arctan\left(\frac{\sqrt{(ex+d)cde-cd^2e+ae^3}}{\sqrt{cd^2e-ae^3}}\right)}{\sqrt{cd^2e-ae^3}\,e} - \frac{3\left(\sqrt{(ex+d\cdots\right.}{\cdots}\right.}{\cdots}$$

input
```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(e*x+d)^(9/2),x, algorit
hm="giac")
```

output
```
1/3*c*d*(15*(c^2*d^4*abs(e) - 2*a*c*d^2*e^2*abs(e) + a^2*e^4*abs(e))*arcta
n(sqrt((e*x + d)*c*d*e - c*d^2*e + a*e^3)/sqrt(c*d^2*e - a*e^3))/(sqrt(c*d
^2*e - a*e^3)*e) - 3*(sqrt((e*x + d)*c*d*e - c*d^2*e + a*e^3)*c^2*d^4*abs(
e) - 2*sqrt((e*x + d)*c*d*e - c*d^2*e + a*e^3)*a*c*d^2*e^2*abs(e) + sqrt((
e*x + d)*c*d*e - c*d^2*e + a*e^3)*a^2*e^4*abs(e))/((e*x + d)*c*d*e^2) - 2*
(6*sqrt((e*x + d)*c*d*e - c*d^2*e + a*e^3)*c*d^2*e^7*abs(e) - 6*sqrt((e*x
+ d)*c*d*e - c*d^2*e + a*e^3)*a*e^9*abs(e) - ((e*x + d)*c*d*e - c*d^2*e +
a*e^3)^(3/2)*e^6*abs(e))/e^9)/e^3
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{(d+ex)^{9/2}}\,dx = \int \frac{{\left(c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e\right)}^{5/2}}{{\left(d+e\,x\right)}^{9/2}} \,d x$$

input
```
int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(5/2)/(d + e*x)^(9/2),x)
```

output
```
int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(5/2)/(d + e*x)^(9/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 350, normalized size of antiderivative = 1.50

$$\int \frac{(ade + (cd^2 + ae^2)x + cdex^2)^{5/2}}{(d+ex)^{9/2}}\,dx = \frac{-15\sqrt{e}\,\sqrt{-a\,e^2+c\,d^2}\,atan\left(\frac{\sqrt{cdx+ae}\,e}{\sqrt{e}\,\sqrt{-a\,e^2+c\,d^2}}\right)ac\,d^2e^2 - 15\sqrt{e}\,\sqrt{-a\,e}}{}$$

input
```
int((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(e*x+d)^(9/2),x)
```

output
```
( - 15*sqrt(e)*sqrt( - a*e**2 + c*d**2)*atan((sqrt(a*e + c*d*x)*e)/(sqrt(e
)*sqrt( - a*e**2 + c*d**2)))*a*c*d**2*e**2 - 15*sqrt(e)*sqrt( - a*e**2 + c
*d**2)*atan((sqrt(a*e + c*d*x)*e)/(sqrt(e)*sqrt( - a*e**2 + c*d**2)))*a*c*
d*e**3*x + 15*sqrt(e)*sqrt( - a*e**2 + c*d**2)*atan((sqrt(a*e + c*d*x)*e)/
(sqrt(e)*sqrt( - a*e**2 + c*d**2)))*c**2*d**4 + 15*sqrt(e)*sqrt( - a*e**2
+ c*d**2)*atan((sqrt(a*e + c*d*x)*e)/(sqrt(e)*sqrt( - a*e**2 + c*d**2)))*c
**2*d**3*e*x - 3*sqrt(a*e + c*d*x)*a**2*e**5 + 20*sqrt(a*e + c*d*x)*a*c*d*
*2*e**3 + 14*sqrt(a*e + c*d*x)*a*c*d*e**4*x - 15*sqrt(a*e + c*d*x)*c**2*d*
*4*e - 10*sqrt(a*e + c*d*x)*c**2*d**3*e**2*x + 2*sqrt(a*e + c*d*x)*c**2*d*
*2*e**3*x**2)/(3*e**4*(d + e*x))
```

3.304 $\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{(d+ex)^{11/2}}\,dx$

Optimal result

Integrand size = 39, antiderivative size = 236

$$\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{(d+ex)^{11/2}}\,dx=\frac{15c^2d^2\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{4e^3\sqrt{d+ex}}-\frac{5cd\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{4e^2(d+ex)^{5/2}}-\frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{2e(d+ex)^{9/2}}-\frac{15c^2d^2\sqrt{cd^2-ae^2}\arctan\left(\frac{\sqrt{e}\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{\sqrt{cd^2-ae^2}\sqrt{d+ex}}\right)}{4e^{7/2}}$$

output
```
15/4*c^2*d^2*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/e^3/(e*x+d)^(1/2)-5/4
*c*d*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/e^2/(e*x+d)^(5/2)-1/2*(a*d*e+
(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/e/(e*x+d)^(9/2)-15/4*c^2*d^2*(-a*e^2+c*d^
2)^(1/2)*arctan(e^(1/2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(-a*e^2+c*
d^2)^(1/2)/(e*x+d)^(1/2))/e^(7/2)
```

Mathematica [A] (verified)

Time = 0.47 (sec) , antiderivative size = 184, normalized size of antiderivative = 0.78

$$\int \frac{(ade+(cd^2+ae^2)x+cdex^2)^{5/2}}{(d+ex)^{11/2}}\,dx = \frac{\sqrt{(ae+cdx)(d+ex)}\left(\sqrt{e}\sqrt{ae+cdx}(-2a^2e^4-acde^2(5d+9ex)\right.}{4e^{7/}}$$

input
```
Integrate[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2)/(d + e*x)^(11/2),x
]
```

output
```
(Sqrt[(a*e + c*d*x)*(d + e*x)]*(Sqrt[e]*Sqrt[a*e + c*d*x]*(-2*a^2*e^4 - a*
c*d*e^2*(5*d + 9*e*x) + c^2*d^2*(15*d^2 + 25*d*e*x + 8*e^2*x^2)) - 15*c^2*
d^2*Sqrt[c*d^2 - a*e^2]*(d + e*x)^2*ArcTan[(Sqrt[e]*Sqrt[a*e + c*d*x])/Sqr
t[c*d^2 - a*e^2]]))/(4*e^(7/2)*Sqrt[a*e + c*d*x]*(d + e*x)^(5/2))
```

Rubi [A] (verified)

Time = 0.63 (sec) , antiderivative size = 236, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.128$, Rules used = {1130, 1130, 1131, 1136, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/2}}{(d+ex)^{11/2}}dx$$

$$\Big\downarrow \text{1130}$$

$$\frac{5cd\int \frac{(cdex^2+(cd^2+ae^2)x+ade)^{3/2}}{(d+ex)^{7/2}}dx}{4e}-\frac{\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/2}}{2e(d+ex)^{9/2}}$$

$$\Big\downarrow \text{1130}$$

$$\frac{5cd\left(\frac{3cd\int \frac{\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}{(d+ex)^{3/2}}dx}{2e}-\frac{\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{e(d+ex)^{5/2}}\right)}{4e}-\frac{\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/2}}{2e(d+ex)^{9/2}}$$

$\Big\downarrow$ 1131

$$\frac{5cd\left(\frac{3cd\left(\frac{2\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{e\sqrt{d+ex}}-\frac{\left(cd^2-ae^2\right)\int \frac{1}{\sqrt{d+ex}\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}dx}{e}\right)}{2e}-\frac{\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{e(d+ex)^{5/2}}\right)}{4e}-\frac{\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/2}}{2e(d+ex)^{9/2}}$$

$\Big\downarrow$ 1136

$$\frac{5cd\left(\frac{3cd\left(\frac{2\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{e\sqrt{d+ex}}-2\left(cd^2-ae^2\right)\int \frac{1}{\frac{\left(cdex^2+\left(cd^2+ae^2\right)x+ade\right)e^2}{d+ex}+\left(cd^2-ae^2\right)e}d\frac{\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}{\sqrt{d+ex}}\right)}{2e}-\frac{\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{e(d+ex)^{5/2}}\right)}{4e}-\frac{\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/2}}{2e(d+ex)^{9/2}}$$

$\Big\downarrow$ 218

$$\frac{5cd\left(\frac{3cd\left(\frac{2\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{e\sqrt{d+ex}}-\frac{2\sqrt{cd^2-ae^2}\arctan\left(\frac{\sqrt{e}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{\sqrt{d+ex}\sqrt{cd^2-ae^2}}\right)}{e^{3/2}}\right)}{2e}-\frac{\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{e(d+ex)^{5/2}}\right)}{4e}-\frac{\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/2}}{2e(d+ex)^{9/2}}$$

input
```
Int[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2)/(d + e*x)^(11/2),x]
```

output
```
-1/2*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2)/(e*(d + e*x)^(9/2)) + (
5*c*d*(-((a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2)/(e*(d + e*x)^(5/2))
) + (3*c*d*((2*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(e*Sqrt[d + e*
x]) - (2*Sqrt[c*d^2 - a*e^2]*ArcTan[(Sqrt[e]*Sqrt[a*d*e + (c*d^2 + a*e^2)*
x + c*d*e*x^2])/(Sqrt[c*d^2 - a*e^2]*Sqrt[d + e*x])])/e^(3/2)))/(2*e)))/(4
*e)
```

Defintions of rubi rules used

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 1130
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(d + e*x)^(m + 1)*((a + b*x + c*x^2)^p/(e*(m + p + 1))), x]
- Simp[c*(p/(e^2*(m + p + 1)))   Int[(d + e*x)^(m + 2)*(a + b*x + c*x^2)^(p
 - 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0]
 && GtQ[p, 0] && (LtQ[m, -2] || EqQ[m + 2*p + 1, 0]) && NeQ[m + p + 1, 0] &
& IntegerQ[2*p]
```

rule 1131
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(d + e*x)^(m + 1)*((a + b*x + c*x^2)^p/(e*(m + 2*p + 1))), x
] - Simp[p*((2*c*d - b*e)/(e^2*(m + 2*p + 1)))   Int[(d + e*x)^(m + 1)*(a +
 b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c*d^2 - b
*d*e + a*e^2, 0] && GtQ[p, 0] && (LeQ[-2, m, 0] || EqQ[m + p + 1, 0]) && Ne
Q[m + 2*p + 1, 0] && IntegerQ[2*p]
```

rule 1136
```
Int[1/(Sqrt[(d_.) + (e_.)*(x_)]*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x
_Symbol] :> Simp[2*e   Subst[Int[1/(2*c*d - b*e + e^2*x^2), x], x, Sqrt[a +
 b*x + c*x^2]/Sqrt[d + e*x]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c*d^2
- b*d*e + a*e^2, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 516 vs. 2(204) = 408.

Time = 1.08 (sec) , antiderivative size = 517, normalized size of antiderivative = 2.19

method	result
default	$-\frac{\sqrt{(ex+d)(cdx+ae)}\left(15\,\mathrm{arctanh}\left(\frac{e\sqrt{cdx+ae}}{\sqrt{\left(a\,e^{2}-c\,d^{2}\right)e}}\right)a\,c^{2}d^{2}e^{4}x^{2}-15\,\mathrm{arctanh}\left(\frac{e\sqrt{cdx+ae}}{\sqrt{\left(a\,e^{2}-c\,d^{2}\right)e}}\right)c^{3}d^{4}e^{2}x^{2}+30\,\mathrm{arctanh}\left(\frac{e\sqrt{cdx+}}{\sqrt{\left(a\,e^{2}-c\right.}}\right.\right.}{}$

input
```
int((a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(5/2)/(e*x+d)^(11/2),x,method=_RETUR
NVERBOSE)
```

output
```
-1/4*((e*x+d)*(c*d*x+a*e))^(1/2)*(15*arctanh(e*(c*d*x+a*e)^(1/2)/((a*e^2-c
*d^2)*e)^(1/2))*a*c^2*d^2*e^4*x^2-15*arctanh(e*(c*d*x+a*e)^(1/2)/((a*e^2-c
*d^2)*e)^(1/2))*c^3*d^4*e^2*x^2+30*arctanh(e*(c*d*x+a*e)^(1/2)/((a*e^2-c*d
^2)*e)^(1/2))*a*c^2*d^3*e^3*x-30*arctanh(e*(c*d*x+a*e)^(1/2)/((a*e^2-c*d^2
)*e)^(1/2))*c^3*d^5*e*x+15*arctanh(e*(c*d*x+a*e)^(1/2)/((a*e^2-c*d^2)*e)^(
1/2))*a*c^2*d^4*e^2-15*arctanh(e*(c*d*x+a*e)^(1/2)/((a*e^2-c*d^2)*e)^(1/2)
)*c^3*d^6-8*c^2*d^2*e^2*x^2*(c*d*x+a*e)^(1/2)*((a*e^2-c*d^2)*e)^(1/2)+9*a*
c*d*e^3*x*(c*d*x+a*e)^(1/2)*((a*e^2-c*d^2)*e)^(1/2)-25*c^2*d^3*e*x*(c*d*x+
a*e)^(1/2)*((a*e^2-c*d^2)*e)^(1/2)+2*((a*e^2-c*d^2)*e)^(1/2)*(c*d*x+a*e)^(
1/2)*a^2*e^4+5*((a*e^2-c*d^2)*e)^(1/2)*(c*d*x+a*e)^(1/2)*a*c*d^2*e^2-15*((
a*e^2-c*d^2)*e)^(1/2)*(c*d*x+a*e)^(1/2)*c^2*d^4)/(e*x+d)^(5/2)/(c*d*x+a*e)
^(1/2)/e^3/((a*e^2-c*d^2)*e)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 594, normalized size of antiderivative = 2.52

$$\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{(d+ex)^{11/2}}\,dx=\left[\frac{15\left(c^2d^2e^3x^3+3\,c^2d^3e^2x^2+3\,c^2d^4ex+c^2d^5\right)\sqrt{-\frac{cd^2-ae^2}{e}}\log\left(-\right.}{}\right.$$

input
```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(e*x+d)^(11/2),x, algori
thm="fricas")
```

output
```
[1/8*(15*(c^2*d^2*e^3*x^3 + 3*c^2*d^3*e^2*x^2 + 3*c^2*d^4*e*x + c^2*d^5)*s
qrt(-(c*d^2 - a*e^2)/e)*log(-(c*d*e^2*x^2 + 2*a*e^3*x - c*d^3 + 2*a*d*e^2
- 2*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(e*x + d)*e*sqrt(-(c*d
^2 - a*e^2)/e))/(e^2*x^2 + 2*d*e*x + d^2)) + 2*(8*c^2*d^2*e^2*x^2 + 15*c^2
*d^4 - 5*a*c*d^2*e^2 - 2*a^2*e^4 + (25*c^2*d^3*e - 9*a*c*d*e^3)*x)*sqrt(c*
d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(e*x + d))/(e^6*x^3 + 3*d*e^5*x^2
 + 3*d^2*e^4*x + d^3*e^3), 1/4*(15*(c^2*d^2*e^3*x^3 + 3*c^2*d^3*e^2*x^2 +
3*c^2*d^4*e*x + c^2*d^5)*sqrt((c*d^2 - a*e^2)/e)*arctan(-sqrt(c*d*e*x^2 +
a*d*e + (c*d^2 + a*e^2)*x)*sqrt(e*x + d)*e*sqrt((c*d^2 - a*e^2)/e)/(c*d^3
- a*d*e^2 + (c*d^2*e - a*e^3)*x)) + (8*c^2*d^2*e^2*x^2 + 15*c^2*d^4 - 5*a*
c*d^2*e^2 - 2*a^2*e^4 + (25*c^2*d^3*e - 9*a*c*d*e^3)*x)*sqrt(c*d*e*x^2 + a
*d*e + (c*d^2 + a*e^2)*x)*sqrt(e*x + d))/(e^6*x^3 + 3*d*e^5*x^2 + 3*d^2*e^
4*x + d^3*e^3)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(ade + (cd^2 + ae^2)\,x + cdex^2)^{5/2}}{(d + ex)^{11/2}}\,dx = \text{Timed out}$$

input
```
integrate((a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(5/2)/(e*x+d)**(11/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{(ade + (cd^2 + ae^2)\,x + cdex^2)^{5/2}}{(d + ex)^{11/2}}\,dx = \int \frac{(cdex^2 + ade + (cd^2 + ae^2)x)^{\frac{5}{2}}}{(ex + d)^{\frac{11}{2}}}\,dx$$

input
```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(e*x+d)^(11/2),x, algori
thm="maxima")
```

output
```
integrate((c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^(5/2)/(e*x + d)^(11/2),
x)
```

Giac [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 335, normalized size of antiderivative = 1.42

$$\int \frac{(ade + (cd^2 + ae^2)x + cdex^2)^{5/2}}{(d + ex)^{11/2}}\,dx = \frac{8\sqrt{(ex+d)cde - cd^2e + ae^3}c^3d^3|e| - \frac{15\,(c^4d^5e|e| - ac^3d^3e^3|e|)\arctan\left(\cdots\right.}{\sqrt{cd^2e - ae^3}}}{\cdots}$$

input
```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(e*x+d)^(11/2),x, algori
thm="giac")
```

output
```
1/4*(8*sqrt((e*x + d)*c*d*e - c*d^2*e + a*e^3)*c^3*d^3*abs(e) - 15*(c^4*d^
5*e*abs(e) - a*c^3*d^3*e^3*abs(e))*arctan(sqrt((e*x + d)*c*d*e - c*d^2*e +
 a*e^3)/sqrt(c*d^2*e - a*e^3))/sqrt(c*d^2*e - a*e^3) + (7*sqrt((e*x + d)*c
*d*e - c*d^2*e + a*e^3)*c^5*d^7*e^2*abs(e) - 14*sqrt((e*x + d)*c*d*e - c*d
^2*e + a*e^3)*a*c^4*d^5*e^4*abs(e) + 7*sqrt((e*x + d)*c*d*e - c*d^2*e + a*
e^3)*a^2*c^3*d^3*e^6*abs(e) + 9*((e*x + d)*c*d*e - c*d^2*e + a*e^3)^(3/2)*
c^4*d^5*e*abs(e) - 9*((e*x + d)*c*d*e - c*d^2*e + a*e^3)^(3/2)*a*c^3*d^3*e
^3*abs(e))/((e*x + d)^2*c^2*d^2*e^2))/(c*d*e^5)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(ade + (cd^2 + ae^2)x + cdex^2)^{5/2}}{(d + ex)^{11/2}}\,dx = \int \frac{\left(c\,d\,e\,x^2 + \left(c\,d^2 + a\,e^2\right)\,x + a\,d\,e\right)^{5/2}}{\left(d + e\,x\right)^{11/2}}\,dx$$

input
```
int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(5/2)/(d + e*x)^(11/2),x)
```

output
```
int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(5/2)/(d + e*x)^(11/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 310, normalized size of antiderivative = 1.31

$$\int \frac{(ade + (cd^2 + ae^2)\,x + cdex^2)^{5/2}}{(d+ex)^{11/2}}\,dx = \frac{-15\sqrt{e}\,\sqrt{-a\,e^2+c\,d^2}\,atan\left(\frac{\sqrt{cdx+ae}\,e}{\sqrt{e}\,\sqrt{-a\,e^2+c\,d^2}}\right)c^2d^4 - 30\sqrt{e}\,\sqrt{-a\,e^2}}{}$$

input
```
int((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(e*x+d)^(11/2),x)
```

output
```
( - 15*sqrt(e)*sqrt( - a*e**2 + c*d**2)*atan((sqrt(a*e + c*d*x)*e)/(sqrt(e
)*sqrt( - a*e**2 + c*d**2)))*c**2*d**4 - 30*sqrt(e)*sqrt( - a*e**2 + c*d**
2)*atan((sqrt(a*e + c*d*x)*e)/(sqrt(e)*sqrt( - a*e**2 + c*d**2)))*c**2*d**
3*e*x - 15*sqrt(e)*sqrt( - a*e**2 + c*d**2)*atan((sqrt(a*e + c*d*x)*e)/(sq
rt(e)*sqrt( - a*e**2 + c*d**2)))*c**2*d**2*e**2*x**2 - 2*sqrt(a*e + c*d*x)
*a**2*e**5 - 5*sqrt(a*e + c*d*x)*a*c*d**2*e**3 - 9*sqrt(a*e + c*d*x)*a*c*d
*e**4*x + 15*sqrt(a*e + c*d*x)*c**2*d**4*e + 25*sqrt(a*e + c*d*x)*c**2*d**
3*e**2*x + 8*sqrt(a*e + c*d*x)*c**2*d**2*e**3*x**2)/(4*e**4*(d**2 + 2*d*e*
x + e**2*x**2))
```

3.305 $\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{(d+ex)^{13/2}}\,dx$

Optimal result

Integrand size = 39, antiderivative size = 236

$$\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{(d+ex)^{13/2}}\,dx = -\frac{5c^2d^2\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{8e^3(d+ex)^{3/2}}-\frac{5cd\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{12e^2(d+ex)^{7/2}}-\frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{3e(d+ex)^{11/2}}+\frac{5c^3d^3\arctan\left(\frac{\sqrt{e}\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{\sqrt{cd^2-ae^2}\sqrt{d+ex}}\right)}{8e^{7/2}\sqrt{cd^2-ae^2}}$$

output
```
-5/8*c^2*d^2*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/e^3/(e*x+d)^(3/2)-5/1
2*c*d*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/e^2/(e*x+d)^(7/2)-1/3*(a*d*e
+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/e/(e*x+d)^(11/2)+5/8*c^3*d^3*arctan(e^(1
/2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(-a*e^2+c*d^2)^(1/2)/(e*x+d)^(
1/2))/e^(7/2)/(-a*e^2+c*d^2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.76 (sec) , antiderivative size = 173, normalized size of antiderivative = 0.73

$$\int \frac{(ade+(cd^2+ae^2)x+cdex^2)^{5/2}}{(d+ex)^{13/2}}\,dx = \frac{\sqrt{(ae+cdx)(d+ex)}\left(-\sqrt{e}(8a^2e^4+2acde^2(5d+13ex)+c^2d^2(\right.}{24e^{7/2}(d+e}$$

input
```
Integrate[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2)/(d + e*x)^(13/2),x
]
```

output
```
(Sqrt[(a*e + c*d*x)*(d + e*x)]*(-(Sqrt[e]*(8*a^2*e^4 + 2*a*c*d*e^2*(5*d +
13*e*x) + c^2*d^2*(15*d^2 + 40*d*e*x + 33*e^2*x^2))) + (15*c^3*d^3*(d + e*
x)^3*ArcTan[(Sqrt[e]*Sqrt[a*e + c*d*x])/Sqrt[c*d^2 - a*e^2]])/(Sqrt[c*d^2
- a*e^2]*Sqrt[a*e + c*d*x])))/(24*e^(7/2)*(d + e*x)^(7/2))
```

Rubi [A] (verified)

Time = 0.62 (sec) , antiderivative size = 239, normalized size of antiderivative = 1.01, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.128$, Rules used = {1130, 1130, 1130, 1136, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/2}}{(d+ex)^{13/2}}dx$$

$$\Big\downarrow \text{ 1130}$$

$$\frac{5cd\int \frac{(cdex^2+(cd^2+ae^2)x+ade)^{3/2}}{(d+ex)^{9/2}}dx}{6e}-\frac{\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/2}}{3e(d+ex)^{11/2}}$$

$$\Big\downarrow \text{ 1130}$$

$$\frac{5cd\left(\frac{3cd\int \frac{\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}{(d+ex)^{5/2}}dx}{4e}-\frac{\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{2e(d+ex)^{7/2}}\right)}{6e}-\frac{\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/2}}{3e(d+ex)^{11/2}}$$

$$\Big\downarrow \ 1130$$

$$\frac{5cd\left(\frac{3cd\left(\frac{cd\int \frac{1}{\sqrt{d+ex}\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}dx}{2e}-\frac{\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{e(d+ex)^{3/2}}\right)}{4e}-\frac{\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{2e(d+ex)^{7/2}}\right)}{6e}-\frac{\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/2}}{3e(d+ex)^{11/2}}$$

$$\Big\downarrow \ 1136$$

$$\frac{5cd\left(\frac{3cd\left(cd\int \frac{1}{\frac{\left(cdex^2+\left(cd^2+ae^2\right)x+ade\right)e^2}{d+ex}+\left(cd^2-ae^2\right)e}d\frac{\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}{\sqrt{d+ex}}-\frac{\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{e(d+ex)^{3/2}}\right)}{4e}-\frac{\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{2e(d+ex)^{7/2}}\right)}{6e}-\frac{\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/2}}{3e(d+ex)^{11/2}}$$

$$\Big\downarrow \ 218$$

$$\frac{5cd\left(\frac{3cd\left(\frac{cd\arctan\left(\frac{\sqrt{e}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{\sqrt{d+ex}\sqrt{cd^2-ae^2}}\right)}{e^{3/2}\sqrt{cd^2-ae^2}}-\frac{\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{e(d+ex)^{3/2}}\right)}{4e}-\frac{\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{2e(d+ex)^{7/2}}\right)}{6e}-\frac{\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/2}}{3e(d+ex)^{11/2}}$$

input
```
Int[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2)/(d + e*x)^(13/2),x]
```

output
```
-1/3*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2)/(e*(d + e*x)^(11/2)) +
(5*c*d*(-1/2*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2)/(e*(d + e*x)^(7
/2)) + (3*c*d*(-(Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]/(e*(d + e*x)^
(3/2))) + (c*d*ArcTan[(Sqrt[e]*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]
)/(Sqrt[c*d^2 - a*e^2]*Sqrt[d + e*x])])/(e^(3/2)*Sqrt[c*d^2 - a*e^2])))/(4
*e)))/(6*e)
```

Defintions of rubi rules used

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 1130
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(d + e*x)^(m + 1)*((a + b*x + c*x^2)^p/(e*(m + p + 1))), x]
- Simp[c*(p/(e^2*(m + p + 1)))   Int[(d + e*x)^(m + 2)*(a + b*x + c*x^2)^(p
 - 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0]
 && GtQ[p, 0] && (LtQ[m, -2] || EqQ[m + 2*p + 1, 0]) && NeQ[m + p + 1, 0] &
& IntegerQ[2*p]
```

rule 1136
```
Int[1/(Sqrt[(d_.) + (e_.)*(x_)]*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x
_Symbol] :> Simp[2*e   Subst[Int[1/(2*c*d - b*e + e^2*x^2), x], x, Sqrt[a +
 b*x + c*x^2]/Sqrt[d + e*x]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c*d^2
- b*d*e + a*e^2, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 432 vs. $2(204) = 408$.

Time = 1.08 (sec) , antiderivative size = 433, normalized size of antiderivative = 1.83

method	result
default	$-\frac{\sqrt{(ex+d)(cdx+ae)}\left(15 \operatorname{arctanh}\left(\frac{e\sqrt{cdx+ae}}{\sqrt{(a\,e^2-c\,d^2)e}}\right)c^3d^3e^3x^3+45 \operatorname{arctanh}\left(\frac{e\sqrt{cdx+ae}}{\sqrt{(a\,e^2-c\,d^2)e}}\right)c^3d^4e^2x^2+45 \operatorname{arctanh}\left(\frac{e\sqrt{cdx+a}}{\sqrt{(a\,e^2-c\,d}}\right.\right.}{}$

input
```
int((a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(5/2)/(e*x+d)^(13/2),x,method=_RETUR
NVERBOSE)
```

output
```
-1/24*((e*x+d)*(c*d*x+a*e))^(1/2)*(15*arctanh(e*(c*d*x+a*e)^(1/2)/((a*e^2-
c*d^2)*e)^(1/2))*c^3*d^3*e^3*x^3+45*arctanh(e*(c*d*x+a*e)^(1/2)/((a*e^2-c*
d^2)*e)^(1/2))*c^3*d^4*e^2*x^2+45*arctanh(e*(c*d*x+a*e)^(1/2)/((a*e^2-c*d^
2)*e)^(1/2))*c^3*d^5*e*x+15*arctanh(e*(c*d*x+a*e)^(1/2)/((a*e^2-c*d^2)*e)^
(1/2))*c^3*d^6+33*c^2*d^2*e^2*x^2*(c*d*x+a*e)^(1/2)*((a*e^2-c*d^2)*e)^(1/2
)+26*a*c*d*e^3*x*(c*d*x+a*e)^(1/2)*((a*e^2-c*d^2)*e)^(1/2)+40*c^2*d^3*e*x*
(c*d*x+a*e)^(1/2)*((a*e^2-c*d^2)*e)^(1/2)+8*((a*e^2-c*d^2)*e)^(1/2)*(c*d*x
+a*e)^(1/2)*a^2*e^4+10*((a*e^2-c*d^2)*e)^(1/2)*(c*d*x+a*e)^(1/2)*a*c*d^2*e
^2+15*((a*e^2-c*d^2)*e)^(1/2)*(c*d*x+a*e)^(1/2)*c^2*d^4)/(e*x+d)^(7/2)/(c*
d*x+a*e)^(1/2)/e^3/((a*e^2-c*d^2)*e)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 836, normalized size of antiderivative = 3.54

$$\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{(d+ex)^{13/2}}\,dx = \text{Too large to display}$$

input
```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(e*x+d)^(13/2),x, algori
thm="fricas")
```

output

```
[-1/48*(15*(c^3*d^3*e^4*x^4 + 4*c^3*d^4*e^3*x^3 + 6*c^3*d^5*e^2*x^2 + 4*c^
3*d^6*e*x + c^3*d^7)*sqrt(-c*d^2*e + a*e^3)*log(-(c*d*e^2*x^2 + 2*a*e^3*x
- c*d^3 + 2*a*d*e^2 - 2*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(-
c*d^2*e + a*e^3)*sqrt(e*x + d))/(e^2*x^2 + 2*d*e*x + d^2)) + 2*(15*c^3*d^6
*e - 5*a*c^2*d^4*e^3 - 2*a^2*c*d^2*e^5 - 8*a^3*e^7 + 33*(c^3*d^4*e^3 - a*c
^2*d^2*e^5)*x^2 + 2*(20*c^3*d^5*e^2 - 7*a*c^2*d^3*e^4 - 13*a^2*c*d*e^6)*x)
*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(e*x + d))/(c*d^6*e^4 - a
*d^4*e^6 + (c*d^2*e^8 - a*e^10)*x^4 + 4*(c*d^3*e^7 - a*d*e^9)*x^3 + 6*(c*d
^4*e^6 - a*d^2*e^8)*x^2 + 4*(c*d^5*e^5 - a*d^3*e^7)*x), -1/24*(15*(c^3*d^3
*e^4*x^4 + 4*c^3*d^4*e^3*x^3 + 6*c^3*d^5*e^2*x^2 + 4*c^3*d^6*e*x + c^3*d^7
)*sqrt(c*d^2*e - a*e^3)*arctan(-sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x
)*sqrt(c*d^2*e - a*e^3)*sqrt(e*x + d)/(c*d^3 - a*d*e^2 + (c*d^2*e - a*e^3)
*x)) + (15*c^3*d^6*e - 5*a*c^2*d^4*e^3 - 2*a^2*c*d^2*e^5 - 8*a^3*e^7 + 33*
(c^3*d^4*e^3 - a*c^2*d^2*e^5)*x^2 + 2*(20*c^3*d^5*e^2 - 7*a*c^2*d^3*e^4 -
13*a^2*c*d*e^6)*x)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(e*x +
d))/(c*d^6*e^4 - a*d^4*e^6 + (c*d^2*e^8 - a*e^10)*x^4 + 4*(c*d^3*e^7 - a*d
*e^9)*x^3 + 6*(c*d^4*e^6 - a*d^2*e^8)*x^2 + 4*(c*d^5*e^5 - a*d^3*e^7)*x)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(ade + (cd^2 + ae^2)\,x + cdex^2)^{5/2}}{(d+ex)^{13/2}}\,dx = \text{Timed out}$$

input

```
integrate((a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(5/2)/(e*x+d)**(13/2),x)
```

output

```
Timed out
```

Maxima [F]

$$\int \frac{(ade + (cd^2 + ae^2)\,x + cdex^2)^{5/2}}{(d+ex)^{13/2}}\,dx = \int \frac{(cdex^2 + ade + (cd^2 + ae^2)x)^{\frac{5}{2}}}{(ex+d)^{\frac{13}{2}}}\,dx$$

input

```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(e*x+d)^(13/2),x, algori
thm="maxima")
```

output
```
integrate((c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^(5/2)/(e*x + d)^(13/2),
x)
```

Giac [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 294, normalized size of antiderivative = 1.25

$$\int \frac{(ade+(cd^2+ae^2)x+cdex^2)^{5/2}}{(d+ex)^{13/2}}\,dx = \frac{c^3d^3\left(\frac{15\,|e|\arctan\left(\frac{\sqrt{(ex+d)cde-cd^2e+ae^3}}{\sqrt{cd^2e-ae^3}}\right)}{\sqrt{cd^2e-ae^3}e^3} - \frac{15\,\sqrt{(ex+d)cde-cd^2e+ae^3}c^2d^4e^2|e|\cdots}{\cdots}\right)}{\cdots}$$

input
```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(e*x+d)^(13/2),x, algori
thm="giac")
```

output
```
1/24*c^3*d^3*(15*abs(e)*arctan(sqrt((e*x + d)*c*d*e - c*d^2*e + a*e^3)/sqr
t(c*d^2*e - a*e^3))/(sqrt(c*d^2*e - a*e^3)*e^3) - (15*sqrt((e*x + d)*c*d*e
 - c*d^2*e + a*e^3)*c^2*d^4*e^2*abs(e) - 30*sqrt((e*x + d)*c*d*e - c*d^2*e
 + a*e^3)*a*c*d^2*e^4*abs(e) + 15*sqrt((e*x + d)*c*d*e - c*d^2*e + a*e^3)*
a^2*e^6*abs(e) + 40*((e*x + d)*c*d*e - c*d^2*e + a*e^3)^(3/2)*c*d^2*e*abs(
e) - 40*((e*x + d)*c*d*e - c*d^2*e + a*e^3)^(3/2)*a*e^3*abs(e) + 33*((e*x
+ d)*c*d*e - c*d^2*e + a*e^3)^(5/2)*abs(e))/((e*x + d)^3*c^3*d^3*e^6))/e
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(ade+(cd^2+ae^2)x+cdex^2)^{5/2}}{(d+ex)^{13/2}}\,dx = \int \frac{{\left(c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e\right)}^{5/2}}{{\left(d+e\,x\right)}^{13/2}} \,d x$$

input
```
int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(5/2)/(d + e*x)^(13/2),x)
```

output
```
int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(5/2)/(d + e*x)^(13/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 500, normalized size of antiderivative = 2.12

$$\int \frac{(ade + (cd^2 + ae^2)x + cdex^2)^{5/2}}{(d + ex)^{13/2}}\,dx = \frac{-15\sqrt{e}\,\sqrt{-a\,e^2 + c\,d^2}\,atan\left(\frac{\sqrt{cdx+ae}\,e}{\sqrt{e}\,\sqrt{-a\,e^2+c\,d^2}}\right)c^3d^6 - 45\sqrt{e}\,\sqrt{-a\,e^2 -}}{}$$

input
```
int((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(e*x+d)^(13/2),x)
```

output
```
( - 15*sqrt(e)*sqrt( - a*e**2 + c*d**2)*atan((sqrt(a*e + c*d*x)*e)/(sqrt(e
)*sqrt( - a*e**2 + c*d**2)))*c**3*d**6 - 45*sqrt(e)*sqrt( - a*e**2 + c*d**
2)*atan((sqrt(a*e + c*d*x)*e)/(sqrt(e)*sqrt( - a*e**2 + c*d**2)))*c**3*d**
5*e*x - 45*sqrt(e)*sqrt( - a*e**2 + c*d**2)*atan((sqrt(a*e + c*d*x)*e)/(sq
rt(e)*sqrt( - a*e**2 + c*d**2)))*c**3*d**4*e**2*x**2 - 15*sqrt(e)*sqrt( -
a*e**2 + c*d**2)*atan((sqrt(a*e + c*d*x)*e)/(sqrt(e)*sqrt( - a*e**2 + c*d*
*2)))*c**3*d**3*e**3*x**3 - 8*sqrt(a*e + c*d*x)*a**3*e**7 - 2*sqrt(a*e + c
*d*x)*a**2*c*d**2*e**5 - 26*sqrt(a*e + c*d*x)*a**2*c*d*e**6*x - 5*sqrt(a*e
 + c*d*x)*a*c**2*d**4*e**3 - 14*sqrt(a*e + c*d*x)*a*c**2*d**3*e**4*x - 33*
sqrt(a*e + c*d*x)*a*c**2*d**2*e**5*x**2 + 15*sqrt(a*e + c*d*x)*c**3*d**6*e
 + 40*sqrt(a*e + c*d*x)*c**3*d**5*e**2*x + 33*sqrt(a*e + c*d*x)*c**3*d**4*
e**3*x**2)/(24*e**4*(a*d**3*e**2 + 3*a*d**2*e**3*x + 3*a*d*e**4*x**2 + a*e
**5*x**3 - c*d**5 - 3*c*d**4*e*x - 3*c*d**3*e**2*x**2 - c*d**2*e**3*x**3))
```

3.306 $\int \frac{(ade+(cd^2+ae^2)x+cdex^2)^{5/2}}{(d+ex)^{15/2}} dx$

Optimal result

Integrand size = 39, antiderivative size = 301

$$\int \frac{(ade+(cd^2+ae^2)x+cdex^2)^{5/2}}{(d+ex)^{15/2}} dx = -\frac{5c^2d^2\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{32e^3(d+ex)^{5/2}} + \frac{5c^3d^3\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{64e^3(cd^2-ae^2)(d+ex)^{3/2}} - \frac{5cd(ade+(cd^2+ae^2)x+cdex^2)^{3/2}}{24e^2(d+ex)^{9/2}} - \frac{(ade+(cd^2+ae^2)x+cdex^2)^{5/2}}{4e(d+ex)^{13/2}} + \frac{5c^4d^4\arctan\left(\frac{\sqrt{e}\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{\sqrt{cd^2-ae^2}\sqrt{d+ex}}\right)}{64e^{7/2}(cd^2-ae^2)^{3/2}}$$

output
```
-5/32*c^2*d^2*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/e^3/(e*x+d)^(5/2)+5/
64*c^3*d^3*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/e^3/(-a*e^2+c*d^2)/(e*x
+d)^(3/2)-5/24*c*d*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/e^2/(e*x+d)^(9/
2)-1/4*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/e/(e*x+d)^(13/2)+5/64*c^4*d
^4*arctan(e^(1/2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(-a*e^2+c*d^2)^(
1/2)/(e*x+d)^(1/2))/e^(7/2)/(-a*e^2+c*d^2)^(3/2)
```

Mathematica [A] (verified)

Time = 1.24 (sec) , antiderivative size = 245, normalized size of antiderivative = 0.81

$$\int \frac{\left(ade + \left(cd^2 + ae^2\right)x + cdex^2\right)^{5/2}}{(d+ex)^{15/2}}\,dx = \frac{c^4d^4((ae+cdx)(d+ex))^{5/2}\left(\frac{\sqrt{e}\left(48a^3e^6 - 8a^2cde^4(d-17ex) - 2ac^2d^2e^2(5d^2 - \cdots}{c^4d^4(cd^2 - \cdots}\right.}{192e^{7/2} \cdots}$$

input
```
Integrate[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2)/(d + e*x)^(15/2),x
]
```

output
```
(c^4*d^4*((a*e + c*d*x)*(d + e*x))^(5/2)*((Sqrt[e]*(48*a^3*e^6 - 8*a^2*c*d
*e^4*(d - 17*e*x) - 2*a*c^2*d^2*e^2*(5*d^2 + 18*d*e*x - 59*e^2*x^2) - c^3*
d^3*(15*d^3 + 55*d^2*e*x + 73*d*e^2*x^2 - 15*e^3*x^3)))/(c^4*d^4*(c*d^2 -
a*e^2)*(a*e + c*d*x)^2*(d + e*x)^4) + (15*ArcTan[(Sqrt[e]*Sqrt[a*e + c*d*x
])/Sqrt[c*d^2 - a*e^2]])/((c*d^2 - a*e^2)^(3/2)*(a*e + c*d*x)^(5/2))))/(19
2*e^(7/2)*(d + e*x)^(5/2))
```

Rubi [A] (verified)

Time = 0.78 (sec) , antiderivative size = 304, normalized size of antiderivative = 1.01, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {1130, 1130, 1130, 1135, 1136, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/2}}{(d+ex)^{15/2}}dx$$

$$\Big\downarrow \text{1130}$$

$$\frac{5cd\int \frac{\left(cdex^2+\left(cd^2+ae^2\right)x+ade\right)^{3/2}}{(d+ex)^{11/2}}dx}{8e} - \frac{\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/2}}{4e(d+ex)^{13/2}}$$

$$\Big\downarrow \text{1130}$$

$$\frac{5cd\left(\frac{cd\int \frac{\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}{(d+ex)^{7/2}}dx}{2e}-\frac{\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{3e(d+ex)^{9/2}}\right)}{8e}-\frac{\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/2}}{4e(d+ex)^{13/2}}$$

$\Big\downarrow$ 1130

$$\frac{5cd\left(\frac{cd\left(\frac{cd\int \frac{1}{(d+ex)^{3/2}\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}dx}{4e}-\frac{\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{2e(d+ex)^{5/2}}\right)}{2e}-\frac{\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{3e(d+ex)^{9/2}}\right)}{8e}-\frac{\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/2}}{4e(d+ex)^{13/2}}$$

$\Big\downarrow$ 1135

$$\frac{5cd\left(\frac{cd\left(\frac{cd\left(\frac{cd\int \frac{1}{\sqrt{d+ex}\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}dx}{2\left(cd^2-ae^2\right)}+\frac{\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{(d+ex)^{3/2}\left(cd^2-ae^2\right)}\right)}{4e}-\frac{\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{2e(d+ex)^{5/2}}\right)}{2e}-\frac{\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)}{3e(d+ex)^{9/2}}\right)}{8e}$$

$$\frac{\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/2}}{4e(d+ex)^{13/2}}$$

$\Big\downarrow$ 1136

$$\frac{5cd\left(\frac{cd\left(\frac{cd\left(\frac{cde\int \frac{1}{\frac{\left(cdex^2+\left(cd^2+ae^2\right)x+ade\right)e^2}{d+ex}+\left(cd^2-ae^2\right)e}d\frac{\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}{\sqrt{d+ex}}}{cd^2-ae^2}+\frac{\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{(d+ex)^{3/2}\left(cd^2-ae^2\right)}\right)}{4e}-\frac{\sqrt{x\left(ae^2+cd^2\right)+ade+cdex}}{2e(d+ex)^{5/2}}}{2e}}{8e}$$

$$\frac{\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/2}}{4e(d+ex)^{13/2}}$$

$\downarrow$ 218

$$\frac{5cd\left(\frac{cd\left(\frac{cd\left(\frac{cd\arctan\left(\frac{\sqrt{e}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{\sqrt{d+ex}\sqrt{cd^2-ae^2}}\right)}{\sqrt{e}\left(cd^2-ae^2\right)^{3/2}}+\frac{\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{(d+ex)^{3/2}\left(cd^2-ae^2\right)}\right)}{4e}-\frac{\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{2e(d+ex)^{5/2}}\right)}{2e}-\frac{\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{3e(d+ex)^{9/2}}\right)}{8e}-\frac{\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/2}}{4e(d+ex)^{13/2}}$$

input
```
Int[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2)/(d + e*x)^(15/2),x]
```

output
```
-1/4*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2)/(e*(d + e*x)^(13/2)) +
(5*c*d*(-1/3*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2)/(e*(d + e*x)^(9
/2)) + (c*d*(-1/2*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]/(e*(d + e*x)
^(5/2)) + (c*d*(Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]/((c*d^2 - a*e^
2)*(d + e*x)^(3/2)) + (c*d*ArcTan[(Sqrt[e]*Sqrt[a*d*e + (c*d^2 + a*e^2)*x
+ c*d*e*x^2])/(Sqrt[c*d^2 - a*e^2]*Sqrt[d + e*x])])/(Sqrt[e]*(c*d^2 - a*e^
2)^(3/2))))/(4*e)))/(2*e)))/(8*e)
```

Defintions of rubi rules used

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 1130
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(d + e*x)^(m + 1)*((a + b*x + c*x^2)^p/(e*(m + p + 1))), x]
- Simp[c*(p/(e^2*(m + p + 1)))   Int[(d + e*x)^(m + 2)*(a + b*x + c*x^2)^(p
 - 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0]
 && GtQ[p, 0] && (LtQ[m, -2] || EqQ[m + 2*p + 1, 0]) && NeQ[m + p + 1, 0] &
& IntegerQ[2*p]
```

rule 1135
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(-e)*(d + e*x)^m*((a + b*x + c*x^2)^(p + 1)/((m + p + 1)*(2*
c*d - b*e))), x] + Simp[c*((m + 2*p + 2)/((m + p + 1)*(2*c*d - b*e)))   Int
[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p},
 x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[m, 0] && NeQ[m + p + 1, 0] && I
ntegerQ[2*p]
```

rule 1136
```
Int[1/(Sqrt[(d_.) + (e_.)*(x_)]*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x
_Symbol] :> Simp[2*e   Subst[Int[1/(2*c*d - b*e + e^2*x^2), x], x, Sqrt[a +
 b*x + c*x^2]/Sqrt[d + e*x]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c*d^2
- b*d*e + a*e^2, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 651 vs. $2(263) = 526$.

Time = 1.07 (sec) , antiderivative size = 652, normalized size of antiderivative = 2.17

method	result
default	$\frac{\sqrt{(ex+d)(cdx+ae)}\left(15\,\operatorname{arctanh}\left(\frac{e\sqrt{cdx+ae}}{\sqrt{(a\,e^2-c\,d^2)e}}\right)c^4d^4e^4x^4+60\,\operatorname{arctanh}\left(\frac{e\sqrt{cdx+ae}}{\sqrt{(a\,e^2-c\,d^2)e}}\right)c^4d^5e^3x^3+90\,\operatorname{arctanh}\left(\frac{e\sqrt{cdx+ae}}{\sqrt{(a\,e^2-c\,d^2)e}}\right.\right.}{}$

input
```
int((a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(5/2)/(e*x+d)^(15/2),x,method=_RETUR
NVERBOSE)
```

output
```
1/192*((e*x+d)*(c*d*x+a*e))^(1/2)*(15*arctanh(e*(c*d*x+a*e)^(1/2)/((a*e^2-
c*d^2)*e)^(1/2))*c^4*d^4*e^4*x^4+60*arctanh(e*(c*d*x+a*e)^(1/2)/((a*e^2-c*
d^2)*e)^(1/2))*c^4*d^5*e^3*x^3+90*arctanh(e*(c*d*x+a*e)^(1/2)/((a*e^2-c*d^
2)*e)^(1/2))*c^4*d^6*e^2*x^2+60*arctanh(e*(c*d*x+a*e)^(1/2)/((a*e^2-c*d^2)
*e)^(1/2))*c^4*d^7*e*x-15*c^3*d^3*e^3*x^3*(c*d*x+a*e)^(1/2)*((a*e^2-c*d^2)
*e)^(1/2)+15*arctanh(e*(c*d*x+a*e)^(1/2)/((a*e^2-c*d^2)*e)^(1/2))*c^4*d^8-
118*a*c^2*d^2*e^4*x^2*(c*d*x+a*e)^(1/2)*((a*e^2-c*d^2)*e)^(1/2)+73*c^3*d^4
*e^2*x^2*(c*d*x+a*e)^(1/2)*((a*e^2-c*d^2)*e)^(1/2)-136*a^2*c*d*e^5*x*(c*d*
x+a*e)^(1/2)*((a*e^2-c*d^2)*e)^(1/2)+36*a*c^2*d^3*e^3*x*(c*d*x+a*e)^(1/2)*
((a*e^2-c*d^2)*e)^(1/2)+55*c^3*d^5*e*x*(c*d*x+a*e)^(1/2)*((a*e^2-c*d^2)*e)
^(1/2)-48*((a*e^2-c*d^2)*e)^(1/2)*(c*d*x+a*e)^(1/2)*a^3*e^6+8*((a*e^2-c*d^
2)*e)^(1/2)*(c*d*x+a*e)^(1/2)*a^2*c*d^2*e^4+10*((a*e^2-c*d^2)*e)^(1/2)*(c*
d*x+a*e)^(1/2)*a*c^2*d^4*e^2+15*((a*e^2-c*d^2)*e)^(1/2)*(c*d*x+a*e)^(1/2)*
c^3*d^6)/(e*x+d)^(9/2)/(c*d*x+a*e)^(1/2)/(a*e^2-c*d^2)/e^3/((a*e^2-c*d^2)*
e)^(1/2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 591 vs. $2(263) = 526$.

Time = 0.13 (sec) , antiderivative size = 1204, normalized size of antiderivative = 4.00

$$\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{(d+ex)^{15/2}}\,dx=\text{Too large to display}$$

input
```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(e*x+d)^(15/2),x, algori
thm="fricas")
```

output
```
[1/384*(15*(c^4*d^4*e^5*x^5 + 5*c^4*d^5*e^4*x^4 + 10*c^4*d^6*e^3*x^3 + 10*
c^4*d^7*e^2*x^2 + 5*c^4*d^8*e*x + c^4*d^9)*sqrt(-c*d^2*e + a*e^3)*log(-(c*
d*e^2*x^2 + 2*a*e^3*x - c*d^3 + 2*a*d*e^2 + 2*sqrt(c*d*e*x^2 + a*d*e + (c*
d^2 + a*e^2)*x)*sqrt(-c*d^2*e + a*e^3)*sqrt(e*x + d))/(e^2*x^2 + 2*d*e*x +
 d^2)) - 2*(15*c^4*d^8*e - 5*a*c^3*d^6*e^3 - 2*a^2*c^2*d^4*e^5 - 56*a^3*c*
d^2*e^7 + 48*a^4*e^9 - 15*(c^4*d^5*e^4 - a*c^3*d^3*e^6)*x^3 + (73*c^4*d^6*
e^3 - 191*a*c^3*d^4*e^5 + 118*a^2*c^2*d^2*e^7)*x^2 + (55*c^4*d^7*e^2 - 19*
a*c^3*d^5*e^4 - 172*a^2*c^2*d^3*e^6 + 136*a^3*c*d*e^8)*x)*sqrt(c*d*e*x^2 +
 a*d*e + (c*d^2 + a*e^2)*x)*sqrt(e*x + d))/(c^2*d^9*e^4 - 2*a*c*d^7*e^6 +
a^2*d^5*e^8 + (c^2*d^4*e^9 - 2*a*c*d^2*e^11 + a^2*e^13)*x^5 + 5*(c^2*d^5*e
^8 - 2*a*c*d^3*e^10 + a^2*d*e^12)*x^4 + 10*(c^2*d^6*e^7 - 2*a*c*d^4*e^9 +
a^2*d^2*e^11)*x^3 + 10*(c^2*d^7*e^6 - 2*a*c*d^5*e^8 + a^2*d^3*e^10)*x^2 +
5*(c^2*d^8*e^5 - 2*a*c*d^6*e^7 + a^2*d^4*e^9)*x), -1/192*(15*(c^4*d^4*e^5*
x^5 + 5*c^4*d^5*e^4*x^4 + 10*c^4*d^6*e^3*x^3 + 10*c^4*d^7*e^2*x^2 + 5*c^4*
d^8*e*x + c^4*d^9)*sqrt(c*d^2*e - a*e^3)*arctan(-sqrt(c*d*e*x^2 + a*d*e +
(c*d^2 + a*e^2)*x)*sqrt(c*d^2*e - a*e^3)*sqrt(e*x + d)/(c*d^3 - a*d*e^2 +
(c*d^2*e - a*e^3)*x)) + (15*c^4*d^8*e - 5*a*c^3*d^6*e^3 - 2*a^2*c^2*d^4*e^
5 - 56*a^3*c*d^2*e^7 + 48*a^4*e^9 - 15*(c^4*d^5*e^4 - a*c^3*d^3*e^6)*x^3 +
 (73*c^4*d^6*e^3 - 191*a*c^3*d^4*e^5 + 118*a^2*c^2*d^2*e^7)*x^2 + (55*c^4*
d^7*e^2 - 19*a*c^3*d^5*e^4 - 172*a^2*c^2*d^3*e^6 + 136*a^3*c*d*e^8)*x)*...
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(ade + (cd^2 + ae^2)\,x + cdex^2)^{5/2}}{(d+ex)^{15/2}}\,dx = \text{Timed out}$$

input
```
integrate((a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(5/2)/(e*x+d)**(15/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{(ade + (cd^2 + ae^2)x + cdex^2)^{5/2}}{(d + ex)^{15/2}}\,dx = \int \frac{(cdex^2 + ade + (cd^2 + ae^2)x)^{\frac{5}{2}}}{(ex + d)^{\frac{15}{2}}}\,dx$$

input
```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(e*x+d)^(15/2),x, algori
thm="maxima")
```

output
```
integrate((c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^(5/2)/(e*x + d)^(15/2),
x)
```

Giac [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 506, normalized size of antiderivative = 1.68

$$\int \frac{(ade + (cd^2 + ae^2)x + cdex^2)^{5/2}}{(d + ex)^{15/2}}\,dx = \frac{\frac{15\,c^5 d^5 e|e| \arctan\left(\frac{\sqrt{(ex+d)cde-cd^2e+ae^3}}{\sqrt{cd^2e-ae^3}}\right)}{\sqrt{cd^2e-ae^3}(cd^2-ae^2)} - \frac{15\,\sqrt{(ex+d)cde-cd^2e+ae^3}\,c^8 d^{11} e^4 |e| - \cdots}{\cdots}}{\cdots}$$

input
```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(e*x+d)^(15/2),x, algori
thm="giac")
```

output
```
1/192*(15*c^5*d^5*e*abs(e)*arctan(sqrt((e*x + d)*c*d*e - c*d^2*e + a*e^3)/
sqrt(c*d^2*e - a*e^3))/(sqrt(c*d^2*e - a*e^3)*(c*d^2 - a*e^2)) - (15*sqrt(
(e*x + d)*c*d*e - c*d^2*e + a*e^3)*c^8*d^11*e^4*abs(e) - 45*sqrt((e*x + d)
*c*d*e - c*d^2*e + a*e^3)*a*c^7*d^9*e^6*abs(e) + 45*sqrt((e*x + d)*c*d*e -
 c*d^2*e + a*e^3)*a^2*c^6*d^7*e^8*abs(e) - 15*sqrt((e*x + d)*c*d*e - c*d^2
*e + a*e^3)*a^3*c^5*d^5*e^10*abs(e) + 55*((e*x + d)*c*d*e - c*d^2*e + a*e^
3)^(3/2)*c^7*d^9*e^3*abs(e) - 110*((e*x + d)*c*d*e - c*d^2*e + a*e^3)^(3/2
)*a*c^6*d^7*e^5*abs(e) + 55*((e*x + d)*c*d*e - c*d^2*e + a*e^3)^(3/2)*a^2*
c^5*d^5*e^7*abs(e) + 73*((e*x + d)*c*d*e - c*d^2*e + a*e^3)^(5/2)*c^6*d^7*
e^2*abs(e) - 73*((e*x + d)*c*d*e - c*d^2*e + a*e^3)^(5/2)*a*c^5*d^5*e^4*ab
s(e) - 15*((e*x + d)*c*d*e - c*d^2*e + a*e^3)^(7/2)*c^5*d^5*e*abs(e))/((c*
d^2 - a*e^2)*(e*x + d)^4*c^4*d^4*e^4))/(c*d*e^5)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(ade + (cd^2 + ae^2)x + cdex^2)^{5/2}}{(d + ex)^{15/2}}\,dx = \int \frac{\left(c\,d\,e\,x^2 + \left(c\,d^2 + a\,e^2\right)\,x + a\,d\,e\right)^{5/2}}{\left(d + e\,x\right)^{15/2}}\,dx$$

input
```
int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(5/2)/(d + e*x)^(15/2),x)
```

output
```
int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(5/2)/(d + e*x)^(15/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 782, normalized size of antiderivative = 2.60

$$\int \frac{(ade + (cd^2 + ae^2)x + cdex^2)^{5/2}}{(d + ex)^{15/2}}\,dx = \frac{15\sqrt{e}\,\sqrt{-a\,e^2 + c\,d^2}\,atan\left(\frac{\sqrt{cdx+ae}\,e}{\sqrt{e}\,\sqrt{-a\,e^2+c\,d^2}}\right) c^4 d^8 + 60\sqrt{e}\,\sqrt{-a\,e^2 +}}{}$$

input
```
int((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(e*x+d)^(15/2),x)
```

output

```
(15*sqrt(e)*sqrt( - a*e**2 + c*d**2)*atan((sqrt(a*e + c*d*x)*e)/(sqrt(e)*s
qrt( - a*e**2 + c*d**2)))*c**4*d**8 + 60*sqrt(e)*sqrt( - a*e**2 + c*d**2)*
atan((sqrt(a*e + c*d*x)*e)/(sqrt(e)*sqrt( - a*e**2 + c*d**2)))*c**4*d**7*e
*x + 90*sqrt(e)*sqrt( - a*e**2 + c*d**2)*atan((sqrt(a*e + c*d*x)*e)/(sqrt(
e)*sqrt( - a*e**2 + c*d**2)))*c**4*d**6*e**2*x**2 + 60*sqrt(e)*sqrt( - a*e
**2 + c*d**2)*atan((sqrt(a*e + c*d*x)*e)/(sqrt(e)*sqrt( - a*e**2 + c*d**2)
))*c**4*d**5*e**3*x**3 + 15*sqrt(e)*sqrt( - a*e**2 + c*d**2)*atan((sqrt(a*
e + c*d*x)*e)/(sqrt(e)*sqrt( - a*e**2 + c*d**2)))*c**4*d**4*e**4*x**4 - 48
*sqrt(a*e + c*d*x)*a**4*e**9 + 56*sqrt(a*e + c*d*x)*a**3*c*d**2*e**7 - 136
*sqrt(a*e + c*d*x)*a**3*c*d*e**8*x + 2*sqrt(a*e + c*d*x)*a**2*c**2*d**4*e*
*5 + 172*sqrt(a*e + c*d*x)*a**2*c**2*d**3*e**6*x - 118*sqrt(a*e + c*d*x)*a
**2*c**2*d**2*e**7*x**2 + 5*sqrt(a*e + c*d*x)*a*c**3*d**6*e**3 + 19*sqrt(a
*e + c*d*x)*a*c**3*d**5*e**4*x + 191*sqrt(a*e + c*d*x)*a*c**3*d**4*e**5*x*
*2 - 15*sqrt(a*e + c*d*x)*a*c**3*d**3*e**6*x**3 - 15*sqrt(a*e + c*d*x)*c**
4*d**8*e - 55*sqrt(a*e + c*d*x)*c**4*d**7*e**2*x - 73*sqrt(a*e + c*d*x)*c*
*4*d**6*e**3*x**2 + 15*sqrt(a*e + c*d*x)*c**4*d**5*e**4*x**3)/(192*e**4*(a
**2*d**4*e**4 + 4*a**2*d**3*e**5*x + 6*a**2*d**2*e**6*x**2 + 4*a**2*d*e**7
*x**3 + a**2*e**8*x**4 - 2*a*c*d**6*e**2 - 8*a*c*d**5*e**3*x - 12*a*c*d**4
*e**4*x**2 - 8*a*c*d**3*e**5*x**3 - 2*a*c*d**2*e**6*x**4 + c**2*d**8 + 4*c
**2*d**7*e*x + 6*c**2*d**6*e**2*x**2 + 4*c**2*d**5*e**3*x**3 + c**2*d**...
```

3.307 $\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{(d+ex)^{17/2}}\,dx$

Optimal result

Integrand size = 39, antiderivative size = 366

$$\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{(d+ex)^{17/2}}\,dx = -\frac{c^2d^2\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{16e^3(d+ex)^{7/2}}+\frac{c^3d^3\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{64e^3\left(cd^2-ae^2\right)(d+ex)^{5/2}}+\frac{3c^4d^4\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{128e^3\left(cd^2-ae^2\right)^2(d+ex)^{3/2}}-\frac{cd\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{8e^2(d+ex)^{11/2}}-\frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{5e(d+ex)^{15/2}}+\frac{3c^5d^5\arctan\left(\frac{\sqrt{e}\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{\sqrt{cd^2-ae^2}\sqrt{d+ex}}\right)}{128e^{7/2}\left(cd^2-ae^2\right)^{5/2}}$$

output
```
-1/16*c^2*d^2*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/e^3/(e*x+d)^(7/2)+1/
64*c^3*d^3*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/e^3/(-a*e^2+c*d^2)/(e*x
+d)^(5/2)+3/128*c^4*d^4*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/e^3/(-a*e^
2+c*d^2)^2/(e*x+d)^(3/2)-1/8*c*d*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/e
^2/(e*x+d)^(11/2)-1/5*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/e/(e*x+d)^(1
5/2)+3/128*c^5*d^5*arctan(e^(1/2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/
(-a*e^2+c*d^2)^(1/2)/(e*x+d)^(1/2))/e^(7/2)/(-a*e^2+c*d^2)^(5/2)
```

Mathematica [A] (verified)

Time = 1.67 (sec) , antiderivative size = 300, normalized size of antiderivative = 0.82

$$\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{(d+ex)^{17/2}}\,dx = \frac{c^5d^5((ae+cdx)(d+ex))^{5/2}\left(-\frac{\sqrt{e}(128a^4e^8+16a^3cde^6(-11d+21ex)+8a^2c^2}{}\right.}{}$$

input
```
Integrate[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2)/(d + e*x)^(17/2),x
]
```

output
```
(c^5*d^5*((a*e + c*d*x)*(d + e*x))^(5/2)*(-((Sqrt[e]*(128*a^4*e^8 + 16*a^3
*c*d*e^6*(-11*d + 21*e*x) + 8*a^2*c^2*d^2*e^4*(d^2 - 64*d*e*x + 31*e^2*x^2
) + 2*a*c^3*d^3*e^2*(5*d^3 + 23*d^2*e*x - 233*d*e^2*x^2 + 5*e^3*x^3) + c^4
*d^4*(15*d^4 + 70*d^3*e*x + 128*d^2*e^2*x^2 - 70*d*e^3*x^3 - 15*e^4*x^4)))
/(c^5*d^5*(c*d^2 - a*e^2)^2*(a*e + c*d*x)^2*(d + e*x)^5)) + (15*ArcTan[(Sq
rt[e]*Sqrt[a*e + c*d*x])/Sqrt[c*d^2 - a*e^2]])/((c*d^2 - a*e^2)^(5/2)*(a*e
 + c*d*x)^(5/2))))/(640*e^(7/2)*(d + e*x)^(5/2))
```

Rubi [A] (verified)

Time = 0.94 (sec) , antiderivative size = 381, normalized size of antiderivative = 1.04, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.179$, Rules used = {1130, 1130, 1130, 1135, 1135, 1136, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/2}}{(d+ex)^{17/2}}dx$$

$$\Big\downarrow \text{ 1130}$$

$$\frac{cd\int \frac{\left(cdex^2+\left(cd^2+ae^2\right)x+ade\right)^{3/2}}{(d+ex)^{13/2}}dx}{2e}-\frac{\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/2}}{5e(d+ex)^{15/2}}$$

$$\Big\downarrow \text{ 1130}$$

$$\frac{cd\left(\frac{3cd\int \frac{\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}{(d+ex)^{9/2}}dx}{8e}-\frac{\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{4e(d+ex)^{11/2}}\right)}{2e}-\frac{\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/2}}{5e(d+ex)^{15/2}}$$

$$\Big\downarrow \; 1130$$

$$\frac{cd\left(\frac{3cd\left(\frac{cd\int \frac{1}{(d+ex)^{5/2}\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}dx}{6e}-\frac{\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{3e(d+ex)^{7/2}}\right)}{8e}-\frac{\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{4e(d+ex)^{11/2}}\right)}{2e}-\frac{\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/2}}{5e(d+ex)^{15/2}}$$

$$\Big\downarrow \; 1135$$

$$\frac{cd\left(\frac{3cd\left(\frac{cd\left(\frac{3cd\int \frac{1}{(d+ex)^{3/2}\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}dx}{4\left(cd^2-ae^2\right)}+\frac{\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{2(d+ex)^{5/2}\left(cd^2-ae^2\right)}\right)}{6e}-\frac{\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{3e(d+ex)^{7/2}}\right)}{8e}-\frac{\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{4e(d+ex)^{11/2}}\right)}{2e}-\frac{\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/2}}{5e(d+ex)^{15/2}}$$

$$\Big\downarrow \; 1135$$

$$cd\left(\frac{3cd\left(\frac{cd\left(\frac{3cd\left(\frac{cd\int\frac{1}{\sqrt{d+ex}\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}dx}{2\left(cd^2-ae^2\right)}+\frac{\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{(d+ex)^{3/2}\left(cd^2-ae^2\right)}\right)}{4\left(cd^2-ae^2\right)}+\frac{\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{2(d+ex)^{5/2}\left(cd^2-ae^2\right)}\right)}{6e}-\frac{\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{3e(d+ex)^{7/2}}\right)}{8e}\right.$$

$$\frac{}{2e}$$

$$\frac{\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/2}}{5e(d+ex)^{15/2}}$$

$\downarrow$ 1136

$$\frac{cd\left(\frac{3cd\left(\frac{cd\left(\frac{3cd\left(\frac{cde\int \frac{1}{\frac{\left(cdex^2+\left(cd^2+ae^2\right)x+ade\right)e^2}{d+ex}+\left(cd^2-ae^2\right)e}d\frac{\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}{\sqrt{d+ex}}}{cd^2-ae^2}+\frac{\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{(d+ex)^{3/2}\left(cd^2-ae^2\right)}\right)}{4\left(cd^2-ae^2\right)}+\frac{\sqrt{x\left(ae^2+cd^2\right)+ade+}}{2(d+ex)^{5/2}\left(cd^2-c\right.}}{6e}}{8e}}{2e}$$

$$\frac{\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/2}}{5e(d+ex)^{15/2}}$$

$\downarrow$ 218

$$\frac{cd\left(\frac{3cd\left(\frac{cd\left(\frac{3cd\left(\frac{cd\arctan\left(\frac{\sqrt{e}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{\sqrt{d+ex}\sqrt{cd^2-ae^2}}\right)}{\sqrt{e}\left(cd^2-ae^2\right)^{3/2}}+\frac{\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{(d+ex)^{3/2}\left(cd^2-ae^2\right)}\right)}{4\left(cd^2-ae^2\right)}+\frac{\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{2(d+ex)^{5/2}\left(cd^2-ae^2\right)}\right)}{6e}-\frac{\sqrt{x\left(ae^2+cd^2\right)+ade+cdex\cdots}}{3e(d+ex)^{7/2}}\cdots}{8e}\cdots\right)}{2e}\cdots$$

$$\frac{\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/2}}{5e(d+ex)^{15/2}}$$

input
```
Int[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2)/(d + e*x)^(17/2),x]
```

output
```
-1/5*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2)/(e*(d + e*x)^(15/2)) +
(c*d*(-1/4*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2)/(e*(d + e*x)^(11/
2)) + (3*c*d*(-1/3*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]/(e*(d + e*x
)^(7/2)) + (c*d*(Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]/(2*(c*d^2 - a
*e^2)*(d + e*x)^(5/2)) + (3*c*d*(Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^
2]/((c*d^2 - a*e^2)*(d + e*x)^(3/2)) + (c*d*ArcTan[(Sqrt[e]*Sqrt[a*d*e + (
c*d^2 + a*e^2)*x + c*d*e*x^2])/(Sqrt[c*d^2 - a*e^2]*Sqrt[d + e*x])])/(Sqrt
[e]*(c*d^2 - a*e^2)^(3/2))))/(4*(c*d^2 - a*e^2))))/(6*e)))/(8*e)))/(2*e)
```

Defintions of rubi rules used

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 1130
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(d + e*x)^(m + 1)*((a + b*x + c*x^2)^p/(e*(m + p + 1))), x]
- Simp[c*(p/(e^2*(m + p + 1)))   Int[(d + e*x)^(m + 2)*(a + b*x + c*x^2)^(p
 - 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0]
 && GtQ[p, 0] && (LtQ[m, -2] || EqQ[m + 2*p + 1, 0]) && NeQ[m + p + 1, 0] &
& IntegerQ[2*p]
```

rule 1135
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(-e)*(d + e*x)^m*((a + b*x + c*x^2)^(p + 1)/((m + p + 1)*(2*
c*d - b*e))), x] + Simp[c*((m + 2*p + 2)/((m + p + 1)*(2*c*d - b*e)))   Int
[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p},
 x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[m, 0] && NeQ[m + p + 1, 0] && I
ntegerQ[2*p]
```

rule 1136
```
Int[1/(Sqrt[(d_.) + (e_.)*(x_)]*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x
_Symbol] :> Simp[2*e   Subst[Int[1/(2*c*d - b*e + e^2*x^2), x], x, Sqrt[a +
 b*x + c*x^2]/Sqrt[d + e*x]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c*d^2
- b*d*e + a*e^2, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 899 vs. $2(322) = 644$.

Time = 1.09 (sec) , antiderivative size = 900, normalized size of antiderivative = 2.46

method	result
default	$-\frac{\sqrt{(ex+d)(cdx+ae)}\left(15 \operatorname{arctanh}\left(\frac{e\sqrt{cdx+ae}}{\sqrt{\left(a\,e^2-c\,d^2\right)e}}\right)c^5d^5e^5x^5+75 \operatorname{arctanh}\left(\frac{e\sqrt{cdx+ae}}{\sqrt{\left(a\,e^2-c\,d^2\right)e}}\right)c^5d^6e^4x^4+150 \operatorname{arctanh}\left(\frac{e\sqrt{cdx+}}{\sqrt{\left(a\,e^2-c\right.}}\right.\right.}{}$

input
```
int((a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(5/2)/(e*x+d)^(17/2),x,method=_RETUR
NVERBOSE)
```

output
```
-1/640*((e*x+d)*(c*d*x+a*e))^(1/2)*(15*arctanh(e*(c*d*x+a*e)^(1/2)/((a*e^2
-c*d^2)*e)^(1/2))*c^5*d^5*e^5*x^5+75*arctanh(e*(c*d*x+a*e)^(1/2)/((a*e^2-c
*d^2)*e)^(1/2))*c^5*d^6*e^4*x^4+150*arctanh(e*(c*d*x+a*e)^(1/2)/((a*e^2-c*
d^2)*e)^(1/2))*c^5*d^7*e^3*x^3+150*arctanh(e*(c*d*x+a*e)^(1/2)/((a*e^2-c*d
^2)*e)^(1/2))*c^5*d^8*e^2*x^2-15*c^4*d^4*e^4*x^4*(c*d*x+a*e)^(1/2)*((a*e^2
-c*d^2)*e)^(1/2)+75*arctanh(e*(c*d*x+a*e)^(1/2)/((a*e^2-c*d^2)*e)^(1/2))*c
^5*d^9*e*x+10*a*c^3*d^3*e^5*x^3*(c*d*x+a*e)^(1/2)*((a*e^2-c*d^2)*e)^(1/2)-
70*c^4*d^5*e^3*x^3*(c*d*x+a*e)^(1/2)*((a*e^2-c*d^2)*e)^(1/2)+15*arctanh(e*
(c*d*x+a*e)^(1/2)/((a*e^2-c*d^2)*e)^(1/2))*c^5*d^10+248*a^2*c^2*d^2*e^6*x^
2*(c*d*x+a*e)^(1/2)*((a*e^2-c*d^2)*e)^(1/2)-466*a*c^3*d^4*e^4*x^2*(c*d*x+a
*e)^(1/2)*((a*e^2-c*d^2)*e)^(1/2)+128*c^4*d^6*e^2*x^2*(c*d*x+a*e)^(1/2)*((
a*e^2-c*d^2)*e)^(1/2)+336*a^3*c*d*e^7*x*(c*d*x+a*e)^(1/2)*((a*e^2-c*d^2)*e
)^(1/2)-512*a^2*c^2*d^3*e^5*x*(c*d*x+a*e)^(1/2)*((a*e^2-c*d^2)*e)^(1/2)+46
*a*c^3*d^5*e^3*x*(c*d*x+a*e)^(1/2)*((a*e^2-c*d^2)*e)^(1/2)+70*c^4*d^7*e*x*
(c*d*x+a*e)^(1/2)*((a*e^2-c*d^2)*e)^(1/2)+128*(c*d*x+a*e)^(1/2)*((a*e^2-c*
d^2)*e)^(1/2)*a^4*e^8-176*(c*d*x+a*e)^(1/2)*((a*e^2-c*d^2)*e)^(1/2)*a^3*c*
d^2*e^6+8*(c*d*x+a*e)^(1/2)*((a*e^2-c*d^2)*e)^(1/2)*a^2*c^2*d^4*e^4+10*(c*
d*x+a*e)^(1/2)*((a*e^2-c*d^2)*e)^(1/2)*a*c^3*d^6*e^2+15*(c*d*x+a*e)^(1/2)*
((a*e^2-c*d^2)*e)^(1/2)*c^4*d^8)/(e*x+d)^(11/2)/((a*e^2-c*d^2)*e)^(1/2)/e^
3/(a*e^2-c*d^2)^2/(c*d*x+a*e)^(1/2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 832 vs. $2(322) = 644$.

Time = 0.17 (sec) , antiderivative size = 1686, normalized size of antiderivative = 4.61

$$\int \frac{(ade + (cd^2 + ae^2)x + cdex^2)^{5/2}}{(d+ex)^{17/2}}\,dx = \text{Too large to display}$$

input
```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(e*x+d)^(17/2),x, algori
thm="fricas")
```

output
```
[-1/1280*(15*(c^5*d^5*e^6*x^6 + 6*c^5*d^6*e^5*x^5 + 15*c^5*d^7*e^4*x^4 + 2
0*c^5*d^8*e^3*x^3 + 15*c^5*d^9*e^2*x^2 + 6*c^5*d^10*e*x + c^5*d^11)*sqrt(-
c*d^2*e + a*e^3)*log(-(c*d*e^2*x^2 + 2*a*e^3*x - c*d^3 + 2*a*d*e^2 - 2*sqr
t(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(-c*d^2*e + a*e^3)*sqrt(e*x +
 d))/(e^2*x^2 + 2*d*e*x + d^2)) + 2*(15*c^5*d^10*e - 5*a*c^4*d^8*e^3 - 2*a
^2*c^3*d^6*e^5 - 184*a^3*c^2*d^4*e^7 + 304*a^4*c*d^2*e^9 - 128*a^5*e^11 -
15*(c^5*d^6*e^5 - a*c^4*d^4*e^7)*x^4 - 10*(7*c^5*d^7*e^4 - 8*a*c^4*d^5*e^6
 + a^2*c^3*d^3*e^8)*x^3 + 2*(64*c^5*d^8*e^3 - 297*a*c^4*d^6*e^5 + 357*a^2*
c^3*d^4*e^7 - 124*a^3*c^2*d^2*e^9)*x^2 + 2*(35*c^5*d^9*e^2 - 12*a*c^4*d^7*
e^4 - 279*a^2*c^3*d^5*e^6 + 424*a^3*c^2*d^3*e^8 - 168*a^4*c*d*e^10)*x)*sqr
t(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(e*x + d))/(c^3*d^12*e^4 - 3*
a*c^2*d^10*e^6 + 3*a^2*c*d^8*e^8 - a^3*d^6*e^10 + (c^3*d^6*e^10 - 3*a*c^2*
d^4*e^12 + 3*a^2*c*d^2*e^14 - a^3*e^16)*x^6 + 6*(c^3*d^7*e^9 - 3*a*c^2*d^5
*e^11 + 3*a^2*c*d^3*e^13 - a^3*d*e^15)*x^5 + 15*(c^3*d^8*e^8 - 3*a*c^2*d^6
*e^10 + 3*a^2*c*d^4*e^12 - a^3*d^2*e^14)*x^4 + 20*(c^3*d^9*e^7 - 3*a*c^2*d
^7*e^9 + 3*a^2*c*d^5*e^11 - a^3*d^3*e^13)*x^3 + 15*(c^3*d^10*e^6 - 3*a*c^2
*d^8*e^8 + 3*a^2*c*d^6*e^10 - a^3*d^4*e^12)*x^2 + 6*(c^3*d^11*e^5 - 3*a*c^
2*d^9*e^7 + 3*a^2*c*d^7*e^9 - a^3*d^5*e^11)*x), -1/640*(15*(c^5*d^5*e^6*x^
6 + 6*c^5*d^6*e^5*x^5 + 15*c^5*d^7*e^4*x^4 + 20*c^5*d^8*e^3*x^3 + 15*c^5*d
^9*e^2*x^2 + 6*c^5*d^10*e*x + c^5*d^11)*sqrt(c*d^2*e - a*e^3)*arctan(-s...
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(ade + (cd^2 + ae^2)x + cdex^2)^{5/2}}{(d+ex)^{17/2}}\,dx = \text{Timed out}$$

input
```
integrate((a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(5/2)/(e*x+d)**(17/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{(ade + (cd^2 + ae^2)x + cdex^2)^{5/2}}{(d+ex)^{17/2}}\,dx = \int \frac{(cdex^2 + ade + (cd^2 + ae^2)x)^{\frac{5}{2}}}{(ex+d)^{\frac{17}{2}}}\,dx$$

input
```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(e*x+d)^(17/2),x, algori
thm="maxima")
```

output
```
integrate((c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^(5/2)/(e*x + d)^(17/2),
x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 683 vs. $2(322) = 644$.

Time = 0.22 (sec) , antiderivative size = 683, normalized size of antiderivative = 1.87

$$\int \frac{(ade + (cd^2 + ae^2)x + cdex^2)^{5/2}}{(d+ex)^{17/2}}\,dx = \text{Too large to display}$$

input
```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(e*x+d)^(17/2),x, algori
thm="giac")
```

output
```
1/640*c^5*d^5*e*(15*abs(e)*arctan(sqrt((e*x + d)*c*d*e - c*d^2*e + a*e^3)/
sqrt(c*d^2*e - a*e^3))/((c^2*d^4*e^5 - 2*a*c*d^2*e^7 + a^2*e^9)*sqrt(c*d^2
*e - a*e^3)) - (15*sqrt((e*x + d)*c*d*e - c*d^2*e + a*e^3)*c^4*d^8*e^4*abs
(e) - 60*sqrt((e*x + d)*c*d*e - c*d^2*e + a*e^3)*a*c^3*d^6*e^6*abs(e) + 90
*sqrt((e*x + d)*c*d*e - c*d^2*e + a*e^3)*a^2*c^2*d^4*e^8*abs(e) - 60*sqrt(
(e*x + d)*c*d*e - c*d^2*e + a*e^3)*a^3*c*d^2*e^10*abs(e) + 15*sqrt((e*x +
d)*c*d*e - c*d^2*e + a*e^3)*a^4*e^12*abs(e) + 70*((e*x + d)*c*d*e - c*d^2*
e + a*e^3)^(3/2)*c^3*d^6*e^3*abs(e) - 210*((e*x + d)*c*d*e - c*d^2*e + a*e
^3)^(3/2)*a*c^2*d^4*e^5*abs(e) + 210*((e*x + d)*c*d*e - c*d^2*e + a*e^3)^(
3/2)*a^2*c*d^2*e^7*abs(e) - 70*((e*x + d)*c*d*e - c*d^2*e + a*e^3)^(3/2)*a
^3*e^9*abs(e) + 128*((e*x + d)*c*d*e - c*d^2*e + a*e^3)^(5/2)*c^2*d^4*e^2*
abs(e) - 256*((e*x + d)*c*d*e - c*d^2*e + a*e^3)^(5/2)*a*c*d^2*e^4*abs(e)
+ 128*((e*x + d)*c*d*e - c*d^2*e + a*e^3)^(5/2)*a^2*e^6*abs(e) - 70*((e*x
+ d)*c*d*e - c*d^2*e + a*e^3)^(7/2)*c*d^2*e*abs(e) + 70*((e*x + d)*c*d*e -
 c*d^2*e + a*e^3)^(7/2)*a*e^3*abs(e) - 15*((e*x + d)*c*d*e - c*d^2*e + a*e
^3)^(9/2)*abs(e))/((c^2*d^4*e^5 - 2*a*c*d^2*e^7 + a^2*e^9)*(e*x + d)^5*c^5
*d^5*e^5))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(ade + (cd^2 + ae^2)x + cdex^2)^{5/2}}{(d + ex)^{17/2}}\,dx = \int \frac{(c\,d\,e\,x^2 + (c\,d^2 + a\,e^2)\,x + a\,d\,e)^{5/2}}{(d + e\,x)^{17/2}}\,dx$$

input
```
int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(5/2)/(d + e*x)^(17/2),x)
```

output
```
int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(5/2)/(d + e*x)^(17/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 1128, normalized size of antiderivative = 3.08

$$\int \frac{(ade + (cd^2 + ae^2) x + cdex^2)^{5/2}}{(d + ex)^{17/2}} dx = \text{Too large to display}$$

input
```
int((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(e*x+d)^(17/2),x)
```

output
```
( - 15*sqrt(e)*sqrt( - a*e**2 + c*d**2)*atan((sqrt(a*e + c*d*x)*e)/(sqrt(e
)*sqrt( - a*e**2 + c*d**2)))*c**5*d**10 - 75*sqrt(e)*sqrt( - a*e**2 + c*d*
*2)*atan((sqrt(a*e + c*d*x)*e)/(sqrt(e)*sqrt( - a*e**2 + c*d**2)))*c**5*d*
*9*e*x - 150*sqrt(e)*sqrt( - a*e**2 + c*d**2)*atan((sqrt(a*e + c*d*x)*e)/(
sqrt(e)*sqrt( - a*e**2 + c*d**2)))*c**5*d**8*e**2*x**2 - 150*sqrt(e)*sqrt(
 - a*e**2 + c*d**2)*atan((sqrt(a*e + c*d*x)*e)/(sqrt(e)*sqrt( - a*e**2 + c
*d**2)))*c**5*d**7*e**3*x**3 - 75*sqrt(e)*sqrt( - a*e**2 + c*d**2)*atan((s
qrt(a*e + c*d*x)*e)/(sqrt(e)*sqrt( - a*e**2 + c*d**2)))*c**5*d**6*e**4*x**
4 - 15*sqrt(e)*sqrt( - a*e**2 + c*d**2)*atan((sqrt(a*e + c*d*x)*e)/(sqrt(e
)*sqrt( - a*e**2 + c*d**2)))*c**5*d**5*e**5*x**5 - 128*sqrt(a*e + c*d*x)*a
**5*e**11 + 304*sqrt(a*e + c*d*x)*a**4*c*d**2*e**9 - 336*sqrt(a*e + c*d*x)
*a**4*c*d*e**10*x - 184*sqrt(a*e + c*d*x)*a**3*c**2*d**4*e**7 + 848*sqrt(a
*e + c*d*x)*a**3*c**2*d**3*e**8*x - 248*sqrt(a*e + c*d*x)*a**3*c**2*d**2*e
**9*x**2 - 2*sqrt(a*e + c*d*x)*a**2*c**3*d**6*e**5 - 558*sqrt(a*e + c*d*x)
*a**2*c**3*d**5*e**6*x + 714*sqrt(a*e + c*d*x)*a**2*c**3*d**4*e**7*x**2 -
10*sqrt(a*e + c*d*x)*a**2*c**3*d**3*e**8*x**3 - 5*sqrt(a*e + c*d*x)*a*c**4
*d**8*e**3 - 24*sqrt(a*e + c*d*x)*a*c**4*d**7*e**4*x - 594*sqrt(a*e + c*d*
x)*a*c**4*d**6*e**5*x**2 + 80*sqrt(a*e + c*d*x)*a*c**4*d**5*e**6*x**3 + 15
*sqrt(a*e + c*d*x)*a*c**4*d**4*e**7*x**4 + 15*sqrt(a*e + c*d*x)*c**5*d**10
*e + 70*sqrt(a*e + c*d*x)*c**5*d**9*e**2*x + 128*sqrt(a*e + c*d*x)*c**5...
```

3.308 $\int \frac{(d+ex)^{7/2}}{\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\,dx$

Optimal result

Integrand size = 39, antiderivative size = 236

$$\int \frac{(d+ex)^{7/2}}{\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\,dx = \frac{2(cd^2-ae^2)^3\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{c^4d^4\sqrt{d+ex}} + \frac{2e(cd^2-ae^2)^2(ade+(cd^2+ae^2)x+cdex^2)^{3/2}}{c^4d^4(d+ex)^{3/2}} + \frac{6e^2(cd^2-ae^2)(ade+(cd^2+ae^2)x+cdex^2)^{5/2}}{5c^4d^4(d+ex)^{5/2}} + \frac{2e^3(ade+(cd^2+ae^2)x+cdex^2)^{7/2}}{7c^4d^4(d+ex)^{7/2}}$$

output
```
2*(-a*e^2+c*d^2)^3*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/c^4/d^4/(e*x+d)
^(1/2)+2*e*(-a*e^2+c*d^2)^2*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/c^4/d^
4/(e*x+d)^(3/2)+6/5*e^2*(-a*e^2+c*d^2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(
5/2)/c^4/d^4/(e*x+d)^(5/2)+2/7*e^3*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(7/2)
/c^4/d^4/(e*x+d)^(7/2)
```

Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 131, normalized size of antiderivative = 0.56

$$\int \frac{(d+ex)^{7/2}}{\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\,dx = \frac{2\sqrt{(ae+cdx)(d+ex)}(-16a^3e^6+8a^2cde^4(7d+ex)-2ac^2d^2e^2(35}{35c^4d^4\sqrt{}}$$

input
```
Integrate[(d + e*x)^(7/2)/Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2],x]
```

output
```
(2*Sqrt[(a*e + c*d*x)*(d + e*x)]*(-16*a^3*e^6 + 8*a^2*c*d*e^4*(7*d + e*x)
- 2*a*c^2*d^2*e^2*(35*d^2 + 14*d*e*x + 3*e^2*x^2) + c^3*d^3*(35*d^3 + 35*d
^2*e*x + 21*d*e^2*x^2 + 5*e^3*x^3)))/(35*c^4*d^4*Sqrt[d + e*x])
```

Rubi [A] (verified)

Time = 0.66 (sec) , antiderivative size = 248, normalized size of antiderivative = 1.05, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.103$, Rules used = {1128, 1128, 1128, 1122}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex)^{7/2}}{\sqrt{x(ae^2+cd^2)+ade+cdex^2}}dx$$

$$\Big\downarrow \text{ 1128}$$

$$\frac{6\left(d^2-\frac{ae^2}{c}\right)\int \frac{(d+ex)^{5/2}}{\sqrt{cdex^2+(cd^2+ae^2)x+ade}}dx}{7d}+\frac{2(d+ex)^{5/2}\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{7cd}$$

$$\Big\downarrow \text{ 1128}$$

$$\frac{6\left(d^2-\frac{ae^2}{c}\right)\left(\frac{4\left(d^2-\frac{ae^2}{c}\right)\int \frac{(d+ex)^{3/2}}{\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}dx}{5d}+\frac{2(d+ex)^{3/2}\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{5cd}\right)}{7d}+$$
$$\frac{2(d+ex)^{5/2}\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{7cd}$$

$$\Big\downarrow \text{1128}$$

$$\frac{6\left(d^2-\frac{ae^2}{c}\right)\left(\frac{4\left(d^2-\frac{ae^2}{c}\right)\left(\frac{2\left(d^2-\frac{ae^2}{c}\right)\int \frac{\sqrt{d+ex}}{\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}dx}{3d}+\frac{2\sqrt{d+ex}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{3cd}\right)}{5d}+\frac{2(d+ex)^{3/2}\sqrt{x(ae^2+cd^2)+\cdots}}{5cd}\right)}{7d}$$

$$\frac{2(d+ex)^{5/2}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{7cd}$$

$$\Big\downarrow \text{1122}$$

$$\frac{2(d+ex)^{5/2}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{7cd}+$$

$$\frac{6\left(d^2-\frac{ae^2}{c}\right)\left(\frac{2(d+ex)^{3/2}\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{5cd}+\frac{4\left(d^2-\frac{ae^2}{c}\right)\left(\frac{4\left(d^2-\frac{ae^2}{c}\right)\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{3cd^2\sqrt{d+ex}}+\frac{2\sqrt{d+ex}\sqrt{x\left(ae^2+cd^2\right)+ade+c\cdots}}{3cd}\right)}{5d}\right)}{7d}$$

input
```
Int[(d + e*x)^(7/2)/Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2],x]
```

output
```
(2*(d + e*x)^(5/2)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(7*c*d) +
(6*(d^2 - (a*e^2)/c)*((2*(d + e*x)^(3/2)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x +
c*d*e*x^2])/(5*c*d) + (4*(d^2 - (a*e^2)/c)*((4*(d^2 - (a*e^2)/c)*Sqrt[a*d*
e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(3*c*d^2*Sqrt[d + e*x]) + (2*Sqrt[d +
e*x]*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(3*c*d)))/(5*d)))/(7*d)
```

Defintions of rubi rules used

rule 1122
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[e*(d + e*x)^(m - 1)*((a + b*x + c*x^2)^(p + 1)/(c*(p + 1))),
 x] /; FreeQ[{a, b, c, d, e, m, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&
EqQ[m + p, 0]
```

rule 1128
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[e*(d + e*x)^(m - 1)*((a + b*x + c*x^2)^(p + 1)/(c*(m + 2*p +
 1))), x] + Simp[Simplify[m + p]*((2*c*d - b*e)/(c*(m + 2*p + 1)))   Int[(d
 + e*x)^(m - 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p},
 x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && IGtQ[Simplify[m + p], 0]
```

Maple [A] (verified)

Time = 1.06 (sec) , antiderivative size = 150, normalized size of antiderivative = 0.64

method	result
default	$-\frac{2\sqrt{(ex+d)(cdx+ae)}\,(-5c^3d^3e^3x^3+6x^2a\,c^2d^2e^4-21c^3d^4e^2x^2-8x\,a^2cd\,e^5+28xa\,c^2d^3e^3-35c^3d^5ex+16e^6a^3-56d^2e^4a^2c+70d^4e}{35\sqrt{ex+d}\,d^4c^4}$
gosper	$-\frac{2(cdx+ae)(-5c^3d^3e^3x^3+6x^2a\,c^2d^2e^4-21c^3d^4e^2x^2-8x\,a^2cd\,e^5+28xa\,c^2d^3e^3-35c^3d^5ex+16e^6a^3-56d^2e^4a^2c+70d^4e^2a\,c^2-35a}{35d^4c^4\sqrt{cd\,x^2e+a\,e^2x+c\,d^2x+ade}}$
orering	$-\frac{2(-5c^3d^3e^3x^3+6x^2a\,c^2d^2e^4-21c^3d^4e^2x^2-8x\,a^2cd\,e^5+28xa\,c^2d^3e^3-35c^3d^5ex+16e^6a^3-56d^2e^4a^2c+70d^4e^2a\,c^2-35d^6c^3)(cdx}{35d^4c^4\sqrt{ade+(a\,e^2+c\,d^2)x+cd\,x^2e}}$

input
```
int((e*x+d)^(7/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(1/2),x,method=_RETURN
VERBOSE)
```

output
```
-2/35/(e*x+d)^(1/2)*((e*x+d)*(c*d*x+a*e))^(1/2)*(-5*c^3*d^3*e^3*x^3+6*a*c^
2*d^2*e^4*x^2-21*c^3*d^4*e^2*x^2-8*a^2*c*d*e^5*x+28*a*c^2*d^3*e^3*x-35*c^3
*d^5*e*x+16*a^3*e^6-56*a^2*c*d^2*e^4+70*a*c^2*d^4*e^2-35*c^3*d^6)/d^4/c^4
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 173, normalized size of antiderivative = 0.73

$$\int \frac{(d+ex)^{7/2}}{\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}\,dx = \frac{2\left(5\,c^3d^3e^3x^3+35\,c^3d^6-70\,ac^2d^4e^2+56\,a^2cd^2e^4-16\,a^3e^6+3\,(7\,c\right.}{}$$

input
```
integrate((e*x+d)^(7/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x, algorit
hm="fricas")
```

output
```
2/35*(5*c^3*d^3*e^3*x^3 + 35*c^3*d^6 - 70*a*c^2*d^4*e^2 + 56*a^2*c*d^2*e^4
 - 16*a^3*e^6 + 3*(7*c^3*d^4*e^2 - 2*a*c^2*d^2*e^4)*x^2 + (35*c^3*d^5*e -
28*a*c^2*d^3*e^3 + 8*a^2*c*d*e^5)*x)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e
^2)*x)*sqrt(e*x + d)/(c^4*d^4*e*x + c^4*d^5)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(d+ex)^{7/2}}{\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}\,dx = \text{Timed out}$$

input
```
integrate((e*x+d)**(7/2)/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(1/2),x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 192, normalized size of antiderivative = 0.81

$$\int \frac{(d+ex)^{7/2}}{\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}\,dx = \frac{2\,(5\,c^4d^4e^3x^4+35\,ac^3d^6e-70\,a^2c^2d^4e^3+56\,a^3cd^2e^5-16\,a^4e^7+(2$$

input
```
integrate((e*x+d)^(7/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x, algorit
hm="maxima")
```

output
```
2/35*(5*c^4*d^4*e^3*x^4 + 35*a*c^3*d^6*e - 70*a^2*c^2*d^4*e^3 + 56*a^3*c*d
^2*e^5 - 16*a^4*e^7 + (21*c^4*d^5*e^2 - a*c^3*d^3*e^4)*x^3 + (35*c^4*d^6*e
 - 7*a*c^3*d^4*e^3 + 2*a^2*c^2*d^2*e^5)*x^2 + (35*c^4*d^7 - 35*a*c^3*d^5*e
^2 + 28*a^2*c^2*d^3*e^4 - 8*a^3*c*d*e^6)*x)/(sqrt(c*d*x + a*e)*c^4*d^4)
```

Giac [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 282, normalized size of antiderivative = 1.19

$$\int \frac{(d+ex)^{7/2}}{\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\,dx = \frac{2\,e\left(\frac{35\,(c^3d^6-3\,ac^2d^4e^2+3\,a^2cd^2e^4-a^3e^6)\sqrt{(ex+d)cde-cd^2e+ae^3}}{c^4d^4e}+\frac{35\,((ex+d)cde-c}{}\right.}{}$$

input
```
integrate((e*x+d)^(7/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x, algorit
hm="giac")
```

output
```
2/35*e*(35*(c^3*d^6 - 3*a*c^2*d^4*e^2 + 3*a^2*c*d^2*e^4 - a^3*e^6)*sqrt((e
*x + d)*c*d*e - c*d^2*e + a*e^3)/(c^4*d^4*e) + (35*((e*x + d)*c*d*e - c*d^
2*e + a*e^3)^(3/2)*c^2*d^4*e^2 - 70*((e*x + d)*c*d*e - c*d^2*e + a*e^3)^(3
/2)*a*c*d^2*e^4 + 35*((e*x + d)*c*d*e - c*d^2*e + a*e^3)^(3/2)*a^2*e^6 + 2
1*((e*x + d)*c*d*e - c*d^2*e + a*e^3)^(5/2)*c*d^2*e - 21*((e*x + d)*c*d*e
- c*d^2*e + a*e^3)^(5/2)*a*e^3 + 5*((e*x + d)*c*d*e - c*d^2*e + a*e^3)^(7/
2))/(c^4*d^4*e^4))/abs(e)
```

Mupad [B] (verification not implemented)

Time = 5.75 (sec) , antiderivative size = 194, normalized size of antiderivative = 0.82

$$\int \frac{(d+ex)^{7/2}}{\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\,dx =$$
$$-\frac{\sqrt{c\,d\,e\,x^2+(c\,d^2+a\,e^2)\,x+a\,d\,e}\left(\frac{\sqrt{d+e\,x}\,(32\,a^3\,e^6-112\,a^2\,c\,d^2\,e^4+140\,a\,c^2\,d^4\,e^2-70\,c^3\,d^6)}{35\,c^4\,d^4\,e}-\frac{2\,x\,\sqrt{d+e\,x}\,(8\,a^2\,e^4-28\,a\,c\,d^2\,e^2}{35\,c^3\,d^3}\right.}{x+\frac{d}{e}}$$

input
```
int((d + e*x)^(7/2)/(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2),x)
```

output
```
-((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2)*(((d + e*x)^(1/2)*(32*a^3*
e^6 - 70*c^3*d^6 + 140*a*c^2*d^4*e^2 - 112*a^2*c*d^2*e^4))/(35*c^4*d^4*e)
- (2*x*(d + e*x)^(1/2)*(8*a^2*e^4 + 35*c^2*d^4 - 28*a*c*d^2*e^2))/(35*c^3*
d^3) - (2*e^2*x^3*(d + e*x)^(1/2))/(7*c*d) + (6*e*x^2*(2*a*e^2 - 7*c*d^2)*
(d + e*x)^(1/2))/(35*c^2*d^2)))/(x + d/e)
```

Reduce [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 135, normalized size of antiderivative = 0.57

$$\int \frac{(d+ex)^{7/2}}{\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}\,dx = \frac{2\sqrt{cdx+ae}\,(5c^3d^3e^3x^3-6a\,c^2d^2e^4x^2+21c^3d^4e^2x^2+8a^2cd\,e^5x-}{35}$$

input
```
int((e*x+d)^(7/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x)
```

output
```
(2*sqrt(a*e + c*d*x)*( - 16*a**3*e**6 + 56*a**2*c*d**2*e**4 + 8*a**2*c*d*e
**5*x - 70*a*c**2*d**4*e**2 - 28*a*c**2*d**3*e**3*x - 6*a*c**2*d**2*e**4*x
**2 + 35*c**3*d**6 + 35*c**3*d**5*e*x + 21*c**3*d**4*e**2*x**2 + 5*c**3*d*
*3*e**3*x**3))/(35*c**4*d**4)
```

3.309 $\int \frac{(d+ex)^{5/2}}{\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\,dx$

Optimal result

Integrand size = 39, antiderivative size = 173

$$\int \frac{(d+ex)^{5/2}}{\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\,dx=\frac{2(cd^2-ae^2)^2\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{c^3d^3\sqrt{d+ex}}+\frac{4e(cd^2-ae^2)\left(ade+(cd^2+ae^2)x+cdex^2\right)^{3/2}}{3c^3d^3(d+ex)^{3/2}}+\frac{2e^2\left(ade+(cd^2+ae^2)x+cdex^2\right)^{5/2}}{5c^3d^3(d+ex)^{5/2}}$$

output
```
2*(-a*e^2+c*d^2)^2*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/c^3/d^3/(e*x+d)
^(1/2)+4/3*e*(-a*e^2+c*d^2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/c^3/d^
3/(e*x+d)^(3/2)+2/5*e^2*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/c^3/d^3/(e
*x+d)^(5/2)
```

Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.50

$$\int \frac{(d+ex)^{5/2}}{\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\,dx=\frac{2\sqrt{(ae+cdx)(d+ex)}\left(8a^2e^4-4acde^2(5d+ex)+c^2d^2(15d^2+10d\right.}{15c^3d^3\sqrt{d+ex}}$$

input
```
Integrate[(d + e*x)^(5/2)/Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2],x]
```

output
```
(2*Sqrt[(a*e + c*d*x)*(d + e*x)]*(8*a^2*e^4 - 4*a*c*d*e^2*(5*d + e*x) + c^
2*d^2*(15*d^2 + 10*d*e*x + 3*e^2*x^2)))/(15*c^3*d^3*Sqrt[d + e*x])
```

Rubi [A] (verified)

Time = 0.51 (sec) , antiderivative size = 179, normalized size of antiderivative = 1.03, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {1128, 1128, 1122}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex)^{5/2}}{\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}dx$$

$$\Big\downarrow \text{ 1128}$$

$$\frac{4\left(d^2-\frac{ae^2}{c}\right)\int \frac{(d+ex)^{3/2}}{\sqrt{cdex^2+(cd^2+ae^2)x+ade}}dx}{5d}+\frac{2(d+ex)^{3/2}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{5cd}$$

$$\Big\downarrow \text{ 1128}$$

$$\frac{4\left(d^2-\frac{ae^2}{c}\right)\left(\frac{2\left(d^2-\frac{ae^2}{c}\right)\int \frac{\sqrt{d+ex}}{\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}dx}{3d}+\frac{2\sqrt{d+ex}\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{3cd}\right)}{5d}+\frac{2(d+ex)^{3/2}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{5cd}$$

$$\Big\downarrow \text{ 1122}$$

$$\frac{2(d+ex)^{3/2}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{5cd}+\frac{4\left(d^2-\frac{ae^2}{c}\right)\left(\frac{4\left(d^2-\frac{ae^2}{c}\right)\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{3cd^2\sqrt{d+ex}}+\frac{2\sqrt{d+ex}\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{3cd}\right)}{5d}$$

input
```
Int[(d + e*x)^(5/2)/Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2],x]
```

output
```
(2*(d + e*x)^(3/2)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(5*c*d) +
(4*(d^2 - (a*e^2)/c)*((4*(d^2 - (a*e^2)/c)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x
+ c*d*e*x^2])/(3*c*d^2*Sqrt[d + e*x]) + (2*Sqrt[d + e*x]*Sqrt[a*d*e + (c*d
^2 + a*e^2)*x + c*d*e*x^2])/(3*c*d)))/(5*d)
```

Defintions of rubi rules used

rule 1122
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[e*(d + e*x)^(m - 1)*((a + b*x + c*x^2)^(p + 1)/(c*(p + 1))),
 x] /; FreeQ[{a, b, c, d, e, m, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&
EqQ[m + p, 0]
```

rule 1128
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[e*(d + e*x)^(m - 1)*((a + b*x + c*x^2)^(p + 1)/(c*(m + 2*p +
 1))), x] + Simp[Simplify[m + p]*((2*c*d - b*e)/(c*(m + 2*p + 1)))   Int[(d
 + e*x)^(m - 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p},
 x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && IGtQ[Simplify[m + p], 0]
```

Maple [A] (verified)

Time = 1.07 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.53

method	result	size
default	$\frac{2\sqrt{(ex+d)(cdx+ae)}\,(3x^2c^2d^2e^2-4xacd\,e^3+10x\,c^2d^3e+8a^2e^4-20ac\,d^2e^2+15c^2d^4)}{15\sqrt{ex+d}\,d^3c^3}$	92
gosper	$\frac{2(cdx+ae)(3x^2c^2d^2e^2-4xacd\,e^3+10x\,c^2d^3e+8a^2e^4-20ac\,d^2e^2+15c^2d^4)\sqrt{ex+d}}{15d^3c^3\sqrt{cd\,x^2e+a\,e^2x+c\,d^2x+ade}}$	110
orering	$\frac{2(3x^2c^2d^2e^2-4xacd\,e^3+10x\,c^2d^3e+8a^2e^4-20ac\,d^2e^2+15c^2d^4)(cdx+ae)\sqrt{ex+d}}{15d^3c^3\sqrt{ade+(a\,e^2+c\,d^2)x+cd\,x^2e}}$	111

input
```
int((e*x+d)^(5/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(1/2),x,method=_RETURN
VERBOSE)
```

output
```
2/15/(e*x+d)^(1/2)*((e*x+d)*(c*d*x+a*e))^(1/2)*(3*c^2*d^2*e^2*x^2-4*a*c*d*
e^3*x+10*c^2*d^3*e*x+8*a^2*e^4-20*a*c*d^2*e^2+15*c^2*d^4)/d^3/c^3
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 117, normalized size of antiderivative = 0.68

$$\int \frac{(d+ex)^{5/2}}{\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}\,dx = \frac{2\,(3\,c^2d^2e^2x^2+15\,c^2d^4-20\,acd^2e^2+8\,a^2e^4+2\,(5\,c^2d^3e-2\,acde^3)}{15\,(c^3d^3ex+c^3d^4)}$$

input
```
integrate((e*x+d)^(5/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x, algorit
hm="fricas")
```

output
```
2/15*(3*c^2*d^2*e^2*x^2 + 15*c^2*d^4 - 20*a*c*d^2*e^2 + 8*a^2*e^4 + 2*(5*c
^2*d^3*e - 2*a*c*d*e^3)*x)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*sqr
t(e*x + d)/(c^3*d^3*e*x + c^3*d^4)
```

Sympy [F]

$$\int \frac{(d+ex)^{5/2}}{\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}\,dx = \int \frac{(d+ex)^{\frac{5}{2}}}{\sqrt{(d+ex)\,(ae+cdx)}}\,dx$$

input
```
integrate((e*x+d)**(5/2)/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(1/2),x)
```

output
```
Integral((d + e*x)**(5/2)/sqrt((d + e*x)*(a*e + c*d*x)), x)
```

Maxima [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 122, normalized size of antiderivative = 0.71

$$\int \frac{(d+ex)^{5/2}}{\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}\,dx = \frac{2\,(3\,c^3d^3e^2x^3+15\,ac^2d^4e-20\,a^2cd^2e^3+8\,a^3e^5+(10\,c^3d^4e-ac^2d^2}{15\,\sqrt{cdx+ae}c^3d^3}$$

input
```
integrate((e*x+d)^(5/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x, algorit
hm="maxima")
```

output
```
2/15*(3*c^3*d^3*e^2*x^3 + 15*a*c^2*d^4*e - 20*a^2*c*d^2*e^3 + 8*a^3*e^5 +
(10*c^3*d^4*e - a*c^2*d^2*e^3)*x^2 + (15*c^3*d^5 - 10*a*c^2*d^3*e^2 + 4*a^
2*c*d*e^4)*x)/(sqrt(c*d*x + a*e)*c^3*d^3)
```

Giac [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 166, normalized size of antiderivative = 0.96

$$\int \frac{(d+ex)^{5/2}}{\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}\,dx = \frac{2\,e\left(\frac{15\,(c^2d^4-2\,acd^2e^2+a^2e^4)\,\sqrt{(ex+d)cde-cd^2e+ae^3}}{c^3d^3e}+\frac{10\,((ex+d)cde-cd^2e+ae^3)^{\frac{3}{2}}\,c}{}\right.}{15\,|e|}$$

input
```
integrate((e*x+d)^(5/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x, algorit
hm="giac")
```

output
```
2/15*e*(15*(c^2*d^4 - 2*a*c*d^2*e^2 + a^2*e^4)*sqrt((e*x + d)*c*d*e - c*d^
2*e + a*e^3)/(c^3*d^3*e) + (10*((e*x + d)*c*d*e - c*d^2*e + a*e^3)^(3/2)*c
*d^2*e - 10*((e*x + d)*c*d*e - c*d^2*e + a*e^3)^(3/2)*a*e^3 + 3*((e*x + d)
*c*d*e - c*d^2*e + a*e^3)^(5/2))/(c^3*d^3*e^3))/abs(e)
```

Mupad [B] (verification not implemented)

Time = 5.95 (sec) , antiderivative size = 131, normalized size of antiderivative = 0.76

$$\int \frac{(d+ex)^{5/2}}{\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}\,dx = \frac{\sqrt{c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e}\,\left(\frac{2\,e\,x^2\,\sqrt{d+e\,x}}{5\,c\,d}-\frac{4\,x\,\left(2\,a\,e^2-5\,c\,d^2\right)\,\sqrt{d+e}}{15\,c^2\,d^2}\right.}{x+\frac{d}{e}}$$

input
```
int((d + e*x)^(5/2)/(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2),x)
```

output
```
((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2)*((2*e*x^2*(d + e*x)^(1/2))/
(5*c*d) - (4*x*(2*a*e^2 - 5*c*d^2)*(d + e*x)^(1/2))/(15*c^2*d^2) + ((d + e
*x)^(1/2)*(16*a^2*e^4 + 30*c^2*d^4 - 40*a*c*d^2*e^2))/(15*c^3*d^3*e)))/(x
+ d/e)
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.45

$$\int \frac{(d+ex)^{5/2}}{\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}\,dx = \frac{2\sqrt{cdx+ae}\,(3c^2d^2e^2x^2-4acd\,e^3x+10c^2d^3ex+8a^2e^4-20ac\,d^2e^2}{15c^3d^3}$$

input
```
int((e*x+d)^(5/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x)
```

output
```
(2*sqrt(a*e + c*d*x)*(8*a**2*e**4 - 20*a*c*d**2*e**2 - 4*a*c*d*e**3*x + 15
*c**2*d**4 + 10*c**2*d**3*e*x + 3*c**2*d**2*e**2*x**2))/(15*c**3*d**3)
```

3.310 $\int \frac{(d+ex)^{3/2}}{\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\,dx$

Optimal result

Integrand size = 39, antiderivative size = 108

$$\int \frac{(d+ex)^{3/2}}{\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}\,dx = \frac{2(cd^2-ae^2)\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{c^2d^2\sqrt{d+ex}} + \frac{2e(ade+(cd^2+ae^2)\,x+cdex^2)^{3/2}}{3c^2d^2(d+ex)^{3/2}}$$

output
```
2*(-a*e^2+c*d^2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/c^2/d^2/(e*x+d)^(
1/2)+2/3*e*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/c^2/d^2/(e*x+d)^(3/2)
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.50

$$\int \frac{(d+ex)^{3/2}}{\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}\,dx = \frac{2\sqrt{(ae+cdx)(d+ex)}(-2ae^2+cd(3d+ex))}{3c^2d^2\sqrt{d+ex}}$$

input
```
Integrate[(d + e*x)^(3/2)/Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2],x]
```

output
```
(2*Sqrt[(a*e + c*d*x)*(d + e*x)]*(-2*a*e^2 + c*d*(3*d + e*x)))/(3*c^2*d^2*
Sqrt[d + e*x])
```

Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 110, normalized size of antiderivative = 1.02, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.051, Rules used = {1128, 1122}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex)^{3/2}}{\sqrt{x(ae^2+cd^2)+ade+cdex^2}}dx$$

$$\Big\downarrow \text{ 1128}$$

$$\frac{2\left(d^2-\frac{ae^2}{c}\right)\int \frac{\sqrt{d+ex}}{\sqrt{cdex^2+(cd^2+ae^2)x+ade}}dx}{3d}+\frac{2\sqrt{d+ex}\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{3cd}$$

$$\Big\downarrow \text{ 1122}$$

$$\frac{4\left(d^2-\frac{ae^2}{c}\right)\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{3cd^2\sqrt{d+ex}}+\frac{2\sqrt{d+ex}\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{3cd}$$

input
```
Int[(d + e*x)^(3/2)/Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2],x]
```

output
```
(4*(d^2 - (a*e^2)/c)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(3*c*d^2
*Sqrt[d + e*x]) + (2*Sqrt[d + e*x]*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*
x^2])/(3*c*d)
```

Defintions of rubi rules used

rule 1122
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[e*(d + e*x)^(m - 1)*((a + b*x + c*x^2)^(p + 1)/(c*(p + 1))),
 x] /; FreeQ[{a, b, c, d, e, m, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&
EqQ[m + p, 0]
```

rule 1128
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[e*(d + e*x)^(m - 1)*((a + b*x + c*x^2)^(p + 1)/(c*(m + 2*p +
 1))), x] + Simp[Simplify[m + p]*((2*c*d - b*e)/(c*(m + 2*p + 1)))   Int[(d
 + e*x)^(m - 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p},
 x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && IGtQ[Simplify[m + p], 0]
```

Maple [A] (verified)

Time = 1.06 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.47

method	result	size
default	$-\frac{2\sqrt{(ex+d)(cdx+ae)}\,(-cdxe+2a\,e^2-3c\,d^2)}{3\sqrt{ex+d}\,c^2d^2}$	51
gosper	$-\frac{2(cdx+ae)(-cdxe+2a\,e^2-3c\,d^2)\sqrt{ex+d}}{3c^2d^2\sqrt{cd\,x^2e+a\,e^2x+c\,d^2x+ade}}$	69
orering	$-\frac{2(-cdxe+2a\,e^2-3c\,d^2)(cdx+ae)\sqrt{ex+d}}{3c^2d^2\sqrt{ade+(a\,e^2+c\,d^2)x+cd\,x^2e}}$	70

input
```
int((e*x+d)^(3/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(1/2),x,method=_RETURN
VERBOSE)
```

output
```
-2/3/(e*x+d)^(1/2)*((e*x+d)*(c*d*x+a*e))^(1/2)*(-c*d*e*x+2*a*e^2-3*c*d^2)/
c^2/d^2
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.68

$$\int \frac{(d+ex)^{3/2}}{\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}\,dx = \frac{2\,\sqrt{cdex^2+ade+(cd^2+ae^2)x}(cdex+3\,cd^2-2\,ae^2)\sqrt{ex+d}}{3\,(c^2d^2ex+c^2d^3)}$$

input
```
integrate((e*x+d)^(3/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x, algorit
hm="fricas")
```

output
```
2/3*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(c*d*e*x + 3*c*d^2 - 2*a*e
^2)*sqrt(e*x + d)/(c^2*d^2*e*x + c^2*d^3)
```

Sympy [F]

$$\int \frac{(d+ex)^{3/2}}{\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}\,dx = \int \frac{(d+ex)^{\frac{3}{2}}}{\sqrt{(d+ex)\,(ae+cdx)}}\,dx$$

input
```
integrate((e*x+d)**(3/2)/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(1/2),x)
```

output
```
Integral((d + e*x)**(3/2)/sqrt((d + e*x)*(a*e + c*d*x)), x)
```

Maxima [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.60

$$\int \frac{(d+ex)^{3/2}}{\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}\,dx = \frac{2\,(c^2d^2ex^2+3\,acd^2e-2\,a^2e^3+(3\,c^2d^3-acde^2)x)}{3\,\sqrt{cdx+ae}c^2d^2}$$

input
```
integrate((e*x+d)^(3/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x, algorit
hm="maxima")
```

output
```
2/3*(c^2*d^2*e*x^2 + 3*a*c*d^2*e - 2*a^2*e^3 + (3*c^2*d^3 - a*c*d*e^2)*x)/
(sqrt(c*d*x + a*e)*c^2*d^2)
```

Giac [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.82

$$\int \frac{(d+ex)^{3/2}}{\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\,dx = \frac{2\,e\left(\frac{3\,\sqrt{(ex+d)cde-cd^2e+ae^3}\,(cd^2-ae^2)}{c^2d^2e}+\frac{((ex+d)cde-cd^2e+ae^3)^{\frac{3}{2}}}{c^2d^2e^2}\right)}{3\,|e|}$$

input
```
integrate((e*x+d)^(3/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x, algorit
hm="giac")
```

output
```
2/3*e*(3*sqrt((e*x + d)*c*d*e - c*d^2*e + a*e^3)*(c*d^2 - a*e^2)/(c^2*d^2*
e) + ((e*x + d)*c*d*e - c*d^2*e + a*e^3)^(3/2)/(c^2*d^2*e^2))/abs(e)
```

Mupad [B] (verification not implemented)

Time = 5.78 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.79

$$\int \frac{(d+ex)^{3/2}}{\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\,dx = \frac{\left(\frac{2\,x\,\sqrt{d+e\,x}}{3\,c\,d}-\frac{(4\,a\,e^2-6\,c\,d^2)\,\sqrt{d+e\,x}}{3\,c^2\,d^2\,e}\right)\,\sqrt{c\,d\,e\,x^2+(c\,d^2+a\,e^2)\,x+a\,d\,e}}{x+\frac{d}{e}}$$

input
```
int((d + e*x)^(3/2)/(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2),x)
```

output
```
(((2*x*(d + e*x)^(1/2))/(3*c*d) - ((4*a*e^2 - 6*c*d^2)*(d + e*x)^(1/2))/(3
*c^2*d^2*e))*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2))/(x + d/e)
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.32

$$\int \frac{(d+ex)^{3/2}}{\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}\,dx = \frac{2\sqrt{cdx+ae}\,(cdex-2a\,e^2+3c\,d^2)}{3c^2d^2}$$

input
```
int((e*x+d)^(3/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x)
```

output
```
(2*sqrt(a*e + c*d*x)*( - 2*a*e**2 + 3*c*d**2 + c*d*e*x))/(3*c**2*d**2)
```

3.311 $\int \frac{\sqrt{d+ex}}{\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\,dx$

Optimal result

Integrand size = 39, antiderivative size = 46

$$\int \frac{\sqrt{d+ex}}{\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}\,dx=\frac{2\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{cd\sqrt{d+ex}}$$

output
```
2*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/c/d/(e*x+d)^(1/2)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.76

$$\int \frac{\sqrt{d+ex}}{\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}\,dx=\frac{2\sqrt{(ae+cdx)(d+ex)}}{cd\sqrt{d+ex}}$$

input
```
Integrate[Sqrt[d + e*x]/Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2],x]
```

output
```
(2*Sqrt[(a*e + c*d*x)*(d + e*x)])/(c*d*Sqrt[d + e*x])
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.026$, Rules used = {1122}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{d+ex}}{\sqrt{x(ae^2+cd^2)+ade+cdex^2}}\,dx$$

$$\Big\downarrow \; 1122$$

$$\frac{2\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{cd\sqrt{d+ex}}$$

input
```
Int[Sqrt[d + e*x]/Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2],x]
```

output
```
(2*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(c*d*Sqrt[d + e*x])
```

Defintions of rubi rules used

rule 1122
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[e*(d + e*x)^(m - 1)*((a + b*x + c*x^2)^(p + 1)/(c*(p + 1))),
 x] /; FreeQ[{a, b, c, d, e, m, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&
EqQ[m + p, 0]
```

Maple [A] (verified)

Time = 1.03 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.70

method	result	size
default	$\frac{2\sqrt{(ex+d)(cdx+ae)}}{\sqrt{ex+d}\,cd}$	32
gosper	$\frac{2(cdx+ae)\sqrt{ex+d}}{cd\sqrt{cd\,x^{2}e+a\,e^{2}x+c\,d^{2}x+ade}}$	50
orering	$\frac{2(cdx+ae)\sqrt{ex+d}}{cd\sqrt{ade+(a\,e^{2}+c\,d^{2})x+cd\,x^{2}e}}$	51

input
```
int((e*x+d)^(1/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(1/2),x,method=_RETURN
VERBOSE)
```

output
```
2/(e*x+d)^(1/2)*((e*x+d)*(c*d*x+a*e))^(1/2)/c/d
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.07

$$\int \frac{\sqrt{d+ex}}{\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}\,dx = \frac{2\,\sqrt{cdex^2+ade+(cd^2+ae^2)x}\sqrt{ex+d}}{cdex+cd^2}$$

input
```
integrate((e*x+d)^(1/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x, algorit
hm="fricas")
```

output
```
2*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(e*x + d)/(c*d*e*x + c*d
^2)
```

Sympy [F]

$$\int \frac{\sqrt{d+ex}}{\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}\,dx = \int \frac{\sqrt{d+ex}}{\sqrt{(d+ex)\,(ae+cdx)}}\,dx$$

input
```
integrate((e*x+d)**(1/2)/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(1/2),x)
```

output
```
Integral(sqrt(d + e*x)/sqrt((d + e*x)*(a*e + c*d*x)), x)
```

Maxima [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.39

$$\int \frac{\sqrt{d+ex}}{\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}\,dx=\frac{2\sqrt{cdx+ae}}{cd}$$

input
```
integrate((e*x+d)^(1/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x, algorit
hm="maxima")
```

output
```
2*sqrt(c*d*x + a*e)/(c*d)
```

Giac [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.78

$$\int \frac{\sqrt{d+ex}}{\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}\,dx=\frac{2\sqrt{(ex+d)cde-cd^2e+ae^3}}{cd|e|}$$

input
```
integrate((e*x+d)^(1/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x, algorit
hm="giac")
```

output
```
2*sqrt((e*x + d)*c*d*e - c*d^2*e + a*e^3)/(c*d*abs(e))
```

Mupad [B] (verification not implemented)

Time = 5.56 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.17

$$\int \frac{\sqrt{d+ex}}{\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}\,dx = \frac{2\,\sqrt{d+e\,x}\,\sqrt{c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e}}{c\,d\,e\,\left(x+\frac{d}{e}\right)}$$

input
```
int((d + e*x)^(1/2)/(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2),x)
```

output
```
(2*(d + e*x)^(1/2)*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2))/(c*d*e*(
x + d/e))
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.37

$$\int \frac{\sqrt{d+ex}}{\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}\,dx = \frac{2\sqrt{cdx+ae}}{cd}$$

input
```
int((e*x+d)^(1/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x)
```

output
```
(2*sqrt(a*e + c*d*x))/(c*d)
```

3.312 $\int \frac{1}{\sqrt{d+ex}\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\,dx$

Optimal result

Integrand size = 39, antiderivative size = 84

$$\int \frac{1}{\sqrt{d+ex}\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}\,dx = -\frac{2\arctan\left(\frac{\sqrt{cd^2-ae^2}\sqrt{d+ex}}{\sqrt{e}\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\right)}{\sqrt{e}\sqrt{cd^2-ae^2}}$$

output
```
-2*arctan(1/e^(1/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)*(-a*e^2+c*d^2)
^(1/2)*(e*x+d)^(1/2))/e^(1/2)/(-a*e^2+c*d^2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.15

$$\int \frac{1}{\sqrt{d+ex}\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}\,dx = \frac{2\sqrt{ae+cdx}\sqrt{d+ex}\arctan\left(\frac{\sqrt{e}\sqrt{ae+cdx}}{\sqrt{cd^2-ae^2}}\right)}{\sqrt{e}\sqrt{cd^2-ae^2}\sqrt{(ae+cdx)(d+ex)}}$$

input
```
Integrate[1/(Sqrt[d + e*x]*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]),x]
```

output
```
(2*Sqrt[a*e + c*d*x]*Sqrt[d + e*x]*ArcTan[(Sqrt[e]*Sqrt[a*e + c*d*x])/Sqrt
[c*d^2 - a*e^2]])/(Sqrt[e]*Sqrt[c*d^2 - a*e^2]*Sqrt[(a*e + c*d*x)*(d + e*x
)])
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 84, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.051$, Rules used = {1136, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{d+ex}\sqrt{x(ae^2+cd^2)+ade+cdex^2}}dx$$

$$\Big\downarrow \text{ 1136}$$

$$2e\int \frac{1}{\frac{(cdex^2+(cd^2+ae^2)x+ade)e^2}{d+ex}+(cd^2-ae^2)e}d\frac{\sqrt{cdex^2+(cd^2+ae^2)x+ade}}{\sqrt{d+ex}}$$

$$\Big\downarrow \text{ 218}$$

$$\frac{2\arctan\left(\frac{\sqrt{e}\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{\sqrt{d+ex}\sqrt{cd^2-ae^2}}\right)}{\sqrt{e}\sqrt{cd^2-ae^2}}$$

input
```
Int[1/(Sqrt[d + e*x]*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]),x]
```

output
```
(2*ArcTan[(Sqrt[e]*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(Sqrt[c*d^
2 - a*e^2]*Sqrt[d + e*x])])/(Sqrt[e]*Sqrt[c*d^2 - a*e^2])
```

Defintions of rubi rules used

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 1136
```
Int[1/(Sqrt[(d_.) + (e_.)*(x_)]*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x
_Symbol] :> Simp[2*e   Subst[Int[1/(2*c*d - b*e + e^2*x^2), x], x, Sqrt[a +
 b*x + c*x^2]/Sqrt[d + e*x]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c*d^2
- b*d*e + a*e^2, 0]
```

Maple [A] (verified)

Time = 1.04 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.96

method	result	size
default	$-\frac{2\sqrt{(ex+d)(cdx+ae)}\,\operatorname{arctanh}\left(\frac{e\sqrt{cdx+ae}}{\sqrt{\left(a\,e^{2}-c\,d^{2}\right)e}}\right)}{\sqrt{ex+d}\,\sqrt{cdx+ae}\,\sqrt{(a\,e^{2}-c\,d^{2})e}}$	81

input
```
int(1/(e*x+d)^(1/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(1/2),x,method=_RETU
RNVERBOSE)
```

output
```
-2/(e*x+d)^(1/2)*((e*x+d)*(c*d*x+a*e))^(1/2)/(c*d*x+a*e)^(1/2)/((a*e^2-c*d
^2)*e)^(1/2)*arctanh(e*(c*d*x+a*e)^(1/2)/((a*e^2-c*d^2)*e)^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 234, normalized size of antiderivative = 2.79

$$\int \frac{1}{\sqrt{d+ex}\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}\,dx$$
$$= \left[-\frac{\sqrt{-cd^2e+ae^3}\log\left(-\frac{cde^2x^2+2\,ae^3x-cd^3+2\,ade^2-2\,\sqrt{cdex^2+ade+(cd^2+ae^2)x}\sqrt{-cd^2e+ae^3}\sqrt{ex+d}}{e^2x^2+2\,dex+d^2}\right)}{cd^2e-ae^3},\right.$$
$$\left.-\frac{2\,\arctan\left(-\frac{\sqrt{cdex^2+ade+(cd^2+ae^2)x}\sqrt{cd^2e-ae^3}\sqrt{ex+d}}{cd^3-ade^2+(cd^2e-ae^3)x}\right)}{\sqrt{cd^2e-ae^3}}\right]$$

input
```
integrate(1/(e*x+d)^(1/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x, algor
ithm="fricas")
```

output
```
[-sqrt(-c*d^2*e + a*e^3)*log(-(c*d*e^2*x^2 + 2*a*e^3*x - c*d^3 + 2*a*d*e^2
 - 2*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(-c*d^2*e + a*e^3)*sq
rt(e*x + d))/(e^2*x^2 + 2*d*e*x + d^2))/(c*d^2*e - a*e^3), -2*arctan(-sqrt
(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(c*d^2*e - a*e^3)*sqrt(e*x + d
)/(c*d^3 - a*d*e^2 + (c*d^2*e - a*e^3)*x))/sqrt(c*d^2*e - a*e^3)]
```

Sympy [F]

$$\int \frac{1}{\sqrt{d+ex}\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}\,dx = \int \frac{1}{\sqrt{(d+ex)\,(ae+cdx)}\sqrt{d+ex}}\,dx$$

input
```
integrate(1/(e*x+d)**(1/2)/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(1/2),x)
```

output
```
Integral(1/(sqrt((d + e*x)*(a*e + c*d*x))*sqrt(d + e*x)), x)
```

Maxima [F]

$$\int \frac{1}{\sqrt{d+ex}\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}\,dx = \int \frac{1}{\sqrt{cdex^2+ade+(cd^2+ae^2)x}\sqrt{ex+d}}\,dx$$

input
```
integrate(1/(e*x+d)^(1/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x, algor
ithm="maxima")
```

output
```
integrate(1/(sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(e*x + d)), x
)
```

Giac [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.75

$$\int \frac{1}{\sqrt{d+ex}\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}\,dx = \frac{2\,e\arctan\left(\frac{\sqrt{(ex+d)cde-cd^2e+ae^3}}{\sqrt{cd^2e-ae^3}}\right)}{\sqrt{cd^2e-ae^3}|e|}$$

input
```
integrate(1/(e*x+d)^(1/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x, algor
ithm="giac")
```

output
```
2*e*arctan(sqrt((e*x + d)*c*d*e - c*d^2*e + a*e^3)/sqrt(c*d^2*e - a*e^3))/
(sqrt(c*d^2*e - a*e^3)*abs(e))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{d+ex}\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}\,dx = \int \frac{1}{\sqrt{d+e\,x}\,\sqrt{c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e}}\,dx$$

input
```
int(1/((d + e*x)^(1/2)*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2)),x)
```

output
```
int(1/((d + e*x)^(1/2)*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2)), x)
```

Reduce [B] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.77

$$\int \frac{1}{\sqrt{d+ex}\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}\,dx = -\frac{2\sqrt{e}\,\sqrt{-a\,e^2+c\,d^2}\,atan\left(\frac{\sqrt{cdx+ae}\,e}{\sqrt{e}\,\sqrt{-a\,e^2+c\,d^2}}\right)}{e\,(a\,e^2-c\,d^2)}$$

input
```
int(1/(e*x+d)^(1/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x)
```

output
```
( - 2*sqrt(e)*sqrt( - a*e**2 + c*d**2)*atan((sqrt(a*e + c*d*x)*e)/(sqrt(e)
*sqrt( - a*e**2 + c*d**2))))/(e*(a*e**2 - c*d**2))
```

3.313 $\int \frac{1}{(d+ex)^{3/2}\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\,dx$

Optimal result

Integrand size = 39, antiderivative size = 140

$$\int \frac{1}{(d+ex)^{3/2}\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\,dx = \frac{\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{(cd^2-ae^2)(d+ex)^{3/2}} - \frac{cd\arctan\left(\frac{\sqrt{cd^2-ae^2}\sqrt{d+ex}}{\sqrt{e}\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\right)}{\sqrt{e}\,(cd^2-ae^2)^{3/2}}$$

output
```
(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(-a*e^2+c*d^2)/(e*x+d)^(3/2)-c*d*a
rctan(1/e^(1/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)*(-a*e^2+c*d^2)^(1/
2)*(e*x+d)^(1/2))/e^(1/2)/(-a*e^2+c*d^2)^(3/2)
```

Mathematica [A] (verified)

Time = 0.20 (sec) , antiderivative size = 135, normalized size of antiderivative = 0.96

$$\int \frac{1}{(d+ex)^{3/2}\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\,dx = \frac{\sqrt{e}\sqrt{cd^2-ae^2}(ae+cdx)+cd\sqrt{ae+cdx}(d+ex)\,\mathrm{arcta}\ldots}{\sqrt{e}\,(cd^2-ae^2)^{3/2}\sqrt{d+ex}\sqrt{(ae+cdx)(d+\ldots}}$$

input
```
Integrate[1/((d + e*x)^(3/2)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]),
x]
```

output
```
(Sqrt[e]*Sqrt[c*d^2 - a*e^2]*(a*e + c*d*x) + c*d*Sqrt[a*e + c*d*x]*(d + e*
x)*ArcTan[(Sqrt[e]*Sqrt[a*e + c*d*x])/Sqrt[c*d^2 - a*e^2]])/(Sqrt[e]*(c*d^
2 - a*e^2)^(3/2)*Sqrt[d + e*x]*Sqrt[(a*e + c*d*x)*(d + e*x)])
```

Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 139, normalized size of antiderivative = 0.99, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {1135, 1136, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(d+ex)^{3/2}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}dx$$

$$\Big\downarrow \text{1135}$$

$$\frac{cd\int \frac{1}{\sqrt{d+ex}\sqrt{cdex^2+(cd^2+ae^2)x+ade}}dx}{2\left(cd^2-ae^2\right)}+\frac{\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{(d+ex)^{3/2}\left(cd^2-ae^2\right)}$$

$$\Big\downarrow \text{1136}$$

$$\frac{cde\int \frac{1}{\frac{\left(cdex^2+\left(cd^2+ae^2\right)x+ade\right)e^2}{d+ex}+(cd^2-ae^2)e}d\frac{\sqrt{cdex^2+(cd^2+ae^2)x+ade}}{\sqrt{d+ex}}}{cd^2-ae^2}+\frac{\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{(d+ex)^{3/2}\left(cd^2-ae^2\right)}$$

$$\Big\downarrow \text{218}$$

$$\frac{cd\arctan\left(\frac{\sqrt{e}\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{\sqrt{d+ex}\sqrt{cd^2-ae^2}}\right)}{\sqrt{e}\left(cd^2-ae^2\right)^{3/2}}+\frac{\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{(d+ex)^{3/2}\left(cd^2-ae^2\right)}$$

input
```
Int[1/((d + e*x)^(3/2)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]),x]
```

output
```
Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]/((c*d^2 - a*e^2)*(d + e*x)^(3/
2)) + (c*d*ArcTan[(Sqrt[e]*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(S
qrt[c*d^2 - a*e^2]*Sqrt[d + e*x])])/(Sqrt[e]*(c*d^2 - a*e^2)^(3/2))
```

Defintions of rubi rules used

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 1135
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(-e)*(d + e*x)^m*((a + b*x + c*x^2)^(p + 1)/((m + p + 1)*(2*
c*d - b*e))), x] + Simp[c*((m + 2*p + 2)/((m + p + 1)*(2*c*d - b*e)))   Int
[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p},
 x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[m, 0] && NeQ[m + p + 1, 0] && I
ntegerQ[2*p]
```

rule 1136
```
Int[1/(Sqrt[(d_.) + (e_.)*(x_)]*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x
_Symbol] :> Simp[2*e   Subst[Int[1/(2*c*d - b*e + e^2*x^2), x], x, Sqrt[a +
 b*x + c*x^2]/Sqrt[d + e*x]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c*d^2
- b*d*e + a*e^2, 0]
```

Maple [A] (verified)

Time = 1.06 (sec) , antiderivative size = 162, normalized size of antiderivative = 1.16

method	result	size
default	$\frac{\sqrt{(ex+d)(cdx+ae)}\left(\operatorname{arctanh}\left(\frac{e\sqrt{cdx+ae}}{\sqrt{\left(a\,e^{2}-c\,d^{2}\right)e}}\right)cdex+\operatorname{arctanh}\left(\frac{e\sqrt{cdx+ae}}{\sqrt{\left(a\,e^{2}-c\,d^{2}\right)e}}\right)c\,d^{2}-\sqrt{cdx+ae}\,\sqrt{(a\,e^{2}-c\,d^{2})e}\right)}{(ex+d)^{\frac{3}{2}}\sqrt{cdx+ae}\,(a\,e^{2}-c\,d^{2})\sqrt{(a\,e^{2}-c\,d^{2})e}}$	162

input
```
int(1/(e*x+d)^(3/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(1/2),x,method=_RETU
RNVERBOSE)
```

output
```
((e*x+d)*(c*d*x+a*e))^(1/2)*(arctanh(e*(c*d*x+a*e)^(1/2)/((a*e^2-c*d^2)*e)
^(1/2))*c*d*e*x+arctanh(e*(c*d*x+a*e)^(1/2)/((a*e^2-c*d^2)*e)^(1/2))*c*d^2
-(c*d*x+a*e)^(1/2)*((a*e^2-c*d^2)*e)^(1/2))/(e*x+d)^(3/2)/(c*d*x+a*e)^(1/2
)/(a*e^2-c*d^2)/((a*e^2-c*d^2)*e)^(1/2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 268 vs. $2(124) = 248$.

Time = 0.10 (sec) , antiderivative size = 557, normalized size of antiderivative = 3.98

$$\int \frac{1}{(d+ex)^{3/2}\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}\,dx = \left[\frac{(cde^2x^2+2\,cd^2ex+cd^3)\sqrt{-cd^2e+ae^3}\log\left(-\frac{cde^2x^2+2}{}\right.}{2\,(c^2d^6e-2\,acd^4e^3}\right.$$
$$-\frac{(cde^2x^2+2\,cd^2ex+cd^3)\sqrt{cd^2e-ae^3}\arctan\left(-\frac{\sqrt{cdex^2+ade+(cd^2+ae^2)x}\sqrt{cd^2e-ae^3}\sqrt{ex+d}}{cd^3-ade^2+(cd^2e-ae^3)x}\right)-\sqrt{cdex^2+ade+}}{c^2d^6e-2\,acd^4e^3+a^2d^2e^5+(c^2d^4e^3-2\,acd^2e^5+a^2e^7)x^2+2\,(c^2d^5e^2-2\,acd^3e^4}$$

input
```
integrate(1/(e*x+d)^(3/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x, algor
ithm="fricas")
```

output
```
[1/2*((c*d*e^2*x^2 + 2*c*d^2*e*x + c*d^3)*sqrt(-c*d^2*e + a*e^3)*log(-(c*d
*e^2*x^2 + 2*a*e^3*x - c*d^3 + 2*a*d*e^2 + 2*sqrt(c*d*e*x^2 + a*d*e + (c*d
^2 + a*e^2)*x)*sqrt(-c*d^2*e + a*e^3)*sqrt(e*x + d))/(e^2*x^2 + 2*d*e*x +
d^2)) + 2*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(c*d^2*e - a*e^3)*sq
rt(e*x + d))/(c^2*d^6*e - 2*a*c*d^4*e^3 + a^2*d^2*e^5 + (c^2*d^4*e^3 - 2*a
*c*d^2*e^5 + a^2*e^7)*x^2 + 2*(c^2*d^5*e^2 - 2*a*c*d^3*e^4 + a^2*d*e^6)*x)
, -((c*d*e^2*x^2 + 2*c*d^2*e*x + c*d^3)*sqrt(c*d^2*e - a*e^3)*arctan(-sqrt
(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(c*d^2*e - a*e^3)*sqrt(e*x + d
)/(c*d^3 - a*d*e^2 + (c*d^2*e - a*e^3)*x)) - sqrt(c*d*e*x^2 + a*d*e + (c*d
^2 + a*e^2)*x)*(c*d^2*e - a*e^3)*sqrt(e*x + d))/(c^2*d^6*e - 2*a*c*d^4*e^3
 + a^2*d^2*e^5 + (c^2*d^4*e^3 - 2*a*c*d^2*e^5 + a^2*e^7)*x^2 + 2*(c^2*d^5*
e^2 - 2*a*c*d^3*e^4 + a^2*d*e^6)*x)]
```

Sympy [F]

$$\int \frac{1}{(d+ex)^{3/2}\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}\,dx = \int \frac{1}{\sqrt{(d+ex)\,(ae+cdx)}\,(d+ex)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(e*x+d)**(3/2)/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(1/2),x)
```

output
```
Integral(1/(sqrt((d + e*x)*(a*e + c*d*x))*(d + e*x)**(3/2)), x)
```

Maxima [F]

$$\int \frac{1}{(d+ex)^{3/2}\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}\,dx=\int \frac{1}{\sqrt{cdex^2+ade+(cd^2+ae^2)x}(ex+d)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(e*x+d)^(3/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x, algor
ithm="maxima")
```

output
```
integrate(1/(sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(e*x + d)^(3/2)),
 x)
```

Giac [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 124, normalized size of antiderivative = 0.89

$$\int \frac{1}{(d+ex)^{3/2}\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}\,dx=\frac{cde^2\left(\frac{\arctan\left(\frac{\sqrt{(ex+d)cde-cd^2e+ae^3}}{\sqrt{cd^2e-ae^3}}\right)}{(cd^2e-ae^3)^{\frac{3}{2}}}+\frac{\sqrt{(ex+d)cde-cd^2e+ae^3}}{(cd^2e-ae^3)(ex+d)cde}\right)}{|e|}$$

input
```
integrate(1/(e*x+d)^(3/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x, algor
ithm="giac")
```

output
```
c*d*e^2*(arctan(sqrt((e*x + d)*c*d*e - c*d^2*e + a*e^3)/sqrt(c*d^2*e - a*e
^3))/(c*d^2*e - a*e^3)^(3/2) + sqrt((e*x + d)*c*d*e - c*d^2*e + a*e^3)/((c
*d^2*e - a*e^3)*(e*x + d)*c*d*e))/abs(e)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(d+ex)^{3/2}\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}\,dx=\int \frac{1}{(d+e\,x)^{3/2}\,\sqrt{c\,d\,e\,x^2+(c\,d^2+a\,e^2)\,x+a\,d\,e}}\,dx$$

input
```
int(1/((d + e*x)^(3/2)*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2)),x)
```

output
```
int(1/((d + e*x)^(3/2)*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2)), x)
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 193, normalized size of antiderivative = 1.38

$$\int \frac{1}{(d+ex)^{3/2}\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}\,dx=\frac{\sqrt{e}\,\sqrt{-a\,e^2+c\,d^2}\;atan\left(\frac{\sqrt{cdx+ae}\,e}{\sqrt{e}\,\sqrt{-a\,e^2+c\,d^2}}\right)c\,d^2+\sqrt{e}\,\sqrt{-a\,e^2}}{e\left(a^2e^5x-2ac\,d^2e^3x-\right.}$$

input
```
int(1/(e*x+d)^(3/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x)
```

output
```
(sqrt(e)*sqrt( - a*e**2 + c*d**2)*atan((sqrt(a*e + c*d*x)*e)/(sqrt(e)*sqrt
( - a*e**2 + c*d**2)))*c*d**2 + sqrt(e)*sqrt( - a*e**2 + c*d**2)*atan((sqr
t(a*e + c*d*x)*e)/(sqrt(e)*sqrt( - a*e**2 + c*d**2)))*c*d*e*x - sqrt(a*e +
 c*d*x)*a*e**3 + sqrt(a*e + c*d*x)*c*d**2*e)/(e*(a**2*d*e**4 + a**2*e**5*x
 - 2*a*c*d**3*e**2 - 2*a*c*d**2*e**3*x + c**2*d**5 + c**2*d**4*e*x))
```

3.314 $\int \frac{1}{(d+ex)^{5/2}\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\,dx$

Optimal result

Integrand size = 39, antiderivative size = 207

$$\int \frac{1}{(d+ex)^{5/2}\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\,dx=\frac{\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{2(cd^2-ae^2)(d+ex)^{5/2}}+\frac{3cd\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{4(cd^2-ae^2)^2(d+ex)^{3/2}}-\frac{3c^2d^2\arctan\left(\frac{\sqrt{cd^2-ae^2}\sqrt{d+ex}}{\sqrt{e}\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\right)}{4\sqrt{e}(cd^2-ae^2)^{5/2}}$$

output
```
1/2*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(-a*e^2+c*d^2)/(e*x+d)^(5/2)+3
/4*c*d*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(-a*e^2+c*d^2)^2/(e*x+d)^(3
/2)-3/4*c^2*d^2*arctan(1/e^(1/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)*(
-a*e^2+c*d^2)^(1/2)*(e*x+d)^(1/2))/e^(1/2)/(-a*e^2+c*d^2)^(5/2)
```

Mathematica [A] (verified)

Time = 0.29 (sec) , antiderivative size = 174, normalized size of antiderivative = 0.84

$$\int \frac{1}{(d+ex)^{5/2}\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\,dx=\frac{\sqrt{e}\sqrt{cd^2-ae^2}(-2a^2e^3+acde(5d+ex)+c^2d^2x(5d+3}{4\sqrt{e}(cd^2-ae^2)^{5/2}(d+ex)}$$

input
```
Integrate[1/((d + e*x)^(5/2)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]),
x]
```

output
```
(Sqrt[e]*Sqrt[c*d^2 - a*e^2]*(-2*a^2*e^3 + a*c*d*e*(5*d + e*x) + c^2*d^2*x
*(5*d + 3*e*x)) + 3*c^2*d^2*Sqrt[a*e + c*d*x]*(d + e*x)^2*ArcTan[(Sqrt[e]*
Sqrt[a*e + c*d*x])/Sqrt[c*d^2 - a*e^2]])/(4*Sqrt[e]*(c*d^2 - a*e^2)^(5/2)*
(d + e*x)^(3/2)*Sqrt[(a*e + c*d*x)*(d + e*x)])
```

Rubi [A] (verified)

Time = 0.57 (sec) , antiderivative size = 216, normalized size of antiderivative = 1.04, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.103$, Rules used = {1135, 1135, 1136, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(d+ex)^{5/2}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}dx$$

$$\Big\downarrow \text{1135}$$

$$\frac{3cd\int \frac{1}{(d+ex)^{3/2}\sqrt{cdex^2+(cd^2+ae^2)x+ade}}dx}{4\left(cd^2-ae^2\right)}+\frac{\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{2(d+ex)^{5/2}\left(cd^2-ae^2\right)}$$

$$\Big\downarrow \text{1135}$$

$$\frac{3cd\left(\frac{cd\int \frac{1}{\sqrt{d+ex}\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}dx}{2(cd^2-ae^2)}+\frac{\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{(d+ex)^{3/2}(cd^2-ae^2)}\right)}{4\left(cd^2-ae^2\right)}+\frac{\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{2(d+ex)^{5/2}\left(cd^2-ae^2\right)}$$

$$\Big\downarrow \text{1136}$$

$$\frac{3cd\left(\frac{cde\int \frac{1}{\frac{\left(cdex^2+\left(cd^2+ae^2\right)x+ade\right)e^2}{d+ex}+\left(cd^2-ae^2\right)e}d\frac{\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}{\sqrt{d+ex}}}{cd^2-ae^2}+\frac{\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{(d+ex)^{3/2}(cd^2-ae^2)}\right)}{4\left(cd^2-ae^2\right)}+\frac{\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{2(d+ex)^{5/2}\left(cd^2-ae^2\right)}$$

$$\Big\downarrow \ 218$$

$$\frac{3cd\left(\frac{cd\arctan\left(\frac{\sqrt{e}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{\sqrt{d+ex}\sqrt{cd^2-ae^2}}\right)}{\sqrt{e}(cd^2-ae^2)^{3/2}}+\frac{\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{(d+ex)^{3/2}(cd^2-ae^2)}\right)}{4\left(cd^2-ae^2\right)}+\frac{\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{2(d+ex)^{5/2}\left(cd^2-ae^2\right)}$$

input
```
Int[1/((d + e*x)^(5/2)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]),x]
```

output
```
Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]/(2*(c*d^2 - a*e^2)*(d + e*x)^(
5/2)) + (3*c*d*(Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]/((c*d^2 - a*e^
2)*(d + e*x)^(3/2)) + (c*d*ArcTan[(Sqrt[e]*Sqrt[a*d*e + (c*d^2 + a*e^2)*x
+ c*d*e*x^2])/(Sqrt[c*d^2 - a*e^2]*Sqrt[d + e*x])])/(Sqrt[e]*(c*d^2 - a*e^
2)^(3/2))))/(4*(c*d^2 - a*e^2))
```

Defintions of rubi rules used

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 1135
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(-e)*(d + e*x)^m*((a + b*x + c*x^2)^(p + 1)/((m + p + 1)*(2*
c*d - b*e))), x] + Simp[c*((m + 2*p + 2)/((m + p + 1)*(2*c*d - b*e)))   Int
[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p},
 x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[m, 0] && NeQ[m + p + 1, 0] && I
ntegerQ[2*p]
```

rule 1136
```
Int[1/(Sqrt[(d_.) + (e_.)*(x_)]*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x
_Symbol] :> Simp[2*e   Subst[Int[1/(2*c*d - b*e + e^2*x^2), x], x, Sqrt[a +
 b*x + c*x^2]/Sqrt[d + e*x]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c*d^2
- b*d*e + a*e^2, 0]
```

Maple [A] (verified)

Time = 1.09 (sec) , antiderivative size = 282, normalized size of antiderivative = 1.36

method	result
default	$-\frac{\sqrt{(ex+d)(cdx+ae)}\left(3\,\mathrm{arctanh}\left(\frac{e\sqrt{cdx+ae}}{\sqrt{\left(a\,e^{2}-c\,d^{2}\right)e}}\right)c^{2}d^{2}e^{2}x^{2}+6\,\mathrm{arctanh}\left(\frac{e\sqrt{cdx+ae}}{\sqrt{\left(a\,e^{2}-c\,d^{2}\right)e}}\right)c^{2}d^{3}ex+3\,\mathrm{arctanh}\left(\frac{e\sqrt{cdx+ae}}{\sqrt{\left(a\,e^{2}-c\,d^{2}\right)e}}\right)\right.}{4(ex+d)^{\frac{5}{2}}\sqrt{cdx+ae}\,\left(a\,e^{2}-c\,d^{2}\right)^{2}\sqrt{(}}$

input
```
int(1/(e*x+d)^(5/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(1/2),x,method=_RETU
RNVERBOSE)
```

output
```
-1/4/(e*x+d)^(5/2)*((e*x+d)*(c*d*x+a*e))^(1/2)*(3*arctanh(e*(c*d*x+a*e)^(1
/2)/((a*e^2-c*d^2)*e)^(1/2))*c^2*d^2*e^2*x^2+6*arctanh(e*(c*d*x+a*e)^(1/2)
/((a*e^2-c*d^2)*e)^(1/2))*c^2*d^3*e*x+3*arctanh(e*(c*d*x+a*e)^(1/2)/((a*e^
2-c*d^2)*e)^(1/2))*c^2*d^4-3*c*d*e*x*(c*d*x+a*e)^(1/2)*((a*e^2-c*d^2)*e)^(
1/2)+2*((a*e^2-c*d^2)*e)^(1/2)*(c*d*x+a*e)^(1/2)*a*e^2-5*((a*e^2-c*d^2)*e)
^(1/2)*(c*d*x+a*e)^(1/2)*c*d^2)/(c*d*x+a*e)^(1/2)/(a*e^2-c*d^2)^2/((a*e^2-
c*d^2)*e)^(1/2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 424 vs. 2(181) = 362.

Time = 0.11 (sec) , antiderivative size = 869, normalized size of antiderivative = 4.20

$$\int \frac{1}{(d+ex)^{5/2}\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}\,dx = \text{Too large to display}$$

input
```
integrate(1/(e*x+d)^(5/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x, algor
ithm="fricas")
```

output
```
[-1/8*(3*(c^2*d^2*e^3*x^3 + 3*c^2*d^3*e^2*x^2 + 3*c^2*d^4*e*x + c^2*d^5)*s
qrt(-c*d^2*e + a*e^3)*log(-(c*d*e^2*x^2 + 2*a*e^3*x - c*d^3 + 2*a*d*e^2 -
2*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(-c*d^2*e + a*e^3)*sqrt(
e*x + d))/(e^2*x^2 + 2*d*e*x + d^2)) - 2*(5*c^2*d^4*e - 7*a*c*d^2*e^3 + 2*
a^2*e^5 + 3*(c^2*d^3*e^2 - a*c*d*e^4)*x)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 +
 a*e^2)*x)*sqrt(e*x + d))/(c^3*d^9*e - 3*a*c^2*d^7*e^3 + 3*a^2*c*d^5*e^5 -
 a^3*d^3*e^7 + (c^3*d^6*e^4 - 3*a*c^2*d^4*e^6 + 3*a^2*c*d^2*e^8 - a^3*e^10
)*x^3 + 3*(c^3*d^7*e^3 - 3*a*c^2*d^5*e^5 + 3*a^2*c*d^3*e^7 - a^3*d*e^9)*x^
2 + 3*(c^3*d^8*e^2 - 3*a*c^2*d^6*e^4 + 3*a^2*c*d^4*e^6 - a^3*d^2*e^8)*x),
-1/4*(3*(c^2*d^2*e^3*x^3 + 3*c^2*d^3*e^2*x^2 + 3*c^2*d^4*e*x + c^2*d^5)*sq
rt(c*d^2*e - a*e^3)*arctan(-sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*sq
rt(c*d^2*e - a*e^3)*sqrt(e*x + d)/(c*d^3 - a*d*e^2 + (c*d^2*e - a*e^3)*x))
 - (5*c^2*d^4*e - 7*a*c*d^2*e^3 + 2*a^2*e^5 + 3*(c^2*d^3*e^2 - a*c*d*e^4)*
x)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(e*x + d))/(c^3*d^9*e -
 3*a*c^2*d^7*e^3 + 3*a^2*c*d^5*e^5 - a^3*d^3*e^7 + (c^3*d^6*e^4 - 3*a*c^2*
d^4*e^6 + 3*a^2*c*d^2*e^8 - a^3*e^10)*x^3 + 3*(c^3*d^7*e^3 - 3*a*c^2*d^5*e
^5 + 3*a^2*c*d^3*e^7 - a^3*d*e^9)*x^2 + 3*(c^3*d^8*e^2 - 3*a*c^2*d^6*e^4 +
 3*a^2*c*d^4*e^6 - a^3*d^2*e^8)*x)]
```

Sympy [F]

$$\int \frac{1}{(d+ex)^{5/2}\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}\,dx=\int \frac{1}{\sqrt{(d+ex)\,(ae+cdx)}\,(d+ex)^{\frac{5}{2}}}\,dx$$

input
```
integrate(1/(e*x+d)**(5/2)/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(1/2),x)
```

output
```
Integral(1/(sqrt((d + e*x)*(a*e + c*d*x))*(d + e*x)**(5/2)), x)
```

Maxima [F]

$$\int \frac{1}{(d+ex)^{5/2}\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}\,dx = \int \frac{1}{\sqrt{cdex^2+ade+(cd^2+ae^2)x}(ex+d)^{\frac{5}{2}}}\,dx$$

input
```
integrate(1/(e*x+d)^(5/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x, algor
ithm="maxima")
```

output
```
integrate(1/(sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(e*x + d)^(5/2)),
 x)
```

Giac [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 254, normalized size of antiderivative = 1.23

$$\int \frac{1}{(d+ex)^{5/2}\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}\,dx = \frac{\frac{3\,c^3d^3e\arctan\left(\frac{\sqrt{(ex+d)cde-cd^2e+ae^3}}{\sqrt{cd^2e-ae^3}}\right)}{(c^2d^4-2\,acd^2e^2+a^2e^4)\sqrt{cd^2e-ae^3}} + \frac{5\,\sqrt{(ex+d)cde-cd^2e+ae^3}c^4d^5 \cdots}{\cdots}}{4\,cd|\cdots}$$

input
```
integrate(1/(e*x+d)^(5/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x, algor
ithm="giac")
```

output
```
1/4*(3*c^3*d^3*e*arctan(sqrt((e*x + d)*c*d*e - c*d^2*e + a*e^3)/sqrt(c*d^2
*e - a*e^3))/((c^2*d^4 - 2*a*c*d^2*e^2 + a^2*e^4)*sqrt(c*d^2*e - a*e^3)) +
 (5*sqrt((e*x + d)*c*d*e - c*d^2*e + a*e^3)*c^4*d^5*e^2 - 5*sqrt((e*x + d)
*c*d*e - c*d^2*e + a*e^3)*a*c^3*d^3*e^4 + 3*((e*x + d)*c*d*e - c*d^2*e + a
*e^3)^(3/2)*c^3*d^3*e)/((c^2*d^4 - 2*a*c*d^2*e^2 + a^2*e^4)*(e*x + d)^2*c^
2*d^2*e^2))/(c*d*abs(e))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(d+ex)^{5/2}\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}\,dx=\int \frac{1}{{\left(d+e\,x\right)}^{5/2}\,\sqrt{c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e}}\,dx$$

input
```
int(1/((d + e*x)^(5/2)*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2)),x)
```

output
```
int(1/((d + e*x)^(5/2)*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2)), x)
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 414, normalized size of antiderivative = 2.00

$$\int \frac{1}{(d+ex)^{5/2}\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}\,dx=\frac{-3\sqrt{e}\,\sqrt{-a\,e^2+c\,d^2}\,atan\left(\frac{\sqrt{cdx+ae}\,e}{\sqrt{e}\,\sqrt{-a\,e^2+c\,d^2}}\right)c^2d^4-6\sqrt{e}\,\sqrt{}$$

input
```
int(1/(e*x+d)^(5/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x)
```

output
```
( - 3*sqrt(e)*sqrt( - a*e**2 + c*d**2)*atan((sqrt(a*e + c*d*x)*e)/(sqrt(e)
*sqrt( - a*e**2 + c*d**2)))*c**2*d**4 - 6*sqrt(e)*sqrt( - a*e**2 + c*d**2)
*atan((sqrt(a*e + c*d*x)*e)/(sqrt(e)*sqrt( - a*e**2 + c*d**2)))*c**2*d**3*
e*x - 3*sqrt(e)*sqrt( - a*e**2 + c*d**2)*atan((sqrt(a*e + c*d*x)*e)/(sqrt(
e)*sqrt( - a*e**2 + c*d**2)))*c**2*d**2*e**2*x**2 - 2*sqrt(a*e + c*d*x)*a*
*2*e**5 + 7*sqrt(a*e + c*d*x)*a*c*d**2*e**3 + 3*sqrt(a*e + c*d*x)*a*c*d*e*
*4*x - 5*sqrt(a*e + c*d*x)*c**2*d**4*e - 3*sqrt(a*e + c*d*x)*c**2*d**3*e**
2*x)/(4*e*(a**3*d**2*e**6 + 2*a**3*d*e**7*x + a**3*e**8*x**2 - 3*a**2*c*d*
*4*e**4 - 6*a**2*c*d**3*e**5*x - 3*a**2*c*d**2*e**6*x**2 + 3*a*c**2*d**6*e
**2 + 6*a*c**2*d**5*e**3*x + 3*a*c**2*d**4*e**4*x**2 - c**3*d**8 - 2*c**3*
d**7*e*x - c**3*d**6*e**2*x**2))
```

3.315 $\int \frac{(d+ex)^{7/2}}{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}\,dx$

Optimal result

Integrand size = 39, antiderivative size = 171

$$\int \frac{(d+ex)^{7/2}}{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}\,dx=-\frac{2\left(cd^2-ae^2\right)^2\sqrt{d+ex}}{c^3d^3\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}+\frac{4e\left(cd^2-ae^2\right)\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{c^3d^3\sqrt{d+ex}}+\frac{2e^2\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{3c^3d^3(d+ex)^{3/2}}$$

output
```
-2*(-a*e^2+c*d^2)^2*(e*x+d)^(1/2)/c^3/d^3/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2
)^(1/2)+4*e*(-a*e^2+c*d^2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/c^3/d^3
/(e*x+d)^(1/2)+2/3*e^2*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/c^3/d^3/(e*
x+d)^(3/2)
```

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.51

$$\int \frac{(d+ex)^{7/2}}{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}\,dx=-\frac{2\sqrt{d+ex}\left(8a^2e^4+4acde^2(-3d+ex)+c^2d^2\left(3d^2-6dex-e^2x^2\right)\right)}{3c^3d^3\sqrt{(ae+cdx)(d+ex)}}$$

input
```
Integrate[(d + e*x)^(7/2)/(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2),x]
```

output
```
(-2*Sqrt[d + e*x]*(8*a^2*e^4 + 4*a*c*d*e^2*(-3*d + e*x) + c^2*d^2*(3*d^2 -
 6*d*e*x - e^2*x^2)))/(3*c^3*d^3*Sqrt[(a*e + c*d*x)*(d + e*x)])
```

Rubi [A] (verified)

Time = 0.51 (sec) , antiderivative size = 175, normalized size of antiderivative = 1.02, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {1128, 1128, 1122}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex)^{7/2}}{(x(ae^2+cd^2)+ade+cdex^2)^{3/2}}dx$$

$$\Big\downarrow \text{1128}$$

$$\frac{4\left(d^2-\frac{ae^2}{c}\right)\int \frac{(d+ex)^{5/2}}{(cdex^2+(cd^2+ae^2)x+ade)^{3/2}}dx}{3d}+\frac{2(d+ex)^{5/2}}{3cd\sqrt{x(ae^2+cd^2)+ade+cdex^2}}$$

$$\Big\downarrow \text{1128}$$

$$\frac{4\left(d^2-\frac{ae^2}{c}\right)\left(\frac{2\left(d^2-\frac{ae^2}{c}\right)\int \frac{(d+ex)^{3/2}}{\left(cdex^2+\left(cd^2+ae^2\right)x+ade\right)^{3/2}}dx}{d}+\frac{2(d+ex)^{3/2}}{cd\sqrt{x(ae^2+cd^2)+ade+cdex^2}}\right)}{3d}+\frac{2(d+ex)^{5/2}}{3cd\sqrt{x(ae^2+cd^2)+ade+cdex^2}}$$

$$\Big\downarrow \text{1122}$$

$$\frac{2(d+ex)^{5/2}}{3cd\sqrt{x(ae^2+cd^2)+ade+cdex^2}}+\frac{4\left(d^2-\frac{ae^2}{c}\right)\left(\frac{2(d+ex)^{3/2}}{cd\sqrt{x(ae^2+cd^2)+ade+cdex^2}}-\frac{4\sqrt{d+ex}\left(d^2-\frac{ae^2}{c}\right)}{cd^2\sqrt{x(ae^2+cd^2)+ade+cdex^2}}\right)}{3d}$$

input
```
Int[(d + e*x)^(7/2)/(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2),x]
```

output
```
(2*(d + e*x)^(5/2))/(3*c*d*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]) +
(4*(d^2 - (a*e^2)/c)*((-4*(d^2 - (a*e^2)/c)*Sqrt[d + e*x])/(c*d^2*Sqrt[a*d
*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]) + (2*(d + e*x)^(3/2))/(c*d*Sqrt[a*d*e
 + (c*d^2 + a*e^2)*x + c*d*e*x^2])))/(3*d)
```

Defintions of rubi rules used

rule 1122
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[e*(d + e*x)^(m - 1)*((a + b*x + c*x^2)^(p + 1)/(c*(p + 1))),
 x] /; FreeQ[{a, b, c, d, e, m, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&
EqQ[m + p, 0]
```

rule 1128
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[e*(d + e*x)^(m - 1)*((a + b*x + c*x^2)^(p + 1)/(c*(m + 2*p +
 1))), x] + Simp[Simplify[m + p]*((2*c*d - b*e)/(c*(m + 2*p + 1)))   Int[(d
 + e*x)^(m - 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p},
 x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && IGtQ[Simplify[m + p], 0]
```

Maple [A] (verified)

Time = 1.04 (sec) , antiderivative size = 102, normalized size of antiderivative = 0.60

method	result	size
default	$-\frac{2\sqrt{(ex+d)(cdx+ae)}\,(-x^2c^2d^2e^2+4xacd\,e^3-6x\,c^2d^3e+8a^2e^4-12ac\,d^2e^2+3c^2d^4)}{3\sqrt{ex+d}\,(cdx+ae)c^3d^3}$	102
gosper	$-\frac{2(cdx+ae)(-x^2c^2d^2e^2+4xacd\,e^3-6x\,c^2d^3e+8a^2e^4-12ac\,d^2e^2+3c^2d^4)(ex+d)^{\frac{3}{2}}}{3d^3c^3(cd\,x^2e+a\,e^2x+c\,d^2x+ade)^{\frac{3}{2}}}$	110
orering	$-\frac{2(-x^2c^2d^2e^2+4xacd\,e^3-6x\,c^2d^3e+8a^2e^4-12ac\,d^2e^2+3c^2d^4)(cdx+ae)(ex+d)^{\frac{3}{2}}}{3d^3c^3(ade+(a\,e^2+c\,d^2)x+cd\,x^2e)^{\frac{3}{2}}}$	111

input
```
int((e*x+d)^(7/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(3/2),x,method=_RETURN
VERBOSE)
```

output
```
-2/3/(e*x+d)^(1/2)*((e*x+d)*(c*d*x+a*e))^(1/2)*(-c^2*d^2*e^2*x^2+4*a*c*d*e
^3*x-6*c^2*d^3*e*x+8*a^2*e^4-12*a*c*d^2*e^2+3*c^2*d^4)/(c*d*x+a*e)/c^3/d^3
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 141, normalized size of antiderivative = 0.82

$$\int \frac{(d+ex)^{7/2}}{(ade+(cd^2+ae^2)\,x+cdex^2)^{3/2}}\,dx = \frac{2\,(c^2d^2e^2x^2-3\,c^2d^4+12\,acd^2e^2-8\,a^2e^4+2\,(3\,c^2d^3e-2\,acde^3)x}{3\,(c^4d^4ex^2+ac^3d^4e+(c^4d^5+ac}$$

input
```
integrate((e*x+d)^(7/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),x, algorit
hm="fricas")
```

output
```
2/3*(c^2*d^2*e^2*x^2 - 3*c^2*d^4 + 12*a*c*d^2*e^2 - 8*a^2*e^4 + 2*(3*c^2*d
^3*e - 2*a*c*d*e^3)*x)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(e*
x + d)/(c^4*d^4*e*x^2 + a*c^3*d^4*e + (c^4*d^5 + a*c^3*d^3*e^2)*x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(d+ex)^{7/2}}{(ade+(cd^2+ae^2)\,x+cdex^2)^{3/2}}\,dx = \text{Timed out}$$

input
```
integrate((e*x+d)**(7/2)/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(3/2),x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.46

$$\int \frac{(d+ex)^{7/2}}{(ade+(cd^2+ae^2)x+cdex^2)^{3/2}}\,dx = \frac{2\,(c^2d^2e^2x^2 - 3\,c^2d^4 + 12\,acd^2e^2 - 8\,a^2e^4 + 2\,(3\,c^2d^3e - 2\,acde^3)x}{3\,\sqrt{cdx+ae}c^3d^3}$$

input
```
integrate((e*x+d)^(7/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),x, algorit
hm="maxima")
```

output
```
2/3*(c^2*d^2*e^2*x^2 - 3*c^2*d^4 + 12*a*c*d^2*e^2 - 8*a^2*e^4 + 2*(3*c^2*d
^3*e - 2*a*c*d*e^3)*x)/(sqrt(c*d*x + a*e)*c^3*d^3)
```

Giac [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 114, normalized size of antiderivative = 0.67

$$\int \frac{(d+ex)^{7/2}}{(ade+(cd^2+ae^2)x+cdex^2)^{3/2}}\,dx = -\frac{2\,(c^2d^4 - 2\,acd^2e^2 + a^2e^4)}{\sqrt{cdx+ae}c^3d^3} + \frac{2\left(6\,\sqrt{cdx+ae}c^7d^8e - 6\,\sqrt{cdx+ae}ac^6d^6e^3 + (cdx+ae)^{\frac{3}{2}}c^6d^6e^2\right)}{3\,c^9d^9}$$

input
```
integrate((e*x+d)^(7/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),x, algorit
hm="giac")
```

output
```
-2*(c^2*d^4 - 2*a*c*d^2*e^2 + a^2*e^4)/(sqrt(c*d*x + a*e)*c^3*d^3) + 2/3*(
6*sqrt(c*d*x + a*e)*c^7*d^8*e - 6*sqrt(c*d*x + a*e)*a*c^6*d^6*e^3 + (c*d*x
 + a*e)^(3/2)*c^6*d^6*e^2)/(c^9*d^9)
```

Mupad [B] (verification not implemented)

Time = 5.85 (sec) , antiderivative size = 174, normalized size of antiderivative = 1.02

$$\int \frac{(d+ex)^{7/2}}{\left(ade+(cd^2+ae^2)\,x+cdex^2\right)^{3/2}}\,dx = \frac{\left(\frac{2\,e\,x^2\,\sqrt{d+e\,x}}{3\,c^2\,d^2} - \frac{\sqrt{d+e\,x}\,\left(16\,a^2\,e^4-24\,a\,c\,d^2\,e^2+6\,c^2\,d^4\right)}{3\,c^4\,d^4\,e} + \frac{x\,\left(12\,c^2\,d^3\,e-8\,a\,c\,d\,e^3\right)}{3\,c^4\,d^4\,e}\right.}{\frac{a}{c}+x^2+\frac{x\,\left(3\,c^4\,d^5+3\,a\,c^3\,d^3\,e\right.}{3\,c^4\,d^4\,e}}$$

input
```
int((d + e*x)^(7/2)/(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(3/2),x)
```

output
```
(((2*e*x^2*(d + e*x)^(1/2))/(3*c^2*d^2) - ((d + e*x)^(1/2)*(16*a^2*e^4 + 6
*c^2*d^4 - 24*a*c*d^2*e^2))/(3*c^4*d^4*e) + (x*(12*c^2*d^3*e - 8*a*c*d*e^3
)*(d + e*x)^(1/2))/(3*c^4*d^4*e))*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^
(1/2))/(a/c + x^2 + (x*(3*c^4*d^5 + 3*a*c^3*d^3*e^2))/(3*c^4*d^4*e))
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.46

$$\int \frac{(d+ex)^{7/2}}{\left(ade+(cd^2+ae^2)\,x+cdex^2\right)^{3/2}}\,dx = \frac{\frac{2}{3}c^2d^2e^2x^2-\frac{8}{3}acd\,e^3x+4c^2d^3ex-\frac{16}{3}a^2e^4+8ac\,d^2e^2-2c^2d^4}{\sqrt{cdx+ae}\,c^3d^3}$$

input
```
int((e*x+d)^(7/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),x)
```

output
```
(2*( - 8*a**2*e**4 + 12*a*c*d**2*e**2 - 4*a*c*d*e**3*x - 3*c**2*d**4 + 6*c
**2*d**3*e*x + c**2*d**2*e**2*x**2))/(3*sqrt(a*e + c*d*x)*c**3*d**3)
```

3.316 $\int \frac{(d+ex)^{5/2}}{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}\,dx$

Optimal result

Integrand size = 39, antiderivative size = 106

$$\int \frac{(d+ex)^{5/2}}{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}\,dx = -\frac{2(cd^2-ae^2)\sqrt{d+ex}}{c^2d^2\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}+\frac{2e\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{c^2d^2\sqrt{d+ex}}$$

output
```
-2*(-a*e^2+c*d^2)*(e*x+d)^(1/2)/c^2/d^2/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^
(1/2)+2*e*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/c^2/d^2/(e*x+d)^(1/2)
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.48

$$\int \frac{(d+ex)^{5/2}}{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}\,dx = -\frac{2\sqrt{d+ex}(-2ae^2+cd(d-ex))}{c^2d^2\sqrt{(ae+cdx)(d+ex)}}$$

input
```
Integrate[(d + e*x)^(5/2)/(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2),x]
```

output
```
(-2*Sqrt[d + e*x]*(-2*a*e^2 + c*d*(d - e*x)))/(c^2*d^2*Sqrt[(a*e + c*d*x)*
(d + e*x)])
```

Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 106, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.051$, Rules used = {1128, 1122}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex)^{5/2}}{\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}dx$$

$$\Big\downarrow \text{ 1128}$$

$$\frac{2\left(d^2-\frac{ae^2}{c}\right)\int \frac{(d+ex)^{3/2}}{\left(cdex^2+\left(cd^2+ae^2\right)x+ade\right)^{3/2}}dx}{d}+\frac{2(d+ex)^{3/2}}{cd\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}$$

$$\Big\downarrow \text{ 1122}$$

$$\frac{2(d+ex)^{3/2}}{cd\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}-\frac{4\sqrt{d+ex}\left(d^2-\frac{ae^2}{c}\right)}{cd^2\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}$$

input
```
Int[(d + e*x)^(5/2)/(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2),x]
```

output
```
(-4*(d^2 - (a*e^2)/c)*Sqrt[d + e*x])/(c*d^2*Sqrt[a*d*e + (c*d^2 + a*e^2)*x
 + c*d*e*x^2]) + (2*(d + e*x)^(3/2))/(c*d*Sqrt[a*d*e + (c*d^2 + a*e^2)*x +
 c*d*e*x^2])
```

Defintions of rubi rules used

rule 1122
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[e*(d + e*x)^(m - 1)*((a + b*x + c*x^2)^(p + 1)/(c*(p + 1))),
 x] /; FreeQ[{a, b, c, d, e, m, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&
EqQ[m + p, 0]
```

rule 1128
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[e*(d + e*x)^(m - 1)*((a + b*x + c*x^2)^(p + 1)/(c*(m + 2*p +
 1))), x] + Simp[Simplify[m + p]*((2*c*d - b*e)/(c*(m + 2*p + 1)))   Int[(d
 + e*x)^(m - 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p},
 x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && IGtQ[Simplify[m + p], 0]
```

Maple [A] (verified)

Time = 1.04 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.57

method	result	size
default	$\frac{2\sqrt{(ex+d)(cdx+ae)}\,(cdxe+2a\,e^{2}-c\,d^{2})}{\sqrt{ex+d}\,(cdx+ae)c^{2}d^{2}}$	60
gosper	$\frac{2(cdx+ae)(cdxe+2a\,e^{2}-c\,d^{2})(ex+d)^{\frac{3}{2}}}{c^{2}d^{2}(cd\,x^{2}e+a\,e^{2}x+c\,d^{2}x+ade)^{\frac{3}{2}}}$	68
orering	$\frac{2(cdxe+2a\,e^{2}-c\,d^{2})(cdx+ae)(ex+d)^{\frac{3}{2}}}{c^{2}d^{2}(ade+(a\,e^{2}+c\,d^{2})x+cd\,x^{2}e)^{\frac{3}{2}}}$	69

input
```
int((e*x+d)^(5/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(3/2),x,method=_RETURN
VERBOSE)
```

output
```
2/(e*x+d)^(1/2)*((e*x+d)*(c*d*x+a*e))^(1/2)*(c*d*e*x+2*a*e^2-c*d^2)/(c*d*x
+a*e)/c^2/d^2
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 98, normalized size of antiderivative = 0.92

$$\int \frac{(d+ex)^{5/2}}{(ade+(cd^2+ae^2)\,x+cdex^2)^{3/2}}\,dx = \frac{2\,\sqrt{cdex^2+ade+(cd^2+ae^2)x}(cdex-cd^2+2\,ae^2)\sqrt{ex+d}}{c^3d^3ex^2+ac^2d^3e+(c^3d^4+ac^2d^2e^2)x}$$

input
```
integrate((e*x+d)^(5/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),x, algorit
hm="fricas")
```

output
```
2*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(c*d*e*x - c*d^2 + 2*a*e^2)*
sqrt(e*x + d)/(c^3*d^3*e*x^2 + a*c^2*d^3*e + (c^3*d^4 + a*c^2*d^2*e^2)*x)
```

Sympy [F]

$$\int \frac{(d+ex)^{5/2}}{(ade+(cd^2+ae^2)\,x+cdex^2)^{3/2}}\,dx = \int \frac{(d+ex)^{\frac{5}{2}}}{((d+ex)\,(ae+cdx))^{\frac{3}{2}}}\,dx$$

input
```
integrate((e*x+d)**(5/2)/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(3/2),x)
```

output
```
Integral((d + e*x)**(5/2)/((d + e*x)*(a*e + c*d*x))**(3/2), x)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.34

$$\int \frac{(d+ex)^{5/2}}{(ade+(cd^2+ae^2)\,x+cdex^2)^{3/2}}\,dx = \frac{2\,(cdex-cd^2+2\,ae^2)}{\sqrt{cdx+ae}c^2d^2}$$

input
```
integrate((e*x+d)^(5/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),x, algorit
hm="maxima")
```

output
```
2*(c*d*e*x - c*d^2 + 2*a*e^2)/(sqrt(c*d*x + a*e)*c^2*d^2)
```

Giac [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.47

$$\int \frac{(d+ex)^{5/2}}{(ade+(cd^2+ae^2)x+cdex^2)^{3/2}}\,dx = \frac{2\sqrt{cdx+ae}e}{c^2d^2} - \frac{2(cd^2-ae^2)}{\sqrt{cdx+ae}c^2d^2}$$

input
```
integrate((e*x+d)^(5/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),x, algorit
hm="giac")
```

output
```
2*sqrt(c*d*x + a*e)*e/(c^2*d^2) - 2*(c*d^2 - a*e^2)/(sqrt(c*d*x + a*e)*c^2
*d^2)
```

Mupad [B] (verification not implemented)

Time = 5.86 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.09

$$\int \frac{(d+ex)^{5/2}}{(ade+(cd^2+ae^2)x+cdex^2)^{3/2}}\,dx = \frac{\left(\frac{2x\sqrt{d+ex}}{c^2d^2}+\frac{(4ae^2-2cd^2)\sqrt{d+ex}}{c^3d^3e}\right)\sqrt{cdex^2+(cd^2+ae^2)x+ad}}{\frac{a}{c}+x^2+\frac{x(c^3d^4+ac^2d^2e^2)}{c^3d^3e}}$$

input
```
int((d + e*x)^(5/2)/(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(3/2),x)
```

output
```
(((2*x*(d + e*x)^(1/2))/(c^2*d^2) + ((4*a*e^2 - 2*c*d^2)*(d + e*x)^(1/2))/
(c^3*d^3*e))*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2))/(a/c + x^2 + (
x*(c^3*d^4 + a*c^2*d^2*e^2))/(c^3*d^3*e))
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.35

$$\int \frac{(d+ex)^{5/2}}{(ade+(cd^2+ae^2)x+cdex^2)^{3/2}}\,dx = \frac{2cdex+4ae^2-2cd^2}{\sqrt{cdx+ae}\,c^2d^2}$$

input
```
int((e*x+d)^(5/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),x)
```

output
```
(2*(2*a*e**2 - c*d**2 + c*d*e*x))/(sqrt(a*e + c*d*x)*c**2*d**2)
```

3.317 $\int \frac{(d+ex)^{3/2}}{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}\,dx$

Optimal result

Integrand size = 39, antiderivative size = 46

$$\int \frac{(d+ex)^{3/2}}{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}\,dx=-\frac{2\sqrt{d+ex}}{cd\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}$$

output
```
-2*(e*x+d)^(1/2)/c/d/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.76

$$\int \frac{(d+ex)^{3/2}}{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}\,dx=-\frac{2\sqrt{d+ex}}{cd\sqrt{(ae+cdx)(d+ex)}}$$

input
```
Integrate[(d + e*x)^(3/2)/(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2),x]
```

output
```
(-2*Sqrt[d + e*x])/(c*d*Sqrt[(a*e + c*d*x)*(d + e*x)])
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.026$, Rules used = {1122}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex)^{3/2}}{(x(ae^2+cd^2)+ade+cdex^2)^{3/2}}dx$$
$$\Big\downarrow \text{1122}$$
$$-\frac{2\sqrt{d+ex}}{cd\sqrt{x(ae^2+cd^2)+ade+cdex^2}}$$

input
```
Int[(d + e*x)^(3/2)/(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2),x]
```

output
```
(-2*Sqrt[d + e*x])/(c*d*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])
```

Defintions of rubi rules used

rule 1122
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[e*(d + e*x)^(m - 1)*((a + b*x + c*x^2)^(p + 1)/(c*(p + 1))),
 x] /; FreeQ[{a, b, c, d, e, m, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&
EqQ[m + p, 0]
```

Maple [A] (verified)

Time = 1.04 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.91

method	result	size
default	$-\frac{2\sqrt{(ex+d)(cdx+ae)}}{\sqrt{ex+d}\,(cdx+ae)cd}$	42
gosper	$-\frac{2(cdx+ae)(ex+d)^{\frac{3}{2}}}{cd(cd\,x^{2}e+a\,e^{2}x+c\,d^{2}x+ade)^{\frac{3}{2}}}$	50
orering	$-\frac{2(cdx+ae)(ex+d)^{\frac{3}{2}}}{cd(ade+(a\,e^{2}+c\,d^{2})x+cd\,x^{2}e)^{\frac{3}{2}}}$	51

input
```
int((e*x+d)^(3/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(3/2),x,method=_RETURN
VERBOSE)
```

output
```
-2/(e*x+d)^(1/2)*((e*x+d)*(c*d*x+a*e))^(1/2)/(c*d*x+a*e)/c/d
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.61

$$\int \frac{(d+ex)^{3/2}}{(ade+(cd^2+ae^2)\,x+cdex^2)^{3/2}}\,dx = -\frac{2\,\sqrt{cdex^2+ade+(cd^2+ae^2)x}\sqrt{ex+d}}{c^2d^2ex^2+acd^2e+(c^2d^3+acde^2)x}$$

input
```
integrate((e*x+d)^(3/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),x, algorit
hm="fricas")
```

output
```
-2*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(e*x + d)/(c^2*d^2*e*x^
2 + a*c*d^2*e + (c^2*d^3 + a*c*d*e^2)*x)
```

Sympy [F]

$$\int \frac{(d+ex)^{3/2}}{(ade+(cd^2+ae^2)\,x+cdex^2)^{3/2}}\,dx = \int \frac{(d+ex)^{\frac{3}{2}}}{((d+ex)\,(ae+cdx))^{\frac{3}{2}}}\,dx$$

input
```
integrate((e*x+d)**(3/2)/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(3/2),x)
```

output
```
Integral((d + e*x)**(3/2)/((d + e*x)*(a*e + c*d*x))**(3/2), x)
```

Maxima [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.39

$$\int \frac{(d+ex)^{3/2}}{(ade+(cd^2+ae^2)\,x+cdex^2)^{3/2}}\,dx=-\frac{2}{\sqrt{cdx+ae}cd}$$

input
```
integrate((e*x+d)^(3/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),x, algorit
hm="maxima")
```

output
```
-2/(sqrt(c*d*x + a*e)*c*d)
```

Giac [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.39

$$\int \frac{(d+ex)^{3/2}}{(ade+(cd^2+ae^2)\,x+cdex^2)^{3/2}}\,dx=-\frac{2}{\sqrt{cdx+ae}cd}$$

input
```
integrate((e*x+d)^(3/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),x, algorit
hm="giac")
```

output
```
-2/(sqrt(c*d*x + a*e)*c*d)
```

Mupad [B] (verification not implemented)

Time = 5.91 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.78

$$\int \frac{(d+ex)^{3/2}}{(ade+(cd^2+ae^2)x+cdex^2)^{3/2}}\,dx = -\frac{2\sqrt{d+e\,x}\,\sqrt{c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e}}{c^2\,d^2\,e\,\left(\frac{a}{c}+x^2+\frac{x\,\left(c^2\,d^3+a\,c\,d\,e^2\right)}{c^2\,d^2\,e}\right)}$$

input
```
int((d + e*x)^(3/2)/(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(3/2),x)
```

output
```
-(2*(d + e*x)^(1/2)*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2))/(c^2*d^
2*e*(a/c + x^2 + (x*(c^2*d^3 + a*c*d*e^2))/(c^2*d^2*e)))
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.41

$$\int \frac{(d+ex)^{3/2}}{(ade+(cd^2+ae^2)x+cdex^2)^{3/2}}\,dx = -\frac{2}{\sqrt{cdx+ae}\,cd}$$

input
```
int((e*x+d)^(3/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),x)
```

output
```
( - 2)/(sqrt(a*e + c*d*x)*c*d)
```

3.318 $\int \frac{\sqrt{d+ex}}{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}\,dx$

Optimal result

Integrand size = 39, antiderivative size = 139

$$\int \frac{\sqrt{d+ex}}{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}\,dx = -\frac{2\sqrt{d+ex}}{\left(cd^2-ae^2\right)\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}} + \frac{2\sqrt{e}\arctan\left(\frac{\sqrt{cd^2-ae^2}\sqrt{d+ex}}{\sqrt{e}\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\right)}{\left(cd^2-ae^2\right)^{3/2}}$$

output
```
-2*(e*x+d)^(1/2)/(-a*e^2+c*d^2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)+2*
e^(1/2)*arctan(1/e^(1/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)*(-a*e^2+c
*d^2)^(1/2)*(e*x+d)^(1/2))/(-a*e^2+c*d^2)^(3/2)
```

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 115, normalized size of antiderivative = 0.83

$$\int \frac{\sqrt{d+ex}}{(ade+(cd^2+ae^2)x+cdex^2)^{3/2}}\,dx = -\frac{2\sqrt{d+ex}\left(\sqrt{cd^2-ae^2}+\sqrt{e}\sqrt{ae+cdx}\arctan\left(\frac{\sqrt{e}\sqrt{ae+cdx}}{\sqrt{cd^2-ae^2}}\right)\right)}{(cd^2-ae^2)^{3/2}\sqrt{(ae+cdx)(d+ex)}}$$

input
```
Integrate[Sqrt[d + e*x]/(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2),x]
```

output
```
(-2*Sqrt[d + e*x]*(Sqrt[c*d^2 - a*e^2] + Sqrt[e]*Sqrt[a*e + c*d*x]*ArcTan[
(Sqrt[e]*Sqrt[a*e + c*d*x])/Sqrt[c*d^2 - a*e^2]]))/((c*d^2 - a*e^2)^(3/2)*
Sqrt[(a*e + c*d*x)*(d + e*x)])
```

Rubi [A] (verified)

Time = 0.48 (sec) , antiderivative size = 139, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {1132, 1136, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{d+ex}}{(x(ae^2+cd^2)+ade+cdex^2)^{3/2}}dx$$

$$\Big\downarrow \ 1132$$

$$-\frac{e\int \frac{1}{\sqrt{d+ex}\sqrt{cdex^2+(cd^2+ae^2)x+ade}}dx}{cd^2-ae^2}-\frac{2\sqrt{d+ex}}{(cd^2-ae^2)\sqrt{x(ae^2+cd^2)+ade+cdex^2}}$$

$$\Big\downarrow \ 1136$$

$$-\frac{2e^2\int \frac{1}{\frac{\left(cdex^2+\left(cd^2+ae^2\right)x+ade\right)e^2}{d+ex}+\left(cd^2-ae^2\right)e}d\frac{\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}{\sqrt{d+ex}}}{cd^2-ae^2}-\frac{2\sqrt{d+ex}}{\left(cd^2-ae^2\right)\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}$$

$$\Big\downarrow \ 218$$

$$-\frac{2\sqrt{e}\arctan\left(\frac{\sqrt{e}\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{\sqrt{d+ex}\sqrt{cd^2-ae^2}}\right)}{\left(cd^2-ae^2\right)^{3/2}}-\frac{2\sqrt{d+ex}}{\left(cd^2-ae^2\right)\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}$$

input
```
Int[Sqrt[d + e*x]/(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2),x]
```

output
```
(-2*Sqrt[d + e*x])/((c*d^2 - a*e^2)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e
*x^2]) - (2*Sqrt[e]*ArcTan[(Sqrt[e]*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e
*x^2])/(Sqrt[c*d^2 - a*e^2]*Sqrt[d + e*x])])/(c*d^2 - a*e^2)^(3/2)
```

Defintions of rubi rules used

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 1132
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(2*c*d - b*e)*(d + e*x)^m*((a + b*x + c*x^2)^(p + 1)/(e*(p +
 1)*(b^2 - 4*a*c))), x] - Simp[(2*c*d - b*e)*((m + 2*p + 2)/((p + 1)*(b^2 -
 4*a*c)))   Int[(d + e*x)^(m - 1)*(a + b*x + c*x^2)^(p + 1), x], x] /; Free
Q[{a, b, c, d, e}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[p, -1] && LtQ
[0, m, 1] && IntegerQ[2*p]
```

rule 1136
```
Int[1/(Sqrt[(d_.) + (e_.)*(x_)]*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x
_Symbol] :> Simp[2*e   Subst[Int[1/(2*c*d - b*e + e^2*x^2), x], x, Sqrt[a +
 b*x + c*x^2]/Sqrt[d + e*x]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c*d^2
- b*d*e + a*e^2, 0]
```

Maple [A] (verified)

Time = 1.07 (sec) , antiderivative size = 126, normalized size of antiderivative = 0.91

method	result	size
default	$-\frac{2\sqrt{(ex+d)(cdx+ae)}\left(e\,\mathrm{arctanh}\left(\frac{e\sqrt{cdx+ae}}{\sqrt{(a\,e^2-c\,d^2)e}}\right)\sqrt{cdx+ae}-\sqrt{(a\,e^2-c\,d^2)e}\right)}{\sqrt{ex+d}\,(cdx+ae)(a\,e^2-c\,d^2)\sqrt{(a\,e^2-c\,d^2)e}}$	126

input
```
int((e*x+d)^(1/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(3/2),x,method=_RETURN
VERBOSE)
```

output
```
-2/(e*x+d)^(1/2)*((e*x+d)*(c*d*x+a*e))^(1/2)*(e*arctanh(e*(c*d*x+a*e)^(1/2
)/((a*e^2-c*d^2)*e)^(1/2))*(c*d*x+a*e)^(1/2)-((a*e^2-c*d^2)*e)^(1/2))/(c*d
*x+a*e)/(a*e^2-c*d^2)/((a*e^2-c*d^2)*e)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 500, normalized size of antiderivative = 3.60

$$\int \frac{\sqrt{d+ex}}{(ade+(cd^2+ae^2)\,x+cdex^2)^{3/2}}\,dx = \left[-\frac{(cdex^2+ade+(cd^2+ae^2)x)\sqrt{-\frac{e}{cd^2-ae^2}}\log\left(-\frac{cde^2x^2+2\,ae^3x-c}{}\right.}{acd^3e-}\right.$$

$$-\frac{2\left(\left(cdex^2+ade+(cd^2+ae^2)x\right)\sqrt{\frac{e}{cd^2-ae^2}}\arctan\left(-\frac{\sqrt{cdex^2+ade+(cd^2+ae^2)x}\,(cd^2-ae^2)\sqrt{ex+d}\sqrt{\frac{e}{cd^2-ae^2}}}{cde^2x^2+ade^2+(cd^2e+ae^3)x}\right)+\sqrt{cd}\right.}{acd^3e-a^2de^3+(c^2d^3e-acde^3)x^2+(c^2d^4-a^2e^4)x}$$

input
```
integrate((e*x+d)^(1/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),x, algorit
hm="fricas")
```

output
```
[-((c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(-e/(c*d^2 - a*e^2))*log(-(
c*d*e^2*x^2 + 2*a*e^3*x - c*d^3 + 2*a*d*e^2 + 2*sqrt(c*d*e*x^2 + a*d*e + (
c*d^2 + a*e^2)*x)*(c*d^2 - a*e^2)*sqrt(e*x + d)*sqrt(-e/(c*d^2 - a*e^2)))/
(e^2*x^2 + 2*d*e*x + d^2)) + 2*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)
*sqrt(e*x + d))/(a*c*d^3*e - a^2*d*e^3 + (c^2*d^3*e - a*c*d*e^3)*x^2 + (c^
2*d^4 - a^2*e^4)*x), -2*((c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(e/(c
*d^2 - a*e^2))*arctan(-sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(c*d^2
- a*e^2)*sqrt(e*x + d)*sqrt(e/(c*d^2 - a*e^2))/(c*d*e^2*x^2 + a*d*e^2 + (c
*d^2*e + a*e^3)*x)) + sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(e*x
 + d))/(a*c*d^3*e - a^2*d*e^3 + (c^2*d^3*e - a*c*d*e^3)*x^2 + (c^2*d^4 - a
^2*e^4)*x)]
```

Sympy [F]

$$\int \frac{\sqrt{d+ex}}{(ade+(cd^2+ae^2)\,x+cdex^2)^{3/2}}\,dx = \int \frac{\sqrt{d+ex}}{((d+ex)\,(ae+cdx))^{\frac{3}{2}}}\,dx$$

input
```
integrate((e*x+d)**(1/2)/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(3/2),x)
```

output
```
Integral(sqrt(d + e*x)/((d + e*x)*(a*e + c*d*x))**(3/2), x)
```

Maxima [F]

$$\int \frac{\sqrt{d+ex}}{(ade+(cd^2+ae^2)\,x+cdex^2)^{3/2}}\,dx = \int \frac{\sqrt{ex+d}}{(cdex^2+ade+(cd^2+ae^2)x)^{\frac{3}{2}}}\,dx$$

input
```
integrate((e*x+d)^(1/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),x, algorit
hm="maxima")
```

output
```
integrate(sqrt(e*x + d)/(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^(3/2), x)
```

Giac [A] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 124, normalized size of antiderivative = 0.89

$$\int \frac{\sqrt{d+ex}}{\left(ade+(cd^2+ae^2)\,x+cdex^2\right)^{3/2}}\,dx =$$
$$-2\,e\left(\frac{e\arctan\left(\frac{\sqrt{(ex+d)cde-cd^2e+ae^3}}{\sqrt{cd^2e-ae^3}}\right)}{\sqrt{cd^2e-ae^3}(cd^2|e|-ae^2|e|)}+\frac{e}{\sqrt{(ex+d)cde-cd^2e+ae^3}(cd^2|e|-ae^2|e|)}\right)$$

input
```
integrate((e*x+d)^(1/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),x, algorit
hm="giac")
```

output
```
-2*e*(e*arctan(sqrt((e*x + d)*c*d*e - c*d^2*e + a*e^3)/sqrt(c*d^2*e - a*e^
3))/(sqrt(c*d^2*e - a*e^3)*(c*d^2*abs(e) - a*e^2*abs(e))) + e/(sqrt((e*x +
 d)*c*d*e - c*d^2*e + a*e^3)*(c*d^2*abs(e) - a*e^2*abs(e))))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{d+ex}}{\left(ade+(cd^2+ae^2)\,x+cdex^2\right)^{3/2}}\,dx = \int \frac{\sqrt{d+e\,x}}{{\left(c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e\right)}^{3/2}}\,dx$$

input
```
int((d + e*x)^(1/2)/(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(3/2),x)
```

output
```
int((d + e*x)^(1/2)/(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(3/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 109, normalized size of antiderivative = 0.78

$$\int \frac{\sqrt{d+ex}}{(ade+(cd^2+ae^2)\,x+cdex^2)^{3/2}}\,dx = \frac{-2\sqrt{e}\,\sqrt{cdx+ae}\,\sqrt{-a\,e^2+c\,d^2}\,atan\left(\frac{\sqrt{cdx+ae}\,e}{\sqrt{e}\,\sqrt{-a\,e^2+c\,d^2}}\right)+2a\,e^2-2}{\sqrt{cdx+ae}\,(a^2e^4-2ac\,d^2e^2+c^2d^4)}$$

input
```
int((e*x+d)^(1/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),x)
```

output
```
(2*( - sqrt(e)*sqrt(a*e + c*d*x)*sqrt( - a*e**2 + c*d**2)*atan((sqrt(a*e +
 c*d*x)*e)/(sqrt(e)*sqrt( - a*e**2 + c*d**2))) + a*e**2 - c*d**2))/(sqrt(a
*e + c*d*x)*(a**2*e**4 - 2*a*c*d**2*e**2 + c**2*d**4))
```

3.319 $\int \frac{1}{\sqrt{d+ex}\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}\,dx$

Optimal result

Integrand size = 39, antiderivative size = 196

$$\int \frac{1}{\sqrt{d+ex}\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}\,dx = \frac{1}{\left(cd^2-ae^2\right)\sqrt{d+ex}\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}} - \frac{3cd\sqrt{d+ex}}{\left(cd^2-ae^2\right)^2\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}} + \frac{3cd\sqrt{e}\arctan\left(\frac{\sqrt{cd^2-ae^2}\sqrt{d+ex}}{\sqrt{e}\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\right)}{\left(cd^2-ae^2\right)^{5/2}}$$

output
```
1/(-a*e^2+c*d^2)/(e*x+d)^(1/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)-3*c
*d*(e*x+d)^(1/2)/(-a*e^2+c*d^2)^2/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)+
3*c*d*e^(1/2)*arctan(1/e^(1/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)*(-a
*e^2+c*d^2)^(1/2)*(e*x+d)^(1/2))/(-a*e^2+c*d^2)^(5/2)
```

Mathematica [A] (verified)

Time = 0.36 (sec) , antiderivative size = 141, normalized size of antiderivative = 0.72

$$\int \frac{1}{\sqrt{d+ex}\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}\,dx = \frac{-\sqrt{cd^2-ae^2}\left(ae^2+cd(2d+3ex)\right)-3cd\sqrt{e}\sqrt{ae+cdx}(}{\left(cd^2-ae^2\right)^{5/2}\sqrt{d+ex}\sqrt{(ae+cdx)}}$$

input
```
Integrate[1/(Sqrt[d + e*x]*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2)),
x]
```

output
```
(-(Sqrt[c*d^2 - a*e^2]*(a*e^2 + c*d*(2*d + 3*e*x))) - 3*c*d*Sqrt[e]*Sqrt[a
*e + c*d*x]*(d + e*x)*ArcTan[(Sqrt[e]*Sqrt[a*e + c*d*x])/Sqrt[c*d^2 - a*e^
2]])/((c*d^2 - a*e^2)^(5/2)*Sqrt[d + e*x]*Sqrt[(a*e + c*d*x)*(d + e*x)])
```

Rubi [A] (verified)

Time = 0.60 (sec) , antiderivative size = 213, normalized size of antiderivative = 1.09, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.103$, Rules used = {1135, 1132, 1136, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{d+ex}\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}dx$$

$$\Big\downarrow \text{1135}$$

$$\frac{3cd\int \frac{\sqrt{d+ex}}{\left(cdex^2+\left(cd^2+ae^2\right)x+ade\right)^{3/2}}dx}{2\left(cd^2-ae^2\right)}+\frac{1}{\sqrt{d+ex}\left(cd^2-ae^2\right)\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}$$

$$\Big\downarrow \text{1132}$$

$$\frac{3cd\left(-\frac{e\int \frac{1}{\sqrt{d+ex}\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}dx}{cd^2-ae^2}-\frac{2\sqrt{d+ex}}{\left(cd^2-ae^2\right)\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}\right)}{2\left(cd^2-ae^2\right)}+\frac{1}{\sqrt{d+ex}\left(cd^2-ae^2\right)\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}$$

$$\Big\downarrow \text{1136}$$

$$\frac{3cd\left(-\frac{2e^2\int \frac{1}{\frac{\left(cdex^2+\left(cd^2+ae^2\right)x+ade\right)e^2}{d+ex}+\left(cd^2-ae^2\right)e}d\frac{\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}{\sqrt{d+ex}}}{cd^2-ae^2}-\frac{2\sqrt{d+ex}}{(cd^2-ae^2)\sqrt{x(ae^2+cd^2)+ade+cdex^2}}\right)}{2\left(cd^2-ae^2\right)}+\frac{1}{\sqrt{d+ex}\left(cd^2-ae^2\right)\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}$$

$$\Big\downarrow \; 218$$

$$\frac{3cd\left(-\frac{2\sqrt{e}\arctan\left(\frac{\sqrt{e}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{\sqrt{d+ex}\sqrt{cd^2-ae^2}}\right)}{\left(cd^2-ae^2\right)^{3/2}}-\frac{2\sqrt{d+ex}}{(cd^2-ae^2)\sqrt{x(ae^2+cd^2)+ade+cdex^2}}\right)}{2\left(cd^2-ae^2\right)}+\frac{1}{\sqrt{d+ex}\left(cd^2-ae^2\right)\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}$$

input
```
Int[1/(Sqrt[d + e*x]*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2)),x]
```

output
```
1/((c*d^2 - a*e^2)*Sqrt[d + e*x]*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^
2]) + (3*c*d*((-2*Sqrt[d + e*x])/((c*d^2 - a*e^2)*Sqrt[a*d*e + (c*d^2 + a*
e^2)*x + c*d*e*x^2]) - (2*Sqrt[e]*ArcTan[(Sqrt[e]*Sqrt[a*d*e + (c*d^2 + a*
e^2)*x + c*d*e*x^2])/(Sqrt[c*d^2 - a*e^2]*Sqrt[d + e*x])])/(c*d^2 - a*e^2)
^(3/2)))/(2*(c*d^2 - a*e^2))
```

Defintions of rubi rules used

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 1132
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(2*c*d - b*e)*(d + e*x)^m*((a + b*x + c*x^2)^(p + 1)/(e*(p +
 1)*(b^2 - 4*a*c))), x] - Simp[(2*c*d - b*e)*((m + 2*p + 2)/((p + 1)*(b^2 -
 4*a*c)))   Int[(d + e*x)^(m - 1)*(a + b*x + c*x^2)^(p + 1), x], x] /; Free
Q[{a, b, c, d, e}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[p, -1] && LtQ
[0, m, 1] && IntegerQ[2*p]
```

rule 1135
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(-e)*(d + e*x)^m*((a + b*x + c*x^2)^(p + 1)/((m + p + 1)*(2*
c*d - b*e))), x] + Simp[c*((m + 2*p + 2)/((m + p + 1)*(2*c*d - b*e)))   Int
[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p},
 x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[m, 0] && NeQ[m + p + 1, 0] && I
ntegerQ[2*p]
```

rule 1136
```
Int[1/(Sqrt[(d_.) + (e_.)*(x_)]*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x
_Symbol] :> Simp[2*e   Subst[Int[1/(2*c*d - b*e + e^2*x^2), x], x, Sqrt[a +
 b*x + c*x^2]/Sqrt[d + e*x]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c*d^2
- b*d*e + a*e^2, 0]
```

Maple [A] (verified)

Time = 1.10 (sec) , antiderivative size = 225, normalized size of antiderivative = 1.15

method	result
default	$\frac{\sqrt{(ex+d)(cdx+ae)}\left(3\,\mathrm{arctanh}\left(\frac{e\sqrt{cdx+ae}}{\sqrt{\left(a\,e^{2}-c\,d^{2}\right)e}}\right)\sqrt{cdx+ae}\,cd\,e^{2}x+3\,\mathrm{arctanh}\left(\frac{e\sqrt{cdx+ae}}{\sqrt{\left(a\,e^{2}-c\,d^{2}\right)e}}\right)\sqrt{cdx+ae}\,c\,d^{2}e-3\sqrt{(a\,e^{2}-c\,d^{2})e}\right.}{(ex+d)^{\frac{3}{2}}(cdx+ae)(a\,e^{2}-c\,d^{2})^{2}\sqrt{(a\,e^{2}-c\,d^{2})e}}$

input
```
int(1/(e*x+d)^(1/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(3/2),x,method=_RETU
RNVERBOSE)
```

output
```
((e*x+d)*(c*d*x+a*e))^(1/2)*(3*arctanh(e*(c*d*x+a*e)^(1/2)/((a*e^2-c*d^2)*
e)^(1/2))*(c*d*x+a*e)^(1/2)*c*d*e^2*x+3*arctanh(e*(c*d*x+a*e)^(1/2)/((a*e^
2-c*d^2)*e)^(1/2))*(c*d*x+a*e)^(1/2)*c*d^2*e-3*((a*e^2-c*d^2)*e)^(1/2)*c*d
*e*x-((a*e^2-c*d^2)*e)^(1/2)*a*e^2-2*((a*e^2-c*d^2)*e)^(1/2)*c*d^2)/(e*x+d
)^(3/2)/(c*d*x+a*e)/(a*e^2-c*d^2)^2/((a*e^2-c*d^2)*e)^(1/2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 371 vs. $2(176) = 352$.

Time = 0.12 (sec) , antiderivative size = 764, normalized size of antiderivative = 3.90

$$\int \frac{1}{\sqrt{d+ex}\,(ade+(cd^2+ae^2)\,x+cdex^2)^{3/2}}\,dx = \left[\frac{3\,(c^2d^2e^2x^3+acd^3e+(2\,c^2d^3e+acde^3)x^2+(c^2d^4+2}{2\,(ac^2d^6e-}\right.$$

$$-\frac{3\,(c^2d^2e^2x^3+acd^3e+(2\,c^2d^3e+acde^3)x^2+(c^2d^4+2\,acd^2e^2)x)\sqrt{\frac{e}{cd^2-ae^2}}\arctan\left(-\frac{\sqrt{cdex^2+ade+(cd^2+ae^2)x}}{cde^2x^2+ade}\right.}{ac^2d^6e-2\,a^2cd^4e^3+a^3d^2e^5+(c^3d^5e^2-2\,ac^2d^3e^4+a^2cde^6)x^3+(2\,c^3d^6}$$

input
```
integrate(1/(e*x+d)^(1/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),x, algor
ithm="fricas")
```

output
```
[1/2*(3*(c^2*d^2*e^2*x^3 + a*c*d^3*e + (2*c^2*d^3*e + a*c*d*e^3)*x^2 + (c^
2*d^4 + 2*a*c*d^2*e^2)*x)*sqrt(-e/(c*d^2 - a*e^2))*log(-(c*d*e^2*x^2 + 2*a
*e^3*x - c*d^3 + 2*a*d*e^2 - 2*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)
*(c*d^2 - a*e^2)*sqrt(e*x + d)*sqrt(-e/(c*d^2 - a*e^2)))/(e^2*x^2 + 2*d*e*
x + d^2)) - 2*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(3*c*d*e*x + 2*c
*d^2 + a*e^2)*sqrt(e*x + d))/(a*c^2*d^6*e - 2*a^2*c*d^4*e^3 + a^3*d^2*e^5
+ (c^3*d^5*e^2 - 2*a*c^2*d^3*e^4 + a^2*c*d*e^6)*x^3 + (2*c^3*d^6*e - 3*a*c
^2*d^4*e^3 + a^3*e^7)*x^2 + (c^3*d^7 - 3*a^2*c*d^3*e^4 + 2*a^3*d*e^6)*x),
-(3*(c^2*d^2*e^2*x^3 + a*c*d^3*e + (2*c^2*d^3*e + a*c*d*e^3)*x^2 + (c^2*d^
4 + 2*a*c*d^2*e^2)*x)*sqrt(e/(c*d^2 - a*e^2))*arctan(-sqrt(c*d*e*x^2 + a*d
*e + (c*d^2 + a*e^2)*x)*(c*d^2 - a*e^2)*sqrt(e*x + d)*sqrt(e/(c*d^2 - a*e^
2))/(c*d*e^2*x^2 + a*d*e^2 + (c*d^2*e + a*e^3)*x)) + sqrt(c*d*e*x^2 + a*d*
e + (c*d^2 + a*e^2)*x)*(3*c*d*e*x + 2*c*d^2 + a*e^2)*sqrt(e*x + d))/(a*c^2
*d^6*e - 2*a^2*c*d^4*e^3 + a^3*d^2*e^5 + (c^3*d^5*e^2 - 2*a*c^2*d^3*e^4 +
a^2*c*d*e^6)*x^3 + (2*c^3*d^6*e - 3*a*c^2*d^4*e^3 + a^3*e^7)*x^2 + (c^3*d^
7 - 3*a^2*c*d^3*e^4 + 2*a^3*d*e^6)*x)]
```

Sympy [F]

$$\int \frac{1}{\sqrt{d+ex}\,(ade+(cd^2+ae^2)\,x+cdex^2)^{3/2}}\,dx = \int \frac{1}{((d+ex)\,(ae+cdx))^{\frac{3}{2}}\,\sqrt{d+ex}}\,dx$$

input
```
integrate(1/(e*x+d)**(1/2)/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(3/2),x)
```

output
```
Integral(1/(((d + e*x)*(a*e + c*d*x))**(3/2)*sqrt(d + e*x)), x)
```

Maxima [F]

$$\int \frac{1}{\sqrt{d+ex}\,(ade+(cd^2+ae^2)\,x+cdex^2)^{3/2}}\,dx = \int \frac{1}{(cdex^2+ade+(cd^2+ae^2)x)^{\frac{3}{2}}\sqrt{ex+d}}\,dx$$

input
```
integrate(1/(e*x+d)^(1/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),x, algor
ithm="maxima")
```

output
```
integrate(1/((c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^(3/2)*sqrt(e*x + d)),
 x)
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 266, normalized size of antiderivative = 1.36

$$\int \frac{1}{\sqrt{d+ex}\,(ade+(cd^2+ae^2)\,x+cdex^2)^{3/2}}\,dx =$$

$$-\left(\frac{3\,cde\arctan\left(\frac{\sqrt{(ex+d)cde-cd^2e+ae^3}}{\sqrt{cd^2e-ae^3}}\right)}{(c^2d^4|e|-2\,acd^2e^2|e|+a^2e^4|e|)\sqrt{cd^2e-ae^3}}+\frac{2\,c^2d^3e^2-2\,ac}{(c^2d^4|e|-2\,acd^2e^2|e|+a^2e^4|e|)\left(\sqrt{(ex+d)cde-cd^2e}\right.}\right.$$

input
```
integrate(1/(e*x+d)^(1/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),x, algor
ithm="giac")
```

output
```
-(3*c*d*e*arctan(sqrt((e*x + d)*c*d*e - c*d^2*e + a*e^3)/sqrt(c*d^2*e - a*
e^3))/((c^2*d^4*abs(e) - 2*a*c*d^2*e^2*abs(e) + a^2*e^4*abs(e))*sqrt(c*d^2
*e - a*e^3)) + (2*c^2*d^3*e^2 - 2*a*c*d*e^4 + 3*((e*x + d)*c*d*e - c*d^2*e
 + a*e^3)*c*d*e)/((c^2*d^4*abs(e) - 2*a*c*d^2*e^2*abs(e) + a^2*e^4*abs(e))
*(sqrt((e*x + d)*c*d*e - c*d^2*e + a*e^3)*c*d^2*e - sqrt((e*x + d)*c*d*e -
 c*d^2*e + a*e^3)*a*e^3 + ((e*x + d)*c*d*e - c*d^2*e + a*e^3)^(3/2))))*e
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{d+ex}\,(ade+(cd^2+ae^2)\,x+cdex^2)^{3/2}}\,dx = \int \frac{1}{\sqrt{d+e\,x}\,\left(c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e\right)^{3/2}}\,dx$$

input
```
int(1/((d + e*x)^(1/2)*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(3/2)),x)
```

output
```
int(1/((d + e*x)^(1/2)*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(3/2)), x)
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 267, normalized size of antiderivative = 1.36

$$\int \frac{1}{\sqrt{d+ex}\,(ade+(cd^2+ae^2)\,x+cdex^2)^{3/2}}\,dx = \frac{3\sqrt{e}\,\sqrt{cdx+ae}\,\sqrt{-a\,e^2+c\,d^2}\,atan\left(\frac{\sqrt{cdx+ae}\,e}{\sqrt{e}\,\sqrt{-a\,e^2+c\,d^2}}\right)c\,d^2 +}{\sqrt{cdx+ae}\,(a^3e^7x-3a}$$

input
```
int(1/(e*x+d)^(1/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),x)
```

output
```
(3*sqrt(e)*sqrt(a*e + c*d*x)*sqrt( - a*e**2 + c*d**2)*atan((sqrt(a*e + c*d
*x)*e)/(sqrt(e)*sqrt( - a*e**2 + c*d**2)))*c*d**2 + 3*sqrt(e)*sqrt(a*e + c
*d*x)*sqrt( - a*e**2 + c*d**2)*atan((sqrt(a*e + c*d*x)*e)/(sqrt(e)*sqrt( -
 a*e**2 + c*d**2)))*c*d*e*x - a**2*e**4 - a*c*d**2*e**2 - 3*a*c*d*e**3*x +
 2*c**2*d**4 + 3*c**2*d**3*e*x)/(sqrt(a*e + c*d*x)*(a**3*d*e**6 + a**3*e**
7*x - 3*a**2*c*d**3*e**4 - 3*a**2*c*d**2*e**5*x + 3*a*c**2*d**5*e**2 + 3*a
*c**2*d**4*e**3*x - c**3*d**7 - c**3*d**6*e*x))
```

3.320 $\int \frac{1}{(d+ex)^{3/2}\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}\,dx$

Optimal result

Integrand size = 39, antiderivative size = 269

$$\int \frac{1}{(d+ex)^{3/2}\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}\,dx = \frac{1}{2\left(cd^2-ae^2\right)(d+ex)^{3/2}\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}$$
$$+\frac{5cd}{4\left(cd^2-ae^2\right)^2\sqrt{d+ex}\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}$$
$$-\frac{15c^2d^2\sqrt{d+ex}}{4\left(cd^2-ae^2\right)^3\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}$$
$$+\frac{15c^2d^2\sqrt{e}\arctan\left(\frac{\sqrt{cd^2-ae^2}\sqrt{d+ex}}{\sqrt{e}\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\right)}{4\left(cd^2-ae^2\right)^{7/2}}$$

output
```
1/2/(-a*e^2+c*d^2)/(e*x+d)^(3/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)+5
/4*c*d/(-a*e^2+c*d^2)^2/(e*x+d)^(1/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1
/2)-15/4*c^2*d^2*(e*x+d)^(1/2)/(-a*e^2+c*d^2)^3/(a*d*e+(a*e^2+c*d^2)*x+c*d
*e*x^2)^(1/2)+15/4*c^2*d^2*e^(1/2)*arctan(1/e^(1/2)/(a*d*e+(a*e^2+c*d^2)*x
+c*d*e*x^2)^(1/2)*(-a*e^2+c*d^2)^(1/2)*(e*x+d)^(1/2))/(-a*e^2+c*d^2)^(7/2)
```

Mathematica [A] (verified)

Time = 0.50 (sec) , antiderivative size = 183, normalized size of antiderivative = 0.68

$$\int \frac{1}{(d+ex)^{3/2}\left(ade+(cd^2+ae^2)\,x+cdex^2\right)^{3/2}}\,dx = \frac{-\sqrt{cd^2-ae^2}(-2a^2e^4+acde^2(9d+5ex)+c^2d^2(8d^2\cdots}{4\,(cd^2-ae^2)^{7/\cdots}}$$

input
```
Integrate[1/((d + e*x)^(3/2)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2)
),x]
```

output
```
(-(Sqrt[c*d^2 - a*e^2]*(-2*a^2*e^4 + a*c*d*e^2*(9*d + 5*e*x) + c^2*d^2*(8*
d^2 + 25*d*e*x + 15*e^2*x^2))) - 15*c^2*d^2*Sqrt[e]*Sqrt[a*e + c*d*x]*(d +
 e*x)^2*ArcTan[(Sqrt[e]*Sqrt[a*e + c*d*x])/Sqrt[c*d^2 - a*e^2]])/(4*(c*d^2
 - a*e^2)^(7/2)*(d + e*x)^(3/2)*Sqrt[(a*e + c*d*x)*(d + e*x)])
```

Rubi [A] (verified)

Time = 0.78 (sec) , antiderivative size = 290, normalized size of antiderivative = 1.08, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.128$, Rules used = {1135, 1135, 1132, 1136, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(d+ex)^{3/2}\left(x\,(ae^2+cd^2)+ade+cdex^2\right)^{3/2}}\,dx$$

$$\Big\downarrow \text{1135}$$

$$\frac{5cd\int \frac{1}{\sqrt{d+ex}\left(cdex^2+(cd^2+ae^2)x+ade\right)^{3/2}}dx}{4\,(cd^2-ae^2)}+\frac{1}{2(d+ex)^{3/2}\,(cd^2-ae^2)\,\sqrt{x\,(ae^2+cd^2)+ade+cdex^2}}$$

$$\Big\downarrow \text{1135}$$

$$\frac{5cd\left(\frac{3cd\int \frac{\sqrt{d+ex}}{\left(cdex^2+\left(cd^2+ae^2\right)x+ade\right)^{3/2}}dx}{2(cd^2-ae^2)}+\frac{1}{\sqrt{d+ex}(cd^2-ae^2)\sqrt{x(ae^2+cd^2)+ade+cdex^2}}\right)}{4\left(cd^2-ae^2\right)}+\frac{1}{2(d+ex)^{3/2}\left(cd^2-ae^2\right)\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}$$

$\downarrow$ 1132

$$\frac{5cd\left(\frac{3cd\left(-\frac{e\int \frac{1}{\sqrt{d+ex}\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}dx}{cd^2-ae^2}-\frac{2\sqrt{d+ex}}{\left(cd^2-ae^2\right)\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}\right)}{2(cd^2-ae^2)}+\frac{1}{\sqrt{d+ex}(cd^2-ae^2)\sqrt{x(ae^2+cd^2)+ade+cdex^2}}\right)}{4\left(cd^2-ae^2\right)}+\frac{1}{2(d+ex)^{3/2}\left(cd^2-ae^2\right)\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}$$

$\downarrow$ 1136

$$\frac{5cd\left(\frac{3cd\left(-\frac{2e^2\int \frac{1}{\frac{\left(cdex^2+\left(cd^2+ae^2\right)x+ade\right)e^2}{d+ex}+\left(cd^2-ae^2\right)e}d\frac{\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}{\sqrt{d+ex}}}{cd^2-ae^2}-\frac{2\sqrt{d+ex}}{\left(cd^2-ae^2\right)\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}\right)}{2(cd^2-ae^2)}+\frac{}{\sqrt{d+ex}(cd^2}\right.}{4\left(cd^2-ae^2\right)}$$

$$\frac{1}{2(d+ex)^{3/2}\left(cd^2-ae^2\right)\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}$$

$\downarrow$ 218

$$\frac{5cd\left(\frac{3cd\left(-\frac{2\sqrt{e}\arctan\left(\frac{\sqrt{e}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{\sqrt{d+ex}\sqrt{cd^2-ae^2}}\right)}{\left(cd^2-ae^2\right)^{3/2}}-\frac{2\sqrt{d+ex}}{\left(cd^2-ae^2\right)\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}\right)}{2(cd^2-ae^2)}+\frac{1}{\sqrt{d+ex}(cd^2-ae^2)\sqrt{x(ae^2+cd^2)+ade+cdex^2}}\right.}{4\left(cd^2-ae^2\right)}$$

$$\frac{1}{2(d+ex)^{3/2}\left(cd^2-ae^2\right)\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}$$

input
```
Int[1/((d + e*x)^(3/2)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2)),x]
```

output
```
1/(2*(c*d^2 - a*e^2)*(d + e*x)^(3/2)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*
e*x^2]) + (5*c*d*(1/((c*d^2 - a*e^2)*Sqrt[d + e*x]*Sqrt[a*d*e + (c*d^2 + a
*e^2)*x + c*d*e*x^2]) + (3*c*d*((-2*Sqrt[d + e*x])/((c*d^2 - a*e^2)*Sqrt[a
*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]) - (2*Sqrt[e]*ArcTan[(Sqrt[e]*Sqrt[a
*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(Sqrt[c*d^2 - a*e^2]*Sqrt[d + e*x])
])/(c*d^2 - a*e^2)^(3/2)))/(2*(c*d^2 - a*e^2))))/(4*(c*d^2 - a*e^2))
```

Defintions of rubi rules used

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 1132
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(2*c*d - b*e)*(d + e*x)^m*((a + b*x + c*x^2)^(p + 1)/(e*(p +
 1)*(b^2 - 4*a*c))), x] - Simp[(2*c*d - b*e)*((m + 2*p + 2)/((p + 1)*(b^2 -
 4*a*c)))   Int[(d + e*x)^(m - 1)*(a + b*x + c*x^2)^(p + 1), x], x] /; Free
Q[{a, b, c, d, e}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[p, -1] && LtQ
[0, m, 1] && IntegerQ[2*p]
```

rule 1135
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(-e)*(d + e*x)^m*((a + b*x + c*x^2)^(p + 1)/((m + p + 1)*(2*
c*d - b*e))), x] + Simp[c*((m + 2*p + 2)/((m + p + 1)*(2*c*d - b*e)))   Int
[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p},
 x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[m, 0] && NeQ[m + p + 1, 0] && I
ntegerQ[2*p]
```

rule 1136
```
Int[1/(Sqrt[(d_.) + (e_.)*(x_)]*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x
_Symbol] :> Simp[2*e   Subst[Int[1/(2*c*d - b*e + e^2*x^2), x], x, Sqrt[a +
 b*x + c*x^2]/Sqrt[d + e*x]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c*d^2
- b*d*e + a*e^2, 0]
```

Maple [A] (verified)

Time = 1.27 (sec) , antiderivative size = 374, normalized size of antiderivative = 1.39

method	result
default	$-\frac{\sqrt{(ex+d)(cdx+ae)}\left(15\sqrt{cdx+ae}\,\operatorname{arctanh}\left(\frac{e\sqrt{cdx+ae}}{\sqrt{\left(a e^{2}-c d^{2}\right)e}}\right)c^{2}d^{2}e^{3}x^{2}+30\sqrt{cdx+ae}\,\operatorname{arctanh}\left(\frac{e\sqrt{cdx+ae}}{\sqrt{\left(a e^{2}-c d^{2}\right)e}}\right)c^{2}d^{3}e^{2}x+15\sqrt{cd \cdots}\right.}{\cdots}$

input
```
int(1/(e*x+d)^(3/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(3/2),x,method=_RETU
RNVERBOSE)
```

output
```
-1/4/(e*x+d)^(5/2)*((e*x+d)*(c*d*x+a*e))^(1/2)*(15*(c*d*x+a*e)^(1/2)*arcta
nh(e*(c*d*x+a*e)^(1/2)/((a*e^2-c*d^2)*e)^(1/2))*c^2*d^2*e^3*x^2+30*(c*d*x+
a*e)^(1/2)*arctanh(e*(c*d*x+a*e)^(1/2)/((a*e^2-c*d^2)*e)^(1/2))*c^2*d^3*e^
2*x+15*(c*d*x+a*e)^(1/2)*arctanh(e*(c*d*x+a*e)^(1/2)/((a*e^2-c*d^2)*e)^(1/
2))*c^2*d^4*e-15*((a*e^2-c*d^2)*e)^(1/2)*c^2*d^2*e^2*x^2-5*((a*e^2-c*d^2)*
e)^(1/2)*a*c*d*e^3*x-25*((a*e^2-c*d^2)*e)^(1/2)*c^2*d^3*e*x+2*((a*e^2-c*d^
2)*e)^(1/2)*a^2*e^4-9*((a*e^2-c*d^2)*e)^(1/2)*a*c*d^2*e^2-8*((a*e^2-c*d^2)
*e)^(1/2)*c^2*d^4)/(c*d*x+a*e)/(a*e^2-c*d^2)^3/((a*e^2-c*d^2)*e)^(1/2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 559 vs. $2(237) = 474$.

Time = 0.15 (sec) , antiderivative size = 1140, normalized size of antiderivative = 4.24

$$\int \frac{1}{(d+ex)^{3/2}\left(ade+(cd^2+ae^2)\,x+cdex^2\right)^{3/2}}\,dx = \text{Too large to display}$$

input
```
integrate(1/(e*x+d)^(3/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),x, algor
ithm="fricas")
```

output
```
[-1/8*(15*(c^3*d^3*e^3*x^4 + a*c^2*d^5*e + (3*c^3*d^4*e^2 + a*c^2*d^2*e^4)
*x^3 + 3*(c^3*d^5*e + a*c^2*d^3*e^3)*x^2 + (c^3*d^6 + 3*a*c^2*d^4*e^2)*x)*
sqrt(-e/(c*d^2 - a*e^2))*log(-(c*d*e^2*x^2 + 2*a*e^3*x - c*d^3 + 2*a*d*e^2
 + 2*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(c*d^2 - a*e^2)*sqrt(e*x
+ d)*sqrt(-e/(c*d^2 - a*e^2)))/(e^2*x^2 + 2*d*e*x + d^2)) + 2*(15*c^2*d^2*
e^2*x^2 + 8*c^2*d^4 + 9*a*c*d^2*e^2 - 2*a^2*e^4 + 5*(5*c^2*d^3*e + a*c*d*e
^3)*x)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(e*x + d))/(a*c^3*d
^9*e - 3*a^2*c^2*d^7*e^3 + 3*a^3*c*d^5*e^5 - a^4*d^3*e^7 + (c^4*d^7*e^3 -
3*a*c^3*d^5*e^5 + 3*a^2*c^2*d^3*e^7 - a^3*c*d*e^9)*x^4 + (3*c^4*d^8*e^2 -
8*a*c^3*d^6*e^4 + 6*a^2*c^2*d^4*e^6 - a^4*e^10)*x^3 + 3*(c^4*d^9*e - 2*a*c
^3*d^7*e^3 + 2*a^3*c*d^3*e^7 - a^4*d*e^9)*x^2 + (c^4*d^10 - 6*a^2*c^2*d^6*
e^4 + 8*a^3*c*d^4*e^6 - 3*a^4*d^2*e^8)*x), -1/4*(15*(c^3*d^3*e^3*x^4 + a*c
^2*d^5*e + (3*c^3*d^4*e^2 + a*c^2*d^2*e^4)*x^3 + 3*(c^3*d^5*e + a*c^2*d^3*
e^3)*x^2 + (c^3*d^6 + 3*a*c^2*d^4*e^2)*x)*sqrt(e/(c*d^2 - a*e^2))*arctan(-
sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(c*d^2 - a*e^2)*sqrt(e*x + d)*
sqrt(e/(c*d^2 - a*e^2))/(c*d*e^2*x^2 + a*d*e^2 + (c*d^2*e + a*e^3)*x)) + (
15*c^2*d^2*e^2*x^2 + 8*c^2*d^4 + 9*a*c*d^2*e^2 - 2*a^2*e^4 + 5*(5*c^2*d^3*
e + a*c*d*e^3)*x)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(e*x + d
))/(a*c^3*d^9*e - 3*a^2*c^2*d^7*e^3 + 3*a^3*c*d^5*e^5 - a^4*d^3*e^7 + (c^4
*d^7*e^3 - 3*a*c^3*d^5*e^5 + 3*a^2*c^2*d^3*e^7 - a^3*c*d*e^9)*x^4 + (3*...
```

Sympy [F]

$$\int \frac{1}{(d+ex)^{3/2}\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}\,dx=\int \frac{1}{\left((d+ex)\left(ae+cdx\right)\right)^{\frac{3}{2}}\left(d+ex\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(e*x+d)**(3/2)/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(3/2),x)
```

output
```
Integral(1/(((d + e*x)*(a*e + c*d*x))**(3/2)*(d + e*x)**(3/2)), x)
```

Maxima [F]

$$\int \frac{1}{(d+ex)^{3/2}\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}\,dx=\int \frac{1}{\left(cdex^2+ade+(cd^2+ae^2)x\right)^{\frac{3}{2}}(ex+d)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(e*x+d)^(3/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),x, algor
ithm="maxima")
```

output
```
integrate(1/((c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^(3/2)*(e*x + d)^(3/2)
), x)
```

Giac [A] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 374, normalized size of antiderivative = 1.39

$$\int \frac{1}{(d+ex)^{3/2}\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}\,dx=$$
$$-\frac{1}{4}\left(\frac{15\,c^2d^2e\arctan\left(\frac{\sqrt{(ex+d)cde-cd^2e+ae^3}}{\sqrt{cd^2e-ae^3}}\right)}{\left(c^3d^6|e|-3\,ac^2d^4e^2|e|+3\,a^2cd^2e^4|e|-a^3e^6|e|\right)\sqrt{cd^2e-ae^3}}+\frac{8\,c^2}{\left(c^3d^6|e|-3\,ac^2d^4e^2|e|+3\,a^2cd^2e^4|e|\right.}\right.$$

input
```
integrate(1/(e*x+d)^(3/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),x, algor
ithm="giac")
```

output
```
-1/4*(15*c^2*d^2*e*arctan(sqrt((e*x + d)*c*d*e - c*d^2*e + a*e^3)/sqrt(c*d
^2*e - a*e^3))/((c^3*d^6*abs(e) - 3*a*c^2*d^4*e^2*abs(e) + 3*a^2*c*d^2*e^4
*abs(e) - a^3*e^6*abs(e))*sqrt(c*d^2*e - a*e^3)) + 8*c^2*d^2*e/((c^3*d^6*a
bs(e) - 3*a*c^2*d^4*e^2*abs(e) + 3*a^2*c*d^2*e^4*abs(e) - a^3*e^6*abs(e))*
sqrt((e*x + d)*c*d*e - c*d^2*e + a*e^3)) + (9*sqrt((e*x + d)*c*d*e - c*d^2
*e + a*e^3)*c^3*d^4*e^2 - 9*sqrt((e*x + d)*c*d*e - c*d^2*e + a*e^3)*a*c^2*
d^2*e^4 + 7*((e*x + d)*c*d*e - c*d^2*e + a*e^3)^(3/2)*c^2*d^2*e)/((c^3*d^6
*abs(e) - 3*a*c^2*d^4*e^2*abs(e) + 3*a^2*c*d^2*e^4*abs(e) - a^3*e^6*abs(e)
)*(e*x + d)^2*c^2*d^2*e^2))*e
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(d+ex)^{3/2}\left(ade+(cd^2+ae^2)x+cdex^2\right)^{3/2}}\,dx=\int \frac{1}{(d+e\,x)^{3/2}\,\left(c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e\right)^{3/2}}\,dx$$

input
```
int(1/((d + e*x)^(3/2)*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(3/2)),x)
```

output
```
int(1/((d + e*x)^(3/2)*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(3/2)), x)
```

Reduce [B] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 503, normalized size of antiderivative = 1.87

$$\int \frac{1}{(d+ex)^{3/2}\left(ade+(cd^2+ae^2)x+cdex^2\right)^{3/2}}\,dx=\frac{-15\sqrt{e}\,\sqrt{cdx+ae}\,\sqrt{-a\,e^2+c\,d^2}\,atan\left(\frac{\sqrt{cdx+ae}\,e}{\sqrt{e}\,\sqrt{-a\,e^2+c\,d^2}}\right)}{4\sqrt{c}}$$

input
```
int(1/(e*x+d)^(3/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),x)
```

output

```
( - 15*sqrt(e)*sqrt(a*e + c*d*x)*sqrt( - a*e**2 + c*d**2)*atan((sqrt(a*e +
 c*d*x)*e)/(sqrt(e)*sqrt( - a*e**2 + c*d**2)))*c**2*d**4 - 30*sqrt(e)*sqrt
(a*e + c*d*x)*sqrt( - a*e**2 + c*d**2)*atan((sqrt(a*e + c*d*x)*e)/(sqrt(e)
*sqrt( - a*e**2 + c*d**2)))*c**2*d**3*e*x - 15*sqrt(e)*sqrt(a*e + c*d*x)*s
qrt( - a*e**2 + c*d**2)*atan((sqrt(a*e + c*d*x)*e)/(sqrt(e)*sqrt( - a*e**2
 + c*d**2)))*c**2*d**2*e**2*x**2 - 2*a**3*e**6 + 11*a**2*c*d**2*e**4 + 5*a
**2*c*d*e**5*x - a*c**2*d**4*e**2 + 20*a*c**2*d**3*e**3*x + 15*a*c**2*d**2
*e**4*x**2 - 8*c**3*d**6 - 25*c**3*d**5*e*x - 15*c**3*d**4*e**2*x**2)/(4*s
qrt(a*e + c*d*x)*(a**4*d**2*e**8 + 2*a**4*d*e**9*x + a**4*e**10*x**2 - 4*a
**3*c*d**4*e**6 - 8*a**3*c*d**3*e**7*x - 4*a**3*c*d**2*e**8*x**2 + 6*a**2*
c**2*d**6*e**4 + 12*a**2*c**2*d**5*e**5*x + 6*a**2*c**2*d**4*e**6*x**2 - 4
*a*c**3*d**8*e**2 - 8*a*c**3*d**7*e**3*x - 4*a*c**3*d**6*e**4*x**2 + c**4*
d**10 + 2*c**4*d**9*e*x + c**4*d**8*e**2*x**2))
```

3.321 $\int \frac{(d+ex)^{9/2}}{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}\,dx$

Optimal result

Integrand size = 39, antiderivative size = 171

$$\int \frac{(d+ex)^{9/2}}{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}\,dx = -\frac{2\left(cd^2-ae^2\right)^2(d+ex)^{3/2}}{3c^3d^3\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}} - \frac{4e\left(cd^2-ae^2\right)\sqrt{d+ex}}{c^3d^3\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}} + \frac{2e^2\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{c^3d^3\sqrt{d+ex}}$$

output
```
-2/3*(-a*e^2+c*d^2)^2*(e*x+d)^(3/2)/c^3/d^3/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x
^2)^(3/2)-4*e*(-a*e^2+c*d^2)*(e*x+d)^(1/2)/c^3/d^3/(a*d*e+(a*e^2+c*d^2)*x+
c*d*e*x^2)^(1/2)+2*e^2*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/c^3/d^3/(e*
x+d)^(1/2)
```

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.49

$$\int \frac{(d+ex)^{9/2}}{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}\,dx = -\frac{2(d+ex)^{3/2}\left(-8a^2e^4+4acde^2(d-3ex)+c^2d^2(d^2+6dex-3e^2x^2)\right)}{3c^3d^3((ae+cdx)(d+ex))^{3/2}}$$

input
```
Integrate[(d + e*x)^(9/2)/(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2),x]
```

output
```
(-2*(d + e*x)^(3/2)*(-8*a^2*e^4 + 4*a*c*d*e^2*(d - 3*e*x) + c^2*d^2*(d^2 +
 6*d*e*x - 3*e^2*x^2)))/(3*c^3*d^3*((a*e + c*d*x)*(d + e*x))^(3/2))
```

Rubi [A] (verified)

Time = 0.52 (sec) , antiderivative size = 173, normalized size of antiderivative = 1.01, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {1128, 1128, 1122}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex)^{9/2}}{(x(ae^2+cd^2)+ade+cdex^2)^{5/2}}dx$$

$$\Big\downarrow \text{1128}$$

$$\frac{4\left(d^2-\frac{ae^2}{c}\right)\int \frac{(d+ex)^{7/2}}{(cdex^2+(cd^2+ae^2)x+ade)^{5/2}}dx}{d}+\frac{2(d+ex)^{7/2}}{cd(x(ae^2+cd^2)+ade+cdex^2)^{3/2}}$$

$$\Big\downarrow \text{1128}$$

$$\frac{4\left(d^2-\frac{ae^2}{c}\right)\left(-\frac{2\left(d^2-\frac{ae^2}{c}\right)\int \frac{(d+ex)^{5/2}}{\left(cdex^2+\left(cd^2+ae^2\right)x+ade\right)^{5/2}}dx}{d}-\frac{2(d+ex)^{5/2}}{cd(x(ae^2+cd^2)+ade+cdex^2)^{3/2}}\right)}{d}+\frac{2(d+ex)^{7/2}}{cd(x(ae^2+cd^2)+ade+cdex^2)^{3/2}}$$

$$\Big\downarrow \text{1122}$$

$$\frac{2(d+ex)^{7/2}}{cd(x(ae^2+cd^2)+ade+cdex^2)^{3/2}}+\frac{4\left(d^2-\frac{ae^2}{c}\right)\left(\frac{4(d+ex)^{3/2}\left(d^2-\frac{ae^2}{c}\right)}{3cd^2(x(ae^2+cd^2)+ade+cdex^2)^{3/2}}-\frac{2(d+ex)^{5/2}}{cd(x(ae^2+cd^2)+ade+cdex^2)^{3/2}}\right)}{d}$$

input
```
Int[(d + e*x)^(9/2)/(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2),x]
```

output
```
(2*(d + e*x)^(7/2))/(c*d*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2)) +
(4*(d^2 - (a*e^2)/c)*((4*(d^2 - (a*e^2)/c)*(d + e*x)^(3/2))/(3*c*d^2*(a*d*
e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2)) - (2*(d + e*x)^(5/2))/(c*d*(a*d*
e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2))))/d
```

Defintions of rubi rules used

rule 1122
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[e*(d + e*x)^(m - 1)*((a + b*x + c*x^2)^(p + 1)/(c*(p + 1))),
 x] /; FreeQ[{a, b, c, d, e, m, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&
EqQ[m + p, 0]
```

rule 1128
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[e*(d + e*x)^(m - 1)*((a + b*x + c*x^2)^(p + 1)/(c*(m + 2*p +
 1))), x] + Simp[Simplify[m + p]*((2*c*d - b*e)/(c*(m + 2*p + 1)))   Int[(d
 + e*x)^(m - 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p},
 x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && IGtQ[Simplify[m + p], 0]
```

Maple [A] (verified)

Time = 1.08 (sec) , antiderivative size = 102, normalized size of antiderivative = 0.60

method	result	size
default	$\frac{2\sqrt{(ex+d)(cdx+ae)}\,(3x^2c^2d^2e^2+12xacd\,e^3-6x\,c^2d^3e+8a^2e^4-4ac\,d^2e^2-c^2d^4)}{3\sqrt{ex+d}\,(cdx+ae)^2c^3d^3}$	102
gosper	$\frac{2(cdx+ae)(3x^2c^2d^2e^2+12xacd\,e^3-6x\,c^2d^3e+8a^2e^4-4ac\,d^2e^2-c^2d^4)(ex+d)^{\frac{5}{2}}}{3d^3c^3(cd\,x^2e+a\,e^2x+c\,d^2x+ade)^{\frac{5}{2}}}$	110
orering	$\frac{2(3x^2c^2d^2e^2+12xacd\,e^3-6x\,c^2d^3e+8a^2e^4-4ac\,d^2e^2-c^2d^4)(cdx+ae)(ex+d)^{\frac{5}{2}}}{3d^3c^3(ade+(a\,e^2+c\,d^2)x+cd\,x^2e)^{\frac{5}{2}}}$	111

input
```
int((e*x+d)^(9/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(5/2),x,method=_RETURN
VERBOSE)
```

output
```
2/3/(e*x+d)^(1/2)*((e*x+d)*(c*d*x+a*e))^(1/2)*(3*c^2*d^2*e^2*x^2+12*a*c*d*
e^3*x-6*c^2*d^3*e*x+8*a^2*e^4-4*a*c*d^2*e^2-c^2*d^4)/(c*d*x+a*e)^2/c^3/d^3
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 174, normalized size of antiderivative = 1.02

$$\int \frac{(d+ex)^{9/2}}{(ade+(cd^2+ae^2)x+cdex^2)^{5/2}}\,dx = \frac{2\,(3\,c^2d^2e^2x^2 - c^2d^4 - 4\,acd^2e^2 + 8\,a^2e^4 - 6\,(c^2d^3e - 2\,acde^3)x)\,\sqrt{}}{3\,(c^5d^5ex^3 + a^2c^3d^4e^2 + (c^5d^6 + 2\,ac^4d^4e^2)x^2 + (2}$$

input
```
integrate((e*x+d)^(9/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2),x, algorit
hm="fricas")
```

output
```
2/3*(3*c^2*d^2*e^2*x^2 - c^2*d^4 - 4*a*c*d^2*e^2 + 8*a^2*e^4 - 6*(c^2*d^3*
e - 2*a*c*d*e^3)*x)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(e*x +
 d)/(c^5*d^5*e*x^3 + a^2*c^3*d^4*e^2 + (c^5*d^6 + 2*a*c^4*d^4*e^2)*x^2 + (
2*a*c^4*d^5*e + a^2*c^3*d^3*e^3)*x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(d+ex)^{9/2}}{(ade+(cd^2+ae^2)x+cdex^2)^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate((e*x+d)**(9/2)/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(5/2),x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 93, normalized size of antiderivative = 0.54

$$\int \frac{(d+ex)^{9/2}}{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}\,dx=\frac{2\left(3\,c^2d^2e^2x^2-c^2d^4-4\,acd^2e^2+8\,a^2e^4-6\,\left(c^2d^3e-2\,acde^3\right)x\right)}{3\,\left(c^4d^4x+ac^3d^3e\right)\sqrt{cdx+ae}}$$

input
```
integrate((e*x+d)^(9/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2),x, algorit
hm="maxima")
```

output
```
2/3*(3*c^2*d^2*e^2*x^2 - c^2*d^4 - 4*a*c*d^2*e^2 + 8*a^2*e^4 - 6*(c^2*d^3*
e - 2*a*c*d*e^3)*x)/((c^4*d^4*x + a*c^3*d^3*e)*sqrt(c*d*x + a*e))
```

Giac [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 171, normalized size of antiderivative = 1.00

$$\int \frac{(d+ex)^{9/2}}{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}\,dx=\frac{2\left(\frac{3\,\sqrt{(ex+d)cde-cd^2e+ae^3}e^2}{cd|e|}-\frac{c^2d^4e^4-2\,acd^2e^6+a^2e^8+6\,\left((ex+d)cde-cd^2e+ae^3\right)\cdots}{\left((ex+d)cde-cd^2e+ae\cdots\right.}\cdots\right.}{3\,c^2d^2}$$

input
```
integrate((e*x+d)^(9/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2),x, algorit
hm="giac")
```

output
```
2/3*(3*sqrt((e*x + d)*c*d*e - c*d^2*e + a*e^3)*e^2/(c*d*abs(e)) - (c^2*d^4
*e^4 - 2*a*c*d^2*e^6 + a^2*e^8 + 6*((e*x + d)*c*d*e - c*d^2*e + a*e^3)*c*d
^2*e^3 - 6*((e*x + d)*c*d*e - c*d^2*e + a*e^3)*a*e^5)/(((e*x + d)*c*d*e -
c*d^2*e + a*e^3)^(3/2)*c*d*abs(e)))/(c^2*d^2)
```

Mupad [B] (verification not implemented)

Time = 5.54 (sec) , antiderivative size = 195, normalized size of antiderivative = 1.14

$$\int \frac{(d+ex)^{9/2}}{\left(ade+(cd^2+ae^2)\,x+cdex^2\right)^{5/2}}\,dx = \frac{\sqrt{c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e}\,\left(\frac{4\,x\,\left(2\,a\,e^2-c\,d^2\right)\,\sqrt{d+e\,x}}{c^4\,d^4}+\frac{2\,e\,x^2\,\sqrt{d+e\,x}}{c^3\,d^3}\right.}{x^3+\frac{a^2\,e}{c^2\,d}+\frac{a\,x\,\left(2\,c\,d^2+a\,e^2\right)}{c^2\,d^2}+\frac{x^2\,\left(3\,c^5\,d^6+6\right.}{3\,c^5\,d}}$$

input
```
int((d + e*x)^(9/2)/(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(5/2),x)
```

output
```
((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2)*((4*x*(2*a*e^2 - c*d^2)*(d
+ e*x)^(1/2))/(c^4*d^4) + (2*e*x^2*(d + e*x)^(1/2))/(c^3*d^3) - ((d + e*x)
^(1/2)*(2*c^2*d^4 - 16*a^2*e^4 + 8*a*c*d^2*e^2))/(3*c^5*d^5*e)))/(x^3 + (a
^2*e)/(c^2*d) + (a*x*(a*e^2 + 2*c*d^2))/(c^2*d^2) + (x^2*(3*c^5*d^6 + 6*a*
c^4*d^4*e^2))/(3*c^5*d^5*e))
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.52

$$\int \frac{(d+ex)^{9/2}}{\left(ade+(cd^2+ae^2)\,x+cdex^2\right)^{5/2}}\,dx = \frac{2c^2d^2e^2x^2+8acd\,e^3x-4c^2d^3ex+\frac{16}{3}a^2e^4-\frac{8}{3}ac\,d^2e^2-\frac{2}{3}c^2d^4}{\sqrt{cdx+ae}\,c^3d^3\,(cdx+ae)}$$

input
```
int((e*x+d)^(9/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2),x)
```

output
```
(2*(8*a**2*e**4 - 4*a*c*d**2*e**2 + 12*a*c*d*e**3*x - c**2*d**4 - 6*c**2*d
**3*e*x + 3*c**2*d**2*e**2*x**2))/(3*sqrt(a*e + c*d*x)*c**3*d**3*(a*e + c*
d*x))
```

3.322 $\int \frac{(d+ex)^{7/2}}{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}\,dx$

Optimal result

Integrand size = 39, antiderivative size = 108

$$\int \frac{(d+ex)^{7/2}}{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}\,dx = -\frac{2(cd^2-ae^2)\,(d+ex)^{3/2}}{3c^2d^2\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}-\frac{2e\sqrt{d+ex}}{c^2d^2\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}$$

output
```
-2/3*(-a*e^2+c*d^2)*(e*x+d)^(3/2)/c^2/d^2/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2
)^(3/2)-2*e*(e*x+d)^(1/2)/c^2/d^2/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.49

$$\int \frac{(d+ex)^{7/2}}{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}\,dx = -\frac{2(d+ex)^{3/2}\left(2ae^2+cd(d+3ex)\right)}{3c^2d^2((ae+cdx)(d+ex))^{3/2}}$$

input
```
Integrate[(d + e*x)^(7/2)/(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2),x]
```

output
```
(-2*(d + e*x)^(3/2)*(2*a*e^2 + c*d*(d + 3*e*x)))/(3*c^2*d^2*((a*e + c*d*x)
*(d + e*x))^(3/2))
```

Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.051$, Rules used = {1128, 1122}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex)^{7/2}}{(x(ae^2+cd^2)+ade+cdex^2)^{5/2}}dx$$

$$\Big\downarrow \text{ 1128}$$

$$-\frac{2\left(d^2-\frac{ae^2}{c}\right)\int \frac{(d+ex)^{5/2}}{(cdex^2+(cd^2+ae^2)x+ade)^{5/2}}dx}{d}-\frac{2(d+ex)^{5/2}}{cd\left(x(ae^2+cd^2)+ade+cdex^2\right)^{3/2}}$$

$$\Big\downarrow \text{ 1122}$$

$$\frac{4(d+ex)^{3/2}\left(d^2-\frac{ae^2}{c}\right)}{3cd^2\left(x(ae^2+cd^2)+ade+cdex^2\right)^{3/2}}-\frac{2(d+ex)^{5/2}}{cd\left(x(ae^2+cd^2)+ade+cdex^2\right)^{3/2}}$$

input
```
Int[(d + e*x)^(7/2)/(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2),x]
```

output
```
(4*(d^2 - (a*e^2)/c)*(d + e*x)^(3/2))/(3*c*d^2*(a*d*e + (c*d^2 + a*e^2)*x
+ c*d*e*x^2)^(3/2)) - (2*(d + e*x)^(5/2))/(c*d*(a*d*e + (c*d^2 + a*e^2)*x
+ c*d*e*x^2)^(3/2))
```

Defintions of rubi rules used

rule 1122
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[e*(d + e*x)^(m - 1)*((a + b*x + c*x^2)^(p + 1)/(c*(p + 1))),
 x] /; FreeQ[{a, b, c, d, e, m, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&
EqQ[m + p, 0]
```

rule 1128
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[e*(d + e*x)^(m - 1)*((a + b*x + c*x^2)^(p + 1)/(c*(m + 2*p +
 1))), x] + Simp[Simplify[m + p]*((2*c*d - b*e)/(c*(m + 2*p + 1)))   Int[(d
 + e*x)^(m - 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p},
 x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && IGtQ[Simplify[m + p], 0]
```

Maple [A] (verified)

Time = 1.07 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.56

method	result	size
default	$-\frac{2\sqrt{\left(ex+d\right)\left(cdx+ae\right)}\,\left(3cdxe+2a\,e^{2}+c\,d^{2}\right)}{3\sqrt{ex+d}\,\left(cdx+ae\right)^{2}c^{2}d^{2}}$	60
gosper	$-\frac{2\left(cdx+ae\right)\left(3cdxe+2a\,e^{2}+c\,d^{2}\right)\left(ex+d\right)^{\frac{5}{2}}}{3c^{2}d^{2}\left(cd\,x^{2}e+a\,e^{2}x+c\,d^{2}x+ade\right)^{\frac{5}{2}}}$	68
orering	$-\frac{2\left(3cdxe+2a\,e^{2}+c\,d^{2}\right)\left(cdx+ae\right)\left(ex+d\right)^{\frac{5}{2}}}{3c^{2}d^{2}\left(ade+\left(a\,e^{2}+c\,d^{2}\right)x+cd\,x^{2}e\right)^{\frac{5}{2}}}$	69

input
```
int((e*x+d)^(7/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(5/2),x,method=_RETURN
VERBOSE)
```

output
```
-2/3/(e*x+d)^(1/2)*((e*x+d)*(c*d*x+a*e))^(1/2)*(3*c*d*e*x+2*a*e^2+c*d^2)/(
c*d*x+a*e)^2/c^2/d^2
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 131, normalized size of antiderivative = 1.21

$$\int \frac{(d+ex)^{7/2}}{(ade+(cd^2+ae^2)\,x+cdex^2)^{5/2}}\,dx = -\frac{2\,\sqrt{cdex^2+ade+(cd^2+ae^2)x}(3\,cdex+cd^2+2\,ae^2)\sqrt{ex+d}}{3\,(c^4d^4ex^3+a^2c^2d^3e^2+(c^4d^5+2\,ac^3d^3e^2)x^2+(2\,ac^3d^4e+a^2c^2d^2e^3)x)}$$

input
```
integrate((e*x+d)^(7/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2),x, algorit
hm="fricas")
```

output
```
-2/3*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(3*c*d*e*x + c*d^2 + 2*a*
e^2)*sqrt(e*x + d)/(c^4*d^4*e*x^3 + a^2*c^2*d^3*e^2 + (c^4*d^5 + 2*a*c^3*d
^3*e^2)*x^2 + (2*a*c^3*d^4*e + a^2*c^2*d^2*e^3)*x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(d+ex)^{7/2}}{(ade+(cd^2+ae^2)\,x+cdex^2)^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate((e*x+d)**(7/2)/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(5/2),x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.46

$$\int \frac{(d+ex)^{7/2}}{(ade+(cd^2+ae^2)\,x+cdex^2)^{5/2}}\,dx = -\frac{2\,(3\,cdex+cd^2+2\,ae^2)}{3\,(c^3d^3x+ac^2d^2e)\sqrt{cdx+ae}}$$

input
```
integrate((e*x+d)^(7/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2),x, algorit
hm="maxima")
```

output
```
-2/3*(3*c*d*e*x + c*d^2 + 2*a*e^2)/((c^3*d^3*x + a*c^2*d^2*e)*sqrt(c*d*x +
 a*e))
```

Giac [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.73

$$\int \frac{(d+ex)^{7/2}}{(ade+(cd^2+ae^2)x+cdex^2)^{5/2}}\,dx = -\frac{2\left(cd^2e^3-ae^5+3\left((ex+d)cde-cd^2e+ae^3\right)e^2\right)e}{3\left((ex+d)cde-cd^2e+ae^3\right)^{\frac{3}{2}}c^2d^2|e|}$$

input
```
integrate((e*x+d)^(7/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2),x, algorit
hm="giac")
```

output
```
-2/3*(c*d^2*e^3 - a*e^5 + 3*((e*x + d)*c*d*e - c*d^2*e + a*e^3)*e^2)*e/(((
e*x + d)*c*d*e - c*d^2*e + a*e^3)^(3/2)*c^2*d^2*abs(e))
```

Mupad [B] (verification not implemented)

Time = 5.52 (sec) , antiderivative size = 147, normalized size of antiderivative = 1.36

$$\int \frac{(d+ex)^{7/2}}{(ade+(cd^2+ae^2)x+cdex^2)^{5/2}}\,dx = -\frac{\left(\frac{2\,x\,\sqrt{d+e\,x}}{c^3\,d^3}+\frac{\left(\frac{2\,c\,d^2}{3}+\frac{4\,a\,e^2}{3}\right)\sqrt{d+e\,x}}{c^4\,d^4\,e}\right)\sqrt{c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e}}{x^3+\frac{a^2\,e}{c^2\,d}+\frac{a\,x\,\left(2\,c\,d^2+a\,e^2\right)}{c^2\,d^2}+\frac{x^2\,\left(c^4\,d^5+2\,a\,c^3\,d^3\,e^2\right)}{c^4\,d^4\,e}}$$

input
```
int((d + e*x)^(7/2)/(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(5/2),x)
```

output
```
-(((2*x*(d + e*x)^(1/2))/(c^3*d^3) + (((4*a*e^2)/3 + (2*c*d^2)/3)*(d + e*x
)^(1/2))/(c^4*d^4*e))*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2))/(x^3
+ (a^2*e)/(c^2*d) + (a*x*(a*e^2 + 2*c*d^2))/(c^2*d^2) + (x^2*(c^4*d^5 + 2*
a*c^3*d^3*e^2))/(c^4*d^4*e))
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.44

$$\int \frac{(d+ex)^{7/2}}{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}\,dx=\frac{-2cdex-\frac{4}{3}a\,e^2-\frac{2}{3}c\,d^2}{\sqrt{cdx+ae}\,c^2d^2\,(cdx+ae)}$$

input
```
int((e*x+d)^(7/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2),x)
```

output
```
(2*( - 2*a*e**2 - c*d**2 - 3*c*d*e*x))/(3*sqrt(a*e + c*d*x)*c**2*d**2*(a*e
 + c*d*x))
```

3.323 $\int \frac{(d+ex)^{5/2}}{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}\,dx$

Optimal result

Integrand size = 39, antiderivative size = 48

$$\int \frac{(d+ex)^{5/2}}{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}\,dx=-\frac{2(d+ex)^{3/2}}{3cd\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}$$

output
```
-2/3*(e*x+d)^(3/2)/c/d/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.77

$$\int \frac{(d+ex)^{5/2}}{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}\,dx=-\frac{2(d+ex)^{3/2}}{3cd((ae+cdx)(d+ex))^{3/2}}$$

input
```
Integrate[(d + e*x)^(5/2)/(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2),x]
```

output
```
(-2*(d + e*x)^(3/2))/(3*c*d*((a*e + c*d*x)*(d + e*x))^(3/2))
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.026$, Rules used = {1122}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex)^{5/2}}{\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/2}}dx$$
$$\Big\downarrow \text{1122}$$
$$-\frac{2(d+ex)^{3/2}}{3cd\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}$$

input
```
Int[(d + e*x)^(5/2)/(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2),x]
```

output
```
(-2*(d + e*x)^(3/2))/(3*c*d*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2))
```

Defintions of rubi rules used

rule 1122
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[e*(d + e*x)^(m - 1)*((a + b*x + c*x^2)^(p + 1)/(c*(p + 1))),
 x] /; FreeQ[{a, b, c, d, e, m, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&
EqQ[m + p, 0]
```

Maple [A] (verified)

Time = 1.07 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.88

method	result	size
default	$-\frac{2\sqrt{(ex+d)(cdx+ae)}}{3\sqrt{ex+d}\,(cdx+ae)^{2}cd}$	42
gosper	$-\frac{2(cdx+ae)(ex+d)^{\frac{5}{2}}}{3cd(cd\,x^{2}e+a\,e^{2}x+c\,d^{2}x+ade)^{\frac{5}{2}}}$	50
orering	$-\frac{2(cdx+ae)(ex+d)^{\frac{5}{2}}}{3cd(ade+(a\,e^{2}+c\,d^{2})x+cd\,x^{2}e)^{\frac{5}{2}}}$	51

input
```
int((e*x+d)^(5/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(5/2),x,method=_RETURN
VERBOSE)
```

output
```
-2/3/(e*x+d)^(1/2)*((e*x+d)*(c*d*x+a*e))^(1/2)/(c*d*x+a*e)^2/c/d
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 107 vs. $2(42) = 84$.

Time = 0.08 (sec) , antiderivative size = 107, normalized size of antiderivative = 2.23

$$\int \frac{(d+ex)^{5/2}}{(ade+(cd^2+ae^2)\,x+cdex^2)^{5/2}}\,dx = -\frac{2\,\sqrt{cdex^2+ade+(cd^2+ae^2)x}\sqrt{ex+d}}{3\,(c^3d^3ex^3+a^2cd^2e^2+(c^3d^4+2\,ac^2d^2e^2)x^2+(2\,ac^2d^3e+a^2cde^3)x)}$$

input
```
integrate((e*x+d)^(5/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2),x, algorit
hm="fricas")
```

output
```
-2/3*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(e*x + d)/(c^3*d^3*e*
x^3 + a^2*c*d^2*e^2 + (c^3*d^4 + 2*a*c^2*d^2*e^2)*x^2 + (2*a*c^2*d^3*e + a
^2*c*d*e^3)*x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(d+ex)^{5/2}}{(ade+(cd^2+ae^2)\,x+cdex^2)^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate((e*x+d)**(5/2)/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(5/2),x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.58

$$\int \frac{(d+ex)^{5/2}}{(ade+(cd^2+ae^2)\,x+cdex^2)^{5/2}}\,dx = -\frac{2}{3\,(c^2d^2x+acde)\sqrt{cdx+ae}}$$

input
```
integrate((e*x+d)^(5/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2),x, algorit
hm="maxima")
```

output
```
-2/3/((c^2*d^2*x + a*c*d*e)*sqrt(c*d*x + a*e))
```

Giac [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.81

$$\int \frac{(d+ex)^{5/2}}{(ade+(cd^2+ae^2)\,x+cdex^2)^{5/2}}\,dx = -\frac{2\,e^4}{3\,((ex+d)cde-cd^2e+ae^3)^{\frac{3}{2}}cd|e|}$$

input
```
integrate((e*x+d)^(5/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2),x, algorit
hm="giac")
```

output
```
-2/3*e^4/(((e*x + d)*c*d*e - c*d^2*e + a*e^3)^(3/2)*c*d*abs(e))
```

Mupad [B] (verification not implemented)

Time = 5.46 (sec) , antiderivative size = 110, normalized size of antiderivative = 2.29

$$\int \frac{(d+ex)^{5/2}}{(ade+(cd^2+ae^2)\,x+cdex^2)^{5/2}}\,dx = -\frac{2\sqrt{d+e\,x}\,\sqrt{c\,d^2\,x+c\,d\,e\,x^2+a\,d\,e+a\,e^2\,x}}{3\,\left(a^2\,c\,d^2\,e^2+a^2\,c\,d\,e^3\,x+2\,a\,c^2\,d^3\,e\,x+2\,a\,c^2\,d^2\,e^2\,x^2+c^3\,d^4\,x^2+c^3\,d^3\,e\,x^3\right)}$$

input
```
int((d + e*x)^(5/2)/(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(5/2),x)
```

output
```
-(2*(d + e*x)^(1/2)*(a*d*e + a*e^2*x + c*d^2*x + c*d*e*x^2)^(1/2))/(3*(c^3
*d^4*x^2 + a^2*c*d^2*e^2 + c^3*d^3*e*x^3 + 2*a*c^2*d^3*e*x + a^2*c*d*e^3*x
 + 2*a*c^2*d^2*e^2*x^2))
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.60

$$\int \frac{(d+ex)^{5/2}}{(ade+(cd^2+ae^2)\,x+cdex^2)^{5/2}}\,dx = -\frac{2}{3\sqrt{cdx+ae}\,cd\,(cdx+ae)}$$

input
```
int((e*x+d)^(5/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2),x)
```

output
```
( - 2)/(3*sqrt(a*e + c*d*x)*c*d*(a*e + c*d*x))
```

3.324 $\int \frac{(d+ex)^{3/2}}{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}\,dx$

Optimal result

Integrand size = 39, antiderivative size = 196

$$\int \frac{(d+ex)^{3/2}}{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}\,dx = -\frac{2(d+ex)^{3/2}}{3\left(cd^2-ae^2\right)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}+\frac{2e\sqrt{d+ex}}{\left(cd^2-ae^2\right)^2\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}-\frac{2e^{3/2}\arctan\left(\frac{\sqrt{cd^2-ae^2}\sqrt{d+ex}}{\sqrt{e}\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\right)}{\left(cd^2-ae^2\right)^{5/2}}$$

output
```
-2/3*(e*x+d)^(3/2)/(-a*e^2+c*d^2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)+
2*e*(e*x+d)^(1/2)/(-a*e^2+c*d^2)^2/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)
-2*e^(3/2)*arctan(1/e^(1/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)*(-a*e^
2+c*d^2)^(1/2)*(e*x+d)^(1/2))/(-a*e^2+c*d^2)^(5/2)
```

Mathematica [A] (verified)

Time = 0.21 (sec) , antiderivative size = 135, normalized size of antiderivative = 0.69

$$\int \frac{(d+ex)^{3/2}}{\left(ade+(cd^2+ae^2)\,x+cdex^2\right)^{5/2}}\,dx = -\frac{2(d+ex)^{3/2}\left(\sqrt{cd^2-ae^2}(-4ae^2+cd(d-3ex))-3e^{3/2}(ae+cdx)^{3/2}\arctan\left(\frac{\sqrt{e}\sqrt{ae+cdx}}{\sqrt{cd^2-ae^2}}\right)\right)}{3\left(cd^2-ae^2\right)^{5/2}((ae+cdx)(d+ex))^{3/2}}$$

input
```
Integrate[(d + e*x)^(3/2)/(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2),x]
```

output
```
(-2*(d + e*x)^(3/2)*(Sqrt[c*d^2 - a*e^2]*(-4*a*e^2 + c*d*(d - 3*e*x)) - 3*
e^(3/2)*(a*e + c*d*x)^(3/2)*ArcTan[(Sqrt[e]*Sqrt[a*e + c*d*x])/Sqrt[c*d^2
- a*e^2]]))/(3*(c*d^2 - a*e^2)^(5/2)*((a*e + c*d*x)*(d + e*x))^(3/2))
```

Rubi [A] (verified)

Time = 0.74 (sec) , antiderivative size = 273, normalized size of antiderivative = 1.39, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.128$, Rules used = {1133, 1135, 1132, 1136, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex)^{3/2}}{\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/2}}dx$$

$$\Big\downarrow \text{ 1133}$$

$$-\frac{2e\int \frac{1}{\sqrt{d+ex}\left(cdex^2+(cd^2+ae^2)x+ade\right)^{3/2}}dx}{3cd}-\frac{2\sqrt{d+ex}}{3cd\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}$$

$$\Big\downarrow \text{ 1135}$$

$$-\frac{2e\left(\frac{3cd\int \frac{\sqrt{d+ex}}{\left(cdex^2+\left(cd^2+ae^2\right)x+ade\right)^{3/2}}dx}{2(cd^2-ae^2)}+\frac{1}{\sqrt{d+ex}(cd^2-ae^2)\sqrt{x(ae^2+cd^2)+ade+cdex^2}}\right)}{3cd}-\frac{2\sqrt{d+ex}}{3cd\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}$$

$$\Big\downarrow \text{ 1132}$$

$$-\frac{2e\left(\frac{3cd\left(-\frac{e\int \frac{1}{\sqrt{d+ex}\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}dx}{cd^2-ae^2}-\frac{2\sqrt{d+ex}}{\left(cd^2-ae^2\right)\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}\right)}{2(cd^2-ae^2)}+\frac{1}{\sqrt{d+ex}(cd^2-ae^2)\sqrt{x(ae^2+cd^2)+ade+cdex^2}}\right)}{3cd}-\frac{2\sqrt{d+ex}}{3cd\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}$$

$$\Big\downarrow \text{ 1136}$$

$$-\frac{2e\left(\frac{3cd\left(-\frac{2e^2\int \frac{1}{\frac{\left(cdex^2+\left(cd^2+ae^2\right)x+ade\right)e^2}{d+ex}+\left(cd^2-ae^2\right)e}d\frac{\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}{\sqrt{d+ex}}}{cd^2-ae^2}-\frac{2\sqrt{d+ex}}{\left(cd^2-ae^2\right)\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}\right)}{2(cd^2-ae^2)}+\frac{}{\sqrt{d+ex}(cd}\right.}{3cd}-\frac{2\sqrt{d+ex}}{3cd\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}$$

$$\Big\downarrow \text{ 218}$$

$$-\frac{2e\left(\frac{3cd\left(-\frac{2\sqrt{e}\arctan\left(\frac{\sqrt{e}\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{\sqrt{d+ex}\sqrt{cd^2-ae^2}}\right)}{(cd^2-ae^2)^{3/2}}-\frac{2\sqrt{d+ex}}{(cd^2-ae^2)\sqrt{x(ae^2+cd^2)+ade+cdex^2}}\right)}{2(cd^2-ae^2)}+\frac{1}{\sqrt{d+ex}(cd^2-ae^2)\sqrt{x(ae^2+cd^2)+ade+cde}}\right.}{3cd}$$

$$\frac{2\sqrt{d+ex}}{3cd\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}$$

input
```
Int[(d + e*x)^(3/2)/(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2),x]
```

output
```
(-2*Sqrt[d + e*x])/(3*c*d*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2)) -
 (2*e*(1/((c*d^2 - a*e^2)*Sqrt[d + e*x]*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c
*d*e*x^2]) + (3*c*d*((-2*Sqrt[d + e*x])/((c*d^2 - a*e^2)*Sqrt[a*d*e + (c*d
^2 + a*e^2)*x + c*d*e*x^2]) - (2*Sqrt[e]*ArcTan[(Sqrt[e]*Sqrt[a*d*e + (c*d
^2 + a*e^2)*x + c*d*e*x^2])/(Sqrt[c*d^2 - a*e^2]*Sqrt[d + e*x])])/(c*d^2 -
 a*e^2)^(3/2)))/(2*(c*d^2 - a*e^2))))/(3*c*d)
```

Defintions of rubi rules used

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 1132
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(2*c*d - b*e)*(d + e*x)^m*((a + b*x + c*x^2)^(p + 1)/(e*(p +
 1)*(b^2 - 4*a*c))), x] - Simp[(2*c*d - b*e)*((m + 2*p + 2)/((p + 1)*(b^2 -
 4*a*c)))   Int[(d + e*x)^(m - 1)*(a + b*x + c*x^2)^(p + 1), x], x] /; Free
Q[{a, b, c, d, e}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[p, -1] && LtQ
[0, m, 1] && IntegerQ[2*p]
```

rule 1133
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[e*(d + e*x)^(m - 1)*((a + b*x + c*x^2)^(p + 1)/(c*(p + 1))), x] - Simp[e^2*((m + p)/(c*(p + 1)))   Int[(d + e*x)^(m - 2)*(a + b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 1] && IntegerQ[2*p]
```

rule 1135
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-e)*(d + e*x)^m*((a + b*x + c*x^2)^(p + 1)/((m + p + 1)*(2*c*d - b*e))), x] + Simp[c*((m + 2*p + 2)/((m + p + 1)*(2*c*d - b*e)))   Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[m, 0] && NeQ[m + p + 1, 0] && IntegerQ[2*p]
```

rule 1136
```
Int[1/(Sqrt[(d_.) + (e_.)*(x_)]*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Symbol] :> Simp[2*e   Subst[Int[1/(2*c*d - b*e + e^2*x^2), x], x, Sqrt[a + b*x + c*x^2]/Sqrt[d + e*x]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0]
```

Maple [A] (verified)

Time = 1.07 (sec) , antiderivative size = 224, normalized size of antiderivative = 1.14

method	result
default	$-\frac{2\sqrt{(ex+d)(cdx+ae)}\left(3\,\operatorname{arctanh}\left(\frac{e\sqrt{cdx+ae}}{\sqrt{\left(a\,e^{2}-c\,d^{2}\right)e}}\right)\sqrt{cdx+ae}\,cd\,e^{2}x+3\,\operatorname{arctanh}\left(\frac{e\sqrt{cdx+ae}}{\sqrt{\left(a\,e^{2}-c\,d^{2}\right)e}}\right)a\,e^{3}\sqrt{cdx+ae}-3\sqrt{(a\,e^{2}-c\,d^{2}}}{3\sqrt{ex+d}\,(cdx+ae)^{2}(a\,e^{2}-c\,d^{2})^{2}\sqrt{(a\,e^{2}-c\,d^{2})e}}$

input
```
int((e*x+d)^(3/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
-2/3*((e*x+d)*(c*d*x+a*e))^(1/2)*(3*arctanh(e*(c*d*x+a*e)^(1/2)/((a*e^2-c*d^2)*e)^(1/2))*(c*d*x+a*e)^(1/2)*c*d*e^2*x+3*arctanh(e*(c*d*x+a*e)^(1/2)/((a*e^2-c*d^2)*e)^(1/2))*a*e^3*(c*d*x+a*e)^(1/2)-3*((a*e^2-c*d^2)*e)^(1/2)*c*d*e*x-4*((a*e^2-c*d^2)*e)^(1/2)*a*e^2+((a*e^2-c*d^2)*e)^(1/2)*c*d^2)/(e*x+d)^(1/2)/(c*d*x+a*e)^2/(a*e^2-c*d^2)^2/((a*e^2-c*d^2)*e)^(1/2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 383 vs. $2(174) = 348$.

Time = 0.14 (sec) , antiderivative size = 788, normalized size of antiderivative = 4.02

$$\int \frac{(d+ex)^{3/2}}{(ade+(cd^2+ae^2)x+cdex^2)^{5/2}}\,dx = \left[\frac{3\,(c^2d^2e^2x^3+a^2de^3+(c^2d^3e+2\,acde^3)x^2+(2\,acd^2e^2+a^2e^4)x}{3\,(a^2c^2d^5e^2-2\,a^3cd^3e^4}\right.$$

input
```
integrate((e*x+d)^(3/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2),x, algorit
hm="fricas")
```

output
```
[1/3*(3*(c^2*d^2*e^2*x^3 + a^2*d*e^3 + (c^2*d^3*e + 2*a*c*d*e^3)*x^2 + (2*
a*c*d^2*e^2 + a^2*e^4)*x)*sqrt(-e/(c*d^2 - a*e^2))*log(-(c*d*e^2*x^2 + 2*a
*e^3*x - c*d^3 + 2*a*d*e^2 + 2*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)
*(c*d^2 - a*e^2)*sqrt(e*x + d)*sqrt(-e/(c*d^2 - a*e^2)))/(e^2*x^2 + 2*d*e*
x + d^2)) + 2*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(3*c*d*e*x - c*d
^2 + 4*a*e^2)*sqrt(e*x + d))/(a^2*c^2*d^5*e^2 - 2*a^3*c*d^3*e^4 + a^4*d*e^
6 + (c^4*d^6*e - 2*a*c^3*d^4*e^3 + a^2*c^2*d^2*e^5)*x^3 + (c^4*d^7 - 3*a^2
*c^2*d^3*e^4 + 2*a^3*c*d*e^6)*x^2 + (2*a*c^3*d^6*e - 3*a^2*c^2*d^4*e^3 + a
^4*e^7)*x), 2/3*(3*(c^2*d^2*e^2*x^3 + a^2*d*e^3 + (c^2*d^3*e + 2*a*c*d*e^3
)*x^2 + (2*a*c*d^2*e^2 + a^2*e^4)*x)*sqrt(e/(c*d^2 - a*e^2))*arctan(-sqrt(
c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(c*d^2 - a*e^2)*sqrt(e*x + d)*sqrt(
e/(c*d^2 - a*e^2))/(c*d*e^2*x^2 + a*d*e^2 + (c*d^2*e + a*e^3)*x)) + sqrt(c
*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(3*c*d*e*x - c*d^2 + 4*a*e^2)*sqrt(e
*x + d))/(a^2*c^2*d^5*e^2 - 2*a^3*c*d^3*e^4 + a^4*d*e^6 + (c^4*d^6*e - 2*a
*c^3*d^4*e^3 + a^2*c^2*d^2*e^5)*x^3 + (c^4*d^7 - 3*a^2*c^2*d^3*e^4 + 2*a^3
*c*d*e^6)*x^2 + (2*a*c^3*d^6*e - 3*a^2*c^2*d^4*e^3 + a^4*e^7)*x)]
```

Sympy [F]

$$\int \frac{(d+ex)^{3/2}}{(ade+(cd^2+ae^2)\,x+cdex^2)^{5/2}}\,dx = \int \frac{(d+ex)^{\frac{3}{2}}}{((d+ex)\,(ae+cdx))^{\frac{5}{2}}}\,dx$$

input
```
integrate((e*x+d)**(3/2)/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(5/2),x)
```

output
```
Integral((d + e*x)**(3/2)/((d + e*x)*(a*e + c*d*x))**(5/2), x)
```

Maxima [F]

$$\int \frac{(d+ex)^{3/2}}{(ade+(cd^2+ae^2)\,x+cdex^2)^{5/2}}\,dx = \int \frac{(ex+d)^{\frac{3}{2}}}{(cdex^2+ade+(cd^2+ae^2)x)^{\frac{5}{2}}}\,dx$$

input
```
integrate((e*x+d)^(3/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2),x, algorit
hm="maxima")
```

output
```
integrate((e*x + d)^(3/2)/(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^(5/2), x
)
```

Giac [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 197, normalized size of antiderivative = 1.01

$$\int \frac{(d+ex)^{3/2}}{(ade+(cd^2+ae^2)\,x+cdex^2)^{5/2}}\,dx = \frac{2}{3}\,e^2\left(\frac{3\,e\arctan\left(\frac{\sqrt{(ex+d)cde-cd^2e+ae^3}}{\sqrt{cd^2e-ae^3}}\right)}{(c^2d^4|e|-2\,acd^2e^2|e|+a^2e^4|e|)\sqrt{cd^2e-ae^3}} - \frac{cd^2}{(c^2d^4|e|-2}\right.$$

input
```
integrate((e*x+d)^(3/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2),x, algorit
hm="giac")
```

output
```
2/3*e^2*(3*e*arctan(sqrt((e*x + d)*c*d*e - c*d^2*e + a*e^3)/sqrt(c*d^2*e -
 a*e^3))/((c^2*d^4*abs(e) - 2*a*c*d^2*e^2*abs(e) + a^2*e^4*abs(e))*sqrt(c*
d^2*e - a*e^3)) - (c*d^2*e^2 - a*e^4 - 3*((e*x + d)*c*d*e - c*d^2*e + a*e^
3)*e)/((c^2*d^4*abs(e) - 2*a*c*d^2*e^2*abs(e) + a^2*e^4*abs(e))*((e*x + d)
*c*d*e - c*d^2*e + a*e^3)^(3/2)))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d+ex)^{3/2}}{(ade+(cd^2+ae^2)\,x+cdex^2)^{5/2}}\,dx = \int \frac{(d+e\,x)^{3/2}}{(c\,d\,e\,x^2+(c\,d^2+a\,e^2)\;x+a\,d\,e)^{5/2}}\,dx$$

input
```
int((d + e*x)^(3/2)/(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(5/2),x)
```

output
```
int((d + e*x)^(3/2)/(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(5/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 273, normalized size of antiderivative = 1.39

$$\int \frac{(d+ex)^{3/2}}{(ade+(cd^2+ae^2)\,x+cdex^2)^{5/2}}\,dx = \frac{-2\sqrt{e}\,\sqrt{cdx+ae}\,\sqrt{-a\,e^2+c\,d^2}\;atan\left(\frac{\sqrt{cdx+ae}\,e}{\sqrt{e}\,\sqrt{-a\,e^2+c\,d^2}}\right)a\,e^2-2\sqrt{e}}{\sqrt{cdx+ae}\;(a^3cd\,e^6x-3a^2c^2d^3e}$$

input
```
int((e*x+d)^(3/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2),x)
```

output
```
(2*( - 3*sqrt(e)*sqrt(a*e + c*d*x)*sqrt( - a*e**2 + c*d**2)*atan((sqrt(a*e
 + c*d*x)*e)/(sqrt(e)*sqrt( - a*e**2 + c*d**2)))*a*e**2 - 3*sqrt(e)*sqrt(a
*e + c*d*x)*sqrt( - a*e**2 + c*d**2)*atan((sqrt(a*e + c*d*x)*e)/(sqrt(e)*s
qrt( - a*e**2 + c*d**2)))*c*d*e*x + 4*a**2*e**4 - 5*a*c*d**2*e**2 + 3*a*c*
d*e**3*x + c**2*d**4 - 3*c**2*d**3*e*x))/(3*sqrt(a*e + c*d*x)*(a**4*e**7 -
 3*a**3*c*d**2*e**5 + a**3*c*d*e**6*x + 3*a**2*c**2*d**4*e**3 - 3*a**2*c**
2*d**3*e**4*x - a*c**3*d**6*e + 3*a*c**3*d**5*e**2*x - c**4*d**7*x))
```

3.325 $\int \frac{\sqrt{d+ex}}{(ade+(cd^2+ae^2)x+cdex^2)^{5/2}}\,dx$

Optimal result

Integrand size = 39, antiderivative size = 255

$$\int \frac{\sqrt{d+ex}}{(ade+(cd^2+ae^2)\,x+cdex^2)^{5/2}}\,dx = \frac{\sqrt{d+ex}}{(cd^2-ae^2)\,(ade+(cd^2+ae^2)\,x+cdex^2)^{3/2}} - \frac{5cd(d+ex)^{3/2}}{3\,(cd^2-ae^2)^2\,(ade+(cd^2+ae^2)\,x+cdex^2)^{3/2}} + \frac{5cde\sqrt{d+ex}}{(cd^2-ae^2)^3\,\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}} - \frac{5cde^{3/2}\arctan\left(\frac{\sqrt{cd^2-ae^2}\sqrt{d+ex}}{\sqrt{e}\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\right)}{(cd^2-ae^2)^{7/2}}$$

output

```
(e*x+d)^(1/2)/(-a*e^2+c*d^2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)-5/3*c
*d*(e*x+d)^(3/2)/(-a*e^2+c*d^2)^2/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)+
5*c*d*e*(e*x+d)^(1/2)/(-a*e^2+c*d^2)^3/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(
1/2)-5*c*d*e^(3/2)*arctan(1/e^(1/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2
)*(-a*e^2+c*d^2)^(1/2)*(e*x+d)^(1/2))/(-a*e^2+c*d^2)^(7/2)
```

Mathematica [A] (verified)

Time = 0.39 (sec) , antiderivative size = 177, normalized size of antiderivative = 0.69

$$\int \frac{\sqrt{d+ex}}{(ade+(cd^2+ae^2)x+cdex^2)^{5/2}}\,dx = \frac{\sqrt{d+ex}\left(\sqrt{cd^2-ae^2}(3a^2e^4+2acde^2(7d+10ex)+c^2d^2(-2d^2+\right.}{3(cd^2-ae^2)^{7/2}}$$

input
```
Integrate[Sqrt[d + e*x]/(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2),x]
```

output
```
(Sqrt[d + e*x]*(Sqrt[c*d^2 - a*e^2]*(3*a^2*e^4 + 2*a*c*d*e^2*(7*d + 10*e*x
) + c^2*d^2*(-2*d^2 + 10*d*e*x + 15*e^2*x^2)) + 15*c*d*e^(3/2)*(a*e + c*d*
x)^(3/2)*(d + e*x)*ArcTan[(Sqrt[e]*Sqrt[a*e + c*d*x])/Sqrt[c*d^2 - a*e^2]]
))/(3*(c*d^2 - a*e^2)^(7/2)*((a*e + c*d*x)*(d + e*x))^(3/2))
```

Rubi [A] (verified)

Time = 0.77 (sec) , antiderivative size = 289, normalized size of antiderivative = 1.13, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.128$, Rules used = {1132, 1135, 1132, 1136, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{d+ex}}{(x(ae^2+cd^2)+ade+cdex^2)^{5/2}}\,dx$$

$$\Big\downarrow \ 1132$$

$$-\frac{5e\int \frac{1}{\sqrt{d+ex}(cdex^2+(cd^2+ae^2)x+ade)^{3/2}}dx}{3(cd^2-ae^2)}-\frac{2\sqrt{d+ex}}{3(cd^2-ae^2)(x(ae^2+cd^2)+ade+cdex^2)^{3/2}}$$

$$\Big\downarrow \ 1135$$

$$-\frac{5e\left(\frac{3cd\int \frac{\sqrt{d+ex}}{\left(cdex^2+\left(cd^2+ae^2\right)x+ade\right)^{3/2}}dx}{2(cd^2-ae^2)}+\frac{1}{\sqrt{d+ex}(cd^2-ae^2)\sqrt{x(ae^2+cd^2)+ade+cdex^2}}\right)}{3(cd^2-ae^2)}-\frac{2\sqrt{d+ex}}{3(cd^2-ae^2)(x(ae^2+cd^2)+ade+cdex^2)^{3/2}}$$

$\downarrow$ 1132

$$-\frac{5e\left(\frac{3cd\left(-\frac{e\int \frac{1}{\sqrt{d+ex}\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}dx}{cd^2-ae^2}-\frac{2\sqrt{d+ex}}{\left(cd^2-ae^2\right)\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}\right)}{2(cd^2-ae^2)}+\frac{1}{\sqrt{d+ex}(cd^2-ae^2)\sqrt{x(ae^2+cd^2)+ade+cdex^2}}\right.}{3\left(cd^2-ae^2\right)}$$

$$\frac{2\sqrt{d+ex}}{3\left(cd^2-ae^2\right)\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}$$

$\downarrow$ 1136

$$-\frac{5e\left(\frac{3cd\left(-\frac{2e^2\int \frac{1}{\frac{\left(cdex^2+\left(cd^2+ae^2\right)x+ade\right)e^2}{d+ex}+\left(cd^2-ae^2\right)e}d\frac{\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}{\sqrt{d+ex}}}{cd^2-ae^2}-\frac{2\sqrt{d+ex}}{\left(cd^2-ae^2\right)\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}\right)}{2(cd^2-ae^2)}+\frac{}{\sqrt{d+ex}(cd}\right.}{3\left(cd^2-ae^2\right)}$$

$$\frac{2\sqrt{d+ex}}{3\left(cd^2-ae^2\right)\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}$$

$\downarrow$ 218

$$-\frac{5e\left(\frac{3cd\left(-\frac{2\sqrt{e}\arctan\left(\frac{\sqrt{e}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{\sqrt{d+ex}\sqrt{cd^2-ae^2}}\right)}{\left(cd^2-ae^2\right)^{3/2}}-\frac{2\sqrt{d+ex}}{\left(cd^2-ae^2\right)\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}\right)}{2(cd^2-ae^2)}+\frac{1}{\sqrt{d+ex}(cd^2-ae^2)\sqrt{x(ae^2+cd^2)+ade+cde}}\right.}{3\left(cd^2-ae^2\right)}$$

$$\frac{2\sqrt{d+ex}}{3\left(cd^2-ae^2\right)\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}$$

input
```
Int[Sqrt[d + e*x]/(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2),x]
```

output
```
(-2*Sqrt[d + e*x])/(3*(c*d^2 - a*e^2)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x
^2)^(3/2)) - (5*e*(1/((c*d^2 - a*e^2)*Sqrt[d + e*x]*Sqrt[a*d*e + (c*d^2 +
a*e^2)*x + c*d*e*x^2]) + (3*c*d*((-2*Sqrt[d + e*x])/((c*d^2 - a*e^2)*Sqrt[
a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]) - (2*Sqrt[e]*ArcTan[(Sqrt[e]*Sqrt[
a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(Sqrt[c*d^2 - a*e^2]*Sqrt[d + e*x]
)])/(c*d^2 - a*e^2)^(3/2)))/(2*(c*d^2 - a*e^2))))/(3*(c*d^2 - a*e^2))
```

Defintions of rubi rules used

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 1132
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(2*c*d - b*e)*(d + e*x)^m*((a + b*x + c*x^2)^(p + 1)/(e*(p +
 1)*(b^2 - 4*a*c))), x] - Simp[(2*c*d - b*e)*((m + 2*p + 2)/((p + 1)*(b^2 -
 4*a*c)))   Int[(d + e*x)^(m - 1)*(a + b*x + c*x^2)^(p + 1), x], x] /; Free
Q[{a, b, c, d, e}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[p, -1] && LtQ
[0, m, 1] && IntegerQ[2*p]
```

rule 1135
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(-e)*(d + e*x)^m*((a + b*x + c*x^2)^(p + 1)/((m + p + 1)*(2*
c*d - b*e))), x] + Simp[c*((m + 2*p + 2)/((m + p + 1)*(2*c*d - b*e)))   Int
[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p},
 x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[m, 0] && NeQ[m + p + 1, 0] && I
ntegerQ[2*p]
```

rule 1136
```
Int[1/(Sqrt[(d_.) + (e_.)*(x_)]*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x
_Symbol] :> Simp[2*e   Subst[Int[1/(2*c*d - b*e + e^2*x^2), x], x, Sqrt[a +
 b*x + c*x^2]/Sqrt[d + e*x]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c*d^2
- b*d*e + a*e^2, 0]
```

Maple [A] (verified)

Time = 1.06 (sec) , antiderivative size = 423, normalized size of antiderivative = 1.66

method	result
default	$\frac{\sqrt{(ex+d)(cdx+ae)}\left(15\sqrt{cdx+ae}\,\operatorname{arctanh}\left(\frac{e\sqrt{cdx+ae}}{\sqrt{\left(a\,e^2-c\,d^2\right)e}}\right)c^2d^2e^3x^2+15\operatorname{arctanh}\left(\frac{e\sqrt{cdx+ae}}{\sqrt{\left(a\,e^2-c\,d^2\right)e}}\right)acd\,e^4x\sqrt{cdx+ae}+15\sqrt{cdx+\cdots}\right.}{}$

input
```
int((e*x+d)^(1/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(5/2),x,method=_RETURN
VERBOSE)
```

output
```
1/3*((e*x+d)*(c*d*x+a*e))^(1/2)*(15*(c*d*x+a*e)^(1/2)*arctanh(e*(c*d*x+a*e
)^(1/2)/((a*e^2-c*d^2)*e)^(1/2))*c^2*d^2*e^3*x^2+15*arctanh(e*(c*d*x+a*e)^
(1/2)/((a*e^2-c*d^2)*e)^(1/2))*a*c*d*e^4*x*(c*d*x+a*e)^(1/2)+15*(c*d*x+a*e
)^(1/2)*arctanh(e*(c*d*x+a*e)^(1/2)/((a*e^2-c*d^2)*e)^(1/2))*c^2*d^3*e^2*x
+15*arctanh(e*(c*d*x+a*e)^(1/2)/((a*e^2-c*d^2)*e)^(1/2))*a*c*d^2*e^3*(c*d*
x+a*e)^(1/2)-15*((a*e^2-c*d^2)*e)^(1/2)*c^2*d^2*e^2*x^2-20*((a*e^2-c*d^2)*
e)^(1/2)*a*c*d*e^3*x-10*((a*e^2-c*d^2)*e)^(1/2)*c^2*d^3*e*x-3*((a*e^2-c*d^
2)*e)^(1/2)*a^2*e^4-14*((a*e^2-c*d^2)*e)^(1/2)*a*c*d^2*e^2+2*((a*e^2-c*d^2
)*e)^(1/2)*c^2*d^4)/(e*x+d)^(3/2)/(c*d*x+a*e)^2/(a*e^2-c*d^2)^3/((a*e^2-c*
d^2)*e)^(1/2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 607 vs. $2(229) = 458$.

Time = 0.20 (sec) , antiderivative size = 1236, normalized size of antiderivative = 4.85

$$\int \frac{\sqrt{d+ex}}{\left(ade+(cd^2+ae^2)\,x+cdex^2\right)^{5/2}}\,dx = \text{Too large to display}$$

input
```
integrate((e*x+d)^(1/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2),x, algorit
hm="fricas")
```

output
```
[-1/6*(15*(c^3*d^3*e^3*x^4 + a^2*c*d^3*e^3 + 2*(c^3*d^4*e^2 + a*c^2*d^2*e^
4)*x^3 + (c^3*d^5*e + 4*a*c^2*d^3*e^3 + a^2*c*d*e^5)*x^2 + 2*(a*c^2*d^4*e^
2 + a^2*c*d^2*e^4)*x)*sqrt(-e/(c*d^2 - a*e^2))*log(-(c*d*e^2*x^2 + 2*a*e^3
*x - c*d^3 + 2*a*d*e^2 - 2*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(c*
d^2 - a*e^2)*sqrt(e*x + d)*sqrt(-e/(c*d^2 - a*e^2)))/(e^2*x^2 + 2*d*e*x +
d^2)) - 2*(15*c^2*d^2*e^2*x^2 - 2*c^2*d^4 + 14*a*c*d^2*e^2 + 3*a^2*e^4 + 1
0*(c^2*d^3*e + 2*a*c*d*e^3)*x)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)
*sqrt(e*x + d))/(a^2*c^3*d^8*e^2 - 3*a^3*c^2*d^6*e^4 + 3*a^4*c*d^4*e^6 - a
^5*d^2*e^8 + (c^5*d^8*e^2 - 3*a*c^4*d^6*e^4 + 3*a^2*c^3*d^4*e^6 - a^3*c^2*
d^2*e^8)*x^4 + 2*(c^5*d^9*e - 2*a*c^4*d^7*e^3 + 2*a^3*c^2*d^3*e^7 - a^4*c*
d*e^9)*x^3 + (c^5*d^10 + a*c^4*d^8*e^2 - 8*a^2*c^3*d^6*e^4 + 8*a^3*c^2*d^4
*e^6 - a^4*c*d^2*e^8 - a^5*e^10)*x^2 + 2*(a*c^4*d^9*e - 2*a^2*c^3*d^7*e^3
+ 2*a^4*c*d^3*e^7 - a^5*d*e^9)*x), 1/3*(15*(c^3*d^3*e^3*x^4 + a^2*c*d^3*e^
3 + 2*(c^3*d^4*e^2 + a*c^2*d^2*e^4)*x^3 + (c^3*d^5*e + 4*a*c^2*d^3*e^3 + a
^2*c*d*e^5)*x^2 + 2*(a*c^2*d^4*e^2 + a^2*c*d^2*e^4)*x)*sqrt(e/(c*d^2 - a*e
^2))*arctan(-sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(c*d^2 - a*e^2)*s
qrt(e*x + d)*sqrt(e/(c*d^2 - a*e^2))/(c*d*e^2*x^2 + a*d*e^2 + (c*d^2*e + a
*e^3)*x)) + (15*c^2*d^2*e^2*x^2 - 2*c^2*d^4 + 14*a*c*d^2*e^2 + 3*a^2*e^4 +
 10*(c^2*d^3*e + 2*a*c*d*e^3)*x)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*
x)*sqrt(e*x + d))/(a^2*c^3*d^8*e^2 - 3*a^3*c^2*d^6*e^4 + 3*a^4*c*d^4*e^...
```

Sympy [F]

$$\int \frac{\sqrt{d+ex}}{(ade+(cd^2+ae^2)\,x+cdex^2)^{5/2}}\,dx=\int \frac{\sqrt{d+ex}}{((d+ex)\,(ae+cdx))^{\frac{5}{2}}}\,dx$$

input
```
integrate((e*x+d)**(1/2)/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(5/2),x)
```

output
```
Integral(sqrt(d + e*x)/((d + e*x)*(a*e + c*d*x))**(5/2), x)
```

Maxima [F]

$$\int \frac{\sqrt{d+ex}}{(ade+(cd^2+ae^2)x+cdex^2)^{5/2}}\,dx = \int \frac{\sqrt{ex+d}}{(cdex^2+ade+(cd^2+ae^2)x)^{\frac{5}{2}}}\,dx$$

input
```
integrate((e*x+d)^(1/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2),x, algorit
hm="maxima")
```

output
```
integrate(sqrt(e*x + d)/(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^(5/2), x)
```

Giac [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 322, normalized size of antiderivative = 1.26

$$\int \frac{\sqrt{d+ex}}{(ade+(cd^2+ae^2)x+cdex^2)^{5/2}}\,dx = \frac{1}{3}\left(\frac{15\,cde\arctan\left(\frac{\sqrt{(ex+d)cde-cd^2e+ae^3}}{\sqrt{cd^2e-ae^3}}\right)}{(c^3d^6|e|-3\,ac^2d^4e^2|e|+3\,a^2cd^2e^4|e|-a^3e^6|e|)\sqrt{cd^2e-ae^3}}\right.$$

input
```
integrate((e*x+d)^(1/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2),x, algorit
hm="giac")
```

output
```
1/3*(15*c*d*e*arctan(sqrt((e*x + d)*c*d*e - c*d^2*e + a*e^3)/sqrt(c*d^2*e
- a*e^3))/((c^3*d^6*abs(e) - 3*a*c^2*d^4*e^2*abs(e) + 3*a^2*c*d^2*e^4*abs(
e) - a^3*e^6*abs(e))*sqrt(c*d^2*e - a*e^3)) - 2*(c^2*d^3*e^2 - a*c*d*e^4 -
 6*((e*x + d)*c*d*e - c*d^2*e + a*e^3)*c*d*e)/((c^3*d^6*abs(e) - 3*a*c^2*d
^4*e^2*abs(e) + 3*a^2*c*d^2*e^4*abs(e) - a^3*e^6*abs(e))*((e*x + d)*c*d*e
- c*d^2*e + a*e^3)^(3/2)) + 3*sqrt((e*x + d)*c*d*e - c*d^2*e + a*e^3)/((c^
3*d^6*abs(e) - 3*a*c^2*d^4*e^2*abs(e) + 3*a^2*c*d^2*e^4*abs(e) - a^3*e^6*a
bs(e))*(e*x + d))*e^2
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{d+ex}}{\left(ade+(cd^2+ae^2)\,x+cdex^2\right)^{5/2}}\,dx = \int \frac{\sqrt{d+e\,x}}{\left(c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e\right)^{5/2}}\,dx$$

input
```
int((d + e*x)^(1/2)/(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(5/2),x)
```

output
```
int((d + e*x)^(1/2)/(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(5/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 584, normalized size of antiderivative = 2.29

$$\int \frac{\sqrt{d+ex}}{\left(ade+(cd^2+ae^2)\,x+cdex^2\right)^{5/2}}\,dx = \frac{15\sqrt{e}\,\sqrt{cdx+ae}\,\sqrt{-a\,e^2+c\,d^2}\,atan\left(\frac{\sqrt{cdx+ae}\,e}{\sqrt{e}\,\sqrt{-a\,e^2+c\,d^2}}\right)ac\,d^2e^2+15}{}$$

input
```
int((e*x+d)^(1/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2),x)
```

output
```
(15*sqrt(e)*sqrt(a*e + c*d*x)*sqrt( - a*e**2 + c*d**2)*atan((sqrt(a*e + c*
d*x)*e)/(sqrt(e)*sqrt( - a*e**2 + c*d**2)))*a*c*d**2*e**2 + 15*sqrt(e)*sqr
t(a*e + c*d*x)*sqrt( - a*e**2 + c*d**2)*atan((sqrt(a*e + c*d*x)*e)/(sqrt(e
)*sqrt( - a*e**2 + c*d**2)))*a*c*d*e**3*x + 15*sqrt(e)*sqrt(a*e + c*d*x)*s
qrt( - a*e**2 + c*d**2)*atan((sqrt(a*e + c*d*x)*e)/(sqrt(e)*sqrt( - a*e**2
 + c*d**2)))*c**2*d**3*e*x + 15*sqrt(e)*sqrt(a*e + c*d*x)*sqrt( - a*e**2 +
 c*d**2)*atan((sqrt(a*e + c*d*x)*e)/(sqrt(e)*sqrt( - a*e**2 + c*d**2)))*c*
*2*d**2*e**2*x**2 - 3*a**3*e**6 - 11*a**2*c*d**2*e**4 - 20*a**2*c*d*e**5*x
 + 16*a*c**2*d**4*e**2 + 10*a*c**2*d**3*e**3*x - 15*a*c**2*d**2*e**4*x**2
- 2*c**3*d**6 + 10*c**3*d**5*e*x + 15*c**3*d**4*e**2*x**2)/(3*sqrt(a*e + c
*d*x)*(a**5*d*e**9 + a**5*e**10*x - 4*a**4*c*d**3*e**7 - 3*a**4*c*d**2*e**
8*x + a**4*c*d*e**9*x**2 + 6*a**3*c**2*d**5*e**5 + 2*a**3*c**2*d**4*e**6*x
 - 4*a**3*c**2*d**3*e**7*x**2 - 4*a**2*c**3*d**7*e**3 + 2*a**2*c**3*d**6*e
**4*x + 6*a**2*c**3*d**5*e**5*x**2 + a*c**4*d**9*e - 3*a*c**4*d**8*e**2*x
- 4*a*c**4*d**7*e**3*x**2 + c**5*d**10*x + c**5*d**9*e*x**2))
```

3.326 $\int \frac{1}{\sqrt{d+ex}\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}\,dx$

Optimal result

Integrand size = 39, antiderivative size = 332

$$\int \frac{1}{\sqrt{d+ex}\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}\,dx = \frac{1}{2\left(cd^2-ae^2\right)\sqrt{d+ex}\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}} + \frac{7cd\sqrt{d+ex}}{4\left(cd^2-ae^2\right)^2\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}} - \frac{35c^2d^2(d+ex)^{3/2}}{12\left(cd^2-ae^2\right)^3\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}} + \frac{35c^2d^2e\sqrt{d+ex}}{4\left(cd^2-ae^2\right)^4\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}} - \frac{35c^2d^2e^{3/2}\arctan\left(\frac{\sqrt{cd^2-ae^2}\sqrt{d+ex}}{\sqrt{e}\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}\right)}{4\left(cd^2-ae^2\right)^{9/2}}$$

output
```
1/2/(-a*e^2+c*d^2)/(e*x+d)^(1/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)+7
/4*c*d*(e*x+d)^(1/2)/(-a*e^2+c*d^2)^2/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3
/2)-35/12*c^2*d^2*(e*x+d)^(3/2)/(-a*e^2+c*d^2)^3/(a*d*e+(a*e^2+c*d^2)*x+c*
d*e*x^2)^(3/2)+35/4*c^2*d^2*e*(e*x+d)^(1/2)/(-a*e^2+c*d^2)^4/(a*d*e+(a*e^2
+c*d^2)*x+c*d*e*x^2)^(1/2)-35/4*c^2*d^2*e^(3/2)*arctan(1/e^(1/2)/(a*d*e+(a
*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)*(-a*e^2+c*d^2)^(1/2)*(e*x+d)^(1/2))/(-a*e^2
+c*d^2)^(9/2)
```

Mathematica [A] (verified)

Time = 0.74 (sec) , antiderivative size = 240, normalized size of antiderivative = 0.72

$$\int \frac{1}{\sqrt{d+ex}\,(ade+(cd^2+ae^2)\,x+cdex^2)^{5/2}}\,dx = \frac{c^2d^2(d+ex)^{5/2}\left(-\frac{(ae+cdx)(6a^3e^6-3a^2cde^4(13d+7ex)-2ac^2d^2e^2(}{c^2d}\right.}{}$$

input
```
Integrate[1/(Sqrt[d + e*x]*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2)),
x]
```

output
```
(c^2*d^2*(d + e*x)^(5/2)*(-(((a*e + c*d*x)*(6*a^3*e^6 - 3*a^2*c*d*e^4*(13*
d + 7*e*x) - 2*a*c^2*d^2*e^2*(40*d^2 + 119*d*e*x + 70*e^2*x^2) + c^3*d^3*(
8*d^3 - 56*d^2*e*x - 175*d*e^2*x^2 - 105*e^3*x^3)))/(c^2*d^2*(c*d^2 - a*e^
2)^4*(d + e*x)^2)) + (105*e^(3/2)*(a*e + c*d*x)^(5/2)*ArcTan[(Sqrt[e]*Sqrt
[a*e + c*d*x])/Sqrt[c*d^2 - a*e^2]])/(c*d^2 - a*e^2)^(9/2)))/(12*((a*e + c
*d*x)*(d + e*x))^(5/2))
```

Rubi [A] (verified)

Time = 0.94 (sec) , antiderivative size = 366, normalized size of antiderivative = 1.10, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {1135, 1132, 1135, 1132, 1136, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{d+ex}\,(x\,(ae^2+cd^2)+ade+cdex^2)^{5/2}}\,dx$$

$$\Big\downarrow \; 1135$$

$$\frac{7cd\int \frac{\sqrt{d+ex}}{(cdex^2+(cd^2+ae^2)x+ade)^{5/2}}dx}{4\,(cd^2-ae^2)}+\frac{1}{2\sqrt{d+ex}\,(cd^2-ae^2)\,(x\,(ae^2+cd^2)+ade+cdex^2)^{3/2}}$$

$$\Big\downarrow \; 1132$$

$$\frac{7cd\left(-\frac{5e\int \frac{1}{\sqrt{d+ex}\left(cdex^2+\left(cd^2+ae^2\right)x+ade\right)^{3/2}}dx}{3(cd^2-ae^2)}-\frac{2\sqrt{d+ex}}{3(cd^2-ae^2)(x(ae^2+cd^2)+ade+cdex^2)^{3/2}}\right)}{4\left(cd^2-ae^2\right)}+\frac{1}{2\sqrt{d+ex}\left(cd^2-ae^2\right)\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}$$

$$\Big\downarrow \text{1135}$$

$$\frac{7cd\left(-\frac{5e\left(\frac{3cd\int \frac{\sqrt{d+ex}}{\left(cdex^2+\left(cd^2+ae^2\right)x+ade\right)^{3/2}}dx}{2\left(cd^2-ae^2\right)}+\frac{1}{\sqrt{d+ex}\left(cd^2-ae^2\right)\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}\right)}{3(cd^2-ae^2)}-\frac{2\sqrt{d+ex}}{3(cd^2-ae^2)(x(ae^2+cd^2)+ade+cdex^2)^{3/2}}\right)}{4\left(cd^2-ae^2\right)}$$

$$\frac{1}{2\sqrt{d+ex}\left(cd^2-ae^2\right)\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}$$

$$\Big\downarrow \text{1132}$$

$$\frac{7cd\left(-\frac{5e\left(\frac{3cd\left(-\frac{e\int \frac{1}{\sqrt{d+ex}\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}dx}{cd^2-ae^2}-\frac{2\sqrt{d+ex}}{\left(cd^2-ae^2\right)\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}\right)}{2\left(cd^2-ae^2\right)}+\frac{1}{\sqrt{d+ex}\left(cd^2-ae^2\right)\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}\right)}{3(cd^2-ae^2)}\right)}{4\left(cd^2-ae^2\right)}$$

$$\frac{1}{2\sqrt{d+ex}\left(cd^2-ae^2\right)\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}$$

$$\Big\downarrow \text{1136}$$

$$\frac{7cd\left(-\frac{5e\left(\frac{3cd\left(-\frac{2e^2\int \frac{1}{\frac{\left(cdex^2+\left(cd^2+ae^2\right)x+ade\right)e^2}{d+ex}+\left(cd^2-ae^2\right)e}d\frac{\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}{\sqrt{d+ex}}}{cd^2-ae^2}-\frac{2\sqrt{d+ex}}{\left(cd^2-ae^2\right)\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}\right)}{2\left(cd^2-ae^2\right)}+\frac{\cdots}{\sqrt{d+ex}\left(\cdots\right.}\right)}{3(cd^2-ae^2)}\cdots\right.}{4\left(cd^2-ae^2\right)\cdots}$$

$$\frac{1}{2\sqrt{d+ex}\left(cd^2-ae^2\right)\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}$$

$$\Big\downarrow \; 218$$

$$\frac{7cd\left(-\frac{5e\left(\frac{3cd\left(-\frac{2\sqrt{e}\arctan\left(\frac{\sqrt{e}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{\sqrt{d+ex}\sqrt{cd^2-ae^2}}\right)}{\left(cd^2-ae^2\right)^{3/2}}-\frac{2\sqrt{d+ex}}{\left(cd^2-ae^2\right)\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}\right)}{2\left(cd^2-ae^2\right)}+\frac{1}{\sqrt{d+ex}\left(cd^2-ae^2\right)\sqrt{x\left(ae^2+cd^2\right)+ade+cdex}}\right.}{3(cd^2-ae^2)}\right.}{4\left(cd^2-ae^2\right)}$$

$$\frac{1}{2\sqrt{d+ex}\left(cd^2-ae^2\right)\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}$$

input
```
Int[1/(Sqrt[d + e*x]*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2)),x]
```

output
```
1/(2*(c*d^2 - a*e^2)*Sqrt[d + e*x]*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)
^(3/2)) + (7*c*d*((-2*Sqrt[d + e*x])/(3*(c*d^2 - a*e^2)*(a*d*e + (c*d^2 +
a*e^2)*x + c*d*e*x^2)^(3/2)) - (5*e*(1/((c*d^2 - a*e^2)*Sqrt[d + e*x]*Sqrt
[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]) + (3*c*d*((-2*Sqrt[d + e*x])/((c*
d^2 - a*e^2)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]) - (2*Sqrt[e]*Arc
Tan[(Sqrt[e]*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(Sqrt[c*d^2 - a*
e^2]*Sqrt[d + e*x])])/(c*d^2 - a*e^2)^(3/2)))/(2*(c*d^2 - a*e^2))))/(3*(c*
d^2 - a*e^2))))/(4*(c*d^2 - a*e^2))
```

Defintions of rubi rules used

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 1132
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(2*c*d - b*e)*(d + e*x)^m*((a + b*x + c*x^2)^(p + 1)/(e*(p +
 1)*(b^2 - 4*a*c))), x] - Simp[(2*c*d - b*e)*((m + 2*p + 2)/((p + 1)*(b^2 -
 4*a*c)))   Int[(d + e*x)^(m - 1)*(a + b*x + c*x^2)^(p + 1), x], x] /; Free
Q[{a, b, c, d, e}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[p, -1] && LtQ
[0, m, 1] && IntegerQ[2*p]
```

rule 1135
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(-e)*(d + e*x)^m*((a + b*x + c*x^2)^(p + 1)/((m + p + 1)*(2*
c*d - b*e))), x] + Simp[c*((m + 2*p + 2)/((m + p + 1)*(2*c*d - b*e)))   Int
[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p},
 x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[m, 0] && NeQ[m + p + 1, 0] && I
ntegerQ[2*p]
```

rule 1136
```
Int[1/(Sqrt[(d_.) + (e_.)*(x_)]*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x
_Symbol] :> Simp[2*e   Subst[Int[1/(2*c*d - b*e + e^2*x^2), x], x, Sqrt[a +
 b*x + c*x^2]/Sqrt[d + e*x]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c*d^2
- b*d*e + a*e^2, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 657 vs. $2(294) = 588$.

Time = 1.08 (sec) , antiderivative size = 658, normalized size of antiderivative = 1.98

method	result
default	$-\frac{\sqrt{(ex+d)(cdx+ae)}\left(105 \operatorname{arctanh}\left(\frac{e\sqrt{cdx+ae}}{\sqrt{\left(a e^2-c d^2\right) e}}\right) c^3 d^3 e^4 x^3 \sqrt{cdx+ae}+105 \operatorname{arctanh}\left(\frac{e\sqrt{cdx+ae}}{\sqrt{\left(a e^2-c d^2\right) e}}\right) a c^2 d^2 e^5 x^2 \sqrt{cdx+ae}+2\right.}{}$

input
```
int(1/(e*x+d)^(1/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(5/2),x,method=_RETU
RNVERBOSE)
```

output
```
-1/12*((e*x+d)*(c*d*x+a*e))^(1/2)*(105*arctanh(e*(c*d*x+a*e)^(1/2)/((a*e^2
-c*d^2)*e)^(1/2))*c^3*d^3*e^4*x^3*(c*d*x+a*e)^(1/2)+105*arctanh(e*(c*d*x+a
*e)^(1/2)/((a*e^2-c*d^2)*e)^(1/2))*a*c^2*d^2*e^5*x^2*(c*d*x+a*e)^(1/2)+210
*arctanh(e*(c*d*x+a*e)^(1/2)/((a*e^2-c*d^2)*e)^(1/2))*c^3*d^4*e^3*x^2*(c*d
*x+a*e)^(1/2)+210*arctanh(e*(c*d*x+a*e)^(1/2)/((a*e^2-c*d^2)*e)^(1/2))*a*c
^2*d^3*e^4*x*(c*d*x+a*e)^(1/2)+105*arctanh(e*(c*d*x+a*e)^(1/2)/((a*e^2-c*d
^2)*e)^(1/2))*c^3*d^5*e^2*x*(c*d*x+a*e)^(1/2)-105*((a*e^2-c*d^2)*e)^(1/2)*
c^3*d^3*e^3*x^3+105*arctanh(e*(c*d*x+a*e)^(1/2)/((a*e^2-c*d^2)*e)^(1/2))*a
*c^2*d^4*e^3*(c*d*x+a*e)^(1/2)-140*((a*e^2-c*d^2)*e)^(1/2)*a*c^2*d^2*e^4*x
^2-175*((a*e^2-c*d^2)*e)^(1/2)*c^3*d^4*e^2*x^2-21*((a*e^2-c*d^2)*e)^(1/2)*
a^2*c*d*e^5*x-238*((a*e^2-c*d^2)*e)^(1/2)*a*c^2*d^3*e^3*x-56*((a*e^2-c*d^2
)*e)^(1/2)*c^3*d^5*e*x+6*((a*e^2-c*d^2)*e)^(1/2)*a^3*e^6-39*((a*e^2-c*d^2)
*e)^(1/2)*a^2*c*d^2*e^4-80*((a*e^2-c*d^2)*e)^(1/2)*a*c^2*d^4*e^2+8*((a*e^2
-c*d^2)*e)^(1/2)*c^3*d^6)/(e*x+d)^(5/2)/(c*d*x+a*e)^2/(a*e^2-c*d^2)^4/((a*
e^2-c*d^2)*e)^(1/2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 878 vs. $2(294) = 588$.

Time = 0.37 (sec) , antiderivative size = 1778, normalized size of antiderivative = 5.36

$$\int \frac{1}{\sqrt{d+ex}\,(ade+(cd^2+ae^2)\,x+cdex^2)^{5/2}}\,dx = \text{Too large to display}$$

input
```
integrate(1/(e*x+d)^(1/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2),x, algor
ithm="fricas")
```

output
```
[1/24*(105*(c^4*d^4*e^4*x^5 + a^2*c^2*d^5*e^3 + (3*c^4*d^5*e^3 + 2*a*c^3*d
^3*e^5)*x^4 + (3*c^4*d^6*e^2 + 6*a*c^3*d^4*e^4 + a^2*c^2*d^2*e^6)*x^3 + (c
^4*d^7*e + 6*a*c^3*d^5*e^3 + 3*a^2*c^2*d^3*e^5)*x^2 + (2*a*c^3*d^6*e^2 + 3
*a^2*c^2*d^4*e^4)*x)*sqrt(-e/(c*d^2 - a*e^2))*log(-(c*d*e^2*x^2 + 2*a*e^3*
x - c*d^3 + 2*a*d*e^2 + 2*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(c*d
^2 - a*e^2)*sqrt(e*x + d)*sqrt(-e/(c*d^2 - a*e^2)))/(e^2*x^2 + 2*d*e*x + d
^2)) + 2*(105*c^3*d^3*e^3*x^3 - 8*c^3*d^6 + 80*a*c^2*d^4*e^2 + 39*a^2*c*d^
2*e^4 - 6*a^3*e^6 + 35*(5*c^3*d^4*e^2 + 4*a*c^2*d^2*e^4)*x^2 + 7*(8*c^3*d^
5*e + 34*a*c^2*d^3*e^3 + 3*a^2*c*d*e^5)*x)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2
 + a*e^2)*x)*sqrt(e*x + d))/(a^2*c^4*d^11*e^2 - 4*a^3*c^3*d^9*e^4 + 6*a^4*
c^2*d^7*e^6 - 4*a^5*c*d^5*e^8 + a^6*d^3*e^10 + (c^6*d^10*e^3 - 4*a*c^5*d^8
*e^5 + 6*a^2*c^4*d^6*e^7 - 4*a^3*c^3*d^4*e^9 + a^4*c^2*d^2*e^11)*x^5 + (3*
c^6*d^11*e^2 - 10*a*c^5*d^9*e^4 + 10*a^2*c^4*d^7*e^6 - 5*a^4*c^2*d^3*e^10
+ 2*a^5*c*d*e^12)*x^4 + (3*c^6*d^12*e - 6*a*c^5*d^10*e^3 - 5*a^2*c^4*d^8*e
^5 + 20*a^3*c^3*d^6*e^7 - 15*a^4*c^2*d^4*e^9 + 2*a^5*c*d^2*e^11 + a^6*e^13
)*x^3 + (c^6*d^13 + 2*a*c^5*d^11*e^2 - 15*a^2*c^4*d^9*e^4 + 20*a^3*c^3*d^7
*e^6 - 5*a^4*c^2*d^5*e^8 - 6*a^5*c*d^3*e^10 + 3*a^6*d*e^12)*x^2 + (2*a*c^5
*d^12*e - 5*a^2*c^4*d^10*e^3 + 10*a^4*c^2*d^6*e^7 - 10*a^5*c*d^4*e^9 + 3*a
^6*d^2*e^11)*x), 1/12*(105*(c^4*d^4*e^4*x^5 + a^2*c^2*d^5*e^3 + (3*c^4*d^5
*e^3 + 2*a*c^3*d^3*e^5)*x^4 + (3*c^4*d^6*e^2 + 6*a*c^3*d^4*e^4 + a^2*c^...
```

Sympy [F]

$$\int \frac{1}{\sqrt{d+ex}\,(ade+(cd^2+ae^2)\,x+cdex^2)^{5/2}}\,dx = \int \frac{1}{((d+ex)\,(ae+cdx))^{\frac{5}{2}}\,\sqrt{d+ex}}\,dx$$

input
```
integrate(1/(e*x+d)**(1/2)/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(5/2),x)
```

output
```
Integral(1/(((d + e*x)*(a*e + c*d*x))**(5/2)*sqrt(d + e*x)), x)
```

Maxima [F]

$$\int \frac{1}{\sqrt{d+ex}\left(ade+(cd^2+ae^2)\,x+cdex^2\right)^{5/2}}\,dx=\int \frac{1}{(cdex^2+ade+(cd^2+ae^2)x)^{\frac{5}{2}}\sqrt{ex+d}}\,dx$$

input
```
integrate(1/(e*x+d)^(1/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2),x, algor
ithm="maxima")
```

output
```
integrate(1/((c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^(5/2)*sqrt(e*x + d)),
 x)
```

Giac [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 469, normalized size of antiderivative = 1.41

$$\int \frac{1}{\sqrt{d+ex}\left(ade+(cd^2+ae^2)\,x+cdex^2\right)^{5/2}}\,dx=\frac{1}{12}\left(\frac{105\,c^2d^2e\arctan\left(\frac{\sqrt{(ex+d)cde-cd^2e}}{\sqrt{cd^2e-ae^3}}\right)}{(c^4d^8|e|-4\,ac^3d^6e^2|e|+6\,a^2c^2d^4e^4|e|-4\,a^3cd^2e^6|e|}\right.$$

input
```
integrate(1/(e*x+d)^(1/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2),x, algor
ithm="giac")
```

output
```
1/12*(105*c^2*d^2*e*arctan(sqrt((e*x + d)*c*d*e - c*d^2*e + a*e^3)/sqrt(c*
d^2*e - a*e^3))/((c^4*d^8*abs(e) - 4*a*c^3*d^6*e^2*abs(e) + 6*a^2*c^2*d^4*
e^4*abs(e) - 4*a^3*c*d^2*e^6*abs(e) + a^4*e^8*abs(e))*sqrt(c*d^2*e - a*e^3
)) - 8*(c^3*d^4*e^2 - a*c^2*d^2*e^4 - 9*((e*x + d)*c*d*e - c*d^2*e + a*e^3
)*c^2*d^2*e)/((c^4*d^8*abs(e) - 4*a*c^3*d^6*e^2*abs(e) + 6*a^2*c^2*d^4*e^4
*abs(e) - 4*a^3*c*d^2*e^6*abs(e) + a^4*e^8*abs(e))*((e*x + d)*c*d*e - c*d^
2*e + a*e^3)^(3/2)) + 3*(13*sqrt((e*x + d)*c*d*e - c*d^2*e + a*e^3)*c^3*d^
4*e^2 - 13*sqrt((e*x + d)*c*d*e - c*d^2*e + a*e^3)*a*c^2*d^2*e^4 + 11*((e*
x + d)*c*d*e - c*d^2*e + a*e^3)^(3/2)*c^2*d^2*e)/((c^4*d^8*abs(e) - 4*a*c^
3*d^6*e^2*abs(e) + 6*a^2*c^2*d^4*e^4*abs(e) - 4*a^3*c*d^2*e^6*abs(e) + a^4
*e^8*abs(e))*(e*x + d)^2*c^2*d^2*e^2))*e^2
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{d+ex}\,(ade+(cd^2+ae^2)\,x+cdex^2)^{5/2}}\,dx=\int \frac{1}{\sqrt{d+e\,x}\,(c\,d\,e\,x^2+(c\,d^2+a\,e^2)\;x+a\,d\,e)^{5/2}}\,dx$$

input
```
int(1/((d + e*x)^(1/2)*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(5/2)),x)
```

output
```
int(1/((d + e*x)^(1/2)*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(5/2)), x)
```

Reduce [B] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 930, normalized size of antiderivative = 2.80

$$\int \frac{1}{\sqrt{d+ex}\,(ade+(cd^2+ae^2)\,x+cdex^2)^{5/2}}\,dx=\text{Too large to display}$$

input
```
int(1/(e*x+d)^(1/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2),x)
```

output
```
( - 105*sqrt(e)*sqrt(a*e + c*d*x)*sqrt( - a*e**2 + c*d**2)*atan((sqrt(a*e
+ c*d*x)*e)/(sqrt(e)*sqrt( - a*e**2 + c*d**2)))*a*c**2*d**4*e**2 - 210*sqr
t(e)*sqrt(a*e + c*d*x)*sqrt( - a*e**2 + c*d**2)*atan((sqrt(a*e + c*d*x)*e)
/(sqrt(e)*sqrt( - a*e**2 + c*d**2)))*a*c**2*d**3*e**3*x - 105*sqrt(e)*sqrt
(a*e + c*d*x)*sqrt( - a*e**2 + c*d**2)*atan((sqrt(a*e + c*d*x)*e)/(sqrt(e)
*sqrt( - a*e**2 + c*d**2)))*a*c**2*d**2*e**4*x**2 - 105*sqrt(e)*sqrt(a*e +
 c*d*x)*sqrt( - a*e**2 + c*d**2)*atan((sqrt(a*e + c*d*x)*e)/(sqrt(e)*sqrt(
 - a*e**2 + c*d**2)))*c**3*d**5*e*x - 210*sqrt(e)*sqrt(a*e + c*d*x)*sqrt(
- a*e**2 + c*d**2)*atan((sqrt(a*e + c*d*x)*e)/(sqrt(e)*sqrt( - a*e**2 + c*
d**2)))*c**3*d**4*e**2*x**2 - 105*sqrt(e)*sqrt(a*e + c*d*x)*sqrt( - a*e**2
 + c*d**2)*atan((sqrt(a*e + c*d*x)*e)/(sqrt(e)*sqrt( - a*e**2 + c*d**2)))*
c**3*d**3*e**3*x**3 - 6*a**4*e**8 + 45*a**3*c*d**2*e**6 + 21*a**3*c*d*e**7
*x + 41*a**2*c**2*d**4*e**4 + 217*a**2*c**2*d**3*e**5*x + 140*a**2*c**2*d*
*2*e**6*x**2 - 88*a*c**3*d**6*e**2 - 182*a*c**3*d**5*e**3*x + 35*a*c**3*d*
*4*e**4*x**2 + 105*a*c**3*d**3*e**5*x**3 + 8*c**4*d**8 - 56*c**4*d**7*e*x
- 175*c**4*d**6*e**2*x**2 - 105*c**4*d**5*e**3*x**3)/(12*sqrt(a*e + c*d*x)
*(a**6*d**2*e**11 + 2*a**6*d*e**12*x + a**6*e**13*x**2 - 5*a**5*c*d**4*e**
9 - 9*a**5*c*d**3*e**10*x - 3*a**5*c*d**2*e**11*x**2 + a**5*c*d*e**12*x**3
 + 10*a**4*c**2*d**6*e**7 + 15*a**4*c**2*d**5*e**8*x - 5*a**4*c**2*d**3*e*
*10*x**3 - 10*a**3*c**3*d**8*e**5 - 10*a**3*c**3*d**7*e**6*x + 10*a**3*...
```

3.327 $\int \frac{1}{(d+ex)^{3/2}\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}\,dx$

Optimal result

Integrand size = 39, antiderivative size = 394

$$\int \frac{1}{(d+ex)^{3/2}\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}\,dx = \frac{1}{3\left(cd^2-ae^2\right)(d+ex)^{3/2}\left(ade+\left(cd^2+ae^2\right)x+cdex\right.}$$
$$+\frac{3cd}{4\left(cd^2-ae^2\right)^2\sqrt{d+ex}\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}$$
$$+\frac{21c^2d^2\sqrt{d+ex}}{8\left(cd^2-ae^2\right)^3\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}$$
$$-\frac{35c^3d^3(d+ex)^{3/2}}{8\left(cd^2-ae^2\right)^4\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}$$
$$+\frac{105c^3d^3e\sqrt{d+ex}}{8\left(cd^2-ae^2\right)^5\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}$$
$$-\frac{105c^3d^3e^{3/2}\arctan\left(\frac{\sqrt{cd^2-ae^2}\sqrt{d+ex}}{\sqrt{e}\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\right)}{8\left(cd^2-ae^2\right)^{11/2}}$$

output
```
1/3/(-a*e^2+c*d^2)/(e*x+d)^(3/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)+3
/4*c*d/(-a*e^2+c*d^2)^2/(e*x+d)^(1/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3
/2)+21/8*c^2*d^2*(e*x+d)^(1/2)/(-a*e^2+c*d^2)^3/(a*d*e+(a*e^2+c*d^2)*x+c*d
*e*x^2)^(3/2)-35/8*c^3*d^3*(e*x+d)^(3/2)/(-a*e^2+c*d^2)^4/(a*d*e+(a*e^2+c*
d^2)*x+c*d*e*x^2)^(3/2)+105/8*c^3*d^3*e*(e*x+d)^(1/2)/(-a*e^2+c*d^2)^5/(a*
d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)-105/8*c^3*d^3*e^(3/2)*arctan(1/e^(1/2
)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)*(-a*e^2+c*d^2)^(1/2)*(e*x+d)^(1/
2))/(-a*e^2+c*d^2)^(11/2)
```

Mathematica [A] (verified)

Time = 1.33 (sec) , antiderivative size = 294, normalized size of antiderivative = 0.75

$$\int \frac{1}{(d+ex)^{3/2}\left(ade+(cd^2+ae^2)x+cdex^2\right)^{5/2}}\,dx = \frac{c^3d^3(d+ex)^{5/2}\left(\frac{(ae+cdx)\left(8a^4e^8-2a^3cde^6(25d+9ex)+3a^2c^2d^2e^4\cdots\right.}{\cdots}\right.}{\cdots}$$

input
```
Integrate[1/((d + e*x)^(3/2)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2)
),x]
```

output
```
(c^3*d^3*(d + e*x)^(5/2)*(((a*e + c*d*x)*(8*a^4*e^8 - 2*a^3*c*d*e^6*(25*d
+ 9*e*x) + 3*a^2*c^2*d^2*e^4*(55*d^2 + 60*d*e*x + 21*e^2*x^2) + 2*a*c^3*d^
3*e^2*(104*d^3 + 477*d^2*e*x + 567*d*e^2*x^2 + 210*e^3*x^3) + c^4*d^4*(-16
*d^4 + 144*d^3*e*x + 693*d^2*e^2*x^2 + 840*d*e^3*x^3 + 315*e^4*x^4)))/(c^3
*d^3*(c*d^2 - a*e^2)^5*(d + e*x)^3) + (315*e^(3/2)*(a*e + c*d*x)^(5/2)*Arc
Tan[(Sqrt[e]*Sqrt[a*e + c*d*x])/Sqrt[c*d^2 - a*e^2]])/(c*d^2 - a*e^2)^(11/
2)))/(24*((a*e + c*d*x)*(d + e*x))^(5/2))
```

Rubi [A] (verified)

Time = 1.07 (sec) , antiderivative size = 443, normalized size of antiderivative = 1.12, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.179$, Rules used = {1135, 1135, 1132, 1135, 1132, 1136, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(d+ex)^{3/2}\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/2}}dx$$

$$\Big\downarrow \; 1135$$

$$\frac{3cd\int \frac{1}{\sqrt{d+ex}(cdex^2+(cd^2+ae^2)x+ade)^{5/2}}dx}{2\left(cd^2-ae^2\right)}+\frac{1}{3(d+ex)^{3/2}\left(cd^2-ae^2\right)\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}$$

$$\Big\downarrow \; 1135$$

$$\frac{3cd\left(\frac{7cd\int \frac{\sqrt{d+ex}}{\left(cdex^2+\left(cd^2+ae^2\right)x+ade\right)^{5/2}}dx}{4(cd^2-ae^2)}+\frac{1}{2\sqrt{d+ex}(cd^2-ae^2)(x(ae^2+cd^2)+ade+cdex^2)^{3/2}}\right)}{2\left(cd^2-ae^2\right)}+\frac{1}{3(d+ex)^{3/2}\left(cd^2-ae^2\right)\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}$$

$$\Big\downarrow \; 1132$$

$$\frac{3cd\left(\frac{7cd\left(-\frac{5e\int \frac{1}{\sqrt{d+ex}\left(cdex^2+\left(cd^2+ae^2\right)x+ade\right)^{3/2}}dx}{3\left(cd^2-ae^2\right)}-\frac{2\sqrt{d+ex}}{3\left(cd^2-ae^2\right)\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}\right)}{4(cd^2-ae^2)}+\frac{1}{2\sqrt{d+ex}(cd^2-ae^2)(x(ae^2+cd^2)+ade+\cdots}\right)}{2\left(cd^2-ae^2\right)}+\frac{1}{3(d+ex)^{3/2}\left(cd^2-ae^2\right)\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}$$

$$\Big\downarrow \; 1135$$

$$\frac{3cd\Bigg(\frac{7cd\Bigg(-\frac{5e\Big(\frac{3cd\int\frac{\sqrt{d+ex}}{(cdex^2+(cd^2+ae^2)x+ade)^{3/2}}dx}{2(cd^2-ae^2)}+\frac{1}{\sqrt{d+ex}(cd^2-ae^2)\sqrt{x(ae^2+cd^2)+ade+cdex^2}}\Big)}{3(cd^2-ae^2)}-\frac{2\sqrt{d+ex}}{3(cd^2-ae^2)(x(ae^2+cd^2)+ade+cdex^2)^{3/2}}}{4(cd^2-ae^2)}}{2(cd^2-ae^2)}$$

$$\frac{1}{3(d+ex)^{3/2}(cd^2-ae^2)(x(ae^2+cd^2)+ade+cdex^2)^{3/2}}$$

$\downarrow$ 1132

$$\frac{3cd\Bigg(\frac{7cd\Bigg(-\frac{5e\Big(\frac{3cd\Big(-\frac{e\int\frac{1}{\sqrt{d+ex}\sqrt{cdex^2+(cd^2+ae^2)x+ade}}dx}{cd^2-ae^2}-\frac{2\sqrt{d+ex}}{(cd^2-ae^2)\sqrt{x(ae^2+cd^2)+ade+cdex^2}}\Big)}{2(cd^2-ae^2)}+\frac{1}{\sqrt{d+ex}(cd^2-ae^2)\sqrt{x(ae^2+cd^2)+ade+cdex^2}}}{3(cd^2-ae^2)}}{4(cd^2-ae^2)}}{2(cd^2-ae^2)}$$

$$\frac{1}{3(d+ex)^{3/2}(cd^2-ae^2)(x(ae^2+cd^2)+ade+cdex^2)^{3/2}}$$

$\Big\downarrow$ 1136

$$\frac{3cd\left(\frac{7cd\left(-\frac{5e\left(\frac{3cd\left(-\frac{2e^2\int \frac{1}{\frac{\left(cdex^2+\left(cd^2+ae^2\right)x+ade\right)e^2}{d+ex}+\left(cd^2-ae^2\right)e}d\frac{\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}{\sqrt{d+ex}}}{cd^2-ae^2}-\frac{2\sqrt{d+ex}}{\left(cd^2-ae^2\right)\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}\right)}{2\left(cd^2-ae^2\right)}+\frac{\cdots}{\sqrt{d+\cdots}}\right)}{3\left(cd^2-ae^2\right)}\cdots\right)}{4(cd^2-ae^2)}\cdots\right)}{2\left(cd^2-\cdots\right.}$$

$$\frac{1}{3(d+ex)^{3/2}\left(cd^2-ae^2\right)\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}$$

$\Big\downarrow$ 218

$$\frac{3cd\left(\frac{7cd\left(-\frac{5e\left(\frac{3cd\left(-\frac{2\sqrt{e}\arctan\left(\frac{\sqrt{e}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{\sqrt{d+ex}\sqrt{cd^2-ae^2}}\right)}{\left(cd^2-ae^2\right)^{3/2}}-\frac{2\sqrt{d+ex}}{\left(cd^2-ae^2\right)\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}\right)}{2\left(cd^2-ae^2\right)}+\frac{1}{\sqrt{d+ex}\left(cd^2-ae^2\right)\sqrt{x\left(ae^2+cd^2\right)+ade+\cdots}}\cdots\right)}{3\left(cd^2-ae^2\right)}\cdots\right)}{4(cd^2-ae^2)}\cdots\right)}{2\left(cd^2-ae^2\right)\cdots}$$

$$\frac{1}{3(d+ex)^{3/2}\left(cd^2-ae^2\right)\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}$$

input
```
Int[1/((d + e*x)^(3/2)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2)),x]
```

output
```
1/(3*(c*d^2 - a*e^2)*(d + e*x)^(3/2)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^
2)^(3/2)) + (3*c*d*(1/(2*(c*d^2 - a*e^2)*Sqrt[d + e*x]*(a*d*e + (c*d^2 + a
*e^2)*x + c*d*e*x^2)^(3/2)) + (7*c*d*((-2*Sqrt[d + e*x])/(3*(c*d^2 - a*e^2
)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2)) - (5*e*(1/((c*d^2 - a*e^2
)*Sqrt[d + e*x]*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]) + (3*c*d*((-2
*Sqrt[d + e*x])/((c*d^2 - a*e^2)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^
2]) - (2*Sqrt[e]*ArcTan[(Sqrt[e]*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^
2])/(Sqrt[c*d^2 - a*e^2]*Sqrt[d + e*x])])/(c*d^2 - a*e^2)^(3/2)))/(2*(c*d^
2 - a*e^2))))/(3*(c*d^2 - a*e^2))))/(4*(c*d^2 - a*e^2))))/(2*(c*d^2 - a*e^
2))
```

Defintions of rubi rules used

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 1132
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(2*c*d - b*e)*(d + e*x)^m*((a + b*x + c*x^2)^(p + 1)/(e*(p +
 1)*(b^2 - 4*a*c))), x] - Simp[(2*c*d - b*e)*((m + 2*p + 2)/((p + 1)*(b^2 -
 4*a*c)))   Int[(d + e*x)^(m - 1)*(a + b*x + c*x^2)^(p + 1), x], x] /; Free
Q[{a, b, c, d, e}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[p, -1] && LtQ
[0, m, 1] && IntegerQ[2*p]
```

rule 1135
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(-e)*(d + e*x)^m*((a + b*x + c*x^2)^(p + 1)/((m + p + 1)*(2*
c*d - b*e))), x] + Simp[c*((m + 2*p + 2)/((m + p + 1)*(2*c*d - b*e)))   Int
[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p},
 x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[m, 0] && NeQ[m + p + 1, 0] && I
ntegerQ[2*p]
```

rule 1136
```
Int[1/(Sqrt[(d_.) + (e_.)*(x_)]*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x
_Symbol] :> Simp[2*e   Subst[Int[1/(2*c*d - b*e + e^2*x^2), x], x, Sqrt[a +
 b*x + c*x^2]/Sqrt[d + e*x]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c*d^2
- b*d*e + a*e^2, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 919 vs. $2(350) = 700$.

Time = 1.08 (sec) , antiderivative size = 920, normalized size of antiderivative = 2.34

method	result
default	$\frac{\sqrt{(ex+d)(cdx+ae)}\left(315 \operatorname{arctanh}\left(\frac{e\sqrt{cdx+ae}}{\sqrt{\left(a\,e^{2}-c\,d^{2}\right)e}}\right)c^{4}d^{4}e^{5}x^{4}\sqrt{cdx+ae}+315 \operatorname{arctanh}\left(\frac{e\sqrt{cdx+ae}}{\sqrt{\left(a\,e^{2}-c\,d^{2}\right)e}}\right)a\,c^{3}d^{3}e^{6}x^{3}\sqrt{cdx+ae}+945\right.}{}$

input
```
int(1/(e*x+d)^(3/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(5/2),x,method=_RETU
RNVERBOSE)
```

output
```
1/24*((e*x+d)*(c*d*x+a*e))^(1/2)*(315*arctanh(e*(c*d*x+a*e)^(1/2)/((a*e^2-
c*d^2)*e)^(1/2))*c^4*d^4*e^5*x^4*(c*d*x+a*e)^(1/2)+315*arctanh(e*(c*d*x+a*
e)^(1/2)/((a*e^2-c*d^2)*e)^(1/2))*a*c^3*d^3*e^6*x^3*(c*d*x+a*e)^(1/2)+945*
arctanh(e*(c*d*x+a*e)^(1/2)/((a*e^2-c*d^2)*e)^(1/2))*c^4*d^5*e^4*x^3*(c*d*
x+a*e)^(1/2)+945*arctanh(e*(c*d*x+a*e)^(1/2)/((a*e^2-c*d^2)*e)^(1/2))*a*c^
3*d^4*e^5*x^2*(c*d*x+a*e)^(1/2)+945*arctanh(e*(c*d*x+a*e)^(1/2)/((a*e^2-c*
d^2)*e)^(1/2))*c^4*d^6*e^3*x^2*(c*d*x+a*e)^(1/2)-315*((a*e^2-c*d^2)*e)^(1/
2)*c^4*d^4*e^4*x^4+945*arctanh(e*(c*d*x+a*e)^(1/2)/((a*e^2-c*d^2)*e)^(1/2)
)*a*c^3*d^5*e^4*x*(c*d*x+a*e)^(1/2)+315*arctanh(e*(c*d*x+a*e)^(1/2)/((a*e^
2-c*d^2)*e)^(1/2))*c^4*d^7*e^2*x*(c*d*x+a*e)^(1/2)-420*((a*e^2-c*d^2)*e)^(
1/2)*a*c^3*d^3*e^5*x^3-840*((a*e^2-c*d^2)*e)^(1/2)*c^4*d^5*e^3*x^3+315*arc
tanh(e*(c*d*x+a*e)^(1/2)/((a*e^2-c*d^2)*e)^(1/2))*a*c^3*d^6*e^3*(c*d*x+a*e
)^(1/2)-63*((a*e^2-c*d^2)*e)^(1/2)*a^2*c^2*d^2*e^6*x^2-1134*((a*e^2-c*d^2)
*e)^(1/2)*a*c^3*d^4*e^4*x^2-693*((a*e^2-c*d^2)*e)^(1/2)*c^4*d^6*e^2*x^2+18
*((a*e^2-c*d^2)*e)^(1/2)*a^3*c*d*e^7*x-180*((a*e^2-c*d^2)*e)^(1/2)*a^2*c^2
*d^3*e^5*x-954*((a*e^2-c*d^2)*e)^(1/2)*a*c^3*d^5*e^3*x-144*((a*e^2-c*d^2)*
e)^(1/2)*c^4*d^7*e*x-8*((a*e^2-c*d^2)*e)^(1/2)*a^4*e^8+50*((a*e^2-c*d^2)*e
)^(1/2)*a^3*c*d^2*e^6-165*((a*e^2-c*d^2)*e)^(1/2)*a^2*c^2*d^4*e^4-208*((a*
e^2-c*d^2)*e)^(1/2)*a*c^3*d^6*e^2+16*((a*e^2-c*d^2)*e)^(1/2)*c^4*d^8)/(e*x
+d)^(7/2)/(c*d*x+a*e)^2/(a*e^2-c*d^2)^5/((a*e^2-c*d^2)*e)^(1/2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1183 vs. $2(350) = 700$.

Time = 0.84 (sec) , antiderivative size = 2388, normalized size of antiderivative = 6.06

$$\int \frac{1}{(d+ex)^{3/2} \left(ade+(cd^2+ae^2)\,x+cdex^2\right)^{5/2}}\,dx = \text{Too large to display}$$

input
```
integrate(1/(e*x+d)^(3/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2),x, algor
ithm="fricas")
```

output
```
[-1/48*(315*(c^5*d^5*e^5*x^6 + a^2*c^3*d^7*e^3 + 2*(2*c^5*d^6*e^4 + a*c^4*
d^4*e^6)*x^5 + (6*c^5*d^7*e^3 + 8*a*c^4*d^5*e^5 + a^2*c^3*d^3*e^7)*x^4 + 4
*(c^5*d^8*e^2 + 3*a*c^4*d^6*e^4 + a^2*c^3*d^4*e^6)*x^3 + (c^5*d^9*e + 8*a*
c^4*d^7*e^3 + 6*a^2*c^3*d^5*e^5)*x^2 + 2*(a*c^4*d^8*e^2 + 2*a^2*c^3*d^6*e^
4)*x)*sqrt(-e/(c*d^2 - a*e^2))*log(-(c*d*e^2*x^2 + 2*a*e^3*x - c*d^3 + 2*a
*d*e^2 - 2*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(c*d^2 - a*e^2)*sqr
t(e*x + d)*sqrt(-e/(c*d^2 - a*e^2)))/(e^2*x^2 + 2*d*e*x + d^2)) - 2*(315*c
^4*d^4*e^4*x^4 - 16*c^4*d^8 + 208*a*c^3*d^6*e^2 + 165*a^2*c^2*d^4*e^4 - 50
*a^3*c*d^2*e^6 + 8*a^4*e^8 + 420*(2*c^4*d^5*e^3 + a*c^3*d^3*e^5)*x^3 + 63*
(11*c^4*d^6*e^2 + 18*a*c^3*d^4*e^4 + a^2*c^2*d^2*e^6)*x^2 + 18*(8*c^4*d^7*
e + 53*a*c^3*d^5*e^3 + 10*a^2*c^2*d^3*e^5 - a^3*c*d*e^7)*x)*sqrt(c*d*e*x^2
 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(e*x + d))/(a^2*c^5*d^14*e^2 - 5*a^3*c^4
*d^12*e^4 + 10*a^4*c^3*d^10*e^6 - 10*a^5*c^2*d^8*e^8 + 5*a^6*c*d^6*e^10 -
a^7*d^4*e^12 + (c^7*d^12*e^4 - 5*a*c^6*d^10*e^6 + 10*a^2*c^5*d^8*e^8 - 10*
a^3*c^4*d^6*e^10 + 5*a^4*c^3*d^4*e^12 - a^5*c^2*d^2*e^14)*x^6 + 2*(2*c^7*d
^13*e^3 - 9*a*c^6*d^11*e^5 + 15*a^2*c^5*d^9*e^7 - 10*a^3*c^4*d^7*e^9 + 3*a
^5*c^2*d^3*e^13 - a^6*c*d*e^15)*x^5 + (6*c^7*d^14*e^2 - 22*a*c^6*d^12*e^4
+ 21*a^2*c^5*d^10*e^6 + 15*a^3*c^4*d^8*e^8 - 40*a^4*c^3*d^6*e^10 + 24*a^5*
c^2*d^4*e^12 - 3*a^6*c*d^2*e^14 - a^7*e^16)*x^4 + 4*(c^7*d^15*e - 2*a*c^6*
d^13*e^3 - 4*a^2*c^5*d^11*e^5 + 15*a^3*c^4*d^9*e^7 - 15*a^4*c^3*d^7*e^9...
```

Sympy [F]

$$\int \frac{1}{(d+ex)^{3/2} \left(ade+(cd^2+ae^2)x+cdex^2\right)^{5/2}} \, dx = \int \frac{1}{((d+ex)(ae+cdx))^{\frac{5}{2}} (d+ex)^{\frac{3}{2}}} \, dx$$

input
```
integrate(1/(e*x+d)**(3/2)/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(5/2),x)
```

output
```
Integral(1/(((d + e*x)*(a*e + c*d*x))**(5/2)*(d + e*x)**(3/2)), x)
```

Maxima [F]

$$\int \frac{1}{(d+ex)^{3/2} \left(ade+(cd^2+ae^2)x+cdex^2\right)^{5/2}} \, dx = \int \frac{1}{(cdex^2+ade+(cd^2+ae^2)x)^{\frac{5}{2}} (ex+d)^{\frac{3}{2}}} \, dx$$

input
```
integrate(1/(e*x+d)^(3/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2),x, algor
ithm="maxima")
```

output
```
integrate(1/((c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^(5/2)*(e*x + d)^(3/2)
), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 742 vs. $2(350) = 700$.

Time = 0.19 (sec) , antiderivative size = 742, normalized size of antiderivative = 1.88

$$\int \frac{1}{(d+ex)^{3/2} \left(ade+(cd^2+ae^2)x+cdex^2\right)^{5/2}} \, dx = \text{Too large to display}$$

input
```
integrate(1/(e*x+d)^(3/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2),x, algor
ithm="giac")
```

output
```
1/24*(315*c^3*d^3*e*arctan(sqrt((e*x + d)*c*d*e - c*d^2*e + a*e^3)/sqrt(c*
d^2*e - a*e^3))/((c^5*d^10*abs(e) - 5*a*c^4*d^8*e^2*abs(e) + 10*a^2*c^3*d^
6*e^4*abs(e) - 10*a^3*c^2*d^4*e^6*abs(e) + 5*a^4*c*d^2*e^8*abs(e) - a^5*e^
10*abs(e))*sqrt(c*d^2*e - a*e^3)) - (16*c^7*d^11*e^5 - 64*a*c^6*d^9*e^7 +
96*a^2*c^5*d^7*e^9 - 64*a^3*c^4*d^5*e^11 + 16*a^4*c^3*d^3*e^13 - 144*((e*x
 + d)*c*d*e - c*d^2*e + a*e^3)*c^6*d^9*e^4 + 432*((e*x + d)*c*d*e - c*d^2*
e + a*e^3)*a*c^5*d^7*e^6 - 432*((e*x + d)*c*d*e - c*d^2*e + a*e^3)*a^2*c^4
*d^5*e^8 + 144*((e*x + d)*c*d*e - c*d^2*e + a*e^3)*a^3*c^3*d^3*e^10 - 693*
((e*x + d)*c*d*e - c*d^2*e + a*e^3)^2*c^5*d^7*e^3 + 1386*((e*x + d)*c*d*e
- c*d^2*e + a*e^3)^2*a*c^4*d^5*e^5 - 693*((e*x + d)*c*d*e - c*d^2*e + a*e^
3)^2*a^2*c^3*d^3*e^7 - 840*((e*x + d)*c*d*e - c*d^2*e + a*e^3)^3*c^4*d^5*e
^2 + 840*((e*x + d)*c*d*e - c*d^2*e + a*e^3)^3*a*c^3*d^3*e^4 - 315*((e*x +
 d)*c*d*e - c*d^2*e + a*e^3)^4*c^3*d^3*e)/((c^5*d^10*abs(e) - 5*a*c^4*d^8*
e^2*abs(e) + 10*a^2*c^3*d^6*e^4*abs(e) - 10*a^3*c^2*d^4*e^6*abs(e) + 5*a^4
*c*d^2*e^8*abs(e) - a^5*e^10*abs(e))*(sqrt((e*x + d)*c*d*e - c*d^2*e + a*e
^3)*c*d^2*e - sqrt((e*x + d)*c*d*e - c*d^2*e + a*e^3)*a*e^3 + ((e*x + d)*c
*d*e - c*d^2*e + a*e^3)^(3/2))^3))*e^2
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(d+ex)^{3/2}\left(ade+(cd^2+ae^2)\,x+cdex^2\right)^{5/2}}\,dx=\int \frac{1}{(d+e\,x)^{3/2}\left(c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e\right)^{5/2}}\,dx$$

input
```
int(1/((d + e*x)^(3/2)*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(5/2)),x)
```

output
```
int(1/((d + e*x)^(3/2)*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(5/2)), x)
```

Reduce [B] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 1379, normalized size of antiderivative = 3.50

$$\int \frac{1}{(d+ex)^{3/2}\left(ade+(cd^2+ae^2)\,x+cdex^2\right)^{5/2}}\,dx=\text{Too large to display}$$

input
```
int(1/(e*x+d)^(3/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2),x)
```

output

```
(315*sqrt(e)*sqrt(a*e + c*d*x)*sqrt( - a*e**2 + c*d**2)*atan((sqrt(a*e + c
*d*x)*e)/(sqrt(e)*sqrt( - a*e**2 + c*d**2)))*a*c**3*d**6*e**2 + 945*sqrt(e
)*sqrt(a*e + c*d*x)*sqrt( - a*e**2 + c*d**2)*atan((sqrt(a*e + c*d*x)*e)/(s
qrt(e)*sqrt( - a*e**2 + c*d**2)))*a*c**3*d**5*e**3*x + 945*sqrt(e)*sqrt(a*
e + c*d*x)*sqrt( - a*e**2 + c*d**2)*atan((sqrt(a*e + c*d*x)*e)/(sqrt(e)*sq
rt( - a*e**2 + c*d**2)))*a*c**3*d**4*e**4*x**2 + 315*sqrt(e)*sqrt(a*e + c*
d*x)*sqrt( - a*e**2 + c*d**2)*atan((sqrt(a*e + c*d*x)*e)/(sqrt(e)*sqrt( -
a*e**2 + c*d**2)))*a*c**3*d**3*e**5*x**3 + 315*sqrt(e)*sqrt(a*e + c*d*x)*s
qrt( - a*e**2 + c*d**2)*atan((sqrt(a*e + c*d*x)*e)/(sqrt(e)*sqrt( - a*e**2
 + c*d**2)))*c**4*d**7*e*x + 945*sqrt(e)*sqrt(a*e + c*d*x)*sqrt( - a*e**2
+ c*d**2)*atan((sqrt(a*e + c*d*x)*e)/(sqrt(e)*sqrt( - a*e**2 + c*d**2)))*c
**4*d**6*e**2*x**2 + 945*sqrt(e)*sqrt(a*e + c*d*x)*sqrt( - a*e**2 + c*d**2
)*atan((sqrt(a*e + c*d*x)*e)/(sqrt(e)*sqrt( - a*e**2 + c*d**2)))*c**4*d**5
*e**3*x**3 + 315*sqrt(e)*sqrt(a*e + c*d*x)*sqrt( - a*e**2 + c*d**2)*atan((
sqrt(a*e + c*d*x)*e)/(sqrt(e)*sqrt( - a*e**2 + c*d**2)))*c**4*d**4*e**4*x*
*4 - 8*a**5*e**10 + 58*a**4*c*d**2*e**8 + 18*a**4*c*d*e**9*x - 215*a**3*c*
*2*d**4*e**6 - 198*a**3*c**2*d**3*e**7*x - 63*a**3*c**2*d**2*e**8*x**2 - 4
3*a**2*c**3*d**6*e**4 - 774*a**2*c**3*d**5*e**5*x - 1071*a**2*c**3*d**4*e*
*6*x**2 - 420*a**2*c**3*d**3*e**7*x**3 + 224*a*c**4*d**8*e**2 + 810*a*c**4
*d**7*e**3*x + 441*a*c**4*d**6*e**4*x**2 - 420*a*c**4*d**5*e**5*x**3 - ...
```

3.328 $\int \frac{1}{\sqrt{d+ex}\sqrt{d^2-e^2x^2}}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 52

$$\int \frac{1}{\sqrt{d+ex}\sqrt{d^2-e^2x^2}}\,dx = -\frac{\sqrt{2}\text{arctanh}\left(\frac{\sqrt{d^2-e^2x^2}}{\sqrt{2}\sqrt{d}\sqrt{d+ex}}\right)}{\sqrt{d}e}$$

output
```
-2^(1/2)*arctanh(1/2*(-e^2*x^2+d^2)^(1/2)*2^(1/2)/d^(1/2)/(e*x+d)^(1/2))/d
^(1/2)/e
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.00

$$\int \frac{1}{\sqrt{d+ex}\sqrt{d^2-e^2x^2}}\,dx = -\frac{\sqrt{2}\text{arctanh}\left(\frac{\sqrt{2}\sqrt{d}\sqrt{d+ex}}{\sqrt{d^2-e^2x^2}}\right)}{\sqrt{d}e}$$

input
```
Integrate[1/(Sqrt[d + e*x]*Sqrt[d^2 - e^2*x^2]),x]
```

output
```
-((Sqrt[2]*ArcTanh[(Sqrt[2]*Sqrt[d]*Sqrt[d + e*x])/Sqrt[d^2 - e^2*x^2]])/(
Sqrt[d]*e))
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {471, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{d+ex}\sqrt{d^2-e^2x^2}}\,dx$$

$$\Big\downarrow \ 471$$

$$2e\int \frac{1}{\frac{e^2(d^2-e^2x^2)}{d+ex}-2de^2}d\frac{\sqrt{d^2-e^2x^2}}{\sqrt{d+ex}}$$

$$\Big\downarrow \ 221$$

$$-\frac{\sqrt{2}\text{arctanh}\left(\frac{\sqrt{d^2-e^2x^2}}{\sqrt{2}\sqrt{d}\sqrt{d+ex}}\right)}{\sqrt{d}e}$$

input
```
Int[1/(Sqrt[d + e*x]*Sqrt[d^2 - e^2*x^2]),x]
```

output
```
-((Sqrt[2]*ArcTanh[Sqrt[d^2 - e^2*x^2]/(Sqrt[2]*Sqrt[d]*Sqrt[d + e*x])])/(
Sqrt[d]*e))
```

Defintions of rubi rules used

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 471
```
Int[1/(Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> Sim
p[2*d   Subst[Int[1/(2*b*c + d^2*x^2), x], x, Sqrt[a + b*x^2]/Sqrt[c + d*x]
], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b*c^2 + a*d^2, 0]
```

Maple [A] (verified)

Time = 0.65 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.12

method	result	size
default	$-\frac{\sqrt{-e^{2} x^{2}+d^{2}}\, \sqrt{2}\, \operatorname{arctanh}\left(\frac{\sqrt{-e x +d}\, \sqrt{2}}{2 \sqrt{d}}\right)}{\sqrt{e x +d}\, \sqrt{-e x +d}\, e \sqrt{d}}$	58

input
```
int(1/(e*x+d)^(1/2)/(-e^2*x^2+d^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/(e*x+d)^(1/2)*(-e^2*x^2+d^2)^(1/2)/(-e*x+d)^(1/2)/e*2^(1/2)/d^(1/2)*arc
tanh(1/2*(-e*x+d)^(1/2)*2^(1/2)/d^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 144, normalized size of antiderivative = 2.77

$$\int \frac{1}{\sqrt{d+ex}\sqrt{d^2-e^2x^2}}\,dx = \left[\frac{\sqrt{2}\log\left(-\frac{e^2x^2-2\,dex+2\,\sqrt{2}\sqrt{-e^2x^2+d^2}\sqrt{ex+d}\sqrt{d}-3\,d^2}{e^2x^2+2\,dex+d^2}\right)}{2\,\sqrt{d}e}, -\frac{\sqrt{2}\sqrt{-\frac{1}{d}}\arctan\left(\frac{\sqrt{2}\sqrt{-e^2x^2+d^2}\sqrt{ex+d}d\sqrt{-\frac{1}{d}}}{e^2x^2-d^2}\right)}{e}\right]$$

input
```
integrate(1/(e*x+d)^(1/2)/(-e^2*x^2+d^2)^(1/2),x, algorithm="fricas")
```

output
```
[1/2*sqrt(2)*log(-(e^2*x^2 - 2*d*e*x + 2*sqrt(2)*sqrt(-e^2*x^2 + d^2)*sqrt
(e*x + d)*sqrt(d) - 3*d^2)/(e^2*x^2 + 2*d*e*x + d^2))/(sqrt(d)*e), -sqrt(2
)*sqrt(-1/d)*arctan(sqrt(2)*sqrt(-e^2*x^2 + d^2)*sqrt(e*x + d)*d*sqrt(-1/d
)/(e^2*x^2 - d^2))/e]
```

Sympy [F]

$$\int \frac{1}{\sqrt{d+ex}\sqrt{d^2-e^2x^2}}\,dx = \int \frac{1}{\sqrt{-\left(-d+ex\right)\left(d+ex\right)}\sqrt{d+ex}}\,dx$$

input
```
integrate(1/(e*x+d)**(1/2)/(-e**2*x**2+d**2)**(1/2),x)
```

output
```
Integral(1/(sqrt(-(-d + e*x)*(d + e*x))*sqrt(d + e*x)), x)
```

Maxima [F]

$$\int \frac{1}{\sqrt{d+ex}\sqrt{d^2-e^2x^2}}\,dx = \int \frac{1}{\sqrt{-e^2x^2+d^2}\sqrt{ex+d}}\,dx$$

input
```
integrate(1/(e*x+d)^(1/2)/(-e^2*x^2+d^2)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/(sqrt(-e^2*x^2 + d^2)*sqrt(e*x + d)), x)
```

Giac [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.60

$$\int \frac{1}{\sqrt{d+ex}\sqrt{d^2-e^2x^2}}\,dx = \frac{\sqrt{2}\arctan\left(\frac{\sqrt{2}\sqrt{-ex+d}}{2\sqrt{-d}}\right)}{\sqrt{-d}e}$$

input
```
integrate(1/(e*x+d)^(1/2)/(-e^2*x^2+d^2)^(1/2),x, algorithm="giac")
```

output
```
sqrt(2)*arctan(1/2*sqrt(2)*sqrt(-e*x + d)/sqrt(-d))/(sqrt(-d)*e)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{d+ex}\sqrt{d^2-e^2x^2}}\,dx=\int \frac{1}{\sqrt{d^2-e^2\,x^2}\,\sqrt{d+e\,x}}\,dx$$

input
```
int(1/((d^2 - e^2*x^2)^(1/2)*(d + e*x)^(1/2)),x)
```

output
```
int(1/((d^2 - e^2*x^2)^(1/2)*(d + e*x)^(1/2)), x)
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.85

$$\int \frac{1}{\sqrt{d+ex}\sqrt{d^2-e^2x^2}}\,dx = \frac{\sqrt{d}\,\sqrt{2}\left(\log\left(\sqrt{-ex+d}-\sqrt{d}\,\sqrt{2}\right)-\log\left(\sqrt{-ex+d}+\sqrt{d}\,\sqrt{2}\right)\right)}{2de}$$

input
```
int(1/(e*x+d)^(1/2)/(-e^2*x^2+d^2)^(1/2),x)
```

output
```
(sqrt(d)*sqrt(2)*(log(sqrt(d - e*x) - sqrt(d)*sqrt(2)) - log(sqrt(d - e*x)
 + sqrt(d)*sqrt(2))))/(2*d*e)
```

3.329 $\int \frac{1}{\sqrt{-d+ex}\sqrt{d^2-e^2x^2}}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 53

$$\int \frac{1}{\sqrt{-d+ex}\sqrt{d^2-e^2x^2}}\,dx = \frac{\sqrt{2}\arctan\left(\frac{\sqrt{d^2-e^2x^2}}{\sqrt{2}\sqrt{d}\sqrt{-d+ex}}\right)}{\sqrt{d}e}$$

output
```
2^(1/2)*arctan(1/2*(-e^2*x^2+d^2)^(1/2)*2^(1/2)/d^(1/2)/(e*x-d)^(1/2))/d^(
1/2)/e
```

Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.02

$$\int \frac{1}{\sqrt{-d+ex}\sqrt{d^2-e^2x^2}}\,dx = -\frac{\sqrt{2}\arctan\left(\frac{\sqrt{2}\sqrt{d}\sqrt{-d+ex}}{\sqrt{d^2-e^2x^2}}\right)}{\sqrt{d}e}$$

input
```
Integrate[1/(Sqrt[-d + e*x]*Sqrt[d^2 - e^2*x^2]),x]
```

output
```
-((Sqrt[2]*ArcTan[(Sqrt[2]*Sqrt[d]*Sqrt[-d + e*x])/Sqrt[d^2 - e^2*x^2]])/(
Sqrt[d]*e))
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071, Rules used = {471, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{ex-d}\sqrt{d^2-e^2x^2}}dx$$

$$\Big\downarrow \ 471$$

$$2e\int \frac{1}{2de^2+\frac{(d^2-e^2x^2)e^2}{ex-d}}d\frac{\sqrt{d^2-e^2x^2}}{\sqrt{ex-d}}$$

$$\Big\downarrow \ 218$$

$$\frac{\sqrt{2}\arctan\left(\frac{\sqrt{d^2-e^2x^2}}{\sqrt{2}\sqrt{d}\sqrt{ex-d}}\right)}{\sqrt{d}e}$$

input
```
Int[1/(Sqrt[-d + e*x]*Sqrt[d^2 - e^2*x^2]),x]
```

output
```
(Sqrt[2]*ArcTan[Sqrt[d^2 - e^2*x^2]/(Sqrt[2]*Sqrt[d]*Sqrt[-d + e*x])])/(Sq
rt[d]*e)
```

Defintions of rubi rules used

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 471
```
Int[1/(Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> Sim
p[2*d   Subst[Int[1/(2*b*c + d^2*x^2), x], x, Sqrt[a + b*x^2]/Sqrt[c + d*x]
], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b*c^2 + a*d^2, 0]
```

Maple [A] (verified)

Time = 0.64 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.19

method	result	size
default	$\frac{\sqrt{-e^{2} x^{2}+d^{2}}\, \sqrt{2}\, \arctan\left(\frac{\sqrt{-e x -d}\, \sqrt{2}}{2 \sqrt{d}}\right)}{\sqrt{e x -d}\, e \sqrt{-e x -d}\, \sqrt{d}}$	63

input
```
int(1/(e*x-d)^(1/2)/(-e^2*x^2+d^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/(e*x-d)^(1/2)/e*(-e^2*x^2+d^2)^(1/2)/(-e*x-d)^(1/2)*2^(1/2)/d^(1/2)*arct
an(1/2*(-e*x-d)^(1/2)*2^(1/2)/d^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 147, normalized size of antiderivative = 2.77

$$\int \frac{1}{\sqrt{-d+ex}\sqrt{d^2-e^2x^2}}\,dx = \left[\frac{\sqrt{2}\sqrt{-\frac{1}{d}}\log\left(-\frac{e^{2} x^{2}+2\,dex-2\,\sqrt{2}\sqrt{-e^{2} x^{2}+d^{2}}\sqrt{ex-d}d\sqrt{-\frac{1}{d}}-3\,d^{2}}{e^{2} x^{2}-2\,dex+d^{2}}\right)}{2\,e}, \frac{\sqrt{2}\arctan\left(\frac{\sqrt{2}\sqrt{-e^{2} x^{2}+d^{2}}\sqrt{ex-d}\sqrt{d}}{e^{2} x^{2}-d^{2}}\right)}{\sqrt{d}e}\right]$$

input
```
integrate(1/(e*x-d)^(1/2)/(-e^2*x^2+d^2)^(1/2),x, algorithm="fricas")
```

output
```
[1/2*sqrt(2)*sqrt(-1/d)*log(-(e^2*x^2 + 2*d*e*x - 2*sqrt(2)*sqrt(-e^2*x^2
+ d^2)*sqrt(e*x - d)*d*sqrt(-1/d) - 3*d^2)/(e^2*x^2 - 2*d*e*x + d^2))/e, s
qrt(2)*arctan(sqrt(2)*sqrt(-e^2*x^2 + d^2)*sqrt(e*x - d)*sqrt(d)/(e^2*x^2
- d^2))/(sqrt(d)*e)]
```

Sympy [F]

$$\int \frac{1}{\sqrt{-d+ex}\sqrt{d^2-e^2x^2}}\,dx = \int \frac{1}{\sqrt{-(-d+ex)(d+ex)}\sqrt{-d+ex}}\,dx$$

input
```
integrate(1/(e*x-d)**(1/2)/(-e**2*x**2+d**2)**(1/2),x)
```

output
```
Integral(1/(sqrt(-(-d + e*x)*(d + e*x))*sqrt(-d + e*x)), x)
```

Maxima [F]

$$\int \frac{1}{\sqrt{-d+ex}\sqrt{d^2-e^2x^2}}\,dx = \int \frac{1}{\sqrt{-e^2x^2+d^2}\sqrt{ex-d}}\,dx$$

input
```
integrate(1/(e*x-d)^(1/2)/(-e^2*x^2+d^2)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/(sqrt(-e^2*x^2 + d^2)*sqrt(e*x - d)), x)
```

Giac [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.55

$$\int \frac{1}{\sqrt{-d+ex}\sqrt{d^2-e^2x^2}}\,dx = \frac{\sqrt{2}\arctan\left(\frac{\sqrt{2}\sqrt{-ex-d}}{2\sqrt{d}}\right)}{\sqrt{d}e}$$

input
```
integrate(1/(e*x-d)^(1/2)/(-e^2*x^2+d^2)^(1/2),x, algorithm="giac")
```

output
```
sqrt(2)*arctan(1/2*sqrt(2)*sqrt(-e*x - d)/sqrt(d))/(sqrt(d)*e)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{-d+ex}\sqrt{d^2-e^2x^2}}\,dx = \int \frac{1}{\sqrt{d^2-e^2\,x^2}\,\sqrt{e\,x-d}}\,dx$$

input
```
int(1/((d^2 - e^2*x^2)^(1/2)*(e*x - d)^(1/2)),x)
```

output
```
int(1/((d^2 - e^2*x^2)^(1/2)*(e*x - d)^(1/2)), x)
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.53

$$\int \frac{1}{\sqrt{-d+ex}\sqrt{d^2-e^2x^2}}\,dx = \frac{\sqrt{d}\,\sqrt{2}\,atan\left(\frac{\sqrt{ex+d}\,i}{\sqrt{d}\,\sqrt{2}}\right)}{de}$$

input
```
int(1/(e*x-d)^(1/2)/(-e^2*x^2+d^2)^(1/2),x)
```

output
```
(sqrt(d)*sqrt(2)*atan((sqrt(d + e*x)*i)/(sqrt(d)*sqrt(2))))/(d*e)
```

3.330 $\int \frac{(d+ex)^{2/3}}{\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\,dx$

Optimal result

Integrand size = 39, antiderivative size = 502

$$\int \frac{(d+ex)^{2/3}}{\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\,dx = \frac{3\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{2cd\sqrt[3]{d+ex}}$$

$$+\frac{3^{3/4}(cd^2-ae^2)^{2/3}(d+ex)^{2/3}\left(\sqrt[3]{cd^2-ae^2}-\sqrt[3]{c}\sqrt[3]{d}\sqrt[3]{d+ex}\right)\sqrt{\frac{(cd^2-ae^2)^{2/3}+\sqrt[3]{c}\sqrt[3]{d}\sqrt[3]{cd^2-ae^2}\sqrt[3]{d+ex}}{\left(\sqrt[3]{cd^2-ae^2}-\left(1+\sqrt{3}\right)\sqrt[3]{c}\sqrt[3]{d}\sqrt[3]{d}\right.}}}{4cde\sqrt{ade+(cd^2+ae^2)x+cdex^2}\sqrt{-\frac{\sqrt[3]{c}\sqrt[3]{d}\sqrt[3]{d+}}{\left(\sqrt[3]{cd^2-\right.}}}}$$

output
```
3/2*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/c/d/(e*x+d)^(1/3)+1/4*3^(3/4)*
(-a*e^2+c*d^2)^(2/3)*(e*x+d)^(2/3)*((-a*e^2+c*d^2)^(1/3)-c^(1/3)*d^(1/3)*(
e*x+d)^(1/3))*(((-a*e^2+c*d^2)^(2/3)+c^(1/3)*d^(1/3)*(-a*e^2+c*d^2)^(1/3)*
(e*x+d)^(1/3)+c^(2/3)*d^(2/3)*(e*x+d)^(2/3))/((-a*e^2+c*d^2)^(1/3)-(1+3^(1
/2))*c^(1/3)*d^(1/3)*(e*x+d)^(1/3))^2)^(1/2)*InverseJacobiAM(arccos(((-a*e
^2+c*d^2)^(1/3)-(1-3^(1/2))*c^(1/3)*d^(1/3)*(e*x+d)^(1/3))/((-a*e^2+c*d^2)
^(1/3)-(1+3^(1/2))*c^(1/3)*d^(1/3)*(e*x+d)^(1/3))),1/4*6^(1/2)+1/4*2^(1/2)
)/c/d/e/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(-c^(1/3)*d^(1/3)*(e*x+d)^
(1/3)*((-a*e^2+c*d^2)^(1/3)-c^(1/3)*d^(1/3)*(e*x+d)^(1/3))/((-a*e^2+c*d^2)
^(1/3)-(1+3^(1/2))*c^(1/3)*d^(1/3)*(e*x+d)^(1/3))^2)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.07 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.19

$$\int \frac{(d+ex)^{2/3}}{\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}\,dx=\frac{2\sqrt{(ae+cdx)(d+ex)}\,\text{Hypergeometric2F1}\left(-\frac{1}{6},\frac{1}{2},\frac{3}{2},\frac{e(ae+cdx)}{-cd^2+ae^2}\right)}{cd\sqrt[3]{d+ex}\sqrt[6]{\frac{cd(d+ex)}{cd^2-ae^2}}}$$

input
```
Integrate[(d + e*x)^(2/3)/Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2],x]
```

output
```
(2*Sqrt[(a*e + c*d*x)*(d + e*x)]*Hypergeometric2F1[-1/6, 1/2, 3/2, (e*(a*e
 + c*d*x))/(-(c*d^2) + a*e^2)])/(c*d*(d + e*x)^(1/3)*((c*d*(d + e*x))/(c*d
^2 - a*e^2))^(1/6))
```

Rubi [A] (warning: unable to verify)

Time = 0.97 (sec) , antiderivative size = 604, normalized size of antiderivative = 1.20, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.128$, Rules used = {1139, 1138, 60, 73, 766}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex)^{2/3}}{\sqrt{x\,(ae^2+cd^2)+ade+cdex^2}}dx$$

$$\Big\downarrow \text{1139}$$

$$\frac{(d+ex)^{2/3}\int \frac{\left(\frac{ex}{d}+1\right)^{2/3}}{\sqrt{cdex^2+(cd^2+ae^2)x+ade}}dx}{\left(\frac{ex}{d}+1\right)^{2/3}}$$

$$\Big\downarrow \text{1138}$$

$$\frac{(d+ex)^{2/3}\sqrt{ade+cd^2x}\int \frac{\sqrt[6]{\frac{ex}{d}+1}}{\sqrt{cxd^2+aed}}dx}{\sqrt[6]{\frac{ex}{d}+1}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}$$

$\Big\downarrow$ 60

$$\frac{(d+ex)^{2/3}\sqrt{ade+cd^2x}\left(\frac{1}{4}\left(1-\frac{ae^2}{cd^2}\right)\int \frac{1}{\sqrt{cxd^2+aed}\left(\frac{ex}{d}+1\right)^{5/6}}dx+\frac{3\sqrt[6]{\frac{ex}{d}+1}\sqrt{ade+cd^2x}}{2cd^2}\right)}{\sqrt[6]{\frac{ex}{d}+1}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}$$

$\Big\downarrow$ 73

$$\frac{(d+ex)^{2/3}\sqrt{ade+cd^2x}\left(\frac{3d\left(1-\frac{ae^2}{cd^2}\right)\int \frac{1}{\sqrt{\frac{cd^3\left(\frac{ex}{d}+1\right)}{e}-d\left(\frac{cd^2}{e}-ae\right)}}d\sqrt[6]{\frac{ex}{d}+1}}{2e}+\frac{3\sqrt[6]{\frac{ex}{d}+1}\sqrt{ade+cd^2x}}{2cd^2}\right)}{\sqrt[6]{\frac{ex}{d}+1}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}$$

$\Big\downarrow$ 766

$$\frac{(d+ex)^{2/3}\sqrt{ade+cd^2x}\left(\frac{3^{3/4}d\sqrt[6]{\frac{ex}{d}+1}\left(1-\frac{ae^2}{cd^2}\right)\left(\sqrt[3]{cd^2-ae^2}-\sqrt[3]{c}d^{2/3}\sqrt[3]{\frac{ex}{d}+1}\right)\sqrt{\frac{\left(cd^2-ae^2\right)^{2/3}+\sqrt[3]{c}d^{2/3}\sqrt[3]{\frac{ex}{d}+1}}{\left(\sqrt[3]{cd^2-ae^2}-\left(1+\sqrt{3}\right.\right.}}}{4e\sqrt[3]{cd^2-ae^2}\sqrt{-\frac{\sqrt[3]{c}d^{2/3}\sqrt[3]{\frac{ex}{d}+1}\left(\sqrt[3]{cd^2}\right.}{\left(\sqrt[3]{cd^2-ae^2}-\left(1+\sqrt{\right.\right.}}}}\right.}{\sqrt[6]{\frac{ex}{d}+1}\sqrt{x\left(ae^2\right.}}$$

input
```
Int[(d + e*x)^(2/3)/Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2],x]
```

output
```
(Sqrt[a*d*e + c*d^2*x]*(d + e*x)^(2/3)*((3*Sqrt[a*d*e + c*d^2*x]*(1 + (e*x
)/d)^(1/6))/(2*c*d^2) + (3^(3/4)*d*(1 - (a*e^2)/(c*d^2))*(1 + (e*x)/d)^(1/
6)*((c*d^2 - a*e^2)^(1/3) - c^(1/3)*d^(2/3)*(1 + (e*x)/d)^(1/3))*Sqrt[((c*
d^2 - a*e^2)^(2/3) + c^(1/3)*d^(2/3)*(c*d^2 - a*e^2)^(1/3)*(1 + (e*x)/d)^(
1/3) + c^(2/3)*d^(4/3)*(1 + (e*x)/d)^(2/3))/((c*d^2 - a*e^2)^(1/3) - (1 +
Sqrt[3])*c^(1/3)*d^(2/3)*(1 + (e*x)/d)^(1/3))^2]*EllipticF[ArcCos[((c*d^2
- a*e^2)^(1/3) - (1 - Sqrt[3])*c^(1/3)*d^(2/3)*(1 + (e*x)/d)^(1/3))/((c*d^
2 - a*e^2)^(1/3) - (1 + Sqrt[3])*c^(1/3)*d^(2/3)*(1 + (e*x)/d)^(1/3))], (2
 + Sqrt[3])/4])/(4*e*(c*d^2 - a*e^2)^(1/3)*Sqrt[-((c^(1/3)*d^(2/3)*(1 + (e
*x)/d)^(1/3)*((c*d^2 - a*e^2)^(1/3) - c^(1/3)*d^(2/3)*(1 + (e*x)/d)^(1/3))
)/((c*d^2 - a*e^2)^(1/3) - (1 + Sqrt[3])*c^(1/3)*d^(2/3)*(1 + (e*x)/d)^(1/
3))^2)]*Sqrt[-(d*((c*d^2)/e - a*e)) + (c*d^3*(1 + (e*x)/d))/e])))/((1 + (e
*x)/d)^(1/6)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])
```

Defintions of rubi rules used

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 766
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[x*(s + r*x^2)*(Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/
(s + (1 + Sqrt[3])*r*x^2)^2]/(2*3^(1/4)*s*Sqrt[a + b*x^6]*Sqrt[r*x^2*((s +
r*x^2)/(s + (1 + Sqrt[3])*r*x^2)^2)]))*EllipticF[ArcCos[(s + (1 - Sqrt[3])*
r*x^2)/(s + (1 + Sqrt[3])*r*x^2)], (2 + Sqrt[3])/4], x]] /; FreeQ[{a, b}, x
]
```

rule 1138
```
Int[((d_) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Sy
mbol] :> Simp[d^m*((a + b*x + c*x^2)^FracPart[p]/((1 + e*(x/d))^FracPart[p]
*(a/d + (c*x)/e)^FracPart[p]))   Int[(1 + e*(x/d))^(m + p)*(a/d + (c/e)*x)^
p, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0]
&& (IntegerQ[m] || GtQ[d, 0]) &&  !(IGtQ[m, 0] && (IntegerQ[3*p] || Integer
Q[4*p]))
```

rule 1139
```
Int[((d_) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Sy
mbol] :> Simp[d^IntPart[m]*((d + e*x)^FracPart[m]/(1 + e*(x/d))^FracPart[m]
)   Int[(1 + e*(x/d))^m*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e
, m}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&  !(IntegerQ[m] || GtQ[d, 0])
```

Maple [F]

$$\int \frac{(ex+d)^{\frac{2}{3}}}{\sqrt{ade+(a\,e^2+c\,d^2)\,x+cd\,x^2e}}dx$$

input
```
int((e*x+d)^(2/3)/(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(1/2),x)
```

output
```
int((e*x+d)^(2/3)/(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(1/2),x)
```

Fricas [F]

$$\int \frac{(d+ex)^{2/3}}{\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}\,dx = \int \frac{(ex+d)^{\frac{2}{3}}}{\sqrt{cdex^2+ade+(cd^2+ae^2)x}}\,dx$$

input
```
integrate((e*x+d)^(2/3)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x, algorit
hm="fricas")
```

output
```
integral((e*x + d)^(2/3)/sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x), x)
```

Sympy [F]

$$\int \frac{(d+ex)^{2/3}}{\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}\,dx = \int \frac{(d+ex)^{\frac{2}{3}}}{\sqrt{(d+ex)\,(ae+cdx)}}\,dx$$

input
```
integrate((e*x+d)**(2/3)/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(1/2),x)
```

output
```
Integral((d + e*x)**(2/3)/sqrt((d + e*x)*(a*e + c*d*x)), x)
```

Maxima [F]

$$\int \frac{(d+ex)^{2/3}}{\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}\,dx = \int \frac{(ex+d)^{\frac{2}{3}}}{\sqrt{cdex^2+ade+(cd^2+ae^2)x}}\,dx$$

input
```
integrate((e*x+d)^(2/3)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x, algorit
hm="maxima")
```

output
```
integrate((e*x + d)^(2/3)/sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x), x)
```

Giac [F]

$$\int \frac{(d+ex)^{2/3}}{\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}\,dx = \int \frac{(ex+d)^{\frac{2}{3}}}{\sqrt{cdex^2+ade+(cd^2+ae^2)x}}\,dx$$

input
```
integrate((e*x+d)^(2/3)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x, algorit
hm="giac")
```

output
```
integrate((e*x + d)^(2/3)/sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d+ex)^{2/3}}{\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}\,dx = \int \frac{(d+e\,x)^{2/3}}{\sqrt{c\,d\,e\,x^2+(c\,d^2+a\,e^2)\,x+a\,d\,e}}\,dx$$

input
```
int((d + e*x)^(2/3)/(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2),x)
```

output
```
int((d + e*x)^(2/3)/(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2), x)
```

Reduce [F]

$$\int \frac{(d+ex)^{2/3}}{\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}\,dx = \frac{6(ex+d)^{\frac{1}{6}}\sqrt{cdx+ae}-\left(\int \frac{(ex+d)^{\frac{1}{6}}\sqrt{cdx+ae}}{cde\,x^2+a\,e^2x+c\,d^2x+ade}dx\right)a\,e^2+\left(\int \frac{(}{cde\,x}\right.}{4cd}$$

input
```
int((e*x+d)^(2/3)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x)
```

output
```
(6*(d + e*x)**(1/6)*sqrt(a*e + c*d*x) - int(((d + e*x)**(1/6)*sqrt(a*e + c
*d*x))/(a*d*e + a*e**2*x + c*d**2*x + c*d*e*x**2),x)*a*e**2 + int(((d + e*
x)**(1/6)*sqrt(a*e + c*d*x))/(a*d*e + a*e**2*x + c*d**2*x + c*d*e*x**2),x)
*c*d**2)/(4*c*d)
```

3.331 $\int \frac{(d+ex)^3}{\sqrt[3]{ade+(cd^2+ae^2)\,x+cdex^2}}\,dx$

Optimal result

Integrand size = 37, antiderivative size = 84

$$\int \frac{(d+ex)^3}{\sqrt[3]{ade+(cd^2+ae^2)\,x+cdex^2}}\,dx = -\frac{3(d+ex)^3\left(ade+(cd^2+ae^2)\,x+cdex^2\right)^{2/3}\text{Hypergeometric2F1}\left(1,\frac{13}{3},\frac{14}{3},\frac{cd(d+ex)}{cd^2-ae^2}\right)}{11\,(cd^2-ae^2)}$$

output
```
-3*(e*x+d)^3*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(2/3)*hypergeom([1, 13/3],[
14/3],c*d*(e*x+d)/(-a*e^2+c*d^2))/(-11*a*e^2+11*c*d^2)
```

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 102, normalized size of antiderivative = 1.21

$$\int \frac{(d+ex)^3}{\sqrt[3]{ade+(cd^2+ae^2)\,x+cdex^2}}\,dx = \frac{3(cd^2-ae^2)^2\,((ae+cdx)(d+ex))^{2/3}\,\text{Hypergeometric2F1}\left(-\frac{8}{3},\frac{2}{3},\frac{5}{3},\frac{e(ae+cdx)}{-cd^2+ae^2}\right)}{2c^3d^3\left(\frac{cd(d+ex)}{cd^2-ae^2}\right)^{2/3}}$$

input
```
Integrate[(d + e*x)^3/(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(1/3),x]
```

output
```
(3*(c*d^2 - a*e^2)^2*((a*e + c*d*x)*(d + e*x))^(2/3)*Hypergeometric2F1[-8/
3, 2/3, 5/3, (e*(a*e + c*d*x))/(-(c*d^2) + a*e^2)])/(2*c^3*d^3*((c*d*(d +
e*x))/(c*d^2 - a*e^2))^(2/3))
```

Rubi [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 1855 vs. $2(84) = 168$.

Time = 2.38 (sec) , antiderivative size = 1855, normalized size of antiderivative = 22.08, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.243$, Rules used = {1166, 27, 1166, 27, 1160, 1095, 832, 759, 2416}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex)^3}{\sqrt[3]{x(ae^2+cd^2)+ade+cdex^2}}dx$$

$$\Big\downarrow \text{1166}$$

$$\frac{3\int \frac{8e(cd^2-ae^2)(d+ex)^2}{3\sqrt[3]{cdex^2+(cd^2+ae^2)x+ade}}dx}{10cde}+\frac{3(d+ex)^2\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{2/3}}{10cd}$$

$$\Big\downarrow \text{27}$$

$$\frac{4\left(cd^2-ae^2\right)\int \frac{(d+ex)^2}{\sqrt[3]{cdex^2+(cd^2+ae^2)x+ade}}dx}{5cd}+\frac{3(d+ex)^2\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{2/3}}{10cd}$$

$$\Big\downarrow \text{1166}$$

$$\frac{4\left(cd^2-ae^2\right)\left(\frac{3\int \frac{5e(cd^2-ae^2)(d+ex)}{3\sqrt[3]{cdex^2+(cd^2+ae^2)x+ade}}dx}{7cde}+\frac{3(d+ex)\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{2/3}}{7cd}\right)}{5cd}+\frac{3(d+ex)^2\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{2/3}}{10cd}$$

$\Big\downarrow$ 27

$$\frac{4\left(cd^2-ae^2\right)\left(\frac{5\left(cd^2-ae^2\right)\int \frac{d+ex}{\sqrt[3]{cdex^2+\left(cd^2+ae^2\right)x+ade}}dx}{7cd}+\frac{3(d+ex)\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{2/3}}{7cd}\right)}{5cd}+\frac{3(d+ex)^2\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{2/3}}{10cd}$$

$\Big\downarrow$ 1160

$$\frac{4\left(cd^2-ae^2\right)\left(\frac{5\left(cd^2-ae^2\right)\left(\frac{\left(d^2-\frac{ae^2}{c}\right)\int \frac{1}{\sqrt[3]{cdex^2+\left(cd^2+ae^2\right)x+ade}}dx}{2d}+\frac{3\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{2/3}}{4cd}\right)}{7cd}+\frac{3(d+ex)\left(x\left(ae^2\right.\right.}{}\right.}{5cd}+\frac{3(d+ex)^2\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{2/3}}{10cd}$$

$\Big\downarrow$ 1095

$$\frac{4\left(cd^2-ae^2\right)\left(\frac{5\left(cd^2-ae^2\right)\left(\frac{3\left(d^2-\frac{ae^2}{c}\right)\sqrt{\left(ae^2+cd^2+2cdex\right)^2}\int \frac{\sqrt[3]{cdex^2+\left(cd^2+ae^2\right)x+ade}}{\sqrt{\left(cd^2-ae^2\right)^2+4cde\left(cdex^2+\left(cd^2+ae^2\right)x+ade\right)}}d\sqrt[3]{cdex^2+\left(cd^2+ae^2\right)x}}{2d\left(ae^2+cd^2+2cdex\right)}\right.}{7cd}\right.}{5cd}$$

$$\frac{3(d+ex)^2\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{2/3}}{10cd}$$

$\Big\downarrow$ 832

$$4\left(cd^2-ae^2\right)\left(\frac{5\left(cd^2-ae^2\right)\left(\frac{3\left(d^2-\frac{ae^2}{c}\right)\sqrt{\left(ae^2+cd^2+2cdex\right)^2}\left(\frac{\int\frac{\left(1-\sqrt{3}\right)\left(cd^2-ae^2\right)^{2/3}+2^{2/3}\sqrt[3]{c}\sqrt[3]{d}\sqrt[3]{e}\sqrt[3]{cdex^2+(cd^2+ae^2)x+}}{\sqrt{\left(cd^2-ae^2\right)^2+4cde\left(cdex^2+\left(cd^2+ae^2\right)x+ade\right)}}}{2^{2/3}\sqrt[3]{c}\sqrt[3]{d}\sqrt[3]{e}}\right.}{}\right.}{}\right.$$

$$\frac{3(d+ex)^2\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{2/3}}{10cd}$$

$\downarrow$ 759

$$\frac{3\left(cdex^2+\left(cd^2+ae^2\right)x+ade\right)^{2/3}(d+ex)^2}{10cd}+$$

$$4\left(cd^2-ae^2\right)\left(\frac{3\left(cdex^2+\left(cd^2+ae^2\right)x+ade\right)^{2/3}(d+ex)}{7cd}+\frac{5\left(cd^2-ae^2\right)\left(\frac{3\left(d^2-\frac{ae^2}{c}\right)\sqrt{\left(cd^2+2cexd+ae^2\right)^2}\left(\int\frac{\left(1-\sqrt{3}\right)\left(cd^2-ae^2\right)^{2/3}+2^{2/3}\sqrt[3]{c}}{\sqrt{\left(cd^2-ae^2\right)^2}}\right.}{}\right.}{}\right.$$

$\downarrow$ 2416

$$\frac{3\left(cdex^2+\left(cd^2+ae^2\right)x+ade\right)^{2/3}(d+ex)^2}{10cd}+$$

$$\sqrt[3]{2}\sqrt{(c\ldots}$$

$$\sqrt[3]{c}\sqrt[3]{d}\sqrt[3]{e}\left(\left(1+\sqrt{3}\right)\left(cd^2-ae^2\right)\ldots\right.$$

$$3\left(d^2-\frac{ae^2}{c}\right)\sqrt{\left(cd^2+2cexd+ae^2\right)^2}$$

$$5\left(cd^2-ae^2\right)$$

$$4\left(cd^2-ae^2\right)$$

$$\frac{3\left(cdex^2+\left(cd^2+ae^2\right)x+ade\right)^{2/3}(d+ex)}{7cd}+$$

input
```
Int[(d + e*x)^3/(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(1/3),x]
```

output
```
(3*(d + e*x)^2*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(2/3))/(10*c*d) + (
4*(c*d^2 - a*e^2)*((3*(d + e*x)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(2
/3))/(7*c*d) + (5*(c*d^2 - a*e^2)*((3*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x
^2)^(2/3))/(4*c*d) + (3*(d^2 - (a*e^2)/c)*Sqrt[(c*d^2 + a*e^2 + 2*c*d*e*x)
^2]*(((2^(1/3)*Sqrt[(c*d^2 - a*e^2)^2 + 4*c*d*e*(a*d*e + (c*d^2 + a*e^2)*x
 + c*d*e*x^2)])/(c^(1/3)*d^(1/3)*e^(1/3)*((1 + Sqrt[3])*(c*d^2 - a*e^2)^(2
/3) + 2^(2/3)*c^(1/3)*d^(1/3)*e^(1/3)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x
^2)^(1/3))) - (3^(1/4)*Sqrt[2 - Sqrt[3]]*(c*d^2 - a*e^2)^(2/3)*((c*d^2 - a
*e^2)^(2/3) + 2^(2/3)*c^(1/3)*d^(1/3)*e^(1/3)*(a*d*e + (c*d^2 + a*e^2)*x +
 c*d*e*x^2)^(1/3))*Sqrt[((c*d^2 - a*e^2)^(4/3) - 2^(2/3)*c^(1/3)*d^(1/3)*e
^(1/3)*(c*d^2 - a*e^2)^(2/3)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(1/3)
 + 2*2^(1/3)*c^(2/3)*d^(2/3)*e^(2/3)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^
2)^(2/3))/((1 + Sqrt[3])*(c*d^2 - a*e^2)^(2/3) + 2^(2/3)*c^(1/3)*d^(1/3)*e
^(1/3)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(1/3))^2]*EllipticE[ArcSin[
((1 - Sqrt[3])*(c*d^2 - a*e^2)^(2/3) + 2^(2/3)*c^(1/3)*d^(1/3)*e^(1/3)*(a*
d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(1/3))/((1 + Sqrt[3])*(c*d^2 - a*e^2)
^(2/3) + 2^(2/3)*c^(1/3)*d^(1/3)*e^(1/3)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*
e*x^2)^(1/3))], -7 - 4*Sqrt[3]])/(2^(2/3)*c^(1/3)*d^(1/3)*e^(1/3)*Sqrt[((c
*d^2 - a*e^2)^(2/3)*((c*d^2 - a*e^2)^(2/3) + 2^(2/3)*c^(1/3)*d^(1/3)*e^(1/
3)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(1/3)))/((1 + Sqrt[3])*(c*d^...
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 759
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

rule 832
```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Simp[(-(1 - Sqrt[3]))*(s/r)   Int[1/Sqrt[a + b*x
^3], x], x] + Simp[1/r   Int[((1 - Sqrt[3])*s + r*x)/Sqrt[a + b*x^3], x], x
]] /; FreeQ[{a, b}, x] && PosQ[a]
```

rule 1095
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[3*(Sqrt[(b
+ 2*c*x)^2]/(b + 2*c*x))   Subst[Int[x^(3*(p + 1) - 1)/Sqrt[b^2 - 4*a*c + 4
*c*x^3], x], x, (a + b*x + c*x^2)^(1/3)], x] /; FreeQ[{a, b, c}, x] && Inte
gerQ[3*p]
```

rule 1160
```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[e*((a + b*x + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Simp[(2*c*d - b
*e)/(2*c)   Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
 && NeQ[p, -1]
```

rule 1166
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[e*(d + e*x)^(m - 1)*((a + b*x + c*x^2)^(p + 1)/(c*(m + 2*p +
 1))), x] + Simp[1/(c*(m + 2*p + 1))   Int[(d + e*x)^(m - 2)*Simp[c*d^2*(m
+ 2*p + 1) - e*(a*e*(m - 1) + b*d*(p + 1)) + e*(2*c*d - b*e)*(m + p)*x, x]*
(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && If[Ration
alQ[m], GtQ[m, 1], SumSimplerQ[m, -2]] && NeQ[m + 2*p + 1, 0] && IntQuadrat
icQ[a, b, c, d, e, m, p, x]
```

rule 2416
```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 - Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 - Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 + Sqrt[3])*s + r*x))), x] - S
imp[3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[s*((s + r*x)/((1 + Sqrt
[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sqrt[3])
*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Maple [F]

$$\int \frac{(ex+d)^3}{(ade+(a\,e^2+c\,d^2)\,x+cd\,x^2e)^{\frac{1}{3}}}dx$$

input
```
int((e*x+d)^3/(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(1/3),x)
```

output
```
int((e*x+d)^3/(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(1/3),x)
```

Fricas [F]

$$\int \frac{(d+ex)^3}{\sqrt[3]{ade+(cd^2+ae^2)\,x+cdex^2}}\,dx = \int \frac{(ex+d)^3}{(cdex^2+ade+(cd^2+ae^2)x)^{\frac{1}{3}}}\,dx$$

input
```
integrate((e*x+d)^3/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/3),x, algorithm="
fricas")
```

output
```
integral((c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^(2/3)*(e^2*x^2 + 2*d*e*x
+ d^2)/(c*d*x + a*e), x)
```

Sympy [F]

$$\int \frac{(d+ex)^3}{\sqrt[3]{ade+(cd^2+ae^2)\,x+cdex^2}}\,dx = \int \frac{(d+ex)^3}{\sqrt[3]{(d+ex)\,(ae+cdx)}}\,dx$$

input
```
integrate((e*x+d)**3/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(1/3),x)
```

output
```
Integral((d + e*x)**3/((d + e*x)*(a*e + c*d*x))**(1/3), x)
```

Maxima [F]

$$\int \frac{(d+ex)^3}{\sqrt[3]{ade+(cd^2+ae^2)\,x+cdex^2}}\,dx=\int \frac{(ex+d)^3}{(cdex^2+ade+(cd^2+ae^2)x)^{\frac{1}{3}}}\,dx$$

input
```
integrate((e*x+d)^3/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/3),x, algorithm="
maxima")
```

output
```
integrate((e*x + d)^3/(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^(1/3), x)
```

Giac [F]

$$\int \frac{(d+ex)^3}{\sqrt[3]{ade+(cd^2+ae^2)\,x+cdex^2}}\,dx=\int \frac{(ex+d)^3}{(cdex^2+ade+(cd^2+ae^2)x)^{\frac{1}{3}}}\,dx$$

input
```
integrate((e*x+d)^3/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/3),x, algorithm="
giac")
```

output
```
integrate((e*x + d)^3/(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^(1/3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d+ex)^3}{\sqrt[3]{ade+(cd^2+ae^2)\,x+cdex^2}}\,dx=\int \frac{(d+e\,x)^3}{\left(c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e\right)^{1/3}}\,dx$$

input
```
int((d + e*x)^3/(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/3),x)
```

output
```
int((d + e*x)^3/(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/3), x)
```

Reduce [F]

$$\int \frac{(d+ex)^3}{\sqrt[3]{ade+(cd^2+ae^2)\,x+cdex^2}}\,dx = \left(\int \frac{x^3}{(cde\,x^2+a\,e^2x+c\,d^2x+ade)^{\frac{1}{3}}}dx\right)e^3 + 3\left(\int \frac{x^2}{(cde\,x^2+a\,e^2x+c\,d^2x+ade)^{\frac{1}{3}}}dx\right)d\,e^2 + 3\left(\int \frac{x}{(cde\,x^2+a\,e^2x+c\,d^2x+ade)^{\frac{1}{3}}}dx\right)d^2e + \left(\int \frac{1}{(cde\,x^2+a\,e^2x+c\,d^2x+ade)^{\frac{1}{3}}}dx\right)d^3$$

input
```
int((e*x+d)^3/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/3),x)
```

output
```
int(x**3/(a*d*e + a*e**2*x + c*d**2*x + c*d*e*x**2)**(1/3),x)*e**3 + 3*int
(x**2/(a*d*e + a*e**2*x + c*d**2*x + c*d*e*x**2)**(1/3),x)*d*e**2 + 3*int(
x/(a*d*e + a*e**2*x + c*d**2*x + c*d*e*x**2)**(1/3),x)*d**2*e + int(1/(a*d
*e + a*e**2*x + c*d**2*x + c*d*e*x**2)**(1/3),x)*d**3
```

3.332 $\int \frac{(d+ex)^2}{\sqrt[3]{ade+(cd^2+ae^2)x+cdex^2}}\,dx$

Optimal result

Integrand size = 37, antiderivative size = 84

$$\int \frac{(d+ex)^2}{\sqrt[3]{ade+(cd^2+ae^2)x+cdex^2}}\,dx = -\frac{3(d+ex)^2(ade+(cd^2+ae^2)x+cdex^2)^{2/3}\,\text{Hypergeometric2F1}\left(1,\frac{10}{3},\frac{11}{3},\frac{cd(d+ex)}{cd^2-ae^2}\right)}{8(cd^2-ae^2)}$$

output
```
-3*(e*x+d)^2*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(2/3)*hypergeom([1, 10/3],[
11/3],c*d*(e*x+d)/(-a*e^2+c*d^2))/(-8*a*e^2+8*c*d^2)
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 93, normalized size of antiderivative = 1.11

$$\int \frac{(d+ex)^2}{\sqrt[3]{ade+(cd^2+ae^2)x+cdex^2}}\,dx = \frac{3(d+ex)((ae+cdx)(d+ex))^{2/3}\,\text{Hypergeometric2F1}\left(-\frac{5}{3},\frac{2}{3},\frac{5}{3},\frac{e(ae+cdx)}{-cd^2+ae^2}\right)}{2cd\left(\frac{cd(d+ex)}{cd^2-ae^2}\right)^{5/3}}$$

input
```
Integrate[(d + e*x)^2/(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(1/3),x]
```

output
```
(3*(d + e*x)*((a*e + c*d*x)*(d + e*x))^(2/3)*Hypergeometric2F1[-5/3, 2/3,
5/3, (e*(a*e + c*d*x))/(-(c*d^2) + a*e^2)])/(2*c*d*((c*d*(d + e*x))/(c*d^2
 - a*e^2))^(5/3))
```

Rubi [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 1786 vs. $2(84) = 168$.

Time = 2.01 (sec) , antiderivative size = 1786, normalized size of antiderivative = 21.26, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.189$, Rules used = {1166, 27, 1160, 1095, 832, 759, 2416}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex)^2}{\sqrt[3]{x(ae^2+cd^2)+ade+cdex^2}}dx$$

$$\Big\downarrow \text{1166}$$

$$\frac{3\int \frac{5e(cd^2-ae^2)(d+ex)}{3\sqrt[3]{cdex^2+(cd^2+ae^2)x+ade}}dx}{7cde}+\frac{3(d+ex)\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{2/3}}{7cd}$$

$$\Big\downarrow \text{27}$$

$$\frac{5\left(cd^2-ae^2\right)\int \frac{d+ex}{\sqrt[3]{cdex^2+(cd^2+ae^2)x+ade}}dx}{7cd}+\frac{3(d+ex)\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{2/3}}{7cd}$$

$$\Big\downarrow \text{1160}$$

$$\frac{5\left(cd^2-ae^2\right)\left(\frac{\left(d^2-\frac{ae^2}{c}\right)\int \frac{1}{\sqrt[3]{cdex^2+(cd^2+ae^2)x+ade}}dx}{2d}+\frac{3\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{2/3}}{4cd}\right)}{7cd}+\frac{3(d+ex)\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{2/3}}{7cd}$$

$$\Big\downarrow \; 1095$$

$$\frac{5(cd^2-ae^2)\left(\frac{3\left(d^2-\frac{ae^2}{c}\right)\sqrt{(ae^2+cd^2+2cdex)^2}\int \frac{\sqrt[3]{cdex^2+(cd^2+ae^2)x+ade}}{\sqrt{\left(cd^2-ae^2\right)^2+4cde\left(cdex^2+\left(cd^2+ae^2\right)x+ade\right)}}d\sqrt[3]{cdex^2+(cd^2+ae^2)x+ade}}{2d(ae^2+cd^2+2cdex)}+\right.}{7cd}$$

$$\frac{3(d+ex)\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{2/3}}{7cd}$$

$$\Big\downarrow \; 832$$

$$\frac{5(cd^2-ae^2)\left(\frac{3\left(d^2-\frac{ae^2}{c}\right)\sqrt{(ae^2+cd^2+2cdex)^2}\left(\frac{\int \frac{\left(1-\sqrt{3}\right)\left(cd^2-ae^2\right)^{2/3}+2^{2/3}\sqrt[3]{c}\sqrt[3]{d}\sqrt[3]{e}\sqrt[3]{cdex^2+(cd^2+ae^2)x+ade}}{\sqrt{\left(cd^2-ae^2\right)^2+4cde\left(cdex^2+\left(cd^2+ae^2\right)x+ade\right)}}d\sqrt[3]{c}}{2^{2/3}\sqrt[3]{c}\sqrt[3]{d}\sqrt[3]{e}}\right.}{}\right.}{}$$

$$\frac{3(d+ex)\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{2/3}}{7cd}$$

$$\Big\downarrow \; 759$$

$$\frac{3\left(cdex^2+\left(cd^2+ae^2\right)x+ade\right)^{2/3}(d+ex)}{7cd}+$$

$$\frac{5(cd^2-ae^2)\left(\frac{3\left(d^2-\frac{ae^2}{c}\right)\sqrt{(cd^2+2cexd+ae^2)^2}\left(\frac{\int \frac{\left(1-\sqrt{3}\right)\left(cd^2-ae^2\right)^{2/3}+2^{2/3}\sqrt[3]{c}\sqrt[3]{d}\sqrt[3]{e}\sqrt[3]{cdex^2+(cd^2+ae^2)x+ade}}{\sqrt{\left(cd^2-ae^2\right)^2+4cde\left(cdex^2+\left(cd^2+ae^2\right)x+ade\right)}}d\sqrt[3]{c}}{2^{2/3}\sqrt[3]{c}\sqrt[3]{d}\sqrt[3]{e}}\right.}{}\right.}{}$$

$$\Big\downarrow \; 2416$$

$$\frac{3\left(cdex^2+\left(cd^2+ae^2\right)x+ade\right)^{2/3}(d+ex)}{7cd}+$$

$$5\left(cd^2-ae^2\right)\frac{3\left(d^2-\frac{ae^2}{c}\right)\sqrt{\left(cd^2+2cexd+ae^2\right)^2}\left(\frac{\sqrt[3]{2}\sqrt{\left(cd^2-ae^2\right)^2+4cde\left(cdex^2+\left(cd^2+ae^2\right)x+ade\right)}}{\sqrt[3]{c}\sqrt[3]{d}\sqrt[3]{e}\left(\left(1+\sqrt{3}\right)\left(cd^2-ae^2\right)^{2/3}+2^{2/3}\sqrt[3]{c}\sqrt[3]{d}\sqrt[3]{e}\sqrt[3]{cdex^2+\left(cd^2+ae^2\right)x\ldots}\right.}\right.}{}$$

input `Int[(d + e*x)^2/(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(1/3),x]`

output
```
(3*(d + e*x)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(2/3))/(7*c*d) + (5*(
c*d^2 - a*e^2)*((3*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(2/3))/(4*c*d)
+ (3*(d^2 - (a*e^2)/c)*Sqrt[(c*d^2 + a*e^2 + 2*c*d*e*x)^2]*(((2^(1/3)*Sqrt
[(c*d^2 - a*e^2)^2 + 4*c*d*e*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)])/(c^
(1/3)*d^(1/3)*e^(1/3)*((1 + Sqrt[3])*(c*d^2 - a*e^2)^(2/3) + 2^(2/3)*c^(1/
3)*d^(1/3)*e^(1/3)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(1/3))) - (3^(1
/4)*Sqrt[2 - Sqrt[3]]*(c*d^2 - a*e^2)^(2/3)*((c*d^2 - a*e^2)^(2/3) + 2^(2/
3)*c^(1/3)*d^(1/3)*e^(1/3)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(1/3))*
Sqrt[((c*d^2 - a*e^2)^(4/3) - 2^(2/3)*c^(1/3)*d^(1/3)*e^(1/3)*(c*d^2 - a*e
^2)^(2/3)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(1/3) + 2*2^(1/3)*c^(2/3
)*d^(2/3)*e^(2/3)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(2/3))/((1 + Sqr
t[3])*(c*d^2 - a*e^2)^(2/3) + 2^(2/3)*c^(1/3)*d^(1/3)*e^(1/3)*(a*d*e + (c*
d^2 + a*e^2)*x + c*d*e*x^2)^(1/3))^2]*EllipticE[ArcSin[((1 - Sqrt[3])*(c*d
^2 - a*e^2)^(2/3) + 2^(2/3)*c^(1/3)*d^(1/3)*e^(1/3)*(a*d*e + (c*d^2 + a*e^
2)*x + c*d*e*x^2)^(1/3))/((1 + Sqrt[3])*(c*d^2 - a*e^2)^(2/3) + 2^(2/3)*c^
(1/3)*d^(1/3)*e^(1/3)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(1/3))], -7
- 4*Sqrt[3]])/(2^(2/3)*c^(1/3)*d^(1/3)*e^(1/3)*Sqrt[((c*d^2 - a*e^2)^(2/3)
*((c*d^2 - a*e^2)^(2/3) + 2^(2/3)*c^(1/3)*d^(1/3)*e^(1/3)*(a*d*e + (c*d^2
+ a*e^2)*x + c*d*e*x^2)^(1/3)))/((1 + Sqrt[3])*(c*d^2 - a*e^2)^(2/3) + 2^(
2/3)*c^(1/3)*d^(1/3)*e^(1/3)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(1...
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 759
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

rule 832
```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Simp[(-(1 - Sqrt[3]))*(s/r)   Int[1/Sqrt[a + b*x
^3], x], x] + Simp[1/r   Int[((1 - Sqrt[3])*s + r*x)/Sqrt[a + b*x^3], x], x
]] /; FreeQ[{a, b}, x] && PosQ[a]
```

rule 1095
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[3*(Sqrt[(b
+ 2*c*x)^2]/(b + 2*c*x))   Subst[Int[x^(3*(p + 1) - 1)/Sqrt[b^2 - 4*a*c + 4
*c*x^3], x], x, (a + b*x + c*x^2)^(1/3)], x] /; FreeQ[{a, b, c}, x] && Inte
gerQ[3*p]
```

rule 1160
```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[e*((a + b*x + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Simp[(2*c*d - b
*e)/(2*c)   Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
 && NeQ[p, -1]
```

rule 1166
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[e*(d + e*x)^(m - 1)*((a + b*x + c*x^2)^(p + 1)/(c*(m + 2*p +
 1))), x] + Simp[1/(c*(m + 2*p + 1))   Int[(d + e*x)^(m - 2)*Simp[c*d^2*(m
+ 2*p + 1) - e*(a*e*(m - 1) + b*d*(p + 1)) + e*(2*c*d - b*e)*(m + p)*x, x]*
(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && If[Ration
alQ[m], GtQ[m, 1], SumSimplerQ[m, -2]] && NeQ[m + 2*p + 1, 0] && IntQuadrat
icQ[a, b, c, d, e, m, p, x]
```

rule 2416
```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 - Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 - Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 + Sqrt[3])*s + r*x))), x] - S
imp[3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[s*((s + r*x)/((1 + Sqrt
[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sqrt[3])
*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Maple [F]

$$\int \frac{(ex+d)^2}{\left(ade+\left(a\,e^2+c\,d^2\right)x+cd\,x^2e\right)^{\frac{1}{3}}}dx$$

input
```
int((e*x+d)^2/(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(1/3),x)
```

output
```
int((e*x+d)^2/(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(1/3),x)
```

Fricas [F]

$$\int \frac{(d+ex)^2}{\sqrt[3]{ade+(cd^2+ae^2)\,x+cdex^2}}\,dx = \int \frac{(ex+d)^2}{(cdex^2+ade+(cd^2+ae^2)x)^{\frac{1}{3}}}\,dx$$

input
```
integrate((e*x+d)^2/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/3),x, algorithm="
fricas")
```

output
```
integral((c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^(2/3)*(e*x + d)/(c*d*x +
a*e), x)
```

Sympy [F]

$$\int \frac{(d+ex)^2}{\sqrt[3]{ade+(cd^2+ae^2)\,x+cdex^2}}\,dx = \int \frac{(d+ex)^2}{\sqrt[3]{(d+ex)\,(ae+cdx)}}\,dx$$

input
```
integrate((e*x+d)**2/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(1/3),x)
```

output
```
Integral((d + e*x)**2/((d + e*x)*(a*e + c*d*x))**(1/3), x)
```

Maxima [F]

$$\int \frac{(d+ex)^2}{\sqrt[3]{ade+(cd^2+ae^2)\,x+cdex^2}}\,dx = \int \frac{(ex+d)^2}{(cdex^2+ade+(cd^2+ae^2)x)^{\frac{1}{3}}}\,dx$$

input
```
integrate((e*x+d)^2/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/3),x, algorithm="
maxima")
```

output
```
integrate((e*x + d)^2/(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^(1/3), x)
```

Giac [F]

$$\int \frac{(d+ex)^2}{\sqrt[3]{ade+(cd^2+ae^2)\,x+cdex^2}}\,dx=\int \frac{(ex+d)^2}{(cdex^2+ade+(cd^2+ae^2)x)^{\frac{1}{3}}}\,dx$$

input
```
integrate((e*x+d)^2/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/3),x, algorithm="
giac")
```

output
```
integrate((e*x + d)^2/(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^(1/3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d+ex)^2}{\sqrt[3]{ade+(cd^2+ae^2)\,x+cdex^2}}\,dx=\int \frac{(d+e\,x)^2}{\left(c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e\right)^{1/3}}\,dx$$

input
```
int((d + e*x)^2/(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/3),x)
```

output
```
int((d + e*x)^2/(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/3), x)
```

Reduce [F]

$$\int \frac{(d+ex)^2}{\sqrt[3]{ade+(cd^2+ae^2)\,x+cdex^2}}\,dx=\left(\int \frac{x^2}{(cde\,x^2+a\,e^2x+c\,d^2x+ade)^{\frac{1}{3}}}dx\right)e^2+2\left(\int \frac{x}{(cde\,x^2+a\,e^2x+c\,d^2x+ade)^{\frac{1}{3}}}dx\right)de+\left(\int \frac{1}{(cde\,x^2+a\,e^2x+c\,d^2x+ade)^{\frac{1}{3}}}dx\right)d^2$$

input
```
int((e*x+d)^2/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/3),x)
```

output
```
int(x**2/(a*d*e + a*e**2*x + c*d**2*x + c*d*e*x**2)**(1/3),x)*e**2 + 2*int
(x/(a*d*e + a*e**2*x + c*d**2*x + c*d*e*x**2)**(1/3),x)*d*e + int(1/(a*d*e
 + a*e**2*x + c*d**2*x + c*d*e*x**2)**(1/3),x)*d**2
```

3.333 $\int \frac{d+ex}{\sqrt[3]{ade+(cd^2+ae^2)x+cdex^2}}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 82

$$\int \frac{d+ex}{\sqrt[3]{ade+(cd^2+ae^2)x+cdex^2}}\,dx = -\frac{3(d+ex)\left(ade+(cd^2+ae^2)x+cdex^2\right)^{2/3}\text{Hypergeometric2F1}\left(1,\frac{7}{3},\frac{8}{3},\frac{cd(d+ex)}{cd^2-ae^2}\right)}{5(cd^2-ae^2)}$$

output
```
-3*(e*x+d)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(2/3)*hypergeom([1, 7/3],[8/3
],c*d*(e*x+d)/(-a*e^2+c*d^2))/(-5*a*e^2+5*c*d^2)
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.07

$$\int \frac{d+ex}{\sqrt[3]{ade+(cd^2+ae^2)x+cdex^2}}\,dx = \frac{3((ae+cdx)(d+ex))^{2/3}\text{Hypergeometric2F1}\left(-\frac{2}{3},\frac{2}{3},\frac{5}{3},\frac{e(ae+cdx)}{-cd^2+ae^2}\right)}{2cd\left(\frac{cd(d+ex)}{cd^2-ae^2}\right)^{2/3}}$$

input
```
Integrate[(d + e*x)/(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(1/3),x]
```

output
```
(3*((a*e + c*d*x)*(d + e*x))^(2/3)*Hypergeometric2F1[-2/3, 2/3, 5/3, (e*(a
*e + c*d*x))/(-(c*d^2) + a*e^2)])/(2*c*d*((c*d*(d + e*x))/(c*d^2 - a*e^2))
^(2/3))
```

Rubi [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 1719 vs. $2(82) = 164$.

Time = 1.78 (sec) , antiderivative size = 1719, normalized size of antiderivative = 20.96, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {1160, 1095, 832, 759, 2416}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{d+ex}{\sqrt[3]{x\left(ae^2+cd^2\right)+ade+cdex^2}}dx$$

$$\Big\downarrow \text{1160}$$

$$\frac{\left(d^2-\frac{ae^2}{c}\right)\int \frac{1}{\sqrt[3]{cdex^2+\left(cd^2+ae^2\right)x+ade}}dx}{2d}+\frac{3\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{2/3}}{4cd}$$

$$\Big\downarrow \text{1095}$$

$$\frac{3\left(d^2-\frac{ae^2}{c}\right)\sqrt{\left(ae^2+cd^2+2cdex\right)^2}\int \frac{\sqrt[3]{cdex^2+\left(cd^2+ae^2\right)x+ade}}{\sqrt{\left(cd^2-ae^2\right)^2+4cde\left(cdex^2+\left(cd^2+ae^2\right)x+ade\right)}}d\sqrt[3]{cdex^2+\left(cd^2+ae^2\right)x+ade}}{2d\left(ae^2+cd^2+2cdex\right)}+\frac{3\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{2/3}}{4cd}$$

$$\Big\downarrow \text{832}$$

$$3\left(d^2-\frac{ae^2}{c}\right)\sqrt{\left(ae^2+cd^2+2cdex\right)^2}\left(\frac{\int \frac{\left(1-\sqrt{3}\right)\left(cd^2-ae^2\right)^{2/3}+2^{2/3}\sqrt[3]{c}\sqrt[3]{d}\sqrt[3]{e}\sqrt[3]{cdex^2+\left(cd^2+ae^2\right)x+ade}}{\sqrt{\left(cd^2-ae^2\right)^2+4cde\left(cdex^2+\left(cd^2+ae^2\right)x+ade\right)}}d\sqrt[3]{cdex^2}}{2^{2/3}\sqrt[3]{c}\sqrt[3]{d}\sqrt[3]{e}}\right.$$

$$2d\left(ae^2+\right.$$

$$\frac{3\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{2/3}}{4cd}$$

$$\Big\downarrow \; 759$$

$$3\left(d^2-\frac{ae^2}{c}\right)\sqrt{\left(cd^2+2cexd+ae^2\right)^2}\left(\frac{\int \frac{\left(1-\sqrt{3}\right)\left(cd^2-ae^2\right)^{2/3}+2^{2/3}\sqrt[3]{c}\sqrt[3]{d}\sqrt[3]{e}\sqrt[3]{cdex^2+\left(cd^2+ae^2\right)x+ade}}{\sqrt{\left(cd^2-ae^2\right)^2+4cde\left(cdex^2+\left(cd^2+ae^2\right)x+ade\right)}}d\sqrt[3]{cdex^2}}{2^{2/3}\sqrt[3]{c}\sqrt[3]{d}\sqrt[3]{e}}\right.$$

$$\frac{3\left(cdex^2+\left(cd^2+ae^2\right)x+ade\right)^{2/3}}{4cd}$$

$$\Big\downarrow \; 2416$$

$$3\left(d^2-\frac{ae^2}{c}\right)\sqrt{\left(cd^2+2cexd+ae^2\right)^2}\left(\frac{\frac{\sqrt[3]{2}\sqrt{\left(cd^2-ae^2\right)^2+4cde\left(cdex^2+\left(cd^2+ae^2\right)x+ade\right)}}{\sqrt[3]{c}\sqrt[3]{d}\sqrt[3]{e}\left(\left(1+\sqrt{3}\right)\left(cd^2-ae^2\right)^{2/3}+2^{2/3}\sqrt[3]{c}\sqrt[3]{d}\sqrt[3]{e}\sqrt[3]{cdex^2+\left(cd^2+ae^2\right)x+ade}\right.}}{}\right.$$

$$\frac{3\left(cdex^2+\left(cd^2+ae^2\right)x+ade\right)^{2/3}}{4cd}$$

input
```
Int[(d + e*x)/(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(1/3),x]
```

output
```
(3*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(2/3))/(4*c*d) + (3*(d^2 - (a*e
^2)/c)*Sqrt[(c*d^2 + a*e^2 + 2*c*d*e*x)^2]*(((2^(1/3)*Sqrt[(c*d^2 - a*e^2)
^2 + 4*c*d*e*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)])/(c^(1/3)*d^(1/3)*e^
(1/3)*((1 + Sqrt[3])*(c*d^2 - a*e^2)^(2/3) + 2^(2/3)*c^(1/3)*d^(1/3)*e^(1/
3)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(1/3))) - (3^(1/4)*Sqrt[2 - Sqr
t[3]]*(c*d^2 - a*e^2)^(2/3)*((c*d^2 - a*e^2)^(2/3) + 2^(2/3)*c^(1/3)*d^(1/
3)*e^(1/3)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(1/3))*Sqrt[((c*d^2 - a
*e^2)^(4/3) - 2^(2/3)*c^(1/3)*d^(1/3)*e^(1/3)*(c*d^2 - a*e^2)^(2/3)*(a*d*e
 + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(1/3) + 2*2^(1/3)*c^(2/3)*d^(2/3)*e^(2/3
)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(2/3))/((1 + Sqrt[3])*(c*d^2 - a
*e^2)^(2/3) + 2^(2/3)*c^(1/3)*d^(1/3)*e^(1/3)*(a*d*e + (c*d^2 + a*e^2)*x +
 c*d*e*x^2)^(1/3))^2]*EllipticE[ArcSin[((1 - Sqrt[3])*(c*d^2 - a*e^2)^(2/3
) + 2^(2/3)*c^(1/3)*d^(1/3)*e^(1/3)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2
)^(1/3))/((1 + Sqrt[3])*(c*d^2 - a*e^2)^(2/3) + 2^(2/3)*c^(1/3)*d^(1/3)*e^
(1/3)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(1/3))], -7 - 4*Sqrt[3]])/(2
^(2/3)*c^(1/3)*d^(1/3)*e^(1/3)*Sqrt[((c*d^2 - a*e^2)^(2/3)*((c*d^2 - a*e^2
)^(2/3) + 2^(2/3)*c^(1/3)*d^(1/3)*e^(1/3)*(a*d*e + (c*d^2 + a*e^2)*x + c*d
*e*x^2)^(1/3)))/((1 + Sqrt[3])*(c*d^2 - a*e^2)^(2/3) + 2^(2/3)*c^(1/3)*d^(
1/3)*e^(1/3)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(1/3))^2]*Sqrt[(c*d^2
 - a*e^2)^2 + 4*c*d*e*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)])))/(2^(2/...
```

Defintions of rubi rules used

rule 759
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

rule 832
```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Simp[(-(1 - Sqrt[3]))*(s/r)   Int[1/Sqrt[a + b*x
^3], x], x] + Simp[1/r   Int[((1 - Sqrt[3])*s + r*x)/Sqrt[a + b*x^3], x], x
]] /; FreeQ[{a, b}, x] && PosQ[a]
```

rule 1095
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[3*(Sqrt[(b
 + 2*c*x)^2]/(b + 2*c*x))   Subst[Int[x^(3*(p + 1) - 1)/Sqrt[b^2 - 4*a*c + 4
*c*x^3], x], x, (a + b*x + c*x^2)^(1/3)], x] /; FreeQ[{a, b, c}, x] && Inte
gerQ[3*p]
```

rule 1160
```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[e*((a + b*x + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Simp[(2*c*d - b
*e)/(2*c)   Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
 && NeQ[p, -1]
```

rule 2416
```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 - Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 - Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 + Sqrt[3])*s + r*x))), x] - S
imp[3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[s*((s + r*x)/((1 + Sqrt
[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sqrt[3])
*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Maple [F]

$$\int \frac{ex+d}{\left(ade+\left(a\,e^{2}+c\,d^{2}\right)x+cd\,x^{2}e\right)^{\frac{1}{3}}}dx$$

input
```
int((e*x+d)/(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(1/3),x)
```

output
```
int((e*x+d)/(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(1/3),x)
```

Fricas [F]

$$\int \frac{d+ex}{\sqrt[3]{ade+(cd^2+ae^2)x+cdex^2}}\,dx=\int \frac{ex+d}{(cdex^2+ade+(cd^2+ae^2)x)^{\frac{1}{3}}}\,dx$$

input
```
integrate((e*x+d)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/3),x, algorithm="fr
icas")
```

output
```
integral((c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^(2/3)/(c*d*x + a*e), x)
```

Sympy [F]

$$\int \frac{d+ex}{\sqrt[3]{ade+(cd^2+ae^2)x+cdex^2}}\,dx=\int \frac{d+ex}{\sqrt[3]{(d+ex)(ae+cdx)}}\,dx$$

input
```
integrate((e*x+d)/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(1/3),x)
```

output
```
Integral((d + e*x)/((d + e*x)*(a*e + c*d*x))**(1/3), x)
```

Maxima [F]

$$\int \frac{d+ex}{\sqrt[3]{ade+(cd^2+ae^2)x+cdex^2}}\,dx=\int \frac{ex+d}{(cdex^2+ade+(cd^2+ae^2)x)^{\frac{1}{3}}}\,dx$$

input
```
integrate((e*x+d)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/3),x, algorithm="ma
xima")
```

output
```
integrate((e*x + d)/(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^(1/3), x)
```

Giac [F]

$$\int \frac{d+ex}{\sqrt[3]{ade+(cd^2+ae^2)\,x+cdex^2}}\,dx=\int \frac{ex+d}{(cdex^2+ade+(cd^2+ae^2)x)^{\frac{1}{3}}}\,dx$$

input
```
integrate((e*x+d)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/3),x, algorithm="gi
ac")
```

output
```
integrate((e*x + d)/(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^(1/3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{d+ex}{\sqrt[3]{ade+(cd^2+ae^2)\,x+cdex^2}}\,dx=\int \frac{d+e\,x}{\left(c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e\right)^{1/3}}\,dx$$

input
```
int((d + e*x)/(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/3),x)
```

output
```
int((d + e*x)/(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/3), x)
```

Reduce [F]

$$\int \frac{d+ex}{\sqrt[3]{ade+(cd^2+ae^2)\,x+cdex^2}}\,dx=\left(\int \frac{x}{(cde\,x^2+a\,e^2x+c\,d^2x+ade)^{\frac{1}{3}}}dx\right)e+\left(\int \frac{1}{(cde\,x^2+a\,e^2x+c\,d^2x+ade)^{\frac{1}{3}}}dx\right)d$$

input
```
int((e*x+d)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/3),x)
```

output
```
int(x/(a*d*e + a*e**2*x + c*d**2*x + c*d*e*x**2)**(1/3),x)*e + int(1/(a*d*
e + a*e**2*x + c*d**2*x + c*d*e*x**2)**(1/3),x)*d
```

3.334 $\int \frac{1}{\sqrt[3]{ade+(cd^2+ae^2)x+cdex^2}}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 108

$$\int \frac{1}{\sqrt[3]{ade+(cd^2+ae^2)x+cdex^2}}\,dx = -\frac{3(ade+(cd^2+ae^2)x+cdex^2)^{2/3}\,\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{2}{3},\frac{5}{3},\frac{cd(d+ex)}{cd^2-ae^2}\right)}{2(cd^2-ae^2)\left(-\frac{e(ae+cdx)}{cd^2-ae^2}\right)^{2/3}}$$

output
```
-3/2*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(2/3)*hypergeom([1/3, 2/3],[5/3],c*
d*(e*x+d)/(-a*e^2+c*d^2))/(-a*e^2+c*d^2)/(-e*(c*d*x+a*e)/(-a*e^2+c*d^2))^(
2/3)
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.88

$$\int \frac{1}{\sqrt[3]{ade+(cd^2+ae^2)x+cdex^2}}\,dx = \frac{3\sqrt[3]{\frac{cd(d+ex)}{cd^2-ae^2}}((ae+cdx)(d+ex))^{2/3}\,\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{2}{3},\frac{5}{3},\frac{e(ae+cdx)}{-cd^2+ae^2}\right)}{2cd(d+ex)}$$

input
```
Integrate[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(-1/3),x]
```

output
```
(3*((c*d*(d + e*x))/(c*d^2 - a*e^2))^(1/3)*((a*e + c*d*x)*(d + e*x))^(2/3)
*Hypergeometric2F1[1/3, 2/3, 5/3, (e*(a*e + c*d*x))/(-(c*d^2) + a*e^2)])/(
2*c*d*(d + e*x))
```

Rubi [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 1661 vs. $2(108) = 216$.

Time = 1.71 (sec) , antiderivative size = 1661, normalized size of antiderivative = 15.38, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.138$, Rules used = {1095, 832, 759, 2416}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt[3]{x\left(ae^2+cd^2\right)+ade+cdex^2}}dx$$

$$\Big\downarrow \text{1095}$$

$$\frac{3\sqrt{\left(ae^2+cd^2+2cdex\right)^2}\int \frac{\sqrt[3]{cdex^2+\left(cd^2+ae^2\right)x+ade}}{\sqrt{\left(cd^2-ae^2\right)^2+4cde\left(cdex^2+\left(cd^2+ae^2\right)x+ade\right)}}d\sqrt[3]{cdex^2+\left(cd^2+ae^2\right)x+ade}}{ae^2+cd^2+2cdex}$$

$$\Big\downarrow \text{832}$$

$$\frac{3\sqrt{\left(ae^2+cd^2+2cdex\right)^2}\left(\frac{\int \frac{\left(1-\sqrt{3}\right)\left(cd^2-ae^2\right)^{2/3}+2^{2/3}\sqrt[3]{c}\sqrt[3]{d}\sqrt[3]{e}\sqrt[3]{cdex^2+\left(cd^2+ae^2\right)x+ade}}{\sqrt{\left(cd^2-ae^2\right)^2+4cde\left(cdex^2+\left(cd^2+ae^2\right)x+ade\right)}}d\sqrt[3]{cdex^2+\left(cd^2+ae\right.}}{2^{2/3}\sqrt[3]{c}\sqrt[3]{d}\sqrt[3]{e}}\right.}{ae^2+cd^2+2c}$$

$$\Big\downarrow \text{759}$$

$$3\sqrt{\left(cd^2+2cexd+ae^2\right)^2}\left(\frac{\int \frac{\left(1-\sqrt{3}\right)\left(cd^2-ae^2\right)^{2/3}+2^{2/3}\sqrt[3]{c}\sqrt[3]{d}\sqrt[3]{e}\sqrt[3]{cdex^2+\left(cd^2+ae^2\right)x+ade}}{\sqrt{\left(cd^2-ae^2\right)^2+4cde\left(cdex^2+\left(cd^2+ae^2\right)x+ade\right)}}d\sqrt[3]{cdex^2+\left(cd^2+ae\right.}}{2^{2/3}\sqrt[3]{c}\sqrt[3]{d}\sqrt[3]{e}}\right.$$

$\big\downarrow$ 2416

$$3\sqrt{\left(cd^2+2cexd+ae^2\right)^2}\left(\frac{\frac{\sqrt[3]{2}\sqrt{\left(cd^2-ae^2\right)^2+4cde\left(cdex^2+\left(cd^2+ae^2\right)x+ade\right)}}{\sqrt[3]{c}\sqrt[3]{d}\sqrt[3]{e}\left(\left(1+\sqrt{3}\right)\left(cd^2-ae^2\right)^{2/3}+2^{2/3}\sqrt[3]{c}\sqrt[3]{d}\sqrt[3]{e}\sqrt[3]{cdex^2+\left(cd^2+ae^2\right)x+ade}\right)}-\frac{\sqrt[4]{3}\sqrt{2-\sqrt{}}}{}}{}\right.$$

input
```
Int[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(-1/3),x]
```

output
```
(3*Sqrt[(c*d^2 + a*e^2 + 2*c*d*e*x)^2]*(((2^(1/3)*Sqrt[(c*d^2 - a*e^2)^2 +
 4*c*d*e*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)])/(c^(1/3)*d^(1/3)*e^(1/3
)*((1 + Sqrt[3])*(c*d^2 - a*e^2)^(2/3) + 2^(2/3)*c^(1/3)*d^(1/3)*e^(1/3)*(
a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(1/3))) - (3^(1/4)*Sqrt[2 - Sqrt[3]
]*(c*d^2 - a*e^2)^(2/3)*((c*d^2 - a*e^2)^(2/3) + 2^(2/3)*c^(1/3)*d^(1/3)*e
^(1/3)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(1/3))*Sqrt[((c*d^2 - a*e^2
)^(4/3) - 2^(2/3)*c^(1/3)*d^(1/3)*e^(1/3)*(c*d^2 - a*e^2)^(2/3)*(a*d*e + (
c*d^2 + a*e^2)*x + c*d*e*x^2)^(1/3) + 2*2^(1/3)*c^(2/3)*d^(2/3)*e^(2/3)*(a
*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(2/3))/((1 + Sqrt[3])*(c*d^2 - a*e^2
)^(2/3) + 2^(2/3)*c^(1/3)*d^(1/3)*e^(1/3)*(a*d*e + (c*d^2 + a*e^2)*x + c*d
*e*x^2)^(1/3))^2]*EllipticE[ArcSin[((1 - Sqrt[3])*(c*d^2 - a*e^2)^(2/3) +
2^(2/3)*c^(1/3)*d^(1/3)*e^(1/3)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(1
/3))/((1 + Sqrt[3])*(c*d^2 - a*e^2)^(2/3) + 2^(2/3)*c^(1/3)*d^(1/3)*e^(1/3
)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(1/3))], -7 - 4*Sqrt[3]])/(2^(2/
3)*c^(1/3)*d^(1/3)*e^(1/3)*Sqrt[((c*d^2 - a*e^2)^(2/3)*((c*d^2 - a*e^2)^(2
/3) + 2^(2/3)*c^(1/3)*d^(1/3)*e^(1/3)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x
^2)^(1/3)))/((1 + Sqrt[3])*(c*d^2 - a*e^2)^(2/3) + 2^(2/3)*c^(1/3)*d^(1/3)
*e^(1/3)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(1/3))^2]*Sqrt[(c*d^2 - a
*e^2)^2 + 4*c*d*e*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)]))/(2^(2/3)*c^(1
/3)*d^(1/3)*e^(1/3)) - ((1 - Sqrt[3])*Sqrt[2 + Sqrt[3]]*(c*d^2 - a*e^2)...
```

Defintions of rubi rules used

rule 759
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

rule 832
```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Simp[(-(1 - Sqrt[3]))*(s/r)   Int[1/Sqrt[a + b*x
^3], x], x] + Simp[1/r   Int[((1 - Sqrt[3])*s + r*x)/Sqrt[a + b*x^3], x], x
]] /; FreeQ[{a, b}, x] && PosQ[a]
```

rule 1095
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[3*(Sqrt[(b
+ 2*c*x)^2]/(b + 2*c*x))   Subst[Int[x^(3*(p + 1) - 1)/Sqrt[b^2 - 4*a*c + 4
*c*x^3], x], x, (a + b*x + c*x^2)^(1/3)], x] /; FreeQ[{a, b, c}, x] && Inte
gerQ[3*p]
```

rule 2416
```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 - Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 - Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 + Sqrt[3])*s + r*x))), x] - S
imp[3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[s*((s + r*x)/((1 + Sqrt
[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sqrt[3])
*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Maple [F]

$$\int \frac{1}{\left(ade+\left(a\,e^{2}+c\,d^{2}\right)x+cd\,x^{2}e\right)^{\frac{1}{3}}}dx$$

input
```
int(1/(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(1/3),x)
```

output
```
int(1/(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(1/3),x)
```

Fricas [F]

$$\int \frac{1}{\sqrt[3]{ade+\left(cd^2+ae^2\right)x+cdex^2}}\,dx=\int \frac{1}{\left(cdex^2+ade+\left(cd^2+ae^2\right)x\right)^{\frac{1}{3}}}\,dx$$

input
```
integrate(1/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/3),x, algorithm="fricas")
```

output
```
integral((c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^(-1/3), x)
```

Sympy [F]

$$\int \frac{1}{\sqrt[3]{ade + (cd^2 + ae^2)\,x + cdex^2}}\,dx = \int \frac{1}{\sqrt[3]{ade + cdex^2 + x\,(ae^2 + cd^2)}}\,dx$$

input
```
integrate(1/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(1/3),x)
```

output
```
Integral((a*d*e + c*d*e*x**2 + x*(a*e**2 + c*d**2))**(-1/3), x)
```

Maxima [F]

$$\int \frac{1}{\sqrt[3]{ade + (cd^2 + ae^2)\,x + cdex^2}}\,dx = \int \frac{1}{(cdex^2 + ade + (cd^2 + ae^2)x)^{\frac{1}{3}}}\,dx$$

input
```
integrate(1/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/3),x, algorithm="maxima")
```

output
```
integrate((c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^(-1/3), x)
```

Giac [F]

$$\int \frac{1}{\sqrt[3]{ade + (cd^2 + ae^2)\,x + cdex^2}}\,dx = \int \frac{1}{(cdex^2 + ade + (cd^2 + ae^2)x)^{\frac{1}{3}}}\,dx$$

input
```
integrate(1/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/3),x, algorithm="giac")
```

output
```
integrate((c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^(-1/3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt[3]{ade + (cd^2 + ae^2)\,x + cdex^2}}\,dx = \int \frac{1}{\left(c\,d\,e\,x^2 + (c\,d^2 + a\,e^2)\,\,x + a\,d\,e\right)^{1/3}}\,dx$$

input
```
int(1/(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/3),x)
```

output
```
int(1/(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/3), x)
```

Reduce [F]

$$\int \frac{1}{\sqrt[3]{ade + (cd^2 + ae^2)\,x + cdex^2}}\,dx = \int \frac{1}{(cde\,x^2 + a\,e^2x + c\,d^2x + ade)^{\frac{1}{3}}}dx$$

input
```
int(1/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/3),x)
```

output
```
int(1/(a*d*e + a*e**2*x + c*d**2*x + c*d*e*x**2)**(1/3),x)
```

3.335 $\int \frac{1}{(d+ex)\sqrt[3]{ade+(cd^2+ae^2)x+cdex^2}}\,dx$

Optimal result

Integrand size = 37, antiderivative size = 82

$$\int \frac{1}{(d+ex)\sqrt[3]{ade+(cd^2+ae^2)x+cdex^2}}\,dx = \frac{3(ade+(cd^2+ae^2)x+cdex^2)^{2/3}\,\text{Hypergeometric2F1}\left(\frac{1}{3},1,\frac{2}{3},\frac{cd(d+ex)}{cd^2-ae^2}\right)}{(cd^2-ae^2)(d+ex)}$$

output
```
3*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(2/3)*hypergeom([1/3, 1],[2/3],c*d*(e*
x+d)/(-a*e^2+c*d^2))/(-a*e^2+c*d^2)/(e*x+d)
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 98, normalized size of antiderivative = 1.20

$$\int \frac{1}{(d+ex)\sqrt[3]{ade+(cd^2+ae^2)x+cdex^2}}\,dx = \frac{3cd((ae+cdx)(d+ex))^{2/3}\,\text{Hypergeometric2F1}\left(\frac{2}{3},\frac{4}{3},\frac{5}{3},\frac{e(ae+cdx)}{-cd^2+ae^2}\right)}{2(cd^2-ae^2)^2\left(\frac{cd(d+ex)}{cd^2-ae^2}\right)^{2/3}}$$

input
```
Integrate[1/((d + e*x)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(1/3)),x]
```

output
```
(3*c*d*((a*e + c*d*x)*(d + e*x))^(2/3)*Hypergeometric2F1[2/3, 4/3, 5/3, (e
*(a*e + c*d*x))/(-(c*d^2) + a*e^2)])/(2*(c*d^2 - a*e^2)^2*((c*d*(d + e*x))
/(c*d^2 - a*e^2))^(2/3))
```

Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.41, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.081$, Rules used = {1138, 80, 79}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(d+ex)\sqrt[3]{x(ae^2+cd^2)+ade+cdex^2}}dx$$

$$\Big\downarrow \text{1138}$$

$$\frac{\sqrt[3]{\frac{ex}{d}+1}\sqrt[3]{ae+cdx}\int \frac{1}{\sqrt[3]{ae+cdx}\left(\frac{ex}{d}+1\right)^{4/3}}dx}{d\sqrt[3]{x(ae^2+cd^2)+ade+cdex^2}}$$

$$\Big\downarrow \text{80}$$

$$\frac{cd\sqrt[3]{ae+cdx}\sqrt[3]{\frac{cd(d+ex)}{cd^2-ae^2}}\int \frac{1}{\sqrt[3]{ae+cdx}\left(\frac{cd^2}{cd^2-ae^2}+\frac{cexd}{cd^2-ae^2}\right)^{4/3}}dx}{(cd^2-ae^2)\sqrt[3]{x(ae^2+cd^2)+ade+cdex^2}}$$

$$\Big\downarrow \text{79}$$

$$\frac{3(ae+cdx)\sqrt[3]{\frac{cd(d+ex)}{cd^2-ae^2}}\text{Hypergeometric2F1}\left(\frac{2}{3},\frac{4}{3},\frac{5}{3},-\frac{e(ae+cdx)}{cd^2-ae^2}\right)}{2(cd^2-ae^2)\sqrt[3]{x(ae^2+cd^2)+ade+cdex^2}}$$

input
```
Int[1/((d + e*x)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(1/3)),x]
```

output
```
(3*(a*e + c*d*x)*((c*d*(d + e*x))/(c*d^2 - a*e^2))^(1/3)*Hypergeometric2F1
[2/3, 4/3, 5/3, -((e*(a*e + c*d*x))/(c*d^2 - a*e^2))])/(2*(c*d^2 - a*e^2)*
(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(1/3))
```

Defintions of rubi rules used

rule 79
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d), 0] && (RationalQ[m]
 ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

rule 80
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*(b*((c + d*x)/(b*c - a*d)))
^FracPart[n])   Int[(a + b*x)^m*Simp[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d)
), x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] &&  !IntegerQ[m] &&  !Integ
erQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

rule 1138
```
Int[((d_) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Sy
mbol] :> Simp[d^m*((a + b*x + c*x^2)^FracPart[p]/((1 + e*(x/d))^FracPart[p]
*(a/d + (c*x)/e)^FracPart[p]))   Int[(1 + e*(x/d))^(m + p)*(a/d + (c/e)*x)^
p, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0]
&& (IntegerQ[m] || GtQ[d, 0]) &&  !(IGtQ[m, 0] && (IntegerQ[3*p] || Integer
Q[4*p]))
```

Maple [F]

$$\int \frac{1}{(ex+d)\left(ade+\left(a\,e^{2}+c\,d^{2}\right)x+cd\,x^{2}e\right)^{\frac{1}{3}}}dx$$

input
```
int(1/(e*x+d)/(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(1/3),x)
```

output
```
int(1/(e*x+d)/(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(1/3),x)
```

Fricas [F]

$$\int \frac{1}{(d+ex)\sqrt[3]{ade+(cd^2+ae^2)\,x+cdex^2}}\,dx$$
$$=\int \frac{1}{(cdex^2+ade+(cd^2+ae^2)x)^{\frac{1}{3}}(ex+d)}\,dx$$

input
```
integrate(1/(e*x+d)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/3),x, algorithm="
fricas")
```

output
```
integral((c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^(2/3)/(c*d*e^2*x^3 + a*d^
2*e + (2*c*d^2*e + a*e^3)*x^2 + (c*d^3 + 2*a*d*e^2)*x), x)
```

Sympy [F]

$$\int \frac{1}{(d+ex)\sqrt[3]{ade+(cd^2+ae^2)\,x+cdex^2}}\,dx=\int \frac{1}{\sqrt[3]{(d+ex)\,(ae+cdx)}\,(d+ex)}\,dx$$

input
```
integrate(1/(e*x+d)/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(1/3),x)
```

output
```
Integral(1/(((d + e*x)*(a*e + c*d*x))**(1/3)*(d + e*x)), x)
```

Maxima [F]

$$\int \frac{1}{(d+ex)\sqrt[3]{ade+(cd^2+ae^2)\,x+cdex^2}}\,dx$$
$$=\int \frac{1}{(cdex^2+ade+(cd^2+ae^2)x)^{\frac{1}{3}}(ex+d)}\,dx$$

input
```
integrate(1/(e*x+d)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/3),x, algorithm="
maxima")
```

output
```
integrate(1/((c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^(1/3)*(e*x + d)), x)
```

Giac [F]

$$\int \frac{1}{(d+ex)\sqrt[3]{ade+(cd^2+ae^2)\,x+cdex^2}}\,dx = \int \frac{1}{(cdex^2+ade+(cd^2+ae^2)x)^{\frac{1}{3}}(ex+d)}\,dx$$

input
```
integrate(1/(e*x+d)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/3),x, algorithm="
giac")
```

output
```
integrate(1/((c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^(1/3)*(e*x + d)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(d+ex)\sqrt[3]{ade+(cd^2+ae^2)\,x+cdex^2}}\,dx = \int \frac{1}{(d+e\,x)\,\left(c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e\right)^{1/3}}\,dx$$

input
```
int(1/((d + e*x)*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/3)),x)
```

output
```
int(1/((d + e*x)*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/3)), x)
```

Reduce [F]

$$\int \frac{1}{(d+ex)\sqrt[3]{ade+(cd^2+ae^2)\,x+cdex^2}}\,dx$$
$$=\int \frac{1}{\left(cde\,x^2+a\,e^2x+c\,d^2x+ade\right)^{\frac{1}{3}}d+\left(cde\,x^2+a\,e^2x+c\,d^2x+ade\right)^{\frac{1}{3}}ex}dx$$

input
```
int(1/(e*x+d)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/3),x)
```

output
```
int(1/((a*d*e + a*e**2*x + c*d**2*x + c*d*e*x**2)**(1/3)*d + (a*d*e + a*e*
*2*x + c*d**2*x + c*d*e*x**2)**(1/3)*e*x),x)
```

3.336 $\int \frac{1}{(d+ex)^2 \sqrt[3]{ade+(cd^2+ae^2)x+cdex^2}}\,dx$

Optimal result

Integrand size = 37, antiderivative size = 84

$$\int \frac{1}{(d+ex)^2 \sqrt[3]{ade+(cd^2+ae^2)x+cdex^2}}\,dx = \frac{3(ade+(cd^2+ae^2)x+cdex^2)^{2/3}\,\text{Hypergeometric2F1}\left(-\frac{2}{3},1,-\frac{1}{3},\frac{cd(d+ex)}{cd^2-ae^2}\right)}{4(cd^2-ae^2)(d+ex)^2}$$

output
```
3/4*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(2/3)*hypergeom([-2/3, 1],[-1/3],c*d
*(e*x+d)/(-a*e^2+c*d^2))/(-a*e^2+c*d^2)/(e*x+d)^2
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 102, normalized size of antiderivative = 1.21

$$\int \frac{1}{(d+ex)^2 \sqrt[3]{ade+(cd^2+ae^2)x+cdex^2}}\,dx = \frac{3c^2d^2((ae+cdx)(d+ex))^{2/3}\,\text{Hypergeometric2F1}\left(\frac{2}{3},\frac{7}{3},\frac{5}{3},\frac{e(ae+cdx)}{-cd^2+ae^2}\right)}{2(cd^2-ae^2)^3\left(\frac{cd(d+ex)}{cd^2-ae^2}\right)^{2/3}}$$

input
```
Integrate[1/((d + e*x)^2*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(1/3)),x]
```

output
```
(3*c^2*d^2*((a*e + c*d*x)*(d + e*x))^(2/3)*Hypergeometric2F1[2/3, 7/3, 5/3
, (e*(a*e + c*d*x))/(-(c*d^2) + a*e^2)])/(2*(c*d^2 - a*e^2)^3*((c*d*(d + e
*x))/(c*d^2 - a*e^2))^(2/3))
```

Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 118, normalized size of antiderivative = 1.40, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.081, Rules used = {1138, 80, 79}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(d+ex)^2\sqrt[3]{x(ae^2+cd^2)+ade+cdex^2}}dx$$

$$\Big\downarrow \text{1138}$$

$$\frac{\sqrt[3]{\frac{ex}{d}+1}\sqrt[3]{ae+cdx}\int \frac{1}{\sqrt[3]{ae+cdx}\left(\frac{ex}{d}+1\right)^{7/3}}dx}{d^2\sqrt[3]{x(ae^2+cd^2)+ade+cdex^2}}$$

$$\Big\downarrow \text{80}$$

$$\frac{c^2d^2\sqrt[3]{ae+cdx}\sqrt[3]{\frac{cd(d+ex)}{cd^2-ae^2}}\int \frac{1}{\sqrt[3]{ae+cdx}\left(\frac{cd^2}{cd^2-ae^2}+\frac{cexd}{cd^2-ae^2}\right)^{7/3}}dx}{(cd^2-ae^2)^2\sqrt[3]{x(ae^2+cd^2)+ade+cdex^2}}$$

$$\Big\downarrow \text{79}$$

$$\frac{3cd(ae+cdx)\sqrt[3]{\frac{cd(d+ex)}{cd^2-ae^2}}\text{Hypergeometric2F1}\left(\frac{2}{3},\frac{7}{3},\frac{5}{3},-\frac{e(ae+cdx)}{cd^2-ae^2}\right)}{2(cd^2-ae^2)^2\sqrt[3]{x(ae^2+cd^2)+ade+cdex^2}}$$

input
```
Int[1/((d + e*x)^2*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(1/3)),x]
```

output
```
(3*c*d*(a*e + c*d*x)*((c*d*(d + e*x))/(c*d^2 - a*e^2))^(1/3)*Hypergeometri
c2F1[2/3, 7/3, 5/3, -((e*(a*e + c*d*x))/(c*d^2 - a*e^2))])/(2*(c*d^2 - a*e
^2)^2*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(1/3))
```

Defintions of rubi rules used

rule 79
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d), 0] && (RationalQ[m]
 ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

rule 80
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*(b*((c + d*x)/(b*c - a*d)))
^FracPart[n])   Int[(a + b*x)^m*Simp[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d)
), x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] &&  !IntegerQ[m] &&  !Integ
erQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

rule 1138
```
Int[((d_) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Sy
mbol] :> Simp[d^m*((a + b*x + c*x^2)^FracPart[p]/((1 + e*(x/d))^FracPart[p]
*(a/d + (c*x)/e)^FracPart[p]))   Int[(1 + e*(x/d))^(m + p)*(a/d + (c/e)*x)^
p, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0]
&& (IntegerQ[m] || GtQ[d, 0]) &&  !(IGtQ[m, 0] && (IntegerQ[3*p] || Integer
Q[4*p]))
```

Maple [F]

$$\int \frac{1}{(ex+d)^2\,(ade+(a\,e^2+c\,d^2)\,x+cd\,x^2e)^{\frac{1}{3}}}dx$$

input
```
int(1/(e*x+d)^2/(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(1/3),x)
```

output
```
int(1/(e*x+d)^2/(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(1/3),x)
```

Fricas [F]

$$\int \frac{1}{(d+ex)^2 \sqrt[3]{ade+(cd^2+ae^2)\,x+cdex^2}}\,dx = \int \frac{1}{(cdex^2+ade+(cd^2+ae^2)x)^{\frac{1}{3}}\,(ex+d)^2}\,dx$$

input
```
integrate(1/(e*x+d)^2/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/3),x, algorithm
="fricas")
```

output
```
integral((c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^(2/3)/(c*d*e^3*x^4 + a*d^
3*e + (3*c*d^2*e^2 + a*e^4)*x^3 + 3*(c*d^3*e + a*d*e^3)*x^2 + (c*d^4 + 3*a
*d^2*e^2)*x), x)
```

Sympy [F]

$$\int \frac{1}{(d+ex)^2 \sqrt[3]{ade+(cd^2+ae^2)\,x+cdex^2}}\,dx = \int \frac{1}{\sqrt[3]{(d+ex)\,(ae+cdx)}\,(d+ex)^2}\,dx$$

input
```
integrate(1/(e*x+d)**2/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(1/3),x)
```

output
```
Integral(1/(((d + e*x)*(a*e + c*d*x))**(1/3)*(d + e*x)**2), x)
```

Maxima [F]

$$\int \frac{1}{(d+ex)^2 \sqrt[3]{ade+(cd^2+ae^2)\,x+cdex^2}}\,dx = \int \frac{1}{(cdex^2+ade+(cd^2+ae^2)x)^{\frac{1}{3}}\,(ex+d)^2}\,dx$$

input
```
integrate(1/(e*x+d)^2/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/3),x, algorithm
="maxima")
```

output
```
integrate(1/((c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^(1/3)*(e*x + d)^2), x
)
```

Giac [F]

$$\int \frac{1}{(d+ex)^2\sqrt[3]{ade+(cd^2+ae^2)\,x+cdex^2}}\,dx = \int \frac{1}{(cdex^2+ade+(cd^2+ae^2)x)^{\frac{1}{3}}(ex+d)^2}\,dx$$

input
```
integrate(1/(e*x+d)^2/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/3),x, algorithm
="giac")
```

output
```
integrate(1/((c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^(1/3)*(e*x + d)^2), x
)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(d+ex)^2\sqrt[3]{ade+(cd^2+ae^2)\,x+cdex^2}}\,dx = \int \frac{1}{(d+e\,x)^2\,\left(c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e\right)^{1/3}}\,dx$$

input
```
int(1/((d + e*x)^2*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/3)),x)
```

output
```
int(1/((d + e*x)^2*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/3)), x)
```

Reduce [F]

$$\int \frac{1}{(d+ex)^2\sqrt[3]{ade+(cd^2+ae^2)\,x+cdex^2}}\,dx$$
$$=\int \frac{1}{(cde\,x^2+a\,e^2x+c\,d^2x+ade)^{\frac{1}{3}}\,d^2+2\,(cde\,x^2+a\,e^2x+c\,d^2x+ade)^{\frac{1}{3}}\,dex+(cde\,x^2+a\,e^2x+c\,d^2x\,\cdots}$$

input
```
int(1/(e*x+d)^2/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/3),x)
```

output
```
int(1/((a*d*e + a*e**2*x + c*d**2*x + c*d*e*x**2)**(1/3)*d**2 + 2*(a*d*e +
 a*e**2*x + c*d**2*x + c*d*e*x**2)**(1/3)*d*e*x + (a*d*e + a*e**2*x + c*d*
*2*x + c*d*e*x**2)**(1/3)*e**2*x**2),x)
```

3.337 $\int (d+ex)^2 \sqrt[4]{ade+(cd^2+ae^2)x+cdex^2}\,dx$

Optimal result

Integrand size = 37, antiderivative size = 350

$$\begin{aligned}\int (d+ex)^2 \sqrt[4]{ade+(cd^2+ae^2)x+cdex^2}\,dx &= \frac{3(cd^2-ae^2)^3\sqrt[4]{ade+(cd^2+ae^2)x+cdex^2}}{28c^3d^3e} \\ &+ \frac{9(cd^2-ae^2)(ade+(cd^2+ae^2)x+cdex^2)^{5/4}}{35c^2d^2} \\ &+ \frac{3(cd^2-ae^2)^2(ade+(cd^2+ae^2)x+cdex^2)^{5/4}}{14c^3d^3(d+ex)} \\ &+ \frac{2(d+ex)(ade+(cd^2+ae^2)x+cdex^2)^{5/4}}{7cd} \\ &+ \frac{3(cd^2-ae^2)^{7/2}(ae+cdx)^{3/2}\left(\frac{cd(d+ex)}{e(ae+cdx)}\right)^{3/4}\text{EllipticF}\left(\frac{1}{2}\arctan\left(\frac{\sqrt{cd^2-ae^2}}{\sqrt{e}\sqrt{ae+cdx}}\right),2\right)}{28c^4d^4\sqrt{e}\,(ade+(cd^2+ae^2)x+cdex^2)^{3/4}}\end{aligned}$$

output

```
3/28*(-a*e^2+c*d^2)^3*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/4)/c^3/d^3/e+9/
35*(-a*e^2+c*d^2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/4)/c^2/d^2+3/14*(-a
*e^2+c*d^2)^2*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/4)/c^3/d^3/(e*x+d)+2/7*
(e*x+d)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/4)/c/d+3/28*(-a*e^2+c*d^2)^(7
/2)*(c*d*x+a*e)^(3/2)*(c*d*(e*x+d)/e/(c*d*x+a*e))^(3/4)*InverseJacobiAM(1/
2*arctan((-a*e^2+c*d^2)^(1/2)/e^(1/2)/(c*d*x+a*e)^(1/2)),2^(1/2))/c^4/d^4/
e^(1/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/4)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.05 (sec) , antiderivative size = 110, normalized size of antiderivative = 0.31

$$\int (d+ex)^2\sqrt[4]{ade+(cd^2+ae^2)\,x+cdex^2}\,dx$$
$$=\frac{4(cd^2-ae^2)^2\,(ae+cdx)\sqrt[4]{(ae+cdx)(d+ex)}\,\text{Hypergeometric2F1}\left(-\frac{9}{4},\frac{5}{4},\frac{9}{4},\frac{e(ae+cdx)}{-cd^2+ae^2}\right)}{5c^3d^3\sqrt[4]{\frac{cd(d+ex)}{cd^2-ae^2}}}$$

input
```
Integrate[(d + e*x)^2*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(1/4),x]
```

output
```
(4*(c*d^2 - a*e^2)^2*(a*e + c*d*x)*((a*e + c*d*x)*(d + e*x))^(1/4)*Hyperge
ometric2F1[-9/4, 5/4, 9/4, (e*(a*e + c*d*x))/(-(c*d^2) + a*e^2)])/(5*c^3*d
^3*((c*d*(d + e*x))/(c*d^2 - a*e^2))^(1/4))
```

Rubi [A] (warning: unable to verify)

Time = 0.98 (sec) , antiderivative size = 581, normalized size of antiderivative = 1.66, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.162, Rules used = {1166, 27, 1160, 1087, 1094, 761}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (d+ex)^2\sqrt[4]{x\,(ae^2+cd^2)+ade+cdex^2}\,dx$$
$$\Big\downarrow \text{1166}$$
$$\frac{2\int \frac{9}{4}e\left(cd^2-ae^2\right)(d+ex)\sqrt[4]{cdex^2+(cd^2+ae^2)\,x+ade}dx}{7cde}+\frac{2(d+ex)\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/4}}{7cd}$$
$$\Big\downarrow \text{27}$$

$$\frac{9(cd^2-ae^2)\int (d+ex)\sqrt[4]{cdex^2+(cd^2+ae^2)x+ade}dx}{14cd}+\frac{2(d+ex)\left(x(ae^2+cd^2)+ade+cdex^2\right)^{5/4}}{7cd}$$

$\big\downarrow$ 1160

$$\frac{9(cd^2-ae^2)\left(\frac{\left(d^2-\frac{ae^2}{c}\right)\int \sqrt[4]{cdex^2+(cd^2+ae^2)x+ade}dx}{2d}+\frac{2\left(x(ae^2+cd^2)+ade+cdex^2\right)^{5/4}}{5cd}\right)}{14cd}+\frac{2(d+ex)\left(x(ae^2+cd^2)+ade+cdex^2\right)^{5/4}}{7cd}$$

$\big\downarrow$ 1087

$$\frac{9(cd^2-ae^2)\left(\frac{\left(d^2-\frac{ae^2}{c}\right)\left(\frac{(ae^2+cd^2+2cdex)\sqrt[4]{x(ae^2+cd^2)+ade+cdex^2}}{3cde}-\frac{(cd^2-ae^2)^2\int \frac{1}{\left(cdex^2+(cd^2+ae^2)x+ade\right)^{3/4}}dx}{12cde}\right)}{2d}+\right.}{14cd}$$

$$\frac{2(d+ex)\left(x(ae^2+cd^2)+ade+cdex^2\right)^{5/4}}{7cd}$$

$\big\downarrow$ 1094

$$\frac{9(cd^2-ae^2)\left(\frac{\left(d^2-\frac{ae^2}{c}\right)\left(\frac{(ae^2+cd^2+2cdex)\sqrt[4]{x(ae^2+cd^2)+ade+cdex^2}}{3cde}-\frac{(cd^2-ae^2)^2\sqrt{(ae^2+cd^2+2cdex)^2}\int \frac{}{\sqrt{(cd^2-ae^2)^2+4cd}}}{3cd}\right.}{2d}\right.}{14cd}$$

$$\frac{2(d+ex)\left(x(ae^2+cd^2)+ade+cdex^2\right)^{5/4}}{7cd}$$

$\big\downarrow$ 761

$$9\left(cd^2-ae^2\right)\left(\frac{\left(d^2-\frac{ae^2}{c}\right)\left(\frac{\left(ae^2+cd^2+2cdex\right)\sqrt[4]{x\left(ae^2+cd^2\right)+ade+cdex^2}}{3cde}-\frac{\left(cd^2-ae^2\right)^{5/2}\sqrt{\left(ae^2+cd^2+2cdex\right)^2}\left(\frac{2\sqrt{c}\sqrt{d}\sqrt{e}\sqrt{x\left(ae^2\right.}}{cd^2}\right.}{}\right.}{}\right.$$

$$\frac{2(d+ex)\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/4}}{7cd}$$

input
```
Int[(d + e*x)^2*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(1/4),x]
```

output
```
(2*(d + e*x)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/4))/(7*c*d) + (9*(
c*d^2 - a*e^2)*((2*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/4))/(5*c*d)
+ ((d^2 - (a*e^2)/c)*(((c*d^2 + a*e^2 + 2*c*d*e*x)*(a*d*e + (c*d^2 + a*e^2
)*x + c*d*e*x^2)^(1/4))/(3*c*d*e) - ((c*d^2 - a*e^2)^(5/2)*Sqrt[(c*d^2 + a
*e^2 + 2*c*d*e*x)^2]*(1 + (2*Sqrt[c]*Sqrt[d]*Sqrt[e]*Sqrt[a*d*e + (c*d^2 +
 a*e^2)*x + c*d*e*x^2])/(c*d^2 - a*e^2))*Sqrt[((c*d^2 - a*e^2)^2 + 4*c*d*e
*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2))/((c*d^2 - a*e^2)^2*(1 + (2*Sqrt[
c]*Sqrt[d]*Sqrt[e]*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(c*d^2 - a
*e^2))^2)]*EllipticF[2*ArcTan[(Sqrt[2]*c^(1/4)*d^(1/4)*e^(1/4)*(a*d*e + (c
*d^2 + a*e^2)*x + c*d*e*x^2)^(1/4))/Sqrt[c*d^2 - a*e^2]], 1/2])/(6*Sqrt[2]
*c^(5/4)*d^(5/4)*e^(5/4)*(c*d^2 + a*e^2 + 2*c*d*e*x)*Sqrt[(c*d^2 - a*e^2)^
2 + 4*c*d*e*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)])))/(2*d)))/(14*c*d)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 761
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 1087
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^p/(2*c*(2*p + 1))), x] - Simp[p*((b^2 - 4*a*c)/(2*c*(2*
p + 1)))   Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] &&
GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[3*p])
```

rule 1094
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[4*(Sqrt[(b
+ 2*c*x)^2]/(b + 2*c*x))   Subst[Int[x^(4*(p + 1) - 1)/Sqrt[b^2 - 4*a*c + 4
*c*x^4], x], x, (a + b*x + c*x^2)^(1/4)], x] /; FreeQ[{a, b, c}, x] && Inte
gerQ[4*p]
```

rule 1160
```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[e*((a + b*x + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Simp[(2*c*d - b
*e)/(2*c)   Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
 && NeQ[p, -1]
```

rule 1166
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[e*(d + e*x)^(m - 1)*((a + b*x + c*x^2)^(p + 1)/(c*(m + 2*p +
 1))), x] + Simp[1/(c*(m + 2*p + 1))   Int[(d + e*x)^(m - 2)*Simp[c*d^2*(m
+ 2*p + 1) - e*(a*e*(m - 1) + b*d*(p + 1)) + e*(2*c*d - b*e)*(m + p)*x, x]*
(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && If[Ration
alQ[m], GtQ[m, 1], SumSimplerQ[m, -2]] && NeQ[m + 2*p + 1, 0] && IntQuadrat
icQ[a, b, c, d, e, m, p, x]
```

Maple [F]

$$\int (ex+d)^2 \left(ade+\left(a\,e^2+c\,d^2\right)x+cd\,x^2e\right)^{\frac{1}{4}}dx$$

input
```
int((e*x+d)^2*(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(1/4),x)
```

output
```
int((e*x+d)^2*(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(1/4),x)
```

Fricas [F]

$$\int (d+ex)^2 \sqrt[4]{ade+(cd^2+ae^2)\,x+cdex^2}\,dx = \int \left(cdex^2+ade+\left(cd^2+ae^2\right)x\right)^{\frac{1}{4}}(ex+d)^2\,dx$$

input
```
integrate((e*x+d)^2*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/4),x, algorithm="
fricas")
```

output
```
integral((c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^(1/4)*(e^2*x^2 + 2*d*e*x
+ d^2), x)
```

Sympy [F]

$$\int (d+ex)^2 \sqrt[4]{ade+(cd^2+ae^2)\,x+cdex^2}\,dx = \int \sqrt[4]{(d+ex)\,(ae+cdx)}(d+ex)^2\,dx$$

input
```
integrate((e*x+d)**2*(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(1/4),x)
```

output
```
Integral(((d + e*x)*(a*e + c*d*x))**(1/4)*(d + e*x)**2, x)
```

Maxima [F]

$$\int (d+ex)^2 \sqrt[4]{ade+(cd^2+ae^2)\,x+cdex^2}\,dx$$
$$= \int \left(cdex^2+ade+\left(cd^2+ae^2\right)x\right)^{\frac{1}{4}}(ex+d)^2\,dx$$

input
```
integrate((e*x+d)^2*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/4),x, algorithm="
maxima")
```

output
```
integrate((c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^(1/4)*(e*x + d)^2, x)
```

Giac [F]

$$\int (d+ex)^2 \sqrt[4]{ade+(cd^2+ae^2)\,x+cdex^2}\,dx$$
$$= \int \left(cdex^2+ade+\left(cd^2+ae^2\right)x\right)^{\frac{1}{4}}(ex+d)^2\,dx$$

input
```
integrate((e*x+d)^2*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/4),x, algorithm="
giac")
```

output
```
integrate((c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^(1/4)*(e*x + d)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int (d+ex)^2 \sqrt[4]{ade+(cd^2+ae^2)\,x+cdex^2}\,dx$$
$$= \int (d+e\,x)^2\,\left(c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e\right)^{1/4}\,dx$$

input
```
int((d + e*x)^2*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/4),x)
```

output
```
int((d + e*x)^2*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/4), x)
```

Reduce [F]

$$\int (d+ex)^2 \sqrt[4]{ade+\left(cd^2+ae^2\right)x+cdex^2}\,dx = \text{Too large to display}$$

input
```
int((e*x+d)^2*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/4),x)
```

output
```
(48*(a*d*e + a*e**2*x + c*d**2*x + c*d*e*x**2)**(1/4)*a**3*d*e**5 - 12*(a*
d*e + a*e**2*x + c*d**2*x + c*d*e*x**2)**(1/4)*a**3*e**6*x - 160*(a*d*e +
a*e**2*x + c*d**2*x + c*d*e*x**2)**(1/4)*a**2*c*d**3*e**3 + 28*(a*d*e + a*
e**2*x + c*d**2*x + c*d*e*x**2)**(1/4)*a**2*c*d**2*e**4*x + 8*(a*d*e + a*e
**2*x + c*d**2*x + c*d*e*x**2)**(1/4)*a**2*c*d*e**5*x**2 + 272*(a*d*e + a*
e**2*x + c*d**2*x + c*d*e*x**2)**(1/4)*a*c**2*d**5*e + 252*(a*d*e + a*e**2
*x + c*d**2*x + c*d*e*x**2)**(1/4)*a*c**2*d**4*e**2*x + 240*(a*d*e + a*e**
2*x + c*d**2*x + c*d*e*x**2)**(1/4)*a*c**2*d**3*e**3*x**2 + 80*(a*d*e + a*
e**2*x + c*d**2*x + c*d*e*x**2)**(1/4)*a*c**2*d**2*e**4*x**3 + 212*(a*d*e
+ a*e**2*x + c*d**2*x + c*d*e*x**2)**(1/4)*c**3*d**6*x + 232*(a*d*e + a*e*
*2*x + c*d**2*x + c*d*e*x**2)**(1/4)*c**3*d**5*e*x**2 + 80*(a*d*e + a*e**2
*x + c*d**2*x + c*d*e*x**2)**(1/4)*c**3*d**4*e**2*x**3 + 15*int(((a*d*e +
a*e**2*x + c*d**2*x + c*d*e*x**2)**(1/4)*x)/(a**2*d*e**3 + a**2*e**4*x + a
*c*d**3*e + 2*a*c*d**2*e**2*x + a*c*d*e**3*x**2 + c**2*d**4*x + c**2*d**3*
e*x**2),x)*a**5*e**10 - 45*int(((a*d*e + a*e**2*x + c*d**2*x + c*d*e*x**2)
**(1/4)*x)/(a**2*d*e**3 + a**2*e**4*x + a*c*d**3*e + 2*a*c*d**2*e**2*x + a
*c*d*e**3*x**2 + c**2*d**4*x + c**2*d**3*e*x**2),x)*a**4*c*d**2*e**8 + 30*
int(((a*d*e + a*e**2*x + c*d**2*x + c*d*e*x**2)**(1/4)*x)/(a**2*d*e**3 + a
**2*e**4*x + a*c*d**3*e + 2*a*c*d**2*e**2*x + a*c*d*e**3*x**2 + c**2*d**4*
x + c**2*d**3*e*x**2),x)*a**3*c**2*d**4*e**6 + 30*int(((a*d*e + a*e**2*...
```

3.338 $\int (d+ex) \sqrt[4]{ade+(cd^2+ae^2)\,x+cdex^2}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 292

$$\begin{aligned}\int (d+ex) \sqrt[4]{ade+(cd^2+ae^2)\,x+cdex^2}\,dx &= \frac{(cd^2-ae^2)^2 \sqrt[4]{ade+(cd^2+ae^2)\,x+cdex^2}}{6c^2d^2e} + \frac{2(ade+(cd^2+ae^2)\,x+cdex^2)^{5/4}}{5cd} \\ &+ \frac{(cd^2-ae^2)\,(ade+(cd^2+ae^2)\,x+cdex^2)^{5/4}}{3c^2d^2(d+ex)} \\ &+ \frac{(cd^2-ae^2)^{5/2}\,(ae+cdx)^{3/2}\left(\frac{cd(d+ex)}{e(ae+cdx)}\right)^{3/4} \operatorname{EllipticF}\left(\frac{1}{2}\arctan\left(\frac{\sqrt{cd^2-ae^2}}{\sqrt{e}\sqrt{ae+cdx}}\right),2\right)}{6c^3d^3\sqrt{e}\,(ade+(cd^2+ae^2)\,x+cdex^2)^{3/4}}\end{aligned}$$

output
```
1/6*(-a*e^2+c*d^2)^2*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/4)/c^2/d^2/e+2/5
*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/4)/c/d+1/3*(-a*e^2+c*d^2)*(a*d*e+(a*
e^2+c*d^2)*x+c*d*e*x^2)^(5/4)/c^2/d^2/(e*x+d)+1/6*(-a*e^2+c*d^2)^(5/2)*(c*
d*x+a*e)^(3/2)*(c*d*(e*x+d)/e/(c*d*x+a*e))^(3/4)*InverseJacobiAM(1/2*arcta
n((-a*e^2+c*d^2)^(1/2)/e^(1/2)/(c*d*x+a*e)^(1/2)),2^(1/2))/c^3/d^3/e^(1/2)
/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/4)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.07 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.30

$$\int (d+ex)\sqrt[4]{ade+(cd^2+ae^2)x+cdex^2}\,dx = \frac{4((ae+cdx)(d+ex))^{5/4}\,\text{Hypergeometric2F1}\left(-\frac{5}{4},\frac{5}{4},\frac{9}{4},\frac{e(ae+cdx)}{-cd^2+ae^2}\right)}{5cd\left(\frac{cd(d+ex)}{cd^2-ae^2}\right)^{5/4}}$$

input
```
Integrate[(d + e*x)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(1/4),x]
```

output
```
(4*((a*e + c*d*x)*(d + e*x))^(5/4)*Hypergeometric2F1[-5/4, 5/4, 9/4, (e*(a
*e + c*d*x))/(-(c*d^2) + a*e^2)])/(5*c*d*((c*d*(d + e*x))/(c*d^2 - a*e^2))
^(5/4))
```

Rubi [A] (warning: unable to verify)

Time = 0.79 (sec) , antiderivative size = 514, normalized size of antiderivative = 1.76, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.114$, Rules used = {1160, 1087, 1094, 761}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (d+ex)\sqrt[4]{x(ae^2+cd^2)+ade+cdex^2}\,dx$$

$$\Big\downarrow \; 1160$$

$$\frac{\left(d^2-\frac{ae^2}{c}\right)\int \sqrt[4]{cdex^2+(cd^2+ae^2)x+ade}dx}{2d}+\frac{2\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/4}}{5cd}$$

$$\Big\downarrow \; 1087$$

$$\frac{\left(d^2-\frac{ae^2}{c}\right)\left(\frac{(ae^2+cd^2+2cdex)\sqrt[4]{x\left(ae^2+cd^2\right)+ade+cdex^2}}{3cde}-\frac{(cd^2-ae^2)^2\int\frac{1}{\left(cdex^2+\left(cd^2+ae^2\right)x+ade\right)^{3/4}}dx}{12cde}\right)}{2d}+$$

$$\frac{2\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/4}}{5cd}$$

$\Big\downarrow$ 1094

$$\frac{\left(d^2-\frac{ae^2}{c}\right)\left(\frac{(ae^2+cd^2+2cdex)\sqrt[4]{x\left(ae^2+cd^2\right)+ade+cdex^2}}{3cde}-\frac{(cd^2-ae^2)^2\sqrt{(ae^2+cd^2+2cdex)^2}\int\frac{1}{\sqrt{\left(cd^2-ae^2\right)^2+4cde\left(cdex^2+(\cdots\right.}}}{3cde(ae^2+cd^2\cdots}\right.}{2d}$$

$$\frac{2\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/4}}{5cd}$$

$\Big\downarrow$ 761

$$\frac{\left(d^2-\frac{ae^2}{c}\right)\left(\frac{(ae^2+cd^2+2cdex)\sqrt[4]{x\left(ae^2+cd^2\right)+ade+cdex^2}}{3cde}-\frac{(cd^2-ae^2)^{5/2}\sqrt{(ae^2+cd^2+2cdex)^2}\left(\frac{2\sqrt{c}\sqrt{d}\sqrt{e}\sqrt{x\left(ae^2+cd^2\right)+a\cdots}}{cd^2-ae^2}\right.}{\cdots}\right.}{\cdots}$$

$$\frac{2\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/4}}{5cd}$$

input
```
Int[(d + e*x)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(1/4),x]
```

output
```
(2*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/4))/(5*c*d) + ((d^2 - (a*e^2
)/c)*(((c*d^2 + a*e^2 + 2*c*d*e*x)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)
^(1/4))/(3*c*d*e) - ((c*d^2 - a*e^2)^(5/2)*Sqrt[(c*d^2 + a*e^2 + 2*c*d*e*x
)^2]*(1 + (2*Sqrt[c]*Sqrt[d]*Sqrt[e]*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*
e*x^2])/(c*d^2 - a*e^2))*Sqrt[((c*d^2 - a*e^2)^2 + 4*c*d*e*(a*d*e + (c*d^2
 + a*e^2)*x + c*d*e*x^2))/((c*d^2 - a*e^2)^2*(1 + (2*Sqrt[c]*Sqrt[d]*Sqrt[
e]*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(c*d^2 - a*e^2))^2)]*Ellip
ticF[2*ArcTan[(Sqrt[2]*c^(1/4)*d^(1/4)*e^(1/4)*(a*d*e + (c*d^2 + a*e^2)*x
+ c*d*e*x^2)^(1/4))/Sqrt[c*d^2 - a*e^2]], 1/2])/(6*Sqrt[2]*c^(5/4)*d^(5/4)
*e^(5/4)*(c*d^2 + a*e^2 + 2*c*d*e*x)*Sqrt[(c*d^2 - a*e^2)^2 + 4*c*d*e*(a*d
*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)])))/(2*d)
```

Defintions of rubi rules used

rule 761
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 1087
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^p/(2*c*(2*p + 1))), x] - Simp[p*((b^2 - 4*a*c)/(2*c*(2*
p + 1)))   Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] &&
GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[3*p])
```

rule 1094
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[4*(Sqrt[(b
+ 2*c*x)^2]/(b + 2*c*x))   Subst[Int[x^(4*(p + 1) - 1)/Sqrt[b^2 - 4*a*c + 4
*c*x^4], x], x, (a + b*x + c*x^2)^(1/4)], x] /; FreeQ[{a, b, c}, x] && Inte
gerQ[4*p]
```

rule 1160
```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[e*((a + b*x + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Simp[(2*c*d - b
*e)/(2*c)   Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
 && NeQ[p, -1]
```

Maple [F]

$$\int (ex+d)\left(ade+\left(a\,e^{2}+c\,d^{2}\right)x+cd\,x^{2}e\right)^{\frac{1}{4}}dx$$

input
```
int((e*x+d)*(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(1/4),x)
```

output
```
int((e*x+d)*(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(1/4),x)
```

Fricas [F]

$$\int (d+ex)\sqrt[4]{ade+(cd^2+ae^2)\,x+cdex^2}\,dx \\ =\int \left(cdex^2+ade+\left(cd^2+ae^2\right)x\right)^{\frac{1}{4}}(ex+d)\,dx$$

input
```
integrate((e*x+d)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/4),x, algorithm="fr
icas")
```

output
```
integral((c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^(1/4)*(e*x + d), x)
```

Sympy [F]

$$\int (d+ex)\sqrt[4]{ade+(cd^2+ae^2)\,x+cdex^2}\,dx=\int \sqrt[4]{(d+ex)\,(ae+cdx)}(d+ex)\,dx$$

input
```
integrate((e*x+d)*(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(1/4),x)
```

output
```
Integral(((d + e*x)*(a*e + c*d*x))**(1/4)*(d + e*x), x)
```

Maxima [F]

$$\int (d+ex)\sqrt[4]{ade+(cd^2+ae^2)\,x+cdex^2}\,dx$$
$$=\int \left(cdex^2+ade+\left(cd^2+ae^2\right)x\right)^{\frac{1}{4}}(ex+d)\,dx$$

input
```
integrate((e*x+d)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/4),x, algorithm="ma
xima")
```

output
```
integrate((c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^(1/4)*(e*x + d), x)
```

Giac [F]

$$\int (d+ex)\sqrt[4]{ade+(cd^2+ae^2)\,x+cdex^2}\,dx$$
$$=\int \left(cdex^2+ade+\left(cd^2+ae^2\right)x\right)^{\frac{1}{4}}(ex+d)\,dx$$

input
```
integrate((e*x+d)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/4),x, algorithm="gi
ac")
```

output
```
integrate((c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^(1/4)*(e*x + d), x)
```

Mupad [F(-1)]

Timed out.

$$\int (d+ex)\sqrt[4]{ade+(cd^2+ae^2)\,x+cdex^2}\,dx$$
$$=\int (d+e\,x)\,\left(c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e\right)^{1/4}\,dx$$

input
```
int((d + e*x)*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/4),x)
```

output
```
int((d + e*x)*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/4), x)
```

Reduce [F]

$$\int (d+ex)\sqrt[4]{ade+(cd^2+ae^2)\,x+cdex^2}\,dx$$

$$= \frac{-16(cde\,x^2+a\,e^2x+c\,d^2x+ade)^{\frac{1}{4}}\,a^2d\,e^3+4(cde\,x^2+a\,e^2x+c\,d^2x+ade)^{\frac{1}{4}}\,a^2e^4x+64(cde\,x^2+a\,e^2x}{}$$

input
```
int((e*x+d)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/4),x)
```

output
```
( - 16*(a*d*e + a*e**2*x + c*d**2*x + c*d*e*x**2)**(1/4)*a**2*d*e**3 + 4*(
a*d*e + a*e**2*x + c*d**2*x + c*d*e*x**2)**(1/4)*a**2*e**4*x + 64*(a*d*e +
 a*e**2*x + c*d**2*x + c*d*e*x**2)**(1/4)*a*c*d**3*e + 48*(a*d*e + a*e**2*
x + c*d**2*x + c*d*e*x**2)**(1/4)*a*c*d**2*e**2*x + 24*(a*d*e + a*e**2*x +
 c*d**2*x + c*d*e*x**2)**(1/4)*a*c*d*e**3*x**2 + 44*(a*d*e + a*e**2*x + c*
d**2*x + c*d*e*x**2)**(1/4)*c**2*d**4*x + 24*(a*d*e + a*e**2*x + c*d**2*x
+ c*d*e*x**2)**(1/4)*c**2*d**3*e*x**2 - 5*int(((a*d*e + a*e**2*x + c*d**2*
x + c*d*e*x**2)**(1/4)*x)/(a**2*d*e**3 + a**2*e**4*x + a*c*d**3*e + 2*a*c*
d**2*e**2*x + a*c*d*e**3*x**2 + c**2*d**4*x + c**2*d**3*e*x**2),x)*a**4*e*
*8 + 10*int(((a*d*e + a*e**2*x + c*d**2*x + c*d*e*x**2)**(1/4)*x)/(a**2*d*
e**3 + a**2*e**4*x + a*c*d**3*e + 2*a*c*d**2*e**2*x + a*c*d*e**3*x**2 + c*
*2*d**4*x + c**2*d**3*e*x**2),x)*a**3*c*d**2*e**6 - 10*int(((a*d*e + a*e**
2*x + c*d**2*x + c*d*e*x**2)**(1/4)*x)/(a**2*d*e**3 + a**2*e**4*x + a*c*d*
*3*e + 2*a*c*d**2*e**2*x + a*c*d*e**3*x**2 + c**2*d**4*x + c**2*d**3*e*x**
2),x)*a*c**3*d**6*e**2 + 5*int(((a*d*e + a*e**2*x + c*d**2*x + c*d*e*x**2)
**(1/4)*x)/(a**2*d*e**3 + a**2*e**4*x + a*c*d**3*e + 2*a*c*d**2*e**2*x + a
*c*d*e**3*x**2 + c**2*d**4*x + c**2*d**3*e*x**2),x)*c**4*d**8)/(60*c*d*(a*
e**2 + c*d**2))
```

3.339 $\int \sqrt[4]{ade+(cd^2+ae^2)x+cdex^2}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 206

$$\int \sqrt[4]{ade+(cd^2+ae^2)x+cdex^2}\,dx = \frac{(cd^2+ae^2+2cdex)\sqrt[4]{ade+(cd^2+ae^2)x+cdex^2}}{3cde} - \frac{(cd^2-ae^2)^3\left(-\frac{cde(ade+(cd^2+ae^2)x+cdex^2)}{(cd^2-ae^2)^2}\right)^{3/4}\text{EllipticF}\left(\frac{1}{2}\arcsin\left(\frac{ae^2+cd(d+2ex)}{cd^2-ae^2}\right),2\right)}{3\sqrt{2}c^2d^2e^2(ade+(cd^2+ae^2)x+cdex^2)^{3/4}}$$

output
```
1/3*(2*c*d*e*x+a*e^2+c*d^2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/4)/c/d/e-
1/6*(-a*e^2+c*d^2)^3*(-c*d*e*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)/(-a*e^2+c*d
^2)^2)^(3/4)*InverseJacobiAM(1/2*arcsin((a*e^2+c*d*(2*e*x+d))/(-a*e^2+c*d^
2)),2^(1/2))*2^(1/2)/c^2/d^2/e^2/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/4)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.03 (sec) , antiderivative size = 96, normalized size of antiderivative = 0.47

$$\int \sqrt[4]{ade+(cd^2+ae^2)\,x+cdex^2}\,dx = \frac{4(ae+cdx)\sqrt[4]{(ae+cdx)(d+ex)}\,\text{Hypergeometric2F1}\left(-\frac{1}{4},\frac{5}{4},\frac{9}{4},\frac{e(ae+cdx)}{-cd^2+ae^2}\right)}{5cd\sqrt[4]{\frac{cd(d+ex)}{cd^2-ae^2}}}$$

input
```
Integrate[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(1/4),x]
```

output
```
(4*(a*e + c*d*x)*((a*e + c*d*x)*(d + e*x))^(1/4)*Hypergeometric2F1[-1/4, 5
/4, 9/4, (e*(a*e + c*d*x))/(-(c*d^2) + a*e^2)])/(5*c*d*((c*d*(d + e*x))/(c
*d^2 - a*e^2))^(1/4))
```

Rubi [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 454 vs. 2(206) = 412.

Time = 0.71 (sec) , antiderivative size = 454, normalized size of antiderivative = 2.20, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.103, Rules used = {1087, 1094, 761}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt[4]{x\,(ae^2+cd^2)+ade+cdex^2}\,dx$$

$$\Big\downarrow \text{1087}$$

$$\frac{(ae^2+cd^2+2cdex)\sqrt[4]{x\,(ae^2+cd^2)+ade+cdex^2}}{3cde}-\frac{(cd^2-ae^2)^2\int\frac{1}{(cdex^2+(cd^2+ae^2)x+ade)^{3/4}}dx}{12cde}$$

$$\Big\downarrow \text{1094}$$

$$\frac{(ae^2+cd^2+2cdex)\sqrt[4]{x(ae^2+cd^2)+ade+cdex^2}}{3cde}-\frac{(cd^2-ae^2)^2\sqrt{(ae^2+cd^2+2cdex)^2}\int \frac{1}{\sqrt{(cd^2-ae^2)^2+4cde(cdex^2+(cd^2+ae^2)x+ade)}}d\sqrt[4]{cdex^2+(cd^2+ae^2)x+ade}}{3cde(ae^2+cd^2+2cdex)}$$

$$\Big\downarrow \text{761}$$

$$\frac{(ae^2+cd^2+2cdex)\sqrt[4]{x(ae^2+cd^2)+ade+cdex^2}}{3cde}-\frac{(cd^2-ae^2)^{5/2}\sqrt{(ae^2+cd^2+2cdex)^2}\left(\frac{2\sqrt{c}\sqrt{d}\sqrt{e}\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{cd^2-ae^2}+1\right)\sqrt{\frac{4cde(x(ae^2+cd^2)+ade+cdex^2)+(cd^2-}{(cd^2-ae^2)^2\left(\frac{2\sqrt{c}\sqrt{d}\sqrt{e}\sqrt{x\left(ae^2+cd^2\right)+ade+c}}{cd^2-ae^2}\right.}}}{6\sqrt{2}c^{5/4}d^{5/4}e^{5/4}(ae^2+cd^2+2cdex)\sqrt{4cde(x(ae^2+cd^2)+}}$$

input

```
Int[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(1/4),x]
```

output

```
((c*d^2 + a*e^2 + 2*c*d*e*x)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(1/4)
)/(3*c*d*e) - ((c*d^2 - a*e^2)^(5/2)*Sqrt[(c*d^2 + a*e^2 + 2*c*d*e*x)^2]*(
1 + (2*Sqrt[c]*Sqrt[d]*Sqrt[e]*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]
)/(c*d^2 - a*e^2))*Sqrt[((c*d^2 - a*e^2)^2 + 4*c*d*e*(a*d*e + (c*d^2 + a*e
^2)*x + c*d*e*x^2))/((c*d^2 - a*e^2)^2*(1 + (2*Sqrt[c]*Sqrt[d]*Sqrt[e]*Sqr
t[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(c*d^2 - a*e^2))^2)]*EllipticF[2
*ArcTan[(Sqrt[2]*c^(1/4)*d^(1/4)*e^(1/4)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*
e*x^2)^(1/4))/Sqrt[c*d^2 - a*e^2]], 1/2])/(6*Sqrt[2]*c^(5/4)*d^(5/4)*e^(5/
4)*(c*d^2 + a*e^2 + 2*c*d*e*x)*Sqrt[(c*d^2 - a*e^2)^2 + 4*c*d*e*(a*d*e + (
c*d^2 + a*e^2)*x + c*d*e*x^2)])
```

Defintions of rubi rules used

rule 761

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 1087
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^p/(2*c*(2*p + 1))), x] - Simp[p*((b^2 - 4*a*c)/(2*c*(2*
p + 1)))   Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] &&
GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[3*p])
```

rule 1094
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[4*(Sqrt[(b
+ 2*c*x)^2]/(b + 2*c*x))   Subst[Int[x^(4*(p + 1) - 1)/Sqrt[b^2 - 4*a*c + 4
*c*x^4], x], x, (a + b*x + c*x^2)^(1/4)], x] /; FreeQ[{a, b, c}, x] && Inte
gerQ[4*p]
```

Maple [F]

$$\int \left(ade+\left(a\,e^{2}+c\,d^{2}\right)x+cd\,x^{2}e\right)^{\frac{1}{4}}dx$$

input
```
int((a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(1/4),x)
```

output
```
int((a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(1/4),x)
```

Fricas [F]

$$\int \sqrt[4]{ade+(cd^2+ae^2)\,x+cdex^2}\,dx=\int \left(cdex^2+ade+\left(cd^2+ae^2\right)x\right)^{\frac{1}{4}}dx$$

input
```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/4),x, algorithm="fricas")
```

output
```
integral((c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^(1/4), x)
```

Sympy [F]

$$\int \sqrt[4]{ade + (cd^2 + ae^2)\,x + cdex^2}\,dx = \int \sqrt[4]{ade + cdex^2 + x\,(ae^2 + cd^2)}\,dx$$

input
```
integrate((a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(1/4),x)
```

output
```
Integral((a*d*e + c*d*e*x**2 + x*(a*e**2 + c*d**2))**(1/4), x)
```

Maxima [F]

$$\int \sqrt[4]{ade + (cd^2 + ae^2)\,x + cdex^2}\,dx = \int \left(cdex^2 + ade + \left(cd^2 + ae^2\right)x\right)^{\frac{1}{4}}\,dx$$

input
```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/4),x, algorithm="maxima")
```

output
```
integrate((c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^(1/4), x)
```

Giac [F]

$$\int \sqrt[4]{ade + (cd^2 + ae^2)\,x + cdex^2}\,dx = \int \left(cdex^2 + ade + \left(cd^2 + ae^2\right)x\right)^{\frac{1}{4}}\,dx$$

input
```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/4),x, algorithm="giac")
```

output
```
integrate((c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^(1/4), x)
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt[4]{ade+(cd^2+ae^2)\,x+cdex^2}\,dx=\int \left(c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e\right)^{1/4}dx$$

input
```
int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/4),x)
```

output
```
int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/4), x)
```

Reduce [F]

$$\int \sqrt[4]{ade+(cd^2+ae^2)\,x+cdex^2}\,dx$$

$$= \frac{8(cde\,x^2+a\,e^2x+c\,d^2x+ade)^{\frac{1}{4}}\,ade+4(cde\,x^2+a\,e^2x+c\,d^2x+ade)^{\frac{1}{4}}\,a\,e^2x+4(cde\,x^2+a\,e^2x+c\,d^2}{}$$

input
```
int((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/4),x)
```

output
```
(8*(a*d*e + a*e**2*x + c*d**2*x + c*d*e*x**2)**(1/4)*a*d*e + 4*(a*d*e + a*
e**2*x + c*d**2*x + c*d*e*x**2)**(1/4)*a*e**2*x + 4*(a*d*e + a*e**2*x + c*
d**2*x + c*d*e*x**2)**(1/4)*c*d**2*x + int(((a*d*e + a*e**2*x + c*d**2*x +
 c*d*e*x**2)**(1/4)*x)/(a**2*d*e**3 + a**2*e**4*x + a*c*d**3*e + 2*a*c*d**
2*e**2*x + a*c*d*e**3*x**2 + c**2*d**4*x + c**2*d**3*e*x**2),x)*a**3*e**6
- int(((a*d*e + a*e**2*x + c*d**2*x + c*d*e*x**2)**(1/4)*x)/(a**2*d*e**3 +
 a**2*e**4*x + a*c*d**3*e + 2*a*c*d**2*e**2*x + a*c*d*e**3*x**2 + c**2*d**
4*x + c**2*d**3*e*x**2),x)*a**2*c*d**2*e**4 - int(((a*d*e + a*e**2*x + c*d
**2*x + c*d*e*x**2)**(1/4)*x)/(a**2*d*e**3 + a**2*e**4*x + a*c*d**3*e + 2*
a*c*d**2*e**2*x + a*c*d*e**3*x**2 + c**2*d**4*x + c**2*d**3*e*x**2),x)*a*c
**2*d**4*e**2 + int(((a*d*e + a*e**2*x + c*d**2*x + c*d*e*x**2)**(1/4)*x)/
(a**2*d*e**3 + a**2*e**4*x + a*c*d**3*e + 2*a*c*d**2*e**2*x + a*c*d*e**3*x
**2 + c**2*d**4*x + c**2*d**3*e*x**2),x)*c**3*d**6)/(6*(a*e**2 + c*d**2))
```

3.340 $\int \frac{\sqrt[4]{ade+(cd^2+ae^2)x+cdex^2}}{d+ex}\,dx$

Optimal result

Integrand size = 37, antiderivative size = 171

$$\int \frac{\sqrt[4]{ade+(cd^2+ae^2)x+cdex^2}}{d+ex}\,dx = \frac{2\sqrt[4]{ade+(cd^2+ae^2)x+cdex^2}}{e} + \frac{2\sqrt{cd^2-ae^2}(ae+cdx)^{3/2}\left(\frac{cd(d+ex)}{e(ae+cdx)}\right)^{3/4}\operatorname{EllipticF}\left(\frac{1}{2}\arctan\left(\frac{\sqrt{cd^2-ae^2}}{\sqrt{e}\sqrt{ae+cdx}}\right),2\right)}{cd\sqrt{e}\left(ade+(cd^2+ae^2)x+cdex^2\right)^{3/4}}$$

output

```
2*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/4)/e+2*(-a*e^2+c*d^2)^(1/2)*(c*d*x+
a*e)^(3/2)*(c*d*(e*x+d)/e/(c*d*x+a*e))^(3/4)*InverseJacobiAM(1/2*arctan((-
a*e^2+c*d^2)^(1/2)/e^(1/2)/(c*d*x+a*e)^(1/2)),2^(1/2))/c/d/e^(1/2)/(a*d*e+
(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/4)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.04 (sec) , antiderivative size = 98, normalized size of antiderivative = 0.57

$$\int \frac{\sqrt[4]{ade+(cd^2+ae^2)x+cdex^2}}{d+ex}\,dx = \frac{4(ae+cdx)^2\left(\frac{cd(d+ex)}{cd^2-ae^2}\right)^{3/4}\text{Hypergeometric2F1}\left(\frac{3}{4},\frac{5}{4},\frac{9}{4},\frac{e(ae+cdx)}{-cd^2+ae^2}\right)}{5cd((ae+cdx)(d+ex))^{3/4}}$$

input
```
Integrate[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(1/4)/(d + e*x),x]
```

output
```
(4*(a*e + c*d*x)^2*((c*d*(d + e*x))/(c*d^2 - a*e^2))^(3/4)*Hypergeometric2
F1[3/4, 5/4, 9/4, (e*(a*e + c*d*x))/(-(c*d^2) + a*e^2)])/(5*c*d*((a*e + c*
d*x)*(d + e*x))^(3/4))
```

Rubi [A] (warning: unable to verify)

Time = 0.67 (sec) , antiderivative size = 236, normalized size of antiderivative = 1.38, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.189$, Rules used = {1138, 60, 73, 768, 858, 807, 229}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt[4]{x(ae^2+cd^2)+ade+cdex^2}}{d+ex}\,dx$$

$$\Big\downarrow \; 1138$$

$$\frac{\sqrt[4]{x(ae^2+cd^2)+ade+cdex^2}\int \frac{\sqrt[4]{ae+cdx}}{\left(\frac{ex}{d}+1\right)^{3/4}}dx}{d\sqrt[4]{\frac{ex}{d}+1}\sqrt[4]{ae+cdx}}$$

$$\Big\downarrow \; 60$$

$$\frac{\sqrt[4]{x\left(ae^2+cd^2\right)+ade+cdex^2}\left(\frac{2d\sqrt[4]{\frac{ex}{d}+1}\sqrt[4]{ae+cdx}}{e}-\frac{\left(cd^2-ae^2\right)\int\frac{1}{(ae+cdx)^{3/4}\left(\frac{ex}{d}+1\right)^{3/4}}dx}{2e}\right)}{d\sqrt[4]{\frac{ex}{d}+1}\sqrt[4]{ae+cdx}}$$

$$\Big\downarrow \text{73}$$

$$\frac{\sqrt[4]{x\left(ae^2+cd^2\right)+ade+cdex^2}\left(\frac{2d\sqrt[4]{\frac{ex}{d}+1}\sqrt[4]{ae+cdx}}{e}-\frac{2\left(cd^2-ae^2\right)\int\frac{1}{\left(-\frac{ae^2}{cd^2}+\frac{(ae+cdx)e}{cd^2}+1\right)^{3/4}}d\sqrt[4]{ae+cdx}}{cde}\right)}{d\sqrt[4]{\frac{ex}{d}+1}\sqrt[4]{ae+cdx}}$$

$$\Big\downarrow \text{768}$$

$$\frac{\sqrt[4]{x\left(ae^2+cd^2\right)+ade+cdex^2}\left(\frac{2d\sqrt[4]{\frac{ex}{d}+1}\sqrt[4]{ae+cdx}}{e}-\frac{2\left(cd^2-ae^2\right)(ae+cdx)^{3/4}\left(\frac{cd^2-ae^2}{e(ae+cdx)}+1\right)^{3/4}\int\frac{1}{(ae+cdx)^{3/4}\left(\frac{cd^2-ae^2}{e(ae+cdx}\right.}}{cde\left(-\frac{ae^2}{cd^2}+\frac{e(ae+cdx)}{cd^2}+1\right)^{3/4}}\right.}{d\sqrt[4]{\frac{ex}{d}+1}\sqrt[4]{ae+cdx}}$$

$$\Big\downarrow \text{858}$$

$$\frac{\sqrt[4]{x\left(ae^2+cd^2\right)+ade+cdex^2}\left(\frac{2\left(cd^2-ae^2\right)(ae+cdx)^{3/4}\left(\frac{cd^2-ae^2}{e(ae+cdx)}+1\right)^{3/4}\int\frac{1}{\sqrt[4]{ae+cdx}\left(\frac{\left(cd^2-ae^2\right)(ae+cdx)}{e}+1\right)^{3/4}}d\frac{1}{\sqrt[4]{ae+c}}}{cde\left(-\frac{ae^2}{cd^2}+\frac{e(ae+cdx)}{cd^2}+1\right)^{3/4}}\right.}{d\sqrt[4]{\frac{ex}{d}+1}\sqrt[4]{ae+cdx}}$$

$$\Big\downarrow \text{807}$$

$$\frac{\sqrt[4]{x\left(ae^2+cd^2\right)+ade+cdex^2}\left(\frac{\left(cd^2-ae^2\right)(ae+cdx)^{3/4}\left(\frac{cd^2-ae^2}{e(ae+cdx)}+1\right)^{3/4}\int\frac{1}{\left(\frac{\sqrt{ae+cdx}\left(cd^2-ae^2\right)}{e}+1\right)^{3/4}}d\sqrt{ae+cdx}}{cde\left(-\frac{ae^2}{cd^2}+\frac{e(ae+cdx)}{cd^2}+1\right)^{3/4}}+\frac{2d\sqrt[4]{\frac{ex}{d}+\dots}}{\dots}\right.}{d\sqrt[4]{\frac{ex}{d}+1}\sqrt[4]{ae+cdx}}$$

$$\Big\downarrow 229$$

$$\frac{\sqrt[4]{x\left(ae^2+cd^2\right)+ade+cdex^2}\left(\frac{2\sqrt{cd^2-ae^2}(ae+cdx)^{3/4}\left(\frac{cd^2-ae^2}{e(ae+cdx)}+1\right)^{3/4}\text{EllipticF}\left(\frac{1}{2}\arctan\left(\frac{\sqrt{cd^2-ae^2}\sqrt{ae+cdx}}{\sqrt{e}}\right),2\right)}{cd\sqrt{e}\left(-\frac{ae^2}{cd^2}+\frac{e(ae+cdx)}{cd^2}+1\right)^{3/4}}+\frac{2d\sqrt[4]{\frac{e}{d}}}{}\right.}{d\sqrt[4]{\frac{ex}{d}+1}\sqrt[4]{ae+cdx}}$$

input
```
Int[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(1/4)/(d + e*x),x]
```

output
```
((a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(1/4)*((2*d*(a*e + c*d*x)^(1/4)*(
1 + (e*x)/d)^(1/4))/e + (2*Sqrt[c*d^2 - a*e^2]*(a*e + c*d*x)^(3/4)*(1 + (c
*d^2 - a*e^2)/(e*(a*e + c*d*x)))^(3/4)*EllipticF[ArcTan[(Sqrt[c*d^2 - a*e^
2]*Sqrt[a*e + c*d*x])/Sqrt[e]]/2, 2])/(c*d*Sqrt[e]*(1 - (a*e^2)/(c*d^2) +
(e*(a*e + c*d*x))/(c*d^2))^(3/4))))/(d*(a*e + c*d*x)^(1/4)*(1 + (e*x)/d)^(
1/4))
```

Defintions of rubi rules used

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 229
```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(2/(a^(3/4)*Rt[b/a, 2])
)*EllipticF[(1/2)*ArcTan[Rt[b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

rule 768
```
Int[((a_) + (b_.)*(x_)^4)^(-3/4), x_Symbol] :> Simp[x^3*((1 + a/(b*x^4))^(3
/4)/(a + b*x^4)^(3/4))   Int[1/(x^3*(1 + a/(b*x^4))^(3/4)), x], x] /; FreeQ
[{a, b}, x]
```

rule 807
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Simp[1/k   Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x,
x^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

rule 858
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

rule 1138
```
Int[((d_) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Sy
mbol] :> Simp[d^m*((a + b*x + c*x^2)^FracPart[p]/((1 + e*(x/d))^FracPart[p]
*(a/d + (c*x)/e)^FracPart[p]))   Int[(1 + e*(x/d))^(m + p)*(a/d + (c/e)*x)^
p, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0]
&& (IntegerQ[m] || GtQ[d, 0]) &&  !(IGtQ[m, 0] && (IntegerQ[3*p] || Integer
Q[4*p]))
```

Maple [F]

$$\int \frac{\left(ade+\left(a\,e^{2}+c\,d^{2}\right)x+cd\,x^{2}e\right)^{\frac{1}{4}}}{ex+d}dx$$

input
```
int((a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(1/4)/(e*x+d),x)
```

output
```
int((a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(1/4)/(e*x+d),x)
```

Fricas [F]

$$\int \frac{\sqrt[4]{ade + (cd^2 + ae^2)\,x + cdex^2}}{d + ex}\,dx = \int \frac{(cdex^2 + ade + (cd^2 + ae^2)x)^{\frac{1}{4}}}{ex + d}\,dx$$

input
```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/4)/(e*x+d),x, algorithm="fr
icas")
```

output
```
integral((c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^(1/4)/(e*x + d), x)
```

Sympy [F]

$$\int \frac{\sqrt[4]{ade + (cd^2 + ae^2)\,x + cdex^2}}{d + ex}\,dx = \int \frac{\sqrt[4]{(d + ex)\,(ae + cdx)}}{d + ex}\,dx$$

input
```
integrate((a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(1/4)/(e*x+d),x)
```

output
```
Integral(((d + e*x)*(a*e + c*d*x))**(1/4)/(d + e*x), x)
```

Maxima [F]

$$\int \frac{\sqrt[4]{ade + (cd^2 + ae^2)\,x + cdex^2}}{d + ex}\,dx = \int \frac{(cdex^2 + ade + (cd^2 + ae^2)x)^{\frac{1}{4}}}{ex + d}\,dx$$

input
```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/4)/(e*x+d),x, algorithm="ma
xima")
```

output
```
integrate((c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^(1/4)/(e*x + d), x)
```

Giac [F]

$$\int \frac{\sqrt[4]{ade + (cd^2 + ae^2)x + cdex^2}}{d + ex}\,dx = \int \frac{(cdex^2 + ade + (cd^2 + ae^2)x)^{\frac{1}{4}}}{ex + d}\,dx$$

input
```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/4)/(e*x+d),x, algorithm="gi
ac")
```

output
```
integrate((c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^(1/4)/(e*x + d), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt[4]{ade + (cd^2 + ae^2)x + cdex^2}}{d + ex}\,dx = \int \frac{(c\,d\,e\,x^2 + (c\,d^2 + a\,e^2)\,x + a\,d\,e)^{1/4}}{d + e\,x}\,dx$$

input
```
int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/4)/(d + e*x),x)
```

output
```
int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/4)/(d + e*x), x)
```

Reduce [F]

$$\int \frac{\sqrt[4]{ade + (cd^2 + ae^2)x + cdex^2}}{d + ex}\,dx$$
$$= \frac{4(cde\,x^2 + a\,e^2x + c\,d^2x + ade)^{\frac{1}{4}}\,ae - \left(\int \frac{(cde\,x^2 + a\,e^2x + c\,d^2x + ade)^{\frac{1}{4}}\,x}{acd\,e^3x^2 + c^2d^3e\,x^2 + a^2e^4x + 2ac\,d^2e^2x + c^2d^4x + a^2d\,e^3 + ac\,d^3e}dx\right)a^2cd\,e^4 + \Big(}{a\,e^2 + c\,d^2}$$

input
```
int((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/4)/(e*x+d),x)
```

output

```
(4*(a*d*e + a*e**2*x + c*d**2*x + c*d*e*x**2)**(1/4)*a*e - int(((a*d*e + a
*e**2*x + c*d**2*x + c*d*e*x**2)**(1/4)*x)/(a**2*d*e**3 + a**2*e**4*x + a*
c*d**3*e + 2*a*c*d**2*e**2*x + a*c*d*e**3*x**2 + c**2*d**4*x + c**2*d**3*e
*x**2),x)*a**2*c*d*e**4 + int(((a*d*e + a*e**2*x + c*d**2*x + c*d*e*x**2)*
*(1/4)*x)/(a**2*d*e**3 + a**2*e**4*x + a*c*d**3*e + 2*a*c*d**2*e**2*x + a*
c*d*e**3*x**2 + c**2*d**4*x + c**2*d**3*e*x**2),x)*c**3*d**5)/(a*e**2 + c*
d**2)
```

3.341 $\int \frac{\sqrt[4]{ade+(cd^2+ae^2)x+cdex^2}}{(d+ex)^2}\,dx$

Optimal result

Integrand size = 37, antiderivative size = 176

$$\begin{aligned}\int \frac{\sqrt[4]{ade+(cd^2+ae^2)x+cdex^2}}{(d+ex)^2}\,dx &= -\frac{4\sqrt[4]{ade+(cd^2+ae^2)x+cdex^2}}{3e(d+ex)} \\ &\quad -\frac{4(ae+cdx)^{3/2}\left(\frac{cd(d+ex)}{e(ae+cdx)}\right)^{3/4}\operatorname{EllipticF}\left(\frac{1}{2}\arctan\left(\frac{\sqrt{cd^2-ae^2}}{\sqrt{e}\sqrt{ae+cdx}}\right),2\right)}{3\sqrt{e}\sqrt{cd^2-ae^2}\,(ade+(cd^2+ae^2)x+cdex^2)^{3/4}}\end{aligned}$$

output
```
-4/3*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/4)/e/(e*x+d)-4/3*(c*d*x+a*e)^(3/
2)*(c*d*(e*x+d)/e/(c*d*x+a*e))^(3/4)*InverseJacobiAM(1/2*arctan((-a*e^2+c*
d^2)^(1/2)/e^(1/2)/(c*d*x+a*e)^(1/2)),2^(1/2))/e^(1/2)/(-a*e^2+c*d^2)^(1/2
)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/4)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.06 (sec) , antiderivative size = 106, normalized size of antiderivative = 0.60

$$\int \frac{\sqrt[4]{ade+(cd^2+ae^2)\,x+cdex^2}}{(d+ex)^2}\,dx = \frac{4cd(ae+cdx)\sqrt[4]{(ae+cdx)(d+ex)}\,\text{Hypergeometric2F1}\left(\frac{5}{4},\frac{7}{4},\frac{9}{4},\frac{e(ae+cdx)}{-cd^2+ae^2}\right)}{5\,(cd^2-ae^2)^2\sqrt[4]{\frac{cd(d+ex)}{cd^2-ae^2}}}$$

input
```
Integrate[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(1/4)/(d + e*x)^2,x]
```

output
```
(4*c*d*(a*e + c*d*x)*((a*e + c*d*x)*(d + e*x))^(1/4)*Hypergeometric2F1[5/4
, 7/4, 9/4, (e*(a*e + c*d*x))/(-(c*d^2) + a*e^2)])/(5*(c*d^2 - a*e^2)^2*((
c*d*(d + e*x))/(c*d^2 - a*e^2))^(1/4))
```

Rubi [A] (warning: unable to verify)

Time = 0.66 (sec) , antiderivative size = 235, normalized size of antiderivative = 1.34, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.189$, Rules used = {1138, 57, 73, 768, 858, 807, 229}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt[4]{x\,(ae^2+cd^2)+ade+cdex^2}}{(d+ex)^2}\,dx$$

$$\Big\downarrow \text{1138}$$

$$\frac{\sqrt[4]{x\,(ae^2+cd^2)+ade+cdex^2}\int \frac{\sqrt[4]{ae+cdx}}{\left(\frac{ex}{d}+1\right)^{7/4}}dx}{d^2\sqrt[4]{\frac{ex}{d}+1}\sqrt[4]{ae+cdx}}$$

$$\Big\downarrow \text{57}$$

$$\frac{\sqrt[4]{x\left(ae^2+cd^2\right)+ade+cdex^2}\left(\frac{cd^2\int \frac{1}{(ae+cdx)^{3/4}\left(\frac{ex}{d}+1\right)^{3/4}}dx}{3e}-\frac{4d\sqrt[4]{ae+cdx}}{3e\left(\frac{ex}{d}+1\right)^{3/4}}\right)}{d^2\sqrt[4]{\frac{ex}{d}+1}\sqrt[4]{ae+cdx}}$$

$\downarrow$ 73

$$\frac{\sqrt[4]{x\left(ae^2+cd^2\right)+ade+cdex^2}\left(\frac{4d\int \frac{1}{\left(-\frac{ae^2}{cd^2}+\frac{(ae+cdx)e}{cd^2}+1\right)^{3/4}}d\sqrt[4]{ae+cdx}}{3e}-\frac{4d\sqrt[4]{ae+cdx}}{3e\left(\frac{ex}{d}+1\right)^{3/4}}\right)}{d^2\sqrt[4]{\frac{ex}{d}+1}\sqrt[4]{ae+cdx}}$$

$\downarrow$ 768

$$\frac{\sqrt[4]{x\left(ae^2+cd^2\right)+ade+cdex^2}\left(\frac{4d(ae+cdx)^{3/4}\left(\frac{cd^2-ae^2}{e(ae+cdx)}+1\right)^{3/4}\int \frac{1}{(ae+cdx)^{3/4}\left(\frac{cd^2-ae^2}{e(ae+cdx)}+1\right)^{3/4}}d\sqrt[4]{ae+cdx}}{3e\left(-\frac{ae^2}{cd^2}+\frac{e(ae+cdx)}{cd^2}+1\right)^{3/4}}-\frac{4d\sqrt[4]{ae+cd\cdots}}{3e\left(\frac{ex}{d}+1\right)^{3/4}}\cdots\right)}{d^2\sqrt[4]{\frac{ex}{d}+1}\sqrt[4]{ae+cdx}}$$

$\downarrow$ 858

$$\frac{\sqrt[4]{x\left(ae^2+cd^2\right)+ade+cdex^2}\left(-\frac{4d(ae+cdx)^{3/4}\left(\frac{cd^2-ae^2}{e(ae+cdx)}+1\right)^{3/4}\int \frac{1}{\sqrt[4]{ae+cdx}\left(\frac{\left(cd^2-ae^2\right)(ae+cdx)}{e}+1\right)^{3/4}}d\frac{1}{\sqrt[4]{ae+cdx}}}{3e\left(-\frac{ae^2}{cd^2}+\frac{e(ae+cdx)}{cd^2}+1\right)^{3/4}}-\cdots\right)}{d^2\sqrt[4]{\frac{ex}{d}+1}\sqrt[4]{ae+cdx}}$$

$\downarrow$ 807

$$\frac{\sqrt[4]{x\left(ae^2+cd^2\right)+ade+cdex^2}\left(-\frac{2d(ae+cdx)^{3/4}\left(\frac{cd^2-ae^2}{e(ae+cdx)}+1\right)^{3/4}\int \frac{1}{\left(\frac{\sqrt{ae+cdx}\left(cd^2-ae^2\right)}{e}+1\right)^{3/4}}d\sqrt{ae+cdx}}{3e\left(-\frac{ae^2}{cd^2}+\frac{e(ae+cdx)}{cd^2}+1\right)^{3/4}}-\frac{4d\sqrt[4]{ae+cdx}}{3e\left(\frac{ex}{d}+1\right)^{3/4}}\right)}{d^2\sqrt[4]{\frac{ex}{d}+1}\sqrt[4]{ae+cdx}}$$

$\downarrow$ 229

$$\frac{\sqrt[4]{x\left(ae^2+cd^2\right)+ade+cdex^2}\left(-\frac{4d(ae+cdx)^{3/4}\left(\frac{cd^2-ae^2}{e(ae+cdx)}+1\right)^{3/4}\text{EllipticF}\left(\frac{1}{2}\arctan\left(\frac{\sqrt{cd^2-ae^2}\sqrt{ae+cdx}}{\sqrt{e}}\right),2\right)}{3\sqrt{e}\sqrt{cd^2-ae^2}\left(-\frac{ae^2}{cd^2}+\frac{e(ae+cdx)}{cd^2}+1\right)^{3/4}}-\frac{4d\sqrt[4]{ae+cd}}{3e\left(\frac{ex}{d}+1\right)^{3/4}}\right.}{d^2\sqrt[4]{\frac{ex}{d}+1}\sqrt[4]{ae+cdx}}$$

input
```
Int[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(1/4)/(d + e*x)^2,x]
```

output
```
((a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(1/4)*((-4*d*(a*e + c*d*x)^(1/4))
/(3*e*(1 + (e*x)/d)^(3/4)) - (4*d*(a*e + c*d*x)^(3/4)*(1 + (c*d^2 - a*e^2)
/(e*(a*e + c*d*x)))^(3/4)*EllipticF[ArcTan[(Sqrt[c*d^2 - a*e^2]*Sqrt[a*e +
 c*d*x])/Sqrt[e]]/2, 2])/(3*Sqrt[e]*Sqrt[c*d^2 - a*e^2]*(1 - (a*e^2)/(c*d^
2) + (e*(a*e + c*d*x))/(c*d^2))^(3/4))))/(d^2*(a*e + c*d*x)^(1/4)*(1 + (e*
x)/d)^(1/4))
```

Defintions of rubi rules used

rule 57
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Simp[d*(n/(b*(m + 1)))
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &
& GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !IntegerQ[m]) &&  !(ILeQ[m
 + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) && IntLinearQ[a, b, c
, d, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 229
```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(2/(a^(3/4)*Rt[b/a, 2])
)*EllipticF[(1/2)*ArcTan[Rt[b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

rule 768
```
Int[((a_) + (b_.)*(x_)^4)^(-3/4), x_Symbol] :> Simp[x^3*((1 + a/(b*x^4))^(3
/4)/(a + b*x^4)^(3/4))   Int[1/(x^3*(1 + a/(b*x^4))^(3/4)), x], x] /; FreeQ
[{a, b}, x]
```

rule 807
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Simp[1/k   Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x,
x^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

rule 858
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

rule 1138
```
Int[((d_) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Sy
mbol] :> Simp[d^m*((a + b*x + c*x^2)^FracPart[p]/((1 + e*(x/d))^FracPart[p]
*(a/d + (c*x)/e)^FracPart[p]))   Int[(1 + e*(x/d))^(m + p)*(a/d + (c/e)*x)^
p, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0]
&& (IntegerQ[m] || GtQ[d, 0]) &&  !(IGtQ[m, 0] && (IntegerQ[3*p] || Integer
Q[4*p]))
```

Maple [F]

$$\int \frac{\left(ade+\left(a\,e^{2}+c\,d^{2}\right)x+cd\,x^{2}e\right)^{\frac{1}{4}}}{\left(ex+d\right)^{2}}dx$$

input
```
int((a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(1/4)/(e*x+d)^2,x)
```

output
```
int((a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(1/4)/(e*x+d)^2,x)
```

Fricas [F]

$$\int \frac{\sqrt[4]{ade+(cd^2+ae^2)\,x+cdex^2}}{(d+ex)^2}\,dx = \int \frac{(cdex^2+ade+(cd^2+ae^2)x)^{\frac{1}{4}}}{(ex+d)^2}\,dx$$

input
```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/4)/(e*x+d)^2,x, algorithm="
fricas")
```

output
```
integral((c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^(1/4)/(e^2*x^2 + 2*d*e*x
+ d^2), x)
```

Sympy [F]

$$\int \frac{\sqrt[4]{ade+(cd^2+ae^2)\,x+cdex^2}}{(d+ex)^2}\,dx = \int \frac{\sqrt[4]{(d+ex)\,(ae+cdx)}}{(d+ex)^2}\,dx$$

input
```
integrate((a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(1/4)/(e*x+d)**2,x)
```

output
```
Integral(((d + e*x)*(a*e + c*d*x))**(1/4)/(d + e*x)**2, x)
```

Maxima [F]

$$\int \frac{\sqrt[4]{ade+(cd^2+ae^2)\,x+cdex^2}}{(d+ex)^2}\,dx = \int \frac{(cdex^2+ade+(cd^2+ae^2)x)^{\frac{1}{4}}}{(ex+d)^2}\,dx$$

input
```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/4)/(e*x+d)^2,x, algorithm="
maxima")
```

output
```
integrate((c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^(1/4)/(e*x + d)^2, x)
```

Giac [F]

$$\int \frac{\sqrt[4]{ade + (cd^2 + ae^2)\,x + cdex^2}}{(d + ex)^2}\,dx = \int \frac{(cdex^2 + ade + (cd^2 + ae^2)x)^{\frac{1}{4}}}{(ex + d)^2}\,dx$$

input
```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/4)/(e*x+d)^2,x, algorithm="
giac")
```

output
```
integrate((c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^(1/4)/(e*x + d)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt[4]{ade + (cd^2 + ae^2)\,x + cdex^2}}{(d + ex)^2}\,dx = \int \frac{(c\,d\,e\,x^2 + (c\,d^2 + a\,e^2)\,x + a\,d\,e)^{1/4}}{(d + e\,x)^2}\,dx$$

input
```
int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/4)/(d + e*x)^2,x)
```

output
```
int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/4)/(d + e*x)^2, x)
```

Reduce [F]

$$\int \frac{\sqrt[4]{ade + (cd^2 + ae^2)\,x + cdex^2}}{(d + ex)^2}\,dx = \text{Too large to display}$$

input
```
int((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/4)/(e*x+d)^2,x)
```

output

```
( - 4*(a*d*e + a*e**2*x + c*d**2*x + c*d*e*x**2)**(1/4)*a*e + 3*int(((a*d*
e + a*e**2*x + c*d**2*x + c*d*e*x**2)**(1/4)*x)/(3*a**2*d**2*e**3 + 6*a**2
*d*e**4*x + 3*a**2*e**5*x**2 - a*c*d**4*e + a*c*d**3*e**2*x + 5*a*c*d**2*e
**3*x**2 + 3*a*c*d*e**4*x**3 - c**2*d**5*x - 2*c**2*d**4*e*x**2 - c**2*d**
3*e**2*x**3),x)*a**2*c*d**2*e**4 + 3*int(((a*d*e + a*e**2*x + c*d**2*x + c
*d*e*x**2)**(1/4)*x)/(3*a**2*d**2*e**3 + 6*a**2*d*e**4*x + 3*a**2*e**5*x**
2 - a*c*d**4*e + a*c*d**3*e**2*x + 5*a*c*d**2*e**3*x**2 + 3*a*c*d*e**4*x**
3 - c**2*d**5*x - 2*c**2*d**4*e*x**2 - c**2*d**3*e**2*x**3),x)*a**2*c*d*e*
*5*x - 4*int(((a*d*e + a*e**2*x + c*d**2*x + c*d*e*x**2)**(1/4)*x)/(3*a**2
*d**2*e**3 + 6*a**2*d*e**4*x + 3*a**2*e**5*x**2 - a*c*d**4*e + a*c*d**3*e*
*2*x + 5*a*c*d**2*e**3*x**2 + 3*a*c*d*e**4*x**3 - c**2*d**5*x - 2*c**2*d**
4*e*x**2 - c**2*d**3*e**2*x**3),x)*a*c**2*d**4*e**2 - 4*int(((a*d*e + a*e*
*2*x + c*d**2*x + c*d*e*x**2)**(1/4)*x)/(3*a**2*d**2*e**3 + 6*a**2*d*e**4*
x + 3*a**2*e**5*x**2 - a*c*d**4*e + a*c*d**3*e**2*x + 5*a*c*d**2*e**3*x**2
 + 3*a*c*d*e**4*x**3 - c**2*d**5*x - 2*c**2*d**4*e*x**2 - c**2*d**3*e**2*x
**3),x)*a*c**2*d**3*e**3*x + int(((a*d*e + a*e**2*x + c*d**2*x + c*d*e*x**
2)**(1/4)*x)/(3*a**2*d**2*e**3 + 6*a**2*d*e**4*x + 3*a**2*e**5*x**2 - a*c*
d**4*e + a*c*d**3*e**2*x + 5*a*c*d**2*e**3*x**2 + 3*a*c*d*e**4*x**3 - c**2
*d**5*x - 2*c**2*d**4*e*x**2 - c**2*d**3*e**2*x**3),x)*c**3*d**6 + int(((a
*d*e + a*e**2*x + c*d**2*x + c*d*e*x**2)**(1/4)*x)/(3*a**2*d**2*e**3 + ...
```

3.342 $\int \frac{\sqrt[4]{ade+(cd^2+ae^2)x+cdex^2}}{(d+ex)^3}\,dx$

Optimal result

Integrand size = 37, antiderivative size = 237

$$\int \frac{\sqrt[4]{ade+(cd^2+ae^2)x+cdex^2}}{(d+ex)^3}\,dx = -\frac{4\sqrt[4]{ade+(cd^2+ae^2)x+cdex^2}}{7e(d+ex)^2}+\frac{4cd\sqrt[4]{ade+(cd^2+ae^2)x+cdex^2}}{21e(cd^2-ae^2)(d+ex)} - \frac{8cd(ae+cdx)^{3/2}\left(\frac{cd(d+ex)}{e(ae+cdx)}\right)^{3/4}\text{EllipticF}\left(\frac{1}{2}\arctan\left(\frac{\sqrt{cd^2-ae^2}}{\sqrt{e}\sqrt{ae+cdx}}\right),2\right)}{21\sqrt{e}(cd^2-ae^2)^{3/2}\left(ade+(cd^2+ae^2)x+cdex^2\right)^{3/4}}$$

output
```
-4/7*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/4)/e/(e*x+d)^2+4/21*c*d*(a*d*e+(
a*e^2+c*d^2)*x+c*d*e*x^2)^(1/4)/e/(-a*e^2+c*d^2)/(e*x+d)-8/21*c*d*(c*d*x+a
*e)^(3/2)*(c*d*(e*x+d)/e/(c*d*x+a*e))^(3/4)*InverseJacobiAM(1/2*arctan((-a
*e^2+c*d^2)^(1/2)/e^(1/2)/(c*d*x+a*e)^(1/2)),2^(1/2))/e^(1/2)/(-a*e^2+c*d^
2)^(3/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/4)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.05 (sec) , antiderivative size = 110, normalized size of antiderivative = 0.46

$$\int \frac{\sqrt[4]{ade+(cd^2+ae^2)x+cdex^2}}{(d+ex)^3}\,dx = \frac{4c^2d^2(ae+cdx)\sqrt[4]{(ae+cdx)(d+ex)}\,\text{Hypergeometric2F1}\left(\frac{5}{4},\frac{11}{4},\frac{9}{4},\frac{e(ae+cdx)}{-cd^2+ae^2}\right)}{5\,(cd^2-ae^2)^3\sqrt[4]{\frac{cd(d+ex)}{cd^2-ae^2}}}$$

input
```
Integrate[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(1/4)/(d + e*x)^3,x]
```

output
```
(4*c^2*d^2*(a*e + c*d*x)*((a*e + c*d*x)*(d + e*x))^(1/4)*Hypergeometric2F1
[5/4, 11/4, 9/4, (e*(a*e + c*d*x))/(-(c*d^2) + a*e^2)])/(5*(c*d^2 - a*e^2)
^3*((c*d*(d + e*x))/(c*d^2 - a*e^2))^(1/4))
```

Rubi [A] (warning: unable to verify)

Time = 0.75 (sec) , antiderivative size = 304, normalized size of antiderivative = 1.28, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.216$, Rules used = {1138, 57, 61, 73, 768, 858, 807, 229}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt[4]{x(ae^2+cd^2)+ade+cdex^2}}{(d+ex)^3}dx$$

$$\Big\downarrow \ 1138$$

$$\frac{\sqrt[4]{x(ae^2+cd^2)+ade+cdex^2}\int \frac{\sqrt[4]{ae+cdx}}{\left(\frac{ex}{d}+1\right)^{11/4}}dx}{d^3\sqrt[4]{\frac{ex}{d}+1}\sqrt[4]{ae+cdx}}$$

$$\Big\downarrow \ 57$$

$$\frac{\sqrt[4]{x\left(ae^2+cd^2\right)+ade+cdex^2}\left(\frac{cd^2\int \frac{1}{(ae+cdx)^{3/4}\left(\frac{ex}{d}+1\right)^{7/4}}dx}{7e}-\frac{4d\sqrt[4]{ae+cdx}}{7e\left(\frac{ex}{d}+1\right)^{7/4}}\right)}{d^3\sqrt[4]{\frac{ex}{d}+1}\sqrt[4]{ae+cdx}}$$

$\downarrow$ 61

$$\frac{\sqrt[4]{x\left(ae^2+cd^2\right)+ade+cdex^2}\left(\frac{cd^2\left(\frac{2cd\int \frac{1}{(ae+cdx)^{3/4}\left(\frac{ex}{d}+1\right)^{3/4}}dx}{3\left(cd-\frac{ae^2}{d}\right)}+\frac{4\sqrt[4]{ae+cdx}}{3\left(\frac{ex}{d}+1\right)^{3/4}\left(cd-\frac{ae^2}{d}\right)}\right)}{7e}-\frac{4d\sqrt[4]{ae+cdx}}{7e\left(\frac{ex}{d}+1\right)^{7/4}}\right)}{d^3\sqrt[4]{\frac{ex}{d}+1}\sqrt[4]{ae+cdx}}$$

$\downarrow$ 73

$$\frac{\sqrt[4]{x\left(ae^2+cd^2\right)+ade+cdex^2}\left(\frac{cd^2\left(\frac{8\int \frac{1}{\left(-\frac{ae^2}{cd^2}+\frac{(ae+cdx)e}{cd^2}+1\right)^{3/4}}d\sqrt[4]{ae+cdx}}{3\left(cd-\frac{ae^2}{d}\right)}+\frac{4\sqrt[4]{ae+cdx}}{3\left(\frac{ex}{d}+1\right)^{3/4}\left(cd-\frac{ae^2}{d}\right)}\right)}{7e}-\frac{4d\sqrt[4]{ae+cdx}}{7e\left(\frac{ex}{d}+1\right)^{7/4}}\right)}{d^3\sqrt[4]{\frac{ex}{d}+1}\sqrt[4]{ae+cdx}}$$

$\downarrow$ 768

$$\frac{\sqrt[4]{x\left(ae^2+cd^2\right)+ade+cdex^2}\left(\frac{cd^2\left(\frac{8(ae+cdx)^{3/4}\left(\frac{cd^2-ae^2}{e(ae+cdx)}+1\right)^{3/4}\int \frac{1}{(ae+cdx)^{3/4}\left(\frac{cd^2-ae^2}{e(ae+cdx)}+1\right)^{3/4}}d\sqrt[4]{ae+cdx}}{3\left(cd-\frac{ae^2}{d}\right)\left(-\frac{ae^2}{cd^2}+\frac{e(ae+cdx)}{cd^2}+1\right)^{3/4}}+\frac{4\sqrt[4]{ae+\cdots}}{3\left(\frac{ex}{d}+1\right)^{3/4}\left(\cdots\right.}\cdots\right.}{7e}\cdots\right.}{d^3\sqrt[4]{\frac{ex}{d}+1}\sqrt[4]{ae+cdx}}$$

$\downarrow$ 858

$$\frac{\sqrt[4]{x\left(ae^2+cd^2\right)+ade+cdex^2}\left(\frac{cd^2\left(\frac{4\sqrt[4]{ae+cdx}}{3\left(\frac{ex}{d}+1\right)^{3/4}\left(cd-\frac{ae^2}{d}\right)}-\frac{8(ae+cdx)^{3/4}\left(\frac{cd^2-ae^2}{e(ae+cdx)}+1\right)^{3/4}\int \frac{1}{\sqrt[4]{ae+cdx}\left(\frac{\left(cd^2-ae^2\right)(ae+cdx)}{e}+1\right.}}{3\left(cd-\frac{ae^2}{d}\right)\left(-\frac{ae^2}{cd^2}+\frac{e(ae+cdx)}{cd^2}+1\right)^{3/4}}\right.}{7e}\right.}{d^3\sqrt[4]{\frac{ex}{d}+1}\sqrt[4]{ae+cdx}}$$

$\downarrow$ 807

$$\frac{\sqrt[4]{x\left(ae^2+cd^2\right)+ade+cdex^2}\left(\frac{cd^2\left(\frac{4\sqrt[4]{ae+cdx}}{3\left(\frac{ex}{d}+1\right)^{3/4}\left(cd-\frac{ae^2}{d}\right)}-\frac{4(ae+cdx)^{3/4}\left(\frac{cd^2-ae^2}{e(ae+cdx)}+1\right)^{3/4}\int \frac{1}{\left(\frac{\sqrt{ae+cdx}\left(cd^2-ae^2\right)}{e}+1\right)^{3/4}}d\sqrt{ae+cdx}}{3\left(cd-\frac{ae^2}{d}\right)\left(-\frac{ae^2}{cd^2}+\frac{e(ae+cdx)}{cd^2}+1\right)^{3/4}}\right.}{7e}\right.}{d^3\sqrt[4]{\frac{ex}{d}+1}\sqrt[4]{ae+cdx}}$$

$\downarrow$ 229

$$\frac{\sqrt[4]{x\left(ae^2+cd^2\right)+ade+cdex^2}\left(\frac{cd^2\left(\frac{4\sqrt[4]{ae+cdx}}{3\left(\frac{ex}{d}+1\right)^{3/4}\left(cd-\frac{ae^2}{d}\right)}-\frac{8\sqrt{e}(ae+cdx)^{3/4}\left(\frac{cd^2-ae^2}{e(ae+cdx)}+1\right)^{3/4}\text{EllipticF}\left(\frac{1}{2}\arctan\left(\frac{\sqrt{cd^2-ae^2}\sqrt{ae+cdx}}{\sqrt{e}}\right.\right.}{3\sqrt{cd^2-ae^2}\left(cd-\frac{ae^2}{d}\right)\left(-\frac{ae^2}{cd^2}+\frac{e(ae+cdx)}{cd^2}+1\right)^{3/4}}\right.}{7e}\right.}{d^3\sqrt[4]{\frac{ex}{d}+1}\sqrt[4]{ae+cdx}}$$

input
```
Int[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(1/4)/(d + e*x)^3,x]
```

output
```
((a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(1/4)*((-4*d*(a*e + c*d*x)^(1/4))
/(7*e*(1 + (e*x)/d)^(7/4)) + (c*d^2*((4*(a*e + c*d*x)^(1/4))/(3*(c*d - (a*
e^2)/d)*(1 + (e*x)/d)^(3/4)) - (8*Sqrt[e]*(a*e + c*d*x)^(3/4)*(1 + (c*d^2
- a*e^2)/(e*(a*e + c*d*x)))^(3/4)*EllipticF[ArcTan[(Sqrt[c*d^2 - a*e^2]*Sq
rt[a*e + c*d*x])/Sqrt[e]]/2, 2])/(3*Sqrt[c*d^2 - a*e^2]*(c*d - (a*e^2)/d)*
(1 - (a*e^2)/(c*d^2) + (e*(a*e + c*d*x))/(c*d^2))^(3/4))))/(7*e)))/(d^3*(a
*e + c*d*x)^(1/4)*(1 + (e*x)/d)^(1/4))
```

Defintions of rubi rules used

rule 57
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Simp[d*(n/(b*(m + 1)))
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &
& GtQ[n, 0] && LtQ[m, -1] && !(IntegerQ[n] && !IntegerQ[m]) && !(ILeQ[m
+ n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) && IntLinearQ[a, b, c
, d, m, n, x]
```

rule 61
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && LtQ[m, -1] && !(LtQ[n, -1] && (EqQ[a, 0
] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && IntLinearQ[a, b, c, d
, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 229
```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(2/(a^(3/4)*Rt[b/a, 2])
)*EllipticF[(1/2)*ArcTan[Rt[b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

rule 768
```
Int[((a_) + (b_.)*(x_)^4)^(-3/4), x_Symbol] :> Simp[x^3*((1 + a/(b*x^4))^(3
/4)/(a + b*x^4)^(3/4))   Int[1/(x^3*(1 + a/(b*x^4))^(3/4)), x], x] /; FreeQ
[{a, b}, x]
```

rule 807
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Simp[1/k   Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x,
x^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

rule 858
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

rule 1138
```
Int[((d_) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Sy
mbol] :> Simp[d^m*((a + b*x + c*x^2)^FracPart[p]/((1 + e*(x/d))^FracPart[p]
*(a/d + (c*x)/e)^FracPart[p]))   Int[(1 + e*(x/d))^(m + p)*(a/d + (c/e)*x)^
p, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0]
&& (IntegerQ[m] || GtQ[d, 0]) &&  !(IGtQ[m, 0] && (IntegerQ[3*p] || Integer
Q[4*p]))
```

Maple [F]

$$\int \frac{(ade + (a\,e^2 + c\,d^2)\,x + cd\,x^2 e)^{\frac{1}{4}}}{(ex + d)^3}dx$$

input
```
int((a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(1/4)/(e*x+d)^3,x)
```

output
```
int((a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(1/4)/(e*x+d)^3,x)
```

Fricas [F]

$$\int \frac{\sqrt[4]{ade+(cd^2+ae^2)\,x+cdex^2}}{(d+ex)^3}\,dx = \int \frac{(cdex^2+ade+(cd^2+ae^2)x)^{\frac{1}{4}}}{(ex+d)^3}\,dx$$

input
```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/4)/(e*x+d)^3,x, algorithm="
fricas")
```

output
```
integral((c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^(1/4)/(e^3*x^3 + 3*d*e^2*
x^2 + 3*d^2*e*x + d^3), x)
```

Sympy [F]

$$\int \frac{\sqrt[4]{ade+(cd^2+ae^2)\,x+cdex^2}}{(d+ex)^3}\,dx = \int \frac{\sqrt[4]{(d+ex)\,(ae+cdx)}}{(d+ex)^3}\,dx$$

input
```
integrate((a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(1/4)/(e*x+d)**3,x)
```

output
```
Integral(((d + e*x)*(a*e + c*d*x))**(1/4)/(d + e*x)**3, x)
```

Maxima [F]

$$\int \frac{\sqrt[4]{ade+(cd^2+ae^2)\,x+cdex^2}}{(d+ex)^3}\,dx = \int \frac{(cdex^2+ade+(cd^2+ae^2)x)^{\frac{1}{4}}}{(ex+d)^3}\,dx$$

input
```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/4)/(e*x+d)^3,x, algorithm="
maxima")
```

output
```
integrate((c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^(1/4)/(e*x + d)^3, x)
```

Giac [F]

$$\int \frac{\sqrt[4]{ade + (cd^2 + ae^2)\,x + cdex^2}}{(d + ex)^3}\,dx = \int \frac{(cdex^2 + ade + (cd^2 + ae^2)x)^{\frac{1}{4}}}{(ex + d)^3}\,dx$$

input
```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/4)/(e*x+d)^3,x, algorithm="
giac")
```

output
```
integrate((c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^(1/4)/(e*x + d)^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt[4]{ade + (cd^2 + ae^2)\,x + cdex^2}}{(d + ex)^3}\,dx = \int \frac{(c\,d\,e\,x^2 + (c\,d^2 + a\,e^2)\,x + a\,d\,e)^{1/4}}{(d + e\,x)^3}\,dx$$

input
```
int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/4)/(d + e*x)^3,x)
```

output
```
int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/4)/(d + e*x)^3, x)
```

Reduce [F]

$$\int \frac{\sqrt[4]{ade + (cd^2 + ae^2)\,x + cdex^2}}{(d + ex)^3}\,dx = \text{too large to display}$$

input
```
int((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/4)/(e*x+d)^3,x)
```

output

```
( - 4*(a*d*e + a*e**2*x + c*d**2*x + c*d*e*x**2)**(1/4)*a*e + 7*int(((a*d*
e + a*e**2*x + c*d**2*x + c*d*e*x**2)**(1/4)*x)/(7*a**2*d**3*e**3 + 21*a**
2*d**2*e**4*x + 21*a**2*d*e**5*x**2 + 7*a**2*e**6*x**3 - a*c*d**5*e + 4*a*
c*d**4*e**2*x + 18*a*c*d**3*e**3*x**2 + 20*a*c*d**2*e**4*x**3 + 7*a*c*d*e*
*5*x**4 - c**2*d**6*x - 3*c**2*d**5*e*x**2 - 3*c**2*d**4*e**2*x**3 - c**2*
d**3*e**3*x**4),x)*a**2*c*d**3*e**4 + 14*int(((a*d*e + a*e**2*x + c*d**2*x
 + c*d*e*x**2)**(1/4)*x)/(7*a**2*d**3*e**3 + 21*a**2*d**2*e**4*x + 21*a**2
*d*e**5*x**2 + 7*a**2*e**6*x**3 - a*c*d**5*e + 4*a*c*d**4*e**2*x + 18*a*c*
d**3*e**3*x**2 + 20*a*c*d**2*e**4*x**3 + 7*a*c*d*e**5*x**4 - c**2*d**6*x -
 3*c**2*d**5*e*x**2 - 3*c**2*d**4*e**2*x**3 - c**2*d**3*e**3*x**4),x)*a**2
*c*d**2*e**5*x + 7*int(((a*d*e + a*e**2*x + c*d**2*x + c*d*e*x**2)**(1/4)*
x)/(7*a**2*d**3*e**3 + 21*a**2*d**2*e**4*x + 21*a**2*d*e**5*x**2 + 7*a**2*
e**6*x**3 - a*c*d**5*e + 4*a*c*d**4*e**2*x + 18*a*c*d**3*e**3*x**2 + 20*a*
c*d**2*e**4*x**3 + 7*a*c*d*e**5*x**4 - c**2*d**6*x - 3*c**2*d**5*e*x**2 -
3*c**2*d**4*e**2*x**3 - c**2*d**3*e**3*x**4),x)*a**2*c*d*e**6*x**2 - 8*int
(((a*d*e + a*e**2*x + c*d**2*x + c*d*e*x**2)**(1/4)*x)/(7*a**2*d**3*e**3 +
 21*a**2*d**2*e**4*x + 21*a**2*d*e**5*x**2 + 7*a**2*e**6*x**3 - a*c*d**5*e
 + 4*a*c*d**4*e**2*x + 18*a*c*d**3*e**3*x**2 + 20*a*c*d**2*e**4*x**3 + 7*a
*c*d*e**5*x**4 - c**2*d**6*x - 3*c**2*d**5*e*x**2 - 3*c**2*d**4*e**2*x**3
- c**2*d**3*e**3*x**4),x)*a*c**2*d**5*e**2 - 16*int(((a*d*e + a*e**2*x ...
```

3.343 $\int \frac{\sqrt[4]{ade+(cd^2+ae^2)x+cdex^2}}{(d+ex)^4}\,dx$

Optimal result

Integrand size = 37, antiderivative size = 304

$$\begin{aligned}\int \frac{\sqrt[4]{ade+(cd^2+ae^2)x+cdex^2}}{(d+ex)^4}\,dx &= -\frac{4\sqrt[4]{ade+(cd^2+ae^2)x+cdex^2}}{11e(d+ex)^3}+\frac{4cd\sqrt[4]{ade+(cd^2+ae^2)x+cdex^2}}{77e(cd^2-ae^2)(d+ex)^2}\\&\quad+\frac{8c^2d^2\sqrt[4]{ade+(cd^2+ae^2)x+cdex^2}}{77e(cd^2-ae^2)^2(d+ex)}\\&\quad-\frac{16c^2d^2(ae+cdx)^{3/2}\left(\frac{cd(d+ex)}{e(ae+cdx)}\right)^{3/4}\operatorname{EllipticF}\left(\frac{1}{2}\arctan\left(\frac{\sqrt{cd^2-ae^2}}{\sqrt{e}\sqrt{ae+cdx}}\right),2\right)}{77\sqrt{e}(cd^2-ae^2)^{5/2}(ade+(cd^2+ae^2)x+cdex^2)^{3/4}}\end{aligned}$$

output
```
-4/11*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/4)/e/(e*x+d)^3+4/77*c*d*(a*d*e+
(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/4)/e/(-a*e^2+c*d^2)/(e*x+d)^2+8/77*c^2*d^2*(
a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/4)/e/(-a*e^2+c*d^2)^2/(e*x+d)-16/77*c^
2*d^2*(c*d*x+a*e)^(3/2)*(c*d*(e*x+d)/e/(c*d*x+a*e))^(3/4)*InverseJacobiAM(
1/2*arctan((-a*e^2+c*d^2)^(1/2)/e^(1/2)/(c*d*x+a*e)^(1/2)),2^(1/2))/e^(1/2
)/(-a*e^2+c*d^2)^(5/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/4)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.05 (sec) , antiderivative size = 110, normalized size of antiderivative = 0.36

$$\int \frac{\sqrt[4]{ade+(cd^2+ae^2)\,x+cdex^2}}{(d+ex)^4}\,dx = \frac{4c^3d^3(ae+cdx)\sqrt[4]{(ae+cdx)(d+ex)}\,\text{Hypergeometric2F1}\left(\frac{5}{4},\frac{15}{4},\frac{9}{4},\frac{e(ae+cdx)}{-cd^2+ae^2}\right)}{5\,(cd^2-ae^2)^4\sqrt[4]{\frac{cd(d+ex)}{cd^2-ae^2}}}$$

input
```
Integrate[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(1/4)/(d + e*x)^4,x]
```

output
```
(4*c^3*d^3*(a*e + c*d*x)*((a*e + c*d*x)*(d + e*x))^(1/4)*Hypergeometric2F1
[5/4, 15/4, 9/4, (e*(a*e + c*d*x))/(-(c*d^2) + a*e^2)])/(5*(c*d^2 - a*e^2)
^4*((c*d*(d + e*x))/(c*d^2 - a*e^2))^(1/4))
```

Rubi [A] (warning: unable to verify)

Time = 0.84 (sec) , antiderivative size = 369, normalized size of antiderivative = 1.21, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.243$, Rules used = {1138, 57, 61, 61, 73, 768, 858, 807, 229}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt[4]{x\,(ae^2+cd^2)+ade+cdex^2}}{(d+ex)^4}\,dx$$

$$\Big\downarrow \text{1138}$$

$$\frac{\sqrt[4]{x\,(ae^2+cd^2)+ade+cdex^2}\int \frac{\sqrt[4]{ae+cdx}}{\left(\frac{ex}{d}+1\right)^{15/4}}dx}{d^4\sqrt[4]{\frac{ex}{d}+1}\sqrt[4]{ae+cdx}}$$

$$\Big\downarrow \text{57}$$

$$\frac{\sqrt[4]{x\left(ae^2+cd^2\right)+ade+cdex^2}\left(\frac{cd^2\int\frac{1}{(ae+cdx)^{3/4}\left(\frac{ex}{d}+1\right)^{11/4}}dx}{11e}-\frac{4d\sqrt[4]{ae+cdx}}{11e\left(\frac{ex}{d}+1\right)^{11/4}}\right)}{d^4\sqrt[4]{\frac{ex}{d}+1}\sqrt[4]{ae+cdx}}$$

$\downarrow$ 61

$$\frac{\sqrt[4]{x\left(ae^2+cd^2\right)+ade+cdex^2}\left(\frac{cd^2\left(\frac{6cd\int\frac{1}{(ae+cdx)^{3/4}\left(\frac{ex}{d}+1\right)^{7/4}}dx}{7\left(cd-\frac{ae^2}{d}\right)}+\frac{4\sqrt[4]{ae+cdx}}{7\left(\frac{ex}{d}+1\right)^{7/4}\left(cd-\frac{ae^2}{d}\right)}\right)}{11e}-\frac{4d\sqrt[4]{ae+cdx}}{11e\left(\frac{ex}{d}+1\right)^{11/4}}\right)}{d^4\sqrt[4]{\frac{ex}{d}+1}\sqrt[4]{ae+cdx}}$$

$\downarrow$ 61

$$\frac{\sqrt[4]{x\left(ae^2+cd^2\right)+ade+cdex^2}\left(\frac{cd^2\left(\frac{6cd\left(\frac{2cd\int\frac{1}{(ae+cdx)^{3/4}\left(\frac{ex}{d}+1\right)^{3/4}}dx}{3\left(cd-\frac{ae^2}{d}\right)}+\frac{4\sqrt[4]{ae+cdx}}{3\left(\frac{ex}{d}+1\right)^{3/4}\left(cd-\frac{ae^2}{d}\right)}\right)}{7\left(cd-\frac{ae^2}{d}\right)}+\frac{4\sqrt[4]{ae+cdx}}{7\left(\frac{ex}{d}+1\right)^{7/4}\left(cd-\frac{ae^2}{d}\right)}\right)}{11e}-\frac{4d}{11}\right.}{d^4\sqrt[4]{\frac{ex}{d}+1}\sqrt[4]{ae+cdx}}$$

$\downarrow$ 73

$$\frac{\sqrt[4]{x\left(ae^2+cd^2\right)+ade+cdex^2}\left(\frac{cd^2\left(\frac{6cd\left(\frac{8\int \frac{1}{\left(-\frac{ae^2}{cd^2}+\frac{(ae+cdx)e}{cd^2}+1\right)^{3/4}}d\sqrt[4]{ae+cdx}}{3\left(cd-\frac{ae^2}{d}\right)}+\frac{4\sqrt[4]{ae+cdx}}{3\left(\frac{ex}{d}+1\right)^{3/4}\left(cd-\frac{ae^2}{d}\right)}\right)}{7\left(cd-\frac{ae^2}{d}\right)}+\frac{4\sqrt[4]{ae+cd\cdots}}{7\left(\frac{ex}{d}+1\right)^{7/4}\left(cd-\cdots\right.}\cdots\right)}{11e}\cdots\right.}{d^4\sqrt[4]{\frac{ex}{d}+1}\sqrt[4]{ae+cdx}}$$

$\downarrow$ 768

$$\frac{\sqrt[4]{x\left(ae^2+cd^2\right)+ade+cdex^2}\left(\frac{cd^2\left(\frac{6cd\left(\frac{8(ae+cdx)^{3/4}\left(\frac{cd^2-ae^2}{e(ae+cdx)}+1\right)^{3/4}\int \frac{1}{(ae+cdx)^{3/4}\left(\frac{cd^2-ae^2}{e(ae+cdx)}+1\right)^{3/4}}d\sqrt[4]{ae+cdx}}{3\left(cd-\frac{ae^2}{d}\right)\left(-\frac{ae^2}{cd^2}+\frac{e(ae+cdx)}{cd^2}+1\right)^{3/4}}+\frac{4\sqrt[4]{a\cdots}}{3\left(\frac{ex}{d}+1\right)^{\cdots}}\cdots\right.}{7\left(cd-\frac{ae^2}{d}\right)}\cdots\right)}{11e}\cdots\right.}{d^4\sqrt[4]{\frac{ex}{d}+1}\sqrt[4]{ae+cdx}}$$

$\downarrow$ 858

$$\frac{\sqrt[4]{x\left(ae^2+cd^2\right)+ade+cdex^2}\left(\frac{cd^2\left(\frac{6cd\left(\frac{4\sqrt[4]{ae+cdx}}{3\left(\frac{ex}{d}+1\right)^{3/4}\left(cd-\frac{ae^2}{d}\right)}-\frac{8(ae+cdx)^{3/4}\left(\frac{cd^2-ae^2}{e(ae+cdx)}+1\right)^{3/4}\int\frac{1}{\sqrt[4]{ae+cdx}\left(\frac{\left(cd^2-ae^2\right)(ae+cd}{e}\right.}}{3\left(cd-\frac{ae^2}{d}\right)\left(-\frac{ae^2}{cd^2}+\frac{e(ae+cdx)}{cd^2}+1\right)^{3/4}}\right.}{7\left(cd-\frac{ae^2}{d}\right)}}{11e}\right.}{d^4\sqrt[4]{\frac{ex}{d}+1}\sqrt[4]{ae+cdx}}$$

$\downarrow$ 807

$$\frac{\sqrt[4]{x\left(ae^2+cd^2\right)+ade+cdex^2}\left(\frac{cd^2\left(\frac{6cd\left(\frac{4\sqrt[4]{ae+cdx}}{3\left(\frac{ex}{d}+1\right)^{3/4}\left(cd-\frac{ae^2}{d}\right)}-\frac{4(ae+cdx)^{3/4}\left(\frac{cd^2-ae^2}{e(ae+cdx)}+1\right)^{3/4}\int\frac{1}{\left(\frac{\sqrt{ae+cdx}\left(cd^2-ae^2\right)}{e}+1\right)^{3/4}}d\sqrt{a}}{3\left(cd-\frac{ae^2}{d}\right)\left(-\frac{ae^2}{cd^2}+\frac{e(ae+cdx)}{cd^2}+1\right)^{3/4}}\right)}{7\left(cd-\frac{ae^2}{d}\right)}}{11e}\right.}{d^4\sqrt[4]{\frac{ex}{d}+1}\sqrt[4]{ae+cdx}}$$

$\downarrow$ 229

$$\frac{\sqrt[4]{x\left(ae^2+cd^2\right)+ade+cdex^2}\left(\frac{cd^2\left(\frac{6cd\left(\frac{4\sqrt[4]{ae+cdx}}{3\left(\frac{ex}{d}+1\right)^{3/4}\left(cd-\frac{ae^2}{d}\right)}-\frac{8\sqrt{e}(ae+cdx)^{3/4}\left(\frac{cd^2-ae^2}{e(ae+cdx)}+1\right)^{3/4}\text{EllipticF}\left(\frac{1}{2}\arctan\left(\frac{\sqrt{cd^2-ae^2}\sqrt{ae}}{\sqrt{e}}\right.\right.}{3\sqrt{cd^2-ae^2}\left(cd-\frac{ae^2}{d}\right)\left(-\frac{ae^2}{cd^2}+\frac{e(ae+cdx)}{cd^2}+1\right)^{3/4}}\right)}{7\left(cd-\frac{ae^2}{d}\right)}}{11e}\right.}{d^4\sqrt[4]{\frac{ex}{d}+1}\sqrt[4]{ae+cdx}}$$

input
```
Int[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(1/4)/(d + e*x)^4,x]
```

output
```
((a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(1/4)*((-4*d*(a*e + c*d*x)^(1/4))
/(11*e*(1 + (e*x)/d)^(11/4)) + (c*d^2*((4*(a*e + c*d*x)^(1/4))/(7*(c*d - (
a*e^2)/d)*(1 + (e*x)/d)^(7/4)) + (6*c*d*((4*(a*e + c*d*x)^(1/4))/(3*(c*d -
 (a*e^2)/d)*(1 + (e*x)/d)^(3/4)) - (8*Sqrt[e]*(a*e + c*d*x)^(3/4)*(1 + (c*
d^2 - a*e^2)/(e*(a*e + c*d*x)))^(3/4)*EllipticF[ArcTan[(Sqrt[c*d^2 - a*e^2
]*Sqrt[a*e + c*d*x])/Sqrt[e]]/2, 2])/(3*Sqrt[c*d^2 - a*e^2]*(c*d - (a*e^2)
/d)*(1 - (a*e^2)/(c*d^2) + (e*(a*e + c*d*x))/(c*d^2))^(3/4))))/(7*(c*d - (
a*e^2)/d))))/(11*e)))/(d^4*(a*e + c*d*x)^(1/4)*(1 + (e*x)/d)^(1/4))
```

Defintions of rubi rules used

rule 57
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Simp[d*(n/(b*(m + 1)))
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &
& GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !IntegerQ[m]) &&  !(ILeQ[m
+ n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) && IntLinearQ[a, b, c
, d, m, n, x]
```

rule 61
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && LtQ[m, -1] &&  !(LtQ[n, -1] && (EqQ[a, 0
] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && IntLinearQ[a, b, c, d
, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 229
```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(2/(a^(3/4)*Rt[b/a, 2])
)*EllipticF[(1/2)*ArcTan[Rt[b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

rule 768
```
Int[((a_) + (b_.)*(x_)^4)^(-3/4), x_Symbol] :> Simp[x^3*((1 + a/(b*x^4))^(3
/4)/(a + b*x^4)^(3/4))   Int[1/(x^3*(1 + a/(b*x^4))^(3/4)), x], x] /; FreeQ
[{a, b}, x]
```

rule 807
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Simp[1/k   Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x,
x^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

rule 858
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

rule 1138
```
Int[((d_) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Sy
mbol] :> Simp[d^m*((a + b*x + c*x^2)^FracPart[p]/((1 + e*(x/d))^FracPart[p]
*(a/d + (c*x)/e)^FracPart[p]))   Int[(1 + e*(x/d))^(m + p)*(a/d + (c/e)*x)^
p, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0]
&& (IntegerQ[m] || GtQ[d, 0]) &&  !(IGtQ[m, 0] && (IntegerQ[3*p] || Integer
Q[4*p]))
```

Maple [F]

$$\int \frac{(ade + (a\,e^2 + c\,d^2)\,x + cd\,x^2e)^{\frac{1}{4}}}{(ex + d)^4}dx$$

input
```
int((a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(1/4)/(e*x+d)^4,x)
```

output
```
int((a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(1/4)/(e*x+d)^4,x)
```

Fricas [F]

$$\int \frac{\sqrt[4]{ade + (cd^2 + ae^2)x + cdex^2}}{(d + ex)^4} dx = \int \frac{(cdex^2 + ade + (cd^2 + ae^2)x)^{\frac{1}{4}}}{(ex + d)^4} dx$$

input
```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/4)/(e*x+d)^4,x, algorithm="
fricas")
```

output
```
integral((c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^(1/4)/(e^4*x^4 + 4*d*e^3*
x^3 + 6*d^2*e^2*x^2 + 4*d^3*e*x + d^4), x)
```

Sympy [F]

$$\int \frac{\sqrt[4]{ade + (cd^2 + ae^2)x + cdex^2}}{(d + ex)^4} dx = \int \frac{\sqrt[4]{(d + ex)(ae + cdx)}}{(d + ex)^4} dx$$

input
```
integrate((a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(1/4)/(e*x+d)**4,x)
```

output
```
Integral(((d + e*x)*(a*e + c*d*x))**(1/4)/(d + e*x)**4, x)
```

Maxima [F]

$$\int \frac{\sqrt[4]{ade + (cd^2 + ae^2)x + cdex^2}}{(d + ex)^4} dx = \int \frac{(cdex^2 + ade + (cd^2 + ae^2)x)^{\frac{1}{4}}}{(ex + d)^4} dx$$

input
```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/4)/(e*x+d)^4,x, algorithm="
maxima")
```

output
```
integrate((c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^(1/4)/(e*x + d)^4, x)
```

Giac [F]

$$\int \frac{\sqrt[4]{ade + (cd^2 + ae^2)\,x + cdex^2}}{(d + ex)^4}\,dx = \int \frac{(cdex^2 + ade + (cd^2 + ae^2)x)^{\frac{1}{4}}}{(ex + d)^4}\,dx$$

input
```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/4)/(e*x+d)^4,x, algorithm="
giac")
```

output
```
integrate((c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^(1/4)/(e*x + d)^4, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt[4]{ade + (cd^2 + ae^2)\,x + cdex^2}}{(d + ex)^4}\,dx = \int \frac{(c\,d\,e\,x^2 + (c\,d^2 + a\,e^2)\,x + a\,d\,e)^{1/4}}{(d + e\,x)^4}\,dx$$

input
```
int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/4)/(d + e*x)^4,x)
```

output
```
int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/4)/(d + e*x)^4, x)
```

Reduce [F]

$$\int \frac{\sqrt[4]{ade + (cd^2 + ae^2)\,x + cdex^2}}{(d + ex)^4}\,dx = \text{too large to display}$$

input
```
int((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/4)/(e*x+d)^4,x)
```

output
```
( - 4*(a*d*e + a*e**2*x + c*d**2*x + c*d*e*x**2)**(1/4)*a*e + 11*int(((a*d
*e + a*e**2*x + c*d**2*x + c*d*e*x**2)**(1/4)*x)/(11*a**2*d**4*e**3 + 44*a
**2*d**3*e**4*x + 66*a**2*d**2*e**5*x**2 + 44*a**2*d*e**6*x**3 + 11*a**2*e
**7*x**4 - a*c*d**6*e + 7*a*c*d**5*e**2*x + 38*a*c*d**4*e**3*x**2 + 62*a*c
*d**3*e**4*x**3 + 43*a*c*d**2*e**5*x**4 + 11*a*c*d*e**6*x**5 - c**2*d**7*x
 - 4*c**2*d**6*e*x**2 - 6*c**2*d**5*e**2*x**3 - 4*c**2*d**4*e**3*x**4 - c*
*2*d**3*e**4*x**5),x)*a**2*c*d**4*e**4 + 33*int(((a*d*e + a*e**2*x + c*d**
2*x + c*d*e*x**2)**(1/4)*x)/(11*a**2*d**4*e**3 + 44*a**2*d**3*e**4*x + 66*
a**2*d**2*e**5*x**2 + 44*a**2*d*e**6*x**3 + 11*a**2*e**7*x**4 - a*c*d**6*e
 + 7*a*c*d**5*e**2*x + 38*a*c*d**4*e**3*x**2 + 62*a*c*d**3*e**4*x**3 + 43*
a*c*d**2*e**5*x**4 + 11*a*c*d*e**6*x**5 - c**2*d**7*x - 4*c**2*d**6*e*x**2
 - 6*c**2*d**5*e**2*x**3 - 4*c**2*d**4*e**3*x**4 - c**2*d**3*e**4*x**5),x)
*a**2*c*d**3*e**5*x + 33*int(((a*d*e + a*e**2*x + c*d**2*x + c*d*e*x**2)**
(1/4)*x)/(11*a**2*d**4*e**3 + 44*a**2*d**3*e**4*x + 66*a**2*d**2*e**5*x**2
 + 44*a**2*d*e**6*x**3 + 11*a**2*e**7*x**4 - a*c*d**6*e + 7*a*c*d**5*e**2*
x + 38*a*c*d**4*e**3*x**2 + 62*a*c*d**3*e**4*x**3 + 43*a*c*d**2*e**5*x**4
+ 11*a*c*d*e**6*x**5 - c**2*d**7*x - 4*c**2*d**6*e*x**2 - 6*c**2*d**5*e**2
*x**3 - 4*c**2*d**4*e**3*x**4 - c**2*d**3*e**4*x**5),x)*a**2*c*d**2*e**6*x
**2 + 11*int(((a*d*e + a*e**2*x + c*d**2*x + c*d*e*x**2)**(1/4)*x)/(11*a**
2*d**4*e**3 + 44*a**2*d**3*e**4*x + 66*a**2*d**2*e**5*x**2 + 44*a**2*d*...
```

3.344 $\int (d+ex)^2 \left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/4} dx$

Optimal result

Integrand size = 37, antiderivative size = 346

$$\begin{aligned}\int (d+ex)^2 \left(ade+\left(cd^2+ae^2\right)x\right.&\left.+cdex^2\right)^{3/4} dx = \frac{11\left(cd^2-ae^2\right)^3 \left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/4}}{180c^3d^3e}\\&+\frac{11\left(cd^2-ae^2\right)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{7/4}}{63c^2d^2}\\&+\frac{11\left(cd^2-ae^2\right)^2 \left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{7/4}}{90c^3d^3(d+ex)}\\&+\frac{2(d+ex)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{7/4}}{9cd}\\&-\frac{11\left(cd^2-ae^2\right)^5 \sqrt[4]{-\frac{cde(ae+cdx)(d+ex)}{\left(cd^2-ae^2\right)^2}}E\left(\frac{1}{2}\arcsin\left(\frac{ae^2+cd(d+2ex)}{cd^2-ae^2}\right)\middle|2\right)}{120\sqrt{2}c^4d^4e^2\sqrt[4]{ade+\left(cd^2+ae^2\right)x+cdex^2}}\end{aligned}$$

output
```
11/180*(-a*e^2+c*d^2)^3*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/4)/c^3/d^3/e+
11/63*(-a*e^2+c*d^2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(7/4)/c^2/d^2+11/90
*(-a*e^2+c*d^2)^2*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(7/4)/c^3/d^3/(e*x+d)+
2/9*(e*x+d)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(7/4)/c/d-11/240*(-a*e^2+c*d
^2)^5*(-c*d*e*(c*d*x+a*e)*(e*x+d)/(-a*e^2+c*d^2)^2)^(1/4)*EllipticE(sin(1/
2*arcsin((a*e^2+c*d*(2*e*x+d))/(-a*e^2+c*d^2))),2^(1/2))*2^(1/2)/c^4/d^4/e
^2/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/4)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.05 (sec) , antiderivative size = 110, normalized size of antiderivative = 0.32

$$\int (d+ex)^2 \left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/4} dx = \frac{4\left(cd^2-ae^2\right)^2 (ae+cdx)((ae+cdx)(d+ex))^{3/4}\,\text{Hypergeometric2F1}\left(-\frac{11}{4},\frac{7}{4},\frac{11}{4},\frac{e(ae+cdx)}{-cd^2+ae^2}\right)}{7c^3d^3\left(\frac{cd(d+ex)}{cd^2-ae^2}\right)^{3/4}}$$

input
```
Integrate[(d + e*x)^2*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/4),x]
```

output
```
(4*(c*d^2 - a*e^2)^2*(a*e + c*d*x)*((a*e + c*d*x)*(d + e*x))^(3/4)*Hyperge
ometric2F1[-11/4, 7/4, 11/4, (e*(a*e + c*d*x))/(-(c*d^2) + a*e^2)])/(7*c^3
*d^3*((c*d*(d + e*x))/(c*d^2 - a*e^2))^(3/4))
```

Rubi [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 1148 vs. 2(346) = 692.

Time = 1.53 (sec) , antiderivative size = 1148, normalized size of antiderivative = 3.32, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.216$, Rules used = {1166, 27, 1160, 1087, 1094, 834, 761, 1510}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (d+ex)^2 \left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/4} dx$$

$$\Big\downarrow \text{1166}$$

$$\frac{2\int \frac{11}{4}e\left(cd^2-ae^2\right)(d+ex)\left(cdex^2+\left(cd^2+ae^2\right)x+ade\right)^{3/4}dx}{9cde}+\frac{2(d+ex)\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{7/4}}{9cd}$$

$$\Big\downarrow \text{27}$$

$$\frac{11\left(cd^2-ae^2\right)\int (d+ex)\left(cdex^2+\left(cd^2+ae^2\right)x+ade\right)^{3/4}dx}{18cd}+\frac{2(d+ex)\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{7/4}}{9cd}$$

$$\Big\downarrow \text{1160}$$

$$\frac{11\left(cd^2-ae^2\right)\left(\frac{\left(d^2-\frac{ae^2}{c}\right)\int \left(cdex^2+\left(cd^2+ae^2\right)x+ade\right)^{3/4}dx}{2d}+\frac{2\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{7/4}}{7cd}\right)}{18cd}+\frac{2(d+ex)\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{7/4}}{9cd}$$

$$\Big\downarrow \text{1087}$$

$$\frac{11\left(cd^2-ae^2\right)\left(\frac{\left(d^2-\frac{ae^2}{c}\right)\left(\frac{\left(ae^2+cd^2+2cdex\right)\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/4}}{5cde}-\frac{3\left(cd^2-ae^2\right)^2\int \frac{1}{\sqrt[4]{cdex^2+\left(cd^2+ae^2\right)x+ade}}dx}{20cde}\right)}{2d}+\cdots\right)}{18cd}+\frac{2(d+ex)\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{7/4}}{9cd}$$

$$\Big\downarrow \text{1094}$$

$$\frac{11\left(cd^2-ae^2\right)\left(\frac{\left(d^2-\frac{ae^2}{c}\right)\left(\frac{\left(ae^2+cd^2+2cdex\right)\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/4}}{5cde}-\frac{3\left(cd^2-ae^2\right)^2\sqrt{\left(ae^2+cd^2+2cdex\right)^2}\int\frac{\sqrt{cdex^2+\left(cd^2+ae\cdots\right.}}{\sqrt{\left(cd^2-ae^2\right)^2+4cde\left(cdex^2+\cdots\right.}}}{5cde\left(ae^2+cd\cdots\right.}\cdots\right.}{2d}\cdots\right.}{18cd}\cdots$$

$$\frac{2(d+ex)\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{7/4}}{9cd}$$

$\downarrow$ 834

$$\frac{11\left(cd^2-ae^2\right)\left(\frac{\left(d^2-\frac{ae^2}{c}\right)\left(\frac{\left(ae^2+cd^2+2cdex\right)\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/4}}{5cde}-\frac{3\left(cd^2-ae^2\right)^2\sqrt{\left(ae^2+cd^2+2cdex\right)^2}\left(\frac{\left(cd^2-ae^2\right)\int\frac{\cdots}{\sqrt{\left(cd^2-ae^2\right)^2+\cdots}}}{\cdots}\cdots\right.}{\cdots}\cdots\right.}{\cdots}\cdots\right.}{\cdots}\cdots$$

$$\frac{2(d+ex)\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{7/4}}{9cd}$$

$\downarrow$ 761

$$\frac{\left(d^2-\frac{ae^2}{c}\right)\left(\frac{\left(ae^2+cd^2+2cdex\right)\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/4}}{5cde}-\frac{\left(cd^2-ae^2\right)^{3/2}\left(\frac{2\sqrt{c}\sqrt{d}\sqrt{e}\sqrt{\cdots}}{\cdots}\cdots\right.}{3\left(cd^2-ae^2\right)^2\sqrt{\left(ae^2+cd^2+2cdex\right)^2}\cdots}\cdots\right.}{11\left(cd^2-ae^2\right)\cdots}$$

$$\frac{2(d+ex)\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{7/4}}{9cd}$$

$\downarrow$ 1510

$$\frac{2(d+ex)\left(cdex^2+\left(cd^2+ae^2\right)x+ade\right)^{7/4}}{9cd}+$$

$$3\left(cd^2-ae^2\right)^2\sqrt{\left(cd^2+2cexd\right.}$$

$$\left(d^2-\frac{ae^2}{c}\right)\left(\frac{\left(cd^2+2cexd+ae^2\right)\left(cdex^2+\left(cd^2+ae^2\right)x+ade\right)^{3/4}}{5cde}-\right.$$

$$11\left(cd^2-ae^2\right)\left(\frac{2\left(cdex^2+\left(cd^2+ae^2\right)x+ade\right)^{7/4}}{7cd}+\right.$$

input
```
Int[(d + e*x)^2*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/4),x]
```

output
```
(2*(d + e*x)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(7/4))/(9*c*d) + (11*
(c*d^2 - a*e^2)*((2*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(7/4))/(7*c*d)
 + ((d^2 - (a*e^2)/c)*(((c*d^2 + a*e^2 + 2*c*d*e*x)*(a*d*e + (c*d^2 + a*e^
2)*x + c*d*e*x^2)^(3/4))/(5*c*d*e) - (3*(c*d^2 - a*e^2)^2*Sqrt[(c*d^2 + a*
e^2 + 2*c*d*e*x)^2]*(-1/2*((c*d^2 - a*e^2)*(-(((a*d*e + (c*d^2 + a*e^2)*x
+ c*d*e*x^2)^(1/4)*Sqrt[(c*d^2 - a*e^2)^2 + 4*c*d*e*(a*d*e + (c*d^2 + a*e^
2)*x + c*d*e*x^2)])/((c*d^2 - a*e^2)^2*(1 + (2*Sqrt[c]*Sqrt[d]*Sqrt[e]*Sqr
t[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(c*d^2 - a*e^2)))) + (Sqrt[c*d^2
 - a*e^2]*(1 + (2*Sqrt[c]*Sqrt[d]*Sqrt[e]*Sqrt[a*d*e + (c*d^2 + a*e^2)*x +
 c*d*e*x^2])/(c*d^2 - a*e^2))*Sqrt[((c*d^2 - a*e^2)^2 + 4*c*d*e*(a*d*e + (
c*d^2 + a*e^2)*x + c*d*e*x^2))/((c*d^2 - a*e^2)^2*(1 + (2*Sqrt[c]*Sqrt[d]*
Sqrt[e]*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(c*d^2 - a*e^2))^2)]*
EllipticE[2*ArcTan[(Sqrt[2]*c^(1/4)*d^(1/4)*e^(1/4)*(a*d*e + (c*d^2 + a*e^
2)*x + c*d*e*x^2)^(1/4))/Sqrt[c*d^2 - a*e^2]], 1/2])/(Sqrt[2]*c^(1/4)*d^(1
/4)*e^(1/4)*Sqrt[(c*d^2 - a*e^2)^2 + 4*c*d*e*(a*d*e + (c*d^2 + a*e^2)*x +
c*d*e*x^2)])))/(Sqrt[c]*Sqrt[d]*Sqrt[e]) + ((c*d^2 - a*e^2)^(3/2)*(1 + (2*
Sqrt[c]*Sqrt[d]*Sqrt[e]*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(c*d^
2 - a*e^2))*Sqrt[((c*d^2 - a*e^2)^2 + 4*c*d*e*(a*d*e + (c*d^2 + a*e^2)*x +
 c*d*e*x^2))/((c*d^2 - a*e^2)^2*(1 + (2*Sqrt[c]*Sqrt[d]*Sqrt[e]*Sqrt[a*d*e
 + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(c*d^2 - a*e^2))^2)]*EllipticF[2*Arc...
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 761
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 834
```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, S
imp[1/q   Int[1/Sqrt[a + b*x^4], x], x] - Simp[1/q   Int[(1 - q*x^2)/Sqrt[a
 + b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 1087
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^p/(2*c*(2*p + 1))), x] - Simp[p*((b^2 - 4*a*c)/(2*c*(2*
p + 1)))   Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] &&
GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[3*p])
```

rule 1094
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[4*(Sqrt[(b
+ 2*c*x)^2]/(b + 2*c*x))   Subst[Int[x^(4*(p + 1) - 1)/Sqrt[b^2 - 4*a*c + 4
*c*x^4], x], x, (a + b*x + c*x^2)^(1/4)], x] /; FreeQ[{a, b, c}, x] && Inte
gerQ[4*p]
```

rule 1160
```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[e*((a + b*x + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Simp[(2*c*d - b
*e)/(2*c)   Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
 && NeQ[p, -1]
```

rule 1166
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[e*(d + e*x)^(m - 1)*((a + b*x + c*x^2)^(p + 1)/(c*(m + 2*p +
 1))), x] + Simp[1/(c*(m + 2*p + 1))   Int[(d + e*x)^(m - 2)*Simp[c*d^2*(m
+ 2*p + 1) - e*(a*e*(m - 1) + b*d*(p + 1)) + e*(2*c*d - b*e)*(m + p)*x, x]*
(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && If[Ration
alQ[m], GtQ[m, 1], SumSimplerQ[m, -2]] && NeQ[m + 2*p + 1, 0] && IntQuadrat
icQ[a, b, c, d, e, m, p, x]
```

rule 1510
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

Maple [F]

$$\int (ex+d)^2 \left(ade+\left(a\,e^2+c\,d^2\right)x+cd\,x^2e\right)^{\frac{3}{4}}dx$$

input
```
int((e*x+d)^2*(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(3/4),x)
```

output
```
int((e*x+d)^2*(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(3/4),x)
```

Fricas [F]

$$\int (d+ex)^2 \left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/4}\,dx=\int \left(cdex^2+ade+\left(cd^2+ae^2\right)x\right)^{\frac{3}{4}}(ex+d)^2\,dx$$

input
```
integrate((e*x+d)^2*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/4),x, algorithm="
fricas")
```

output
```
integral((c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^(3/4)*(e^2*x^2 + 2*d*e*x
+ d^2), x)
```

Sympy [F]

$$\int (d+ex)^2 \left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/4}\,dx=\int \left((d+ex)\,(ae+cdx)\right)^{\frac{3}{4}}(d+ex)^2\,dx$$

input
```
integrate((e*x+d)**2*(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(3/4),x)
```

output
```
Integral(((d + e*x)*(a*e + c*d*x))**(3/4)*(d + e*x)**2, x)
```

Maxima [F]

$$\int (d+ex)^2 \left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/4} dx = \int \left(cdex^2+ade+\left(cd^2+ae^2\right)x\right)^{\frac{3}{4}}(ex+d)^2\, dx$$

input
```
integrate((e*x+d)^2*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/4),x, algorithm="
maxima")
```

output
```
integrate((c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^(3/4)*(e*x + d)^2, x)
```

Giac [F]

$$\int (d+ex)^2 \left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/4} dx = \int \left(cdex^2+ade+\left(cd^2+ae^2\right)x\right)^{\frac{3}{4}}(ex+d)^2\, dx$$

input
```
integrate((e*x+d)^2*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/4),x, algorithm="
giac")
```

output
```
integrate((c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^(3/4)*(e*x + d)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int (d+ex)^2 \left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/4} dx = \int (d+e\,x)^2\,\left(c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e\right)^{3/4}\, dx$$

input
```
int((d + e*x)^2*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(3/4),x)
```

output
```
int((d + e*x)^2*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(3/4), x)
```

Reduce [F]

$$\int (d+ex)^2 \left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/4} dx = \text{Too large to display}$$

input
```
int((e*x+d)^2*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/4),x)
```

output
```
(176*(a*d*e + a*e**2*x + c*d**2*x + c*d*e*x**2)**(3/4)*a**3*d*e**5 - 132*(
a*d*e + a*e**2*x + c*d**2*x + c*d*e*x**2)**(3/4)*a**3*e**6*x - 672*(a*d*e
+ a*e**2*x + c*d**2*x + c*d*e*x**2)**(3/4)*a**2*c*d**3*e**3 + 372*(a*d*e +
 a*e**2*x + c*d**2*x + c*d*e*x**2)**(3/4)*a**2*c*d**2*e**4*x + 120*(a*d*e
+ a*e**2*x + c*d**2*x + c*d*e*x**2)**(3/4)*a**2*c*d*e**5*x**2 + 1616*(a*d*
e + a*e**2*x + c*d**2*x + c*d*e*x**2)**(3/4)*a*c**2*d**5*e + 1812*(a*d*e +
 a*e**2*x + c*d**2*x + c*d*e*x**2)**(3/4)*a*c**2*d**4*e**2*x + 1680*(a*d*e
 + a*e**2*x + c*d**2*x + c*d*e*x**2)**(3/4)*a*c**2*d**3*e**3*x**2 + 560*(a
*d*e + a*e**2*x + c*d**2*x + c*d*e*x**2)**(3/4)*a*c**2*d**2*e**4*x**3 + 13
08*(a*d*e + a*e**2*x + c*d**2*x + c*d*e*x**2)**(3/4)*c**3*d**6*x + 1560*(a
*d*e + a*e**2*x + c*d**2*x + c*d*e*x**2)**(3/4)*c**3*d**5*e*x**2 + 560*(a*
d*e + a*e**2*x + c*d**2*x + c*d*e*x**2)**(3/4)*c**3*d**4*e**2*x**3 + 231*i
nt(((a*d*e + a*e**2*x + c*d**2*x + c*d*e*x**2)**(3/4)*x)/(a**2*d*e**3 + a*
*2*e**4*x + a*c*d**3*e + 2*a*c*d**2*e**2*x + a*c*d*e**3*x**2 + c**2*d**4*x
 + c**2*d**3*e*x**2),x)*a**5*e**10 - 693*int(((a*d*e + a*e**2*x + c*d**2*x
 + c*d*e*x**2)**(3/4)*x)/(a**2*d*e**3 + a**2*e**4*x + a*c*d**3*e + 2*a*c*d
**2*e**2*x + a*c*d*e**3*x**2 + c**2*d**4*x + c**2*d**3*e*x**2),x)*a**4*c*d
**2*e**8 + 462*int(((a*d*e + a*e**2*x + c*d**2*x + c*d*e*x**2)**(3/4)*x)/(
a**2*d*e**3 + a**2*e**4*x + a*c*d**3*e + 2*a*c*d**2*e**2*x + a*c*d*e**3*x*
*2 + c**2*d**4*x + c**2*d**3*e*x**2),x)*a**3*c**2*d**4*e**6 + 462*int((...
```

3.345 $\int (d+ex) \left(ade+\left(cd^2+ae^2\right) x+cdex^2\right)^{3/4} \, dx$

Optimal result

Integrand size = 35, antiderivative size = 288

$$\int (d+ex) \left(ade+\left(cd^2+ae^2\right) x+cdex^2\right)^{3/4} \, dx = \frac{\left(cd^2-ae^2\right)^2 \left(ade+\left(cd^2+ae^2\right) x+cdex^2\right)^{3/4}}{10c^2d^2e} + \frac{2(ade+\left(cd^2+ae^2\right) x+cdex^2)^{7/4}}{7cd} + \frac{\left(cd^2-ae^2\right) \left(ade+\left(cd^2+ae^2\right) x+cdex^2\right)^{7/4}}{5c^2d^2(d+ex)} - \frac{3(cd^2-ae^2)^4 \sqrt[4]{-\frac{cde(ae+cdx)(d+ex)}{\left(cd^2-ae^2\right)^2}} E\left(\frac{1}{2}\arcsin\left(\frac{ae^2+cd(d+2ex)}{cd^2-ae^2}\right)\middle|2\right)}{20\sqrt{2}c^3d^3e^2 \sqrt[4]{ade+\left(cd^2+ae^2\right) x+cdex^2}}$$

output
```
1/10*(-a*e^2+c*d^2)^2*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/4)/c^2/d^2/e+2/
7*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(7/4)/c/d+1/5*(-a*e^2+c*d^2)*(a*d*e+(a
*e^2+c*d^2)*x+c*d*e*x^2)^(7/4)/c^2/d^2/(e*x+d)-3/40*(-a*e^2+c*d^2)^4*(-c*d
*e*(c*d*x+a*e)*(e*x+d)/(-a*e^2+c*d^2)^2)^(1/4)*EllipticE(sin(1/2*arcsin((a
*e^2+c*d*(2*e*x+d))/(-a*e^2+c*d^2))),2^(1/2))*2^(1/2)/c^3/d^3/e^2/(a*d*e+(
a*e^2+c*d^2)*x+c*d*e*x^2)^(1/4)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.06 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.31

$$\int (d+ex)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/4}\,dx=\frac{4((ae+cdx)(d+ex))^{7/4}\,\text{Hypergeometric2F1}\left(-\frac{7}{4},\frac{7}{4},\frac{11}{4},\frac{e(ae+cdx)}{-cd^2+ae^2}\right)}{7cd\left(\frac{cd(d+ex)}{cd^2-ae^2}\right)^{7/4}}$$

input
```
Integrate[(d + e*x)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/4),x]
```

output
```
(4*((a*e + c*d*x)*(d + e*x))^(7/4)*Hypergeometric2F1[-7/4, 7/4, 11/4, (e*(
a*e + c*d*x))/(-(c*d^2) + a*e^2)])/(7*c*d*((c*d*(d + e*x))/(c*d^2 - a*e^2)
)^(7/4))
```

Rubi [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 1081 vs. 2(288) = 576.

Time = 1.28 (sec) , antiderivative size = 1081, normalized size of antiderivative = 3.75, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.171, Rules used = {1160, 1087, 1094, 834, 761, 1510}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (d+ex)\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/4}dx$$

$$\Big\downarrow \ 1160$$

$$\frac{\left(d^2-\frac{ae^2}{c}\right)\int \left(cdex^2+\left(cd^2+ae^2\right)x+ade\right)^{3/4}dx}{2d}+\frac{2\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{7/4}}{7cd}$$

$$\Big\downarrow \ 1087$$

$$\frac{\left(d^2-\frac{ae^2}{c}\right)\left(\frac{(ae^2+cd^2+2cdex)\left(x(ae^2+cd^2)+ade+cdex^2\right)^{3/4}}{5cde}-\frac{3\left(cd^2-ae^2\right)^2\int\frac{1}{\sqrt[4]{cdex^2+(cd^2+ae^2)x+ade}}dx}{20cde}\right)}{2d}+\frac{2\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{7/4}}{7cd}$$

$\Big\downarrow$ 1094

$$\frac{\left(d^2-\frac{ae^2}{c}\right)\left(\frac{(ae^2+cd^2+2cdex)\left(x(ae^2+cd^2)+ade+cdex^2\right)^{3/4}}{5cde}-\frac{3\left(cd^2-ae^2\right)^2\sqrt{(ae^2+cd^2+2cdex)^2}\int\frac{\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}{\sqrt{\left(cd^2-ae^2\right)^2+4cde\left(cdex^2+\left(cd^2+ae^2\right)x+ade\right)}}}{5cde(ae^2+cd^2+2cdex)}\right)}{2d}$$

$$\frac{2\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{7/4}}{7cd}$$

$\Big\downarrow$ 834

$$\frac{\left(d^2-\frac{ae^2}{c}\right)\left(\frac{(ae^2+cd^2+2cdex)\left(x(ae^2+cd^2)+ade+cdex^2\right)^{3/4}}{5cde}-\frac{3\left(cd^2-ae^2\right)^2\sqrt{(ae^2+cd^2+2cdex)^2}\left(\frac{\left(cd^2-ae^2\right)\int\frac{1}{\sqrt{\left(cd^2-ae^2\right)^2+4cde\left(cde\ldots\right.}}}{\ldots}\right.}{\ldots}\right.}{\ldots}$$

$$\frac{2\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{7/4}}{7cd}$$

$\Big\downarrow$ 761

$$\left(d^2-\frac{ae^2}{c}\right)\left(\frac{(ae^2+cd^2+2cdex)\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/4}}{5cde}-\frac{\left(cd^2-ae^2\right)^{3/2}\left(\frac{2\sqrt{c}\sqrt{d}\sqrt{e}\sqrt{x\left(ae^2+cd\right.}}{cd^2-ae}\right.}{3\left(cd^2-ae^2\right)^2\sqrt{(ae^2+cd^2+2cdex)^2}}\right.$$

$$\frac{2\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{7/4}}{7cd}$$

$\downarrow$ 1510

$$\frac{2\left(cdex^2+\left(cd^2+ae^2\right)x+ade\right)^{7/4}}{7cd}+$$

$$\left(d^2-\frac{ae^2}{c}\right)\left(\frac{\left(cd^2+2cexd+ae^2\right)\left(cdex^2+\left(cd^2+ae^2\right)x+ade\right)^{3/4}}{5cde}-\frac{3\left(cd^2-ae^2\right)^2\sqrt{\left(cd^2+2cexd+ae^2\right)^2}\left(\frac{\left(cd^2-ae^2\right)^{3/2}\left(\frac{2\sqrt{c}\sqrt{d}\sqrt{e}\sqrt{cdex^2+(c\cdots}}{cd^2-ae\cdots}\right.}{\cdots}\right.}{\cdots}\right.$$

input
```
Int[(d + e*x)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/4),x]
```

output
```
(2*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(7/4))/(7*c*d) + ((d^2 - (a*e^2
)/c)*(((c*d^2 + a*e^2 + 2*c*d*e*x)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)
^(3/4))/(5*c*d*e) - (3*(c*d^2 - a*e^2)^2*Sqrt[(c*d^2 + a*e^2 + 2*c*d*e*x)^
2]*(-1/2*((c*d^2 - a*e^2)*(-(((a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(1/4
)*Sqrt[(c*d^2 - a*e^2)^2 + 4*c*d*e*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)
])/((c*d^2 - a*e^2)^2*(1 + (2*Sqrt[c]*Sqrt[d]*Sqrt[e]*Sqrt[a*d*e + (c*d^2
+ a*e^2)*x + c*d*e*x^2])/(c*d^2 - a*e^2)))) + (Sqrt[c*d^2 - a*e^2]*(1 + (2
*Sqrt[c]*Sqrt[d]*Sqrt[e]*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(c*d
^2 - a*e^2))*Sqrt[((c*d^2 - a*e^2)^2 + 4*c*d*e*(a*d*e + (c*d^2 + a*e^2)*x
+ c*d*e*x^2))/((c*d^2 - a*e^2)^2*(1 + (2*Sqrt[c]*Sqrt[d]*Sqrt[e]*Sqrt[a*d*
e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(c*d^2 - a*e^2))^2)]*EllipticE[2*ArcTa
n[(Sqrt[2]*c^(1/4)*d^(1/4)*e^(1/4)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)
^(1/4))/Sqrt[c*d^2 - a*e^2]], 1/2])/(Sqrt[2]*c^(1/4)*d^(1/4)*e^(1/4)*Sqrt[
(c*d^2 - a*e^2)^2 + 4*c*d*e*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)])))/(S
qrt[c]*Sqrt[d]*Sqrt[e]) + ((c*d^2 - a*e^2)^(3/2)*(1 + (2*Sqrt[c]*Sqrt[d]*S
qrt[e]*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(c*d^2 - a*e^2))*Sqrt[
((c*d^2 - a*e^2)^2 + 4*c*d*e*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2))/((c*
d^2 - a*e^2)^2*(1 + (2*Sqrt[c]*Sqrt[d]*Sqrt[e]*Sqrt[a*d*e + (c*d^2 + a*e^2
)*x + c*d*e*x^2])/(c*d^2 - a*e^2))^2)]*EllipticF[2*ArcTan[(Sqrt[2]*c^(1/4)
*d^(1/4)*e^(1/4)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(1/4))/Sqrt[c*...
```

Defintions of rubi rules used

rule 761
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 834
```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, S
imp[1/q   Int[1/Sqrt[a + b*x^4], x], x] - Simp[1/q   Int[(1 - q*x^2)/Sqrt[a
 + b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 1087
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^p/(2*c*(2*p + 1))), x] - Simp[p*((b^2 - 4*a*c)/(2*c*(2*
p + 1)))   Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] &&
GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[3*p])
```

rule 1094
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[4*(Sqrt[(b
+ 2*c*x)^2]/(b + 2*c*x))   Subst[Int[x^(4*(p + 1) - 1)/Sqrt[b^2 - 4*a*c + 4
*c*x^4], x], x, (a + b*x + c*x^2)^(1/4)], x] /; FreeQ[{a, b, c}, x] && Inte
gerQ[4*p]
```

rule 1160
```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[e*((a + b*x + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Simp[(2*c*d - b
*e)/(2*c)   Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
 && NeQ[p, -1]
```

rule 1510
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

Maple [F]

$$\int (ex+d)\left(ade+\left(a\,e^{2}+c\,d^{2}\right)x+cd\,x^{2}e\right)^{\frac{3}{4}}dx$$

input
```
int((e*x+d)*(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(3/4),x)
```

output
```
int((e*x+d)*(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(3/4),x)
```

Fricas [F]

$$\int (d+ex)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/4}\,dx=\int \left(cdex^2+ade+\left(cd^2+ae^2\right)x\right)^{\frac{3}{4}}(ex+d)\,dx$$

input
```
integrate((e*x+d)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/4),x, algorithm="fr
icas")
```

output
```
integral((c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^(3/4)*(e*x + d), x)
```

Sympy [F]

$$\int (d+ex)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/4}\,dx=\int ((d+ex)\,(ae+cdx))^{\frac{3}{4}}\,(d+ex)\,dx$$

input
```
integrate((e*x+d)*(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(3/4),x)
```

output
```
Integral(((d + e*x)*(a*e + c*d*x))**(3/4)*(d + e*x), x)
```

Maxima [F]

$$\int (d+ex)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/4}\,dx=\int \left(cdex^2+ade+\left(cd^2+ae^2\right)x\right)^{\frac{3}{4}}(ex+d)\,dx$$

input
```
integrate((e*x+d)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/4),x, algorithm="ma
xima")
```

output
```
integrate((c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^(3/4)*(e*x + d), x)
```

Giac [F]

$$\int (d+ex)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/4}\,dx=\int \left(cdex^2+ade+\left(cd^2+ae^2\right)x\right)^{\frac{3}{4}}(ex+d)\,dx$$

input
```
integrate((e*x+d)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/4),x, algorithm="gi
ac")
```

output
```
integrate((c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^(3/4)*(e*x + d), x)
```

Mupad [F(-1)]

Timed out.

$$\int (d+ex)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/4}\,dx=\int (d+e\,x)\,\left(c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e\right)^{3/4}\,dx$$

input
```
int((d + e*x)*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(3/4),x)
```

output
```
int((d + e*x)*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(3/4), x)
```

Reduce [F]

$$\int (d+ex)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/4}\,dx=\frac{-16(cde\,x^2+a\,e^2x+c\,d^2x+ade)^{\frac{3}{4}}\,a^2d\,e^3+12(cde\,x^2+a\,e^2x+c\,d^2x+ade)^{\frac{3}{4}}\,a^2e^4x+9}{}$$

input
```
int((e*x+d)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/4),x)
```

output
```
( - 16*(a*d*e + a*e**2*x + c*d**2*x + c*d*e*x**2)**(3/4)*a**2*d*e**3 + 12*
(a*d*e + a*e**2*x + c*d**2*x + c*d*e*x**2)**(3/4)*a**2*e**4*x + 96*(a*d*e
+ a*e**2*x + c*d**2*x + c*d*e*x**2)**(3/4)*a*c*d**3*e + 80*(a*d*e + a*e**2
*x + c*d**2*x + c*d*e*x**2)**(3/4)*a*c*d**2*e**2*x + 40*(a*d*e + a*e**2*x
+ c*d**2*x + c*d*e*x**2)**(3/4)*a*c*d*e**3*x**2 + 68*(a*d*e + a*e**2*x + c
*d**2*x + c*d*e*x**2)**(3/4)*c**2*d**4*x + 40*(a*d*e + a*e**2*x + c*d**2*x
 + c*d*e*x**2)**(3/4)*c**2*d**3*e*x**2 - 21*int(((a*d*e + a*e**2*x + c*d**
2*x + c*d*e*x**2)**(3/4)*x)/(a**2*d*e**3 + a**2*e**4*x + a*c*d**3*e + 2*a*
c*d**2*e**2*x + a*c*d*e**3*x**2 + c**2*d**4*x + c**2*d**3*e*x**2),x)*a**4*
e**8 + 42*int(((a*d*e + a*e**2*x + c*d**2*x + c*d*e*x**2)**(3/4)*x)/(a**2*
d*e**3 + a**2*e**4*x + a*c*d**3*e + 2*a*c*d**2*e**2*x + a*c*d*e**3*x**2 +
c**2*d**4*x + c**2*d**3*e*x**2),x)*a**3*c*d**2*e**6 - 42*int(((a*d*e + a*e
**2*x + c*d**2*x + c*d*e*x**2)**(3/4)*x)/(a**2*d*e**3 + a**2*e**4*x + a*c*
d**3*e + 2*a*c*d**2*e**2*x + a*c*d*e**3*x**2 + c**2*d**4*x + c**2*d**3*e*x
**2),x)*a*c**3*d**6*e**2 + 21*int(((a*d*e + a*e**2*x + c*d**2*x + c*d*e*x*
*2)**(3/4)*x)/(a**2*d*e**3 + a**2*e**4*x + a*c*d**3*e + 2*a*c*d**2*e**2*x
+ a*c*d*e**3*x**2 + c**2*d**4*x + c**2*d**3*e*x**2),x)*c**4*d**8)/(140*c*d
*(a*e**2 + c*d**2))
```

3.346 $\int \left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/4}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 206

$$\int \left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/4}\,dx=\frac{\left(cd^2+ae^2+2cdex\right)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/4}}{5cde}-\frac{3\left(cd^2-ae^2\right)^3\sqrt[4]{-\frac{cde\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)}{\left(cd^2-ae^2\right)^2}}\,E\left(\frac{1}{2}\arcsin\left(\frac{ae^2+cd(d+2ex)}{cd^2-ae^2}\right)\middle|2\right)}{10\sqrt{2}c^2d^2e^2\sqrt[4]{ade+\left(cd^2+ae^2\right)x+cdex^2}}$$

output
```
1/5*(2*c*d*e*x+a*e^2+c*d^2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/4)/c/d/e-
3/20*(-a*e^2+c*d^2)^3*(-c*d*e*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)/(-a*e^2+c*
d^2)^2)^(1/4)*EllipticE(sin(1/2*arcsin((a*e^2+c*d*(2*e*x+d))/(-a*e^2+c*d^2
))),2^(1/2))*2^(1/2)/c^2/d^2/e^2/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/4)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.03 (sec) , antiderivative size = 96, normalized size of antiderivative = 0.47

$$\int \left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/4}\,dx=\frac{4(ae+cdx)((ae+cdx)(d+ex))^{3/4}\,\text{Hypergeometric2F1}\left(-\frac{3}{4},\frac{7}{4},\frac{11}{4},\frac{e(ae+cdx)}{-cd^2+ae^2}\right)}{7cd\left(\frac{cd(d+ex)}{cd^2-ae^2}\right)^{3/4}}$$

input
```
Integrate[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/4),x]
```

output
```
(4*(a*e + c*d*x)*((a*e + c*d*x)*(d + e*x))^(3/4)*Hypergeometric2F1[-3/4, 7
/4, 11/4, (e*(a*e + c*d*x))/(-(c*d^2) + a*e^2)])/(7*c*d*((c*d*(d + e*x))/(
c*d^2 - a*e^2))^(3/4))
```

Rubi [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 1021 vs. 2(206) = 412.

Time = 1.21 (sec) , antiderivative size = 1021, normalized size of antiderivative = 4.96, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.172, Rules used = {1087, 1094, 834, 761, 1510}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/4}dx$$

$$\Big\downarrow \ 1087$$

$$\frac{\left(ae^2+cd^2+2cdex\right)\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/4}}{5cde}-\frac{3\left(cd^2-ae^2\right)^2\int \frac{1}{\sqrt[4]{cdex^2+\left(cd^2+ae^2\right)x+ade}}dx}{20cde}$$

$$\Big\downarrow \ 1094$$

$$\frac{\left(ae^2+cd^2+2cdex\right)\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/4}}{5cde}-\frac{3\left(cd^2-ae^2\right)^2\sqrt{\left(ae^2+cd^2+2cdex\right)^2}\int \frac{\sqrt{cdex^2+(cd^2+ae^2)x+ade}}{\sqrt{(cd^2-ae^2)^2+4cde(cdex^2+(cd^2+ae^2)x+ade)}}d\sqrt[4]{cdex^2+\left(cd^2+ae^2\right)x+ade}}{5cde\left(ae^2+cd^2+2cdex\right)}$$

$$\Big\downarrow \ 834$$

$$\frac{\left(ae^2+cd^2+2cdex\right)\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/4}}{5cde}-\frac{3\left(cd^2-ae^2\right)^2\sqrt{\left(ae^2+cd^2+2cdex\right)^2}\left(\frac{\left(cd^2-ae^2\right)\int \frac{1}{\sqrt{\left(cd^2-ae^2\right)^2+4cde\left(cdex^2+\left(cd^2+ae^2\right)x+ade\right)}}d\sqrt[4]{cdex^2+\left(cd^2+ae^2\right)x+}}{2\sqrt{c}\sqrt{d}\sqrt{e}}\right.}{5cde\left(ae^2+cd^2+2cdex\right)}$$

$$\Big\downarrow \ 761$$

$$\frac{\left(ae^2+cd^2+2cdex\right)\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/4}}{5cde}-\frac{3\left(cd^2-ae^2\right)^2\sqrt{\left(ae^2+cd^2+2cdex\right)^2}\left(\frac{\left(cd^2-ae^2\right)^{3/2}\left(\frac{2\sqrt{c}\sqrt{d}\sqrt{e}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{cd^2-ae^2}+1\right)\sqrt{\frac{4cde\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)}{\left(cd^2-ae^2\right)^2\left(\frac{2\sqrt{c}\sqrt{d}\sqrt{e}\sqrt{x\left(ae^2+cd^2\right.}}{cd^2-ae^2}\right.}}}{4\sqrt{2}c^{3/4}d^{3/4}e^{3/4}\sqrt{4cde(x}}\right.}{}$$

$$\Big\downarrow \ 1510$$

$$\frac{(cd^2+2cexd+ae^2)\,(cdex^2+(cd^2+ae^2)\,x+ade)^{3/4}}{5cde}-$$

$$\frac{3(cd^2-ae^2)^2\sqrt{(cd^2+2cexd+ae^2)^2}\left(\frac{(cd^2-ae^2)^{3/2}\left(\frac{2\sqrt{c}\sqrt{d}\sqrt{e}\sqrt{cdex^2+(cd^2+ae^2)x+ade}}{cd^2-ae^2}+1\right)\sqrt{\frac{(cd^2-ae^2)^2+4cde\,(cdex^2+(cd^2}{(cd^2-ae^2)^2\left(\frac{2\sqrt{c}\sqrt{d}\sqrt{e}\sqrt{cdex^2+(cd}{cd^2-ae^2}\right.}}}{4\sqrt{2}c^{3/4}d^{3/4}e^{3/4}\sqrt{(cd^2-a}}\right.}{}$$

input
```
Int[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/4),x]
```

output
```
((c*d^2 + a*e^2 + 2*c*d*e*x)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/4)
)/(5*c*d*e) - (3*(c*d^2 - a*e^2)^2*Sqrt[(c*d^2 + a*e^2 + 2*c*d*e*x)^2]*(-1
/2*((c*d^2 - a*e^2)*(-(((a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(1/4)*Sqrt
[(c*d^2 - a*e^2)^2 + 4*c*d*e*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)])/((c
*d^2 - a*e^2)^2*(1 + (2*Sqrt[c]*Sqrt[d]*Sqrt[e]*Sqrt[a*d*e + (c*d^2 + a*e^
2)*x + c*d*e*x^2])/(c*d^2 - a*e^2)))) + (Sqrt[c*d^2 - a*e^2]*(1 + (2*Sqrt[
c]*Sqrt[d]*Sqrt[e]*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(c*d^2 - a
*e^2))*Sqrt[((c*d^2 - a*e^2)^2 + 4*c*d*e*(a*d*e + (c*d^2 + a*e^2)*x + c*d*
e*x^2))/((c*d^2 - a*e^2)^2*(1 + (2*Sqrt[c]*Sqrt[d]*Sqrt[e]*Sqrt[a*d*e + (c
*d^2 + a*e^2)*x + c*d*e*x^2])/(c*d^2 - a*e^2))^2)]*EllipticE[2*ArcTan[(Sqr
t[2]*c^(1/4)*d^(1/4)*e^(1/4)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(1/4)
)/Sqrt[c*d^2 - a*e^2]], 1/2])/(Sqrt[2]*c^(1/4)*d^(1/4)*e^(1/4)*Sqrt[(c*d^2
 - a*e^2)^2 + 4*c*d*e*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)])))/(Sqrt[c]
*Sqrt[d]*Sqrt[e]) + ((c*d^2 - a*e^2)^(3/2)*(1 + (2*Sqrt[c]*Sqrt[d]*Sqrt[e]
*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(c*d^2 - a*e^2))*Sqrt[((c*d^
2 - a*e^2)^2 + 4*c*d*e*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2))/((c*d^2 -
a*e^2)^2*(1 + (2*Sqrt[c]*Sqrt[d]*Sqrt[e]*Sqrt[a*d*e + (c*d^2 + a*e^2)*x +
c*d*e*x^2])/(c*d^2 - a*e^2))^2)]*EllipticF[2*ArcTan[(Sqrt[2]*c^(1/4)*d^(1/
4)*e^(1/4)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(1/4))/Sqrt[c*d^2 - a*e
^2]], 1/2])/(4*Sqrt[2]*c^(3/4)*d^(3/4)*e^(3/4)*Sqrt[(c*d^2 - a*e^2)^2 +...
```

Defintions of rubi rules used

rule 761
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 834
```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, S
imp[1/q   Int[1/Sqrt[a + b*x^4], x], x] - Simp[1/q   Int[(1 - q*x^2)/Sqrt[a
 + b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 1087
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^p/(2*c*(2*p + 1))), x] - Simp[p*((b^2 - 4*a*c)/(2*c*(2*
p + 1)))   Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] &&
GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[3*p])
```

rule 1094
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[4*(Sqrt[(b
+ 2*c*x)^2]/(b + 2*c*x))   Subst[Int[x^(4*(p + 1) - 1)/Sqrt[b^2 - 4*a*c + 4
*c*x^4], x], x, (a + b*x + c*x^2)^(1/4)], x] /; FreeQ[{a, b, c}, x] && Inte
gerQ[4*p]
```

rule 1510
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

Maple [F]

$$\int \left(ade+\left(a\,e^{2}+c\,d^{2}\right)x+cd\,x^{2}e\right)^{\frac{3}{4}}dx$$

input
```
int((a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(3/4),x)
```

output
```
int((a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(3/4),x)
```

Fricas [F]

$$\int \left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/4}\,dx=\int \left(cdex^2+ade+\left(cd^2+ae^2\right)x\right)^{\frac{3}{4}}\,dx$$

input
```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/4),x, algorithm="fricas")
```

output
```
integral((c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^(3/4), x)
```

Sympy [F]

$$\int \left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/4}\,dx=\int \left(ade+cdex^2+x\left(ae^2+cd^2\right)\right)^{\frac{3}{4}}\,dx$$

input
```
integrate((a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(3/4),x)
```

output
```
Integral((a*d*e + c*d*e*x**2 + x*(a*e**2 + c*d**2))**(3/4), x)
```

Maxima [F]

$$\int \left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/4}\,dx=\int \left(cdex^2+ade+\left(cd^2+ae^2\right)x\right)^{\frac{3}{4}}\,dx$$

input
```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/4),x, algorithm="maxima")
```

output
```
integrate((c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^(3/4), x)
```

Giac [F]

$$\int \left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/4}\,dx=\int \left(cdex^2+ade+\left(cd^2+ae^2\right)x\right)^{\frac{3}{4}}\,dx$$

input
```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/4),x, algorithm="giac")
```

output
```
integrate((c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^(3/4), x)
```

Mupad [F(-1)]

Timed out.

$$\int \left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/4}\,dx=\int \left(c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e\right)^{3/4}\,dx$$

input
```
int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(3/4),x)
```

output
```
int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(3/4), x)
```

Reduce [F]

$$\int \left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/4}\,dx=\frac{8(cde\,x^2+a\,e^2x+c\,d^2x+ade)^{\frac{3}{4}}\,ade+4(cde\,x^2+a\,e^2x+c\,d^2x+ade)^{\frac{3}{4}}\,a\,e^2x+4(cde\,x^2}{}$$

input
```
int((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/4),x)
```

output
```
(8*(a*d*e + a*e**2*x + c*d**2*x + c*d*e*x**2)**(3/4)*a*d*e + 4*(a*d*e + a*
e**2*x + c*d**2*x + c*d*e*x**2)**(3/4)*a*e**2*x + 4*(a*d*e + a*e**2*x + c*
d**2*x + c*d*e*x**2)**(3/4)*c*d**2*x + 3*int(((a*d*e + a*e**2*x + c*d**2*x
 + c*d*e*x**2)**(3/4)*x)/(a**2*d*e**3 + a**2*e**4*x + a*c*d**3*e + 2*a*c*d
**2*e**2*x + a*c*d*e**3*x**2 + c**2*d**4*x + c**2*d**3*e*x**2),x)*a**3*e**
6 - 3*int(((a*d*e + a*e**2*x + c*d**2*x + c*d*e*x**2)**(3/4)*x)/(a**2*d*e*
*3 + a**2*e**4*x + a*c*d**3*e + 2*a*c*d**2*e**2*x + a*c*d*e**3*x**2 + c**2
*d**4*x + c**2*d**3*e*x**2),x)*a**2*c*d**2*e**4 - 3*int(((a*d*e + a*e**2*x
 + c*d**2*x + c*d*e*x**2)**(3/4)*x)/(a**2*d*e**3 + a**2*e**4*x + a*c*d**3*
e + 2*a*c*d**2*e**2*x + a*c*d*e**3*x**2 + c**2*d**4*x + c**2*d**3*e*x**2),
x)*a*c**2*d**4*e**2 + 3*int(((a*d*e + a*e**2*x + c*d**2*x + c*d*e*x**2)**(
3/4)*x)/(a**2*d*e**3 + a**2*e**4*x + a*c*d**3*e + 2*a*c*d**2*e**2*x + a*c*
d*e**3*x**2 + c**2*d**4*x + c**2*d**3*e*x**2),x)*c**3*d**6)/(10*(a*e**2 +
c*d**2))
```

3.347 $\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/4}}{d+ex}\,dx$

Optimal result

Integrand size = 37, antiderivative size = 169

$$\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/4}}{d+ex}\,dx=\frac{2\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/4}}{3e}-\frac{\left(cd^2-ae^2\right)^2\sqrt[4]{-\frac{cde(ae+cdx)(d+ex)}{\left(cd^2-ae^2\right)^2}}E\left(\frac{1}{2}\arcsin\left(\frac{ae^2+cd(d+2ex)}{cd^2-ae^2}\right)\middle|2\right)}{\sqrt{2}cde^2\sqrt[4]{ade+\left(cd^2+ae^2\right)x+cdex^2}}$$

output
```
2/3*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/4)/e-1/2*(-a*e^2+c*d^2)^2*(-c*d*e
*(c*d*x+a*e)*(e*x+d)/(-a*e^2+c*d^2)^2)^(1/4)*EllipticE(sin(1/2*arcsin((a*e
^2+c*d*(2*e*x+d))/(-a*e^2+c*d^2))),2^(1/2))*2^(1/2)/c/d/e^2/(a*d*e+(a*e^2+
c*d^2)*x+c*d*e*x^2)^(1/4)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.04 (sec) , antiderivative size = 98, normalized size of antiderivative = 0.58

$$\int \frac{(ade+(cd^2+ae^2)x+cdex^2)^{3/4}}{d+ex}\,dx=\frac{4(ae+cdx)^2\sqrt[4]{\frac{cd(d+ex)}{cd^2-ae^2}}\,\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{7}{4},\frac{11}{4},\frac{e(ae+cdx)}{-cd^2+ae^2}\right)}{7cd\sqrt[4]{(ae+cdx)(d+ex)}}$$

input
```
Integrate[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/4)/(d + e*x),x]
```

output
```
(4*(a*e + c*d*x)^2*((c*d*(d + e*x))/(c*d^2 - a*e^2))^(1/4)*Hypergeometric2
F1[1/4, 7/4, 11/4, (e*(a*e + c*d*x))/(-(c*d^2) + a*e^2)])/(7*c*d*((a*e + c
*d*x)*(d + e*x))^(1/4))
```

Rubi [A] (warning: unable to verify)

Time = 0.75 (sec) , antiderivative size = 326, normalized size of antiderivative = 1.93, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.216, Rules used = {1138, 60, 73, 839, 813, 858, 807, 212}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(x(ae^2+cd^2)+ade+cdex^2)^{3/4}}{d+ex}dx$$

$$\Big\downarrow \text{1138}$$

$$\frac{(x(ae^2+cd^2)+ade+cdex^2)^{3/4}\int \frac{(ae+cdx)^{3/4}}{\sqrt[4]{\frac{ex}{d}+1}}dx}{d\left(\frac{ex}{d}+1\right)^{3/4}(ae+cdx)^{3/4}}$$

$$\Big\downarrow \text{60}$$

$$\frac{\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/4}\left(\frac{2d\left(\frac{ex}{d}+1\right)^{3/4}(ae+cdx)^{3/4}}{3e}-\frac{\left(cd^2-ae^2\right)\int\frac{1}{\sqrt[4]{ae+cdx}\sqrt[4]{\frac{ex}{d}+1}}dx}{2e}\right)}{d\left(\frac{ex}{d}+1\right)^{3/4}(ae+cdx)^{3/4}}$$

$\big\downarrow$ 73

$$\frac{\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/4}\left(\frac{2d\left(\frac{ex}{d}+1\right)^{3/4}(ae+cdx)^{3/4}}{3e}-\frac{2\left(cd^2-ae^2\right)\int\frac{\sqrt{ae+cdx}}{\sqrt[4]{-\frac{ae^2}{cd^2}+\frac{(ae+cdx)e}{cd^2}+1}}d\sqrt[4]{ae+cdx}}{cde}\right)}{d\left(\frac{ex}{d}+1\right)^{3/4}(ae+cdx)^{3/4}}$$

$\big\downarrow$ 839

$$\frac{\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/4}\left(\frac{2d\left(\frac{ex}{d}+1\right)^{3/4}(ae+cdx)^{3/4}}{3e}-\frac{2\left(cd^2-ae^2\right)\left(\frac{(ae+cdx)^{3/4}}{2\sqrt[4]{-\frac{ae^2}{cd^2}+\frac{e(ae+cdx)}{cd^2}+1}}-\frac{1}{2}\left(1-\frac{ae^2}{cd^2}\right)\int\right.}{cde}\right)}{d\left(\frac{ex}{d}+1\right)^{3/4}(ae+cdx)^{3/4}}$$

$\big\downarrow$ 813

$$\frac{\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/4}\left(\frac{2d\left(\frac{ex}{d}+1\right)^{3/4}(ae+cdx)^{3/4}}{3e}-\frac{2\left(cd^2-ae^2\right)\left(\frac{(ae+cdx)^{3/4}}{2\sqrt[4]{-\frac{ae^2}{cd^2}+\frac{e(ae+cdx)}{cd^2}+1}}-\frac{cd^2\left(1-\frac{ae^2}{cd^2}\right)\sqrt[4]{}}{}\right.}{}\right)}{d\left(\frac{ex}{d}+1\right)^{3/4}(ae+cdx)^{3/4}}$$

$\big\downarrow$ 858

$$\frac{\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/4}\left(\frac{2d\left(\frac{ex}{d}+1\right)^{3/4}(ae+cdx)^{3/4}}{3e}-\frac{2\left(cd^2-ae^2\right)\left(\frac{cd^2\left(1-\frac{ae^2}{cd^2}\right)\sqrt[4]{ae+cdx}\sqrt[4]{\frac{cd^2-ae^2}{e(ae+cdx)}+1}\int}{2e\sqrt[4]{-\frac{ae^2}{cd^2}+}}\right)}{}\right)}{d\left(\frac{ex}{d}+1\right)^{3/4}(ae+cdx)^{3/4}}$$

$\big\downarrow$ 807

$$\frac{\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/4}\left(\frac{2d\left(\frac{ex}{d}+1\right)^{3/4}(ae+cdx)^{3/4}}{3e}-\frac{2\left(cd^2-ae^2\right)\left(\frac{cd^2\left(1-\frac{ae^2}{cd^2}\right)\sqrt[4]{ae+cdx}\sqrt[4]{\frac{cd^2-ae^2}{e(ae+cdx)}+1}\int}{4e\sqrt[4]{-\frac{ae^2}{cd^2}+\frac{e(ae+}{cd^2}}}\right)}{}\right)}{d\left(\frac{ex}{d}+1\right)^{3/4}(ae+cdx)^{3/4}}$$

$\big\downarrow$ 212

$$\frac{\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/4}\left(\frac{2d\left(\frac{ex}{d}+1\right)^{3/4}(ae+cdx)^{3/4}}{3e}-\frac{2\left(cd^2-ae^2\right)\left(\frac{cd^2\left(1-\frac{ae^2}{cd^2}\right)\sqrt[4]{ae+cdx}\sqrt[4]{\frac{cd^2-ae^2}{e(ae+cdx)}+1}E}{2\sqrt{e}\sqrt{cd^2-ae^2}\sqrt[4]{-\frac{ae^2}{cd^2}+\frac{e(ae}{}}}\right.}{}\right.}{d\left(\frac{ex}{d}+1\right)^{3/4}(ae+cdx)^{3/4}}$$

input
```
Int[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/4)/(d + e*x),x]
```

output
```
((a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/4)*((2*d*(a*e + c*d*x)^(3/4)*(
1 + (e*x)/d)^(3/4))/(3*e) - (2*(c*d^2 - a*e^2)*((a*e + c*d*x)^(3/4)/(2*(1
- (a*e^2)/(c*d^2) + (e*(a*e + c*d*x))/(c*d^2))^(1/4)) + (c*d^2*(1 - (a*e^2
)/(c*d^2))*(a*e + c*d*x)^(1/4)*(1 + (c*d^2 - a*e^2)/(e*(a*e + c*d*x)))^(1/
4)*EllipticE[ArcTan[(Sqrt[c*d^2 - a*e^2]*Sqrt[a*e + c*d*x])/Sqrt[e]]/2, 2]
)/(2*Sqrt[e]*Sqrt[c*d^2 - a*e^2]*(1 - (a*e^2)/(c*d^2) + (e*(a*e + c*d*x))/
(c*d^2))^(1/4))))/(c*d*e)))/(d*(a*e + c*d*x)^(3/4)*(1 + (e*x)/d)^(3/4))
```

Defintions of rubi rules used

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 212
```
Int[((a_) + (b_.)*(x_)^2)^(-5/4), x_Symbol] :> Simp[(2/(a^(5/4)*Rt[b/a, 2])
)*EllipticE[(1/2)*ArcTan[Rt[b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

rule 807
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Simp[1/k   Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x,
x^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

rule 813
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4)^(5/4), x_Symbol] :> Simp[x*((1 + a/(b*x^4)
)^(1/4)/(b*(a + b*x^4)^(1/4)))   Int[1/(x^3*(1 + a/(b*x^4))^(5/4)), x], x]
/; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 839
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4)^(1/4), x_Symbol] :> Simp[x^3/(2*(a + b*x^4
)^(1/4)), x] - Simp[a/2   Int[x^2/(a + b*x^4)^(5/4), x], x] /; FreeQ[{a, b}
, x] && PosQ[b/a]
```

rule 858
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

rule 1138
```
Int[((d_) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Sy
mbol] :> Simp[d^m*((a + b*x + c*x^2)^FracPart[p]/((1 + e*(x/d))^FracPart[p]
*(a/d + (c*x)/e)^FracPart[p]))   Int[(1 + e*(x/d))^(m + p)*(a/d + (c/e)*x)^
p, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0]
&& (IntegerQ[m] || GtQ[d, 0]) &&  !(IGtQ[m, 0] && (IntegerQ[3*p] || Integer
Q[4*p]))
```

Maple [F]

$$\int \frac{\left(ade+\left(a\,e^{2}+c\,d^{2}\right)x+cd\,x^{2}e\right)^{\frac{3}{4}}}{ex+d}dx$$

input
```
int((a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(3/4)/(e*x+d),x)
```

output
```
int((a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(3/4)/(e*x+d),x)
```

Fricas [F]

$$\int \frac{(ade+(cd^2+ae^2)\,x+cdex^2)^{3/4}}{d+ex}\,dx=\int \frac{(cdex^2+ade+(cd^2+ae^2)x)^{\frac{3}{4}}}{ex+d}\,dx$$

input
```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/4)/(e*x+d),x, algorithm="fr
icas")
```

output
```
integral((c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^(3/4)/(e*x + d), x)
```

Sympy [F]

$$\int \frac{(ade+(cd^2+ae^2)\,x+cdex^2)^{3/4}}{d+ex}\,dx=\int \frac{((d+ex)\,(ae+cdx))^{\frac{3}{4}}}{d+ex}\,dx$$

input
```
integrate((a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(3/4)/(e*x+d),x)
```

output
```
Integral(((d + e*x)*(a*e + c*d*x))**(3/4)/(d + e*x), x)
```

Maxima [F]

$$\int \frac{(ade + (cd^2 + ae^2)x + cdex^2)^{3/4}}{d + ex}\,dx = \int \frac{(cdex^2 + ade + (cd^2 + ae^2)x)^{\frac{3}{4}}}{ex + d}\,dx$$

input
```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/4)/(e*x+d),x, algorithm="ma
xima")
```

output
```
integrate((c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^(3/4)/(e*x + d), x)
```

Giac [F]

$$\int \frac{(ade + (cd^2 + ae^2)x + cdex^2)^{3/4}}{d + ex}\,dx = \int \frac{(cdex^2 + ade + (cd^2 + ae^2)x)^{\frac{3}{4}}}{ex + d}\,dx$$

input
```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/4)/(e*x+d),x, algorithm="gi
ac")
```

output
```
integrate((c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^(3/4)/(e*x + d), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(ade + (cd^2 + ae^2)x + cdex^2)^{3/4}}{d + ex}\,dx = \int \frac{(c\,d\,e\,x^2 + (c\,d^2 + a\,e^2)\,x + a\,d\,e)^{3/4}}{d + e\,x}\,dx$$

input
```
int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(3/4)/(d + e*x),x)
```

output
```
int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(3/4)/(d + e*x), x)
```

Reduce [F]

$$\int \frac{(ade + (cd^2 + ae^2)\,x + cdex^2)^{3/4}}{d + ex}\,dx = \frac{4(cde\,x^2 + a\,e^2x + c\,d^2x + ade)^{\frac{3}{4}}\,ae - 3\Bigg(\int \frac{(cde\,x^2+a\,e^2}{acd\,e^3x^2+c^2d^3e\,x^2+a^2e^4x+}$$

input

```
int((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/4)/(e*x+d),x)
```

output

```
(4*(a*d*e + a*e**2*x + c*d**2*x + c*d*e*x**2)**(3/4)*a*e - 3*int(((a*d*e +
 a*e**2*x + c*d**2*x + c*d*e*x**2)**(3/4)*x)/(a**2*d*e**3 + a**2*e**4*x +
a*c*d**3*e + 2*a*c*d**2*e**2*x + a*c*d*e**3*x**2 + c**2*d**4*x + c**2*d**3
*e*x**2),x)*a**2*c*d*e**4 + 3*int(((a*d*e + a*e**2*x + c*d**2*x + c*d*e*x*
*2)**(3/4)*x)/(a**2*d*e**3 + a**2*e**4*x + a*c*d**3*e + 2*a*c*d**2*e**2*x
+ a*c*d*e**3*x**2 + c**2*d**4*x + c**2*d**3*e*x**2),x)*c**3*d**5)/(3*(a*e*
*2 + c*d**2))
```

3.348 $\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/4}}{(d+ex)^2}\,dx$

Optimal result

Integrand size = 37, antiderivative size = 166

$$\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/4}}{(d+ex)^2}\,dx=-\frac{4\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/4}}{e(d+ex)}+\frac{3\sqrt{2}\left(cd^2-ae^2\right)\sqrt[4]{-\frac{cde(ae+cdx)(d+ex)}{\left(cd^2-ae^2\right)^2}}\,E\left(\frac{1}{2}\arcsin\left(\frac{ae^2+cd(d+2ex)}{cd^2-ae^2}\right)\middle|2\right)}{e^2\sqrt[4]{ade+\left(cd^2+ae^2\right)x+cdex^2}}$$

output
```
-4*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/4)/e/(e*x+d)+3*2^(1/2)*(-a*e^2+c*d
^2)*(-c*d*e*(c*d*x+a*e)*(e*x+d)/(-a*e^2+c*d^2)^2)^(1/4)*EllipticE(sin(1/2*
arcsin((a*e^2+c*d*(2*e*x+d))/(-a*e^2+c*d^2))),2^(1/2))/e^2/(a*d*e+(a*e^2+c
*d^2)*x+c*d*e*x^2)^(1/4)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.05 (sec) , antiderivative size = 106, normalized size of antiderivative = 0.64

$$\int \frac{(ade + (cd^2 + ae^2)x + cdex^2)^{3/4}}{(d+ex)^2} dx = \frac{4cd(ae+cdx)((ae+cdx)(d+ex))^{3/4}\text{Hypergeometric2F1}\left(\frac{5}{4}, \frac{7}{4}, \ldots\right)}{7(cd^2-ae^2)^2\left(\frac{cd(d+ex)}{cd^2-ae^2}\right)^{3/4}}$$

input
```
Integrate[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/4)/(d + e*x)^2,x]
```

output
```
(4*c*d*(a*e + c*d*x)*((a*e + c*d*x)*(d + e*x))^(3/4)*Hypergeometric2F1[5/4
, 7/4, 11/4, (e*(a*e + c*d*x))/(-(c*d^2) + a*e^2)])/(7*(c*d^2 - a*e^2)^2*(
(c*d*(d + e*x))/(c*d^2 - a*e^2))^(3/4))
```

Rubi [A] (warning: unable to verify)

Time = 0.74 (sec) , antiderivative size = 307, normalized size of antiderivative = 1.85, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.216$, Rules used = {1138, 57, 73, 839, 813, 858, 807, 212}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(x(ae^2+cd^2)+ade+cdex^2)^{3/4}}{(d+ex)^2} dx$$

$$\Big\downarrow \; 1138$$

$$\frac{(x(ae^2+cd^2)+ade+cdex^2)^{3/4}\int \frac{(ae+cdx)^{3/4}}{\left(\frac{ex}{d}+1\right)^{5/4}}dx}{d^2\left(\frac{ex}{d}+1\right)^{3/4}(ae+cdx)^{3/4}}$$

$$\Big\downarrow \; 57$$

$$\frac{\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/4}\left(\frac{3cd^2\int\frac{1}{\sqrt[4]{ae+cdx}\sqrt[4]{\frac{ex}{d}+1}}dx}{e}-\frac{4d(ae+cdx)^{3/4}}{e\sqrt[4]{\frac{ex}{d}+1}}\right)}{d^2\left(\frac{ex}{d}+1\right)^{3/4}(ae+cdx)^{3/4}}$$

$\Big\downarrow$ 73

$$\frac{\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/4}\left(\frac{12d\int\frac{\sqrt{ae+cdx}}{\sqrt[4]{-\frac{ae^2}{cd^2}+\frac{(ae+cdx)e}{cd^2}+1}}d\sqrt[4]{ae+cdx}}{e}-\frac{4d(ae+cdx)^{3/4}}{e\sqrt[4]{\frac{ex}{d}+1}}\right)}{d^2\left(\frac{ex}{d}+1\right)^{3/4}(ae+cdx)^{3/4}}$$

$\Big\downarrow$ 839

$$\frac{\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/4}\left(\frac{12d\left(\frac{(ae+cdx)^{3/4}}{2\sqrt[4]{-\frac{ae^2}{cd^2}+\frac{e(ae+cdx)}{cd^2}+1}}-\frac{1}{2}\left(1-\frac{ae^2}{cd^2}\right)\int\frac{\sqrt{ae+cdx}}{\left(-\frac{ae^2}{cd^2}+\frac{(ae+cdx)e}{cd^2}+1\right)^{5/4}}d\sqrt[4]{ae+ca}\right.}{e}\right.}{d^2\left(\frac{ex}{d}+1\right)^{3/4}(ae+cdx)^{3/4}}$$

$\Big\downarrow$ 813

$$\frac{\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/4}\left(\frac{12d\left(\frac{(ae+cdx)^{3/4}}{2\sqrt[4]{-\frac{ae^2}{cd^2}+\frac{e(ae+cdx)}{cd^2}+1}}-\frac{cd^2\left(1-\frac{ae^2}{cd^2}\right)\sqrt[4]{ae+cdx}\sqrt[4]{\frac{cd^2-ae^2}{e(ae+cdx)}+1}\int}{2e\sqrt[4]{-\frac{ae^2}{cd^2}+\frac{e(ae}{}}}\right.}{e}\right.}{d^2\left(\frac{ex}{d}+1\right)^{3/4}(ae+cdx)^{3/4}}$$

$\downarrow$ 858

$$\frac{\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/4}\left(\frac{12d\left(\frac{cd^2\left(1-\frac{ae^2}{cd^2}\right)\sqrt[4]{ae+cdx}\sqrt[4]{\frac{cd^2-ae^2}{e(ae+cdx)}+1}\int\frac{1}{\sqrt[4]{ae+cdx}\left(\frac{\left(cd^2-ae^2\right)(ae+cdx)}{e}+1\right.}}{2e\sqrt[4]{-\frac{ae^2}{cd^2}+\frac{e(ae+cdx)}{cd^2}+1}}\right.}{e}\right.}{d^2\left(\frac{ex}{d}+1\right)^{3/4}(ae+cdx)^{3/4}}$$

$\downarrow$ 807

$$\frac{\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/4}\left(\frac{12d\left(\frac{cd^2\left(1-\frac{ae^2}{cd^2}\right)\sqrt[4]{ae+cdx}\sqrt[4]{\frac{cd^2-ae^2}{e(ae+cdx)}+1}\int\frac{1}{\left(\frac{\sqrt{ae+cdx}\left(cd^2-ae^2\right)}{e}+1\right)^{5/4}}d\sqrt{ae+cdx}}{4e\sqrt[4]{-\frac{ae^2}{cd^2}+\frac{e(ae+cdx)}{cd^2}+1}}\right.}{e}\right.}{d^2\left(\frac{ex}{d}+1\right)^{3/4}(ae+cdx)^{3/4}}$$

$\downarrow$ 212

$$\frac{\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/4}\left(\frac{12d\left(\frac{cd^2\left(1-\frac{ae^2}{cd^2}\right)\sqrt[4]{ae+cdx}\sqrt[4]{\frac{cd^2-ae^2}{e(ae+cdx)}+1}\,E\left(\frac{1}{2}\arctan\left(\frac{\sqrt{cd^2-ae^2}\sqrt{ae+cdx}}{\sqrt{e}}\right)\middle|2\right)}{2\sqrt{e}\sqrt{cd^2-ae^2}\sqrt[4]{-\frac{ae^2}{cd^2}+\frac{e(ae+cdx)}{cd^2}+1}}+\cdots\right)}{e}\cdots\right)}{d^2\left(\frac{ex}{d}+1\right)^{3/4}(ae+cdx)^{3/4}}$$

input
```
Int[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/4)/(d + e*x)^2,x]
```

output
```
((a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/4)*((-4*d*(a*e + c*d*x)^(3/4))
/(e*(1 + (e*x)/d)^(1/4)) + (12*d*((a*e + c*d*x)^(3/4)/(2*(1 - (a*e^2)/(c*d
^2) + (e*(a*e + c*d*x))/(c*d^2))^(1/4)) + (c*d^2*(1 - (a*e^2)/(c*d^2))*(a*
e + c*d*x)^(1/4)*(1 + (c*d^2 - a*e^2)/(e*(a*e + c*d*x)))^(1/4)*EllipticE[A
rcTan[(Sqrt[c*d^2 - a*e^2]*Sqrt[a*e + c*d*x])/Sqrt[e]]/2, 2])/(2*Sqrt[e]*S
qrt[c*d^2 - a*e^2]*(1 - (a*e^2)/(c*d^2) + (e*(a*e + c*d*x))/(c*d^2))^(1/4)
)))/e))/(d^2*(a*e + c*d*x)^(3/4)*(1 + (e*x)/d)^(3/4))
```

Defintions of rubi rules used

rule 57
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Simp[d*(n/(b*(m + 1)))
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &
& GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !IntegerQ[m]) &&  !(ILeQ[m
+ n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) && IntLinearQ[a, b, c
, d, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 212
```
Int[((a_) + (b_.)*(x_)^2)^(-5/4), x_Symbol] :> Simp[(2/(a^(5/4)*Rt[b/a, 2])
)*EllipticE[(1/2)*ArcTan[Rt[b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

rule 807
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Simp[1/k   Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x,
x^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

rule 813
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4)^(5/4), x_Symbol] :> Simp[x*((1 + a/(b*x^4)
)^(1/4)/(b*(a + b*x^4)^(1/4)))   Int[1/(x^3*(1 + a/(b*x^4))^(5/4)), x], x]
/; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 839
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4)^(1/4), x_Symbol] :> Simp[x^3/(2*(a + b*x^4
)^(1/4)), x] - Simp[a/2   Int[x^2/(a + b*x^4)^(5/4), x], x] /; FreeQ[{a, b}
, x] && PosQ[b/a]
```

rule 858
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

rule 1138
```
Int[((d_) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Sy
mbol] :> Simp[d^m*((a + b*x + c*x^2)^FracPart[p]/((1 + e*(x/d))^FracPart[p]
*(a/d + (c*x)/e)^FracPart[p]))   Int[(1 + e*(x/d))^(m + p)*(a/d + (c/e)*x)^
p, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0]
&& (IntegerQ[m] || GtQ[d, 0]) &&  !(IGtQ[m, 0] && (IntegerQ[3*p] || Integer
Q[4*p]))
```

Maple [F]

$$\int \frac{\left(ade + \left(a\,e^{2} + c\,d^{2}\right)x + cd\,x^{2}e\right)^{\frac{3}{4}}}{\left(ex+d\right)^{2}}dx$$

input
```
int((a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(3/4)/(e*x+d)^2,x)
```

output
```
int((a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(3/4)/(e*x+d)^2,x)
```

Fricas [F]

$$\int \frac{\left(ade + \left(cd^2 + ae^2\right)x + cdex^2\right)^{3/4}}{(d+ex)^2}\,dx = \int \frac{\left(cdex^2 + ade + (cd^2 + ae^2)x\right)^{\frac{3}{4}}}{\left(ex+d\right)^2}\,dx$$

input
```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/4)/(e*x+d)^2,x, algorithm="
fricas")
```

output
```
integral((c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^(3/4)/(e^2*x^2 + 2*d*e*x
+ d^2), x)
```

Sympy [F]

$$\int \frac{\left(ade + \left(cd^2 + ae^2\right)x + cdex^2\right)^{3/4}}{(d+ex)^2}\,dx = \int \frac{\left((d+ex)\left(ae+cdx\right)\right)^{\frac{3}{4}}}{\left(d+ex\right)^2}\,dx$$

input
```
integrate((a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(3/4)/(e*x+d)**2,x)
```

output
```
Integral(((d + e*x)*(a*e + c*d*x))**(3/4)/(d + e*x)**2, x)
```

Maxima [F]

$$\int \frac{(ade + (cd^2 + ae^2)x + cdex^2)^{3/4}}{(d+ex)^2} dx = \int \frac{(cdex^2 + ade + (cd^2 + ae^2)x)^{\frac{3}{4}}}{(ex+d)^2} dx$$

input
```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/4)/(e*x+d)^2,x, algorithm="
maxima")
```

output
```
integrate((c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^(3/4)/(e*x + d)^2, x)
```

Giac [F]

$$\int \frac{(ade + (cd^2 + ae^2)x + cdex^2)^{3/4}}{(d+ex)^2} dx = \int \frac{(cdex^2 + ade + (cd^2 + ae^2)x)^{\frac{3}{4}}}{(ex+d)^2} dx$$

input
```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/4)/(e*x+d)^2,x, algorithm="
giac")
```

output
```
integrate((c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^(3/4)/(e*x + d)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(ade + (cd^2 + ae^2)x + cdex^2)^{3/4}}{(d+ex)^2} dx = \int \frac{(c\,d\,e\,x^2 + (c\,d^2 + a\,e^2)\,x + a\,d\,e)^{3/4}}{(d+e\,x)^2} dx$$

input
```
int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(3/4)/(d + e*x)^2,x)
```

output
```
int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(3/4)/(d + e*x)^2, x)
```

Reduce [F]

$$\int \frac{(ade + (cd^2 + ae^2)x + cdex^2)^{3/4}}{(d+ex)^2}\,dx = \text{Too large to display}$$

input
```
int((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/4)/(e*x+d)^2,x)
```

output
```
( - 4*(a*d*e + a*e**2*x + c*d**2*x + c*d*e*x**2)**(3/4)*a*e + 3*int(((a*d*
e + a*e**2*x + c*d**2*x + c*d*e*x**2)**(3/4)*x)/(a**2*d**2*e**3 + 2*a**2*d
*e**4*x + a**2*e**5*x**2 - 3*a*c*d**4*e - 5*a*c*d**3*e**2*x - a*c*d**2*e**
3*x**2 + a*c*d*e**4*x**3 - 3*c**2*d**5*x - 6*c**2*d**4*e*x**2 - 3*c**2*d**
3*e**2*x**3),x)*a**2*c*d**2*e**4 + 3*int(((a*d*e + a*e**2*x + c*d**2*x + c
*d*e*x**2)**(3/4)*x)/(a**2*d**2*e**3 + 2*a**2*d*e**4*x + a**2*e**5*x**2 -
3*a*c*d**4*e - 5*a*c*d**3*e**2*x - a*c*d**2*e**3*x**2 + a*c*d*e**4*x**3 -
3*c**2*d**5*x - 6*c**2*d**4*e*x**2 - 3*c**2*d**3*e**2*x**3),x)*a**2*c*d*e*
*5*x - 12*int(((a*d*e + a*e**2*x + c*d**2*x + c*d*e*x**2)**(3/4)*x)/(a**2*
d**2*e**3 + 2*a**2*d*e**4*x + a**2*e**5*x**2 - 3*a*c*d**4*e - 5*a*c*d**3*e
**2*x - a*c*d**2*e**3*x**2 + a*c*d*e**4*x**3 - 3*c**2*d**5*x - 6*c**2*d**4
*e*x**2 - 3*c**2*d**3*e**2*x**3),x)*a*c**2*d**4*e**2 - 12*int(((a*d*e + a*
e**2*x + c*d**2*x + c*d*e*x**2)**(3/4)*x)/(a**2*d**2*e**3 + 2*a**2*d*e**4*
x + a**2*e**5*x**2 - 3*a*c*d**4*e - 5*a*c*d**3*e**2*x - a*c*d**2*e**3*x**2
 + a*c*d*e**4*x**3 - 3*c**2*d**5*x - 6*c**2*d**4*e*x**2 - 3*c**2*d**3*e**2
*x**3),x)*a*c**2*d**3*e**3*x + 9*int(((a*d*e + a*e**2*x + c*d**2*x + c*d*e
*x**2)**(3/4)*x)/(a**2*d**2*e**3 + 2*a**2*d*e**4*x + a**2*e**5*x**2 - 3*a*
c*d**4*e - 5*a*c*d**3*e**2*x - a*c*d**2*e**3*x**2 + a*c*d*e**4*x**3 - 3*c*
*2*d**5*x - 6*c**2*d**4*e*x**2 - 3*c**2*d**3*e**2*x**3),x)*c**3*d**6 + 9*i
nt(((a*d*e + a*e**2*x + c*d**2*x + c*d*e*x**2)**(3/4)*x)/(a**2*d**2*e**...
```

3.349 $\int \frac{(ade+(cd^2+ae^2)x+cdex^2)^{3/4}}{(d+ex)^3}\,dx$

Optimal result

Integrand size = 37, antiderivative size = 178

$$\int \frac{(ade+(cd^2+ae^2)x+cdex^2)^{3/4}}{(d+ex)^3}\,dx = -\frac{4(ade+(cd^2+ae^2)x+cdex^2)^{3/4}}{5e(d+ex)^2} - \frac{12cd\sqrt{ae+cdx}\sqrt[4]{\frac{cd(d+ex)}{e(ae+cdx)}}E\left(\frac{1}{2}\arctan\left(\frac{\sqrt{cd^2-ae^2}}{\sqrt{e}\sqrt{ae+cdx}}\right)\middle|2\right)}{5e^{3/2}\sqrt{cd^2-ae^2}\sqrt[4]{ade+(cd^2+ae^2)x+cdex^2}}$$

output
```
-4/5*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/4)/e/(e*x+d)^2-12/5*c*d*(c*d*x+a
*e)^(1/2)*(c*d*(e*x+d)/e/(c*d*x+a*e))^(1/4)*EllipticE(sin(1/2*arctan((-a*e
^2+c*d^2)^(1/2)/e^(1/2)/(c*d*x+a*e)^(1/2))),2^(1/2))/e^(3/2)/(-a*e^2+c*d^2
)^(1/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/4)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.05 (sec) , antiderivative size = 110, normalized size of antiderivative = 0.62

$$\int \frac{(ade+(cd^2+ae^2)x+cdex^2)^{3/4}}{(d+ex)^3}\,dx=\frac{4c^2d^2(ae+cdx)((ae+cdx)(d+ex))^{3/4}\,\text{Hypergeometric2F1}\left(\frac{7}{4},\right.}{7(cd^2-ae^2)^3\left(\frac{cd(d+ex)}{cd^2-ae^2}\right)^{3/4}}$$

input
```
Integrate[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/4)/(d + e*x)^3,x]
```

output
```
(4*c^2*d^2*(a*e + c*d*x)*((a*e + c*d*x)*(d + e*x))^(3/4)*Hypergeometric2F1
[7/4, 9/4, 11/4, (e*(a*e + c*d*x))/(-(c*d^2) + a*e^2)])/(7*(c*d^2 - a*e^2)
^3*((c*d*(d + e*x))/(c*d^2 - a*e^2))^(3/4))
```

Rubi [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 373 vs. 2(178) = 356.

Time = 0.86 (sec) , antiderivative size = 373, normalized size of antiderivative = 2.10, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.243, Rules used = {1138, 57, 61, 73, 839, 813, 858, 807, 212}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/4}}{(d+ex)^3}dx$$

$$\Big\downarrow \text{1138}$$

$$\frac{\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/4}\int \frac{(ae+cdx)^{3/4}}{\left(\frac{ex}{d}+1\right)^{9/4}}dx}{d^3\left(\frac{ex}{d}+1\right)^{3/4}(ae+cdx)^{3/4}}$$

$$\Big\downarrow \text{57}$$

$$\frac{\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/4}\left(\frac{3cd^2\int\frac{1}{\sqrt[4]{ae+cdx}\left(\frac{ex}{d}+1\right)^{5/4}}dx}{5e}-\frac{4d(ae+cdx)^{3/4}}{5e\left(\frac{ex}{d}+1\right)^{5/4}}\right)}{d^3\left(\frac{ex}{d}+1\right)^{3/4}(ae+cdx)^{3/4}}$$

$\downarrow$ 61

$$\frac{\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/4}\left(\frac{3cd^2\left(\frac{4d(ae+cdx)^{3/4}}{\sqrt[4]{\frac{ex}{d}+1}\left(cd^2-ae^2\right)}-\frac{2cd^2\int\frac{1}{\sqrt[4]{ae+cdx}\sqrt[4]{\frac{ex}{d}+1}}dx}{cd^2-ae^2}\right)}{5e}-\frac{4d(ae+cdx)^{3/4}}{5e\left(\frac{ex}{d}+1\right)^{5/4}}\right)}{d^3\left(\frac{ex}{d}+1\right)^{3/4}(ae+cdx)^{3/4}}$$

$\downarrow$ 73

$$\frac{\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/4}\left(\frac{3cd^2\left(\frac{4d(ae+cdx)^{3/4}}{\sqrt[4]{\frac{ex}{d}+1}\left(cd^2-ae^2\right)}-\frac{8d\int\frac{\sqrt{ae+cdx}}{\sqrt[4]{-\frac{ae^2}{cd^2}+\frac{(ae+cdx)e}{cd^2}+1}}d\sqrt[4]{ae+cdx}}{cd^2-ae^2}\right)}{5e}-\frac{4d(}{5e}\right.}{d^3\left(\frac{ex}{d}+1\right)^{3/4}(ae+cdx)^{3/4}}$$

$\downarrow$ 839

$$\frac{\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/4}\left(\frac{3cd^2\left(\frac{4d(ae+cdx)^{3/4}}{\sqrt[4]{\frac{ex}{d}+1}\left(cd^2-ae^2\right)}-\frac{8d\left(\frac{(ae+cdx)^{3/4}}{2\sqrt[4]{-\frac{ae^2}{cd^2}+\frac{e(ae+cdx)}{cd^2}+1}}-\frac{1}{2}\left(1-\frac{ae^2}{cd^2}\right)\int\frac{\sqrt{\cdots}}{\left(-\frac{ae^2}{cd^2}+\frac{(\cdots}{\cdots}\right)\cdots}\cdots\right)}{cd^2-ae^2}\cdots\right)}{5e}\cdots\right)}{d^3\left(\frac{ex}{d}+1\right)^{3/4}(ae+cdx)^{3/4}\cdots}$$

$\downarrow$ 813

$$\frac{\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/4}\left(\frac{3cd^2\left(\frac{4d(ae+cdx)^{3/4}}{\sqrt[4]{\frac{ex}{d}+1}\left(cd^2-ae^2\right)}-\frac{8d\left(\frac{(ae+cdx)^{3/4}}{2\sqrt[4]{-\frac{ae^2}{cd^2}+\frac{e(ae+cdx)}{cd^2}+1}}-\frac{cd^2\left(1-\frac{ae^2}{cd^2}\right)\sqrt[4]{ae+c\cdots}}{\cdots}\right)}{\cdots}\right)}{5e}\cdots\right)}{d^3\left(\frac{ex}{d}+1\right)^{3/4}(ae+cdx)^{3/4}\cdots}$$

$\downarrow$ 858

$$\frac{\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/4}\left(\frac{3cd^2\left(\frac{4d(ae+cdx)^{3/4}}{\sqrt[4]{\frac{ex}{d}+1}\left(cd^2-ae^2\right)}-\frac{8d\left(\frac{cd^2\left(1-\frac{ae^2}{cd^2}\right)\sqrt[4]{ae+cdx}\sqrt[4]{\frac{cd^2-ae^2}{e(ae+cdx)}+1}\int\frac{}{\sqrt[4]{ae+\cdots}}}{2e\sqrt[4]{-\frac{ae^2}{cd^2}+\frac{e(ae-\cdots}{c\cdots}}}\right)}{\cdots}\right)}{5e\cdots}\right)}{d^3\left(\frac{ex}{d}+1\right)^{3/4}(ae+cdx)\cdots}$$

$\downarrow$ 807

$$\frac{\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/4}\left(\frac{3cd^2\left(\frac{4d(ae+cdx)^{3/4}}{\sqrt[4]{\frac{ex}{d}+1}\left(cd^2-ae^2\right)}-\frac{8d\left(\frac{cd^2\left(1-\frac{ae^2}{cd^2}\right)\sqrt[4]{ae+cdx}\sqrt[4]{\frac{cd^2-ae^2}{e(ae+cdx)}+1}\int\frac{}{\left(\frac{\sqrt{ae+c}}{}\right.}}{4e\sqrt[4]{-\frac{ae^2}{cd^2}+\frac{e(ae+cdx)}{cd^2}+}}\right)}{c}\right)}{5e}\right.}{d^3\left(\frac{ex}{d}+1\right)^{3/4}(ae+cdx)^{3/4}}$$

$\downarrow$ 212

$$\frac{\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/4}\left(\frac{3cd^2\left(\frac{4d(ae+cdx)^{3/4}}{\sqrt[4]{\frac{ex}{d}+1}\left(cd^2-ae^2\right)}-\frac{8d\left(\frac{cd^2\left(1-\frac{ae^2}{cd^2}\right)\sqrt[4]{ae+cdx}\sqrt[4]{\frac{cd^2-ae^2}{e(ae+cdx)}+1}E\left(\frac{1}{2}\arctan\right.}{2\sqrt{e}\sqrt{cd^2-ae^2}\sqrt[4]{-\frac{ae^2}{cd^2}+\frac{e(ae+cdx}{cd^2}}}\right.}{cd^2}\right)}{5e}\right.}{d^3\left(\frac{ex}{d}+1\right)^{3/4}(ae+cdx)^{3/4}}$$

input
```
Int[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/4)/(d + e*x)^3,x]
```

output
```
((a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/4)*((-4*d*(a*e + c*d*x)^(3/4))
/(5*e*(1 + (e*x)/d)^(5/4)) + (3*c*d^2*((4*d*(a*e + c*d*x)^(3/4))/((c*d^2 -
 a*e^2)*(1 + (e*x)/d)^(1/4)) - (8*d*((a*e + c*d*x)^(3/4)/(2*(1 - (a*e^2)/(
c*d^2) + (e*(a*e + c*d*x))/(c*d^2))^(1/4)) + (c*d^2*(1 - (a*e^2)/(c*d^2))*
(a*e + c*d*x)^(1/4)*(1 + (c*d^2 - a*e^2)/(e*(a*e + c*d*x)))^(1/4)*Elliptic
E[ArcTan[(Sqrt[c*d^2 - a*e^2]*Sqrt[a*e + c*d*x])/Sqrt[e]]/2, 2])/(2*Sqrt[e
]*Sqrt[c*d^2 - a*e^2]*(1 - (a*e^2)/(c*d^2) + (e*(a*e + c*d*x))/(c*d^2))^(1
/4))))/(c*d^2 - a*e^2)))/(5*e)))/(d^3*(a*e + c*d*x)^(3/4)*(1 + (e*x)/d)^(3
/4))
```

Defintions of rubi rules used

rule 57
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Simp[d*(n/(b*(m + 1)))
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &
& GtQ[n, 0] && LtQ[m, -1] && !(IntegerQ[n] && !IntegerQ[m]) && !(ILeQ[m
+ n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) && IntLinearQ[a, b, c
, d, m, n, x]
```

rule 61
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && LtQ[m, -1] && !(LtQ[n, -1] && (EqQ[a, 0
] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && IntLinearQ[a, b, c, d
, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 212
```
Int[((a_) + (b_.)*(x_)^2)^(-5/4), x_Symbol] :> Simp[(2/(a^(5/4)*Rt[b/a, 2])
)*EllipticE[(1/2)*ArcTan[Rt[b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

rule 807
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Simp[1/k   Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x,
x^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

rule 813
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4)^(5/4), x_Symbol] :> Simp[x*((1 + a/(b*x^4)
)^(1/4)/(b*(a + b*x^4)^(1/4)))   Int[1/(x^3*(1 + a/(b*x^4))^(5/4)), x], x]
/; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 839
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4)^(1/4), x_Symbol] :> Simp[x^3/(2*(a + b*x^4
)^(1/4)), x] - Simp[a/2   Int[x^2/(a + b*x^4)^(5/4), x], x] /; FreeQ[{a, b}
, x] && PosQ[b/a]
```

rule 858
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

rule 1138
```
Int[((d_) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Sy
mbol] :> Simp[d^m*((a + b*x + c*x^2)^FracPart[p]/((1 + e*(x/d))^FracPart[p]
*(a/d + (c*x)/e)^FracPart[p]))   Int[(1 + e*(x/d))^(m + p)*(a/d + (c/e)*x)^
p, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0]
&& (IntegerQ[m] || GtQ[d, 0]) &&  !(IGtQ[m, 0] && (IntegerQ[3*p] || Integer
Q[4*p]))
```

Maple [F]

$$\int \frac{(ade + (a\,e^2 + c\,d^2)\,x + cd\,x^2 e)^{\frac{3}{4}}}{(ex+d)^3}dx$$

input
```
int((a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(3/4)/(e*x+d)^3,x)
```

output
```
int((a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(3/4)/(e*x+d)^3,x)
```

Fricas [F]

$$\int \frac{(ade + (cd^2 + ae^2)\,x + cdex^2)^{3/4}}{(d+ex)^3}\,dx = \int \frac{(cdex^2 + ade + (cd^2 + ae^2)x)^{\frac{3}{4}}}{(ex+d)^3}\,dx$$

input
```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/4)/(e*x+d)^3,x, algorithm="
fricas")
```

output
```
integral((c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^(3/4)/(e^3*x^3 + 3*d*e^2*
x^2 + 3*d^2*e*x + d^3), x)
```

Sympy [F]

$$\int \frac{(ade + (cd^2 + ae^2)\, x + cdex^2)^{3/4}}{(d + ex)^3}\, dx = \int \frac{((d + ex)\,(ae + cdx))^{\frac{3}{4}}}{(d + ex)^3}\, dx$$

input
```
integrate((a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(3/4)/(e*x+d)**3,x)
```

output
```
Integral(((d + e*x)*(a*e + c*d*x))**(3/4)/(d + e*x)**3, x)
```

Maxima [F]

$$\int \frac{(ade + (cd^2 + ae^2)\, x + cdex^2)^{3/4}}{(d + ex)^3}\, dx = \int \frac{(cdex^2 + ade + (cd^2 + ae^2)x)^{\frac{3}{4}}}{(ex + d)^3}\, dx$$

input
```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/4)/(e*x+d)^3,x, algorithm="
maxima")
```

output
```
integrate((c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^(3/4)/(e*x + d)^3, x)
```

Giac [F]

$$\int \frac{(ade + (cd^2 + ae^2)\, x + cdex^2)^{3/4}}{(d + ex)^3}\, dx = \int \frac{(cdex^2 + ade + (cd^2 + ae^2)x)^{\frac{3}{4}}}{(ex + d)^3}\, dx$$

input
```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/4)/(e*x+d)^3,x, algorithm="
giac")
```

output
```
integrate((c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^(3/4)/(e*x + d)^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/4}}{(d+ex)^3}\,dx=\int \frac{{\left(c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e\right)}^{3/4}}{{\left(d+e\,x\right)}^3}\,dx$$

input
```
int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(3/4)/(d + e*x)^3,x)
```

output
```
int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(3/4)/(d + e*x)^3, x)
```

Reduce [F]

$$\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/4}}{(d+ex)^3}\,dx=\text{too large to display}$$

input
```
int((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/4)/(e*x+d)^3,x)
```

output

```
( - 4*(a*d*e + a*e**2*x + c*d**2*x + c*d*e*x**2)**(3/4)*a*e + 15*int(((a*d
*e + a*e**2*x + c*d**2*x + c*d*e*x**2)**(3/4)*x)/(5*a**2*d**3*e**3 + 15*a*
*2*d**2*e**4*x + 15*a**2*d*e**5*x**2 + 5*a**2*e**6*x**3 - 3*a*c*d**5*e - 4
*a*c*d**4*e**2*x + 6*a*c*d**3*e**3*x**2 + 12*a*c*d**2*e**4*x**3 + 5*a*c*d*
e**5*x**4 - 3*c**2*d**6*x - 9*c**2*d**5*e*x**2 - 9*c**2*d**4*e**2*x**3 - 3
*c**2*d**3*e**3*x**4),x)*a**2*c*d**3*e**4 + 30*int(((a*d*e + a*e**2*x + c*
d**2*x + c*d*e*x**2)**(3/4)*x)/(5*a**2*d**3*e**3 + 15*a**2*d**2*e**4*x + 1
5*a**2*d*e**5*x**2 + 5*a**2*e**6*x**3 - 3*a*c*d**5*e - 4*a*c*d**4*e**2*x +
 6*a*c*d**3*e**3*x**2 + 12*a*c*d**2*e**4*x**3 + 5*a*c*d*e**5*x**4 - 3*c**2
*d**6*x - 9*c**2*d**5*e*x**2 - 9*c**2*d**4*e**2*x**3 - 3*c**2*d**3*e**3*x*
*4),x)*a**2*c*d**2*e**5*x + 15*int(((a*d*e + a*e**2*x + c*d**2*x + c*d*e*x
**2)**(3/4)*x)/(5*a**2*d**3*e**3 + 15*a**2*d**2*e**4*x + 15*a**2*d*e**5*x*
*2 + 5*a**2*e**6*x**3 - 3*a*c*d**5*e - 4*a*c*d**4*e**2*x + 6*a*c*d**3*e**3
*x**2 + 12*a*c*d**2*e**4*x**3 + 5*a*c*d*e**5*x**4 - 3*c**2*d**6*x - 9*c**2
*d**5*e*x**2 - 9*c**2*d**4*e**2*x**3 - 3*c**2*d**3*e**3*x**4),x)*a**2*c*d*
e**6*x**2 - 24*int(((a*d*e + a*e**2*x + c*d**2*x + c*d*e*x**2)**(3/4)*x)/(
5*a**2*d**3*e**3 + 15*a**2*d**2*e**4*x + 15*a**2*d*e**5*x**2 + 5*a**2*e**6
*x**3 - 3*a*c*d**5*e - 4*a*c*d**4*e**2*x + 6*a*c*d**3*e**3*x**2 + 12*a*c*d
**2*e**4*x**3 + 5*a*c*d*e**5*x**4 - 3*c**2*d**6*x - 9*c**2*d**5*e*x**2 - 9
*c**2*d**4*e**2*x**3 - 3*c**2*d**3*e**3*x**4),x)*a*c**2*d**5*e**2 - 48*...
```

3.350 $\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/4}}{(d+ex)^4}\,dx$

Optimal result

Integrand size = 37, antiderivative size = 241

$$\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/4}}{(d+ex)^4}\,dx = -\frac{4\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/4}}{9e(d+ex)^3}+\frac{4cd\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/4}}{15e\left(cd^2-ae^2\right)(d+ex)^2}-\frac{8c^2d^2\sqrt{ae+cdx}\sqrt[4]{\frac{cd(d+ex)}{e(ae+cdx)}}E\left(\frac{1}{2}\arctan\left(\frac{\sqrt{cd^2-ae^2}}{\sqrt{e}\sqrt{ae+cdx}}\right)\middle|2\right)}{15e^{3/2}\left(cd^2-ae^2\right)^{3/2}\sqrt[4]{ade+\left(cd^2+ae^2\right)x+cdex^2}}$$

output
```
-4/9*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/4)/e/(e*x+d)^3+4/15*c*d*(a*d*e+(
a*e^2+c*d^2)*x+c*d*e*x^2)^(3/4)/e/(-a*e^2+c*d^2)/(e*x+d)^2-8/15*c^2*d^2*(c
*d*x+a*e)^(1/2)*(c*d*(e*x+d)/e/(c*d*x+a*e))^(1/4)*EllipticE(sin(1/2*arctan
((-a*e^2+c*d^2)^(1/2)/e^(1/2)/(c*d*x+a*e)^(1/2))),2^(1/2))/e^(3/2)/(-a*e^2
+c*d^2)^(3/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/4)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.09 (sec) , antiderivative size = 110, normalized size of antiderivative = 0.46

$$\int \frac{(ade+(cd^2+ae^2)\,x+cdex^2)^{3/4}}{(d+ex)^4}\,dx = \frac{4c^3d^3(ae+cdx)((ae+cdx)(d+ex))^{3/4}\,\text{Hypergeometric2F1}\left(\frac{7}{4},\right.}{7\,(cd^2-ae^2)^4\left(\frac{cd(d+ex)}{cd^2-ae^2}\right)^{3/4}}$$

input
```
Integrate[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/4)/(d + e*x)^4,x]
```

output
```
(4*c^3*d^3*(a*e + c*d*x)*((a*e + c*d*x)*(d + e*x))^(3/4)*Hypergeometric2F1
[7/4, 13/4, 11/4, (e*(a*e + c*d*x))/(-(c*d^2) + a*e^2)])/(7*(c*d^2 - a*e^2
)^4*((c*d*(d + e*x))/(c*d^2 - a*e^2))^(3/4))
```

Rubi [A] (warning: unable to verify)

Time = 0.96 (sec) , antiderivative size = 438, normalized size of antiderivative = 1.82, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.270$, Rules used = {1138, 57, 61, 61, 73, 839, 813, 858, 807, 212}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(x(ae^2+cd^2)+ade+cdex^2)^{3/4}}{(d+ex)^4}dx$$

$$\Big\downarrow \text{1138}$$

$$\frac{(x(ae^2+cd^2)+ade+cdex^2)^{3/4}\int \frac{(ae+cdx)^{3/4}}{\left(\frac{ex}{d}+1\right)^{13/4}}dx}{d^4\left(\frac{ex}{d}+1\right)^{3/4}(ae+cdx)^{3/4}}$$

$$\Big\downarrow \text{57}$$

$$\frac{(x(ae^2+cd^2)+ade+cdex^2)^{3/4}\left(\frac{cd^2\int \frac{1}{\sqrt[4]{ae+cdx}\left(\frac{ex}{d}+1\right)^{9/4}}dx}{3e}-\frac{4d(ae+cdx)^{3/4}}{9e\left(\frac{ex}{d}+1\right)^{9/4}}\right)}{d^4\left(\frac{ex}{d}+1\right)^{3/4}(ae+cdx)^{3/4}}$$

$\downarrow$ 61

$$\frac{\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/4}\left(\frac{cd^2\left(\frac{2cd\int\frac{1}{\sqrt[4]{ae+cdx}\left(\frac{ex}{d}+1\right)^{5/4}}dx}{5\left(cd-\frac{ae^2}{d}\right)}+\frac{4(ae+cdx)^{3/4}}{5\left(\frac{ex}{d}+1\right)^{5/4}\left(cd-\frac{ae^2}{d}\right)}\right)}{3e}-\frac{4d(ae+cdx)^{3/4}}{9e\left(\frac{ex}{d}+1\right)^{9/4}}\right)}{d^4\left(\frac{ex}{d}+1\right)^{3/4}(ae+cdx)^{3/4}}$$

$\downarrow$ 61

$$\frac{\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/4}\left(\frac{cd^2\left(\frac{2cd\left(\frac{4d(ae+cdx)^{3/4}}{\sqrt[4]{\frac{ex}{d}+1}\left(cd^2-ae^2\right)}-\frac{2cd^2\int\frac{1}{\sqrt[4]{ae+cdx}\sqrt[4]{\frac{ex}{d}+1}}dx}{cd^2-ae^2}\right)}{5\left(cd-\frac{ae^2}{d}\right)}+\frac{4(ae+cdx)^{3/4}}{5\left(\frac{ex}{d}+1\right)^{5/4}\left(cd-\frac{ae^2}{d}\right)}\right)}{3e}\right.}{d^4\left(\frac{ex}{d}+1\right)^{3/4}(ae+cdx)^{3/4}}$$

$\downarrow$ 73

$$\frac{\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/4}\left(\frac{cd^2\left(\frac{2cd\left(\frac{4d(ae+cdx)^{3/4}}{\sqrt[4]{\frac{ex}{d}+1}\left(cd^2-ae^2\right)}-\frac{8d\int\frac{\sqrt{ae+cdx}}{\sqrt[4]{-\frac{ae^2}{cd^2}+\frac{(ae+cdx)e}{cd^2}+1}}d\sqrt[4]{ae+cdx}}{cd^2-ae^2}\right)}{5\left(cd-\frac{ae^2}{d}\right)}+\frac{}{5\left(\right.}}{3e}\right.}{d^4\left(\frac{ex}{d}+1\right)^{3/4}(ae+cdx)^{3/4}}$$

$\downarrow$ 839

$$\frac{\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/4}\left(\frac{cd^2\left(\frac{2cd\left(\frac{4d(ae+cdx)^{3/4}}{\sqrt[4]{\frac{ex}{d}+1}\left(cd^2-ae^2\right)}-\frac{8d\left(\frac{(ae+cdx)^{3/4}}{2\sqrt[4]{-\frac{ae^2}{cd^2}+\frac{e(ae+cdx)}{cd^2}+1}}-\frac{1}{2}\left(1-\frac{ae^2}{cd^2}\right)\int\cdots\right)}{cd^2-ae^2}\cdots\right)}{5\left(cd-\frac{ae^2}{d}\right)}\cdots\right)}{3e}\cdots\right)}{d^4\left(\frac{ex}{d}+1\right)^{3/4}(ae+cdx)^{3/4}\cdots}$$

$$\Big\downarrow \; 813$$

$$\frac{\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/4}\left(\frac{cd^2\left(2cd\left(\frac{4d(ae+cdx)^{3/4}}{\sqrt[4]{\frac{ex}{d}+1}\left(cd^2-ae^2\right)}-\frac{8d\left(\frac{(ae+cdx)^{3/4}}{2\sqrt[4]{-\frac{ae^2}{cd^2}+\frac{e(ae+cdx)}{cd^2}+1}}-\frac{cd^2\left(1-\frac{ae^2}{cd^2}\right)\sqrt[4]{ae\cdots}}{\cdots}\right)}{5\left(cd-\frac{ae^2}{d}\cdots\right)}\right)\cdots}{\cdots}\cdots\right)}{d^4\left(\frac{ex}{d}+1\right)^{3/4}(ae\cdots}$$

↓ 858

$$\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/4}$$

$$cd^2 \quad 2cd \left(\frac{4d(ae+cdx)^{3/4}}{\sqrt[4]{\frac{ex}{d}+1}\left(cd^2-ae^2\right)} - \frac{8d\left(\frac{cd^2\left(1-\frac{ae^2}{cd^2}\right)\sqrt[4]{ae+cdx}\sqrt[4]{\frac{cd^2-ae^2}{e(ae+cdx)}+1}\int \frac{}{\sqrt[4]{a}}}{2e\sqrt[4]{-\frac{ae^2}{cd^2}+\frac{e(a}{}}}\right.}{} \right. \quad 5\left(c\right.$$

↓ 807

$$\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/4} \qquad cd^2 \qquad 2cd\left(\frac{4d(ae+cdx)^{3/4}}{\sqrt[4]{\frac{ex}{d}+1}\left(cd^2-ae^2\right)} - \frac{8d\left(\frac{cd^2\left(1-\frac{ae^2}{cd^2}\right)\sqrt[4]{ae+cdx}\sqrt[4]{\frac{cd^2-ae^2}{e(ae+cdx)}+1}\int \frac{}{\left(\frac{\sqrt{a}}{}\right.}}{4e\sqrt[4]{-\frac{ae^2}{cd^2}+\frac{e(ae+cdx}{cd^2}}}\right.}{5\left(cd-\frac{ae^2}{d}\right)}\right.$$

↓ 212

$$\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/4}\left(\frac{cd^2\left(\frac{2cd\left(\frac{4d(ae+cdx)^{3/4}}{\sqrt[4]{\frac{ex}{d}+1}\left(cd^2-ae^2\right)}-\frac{8d\left(\frac{cd^2\left(1-\frac{ae^2}{cd^2}\right)\sqrt[4]{ae+cdx}\sqrt[4]{\frac{cd^2-ae^2}{e(ae+cdx)}+1}E\left(\frac{1}{2}\text{ar}\right.}{2\sqrt{e}\sqrt{cd^2-ae^2}\sqrt[4]{-\frac{ae^2}{cd^2}+\frac{e(ae+c}{cd^2}}}\right.}{}\right)}{5\left(cd-\frac{ae^2}{d}\right)}}{}\right.}{d^4\left(\frac{ex}{d}+1\right)^{3/4}(ae+cd}$$

input
```
Int[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/4)/(d + e*x)^4,x]
```

output
```
((a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/4)*((-4*d*(a*e + c*d*x)^(3/4))
/(9*e*(1 + (e*x)/d)^(9/4)) + (c*d^2*((4*(a*e + c*d*x)^(3/4))/(5*(c*d - (a*
e^2)/d)*(1 + (e*x)/d)^(5/4)) + (2*c*d*((4*d*(a*e + c*d*x)^(3/4))/((c*d^2 -
 a*e^2)*(1 + (e*x)/d)^(1/4)) - (8*d*((a*e + c*d*x)^(3/4)/(2*(1 - (a*e^2)/(
c*d^2) + (e*(a*e + c*d*x))/(c*d^2))^(1/4)) + (c*d^2*(1 - (a*e^2)/(c*d^2))*
(a*e + c*d*x)^(1/4)*(1 + (c*d^2 - a*e^2)/(e*(a*e + c*d*x)))^(1/4)*Elliptic
E[ArcTan[(Sqrt[c*d^2 - a*e^2]*Sqrt[a*e + c*d*x])/Sqrt[e]]/2, 2])/(2*Sqrt[e
]*Sqrt[c*d^2 - a*e^2]*(1 - (a*e^2)/(c*d^2) + (e*(a*e + c*d*x))/(c*d^2))^(1
/4))))/(c*d^2 - a*e^2)))/(5*(c*d - (a*e^2)/d))))/(3*e)))/(d^4*(a*e + c*d*x
)^(3/4)*(1 + (e*x)/d)^(3/4))
```

Defintions of rubi rules used

rule 57
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Simp[d*(n/(b*(m + 1)))
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &
& GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !IntegerQ[m]) &&  !(ILeQ[m
+ n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) && IntLinearQ[a, b, c
, d, m, n, x]
```

rule 61
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && LtQ[m, -1] &&  !(LtQ[n, -1] && (EqQ[a, 0
] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && IntLinearQ[a, b, c, d
, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 212
```
Int[((a_) + (b_.)*(x_)^2)^(-5/4), x_Symbol] :> Simp[(2/(a^(5/4)*Rt[b/a, 2])
)*EllipticE[(1/2)*ArcTan[Rt[b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

rule 807
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Simp[1/k   Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x,
x^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

rule 813
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4)^(5/4), x_Symbol] :> Simp[x*((1 + a/(b*x^4)
)^(1/4)/(b*(a + b*x^4)^(1/4)))   Int[1/(x^3*(1 + a/(b*x^4))^(5/4)), x], x]
/; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 839
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4)^(1/4), x_Symbol] :> Simp[x^3/(2*(a + b*x^4
)^(1/4)), x] - Simp[a/2   Int[x^2/(a + b*x^4)^(5/4), x], x] /; FreeQ[{a, b}
, x] && PosQ[b/a]
```

rule 858
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

rule 1138
```
Int[((d_) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Sy
mbol] :> Simp[d^m*((a + b*x + c*x^2)^FracPart[p]/((1 + e*(x/d))^FracPart[p]
*(a/d + (c*x)/e)^FracPart[p]))   Int[(1 + e*(x/d))^(m + p)*(a/d + (c/e)*x)^
p, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0]
&& (IntegerQ[m] || GtQ[d, 0]) &&  !(IGtQ[m, 0] && (IntegerQ[3*p] || Integer
Q[4*p]))
```

Maple [F]

$$\int \frac{(ade + (a\,e^2 + c\,d^2)\,x + cd\,x^2 e)^{\frac{3}{4}}}{(ex+d)^4}dx$$

input
```
int((a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(3/4)/(e*x+d)^4,x)
```

output
```
int((a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(3/4)/(e*x+d)^4,x)
```

Fricas [F]

$$\int \frac{(ade + (cd^2 + ae^2)\,x + cdex^2)^{3/4}}{(d+ex)^4}\,dx = \int \frac{(cdex^2 + ade + (cd^2 + ae^2)x)^{\frac{3}{4}}}{(ex+d)^4}\,dx$$

input
```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/4)/(e*x+d)^4,x, algorithm="
fricas")
```

output
```
integral((c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^(3/4)/(e^4*x^4 + 4*d*e^3*
x^3 + 6*d^2*e^2*x^2 + 4*d^3*e*x + d^4), x)
```

Sympy [F]

$$\int \frac{(ade + (cd^2 + ae^2)\,x + cdex^2)^{3/4}}{(d+ex)^4}\,dx = \int \frac{((d+ex)\,(ae+cdx))^{\frac{3}{4}}}{(d+ex)^4}\,dx$$

input
```
integrate((a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(3/4)/(e*x+d)**4,x)
```

output
```
Integral(((d + e*x)*(a*e + c*d*x))**(3/4)/(d + e*x)**4, x)
```

Maxima [F]

$$\int \frac{(ade + (cd^2 + ae^2)\,x + cdex^2)^{3/4}}{(d + ex)^4}\,dx = \int \frac{(cdex^2 + ade + (cd^2 + ae^2)x)^{\frac{3}{4}}}{(ex + d)^4}\,dx$$

input
```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/4)/(e*x+d)^4,x, algorithm="
maxima")
```

output
```
integrate((c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^(3/4)/(e*x + d)^4, x)
```

Giac [F]

$$\int \frac{(ade + (cd^2 + ae^2)\,x + cdex^2)^{3/4}}{(d + ex)^4}\,dx = \int \frac{(cdex^2 + ade + (cd^2 + ae^2)x)^{\frac{3}{4}}}{(ex + d)^4}\,dx$$

input
```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/4)/(e*x+d)^4,x, algorithm="
giac")
```

output
```
integrate((c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^(3/4)/(e*x + d)^4, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(ade + (cd^2 + ae^2)\,x + cdex^2)^{3/4}}{(d + ex)^4}\,dx = \int \frac{(c\,d\,e\,x^2 + (c\,d^2 + a\,e^2)\;x + a\,d\,e)^{3/4}}{(d + e\,x)^4}\,dx$$

input
```
int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(3/4)/(d + e*x)^4,x)
```

output
```
int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(3/4)/(d + e*x)^4, x)
```

Reduce [F]

$$\int \frac{(ade + (cd^2 + ae^2)\,x + cdex^2)^{3/4}}{(d + ex)^4}\,dx = \text{too large to display}$$

input

```
int((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/4)/(e*x+d)^4,x)
```

output

```
( - 4*(a*d*e + a*e**2*x + c*d**2*x + c*d*e*x**2)**(3/4)*a*e + 9*int(((a*d*
e + a*e**2*x + c*d**2*x + c*d*e*x**2)**(3/4)*x)/(3*a**2*d**4*e**3 + 12*a**
2*d**3*e**4*x + 18*a**2*d**2*e**5*x**2 + 12*a**2*d*e**6*x**3 + 3*a**2*e**7
*x**4 - a*c*d**6*e - a*c*d**5*e**2*x + 6*a*c*d**4*e**3*x**2 + 14*a*c*d**3*
e**4*x**3 + 11*a*c*d**2*e**5*x**4 + 3*a*c*d*e**6*x**5 - c**2*d**7*x - 4*c*
*2*d**6*e*x**2 - 6*c**2*d**5*e**2*x**3 - 4*c**2*d**4*e**3*x**4 - c**2*d**3
*e**4*x**5),x)*a**2*c*d**4*e**4 + 27*int(((a*d*e + a*e**2*x + c*d**2*x + c
*d*e*x**2)**(3/4)*x)/(3*a**2*d**4*e**3 + 12*a**2*d**3*e**4*x + 18*a**2*d**
2*e**5*x**2 + 12*a**2*d*e**6*x**3 + 3*a**2*e**7*x**4 - a*c*d**6*e - a*c*d*
*5*e**2*x + 6*a*c*d**4*e**3*x**2 + 14*a*c*d**3*e**4*x**3 + 11*a*c*d**2*e**
5*x**4 + 3*a*c*d*e**6*x**5 - c**2*d**7*x - 4*c**2*d**6*e*x**2 - 6*c**2*d**
5*e**2*x**3 - 4*c**2*d**4*e**3*x**4 - c**2*d**3*e**4*x**5),x)*a**2*c*d**3*
e**5*x + 27*int(((a*d*e + a*e**2*x + c*d**2*x + c*d*e*x**2)**(3/4)*x)/(3*a
**2*d**4*e**3 + 12*a**2*d**3*e**4*x + 18*a**2*d**2*e**5*x**2 + 12*a**2*d*e
**6*x**3 + 3*a**2*e**7*x**4 - a*c*d**6*e - a*c*d**5*e**2*x + 6*a*c*d**4*e*
*3*x**2 + 14*a*c*d**3*e**4*x**3 + 11*a*c*d**2*e**5*x**4 + 3*a*c*d*e**6*x**
5 - c**2*d**7*x - 4*c**2*d**6*e*x**2 - 6*c**2*d**5*e**2*x**3 - 4*c**2*d**4
*e**3*x**4 - c**2*d**3*e**4*x**5),x)*a**2*c*d**2*e**6*x**2 + 9*int(((a*d*e
 + a*e**2*x + c*d**2*x + c*d*e*x**2)**(3/4)*x)/(3*a**2*d**4*e**3 + 12*a**2
*d**3*e**4*x + 18*a**2*d**2*e**5*x**2 + 12*a**2*d*e**6*x**3 + 3*a**2*e*...
```

3.351 $\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/4}}{(d+ex)^5}\,dx$

Optimal result

Integrand size = 37, antiderivative size = 304

$$\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/4}}{(d+ex)^5}\,dx = -\frac{4\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/4}}{13e(d+ex)^4} + \frac{4cd\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/4}}{39e\left(cd^2-ae^2\right)(d+ex)^3} + \frac{8c^2d^2\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/4}}{65e\left(cd^2-ae^2\right)^2(d+ex)^2} - \frac{16c^3d^3\sqrt{ae+cdx}\sqrt[4]{\frac{cd(d+ex)}{e(ae+cdx)}}\,E\left(\frac{1}{2}\arctan\left(\frac{\sqrt{cd^2-ae^2}}{\sqrt{e}\sqrt{ae+cdx}}\right)\middle|2\right)}{65e^{3/2}\left(cd^2-ae^2\right)^{5/2}\sqrt[4]{ade+\left(cd^2+ae^2\right)x+cdex^2}}$$

output
```
-4/13*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/4)/e/(e*x+d)^4+4/39*c*d*(a*d*e+
(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/4)/e/(-a*e^2+c*d^2)/(e*x+d)^3+8/65*c^2*d^2*(
a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/4)/e/(-a*e^2+c*d^2)^2/(e*x+d)^2-16/65*
c^3*d^3*(c*d*x+a*e)^(1/2)*(c*d*(e*x+d)/e/(c*d*x+a*e))^(1/4)*EllipticE(sin(
1/2*arctan((-a*e^2+c*d^2)^(1/2)/e^(1/2)/(c*d*x+a*e)^(1/2))),2^(1/2))/e^(3/
2)/(-a*e^2+c*d^2)^(5/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/4)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.05 (sec) , antiderivative size = 110, normalized size of antiderivative = 0.36

$$\int \frac{(ade+(cd^2+ae^2)x+cdex^2)^{3/4}}{(d+ex)^5}\,dx = \frac{4c^4d^4(ae+cdx)((ae+cdx)(d+ex))^{3/4}\,\text{Hypergeometric2F1}\left(\frac{7}{4},\right.}{7(cd^2-ae^2)^5\left(\frac{cd(d+ex)}{cd^2-ae^2}\right)^{3/4}}$$

input
```
Integrate[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/4)/(d + e*x)^5,x]
```

output
```
(4*c^4*d^4*(a*e + c*d*x)*((a*e + c*d*x)*(d + e*x))^(3/4)*Hypergeometric2F1
[7/4, 17/4, 11/4, (e*(a*e + c*d*x))/(-(c*d^2) + a*e^2)])/(7*(c*d^2 - a*e^2
)^5*((c*d*(d + e*x))/(c*d^2 - a*e^2))^(3/4))
```

Rubi [A] (warning: unable to verify)

Time = 1.03 (sec) , antiderivative size = 503, normalized size of antiderivative = 1.65, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.297$, Rules used = {1138, 57, 61, 61, 61, 73, 839, 813, 858, 807, 212}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(x(ae^2+cd^2)+ade+cdex^2)^{3/4}}{(d+ex)^5}\,dx$$

$$\Big\downarrow \text{1138}$$

$$\frac{(x(ae^2+cd^2)+ade+cdex^2)^{3/4}\int \frac{(ae+cdx)^{3/4}}{\left(\frac{ex}{d}+1\right)^{17/4}}dx}{d^5\left(\frac{ex}{d}+1\right)^{3/4}(ae+cdx)^{3/4}}$$

$$\Big\downarrow \text{57}$$

$$\frac{(x(ae^2+cd^2)+ade+cdex^2)^{3/4}\left(\frac{3cd^2\int \frac{1}{\sqrt[4]{ae+cdx}\left(\frac{ex}{d}+1\right)^{13/4}}dx}{13e}-\frac{4d(ae+cdx)^{3/4}}{13e\left(\frac{ex}{d}+1\right)^{13/4}}\right)}{d^5\left(\frac{ex}{d}+1\right)^{3/4}(ae+cdx)^{3/4}}$$

$\big\downarrow$ 61

$$\frac{\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/4}\left(\frac{3cd^2\left(\frac{2cd\int \frac{1}{\sqrt[4]{ae+cdx}\left(\frac{ex}{d}+1\right)^{9/4}}dx}{3\left(cd-\frac{ae^2}{d}\right)}+\frac{4(ae+cdx)^{3/4}}{9\left(\frac{ex}{d}+1\right)^{9/4}\left(cd-\frac{ae^2}{d}\right)}\right)}{13e}-\frac{4d(ae+cdx)^{3/4}}{13e\left(\frac{ex}{d}+1\right)^{13/4}}\right)}{d^5\left(\frac{ex}{d}+1\right)^{3/4}(ae+cdx)^{3/4}}$$

$\big\downarrow$ 61

$$\frac{\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/4}\left(\frac{3cd^2\left(\frac{2cd\left(\frac{2cd\int \frac{1}{\sqrt[4]{ae+cdx}\left(\frac{ex}{d}+1\right)^{5/4}}dx}{5\left(cd-\frac{ae^2}{d}\right)}+\frac{4(ae+cdx)^{3/4}}{5\left(\frac{ex}{d}+1\right)^{5/4}\left(cd-\frac{ae^2}{d}\right)}\right)}{3\left(cd-\frac{ae^2}{d}\right)}+\frac{4(ae+cdx)^{3/4}}{9\left(\frac{ex}{d}+1\right)^{9/4}\left(cd-\frac{ae^2}{d}\right)}\right)}{13e}\right.}{d^5\left(\frac{ex}{d}+1\right)^{3/4}(ae+cdx)^{3/4}}$$

$\big\downarrow$ 61

$$\frac{\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/4}\left(\frac{3cd^2\left(\frac{2cd\left(\frac{2cd\left(\frac{4d(ae+cdx)^{3/4}}{\sqrt[4]{\frac{ex}{d}+1}\left(cd^2-ae^2\right)}-\frac{2cd^2\int\frac{1}{\sqrt[4]{ae+cdx}\sqrt[4]{\frac{ex}{d}+1}}dx}{cd^2-ae^2}\right)}{5\left(cd-\frac{ae^2}{d}\right)}+\frac{4(ae+cdx)^{3/}}{5\left(\frac{ex}{d}+1\right)^{5/4}\left(cd-\right.}}{3\left(cd-\frac{ae^2}{d}\right)}}{13e}\right.}{d^5\left(\frac{ex}{d}+1\right)^{3/4}(ae+cdx)^{3/4}}$$

↓ 73

$$\frac{\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/4}\left(\frac{3cd^2\left(\frac{2cd\left(\frac{2cd\left(\frac{4d(ae+cdx)^{3/4}}{\sqrt[4]{\frac{ex}{d}+1}\left(cd^2-ae^2\right)}-\frac{8d\int\frac{\sqrt{ae+cdx}}{\sqrt[4]{-\frac{ae^2}{cd^2}+\frac{(ae+cdx)e}{cd^2}+1}}d\sqrt[4]{ae+cdx}}{cd^2-ae^2}\right.}{5\left(cd-\frac{ae^2}{d}\right)}\right.}{3\left(cd-\frac{ae^2}{d}\right)}\right.}{13e}\right.}{d^5\left(\frac{ex}{d}+1\right)^{3/4}(ae+cdx)^{3/4}}$$

↓ 839

$$\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/4} \left(\frac{3cd^2 \left(\frac{2cd \left(\frac{2cd \left(\frac{4d(ae+cdx)^{3/4}}{\sqrt[4]{\frac{ex}{d}+1}\left(cd^2-ae^2\right)} - \frac{8d \left(\frac{(ae+cdx)^{3/4}}{2\sqrt[4]{-\frac{ae^2}{cd^2}+\frac{e(ae+cdx)}{cd^2}+1}} - \frac{1}{2}\left(1-\frac{ae^2}{cd^2}\right) \int \cdots \right.}{cd^2-ae^2} \cdots \right.}{5\left(cd-\frac{ae^2}{d}\right)} \cdots \right.}{3\left(cd-\frac{ae^2}{d}\right)} \cdots \right.}{13 \cdots} \cdots \right.$$

↓ 813

$$cd^2\left(1-\frac{ae^2}{cd^2}\right)$$

$$8d\left(\frac{(ae+cdx)^{3/4}}{2\sqrt[4]{-\frac{ae^2}{cd^2}+\frac{e(ae+cdx)}{cd^2}+1}}-\right.$$

$$2cd\left(\frac{4d(ae+cdx)^{3/4}}{\sqrt[4]{\frac{ex}{d}+1}\left(cd^2-ae^2\right)}-\right.$$

$$2cd$$

$$5\left(c\right.$$

$$3cd^2$$

$$\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/4}$$

↓ 858

$$3cd^2 \Bigg(2cd \Bigg(2cd \Bigg(\frac{4d(ae+cdx)^{3/4}}{\sqrt[4]{\frac{ex}{d}+1}\left(cd^2-ae^2\right)} - \frac{8d \left(\frac{cd^2\left(1-\frac{ae^2}{cd^2}\right)\sqrt[4]{ae+cdx}\sqrt[4]{\frac{cd^2-ae^2}{e(ae+cdx)}+1}}{2e\sqrt[4]{-\frac{ae^2}{cd^2}}} \right)}{}$$

↓ 807

$$3cd^2 \left(2cd \left(2cd \left(\frac{4d(ae+cdx)^{3/4}}{\sqrt[4]{\frac{ex}{d}+1}\,(cd^2-ae^2)} - \frac{8d \left(\frac{cd^2\left(1-\frac{ae^2}{cd^2}\right)\sqrt[4]{ae+cdx}\sqrt[4]{\frac{cd^2-ae^2}{e(ae+cdx)}+1}}{4e\sqrt[4]{-\frac{ae^2}{cd^2}+\frac{e(ae+}{cd}}} \right.}{} \right. \right. \right.$$

$$5\left(cd-\frac{a}{}\right.$$

$$\left(\ (\ ^2+\ d^2)+\ d+\ d\ ^2\right)^{3/4}$$

↓ 212

$$\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/4}\left(\frac{3cd^2\left(\frac{2cd\left(\frac{2cd\left(\frac{4d(ae+cdx)^{3/4}}{\sqrt[4]{\frac{ex}{d}+1}\left(cd^2-ae^2\right)}-\frac{8d\left(\frac{cd^2\left(1-\frac{ae^2}{cd^2}\right)\sqrt[4]{ae+cdx}\sqrt[4]{\frac{cd^2-ae^2}{e(ae+cdx)}+1\ldots}}{2\sqrt{e}\sqrt{cd^2-ae^2}\sqrt[4]{-\frac{ae^2}{cd^2}+\frac{e(a\ldots}{}}}\right)}{}\right)}{5\left(cd-\frac{ae^2}{d}\ldots\right)}\right)}{}\right)}{}\right.$$

input
```
Int[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/4)/(d + e*x)^5,x]
```

output
```
((a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/4)*((-4*d*(a*e + c*d*x)^(3/4))
/(13*e*(1 + (e*x)/d)^(13/4)) + (3*c*d^2*((4*(a*e + c*d*x)^(3/4))/(9*(c*d -
 (a*e^2)/d)*(1 + (e*x)/d)^(9/4)) + (2*c*d*((4*(a*e + c*d*x)^(3/4))/(5*(c*d
 - (a*e^2)/d)*(1 + (e*x)/d)^(5/4)) + (2*c*d*((4*d*(a*e + c*d*x)^(3/4))/((c
*d^2 - a*e^2)*(1 + (e*x)/d)^(1/4)) - (8*d*((a*e + c*d*x)^(3/4)/(2*(1 - (a*
e^2)/(c*d^2) + (e*(a*e + c*d*x))/(c*d^2))^(1/4)) + (c*d^2*(1 - (a*e^2)/(c*
d^2))*(a*e + c*d*x)^(1/4)*(1 + (c*d^2 - a*e^2)/(e*(a*e + c*d*x)))^(1/4)*El
lipticE[ArcTan[(Sqrt[c*d^2 - a*e^2]*Sqrt[a*e + c*d*x])/Sqrt[e]]/2, 2])/(2*
Sqrt[e]*Sqrt[c*d^2 - a*e^2]*(1 - (a*e^2)/(c*d^2) + (e*(a*e + c*d*x))/(c*d^
2))^(1/4))))/(c*d^2 - a*e^2)))/(5*(c*d - (a*e^2)/d))))/(3*(c*d - (a*e^2)/d
))))/(13*e)))/(d^5*(a*e + c*d*x)^(3/4)*(1 + (e*x)/d)^(3/4))
```

Defintions of rubi rules used

rule 57
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Simp[d*(n/(b*(m + 1)))
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &
& GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !IntegerQ[m]) &&  !(ILeQ[m
+ n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) && IntLinearQ[a, b, c
, d, m, n, x]
```

rule 61
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && LtQ[m, -1] &&  !(LtQ[n, -1] && (EqQ[a, 0
] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && IntLinearQ[a, b, c, d
, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-5/4), x_Symbol] :> Simp[(2/(a^(5/4)*Rt[b/a, 2])
)*EllipticE[(1/2)*ArcTan[Rt[b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

rule 807

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Simp[1/k   Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x,
x^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

rule 813

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4)^(5/4), x_Symbol] :> Simp[x*((1 + a/(b*x^4)
)^(1/4)/(b*(a + b*x^4)^(1/4)))   Int[1/(x^3*(1 + a/(b*x^4))^(5/4)), x], x]
/; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 839

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4)^(1/4), x_Symbol] :> Simp[x^3/(2*(a + b*x^4
)^(1/4)), x] - Simp[a/2   Int[x^2/(a + b*x^4)^(5/4), x], x] /; FreeQ[{a, b}
, x] && PosQ[b/a]
```

rule 858

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

rule 1138

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Sy
mbol] :> Simp[d^m*((a + b*x + c*x^2)^FracPart[p]/((1 + e*(x/d))^FracPart[p]
*(a/d + (c*x)/e)^FracPart[p]))   Int[(1 + e*(x/d))^(m + p)*(a/d + (c/e)*x)^
p, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0]
&& (IntegerQ[m] || GtQ[d, 0]) &&  !(IGtQ[m, 0] && (IntegerQ[3*p] || Integer
Q[4*p]))
```

Maple [F]

$$\int \frac{\left(ade+\left(a\,e^{2}+c\,d^{2}\right)x+cd\,x^{2}e\right)^{\frac{3}{4}}}{\left(ex+d\right)^{5}}dx$$

input
```
int((a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(3/4)/(e*x+d)^5,x)
```

output
```
int((a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(3/4)/(e*x+d)^5,x)
```

Fricas [F]

$$\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/4}}{(d+ex)^5}\,dx=\int \frac{\left(cdex^2+ade+\left(cd^2+ae^2\right)x\right)^{\frac{3}{4}}}{\left(ex+d\right)^{5}}\,dx$$

input
```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/4)/(e*x+d)^5,x, algorithm="
fricas")
```

output
```
integral((c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^(3/4)/(e^5*x^5 + 5*d*e^4*
x^4 + 10*d^2*e^3*x^3 + 10*d^3*e^2*x^2 + 5*d^4*e*x + d^5), x)
```

Sympy [F]

$$\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/4}}{(d+ex)^5}\,dx=\int \frac{\left(\left(d+ex\right)\left(ae+cdx\right)\right)^{\frac{3}{4}}}{\left(d+ex\right)^{5}}\,dx$$

input
```
integrate((a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(3/4)/(e*x+d)**5,x)
```

output
```
Integral(((d + e*x)*(a*e + c*d*x))**(3/4)/(d + e*x)**5, x)
```

Maxima [F]

$$\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/4}}{(d+ex)^5}\,dx=\int \frac{\left(cdex^2+ade+\left(cd^2+ae^2\right)x\right)^{\frac{3}{4}}}{\left(ex+d\right)^5}\,dx$$

input
```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/4)/(e*x+d)^5,x, algorithm="
maxima")
```

output
```
integrate((c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^(3/4)/(e*x + d)^5, x)
```

Giac [F]

$$\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/4}}{(d+ex)^5}\,dx=\int \frac{\left(cdex^2+ade+\left(cd^2+ae^2\right)x\right)^{\frac{3}{4}}}{\left(ex+d\right)^5}\,dx$$

input
```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/4)/(e*x+d)^5,x, algorithm="
giac")
```

output
```
integrate((c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^(3/4)/(e*x + d)^5, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/4}}{(d+ex)^5}\,dx=\int \frac{\left(c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e\right)^{3/4}}{\left(d+e\,x\right)^5}\,dx$$

input
```
int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(3/4)/(d + e*x)^5,x)
```

output
```
int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(3/4)/(d + e*x)^5, x)
```

Reduce [F]

$$\int \frac{(ade + (cd^2 + ae^2)x + cdex^2)^{3/4}}{(d + ex)^5} \, dx = \text{too large to display}$$

input
```
int((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/4)/(e*x+d)^5,x)
```

output
```
( - 4*(a*d*e + a*e**2*x + c*d**2*x + c*d*e*x**2)**(3/4)*a*e + 39*int(((a*d
*e + a*e**2*x + c*d**2*x + c*d*e*x**2)**(3/4)*x)/(13*a**2*d**5*e**3 + 65*a
**2*d**4*e**4*x + 130*a**2*d**3*e**5*x**2 + 130*a**2*d**2*e**6*x**3 + 65*a
**2*d*e**7*x**4 + 13*a**2*e**8*x**5 - 3*a*c*d**7*e - 2*a*c*d**6*e**2*x + 3
5*a*c*d**5*e**3*x**2 + 100*a*c*d**4*e**4*x**3 + 115*a*c*d**3*e**5*x**4 + 6
2*a*c*d**2*e**6*x**5 + 13*a*c*d*e**7*x**6 - 3*c**2*d**8*x - 15*c**2*d**7*e
*x**2 - 30*c**2*d**6*e**2*x**3 - 30*c**2*d**5*e**3*x**4 - 15*c**2*d**4*e**
4*x**5 - 3*c**2*d**3*e**5*x**6),x)*a**2*c*d**5*e**4 + 156*int(((a*d*e + a*
e**2*x + c*d**2*x + c*d*e*x**2)**(3/4)*x)/(13*a**2*d**5*e**3 + 65*a**2*d**
4*e**4*x + 130*a**2*d**3*e**5*x**2 + 130*a**2*d**2*e**6*x**3 + 65*a**2*d*e
**7*x**4 + 13*a**2*e**8*x**5 - 3*a*c*d**7*e - 2*a*c*d**6*e**2*x + 35*a*c*d
**5*e**3*x**2 + 100*a*c*d**4*e**4*x**3 + 115*a*c*d**3*e**5*x**4 + 62*a*c*d
**2*e**6*x**5 + 13*a*c*d*e**7*x**6 - 3*c**2*d**8*x - 15*c**2*d**7*e*x**2 -
 30*c**2*d**6*e**2*x**3 - 30*c**2*d**5*e**3*x**4 - 15*c**2*d**4*e**4*x**5
- 3*c**2*d**3*e**5*x**6),x)*a**2*c*d**4*e**5*x + 234*int(((a*d*e + a*e**2*
x + c*d**2*x + c*d*e*x**2)**(3/4)*x)/(13*a**2*d**5*e**3 + 65*a**2*d**4*e**
4*x + 130*a**2*d**3*e**5*x**2 + 130*a**2*d**2*e**6*x**3 + 65*a**2*d*e**7*x
**4 + 13*a**2*e**8*x**5 - 3*a*c*d**7*e - 2*a*c*d**6*e**2*x + 35*a*c*d**5*e
**3*x**2 + 100*a*c*d**4*e**4*x**3 + 115*a*c*d**3*e**5*x**4 + 62*a*c*d**2*e
**6*x**5 + 13*a*c*d*e**7*x**6 - 3*c**2*d**8*x - 15*c**2*d**7*e*x**2 - 3...
```

3.352 $\int (d+ex)^2 \left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/4} dx$

Optimal result

Integrand size = 37, antiderivative size = 473

$$\int (d+ex)^2 \left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/4} dx = -\frac{65\left(cd^2-ae^2\right)^5 \sqrt[4]{ade+\left(cd^2+ae^2\right)x+cdex^2}}{3696c^4d^4e^2} + \frac{13\left(cd^2-ae^2\right)^4 \left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/4}}{1848c^4d^4e(d+ex)} + \frac{13\left(cd^2-ae^2\right)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{9/4}}{99c^2d^2} + \frac{13\left(cd^2-ae^2\right)^3 \left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{9/4}}{308c^4d^4(d+ex)^2} + \frac{13\left(cd^2-ae^2\right)^2 \left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{9/4}}{154c^3d^3(d+ex)} + \frac{2(d+ex)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{9/4}}{11cd} - \frac{65\left(cd^2-ae^2\right)^{11/2}(ae+cdx)^{3/2}\left(\frac{cd(d+ex)}{e(ae+cdx)}\right)^{3/4} \operatorname{EllipticF}\left(\frac{1}{2}\arctan\left(\frac{\sqrt{cd^2-ae^2}}{\sqrt{e}\sqrt{ae+cdx}}\right),2\right)}{3696c^5d^5e^{3/2}\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/4}}$$

output
```
-65/3696*(-a*e^2+c*d^2)^5*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/4)/c^4/d^4/
e^2+13/1848*(-a*e^2+c*d^2)^4*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/4)/c^4/d
^4/e/(e*x+d)+13/99*(-a*e^2+c*d^2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(9/4)/
c^2/d^2+13/308*(-a*e^2+c*d^2)^3*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(9/4)/c^
4/d^4/(e*x+d)^2+13/154*(-a*e^2+c*d^2)^2*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^
(9/4)/c^3/d^3/(e*x+d)+2/11*(e*x+d)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(9/4)
/c/d-65/3696*(-a*e^2+c*d^2)^(11/2)*(c*d*x+a*e)^(3/2)*(c*d*(e*x+d)/e/(c*d*x
+a*e))^(3/4)*InverseJacobiAM(1/2*arctan((-a*e^2+c*d^2)^(1/2)/e^(1/2)/(c*d*
x+a*e)^(1/2)),2^(1/2))/c^5/d^5/e^(3/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(
3/4)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.07 (sec) , antiderivative size = 112, normalized size of antiderivative = 0.24

$$\int (d+ex)^2 \left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/4} dx = \frac{4(cd^2-ae^2)^3 (ae+cdx)^2 \sqrt[4]{(ae+cdx)(d+ex)}\,\text{Hypergeometric2F1}\left(-\frac{13}{4},\frac{9}{4},\frac{13}{4},\frac{e(ae+cdx)}{-cd^2+ae^2}\right)}{9c^4d^4\sqrt[4]{\frac{cd(d+ex)}{cd^2-ae^2}}}$$

input
```
Integrate[(d + e*x)^2*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/4),x]
```

output
```
(4*(c*d^2 - a*e^2)^3*(a*e + c*d*x)^2*((a*e + c*d*x)*(d + e*x))^(1/4)*Hyper
geometric2F1[-13/4, 9/4, 13/4, (e*(a*e + c*d*x))/(-(c*d^2) + a*e^2)])/(9*c
^4*d^4*((c*d*(d + e*x))/(c*d^2 - a*e^2))^(1/4))
```

Rubi [A] (warning: unable to verify)

Time = 1.17 (sec) , antiderivative size = 668, normalized size of antiderivative = 1.41, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.189$, Rules used = {1166, 27, 1160, 1087, 1087, 1094, 761}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (d+ex)^2 \left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/4} dx$$

$$\Big\downarrow \text{1166}$$

$$\frac{2\int \frac{13}{4}e\left(cd^2-ae^2\right)(d+ex)\left(cdex^2+\left(cd^2+ae^2\right)x+ade\right)^{5/4}dx}{11cde}+\frac{2(d+ex)\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{9/4}}{11cd}$$

$$\Big\downarrow \text{27}$$

$$\frac{13\left(cd^2-ae^2\right)\int (d+ex)\left(cdex^2+\left(cd^2+ae^2\right)x+ade\right)^{5/4}dx}{22cd}+\frac{2(d+ex)\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{9/4}}{11cd}$$

$$\Big\downarrow \text{1160}$$

$$\frac{13\left(cd^2-ae^2\right)\left(\frac{\left(d^2-\frac{ae^2}{c}\right)\int \left(cdex^2+\left(cd^2+ae^2\right)x+ade\right)^{5/4}dx}{2d}+\frac{2\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{9/4}}{9cd}\right)}{22cd}+\frac{2(d+ex)\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{9/4}}{11cd}$$

$$\Big\downarrow \text{1087}$$

$$\frac{13\left(cd^2-ae^2\right)\left(\frac{\left(d^2-\frac{ae^2}{c}\right)\left(\frac{\left(ae^2+cd^2+2cdex\right)\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/4}}{7cde}-\frac{5\left(cd^2-ae^2\right)^2\int \sqrt[4]{cdex^2+\left(cd^2+ae^2\right)x+ade}\,dx}{28cde}\right)}{2d}\right)}{22cd}+\frac{2(d+ex)\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{9/4}}{11cd}$$

$$\Big\downarrow \text{1087}$$

$$\frac{13\left(cd^2-ae^2\right)\left(\frac{\left(d^2-\frac{ae^2}{c}\right)\left(\frac{\left(ae^2+cd^2+2cdex\right)\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/4}}{7cde}-\frac{5\left(cd^2-ae^2\right)^2\left(\frac{\left(ae^2+cd^2+2cdex\right)\sqrt[4]{x\left(ae^2+cd^2\right)+ade+\cdots}}{3cde}\cdots\right)}{28cd\cdots}\right)}{2d}\cdots\right)}{22cd}\cdots$$

$$\frac{2(d+ex)\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{9/4}}{11cd}$$

$\Big\downarrow$ 1094

$$13\left(cd^2-ae^2\right)\left(\frac{\left(d^2-\frac{ae^2}{c}\right)\left(\frac{\left(ae^2+cd^2+2cdex\right)\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/4}}{7cde}-\frac{5\left(cd^2-ae^2\right)^2\left(\frac{\left(ae^2+cd^2+2cdex\right)\sqrt[4]{x\left(ae^2+cd^2\right)+ade+\cdots}}{3cde}\cdots\right)}{\cdots}\right)}{\cdots}\cdots\right)$$

$$\frac{2(d+ex)\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{9/4}}{11cd}$$

$\Big\downarrow$ 761

$$13\left(cd^2-ae^2\right)\left(\frac{\left(d^2-\frac{ae^2}{c}\right)\left(\frac{\left(ae^2+cd^2+2cdex\right)\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/4}}{7cde}-\frac{5\left(cd^2-ae^2\right)^2\left(\frac{\left(ae^2+cd^2+2cdex\right)\sqrt[4]{x\left(ae^2+cd^2\right)+ade\cdots}}{3cde}\cdots\right)}{\cdots}\right)}{\cdots}\cdots\right)$$

$$\frac{2(d+ex)\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{9/4}}{11cd}$$

input
```
Int[(d + e*x)^2*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/4),x]
```

output
```
(2*(d + e*x)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(9/4))/(11*c*d) + (13
*(c*d^2 - a*e^2)*((2*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(9/4))/(9*c*d
) + ((d^2 - (a*e^2)/c)*(((c*d^2 + a*e^2 + 2*c*d*e*x)*(a*d*e + (c*d^2 + a*e
^2)*x + c*d*e*x^2)^(5/4))/(7*c*d*e) - (5*(c*d^2 - a*e^2)^2*(((c*d^2 + a*e^
2 + 2*c*d*e*x)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(1/4))/(3*c*d*e) -
((c*d^2 - a*e^2)^(5/2)*Sqrt[(c*d^2 + a*e^2 + 2*c*d*e*x)^2]*(1 + (2*Sqrt[c]
*Sqrt[d]*Sqrt[e]*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(c*d^2 - a*e
^2))*Sqrt[((c*d^2 - a*e^2)^2 + 4*c*d*e*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*
x^2))/((c*d^2 - a*e^2)^2*(1 + (2*Sqrt[c]*Sqrt[d]*Sqrt[e]*Sqrt[a*d*e + (c*d
^2 + a*e^2)*x + c*d*e*x^2])/(c*d^2 - a*e^2))^2)]*EllipticF[2*ArcTan[(Sqrt[
2]*c^(1/4)*d^(1/4)*e^(1/4)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(1/4))/
Sqrt[c*d^2 - a*e^2]], 1/2])/(6*Sqrt[2]*c^(5/4)*d^(5/4)*e^(5/4)*(c*d^2 + a*
e^2 + 2*c*d*e*x)*Sqrt[(c*d^2 - a*e^2)^2 + 4*c*d*e*(a*d*e + (c*d^2 + a*e^2)
*x + c*d*e*x^2)])))/(28*c*d*e)))/(2*d)))/(22*c*d)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 761
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 1087
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^p/(2*c*(2*p + 1))), x] - Simp[p*((b^2 - 4*a*c)/(2*c*(2*
p + 1)))   Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] &&
GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[3*p])
```

rule 1094
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[4*(Sqrt[(b
+ 2*c*x)^2]/(b + 2*c*x))   Subst[Int[x^(4*(p + 1) - 1)/Sqrt[b^2 - 4*a*c + 4
*c*x^4], x], x, (a + b*x + c*x^2)^(1/4)], x] /; FreeQ[{a, b, c}, x] && Inte
gerQ[4*p]
```

rule 1160
```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[e*((a + b*x + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Simp[(2*c*d - b
*e)/(2*c)   Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
 && NeQ[p, -1]
```

rule 1166
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[e*(d + e*x)^(m - 1)*((a + b*x + c*x^2)^(p + 1)/(c*(m + 2*p +
 1))), x] + Simp[1/(c*(m + 2*p + 1))   Int[(d + e*x)^(m - 2)*Simp[c*d^2*(m
+ 2*p + 1) - e*(a*e*(m - 1) + b*d*(p + 1)) + e*(2*c*d - b*e)*(m + p)*x, x]*
(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && If[Ration
alQ[m], GtQ[m, 1], SumSimplerQ[m, -2]] && NeQ[m + 2*p + 1, 0] && IntQuadrat
icQ[a, b, c, d, e, m, p, x]
```

Maple [F]

$$\int (ex+d)^2 \left(ade+\left(a\,e^2+c\,d^2\right)x+cd\,x^2e\right)^{\frac{5}{4}}dx$$

input
```
int((e*x+d)^2*(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(5/4),x)
```

output
```
int((e*x+d)^2*(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(5/4),x)
```

Fricas [F]

$$\int (d+ex)^2 \left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/4}\,dx=\int \left(cdex^2+ade+\left(cd^2+ae^2\right)x\right)^{\frac{5}{4}}(ex+d)^2\,dx$$

input
```
integrate((e*x+d)^2*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/4),x, algorithm="
fricas")
```

output
```
integral((c*d*e^3*x^4 + a*d^3*e + (3*c*d^2*e^2 + a*e^4)*x^3 + 3*(c*d^3*e +
 a*d*e^3)*x^2 + (c*d^4 + 3*a*d^2*e^2)*x)*(c*d*e*x^2 + a*d*e + (c*d^2 + a*e
^2)*x)^(1/4), x)
```

Sympy [F]

$$\int (d+ex)^2 \left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/4}\,dx=\int \left((d+ex)\,(ae+cdx)\right)^{\frac{5}{4}}(d+ex)^2\,dx$$

input
```
integrate((e*x+d)**2*(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(5/4),x)
```

output
```
Integral(((d + e*x)*(a*e + c*d*x))**(5/4)*(d + e*x)**2, x)
```

Maxima [F]

$$\int (d+ex)^2 \left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/4}\,dx=\int \left(cdex^2+ade+\left(cd^2+ae^2\right)x\right)^{\frac{5}{4}}(ex+d)^2\,dx$$

input
```
integrate((e*x+d)^2*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/4),x, algorithm="
maxima")
```

output
```
integrate((c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^(5/4)*(e*x + d)^2, x)
```

Giac [F]

$$\int (d+ex)^2 \left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/4}\,dx=\int \left(cdex^2+ade+\left(cd^2+ae^2\right)x\right)^{\frac{5}{4}}(ex+d)^2\,dx$$

input
```
integrate((e*x+d)^2*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/4),x, algorithm="
giac")
```

output
```
integrate((c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^(5/4)*(e*x + d)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int (d+ex)^2 \left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/4}\,dx=\int (d+e\,x)^2\,\left(c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e\right)^{5/4}\,dx$$

input
```
int((d + e*x)^2*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(5/4),x)
```

output
```
int((d + e*x)^2*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(5/4), x)
```

Reduce [F]

$$\int (d+ex)^2 \left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/4}\,dx=\text{too large to display}$$

input
```
int((e*x+d)^2*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/4),x)
```

output

```
( - 624*(a*d*e + a*e**2*x + c*d**2*x + c*d*e*x**2)**(1/4)*a**5*d*e**9 + 15
6*(a*d*e + a*e**2*x + c*d**2*x + c*d*e*x**2)**(1/4)*a**5*e**10*x + 3328*(a
*d*e + a*e**2*x + c*d**2*x + c*d*e*x**2)**(1/4)*a**4*c*d**3*e**7 - 676*(a*
d*e + a*e**2*x + c*d**2*x + c*d*e*x**2)**(1/4)*a**4*c*d**2*e**8*x - 104*(a
*d*e + a*e**2*x + c*d**2*x + c*d*e*x**2)**(1/4)*a**4*c*d*e**9*x**2 - 7200*
(a*d*e + a*e**2*x + c*d**2*x + c*d*e*x**2)**(1/4)*a**3*c**2*d**5*e**5 + 96
8*(a*d*e + a*e**2*x + c*d**2*x + c*d*e*x**2)**(1/4)*a**3*c**2*d**4*e**6*x
+ 448*(a*d*e + a*e**2*x + c*d**2*x + c*d*e*x**2)**(1/4)*a**3*c**2*d**3*e**
7*x**2 + 80*(a*d*e + a*e**2*x + c*d**2*x + c*d*e*x**2)**(1/4)*a**3*c**2*d*
*2*e**8*x**3 + 13184*(a*d*e + a*e**2*x + c*d**2*x + c*d*e*x**2)**(1/4)*a**
2*c**3*d**7*e**3 + 20680*(a*d*e + a*e**2*x + c*d**2*x + c*d*e*x**2)**(1/4)
*a**2*c**3*d**6*e**4*x + 30672*(a*d*e + a*e**2*x + c*d**2*x + c*d*e*x**2)*
*(1/4)*a**2*c**3*d**5*e**5*x**2 + 20528*(a*d*e + a*e**2*x + c*d**2*x + c*d
*e*x**2)**(1/4)*a**2*c**3*d**4*e**6*x**3 + 5152*(a*d*e + a*e**2*x + c*d**2
*x + c*d*e*x**2)**(1/4)*a**2*c**3*d**3*e**7*x**4 - 624*(a*d*e + a*e**2*x +
 c*d**2*x + c*d*e*x**2)**(1/4)*a*c**4*d**9*e + 19036*(a*d*e + a*e**2*x + c
*d**2*x + c*d*e*x**2)**(1/4)*a*c**4*d**8*e**2*x + 39872*(a*d*e + a*e**2*x
+ c*d**2*x + c*d*e*x**2)**(1/4)*a*c**4*d**7*e**3*x**2 + 40240*(a*d*e + a*e
**2*x + c*d**2*x + c*d*e*x**2)**(1/4)*a*c**4*d**6*e**4*x**3 + 20160*(a*d*e
 + a*e**2*x + c*d**2*x + c*d*e*x**2)**(1/4)*a*c**4*d**5*e**5*x**4 + 403...
```

3.353 $\int (d+ex) \left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/4} dx$

Optimal result

Integrand size = 35, antiderivative size = 415

$$\begin{aligned}
&\int (d+ex) \left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/4} dx = \\
&-\frac{5(cd^2-ae^2)^4 \sqrt[4]{ade+\left(cd^2+ae^2\right)x+cdex^2}}{168c^3d^3e^2} \\
&+\frac{\left(cd^2-ae^2\right)^3 \left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/4}}{84c^3d^3e(d+ex)} \\
&+\frac{2(ade+\left(cd^2+ae^2\right)x+cdex^2)^{9/4}}{9cd}+\frac{\left(cd^2-ae^2\right)^2 \left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{9/4}}{14c^3d^3(d+ex)^2} \\
&+\frac{\left(cd^2-ae^2\right) \left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{9/4}}{7c^2d^2(d+ex)} \\
&-\frac{5(cd^2-ae^2)^{9/2}(ae+cdx)^{3/2}\left(\frac{cd(d+ex)}{e(ae+cdx)}\right)^{3/4}\text{EllipticF}\left(\frac{1}{2}\arctan\left(\frac{\sqrt{cd^2-ae^2}}{\sqrt{e}\sqrt{ae+cdx}}\right),2\right)}{168c^4d^4e^{3/2}\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/4}}
\end{aligned}$$

output
```
-5/168*(-a*e^2+c*d^2)^4*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/4)/c^3/d^3/e^
2+1/84*(-a*e^2+c*d^2)^3*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/4)/c^3/d^3/e/
(e*x+d)+2/9*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(9/4)/c/d+1/14*(-a*e^2+c*d^2
)^2*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(9/4)/c^3/d^3/(e*x+d)^2+1/7*(-a*e^2+
c*d^2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(9/4)/c^2/d^2/(e*x+d)-5/168*(-a*e
^2+c*d^2)^(9/2)*(c*d*x+a*e)^(3/2)*(c*d*(e*x+d)/e/(c*d*x+a*e))^(3/4)*Invers
eJacobiAM(1/2*arctan((-a*e^2+c*d^2)^(1/2)/e^(1/2)/(c*d*x+a*e)^(1/2)),2^(1/
2))/c^4/d^4/e^(3/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/4)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.05 (sec) , antiderivative size = 112, normalized size of antiderivative = 0.27

$$\int (d+ex) \left(ade+\left(cd^2+ae^2\right) x+cdex^2\right)^{5/4} dx=\frac{4(cd^2-ae^2)^2 (ae+cdx)^2 \sqrt[4]{(ae+cdx)(d+ex)}\, \text{Hypergeometric2F1}\left(-\frac{9}{4},\frac{9}{4},\frac{13}{4},\frac{e(ae+cdx)}{-cd^2+ae^2}\right)}{9c^3d^3\sqrt[4]{\frac{cd(d+ex)}{cd^2-ae^2}}}$$

input
```
Integrate[(d + e*x)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/4),x]
```

output
```
(4*(c*d^2 - a*e^2)^2*(a*e + c*d*x)^2*((a*e + c*d*x)*(d + e*x))^(1/4)*Hyper
geometric2F1[-9/4, 9/4, 13/4, (e*(a*e + c*d*x))/(-(c*d^2) + a*e^2)])/(9*c^
3*d^3*((c*d*(d + e*x))/(c*d^2 - a*e^2))^(1/4))
```

Rubi [A] (warning: unable to verify)

Time = 0.92 (sec) , antiderivative size = 601, normalized size of antiderivative = 1.45, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {1160, 1087, 1087, 1094, 761}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (d+ex)\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/4}dx$$

$\big\downarrow$ 1160

$$\frac{\left(d^2-\frac{ae^2}{c}\right)\int \left(cdex^2+\left(cd^2+ae^2\right)x+ade\right)^{5/4}dx}{2d}+\frac{2\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{9/4}}{9cd}$$

$\big\downarrow$ 1087

$$\frac{\left(d^2-\frac{ae^2}{c}\right)\left(\frac{\left(ae^2+cd^2+2cdex\right)\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/4}}{7cde}-\frac{5\left(cd^2-ae^2\right)^2\int \sqrt[4]{cdex^2+\left(cd^2+ae^2\right)x+ade}dx}{28cde}\right)}{2d}+\frac{2\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{9/4}}{9cd}$$

$\big\downarrow$ 1087

$$\frac{\left(d^2-\frac{ae^2}{c}\right)\left(\frac{\left(ae^2+cd^2+2cdex\right)\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/4}}{7cde}-\frac{5\left(cd^2-ae^2\right)^2\left(\frac{\left(ae^2+cd^2+2cdex\right)\sqrt[4]{x\left(ae^2+cd^2\right)+ade+cdex^2}}{3cde}\right.}{28cde}\right.}{2d}$$

$$\frac{2\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{9/4}}{9cd}$$

$\big\downarrow$ 1094

$$\frac{\left(d^2-\frac{ae^2}{c}\right)\left(\frac{\left(ae^2+cd^2+2cdex\right)\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/4}}{7cde}-\frac{5\left(cd^2-ae^2\right)^2\left(\frac{\left(ae^2+cd^2+2cdex\right)\sqrt[4]{x\left(ae^2+cd^2\right)+ade+cdex^2}}{3cde}\right.}{}\right.}{2d}$$

$$\frac{2\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{9/4}}{9cd}$$

$\big\downarrow$ 761

$$\frac{\left(d^2-\frac{ae^2}{c}\right)\left(\frac{(ae^2+cd^2+2cdex)\left(x(ae^2+cd^2)+ade+cdex^2\right)^{5/4}}{7cde}-\frac{5\left(cd^2-ae^2\right)^2\left(\frac{(ae^2+cd^2+2cdex)\sqrt[4]{x\left(ae^2+cd^2\right)+ade+cdex^2}}{3cde}\right.}{}\right.}{}$$

$$\frac{2\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{9/4}}{9cd}$$

input
```
Int[(d + e*x)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/4),x]
```

output
```
(2*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(9/4))/(9*c*d) + ((d^2 - (a*e^2
)/c)*(((c*d^2 + a*e^2 + 2*c*d*e*x)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)
^(5/4))/(7*c*d*e) - (5*(c*d^2 - a*e^2)^2*(((c*d^2 + a*e^2 + 2*c*d*e*x)*(a*
d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(1/4))/(3*c*d*e) - ((c*d^2 - a*e^2)^(
5/2)*Sqrt[(c*d^2 + a*e^2 + 2*c*d*e*x)^2]*(1 + (2*Sqrt[c]*Sqrt[d]*Sqrt[e]*S
qrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(c*d^2 - a*e^2))*Sqrt[((c*d^2
- a*e^2)^2 + 4*c*d*e*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2))/((c*d^2 - a*
e^2)^2*(1 + (2*Sqrt[c]*Sqrt[d]*Sqrt[e]*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*
d*e*x^2])/(c*d^2 - a*e^2))^2)]*EllipticF[2*ArcTan[(Sqrt[2]*c^(1/4)*d^(1/4)
*e^(1/4)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(1/4))/Sqrt[c*d^2 - a*e^2
]], 1/2])/(6*Sqrt[2]*c^(5/4)*d^(5/4)*e^(5/4)*(c*d^2 + a*e^2 + 2*c*d*e*x)*S
qrt[(c*d^2 - a*e^2)^2 + 4*c*d*e*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)]))
)/(28*c*d*e)))/(2*d)
```

Defintions of rubi rules used

rule 761
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 1087
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^p/(2*c*(2*p + 1))), x] - Simp[p*((b^2 - 4*a*c)/(2*c*(2*
p + 1)))   Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] &&
GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[3*p])
```

rule 1094
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[4*(Sqrt[(b
+ 2*c*x)^2]/(b + 2*c*x))   Subst[Int[x^(4*(p + 1) - 1)/Sqrt[b^2 - 4*a*c + 4
*c*x^4], x], x, (a + b*x + c*x^2)^(1/4)], x] /; FreeQ[{a, b, c}, x] && Inte
gerQ[4*p]
```

rule 1160
```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[e*((a + b*x + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Simp[(2*c*d - b
*e)/(2*c)   Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
 && NeQ[p, -1]
```

Maple [F]

$$\int (ex+d)\left(ade+\left(a\,e^{2}+c\,d^{2}\right)x+cd\,x^{2}e\right)^{\frac{5}{4}}dx$$

input
```
int((e*x+d)*(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(5/4),x)
```

output
```
int((e*x+d)*(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(5/4),x)
```

Fricas [F]

$$\int (d+ex) \left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/4} dx = \int \left(cdex^2+ade+\left(cd^2+ae^2\right)x\right)^{\frac{5}{4}}(ex+d)\,dx$$

input
```
integrate((e*x+d)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/4),x, algorithm="fr
icas")
```

output
```
integral((c*d*e^2*x^3 + a*d^2*e + (2*c*d^2*e + a*e^3)*x^2 + (c*d^3 + 2*a*d
*e^2)*x)*(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^(1/4), x)
```

Sympy [F]

$$\int (d+ex) \left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/4} dx = \int ((d+ex)(ae+cdx))^{\frac{5}{4}}(d+ex)\,dx$$

input
```
integrate((e*x+d)*(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(5/4),x)
```

output
```
Integral(((d + e*x)*(a*e + c*d*x))**(5/4)*(d + e*x), x)
```

Maxima [F]

$$\int (d+ex) \left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/4} dx = \int \left(cdex^2+ade+\left(cd^2+ae^2\right)x\right)^{\frac{5}{4}}(ex+d)\,dx$$

input
```
integrate((e*x+d)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/4),x, algorithm="ma
xima")
```

output
```
integrate((c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^(5/4)*(e*x + d), x)
```

Giac [F]

$$\int (d+ex)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/4}\,dx=\int \left(cdex^2+ade+\left(cd^2+ae^2\right)x\right)^{\frac{5}{4}}(ex+d)\,dx$$

input
```
integrate((e*x+d)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/4),x, algorithm="gi
ac")
```

output
```
integrate((c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^(5/4)*(e*x + d), x)
```

Mupad [F(-1)]

Timed out.

$$\int (d+ex)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/4}\,dx=\int (d+e\,x)\,\left(c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e\right)^{5/4}\,dx$$

input
```
int((d + e*x)*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(5/4),x)
```

output
```
int((d + e*x)*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(5/4), x)
```

Reduce [F]

$$\int (d+ex)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/4}\,dx=\text{Too large to display}$$

input
```
int((e*x+d)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/4),x)
```

output

```
(48*(a*d*e + a*e**2*x + c*d**2*x + c*d*e*x**2)**(1/4)*a**4*d*e**7 - 12*(a*
d*e + a*e**2*x + c*d**2*x + c*d*e*x**2)**(1/4)*a**4*e**8*x - 208*(a*d*e +
a*e**2*x + c*d**2*x + c*d*e*x**2)**(1/4)*a**3*c*d**3*e**5 + 40*(a*d*e + a*
e**2*x + c*d**2*x + c*d*e*x**2)**(1/4)*a**3*c*d**2*e**6*x + 8*(a*d*e + a*e
**2*x + c*d**2*x + c*d*e*x**2)**(1/4)*a**3*c*d*e**7*x**2 + 656*(a*d*e + a*
e**2*x + c*d**2*x + c*d*e*x**2)**(1/4)*a**2*c**2*d**5*e**3 + 896*(a*d*e +
a*e**2*x + c*d**2*x + c*d*e*x**2)**(1/4)*a**2*c**2*d**4*e**4*x + 904*(a*d*
e + a*e**2*x + c*d**2*x + c*d*e*x**2)**(1/4)*a**2*c**2*d**3*e**5*x**2 + 30
4*(a*d*e + a*e**2*x + c*d**2*x + c*d*e*x**2)**(1/4)*a**2*c**2*d**2*e**6*x*
*3 - 48*(a*d*e + a*e**2*x + c*d**2*x + c*d*e*x**2)**(1/4)*a*c**3*d**7*e +
856*(a*d*e + a*e**2*x + c*d**2*x + c*d*e*x**2)**(1/4)*a*c**3*d**6*e**2*x +
 1336*(a*d*e + a*e**2*x + c*d**2*x + c*d*e*x**2)**(1/4)*a*c**3*d**5*e**3*x
**2 + 896*(a*d*e + a*e**2*x + c*d**2*x + c*d*e*x**2)**(1/4)*a*c**3*d**4*e*
*4*x**3 + 224*(a*d*e + a*e**2*x + c*d**2*x + c*d*e*x**2)**(1/4)*a*c**3*d**
3*e**5*x**4 + 12*(a*d*e + a*e**2*x + c*d**2*x + c*d*e*x**2)**(1/4)*c**4*d*
*8*x + 440*(a*d*e + a*e**2*x + c*d**2*x + c*d*e*x**2)**(1/4)*c**4*d**7*e*x
**2 + 592*(a*d*e + a*e**2*x + c*d**2*x + c*d*e*x**2)**(1/4)*c**4*d**6*e**2
*x**3 + 224*(a*d*e + a*e**2*x + c*d**2*x + c*d*e*x**2)**(1/4)*c**4*d**5*e*
*3*x**4 + 15*int(((a*d*e + a*e**2*x + c*d**2*x + c*d*e*x**2)**(1/4)*x)/(a*
*2*d*e**3 + a**2*e**4*x + a*c*d**3*e + 2*a*c*d**2*e**2*x + a*c*d*e**3*x...
```

3.354 $\int \left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/4}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 279

$$\begin{aligned}\int \left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/4}\,dx=&-\frac{5\left(cd^2-ae^2\right)^2\left(cd^2+ae^2+2cdex\right)\sqrt[4]{ade+\left(cd^2+ae^2\right)x+cdex^2}}{84c^2d^2e^2}\\&+\frac{\left(cd^2+ae^2+2cdex\right)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/4}}{7cde}\\&+\frac{5\left(cd^2-ae^2\right)^5\left(-\frac{cde\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)}{\left(cd^2-ae^2\right)^2}\right)^{3/4}\text{EllipticF}\left(\frac{1}{2}\arcsin\left(\frac{ae^2+cd(d+2ex)}{cd^2-ae^2}\right),2\right)}{84\sqrt{2}c^3d^3e^3\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/4}}\end{aligned}$$

output
```
-5/84*(-a*e^2+c*d^2)^2*(2*c*d*e*x+a*e^2+c*d^2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*
e*x^2)^(1/4)/c^2/d^2/e^2+1/7*(2*c*d*e*x+a*e^2+c*d^2)*(a*d*e+(a*e^2+c*d^2)*
x+c*d*e*x^2)^(5/4)/c/d/e+5/168*(-a*e^2+c*d^2)^5*(-c*d*e*(a*d*e+(a*e^2+c*d^
2)*x+c*d*e*x^2)/(-a*e^2+c*d^2)^2)^(3/4)*InverseJacobiAM(1/2*arcsin((a*e^2+
c*d*(2*e*x+d))/(-a*e^2+c*d^2)),2^(1/2))*2^(1/2)/c^3/d^3/e^3/(a*d*e+(a*e^2+
c*d^2)*x+c*d*e*x^2)^(3/4)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.06 (sec) , antiderivative size = 96, normalized size of antiderivative = 0.34

$$\int (ade + (cd^2 + ae^2) x + cdex^2)^{5/4} dx = \frac{4(ae + cdx)((ae + cdx)(d + ex))^{5/4} \text{Hypergeometric2F1}\left(-\frac{5}{4}, \frac{9}{4}, \frac{13}{4}, \frac{e(ae+cdx)}{-cd^2+ae^2}\right)}{9cd\left(\frac{cd(d+ex)}{cd^2-ae^2}\right)^{5/4}}$$

input
```
Integrate[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/4),x]
```

output
```
(4*(a*e + c*d*x)*((a*e + c*d*x)*(d + e*x))^(5/4)*Hypergeometric2F1[-5/4, 9
/4, 13/4, (e*(a*e + c*d*x))/(-(c*d^2) + a*e^2)])/(9*c*d*((c*d*(d + e*x))/(
c*d^2 - a*e^2))^(5/4))
```

Rubi [A] (warning: unable to verify)

Time = 0.79 (sec) , antiderivative size = 541, normalized size of antiderivative = 1.94, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.138, Rules used = {1087, 1087, 1094, 761}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (x(ae^2 + cd^2) + ade + cdex^2)^{5/4} dx$$

$$\Big\downarrow \text{1087}$$

$$\frac{(ae^2 + cd^2 + 2cdex)(x(ae^2 + cd^2) + ade + cdex^2)^{5/4}}{7cde} - \frac{5(cd^2 - ae^2)^2 \int \sqrt[4]{cdex^2 + (cd^2 + ae^2) x + ade}dx}{28cde}$$

$$\Big\downarrow \text{1087}$$

$$\frac{\left(ae^2+cd^2+2cdex\right)\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/4}}{7cde}-\frac{5\left(cd^2-ae^2\right)^2\left(\frac{\left(ae^2+cd^2+2cdex\right)\sqrt[4]{x\left(ae^2+cd^2\right)+ade+cdex^2}}{3cde}-\frac{\left(cd^2-ae^2\right)^2\int\frac{1}{\left(cdex^2+\left(cd^2+ae^2\right)x+ade\right)^{3/4}}dx}{12cde}\right)}{28cde}$$

$$\Big\downarrow \; 1094$$

$$\frac{\left(ae^2+cd^2+2cdex\right)\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/4}}{7cde}-\frac{5\left(cd^2-ae^2\right)^2\left(\frac{\left(ae^2+cd^2+2cdex\right)\sqrt[4]{x\left(ae^2+cd^2\right)+ade+cdex^2}}{3cde}-\frac{\left(cd^2-ae^2\right)^2\sqrt{\left(ae^2+cd^2+2cdex\right)^2}\int\frac{1}{\sqrt{\left(cd^2-ae^2\right)^2+4cde\left(cdex\right.}}}{3cde(ae^2+}\right.}{28cde}$$

$$\Big\downarrow \; 761$$

$$\frac{\left(ae^2+cd^2+2cdex\right)\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/4}}{7cde}-\frac{5\left(cd^2-ae^2\right)^2\left(\frac{\left(ae^2+cd^2+2cdex\right)\sqrt[4]{x\left(ae^2+cd^2\right)+ade+cdex^2}}{3cde}-\frac{\left(cd^2-ae^2\right)^{5/2}\sqrt{\left(ae^2+cd^2+2cdex\right)^2}\left(\frac{2\sqrt{c}\sqrt{d}\sqrt{e}\sqrt{x\left(ae^2+cd^2\right)}}{cd^2-ae^2}\right.}{}\right.}{}$$

input
```
Int[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/4),x]
```

output
```
((c*d^2 + a*e^2 + 2*c*d*e*x)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/4)
)/(7*c*d*e) - (5*(c*d^2 - a*e^2)^2*(((c*d^2 + a*e^2 + 2*c*d*e*x)*(a*d*e +
(c*d^2 + a*e^2)*x + c*d*e*x^2)^(1/4))/(3*c*d*e) - ((c*d^2 - a*e^2)^(5/2)*S
qrt[(c*d^2 + a*e^2 + 2*c*d*e*x)^2]*(1 + (2*Sqrt[c]*Sqrt[d]*Sqrt[e]*Sqrt[a*
d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(c*d^2 - a*e^2))*Sqrt[((c*d^2 - a*e^
2)^2 + 4*c*d*e*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2))/((c*d^2 - a*e^2)^2
*(1 + (2*Sqrt[c]*Sqrt[d]*Sqrt[e]*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^
2])/(c*d^2 - a*e^2))^2)]*EllipticF[2*ArcTan[(Sqrt[2]*c^(1/4)*d^(1/4)*e^(1/
4)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(1/4))/Sqrt[c*d^2 - a*e^2]], 1/
2])/(6*Sqrt[2]*c^(5/4)*d^(5/4)*e^(5/4)*(c*d^2 + a*e^2 + 2*c*d*e*x)*Sqrt[(c
*d^2 - a*e^2)^2 + 4*c*d*e*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)])))/(28*
c*d*e)
```

Defintions of rubi rules used

rule 761
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 1087
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^p/(2*c*(2*p + 1))), x] - Simp[p*((b^2 - 4*a*c)/(2*c*(2*
p + 1)))   Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] &&
GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[3*p])
```

rule 1094
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[4*(Sqrt[(b
+ 2*c*x)^2]/(b + 2*c*x))   Subst[Int[x^(4*(p + 1) - 1)/Sqrt[b^2 - 4*a*c + 4
*c*x^4], x], x, (a + b*x + c*x^2)^(1/4)], x] /; FreeQ[{a, b, c}, x] && Inte
gerQ[4*p]
```

Maple [F]

$$\int \left(ade+\left(a\,e^{2}+c\,d^{2}\right)x+cd\,x^{2}e\right)^{\frac{5}{4}}dx$$

input
```
int((a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(5/4),x)
```

output
```
int((a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(5/4),x)
```

Fricas [F]

$$\int \left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/4}\,dx=\int \left(cdex^2+ade+\left(cd^2+ae^2\right)x\right)^{\frac{5}{4}}\,dx$$

input
```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/4),x, algorithm="fricas")
```

output
```
integral((c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^(5/4), x)
```

Sympy [F]

$$\int \left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/4}\,dx=\int \left(ade+cdex^2+x\left(ae^2+cd^2\right)\right)^{\frac{5}{4}}\,dx$$

input
```
integrate((a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(5/4),x)
```

output
```
Integral((a*d*e + c*d*e*x**2 + x*(a*e**2 + c*d**2))**(5/4), x)
```

Maxima [F]

$$\int \left(ade + \left(cd^2 + ae^2\right)x + cdex^2\right)^{5/4} dx = \int \left(cdex^2 + ade + \left(cd^2 + ae^2\right)x\right)^{\frac{5}{4}} dx$$

input
```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/4),x, algorithm="maxima")
```

output
```
integrate((c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^(5/4), x)
```

Giac [F]

$$\int \left(ade + \left(cd^2 + ae^2\right)x + cdex^2\right)^{5/4} dx = \int \left(cdex^2 + ade + \left(cd^2 + ae^2\right)x\right)^{\frac{5}{4}} dx$$

input
```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/4),x, algorithm="giac")
```

output
```
integrate((c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^(5/4), x)
```

Mupad [F(-1)]

Timed out.

$$\int \left(ade + \left(cd^2 + ae^2\right)x + cdex^2\right)^{5/4} dx = \int \left(c\,d\,e\,x^2 + \left(c\,d^2 + a\,e^2\right)\,x + a\,d\,e\right)^{5/4} dx$$

input
```
int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(5/4),x)
```

output
```
int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(5/4), x)
```

Reduce [F]

$$\int \left(ade + \left(cd^2 + ae^2\right)x + cdex^2\right)^{5/4} dx = \text{Too large to display}$$

input
```
int((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/4),x)
```

output
```
( - 16*(a*d*e + a*e**2*x + c*d**2*x + c*d*e*x**2)**(1/4)*a**3*d*e**5 + 4*(
a*d*e + a*e**2*x + c*d**2*x + c*d*e*x**2)**(1/4)*a**3*e**6*x + 128*(a*d*e
+ a*e**2*x + c*d**2*x + c*d*e*x**2)**(1/4)*a**2*c*d**3*e**3 + 140*(a*d*e +
 a*e**2*x + c*d**2*x + c*d*e*x**2)**(1/4)*a**2*c*d**2*e**4*x + 72*(a*d*e +
 a*e**2*x + c*d**2*x + c*d*e*x**2)**(1/4)*a**2*c*d*e**5*x**2 - 16*(a*d*e +
 a*e**2*x + c*d**2*x + c*d*e*x**2)**(1/4)*a*c**2*d**5*e + 140*(a*d*e + a*e
**2*x + c*d**2*x + c*d*e*x**2)**(1/4)*a*c**2*d**4*e**2*x + 144*(a*d*e + a*
e**2*x + c*d**2*x + c*d*e*x**2)**(1/4)*a*c**2*d**3*e**3*x**2 + 48*(a*d*e +
 a*e**2*x + c*d**2*x + c*d*e*x**2)**(1/4)*a*c**2*d**2*e**4*x**3 + 4*(a*d*e
 + a*e**2*x + c*d**2*x + c*d*e*x**2)**(1/4)*c**3*d**6*x + 72*(a*d*e + a*e*
*2*x + c*d**2*x + c*d*e*x**2)**(1/4)*c**3*d**5*e*x**2 + 48*(a*d*e + a*e**2
*x + c*d**2*x + c*d*e*x**2)**(1/4)*c**3*d**4*e**2*x**3 - 5*int(((a*d*e + a
*e**2*x + c*d**2*x + c*d*e*x**2)**(1/4)*x)/(a**2*d*e**3 + a**2*e**4*x + a*
c*d**3*e + 2*a*c*d**2*e**2*x + a*c*d*e**3*x**2 + c**2*d**4*x + c**2*d**3*e
*x**2),x)*a**5*e**10 + 15*int(((a*d*e + a*e**2*x + c*d**2*x + c*d*e*x**2)*
*(1/4)*x)/(a**2*d*e**3 + a**2*e**4*x + a*c*d**3*e + 2*a*c*d**2*e**2*x + a*
c*d*e**3*x**2 + c**2*d**4*x + c**2*d**3*e*x**2),x)*a**4*c*d**2*e**8 - 10*i
nt(((a*d*e + a*e**2*x + c*d**2*x + c*d*e*x**2)**(1/4)*x)/(a**2*d*e**3 + a*
*2*e**4*x + a*c*d**3*e + 2*a*c*d**2*e**2*x + a*c*d*e**3*x**2 + c**2*d**4*x
 + c**2*d**3*e*x**2),x)*a**3*c**2*d**4*e**6 - 10*int(((a*d*e + a*e**2*x...
```

3.355 $\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/4}}{d+ex}\,dx$

Optimal result

Integrand size = 37, antiderivative size = 297

$$\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/4}}{d+ex}\,dx = -\frac{\left(cd^2-ae^2\right)^2\sqrt[4]{ade+\left(cd^2+ae^2\right)x+cdex^2}}{6cde^2} + \frac{\left(\frac{d}{e}-\frac{ae}{cd}\right)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/4}}{15(d+ex)} + \frac{2\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{9/4}}{5cd(d+ex)^2} - \frac{\left(cd^2-ae^2\right)^{5/2}(ae+cdx)^{3/2}\left(\frac{cd(d+ex)}{e(ae+cdx)}\right)^{3/4}\text{EllipticF}\left(\frac{1}{2}\arctan\left(\frac{\sqrt{cd^2-ae^2}}{\sqrt{e}\sqrt{ae+cdx}}\right),2\right)}{6c^2d^2e^{3/2}\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/4}}$$

output
```
-1/6*(-a*e^2+c*d^2)^2*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/4)/c/d/e^2+(d/e
-a*e/c/d)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/4)/(15*e*x+15*d)+2/5*(a*d*e
+(a*e^2+c*d^2)*x+c*d*e*x^2)^(9/4)/c/d/(e*x+d)^2-1/6*(-a*e^2+c*d^2)^(5/2)*(
c*d*x+a*e)^(3/2)*(c*d*(e*x+d)/e/(c*d*x+a*e))^(3/4)*InverseJacobiAM(1/2*arc
tan((-a*e^2+c*d^2)^(1/2)/e^(1/2)/(c*d*x+a*e)^(1/2)),2^(1/2))/c^2/d^2/e^(3/
2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/4)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.04 (sec) , antiderivative size = 98, normalized size of antiderivative = 0.33

$$\int \frac{(ade+(cd^2+ae^2)\,x+cdex^2)^{5/4}}{d+ex}\,dx = \frac{4(ae+cdx)^2\sqrt[4]{(ae+cdx)(d+ex)}\,\text{Hypergeometric2F1}\left(-\frac{1}{4},\frac{9}{4},\frac{1}{4}\right.}{9cd\sqrt[4]{\frac{cd(d+ex)}{cd^2-ae^2}}}$$

input
```
Integrate[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/4)/(d + e*x),x]
```

output
```
(4*(a*e + c*d*x)^2*((a*e + c*d*x)*(d + e*x))^(1/4)*Hypergeometric2F1[-1/4,
 9/4, 13/4, (e*(a*e + c*d*x))/(-(c*d^2) + a*e^2)])/(9*c*d*((c*d*(d + e*x))
/(c*d^2 - a*e^2))^(1/4))
```

Rubi [A] (warning: unable to verify)

Time = 0.81 (sec) , antiderivative size = 341, normalized size of antiderivative = 1.15, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.243$, Rules used = {1138, 60, 60, 60, 73, 768, 858, 807, 229}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(x(ae^2+cd^2)+ade+cdex^2)^{5/4}}{d+ex}\,dx$$

$$\Big\downarrow \text{1138}$$

$$\frac{\sqrt[4]{x\,(ae^2+cd^2)+ade+cdex^2}\int (ae+cdx)^{5/4}\sqrt[4]{\frac{ex}{d}+1}dx}{d\sqrt[4]{\frac{ex}{d}+1}\sqrt[4]{ae+cdx}}$$

$$\Big\downarrow \text{60}$$

$$\frac{\sqrt[4]{x\left(ae^2+cd^2\right)+ade+cdex^2}\left(\frac{1}{10}\left(1-\frac{ae^2}{cd^2}\right)\int\frac{(ae+cdx)^{5/4}}{\left(\frac{ex}{d}+1\right)^{3/4}}dx+\frac{2\sqrt[4]{\frac{ex}{d}+1}(ae+cdx)^{9/4}}{5cd}\right)}{d\sqrt[4]{\frac{ex}{d}+1}\sqrt[4]{ae+cdx}}$$

$\downarrow$ 60

$$\frac{\sqrt[4]{x\left(ae^2+cd^2\right)+ade+cdex^2}\left(\frac{1}{10}\left(1-\frac{ae^2}{cd^2}\right)\left(\frac{2d\sqrt[4]{\frac{ex}{d}+1}(ae+cdx)^{5/4}}{3e}-\frac{5\left(cd^2-ae^2\right)\int\frac{\sqrt[4]{ae+cdx}}{\left(\frac{ex}{d}+1\right)^{3/4}}dx}{6e}\right)+\frac{2\sqrt[4]{\frac{ex}{d}+\ldots}}{\ldots}\right.}{d\sqrt[4]{\frac{ex}{d}+1}\sqrt[4]{ae+cdx}}$$

$\downarrow$ 60

$$\frac{\sqrt[4]{x\left(ae^2+cd^2\right)+ade+cdex^2}\left(\frac{1}{10}\left(1-\frac{ae^2}{cd^2}\right)\left(\frac{2d\sqrt[4]{\frac{ex}{d}+1}(ae+cdx)^{5/4}}{3e}-\frac{5\left(cd^2-ae^2\right)\left(\frac{2d\sqrt[4]{\frac{ex}{d}+1}\sqrt[4]{ae+cdx}}{e}-\frac{\left(cd^2\ldots\right.}{\ldots}\right.}{6e}\right.\right.}{d\sqrt[4]{\frac{ex}{d}+1}\sqrt[4]{ae+cdx}}$$

$\downarrow$ 73

$$\frac{\sqrt[4]{x\left(ae^2+cd^2\right)+ade+cdex^2}\left(\frac{1}{10}\left(1-\frac{ae^2}{cd^2}\right)\left(\frac{2d\sqrt[4]{\frac{ex}{d}+1}(ae+cdx)^{5/4}}{3e}-\frac{5\left(cd^2-ae^2\right)\left(\frac{2d\sqrt[4]{\frac{ex}{d}+1}\sqrt[4]{ae+cdx}}{e}-\frac{2\left(cd^2\ldots\right.}{\ldots}\right.}{\ldots}\right.\right.}{d\sqrt[4]{\frac{ex}{d}+1}\sqrt[4]{ae+cdx}}$$

$\downarrow$ 768

$$\frac{\sqrt[4]{x\left(ae^2+cd^2\right)+ade+cdex^2}\left(\frac{1}{10}\left(1-\frac{ae^2}{cd^2}\right)\left(\frac{2d\sqrt[4]{\frac{ex}{d}+1}(ae+cdx)^{5/4}}{3e}-\frac{5\left(cd^2-ae^2\right)\left(\frac{2d\sqrt[4]{\frac{ex}{d}+1}\sqrt[4]{ae+cdx}}{e}-\frac{2\left(cd^2\right.}{}\right.}{}\right.\right.}{d\sqrt[4]{\frac{ex}{d}+1}\sqrt[4]{ae+c}}$$

$\downarrow$ 858

$$\frac{\sqrt[4]{x\left(ae^2+cd^2\right)+ade+cdex^2}\left(\frac{1}{10}\left(1-\frac{ae^2}{cd^2}\right)\left(\frac{2d\sqrt[4]{\frac{ex}{d}+1}(ae+cdx)^{5/4}}{3e}-\frac{5\left(cd^2-ae^2\right)\left(\frac{2\left(cd^2-ae^2\right)(ae+cdx)^{3/4}\left(\frac{cd^2-ae^2}{e(ae+cdx)}+\right.}{}\right.}{}\right.\right.}{d\sqrt[4]{\frac{ex}{d}+1}\sqrt[4]{a}}$$

$\downarrow$ 807

$$\frac{\sqrt[4]{x\left(ae^2+cd^2\right)+ade+cdex^2}\left(\frac{1}{10}\left(1-\frac{ae^2}{cd^2}\right)\left(\frac{2d\sqrt[4]{\frac{ex}{d}+1}(ae+cdx)^{5/4}}{3e}-\frac{5\left(cd^2-ae^2\right)\left(\frac{\left(cd^2-ae^2\right)(ae+cdx)^{3/4}\left(\frac{cd^2-ae^2}{e(ae+cdx)}+1\right.}{cde\left(-\frac{ae^2}{cd^2}\right.}\right.}{}\right.\right.}{d\sqrt[4]{\frac{ex}{d}+1}\sqrt[4]{ae+cdx}}$$

$\downarrow$ 229

$$\frac{\sqrt[4]{x\left(ae^2+cd^2\right)+ade+cdex^2}\left(\frac{1}{10}\left(1-\frac{ae^2}{cd^2}\right)\left(\frac{2d\sqrt[4]{\frac{ex}{d}+1}(ae+cdx)^{5/4}}{3e}-\frac{5\left(cd^2-ae^2\right)\left(\frac{2\sqrt{cd^2-ae^2}(ae+cdx)^{3/4}\left(\frac{cd^2-ae^2}{e(ae+cdx)}+1\cdots\right.}{cd\sqrt{e}\left(-\frac{ae\cdots}{cd\cdots}\right.}\right.}{\cdots}\right.\right.}{d\sqrt[4]{\frac{ex}{d}+1}\sqrt[4]{ae+cdx\cdots}}$$

input

```
Int[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/4)/(d + e*x),x]
```

output

```
((a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(1/4)*((2*(a*e + c*d*x)^(9/4)*(1
+ (e*x)/d)^(1/4))/(5*c*d) + ((1 - (a*e^2)/(c*d^2))*((2*d*(a*e + c*d*x)^(5/
4)*(1 + (e*x)/d)^(1/4))/(3*e) - (5*(c*d^2 - a*e^2)*((2*d*(a*e + c*d*x)^(1/
4)*(1 + (e*x)/d)^(1/4))/e + (2*Sqrt[c*d^2 - a*e^2]*(a*e + c*d*x)^(3/4)*(1
+ (c*d^2 - a*e^2)/(e*(a*e + c*d*x)))^(3/4)*EllipticF[ArcTan[(Sqrt[c*d^2 -
a*e^2]*Sqrt[a*e + c*d*x])/Sqrt[e]]/2, 2])/(c*d*Sqrt[e]*(1 - (a*e^2)/(c*d^2
) + (e*(a*e + c*d*x))/(c*d^2))^(3/4))))/(6*e)))/10))/(d*(a*e + c*d*x)^(1/4
)*(1 + (e*x)/d)^(1/4))
```

Defintions of rubi rules used

rule 60

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 73

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 229
```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(2/(a^(3/4)*Rt[b/a, 2])
)*EllipticF[(1/2)*ArcTan[Rt[b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

rule 768
```
Int[((a_) + (b_.)*(x_)^4)^(-3/4), x_Symbol] :> Simp[x^3*((1 + a/(b*x^4))^(3
/4)/(a + b*x^4)^(3/4))   Int[1/(x^3*(1 + a/(b*x^4))^(3/4)), x], x] /; FreeQ
[{a, b}, x]
```

rule 807
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Simp[1/k   Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x,
x^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

rule 858
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

rule 1138
```
Int[((d_) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Sy
mbol] :> Simp[d^m*((a + b*x + c*x^2)^FracPart[p]/((1 + e*(x/d))^FracPart[p]
*(a/d + (c*x)/e)^FracPart[p]))   Int[(1 + e*(x/d))^(m + p)*(a/d + (c/e)*x)^
p, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0]
&& (IntegerQ[m] || GtQ[d, 0]) &&  !(IGtQ[m, 0] && (IntegerQ[3*p] || Integer
Q[4*p]))
```

Maple [F]

$$\int \frac{(ade + (a\,e^2 + c\,d^2)\,x + cd\,x^2e)^{\frac{5}{4}}}{ex + d}dx$$

input
```
int((a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(5/4)/(e*x+d),x)
```

output
```
int((a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(5/4)/(e*x+d),x)
```

Fricas [F]

$$\int \frac{(ade + (cd^2 + ae^2)x + cdex^2)^{5/4}}{d + ex}\,dx = \int \frac{(cdex^2 + ade + (cd^2 + ae^2)x)^{\frac{5}{4}}}{ex + d}\,dx$$

input
```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/4)/(e*x+d),x, algorithm="fr
icas")
```

output
```
integral((c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^(1/4)*(c*d*x + a*e), x)
```

Sympy [F]

$$\int \frac{(ade + (cd^2 + ae^2)x + cdex^2)^{5/4}}{d + ex}\,dx = \int \frac{((d + ex)(ae + cdx))^{\frac{5}{4}}}{d + ex}\,dx$$

input
```
integrate((a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(5/4)/(e*x+d),x)
```

output
```
Integral(((d + e*x)*(a*e + c*d*x))**(5/4)/(d + e*x), x)
```

Maxima [F]

$$\int \frac{(ade + (cd^2 + ae^2)x + cdex^2)^{5/4}}{d + ex}\,dx = \int \frac{(cdex^2 + ade + (cd^2 + ae^2)x)^{\frac{5}{4}}}{ex + d}\,dx$$

input
```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/4)/(e*x+d),x, algorithm="ma
xima")
```

output
```
integrate((c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^(5/4)/(e*x + d), x)
```

Giac [F]

$$\int \frac{(ade + (cd^2 + ae^2)x + cdex^2)^{5/4}}{d + ex} dx = \int \frac{(cdex^2 + ade + (cd^2 + ae^2)x)^{\frac{5}{4}}}{ex + d} dx$$

input
```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/4)/(e*x+d),x, algorithm="gi
ac")
```

output
```
integrate((c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^(5/4)/(e*x + d), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(ade + (cd^2 + ae^2)x + cdex^2)^{5/4}}{d + ex} dx = \int \frac{(c\,d\,e\,x^2 + (c\,d^2 + a\,e^2)\,x + a\,d\,e)^{5/4}}{d + e\,x} dx$$

input
```
int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(5/4)/(d + e*x),x)
```

output
```
int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(5/4)/(d + e*x), x)
```

Reduce [F]

$$\int \frac{(ade + (cd^2 + ae^2)x + cdex^2)^{5/4}}{d + ex} dx = \frac{64(cde\,x^2 + a\,e^2x + c\,d^2x + ade)^{\frac{1}{4}}\,a^2d\,e^3 + 44(cde\,x^2 + a\,e^2x + c}{}$$

input
```
int((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/4)/(e*x+d),x)
```

output

```
(64*(a*d*e + a*e**2*x + c*d**2*x + c*d*e*x**2)**(1/4)*a**2*d*e**3 + 44*(a*
d*e + a*e**2*x + c*d**2*x + c*d*e*x**2)**(1/4)*a**2*e**4*x - 16*(a*d*e + a
*e**2*x + c*d**2*x + c*d*e*x**2)**(1/4)*a*c*d**3*e + 48*(a*d*e + a*e**2*x
+ c*d**2*x + c*d*e*x**2)**(1/4)*a*c*d**2*e**2*x + 24*(a*d*e + a*e**2*x + c
*d**2*x + c*d*e*x**2)**(1/4)*a*c*d*e**3*x**2 + 4*(a*d*e + a*e**2*x + c*d**
2*x + c*d*e*x**2)**(1/4)*c**2*d**4*x + 24*(a*d*e + a*e**2*x + c*d**2*x + c
*d*e*x**2)**(1/4)*c**2*d**3*e*x**2 + 5*int(((a*d*e + a*e**2*x + c*d**2*x +
 c*d*e*x**2)**(1/4)*x)/(a**2*d*e**3 + a**2*e**4*x + a*c*d**3*e + 2*a*c*d**
2*e**2*x + a*c*d*e**3*x**2 + c**2*d**4*x + c**2*d**3*e*x**2),x)*a**4*e**8
- 10*int(((a*d*e + a*e**2*x + c*d**2*x + c*d*e*x**2)**(1/4)*x)/(a**2*d*e**
3 + a**2*e**4*x + a*c*d**3*e + 2*a*c*d**2*e**2*x + a*c*d*e**3*x**2 + c**2*
d**4*x + c**2*d**3*e*x**2),x)*a**3*c*d**2*e**6 + 10*int(((a*d*e + a*e**2*x
 + c*d**2*x + c*d*e*x**2)**(1/4)*x)/(a**2*d*e**3 + a**2*e**4*x + a*c*d**3*
e + 2*a*c*d**2*e**2*x + a*c*d*e**3*x**2 + c**2*d**4*x + c**2*d**3*e*x**2),
x)*a*c**3*d**6*e**2 - 5*int(((a*d*e + a*e**2*x + c*d**2*x + c*d*e*x**2)**(
1/4)*x)/(a**2*d*e**3 + a**2*e**4*x + a*c*d**3*e + 2*a*c*d**2*e**2*x + a*c*
d*e**3*x**2 + c**2*d**4*x + c**2*d**3*e*x**2),x)*c**4*d**8)/(60*e*(a*e**2
+ c*d**2))
```

3.356 $\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/4}}{(d+ex)^2}\,dx$

Optimal result

Integrand size = 37, antiderivative size = 226

$$\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/4}}{(d+ex)^2}\,dx=\frac{5}{3}\left(a-\frac{cd^2}{e^2}\right)\sqrt[4]{ade+\left(cd^2+ae^2\right)x+cdex^2}+\frac{2\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/4}}{3e(d+ex)}-\frac{5\left(cd^2-ae^2\right)^{3/2}(ae+cdx)^{3/2}\left(\frac{cd(d+ex)}{e(ae+cdx)}\right)^{3/4}\text{EllipticF}\left(\frac{1}{2}\arctan\left(\frac{\sqrt{cd^2-ae^2}}{\sqrt{e}\sqrt{ae+cdx}}\right),2\right)}{3cde^{3/2}\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/4}}$$

output
```
5/3*(a-c*d^2/e^2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/4)+2/3*(a*d*e+(a*e^
2+c*d^2)*x+c*d*e*x^2)^(5/4)/e/(e*x+d)-5/3*(-a*e^2+c*d^2)^(3/2)*(c*d*x+a*e)
^(3/2)*(c*d*(e*x+d)/e/(c*d*x+a*e))^(3/4)*InverseJacobiAM(1/2*arctan((-a*e^
2+c*d^2)^(1/2)/e^(1/2)/(c*d*x+a*e)^(1/2)),2^(1/2))/c/d/e^(3/2)/(a*d*e+(a*e
^2+c*d^2)*x+c*d*e*x^2)^(3/4)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.04 (sec) , antiderivative size = 98, normalized size of antiderivative = 0.43

$$\int \frac{(ade + (cd^2 + ae^2)\,x + cdex^2)^{5/4}}{(d+ex)^2}\,dx = \frac{4(ae+cdx)^3\left(\frac{cd(d+ex)}{cd^2-ae^2}\right)^{3/4}\text{Hypergeometric2F1}\left(\frac{3}{4},\frac{9}{4},\frac{13}{4},\frac{e(ae+cdx)}{-cd^2+ae^2}\right)}{9cd((ae+cdx)(d+ex))^{3/4}}$$

input
```
Integrate[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/4)/(d + e*x)^2,x]
```

output
```
(4*(a*e + c*d*x)^3*((c*d*(d + e*x))/(c*d^2 - a*e^2))^(3/4)*Hypergeometric2
F1[3/4, 9/4, 13/4, (e*(a*e + c*d*x))/(-(c*d^2) + a*e^2)])/(9*c*d*((a*e + c
*d*x)*(d + e*x))^(3/4))
```

Rubi [A] (warning: unable to verify)

Time = 0.69 (sec) , antiderivative size = 288, normalized size of antiderivative = 1.27, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.216$, Rules used = {1138, 60, 60, 73, 768, 858, 807, 229}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/4}}{(d+ex)^2}dx$$

$$\Big\downarrow \text{1138}$$

$$\frac{\sqrt[4]{x\left(ae^2+cd^2\right)+ade+cdex^2}\int \frac{(ae+cdx)^{5/4}}{\left(\frac{ex}{d}+1\right)^{3/4}}dx}{d^2\sqrt[4]{\frac{ex}{d}+1}\sqrt[4]{ae+cdx}}$$

$$\Big\downarrow \text{60}$$

$$\frac{\sqrt[4]{x\left(ae^2+cd^2\right)+ade+cdex^2}\left(\frac{2d\sqrt[4]{\frac{ex}{d}+1}(ae+cdx)^{5/4}}{3e}-\frac{5\left(cd^2-ae^2\right)\int\frac{\sqrt[4]{ae+cdx}}{\left(\frac{ex}{d}+1\right)^{3/4}}dx}{6e}\right)}{d^2\sqrt[4]{\frac{ex}{d}+1}\sqrt[4]{ae+cdx}}$$

$\downarrow$ 60

$$\frac{\sqrt[4]{x\left(ae^2+cd^2\right)+ade+cdex^2}\left(\frac{2d\sqrt[4]{\frac{ex}{d}+1}(ae+cdx)^{5/4}}{3e}-\frac{5\left(cd^2-ae^2\right)\left(\frac{2d\sqrt[4]{\frac{ex}{d}+1}\sqrt[4]{ae+cdx}}{e}-\frac{\left(cd^2-ae^2\right)\int\frac{1}{(ae+cdx)^{3/4}\cdots}}{2e}\cdots\right.}{6e}\cdots\right.}{d^2\sqrt[4]{\frac{ex}{d}+1}\sqrt[4]{ae+cdx}}$$

$\downarrow$ 73

$$\frac{\sqrt[4]{x\left(ae^2+cd^2\right)+ade+cdex^2}\left(\frac{2d\sqrt[4]{\frac{ex}{d}+1}(ae+cdx)^{5/4}}{3e}-\frac{5\left(cd^2-ae^2\right)\left(\frac{2d\sqrt[4]{\frac{ex}{d}+1}\sqrt[4]{ae+cdx}}{e}-\frac{2\left(cd^2-ae^2\right)\int\frac{\cdots}{\left(-\frac{ae^2}{cd^2}+\cdots\right.}}{\cdots}\cdots\right.}{6e}\cdots\right.}{d^2\sqrt[4]{\frac{ex}{d}+1}\sqrt[4]{ae+cdx}}$$

$\downarrow$ 768

$$\frac{\sqrt[4]{x\left(ae^2+cd^2\right)+ade+cdex^2}\left(\frac{2d\sqrt[4]{\frac{ex}{d}+1}(ae+cdx)^{5/4}}{3e}-\frac{5\left(cd^2-ae^2\right)\left(\frac{2d\sqrt[4]{\frac{ex}{d}+1}\sqrt[4]{ae+cdx}}{e}-\frac{2\left(cd^2-ae^2\right)(ae+cdx)^{3/4}\left(\cdots\right.}{\cdots}\cdots\right.}{6e\cdots}\cdots\right.}{d^2\sqrt[4]{\frac{ex}{d}+1}\sqrt[4]{ae+cdx}\cdots}$$

$\downarrow$ 858

$$\frac{\sqrt[4]{x\left(ae^2+cd^2\right)+ade+cdex^2}\left(\frac{2d\sqrt[4]{\frac{ex}{d}+1}(ae+cdx)^{5/4}}{3e}-\frac{5\left(cd^2-ae^2\right)\left(\frac{2\left(cd^2-ae^2\right)(ae+cdx)^{3/4}\left(\frac{cd^2-ae^2}{e(ae+cdx)}+1\right)^{3/4}\int\frac{\cdots}{\sqrt[4]{ae+c\cdots}}}{cde\left(-\frac{ae^2}{cd^2}+\frac{e(ae+c\cdots}{cd^2}\cdots\right.}\cdots\right.}{\cdots}\cdots\right.}{d^2\sqrt[4]{\frac{ex}{d}+1}\sqrt[4]{ae+cdx}\cdots}$$

$\downarrow$ 807

$$\frac{\sqrt[4]{x\left(ae^2+cd^2\right)+ade+cdex^2}\left(\frac{2d\sqrt[4]{\frac{ex}{d}+1}(ae+cdx)^{5/4}}{3e}-\frac{5\left(cd^2-ae^2\right)\left(\frac{\left(cd^2-ae^2\right)(ae+cdx)^{3/4}\left(\frac{cd^2-ae^2}{e(ae+cdx)}+1\right)^{3/4}\int\frac{\cdots}{\left(\frac{\sqrt{ae+cdx}\left(c\cdots\right.}{e}\cdots\right.}}{cde\left(-\frac{ae^2}{cd^2}+\frac{e(ae+cdx)}{cd^2}+1\right)^{3/4}\cdots}\cdots\right.}{6e\cdots}\cdots\right.}{d^2\sqrt[4]{\frac{ex}{d}+1}\sqrt[4]{ae+cdx}\cdots}$$

$\downarrow$ 229

$$\frac{\sqrt[4]{x\left(ae^2+cd^2\right)+ade+cdex^2}\left(\frac{2d\sqrt[4]{\frac{ex}{d}+1}(ae+cdx)^{5/4}}{3e}-\frac{5\left(cd^2-ae^2\right)\left(\frac{2\sqrt{cd^2-ae^2}(ae+cdx)^{3/4}\left(\frac{cd^2-ae^2}{e(ae+cdx)}+1\right)^{3/4}\text{EllipticF}\left(\frac{1}{2}\text{arc}\right.}{cd\sqrt{e}\left(-\frac{ae^2}{cd^2}+\frac{e(ae+cdx)}{cd^2}+1\right)^{3/}}\right.}{6e}\right.}{d^2\sqrt[4]{\frac{ex}{d}+1}\sqrt[4]{ae+cdx}}$$

input
```
Int[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/4)/(d + e*x)^2,x]
```

output
```
((a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(1/4)*((2*d*(a*e + c*d*x)^(5/4)*(
1 + (e*x)/d)^(1/4))/(3*e) - (5*(c*d^2 - a*e^2)*((2*d*(a*e + c*d*x)^(1/4)*(
1 + (e*x)/d)^(1/4))/e + (2*Sqrt[c*d^2 - a*e^2]*(a*e + c*d*x)^(3/4)*(1 + (c
*d^2 - a*e^2)/(e*(a*e + c*d*x)))^(3/4)*EllipticF[ArcTan[(Sqrt[c*d^2 - a*e^
2]*Sqrt[a*e + c*d*x])/Sqrt[e]]/2, 2])/(c*d*Sqrt[e]*(1 - (a*e^2)/(c*d^2) +
(e*(a*e + c*d*x))/(c*d^2))^(3/4))))/(6*e)))/(d^2*(a*e + c*d*x)^(1/4)*(1 +
(e*x)/d)^(1/4))
```

Defintions of rubi rules used

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 229
```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(2/(a^(3/4)*Rt[b/a, 2])
)*EllipticF[(1/2)*ArcTan[Rt[b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

rule 768
```
Int[((a_) + (b_.)*(x_)^4)^(-3/4), x_Symbol] :> Simp[x^3*((1 + a/(b*x^4))^(3
/4)/(a + b*x^4)^(3/4))   Int[1/(x^3*(1 + a/(b*x^4))^(3/4)), x], x] /; FreeQ
[{a, b}, x]
```

rule 807
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Simp[1/k   Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x,
x^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

rule 858
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

rule 1138
```
Int[((d_) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Sy
mbol] :> Simp[d^m*((a + b*x + c*x^2)^FracPart[p]/((1 + e*(x/d))^FracPart[p]
*(a/d + (c*x)/e)^FracPart[p]))   Int[(1 + e*(x/d))^(m + p)*(a/d + (c/e)*x)^
p, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0]
&& (IntegerQ[m] || GtQ[d, 0]) &&  !(IGtQ[m, 0] && (IntegerQ[3*p] || Integer
Q[4*p]))
```

Maple [F]

$$\int \frac{(ade + (a\,e^2 + c\,d^2)\,x + cd\,x^2 e)^{\frac{5}{4}}}{(ex + d)^2}dx$$

input
```
int((a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(5/4)/(e*x+d)^2,x)
```

output
```
int((a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(5/4)/(e*x+d)^2,x)
```

Fricas [F]

$$\int \frac{(ade + (cd^2 + ae^2)\,x + cdex^2)^{5/4}}{(d + ex)^2}\,dx = \int \frac{(cdex^2 + ade + (cd^2 + ae^2)x)^{\frac{5}{4}}}{(ex + d)^2}\,dx$$

input
```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/4)/(e*x+d)^2,x, algorithm="
fricas")
```

output
```
integral((c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^(1/4)*(c*d*x + a*e)/(e*x
+ d), x)
```

Sympy [F]

$$\int \frac{(ade + (cd^2 + ae^2)\,x + cdex^2)^{5/4}}{(d + ex)^2}\,dx = \int \frac{((d + ex)\,(ae + cdx))^{\frac{5}{4}}}{(d + ex)^2}\,dx$$

input
```
integrate((a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(5/4)/(e*x+d)**2,x)
```

output
```
Integral(((d + e*x)*(a*e + c*d*x))**(5/4)/(d + e*x)**2, x)
```

Maxima [F]

$$\int \frac{(ade + (cd^2 + ae^2)\,x + cdex^2)^{5/4}}{(d + ex)^2}\,dx = \int \frac{(cdex^2 + ade + (cd^2 + ae^2)x)^{\frac{5}{4}}}{(ex + d)^2}\,dx$$

input
```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/4)/(e*x+d)^2,x, algorithm="
maxima")
```

output
```
integrate((c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^(5/4)/(e*x + d)^2, x)
```

Giac [F]

$$\int \frac{(ade + (cd^2 + ae^2)x + cdex^2)^{5/4}}{(d + ex)^2}\,dx = \int \frac{(cdex^2 + ade + (cd^2 + ae^2)x)^{\frac{5}{4}}}{(ex + d)^2}\,dx$$

input
```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/4)/(e*x+d)^2,x, algorithm="
giac")
```

output
```
integrate((c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^(5/4)/(e*x + d)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(ade + (cd^2 + ae^2)x + cdex^2)^{5/4}}{(d + ex)^2}\,dx = \int \frac{(c\,d\,e\,x^2 + (c\,d^2 + a\,e^2)\,x + a\,d\,e)^{5/4}}{(d + e\,x)^2}\,dx$$

input
```
int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(5/4)/(d + e*x)^2,x)
```

output
```
int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(5/4)/(d + e*x)^2, x)
```

Reduce [F]

$$\int \frac{(ade + (cd^2 + ae^2)x + cdex^2)^{5/4}}{(d + ex)^2}\,dx = \frac{24(cde\,x^2 + a\,e^2x + c\,d^2x + ade)^{\frac{1}{4}}\,a^2e^3 - 16(cde\,x^2 + a\,e^2x + c\,d}{}$$

input
```
int((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/4)/(e*x+d)^2,x)
```

output
```
(24*(a*d*e + a*e**2*x + c*d**2*x + c*d*e*x**2)**(1/4)*a**2*e**3 - 16*(a*d*
e + a*e**2*x + c*d**2*x + c*d*e*x**2)**(1/4)*a*c*d**2*e + 4*(a*d*e + a*e**
2*x + c*d**2*x + c*d*e*x**2)**(1/4)*a*c*d*e**2*x + 4*(a*d*e + a*e**2*x + c
*d**2*x + c*d*e*x**2)**(1/4)*c**2*d**3*x - 5*int(((a*d*e + a*e**2*x + c*d*
*2*x + c*d*e*x**2)**(1/4)*x)/(a**2*d*e**3 + a**2*e**4*x + a*c*d**3*e + 2*a
*c*d**2*e**2*x + a*c*d*e**3*x**2 + c**2*d**4*x + c**2*d**3*e*x**2),x)*a**3
*c*d*e**6 + 5*int(((a*d*e + a*e**2*x + c*d**2*x + c*d*e*x**2)**(1/4)*x)/(a
**2*d*e**3 + a**2*e**4*x + a*c*d**3*e + 2*a*c*d**2*e**2*x + a*c*d*e**3*x**
2 + c**2*d**4*x + c**2*d**3*e*x**2),x)*a**2*c**2*d**3*e**4 + 5*int(((a*d*e
 + a*e**2*x + c*d**2*x + c*d*e*x**2)**(1/4)*x)/(a**2*d*e**3 + a**2*e**4*x
+ a*c*d**3*e + 2*a*c*d**2*e**2*x + a*c*d*e**3*x**2 + c**2*d**4*x + c**2*d*
*3*e*x**2),x)*a*c**3*d**5*e**2 - 5*int(((a*d*e + a*e**2*x + c*d**2*x + c*d
*e*x**2)**(1/4)*x)/(a**2*d*e**3 + a**2*e**4*x + a*c*d**3*e + 2*a*c*d**2*e*
*2*x + a*c*d*e**3*x**2 + c**2*d**4*x + c**2*d**3*e*x**2),x)*c**4*d**7)/(6*
e*(a*e**2 + c*d**2))
```

3.357 $\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/4}}{(d+ex)^3}\,dx$

Optimal result

Integrand size = 37, antiderivative size = 214

$$\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/4}}{(d+ex)^3}\,dx=\frac{10cd\sqrt[4]{ade+\left(cd^2+ae^2\right)x+cdex^2}}{3e^2}-\frac{4\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/4}}{3e(d+ex)^2}+\frac{10\sqrt{cd^2-ae^2}(ae+cdx)^{3/2}\left(\frac{cd(d+ex)}{e(ae+cdx)}\right)^{3/4}\operatorname{EllipticF}\left(\frac{1}{2}\arctan\left(\frac{\sqrt{cd^2-ae^2}}{\sqrt{e}\sqrt{ae+cdx}}\right),2\right)}{3e^{3/2}\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/4}}$$

output
```
10/3*c*d*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/4)/e^2-4/3*(a*d*e+(a*e^2+c*d
^2)*x+c*d*e*x^2)^(5/4)/e/(e*x+d)^2+10/3*(-a*e^2+c*d^2)^(1/2)*(c*d*x+a*e)^(
3/2)*(c*d*(e*x+d)/e/(c*d*x+a*e))^(3/4)*InverseJacobiAM(1/2*arctan((-a*e^2+
c*d^2)^(1/2)/e^(1/2)/(c*d*x+a*e)^(1/2)),2^(1/2))/e^(3/2)/(a*d*e+(a*e^2+c*d
^2)*x+c*d*e*x^2)^(3/4)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.05 (sec) , antiderivative size = 108, normalized size of antiderivative = 0.50

$$\int \frac{(ade + (cd^2 + ae^2)\,x + cdex^2)^{5/4}}{(d+ex)^3}\,dx = \frac{4cd(ae+cdx)^2\sqrt[4]{(ae+cdx)(d+ex)}\,\text{Hypergeometric2F1}\left(\frac{7}{4},\frac{9}{4},\right.}{9\left(cd^2-ae^2\right)^2\sqrt[4]{\frac{cd(d+ex)}{cd^2-ae^2}}}$$

input
```
Integrate[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/4)/(d + e*x)^3,x]
```

output
```
(4*c*d*(a*e + c*d*x)^2*((a*e + c*d*x)*(d + e*x))^(1/4)*Hypergeometric2F1[7
/4, 9/4, 13/4, (e*(a*e + c*d*x))/(-(c*d^2) + a*e^2)])/(9*(c*d^2 - a*e^2)^2
*((c*d*(d + e*x))/(c*d^2 - a*e^2))^(1/4))
```

Rubi [A] (warning: unable to verify)

Time = 0.69 (sec) , antiderivative size = 280, normalized size of antiderivative = 1.31, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.216$, Rules used = {1138, 57, 60, 73, 768, 858, 807, 229}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/4}}{(d+ex)^3}dx$$

$$\Big\downarrow \text{1138}$$

$$\frac{\sqrt[4]{x\left(ae^2+cd^2\right)+ade+cdex^2}\int \frac{(ae+cdx)^{5/4}}{\left(\frac{ex}{d}+1\right)^{7/4}}dx}{d^3\sqrt[4]{\frac{ex}{d}+1}\sqrt[4]{ae+cdx}}$$

$$\Big\downarrow \text{57}$$

$$\frac{\sqrt[4]{x\left(ae^2+cd^2\right)+ade+cdex^2}\left(\frac{5cd^2\int \frac{\sqrt[4]{ae+cdx}}{\left(\frac{ex}{d}+1\right)^{3/4}}dx}{3e}-\frac{4d(ae+cdx)^{5/4}}{3e\left(\frac{ex}{d}+1\right)^{3/4}}\right)}{d^3\sqrt[4]{\frac{ex}{d}+1}\sqrt[4]{ae+cdx}}$$

$\downarrow$ 60

$$\frac{\sqrt[4]{x\left(ae^2+cd^2\right)+ade+cdex^2}\left(\frac{5cd^2\left(\frac{2d\sqrt[4]{\frac{ex}{d}+1}\sqrt[4]{ae+cdx}}{e}-\frac{\left(cd^2-ae^2\right)\int \frac{1}{(ae+cdx)^{3/4}\left(\frac{ex}{d}+1\right)^{3/4}}dx}{2e}\right)}{3e}-\frac{4d(ae+cdx)^{5/4}}{3e\left(\frac{ex}{d}+1\right)^{3/4}}\right)}{d^3\sqrt[4]{\frac{ex}{d}+1}\sqrt[4]{ae+cdx}}$$

$\downarrow$ 73

$$\frac{\sqrt[4]{x\left(ae^2+cd^2\right)+ade+cdex^2}\left(\frac{5cd^2\left(\frac{2d\sqrt[4]{\frac{ex}{d}+1}\sqrt[4]{ae+cdx}}{e}-\frac{2\left(cd^2-ae^2\right)\int \frac{1}{\left(-\frac{ae^2}{cd^2}+\frac{(ae+cdx)e}{cd^2}+1\right)^{3/4}}d\sqrt[4]{ae+cdx}}{cde}\right)}{3e}-\cdots\right)}{d^3\sqrt[4]{\frac{ex}{d}+1}\sqrt[4]{ae+cdx}}$$

$\downarrow$ 768

$$\frac{\sqrt[4]{x\left(ae^2+cd^2\right)+ade+cdex^2}\left(\frac{5cd^2\left(\frac{2d\sqrt[4]{\frac{ex}{d}+1}\sqrt[4]{ae+cdx}}{e}-\frac{2\left(cd^2-ae^2\right)(ae+cdx)^{3/4}\left(\frac{cd^2-ae^2}{e(ae+cdx)}+1\right)^{3/4}\int\frac{1}{(ae+cdx)^{3/4}\left(\frac{cd^2-a}{e(ae+c}\right.}}{cde\left(-\frac{ae^2}{cd^2}+\frac{e(ae+cdx)}{cd^2}+1\right)^{3/4}}\right)}{3e}\right.}{d^3\sqrt[4]{\frac{ex}{d}+1}\sqrt[4]{ae+cdx}}$$

$\downarrow$ 858

$$\frac{\sqrt[4]{x\left(ae^2+cd^2\right)+ade+cdex^2}\left(\frac{5cd^2\left(\frac{2\left(cd^2-ae^2\right)(ae+cdx)^{3/4}\left(\frac{cd^2-ae^2}{e(ae+cdx)}+1\right)^{3/4}\int\frac{1}{\sqrt[4]{ae+cdx}\left(\frac{\left(cd^2-ae^2\right)(ae+cdx)}{e}+1\right)^{3/4}}d\sqrt[4]{ae}}{cde\left(-\frac{ae^2}{cd^2}+\frac{e(ae+cdx)}{cd^2}+1\right)^{3/4}}\right.}{3e}\right.}{d^3\sqrt[4]{\frac{ex}{d}+1}\sqrt[4]{ae+cdx}}$$

$\downarrow$ 807

$$\frac{\sqrt[4]{x\left(ae^2+cd^2\right)+ade+cdex^2}\left(\frac{5cd^2\left(\frac{\left(cd^2-ae^2\right)(ae+cdx)^{3/4}\left(\frac{cd^2-ae^2}{e(ae+cdx)}+1\right)^{3/4}\int\frac{1}{\left(\frac{\sqrt{ae+cdx}\left(cd^2-ae^2\right)}{e}+1\right)^{3/4}}d\sqrt{ae+cdx}}{cde\left(-\frac{ae^2}{cd^2}+\frac{e(ae+cdx)}{cd^2}+1\right)^{3/4}}+\frac{2d\sqrt[4]{\frac{ex}{d}}}{}\right.}{3e}\right.}{d^3\sqrt[4]{\frac{ex}{d}+1}\sqrt[4]{ae+cdx}}$$

$\downarrow$ 229

$$\frac{\sqrt[4]{x\left(ae^2+cd^2\right)+ade+cdex^2}\left(\frac{5cd^2\left(\frac{2\sqrt{cd^2-ae^2}(ae+cdx)^{3/4}\left(\frac{cd^2-ae^2}{e(ae+cdx)}+1\right)^{3/4}\text{EllipticF}\left(\frac{1}{2}\arctan\left(\frac{\sqrt{cd^2-ae^2}\sqrt{ae+cdx}}{\sqrt{e}}\right),2\right)}{cd\sqrt{e}\left(-\frac{ae^2}{cd^2}+\frac{e(ae+cdx)}{cd^2}+1\right)^{3/4}}+\frac{2d\sqrt[4]{\frac{ex}{d}}}{}\right)}{3e}\right)}{d^3\sqrt[4]{\frac{ex}{d}+1}\sqrt[4]{ae+cdx}}$$

input
```
Int[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/4)/(d + e*x)^3,x]
```

output
```
((a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(1/4)*((-4*d*(a*e + c*d*x)^(5/4))
/(3*e*(1 + (e*x)/d)^(3/4)) + (5*c*d^2*((2*d*(a*e + c*d*x)^(1/4)*(1 + (e*x)
/d)^(1/4))/e + (2*Sqrt[c*d^2 - a*e^2]*(a*e + c*d*x)^(3/4)*(1 + (c*d^2 - a*
e^2)/(e*(a*e + c*d*x)))^(3/4)*EllipticF[ArcTan[(Sqrt[c*d^2 - a*e^2]*Sqrt[a
*e + c*d*x])/Sqrt[e]]/2, 2])/(c*d*Sqrt[e]*(1 - (a*e^2)/(c*d^2) + (e*(a*e +
 c*d*x))/(c*d^2))^(3/4))))/(3*e)))/(d^3*(a*e + c*d*x)^(1/4)*(1 + (e*x)/d)^
(1/4))
```

Defintions of rubi rules used

rule 57
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Simp[d*(n/(b*(m + 1)))
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &
& GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !IntegerQ[m]) &&  !(ILeQ[m
+ n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) && IntLinearQ[a, b, c
, d, m, n, x]
```

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 229
```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(2/(a^(3/4)*Rt[b/a, 2])
)*EllipticF[(1/2)*ArcTan[Rt[b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

rule 768
```
Int[((a_) + (b_.)*(x_)^4)^(-3/4), x_Symbol] :> Simp[x^3*((1 + a/(b*x^4))^(3
/4)/(a + b*x^4)^(3/4))   Int[1/(x^3*(1 + a/(b*x^4))^(3/4)), x], x] /; FreeQ
[{a, b}, x]
```

rule 807
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Simp[1/k   Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x,
x^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

rule 858
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

rule 1138
```
Int[((d_) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Sy
mbol] :> Simp[d^m*((a + b*x + c*x^2)^FracPart[p]/((1 + e*(x/d))^FracPart[p]
*(a/d + (c*x)/e)^FracPart[p]))   Int[(1 + e*(x/d))^(m + p)*(a/d + (c/e)*x)^
p, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0]
&& (IntegerQ[m] || GtQ[d, 0]) &&  !(IGtQ[m, 0] && (IntegerQ[3*p] || Integer
Q[4*p]))
```

Maple [F]

$$\int \frac{\left(ade+\left(a\,e^{2}+c\,d^{2}\right)x+cd\,x^{2}e\right)^{\frac{5}{4}}}{\left(ex+d\right)^{3}}dx$$

input
```
int((a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(5/4)/(e*x+d)^3,x)
```

output
```
int((a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(5/4)/(e*x+d)^3,x)
```

Fricas [F]

$$\int \frac{(ade+(cd^2+ae^2)\,x+cdex^2)^{5/4}}{(d+ex)^3}\,dx=\int \frac{(cdex^2+ade+(cd^2+ae^2)x)^{\frac{5}{4}}}{(ex+d)^3}\,dx$$

input
```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/4)/(e*x+d)^3,x, algorithm="
fricas")
```

output
```
integral((c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^(1/4)*(c*d*x + a*e)/(e^2*
x^2 + 2*d*e*x + d^2), x)
```

Sympy [F]

$$\int \frac{(ade+(cd^2+ae^2)\,x+cdex^2)^{5/4}}{(d+ex)^3}\,dx=\int \frac{((d+ex)\,(ae+cdx))^{\frac{5}{4}}}{(d+ex)^3}\,dx$$

input
```
integrate((a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(5/4)/(e*x+d)**3,x)
```

output
```
Integral(((d + e*x)*(a*e + c*d*x))**(5/4)/(d + e*x)**3, x)
```

Maxima [F]

$$\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/4}}{(d+ex)^3}\,dx=\int \frac{\left(cdex^2+ade+\left(cd^2+ae^2\right)x\right)^{\frac{5}{4}}}{(ex+d)^3}\,dx$$

input
```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/4)/(e*x+d)^3,x, algorithm="
maxima")
```

output
```
integrate((c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^(5/4)/(e*x + d)^3, x)
```

Giac [F]

$$\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/4}}{(d+ex)^3}\,dx=\int \frac{\left(cdex^2+ade+\left(cd^2+ae^2\right)x\right)^{\frac{5}{4}}}{(ex+d)^3}\,dx$$

input
```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/4)/(e*x+d)^3,x, algorithm="
giac")
```

output
```
integrate((c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^(5/4)/(e*x + d)^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/4}}{(d+ex)^3}\,dx=\int \frac{\left(c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e\right)^{5/4}}{(d+e\,x)^3}\,dx$$

input
```
int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(5/4)/(d + e*x)^3,x)
```

output
```
int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(5/4)/(d + e*x)^3, x)
```

Reduce [F]

$$\int \frac{(ade + (cd^2 + ae^2)\,x + cdex^2)^{5/4}}{(d + ex)^3}\,dx = \text{Too large to display}$$

input
```
int((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/4)/(e*x+d)^3,x)
```

output
```
( - 8*(a*d*e + a*e**2*x + c*d**2*x + c*d*e*x**2)**(1/4)*a**2*e**3 + 16*(a*
d*e + a*e**2*x + c*d**2*x + c*d*e*x**2)**(1/4)*a*c*d**2*e + 12*(a*d*e + a*
e**2*x + c*d**2*x + c*d*e*x**2)**(1/4)*a*c*d*e**2*x - 4*(a*d*e + a*e**2*x
+ c*d**2*x + c*d*e*x**2)**(1/4)*c**2*d**3*x + 15*int(((a*d*e + a*e**2*x +
c*d**2*x + c*d*e*x**2)**(1/4)*x)/(3*a**2*d**2*e**3 + 6*a**2*d*e**4*x + 3*a
**2*e**5*x**2 - a*c*d**4*e + a*c*d**3*e**2*x + 5*a*c*d**2*e**3*x**2 + 3*a*
c*d*e**4*x**3 - c**2*d**5*x - 2*c**2*d**4*e*x**2 - c**2*d**3*e**2*x**3),x)
*a**3*c*d**2*e**6 + 15*int(((a*d*e + a*e**2*x + c*d**2*x + c*d*e*x**2)**(1
/4)*x)/(3*a**2*d**2*e**3 + 6*a**2*d*e**4*x + 3*a**2*e**5*x**2 - a*c*d**4*e
 + a*c*d**3*e**2*x + 5*a*c*d**2*e**3*x**2 + 3*a*c*d*e**4*x**3 - c**2*d**5*
x - 2*c**2*d**4*e*x**2 - c**2*d**3*e**2*x**3),x)*a**3*c*d*e**7*x - 35*int(
((a*d*e + a*e**2*x + c*d**2*x + c*d*e*x**2)**(1/4)*x)/(3*a**2*d**2*e**3 +
6*a**2*d*e**4*x + 3*a**2*e**5*x**2 - a*c*d**4*e + a*c*d**3*e**2*x + 5*a*c*
d**2*e**3*x**2 + 3*a*c*d*e**4*x**3 - c**2*d**5*x - 2*c**2*d**4*e*x**2 - c*
*2*d**3*e**2*x**3),x)*a**2*c**2*d**4*e**4 - 35*int(((a*d*e + a*e**2*x + c*
d**2*x + c*d*e*x**2)**(1/4)*x)/(3*a**2*d**2*e**3 + 6*a**2*d*e**4*x + 3*a**
2*e**5*x**2 - a*c*d**4*e + a*c*d**3*e**2*x + 5*a*c*d**2*e**3*x**2 + 3*a*c*
d*e**4*x**3 - c**2*d**5*x - 2*c**2*d**4*e*x**2 - c**2*d**3*e**2*x**3),x)*a
**2*c**2*d**3*e**5*x + 25*int(((a*d*e + a*e**2*x + c*d**2*x + c*d*e*x**2)*
*(1/4)*x)/(3*a**2*d**2*e**3 + 6*a**2*d*e**4*x + 3*a**2*e**5*x**2 - a*c*...
```

3.358 $\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/4}}{(d+ex)^4}\,dx$

Optimal result

Integrand size = 37, antiderivative size = 223

$$\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/4}}{(d+ex)^4}\,dx = -\frac{20cd\sqrt[4]{ade+\left(cd^2+ae^2\right)x+cdex^2}}{21e^2(d+ex)}-\frac{4\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/4}}{7e(d+ex)^3}-\frac{20cd(ae+cdx)^{3/2}\left(\frac{cd(d+ex)}{e(ae+cdx)}\right)^{3/4}\operatorname{EllipticF}\left(\frac{1}{2}\arctan\left(\frac{\sqrt{cd^2-ae^2}}{\sqrt{e}\sqrt{ae+cdx}}\right),2\right)}{21e^{3/2}\sqrt{cd^2-ae^2}\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/4}}$$

output
```
-20/21*c*d*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/4)/e^2/(e*x+d)-4/7*(a*d*e+
(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/4)/e/(e*x+d)^3-20/21*c*d*(c*d*x+a*e)^(3/2)*(
c*d*(e*x+d)/e/(c*d*x+a*e))^(3/4)*InverseJacobiAM(1/2*arctan((-a*e^2+c*d^2)
^(1/2)/e^(1/2)/(c*d*x+a*e)^(1/2)),2^(1/2))/e^(3/2)/(-a*e^2+c*d^2)^(1/2)/(a
*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/4)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.06 (sec) , antiderivative size = 112, normalized size of antiderivative = 0.50

$$\int \frac{(ade+(cd^2+ae^2)x+cdex^2)^{5/4}}{(d+ex)^4}\,dx = \frac{4c^2d^2(ae+cdx)^2\sqrt[4]{(ae+cdx)(d+ex)}\,\text{Hypergeometric2F1}\left(\frac{9}{4},\frac{1}{4}\right.}{9\left(cd^2-ae^2\right)^3\sqrt[4]{\frac{cd(d+ex)}{cd^2-ae^2}}}$$

input
```
Integrate[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/4)/(d + e*x)^4,x]
```

output
```
(4*c^2*d^2*(a*e + c*d*x)^2*((a*e + c*d*x)*(d + e*x))^(1/4)*Hypergeometric2
F1[9/4, 11/4, 13/4, (e*(a*e + c*d*x))/(-(c*d^2) + a*e^2)])/(9*(c*d^2 - a*e
^2)^3*((c*d*(d + e*x))/(c*d^2 - a*e^2))^(1/4))
```

Rubi [A] (warning: unable to verify)

Time = 0.74 (sec) , antiderivative size = 279, normalized size of antiderivative = 1.25, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.216$, Rules used = {1138, 57, 57, 73, 768, 858, 807, 229}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/4}}{(d+ex)^4}dx$$

$$\Big\downarrow \; 1138$$

$$\frac{\sqrt[4]{x\left(ae^2+cd^2\right)+ade+cdex^2}\int \frac{(ae+cdx)^{5/4}}{\left(\frac{ex}{d}+1\right)^{11/4}}dx}{d^4\sqrt[4]{\frac{ex}{d}+1}\sqrt[4]{ae+cdx}}$$

$$\Big\downarrow \; 57$$

$$\frac{\sqrt[4]{x\left(ae^2+cd^2\right)+ade+cdex^2}\left(\frac{5cd^2\int \frac{\sqrt[4]{ae+cdx}}{\left(\frac{ex}{d}+1\right)^{7/4}}dx}{7e}-\frac{4d(ae+cdx)^{5/4}}{7e\left(\frac{ex}{d}+1\right)^{7/4}}\right)}{d^4\sqrt[4]{\frac{ex}{d}+1}\sqrt[4]{ae+cdx}}$$

$\big\downarrow$ 57

$$\frac{\sqrt[4]{x\left(ae^2+cd^2\right)+ade+cdex^2}\left(\frac{5cd^2\left(\frac{cd^2\int \frac{1}{(ae+cdx)^{3/4}\left(\frac{ex}{d}+1\right)^{3/4}}dx}{3e}-\frac{4d\sqrt[4]{ae+cdx}}{3e\left(\frac{ex}{d}+1\right)^{3/4}}\right)}{7e}-\frac{4d(ae+cdx)^{5/4}}{7e\left(\frac{ex}{d}+1\right)^{7/4}}\right)}{d^4\sqrt[4]{\frac{ex}{d}+1}\sqrt[4]{ae+cdx}}$$

$\big\downarrow$ 73

$$\frac{\sqrt[4]{x\left(ae^2+cd^2\right)+ade+cdex^2}\left(\frac{5cd^2\left(\frac{4d\int \frac{1}{\left(-\frac{ae^2}{cd^2}+\frac{(ae+cdx)e}{cd^2}+1\right)^{3/4}}d\sqrt[4]{ae+cdx}}{3e}-\frac{4d\sqrt[4]{ae+cdx}}{3e\left(\frac{ex}{d}+1\right)^{3/4}}\right)}{7e}-\frac{4d(ae+cdx)^{5/4}}{7e\left(\frac{ex}{d}+1\right)^{7/4}}\right)}{d^4\sqrt[4]{\frac{ex}{d}+1}\sqrt[4]{ae+cdx}}$$

$\big\downarrow$ 768

$$\frac{\sqrt[4]{x\left(ae^2+cd^2\right)+ade+cdex^2}\left(\frac{5cd^2\left(\frac{4d(ae+cdx)^{3/4}\left(\frac{cd^2-ae^2}{e(ae+cdx)}+1\right)^{3/4}\int \frac{1}{(ae+cdx)^{3/4}\left(\frac{cd^2-ae^2}{e(ae+cdx)}+1\right)^{3/4}}d\sqrt[4]{ae+cdx}}{3e\left(-\frac{ae^2}{cd^2}+\frac{e(ae+cdx)}{cd^2}+1\right)^{3/4}}-\frac{4d\sqrt[4]{ae+\cdots}}{3e\left(\frac{ex}{d}+1\right)\cdots}\right)}{7e}\cdots\right)}{d^4\sqrt[4]{\frac{ex}{d}+1}\sqrt[4]{ae+cdx}}$$

$\big\downarrow$ 858

$$\frac{\sqrt[4]{x\left(ae^2+cd^2\right)+ade+cdex^2}\left(\frac{5cd^2\left(-\frac{4d(ae+cdx)^{3/4}\left(\frac{cd^2-ae^2}{e(ae+cdx)}+1\right)^{3/4}\int \frac{1}{\sqrt[4]{ae+cdx}\left(\frac{\left(cd^2-ae^2\right)(ae+cdx)}{e}+1\right)^{3/4}}d\frac{1}{\sqrt[4]{ae+cdx}}}{3e\left(-\frac{ae^2}{cd^2}+\frac{e(ae+cdx)}{cd^2}+1\right)^{3/4}}\right.}{7e}\right.}{d^4\sqrt[4]{\frac{ex}{d}+1}\sqrt[4]{ae+cdx}}$$

$\downarrow$ 807

$$\frac{\sqrt[4]{x\left(ae^2+cd^2\right)+ade+cdex^2}\left(\frac{5cd^2\left(-\frac{2d(ae+cdx)^{3/4}\left(\frac{cd^2-ae^2}{e(ae+cdx)}+1\right)^{3/4}\int \frac{1}{\left(\frac{\sqrt{ae+cdx}\left(cd^2-ae^2\right)}{e}+1\right)^{3/4}}d\sqrt{ae+cdx}}{3e\left(-\frac{ae^2}{cd^2}+\frac{e(ae+cdx)}{cd^2}+1\right)^{3/4}}-\frac{4d\sqrt[4]{ae+cdx}}{3e\left(\frac{ex}{d}+1\right)^{3/4}}\right.}{7e}\right.}{d^4\sqrt[4]{\frac{ex}{d}+1}\sqrt[4]{ae+cdx}}$$

$\downarrow$ 229

$$\frac{\sqrt[4]{x\left(ae^2+cd^2\right)+ade+cdex^2}\left(\frac{5cd^2\left(-\frac{4d(ae+cdx)^{3/4}\left(\frac{cd^2-ae^2}{e(ae+cdx)}+1\right)^{3/4}\text{EllipticF}\left(\frac{1}{2}\arctan\left(\frac{\sqrt{cd^2-ae^2}\sqrt{ae+cdx}}{\sqrt{e}}\right),2\right)}{3\sqrt{e}\sqrt{cd^2-ae^2}\left(-\frac{ae^2}{cd^2}+\frac{e(ae+cdx)}{cd^2}+1\right)^{3/4}}-\frac{4d\sqrt[4]{ae+cdx}}{3e\left(\frac{ex}{d}+1\right)^{3/4}}\right.}{7e}\right.}{d^4\sqrt[4]{\frac{ex}{d}+1}\sqrt[4]{ae+cdx}}$$

input
```
Int[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/4)/(d + e*x)^4,x]
```

output
```
((a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(1/4)*((-4*d*(a*e + c*d*x)^(5/4))
/(7*e*(1 + (e*x)/d)^(7/4)) + (5*c*d^2*((-4*d*(a*e + c*d*x)^(1/4))/(3*e*(1
+ (e*x)/d)^(3/4)) - (4*d*(a*e + c*d*x)^(3/4)*(1 + (c*d^2 - a*e^2)/(e*(a*e
+ c*d*x)))^(3/4)*EllipticF[ArcTan[(Sqrt[c*d^2 - a*e^2]*Sqrt[a*e + c*d*x])/
Sqrt[e]]/2, 2])/(3*Sqrt[e]*Sqrt[c*d^2 - a*e^2]*(1 - (a*e^2)/(c*d^2) + (e*(
a*e + c*d*x))/(c*d^2))^(3/4))))/(7*e)))/(d^4*(a*e + c*d*x)^(1/4)*(1 + (e*x
)/d)^(1/4))
```

Defintions of rubi rules used

rule 57
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Simp[d*(n/(b*(m + 1)))
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &
& GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !IntegerQ[m]) &&  !(ILeQ[m
+ n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) && IntLinearQ[a, b, c
, d, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 229
```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(2/(a^(3/4)*Rt[b/a, 2])
)*EllipticF[(1/2)*ArcTan[Rt[b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

rule 768
```
Int[((a_) + (b_.)*(x_)^4)^(-3/4), x_Symbol] :> Simp[x^3*((1 + a/(b*x^4))^(3
/4)/(a + b*x^4)^(3/4))   Int[1/(x^3*(1 + a/(b*x^4))^(3/4)), x], x] /; FreeQ
[{a, b}, x]
```

rule 807
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Simp[1/k   Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x,
x^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

rule 858
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

rule 1138
```
Int[((d_) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Sy
mbol] :> Simp[d^m*((a + b*x + c*x^2)^FracPart[p]/((1 + e*(x/d))^FracPart[p]
*(a/d + (c*x)/e)^FracPart[p]))   Int[(1 + e*(x/d))^(m + p)*(a/d + (c/e)*x)^
p, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0]
&& (IntegerQ[m] || GtQ[d, 0]) &&  !(IGtQ[m, 0] && (IntegerQ[3*p] || Integer
Q[4*p]))
```

Maple [F]

$$\int \frac{\left(ade+\left(a\,e^{2}+c\,d^{2}\right)x+cd\,x^{2}e\right)^{\frac{5}{4}}}{\left(ex+d\right)^{4}}dx$$

input
```
int((a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(5/4)/(e*x+d)^4,x)
```

output
```
int((a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(5/4)/(e*x+d)^4,x)
```

Fricas [F]

$$\int \frac{(ade+(cd^2+ae^2)\,x+cdex^2)^{5/4}}{(d+ex)^4}\,dx=\int \frac{(cdex^2+ade+(cd^2+ae^2)x)^{\frac{5}{4}}}{(ex+d)^4}\,dx$$

input
```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/4)/(e*x+d)^4,x, algorithm="
fricas")
```

output
```
integral((c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^(1/4)*(c*d*x + a*e)/(e^3*
x^3 + 3*d*e^2*x^2 + 3*d^2*e*x + d^3), x)
```

Sympy [F]

$$\int \frac{(ade + (cd^2 + ae^2)\,x + cdex^2)^{5/4}}{(d + ex)^4}\,dx = \int \frac{((d + ex)\,(ae + cdx))^{\frac{5}{4}}}{(d + ex)^4}\,dx$$

input
```
integrate((a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(5/4)/(e*x+d)**4,x)
```

output
```
Integral(((d + e*x)*(a*e + c*d*x))**(5/4)/(d + e*x)**4, x)
```

Maxima [F]

$$\int \frac{(ade + (cd^2 + ae^2)\,x + cdex^2)^{5/4}}{(d + ex)^4}\,dx = \int \frac{(cdex^2 + ade + (cd^2 + ae^2)x)^{\frac{5}{4}}}{(ex + d)^4}\,dx$$

input
```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/4)/(e*x+d)^4,x, algorithm="
maxima")
```

output
```
integrate((c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^(5/4)/(e*x + d)^4, x)
```

Giac [F]

$$\int \frac{(ade + (cd^2 + ae^2)\,x + cdex^2)^{5/4}}{(d + ex)^4}\,dx = \int \frac{(cdex^2 + ade + (cd^2 + ae^2)x)^{\frac{5}{4}}}{(ex + d)^4}\,dx$$

input
```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/4)/(e*x+d)^4,x, algorithm="
giac")
```

output
```
integrate((c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^(5/4)/(e*x + d)^4, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(ade + (cd^2 + ae^2)x + cdex^2)^{5/4}}{(d + ex)^4} dx = \int \frac{\left(c\,d\,e\,x^2 + \left(c\,d^2 + a\,e^2\right)\,x + a\,d\,e\right)^{5/4}}{\left(d + e\,x\right)^4} dx$$

input
```
int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(5/4)/(d + e*x)^4,x)
```

output
```
int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(5/4)/(d + e*x)^4, x)
```

Reduce [F]

$$\int \frac{(ade + (cd^2 + ae^2)x + cdex^2)^{5/4}}{(d + ex)^4} dx = \text{too large to display}$$

input
```
int((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/4)/(e*x+d)^4,x)
```

output

```
( - 8*(a*d*e + a*e**2*x + c*d**2*x + c*d*e*x**2)**(1/4)*a**2*e**3 - 16*(a*
d*e + a*e**2*x + c*d**2*x + c*d*e*x**2)**(1/4)*a*c*d**2*e - 28*(a*d*e + a*
e**2*x + c*d**2*x + c*d*e*x**2)**(1/4)*a*c*d*e**2*x + 4*(a*d*e + a*e**2*x
+ c*d**2*x + c*d*e*x**2)**(1/4)*c**2*d**3*x - 35*int(((a*d*e + a*e**2*x +
c*d**2*x + c*d*e*x**2)**(1/4)*x)/(7*a**2*d**3*e**3 + 21*a**2*d**2*e**4*x +
 21*a**2*d*e**5*x**2 + 7*a**2*e**6*x**3 - a*c*d**5*e + 4*a*c*d**4*e**2*x +
 18*a*c*d**3*e**3*x**2 + 20*a*c*d**2*e**4*x**3 + 7*a*c*d*e**5*x**4 - c**2*
d**6*x - 3*c**2*d**5*e*x**2 - 3*c**2*d**4*e**2*x**3 - c**2*d**3*e**3*x**4)
,x)*a**3*c*d**3*e**6 - 70*int(((a*d*e + a*e**2*x + c*d**2*x + c*d*e*x**2)*
*(1/4)*x)/(7*a**2*d**3*e**3 + 21*a**2*d**2*e**4*x + 21*a**2*d*e**5*x**2 +
7*a**2*e**6*x**3 - a*c*d**5*e + 4*a*c*d**4*e**2*x + 18*a*c*d**3*e**3*x**2
+ 20*a*c*d**2*e**4*x**3 + 7*a*c*d*e**5*x**4 - c**2*d**6*x - 3*c**2*d**5*e*
x**2 - 3*c**2*d**4*e**2*x**3 - c**2*d**3*e**3*x**4),x)*a**3*c*d**2*e**7*x
- 35*int(((a*d*e + a*e**2*x + c*d**2*x + c*d*e*x**2)**(1/4)*x)/(7*a**2*d**
3*e**3 + 21*a**2*d**2*e**4*x + 21*a**2*d*e**5*x**2 + 7*a**2*e**6*x**3 - a*
c*d**5*e + 4*a*c*d**4*e**2*x + 18*a*c*d**3*e**3*x**2 + 20*a*c*d**2*e**4*x*
*3 + 7*a*c*d*e**5*x**4 - c**2*d**6*x - 3*c**2*d**5*e*x**2 - 3*c**2*d**4*e*
*2*x**3 - c**2*d**3*e**3*x**4),x)*a**3*c*d*e**8*x**2 + 75*int(((a*d*e + a*
e**2*x + c*d**2*x + c*d*e*x**2)**(1/4)*x)/(7*a**2*d**3*e**3 + 21*a**2*d**2
*e**4*x + 21*a**2*d*e**5*x**2 + 7*a**2*e**6*x**3 - a*c*d**5*e + 4*a*c*d...
```

3.359 $\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/4}}{(d+ex)^5}\,dx$

Optimal result

Integrand size = 37, antiderivative size = 290

$$\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/4}}{(d+ex)^5}\,dx = -\frac{20cd\sqrt[4]{ade+\left(cd^2+ae^2\right)x+cdex^2}}{77e^2(d+ex)^2} + \frac{20c^2d^2\sqrt[4]{ade+\left(cd^2+ae^2\right)x+cdex^2}}{231e^2\left(cd^2-ae^2\right)(d+ex)} - \frac{4\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/4}}{11e(d+ex)^4} - \frac{40c^2d^2(ae+cdx)^{3/2}\left(\frac{cd(d+ex)}{e(ae+cdx)}\right)^{3/4}\operatorname{EllipticF}\left(\frac{1}{2}\arctan\left(\frac{\sqrt{cd^2-ae^2}}{\sqrt{e}\sqrt{ae+cdx}}\right),2\right)}{231e^{3/2}\left(cd^2-ae^2\right)^{3/2}\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/4}}$$

output
```
-20/77*c*d*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/4)/e^2/(e*x+d)^2+20/231*c^
2*d^2*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/4)/e^2/(-a*e^2+c*d^2)/(e*x+d)-4
/11*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/4)/e/(e*x+d)^4-40/231*c^2*d^2*(c*
d*x+a*e)^(3/2)*(c*d*(e*x+d)/e/(c*d*x+a*e))^(3/4)*InverseJacobiAM(1/2*arcta
n((-a*e^2+c*d^2)^(1/2)/e^(1/2)/(c*d*x+a*e)^(1/2)),2^(1/2))/e^(3/2)/(-a*e^2
+c*d^2)^(3/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/4)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.06 (sec) , antiderivative size = 112, normalized size of antiderivative = 0.39

$$\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/4}}{(d+ex)^5}\,dx=\frac{4c^3d^3(ae+cdx)^2\sqrt[4]{(ae+cdx)(d+ex)}\,\text{Hypergeometric2F1}\left(\frac{9}{4},\ldots\right.}{9\left(cd^2-ae^2\right)^4\sqrt[4]{\frac{cd(d+ex)}{cd^2-ae^2}}}$$

input
```
Integrate[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/4)/(d + e*x)^5,x]
```

output
```
(4*c^3*d^3*(a*e + c*d*x)^2*((a*e + c*d*x)*(d + e*x))^(1/4)*Hypergeometric2
F1[9/4, 15/4, 13/4, (e*(a*e + c*d*x))/(-(c*d^2) + a*e^2)])/(9*(c*d^2 - a*e
^2)^4*((c*d*(d + e*x))/(c*d^2 - a*e^2))^(1/4))
```

Rubi [A] (warning: unable to verify)

Time = 0.82 (sec) , antiderivative size = 348, normalized size of antiderivative = 1.20, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.243, Rules used = {1138, 57, 57, 61, 73, 768, 858, 807, 229}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/4}}{(d+ex)^5}dx$$

$$\Big\downarrow \text{ 1138}$$

$$\frac{\sqrt[4]{x\left(ae^2+cd^2\right)+ade+cdex^2}\int \frac{(ae+cdx)^{5/4}}{\left(\frac{ex}{d}+1\right)^{15/4}}dx}{d^5\sqrt[4]{\frac{ex}{d}+1}\sqrt[4]{ae+cdx}}$$

$$\Big\downarrow \text{ 57}$$

$$\frac{\sqrt[4]{x\left(ae^2+cd^2\right)+ade+cdex^2}\left(\frac{5cd^2\int \frac{\sqrt[4]{ae+cdx}}{\left(\frac{ex}{d}+1\right)^{11/4}}dx}{11e}-\frac{4d(ae+cdx)^{5/4}}{11e\left(\frac{ex}{d}+1\right)^{11/4}}\right)}{d^5\sqrt[4]{\frac{ex}{d}+1}\sqrt[4]{ae+cdx}}$$

$\downarrow$ 57

$$\frac{\sqrt[4]{x\left(ae^2+cd^2\right)+ade+cdex^2}\left(\frac{5cd^2\left(\frac{cd^2\int \frac{1}{(ae+cdx)^{3/4}\left(\frac{ex}{d}+1\right)^{7/4}}dx}{7e}-\frac{4d\sqrt[4]{ae+cdx}}{7e\left(\frac{ex}{d}+1\right)^{7/4}}\right)}{11e}-\frac{4d(ae+cdx)^{5/4}}{11e\left(\frac{ex}{d}+1\right)^{11/4}}\right)}{d^5\sqrt[4]{\frac{ex}{d}+1}\sqrt[4]{ae+cdx}}$$

$\downarrow$ 61

$$\frac{\sqrt[4]{x\left(ae^2+cd^2\right)+ade+cdex^2}\left(\frac{5cd^2\left(\frac{cd^2\left(\frac{2cd\int \frac{1}{(ae+cdx)^{3/4}\left(\frac{ex}{d}+1\right)^{3/4}}dx}{3\left(cd-\frac{ae^2}{d}\right)}+\frac{4\sqrt[4]{ae+cdx}}{3\left(\frac{ex}{d}+1\right)^{3/4}\left(cd-\frac{ae^2}{d}\right)}\right)}{7e}-\frac{4d\sqrt[4]{ae+cdx}}{7e\left(\frac{ex}{d}+1\right)^{7/4}}\right)}{11e}-\frac{4d(ae+cdx)^{5/4}}{11e\left(\frac{ex}{d}+1\right)^{11/4}}\right)}{d^5\sqrt[4]{\frac{ex}{d}+1}\sqrt[4]{ae+cdx}}$$

$\downarrow$ 73

$$\frac{\sqrt[4]{x\left(ae^2+cd^2\right)+ade+cdex^2}\left(\frac{5cd^2\left(\frac{cd^2\left(\frac{8\int\frac{1}{\left(-\frac{ae^2}{cd^2}+\frac{(ae+cdx)e}{cd^2}+1\right)^{3/4}}d\sqrt[4]{ae+cdx}}{3\left(cd-\frac{ae^2}{d}\right)}+\frac{4\sqrt[4]{ae+cdx}}{3\left(\frac{ex}{d}+1\right)^{3/4}\left(cd-\frac{ae^2}{d}\right)}\right)}{7e}-\frac{4d\sqrt[4]{ae+cdx}}{7e\left(\frac{ex}{d}+1\right)^{7/4}}\right.}{11e}\right.}{d^5\sqrt[4]{\frac{ex}{d}+1}\sqrt[4]{ae+cdx}}$$

$\downarrow$ 768

$$\frac{\sqrt[4]{x\left(ae^2+cd^2\right)+ade+cdex^2}\left(\frac{5cd^2\left(\frac{cd^2\left(\frac{8(ae+cdx)^{3/4}\left(\frac{cd^2-ae^2}{e(ae+cdx)}+1\right)^{3/4}\int\frac{1}{(ae+cdx)^{3/4}\left(\frac{cd^2-ae^2}{e(ae+cdx)}+1\right)^{3/4}}d\sqrt[4]{ae+cdx}}{3\left(cd-\frac{ae^2}{d}\right)\left(-\frac{ae^2}{cd^2}+\frac{e(ae+cdx)}{cd^2}+1\right)^{3/4}}+\frac{4\sqrt[4]{\ldots}}{3\left(\frac{ex}{d}+1\ldots\right.}\right.}{7e}\right.}{11e}\right.}{d^5\sqrt[4]{\frac{ex}{d}+1}\sqrt[4]{ae+cdx}}$$

$\downarrow$ 858

$$\frac{\sqrt[4]{x(ae^2+cd^2)+ade+cdex^2}\left(\frac{5cd^2\left(\frac{cd^2\left(\frac{4\sqrt[4]{ae+cdx}}{3\left(\frac{ex}{d}+1\right)^{3/4}\left(cd-\frac{ae^2}{d}\right)}-\frac{8(ae+cdx)^{3/4}\left(\frac{cd^2-ae^2}{e(ae+cdx)}+1\right)^{3/4}\int\frac{1}{\sqrt[4]{ae+cdx}\left(\frac{\left(cd^2-ae^2\right)(ae+c}{e}\right.}}{3\left(cd-\frac{ae^2}{d}\right)\left(-\frac{ae^2}{cd^2}+\frac{e(ae+cdx)}{cd^2}+1\right)^{3/4}}\right)}{7e}}{11e}\right.}{d^5\sqrt[4]{\frac{ex}{d}+1}\sqrt[4]{ae+cdx}}$$

$\downarrow$ 807

$$\frac{\sqrt[4]{x\left(ae^2+cd^2\right)+ade+cdex^2}\left(\frac{5cd^2\left(\frac{cd^2\left(\frac{4\sqrt[4]{ae+cdx}}{3\left(\frac{ex}{d}+1\right)^{3/4}\left(cd-\frac{ae^2}{d}\right)}-\frac{4(ae+cdx)^{3/4}\left(\frac{cd^2-ae^2}{e(ae+cdx)}+1\right)^{3/4}\int\frac{1}{\left(\frac{\sqrt{ae+cdx}\left(cd^2-ae^2\right)}{e}+1\right)^{3/4}}d\sqrt{}}{3\left(cd-\frac{ae^2}{d}\right)\left(-\frac{ae^2}{cd^2}+\frac{e(ae+cdx)}{cd^2}+1\right)^{3/4}}\right)}{7e}}{11e}\right)}{d^5\sqrt[4]{\frac{ex}{d}+1}\sqrt[4]{ae+cdx}}$$

$$\Big\downarrow \; 229$$

$$\frac{\sqrt[4]{x\left(ae^2+cd^2\right)+ade+cdex^2}\left(\frac{5cd^2\left(\frac{cd^2\left(\frac{4\sqrt[4]{ae+cdx}}{3\left(\frac{ex}{d}+1\right)^{3/4}\left(cd-\frac{ae^2}{d}\right)}-\frac{8\sqrt{e}(ae+cdx)^{3/4}\left(\frac{cd^2-ae^2}{e(ae+cdx)}+1\right)^{3/4}\text{EllipticF}\left(\frac{1}{2}\arctan\left(\frac{\sqrt{cd^2-ae^2}\sqrt{}}{\sqrt{e}}\right.\right.}{3\sqrt{cd^2-ae^2}\left(cd-\frac{ae^2}{d}\right)\left(-\frac{ae^2}{cd^2}+\frac{e(ae+cdx)}{cd^2}+1\right)^{3/4}}\right)}{7e}}{11e}\right)}{d^5\sqrt[4]{\frac{ex}{d}+1}\sqrt[4]{ae+cdx}}$$

input
```
Int[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/4)/(d + e*x)^5,x]
```

output
```
((a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(1/4)*((-4*d*(a*e + c*d*x)^(5/4))
/(11*e*(1 + (e*x)/d)^(11/4)) + (5*c*d^2*((-4*d*(a*e + c*d*x)^(1/4))/(7*e*(
1 + (e*x)/d)^(7/4)) + (c*d^2*((4*(a*e + c*d*x)^(1/4))/(3*(c*d - (a*e^2)/d)
*(1 + (e*x)/d)^(3/4)) - (8*Sqrt[e]*(a*e + c*d*x)^(3/4)*(1 + (c*d^2 - a*e^2
)/(e*(a*e + c*d*x)))^(3/4)*EllipticF[ArcTan[(Sqrt[c*d^2 - a*e^2]*Sqrt[a*e
+ c*d*x])/Sqrt[e]]/2, 2])/(3*Sqrt[c*d^2 - a*e^2]*(c*d - (a*e^2)/d)*(1 - (a
*e^2)/(c*d^2) + (e*(a*e + c*d*x))/(c*d^2))^(3/4))))/(7*e)))/(11*e)))/(d^5*
(a*e + c*d*x)^(1/4)*(1 + (e*x)/d)^(1/4))
```

Defintions of rubi rules used

rule 57
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Simp[d*(n/(b*(m + 1)))
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &
& GtQ[n, 0] && LtQ[m, -1] && !(IntegerQ[n] && !IntegerQ[m]) && !(ILeQ[m
+ n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) && IntLinearQ[a, b, c
, d, m, n, x]
```

rule 61
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && LtQ[m, -1] && !(LtQ[n, -1] && (EqQ[a, 0
] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && IntLinearQ[a, b, c, d
, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 229
```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(2/(a^(3/4)*Rt[b/a, 2])
)*EllipticF[(1/2)*ArcTan[Rt[b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

rule 768
```
Int[((a_) + (b_.)*(x_)^4)^(-3/4), x_Symbol] :> Simp[x^3*((1 + a/(b*x^4))^(3
/4)/(a + b*x^4)^(3/4))   Int[1/(x^3*(1 + a/(b*x^4))^(3/4)), x], x] /; FreeQ
[{a, b}, x]
```

rule 807
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Simp[1/k   Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x,
x^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

rule 858
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

rule 1138
```
Int[((d_) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Sy
mbol] :> Simp[d^m*((a + b*x + c*x^2)^FracPart[p]/((1 + e*(x/d))^FracPart[p]
*(a/d + (c*x)/e)^FracPart[p]))   Int[(1 + e*(x/d))^(m + p)*(a/d + (c/e)*x)^
p, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0]
&& (IntegerQ[m] || GtQ[d, 0]) &&  !(IGtQ[m, 0] && (IntegerQ[3*p] || Integer
Q[4*p]))
```

Maple [F]

$$\int \frac{(ade + (a\,e^2 + c\,d^2)\,x + cd\,x^2 e)^{\frac{5}{4}}}{(ex + d)^5}dx$$

input
```
int((a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(5/4)/(e*x+d)^5,x)
```

output
```
int((a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(5/4)/(e*x+d)^5,x)
```

Fricas [F]

$$\int \frac{(ade + (cd^2 + ae^2)x + cdex^2)^{5/4}}{(d + ex)^5} dx = \int \frac{(cdex^2 + ade + (cd^2 + ae^2)x)^{\frac{5}{4}}}{(ex + d)^5} dx$$

input
```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/4)/(e*x+d)^5,x, algorithm="
fricas")
```

output
```
integral((c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^(1/4)*(c*d*x + a*e)/(e^4*
x^4 + 4*d*e^3*x^3 + 6*d^2*e^2*x^2 + 4*d^3*e*x + d^4), x)
```

Sympy [F]

$$\int \frac{(ade + (cd^2 + ae^2)x + cdex^2)^{5/4}}{(d + ex)^5} dx = \int \frac{((d + ex)(ae + cdx))^{\frac{5}{4}}}{(d + ex)^5} dx$$

input
```
integrate((a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(5/4)/(e*x+d)**5,x)
```

output
```
Integral(((d + e*x)*(a*e + c*d*x))**(5/4)/(d + e*x)**5, x)
```

Maxima [F]

$$\int \frac{(ade + (cd^2 + ae^2)x + cdex^2)^{5/4}}{(d + ex)^5} dx = \int \frac{(cdex^2 + ade + (cd^2 + ae^2)x)^{\frac{5}{4}}}{(ex + d)^5} dx$$

input
```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/4)/(e*x+d)^5,x, algorithm="
maxima")
```

output
```
integrate((c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^(5/4)/(e*x + d)^5, x)
```

Giac [F]

$$\int \frac{(ade + (cd^2 + ae^2)\,x + cdex^2)^{5/4}}{(d + ex)^5}\,dx = \int \frac{(cdex^2 + ade + (cd^2 + ae^2)x)^{\frac{5}{4}}}{(ex + d)^5}\,dx$$

input
```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/4)/(e*x+d)^5,x, algorithm="
giac")
```

output
```
integrate((c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^(5/4)/(e*x + d)^5, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(ade + (cd^2 + ae^2)\,x + cdex^2)^{5/4}}{(d + ex)^5}\,dx = \int \frac{(c\,d\,e\,x^2 + (c\,d^2 + a\,e^2)\,x + a\,d\,e)^{5/4}}{(d + e\,x)^5}\,dx$$

input
```
int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(5/4)/(d + e*x)^5,x)
```

output
```
int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(5/4)/(d + e*x)^5, x)
```

Reduce [F]

$$\int \frac{(ade + (cd^2 + ae^2)\,x + cdex^2)^{5/4}}{(d + ex)^5}\,dx = \text{too large to display}$$

input
```
int((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/4)/(e*x+d)^5,x)
```

output
```
( - 24*(a*d*e + a*e**2*x + c*d**2*x + c*d*e*x**2)**(1/4)*a**2*e**3 - 16*(a
*d*e + a*e**2*x + c*d**2*x + c*d*e*x**2)**(1/4)*a*c*d**2*e - 44*(a*d*e + a
*e**2*x + c*d**2*x + c*d*e*x**2)**(1/4)*a*c*d*e**2*x + 4*(a*d*e + a*e**2*x
 + c*d**2*x + c*d*e*x**2)**(1/4)*c**2*d**3*x - 55*int(((a*d*e + a*e**2*x +
 c*d**2*x + c*d*e*x**2)**(1/4)*x)/(11*a**2*d**4*e**3 + 44*a**2*d**3*e**4*x
 + 66*a**2*d**2*e**5*x**2 + 44*a**2*d*e**6*x**3 + 11*a**2*e**7*x**4 - a*c*
d**6*e + 7*a*c*d**5*e**2*x + 38*a*c*d**4*e**3*x**2 + 62*a*c*d**3*e**4*x**3
 + 43*a*c*d**2*e**5*x**4 + 11*a*c*d*e**6*x**5 - c**2*d**7*x - 4*c**2*d**6*
e*x**2 - 6*c**2*d**5*e**2*x**3 - 4*c**2*d**4*e**3*x**4 - c**2*d**3*e**4*x*
*5),x)*a**3*c*d**4*e**6 - 165*int(((a*d*e + a*e**2*x + c*d**2*x + c*d*e*x*
*2)**(1/4)*x)/(11*a**2*d**4*e**3 + 44*a**2*d**3*e**4*x + 66*a**2*d**2*e**5
*x**2 + 44*a**2*d*e**6*x**3 + 11*a**2*e**7*x**4 - a*c*d**6*e + 7*a*c*d**5*
e**2*x + 38*a*c*d**4*e**3*x**2 + 62*a*c*d**3*e**4*x**3 + 43*a*c*d**2*e**5*
x**4 + 11*a*c*d*e**6*x**5 - c**2*d**7*x - 4*c**2*d**6*e*x**2 - 6*c**2*d**5
*e**2*x**3 - 4*c**2*d**4*e**3*x**4 - c**2*d**3*e**4*x**5),x)*a**3*c*d**3*e
**7*x - 165*int(((a*d*e + a*e**2*x + c*d**2*x + c*d*e*x**2)**(1/4)*x)/(11*
a**2*d**4*e**3 + 44*a**2*d**3*e**4*x + 66*a**2*d**2*e**5*x**2 + 44*a**2*d*
e**6*x**3 + 11*a**2*e**7*x**4 - a*c*d**6*e + 7*a*c*d**5*e**2*x + 38*a*c*d*
*4*e**3*x**2 + 62*a*c*d**3*e**4*x**3 + 43*a*c*d**2*e**5*x**4 + 11*a*c*d*e*
*6*x**5 - c**2*d**7*x - 4*c**2*d**6*e*x**2 - 6*c**2*d**5*e**2*x**3 - 4*...
```

3.360 $\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/4}}{(d+ex)^6}\,dx$

Optimal result

Integrand size = 37, antiderivative size = 353

$$\begin{aligned}\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/4}}{(d+ex)^6}\,dx = &-\frac{4cd\sqrt[4]{ade+\left(cd^2+ae^2\right)x+cdex^2}}{33e^2(d+ex)^3}+\frac{4c^2d^2\sqrt[4]{ade+\left(cd^2+ae^2\right)x+cdex^2}}{231e^2\left(cd^2-ae^2\right)(d+ex)^2}\\ &+\frac{8c^3d^3\sqrt[4]{ade+\left(cd^2+ae^2\right)x+cdex^2}}{231e^2\left(cd^2-ae^2\right)^2(d+ex)}-\frac{4\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/4}}{15e(d+ex)^5}\\ &-\frac{16c^3d^3(ae+cdx)^{3/2}\left(\frac{cd(d+ex)}{e(ae+cdx)}\right)^{3/4}\operatorname{EllipticF}\left(\frac{1}{2}\arctan\left(\frac{\sqrt{cd^2-ae^2}}{\sqrt{e}\sqrt{ae+cdx}}\right),2\right)}{231e^{3/2}\left(cd^2-ae^2\right)^{5/2}\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/4}}\end{aligned}$$

output

```
-4/33*c*d*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/4)/e^2/(e*x+d)^3+4/231*c^2*
d^2*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/4)/e^2/(-a*e^2+c*d^2)/(e*x+d)^2+8
/231*c^3*d^3*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/4)/e^2/(-a*e^2+c*d^2)^2/
(e*x+d)-4/15*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/4)/e/(e*x+d)^5-16/231*c^
3*d^3*(c*d*x+a*e)^(3/2)*(c*d*(e*x+d)/e/(c*d*x+a*e))^(3/4)*InverseJacobiAM(
1/2*arctan((-a*e^2+c*d^2)^(1/2)/e^(1/2)/(c*d*x+a*e)^(1/2)),2^(1/2))/e^(3/2
)/(-a*e^2+c*d^2)^(5/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/4)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.07 (sec) , antiderivative size = 112, normalized size of antiderivative = 0.32

$$\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/4}}{(d+ex)^6}\,dx=\frac{4c^4d^4(ae+cdx)^2\sqrt[4]{(ae+cdx)(d+ex)}\,\text{Hypergeometric2F1}\left(\frac{9}{4},\frac{1}{4}\right.}{9\left(cd^2-ae^2\right)^5\sqrt[4]{\frac{cd(d+ex)}{cd^2-ae^2}}}$$

input
```
Integrate[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/4)/(d + e*x)^6,x]
```

output
```
(4*c^4*d^4*(a*e + c*d*x)^2*((a*e + c*d*x)*(d + e*x))^(1/4)*Hypergeometric2
F1[9/4, 19/4, 13/4, (e*(a*e + c*d*x))/(-(c*d^2) + a*e^2)])/(9*(c*d^2 - a*e
^2)^5*((c*d*(d + e*x))/(c*d^2 - a*e^2))^(1/4))
```

Rubi [A] (warning: unable to verify)

Time = 0.90 (sec) , antiderivative size = 413, normalized size of antiderivative = 1.17, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.270, Rules used = {1138, 57, 57, 61, 61, 73, 768, 858, 807, 229}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/4}}{(d+ex)^6}dx$$

$$\Big\downarrow \text{ 1138}$$

$$\frac{\sqrt[4]{x\left(ae^2+cd^2\right)+ade+cdex^2}\int \frac{(ae+cdx)^{5/4}}{\left(\frac{ex}{d}+1\right)^{19/4}}dx}{d^6\sqrt[4]{\frac{ex}{d}+1}\sqrt[4]{ae+cdx}}$$

$$\Big\downarrow \text{ 57}$$

$$\frac{\sqrt[4]{x\left(ae^2+cd^2\right)+ade+cdex^2}\left(\frac{cd^2\int \frac{\sqrt[4]{ae+cdx}}{\left(\frac{ex}{d}+1\right)^{15/4}}dx}{3e}-\frac{4d(ae+cdx)^{5/4}}{15e\left(\frac{ex}{d}+1\right)^{15/4}}\right)}{d^6\sqrt[4]{\frac{ex}{d}+1}\sqrt[4]{ae+cdx}}$$

$\Big\downarrow$ 57

$$\frac{\sqrt[4]{x\left(ae^2+cd^2\right)+ade+cdex^2}\left(\frac{cd^2\left(\frac{cd^2\int \frac{1}{(ae+cdx)^{3/4}\left(\frac{ex}{d}+1\right)^{11/4}}dx}{11e}-\frac{4d\sqrt[4]{ae+cdx}}{11e\left(\frac{ex}{d}+1\right)^{11/4}}\right)}{3e}-\frac{4d(ae+cdx)^{5/4}}{15e\left(\frac{ex}{d}+1\right)^{15/4}}\right)}{d^6\sqrt[4]{\frac{ex}{d}+1}\sqrt[4]{ae+cdx}}$$

$\Big\downarrow$ 61

$$\frac{\sqrt[4]{x\left(ae^2+cd^2\right)+ade+cdex^2}\left(\frac{cd^2\left(\frac{cd^2\left(\frac{6cd\int \frac{1}{(ae+cdx)^{3/4}\left(\frac{ex}{d}+1\right)^{7/4}}dx}{7\left(cd-\frac{ae^2}{d}\right)}+\frac{4\sqrt[4]{ae+cdx}}{7\left(\frac{ex}{d}+1\right)^{7/4}\left(cd-\frac{ae^2}{d}\right)}\right)}{11e}-\frac{4d\sqrt[4]{ae+cdx}}{11e\left(\frac{ex}{d}+1\right)^{11/4}}\right)}{3e}-\frac{4d(ae+cdx)^{5/4}}{15e\left(\frac{ex}{d}+1\right)^{15/4}}\right)}{d^6\sqrt[4]{\frac{ex}{d}+1}\sqrt[4]{ae+cdx}}$$

$\Big\downarrow$ 61

$$\frac{\sqrt[4]{x\left(ae^2+cd^2\right)+ade+cdex^2}\left(\frac{cd^2\left(\frac{cd^2\left(\frac{6cd\left(\frac{2cd\int\frac{1}{(ae+cdx)^{3/4}\left(\frac{ex}{d}+1\right)^{3/4}}dx}{3\left(cd-\frac{ae^2}{d}\right)}+\frac{4\sqrt[4]{ae+cdx}}{3\left(\frac{ex}{d}+1\right)^{3/4}\left(cd-\frac{ae^2}{d}\right)}\right)}{7\left(cd-\frac{ae^2}{d}\right)}+\frac{4\sqrt[4]{ae+cdx}}{7\left(\frac{ex}{d}+1\right)^{7/4}\left(cd-\frac{ae^2}{d}\right)}\right)}{11e}-\cdots\right)}{3e}\cdots\right)}{d^6\sqrt[4]{\frac{ex}{d}+1}\sqrt[4]{ae+cdx}}$$

$\downarrow$ 73

$$\frac{\sqrt[4]{x\left(ae^2+cd^2\right)+ade+cdex^2}\left(\frac{cd^2\left(\frac{cd^2\left(\frac{6cd\left(\frac{8\int\frac{1}{\left(-\frac{ae^2}{cd^2}+\frac{(ae+cdx)e}{cd^2}+1\right)^{3/4}}d\sqrt[4]{ae+cdx}}{3\left(cd-\frac{ae^2}{d}\right)}+\frac{4\sqrt[4]{ae+cdx}}{3\left(\frac{ex}{d}+1\right)^{3/4}\left(cd-\frac{ae^2}{d}\right)}\right)}{7\left(cd-\frac{ae^2}{d}\right)}+\frac{4\sqrt[4]{ae+}}{7\left(\frac{ex}{d}+1\right)^{7/4}}}{11e}}{3e}}{d^6\sqrt[4]{\frac{ex}{d}+1}\sqrt[4]{ae+cdx}}$$

$\downarrow$ 768

$$\frac{\sqrt[4]{x\left(ae^2+cd^2\right)+ade+cdex^2}\left(\frac{cd^2\left(\frac{cd^2\left(\frac{6cd\left(\frac{8(ae+cdx)^{3/4}\left(\frac{cd^2-ae^2}{e(ae+cdx)}+1\right)^{3/4}\int\frac{1}{(ae+cdx)^{3/4}\left(\frac{cd^2-ae^2}{e(ae+cdx)}+1\right)^{3/4}}d\sqrt[4]{ae+cdx}}{3\left(cd-\frac{ae^2}{d}\right)\left(-\frac{ae^2}{cd^2}+\frac{e(ae+cdx)}{cd^2}+1\right)^{3/4}}+\cdots}{7\left(cd-\frac{ae^2}{d}\right)}\cdots}{11e}\cdots}{3e}\cdots\right)}{d^6\sqrt[4]{\frac{ex}{d}+1}\sqrt[4]{ae+cdx}\cdots}$$

$\downarrow$ 858

$$\frac{\sqrt[4]{x\left(ae^2+cd^2\right)+ade+cdex^2}\left(\frac{cd^2\left(\frac{cd^2\left(\frac{cd^2\left(6cd\left(\frac{4\sqrt[4]{ae+cdx}}{3\left(\frac{ex}{d}+1\right)^{3/4}\left(cd-\frac{ae^2}{d}\right)}-\frac{8(ae+cdx)^{3/4}\left(\frac{cd^2-ae^2}{e(ae+cdx)}+1\right)^{3/4}\int\frac{1}{\sqrt[4]{ae+cdx}\left(\frac{\left(cd^2-ae^2\right)(\cdots}{e}\cdots\right.}}{3\left(cd-\frac{ae^2}{d}\right)\left(-\frac{ae^2}{cd^2}+\frac{e(ae+cdx)}{cd^2}+1\right)\cdots}\right.\cdots}{7\left(cd-\frac{ae^2}{d}\right)\cdots}\cdots}{11e\cdots}\cdots}{3e\cdots}\cdots\right.}{d^6\sqrt[4]{\frac{ex}{d}+1}\sqrt[4]{ae+cdx}\cdots}$$

↓ 807

$$\frac{\sqrt[4]{x\left(ae^2+cd^2\right)+ade+cdex^2}\left(\frac{cd^2\left(\frac{cd^2\left(\frac{6cd\left(\frac{4\sqrt[4]{ae+cdx}}{3\left(\frac{ex}{d}+1\right)^{3/4}\left(cd-\frac{ae^2}{d}\right)}-\frac{4(ae+cdx)^{3/4}\left(\frac{cd^2-ae^2}{e(ae+cdx)}+1\right)^{3/4}\int\frac{1}{\left(\frac{\sqrt{ae+cdx}\left(cd^2-ae^2\right)}{e}+1\right)^{3/4}}}{3\left(cd-\frac{ae^2}{d}\right)\left(-\frac{ae^2}{cd^2}+\frac{e(ae+cdx)}{cd^2}+1\right)^{3/4}}\right)}{7\left(cd-\frac{ae^2}{d}\right)}\right)}{11e}\right)}{3e}\right)}{d^6\sqrt[4]{\frac{ex}{d}+1}\sqrt[4]{ae+cdx}}$$

↓ 229

$$\frac{\sqrt[4]{x\left(ae^2+cd^2\right)+ade+cdex^2}\left(\frac{cd^2\left(\frac{cd^2\left(\frac{6cd\left(\frac{4\sqrt[4]{ae+cdx}}{3\left(\frac{ex}{d}+1\right)^{3/4}\left(cd-\frac{ae^2}{d}\right)}-\frac{8\sqrt{e}(ae+cdx)^{3/4}\left(\frac{cd^2-ae^2}{e(ae+cdx)}+1\right)^{3/4}\text{EllipticF}\left(\frac{1}{2}\arctan\left(\frac{\sqrt{cd^2-a\cdots}}{\cdots}\right)\cdots\right)}{3\sqrt{cd^2-ae^2}\left(cd-\frac{ae^2}{d}\right)\left(-\frac{ae^2}{cd^2}+\frac{e(ae+cdx)}{cd^2}+1\right)^{3/4}\cdots}\right)}{7\left(cd-\frac{ae^2}{d}\right)}\cdots\right)}{11e}\cdots\right)}{3e}\cdots\right)}{d^6\sqrt[4]{\frac{ex}{d}+1}\sqrt[4]{ae+cdx}\cdots}$$

input
```
Int[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/4)/(d + e*x)^6,x]
```

output
```
((a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(1/4)*((-4*d*(a*e + c*d*x)^(5/4))
/(15*e*(1 + (e*x)/d)^(15/4)) + (c*d^2*((-4*d*(a*e + c*d*x)^(1/4))/(11*e*(1
 + (e*x)/d)^(11/4)) + (c*d^2*((4*(a*e + c*d*x)^(1/4))/(7*(c*d - (a*e^2)/d)
*(1 + (e*x)/d)^(7/4)) + (6*c*d*((4*(a*e + c*d*x)^(1/4))/(3*(c*d - (a*e^2)/
d)*(1 + (e*x)/d)^(3/4)) - (8*Sqrt[e]*(a*e + c*d*x)^(3/4)*(1 + (c*d^2 - a*e
^2)/(e*(a*e + c*d*x)))^(3/4)*EllipticF[ArcTan[(Sqrt[c*d^2 - a*e^2]*Sqrt[a*
e + c*d*x])/Sqrt[e]]/2, 2])/(3*Sqrt[c*d^2 - a*e^2]*(c*d - (a*e^2)/d)*(1 -
(a*e^2)/(c*d^2) + (e*(a*e + c*d*x))/(c*d^2))^(3/4))))/(7*(c*d - (a*e^2)/d)
)))/(11*e)))/(3*e)))/(d^6*(a*e + c*d*x)^(1/4)*(1 + (e*x)/d)^(1/4))
```

Defintions of rubi rules used

rule 57
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Simp[d*(n/(b*(m + 1)))
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &
& GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !IntegerQ[m]) &&  !(ILeQ[m
+ n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) && IntLinearQ[a, b, c
, d, m, n, x]
```

rule 61
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && LtQ[m, -1] &&  !(LtQ[n, -1] && (EqQ[a, 0
] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && IntLinearQ[a, b, c, d
, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 229
```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(2/(a^(3/4)*Rt[b/a, 2])
)*EllipticF[(1/2)*ArcTan[Rt[b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

rule 768
```
Int[((a_) + (b_.)*(x_)^4)^(-3/4), x_Symbol] :> Simp[x^3*((1 + a/(b*x^4))^(3
/4)/(a + b*x^4)^(3/4))   Int[1/(x^3*(1 + a/(b*x^4))^(3/4)), x], x] /; FreeQ
[{a, b}, x]
```

rule 807
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Simp[1/k   Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x,
x^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

rule 858
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

rule 1138
```
Int[((d_) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Sy
mbol] :> Simp[d^m*((a + b*x + c*x^2)^FracPart[p]/((1 + e*(x/d))^FracPart[p]
*(a/d + (c*x)/e)^FracPart[p]))   Int[(1 + e*(x/d))^(m + p)*(a/d + (c/e)*x)^
p, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0]
&& (IntegerQ[m] || GtQ[d, 0]) &&  !(IGtQ[m, 0] && (IntegerQ[3*p] || Integer
Q[4*p]))
```

Maple [F]

$$\int \frac{(ade + (a\,e^2 + c\,d^2)\,x + cd\,x^2 e)^{\frac{5}{4}}}{(ex + d)^6}dx$$

input
```
int((a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(5/4)/(e*x+d)^6,x)
```

output
```
int((a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(5/4)/(e*x+d)^6,x)
```

Fricas [F]

$$\int \frac{(ade + (cd^2 + ae^2)\,x + cdex^2)^{5/4}}{(d + ex)^6}\,dx = \int \frac{(cdex^2 + ade + (cd^2 + ae^2)x)^{\frac{5}{4}}}{(ex + d)^6}\,dx$$

input
```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/4)/(e*x+d)^6,x, algorithm="
fricas")
```

output
```
integral((c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^(1/4)*(c*d*x + a*e)/(e^5*
x^5 + 5*d*e^4*x^4 + 10*d^2*e^3*x^3 + 10*d^3*e^2*x^2 + 5*d^4*e*x + d^5), x)
```

Sympy [F]

$$\int \frac{(ade + (cd^2 + ae^2)x + cdex^2)^{5/4}}{(d + ex)^6} dx = \int \frac{((d + ex)(ae + cdx))^{\frac{5}{4}}}{(d + ex)^6} dx$$

input
```
integrate((a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(5/4)/(e*x+d)**6,x)
```

output
```
Integral(((d + e*x)*(a*e + c*d*x))**(5/4)/(d + e*x)**6, x)
```

Maxima [F]

$$\int \frac{(ade + (cd^2 + ae^2)x + cdex^2)^{5/4}}{(d + ex)^6} dx = \int \frac{(cdex^2 + ade + (cd^2 + ae^2)x)^{\frac{5}{4}}}{(ex + d)^6} dx$$

input
```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/4)/(e*x+d)^6,x, algorithm="
maxima")
```

output
```
integrate((c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^(5/4)/(e*x + d)^6, x)
```

Giac [F]

$$\int \frac{(ade + (cd^2 + ae^2)x + cdex^2)^{5/4}}{(d + ex)^6} dx = \int \frac{(cdex^2 + ade + (cd^2 + ae^2)x)^{\frac{5}{4}}}{(ex + d)^6} dx$$

input
```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/4)/(e*x+d)^6,x, algorithm="
giac")
```

output
```
integrate((c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^(5/4)/(e*x + d)^6, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(ade+(cd^2+ae^2)x+cdex^2)^{5/4}}{(d+ex)^6}\,dx=\int \frac{\left(c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e\right)^{5/4}}{\left(d+e\,x\right)^6}\,dx$$

input
```
int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(5/4)/(d + e*x)^6,x)
```

output
```
int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(5/4)/(d + e*x)^6, x)
```

Reduce [F]

$$\int \frac{(ade+(cd^2+ae^2)x+cdex^2)^{5/4}}{(d+ex)^6}\,dx=\text{too large to display}$$

input
```
int((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/4)/(e*x+d)^6,x)
```

output
```
( - 40*(a*d*e + a*e**2*x + c*d**2*x + c*d*e*x**2)**(1/4)*a**2*e**3 - 16*(a
*d*e + a*e**2*x + c*d**2*x + c*d*e*x**2)**(1/4)*a*c*d**2*e - 60*(a*d*e + a
*e**2*x + c*d**2*x + c*d*e*x**2)**(1/4)*a*c*d*e**2*x + 4*(a*d*e + a*e**2*x
 + c*d**2*x + c*d*e*x**2)**(1/4)*c**2*d**3*x - 75*int(((a*d*e + a*e**2*x +
 c*d**2*x + c*d*e*x**2)**(1/4)*x)/(15*a**2*d**5*e**3 + 75*a**2*d**4*e**4*x
 + 150*a**2*d**3*e**5*x**2 + 150*a**2*d**2*e**6*x**3 + 75*a**2*d*e**7*x**4
 + 15*a**2*e**8*x**5 - a*c*d**7*e + 10*a*c*d**6*e**2*x + 65*a*c*d**5*e**3*
x**2 + 140*a*c*d**4*e**4*x**3 + 145*a*c*d**3*e**5*x**4 + 74*a*c*d**2*e**6*
x**5 + 15*a*c*d*e**7*x**6 - c**2*d**8*x - 5*c**2*d**7*e*x**2 - 10*c**2*d**
6*e**2*x**3 - 10*c**2*d**5*e**3*x**4 - 5*c**2*d**4*e**4*x**5 - c**2*d**3*e
**5*x**6),x)*a**3*c*d**5*e**6 - 300*int(((a*d*e + a*e**2*x + c*d**2*x + c*
d*e*x**2)**(1/4)*x)/(15*a**2*d**5*e**3 + 75*a**2*d**4*e**4*x + 150*a**2*d*
*3*e**5*x**2 + 150*a**2*d**2*e**6*x**3 + 75*a**2*d*e**7*x**4 + 15*a**2*e**
8*x**5 - a*c*d**7*e + 10*a*c*d**6*e**2*x + 65*a*c*d**5*e**3*x**2 + 140*a*c
*d**4*e**4*x**3 + 145*a*c*d**3*e**5*x**4 + 74*a*c*d**2*e**6*x**5 + 15*a*c*
d*e**7*x**6 - c**2*d**8*x - 5*c**2*d**7*e*x**2 - 10*c**2*d**6*e**2*x**3 -
10*c**2*d**5*e**3*x**4 - 5*c**2*d**4*e**4*x**5 - c**2*d**3*e**5*x**6),x)*a
**3*c*d**4*e**7*x - 450*int(((a*d*e + a*e**2*x + c*d**2*x + c*d*e*x**2)**(
1/4)*x)/(15*a**2*d**5*e**3 + 75*a**2*d**4*e**4*x + 150*a**2*d**3*e**5*x**2
 + 150*a**2*d**2*e**6*x**3 + 75*a**2*d*e**7*x**4 + 15*a**2*e**8*x**5 - ...
```

3.361 $\int \frac{(d+ex)^3}{\sqrt[4]{ade+(cd^2+ae^2)x+cdex^2}}\,dx$

Optimal result

Integrand size = 37, antiderivative size = 290

$$\begin{aligned}
\int \frac{(d+ex)^3}{\sqrt[4]{ade+(cd^2+ae^2)x+cdex^2}}\,dx
&= \frac{11(cd^2-ae^2)^2\left(ade+(cd^2+ae^2)x+cdex^2\right)^{3/4}}{30c^3d^3} \\
&+ \frac{11(cd^2-ae^2)(d+ex)\left(ade+(cd^2+ae^2)x+cdex^2\right)^{3/4}}{35c^2d^2} \\
&+ \frac{2(d+ex)^2\left(ade+(cd^2+ae^2)x+cdex^2\right)^{3/4}}{7cd} \\
&+ \frac{11(cd^2-ae^2)^4\sqrt[4]{-\frac{cde(ae+cdx)(d+ex)}{(cd^2-ae^2)^2}}\,E\left(\frac{1}{2}\arcsin\left(\frac{ae^2+cd(d+2ex)}{cd^2-ae^2}\right)\middle|2\right)}{20\sqrt{2}c^4d^4e\sqrt[4]{ade+(cd^2+ae^2)x+cdex^2}}
\end{aligned}$$

output
```
11/30*(-a*e^2+c*d^2)^2*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/4)/c^3/d^3+11/
35*(-a*e^2+c*d^2)*(e*x+d)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/4)/c^2/d^2+
2/7*(e*x+d)^2*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/4)/c/d+11/40*(-a*e^2+c*
d^2)^4*(-c*d*e*(c*d*x+a*e)*(e*x+d)/(-a*e^2+c*d^2)^2)^(1/4)*EllipticE(sin(1
/2*arcsin((a*e^2+c*d*(2*e*x+d))/(-a*e^2+c*d^2))),2^(1/2))*2^(1/2)/c^4/d^4/
e/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/4)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.07 (sec) , antiderivative size = 102, normalized size of antiderivative = 0.35

$$\int \frac{(d+ex)^3}{\sqrt[4]{ade+(cd^2+ae^2)\,x+cdex^2}}\,dx = \frac{4(cd^2-ae^2)^2\,((ae+cdx)(d+ex))^{3/4}\,\text{Hypergeometric2F1}\left(-\frac{11}{4},\frac{3}{4},\frac{7}{4},\frac{e(ae+cdx)}{-cd^2+ae^2}\right)}{3c^3d^3\left(\frac{cd(d+ex)}{cd^2-ae^2}\right)^{3/4}}$$

input
```
Integrate[(d + e*x)^3/(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(1/4),x]
```

output
```
(4*(c*d^2 - a*e^2)^2*((a*e + c*d*x)*(d + e*x))^(3/4)*Hypergeometric2F1[-11
/4, 3/4, 7/4, (e*(a*e + c*d*x))/(-(c*d^2) + a*e^2)])/(3*c^3*d^3*((c*d*(d +
 e*x))/(c*d^2 - a*e^2))^(3/4))
```

Rubi [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 1128 vs. $2(290) = 580$.

Time = 1.55 (sec) , antiderivative size = 1128, normalized size of antiderivative = 3.89, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.243$, Rules used = {1166, 27, 1166, 27, 1160, 1094, 834, 761, 1510}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex)^3}{\sqrt[4]{x\left(ae^2+cd^2\right)+ade+cdex^2}}\,dx$$

$$\Big\downarrow \text{1166}$$

$$\frac{2\int \frac{11e\left(cd^2-ae^2\right)(d+ex)^2}{4\sqrt[4]{cdex^2+\left(cd^2+ae^2\right)x+ade}}dx}{7cde}+\frac{2(d+ex)^2\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/4}}{7cd}$$

$$\Big\downarrow \text{27}$$

$$\frac{11\left(cd^2-ae^2\right)\int \frac{(d+ex)^2}{\sqrt[4]{cdex^2+\left(cd^2+ae^2\right)x+ade}}dx}{14cd}+\frac{2(d+ex)^2\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/4}}{7cd}$$

$\Big\downarrow$ 1166

$$\frac{11\left(cd^2-ae^2\right)\left(\frac{2\int \frac{7e\left(cd^2-ae^2\right)(d+ex)}{4\sqrt[4]{cdex^2+\left(cd^2+ae^2\right)x+ade}}dx}{5cde}+\frac{2(d+ex)\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/4}}{5cd}\right)}{14cd}+\frac{2(d+ex)^2\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/4}}{7cd}$$

$\Big\downarrow$ 27

$$\frac{11\left(cd^2-ae^2\right)\left(\frac{7\left(cd^2-ae^2\right)\int \frac{d+ex}{\sqrt[4]{cdex^2+\left(cd^2+ae^2\right)x+ade}}dx}{10cd}+\frac{2(d+ex)\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/4}}{5cd}\right)}{14cd}+\frac{2(d+ex)^2\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/4}}{7cd}$$

$\Big\downarrow$ 1160

$$\frac{11\left(cd^2-ae^2\right)\left(\frac{7\left(cd^2-ae^2\right)\left(\frac{\left(d^2-\frac{ae^2}{c}\right)\int \frac{1}{\sqrt[4]{cdex^2+\left(cd^2+ae^2\right)x+ade}}dx}{2d}+\frac{2\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/4}}{3cd}\right)}{10cd}+\frac{2(d+ex)(x(ae}{}\right.}{14cd}+\frac{2(d+ex)^2\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/4}}{7cd}$$

$\Big\downarrow$ 1094

$$\frac{11\left(cd^2-ae^2\right)\left(\frac{7\left(cd^2-ae^2\right)\left(\frac{2\left(d^2-\frac{ae^2}{c}\right)\sqrt{\left(ae^2+cd^2+2cdex\right)^2}\int \frac{\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}{\sqrt{\left(cd^2-ae^2\right)^2+4cde\left(cdex^2+\left(cd^2+ae^2\right)x+ade\right)}}d\sqrt[4]{cdex^2+(cd^2+ae^2)}}{d\left(ae^2+cd^2+2cdex\right)}\right.}{10cd}\right.}{14cd}$$

$$\frac{2(d+ex)^2\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/4}}{7cd}$$

$\downarrow$ 834

$$11\left(cd^2-ae^2\right)\left(\frac{7\left(cd^2-ae^2\right)\left(\frac{2\left(d^2-\frac{ae^2}{c}\right)\sqrt{\left(ae^2+cd^2+2cdex\right)^2}\left(\frac{\left(cd^2-ae^2\right)\int \frac{1}{\sqrt{\left(cd^2-ae^2\right)^2+4cde\left(cdex^2+\left(cd^2+ae^2\right)x+ade\right)}}d\sqrt[4]{cdex^2+(c}}{2\sqrt{c}\sqrt{d}\sqrt{e}}\right.}{}\right.}{}\right.$$

$$\frac{2(d+ex)^2\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/4}}{7cd}$$

$\downarrow$ 761

$$\frac{11\left(cd^2-ae^2\right)\left(\frac{7\left(cd^2-ae^2\right)\left(\frac{2\left(d^2-\frac{ae^2}{c}\right)\sqrt{\left(ae^2+cd^2+2cdex\right)^2}\left(\frac{\left(cd^2-ae^2\right)^{3/2}\left(\frac{2\sqrt{c}\sqrt{d}\sqrt{e}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{cd^2-ae^2}+1\right)\sqrt{\frac{4cde\left(x\left(ae^2+c\cdots\right.\right.}{\left(cd^2-ae^2\right)^2\left(\frac{2\sqrt{c}\sqrt{\cdots}}{\cdots}\right.}}}{4\sqrt{2}c^{3/4}d^3\cdots}\cdots\right.}{\cdots}\cdots\right.}{\cdots}\cdots\right.}{\cdots}$$

$$\frac{2(d+ex)^2\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/4}}{7cd}$$

$\downarrow$ 1510

$$\frac{2\left(cdex^2+\left(cd^2+ae^2\right)x+ade\right)^{3/4}(d+ex)^2}{7cd}+$$

$$11\left(cd^2-ae^2\right)\left(\frac{2\left(cdex^2+\left(cd^2+ae^2\right)x+ade\right)^{3/4}(d+ex)}{5cd}+\frac{7\left(cd^2-ae^2\right)\left(\frac{2\left(d^2-\frac{ae^2}{c}\right)\sqrt{\left(cd^2+2cexd+ae^2\right)^2}\left(\frac{\left(cd^2-ae^2\right)^{3/2}\left(\frac{2\sqrt{c}\sqrt{d}\sqrt{e}\sqrt{cdex^2}}{cd^2}\right.}{}\right.}{}\right.}{}\right.$$

input
```
Int[(d + e*x)^3/(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(1/4),x]
```

output
```
(2*(d + e*x)^2*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/4))/(7*c*d) + (1
1*(c*d^2 - a*e^2)*((2*(d + e*x)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3
/4))/(5*c*d) + (7*(c*d^2 - a*e^2)*((2*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x
^2)^(3/4))/(3*c*d) + (2*(d^2 - (a*e^2)/c)*Sqrt[(c*d^2 + a*e^2 + 2*c*d*e*x)
^2]*(-1/2*((c*d^2 - a*e^2)*(-(((a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(1/
4)*Sqrt[(c*d^2 - a*e^2)^2 + 4*c*d*e*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2
)])/((c*d^2 - a*e^2)^2*(1 + (2*Sqrt[c]*Sqrt[d]*Sqrt[e]*Sqrt[a*d*e + (c*d^2
 + a*e^2)*x + c*d*e*x^2])/(c*d^2 - a*e^2)))) + (Sqrt[c*d^2 - a*e^2]*(1 + (
2*Sqrt[c]*Sqrt[d]*Sqrt[e]*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(c*
d^2 - a*e^2))*Sqrt[((c*d^2 - a*e^2)^2 + 4*c*d*e*(a*d*e + (c*d^2 + a*e^2)*x
 + c*d*e*x^2))/((c*d^2 - a*e^2)^2*(1 + (2*Sqrt[c]*Sqrt[d]*Sqrt[e]*Sqrt[a*d
*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(c*d^2 - a*e^2))^2)]*EllipticE[2*ArcT
an[(Sqrt[2]*c^(1/4)*d^(1/4)*e^(1/4)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2
)^(1/4))/Sqrt[c*d^2 - a*e^2]], 1/2])/(Sqrt[2]*c^(1/4)*d^(1/4)*e^(1/4)*Sqrt
[(c*d^2 - a*e^2)^2 + 4*c*d*e*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)])))/(
Sqrt[c]*Sqrt[d]*Sqrt[e]) + ((c*d^2 - a*e^2)^(3/2)*(1 + (2*Sqrt[c]*Sqrt[d]*
Sqrt[e]*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(c*d^2 - a*e^2))*Sqrt
[((c*d^2 - a*e^2)^2 + 4*c*d*e*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2))/((c
*d^2 - a*e^2)^2*(1 + (2*Sqrt[c]*Sqrt[d]*Sqrt[e]*Sqrt[a*d*e + (c*d^2 + a*e^
2)*x + c*d*e*x^2])/(c*d^2 - a*e^2))^2)]*EllipticF[2*ArcTan[(Sqrt[2]*c^(...
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 761
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 834
```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, S
imp[1/q   Int[1/Sqrt[a + b*x^4], x], x] - Simp[1/q   Int[(1 - q*x^2)/Sqrt[a
 + b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 1094
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[4*(Sqrt[(b
 + 2*c*x)^2]/(b + 2*c*x))   Subst[Int[x^(4*(p + 1) - 1)/Sqrt[b^2 - 4*a*c + 4
*c*x^4], x], x, (a + b*x + c*x^2)^(1/4)], x] /; FreeQ[{a, b, c}, x] && Inte
gerQ[4*p]
```

rule 1160
```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[e*((a + b*x + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Simp[(2*c*d - b
*e)/(2*c)   Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
 && NeQ[p, -1]
```

rule 1166
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[e*(d + e*x)^(m - 1)*((a + b*x + c*x^2)^(p + 1)/(c*(m + 2*p +
 1))), x] + Simp[1/(c*(m + 2*p + 1))   Int[(d + e*x)^(m - 2)*Simp[c*d^2*(m
+ 2*p + 1) - e*(a*e*(m - 1) + b*d*(p + 1)) + e*(2*c*d - b*e)*(m + p)*x, x]*
(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && If[Ration
alQ[m], GtQ[m, 1], SumSimplerQ[m, -2]] && NeQ[m + 2*p + 1, 0] && IntQuadrat
icQ[a, b, c, d, e, m, p, x]
```

rule 1510
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

Maple [F]

$$\int \frac{(ex+d)^3}{(ade+(a\,e^2+c\,d^2)\,x+cd\,x^2e)^{\frac{1}{4}}}dx$$

input
```
int((e*x+d)^3/(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(1/4),x)
```

output
```
int((e*x+d)^3/(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(1/4),x)
```

Fricas [F]

$$\int \frac{(d+ex)^3}{\sqrt[4]{ade+(cd^2+ae^2)\,x+cdex^2}}\,dx = \int \frac{(ex+d)^3}{(cdex^2+ade+(cd^2+ae^2)x)^{\frac{1}{4}}}\,dx$$

input
```
integrate((e*x+d)^3/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/4),x, algorithm="
fricas")
```

output
```
integral((c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^(3/4)*(e^2*x^2 + 2*d*e*x
+ d^2)/(c*d*x + a*e), x)
```

Sympy [F]

$$\int \frac{(d+ex)^3}{\sqrt[4]{ade+(cd^2+ae^2)\,x+cdex^2}}\,dx = \int \frac{(d+ex)^3}{\sqrt[4]{(d+ex)\,(ae+cdx)}}\,dx$$

input
```
integrate((e*x+d)**3/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(1/4),x)
```

output
```
Integral((d + e*x)**3/((d + e*x)*(a*e + c*d*x))**(1/4), x)
```

Maxima [F]

$$\int \frac{(d+ex)^3}{\sqrt[4]{ade+(cd^2+ae^2)\,x+cdex^2}}\,dx = \int \frac{(ex+d)^3}{(cdex^2+ade+(cd^2+ae^2)x)^{\frac{1}{4}}}\,dx$$

input
```
integrate((e*x+d)^3/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/4),x, algorithm="
maxima")
```

output
```
integrate((e*x + d)^3/(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^(1/4), x)
```

Giac [F]

$$\int \frac{(d+ex)^3}{\sqrt[4]{ade+(cd^2+ae^2)\,x+cdex^2}}\,dx=\int \frac{(ex+d)^3}{(cdex^2+ade+(cd^2+ae^2)x)^{\frac{1}{4}}}\,dx$$

input
```
integrate((e*x+d)^3/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/4),x, algorithm="
giac")
```

output
```
integrate((e*x + d)^3/(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^(1/4), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d+ex)^3}{\sqrt[4]{ade+(cd^2+ae^2)\,x+cdex^2}}\,dx=\int \frac{(d+e\,x)^3}{\left(c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e\right)^{1/4}}\,dx$$

input
```
int((d + e*x)^3/(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/4),x)
```

output
```
int((d + e*x)^3/(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/4), x)
```

Reduce [F]

$$\begin{aligned}\int \frac{(d+ex)^3}{\sqrt[4]{ade+(cd^2+ae^2)\,x+cdex^2}}\,dx&=\left(\int \frac{x^3}{(cde\,x^2+a\,e^2x+c\,d^2x+ade)^{\frac{1}{4}}}dx\right)e^3\\&+3\left(\int \frac{x^2}{(cde\,x^2+a\,e^2x+c\,d^2x+ade)^{\frac{1}{4}}}dx\right)d\,e^2\\&+3\left(\int \frac{x}{(cde\,x^2+a\,e^2x+c\,d^2x+ade)^{\frac{1}{4}}}dx\right)d^2e\\&+\left(\int \frac{1}{(cde\,x^2+a\,e^2x+c\,d^2x+ade)^{\frac{1}{4}}}dx\right)d^3\end{aligned}$$

input
```
int((e*x+d)^3/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/4),x)
```

output
```
int(x**3/(a*d*e + a*e**2*x + c*d**2*x + c*d*e*x**2)**(1/4),x)*e**3 + 3*int
(x**2/(a*d*e + a*e**2*x + c*d**2*x + c*d*e*x**2)**(1/4),x)*d*e**2 + 3*int(
x/(a*d*e + a*e**2*x + c*d**2*x + c*d*e*x**2)**(1/4),x)*d**2*e + int(1/(a*d
*e + a*e**2*x + c*d**2*x + c*d*e*x**2)**(1/4),x)*d**3
```

3.362 $\int \frac{(d+ex)^2}{\sqrt[4]{ade+(cd^2+ae^2)\,x+cdex^2}}\,dx$

Optimal result

Integrand size = 37, antiderivative size = 230

$$\int \frac{(d+ex)^2}{\sqrt[4]{ade+(cd^2+ae^2)\,x+cdex^2}}\,dx = \frac{7(cd^2-ae^2)\left(ade+(cd^2+ae^2)\,x+cdex^2\right)^{3/4}}{15c^2d^2} + \frac{2(d+ex)\left(ade+(cd^2+ae^2)\,x+cdex^2\right)^{3/4}}{5cd} + \frac{7(cd^2-ae^2)^3\sqrt[4]{-\frac{cde(ae+cdx)(d+ex)}{(cd^2-ae^2)^2}}\,E\left(\frac{1}{2}\arcsin\left(\frac{ae^2+cd(d+2ex)}{cd^2-ae^2}\right)\middle|2\right)}{10\sqrt{2}c^3d^3e\sqrt[4]{ade+(cd^2+ae^2)\,x+cdex^2}}$$

output

```
7/15*(-a*e^2+c*d^2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/4)/c^2/d^2+2/5*(e
*x+d)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/4)/c/d+7/20*(-a*e^2+c*d^2)^3*(-
c*d*e*(c*d*x+a*e)*(e*x+d)/(-a*e^2+c*d^2)^2)^(1/4)*EllipticE(sin(1/2*arcsin
((a*e^2+c*d*(2*e*x+d))/(-a*e^2+c*d^2))),2^(1/2))*2^(1/2)/c^3/d^3/e/(a*d*e+
(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/4)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.05 (sec) , antiderivative size = 93, normalized size of antiderivative = 0.40

$$\int \frac{(d+ex)^2}{\sqrt[4]{ade+(cd^2+ae^2)\,x+cdex^2}}\,dx = \frac{4(d+ex)((ae+cdx)(d+ex))^{3/4}\,\text{Hypergeometric2F1}\left(-\frac{7}{4},\frac{3}{4},\frac{7}{4},\frac{e(ae+cdx)}{-cd^2+ae^2}\right)}{3cd\left(\frac{cd(d+ex)}{cd^2-ae^2}\right)^{7/4}}$$

input
```
Integrate[(d + e*x)^2/(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(1/4),x]
```

output
```
(4*(d + e*x)*((a*e + c*d*x)*(d + e*x))^(3/4)*Hypergeometric2F1[-7/4, 3/4,
7/4, (e*(a*e + c*d*x))/(-(c*d^2) + a*e^2)])/(3*c*d*((c*d*(d + e*x))/(c*d^2
 - a*e^2))^(7/4))
```

Rubi [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 1059 vs. 2(230) = 460.

Time = 0.77 (sec) , antiderivative size = 1059, normalized size of antiderivative = 4.60, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.189$, Rules used = {1166, 27, 1160, 1094, 834, 761, 1510}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex)^2}{\sqrt[4]{x\,(ae^2+cd^2)+ade+cdex^2}}\,dx$$

$$\Big\downarrow \text{1166}$$

$$\frac{2\int \frac{7e(cd^2-ae^2)(d+ex)}{4\sqrt[4]{cdex^2+(cd^2+ae^2)\,x+ade}}dx}{5cde}+\frac{2(d+ex)\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/4}}{5cd}$$

$$\Big\downarrow \text{27}$$

$$\frac{7\left(cd^2-ae^2\right)\int \frac{d+ex}{\sqrt[4]{cdex^2+\left(cd^2+ae^2\right)x+ade}}dx}{10cd}+\frac{2(d+ex)\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/4}}{5cd}$$

$$\Big\downarrow \; 1160$$

$$\frac{7\left(cd^2-ae^2\right)\left(\frac{\left(d^2-\frac{ae^2}{c}\right)\int \frac{1}{\sqrt[4]{cdex^2+\left(cd^2+ae^2\right)x+ade}}dx}{2d}+\frac{2\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/4}}{3cd}\right)}{10cd}+\frac{2(d+ex)\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/4}}{5cd}$$

$$\Big\downarrow \; 1094$$

$$\frac{7\left(cd^2-ae^2\right)\left(\frac{2\left(d^2-\frac{ae^2}{c}\right)\sqrt{\left(ae^2+cd^2+2cdex\right)^2}\int \frac{\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}{\sqrt{\left(cd^2-ae^2\right)^2+4cde\left(cdex^2+\left(cd^2+ae^2\right)x+ade\right)}}d\sqrt[4]{cdex^2+\left(cd^2+ae^2\right)x+ade}}{d\left(ae^2+cd^2+2cdex\right)}+\right.}{10cd}$$

$$\frac{2(d+ex)\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/4}}{5cd}$$

$$\Big\downarrow \; 834$$

$$\frac{7\left(cd^2-ae^2\right)\left(\frac{2\left(d^2-\frac{ae^2}{c}\right)\sqrt{\left(ae^2+cd^2+2cdex\right)^2}\left(\frac{\left(cd^2-ae^2\right)\int \frac{1}{\sqrt{\left(cd^2-ae^2\right)^2+4cde\left(cdex^2+\left(cd^2+ae^2\right)x+ade\right)}}d\sqrt[4]{cdex^2+\left(cd^2+ae^2\right)}}{2\sqrt{c}\sqrt{d}\sqrt{e}}\right.}{d\left(ae^2+cd^2+2cd\right.}\right.}{}$$

$$\frac{2(d+ex)\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/4}}{5cd}$$

$$\Big\downarrow \; 761$$

$$7\left(cd^2-ae^2\right)\left(\frac{2\left(d^2-\frac{ae^2}{c}\right)\sqrt{(ae^2+cd^2+2cdex)^2}\left(\frac{\left(cd^2-ae^2\right)^{3/2}\left(\frac{2\sqrt{c}\sqrt{d}\sqrt{e}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{cd^2-ae^2}+1\right)\sqrt{\frac{4cde\left(x\left(ae^2+cd^2\right)+ade+cdex\right.}{\left(cd^2-ae^2\right)^2\left(\frac{2\sqrt{c}\sqrt{d}\sqrt{e}\sqrt{x\left(ae^2+\right.}}{cd^2-a}\right.}}}{4\sqrt{2}c^{3/4}d^{3/4}e^{3/4}\sqrt{4cde\left(\right.}}\right.}{}\right.$$

$$\frac{2(d+ex)\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/4}}{5cd}$$

$$\Big\downarrow \; 1510$$

$$\frac{2\left(cdex^2+\left(cd^2+ae^2\right)x+ade\right)^{3/4}(d+ex)}{5cd}+$$

$$7\left(cd^2-ae^2\right)\left(\frac{2\left(d^2-\frac{ae^2}{c}\right)\sqrt{\left(cd^2+2cexd+ae^2\right)^2}\left(\frac{\left(cd^2-ae^2\right)^{3/2}\left(\frac{2\sqrt{c}\sqrt{d}\sqrt{e}\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}{cd^2-ae^2}+1\right)\sqrt{\frac{\left(cd^2-ae^2\right)^2+4cde\left(cdex^2+\left(\right.\right.}{\left(cd^2-ae^2\right)^2\left(\frac{2\sqrt{c}\sqrt{d}\sqrt{e}\sqrt{cdex^2+\left(\right.}}{cd^2-\right.}}}}{4\sqrt{2}c^{3/4}d^{3/4}e^{3/4}\sqrt{\left(cd^2-\right.}}\right.}{}\right.$$

input

```
Int[(d + e*x)^2/(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(1/4),x]
```

output
```
(2*(d + e*x)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/4))/(5*c*d) + (7*(
c*d^2 - a*e^2)*((2*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/4))/(3*c*d)
+ (2*(d^2 - (a*e^2)/c)*Sqrt[(c*d^2 + a*e^2 + 2*c*d*e*x)^2]*(-1/2*((c*d^2 -
 a*e^2)*(-(((a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(1/4)*Sqrt[(c*d^2 - a*
e^2)^2 + 4*c*d*e*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)])/((c*d^2 - a*e^2
)^2*(1 + (2*Sqrt[c]*Sqrt[d]*Sqrt[e]*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e
*x^2])/(c*d^2 - a*e^2)))) + (Sqrt[c*d^2 - a*e^2]*(1 + (2*Sqrt[c]*Sqrt[d]*S
qrt[e]*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(c*d^2 - a*e^2))*Sqrt[
((c*d^2 - a*e^2)^2 + 4*c*d*e*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2))/((c*
d^2 - a*e^2)^2*(1 + (2*Sqrt[c]*Sqrt[d]*Sqrt[e]*Sqrt[a*d*e + (c*d^2 + a*e^2
)*x + c*d*e*x^2])/(c*d^2 - a*e^2))^2)]*EllipticE[2*ArcTan[(Sqrt[2]*c^(1/4)
*d^(1/4)*e^(1/4)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(1/4))/Sqrt[c*d^2
 - a*e^2]], 1/2])/(Sqrt[2]*c^(1/4)*d^(1/4)*e^(1/4)*Sqrt[(c*d^2 - a*e^2)^2
+ 4*c*d*e*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)])))/(Sqrt[c]*Sqrt[d]*Sqr
t[e]) + ((c*d^2 - a*e^2)^(3/2)*(1 + (2*Sqrt[c]*Sqrt[d]*Sqrt[e]*Sqrt[a*d*e
+ (c*d^2 + a*e^2)*x + c*d*e*x^2])/(c*d^2 - a*e^2))*Sqrt[((c*d^2 - a*e^2)^2
 + 4*c*d*e*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2))/((c*d^2 - a*e^2)^2*(1
+ (2*Sqrt[c]*Sqrt[d]*Sqrt[e]*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/
(c*d^2 - a*e^2))^2)]*EllipticF[2*ArcTan[(Sqrt[2]*c^(1/4)*d^(1/4)*e^(1/4)*(
a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(1/4))/Sqrt[c*d^2 - a*e^2]], 1/2...
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 761
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 834
```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, S
imp[1/q   Int[1/Sqrt[a + b*x^4], x], x] - Simp[1/q   Int[(1 - q*x^2)/Sqrt[a
 + b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 1094
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[4*(Sqrt[(b
+ 2*c*x)^2]/(b + 2*c*x))   Subst[Int[x^(4*(p + 1) - 1)/Sqrt[b^2 - 4*a*c + 4
*c*x^4], x], x, (a + b*x + c*x^2)^(1/4)], x] /; FreeQ[{a, b, c}, x] && Inte
gerQ[4*p]
```

rule 1160
```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[e*((a + b*x + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Simp[(2*c*d - b
*e)/(2*c)   Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
 && NeQ[p, -1]
```

rule 1166
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[e*(d + e*x)^(m - 1)*((a + b*x + c*x^2)^(p + 1)/(c*(m + 2*p +
 1))), x] + Simp[1/(c*(m + 2*p + 1))   Int[(d + e*x)^(m - 2)*Simp[c*d^2*(m
+ 2*p + 1) - e*(a*e*(m - 1) + b*d*(p + 1)) + e*(2*c*d - b*e)*(m + p)*x, x]*
(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && If[Ration
alQ[m], GtQ[m, 1], SumSimplerQ[m, -2]] && NeQ[m + 2*p + 1, 0] && IntQuadrat
icQ[a, b, c, d, e, m, p, x]
```

rule 1510
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

Maple [F]

$$\int \frac{(ex+d)^2}{(ade+(a\,e^2+c\,d^2)\,x+cd\,x^2e)^{\frac{1}{4}}}dx$$

input
```
int((e*x+d)^2/(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(1/4),x)
```

output
```
int((e*x+d)^2/(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(1/4),x)
```

Fricas [F]

$$\int \frac{(d+ex)^2}{\sqrt[4]{ade+(cd^2+ae^2)\,x+cdex^2}}\,dx = \int \frac{(ex+d)^2}{(cdex^2+ade+(cd^2+ae^2)x)^{\frac{1}{4}}}\,dx$$

input
```
integrate((e*x+d)^2/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/4),x, algorithm="
fricas")
```

output
```
integral((c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^(3/4)*(e*x + d)/(c*d*x +
a*e), x)
```

Sympy [F]

$$\int \frac{(d+ex)^2}{\sqrt[4]{ade+(cd^2+ae^2)\,x+cdex^2}}\,dx = \int \frac{(d+ex)^2}{\sqrt[4]{(d+ex)\,(ae+cdx)}}\,dx$$

input
```
integrate((e*x+d)**2/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(1/4),x)
```

output
```
Integral((d + e*x)**2/((d + e*x)*(a*e + c*d*x))**(1/4), x)
```

Maxima [F]

$$\int \frac{(d+ex)^2}{\sqrt[4]{ade+(cd^2+ae^2)\,x+cdex^2}}\,dx = \int \frac{(ex+d)^2}{(cdex^2+ade+(cd^2+ae^2)x)^{\frac{1}{4}}}\,dx$$

input
```
integrate((e*x+d)^2/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/4),x, algorithm="
maxima")
```

output
```
integrate((e*x + d)^2/(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^(1/4), x)
```

Giac [F]

$$\int \frac{(d+ex)^2}{\sqrt[4]{ade+(cd^2+ae^2)\,x+cdex^2}}\,dx=\int \frac{(ex+d)^2}{(cdex^2+ade+(cd^2+ae^2)x)^{\frac{1}{4}}}\,dx$$

input
```
integrate((e*x+d)^2/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/4),x, algorithm="
giac")
```

output
```
integrate((e*x + d)^2/(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^(1/4), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d+ex)^2}{\sqrt[4]{ade+(cd^2+ae^2)\,x+cdex^2}}\,dx=\int \frac{(d+e\,x)^2}{(c\,d\,e\,x^2+(c\,d^2+a\,e^2)\,x+a\,d\,e)^{1/4}}\,dx$$

input
```
int((d + e*x)^2/(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/4),x)
```

output
```
int((d + e*x)^2/(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/4), x)
```

Reduce [F]

$$\int \frac{(d+ex)^2}{\sqrt[4]{ade+(cd^2+ae^2)\,x+cdex^2}}\,dx=\left(\int \frac{x^2}{(cde\,x^2+a\,e^2x+c\,d^2x+ade)^{\frac{1}{4}}}dx\right)e^2+2\left(\int \frac{x}{(cde\,x^2+a\,e^2x+c\,d^2x+ade)^{\frac{1}{4}}}dx\right)de+\left(\int \frac{1}{(cde\,x^2+a\,e^2x+c\,d^2x+ade)^{\frac{1}{4}}}dx\right)d^2$$

input
```
int((e*x+d)^2/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/4),x)
```

output
```
int(x**2/(a*d*e + a*e**2*x + c*d**2*x + c*d*e*x**2)**(1/4),x)*e**2 + 2*int
(x/(a*d*e + a*e**2*x + c*d**2*x + c*d*e*x**2)**(1/4),x)*d*e + int(1/(a*d*e
 + a*e**2*x + c*d**2*x + c*d*e*x**2)**(1/4),x)*d**2
```

3.363 $\int \frac{d+ex}{\sqrt[4]{ade+(cd^2+ae^2)x+cdex^2}}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 171

$$\int \frac{d+ex}{\sqrt[4]{ade+(cd^2+ae^2)x+cdex^2}}\,dx = \frac{2\left(ade+(cd^2+ae^2)x+cdex^2\right)^{3/4}}{3cd} + \frac{(cd^2-ae^2)^2\sqrt[4]{-\frac{cde(ae+cdx)(d+ex)}{(cd^2-ae^2)^2}}\,E\left(\frac{1}{2}\arcsin\left(\frac{ae^2+cd(d+2ex)}{cd^2-ae^2}\right)\middle|2\right)}{\sqrt{2}c^2d^2e\sqrt[4]{ade+(cd^2+ae^2)x+cdex^2}}$$

output
```
2/3*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/4)/c/d+1/2*(-a*e^2+c*d^2)^2*(-c*d
*e*(c*d*x+a*e)*(e*x+d)/(-a*e^2+c*d^2)^2)^(1/4)*EllipticE(sin(1/2*arcsin((a
*e^2+c*d*(2*e*x+d))/(-a*e^2+c*d^2))),2^(1/2))*2^(1/2)/c^2/d^2/e/(a*d*e+(a*
e^2+c*d^2)*x+c*d*e*x^2)^(1/4)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.03 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.51

$$\int \frac{d+ex}{\sqrt[4]{ade+(cd^2+ae^2)x+cdex^2}}\,dx = \frac{4((ae+cdx)(d+ex))^{3/4}\,\text{Hypergeometric2F1}\left(-\frac{3}{4},\frac{3}{4},\frac{7}{4},\frac{e(ae+cdx)}{-cd^2+ae^2}\right)}{3cd\left(\frac{cd(d+ex)}{cd^2-ae^2}\right)^{3/4}}$$

input
```
Integrate[(d + e*x)/(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(1/4),x]
```

output
```
(4*((a*e + c*d*x)*(d + e*x))^(3/4)*Hypergeometric2F1[-3/4, 3/4, 7/4, (e*(a
*e + c*d*x))/(-(c*d^2) + a*e^2)])/(3*c*d*((c*d*(d + e*x))/(c*d^2 - a*e^2))
^(3/4))
```

Rubi [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 992 vs. 2(171) = 342.

Time = 0.68 (sec) , antiderivative size = 992, normalized size of antiderivative = 5.80, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {1160, 1094, 834, 761, 1510}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{d+ex}{\sqrt[4]{x(ae^2+cd^2)+ade+cdex^2}}dx$$

$$\Big\downarrow \text{1160}$$

$$\frac{\left(d^2-\frac{ae^2}{c}\right)\int \frac{1}{\sqrt[4]{cdex^2+(cd^2+ae^2)x+ade}}dx}{2d}+\frac{2\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/4}}{3cd}$$

$$\Big\downarrow \text{1094}$$

$$\frac{2\left(d^2-\frac{ae^2}{c}\right)\sqrt{(ae^2+cd^2+2cdex)^2}\int\frac{\sqrt{cdex^2+(cd^2+ae^2)x+ade}}{\sqrt{(cd^2-ae^2)^2+4cde(cdex^2+(cd^2+ae^2)x+ade)}}d\sqrt[4]{cdex^2+(cd^2+ae^2)x+ade}}{d\,(ae^2+cd^2+2cdex)}+\frac{2\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/4}}{3cd}$$

$\Big\downarrow$ 834

$$\frac{2\left(d^2-\frac{ae^2}{c}\right)\sqrt{(ae^2+cd^2+2cdex)^2}\left(\frac{(cd^2-ae^2)\int\frac{1}{\sqrt{\left(cd^2-ae^2\right)^2+4cde\left(cdex^2+\left(cd^2+ae^2\right)x+ade\right)}}d\sqrt[4]{cdex^2+(cd^2+ae^2)x+}}{2\sqrt{c}\sqrt{d}\sqrt{e}}\right.}{d\,(ae^2+cd^2+2cdex)}$$

$$\frac{2\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/4}}{3cd}$$

$\Big\downarrow$ 761

$$2\left(d^2-\frac{ae^2}{c}\right)\sqrt{(ae^2+cd^2+2cdex)^2}\left(\frac{(cd^2-ae^2)^{3/2}\left(\frac{2\sqrt{c}\sqrt{d}\sqrt{e}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{cd^2-ae^2}+1\right)\sqrt{\frac{4cde\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)+}{\left(cd^2-ae^2\right)^2\left(\frac{2\sqrt{c}\sqrt{d}\sqrt{e}\sqrt{x\left(ae^2+cd^2\right)}}{cd^2-ae^2}\right.}}}{4\sqrt{2}c^{3/4}d^{3/4}e^{3/4}\sqrt{4cde(x(a}}\right.$$

$$\frac{2\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/4}}{3cd}$$

$\Big\downarrow$ 1510

$$\frac{2\left(d^2-\frac{ae^2}{c}\right)\sqrt{(cd^2+2cexd+ae^2)^2}\left(\frac{(cd^2-ae^2)^{3/2}\left(\frac{2\sqrt{c}\sqrt{d}\sqrt{e}\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}{cd^2-ae^2}+1\right)\sqrt{\frac{\left(cd^2-ae^2\right)^2+4cde\left(cdex^2+\left(cd^2+\right.\right.}{\left(cd^2-ae^2\right)^2\left(\frac{2\sqrt{c}\sqrt{d}\sqrt{e}\sqrt{cdex^2+\left(cd^2-\right.}}{cd^2-ae^2}\right.}}}{4\sqrt{2}c^{3/4}d^{3/4}e^{3/4}\sqrt{(cd^2-ae^2}}\right.}{}$$

$$\frac{2\left(cdex^2+\left(cd^2+ae^2\right)x+ade\right)^{3/4}}{3cd}$$

input
```
Int[(d + e*x)/(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(1/4),x]
```

output

```
(2*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/4))/(3*c*d) + (2*(d^2 - (a*e
^2)/c)*Sqrt[(c*d^2 + a*e^2 + 2*c*d*e*x)^2]*(-1/2*((c*d^2 - a*e^2)*(-(((a*d
*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(1/4)*Sqrt[(c*d^2 - a*e^2)^2 + 4*c*d*e
*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)])/((c*d^2 - a*e^2)^2*(1 + (2*Sqrt
[c]*Sqrt[d]*Sqrt[e]*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(c*d^2 -
a*e^2)))) + (Sqrt[c*d^2 - a*e^2]*(1 + (2*Sqrt[c]*Sqrt[d]*Sqrt[e]*Sqrt[a*d*
e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(c*d^2 - a*e^2))*Sqrt[((c*d^2 - a*e^2)
^2 + 4*c*d*e*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2))/((c*d^2 - a*e^2)^2*(
1 + (2*Sqrt[c]*Sqrt[d]*Sqrt[e]*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]
)/(c*d^2 - a*e^2))^2)]*EllipticE[2*ArcTan[(Sqrt[2]*c^(1/4)*d^(1/4)*e^(1/4)
*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(1/4))/Sqrt[c*d^2 - a*e^2]], 1/2]
)/(Sqrt[2]*c^(1/4)*d^(1/4)*e^(1/4)*Sqrt[(c*d^2 - a*e^2)^2 + 4*c*d*e*(a*d*e
 + (c*d^2 + a*e^2)*x + c*d*e*x^2)])))/(Sqrt[c]*Sqrt[d]*Sqrt[e]) + ((c*d^2
- a*e^2)^(3/2)*(1 + (2*Sqrt[c]*Sqrt[d]*Sqrt[e]*Sqrt[a*d*e + (c*d^2 + a*e^2
)*x + c*d*e*x^2])/(c*d^2 - a*e^2))*Sqrt[((c*d^2 - a*e^2)^2 + 4*c*d*e*(a*d*
e + (c*d^2 + a*e^2)*x + c*d*e*x^2))/((c*d^2 - a*e^2)^2*(1 + (2*Sqrt[c]*Sqr
t[d]*Sqrt[e]*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(c*d^2 - a*e^2))
^2)]*EllipticF[2*ArcTan[(Sqrt[2]*c^(1/4)*d^(1/4)*e^(1/4)*(a*d*e + (c*d^2 +
 a*e^2)*x + c*d*e*x^2)^(1/4))/Sqrt[c*d^2 - a*e^2]], 1/2])/(4*Sqrt[2]*c^(3/
4)*d^(3/4)*e^(3/4)*Sqrt[(c*d^2 - a*e^2)^2 + 4*c*d*e*(a*d*e + (c*d^2 + a...
```

Defintions of rubi rules used

rule 761

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 834

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, S
imp[1/q   Int[1/Sqrt[a + b*x^4], x], x] - Simp[1/q   Int[(1 - q*x^2)/Sqrt[a
 + b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 1094

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[4*(Sqrt[(b
+ 2*c*x)^2]/(b + 2*c*x))   Subst[Int[x^(4*(p + 1) - 1)/Sqrt[b^2 - 4*a*c + 4
*c*x^4], x], x, (a + b*x + c*x^2)^(1/4)], x] /; FreeQ[{a, b, c}, x] && Inte
gerQ[4*p]
```

rule 1160
```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[e*((a + b*x + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Simp[(2*c*d - b
*e)/(2*c)   Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
 && NeQ[p, -1]
```

rule 1510
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

Maple [F]

$$\int \frac{ex + d}{(ade + (a\,e^2 + c\,d^2)\,x + cd\,x^2e)^{\frac{1}{4}}}dx$$

input
```
int((e*x+d)/(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(1/4),x)
```

output
```
int((e*x+d)/(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(1/4),x)
```

Fricas [F]

$$\int \frac{d + ex}{\sqrt[4]{ade + (cd^2 + ae^2)\,x + cdex^2}}\,dx = \int \frac{ex + d}{(cdex^2 + ade + (cd^2 + ae^2)x)^{\frac{1}{4}}}\,dx$$

input
```
integrate((e*x+d)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/4),x, algorithm="fr
icas")
```

output
```
integral((c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^(3/4)/(c*d*x + a*e), x)
```

Sympy [F]

$$\int \frac{d+ex}{\sqrt[4]{ade+(cd^2+ae^2)\,x+cdex^2}}\,dx=\int \frac{d+ex}{\sqrt[4]{(d+ex)\,(ae+cdx)}}\,dx$$

input
```
integrate((e*x+d)/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(1/4),x)
```

output
```
Integral((d + e*x)/((d + e*x)*(a*e + c*d*x))**(1/4), x)
```

Maxima [F]

$$\int \frac{d+ex}{\sqrt[4]{ade+(cd^2+ae^2)\,x+cdex^2}}\,dx=\int \frac{ex+d}{(cdex^2+ade+(cd^2+ae^2)x)^{\frac{1}{4}}}\,dx$$

input
```
integrate((e*x+d)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/4),x, algorithm="ma
xima")
```

output
```
integrate((e*x + d)/(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^(1/4), x)
```

Giac [F]

$$\int \frac{d+ex}{\sqrt[4]{ade+(cd^2+ae^2)\,x+cdex^2}}\,dx=\int \frac{ex+d}{(cdex^2+ade+(cd^2+ae^2)x)^{\frac{1}{4}}}\,dx$$

input
```
integrate((e*x+d)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/4),x, algorithm="gi
ac")
```

output
```
integrate((e*x + d)/(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^(1/4), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{d+ex}{\sqrt[4]{ade+(cd^2+ae^2)\,x+cdex^2}}\,dx=\int \frac{d+e\,x}{\left(c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e\right)^{1/4}}\,dx$$

input
```
int((d + e*x)/(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/4),x)
```

output
```
int((d + e*x)/(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/4), x)
```

Reduce [F]

$$\int \frac{d+ex}{\sqrt[4]{ade+(cd^2+ae^2)\,x+cdex^2}}\,dx=\left(\int \frac{x}{(cde\,x^2+a\,e^2x+c\,d^2x+ade)^{\frac{1}{4}}}dx\right)e+\left(\int \frac{1}{(cde\,x^2+a\,e^2x+c\,d^2x+ade)^{\frac{1}{4}}}dx\right)d$$

input
```
int((e*x+d)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/4),x)
```

output
```
int(x/(a*d*e + a*e**2*x + c*d**2*x + c*d*e*x**2)**(1/4),x)*e + int(1/(a*d*
e + a*e**2*x + c*d**2*x + c*d*e*x**2)**(1/4),x)*d
```

3.364 $\int \frac{1}{\sqrt[4]{ade + (cd^2 + ae^2)x + cdex^2}} \, dx$

Optimal result

Integrand size = 29, antiderivative size = 141

$$\int \frac{1}{\sqrt[4]{ade + (cd^2 + ae^2)x + cdex^2}} \, dx = \frac{\sqrt{2}(cd^2 - ae^2)\sqrt[4]{-\frac{cde(ade + (cd^2 + ae^2)x + cdex^2)}{(cd^2 - ae^2)^2}} E\left(\frac{1}{2}\arcsin\left(\frac{ae^2 + cd(d + 2ex)}{cd^2 - ae^2}\right) \middle| 2\right)}{cde\sqrt[4]{ade + (cd^2 + ae^2)x + cdex^2}}$$

output
```
2^(1/2)*(-a*e^2+c*d^2)*(-c*d*e*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)/(-a*e^2+c
*d^2)^2)^(1/4)*EllipticE(sin(1/2*arcsin((a*e^2+c*d*(2*e*x+d))/(-a*e^2+c*d^
2))),2^(1/2))/c/d/e/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/4)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.05 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.67

$$\int \frac{1}{\sqrt[4]{ade+(cd^2+ae^2)x+cdex^2}}\,dx = \frac{4\sqrt[4]{\frac{cd(d+ex)}{cd^2-ae^2}}((ae+cdx)(d+ex))^{3/4}\,\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{3}{4},\frac{7}{4},\frac{e(ae+cdx)}{-cd^2+ae^2}\right)}{3cd(d+ex)}$$

input
```
Integrate[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(-1/4),x]
```

output
```
(4*((c*d*(d + e*x))/(c*d^2 - a*e^2))^(1/4)*((a*e + c*d*x)*(d + e*x))^(3/4)
*Hypergeometric2F1[1/4, 3/4, 7/4, (e*(a*e + c*d*x))/(-(c*d^2) + a*e^2)])/(
3*c*d*(d + e*x))
```

Rubi [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 936 vs. $2(141) = 282$.

Time = 0.60 (sec) , antiderivative size = 936, normalized size of antiderivative = 6.64, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.138$, Rules used = {1094, 834, 761, 1510}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt[4]{x(ae^2+cd^2)+ade+cdex^2}}\,dx$$

$$\Big\downarrow \text{1094}$$

$$\frac{4\sqrt{(ae^2+cd^2+2cdex)^2}\int \frac{\sqrt{cdex^2+(cd^2+ae^2)x+ade}}{\sqrt{(cd^2-ae^2)^2+4cde(cdex^2+(cd^2+ae^2)x+ade)}}d\sqrt[4]{cdex^2+(cd^2+ae^2)x+ade}}{ae^2+cd^2+2cdex}$$

$$\Big\downarrow \text{834}$$

$$\frac{4\sqrt{(ae^2+cd^2+2cdex)^2}\left(\frac{(cd^2-ae^2)\int\frac{1}{\sqrt{\left(cd^2-ae^2\right)^2+4cde\left(cdex^2+\left(cd^2+ae^2\right)x+ade\right)}}d\sqrt[4]{cdex^2+(cd^2+ae^2)\,x+ade}}{2\sqrt{c}\sqrt{d}\sqrt{e}}-\frac{(cd^2-}{}\right.}{ae^2+cd^2+2cdex}$$

$\downarrow$ 761

$$\frac{4\sqrt{(ae^2+cd^2+2cdex)^2}\left(\frac{(cd^2-ae^2)^{3/2}\left(\frac{2\sqrt{c}\sqrt{d}\sqrt{e}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{cd^2-ae^2}+1\right)\sqrt{\frac{4cde\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)+\left(cd^2-ae^2\right)^2}{\left(cd^2-ae^2\right)^2\left(\frac{2\sqrt{c}\sqrt{d}\sqrt{e}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{cd^2-ae^2}+1\right.}}}{4\sqrt{2}c^{3/4}d^{3/4}e^{3/4}\sqrt{4cde(x(ae^2+cd^2)+ade+}}\right.}{}$$

$\downarrow$ 1510

$$\frac{4\sqrt{(cd^2+2cexd+ae^2)^2}\left(\frac{(cd^2-ae^2)^{3/2}\left(\frac{2\sqrt{c}\sqrt{d}\sqrt{e}\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}{cd^2-ae^2}+1\right)\sqrt{\frac{\left(cd^2-ae^2\right)^2+4cde\left(cdex^2+\left(cd^2+ae^2\right)x+ade\right)}{\left(cd^2-ae^2\right)^2\left(\frac{2\sqrt{c}\sqrt{d}\sqrt{e}\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}{cd^2-ae^2}+1\right.}}}{4\sqrt{2}c^{3/4}d^{3/4}e^{3/4}\sqrt{(cd^2-ae^2)^2+4cde(cdex}}\right.}{}$$

input

```
Int[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(-1/4),x]
```

output
```
(4*Sqrt[(c*d^2 + a*e^2 + 2*c*d*e*x)^2]*(-1/2*((c*d^2 - a*e^2)*(-(((a*d*e +
 (c*d^2 + a*e^2)*x + c*d*e*x^2)^(1/4)*Sqrt[(c*d^2 - a*e^2)^2 + 4*c*d*e*(a*
d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)])/((c*d^2 - a*e^2)^2*(1 + (2*Sqrt[c]*
Sqrt[d]*Sqrt[e]*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(c*d^2 - a*e^
2)))) + (Sqrt[c*d^2 - a*e^2]*(1 + (2*Sqrt[c]*Sqrt[d]*Sqrt[e]*Sqrt[a*d*e +
(c*d^2 + a*e^2)*x + c*d*e*x^2])/(c*d^2 - a*e^2))*Sqrt[((c*d^2 - a*e^2)^2 +
 4*c*d*e*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2))/((c*d^2 - a*e^2)^2*(1 +
(2*Sqrt[c]*Sqrt[d]*Sqrt[e]*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(c
*d^2 - a*e^2))^2)]*EllipticE[2*ArcTan[(Sqrt[2]*c^(1/4)*d^(1/4)*e^(1/4)*(a*
d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(1/4))/Sqrt[c*d^2 - a*e^2]], 1/2])/(S
qrt[2]*c^(1/4)*d^(1/4)*e^(1/4)*Sqrt[(c*d^2 - a*e^2)^2 + 4*c*d*e*(a*d*e + (
c*d^2 + a*e^2)*x + c*d*e*x^2)])))/(Sqrt[c]*Sqrt[d]*Sqrt[e]) + ((c*d^2 - a*
e^2)^(3/2)*(1 + (2*Sqrt[c]*Sqrt[d]*Sqrt[e]*Sqrt[a*d*e + (c*d^2 + a*e^2)*x
+ c*d*e*x^2])/(c*d^2 - a*e^2))*Sqrt[((c*d^2 - a*e^2)^2 + 4*c*d*e*(a*d*e +
(c*d^2 + a*e^2)*x + c*d*e*x^2))/((c*d^2 - a*e^2)^2*(1 + (2*Sqrt[c]*Sqrt[d]
*Sqrt[e]*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(c*d^2 - a*e^2))^2)]
*EllipticF[2*ArcTan[(Sqrt[2]*c^(1/4)*d^(1/4)*e^(1/4)*(a*d*e + (c*d^2 + a*e
^2)*x + c*d*e*x^2)^(1/4))/Sqrt[c*d^2 - a*e^2]], 1/2])/(4*Sqrt[2]*c^(3/4)*d
^(3/4)*e^(3/4)*Sqrt[(c*d^2 - a*e^2)^2 + 4*c*d*e*(a*d*e + (c*d^2 + a*e^2)*x
 + c*d*e*x^2)])))/(c*d^2 + a*e^2 + 2*c*d*e*x)
```

Defintions of rubi rules used

rule 761
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 834
```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, S
imp[1/q   Int[1/Sqrt[a + b*x^4], x], x] - Simp[1/q   Int[(1 - q*x^2)/Sqrt[a
 + b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 1094
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[4*(Sqrt[(b
+ 2*c*x)^2]/(b + 2*c*x))   Subst[Int[x^(4*(p + 1) - 1)/Sqrt[b^2 - 4*a*c + 4
*c*x^4], x], x, (a + b*x + c*x^2)^(1/4)], x] /; FreeQ[{a, b, c}, x] && Inte
gerQ[4*p]
```

rule 1510
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

Maple [F]

$$\int \frac{1}{\left(ade+\left(a\,e^{2}+c\,d^{2}\right)x+cd\,x^{2}e\right)^{\frac{1}{4}}}dx$$

input
```
int(1/(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(1/4),x)
```

output
```
int(1/(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(1/4),x)
```

Fricas [F]

$$\int \frac{1}{\sqrt[4]{ade+(cd^2+ae^2)\,x+cdex^2}}\,dx=\int \frac{1}{\left(cdex^2+ade+(cd^2+ae^2)x\right)^{\frac{1}{4}}}\,dx$$

input
```
integrate(1/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/4),x, algorithm="fricas")
```

output
```
integral((c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^(-1/4), x)
```

Sympy [F]

$$\int \frac{1}{\sqrt[4]{ade+(cd^2+ae^2)\,x+cdex^2}}\,dx=\int \frac{1}{\sqrt[4]{ade+cdex^2+x\,(ae^2+cd^2)}}\,dx$$

input
```
integrate(1/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(1/4),x)
```

output
```
Integral((a*d*e + c*d*e*x**2 + x*(a*e**2 + c*d**2))**(-1/4), x)
```

Maxima [F]

$$\int \frac{1}{\sqrt[4]{ade+(cd^2+ae^2)\,x+cdex^2}}\,dx=\int \frac{1}{(cdex^2+ade+(cd^2+ae^2)x)^{\frac{1}{4}}}\,dx$$

input
```
integrate(1/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/4),x, algorithm="maxima")
```

output
```
integrate((c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^(-1/4), x)
```

Giac [F]

$$\int \frac{1}{\sqrt[4]{ade+(cd^2+ae^2)\,x+cdex^2}}\,dx=\int \frac{1}{(cdex^2+ade+(cd^2+ae^2)x)^{\frac{1}{4}}}\,dx$$

input
```
integrate(1/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/4),x, algorithm="giac")
```

output
```
integrate((c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^(-1/4), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt[4]{ade+(cd^2+ae^2)\,x+cdex^2}}\,dx=\int \frac{1}{\left(c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e\right)^{1/4}}\,dx$$

input
```
int(1/(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/4),x)
```

output
```
int(1/(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/4), x)
```

Reduce [F]

$$\int \frac{1}{\sqrt[4]{ade + (cd^2 + ae^2)\,x + cdex^2}}\,dx = \int \frac{1}{(cde\,x^2 + a\,e^2x + c\,d^2x + ade)^{\frac{1}{4}}}dx$$

input
```
int(1/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/4),x)
```

output
```
int(1/(a*d*e + a*e**2*x + c*d**2*x + c*d*e*x**2)**(1/4),x)
```

3.365 $\int \frac{1}{(d+ex)\sqrt[4]{ade+(cd^2+ae^2)\,x+cdex^2}}\,dx$

Optimal result

Integrand size = 37, antiderivative size = 130

$$\int \frac{1}{(d+ex)\sqrt[4]{ade+(cd^2+ae^2)\,x+cdex^2}}\,dx = -\frac{4\sqrt{ae+cdx}\sqrt[4]{\frac{cd(d+ex)}{e(ae+cdx)}}E\left(\frac{1}{2}\arctan\left(\frac{\sqrt{cd^2-ae^2}}{\sqrt{e}\sqrt{ae+cdx}}\right)\middle|2\right)}{\sqrt{e}\sqrt{cd^2-ae^2}\sqrt[4]{ade+(cd^2+ae^2)\,x+cdex^2}}$$

output
```
-4*(c*d*x+a*e)^(1/2)*(c*d*(e*x+d)/e/(c*d*x+a*e))^(1/4)*EllipticE(sin(1/2*a
rctan((-a*e^2+c*d^2)^(1/2)/e^(1/2)/(c*d*x+a*e)^(1/2))),2^(1/2))/e^(1/2)/(-
a*e^2+c*d^2)^(1/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/4)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.05 (sec) , antiderivative size = 98, normalized size of antiderivative = 0.75

$$\int \frac{1}{(d+ex)\sqrt[4]{ade+(cd^2+ae^2)x+cdex^2}}\,dx = \frac{4cd((ae+cdx)(d+ex))^{3/4}\,\text{Hypergeometric2F1}\left(\frac{3}{4},\frac{5}{4},\frac{7}{4},\frac{e(ae+cdx)}{-cd^2+ae^2}\right)}{3\,(cd^2-ae^2)^2\left(\frac{cd(d+ex)}{cd^2-ae^2}\right)^{3/4}}$$

input
```
Integrate[1/((d + e*x)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(1/4)),x]
```

output
```
(4*c*d*((a*e + c*d*x)*(d + e*x))^(3/4)*Hypergeometric2F1[3/4, 5/4, 7/4, (e
*(a*e + c*d*x))/(-(c*d^2) + a*e^2)])/(3*(c*d^2 - a*e^2)^2*((c*d*(d + e*x))
/(c*d^2 - a*e^2))^(3/4))
```

Rubi [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 329 vs. 2(130) = 260.

Time = 0.51 (sec) , antiderivative size = 329, normalized size of antiderivative = 2.53, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.216$, Rules used = {1138, 61, 73, 839, 813, 858, 807, 212}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(d+ex)\sqrt[4]{x(ae^2+cd^2)+ade+cdex^2}}dx$$

$$\Big\downarrow \; 1138$$

$$\frac{\sqrt[4]{\frac{ex}{d}+1}\sqrt[4]{ae+cdx}\int \frac{1}{\sqrt[4]{ae+cdx}\left(\frac{ex}{d}+1\right)^{5/4}}dx}{d\sqrt[4]{x(ae^2+cd^2)+ade+cdex^2}}$$

$$\Big\downarrow \; 61$$

$$\frac{\sqrt[4]{\frac{ex}{d}+1}\sqrt[4]{ae+cdx}\left(\frac{4d(ae+cdx)^{3/4}}{\sqrt[4]{\frac{ex}{d}+1}(cd^2-ae^2)}-\frac{2cd^2\int\frac{1}{\sqrt[4]{ae+cdx}\sqrt[4]{\frac{ex}{d}+1}}dx}{cd^2-ae^2}\right)}{d\sqrt[4]{x(ae^2+cd^2)+ade+cdex^2}}$$

$\Big\downarrow$ 73

$$\frac{\sqrt[4]{\frac{ex}{d}+1}\sqrt[4]{ae+cdx}\left(\frac{4d(ae+cdx)^{3/4}}{\sqrt[4]{\frac{ex}{d}+1}(cd^2-ae^2)}-\frac{8d\int\frac{\sqrt{ae+cdx}}{\sqrt[4]{-\frac{ae^2}{cd^2}+\frac{(ae+cdx)e}{cd^2}+1}}d\sqrt[4]{ae+cdx}}{cd^2-ae^2}\right)}{d\sqrt[4]{x(ae^2+cd^2)+ade+cdex^2}}$$

$\Big\downarrow$ 839

$$\frac{\sqrt[4]{\frac{ex}{d}+1}\sqrt[4]{ae+cdx}\left(\frac{4d(ae+cdx)^{3/4}}{\sqrt[4]{\frac{ex}{d}+1}(cd^2-ae^2)}-\frac{8d\left(\frac{(ae+cdx)^{3/4}}{2\sqrt[4]{-\frac{ae^2}{cd^2}+\frac{e(ae+cdx)}{cd^2}+1}}-\frac{1}{2}\left(1-\frac{ae^2}{cd^2}\right)\int\frac{\sqrt{ae+cdx}}{\left(-\frac{ae^2}{cd^2}+\frac{(ae+cdx)e}{cd^2}+1\right)^{5/4}}d\sqrt[4]{\cdots}\right.}{cd^2-ae^2}\right.}{d\sqrt[4]{x(ae^2+cd^2)+ade+cdex^2}}$$

$\Big\downarrow$ 813

$$\frac{\sqrt[4]{\frac{ex}{d}+1}\sqrt[4]{ae+cdx}\left(\frac{4d(ae+cdx)^{3/4}}{\sqrt[4]{\frac{ex}{d}+1}(cd^2-ae^2)}-\frac{8d\left(\frac{(ae+cdx)^{3/4}}{2\sqrt[4]{-\frac{ae^2}{cd^2}+\frac{e(ae+cdx)}{cd^2}+1}}-\frac{cd^2\left(1-\frac{ae^2}{cd^2}\right)\sqrt[4]{ae+cdx}\sqrt[4]{\frac{cd^2-ae^2}{e(ae+cd\cdots}}}{2e\sqrt[4]{-\frac{ae^2}{cd^2}\cdots}}\right.}{cd^2-ae^2}\right.}{d\sqrt[4]{x(ae^2+cd^2)+ade+cdex^2}}$$

$\Big\downarrow$ 858

$$\frac{\sqrt[4]{\frac{ex}{d}+1}\sqrt[4]{ae+cdx}\left(\frac{4d(ae+cdx)^{3/4}}{\sqrt[4]{\frac{ex}{d}+1}(cd^2-ae^2)}-\frac{8d\left(\frac{cd^2\left(1-\frac{ae^2}{cd^2}\right)\sqrt[4]{ae+cdx}\sqrt[4]{\frac{cd^2-ae^2}{e(ae+cdx)}+1}\int\frac{1}{\sqrt[4]{ae+cdx}\left(\frac{\left(cd^2-ae^2\right)(ae\cdots}{e}\cdots\right.}\cdots}{2e\sqrt[4]{-\frac{ae^2}{cd^2}+\frac{e(ae+cdx)}{cd^2}+1}}\cdots\right.}{cd^2-ae^2\cdots}\cdots\right.}{d\sqrt[4]{x\left(ae^2+cd^2\right)+ade+cdex^2}\cdots}$$

$\downarrow$ 807

$$\frac{\sqrt[4]{\frac{ex}{d}+1}\sqrt[4]{ae+cdx}\left(\frac{4d(ae+cdx)^{3/4}}{\sqrt[4]{\frac{ex}{d}+1}(cd^2-ae^2)}-\frac{8d\left(\frac{cd^2\left(1-\frac{ae^2}{cd^2}\right)\sqrt[4]{ae+cdx}\sqrt[4]{\frac{cd^2-ae^2}{e(ae+cdx)}+1}\int\frac{1}{\left(\frac{\sqrt{ae+cdx}\left(cd^2-ae^2\right)}{e}+1\right)^{5/4}}\cdots}{4e\sqrt[4]{-\frac{ae^2}{cd^2}+\frac{e(ae+cdx)}{cd^2}+1}}\cdots\right.}{cd^2-ae^2\cdots}\cdots\right.}{d\sqrt[4]{x\left(ae^2+cd^2\right)+ade+cdex^2}\cdots}$$

$\downarrow$ 212

$$\frac{\sqrt[4]{\frac{ex}{d}+1}\sqrt[4]{ae+cdx}\Bigg(\frac{4d(ae+cdx)^{3/4}}{\sqrt[4]{\frac{ex}{d}+1}(cd^2-ae^2)}-\frac{8d\Bigg(\frac{cd^2\left(1-\frac{ae^2}{cd^2}\right)\sqrt[4]{ae+cdx}\sqrt[4]{\frac{cd^2-ae^2}{e(ae+cdx)}+1}E\Big(\frac{1}{2}\arctan\Big(\frac{\sqrt{cd^2-ae^2}\sqrt{ae+cdx}}{\sqrt{e}}}{2\sqrt{e}\sqrt{cd^2-ae^2}\sqrt[4]{-\frac{ae^2}{cd^2}+\frac{e(ae+cdx)}{cd^2}+1}}}{cd^2-ae^2}}{d\sqrt[4]{x\left(ae^2+cd^2\right)+ade+cdex^2}}$$

input
```
Int[1/((d + e*x)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(1/4)),x]
```

output
```
((a*e + c*d*x)^(1/4)*(1 + (e*x)/d)^(1/4)*((4*d*(a*e + c*d*x)^(3/4))/((c*d^
2 - a*e^2)*(1 + (e*x)/d)^(1/4)) - (8*d*((a*e + c*d*x)^(3/4)/(2*(1 - (a*e^2
)/(c*d^2) + (e*(a*e + c*d*x))/(c*d^2))^(1/4)) + (c*d^2*(1 - (a*e^2)/(c*d^2
))*(a*e + c*d*x)^(1/4)*(1 + (c*d^2 - a*e^2)/(e*(a*e + c*d*x)))^(1/4)*Ellip
ticE[ArcTan[(Sqrt[c*d^2 - a*e^2]*Sqrt[a*e + c*d*x])/Sqrt[e]]/2, 2])/(2*Sqr
t[e]*Sqrt[c*d^2 - a*e^2]*(1 - (a*e^2)/(c*d^2) + (e*(a*e + c*d*x))/(c*d^2))
^(1/4))))/(c*d^2 - a*e^2)))/(d*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(1/
4))
```

Defintions of rubi rules used

rule 61
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && LtQ[m, -1] &&  !(LtQ[n, -1] && (EqQ[a, 0
] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && IntLinearQ[a, b, c, d
, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 212
```
Int[((a_) + (b_.)*(x_)^2)^(-5/4), x_Symbol] :> Simp[(2/(a^(5/4)*Rt[b/a, 2])
)*EllipticE[(1/2)*ArcTan[Rt[b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

rule 807
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Simp[1/k   Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x,
x^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

rule 813
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4)^(5/4), x_Symbol] :> Simp[x*((1 + a/(b*x^4)
)^(1/4)/(b*(a + b*x^4)^(1/4)))   Int[1/(x^3*(1 + a/(b*x^4))^(5/4)), x], x]
/; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 839
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4)^(1/4), x_Symbol] :> Simp[x^3/(2*(a + b*x^4
)^(1/4)), x] - Simp[a/2   Int[x^2/(a + b*x^4)^(5/4), x], x] /; FreeQ[{a, b}
, x] && PosQ[b/a]
```

rule 858
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

rule 1138
```
Int[((d_) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Sy
mbol] :> Simp[d^m*((a + b*x + c*x^2)^FracPart[p]/((1 + e*(x/d))^FracPart[p]
*(a/d + (c*x)/e)^FracPart[p]))   Int[(1 + e*(x/d))^(m + p)*(a/d + (c/e)*x)^
p, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0]
&& (IntegerQ[m] || GtQ[d, 0]) &&  !(IGtQ[m, 0] && (IntegerQ[3*p] || Integer
Q[4*p]))
```

Maple [F]

$$\int \frac{1}{(ex+d)\left(ade+\left(a\,e^{2}+c\,d^{2}\right)x+cd\,x^{2}e\right)^{\frac{1}{4}}}dx$$

input
```
int(1/(e*x+d)/(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(1/4),x)
```

output
```
int(1/(e*x+d)/(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(1/4),x)
```

Fricas [F]

$$\int \frac{1}{(d+ex)\sqrt[4]{ade+(cd^2+ae^2)\,x+cdex^2}}\,dx = \int \frac{1}{(cdex^2+ade+(cd^2+ae^2)x)^{\frac{1}{4}}(ex+d)}\,dx$$

input
```
integrate(1/(e*x+d)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/4),x, algorithm="
fricas")
```

output
```
integral((c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^(3/4)/(c*d*e^2*x^3 + a*d^
2*e + (2*c*d^2*e + a*e^3)*x^2 + (c*d^3 + 2*a*d*e^2)*x), x)
```

Sympy [F]

$$\int \frac{1}{(d+ex)\sqrt[4]{ade+(cd^2+ae^2)\,x+cdex^2}}\,dx = \int \frac{1}{\sqrt[4]{(d+ex)\,(ae+cdx)}\,(d+ex)}\,dx$$

input
```
integrate(1/(e*x+d)/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(1/4),x)
```

output
```
Integral(1/(((d + e*x)*(a*e + c*d*x))**(1/4)*(d + e*x)), x)
```

Maxima [F]

$$\int \frac{1}{(d+ex)\sqrt[4]{ade+(cd^2+ae^2)\,x+cdex^2}}\,dx = \int \frac{1}{(cdex^2+ade+(cd^2+ae^2)x)^{\frac{1}{4}}(ex+d)}\,dx$$

input
```
integrate(1/(e*x+d)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/4),x, algorithm="
maxima")
```

output
```
integrate(1/((c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^(1/4)*(e*x + d)), x)
```

Giac [F]

$$\int \frac{1}{(d+ex)\sqrt[4]{ade+(cd^2+ae^2)\,x+cdex^2}}\,dx = \int \frac{1}{(cdex^2+ade+(cd^2+ae^2)x)^{\frac{1}{4}}(ex+d)}\,dx$$

input
```
integrate(1/(e*x+d)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/4),x, algorithm="
giac")
```

output
```
integrate(1/((c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^(1/4)*(e*x + d)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(d+ex)\sqrt[4]{ade+(cd^2+ae^2)\,x+cdex^2}}\,dx = \int \frac{1}{(d+e\,x)\,\left(c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e\right)^{1/4}}\,dx$$

input
```
int(1/((d + e*x)*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/4)),x)
```

output
```
int(1/((d + e*x)*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/4)), x)
```

Reduce [F]

$$\int \frac{1}{(d+ex)\sqrt[4]{ade+(cd^2+ae^2)\,x+cdex^2}}\,dx$$
$$=\int \frac{1}{(cde\,x^2+a\,e^2x+c\,d^2x+ade)^{\frac{1}{4}}\,d+(cde\,x^2+a\,e^2x+c\,d^2x+ade)^{\frac{1}{4}}\,ex}dx$$

input
```
int(1/(e*x+d)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/4),x)
```

output
```
int(1/((a*d*e + a*e**2*x + c*d**2*x + c*d*e*x**2)**(1/4)*d + (a*d*e + a*e*
*2*x + c*d**2*x + c*d*e*x**2)**(1/4)*e*x),x)
```

3.366 $\int \frac{1}{(d+ex)^2 \sqrt[4]{ade+(cd^2+ae^2)x+cdex^2}}\,dx$

Optimal result

Integrand size = 37, antiderivative size = 189

$$\int \frac{1}{(d+ex)^2 \sqrt[4]{ade+(cd^2+ae^2)x+cdex^2}}\,dx = \frac{4(ade+(cd^2+ae^2)x+cdex^2)^{3/4}}{5(cd^2-ae^2)(d+ex)^2} - \frac{8cd\sqrt{ae+cdx}\sqrt[4]{\frac{cd(d+ex)}{e(ae+cdx)}}E\left(\frac{1}{2}\arctan\left(\frac{\sqrt{cd^2-ae^2}}{\sqrt{e}\sqrt{ae+cdx}}\right)\middle|2\right)}{5\sqrt{e}(cd^2-ae^2)^{3/2}\sqrt[4]{ade+(cd^2+ae^2)x+cdex^2}}$$

output
```
4/5*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/4)/(-a*e^2+c*d^2)/(e*x+d)^2-8/5*c
*d*(c*d*x+a*e)^(1/2)*(c*d*(e*x+d)/e/(c*d*x+a*e))^(1/4)*EllipticE(sin(1/2*a
rctan((-a*e^2+c*d^2)^(1/2)/e^(1/2)/(c*d*x+a*e)^(1/2))),2^(1/2))/e^(1/2)/(-
a*e^2+c*d^2)^(3/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/4)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.05 (sec) , antiderivative size = 102, normalized size of antiderivative = 0.54

$$\int \frac{1}{(d+ex)^2\sqrt[4]{ade+(cd^2+ae^2)\,x+cdex^2}}\,dx = \frac{4c^2d^2((ae+cdx)(d+ex))^{3/4}\,\text{Hypergeometric2F1}\left(\frac{3}{4},\frac{9}{4},\frac{7}{4},\frac{e(ae+cdx)}{-cd^2+ae^2}\right)}{3\,(cd^2-ae^2)^3\left(\frac{cd(d+ex)}{cd^2-ae^2}\right)^{3/4}}$$

input
```
Integrate[1/((d + e*x)^2*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(1/4)),x]
```

output
```
(4*c^2*d^2*((a*e + c*d*x)*(d + e*x))^(3/4)*Hypergeometric2F1[3/4, 9/4, 7/4
, (e*(a*e + c*d*x))/(-(c*d^2) + a*e^2)])/(3*(c*d^2 - a*e^2)^3*((c*d*(d + e
*x))/(c*d^2 - a*e^2))^(3/4))
```

Rubi [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 394 vs. 2(189) = 378.

Time = 0.53 (sec) , antiderivative size = 394, normalized size of antiderivative = 2.08, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.243$, Rules used = {1138, 61, 61, 73, 839, 813, 858, 807, 212}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(d+ex)^2\sqrt[4]{x\,(ae^2+cd^2)+ade+cdex^2}}\,dx$$

$$\Big\downarrow \text{ 1138}$$

$$\frac{\sqrt[4]{\frac{ex}{d}+1}\sqrt[4]{ae+cdx}\int \frac{1}{\sqrt[4]{ae+cdx}\left(\frac{ex}{d}+1\right)^{9/4}}dx}{d^2\sqrt[4]{x\,(ae^2+cd^2)+ade+cdex^2}}$$

$$\Big\downarrow \text{ 61}$$

$$\frac{\sqrt[4]{\frac{ex}{d}+1}\sqrt[4]{ae+cdx}\left(\frac{2cd\int\frac{1}{\sqrt[4]{ae+cdx}\left(\frac{ex}{d}+1\right)^{5/4}}dx}{5\left(cd-\frac{ae^2}{d}\right)}+\frac{4(ae+cdx)^{3/4}}{5\left(\frac{ex}{d}+1\right)^{5/4}\left(cd-\frac{ae^2}{d}\right)}\right)}{d^2\sqrt[4]{x\left(ae^2+cd^2\right)+ade+cdex^2}}$$

$\Big\downarrow$ 61

$$\frac{\sqrt[4]{\frac{ex}{d}+1}\sqrt[4]{ae+cdx}\left(\frac{2cd\left(\frac{4d(ae+cdx)^{3/4}}{\sqrt[4]{\frac{ex}{d}+1}\left(cd^2-ae^2\right)}-\frac{2cd^2\int\frac{1}{\sqrt[4]{ae+cdx}\sqrt[4]{\frac{ex}{d}+1}}dx}{cd^2-ae^2}\right)}{5\left(cd-\frac{ae^2}{d}\right)}+\frac{4(ae+cdx)^{3/4}}{5\left(\frac{ex}{d}+1\right)^{5/4}\left(cd-\frac{ae^2}{d}\right)}\right)}{d^2\sqrt[4]{x\left(ae^2+cd^2\right)+ade+cdex^2}}$$

$\Big\downarrow$ 73

$$\frac{\sqrt[4]{\frac{ex}{d}+1}\sqrt[4]{ae+cdx}\left(\frac{2cd\left(\frac{4d(ae+cdx)^{3/4}}{\sqrt[4]{\frac{ex}{d}+1}\left(cd^2-ae^2\right)}-\frac{8d\int\frac{\sqrt{ae+cdx}}{\sqrt[4]{-\frac{ae^2}{cd^2}+\frac{(ae+cdx)e}{cd^2}+1}}d\sqrt[4]{ae+cdx}}{cd^2-ae^2}\right)}{5\left(cd-\frac{ae^2}{d}\right)}+\frac{4(ae+cdx)^{3/4}}{5\left(\frac{ex}{d}+1\right)^{5/4}\left(cd-\frac{ae^2}{d}\right)}\right)}{d^2\sqrt[4]{x\left(ae^2+cd^2\right)+ade+cdex^2}}$$

$\Big\downarrow$ 839

$$\frac{\sqrt[4]{\frac{ex}{d}+1}\sqrt[4]{ae+cdx}\left(\frac{2cd\left(\frac{4d(ae+cdx)^{3/4}}{\sqrt[4]{\frac{ex}{d}+1}(cd^2-ae^2)}-\frac{8d\left(\frac{(ae+cdx)^{3/4}}{2\sqrt[4]{-\frac{ae^2}{cd^2}+\frac{e(ae+cdx)}{cd^2}+1}}-\frac{1}{2}\left(1-\frac{ae^2}{cd^2}\right)\int \frac{\sqrt{ae+cdx}}{\left(-\frac{ae^2}{cd^2}+\frac{(ae+cdx)e}{cd^2}+1\right)^{5/4}}d\right.}{cd^2-ae^2}\right)}{5\left(cd-\frac{ae^2}{d}\right)}\right.}{d^2\sqrt[4]{x\left(ae^2+cd^2\right)+ade+cdex^2}}$$

↓ 813

$$\frac{\sqrt[4]{\frac{ex}{d}+1}\sqrt[4]{ae+cdx}\left(\frac{2cd\left(\frac{4d(ae+cdx)^{3/4}}{\sqrt[4]{\frac{ex}{d}+1}(cd^2-ae^2)}-\frac{8d\left(\frac{(ae+cdx)^{3/4}}{2\sqrt[4]{-\frac{ae^2}{cd^2}+\frac{e(ae+cdx)}{cd^2}+1}}-\frac{cd^2\left(1-\frac{ae^2}{cd^2}\right)\sqrt[4]{ae+cdx}\sqrt[4]{\frac{cd^2-c}{e(ae+c}}}{2e\sqrt[4]{-\frac{a}{c}}}\right.}{cd^2-ae^2}\right)}{5\left(cd-\frac{ae^2}{d}\right)}\right.}{d^2\sqrt[4]{x\left(ae^2+cd^2\right)+ade+cdex^2}}$$

↓ 858

$$\frac{\sqrt[4]{\frac{ex}{d}+1}\sqrt[4]{ae+cdx}\left(\frac{2cd\left(\frac{4d(ae+cdx)^{3/4}}{\sqrt[4]{\frac{ex}{d}+1}(cd^2-ae^2)}-\frac{8d\left(\frac{cd^2\left(1-\frac{ae^2}{cd^2}\right)\sqrt[4]{ae+cdx}\sqrt[4]{\frac{cd^2-ae^2}{e(ae+cdx)}+1}\int\frac{1}{\sqrt[4]{ae+cdx}\left(\frac{(cd^2-ae^2)}{e}\cdots\right.}}{2e\sqrt[4]{-\frac{ae^2}{cd^2}+\frac{e(ae+cdx)}{cd^2}+1}}\cdots\right.}{cd^2-ae^2\cdots}\right.}{5\left(cd-\frac{ae^2}{d}\right)\cdots}\right.}{d^2\sqrt[4]{x(ae^2+cd^2)+ade+cde\cdots}}$$

$\Big\downarrow$ 807

$$\frac{\sqrt[4]{\frac{ex}{d}+1}\sqrt[4]{ae+cdx}\left(\frac{2cd\left(\frac{4d(ae+cdx)^{3/4}}{\sqrt[4]{\frac{ex}{d}+1}\left(cd^2-ae^2\right)}-\frac{8d\left(\frac{cd^2\left(1-\frac{ae^2}{cd^2}\right)\sqrt[4]{ae+cdx}\sqrt[4]{\frac{cd^2-ae^2}{e(ae+cdx)}+1}\int\frac{1}{\left(\frac{\sqrt{ae+cdx}\left(cd^2-ae^2\right)}{e}+1\right)^{5}\cdots}}{4e\sqrt[4]{-\frac{ae^2}{cd^2}+\frac{e(ae+cdx)}{cd^2}+1}}\cdots\right)}{cd^2-ae^2\cdots}\right)}{5\left(cd-\frac{ae^2}{d}\right)\cdots}\cdots\right)}{d^2\sqrt[4]{x\left(ae^2+cd^2\right)+ade+cdex^2}\cdots}$$

$\downarrow$ 212

$$\frac{\sqrt[4]{\frac{ex}{d}+1}\sqrt[4]{ae+cdx}\left(\frac{2cd\left(\frac{4d(ae+cdx)^{3/4}}{\sqrt[4]{\frac{ex}{d}+1}(cd^2-ae^2)}-\frac{8d\left(\frac{cd^2\left(1-\frac{ae^2}{cd^2}\right)\sqrt[4]{ae+cdx}\sqrt[4]{\frac{cd^2-ae^2}{e(ae+cdx)}+1}E\left(\frac{1}{2}\arctan\left(\frac{\sqrt{cd^2-ae^2}\sqrt{ae+c}}{\sqrt{e}}\right.\right.}{2\sqrt{e}\sqrt{cd^2-ae^2}\sqrt[4]{-\frac{ae^2}{cd^2}+\frac{e(ae+cdx)}{cd^2}+1}}\right.}{cd^2-ae^2}\right)}{5\left(cd-\frac{ae^2}{d}\right)}\right.}{d^2\sqrt[4]{x\left(ae^2+cd^2\right)+ade+cdex^2}}$$

input
```
Int[1/((d + e*x)^2*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(1/4)),x]
```

output
```
((a*e + c*d*x)^(1/4)*(1 + (e*x)/d)^(1/4)*((4*(a*e + c*d*x)^(3/4))/(5*(c*d
 - (a*e^2)/d)*(1 + (e*x)/d)^(5/4)) + (2*c*d*((4*d*(a*e + c*d*x)^(3/4))/((c*
d^2 - a*e^2)*(1 + (e*x)/d)^(1/4)) - (8*d*((a*e + c*d*x)^(3/4)/(2*(1 - (a*e
^2)/(c*d^2) + (e*(a*e + c*d*x))/(c*d^2))^(1/4)) + (c*d^2*(1 - (a*e^2)/(c*d
^2))*(a*e + c*d*x)^(1/4)*(1 + (c*d^2 - a*e^2)/(e*(a*e + c*d*x)))^(1/4)*Ell
ipticE[ArcTan[(Sqrt[c*d^2 - a*e^2]*Sqrt[a*e + c*d*x])/Sqrt[e]]/2, 2])/(2*S
qrt[e]*Sqrt[c*d^2 - a*e^2]*(1 - (a*e^2)/(c*d^2) + (e*(a*e + c*d*x))/(c*d^2
))^(1/4))))/(c*d^2 - a*e^2)))/(5*(c*d - (a*e^2)/d))))/(d^2*(a*d*e + (c*d^2
 + a*e^2)*x + c*d*e*x^2)^(1/4))
```

Defintions of rubi rules used

rule 61
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && LtQ[m, -1] &&  !(LtQ[n, -1] && (EqQ[a, 0
] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && IntLinearQ[a, b, c, d
, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 212
```
Int[((a_) + (b_.)*(x_)^2)^(-5/4), x_Symbol] :> Simp[(2/(a^(5/4)*Rt[b/a, 2])
)*EllipticE[(1/2)*ArcTan[Rt[b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

rule 807
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Simp[1/k   Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x,
x^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

rule 813
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4)^(5/4), x_Symbol] :> Simp[x*((1 + a/(b*x^4)
)^(1/4)/(b*(a + b*x^4)^(1/4)))   Int[1/(x^3*(1 + a/(b*x^4))^(5/4)), x], x]
/; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 839
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4)^(1/4), x_Symbol] :> Simp[x^3/(2*(a + b*x^4
)^(1/4)), x] - Simp[a/2   Int[x^2/(a + b*x^4)^(5/4), x], x] /; FreeQ[{a, b}
, x] && PosQ[b/a]
```

rule 858
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

rule 1138
```
Int[((d_) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Sy
mbol] :> Simp[d^m*((a + b*x + c*x^2)^FracPart[p]/((1 + e*(x/d))^FracPart[p]
*(a/d + (c*x)/e)^FracPart[p]))   Int[(1 + e*(x/d))^(m + p)*(a/d + (c/e)*x)^
p, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0]
&& (IntegerQ[m] || GtQ[d, 0]) &&  !(IGtQ[m, 0] && (IntegerQ[3*p] || Integer
Q[4*p]))
```

Maple [F]

$$\int \frac{1}{\left(ex+d\right)^{2} \left(ade+\left(a\,e^{2}+c\,d^{2}\right) x+cd\,x^{2}e\right)^{\frac{1}{4}}}dx$$

input
```
int(1/(e*x+d)^2/(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(1/4),x)
```

output
```
int(1/(e*x+d)^2/(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(1/4),x)
```

Fricas [F]

$$\int \frac{1}{(d+ex)^2\sqrt[4]{ade+(cd^2+ae^2)\,x+cdex^2}}\,dx = \int \frac{1}{(cdex^2+ade+(cd^2+ae^2)x)^{\frac{1}{4}}(ex+d)^2}\,dx$$

input
```
integrate(1/(e*x+d)^2/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/4),x, algorithm
="fricas")
```

output
```
integral((c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^(3/4)/(c*d*e^3*x^4 + a*d^
3*e + (3*c*d^2*e^2 + a*e^4)*x^3 + 3*(c*d^3*e + a*d*e^3)*x^2 + (c*d^4 + 3*a
*d^2*e^2)*x), x)
```

Sympy [F]

$$\int \frac{1}{(d+ex)^2 \sqrt[4]{ade+(cd^2+ae^2)\,x+cdex^2}}\,dx = \int \frac{1}{\sqrt[4]{(d+ex)\,(ae+cdx)}\,(d+ex)^2}\,dx$$

input
```
integrate(1/(e*x+d)**2/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(1/4),x)
```

output
```
Integral(1/(((d + e*x)*(a*e + c*d*x))**(1/4)*(d + e*x)**2), x)
```

Maxima [F]

$$\int \frac{1}{(d+ex)^2 \sqrt[4]{ade+(cd^2+ae^2)\,x+cdex^2}}\,dx = \int \frac{1}{(cdex^2+ade+(cd^2+ae^2)x)^{\frac{1}{4}}(ex+d)^2}\,dx$$

input
```
integrate(1/(e*x+d)^2/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/4),x, algorithm
="maxima")
```

output
```
integrate(1/((c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^(1/4)*(e*x + d)^2), x
)
```

Giac [F]

$$\int \frac{1}{(d+ex)^2 \sqrt[4]{ade+(cd^2+ae^2)\,x+cdex^2}}\,dx = \int \frac{1}{(cdex^2+ade+(cd^2+ae^2)x)^{\frac{1}{4}}(ex+d)^2}\,dx$$

input
```
integrate(1/(e*x+d)^2/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/4),x, algorithm
="giac")
```

output
```
integrate(1/((c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^(1/4)*(e*x + d)^2), x
)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(d+ex)^2\sqrt[4]{ade+(cd^2+ae^2)\,x+cdex^2}}\,dx$$
$$=\int \frac{1}{(d+e\,x)^2\,\left(c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e\right)^{1/4}}\,dx$$

input
```
int(1/((d + e*x)^2*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/4)),x)
```

output
```
int(1/((d + e*x)^2*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/4)), x)
```

Reduce [F]

$$\int \frac{1}{(d+ex)^2\sqrt[4]{ade+(cd^2+ae^2)\,x+cdex^2}}\,dx$$
$$=\int \frac{1}{(cde\,x^2+a\,e^2x+c\,d^2x+ade)^{\frac{1}{4}}\,d^2+2\,(cde\,x^2+a\,e^2x+c\,d^2x+ade)^{\frac{1}{4}}\,dex+(cde\,x^2+a\,e^2x+c\,d^2x}$$

input
```
int(1/(e*x+d)^2/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/4),x)
```

output
```
int(1/((a*d*e + a*e**2*x + c*d**2*x + c*d*e*x**2)**(1/4)*d**2 + 2*(a*d*e +
 a*e**2*x + c*d**2*x + c*d*e*x**2)**(1/4)*d*e*x + (a*d*e + a*e**2*x + c*d*
*2*x + c*d*e*x**2)**(1/4)*e**2*x**2),x)
```

3.367 $\int \frac{1}{(d+ex)^3 \sqrt[4]{ade+(cd^2+ae^2)x+cdex^2}}\,dx$

Optimal result

Integrand size = 37, antiderivative size = 249

$$\int \frac{1}{(d+ex)^3 \sqrt[4]{ade+(cd^2+ae^2)x+cdex^2}}\,dx = \frac{4(ade+(cd^2+ae^2)x+cdex^2)^{3/4}}{9(cd^2-ae^2)(d+ex)^3}+\frac{8cd(ade+(cd^2+ae^2)x+cdex^2)^{3/4}}{15(cd^2-ae^2)^2(d+ex)^2} - \frac{16c^2d^2\sqrt{ae+cdx}\sqrt[4]{\frac{cd(d+ex)}{e(ae+cdx)}}E\left(\frac{1}{2}\arctan\left(\frac{\sqrt{cd^2-ae^2}}{\sqrt{e}\sqrt{ae+cdx}}\right)\middle|2\right)}{15\sqrt{e}(cd^2-ae^2)^{5/2}\sqrt[4]{ade+(cd^2+ae^2)x+cdex^2}}$$

output

```
4/9*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/4)/(-a*e^2+c*d^2)/(e*x+d)^3+8/15*
c*d*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/4)/(-a*e^2+c*d^2)^2/(e*x+d)^2-16/
15*c^2*d^2*(c*d*x+a*e)^(1/2)*(c*d*(e*x+d)/e/(c*d*x+a*e))^(1/4)*EllipticE(s
in(1/2*arctan((-a*e^2+c*d^2)^(1/2)/e^(1/2)/(c*d*x+a*e)^(1/2))),2^(1/2))/e^
(1/2)/(-a*e^2+c*d^2)^(5/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/4)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.05 (sec) , antiderivative size = 102, normalized size of antiderivative = 0.41

$$\int \frac{1}{(d+ex)^3 \sqrt[4]{ade+(cd^2+ae^2)x+cdex^2}}\,dx = \frac{4c^3d^3((ae+cdx)(d+ex))^{3/4}\,\text{Hypergeometric2F1}\left(\frac{3}{4},\frac{13}{4},\frac{7}{4},\frac{e(ae+cdx)}{-cd^2+ae^2}\right)}{3\,(cd^2-ae^2)^4\left(\frac{cd(d+ex)}{cd^2-ae^2}\right)^{3/4}}$$

input
```
Integrate[1/((d + e*x)^3*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(1/4)),x]
```

output
```
(4*c^3*d^3*((a*e + c*d*x)*(d + e*x))^(3/4)*Hypergeometric2F1[3/4, 13/4, 7/
4, (e*(a*e + c*d*x))/(-(c*d^2) + a*e^2)])/(3*(c*d^2 - a*e^2)^4*((c*d*(d +
e*x))/(c*d^2 - a*e^2))^(3/4))
```

Rubi [A] (warning: unable to verify)

Time = 0.60 (sec) , antiderivative size = 459, normalized size of antiderivative = 1.84, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.270$, Rules used = {1138, 61, 61, 61, 73, 839, 813, 858, 807, 212}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(d+ex)^3 \sqrt[4]{x(ae^2+cd^2)+ade+cdex^2}}\,dx$$

$$\Big\downarrow \text{1138}$$

$$\frac{\sqrt[4]{\frac{ex}{d}+1}\sqrt[4]{ae+cdx}\int \frac{1}{\sqrt[4]{ae+cdx}\left(\frac{ex}{d}+1\right)^{13/4}}dx}{d^3\sqrt[4]{x(ae^2+cd^2)+ade+cdex^2}}$$

$$\Big\downarrow \text{61}$$

$$\frac{\sqrt[4]{\frac{ex}{d}+1}\sqrt[4]{ae+cdx}\left(\frac{2cd\int \frac{1}{\sqrt[4]{ae+cdx}\left(\frac{ex}{d}+1\right)^{9/4}}dx}{3\left(cd-\frac{ae^2}{d}\right)}+\frac{4(ae+cdx)^{3/4}}{9\left(\frac{ex}{d}+1\right)^{9/4}\left(cd-\frac{ae^2}{d}\right)}\right)}{d^3\sqrt[4]{x\left(ae^2+cd^2\right)+ade+cdex^2}}$$

$$\Big\downarrow \; 61$$

$$\frac{\sqrt[4]{\frac{ex}{d}+1}\sqrt[4]{ae+cdx}\left(\frac{2cd\left(\frac{2cd\int \frac{1}{\sqrt[4]{ae+cdx}\left(\frac{ex}{d}+1\right)^{5/4}}dx}{5\left(cd-\frac{ae^2}{d}\right)}+\frac{4(ae+cdx)^{3/4}}{5\left(\frac{ex}{d}+1\right)^{5/4}\left(cd-\frac{ae^2}{d}\right)}\right)}{3\left(cd-\frac{ae^2}{d}\right)}+\frac{4(ae+cdx)^{3/4}}{9\left(\frac{ex}{d}+1\right)^{9/4}\left(cd-\frac{ae^2}{d}\right)}\right)}{d^3\sqrt[4]{x\left(ae^2+cd^2\right)+ade+cdex^2}}$$

$$\Big\downarrow \; 61$$

$$\frac{\sqrt[4]{\frac{ex}{d}+1}\sqrt[4]{ae+cdx}\left(\frac{2cd\left(\frac{2cd\left(\frac{4d(ae+cdx)^{3/4}}{\sqrt[4]{\frac{ex}{d}+1}\left(cd^2-ae^2\right)}-\frac{2cd^2\int \frac{1}{\sqrt[4]{ae+cdx}\sqrt[4]{\frac{ex}{d}+1}}dx}{cd^2-ae^2}\right)}{5\left(cd-\frac{ae^2}{d}\right)}+\frac{4(ae+cdx)^{3/4}}{5\left(\frac{ex}{d}+1\right)^{5/4}\left(cd-\frac{ae^2}{d}\right)}\right)}{3\left(cd-\frac{ae^2}{d}\right)}+\frac{4(ae+cdx)^{3/4}}{9\left(\frac{ex}{d}+1\right)^{9/4}\left(cd-\frac{ae^2}{d}\right)}\right)}{d^3\sqrt[4]{x\left(ae^2+cd^2\right)+ade+cdex^2}}$$

$$\Big\downarrow \; 73$$

$$\frac{\sqrt[4]{\frac{ex}{d}+1}\sqrt[4]{ae+cdx}\left(\frac{2cd\left(\frac{2cd\left(\frac{4d(ae+cdx)^{3/4}}{\sqrt[4]{\frac{ex}{d}+1}\left(cd^2-ae^2\right)}-\frac{8d\int \frac{\sqrt{ae+cdx}}{\sqrt[4]{-\frac{ae^2}{cd^2}+\frac{(ae+cdx)e}{cd^2}+1}}d\sqrt[4]{ae+cdx}}{cd^2-ae^2}\right)}{5\left(cd-\frac{ae^2}{d}\right)}+\frac{4(ae+cdx)^{3/4}}{5\left(\frac{ex}{d}+1\right)^{5/4}\left(cd-\frac{ae^2}{d}\right)}\right)}{3\left(cd-\frac{ae^2}{d}\right)}\right)}{d^3\sqrt[4]{x\left(ae^2+cd^2\right)+ade+cdex^2}}$$

$\downarrow$ 839

$$\frac{\sqrt[4]{\frac{ex}{d}+1}\sqrt[4]{ae+cdx}\left(\frac{2cd\left(\frac{2cd\left(\frac{4d(ae+cdx)^{3/4}}{\sqrt[4]{\frac{ex}{d}+1}\left(cd^2-ae^2\right)}-\frac{8d\left(\frac{(ae+cdx)^{3/4}}{2\sqrt[4]{-\frac{ae^2}{cd^2}+\frac{e(ae+cdx)}{cd^2}+1}}-\frac{1}{2}\left(1-\frac{ae^2}{cd^2}\right)\int\frac{\sqrt{ae+cdx}}{\left(-\frac{ae^2}{cd^2}+\frac{(ae+cdx)e}{cd^2}+1\right)^{5}\cdots}\cdots\right.}{cd^2-ae^2}\cdots\right.}{5\left(cd-\frac{ae^2}{d}\right)}\cdots\right.}{3\left(cd-\frac{ae^2}{d}\right)}\cdots\right.}{d^3\sqrt[4]{x\left(ae^2+cd^2\right)+ade+cdex^2}\cdots}$$

$$\Big\downarrow\ 813$$

$$\frac{\sqrt[4]{\frac{ex}{d}+1}\sqrt[4]{ae+cdx}\left(\frac{2cd\left(\frac{2cd\left(\frac{4d(ae+cdx)^{3/4}}{\sqrt[4]{\frac{ex}{d}+1}\left(cd^2-ae^2\right)}-\frac{8d\left(\frac{(ae+cdx)^{3/4}}{2\sqrt[4]{-\frac{ae^2}{cd^2}+\frac{e(ae+cdx)}{cd^2}+1}}-\frac{cd^2\left(1-\frac{ae^2}{cd^2}\right)\sqrt[4]{ae+cdx}\sqrt[4]{\frac{cd^2}{e(ae}}}{2e\sqrt[4]{}}\right)}{cd^2-ae^2}\right)}{5\left(cd-\frac{ae^2}{d}\right)}}{3\left(cd-\frac{ae^2}{d}\right)}\right)}{d^3\sqrt[4]{x\left(ae^2+cd^2\right)+ade}}$$

↓ 858

$$cd^2\left(1-\frac{ae^2}{cd^2}\right)\sqrt[4]{ae+cdx}\sqrt[4]{\frac{cd^2-ae^2}{e(ae+cdx)}+1}\int\frac{}{\sqrt[4]{ae+cdx}\left(\frac{(cd^2-}{}\right.}$$

$$8d\left(\frac{}{2e\sqrt[4]{-\frac{ae^2}{cd^2}+\frac{e(ae+cdx)}{cd^2}+1}}\right.$$

$$2cd\left(\frac{4d(ae+cdx)^{3/4}}{\sqrt[4]{\frac{ex}{d}+1}\left(cd^2-ae^2\right)}-\frac{}{cd^2-}\right.$$

$$2cd\left(\frac{}{5\left(cd-\frac{ae^2}{d}\right)}\right.$$

$$\sqrt[4]{\frac{ex}{d}+1}\sqrt[4]{ae+cdx}\left(\frac{}{3\left(cd-\frac{ae^2}{d}\right.}\right.$$

↓ 807

$$\sqrt[4]{\frac{ex}{d}+1}\sqrt[4]{ae+cdx}\left(\frac{2cd\left(\frac{2cd\left(\frac{4d(ae+cdx)^{3/4}}{\sqrt[4]{\frac{ex}{d}+1}\left(cd^2-ae^2\right)}-\frac{8d\left(\frac{cd^2\left(1-\frac{ae^2}{cd^2}\right)\sqrt[4]{ae+cdx}\sqrt[4]{\frac{cd^2-ae^2}{e(ae+cdx)}+1}\int\frac{1}{\left(\frac{\sqrt{ae+cdx}\left(cd^2-ae^2\right)}{e}\cdots\right.}}{4e\sqrt[4]{-\frac{ae^2}{cd^2}+\frac{e(ae+cdx)}{cd^2}+1}}\cdots\right.}{cd^2-ae^2}\cdots\right.}{5\left(cd-\frac{ae^2}{d}\right)}\cdots\right.}{3\left(cd-\frac{ae^2}{d}\right)}\cdots\right.$$

$\downarrow$ 212

$$\frac{\sqrt[4]{\frac{ex}{d}+1}\sqrt[4]{ae+cdx}\left(\frac{2cd\left(\frac{2cd\left(\frac{4d(ae+cdx)^{3/4}}{\sqrt[4]{\frac{ex}{d}+1}(cd^2-ae^2)}-\frac{8d\left(\frac{cd^2\left(1-\frac{ae^2}{cd^2}\right)\sqrt[4]{ae+cdx}\sqrt[4]{\frac{cd^2-ae^2}{e(ae+cdx)}+1}E\left(\frac{1}{2}\arctan\left(\frac{\sqrt{cd^2-ae^2}\cdots}{\sqrt{e}\cdots}\right.\right.}{2\sqrt{e}\sqrt{cd^2-ae^2}\sqrt[4]{-\frac{ae^2}{cd^2}+\frac{e(ae+cdx)}{cd^2}+1}}\right.}{cd^2-ae^2}\right.}{5\left(cd-\frac{ae^2}{d}\right)}\right.}{3\left(cd-\frac{ae^2}{d}\right)}\right.}{d^3\sqrt[4]{x(ae^2+cd^2)+ade+c\cdots}}$$

input
```
Int[1/((d + e*x)^3*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(1/4)),x]
```

output
```
((a*e + c*d*x)^(1/4)*(1 + (e*x)/d)^(1/4)*((4*(a*e + c*d*x)^(3/4))/(9*(c*d
- (a*e^2)/d)*(1 + (e*x)/d)^(9/4)) + (2*c*d*((4*(a*e + c*d*x)^(3/4))/(5*(c*
d - (a*e^2)/d)*(1 + (e*x)/d)^(5/4)) + (2*c*d*((4*d*(a*e + c*d*x)^(3/4))/((
c*d^2 - a*e^2)*(1 + (e*x)/d)^(1/4)) - (8*d*((a*e + c*d*x)^(3/4)/(2*(1 - (a
*e^2)/(c*d^2) + (e*(a*e + c*d*x))/(c*d^2))^(1/4)) + (c*d^2*(1 - (a*e^2)/(c
*d^2))*(a*e + c*d*x)^(1/4)*(1 + (c*d^2 - a*e^2)/(e*(a*e + c*d*x)))^(1/4)*E
llipticE[ArcTan[(Sqrt[c*d^2 - a*e^2]*Sqrt[a*e + c*d*x])/Sqrt[e]]/2, 2])/(2
*Sqrt[e]*Sqrt[c*d^2 - a*e^2]*(1 - (a*e^2)/(c*d^2) + (e*(a*e + c*d*x))/(c*d
^2))^(1/4))))/(c*d^2 - a*e^2)))/(5*(c*d - (a*e^2)/d))))/(3*(c*d - (a*e^2)/
d))))/(d^3*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(1/4))
```

Defintions of rubi rules used

rule 61
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && LtQ[m, -1] &&  !(LtQ[n, -1] && (EqQ[a, 0
] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && IntLinearQ[a, b, c, d
, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 212
```
Int[((a_) + (b_.)*(x_)^2)^(-5/4), x_Symbol] :> Simp[(2/(a^(5/4)*Rt[b/a, 2])
)*EllipticE[(1/2)*ArcTan[Rt[b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

rule 807
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Simp[1/k   Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x,
x^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

rule 813
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4)^(5/4), x_Symbol] :> Simp[x*((1 + a/(b*x^4)
)^(1/4)/(b*(a + b*x^4)^(1/4)))   Int[1/(x^3*(1 + a/(b*x^4))^(5/4)), x], x]
/; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 839
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4)^(1/4), x_Symbol] :> Simp[x^3/(2*(a + b*x^4
)^(1/4)), x] - Simp[a/2   Int[x^2/(a + b*x^4)^(5/4), x], x] /; FreeQ[{a, b}
, x] && PosQ[b/a]
```

rule 858
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

rule 1138
```
Int[((d_) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Sy
mbol] :> Simp[d^m*((a + b*x + c*x^2)^FracPart[p]/((1 + e*(x/d))^FracPart[p]
*(a/d + (c*x)/e)^FracPart[p]))   Int[(1 + e*(x/d))^(m + p)*(a/d + (c/e)*x)^
p, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0]
&& (IntegerQ[m] || GtQ[d, 0]) &&  !(IGtQ[m, 0] && (IntegerQ[3*p] || Integer
Q[4*p]))
```

Maple [F]

$$\int \frac{1}{\left(ex+d\right)^{3} \left(ade+\left(a\,e^{2}+c\,d^{2}\right)x+cd\,x^{2}e\right)^{\frac{1}{4}}}dx$$

input
```
int(1/(e*x+d)^3/(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(1/4),x)
```

output
```
int(1/(e*x+d)^3/(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(1/4),x)
```

Fricas [F]

$$\int \frac{1}{(d+ex)^3 \sqrt[4]{ade+(cd^2+ae^2)\,x+cdex^2}}\,dx = \int \frac{1}{(cdex^2+ade+(cd^2+ae^2)x)^{\frac{1}{4}}(ex+d)^3}\,dx$$

input
```
integrate(1/(e*x+d)^3/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/4),x, algorithm
="fricas")
```

output
```
integral((c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^(3/4)/(c*d*e^4*x^5 + a*d^
4*e + (4*c*d^2*e^3 + a*e^5)*x^4 + 2*(3*c*d^3*e^2 + 2*a*d*e^4)*x^3 + 2*(2*c
*d^4*e + 3*a*d^2*e^3)*x^2 + (c*d^5 + 4*a*d^3*e^2)*x), x)
```

Sympy [F]

$$\int \frac{1}{(d+ex)^3 \sqrt[4]{ade+(cd^2+ae^2)\,x+cdex^2}}\,dx = \int \frac{1}{\sqrt[4]{(d+ex)\,(ae+cdx)}\,(d+ex)^3}\,dx$$

input
```
integrate(1/(e*x+d)**3/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(1/4),x)
```

output
```
Integral(1/(((d + e*x)*(a*e + c*d*x))**(1/4)*(d + e*x)**3), x)
```

Maxima [F]

$$\int \frac{1}{(d+ex)^3 \sqrt[4]{ade+(cd^2+ae^2)\,x+cdex^2}}\,dx = \int \frac{1}{(cdex^2+ade+(cd^2+ae^2)x)^{\frac{1}{4}}(ex+d)^3}\,dx$$

input
```
integrate(1/(e*x+d)^3/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/4),x, algorithm
="maxima")
```

output
```
integrate(1/((c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^(1/4)*(e*x + d)^3), x
)
```

Giac [F]

$$\int \frac{1}{(d+ex)^3 \sqrt[4]{ade+(cd^2+ae^2)\,x+cdex^2}}\,dx$$
$$=\int \frac{1}{(cdex^2+ade+(cd^2+ae^2)x)^{\frac{1}{4}}(ex+d)^3}\,dx$$

input
```
integrate(1/(e*x+d)^3/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/4),x, algorithm
="giac")
```

output
```
integrate(1/((c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^(1/4)*(e*x + d)^3), x
)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(d+ex)^3 \sqrt[4]{ade+(cd^2+ae^2)\,x+cdex^2}}\,dx$$
$$=\int \frac{1}{(d+e\,x)^3\,(c\,d\,e\,x^2+(c\,d^2+a\,e^2)\,x+a\,d\,e)^{1/4}}\,dx$$

input
```
int(1/((d + e*x)^3*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/4)),x)
```

output
```
int(1/((d + e*x)^3*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/4)), x)
```

Reduce [F]

$$\int \frac{1}{(d+ex)^3 \sqrt[4]{ade+(cd^2+ae^2)\,x+cdex^2}}\,dx$$
$$= \int \frac{1}{(cde\,x^2+a\,e^2x+c\,d^2x+ade)^{\frac{1}{4}}\,d^3+3\,(cde\,x^2+a\,e^2x+c\,d^2x+ade)^{\frac{1}{4}}\,d^2ex+3\,(cde\,x^2+a\,e^2x+c\,d^2}$$

input
```
int(1/(e*x+d)^3/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/4),x)
```

output
```
int(1/((a*d*e + a*e**2*x + c*d**2*x + c*d*e*x**2)**(1/4)*d**3 + 3*(a*d*e +
 a*e**2*x + c*d**2*x + c*d*e*x**2)**(1/4)*d**2*e*x + 3*(a*d*e + a*e**2*x +
 c*d**2*x + c*d*e*x**2)**(1/4)*d*e**2*x**2 + (a*d*e + a*e**2*x + c*d**2*x
+ c*d*e*x**2)**(1/4)*e**3*x**3),x)
```

3.368 $\int \frac{(d+ex)^3}{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/4}}\,dx$

Optimal result

Integrand size = 37, antiderivative size = 294

$$\int \frac{(d+ex)^3}{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/4}}\,dx = \frac{3\left(cd^2-ae^2\right)^2\sqrt[4]{ade+\left(cd^2+ae^2\right)x+cdex^2}}{2c^3d^3} + \frac{3\left(cd^2-ae^2\right)(d+ex)\sqrt[4]{ade+\left(cd^2+ae^2\right)x+cdex^2}}{5c^2d^2} + \frac{2(d+ex)^2\sqrt[4]{ade+\left(cd^2+ae^2\right)x+cdex^2}}{5cd} - \frac{3\sqrt{e}\left(cd^2-ae^2\right)^{5/2}(ae+cdx)^{3/2}\left(\frac{cd(d+ex)}{e(ae+cdx)}\right)^{3/4}\operatorname{EllipticF}\left(\frac{1}{2}\arctan\left(\frac{\sqrt{cd^2-ae^2}}{\sqrt{e}\sqrt{ae+cdx}}\right),2\right)}{2c^4d^4\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/4}}$$

output
```
3/2*(-a*e^2+c*d^2)^2*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/4)/c^3/d^3+3/5*(
-a*e^2+c*d^2)*(e*x+d)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/4)/c^2/d^2+2/5*
(e*x+d)^2*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/4)/c/d-3/2*e^(1/2)*(-a*e^2+
c*d^2)^(5/2)*(c*d*x+a*e)^(3/2)*(c*d*(e*x+d)/e/(c*d*x+a*e))^(3/4)*InverseJa
cobiAM(1/2*arctan((-a*e^2+c*d^2)^(1/2)/e^(1/2)/(c*d*x+a*e)^(1/2)),2^(1/2))
/c^4/d^4/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/4)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.06 (sec) , antiderivative size = 100, normalized size of antiderivative = 0.34

$$\int \frac{(d+ex)^3}{(ade+(cd^2+ae^2)x+cdex^2)^{3/4}}\,dx = \frac{4(cd^2-ae^2)^2\sqrt[4]{(ae+cdx)(d+ex)}\,\text{Hypergeometric2F1}\left(-\frac{9}{4},\frac{1}{4},\right.}{c^3d^3\sqrt[4]{\frac{cd(d+ex)}{cd^2-ae^2}}}$$

input
```
Integrate[(d + e*x)^3/(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/4),x]
```

output
```
(4*(c*d^2 - a*e^2)^2*((a*e + c*d*x)*(d + e*x))^(1/4)*Hypergeometric2F1[-9/
4, 1/4, 5/4, (e*(a*e + c*d*x))/(-(c*d^2) + a*e^2)])/(c^3*d^3*((c*d*(d + e*
x))/(c*d^2 - a*e^2))^(1/4))
```

Rubi [A] (warning: unable to verify)

Time = 0.65 (sec) , antiderivative size = 578, normalized size of antiderivative = 1.97, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.189$, Rules used = {1166, 27, 1166, 27, 1160, 1094, 761}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex)^3}{(x(ae^2+cd^2)+ade+cdex^2)^{3/4}}dx$$

$$\Big\downarrow \; 1166$$

$$\frac{2\int \frac{9e(cd^2-ae^2)(d+ex)^2}{4(cdex^2+(cd^2+ae^2)x+ade)^{3/4}}dx}{5cde}+\frac{2(d+ex)^2\sqrt[4]{x(ae^2+cd^2)+ade+cdex^2}}{5cd}$$

$$\Big\downarrow \; 27$$

$$\frac{9(cd^2-ae^2)\int \frac{(d+ex)^2}{(cdex^2+(cd^2+ae^2)x+ade)^{3/4}}dx}{10cd}+\frac{2(d+ex)^2\sqrt[4]{x(ae^2+cd^2)+ade+cdex^2}}{5cd}$$

$$\Big\downarrow \; 1166$$

$$\frac{9(cd^2-ae^2)\left(\frac{2\int \frac{5e(cd^2-ae^2)(d+ex)}{4\left(cdex^2+(cd^2+ae^2)x+ade\right)^{3/4}}dx}{3cde}+\frac{2(d+ex)\sqrt[4]{x(ae^2+cd^2)+ade+cdex^2}}{3cd}\right)}{10cd}+\frac{2(d+ex)^2\sqrt[4]{x(ae^2+cd^2)+ade+cdex^2}}{5cd}$$

$\downarrow$ 27

$$\frac{9(cd^2-ae^2)\left(\frac{5(cd^2-ae^2)\int \frac{d+ex}{\left(cdex^2+(cd^2+ae^2)x+ade\right)^{3/4}}dx}{6cd}+\frac{2(d+ex)\sqrt[4]{x(ae^2+cd^2)+ade+cdex^2}}{3cd}\right)}{10cd}+\frac{2(d+ex)^2\sqrt[4]{x(ae^2+cd^2)+ade+cdex^2}}{5cd}$$

$\downarrow$ 1160

$$\frac{9(cd^2-ae^2)\left(\frac{5(cd^2-ae^2)\left(\frac{\left(d^2-\frac{ae^2}{c}\right)\int \frac{1}{\left(cdex^2+(cd^2+ae^2)x+ade\right)^{3/4}}dx}{2d}+\frac{2\sqrt[4]{x(ae^2+cd^2)+ade+cdex^2}}{cd}\right)}{6cd}+\frac{2(d+ex)\sqrt[4]{x(\ldots}}{\ldots}\right)}{10cd}+\frac{2(d+ex)^2\sqrt[4]{x(ae^2+cd^2)+ade+cdex^2}}{5cd}$$

$\downarrow$ 1094

$$\frac{9(cd^2-ae^2)\left(\frac{5(cd^2-ae^2)\left(\frac{2\left(d^2-\frac{ae^2}{c}\right)\sqrt{(ae^2+cd^2+2cdex)^2}\int \frac{1}{\sqrt{(cd^2-ae^2)^2+4cde\left(cdex^2+(cd^2+ae^2)x+ade\right)}}d\sqrt[4]{cdex^2+(cd^2+ae^2)\ldots}}{d(ae^2+cd^2+2cdex)}\ldots\right)}{6cd}\ldots\right)}{10cd}+\frac{2(d+ex)^2\sqrt[4]{x(ae^2+cd^2)+ade+cdex^2}}{5cd}$$

$\downarrow$ 761

$$\frac{9\left(cd^2-ae^2\right)\left(\frac{5\left(cd^2-ae^2\right)\left(\frac{\sqrt{cd^2-ae^2}\left(d^2-\frac{ae^2}{c}\right)\sqrt{\left(ae^2+cd^2+2cdex\right)^2}\left(\frac{2\sqrt{c}\sqrt{d}\sqrt{e}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{cd^2-ae^2}+1\right)\sqrt{\frac{4cde\left(x\left(ae^2+cd^2\right)+ade\right.}{\left(cd^2-ae^2\right)^2\left(\frac{2\sqrt{c}\sqrt{d}\sqrt{e}\sqrt{x\left(\right.}}{}\right.}}}{\sqrt{2}\sqrt[4]{c}d^{5/4}\sqrt[4]{e}\left(ae^2+cd^2+2cdex\right)\sqrt{4cde\left(\right.}}\right.}{}\right.}{}$$

$$\frac{2(d+ex)^2\sqrt[4]{x\left(ae^2+cd^2\right)+ade+cdex^2}}{5cd}$$

input
```
Int[(d + e*x)^3/(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/4),x]
```

output
```
(2*(d + e*x)^2*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(1/4))/(5*c*d) + (9
*(c*d^2 - a*e^2)*((2*(d + e*x)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(1/
4))/(3*c*d) + (5*(c*d^2 - a*e^2)*((2*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^
2)^(1/4))/(c*d) + (Sqrt[c*d^2 - a*e^2]*(d^2 - (a*e^2)/c)*Sqrt[(c*d^2 + a*e
^2 + 2*c*d*e*x)^2]*(1 + (2*Sqrt[c]*Sqrt[d]*Sqrt[e]*Sqrt[a*d*e + (c*d^2 + a
*e^2)*x + c*d*e*x^2])/(c*d^2 - a*e^2))*Sqrt[((c*d^2 - a*e^2)^2 + 4*c*d*e*(
a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2))/((c*d^2 - a*e^2)^2*(1 + (2*Sqrt[c]
*Sqrt[d]*Sqrt[e]*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(c*d^2 - a*e
^2))^2)]*EllipticF[2*ArcTan[(Sqrt[2]*c^(1/4)*d^(1/4)*e^(1/4)*(a*d*e + (c*d
^2 + a*e^2)*x + c*d*e*x^2)^(1/4))/Sqrt[c*d^2 - a*e^2]], 1/2])/(Sqrt[2]*c^(
1/4)*d^(5/4)*e^(1/4)*(c*d^2 + a*e^2 + 2*c*d*e*x)*Sqrt[(c*d^2 - a*e^2)^2 +
4*c*d*e*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)])))/(6*c*d)))/(10*c*d)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 761
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 1094
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[4*(Sqrt[(b
+ 2*c*x)^2]/(b + 2*c*x))   Subst[Int[x^(4*(p + 1) - 1)/Sqrt[b^2 - 4*a*c + 4
*c*x^4], x], x, (a + b*x + c*x^2)^(1/4)], x] /; FreeQ[{a, b, c}, x] && Inte
gerQ[4*p]
```

rule 1160
```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[e*((a + b*x + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Simp[(2*c*d - b
*e)/(2*c)   Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
 && NeQ[p, -1]
```

rule 1166
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[e*(d + e*x)^(m - 1)*((a + b*x + c*x^2)^(p + 1)/(c*(m + 2*p +
 1))), x] + Simp[1/(c*(m + 2*p + 1))   Int[(d + e*x)^(m - 2)*Simp[c*d^2*(m
+ 2*p + 1) - e*(a*e*(m - 1) + b*d*(p + 1)) + e*(2*c*d - b*e)*(m + p)*x, x]*
(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && If[Ration
alQ[m], GtQ[m, 1], SumSimplerQ[m, -2]] && NeQ[m + 2*p + 1, 0] && IntQuadrat
icQ[a, b, c, d, e, m, p, x]
```

Maple [F]

$$\int \frac{(ex+d)^3}{(ade+(a\,e^2+c\,d^2)\,x+cd\,x^2e)^{\frac{3}{4}}}dx$$

input
```
int((e*x+d)^3/(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(3/4),x)
```

output
```
int((e*x+d)^3/(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(3/4),x)
```

Fricas [F]

$$\int \frac{(d+ex)^3}{(ade+(cd^2+ae^2)\,x+cdex^2)^{3/4}}\,dx=\int \frac{(ex+d)^3}{(cdex^2+ade+(cd^2+ae^2)x)^{\frac{3}{4}}}\,dx$$

input
```
integrate((e*x+d)^3/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/4),x, algorithm="
fricas")
```

output
```
integral((c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^(1/4)*(e^2*x^2 + 2*d*e*x
+ d^2)/(c*d*x + a*e), x)
```

Sympy [F]

$$\int \frac{(d+ex)^3}{(ade+(cd^2+ae^2)\,x+cdex^2)^{3/4}}\,dx=\int \frac{(d+ex)^3}{((d+ex)\,(ae+cdx))^{\frac{3}{4}}}\,dx$$

input
```
integrate((e*x+d)**3/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(3/4),x)
```

output
```
Integral((d + e*x)**3/((d + e*x)*(a*e + c*d*x))**(3/4), x)
```

Maxima [F]

$$\int \frac{(d+ex)^3}{(ade+(cd^2+ae^2)\,x+cdex^2)^{3/4}}\,dx=\int \frac{(ex+d)^3}{(cdex^2+ade+(cd^2+ae^2)x)^{\frac{3}{4}}}\,dx$$

input
```
integrate((e*x+d)^3/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/4),x, algorithm="
maxima")
```

output
```
integrate((e*x + d)^3/(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^(3/4), x)
```

Giac [F]

$$\int \frac{(d+ex)^3}{(ade+(cd^2+ae^2)\,x+cdex^2)^{3/4}}\,dx = \int \frac{(ex+d)^3}{(cdex^2+ade+(cd^2+ae^2)x)^{\frac{3}{4}}}\,dx$$

input
```
integrate((e*x+d)^3/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/4),x, algorithm="
giac")
```

output
```
integrate((e*x + d)^3/(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^(3/4), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d+ex)^3}{(ade+(cd^2+ae^2)\,x+cdex^2)^{3/4}}\,dx = \int \frac{(d+e\,x)^3}{(c\,d\,e\,x^2+(c\,d^2+a\,e^2)\,x+a\,d\,e)^{3/4}}\,dx$$

input
```
int((d + e*x)^3/(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(3/4),x)
```

output
```
int((d + e*x)^3/(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(3/4), x)
```

Reduce [F]

$$\int \frac{(d+ex)^3}{(ade+(cd^2+ae^2)\,x+cdex^2)^{3/4}}\,dx = \left(\int \frac{x^3}{(cde\,x^2+a\,e^2x+c\,d^2x+ade)^{\frac{3}{4}}}dx\right)e^3 + 3\left(\int \frac{x^2}{(cde\,x^2+a\,e^2x+c\,d^2x+ade)^{\frac{3}{4}}}dx\right)d\,e^2 + 3\left(\int \frac{x}{(cde\,x^2+a\,e^2x+c\,d^2x+ade)^{\frac{3}{4}}}dx\right)d^2e + \left(\int \frac{1}{(cde\,x^2+a\,e^2x+c\,d^2x+ade)^{\frac{3}{4}}}dx\right)d^3$$

input
```
int((e*x+d)^3/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/4),x)
```

output
```
int(x**3/(a*d*e + a*e**2*x + c*d**2*x + c*d*e*x**2)**(3/4),x)*e**3 + 3*int
(x**2/(a*d*e + a*e**2*x + c*d**2*x + c*d*e*x**2)**(3/4),x)*d*e**2 + 3*int(
x/(a*d*e + a*e**2*x + c*d**2*x + c*d*e*x**2)**(3/4),x)*d**2*e + int(1/(a*d
*e + a*e**2*x + c*d**2*x + c*d*e*x**2)**(3/4),x)*d**3
```

3.369 $\int \frac{(d+ex)^2}{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/4}}\,dx$

Optimal result

Integrand size = 37, antiderivative size = 234

$$\int \frac{(d+ex)^2}{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/4}}\,dx = \frac{5\left(cd^2-ae^2\right)\sqrt[4]{ade+\left(cd^2+ae^2\right)x+cdex^2}}{3c^2d^2} + \frac{2(d+ex)\sqrt[4]{ade+\left(cd^2+ae^2\right)x+cdex^2}}{3cd} - \frac{5\sqrt{e}\left(cd^2-ae^2\right)^{3/2}(ae+cdx)^{3/2}\left(\frac{cd(d+ex)}{e(ae+cdx)}\right)^{3/4}\operatorname{EllipticF}\left(\frac{1}{2}\arctan\left(\frac{\sqrt{cd^2-ae^2}}{\sqrt{e}\sqrt{ae+cdx}}\right),2\right)}{3c^3d^3\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/4}}$$

output
```
5/3*(-a*e^2+c*d^2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/4)/c^2/d^2+2/3*(e*
x+d)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/4)/c/d-5/3*e^(1/2)*(-a*e^2+c*d^2
)^(3/2)*(c*d*x+a*e)^(3/2)*(c*d*(e*x+d)/e/(c*d*x+a*e))^(3/4)*InverseJacobiA
M(1/2*arctan((-a*e^2+c*d^2)^(1/2)/e^(1/2)/(c*d*x+a*e)^(1/2)),2^(1/2))/c^3/
d^3/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/4)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.05 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.39

$$\int \frac{(d+ex)^2}{(ade+(cd^2+ae^2)x+cdex^2)^{3/4}}\,dx=\frac{4(d+ex)\sqrt[4]{(ae+cdx)(d+ex)}\,\text{Hypergeometric2F1}\left(-\frac{5}{4},\frac{1}{4},\frac{5}{4},\frac{e(}{-}\right.}{cd\left(\frac{cd(d+ex)}{cd^2-ae^2}\right)^{5/4}}$$

input
```
Integrate[(d + e*x)^2/(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/4),x]
```

output
```
(4*(d + e*x)*((a*e + c*d*x)*(d + e*x))^(1/4)*Hypergeometric2F1[-5/4, 1/4,
5/4, (e*(a*e + c*d*x))/(-(c*d^2) + a*e^2)])/(c*d*((c*d*(d + e*x))/(c*d^2 -
 a*e^2))^(5/4))
```

Rubi [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 509 vs. 2(234) = 468.

Time = 0.51 (sec) , antiderivative size = 509, normalized size of antiderivative = 2.18, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.135, Rules used = {1166, 27, 1160, 1094, 761}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex)^2}{(x(ae^2+cd^2)+ade+cdex^2)^{3/4}}dx$$

$$\Big\downarrow \text{1166}$$

$$\frac{2\int \frac{5e(cd^2-ae^2)(d+ex)}{4(cdex^2+(cd^2+ae^2)x+ade)^{3/4}}dx}{3cde}+\frac{2(d+ex)\sqrt[4]{x(ae^2+cd^2)+ade+cdex^2}}{3cd}$$

$$\Big\downarrow \text{27}$$

$$\frac{5(cd^2-ae^2)\int \frac{d+ex}{(cdex^2+(cd^2+ae^2)x+ade)^{3/4}}dx}{6cd}+\frac{2(d+ex)\sqrt[4]{x(ae^2+cd^2)+ade+cdex^2}}{3cd}$$

$$\Big\downarrow \; 1160$$

$$\frac{5\left(cd^2-ae^2\right)\left(\frac{\left(d^2-\frac{ae^2}{c}\right)\int \frac{1}{\left(cdex^2+\left(cd^2+ae^2\right)x+ade\right)^{3/4}}dx}{2d}+\frac{2\sqrt[4]{x\left(ae^2+cd^2\right)+ade+cdex^2}}{cd}\right)}{6cd}+\frac{2(d+ex)\sqrt[4]{x\left(ae^2+cd^2\right)+ade+cdex^2}}{3cd}$$

$$\Big\downarrow \; 1094$$

$$\frac{5\left(cd^2-ae^2\right)\left(\frac{2\left(d^2-\frac{ae^2}{c}\right)\sqrt{(ae^2+cd^2+2cdex)^2}\int \frac{1}{\sqrt{\left(cd^2-ae^2\right)^2+4cde\left(cdex^2+\left(cd^2+ae^2\right)x+ade\right)}}d\sqrt[4]{cdex^2+\left(cd^2+ae^2\right)x+ade}}{d(ae^2+cd^2+2cdex)}+\cdots\right.}{6cd}$$

$$\frac{2(d+ex)\sqrt[4]{x\left(ae^2+cd^2\right)+ade+cdex^2}}{3cd}$$

$$\Big\downarrow \; 761$$

$$5\left(cd^2-ae^2\right)\left(\frac{\sqrt{cd^2-ae^2}\left(d^2-\frac{ae^2}{c}\right)\sqrt{(ae^2+cd^2+2cdex)^2}\left(\frac{2\sqrt{c}\sqrt{d}\sqrt{e}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{cd^2-ae^2}+1\right)\sqrt{\frac{4cde\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)+\cdots}{\left(cd^2-ae^2\right)^2\left(\frac{2\sqrt{c}\sqrt{d}\sqrt{e}\sqrt{x\left(ae^2+cd^2\right)\cdots}}{cd^2-ae^2}\cdots\right.}}}{\sqrt{2}\sqrt[4]{c}d^{5/4}\sqrt[4]{e}(ae^2+cd^2+2cdex)\sqrt{4cde(x(ae^2+cd\cdots}}\cdots\right.$$

$$\frac{2(d+ex)\sqrt[4]{x\left(ae^2+cd^2\right)+ade+cdex^2}}{3cd}$$

input

```
Int[(d + e*x)^2/(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/4),x]
```

output
```
(2*(d + e*x)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(1/4))/(3*c*d) + (5*(
c*d^2 - a*e^2)*((2*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(1/4))/(c*d) +
(Sqrt[c*d^2 - a*e^2]*(d^2 - (a*e^2)/c)*Sqrt[(c*d^2 + a*e^2 + 2*c*d*e*x)^2]
*(1 + (2*Sqrt[c]*Sqrt[d]*Sqrt[e]*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^
2])/(c*d^2 - a*e^2))*Sqrt[((c*d^2 - a*e^2)^2 + 4*c*d*e*(a*d*e + (c*d^2 + a
*e^2)*x + c*d*e*x^2))/((c*d^2 - a*e^2)^2*(1 + (2*Sqrt[c]*Sqrt[d]*Sqrt[e]*S
qrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(c*d^2 - a*e^2))^2)]*EllipticF
[2*ArcTan[(Sqrt[2]*c^(1/4)*d^(1/4)*e^(1/4)*(a*d*e + (c*d^2 + a*e^2)*x + c*
d*e*x^2)^(1/4))/Sqrt[c*d^2 - a*e^2]], 1/2])/(Sqrt[2]*c^(1/4)*d^(5/4)*e^(1/
4)*(c*d^2 + a*e^2 + 2*c*d*e*x)*Sqrt[(c*d^2 - a*e^2)^2 + 4*c*d*e*(a*d*e + (
c*d^2 + a*e^2)*x + c*d*e*x^2)])))/(6*c*d)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 761
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 1094
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[4*(Sqrt[(b
+ 2*c*x)^2]/(b + 2*c*x))   Subst[Int[x^(4*(p + 1) - 1)/Sqrt[b^2 - 4*a*c + 4
*c*x^4], x], x, (a + b*x + c*x^2)^(1/4)], x] /; FreeQ[{a, b, c}, x] && Inte
gerQ[4*p]
```

rule 1160
```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[e*((a + b*x + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Simp[(2*c*d - b
*e)/(2*c)   Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
 && NeQ[p, -1]
```

rule 1166
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[e*(d + e*x)^(m - 1)*((a + b*x + c*x^2)^(p + 1)/(c*(m + 2*p +
 1))), x] + Simp[1/(c*(m + 2*p + 1))   Int[(d + e*x)^(m - 2)*Simp[c*d^2*(m
+ 2*p + 1) - e*(a*e*(m - 1) + b*d*(p + 1)) + e*(2*c*d - b*e)*(m + p)*x, x]*
(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && If[Ration
alQ[m], GtQ[m, 1], SumSimplerQ[m, -2]] && NeQ[m + 2*p + 1, 0] && IntQuadrat
icQ[a, b, c, d, e, m, p, x]
```

Maple [F]

$$\int \frac{(ex+d)^2}{(ade+(a\,e^2+c\,d^2)\,x+cd\,x^2e)^{\frac{3}{4}}}dx$$

input
```
int((e*x+d)^2/(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(3/4),x)
```

output
```
int((e*x+d)^2/(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(3/4),x)
```

Fricas [F]

$$\int \frac{(d+ex)^2}{(ade+(cd^2+ae^2)\,x+cdex^2)^{3/4}}\,dx = \int \frac{(ex+d)^2}{(cdex^2+ade+(cd^2+ae^2)x)^{\frac{3}{4}}}\,dx$$

input
```
integrate((e*x+d)^2/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/4),x, algorithm="
fricas")
```

output
```
integral((c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^(1/4)*(e*x + d)/(c*d*x +
a*e), x)
```

Sympy [F]

$$\int \frac{(d+ex)^2}{(ade+(cd^2+ae^2)\,x+cdex^2)^{3/4}}\,dx = \int \frac{(d+ex)^2}{((d+ex)\,(ae+cdx))^{\frac{3}{4}}}\,dx$$

input
```
integrate((e*x+d)**2/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(3/4),x)
```

output
```
Integral((d + e*x)**2/((d + e*x)*(a*e + c*d*x))**(3/4), x)
```

Maxima [F]

$$\int \frac{(d+ex)^2}{(ade+(cd^2+ae^2)\,x+cdex^2)^{3/4}}\,dx = \int \frac{(ex+d)^2}{(cdex^2+ade+(cd^2+ae^2)x)^{\frac{3}{4}}}\,dx$$

input
```
integrate((e*x+d)^2/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/4),x, algorithm="
maxima")
```

output
```
integrate((e*x + d)^2/(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^(3/4), x)
```

Giac [F]

$$\int \frac{(d+ex)^2}{(ade+(cd^2+ae^2)\,x+cdex^2)^{3/4}}\,dx = \int \frac{(ex+d)^2}{(cdex^2+ade+(cd^2+ae^2)x)^{\frac{3}{4}}}\,dx$$

input
```
integrate((e*x+d)^2/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/4),x, algorithm="
giac")
```

output
```
integrate((e*x + d)^2/(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^(3/4), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d+ex)^2}{(ade+(cd^2+ae^2)x+cdex^2)^{3/4}}\,dx = \int \frac{(d+e\,x)^2}{(c\,d\,e\,x^2+(c\,d^2+a\,e^2)\,x+a\,d\,e)^{3/4}}\,dx$$

input
```
int((d + e*x)^2/(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(3/4),x)
```

output
```
int((d + e*x)^2/(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(3/4), x)
```

Reduce [F]

$$\int \frac{(d+ex)^2}{(ade+(cd^2+ae^2)x+cdex^2)^{3/4}}\,dx = \left(\int \frac{x^2}{(cde\,x^2+a\,e^2x+c\,d^2x+ade)^{\frac{3}{4}}}dx\right)e^2 + 2\left(\int \frac{x}{(cde\,x^2+a\,e^2x+c\,d^2x+ade)^{\frac{3}{4}}}dx\right)de + \left(\int \frac{1}{(cde\,x^2+a\,e^2x+c\,d^2x+ade)^{\frac{3}{4}}}dx\right)d^2$$

input
```
int((e*x+d)^2/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/4),x)
```

output
```
int(x**2/(a*d*e + a*e**2*x + c*d**2*x + c*d*e*x**2)**(3/4),x)*e**2 + 2*int
(x/(a*d*e + a*e**2*x + c*d**2*x + c*d*e*x**2)**(3/4),x)*d*e + int(1/(a*d*e
 + a*e**2*x + c*d**2*x + c*d*e*x**2)**(3/4),x)*d**2
```

3.370 $\int \frac{d+ex}{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/4}}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 174

$$\int \frac{d+ex}{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/4}}\,dx=\frac{2\sqrt[4]{ade+\left(cd^2+ae^2\right)x+cdex^2}}{cd}-\frac{2\sqrt{e}\sqrt{cd^2-ae^2}(ae+cdx)^{3/2}\left(\frac{cd(d+ex)}{e(ae+cdx)}\right)^{3/4}\operatorname{EllipticF}\left(\frac{1}{2}\arctan\left(\frac{\sqrt{cd^2-ae^2}}{\sqrt{e}\sqrt{ae+cdx}}\right),2\right)}{c^2d^2\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/4}}$$

output
```
2*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/4)/c/d-2*e^(1/2)*(-a*e^2+c*d^2)^(1/
2)*(c*d*x+a*e)^(3/2)*(c*d*(e*x+d)/e/(c*d*x+a*e))^(3/4)*InverseJacobiAM(1/2
*arctan((-a*e^2+c*d^2)^(1/2)/e^(1/2)/(c*d*x+a*e)^(1/2)),2^(1/2))/c^2/d^2/(
a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/4)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.03 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.49

$$\int \frac{d+ex}{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/4}}\,dx=\frac{4\sqrt[4]{(ae+cdx)(d+ex)}\operatorname{Hypergeometric2F1}\left(-\frac{1}{4},\frac{1}{4},\frac{5}{4},\frac{e(ae+cdx)}{-cd^2+ae^2}\right)}{cd\sqrt[4]{\frac{cd(d+ex)}{cd^2-ae^2}}}$$

input
```
Integrate[(d + e*x)/(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/4),x]
```

output
```
(4*((a*e + c*d*x)*(d + e*x))^(1/4)*Hypergeometric2F1[-1/4, 1/4, 5/4, (e*(a
*e + c*d*x))/(-(c*d^2) + a*e^2)])/(c*d*((c*d*(d + e*x))/(c*d^2 - a*e^2))^(
1/4))
```

Rubi [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 442 vs. $2(174) = 348$.

Time = 0.41 (sec) , antiderivative size = 442, normalized size of antiderivative = 2.54, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.086$, Rules used = {1160, 1094, 761}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{d+ex}{(x(ae^2+cd^2)+ade+cdex^2)^{3/4}}dx$$

$$\Big\downarrow \text{1160}$$

$$\frac{\left(d^2-\frac{ae^2}{c}\right)\int \frac{1}{(cdex^2+(cd^2+ae^2)x+ade)^{3/4}}dx}{2d}+\frac{2\sqrt[4]{x(ae^2+cd^2)+ade+cdex^2}}{cd}$$

$$\Big\downarrow \text{1094}$$

$$\frac{2\left(d^2-\frac{ae^2}{c}\right)\sqrt{(ae^2+cd^2+2cdex)^2}\int \frac{1}{\sqrt{(cd^2-ae^2)^2+4cde(cdex^2+(cd^2+ae^2)x+ade)}}d\sqrt[4]{cdex^2+(cd^2+ae^2)x+ade}}{d(ae^2+cd^2+2cdex)}+\frac{2\sqrt[4]{x(ae^2+cd^2)+ade+cdex^2}}{cd}$$

$$\Big\downarrow \text{761}$$

$$\frac{\sqrt{cd^2-ae^2}\left(d^2-\frac{ae^2}{c}\right)\sqrt{(ae^2+cd^2+2cdex)^2}\left(\frac{2\sqrt{c}\sqrt{d}\sqrt{e}\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{cd^2-ae^2}+1\right)\sqrt{\frac{4cde(x(ae^2+cd^2)+ade+cdex}{(cd^2-ae^2)^2\left(\frac{2\sqrt{c}\sqrt{d}\sqrt{e}\sqrt{x\left(ae^2+c\right.}}{cd^2-a}\right.}}}{\sqrt{2}\sqrt[4]{c}d^{5/4}\sqrt[4]{e}\,(ae^2+cd^2+2cdex)\sqrt{4cde\,(x\,(ae^2+cd^2)}}$$

$$\frac{2\sqrt[4]{x\,(ae^2+cd^2)+ade+cdex^2}}{cd}$$

input
```
Int[(d + e*x)/(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/4),x]
```

output
```
(2*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(1/4))/(c*d) + (Sqrt[c*d^2 - a*
e^2]*(d^2 - (a*e^2)/c)*Sqrt[(c*d^2 + a*e^2 + 2*c*d*e*x)^2]*(1 + (2*Sqrt[c]
*Sqrt[d]*Sqrt[e]*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(c*d^2 - a*e
^2))*Sqrt[((c*d^2 - a*e^2)^2 + 4*c*d*e*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*
x^2))/((c*d^2 - a*e^2)^2*(1 + (2*Sqrt[c]*Sqrt[d]*Sqrt[e]*Sqrt[a*d*e + (c*d
^2 + a*e^2)*x + c*d*e*x^2])/(c*d^2 - a*e^2))^2)]*EllipticF[2*ArcTan[(Sqrt[
2]*c^(1/4)*d^(1/4)*e^(1/4)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(1/4))/
Sqrt[c*d^2 - a*e^2]], 1/2])/(Sqrt[2]*c^(1/4)*d^(5/4)*e^(1/4)*(c*d^2 + a*e^
2 + 2*c*d*e*x)*Sqrt[(c*d^2 - a*e^2)^2 + 4*c*d*e*(a*d*e + (c*d^2 + a*e^2)*x
 + c*d*e*x^2)])
```

Defintions of rubi rules used

rule 761
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 1094
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[4*(Sqrt[(b
+ 2*c*x)^2]/(b + 2*c*x))   Subst[Int[x^(4*(p + 1) - 1)/Sqrt[b^2 - 4*a*c + 4
*c*x^4], x], x, (a + b*x + c*x^2)^(1/4)], x] /; FreeQ[{a, b, c}, x] && Inte
gerQ[4*p]
```

rule 1160
```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[e*((a + b*x + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Simp[(2*c*d - b
*e)/(2*c)   Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
 && NeQ[p, -1]
```

Maple [F]

$$\int \frac{ex+d}{\left(ade+\left(a\,e^{2}+c\,d^{2}\right)x+cd\,x^{2}e\right)^{\frac{3}{4}}}dx$$

input
```
int((e*x+d)/(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(3/4),x)
```

output
```
int((e*x+d)/(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(3/4),x)
```

Fricas [F]

$$\int \frac{d+ex}{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/4}}\,dx=\int \frac{ex+d}{\left(cdex^2+ade+\left(cd^2+ae^2\right)x\right)^{\frac{3}{4}}}\,dx$$

input
```
integrate((e*x+d)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/4),x, algorithm="fr
icas")
```

output
```
integral((c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^(1/4)/(c*d*x + a*e), x)
```

Sympy [F]

$$\int \frac{d+ex}{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/4}}\,dx=\int \frac{d+ex}{\left(\left(d+ex\right)\left(ae+cdx\right)\right)^{\frac{3}{4}}}\,dx$$

input
```
integrate((e*x+d)/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(3/4),x)
```

output
```
Integral((d + e*x)/((d + e*x)*(a*e + c*d*x))**(3/4), x)
```

Maxima [F]

$$\int \frac{d+ex}{(ade+(cd^2+ae^2)\,x+cdex^2)^{3/4}}\,dx=\int \frac{ex+d}{(cdex^2+ade+(cd^2+ae^2)x)^{\frac{3}{4}}}\,dx$$

input
```
integrate((e*x+d)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/4),x, algorithm="ma
xima")
```

output
```
integrate((e*x + d)/(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^(3/4), x)
```

Giac [F]

$$\int \frac{d+ex}{(ade+(cd^2+ae^2)\,x+cdex^2)^{3/4}}\,dx=\int \frac{ex+d}{(cdex^2+ade+(cd^2+ae^2)x)^{\frac{3}{4}}}\,dx$$

input
```
integrate((e*x+d)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/4),x, algorithm="gi
ac")
```

output
```
integrate((e*x + d)/(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^(3/4), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{d+ex}{(ade+(cd^2+ae^2)\,x+cdex^2)^{3/4}}\,dx=\int \frac{d+e\,x}{{\left(c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e\right)}^{3/4}}\,dx$$

input
```
int((d + e*x)/(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(3/4),x)
```

output
```
int((d + e*x)/(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(3/4), x)
```

Reduce [F]

$$\int \frac{d+ex}{(ade+(cd^2+ae^2)\,x+cdex^2)^{3/4}}\,dx = \left(\int \frac{x}{(cde\,x^2+a\,e^2x+c\,d^2x+ade)^{\frac{3}{4}}}dx\right)e + \left(\int \frac{1}{(cde\,x^2+a\,e^2x+c\,d^2x+ade)^{\frac{3}{4}}}dx\right)d$$

input
```
int((e*x+d)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/4),x)
```

output
```
int(x/(a*d*e + a*e**2*x + c*d**2*x + c*d*e*x**2)**(3/4),x)*e + int(1/(a*d*
e + a*e**2*x + c*d**2*x + c*d*e*x**2)**(3/4),x)*d
```

3.371 $\int \frac{1}{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/4}}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 142

$$\int \frac{1}{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/4}}\,dx=\frac{2\sqrt{2}(cd^2-ae^2)\left(-\frac{cde\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)}{\left(cd^2-ae^2\right)^2}\right)^{3/4}\text{EllipticF}\left(\frac{1}{2}\text{arcsi}\right)}{cde\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/4}}$$

output
```
2*2^(1/2)*(-a*e^2+c*d^2)*(-c*d*e*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)/(-a*e^2
+c*d^2)^2)^(3/4)*InverseJacobiAM(1/2*arcsin((a*e^2+c*d*(2*e*x+d))/(-a*e^2+
c*d^2)),2^(1/2))/c/d/e/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/4)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.03 (sec) , antiderivative size = 93, normalized size of antiderivative = 0.65

$$\int \frac{1}{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/4}}\,dx=\frac{4\left(\frac{cd(d+ex)}{cd^2-ae^2}\right)^{3/4}\sqrt[4]{(ae+cdx)(d+ex)}\,\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{3}{4},\frac{5}{4}\right)}{cd(d+ex)}$$

input
```
Integrate[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(-3/4),x]
```

output
```
(4*((c*d*(d + e*x))/(c*d^2 - a*e^2))^(3/4)*((a*e + c*d*x)*(d + e*x))^(1/4)
*Hypergeometric2F1[1/4, 3/4, 5/4, (e*(a*e + c*d*x))/(-(c*d^2) + a*e^2)])/(
c*d*(d + e*x))
```

Rubi [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 391 vs. 2(142) = 284.

Time = 0.35 (sec) , antiderivative size = 391, normalized size of antiderivative = 2.75, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.069$, Rules used = {1094, 761}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(x(ae^2+cd^2)+ade+cdex^2)^{3/4}}\,dx$$

$$\Big\downarrow \text{1094}$$

$$\frac{4\sqrt{(ae^2+cd^2+2cdex)^2}\int \frac{1}{\sqrt{(cd^2-ae^2)^2+4cde(cdex^2+(cd^2+ae^2)x+ade)}}d\sqrt[4]{cdex^2+(cd^2+ae^2)x+ade}}{ae^2+cd^2+2cdex}$$

$$\Big\downarrow \text{761}$$

$$\frac{\sqrt{2}\sqrt{cd^2-ae^2}\sqrt{(ae^2+cd^2+2cdex)^2}\left(\frac{2\sqrt{c}\sqrt{d}\sqrt{e}\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{cd^2-ae^2}+1\right)\sqrt{\frac{4cde(x(ae^2+cd^2)+ade+cdex^2)+(cd^2-}{(cd^2-ae^2)^2\left(\frac{2\sqrt{c}\sqrt{d}\sqrt{e}\sqrt{x\left(ae^2+cd^2\right)+ade+c}}{cd^2-ae^2}\right.}}}{\sqrt[4]{c}\sqrt[4]{d}\sqrt[4]{e}\,(ae^2+cd^2+2cdex)\sqrt{4cde\,(x\,(ae^2+cd^2)+ad}}$$

input
```
Int[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(-3/4),x]
```

output
```
(Sqrt[2]*Sqrt[c*d^2 - a*e^2]*Sqrt[(c*d^2 + a*e^2 + 2*c*d*e*x)^2]*(1 + (2*S
qrt[c]*Sqrt[d]*Sqrt[e]*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(c*d^2
 - a*e^2))*Sqrt[((c*d^2 - a*e^2)^2 + 4*c*d*e*(a*d*e + (c*d^2 + a*e^2)*x +
c*d*e*x^2))/((c*d^2 - a*e^2)^2*(1 + (2*Sqrt[c]*Sqrt[d]*Sqrt[e]*Sqrt[a*d*e
+ (c*d^2 + a*e^2)*x + c*d*e*x^2])/(c*d^2 - a*e^2))^2)]*EllipticF[2*ArcTan[
(Sqrt[2]*c^(1/4)*d^(1/4)*e^(1/4)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(
1/4))/Sqrt[c*d^2 - a*e^2]], 1/2])/(c^(1/4)*d^(1/4)*e^(1/4)*(c*d^2 + a*e^2
+ 2*c*d*e*x)*Sqrt[(c*d^2 - a*e^2)^2 + 4*c*d*e*(a*d*e + (c*d^2 + a*e^2)*x +
 c*d*e*x^2)])
```

Defintions of rubi rules used

rule 761
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 1094
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[4*(Sqrt[(b
+ 2*c*x)^2]/(b + 2*c*x))   Subst[Int[x^(4*(p + 1) - 1)/Sqrt[b^2 - 4*a*c + 4
*c*x^4], x], x, (a + b*x + c*x^2)^(1/4)], x] /; FreeQ[{a, b, c}, x] && Inte
gerQ[4*p]
```

Maple [F]

$$\int \frac{1}{\left(ade+\left(a\,e^{2}+c\,d^{2}\right)x+cd\,x^{2}e\right)^{\frac{3}{4}}}dx$$

input
```
int(1/(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(3/4),x)
```

output
```
int(1/(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(3/4),x)
```

Fricas [F]

$$\int \frac{1}{(ade + (cd^2 + ae^2)\,x + cdex^2)^{3/4}}\,dx = \int \frac{1}{(cdex^2 + ade + (cd^2 + ae^2)x)^{\frac{3}{4}}}\,dx$$

input
```
integrate(1/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/4),x, algorithm="fricas")
```

output
```
integral((c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^(-3/4), x)
```

Sympy [F]

$$\int \frac{1}{(ade + (cd^2 + ae^2)\,x + cdex^2)^{3/4}}\,dx = \int \frac{1}{(ade + cdex^2 + x\,(ae^2 + cd^2))^{\frac{3}{4}}}\,dx$$

input
```
integrate(1/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(3/4),x)
```

output
```
Integral((a*d*e + c*d*e*x**2 + x*(a*e**2 + c*d**2))**(-3/4), x)
```

Maxima [F]

$$\int \frac{1}{(ade + (cd^2 + ae^2)\,x + cdex^2)^{3/4}}\,dx = \int \frac{1}{(cdex^2 + ade + (cd^2 + ae^2)x)^{\frac{3}{4}}}\,dx$$

input
```
integrate(1/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/4),x, algorithm="maxima")
```

output
```
integrate((c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^(-3/4), x)
```

Giac [F]

$$\int \frac{1}{\left(ade+(cd^2+ae^2)x+cdex^2\right)^{3/4}}\,dx = \int \frac{1}{(cdex^2+ade+(cd^2+ae^2)x)^{\frac{3}{4}}}\,dx$$

input
```
integrate(1/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/4),x, algorithm="giac")
```

output
```
integrate((c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^(-3/4), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\left(ade+(cd^2+ae^2)x+cdex^2\right)^{3/4}}\,dx = \int \frac{1}{\left(c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e\right)^{3/4}}\,dx$$

input
```
int(1/(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(3/4),x)
```

output
```
int(1/(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(3/4), x)
```

Reduce [F]

$$\int \frac{1}{\left(ade+(cd^2+ae^2)x+cdex^2\right)^{3/4}}\,dx = \int \frac{1}{(cde\,x^2+a\,e^2x+c\,d^2x+ade)^{\frac{3}{4}}}dx$$

input
```
int(1/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/4),x)
```

output
```
int(1/(a*d*e + a*e**2*x + c*d**2*x + c*d*e*x**2)**(3/4),x)
```

3.372 $\int \frac{1}{(d+ex)\left(ade+(cd^2+ae^2)x+cdex^2\right)^{3/4}}\,dx$

Optimal result

Integrand size = 37, antiderivative size = 187

$$\int \frac{1}{(d+ex)\left(ade+(cd^2+ae^2)x+cdex^2\right)^{3/4}}\,dx = \frac{4\sqrt[4]{ade+(cd^2+ae^2)x+cdex^2}}{3(cd^2-ae^2)(d+ex)} - \frac{8\sqrt{e}(ae+cdx)^{3/2}\left(\frac{cd(d+ex)}{e(ae+cdx)}\right)^{3/4}\text{EllipticF}\left(\frac{1}{2}\arctan\left(\frac{\sqrt{cd^2-ae^2}}{\sqrt{e}\sqrt{ae+cdx}}\right),2\right)}{3(cd^2-ae^2)^{3/2}\left(ade+(cd^2+ae^2)x+cdex^2\right)^{3/4}}$$

output
```
4/3*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/4)/(-a*e^2+c*d^2)/(e*x+d)-8/3*e^(
1/2)*(c*d*x+a*e)^(3/2)*(c*d*(e*x+d)/e/(c*d*x+a*e))^(3/4)*InverseJacobiAM(1
/2*arctan((-a*e^2+c*d^2)^(1/2)/e^(1/2)/(c*d*x+a*e)^(1/2)),2^(1/2))/(-a*e^2
+c*d^2)^(3/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/4)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.05 (sec) , antiderivative size = 96, normalized size of antiderivative = 0.51

$$\int \frac{1}{(d+ex)\left(ade+(cd^2+ae^2)x+cdex^2\right)^{3/4}}\,dx = \frac{4cd\sqrt[4]{(ae+cdx)(d+ex)}\,\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{7}{4},\frac{5}{4},\frac{e}{-}\right.}{(cd^2-ae^2)^2\sqrt[4]{\frac{cd(d+ex)}{cd^2-ae^2}}}$$

input
```
Integrate[1/((d + e*x)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/4)),x]
```

output
```
(4*c*d*((a*e + c*d*x)*(d + e*x))^(1/4)*Hypergeometric2F1[1/4, 7/4, 5/4, (e
*(a*e + c*d*x))/(-(c*d^2) + a*e^2)])/((c*d^2 - a*e^2)^2*((c*d*(d + e*x))/(
c*d^2 - a*e^2))^(1/4))
```

Rubi [A] (warning: unable to verify)

Time = 0.42 (sec) , antiderivative size = 260, normalized size of antiderivative = 1.39, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.189, Rules used = {1138, 61, 73, 768, 858, 807, 229}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(d+ex)\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/4}}dx$$

$$\Big\downarrow \ 1138$$

$$\frac{\left(\frac{ex}{d}+1\right)^{3/4}(ae+cdx)^{3/4}\int \frac{1}{(ae+cdx)^{3/4}\left(\frac{ex}{d}+1\right)^{7/4}}dx}{d\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/4}}$$

$$\Big\downarrow \ 61$$

$$\frac{\left(\frac{ex}{d}+1\right)^{3/4}(ae+cdx)^{3/4}\left(\frac{2cd\int \frac{1}{(ae+cdx)^{3/4}\left(\frac{ex}{d}+1\right)^{3/4}}dx}{3\left(cd-\frac{ae^2}{d}\right)}+\frac{4\sqrt[4]{ae+cdx}}{3\left(\frac{ex}{d}+1\right)^{3/4}\left(cd-\frac{ae^2}{d}\right)}\right)}{d\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/4}}$$

$$\Big\downarrow \ 73$$

$$\frac{\left(\frac{ex}{d}+1\right)^{3/4}(ae+cdx)^{3/4}\left(\frac{8\int \frac{1}{\left(-\frac{ae^2}{cd^2}+\frac{(ae+cdx)e}{cd^2}+1\right)^{3/4}}d\sqrt[4]{ae+cdx}}{3\left(cd-\frac{ae^2}{d}\right)}+\frac{4\sqrt[4]{ae+cdx}}{3\left(\frac{ex}{d}+1\right)^{3/4}\left(cd-\frac{ae^2}{d}\right)}\right)}{d\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/4}}$$

$$\Big\downarrow \ 768$$

$$\frac{\left(\frac{ex}{d}+1\right)^{3/4}(ae+cdx)^{3/4}\left(\frac{8(ae+cdx)^{3/4}\left(\frac{cd^2-ae^2}{e(ae+cdx)}+1\right)^{3/4}\int \frac{1}{(ae+cdx)^{3/4}\left(\frac{cd^2-ae^2}{e(ae+cdx)}+1\right)^{3/4}}d\sqrt[4]{ae+cdx}}{3\left(cd-\frac{ae^2}{d}\right)\left(-\frac{ae^2}{cd^2}+\frac{e(ae+cdx)}{cd^2}+1\right)^{3/4}}+\frac{4\sqrt[4]{ae+cdx}}{3\left(\frac{ex}{d}+1\right)^{3/4}\left(cd-\frac{ae^2}{d}\right)}\right)}{d\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/4}}$$

$$\Big\downarrow \; 858$$

$$\frac{\left(\frac{ex}{d}+1\right)^{3/4}(ae+cdx)^{3/4}\left(\frac{4\sqrt[4]{ae+cdx}}{3\left(\frac{ex}{d}+1\right)^{3/4}\left(cd-\frac{ae^2}{d}\right)}-\frac{8(ae+cdx)^{3/4}\left(\frac{cd^2-ae^2}{e(ae+cdx)}+1\right)^{3/4}\int \frac{1}{\sqrt[4]{ae+cdx}\left(\frac{\left(cd^2-ae^2\right)(ae+cdx)}{e}+1\right)^{3/4}}d\sqrt[4]{ae+cdx}}{3\left(cd-\frac{ae^2}{d}\right)\left(-\frac{ae^2}{cd^2}+\frac{e(ae+cdx)}{cd^2}+1\right)^{3/4}}\right)}{d\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/4}}$$

$$\Big\downarrow \; 807$$

$$\frac{\left(\frac{ex}{d}+1\right)^{3/4}(ae+cdx)^{3/4}\left(\frac{4\sqrt[4]{ae+cdx}}{3\left(\frac{ex}{d}+1\right)^{3/4}\left(cd-\frac{ae^2}{d}\right)}-\frac{4(ae+cdx)^{3/4}\left(\frac{cd^2-ae^2}{e(ae+cdx)}+1\right)^{3/4}\int \frac{1}{\left(\frac{\sqrt{ae+cdx}\left(cd^2-ae^2\right)}{e}+1\right)^{3/4}}d\sqrt{ae+cdx}}{3\left(cd-\frac{ae^2}{d}\right)\left(-\frac{ae^2}{cd^2}+\frac{e(ae+cdx)}{cd^2}+1\right)^{3/4}}\right)}{d\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/4}}$$

$$\Big\downarrow \; 229$$

$$\frac{\left(\frac{ex}{d}+1\right)^{3/4}(ae+cdx)^{3/4}\left(\frac{4\sqrt[4]{ae+cdx}}{3\left(\frac{ex}{d}+1\right)^{3/4}\left(cd-\frac{ae^2}{d}\right)}-\frac{8\sqrt{e}(ae+cdx)^{3/4}\left(\frac{cd^2-ae^2}{e(ae+cdx)}+1\right)^{3/4}\text{EllipticF}\left(\frac{1}{2}\arctan\left(\frac{\sqrt{cd^2-ae^2}\sqrt{ae+cdx}}{\sqrt{e}}\right),2\right)}{3\sqrt{cd^2-ae^2}\left(cd-\frac{ae^2}{d}\right)\left(-\frac{ae^2}{cd^2}+\frac{e(ae+cdx)}{cd^2}+1\right)^{3/4}}\right)}{d\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/4}}$$

input
```
Int[1/((d + e*x)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/4)),x]
```

output
```
((a*e + c*d*x)^(3/4)*(1 + (e*x)/d)^(3/4)*((4*(a*e + c*d*x)^(1/4))/(3*(c*d
- (a*e^2)/d)*(1 + (e*x)/d)^(3/4)) - (8*Sqrt[e]*(a*e + c*d*x)^(3/4)*(1 + (c
*d^2 - a*e^2)/(e*(a*e + c*d*x)))^(3/4)*EllipticF[ArcTan[(Sqrt[c*d^2 - a*e^
2]*Sqrt[a*e + c*d*x])/Sqrt[e]]/2, 2])/(3*Sqrt[c*d^2 - a*e^2]*(c*d - (a*e^2
)/d)*(1 - (a*e^2)/(c*d^2) + (e*(a*e + c*d*x))/(c*d^2))^(3/4))))/(d*(a*d*e
+ (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/4))
```

Defintions of rubi rules used

rule 61
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && LtQ[m, -1] &&  !(LtQ[n, -1] && (EqQ[a, 0
] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && IntLinearQ[a, b, c, d
, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 229
```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(2/(a^(3/4)*Rt[b/a, 2])
)*EllipticF[(1/2)*ArcTan[Rt[b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

rule 768
```
Int[((a_) + (b_.)*(x_)^4)^(-3/4), x_Symbol] :> Simp[x^3*((1 + a/(b*x^4))^(3
/4)/(a + b*x^4)^(3/4))   Int[1/(x^3*(1 + a/(b*x^4))^(3/4)), x], x] /; FreeQ
[{a, b}, x]
```

rule 807
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Simp[1/k   Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x,
x^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

rule 858
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

rule 1138
```
Int[((d_) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Sy
mbol] :> Simp[d^m*((a + b*x + c*x^2)^FracPart[p]/((1 + e*(x/d))^FracPart[p]
*(a/d + (c*x)/e)^FracPart[p]))   Int[(1 + e*(x/d))^(m + p)*(a/d + (c/e)*x)^
p, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0]
&& (IntegerQ[m] || GtQ[d, 0]) &&  !(IGtQ[m, 0] && (IntegerQ[3*p] || Integer
Q[4*p]))
```

Maple [F]

$$\int \frac{1}{(ex+d)\left(ade+\left(a\,e^{2}+c\,d^{2}\right)x+cd\,x^{2}e\right)^{\frac{3}{4}}}dx$$

input
```
int(1/(e*x+d)/(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(3/4),x)
```

output
```
int(1/(e*x+d)/(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(3/4),x)
```

Fricas [F]

$$\int \frac{1}{(d+ex)\left(ade+(cd^2+ae^2)\,x+cdex^2\right)^{3/4}}\,dx=\int \frac{1}{(cdex^2+ade+(cd^2+ae^2)x)^{\frac{3}{4}}(ex+d)}\,dx$$

input
```
integrate(1/(e*x+d)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/4),x, algorithm="
fricas")
```

output
```
integral((c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^(1/4)/(c*d*e^2*x^3 + a*d^
2*e + (2*c*d^2*e + a*e^3)*x^2 + (c*d^3 + 2*a*d*e^2)*x), x)
```

Sympy [F]

$$\int \frac{1}{(d+ex)\left(ade+(cd^2+ae^2)\,x+cdex^2\right)^{3/4}}\,dx=\int \frac{1}{\left((d+ex)\,(ae+cdx)\right)^{\frac{3}{4}}(d+ex)}\,dx$$

input
```
integrate(1/(e*x+d)/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(3/4),x)
```

output
```
Integral(1/(((d + e*x)*(a*e + c*d*x))**(3/4)*(d + e*x)), x)
```

Maxima [F]

$$\int \frac{1}{(d+ex)\left(ade+(cd^2+ae^2)\,x+cdex^2\right)^{3/4}}\,dx=\int \frac{1}{(cdex^2+ade+(cd^2+ae^2)x)^{\frac{3}{4}}(ex+d)}\,dx$$

input
```
integrate(1/(e*x+d)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/4),x, algorithm="
maxima")
```

output
```
integrate(1/((c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^(3/4)*(e*x + d)), x)
```

Giac [F]

$$\int \frac{1}{(d+ex)\left(ade+(cd^2+ae^2)\,x+cdex^2\right)^{3/4}}\,dx=\int \frac{1}{(cdex^2+ade+(cd^2+ae^2)x)^{\frac{3}{4}}(ex+d)}\,dx$$

input
```
integrate(1/(e*x+d)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/4),x, algorithm="
giac")
```

output
```
integrate(1/((c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^(3/4)*(e*x + d)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(d+ex)\,(ade+(cd^2+ae^2)\,x+cdex^2)^{3/4}}\,dx=\int \frac{1}{(d+e\,x)\,\left(c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e\right)^{3/4}}\,dx$$

input
```
int(1/((d + e*x)*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(3/4)),x)
```

output
```
int(1/((d + e*x)*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(3/4)), x)
```

Reduce [F]

$$\int \frac{1}{(d+ex)\,(ade+(cd^2+ae^2)\,x+cdex^2)^{3/4}}\,dx=\int \frac{1}{(cde\,x^2+a\,e^2x+c\,d^2x+ade)^{\frac{3}{4}}\,d+(cde\,x^2+a\,e^2x+$$

input
```
int(1/(e*x+d)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/4),x)
```

output
```
int(1/((a*d*e + a*e**2*x + c*d**2*x + c*d*e*x**2)**(3/4)*d + (a*d*e + a*e*
*2*x + c*d**2*x + c*d*e*x**2)**(3/4)*e*x),x)
```

3.373 $\int \frac{1}{(d+ex)^2\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/4}}\,dx$

Optimal result

Integrand size = 37, antiderivative size = 245

$$\int \frac{1}{(d+ex)^2\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/4}}\,dx=\frac{4\sqrt[4]{ade+\left(cd^2+ae^2\right)x+cdex^2}}{7\left(cd^2-ae^2\right)(d+ex)^2}$$
$$+\frac{8cd\sqrt[4]{ade+\left(cd^2+ae^2\right)x+cdex^2}}{7\left(cd^2-ae^2\right)^2(d+ex)}$$
$$-\frac{16cd\sqrt{e}(ae+cdx)^{3/2}\left(\frac{cd(d+ex)}{e(ae+cdx)}\right)^{3/4}\operatorname{EllipticF}\left(\frac{1}{2}\arctan\left(\frac{\sqrt{cd^2-ae^2}}{\sqrt{e}\sqrt{ae+cdx}}\right),2\right)}{7\left(cd^2-ae^2\right)^{5/2}\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/4}}$$

output
```
4/7*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/4)/(-a*e^2+c*d^2)/(e*x+d)^2+8/7*c
*d*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/4)/(-a*e^2+c*d^2)^2/(e*x+d)-16/7*c
*d*e^(1/2)*(c*d*x+a*e)^(3/2)*(c*d*(e*x+d)/e/(c*d*x+a*e))^(3/4)*InverseJaco
biAM(1/2*arctan((-a*e^2+c*d^2)^(1/2)/e^(1/2)/(c*d*x+a*e)^(1/2)),2^(1/2))/(
-a*e^2+c*d^2)^(5/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/4)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.05 (sec) , antiderivative size = 100, normalized size of antiderivative = 0.41

$$\int \frac{1}{(d+ex)^2\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/4}}\,dx=\frac{4c^2d^2\sqrt[4]{(ae+cdx)(d+ex)}\,\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{11}{4},\right.}{\left(cd^2-ae^2\right)^3\sqrt[4]{\frac{cd(d+ex)}{cd^2-ae^2}}}$$

input

```
Integrate[1/((d + e*x)^2*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/4)),x]
```

output

```
(4*c^2*d^2*((a*e + c*d*x)*(d + e*x))^(1/4)*Hypergeometric2F1[1/4, 11/4, 5/
4, (e*(a*e + c*d*x))/(-(c*d^2) + a*e^2)])/((c*d^2 - a*e^2)^3*((c*d*(d + e*
x))/(c*d^2 - a*e^2))^(1/4))
```

Rubi [A] (warning: unable to verify)

Time = 0.47 (sec) , antiderivative size = 325, normalized size of antiderivative = 1.33, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.216, Rules used = {1138, 61, 61, 73, 768, 858, 807, 229}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(d+ex)^2\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/4}}dx$$

$$\Big\downarrow \text{1138}$$

$$\frac{\left(\frac{ex}{d}+1\right)^{3/4}(ae+cdx)^{3/4}\int \frac{1}{(ae+cdx)^{3/4}\left(\frac{ex}{d}+1\right)^{11/4}}dx}{d^2\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/4}}$$

$$\Big\downarrow \text{61}$$

$$\frac{\left(\frac{ex}{d}+1\right)^{3/4}(ae+cdx)^{3/4}\left(\frac{6cd\int \frac{1}{(ae+cdx)^{3/4}\left(\frac{ex}{d}+1\right)^{7/4}}dx}{7\left(cd-\frac{ae^2}{d}\right)}+\frac{4\sqrt[4]{ae+cdx}}{7\left(\frac{ex}{d}+1\right)^{7/4}\left(cd-\frac{ae^2}{d}\right)}\right)}{d^2\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/4}}$$

$\downarrow$ 61

$$\frac{\left(\frac{ex}{d}+1\right)^{3/4}(ae+cdx)^{3/4}\left(\frac{6cd\left(\frac{2cd\int \frac{1}{(ae+cdx)^{3/4}\left(\frac{ex}{d}+1\right)^{3/4}}dx}{3\left(cd-\frac{ae^2}{d}\right)}+\frac{4\sqrt[4]{ae+cdx}}{3\left(\frac{ex}{d}+1\right)^{3/4}\left(cd-\frac{ae^2}{d}\right)}\right)}{7\left(cd-\frac{ae^2}{d}\right)}+\frac{4\sqrt[4]{ae+cdx}}{7\left(\frac{ex}{d}+1\right)^{7/4}\left(cd-\frac{ae^2}{d}\right)}\right)}{d^2\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/4}}$$

$\downarrow$ 73

$$\frac{\left(\frac{ex}{d}+1\right)^{3/4}(ae+cdx)^{3/4}\left(\frac{6cd\left(\frac{8\int \frac{1}{\left(-\frac{ae^2}{cd^2}+\frac{(ae+cdx)e}{cd^2}+1\right)^{3/4}}d\sqrt[4]{ae+cdx}}{3\left(cd-\frac{ae^2}{d}\right)}+\frac{4\sqrt[4]{ae+cdx}}{3\left(\frac{ex}{d}+1\right)^{3/4}\left(cd-\frac{ae^2}{d}\right)}\right)}{7\left(cd-\frac{ae^2}{d}\right)}+\frac{4\sqrt[4]{ae+cdx}}{7\left(\frac{ex}{d}+1\right)^{7/4}\left(cd-\frac{ae^2}{d}\right)}\right)}{d^2\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/4}}$$

$\downarrow$ 768

$$\frac{\left(\frac{ex}{d}+1\right)^{3/4}(ae+cdx)^{3/4}\left(\frac{6cd\left(\frac{8(ae+cdx)^{3/4}\left(\frac{cd^2-ae^2}{e(ae+cdx)}+1\right)^{3/4}\int \frac{1}{(ae+cdx)^{3/4}\left(\frac{cd^2-ae^2}{e(ae+cdx)}+1\right)^{3/4}}d\sqrt[4]{ae+cdx}}{3\left(cd-\frac{ae^2}{d}\right)\left(-\frac{ae^2}{cd^2}+\frac{e(ae+cdx)}{cd^2}+1\right)^{3/4}}+\frac{4\sqrt[4]{ae+cdx}}{3\left(\frac{ex}{d}+1\right)^{3/4}\left(cd-\frac{ae^2}{d} \cdots\right)}\cdots\right)}{7\left(cd-\frac{ae^2}{d}\right)}\cdots\right)}{d^2\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/4}}$$

$\downarrow$ 858

$$\frac{\left(\frac{ex}{d}+1\right)^{3/4}(ae+cdx)^{3/4}\left(\frac{6cd\left(\frac{4\sqrt[4]{ae+cdx}}{3\left(\frac{ex}{d}+1\right)^{3/4}\left(cd-\frac{ae^2}{d}\right)}-\frac{8(ae+cdx)^{3/4}\left(\frac{cd^2-ae^2}{e(ae+cdx)}+1\right)^{3/4}\int\frac{1}{\sqrt[4]{ae+cdx}\left(\frac{\left(cd^2-ae^2\right)(ae+cdx)}{e}+1\right)^{3/4}}d}{3\left(cd-\frac{ae^2}{d}\right)\left(-\frac{ae^2}{cd^2}+\frac{e(ae+cdx)}{cd^2}+1\right)^{3/4}}\right.}{7\left(cd-\frac{ae^2}{d}\right)}\right.}{d^2\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/4}}$$

$\downarrow$ 807

$$\frac{\left(\frac{ex}{d}+1\right)^{3/4}(ae+cdx)^{3/4}\left(\frac{6cd\left(\frac{4\sqrt[4]{ae+cdx}}{3\left(\frac{ex}{d}+1\right)^{3/4}\left(cd-\frac{ae^2}{d}\right)}-\frac{4(ae+cdx)^{3/4}\left(\frac{cd^2-ae^2}{e(ae+cdx)}+1\right)^{3/4}\int\frac{1}{\left(\frac{\sqrt{ae+cdx}\left(cd^2-ae^2\right)}{e}+1\right)^{3/4}}d\sqrt{ae+cdx}}{3\left(cd-\frac{ae^2}{d}\right)\left(-\frac{ae^2}{cd^2}+\frac{e(ae+cdx)}{cd^2}+1\right)^{3/4}}\right)}{7\left(cd-\frac{ae^2}{d}\right)}+\right.}{d^2\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/4}}$$

$\downarrow$ 229

$$\frac{\left(\frac{ex}{d}+1\right)^{3/4}(ae+cdx)^{3/4}\left(\frac{6cd\left(\frac{4\sqrt[4]{ae+cdx}}{3\left(\frac{ex}{d}+1\right)^{3/4}\left(cd-\frac{ae^2}{d}\right)}-\frac{8\sqrt{e}(ae+cdx)^{3/4}\left(\frac{cd^2-ae^2}{e(ae+cdx)}+1\right)^{3/4}\text{EllipticF}\left(\frac{1}{2}\arctan\left(\frac{\sqrt{cd^2-ae^2}\sqrt{ae+cdx}}{\sqrt{e}}\right),2\right)}{3\sqrt{cd^2-ae^2}\left(cd-\frac{ae^2}{d}\right)\left(-\frac{ae^2}{cd^2}+\frac{e(ae+cdx)}{cd^2}+1\right)^{3/4}}\right.}{7\left(cd-\frac{ae^2}{d}\right)}\right.}{d^2\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/4}}$$

input
```
Int[1/((d + e*x)^2*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/4)),x]
```

output
```
((a*e + c*d*x)^(3/4)*(1 + (e*x)/d)^(3/4)*((4*(a*e + c*d*x)^(1/4))/(7*(c*d
- (a*e^2)/d)*(1 + (e*x)/d)^(7/4)) + (6*c*d*((4*(a*e + c*d*x)^(1/4))/(3*(c*
d - (a*e^2)/d)*(1 + (e*x)/d)^(3/4)) - (8*Sqrt[e]*(a*e + c*d*x)^(3/4)*(1 +
(c*d^2 - a*e^2)/(e*(a*e + c*d*x)))^(3/4)*EllipticF[ArcTan[(Sqrt[c*d^2 - a*
e^2]*Sqrt[a*e + c*d*x])/Sqrt[e]]/2, 2])/(3*Sqrt[c*d^2 - a*e^2]*(c*d - (a*e
^2)/d)*(1 - (a*e^2)/(c*d^2) + (e*(a*e + c*d*x))/(c*d^2))^(3/4))))/(7*(c*d
- (a*e^2)/d))))/(d^2*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/4))
```

Defintions of rubi rules used

rule 61
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && LtQ[m, -1] &&  !(LtQ[n, -1] && (EqQ[a, 0
] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && IntLinearQ[a, b, c, d
, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 229
```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(2/(a^(3/4)*Rt[b/a, 2])
)*EllipticF[(1/2)*ArcTan[Rt[b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

rule 768
```
Int[((a_) + (b_.)*(x_)^4)^(-3/4), x_Symbol] :> Simp[x^3*((1 + a/(b*x^4))^(3
/4)/(a + b*x^4)^(3/4))   Int[1/(x^3*(1 + a/(b*x^4))^(3/4)), x], x] /; FreeQ
[{a, b}, x]
```

rule 807
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Simp[1/k   Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x,
x^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

rule 858
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

rule 1138
```
Int[((d_) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Sy
mbol] :> Simp[d^m*((a + b*x + c*x^2)^FracPart[p]/((1 + e*(x/d))^FracPart[p]
*(a/d + (c*x)/e)^FracPart[p]))   Int[(1 + e*(x/d))^(m + p)*(a/d + (c/e)*x)^
p, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0]
&& (IntegerQ[m] || GtQ[d, 0]) &&  !(IGtQ[m, 0] && (IntegerQ[3*p] || Integer
Q[4*p]))
```

Maple [F]

$$\int \frac{1}{\left(ex+d\right)^{2} \left(ade+\left(a\,e^{2}+c\,d^{2}\right)x+cd\,x^{2}e\right)^{\frac{3}{4}}}dx$$

input
```
int(1/(e*x+d)^2/(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(3/4),x)
```

output
```
int(1/(e*x+d)^2/(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(3/4),x)
```

Fricas [F]

$$\int \frac{1}{(d+ex)^2 (ade+(cd^2+ae^2)x+cdex^2)^{3/4}}\,dx = \int \frac{1}{(cdex^2+ade+(cd^2+ae^2)x)^{\frac{3}{4}}(ex+d)^2}\,dx$$

input
```
integrate(1/(e*x+d)^2/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/4),x, algorithm
="fricas")
```

output
```
integral((c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^(1/4)/(c*d*e^3*x^4 + a*d^
3*e + (3*c*d^2*e^2 + a*e^4)*x^3 + 3*(c*d^3*e + a*d*e^3)*x^2 + (c*d^4 + 3*a
*d^2*e^2)*x), x)
```

Sympy [F]

$$\int \frac{1}{(d+ex)^2\left(ade+(cd^2+ae^2)x+cdex^2\right)^{3/4}}\,dx = \int \frac{1}{\left((d+ex)(ae+cdx)\right)^{\frac{3}{4}}(d+ex)^2}\,dx$$

input
```
integrate(1/(e*x+d)**2/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(3/4),x)
```

output
```
Integral(1/(((d + e*x)*(a*e + c*d*x))**(3/4)*(d + e*x)**2), x)
```

Maxima [F]

$$\int \frac{1}{(d+ex)^2\left(ade+(cd^2+ae^2)x+cdex^2\right)^{3/4}}\,dx = \int \frac{1}{(cdex^2+ade+(cd^2+ae^2)x)^{\frac{3}{4}}(ex+d)^2}\,dx$$

input
```
integrate(1/(e*x+d)^2/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/4),x, algorithm
="maxima")
```

output
```
integrate(1/((c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^(3/4)*(e*x + d)^2), x
)
```

Giac [F]

$$\int \frac{1}{(d+ex)^2\left(ade+(cd^2+ae^2)x+cdex^2\right)^{3/4}}\,dx = \int \frac{1}{(cdex^2+ade+(cd^2+ae^2)x)^{\frac{3}{4}}(ex+d)^2}\,dx$$

input
```
integrate(1/(e*x+d)^2/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/4),x, algorithm
="giac")
```

output
```
integrate(1/((c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^(3/4)*(e*x + d)^2), x
)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(d+ex)^2 \left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/4}}\,dx = \int \frac{1}{{\left(d+e\,x\right)}^2\,{\left(c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e\right)}^{3/4}} \,d x$$

input
```
int(1/((d + e*x)^2*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(3/4)),x)
```

output
```
int(1/((d + e*x)^2*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(3/4)), x)
```

Reduce [F]

$$\int \frac{1}{(d+ex)^2 \left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/4}}\,dx = \int \frac{}{\left(cde\,x^2+a\,e^2x+c\,d^2x+ade\right)^{\frac{3}{4}} d^2+2\left(cde\,x^2+a\,e^2\right.}$$

input
```
int(1/(e*x+d)^2/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/4),x)
```

output
```
int(1/((a*d*e + a*e**2*x + c*d**2*x + c*d*e*x**2)**(3/4)*d**2 + 2*(a*d*e +
 a*e**2*x + c*d**2*x + c*d*e*x**2)**(3/4)*d*e*x + (a*d*e + a*e**2*x + c*d*
*2*x + c*d*e*x**2)**(3/4)*e**2*x**2),x)
```

3.374 $\int \frac{1}{(d+ex)^3\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/4}}\,dx$

Optimal result

Integrand size = 37, antiderivative size = 309

$$\int \frac{1}{(d+ex)^3\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/4}}\,dx=\frac{4\sqrt[4]{ade+\left(cd^2+ae^2\right)x+cdex^2}}{11\left(cd^2-ae^2\right)(d+ex)^3}+\frac{40cd\sqrt[4]{ade+\left(cd^2+ae^2\right)x+cdex^2}}{77\left(cd^2-ae^2\right)^2(d+ex)^2}+\frac{80c^2d^2\sqrt[4]{ade+\left(cd^2+ae^2\right)x+cdex^2}}{77\left(cd^2-ae^2\right)^3(d+ex)}-\frac{160c^2d^2\sqrt{e}(ae+cdx)^{3/2}\left(\frac{cd(d+ex)}{e(ae+cdx)}\right)^{3/4}\operatorname{EllipticF}\left(\frac{1}{2}\arctan\left(\frac{\sqrt{cd^2-ae^2}}{\sqrt{e}\sqrt{ae+cdx}}\right),2\right)}{77\left(cd^2-ae^2\right)^{7/2}\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/4}}$$

output
```
4/11*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/4)/(-a*e^2+c*d^2)/(e*x+d)^3+40/7
7*c*d*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/4)/(-a*e^2+c*d^2)^2/(e*x+d)^2+8
0/77*c^2*d^2*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/4)/(-a*e^2+c*d^2)^3/(e*x
+d)-160/77*c^2*d^2*e^(1/2)*(c*d*x+a*e)^(3/2)*(c*d*(e*x+d)/e/(c*d*x+a*e))^(
3/4)*InverseJacobiAM(1/2*arctan((-a*e^2+c*d^2)^(1/2)/e^(1/2)/(c*d*x+a*e)^(
1/2)),2^(1/2))/(-a*e^2+c*d^2)^(7/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/4
)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.06 (sec) , antiderivative size = 100, normalized size of antiderivative = 0.32

$$\int \frac{1}{(d+ex)^3\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/4}}\,dx=\frac{4c^3d^3\sqrt[4]{(ae+cdx)(d+ex)}\,\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{15}{4},\right.}{\left(cd^2-ae^2\right)^4\sqrt[4]{\frac{cd(d+ex)}{cd^2-ae^2}}}$$

input
```
Integrate[1/((d + e*x)^3*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/4)),x]
```

output
```
(4*c^3*d^3*((a*e + c*d*x)*(d + e*x))^(1/4)*Hypergeometric2F1[1/4, 15/4, 5/
4, (e*(a*e + c*d*x))/(-(c*d^2) + a*e^2)])/((c*d^2 - a*e^2)^4*((c*d*(d + e*
x))/(c*d^2 - a*e^2))^(1/4))
```

Rubi [A] (warning: unable to verify)

Time = 0.53 (sec) , antiderivative size = 390, normalized size of antiderivative = 1.26, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.243$, Rules used = {1138, 61, 61, 61, 73, 768, 858, 807, 229}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(d+ex)^3\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/4}}dx$$

$$\Big\downarrow \text{1138}$$

$$\frac{\left(\frac{ex}{d}+1\right)^{3/4}(ae+cdx)^{3/4}\int \frac{1}{(ae+cdx)^{3/4}\left(\frac{ex}{d}+1\right)^{15/4}}dx}{d^3\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/4}}$$

$$\Big\downarrow \text{61}$$

$$\frac{\left(\frac{ex}{d}+1\right)^{3/4}(ae+cdx)^{3/4}\left(\frac{10cd\int \frac{1}{(ae+cdx)^{3/4}\left(\frac{ex}{d}+1\right)^{11/4}}dx}{11\left(cd-\frac{ae^2}{d}\right)}+\frac{4\sqrt[4]{ae+cdx}}{11\left(\frac{ex}{d}+1\right)^{11/4}\left(cd-\frac{ae^2}{d}\right)}\right)}{d^3\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/4}}$$

$\downarrow$ 61

$$\frac{\left(\frac{ex}{d}+1\right)^{3/4}(ae+cdx)^{3/4}\left(\frac{10cd\left(\frac{6cd\int\frac{1}{(ae+cdx)^{3/4}\left(\frac{ex}{d}+1\right)^{7/4}}dx}{7\left(cd-\frac{ae^2}{d}\right)}+\frac{4\sqrt[4]{ae+cdx}}{7\left(\frac{ex}{d}+1\right)^{7/4}\left(cd-\frac{ae^2}{d}\right)}\right)}{11\left(cd-\frac{ae^2}{d}\right)}+\frac{4\sqrt[4]{ae+cdx}}{11\left(\frac{ex}{d}+1\right)^{11/4}\left(cd-\frac{ae^2}{d}\right)}\right)}{d^3\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/4}}$$

$\downarrow$ 61

$$\frac{\left(\frac{ex}{d}+1\right)^{3/4}(ae+cdx)^{3/4}\left(\frac{10cd\left(\frac{6cd\left(\frac{2cd\int\frac{1}{(ae+cdx)^{3/4}\left(\frac{ex}{d}+1\right)^{3/4}}dx}{3\left(cd-\frac{ae^2}{d}\right)}+\frac{4\sqrt[4]{ae+cdx}}{3\left(\frac{ex}{d}+1\right)^{3/4}\left(cd-\frac{ae^2}{d}\right)}\right)}{7\left(cd-\frac{ae^2}{d}\right)}+\frac{4\sqrt[4]{ae+cdx}}{7\left(\frac{ex}{d}+1\right)^{7/4}\left(cd-\frac{ae^2}{d}\right)}\right)}{11\left(cd-\frac{ae^2}{d}\right)}+\frac{4\sqrt[4]{}}{11\left(\frac{ex}{d}+\right.}\right.}{d^3\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/4}}$$

$\downarrow$ 73

$$\frac{\left(\frac{ex}{d}+1\right)^{3/4}(ae+cdx)^{3/4}\left(\frac{10cd\left(\frac{6cd\left(\frac{8\int\frac{1}{\left(-\frac{ae^2}{cd^2}+\frac{(ae+cdx)e}{cd^2}+1\right)^{3/4}}d\sqrt[4]{ae+cdx}}{3\left(cd-\frac{ae^2}{d}\right)}+\frac{4\sqrt[4]{ae+cdx}}{3\left(\frac{ex}{d}+1\right)^{3/4}\left(cd-\frac{ae^2}{d}\right)}\right)}{7\left(cd-\frac{ae^2}{d}\right)}+\frac{4\sqrt[4]{ae+cdx}}{7\left(\frac{ex}{d}+1\right)^{7/4}\left(cd-\frac{ae^2}{d}\right)}\right.}{11\left(cd-\frac{ae^2}{d}\right)}\right.}{d^3\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/4}}$$

$\downarrow$ 768

$$\frac{\left(\frac{ex}{d}+1\right)^{3/4}(ae+cdx)^{3/4}\left(\frac{10cd\left(\frac{6cd\left(\frac{8(ae+cdx)^{3/4}\left(\frac{cd^2-ae^2}{e(ae+cdx)}+1\right)^{3/4}\int\frac{1}{(ae+cdx)^{3/4}\left(\frac{cd^2-ae^2}{e(ae+cdx)}+1\right)^{3/4}}d\sqrt[4]{ae+cdx}}{3\left(cd-\frac{ae^2}{d}\right)\left(-\frac{ae^2}{cd^2}+\frac{e(ae+cdx)}{cd^2}+1\right)^{3/4}}+\frac{4\sqrt[4]{ae+}}{3\left(\frac{ex}{d}+1\right)^{3/4}(c}}{7\left(cd-\frac{ae^2}{d}\right)}}{11\left(cd-\frac{ae^2}{d}\right)}}{d^3\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/4}}$$

$\downarrow$ 858

$$\frac{\left(\frac{ex}{d}+1\right)^{3/4}(ae+cdx)^{3/4}\left(\frac{10cd\left(\frac{6cd\left(\frac{4\sqrt[4]{ae+cdx}}{3\left(\frac{ex}{d}+1\right)^{3/4}\left(cd-\frac{ae^2}{d}\right)}-\frac{8(ae+cdx)^{3/4}\left(\frac{cd^2-ae^2}{e(ae+cdx)}+1\right)^{3/4}\int\frac{1}{\sqrt[4]{ae+cdx}\left(\frac{\left(cd^2-ae^2\right)(ae+cdx)}{e}+1\right.}}{3\left(cd-\frac{ae^2}{d}\right)\left(-\frac{ae^2}{cd^2}+\frac{e(ae+cdx)}{cd^2}+1\right)^{3/4}}}{7\left(cd-\frac{ae^2}{d}\right)}}{11\left(cd-\frac{ae^2}{d}\right)}}{d^3\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/4}}$$

$\downarrow$ 807

$$\frac{\left(\frac{ex}{d}+1\right)^{3/4}(ae+cdx)^{3/4}\left(\frac{10cd\left(\frac{6cd\left(\frac{4\sqrt[4]{ae+cdx}}{3\left(\frac{ex}{d}+1\right)^{3/4}\left(cd-\frac{ae^2}{d}\right)}-\frac{4(ae+cdx)^{3/4}\left(\frac{cd^2-ae^2}{e(ae+cdx)}+1\right)^{3/4}\int\frac{1}{\left(\frac{\sqrt{ae+cdx}\left(cd^2-ae^2\right)}{e}+1\right)^{3/4}}d\sqrt{ae+cdx}}{3\left(cd-\frac{ae^2}{d}\right)\left(-\frac{ae^2}{cd^2}+\frac{e(ae+cdx)}{cd^2}+1\right)^{3/4}}\right)}{7\left(cd-\frac{ae^2}{d}\right)}}{11\left(cd-\frac{ae^2}{d}\right)}}{d^3\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/4}}$$

$\big\downarrow$ 229

$$\frac{\left(\frac{ex}{d}+1\right)^{3/4}(ae+cdx)^{3/4}\left(\frac{10cd\left(\frac{6cd\left(\frac{4\sqrt[4]{ae+cdx}}{3\left(\frac{ex}{d}+1\right)^{3/4}\left(cd-\frac{ae^2}{d}\right)}-\frac{8\sqrt{e}(ae+cdx)^{3/4}\left(\frac{cd^2-ae^2}{e(ae+cdx)}+1\right)^{3/4}\text{EllipticF}\left(\frac{1}{2}\arctan\left(\frac{\sqrt{cd^2-ae^2}\sqrt{ae+cdx}}{\sqrt{e}}\right.\right.}{3\sqrt{cd^2-ae^2}\left(cd-\frac{ae^2}{d}\right)\left(-\frac{ae^2}{cd^2}+\frac{e(ae+cdx)}{cd^2}+1\right)^{3/4}}\right)}{7\left(cd-\frac{ae^2}{d}\right)}}{11\left(cd-\frac{ae^2}{d}\right)}}{d^3\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/4}}$$

input
```
Int[1/((d + e*x)^3*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/4)),x]
```

output
```
((a*e + c*d*x)^(3/4)*(1 + (e*x)/d)^(3/4)*((4*(a*e + c*d*x)^(1/4))/(11*(c*d
 - (a*e^2)/d)*(1 + (e*x)/d)^(11/4)) + (10*c*d*((4*(a*e + c*d*x)^(1/4))/(7*
(c*d - (a*e^2)/d)*(1 + (e*x)/d)^(7/4)) + (6*c*d*((4*(a*e + c*d*x)^(1/4))/(
3*(c*d - (a*e^2)/d)*(1 + (e*x)/d)^(3/4)) - (8*Sqrt[e]*(a*e + c*d*x)^(3/4)*
(1 + (c*d^2 - a*e^2)/(e*(a*e + c*d*x)))^(3/4)*EllipticF[ArcTan[(Sqrt[c*d^2
 - a*e^2]*Sqrt[a*e + c*d*x])/Sqrt[e]]/2, 2])/(3*Sqrt[c*d^2 - a*e^2]*(c*d -
 (a*e^2)/d)*(1 - (a*e^2)/(c*d^2) + (e*(a*e + c*d*x))/(c*d^2))^(3/4))))/(7*
(c*d - (a*e^2)/d))))/(11*(c*d - (a*e^2)/d))))/(d^3*(a*d*e + (c*d^2 + a*e^2
)*x + c*d*e*x^2)^(3/4))
```

Defintions of rubi rules used

rule 61
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && LtQ[m, -1] &&  !(LtQ[n, -1] && (EqQ[a, 0
] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && IntLinearQ[a, b, c, d
, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 229
```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(2/(a^(3/4)*Rt[b/a, 2])
)*EllipticF[(1/2)*ArcTan[Rt[b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

rule 768
```
Int[((a_) + (b_.)*(x_)^4)^(-3/4), x_Symbol] :> Simp[x^3*((1 + a/(b*x^4))^(3
/4)/(a + b*x^4)^(3/4))   Int[1/(x^3*(1 + a/(b*x^4))^(3/4)), x], x] /; FreeQ
[{a, b}, x]
```

rule 807
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Simp[1/k   Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x,
x^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

rule 858
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

rule 1138
```
Int[((d_) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Sy
mbol] :> Simp[d^m*((a + b*x + c*x^2)^FracPart[p]/((1 + e*(x/d))^FracPart[p]
*(a/d + (c*x)/e)^FracPart[p]))   Int[(1 + e*(x/d))^(m + p)*(a/d + (c/e)*x)^
p, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0]
&& (IntegerQ[m] || GtQ[d, 0]) &&  !(IGtQ[m, 0] && (IntegerQ[3*p] || Integer
Q[4*p]))
```

Maple [F]

$$\int \frac{1}{(ex+d)^{3} \left(ade+\left(a\,e^{2}+c\,d^{2}\right)x+cd\,x^{2}e\right)^{\frac{3}{4}}}dx$$

input
```
int(1/(e*x+d)^3/(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(3/4),x)
```

output
```
int(1/(e*x+d)^3/(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(3/4),x)
```

Fricas [F]

$$\int \frac{1}{(d+ex)^3\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/4}}\,dx=\int \frac{1}{\left(cdex^2+ade+(cd^2+ae^2)x\right)^{\frac{3}{4}}(ex+d)^3}\,dx$$

input
```
integrate(1/(e*x+d)^3/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/4),x, algorithm
="fricas")
```

output
```
integral((c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^(1/4)/(c*d*e^4*x^5 + a*d^
4*e + (4*c*d^2*e^3 + a*e^5)*x^4 + 2*(3*c*d^3*e^2 + 2*a*d*e^4)*x^3 + 2*(2*c
*d^4*e + 3*a*d^2*e^3)*x^2 + (c*d^5 + 4*a*d^3*e^2)*x), x)
```

Sympy [F]

$$\int \frac{1}{(d+ex)^3 (ade+(cd^2+ae^2)x+cdex^2)^{3/4}} dx = \int \frac{1}{((d+ex)(ae+cdx))^{\frac{3}{4}} (d+ex)^3} dx$$

input
```
integrate(1/(e*x+d)**3/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(3/4),x)
```

output
```
Integral(1/(((d + e*x)*(a*e + c*d*x))**(3/4)*(d + e*x)**3), x)
```

Maxima [F]

$$\int \frac{1}{(d+ex)^3 (ade+(cd^2+ae^2)x+cdex^2)^{3/4}} dx = \int \frac{1}{(cdex^2+ade+(cd^2+ae^2)x)^{\frac{3}{4}} (ex+d)^3} dx$$

input
```
integrate(1/(e*x+d)^3/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/4),x, algorithm
="maxima")
```

output
```
integrate(1/((c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^(3/4)*(e*x + d)^3), x
)
```

Giac [F]

$$\int \frac{1}{(d+ex)^3\left(ade+(cd^2+ae^2)x+cdex^2\right)^{3/4}}\,dx = \int \frac{1}{(cdex^2+ade+(cd^2+ae^2)x)^{\frac{3}{4}}(ex+d)^3}\,dx$$

input
```
integrate(1/(e*x+d)^3/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/4),x, algorithm
="giac")
```

output
```
integrate(1/((c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^(3/4)*(e*x + d)^3), x
)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(d+ex)^3\left(ade+(cd^2+ae^2)x+cdex^2\right)^{3/4}}\,dx = \int \frac{1}{(d+e\,x)^3\,\left(c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e\right)^{3/4}}\,dx$$

input
```
int(1/((d + e*x)^3*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(3/4)),x)
```

output
```
int(1/((d + e*x)^3*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(3/4)), x)
```

Reduce [F]

$$\int \frac{1}{(d+ex)^3\left(ade+(cd^2+ae^2)x+cdex^2\right)^{3/4}}\,dx = \int \frac{}{(cde\,x^2+a\,e^2x+c\,d^2x+ade)^{\frac{3}{4}}\,d^3+3\,(cde\,x^2+a\,e^2}$$

input
```
int(1/(e*x+d)^3/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/4),x)
```

output
```
int(1/((a*d*e + a*e**2*x + c*d**2*x + c*d*e*x**2)**(3/4)*d**3 + 3*(a*d*e +
 a*e**2*x + c*d**2*x + c*d*e*x**2)**(3/4)*d**2*e*x + 3*(a*d*e + a*e**2*x +
 c*d**2*x + c*d*e*x**2)**(3/4)*d*e**2*x**2 + (a*d*e + a*e**2*x + c*d**2*x
+ c*d*e*x**2)**(3/4)*e**3*x**3),x)
```

3.375 $\int \frac{(d+ex)^4}{(ade+(cd^2+ae^2)x+cdex^2)^{5/4}}\,dx$

Optimal result

Integrand size = 37, antiderivative size = 285

$$\int \frac{(d+ex)^4}{(ade+(cd^2+ae^2)x+cdex^2)^{5/4}}\,dx = -\frac{4(cd^2-ae^2)^2(d+ex)}{c^3d^3\sqrt[4]{ade+(cd^2+ae^2)x+cdex^2}} + \frac{17e(cd^2-ae^2)(ade+(cd^2+ae^2)x+cdex^2)^{3/4}}{15c^3d^3} + \frac{2e(d+ex)(ade+(cd^2+ae^2)x+cdex^2)^{3/4}}{5c^2d^2} + \frac{77(cd^2-ae^2)^3\sqrt[4]{-\frac{cde(ae+cdx)(d+ex)}{(cd^2-ae^2)^2}}E\left(\frac{1}{2}\arcsin\left(\frac{ae^2+cd(d+2ex)}{cd^2-ae^2}\right)\middle|2\right)}{10\sqrt{2}c^4d^4\sqrt[4]{ade+(cd^2+ae^2)x+cdex^2}}$$

output
```
-4*(-a*e^2+c*d^2)^2*(e*x+d)/c^3/d^3/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/4
)+17/15*e*(-a*e^2+c*d^2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/4)/c^3/d^3+2
/5*e*(e*x+d)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/4)/c^2/d^2+77/20*(-a*e^2
+c*d^2)^3*(-c*d*e*(c*d*x+a*e)*(e*x+d)/(-a*e^2+c*d^2)^2)^(1/4)*EllipticE(si
n(1/2*arcsin((a*e^2+c*d*(2*e*x+d))/(-a*e^2+c*d^2))),2^(1/2))*2^(1/2)/c^4/d
^4/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/4)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.05 (sec) , antiderivative size = 100, normalized size of antiderivative = 0.35

$$\int \frac{(d+ex)^4}{(ade+(cd^2+ae^2)\,x+cdex^2)^{5/4}}\,dx = -\frac{4(cd^2-ae^2)^3\sqrt[4]{\frac{cd(d+ex)}{cd^2-ae^2}}\,\text{Hypergeometric2F1}\left(-\frac{11}{4},-\frac{1}{4},\frac{3}{4},\frac{e(ae+cdx)}{-cd^2+ae^2}\right)}{c^4d^4\sqrt[4]{(ae+cdx)(d+ex)}}$$

input
```
Integrate[(d + e*x)^4/(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/4),x]
```

output
```
(-4*(c*d^2 - a*e^2)^3*((c*d*(d + e*x))/(c*d^2 - a*e^2))^(1/4)*Hypergeometr
ic2F1[-11/4, -1/4, 3/4, (e*(a*e + c*d*x))/(-(c*d^2) + a*e^2)])/(c^4*d^4*((
a*e + c*d*x)*(d + e*x))^(1/4))
```

Rubi [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 1198 vs. 2(285) = 570.

Time = 1.10 (sec) , antiderivative size = 1198, normalized size of antiderivative = 4.20, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.324, Rules used = {1164, 27, 27, 1166, 27, 1166, 27, 1160, 1094, 834, 761, 1510}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex)^4}{(x(ae^2+cd^2)+ade+cdex^2)^{5/4}}dx$$

$$\Big\downarrow \ 1164$$

$$-\frac{4\int -\frac{7e(d+ex)^2(d(cd^2-ae^2)+ex(cd^2-ae^2))}{2\sqrt[4]{cdex^2+(cd^2+ae^2)\,x+ade}}dx}{(cd^2-ae^2)^2}-\frac{4(d+ex)^4}{(cd^2-ae^2)\sqrt[4]{x(ae^2+cd^2)+ade+cdex^2}}$$

$$\Big\downarrow \ 27$$

$$\frac{14e\int \frac{(cd^2-ae^2)(d+ex)^3}{\sqrt[4]{cdex^2+(cd^2+ae^2)x+ade}}dx}{(cd^2-ae^2)^2}-\frac{4(d+ex)^4}{(cd^2-ae^2)\sqrt[4]{x(ae^2+cd^2)+ade+cdex^2}}$$

$\big\downarrow$ 27

$$\frac{14e\int \frac{(d+ex)^3}{\sqrt[4]{cdex^2+(cd^2+ae^2)x+ade}}dx}{cd^2-ae^2}-\frac{4(d+ex)^4}{(cd^2-ae^2)\sqrt[4]{x(ae^2+cd^2)+ade+cdex^2}}$$

$\big\downarrow$ 1166

$$\frac{14e\left(\frac{2\int \frac{11e\left(cd^2-ae^2\right)(d+ex)^2}{4\sqrt[4]{cdex^2+(cd^2+ae^2)x+ade}}dx}{7cde}+\frac{2(d+ex)^2\left(x(ae^2+cd^2)+ade+cdex^2\right)^{3/4}}{7cd}\right)}{cd^2-ae^2}-\frac{4(d+ex)^4}{(cd^2-ae^2)\sqrt[4]{x(ae^2+cd^2)+ade+cdex^2}}$$

$\big\downarrow$ 27

$$\frac{14e\left(\frac{11(cd^2-ae^2)\int \frac{(d+ex)^2}{\sqrt[4]{cdex^2+(cd^2+ae^2)x+ade}}dx}{14cd}+\frac{2(d+ex)^2\left(x(ae^2+cd^2)+ade+cdex^2\right)^{3/4}}{7cd}\right)}{cd^2-ae^2}-\frac{4(d+ex)^4}{(cd^2-ae^2)\sqrt[4]{x(ae^2+cd^2)+ade+cdex^2}}$$

$\big\downarrow$ 1166

$$\frac{14e\left(\frac{11\left(cd^2-ae^2\right)\left(\frac{2\int \frac{7e\left(cd^2-ae^2\right)(d+ex)}{4\sqrt[4]{cdex^2+(cd^2+ae^2)x+ade}}dx}{5cde}+\frac{2(d+ex)\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/4}}{5cd}\right)}{14cd}+\frac{2(d+ex)^2(x(ae^2+cd^2)+ade+}{7cd}\right.}{cd^2-ae^2}$$

$$\frac{4(d+ex)^4}{(cd^2-ae^2)\sqrt[4]{x(ae^2+cd^2)+ade+cdex^2}}$$

$\big\downarrow$ 27

$$14e\left(\frac{11\left(cd^2-ae^2\right)\left(\frac{7\left(cd^2-ae^2\right)\int\frac{d+ex}{\sqrt[4]{cdex^2+\left(cd^2+ae^2\right)x+ade}}dx}{10cd}+\frac{2(d+ex)\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/4}}{5cd}\right)}{14cd}+\frac{2(d+ex)^2\left(x\left(ae^2+c\right.\right.}{7c}\right.$$

$$\overline{\qquad cd^2-ae^2 \qquad}$$

$$\frac{4(d+ex)^4}{\left(cd^2-ae^2\right)\sqrt[4]{x\left(ae^2+cd^2\right)+ade+cdex^2}}$$

$\downarrow$ 1160

$$14e\left(\frac{11\left(cd^2-ae^2\right)\left(\frac{7\left(cd^2-ae^2\right)\left(\frac{\left(d^2-\frac{ae^2}{c}\right)\int\frac{1}{\sqrt[4]{cdex^2+\left(cd^2+ae^2\right)x+ade}}dx}{2d}+\frac{2\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/4}}{3cd}\right)}{10cd}+\frac{2(d+ex)\left(x\left(ae^2+c\right.\right.}{}\right.}{14cd}\right.$$

$$\overline{\qquad cd^2-ae^2 \qquad}$$

$$\frac{4(d+ex)^4}{\left(cd^2-ae^2\right)\sqrt[4]{x\left(ae^2+cd^2\right)+ade+cdex^2}}$$

$\downarrow$ 1094

$$14e\left(\frac{11\left(cd^2-ae^2\right)\left(\frac{7\left(cd^2-ae^2\right)\left(\frac{2\left(d^2-\frac{ae^2}{c}\right)\sqrt{\left(ae^2+cd^2+2cdex\right)^2}\int\frac{\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}{\sqrt{\left(cd^2-ae^2\right)^2+4cde\left(cdex^2+\left(cd^2+ae^2\right)x+ade\right)}}d\sqrt[4]{cdex^2+(cd^2+ae^2)}}{d\left(ae^2+cd^2+2cdex\right)}\right.}{10cd}}{14cd}\right.}{cd^2-ae^2}$$

$$\frac{4(d+ex)^4}{(cd^2-ae^2)\sqrt[4]{x\left(ae^2+cd^2\right)+ade+cdex^2}}$$

$\downarrow$ 834

$$\frac{14e}{\left(\frac{11(cd^2-ae^2)}{\left(\frac{7(cd^2-ae^2)}{\left(\frac{2\left(d^2-\frac{ae^2}{c}\right)\sqrt{\left(ae^2+cd^2+2cdex\right)^2}}{\left(\frac{\left(cd^2-ae^2\right)\int \frac{1}{\sqrt{\left(cd^2-ae^2\right)^2+4cde\left(cdex^2+\left(cd^2+ae^2\right)x+ade\right)}}d\sqrt[4]{cdex^2+(\cdots}}{2\sqrt{c}\sqrt{d}\sqrt{e}}\cdots\right.}\cdots\right.}\cdots\right.}\cdots\right.}\cdots\right.}$$

$$\frac{4(d+ex)^4}{(cd^2-ae^2)\sqrt[4]{x\left(ae^2+cd^2\right)+ade+cdex^2}}$$

↓ 761

$$2\left(d^2-\frac{ae^2}{c}\right)\sqrt{\left(cd^2+2cexd\right.}$$

$$7\left(cd^2-ae^2\right)\left(\vphantom{\frac{a}{b}}\right.$$

$$11\left(cd^2-ae^2\right)\left(\frac{2\left(cdex^2+\left(cd^2+ae^2\right)x+ade\right)^{3/4}(d+ex)}{5cd}+\right.$$

$$14e\left(\frac{2(cdex^2+(cd^2+ae^2)x+ade)^{3/4}(d+ex)^2}{7cd}+\right.$$

↓ 1510

$$2\left(d^2-\frac{ae^2}{c}\right)\sqrt{\left(cd^2+2cexd\right.}$$

$$7\left(cd^2-ae^2\right)$$

$$11(cd^2-ae^2)\left(\frac{2\left(cdex^2+\left(cd^2+ae^2\right)x+ade\right)^{3/4}(d+ex)}{5cd}+\right.$$

input
```
Int[(d + e*x)^4/(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/4),x]
```

output
```
(-4*(d + e*x)^4)/((c*d^2 - a*e^2)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^
(1/4)) + (14*e*((2*(d + e*x)^2*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/
4))/(7*c*d) + (11*(c*d^2 - a*e^2)*((2*(d + e*x)*(a*d*e + (c*d^2 + a*e^2)*x
 + c*d*e*x^2)^(3/4))/(5*c*d) + (7*(c*d^2 - a*e^2)*((2*(a*d*e + (c*d^2 + a*
e^2)*x + c*d*e*x^2)^(3/4))/(3*c*d) + (2*(d^2 - (a*e^2)/c)*Sqrt[(c*d^2 + a*
e^2 + 2*c*d*e*x)^2]*(-1/2*((c*d^2 - a*e^2)*(-(((a*d*e + (c*d^2 + a*e^2)*x
+ c*d*e*x^2)^(1/4)*Sqrt[(c*d^2 - a*e^2)^2 + 4*c*d*e*(a*d*e + (c*d^2 + a*e^
2)*x + c*d*e*x^2)])/((c*d^2 - a*e^2)^2*(1 + (2*Sqrt[c]*Sqrt[d]*Sqrt[e]*Sqr
t[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(c*d^2 - a*e^2)))) + (Sqrt[c*d^2
 - a*e^2]*(1 + (2*Sqrt[c]*Sqrt[d]*Sqrt[e]*Sqrt[a*d*e + (c*d^2 + a*e^2)*x +
 c*d*e*x^2])/(c*d^2 - a*e^2))*Sqrt[((c*d^2 - a*e^2)^2 + 4*c*d*e*(a*d*e + (
c*d^2 + a*e^2)*x + c*d*e*x^2))/((c*d^2 - a*e^2)^2*(1 + (2*Sqrt[c]*Sqrt[d]*
Sqrt[e]*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(c*d^2 - a*e^2))^2)]*
EllipticE[2*ArcTan[(Sqrt[2]*c^(1/4)*d^(1/4)*e^(1/4)*(a*d*e + (c*d^2 + a*e^
2)*x + c*d*e*x^2)^(1/4))/Sqrt[c*d^2 - a*e^2]], 1/2])/(Sqrt[2]*c^(1/4)*d^(1
/4)*e^(1/4)*Sqrt[(c*d^2 - a*e^2)^2 + 4*c*d*e*(a*d*e + (c*d^2 + a*e^2)*x +
c*d*e*x^2)]))))/(Sqrt[c]*Sqrt[d]*Sqrt[e]) + ((c*d^2 - a*e^2)^(3/2)*(1 + (2*
Sqrt[c]*Sqrt[d]*Sqrt[e]*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(c*d^
2 - a*e^2))*Sqrt[((c*d^2 - a*e^2)^2 + 4*c*d*e*(a*d*e + (c*d^2 + a*e^2)*x +
 c*d*e*x^2))/((c*d^2 - a*e^2)^2*(1 + (2*Sqrt[c]*Sqrt[d]*Sqrt[e]*Sqrt[a*...
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 761
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 834
```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, S
imp[1/q   Int[1/Sqrt[a + b*x^4], x], x] - Simp[1/q   Int[(1 - q*x^2)/Sqrt[a
 + b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 1094
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[4*(Sqrt[(b
+ 2*c*x)^2]/(b + 2*c*x))   Subst[Int[x^(4*(p + 1) - 1)/Sqrt[b^2 - 4*a*c + 4
*c*x^4], x], x, (a + b*x + c*x^2)^(1/4)], x] /; FreeQ[{a, b, c}, x] && Inte
gerQ[4*p]
```

rule 1160
```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[e*((a + b*x + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Simp[(2*c*d - b
*e)/(2*c)   Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
 && NeQ[p, -1]
```

rule 1164
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(d + e*x)^(m - 1)*(d*b - 2*a*e + (2*c*d - b*e)*x)*((a + b*x
+ c*x^2)^(p + 1)/((p + 1)*(b^2 - 4*a*c))), x] + Simp[1/((p + 1)*(b^2 - 4*a*
c))   Int[(d + e*x)^(m - 2)*Simp[e*(2*a*e*(m - 1) + b*d*(2*p - m + 4)) - 2*
c*d^2*(2*p + 3) + e*(b*e - 2*d*c)*(m + 2*p + 2)*x, x]*(a + b*x + c*x^2)^(p
+ 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && LtQ[p, -1] && GtQ[m, 1] && Int
QuadraticQ[a, b, c, d, e, m, p, x]
```

rule 1166
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[e*(d + e*x)^(m - 1)*((a + b*x + c*x^2)^(p + 1)/(c*(m + 2*p +
 1))), x] + Simp[1/(c*(m + 2*p + 1))   Int[(d + e*x)^(m - 2)*Simp[c*d^2*(m
+ 2*p + 1) - e*(a*e*(m - 1) + b*d*(p + 1)) + e*(2*c*d - b*e)*(m + p)*x, x]*
(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && If[Ration
alQ[m], GtQ[m, 1], SumSimplerQ[m, -2]] && NeQ[m + 2*p + 1, 0] && IntQuadrat
icQ[a, b, c, d, e, m, p, x]
```

rule 1510
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

Maple [F]

$$\int \frac{(ex+d)^4}{(ade+(a\,e^2+c\,d^2)\,x+cd\,x^2e)^{\frac{5}{4}}}dx$$

input
```
int((e*x+d)^4/(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(5/4),x)
```

output
```
int((e*x+d)^4/(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(5/4),x)
```

Fricas [F]

$$\int \frac{(d+ex)^4}{(ade+(cd^2+ae^2)\,x+cdex^2)^{5/4}}\,dx = \int \frac{(ex+d)^4}{(cdex^2+ade+(cd^2+ae^2)x)^{\frac{5}{4}}}\,dx$$

input
```
integrate((e*x+d)^4/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/4),x, algorithm="
fricas")
```

output
```
integral((c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^(3/4)*(e^2*x^2 + 2*d*e*x
+ d^2)/(c^2*d^2*x^2 + 2*a*c*d*e*x + a^2*e^2), x)
```

Sympy [F]

$$\int \frac{(d+ex)^4}{(ade+(cd^2+ae^2)\,x+cdex^2)^{5/4}}\,dx = \int \frac{(d+ex)^4}{((d+ex)\,(ae+cdx))^{\frac{5}{4}}}\,dx$$

input
```
integrate((e*x+d)**4/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(5/4),x)
```

output
```
Integral((d + e*x)**4/((d + e*x)*(a*e + c*d*x))**(5/4), x)
```

Maxima [F]

$$\int \frac{(d+ex)^4}{(ade+(cd^2+ae^2)\,x+cdex^2)^{5/4}}\,dx = \int \frac{(ex+d)^4}{(cdex^2+ade+(cd^2+ae^2)x)^{\frac{5}{4}}}\,dx$$

input
```
integrate((e*x+d)^4/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/4),x, algorithm="
maxima")
```

output
```
integrate((e*x + d)^4/(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^(5/4), x)
```

Giac [F]

$$\int \frac{(d+ex)^4}{(ade+(cd^2+ae^2)\,x+cdex^2)^{5/4}}\,dx = \int \frac{(ex+d)^4}{(cdex^2+ade+(cd^2+ae^2)x)^{\frac{5}{4}}}\,dx$$

input
```
integrate((e*x+d)^4/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/4),x, algorithm="
giac")
```

output
```
integrate((e*x + d)^4/(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^(5/4), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d+ex)^4}{(ade+(cd^2+ae^2)\,x+cdex^2)^{5/4}}\,dx = \int \frac{(d+e\,x)^4}{(c\,d\,e\,x^2+(c\,d^2+a\,e^2)\,x+a\,d\,e)^{5/4}}\,dx$$

input
```
int((d + e*x)^4/(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(5/4),x)
```

output
```
int((d + e*x)^4/(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(5/4), x)
```

Reduce [F]

$$\int \frac{(d+ex)^4}{\left(ade+(cd^2+ae^2)\,x+cdex^2\right)^{5/4}}\,dx = \left(\int \frac{x^3}{\left(cde\,x^2+a\,e^2x+c\,d^2x+ade\right)^{\frac{1}{4}} ae+\left(cde\,x^2+a\,e^2x+c\,d^2x\right.}\right.$$

$$+3\left(\int \frac{x^2}{\left(cde\,x^2+a\,e^2x+c\,d^2x+ade\right)^{\frac{1}{4}} ae+\left(cde\,x^2+a\,e^2x+c\,d^2x+ade\right)^{\frac{1}{4}} cdx}dx\right) d\,e^2$$

$$+3\left(\int \frac{x}{\left(cde\,x^2+a\,e^2x+c\,d^2x+ade\right)^{\frac{1}{4}} ae+\left(cde\,x^2+a\,e^2x+c\,d^2x+ade\right)^{\frac{1}{4}} cdx}dx\right) d^2e$$

$$+\left(\int \frac{1}{\left(cde\,x^2+a\,e^2x+c\,d^2x+ade\right)^{\frac{1}{4}} ae+\left(cde\,x^2+a\,e^2x+c\,d^2x+ade\right)^{\frac{1}{4}} cdx}dx\right) d^3$$

input
```
int((e*x+d)^4/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/4),x)
```

output
```
int(x**3/((a*d*e + a*e**2*x + c*d**2*x + c*d*e*x**2)**(1/4)*a*e + (a*d*e +
 a*e**2*x + c*d**2*x + c*d*e*x**2)**(1/4)*c*d*x),x)*e**3 + 3*int(x**2/((a*
d*e + a*e**2*x + c*d**2*x + c*d*e*x**2)**(1/4)*a*e + (a*d*e + a*e**2*x + c
*d**2*x + c*d*e*x**2)**(1/4)*c*d*x),x)*d*e**2 + 3*int(x/((a*d*e + a*e**2*x
 + c*d**2*x + c*d*e*x**2)**(1/4)*a*e + (a*d*e + a*e**2*x + c*d**2*x + c*d*
e*x**2)**(1/4)*c*d*x),x)*d**2*e + int(1/((a*d*e + a*e**2*x + c*d**2*x + c*
d*e*x**2)**(1/4)*a*e + (a*d*e + a*e**2*x + c*d**2*x + c*d*e*x**2)**(1/4)*c
*d*x),x)*d**3
```

3.376 $\int \frac{(d+ex)^3}{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/4}}\,dx$

Optimal result

Integrand size = 37, antiderivative size = 224

$$\int \frac{(d+ex)^3}{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/4}}\,dx = -\frac{4\left(cd^2-ae^2\right)(d+ex)}{c^2d^2\sqrt[4]{ade+\left(cd^2+ae^2\right)x+cdex^2}}+\frac{2e\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/4}}{3c^2d^2} +\frac{7\left(cd^2-ae^2\right)^2\sqrt[4]{-\frac{cde(ae+cdx)(d+ex)}{\left(cd^2-ae^2\right)^2}}E\left(\frac{1}{2}\arcsin\left(\frac{ae^2+cd(d+2ex)}{cd^2-ae^2}\right)\middle|2\right)}{\sqrt{2}c^3d^3\sqrt[4]{ade+\left(cd^2+ae^2\right)x+cdex^2}}$$

output

```
-4*(-a*e^2+c*d^2)*(e*x+d)/c^2/d^2/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/4)+
2/3*e*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/4)/c^2/d^2+7/2*(-a*e^2+c*d^2)^2
*(-c*d*e*(c*d*x+a*e)*(e*x+d)/(-a*e^2+c*d^2)^2)^(1/4)*EllipticE(sin(1/2*arc
sin((a*e^2+c*d*(2*e*x+d))/(-a*e^2+c*d^2))),2^(1/2))*2^(1/2)/c^3/d^3/(a*d*e
+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/4)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.04 (sec) , antiderivative size = 93, normalized size of antiderivative = 0.42

$$\int \frac{(d+ex)^3}{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/4}}\,dx = -\frac{4(d+ex)^2\,\text{Hypergeometric2F1}\left(-\frac{7}{4},-\frac{1}{4},\frac{3}{4},\frac{e(ae+cdx)}{-cd^2+ae^2}\right)}{cd\left(\frac{cd(d+ex)}{cd^2-ae^2}\right)^{7/4}\sqrt[4]{(ae+cdx)(d+ex)}}$$

input
```
Integrate[(d + e*x)^3/(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/4),x]
```

output
```
(-4*(d + e*x)^2*Hypergeometric2F1[-7/4, -1/4, 3/4, (e*(a*e + c*d*x))/(-(c*
d^2) + a*e^2)])/(c*d*((c*d*(d + e*x))/(c*d^2 - a*e^2))^(7/4)*((a*e + c*d*x
)*(d + e*x))^(1/4))
```

Rubi [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 1129 vs. 2(224) = 448.

Time = 0.94 (sec) , antiderivative size = 1129, normalized size of antiderivative = 5.04, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.270$, Rules used = {1164, 27, 27, 1166, 27, 1160, 1094, 834, 761, 1510}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex)^3}{\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/4}}dx$$

$$\Big\downarrow \ 1164$$

$$-\frac{4\int -\frac{5e(d+ex)\left(d\left(cd^2-ae^2\right)+ex\left(cd^2-ae^2\right)\right)}{2\sqrt[4]{cdex^2+\left(cd^2+ae^2\right)x+ade}}dx}{\left(cd^2-ae^2\right)^2}-\frac{4(d+ex)^3}{\left(cd^2-ae^2\right)\sqrt[4]{x\left(ae^2+cd^2\right)+ade+cdex^2}}$$

$$\Big\downarrow \ 27$$

$$\frac{10e\int \frac{(cd^2-ae^2)(d+ex)^2}{\sqrt[4]{cdex^2+(cd^2+ae^2)x+ade}}dx}{(cd^2-ae^2)^2}-\frac{4(d+ex)^3}{(cd^2-ae^2)\sqrt[4]{x(ae^2+cd^2)+ade+cdex^2}}$$

$\big\downarrow$ 27

$$\frac{10e\int \frac{(d+ex)^2}{\sqrt[4]{cdex^2+(cd^2+ae^2)x+ade}}dx}{cd^2-ae^2}-\frac{4(d+ex)^3}{(cd^2-ae^2)\sqrt[4]{x(ae^2+cd^2)+ade+cdex^2}}$$

$\big\downarrow$ 1166

$$\frac{10e\left(\frac{2\int \frac{7e\left(cd^2-ae^2\right)(d+ex)}{4\sqrt[4]{cdex^2+(cd^2+ae^2)x+ade}}dx}{5cde}+\frac{2(d+ex)\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/4}}{5cd}\right)}{cd^2-ae^2}-\frac{4(d+ex)^3}{(cd^2-ae^2)\sqrt[4]{x(ae^2+cd^2)+ade+cdex^2}}$$

$\big\downarrow$ 27

$$\frac{10e\left(\frac{7\left(cd^2-ae^2\right)\int \frac{d+ex}{\sqrt[4]{cdex^2+(cd^2+ae^2)x+ade}}dx}{10cd}+\frac{2(d+ex)\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/4}}{5cd}\right)}{cd^2-ae^2}-\frac{4(d+ex)^3}{(cd^2-ae^2)\sqrt[4]{x(ae^2+cd^2)+ade+cdex^2}}$$

$\big\downarrow$ 1160

$$\frac{10e\left(\frac{7\left(cd^2-ae^2\right)\left(\frac{\left(d^2-\frac{ae^2}{c}\right)\int \frac{1}{\sqrt[4]{cdex^2+(cd^2+ae^2)x+ade}}dx}{2d}+\frac{2\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/4}}{3cd}\right)}{10cd}+\frac{2(d+ex)\left(x\left(ae^2+cd^2\right)+ade+\right.}{5cd}\right.}{cd^2-ae^2}$$
$$\frac{4(d+ex)^3}{(cd^2-ae^2)\sqrt[4]{x(ae^2+cd^2)+ade+cdex^2}}$$

$\big\downarrow$ 1094

$$10e\left(\frac{7\left(cd^2-ae^2\right)\left(\frac{2\left(d^2-\frac{ae^2}{c}\right)\sqrt{\left(ae^2+cd^2+2cdex\right)^2}\int\frac{\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}{\sqrt{\left(cd^2-ae^2\right)^2+4cde\left(cdex^2+\left(cd^2+ae^2\right)x+ade\right)}}d\sqrt[4]{cdex^2+(cd^2+ae^2)\,x+ade}}{d\left(ae^2+cd^2+2cdex\right)}+\cdots\right)}{10cd}\right.$$

$$cd^2-ae^2$$

$$\frac{4(d+ex)^3}{(cd^2-ae^2)\sqrt[4]{x\,(ae^2+cd^2)+ade+cdex^2}}$$

$\downarrow$ 834

$$10e\left(\frac{7\left(cd^2-ae^2\right)\left(\frac{2\left(d^2-\frac{ae^2}{c}\right)\sqrt{\left(ae^2+cd^2+2cdex\right)^2}\left(\frac{\left(cd^2-ae^2\right)\int\frac{1}{\sqrt{\left(cd^2-ae^2\right)^2+4cde\left(cdex^2+\left(cd^2+ae^2\right)x+ade\right)}}d\sqrt[4]{cdex^2+(cd^2+ae^2)\,x\cdots}}{2\sqrt{c}\sqrt{d}\sqrt{e}}\cdots\right.}{d\left(ae^2+cd^2+2cd\cdots\right.}\cdots\right.}{\cdots}\right.$$

$$\frac{4(d+ex)^3}{(cd^2-ae^2)\sqrt[4]{x\,(ae^2+cd^2)+ade+cdex^2}}$$

$\downarrow$ 761

$$10e\left(\frac{7(cd^2-ae^2)\left(\frac{2\left(d^2-\frac{ae^2}{c}\right)\sqrt{\left(ae^2+cd^2+2cdex\right)^2}\left(\frac{\left(cd^2-ae^2\right)^{3/2}\left(\frac{2\sqrt{c}\sqrt{d}\sqrt{e}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{cd^2-ae^2}+1\right)\sqrt{\frac{4cde\left(x\left(ae^2+cd^2\right)+ade+cdex\right.}{\left(cd^2-ae^2\right)^2\left(\frac{2\sqrt{c}\sqrt{d}\sqrt{e}\sqrt{x\left(ae^2+c\right.}}{cd^2-a\cdots}\right.}}}{4\sqrt{2}c^{3/4}d^{3/4}e^{3/4}\sqrt{4cde\left(x\right.}\cdots}\right.}{\cdots}\right.}{\cdots}\right.$$

$$\frac{4(d+ex)^3}{(cd^2-ae^2)\sqrt[4]{x(ae^2+cd^2)+ade+cdex^2}}$$

$\downarrow$ 1510

$$\left(cd^2-ae^2\right)^{3/2}\left(\frac{2\sqrt{c}\sqrt{d}\sqrt{e}\sqrt{cdex^2+\left(cd^2+ae^2\right)x\cdots}}{cd^2-ae^2}\right.$$

$$2\left(d^2-\frac{ae^2}{c}\right)\sqrt{\left(cd^2+2cexd+ae^2\right)^2}$$

$$7\left(cd^2-ae^2\right)$$

$$10e\left(\frac{2\left(cdex^2+\left(cd^2+ae^2\right)x+ade\right)^{3/4}(d+ex)}{5cd}+\cdots\right.$$

input
```
Int[(d + e*x)^3/(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/4),x]
```

output
```
(-4*(d + e*x)^3)/((c*d^2 - a*e^2)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^
(1/4)) + (10*e*((2*(d + e*x)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/4)
)/(5*c*d) + (7*(c*d^2 - a*e^2)*((2*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)
^(3/4))/(3*c*d) + (2*(d^2 - (a*e^2)/c)*Sqrt[(c*d^2 + a*e^2 + 2*c*d*e*x)^2]
*(-1/2*((c*d^2 - a*e^2)*(-(((a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(1/4)*
Sqrt[(c*d^2 - a*e^2)^2 + 4*c*d*e*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)])
/((c*d^2 - a*e^2)^2*(1 + (2*Sqrt[c]*Sqrt[d]*Sqrt[e]*Sqrt[a*d*e + (c*d^2 +
a*e^2)*x + c*d*e*x^2])/(c*d^2 - a*e^2)))) + (Sqrt[c*d^2 - a*e^2]*(1 + (2*S
qrt[c]*Sqrt[d]*Sqrt[e]*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(c*d^2
 - a*e^2))*Sqrt[((c*d^2 - a*e^2)^2 + 4*c*d*e*(a*d*e + (c*d^2 + a*e^2)*x +
c*d*e*x^2))/((c*d^2 - a*e^2)^2*(1 + (2*Sqrt[c]*Sqrt[d]*Sqrt[e]*Sqrt[a*d*e
+ (c*d^2 + a*e^2)*x + c*d*e*x^2])/(c*d^2 - a*e^2))^2)]*EllipticE[2*ArcTan[
(Sqrt[2]*c^(1/4)*d^(1/4)*e^(1/4)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(
1/4))/Sqrt[c*d^2 - a*e^2]], 1/2])/(Sqrt[2]*c^(1/4)*d^(1/4)*e^(1/4)*Sqrt[(c
*d^2 - a*e^2)^2 + 4*c*d*e*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)])))/(Sqr
t[c]*Sqrt[d]*Sqrt[e]) + ((c*d^2 - a*e^2)^(3/2)*(1 + (2*Sqrt[c]*Sqrt[d]*Sqr
t[e]*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(c*d^2 - a*e^2))*Sqrt[((
c*d^2 - a*e^2)^2 + 4*c*d*e*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2))/((c*d^
2 - a*e^2)^2*(1 + (2*Sqrt[c]*Sqrt[d]*Sqrt[e]*Sqrt[a*d*e + (c*d^2 + a*e^2)*
x + c*d*e*x^2])/(c*d^2 - a*e^2))^2)]*EllipticF[2*ArcTan[(Sqrt[2]*c^(1/4...
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 761
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 834
```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, S
imp[1/q   Int[1/Sqrt[a + b*x^4], x], x] - Simp[1/q   Int[(1 - q*x^2)/Sqrt[a
 + b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 1094
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[4*(Sqrt[(b
+ 2*c*x)^2]/(b + 2*c*x))   Subst[Int[x^(4*(p + 1) - 1)/Sqrt[b^2 - 4*a*c + 4
*c*x^4], x], x, (a + b*x + c*x^2)^(1/4)], x] /; FreeQ[{a, b, c}, x] && Inte
gerQ[4*p]
```

rule 1160
```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[e*((a + b*x + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Simp[(2*c*d - b
*e)/(2*c)   Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
 && NeQ[p, -1]
```

rule 1164
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(d + e*x)^(m - 1)*(d*b - 2*a*e + (2*c*d - b*e)*x)*((a + b*x
+ c*x^2)^(p + 1)/((p + 1)*(b^2 - 4*a*c))), x] + Simp[1/((p + 1)*(b^2 - 4*a*
c))   Int[(d + e*x)^(m - 2)*Simp[e*(2*a*e*(m - 1) + b*d*(2*p - m + 4)) - 2*
c*d^2*(2*p + 3) + e*(b*e - 2*d*c)*(m + 2*p + 2)*x, x]*(a + b*x + c*x^2)^(p
+ 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && LtQ[p, -1] && GtQ[m, 1] && Int
QuadraticQ[a, b, c, d, e, m, p, x]
```

rule 1166
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[e*(d + e*x)^(m - 1)*((a + b*x + c*x^2)^(p + 1)/(c*(m + 2*p +
 1))), x] + Simp[1/(c*(m + 2*p + 1))   Int[(d + e*x)^(m - 2)*Simp[c*d^2*(m
+ 2*p + 1) - e*(a*e*(m - 1) + b*d*(p + 1)) + e*(2*c*d - b*e)*(m + p)*x, x]*
(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && If[Ration
alQ[m], GtQ[m, 1], SumSimplerQ[m, -2]] && NeQ[m + 2*p + 1, 0] && IntQuadrat
icQ[a, b, c, d, e, m, p, x]
```

rule 1510
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

Maple [F]

$$\int \frac{(ex+d)^3}{\left(ade+\left(a\,e^2+c\,d^2\right)x+cd\,x^2e\right)^{\frac{5}{4}}}dx$$

input
```
int((e*x+d)^3/(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(5/4),x)
```

output
```
int((e*x+d)^3/(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(5/4),x)
```

Fricas [F]

$$\int \frac{(d+ex)^3}{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/4}}\,dx=\int \frac{(ex+d)^3}{\left(cdex^2+ade+\left(cd^2+ae^2\right)x\right)^{\frac{5}{4}}}\,dx$$

input
```
integrate((e*x+d)^3/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/4),x, algorithm="
fricas")
```

output
```
integral((c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^(3/4)*(e*x + d)/(c^2*d^2*
x^2 + 2*a*c*d*e*x + a^2*e^2), x)
```

Sympy [F]

$$\int \frac{(d+ex)^3}{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/4}}\,dx=\int \frac{(d+ex)^3}{\left((d+ex)\,(ae+cdx)\right)^{\frac{5}{4}}}\,dx$$

input
```
integrate((e*x+d)**3/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(5/4),x)
```

output
```
Integral((d + e*x)**3/((d + e*x)*(a*e + c*d*x))**(5/4), x)
```

Maxima [F]

$$\int \frac{(d+ex)^3}{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/4}}\,dx = \int \frac{(ex+d)^3}{\left(cdex^2+ade+(cd^2+ae^2)x\right)^{\frac{5}{4}}}\,dx$$

input
```
integrate((e*x+d)^3/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/4),x, algorithm="
maxima")
```

output
```
integrate((e*x + d)^3/(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^(5/4), x)
```

Giac [F]

$$\int \frac{(d+ex)^3}{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/4}}\,dx = \int \frac{(ex+d)^3}{\left(cdex^2+ade+(cd^2+ae^2)x\right)^{\frac{5}{4}}}\,dx$$

input
```
integrate((e*x+d)^3/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/4),x, algorithm="
giac")
```

output
```
integrate((e*x + d)^3/(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^(5/4), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d+ex)^3}{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/4}}\,dx = \int \frac{(d+e\,x)^3}{\left(c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e\right)^{5/4}}\,dx$$

input
```
int((d + e*x)^3/(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(5/4),x)
```

output
```
int((d + e*x)^3/(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(5/4), x)
```

Reduce [F]

$$\int \frac{(d+ex)^3}{(ade+(cd^2+ae^2)\,x+cdex^2)^{5/4}}\,dx = \left(\int \frac{x^2}{(cde\,x^2+a\,e^2x+c\,d^2x+ade)^{\frac{1}{4}}\,ae+(cde\,x^2+a\,e^2x+c\,d^2x$$

$$+2\left(\int \frac{x}{(cde\,x^2+a\,e^2x+c\,d^2x+ade)^{\frac{1}{4}}\,ae+(cde\,x^2+a\,e^2x+c\,d^2x+ade)^{\frac{1}{4}}\,cdx}dx\right)de$$

$$+\left(\int \frac{1}{(cde\,x^2+a\,e^2x+c\,d^2x+ade)^{\frac{1}{4}}\,ae+(cde\,x^2+a\,e^2x+c\,d^2x+ade)^{\frac{1}{4}}\,cdx}dx\right)d^2$$

input
```
int((e*x+d)^3/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/4),x)
```

output
```
int(x**2/((a*d*e + a*e**2*x + c*d**2*x + c*d*e*x**2)**(1/4)*a*e + (a*d*e +
 a*e**2*x + c*d**2*x + c*d*e*x**2)**(1/4)*c*d*x),x)*e**2 + 2*int(x/((a*d*e
 + a*e**2*x + c*d**2*x + c*d*e*x**2)**(1/4)*a*e + (a*d*e + a*e**2*x + c*d*
*2*x + c*d*e*x**2)**(1/4)*c*d*x),x)*d*e + int(1/((a*d*e + a*e**2*x + c*d**
2*x + c*d*e*x**2)**(1/4)*a*e + (a*d*e + a*e**2*x + c*d**2*x + c*d*e*x**2)*
*(1/4)*c*d*x),x)*d**2
```

3.377 $\int \frac{(d+ex)^2}{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/4}}\,dx$

Optimal result

Integrand size = 37, antiderivative size = 170

$$\int \frac{(d+ex)^2}{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/4}}\,dx = -\frac{4(d+ex)}{cd\sqrt[4]{ade+\left(cd^2+ae^2\right)x+cdex^2}} + \frac{3\sqrt{2}(cd^2-ae^2)\sqrt[4]{-\frac{cde(ae+cdx)(d+ex)}{(cd^2-ae^2)^2}}E\left(\frac{1}{2}\arcsin\left(\frac{ae^2+cd(d+2ex)}{cd^2-ae^2}\right)\middle|2\right)}{c^2d^2\sqrt[4]{ade+\left(cd^2+ae^2\right)x+cdex^2}}$$

output

```
(-4*e*x-4*d)/c/d/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/4)+3*2^(1/2)*(-a*e^2
+c*d^2)*(-c*d*e*(c*d*x+a*e)*(e*x+d)/(-a*e^2+c*d^2)^2)^(1/4)*EllipticE(sin(
1/2*arcsin((a*e^2+c*d*(2*e*x+d))/(-a*e^2+c*d^2))),2^(1/2))/c^2/d^2/(a*d*e+
(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/4)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.03 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.54

$$\int \frac{(d+ex)^2}{(ade+(cd^2+ae^2)x+cdex^2)^{5/4}}\,dx = -\frac{4(d+ex)\,\text{Hypergeometric2F1}\left(-\frac{3}{4},-\frac{1}{4},\frac{3}{4},\frac{e(ae+cdx)}{-cd^2+ae^2}\right)}{cd\left(\frac{cd(d+ex)}{cd^2-ae^2}\right)^{3/4}\sqrt[4]{(ae+cdx)(d+ex)}}$$

input
```
Integrate[(d + e*x)^2/(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/4),x]
```

output
```
(-4*(d + e*x)*Hypergeometric2F1[-3/4, -1/4, 3/4, (e*(a*e + c*d*x))/(-(c*d^
2) + a*e^2)])/(c*d*((c*d*(d + e*x))/(c*d^2 - a*e^2))^(3/4)*((a*e + c*d*x)*
(d + e*x))^(1/4))
```

Rubi [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 986 vs. 2(170) = 340.

Time = 0.69 (sec) , antiderivative size = 986, normalized size of antiderivative = 5.80, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.135$, Rules used = {1126, 1094, 834, 761, 1510}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex)^2}{(x(ae^2+cd^2)+ade+cdex^2)^{5/4}}dx$$

$$\Big\downarrow \text{1126}$$

$$\frac{3e\int \frac{1}{\sqrt[4]{cdex^2+(cd^2+ae^2)x+ade}}dx}{cd}-\frac{4(d+ex)}{cd\sqrt[4]{x(ae^2+cd^2)+ade+cdex^2}}$$

$$\Big\downarrow \text{1094}$$

$$\frac{12e\sqrt{\left(ae^2+cd^2+2cdex\right)^2}\int \frac{\sqrt{cdex^2+(cd^2+ae^2)x+ade}}{\sqrt{(cd^2-ae^2)^2+4cde(cdex^2+(cd^2+ae^2)x+ade)}}d\sqrt[4]{cdex^2+\left(cd^2+ae^2\right)x+ade}}{cd\left(ae^2+cd^2+2cdex\right)}-\frac{4(d+ex)}{cd\sqrt[4]{x\left(ae^2+cd^2\right)+ade+cdex^2}}$$

$\Big\downarrow$ 834

$$\frac{12e\sqrt{\left(ae^2+cd^2+2cdex\right)^2}\left(\frac{\left(cd^2-ae^2\right)\int \frac{1}{\sqrt{\left(cd^2-ae^2\right)^2+4cde\left(cdex^2+\left(cd^2+ae^2\right)x+ade\right)}}d\sqrt[4]{cdex^2+\left(cd^2+ae^2\right)x+ade}}{2\sqrt{c}\sqrt{d}\sqrt{e}}-\frac{(cd}{}\right.}{cd\left(ae^2+cd^2+2cdex\right)}$$

$$\frac{4(d+ex)}{cd\sqrt[4]{x\left(ae^2+cd^2\right)+ade+cdex^2}}$$

$\Big\downarrow$ 761

$$\frac{12e\sqrt{\left(ae^2+cd^2+2cdex\right)^2}\left(\frac{\left(cd^2-ae^2\right)^{3/2}\left(\frac{2\sqrt{c}\sqrt{d}\sqrt{e}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{cd^2-ae^2}+1\right)\sqrt{\frac{4cde\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)+\left(cd^2-ae^2\right)^2}{\left(cd^2-ae^2\right)^2\left(\frac{2\sqrt{c}\sqrt{d}\sqrt{e}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{cd^2-ae^2}\right.}}}{4\sqrt{2}c^{3/4}d^{3/4}e^{3/4}\sqrt{4cde(x(ae^2+cd^2)+ad}}\right.}{}$$

$$\frac{4(d+ex)}{cd\sqrt[4]{x\left(ae^2+cd^2\right)+ade+cdex^2}}$$

$\Big\downarrow$ 1510

$$12e\sqrt{(cd^2+2cexd+ae^2)^2}\left(\frac{(cd^2-ae^2)^{3/2}\left(\frac{2\sqrt{c}\sqrt{d}\sqrt{e}\sqrt{cdex^2+(cd^2+ae^2)x+ade}}{cd^2-ae^2}+1\right)\sqrt{\frac{(cd^2-ae^2)^2+4cde(cdex^2+(cd^2+ae^2)x+ade)}{(cd^2-ae^2)^2\left(\frac{2\sqrt{c}\sqrt{d}\sqrt{e}\sqrt{cdex^2+(cd^2+ae^2)x+ade}}{cd^2-ae^2}\right.}}}{4\sqrt{2}c^{3/4}d^{3/4}e^{3/4}\sqrt{(cd^2-ae^2)^2+4cde(cd}}\right.$$

$$\frac{4(d+ex)}{cd\sqrt[4]{cdex^2+(cd^2+ae^2)x+ade}}$$

input
```
Int[(d + e*x)^2/(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/4),x]
```

output
```
(-4*(d + e*x))/(c*d*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(1/4)) + (12*e
*Sqrt[(c*d^2 + a*e^2 + 2*c*d*e*x)^2]*(-1/2*((c*d^2 - a*e^2)*(-(((a*d*e + (
c*d^2 + a*e^2)*x + c*d*e*x^2)^(1/4)*Sqrt[(c*d^2 - a*e^2)^2 + 4*c*d*e*(a*d*
e + (c*d^2 + a*e^2)*x + c*d*e*x^2)])/((c*d^2 - a*e^2)^2*(1 + (2*Sqrt[c]*Sq
rt[d]*Sqrt[e]*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(c*d^2 - a*e^2)
))) + (Sqrt[c*d^2 - a*e^2]*(1 + (2*Sqrt[c]*Sqrt[d]*Sqrt[e]*Sqrt[a*d*e + (c
*d^2 + a*e^2)*x + c*d*e*x^2])/(c*d^2 - a*e^2))*Sqrt[((c*d^2 - a*e^2)^2 + 4
*c*d*e*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2))/((c*d^2 - a*e^2)^2*(1 + (2
*Sqrt[c]*Sqrt[d]*Sqrt[e]*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(c*d
^2 - a*e^2))^2)]*EllipticE[2*ArcTan[(Sqrt[2]*c^(1/4)*d^(1/4)*e^(1/4)*(a*d*
e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(1/4))/Sqrt[c*d^2 - a*e^2]], 1/2])/(Sqr
t[2]*c^(1/4)*d^(1/4)*e^(1/4)*Sqrt[(c*d^2 - a*e^2)^2 + 4*c*d*e*(a*d*e + (c*
d^2 + a*e^2)*x + c*d*e*x^2)])))/(Sqrt[c]*Sqrt[d]*Sqrt[e]) + ((c*d^2 - a*e^
2)^(3/2)*(1 + (2*Sqrt[c]*Sqrt[d]*Sqrt[e]*Sqrt[a*d*e + (c*d^2 + a*e^2)*x +
c*d*e*x^2])/(c*d^2 - a*e^2))*Sqrt[((c*d^2 - a*e^2)^2 + 4*c*d*e*(a*d*e + (c
*d^2 + a*e^2)*x + c*d*e*x^2))/((c*d^2 - a*e^2)^2*(1 + (2*Sqrt[c]*Sqrt[d]*S
qrt[e]*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(c*d^2 - a*e^2))^2)]*E
llipticF[2*ArcTan[(Sqrt[2]*c^(1/4)*d^(1/4)*e^(1/4)*(a*d*e + (c*d^2 + a*e^2
)*x + c*d*e*x^2)^(1/4))/Sqrt[c*d^2 - a*e^2]], 1/2])/(4*Sqrt[2]*c^(3/4)*d^(
3/4)*e^(3/4)*Sqrt[(c*d^2 - a*e^2)^2 + 4*c*d*e*(a*d*e + (c*d^2 + a*e^2)*...
```

Defintions of rubi rules used

rule 761
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 834
```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, S
imp[1/q   Int[1/Sqrt[a + b*x^4], x], x] - Simp[1/q   Int[(1 - q*x^2)/Sqrt[a
 + b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 1094
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[4*(Sqrt[(b
+ 2*c*x)^2]/(b + 2*c*x))   Subst[Int[x^(4*(p + 1) - 1)/Sqrt[b^2 - 4*a*c + 4
*c*x^4], x], x, (a + b*x + c*x^2)^(1/4)], x] /; FreeQ[{a, b, c}, x] && Inte
gerQ[4*p]
```

rule 1126
```
Int[((d_.) + (e_.)*(x_))^2*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symb
ol] :> Simp[e*(d + e*x)*((a + b*x + c*x^2)^(p + 1)/(c*(p + 1))), x] - Simp[
e^2*((p + 2)/(c*(p + 1)))   Int[(a + b*x + c*x^2)^(p + 1), x], x] /; FreeQ[
{a, b, c, d, e, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[p, -1]
```

rule 1510
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

Maple [F]

$$\int \frac{(ex+d)^2}{\left(ade+(a\,e^2+c\,d^2)\,x+cd\,x^2e\right)^{\frac{5}{4}}}dx$$

input
```
int((e*x+d)^2/(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(5/4),x)
```

output
```
int((e*x+d)^2/(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(5/4),x)
```

Fricas [F]

$$\int \frac{(d+ex)^2}{(ade+(cd^2+ae^2)\,x+cdex^2)^{5/4}}\,dx=\int \frac{(ex+d)^2}{(cdex^2+ade+(cd^2+ae^2)x)^{\frac{5}{4}}}\,dx$$

input
```
integrate((e*x+d)^2/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/4),x, algorithm="
fricas")
```

output
```
integral((c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^(3/4)/(c^2*d^2*x^2 + 2*a*
c*d*e*x + a^2*e^2), x)
```

Sympy [F]

$$\int \frac{(d+ex)^2}{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/4}}\,dx = \int \frac{\left(d+ex\right)^2}{\left(\left(d+ex\right)\left(ae+cdx\right)\right)^{\frac{5}{4}}}\,dx$$

input
```
integrate((e*x+d)**2/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(5/4),x)
```

output
```
Integral((d + e*x)**2/((d + e*x)*(a*e + c*d*x))**(5/4), x)
```

Maxima [F]

$$\int \frac{(d+ex)^2}{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/4}}\,dx = \int \frac{\left(ex+d\right)^2}{\left(cdex^2+ade+\left(cd^2+ae^2\right)x\right)^{\frac{5}{4}}}\,dx$$

input
```
integrate((e*x+d)^2/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/4),x, algorithm="
maxima")
```

output
```
integrate((e*x + d)^2/(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^(5/4), x)
```

Giac [F]

$$\int \frac{(d+ex)^2}{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/4}}\,dx = \int \frac{\left(ex+d\right)^2}{\left(cdex^2+ade+\left(cd^2+ae^2\right)x\right)^{\frac{5}{4}}}\,dx$$

input
```
integrate((e*x+d)^2/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/4),x, algorithm="
giac")
```

output
```
integrate((e*x + d)^2/(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^(5/4), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d+ex)^2}{(ade+(cd^2+ae^2)\,x+cdex^2)^{5/4}}\,dx = \int \frac{(d+e\,x)^2}{\left(c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e\right)^{5/4}}\,dx$$

input
```
int((d + e*x)^2/(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(5/4),x)
```

output
```
int((d + e*x)^2/(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(5/4), x)
```

Reduce [F]

$$\int \frac{(d+ex)^2}{(ade+(cd^2+ae^2)\,x+cdex^2)^{5/4}}\,dx = \left(\int \frac{x}{(cde\,x^2+a\,e^2x+c\,d^2x+ade)^{\frac{1}{4}}\,ae+(cde\,x^2+a\,e^2x+c\,d^2x}\right.$$
$$+\left(\int \frac{1}{(cde\,x^2+a\,e^2x+c\,d^2x+ade)^{\frac{1}{4}}\,ae+(cde\,x^2+a\,e^2x+c\,d^2x+ade)^{\frac{1}{4}}\,cdx}dx\right)d$$

input
```
int((e*x+d)^2/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/4),x)
```

output
```
int(x/((a*d*e + a*e**2*x + c*d**2*x + c*d*e*x**2)**(1/4)*a*e + (a*d*e + a*
e**2*x + c*d**2*x + c*d*e*x**2)**(1/4)*c*d*x),x)*e + int(1/((a*d*e + a*e**
2*x + c*d**2*x + c*d*e*x**2)**(1/4)*a*e + (a*d*e + a*e**2*x + c*d**2*x + c
*d*e*x**2)**(1/4)*c*d*x),x)*d
```

3.378 $\int \frac{d+ex}{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/4}}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 136

$$\int \frac{d+ex}{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/4}}\,dx = -\frac{4\sqrt[4]{\frac{e(ae+cdx)}{cd(d+ex)}}\sqrt{d+ex}\,E\left(\frac{1}{2}\arctan\left(\frac{\sqrt{-cd^2+ae^2}}{\sqrt{c}\sqrt{d}\sqrt{d+ex}}\right)\middle|2\right)}{\sqrt{c}\sqrt{d}\sqrt{-cd^2+ae^2}\,\sqrt[4]{ade+\left(cd^2+ae^2\right)x+cdex^2}}$$

output
```
-4*(e*(c*d*x+a*e)/c/d/(e*x+d))^(1/4)*(e*x+d)^(1/2)*EllipticE(sin(1/2*arcta
n((a*e^2-c*d^2)^(1/2)/c^(1/2)/d^(1/2)/(e*x+d)^(1/2))),2^(1/2))/c^(1/2)/d^(
1/2)/(a*e^2-c*d^2)^(1/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/4)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.03 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.63

$$\int \frac{d+ex}{(ade+(cd^2+ae^2)x+cdex^2)^{5/4}}\,dx = -\frac{4\sqrt[4]{\frac{cd(d+ex)}{cd^2-ae^2}}\,\text{Hypergeometric2F1}\left(-\frac{1}{4},\frac{1}{4},\frac{3}{4},\frac{e(ae+cdx)}{-cd^2+ae^2}\right)}{cd\sqrt[4]{(ae+cdx)(d+ex)}}$$

input
```
Integrate[(d + e*x)/(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/4),x]
```

output
```
(-4*((c*d*(d + e*x))/(c*d^2 - a*e^2))^(1/4)*Hypergeometric2F1[-1/4, 1/4, 3
/4, (e*(a*e + c*d*x))/(-(c*d^2) + a*e^2)])/(c*d*((a*e + c*d*x)*(d + e*x))^
(1/4))
```

Rubi [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 1002 vs. $2(136) = 272$.

Time = 0.71 (sec) , antiderivative size = 1002, normalized size of antiderivative = 7.37, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {1159, 1094, 834, 761, 1510}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{d+ex}{(x(ae^2+cd^2)+ade+cdex^2)^{5/4}}dx$$

$$\Big\downarrow \text{1159}$$

$$\frac{2e\int \frac{1}{\sqrt[4]{cdex^2+(cd^2+ae^2)x+ade}}dx}{cd^2-ae^2}-\frac{4(d+ex)}{(cd^2-ae^2)\sqrt[4]{x(ae^2+cd^2)+ade+cdex^2}}$$

$$\Big\downarrow \text{1094}$$

$$\frac{8e\sqrt{(ae^2+cd^2+2cdex)^2}\int \frac{\sqrt{cdex^2+(cd^2+ae^2)x+ade}}{\sqrt{(cd^2-ae^2)^2+4cde(cdex^2+(cd^2+ae^2)x+ade)}}d\sqrt[4]{cdex^2+(cd^2+ae^2)\,x+ade}}{(cd^2-ae^2)\,(ae^2+cd^2+2cdex)}-\frac{4(d+ex)}{(cd^2-ae^2)\sqrt[4]{x\,(ae^2+cd^2)+ade+cdex^2}}$$

$\big\downarrow$ 834

$$\frac{8e\sqrt{(ae^2+cd^2+2cdex)^2}\left(\frac{(cd^2-ae^2)\int \frac{1}{\sqrt{\left(cd^2-ae^2\right)^2+4cde\left(cdex^2+\left(cd^2+ae^2\right)x+ade\right)}}d\sqrt[4]{cdex^2+(cd^2+ae^2)\,x+ade}}{2\sqrt{c}\sqrt{d}\sqrt{e}}-\frac{(cd^2 \cdots}{\cdots}\right.}{(cd^2-ae^2)\,(ae^2+cd^2+2cdex)}$$

$$\frac{4(d+ex)}{(cd^2-ae^2)\sqrt[4]{x\,(ae^2+cd^2)+ade+cdex^2}}$$

$\big\downarrow$ 761

$$\frac{8e\sqrt{(ae^2+cd^2+2cdex)^2}\left(\frac{(cd^2-ae^2)^{3/2}\left(\frac{2\sqrt{c}\sqrt{d}\sqrt{e}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{cd^2-ae^2}+1\right)\sqrt{\frac{4cde\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)+\left(cd^2-ae^2\right)^2}{\left(cd^2-ae^2\right)^2\left(\frac{2\sqrt{c}\sqrt{d}\sqrt{e}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{cd^2-ae^2}+\cdots\right.}}}{4\sqrt{2}c^{3/4}d^{3/4}e^{3/4}\sqrt{4cde(x(ae^2+cd^2)+ade\cdots}}\right.}{\cdots}$$

$$\frac{4(d+ex)}{(cd^2-ae^2)\sqrt[4]{x\,(ae^2+cd^2)+ade+cdex^2}}$$

$\big\downarrow$ 1510

$$\frac{8e\sqrt{\left(cd^2+2cexd+ae^2\right)^2}\left(\frac{\left(cd^2-ae^2\right)^{3/2}\left(\frac{2\sqrt{c}\sqrt{d}\sqrt{e}\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}{cd^2-ae^2}+1\right)\sqrt{\frac{\left(cd^2-ae^2\right)^2+4cde\left(cdex^2+\left(cd^2+ae^2\right)x+ade\right)}{\left(cd^2-ae^2\right)^2\left(\frac{2\sqrt{c}\sqrt{d}\sqrt{e}\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}{cd^2-ae^2}+\cdots\right.}}}{4\sqrt{2}c^{3/4}d^{3/4}e^{3/4}\sqrt{\left(cd^2-ae^2\right)^2+4cde(cde\cdots}}\right.}{}$$

$$\frac{4(d+ex)}{\left(cd^2-ae^2\right)\sqrt[4]{cdex^2+\left(cd^2+ae^2\right)x+ade}}$$

input

```
Int[(d + e*x)/(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/4),x]
```

output
```
(-4*(d + e*x))/((c*d^2 - a*e^2)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(1
/4)) + (8*e*Sqrt[(c*d^2 + a*e^2 + 2*c*d*e*x)^2]*(-1/2*((c*d^2 - a*e^2)*(-(
((a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(1/4)*Sqrt[(c*d^2 - a*e^2)^2 + 4*
c*d*e*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)])/((c*d^2 - a*e^2)^2*(1 + (2
*Sqrt[c]*Sqrt[d]*Sqrt[e]*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(c*d
^2 - a*e^2)))) + (Sqrt[c*d^2 - a*e^2]*(1 + (2*Sqrt[c]*Sqrt[d]*Sqrt[e]*Sqrt
[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(c*d^2 - a*e^2))*Sqrt[((c*d^2 - a
*e^2)^2 + 4*c*d*e*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2))/((c*d^2 - a*e^2
)^2*(1 + (2*Sqrt[c]*Sqrt[d]*Sqrt[e]*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e
*x^2])/(c*d^2 - a*e^2))^2)]*EllipticE[2*ArcTan[(Sqrt[2]*c^(1/4)*d^(1/4)*e^
(1/4)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(1/4))/Sqrt[c*d^2 - a*e^2]],
 1/2])/(Sqrt[2]*c^(1/4)*d^(1/4)*e^(1/4)*Sqrt[(c*d^2 - a*e^2)^2 + 4*c*d*e*(
a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)])))/(Sqrt[c]*Sqrt[d]*Sqrt[e]) + ((c
*d^2 - a*e^2)^(3/2)*(1 + (2*Sqrt[c]*Sqrt[d]*Sqrt[e]*Sqrt[a*d*e + (c*d^2 +
a*e^2)*x + c*d*e*x^2])/(c*d^2 - a*e^2))*Sqrt[((c*d^2 - a*e^2)^2 + 4*c*d*e*
(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2))/((c*d^2 - a*e^2)^2*(1 + (2*Sqrt[c
]*Sqrt[d]*Sqrt[e]*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(c*d^2 - a*
e^2))^2)]*EllipticF[2*ArcTan[(Sqrt[2]*c^(1/4)*d^(1/4)*e^(1/4)*(a*d*e + (c*
d^2 + a*e^2)*x + c*d*e*x^2)^(1/4))/Sqrt[c*d^2 - a*e^2]], 1/2])/(4*Sqrt[2]*
c^(3/4)*d^(3/4)*e^(3/4)*Sqrt[(c*d^2 - a*e^2)^2 + 4*c*d*e*(a*d*e + (c*d^...
```

Defintions of rubi rules used

rule 761
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 834
```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, S
imp[1/q   Int[1/Sqrt[a + b*x^4], x], x] - Simp[1/q   Int[(1 - q*x^2)/Sqrt[a
 + b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 1094
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[4*(Sqrt[(b
+ 2*c*x)^2]/(b + 2*c*x))   Subst[Int[x^(4*(p + 1) - 1)/Sqrt[b^2 - 4*a*c + 4
*c*x^4], x], x, (a + b*x + c*x^2)^(1/4)], x] /; FreeQ[{a, b, c}, x] && Inte
gerQ[4*p]
```

rule 1159
```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[((b*d - 2*a*e + (2*c*d - b*e)*x)/((p + 1)*(b^2 - 4*a*c)))*(a + b*
x + c*x^2)^(p + 1), x] - Simp[(2*p + 3)*((2*c*d - b*e)/((p + 1)*(b^2 - 4*a*
c)))   Int[(a + b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] &
& LtQ[p, -1] && NeQ[p, -3/2]
```

rule 1510
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

Maple [F]

$$\int \frac{ex+d}{(ade+(a\,e^2+c\,d^2)\,x+cd\,x^2e)^{\frac{5}{4}}}dx$$

input
```
int((e*x+d)/(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(5/4),x)
```

output
```
int((e*x+d)/(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(5/4),x)
```

Fricas [F]

$$\int \frac{d+ex}{(ade+(cd^2+ae^2)\,x+cdex^2)^{5/4}}\,dx = \int \frac{ex+d}{(cdex^2+ade+(cd^2+ae^2)x)^{\frac{5}{4}}}\,dx$$

input
```
integrate((e*x+d)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/4),x, algorithm="fr
icas")
```

output
```
integral((c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^(3/4)/(c^2*d^2*e*x^3 + a^
2*d*e^2 + (c^2*d^3 + 2*a*c*d*e^2)*x^2 + (2*a*c*d^2*e + a^2*e^3)*x), x)
```

Sympy [F]

$$\int \frac{d+ex}{(ade+(cd^2+ae^2)\,x+cdex^2)^{5/4}}\,dx = \int \frac{d+ex}{((d+ex)\,(ae+cdx))^{\frac{5}{4}}}\,dx$$

input
```
integrate((e*x+d)/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(5/4),x)
```

output
```
Integral((d + e*x)/((d + e*x)*(a*e + c*d*x))**(5/4), x)
```

Maxima [F]

$$\int \frac{d+ex}{(ade+(cd^2+ae^2)\,x+cdex^2)^{5/4}}\,dx = \int \frac{ex+d}{(cdex^2+ade+(cd^2+ae^2)x)^{\frac{5}{4}}}\,dx$$

input
```
integrate((e*x+d)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/4),x, algorithm="ma
xima")
```

output
```
integrate((e*x + d)/(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^(5/4), x)
```

Giac [F]

$$\int \frac{d+ex}{(ade+(cd^2+ae^2)\,x+cdex^2)^{5/4}}\,dx = \int \frac{ex+d}{(cdex^2+ade+(cd^2+ae^2)x)^{\frac{5}{4}}}\,dx$$

input
```
integrate((e*x+d)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/4),x, algorithm="gi
ac")
```

output
```
integrate((e*x + d)/(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^(5/4), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{d+ex}{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/4}}\,dx=\int \frac{d+e\,x}{\left(c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e\right)^{5/4}}\,dx$$

input
```
int((d + e*x)/(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(5/4),x)
```

output
```
int((d + e*x)/(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(5/4), x)
```

Reduce [F]

$$\int \frac{d+ex}{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/4}}\,dx=\int \frac{1}{\left(cde\,x^2+a\,e^2x+c\,d^2x+ade\right)^{\frac{1}{4}}ae+\left(cde\,x^2+a\,e^2x+c\,d^2x+\right.}$$

input
```
int((e*x+d)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/4),x)
```

output
```
int(1/((a*d*e + a*e**2*x + c*d**2*x + c*d*e*x**2)**(1/4)*a*e + (a*d*e + a*
e**2*x + c*d**2*x + c*d*e*x**2)**(1/4)*c*d*x),x)
```

3.379 $\int \frac{1}{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/4}}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 198

$$\int \frac{1}{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/4}}\,dx = -\frac{4\left(cd^2+ae^2+2cdex\right)}{\left(cd^2-ae^2\right)^2\sqrt[4]{ade+\left(cd^2+ae^2\right)x+cdex^2}} + \frac{4\sqrt{2}\sqrt[4]{-\frac{cde\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)}{\left(cd^2-ae^2\right)^2}}\,E\left(\frac{1}{2}\arcsin\left(\frac{ae^2+cd(d+2ex)}{cd^2-ae^2}\right)\middle|2\right)}{\left(cd^2-ae^2\right)\sqrt[4]{ade+\left(cd^2+ae^2\right)x+cdex^2}}$$

output
```
(-8*c*d*e*x-4*a*e^2-4*c*d^2)/(-a*e^2+c*d^2)^2/(a*d*e+(a*e^2+c*d^2)*x+c*d*e
*x^2)^(1/4)+4*2^(1/2)*(-c*d*e*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)/(-a*e^2+c*
d^2)^2)^(1/4)*EllipticE(sin(1/2*arcsin((a*e^2+c*d*(2*e*x+d))/(-a*e^2+c*d^2
))),2^(1/2))/(-a*e^2+c*d^2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/4)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.03 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.47

$$\int \frac{1}{(ade+(cd^2+ae^2)\,x+cdex^2)^{5/4}}\,dx = -\frac{4\sqrt[4]{\frac{cd(d+ex)}{cd^2-ae^2}}\,\text{Hypergeometric2F1}\left(-\frac{1}{4},\frac{5}{4},\frac{3}{4},\frac{e(ae+cdx)}{-cd^2+ae^2}\right)}{(cd^2-ae^2)\,\sqrt[4]{(ae+cdx)(d+ex)}}$$

input
```
Integrate[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(-5/4),x]
```

output
```
(-4*((c*d*(d + e*x))/(c*d^2 - a*e^2))^(1/4)*Hypergeometric2F1[-1/4, 5/4, 3
/4, (e*(a*e + c*d*x))/(-(c*d^2) + a*e^2)])/((c*d^2 - a*e^2)*((a*e + c*d*x)
*(d + e*x))^(1/4))
```

Rubi [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 1016 vs. 2(198) = 396.

Time = 0.67 (sec) , antiderivative size = 1016, normalized size of antiderivative = 5.13, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.172$, Rules used = {1089, 1094, 834, 761, 1510}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(x\,(ae^2+cd^2)+ade+cdex^2)^{5/4}}dx$$

$$\Big\downarrow \text{1089}$$

$$\frac{4cde\int \frac{1}{\sqrt[4]{cdex^2+(cd^2+ae^2)\,x+ade}}dx}{(cd^2-ae^2)^2}-\frac{4(ae^2+cd^2+2cdex)}{(cd^2-ae^2)^2\,\sqrt[4]{x\,(ae^2+cd^2)+ade+cdex^2}}$$

$$\Big\downarrow \text{1094}$$

$$\frac{16cde\sqrt{(ae^2+cd^2+2cdex)^2}\int \frac{\sqrt{cdex^2+(cd^2+ae^2)x+ade}}{\sqrt{(cd^2-ae^2)^2+4cde(cdex^2+(cd^2+ae^2)x+ade)}}d\sqrt[4]{cdex^2+(cd^2+ae^2)x+ade}}{(cd^2-ae^2)^2(ae^2+cd^2+2cdex)}-\frac{4(ae^2+cd^2+2cdex)}{(cd^2-ae^2)^2\sqrt[4]{x(ae^2+cd^2)+ade+cdex^2}}$$

$\downarrow$ 834

$$\frac{16cde\sqrt{(ae^2+cd^2+2cdex)^2}\left(\frac{(cd^2-ae^2)\int \frac{1}{\sqrt{\left(cd^2-ae^2\right)^2+4cde\left(cdex^2+\left(cd^2+ae^2\right)x+ade\right)}}d\sqrt[4]{cdex^2+(cd^2+ae^2)x+ade}}{2\sqrt{c}\sqrt{d}\sqrt{e}}-\cdots\right.}{(cd^2-ae^2)^2(ae^2+cd^2+2cdex)}-\frac{4(ae^2+cd^2+2cdex)}{(cd^2-ae^2)^2\sqrt[4]{x(ae^2+cd^2)+ade+cdex^2}}$$

$\downarrow$ 761

$$\frac{16cde\sqrt{(ae^2+cd^2+2cdex)^2}\left(\frac{(cd^2-ae^2)^{3/2}\left(\frac{2\sqrt{c}\sqrt{d}\sqrt{e}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{cd^2-ae^2}+1\right)\sqrt{\frac{4cde\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)+\left(cd^2-ae^2\cdots\right.}{\left(cd^2-ae^2\right)^2\left(\frac{2\sqrt{c}\sqrt{d}\sqrt{e}\sqrt{x\left(ae^2+cd^2\right)+ade+cde\cdots}}{cd^2-ae^2}\cdots\right.}}}{4\sqrt{2}c^{3/4}d^{3/4}e^{3/4}\sqrt{4cde(x(ae^2+cd^2)\cdots}}\right.}{\cdots}-\frac{4(ae^2+cd^2+2cdex)}{(cd^2-ae^2)^2\sqrt[4]{x(ae^2+cd^2)+ade+cdex^2}}$$

$\downarrow$ 1510

$$\frac{16cde\sqrt{\left(cd^2+2cexd+ae^2\right)^2}\left(\frac{\left(cd^2-ae^2\right)^{3/2}\left(\frac{2\sqrt{c}\sqrt{d}\sqrt{e}\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}{cd^2-ae^2}+1\right)\sqrt{\frac{\left(cd^2-ae^2\right)^2+4cde\left(cdex^2+\left(cd^2+ae^2\right)x+a\cdots\right.}{\left(cd^2-ae^2\right)^2\left(\frac{2\sqrt{c}\sqrt{d}\sqrt{e}\sqrt{cdex^2+\left(cd^2+ae^2\right)x+a\cdots}}{cd^2-ae^2}\cdots\right.}}}{4\sqrt{2}c^{3/4}d^{3/4}e^{3/4}\sqrt{\left(cd^2-ae^2\right)^2+4cde\cdots}}\cdots\right.}{\cdots}$$

$$\frac{4\left(cd^2+2cexd+ae^2\right)}{\left(cd^2-ae^2\right)^2\sqrt[4]{cdex^2+\left(cd^2+ae^2\right)x+ade}}$$

input
```
Int[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(-5/4),x]
```

output
```
(-4*(c*d^2 + a*e^2 + 2*c*d*e*x))/((c*d^2 - a*e^2)^2*(a*d*e + (c*d^2 + a*e^
2)*x + c*d*e*x^2)^(1/4)) + (16*c*d*e*Sqrt[(c*d^2 + a*e^2 + 2*c*d*e*x)^2]*(
-1/2*((c*d^2 - a*e^2)*(-(((a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(1/4)*Sq
rt[(c*d^2 - a*e^2)^2 + 4*c*d*e*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)])/(
(c*d^2 - a*e^2)^2*(1 + (2*Sqrt[c]*Sqrt[d]*Sqrt[e]*Sqrt[a*d*e + (c*d^2 + a*
e^2)*x + c*d*e*x^2])/(c*d^2 - a*e^2)))) + (Sqrt[c*d^2 - a*e^2]*(1 + (2*Sqr
t[c]*Sqrt[d]*Sqrt[e]*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(c*d^2 -
 a*e^2))*Sqrt[((c*d^2 - a*e^2)^2 + 4*c*d*e*(a*d*e + (c*d^2 + a*e^2)*x + c*
d*e*x^2))/((c*d^2 - a*e^2)^2*(1 + (2*Sqrt[c]*Sqrt[d]*Sqrt[e]*Sqrt[a*d*e +
(c*d^2 + a*e^2)*x + c*d*e*x^2])/(c*d^2 - a*e^2))^2)]*EllipticE[2*ArcTan[(S
qrt[2]*c^(1/4)*d^(1/4)*e^(1/4)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(1/
4))/Sqrt[c*d^2 - a*e^2]], 1/2])/(Sqrt[2]*c^(1/4)*d^(1/4)*e^(1/4)*Sqrt[(c*d
^2 - a*e^2)^2 + 4*c*d*e*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)])))/(Sqrt[
c]*Sqrt[d]*Sqrt[e]) + ((c*d^2 - a*e^2)^(3/2)*(1 + (2*Sqrt[c]*Sqrt[d]*Sqrt[
e]*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(c*d^2 - a*e^2))*Sqrt[((c*
d^2 - a*e^2)^2 + 4*c*d*e*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2))/((c*d^2
- a*e^2)^2*(1 + (2*Sqrt[c]*Sqrt[d]*Sqrt[e]*Sqrt[a*d*e + (c*d^2 + a*e^2)*x
+ c*d*e*x^2])/(c*d^2 - a*e^2))^2)]*EllipticF[2*ArcTan[(Sqrt[2]*c^(1/4)*d^(
1/4)*e^(1/4)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(1/4))/Sqrt[c*d^2 - a
*e^2]], 1/2])/(4*Sqrt[2]*c^(3/4)*d^(3/4)*e^(3/4)*Sqrt[(c*d^2 - a*e^2)^2...
```

Defintions of rubi rules used

rule 761
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 834
```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, S
imp[1/q   Int[1/Sqrt[a + b*x^4], x], x] - Simp[1/q   Int[(1 - q*x^2)/Sqrt[a
 + b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 1089
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^(p + 1)/((p + 1)*(b^2 - 4*a*c))), x] - Simp[2*c*((2*p +
 3)/((p + 1)*(b^2 - 4*a*c)))   Int[(a + b*x + c*x^2)^(p + 1), x], x] /; Fre
eQ[{a, b, c}, x] && LtQ[p, -1] && (IntegerQ[4*p] || IntegerQ[3*p])
```

rule 1094
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[4*(Sqrt[(b
+ 2*c*x)^2]/(b + 2*c*x))   Subst[Int[x^(4*(p + 1) - 1)/Sqrt[b^2 - 4*a*c + 4
*c*x^4], x], x, (a + b*x + c*x^2)^(1/4)], x] /; FreeQ[{a, b, c}, x] && Inte
gerQ[4*p]
```

rule 1510
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

Maple [F]

$$\int \frac{1}{\left(ade+\left(a\,e^{2}+c\,d^{2}\right)x+cd\,x^{2}e\right)^{\frac{5}{4}}}dx$$

input
```
int(1/(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(5/4),x)
```

output
```
int(1/(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(5/4),x)
```

Fricas [F]

$$\int \frac{1}{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/4}}\,dx=\int \frac{1}{\left(cdex^2+ade+\left(cd^2+ae^2\right)x\right)^{\frac{5}{4}}}\,dx$$

input
```
integrate(1/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/4),x, algorithm="fricas")
```

output
```
integral((c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^(3/4)/(c^2*d^2*e^2*x^4 +
a^2*d^2*e^2 + 2*(c^2*d^3*e + a*c*d*e^3)*x^3 + (c^2*d^4 + 4*a*c*d^2*e^2 + a
^2*e^4)*x^2 + 2*(a*c*d^3*e + a^2*d*e^3)*x), x)
```

Sympy [F]

$$\int \frac{1}{(ade + (cd^2 + ae^2)\, x + cdex^2)^{5/4}}\, dx = \int \frac{1}{(ade + cdex^2 + x\, (ae^2 + cd^2))^{\frac{5}{4}}}\, dx$$

input
```
integrate(1/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(5/4),x)
```

output
```
Integral((a*d*e + c*d*e*x**2 + x*(a*e**2 + c*d**2))**(-5/4), x)
```

Maxima [F]

$$\int \frac{1}{(ade + (cd^2 + ae^2)\, x + cdex^2)^{5/4}}\, dx = \int \frac{1}{(cdex^2 + ade + (cd^2 + ae^2)x)^{\frac{5}{4}}}\, dx$$

input
```
integrate(1/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/4),x, algorithm="maxima")
```

output
```
integrate((c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^(-5/4), x)
```

Giac [F]

$$\int \frac{1}{(ade + (cd^2 + ae^2)\, x + cdex^2)^{5/4}}\, dx = \int \frac{1}{(cdex^2 + ade + (cd^2 + ae^2)x)^{\frac{5}{4}}}\, dx$$

input
```
integrate(1/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/4),x, algorithm="giac")
```

output
```
integrate((c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^(-5/4), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\left(ade+(cd^2+ae^2)\,x+cdex^2\right)^{5/4}}\,dx=\int \frac{1}{\left(c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e\right)^{5/4}}\,dx$$

input
```
int(1/(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(5/4),x)
```

output
```
int(1/(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(5/4), x)
```

Reduce [F]

$$\int \frac{1}{\left(ade+(cd^2+ae^2)\,x+cdex^2\right)^{5/4}}\,dx=\int \frac{}{(cde\,x^2+a\,e^2x+c\,d^2x+ade)^{\frac{1}{4}}\,ade+(cde\,x^2+a\,e^2x+c\,d^2x}$$

input
```
int(1/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/4),x)
```

output
```
int(1/((a*d*e + a*e**2*x + c*d**2*x + c*d*e*x**2)**(1/4)*a*d*e + (a*d*e +
a*e**2*x + c*d**2*x + c*d*e*x**2)**(1/4)*a*e**2*x + (a*d*e + a*e**2*x + c*
d**2*x + c*d*e*x**2)**(1/4)*c*d**2*x + (a*d*e + a*e**2*x + c*d**2*x + c*d*
e*x**2)**(1/4)*c*d*e*x**2),x)
```

3.380 $\int \frac{1}{(d+ex)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/4}}\,dx$

Optimal result

Integrand size = 37, antiderivative size = 242

$$\begin{aligned}\int \frac{1}{(d+ex)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/4}}\,dx = & -\frac{4}{\left(cd^2-ae^2\right)(d+ex)\sqrt[4]{ade+\left(cd^2+ae^2\right)x+cdex^2}} \\ & -\frac{24e\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/4}}{5\left(cd^2-ae^2\right)^2(d+ex)^2} \\ & +\frac{48cd\sqrt{e}\sqrt{ae+cdx}\sqrt[4]{\frac{cd(d+ex)}{e(ae+cdx)}}E\left(\frac{1}{2}\arctan\left(\frac{\sqrt{cd^2-ae^2}}{\sqrt{e}\sqrt{ae+cdx}}\right)\middle|2\right)}{5\left(cd^2-ae^2\right)^{5/2}\sqrt[4]{ade+\left(cd^2+ae^2\right)x+cdex^2}}\end{aligned}$$

output
```
-4/(-a*e^2+c*d^2)/(e*x+d)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/4)-24/5*e*(
a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/4)/(-a*e^2+c*d^2)^2/(e*x+d)^2+48/5*c*d
*e^(1/2)*(c*d*x+a*e)^(1/2)*(c*d*(e*x+d)/e/(c*d*x+a*e))^(1/4)*EllipticE(sin
(1/2*arctan((-a*e^2+c*d^2)^(1/2)/e^(1/2)/(c*d*x+a*e)^(1/2))),2^(1/2))/(-a*
e^2+c*d^2)^(5/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/4)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.05 (sec) , antiderivative size = 96, normalized size of antiderivative = 0.40

$$\int \frac{1}{(d+ex)\left(ade+(cd^2+ae^2)x+cdex^2\right)^{5/4}}\,dx = -\frac{4cd\sqrt[4]{\frac{cd(d+ex)}{cd^2-ae^2}}\,\text{Hypergeometric2F1}\left(-\frac{1}{4},\frac{9}{4},\frac{3}{4},\frac{e(ae+cdx)}{-cd^2+ae^2}\right)}{(cd^2-ae^2)^2\sqrt[4]{(ae+cdx)(d+ex)}}$$

input
```
Integrate[1/((d + e*x)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/4)),x]
```

output
```
(-4*c*d*((c*d*(d + e*x))/(c*d^2 - a*e^2))^(1/4)*Hypergeometric2F1[-1/4, 9/
4, 3/4, (e*(a*e + c*d*x))/(-(c*d^2) + a*e^2)])/((c*d^2 - a*e^2)^2*((a*e +
c*d*x)*(d + e*x))^(1/4))
```

Rubi [A] (warning: unable to verify)

Time = 0.58 (sec) , antiderivative size = 453, normalized size of antiderivative = 1.87, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.270$, Rules used = {1138, 61, 61, 61, 73, 839, 813, 858, 807, 212}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(d+ex)\left(x(ae^2+cd^2)+ade+cdex^2\right)^{5/4}}dx$$

$$\Big\downarrow \ 1138$$

$$\frac{\sqrt[4]{\frac{ex}{d}+1}\sqrt[4]{ae+cdx}\int \frac{1}{(ae+cdx)^{5/4}\left(\frac{ex}{d}+1\right)^{9/4}}dx}{d\sqrt[4]{x(ae^2+cd^2)+ade+cdex^2}}$$

$$\Big\downarrow \ 61$$

$$\frac{\sqrt[4]{\frac{ex}{d}+1}\sqrt[4]{ae+cdx}\left(-\frac{6e\int\frac{1}{\sqrt[4]{ae+cdx}\left(\frac{ex}{d}+1\right)^{9/4}}dx}{cd^2-ae^2}-\frac{4}{\left(\frac{ex}{d}+1\right)^{5/4}\left(cd-\frac{ae^2}{d}\right)\sqrt[4]{ae+cdx}}\right)}{d\sqrt[4]{x\left(ae^2+cd^2\right)+ade+cdex^2}}$$

$$\downarrow \ 61$$

$$\frac{\sqrt[4]{\frac{ex}{d}+1}\sqrt[4]{ae+cdx}\left(-\frac{6e\left(\frac{2cd\int\frac{1}{\sqrt[4]{ae+cdx}\left(\frac{ex}{d}+1\right)^{5/4}}dx}{5\left(cd-\frac{ae^2}{d}\right)}+\frac{4(ae+cdx)^{3/4}}{5\left(\frac{ex}{d}+1\right)^{5/4}\left(cd-\frac{ae^2}{d}\right)}\right)}{cd^2-ae^2}-\frac{4}{\left(\frac{ex}{d}+1\right)^{5/4}\left(cd-\frac{ae^2}{d}\right)\sqrt[4]{ae+cdx}}\right)}{d\sqrt[4]{x\left(ae^2+cd^2\right)+ade+cdex^2}}$$

$$\downarrow \ 61$$

$$\frac{\sqrt[4]{\frac{ex}{d}+1}\sqrt[4]{ae+cdx}\left(-\frac{6e\left(\frac{2cd\left(\frac{4d(ae+cdx)^{3/4}}{\sqrt[4]{\frac{ex}{d}+1}\left(cd^2-ae^2\right)}-\frac{2cd^2\int\frac{1}{\sqrt[4]{ae+cdx}\sqrt[4]{\frac{ex}{d}+1}}dx}{cd^2-ae^2}\right)}{5\left(cd-\frac{ae^2}{d}\right)}+\frac{4(ae+cdx)^{3/4}}{5\left(\frac{ex}{d}+1\right)^{5/4}\left(cd-\frac{ae^2}{d}\right)}\right)}{cd^2-ae^2}-\frac{}{\left(\frac{ex}{d}+1\right)^{5}}\right.}{d\sqrt[4]{x\left(ae^2+cd^2\right)+ade+cdex^2}}$$

$$\downarrow \ 73$$

$$\frac{\sqrt[4]{\frac{ex}{d}+1}\sqrt[4]{ae+cdx}\left(-\frac{6e\left(\frac{2cd\left(\frac{4d(ae+cdx)^{3/4}}{\sqrt[4]{\frac{ex}{d}+1}\left(cd^2-ae^2\right)}-\frac{8d\int\frac{\sqrt{ae+cdx}}{\sqrt[4]{-\frac{ae^2}{cd^2}+\frac{(ae+cdx)e}{cd^2}+1}}d\sqrt[4]{ae+cdx}}{cd^2-ae^2}\right)}{5\left(cd-\frac{ae^2}{d}\right)}+\frac{4(ae+cdx)^{3/4}}{5\left(\frac{ex}{d}+1\right)^{5/4}\left(cd-\cdots\right.}\cdots\right)}{cd^2-ae^2}\cdots\right.}{d\sqrt[4]{x\left(ae^2+cd^2\right)+ade+cdex^2}}$$

$\downarrow$ 839

$$\frac{\sqrt[4]{\frac{ex}{d}+1}\sqrt[4]{ae+cdx}\left(-\frac{6e\left(\frac{2cd\left(\frac{4d(ae+cdx)^{3/4}}{\sqrt[4]{\frac{ex}{d}+1}\left(cd^2-ae^2\right)}-\frac{8d\left(\frac{(ae+cdx)^{3/4}}{2\sqrt[4]{-\frac{ae^2}{cd^2}+\frac{e(ae+cdx)}{cd^2}+1}}-\frac{1}{2}\left(1-\frac{ae^2}{cd^2}\right)\int\frac{\sqrt{ae+cdx}}{\left(-\frac{ae^2}{cd^2}+\frac{(ae+cdx)e}{cd^2}+1\right)\cdots}\cdots\right)}{cd^2-ae^2}\cdots\right)}{5\left(cd-\frac{ae^2}{d}\right)}\cdots\right)}{cd^2-ae^2}\cdots\right)}{d\sqrt[4]{x\left(ae^2+cd^2\right)+ade+cdex^2}\cdots}$$

$\downarrow$ 813

$$\frac{\sqrt[4]{\frac{ex}{d}+1}\sqrt[4]{ae+cdx}\left(-\frac{6e\left(\frac{2cd\left(\frac{4d(ae+cdx)^{3/4}}{\sqrt[4]{\frac{ex}{d}+1}\left(cd^2-ae^2\right)}-\frac{8d\left(\frac{(ae+cdx)^{3/4}}{2\sqrt[4]{-\frac{ae^2}{cd^2}+\frac{e(ae+cdx)}{cd^2}+1}}-\frac{cd^2\left(1-\frac{ae^2}{cd^2}\right)\sqrt[4]{ae+cdx}\sqrt[4]{\frac{cd\ldots}{e(a\ldots}}}{2e\sqrt[4]{\ldots}}\right)}{cd^2-ae^2\ldots}\right)}{5\left(cd-\frac{ae^2}{d}\right)\ldots}\ldots\right)}{cd^2-ae^2\ldots}\ldots\right)}{d\sqrt[4]{x\left(ae^2+cd^2\right)\ldots}}$$

↓ 858

$$\sqrt[4]{\frac{ex}{d}+1}\,\sqrt[4]{ae+cdx}\left(-\frac{6e\left(2cd\left(\frac{4d(ae+cdx)^{3/4}}{\sqrt[4]{\frac{ex}{d}+1}\left(cd^2-ae^2\right)}-\frac{8d\left(\frac{cd^2\left(1-\frac{ae^2}{cd^2}\right)\sqrt[4]{ae+cdx}\,\sqrt[4]{\frac{cd^2-ae^2}{e(ae+cdx)}+1}\int\frac{\ldots}{\sqrt[4]{ae+cdx}\left(\frac{\left(cd^2\ldots\right.}{\ldots}\right.}}{2e\sqrt[4]{-\frac{ae^2}{cd^2}+\frac{e(ae+cdx)}{cd^2}+\ldots}}\ldots\right)}{cd^2\ldots}\right)\ldots\right)}{5\left(cd-\frac{ae^2}{d}\right)\ldots}\ldots}{cd^2-a\ldots}\ldots\right.$$

↓ 807

$$\sqrt[4]{\frac{ex}{d}+1}\sqrt[4]{ae+cdx}\left(-\frac{6e\left(\frac{2cd\left(\frac{4d(ae+cdx)^{3/4}}{\sqrt[4]{\frac{ex}{d}+1}\left(cd^2-ae^2\right)}-\frac{8d\left(\frac{cd^2\left(1-\frac{ae^2}{cd^2}\right)\sqrt[4]{ae+cdx}\sqrt[4]{\frac{cd^2-ae^2}{e(ae+cdx)}+1}\int\frac{1}{\left(\frac{\sqrt{ae+cdx}\left(cd^2-ae^2\right)}{e}\cdots\right.}}{4e\sqrt[4]{-\frac{ae^2}{cd^2}+\frac{e(ae+cdx)}{cd^2}+1}}\cdots\right)}{cd^2-ae^2}\cdots\right)}{5\left(cd-\frac{ae^2}{d}\right)}\cdots\right)}{cd^2-ae^2}\cdots\right.$$

$$\Big\downarrow 212$$

$$\frac{\sqrt[4]{\frac{ex}{d}+1}\sqrt[4]{ae+cdx}\,\Bigg(-\frac{6e\Bigg(\frac{2cd\Bigg(\frac{4d(ae+cdx)^{3/4}}{\sqrt[4]{\frac{ex}{d}+1}\,(cd^2-ae^2)}-\frac{8d\Bigg(\frac{cd^2\left(1-\frac{ae^2}{cd^2}\right)\sqrt[4]{ae+cdx}\sqrt[4]{\frac{cd^2-ae^2}{e(ae+cdx)}+1}\,E\Big(\frac{1}{2}\arctan\Big(\frac{\sqrt{cd^2-ae^2}\cdots}{\sqrt{\cdots}}\cdots}{2\sqrt{e}\sqrt{cd^2-ae^2}\sqrt[4]{-\frac{ae^2}{cd^2}+\frac{e(ae+cdx)}{cd^2}+1}}\cdots}{cd^2-ae^2}\cdots}{5\left(cd-\frac{ae^2}{d}\right)}\cdots}{cd^2-ae^2}\cdots}{d\sqrt[4]{x\left(ae^2+cd^2\right)+ad\cdots}}$$

input
```
Int[1/((d + e*x)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/4)),x]
```

output
```
((a*e + c*d*x)^(1/4)*(1 + (e*x)/d)^(1/4)*(-4/((c*d - (a*e^2)/d)*(a*e + c*d
*x)^(1/4)*(1 + (e*x)/d)^(5/4)) - (6*e*((4*(a*e + c*d*x)^(3/4))/(5*(c*d - (
a*e^2)/d)*(1 + (e*x)/d)^(5/4)) + (2*c*d*((4*d*(a*e + c*d*x)^(3/4))/((c*d^2
 - a*e^2)*(1 + (e*x)/d)^(1/4)) - (8*d*((a*e + c*d*x)^(3/4)/(2*(1 - (a*e^2)
/(c*d^2) + (e*(a*e + c*d*x))/(c*d^2))^(1/4)) + (c*d^2*(1 - (a*e^2)/(c*d^2)
)*(a*e + c*d*x)^(1/4)*(1 + (c*d^2 - a*e^2)/(e*(a*e + c*d*x)))^(1/4)*Ellipt
icE[ArcTan[(Sqrt[c*d^2 - a*e^2]*Sqrt[a*e + c*d*x])/Sqrt[e]]/2, 2])/(2*Sqrt
[e]*Sqrt[c*d^2 - a*e^2]*(1 - (a*e^2)/(c*d^2) + (e*(a*e + c*d*x))/(c*d^2))^
(1/4))))/(c*d^2 - a*e^2)))/(5*(c*d - (a*e^2)/d))))/(c*d^2 - a*e^2)))/(d*(a
*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(1/4))
```

Defintions of rubi rules used

rule 61
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && LtQ[m, -1] &&  !(LtQ[n, -1] && (EqQ[a, 0
] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && IntLinearQ[a, b, c, d
, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 212
```
Int[((a_) + (b_.)*(x_)^2)^(-5/4), x_Symbol] :> Simp[(2/(a^(5/4)*Rt[b/a, 2])
)*EllipticE[(1/2)*ArcTan[Rt[b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

rule 807
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Simp[1/k   Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x,
x^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

rule 813
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4)^(5/4), x_Symbol] :> Simp[x*((1 + a/(b*x^4)
)^(1/4)/(b*(a + b*x^4)^(1/4)))   Int[1/(x^3*(1 + a/(b*x^4))^(5/4)), x], x]
/; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 839
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4)^(1/4), x_Symbol] :> Simp[x^3/(2*(a + b*x^4
)^(1/4)), x] - Simp[a/2   Int[x^2/(a + b*x^4)^(5/4), x], x] /; FreeQ[{a, b}
, x] && PosQ[b/a]
```

rule 858
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

rule 1138
```
Int[((d_) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Sy
mbol] :> Simp[d^m*((a + b*x + c*x^2)^FracPart[p]/((1 + e*(x/d))^FracPart[p]
*(a/d + (c*x)/e)^FracPart[p]))   Int[(1 + e*(x/d))^(m + p)*(a/d + (c/e)*x)^
p, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0]
&& (IntegerQ[m] || GtQ[d, 0]) &&  !(IGtQ[m, 0] && (IntegerQ[3*p] || Integer
Q[4*p]))
```

Maple [F]

$$\int \frac{1}{(ex+d)\left(ade+\left(a\,e^{2}+c\,d^{2}\right)x+cd\,x^{2}e\right)^{\frac{5}{4}}}dx$$

input
```
int(1/(e*x+d)/(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(5/4),x)
```

output
```
int(1/(e*x+d)/(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(5/4),x)
```

Fricas [F]

$$\int \frac{1}{(d+ex)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/4}}\,dx=\int \frac{1}{\left(cdex^2+ade+\left(cd^2+ae^2\right)x\right)^{\frac{5}{4}}(ex+d)}\,dx$$

input
```
integrate(1/(e*x+d)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/4),x, algorithm="
fricas")
```

output
```
integral((c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^(3/4)/(c^2*d^2*e^3*x^5 +
a^2*d^3*e^2 + (3*c^2*d^3*e^2 + 2*a*c*d*e^4)*x^4 + (3*c^2*d^4*e + 6*a*c*d^2
*e^3 + a^2*e^5)*x^3 + (c^2*d^5 + 6*a*c*d^3*e^2 + 3*a^2*d*e^4)*x^2 + (2*a*c
*d^4*e + 3*a^2*d^2*e^3)*x), x)
```

Sympy [F]

$$\int \frac{1}{(d+ex)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/4}}\,dx=\int \frac{1}{\left((d+ex)\left(ae+cdx\right)\right)^{\frac{5}{4}}(d+ex)}\,dx$$

input
```
integrate(1/(e*x+d)/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(5/4),x)
```

output
```
Integral(1/(((d + e*x)*(a*e + c*d*x))**(5/4)*(d + e*x)), x)
```

Maxima [F]

$$\int \frac{1}{(d+ex)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/4}}\,dx=\int \frac{1}{\left(cdex^2+ade+\left(cd^2+ae^2\right)x\right)^{\frac{5}{4}}(ex+d)}\,dx$$

input
```
integrate(1/(e*x+d)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/4),x, algorithm="
maxima")
```

output
```
integrate(1/((c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^(5/4)*(e*x + d)), x)
```

Giac [F]

$$\int \frac{1}{(d+ex)\left(ade+(cd^2+ae^2)x+cdex^2\right)^{5/4}}\,dx=\int \frac{1}{(cdex^2+ade+(cd^2+ae^2)x)^{\frac{5}{4}}(ex+d)}\,dx$$

input
```
integrate(1/(e*x+d)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/4),x, algorithm="
giac")
```

output
```
integrate(1/((c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^(5/4)*(e*x + d)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(d+ex)\left(ade+(cd^2+ae^2)x+cdex^2\right)^{5/4}}\,dx=\int \frac{1}{(d+e\,x)\,\left(c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e\right)^{5/4}}\,dx$$

input
```
int(1/((d + e*x)*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(5/4)),x)
```

output
```
int(1/((d + e*x)*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(5/4)), x)
```

Reduce [F]

$$\int \frac{1}{(d+ex)\left(ade+(cd^2+ae^2)x+cdex^2\right)^{5/4}}\,dx=\int \frac{}{(cde\,x^2+a\,e^2x+c\,d^2x+ade)^{\frac{1}{4}}\,a\,d^2e+2\,(cde\,x^2+a$$

input
```
int(1/(e*x+d)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/4),x)
```

output
```
int(1/((a*d*e + a*e**2*x + c*d**2*x + c*d*e*x**2)**(1/4)*a*d**2*e + 2*(a*d
*e + a*e**2*x + c*d**2*x + c*d*e*x**2)**(1/4)*a*d*e**2*x + (a*d*e + a*e**2
*x + c*d**2*x + c*d*e*x**2)**(1/4)*a*e**3*x**2 + (a*d*e + a*e**2*x + c*d**
2*x + c*d*e*x**2)**(1/4)*c*d**3*x + 2*(a*d*e + a*e**2*x + c*d**2*x + c*d*e
*x**2)**(1/4)*c*d**2*e*x**2 + (a*d*e + a*e**2*x + c*d**2*x + c*d*e*x**2)**
(1/4)*c*d*e**2*x**3),x)
```

3.381 $\int \frac{1}{(d+ex)^2\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/4}}\,dx$

Optimal result

Integrand size = 37, antiderivative size = 303

$$\int \frac{1}{(d+ex)^2\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/4}}\,dx = -\frac{4}{\left(cd^2-ae^2\right)(d+ex)^2\sqrt[4]{ade+\left(cd^2+ae^2\right)x+cdex^2}} - \frac{40e\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/4}}{9\left(cd^2-ae^2\right)^2(d+ex)^3} - \frac{16cde\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/4}}{3\left(cd^2-ae^2\right)^3(d+ex)^2} + \frac{32c^2d^2\sqrt{e}\sqrt{ae+cdx}\sqrt[4]{\frac{cd(d+ex)}{e(ae+cdx)}}E\left(\frac{1}{2}\arctan\left(\frac{\sqrt{cd^2-ae^2}}{\sqrt{e}\sqrt{ae+cdx}}\right)\middle|2\right)}{3\left(cd^2-ae^2\right)^{7/2}\sqrt[4]{ade+\left(cd^2+ae^2\right)x+cdex^2}}$$

output
```
-4/(-a*e^2+c*d^2)/(e*x+d)^2/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/4)-40/9*e
*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/4)/(-a*e^2+c*d^2)^2/(e*x+d)^3-16/3*c
*d*e*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/4)/(-a*e^2+c*d^2)^3/(e*x+d)^2+32
/3*c^2*d^2*e^(1/2)*(c*d*x+a*e)^(1/2)*(c*d*(e*x+d)/e/(c*d*x+a*e))^(1/4)*Ell
ipticE(sin(1/2*arctan((-a*e^2+c*d^2)^(1/2)/e^(1/2)/(c*d*x+a*e)^(1/2))),2^(
1/2))/(-a*e^2+c*d^2)^(7/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/4)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.05 (sec) , antiderivative size = 100, normalized size of antiderivative = 0.33

$$\int \frac{1}{(d+ex)^2 (ade+(cd^2+ae^2)x+cdex^2)^{5/4}}\,dx = -\frac{4c^2d^2\sqrt[4]{\frac{cd(d+ex)}{cd^2-ae^2}}\,\text{Hypergeometric2F1}\left(-\frac{1}{4},\frac{13}{4},\frac{3}{4},\frac{e(ae+cdx)}{-cd^2+ae^2}\right)}{(cd^2-ae^2)^3\sqrt[4]{(ae+cdx)(d+ex)}}$$

input
```
Integrate[1/((d + e*x)^2*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/4)),x]
```

output
```
(-4*c^2*d^2*((c*d*(d + e*x))/(c*d^2 - a*e^2))^(1/4)*Hypergeometric2F1[-1/4
, 13/4, 3/4, (e*(a*e + c*d*x))/(-(c*d^2) + a*e^2)])/((c*d^2 - a*e^2)^3*((a
*e + c*d*x)*(d + e*x))^(1/4))
```

Rubi [A] (warning: unable to verify)

Time = 0.64 (sec) , antiderivative size = 518, normalized size of antiderivative = 1.71, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.297$, Rules used = {1138, 61, 61, 61, 61, 73, 839, 813, 858, 807, 212}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(d+ex)^2 (x(ae^2+cd^2)+ade+cdex^2)^{5/4}}dx$$

$$\Big\downarrow \text{1138}$$

$$\frac{\sqrt[4]{\frac{ex}{d}+1}\sqrt[4]{ae+cdx}\int \frac{1}{(ae+cdx)^{5/4}\left(\frac{ex}{d}+1\right)^{13/4}}dx}{d^2\sqrt[4]{x(ae^2+cd^2)+ade+cdex^2}}$$

$$\Big\downarrow \text{61}$$

$$\frac{\sqrt[4]{\frac{ex}{d}+1}\sqrt[4]{ae+cdx}\left(-\frac{10e\int \frac{1}{\sqrt[4]{ae+cdx}\left(\frac{ex}{d}+1\right)^{13/4}}dx}{cd^2-ae^2}-\frac{4}{\left(\frac{ex}{d}+1\right)^{9/4}\left(cd-\frac{ae^2}{d}\right)\sqrt[4]{ae+cdx}}\right)}{d^2\sqrt[4]{x\left(ae^2+cd^2\right)+ade+cdex^2}}$$

$$\Big\downarrow \ 61$$

$$\frac{\sqrt[4]{\frac{ex}{d}+1}\sqrt[4]{ae+cdx}\left(-\frac{10e\left(\frac{2cd\int \frac{1}{\sqrt[4]{ae+cdx}\left(\frac{ex}{d}+1\right)^{9/4}}dx}{3\left(cd-\frac{ae^2}{d}\right)}+\frac{4(ae+cdx)^{3/4}}{9\left(\frac{ex}{d}+1\right)^{9/4}\left(cd-\frac{ae^2}{d}\right)}\right)}{cd^2-ae^2}-\frac{4}{\left(\frac{ex}{d}+1\right)^{9/4}\left(cd-\frac{ae^2}{d}\right)\sqrt[4]{ae+cdx}}\right)}{d^2\sqrt[4]{x\left(ae^2+cd^2\right)+ade+cdex^2}}$$

$$\Big\downarrow \ 61$$

$$\frac{\sqrt[4]{\frac{ex}{d}+1}\sqrt[4]{ae+cdx}\left(-\frac{10e\left(\frac{2cd\left(\frac{2cd\int \frac{1}{\sqrt[4]{ae+cdx}\left(\frac{ex}{d}+1\right)^{5/4}}dx}{5\left(cd-\frac{ae^2}{d}\right)}+\frac{4(ae+cdx)^{3/4}}{5\left(\frac{ex}{d}+1\right)^{5/4}\left(cd-\frac{ae^2}{d}\right)}\right)}{3\left(cd-\frac{ae^2}{d}\right)}+\frac{4(ae+cdx)^{3/4}}{9\left(\frac{ex}{d}+1\right)^{9/4}\left(cd-\frac{ae^2}{d}\right)}\right)}{cd^2-ae^2}-\frac{\ldots}{\left(\frac{ex}{d}+1\right)^{9/4}\ldots}\right.}{d^2\sqrt[4]{x\left(ae^2+cd^2\right)+ade+cdex^2}}$$

$$\Big\downarrow \ 61$$

$$\frac{\sqrt[4]{\frac{ex}{d}+1}\sqrt[4]{ae+cdx}\left(-\frac{10e\left(\frac{2cd\left(\frac{2cd\left(\frac{4d(ae+cdx)^{3/4}}{\sqrt[4]{\frac{ex}{d}+1}(cd^2-ae^2)}-\frac{2cd^2\int\frac{1}{\sqrt[4]{ae+cdx}\sqrt[4]{\frac{ex}{d}+1}}dx}{cd^2-ae^2}\right)}{5\left(cd-\frac{ae^2}{d}\right)}+\frac{4(ae+cdx)^{3/4}}{5\left(\frac{ex}{d}+1\right)^{5/4}\left(cd-\frac{ae^2}{d}\right)}\right)}{3\left(cd-\frac{ae^2}{d}\right)}+\frac{\cdots}{9\left(\frac{ex}{d}\cdots\right.}\cdots\right)}{cd^2-ae^2}\cdots\right.}{d^2\sqrt[4]{x\left(ae^2+cd^2\right)+ade+cdex^2}}$$

$\downarrow$ 73

$$\frac{\sqrt[4]{\frac{ex}{d}+1}\sqrt[4]{ae+cdx}\left(-\frac{10e\left(\frac{2cd\left(\frac{2cd\left(\frac{4d(ae+cdx)^{3/4}}{\sqrt[4]{\frac{ex}{d}+1}(cd^2-ae^2)}-\frac{8d\int\frac{\sqrt{ae+cdx}}{\sqrt[4]{-\frac{ae^2}{cd^2}+\frac{(ae+cdx)e}{cd^2}+1}}d\sqrt[4]{ae+cdx}}{cd^2-ae^2}\right)}{5\left(cd-\frac{ae^2}{d}\right)}+\frac{4(ae+cd\ldots}{5\left(\frac{ex}{d}+1\right)^{5/4}\ldots}}{3\left(cd-\frac{ae^2}{d}\right)}\ldots}{cd^2-ae^2}\ldots\right)}{d^2\sqrt[4]{x\left(ae^2+cd^2\right)+ade+cdex^2}}$$

$\downarrow$ 839

$$\sqrt[4]{\frac{ex}{d}+1}\sqrt[4]{ae+cdx}\left(-\frac{10e\left(\frac{2cd\left(\frac{2cd\left(\frac{4d(ae+cdx)^{3/4}}{\sqrt[4]{\frac{ex}{d}+1}\left(cd^2-ae^2\right)}-\frac{8d\left(\frac{(ae+cdx)^{3/4}}{2\sqrt[4]{-\frac{ae^2}{cd^2}+\frac{e(ae+cdx)}{cd^2}+1}}-\frac{1}{2}\left(1-\frac{ae^2}{cd^2}\right)\int\frac{\sqrt{ae+cd\cdots}}{\left(-\frac{ae^2}{cd^2}+\frac{(ae+cd\cdots}{cd^2}\cdots\right)}\cdots\right)}{cd^2-ae^2}\cdots\right)}{5\left(cd-\frac{ae^2}{d}\right)}\cdots\right)}{3\left(cd-\frac{ae^2}{d}\right)}\cdots\right)}{cd^2-ae^2}\cdots\right)$$

↓ 813

$$\sqrt[4]{\frac{ex}{d}+1}\sqrt[4]{ae+cdx}\left(-\frac{10e\left(\frac{2cd\left(\frac{2cd\left(\frac{4d(ae+cdx)^{3/4}}{\sqrt[4]{\frac{ex}{d}+1}\left(cd^2-ae^2\right)}-\frac{8d\left(\frac{(ae+cdx)^{3/4}}{2\sqrt[4]{-\frac{ae^2}{cd^2}+\frac{e(ae+cdx)}{cd^2}+1}}-\frac{cd^2\left(1-\frac{ae^2}{cd^2}\right)\sqrt[4]{ae+cdx}\,\sqrt[4]{\cdots}}{\cdots}\right)}{cd^2\cdots}\right)}{5\left(cd-\frac{ae^2}{d}\right)\cdots}\cdots\right)}{3\left(cd-\frac{ae^2}{d}\right)\cdots}\cdots\right)}{\cdots}\cdots\right.$$

↓ 858

$$10e\left(2cd\left(2cd\left(\frac{4d(ae+cdx)^{3/4}}{\sqrt[4]{\frac{ex}{d}+1}\left(cd^2-ae^2\right)}-\frac{8d\left(\frac{cd^2\left(1-\frac{ae^2}{cd^2}\right)\sqrt[4]{ae+cdx}\sqrt[4]{\frac{cd^2-ae^2}{e(ae+cdx)}+1}\int\frac{}{\sqrt[4]{ae+cdx}}}{2e\sqrt[4]{-\frac{ae^2}{cd^2}+\frac{e(ae+cdx)}{cd^2}}}\right)}{}\right)}{5\left(cd-\frac{ae^2}{d}\right)}\right)}{3(}\right)$$

$$\sqrt[4]{\frac{ex}{d}+1}\sqrt[4]{}$$

↓ 807

$$10e\left(\frac{2cd\left(\frac{2cd\left(\frac{4d(ae+cdx)^{3/4}}{\sqrt[4]{\frac{ex}{d}+1}\left(cd^2-ae^2\right)}-\frac{8d\left(\frac{cd^2\left(1-\frac{ae^2}{cd^2}\right)\sqrt[4]{ae+cdx}\sqrt[4]{\frac{cd^2-ae^2}{e(ae+cdx)}+1}\int\frac{}{\left(\frac{\sqrt{ae+cdx}\left(cd^2\cdots\right)}{e}\cdots\right)}}{4e\sqrt[4]{-\frac{ae^2}{cd^2}+\frac{e(ae+cdx)}{cd^2}+1}}\cdots\right)}{cd^2-ae^2\cdots}\right)}{5\left(cd-\frac{ae^2}{d}\right)\cdots}\right)}{3\left(cd-\frac{ae^2}{d}\right)\cdots}\cdots\right)$$

$$\sqrt[4]{\frac{ex}{d}+1}\sqrt[4]{\cdots}$$

↓ 212

$$\sqrt[4]{\frac{ex}{d}+1}\sqrt[4]{ae+cdx}\left(-\frac{10e\left(\frac{2cd\left(\frac{2cd\left(\frac{4d(ae+cdx)^{3/4}}{\sqrt[4]{\frac{ex}{d}+1}\left(cd^2-ae^2\right)}-\frac{8d\left(\frac{cd^2\left(1-\frac{ae^2}{cd^2}\right)\sqrt[4]{ae+cdx}\sqrt[4]{\frac{cd^2-ae^2}{e(ae+cdx)}+1}E\left(\frac{1}{2}\arctan\left(\frac{\sqrt{c}\ldots}{\ldots}\right)\ldots\right.}{2\sqrt{e}\sqrt{cd^2-ae^2}\sqrt[4]{-\frac{ae^2}{cd^2}+\frac{e(ae+cdx)}{cd^2}+1}\ldots}\ldots\right)}{cd^2-ae^2\ldots}\right)}{5\left(cd-\frac{ae^2}{d}\right)\ldots}\ldots\right)}{3\left(cd-\frac{ae^2}{d}\right)\ldots}\ldots\right)}{cd\ldots}\ldots\right.$$

input
```
Int[1/((d + e*x)^2*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/4)),x]
```

output
```
((a*e + c*d*x)^(1/4)*(1 + (e*x)/d)^(1/4)*(-4/((c*d - (a*e^2)/d)*(a*e + c*d
*x)^(1/4)*(1 + (e*x)/d)^(9/4)) - (10*e*((4*(a*e + c*d*x)^(3/4))/(9*(c*d -
(a*e^2)/d)*(1 + (e*x)/d)^(9/4)) + (2*c*d*((4*(a*e + c*d*x)^(3/4))/(5*(c*d
- (a*e^2)/d)*(1 + (e*x)/d)^(5/4)) + (2*c*d*((4*d*(a*e + c*d*x)^(3/4))/((c*
d^2 - a*e^2)*(1 + (e*x)/d)^(1/4)) - (8*d*((a*e + c*d*x)^(3/4)/(2*(1 - (a*e
^2)/(c*d^2) + (e*(a*e + c*d*x))/(c*d^2))^(1/4)) + (c*d^2*(1 - (a*e^2)/(c*d
^2))*(a*e + c*d*x)^(1/4)*(1 + (c*d^2 - a*e^2)/(e*(a*e + c*d*x)))^(1/4)*Ell
ipticE[ArcTan[(Sqrt[c*d^2 - a*e^2]*Sqrt[a*e + c*d*x])/Sqrt[e]]/2, 2])/(2*S
qrt[e]*Sqrt[c*d^2 - a*e^2]*(1 - (a*e^2)/(c*d^2) + (e*(a*e + c*d*x))/(c*d^2
))^(1/4))))/(c*d^2 - a*e^2)))/(5*(c*d - (a*e^2)/d))))/(3*(c*d - (a*e^2)/d)
)))/(c*d^2 - a*e^2)))/(d^2*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(1/4))
```

Defintions of rubi rules used

rule 61
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && LtQ[m, -1] &&  !(LtQ[n, -1] && (EqQ[a, 0
] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && IntLinearQ[a, b, c, d
, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 212
```
Int[((a_) + (b_.)*(x_)^2)^(-5/4), x_Symbol] :> Simp[(2/(a^(5/4)*Rt[b/a, 2])
)*EllipticE[(1/2)*ArcTan[Rt[b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

rule 807
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Simp[1/k   Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x,
x^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

rule 813
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4)^(5/4), x_Symbol] :> Simp[x*((1 + a/(b*x^4)
)^(1/4)/(b*(a + b*x^4)^(1/4)))   Int[1/(x^3*(1 + a/(b*x^4))^(5/4)), x], x]
/; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 839
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4)^(1/4), x_Symbol] :> Simp[x^3/(2*(a + b*x^4
)^(1/4)), x] - Simp[a/2   Int[x^2/(a + b*x^4)^(5/4), x], x] /; FreeQ[{a, b}
, x] && PosQ[b/a]
```

rule 858
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

rule 1138
```
Int[((d_) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Sy
mbol] :> Simp[d^m*((a + b*x + c*x^2)^FracPart[p]/((1 + e*(x/d))^FracPart[p]
*(a/d + (c*x)/e)^FracPart[p]))   Int[(1 + e*(x/d))^(m + p)*(a/d + (c/e)*x)^
p, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0]
&& (IntegerQ[m] || GtQ[d, 0]) &&  !(IGtQ[m, 0] && (IntegerQ[3*p] || Integer
Q[4*p]))
```

Maple [F]

$$\int \frac{1}{(ex+d)^2\,(ade+(a\,e^2+c\,d^2)\,x+cd\,x^2e)^{\frac{5}{4}}}dx$$

input
```
int(1/(e*x+d)^2/(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(5/4),x)
```

output
```
int(1/(e*x+d)^2/(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(5/4),x)
```

Fricas [F]

$$\int \frac{1}{(d+ex)^2 \left(ade+(cd^2+ae^2)x+cdex^2\right)^{5/4}}\,dx = \int \frac{1}{(cdex^2+ade+(cd^2+ae^2)x)^{\frac{5}{4}}(ex+d)^2}\,dx$$

input
```
integrate(1/(e*x+d)^2/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/4),x, algorithm
="fricas")
```

output
```
integral((c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^(3/4)/(c^2*d^2*e^4*x^6 +
a^2*d^4*e^2 + 2*(2*c^2*d^3*e^3 + a*c*d*e^5)*x^5 + (6*c^2*d^4*e^2 + 8*a*c*d
^2*e^4 + a^2*e^6)*x^4 + 4*(c^2*d^5*e + 3*a*c*d^3*e^3 + a^2*d*e^5)*x^3 + (c
^2*d^6 + 8*a*c*d^4*e^2 + 6*a^2*d^2*e^4)*x^2 + 2*(a*c*d^5*e + 2*a^2*d^3*e^3
)*x), x)
```

Sympy [F]

$$\int \frac{1}{(d+ex)^2 \left(ade+(cd^2+ae^2)x+cdex^2\right)^{5/4}}\,dx = \int \frac{1}{\left((d+ex)(ae+cdx)\right)^{\frac{5}{4}}(d+ex)^2}\,dx$$

input
```
integrate(1/(e*x+d)**2/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(5/4),x)
```

output
```
Integral(1/(((d + e*x)*(a*e + c*d*x))**(5/4)*(d + e*x)**2), x)
```

Maxima [F]

$$\int \frac{1}{(d+ex)^2 \left(ade+(cd^2+ae^2)x+cdex^2\right)^{5/4}}\,dx = \int \frac{1}{(cdex^2+ade+(cd^2+ae^2)x)^{\frac{5}{4}}(ex+d)^2}\,dx$$

input
```
integrate(1/(e*x+d)^2/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/4),x, algorithm
="maxima")
```

output
```
integrate(1/((c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^(5/4)*(e*x + d)^2), x
)
```

Giac [F]

$$\int \frac{1}{(d+ex)^2 \left(ade+(cd^2+ae^2)\,x+cdex^2\right)^{5/4}}\,dx = \int \frac{1}{(cdex^2+ade+(cd^2+ae^2)x)^{\frac{5}{4}}(ex+d)^2}\,dx$$

input
```
integrate(1/(e*x+d)^2/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/4),x, algorithm
="giac")
```

output
```
integrate(1/((c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^(5/4)*(e*x + d)^2), x
)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(d+ex)^2 \left(ade+(cd^2+ae^2)\,x+cdex^2\right)^{5/4}}\,dx = \int \frac{1}{(d+e\,x)^2\,\left(c\,d\,e\,x^2+(c\,d^2+a\,e^2)\,x+a\,d\,e\right)^{5/4}}\,dx$$

input
```
int(1/((d + e*x)^2*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(5/4)),x)
```

output
```
int(1/((d + e*x)^2*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(5/4)), x)
```

Reduce [F]

$$\int \frac{1}{(d+ex)^2 \left(ade+(cd^2+ae^2)\,x+cdex^2\right)^{5/4}}\,dx = \int \frac{}{(cde\,x^2+a\,e^2x+c\,d^2x+ade)^{\frac{1}{4}}\,a\,d^3e+3\,(cde\,x^2+c}$$

input
```
int(1/(e*x+d)^2/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/4),x)
```

output
```
int(1/((a*d*e + a*e**2*x + c*d**2*x + c*d*e*x**2)**(1/4)*a*d**3*e + 3*(a*d
*e + a*e**2*x + c*d**2*x + c*d*e*x**2)**(1/4)*a*d**2*e**2*x + 3*(a*d*e + a
*e**2*x + c*d**2*x + c*d*e*x**2)**(1/4)*a*d*e**3*x**2 + (a*d*e + a*e**2*x
+ c*d**2*x + c*d*e*x**2)**(1/4)*a*e**4*x**3 + (a*d*e + a*e**2*x + c*d**2*x
 + c*d*e*x**2)**(1/4)*c*d**4*x + 3*(a*d*e + a*e**2*x + c*d**2*x + c*d*e*x*
*2)**(1/4)*c*d**3*e*x**2 + 3*(a*d*e + a*e**2*x + c*d**2*x + c*d*e*x**2)**(
1/4)*c*d**2*e**2*x**3 + (a*d*e + a*e**2*x + c*d**2*x + c*d*e*x**2)**(1/4)*
c*d*e**3*x**4),x)
```

3.382 $\int (d+ex)^m \left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^3 dx$

Optimal result

Integrand size = 35, antiderivative size = 130

$$\begin{aligned}\int (d+ex)^m \left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^3 dx &= -\frac{\left(cd^2-ae^2\right)^3 (d+ex)^{4+m}}{e^4(4+m)}+\frac{3cd\left(cd^2-ae^2\right)^2 (d+ex)^{5+m}}{e^4(5+m)}\\ &\quad -\frac{3c^2d^2\left(cd^2-ae^2\right)(d+ex)^{6+m}}{e^4(6+m)}+\frac{c^3d^3(d+ex)^{7+m}}{e^4(7+m)}\end{aligned}$$

output
```
-(-a*e^2+c*d^2)^3*(e*x+d)^(4+m)/e^4/(4+m)+3*c*d*(-a*e^2+c*d^2)^2*(e*x+d)^(
5+m)/e^4/(5+m)-3*c^2*d^2*(-a*e^2+c*d^2)*(e*x+d)^(6+m)/e^4/(6+m)+c^3*d^3*(e
*x+d)^(7+m)/e^4/(7+m)
```

Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 114, normalized size of antiderivative = 0.88

$$\int (d+ex)^m \left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^3 dx = \frac{(d+ex)^{4+m}\left(-\frac{\left(cd^2-ae^2\right)^3}{4+m}+\frac{3cd\left(cd^2-ae^2\right)^2(d+ex)}{5+m}-\frac{3c^2d^2\left(cd^2-ae^2\right)(d+ex)^2}{6+m}+\frac{c^3d^3(d+ex)^3}{7+m}\right)}{e^4}$$

input
```
Integrate[(d + e*x)^m*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^3,x]
```

output
```
((d + e*x)^(4 + m)*(-((c*d^2 - a*e^2)^3/(4 + m)) + (3*c*d*(c*d^2 - a*e^2)^
2*(d + e*x))/(5 + m) - (3*c^2*d^2*(c*d^2 - a*e^2)*(d + e*x)^2)/(6 + m) + (
c^3*d^3*(d + e*x)^3)/(7 + m)))/e^4
```

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 130, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.057$, Rules used = {1121, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (d+ex)^m \left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^3 dx$$

$$\Big\downarrow \text{1121}$$

$$\int \left(-\frac{3c^2d^2\left(cd^2-ae^2\right)(d+ex)^{m+5}}{e^3}+\frac{\left(ae^2-cd^2\right)^3(d+ex)^{m+3}}{e^3}+\frac{3cd\left(cd^2-ae^2\right)^2(d+ex)^{m+4}}{e^3}+\frac{c^3d^3(d+e}{e^3}\right.$$

$$\Big\downarrow \text{2009}$$

$$-\frac{3c^2d^2\left(cd^2-ae^2\right)(d+ex)^{m+6}}{e^4(m+6)}-\frac{\left(cd^2-ae^2\right)^3(d+ex)^{m+4}}{e^4(m+4)}+\frac{3cd\left(cd^2-ae^2\right)^2(d+ex)^{m+5}}{e^4(m+5)}+\frac{c^3d^3(d+ex)^{m+7}}{e^4(m+7)}$$

input
```
Int[(d + e*x)^m*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^3,x]
```

output
```
-(((c*d^2 - a*e^2)^3*(d + e*x)^(4 + m))/(e^4*(4 + m))) + (3*c*d*(c*d^2 - a
*e^2)^2*(d + e*x)^(5 + m))/(e^4*(5 + m)) - (3*c^2*d^2*(c*d^2 - a*e^2)*(d +
 e*x)^(6 + m))/(e^4*(6 + m)) + (c^3*d^3*(d + e*x)^(7 + m))/(e^4*(7 + m))
```

Defintions of rubi rules used

rule 1121
```
Int[((d_) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Int[ExpandIntegrand[(d + e*x)^(m + p)*(a/d + (c/e)*x)^p, x], x]
/; FreeQ[{a, b, c, d, e, m, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && (Int
egerQ[p] || (GtQ[a, 0] && GtQ[d, 0] && LtQ[c, 0]))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 435 vs. $2(130) = 260$.

Time = 1.08 (sec) , antiderivative size = 436, normalized size of antiderivative = 3.35

method	result
gosper	$\frac{(ex+d)^{4+m}(c^3d^3e^3m^3x^3+3a\,c^2d^2e^4m^3x^2+15c^3d^3e^3m^2x^3+3a^2cd\,e^5m^3x+48a\,c^2d^2e^4m^2x^2-3c^3d^4e^2m^2x^2+74c^3d^3e^3m\,x^3}{}$
orering	$\frac{(c^3d^3e^3m^3x^3+3a\,c^2d^2e^4m^3x^2+15c^3d^3e^3m^2x^3+3a^2cd\,e^5m^3x+48a\,c^2d^2e^4m^2x^2-3c^3d^4e^2m^2x^2+74c^3d^3e^3m\,x^3+a^3e^6m^3+}{}$
norman	Expression too large to display
risch	Expression too large to display
parallelrisch	Expression too large to display

input
```
int((e*x+d)^m*(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^3,x,method=_RETURNVERBOSE)
```

output
```
1/e^4*(e*x+d)^(4+m)/(m^4+22*m^3+179*m^2+638*m+840)*(c^3*d^3*e^3*m^3*x^3+3*
a*c^2*d^2*e^4*m^3*x^2+15*c^3*d^3*e^3*m^2*x^3+3*a^2*c*d*e^5*m^3*x+48*a*c^2*
d^2*e^4*m^2*x^2-3*c^3*d^4*e^2*m^2*x^2+74*c^3*d^3*e^3*m*x^3+a^3*e^6*m^3+51*
a^2*c*d*e^5*m^2*x-6*a*c^2*d^3*e^3*m^2*x+249*a*c^2*d^2*e^4*m*x^2-27*c^3*d^4
*e^2*m*x^2+120*c^3*d^3*e^3*x^3+18*a^3*e^6*m^2-3*a^2*c*d^2*e^4*m^2+282*a^2*
c*d*e^5*m*x-66*a*c^2*d^3*e^3*m*x+420*a*c^2*d^2*e^4*x^2+6*c^3*d^5*e*m*x-60*
c^3*d^4*e^2*x^2+107*a^3*e^6*m-39*a^2*c*d^2*e^4*m+504*a^2*c*d*e^5*x+6*a*c^2
*d^4*e^2*m-168*a*c^2*d^3*e^3*x+24*c^3*d^5*e*x+210*a^3*e^6-126*a^2*c*d^2*e^
4+42*a*c^2*d^4*e^2-6*c^3*d^6)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1156 vs. $2(130) = 260$.

Time = 0.12 (sec) , antiderivative size = 1156, normalized size of antiderivative = 8.89

$$\int (d+ex)^m \left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^3 dx = \text{Too large to display}$$

input
```
integrate((e*x+d)^m*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^3,x, algorithm="fric
as")
```

output
```
(a^3*d^4*e^6*m^3 - 6*c^3*d^10 + 42*a*c^2*d^8*e^2 - 126*a^2*c*d^6*e^4 + 210
*a^3*d^4*e^6 + (c^3*d^3*e^7*m^3 + 15*c^3*d^3*e^7*m^2 + 74*c^3*d^3*e^7*m +
120*c^3*d^3*e^7)*x^7 + (420*c^3*d^4*e^6 + 420*a*c^2*d^2*e^8 + (4*c^3*d^4*e
^6 + 3*a*c^2*d^2*e^8)*m^3 + 3*(19*c^3*d^4*e^6 + 16*a*c^2*d^2*e^8)*m^2 + (2
69*c^3*d^4*e^6 + 249*a*c^2*d^2*e^8)*m)*x^6 + 3*(168*c^3*d^5*e^5 + 504*a*c^
2*d^3*e^7 + 168*a^2*c*d*e^9 + (2*c^3*d^5*e^5 + 4*a*c^2*d^3*e^7 + a^2*c*d*e
^9)*m^3 + (26*c^3*d^5*e^5 + 62*a*c^2*d^3*e^7 + 17*a^2*c*d*e^9)*m^2 + 2*(57
*c^3*d^5*e^5 + 155*a*c^2*d^3*e^7 + 47*a^2*c*d*e^9)*m)*x^5 + (210*c^3*d^6*e
^4 + 1890*a*c^2*d^4*e^6 + 1890*a^2*c*d^2*e^8 + 210*a^3*e^10 + (4*c^3*d^6*e
^4 + 18*a*c^2*d^4*e^6 + 12*a^2*c*d^2*e^8 + a^3*e^10)*m^3 + 3*(14*c^3*d^6*e
^4 + 88*a*c^2*d^4*e^6 + 67*a^2*c*d^2*e^8 + 6*a^3*e^10)*m^2 + (158*c^3*d^6*
e^4 + 1236*a*c^2*d^4*e^6 + 1089*a^2*c*d^2*e^8 + 107*a^3*e^10)*m)*x^4 + (84
0*a*c^2*d^5*e^5 + 2520*a^2*c*d^3*e^7 + 840*a^3*d*e^9 + (c^3*d^7*e^3 + 12*a
*c^2*d^5*e^5 + 18*a^2*c*d^3*e^7 + 4*a^3*d*e^9)*m^3 + 3*(c^3*d^7*e^3 + 52*a
*c^2*d^5*e^5 + 98*a^2*c*d^3*e^7 + 24*a^3*d*e^9)*m^2 + 2*(c^3*d^7*e^3 + 312
*a*c^2*d^5*e^5 + 768*a^2*c*d^3*e^7 + 214*a^3*d*e^9)*m)*x^3 - 3*(a^2*c*d^6*
e^4 - 6*a^3*d^4*e^6)*m^2 + 3*(420*a^2*c*d^4*e^6 + 420*a^3*d^2*e^8 + (a*c^2
*d^6*e^4 + 4*a^2*c*d^4*e^6 + 2*a^3*d^2*e^8)*m^3 - (c^3*d^8*e^2 - 8*a*c^2*d
^6*e^4 - 62*a^2*c*d^4*e^6 - 36*a^3*d^2*e^8)*m^2 - (c^3*d^8*e^2 - 7*a*c^2*d
^6*e^4 - 298*a^2*c*d^4*e^6 - 214*a^3*d^2*e^8)*m)*x^2 + (6*a*c^2*d^8*e^2...
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 7164 vs. $2(114) = 228$.

Time = 2.43 (sec) , antiderivative size = 7164, normalized size of antiderivative = 55.11

$$\int (d+ex)^m \left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^3 dx = \text{Too large to display}$$

input
```
integrate((e*x+d)**m*(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**3,x)
```

output
```
Piecewise((c**3*d**6*d**m*x**4/4, Eq(e, 0)), (-2*a**3*e**6/(6*d**3*e**4 +
18*d**2*e**5*x + 18*d*e**6*x**2 + 6*e**7*x**3) - 3*a**2*c*d**2*e**4/(6*d**
3*e**4 + 18*d**2*e**5*x + 18*d*e**6*x**2 + 6*e**7*x**3) - 9*a**2*c*d*e**5*
x/(6*d**3*e**4 + 18*d**2*e**5*x + 18*d*e**6*x**2 + 6*e**7*x**3) - 6*a*c**2
*d**4*e**2/(6*d**3*e**4 + 18*d**2*e**5*x + 18*d*e**6*x**2 + 6*e**7*x**3) -
 18*a*c**2*d**3*e**3*x/(6*d**3*e**4 + 18*d**2*e**5*x + 18*d*e**6*x**2 + 6*
e**7*x**3) - 18*a*c**2*d**2*e**4*x**2/(6*d**3*e**4 + 18*d**2*e**5*x + 18*d
*e**6*x**2 + 6*e**7*x**3) + 6*c**3*d**6*log(d/e + x)/(6*d**3*e**4 + 18*d**
2*e**5*x + 18*d*e**6*x**2 + 6*e**7*x**3) + 11*c**3*d**6/(6*d**3*e**4 + 18*
d**2*e**5*x + 18*d*e**6*x**2 + 6*e**7*x**3) + 18*c**3*d**5*e*x*log(d/e + x
)/(6*d**3*e**4 + 18*d**2*e**5*x + 18*d*e**6*x**2 + 6*e**7*x**3) + 27*c**3*
d**5*e*x/(6*d**3*e**4 + 18*d**2*e**5*x + 18*d*e**6*x**2 + 6*e**7*x**3) + 1
8*c**3*d**4*e**2*x**2*log(d/e + x)/(6*d**3*e**4 + 18*d**2*e**5*x + 18*d*e*
*6*x**2 + 6*e**7*x**3) + 18*c**3*d**4*e**2*x**2/(6*d**3*e**4 + 18*d**2*e**
5*x + 18*d*e**6*x**2 + 6*e**7*x**3) + 6*c**3*d**3*e**3*x**3*log(d/e + x)/(
6*d**3*e**4 + 18*d**2*e**5*x + 18*d*e**6*x**2 + 6*e**7*x**3), Eq(m, -7)),
(-a**3*e**6/(2*d**2*e**4 + 4*d*e**5*x + 2*e**6*x**2) - 3*a**2*c*d**2*e**4/
(2*d**2*e**4 + 4*d*e**5*x + 2*e**6*x**2) - 6*a**2*c*d*e**5*x/(2*d**2*e**4
+ 4*d*e**5*x + 2*e**6*x**2) + 6*a*c**2*d**4*e**2*log(d/e + x)/(2*d**2*e**4
 + 4*d*e**5*x + 2*e**6*x**2) + 9*a*c**2*d**4*e**2/(2*d**2*e**4 + 4*d*e*...
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1819 vs. 2(130) = 260.

Time = 0.11 (sec) , antiderivative size = 1819, normalized size of antiderivative = 13.99

$$\int (d+ex)^m \left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^3 dx = \text{Too large to display}$$

input
```
integrate((e*x+d)^m*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^3,x, algorithm="maxi
ma")
```

output
```
3*(e^2*(m + 1)*x^2 + d*e*m*x - d^2)*(e*x + d)^m*a^2*c*d^4/(m^2 + 3*m + 2)
+ 3*(e^2*(m + 1)*x^2 + d*e*m*x - d^2)*(e*x + d)^m*a^3*d^2*e^2/(m^2 + 3*m +
 2) + (e*x + d)^(m + 1)*a^3*d^3*e^2/(m + 1) + 9*((m^2 + 3*m + 2)*e^3*x^3 +
 (m^2 + m)*d*e^2*x^2 - 2*d^2*e*m*x + 2*d^3)*(e*x + d)^m*a^2*c*d^3/(m^3 + 6
*m^2 + 11*m + 6) + 3*((m^2 + 3*m + 2)*e^3*x^3 + (m^2 + m)*d*e^2*x^2 - 2*d^
2*e*m*x + 2*d^3)*(e*x + d)^m*a*c^2*d^5/((m^3 + 6*m^2 + 11*m + 6)*e^2) + 3*
((m^2 + 3*m + 2)*e^3*x^3 + (m^2 + m)*d*e^2*x^2 - 2*d^2*e*m*x + 2*d^3)*(e*x
 + d)^m*a^3*d*e^2/(m^3 + 6*m^2 + 11*m + 6) + 9*((m^3 + 6*m^2 + 11*m + 6)*e
^4*x^4 + (m^3 + 3*m^2 + 2*m)*d*e^3*x^3 - 3*(m^2 + m)*d^2*e^2*x^2 + 6*d^3*e
*m*x - 6*d^4)*(e*x + d)^m*a^2*c*d^2/(m^4 + 10*m^3 + 35*m^2 + 50*m + 24) +
((m^3 + 6*m^2 + 11*m + 6)*e^4*x^4 + (m^3 + 3*m^2 + 2*m)*d*e^3*x^3 - 3*(m^2
 + m)*d^2*e^2*x^2 + 6*d^3*e*m*x - 6*d^4)*(e*x + d)^m*c^3*d^6/((m^4 + 10*m^
3 + 35*m^2 + 50*m + 24)*e^4) + 9*((m^3 + 6*m^2 + 11*m + 6)*e^4*x^4 + (m^3
+ 3*m^2 + 2*m)*d*e^3*x^3 - 3*(m^2 + m)*d^2*e^2*x^2 + 6*d^3*e*m*x - 6*d^4)*
(e*x + d)^m*a*c^2*d^4/((m^4 + 10*m^3 + 35*m^2 + 50*m + 24)*e^2) + ((m^3 +
6*m^2 + 11*m + 6)*e^4*x^4 + (m^3 + 3*m^2 + 2*m)*d*e^3*x^3 - 3*(m^2 + m)*d^
2*e^2*x^2 + 6*d^3*e*m*x - 6*d^4)*(e*x + d)^m*a^3*e^2/(m^4 + 10*m^3 + 35*m^
2 + 50*m + 24) + 3*((m^4 + 10*m^3 + 35*m^2 + 50*m + 24)*e^5*x^5 + (m^4 + 6
*m^3 + 11*m^2 + 6*m)*d*e^4*x^4 - 4*(m^3 + 3*m^2 + 2*m)*d^2*e^3*x^3 + 12*(m
^2 + m)*d^3*e^2*x^2 - 24*d^4*e*m*x + 24*d^5)*(e*x + d)^m*a^2*c*d/(m^5 +...
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1999 vs. $2(130) = 260$.

Time = 0.17 (sec) , antiderivative size = 1999, normalized size of antiderivative = 15.38

$$\int (d+ex)^m \left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^3 \, dx = \text{Too large to display}$$

input
```
integrate((e*x+d)^m*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^3,x, algorithm="giac
")
```

output
```
((e*x + d)^m*c^3*d^3*e^7*m^3*x^7 + 4*(e*x + d)^m*c^3*d^4*e^6*m^3*x^6 + 3*(
e*x + d)^m*a*c^2*d^2*e^8*m^3*x^6 + 15*(e*x + d)^m*c^3*d^3*e^7*m^2*x^7 + 6*
(e*x + d)^m*c^3*d^5*e^5*m^3*x^5 + 12*(e*x + d)^m*a*c^2*d^3*e^7*m^3*x^5 + 3
*(e*x + d)^m*a^2*c*d*e^9*m^3*x^5 + 57*(e*x + d)^m*c^3*d^4*e^6*m^2*x^6 + 48
*(e*x + d)^m*a*c^2*d^2*e^8*m^2*x^6 + 74*(e*x + d)^m*c^3*d^3*e^7*m*x^7 + 4*
(e*x + d)^m*c^3*d^6*e^4*m^3*x^4 + 18*(e*x + d)^m*a*c^2*d^4*e^6*m^3*x^4 + 1
2*(e*x + d)^m*a^2*c*d^2*e^8*m^3*x^4 + (e*x + d)^m*a^3*e^10*m^3*x^4 + 78*(e
*x + d)^m*c^3*d^5*e^5*m^2*x^5 + 186*(e*x + d)^m*a*c^2*d^3*e^7*m^2*x^5 + 51
*(e*x + d)^m*a^2*c*d*e^9*m^2*x^5 + 269*(e*x + d)^m*c^3*d^4*e^6*m*x^6 + 249
*(e*x + d)^m*a*c^2*d^2*e^8*m*x^6 + 120*(e*x + d)^m*c^3*d^3*e^7*x^7 + (e*x
+ d)^m*c^3*d^7*e^3*m^3*x^3 + 12*(e*x + d)^m*a*c^2*d^5*e^5*m^3*x^3 + 18*(e*
x + d)^m*a^2*c*d^3*e^7*m^3*x^3 + 4*(e*x + d)^m*a^3*d*e^9*m^3*x^3 + 42*(e*x
 + d)^m*c^3*d^6*e^4*m^2*x^4 + 264*(e*x + d)^m*a*c^2*d^4*e^6*m^2*x^4 + 201*
(e*x + d)^m*a^2*c*d^2*e^8*m^2*x^4 + 18*(e*x + d)^m*a^3*e^10*m^2*x^4 + 342*
(e*x + d)^m*c^3*d^5*e^5*m*x^5 + 930*(e*x + d)^m*a*c^2*d^3*e^7*m*x^5 + 282*
(e*x + d)^m*a^2*c*d*e^9*m*x^5 + 420*(e*x + d)^m*c^3*d^4*e^6*x^6 + 420*(e*x
 + d)^m*a*c^2*d^2*e^8*x^6 + 3*(e*x + d)^m*a*c^2*d^6*e^4*m^3*x^2 + 12*(e*x
+ d)^m*a^2*c*d^4*e^6*m^3*x^2 + 6*(e*x + d)^m*a^3*d^2*e^8*m^3*x^2 + 3*(e*x
+ d)^m*c^3*d^7*e^3*m^2*x^3 + 156*(e*x + d)^m*a*c^2*d^5*e^5*m^2*x^3 + 294*(
e*x + d)^m*a^2*c*d^3*e^7*m^2*x^3 + 72*(e*x + d)^m*a^3*d*e^9*m^2*x^3 + 1...
```

Mupad [B] (verification not implemented)

Time = 6.07 (sec) , antiderivative size = 1202, normalized size of antiderivative = 9.25

$$\int (d+ex)^m \left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^3 \, dx = \text{Too large to display}$$

input
```
int((d + e*x)^m*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^3,x)
```

output
```
(d^4*(d + e*x)^m*(210*a^3*e^6 - 6*c^3*d^6 + 107*a^3*e^6*m + 18*a^3*e^6*m^2
 + a^3*e^6*m^3 + 42*a*c^2*d^4*e^2 - 126*a^2*c*d^2*e^4 + 6*a*c^2*d^4*e^2*m
- 39*a^2*c*d^2*e^4*m - 3*a^2*c*d^2*e^4*m^2))/(e^4*(638*m + 179*m^2 + 22*m^
3 + m^4 + 840)) + (x^4*(d + e*x)^m*(210*a^3*e^10 + 107*a^3*e^10*m + 210*c^
3*d^6*e^4 + 18*a^3*e^10*m^2 + a^3*e^10*m^3 + 1890*a*c^2*d^4*e^6 + 1890*a^2
*c*d^2*e^8 + 158*c^3*d^6*e^4*m + 42*c^3*d^6*e^4*m^2 + 4*c^3*d^6*e^4*m^3 +
1236*a*c^2*d^4*e^6*m + 1089*a^2*c*d^2*e^8*m + 264*a*c^2*d^4*e^6*m^2 + 201*
a^2*c*d^2*e^8*m^2 + 18*a*c^2*d^4*e^6*m^3 + 12*a^2*c*d^2*e^8*m^3))/(e^4*(63
8*m + 179*m^2 + 22*m^3 + m^4 + 840)) + (3*d^2*x^2*(d + e*x)^m*(420*a^3*e^6
 + 214*a^3*e^6*m - c^3*d^6*m + 36*a^3*e^6*m^2 + 2*a^3*e^6*m^3 - c^3*d^6*m^
2 + 420*a^2*c*d^2*e^4 + 7*a*c^2*d^4*e^2*m + 298*a^2*c*d^2*e^4*m + 8*a*c^2*
d^4*e^2*m^2 + 62*a^2*c*d^2*e^4*m^2 + a*c^2*d^4*e^2*m^3 + 4*a^2*c*d^2*e^4*m
^3))/(e^2*(638*m + 179*m^2 + 22*m^3 + m^4 + 840)) + (d^3*x*(d + e*x)^m*(84
0*a^3*e^6 + 428*a^3*e^6*m + 6*c^3*d^6*m + 72*a^3*e^6*m^2 + 4*a^3*e^6*m^3 -
 42*a*c^2*d^4*e^2*m + 126*a^2*c*d^2*e^4*m - 6*a*c^2*d^4*e^2*m^2 + 39*a^2*c
*d^2*e^4*m^2 + 3*a^2*c*d^2*e^4*m^3))/(e^3*(638*m + 179*m^2 + 22*m^3 + m^4
+ 840)) + (d*x^3*(d + e*x)^m*(840*a^3*e^6 + 428*a^3*e^6*m + 2*c^3*d^6*m +
72*a^3*e^6*m^2 + 4*a^3*e^6*m^3 + 3*c^3*d^6*m^2 + c^3*d^6*m^3 + 840*a*c^2*d
^4*e^2 + 2520*a^2*c*d^2*e^4 + 624*a*c^2*d^4*e^2*m + 1536*a^2*c*d^2*e^4*m +
 156*a*c^2*d^4*e^2*m^2 + 294*a^2*c*d^2*e^4*m^2 + 12*a*c^2*d^4*e^2*m^3 +...
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 1376, normalized size of antiderivative = 10.58

$$\int (d+ex)^m \left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^3 \, dx = \text{Too large to display}$$

input
```
int((e*x+d)^m*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^3,x)
```

output
```
((d + e*x)**m*(a**3*d**4*e**6*m**3 + 18*a**3*d**4*e**6*m**2 + 107*a**3*d**
4*e**6*m + 210*a**3*d**4*e**6 + 4*a**3*d**3*e**7*m**3*x + 72*a**3*d**3*e**
7*m**2*x + 428*a**3*d**3*e**7*m*x + 840*a**3*d**3*e**7*x + 6*a**3*d**2*e**
8*m**3*x**2 + 108*a**3*d**2*e**8*m**2*x**2 + 642*a**3*d**2*e**8*m*x**2 + 1
260*a**3*d**2*e**8*x**2 + 4*a**3*d*e**9*m**3*x**3 + 72*a**3*d*e**9*m**2*x*
*3 + 428*a**3*d*e**9*m*x**3 + 840*a**3*d*e**9*x**3 + a**3*e**10*m**3*x**4
+ 18*a**3*e**10*m**2*x**4 + 107*a**3*e**10*m*x**4 + 210*a**3*e**10*x**4 -
3*a**2*c*d**6*e**4*m**2 - 39*a**2*c*d**6*e**4*m - 126*a**2*c*d**6*e**4 + 3
*a**2*c*d**5*e**5*m**3*x + 39*a**2*c*d**5*e**5*m**2*x + 126*a**2*c*d**5*e*
*5*m*x + 12*a**2*c*d**4*e**6*m**3*x**2 + 186*a**2*c*d**4*e**6*m**2*x**2 +
894*a**2*c*d**4*e**6*m*x**2 + 1260*a**2*c*d**4*e**6*x**2 + 18*a**2*c*d**3*
e**7*m**3*x**3 + 294*a**2*c*d**3*e**7*m**2*x**3 + 1536*a**2*c*d**3*e**7*m*
x**3 + 2520*a**2*c*d**3*e**7*x**3 + 12*a**2*c*d**2*e**8*m**3*x**4 + 201*a*
*2*c*d**2*e**8*m**2*x**4 + 1089*a**2*c*d**2*e**8*m*x**4 + 1890*a**2*c*d**2
*e**8*x**4 + 3*a**2*c*d*e**9*m**3*x**5 + 51*a**2*c*d*e**9*m**2*x**5 + 282*
a**2*c*d*e**9*m*x**5 + 504*a**2*c*d*e**9*x**5 + 6*a*c**2*d**8*e**2*m + 42*
a*c**2*d**8*e**2 - 6*a*c**2*d**7*e**3*m**2*x - 42*a*c**2*d**7*e**3*m*x + 3
*a*c**2*d**6*e**4*m**3*x**2 + 24*a*c**2*d**6*e**4*m**2*x**2 + 21*a*c**2*d*
*6*e**4*m*x**2 + 12*a*c**2*d**5*e**5*m**3*x**3 + 156*a*c**2*d**5*e**5*m**2
*x**3 + 624*a*c**2*d**5*e**5*m*x**3 + 840*a*c**2*d**5*e**5*x**3 + 18*a*...
```

3.383 $\int (d+ex)^m \left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^2 dx$

Optimal result

Integrand size = 35, antiderivative size = 90

$$\int (d+ex)^m \left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^2 dx = \frac{\left(cd^2-ae^2\right)^2 (d+ex)^{3+m}}{e^3(3+m)} - \frac{2cd\left(cd^2-ae^2\right)(d+ex)^{4+m}}{e^3(4+m)} + \frac{c^2d^2(d+ex)^{5+m}}{e^3(5+m)}$$

output
```
(-a*e^2+c*d^2)^2*(e*x+d)^(3+m)/e^3/(3+m)-2*c*d*(-a*e^2+c*d^2)*(e*x+d)^(4+m
)/e^3/(4+m)+c^2*d^2*(e*x+d)^(5+m)/e^3/(5+m)
```

Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.88

$$\int (d+ex)^m \left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^2 dx = \frac{(d+ex)^{3+m}\left(\frac{\left(cd^2-ae^2\right)^2}{3+m} - \frac{2cd\left(cd^2-ae^2\right)(d+ex)}{4+m} + \frac{c^2d^2(d+ex)^2}{5+m}\right)}{e^3}$$

input
```
Integrate[(d + e*x)^m*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^2,x]
```

output
```
((d + e*x)^(3 + m)*((c*d^2 - a*e^2)^2/(3 + m) - (2*c*d*(c*d^2 - a*e^2)*(d
+ e*x))/(4 + m) + (c^2*d^2*(d + e*x)^2)/(5 + m)))/e^3
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.057$, Rules used = {1121, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (d+ex)^m \left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^2 dx$$

$$\Big\downarrow \text{1121}$$

$$\int \left(\frac{\left(ae^2-cd^2\right)^2 (d+ex)^{m+2}}{e^2}-\frac{2cd\left(cd^2-ae^2\right)(d+ex)^{m+3}}{e^2}+\frac{c^2d^2(d+ex)^{m+4}}{e^2}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{\left(cd^2-ae^2\right)^2 (d+ex)^{m+3}}{e^3(m+3)}-\frac{2cd\left(cd^2-ae^2\right)(d+ex)^{m+4}}{e^3(m+4)}+\frac{c^2d^2(d+ex)^{m+5}}{e^3(m+5)}$$

input
```
Int[(d + e*x)^m*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^2,x]
```

output
```
((c*d^2 - a*e^2)^2*(d + e*x)^(3 + m))/(e^3*(3 + m)) - (2*c*d*(c*d^2 - a*e^
2)*(d + e*x)^(4 + m))/(e^3*(4 + m)) + (c^2*d^2*(d + e*x)^(5 + m))/(e^3*(5
+ m))
```

Defintions of rubi rules used

rule 1121
```
Int[((d_) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Int[ExpandIntegrand[(d + e*x)^(m + p)*(a/d + (c/e)*x)^p, x], x]
/; FreeQ[{a, b, c, d, e, m, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && (Int
egerQ[p] || (GtQ[a, 0] && GtQ[d, 0] && LtQ[c, 0]))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 182 vs. $2(90) = 180$.

Time = 0.95 (sec) , antiderivative size = 183, normalized size of antiderivative = 2.03

method	result
gosper	$\frac{(ex+d)^{3+m}(c^2d^2e^2m^2x^2+2acd\,e^3m^2x+7c^2d^2e^2m\,x^2+a^2e^4m^2+16acd\,e^3mx-2c^2d^3emx+12x^2c^2d^2e^2+9a^2e^4m-2ac\,d^2e^2m}{e^3(m^3+12m^2+47m+60)}$
orering	$\frac{(c^2d^2e^2m^2x^2+2acd\,e^3m^2x+7c^2d^2e^2m\,x^2+a^2e^4m^2+16acd\,e^3mx-2c^2d^3emx+12x^2c^2d^2e^2+9a^2e^4m-2ac\,d^2e^2m+30xacd\,e}{e^3(m^3+12m^2+47m+60)(cdx+ae)^2}$
norman	$\frac{(a^2e^4m^2+6ac\,d^2e^2m^2+3c^2d^4m^2+9a^2e^4m+46ac\,d^2e^2m+15c^2d^4m+20a^2e^4+80ac\,d^2e^2+20c^2d^4)x^3\mathrm{e}^{m\ln(ex+d)}}{m^3+12m^2+47m+60}+\frac{d^3(a^2e^4}{}$
risch	$\frac{(c^2d^2e^5m^2x^5+2acd\,e^6m^2x^4+3c^2d^3e^4m^2x^4+7c^2d^2e^5m\,x^5+a^2e^7m^2x^3+6ac\,d^2e^5m^2x^3+16acd\,e^6m\,x^4+3c^2d^4e^3m^2x^3+19c^2}{}$
parallelrisch	$\frac{3x^4(ex+d)^mc^2d^4e^4m^2+3x(ex+d)^ma^2d^3e^5m^2-2x(ex+d)^mc^2d^7em+19x^4(ex+d)^mc^2d^4e^4m+x^3(ex+d)^ma^2d\,e^7m^2+3x^3(e}{}$

input
```
int((e*x+d)^m*(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^2,x,method=_RETURNVERBOSE)
```

output
```
1/e^3*(e*x+d)^(3+m)/(m^3+12*m^2+47*m+60)*(c^2*d^2*e^2*m^2*x^2+2*a*c*d*e^3*
m^2*x+7*c^2*d^2*e^2*m*x^2+a^2*e^4*m^2+16*a*c*d*e^3*m*x-2*c^2*d^3*e*m*x+12*
c^2*d^2*e^2*x^2+9*a^2*e^4*m-2*a*c*d^2*e^2*m+30*a*c*d*e^3*x-6*c^2*d^3*e*x+2
0*a^2*e^4-10*a*c*d^2*e^2+2*c^2*d^4)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 479 vs. $2(90) = 180$.

Time = 0.10 (sec) , antiderivative size = 479, normalized size of antiderivative = 5.32

$$\int (d+ex)^m \left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^2 dx = \frac{(a^2d^3e^4m^2 + 2\,c^2d^7 - 10\,acd^5e^2 + 20\,a^2d^3e^4 + (c^2d^2e^5m^2 + 7\,c^2d^2e^5m + 12\,c^2d^2e^5)x^5 + (30\,c^2d^3e^4 + 30\,a}{}$$

input
```
integrate((e*x+d)^m*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^2,x, algorithm="fric
as")
```

output
```
(a^2*d^3*e^4*m^2 + 2*c^2*d^7 - 10*a*c*d^5*e^2 + 20*a^2*d^3*e^4 + (c^2*d^2*
e^5*m^2 + 7*c^2*d^2*e^5*m + 12*c^2*d^2*e^5)*x^5 + (30*c^2*d^3*e^4 + 30*a*c
*d*e^6 + (3*c^2*d^3*e^4 + 2*a*c*d*e^6)*m^2 + (19*c^2*d^3*e^4 + 16*a*c*d*e^
6)*m)*x^4 + (20*c^2*d^4*e^3 + 80*a*c*d^2*e^5 + 20*a^2*e^7 + (3*c^2*d^4*e^3
 + 6*a*c*d^2*e^5 + a^2*e^7)*m^2 + (15*c^2*d^4*e^3 + 46*a*c*d^2*e^5 + 9*a^2
*e^7)*m)*x^3 + (60*a*c*d^3*e^4 + 60*a^2*d*e^6 + (c^2*d^5*e^2 + 6*a*c*d^3*e
^4 + 3*a^2*d*e^6)*m^2 + (c^2*d^5*e^2 + 42*a*c*d^3*e^4 + 27*a^2*d*e^6)*m)*x
^2 - (2*a*c*d^5*e^2 - 9*a^2*d^3*e^4)*m + (60*a^2*d^2*e^5 + (2*a*c*d^4*e^3
+ 3*a^2*d^2*e^5)*m^2 - (2*c^2*d^6*e - 10*a*c*d^4*e^3 - 27*a^2*d^2*e^5)*m)*
x)*(e*x + d)^m/(e^3*m^3 + 12*e^3*m^2 + 47*e^3*m + 60*e^3)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2494 vs. $2(78) = 156$.

Time = 1.02 (sec) , antiderivative size = 2494, normalized size of antiderivative = 27.71

$$\int (d+ex)^m \left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^2 dx = \text{Too large to display}$$

input
```
integrate((e*x+d)**m*(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**2,x)
```

output
```
Piecewise((c**2*d**4*d**m*x**3/3, Eq(e, 0)), (-a**2*e**4/(2*d**2*e**3 + 4*
d*e**4*x + 2*e**5*x**2) - 2*a*c*d**2*e**2/(2*d**2*e**3 + 4*d*e**4*x + 2*e*
*5*x**2) - 4*a*c*d*e**3*x/(2*d**2*e**3 + 4*d*e**4*x + 2*e**5*x**2) + 2*c**
2*d**4*log(d/e + x)/(2*d**2*e**3 + 4*d*e**4*x + 2*e**5*x**2) + 3*c**2*d**4
/(2*d**2*e**3 + 4*d*e**4*x + 2*e**5*x**2) + 4*c**2*d**3*e*x*log(d/e + x)/(
2*d**2*e**3 + 4*d*e**4*x + 2*e**5*x**2) + 4*c**2*d**3*e*x/(2*d**2*e**3 + 4
*d*e**4*x + 2*e**5*x**2) + 2*c**2*d**2*e**2*x**2*log(d/e + x)/(2*d**2*e**3
 + 4*d*e**4*x + 2*e**5*x**2), Eq(m, -5)), (-a**2*e**4/(d*e**3 + e**4*x) +
2*a*c*d**2*e**2*log(d/e + x)/(d*e**3 + e**4*x) + 2*a*c*d**2*e**2/(d*e**3 +
 e**4*x) + 2*a*c*d*e**3*x*log(d/e + x)/(d*e**3 + e**4*x) - 2*c**2*d**4*log
(d/e + x)/(d*e**3 + e**4*x) - 4*c**2*d**4/(d*e**3 + e**4*x) - 2*c**2*d**3*
e*x*log(d/e + x)/(d*e**3 + e**4*x) - 2*c**2*d**3*e*x/(d*e**3 + e**4*x) + c
**2*d**2*e**2*x**2/(d*e**3 + e**4*x), Eq(m, -4)), (a**2*e*log(d/e + x) - 2
*a*c*d**2*log(d/e + x)/e + 2*a*c*d*x + c**2*d**4*log(d/e + x)/e**3 - c**2*
d**3*x/e**2 + c**2*d**2*x**2/(2*e), Eq(m, -3)), (a**2*d**3*e**4*m**2*(d +
e*x)**m/(e**3*m**3 + 12*e**3*m**2 + 47*e**3*m + 60*e**3) + 9*a**2*d**3*e**
4*m*(d + e*x)**m/(e**3*m**3 + 12*e**3*m**2 + 47*e**3*m + 60*e**3) + 20*a**
2*d**3*e**4*(d + e*x)**m/(e**3*m**3 + 12*e**3*m**2 + 47*e**3*m + 60*e**3)
+ 3*a**2*d**2*e**5*m**2*x*(d + e*x)**m/(e**3*m**3 + 12*e**3*m**2 + 47*e**3
*m + 60*e**3) + 27*a**2*d**2*e**5*m*x*(d + e*x)**m/(e**3*m**3 + 12*e**3...
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 691 vs. $2(90) = 180$.

Time = 0.07 (sec) , antiderivative size = 691, normalized size of antiderivative = 7.68

$$\int (d+ex)^m \left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^2 \, dx = \text{Too large to display}$$

input
```
integrate((e*x+d)^m*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^2,x, algorithm="maxi
ma")
```

output
```
2*(e^2*(m + 1)*x^2 + d*e*m*x - d^2)*(e*x + d)^m*a*c*d^3/((m^2 + 3*m + 2)*e
) + 2*(e^2*(m + 1)*x^2 + d*e*m*x - d^2)*(e*x + d)^m*a^2*d*e/(m^2 + 3*m + 2
) + (e*x + d)^(m + 1)*a^2*d^2*e/(m + 1) + ((m^2 + 3*m + 2)*e^3*x^3 + (m^2
+ m)*d*e^2*x^2 - 2*d^2*e*m*x + 2*d^3)*(e*x + d)^m*c^2*d^4/((m^3 + 6*m^2 +
11*m + 6)*e^3) + 4*((m^2 + 3*m + 2)*e^3*x^3 + (m^2 + m)*d*e^2*x^2 - 2*d^2*
e*m*x + 2*d^3)*(e*x + d)^m*a*c*d^2/((m^3 + 6*m^2 + 11*m + 6)*e) + ((m^2 +
3*m + 2)*e^3*x^3 + (m^2 + m)*d*e^2*x^2 - 2*d^2*e*m*x + 2*d^3)*(e*x + d)^m*
a^2*e/(m^3 + 6*m^2 + 11*m + 6) + 2*((m^3 + 6*m^2 + 11*m + 6)*e^4*x^4 + (m^
3 + 3*m^2 + 2*m)*d*e^3*x^3 - 3*(m^2 + m)*d^2*e^2*x^2 + 6*d^3*e*m*x - 6*d^4
)*(e*x + d)^m*c^2*d^3/((m^4 + 10*m^3 + 35*m^2 + 50*m + 24)*e^3) + 2*((m^3
+ 6*m^2 + 11*m + 6)*e^4*x^4 + (m^3 + 3*m^2 + 2*m)*d*e^3*x^3 - 3*(m^2 + m)*
d^2*e^2*x^2 + 6*d^3*e*m*x - 6*d^4)*(e*x + d)^m*a*c*d/((m^4 + 10*m^3 + 35*m
^2 + 50*m + 24)*e) + ((m^4 + 10*m^3 + 35*m^2 + 50*m + 24)*e^5*x^5 + (m^4 +
 6*m^3 + 11*m^2 + 6*m)*d*e^4*x^4 - 4*(m^3 + 3*m^2 + 2*m)*d^2*e^3*x^3 + 12*
(m^2 + m)*d^3*e^2*x^2 - 24*d^4*e*m*x + 24*d^5)*(e*x + d)^m*c^2*d^2/((m^5 +
 15*m^4 + 85*m^3 + 225*m^2 + 274*m + 120)*e^3)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 805 vs. $2(90) = 180$.

Time = 0.13 (sec) , antiderivative size = 805, normalized size of antiderivative = 8.94

$$\int (d+ex)^m \left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^2 dx = \text{Too large to display}$$

input
```
integrate((e*x+d)^m*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^2,x, algorithm="giac
")
```

output
```
((e*x + d)^m*c^2*d^2*e^5*m^2*x^5 + 3*(e*x + d)^m*c^2*d^3*e^4*m^2*x^4 + 2*(
e*x + d)^m*a*c*d*e^6*m^2*x^4 + 7*(e*x + d)^m*c^2*d^2*e^5*m*x^5 + 3*(e*x +
d)^m*c^2*d^4*e^3*m^2*x^3 + 6*(e*x + d)^m*a*c*d^2*e^5*m^2*x^3 + (e*x + d)^m
*a^2*e^7*m^2*x^3 + 19*(e*x + d)^m*c^2*d^3*e^4*m*x^4 + 16*(e*x + d)^m*a*c*d
*e^6*m*x^4 + 12*(e*x + d)^m*c^2*d^2*e^5*x^5 + (e*x + d)^m*c^2*d^5*e^2*m^2*
x^2 + 6*(e*x + d)^m*a*c*d^3*e^4*m^2*x^2 + 3*(e*x + d)^m*a^2*d*e^6*m^2*x^2
+ 15*(e*x + d)^m*c^2*d^4*e^3*m*x^3 + 46*(e*x + d)^m*a*c*d^2*e^5*m*x^3 + 9*
(e*x + d)^m*a^2*e^7*m*x^3 + 30*(e*x + d)^m*c^2*d^3*e^4*x^4 + 30*(e*x + d)^
m*a*c*d*e^6*x^4 + 2*(e*x + d)^m*a*c*d^4*e^3*m^2*x + 3*(e*x + d)^m*a^2*d^2*
e^5*m^2*x + (e*x + d)^m*c^2*d^5*e^2*m*x^2 + 42*(e*x + d)^m*a*c*d^3*e^4*m*x
^2 + 27*(e*x + d)^m*a^2*d*e^6*m*x^2 + 20*(e*x + d)^m*c^2*d^4*e^3*x^3 + 80*
(e*x + d)^m*a*c*d^2*e^5*x^3 + 20*(e*x + d)^m*a^2*e^7*x^3 + (e*x + d)^m*a^2
*d^3*e^4*m^2 - 2*(e*x + d)^m*c^2*d^6*e*m*x + 10*(e*x + d)^m*a*c*d^4*e^3*m*
x + 27*(e*x + d)^m*a^2*d^2*e^5*m*x + 60*(e*x + d)^m*a*c*d^3*e^4*x^2 + 60*(
e*x + d)^m*a^2*d*e^6*x^2 - 2*(e*x + d)^m*a*c*d^5*e^2*m + 9*(e*x + d)^m*a^2
*d^3*e^4*m + 60*(e*x + d)^m*a^2*d^2*e^5*x + 2*(e*x + d)^m*c^2*d^7 - 10*(e*
x + d)^m*a*c*d^5*e^2 + 20*(e*x + d)^m*a^2*d^3*e^4)/(e^3*m^3 + 12*e^3*m^2 +
 47*e^3*m + 60*e^3)
```

Mupad [B] (verification not implemented)

Time = 5.46 (sec) , antiderivative size = 486, normalized size of antiderivative = 5.40

$$\int (d+ex)^m \left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^2 \, dx$$

$$= (d+e\,x)^m \left(\frac{x^3\,(a^2\,e^7\,m^2+9\,a^2\,e^7\,m+20\,a^2\,e^7+6\,a\,c\,d^2\,e^5\,m^2+46\,a\,c\,d^2\,e^5\,m+80\,a\,c\,d^2\,e^5+3\,c^2\,d^4\,e^3\,m}{e^3\,\left(m^3+12\,m^2+47\,m+60\right)} + \frac{d^3\,(a^2\,e^4\,m^2+9\,a^2\,e^4\,m+20\,a^2\,e^4-2\,a\,c\,d^2\,e^2\,m-10\,a\,c\,d^2\,e^2+2\,c^2\,d^4)}{e^3\,\left(m^3+12\,m^2+47\,m+60\right)} + \frac{d^2\,x\,(3\,a^2\,e^4\,m^2+27\,a^2\,e^4\,m+60\,a^2\,e^4+2\,a\,c\,d^2\,e^2\,m^2+10\,a\,c\,d^2\,e^2\,m-2\,c^2\,d^4\,m)}{e^2\,\left(m^3+12\,m^2+47\,m+60\right)} + \frac{d\,x^2\,(3\,a^2\,e^4\,m^2+27\,a^2\,e^4\,m+60\,a^2\,e^4+6\,a\,c\,d^2\,e^2\,m^2+42\,a\,c\,d^2\,e^2\,m+60\,a\,c\,d^2\,e^2+c^2\,d^4\,m^2+c^2\,d^4}{e\,\left(m^3+12\,m^2+47\,m+60\right)} + \frac{c^2\,d^2\,e^2\,x^5\,\left(m^2+7\,m+12\right)}{m^3+12\,m^2+47\,m+60} + \frac{c\,d\,e\,x^4\,\left(m+3\right)\,\left(10\,a\,e^2+10\,c\,d^2+2\,a\,e^2\,m+3\,c\,d^2\,m\right)}{m^3+12\,m^2+47\,m+60} \right)$$

input
```
int((d + e*x)^m*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^2,x)
```

output
```
(d + e*x)^m*((x^3*(20*a^2*e^7 + 9*a^2*e^7*m + 20*c^2*d^4*e^3 + a^2*e^7*m^2
 + 15*c^2*d^4*e^3*m + 3*c^2*d^4*e^3*m^2 + 80*a*c*d^2*e^5 + 46*a*c*d^2*e^5*
m + 6*a*c*d^2*e^5*m^2))/(e^3*(47*m + 12*m^2 + m^3 + 60)) + (d^3*(20*a^2*e^
4 + 2*c^2*d^4 + 9*a^2*e^4*m + a^2*e^4*m^2 - 10*a*c*d^2*e^2 - 2*a*c*d^2*e^2
*m))/(e^3*(47*m + 12*m^2 + m^3 + 60)) + (d^2*x*(60*a^2*e^4 + 27*a^2*e^4*m
- 2*c^2*d^4*m + 3*a^2*e^4*m^2 + 10*a*c*d^2*e^2*m + 2*a*c*d^2*e^2*m^2))/(e^
2*(47*m + 12*m^2 + m^3 + 60)) + (d*x^2*(60*a^2*e^4 + 27*a^2*e^4*m + c^2*d^
4*m + 3*a^2*e^4*m^2 + c^2*d^4*m^2 + 60*a*c*d^2*e^2 + 42*a*c*d^2*e^2*m + 6*
a*c*d^2*e^2*m^2))/(e*(47*m + 12*m^2 + m^3 + 60)) + (c^2*d^2*e^2*x^5*(7*m +
 m^2 + 12))/(47*m + 12*m^2 + m^3 + 60) + (c*d*e*x^4*(m + 3)*(10*a*e^2 + 10
*c*d^2 + 2*a*e^2*m + 3*c*d^2*m))/(47*m + 12*m^2 + m^3 + 60))
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 535, normalized size of antiderivative = 5.94

$$\int (d+ex)^m \left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^2 dx$$
$$= \frac{(ex+d)^m \left(c^2d^2e^5m^2x^5+2acd\,e^6m^2x^4+3c^2d^3e^4m^2x^4+7c^2d^2e^5m\,x^5+a^2e^7m^2x^3+6ac\,d^2e^5m^2x^3+16a\right.}{}$$

input
```
int((e*x+d)^m*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^2,x)
```

output
```
((d + e*x)**m*(a**2*d**3*e**4*m**2 + 9*a**2*d**3*e**4*m + 20*a**2*d**3*e**
4 + 3*a**2*d**2*e**5*m**2*x + 27*a**2*d**2*e**5*m*x + 60*a**2*d**2*e**5*x
+ 3*a**2*d*e**6*m**2*x**2 + 27*a**2*d*e**6*m*x**2 + 60*a**2*d*e**6*x**2 +
a**2*e**7*m**2*x**3 + 9*a**2*e**7*m*x**3 + 20*a**2*e**7*x**3 - 2*a*c*d**5*
e**2*m - 10*a*c*d**5*e**2 + 2*a*c*d**4*e**3*m**2*x + 10*a*c*d**4*e**3*m*x
+ 6*a*c*d**3*e**4*m**2*x**2 + 42*a*c*d**3*e**4*m*x**2 + 60*a*c*d**3*e**4*x
**2 + 6*a*c*d**2*e**5*m**2*x**3 + 46*a*c*d**2*e**5*m*x**3 + 80*a*c*d**2*e*
*5*x**3 + 2*a*c*d*e**6*m**2*x**4 + 16*a*c*d*e**6*m*x**4 + 30*a*c*d*e**6*x*
*4 + 2*c**2*d**7 - 2*c**2*d**6*e*m*x + c**2*d**5*e**2*m**2*x**2 + c**2*d**
5*e**2*m*x**2 + 3*c**2*d**4*e**3*m**2*x**3 + 15*c**2*d**4*e**3*m*x**3 + 20
*c**2*d**4*e**3*x**3 + 3*c**2*d**3*e**4*m**2*x**4 + 19*c**2*d**3*e**4*m*x*
*4 + 30*c**2*d**3*e**4*x**4 + c**2*d**2*e**5*m**2*x**5 + 7*c**2*d**2*e**5*
m*x**5 + 12*c**2*d**2*e**5*x**5))/(e**3*(m**3 + 12*m**2 + 47*m + 60))
```

3.384 $\int (d+ex)^m \left(ade+\left(cd^2+ae^2\right) x+cdex^2\right) \, dx$

Optimal result

Integrand size = 33, antiderivative size = 52

$$\int (d+ex)^m \left(ade+\left(cd^2+ae^2\right) x+cdex^2\right) \, dx = -\frac{\left(cd^2-ae^2\right) (d+ex)^{2+m}}{e^2(2+m)}+\frac{cd(d+ex)^{3+m}}{e^2(3+m)}$$

output
```
-(-a*e^2+c*d^2)*(e*x+d)^(2+m)/e^2/(2+m)+c*d*(e*x+d)^(3+m)/e^2/(3+m)
```

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.87

$$\int (d+ex)^m \left(ade+\left(cd^2+ae^2\right) x+cdex^2\right) \, dx = \frac{(d+ex)^{2+m} \left(ae^2(3+m)+cd(-d+e(2+m)x)\right)}{e^2(2+m)(3+m)}$$

input
```
Integrate[(d + e*x)^m*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2),x]
```

output
```
((d + e*x)^(2 + m)*(a*e^2*(3 + m) + c*d*(-d + e*(2 + m)*x)))/(e^2*(2 + m)*
(3 + m))
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.061$, Rules used = {1121, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (d+ex)^m \left(x\left(ae^2+cd^2\right)+ade+cdex^2\right) \, dx$$

$$\Big\downarrow \text{ 1121}$$

$$\int \left(\frac{\left(ae^2-cd^2\right)(d+ex)^{m+1}}{e}+\frac{cd(d+ex)^{m+2}}{e}\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{cd(d+ex)^{m+3}}{e^2(m+3)}-\frac{\left(cd^2-ae^2\right)(d+ex)^{m+2}}{e^2(m+2)}$$

input
```
Int[(d + e*x)^m*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2),x]
```

output
```
-(((c*d^2 - a*e^2)*(d + e*x)^(2 + m))/(e^2*(2 + m))) + (c*d*(d + e*x)^(3 +
 m))/(e^2*(3 + m))
```

Defintions of rubi rules used

rule 1121
```
Int[((d_) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Int[ExpandIntegrand[(d + e*x)^(m + p)*(a/d + (c/e)*x)^p, x], x]
/; FreeQ[{a, b, c, d, e, m, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && (Int
egerQ[p] || (GtQ[a, 0] && GtQ[d, 0] && LtQ[c, 0]))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.51 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.06

method	result
gosper	$\frac{(ex+d)^{2+m}\left(cdemx+a\,e^{2}m+2cdxe+3a\,e^{2}-c\,d^{2}\right)}{e^{2}\left(m^{2}+5m+6\right)}$
orering	$\frac{\left(cdemx+a\,e^{2}m+2cdxe+3a\,e^{2}-c\,d^{2}\right)(ex+d)(ex+d)^{m}\left(ade+\left(a\,e^{2}+c\,d^{2}\right)x+cd\,x^{2}e\right)}{e^{2}\left(m^{2}+5m+6\right)(cdx+ae)}$
risch	$\frac{\left(cd\,e^{3}m\,x^{3}+a\,e^{4}m\,x^{2}+2c\,d^{2}e^{2}m\,x^{2}+2cd\,e^{3}x^{3}+2ad\,e^{3}mx+3a\,e^{4}x^{2}+c\,d^{3}emx+3c\,d^{2}e^{2}x^{2}+a\,d^{2}e^{2}m+6ad\,e^{3}x+3a\,d^{2}e^{2}-c\,d^{4}\right)}{(2+m)(3+m)e^{2}}$
norman	$\frac{\left(a\,e^{2}m+2c\,d^{2}m+3a\,e^{2}+3c\,d^{2}\right)x^{2}\mathrm{e}^{m\ln(ex+d)}}{m^{2}+5m+6}+\frac{d^{2}\left(a\,e^{2}m+3a\,e^{2}-c\,d^{2}\right)\mathrm{e}^{m\ln(ex+d)}}{e^{2}\left(m^{2}+5m+6\right)}+\frac{dec\,x^{3}\mathrm{e}^{m\ln(ex+d)}}{3+m}+$
parallelrisch	$x^{3}(ex+d)^{m}cd\,e^{3}m+2x^{3}(ex+d)^{m}cd\,e^{3}+x^{2}(ex+d)^{m}a\,e^{4}m+2x^{2}(ex+d)^{m}c\,d^{2}e^{2}m+3x^{2}(ex+d)^{m}a\,e^{4}+3x^{2}(ex+d)^{m}c\,d^{2}e^{2}$

input
```
int((e*x+d)^m*(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e),x,method=_RETURNVERBOSE)
```

output
```
1/e^2*(e*x+d)^(2+m)/(m^2+5*m+6)*(c*d*e*m*x+a*e^2*m+2*c*d*e*x+3*a*e^2-c*d^2
)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 136 vs. $2(52) = 104$.

Time = 0.10 (sec) , antiderivative size = 136, normalized size of antiderivative = 2.62

$$\int (d+ex)^m \left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)\,dx$$
$$= \frac{(ad^2e^2m - cd^4 + 3\,ad^2e^2 + (cde^3m + 2\,cde^3)x^3 + (3\,cd^2e^2 + 3\,ae^4 + (2\,cd^2e^2 + ae^4)m)x^2 + (6\,ade^3 + (c}{e^2m^2 + 5\,e^2m + 6\,e^2}$$

input
```
integrate((e*x+d)^m*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2),x, algorithm="fricas
")
```

output
```
(a*d^2*e^2*m - c*d^4 + 3*a*d^2*e^2 + (c*d*e^3*m + 2*c*d*e^3)*x^3 + (3*c*d^
2*e^2 + 3*a*e^4 + (2*c*d^2*e^2 + a*e^4)*m)*x^2 + (6*a*d*e^3 + (c*d^3*e + 2
*a*d*e^3)*m)*x)*(e*x + d)^m/(e^2*m^2 + 5*e^2*m + 6*e^2)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 556 vs. $2(42) = 84$.

Time = 0.42 (sec) , antiderivative size = 556, normalized size of antiderivative = 10.69

$$\int (d+ex)^m \left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)\,dx$$
$$= \begin{cases} \frac{cd^2d^mx^2}{2} \\ -\frac{ae^2}{de^2+e^3x} + \frac{cd^2\log\left(\frac{d}{e}+x\right)}{de^2+e^3x} + \frac{cd^2}{de^2+e^3x} + \frac{cdex\log\left(\frac{d}{e}+x\right)}{de^2+e^3x} \\ a\log\left(\frac{d}{e}+x\right) - \frac{cd^2\log\left(\frac{d}{e}+x\right)}{e^2} + \frac{cdx}{e} \\ \frac{ad^2e^2m(d+ex)^m}{e^2m^2+5e^2m+6e^2} + \frac{3ad^2e^2(d+ex)^m}{e^2m^2+5e^2m+6e^2} + \frac{2ade^3mx(d+ex)^m}{e^2m^2+5e^2m+6e^2} + \frac{6ade^3x(d+ex)^m}{e^2m^2+5e^2m+6e^2} + \frac{ae^4mx^2(d+ex)^m}{e^2m^2+5e^2m+6e^2} + \frac{3ae^4x^2(d+ex)^m}{e^2m^2+5e^2m+6e^2} - \frac{}{e^2r} \end{cases}$$

input
```
integrate((e*x+d)**m*(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2),x)
```

output
```
Piecewise((c*d**2*d**m*x**2/2, Eq(e, 0)), (-a*e**2/(d*e**2 + e**3*x) + c*d
**2*log(d/e + x)/(d*e**2 + e**3*x) + c*d**2/(d*e**2 + e**3*x) + c*d*e*x*lo
g(d/e + x)/(d*e**2 + e**3*x), Eq(m, -3)), (a*log(d/e + x) - c*d**2*log(d/e
 + x)/e**2 + c*d*x/e, Eq(m, -2)), (a*d**2*e**2*m*(d + e*x)**m/(e**2*m**2 +
 5*e**2*m + 6*e**2) + 3*a*d**2*e**2*(d + e*x)**m/(e**2*m**2 + 5*e**2*m + 6
*e**2) + 2*a*d*e**3*m*x*(d + e*x)**m/(e**2*m**2 + 5*e**2*m + 6*e**2) + 6*a
*d*e**3*x*(d + e*x)**m/(e**2*m**2 + 5*e**2*m + 6*e**2) + a*e**4*m*x**2*(d
+ e*x)**m/(e**2*m**2 + 5*e**2*m + 6*e**2) + 3*a*e**4*x**2*(d + e*x)**m/(e*
*2*m**2 + 5*e**2*m + 6*e**2) - c*d**4*(d + e*x)**m/(e**2*m**2 + 5*e**2*m +
 6*e**2) + c*d**3*e*m*x*(d + e*x)**m/(e**2*m**2 + 5*e**2*m + 6*e**2) + 2*c
*d**2*e**2*m*x**2*(d + e*x)**m/(e**2*m**2 + 5*e**2*m + 6*e**2) + 3*c*d**2*
e**2*x**2*(d + e*x)**m/(e**2*m**2 + 5*e**2*m + 6*e**2) + c*d*e**3*m*x**3*(
d + e*x)**m/(e**2*m**2 + 5*e**2*m + 6*e**2) + 2*c*d*e**3*x**3*(d + e*x)**m
/(e**2*m**2 + 5*e**2*m + 6*e**2), True))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 174 vs. $2(52) = 104$.

Time = 0.04 (sec) , antiderivative size = 174, normalized size of antiderivative = 3.35

$$\int (d+ex)^m \left(ade+\left(cd^2+ae^2\right)x+cdex^2\right) dx$$
$$= \frac{(e^2(m+1)x^2+demx-d^2)(ex+d)^m a}{m^2+3m+2} + \frac{(e^2(m+1)x^2+demx-d^2)(ex+d)^m cd^2}{(m^2+3m+2)e^2} + \frac{(ex+d)^{m+1}ad}{m+1} + \frac{((m^2+3m+2)e^3x^3+(m^2+m)de^2x^2-2d^2emx+2d^3)(ex+d)^m cd}{(m^3+6m^2+11m+6)e^2}$$

input
```
integrate((e*x+d)^m*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2),x, algorithm="maxima
")
```

output
```
(e^2*(m + 1)*x^2 + d*e*m*x - d^2)*(e*x + d)^m*a/(m^2 + 3*m + 2) + (e^2*(m
+ 1)*x^2 + d*e*m*x - d^2)*(e*x + d)^m*c*d^2/((m^2 + 3*m + 2)*e^2) + (e*x +
 d)^(m + 1)*a*d/(m + 1) + ((m^2 + 3*m + 2)*e^3*x^3 + (m^2 + m)*d*e^2*x^2 -
 2*d^2*e*m*x + 2*d^3)*(e*x + d)^m*c*d/((m^3 + 6*m^2 + 11*m + 6)*e^2)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 219 vs. $2(52) = 104$.

Time = 0.12 (sec) , antiderivative size = 219, normalized size of antiderivative = 4.21

$$\int (d+ex)^m \left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)\,dx$$
$$= \frac{(ex+d)^m cde^3mx^3 + 2\,(ex+d)^m cd^2e^2mx^2 + (ex+d)^m ae^4mx^2 + 2\,(ex+d)^m cde^3x^3 + (ex+d)^m cd^3em}{}$$

input
```
integrate((e*x+d)^m*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2),x, algorithm="giac")
```

output
```
((e*x + d)^m*c*d*e^3*m*x^3 + 2*(e*x + d)^m*c*d^2*e^2*m*x^2 + (e*x + d)^m*a
*e^4*m*x^2 + 2*(e*x + d)^m*c*d*e^3*x^3 + (e*x + d)^m*c*d^3*e*m*x + 2*(e*x
+ d)^m*a*d*e^3*m*x + 3*(e*x + d)^m*c*d^2*e^2*x^2 + 3*(e*x + d)^m*a*e^4*x^2
 + (e*x + d)^m*a*d^2*e^2*m + 6*(e*x + d)^m*a*d*e^3*x - (e*x + d)^m*c*d^4 +
 3*(e*x + d)^m*a*d^2*e^2)/(e^2*m^2 + 5*e^2*m + 6*e^2)
```

Mupad [B] (verification not implemented)

Time = 5.38 (sec) , antiderivative size = 141, normalized size of antiderivative = 2.71

$$\int (d+ex)^m \left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)\,dx$$
$$= (d+e\,x)^m \left(\frac{x^2\,(3\,a\,e^2+3\,c\,d^2+a\,e^2\,m+2\,c\,d^2\,m)}{m^2+5\,m+6}+\frac{d^2\,(3\,a\,e^2-c\,d^2+a\,e^2\,m)}{e^2\,(m^2+5\,m+6)} + \frac{d\,x\,(6\,a\,e^2+2\,a\,e^2\,m+c\,d^2\,m)}{e\,(m^2+5\,m+6)}+\frac{c\,d\,e\,x^3\,(m+2)}{m^2+5\,m+6}\right)$$

input
```
int((d + e*x)^m*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2),x)
```

output
```
(d + e*x)^m*((x^2*(3*a*e^2 + 3*c*d^2 + a*e^2*m + 2*c*d^2*m))/(5*m + m^2 +
6) + (d^2*(3*a*e^2 - c*d^2 + a*e^2*m))/(e^2*(5*m + m^2 + 6)) + (d*x*(6*a*e
^2 + 2*a*e^2*m + c*d^2*m))/(e*(5*m + m^2 + 6)) + (c*d*e*x^3*(m + 2))/(5*m
+ m^2 + 6))
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 134, normalized size of antiderivative = 2.58

$$\int (d+ex)^m \left(ade+\left(cd^2+ae^2\right)x+cdex^2\right) dx$$
$$= \frac{(ex+d)^m \left(cd\,e^3m\,x^3+a\,e^4m\,x^2+2c\,d^2e^2m\,x^2+2cd\,e^3x^3+2ad\,e^3mx+3a\,e^4x^2+c\,d^3emx+3c\,d^2e^2x^2\right.}{e^2\left(m^2+5m+6\right)}$$

input
```
int((e*x+d)^m*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2),x)
```

output
```
((d + e*x)**m*(a*d**2*e**2*m + 3*a*d**2*e**2 + 2*a*d*e**3*m*x + 6*a*d*e**3
*x + a*e**4*m*x**2 + 3*a*e**4*x**2 - c*d**4 + c*d**3*e*m*x + 2*c*d**2*e**2
*m*x**2 + 3*c*d**2*e**2*x**2 + c*d*e**3*m*x**3 + 2*c*d*e**3*x**3))/(e**2*(
m**2 + 5*m + 6))
```

3.385 $\int \frac{(d+ex)^m}{ade+(cd^2+ae^2)x+cdex^2}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 54

$$\int \frac{(d+ex)^m}{ade+(cd^2+ae^2)x+cdex^2}\,dx = -\frac{(d+ex)^m \operatorname{Hypergeometric2F1}\left(1,m,1+m,\frac{cd(d+ex)}{cd^2-ae^2}\right)}{(cd^2-ae^2)\,m}$$

output
```
-(e*x+d)^m*hypergeom([1, m],[1+m],c*d*(e*x+d)/(-a*e^2+c*d^2))/(-a*e^2+c*d^
2)/m
```

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.00

$$\int \frac{(d+ex)^m}{ade+(cd^2+ae^2)x+cdex^2}\,dx = -\frac{(d+ex)^m \operatorname{Hypergeometric2F1}\left(1,m,1+m,\frac{cd(d+ex)}{cd^2-ae^2}\right)}{(cd^2-ae^2)\,m}$$

input
```
Integrate[(d + e*x)^m/(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2),x]
```

output
```
-(((d + e*x)^m*Hypergeometric2F1[1, m, 1 + m, (c*d*(d + e*x))/(c*d^2 - a*e
^2)])/((c*d^2 - a*e^2)*m))
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.057, Rules used = {1121, 78}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex)^m}{x(ae^2+cd^2)+ade+cdex^2}dx$$

$$\Big\downarrow \text{ 1121}$$

$$\int \frac{(d+ex)^{m-1}}{ae+cdx}dx$$

$$\Big\downarrow \text{ 78}$$

$$-\frac{(d+ex)^m \text{Hypergeometric2F1}\left(1,m,m+1,\frac{cd(d+ex)}{cd^2-ae^2}\right)}{m(cd^2-ae^2)}$$

input
```
Int[(d + e*x)^m/(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2),x]
```

output

```
-(((d + e*x)^m*Hypergeometric2F1[1, m, 1 + m, (c*d*(d + e*x))/(c*d^2 - a*e
^2)])/((c*d^2 - a*e^2)*m))
```

Defintions of rubi rules used

rule 78
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(b
*c - a*d)^n*((a + b*x)^(m + 1)/(b^(n + 1)*(m + 1)))*Hypergeometric2F1[-n, m
 + 1, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m}, x]
 &&  !IntegerQ[m] && IntegerQ[n]
```

rule 1121
```
Int[((d_) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Int[ExpandIntegrand[(d + e*x)^(m + p)*(a/d + (c/e)*x)^p, x], x]
/; FreeQ[{a, b, c, d, e, m, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && (Int
egerQ[p] || (GtQ[a, 0] && GtQ[d, 0] && LtQ[c, 0]))
```

Maple [F]

$$\int \frac{(ex+d)^m}{ade+(a\,e^2+c\,d^2)\,x+cd\,x^2e}dx$$

input
```
int((e*x+d)^m/(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e),x)
```

output
```
int((e*x+d)^m/(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e),x)
```

Fricas [F]

$$\int \frac{(d+ex)^m}{ade+(cd^2+ae^2)\,x+cdex^2}\,dx=\int \frac{(ex+d)^m}{cdex^2+ade+(cd^2+ae^2)x}\,dx$$

input
```
integrate((e*x+d)^m/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2),x, algorithm="fricas
")
```

output
```
integral((e*x + d)^m/(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x), x)
```

Sympy [F]

$$\int \frac{(d+ex)^m}{ade+(cd^2+ae^2)\,x+cdex^2}\,dx=\int \frac{(d+ex)^m}{(d+ex)\,(ae+cdx)}\,dx$$

input
```
integrate((e*x+d)**m/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2),x)
```

output
```
Integral((d + e*x)**m/((d + e*x)*(a*e + c*d*x)), x)
```

Maxima [F]

$$\int \frac{(d+ex)^m}{ade+(cd^2+ae^2)\,x+cdex^2}\,dx=\int \frac{(ex+d)^m}{cdex^2+ade+(cd^2+ae^2)x}\,dx$$

input
```
integrate((e*x+d)^m/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2),x, algorithm="maxima
")
```

output
```
integrate((e*x + d)^m/(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x), x)
```

Giac [F]

$$\int \frac{(d+ex)^m}{ade+(cd^2+ae^2)\,x+cdex^2}\,dx=\int \frac{(ex+d)^m}{cdex^2+ade+(cd^2+ae^2)x}\,dx$$

input
```
integrate((e*x+d)^m/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2),x, algorithm="giac")
```

output
```
integrate((e*x + d)^m/(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d+ex)^m}{ade+(cd^2+ae^2)\,x+cdex^2}\,dx=\int \frac{(d+e\,x)^m}{c\,d\,e\,x^2+(c\,d^2+a\,e^2)\;x+a\,d\,e}\,dx$$

input
```
int((d + e*x)^m/(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2),x)
```

output
```
int((d + e*x)^m/(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2), x)
```

Reduce [F]

$$\int \frac{(d+ex)^m}{ade+(cd^2+ae^2)\,x+cdex^2}\,dx=\int \frac{(ex+d)^m}{cde\,x^2+a\,e^2x+c\,d^2x+ade}dx$$

input
```
int((e*x+d)^m/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2),x)
```

output
```
int((d + e*x)**m/(a*d*e + a*e**2*x + c*d**2*x + c*d*e*x**2),x)
```

3.386 $\int \frac{(d+ex)^m}{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^2}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 61

$$\int \frac{(d+ex)^m}{(ade+(cd^2+ae^2)\,x+cdex^2)^2}\,dx = -\frac{e(d+ex)^{-1+m}\,\text{Hypergeometric2F1}\left(2,-1+m,m,\frac{cd(d+ex)}{cd^2-ae^2}\right)}{(cd^2-ae^2)^2\,(1-m)}$$

output
```
-e*(e*x+d)^(-1+m)*hypergeom([2, -1+m],[m],c*d*(e*x+d)/(-a*e^2+c*d^2))/(-a*
e^2+c*d^2)^2/(1-m)
```

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.97

$$\int \frac{(d+ex)^m}{(ade+(cd^2+ae^2)\,x+cdex^2)^2}\,dx = \frac{e(d+ex)^{-1+m}\,\text{Hypergeometric2F1}\left(2,-1+m,m,-\frac{cd(d+ex)}{-cd^2+ae^2}\right)}{(-cd^2+ae^2)^2\,(-1+m)}$$

input
```
Integrate[(d + e*x)^m/(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^2,x]
```

output
```
(e*(d + e*x)^(-1 + m)*Hypergeometric2F1[2, -1 + m, m, -((c*d*(d + e*x))/(-
(c*d^2) + a*e^2))])/((-(c*d^2) + a*e^2)^2*(-1 + m))
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.057$, Rules used = $\{1121, 78\}$

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex)^m}{(x(ae^2+cd^2)+ade+cdex^2)^2}dx$$

$$\Big\downarrow \text{1121}$$

$$\int \frac{(d+ex)^{m-2}}{(ae+cdx)^2}dx$$

$$\Big\downarrow \text{78}$$

$$-\frac{e(d+ex)^{m-1}\text{Hypergeometric2F1}\left(2,m-1,m,\frac{cd(d+ex)}{cd^2-ae^2}\right)}{(1-m)(cd^2-ae^2)^2}$$

input
```
Int[(d + e*x)^m/(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^2,x]
```

output
```
-((e*(d + e*x)^(-1 + m)*Hypergeometric2F1[2, -1 + m, m, (c*d*(d + e*x))/(c
*d^2 - a*e^2)])/((c*d^2 - a*e^2)^2*(1 - m)))
```

Defintions of rubi rules used

rule 78
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(b
*c - a*d)^n*((a + b*x)^(m + 1)/(b^(n + 1)*(m + 1)))*Hypergeometric2F1[-n, m
 + 1, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m}, x]
 && !IntegerQ[m] && IntegerQ[n]
```

rule 1121
```
Int[((d_) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Int[ExpandIntegrand[(d + e*x)^(m + p)*(a/d + (c/e)*x)^p, x], x]
/; FreeQ[{a, b, c, d, e, m, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && (Int
egerQ[p] || (GtQ[a, 0] && GtQ[d, 0] && LtQ[c, 0]))
```

Maple [F]

$$\int \frac{(ex+d)^m}{\left(ade+\left(a\,e^2+c\,d^2\right)x+cd\,x^2e\right)^2}dx$$

input
```
int((e*x+d)^m/(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^2,x)
```

output
```
int((e*x+d)^m/(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^2,x)
```

Fricas [F]

$$\int \frac{(d+ex)^m}{(ade+(cd^2+ae^2)\,x+cdex^2)^2}\,dx=\int \frac{(ex+d)^m}{(cdex^2+ade+(cd^2+ae^2)x)^2}\,dx$$

input
```
integrate((e*x+d)^m/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^2,x, algorithm="fric
as")
```

output
```
integral((e*x + d)^m/(c^2*d^2*e^2*x^4 + a^2*d^2*e^2 + 2*(c^2*d^3*e + a*c*d
*e^3)*x^3 + (c^2*d^4 + 4*a*c*d^2*e^2 + a^2*e^4)*x^2 + 2*(a*c*d^3*e + a^2*d
*e^3)*x), x)
```

Sympy [F]

$$\int \frac{(d+ex)^m}{(ade+(cd^2+ae^2)\,x+cdex^2)^2}\,dx = \int \frac{(d+ex)^m}{(d+ex)^2\,(ae+cdx)^2}\,dx$$

input
```
integrate((e*x+d)**m/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**2,x)
```

output
```
Integral((d + e*x)**m/((d + e*x)**2*(a*e + c*d*x)**2), x)
```

Maxima [F]

$$\int \frac{(d+ex)^m}{(ade+(cd^2+ae^2)\,x+cdex^2)^2}\,dx = \int \frac{(ex+d)^m}{(cdex^2+ade+(cd^2+ae^2)x)^2}\,dx$$

input
```
integrate((e*x+d)^m/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^2,x, algorithm="maxi
ma")
```

output
```
integrate((e*x + d)^m/(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^2, x)
```

Giac [F]

$$\int \frac{(d+ex)^m}{(ade+(cd^2+ae^2)\,x+cdex^2)^2}\,dx = \int \frac{(ex+d)^m}{(cdex^2+ade+(cd^2+ae^2)x)^2}\,dx$$

input
```
integrate((e*x+d)^m/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^2,x, algorithm="giac
")
```

output
```
integrate((e*x + d)^m/(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d+ex)^m}{(ade+(cd^2+ae^2)\,x+cdex^2)^2}\,dx = \int \frac{(d+e\,x)^m}{\left(c\,d\,e\,x^2+(c\,d^2+a\,e^2)\,\,x+a\,d\,e\right)^2}\,dx$$

input
```
int((d + e*x)^m/(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^2,x)
```

output
```
int((d + e*x)^m/(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^2, x)
```

Reduce [F]

$$\int \frac{(d+ex)^m}{(ade+(cd^2+ae^2)\,x+cdex^2)^2}\,dx$$
$$= \int \frac{(ex+d)^m}{c^2d^2e^2x^4+2acd\,e^3x^3+2c^2d^3e\,x^3+a^2e^4x^2+4ac\,d^2e^2x^2+c^2d^4x^2+2a^2d\,e^3x+2ac\,d^3ex+a^2d^2e^2}dx$$

input
```
int((e*x+d)^m/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^2,x)
```

output
```
int((d + e*x)**m/(a**2*d**2*e**2 + 2*a**2*d*e**3*x + a**2*e**4*x**2 + 2*a*
c*d**3*e*x + 4*a*c*d**2*e**2*x**2 + 2*a*c*d*e**3*x**3 + c**2*d**4*x**2 + 2
*c**2*d**3*e*x**3 + c**2*d**2*e**2*x**4),x)
```

3.387 $\int \frac{(d+ex)^m}{(ade+(cd^2+ae^2)x+cdex^2)^3}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 64

$$\int \frac{(d+ex)^m}{(ade+(cd^2+ae^2)\,x+cdex^2)^3}\,dx = \frac{e^2(d+ex)^{-2+m}\,\text{Hypergeometric2F1}\left(3,-2+m,-1+m,\frac{cd(d+ex)}{cd^2-ae^2}\right)}{(cd^2-ae^2)^3\,(2-m)}$$

output
```
e^2*(e*x+d)^(-2+m)*hypergeom([3, -2+m],[-1+m],c*d*(e*x+d)/(-a*e^2+c*d^2))/
(-a*e^2+c*d^2)^3/(2-m)
```

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.98

$$\int \frac{(d+ex)^m}{(ade+(cd^2+ae^2)\,x+cdex^2)^3}\,dx = \frac{e^2(d+ex)^{-2+m}\,\text{Hypergeometric2F1}\left(3,-2+m,-1+m,-\frac{cd(d+ex)}{-cd^2+ae^2}\right)}{(-cd^2+ae^2)^3\,(-2+m)}$$

input
```
Integrate[(d + e*x)^m/(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^3,x]
```

output
```
(e^2*(d + e*x)^(-2 + m)*Hypergeometric2F1[3, -2 + m, -1 + m, -((c*d*(d + e
*x))/(-(c*d^2) + a*e^2))])/((-(c*d^2) + a*e^2)^3*(-2 + m))
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.057$, Rules used = $\{1121, 78\}$

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex)^m}{(x(ae^2+cd^2)+ade+cdex^2)^3}dx$$

$$\Big\downarrow \text{1121}$$

$$\int \frac{(d+ex)^{m-3}}{(ae+cdx)^3}dx$$

$$\Big\downarrow \text{78}$$

$$\frac{e^2(d+ex)^{m-2}\text{Hypergeometric2F1}\left(3,m-2,m-1,\frac{cd(d+ex)}{cd^2-ae^2}\right)}{(2-m)(cd^2-ae^2)^3}$$

input
```
Int[(d + e*x)^m/(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^3,x]
```

output
```
(e^2*(d + e*x)^(-2 + m)*Hypergeometric2F1[3, -2 + m, -1 + m, (c*d*(d + e*x
))/(c*d^2 - a*e^2)])/((c*d^2 - a*e^2)^3*(2 - m))
```

Defintions of rubi rules used

rule 78
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(b
*c - a*d)^n*((a + b*x)^(m + 1)/(b^(n + 1)*(m + 1)))*Hypergeometric2F1[-n, m
 + 1, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m}, x]
 &&  !IntegerQ[m] && IntegerQ[n]
```

rule 1121
```
Int[((d_) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Int[ExpandIntegrand[(d + e*x)^(m + p)*(a/d + (c/e)*x)^p, x], x]
/; FreeQ[{a, b, c, d, e, m, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && (Int
egerQ[p] || (GtQ[a, 0] && GtQ[d, 0] && LtQ[c, 0]))
```

Maple [F]

$$\int \frac{(ex+d)^m}{(ade+(a\,e^2+c\,d^2)\,x+cd\,x^2e)^3}dx$$

input
```
int((e*x+d)^m/(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^3,x)
```

output
```
int((e*x+d)^m/(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^3,x)
```

Fricas [F]

$$\int \frac{(d+ex)^m}{(ade+(cd^2+ae^2)\,x+cdex^2)^3}\,dx = \int \frac{(ex+d)^m}{(cdex^2+ade+(cd^2+ae^2)x)^3}\,dx$$

input
```
integrate((e*x+d)^m/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^3,x, algorithm="fric
as")
```

output
```
integral((e*x + d)^m/(c^3*d^3*e^3*x^6 + a^3*d^3*e^3 + 3*(c^3*d^4*e^2 + a*c
^2*d^2*e^4)*x^5 + 3*(c^3*d^5*e + 3*a*c^2*d^3*e^3 + a^2*c*d*e^5)*x^4 + (c^3
*d^6 + 9*a*c^2*d^4*e^2 + 9*a^2*c*d^2*e^4 + a^3*e^6)*x^3 + 3*(a*c^2*d^5*e +
 3*a^2*c*d^3*e^3 + a^3*d*e^5)*x^2 + 3*(a^2*c*d^4*e^2 + a^3*d^2*e^4)*x), x)
```

Sympy [F]

$$\int \frac{(d+ex)^m}{(ade+(cd^2+ae^2)\,x+cdex^2)^3}\,dx = \int \frac{(d+ex)^m}{(d+ex)^3\,(ae+cdx)^3}\,dx$$

input
```
integrate((e*x+d)**m/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**3,x)
```

output
```
Integral((d + e*x)**m/((d + e*x)**3*(a*e + c*d*x)**3), x)
```

Maxima [F]

$$\int \frac{(d+ex)^m}{(ade+(cd^2+ae^2)\,x+cdex^2)^3}\,dx = \int \frac{(ex+d)^m}{(cdex^2+ade+(cd^2+ae^2)x)^3}\,dx$$

input
```
integrate((e*x+d)^m/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^3,x, algorithm="maxi
ma")
```

output
```
integrate((e*x + d)^m/(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^3, x)
```

Giac [F]

$$\int \frac{(d+ex)^m}{(ade+(cd^2+ae^2)\,x+cdex^2)^3}\,dx = \int \frac{(ex+d)^m}{(cdex^2+ade+(cd^2+ae^2)x)^3}\,dx$$

input
```
integrate((e*x+d)^m/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^3,x, algorithm="giac
")
```

output
```
integrate((e*x + d)^m/(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d+ex)^m}{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^3}\,dx=\int \frac{{\left(d+e\,x\right)}^m}{{\left(c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e\right)}^3}\,dx$$

input
```
int((d + e*x)^m/(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^3,x)
```

output
```
int((d + e*x)^m/(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^3, x)
```

Reduce [F]

$$\int \frac{(d+ex)^m}{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^3}\,dx$$
$$=\int \frac{(ex+d}{c^3d^3e^3x^6+3a\,c^2d^2e^4x^5+3c^3d^4e^2x^5+3a^2cd\,e^5x^4+9a\,c^2d^3e^3x^4+3c^3d^5e\,x^4+a^3e^6x^3+9a^2c\,d^2e^4x^3+}$$

input
```
int((e*x+d)^m/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^3,x)
```

output
```
int((d + e*x)**m/(a**3*d**3*e**3 + 3*a**3*d**2*e**4*x + 3*a**3*d*e**5*x**2
 + a**3*e**6*x**3 + 3*a**2*c*d**4*e**2*x + 9*a**2*c*d**3*e**3*x**2 + 9*a**
2*c*d**2*e**4*x**3 + 3*a**2*c*d*e**5*x**4 + 3*a*c**2*d**5*e*x**2 + 9*a*c**
2*d**4*e**2*x**3 + 9*a*c**2*d**3*e**3*x**4 + 3*a*c**2*d**2*e**4*x**5 + c**
3*d**6*x**3 + 3*c**3*d**5*e*x**4 + 3*c**3*d**4*e**2*x**5 + c**3*d**3*e**3*
x**6),x)
```

3.388 $\int \frac{(d+ex)^m}{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^4}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 65

$$\int \frac{(d+ex)^m}{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^4}\,dx = -\frac{e^3(d+ex)^{-3+m}\,\text{Hypergeometric2F1}\left(4,-3+m,-2+m,\frac{cd(d+ex)}{cd^2-ae^2}\right)}{\left(cd^2-ae^2\right)^4(3-m)}$$

output
```
-e^3*(e*x+d)^(-3+m)*hypergeom([4, -3+m],[-2+m],c*d*(e*x+d)/(-a*e^2+c*d^2))
/(-a*e^2+c*d^2)^4/(3-m)
```

Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.97

$$\int \frac{(d+ex)^m}{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^4}\,dx = \frac{e^3(d+ex)^{-3+m}\,\text{Hypergeometric2F1}\left(4,-3+m,-2+m,-\frac{cd(d+ex)}{-cd^2+ae^2}\right)}{\left(-cd^2+ae^2\right)^4(-3+m)}$$

input
```
Integrate[(d + e*x)^m/(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^4,x]
```

output
```
(e^3*(d + e*x)^(-3 + m)*Hypergeometric2F1[4, -3 + m, -2 + m, -((c*d*(d + e
*x))/(-(c*d^2) + a*e^2))])/((-(c*d^2) + a*e^2)^4*(-3 + m))
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.057$, Rules used = $\{1121, 78\}$

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex)^m}{(x(ae^2+cd^2)+ade+cdex^2)^4}dx$$

$$\Big\downarrow \; 1121$$

$$\int \frac{(d+ex)^{m-4}}{(ae+cdx)^4}dx$$

$$\Big\downarrow \; 78$$

$$-\frac{e^3(d+ex)^{m-3}\text{Hypergeometric2F1}\left(4,m-3,m-2,\frac{cd(d+ex)}{cd^2-ae^2}\right)}{(3-m)(cd^2-ae^2)^4}$$

input
```
Int[(d + e*x)^m/(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^4,x]
```

output
```
-((e^3*(d + e*x)^(-3 + m)*Hypergeometric2F1[4, -3 + m, -2 + m, (c*d*(d + e
*x))/(c*d^2 - a*e^2)])/((c*d^2 - a*e^2)^4*(3 - m)))
```

Defintions of rubi rules used

rule 78
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(b
*c - a*d)^n*((a + b*x)^(m + 1)/(b^(n + 1)*(m + 1)))*Hypergeometric2F1[-n, m
 + 1, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m}, x]
 &&  !IntegerQ[m] && IntegerQ[n]
```

rule 1121
```
Int[((d_) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Int[ExpandIntegrand[(d + e*x)^(m + p)*(a/d + (c/e)*x)^p, x], x]
/; FreeQ[{a, b, c, d, e, m, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && (Int
egerQ[p] || (GtQ[a, 0] && GtQ[d, 0] && LtQ[c, 0]))
```

Maple [F]

$$\int \frac{(ex+d)^m}{(ade+(a\,e^2+c\,d^2)\,x+cd\,x^2e)^4}dx$$

input
```
int((e*x+d)^m/(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^4,x)
```

output
```
int((e*x+d)^m/(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^4,x)
```

Fricas [F]

$$\int \frac{(d+ex)^m}{(ade+(cd^2+ae^2)\,x+cdex^2)^4}\,dx=\int \frac{(ex+d)^m}{(cdex^2+ade+(cd^2+ae^2)x)^4}\,dx$$

input
```
integrate((e*x+d)^m/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^4,x, algorithm="fric
as")
```

output
```
integral((e*x + d)^m/(c^4*d^4*e^4*x^8 + a^4*d^4*e^4 + 4*(c^4*d^5*e^3 + a*c
^3*d^3*e^5)*x^7 + 2*(3*c^4*d^6*e^2 + 8*a*c^3*d^4*e^4 + 3*a^2*c^2*d^2*e^6)*
x^6 + 4*(c^4*d^7*e + 6*a*c^3*d^5*e^3 + 6*a^2*c^2*d^3*e^5 + a^3*c*d*e^7)*x^
5 + (c^4*d^8 + 16*a*c^3*d^6*e^2 + 36*a^2*c^2*d^4*e^4 + 16*a^3*c*d^2*e^6 +
a^4*e^8)*x^4 + 4*(a*c^3*d^7*e + 6*a^2*c^2*d^5*e^3 + 6*a^3*c*d^3*e^5 + a^4*
d*e^7)*x^3 + 2*(3*a^2*c^2*d^6*e^2 + 8*a^3*c*d^4*e^4 + 3*a^4*d^2*e^6)*x^2 +
 4*(a^3*c*d^5*e^3 + a^4*d^3*e^5)*x), x)
```

Sympy [F(-2)]

Exception generated.

$$\int \frac{(d+ex)^m}{(ade+(cd^2+ae^2)\,x+cdex^2)^4}\,dx = \text{Exception raised: HeuristicGCDFailed}$$

input
```
integrate((e*x+d)**m/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**4,x)
```

output
```
Exception raised: HeuristicGCDFailed >> no luck
```

Maxima [F]

$$\int \frac{(d+ex)^m}{(ade+(cd^2+ae^2)\,x+cdex^2)^4}\,dx = \int \frac{(ex+d)^m}{(cdex^2+ade+(cd^2+ae^2)x)^4}\,dx$$

input
```
integrate((e*x+d)^m/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^4,x, algorithm="maxi
ma")
```

output
```
integrate((e*x + d)^m/(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^4, x)
```

Giac [F]

$$\int \frac{(d+ex)^m}{(ade+(cd^2+ae^2)\,x+cdex^2)^4}\,dx=\int \frac{(ex+d)^m}{(cdex^2+ade+(cd^2+ae^2)x)^4}\,dx$$

input
```
integrate((e*x+d)^m/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^4,x, algorithm="giac
")
```

output
```
integrate((e*x + d)^m/(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^4, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d+ex)^m}{(ade+(cd^2+ae^2)\,x+cdex^2)^4}\,dx=\int \frac{(d+e\,x)^m}{(c\,d\,e\,x^2+(c\,d^2+a\,e^2)\,x+a\,d\,e)^4}\,dx$$

input
```
int((d + e*x)^m/(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^4,x)
```

output
```
int((d + e*x)^m/(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^4, x)
```

Reduce [F]

$$\int \frac{(d+ex)^m}{(ade+(cd^2+ae^2)\,x+cdex^2)^4}\,dx$$
$$=\int \frac{}{c^4d^4e^4x^8+4a\,c^3d^3e^5x^7+4c^4d^5e^3x^7+6a^2c^2d^2e^6x^6+16a\,c^3d^4e^4x^6+6c^4d^6e^2x^6+4a^3cd\,e^7x^5+24a^2c^2}$$

input
```
int((e*x+d)^m/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^4,x)
```

output

```
int((d + e*x)**m/(a**4*d**4*e**4 + 4*a**4*d**3*e**5*x + 6*a**4*d**2*e**6*x
**2 + 4*a**4*d*e**7*x**3 + a**4*e**8*x**4 + 4*a**3*c*d**5*e**3*x + 16*a**3
*c*d**4*e**4*x**2 + 24*a**3*c*d**3*e**5*x**3 + 16*a**3*c*d**2*e**6*x**4 +
4*a**3*c*d*e**7*x**5 + 6*a**2*c**2*d**6*e**2*x**2 + 24*a**2*c**2*d**5*e**3
*x**3 + 36*a**2*c**2*d**4*e**4*x**4 + 24*a**2*c**2*d**3*e**5*x**5 + 6*a**2
*c**2*d**2*e**6*x**6 + 4*a*c**3*d**7*e*x**3 + 16*a*c**3*d**6*e**2*x**4 + 2
4*a*c**3*d**5*e**3*x**5 + 16*a*c**3*d**4*e**4*x**6 + 4*a*c**3*d**3*e**5*x*
*7 + c**4*d**8*x**4 + 4*c**4*d**7*e*x**5 + 6*c**4*d**6*e**2*x**6 + 4*c**4*
d**5*e**3*x**7 + c**4*d**4*e**4*x**8),x)
```

3.389 $\int (d+ex)^3 \left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^p \, dx$

Optimal result

Integrand size = 35, antiderivative size = 89

$$\int (d+ex)^3 \left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^p \, dx = -\frac{(d+ex)^3 \left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{1+p} \text{Hypergeometric2F1}\left(1,5+2p,5+p,\frac{cd(d+ex)}{cd^2-ae^2}\right)}{\left(cd^2-ae^2\right)(4+p)}$$

output
```
-(e*x+d)^3*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(p+1)*hypergeom([1, 5+2*p],[5
+p],c*d*(e*x+d)/(-a*e^2+c*d^2))/(-a*e^2+c*d^2)/(4+p)
```

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.26

$$\int (d+ex)^3 \left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^p \, dx = \frac{\left(cd^2-ae^2\right)^3 (ae+cdx) \left(\frac{cd(d+ex)}{cd^2-ae^2}\right)^{-p} ((ae+cdx)(d+ex))^p \text{Hypergeometric2F1}\left(-3-p,1+p,2+p,\frac{e(}{-}\right.}{c^4d^4(1+p)}$$

input
```
Integrate[(d + e*x)^3*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^p,x]
```

output
```
((c*d^2 - a*e^2)^3*(a*e + c*d*x)*((a*e + c*d*x)*(d + e*x))^p*Hypergeometri
c2F1[-3 - p, 1 + p, 2 + p, (e*(a*e + c*d*x))/(-(c*d^2) + a*e^2)])/(c^4*d^4
*(1 + p)*((c*d*(d + e*x))/(c*d^2 - a*e^2))^p)
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 124, normalized size of antiderivative = 1.39, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.086, Rules used = {1138, 80, 79}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (d+ex)^3 \left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^p \, dx$$

$$\Big\downarrow \text{ 1138}$$

$$d^3\left(\frac{ex}{d}+1\right)^{-p}(ae+cdx)^{-p}\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^p\int (ae+cdx)^p\left(\frac{ex}{d}+1\right)^{p+3}dx$$

$$\Big\downarrow \text{ 80}$$

$$\frac{\left(cd^2-ae^2\right)^3(ae+cdx)^{-p}\left(\frac{cd(d+ex)}{cd^2-ae^2}\right)^{-p}\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^p\int (ae+cdx)^p\left(\frac{cd^2}{cd^2-ae^2}+\frac{cexd}{cd^2-ae^2}\right)^{p+3}dx}{c^3d^3}$$

$$\Big\downarrow \text{ 79}$$

$$\frac{\left(cd^2-ae^2\right)^3(ae+cdx)\left(\frac{cd(d+ex)}{cd^2-ae^2}\right)^{-p}\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^p\text{Hypergeometric2F1}\left(-p-3,p+1,p+2,\right.}{c^4d^4(p+1)}$$

input
```
Int[(d + e*x)^3*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^p,x]
```

output
```
((c*d^2 - a*e^2)^3*(a*e + c*d*x)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^p
*Hypergeometric2F1[-3 - p, 1 + p, 2 + p, -((e*(a*e + c*d*x))/(c*d^2 - a*e^
2))])/(c^4*d^4*(1 + p)*((c*d*(d + e*x))/(c*d^2 - a*e^2))^p)
```

Defintions of rubi rules used

rule 79
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d), 0] && (RationalQ[m]
 ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

rule 80
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*(b*((c + d*x)/(b*c - a*d)))
^FracPart[n])   Int[(a + b*x)^m*Simp[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d)
), x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] &&  !IntegerQ[m] &&  !Integ
erQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

rule 1138
```
Int[((d_) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Sy
mbol] :> Simp[d^m*((a + b*x + c*x^2)^FracPart[p]/((1 + e*(x/d))^FracPart[p]
*(a/d + (c*x)/e)^FracPart[p]))   Int[(1 + e*(x/d))^(m + p)*(a/d + (c/e)*x)^
p, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0]
&& (IntegerQ[m] || GtQ[d, 0]) &&  !(IGtQ[m, 0] && (IntegerQ[3*p] || Integer
Q[4*p]))
```

Maple [F]

$$\int (ex+d)^3 \left(ade+\left(a\,e^2+c\,d^2\right)x+cd\,x^2e\right)^p dx$$

input
```
int((e*x+d)^3*(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^p,x)
```

output
```
int((e*x+d)^3*(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^p,x)
```

Fricas [F]

$$\int (d+ex)^3 \left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^p\,dx = \int (ex+d)^3 \left(cdex^2+ade+\left(cd^2+ae^2\right)x\right)^p\,dx$$

input
```
integrate((e*x+d)^3*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^p,x, algorithm="fric
as")
```

output
```
integral((e^3*x^3 + 3*d*e^2*x^2 + 3*d^2*e*x + d^3)*(c*d*e*x^2 + a*d*e + (c
*d^2 + a*e^2)*x)^p, x)
```

Sympy [F(-1)]

Timed out.

$$\int (d+ex)^3 \left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^p\,dx = \text{Timed out}$$

input
```
integrate((e*x+d)**3*(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**p,x)
```

output
```
Timed out
```

Maxima [F]

$$\int (d+ex)^3 \left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^p\,dx = \int (ex+d)^3 \left(cdex^2+ade+\left(cd^2+ae^2\right)x\right)^p\,dx$$

input
```
integrate((e*x+d)^3*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^p,x, algorithm="maxi
ma")
```

output
```
integrate((e*x + d)^3*(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^p, x)
```

Giac [F]

$$\int (d+ex)^3 \left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^p\,dx$$
$$=\int (ex+d)^3 \left(cdex^2+ade+\left(cd^2+ae^2\right)x\right)^p\,dx$$

input
```
integrate((e*x+d)^3*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^p,x, algorithm="giac
")
```

output
```
integrate((e*x + d)^3*(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^p, x)
```

Mupad [F(-1)]

Timed out.

$$\int (d+ex)^3 \left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^p\,dx$$
$$=\int (d+e\,x)^3 \left(c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e\right)^p\,dx$$

input
```
int((d + e*x)^3*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^p,x)
```

output
```
int((d + e*x)^3*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^p, x)
```

Reduce [F]

$$\int (d+ex)^3 \left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^p\,dx=\text{too large to display}$$

input
```
int((e*x+d)^3*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^p,x)
```

output
```
( - (a*d*e + a*e**2*x + c*d**2*x + c*d*e*x**2)**p*a**4*d*e**7*p**2 - 5*(a*
d*e + a*e**2*x + c*d**2*x + c*d*e*x**2)**p*a**4*d*e**7*p - 6*(a*d*e + a*e*
*2*x + c*d**2*x + c*d*e*x**2)**p*a**4*d*e**7 + (a*d*e + a*e**2*x + c*d**2*
x + c*d*e*x**2)**p*a**4*e**8*p**3*x + 5*(a*d*e + a*e**2*x + c*d**2*x + c*d
*e*x**2)**p*a**4*e**8*p**2*x + 6*(a*d*e + a*e**2*x + c*d**2*x + c*d*e*x**2
)**p*a**4*e**8*p*x + 7*(a*d*e + a*e**2*x + c*d**2*x + c*d*e*x**2)**p*a**3*
c*d**3*e**5*p**2 + 29*(a*d*e + a*e**2*x + c*d**2*x + c*d*e*x**2)**p*a**3*c
*d**3*e**5*p + 24*(a*d*e + a*e**2*x + c*d**2*x + c*d*e*x**2)**p*a**3*c*d**
3*e**5 - 6*(a*d*e + a*e**2*x + c*d**2*x + c*d*e*x**2)**p*a**3*c*d**2*e**6*
p**3*x - 24*(a*d*e + a*e**2*x + c*d**2*x + c*d*e*x**2)**p*a**3*c*d**2*e**6
*p**2*x - 18*(a*d*e + a*e**2*x + c*d**2*x + c*d*e*x**2)**p*a**3*c*d**2*e**
6*p*x - 2*(a*d*e + a*e**2*x + c*d**2*x + c*d*e*x**2)**p*a**3*c*d*e**7*p**3
*x**2 - 7*(a*d*e + a*e**2*x + c*d**2*x + c*d*e*x**2)**p*a**3*c*d*e**7*p**2
*x**2 - 3*(a*d*e + a*e**2*x + c*d**2*x + c*d*e*x**2)**p*a**3*c*d*e**7*p*x*
*2 - 23*(a*d*e + a*e**2*x + c*d**2*x + c*d*e*x**2)**p*a**2*c**2*d**5*e**3*
p**2 - 61*(a*d*e + a*e**2*x + c*d**2*x + c*d*e*x**2)**p*a**2*c**2*d**5*e**
3*p - 36*(a*d*e + a*e**2*x + c*d**2*x + c*d*e*x**2)**p*a**2*c**2*d**5*e**3
 + 16*(a*d*e + a*e**2*x + c*d**2*x + c*d*e*x**2)**p*a**2*c**2*d**4*e**4*p*
*3*x + 32*(a*d*e + a*e**2*x + c*d**2*x + c*d*e*x**2)**p*a**2*c**2*d**4*e**
4*p**2*x + 12*(a*d*e + a*e**2*x + c*d**2*x + c*d*e*x**2)**p*a**2*c**2*d...
```

3.390 $\int (d+ex)^2 \left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^p \,dx$

Optimal result

Integrand size = 35, antiderivative size = 89

$$\int (d+ex)^2 \left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^p \,dx = -\frac{(d+ex)^2 \left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{1+p} \text{Hypergeometric2F1}\left(1,2(2+p),4+p,\frac{cd(d+ex)}{cd^2-ae^2}\right)}{\left(cd^2-ae^2\right)(3+p)}$$

output
```
-(e*x+d)^2*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(p+1)*hypergeom([1, 4+2*p],[4
+p],c*d*(e*x+d)/(-a*e^2+c*d^2))/(-a*e^2+c*d^2)/(3+p)
```

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.26

$$\int (d+ex)^2 \left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^p \,dx$$
$$= \frac{\left(cd^2-ae^2\right)^2 (ae+cdx)\left(\frac{cd(d+ex)}{cd^2-ae^2}\right)^{-p} ((ae+cdx)(d+ex))^p \text{Hypergeometric2F1}\left(-2-p,1+p,2+p,\frac{e(}{-}\right.}{c^3d^3(1+p)}$$

input
```
Integrate[(d + e*x)^2*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^p,x]
```

output
```
((c*d^2 - a*e^2)^2*(a*e + c*d*x)*((a*e + c*d*x)*(d + e*x))^p*Hypergeometri
c2F1[-2 - p, 1 + p, 2 + p, (e*(a*e + c*d*x))/(-(c*d^2) + a*e^2)])/(c^3*d^3
*(1 + p)*((c*d*(d + e*x))/(c*d^2 - a*e^2))^p)
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 124, normalized size of antiderivative = 1.39, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.086$, Rules used = {1138, 80, 79}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (d+ex)^2 \left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^p dx$$

$$\Big\downarrow \text{1138}$$

$$d^2\left(\frac{ex}{d}+1\right)^{-p}(ae+cdx)^{-p}\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^p\int (ae+cdx)^p\left(\frac{ex}{d}+1\right)^{p+2}dx$$

$$\Big\downarrow \text{80}$$

$$\frac{\left(cd^2-ae^2\right)^2(ae+cdx)^{-p}\left(\frac{cd(d+ex)}{cd^2-ae^2}\right)^{-p}\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^p\int (ae+cdx)^p\left(\frac{cd^2}{cd^2-ae^2}+\frac{cexd}{cd^2-ae^2}\right)^{p+2}dx}{c^2d^2}$$

$$\Big\downarrow \text{79}$$

$$\frac{\left(cd^2-ae^2\right)^2(ae+cdx)\left(\frac{cd(d+ex)}{cd^2-ae^2}\right)^{-p}\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^p\text{Hypergeometric2F1}\left(-p-2,p+1,p+2,\right.}{c^3d^3(p+1)}$$

input
```
Int[(d + e*x)^2*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^p,x]
```

output
```
((c*d^2 - a*e^2)^2*(a*e + c*d*x)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^p
*Hypergeometric2F1[-2 - p, 1 + p, 2 + p, -((e*(a*e + c*d*x))/(c*d^2 - a*e^
2))])/(c^3*d^3*(1 + p)*((c*d*(d + e*x))/(c*d^2 - a*e^2))^p)
```

Defintions of rubi rules used

rule 79
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d), 0] && (RationalQ[m]
 ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

rule 80
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*(b*((c + d*x)/(b*c - a*d)))
^FracPart[n])   Int[(a + b*x)^m*Simp[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d)
), x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] &&  !IntegerQ[m] &&  !Integ
erQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

rule 1138
```
Int[((d_) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Sy
mbol] :> Simp[d^m*((a + b*x + c*x^2)^FracPart[p]/((1 + e*(x/d))^FracPart[p]
*(a/d + (c*x)/e)^FracPart[p]))   Int[(1 + e*(x/d))^(m + p)*(a/d + (c/e)*x)^
p, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0]
&& (IntegerQ[m] || GtQ[d, 0]) &&  !(IGtQ[m, 0] && (IntegerQ[3*p] || Integer
Q[4*p]))
```

Maple [F]

$$\int (ex+d)^2 \left(ade+\left(a\,e^2+c\,d^2\right)x+cd\,x^2e\right)^p dx$$

input
```
int((e*x+d)^2*(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^p,x)
```

output
```
int((e*x+d)^2*(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^p,x)
```

Fricas [F]

$$\int (d+ex)^2 \left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^p\,dx = \int (ex+d)^2 \left(cdex^2+ade+\left(cd^2+ae^2\right)x\right)^p\,dx$$

input
```
integrate((e*x+d)^2*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^p,x, algorithm="fric
as")
```

output
```
integral((e^2*x^2 + 2*d*e*x + d^2)*(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)
^p, x)
```

Sympy [F(-1)]

Timed out.

$$\int (d+ex)^2 \left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^p\,dx = \text{Timed out}$$

input
```
integrate((e*x+d)**2*(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**p,x)
```

output
```
Timed out
```

Maxima [F]

$$\int (d+ex)^2 \left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^p\,dx = \int (ex+d)^2 \left(cdex^2+ade+\left(cd^2+ae^2\right)x\right)^p\,dx$$

input
```
integrate((e*x+d)^2*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^p,x, algorithm="maxi
ma")
```

output
```
integrate((e*x + d)^2*(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^p, x)
```

Giac [F]

$$\int (d+ex)^2 \left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^p \, dx$$
$$= \int (ex+d)^2 \left(cdex^2+ade+\left(cd^2+ae^2\right)x\right)^p \, dx$$

input
```
integrate((e*x+d)^2*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^p,x, algorithm="giac
")
```

output
```
integrate((e*x + d)^2*(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^p, x)
```

Mupad [F(-1)]

Timed out.

$$\int (d+ex)^2 \left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^p \, dx$$
$$= \int (d+e\,x)^2 \left(c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e\right)^p \,dx$$

input
```
int((d + e*x)^2*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^p,x)
```

output
```
int((d + e*x)^2*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^p, x)
```

Reduce [F]

$$\int (d+ex)^2 \left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^p\,dx=\text{too large to display}$$

input
```
int((e*x+d)^2*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^p,x)
```

output
```
((a*d*e + a*e**2*x + c*d**2*x + c*d*e*x**2)**p*a**3*d*e**5*p + 2*(a*d*e +
a*e**2*x + c*d**2*x + c*d*e*x**2)**p*a**3*d*e**5 - (a*d*e + a*e**2*x + c*d
**2*x + c*d*e*x**2)**p*a**3*e**6*p**2*x - 2*(a*d*e + a*e**2*x + c*d**2*x +
 c*d*e*x**2)**p*a**3*e**6*p*x - 6*(a*d*e + a*e**2*x + c*d**2*x + c*d*e*x**
2)**p*a**2*c*d**3*e**3*p - 6*(a*d*e + a*e**2*x + c*d**2*x + c*d*e*x**2)**p
*a**2*c*d**3*e**3 + 5*(a*d*e + a*e**2*x + c*d**2*x + c*d*e*x**2)**p*a**2*c
*d**2*e**4*p**2*x + 4*(a*d*e + a*e**2*x + c*d**2*x + c*d*e*x**2)**p*a**2*c
*d**2*e**4*p*x + 2*(a*d*e + a*e**2*x + c*d**2*x + c*d*e*x**2)**p*a**2*c*d*
e**5*p**2*x**2 + (a*d*e + a*e**2*x + c*d**2*x + c*d*e*x**2)**p*a**2*c*d*e*
*5*p*x**2 + 8*(a*d*e + a*e**2*x + c*d**2*x + c*d*e*x**2)**p*a*c**2*d**5*e*
p**2 + 17*(a*d*e + a*e**2*x + c*d**2*x + c*d*e*x**2)**p*a*c**2*d**5*e*p +
8*(a*d*e + a*e**2*x + c*d**2*x + c*d*e*x**2)**p*a*c**2*d**5*e + 13*(a*d*e
+ a*e**2*x + c*d**2*x + c*d*e*x**2)**p*a*c**2*d**4*e**2*p**2*x + 20*(a*d*e
 + a*e**2*x + c*d**2*x + c*d*e*x**2)**p*a*c**2*d**4*e**2*p*x + 6*(a*d*e +
a*e**2*x + c*d**2*x + c*d*e*x**2)**p*a*c**2*d**4*e**2*x + 12*(a*d*e + a*e*
*2*x + c*d**2*x + c*d*e*x**2)**p*a*c**2*d**3*e**3*p**2*x**2 + 18*(a*d*e +
a*e**2*x + c*d**2*x + c*d*e*x**2)**p*a*c**2*d**3*e**3*p*x**2 + 6*(a*d*e +
a*e**2*x + c*d**2*x + c*d*e*x**2)**p*a*c**2*d**3*e**3*x**2 + 4*(a*d*e + a*
e**2*x + c*d**2*x + c*d*e*x**2)**p*a*c**2*d**2*e**4*p**2*x**3 + 6*(a*d*e +
 a*e**2*x + c*d**2*x + c*d*e*x**2)**p*a*c**2*d**2*e**4*p*x**3 + 2*(a*d*...
```

3.391 $\int (d+ex)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^p\,dx$

Optimal result

Integrand size = 33, antiderivative size = 87

$$\int (d+ex)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^p\,dx = -\frac{(d+ex)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{1+p}\text{Hypergeometric2F1}\left(1,3+2p,3+p,\frac{cd(d+ex)}{cd^2-ae^2}\right)}{\left(cd^2-ae^2\right)(2+p)}$$

output
```
-(e*x+d)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(p+1)*hypergeom([1, 3+2*p],[3+p
],c*d*(e*x+d)/(-a*e^2+c*d^2))/(-a*e^2+c*d^2)/(2+p)
```

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.08

$$\int (d+ex)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^p\,dx = \frac{\left(\frac{cd(d+ex)}{cd^2-ae^2}\right)^{-1-p}((ae+cdx)(d+ex))^{1+p}\text{Hypergeometric2F1}\left(-1-p,1+p,2+p,\frac{e(ae+cdx)}{-cd^2+ae^2}\right)}{cd(1+p)}$$

input
```
Integrate[(d + e*x)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^p,x]
```

output
```
(((c*d*(d + e*x))/(c*d^2 - a*e^2))^(-1 - p)*((a*e + c*d*x)*(d + e*x))^(1 +
 p)*Hypergeometric2F1[-1 - p, 1 + p, 2 + p, (e*(a*e + c*d*x))/(-(c*d^2) +
a*e^2)])/(c*d*(1 + p))
```

Rubi [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 176 vs. 2(87) = 174.

Time = 0.29 (sec) , antiderivative size = 176, normalized size of antiderivative = 2.02, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.061$, Rules used = {1160, 1096}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (d+ex)\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^p dx$$

$$\Big\downarrow \text{1160}$$

$$\frac{\left(d^2-\frac{ae^2}{c}\right)\int \left(cdex^2+\left(cd^2+ae^2\right)x+ade\right)^p dx}{2d}+\frac{\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{p+1}}{2cd(p+1)}$$

$$\Big\downarrow \text{1096}$$

$$\frac{\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{p+1}}{2cd(p+1)}-\frac{\left(d^2-\frac{ae^2}{c}\right)\left(-\frac{e(ae+cdx)}{cd^2-ae^2}\right)^{-p-1}\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{p+1}\text{Hypergeometric2F1}\left(-p,p+1,p+2,\frac{cd(d+ex)}{cd^2-ae^2}\right)}{2d(p+1)\left(cd^2-ae^2\right)}$$

input
```
Int[(d + e*x)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^p,x]
```

output
```
(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(1 + p)/(2*c*d*(1 + p)) - ((d^2 -
(a*e^2)/c)*(-((e*(a*e + c*d*x))/(c*d^2 - a*e^2)))^(-1 - p)*(a*d*e + (c*d^2
 + a*e^2)*x + c*d*e*x^2)^(1 + p)*Hypergeometric2F1[-p, 1 + p, 2 + p, (c*d*
(d + e*x))/(c*d^2 - a*e^2)])/(2*d*(c*d^2 - a*e^2)*(1 + p))
```

Defintions of rubi rules used

rule 1096
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{q = Rt[b^2
 - 4*a*c, 2]}, Simp[(-(a + b*x + c*x^2)^(p + 1)/(q*(p + 1)*((q - b - 2*c*x)
/(2*q))^(p + 1)))*Hypergeometric2F1[-p, p + 1, p + 2, (b + q + 2*c*x)/(2*q)
], x]] /; FreeQ[{a, b, c, p}, x] &&  !IntegerQ[4*p] &&  !IntegerQ[3*p]
```

rule 1160
```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[e*((a + b*x + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Simp[(2*c*d - b
*e)/(2*c)   Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
 && NeQ[p, -1]
```

Maple [F]

$$\int (ex+d)\left(ade+\left(a\,e^{2}+c\,d^{2}\right)x+cd\,x^{2}e\right)^{p}dx$$

input
```
int((e*x+d)*(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^p,x)
```

output
```
int((e*x+d)*(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^p,x)
```

Fricas [F]

$$\int (d+ex)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^p\,dx$$
$$=\int (ex+d)\left(cdex^2+ade+\left(cd^2+ae^2\right)x\right)^p\,dx$$

input
```
integrate((e*x+d)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^p,x, algorithm="fricas
")
```

output
```
integral((e*x + d)*(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^p, x)
```

Sympy [F]

$$\int (d+ex)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^p\,dx=\int ((d+ex)(ae+cdx))^p(d+ex)\,dx$$

input
```
integrate((e*x+d)*(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**p,x)
```

output
```
Integral(((d + e*x)*(a*e + c*d*x))**p*(d + e*x), x)
```

Maxima [F]

$$\int (d+ex)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^p\,dx$$
$$=\int (ex+d)\left(cdex^2+ade+\left(cd^2+ae^2\right)x\right)^p\,dx$$

input
```
integrate((e*x+d)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^p,x, algorithm="maxima
")
```

output
```
integrate((e*x + d)*(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^p, x)
```

Giac [F]

$$\int (d+ex)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^p\,dx$$
$$=\int (ex+d)\left(cdex^2+ade+\left(cd^2+ae^2\right)x\right)^p\,dx$$

input
```
integrate((e*x+d)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^p,x, algorithm="giac")
```

output
```
integrate((e*x + d)*(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^p, x)
```

Mupad [F(-1)]

Timed out.

$$\int (d+ex)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^p\,dx$$
$$=\int (d+e\,x)\,\left(c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e\right)^p\,dx$$

input
```
int((d + e*x)*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^p,x)
```

output
```
int((d + e*x)*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^p, x)
```

Reduce [F]

$$\int (d+ex)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^p\,dx=\text{too large to display}$$

input
```
int((e*x+d)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^p,x)
```

output

```
( - (a*d*e + a*e**2*x + c*d**2*x + c*d*e*x**2)**p*a**2*d*e**3 + (a*d*e + a
*e**2*x + c*d**2*x + c*d*e*x**2)**p*a**2*e**4*p*x + 4*(a*d*e + a*e**2*x +
c*d**2*x + c*d*e*x**2)**p*a*c*d**3*e*p + 3*(a*d*e + a*e**2*x + c*d**2*x +
c*d*e*x**2)**p*a*c*d**3*e + 4*(a*d*e + a*e**2*x + c*d**2*x + c*d*e*x**2)**
p*a*c*d**2*e**2*p*x + 2*(a*d*e + a*e**2*x + c*d**2*x + c*d*e*x**2)**p*a*c*
d**2*e**2*x + 2*(a*d*e + a*e**2*x + c*d**2*x + c*d*e*x**2)**p*a*c*d*e**3*p
*x**2 + (a*d*e + a*e**2*x + c*d**2*x + c*d*e*x**2)**p*a*c*d*e**3*x**2 + 3*
(a*d*e + a*e**2*x + c*d**2*x + c*d*e*x**2)**p*c**2*d**4*p*x + 2*(a*d*e + a
*e**2*x + c*d**2*x + c*d*e*x**2)**p*c**2*d**4*x + 2*(a*d*e + a*e**2*x + c*
d**2*x + c*d*e*x**2)**p*c**2*d**3*e*p*x**2 + (a*d*e + a*e**2*x + c*d**2*x
+ c*d*e*x**2)**p*c**2*d**3*e*x**2 - 2*int(((a*d*e + a*e**2*x + c*d**2*x +
c*d*e*x**2)**p*x)/(2*a**2*d*e**3*p + a**2*d*e**3 + 2*a**2*e**4*p*x + a**2*
e**4*x + 2*a*c*d**3*e*p + a*c*d**3*e + 4*a*c*d**2*e**2*p*x + 2*a*c*d**2*e*
*2*x + 2*a*c*d*e**3*p*x**2 + a*c*d*e**3*x**2 + 2*c**2*d**4*p*x + c**2*d**4
*x + 2*c**2*d**3*e*p*x**2 + c**2*d**3*e*x**2),x)*a**4*e**8*p**3 - 3*int(((
a*d*e + a*e**2*x + c*d**2*x + c*d*e*x**2)**p*x)/(2*a**2*d*e**3*p + a**2*d*
e**3 + 2*a**2*e**4*p*x + a**2*e**4*x + 2*a*c*d**3*e*p + a*c*d**3*e + 4*a*c
*d**2*e**2*p*x + 2*a*c*d**2*e**2*x + 2*a*c*d*e**3*p*x**2 + a*c*d*e**3*x**2
 + 2*c**2*d**4*p*x + c**2*d**4*x + 2*c**2*d**3*e*p*x**2 + c**2*d**3*e*x**2
),x)*a**4*e**8*p**2 - int(((a*d*e + a*e**2*x + c*d**2*x + c*d*e*x**2)**...
```

3.392 $\int (ade + (cd^2 + ae^2) x + cdex^2)^p \, dx$

Optimal result

Integrand size = 27, antiderivative size = 113

$$\int (ade + (cd^2 + ae^2) x + cdex^2)^p \, dx = -\frac{\left(-\frac{e(ae+cdx)}{cd^2-ae^2}\right)^{-1-p} (ade + (cd^2 + ae^2) x + cdex^2)^{1+p} \text{Hypergeometric2F1}\left(-p, 1+p, 2+p, \frac{cd(d+ex)}{cd^2-ae^2}\right)}{(cd^2 - ae^2)(1+p)}$$

output
```
-(-e*(c*d*x+a*e)/(-a*e^2+c*d^2))^(-1-p)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^
(p+1)*hypergeom([-p, p+1],[2+p],c*d*(e*x+d)/(-a*e^2+c*d^2))/(-a*e^2+c*d^2)
/(p+1)
```

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 96, normalized size of antiderivative = 0.85

$$\int (ade + (cd^2 + ae^2) x + cdex^2)^p \, dx = \frac{(ae + cdx) \left(\frac{cd(d+ex)}{cd^2-ae^2}\right)^{-p} ((ae + cdx)(d + ex))^p \text{Hypergeometric2F1}\left(-p, 1+p, 2+p, \frac{e(ae+cdx)}{-cd^2+ae^2}\right)}{cd(1+p)}$$

input
```
Integrate[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^p,x]
```

output
```
((a*e + c*d*x)*((a*e + c*d*x)*(d + e*x))^p*Hypergeometric2F1[-p, 1 + p, 2
+ p, (e*(a*e + c*d*x))/(-(c*d^2) + a*e^2)])/(c*d*(1 + p)*((c*d*(d + e*x))/
(c*d^2 - a*e^2))^p)
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.037$, Rules used = $\{1096\}$

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^p\,dx$$

$$\Big\downarrow \; 1096$$

$$-\frac{\left(-\frac{e(ae+cdx)}{cd^2-ae^2}\right)^{-p-1}\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{p+1}\text{Hypergeometric2F1}\left(-p,p+1,p+2,\frac{cd(d+ex)}{cd^2-ae^2}\right)}{(p+1)\left(cd^2-ae^2\right)}$$

input
```
Int[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^p,x]
```

output
```
-(((-((e*(a*e + c*d*x))/(c*d^2 - a*e^2)))^(-1 - p)*(a*d*e + (c*d^2 + a*e^2
)*x + c*d*e*x^2)^(1 + p)*Hypergeometric2F1[-p, 1 + p, 2 + p, (c*d*(d + e*x
))/(c*d^2 - a*e^2)])/((c*d^2 - a*e^2)*(1 + p)))
```

Defintions of rubi rules used

rule 1096
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{q = Rt[b^2
 - 4*a*c, 2]}, Simp[(-(a + b*x + c*x^2)^(p + 1)/(q*(p + 1)*((q - b - 2*c*x)
/(2*q))^(p + 1)))*Hypergeometric2F1[-p, p + 1, p + 2, (b + q + 2*c*x)/(2*q)
], x]] /; FreeQ[{a, b, c, p}, x] && !IntegerQ[4*p] && !IntegerQ[3*p]
```

Maple [F]

$$\int \left(ade+\left(a\,e^{2}+c\,d^{2}\right)x+cd\,x^{2}e\right)^{p}dx$$

input
```
int((a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^p,x)
```

output
```
int((a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^p,x)
```

Fricas [F]

$$\int \left(ade+\left(cd^{2}+ae^{2}\right)x+cdex^{2}\right)^{p}\,dx=\int \left(cdex^{2}+ade+\left(cd^{2}+ae^{2}\right)x\right)^{p}\,dx$$

input
```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^p,x, algorithm="fricas")
```

output
```
integral((c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^p, x)
```

Sympy [F]

$$\int \left(ade+\left(cd^{2}+ae^{2}\right)x+cdex^{2}\right)^{p}\,dx=\int \left(ade+cdex^{2}+x\left(ae^{2}+cd^{2}\right)\right)^{p}\,dx$$

input
```
integrate((a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**p,x)
```

output
```
Integral((a*d*e + c*d*e*x**2 + x*(a*e**2 + c*d**2))**p, x)
```

Maxima [F]

$$\int \left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^p\,dx=\int \left(cdex^2+ade+\left(cd^2+ae^2\right)x\right)^p dx$$

input
```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^p,x, algorithm="maxima")
```

output
```
integrate((c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^p, x)
```

Giac [F]

$$\int \left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^p\,dx=\int \left(cdex^2+ade+\left(cd^2+ae^2\right)x\right)^p dx$$

input
```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^p,x, algorithm="giac")
```

output
```
integrate((c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^p, x)
```

Mupad [F(-1)]

Timed out.

$$\int \left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^p\,dx=\int \left(c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e\right)^p dx$$

input
```
int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^p,x)
```

output
```
int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^p, x)
```

Reduce [F]

$$\int \left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^p\,dx=\text{Too large to display}$$

input
```
int((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^p,x)
```

output
```
(2*(a*d*e + a*e**2*x + c*d**2*x + c*d*e*x**2)**p*a*d*e + (a*d*e + a*e**2*x
 + c*d**2*x + c*d*e*x**2)**p*a*e**2*x + (a*d*e + a*e**2*x + c*d**2*x + c*d
*e*x**2)**p*c*d**2*x + 2*int(((a*d*e + a*e**2*x + c*d**2*x + c*d*e*x**2)**
p*x)/(2*a**2*d*e**3*p + a**2*d*e**3 + 2*a**2*e**4*p*x + a**2*e**4*x + 2*a*
c*d**3*e*p + a*c*d**3*e + 4*a*c*d**2*e**2*p*x + 2*a*c*d**2*e**2*x + 2*a*c*
d*e**3*p*x**2 + a*c*d*e**3*x**2 + 2*c**2*d**4*p*x + c**2*d**4*x + 2*c**2*d
**3*e*p*x**2 + c**2*d**3*e*x**2),x)*a**3*e**6*p**2 + int(((a*d*e + a*e**2*
x + c*d**2*x + c*d*e*x**2)**p*x)/(2*a**2*d*e**3*p + a**2*d*e**3 + 2*a**2*e
**4*p*x + a**2*e**4*x + 2*a*c*d**3*e*p + a*c*d**3*e + 4*a*c*d**2*e**2*p*x
+ 2*a*c*d**2*e**2*x + 2*a*c*d*e**3*p*x**2 + a*c*d*e**3*x**2 + 2*c**2*d**4*
p*x + c**2*d**4*x + 2*c**2*d**3*e*p*x**2 + c**2*d**3*e*x**2),x)*a**3*e**6*
p - 2*int(((a*d*e + a*e**2*x + c*d**2*x + c*d*e*x**2)**p*x)/(2*a**2*d*e**3
*p + a**2*d*e**3 + 2*a**2*e**4*p*x + a**2*e**4*x + 2*a*c*d**3*e*p + a*c*d*
*3*e + 4*a*c*d**2*e**2*p*x + 2*a*c*d**2*e**2*x + 2*a*c*d*e**3*p*x**2 + a*c
*d*e**3*x**2 + 2*c**2*d**4*p*x + c**2*d**4*x + 2*c**2*d**3*e*p*x**2 + c**2
*d**3*e*x**2),x)*a**2*c*d**2*e**4*p**2 - int(((a*d*e + a*e**2*x + c*d**2*x
 + c*d*e*x**2)**p*x)/(2*a**2*d*e**3*p + a**2*d*e**3 + 2*a**2*e**4*p*x + a*
*2*e**4*x + 2*a*c*d**3*e*p + a*c*d**3*e + 4*a*c*d**2*e**2*p*x + 2*a*c*d**2
*e**2*x + 2*a*c*d*e**3*p*x**2 + a*c*d*e**3*x**2 + 2*c**2*d**4*p*x + c**2*d
**4*x + 2*c**2*d**3*e*p*x**2 + c**2*d**3*e*x**2),x)*a**2*c*d**2*e**4*p ...
```

3.393 $\int \frac{(ade+(cd^2+ae^2)x+cdex^2)^p}{d+ex}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 87

$$\int \frac{(ade+(cd^2+ae^2)x+cdex^2)^p}{d+ex}\,dx = -\frac{(ade+(cd^2+ae^2)x+cdex^2)^{1+p}\,\text{Hypergeometric2F1}\left(1,1+2p,1+p,\frac{cd(d+ex)}{cd^2-ae^2}\right)}{(cd^2-ae^2)\,p(d+ex)}$$

output
```
-(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(p+1)*hypergeom([1, 1+2*p],[p+1],c*d*(e
*x+d)/(-a*e^2+c*d^2))/(-a*e^2+c*d^2)/p/(e*x+d)
```

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.93

$$\int \frac{(ade+(cd^2+ae^2)x+cdex^2)^p}{d+ex}\,dx = \frac{\left(\frac{e(ae+cdx)}{-cd^2+ae^2}\right)^{-p}((ae+cdx)(d+ex))^p\,\text{Hypergeometric2F1}\left(-p,p,1+p,\frac{cd(d+ex)}{cd^2-ae^2}\right)}{ep}$$

input
```
Integrate[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^p/(d + e*x),x]
```

output
```
(((a*e + c*d*x)*(d + e*x))^p*Hypergeometric2F1[-p, p, 1 + p, (c*d*(d + e*x
))/(c*d^2 - a*e^2)])/(e*p*((e*(a*e + c*d*x))/(-(c*d^2) + a*e^2))^p)
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 93, normalized size of antiderivative = 1.07, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.086, Rules used = {1138, 80, 79}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^p}{d+ex}\,dx$$

$$\Big\downarrow \text{1138}$$

$$\frac{\left(\frac{ex}{d}+1\right)^{-p}(ae+cdx)^{-p}\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^p\int (ae+cdx)^p\left(\frac{ex}{d}+1\right)^{p-1}dx}{d}$$

$$\Big\downarrow \text{80}$$

$$\frac{\left(\frac{ex}{d}+1\right)^{-p}\left(-\frac{e(ae+cdx)}{cd^2-ae^2}\right)^{-p}\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^p\int \left(\frac{ex}{d}+1\right)^{p-1}\left(-\frac{ae^2}{cd^2-ae^2}-\frac{cdxe}{cd^2-ae^2}\right)^p dx}{d}$$

$$\Big\downarrow \text{79}$$

$$\frac{\left(-\frac{e(ae+cdx)}{cd^2-ae^2}\right)^{-p}\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^p \text{Hypergeometric2F1}\left(-p,p,p+1,\frac{cd(d+ex)}{cd^2-ae^2}\right)}{ep}$$

input
```
Int[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^p/(d + e*x),x]
```

output
```
((a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^p*Hypergeometric2F1[-p, p, 1 + p,
 (c*d*(d + e*x))/(c*d^2 - a*e^2)])/(e*p*(-((e*(a*e + c*d*x))/(c*d^2 - a*e^
2)))^p)
```

Defintions of rubi rules used

rule 79
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d), 0] && (RationalQ[m]
 ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

rule 80
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*(b*((c + d*x)/(b*c - a*d)))
^FracPart[n])   Int[(a + b*x)^m*Simp[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d)
), x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] &&  !IntegerQ[m] &&  !Integ
erQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

rule 1138
```
Int[((d_) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Sy
mbol] :> Simp[d^m*((a + b*x + c*x^2)^FracPart[p]/((1 + e*(x/d))^FracPart[p]
*(a/d + (c*x)/e)^FracPart[p]))   Int[(1 + e*(x/d))^(m + p)*(a/d + (c/e)*x)^
p, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0]
&& (IntegerQ[m] || GtQ[d, 0]) &&  !(IGtQ[m, 0] && (IntegerQ[3*p] || Integer
Q[4*p]))
```

Maple [F]

$$\int \frac{(ade + (a\,e^2 + c\,d^2)\,x + cd\,x^2e)^p}{ex + d}dx$$

input
```
int((a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^p/(e*x+d),x)
```

output
```
int((a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^p/(e*x+d),x)
```

Fricas [F]

$$\int \frac{(ade + (cd^2 + ae^2)x + cdex^2)^p}{d + ex} dx = \int \frac{(cdex^2 + ade + (cd^2 + ae^2)x)^p}{ex + d} dx$$

input
```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^p/(e*x+d),x, algorithm="fricas
")
```

output
```
integral((c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^p/(e*x + d), x)
```

Sympy [F]

$$\int \frac{(ade + (cd^2 + ae^2)x + cdex^2)^p}{d + ex} dx = \int \frac{((d + ex)(ae + cdx))^p}{d + ex} dx$$

input
```
integrate((a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**p/(e*x+d),x)
```

output
```
Integral(((d + e*x)*(a*e + c*d*x))**p/(d + e*x), x)
```

Maxima [F]

$$\int \frac{(ade + (cd^2 + ae^2)x + cdex^2)^p}{d + ex} dx = \int \frac{(cdex^2 + ade + (cd^2 + ae^2)x)^p}{ex + d} dx$$

input
```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^p/(e*x+d),x, algorithm="maxima
")
```

output
```
integrate((c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^p/(e*x + d), x)
```

Giac [F]

$$\int \frac{(ade+(cd^2+ae^2)\,x+cdex^2)^p}{d+ex}\,dx=\int \frac{(cdex^2+ade+(cd^2+ae^2)x)^p}{ex+d}\,dx$$

input
```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^p/(e*x+d),x, algorithm="giac")
```

output
```
integrate((c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^p/(e*x + d), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(ade+(cd^2+ae^2)\,x+cdex^2)^p}{d+ex}\,dx=\int \frac{\left(c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e\right)^p}{d+e\,x}\,dx$$

input
```
int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^p/(d + e*x),x)
```

output
```
int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^p/(d + e*x), x)
```

Reduce [F]

$$\int \frac{(ade+(cd^2+ae^2)\,x+cdex^2)^p}{d+ex}\,dx$$
$$=\frac{(cde\,x^2+a\,e^2x+c\,d^2x+ade)^p\,ae-\left(\int \frac{(cde\,x^2+a\,e^2x+c\,d^2x+ade)^p\,x}{acd\,e^3x^2+c^2d^3e\,x^2+a^2e^4x+2ac\,d^2e^2x+c^2d^4x+a^2d\,e^3+ac\,d^3e}dx\right)a^2cd\,e^4p+\left(\int\right.}{p\,(a\,e^2+c\,d^2)}$$

input
```
int((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^p/(e*x+d),x)
```

output
```
((a*d*e + a*e**2*x + c*d**2*x + c*d*e*x**2)**p*a*e - int(((a*d*e + a*e**2*
x + c*d**2*x + c*d*e*x**2)**p*x)/(a**2*d*e**3 + a**2*e**4*x + a*c*d**3*e +
 2*a*c*d**2*e**2*x + a*c*d*e**3*x**2 + c**2*d**4*x + c**2*d**3*e*x**2),x)*
a**2*c*d*e**4*p + int(((a*d*e + a*e**2*x + c*d**2*x + c*d*e*x**2)**p*x)/(a
**2*d*e**3 + a**2*e**4*x + a*c*d**3*e + 2*a*c*d**2*e**2*x + a*c*d*e**3*x**
2 + c**2*d**4*x + c**2*d**3*e*x**2),x)*c**3*d**5*p)/(p*(a*e**2 + c*d**2))
```

3.394 $\int \frac{(ade+(cd^2+ae^2)x+cdex^2)^p}{(d+ex)^2} \, dx$

Optimal result

Integrand size = 35, antiderivative size = 86

$$\int \frac{(ade+(cd^2+ae^2)\,x+cdex^2)^p}{(d+ex)^2} \, dx = \frac{(ade+(cd^2+ae^2)\,x+cdex^2)^{1+p}\,\text{Hypergeometric2F1}\left(1,2p,p,\frac{cd(d+ex)}{cd^2-ae^2}\right)}{(cd^2-ae^2)\,(1-p)(d+ex)^2}$$

output
```
(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(p+1)*hypergeom([1, 2*p],[p],c*d*(e*x+d)
/(-a*e^2+c*d^2))/(-a*e^2+c*d^2)/(1-p)/(e*x+d)^2
```

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.26

$$\int \frac{(ade+(cd^2+ae^2)\,x+cdex^2)^p}{(d+ex)^2} \, dx = \frac{cd(ae+cdx)\left(\frac{cd(d+ex)}{cd^2-ae^2}\right)^{-p}((ae+cdx)(d+ex))^p\,\text{Hypergeometric2F1}\left(2-p,1+p,2+p,\frac{e(ae+cdx)}{-cd^2+ae^2}\right)}{(cd^2-ae^2)^2\,(1+p)}$$

input
```
Integrate[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^p/(d + e*x)^2,x]
```

output
```
(c*d*(a*e + c*d*x)*((a*e + c*d*x)*(d + e*x))^p*Hypergeometric2F1[2 - p, 1
+ p, 2 + p, (e*(a*e + c*d*x))/(-(c*d^2) + a*e^2)])/((c*d^2 - a*e^2)^2*(1 +
 p)*((c*d*(d + e*x))/(c*d^2 - a*e^2))^p)
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 120, normalized size of antiderivative = 1.40, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.086$, Rules used = {1138, 80, 79}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^p}{(d+ex)^2}dx$$

$$\Big\downarrow \text{ 1138}$$

$$\frac{\left(\frac{ex}{d}+1\right)^{-p}(ae+cdx)^{-p}\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^p\int (ae+cdx)^p\left(\frac{ex}{d}+1\right)^{p-2}dx}{d^2}$$

$$\Big\downarrow \text{ 80}$$

$$\frac{c^2d^2(ae+cdx)^{-p}\left(\frac{cd(d+ex)}{cd^2-ae^2}\right)^{-p}\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^p\int (ae+cdx)^p\left(\frac{cd^2}{cd^2-ae^2}+\frac{cexd}{cd^2-ae^2}\right)^{p-2}dx}{\left(cd^2-ae^2\right)^2}$$

$$\Big\downarrow \text{ 79}$$

$$\frac{cd(ae+cdx)\left(\frac{cd(d+ex)}{cd^2-ae^2}\right)^{-p}\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^p\text{Hypergeometric2F1}\left(2-p,p+1,p+2,-\frac{e(ae+cdx)}{cd^2-ae^2}\right)}{(p+1)\left(cd^2-ae^2\right)^2}$$

input
```
Int[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^p/(d + e*x)^2,x]
```

output
```
(c*d*(a*e + c*d*x)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^p*Hypergeometri
c2F1[2 - p, 1 + p, 2 + p, -((e*(a*e + c*d*x))/(c*d^2 - a*e^2))]])/((c*d^2 -
 a*e^2)^2*(1 + p)*((c*d*(d + e*x))/(c*d^2 - a*e^2))^p)
```

Defintions of rubi rules used

rule 79
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d), 0] && (RationalQ[m]
 ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

rule 80
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*(b*((c + d*x)/(b*c - a*d)))
^FracPart[n])   Int[(a + b*x)^m*Simp[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d)
), x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] &&  !IntegerQ[m] &&  !Integ
erQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

rule 1138
```
Int[((d_) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Sy
mbol] :> Simp[d^m*((a + b*x + c*x^2)^FracPart[p]/((1 + e*(x/d))^FracPart[p]
*(a/d + (c*x)/e)^FracPart[p]))   Int[(1 + e*(x/d))^(m + p)*(a/d + (c/e)*x)^
p, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0]
&& (IntegerQ[m] || GtQ[d, 0]) &&  !(IGtQ[m, 0] && (IntegerQ[3*p] || Integer
Q[4*p]))
```

Maple [F]

$$\int \frac{(ade + (a\,e^2 + c\,d^2)\,x + cd\,x^2e)^p}{(ex + d)^2}dx$$

input
```
int((a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^p/(e*x+d)^2,x)
```

output
```
int((a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^p/(e*x+d)^2,x)
```

Fricas [F]

$$\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^p}{(d+ex)^2}\,dx=\int \frac{\left(cdex^2+ade+\left(cd^2+ae^2\right)x\right)^p}{\left(ex+d\right)^2}\,dx$$

input
```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^p/(e*x+d)^2,x, algorithm="fric
as")
```

output
```
integral((c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^p/(e^2*x^2 + 2*d*e*x + d^
2), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^p}{(d+ex)^2}\,dx=\text{Timed out}$$

input
```
integrate((a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**p/(e*x+d)**2,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^p}{(d+ex)^2}\,dx=\int \frac{\left(cdex^2+ade+\left(cd^2+ae^2\right)x\right)^p}{\left(ex+d\right)^2}\,dx$$

input
```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^p/(e*x+d)^2,x, algorithm="maxi
ma")
```

output
```
integrate((c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^p/(e*x + d)^2, x)
```

Giac [F]

$$\int \frac{(ade + (cd^2 + ae^2)x + cdex^2)^p}{(d + ex)^2} dx = \int \frac{(cdex^2 + ade + (cd^2 + ae^2)x)^p}{(ex + d)^2} dx$$

input
```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^p/(e*x+d)^2,x, algorithm="giac
")
```

output
```
integrate((c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^p/(e*x + d)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(ade + (cd^2 + ae^2)x + cdex^2)^p}{(d + ex)^2} dx = \int \frac{(c\,d\,e\,x^2 + (c\,d^2 + a\,e^2)\,x + a\,d\,e)^p}{(d + e\,x)^2} dx$$

input
```
int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^p/(d + e*x)^2,x)
```

output
```
int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^p/(d + e*x)^2, x)
```

Reduce [F]

$$\int \frac{(ade + (cd^2 + ae^2)x + cdex^2)^p}{(d + ex)^2} dx = \text{too large to display}$$

input
```
int((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^p/(e*x+d)^2,x)
```

output
```
((a*d*e + a*e**2*x + c*d**2*x + c*d*e*x**2)**p*a*e - int(((a*d*e + a*e**2*
x + c*d**2*x + c*d*e*x**2)**p*x)/(a**2*d**2*e**3*p - a**2*d**2*e**3 + 2*a*
*2*d*e**4*p*x - 2*a**2*d*e**4*x + a**2*e**5*p*x**2 - a**2*e**5*x**2 + a*c*
d**4*e*p + 3*a*c*d**3*e**2*p*x - a*c*d**3*e**2*x + 3*a*c*d**2*e**3*p*x**2
- 2*a*c*d**2*e**3*x**2 + a*c*d*e**4*p*x**3 - a*c*d*e**4*x**3 + c**2*d**5*p
*x + 2*c**2*d**4*e*p*x**2 + c**2*d**3*e**2*p*x**3),x)*a**2*c*d**2*e**4*p**
2 + int(((a*d*e + a*e**2*x + c*d**2*x + c*d*e*x**2)**p*x)/(a**2*d**2*e**3*
p - a**2*d**2*e**3 + 2*a**2*d*e**4*p*x - 2*a**2*d*e**4*x + a**2*e**5*p*x**
2 - a**2*e**5*x**2 + a*c*d**4*e*p + 3*a*c*d**3*e**2*p*x - a*c*d**3*e**2*x
+ 3*a*c*d**2*e**3*p*x**2 - 2*a*c*d**2*e**3*x**2 + a*c*d*e**4*p*x**3 - a*c*
d*e**4*x**3 + c**2*d**5*p*x + 2*c**2*d**4*e*p*x**2 + c**2*d**3*e**2*p*x**3
),x)*a**2*c*d**2*e**4*p - int(((a*d*e + a*e**2*x + c*d**2*x + c*d*e*x**2)*
*p*x)/(a**2*d**2*e**3*p - a**2*d**2*e**3 + 2*a**2*d*e**4*p*x - 2*a**2*d*e*
*4*x + a**2*e**5*p*x**2 - a**2*e**5*x**2 + a*c*d**4*e*p + 3*a*c*d**3*e**2*
p*x - a*c*d**3*e**2*x + 3*a*c*d**2*e**3*p*x**2 - 2*a*c*d**2*e**3*x**2 + a*
c*d*e**4*p*x**3 - a*c*d*e**4*x**3 + c**2*d**5*p*x + 2*c**2*d**4*e*p*x**2 +
 c**2*d**3*e**2*p*x**3),x)*a**2*c*d*e**5*p**2*x + int(((a*d*e + a*e**2*x +
 c*d**2*x + c*d*e*x**2)**p*x)/(a**2*d**2*e**3*p - a**2*d**2*e**3 + 2*a**2*
d*e**4*p*x - 2*a**2*d*e**4*x + a**2*e**5*p*x**2 - a**2*e**5*x**2 + a*c*d**
4*e*p + 3*a*c*d**3*e**2*p*x - a*c*d**3*e**2*x + 3*a*c*d**2*e**3*p*x**2 ...
```

3.395 $\int \frac{(ade+(cd^2+ae^2)x+cdex^2)^p}{(d+ex)^3} dx$

Optimal result

Integrand size = 35, antiderivative size = 90

$$\int \frac{(ade+(cd^2+ae^2)x+cdex^2)^p}{(d+ex)^3} dx = \frac{(ade+(cd^2+ae^2)x+cdex^2)^{1+p}\text{Hypergeometric2F1}\left(1,-1+2p,-1+p,\frac{cd(d+ex)}{cd^2-ae^2}\right)}{(cd^2-ae^2)(2-p)(d+ex)^3}$$

output
```
(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(p+1)*hypergeom([1, -1+2*p],[-1+p],c*d*(
e*x+d)/(-a*e^2+c*d^2))/(-a*e^2+c*d^2)/(2-p)/(e*x+d)^3
```

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.24

$$\int \frac{(ade+(cd^2+ae^2)x+cdex^2)^p}{(d+ex)^3} dx = \frac{c^2d^2(ae+cdx)\left(\frac{cd(d+ex)}{cd^2-ae^2}\right)^{-p}((ae+cdx)(d+ex))^p\text{Hypergeometric2F1}\left(3-p,1+p,2+p,\frac{e(ae+cdx)}{-cd^2+ae^2}\right)}{(cd^2-ae^2)^3(1+p)}$$

input
```
Integrate[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^p/(d + e*x)^3,x]
```

output
```
(c^2*d^2*(a*e + c*d*x)*((a*e + c*d*x)*(d + e*x))^p*Hypergeometric2F1[3 - p
, 1 + p, 2 + p, (e*(a*e + c*d*x))/(-(c*d^2) + a*e^2)])/((c*d^2 - a*e^2)^3*
(1 + p)*((c*d*(d + e*x))/(c*d^2 - a*e^2))^p)
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 124, normalized size of antiderivative = 1.38, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.086$, Rules used = {1138, 80, 79}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^p}{(d+ex)^3}dx$$

$$\Big\downarrow \text{1138}$$

$$\frac{\left(\frac{ex}{d}+1\right)^{-p}(ae+cdx)^{-p}\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^p\int(ae+cdx)^p\left(\frac{ex}{d}+1\right)^{p-3}dx}{d^3}$$

$$\Big\downarrow \text{80}$$

$$\frac{c^3d^3(ae+cdx)^{-p}\left(\frac{cd(d+ex)}{cd^2-ae^2}\right)^{-p}\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^p\int(ae+cdx)^p\left(\frac{cd^2}{cd^2-ae^2}+\frac{cexd}{cd^2-ae^2}\right)^{p-3}dx}{\left(cd^2-ae^2\right)^3}$$

$$\Big\downarrow \text{79}$$

$$\frac{c^2d^2(ae+cdx)\left(\frac{cd(d+ex)}{cd^2-ae^2}\right)^{-p}\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^p\text{Hypergeometric2F1}\left(3-p,p+1,p+2,-\frac{e(ae+cdx)}{cd^2-ae^2}\right)}{(p+1)\left(cd^2-ae^2\right)^3}$$

input
```
Int[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^p/(d + e*x)^3,x]
```

output
```
(c^2*d^2*(a*e + c*d*x)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^p*Hypergeom
etric2F1[3 - p, 1 + p, 2 + p, -((e*(a*e + c*d*x))/(c*d^2 - a*e^2))])/((c*d
^2 - a*e^2)^3*(1 + p)*((c*d*(d + e*x))/(c*d^2 - a*e^2))^p)
```

Defintions of rubi rules used

rule 79
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&& !IntegerQ[m] && !IntegerQ[n] && GtQ[b/(b*c - a*d), 0] && (RationalQ[m]
 || !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

rule 80
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*(b*((c + d*x)/(b*c - a*d)))
^FracPart[n])   Int[(a + b*x)^m*Simp[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d)
), x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && !IntegerQ[m] && !Integ
erQ[n] && (RationalQ[m] || !SimplerQ[n + 1, m + 1])
```

rule 1138
```
Int[((d_) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Sy
mbol] :> Simp[d^m*((a + b*x + c*x^2)^FracPart[p]/((1 + e*(x/d))^FracPart[p]
*(a/d + (c*x)/e)^FracPart[p]))   Int[(1 + e*(x/d))^(m + p)*(a/d + (c/e)*x)^
p, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0]
&& (IntegerQ[m] || GtQ[d, 0]) && !(IGtQ[m, 0] && (IntegerQ[3*p] || Integer
Q[4*p]))
```

Maple [F]

$$\int \frac{(ade + (a\,e^2 + c\,d^2)\,x + cd\,x^2e)^p}{(ex + d)^3}dx$$

input
```
int((a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^p/(e*x+d)^3,x)
```

output
```
int((a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^p/(e*x+d)^3,x)
```

Fricas [F]

$$\int \frac{(ade + (cd^2 + ae^2)x + cdex^2)^p}{(d + ex)^3} dx = \int \frac{(cdex^2 + ade + (cd^2 + ae^2)x)^p}{(ex + d)^3} dx$$

input
```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^p/(e*x+d)^3,x, algorithm="fric
as")
```

output
```
integral((c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^p/(e^3*x^3 + 3*d*e^2*x^2
+ 3*d^2*e*x + d^3), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(ade + (cd^2 + ae^2)x + cdex^2)^p}{(d + ex)^3} dx = \text{Timed out}$$

input
```
integrate((a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**p/(e*x+d)**3,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{(ade + (cd^2 + ae^2)x + cdex^2)^p}{(d + ex)^3} dx = \int \frac{(cdex^2 + ade + (cd^2 + ae^2)x)^p}{(ex + d)^3} dx$$

input
```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^p/(e*x+d)^3,x, algorithm="maxi
ma")
```

output
```
integrate((c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^p/(e*x + d)^3, x)
```

Giac [F]

$$\int \frac{(ade + (cd^2 + ae^2)x + cdex^2)^p}{(d + ex)^3} dx = \int \frac{(cdex^2 + ade + (cd^2 + ae^2)x)^p}{(ex + d)^3} dx$$

input
```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^p/(e*x+d)^3,x, algorithm="giac
")
```

output
```
integrate((c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^p/(e*x + d)^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(ade + (cd^2 + ae^2)x + cdex^2)^p}{(d + ex)^3} dx = \int \frac{(c\,d\,e\,x^2 + (c\,d^2 + a\,e^2)\,x + a\,d\,e)^p}{(d + e\,x)^3} dx$$

input
```
int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^p/(d + e*x)^3,x)
```

output
```
int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^p/(d + e*x)^3, x)
```

Reduce [F]

$$\int \frac{(ade + (cd^2 + ae^2)x + cdex^2)^p}{(d + ex)^3} dx = \text{too large to display}$$

input
```
int((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^p/(e*x+d)^3,x)
```

output

```
((a*d*e + a*e**2*x + c*d**2*x + c*d*e*x**2)**p*a*e - int(((a*d*e + a*e**2*
x + c*d**2*x + c*d*e*x**2)**p*x)/(a**2*d**3*e**3*p - 2*a**2*d**3*e**3 + 3*
a**2*d**2*e**4*p*x - 6*a**2*d**2*e**4*x + 3*a**2*d*e**5*p*x**2 - 6*a**2*d*
e**5*x**2 + a**2*e**6*p*x**3 - 2*a**2*e**6*x**3 + a*c*d**5*e*p + 4*a*c*d**
4*e**2*p*x - 2*a*c*d**4*e**2*x + 6*a*c*d**3*e**3*p*x**2 - 6*a*c*d**3*e**3*
x**2 + 4*a*c*d**2*e**4*p*x**3 - 6*a*c*d**2*e**4*x**3 + a*c*d*e**5*p*x**4 -
 2*a*c*d*e**5*x**4 + c**2*d**6*p*x + 3*c**2*d**5*e*p*x**2 + 3*c**2*d**4*e*
*2*p*x**3 + c**2*d**3*e**3*p*x**4),x)*a**2*c*d**3*e**4*p**2 + 2*int(((a*d*
e + a*e**2*x + c*d**2*x + c*d*e*x**2)**p*x)/(a**2*d**3*e**3*p - 2*a**2*d**
3*e**3 + 3*a**2*d**2*e**4*p*x - 6*a**2*d**2*e**4*x + 3*a**2*d*e**5*p*x**2
- 6*a**2*d*e**5*x**2 + a**2*e**6*p*x**3 - 2*a**2*e**6*x**3 + a*c*d**5*e*p
+ 4*a*c*d**4*e**2*p*x - 2*a*c*d**4*e**2*x + 6*a*c*d**3*e**3*p*x**2 - 6*a*c
*d**3*e**3*x**2 + 4*a*c*d**2*e**4*p*x**3 - 6*a*c*d**2*e**4*x**3 + a*c*d*e*
*5*p*x**4 - 2*a*c*d*e**5*x**4 + c**2*d**6*p*x + 3*c**2*d**5*e*p*x**2 + 3*c
**2*d**4*e**2*p*x**3 + c**2*d**3*e**3*p*x**4),x)*a**2*c*d**3*e**4*p - 2*in
t(((a*d*e + a*e**2*x + c*d**2*x + c*d*e*x**2)**p*x)/(a**2*d**3*e**3*p - 2*
a**2*d**3*e**3 + 3*a**2*d**2*e**4*p*x - 6*a**2*d**2*e**4*x + 3*a**2*d*e**5
*p*x**2 - 6*a**2*d*e**5*x**2 + a**2*e**6*p*x**3 - 2*a**2*e**6*x**3 + a*c*d
**5*e*p + 4*a*c*d**4*e**2*p*x - 2*a*c*d**4*e**2*x + 6*a*c*d**3*e**3*p*x**2
 - 6*a*c*d**3*e**3*x**2 + 4*a*c*d**2*e**4*p*x**3 - 6*a*c*d**2*e**4*x**3...
```

3.396 $\int (d+ex)^m \left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^p \, dx$

Optimal result

Integrand size = 35, antiderivative size = 92

$$\int (d+ex)^m \left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^p \, dx = -\frac{(d+ex)^m \left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{1+p} \text{Hypergeometric2F1}\left(1,2+m+2p,2+m+p,\frac{cd(d+ex)}{cd^2-ae^2}\right)}{\left(cd^2-ae^2\right)(1+m+p)}$$

output
```
-(e*x+d)^m*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(p+1)*hypergeom([1, 2+m+2*p],
[2+m+p],c*d*(e*x+d)/(-a*e^2+c*d^2))/(-a*e^2+c*d^2)/(1+m+p)
```

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.16

$$\int (d+ex)^m \left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^p \, dx = \frac{(d+ex)^{-1+m}\left(\frac{cd(d+ex)}{cd^2-ae^2}\right)^{-m-p}((ae+cdx)(d+ex))^{1+p}\text{Hypergeometric2F1}\left(-m-p,1+p,2+p,\frac{e(ae+c}{-cd^2+}\right.}{cd(1+p)}$$

input
```
Integrate[(d + e*x)^m*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^p,x]
```

output
```
((d + e*x)^(-1 + m)*((c*d*(d + e*x))/(c*d^2 - a*e^2))^(-m - p)*((a*e + c*d
*x)*(d + e*x))^(1 + p)*Hypergeometric2F1[-m - p, 1 + p, 2 + p, (e*(a*e + c
*d*x))/(-(c*d^2) + a*e^2)])/(c*d*(1 + p))
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 126, normalized size of antiderivative = 1.37, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.114, Rules used = {1139, 1138, 80, 79}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (d+ex)^m \left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^p dx$$

$$\Big\downarrow \text{ 1139}$$

$$(d+ex)^m \left(\frac{ex}{d}+1\right)^{-m} \int \left(\frac{ex}{d}+1\right)^m \left(cdex^2+\left(cd^2+ae^2\right)x+ade\right)^p dx$$

$$\Big\downarrow \text{ 1138}$$

$$(d+ex)^m \left(\frac{ex}{d}+1\right)^{-m-p} \left(ade+cd^2x\right)^{-p} \left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^p \int \left(cxd^2+aed\right)^p \left(\frac{ex}{d}+1\right)^{m+p} dx$$

$$\Big\downarrow \text{ 80}$$

$$(d+ex)^m \left(ade+cd^2x\right)^{-p} \left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^p \left(\frac{cd(d+ex)}{cd^2-ae^2}\right)^{-m-p} \int \left(cxd^2+aed\right)^p \left(\frac{cd^2}{cd^2-ae^2}+\frac{cex}{cd^2-}\right.$$

$$\Big\downarrow \text{ 79}$$

$$\frac{(d+ex)^m \left(ade+cd^2x\right) \left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^p \left(\frac{cd(d+ex)}{cd^2-ae^2}\right)^{-m-p} \text{Hypergeometric2F1}\left(-m-p,p+1,p\right.}{cd^2(p+1)}$$

input
```
Int[(d + e*x)^m*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^p,x]
```

output
```
((a*d*e + c*d^2*x)*(d + e*x)^m*((c*d*(d + e*x))/(c*d^2 - a*e^2))^(-m - p)*
(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^p*Hypergeometric2F1[-m - p, 1 + p,
 2 + p, -((e*(a*e + c*d*x))/(c*d^2 - a*e^2))])/(c*d^2*(1 + p))
```

Defintions of rubi rules used

rule 79
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d), 0] && (RationalQ[m]
 ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

rule 80
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*(b*((c + d*x)/(b*c - a*d)))
^FracPart[n])   Int[(a + b*x)^m*Simp[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d)
), x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] &&  !IntegerQ[m] &&  !Integ
erQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

rule 1138
```
Int[((d_) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Sy
mbol] :> Simp[d^m*((a + b*x + c*x^2)^FracPart[p]/((1 + e*(x/d))^FracPart[p]
*(a/d + (c*x)/e)^FracPart[p]))   Int[(1 + e*(x/d))^(m + p)*(a/d + (c/e)*x)^
p, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0]
&& (IntegerQ[m] || GtQ[d, 0]) &&  !(IGtQ[m, 0] && (IntegerQ[3*p] || Integer
Q[4*p]))
```

rule 1139
```
Int[((d_) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Sy
mbol] :> Simp[d^IntPart[m]*((d + e*x)^FracPart[m]/(1 + e*(x/d))^FracPart[m]
)   Int[(1 + e*(x/d))^m*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e
, m}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&  !(IntegerQ[m] || GtQ[d, 0])
```

Maple [F]

$$\int (ex+d)^m \left(ade+\left(a\,e^2+c\,d^2\right)x+cd\,x^2e\right)^p dx$$

input
```
int((e*x+d)^m*(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^p,x)
```

output
```
int((e*x+d)^m*(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^p,x)
```

Fricas [F]

$$\int (d+ex)^m \left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^p \, dx$$
$$= \int \left(cdex^2+ade+\left(cd^2+ae^2\right)x\right)^p (ex+d)^m \, dx$$

input
```
integrate((e*x+d)^m*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^p,x, algorithm="fric
as")
```

output
```
integral((c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^p*(e*x + d)^m, x)
```

Sympy [F(-1)]

Timed out.

$$\int (d+ex)^m \left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^p \, dx = \text{Timed out}$$

input
```
integrate((e*x+d)**m*(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**p,x)
```

output
```
Timed out
```

Maxima [F]

$$\int (d+ex)^m \left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^p\,dx = \int \left(cdex^2+ade+\left(cd^2+ae^2\right)x\right)^p (ex+d)^m\,dx$$

input
```
integrate((e*x+d)^m*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^p,x, algorithm="maxi
ma")
```

output
```
integrate((c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^p*(e*x + d)^m, x)
```

Giac [F]

$$\int (d+ex)^m \left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^p\,dx = \int \left(cdex^2+ade+\left(cd^2+ae^2\right)x\right)^p (ex+d)^m\,dx$$

input
```
integrate((e*x+d)^m*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^p,x, algorithm="giac
")
```

output
```
integrate((c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^p*(e*x + d)^m, x)
```

Mupad [F(-1)]

Timed out.

$$\int (d+ex)^m \left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^p\,dx = \int (d+e\,x)^m \left(c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e\right)^p dx$$

input
```
int((d + e*x)^m*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^p,x)
```

output
```
int((d + e*x)^m*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^p, x)
```

Reduce [F]

$$\int (d+ex)^m \left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^p\,dx = \text{too large to display}$$

input
```
int((e*x+d)^m*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^p,x)
```

output
```
((d + e*x)**m*(a*d*e + a*e**2*x + c*d**2*x + c*d*e*x**2)**p*a*d*e*m + 2*(d
 + e*x)**m*(a*d*e + a*e**2*x + c*d**2*x + c*d*e*x**2)**p*a*d*e*p + (d + e*
x)**m*(a*d*e + a*e**2*x + c*d**2*x + c*d*e*x**2)**p*a*e**2*m*x + (d + e*x)
**m*(a*d*e + a*e**2*x + c*d**2*x + c*d*e*x**2)**p*a*e**2*p*x + (d + e*x)**
m*(a*d*e + a*e**2*x + c*d**2*x + c*d*e*x**2)**p*c*d**2*p*x + int(((d + e*x
)**m*(a*d*e + a*e**2*x + c*d**2*x + c*d*e*x**2)**p*x)/(a**2*d*e**3*m**2 +
3*a**2*d*e**3*m*p + a**2*d*e**3*m + 2*a**2*d*e**3*p**2 + a**2*d*e**3*p + a
**2*e**4*m**2*x + 3*a**2*e**4*m*p*x + a**2*e**4*m*x + 2*a**2*e**4*p**2*x +
 a**2*e**4*p*x + a*c*d**3*e*m*p + 2*a*c*d**3*e*p**2 + a*c*d**3*e*p + a*c*d
**2*e**2*m**2*x + 4*a*c*d**2*e**2*m*p*x + a*c*d**2*e**2*m*x + 4*a*c*d**2*e
**2*p**2*x + 2*a*c*d**2*e**2*p*x + a*c*d*e**3*m**2*x**2 + 3*a*c*d*e**3*m*p
*x**2 + a*c*d*e**3*m*x**2 + 2*a*c*d*e**3*p**2*x**2 + a*c*d*e**3*p*x**2 + c
**2*d**4*m*p*x + 2*c**2*d**4*p**2*x + c**2*d**4*p*x + c**2*d**3*e*m*p*x**2
 + 2*c**2*d**3*e*p**2*x**2 + c**2*d**3*e*p*x**2),x)*a**3*e**6*m**3*p + 4*i
nt(((d + e*x)**m*(a*d*e + a*e**2*x + c*d**2*x + c*d*e*x**2)**p*x)/(a**2*d*
e**3*m**2 + 3*a**2*d*e**3*m*p + a**2*d*e**3*m + 2*a**2*d*e**3*p**2 + a**2*
d*e**3*p + a**2*e**4*m**2*x + 3*a**2*e**4*m*p*x + a**2*e**4*m*x + 2*a**2*e
**4*p**2*x + a**2*e**4*p*x + a*c*d**3*e*m*p + 2*a*c*d**3*e*p**2 + a*c*d**3
*e*p + a*c*d**2*e**2*m**2*x + 4*a*c*d**2*e**2*m*p*x + a*c*d**2*e**2*m*x +
4*a*c*d**2*e**2*p**2*x + 2*a*c*d**2*e**2*p*x + a*c*d*e**3*m**2*x**2 + 3...
```

3.397 $\int (d+ex)^{-5-2p} \left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^p \, dx$

Optimal result

Integrand size = 39, antiderivative size = 288

$$\begin{aligned}
&\int (d+ex)^{-5-2p} \left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^p \, dx \\
&= \frac{(d+ex)^{-5-2p} \left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{1+p}}{\left(cd^2-ae^2\right)(4+p)} \\
&\quad + \frac{6c^2d^2(d+ex)^{-3-2p} \left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{1+p}}{\left(cd^2-ae^2\right)^3(2+p)(3+p)(4+p)} \\
&\quad + \frac{6c^3d^3(d+ex)^{-2(1+p)} \left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{1+p}}{\left(cd^2-ae^2\right)^4(1+p)(2+p)(3+p)(4+p)} \\
&\quad + \frac{3cd(d+ex)^{-2(2+p)} \left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{1+p}}{\left(cd^2-ae^2\right)^2(3+p)(4+p)}
\end{aligned}$$

output
```
(e*x+d)^(-5-2*p)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(p+1)/(-a*e^2+c*d^2)/(4
+p)+6*c^2*d^2*(e*x+d)^(-3-2*p)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(p+1)/(-a
*e^2+c*d^2)^3/(2+p)/(3+p)/(4+p)+6*c^3*d^3*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2
)^(p+1)/(-a*e^2+c*d^2)^4/(p+1)/(2+p)/(3+p)/(4+p)/((e*x+d)^(2*p+2))+3*c*d*(
a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(p+1)/(-a*e^2+c*d^2)^2/(3+p)/(4+p)/((e*x+
d)^(4+2*p))
```

Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 217, normalized size of antiderivative = 0.75

$$\int (d+ex)^{-5-2p}\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^p\,dx$$
$$=\frac{(d+ex)^{-5-2p}((ae+cdx)(d+ex))^{1+p}\left(-a^3e^6(6+11p+6p^2+p^3)+3a^2cde^4(2+3p+p^2)\left(d(4+p)+e\right.\right.}{}$$

input
```
Integrate[(d + e*x)^(-5 - 2*p)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^p,x
]
```

output
```
((d + e*x)^(-5 - 2*p)*((a*e + c*d*x)*(d + e*x))^(1 + p)*(-(a^3*e^6*(6 + 11
*p + 6*p^2 + p^3)) + 3*a^2*c*d*e^4*(2 + 3*p + p^2)*(d*(4 + p) + e*x) - 3*a
*c^2*d^2*e^2*(1 + p)*(d^2*(12 + 7*p + p^2) + 2*d*e*(4 + p)*x + 2*e^2*x^2)
+ c^3*d^3*(d^3*(24 + 26*p + 9*p^2 + p^3) + 3*d^2*e*(12 + 7*p + p^2)*x + 6*
d*e^2*(4 + p)*x^2 + 6*e^3*x^3)))/((c*d^2 - a*e^2)^4*(1 + p)*(2 + p)*(3 + p
)*(4 + p))
```

Rubi [A] (verified)

Time = 0.49 (sec) , antiderivative size = 296, normalized size of antiderivative = 1.03, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.103, Rules used = {1129, 1129, 1129, 1123}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (d+ex)^{-2p-5}\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^p\,dx$$

$$\Big\downarrow \text{1129}$$

$$\frac{3cd\int (d+ex)^{-2(p+2)}\left(cdex^2+\left(cd^2+ae^2\right)x+ade\right)^p\,dx}{(p+4)\left(cd^2-ae^2\right)}+\frac{(d+ex)^{-2p-5}\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{p+1}}{(p+4)\left(cd^2-ae^2\right)}$$

$$\Big\downarrow \text{1129}$$

$$\frac{3cd\left(\frac{2cd\int(d+ex)^{-2p-3}\left(cdex^2+\left(cd^2+ae^2\right)x+ade\right)^p dx}{(p+3)(cd^2-ae^2)}+\frac{(d+ex)^{-2(p+2)}\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{p+1}}{(p+3)(cd^2-ae^2)}\right)}{(p+4)\left(cd^2-ae^2\right)}+\frac{(d+ex)^{-2p-5}\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{p+1}}{(p+4)\left(cd^2-ae^2\right)}$$

$\Big\downarrow$ 1129

$$\frac{3cd\left(\frac{2cd\left(\frac{cd\int(d+ex)^{-2(p+1)}\left(cdex^2+\left(cd^2+ae^2\right)x+ade\right)^p dx}{(p+2)\left(cd^2-ae^2\right)}+\frac{(d+ex)^{-2p-3}\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{p+1}}{(p+2)\left(cd^2-ae^2\right)}\right)}{(p+3)(cd^2-ae^2)}+\frac{(d+ex)^{-2(p+2)}\left(x\left(ae^2+cd^2\right)+ade\right.}{(p+3)(cd^2-ae^2)}\right.}{(p+4)\left(cd^2-ae^2\right)}$$

$$\frac{(d+ex)^{-2p-5}\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{p+1}}{(p+4)\left(cd^2-ae^2\right)}$$

$\Big\downarrow$ 1123

$$\frac{(d+ex)^{-2p-5}\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{p+1}}{(p+4)\left(cd^2-ae^2\right)}+$$

$$\frac{3cd\left(\frac{(d+ex)^{-2(p+2)}\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{p+1}}{(p+3)(cd^2-ae^2)}+\frac{2cd\left(\frac{(d+ex)^{-2p-3}\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{p+1}}{(p+2)\left(cd^2-ae^2\right)}+\frac{cd(d+ex)^{-2(p+1)}\left(x\left(ae^2+cd^2\right)+ade+c\right.}{(p+1)(p+2)\left(cd^2-ae^2\right)^2}\right.}{(p+3)(cd^2-ae^2)}\right.}{(p+4)\left(cd^2-ae^2\right)}$$

input
```
Int[(d + e*x)^(-5 - 2*p)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^p,x]
```

output
```
((d + e*x)^(-5 - 2*p)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(1 + p))/((c
*d^2 - a*e^2)*(4 + p)) + (3*c*d*((a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(
1 + p)/((c*d^2 - a*e^2)*(3 + p)*(d + e*x)^(2*(2 + p))) + (2*c*d*(((d + e*x
)^(-3 - 2*p)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(1 + p))/((c*d^2 - a*
e^2)*(2 + p)) + (c*d*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(1 + p))/((c*
d^2 - a*e^2)^2*(1 + p)*(2 + p)*(d + e*x)^(2*(1 + p)))))/((c*d^2 - a*e^2)*(
3 + p))))/((c*d^2 - a*e^2)*(4 + p))
```

Defintions of rubi rules used

rule 1123
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[e*(d + e*x)^m*((a + b*x + c*x^2)^(p + 1)/((p + 1)*(2*c*d - b
*e))), x] /; FreeQ[{a, b, c, d, e, m, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2,
0] && EqQ[m + 2*p + 2, 0]
```

rule 1129
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(-e)*(d + e*x)^m*((a + b*x + c*x^2)^(p + 1)/((m + p + 1)*(2*
c*d - b*e))), x] + Simp[c*(Simplify[m + 2*p + 2]/((m + p + 1)*(2*c*d - b*e)
))   Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d
, e, m, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && ILtQ[Simplify[m + 2*p +
2], 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 744 vs. $2(292) = 584$.

Time = 1.83 (sec) , antiderivative size = 745, normalized size of antiderivative = 2.59

method	result
gosper	$-\frac{(ex+d)^{-4-2p}(cdx+ae)\left(cd\,x^2e+a\,e^2x+c\,d^2x+ade\right)^p\left(a^3e^6p^3-3a^2c\,d^2e^4p^3-3a^2cd\,e^5p^2x+3a\,c^2d^4e^2p^3+6a\,c^2d^3e^3p^2x+6a\,c^2d^2e\right.}{a^4e^8p^4-4a^3c\,d^2e^6p^4+6a^2c^2d^4e^4p^4-4a\,c^3d^6e^2p^4+c^4d^8p^4+10a^4e^8p^3-40a^3}$
orering	$-\frac{\left(a^3e^6p^3-3a^2c\,d^2e^4p^3-3a^2cd\,e^5p^2x+3a\,c^2d^4e^2p^3+6a\,c^2d^3e^3p^2x+6a\,c^2d^2e^4p\,x^2-c^3d^6p^3-3c^3d^5e\,p^2x-6c^3d^4e^2p\,x^2-6c^3d^3e^3x^3\right.}{a^4e^8p^4-4a^3c\,d^2e^6p^4+6a^2c^2d^4e^4p^4-4a\,c^3d^6e^2p^4+c^4d^8p^4+10a^4e^8p^3-}$

input
```
int((e*x+d)^(-5-2*p)*(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^p,x,method=_RETURNV
ERBOSE)
```

output
```
-(e*x+d)^(-4-2*p)*(c*d*x+a*e)*(c*d*e*x^2+a*e^2*x+c*d^2*x+a*d*e)^p/(a^4*e^8
*p^4-4*a^3*c*d^2*e^6*p^4+6*a^2*c^2*d^4*e^4*p^4-4*a*c^3*d^6*e^2*p^4+c^4*d^8
*p^4+10*a^4*e^8*p^3-40*a^3*c*d^2*e^6*p^3+60*a^2*c^2*d^4*e^4*p^3-40*a*c^3*d
^6*e^2*p^3+10*c^4*d^8*p^3+35*a^4*e^8*p^2-140*a^3*c*d^2*e^6*p^2+210*a^2*c^2
*d^4*e^4*p^2-140*a*c^3*d^6*e^2*p^2+35*c^4*d^8*p^2+50*a^4*e^8*p-200*a^3*c*d
^2*e^6*p+300*a^2*c^2*d^4*e^4*p-200*a*c^3*d^6*e^2*p+50*c^4*d^8*p+24*a^4*e^8
-96*a^3*c*d^2*e^6+144*a^2*c^2*d^4*e^4-96*a*c^3*d^6*e^2+24*c^4*d^8)*(a^3*e^
6*p^3-3*a^2*c*d^2*e^4*p^3-3*a^2*c*d*e^5*p^2*x+3*a*c^2*d^4*e^2*p^3+6*a*c^2*
d^3*e^3*p^2*x+6*a*c^2*d^2*e^4*p*x^2-c^3*d^6*p^3-3*c^3*d^5*e*p^2*x-6*c^3*d^
4*e^2*p*x^2-6*c^3*d^3*e^3*x^3+6*a^3*e^6*p^2-21*a^2*c*d^2*e^4*p^2-9*a^2*c*d
*e^5*p*x+24*a*c^2*d^4*e^2*p^2+30*a*c^2*d^3*e^3*p*x+6*a*c^2*d^2*e^4*x^2-9*c
^3*d^6*p^2-21*c^3*d^5*e*p*x-24*c^3*d^4*e^2*x^2+11*a^3*e^6*p-42*a^2*c*d^2*e
^4*p-6*a^2*c*d*e^5*x+57*a*c^2*d^4*e^2*p+24*a*c^2*d^3*e^3*x-26*c^3*d^6*p-36
*c^3*d^5*e*x+6*a^3*e^6-24*a^2*c*d^2*e^4+36*a*c^2*d^4*e^2-24*c^3*d^6)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1051 vs. $2(292) = 584$.

Time = 0.17 (sec) , antiderivative size = 1051, normalized size of antiderivative = 3.65

$$\int (d+ex)^{-5-2p}\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^p\,dx=\text{Too large to display}$$

input
```
integrate((e*x+d)^(-5-2*p)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^p,x, algorith
m="fricas")
```

output
```
(6*c^4*d^4*e^4*x^5 + 24*a*c^3*d^7*e - 36*a^2*c^2*d^5*e^3 + 24*a^3*c*d^3*e^
5 - 6*a^4*d*e^7 + 6*(5*c^4*d^5*e^3 + (c^4*d^5*e^3 - a*c^3*d^3*e^5)*p)*x^4
+ (a*c^3*d^7*e - 3*a^2*c^2*d^5*e^3 + 3*a^3*c*d^3*e^5 - a^4*d*e^7)*p^3 + 3*
(20*c^4*d^6*e^2 + (c^4*d^6*e^2 - 2*a*c^3*d^4*e^4 + a^2*c^2*d^2*e^6)*p^2 +
(9*c^4*d^6*e^2 - 10*a*c^3*d^4*e^4 + a^2*c^2*d^2*e^6)*p)*x^3 + 3*(3*a*c^3*d
^7*e - 8*a^2*c^2*d^5*e^3 + 7*a^3*c*d^3*e^5 - 2*a^4*d*e^7)*p^2 + (60*c^4*d^
7*e + (c^4*d^7*e - 3*a*c^3*d^5*e^3 + 3*a^2*c^2*d^3*e^5 - a^3*c*d*e^7)*p^3
+ 3*(4*c^4*d^7*e - 9*a*c^3*d^5*e^3 + 6*a^2*c^2*d^3*e^5 - a^3*c*d*e^7)*p^2
+ (47*c^4*d^7*e - 60*a*c^3*d^5*e^3 + 15*a^2*c^2*d^3*e^5 - 2*a^3*c*d*e^7)*p
)*x^2 + (26*a*c^3*d^7*e - 57*a^2*c^2*d^5*e^3 + 42*a^3*c*d^3*e^5 - 11*a^4*d
*e^7)*p + (24*c^4*d^8 + 24*a*c^3*d^6*e^2 - 36*a^2*c^2*d^4*e^4 + 24*a^3*c*d
^2*e^6 - 6*a^4*e^8 + (c^4*d^8 - 2*a*c^3*d^6*e^2 + 2*a^3*c*d^2*e^6 - a^4*e^
8)*p^3 + 3*(3*c^4*d^8 - 4*a*c^3*d^6*e^2 - 3*a^2*c^2*d^4*e^4 + 6*a^3*c*d^2*
e^6 - 2*a^4*e^8)*p^2 + (26*c^4*d^8 - 10*a*c^3*d^6*e^2 - 45*a^2*c^2*d^4*e^4
 + 40*a^3*c*d^2*e^6 - 11*a^4*e^8)*p)*x)*(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^
2)*x)^p*(e*x + d)^(-2*p - 5)/(24*c^4*d^8 - 96*a*c^3*d^6*e^2 + 144*a^2*c^2*
d^4*e^4 - 96*a^3*c*d^2*e^6 + 24*a^4*e^8 + (c^4*d^8 - 4*a*c^3*d^6*e^2 + 6*a
^2*c^2*d^4*e^4 - 4*a^3*c*d^2*e^6 + a^4*e^8)*p^4 + 10*(c^4*d^8 - 4*a*c^3*d^
6*e^2 + 6*a^2*c^2*d^4*e^4 - 4*a^3*c*d^2*e^6 + a^4*e^8)*p^3 + 35*(c^4*d^8 -
 4*a*c^3*d^6*e^2 + 6*a^2*c^2*d^4*e^4 - 4*a^3*c*d^2*e^6 + a^4*e^8)*p^2 +...
```

Sympy [F]

$$\int (d+ex)^{-5-2p}\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^p\,dx$$
$$=\int ((d+ex)(ae+cdx))^p (d+ex)^{-2p-5}\,dx$$

input
```
integrate((e*x+d)**(-5-2*p)*(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**p,x)
```

output
```
Integral(((d + e*x)*(a*e + c*d*x))**p*(d + e*x)**(-2*p - 5), x)
```

Maxima [F]

$$\int (d+ex)^{-5-2p} \left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^p \, dx = \int \left(cdex^2+ade+\left(cd^2+ae^2\right)x\right)^p (ex+d)^{-2\,p-5} \, dx$$

input
```
integrate((e*x+d)^(-5-2*p)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^p,x, algorith
m="maxima")
```

output
```
integrate((c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^p*(e*x + d)^(-2*p - 5),
x)
```

Giac [F]

$$\int (d+ex)^{-5-2p} \left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^p \, dx = \int \left(cdex^2+ade+\left(cd^2+ae^2\right)x\right)^p (ex+d)^{-2\,p-5} \, dx$$

input
```
integrate((e*x+d)^(-5-2*p)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^p,x, algorith
m="giac")
```

output
```
integrate((c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^p*(e*x + d)^(-2*p - 5),
x)
```

Mupad [B] (verification not implemented)

Time = 6.42 (sec) , antiderivative size = 1036, normalized size of antiderivative = 3.60

$$\int (d+ex)^{-5-2p} \left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^p \, dx = \text{Too large to display}$$

input
```
int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^p/(d + e*x)^(2*p + 5),x)
```

output
```
(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^p*((6*c^4*d^4*e^4*x^5)/((a*e^2 - c
*d^2)^4*(d + e*x)^(2*p + 5)*(50*p + 35*p^2 + 10*p^3 + p^4 + 24)) - (x*(6*a
^4*e^8 - 24*c^4*d^8 + 11*a^4*e^8*p - 26*c^4*d^8*p + 6*a^4*e^8*p^2 + a^4*e^
8*p^3 - 9*c^4*d^8*p^2 - c^4*d^8*p^3 - 24*a*c^3*d^6*e^2 - 24*a^3*c*d^2*e^6
+ 36*a^2*c^2*d^4*e^4 + 9*a^2*c^2*d^4*e^4*p^2 + 10*a*c^3*d^6*e^2*p - 40*a^3
*c*d^2*e^6*p + 45*a^2*c^2*d^4*e^4*p + 12*a*c^3*d^6*e^2*p^2 - 18*a^3*c*d^2*
e^6*p^2 + 2*a*c^3*d^6*e^2*p^3 - 2*a^3*c*d^2*e^6*p^3))/((a*e^2 - c*d^2)^4*(
d + e*x)^(2*p + 5)*(50*p + 35*p^2 + 10*p^3 + p^4 + 24)) - (a*d*e*(6*a^3*e^
6 - 24*c^3*d^6 + 11*a^3*e^6*p - 26*c^3*d^6*p + 6*a^3*e^6*p^2 + a^3*e^6*p^3
 - 9*c^3*d^6*p^2 - c^3*d^6*p^3 + 36*a*c^2*d^4*e^2 - 24*a^2*c*d^2*e^4 + 57*
a*c^2*d^4*e^2*p - 42*a^2*c*d^2*e^4*p + 24*a*c^2*d^4*e^2*p^2 - 21*a^2*c*d^2
*e^4*p^2 + 3*a*c^2*d^4*e^2*p^3 - 3*a^2*c*d^2*e^4*p^3))/((a*e^2 - c*d^2)^4*
(d + e*x)^(2*p + 5)*(50*p + 35*p^2 + 10*p^3 + p^4 + 24)) + (6*c^3*d^3*e^3*
x^4*(5*c*d^2 - a*e^2*p + c*d^2*p))/((a*e^2 - c*d^2)^4*(d + e*x)^(2*p + 5)*
(50*p + 35*p^2 + 10*p^3 + p^4 + 24)) + (3*c^2*d^2*e^2*x^3*(20*c^2*d^4 + a^
2*e^4*p + 9*c^2*d^4*p + a^2*e^4*p^2 + c^2*d^4*p^2 - 10*a*c*d^2*e^2*p - 2*a
*c*d^2*e^2*p^2))/((a*e^2 - c*d^2)^4*(d + e*x)^(2*p + 5)*(50*p + 35*p^2 + 1
0*p^3 + p^4 + 24)) + (c*d*e*x^2*(60*c^3*d^6 - 2*a^3*e^6*p + 47*c^3*d^6*p -
 3*a^3*e^6*p^2 - a^3*e^6*p^3 + 12*c^3*d^6*p^2 + c^3*d^6*p^3 - 60*a*c^2*d^4
*e^2*p + 15*a^2*c*d^2*e^4*p - 27*a*c^2*d^4*e^2*p^2 + 18*a^2*c*d^2*e^4*p...
```

Reduce [F]

$$\int (d+ex)^{-5-2p} \left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^p \, dx$$
$$= \int \frac{(cde\,x^2+a\,e^2x+c\,d^2x+ade)^p}{(ex+d)^{2p}\,d^5+5\,(ex+d)^{2p}\,d^4ex+10\,(ex+d)^{2p}\,d^3e^2x^2+10\,(ex+d)^{2p}\,d^2e^3x^3+5\,(ex+d)^{2p}\,d\,e^4x^4+}$$

input
```
int((e*x+d)^(-5-2*p)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^p,x)
```

output
```
int((a*d*e + a*e**2*x + c*d**2*x + c*d*e*x**2)**p/((d + e*x)**(2*p)*d**5 +
 5*(d + e*x)**(2*p)*d**4*e*x + 10*(d + e*x)**(2*p)*d**3*e**2*x**2 + 10*(d
+ e*x)**(2*p)*d**2*e**3*x**3 + 5*(d + e*x)**(2*p)*d*e**4*x**4 + (d + e*x)*
*(2*p)*e**5*x**5),x)
```

3.398 $\int (d+ex)^{-4-2p} \left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^p\,dx$

Optimal result

Integrand size = 39, antiderivative size = 206

$$\begin{aligned}\int (d+ex)^{-4-2p} \left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^p\,dx &= \frac{2cd(d+ex)^{-3-2p}\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{1+p}}{\left(cd^2-ae^2\right)^2(2+p)(3+p)} \\ &+ \frac{2c^2d^2(d+ex)^{-2(1+p)}\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{1+p}}{\left(cd^2-ae^2\right)^3(1+p)(2+p)(3+p)} \\ &+ \frac{(d+ex)^{-2(2+p)}\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{1+p}}{\left(cd^2-ae^2\right)(3+p)}\end{aligned}$$

output
```
2*c*d*(e*x+d)^(-3-2*p)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(p+1)/(-a*e^2+c*d
^2)^2/(2+p)/(3+p)+2*c^2*d^2*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(p+1)/(-a*e^
2+c*d^2)^3/(p+1)/(2+p)/(3+p)/((e*x+d)^(2*p+2))+(a*d*e+(a*e^2+c*d^2)*x+c*d*
e*x^2)^(p+1)/(-a*e^2+c*d^2)/(3+p)/((e*x+d)^(4+2*p))
```

Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 131, normalized size of antiderivative = 0.64

$$\int (d+ex)^{-4-2p}\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^p\,dx$$
$$=\frac{(d+ex)^{-2(2+p)}((ae+cdx)(d+ex))^{1+p}\left(a^2e^4(2+3p+p^2)-2acde^2(1+p)(d(3+p)+ex)+c^2d^2(d^2(6\right.}{\left(cd^2-ae^2\right)^3(1+p)(2+p)(3+p)}$$

input
```
Integrate[(d + e*x)^(-4 - 2*p)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^p,x
]
```

output
```
(((a*e + c*d*x)*(d + e*x))^(1 + p)*(a^2*e^4*(2 + 3*p + p^2) - 2*a*c*d*e^2*
(1 + p)*(d*(3 + p) + e*x) + c^2*d^2*(d^2*(6 + 5*p + p^2) + 2*d*e*(3 + p)*x
 + 2*e^2*x^2)))/((c*d^2 - a*e^2)^3*(1 + p)*(2 + p)*(3 + p)*(d + e*x)^(2*(2
 + p)))
```

Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 212, normalized size of antiderivative = 1.03, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {1129, 1129, 1123}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (d+ex)^{-2p-4}\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^p\,dx$$

$$\Big\downarrow \text{1129}$$

$$\frac{2cd\int (d+ex)^{-2p-3}\left(cdex^2+\left(cd^2+ae^2\right)x+ade\right)^p\,dx}{(p+3)\left(cd^2-ae^2\right)}+\frac{(d+ex)^{-2(p+2)}\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{p+1}}{(p+3)\left(cd^2-ae^2\right)}$$

$$\Big\downarrow \text{1129}$$

$$\frac{2cd\left(\frac{cd\int (d+ex)^{-2(p+1)}\left(cdex^2+\left(cd^2+ae^2\right)x+ade\right)^pdx}{(p+2)(cd^2-ae^2)}+\frac{(d+ex)^{-2p-3}\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{p+1}}{(p+2)(cd^2-ae^2)}\right)}{(p+3)\left(cd^2-ae^2\right)}+\frac{(d+ex)^{-2(p+2)}\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{p+1}}{(p+3)\left(cd^2-ae^2\right)}$$

$$\Big\downarrow \text{ 1123}$$

$$\frac{(d+ex)^{-2(p+2)}\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{p+1}}{(p+3)\left(cd^2-ae^2\right)}+\frac{2cd\left(\frac{(d+ex)^{-2p-3}\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{p+1}}{(p+2)(cd^2-ae^2)}+\frac{cd(d+ex)^{-2(p+1)}\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{p+1}}{(p+1)(p+2)(cd^2-ae^2)^2}\right)}{(p+3)\left(cd^2-ae^2\right)}$$

input
```
Int[(d + e*x)^(-4 - 2*p)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^p,x]
```

output
```
(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(1 + p)/((c*d^2 - a*e^2)*(3 + p)*(
d + e*x)^(2*(2 + p))) + (2*c*d*(((d + e*x)^(-3 - 2*p)*(a*d*e + (c*d^2 + a*
e^2)*x + c*d*e*x^2)^(1 + p))/((c*d^2 - a*e^2)*(2 + p)) + (c*d*(a*d*e + (c*
d^2 + a*e^2)*x + c*d*e*x^2)^(1 + p))/((c*d^2 - a*e^2)^2*(1 + p)*(2 + p)*(d
 + e*x)^(2*(1 + p)))))/((c*d^2 - a*e^2)*(3 + p))
```

Defintions of rubi rules used

rule 1123
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[e*(d + e*x)^m*((a + b*x + c*x^2)^(p + 1)/((p + 1)*(2*c*d - b
*e))), x] /; FreeQ[{a, b, c, d, e, m, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2,
0] && EqQ[m + 2*p + 2, 0]
```

rule 1129
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(-e)*(d + e*x)^m*((a + b*x + c*x^2)^(p + 1)/((m + p + 1)*(2*
c*d - b*e))), x] + Simp[c*(Simplify[m + 2*p + 2]/((m + p + 1)*(2*c*d - b*e)
))   Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d
, e, m, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && ILtQ[Simplify[m + 2*p +
2], 0]
```

Maple [A] (verified)

Time = 1.55 (sec) , antiderivative size = 381, normalized size of antiderivative = 1.85

method	result
gosper	$-\frac{(ex+d)^{-3-2p}(cdx+ae)(cd\,x^2e+a\,e^2x+c\,d^2x+ade)^p\,(a^2e^4p^2-2ac\,d^2e^2p^2-2acd\,e^3px+c^2d^4p^2+2c^2d^3epx+2x^2c^2d^2e^2+3a}{a^3e^6p^3-3a^2c\,d^2e^4p^3+3a\,c^2d^4e^2p^3-c^3d^6p^3+6a^3e^6p^2-18a^2c\,d^2e^4p^2+18a\,c^2d^4e^2p^2-6c^3d^6p^2+11a^3e^6p-33a^2c\,d^2e}$
orering	$-\frac{(a^2e^4p^2-2ac\,d^2e^2p^2-2acd\,e^3px+c^2d^4p^2+2c^2d^3epx+2x^2c^2d^2e^2+3a^2e^4p-8ac\,d^2e^2p-2xacd\,e^3+5c^2d^4p+6x\,c^2d^3e+2a^2e}{a^3e^6p^3-3a^2c\,d^2e^4p^3+3a\,c^2d^4e^2p^3-c^3d^6p^3+6a^3e^6p^2-18a^2c\,d^2e^4p^2+18a\,c^2d^4e^2p^2-6c^3d^6p^2+11a^3e^6p-33a^2c}$
parallelrisch	Expression too large to display

input
```
int((e*x+d)^(-4-2*p)*(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^p,x,method=_RETURNV
ERBOSE)
```

output
```
-(e*x+d)^(-3-2*p)*(c*d*x+a*e)*(c*d*e*x^2+a*e^2*x+c*d^2*x+a*d*e)^p*(a^2*e^4
*p^2-2*a*c*d^2*e^2*p^2-2*a*c*d*e^3*p*x+c^2*d^4*p^2+2*c^2*d^3*e*p*x+2*c^2*d
^2*e^2*x^2+3*a^2*e^4*p-8*a*c*d^2*e^2*p-2*a*c*d*e^3*x+5*c^2*d^4*p+6*c^2*d^3
*e*x+2*a^2*e^4-6*a*c*d^2*e^2+6*c^2*d^4)/(a^3*e^6*p^3-3*a^2*c*d^2*e^4*p^3+3
*a*c^2*d^4*e^2*p^3-c^3*d^6*p^3+6*a^3*e^6*p^2-18*a^2*c*d^2*e^4*p^2+18*a*c^2
*d^4*e^2*p^2-6*c^3*d^6*p^2+11*a^3*e^6*p-33*a^2*c*d^2*e^4*p+33*a*c^2*d^4*e^
2*p-11*c^3*d^6*p+6*a^3*e^6-18*a^2*c*d^2*e^4+18*a*c^2*d^4*e^2-6*c^3*d^6)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 580 vs. 2(210) = 420.

Time = 0.12 (sec) , antiderivative size = 580, normalized size of antiderivative = 2.82

$$\int (d+ex)^{-4-2p}\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^p\,dx$$
$$=\frac{(2\,c^3d^3e^3x^4+6\,ac^2d^5e-6\,a^2cd^3e^3+2\,a^3de^5+2\,(4\,c^3d^4e^2+(c^3d^4e^2-ac^2d^2e^4)p)x^3+(ac^2d^5e-2\,a^2cd^3e}{}$$

input
```
integrate((e*x+d)^(-4-2*p)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^p,x, algorith
m="fricas")
```

output
```
(2*c^3*d^3*e^3*x^4 + 6*a*c^2*d^5*e - 6*a^2*c*d^3*e^3 + 2*a^3*d*e^5 + 2*(4*
c^3*d^4*e^2 + (c^3*d^4*e^2 - a*c^2*d^2*e^4)*p)*x^3 + (a*c^2*d^5*e - 2*a^2*
c*d^3*e^3 + a^3*d*e^5)*p^2 + (12*c^3*d^5*e + (c^3*d^5*e - 2*a*c^2*d^3*e^3
+ a^2*c*d*e^5)*p^2 + (7*c^3*d^5*e - 8*a*c^2*d^3*e^3 + a^2*c*d*e^5)*p)*x^2
+ (5*a*c^2*d^5*e - 8*a^2*c*d^3*e^3 + 3*a^3*d*e^5)*p + (6*c^3*d^6 + 6*a*c^2
*d^4*e^2 - 6*a^2*c*d^2*e^4 + 2*a^3*e^6 + (c^3*d^6 - a*c^2*d^4*e^2 - a^2*c*
d^2*e^4 + a^3*e^6)*p^2 + (5*c^3*d^6 - a*c^2*d^4*e^2 - 7*a^2*c*d^2*e^4 + 3*
a^3*e^6)*p)*x)*(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^p*(e*x + d)^(-2*p -
 4)/(6*c^3*d^6 - 18*a*c^2*d^4*e^2 + 18*a^2*c*d^2*e^4 - 6*a^3*e^6 + (c^3*d^
6 - 3*a*c^2*d^4*e^2 + 3*a^2*c*d^2*e^4 - a^3*e^6)*p^3 + 6*(c^3*d^6 - 3*a*c^
2*d^4*e^2 + 3*a^2*c*d^2*e^4 - a^3*e^6)*p^2 + 11*(c^3*d^6 - 3*a*c^2*d^4*e^2
 + 3*a^2*c*d^2*e^4 - a^3*e^6)*p)
```

Sympy [F]

$$\int (d+ex)^{-4-2p}\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^p\,dx = \int ((d+ex)(ae+cdx))^p(d+ex)^{-2p-4}\,dx$$

input
```
integrate((e*x+d)**(-4-2*p)*(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**p,x)
```

output
```
Integral(((d + e*x)*(a*e + c*d*x))**p*(d + e*x)**(-2*p - 4), x)
```

Maxima [F]

$$\int (d+ex)^{-4-2p}\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^p\,dx = \int \left(cdex^2+ade+\left(cd^2+ae^2\right)x\right)^p(ex+d)^{-2p-4}\,dx$$

input
```
integrate((e*x+d)^(-4-2*p)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^p,x, algorith
m="maxima")
```

output
```
integrate((c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^p*(e*x + d)^(-2*p - 4),
x)
```

Giac [F]

$$\int (d+ex)^{-4-2p} \left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^p \, dx$$
$$= \int \left(cdex^2+ade+\left(cd^2+ae^2\right)x\right)^p (ex+d)^{-2\,p-4} \, dx$$

input
```
integrate((e*x+d)^(-4-2*p)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^p,x, algorith
m="giac")
```

output
```
integrate((c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^p*(e*x + d)^(-2*p - 4),
x)
```

Mupad [B] (verification not implemented)

Time = 6.38 (sec) , antiderivative size = 595, normalized size of antiderivative = 2.89

$$\int (d+ex)^{-4-2p} \left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^p \, dx =$$
$$-\left(c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e\right)^p \left(\frac{x\,\left(a^3\,e^6\,p^2+3\,a^3\,e^6\,p+2\,a^3\,e^6-a^2\,c\,d^2\,e^4\,p^2-7\,a^2\,c\,d^2\,e^4\,p-6\,a}{\left(a\,e^2-c\,d^2\right)^3\,(d+}\right.$$
$$+\frac{2\,c^3\,d^3\,e^3\,x^4}{\left(a\,e^2-c\,d^2\right)^3\,{\left(d+e\,x\right)}^{2\,p+4}\,\left(p^3+6\,p^2+11\,p+6\right)}$$
$$+\frac{a\,d\,e\,\left(a^2\,e^4\,p^2+3\,a^2\,e^4\,p+2\,a^2\,e^4-2\,a\,c\,d^2\,e^2\,p^2-8\,a\,c\,d^2\,e^2\,p-6\,a\,c\,d^2\,e^2+c^2\,d^4\,p^2+5\,c^2\,d^4\,p+6\,c^2}{\left(a\,e^2-c\,d^2\right)^3\,{\left(d+e\,x\right)}^{2\,p+4}\,\left(p^3+6\,p^2+11\,p+6\right)}$$
$$+\frac{c\,d\,e\,x^2\,\left(a^2\,e^4\,p^2+a^2\,e^4\,p-2\,a\,c\,d^2\,e^2\,p^2-8\,a\,c\,d^2\,e^2\,p+c^2\,d^4\,p^2+7\,c^2\,d^4\,p+12\,c^2\,d^4\right)}{\left(a\,e^2-c\,d^2\right)^3\,{\left(d+e\,x\right)}^{2\,p+4}\,\left(p^3+6\,p^2+11\,p+6\right)}$$
$$\left.+\frac{2\,c^2\,d^2\,e^2\,x^3\,\left(4\,c\,d^2-a\,e^2\,p+c\,d^2\,p\right)}{\left(a\,e^2-c\,d^2\right)^3\,{\left(d+e\,x\right)}^{2\,p+4}\,\left(p^3+6\,p^2+11\,p+6\right)}\right)$$

input
```
int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^p/(d + e*x)^(2*p + 4),x)
```

output
```
-(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^p*((x*(2*a^3*e^6 + 6*c^3*d^6 + 3*
a^3*e^6*p + 5*c^3*d^6*p + a^3*e^6*p^2 + c^3*d^6*p^2 + 6*a*c^2*d^4*e^2 - 6*
a^2*c*d^2*e^4 - a*c^2*d^4*e^2*p - 7*a^2*c*d^2*e^4*p - a*c^2*d^4*e^2*p^2 -
a^2*c*d^2*e^4*p^2))/((a*e^2 - c*d^2)^3*(d + e*x)^(2*p + 4)*(11*p + 6*p^2 +
 p^3 + 6)) + (2*c^3*d^3*e^3*x^4)/((a*e^2 - c*d^2)^3*(d + e*x)^(2*p + 4)*(1
1*p + 6*p^2 + p^3 + 6)) + (a*d*e*(2*a^2*e^4 + 6*c^2*d^4 + 3*a^2*e^4*p + 5*
c^2*d^4*p + a^2*e^4*p^2 + c^2*d^4*p^2 - 6*a*c*d^2*e^2 - 8*a*c*d^2*e^2*p -
2*a*c*d^2*e^2*p^2))/((a*e^2 - c*d^2)^3*(d + e*x)^(2*p + 4)*(11*p + 6*p^2 +
 p^3 + 6)) + (c*d*e*x^2*(12*c^2*d^4 + a^2*e^4*p + 7*c^2*d^4*p + a^2*e^4*p^
2 + c^2*d^4*p^2 - 8*a*c*d^2*e^2*p - 2*a*c*d^2*e^2*p^2))/((a*e^2 - c*d^2)^3
*(d + e*x)^(2*p + 4)*(11*p + 6*p^2 + p^3 + 6)) + (2*c^2*d^2*e^2*x^3*(4*c*d
^2 - a*e^2*p + c*d^2*p))/((a*e^2 - c*d^2)^3*(d + e*x)^(2*p + 4)*(11*p + 6*
p^2 + p^3 + 6)))
```

Reduce [F]

$$\int (d+ex)^{-4-2p} \left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^p \, dx$$
$$= \int \frac{(cde\,x^2+a\,e^2x+c\,d^2x+ade)^p}{(ex+d)^{2p}\,d^4+4\,(ex+d)^{2p}\,d^3ex+6\,(ex+d)^{2p}\,d^2e^2x^2+4\,(ex+d)^{2p}\,d\,e^3x^3+(ex+d)^{2p}\,e^4x^4}dx$$

input
```
int((e*x+d)^(-4-2*p)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^p,x)
```

output
```
int((a*d*e + a*e**2*x + c*d**2*x + c*d*e*x**2)**p/((d + e*x)**(2*p)*d**4 +
 4*(d + e*x)**(2*p)*d**3*e*x + 6*(d + e*x)**(2*p)*d**2*e**2*x**2 + 4*(d +
e*x)**(2*p)*d*e**3*x**3 + (d + e*x)**(2*p)*e**4*x**4),x)
```

3.399 $\int (d+ex)^{-3-2p} \left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^p \, dx$

Optimal result

Integrand size = 39, antiderivative size = 128

$$\int (d+ex)^{-3-2p} \left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^p \, dx = \frac{(d+ex)^{-3-2p} \left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{1+p}}{\left(cd^2-ae^2\right)(2+p)} + \frac{cd(d+ex)^{-2(1+p)} \left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{1+p}}{\left(cd^2-ae^2\right)^2 (1+p)(2+p)}$$

output
```
(e*x+d)^(-3-2*p)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(p+1)/(-a*e^2+c*d^2)/(2
+p)+c*d*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(p+1)/(-a*e^2+c*d^2)^2/(p+1)/(2+
p)/((e*x+d)^(2*p+2))
```

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.59

$$\int (d+ex)^{-3-2p} \left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^p \, dx = \frac{(d+ex)^{-3-2p}((ae+cdx)(d+ex))^{1+p} \left(-ae^2(1+p)+cd(d(2+p)+ex)\right)}{\left(cd^2-ae^2\right)^2 (1+p)(2+p)}$$

input
```
Integrate[(d + e*x)^(-3 - 2*p)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^p,x
]
```

output
```
((d + e*x)^(-3 - 2*p)*((a*e + c*d*x)*(d + e*x))^(1 + p)*(-(a*e^2*(1 + p))
+ c*d*(d*(2 + p) + e*x)))/((c*d^2 - a*e^2)^2*(1 + p)*(2 + p))
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 128, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.051$, Rules used = {1129, 1123}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (d+ex)^{-2p-3}\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^p\,dx$$

$$\Big\downarrow \text{1129}$$

$$\frac{cd\int (d+ex)^{-2(p+1)}\left(cdex^2+\left(cd^2+ae^2\right)x+ade\right)^p dx}{(p+2)\left(cd^2-ae^2\right)}+\frac{(d+ex)^{-2p-3}\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{p+1}}{(p+2)\left(cd^2-ae^2\right)}$$

$$\Big\downarrow \text{1123}$$

$$\frac{(d+ex)^{-2p-3}\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{p+1}}{(p+2)\left(cd^2-ae^2\right)}+\frac{cd(d+ex)^{-2(p+1)}\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{p+1}}{(p+1)(p+2)\left(cd^2-ae^2\right)^2}$$

input
```
Int[(d + e*x)^(-3 - 2*p)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^p,x]
```

output
```
((d + e*x)^(-3 - 2*p)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(1 + p))/((c
*d^2 - a*e^2)*(2 + p)) + (c*d*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(1 +
 p))/((c*d^2 - a*e^2)^2*(1 + p)*(2 + p)*(d + e*x)^(2*(1 + p)))
```

Defintions of rubi rules used

rule 1123
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[e*(d + e*x)^m*((a + b*x + c*x^2)^(p + 1)/((p + 1)*(2*c*d - b
*e))), x] /; FreeQ[{a, b, c, d, e, m, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2,
0] && EqQ[m + 2*p + 2, 0]
```

rule 1129
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(-e)*(d + e*x)^m*((a + b*x + c*x^2)^(p + 1)/((m + p + 1)*(2*
c*d - b*e))), x] + Simp[c*(Simplify[m + 2*p + 2]/((m + p + 1)*(2*c*d - b*e)
))   Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d
, e, m, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && ILtQ[Simplify[m + 2*p +
2], 0]
```

Maple [A] (verified)

Time = 1.54 (sec) , antiderivative size = 170, normalized size of antiderivative = 1.33

method	result
gosper	$-\frac{(ex+d)^{-2p-2}(cdx+ae)(cd\,x^2e+a\,e^2x+c\,d^2x+ade)^p(a\,e^2p-c\,d^2p-cdxe+a\,e^2-2c\,d^2)}{a^2e^4p^2-2ac\,d^2e^2p^2+c^2d^4p^2+3a^2e^4p-6ac\,d^2e^2p+3c^2d^4p+2a^2e^4-4ac\,d^2e^2+2c^2d^4}$
orering	$-\frac{(a\,e^2p-c\,d^2p-cdxe+a\,e^2-2c\,d^2)(ex+d)(cdx+ae)(ex+d)^{-3-2p}(ade+(a\,e^2+c\,d^2)x+cd\,x^2e)^p}{a^2e^4p^2-2ac\,d^2e^2p^2+c^2d^4p^2+3a^2e^4p-6ac\,d^2e^2p+3c^2d^4p+2a^2e^4-4ac\,d^2e^2+2c^2d^4}$
parallelrisch	$x^3(ex+d)^{-3-2p}(cd\,x^2e+a\,e^2x+c\,d^2x+ade)^pc^3d^3e^3-x^2(ex+d)^{-3-2p}(cd\,x^2e+a\,e^2x+c\,d^2x+ade)^pa\,c^2d^2e^4p+x^2(ex+d)^{-3-}$

input
```
int((e*x+d)^(-3-2*p)*(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^p,x,method=_RETURNV
ERBOSE)
```

output
```
-(e*x+d)^(-2*p-2)*(c*d*x+a*e)*(c*d*e*x^2+a*e^2*x+c*d^2*x+a*d*e)^p*(a*e^2*p
-c*d^2*p-c*d*e*x+a*e^2-2*c*d^2)/(a^2*e^4*p^2-2*a*c*d^2*e^2*p^2+c^2*d^4*p^2
+3*a^2*e^4*p-6*a*c*d^2*e^2*p+3*c^2*d^4*p+2*a^2*e^4-4*a*c*d^2*e^2+2*c^2*d^4
)
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 255, normalized size of antiderivative = 1.99

$$\int (d+ex)^{-3-2p}\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^p\,dx$$
$$=\frac{(c^2d^2e^2x^3+2\,acd^3e-a^2de^3+(3\,c^2d^3e+(c^2d^3e-acde^3)p)x^2+(acd^3e-a^2de^3)p+(2\,c^2d^4+2\,acd^2e^2-}{2\,c^2d^4-4\,acd^2e^2+2\,a^2e^4+(c^2d^4-2\,acd^2e^2+a^2e^4)p^2+3}$$

input
```
integrate((e*x+d)^(-3-2*p)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^p,x, algorith
m="fricas")
```

output
```
(c^2*d^2*e^2*x^3 + 2*a*c*d^3*e - a^2*d*e^3 + (3*c^2*d^3*e + (c^2*d^3*e - a
*c*d*e^3)*p)*x^2 + (a*c*d^3*e - a^2*d*e^3)*p + (2*c^2*d^4 + 2*a*c*d^2*e^2
- a^2*e^4 + (c^2*d^4 - a^2*e^4)*p)*x)*(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)
*x)^p*(e*x + d)^(-2*p - 3)/(2*c^2*d^4 - 4*a*c*d^2*e^2 + 2*a^2*e^4 + (c^2*d
^4 - 2*a*c*d^2*e^2 + a^2*e^4)*p^2 + 3*(c^2*d^4 - 2*a*c*d^2*e^2 + a^2*e^4)*
p)
```

Sympy [F]

$$\int (d+ex)^{-3-2p}\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^p\,dx$$
$$=\int \left((d+ex)\,(ae+cdx)\right)^p(d+ex)^{-2p-3}\,dx$$

input
```
integrate((e*x+d)**(-3-2*p)*(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**p,x)
```

output
```
Integral(((d + e*x)*(a*e + c*d*x))**p*(d + e*x)**(-2*p - 3), x)
```

Maxima [F]

$$\int (d+ex)^{-3-2p}\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^p\,dx$$
$$=\int \left(cdex^2+ade+\left(cd^2+ae^2\right)x\right)^p(ex+d)^{-2p-3}\,dx$$

input
```
integrate((e*x+d)^(-3-2*p)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^p,x, algorith
m="maxima")
```

output
```
integrate((c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^p*(e*x + d)^(-2*p - 3),
x)
```

Giac [F]

$$\int (d+ex)^{-3-2p}\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^p\,dx$$
$$=\int \left(cdex^2+ade+\left(cd^2+ae^2\right)x\right)^p(ex+d)^{-2p-3}\,dx$$

input
```
integrate((e*x+d)^(-3-2*p)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^p,x, algorith
m="giac")
```

output
```
integrate((c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^p*(e*x + d)^(-2*p - 3),
x)
```

Mupad [B] (verification not implemented)

Time = 5.80 (sec) , antiderivative size = 293, normalized size of antiderivative = 2.29

$$\int (d+ex)^{-3-2p}\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^p\,dx$$
$$=\left(c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e\right)^p\left(\frac{x\,(2\,c^2\,d^4-a^2\,e^4-a^2\,e^4\,p+c^2\,d^4\,p+2\,a\,c\,d^2\,e^2)}{(a\,e^2-c\,d^2)^2\,(d+e\,x)^{2\,p+3}\,(p^2+3\,p+2)}+\frac{c^2\,d^2\,e^2\,x^3}{(a\,e^2-c\,d^2)^2\,(d+e\,x)^{2\,p+3}\,(p^2+3\,p+2)}-\frac{a\,d\,e\,(a\,e^2-2\,c\,d^2+a\,e^2\,p-c\,d^2\,p)}{(a\,e^2-c\,d^2)^2\,(d+e\,x)^{2\,p+3}\,(p^2+3\,p+2)}+\frac{c\,d\,e\,x^2\,(3\,c\,d^2-a\,e^2\,p+c\,d^2\,p)}{(a\,e^2-c\,d^2)^2\,(d+e\,x)^{2\,p+3}\,(p^2+3\,p+2)}\right)$$

input
```
int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^p/(d + e*x)^(2*p + 3),x)
```

output
```
(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^p*((x*(2*c^2*d^4 - a^2*e^4 - a^2*e
^4*p + c^2*d^4*p + 2*a*c*d^2*e^2))/((a*e^2 - c*d^2)^2*(d + e*x)^(2*p + 3)*
(3*p + p^2 + 2)) + (c^2*d^2*e^2*x^3)/((a*e^2 - c*d^2)^2*(d + e*x)^(2*p + 3
)*(3*p + p^2 + 2)) - (a*d*e*(a*e^2 - 2*c*d^2 + a*e^2*p - c*d^2*p))/((a*e^2
 - c*d^2)^2*(d + e*x)^(2*p + 3)*(3*p + p^2 + 2)) + (c*d*e*x^2*(3*c*d^2 - a
*e^2*p + c*d^2*p))/((a*e^2 - c*d^2)^2*(d + e*x)^(2*p + 3)*(3*p + p^2 + 2))
)
```

Reduce [F]

$$\int (d+ex)^{-3-2p}\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^p\,dx$$
$$=\int\frac{(cde\,x^2+a\,e^2x+c\,d^2x+ade)^p}{(ex+d)^{2p}\,d^3+3\,(ex+d)^{2p}\,d^2ex+3\,(ex+d)^{2p}\,d\,e^2x^2+(ex+d)^{2p}\,e^3x^3}dx$$

input
```
int((e*x+d)^(-3-2*p)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^p,x)
```

output
```
int((a*d*e + a*e**2*x + c*d**2*x + c*d*e*x**2)**p/((d + e*x)**(2*p)*d**3 +
 3*(d + e*x)**(2*p)*d**2*e*x + 3*(d + e*x)**(2*p)*d*e**2*x**2 + (d + e*x)*
*(2*p)*e**3*x**3),x)
```

3.400 $\int (d+ex)^{-2-2p} \left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^p \, dx$

Optimal result

Integrand size = 39, antiderivative size = 60

$$\int (d+ex)^{-2-2p} \left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^p \, dx = \frac{(d+ex)^{-2(1+p)} \left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{1+p}}{\left(cd^2-ae^2\right)(1+p)}$$

output
```
(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(p+1)/(-a*e^2+c*d^2)/(p+1)/((e*x+d)^(2*p
+2))
```

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.82

$$\int (d+ex)^{-2-2p} \left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^p \, dx = \frac{(d+ex)^{-2(1+p)}((ae+cdx)(d+ex))^{1+p}}{\left(cd^2-ae^2\right)(1+p)}$$

input
```
Integrate[(d + e*x)^(-2 - 2*p)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^p,x
]
```

output
```
((a*e + c*d*x)*(d + e*x))^(1 + p)/((c*d^2 - a*e^2)*(1 + p)*(d + e*x)^(2*(1
 + p)))
```

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.026, Rules used = {1123}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (d+ex)^{-2p-2} \left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^p dx$$

$$\Big\downarrow \text{1123}$$

$$\frac{(d+ex)^{-2(p+1)} \left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{p+1}}{(p+1)\left(cd^2-ae^2\right)}$$

input
```
Int[(d + e*x)^(-2 - 2*p)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^p,x]
```

output
```
(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(1 + p)/((c*d^2 - a*e^2)*(1 + p)*(
d + e*x)^(2*(1 + p)))
```

Defintions of rubi rules used

rule 1123
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[e*(d + e*x)^m*((a + b*x + c*x^2)^(p + 1)/((p + 1)*(2*c*d - b
*e))), x] /; FreeQ[{a, b, c, d, e, m, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2,
0] && EqQ[m + 2*p + 2, 0]
```

Maple [A] (verified)

Time = 1.52 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.25

method	result
gosper	$-\frac{(ex+d)^{-1-2p}(cdx+ae)(cd\,x^2e+a\,e^2x+c\,d^2x+ade)^p}{a\,e^2p-c\,d^2p+a\,e^2-c\,d^2}$
orering	$-\frac{(ex+d)(cdx+ae)(ex+d)^{-2p-2}(ade+(a\,e^2+c\,d^2)x+cd\,x^2e)^p}{a\,e^2p-c\,d^2p+a\,e^2-c\,d^2}$
parallelrisch	$-\frac{x^2(ex+d)^{-2p-2}(cd\,x^2e+a\,e^2x+c\,d^2x+ade)^p c^2d^2e^2+x(ex+d)^{-2p-2}(cd\,x^2e+a\,e^2x+c\,d^2x+ade)^p acd\,e^3+x(ex+d)^{-2p-2}}{(a\,e^2p-c\,d^2p+a\,e^2-c\,d^2)dec}$

input
```
int((e*x+d)^(-2*p-2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^p,x,method=_RETURNV
ERBOSE)
```

output
```
-(e*x+d)^(-1-2*p)*(c*d*x+a*e)/(a*e^2*p-c*d^2*p+a*e^2-c*d^2)*(c*d*e*x^2+a*e
^2*x+c*d^2*x+a*d*e)^p
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.53

$$\int (d+ex)^{-2-2p}\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^p\,dx = \frac{(cdex^2+ade+(cd^2+ae^2)x)(cdex^2+ade+(cd^2+ae^2)x)^p(ex+d)^{-2\,p-2}}{cd^2-ae^2+(cd^2-ae^2)p}$$

input
```
integrate((e*x+d)^(-2-2*p)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^p,x, algorith
m="fricas")
```

output
```
(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^
2)*x)^p*(e*x + d)^(-2*p - 2)/(c*d^2 - a*e^2 + (c*d^2 - a*e^2)*p)
```

Sympy [F]

$$\int (d+ex)^{-2-2p}\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^p\,dx = \int ((d+ex)(ae+cdx))^p(d+ex)^{-2p-2}\,dx$$

input
```
integrate((e*x+d)**(-2-2*p)*(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**p,x)
```

output
```
Integral(((d + e*x)*(a*e + c*d*x))**p*(d + e*x)**(-2*p - 2), x)
```

Maxima [F]

$$\int (d+ex)^{-2-2p}\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^p\,dx = \int \left(cdex^2+ade+\left(cd^2+ae^2\right)x\right)^p(ex+d)^{-2p-2}\,dx$$

input
```
integrate((e*x+d)^(-2-2*p)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^p,x, algorith
m="maxima")
```

output
```
integrate((c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^p*(e*x + d)^(-2*p - 2),
x)
```

Giac [F]

$$\int (d+ex)^{-2-2p}\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^p\,dx = \int \left(cdex^2+ade+\left(cd^2+ae^2\right)x\right)^p(ex+d)^{-2p-2}\,dx$$

input
```
integrate((e*x+d)^(-2-2*p)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^p,x, algorith
m="giac")
```

output
```
integrate((c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^p*(e*x + d)^(-2*p - 2),
x)
```

Mupad [B] (verification not implemented)

Time = 5.67 (sec) , antiderivative size = 150, normalized size of antiderivative = 2.50

$$\int (d+ex)^{-2-2p}\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^p\,dx = -\left(\frac{x\,\left(c\,d^2+a\,e^2\right)}{\left(a\,e^2-c\,d^2\right)\,\left(p+1\right)\,{\left(d+e\,x\right)}^{2\,p+2}}+\frac{a\,d\,e}{\left(a\,e^2-c\,d^2\right)\,\left(p+1\right)\,{\left(d+e\,x\right)}^{2\,p+2}}+\frac{c\,d\,e\,x^2}{\left(a\,e^2-c\,d^2\right)\,\left(p+1\right)\,{\left(d+e\,x\right)}^{2\,p+2}}\right)\,{\left(c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e\right)}^p$$

input
```
int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^p/(d + e*x)^(2*p + 2),x)
```

output
```
-((x*(a*e^2 + c*d^2))/((a*e^2 - c*d^2)*(p + 1)*(d + e*x)^(2*p + 2)) + (a*d
*e)/((a*e^2 - c*d^2)*(p + 1)*(d + e*x)^(2*p + 2)) + (c*d*e*x^2)/((a*e^2 -
c*d^2)*(p + 1)*(d + e*x)^(2*p + 2)))*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^
2)^p
```

Reduce [F]

$$\int (d+ex)^{-2-2p}\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^p\,dx = \int \frac{\left(cde\,x^2+a\,e^2x+c\,d^2x+ade\right)^p}{\left(ex+d\right)^{2p}d^2+2\left(ex+d\right)^{2p}dex+\left(ex+d\right)^{2p}e^2x^2}dx$$

input
```
int((e*x+d)^(-2-2*p)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^p,x)
```

output
```
int((a*d*e + a*e**2*x + c*d**2*x + c*d*e*x**2)**p/((d + e*x)**(2*p)*d**2 +
 2*(d + e*x)**(2*p)*d*e*x + (d + e*x)**(2*p)*e**2*x**2),x)
```

3.401 $\int (d+ex)^{-1-2p} \left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^p \, dx$

Optimal result

Integrand size = 39, antiderivative size = 88

$$\int (d+ex)^{-1-2p} \left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^p \, dx = \frac{(d+ex)^{-1-2p} \left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{1+p} \text{Hypergeometric2F1}\left(1,1,1-p,\frac{cd(d+ex)}{cd^2-ae^2}\right)}{\left(cd^2-ae^2\right)p}$$

output
```
(e*x+d)^(-1-2*p)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(p+1)*hypergeom([1, 1],
[1-p],c*d*(e*x+d)/(-a*e^2+c*d^2))/(-a*e^2+c*d^2)/p
```

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.08

$$\int (d+ex)^{-1-2p} \left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^p \, dx = -\frac{\left(\frac{e(ae+cdx)}{-cd^2+ae^2}\right)^{-p} (d+ex)^{-2p}((ae+cdx)(d+ex))^p \text{Hypergeometric2F1}\left(-p,-p,1-p,\frac{cd(d+ex)}{cd^2-ae^2}\right)}{ep}$$

input
```
Integrate[(d + e*x)^(-1 - 2*p)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^p,x
]
```

output
```
-((((a*e + c*d*x)*(d + e*x))^p*Hypergeometric2F1[-p, -p, 1 - p, (c*d*(d +
e*x))/(c*d^2 - a*e^2)])/(e*p*((e*(a*e + c*d*x))/(-(c*d^2) + a*e^2))^p*(d +
 e*x)^(2*p)))
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.22, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.103, Rules used = {1139, 1138, 80, 79}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (d+ex)^{-2p-1} \left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^p \, dx$$

$$\Big\downarrow \text{1139}$$

$$\frac{(d+ex)^{-2p}\left(\frac{ex}{d}+1\right)^{2p}\int \left(\frac{ex}{d}+1\right)^{-2p-1}\left(cdex^2+\left(cd^2+ae^2\right)x+ade\right)^p dx}{d}$$

$$\Big\downarrow \text{1138}$$

$$\frac{(d+ex)^{-2p}\left(\frac{ex}{d}+1\right)^{p}\left(ade+cd^2x\right)^{-p}\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^p\int \left(cxd^2+aed\right)^p\left(\frac{ex}{d}+1\right)^{-p-1}dx}{d}$$

$$\Big\downarrow \text{80}$$

$$\frac{(d+ex)^{-2p}\left(\frac{ex}{d}+1\right)^{p}\left(-\frac{e(ae+cdx)}{cd^2-ae^2}\right)^{-p}\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^p\int \left(\frac{ex}{d}+1\right)^{-p-1}\left(-\frac{ae^2}{cd^2-ae^2}-\frac{cdxe}{cd^2-ae^2}\right)^p dx}{d}$$

$$\Big\downarrow \text{79}$$

$$-\frac{(d+ex)^{-2p}\left(-\frac{e(ae+cdx)}{cd^2-ae^2}\right)^{-p}\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^p \text{Hypergeometric2F1}\left(-p,-p,1-p,\frac{cd(d+ex)}{cd^2-ae^2}\right)}{ep}$$

input
```
Int[(d + e*x)^(-1 - 2*p)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^p,x]
```

output
```
-(((a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^p*Hypergeometric2F1[-p, -p, 1 -
 p, (c*d*(d + e*x))/(c*d^2 - a*e^2)])/(e*p*(-((e*(a*e + c*d*x))/(c*d^2 - a
*e^2)))^p*(d + e*x)^(2*p)))
```

Defintions of rubi rules used

rule 79
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d), 0] && (RationalQ[m]
 ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

rule 80
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*(b*((c + d*x)/(b*c - a*d)))
^FracPart[n])   Int[(a + b*x)^m*Simp[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d)
), x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] &&  !IntegerQ[m] &&  !Integ
erQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

rule 1138
```
Int[((d_) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Sy
mbol] :> Simp[d^m*((a + b*x + c*x^2)^FracPart[p]/((1 + e*(x/d))^FracPart[p]
*(a/d + (c*x)/e)^FracPart[p]))   Int[(1 + e*(x/d))^(m + p)*(a/d + (c/e)*x)^
p, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0]
&& (IntegerQ[m] || GtQ[d, 0]) &&  !(IGtQ[m, 0] && (IntegerQ[3*p] || Integer
Q[4*p]))
```

rule 1139
```
Int[((d_) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Sy
mbol] :> Simp[d^IntPart[m]*((d + e*x)^FracPart[m]/(1 + e*(x/d))^FracPart[m]
)   Int[(1 + e*(x/d))^m*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e
, m}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&  !(IntegerQ[m] || GtQ[d, 0])
```

Maple [F]

$$\int (ex+d)^{-1-2p}\left(ade+\left(a\,e^{2}+c\,d^{2}\right)x+cd\,x^{2}e\right)^{p}dx$$

input
```
int((e*x+d)^(-1-2*p)*(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^p,x)
```

output
```
int((e*x+d)^(-1-2*p)*(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^p,x)
```

Fricas [F]

$$\int (d+ex)^{-1-2p}\left(ade+\left(cd^{2}+ae^{2}\right)x+cdex^{2}\right)^{p}\,dx$$
$$=\int \left(cdex^{2}+ade+\left(cd^{2}+ae^{2}\right)x\right)^{p}(ex+d)^{-2\,p-1}\,dx$$

input
```
integrate((e*x+d)^(-1-2*p)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^p,x, algorith
m="fricas")
```

output
```
integral((c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^p*(e*x + d)^(-2*p - 1), x
)
```

Sympy [F]

$$\int (d+ex)^{-1-2p}\left(ade+\left(cd^{2}+ae^{2}\right)x+cdex^{2}\right)^{p}\,dx$$
$$=\int \left((d+ex)\left(ae+cdx\right)\right)^{p}(d+ex)^{-2p-1}\,dx$$

input
```
integrate((e*x+d)**(-1-2*p)*(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**p,x)
```

output
```
Integral(((d + e*x)*(a*e + c*d*x))**p*(d + e*x)**(-2*p - 1), x)
```

Maxima [F]

$$\int (d+ex)^{-1-2p} \left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^p \, dx = \int \left(cdex^2+ade+\left(cd^2+ae^2\right)x\right)^p (ex+d)^{-2\,p-1}\, dx$$

input
```
integrate((e*x+d)^(-1-2*p)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^p,x, algorith
m="maxima")
```

output
```
integrate((c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^p*(e*x + d)^(-2*p - 1),
x)
```

Giac [F]

$$\int (d+ex)^{-1-2p} \left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^p \, dx = \int \left(cdex^2+ade+\left(cd^2+ae^2\right)x\right)^p (ex+d)^{-2\,p-1}\, dx$$

input
```
integrate((e*x+d)^(-1-2*p)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^p,x, algorith
m="giac")
```

output
```
integrate((c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^p*(e*x + d)^(-2*p - 1),
x)
```

Mupad [F(-1)]

Timed out.

$$\int (d+ex)^{-1-2p} \left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^p\,dx = \int \frac{\left(c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e\right)^p}{\left(d+e\,x\right)^{2\,p+1}}\,dx$$

input
```
int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^p/(d + e*x)^(2*p + 1),x)
```

output
```
int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^p/(d + e*x)^(2*p + 1), x)
```

Reduce [F]

$$\int (d+ex)^{-1-2p} \left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^p\,dx = \int \frac{\left(cde\,x^2+a\,e^2x+c\,d^2x+ade\right)^p}{\left(ex+d\right)^{2p}d+\left(ex+d\right)^{2p}ex}dx$$

input
```
int((e*x+d)^(-1-2*p)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^p,x)
```

output
```
int((a*d*e + a*e**2*x + c*d**2*x + c*d*e*x**2)**p/((d + e*x)**(2*p)*d + (d
 + e*x)**(2*p)*e*x),x)
```

3.402 $\int (d+ex)^{-2p} \left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^p \, dx$

Optimal result

Integrand size = 37, antiderivative size = 91

$$\int (d+ex)^{-2p} \left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^p \, dx = -\frac{(d+ex)^{-2p}\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{1+p}\text{Hypergeometric2F1}\left(1,2,2-p,\frac{cd(d+ex)}{cd^2-ae^2}\right)}{\left(cd^2-ae^2\right)(1-p)}$$

output
```
-(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(p+1)*hypergeom([1, 2],[2-p],c*d*(e*x+d
)/(-a*e^2+c*d^2))/(-a*e^2+c*d^2)/(1-p)/((e*x+d)^(2*p))
```

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.07

$$\int (d+ex)^{-2p} \left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^p \, dx = \frac{(d+ex)^{-1-2p}\left(\frac{cd(d+ex)}{cd^2-ae^2}\right)^p((ae+cdx)(d+ex))^{1+p}\text{Hypergeometric2F1}\left(p,1+p,2+p,\frac{e(ae+cdx)}{-cd^2+ae^2}\right)}{cd(1+p)}$$

input
```
Integrate[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^p/(d + e*x)^(2*p),x]
```

output
```
((d + e*x)^(-1 - 2*p)*((c*d*(d + e*x))/(c*d^2 - a*e^2))^p*((a*e + c*d*x)*(
d + e*x))^(1 + p)*Hypergeometric2F1[p, 1 + p, 2 + p, (e*(a*e + c*d*x))/(-(
c*d^2) + a*e^2)])/(c*d*(1 + p))
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.27, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.108$, Rules used = {1139, 1138, 80, 79}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (d+ex)^{-2p}\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^p\,dx$$

$$\Big\downarrow \text{1139}$$

$$(d+ex)^{-2p}\left(\frac{ex}{d}+1\right)^{2p}\int \left(\frac{ex}{d}+1\right)^{-2p}\left(cdex^2+\left(cd^2+ae^2\right)x+ade\right)^p dx$$

$$\Big\downarrow \text{1138}$$

$$(d+ex)^{-2p}\left(\frac{ex}{d}+1\right)^{p}\left(ade+cd^2x\right)^{-p}\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^p\int \left(cxd^2+aed\right)^p\left(\frac{ex}{d}+1\right)^{-p}dx$$

$$\Big\downarrow \text{80}$$

$$(d+ex)^{-2p}\left(ade+cd^2x\right)^{-p}\left(\frac{cd(d+ex)}{cd^2-ae^2}\right)^{p}\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^p\int \left(cxd^2+aed\right)^p\left(\frac{cd^2}{cd^2-ae^2}+\frac{cexd}{cd^2-ae^2}\right.$$

$$\Big\downarrow \text{79}$$

$$\frac{(d+ex)^{-2p}\left(ade+cd^2x\right)\left(\frac{cd(d+ex)}{cd^2-ae^2}\right)^{p}\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^p \text{Hypergeometric2F1}\left(p,p+1,p+2,-\frac{e(ae}{cd^2}\right.}{cd^2(p+1)}$$

input
```
Int[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^p/(d + e*x)^(2*p),x]
```

output
```
((a*d*e + c*d^2*x)*((c*d*(d + e*x))/(c*d^2 - a*e^2))^p*(a*d*e + (c*d^2 + a
*e^2)*x + c*d*e*x^2)^p*Hypergeometric2F1[p, 1 + p, 2 + p, -((e*(a*e + c*d*
x))/(c*d^2 - a*e^2))])/(c*d^2*(1 + p)*(d + e*x)^(2*p))
```

Defintions of rubi rules used

rule 79
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d), 0] && (RationalQ[m]
 ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

rule 80
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*(b*((c + d*x)/(b*c - a*d)))
^FracPart[n])   Int[(a + b*x)^m*Simp[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d)
), x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] &&  !IntegerQ[m] &&  !Integ
erQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

rule 1138
```
Int[((d_) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Sy
mbol] :> Simp[d^m*((a + b*x + c*x^2)^FracPart[p]/((1 + e*(x/d))^FracPart[p]
*(a/d + (c*x)/e)^FracPart[p]))   Int[(1 + e*(x/d))^(m + p)*(a/d + (c/e)*x)^
p, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0]
&& (IntegerQ[m] || GtQ[d, 0]) &&  !(IGtQ[m, 0] && (IntegerQ[3*p] || Integer
Q[4*p]))
```

rule 1139
```
Int[((d_) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Sy
mbol] :> Simp[d^IntPart[m]*((d + e*x)^FracPart[m]/(1 + e*(x/d))^FracPart[m]
)   Int[(1 + e*(x/d))^m*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e
, m}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&  !(IntegerQ[m] || GtQ[d, 0])
```

Maple [F]

$$\int \left(ade + \left(a\,e^{2} + c\,d^{2}\right) x + cd\,x^{2} e\right)^{p} (ex + d)^{-2p}\, dx$$

input
```
int((a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^p/((e*x+d)^(2*p)),x)
```

output
```
int((a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^p/((e*x+d)^(2*p)),x)
```

Fricas [F]

$$\int (d+ex)^{-2p} \left(ade + \left(cd^2 + ae^2\right) x + cdex^2\right)^p\, dx = \int \frac{(cdex^2 + ade + (cd^2 + ae^2)x)^p}{(ex+d)^{2\,p}}\, dx$$

input
```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^p/((e*x+d)^(2*p)),x, algorithm
="fricas")
```

output
```
integral((c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^p/(e*x + d)^(2*p), x)
```

Sympy [F(-1)]

Timed out.

$$\int (d+ex)^{-2p} \left(ade + \left(cd^2 + ae^2\right) x + cdex^2\right)^p\, dx = \text{Timed out}$$

input
```
integrate((a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**p/((e*x+d)**(2*p)),x)
```

output
```
Timed out
```

Maxima [F]

$$\int (d+ex)^{-2p} \left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^p \, dx = \int \frac{(cdex^2+ade+(cd^2+ae^2)x)^p}{(ex+d)^{2p}} \, dx$$

input
```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^p/((e*x+d)^(2*p)),x, algorithm
="maxima")
```

output
```
integrate((c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^p/(e*x + d)^(2*p), x)
```

Giac [F]

$$\int (d+ex)^{-2p} \left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^p \, dx = \int \frac{(cdex^2+ade+(cd^2+ae^2)x)^p}{(ex+d)^{2p}} \, dx$$

input
```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^p/((e*x+d)^(2*p)),x, algorithm
="giac")
```

output
```
integrate((c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^p/(e*x + d)^(2*p), x)
```

Mupad [F(-1)]

Timed out.

$$\int (d+ex)^{-2p} \left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^p \, dx = \int \frac{\left(c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e\right)^p}{{\left(d+e\,x\right)}^{2\,p}} \, dx$$

input
```
int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^p/(d + e*x)^(2*p),x)
```

output
```
int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^p/(d + e*x)^(2*p), x)
```

Reduce [F]

$$\int (d+ex)^{-2p} \left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^p \, dx = \int \frac{(cde\,x^2+a\,e^2x+c\,d^2x+ade)^p}{(ex+d)^{2p}}dx$$

input
```
int((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^p/((e*x+d)^(2*p)),x)
```

output
```
int((a*d*e + a*e**2*x + c*d**2*x + c*d*e*x**2)**p/(d + e*x)**(2*p),x)
```

3.403 $\int (d+ex)^m \left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{-m} dx$

Optimal result

Integrand size = 37, antiderivative size = 54

$$\int (d+ex)^m \left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{-m} dx = \frac{(d+ex)^{-1+m}\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{1-m}}{cd(1-m)}$$

output
```
(e*x+d)^(-1+m)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1-m)/c/d/(1-m)
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.78

$$\int (d+ex)^m \left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{-m} dx = -\frac{(d+ex)^{-1+m}((ae+cdx)(d+ex))^{1-m}}{cd(-1+m)}$$

input
```
Integrate[(d + e*x)^m/(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^m,x]
```

output
```
-(((d + e*x)^(-1 + m)*((a*e + c*d*x)*(d + e*x))^(1 - m))/(c*d*(-1 + m)))
```

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.027$, Rules used = {1122}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (d+ex)^m \left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{-m} dx$$

$$\Big\downarrow \; 1122$$

$$\frac{(d+ex)^{m-1} \left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{1-m}}{cd(1-m)}$$

input
```
Int[(d + e*x)^m/(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^m,x]
```

output
```
((d + e*x)^(-1 + m)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(1 - m))/(c*d*
(1 - m))
```

Defintions of rubi rules used

rule 1122
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[e*(d + e*x)^(m - 1)*((a + b*x + c*x^2)^(p + 1)/(c*(p + 1))),
 x] /; FreeQ[{a, b, c, d, e, m, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&
EqQ[m + p, 0]
```

Maple [A] (verified)

Time = 1.37 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.06

method	result
gosper	$-\frac{(cdx+ae)(ex+d)^{m}\left(cd\,x^{2}e+a\,e^{2}x+c\,d^{2}x+ade\right)^{-m}}{cd(m-1)}$
orering	$-\frac{(cdx+ae)(ex+d)^{m}\left(ade+\left(a\,e^{2}+c\,d^{2}\right)x+cd\,x^{2}e\right)^{-m}}{cd(m-1)}$
parallelrisch	$\frac{\left(-x(ex+d)^{m}cde-(ex+d)^{m}a\,e^{2}\right)\left(cd\,x^{2}e+a\,e^{2}x+c\,d^{2}x+ade\right)^{-m}}{dec(m-1)}$
norman	$\left(-\frac{x\,e^{m\ln(ex+d)}}{m-1}-\frac{ae\,e^{m\ln(ex+d)}}{cd(m-1)}\right)e^{-m\ln\left(ade+\left(a\,e^{2}+c\,d^{2}\right)x+cd\,x^{2}e\right)}$
risch	$-\frac{(cdx+ae)(cdx+ae)^{-m}\,e^{\frac{i\pi\ \mathrm{csgn}(i(ex+d)(cdx+ae))m(-\mathrm{csgn}(i(ex+d)(cdx+ae))+\mathrm{csgn}(i(cdx+ae)))(-\mathrm{csgn}(i(ex+d)(cdx+ae))+\mathrm{csgn}(i}{2}}}{cd(m-1)}$

input
```
int((e*x+d)^m/((a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^m),x,method=_RETURNVERBOS
E)
```

output
```
-(c*d*x+a*e)/c/d/(m-1)*(e*x+d)^m/((c*d*e*x^2+a*e^2*x+c*d^2*x+a*d*e)^m)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.06

$$\int (d+ex)^m \left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{-m}\,dx$$
$$=-\frac{(cdx+ae)(ex+d)^m}{(cdm-cd)(cdex^2+ade+(cd^2+ae^2)x)^m}$$

input
```
integrate((e*x+d)^m/((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^m),x, algorithm="fr
icas")
```

output
```
-(c*d*x + a*e)*(e*x + d)^m/((c*d*m - c*d)*(c*d*e*x^2 + a*d*e + (c*d^2 + a*
e^2)*x)^m)
```

Sympy [F(-2)]

Exception generated.

$$\int (d+ex)^m \left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{-m} dx = \text{Exception raised: TypeError}$$

input
```
integrate((e*x+d)**m/((a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**m),x)
```

output
```
Exception raised: TypeError >> Invalid NaN comparison
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.61

$$\int (d+ex)^m \left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{-m} dx = -\frac{cdx+ae}{(cdx+ae)^m cd(m-1)}$$

input
```
integrate((e*x+d)^m/((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^m),x, algorithm="ma
xima")
```

output
```
-(c*d*x + a*e)/((c*d*x + a*e)^m*c*d*(m - 1))
```

Giac [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.50

$$\int (d+ex)^m \left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{-m} dx = -\frac{(ex+d)^m cdxe^{(-m\log(cdx+ae)-m\log(ex+d))}+(ex+d)^m aee^{(-m\log(cdx+ae)-m\log(ex+d))}}{cdm-cd}$$

input
```
integrate((e*x+d)^m/((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^m),x, algorithm="gi
ac")
```

output
```
-((e*x + d)^m*c*d*x*e^(-m*log(c*d*x + a*e) - m*log(e*x + d)) + (e*x + d)^m
*a*e*e^(-m*log(c*d*x + a*e) - m*log(e*x + d)))/(c*d*m - c*d)
```

Mupad [B] (verification not implemented)

Time = 5.66 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.06

$$\int (d+ex)^m \left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{-m}\,dx = -\frac{(a\,e+c\,d\,x)\,\left(d+e\,x\right)^m}{c\,d\,\left(m-1\right)\,\left(c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e\right)^m}$$

input
```
int((d + e*x)^m/(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^m,x)
```

output
```
-((a*e + c*d*x)*(d + e*x)^m)/(c*d*(m - 1)*(x*(a*e^2 + c*d^2) + a*d*e + c*d
*e*x^2)^m)
```

Reduce [F]

$$\int (d+ex)^m \left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{-m}\,dx = \int \frac{(ex+d)^m}{(cde\,x^2+a\,e^2x+c\,d^2x+ade)^m}dx$$

input
```
int((e*x+d)^m/((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^m),x)
```

output
```
int((d + e*x)**m/(a*d*e + a*e**2*x + c*d**2*x + c*d*e*x**2)**m,x)
```

3.404 $\int (d+ex)^{-p} \left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^p\,dx$

Optimal result

Integrand size = 37, antiderivative size = 52

$$\int (d+ex)^{-p} \left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^p\,dx = \frac{(d+ex)^{-1-p}\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{1+p}}{cd(1+p)}$$

output
```
(e*x+d)^(-1-p)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(p+1)/c/d/(p+1)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.79

$$\int (d+ex)^{-p} \left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^p\,dx = \frac{(d+ex)^{-1-p}((ae+cdx)(d+ex))^{1+p}}{cd(1+p)}$$

input
```
Integrate[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^p/(d + e*x)^p,x]
```

output
```
((d + e*x)^(-1 - p)*((a*e + c*d*x)*(d + e*x))^(1 + p))/(c*d*(1 + p))
```

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.027$, Rules used = {1122}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (d+ex)^{-p}\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^p\,dx$$

$$\Big\downarrow \text{1122}$$

$$\frac{(d+ex)^{-p-1}\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{p+1}}{cd(p+1)}$$

input
```
Int[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^p/(d + e*x)^p,x]
```

output
```
((d + e*x)^(-1 - p)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(1 + p))/(c*d*
(1 + p))
```

Defintions of rubi rules used

rule 1122
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[e*(d + e*x)^(m - 1)*((a + b*x + c*x^2)^(p + 1)/(c*(p + 1))),
 x] /; FreeQ[{a, b, c, d, e, m, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&
EqQ[m + p, 0]
```

Maple [A] (verified)

Time = 1.36 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.08

method	result
gosper	$\frac{(cdx+ae)\left(cd\,x^2e+a\,e^2x+c\,d^2x+ade\right)^p (ex+d)^{-p}}{cd(p+1)}$
orering	$\frac{(cdx+ae)\left(ade+\left(a\,e^2+c\,d^2\right)x+cd\,x^2e\right)^p (ex+d)^{-p}}{cd(p+1)}$
parallelrisch	$\frac{\left(x\left(cd\,x^2e+a\,e^2x+c\,d^2x+ade\right)^p cde+\left(cd\,x^2e+a\,e^2x+c\,d^2x+ade\right)^p a\,e^2\right)(ex+d)^{-p}}{dec(p+1)}$
norman	$\left(\frac{x\,e^{p\ln\left(ade+\left(a\,e^2+c\,d^2\right)x+cd\,x^2e\right)}}{p+1}+\frac{ae\,e^{p\ln\left(ade+\left(a\,e^2+c\,d^2\right)x+cd\,x^2e\right)}}{cd(p+1)}\right)e^{-\ln(ex+d)p}$
risch	$\frac{(cdx+ae)(cdx+ae)^p\,e^{-\frac{i\pi\ \mathrm{csgn}(i(ex+d)(cdx+ae))p(-\mathrm{csgn}(i(ex+d)(cdx+ae))+\mathrm{csgn}(i(cdx+ae)))(-\mathrm{csgn}(i(ex+d)(cdx+ae))+\mathrm{csgn}(i(ex+}{2}}}{cd(p+1)}$

input
```
int((a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^p/((e*x+d)^p),x,method=_RETURNVERBOS
E)
```

output
```
(c*d*x+a*e)/c/d/(p+1)*(c*d*e*x^2+a*e^2*x+c*d^2*x+a*d*e)^p/((e*x+d)^p)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.06

$$\int (d+ex)^{-p}\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^p\,dx=\frac{(cdx+ae)(cdex^2+ade+(cd^2+ae^2)x)^p}{(cdp+cd)(ex+d)^p}$$

input
```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^p/((e*x+d)^p),x, algorithm="fr
icas")
```

output
```
(c*d*x + a*e)*(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^p/((c*d*p + c*d)*(e*
x + d)^p)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 190 vs. $2(44) = 88$.

Time = 134.40 (sec) , antiderivative size = 190, normalized size of antiderivative = 3.65

$$\int (d+ex)^{-p}\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^p\,dx = \begin{cases} \frac{x}{ae} & \text{for}\ c=0\wedge d=0\wedge p=-1 \\ -\frac{d(d+ex)^{-p}(ade+ae^2x)^p}{e}+x(d+ex)^{-p}\left(ade+ae^2x\right)^p & \text{for}\ c=0 \\ x(ex)^{-p}\left(ae^2x\right)^p & \text{for}\ d=0 \\ \frac{\log\left(\frac{ae}{cd}+x\right)}{cd} & \text{for}\ p=-1 \\ \frac{ae(ade+ae^2x+cd^2x+cdex^2)^p}{cdp(d+ex)^p+cd(d+ex)^p}+\frac{cdx(ade+ae^2x+cd^2x+cdex^2)^p}{cdp(d+ex)^p+cd(d+ex)^p} & \text{otherwise} \end{cases}$$

input
```
integrate((a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**p/((e*x+d)**p),x)
```

output
```
Piecewise((x/(a*e), Eq(c, 0) & Eq(d, 0) & Eq(p, -1)), (-d*(a*d*e + a*e**2*
x)**p/(e*(d + e*x)**p) + x*(a*d*e + a*e**2*x)**p/(d + e*x)**p, Eq(c, 0)),
(x*(a*e**2*x)**p/(e*x)**p, Eq(d, 0)), (log(a*e/(c*d) + x)/(c*d), Eq(p, -1)
), (a*e*(a*d*e + a*e**2*x + c*d**2*x + c*d*e*x**2)**p/(c*d*p*(d + e*x)**p
+ c*d*(d + e*x)**p) + c*d*x*(a*d*e + a*e**2*x + c*d**2*x + c*d*e*x**2)**p/
(c*d*p*(d + e*x)**p + c*d*(d + e*x)**p), True))
```

Maxima [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.58

$$\int (d+ex)^{-p}\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^p\,dx = \frac{(cdx+ae)(cdx+ae)^p}{cd(p+1)}$$

input
```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^p/((e*x+d)^p),x, algorithm="ma
xima")
```

output
```
(c*d*x + a*e)*(c*d*x + a*e)^p/(c*d*(p + 1))
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.52

$$\int (d+ex)^{-p}\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^p\,dx = \frac{\frac{cdxe^{(p\log(cdx+ae)+p\log(ex+d))}}{(ex+d)^p}+\frac{aee^{(p\log(cdx+ae)+p\log(ex+d))}}{(ex+d)^p}}{cdp+cd}$$

input
```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^p/((e*x+d)^p),x, algorithm="gi
ac")
```

output
```
(c*d*x*e^(p*log(c*d*x + a*e) + p*log(e*x + d))/(e*x + d)^p + a*e*e^(p*log(
c*d*x + a*e) + p*log(e*x + d))/(e*x + d)^p)/(c*d*p + c*d)
```

Mupad [B] (verification not implemented)

Time = 5.55 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.08

$$\int (d+ex)^{-p}\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^p\,dx = \frac{(a\,e+c\,d\,x)\,\left(c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e\right)^p}{c\,d\,(p+1)\,(d+e\,x)^p}$$

input
```
int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^p/(d + e*x)^p,x)
```

output
```
((a*e + c*d*x)*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^p)/(c*d*(p + 1)*(d
+ e*x)^p)
```

Reduce [F]

$$\int (d+ex)^{-p} \left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^p \, dx = \int \frac{\left(cde\,x^2+a\,e^2x+c\,d^2x+ade\right)^p}{\left(ex+d\right)^p}dx$$

input
```
int((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^p/((e*x+d)^p),x)
```

output
```
int((a*d*e + a*e**2*x + c*d**2*x + c*d*e*x**2)**p/(d + e*x)**p,x)
```

CHAPTER 4

Appendix

4.1 Listing of Grading functions

The following are the current version of the grading functions used for grading the quality of the antiderivative with reference to the optimal antiderivative included in the test suite.

There is a version for Maple and for Mathematica/Rubi. There is a version for grading Sympy and version for use with Sagemath.

The following are links to the current source code.

The following are the listings of source code of the grading functions.

Mathematica and Rubi grading function

```
(* Original version thanks to Albert Rich emailed on 03/21/2017 *)
(* ::Package:: *)

(* Nasser: April 7,2022. add second output which gives reason for the grade *)
(*                       Small rewrite of logic in main function to make it*)
(*                       match Maple's logic. No change in functionality otherwise*)

(* ::Subsection:: *)
(*GradeAntiderivative[result,optimal]*)
```

```
(* ::Text:: *)
(*If result and optimal are mathematical expressions, *)
(*          GradeAntiderivative[result,optimal] returns*)
(*  "F" if the result fails to integrate an expression that*)
(*      is integrable*)
(*  "C" if result involves higher level functions than necessary*)
(*  "B" if result is more than twice the size of the optimal*)
(*      antiderivative*)
(*  "A" if result can be considered optimal*)


GradeAntiderivative[result_,optimal_] := Module[{expnResult,expnOptimal,leafCountResult,leaf
    expnResult  = ExpnType[result];
    expnOptimal = ExpnType[optimal];
    leafCountResult = LeafCount[result];
    leafCountOptimal = LeafCount[optimal];

    (*Print["expnResult=",expnResult," expnOptimal=",expnOptimal];*)
    If[expnResult<=expnOptimal,
        If[Not[FreeQ[result,Complex]], (*result contains complex*)
            If[Not[FreeQ[optimal,Complex]], (*optimal contains complex*)
                If[leafCountResult<=2*leafCountOptimal,
                    finalresult={"A",""}
                    ,(*ELSE*)
                    finalresult={"B","Both result and optimal contain complex but leaf count
                  ]
            ,(*ELSE*)
                finalresult={"C","Result contains complex when optimal does not."}
            ]
        ,(*ELSE*)(*result does not contains complex*)
            If[leafCountResult<=2*leafCountOptimal,
               finalresult={"A",""}
            ,(*ELSE*)
               finalresult={"B","Leaf count is larger than twice the leaf count of optimal.
              ]
        ]
    ,(*ELSE*) (*expnResult>expnOptimal*)
        If[FreeQ[result,Integrate] && FreeQ[result,Int],
            finalresult={"C","Result contains higher order function than in optimal. Order "
            ,
```

```
            finalresult={"F","Contains unresolved integral."}
        ]
    ];

    finalresult
]

(* ::Text:: *)
(*The following summarizes the type number assigned an *)
(*expression based on the functions it involves*)
(*1 = rational function*)
(*2 = algebraic function*)
(*3 = elementary function*)
(*4 = special function*)
(*5 = hyperpergeometric function*)
(*6 = appell function*)
(*7 = rootsum function*)
(*8 = integrate function*)
(*9 = unknown function*)


ExpnType[expn_] :=
  If[AtomQ[expn],
    1,
  If[ListQ[expn],
    Max[Map[ExpnType,expn]],
  If[Head[expn]===Power,
    If[IntegerQ[expn[[2]]],
      ExpnType[expn[[1]]],
    If[Head[expn[[2]]]===Rational,
      If[IntegerQ[expn[[1]]] || Head[expn[[1]]]===Rational,
        1,
      Max[ExpnType[expn[[1]]],2]],
    Max[ExpnType[expn[[1]]],ExpnType[expn[[2]]],3]]],
  If[Head[expn]===Plus || Head[expn]===Times,
    Max[ExpnType[First[expn]],ExpnType[Rest[expn]]],
  If[ElementaryFunctionQ[Head[expn]],
    Max[3,ExpnType[expn[[1]]]],
  If[SpecialFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],4]],
  If[HypergeometricFunctionQ[Head[expn]],
```

```
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],5]],
  If[AppellFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],6]],
  If[Head[expn]===RootSum,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
  If[Head[expn]===Integrate || Head[expn]===Int,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],8]],
  9]]]]]]]]]]


ElementaryFunctionQ[func_] :=
  MemberQ[{
  Exp,Log,
  Sin,Cos,Tan,Cot,Sec,Csc,
  ArcSin,ArcCos,ArcTan,ArcCot,ArcSec,ArcCsc,
  Sinh,Cosh,Tanh,Coth,Sech,Csch,
  ArcSinh,ArcCosh,ArcTanh,ArcCoth,ArcSech,ArcCsch
},func]


SpecialFunctionQ[func_] :=
  MemberQ[{
  Erf, Erfc, Erfi,
  FresnelS, FresnelC,
  ExpIntegralE, ExpIntegralEi, LogIntegral,
  SinIntegral, CosIntegral, SinhIntegral, CoshIntegral,
  Gamma, LogGamma, PolyGamma,
  Zeta, PolyLog, ProductLog,
  EllipticF, EllipticE, EllipticPi
},func]


HypergeometricFunctionQ[func_] :=
  MemberQ[{Hypergeometric1F1,Hypergeometric2F1,HypergeometricPFQ},func]


AppellFunctionQ[func_] :=
  MemberQ[{AppellF1},func]
```

Maple grading function

```
# File: GradeAntiderivative.mpl
# Original version thanks to Albert Rich emailed on 03/21/2017

#Nasser 03/22/2017  Use Maple leaf count instead since buildin
#Nasser 03/23/2017  missing 'ln' for ElementaryFunctionQ added
#Nasser 03/24/2017  corrected the check for complex result
#Nasser 10/27/2017  check for leafsize and do not call ExpnType()
#                   if leaf size is "too large". Set at 500,000
#Nasser 12/22/2019  Added debug flag, added 'dilog' to special functions
#                   see problem 156, file Apostol_Problems
#Nasser 4/07/2022   add second output which gives reason for the grade

GradeAntiderivative := proc(result,optimal)
local leaf_count_result,
        leaf_count_optimal,
        ExpnType_result,
        ExpnType_optimal,
        debug:=false;

        leaf_count_result:=leafcount(result);
        #do NOT call ExpnType() if leaf size is too large. Recursion problem
        if leaf_count_result > 500000 then
            return "B","result has leaf size over 500,000. Avoiding possible recursion issue
        fi;

        leaf_count_optimal := leafcount(optimal);
        ExpnType_result  := ExpnType(result);
        ExpnType_optimal := ExpnType(optimal);

        if debug then
            print("ExpnType_result",ExpnType_result," ExpnType_optimal=",ExpnType_optimal);
        fi;

# If result and optimal are mathematical expressions,
#  GradeAntiderivative[result,optimal] returns
#   "F" if the result fails to integrate an expression that
#       is integrable
#   "C" if result involves higher level functions than necessary
#   "B" if result is more than twice the size of the optimal
```

```
#       antiderivative
#   "A" if result can be considered optimal

    #This check below actually is not needed, since I only
    #call this grading only for passed integrals. i.e. I check
    #for "F" before calling this. But no harm of keeping it here.
    #just in case.


    if not type(result,freeof('int')) then
        return "F","Result contains unresolved integral";
    fi;


    if ExpnType_result<=ExpnType_optimal then
        if debug then
             print("ExpnType_result<=ExpnType_optimal");
        fi;
        if is_contains_complex(result) then
            if is_contains_complex(optimal) then
                if debug then
                        print("both result and optimal complex");
                fi;
                if leaf_count_result<=2*leaf_count_optimal then
                    return "A"," ";
                else
                   return "B",cat("Both result and optimal contain complex but leaf count of
                                convert(leaf_count_result,string)," vs. $2 (",
                                convert(leaf_count_optimal,string)," ) = ",convert(2*leaf
                end if
            else #result contains complex but optimal is not
                if debug then
                        print("result contains complex but optimal is not");
                fi;
                return "C","Result contains complex when optimal does not.";
            fi;
        else # result do not contain complex
             # this assumes optimal do not as well. No check is needed here.
            if debug then
                   print("result do not contain complex, this assumes optimal do not as well
            fi;
```

```
            if leaf_count_result<=2*leaf_count_optimal then
                if debug then
                    print("leaf_count_result<=2*leaf_count_optimal");
                fi;
                return "A"," ";
            else
                if debug then
                    print("leaf_count_result>2*leaf_count_optimal");
                fi;
                return "B",cat("Leaf count of result is larger than twice the leaf count of
                                 convert(leaf_count_result,string),"$ vs. $2(",
                                 convert(leaf_count_optimal,string),")=",convert(2*leaf_co
            fi;
        fi;
    else #ExpnType(result) > ExpnType(optimal)
        if debug then
            print("ExpnType(result) > ExpnType(optimal)");
        fi;
        return "C",cat("Result contains higher order function than in optimal. Order ",
                       convert(ExpnType_result,string)," vs. order ",
                       convert(ExpnType_optimal,string),".");
    fi;

end proc:

#
# is_contains_complex(result)
# takes expressions and returns true if it contains "I" else false
#
#Nasser 032417
is_contains_complex:= proc(expression)
  return (has(expression,I));
end proc:

# The following summarizes the type number assigned an expression
# based on the functions it involves
# 1 = rational function
# 2 = algebraic function
# 3 = elementary function
# 4 = special function
# 5 = hyperpergeometric function
```

```
# 6 = appell function
# 7 = rootsum function
# 8 = integrate function
# 9 = unknown function

ExpnType := proc(expn)
  if type(expn,'atomic') then
    1
  elif type(expn,'list') then
    apply(max,map(ExpnType,expn))
  elif type(expn,'sqrt') then
    if type(op(1,expn),'rational') then
       1
    else
       max(2,ExpnType(op(1,expn)))
    end if
  elif type(expn,'`^`') then
    if type(op(2,expn),'integer') then
      ExpnType(op(1,expn))
    elif type(op(2,expn),'rational') then
      if type(op(1,expn),'rational') then
         1
      else
         max(2,ExpnType(op(1,expn)))
      end if
    else
         max(3,ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    end if
  elif type(expn,'`+`') or type(expn,'`*`') then
    max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
  elif ElementaryFunctionQ(op(0,expn)) then
    max(3,ExpnType(op(1,expn)))
  elif SpecialFunctionQ(op(0,expn)) then
    max(4,apply(max,map(ExpnType,[op(expn)])))
  elif HypergeometricFunctionQ(op(0,expn)) then
    max(5,apply(max,map(ExpnType,[op(expn)])))
  elif AppellFunctionQ(op(0,expn)) then
    max(6,apply(max,map(ExpnType,[op(expn)])))
  elif op(0,expn)='int' then
    max(8,apply(max,map(ExpnType,[op(expn)]))) else
  9
```

```
  end if
end proc:


ElementaryFunctionQ := proc(func)
  member(func,[
        exp,log,ln,
        sin,cos,tan,cot,sec,csc,
        arcsin,arccos,arctan,arccot,arcsec,arccsc,
        sinh,cosh,tanh,coth,sech,csch,
        arcsinh,arccosh,arctanh,arccoth,arcsech,arccsch])
end proc:

SpecialFunctionQ := proc(func)
  member(func,[
        erf,erfc,erfi,
        FresnelS,FresnelC,
        Ei,Ei,Li,Si,Ci,Shi,Chi,
        GAMMA,lnGAMMA,Psi,Zeta,polylog,dilog,LambertW,
        EllipticF,EllipticE,EllipticPi])
end proc:

HypergeometricFunctionQ := proc(func)
  member(func,[Hypergeometric1F1,hypergeom,HypergeometricPFQ])
end proc:

AppellFunctionQ := proc(func)
  member(func,[AppellF1])
end proc:

# u is a sum or product.  rest(u) returns all but the
# first term or factor of u.
rest := proc(u) local v;
  if nops(u)=2 then
     op(2,u)
  else
     apply(op(0,u),op(2..nops(u),u))
  end if
end proc:

#leafcount(u) returns the number of nodes in u.
```

```
#Nasser 3/23/17  Replaced by build-in leafCount from package in Maple
leafcount := proc(u)
   MmaTranslator[Mma][LeafCount](u);
end proc:
```

Sympy grading function

```
#Dec 24, 2019. Nasser M. Abbasi:
#             Port of original Maple grading function by
#             Albert Rich to use with Sympy/Python
#Dec 27, 2019  Nasser. Added `RootSum`. See problem 177, Timofeev file
#             added 'exp_polar'
from sympy import *

def leaf_count(expr):
   #sympy do not have leaf count function. This is approximation
   return round(1.7*count_ops(expr))

def is_sqrt(expr):
   if isinstance(expr,Pow):
      if expr.args[1] == Rational(1,2):
         return True
      else:
         return False
   else:
      return False

def is_elementary_function(func):
   return func in [exp,log,ln,sin,cos,tan,cot,sec,csc,
         asin,acos,atan,acot,asec,acsc,sinh,cosh,tanh,coth,sech,csch,
         asinh,acosh,atanh,acoth,asech,acsch
      ]

def is_special_function(func):
   return func in [ erf,erfc,erfi,
          fresnels,fresnelc,Ei,Ei,Li,Si,Ci,Shi,Chi,
          gamma,loggamma,digamma,zeta,polylog,LambertW,
          elliptic_f,elliptic_e,elliptic_pi,exp_polar
       ]
```

```
def is_hypergeometric_function(func):
   return func in [hyper]

def is_appell_function(func):
   return func in [appellf1]

def is_atom(expn):
   try:
      if expn.isAtom or isinstance(expn,int) or isinstance(expn,float):
        return True
      else:
        return False

   except AttributeError as error:
      return False

def expnType(expn):
   debug=False
   if debug:
      print("expn=",expn,"type(expn)=",type(expn))

   if is_atom(expn):
     return 1
   elif isinstance(expn,list):
      return max(map(expnType, expn))    #apply(max,map(ExpnType,expn))
   elif  is_sqrt(expn):
      if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
         return 1
      else:
         return max(2,expnType(expn.args[0]))   #max(2,ExpnType(op(1,expn)))
   elif isinstance(expn,Pow):    #type(expn,'`^`')
      if isinstance(expn.args[1],Integer):   #type(op(2,expn),'integer')
         return expnType(expn.args[0])    #ExpnType(op(1,expn))
      elif isinstance(expn.args[1],Rational):   #type(op(2,expn),'rational')
         if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
            return 1
         else:
            return max(2,expnType(expn.args[0]))   #max(2,ExpnType(op(1,expn)))
      else:
         return max(3,expnType(expn.args[0]),expnType(expn.args[1])) #max(3,ExpnType(op(1,expn)),Expr
   elif isinstance(expn,Add) or isinstance(expn,Mul): #type(expn,'`+`') or type(expn,'`*`')
```

```
        m1 = expnType(expn.args[0])
        m2 = expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.func):  #ElementaryFunctionQ(op(0,expn))
        return max(3,expnType(expn.args[0]))  #max(3,ExpnType(op(1,expn)))
    elif is_special_function(expn.func): #SpecialFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(4,m1)  #max(4,apply(max,map(ExpnType,[op(expn)])))
    elif is_hypergeometric_function(expn.func): #HypergeometricFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(5,m1)  #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif is_appell_function(expn.func):
        m1 = max(map(expnType, list(expn.args)))
        return max(6,m1)  #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif isinstance(expn,RootSum):
        m1 = max(map(expnType, list(expn.args))) #Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
        return max(7,m1)
    elif str(expn).find("Integral") != -1:
        m1 = max(map(expnType, list(expn.args)))
        return max(8,m1)  #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):

    #print ("Enter grade_antiderivative for sagemath")
    #print("Enter grade_antiderivative, result=",result," optimal=",optimal)

    leaf_count_result  = leaf_count(result)
    leaf_count_optimal = leaf_count(optimal)

    #print("leaf_count_result=",leaf_count_result)
    #print("leaf_count_optimal=",leaf_count_optimal)

    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if str(result).find("Integral") != -1:
        grade = "F"
        grade_annotation =""
```

```
  else:
      if expnType_result <= expnType_optimal:
          if result.has(I):
              if optimal.has(I): #both result and optimal complex
                  if leaf_count_result <= 2*leaf_count_optimal:
                      grade = "A"
                      grade_annotation =""
                  else:
                      grade = "B"
                      grade_annotation ="Both result and optimal contain complex but leaf count of result is lar
              else: #result contains complex but optimal is not
                  grade = "C"
                  grade_annotation ="Result contains complex when optimal does not."
          else: # result do not contain complex, this assumes optimal do not as well
              if leaf_count_result <= 2*leaf_count_optimal:
                  grade = "A"
                  grade_annotation =""
              else:
                  grade = "B"
                  grade_annotation ="Leaf count of result is larger than twice the leaf count of optimal. "+str(
      else:
          grade = "C"
          grade_annotation ="Result contains higher order function than in optimal. Order "+str(ExpnType


  #print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)

  return grade, grade_annotation
```

SageMath grading function

```
#Dec 24, 2019. Nasser: Ported original Maple grading function by
#             Albert Rich to use with Sagemath. This is used to
#             grade Fricas, Giac and Maxima results.
#Dec 24, 2019. Nasser: Added 'exp_integral_e' and 'sng', 'sin_integral'
#             'arctan2','floor','abs','log_integral'
#June 4, 2022  Made default grade_annotation "none" instead of "" due
#             issue later when reading the file.
#July 14, 2022. Added ellipticF. This is until they fix sagemath, then remove it.
```

```
from sage.all import *
from sage.symbolic.operators import add_vararg, mul_vararg

debug=False;

def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    #print("Enter tree_size, expr is ",expr)

    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)

def is_sqrt(expr):
    if expr.operator() == operator.pow:   #isinstance(expr,Pow):
        if expr.operands()[1]==1/2: #expr.args[1] == Rational(1,2):
            if debug: print ("expr is sqrt")
            return True
        else:
            return False
    else:
        return False

def is_elementary_function(func):
    #debug=False
    m = func.name() in ['exp','log','ln',
            'sin','cos','tan','cot','sec','csc',
            'arcsin','arccos','arctan','arccot','arcsec','arccsc',
            'sinh','cosh','tanh','coth','sech','csch',
            'arcsinh','arccosh','arctanh','arccoth','arcsech','arccsch','sgn',
        'arctan2','floor','abs'
        ]
    if debug:
```

```
        if m:
            print ("func ", func , " is elementary_function")
        else:
            print ("func ", func , " is NOT elementary_function")


    return m

def is_special_function(func):
    #debug=False
    if debug:
        print ("type(func)=", type(func))

    m= func.name() in ['erf','erfc','erfi','fresnel_sin','fresnel_cos','Ei',
           'Ei','Li','Si','sin_integral','Ci','cos_integral','Shi','sinh_integral'
           'Chi','cosh_integral','gamma','log_gamma','psi','zeta',
           'polylog','lambert_w','elliptic_f','elliptic_e','ellipticF',
           'elliptic_pi','exp_integral_e','log_integral',
           'weierstrassPInverse','weierstrass','weierstrassP','weierstrassZeta',
           'weierstrassPPrime','weierstrassSigma']

    if debug:
        print ("m=",m)
        if m:
            print ("func ", func ," is special_function")
        else:
            print ("func ", func ," is NOT special_function")


    return m


def is_hypergeometric_function(func):
    return func.name() in ['hypergeometric','hypergeometric_M','hypergeometric_U']

def is_appell_function(func):
    return func.name() in ['hypergeometric']   #[appellf1] can't find this in sagemath

def is_atom(expn):

    #debug=False
```

```
    if debug:
         print ("Enter is_atom, expn=",expn)

    if not hasattr(expn, 'parent'):
        return False


    #thanks to answer at https://ask.sagemath.org/question/49179/what-is-sagemath-equivalent-to-atomic
    try:
        if expn.parent() is SR:
            return expn.operator() is None
        if expn.parent() in (ZZ, QQ, AA, QQbar):
            return expn in expn.parent() # Should always return True
        if hasattr(expn.parent(),"base_ring") and hasattr(expn.parent(),"gens"):
            return expn in expn.parent().base_ring() or expn in expn.parent().gens()

        return False

    except AttributeError as error:
        print("Exception,AttributeError in  is_atom")
        print ("cought exception" , type(error).__name__ )
        return False


def expnType(expn):

    if debug:
        print (">>>>>Enter expnType, expn=", expn)
        print (">>>>>is_atom(expn)=", is_atom(expn))

    if is_atom(expn):
        return 1
    elif type(expn)==list:   #isinstance(expn,list):
        return max(map(expnType, expn))    #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if  type(expn.operands()[0])==Rational:  #type(isinstance(expn.args[0],Rational):
            return 1
        else:
            return max(2,expnType(expn.operands()[0]))   #max(2,expnType(expn.args[0]))
    elif expn.operator() == operator.pow:    #isinstance(expn,Pow)
        if type(expn.operands()[1])==Integer:   #isinstance(expn.args[1],Integer)
```

```
            return expnType(expn.operands()[0])    #expnType(expn.args[0])
        elif type(expn.operands()[1])==Rational:  #isinstance(expn.args[1],Rational)
            if type(expn.operands()[0])==Rational: #isinstance(expn.args[0],Rational)
                return 1
            else:
                return max(2,expnType(expn.operands()[0]))  #max(2,expnType(expn.args[0]))
        else:
            return max(3,expnType(expn.operands()[0]),expnType(expn.operands()[1])) #max(3,expnType(expr
    elif expn.operator() == add_vararg or expn.operator() == mul_vararg: #isinstance(expn,Add) or isins
        m1 = expnType(expn.operands()[0]) #expnType(expn.args[0])
        m2 = expnType(expn.operands()[1:]) #expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.operator()):  #is_elementary_function(expn.func)
        return max(3,expnType(expn.operands()[0]))
    elif is_special_function(expn.operator()): #is_special_function(expn.func)
        m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
        return max(4,m1)   #max(4,m1)
    elif is_hypergeometric_function(expn.operator()): #is_hypergeometric_function(expn.func)
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
        return max(5,m1)   #max(5,m1)
    elif is_appell_function(expn.operator()):
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
        return max(6,m1)   #max(6,m1)
    elif str(expn).find("Integral") != -1: #this will never happen, since it
              #is checked before calling the grading function that is passed.
              #but kept it here.
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
        return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):


    if debug:
        print ("Enter grade_antiderivative for sagemath")
        print("Enter grade_antiderivative, result=",result)
        print("Enter grade_antiderivative, optimal=",optimal)
        print("type(anti)=",type(result))
        print("type(optimal)=",type(optimal))
```

```
leaf_count_result  = tree_size(result) #leaf_count(result)
leaf_count_optimal = tree_size(optimal) #leaf_count(optimal)

#if debug: print ("leaf_count_result=", leaf_count_result, "leaf_count_optimal=",leaf_count_optimal)


expnType_result  = expnType(result)
expnType_optimal = expnType(optimal)

if debug: print ("expnType_result=", expnType_result, "expnType_optimal=",expnType_optimal)

if expnType_result <= expnType_optimal:
    if result.has(I):
        if optimal.has(I): #both result and optimal complex
            if leaf_count_result <= 2*leaf_count_optimal:
                grade = "A"
                grade_annotation ="none"
            else:
                grade = "B"
                grade_annotation ="Both result and optimal contain complex but leaf count of result is larger
        else: #result contains complex but optimal is not
            grade = "C"
            grade_annotation ="Result contains complex when optimal does not."
    else: # result do not contain complex, this assumes optimal do not as well
        if leaf_count_result <= 2*leaf_count_optimal:
            grade = "A"
            grade_annotation ="none"
        else:
            grade = "B"
            grade_annotation ="Leaf count of result is larger than twice the leaf count of optimal. "+str(lea
else:
    grade = "C"
    grade_annotation ="Result contains higher order function than in optimal. Order "+str(expnType_re


print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)

return grade, grade_annotation
```

4.2 Links to plain text integration problems used in this report for each CAS

1. Mathematica integration problems as .m file
2. Maple integration problems as .txt file
3. Sagemath integration problems as .sage file
4. Reduce integration problems as .txt file
5. Mupad integration problems as .txt file
6. Sympy integration problems as .py file